

Testing, simulation and design of steel equal-leg angle section beams

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Abstract

The stability and design of steel equal-leg angle section members subjected to uniaxial bending are studied herein. An experimental investigation, comprising material testing, initial geometric imperfection measurements and physical tests on hot-rolled steel equal-leg angle section beams is first presented. The test results, in combination with existing experimental data on steel equal-leg angle section beams collected from the literature, are then used for the validation of numerical (shell finite element) models, developed within the commercial package ABAQUS. Next, a numerical parametric study is presented considering both hot-rolled and cold-formed steel angle section beams with a wide range of slenderness values. In major-axis bending, both lateral-torsional and local buckling were observed, with the former characterised by lateral deflection and twist of the cross-section along the member length but no cross-section deformation and the latter by relative twist and transverse bending of the outstands. In minor-axis bending, lateral-torsional buckling and Brazier flattening were observed, with the latter characterised by splaying of the outstands. Note that, for equal-leg angles under minor-axis bending, lateral-torsional buckling is dominant when the cross-section tips are in compression, while Brazier flattening is more influential when the cross-section tips are in tension. When designing for major-axis bending according to Eurocode 3, both local and lateral-torsional buckling are considered; it is shown herein that equal-leg angle section beams under major-axis bending can be designed using a normalised slenderness based on the minimum of the local and lateral-torsional elastic buckling moments, while also considering the ratio of the local to the lateral-torsional elastic buckling moments. For minor-axis bending, Eurocode 3 only requires cross-section checks; this is found to result in unsafe predictions in some cases. It is shown that both safer and more accurate resistance predictions can be achieved when lateral-torsional buckling and Brazier flattening are accounted for in the design of equal-leg angle section beams under minor-axis bending, using a normalised slenderness, based on the minimum of the lateral-torsional and Brazier flattening critical buckling moments. Finally, new design proposals for steel equal-leg angle section beams, covering both major- and minor-axis bending, are developed and verified against the experimental and numerical results. The proposed design rules are shown to offer substantially more accurate and consistent resistance predictions compared to existing codified design rules. The reliability of the new design provisions, with a recommended partial safety factor $\gamma_{M1} = 1.0$, is verified following the EN 1990 procedure.

Keywords: angle beams, EC3 design procedure, equal-leg angles, experiments, finite element modelling, lateral-torsional buckling, testing

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1. Introduction

Steel angle section members are widely used in a variety of structures, including power transmission pylons and trusses, as well as in the bracing of buildings and bridges. They can be produced by hot-rolling, cold-forming, extrusion or welding, with the first two methods being predominant. Although research studies into the structural response of angle section members date back at least to the 1920s [1], the behaviour of angles under bending remains a topic of current interest.

Comparisons with experimental [2, 3] and numerical results have demonstrated that existing design provisions are generally conservative in predicting the ultimate resistance of steel angle section members subjected to major-axis bending and often unsafe for minor-axis bending. In the current European design codes, EN 1993-1-1 [4] and EN 1993-1-3 [5] for hot-rolled and cold-formed angles, respectively, local buckling and lateral-torsional buckling are accounted for through the use of effective widths b_{eff} and the lateral-torsional buckling reduction factor χ_{LT} , respectively. The effective width for an outstand element under a triangular stress distribution is determined with reference to EN 1993-1-5 [6], where one edge of the outstand is assumed to be simply supported (at the junction between the two legs of the angle) and the other (i.e. the tips of the angle) free. However, for angles in bending about the major axis the compressed outstand is, in fact, partially restrained against rotation by the other outstand, which is in tension; this partly explains the conservatism in Eurocode 3 (EC3).

For members in bending about the minor axis, lateral-torsional buckling can typically be disregarded [4, 5]. However, Clark and Hill [7] showed that for any cross-section where the shear centre is offset from the axis of bending, i.e. $u_0 \neq 0$ (see Fig. 1), even when bending is about the minor axis, the member can be susceptible to lateral-torsional buckling. Hence, equal-leg angle section members are susceptible to lateral-torsional buckling when bent about their minor axis. A further consideration is that of flange curling or Brazier flattening, which is a geometrically nonlinear phenomenon that causes a gradual reduction in stiffness under increasing bending. The effect was first studied by Brazier [8] in the context of circular tubes while, much more recently, Zhou et al. [9] developed an expression to determine the critical moment for Brazier flattening in angles.

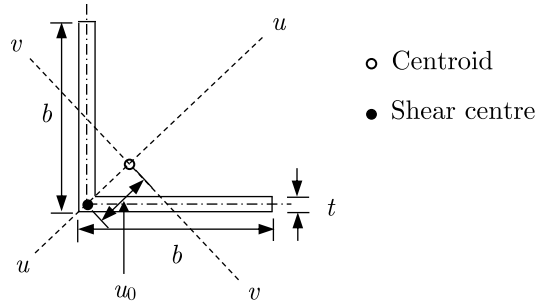


Figure 1: Dimensions and principal axes of equal-leg angles.

The aim of the present paper is to develop a new approach to the design of angle section beams, taking due account of the above phenomena, suitable for incorporation into EC3. A review of existing experimental work undertaken on equal-leg angle section beams is initially described in Section 2. The mechanical behaviour of equal-leg angle section beams is then described in Section 3. In particular, the local and lateral-torsional buckling modes for major- and minor-axis bending are explained. Experiments on equal-leg angle section beams are reported in Section 4, while a numerical simulation programme, conducted using the commercial finite element (FE) software ABAQUS, is presented in Section 5. The FE models, which are validated using

comparisons against the new and existing physical experiments, are utilised to perform a parametric study to generate additional data, considering a wide range of slenderness values and local to lateral-torsional elastic buckling moment ratios; the observations are summarised in Section 6. The experimental and numerical results are used to evaluate the current design rules in EC3, the shortcomings of which are highlighted in Section 7. Subsequently, new proposals for the design of steel equal-leg angle section beams are presented and assessed against the experimental and numerical data. Statistical analyses confirm the accuracy and reliability of the proposed design method.

2. Review of previous research

The behaviour and design of steel equal-leg angle section members under geometric axis bending (i.e. bending in the plane of the legs of the section generating flexure about both principal axes) have been the subject of a number of previous investigations [10, 11, 12, 13, 14, 15, 16, 17, 18]. However, very few investigations have been performed into equal-leg angle section members under uniaxial bending [2, 3, 19, 20, 21]. Leigh and Lay [19], and later Earls and Galambos [20], explored the design of equal-leg angle section beams. Earls [21] performed a numerical study on equal-leg angle section beams under major-axis bending and highlighted excessive conservatism in existing design provisions. Madugula et al. [2] and Spiliopoulos et al. [3] conducted experimental studies on angles bending about the minor axis, as summarised in Table 1.

Table 1: Summary of existing experiments on steel equal-leg angle section members in bending about their principal axes.

Reference	Axis of bending	Manufacturing process	No. of tests
Madugula et al. [2]	Minor	Hot-rolling	6
Spiliopoulos et al. [3]			5
Total			11

There are no experiments available in the literature for equal-leg angle section beams under major-axis bending. Therefore, in the current study, tests are reported on hot-rolled steel angle section beams bending about both the major and minor axes. Additional results are generated numerically and used to underpin the development of improved design rules for equal-leg angle section beams.

3. Mechanical behaviour

The current section presents the different buckling modes involved in the response of equal-leg angle section beams, as well as their corresponding elastic buckling moments.

3.1. Major-axis bending

When equal-leg angle section members are subjected to major-axis bending, one leg is fully in compression, while the other is fully in tension. The result of this is that the compressed outstand leg is not simply-supported along its adjoined longitudinal edge at the intersection with the other outstand, but receives beneficial rotational restraint that leads to an increase in the elastic local buckling moment $M_{cr,l,u}$ [22]. No analytical expression

exists to determine $M_{cr,l,u}$ for angles. Hence, the approximate expression given by Eq. (1), which matches the general form of elastic local plate buckling formulae and incorporates the effect of plate length, was developed on the basis of numerical results and analogies with existing expressions [23].

$$M_{cr,l,u} = W_{el,u} \left[\frac{1}{\phi^2} + \frac{18(1-\nu)}{\pi^2} \right] \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b} \right)^2 \quad (1)$$

where $W_{el,u}$ is the elastic section modulus about the major ($u-u$) axis, $\phi = L/b$ is the plate aspect ratio, in which L is the local buckling length, ν is the Poisson's ratio, E is the Young's modulus, and b and t are the width and thickness of the cross-section legs, respectively. Clark and Hill [7] summarised the results of numerous previous analytical studies on the elastic buckling of beams and provided the following general formula for the elastic lateral-torsional buckling moment $M_{cr,LT,u}$ for a member in bending about the $u-u$ (see Fig. 1) axis:

$$M_{cr,LT,u} = C_1 \frac{\pi^2 E I_v}{L_{cr,v}^2} \left[C_2 g + C_3 k + \sqrt{(C_2 g + C_3 k)^2 + \left(\frac{L_{cr,v}}{L_{cr,T}} \right)^2 \frac{I_w}{I_v} + \frac{L_{cr,v}^2 G J}{\pi^2 E I_v}} \right] \quad (2)$$

in which

$$k = u_0 + \frac{1}{2I_u} \int_A v (u^2 + v^2) dA \quad (3)$$

where I_v is the second moment of area about the $v-v$ (i.e. vertical) axis, $L_{cr,v}$ and $L_{cr,T}$ are the buckling lengths for flexural and torsional buckling, respectively, g is the vertical distance of the point of application of transverse load to the shear centre (positive for loads below the shear centre and negative for loads applied above the shear centre), I_w is the warping constant associated with secondary warping, G is the shear modulus, J is the St. Venant torsion constant and u_0 is the distance between the shear centre and the centroid, (positive when the shear centre is located above the centroid on the compressive side and negative in the opposite case).

For equal-leg angle section members under uniform bending about the major axis ($u-u$) with no transverse loading, $C_1 = 1.0$, $C_2 = 0$, $C_3 = 1.0$ and $k = 0$ (since the section is symmetrical about the axis of bending) and Eq. (2) may be simplified to the following form:

$$M_{cr,LT,u} = \frac{\pi^2 E I_v}{L_{cr,v}^2} \left[\sqrt{\left(\frac{L_{cr,v}}{L_{cr,T}} \right)^2 \frac{I_w}{I_v} + \frac{L_{cr,v}^2 G J}{\pi^2 E I_v}} \right]. \quad (4)$$

The variation of the elastic buckling moment M_{cr} as obtained from the analytical expressions (Eq. (1) for local buckling $M_{cr,l,u}$ and Eq. (4) for lateral-torsional buckling $M_{cr,LT,u}$) and elastic linear buckling analysis (LBA) via ABAQUS, with length L (on a logarithmic scale) is presented in Fig. 2 for a typical equal-leg angle section beam bending about the major axis with simply-supported (forked) boundary conditions with warping prevented. For short-length members, local buckling is critical, whereas for intermediate lengths, the critical buckling mode transitions from local to lateral-torsional. In addition, the lateral-torsional and local buckling moments can be seen to be distinct for short member lengths, unlike the behaviour seen in columns [24] and beams in bending about the minor axis when the cross-section tips are in compression, where the two modes are essentially the same for short member lengths.

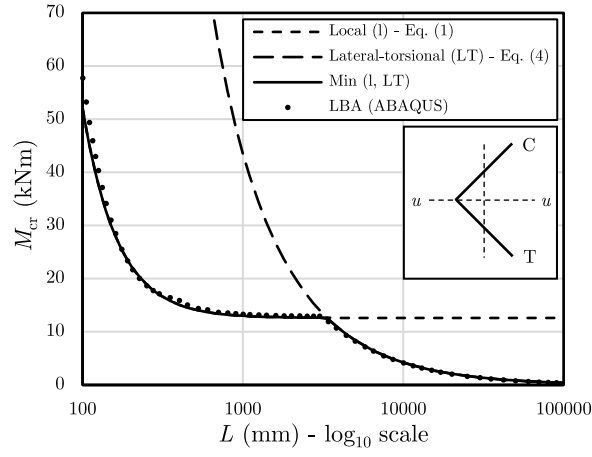


Figure 2: Elastic buckling moments M_{cr} for the different modes plotted against length L for an equal-leg angle section ($100 \times 100 \times 5$) beam bending about the major axis with simply-supported end conditions.

Typical elastic buckling mode shapes for an equal-leg angle section member subjected to uniform major-axis bending are shown in Fig. 3a and Fig. 3b for local and lateral-torsional buckling, respectively. The local buckling mode shape is characterised by pronounced deformation of the compressed leg of the angle that is periodic along the member length, while the lateral-torsional mode shows no cross-section deformation, but out-of-plane displacement and twist of the member in a single half-sine wave.

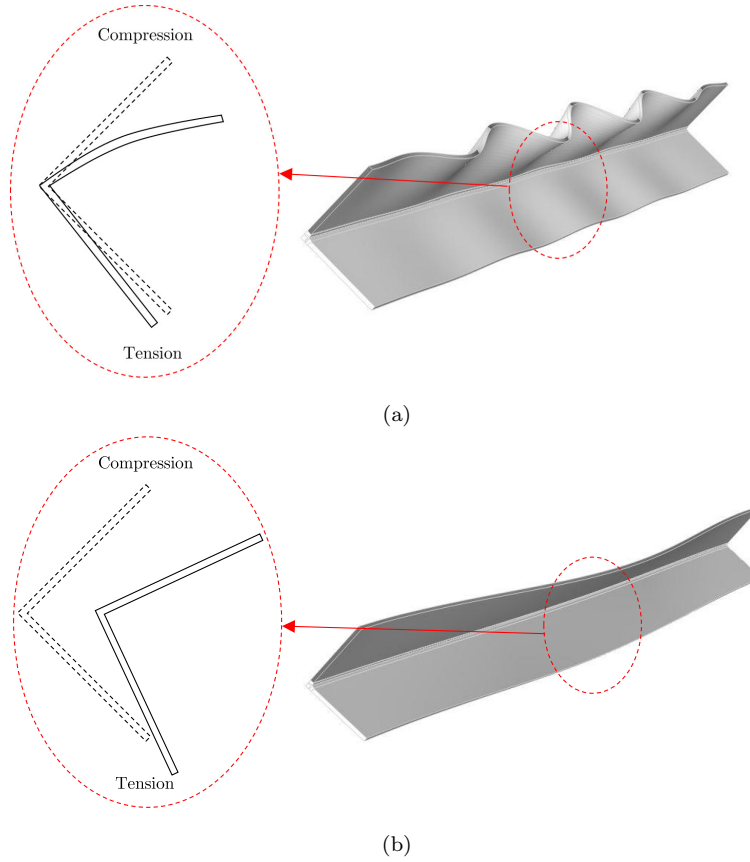


Figure 3: Elastic buckling mode shapes for equal-leg angle section members under uniform major-axis bending for (a) local and (b) lateral-torsional buckling.

3.2. Minor-axis bending

For equal-leg angle section members subjected to uniform bending about the minor axis ($v-v$), since both legs are under the same stress distribution, it is reasonable to assume that they will buckle locally simultaneously and hence exhibit simply-supported conditions along the adjoined plate edges, with the other edge being free. Bulson [23] derived an expression for determining the elastic local buckling stress of plates with such boundary conditions under bending, resulting in the elastic local buckling moment for an equal-leg angle section $M_{cr,l,v}$ given thus:

$$M_{cr,l,v} = W_{el,v} \left[\frac{2}{\phi^2} + \frac{12(1-\nu)}{\pi^2} \right] \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b} \right)^2 \quad (5)$$

where $W_{el,v}$ is the elastic section modulus about the minor ($v-v$) axis. Under minor-axis bending, both legs have the same shape and direction of buckling (i.e. rigid twisting); hence, the local and torsional buckling modes are essentially the same. This can also be shown by assuming that anticlastic plate bending is restrained due to plate continuity, which leads to identical analytical expressions for the elastic torsional and local buckling stresses for a plate simply-supported along one edge and free along the other [24].

The expression for the lateral-torsional buckling moment of angle section members subjected to uniform bending about the minor axis ($v-v$) reduces, by setting $C_1 = 1.0$, $C_2 = 0$ and $C_3 = 1.0$, from Eq. (2) to:

$$M_{cr,LT,v} = \frac{\pi^2 E I_u}{L_{cr,u}^2} \left[k + \sqrt{k^2 + \left(\frac{L_{cr,u}}{L_{cr,T}} \right)^2 \frac{I_w}{I_u} + \frac{L_{cr,u}^2 G J}{\pi^2 E I_u}} \right] \quad (6)$$

in which

$$k = u_0 + \frac{1}{2I_v} \int_A u (u^2 + v^2) dA \quad (7)$$

Rasmussen [25] showed that Eq. (7) can be simplified to $k = -b/\sqrt{2}$ and $k = b/\sqrt{2}$ when the cross-section tips are in compression and tension, respectively.

An additional consideration for angle section beams is that of flange curling or Brazier flattening, which is a geometrically nonlinear effect leading to a progressive reduction in flexural stiffness with increasing deformation. The phenomenon was first studied by Brazier [8] for cylindrical shells. Zhou et al. [9] developed an analytical expression for predicting the Brazier critical moment $M_{cr,Br}$ in angle section as given by:

$$M_{cr,Br} = \frac{1}{8} \sqrt{\frac{35}{39}} \frac{E b t^2}{\sqrt{(1-\nu^2)}} \quad (8)$$

Brazier flattening also depends on the length of the beam, but the length-dependency is negligible for beams with $L/b > 25$ [9]. The variation of the elastic buckling moment M_{cr} , obtained from analytical expressions as well as elastic linear buckling analysis (LBA) and geometrically nonlinear analysis (GNA), to capture Brazier flattening, via ABAQUS, with length L (on a logarithmic scale) is presented in Fig. 4 for a typical equal-leg angle section member in minor-axis bending. A distinction is made between cases where the tips of the cross-section legs are in (a) compression and (b) tension. For angle section beams under minor-axis bending, the direction of bending is important. When the angle is bent such that the cross-section tips are in compression (see Fig. 4a), lateral-torsional buckling is almost always critical; for short beams, the buckling mode is torsional-dominant and plate-like, while for longer beams, flexure becomes more prevalent. On the other hand, when the cross-section tips are in tension (see Fig. 4b), the local/torsional buckling moment is very high, Brazier flattening becomes critical for most practical lengths, while lateral-torsional buckling governs for very long beams. Note that the local/torsional buckling moment is very high when the cross-section tips are in tension and is therefore not shown on the graph.

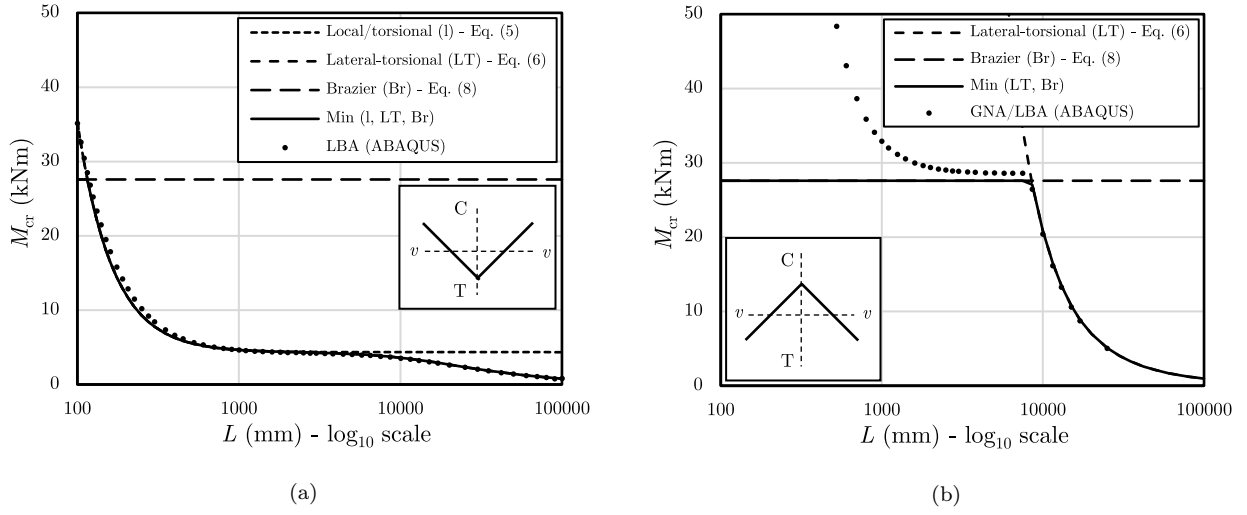


Figure 4: Elastic buckling moments M_{cr} for the different modes plotted against length L for a simply-supported equal-leg angle section ($100 \times 100 \times 5$) member bending about the minor axis where the cross-section tips are in (a) compression and (b) tension.

The cross-section displacement patterns of an equal-leg angle section member subjected to minor-axis bending where (a) local/torsional buckling, (b) lateral-torsional buckling and (c) Brazier flattening is critical are presented in Fig. 5.

4. Experimental investigation

The majority of the existing experiments on steel angles in the literature involve angle section beams bending about their geometric axes and no experiments on major-axis bending have been reported. The experimental programme conducted herein involved material tensile coupon tests, initial geometric imperfection measurements, five tests on steel angle section beams under major-axis bending and two under minor-axis bending with varying lengths and simply-supported boundary conditions with warping prevented. The tested cross-section was a hot-rolled $50 \times 50 \times 6$ equal-leg angle in Grade S275 steel.

4.1. Tensile coupon tests

Prior to the beam tests, material testing was conducted to obtain the material properties of the studied steel angle section. Two coupons were cut from the middle portion of both legs in the longitudinal direction (see Fig. 6), with the coupon dimensions chosen to meet the requirements specified in EN ISO 6892 [26]. The tensile tests were conducted using an Instron 250 kN testing machine. Two linear electrical resistance strain gauges were attached to the centre of the front and back faces of the coupons at mid-height to determine the strains in the longitudinal direction. A clip gauge with a gauge length of 100 mm was placed within the parallel length, and the extension was recorded and used to obtain the average strain over the gauge length. The tensile test set-up is illustrated in Fig. 7. The engineering stress-strain curves from the tensile coupon tests are presented in Fig. 8. The first stage of the stress-strain curves up to the strain value of 10% was obtained using the strain gauge readings, while the second stage of the curve until fracture was derived based on the clip gauge readings. The average obtained mechanical properties are presented in Table 2, where f_u is the ultimate stress, ϵ_u is the strain at the ultimate stress f_u and ϵ_f is the fracture strain measured over the 100 mm gauge length.

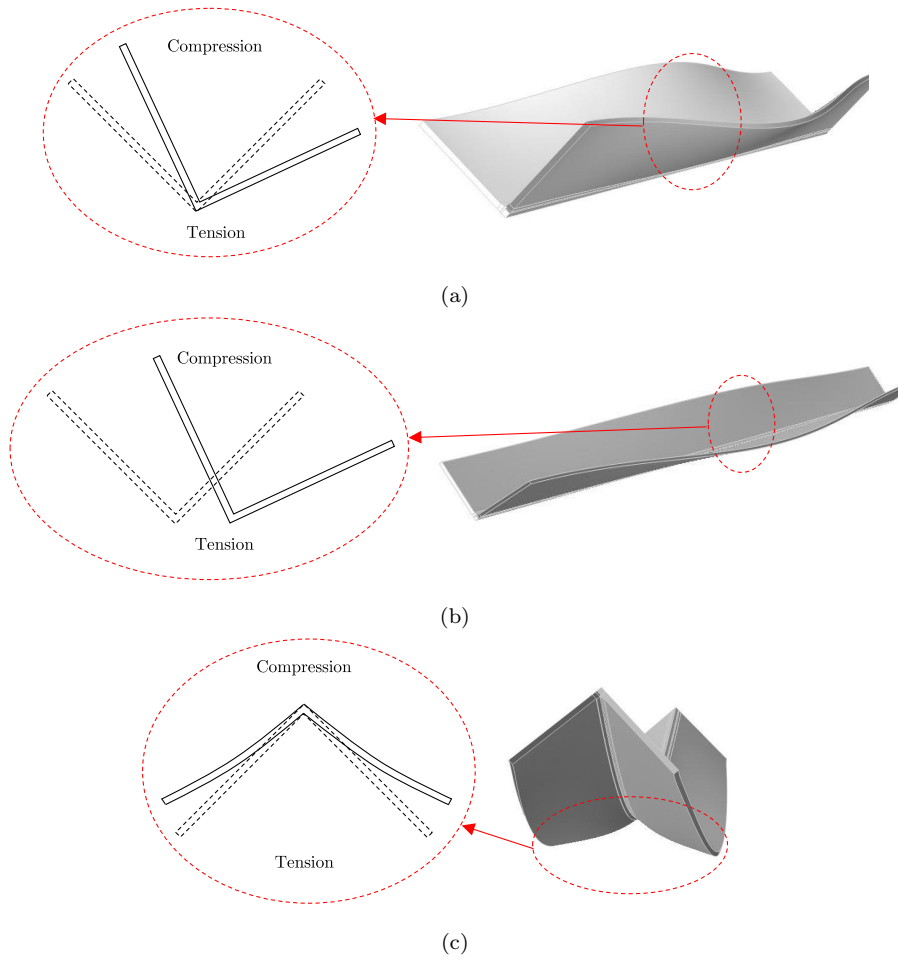


Figure 5: Cross-section displacement patterns for equal-leg angle section members bending about the minor axis when (a) local/torsional buckling, (b) lateral-torsional buckling and (c) the Brazier effect is critical.

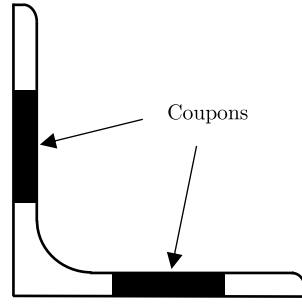


Figure 6: Location of tensile coupons within the angle section.

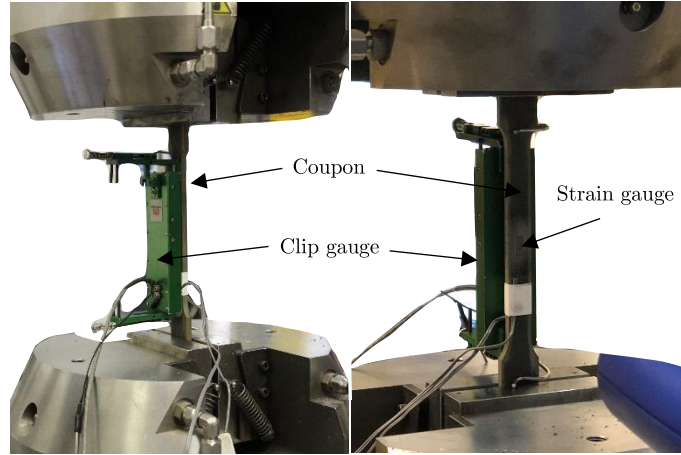


Figure 7: Tensile coupon test set-up.

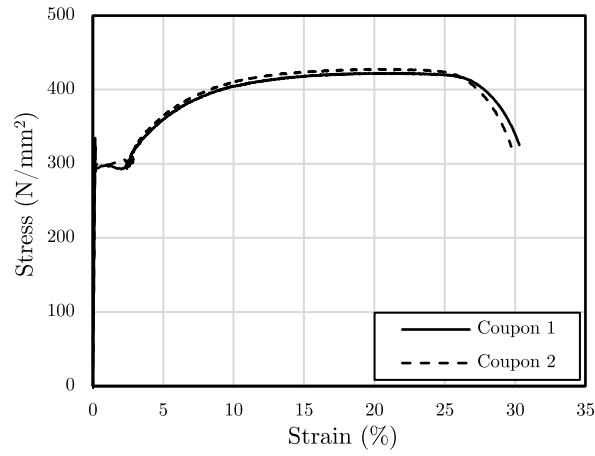


Figure 8: Measured engineering stress-strain curves from tensile coupon tests.

Table 2: Measured material properties of the tested cross-section from tensile coupon tests.

Cross-section	Coupon	E (N/mm ²)	f_y (N/mm ²)	f_u (N/mm ²)	ϵ_u (%)	ϵ_f (%)
50×50×6	Coupon 1	200500	295	423	21.4	30.3
	Coupon 2	203500	299	428	19.6	29.7
	Average	202000	297	425	20.5	30.0

4.2. Initial geometric imperfection measurements

Initial geometric imperfections in the angle section beam specimens, including the bow imperfection about both the major and minor axes, denoted as δ_u and δ_v respectively, together with the twist ϑ along the member length, were measured prior to testing. The three modes of imperfection, presented in Fig. 9, were determined using the same imperfection rig as employed in previous studies [27, 28]. The set-up and approach for measuring and calculating the initial imperfections are presented in detail in [28]. Typical measured longitudinal distributions of initial global and torsional geometric imperfections are presented in Fig. 10, where local imperfections of very low amplitude can also be observed. A half-sine wave was employed to represent the shape of the imperfections, with the amplitude being taken as the maximum measured deviation from straightness. A summary of the measured initial geometric imperfection amplitudes of the specimens relative to the beam length L is provided in Table 3. Note that a positive minor-axis bow imperfection value indicates that the imperfection direction is towards the free edges of the cross-section, as shown in Fig. 9a.

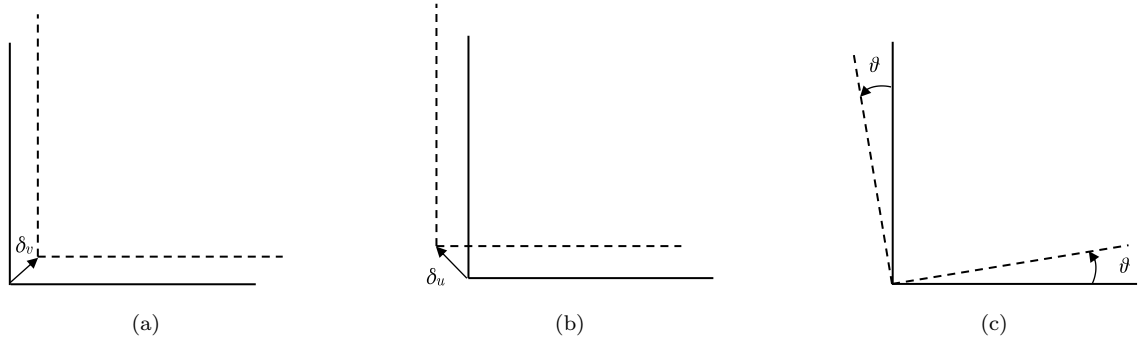


Figure 9: Illustrations of amplitudes of imperfection shapes for (a) minor-axis flexural, (b) major-axis flexural and (c) torsional modes.

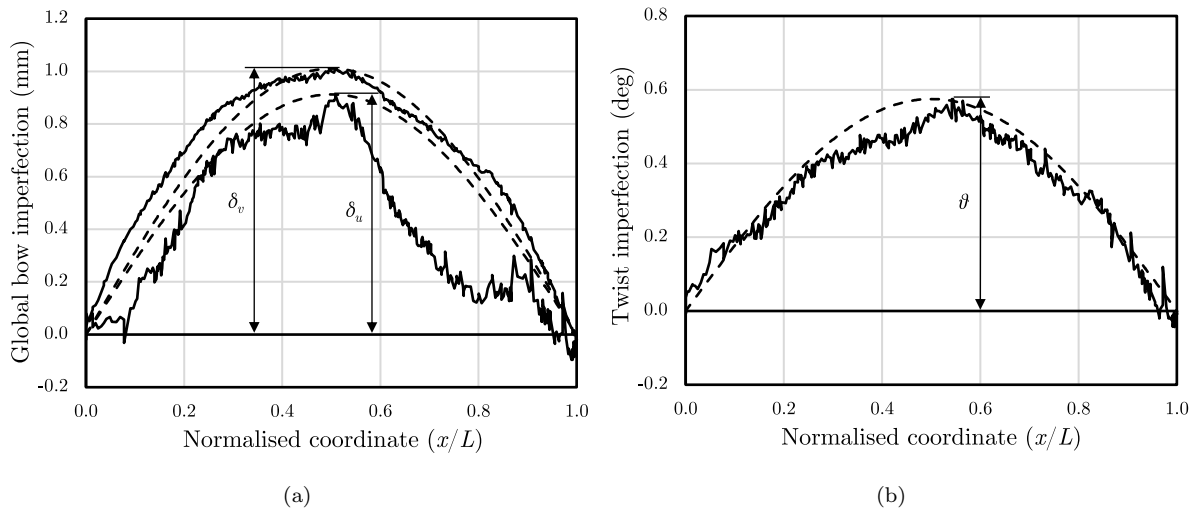


Figure 10: Typical measured initial (a) global bow and (b) twist geometric imperfections versus normalised position along the member length (x/L), with dashed lines marking representative half-sine waves.

Table 3: Summary of initial geometric imperfection amplitude measurements.

ID	L/δ_v	L/δ_u	ϑ/L (deg/m)
1	-4000	8000	0.33
2	-2941	4167	0.10
3	1914	1500	0.31
4	2772	3077	0.20
5	1379	2209	0.19
6	-2439	1754	0.60
7	3030	552	0.10
Mean	2640	3037	0.26
CoV	0.32	0.82	0.67

4.3. Beam buckling tests

Tests on seven hot-rolled Grade S275 steel equal-leg angle section beams were performed, including five beams under major-axis bending and two beams under minor-axis bending, to study their buckling response and moment capacity under uniform bending. For the beams under major-axis bending, the member lengths L were varied to cover a range of member slenderness $\bar{\lambda}_{\max,u}$ (maximum of $\bar{\lambda}_{l,u} = \sqrt{W_{pl,u}f_y/M_{cr,l,u}}$ and $\bar{\lambda}_{LT,u} = \sqrt{W_{pl,u}f_y/M_{cr,LT,u}}$) values. For the beams under minor-axis bending, two beams of the same geometry were tested, but bending was applied in different directions to investigate the influence of the section tips being in compression or tension on the failure mode and ultimate strength. The beam tests were performed under displacement control at a constant rate of 0.5 mm/min. The beam test set-up is presented in Fig. 11; end plates were welded to both ends of each angle section test specimen, which prevented warping, and clamped to two stiff beams (square hollow sections $180 \times 180 \times 10$) at either end. The stiff end beams were supported such that they were free to rotate in the plane of bending. Loading was applied via a spreader beam at two points through rollers; this enabled the application of a uniform bending moment to the specimens. Two lateral restraints were used at each end of the stiff end beams to prevent lateral movement and twisting. Three string pots were used to measure the vertical displacement at the mid-span and the two ends of the angle section members. Four string pots, attached to the corner and the tips of both legs of the angle section beams, were employed to measure the twist and lateral deflection at mid-span. Four electrical resistance strain gauges were affixed to the beam specimens at mid-span to measure the longitudinal strains at the corner and the tips of the cross-sections. The configuration of the string pots and strain gauges is shown in Figs. 11 and 12. The key test outputs, including the applied load and readings from the strain gauges and string pots were recorded at 0.5 s intervals using the data acquisition equipment DATASCAN and logged using the DSLOG software.

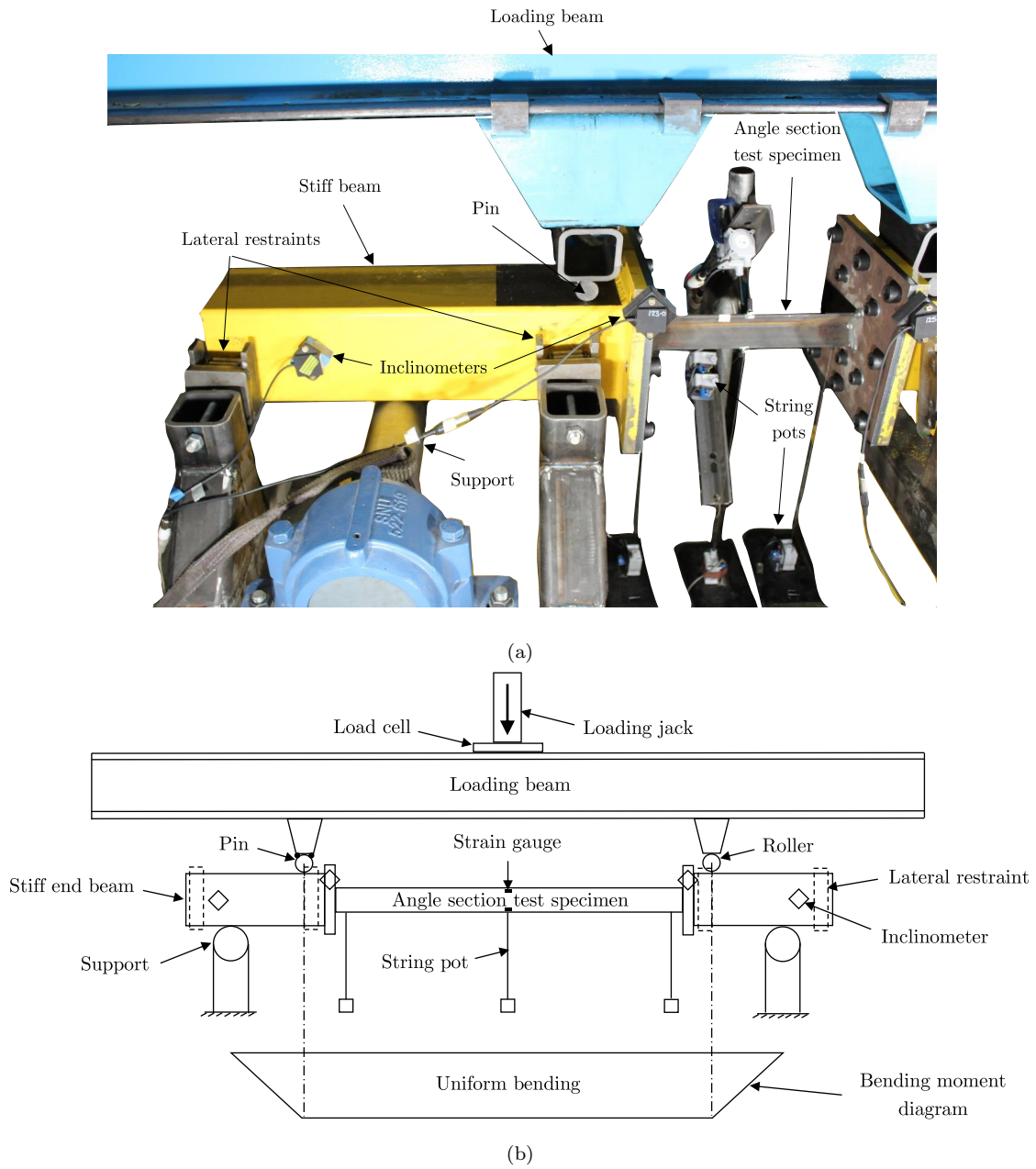


Figure 11: (a) Photographic and (b) schematic representation of beam test set-up.

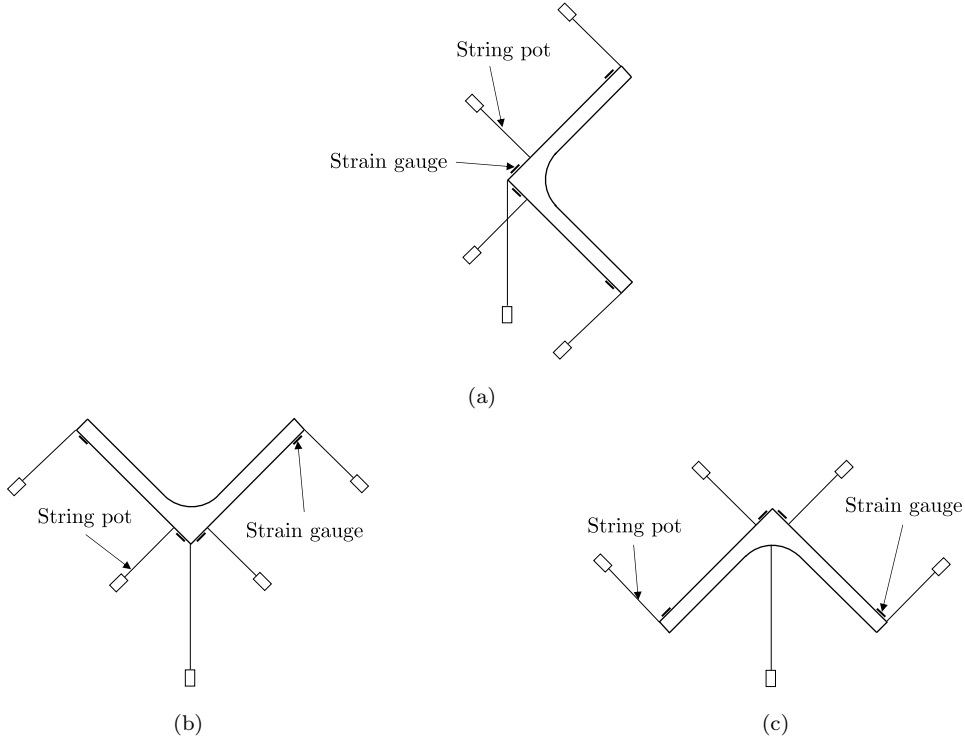


Figure 12: Configuration of strain gauges and string pots at mid-span for beam specimens bending about (a) the major axis, (b) the minor axis with tips in compression and (c) the minor axis with tips in tension.

The geometry and notation for the tested equal-leg angles is presented in Fig. 13, while a summary of the measured dimensions and key test results is presented in Table 4. The bending moment versus (a) end rotation and (b) curvature (calculated using the vertical displacements from the three string pots [29]) are plotted for each test in Figs. 14 and 15 for major- and minor-axis bending, respectively. All tested beams under major-axis bending failed by lateral-torsional buckling, and the direction of buckling was dictated by the direction of the minor-axis initial global imperfections (see Fig. 16). The failure of both beams under minor-axis bending was dominated by yielding and inelastic Brazier flattening (see Fig. 17); this was anticipated since $\bar{\lambda}_{\max,v}$ (i.e. the maximum of $\bar{\lambda}_{LT,v} = \sqrt{W_{pl,v}f_y/M_{cr,LT,v}}$ and $\bar{\lambda}_{Br} = \sqrt{W_{pl,v}f_y/M_{cr,Br}}$) was below 0.4 for both loading cases. A comparison of the bending test data with the current EC3 buckling curve [30], with limiting slenderness $\bar{\lambda}_0 = 0.2$ and imperfection factor $\alpha = 0.76$ for major-axis bending, is presented in Fig. 18a, where $\chi = M_u/(W_{pl}f_y)$ is the buckling reduction factor with M_u being the ultimate bending moment reached during the test. The same buckling curve is assumed for minor-axis bending and a comparison is presented in Fig. 18b.

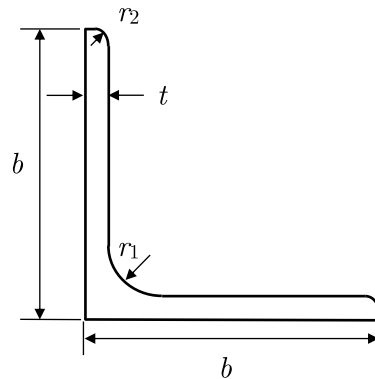


Figure 13: Geometry and notation for hot-rolled equal-leg angle sections.

Table 4: Measured geometric properties and key results from bending test specimens.

Cross-section	Axis of bending	ID	b (mm)	t (mm)	r_1 (mm)	r_2 (mm)	L (mm)	M_u (kNm)
$50 \times 50 \times 6$	Major	1	50.03	5.99	6.0	3.0	400	2.91
		2	50.64	6.00	6.0	3.0	1001	2.76
		3	50.13	5.99	6.0	3.0	1798	2.72
		4	50.04	5.99	6.0	3.0	2799	2.34
		5	50.06	5.99	6.0	3.0	3998	2.16
	Minor	6	50.19	6.01	6.0	3.0	999	1.37
		7	50.66	6.08	6.0	3.0	1001	1.38

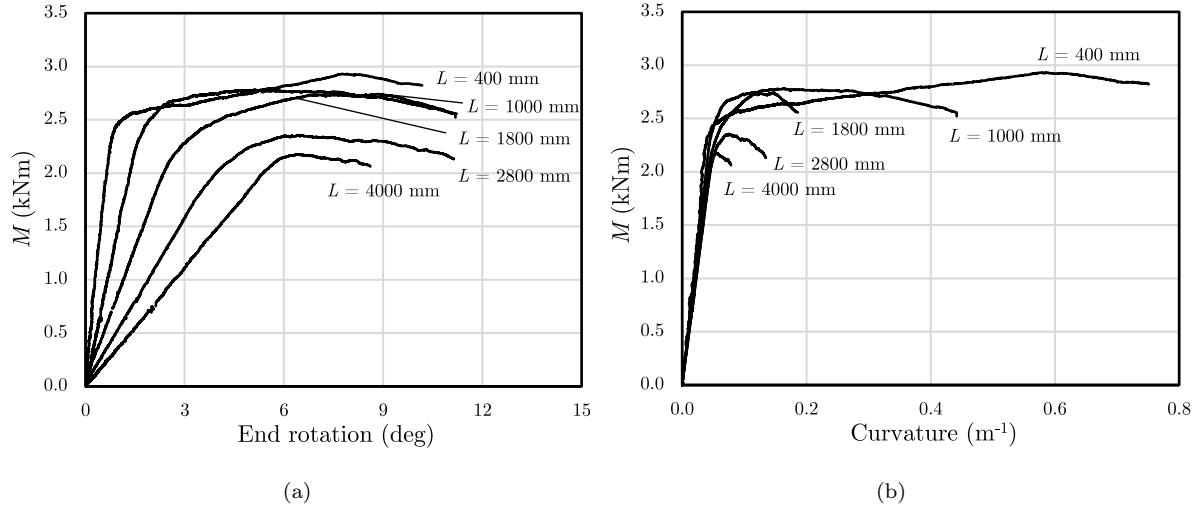


Figure 14: Bending moment M versus (a) end rotation and (b) curvature for the five test specimens under major-axis bending; the nominal lengths of the test specimens are indicated.

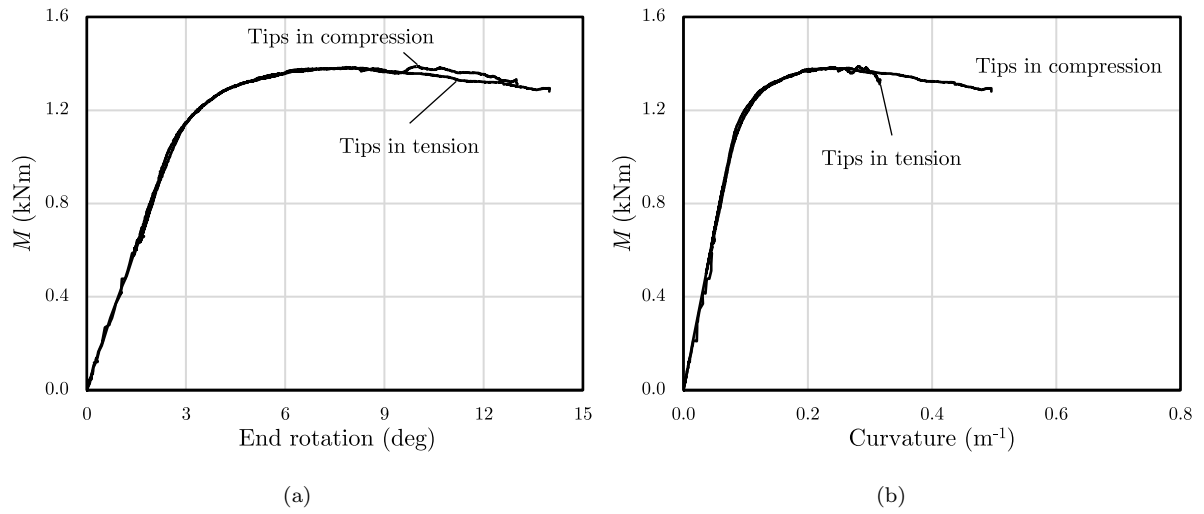


Figure 15: Bending moment M versus (a) end rotation and (b) curvature for the two test specimens under minor-axis bending.

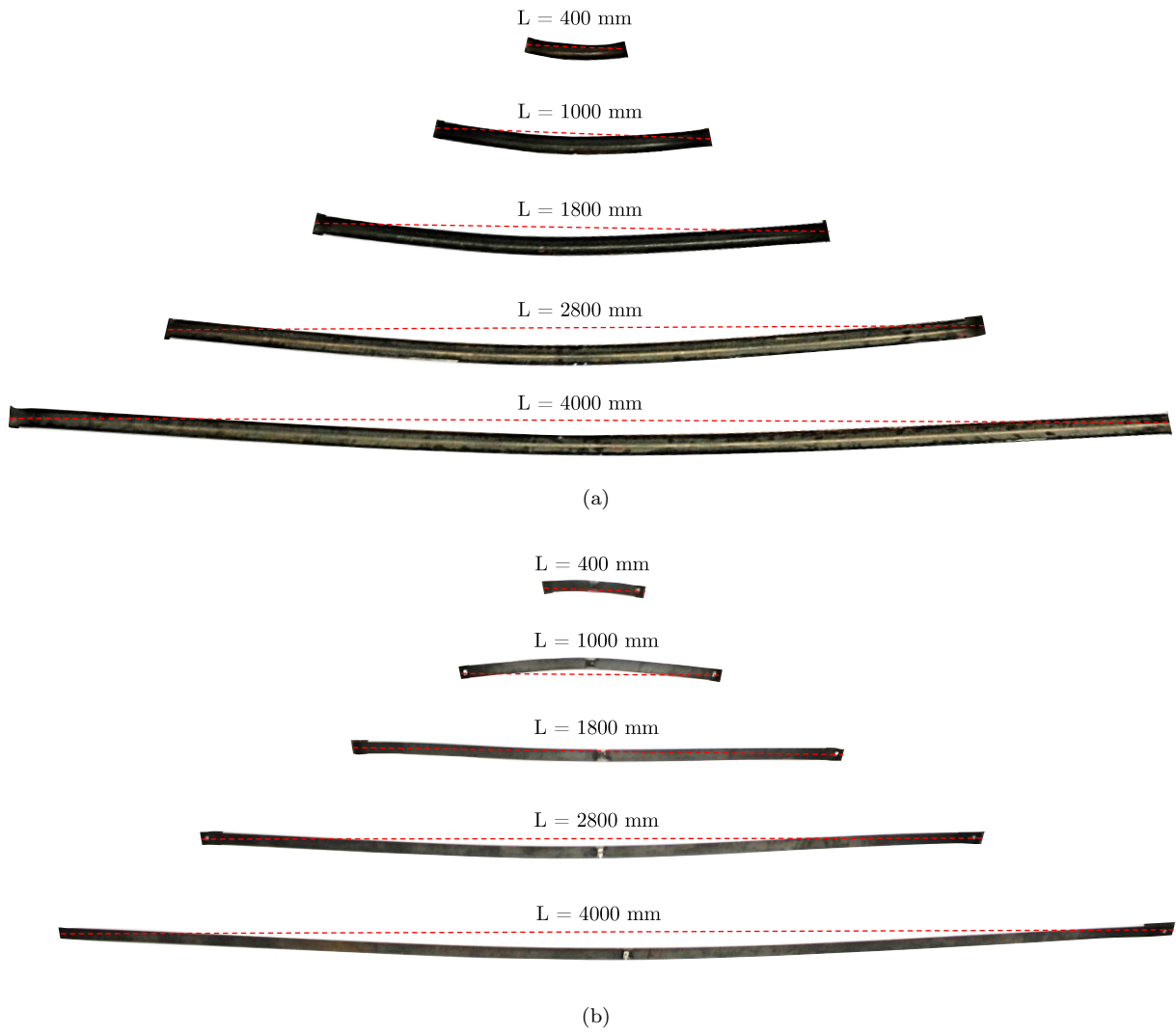


Figure 16: (a) Vertical and (b) lateral deformations of the five specimens in bending about the major axis after failure.

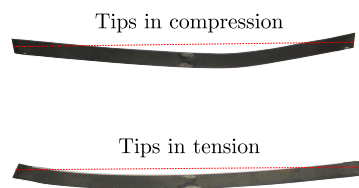


Figure 17: Vertical deformation of the two specimens in bending about the minor axis after failure.

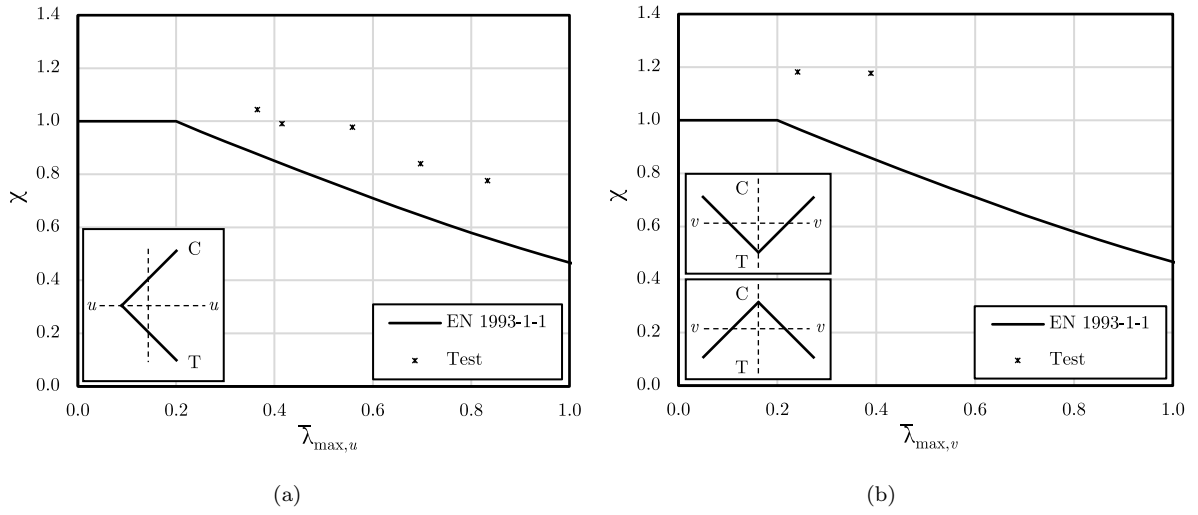


Figure 18: Comparison of bending test data with lateral-torsional buckling curves for (a) major-axis bending, and (b) minor-axis bending.

5. Numerical modelling

The commercial software ABAQUS was used herein to create finite element (FE) models to simulate the mechanical behaviour of steel equal-leg angle section members under uniform bending moment. Validation of the FE models against the newly generated and existing experimental data on steel equal-leg angle section beams is first presented. The model validation is described in Section 5.2. In Section 5.3, a parametric study is presented to investigate the behaviour of steel equal-leg angle section beams, produced by different manufacturing processes – hot-rolling and cold-forming – and considering a range of geometries.

5.1. General modelling assumptions

The general modelling assumptions are described in the current section. A 4-noded shell element with reduced integration, 6 degrees of freedom (3 translational and 3 rotational) at each node and hourglass control, designated S4R in the ABAQUS element library [31], was employed to model the angle section members. The chosen element is suitable for simulating the behaviour of both thin- and thick-walled cross-sections. A mesh size of approximately 5 mm (typically resulting in at least 10 elements across each leg of the angles) was utilised.

Both hot-rolled and cold-formed angles were modelled. For the hot-rolled angles, the root fillet was modelled by introducing thicker elements at the junction between the angle section legs and the thickness was offset to avoid overlapping of the two legs, which were connected using multi-point constraints (MPC) type BEAM, as shown in Fig. 19a. The adopted mesh and MPC result in a section that is symmetrical about the major axis, as confirmed by comparing the stiffness and elastic buckling moments under positive and negative bending, where the results are within 0.05%. The rounded corner in the cold-formed angles was represented by employing a finer mesh in the curved corner region than in the flat regions, as shown in Fig. 19b.

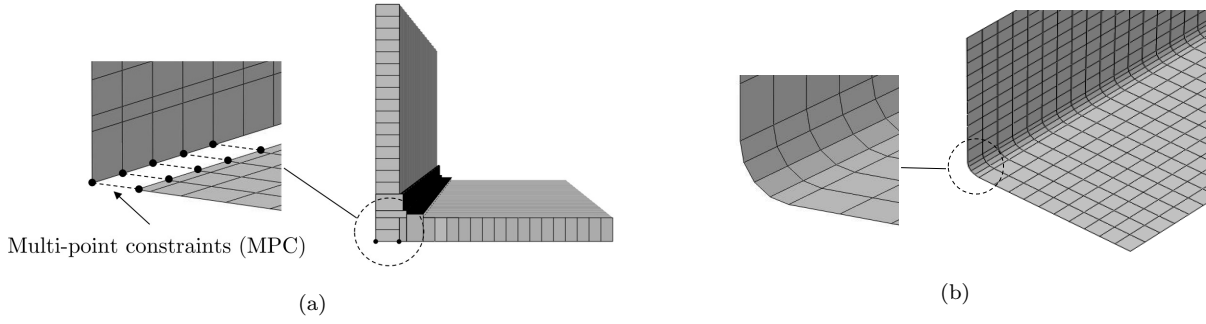


Figure 19: Representation of corner region in: (a) hot-rolled and (b) cold-formed angle section members in the FE models.

To model the uniform bending moment applied to the members, a rotation was imposed at both ends through reference points, one of which was free to move longitudinally. The geometrically and materially nonlinear analyses were executed using the Riks arc-length method [32].

For validation of the FE models, a series of experiments on steel angle section beams were simulated (see Section 5.2). The measured geometric properties and stress-strain curves were employed in these analyses. For the cases in which the full stress-strain curves were not reported, and throughout the parametric study, the material models developed by Yun and Gardner [33] for hot-rolled steel and Gardner and Yun [34] for cold-formed steel were employed. A Poisson's ratio of 0.3 in the elastic range was assumed in all cases. The material properties were input into ABAQUS in the form of true stress σ_{true} and true plastic strain ϵ_{true} , converted from the measured engineering stress σ and strain ϵ using the following well-known relationships:

$$\sigma_{\text{true}} = \sigma (1 + \epsilon), \quad (9)$$

$$\epsilon_{\text{true}} = \ln(1 + \epsilon) - \frac{\sigma_{\text{true}}}{E}. \quad (10)$$

For the hot-rolled steel angles, a bilinear residual stress distribution with a peak value of 70 MPa, constant with thickness and leg width, was employed [24], as illustrated in Fig. 20, where the negative sign indicates compressive stress and the positive sign denotes tensile stress. The residual stresses were input into ABAQUS using the *INITIAL CONDITIONS command. For the cold-formed steel angles, the dominant bending residual stresses were assumed to be inherently captured in the material stress-strain curves [35] and were therefore not explicitly defined.

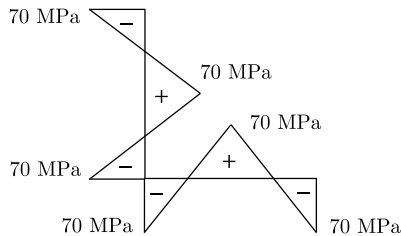


Figure 20: Residual stress distribution employed for hot-rolled steel equal-leg angles in the present FE models.

For the validation study, if reported, the measured initial imperfections (including those reported in Table 3 for the columns tested in the present study) were used. However, where measured data were not available, and throughout the parametric study, a half-sine wave function over the member length with an amplitude of $L/1000$ at mid-span was adopted about both principal axes for the initial bow imperfections. The direction of

the minor-axis bow imperfection was chosen to give the lowest resistance in the parametric study, which was towards the tips of the cross-section legs for the beams under major-axis bending and in the same direction as bending for minor-axis bending. A similar shape was also adopted for the initial twist with an amplitude of $\vartheta = \tan^{-1}(L/1000b)$ at mid-span. Boundary conditions were applied to the end supports through kinematic coupling constraints to link all degrees of freedom (translational and rotational) of the member end-nodes to two reference points. Simply-supported boundary conditions were created by restraining the necessary degrees of freedom at the reference points; warping was also prevented at both ends of the beams.

5.2. Validation

The FE models were validated against the seven tests from the present study, together with a total of 11 existing experiments on steel angle section beams from the literature [2, 3]; a summary of the comparisons between the FE model ultimate bending moments and those obtained experimentally is presented in Table 5 and Fig. 21. Overall, there is good agreement between the test and FE ultimate bending moments, with a mean $M_{u,FE}/M_{u,Test}$ ratio (where $M_{u,FE}$ and $M_{u,Test}$ are the FE and test ultimate bending moments, respectively) of 1.00 and a coefficient of variation (CoV) of 0.04.

Table 5: Comparisons of FE model ultimate moments $M_{u,FE}$ with those obtained experimentally $M_{u,Test}$.

Source	Axis of bending	No. of tests	$\frac{M_{u,FE}}{M_{u,Test}}$	Mean	CoV	Min	Max
Present study	Major	5	0.96	1.00	0.04	0.96	1.05
			0.99				
			0.97				
			1.05				
			1.04				
Madugula et al. [2]	Minor	5	0.95	0.97	0.03	0.94	1.01
			0.94				
			1.00				
			1.01				
			0.96				
Spiliopoulos et al. [3]	Minor	6	0.97	1.02	0.03	0.97	1.07
			1.00				
			1.04				
			1.01				
			1.07				
Present study	Minor	2	1.03	1.03	0.00	1.03	1.03
			1.03				
Total		18		1.00	0.04	0.94	1.07

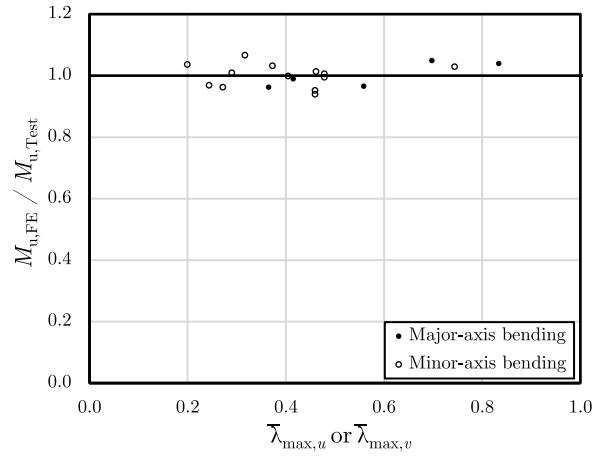


Figure 21: Ratio of FE model to test ultimate bending moments versus the corresponding normalised slenderness (i.e. $\bar{\lambda}_{\max,u}$ and $\bar{\lambda}_{\max,v}$ for beams in major- and minor-axis bending, respectively).

Comparisons of typical bending moment–deformation curves and deformed shapes, shown for Specimens 2 and 6 tested in the current study, are presented in Figs. 22 and 23, and 24 and 25 respectively, illustrating good agreement between the test and FE results. The described comparisons demonstrate the ability of the developed FE models to predict the physical behaviour observed in the experiments on steel angle section beams and the suitability of their use in a parametric study.

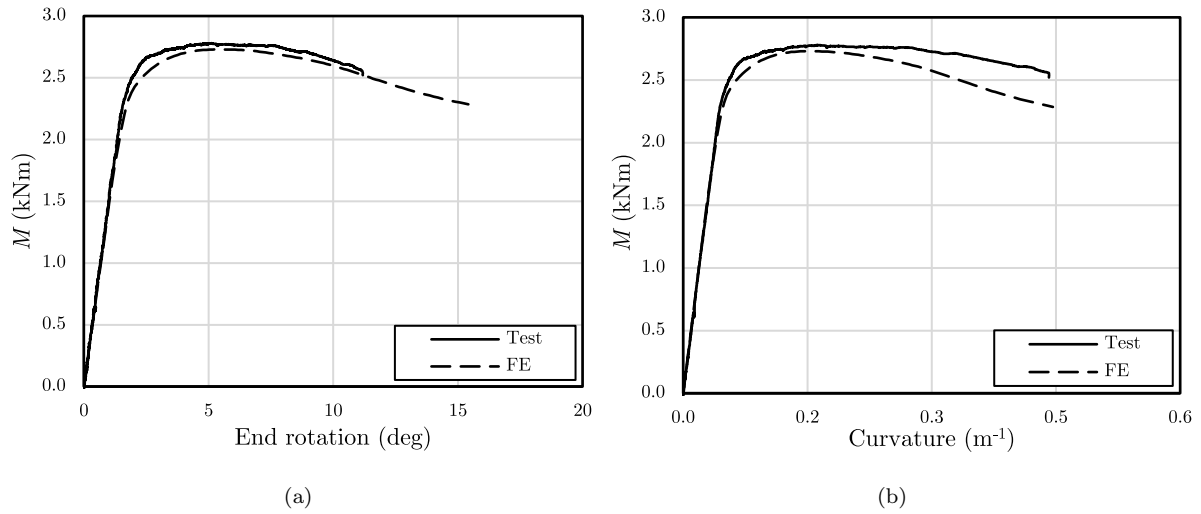


Figure 22: Comparisons of major-axis moment M versus (a) end rotation and (b) curvature curves for Specimen 2 ($L = 1000$ mm) obtained from test and FE model.

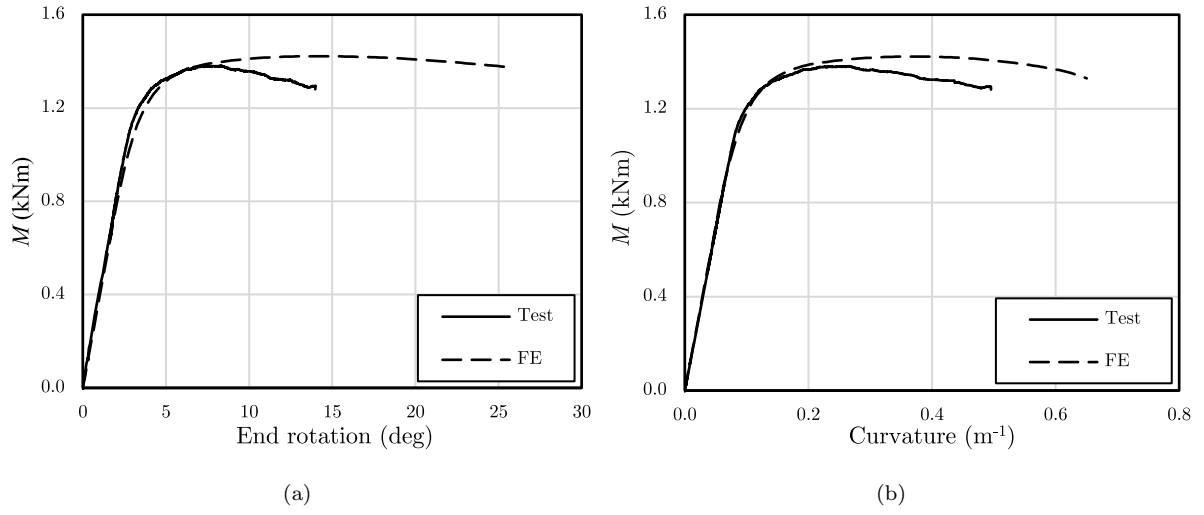


Figure 23: Comparisons of minor-axis moment M versus (a) end rotation and (b) curvature curves for Specimen 6 ($L = 1000$ mm) obtained from test and FE model.



Figure 24: Comparison of deformed beam shape between (a) test and (b) FE model for Specimen 2 ($L = 1000$ mm) subjected to major-axis bending.



Figure 25: Comparison of deformed beam shape between (a) test and (b) FE model for Specimen 6 ($L = 1000$ mm) subjected to minor-axis bending.

5.3. Parametric study

Following validation of the numerical models, a parametric study was conducted, as presented in this section. The FE models were used to generate additional data for hot-rolled and cold-formed steel angle section beams, considering a wide range of cross-section geometries and member lengths. Steel grade S355 with nominal values of $f_y = 355$ N/mm² and $f_u = 490$ N/mm² for hot-rolled and $f_u = 430$ N/mm² for cold-formed from EN 1993-1-1 [4], and a Young's modulus of 210000 N/mm² were used. For the cold-formed members, an enhanced strength was applied to the corner region of the angles using the predictive model developed by Rossi et al. [36]. Leg widths b of 50 mm, 100 mm and 150 mm were analysed, while the thicknesses t and member lengths L were varied to evaluate a spectrum of normalised slendernesses, $\bar{\lambda}_{\max,u}$ or $\bar{\lambda}_{\max,v}$, and elastic buckling moment $M_{cr,l,u}/M_{cr,LT,u}$ ratios for major-axis bending. The b/t ratios ranged between 5 and 50, while the L/b ratios ranged between 0.7 and 100. In total, 1142 FE results were generated. The inner corner radius was assumed to be equal to the cross-section thickness (i.e. $r_1 = t$) for both the hot-rolled and cold-formed sections, and sharp leg tips were adopted for the hot-rolled sections (i.e. $r_2 = 0$). The results for steel equal-leg angle section beams

from the parametric study are presented in Figs. 26 and 27 for major- and minor-axis bending, respectively, where the reduction factor $\chi = M_u/W_{pl}f_y$ is plotted against the corresponding normalised slenderness. The results for the cases where local buckling is critical (i.e. $M_{cr,l,u} \leq M_{cr,LT,u}$) are shown in Fig. 26a, while the results for the cases where lateral-torsional buckling is critical (i.e. $M_{cr,LT,u} < M_{cr,l,u}$) are shown in Fig. 26b. For reference, the local (from EN 1993-1-5 [6] for single outstand flanges) and lateral-torsional (from prEN 1993-1-1 [30] with imperfection factor $\alpha = 0.76$ and limiting slenderness $\bar{\lambda}_0 = 0.2$) buckling curves are also shown in Fig. 26, corresponding to the local and lateral-torsional buckling modes, respectively, for angle section beams under major-axis bending. For the angles in minor-axis bending, the results for the cases where the tips of the cross-section legs are in compression (i.e. with lateral-torsional buckling being critical) are shown in Fig. 27a, while the results for the cases where the tips of the cross-section legs are in tension (i.e. with Brazier flattening being critical) are shown in Fig. 27b. For reference, the current lateral-torsional buckling curves from prEN 1993-1-1 [30] with the imperfection factor $\alpha = 0.76$ and the limiting slenderness $\bar{\lambda}_0 = 0.2$ are also shown in Fig. 27, though note that improved (higher) buckling curves are proposed in Section 8.

Table 6: Summary of ranges of key parameters varied in FE parametric study for major-axis bending.

Critical buckling mode	b/t	L/b	$M_{cr,l,u}/M_{cr,LT,u}$	$\bar{\lambda}_{max,u}$
Local	5 – 50	0.7 – 67.0	0.05 – 1.00	0.25 – 2.25
Lateral-torsional	5 – 50	11.0 – 100.0	1.00 – 6.00	0.35 – 2.25

Table 7: Summary of ranges of key parameters varied in FE parametric study for minor-axis bending.

Critical mode	b/t	L/b	$\bar{\lambda}_{max,v}$
Lateral-torsional	6 – 50	0.6 – 72.0	0.25 – 2.80
Brazier flattening	6 – 50	0.6 – 72.0	0.20 – 0.75

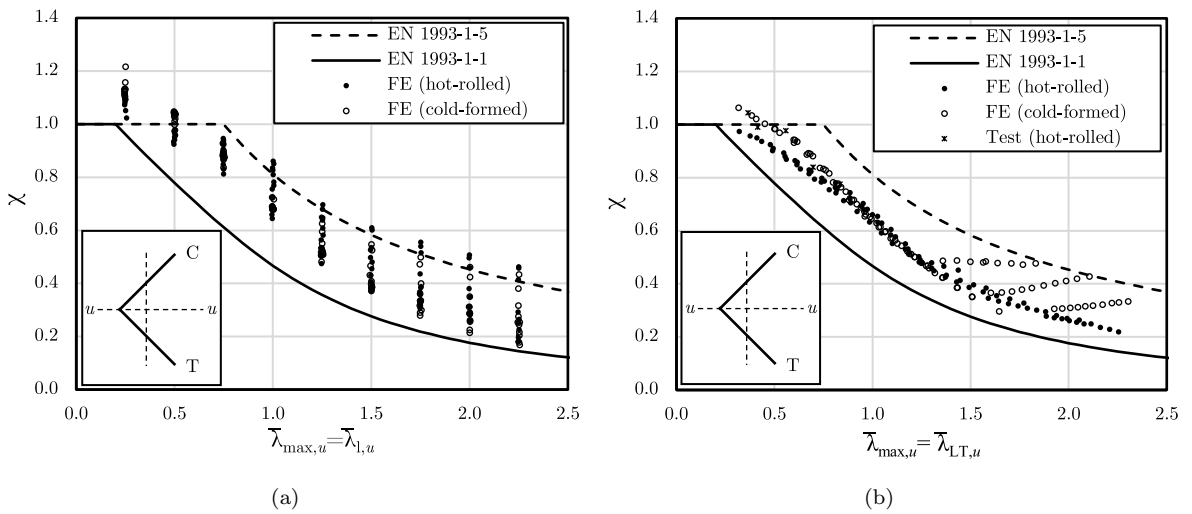


Figure 26: Comparison of test and FE results for steel equal-leg angle section beams bending about the major axis against EC3 buckling curves for the cases where: (a) local buckling is critical (i.e. $M_{cr,l,u} \leq M_{cr,LT,u}$ and $\bar{\lambda}_{max,u} = \bar{\lambda}_{l,u}$ with $\bar{\lambda}_{l,u} = \sqrt{W_{pl,u}f_y/M_{cr,l,u}}$) and (b) lateral-torsional buckling is critical (i.e. $M_{cr,LT,u} < M_{cr,l,u}$ and $\bar{\lambda}_{max,u} = \bar{\lambda}_{LT,u}$ with $\bar{\lambda}_{LT,u} = \sqrt{W_{pl,u}f_y/M_{cr,LT,u}}$).

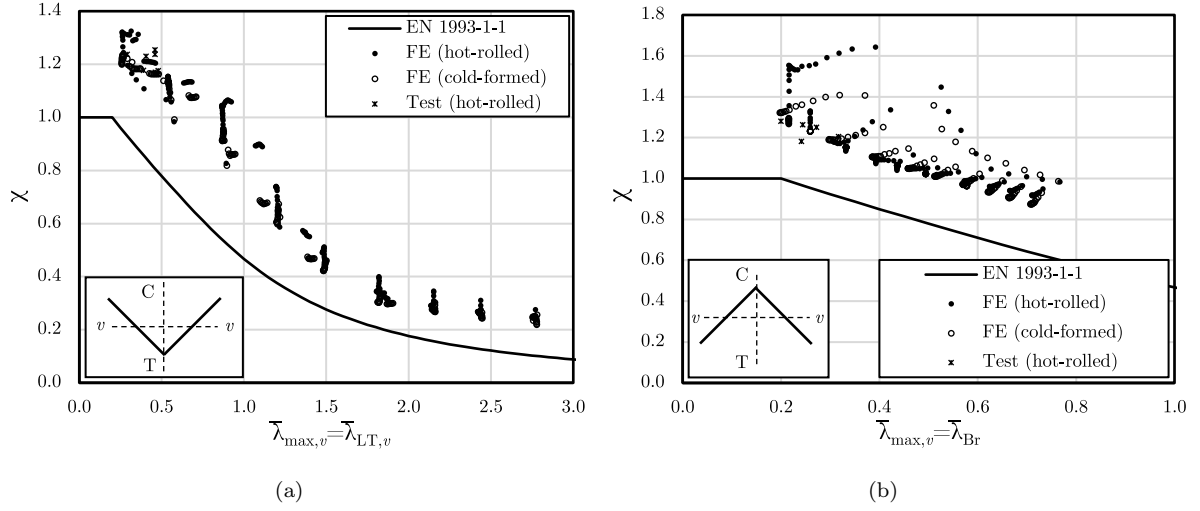


Figure 27: Comparison of test and FE results for steel equal-leg angle section beams bending about the minor axis against EC3 buckling curves for the cases where: (a) the cross-section tips are in compression (i.e. lateral-torsional-buckling is critical: $M_{cr,LT,v} < M_{cr,Br}$ and $\bar{\lambda}_{max,v} = \bar{\lambda}_{LT,v}$ with $\bar{\lambda}_{LT,v} = \sqrt{W_{pl,v} f_y / M_{cr,LT,v}}$) and (b) the cross-section tips are in tension (i.e. Brazier effect is critical: $M_{cr,Br} < M_{cr,LT,v}$ and $\bar{\lambda}_{max,v} = \bar{\lambda}_{Br}$ with $\bar{\lambda}_{Br} = \sqrt{W_{pl,v} f_y / M_{cr,Br}}$).

6. Analysis and discussion of results

6.1. Major-axis bending

According to the existing EC3 [4] design approach, two bending resistance checks are required for angle section beams under major-axis bending. Firstly, a cross-section check is performed, following classification of the cross-section to assess the severity of local buckling and to determine the appropriate choice of section modulus - the plastic $W_{pl,u}$, elastic $W_{el,u}$ and effective $W_{eff,u}$ section moduli are used for Class 1 and 2, 3 and 4 cross-sections, respectively. Secondly, a member buckling check is performed through the use of the lateral-torsional buckling reduction factor χ_{LT} , where beams with the same member slenderness have the same member buckling reduction factor. For members with slender cross-sections, reduction factors for both local buckling (to determine the effective section properties) and lateral-torsional buckling are required. An alternative approach, proposed herein, is to apply a single reduction factor that accounts for both local and lateral-torsional buckling and considers the ratio of the critical moments $M_{cr,l,u} / M_{cr,LT,u}$. As can be observed in Fig. 26a, and further demonstrated in Fig. 28, the numerically obtained buckling reduction factors vary with the $M_{cr,l,u} / M_{cr,LT,u}$ ratio for the same slenderness when local buckling is critical (i.e. $M_{cr,l,u} \leq M_{cr,LT,u}$). The ratio $M_{cr,l,u} / M_{cr,LT,u}$ varies between zero (indicating purely local buckling and corresponding to the higher data points on the graph) and 1.0 (denoting the transition point from local buckling to lateral-torsional buckling and corresponding to the lower data points on the graph). For $M_{cr,l,u} / M_{cr,LT,u}$ ratios greater than 1.0, lateral-torsional buckling is critical. In Fig. 26a, the top data points correspond to beams with the lowest $M_{cr,l,u} / M_{cr,LT,u}$ ratio (0.05) and lie in the region of the plate buckling curve set out in EN 1993-1-5 [6]; as this ratio increases, the buckling reduction factor decreases and converges to the lateral-torsional buckling curve. Similar behaviour was observed for fixed-ended angle section columns in [24, 28]. Therefore, a similar method to that proposed in [24, 28] for fixed-ended equal-leg angle section columns is also proposed herein for equal-leg angle section beams, where the plate buckling and lateral-torsional buckling curves are used as the upper and lower bound limits, respectively, to predict the ultimate resistance of equal-leg angle section members under major-axis bending. Note that the

flat and even gently rising trend in the cold-formed angle section beam data in the high slenderness range has also been observed in previous studies [37, 38], and relates to large deflection effects.

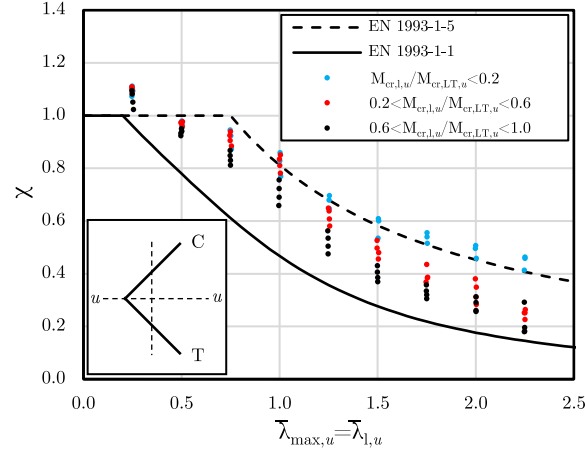


Figure 28: Comparison of FE results for hot-rolled steel equal-leg angle section beams bending about the major axis against EC3 buckling curves for the cases where local buckling is critical, showing variation of buckling reduction factor with $M_{cr,l,w}/M_{cr,L,T,u}$.

6.2. Minor-axis bending

In the current version of EC3 [4], no member buckling check is required for beams under minor-axis bending since it is assumed that they are not susceptible to global instability. However, as shown in Section 3, equal-leg angle section beams under minor-axis bending may experience lateral-torsional buckling. This effect can also be seen in Fig. 27a, where the normalised ultimate strength can be seen to decrease with member slenderness. In addition, when the tips are in tension, Brazier flattening becomes critical, and reduces the ultimate strength of equal-leg angle section beams, as shown in Fig. 27b. The current EC3 provisions for equal-leg angle section beams under minor-axis bending therefore require re-evaluation, as presented in the following section. A new design approach for angle section members in minor-axis bending, giving due consideration to both lateral-torsional buckling and Brazier flattening is therefore presented in Section 8.

7. Design of steel angle section beams to Eurocode 3

In EN 1993-1-1 [4], outstand elements under stress gradients (i.e. the legs of angle sections in bending) are classified according to the following criteria:

$$\text{Tips in compression} \left\{ \begin{array}{ll} \frac{b}{t} \leq \frac{9\epsilon}{\alpha} & \text{Class 1} \\ \frac{b}{t} \leq \frac{10\epsilon}{\alpha} & \text{Class 2} \\ \frac{b}{t} \leq 21\epsilon\sqrt{k_\sigma} & \text{Class 3} \end{array} \right. \quad (11)$$

$$\text{Tips in tension} \left\{ \begin{array}{ll} \frac{b}{t} \leq \frac{9\epsilon}{\alpha\sqrt{\alpha}} & \text{Class 1} \\ \frac{b}{t} \leq \frac{10\epsilon}{\alpha\sqrt{\alpha}} & \text{Class 2} \\ \frac{b}{t} \leq 21\epsilon\sqrt{k_\sigma} & \text{Class 3} \end{array} \right. \quad (12)$$

where b and t are the width and thickness of the legs in equal-leg angles, $\epsilon = \sqrt{235/f_y}$, α is the compressed portion of the element assuming a plastic stress distribution and k_σ is the plate buckling coefficient that depends

on the ratio of stress across the width of the leg assuming an elastic stress distribution. Beyond these limits, cross-sections are Class 4 and the effective width b_{eff} formulae given in EN 1993-1-5 [6] should be used to determine the effective cross-sectional properties.

For beams in bending about the major axis, the calculation of the normalised member slenderness is based on the elastic buckling moment for lateral-torsional buckling in both EN 1993-1-1 [4] and EN 1993-1-3 [5] for hot-rolled and cold-formed angle section beams, respectively. The design major-axis member buckling resistance $M_{b,Rd}$ is given by:

$$M_{b,Rd} = \frac{\chi_{LT} W_u f_y}{\gamma_{M1}} \quad (13)$$

where W_u is the major-axis section modulus; for Class 1 and 2 cross-sections, this is the plastic section modulus $W_{pl,u}$, for Class 3 cross-sections it is the elastic section modulus $W_{el,u}$, and for Class 4 cross-sections it is the effective section modulus $W_{\text{eff},u}$; χ_{LT} is the lateral-torsional buckling reduction factor determined from the relevant member buckling curve and γ_{M1} is the partial factor for the resistance of members to instability, with a recommended value of 1.0 [4] for steel members. The buckling reduction factor χ_{LT} is given by Eq. (14):

$$\chi_{LT} = \frac{1}{\phi_{LT} \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \quad \text{but} \quad \chi_{LT} \leq 1.0 \quad (14)$$

where $\bar{\lambda}_{LT}$ is the normalised slenderness, given by Eq. (15):

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_u f_y}{M_{cr,LT,u}}} \quad (15)$$

and ϕ_{LT} is an auxiliary parameter used to determine the reduction factor χ_{LT} , given by:

$$\phi_{LT} = 0.5 [1 + \alpha_{LT} (\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \bar{\lambda}_{LT}^2] \quad (16)$$

in which $M_{cr,LT,u}$ is the major-axis elastic lateral-torsional buckling moment and α_{LT} is the imperfection factor. The value of the imperfection factor $\alpha_{LT} = 0.76$ for both hot-rolled and cold-formed angles, as specified in prEN 1993-1-1 [30] and prEN 1993-1-3 [39], respectively, while the value for the limiting slenderness $\bar{\lambda}_{LT,0}$ is 0.2. Comparisons of the collected test and generated FE capacities M_u with the resistances predicted by EC3, $M_{b,EC3}$, are presented in Table 8 and Fig. 29, where a distinction is made between the members based on their lowest elastic critical buckling moment (i.e. local or lateral-torsional). The EC3 resistance predictions can be seen to be both conservative and scattered.

Table 8: Summary of comparisons of test and FE capacities with EC3 resistance predictions for steel equal-leg angle section beams in bending about the major axis.

Source	Evaluation parameter	$M_u/M_{b,EC3}$		
		Local critical	Lateral-torsional critical	Total
FE (hot-rolled)	Mean	1.68	1.75	1.72
	CoV	0.16	0.13	0.15
FE (cold-formed)	Mean	1.61	1.97	1.78
	CoV	0.19	0.24	0.24
Test (hot-rolled)	Mean	-	1.32	1.32
	CoV	-	0.08	0.08

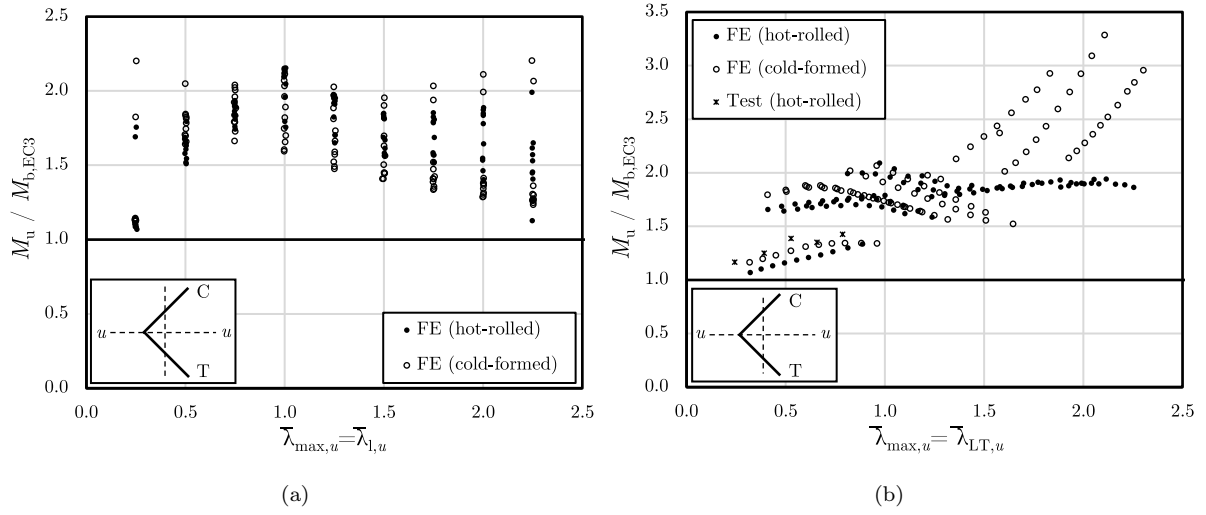


Figure 29: Comparisons of test and FE capacities with EC3 resistance predictions for steel equal-leg angle section beams in bending about the major axis for the cases where: (a) local buckling is critical (i.e. $M_{cr,l,u} \leq M_{cr,LT,u}$) and (b) lateral-torsional buckling is critical (i.e. $M_{cr,LT,u} < M_{cr,l,u}$).

For beams in bending about the minor axis, EN 1993-1-1 states that only the cross-section checks are required. Local buckling is accounted for through cross-section classification and the use of the appropriate section modulus. The design cross-section minor-axis bending resistance $M_{c,Rd}$ is given by:

$$M_{c,Rd} = \frac{W_v f_y}{\gamma_{M0}} \quad (17)$$

where W_v is the section modulus, with the plastic $W_{pl,v}$, elastic $W_{el,v}$ and effective $W_{eff,v}$ section moduli being used for Class 1 and 2, Class 3 and Class 4 cross-sections, respectively. This approach leads to unsafe predictions (see Fig. 30a, where $M_{c,EC3}$ is the EC3 predicted cross-section resistance) for cases where the beam is susceptible to lateral-torsional buckling (i.e. when the cross-section tips are in compression). This is addressed in Section 8.2 by introducing a reduction factor to reflect the loss in capacity due to lateral-torsional buckling.

Table 9: Summary of comparisons of test and FE capacities with EC3 resistance predictions for steel equal-leg angle section beams in bending about the minor axis.

Source	Evaluation parameter	$M_u / M_{c,EC3}$		
		Tips in compression	Tips in tension	Total
FE (hot-rolled)	Mean	1.09	1.42	1.26
	CoV	0.26	0.10	0.22
FE (cold-formed)	Mean	1.05	1.56	1.31
	CoV	0.27	0.11	0.27
Test (hot-rolled)	Mean	1.22	1.44	1.30
	CoV	0.03	0.02	0.09

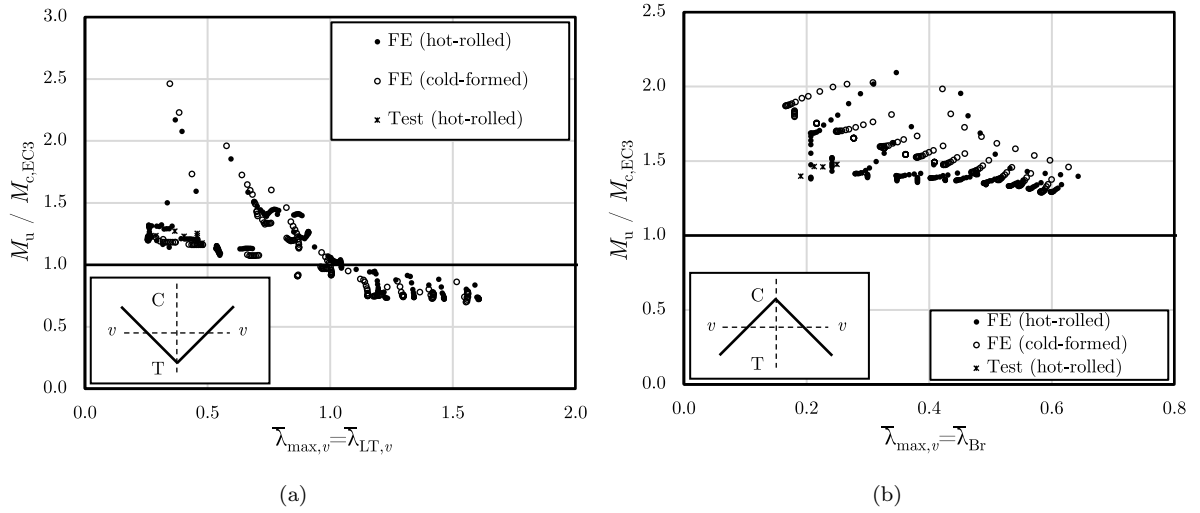


Figure 30: Comparisons of test and FE capacities with EC3 resistance predictions for the design of steel equal-leg angle section beams in bending about the minor axis for the cases where: (a) the cross-section tips are in compression (i.e. lateral-torsional buckling is critical: $M_{cr,LT,v} < M_{cr,Br}$) and (b) the cross-section tips are in tension (i.e. Brazier flattening is critical: $M_{cr,Br} < M_{cr,LT,v}$).

8. New design proposals

In order to address the aforementioned shortcomings in the current EC3 design rules, new design rules are proposed in the subsequent sub-sections, reflecting the observations made in Section 6.

8.1. Bending about the major axis

8.1.1. Method 1 - modified Eurocode approach

A simple improvement that can be made to the existing EC3 design approach for equal-leg angle section members under major-axis bending is to recognise the restraint afforded to the compressed leg of the angle by the leg in tension, by increasing the buckling coefficient for an outstand with zero stress at the supported edge and the maximum stress at the free edge from $k_\sigma = 0.57$ to $k_\sigma = 1.28$ (resulting in a more relaxed Class 3 slenderness limit) and making a commensurate relaxation to the Class 1 and 2 slenderness limits (i.e. by increasing the existing limits by the ratio $\sqrt{1.28/0.57}$). Hence, a more favourable cross-section classification and less severe reduction factors for local buckling are achieved. The revised classification limits for the compressed leg of an equal-leg angle section in pure bending with $\alpha = 1$ and $\psi = 0$, become:

$$\text{Tips in compression} \begin{cases} \frac{b}{t} \leq 13.5\epsilon & \text{Class 1} \\ \frac{b}{t} \leq 15\epsilon & \text{Class 2} \\ \frac{b}{t} \leq 21\epsilon\sqrt{k_\sigma} & \text{Class 3} \end{cases} \quad (18)$$

New imperfection factor α_{LT} and limiting slenderness $\bar{\lambda}_{LT,0}$ values of 0.34 and 0.2, respectively, are also proposed in predicting the bending resistance. A comparison between the test/FE capacities and resistance predictions achieved using this revised approach is shown in Figs. 31a and 31b, for cases where local buckling and lateral-torsional buckling is critical, respectively. Significant improvements in the predictions can be seen compared to Figs. 29a and 29b.

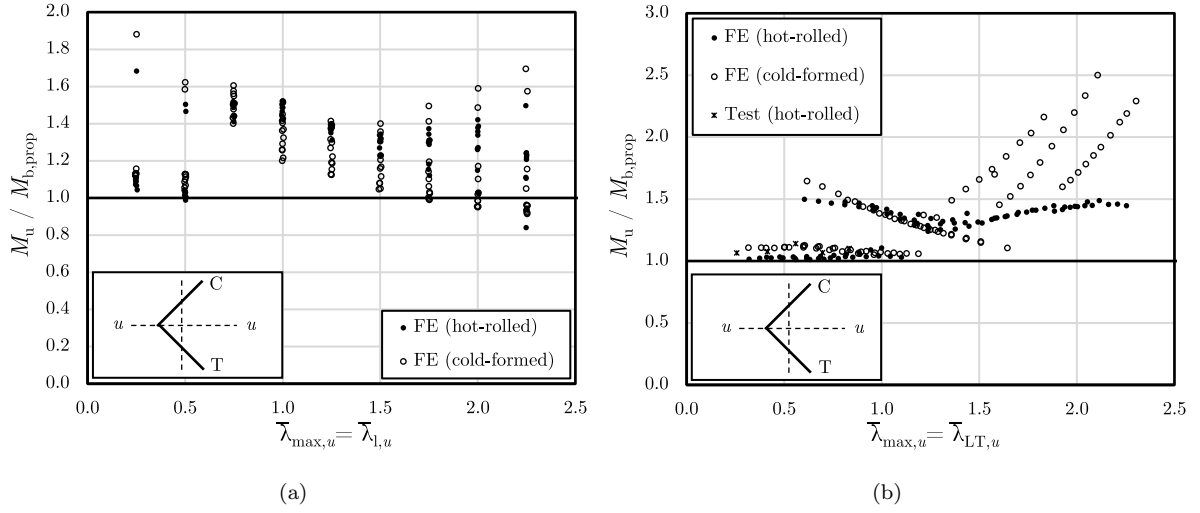


Figure 31: Comparisons of test and FE capacities with resistance predictions obtained using Method 1 for steel equal-leg angle section beams in bending about the major axis for the cases where: (a) local buckling is critical (i.e. $M_{cr,l,u} < M_{cr,LT,u}$) and (b) lateral-torsional buckling is critical (i.e. $M_{cr,LT,u} < M_{cr,l,u}$).

8.1.2. Method 2 - new approach

Similar to the recent proposals [24, 28] for the design of fixed-ended equal-leg angle section columns, new proposals for the design of steel equal-leg angle section beams in bending about the major axis are presented and assessed in the current section. The proposed design buckling resistance $M_{b,u,Rd}$ of steel equal-leg angle section beams in bending about the major axis is:

$$M_{b,u,Rd} = \frac{\chi W_{pl,u} f_y}{\gamma_{M1}} \quad (19)$$

noting that the plastic section modulus $W_{pl,u}$ is used for all classes of cross-section, but, following the same reasoning as described in [24, 28], the influence of both local and lateral-torsional buckling is nonetheless captured, as shown below. In Eq. (19):

$$\chi = \chi_{LT} + \Delta (\chi_1 - \chi_{LT}) \quad (20)$$

in which the local buckling reduction factor χ_1 is given by:

$$\chi_1 = \frac{\bar{\lambda}_{max,u} - 0.188}{\bar{\lambda}_{max,u}^2} \quad \text{but} \quad \chi_1 \leq 1.0, \quad (21)$$

the lateral-torsional buckling reduction factor χ_{LT} is given by:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{max,u}^2}} \quad \text{but} \quad \chi_{LT} \leq 1.0, \quad (22)$$

and Δ is given thus:

$$\Delta = \begin{cases} \left(1 - \frac{M_{cr,l,u}}{M_{cr,LT,u}}\right)^{3.5} & \text{for } \frac{M_{cr,l,u}}{M_{cr,LT,u}} \leq 1 \\ 0 & \text{for } \frac{M_{cr,l,u}}{M_{cr,LT,u}} > 1 \end{cases} \quad (23)$$

with the maximum normalised slenderness $\bar{\lambda}_{max,u}$ and ϕ_{LT} being given by:

$$\bar{\lambda}_{max,u} = \sqrt{\frac{W_{pl,u} f_y}{M_{cr}}} \quad (24)$$

$$\phi_{LT} = 0.5 [1 + \alpha_{LT} (\bar{\lambda}_{max,u} - \bar{\lambda}_{LT,0}) + \bar{\lambda}_{max,u}^2] \quad (25)$$

where M_{cr} is the minimum of $M_{cr,l,u}$ and $M_{cr,LT,u}$. The definitions of $(\chi - \chi_{LT})$ and $(\chi_l - \chi_{LT})$, which are used in determining Δ , are illustrated in Fig. 32, and a comparison between the predicted and numerically obtained $\Delta = (\chi - \chi_{LT})/(\chi_l - \chi_{LT})$ values is presented in Fig. 33. For both hot-rolled and cold-formed steel angles, the proposed values for the imperfection factor α_{LT} and limiting slenderness $\bar{\lambda}_{LT,0}$ are 0.34 and 0.2, respectively.

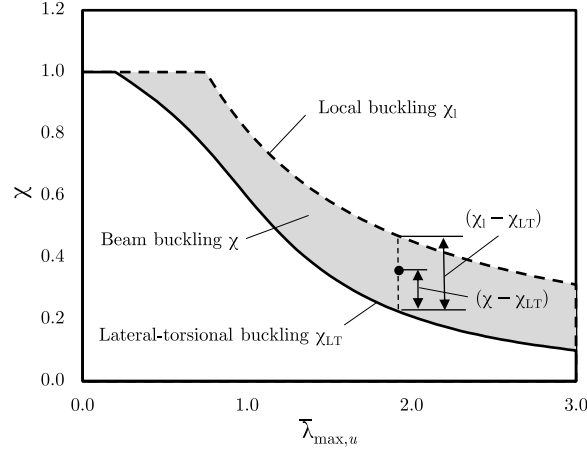


Figure 32: Illustration of $(\chi - \chi_{LT})$ and $(\chi_l - \chi_{LT})$ that are used to determine Δ .

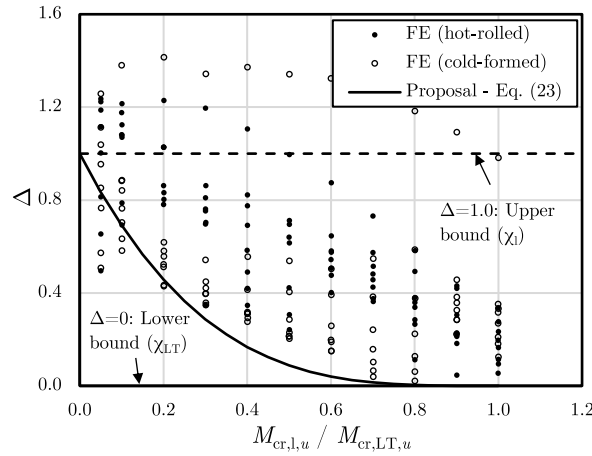


Figure 33: Δ versus $M_{cr,l,u}/M_{cr,LT,u}$ for steel equal-leg angle section beams under major-axis bending.

8.2. Bending about the minor axis

For equal-leg angle section beams in bending about the minor axis, it is proposed that the design buckling resistance $M_{b,v,Rd}$ is obtained thus:

$$M_{b,v,Rd} = \frac{\chi W_{pl,v} f_y}{\gamma_{M1}} \quad (26)$$

where

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}_{max,v}^2}} \quad \text{but} \quad \chi \leq 1.0 \quad (27)$$

in which the maximum normalised slenderness $\bar{\lambda}_{max,v}$ and ϕ are given by:

$$\bar{\lambda}_{max,v} = \sqrt{\frac{W_{pl,v} f_y}{M_{cr}}} \quad (28)$$

$$\phi = 0.5 [1 + \alpha (\bar{\lambda}_{max,v} - \bar{\lambda}_0) + \bar{\lambda}_{max,v}^2] \quad (29)$$

where M_{cr} is the minimum of $M_{cr,LT,v}$ and $M_{cr,Br}$ and α is the imperfection factor. The proposed imperfection factor and limiting slenderness are $\alpha = 0.13$ and $\bar{\lambda}_0 = 0.4$, respectively, for both hot-rolled and cold-formed steel angles.

8.3. Assessment of design proposals

Comparisons of the collected test and generated FE capacities M_u against the resistance predictions according to the new proposals $M_{b,prop}$ are presented in Tables 10, 11 and 12, alongside Figs. 31 and 34, and 35 for beams under major- and minor-axis bending, respectively, where a distinction is made between the beams based on their critical elastic buckling moment. By comparing the results for angles under major-axis bending, it can be observed that Method 2 leads to the most accurate and consistent resistance predictions.

Table 10: Summary of comparisons of test and FE capacities with resistance predictions determined using new design proposals (Method 1) for steel equal-leg angle section beams in bending about the major axis.

Source	Evaluation parameter	$M_u/M_{b,prop}$		
		Local critical	Lateral-torsional critical	Total
FE (hot-rolled)	Mean	1.29	1.27	1.28
	CoV	0.14	0.14	0.14
FE (cold-formed)	Mean	1.22	1.43	1.32
	CoV	0.17	0.26	0.24
Test (hot-rolled)	Mean	-	1.09	1.09
	CoV	-	0.03	0.03

Table 11: Summary of comparisons of test and FE capacities with resistance predictions determined using new design proposals (Method 2) for steel equal-leg angle section beams in bending about the major axis.

Source	Evaluation parameter	$M_u/M_{b,prop}$		
		Local critical	Lateral-torsional critical	Total
FE (hot-rolled)	Mean	1.19	1.12	1.16
	CoV	0.11	0.08	0.11
FE (cold-formed)	Mean	1.12	1.26	1.18
	CoV	0.08	0.24	0.20
Test (hot-rolled)	Mean	-	1.10	1.10
	CoV	-	0.03	0.03

Table 12: Summary of comparisons of test and FE capacities with resistance predictions determined using new design proposals for steel equal-leg angle section beams in bending about the minor axis.

Source	Evaluation parameter	$M_u/M_{b,prop}$		
		Tips in compression	Tips in tension	Total
FE (hot-rolled)	Mean	1.34	1.13	1.23
	CoV	0.19	0.15	0.19
FE (cold-formed)	Mean	1.24	1.10	1.17
	CoV	0.21	0.11	0.18
Test (hot-rolled)	Mean	1.21	1.24	1.22
	CoV	0.03	0.03	0.03

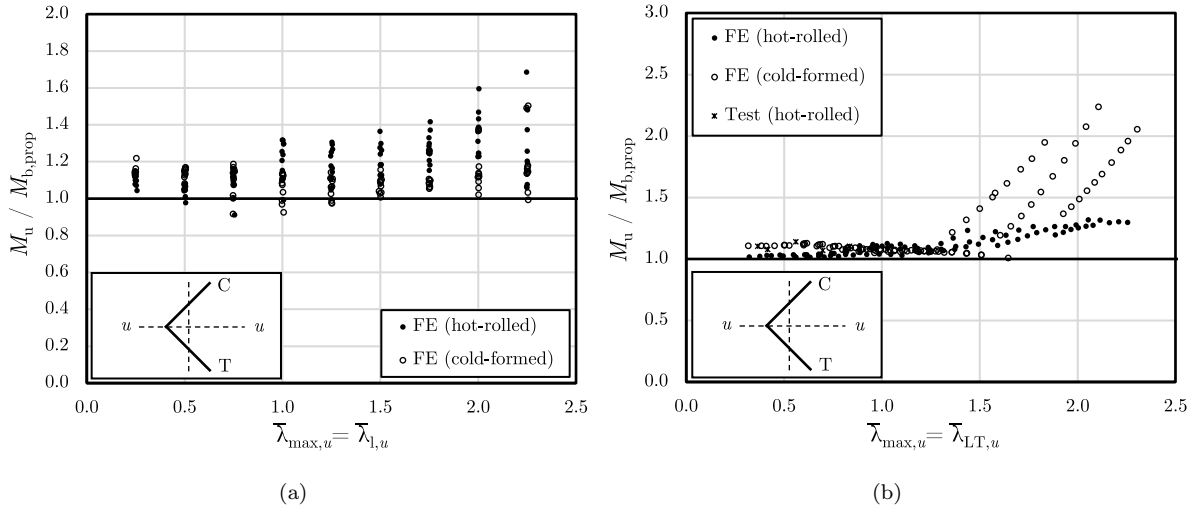


Figure 34: Comparisons of test and FE capacities with resistance predictions obtained using Method 2 for steel equal-leg angle section beams in bending about the major axis for the cases where: (a) local buckling is critical (i.e. $M_{cr,l,u} < M_{cr,LT,u}$) and (b) lateral-torsional buckling is critical (i.e. $M_{cr,LT,u} < M_{cr,l,u}$).

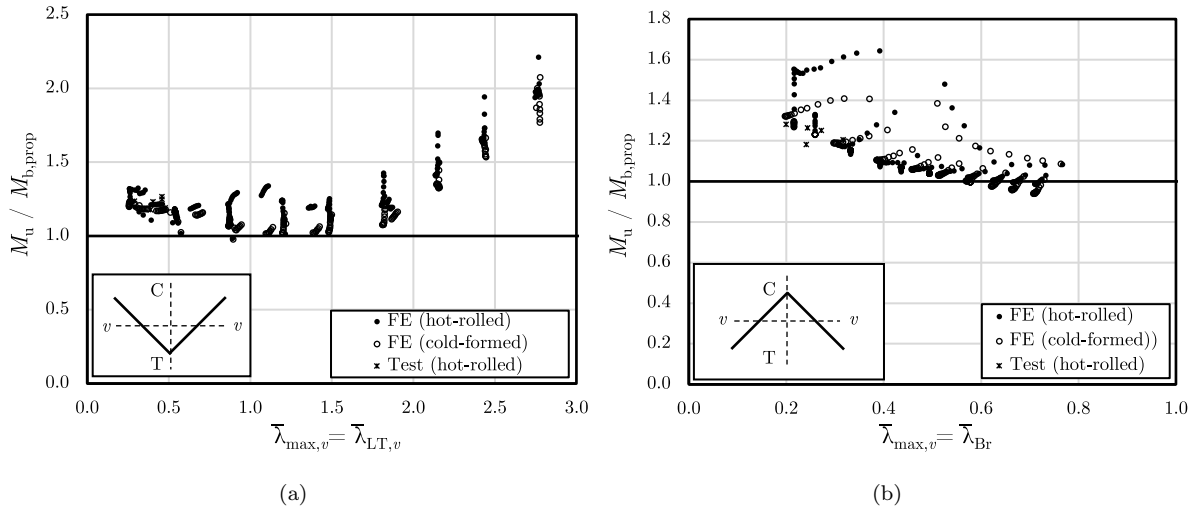


Figure 35: Comparisons of test and FE capacities with resistance predictions obtained using new design proposals for steel equal-leg angle section beams in bending about the minor axis for the cases where: (a) the cross-section tips are in compression (i.e. lateral-torsional buckling is critical: $M_{cr,LT} < M_{cr,Br}$) and (b) the cross-section tips are in tension (i.e. Brazier flattening is critical: $M_{cr,Br} < M_{cr,LT,v}$).

8.4. Reliability analysis

Statistical analyses were performed in accordance with [40] to evaluate the reliability of the current EC3 [4] and proposed design approaches for steel equal-leg angle section beams. As provided in Annex E of [30], the material over-strength factor $f_{y,m}/f_{y,n}$ (i.e. the ratio of the mean to the nominal yield strength) and the corresponding CoV of the yield strength V_{f_y} were taken as 1.2 and 0.05, respectively, while the CoV of the Young's modulus was taken as 0.03. The CoV of the cross-section modulus V_W was calculated based on the parameters of the geometric dimensions provided in Annex E of [30] and following the procedure outlined in [41], resulting in a value of V_W equal to 0.03.

Following Annex E of [30], the mean values of the Young's modulus and all the geometric properties were taken to be equal to the nominal values. A summary of the reliability analysis procedure is provided in [24]; the key results are presented in Tables 13 and 14 for the current EC3 method, and Tables 15 to 17 for the new design proposals, where $k_{d,n}$ is the design fractile factor, b and V_δ are the average and CoV of test and FE resistances relative to predictions from the resistance model, respectively, and V_r is the variability of the resistance model and the basic variables. The required value of the partial safety factor γ_{M1}^* was taken as the mean value for each group of test and FE results and determined following the approach in [41]; γ_{M1} is the target partial safety factor value (equal to 1.0) and f_a is the acceptance limit, given [42] thus:

$$f_a = 1.03 + 0.75(V_r - 0.04) \quad \text{where} \quad 1.03 \leq f_a \leq 1.15 \quad (30)$$

The ratio $\gamma_{M1}^*/\gamma_{M1}$ should satisfy:

$$\gamma_{M1}^*/\gamma_{M1} \leq f_a \quad (31)$$

in order for the reliability of the method to be deemed acceptable. A graphical comparison of $\gamma_{M1}^*/\gamma_{M1}$ and the recommended acceptance limit f_a [42] is presented in Fig. 36 for the new design method. The majority of $(\gamma_{M1}^*/\gamma_{M1})/f_a$ ratios for the new proposals are less than 1.0 for all FE and test results. However, for the FE results on cold-formed angles under major-axis bending, where lateral-torsional buckling is critical and

407 $\bar{\lambda}_{\max,u} > 1.5$, this ratio is 1.06 - this is caused primarily by the ‘very safe’ results arising from the post-critical
408 response of slender beams increasing the scatter of the predictions, and is therefore considered to be tolerable. In
409 addition, Method 2 results in a better ratio of $(\gamma_{M1}^*/\gamma_{M1})/f_a$ of 1.01 than Method 1 (1.12) for cold-formed angles
410 under major-axis bending with local buckling being critical. The current EC3 design expressions clearly lead
411 to unacceptably high γ_{M1}^* values owing principally to the excessively high scatter of the resistance predictions.

Table 13: Reliability analysis results for beams in bending about the major axis for EC3.

Source	Critical mode	$k_{d,n}$	b	V_δ	V_r	γ_{M1}^*	$\gamma_{M1}^*/\gamma_{M1}$	f_a	$(\gamma_{M1}^*/\gamma_{M1})/f_a$
FE (hot-rolled)	Local	3.141	1.685	0.179	0.184	0.96	0.96	1.14	0.85
	LT	3.141	1.755	0.147	0.155	0.88	0.88	1.12	0.79
FE (cold-formed)	Local	3.142	1.612	0.193	0.197	1.05	1.05	1.15	0.91
	LT	3.142	1.972	0.232	0.238	1.03	1.03	1.15	0.90
Test (hot-rolled)	-	3.141	1.316	0.082	0.097	0.89	0.89	1.07	0.83

Table 14: Reliability analysis results for beams in bending about the minor axis for EC3.

Source	Critical mode	$k_{d,n}$	b	V_δ	V_r	γ_{M1}^*	$\gamma_{M1}^*/\gamma_{M1}$	f_a	$(\gamma_{M1}^*/\gamma_{M1})/f_a$
FE (hot-rolled)	LT	3.116	1.090	0.262	0.266	1.90	1.90	1.15	1.65
	Brazier	3.116	1.424	0.089	0.106	0.82	0.82	1.08	0.76
FE (cold-formed)	LT	3.116	1.051	0.254	0.258	1.92	1.92	1.15	1.67
	Brazier	3.116	1.560	0.111	0.125	0.79	0.79	1.09	0.72
Test (hot-rolled)	-	4.078	1.304	0.092	0.107	0.97	0.97	1.08	0.89

Table 15: Reliability analysis results for beams in bending about the major axis for the new design proposals (Method 1).

Source	Critical mode	Slenderness range	$k_{d,n}$	b	V_δ	V_r	γ_{M1}^*	$\gamma_{M1}^*/\gamma_{M1}$	f_a	$(\gamma_{M1}^*/\gamma_{M1})/f_a$
FE (hot-rolled)	Local		3.141	1.287	0.145	0.155	1.13	1.13	1.12	1.01
	LT	$\bar{\lambda}_{\max,u} \leq 1.5$	3.141	1.211	0.144	0.154	1.25	1.25	1.12	1.12
		$\bar{\lambda}_{\max,u} > 1.5$	3.141	1.413	0.033	0.049	0.79	0.79	1.04	0.76
	Local		3.142	1.220	0.166	0.176	1.26	1.26	1.13	1.12
FE (cold-formed)	LT	$\bar{\lambda}_{\max,u} \leq 1.5$	3.142	1.234	0.134	0.144	1.19	1.19	1.11	1.08
		$\bar{\lambda}_{\max,u} > 1.5$	3.142	1.832	0.208	0.211	1.03	1.03	1.15	0.90
	Local		3.141	1.091	0.028	0.060	0.94	0.94	1.05	0.89

Table 16: Reliability analysis results for beams in bending about the major axis for the new design proposals (Method 2).

Source	Critical mode	Slenderness range	$k_{d,n}$	b	V_δ	V_r	γ_{M1}^*	$\gamma_{M1}^*/\gamma_{M1}$	f_a	$(\gamma_{M1}^*/\gamma_{M1})/f_a$
FE (hot-rolled)	Local		3.141	1.194	0.110	0.124	1.11	1.11	1.09	1.02
	LT	$\bar{\lambda}_{\max,u} \leq 1.5$	3.141	1.068	0.040	0.068	1.01	1.01	1.05	0.97
		$\bar{\lambda}_{\max,u} > 1.5$	3.141	1.235	0.046	0.061	0.96	0.96	1.05	0.91
FE (cold-formed)	Local		3.142	1.116	0.079	0.098	1.09	1.09	1.07	1.01
	LT	$\bar{\lambda}_{\max,u} \leq 1.5$	3.142	1.095	0.049	0.073	1.01	1.01	1.06	0.96
		$\bar{\lambda}_{\max,u} > 1.5$	3.142	1.608	0.214	0.218	1.22	1.22	1.15	1.06
Test (hot-rolled)			3.141	1.099	0.025	0.058	0.92	0.92	1.04	0.89

Table 17: Reliability analysis results for beams bending about the minor axis for the new design proposals.

Source	Critical mode	Slenderness range	$k_{d,n}$	b	V_δ	V_r	γ_{M1}^*	$\gamma_{M1}^*/\gamma_{M1}$	f_a	$(\gamma_{M1}^*/\gamma_{M1})/f_a$
FE (hot-rolled)	LT	$\bar{\lambda}_{\max,v} \leq 2$	3.116	1.218	0.061	0.079	0.94	0.94	1.06	0.89
		$\bar{\lambda}_{\max,v} > 2$	3.116	1.724	0.131	0.135	0.89	0.89	1.10	0.80
	Brazier	$\bar{\lambda}_{\max,v} \leq 0.4$	3.116	1.278	0.059	0.078	1.05	1.05	1.06	0.99
		$\bar{\lambda}_{\max,v} > 0.4$	3.116	1.045	0.138	0.149	1.03	1.03	1.11	0.92
FE (cold-formed)	LT	$\bar{\lambda}_{\max,v} \leq 2$	3.116	1.116	0.057	0.077	1.02	1.02	1.06	0.96
		$\bar{\lambda}_{\max,v} > 2$	3.116	1.630	0.148	0.152	0.99	0.99	1.11	0.89
	Brazier	$\bar{\lambda}_{\max,v} \leq 0.4$	3.116	1.216	0.065	0.083	0.92	0.92	1.06	0.87
		$\bar{\lambda}_{\max,v} > 0.4$	3.116	1.034	0.075	0.093	1.05	1.05	1.07	0.98
Test (hot-rolled)			4.078	1.223	0.031	0.066	0.85	0.85	1.05	0.81

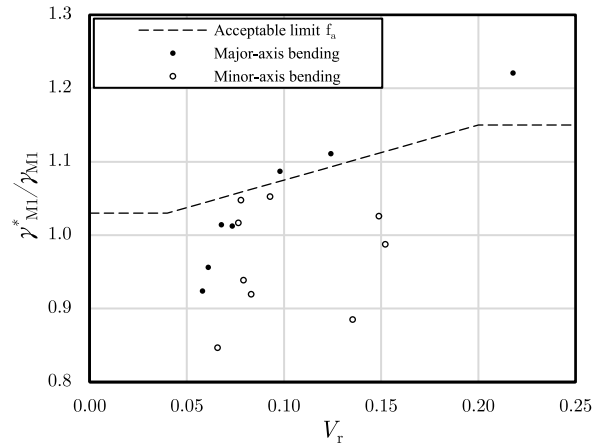


Figure 36: Comparison of the $\gamma_{M1}^*/\gamma_{M1}$ ratios with the acceptance limit f_a for the new design proposals, where Method 2 is used for major-axis bending.

9. Conclusions

A comprehensive study into the behaviour and stability design of steel equal-leg angle section members subjected to uniaxial bending has been presented herein. A series of experiments on steel angle section beams, together with supporting material tests and initial geometric imperfection measurements, was first reported. Finite element models were then developed and validated against the reported test results, as well as existing

experimental data from the literature. The test and numerical results were generally in good agreement. A parametric study was subsequently conducted on both hot-rolled and cold-formed steel angle section beams covering a wide spectrum of cross-section and member geometries with some 1142 numerical results being generated. These results, combined with the new and existing test results, were used to assess the current design provisions in EC3 for steel equal-leg angle section beams. The current EC3 design provisions were found to be overly conservative in cases of bending about the major axis and unsafe in some cases of bending about the minor axis.

For major-axis bending, two new design methods were proposed - a modified EC3 approach and an approach in which, as well as the member slenderness, the ratio of the local to lateral-torsional elastic buckling moments are considered; both led to substantially improved results, but the latter provided better consistency and reliability in the resistance predictions. For angles in bending about the minor axis, only a cross-section resistance check is recommended in the current EC3 methodology. However, it has been shown that angles subjected to minor-axis bending can still be susceptible to lateral-torsional buckling. Additionally, Brazier flattening can become important when the tips of the angle section are in tension. A new design approach for angle section beams in minor-axis bending reflecting these findings has been proposed. Overall, the new design proposals have been shown to provide dramatic improvements in the accuracy and reliability of strength predictions for both hot-rolled and cold-formed steel equal-leg angle section beams.

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