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Essays in financial economics

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Abstract

This thesis investigates moral hazard problems and issues related to banks' risk-taking; it delivers implications with regard to prudential regulation, dynamic incentive contracts, and capital structure.

In the first chapter, I develop a continuous-time model in which a principal hires an agent to run a risky project exposed to unpredictable events that may generate potentially large losses. Although event occurrences are beyond the control of the agent, the distribution of the random losses they generate is shaped by the agent's unobservable risk mitigation effort. I study optimal incentive contracts and show how these contracts strike a delicate balance between supporting and punishing the agent in the face of downside risk.

In the second chapter, I adopt a dynamic contracting framework in order to study the design of incentive-based regulations of a bank engaging in a double moral hazard. In addition to shirking, the bank's manager can engage in risk-taking, which enhances short-term profits while increasing the bank's exposure to tail risk. Under the regulation optimising the bank's private value, excessive risk-taking and regulatory forbearance emerge when the bank is undercapitalised. The chapter then shows how the socially optimal regulator, which internalises the negative externalities of the bank's strategies on the economy, can guarantee the bank's continuation under a safe management.

In the last chapter I propose an empirical study to assess the effects of monetary policy on banks' risk-taking. The study focuses on the impact of monetary shocks on banks' financial distress and lending during the recent financial crisis period characterised by money market rates near the lower bound and unconventional monetary measures. The analysis combines macroeconomic and bank-level data for a set of euro area banks using a factor-augmented autoregressive model. In line with the agency literature, a risk-taking channel arises for banks with low levels of capital.

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Declaration of Originality

'I hereby declare that this thesis represents my own work and that all material that is not my own work has been properly acknowledged.'

Caterina Lepore

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to Marco and Sofia

Chapter 1

Dynamic incentives with event risk

Firms and financial companies are exposed to event risk, which comprises exogenous shocks that may generate large losses. Shocks include disaster events such as industrial accidents, natural catastrophes, macroeconomic downturns, or financial crises. Although the arrival of such shocks is exogenous, the losses generated can be mitigated by managerial effort; this may represent the use of risk monitoring or mitigation tools. This chapter studies the design of dynamic incentive provisions to reduce exposure to event risk.

To this end, I consider a principal-agent problem where a principal hires an agent to run a project exposed to event risk (disaster shocks). Conditional on a disaster occurring, the losses that the shock generates are unpredictable and drawn from a distribution that is shaped by the agent's private effort choice. Since the principal can only observe the cash flow realisations and shock occurrences, but not the loss distribution, a moral hazard problem arises. I am interested in answering the question of how the principal can provide the agents with incentives to exert effort over time.

One may expect that, in order to keep the agent motivated, the principal should force him to cover part of the losses following a disaster event. However, this is possible to a limited extent as the agent is protected by limited liability. This situation was originally studied by Shavell (1986), who showed that the possibility of damages exceeding the injurer's assets makes the injurer effectively "judgment proof", and thus reduces the incentives to mitigate risk. Similar problems arise in delegated asset management, where the inability of investors to make an asset manager share potential losses in full may induce him to take on excessive risk, or in executive compensation, where limited liability makes it difficult to punish managers for the full extent of the downside risk associated with decisions aimed at reaping the rewards of the upside.

To resolve the conflict between the need to discipline the agent against risk-taking and the protection offered by limited liability, in this chapter I study optimal contracts that allow the principal to dynamically reward the agent or fire him depending on the

[0]The content of this chapter has been extracted from the working paper Biffis and Lepore (2015).

project's cash-flow realisations. As is common in similar settings, I find that the optimal contract rewards the agent upon cumulated cash flows reaching a prescribed target. After a long enough period of good performance, the agent receives continuous transfers until a negative shock forces the agent back into a probation period.

What is peculiar to my setting is the provision of incentives on the downside. Upon occurrence of a shock, the agent must be punished by sharing part of the losses. However, when the agent's continuation utility is too low, such contractual incentives become ineffective because of limited liability. Then, the principal can decide whether to support the agent, by allowing him to undergo a further probationary period, or can simply decide to terminate the project. My results suggest that stochastic liquidation occurs only when strictly necessary to guarantee the agent's limited liability, and with probability increasing in the severity of disaster losses. Hence, the principal's threat of termination and support of the agent in the form of insurance of large losses co-exist to ensure that disaster loss realisations can be used to provide the right dynamic incentives.

This chapter explores some of the implications of the optimal contract by considering two possible implementations. In the first implementation the agent's continuation utility is mapped using the firm's cash reserves. I find that the contract entails issuing performance-sensitive debt and compensating the agent upon reserves reaching a high enough threshold. Negative shocks are partially covered by insurance. When cash reserves are depleted and fall below a critical threshold, insurance is ineffective and the firm is either liquidated or bailed out via a cash injection. The termination versus bailout outcome depends on the severity of disaster losses: the larger the realised disaster losses, and the smaller the distance to default, the lower the probability of a bailout.

In an alternative implementation, the agent is endowed with stock options and participates in disaster losses according to his equity share. If disaster losses are severe enough, either the company is shut down and the agent is fired, or stock options are repriced so as to ensure that the agent's incentives are aligned with those of investors, and the firm can be kept alive. The probability of stock option repricing is inversely proportional to the extent to which disaster losses push stock options out of the money.

I. Related literature

This study contributes to the literature on dynamic agency problems, which was pioneered by Holmstrom and Milgrom (1987) and Spear and Srivastava (1987), and recently made considerable progress by making use of martingale techniques along the lines indicated by

Sannikov (2008). Most contributions in the literature study situations in which an agent runs a project exposed to Brownian risk sources. Notable examples[1] are DeMarzo and Sannikov (2006), Biais et al. (2007), He (2009), and DeMarzo et al. (2012). Some studies have enriched the Brownian setting by considering Poisson uncertainty[2] with predictable shock sizes. A strand of literature uses Poisson jumps to capture exogenous shocks that are beyond the control of the agent. Piskorski and Tchistyi (2010), for example, study an optimal mortgage design problem, extending the model of DeMarzo and Sannikov (2006) to stochastic market interest rate. Hoffmann and Pfeil (2010), on the other hand, still model a project's cash flows as a diffusion, but allow for jumps in the drift rate to capture observable shocks affecting a firm's profitability.

A different stream of literature uses Poisson jumps to model negative cash flow shocks whose likelihood of occurrence can be controlled by the agent. For example, in the work of Biais et al. (2010), which is closely related to this chapter, the authors consider a setting where negative shocks have predictable size and are less likely to occur if the agent exerts effort. The agent is motivated by the promise of payments, in case performance is good for a long enough period, and by the threat of project downsizing, should the agent's performance be so poor that limited liability makes contractual incentives ineffective without a reduction in the scale of the project[3]. In a similar setting, Pagès (2013) examines the optimal securitisation of a pool of correlated loans exposed to credit risk, where the default intensity can be decreased by the bank's monitoring activities. The optimal contract prescribes that fees are paid to the bank upon the pool's cash flows reaching a prescribed target. Conversely, when the bank's performance is poor, monitoring incentives are preserved by the threat of termination.

All of the above cited papers adopt Poisson shocks with predictable jump sizes, revealing an important gap in the literature. To the best of my knowledge, the only work considering negative cash flow shocks of random size is that of Capponi and Frei (2015). However, in the authors' jump-diffusion setting, the Poisson component has an exogenous loss distribution, and the agent can only control the jumps' arrival rate. They adopt this framework to study a dynamic multi-tasking principal agent model, in which the agent can

[1]Further contributions include Cadenillas et al. (2007), Cvitanić et al. (2006), Cvitanić et al. (2009), Williams (2013), Williams (2011), who often use the maximum principle instead of the dynamic programming approach used in this chapter.

[2]Among the first contributions to study Poisson shocks is Sung (1997), who extended the continuous time dynamic agency model of Holmstrom and Milgrom (1987) to point processes.

[3]On the other hand, after a long enough period of good performance, investment occurs leading to an increase in the firm's size. Related works on agency models with investments include Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007) and DeMarzo et al. (2012).

exert both effort and accident prevention. Due to the agent's risk aversion, the optimal incentive contract includes non-linear accident penalties, with higher penalties for small relative to large accidents.

As opposed to the studies discussed above, in my model disaster losses are unpredictable and I do not deal with loss prevention but with loss reduction effort. Losses are drawn from a distribution that is shaped by the agent's private risk mitigation effort. The principal is able to contract on loss realisations but will have to pair the threat of termination with some form of insurance, on account of the fact that large losses can occur even under maximal effort, and the agent is protected by limited liability.

The rest of the chapter is organised as follows. In the next section I outline the model, describe the problems faced by the agent and the principal, and address the issues arising from the interaction between limited liability and incentive compatibility constraints. In Section 3, I derive optimal contracts given that the agent exerts maximal risk mitigation effort. I then illustrate the results through a numerical example. In Section 4, I provide a sufficient condition for the optimality of maximal effort. Section 5 explores implementations of the optimal contract. Finally, the last section concludes the chapter, while an appendix contains the proofs.

II. The model

In my model, I consider a risky project that can be undertaken by investors (the principal) only by hiring an agent with unique skills to run the project. The principal has unlimited wealth, whereas the agent has no resources with which to start the project and is protected by limited liability. The project generates cash flow process $Y := (Y_t)_{t\geq 0}$, which is affected by exogenous shocks (disaster events) inducing potentially large losses (disaster losses). Although shock occurrences are beyond the control of the agent, the losses they generate are drawn from a distribution that is shaped by the agent's risk mitigation effort.

The agent's private effort at time t is denoted by e_t. For simplicity, I restrict the model to the case of binary effort, with e_t taking value in the set $\mathcal{E} := \{\underline{e}, \overline{e}\}$, for some $\underline{e}$, $\overline{e} \in \mathbb{R}_+$. A random disaster loss, Z_t, occurring at a generic time t, can take two values: $\underline{z} \geq 0$ (small loss) and $\overline{z}$ (large loss), with $\overline{z} > \underline{z}$. The probability of observing a large loss depends on the agent's effort choice made right before the disaster event, e_{t-}. The probability of a large loss under high effort is denoted by q , and the probability under low effort by $q+\Delta q$ (with $\Delta q > 0$). As is customary in principal agent problems with moral hazard, it is convenient to think of an effort plan $e := (e_t)_{t\geq 0}$ as inducing a probability measure $\mathbb{P}^e$ equivalent to

a reference probability $\mathbb{P}$, so that $\mathbb{P}^e(Z_t = \overline{z}|e_{t-} = \overline{e}) = q$ for every $t \geq 0$. I will at times also write Z_t^e to emphasize dependence of the random loss size on the effort choice e_{t-}. Then define[4] the average loss size under effort e_{t-} by $\mu_t^e := E^e[Z_t|\mathcal{F}_{t-}^Y] = E^e[Z_t|e_{t-}]$ which can take values in $\{\mu(\overline{e}), \mu(\underline{e})\}$. I assume $\mu(\overline{e}) < \mu(\underline{e})$, which means that high-effort losses dominate low-effort losses in the sense of first order stochastic dominance. The cash flow dynamics can be written as

$$\mathrm{d}Y_t^e = \delta \mathrm{d}t - Z_t^e \mathrm{d}N_t = (\delta - \lambda\mu_t^e)\,\mathrm{d}t - \mathrm{d}M_t^e, \tag{II.1}$$

where the compensated jump process $M_t^e = \int_0^t Z_s^e \mathrm{d}N_s - \lambda\mu_s^e \mathrm{d}s$ is an $(\mathbb{F}^Y, \mathbb{P}^e)$-martingale.

A. The agent's and principal's problems

Both the principal and the agent are risk-neutral and can commit to a long-term contract that specifies the payments that the principal will make to the agent, as well as the conditions under which the project will be terminated. Denote by T the process of cumulative transfers to the agent, which is assumed to be non-decreasing, non-negative, right-continuous with left limits, and adapted to $\mathbb{F}^Y$. In other words, I assume the value of T_t to be known based on cashflow information available up to and including time t.

The following timeline describes the sequence of actions taking place over each small time interval $[t, t + \mathrm{d}t]$:

1. The principal prescribes an action recommendation e_t to the agent.
2. The agent makes an effort choice $\hat{e}_t$, incurring a cost $h(\hat{e}_t)$ expressed in units of compensation, with $h(\cdot)$ such that $h(\overline{e}) > h(\underline{e}) \geq 0$.
3. With probability $\lambda \mathrm{d}t$ there is a shock occurrence that results in a loss realisation of size $z \in \{\underline{z}, \overline{z}\}$, drawn from a distribution determined by the agent's private effort choice.
4. Both agent and principal observe the cash flow realisation Y_t.
5. The agent receives a nonnegative transfer $\mathrm{d}T_t \geq 0$.
6. The project is either terminated with probability $1-\theta_t$, or continued with probability θ_t.

[4]Here, $\mathbb{F}^Y = (\mathcal{F}_t^Y)_{t\geq 0}$ denotes the completed natural filtration of the cash flow process, and $E^e[\cdot]$ is the expectation operator under the measure $\mathbb{P}^e$.

The agent discounts the future transfers and effort costs at the rate $\gamma > 0$. Setting the liquidation payoff at the termination time τ equal to zero, it follows that the agent will choose an action plan e so as to maximise the payoff

$$W_0^e = E^e \left[\int_0^\tau \exp(-\gamma t) \left(\mathrm{d}T_t - h(e_t) \mathrm{d}t \right) \right]. \tag{II.2}$$

The principal discounts the future at the rate $\rho \in (0, \gamma)$. She chooses transfers, a liquidation strategy, and a suggested action plan e in order to maximise the expected cash flows from the project, net of any transfers to the agent. Setting the principal's liquidation value to zero, the principal will maximise

$$V_0(W_0) = E^e \left[\int_0^\tau \exp(-\rho t) \left(\mathrm{d}Y_t - \mathrm{d}T_t \right) \right], \tag{II.3}$$

subject to incentive compatibility of action recommendation e, and to delivering at least a nonnegative payoff W_0 to the agent. Since the principal can choose not to hire the agent, I restrict the attention to contracts that result in a nonnegative payoff $V_0(W_0)$.

Denote by $\Delta\mu := \mu(\underline{e}) - \mu(\overline{e}) = \Delta q(\overline{z} - \underline{z})$ the expected increase in disaster losses induced by shirking, and by $\Delta h := h(\overline{e}) - h(\underline{e})$ the cost to the agent of switching from low to high effort.

Assumption II.1. *The following conditions are assumed to hold throughout:*

(i) $\delta - \lambda\mu(\overline{e}) > 0$, *meaning that the project has positive NPV at least when maximal effort is exerted.*

(ii) $\lambda\Delta\mu > \Delta h$, *meaning that in the absence of moral hazard it would be optimal to require the agent to always exercise maximal effort, as the cost of shirking outweighs the cost of risk mitigation effort.*

In the first best contract, maximal effort is implemented at all times, and by Assumption II.1(i) the project is never liquidated. Because the agent is more impatient than the principal, he is paid immediately, and the principal's value function is

$$V^{FB}(W) = \frac{\delta - \lambda\mu(\overline{e})}{\rho} - W.$$

B. Incentive compatibility and limited liability

For a given contract π and action plan e, the agent's continuation utility at time $t \geq 0$ before termination can be written as

$$W_t = E^e \left[\int_t^\tau \exp(-\gamma(s-t)) \left(\mathrm{d}T_s - h(e_s)\mathrm{d}s \right) \middle| \mathcal{G}_t \right], \tag{II.4}$$

where $\mathbb{G} = (\mathcal{G}_t)_{t\geq 0}$ is the minimal filtration containing $\mathbb{F}^Y$, making the liquidation time τ a $\mathbb{G}$-stopping time. The discounted cumulative gain to the agent,

$$H_t = \int_0^{t\wedge\tau} \exp(-\gamma s) \left(\mathrm{d}T_s - h(e_s)\mathrm{d}s \right) + \exp(-\gamma t) W_{t\wedge\tau},$$

is therefore a $\mathbb{G}$-martingale. By a martingale representation theorem and a suitable change of measure (e.g., Jacod and Shiryaev, 1987), there exists a predictable, square-integrable process $\phi := (\phi_t)_{t\geq 0}$ such that

$$\mathrm{d}H_t = -\exp(-\gamma t)\phi_t \mathrm{d}M_t^e - \exp(-\gamma t)\mathrm{d}M_t^R,$$

where $M_t^R = \int_0^t R_s \mathrm{d}N_s - \int_0^t E_{s-}^e[R_s]\lambda \mathrm{d}s$ is a randomisation martingale capturing termination/continuation of the project and taking the form of a compensated jump process with Poisson jumps of random size R_t. For simplicity, I have written $E_{s-}^e[R_s] := E^e[R_s|\mathcal{G}_{s-}]$ in the compensator of M^R and dropped dependence on e in the notation for M^R and R. On $\{\tau \geq t\}$, the dynamics of the agent's continuation utility are then given by

$$\begin{aligned} \mathrm{d}W_t &= (\gamma W_t + h(e_t))\mathrm{d}t - \mathrm{d}T_t - \phi_t \mathrm{d}M_t^e - \mathrm{d}M_t^R \\ &= (\gamma W_t + \phi_t \lambda \mu_t^e + \lambda E_{t-}[R_t])\mathrm{d}t + (h(e_t)\mathrm{d}t - \mathrm{d}T_t) - \phi_t Z_t^e \mathrm{d}N_t - R_t \mathrm{d}N_t. \end{aligned} \tag{II.5}$$

The above shows that ϕ represents the sensitivity of the agent's continuation value to the cash flow variations: following a loss of size $z \in \{\underline{z}, \overline{z}\}$ in the cash flows, the agent's continuation utility is reduced by $\phi_t z$. I now turn to the issue of incentive compatibility, which is summarised in the following proposition, and which will allow me to give an explicit expression to the continuation probability and the jumps of the randomisation martingale.

Proposition II.2 (**Incentive compatibility**). *High effort $\overline{e}$ is optimal if and only if the*

agent's sensitivity ϕ_t satisfies the following condition at each time $t \in [0, \tau)$:

$$\phi_t \geq \beta := \frac{\Delta h}{\lambda \Delta \mu}. \tag{II.6}$$

I now provide an intuitive derivation of the above condition, and refer the reader to the appendix for a formal proof. The agent chooses the effort level maximising the expected change in continuation utility (including randomisation) net of effort costs, that is

$$\lambda \phi_t \mu_t^e + \lambda E_{t-}^e[R_t] - h(e).$$

To induce the agent to exert high risk mitigation effort, the principal should therefore make sure that

$$\lambda \phi_t \mu_t^{\overline{e}} + \lambda E_{t-}^{\overline{e}}[R_t] - h(\overline{e}) \geq \lambda \phi_t \mu_t^{\underline{e}} + \lambda E_{t-}^{\underline{e}}[R_t] - h(\underline{e}),$$

which is equivalent to requiring

$$\phi_t \geq \tilde{\beta}_{t-} := \frac{\Delta h + \lambda \Delta_{t-}^R}{\lambda \Delta \mu},$$

with $\Delta_{t-}^R := E_{t-}^{\overline{e}}[R_t] - E_{t-}^{\underline{e}}[R_t]$. Hence, the agent's continuation utility should be reduced at least by the amount $\tilde{\beta}_{t-} z$, should a loss of size $z \in \{\underline{z}, \overline{z}\}$ materialise at time $t \in [0, \tau)$. However, the agent's continuation utility cannot lie below the threshold $\tilde{\beta}_t \overline{z}$, as otherwise an additional shock would mean that the limited liability constraint, requiring the agent's continuation utility to stay positive at all times, would be violated.

Denote by R_t^C the jump of the randomisation martingale in case the project is continued, and by R_t^T the jump in case the project is terminated. Upon observing a shock ($\mathrm{d}N_t = 1$), the project is terminated with probability $1 - \theta_t$, bringing the agent's continuation utility to level $W_\tau = 0$, so that it must hold

$$R_t^T = W_{t-} - \phi_t Z_t^e = W_t. \tag{II.7}$$

With probability θ_t the project is instead continued, in which case a jump R_t^C takes the agent's continuation utility to level

$$W_{t+} = W_{t-} - \phi_t Z_t^e - R_t^C \geq \tilde{\beta}_t \overline{z},$$

where I have denoted by $W_{t+} \in \mathcal{G}_t$ the agent's continuation utility after the shock is

observed and conditional on public randomisation resulting in project continuation. To find the optimal agent's continuation utility, it is convenient to rewrite W_{t+} as $\tilde{\beta}_t\overline{z} + c_t$ for some nonnegative, $\mathbb{G}$-adapted process c. I can then express the continuation jump as

$$R_t^C = -\left[(\tilde{\beta}_t\overline{z} + c_t) - W_t\right]^+, \tag{II.8}$$

and optimize the principal's utility over the process c to find the optimal jump size R^{C*} and randomisation threshold $\tilde{\beta}_t\overline{z} + c_t^*$. From the "promise-keeping constraint"

$$W_{t-} - \phi_t Z_t^e = W_\tau(1-\theta_t) + (\tilde{\beta}_t\overline{z} + c_t^*)\theta_t = (\tilde{\beta}_t\overline{z} + c_t^*)\theta_t,$$

I obtain that the continuation probability is given by

$$\theta_t = \frac{W_t}{\tilde{\beta}_t\overline{z} + c_t^*} \wedge 1, \tag{II.9}$$

meaning that the project is always continued as long as the agent's continuation utility does not fall below the randomisation threshold. Finally, it should be noted from expressions (II.8), (II.7), and (II.9), that $E_{t-}^e[R_t] = 0$: the agent's continuation utility is neutral to randomisation between continuation and termination, and therefore the sensitivity $\tilde{\beta}_t$ is reduced to the constant β, which is all that is needed for the incentive compatibility condition (II.6). The term β quantifies the magnitude of the moral hazard problem and represents the cost of incentives.

III. Optimal contract under maximal risk mitigation effort

To derive the main features of the optimal contract, I provide a heuristic construction of the principal's value function, $V(w)$, assuming that V is concave. Although this can always be ensured by introducing an additional public randomisation device concavifying the value function, I show in the appendix that this is not needed in this setting. As is customary, in the following section I focus on optimal contracts, given maximal effort $\overline{e}$ exerted by the agent. The principal optimises the set $\mathcal{A}$ of admissible contracts represented by triplets $\pi = (T, \phi, c)$ satisfying the pathwise and measurability conditions indicated in the previous sections.

It is convenient to start by assuming that transfers are absolutely continuous, meaning that they can be represented as $\mathrm{d}T_s = t_s \mathrm{d}s$, with $\overline{t} \geq t_s \geq 0$, for some upper bound $\overline{t}$, which will then be let grow to infinity to encompass my more general setting. Applying

Itô's formula to $V(W_s)$, and taking into account the net cash flows originating from the project, I can write the expected change in the principal's continuation utility as

$$\begin{aligned} E_s^{\overline{e}}(\mathrm{d}V(W_s) + \mathrm{d}Y_s - t_s\mathrm{d}s) =& V(W_s)'(\gamma W_s + h(\overline{e}) - t_s + \phi_s\lambda\mu(\overline{e}))\mathrm{d}s \\ &+ (\delta - \lambda\mu(\overline{e}) - t_s)\,\mathrm{d}s + E_s^{\overline{e}}(V(W_s - \phi_s Z_s - R_s)\mathrm{d}N_s). \end{aligned} \tag{III.1}$$

Noting that at an optimum the contract must deliver at least $\rho V(W_s)$ to the principal, I obtain the following HJB equation for the optimal value function:

$$\begin{aligned} \sup_{\pi\in\mathcal{A}} \big\{ &- t(V'(w)+1) + V'(w)(\gamma w + h(\overline{e}) + \lambda\phi\mu(\overline{e})) \\ &+ \delta - \lambda\mu(\overline{e}) + \lambda E^{\overline{e}}\left[V(w-\phi Z - R)\right] - (\lambda+\rho)V(w)\big\} = 0, \end{aligned}$$

where the last conditional expectation can be given the explicit form

$$\begin{aligned} E^{\overline{e}}\left[V(w-\phi Z - R)\right] \;=\; & q\left(V(w-\phi\overline{z})1_{\{w-\phi\overline{z}>\beta\overline{z}+c\}} + \frac{w-\phi\overline{z}}{\beta\overline{z}+c}V(\beta\overline{z}+c)1_{\{w-\phi\overline{z}\le\beta\overline{z}+c\}}\right) \\ &+ (1-q)\left(V(w-\phi\underline{z})1_{\{w-\phi\underline{z}>\beta\overline{z}+c\}} + \frac{w-\phi\underline{z}}{\beta\overline{z}+c}V(\beta\overline{z}+c)1_{\{w-\phi\underline{z}\le\beta\overline{z}+c\}}\right). \end{aligned} \tag{III.2}$$

From the above it is easy to see that if $V'(w) < -1$, the optimiser t^* would be unbounded when letting $\bar{t}$ go to infinity, and the HJB equation would not have a solution. Restricting attention to the case of $V'(w) \geq -1$, I find that t^* is zero whenever $V'(w) > -1$, and can take any value when $V'(w) = -1$. In the latter case the HJB equation reduces to $\mathcal{K}_V(w) = 0$, with

$$\mathcal{K}_V(w) := \sup_{\pi\in\mathcal{A}} \big\{ - (\gamma w + h(\overline{e}) + \lambda\phi\mu(\overline{e})) + \delta - \lambda\mu(\overline{e}) + \lambda E^{\overline{e}}\left[V(w-\phi Z - R)\right] - (\lambda+\rho)V(w)\big\} = 0.$$

The problem can therefore be summarised by the following HJB variational inequality:

$$\max\{-(1+V'(w)), \mathcal{K}_V(w)\} \leq 0,$$

which is studied in detail in the appendix. As for the optimal transfers, $\overline{w}$ denotes the lowest value such that $V'(w) = -1$. By concavity of V, I then find that for any $w > \overline{w}$ the principal would immediately pay to the agent a lump sum $w - \overline{w}$, bringing his continuation utility down to the threshold $\overline{w}$, where continuous transfers are made until the next event

occurrence. Again by concavity of V, it follows that $V'(w) > -1$ for any $w < \overline{w}$, meaning that the principal will delay payment to the agent, as the marginal benefit of an increase in the agent's continuation utility exceeds the marginal cost of an immediate payment. These considerations are summarised in the following proposition.

Proposition III.1 (**Transfers**). *Transfers to the agent are made only for continuation utility levels $W_t \geq \overline{w} \equiv \inf\{W_t \geq 0 : V'(W_t) = -1\}$. They are lumpy for $W_t > \overline{w}$, and equal to $W_t - \overline{w}$. They are continuous for $W_t = \overline{w}$, with flow of payment equal to $\gamma\overline{w} + h(\overline{e}) + \lambda\beta\mu(\overline{e})$.*

I now turn to the principal's optimal randomisation strategy, which is found by optimising $\lambda E^{\overline{e}}[V(w - \phi Z - R)]$ over c. From the first order condition, it follows

$$q\frac{w - \phi\overline{z}}{\beta\overline{z} + c}\left(V'(\beta\overline{z} + c) - \frac{V(\beta\overline{z} + c)}{\beta\overline{z} + c}\right) \leq 0, \tag{III.3}$$

for $\beta\overline{z} + \phi\underline{z} < w < \phi\overline{z} + \beta\overline{z} + c$, and

$$q\frac{w - \phi\overline{z}}{\beta\overline{z} + c}\left(V'(\beta\overline{z}+c) - \frac{V(\beta\overline{z} + c)}{\beta\overline{z} + c}\right) + (1-q)\frac{w - \phi\underline{z}}{\beta\overline{z} + c}\left(V'(\beta\overline{z}+c) - \frac{V(\beta\overline{z} + c)}{\beta\overline{z} + c}\right) \leq 0 \tag{III.4}$$

for $0 < w < \phi\underline{z} + \beta\overline{z} + c$. The left-hand terms in both (III.3) and (III.4) are negative by concavity of V. Hence, for every w, I must have $c^* = 0$ at an optimum, meaning that randomisation will occur only conditionally on the agent's continuation utility falling below the threshold $\beta\overline{z}$. Randomisation between continuation and termination will therefore be used by the principal only when it is strictly necessary, in order to ensure that the agent is protected by limited liability.

Proposition III.2 (**Termination**). *When $W_t \geq \beta\overline{z}$, the project is continued with probability $\theta_t = 1$. When $W_t < \beta\overline{z}$, the project is stochastically terminated, with continuation probability given by*

$$\theta_t = \frac{W_t}{\beta\overline{z}}. \tag{III.5}$$

The continuation and termination jumps of the randomisation martingale M^R are given by $R^C = -\left[\beta\overline{z} - W_t\right]^+$ and $R^T_t = W_t$.

From the above I see that the threat of termination, as proxied by the termination probability, $(\beta\overline{z} - W_t)/(\beta\overline{z})$, is more severe the larger the drop of the agent's continuation utility below the threshold $\beta\overline{z}$. As the agent's continuation utility will never enter the

interval $[0, \beta\overline{z})$, for $0 \leq w < \beta\overline{z}$ it is convenient to set:

$$V(w) = \frac{w}{\beta\overline{z}} V(\beta\overline{z}).$$

Finally, I study the agent's optimal sensitivity to shock events. Writing the first order condition with respect to ϕ, I obtain:

$$\lambda V'(w)\mu(\overline{e}) - \lambda E^{\overline{e}}\left[V'(w - \phi Z)Z\right] \leq 0.$$

The term on the left-hand side is always non-positive by the concavity of V. Hence it is optimal to set the agent's sensitivity ϕ to the minimal level β inducing the agent to exert high risk mitigation effort, as seen from the incentive compatibility constraint (II.6).

I can now summarise the results of the previous discussion on the principal's optimal value function in the following theorem, which provides conditions under which the contract outlined above is indeed optimal.

Theorem III.3 (**Verification theorem**). *Assume that the following condition is satisfied:*

$$\lambda(\delta - \lambda\mu(\overline{e}) - h(\overline{e})) > (\gamma - \rho)(\lambda\beta\mu(\overline{e}) + \beta\overline{z}(\lambda + \rho)). \tag{III.6}$$

Then the principal's optimal value function is given by the solution $V(w)$ to the following system

$$\begin{cases} V(w) = \frac{w}{\beta\overline{z}} V(\beta\overline{z}) & w \leq \beta\overline{z}, \\ \rho V(w) = V'(w)(\gamma w + h(\overline{e}) + \lambda\beta\mu(\overline{e})) & \\ \qquad\qquad +\delta - \lambda\mu(\overline{e}) + \lambda E^{\overline{e}}[V(w - \beta Z)] - \lambda V(w), & w \in (\beta\overline{z}, \overline{w}), \\ V(w) = V(\overline{w}) - w + \overline{w}, & w \geq \overline{w}, \end{cases} \tag{III.7}$$

where $\overline{w} \equiv \inf\{w \geq 0 : V'(w) = -1\}$. The value function is continuous over $\mathbb{R}_+$ and continuously differentiable in $(\beta\overline{z}, \infty)$. The optimal contract takes the following form

$$\begin{cases} dT_t^* = (W_t - \overline{w})^+ + 1_{\{w=\overline{w}\}}(\gamma\overline{w} + h(\overline{e}) + \lambda\beta\mu(\overline{e}))dt, \\ \phi^* = \beta, \\ \theta^* = \frac{W_t}{\beta\overline{z}} \wedge 1, \\ R^{C*} = -\left[\beta\overline{z} - W_t\right]^+, \\ R^{T*} = W_t, \end{cases} \tag{III.8}$$

for loss realisation $z \in \{\underline{z}, \overline{z}\}$.

Condition (III.6) ensures that the cost of high risk mitigation effort is not too high relative to the NPV of the project under maximal effort. If the condition does not hold, the principal will pay the agent immediately and stochastically terminate the project at the arrival of the first negative shock. If condition (III.6) holds instead, the optimal contract implementing maximal risk mitigation effort will resort to stochastic termination only when the agent's continuation utility falls in the interval $[0, \beta\overline{z})$. Here, the probability of termination is increasing in the undershoot of W below the threshold $\beta\overline{z}$.

As for the upside, after a long enough period of good performance, the agent will be rewarded upon W exiting the probation region $[\beta\overline{z}, \overline{w})$ and hitting the threshold $\overline{w}$. As illustrated in Figure (1.1), the agent will then be rewarded continuously, thus keeping W constant at $\overline{w}$, until occurrence of the next loss event. Upon the arrival of a negative shock of a loss size $Z \in \{\underline{z}, \overline{z}\}$, the agent is punished by having his continuation's utility decreased proportionally by the amount βZ.

< Figure 1.1 about here >

A. Numerical example

To further illustrate the results, I now provide a numerical example of the optimal contract. Figure (1.2) shows the principal's value function for a set of baseline parameters. It should be noted that V is strictly concave for values of the agent's continuation utility in the probation region $(\beta\overline{z}, \overline{w})$, with termination threshold $\beta\overline{z} = 5.20$ and payment threshold $\overline{w} = 14.03$, and V is linear in the liquidation and transfer regions. Under this choice of parameters, the agent's sensitivity to cash flow variations is $\beta = 0.52$. Therefore, whenever there is a loss the agent bears a punishment equal to half of the loss size in terms of his continuation utility.

< Figure 1.2 about here >

In order to assess the impact of the key parameters on the optimal contract, I now perform some comparative statics. In particular I am interested in analysing the sensitivity to the shock's distribution parameters of the main contractual terms, i.e. the payment threshold, the termination threshold and the agent's cash flow sensitivity.

As shown in Table (1.1), less frequent shocks imply a higher agent's cash flow sensitivity and payment threshold. Because it is more difficult to incentivise risk mitigation of very rare disaster events, the contract requires heavier punishments and more back-loaded

compensation schemes. Lower values of Δq and $\overline{z}$ or higher $\underline{z}$ yield a lower $\Delta\mu = \Delta q(\overline{z}-\underline{z})$, which represents the expected increase in disaster loss induced by shirking. A lower $\Delta\mu$ in turn implies that it is more difficult to detect shirking. Hence the need to rely on more back-loaded payments and heavier punishments. Furthermore, a longer compensation's deferral is adopted when there is a higher probability of observing a large shock under high risk mitigation effort.

Table 1.1. Comparative statics of λ, $\overline{z}$, $\underline{z}$, Δq.
The table reports comparative statics with respect to λ, $\overline{z}$, $\underline{z}$, Δq. The remaining parameters are $\gamma = 0.1$, $\rho = 0.15$, $\delta = 12$, $h(e) = 0.5 * e$, $\underline{e} = 0$, $\overline{e} = 1$.

Parameter	**value**	β	$\beta\overline{z}$	$\overline{w}$
λ	0.1	1.56	15.62	31.24
	0.2	0.78	7.81	17.12
	0.6	0.26	2.60	8.34
$\overline{z}$	8	0.69	5.55	14.97
	15	0.32	4.80	12.95
	25	0.18	4.52	12.14
$\underline{z}$	1	0.46	4.62	12.59
	4	0.69	6.94	18.71
	5	0.83	8.33	22.92
Δq	0.3	0.69	6.94	17.9
	0.5	0.41	4.16	11.51
	0.7	0.29	2.97	8.44
q	0.1	0.52	5.20	11.46
	0.3	0.52	5.20	15.14
	0.5	0.52	5.20	16.69

It should be noted that whenever, in order to provide the right incentives, the agent is exposed to higher risks, the likelihood of early termination increases. As a result, the payment boundary increases to provide a long enough period of compensation's deferral. Similar results have been found by DeMarzo et al. (2012) and Wong (2013) within different agency models.

The literature on executive compensation has long debated about the sensitivity of CEOs' wealth to performance and the trade-off between risk and incentives[5]. Moreover

[5]In particular the literature is divided with respect to the relation between incentives and risk. While standard theoretical agency models (like that of Holmstrom and Milgrom (1987) find a negative relationship, the empirical evidence is mixed. I refer to Edmans and Gabaix (2009) for a survey of the existing literature.

the recent financial crisis has sparked renewed attention on the relation between risk and the managers' level of incentives. In particular, policy makers have emphasised the need for a better design of incentive compensation structures in order to ensure appropriate risk management. In this regard my model predicts that, in order to incentivise managers to reduce exposure to event risk, their compensation should be made more sensitive to less frequent shocks. However, when exposed to very large losses the managers' sensitivity must be decreased.

Table 1.2. Comparative statics of $h(\overline{e})$, γ, δ
The table reports comparative statics with respect to $h(\overline{e})$, γ, δ. The remaining parameters are $\rho = 0.15$, $\lambda = 0.3$, $\underline{e} = 0$, $\overline{e} = 1$, $q = 0.2$, $\Delta q = 0.4$, $\overline{z} = 10$, $\underline{z} = 2$.

Parameter	**value**	β	$\beta\overline{z}$	$\overline{w}$
$h(\overline{e})$	0.3	0.31	3.12	8.98
	0.8	0.83	8.33	20.17
	0.9	0.93	9.37	21.93
γ	0.2	0.52	5.20	11.64
	0.5	0.52	5.20	10.42
	0.8	0.52	5.20	6.25
δ	8	0.52	5.20	12.66
	20	0.52	5.20	14.99
	25	0.52	5.20	15.25

Finally Table (1.2) reports comparative statics for some other parameters of the model. In particular, higher cost of effort intensifies the agency problem; therefore the optimal contract relies on more high-powered incentive schemes to motivate the agent. The other two parameters, γ and δ, do not affect the sensitivity β but only the transfers threshold. Specifically, the more impatient the agent is (higher γ), the earlier the principal needs to reward him with some transfers as it becomes costlier to delay payments. On the other hand, a higher δ means that the project is more profitable, and hence it is optimal to delay the agent's transfers longer.

IV. Optimality of maximal risk mitigation effort

In the derivation of the optimal contract of the previous section I focused on contracts inducing maximal effort. However in full generality a contract may give rise to varying levels of effort being exerted depending on the cost-benefit trade-off, i.e. the benefits of risk mitigation effort relative to the costs of incentive provisions.

In line with Biais et al. (2010), I now provide a condition ensuring that the principal finds it optimal to prescribe maximal effort ($e_t^* = \overline{e}$) at all times. For maximal and minimal effort, the agent's continuation utility evolves as follows, respectively:

$$dW_t = \begin{cases} (\gamma W_t + \beta\lambda\mu(\overline{e}))\mathrm{d}t + (h(\overline{e})\mathrm{d}t - dT_t) - \beta Z^{\overline{e}}\mathrm{d}N_t - R_t\mathrm{d}N_t & \text{if } \ e_t = \overline{e}, \\ (\gamma W_t + h(\underline{e}))\mathrm{d}t & \text{if } \ e_t = \underline{e} \end{cases} \tag{IV.1}$$

For high effort $\overline{e}$ to be exerted, the expected reduction in disaster losses given by maximal effort must be large enough to justify the costs incurred by the agent[6]. For given Δh, the larger $\Delta\mu$ is, the larger the principal's benefits and the lower the incentives cost will be. If the difference is sufficiently large, the principal will find it optimal to always induce maximal effort.

Proposition IV.1. *There exists a $\Delta^* > 0$ such that, if*

$$\Delta\mu > \Delta^*, \tag{IV.2}$$

then exerting effort $\overline{e}$ is optimal at all times.

V. Implementation

In this section I demonstrate how the optimal contract derived in the previous section can be implemented through standard financial securities. The firm's cash reserves Q_t are adopted as a record-keeping device to keep track of the agent's continuation utility W_t. I show how debt, insurance and stochastic termination/bailout can be used to replicate the optimal contract.

Cash reserves: The firm's initial funds and the project's cash flows are deposited into a cash account, continuously yielding a rate ρ. The account is used to meet the debt and insurance payment and manager compensation. Hence the accumulated cash reserves represent a measure of the firm's performance.

Debt: The firm issues a corporate bond paying a constant coupon $x_t = \delta - \lambda\mu(\overline{e}) - h(\overline{e})$.

Insurance contract: In exchange for the payment of a premium $IP_t\mathrm{d}t$, the insurance contract offers protection against losses IL_t. In particular, if there is a loss ($\mathrm{d}N_t = 1$) of

[6]It should be noted that in Biais et al. (2010), the optimality of maximal risk prevention effort is ensured by a sufficiently high decrease in the probability of a disaster's occurrence under high effort instead.

size z, the insurance covers a proportional part of the losses

$$IL_t = (1-\beta)z\mathrm{d}N_t,$$

leaving the firm liable for the remaining fraction β (to keep the appropriate incentives). This result is in line with the literature on self-insurance. For the case of loss reduction a coinsurance contract[7], i.e. a contract implying monotonicity of the amount the insured has to bear relative to the size of the loss, has been proven to be optimal under some conditions[8]. The premium paid by the firm in exchange of the insurance coverage is given by a fair premium

$$(1-\beta)\mu(\overline{e})\mathrm{d}t$$

plus a risk-based premium

$$(\rho-\gamma)Q_t\mathrm{d}t.$$

This last component of the insurance premium is decreasing with the level of cash reserves. Hence when the firm is in financial distress the insurance company will require a higher premium, while when the cash reserves increase this component will lower the premium (similarly to a bonus).

Termination/Recapitalisation: When the firm is in financial distress, i.e. the cash reserves fall below the threshold $\underline{q} \equiv \beta\overline{z}$, with probability $\theta_t = \frac{Q_{t-}-\beta z}{\bar{q}}$ the firm is kept alive by a cash injection (bailout) that brings the cash reserves back to the threshold $\underline{q}$, and with probability $1-\theta_t$ it is terminated[9]. It should be noted that the probability of termination is higher the lower the level of cash reserves is (below the threshold $\underline{q}$). Several papers on prudential regulation establish the optimality of having mixed termination strategies. In particular, a stream of literature has supported the "constructive ambiguity" approach to government bailouts in order to mitigate the moral hazard problem facing central banks. I refer for instance to Mishkin (1999), Freixas (1999) and Shim (2011). Specifically, this bailout strategy resembles the one obtained by Shim (2011), which he denominates[10] a form of "structured ambiguity".

[7]For loss prevention, conversely, the optimal insurance contract takes the form of a deductible. This is indeed the type of contract found in Biais et al. (2010). See the working paper version for an implementation of the contract using an insurance contract.

[8]I refer to Rees and Wambach (2008) Chapter 5 for a review of insurance models under moral hazard.

[9]The principal breaks even because when the firm is terminated he can extract $Q_{t-}-\beta z$ of cash and when the firm is kept alive he needs to inject an amount $\beta\overline{z}+\beta z-Q_t$. Therefore in expectation the termination/bailout strategy has a value of $\frac{Q_{t-}-\beta z}{\beta\overline{z}}(Q_{t-}-\beta z-\beta\overline{z})+(1-\frac{Q_{t-}-\beta z}{\beta\overline{z}})(Q_{t-}-\beta z)=0$.

[10]This is due to the fact that the contract does not leave discretion to the principal on the termination but establishes ex ante the termination probability

Managerial Compensation: When the cash reserves reach the upper threshold $\overline{q} \equiv \overline{w}$ the manager is rewarded with cash transfers:

$$(\gamma\overline{q} + h(\overline{e}) + \lambda\beta\mu(\overline{e}))\mathrm{d}t$$

as long as no shocks occur. Alternatively, mapping the agent's continuation utility with the firm's equity, I could think of the manager as holding stock options. Then the cash payment would reflect the profit from exercising the option at the strike price $\overline{q}$. On the other hand, when the equity value drops below $\underline{q}$ the options would be repriced if the project was continued in order to restore the manager's incentives. The probability of stock option repricing is inversely proportional to the extent to which disaster losses push stock options out of the money. Even if subject to heavy criticisms, such as rewarding the agent for poor performance, option repricing after negative shocks has often been documented empirically. As is done in my model, this practice has been advocated as a possible mechanism for restoring the proper manager's incentives after drops in stock prices[11].

VI. Conclusion

In this chapter I considered a continuous-time principal agent problem in which a risky project is exposed to event risk, in the sense of exogenous shocks (disasters) generating potentially large losses. Shock occurrences are natural catastrophes, macroeconomic downturns, or financial crises. Although disasters' occurrence is beyond the control of the agent, the losses they generate are drawn from a distribution that is shaped by the agent's private risk mitigation effort. Realised losses are publicly observable and contractible, but their distribution is not, and hence a moral hazard problem arises.

I show that in this setting the agent must be rewarded for good performance, upon reaching a performance target, but may need support from the principal in bad states of the world. As disaster events are beyond the agent's control, the principal and the agent need to share downside risk to some extent. To prevent the agent from shirking, risk

[11] In particular, option repricings have been justified as a means of protecting employees from industry wide negative shocks that are outside of their control. For instance, Saly (1994) demonstrates that when poor performance is due to a market downturn, it is optimal to cancel underwater options and reissue options with a lower exercise price. The result is confirmed empirically by analyzing the repricing of employee stock options after the 1987 crash. Acharya et al. (2000) show the optimality of resetting in an equilibrium contracting model. They argue that an ex-ante strategy of resetting options after bad outcomes, although associated with a negative anticipation effect, is better than a pre-commitment not to renegotiate, showing that some resetting is always optimal.

sharing is paired with the threat of stochastic termination, with termination probability increasing with the severity of the realised losses, conditional on a disaster event.

The findings are consistent with the use of instruments such as stock option repricing and coinsurance to protect managers against downside risk, as well as with regulatory bailout provisions that are deployed during financial crises.

VII. Appendix: Figures

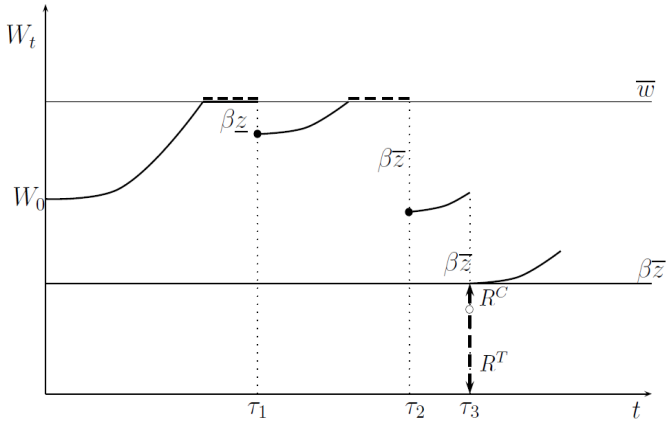


Figure 1.1. The Agent's continuation utility.
The figure shows a sample path of the agent's continuation utility. W_t starts at the initial value W_0 and drifts upward until it reaches the upper threshold $\overline{w}$. At this point, W_t is kept constant by positive transfers until the arrival of a loss, which can be of a small size $\underline{z}$ as at time τ_1 or of a large size at time τ_2. At time τ_3, another large loss is realised, bringing W_t below the lower threshold $\beta\overline{z}$. Below this point the agent's continuation utility is randomised. With probability θ_t, a positive jump R^C brings W_t back to $\beta\overline{z}$ where its evolution restarts, while with probability $1-\theta_t$ a negative jump R^T sets W_t to zero.

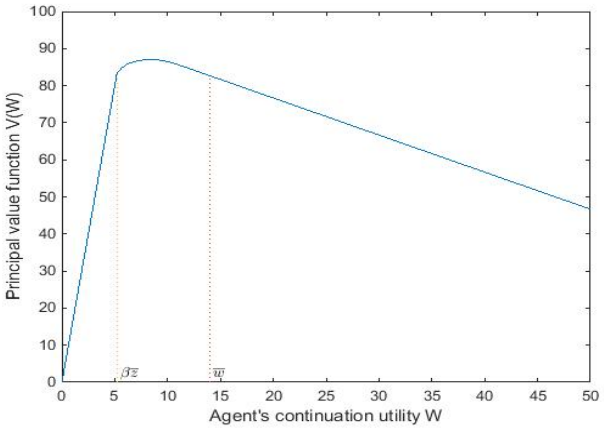


Figure 1.2. The Principal's value function.
Parameter values: $\gamma = 0.1$, $\rho = 0.15$, $\lambda = 0.3$, $\delta = 12$, $h(e) = 0.5 * e$, $\underline{e} = 0$, $\overline{e} = 1$, $q = 0.2$, $\Delta q = 0.4$, $\overline{z} = 10$, $\underline{z} = 2$. Under the optimal contract the liquidation and payment thresholds are equal to $\beta\overline{z} = 5.20$ and $\overline{w} = 14.03$, respectively.

VIII. Appendix: Proofs

A. *Proof of Proposition (II.2)*

I first prove that in general high effort $\overline{e}$ is optimal if and only if

$$h(\overline{e}) - h(\underline{e}) - \lambda\phi_t(\mu(\underline{e}) - \mu(\overline{e})) + \lambda(E^{\overline{e}}_{t-}[R_t] - E^{\underline{e}}_{t-}[R_t]) \leq 0 \tag{VIII.1}$$

Then, since $E^{\overline{e}}_{t-}[R_t] = E^{\underline{e}}_{t-}[R_t] = 0$, (VIII.1) reduces to (II.6). If the agent deviates from maximal effort choice $\overline{e}$ by taking low effort $\underline{e}$ during the time interval $[0, t]$, the discounted cumulated payoffs to the agent are then given by

$$H_t = \int_0^t e^{-\gamma s}(\mathrm{d}T_s - h(\underline{e})\mathrm{d}s) + e^{-\gamma t}W^{\overline{e}}_t.$$

Differentiating, I have

$$\begin{aligned}
\mathrm{d}H_t &= e^{-\gamma t}(\mathrm{d}T_t - h(\underline{e})\mathrm{d}t) - \gamma e^{-\gamma t}W^{\overline{e}}_t\mathrm{d}t + e^{-\gamma t}\mathrm{d}W^{\overline{e}}_t \\
&= e^{-\gamma t}(\mathrm{d}T_t - h(\underline{e})\mathrm{d}t) - \gamma e^{-\gamma t}W^{\overline{e}}_t\mathrm{d}t + e^{-\gamma t}\Big(\gamma W^{\overline{e}}_t + \phi_t\lambda\mu(\overline{e}) + \lambda E^{\overline{e}}_{t-}[R_t]\Big)\mathrm{d}t \\
&\quad + e^{-\gamma t}\Big((h(\overline{e})\mathrm{d}t - \mathrm{d}T_t) - \phi_t Z^{\overline{e}}_t\mathrm{d}N_t - R_t\mathrm{d}N_t\Big) \\
&= e^{-\gamma t}\Big(h(\overline{e}) - h(\underline{e}) - \lambda\phi_t(\mu(\underline{e}) - \mu(\overline{e})) + \lambda(E^{\overline{e}}_{t-}[R_t] - E^{\underline{e}}_{t-}[R_t])\Big)\mathrm{d}t \\
&\quad - e^{-\gamma t}\phi_t(Z^{\underline{e}}_t\mathrm{d}N_t - \lambda\mu(\underline{e})\mathrm{d}t) - e^{-\gamma t}(R_t\mathrm{d}N_t - \lambda E^{\underline{e}}_{t-}[R_t]\mathrm{d}t)
\end{aligned}$$

Assuming that condition (VIII.1) is violated on a set of positive measure, I would then obtain that $E^{\underline{e}}[H_T] \geq W^{\overline{e}}_0$ for some $T > 0$, thus contradicting the optimality of $\overline{e}$. Conversely, assuming that condition (VIII.1) is satisfied, then H is a $\mathbb{P}^{\underline{e}}$-supermartingale for low effort $\underline{e}$. Thus $W^{\overline{e}}_t = H_0 \geq W^{\underline{e}}_t$, showing that $\overline{e}$ is at least as good as low effort $\underline{e}$.

B. *Proof of Theorem (III.3)*

Define ψ_η for $\eta \geq 0$ as the solution to

$$\begin{cases}
\psi_\eta(w) = \eta w, \text{ if } w \in [0, \beta\overline{z}] \\
\rho\psi_\eta(w) = \psi'_\eta(w)(\gamma w + h(\overline{e}) + \lambda\beta\mu(\overline{e})) + \delta - \lambda\mu(\overline{e}) + \lambda E^{\overline{e}}[\psi_\eta(w - \beta Z)] - \lambda\psi_\eta(w), \ w \in (\beta\overline{z}, \infty).
\end{cases} \tag{VIII.2}$$

Following Biais et al. (2010) I proceed as follows. First I show that there exists an η^* such that, under condition (III.6), ψ_{η^*} is strictly concave for $w < \overline{w}$ and that defining the function u as

$$u(w) = \begin{cases} \psi_{\eta^*}(w), & \text{if } 0 \leq w \leq \overline{w} \\ u(\overline{w}) - (w - \overline{w}), & \text{if } w > \overline{w} \end{cases} \tag{VIII.3}$$

it follows that u is the maximal among the solutions of (III.7). I will then proceed to prove that $u(w)$ provides an upper bound for the principal's value function $V(w)$. Finally I will show that the upper bound is attained by the principal's value function, i.e. $u(w) = V(w)$, and hence the optimal contract is given by (III.8). Condition (III.6) is assumed to hold through this section. I start by proving the following proposition.

Proposition VIII.1.

i) $\psi_{\eta_1} \geq \psi_{\eta_2}$ *if and only if* $\eta_1 > \eta_2$.

ii) There exists a maximum value of η, η^*, *such that* $\psi'_\eta(w) = -1$ *has a solution.*

iii) The solution $\overline{w}$ *to* $\psi'_{\eta^*}(\overline{w}) = -1$ *is unique and strictly positive.*

iv) The function ψ_{η^*} *is concave over* $[0, \overline{w}]$ *and strictly concave over* $[\beta\overline{z}, \overline{w}]$.

For convenience I will work with the associated social value function denoted by $g(w) = u(w) + w$ that satisfies

$$\begin{cases} \rho g(w) = g'(w)(\gamma w + h(\overline{e}) + \beta\lambda\mu(\overline{e})) + \delta - \lambda\mu(\overline{e}) + (\rho - \gamma)w - h(\overline{e}) \\ +\lambda E^{\overline{e}}[g(w - \beta Z)] - \lambda g(w), \text{ if } w \in (0, \overline{w}] \\ g(w) = g(\overline{w}), \text{ if } w \geq \overline{w} \end{cases} \tag{VIII.4}$$

Now consider the function G to be the solution of the following delay differential equation:

$$\begin{cases} G(w) = \frac{G(\beta\overline{z})}{\beta\overline{z}} w, \text{if } w \in [0, \beta\overline{z}] \\ \rho G(w) = \delta - \lambda\mu(\overline{e}) + (\rho - \gamma)w - h(\overline{e}) + \mathcal{L}G(w), \text{ if } w \in (\beta\overline{z}, \infty). \end{cases} \tag{VIII.5}$$

where the differential operator $\mathcal{L}G(w)$ is defined for every $w > 0$ as

$$\mathcal{L}G(w) = G'(w)(\gamma w + h(\overline{e}) + \beta\lambda\mu(\overline{e})) + \lambda E^{\overline{e}}[G(w - \beta Z)] - \lambda G(w).$$

I will show that the results of proposition (VIII.1) holds if, for an appropriate choice of $\frac{G(\beta\overline{z})}{\beta\overline{z}}$ and $\overline{w}$, g is defined as

$$g(w) = \begin{cases} G(w), & \text{if } 0 \leq w < \overline{w} \\ g(\overline{w}), & \text{if } \overline{w} \leq w < \infty \end{cases} \tag{VIII.6}$$

Now consider the functions G_1 and G_2 solving respectively

$$\begin{cases} G_1(w) = 0, \text{ if } w \in [0, \beta\overline{z}], \\ \rho G_1(w) = \delta - \lambda\mu(\overline{e}) + (\rho - \gamma)w - h(\overline{e}) + \mathcal{L}G_1(w), \text{ if } w \in (\beta\overline{z}, \infty); \end{cases} \tag{VIII.7}$$

and

$$\begin{cases} G_2(w) = w, \text{ if } w \in [0, \beta\overline{z}], \\ \rho G_2(w) = \mathcal{L}G_2(w), \text{ if } w \in (\beta\overline{z}, \infty). \end{cases} \tag{VIII.8}$$

It should be noted that G_1 and G_2 are continuously differentiable over $(\beta\overline{z}, \infty)$ except for $w = \beta\overline{z}$

$$G'_{1+}(\beta\overline{z}) = \frac{(h(\overline{e}) - \delta + \lambda\mu(\overline{e}) + \beta\overline{z}(\gamma - \rho))}{\gamma\beta\overline{z} + h(\overline{e}) + \lambda\beta\mu(\overline{e})} < 0 = G'_{1-}(\beta\overline{z}) \tag{VIII.9}$$

and

$$G'_{2+}(\beta\overline{z}) = \frac{\rho\beta\overline{z} + \lambda\beta\mu(\overline{e})}{\gamma\beta\overline{z} + h(\overline{e}) + \lambda\beta\mu(\overline{e})} < 1 = G'_{2-}(\beta\overline{z}). \tag{VIII.10}$$

where the inequalities follow by assumption (III.6) and the fact that $\rho < \gamma$. The function G can be rewritten as $G_1 + \kappa G_2$ for an appropriate choice of κ. Functions of this form are increasing in κ and satisfy the analogue of (VIII.5) with κ replacing $\frac{G(\beta\overline{z})}{\beta\overline{z}}$. The former property is due to the fact that G_2 and its derivative are strictly positive, as shown in the following Lemma (VIII.2). Therefore statement (i) of proposition (VIII.1) follows.

Lemma VIII.2. *$G'_2(w) > 0, \forall w > 0$.*

Proof. From (VIII.10) I have $G'_{2+}(\beta\overline{z}) > 0$. Assuming $G'_2(\tilde{w}) = 0$ for some $\tilde{w} \in (\beta\overline{z}, \infty)$, then

$$-\rho G_2(\tilde{w}) - \lambda G_2(\tilde{w}) + \lambda E^{\overline{e}}[G_2(\tilde{w} - \beta z)] = 0.$$

This is a contradiction, since G_2 is strictly increasing and strictly positive over $[0, \tilde{w}]$. □

Now I proceed to show that the result of the following Lemma (VIII.3) holds. As a consequence the ratio $-\frac{G'_1+(w)}{G'_2+(w)}$, is strictly positive at $\beta\overline{z}$ and becomes strictly negative for

a large enough w. Then, since $-\frac{G_1'+(w)}{G_2'+(w)}$ is a continuous function of $w \in [\beta\overline{z}, \infty)$, it attains a maximum over $[\beta\overline{z}, \infty)$.

Lemma VIII.3. $\liminf_{w\to\infty} G_1'(w) \geq 1$.

Proof. The result is demonstrated in two steps. Define $l \equiv \liminf_{w\to\infty} G_1'(w)$. First I prove that $l \neq -\infty$ and then I show that $l \geq 1$. Assume by contradiction that $l = -\infty$. Then there exists an increasing divergent sequence $(w_m)_{m\geq 0}$ such that $\lim_{m\to\infty} G_1'(w_m) = -\infty$ and $w_m = \arg\min_{w\in[0,w_m]} G_{1+}'(w)$. From (VIII.7) it follows that for every $m \geq 0$

$$\begin{aligned}(\gamma w_m + h(\overline{e}) + \beta\lambda\mu(\overline{e}))G_1'(w_m) &= \lambda E^{\overline{e}}[G_1(w_m) - G_1(w_m - \beta Z)] + \rho G_1(w_m) \\ &\quad - \delta + \lambda\mu(\overline{e}) + (\gamma - \rho)w_m + h(\overline{e}) \\ &= \lambda G_1'(\hat{w}_m)\beta\mu(\overline{e}) + \rho G_1(w_m) - \delta + \lambda\mu(\overline{e}) + (\gamma - \rho)w_m + h(\overline{e})\end{aligned}$$

for some $\hat{w}_m \in (w_m - \beta\overline{z}, w_m)$. Rearranging the terms, I obtain

$$G_1'(\hat{w}_m) = \frac{w_m}{\lambda\beta\mu(\overline{e})}\left[\gamma G_1'(w_m) - \frac{\rho}{w_m}G_1(w_m)\right] + \frac{\delta - \lambda\mu(\overline{e}) - h(\overline{e})}{\lambda\beta\mu(\overline{e})} - \frac{(\gamma-\rho)w_m}{\lambda\beta\mu(\overline{e})} + \left(1 + \frac{h(\overline{e})}{\lambda\beta\mu(\overline{e})}\right)G_1'(w_m).$$

By construction, $G_1(w) \geq w_m G_1'(w_m)$ and $G_1'(w_m) < 0$ for a large enough m. Therefore, evaluating the above expression for such an m, I obtain

$$G_1'(\hat{w}_m) \leq \frac{w_m}{\lambda\beta\mu(\overline{e})}[\gamma G_1'(w_m) - \rho G_1'(w_m)] + \frac{\delta - \lambda\mu(\overline{e}) - h(\overline{e})}{\lambda\beta\mu(\overline{e})}.$$

Dividing by $G_1'(w_m) < 0$ the above expression, it follows that

$$\frac{G_1'(\hat{w}_m)}{G_1'(w_m)} \geq \frac{(\gamma-\rho)w_m}{\lambda\beta\mu(\overline{e})} + \frac{\delta - \lambda\mu(\overline{e}) - h(\overline{e})}{\lambda\beta\mu(\overline{e})G_1'(w_m)}.$$

Then, taking the limit for $m \to \infty$, the ratio $\frac{G_1'(\hat{w}_m)}{G_1'(w_m)}$ goes to ∞. Hence, since $G_1'(w_m) < 0$, for a large enough m, $G_1'(\hat{w}_m) < G_1'(w_m)$. However this is not possible, because $\hat{w}_m < w_m$ and by definition $w_m = \arg\min_{w\in[0,w_m]} G_{1+}'(w)$. Hence I obtain a contradiction.

Now I proceed to the second step of the proof. Since $\liminf_{w\to\infty} G_1'(w) = l$, for an increasing divergent sequence $(w_m)_{m\geq 0}$, there exists a constant C such that $G_1(w_m) \geq l w_m + C$. Then for some $\hat{w}_m \in (w_m - \beta\overline{z}, w_m)$ it holds that

$$\gamma G_1'(w_m) + \frac{\lambda\beta\mu(\overline{e})}{w_m}G_1'(w_m) \geq \lambda\mu(\overline{e})\beta\frac{G_1'(\hat{w}_m)}{w_m} + \rho l + \rho\frac{C}{w_m} + \frac{-\delta + \lambda\mu(\overline{e}) + h(\overline{e})}{w_m} + \gamma - \rho.$$

Taking the limit for $m \to \infty$, it follows that

$$(\gamma - \rho)(1 - l) \geq \beta\mu(\overline{e})\lambda \limsup_{m\to\infty} \frac{G_1'(w_m)}{w_m}.$$

Since $l < 1$ would imply that $\limsup_{m\to\infty} \frac{G_1'(w_m)}{w_m} = -\infty$, which contradicts the result of the first step, it must hold that $l \geq 1$, hence the final result. □

I denote by $\overline{w} = \inf\left\{w \geq \beta\overline{z} : w \in \arg\max -\frac{G_1'+(w)}{G_2'+(w)}\right\}$, i.e. the smallest point at which the maximum is attained. Then, choosing $\kappa = -\frac{G_{1+}'(\overline{w})}{G_{2+}'(\overline{w})}$, I obtain that for all $w \geq 0$:

$$G(w) = G_1(w) - \frac{G_{1+}'(\overline{w})}{G_{2+}'(\overline{w})} G_2(w), \tag{VIII.11}$$

implying that $G_+'(\overline{w}) = 0$. Therefore G is the maximal function whose right derivative has a zero in $[\beta\overline{z}, \infty)$. Furthermore, $\frac{G(\beta\overline{z})}{\beta\overline{z}} = -\frac{G_{1+}'(\overline{w})}{G_{2+}'(\overline{w})}$ and G satisfies (VIII.5). Setting $\psi_{\eta^*}(w) = G(w) - w$, with $\eta^* = \frac{G(\beta\overline{z})}{\beta\overline{z}} - 1$, yields $\psi_{\eta^*}'(\overline{w}) = -1$. In order to prove statement (ii) of proposition (VIII.1) I first need to check whether $\psi_{\eta^*}(w)$ is differentiable in $\overline{w}$. This conjecture is ensured by the following Lemma.

Lemma VIII.4. *$\overline{w} > \beta\overline{z}$.*

Proof. It is sufficient to show that the right derivative of $-G_{1+}'/G_{2+}'$ in $\beta\overline{z}$ is strictly positive, i.e.

$$-G_{1+}''(\beta\overline{z})G_{2+}'(\beta\overline{z}) + G_{1+}'(\beta\overline{z})G_{2+}''(\beta\overline{z}) > 0. \tag{VIII.12}$$

Differentiating (VIII.7) and (VIII.8) to the right of $\beta\overline{z}$ I obtain

$$G_{1+}''(\beta\overline{z})(\gamma\beta\overline{z} + h(\overline{e}) + \lambda\beta\mu(\overline{e})) = (\rho - \gamma + \lambda)G_{1+}'(\beta\overline{z}) + \gamma - \rho$$

and

$$G_{2+}''(\beta\overline{z})(\gamma\beta\overline{z} + h(\overline{e}) + \lambda\beta\mu(\overline{e})) = (\rho - \gamma + \lambda)G_{2+}'(\beta\overline{z}) - \lambda.$$

Substituting these values in (VIII.12) I have

$$\frac{1}{(\gamma\beta\overline{z} + h(\overline{e}) + \lambda\beta\mu(\overline{e}))}\left[(\rho - \gamma)G_{2+}'(\beta\overline{z}) - \lambda G_{1+}'(\beta\overline{z})\right] > 0.$$

Using the expressions (VIII.9) and (VIII.10), for $G_{1+}'(\beta\overline{z})$ and $G_{2+}'(\beta\overline{z})$ respectively, the

above condition becomes

$$\frac{(\rho-\gamma)(\rho\beta\overline{z}+\lambda\beta\mu(\overline{e}))-\lambda(h(\overline{e})-\delta+\lambda\mu(\overline{e})+\beta\overline{z}(\gamma-\rho))}{(\gamma\beta\overline{z}+h(\overline{e})+\lambda\beta\mu(\overline{e}))^2}>0,$$

which is guaranteed by assumption (III.6).

□

Lastly I want to verify that the function $G(w)$ is concave in $[0,\overline{w}]$, which yields that $\psi_{\eta^*}(w)$ is concave in $[0,\overline{w}]$ as stated in (iv). In turn, concavity of $\psi_{\eta^*}(w)$ implies that statement (iii) holds. I start by proving the following Lemma.

Lemma VIII.5. $G''_+(\beta\overline{z})<0$.

Proof. Differentiating (VIII.5) to the right for $w\geq\beta\overline{z}$, I obtain

$$G''_+(w)(\gamma w+h(\overline{e})+\beta\lambda\mu(\overline{e}))=\gamma-\rho+G'_+(w)(\rho-\gamma+\lambda)-\lambda E^{\overline{e}}[G'_+(w-\beta Z)].$$

Evaluating this expression at $w=\beta\overline{z}$ yields

$$G''_+(\beta\overline{z})(\gamma\beta\overline{z}+h(\overline{e})+\beta\lambda\mu(\overline{e}))=\lambda(G'_+(\beta\overline{z})-\frac{G(\beta\overline{z})}{\beta\overline{z}})+(\rho-\gamma)(G'_+(\beta\overline{z})-1). \quad \text{(VIII.13)}$$

Now the decomposition of G given by (VIII.11) leads to

$$\begin{aligned}G'_+(\beta\overline{z})&=G'_{1+}(\beta\overline{z})+\frac{G(\beta\overline{z})}{\beta\overline{z}}G'_{2+}(\beta\overline{z})\\&=\frac{1}{\gamma\beta\overline{z}+h(\overline{e})+\beta\lambda\mu(\overline{e})}\left(\beta\overline{z}(\gamma-\rho)-\delta+\lambda\mu(\overline{e})+h(\overline{e})+\frac{G(\beta\overline{z})}{\beta\overline{z}}(\rho\beta\overline{z}+\lambda\beta\mu(\overline{e}))\right)>0,\end{aligned} \quad \text{(VIII.14)}$$

where I have used (VIII.9) and (VIII.10). Then the following inequality holds

$$\frac{G(\beta\overline{z})}{\beta\overline{z}}>\frac{\delta-\lambda\mu(\overline{e})-h(\overline{e})-\beta\overline{z}(\gamma-\rho)}{\rho\beta\overline{z}+\lambda\beta\mu(\overline{e})}. \quad \text{(VIII.15)}$$

Substituting (VIII.14) in (VIII.13) yields

$$\begin{aligned}G''_+(\beta\overline{z})(\gamma\beta\overline{z}+h(\overline{e})+\beta\lambda\mu(\overline{e}))^2&=(\lambda+\rho-\gamma)(\beta\overline{z}(\gamma-\rho)-\delta+\lambda\mu(\overline{e})+h(\overline{e}))\\&\quad+\frac{G(\beta\overline{z})}{\beta\overline{z}}((\rho-\gamma)(\rho\beta\overline{z}+\lambda\beta\overline{z}+\lambda\beta\mu(\overline{z}))-\lambda h(\overline{e}))+\gamma-\rho.\end{aligned} \quad \text{(VIII.16)}$$

Exploiting the inequality (VIII.15) I obtain that

$$G''_+(\beta\overline{z})(\gamma\beta\overline{z}+h(\overline{e})+\beta\lambda\mu(\overline{e}))^2 \leq \lambda(-\delta+\lambda\mu(\overline{e})+h(\overline{e})-\beta\overline{z}(\gamma-\rho))(h(\overline{e})+\beta\overline{z}\gamma+\lambda\beta\mu(\overline{z}))+\gamma-\rho$$

which is negative by assumption (III.6). The result follows. □

The function G is twice continuously differentiable in $(0,\infty)\setminus\{\beta(\overline{z}+\underline{z}), 2\beta\overline{z}\}$. It is possible to verify that $G''_-(\beta(\overline{z}+\underline{z})) < G''_+(\beta(\overline{z}+\underline{z}))$ and $G''_-(2\beta\overline{z}) < G''_+(2\beta\overline{z})$. Hence G''_+ is upper semicontinuous over $[\beta\overline{z},\infty)$ and the set $\left\{w \geq \beta\overline{z} \mid G''_+(w) \geq 0\right\}$ is closed. Therefore I define $w^c = \inf\{w > \beta\overline{z} : G''(w) \geq 0\}$. It can be observed that, by construction, $\overline{w}$ is the smallest point at which G' vanishes. Since G is non-decreasing and $G'(\overline{w}) = 0$, it must hold that $G''_+(\overline{w}) \geq 0$ and hence $\overline{w} \geq w^c$. By Lemma (VIII.5), $w^c > \beta\overline{z}$ and therefore G is strictly concave on $[\beta\overline{z}, w^c)$. Furthermore G is linear in $[0,\beta\overline{z}]$ and $G'_+(\beta\overline{z}) < G'_-(\beta\overline{z})$. Hence G is concave in $[0,w^c]$. I will now show that the two thresholds coincide, $\overline{w} = w^c$. Consequently, G is concave over $[0,\overline{w}]$ and strictly concave in $[\beta\overline{z},\overline{w}]$. First I need the following result.

Lemma VIII.6. $w^c > \beta(\overline{z}+\underline{z})$.

Proof. Assume by contradiction that $w^c \in (\beta\overline{z}, \beta(\overline{z}+\underline{z})]$. Then consider the following two cases.

i) Suppose $w^c = \beta(\overline{z}+\underline{z})$. Then it follows that $G''(w) < 0$ for every $w \in (\beta\overline{z}, \beta(\overline{z}+\underline{z}))$ and $G''_+(\beta(\overline{z}+\underline{z})) \geq 0 > G''_-(\beta(\overline{z}+\underline{z}))$. Differentiating (VIII.5) to the right of $\beta(\overline{z}+\underline{z})$ I have

$$\begin{aligned}
&G''_+(\beta(\overline{z}+\underline{z}))(\gamma\beta(\overline{z}+\underline{z})+h(\overline{e})+\lambda\beta\mu(\overline{e}))\\
&= \gamma-\rho+G'(\beta(\overline{z}+\underline{z}))(\rho-\gamma+\lambda)-\lambda E^{\overline{e}}[G'_+(\beta(\overline{z}+\underline{z})-\beta z)]\\
&= \gamma-\rho+G'(\beta(\overline{z}+\underline{z}))(\rho-\gamma+\lambda)-\lambda q\frac{G(\beta\overline{z})}{\beta\overline{z}}-\lambda(1-q)G'_+(\beta\overline{z})\\
&= (\rho-\gamma)(G'(\beta(\overline{z}+\underline{z}))-1)+\lambda q(G'_+(\beta\overline{z})-\frac{G(\beta\overline{z})}{\beta\overline{z}})+\lambda(G'(\beta(\overline{z}+\underline{z}))-G'_+(\beta\overline{z}))
\end{aligned}$$

where the second equality follows from $G'(\beta\underline{z}) = \frac{G(\beta\overline{z})}{\beta\overline{z}}$. Since G is concave for $w \leq \beta(\overline{z}+\underline{z})$, I find that the last two terms are negative. I will now show that $G'(\beta(\overline{z}+\underline{z})) > 1$, implying that the first term is also negative and therefore $G''_+(\beta(\overline{z}+\underline{z})) < 0$, which is a contradiction. Evaluating (VIII.5) in $\beta(\overline{z}+\underline{z})$ I have

$$\begin{aligned}
\rho G(\beta(\overline{z}+\underline{z})) = {}& \delta-\lambda\mu(\overline{e})+(\rho-\gamma)\beta(\overline{z}+\underline{z})-h(\overline{e})+G'(\beta(\overline{z}+\underline{z}))(\gamma(\beta(\overline{z}+\underline{z}))+h(\overline{e})+\beta\lambda\mu(\overline{e}))\\
&+\lambda qG(\beta\underline{z})+\lambda(1-q)G(\beta\overline{z})-\lambda G(\beta(\overline{z}+\underline{z})).
\end{aligned}$$

Using $G(\beta\underline{z}) = G(\beta\overline{z})\frac{\beta\underline{z}}{\beta\overline{z}}$ and rearranging the terms I obtain

$$G'(\beta(\overline{z}+\underline{z})) = \frac{\gamma(\beta(\overline{z}+\underline{z})) + h(\overline{e}) + \lambda\mu(\overline{e}) - \delta}{\gamma(\beta(\overline{z}+\underline{z})) + h(\overline{e}) + \beta\lambda\mu(\overline{e})} + \frac{\rho(G(\beta(\overline{z}+\underline{z})) - \beta(\overline{z}+\underline{z}))}{\gamma(\beta(\overline{z}+\underline{z})) + h(\overline{e}) + \beta\lambda\mu(\overline{e})}$$
$$+ \frac{\lambda}{\gamma(\beta(\overline{z}+\underline{z})) + h(\overline{e}) + \beta\lambda\mu(\overline{e})}\left(G(\beta(\overline{z}+\underline{z})) - qG(\beta\overline{z})\frac{\beta\underline{z}}{\beta\overline{z}} - (1-q)G(\beta\overline{z})\right).$$

It should be noted that the second term is positive since $G(\beta(\overline{z}+\underline{z})) - \beta(\overline{z}+\underline{z}) = u(\beta(\overline{z}+\underline{z})) \geq 0$. Moreover, because G is increasing I have $G(\beta(\overline{z}+\underline{z})) - G(\beta\overline{z}) \geq 0$. Therefore it follows that

$$G'(\beta(\overline{z}+\underline{z})) \geq \frac{\gamma(\beta(\overline{z}+\underline{z})) + h(\overline{e}) + \lambda\mu(\overline{e}) - \delta + \lambda q\frac{G(\beta\overline{z})}{\beta\overline{z}}\beta(\overline{z}-\underline{z})}{\gamma(\beta(\overline{z}+\underline{z})) + h(\overline{e}) + \beta\lambda\mu(\overline{e})} \geq 1$$

where the second inequality is due to (VIII.15) and assumption (III.6). Hence the result follows.

ii) Suppose instead that $w^c < \beta(\overline{z}+\underline{z})$. Then G is twice continuously differentiable in w^c and it follows that $G''(w^c) = 0$ and $G''(w) < 0$ in $(\beta\overline{z}, w^c)$. Now I will consider three cases.

a) Assume that $\gamma - \rho > \lambda$. Differentiating (VIII.5) in w^c, I have

$$(\rho - \gamma + \lambda)G'(w^c) = \rho - \gamma + \lambda E^{\overline{e}}[G'(w^c - \beta z)].$$

This result implies that

$$0 \geq (\rho - \gamma + \lambda)G'(w^c) = \rho - \gamma + \lambda\frac{G(\beta\overline{z})}{\beta\overline{z}}.$$

By (VIII.15) I obtain

$$\lambda\frac{\delta - \lambda\mu(\overline{e}) - h(\overline{e}) - \beta\overline{z}(\gamma - \rho)}{\rho\beta\overline{z} + \lambda\beta\mu(\overline{e})} \leq \gamma - \rho,$$

contradicting assumption (III.6).

b) Assume that $\lambda \geq 2\gamma - \rho$. Differentiating (VIII.5) twice for $w \in (\beta\overline{z}, \beta(\overline{z}+\underline{z}))$, I have

$$G'''(w)(\gamma w + h(\overline{e}) + \lambda\beta\mu(\overline{e})) = G''(w)(\rho - 2\gamma + \lambda) - \lambda E^{\overline{e}}[G''(w - \beta z)] = G''(w)(\rho - 2\gamma + \lambda)$$

where the second equality is due to the linearity of G over $(0, \beta\overline{z})$. Now using $\lambda \geq 2\gamma - \rho$ and the concavity of G over the interval $(\beta\overline{z}, w^c)$, I find that $G''' \leq 0$ over this interval. Therefore $G''(w^c) \leq G''_+(\beta\overline{z})$. Since $G''(w^c) = 0$ and $G''_+(\beta\overline{z}) < 0$ by Lemma (VIII.5), this leads to a contradiction.

c) Assume that $\gamma - \rho < \lambda < 2\gamma - \rho$. Differentiating (VIII.5) twice for $w \in (\beta\overline{z}, \beta(\overline{z} + \underline{z}))$, I obtain that $G'''(w) \cdot G''(w) < 0$ over this interval. Since $G''(w) < 0$, it follows that $G'''(w) > 0$, implying that $G''(w) > G''(\beta\overline{z})$ for $w \in (\beta\overline{z}, w^c)$. Using $\gamma - \rho < \lambda < 2\gamma - \rho$, I obtain

$$G'''(w) = \frac{(\lambda - 2\gamma + \rho)G''(w)}{\gamma w + h(\overline{e}) + \lambda\beta\mu(\overline{e})} < \frac{(\lambda - 2\gamma + \rho)G''_+(\beta\overline{z})}{\gamma w + h(\overline{e}) + \lambda\beta\mu(\overline{e})},$$

for every $w \in (\beta\overline{z}, w^c)$. Note that, using the concavity of G for $w \leq w^c$, I have

$$\begin{aligned} G''(w^c) &= G''_+(\beta\overline{z}) + \int_{\beta\overline{z}}^{w^c} \frac{(\lambda - 2\gamma + \rho)G''(w)}{\gamma w + h(\overline{e}) + \lambda\beta\mu(\overline{e})}\mathrm{d}w \\ &< \left(1 + \int_{\beta\overline{z}}^{w^c} \frac{(\lambda - 2\gamma + \rho)}{\gamma w + h(\overline{e}) + \lambda\beta\mu(\overline{e})}\mathrm{d}w\right) G''_+(\beta\overline{z}). \end{aligned}$$

Define

$$A \equiv 1 + \int_{\beta\overline{z}}^{w^c} \frac{(\lambda - 2\gamma + \rho)}{\gamma w + h(\overline{e}) + \lambda\beta\mu(\overline{e})}\mathrm{d}w > 0.$$

Observe that, since $w^c < \beta(\overline{z} + \underline{z})$ by assumption, it follows that

$$\int_{\beta\overline{z}}^{w^c} \frac{1}{\gamma w + h(\overline{e}) + \lambda\beta\mu(\overline{e})}\mathrm{d}w < \int_{\beta\overline{z}}^{\beta(\overline{z}+\underline{z})} \frac{1}{\gamma\beta\overline{z} + h(\overline{e}) + \lambda\beta\mu(\overline{e})}\mathrm{d}w = \frac{\beta\underline{z}}{\gamma\beta\overline{z} + h(\overline{e}) + \lambda\beta\mu(\overline{e})}.$$

Then, using $\gamma - \rho < \lambda < 2\gamma - \rho$, I obtain

$$A > 1 + \frac{\beta\underline{z}(\lambda - 2\gamma + \rho)}{\gamma\beta\overline{z} + h(\overline{e}) + \lambda\beta\mu(\overline{e})} = \frac{h(\overline{e}) + \gamma\beta(\overline{z} - \underline{z}) + \lambda\beta\mu(\overline{e}) + \beta\underline{z}(\lambda - \gamma + \rho)}{\gamma\beta\overline{z} + h(\overline{e}) + \lambda\beta\mu(\overline{e})} > 0.$$

By definition of w^c, $G''(w^c) = 0$ and $G''_+(\beta\overline{z}) < 0$. Then a contradiction follows.

□

Finally, I will demonstrate the following Lemma, concluding the proof of proposition (VIII.1).

Lemma VIII.7. $w^c = \overline{w}$.

Proof. First I will show that $G''(w) > 0$ for $w \in (w^c, w^c + \epsilon)$ for some positive ϵ. If $w^c = 2\beta\overline{z}$ and $G''_+(2\beta\overline{z}) > 0$, this is obvious. If instead $G''_+(w^c) = 0$, differentiating twice (VIII.5) to the right of w^c, I have

$$G'''_+(w^c)(\gamma w^c + h(\overline{e}) + \lambda\beta\mu(\overline{e})) = -\lambda E^{\overline{e}}[G''_+(w^c - \beta z)] > 0$$

where the inequality is implied by Lemma (VIII.6), which guarantees that $w^c - \beta\underline{z} > \beta\overline{z}$. Together $G''_+(w^c) = 0$ and $G'''_+(w^c) > 0$, imply that $G''(w) > 0$ in $(w^c, w^c + \epsilon)$.

Now suppose that $w^c \neq \overline{w}$, implying that $\overline{w} > w^c$. Because $G'(w^c) > 0$ and $G'(\overline{w}) = 0$, there must be a point $\tilde{w} = \inf\left\{w^c < w < \overline{w} \mid G''_+(w) < 0\right\}$. If $\tilde{w} = 2\beta\overline{z}$, it follows that $G''_-(2\beta\overline{z}) > 0 \geq G''_+(2\beta\overline{z})$. Then I have

$$\begin{aligned} G''_-(2\beta\overline{z}) = \gamma - \rho + G'(2\beta\overline{z})(\rho - \gamma + \lambda) - \lambda q\tfrac{G(\beta\overline{z})}{\beta\overline{z}} - \lambda(1-q)G'(2\beta\overline{z} - \beta\underline{z}) > \\ > G''_+(2\beta\overline{z}) = \gamma - \rho + G'(2\beta\overline{z})(\rho - \gamma + \lambda) - \lambda q G'_+(\beta\overline{z}) - \lambda(1-q)G'(2\beta\overline{z} - \beta\underline{z}). \end{aligned}$$

It follows that $\frac{G(\beta\overline{z})}{\beta\overline{z}} < G'_+(\beta\overline{z})$, which is a contradiction of Lemma (VIII.5). If instead $\tilde{w} \neq 2\beta\overline{z}$, then because $\tilde{w} > w^c > \beta(\overline{z} + \underline{z})$, G is twice continuously differentiable in $\tilde{w}$ and it follows that $G''(\tilde{w}) = 0$. Differentiating (VIII.5) twice to the right of $\tilde{w}$ and using $G'''_+(\tilde{w}) \leq 0$, I obtain

$$0 \geq G'''_+(\tilde{w})(\gamma\tilde{w} + h(\overline{e}) + \lambda\beta\mu(\overline{e})) = -\lambda E^{\overline{e}}[G''_+(\tilde{w} - \beta z)].$$

This implies that it is not possible to have $\tilde{w} < w^c + \beta\underline{z}$, since otherwise it would hold that $G''_+(\tilde{w} - \beta\overline{z}) < 0$ and $G''_+(w^c - \beta\underline{z}) < 0$, contradicting the above inequality. Therefore there are two possible cases:

i) $\tilde{w} > w^c + \beta\overline{z} > w^c + \beta\underline{z}$, implying that $G''_+(\tilde{w} - \beta\overline{z}) > 0$ and $G''_+(\tilde{w} - \beta\underline{z}) > 0$;

ii) $w^c + \beta\underline{z} < \tilde{w} < w^c + \beta\overline{z}$ implying that $G''_+(\tilde{w} - \beta\overline{z}) < 0 < G''_+(\tilde{w} - \beta\underline{z})$.

In the first case, $G'' > 0$ in $[\tilde{w} - \beta\overline{z}, \tilde{w})$. Then, differentiating (VIII.5) in $\tilde{w}$, I obtain

$$0 = G''_+(\tilde{w})(\gamma\tilde{w} + h(\overline{e}) + \lambda\beta\mu(\overline{e})) = \lambda(G'(\tilde{w}) - E^{\overline{e}}[G'_+(\tilde{w} - \beta z)]) + (\gamma - \rho)(1 - G'(\tilde{w})) \quad \text{(VIII.17)}$$

which implies that $G'(\tilde{w}) \geq 1$. Evaluating (VIII.5) in $\tilde{w}$, I have

$$\begin{aligned}\gamma\tilde{w} + h(\overline{e}) + \lambda\beta\mu(\overline{e})G'(\tilde{w}) &\leq G'(\tilde{w})(\gamma\tilde{w} + h(\overline{e}) + \lambda\beta\mu(\overline{e})) \\ &= \rho G(\tilde{w}) - \delta + \lambda\mu(\overline{e}) - \tilde{w}(\rho-\gamma) + h(\overline{e}) + \lambda(G(\tilde{w}) - E^{\overline{e}}[G(\tilde{w}-\beta z)]) \\ &\leq \rho G(\tilde{w}) - \delta + \lambda\mu(\overline{e}) - \tilde{w}(\rho-\gamma) + h(\overline{e}) + \lambda(G(\tilde{w}) - G(\tilde{w}-\beta\mu(\overline{e}))) \\ &\leq \rho G(\tilde{w}) - \delta + \lambda\mu(\overline{e}) - \tilde{w}(\rho-\gamma) + h(\overline{e}) + \lambda\beta\mu(\overline{e})G'(\tilde{w}),\end{aligned} \tag{VIII.18}$$

where the last two inequalities follow from the convexity of G over $[\tilde{w}-\beta\overline{z}, \tilde{w})$. Therefore I obtain

$$G(\tilde{w}) \geq \frac{\delta - \lambda\mu(\overline{e})}{\rho} + \tilde{w} \geq \frac{\delta - \lambda\mu(\overline{e})}{\rho}.$$

Then, since $\overline{w} > \tilde{w}$ and G is non-decreasing, it must hold that $G(\overline{w}) > \frac{\delta-\lambda\mu(\overline{e})}{\rho}$. It should be noticed that, however, that evaluating (VIII.5) in $\overline{w}$, I obtain

$$0 = G'(\overline{w})(\gamma\overline{w}+h(\overline{e})+\lambda\beta\mu(\overline{e})) = \rho G(\overline{w})+\lambda\mu(\overline{e})-\delta+h(\overline{e})-\overline{w}(\rho-\gamma)+\lambda(G(\overline{w})-E^{\overline{e}}[G(\overline{w}-\beta z)]).$$

Since G is non-decreasing and $\rho < \gamma$, this implies

$$G(\overline{w}) < \frac{\delta - \lambda\mu(\overline{e})}{\rho} \tag{VIII.19}$$

and therefore a contradiction. I will now analyse the second case ii), when $w^c + \beta\underline{z} < \tilde{w} < w^c + \beta\overline{z}$. From (VIII.17) it follows that

$$\lambda(q(G'(\tilde{w}) - G'_+(\tilde{w}-\beta\overline{z})) + (1-q)(G'(\tilde{w}) - G'_+(\tilde{w}-\beta\underline{z}))) = (\rho-\gamma)(1-G'(\tilde{w})).$$

Since $G'(\tilde{w}) \geq G'_+(\tilde{w}-\beta\overline{z})$, the first term on the left-hand side is non-negative. By convexity of G over the interval $(\tilde{w}-\beta\underline{z}, \tilde{w})$, the second term is non-negative as well. It follows that $G'(\tilde{w}) \geq 1$. Analogously to the case i, evaluating (VIII.5) in $\tilde{w}$ yields

$$\begin{aligned}&\gamma\tilde{w} + h(\overline{e}) + \lambda\beta\mu(\overline{e})G'(\tilde{w}) \leq G'(\tilde{w})(\gamma\tilde{w} + h(\overline{e}) + \lambda\beta\mu(\overline{e})) \\ &= \rho G(\tilde{w}) - \delta + \lambda\mu(\overline{e}) - \tilde{w}(\rho-\gamma) + h(\overline{e}) + \lambda q(G(\tilde{w}) - G(\tilde{w}-\beta\overline{z})) + \lambda(1-q)(G(\tilde{w}) - G(\tilde{w}-\beta\underline{z})) \\ &\leq \rho G(\tilde{w}) - \delta + \lambda\mu(\overline{e}) - \tilde{w}(\rho-\gamma) + h(\overline{e}) + \lambda q\beta\overline{z}G'(\tilde{w}) + \lambda(1-q)\beta\underline{z}G'(\tilde{w}) \\ &= \rho G(\tilde{w}) - \delta + \lambda\mu(\overline{e}) - \tilde{w}(\rho-\gamma) + h(\overline{e}) + \lambda\beta\mu(\overline{e})G'(\tilde{w});\end{aligned}$$

where the second inequality follows from the convexity of G over $(\tilde{w} - \beta\underline{z}, \tilde{w})$, and using

$$G(\tilde{w}) - G(\tilde{w} - \beta\overline{z}) \leq G(w^c) - G'(\tilde{w})(w^c - \tilde{w}) - G(w^c) + G'(\tilde{w} - \beta\overline{z})(w^c - \tilde{w} + \beta\overline{z}) \leq G'(\tilde{w})\beta\overline{z}.$$

Then $G(\tilde{w}) \geq \frac{\delta - \lambda\mu(\overline{e})}{\rho}$, contradicting (VIII.19). Hence the result follows. □

Next I prove a result that will be useful in the following.

Lemma VIII.8.

$$\mathcal{L}g(w) - \rho g(w) \leq (\gamma - \rho)w + h(\overline{e}) - \delta + \lambda\mu(\overline{e})$$

for all $w \in (\beta\overline{z}, \infty)$.

Proof. For every $w \in [\beta\overline{z}, \overline{w}]$ the result follows from (VIII.5) and (VIII.6). For $w > \overline{w}$ I have

$$\begin{aligned}
&\mathcal{L}g(w) - \rho g(w) - (\gamma - \rho)w - h(\overline{e}) + \delta - \lambda\mu(\overline{e}) \\
&= -\lambda g(\overline{w}) + \lambda E^{\overline{e}}[g(w - \beta z)] - (\gamma - \rho)w - h(\overline{e}) + \delta - \lambda\mu(\overline{e}) - \rho g(\overline{w}) \\
&= \lambda(E^{\overline{e}}[g(w - \beta z)] - E^{\overline{e}}[g(\overline{w} - \beta z)]) - (\gamma - \rho)(w - \overline{w}) \\
&\leq (\lambda E^{\overline{e}}[g'_+(w - \beta z)] - \gamma + \rho)(w - \overline{w}) \\
&= (\lambda E^{\overline{e}}[G'_+(w - \beta z)] - \gamma + \rho)(w - \overline{w})
\end{aligned}$$

where the first equality has been obtained using that $g(w) = g(\overline{w})$ for $w > \overline{w}$, the second noting that $-\lambda g(\overline{w}) - \rho g(\overline{w}) + \delta - \lambda\mu(\overline{e}) - h(\overline{e}) = (\gamma - \rho)\overline{w} - \lambda E^{\overline{e}}[g(\overline{w} - \beta z)]$. The inequality comes by concavity of g and the last equality by (VIII.6). Differentiating (VIII.5) to the right of $\overline{w}$ and using that $G'(\overline{w}) = 0$ I obtain that the last term is negative:

$$\lambda E^{\overline{e}}[G'_+(w - \beta z)] - \gamma + \rho = -G''_+(\overline{w})(\gamma\overline{w} + h(\overline{e}) + \lambda\beta\mu(\overline{e})) \leq 0$$

since $G''_+(\overline{w}) \geq 0$, implying the result.

□

As a consequence of Lemma (VIII.8), using that $u(w) = g(w) - w$, I obtain for all $w \in [\beta\overline{z}, \infty)$

$$(\gamma w + h(\overline{e}) + \lambda\beta\mu(\overline{e}))u'_+(w) + \lambda E^{\overline{e}}[u(w - \beta z)] - (\lambda + \rho)u(w) \leq -\delta + \lambda\mu(\overline{e}). \quad \text{(VIII.20)}$$

In particular, for $w \in [\beta\overline{z}, \overline{w}]$

$$(\gamma w + h(\overline{e}) + \lambda\beta\mu(\overline{e}))u'_+(w) + \lambda E^{\overline{e}}[u(w-\beta z)] - (\lambda+\rho)u(w) = -\delta + \lambda\mu(\overline{e}). \quad \text{(VIII.21)}$$

Now I proceed to show that, for any incentive compatible contract inducing maximal risk mitigation effort, the principal's value function $V(w)$ is bounded from above by $u(w)$.

Proposition VIII.9. *For any contract $\pi = (T, \phi, c)$ inducing $e^* = \overline{e}$ at all times, and delivering the agent an initial payoff of $W_0^{\overline{e}}$, the following holds*

$$u(W_0^{\overline{e}}) \geq V_0(W_0^{\overline{e}}) = E^{\overline{e}}\left[\int_0^{\tau} \exp(-\rho s)\left(dY_s - dT_s\right)\right].$$

Proof. The agent's continuation utility $W_t^{\overline{e}}$ follows the dynamic in (II.5). For convenience, in the rest of the proof, I will drop the superscript $\overline{e}$. Applying the change of variable formula for bounded variation processes (see Dellacherie and Meyer (1982), Chapter VI, Sect.92) to u gives

$$\begin{aligned} e^{-\rho(t\wedge\tau)}u(W_{t\wedge\tau}) = u(W_{0-}) &- \int_0^{t\wedge\tau} e^{-\rho s}u'(W_{s-})\mathrm{d}T_s^c \\ &+ \int_0^{t\wedge\tau} e^{-\rho s}\Big(u'(W_{s-})(\gamma W_{s-} + h(\overline{e}) + \lambda\phi_s\mu(\overline{e}) + \lambda E^{\overline{e}}[R_s]) - \rho u(W_{s-})\Big)\mathrm{d}s \\ &+ \sum_{0<s\leq t\wedge\tau, W_{s-}\neq W_s} e^{-\rho s}[u(W_s) - u(W_{s-})] + \sum_{0<s<t\wedge\tau, W_s\neq W_{s+}} e^{-\rho s}[u(W_{s+}) - u(W_s)] \end{aligned} \quad \text{(VIII.22)}$$

for all t, where T_s^c refers to the continuous part of T. Since $W_{s-} \neq W_s$ only at the arrival of a jump, I find that

$$\sum_{0<s\leq t\wedge\tau, W_{s-}\neq W_s} e^{-\rho s}[u(W_s) - u(W_{s-})] = \int_0^{t\wedge\tau} e^{-\rho s}[u(W_{s-} - \phi_s Z_s - R_s) - u(W_{s-})]\mathrm{d}N_s.$$

Define, for $t \in [0, \tau)$,

$$A = \int_0^t e^{-\rho s}[u(W_{s-}-\phi_s Z_s - R_s) - u(W_{s-})]\mathrm{d}N_s - \lambda\int_0^t e^{-\rho s}[E^{\overline{e}}[u(W_{s-}-\phi_s Z_s - R_s)] - u(W_{s-})]\mathrm{d}s \quad \text{(VIII.23)}$$

which is a $\mathbb{G}$-martingale. Then expression (VIII.22) can be rewritten as

$$e^{-\rho(t\wedge\tau)}u(W_{t\wedge\tau}) = u(W_{0-}) + A + B + C \quad \text{(VIII.24)}$$

where B is defined as

$$\begin{aligned} B = &\int_0^{t\wedge\tau} \Big(u'(W_{s-})(\gamma W_{s-} + h(\overline{e}) + \lambda\phi_s\mu(\overline{e}) + \lambda E^{\overline{e}}[R_s]) - \rho u(W_{s-}) \Big) \mathrm{d}s \\ &+ \lambda \int_0^t e^{-\rho s}[E^{\overline{e}}[u(W_{s-} - \phi_s Z_s - R_s)] - u(W_{s-})]\mathrm{d}s \end{aligned} \tag{VIII.25}$$

and C accounts for the changes in cumulative transfers,

$$C = -\int_0^{t\wedge\tau} e^{-\rho s} u'(W_{s-})\mathrm{d}T_s^c + \sum_{0<s<t\wedge\tau, W_s\neq W_{s+}} e^{-\rho s}[u(W_{s+}) - u(W_s)]. \tag{VIII.26}$$

Observe that

$$\begin{aligned} B &\leq \int_0^{t\wedge\tau} \Big(u'(W_{s-})(\gamma W_{s-} + h(\overline{e}) + \lambda\beta\mu(\overline{e}) + \lambda E^{\overline{e}}[R_s]) - \rho u(W_{s-}) \Big) \mathrm{d}s \\ &+ \lambda \int_0^t e^{-\rho s}[E^{\overline{e}}[u(W_{s-} - \beta Z_s)] - u(W_{s-})]\mathrm{d}s \\ &\leq \int_0^{t\wedge\tau} e^{-\rho s}(-\delta + \lambda\mu(\overline{e}))\mathrm{d}s \end{aligned} \tag{VIII.27}$$

where the second inequality follows from the promise-keeping constraint, which implies that $E^{\overline{e}}[R_s] = 0$ and (VIII.20). The first inequality was obtained using the concavity of u and the incentive compatibility constraint $\phi_t \geq \beta$. From (III.2) follows

$$\begin{aligned} &E^{\overline{e}}[u(W_{s-} - \phi_s Z_s - R_s) + \phi z u'(w)] \\ &= E^{\overline{e}}[1_{\{w-\phi\overline{z}>\beta\overline{z}+c\}} u(W_{s-} - \phi_s Z_s)] + E^{\overline{e}}[1_{\{w-\phi\overline{z}\leq\beta\overline{z}+c\}} \frac{w-\phi z}{\beta\overline{z}+c} V(\beta z + c)] + E^{\overline{e}}[\phi z u'(w)] \\ &\leq E^{\overline{e}}[u(W_{s-} - \phi_s Z_s) + \phi z u'(w)] \leq E^{\overline{e}}[u(w-\beta z) + u'(w-\beta z)z(\beta-\phi) + \phi z u'(w)] \\ &\leq E^{\overline{e}}[u(w-\beta z) + u'(w)z(\beta-\phi) + \phi z u'(w)] = E^{\overline{e}}[u(w-\beta z) + u'(w)z\beta] \end{aligned}$$

Now consider the last term C. Since $W_s \neq W_{s+}$ only at the jumps of T_s, I have

$$\begin{aligned} \sum_{0<s<t\wedge\tau, W_s\neq W_{s+}} e^{-\rho s}[u(W_{s+}) - u(W_s)] &= -\sum_{0<s<t\wedge\tau, W_s\neq W_{s+}} e^{-\rho s} \int_0^{T_{s+}-T_s} u'_+(W_s - p)\mathrm{d}p \\ &\leq \sum_{0<s<t\wedge\tau, W_s\neq W_{s+}} e^{-\rho s} \int_0^{T_{s+}-T_s} \mathrm{d}p \end{aligned}$$

using $u'_+ \geq -1$. Therefore I obtain

$$C \leq \int_0^{t\wedge\tau} e^{-\rho s} \mathrm{d}T_s^c + \sum_{0<s<t\wedge\tau, W_s \neq W_{s+}} e^{-\rho s} \int_0^{T_{s+}-T_s} \mathrm{d}p = \int_0^{t\wedge\tau} e^{-\rho s} \mathrm{d}T_s. \tag{VIII.28}$$

Finally, using (VIII.25) and (VIII.28), and taking expectations in (VIII.22) yields

$$\begin{aligned} u(W_{0-}) &\geq E\left[e^{-\rho(t\wedge\tau)} u(W_{t\wedge\tau}) + \int_0^{t\wedge\tau} e^{-\rho s}\Big((\delta - \lambda\mu(\overline{e}))\mathrm{d}s - \mathrm{d}T_s\Big)\right] \\ &= E\left[\int_0^{\tau} e^{-\rho s}\Big((\delta - \lambda\mu(\overline{e}))\mathrm{d}s - \mathrm{d}T_s\Big)\right] + E\left[1_{\{t<\tau\}}\left(e^{-\rho t}u(W_t) - \int_t^{\tau} e^{-\rho s}\Big((\delta - \lambda\mu(\overline{e}))\mathrm{d}s - \mathrm{d}T_s\Big)\right)\right] \\ &\geq E\left[\int_0^{\tau} e^{-\rho s}\Big((\delta - \lambda\mu(\overline{e}))\mathrm{d}s - \mathrm{d}T_s\Big)\right] - e^{-\rho t}E\left[1_{\{t<\tau\}}\left(\frac{\delta - \lambda\mu(\overline{e})}{\rho} - W_t - u(W_t)\right)\right]. \end{aligned} \tag{VIII.29}$$

where the inequality comes from

$$E_t\left(\int_t^{\tau} e^{-\rho s}\Big((\delta - \lambda\mu(\overline{e}))\mathrm{d}s - \mathrm{d}T_s\Big)\right) \leq \frac{\delta - \lambda\mu(\overline{e})}{\rho} - W_t = V^{FB}(W_t).$$

Since $V^{FB}(W_t) \geq u(W_t)$ the second term on the right-hand side of (VIII.29) is non-negative and is bounded by $\frac{\delta-\lambda\mu(\overline{e})}{\rho}$. Taking the limit for $t \to \infty$ I obtain the final result

$$u(W_0) \geq E\left[\int_0^{\tau} e^{-\rho s}\Big((\delta - \lambda\mu(\overline{e}))\mathrm{d}s - \mathrm{d}T_s\Big)\right].$$

□

Finally, I will show that the principal's value function $V(w)$ attains the upper bound $u(w)$ under the optimal contract (III.8).

Proposition VIII.10. *For the optimal contract (III.8) inducing $e^* = \overline{e}$ at all times, and delivering the agent an initial payoff of $W_0^{\overline{e}}$, the following holds:*

$$u(W_0^{\overline{e}}) = V_0(W_0^{\overline{e}}) = E^{\overline{e}}\left[\int_0^{\tau} \exp(-\rho s)\left(dY_s - dT_s\right)\right].$$

Proof. Analogously to (VIII.24) it is possible to write for any $t \geq 0$

$$e^{-\rho(t\wedge\tau)} u(W_{t\wedge\tau}) = u(W_{0-}) + A + B + C \tag{VIII.30}$$

where A, B and C are defined as in (VIII.23), (VIII.25) and (VIII.28) respectively, with $\phi_t = \beta$ and R_t taking value R^{C_*} with probability θ^* and R^{T_*} with probability $1-\theta^*$. This in turn implies that $E^{\overline{e}}[R_t] = 0$. I will now analyse B. From (VIII.21) I have

$$\begin{aligned} B &= \int_0^{t\wedge\tau} \Big(u'(W_{s-})(\gamma W_{s-} + h(\overline{e}) + \lambda\beta\mu(\overline{e})) - \rho u(W_{s-}) \Big) \mathrm{d}s \\ &\quad + \lambda \int_0^t e^{-\rho s}[E^{\overline{e}}[u(W_{s-} - \beta Z_s)] - u(W_{s-})]\mathrm{d}s \\ &= \int_0^{t\wedge\tau} e^{-\rho s}(-\delta + \lambda\mu(\overline{e}))\mathrm{d}s. \end{aligned} \tag{VIII.31}$$

Next consider C. Under the optimal contract, the process T can only jump at the initial time 0, hence

$$\begin{aligned} C &= -\int_0^{t\wedge\tau} e^{-\rho s} u'(W_{s-})\mathrm{d}T_s^c + u(W_{0+}) - u(W_0) \\ &= -\int_0^{t\wedge\tau} e^{-\rho s} u'(W_{s-}) 1_{\{w=\overline{w}\}}(\gamma\overline{w} + h(\overline{e}) + \lambda\beta\mu(\overline{e}))\mathrm{d}t + (W_0 - \overline{w})^+ \\ &= \int_0^{t\wedge\tau} e^{-\rho s}\mathrm{d}T_s \end{aligned}$$

where I have used $\mathrm{d}T_t = \mathrm{d}T_t^*$, as defined in (III.8), and $u'(\overline{w}) = -1$. Then taking expectations in (VIII.30) and using these results, and observing that A is a $\mathbb{G}$-martingale yields

$$u(W_{0-}) = E^{\overline{e}}[e^{-\rho(t\wedge\tau)} u(W_{t\wedge\tau}) + \int_0^{t\wedge\tau} e^{-\rho s}[(\delta - \lambda\mu(\overline{e}))\mathrm{d}s - \mathrm{d}T_s].$$

Taking the limit for $t \to \infty$ and using $u(W_\tau) = 0$ leads to the final result

$$u(W_0) = E^{\overline{e}}\left[\int_0^\tau \exp(-\rho s) \left(\mathrm{d}Y_s - \mathrm{d}T_s\right) \right].$$

□

This result concludes the proof of the Verification Theorem (III.3).

C. Proof of Proposition (IV.1)

In order to ensure the optimality of maximal effort $\overline{e}$, the principal's payoff from this strategy should be higher than the one from $\underline{e}$:

$$\rho V(w) \geq V'(w)(\gamma w + h(\underline{e})) + \delta - \lambda\mu(\underline{e}). \tag{VIII.32}$$

It can be shown that

$$\rho V(w) \geq V'(w)(\gamma w + h(\overline{e}) + \lambda\beta\mu(\overline{e})) + \delta - \lambda\mu(\overline{e}) + \lambda E^{\overline{e}}[V(w - \beta Z)] - \lambda V(w) \tag{VIII.33}$$

Therefore a sufficient condition for (VIII.32) to hold is that the right-hand side of (VIII.33) should be larger than the right-hand side of (VIII.32), which is true if

$$V'(w)(h(\overline{e}) - h(\underline{e})) + \lambda(\mu(\underline{e}) - \mu(\overline{e})) \geq \lambda V(w) - V'(w)\lambda\beta\mu(\overline{e}) - \lambda E^{\overline{e}}[V(w - \beta Z)]. \tag{VIII.34}$$

The right hand side of (VIII.34) is bounded and nonnegative by concavity of V. Because $V'(w) \geq -1$ the left hand side is greater than $\lambda\Delta\mu - \Delta h$, which is nonnegative by assumption (II.1) (ii). Therefore, for fixed Δh, (VIII.34) is satisfied for a high enough $\Delta\mu$.

Chapter 2

Risk-taking and dynamic prudential regulation

As shown by the recent financial crisis, banks' excessive risk-taking can impose huge losses on the economy. The roots of the problem are multifold. First, financial innovations led to higher degrees of sophistication and opaqueness of financial products. This, together with investors' limited liability and a misalignment of risk and reward in managerial compensation structures[1], contributed to the generation of excessive risks.

At the same time, regulators failed to prevent risk-taking ex ante, if not exacerbated it through loose monetary policy. Governments granted forbearance to undercapitalised banks and committed to bailout troubled financial institutions ex post. Such regulatory failures provided banks with implicit protections from the consequences of their risky behaviour, which in turn exacerbated systemic risks. The impact on market discipline and moral hazard was significant.

In the crisis aftermath, excessive risk prevention has therefore been recognised as an absolute priority in the regulators' international agenda[2]. New reforms have been developed to foster prudent behaviour and avoid costly government bailouts. In order to produce new effective regulatory policies, a full understanding of the strategic factors that led to excessive risks in the first place is needed.

To this end, I have developed a continuous-time contracting framework to study the optimal regulation of a bank, whose risky assets are exposed to rare events (such as financial crisis) generating unpredictable losses of possibly very large size. The bank's manager, who is protected by limited liability, can engage in two detrimental activities: shirking and risk-taking. In particular, the manager can increase the bank's profitability by either exerting effort or taking large risks, which expose the bank to higher expected losses should a crisis occur. The banker's actions are unobservable, and hence a double moral hazard problem arises. The optimal regulatory contract designs the manager's

[1]Limited liability allows investors and executives to benefit from their risk-taking activities in the upside, while limiting their downside exposure.

[2]Most importantly: the Dodd-Frank Bill in the US, the Liikanen Commission in the EU and the Vickers Commission in the UK.

incentives, striking a delicate balance between inducing effort and preventing risk-taking.

In the baseline model[3] the regulator maximises the bank's private value, accounting for the bank's shareholders and manager payoffs. Under the optimal contract, inducing high effort at all times, the agent is incentivised to refrain from risk-taking by sharing parts of the losses upon the occurrence of a shock. However, when the bank is in financial distress, the manager does not have enough "skin in the game" and could start gambling for resurrection. Even if risk-taking is detrimental, the regulator finds it optimal to let the manager gamble in order to avoid costly liquidation. Furthermore, when inducing prudent risk management is too costly, risky investments are made even when the bank has not yet entered financial distress. Hence, excessive risk-taking arises endogenously in the model along the equilibrium path.

The optimal baseline contract can be implemented through capital regulation and risk-based deposit insurance. In particular, when the bank becomes undercapitalised, government support emerges as a public recapitalisation provision to avoid the bank's closure. Termination is then only used as a measure of last resort. The results are in line with the often-observed practice of regulatory forbearance. In this set up, the absence of prompt corrective actions can be rationalised through the high cost of liquidation and the lack of transparency, which impedes verifiability of the bank's risk-taking strategies[4].

The baseline model highlights that a "micro-prudential" regulation, focused on the soundness of individual financial institutions, is not sufficient to deter excessive risks and makes government support necessary ex post. The model is then extended to analyse "macro-prudential" regulations of systemically important banks. The socially optimal regulator internalises the negative externalities associated with the bank's risk-taking strategies and guarantees its continuation under a safe management.

The socially optimal contract prevents inefficient risky behaviours, forcing shareholders to recapitalise the bank before reaching financial distress. The proposed regulation disciplines the bank, imposing recapitalisation's costs to be shared internally, and in turn avoids the need for government interventions. In particular, the minimum capital requirement serves a double purpose. First, it provides the banker with enough "skin in the game" so as to steer her away from risky strategies. Moreover, it guarantees adequate loss absorbing capacity in the event of a crisis. As a result, the requirement possesses banks' specific

[3]It can be observed that the baseline model is equivalent to the case where the optimal contract maximises the shareholders' payoff and the bank is unregulated.

[4]Forbearance to failing banks has been rationalised from other academic studies, as part of the regulatory architecture, depending on several factors such as reputation concerns (Morrison and White (2013)), collateral value (Kocherlakota and Shim (2007)) or supervisory delegation (Colliard (2015)).

attributes, being stricter for institutions exposed to larger risks and stronger agency problems.

As an alternative discipline device, the regulator may employ a suspension phase during which "strategic shirking" is implemented. During this period, the manager is temporarily requested to stop working, which in turn is sufficient to deter risk-taking. The bank keeps operating, running its core (deposit-oriented) functions, while speculative activities are interrupted. Despite the lower profitability, the bank is able to slowly recover from financial distress. Once the bank has overcome distress, management is reinstated and the bank resumes normal operation. Suspending risky activities after a period of poor performance prevents risk-taking and avoids the bank's liquidation.

I. Related literature

This chapter contributes to two streams of literature: dynamic moral hazard and prudential regulation models. First, the chapter is related to the fast-growing literature on dynamic agency problems in continuous-time settings, adopting the martingale techniques developed by Sannikov (2008). As underlined in the first chapter, most of the studies in this literature focus on frameworks characterized by Brownian uncertainty, such as for instance the study by DeMarzo and Sannikov (2006), or Poisson shocks of constant jump size, such as the works of Biais et al. (2010) and Myerson (2010), among others. This chapter builds on that stream of literature by analysing a double moral hazard problem in a jump-diffusion model. In particular, similarly to Biffis and Lepore (2015), I consider Poisson risk with unpredictable jump size shaped by the agent's actions.

Most of the existing studies on agency problems involving two hidden actions adopt static settings: most notably, Diamond (1998), Biais and Casamatta (1999), and Palomino and Prat (2003). Only recently have double moral hazard models been developed in dynamic frameworks. Among these contributions, DeMarzo et al. (2013) study an agency problem in which a manager can divert funds and undertake risk-taking activities, which expose the firm to a disaster event. To deter the agent from stealing, his continuation utility must be reduced following poor performance. However, this provides poorly performing managers with incentives to gamble.[5] Occurrence of a disaster loss is fully revealing of the agent's risk-taking, leading to the immediate contract termination. Rochet and Roger (2015) adopt a similar setting to develop a theory of "risky utilities", i.e. private firms managing services essential to the economy. Utilities can engage in risky activities that

[5]Even when the disaster state is contractible, paying a bonus contingent on the firm's survival may not prevent risk-taking if the manager's incentives are sufficiently weak.

expose them to catastrophic losses, and are regulated by public authorities. They are interested in the optimal regulation contract minimising the social cost of restructuring[6] and deterring cash flow diversion and speculation at all times.

In contrast, in this study the opacity of the manager's strategies obstructs the principal from prompt intervention and excessive risks arise endogenously. The model is strictly close to that of Wong (2013), who analyses the case in which risk-taking increases the likelihood of negative shocks. In this setting, in contrast, the arrival of a shock is exogenous, while risk-taking exposes the bank to higher expected losses conditional on a crisis occurring. In my model I therefore focus on crisis events that are unavoidable, and whose consequences can be moderated through efficient risk management or amplified by imprudent risky investments.

Because risk-taking incentives are strictly connected to the agency cost of shirking, I then extend the analysis to explore contracts that deter risk-taking by implementing low levels of effort. Most of the agency literature on hidden actions focuses on optimal contracts inducing the first-best optimal action at all times. However, it is not a priori known[7] that, even under the second-best contract, this action remains optimal. Indeed, as shown by Zhu (2012), shirking may constitute an integral part of the solution. In his setting, as in my model, the optimal contract can take different forms[8] including phases when the agent shirks frequently.

The second strand of literature connected to this work is the one studying banking regulation and resolution mechanisms. Closely related to this contribution, Shim (2011) analyses the optimality of the Federal Insurance Corporation Improvement Act (FDICIA) mandated prompt corrective action (PCA), in a discrete-time dynamic agency model. Solving for the optimal contract between a regulator and a banker, he characterises the optimal stochastic liquidation[9] procedure for undercapitalised banks, denoted as a form of "structured ambiguity"[10]. Conversely, Freixas and Rochet (2013) study the optimal

[6]Restructuring in their model stands for selling the firms to new shareholders in order to guarantee the continuation of the services.

[7]Usually some necessary and sufficient conditions are imposed to ensure the optimality of high effort, as for instance in DeMarzo and Sannikov (2006).

[8]In particular, Zhu shows that in addition to the baseline contract, inducing high effort at all times, and the static contract in which the agent shirks forever, two other forms of contracts may arise. He denotes these two new forms the "Quite-Life" contract and the "Renegotiating Baseline" contract. Under the former type of contract, shirking arises as a form of hidden compensation after a long enough period of good performance. On the other hand, in the Renegotiating Baseline model, after sustained poor performance, the principal punishes the agent inducing him to shirk.

[9]Anh (2009) extends Shim's work to include the possibility of recapitalisation in a continuous-time setting. A similar approach is followed in Section (IV) when studying recapitalisation procedures under the socially optimal contract.

[10]Furthermore, he also studies the case in which low effort is first-best optimal. Under this assumption,

regulation of systematically important financial institutions, whose liquidation is never possible. In order to guarantee the continuation of the bank and deter excessive risks, they propose the establishment of a systemic risk authority. The optimal regulation involves restructuring the bank after a crisis by firing the managers and expropriating the shareholders.

In their framework, the bank is exposed to extremely large losses in the event of a crisis, and hence capital regulation has a very limited scope. In contrast, in my model, the bank cannot be wiped out by a single crisis and capital requirements play a central role in the resolution of financial distress[11]. An extensive academic literature has studied bank capital regulation, leading to mixed predictions on the relationship between banks' riskiness and capital standards[12].

My contribution relates to theoretical models on bank behaviour, under capital regulation, in the face of moral hazard. In particular, Milne (2002), Santos (1999), and Morrison and White (2005) advocate the positive incentives effect of capital requirements and their welfare-improving properties. More recently, Klimenko (2014) shows by adopting an incomplete contracting approach, how to design incentive-based capital requirements to prevent risk-taking. As in my model, what is crucial, in order to discourage gambling is making bank shareholders internalise the cost of recapitalisation. As suggested by Milne (2002) and Milne and Whalley (1998), prudential capital regulations should be structured as "an incentive mechanism, with some penalty imposed on the bank in the event of a breach".

The rest of the chapter is organised as follows. In the next section I outline the model. Section 4 presents the baseline regulatory contract maximising the bank's private value, under the outlined limited liability and incentive compatibility constraints. I then show how the contract can be replicated through capital regulation and deposit insurance. In Section 5 the model is extended to study socially optimal regulations deterring risk-taking at all times. The last section concludes the chapter, and proofs are relegated to the appendix.

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low effort arises in the optimal contract when the bank's capital level is high enough. This result is due to the assumption that high effort reduces the variance of realised income.

[11] Alternatively, restructuring is substituted by a suspension phase during which the manager is not fired but is allowed to shirk. This solution eliminates the cost of restructuring, at the expense of having a period of lower productivity.

[12] See for instance VanHoose (2007) for a survey on the topic.

II. The model

There is a single representative financial institution, a bank, operated by a manager. The bank is owned by private investors, the shareholders, who need the manager to run it[13]. Both the banker and the investors are risk-neutral and discount future cash flows at rate γ and ρ respectively, with $\gamma > \rho > 0$.

The bank's investors "transfer" an amount Q_0 to a risk-neutral regulator, collect D units of deposit and hire the manager to invest in a risky portfolio. The regulator provides deposit insurance and is in charge of supervising the bank's activities. The bank's asset consists of a risky portfolio of financial products, such as derivatives and loans.

Define a probability space $(\Omega, \mathbb{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$, where $\mathbb{F}$ is the completed natural filtration of the bank's cash flows $Y := \{Y_t\}_{t\geq 0}$ and $\mathbb{P}$ is the reference probability measure. The cash flows generated by the risky investments Y are exposed to volatility risk in financial markets, as represented by the standard Brownian motion $Z = \{Z_t\}_{t\geq 0}$. In addition, the bank is subject to tail risk, characterised by rare exogenous shocks inducing potentially large losses, such as financial crisis. The occurrence of these losses is modelled as a Poisson process $N = \{N_t\}_{t\geq 0}$ with constant intensity λ. Upon the occurrence of a shock at time t, the random loss generated L_t^r can take two values: $\underline{l}$ (small loss) or $\bar{l}$ (large loss, $\underline{l} < \bar{l}$), with probability depending on the manager's risk choice r as specified below. The processes Z and N are independent. The bank's cash flows evolves as follows

$$\mathrm{d}Y_t = \delta(e_t, r_t)\mathrm{d}t + \sigma \mathrm{d}Z_t - L_t^r \mathrm{d}N_t,$$

with positive drift $\delta(e_t, r_t) > 0$. I denote by $X := (X_t)_{t\geq 0}$ the diffusion component $\mathrm{d}X_t = \delta(e_t, r_t)\mathrm{d}t + \sigma \mathrm{d}Z_t$. The bank's cash flows are publicly observable, but the manager's actions $a_t = (e_t, r_t)$ are not. Hence a double moral hazard problem arises.

Precisely at every time t the banker decides to adopt a level of effort e_t, which takes value in $\left\{e^H, e^L\right\}$. When the manager chooses high effort e^H ("working"), she raises the cash flows' drift by $\Delta\delta > 0$, while incurring a cost $B > 0$ expressed in units of compensation. The cost of low effort e^L ("shirking") is normalised to zero.

Moreover, the manager can engage in risky activities that shape the distribution of losses in the event of a crisis. In particular, the probability of observing a large loss depends on the manager's risk choice $r_t \in \left\{r^H, r^L\right\}$. I denote by q the probability of a large loss under r^H, and by $q - \Delta q$ (with $\Delta q > 0$) under low risk r^L. This in turn implies

[13] In the rest of the chapter I will use the terms manager and banker, and investors and shareholders interchangeably.

that the average[14] loss size under high risk, denoted with $\mu(r^H)$, is greater than the one under low risk $\mu(r^L)$. Therefore, high risk increases the bank's exposure to crisis event while increasing short-term profits by $\alpha > 0$.

Risk-taking is costless for the banker and can be interpreted as imprudent investments in risky assets, increasing leverage and decreasing hedging, leading to faster returns at the cost of poorer risk management. Alternatively, considering a large part of the bank's assets is composed of loans, risk-taking corresponds to poor screening of loan quality and poor monitoring of loan performance. The following table summarises the combined effects of effort and risk choices.

Effort and Risk Effects			
action a=(e,r)	**drift** $\delta(e,r)$	**average loss size** $\mu(r)$	**manager's cost**
$a^S = (e^H, r^L)$	$\delta + \Delta\delta$	$\mu(r^L)$	B
$a^R = (e^H, r^H)$	$\delta + \Delta\delta + \alpha$	$\mu(r^H)$	B
$a^L = (e^L, r^L)$	δ	$\mu(r^L)$	0
$a^I = (e^L, r^H)$	$\delta + \alpha$	$\mu(r^H)$	0

Although risk-taking raises short-term profits, it also increases the cost of crisis events and the resulting overall effect is negative. Risk-taking is detrimental in the sense that it decreases the bank's expected returns:

$$\alpha - \lambda\Delta\mu(r) \leq 0, \tag{II.1}$$

where $\Delta\mu(r) = \mu(r^H) - \mu(r^L) \geq 0$ is the expected increase in losses induced by high-risk strategies. Nonetheless, even if the manager takes high risks, the bank remains profitable if she keeps working:

$$\delta + \Delta\delta + \alpha - \lambda\mu(r^H) - B > \rho R, \tag{II.2}$$

where $R \geq 0$ is the bank's liquidation value. Hence liquidation is inefficient. On the other hand, high effort is socially efficient:

$$\Delta\delta > B, \tag{II.3}$$

i.e. the bank's gains exceed the manager's cost.

In the absence of moral hazard, under the optimal contract the manager would always exert high effort and no risk-taking. Given that the banker is more impatient than the investors, she is paid immediately and the bank operates forever. The first best social

[14]Formally, define $\mu_t^r := E^r[L_t^r|\mathcal{F}_{t-}]$ as the average loss size under the managers' risk choice right before the shock arrival r_{t-}. Then the process μ_t^r takes value in $\{\mu(r^H), \mu(r^L)\}$.

value is:

$$\frac{\delta + \Delta\delta - \lambda\mu(r^L) - B}{\rho}.$$

Observe that the first best value is positive and strictly greater than the liquidation value R, by assumptions (II.1) and (II.2). However, investors and outsiders in general can only observe the bank's cash flows but not the manager's hidden actions. Hence the first best contract is not attainable.

In the following, I first develop a baseline model in which the optimal regulatory contract maximises the bank's private value, given by bankers' and investors' payoffs. This contract exhibits regulatory forbearance and allowance of excessive risk-taking.

The model is then extended in Section (IV) to examine the socially optimal regulatory contract, which takes into account the negative externalities generated by the bank's risk-taking and termination. The socially optimal regulation guarantees the bank's continuation under safe management, imposing a minimum capital requirement. Forcing the shareholders to recapitalise the bank while bearing the associated costs avoids public intervention. Moreover, minimum capital requirements deter risk-taking by providing the manager with enough "skin in the game" at all times.

Alternatively, the regulator can prevent the bank from undertaking excessive risks employing, after a long enough period of poor performance, a suspension phase during which the manager is allowed to shirk. "Strategic shirking" is equivalent to suspending the bank's risky investments. The suspension phase, although temporarily reducing the bank's profitability, allows the bank to recover from financial distress and avoid liquidation.

III. Baseline contract

Based on the realised cash flows, the regulator creates a contract specifying the bank's liquidation time τ, which is an $\mathbb{F}$-stopping time, and the manager payments. I denote by $T = \{T_t\}_{\{0 \leq t \leq \tau\}}$ the cumulative payments, which is assumed to be an $\mathbb{F}$-adapted nonnegative and nondecreasing process. The agents fully commit to a long-term contract. During any time interval $[t, t + \mathrm{d}t]$, with $t < \tau$, the sequence of events can be described as follows:

1. The regulator prescribes an action $a_t = (e_t, r_t)$ that shareholders must recommend to the bank's manager.

2. The manager makes effort and risk choices.

3. The cash flow realisation Y_t is publicly observed.

4. The manager receives a nonnegative transfer $\mathrm{d}T_t \geq 0$ from the shareholders.

5. The regulator decides whether or not liquidate the bank.

Given a contract $\Gamma = (\tau, T)$ and a prescribed action $a = (e, r)$, the manager's initial expected payoff from future payments, net of the cost of effort, is:

$$W_0 = E^a \left[\int_0^\tau \exp(-\gamma t) \left(\mathrm{d}T_t - 1_{\{e_t = e^H\}} B \mathrm{d}t \right) \right]. \tag{III.1}$$

An action $a = (e, r)$ induces a probability measure $\mathbb{P}^a$ equivalent to the reference probability $\mathbb{P}$, and E^a denotes the expectation under this measure. The investors' initial expected payoff is given by the bank's asset value, including the liquidation value, net of the manager's payments:

$$S_0(W_0) = E^a \left[\int_0^\tau \exp(-\rho t) \left(\mathrm{d}Y_t - \mathrm{d}T_t \right) + \exp(-\rho\tau) R \right]. \tag{III.2}$$

In this baseline model, the initial regulatory value is defined as the bank's private value $V_0(W_0) = S_0(W_0) + W_0$, given by the manager's and investors' payoffs.

A. Incentive compatibility and limited liability

At every time $t \leq \tau$, for a given contract (τ, T), the manager's discounted utility is given by:

$$W_t = E^a \left[\int_t^\tau \exp(-\gamma s) \left(\mathrm{d}T_s - 1_{\{e_s = e^H\}} B \mathrm{d}s \right) \right]. \tag{III.3}$$

Following Sannikov (2008), I use martingale techniques to characterise the dynamic of the manager's continuation utility as a function of the past cash flows:

$$\mathrm{d}W_t = (\gamma W_t + 1_{\{e_t = e^H\}} B + \psi_t \lambda \mu(r_t)) \mathrm{d}t - \mathrm{d}T_t + \phi_t (\mathrm{d}X_t - \delta(e_t, r_t) \mathrm{d}t) - \psi_t L_t^\tau \mathrm{d}N_t. \tag{III.4}$$

where $(\phi_t)_{t \geq 0}$ and $(\psi_t)_{t \geq 0}$ are two $\mathbb{F}$-predictable processes. Then, the following proposition provides necessary and sufficient conditions for an action recommendation $a = (e, r)$ to be *incentive compatible.*

Proposition III.1. (Incentive compatibility)

Define $\beta \equiv \frac{B}{\Delta\delta}$ *and* $\theta \equiv \frac{\alpha}{\lambda \Delta\mu(r)}$. *Given a contract* (τ, T), *the safe action* $a^S = (e^H, r^L)$ *is incentive compatible if and only if:*

$$\phi_t \geq \beta \text{ and } \psi_t \geq \phi_t \theta, \tag{III.5}$$

while the risky action $a^R = (e^H, r^H)$ *is incentive compatible if and only if* $\phi_t \geq \beta$ *and* $\psi_t \leq \phi_t \theta$.

Observe that ϕ_t and ψ_t represent the banker's sensitivities to volatility and jump risks, respectively. The incentive compatibility condition states that the manager prefers working to shirking if and only if the increase in her utility from choosing high effort ($\Delta\delta\phi_t$) is greater than the cost of exerting effort (B). Equivalently, her "skin in the game" must be at least equal to β, which represents the agency cost of inducing high effort.

At the same time, the manager chooses low-risk investments if and only if the short-term gain from risk-taking, $\phi_t\alpha$, is lower than its marginal cost represented by $\psi_t\lambda\Delta\mu(r)$, where $\Delta\mu(r)$ is the expected increase in losses induced by risk-taking. Observe that, although shirking and risk-taking are different activities, they are strictly linked via the incentive contract. Specifically, the more attractive shirking is, the more difficult it is to deter risk-taking.

In order to induce the manager to choose safe investments, she must be punished whenever downside risk materialises. By incentive compatibility, given a realised loss of size $l \in \{\underline{l}, \bar{l}\}$ the manager's continuation utility must be reduced by at least $\beta\theta l$. However, the banker is protected by *limited liability*, which limits the amount of feasible punishments, imposing that the manager's continuation utility must be non-negative at all times, and in particular after the realisation of a shock:

$$W_{t-} - \psi_t L_t^r \geq 0. \tag{III.6}$$

As a result in the region $[0, \widetilde{W})$, with $\widetilde{W} \equiv \beta\theta\bar{l}$, inducing low-risk strategies is not possible and the manager starts to gamble. By assumption (II.2), allowing for risk-taking is preferable to liquidating the bank. Furthermore, as I will show in the next section, under the optimal contract risk-taking may arise even when the limited liability constraint is not binding.

B. Derivation of the optimal contract

In this section the optimal incentive compatible contract is characterised. In particular, I focus on contracts inducing high effort e^H at all times[15], while optimising among the different risk strategies (safe r^L or risky r^H investments). In the extensions presented in

[15]In line with DeMarzo and Sannikov (2006) a necessary and sufficient condition for the optimality of high effort is that:

$$s^L \leq \min_{w \geq 0} S(w) - S'(w)\frac{\gamma}{\rho}w, \tag{III.7}$$

Section (IV), I show how the optimal contract changes when allowing for low effort.

Denote by $V(W) = S(W)+W$ the regulator's value function, given by the shareholders' payoff and the manager's continuation utility. In the following heuristic derivation V is assumed to be concave. A formal proof is provided in the appendix.

First I analyse the optimal transfers to the manager. Consider the strategy of making a lump sum payment of $\mathrm{d}T > 0$ immediately, decreasing the manager's continuation utility to the value $W - \mathrm{d}T$. Then, it is immediate to see that the shareholders' payoff satisfies $S(W) \geq S(W-\mathrm{d}T)-\mathrm{d}T$, implying that $S'(W) \geq -1$ for every W. The left-hand side can be interpreted as the marginal benefit from promising future compensation, while the right-hand side represents the shareholder's marginal cost of an immediate payment. The optimal contract delays payments to the manager when $V'(W) = S'(W)+1 > 0$ that is, as long as they are more costly than utility promises. Denote by $\overline{W} \equiv \min\{W \geq 0 \mid V'(W) = 0\}$ the payment threshold. Then, by concavity of V, payments are postponed whenever $W < \overline{W}$. On the other hand, when the manager continuation utility reaches the threshold $\overline{W}$, continuous payments are transferred to the manager. Finally, if W starts from a value above $\overline{W}$, a lump payment of size $W - \overline{W}$ immediately brings the manager's continuation utility down to the payment threshold. Then the regulator's value function can be set at $V(W) = V(\overline{W})$ for $W \geq \overline{W}$.

I will now analyse the regulator's value function for $W \in [0, \overline{W}]$. First I characterise V when the risky action $a^R = (e^H, r^H)$ is implemented. Using the dynamic programming approach it is known that $V(W)$ must satisfy the following equation:

$$\rho V(W) = \delta + \Delta\delta + \alpha - \lambda\mu(r^H) - B + \max_{\phi\geq\beta,\psi\leq\phi\theta} \mathcal{L}_R V(W)$$

where the "risky action's operator" $\mathcal{L}_R$ is defined as:

$$\mathcal{L}_R V(W) = (\rho-\gamma)W + V'(W)(\gamma W + B + \lambda\psi\mu(r^H)) + \frac{1}{2}V''(W)\phi^2\sigma^2 + \lambda E^{a^R}[V(W-\psi l)] - \lambda V(W).$$

By concavity of $V(W)$, the regulator finds it optimal to set the manager's sensitivities to the minimum feasible values implementing the risky action. From the incentive compatibility constraint, these values are equal to $\phi_t = \beta$ and $\psi_t = 0$. Note that, when the risky strategy is implemented, the manager is fully insured against tail risk. It follows that the

where $s^L \equiv \frac{\delta-\lambda\mu(r^L)}{\rho}$ is the shareholders's payoff if the shirking strategy $a^L = (e^L, r^L)$ is implemented forever. Note that condition (III.7) is also sufficient to guarantee the sub-optimality of the strategy $a^I = (e^L, r^H)$.

manager's continuation utility evolves as

$$\mathrm{d}W_t = (\gamma W_t + B)\mathrm{d}t - \mathrm{d}T_t + \beta\sigma\mathrm{d}Z_t, \tag{III.8}$$

and the regulator's value function solves the following equation:

$$\rho V(W) = \delta + \Delta\delta + \alpha - \lambda\mu(r^H) - B + (\rho-\gamma)W + V'(W)(\gamma W + B) + \frac{1}{2}V''(W)\beta^2\sigma^2. \tag{III.9}$$

Now I consider a regulatory contract implementing the safe action $a^S = (e^H, r^L)$. Under the safe strategy, the regulator's value function satisfies:

$$\rho V(W) = \delta + \Delta\delta - \lambda\mu(r^L) - B + \max_{\phi\geq\beta,\psi\geq\phi\theta} \mathcal{L}_S V(W)$$

where the "safe action's operator" $\mathcal{L}_S$ is defined as:

$$\mathcal{L}_S V(W) = (\rho-\gamma)W + V'(W)(\gamma W + B + \lambda\psi\mu(r^L)) + \frac{1}{2}V''(W)\phi^2\sigma^2 + \lambda E^{a^S}[V(W-\psi l)] - \lambda V(W). \tag{III.10}$$

By concavity of $V(W)$, the optimal manager's sensitivities are fixed at the lower bounds given by incentive compatibility: that is $\phi_t = \beta$ and $\psi_t = \beta\theta$. In order to incentivise the safe strategy, the agent must bear part of the losses at every shock occurrence. Then, the solution to the following equation represents the regulator's value function when the safe strategy is implemented:

$$\begin{aligned}\rho V(W) = {} & \delta + \Delta\delta - \lambda\mu(r^L) - B + (\rho-\gamma)W + V'(W)(\gamma W + B + \lambda\beta\theta\mu(r^L)) + \frac{1}{2}V''(W)\beta^2\sigma^2 \\ & + \lambda E^{a^S}[V(W-\beta\theta l)] - \lambda V(W).\end{aligned} \tag{III.11}$$

Given the value from implementing the safe and the risky strategies, (III.11) and (III.9) respectively, the regulator's value function can be characterised as the solution to the following HJB equation:

$$\rho V(W) = \max\left\{\delta + \Delta\delta - \lambda\mu(r^L) - B + \mathcal{L}_S V(W), \delta + \Delta\delta + \alpha - \lambda\mu(r^H) - B + \mathcal{L}_R V(W)\right\}. \tag{III.12}$$

The regulator finds it optimal to prescribe the safe strategy if the expected value from a^S

is greater than the one from the risky strategy a^R. That is, if and only if:

$$\lambda\Delta\mu(r) - \alpha \geq \mathcal{A}(W) \equiv \lambda E^{a^S}[V(W) - V(W - \psi l) - V'(W)\psi l]. \tag{III.13}$$

The left-hand side represents the bank's expected monetary cost associated with risky investments. Since risk-taking is harmful by assumption (II.1), this term is positive. By concavity of V, the operator $\mathcal{A}(W)$ on the right-hand side is positive as well. This term can be interpreted as the regulatory expected cost of inducing the safe action against the risky action. Inducing the safe strategy is costly because the regulator must promise punishments to the manager upon the arrival of a loss of size l, decreasing her continuation utility to $W - \psi l$. Punishments in turn decrease the regulator's value function from $V(W)$ to $V(W - \psi l)$, while increasing the manager promised utility by ψl. As a result, the overall regulatory cost is given by $\mathcal{A}(W)$.

The regulator thus faces a trade-off when optimising over the two strategies. Condition (III.13) states that inducing the safe action is optimal as long as the agency cost of refraining excessive risks does not exceed the monetary cost of risk-taking. I denote by $\underline{W}$ the threshold at which the two costs offset each other, i.e. where condition (III.13) holds as an equality. Then for values of W above $\underline{W}$, the regulator implements the safe action a^S, while for values below $\underline{W}$ the regulator prefers inducing the risky action a^R. By limited liability, the gambling threshold $\underline{W}$ must lie in the region $[\widetilde{W}, \overline{W}]$.

However, it is not a priori obvious that a unique gambling threshold exists. Uniqueness is guaranteed under the assumption specified in the following proposition, which is assumed to hold throughout this section.

Proposition III.2. *Assume* $0 \leq V'(0) \leq 1$. *Then a unique gambling threshold* $\underline{W}$ *exists:*

i) If $\mathcal{A}(\overline{W}) < \lambda\Delta\mu(r) - \alpha < \mathcal{A}(\widetilde{W})$, *then* $\overline{W} > \underline{W} > \beta\theta\bar{l}$;

ii) If $\mathcal{A}(\overline{W}) \geq \lambda\Delta\mu(r) - \alpha$, *then* $\underline{W} = \overline{W}$;

iii) If $\mathcal{A}(\widetilde{W}) \leq \lambda\Delta\mu(r) - \alpha$, *then* $\underline{W} = \widetilde{W}$.

Lastly, because liquidation is socially costly, the regulator avoids terminating the bank unless strictly necessary, which occurs when the manager continuation utility reaches the lower bound $W = 0$. The following proposition provides a complete characterisation of the regulator's value function and the optimal contract in terms of the manager's continuation utility.

Proposition III.3. *The regulator's value function is given by the following concave function:*

$$V(W) = \begin{cases} V_R(W), \ W \in [0, \underline{W}), \\ V_S(W), \ W \in [\underline{W}, \overline{W}], \\ V_S(\overline{W}), \ W > \overline{W}. \end{cases} \tag{III.14}$$

where $V_R(W)$ and $V_S(W)$ respectively solve equations (III.9) and (III.11), with boundary conditions:

$$V(0) = R, \ V'(\overline{W}) = 0, \ V''(\overline{W}) = 0.$$

The optimal contract implements high effort all the time. Define the gambling threshold $\underline{W} \in [\widetilde{W}, \overline{W}]$ such that $\mathcal{A}(\underline{W}) = \lambda \Delta\mu(r) - \alpha$. Then the optimal regulation recommends that the bank' manager chooses the safe action a^S when $W \geq \underline{W}$ and to switch to the risky action a^R when $W < \underline{W}$. The optimal sensitivity to volatility risk is $\phi = \beta$ for every W, while the sensitivity to tail risk is given by $\psi = \beta\theta$, $W \geq \underline{W}$, and $\psi = 0$ when $W < \underline{W}$. The manager is compensated upon reaching the payment threshold $\overline{W} = \min\{W \geq 0 \mid V'(W) = 0\}$. The optimal amount transferred by the shareholders to the manager is equal to:

$$dT = 1_{\{W = \overline{W}\}}(\gamma \overline{W} + B + \lambda\beta\theta\mu(r^L))dt + \max\{W - \overline{W}, 0\}$$

The bank is liquidated the first time $W = 0$. Under the optimal contract the banker's continuation utility evolves as:

$$dW_t = (\gamma W_t + B)dt - dT_t + \beta\sigma\, dZ_t + \beta\theta(\lambda\mu(r^L)dt - L_t^{r^L} dN_t)1_{\{W_t \geq \underline{W}\}}.$$

C. Discussion

The optimal regulation incentivises the banker to work all the time by providing her with enough "skin in the game" and promising future payments. After a long enough period of good performance, when $W \geq \overline{W}$, the banker is compensated with a positive transfer.

On the other hand, in order to stop the manager from taking large risks, punishments must be imposed after every loss. Since the banker is protected by limited liability, however, when her continuation utility is too low ($W < \widetilde{W}$) punishments become unfeasible. The bank enters in financial distress and the regulator is forced to implement risk-taking, providing full insurance to the banker against realised losses ($\psi = 0$). Even if risk-taking is detrimental, in order to avoid an inefficient liquidation the regulator prefers allowing

the manager to gamble for resurrection. Only when the bank is no longer viable does the regulator mandate its closure.

In general, the optimal regulation implements risk-taking even when the bank has not yet entered financial distress and the limited liability constraint is not binding. In fact, unless the monetary cost of risk-taking is extremely large, the regulation allows the bank to take risks in order to avoid bearing the cost of inducing safe management. Therefore, when regulatory contracts are written in the interest of the individual financial institution, excessive risks arise. This baseline model highlights the determinants of rationally chosen "micro-prudential" regulations, which are however sub-optimal under an economy-wide perspective.

It is important to notice that the optimality of risk-taking forbearance is strictly linked to the opacity of the bank's strategy[16]. If risk-taking were fully revealing, as for instance in DeMarzo et al. (2013) and Rochet and Roger (2015), the regulator would immediately close the bank at the arrival of the first crisis. Fully revealing risk-taking can be obtained as a special case of my model, setting the loss size $L_t^r = r_t l$, where $r_t \in \left\{r^L = 0, r^H = 1\right\}$ and l is a positive constant. Under these assumptions, the bank is exposed to tail risk only under high-risk strategies. Hence, the realisation of a loss is fully revealing of the manager's risk-taking and it is therefore optimal to immediately terminate the contract.

In the full generality of my framework, on the other hand, it is not possible to immediately detect the managerial risk choices from the observation of a single loss. The opacity of the banker's actions obstructs the outsiders, such as investors and financial authorities, from properly assessing the bank's risks. The lack of transparency allows the regulator to forgo prompt interventions and the bank to capitalise on excessive risk-taking.

D. Implementation

This section shows how the optimal baseline contract can be implemented in terms of capital regulation and deposit insurance. In line with Shim (2011), I adopt the bank's book-value capital $Q = (Q_t)_{t\in[0,\tau]}$ as record-keeping device to keep track of the banker's continuation utility W_t. In particular, the bank capital level Q_t is required to map the ratio $\frac{W_t}{\beta\theta}$, measuring the bank's financial slackness[17].

[16] Consistent with this result, Gallemore (2013) empirically documents that a bank's opacity is positively associated with forbearance and negatively associated with the probability of closure.

[17] Note that under the optimal contract discussed in the previous section, the bank is closed down when the banker's continuation utility reaches the zero lower bound. Thus the banker's expected payoff indicates the distance to default and can be interpreted as the bank's financial slackness.

Book-value capital regulation. The regulator requires an initial amount of capital Q_0 from investors in order to set up the bank. In exchange, investors receive D units of deposits, which are invested in risky financial products. The initial capital is kept as cash in a reserve account and grows at the risk-free rate ρ. The account is used to meet possible losses due to the arrival of a crisis, to pay a deposit insurance premium, and to distribute dividends. Based on the amount of capital available in the account, which measures the bank's financial slack, the regulator prescribes different sets of actions and restrictions on the bank's activities. The scheme is reminiscent of the US banking regulation system, as introduced by the FDICIA[18]. In particular banks are classified into four different categories:

- well capitalised, if $Q_t > \overline{Q} \equiv \overline{W}/\beta\theta$;
- adequately capitalised, if $\underline{Q} \leq Q_t \leq \overline{Q}$ where the threshold $\underline{Q} \equiv \underline{W}/\beta\theta$;
- undercapitalised, if $\bar{l} \leq Q_t < \underline{Q}$;
- significantly undercapitalised, if $Q_t < \bar{l}$, i.e. only one large loss away from default.

Under the optimal regulatory contract, dividends may be distributed only when the bank is well capitalised, while they are suspended for levels of capital lower than $\overline{Q}$. In order to align the manager's incentives with those of shareholders, the manager is granted a fraction $\beta\theta$ of equity. The fraction β is necessary to incentivise the manager to exert effort, whereas the fraction θ is introduced to deter risk-taking. Note that, since risk-taking is detrimental ($\theta < 1$), the banker's equity stake is lower than in the absence of risk-taking incentives.

Public recapitalisation/forbearance. When the bank capital falls below the threshold $\underline{Q}$, the bank becomes undercapitalised. In this case, should downside risk materialise with a loss of size $l \in \left\{\underline{l}, \bar{l}\right\}$, a public recapitalisation provision will be activated and the regulator makes up for the shortfall. The bank is only terminated when it entirely depletes its capital due to market volatility. Figure (2.1) illustrates a sample path of the bank's capital under this regulation.

< Figure 2.1 about here >

Alternatively, the absence of losses in the book-value capital of undercapitalised banks can be interpreted as regulatory forbearance: that is the act of refraining from forcing banks to recognise their losses. This and more general practices of forbearance to keep

[18]See Table 2 in Shim (2011) for the FDICIA classification of banks as in PCA.

distressed banks operating have been often observed in the past as well as during the recent financial crisis. Some examples are Japan's banking crisis of the 1990s and the US savings and loan crisis in the 1980s, as documented by Nelson and Tanaka (2014) and White (1991), respectively.

Deposit insurance premium. In exchange for deposit insurance, the banker transfers a flow of payments $\mathrm{d}P_t$ to the regulator equal to:

$$\mathrm{d}P_t = \begin{cases} (\rho-\gamma)Q_t\mathrm{d}t + (\frac{\delta+\Delta\delta}{\theta} - \frac{B}{\beta\theta} - \lambda\mu(r^L))\mathrm{d}t + (1-\frac{1}{\theta})\mathrm{d}X_t, & \text{if } Q_t \geq \underline{Q}, \\ (\rho-\gamma)Q_t\mathrm{d}t + (\frac{\delta+\Delta\delta+\alpha}{\theta} - \frac{B}{\beta\theta})\mathrm{d}t + (1-\frac{1}{\theta})\mathrm{d}X_t, & \text{if } Q_t < \underline{Q} \end{cases}$$

This specific implementation involves a strongly risk-based insurance premium. In fact, $\mathrm{d}P_t$ is decreasing both in the bank's level of capital and cash flows. The latter property is due to the assumption that risk-taking is detrimental for the bank's value[19]. Moreover, the deposit insurance premium increases when the bank is significantly undercapitalised and public recapitalisation is in place.

IV. The socially optimal contract

This section presents alternative regulatory contracts under a broader "macro-prudential" perspective. As highlighted by the baseline model previously analysed, a regulation of micro-prudential nature that is focused exclusively on the bank's private value is unable to curb excessive risks.

Bank investors and managers take into account only their private benefits, and not the social costs of risk-taking. Moreover, weak supervisory powers, together with the absence of ex ante resolution mechanisms, leads to socially inefficient bailouts and forbearance of excessive risks. On the other hand, macro-prudential regulations that force individual banks to internalise their contribution to systemic risk provide incentives to choose socially optimal strategies.

Since the main objective of macro-prudential regulation is limiting systemic risk, I will focus on recasting regulation for systemically important financial institutions. In particular, I assume that the bank's risk-taking strategies produce large negative externalities on the economy, and that this externalities are now internalised by the regulator. These negative effects can be interpreted as the impact of the bank's risk-taking on other institutions due to their interconnections and financial linkages[20], or as merely the financial cost

[19] Observe that $1-\frac{1}{\theta} < 0$, since $\theta = \frac{\alpha}{\lambda\Delta\mu(r)} < 1$ by assumption (II.1).

[20] For instance, during the 2008 subprime mortgages crisis banks' interconnections through securitisa-

of bailouts induced by excessive leverage. Specifically, I denote with $C > 0$ the social costs of risk-taking, which materialises every time t a sudden shock arrives $\mathrm{d}N_t = 1$ and the banker has chosen a high-risk strategy r^H. In the remainder of this section C is assumed to be large enough so that it is always socially optimal to prevent risk-taking.

Furthermore, the macro-prudential[21] regulator never liquidates the bank. This is possibly because of adverse spillover effects stemming from contagion risks with others institutions and the economy as a whole[22]. Additionally, the regulator might be concerned with maintaining the bank's critical economic functions. However, I abstain from modelling these channels explicitly.

A. Recapitalisation and optimal capital requirements

In this section I explore the optimal design of incentive-based capital regulation, tailored to curb risk-taking that increases bank's exposure to systemic risk.

I assume that the regulator is now endowed with "strong" supervisory powers and can force the bank to recapitalise by raising equity through the issuance of new shares[23]. Equity injections are lumpy and involve proportional costs. I denote by $\xi_0 > 1$, the cost of equity injections for outside investors. The manager, representing the bank's internal shareholders, also has to bear a proportional cost $\xi_1 > 0$[24].

Formally, define equity injections as the $\mathbb{F}$-adapted non-negative non-decreasing process $I = \{I_t\}_{t\geq 0}$:

$$I_t = \sum_{n\geq 1} i_n 1_{\{t\geq \tau_n\}},$$

where $(i)_{n\geq 1}$ is the sequence of nonnegative random variables representing the amount of capital injected, and $(\tau_n)_{n\geq 1}$ is the sequence of stopping times representing the equity issuance dates. Accounting for recapitalisation' costs, the manager's and shareholders'

tions of badly screened loans led to large credit losses and contributed to amplifying the crisis.

[21]In the remainder of the section I will use macro-prudential and socially optimal as equivalent denominations of the regulatory contract. However, in order to develop a fully fledged macro-prudential regulation, endogenous feedback effects arising from the other sectors of the economy should be taken into account. For this purpose, a general equilibrium approach is needed, which is out of the scope of this chapter.

[22]For instance liquidating Lehman Brothers proved to be damaging economy-wide.

[23]Alternatively, recapitalisation could be realised through the conversion of hybrid debt instruments such as contingent convertible bonds. Recapitalisation could also be interpreted as restructuring the bank, by expropriating shareholders and management and selling the shares of the bank to new investors, as suggested by Freixas and Rochet (2013). However, I will focus on the simple case of new equity issuance.

[24]The manager's costs of equity issuance comprises transaction and asset restructuring costs, while the bank's external shareholders bear dilutions costs. In order to ensure incentive compatibility, it is important that the issuance of new shares does not change the proportion of equity held by the manager.

payoffs become:

$$W_t = E^a\left[\int_t^\infty \exp(-\gamma s)(\mathrm{d}T_s - 1_{\{e_s=e^H\}}B\mathrm{d}s - \xi_1\mathrm{d}I_t)\right],$$

and

$$S(W_t) = E^a\left[\int_t^\infty \exp(-\rho s)\left(\mathrm{d}Y_s + (1-\xi_0)\mathrm{d}I_t - \mathrm{d}T_s\right)\right],$$

respectively. The regulator's value function, which now incorporates the social costs of risk-taking, is given by:

$$F(W_t) = S(W_t) + W_t - E^a\left[\int_t^\infty \exp(-\rho s)C1_{\{r_s=r^H\}}\mathrm{d}N_s\right].$$

The socially optimal regulation maximises social welfare $F(W_t)$ over the injections I_t and transfers T_t strategies, while forcing the banker to implement the safe action $a^S = (e^H, r^L)$ at all times. By incentive compatibility and limited liability constraints (III.5) and (III.6), the banker's continuation utility must always be $W_t \geq \widetilde{W} \equiv \beta\theta\bar{l}$. Otherwise, any further penalty prescribed to preserve incentive compatibility would violate limited liability. In order to avoid terminating the bank, for values of $W_t < \widetilde{W}$ it becomes necessary to require the injection of additional equity. However, as I will analyse in details in the following, it might be optimal to force recapitalisation even before this is strictly necessary.

Let $\epsilon > 0$ and consider the strategy that, for a given initial continuation utility $W - \epsilon$, makes a capital injection immediately to bring the banker's continuation utility up to the value W. Then it follows that $F(W-\epsilon) \geq F(W) - \frac{\epsilon}{\xi_1}(\xi_0 + \xi_1 - 1)$, which implies that $1 + \xi_1 F'(W) \leq \xi_0 + \xi_1$. The left-hand side represents the benefits of recapitalisation, including the marginal benefit from the banker's threat of capital adjustment costs, while the right-hand side is the social cost of an equity injection. The bank will not be recapitalised as long as the social benefits from a capital injection are lower than its costs.

Define the recapitalisation threshold $\widehat{W} \equiv \min\left\{W \geq \widetilde{W} | F'(\widehat{W}) = \frac{\xi_0+\xi_1-1}{\xi_1}\right\}$. Then the optimal injections can be characterized by $i_n = \frac{(\widehat{W}-W_n)}{\xi_1}$ and $\tau_n = \inf\left\{t > \tau_{n-1} | W_n < \widehat{W}\right\}$ for all $n \geq 1$. After a long period of poor performance, when the banker's continuation utility drops below $\widehat{W}$, the bank is recapitalised through a lumpy injection of capital, which brings W back to the value $\widehat{W}$.

The recapitalisation threshold depends on the cost of injections, balancing two opposite effects generated by internal and external costs. In particular, if the banker faces a high cost of equity issuance, increasing the benefits from the threat of mandatory recapitalisation, the requirement becomes tighter. On the other hand, the boundary is decreasing in

the cost of investors' capital. When external financing is expensive, the regulator relaxes the recapitalisation requirement. In particular, if the social cost of equity injections is especially large[25] relative to the manager's cost, recapitalisation is enforced only if strictly necessary to guarantee the continuation of the bank under a safe management. That is the two boundaries coincide $\widehat{W} = \widetilde{W}$.

Finally, it should be observed that, as in the baseline model, the regulatory contract allows for transfers to the manager only at or above the payment threshold $\overline{W} \equiv \min\{W \geq 0 | F'(W) = 0\}$. The above heuristic derivation of the socially optimal regulatory contract is formalised in the next proposition.

Proposition IV.1. *The socially optimal regulatory value function is given by the following concave function:*

$$F(W) = \begin{cases} F(\widehat{W}) + \frac{1-\xi_0-\xi_1}{\xi_1}(\widehat{W} - W), \ W < \widehat{W}, \\ F(W), \ W \in [\widehat{W}, \overline{W}], \\ F(\overline{W}), \ W > \overline{W}, \end{cases} \tag{IV.1}$$

where $F(W)$ solves equation (III.11) with boundary conditions:

$$F'(\widehat{W}) = \frac{\xi_0 + \xi_1 - 1}{\xi_1}, F''(\widehat{W}) = 0, F(\overline{W})' = 0, F''(\overline{W}) = 0.$$

The socially optimal contract implements the safe action $a^S = (e^L, r^L)$ at all the times and the optimal sensitivities are $\phi = \beta$ and $\psi = \beta\theta$ for every W. Recapitalisation is required when W falls below the threshold $\widehat{W}$, and the optimal capital injection strategy is

$$dI = \frac{(\widehat{W} - W)^+}{\xi_1}.$$

The manager is compensated upon reaching the payment threshold $\overline{W}$, with a transfer equal to:

$$dT = 1_{\{W=\overline{W}\}}(\gamma\overline{W} + B + \lambda\beta\theta\mu(r^L))dt + \max\left\{W - \overline{W}, 0\right\}.$$

Under the socially optimal contract, the banker's continuation utility evolves as:

$$dW_t = (\gamma W_t + B)dt - dT_t + \beta\sigma dZ_t + \beta\theta(\lambda\mu(r^L)dt - L_t^{r^L} dN_t) + \xi_1 dI_t$$

and the bank is never terminated.

[25] A sufficient condition for this is $F'(\widetilde{W}) < \frac{\xi_0+\xi_1-1}{\xi_1}$, due to the concavity of the social value function.

I will now discuss how the socially optimal contract differs from the baseline model presented in Section 4 in terms of capital implementation. As under the baseline regulatory contract, dividend' distribution is allowed only when the bank is well capitalised i.e. the level of capital exceeds the boundary $\overline{Q} \equiv \overline{W}/\beta\theta$. What differs from the previous implementation is the resolution of financial distress.

Whenever the bank's capital $Q_t \equiv \frac{W_t}{\beta\theta}$ reaches $\widehat{Q} \equiv \widehat{W}/\beta\theta$, the socially optimal regulation forces shareholders to recapitalise the bank. An amount of capital equal to $(\widehat{Q} - Q_t)$ is injected, bringing the level of capital back to the boundary. Because[26] $\overline{Q} > \widehat{Q}$, dividends are never distributed in the face of a recapitalisation provision.

Notice that, since $\widehat{Q} \geq \bar{l}$, under this regulation the bank never becomes significantly undercapitalised. The bank's capital is restored to a level ensuring a sufficient buffer before capital is completely depleted, as shown in figure (2.2).

< Figure 2.2 about here >

The capital requirement $\widehat{Q}$ guarantees that there is enough capital to absorb subsequent losses, and in turn eliminates the need for public bailouts. Moreover, the requirement ensures that the banker always has enough "skin in the game" and hence the right incentives to refrain from taking excessive risks. The optimal regulation may require an extra buffer of capital, when $\widehat{Q}$ is strictly greater than $\bar{l}$, in addition to the minimum capital requirement $\bar{l}$. The buffer provides an additional layer of capital to absorb losses during periods of distress.

The optimal capital requirement contains some elements that are bank' specific. In particular, the magnitude of the requirement depends on the bank's loss size distribution. The larger the losses to which the bank is exposed, the higher the capital requirement is. Furthermore, the capital requirement increases with the severity of the agency problem. Banks that are more prone to moral hazard, face stricter capital requirements.

The capital regulation proposed resembles some features of the Basel III agreement on capital regulation. In the crisis aftermath, regulatory reforms have been carried out, in an effort to strengthen the effectiveness of capital requirements and improve the resilience of the banking system. As specified in BIS (2011), the overall set of measures produced aim to reduce moral hazard and the negative externalities associated with it. Specifically, as stylised in this model, Basel III prescribes banks to hold a capital conservation buffer in addition to an increased minimum capital requirement, in order to endure future periods of financial and economic distress (BIS (2010)). Furthermore, capital buffers should account for firm-specific requirements, including surcharges for systemically important banks.

[26]Indeed $F'(\widehat{W}) > 0$ and using the concavity of the social value function, it follows that $\overline{W} > \widehat{W}$.

B. Strategic shirking and the suspension phase

So far in this chapter I have focused on contracts, inducing the banker to exert high effort at all times[27]. However, in some cases inducing "strategic shirking", which is equivalent to suspending the banker from working, can constitute part of an optimal strategy.

This section proposes a heuristic description of an alternative socially optimal regulation, considering the implementation of low levels of effort. The regulator still internalises the social costs of risk-taking and liquidation, which are therefore never implemented. As an alternative punishment device to costly recapitalisation, the regulator can employ a suspension phase in which the banker is allowed to shirk. Throughout the section it is assumed that the bank remains profitable when the banker shirks if there is no risk-taking:

$$\delta + \alpha - \lambda\mu(r^H) < \rho R < \delta - \lambda\mu(r^L). \tag{IV.2}$$

This assumption implies that the action $a^I = (e^L, r^H)$ is always inefficient. Analogously to proposition (III.5), it is possible to show that the shirking action $a^L = (e^L, r^L)$, involving low levels of effort and risk, is incentive compatible if and only if:

$$\phi_t \leq \beta \text{ and } \psi_t \geq \phi_t \theta. \tag{IV.3}$$

Notice that the incentives needed to induce low-risk strategies are strictly connected to the agency cost of shirking. This link plays a crucial role in deriving the optimal strategy, as I will show below. Under the shirking action $a^L = (e^L, r^L)$, the regulator's value function satisfies the following equation:

$$\rho F(W) = \delta - \lambda\mu(r^L) + \max_{\phi \leq \beta, \psi \geq \phi\theta} \mathcal{L}_L F(W),$$

where the "shirking operator" $\mathcal{L}_L$ is defined as:

$$\mathcal{L}_L F(W) = (\rho - \gamma)W + F'(W)(\gamma W + \lambda\psi\mu(r^L)) + \frac{1}{2}F''(W)\phi^2\sigma^2 + \lambda E^{a^L}[F(W - \psi l)] - \lambda F(W).$$

Since it is costly to expose the manager to risks, the regulator prefers to minimise the volatility of the agent's continuation utility, setting $\phi_t = 0$, and in turn reducing the probability of the bank's distress. Indeed, when the agent is allowed to shirk, no sensitivity to asset performance is required[28]. It follows that it is optimal to shut down the sensitivity

[27] The optimality of high effort was insured under the condition (III.7).

[28] In terms of implementation, this could be achieved by divesting the manager of her shares.

to tail risk as well, setting $\psi_t = \phi_t\theta = 0$. When the banker is allowed to exert low effort, restraining her from risk-taking does not require any further punishments. Then, the solution to the following equation represents the social value function implementing the shirking strategy a^L:

$$\rho F(W) = \delta - \lambda\mu(r^L) + (\rho - \gamma)W + F'(W)\gamma W. \tag{IV.4}$$

Analogously to the baseline contract, the optimal policy can be characterised comparing the regulator's value function under the two strategies: the safe and the shirking actions in this case. In order to preserve limited liability, it is not possible to incentivise the safe action when the manager's continuation utility becomes lower than $\widetilde{W}$. Then, after a long enough period of poor performance, when $W < \widetilde{W}$, implementing shirking becomes necessary in order to deter risk-taking and avoid costly liquidation[29]. For values of $W \geq \widetilde{W}$, the regulator finds it optimal to incentivise high effort when the social value from the safe action $a^S = (e^H, r^L)$, given by equation (III.11), is greater than the value from shirking $a^L = (e^L, r^L)$, as in equation (IV.4): that is, if and only if:

$$\Delta\delta - B \geq \mathcal{B}(W) \equiv \mathcal{A}(W) - \frac{1}{2}F''(W)\beta^2\sigma^2 - BF'(W) \tag{IV.5}$$

The right-hand side represents the agency cost of incentivising the safe action against the shirking action. As in the baseline model, $\mathcal{A}(W)$ is the regulatory cost of incentivising low risk. The second term, which is positive by concavity of $F(W)$, represents the cost of incentivising high effort; this is the cost of exposing the manager to volatility risk, in order to provide her with enough skin in the game. The remaining term is the marginal benefit, as an increase in the banker's continuation utility, from implementing costly effort. The left-hand side represents the bank's monetary benefit from high effort, net of the banker's cost of effort, and it is positive by assumption (II.3). Condition (IV.5) specifies that as long as this benefit exceeds the incentives' cost, the safe action is optimal.

Define the shirking threshold $W^* \equiv \min\left\{W \geq \widetilde{W} | \mathcal{B}(W^*) = \Delta\delta - B\right\}$. Then for values of $W \geq W^*$, the safe action a^S is implemented and the manager's continuation utility dynamic is:

$$\mathrm{d}W_t = (\gamma W_t + B)\mathrm{d}t - \mathrm{d}T_t + \beta\sigma\mathrm{d}Z_t + \beta\theta(\lambda\mu(r^L) - L_t^{r^L}\mathrm{d}N_t).$$

For values of $W < W^*$, the shirking action a^L is socially optimal and W evolves deter-

[29]Note that by assumption (IV.2) the bank remains profitable under low effort if there is no risk taking.

ministically as:

$$\mathrm{d}W_t = \gamma W_t \mathrm{d}t,$$

with a positive drift. The evolution of the manager's continuation utility is depicted in figure (2.3).

< Figure 2.3 about here >

If W hits the boundary W^* exactly, as long as no shocks occur, the agent's continuation utility sticks around it for a positive period of time[30]. If instead W crosses the shirking boundary with an overshoot, it drifts upwards until the boundary W^* is reached. Once W overcomes the shirking threshold, high effort is implemented again. In general, the shirking period represents a suspension phase, during which risky investments are interrupted and the bank operates at a lower profitability rate. This phase allows for a slow recovery from financial distress, curbing excessive risks and avoiding the bank's termination.

In terms of implementation, the suspension period could be achieved through a special compulsory administration[31], in which the regulator limits the banker's duties or runs the bank's main functions, replacing the manager. During this period, speculative activities would be prohibited, yielding in turn lower but risk-free returns. Regulatory interventions, acting as an early measure, would avoid costly bailouts while protecting the bank's critical functions.

C. Extensions

To conclude, I will discuss some possible extensions of the model.

First, when addressing the socially optimal regulatory problem in my model, risk-taking has been assumed to always be socially inefficient. Instead, under the unrestricted class of contracts, high-risk strategies could still arise under the equilibrium path. In such a model, it would be interesting to analyse the interplay between gambling and recapitalisation strategies.

More generally, one could study the optimal regulatory contract optimising the safe, risky and shirking actions contemporaneously. This framework would generate a larger set of feasible contracts, depending on the values of the shirking payoff and risk-taking cost.

[30] In the absence of jumps, this dynamic is known as Sticky Brownian Motion, see Harrison and Lemoine (1981). This motion of the agent's continuation utility appears under the optimal Quiet-Life and Renegotiating Baseline contracts in Zhu (2012).

[31] Banks' administration procedures constitute part of the regulatory tools of special resolution regimes, as established in the EU bank recovery and resolution directive (BRRD (2014)).

Furthermore, the model could accommodate a broader class of banks' cash flow loss distributions with bounded support. However, the present framework could not include any possible large loss, since it would not be feasible to guarantee the compatibility between the limited liability and incentive compatibility constraints. This setting would depict the case in which disaster losses could eliminate any plausible buffer and capital regulation would have only a limited scope[32].

Finally, in order to study a fully fledged macro-prudential regulation, the current model should be embedded in a general equilibrium framework. These analyses are involving and therefore left for future research.

V. Conclusion

This chapter proposed an incentive-based approach to prudential regulation. In a principal agent model I investigated the design of the optimal regulation of banks whose management engages in inefficient activities: shirking and risk-taking. In particular, shirking reduces the bank's profitability while risk-taking raises short-term profits but increases the bank's exposure to tail risk. The manager's actions depend on the compensation scheme offered by the bank's shareholders, which in turn is shaped by the regulations designed by financial authorities.

In the baseline model, the optimal contract maximises the bank's private value. In this framework, excessive financial risks arise due to several factors. The manager's limited liability, the opacity of the bank's strategies, and the high cost of liquidation make "gambling for resurrection" optimal. Furthermore, because of the agency costs of high-risk prevention, risk-taking occurs when the bank is undercapitalised, even if it has not yet reached financial distress. I then demonstrated how the regulatory contract can be implemented through capital regulation and risk-based deposit insurance. The implementation features regulatory forbearance and public recapitalisation when the bank becomes undercapitalised.

The model was then extended to analyse macro-prudential regulatory contracts for systemically important financial institutions. The socially optimal regulation internalises the social cost of risk-taking and the negative externalities from the bank's closure. In this setting, the regulator is able to curb excessive risks by either forcing the shareholders to recapitalise the bank or suspending the banker's speculative activities after a period of poor performance. In the first case, the optimal capital requirement guarantees enough

[32]The case of losses with unbounded support has been analysed by Hugonnier and Morellec (2015) in a model to assess the effects of prudential regulation on banks' insolvency risk.

loss absorbing capacity and hence "skin in the game" to curtail risk-taking incentives. Alternatively the suspension phase, allowing for "strategic shirking", guarantees the bank's continuation under a safe management. Even though profitability is reduced, the bank eventually overcomes financial distress, thereby avoiding liquidation.

Overall, the chapter highlighted the importance of accounting for agency problems and the associated economy-wide effects, in order to produce welfare-enhancing regulations. Several policy implications were delivered in terms of capital regulation and possible crisis prevention mechanisms.

VI. Appendix: Figures

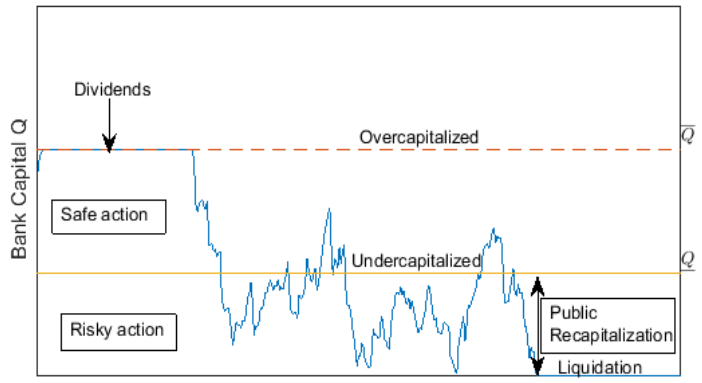


Figure 2.1. Bank's capital under the baseline contract.
The figure shows a sample path of the bank's capital under the baseline regulatory contract. When the bank is overcapitalised, dividends are distributed, keeping the capital value at the boundary $\overline{Q}$. When the bank becomes undercapitalised, public capital is injected after every loss. The bank is liquidated when capital is completely exhausted.

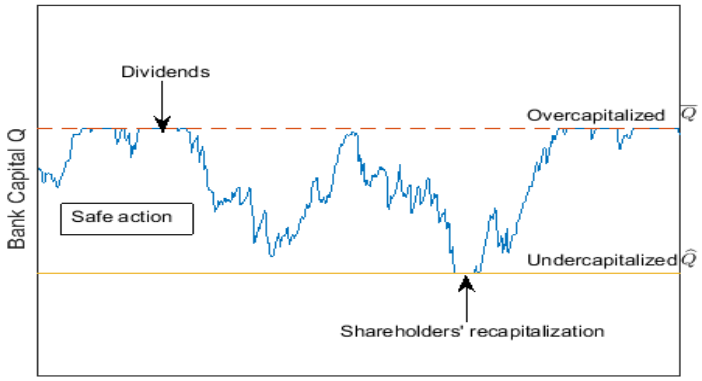


Figure 2.2. Bank's capital under the socially optimal contract.
The figure shows a sample path of the bank's capital under the socially optimal regulatory contract. Under this regulation, when the minimum capital requirement $\widehat{Q}$ is breached, shareholders' equity injections replenish the bank's capital, thereby avoiding liquidation.

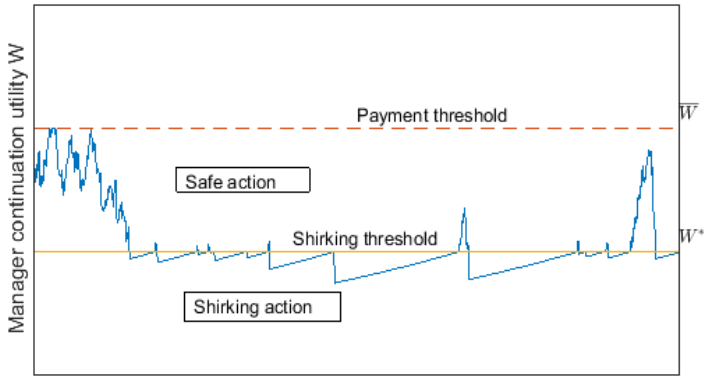


Figure 2.3. Manager's continuation utility under strategic shirking. The figure shows a sample path of the manager's continuation utility under the socially optimal regulatory contract allowing for shirking.

VII. Appendix: Proofs

A. Proof of Proposition (III.1)

The manager's expected payoff conditional on $\mathcal{F}_t$,

$$H_t = \int_0^{t\wedge\tau} \exp(-\gamma s)\left(\mathrm{d}T_s - 1_{\{e_s=e^H\}}B\mathrm{d}s\right) + \exp(-\gamma t)W_{t\wedge\tau},$$

is an $\mathbb{F}$-martingale under $\mathbb{P}^a$. By the Martingale Representation Theorem for jump-diffusion processes (see Theorem 12.11, Hanson (2007)), two $\mathbb{F}$-predictable processes $(\phi_t)_{t\geq 0}$ and $(\psi_t)_{t\geq 0}$ exist such that at any time $\mathrm{d}H_t = -\exp(-\gamma t)\phi_t \mathrm{d}Z_t^a - \exp(-\gamma t)\psi_t \mathrm{d}M_t^a$ where $\mathrm{d}Z_t^a = \mathrm{d}X_t - \delta(e_t, r_t)\mathrm{d}t$ and $\mathrm{d}M_t^a = L_t^r \mathrm{d}N_t - \lambda\mu(r_t)\mathrm{d}t$ are $(\mathbb{F}, \mathbb{P}^a)$-martingales. Differentiating H_t and rearranging the terms yields that the manager's continuation utility evolves following equation (III.4). Suppose that the manager deviates from action $a = (e, r)$ switching to action $\hat{a} = (\hat{e}, \hat{r})$ during the time interval $[0, t]$. The manager's discounted cumulated payoff is given by

$$H_t = \int_0^t e^{-\gamma s}(\mathrm{d}T_s - 1_{\{\hat{e}_s=e^H\}}B\mathrm{d}s) + e^{-\gamma t}W_t^a.$$

By differentiation, it follows that

$$\begin{aligned}
\mathrm{d}H_t &= e^{-\gamma t}(\mathrm{d}T_t - 1_{\{\hat{e}_t=e^H\}}B\mathrm{d}t) - \gamma e^{-\gamma t}W_t^a\mathrm{d}t + e^{-\gamma t}\mathrm{d}W_t^a \\
&= e^{-\gamma t}(\mathrm{d}T_t - 1_{\{\hat{e}_t=e^H\}}B\mathrm{d}t) - \gamma e^{-\gamma t}W_t^a\mathrm{d}t \\
&\quad + e^{-\gamma t}\Big((\gamma W_t^a + \psi_t\lambda\mu(r_t))\mathrm{d}t + 1_{\{e_t=e^H\}}B\mathrm{d}t - \mathrm{d}T_t \\
&\quad + \phi_t(\mathrm{d}X_t - \delta(e_t, r_t)\mathrm{d}t) - \psi_t L_t^r \mathrm{d}N_t\Big) \\
&= e^{-\gamma t}\Big((1_{\{e_t=e^H\}} - 1_{\{\hat{e}_t=e^H\}})B\mathrm{d}t + \phi_t(\mathrm{d}X_t - \delta(e_t, r_t)\mathrm{d}t) - \psi_t\mathrm{d}M_t^a\Big).
\end{aligned}$$

Therefore, if the manager deviates from the safe strategy $a = a^S \equiv (e^H, r^L)$ to shirking $\hat{a} = a^L \equiv (e^L, r^L)$, the above expression becomes

$$\mathrm{d}H_t = e^{-\gamma t}(B - \phi_t\Delta\delta)\mathrm{d}t + e^{-\gamma t}(\phi_t \mathrm{d}Z^{\hat{a}} - \psi_t \mathrm{d}M_t^a).$$

If $\phi_t \geq \frac{B}{\Delta\delta}$ the drift of H_t is non-positive. If instead the manager deviates from the safe strategy $a = a^S \equiv (e^H, r^L)$ to the risky strategy $\hat{a} = a^R \equiv (e^H, r^H)$, it follows that

$$\mathrm{d}H_t = e^{-\gamma t}(\phi_t \alpha - \psi_t \lambda \Delta\mu(r))\mathrm{d}t + e^{-\gamma t}(\phi_t(\mathrm{d}X_t - (\delta + \alpha)\mathrm{d}t) - \psi_t \mathrm{d}M_t^{\hat{a}}).$$

When $\psi_t \geq \frac{\phi_t \alpha}{\lambda \Delta\mu(r)}$ the drift is again non-positive. Therefore the manager's cumulative discounted payoff is a supermartingale, and deviating from action a^S is always sub-optimal. If on the other hand the incentive compatibility constraint (III.5) is violated, that is either $\phi_t < \frac{B}{\Delta\delta}$ or $\psi_t < \frac{\phi_t \alpha}{\lambda \Delta\mu(r)}$, then H_t would be a submartingale contradicting the optimality of action a^S. Analogous arguments lead to the incentive compatibility condition for action $a = a^R \equiv (e^H, r^H)$.

B. Proof of Proposition (III.2)

Differentiating (III.9) follows that the regulator's value function implementing the risky strategy satisfies

$$\frac{1}{2}V'''(W)\beta^2\sigma^2 = (\gamma - \rho)(1 - V'(W)) - V''(W)(\gamma W + B). \tag{VII.1}$$

Evaluating the above expression in $W = 0$ yields

$$\frac{1}{2}V'''(0)\beta^2\sigma^2 = (\gamma - \rho)(1 - V'(0)) - V''(0)B.$$

From the concavity of V and the assumption that $V'(0) \leq 1$, it follows that $V'''(0) > 0$. Moreover, $V'(0) \leq 1$ implies that $V'(W) \leq 1$ for every W. Hence from (VII.1), I obtain $V'''(W) > 0$ for $W \in [0, \underline{W})$. For $W \geq \underline{W}$, the regulatory contract implements the safe strategy. Differentiating (III.11) yields that

$$\frac{1}{2}V'''(W)\beta^2\sigma^2 = (\gamma-\rho)(1-V'(W))-V''(W)(\gamma W+B+\lambda\beta\theta\mu(r^L))+\lambda E^{a^s}[V'(W)-V'(W-\beta\theta l)]. \tag{VII.2}$$

Now suppose that $V'''(W)$ is not always positive. Then there exists a point $W_l = \inf\left\{W \in (\underline{W}, \overline{W}) : V'''(W) \leq 0\right\}$, and by continuity $V'''(W_l) = 0$. Differentiating (VII.2) in W_l and using that $V'''(W_l) = 0$ leads to

$$\frac{1}{2}V^{(4)}(W_l)\beta^2\sigma^2 = -(2\gamma - \rho)V''(W_l) + \lambda E^{a^s}[V''(W_l) - V''(W_l - \beta\theta l)].$$

From the concavity of V, and that $V'''(W) > 0$ for $W \in [0, W_l]$, I obtain $V^{(4)}(W_l) > 0$ contradicting the definition[33] of W_l. Therefore I have proved that $V'''(W) > 0$ for every W. Differentiating $\mathcal{A}(W)$ gives

$$\mathcal{A}'(W) = \lambda E^{a^s}[V'(W) - V'(W - \psi l) - V''(W)\psi l].$$

The convexity of the first order derivative together with the concavity of V imply that $\mathcal{A}'(W) < 0$. Hence $\mathcal{A}(W)$ is strictly decreasing in W. Then $\mathcal{A}(W) = \lambda\Delta\mu(r) - \alpha$ holds at most for one value of W, denoted as $\underline{W}$. If $\mathcal{A}(\overline{W}) < \lambda\Delta\mu(r) - \alpha < \mathcal{A}(\widetilde{W})$, by continuity of $\mathcal{A}$, $\overline{W} > \underline{W} > \widetilde{W}$. If $\mathcal{A}(\overline{W}) \geq \lambda\Delta\mu(r) - \alpha$, then risk-taking is always optimal. That is $a^* = (e^H, r^H)$ for every $W \in [0, \overline{W}]$, with $\underline{W} = \overline{W}$. Finally if $\mathcal{A}(\beta\theta\bar{l}) \leq \lambda\Delta\mu(r) - \alpha$, by limited liability, I must set $\underline{W} = \beta\theta\bar{l}$.

C. Proof of Proposition (III.3)

First I prove that $V(W)$ is strictly concave on $[0, \overline{W})$. Notice that evaluating (III.9) at $W = 0$ yields $\rho V(0) = \delta + \Delta\delta + \alpha - \lambda\mu(r^H) - B + V'(0)B + \frac{1}{2}V''(0)\beta^2\sigma^2$. Using that $V(0) = R$, I obtain $\frac{1}{2}V''(0)\beta^2\sigma^2 = \rho R - (\delta + \Delta\delta + \alpha - \lambda\mu(r^H) - B) - V'(0)B$, which is negative by assumption (II.2). By contradiction, assume that V is not concave on $(0, \overline{W})$. Then, there exists a point $W_c = \inf\left\{W \in (0, \overline{W}) : V''(W) \geq 0\right\}$. By continuity of V, it follows that $V''(W_c) = 0$. There are two possible cases:

i) $W_c < \underline{W}$

ii) $W_c \geq \underline{W}$

I start by analysing the first case. Taking the derivative of the regulator's value function implementing the risky action (III.9) for $W = W_c$ I obtain

$$\frac{1}{2}V'''(W_c)\beta^2\sigma^2 = (\gamma - \rho)(1 - V'(W_c)). \tag{VII.3}$$

Evaluating (III.9) at W_c yields

$$V'(W_c) = 1 + \frac{\rho(V(W_c) - W_c) - (\delta + \Delta\delta + \alpha - \lambda\mu(r^H))}{\gamma W_c + B}.$$

Hence, if (and only if) $V(W_c) - W_c = S(W_c) > \frac{\delta + \Delta\delta + \alpha - \lambda\mu(r^H)}{\rho}$ it holds that $V'(W_c) > 1$. Then, from (VII.3) follows that $V'''(W_c) < 0$, contradicting the definition of W_c.

[33] Take an $\epsilon > 0$, then $V'''(W_l - \epsilon) > V'''(W_l) = 0$, which contradicts the definition of W_l.

Conversely, if $S(W_c) \leq \frac{\delta+\Delta\delta+\alpha-\lambda\mu(r^H)}{\rho}$ then $V'(W_c) \leq 1$, implying that $V'''(W_c) \geq 0$. Then, for a small enough $\epsilon > 0$, $V''(W_c+\epsilon) > V''(W_c)$ and $V'(W_c+\epsilon) > V'(W_c)$. From (III.9), it follows that $V(W_c+\epsilon) + (W_c+\epsilon) = S(W_c+\epsilon) > V(W_c) + W_c = S(W_c)$ contradicting $V'(W_c) \leq 1$.

Now I analyse the second case $W_c \geq \underline{W}$. Differentiating the regulator's value function implementing the safe action (III.11) in W_c leads to

$$\frac{1}{2}V'''(W_c)\beta^2\sigma^2 = (\gamma-\rho)(1-V'(W_c)) + \lambda E^{a^S}[V'(W_c) - V'(W_c-\beta\theta l)]. \tag{VII.4}$$

The second term on the right-hand side is negative because $V''(W) \leq 0$ for $W \in [0, W_c]$. Then, if $V'(W_c) \geq 1$, it follows that $V'''(W_c) < 0$, contradicting the definition of W_c. Assume instead that $V'(W_c) < 1$. By definition of W_c, $V'''(W_c) \geq 0$ and, for a small enough $\epsilon > 0$, $V'(W_c+\epsilon) \geq V'(W_c)$ and $V''(W_c+\epsilon) \geq 0$. Then, by concavity of V on $[0, W_c]$, it follows that

$$\mathcal{A}'V(W_c) = -\lambda E^{a^S}\Big[V''(W_c)\beta\theta l + V'(W_c-\beta\theta l) - V'(W_c)\Big] \leq 0.$$

Therefore $\mathcal{A}V(W_c+\epsilon) \leq \mathcal{A}V(W_c)$. This implies, by (III.11), that $V(W_c+\epsilon) - (W_c+\epsilon) = S(W_c+\epsilon) \geq V(W_c) - W_c = S(W_c)$ contradicting $S'(W_c) = V'(W_c) - 1 < 0$. Thus the concavity of $V(W)$ follows. Consequently, by definition of $\overline{W} = \min\{W \geq 0 | V'(W) = 0\}$, $V'(W) > 0$ for every $W < \overline{W}$.

I will now show that, under any incentive compatible contract, the regulator's utility can achieve at most the value $V(W)$ given by (III.14):

$$V(W_0) \geq E^a\left[\int_0^\tau e^{-\rho t}(\mathrm{d}Y_t - \mathrm{d}T_t) + e^{-\rho\tau}R\right] + E^a\left[\int_0^\tau e^{-\gamma t}(\mathrm{d}T_t - 1_{\{e_t=e^H\}}B\mathrm{d}t)\right].$$

Further, I will prove that the regulator's utility attains the upper bound $V(W_0)$ only under the contract described in proposition (III.3), which is therefore optimal. Define the shareholders' and banker's discounted cumulated payoffs respectively as

$$G_t = \int_0^t e^{-\rho s}(\mathrm{d}Y_s - \mathrm{d}T_s) + e^{-\rho t}(V(W_t) - W_t),$$

and

$$H_t = \int_0^t e^{-\gamma s}(\mathrm{d}T_s - 1_{\{e_s=e^H\}}B\mathrm{d}s) + e^{-\gamma t}W_t.$$

Notice that, by definition, $G_0 = V(W_0) - W_0$ and $H_0 = W_0$. As I showed in the proof of proposition (III.5), under any incentive compatible contract the process H_t is a supermartingale, and it is a martingale under the optimal contract. Under any incentive compatible contract, the banker's continuation utility evolves as follows

$$\mathrm{d}W_t = (\gamma W_t + B + \psi_t \lambda \mu(r_t))\mathrm{d}t - \mathrm{d}T_t + \phi_t(\mathrm{d}X_t - \delta(e_t, r_t)\mathrm{d}t) - \psi_t L_t^r \mathrm{d}N_t.$$

Applying Ito's Lemma gives

$$\begin{aligned} e^{\rho t}\mathrm{d}G_t = \Big(& -\rho V(W_t) + \delta(e_t, r_t) - \lambda\mu(r_t) - B + (\rho - \gamma)W_t \mathrm{d}t + V'(W_t)(\gamma W_t + B + \lambda\psi\mu(r_t)) \\ & + \frac{1}{2}V''(W_t)\phi^2 + \lambda E^a[V(W_t - \psi l) - V(W_t)]\Big)\mathrm{d}t - V'(W_t)\mathrm{d}T_t \\ & + \sigma \mathrm{d}Z_t^a((1-\phi) + \phi V'(W_t)) - (1+\psi)\mathrm{d}M_t^a + \Delta J_t^a \end{aligned}$$

where $\mathrm{d}M_t^a = L_t^r \mathrm{d}N_t - \lambda\mu(r_t)\mathrm{d}t$ and $\Delta J_t^a = [V(W_t - \psi l) - V(W_t)]\mathrm{d}N_t - \lambda E^a[V(W_t - \psi l) - V(W_t)]\mathrm{d}t$ are $(\mathbb{F}, \mathbb{P}^a)$-martingales. Observe that the drift is non-positive by the HJB equation (III.12). Moreover, $V'(W_t)\mathrm{d}T_t \geq 0$ because the banker's transfers are non-negative and $V'(W_t) \geq 0$. Therefore, under any incentive compatible contract, the process G_t is a supermartingale. Under the optimal contract of proposition (III.3), $V'(W_t)\mathrm{d}T_t = 0$ and the drift of G_t is equal to

$$-\rho V(W) + \delta + \Delta\delta + \alpha - \lambda\mu(r^H) - B + (\rho - \gamma)W + V'(W)\gamma W + \frac{1}{2}V''(W)\beta^2\sigma^2$$

for $W_t \in [0, \underline{W})$ and to

$$\begin{aligned} & -\rho V(W) + \delta + \Delta\delta - \lambda\mu(r^L) - B + (\rho - \gamma)W + V'(W)(\gamma W + \lambda\beta\theta\mu(r^L)) + \frac{1}{2}V''(W)\beta^2\sigma^2 \\ & + \lambda E^{a^S}[V(W - \beta\theta l)] - \lambda V(W) \end{aligned}$$

for $W_t \in [\underline{W}, \overline{W}]$. In both cases the drift is zero, from (III.9) and (III.11) respectively, implying that the process G_t is a martingale. Now I will consider the regulator's utility

from any incentive compatible contract

$$
\begin{aligned}
&E^a\left[\int_0^\tau e^{-\rho t}(\mathrm{d}Y_t-\mathrm{d}T_t)+e^{-\rho\tau}R\right]+E^a\left[\int_0^\tau e^{-\gamma t}(\mathrm{d}T_t-1_{\{e=e^H\}}B\mathrm{d}t)\right]\\
&=E^a\left[G_{t\wedge\tau}+H_{t\wedge\tau}\right]+E^a\left[1_{\{t\leq\tau\}}\left(\int_t^\tau e^{-\rho s}(\mathrm{d}Y_s-\mathrm{d}T_s)+e^{-\rho\tau}R-e^{-\rho t}S(W_t)\right)\right]\\
&+E^a\left[1_{\{t\leq\tau\}}\left(\int_t^\tau e^{-\gamma s}(\mathrm{d}T_s-1_{\{e_t=e^H\}}B\mathrm{d}s)-e^{-\gamma t}W_t\right)\right]\\
&=E^a\left[G_{t\wedge\tau}+H_{t\wedge\tau}\right]+e^{-\rho t}E^a\left[1_{\{t\leq\tau\}}\left(E_t^a\left[\int_t^\tau e^{-\rho(s-t)}(\mathrm{d}Y_s-\mathrm{d}T_s)+e^{-\rho\tau}R\right]-S(W_t)\right)\right]\\
&+e^{-\gamma t}E^a\left[1_{\{t\leq\tau\}}\left(E_t^a\left[\int_t^\tau e^{-\gamma(s-t)}(\mathrm{d}T_s-1_{\{e_t=e^H\}}B\mathrm{d}s)\right]-W_t\right)\right].
\end{aligned}
$$

Letting t to ∞ the second and third terms vanish, while the first term tends to:

$$G_\tau+H_\tau\leq G_0+H_0=V(W_0),$$

where the inequality follows from $(G_t)_{t\geq0}$ and $(H_t)_{t\geq0}$ being supermatingales. Analogously, since under the optimal contract $(G_t)_{t\geq0}$ and $(H_t)_{t\geq0}$ are martingales, the upper bound $V(W_0)$ is achieved with equality. Thus the result holds.

D. Proof of Proposition (IV.1)

The social value function $F(W)$ satisfies the following HJB equation

$$\max\left\{\delta+\Delta\delta-\lambda\mu(r^L)-B+\mathcal{L}_SF(W)-\rho F(W),\xi_1F'(W)+(1-\xi_0-\xi_1)\right\}=0 \quad \text{(VII.5)}$$

where $\mathcal{L}_S$ is the "safe operator" defined as (III.10). For $W<\widehat{W}$, F is linear and satisfies the second part of the equation. For $W\geq\widehat{W}$, the social value function solves the first part of the variational inequality (VII.5). As shown in the proof of proposition (III.3), the solution of this equation is concave, given a concave initial history function.

Analogously to the proof of proposition (III.3), I will now show that, under any incentive compatible contract, the regulator's utility can achieve at most the value $F(W)$ given by (IV.1), that is

$$
\begin{aligned}
F(W_0)\geq&E^a\left[\int_0^\infty e^{-\rho t}(\mathrm{d}Y_t+(1-\xi_0)\mathrm{d}I_t-\mathrm{d}T_t)\right]+E^a\left[\int_0^\infty e^{-\gamma t}(\mathrm{d}T_t-1_{\{e_t=e^H\}}B\mathrm{d}t-\xi_1\mathrm{d}I_t)\right]\\
&-E^a\left[\int_0^\infty e^{-\rho t}1_{\{r_t=r^H\}}C\mathrm{d}N_t\right].
\end{aligned}
$$

Define the investors' and the banker's discounted cumulated payoffs respectively as

$$D_t = \int_0^t e^{-\rho s}(\mathrm{d}Y_s + (1-\xi_0)\mathrm{d}I_t - \mathrm{d}T_s) + e^{-\rho t}(F(W_t) - W_t)$$

and

$$E_t = \int_0^t e^{-\gamma s}(\mathrm{d}T_s - 1_{\{e_s = e^H\}}B - \xi_1 \mathrm{d}I_t \mathrm{d}s) + e^{-\gamma t}W_t.$$

Under any incentive compatible contract the banker's continuation utility evolves as follows

$$\mathrm{d}W_t = (\gamma W_t + B + \psi_t \lambda \mu(r_t))\mathrm{d}t - \mathrm{d}T_t + \phi_t(\mathrm{d}X_t - \delta(e_t, r_t)\mathrm{d}t) - \psi_t L_t^r \mathrm{d}N_t + \xi_1 \mathrm{d}I_t.$$

Applying Ito's Lemma gives

$$\begin{aligned} e^{\rho t}\mathrm{d}D_t = \Big(& -\rho F(W_t) + \delta(e_t, r_t) - \lambda\mu(r_t) - B + (\rho - \gamma)W_t \mathrm{d}t + F'(W_t)(\gamma W_t + B + \lambda\psi\mu(r_t)) \\ & + \frac{1}{2}F''(W_t)\phi^2 + \lambda E^a[F(W_t - \psi l) - V(W_t)]\Big)\mathrm{d}t - F'(W_t)\mathrm{d}T_t + \mathrm{d}I_t(\xi_1 F'(W) + 1 - \xi_0 - \xi_1) \\ & + \sigma \mathrm{d}Z_t^a((1-\phi) + \phi F'(W_t)) - (1+\psi)\mathrm{d}M_t^a + \Delta J_t^a, \end{aligned}$$

where $\mathrm{d}M_t^a = L_t^r \mathrm{d}N_t - \lambda\mu(r_t)\mathrm{d}t$ and $\Delta J_t^a = [V(W_t - \psi l) - V(W_t)]\mathrm{d}N_t - \lambda E^a[V(W_t - \psi l) - V(W_t)]\mathrm{d}t$ are $(\mathbb{F}, \mathbb{P}^a)$-martingales. Observe that the drift is non-positive by the HJB equation (VII.5), $F'(W_t)\mathrm{d}T_t \geq 0$ and $\mathrm{d}I_t(\xi_1 F'(W) + 1 - \xi_0 - \xi_1) \geq 0$. The remaining terms are $(\mathbb{F}, \mathbb{P}^a)$-martingales. Hence the process D_t is a supermartingale. Under the optimal contract defined in proposition (IV.1), $F'(W_t)\mathrm{d}T_t = 0$ and $\mathrm{d}I_t(\xi_1 F'(W) + 1 - \xi_0 - \xi_1) = 0$. Then, for $W_t \in [\widehat{W}, \overline{W}]$, the drift of D_t is equal to

$$\begin{aligned} & -\rho F(W) + \delta + \Delta\delta - \lambda\mu(r^L) - B + (\rho - \gamma)W + F'(W)(\gamma W + B + \lambda\beta\theta\mu(r^L)) + \frac{1}{2}F''(W)\beta^2\sigma^2 \\ & + \lambda E^{a^S}[F(W - \beta\theta l)] - \lambda F(W) = 0, \end{aligned}$$

by (III.11). Therefore, under the optimal contract, the process D_t is a martingale. Now I

will consider the regulator's utility from any incentive compatible contract

$$
\begin{aligned}
&E^a\left[\int_0^\infty e^{-\rho t}(\mathrm{d}Y_t + (1-\xi_0)\mathrm{d}I_t - \mathrm{d}T_t)\right] + E^a\left[\int_0^\infty e^{-\gamma t}(\mathrm{d}T_t - 1_{\{e=e^H\}}B\mathrm{d}t - \xi_1\mathrm{d}I_t)\right] \\
&- E^a\left[\int_0^\infty e^{-\rho t}C1_{\{r_t=r^H\}}\mathrm{d}N_t\right] \\
&= E^a\Big[D_t + E_t\Big] + E^a\left[\left(\int_t^\infty e^{-\rho s}(\mathrm{d}Y_s + (1-\xi_0)\mathrm{d}I_t - \mathrm{d}T_s) - e^{-\rho t}S(W_t)\right)\right] \\
&+ E^a\left[\left(\int_t^\infty e^{-\gamma s}(\mathrm{d}T_s - 1_{\{e_t=e^H\}}B\mathrm{d}s - \xi_1\mathrm{d}I_t) - e^{-\gamma t}W_t\right)\right] - E^a\left[\int_0^\infty e^{-\rho t}C1_{\{r_t=r^H\}}\mathrm{d}N_t\right] \\
&\leq E^a\Big[D_t + E_t\Big] + e^{-\rho t}E^a\left[\left(E_t^a\left[\int_t^\infty e^{-\rho(s-t)}(\mathrm{d}Y_s - \mathrm{d}T_s - (1-\xi_0)\mathrm{d}I_s) + e^{-\rho\tau}R\right] - S(W_t)\right)\right] \\
&+ e^{-\gamma t}E^a\left[\left(E_t^a\left[\int_t^\infty e^{-\gamma(s-t)}(\mathrm{d}T_s - 1_{\{e_t=e^H\}}B\mathrm{d}s - \xi_1\mathrm{d}I_s)\right] - W_t\right)\right].
\end{aligned}
$$

When taking the limit for t that goes to ∞, the second and term terms vanish, leaving the regulator's utility bounded above by the value $D_0 + E_0 = F(W_0)$, using that $(D_t)_{t\geq 0}$ and $(E_t)_{t\geq 0}$ are supermatingales. Under the optimal contract instead $(D_t)_{t\geq 0}$ and $(E_t)_{t\geq 0}$ are martingales, hence the upper bound $F(W_0)$ is achieved with equality. Therefore the contract described in proposition (IV.1) is optimal.

Chapter 3

A FAVAR model to explore the impact of monetary policy on euro area banks' risk-taking

This chapter seeks to contribute empirical evidence to the ongoing debate among policy makers, academics, and the public, concerning the extent to which monetary policy and the financial stability of banks depend on each other. Recently, renewed considerable attention has been given to the impact of monetary policy on bank risk-taking, the so-called "risk-taking channel" of monetary transmission. The channel gains particular relevance in the context of a low interest rate environment (LIRE), as in the aftermath of the global financial crisis.

Despite the central role of the banking sector in the European Economic and Monetary Union (EMU), empirical evidence of the intricate relation between monetary policy and bank risk-taking is still limited. Only a few studies employ bank-level indicators of distress and risk-taking to link it to monetary policy. Moreover, all of the existing works have confined the analysis to conventional monetary measures. Due to the interdependence between the behaviour of individual agents and macroeconomic performances, studying the feedback mechanisms between bank-level indicators of risk and monetary policy is essential for models that serve macro-prudential and stress-testing purposes.

I address this research question by using an integrated micro-macro approach, which allows me to use information from a large number of bank-level time-series by distilling it into a small number of estimated factors. The data set comprises banks' expected default frequencies (EDFs) and balance sheet variables from a unique monthly data set of banks operating in the euro area. In particular, I include loan volumes and spreads to households and non-financial corporations, distinguishing between spreads on new business loans to small and large firms. Furthermore, I divide banks into groups depending on their total capital ratio. In this way I am able to disentangle aggregate effects from those that are

[0]This chapter presents the results of the research project on which I worked at the European Central Bank (ECB) under the supervision of Cosimo Pancaro, Costanza Rodriguez D'Acri and Peter Welz.

due to specific banks' characteristics.

The results show that, following an expansionary monetary shock, the median bank's lending rises[1] and loan spreads drop across all loan categories, indicating that supply-driven effects have been identified. At the same time the median bank's EDF decreases. Overall, I have not found conclusive evidence of excessive risks arising in the banking sector considered as a whole.

Heterogeneous results arise when I divide banks based on their degree of capitalisation. Specifically, the probability of distress of banks with low levels of capital does not decrease significantly. Most importantly, these banks increase business lending while tightening spreads on new business loans to small firms more than to large corporates. Due to the higher credit risk involved in small business lending, I conclude that a risk-taking channel arises for poorly capitalized banks.

The results are in line with the literature on the "bank lending channel" of monetary policy transmission, which argues that undercapitalised banks, facing tighter constraints in external uninsured sources of financing, tend to be more exposed to monetary shocks[2]. Moreover, my findings confirm that banks more affected by agency problems, such as poorly capitalized banks, are more prone to risk-taking. Indeed, low levels of capital, providing shareholders with less "skin in the game", increase their limited liability-driven incentives to take excessive risks.

I. Related literature

This chapter contributes to the literature on the impact of monetary policy on banks' risk. Due to the interaction of several pass-through effects, this relation is multifaceted and challenging to characterise. One stream of literature emphasises the fact that a reduction in interest rates leads to an increase in risk-taking. Borio and Zhu (2012) argue that the "risk-taking channel" of monetary policy operates in at least three different ways. First, lower interest rates increase asset prices, collateral values and profits, leading to an increase in risk tolerance[3]. Second, in a low interest rate environment, banks chasing higher returns may switch to riskier investment strategies and expand lending: the so-called "search for yield" effect (Rajan (2005)). Finally, the expectation of accommodative monetary policy

[1]However, the rise in business lending is non-significant and features an initial contraction.

[2]I refer for instance to Kishan and Opiela (2000), Gambacorta and Mistrulli (2004), and Altunbas et al. (2002), who find that lending of banks with low levels of capital responds significantly to interest rate changes.

[3]Empirically Bekaert et al. (2013), using a decomposition of the VIX, document that loose monetary policy significantly decrease risk aversion in the medium and long term.

during bad times can produce an insurance effect exacerbating moral hazard problems and in particular, risk-taking[4]. Furthermore, several additional explanations of why banks' risk-taking could be fuelled by low rates have been suggested, such as a relaxation of value-at-risk constraints (Adrian and Shin (2010)), or through habit formations, as first noticed by Campbell and Cochrane (1999).

On the other hand, under some conditions a lower policy rate can in fact reduce banks' risk. First, a decline in interest rates increases the value of borrowers' collateral and reduces their interest payments. Therefore lower rates increase the probability of repayment, in turn reducing the credit risk of outstanding loans. Second, lower rates increase the value of bank assets and therefore reduce bank's default probability. Finally, a policy rate cut reduces the cost of banks' liabilities and increases the share of inside equity, potentially increasing monitoring and mitigating the moral hazard problem between banks and depositors (Dell'Ariccia et al. (2014)).

The lack of a clear consensus and a sound theoretical foundation for the complex relation between monetary policy and financial stability suggests that the risk-taking channel requires further analysis. Recent studies have tried to empirically assess this channel. However, the countervailing effects of lower rates on banks' risk, i.e. the decrease in realised risk and bank's incentives to take on new risk, substantially complicate the analysis. The choice of risk measures' is therefore of crucial importance.

Some papers focus on the probability of defaults as a proxy of bank risk. For instance, DeGraeve et al. (2008) study distress probabilities of individual German banks. They find an increase in banks' probability of distress following a monetary tightening. Responses differ across banking groups, with larger increases for small cooperative and poorly capitalised banks.

Altunbas et al. (2010) analyse the effect of low policy rates on EDFs, for listed banks in the European Union and the US. To tackle the identification problem, they examine both quarterly changes in policy rate and the monetary policy stance (measured as deviations of the interest rate from a historical benchmark). They find that default probabilities decrease after an interest rate cut, while they increase when interest rates are below the benchmark. The authors conclude that keeping low interest rates for a prolonged period of time spurs banks risk-taking.

In order to identify risk-taking, Buch et al. (2014b) adopt two different indicators of banks' risk. The first measures the credit quality of outstanding loans, and therefore rep-

[4]Diamond and Rajan (2009) show that anticipated central bank interventions, distort the price of liquidity and banks incentives, encouraging an increase in leverage.

resents backward-looking risk. Conversely, the second measures the share of non-interest income in total income and is more forward-looking. Their main results show that, following expansionary monetary shocks, backward-looking risk decreases while forward-looking risk rises.

Another stream of literature focuses alternatively on the impact of low interest rates on credit risk-taking and banks' lending behaviour. Buch et al. (2014a), using the Federal Reserve's Survey of Terms and Business Lending, investigate the response of loans divided by risk categories and banking groups. They show that, following an easing of monetary policy, small domestic banks significantly increase loans to high-risk borrowers without charging higher risk premia. The results do not hold for large domestic and foreign banks.

Using surveys on banks' lending standards for the US and Euro area, Maddaloni and Peydró (2011), find that low short-term interest rates lead to a softening of lending standards, especially when supervision standards for bank capital are weak.

Additional studies show the dependence of the risk-taking effects on bank capital. For instance, Ioannidou et al. (2015) and Jimenez et al. (2014), studying exhaustive loan-level data from Bolivia and Spain credit registers respectively, show that lower interest rates induce poorly capitalized banks to grant more risky loans to lower quality borrowers without compensating with higher collateral requirements or returns. Accordingly, Dell'Ariccia et al. (2013) and Paligorova and Santos (2012) find a stronger risk-taking channel for banks with low levels of capital.

In this chapter I combine these two streams of literature, considering both banks' default probabilities as indicators of distress and outstanding risk, and lending activity to assess credit risk-taking. Similarly[5] to Buch et al. (2014a,b), I adopt a factor-augmented vector autoregressive (FAVAR) model, which allows me to integrate macroeconomic factors with bank-level data. First noticed by Bernanke et al. (2005), this framework can easily accommodate large sets of information, overcoming one of the main criticism of low dimensional vector autoregressive (VAR) models. As a result, I am able to characterise the impulse responses of individual banks' risk variables[6]. A by-product of this estimation methodology is the ability to incorporate the feedback effects between macroeconomic shocks and banking variables, hence accounting for the endogeneity of monetary policy.

Moreover, this setup is well suited for modelling monetary shocks without the short-

[5]However, I recur to a Bayesian estimation of the model by making use of Gibbs sampling. The analysis is repeated using a two-step estimation method, as is done in Bernanke et al. (2005) and Buch et al. (2014a,b). The results, as reported in Section (5.D), remain robust.

[6]Previous works using FAVAR models have mainly focused on macroeconomic data sets. See for instance Boivin et al. (2009) and Mumtaz and Surico (2009).

coming of imposing specific restrictions on the policy instruments. In particular, policy innovations are specified as orthogonal shocks to the shadow rate estimated by Wu and Xia (2015). The shadow rate, taking into account unconventional monetary policy, represents a measure of the monetary stance at the zero lower bound. To the best of my knowledge, this work is among the first to investigate the effects of non-conventional monetary policy on the bank risk-taking channel in the low interest rate period following the crisis[7].

The rest of the chapter is organised as follows. In the next section the dataset is described. Section 3 presents the methodological approach; this is then followed by an overview and discussion of the results in Section 4. The last section concludes the chapter.

II. Data

This analysis relies on the use of a balanced panel dataset composed of monthly series for the period running from March 2009 to March 2015. The dataset comprises both macroeconomic data, extracted from SDW, and bank-level data including banks' EDFs, i.e. market-based forward-looking indicators of risk, provided by Moody's KMV and banks' balance sheet data stemming from the Individual MFI (Monetary Financial Institutions) Balance Sheet Items (IBSI) ECB database[8]. As regards macroeconomic variables, the dataset comprises the euro area industrial production and HICP inflation and the euro area shadow rate produced by Wu and Xia (2015) and made available on Wu's website[9].

Since the onset of the global financial crisis, the ECB has implemented a number of monetary policy measures aimed at stabilising the financial system and avoiding its collapse. In particular, since October 2008 the ECB has combined unconventional monetary measures such as the change in tender procedure to fixed rate full allotment, with standard monetary policy instruments[10]. Consequently, since 2009 the euro area has been characterised by a low interest rate environment.

During the time period that the data span, money market rates have been close to the lower bound, ceasing to constitute an informative policy measure. This dynamic of the euro area policy rate leads to rely on an alternative monetary policy measure that also incorporates the effects of unconventional monetary policies. The shadow rate produced

[7]Few recent studies have analysed the effects of non-conventional monetary policy, focusing on different aspects of interest. For instance Krippner et al. (2015) concentrate on the interest rate pass-through of monetary policy during the sovereign-debt crisis, while Gambacorta et al. (2014) focus on the macroeconomic effects.

[8]The data contained in the Individual MFI Balance Sheet Items ECB database are confidential and are only available at the individual institution level from June 2007.

[9]`http://faculty.chicagobooth.edu/jing.wu/research/data/WX.html`

[10]For instance, the ECB cut the MRO interest rate from 4.25 pre-crisis to 1.5 in March 2009.

in Wu and Xia (2015) is consistent with zero lower bound periods during which the rate becomes negative and continues to display meaningful variation[11]. Figure (3.1) shows the shadow rate and the ECB's main policy rates from September 2004 until March 2015.

< Figure 3.1 about here >

With regard to the banks' variables, the dataset encompasses 40 listed banks operating in 11 euro area countries[12]: Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, the Netherlands and Portugal. In particular, as a measure of bank risk, I include the one-year ahead EDF from Moody's KMV. EDFs are estimated according to Merton's model (Merton (1974)) and provide a measure for the probability of default within a defined interval of time. Specifically, the EDF's value, expressed as a percentage, is determined by combining both market and banks' balance sheet information. EDF measures are widely used by researchers, regulators and financial institutions and their performance is comparable to other risk indicators such as credit default swaps (CDS) spreads. However, as shown in figure (3.2), EDFs have probably underestimated banks' risk in the pre-crisis period. Although they are advocated as a forward-looking indicator, I consider EDFs as a measure of the outstanding banks' risk.

< Figure 3.2 about here >

To capture banks' incentives to take on new risk, I include information on volumes of loans to households (HH) and to non-financial corporations (NFC) together with spreads on newly issued loans. Including both loan volumes and prices allows me to disentangle supply- from demand-driven effects. The data are extracted from the ECB monthly balance sheet statistics for MFIs. All loan series are seasonally adjusted using the X12 seasonal adjustment procedure. To construct a measure of loans' growth rates excluding the effect of non-transaction related changes, I use the rate of change of the index of notional stocks[13].

Loan spreads are measured as the difference between the composite lending rates (per product) and composite deposit rates. Disentangling the bank's lending rates from its funding costs leaves me with a measure of the bank's margin. This measure reflects the

[11]The Wu-Xia shadow rate is based on an analytical approximation of the shadow rate term structure model, first introduced by Black (1995). In Wu and Xia (2015) the authors provide evidence that their estimated shadow rate represents a powerful measure of the monetary stance when the federal funds rate is stuck at the zero lower bound.

[12]The sample coverage of the respective host country is AT 21%, BE 38%, DE 14%, ES 56%, FI 74%, FR 65%, GR 102%, IE 45%, IT 44%, NL 28%, PT 37%.

[13]The index of notional stocks is calculated following Section 4.3 of ECB (2012).

risk premium that banks charge and thus their price setting (risk pricing) behaviour. I differentiate between spreads for small loans (under one million) and large loans (over one millions), which can be assumed as proxies for lending to small and large companies, respectively[14].

Small business lending involves higher credit risk due to stronger asymmetric information and opaqueness, which in turn exacerbate moral hazard and adverse selection problems. As a consequence, small firms face higher credit constraints and are more dependent on bank financing. The relationship lending literature supports this prediction. As is for instance shown by Berger and Udell (1995), Berger and Udell (2002), and Hernández-Cánovas and Martínez-Solano (2010), the bank-borrower relationship, by providing soft information, attenuates issues associated with asymmetric information. The greater borrower risk and informational frictions of small firms are reflected in the design of loans' contract terms, such as lending rates[15]. As shown in figure (3.3), banks tend to demand higher risk premia from small firms compared to large corporations. The difference between the two loan categories is consistent over time.

< Figure 3.3 about here >

In order to gather the aggregate response of the entire banking sector, I include in the data set the median of the entire sample for each of the variables under analysis (EDFs, loan volumes and spreads). Moreover, banks are divided based on their total capital ratio[16] at the beginning of the sample. The data set comprises the medians of every variable for the three groups of banks: high, medium, and low capital. The final sample comprises $N = 264$ series of banking variables. To guarantee stationarity, loans and EDFs are log differenced while loan spreads are first differenced. The stationary series are then standardised. Table (3.1) reports descriptive statistics on EDFs, loans' growth and loans' spreads for the entire sample and the three different groups of banks.

< Table 3.1 about here >

Over the time period analysed, there has been a reduction in loan volumes, with the reaction being stronger for business lending. The responses differ across types of banks,

[14]This is a common assumption in the literature on the lending channel of monetary policy transmission. See for instance Al-Eyd and Berkmen (2013) and Holton and Rodriguez d'Acri (2015) who document the more sluggish transmission of monetary policy changes to small loans compared to larger loans.

[15]Alternatively, banks have also employed non-price elements of the loan contract to mitigate asymmetric information problems. For instance, as documented by Ortiz-Molina and Penas (2008), stricter collateral requirements and shorter loan maturities have been widely adopted for small business lending.

[16]The data on banks' capital ratio were collected from Bloomberg. The results from the analysis are robust to measures of capital made available from alternative data providers such as SNL Financial.

with a higher decrease of business lending by high and medium capital banks. Loan spreads reflect borrower risk with higher spreads for small loans, as expected from the previous discussion. Across the different groups, low capital banks are the ones charging the highest spreads. Understanding the extent to which the observed patterns reflect a response to loose monetary policy is the object of this study.

In particular, the risk-taking identification strategy adopted consists of analysing banks' loan volumes and rates responses to an expansionary monetary shock, comparing the reactions of spreads across loan categories. Following a loosening in monetary policy, an increase in risk propensity should lead to a decrease in loans' spreads. A contemporary increase in loan volumes implies that supply effects dominate. Moreover, if banks' responses differ across loan size categories, with a larger decrease in spreads for small loans, I conclude that excessive risks have been taken.

III. Methodology

In this work, the FAVAR framework, originally proposed by Bernanke et al. (2005), is adopted. The model can efficiently use the information from a large number of time-series by distilling it into a small number of estimated factors. I denote by r_t, i_t and π_t the shadow rate, industrial production, and HICP, respectively. Then the 3×1 vector $Y_t = \begin{bmatrix} i_t & \pi_t & r_t \end{bmatrix}$ represents the observable macroeconomic variables of the model. I assume the joint dynamics of macroeconomic variables and banking factors follow a VAR(p) model, as indicated in the following transition equation:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \epsilon_t, \tag{III.1}$$

where $\phi(L)$ is a conformable lag polynomial of finite order p, and the error term $\epsilon_t \sim N(0, \Omega)$. The K factors F_t are unobservable and are extracted from a panel X_t containing the $N = 264$ euro area banks' EDFs and lending (volumes and spreads) time-series. Therefore, the factors are assumed to capture the dynamics of the euro area banks' risk and lending variables. Thus, they are denominated as "banking factors". The banking data X_t are assumed to be related to the factors F_t and the macroeconomic data Y_t through the following observation equation:

$$X_t = \lambda^F F_t + \lambda^Y Y_t + e_t \tag{III.2}$$

where λ^F and λ^Y are $N \times K$ and $N \times 3$ matrices of factor loadings respectively and the vector $e_t \sim N(0, R)$ contains $N \times 1$ idiosyncratic components.

Equations (III.1) and (III.2) were jointly estimated using Bayesian likelihood and Gibbs sampling techniques[17]. In particular, following Bernanke et al. (2005), I set a Normal prior for the factor loadings and the coefficients of the transition equation, an inverse Gamma for the variance of the idiosyncratic errors e_t, and an independent Normal inverse Wishart prior for the error ϵ_t covariance matrix Ω. The latent factors were initialised using the first K principal components extracted from the banking data X_t. Conditional on the factors, the model parameters were sampled from their posteriors. Using the Carter and Kohn algorithm for state space models, the factors were then sampled conditionally on the most recent draws of the model parameters. The Gibbs sampler was iterated 800 times and, once past the burn-in period (400 iterations), I calculated the impulse responses of the macroeconomic variables and bank-level data.

Although computationally intensive, this method allowed me to fully exploit the factor structure of the data. As the factors were estimated by both the observation and the transition equation, further restrictions had to be imposed in order to uniquely identify the factors against rotational indeterminacy. In particular, a sufficient condition for identification was to require that the first K variables in X_t did not react contemporaneously to changes in Y_t[18].

Following Bernanke et al. (2005), those variables are denoted as "slow-moving" and the remaining ones as "fast-moving". Precisely, in this model, loans to NFC are denoted as the "slow-moving" variables, while the remaining variables (loans to HH, loans spreads and EDFs) are considered as "fast-moving". Expected default frequencies and loan spreads are financial variables and can be therefore considered highly responsive to contemporaneous macroeconomic shocks.

Less intuitive is the reason why volume of lending to non-financial corporations should be considered slow-moving, compared to households lending volumes. Because loans to households are mainly composed of mortgages, it is sensible to assume a strong relationship between households loans' growth and the economy. Stylized facts about private sector loans (e.g. ECB (2013)) confirm that households loans display strong comovements with real GDP growth, while loans to non-financial corporations tend to lag in the business cycle. Furthermore the literature on the credit channel of monetary policy transmission

[17] I refer to the Appendix A of the working paper version of Bernanke et al. (2005) for an in-depth description of the likelihood-based Gibbs sampling.

[18] This is equivalent to setting the upper $K \times K$ part of the λ^F to an identity matrix and the upper $K \times 3$ part of λ^Y to zero.

provides evidence that, although housing loans are highly responsive to monetary shocks, business loans tend to react with a lag or even in the opposite direction that what is expected (the so-called business lending "perverse effect"). I refer for instance to Gertler and Gilchrist (1993) or Christiano et al. (1996).

Innovations were identified in the standard recursive manner using the Cholesky decomposition of the covariance matrix of the reduced-form VAR residuals [19]. More specifically, I ordered the shadow rate last and considered its innovations as monetary policy shocks. It is quite standard in the structural VAR (SVAR) literature to order output and inflation before the interest rate, i.e. to assume that the policy rate can respond contemporaneously to other macroeconomic shocks, while the reverse is not true[20].

Moreover, the specific recursive ordering used in this application also implies that the "banking factors" react with a delay to monetary shocks, while allowing for immediate feedback of the banking shocks into the policy instrument. This assumption is motivated by the fact that the ECB's policy makers are informed about the risks arising in the banking sector when setting monetary policy. I share this choice with other studies using FAVAR or SVAR models with macroeconomic and bank-level data, such as Buch et al. (2014a,b) and Ciccarelli et al. (2014), respectively[21]. It is also important to note that, although this ordering requires the banking factors to react with a delay to monetary shocks, a contemporaneous relationship of individual banks' variables with the policy interest rate is allowed in the observation equation. Furthermore, since monthly data were used, the ordering implies only a limited timing distinction.

The remaining modelling assumptions to be made regard the number of factors K and the number of lags p in the equation (III.1). I included $K = 5$ "banking factors", as indicated by the information criterion proposed by Bai and Ng (2002). Accounting for more factors did not significantly alter the results. Regarding the specification of the VAR model, given the adoption of differenced data and in the sake of parsimony, I set the number of lags $p = 2$.

IV. Results

I start by showing the effects of an expansionary monetary shock on the macroeconomic variables and the median bank. Exploiting the cross-sectional dimension of the data set,

[19] Christiano et al. (1999) provide a detailed discussion of this identification scheme.

[20] See for instance Christiano et al. (1999).

[21] However, for completeness, it is worth mentioning that other works have opted for different ordering assumptions, e.g. Abbate and Thaler (2014) or Christiano et al. (1999), arguing that banks take into account monetary policy rates when making investment decisions.

I then characterise the heterogeneity in the responses according to banks' capitalisation. Subsequently, some robustness checks are explored. As is standard in the VAR literature, the results are reported in terms of impulse response functions. In particular, for each variable of interest, the median response is displayed (blue line), together with 16 and 84 confidence intervals (shaded area), to one standard deviation monetary policy shock.

A. Responses of macroeconomic variables

As shown in figure (3.4), an expansionary monetary shock is characterised by a decrease in the Shadow rate of about 13 basis points, which then reverts to zero. The responses of the euro area macroeconomic variables are of the expected sign. Following an unexpected loosening in monetary policy, both output and prices increase.

However, while output rises substantially, the impact of the monetary shock on the price level is much weaker. Similar results are conveyed by Gambacorta et al. (2014), who study the effect of unconventional monetary policy in eight different economies during the crisis period. They observe that unconventional monetary policy shocks involve larger output and smaller price effects compared to conventional monetary policy shocks. Moreover, the weak price response could be driven by the fact that this model was estimated during a period of recession, in line with results for different recession times[22].

< Figure 3.4 about here >

B. Responses of the banking sector

I now illustrate the reactions of the banking variables to an expansionary monetary shock. Figure (3.5) reports the impulse response functions of the median bank.

The expected default frequency decreases sharply with the maximum reaction reached after about one year. The observed behaviour reflects the fact that the bank's outstanding risk has been lowered by a decrease in interest rates. Low interest rates decrease the credit risk of outstanding loans and increase the value of banks' assets, in turn reducing the banks' default probability. The EDFs' results are in line with previous works (e.g. DeGraeve et al. (2008) and Altunbas et al. (2002)) that emphasize that a more accommodative monetary policy reduces banks' risk intended as banks' probability of distress.

< Figure 3.5 about here >

[22]See for instance Peersman and Smets (2001).

The responses of the median bank's lending are also in line with the literature on the topic. The literature on the credit channel of monetary policy transmission in particular (e.g. Bernanke and Blinder (1992) and Kashyap and Stein (2000)) provides extensive evidence that an expansionary monetary policy shock leads to an increase in lending. Accordingly, my results suggest that following an expansionary monetary policy shock, lending to households and non-financial corporations increases by about 2% and 1%, respectively. However, while lending to households responds quickly, business lending's reaction is slow and displays an initial decrease. The observed pattern, denominated as the "perverse effect" of business loans, is also largely documented in the lending literature[23]. However, the response is statistically insignificant, possibly due to the high heterogeneity of the banks in the sample. Finally, loan spreads of the representative bank drop after a loosening of monetary policy across every loan categories. All the responses are persistent, with a stronger effect for small business loans.

Although the slightly greater tightening of spreads on new business loans to small firms than to large corporates would indicate an increase in risk-taking, the weak reaction in business lending offsets this prediction. At the same time, a significant decrease in the bank's probability of default is observed. Hence, my results do not provide conclusive evidence of excessive risks arising in the banking sector considered as a whole.

In the following section I analyse the role of banks' specific characteristics - particularly capital - in driving these results. If the financial stability of individual banks differs, this is likely to affect the transmission mechanism of monetary policy, too. As I will show in the next section, some interesting effects disappear when examining the aggregate banking sector, i.e. neglecting important microeffects and heterogeneity among financial institutions.

C. Capital and the risk-taking channel

This section presents the ways in which banks' responses to an expansionary monetary shock differ among groups with different degrees of capitalisation. As the bank lending channel literature highlights[24], capital plays a relevant role in the transmission of monetary policy.

The entire sample of banks is divided into three different groups, namely banks with high, medium and low levels of capital, based on their total capital ratio at the beginning of the sample. Specifically, the measure of capital adopted is the total risk-based capital ratio,

[23]Empirical evidence on this effect has been documented by Gertler and Gilchrist (1993) and Christiano et al. (1996).

[24]I refer for instance to Kishan and Opiela (2000) and Altunbas et al. (2002).

defined as the ratio of total risk-based capital (including both core and supplementary capital) to risk-weighted assets.

As is shown in figure (3.6), following an expansionary monetary shock, EDFs of banks with high and medium levels of capital decrease, with a maximum response of 8 basis points after about one year. On the other hand, the probability of default of banks with low levels of capital decreases much less and becomes insignificant after a few months. This result suggests that, although lower interest rates generally lead to a reduction of banks' outstanding risks, the overall market perception of poorly capitalized banks' risk remains almost unchanged. Accordingly, Altunbas et al. (2010) also find a negative effect of capitalisation on banks' risk, as measured by EDFs.

I will now proceed to analyse the changes in lending volumes and loan rates depending on the banks' level of capital. In particular, I focus on business lending, given that the risk-taking identification strategy is constructed around differences in responses across business loan categories. Following a loosening of monetary policy, the lending of poorly capital banks to non-financial corporations rises strongly. Conversely, the responses of banks with medium and high levels of capital are much weaker and statistically non-significant.

In order to better demonstrate the differences across banking groups, figure (3.8) plots the median impulse response function for the three sets of banks together. The rise in business lending is inversely proportional to the level of capitalisation. Lowly capitalised banks lending to non-financial corporations cumulative impulse response amounts to 1.67%, while banks with medium and high levels of capital's lending rises respectively by only 0.67% and 0.16% after about three years.

Finally, I look at the responses of loan spreads across loan size categories and banks' capitalisation, as illustrated in figure (3.9). The combination of a decline in loan spreads with an increase in lending volumes confirms that the observed patterns are driven by supply effects. The central result of the paper is obtained by comparing the reactions of spreads between small and large loans. In this regard, it should be noted that the banks' responses vary depending on the level of capital. Banks with medium and low levels of capital are the ones tightening spreads on small business loans the most, with a maximum reduction of about five basis points reached after one year. Moreover these banks decrease spreads on new business loans to small firms more than to large corporates, with a difference between the two of -0.03 as reported in table (3.2).

On the other hand, well-capitalised banks respond by tightening spreads to large companies more than to small firms, in line with their higher credit risk. To better illustrate the central result of the paper, I plot the median response of loan spreads for small and large

firms together in figure (3.10). Banks with medium and low levels of capital significantly decrease spreads for small business loans compared to spreads for large loans.

This result, together with the strong increase in business lending for low-capital banks, implies that these banks are increasing their exposure to risk. Therefore I conclude that a risk-taking channel arise for poorly capitalized banks.

< FIGURES 3.6, 3.7, 3.8, 3.9, 3.10 ABOUT HERE >

My findings are in line with the literature on the "bank lending channel" of monetary policy transmission. In particular, a stream of this literature emphasises that banks with low levels of capital are more exposed to monetary policy shocks compared to well-capitalised banks, which are better able to shield themselves from these shocks. For example, Kishan and Opiela (2000) demonstrate that the loan supply of poorly capitalised banks reacts more sensitively compared to that of well-capitalised peers[25]. This empirical evidence can be rationalised by the fact that capital, increasing the ability to raise uninsured debt, enables banks to avoid a decline in deposits on lending.

Moreover, as emphasised by the moral hazard literature, capital reduces banks' shareholders' incentives to take excessive risks. The results highlight the importance of agency problems in the transmission of monetary policy. A stronger risk-taking channel arises for banks with more severe agency problems.

D. Robustness checks

In order to test the results presented in the previous section, a number of robustness checks were conducted. First, I adopted alternative measures of bank distress, such as 5-years ahead EDFs and distance to default. The results are in line with the baseline model using EDFs with 1 year horizon.

Second, the analysis was repeated using a two-step estimation method, following Boivin et al. (2009). Bernanke et al. (2005) first introduced the FAVAR model proposing two different estimation techniques, the Bayesian approach I have adopted so far as well as a two-step method[26]. The latter technique, equivalent to the estimation procedure of Stock and Watson (2002), consists of extracting the unobservable factors from the observation

[25] Additional studies confirming this channel are Gambacorta and Mistrulli (2004), Altunbas et al. (2002) and Ehrmann et al. (2003).

[26] Notice that it is not a priori obvious which approach performs better. In Bernanke et al. (2005) a comparison between the two approaches is conducted, analysing the results of a simple macroeconomic model of monetary transmission. The authors observe that the two methods yield similar results, although the ones from the two-step estimation seem more reasonable in their context.

equation through principal component analysis first. Then, in the second step, the transition equation is estimated using standard methods. The Bayesian approach, on the other hand, estimates the observation and transition equations jointly.

One of the crucial differences between these two methods lies in the computation of the unobservable factors. While the Gibbs sampling updates the factor-augmented VAR at every iteration, producing an entire distribution of factors (from which the confidence bands are produced), in the two-step method the factors are estimated via principal components in the first step and the confidence intervals are obtained implementing a bootstrap procedure (based on Kilian (1998)). Figure (3.11) shows the factors obtained from the two-step estimation plotted against the factors estimated via the Bayesian method. The first factor, which accounts for most of the variance share in the data, is very similar. The succeeding factors, which are less significant, differ. However, part of the variation is due to the fact that the sign of the factors is not identified.

I then compare the impulse response functions for the median bank. A general observation to be made is that confidence bands obtained from the two-step estimation are always wider and the responses less smooth. Moreover, in order to obtain very similar responses with the two-step method, the number of factors must be increased up to 10, see figure (3.13), since with five factors some discrepancies arise for the lending variables, as reported in figure (3.12). In general, however, the main results of the paper are preserved.

< Figures 3.11, 3.12 and 3.13 about here >

Furthermore, in order to assess the extent to which the results are dependent on the specific definition of capital, I constructed alternative groupings based on Tier 1 ratios collected from both Bloomberg and SNL. Moreover, I built banks' groups based on total capital ratios over the entire sample period from 2009 to 2015. Across all of these different definitions of capital, individual banks change classification quite infrequently and the results continue to hold, confirming that the original measure of capital is robust[27].

Additionally the findings are robust with respect to the terminal date of the sample. Reducing the sample end period by one year, the main results remain unaffected. However this is no longer true if I vary the start date of the sample, which could go back to June 2007, as determined by the data availability. Including these additional years in the sample drastically changes the outcomes of the model. This is because those times were characterised by financial turmoil and a high degree of market volatility due to the

[27]I also investigated alternative groupings based on size and liquidity variables, as standard in the "bank lending channel" literature. However, none of them produced interesting results and therefore I do not report them for the sake of brevity.

outbreak of the financial crisis. Moreover, as is shown in figure (3.1), money market rates and the shadow rate in turn dropped dramatically at the end of 2008, due to the ECB interventions. Hence, including these data in the sample would substantially complicate the analysis. In fact, disentangling exogenous monetary policy innovations from endogenous reactions to the financial crisis and the greater uncertainty associated would be extremely challenging, possibly leading to biased results.

V. Conclusion

As the recent financial crisis has proven, price stability is not a sufficient condition for financial stability. Policymakers and academics have raised concerns that central banks' loose monetary policy has led to an increase in banks' risk-taking. Nonetheless, in the aftermath of the crisis, interest rates have been kept low in order to enhance credit supply to the economy. Therefore, understanding the extent to which monetary policy might spur "excessive" risk-taking by banks is a critical and challenging question.

This study, employing a FAVAR model, aims to shed some light on the interdependence between monetary policy and financial stability. In contrast to the rest of the literature, I focus on the euro area in a period characterised by a low interest rate environment and unconventional monetary policy measures. The analysis combines macroeconomic and bank-level data, including measures of banks' financial distress and lending. In particular, I compare loan spreads across different credit risk categories, distinguishing between loans to small and large companies. Moreover, I analyse the differential effects arising among groups of banks with different degrees of capitalisation.

Following an expansionary monetary shock, banks with low levels of capital increase business lending and tighten spreads on new business loans to small firms more than to large corporates. At the same time, their expected default frequencies do not fall significantly. The results suggest that poorly capitalized banks are shifting towards higher credit risk strategies. On the other hand, no conclusive evidence of increased risk exposure by well capitalised banks is conveyed. The cross-sectional analysis reveals a risk-taking channel for low-capital banks, underlying the importance of banks' balance sheet characteristics and agency problems in the transmission of monetary policy.

Overall the model provides a useful tool for policy analysis, as it could be adopted for forecasting and scenario analysis. Assessing the ability of the framework to generate meaningful scenarios and effective macro-prudential policy implications, in order to reduce the negative effects of monetary policy on banks' risk-taking, is left for future research.

VI. Appendix: Tables

Table 3.1. Summary statistics.
The table reports summary statistics for the entire banking data set and the different groups of banks. High-capital banks are characterised by a total capital ratio higher than 13.46, medium-capital banks by a capital ratio between 10.8 and 13.46, low-capital banks by a capital ratio lower than 10.8.

	All banks		**High capital**		**Medium capital**		**Low capital**	
	Median	s.d.	Median	s.d.	Median	s.d.	Median	s.d.
EDFs	0.55	0.90	0.53	1.24	0.57	0.90	0.77	0.85
Loans HH Growth	-0.46	2.02	0.23	1.90	-0.69	2.51	-0.55	2.72
Loans NFC Growth	-1.09	2.75	-1.91	4.49	-1.9	3.78	0.68	4.42
Loan Spreads NFC SMALL	2.79	0.28	2.22	0.23	2.58	0.21	3.15	0.42
Loan Spreads NFC LARGE	1.81	0.18	1.48	0.17	1.73	0.24	2.18	0.38
Loan Spreads HP	2.32	0.17	2.57	0.19	2.23	0.22	2.34	0.29

Table 3.2. Loan spreads impulse responses.
The table reports differences between impulse responses of spreads for small and large loans to a monetary policy shock.

Horizon	**High capital**	**Medium capital**	**Low capital**
12 months	0.0057	-0.0334	-0.0293
20 months	0.0065	-0.0355	-0.0333
40 months	0.0072	-0.0357	-0.0344

VII. Appendix: Figures

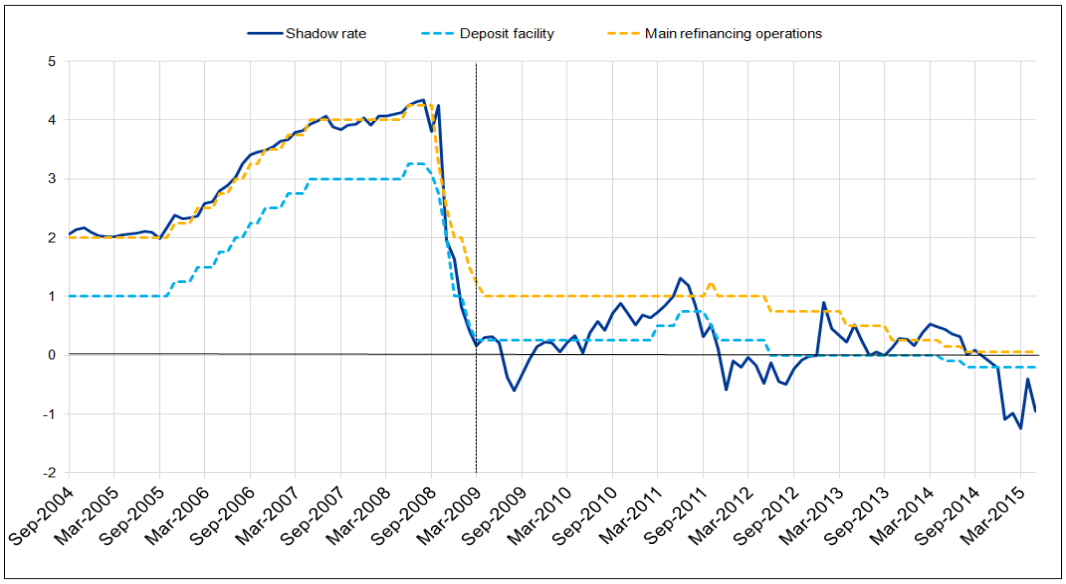


Figure 3.1. Shadow rate and ECB rates.
The figure shows the Wu-Xia shadow rate for the euro area plotted against the ECB deposit facility rate and main refinancing operations rate. Source: ECB and Prof. Wu's website.

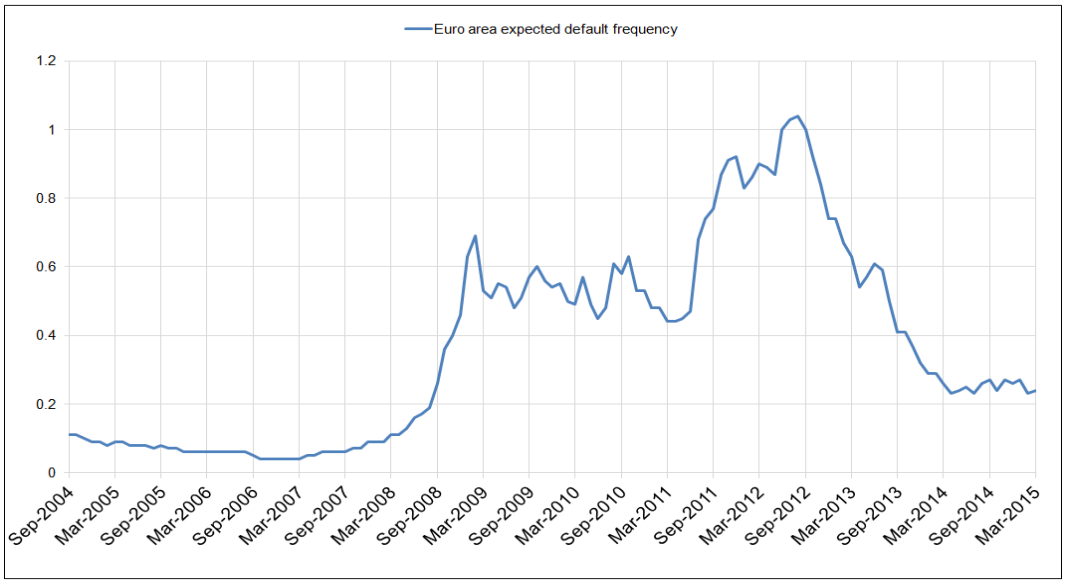


Figure 3.2. Euro area EDF.
The figure shows the euro area EDF, calculated as the median of individual banks' EDFs, of size $30 and above in the euro area. Source: Moody's KMV.

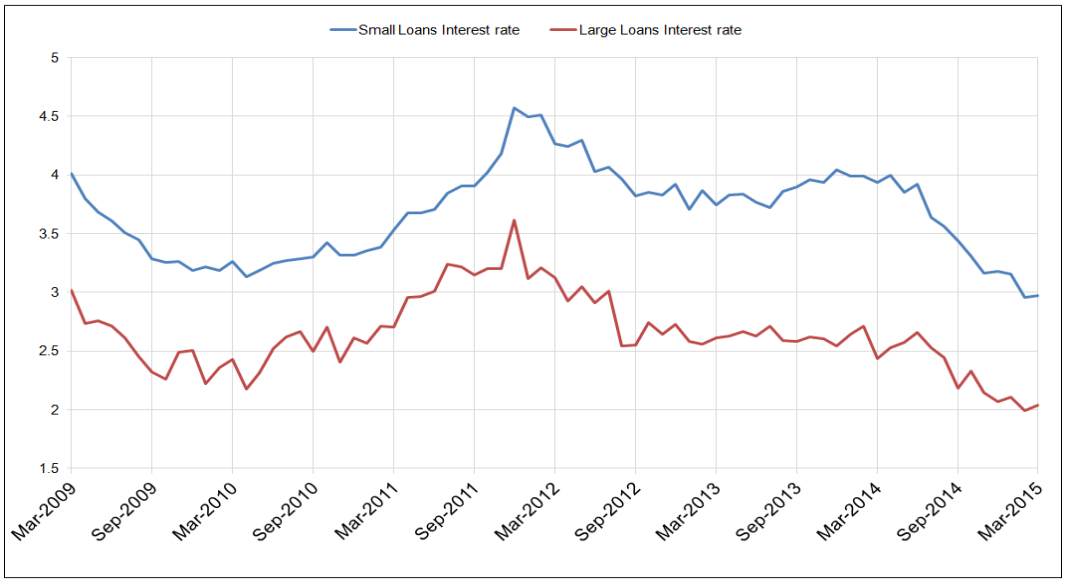


Figure 3.3. Lending rates for small and large loans.
The figure shows the median for the banks in the sample of the lending rates for small and large loans. Source: ECB.

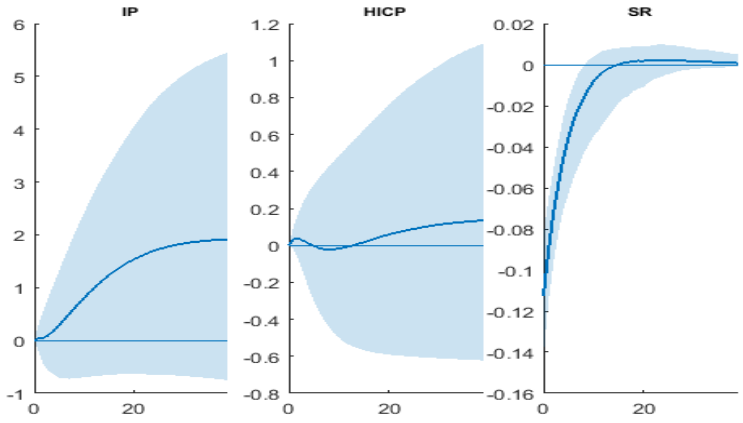


Figure 3.4. Macroeconomic variables' responses.
The figure displays the impulse response functions of the macroeconomic variables of the model to a one standard deviation shock of the shadow rate. The blue lines correspond to the median responses, while the shaded areas correspond to the 68% confidence bands.

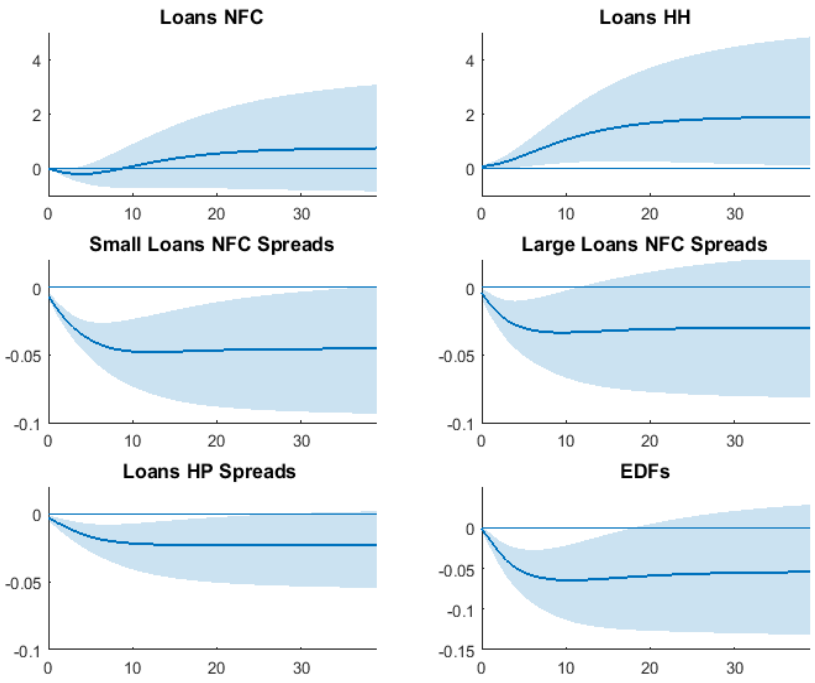


Figure 3.5. Banking sector response

The figure displays the impulse response functions of the median bank's variables: EDFs, loan volumes and spreads, to a one standard deviation shock of the shadow rate. The blue lines correspond to the median response, while the shaded areas correspond to the 68% confidence bands.

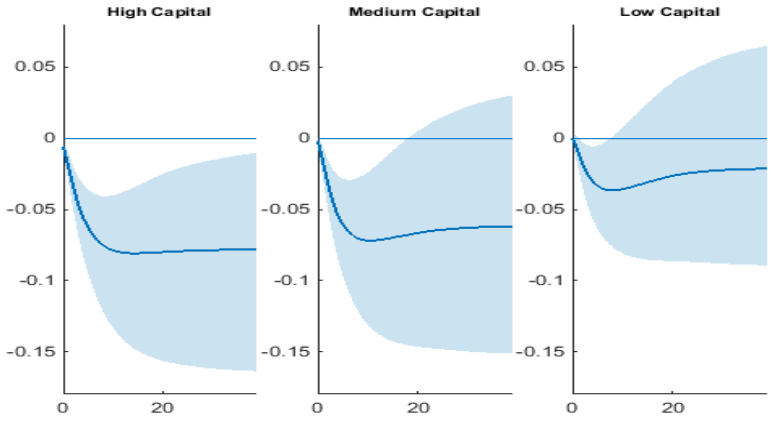


Figure 3.6. Expected default frequencies

The figure displays the impulse response functions of the expected default frequencies to a one standard deviation shock of the shadow rate. Banks have been divided into groups based on their total capital ratio. The blue lines correspond to the median responses, while the shaded areas correspond to the 68% confidence bands.

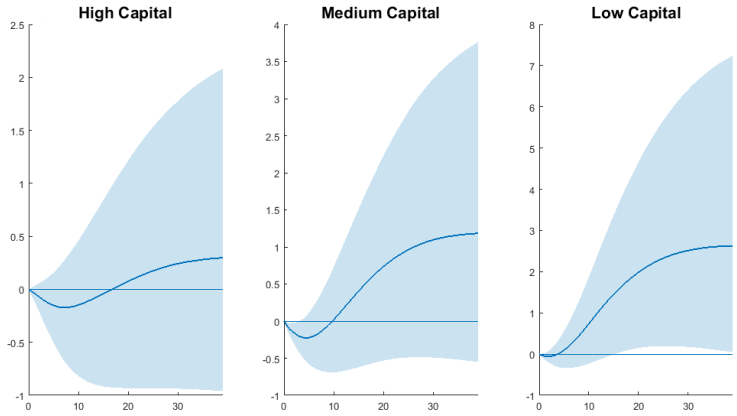


Figure 3.7. Loans to non financial corporations

The figure displays the impulse response functions of the banks' business lending volumes to a one standard deviation shock of the shadow rate. Banks have been divided into groups based on their total capital ratio. The blue lines correspond to the median responses, while the shaded areas correspond to the 68% confidence bands.

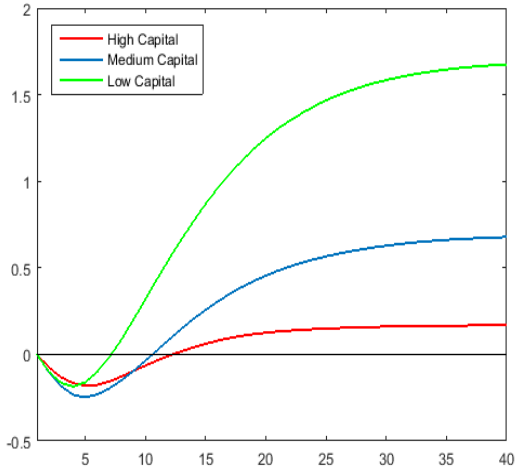


Figure 3.8. Median responses of loans to non-financial corporations

The figure displays the median impulse response functions of the volumes of loans to non-financial corporations to a one standard deviation shock of the shadow rate. Banks have been divided into groups based on their total capital ratio.

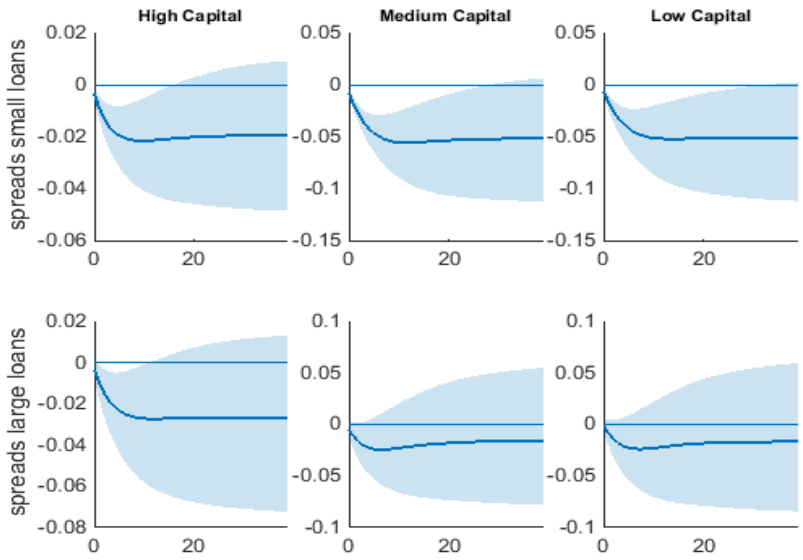


Figure 3.9. Spreads for small and large loans

The figure displays the impulse response functions of the banks' loan spreads for small and large loans to a one standard deviation shock of the shadow rate. Banks have been divided into groups based on their total capital ratio. The blue lines correspond to the median responses, while the shaded areas correspond to the 68% confidence bands.

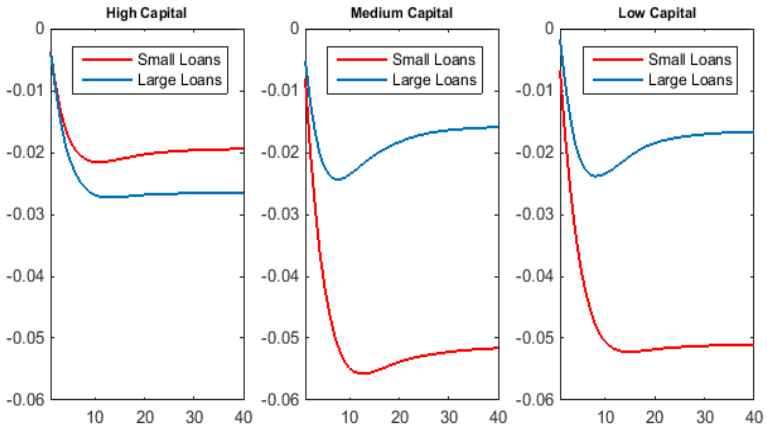


Figure 3.10. Median responses of spreads for small and large loans

The figure displays the median impulse response functions of the small and large loan spreads to a one standard deviation shock of the shadow rate. Banks have been divided into groups based on their total capital ratio.

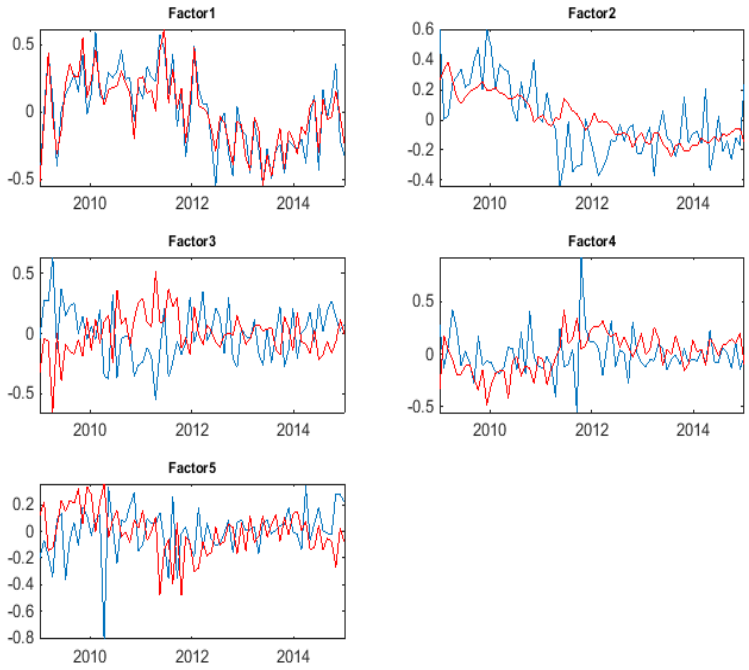


Figure 3.11. Factor comparison
The figure shows the five unobservable factors of the model estimated using the two-step method (blue lines) and the Bayesian approach (red lines).

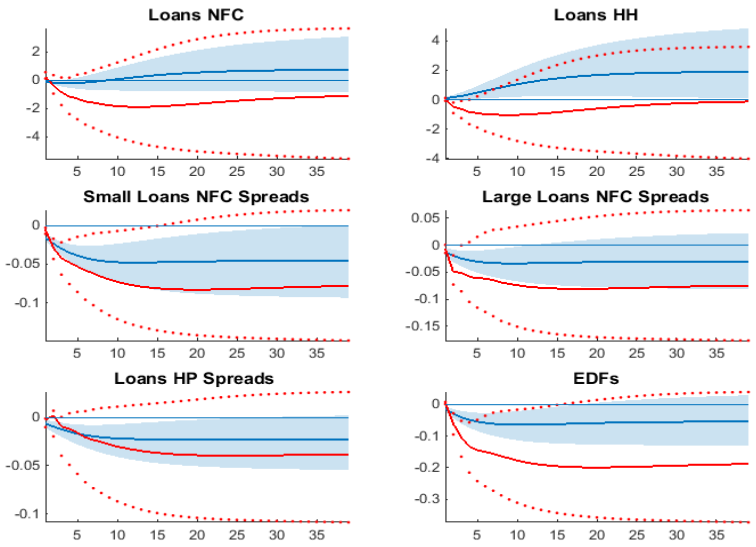


Figure 3.12. Banking sector response two-step estimation with five factors
The figure displays the impulse response functions of the median bank's variables EDFs, loan volumes, and spreads to a one standard deviation shock of the shadow rate. The blue lines and shaded areas correspond to the median responses and the 68% confidence bands, obtained from the Bayesian estimation. The red lines correspond to the median response (solid) and the 68% confidence bands (dotted), obtained from the two-step estimation using five factors.

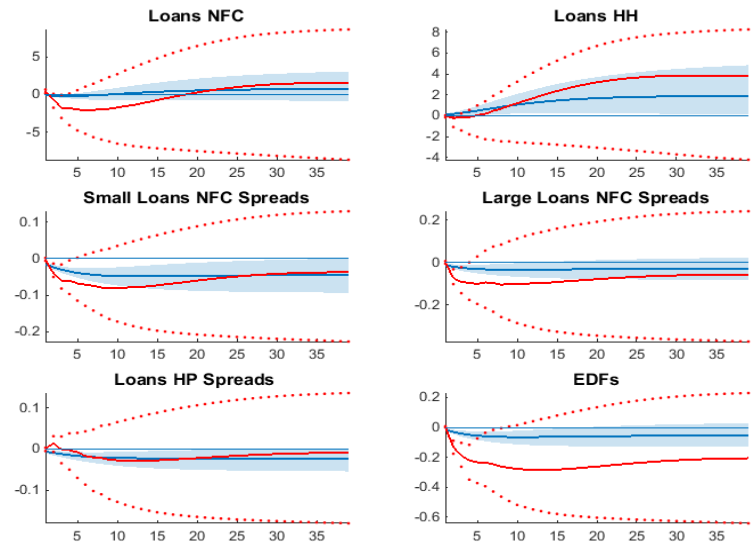


Figure 3.13. Banking sector response two-step estimation with ten factors
The figure displays the impulse response functions of the median bank's variables EDFs, loan volumes, and spreads to a one standard deviation shock of the shadow rate. The blue lines and shaded areas correspond to the median responses and the 68% confidence bands, obtained from the Bayesian estimation. The red lines correspond to the median response (solid) and the 68% confidence bands (dotted), obtained from the two-step estimation using ten factors.

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