

RESEARCH ARTICLE | MARCH 28 2024

Effects of the applied fields' strength on the plasma behavior and processes in $E \times B$ plasma discharges of various propellants. II. Magnetic field

M. Reza ; F. Faraji ; A. Knoll 



Phys. Plasmas 31, 032121 (2024)

<https://doi.org/10.1063/5.0176586>



Effects of the applied fields' strength on the plasma behavior and processes in $E \times B$ plasma discharges of various propellants. II. Magnetic field

Cite as: Phys. Plasmas **31**, 032121 (2024); doi: [10.1063/5.0176586](https://doi.org/10.1063/5.0176586)

Submitted: 14 September 2023 · Accepted: 18 January 2024 ·

Published Online: 28 March 2024



M. Reza,^{a)} F. Faraji, and A. Knoll

AFFILIATIONS

Plasma Propulsion Laboratory, Department of Aeronautics, Imperial College London, Exhibition Road, London SW7 2AZ, United Kingdom

^{a)} Author to whom correspondence should be addressed: m.reza20@imperial.ac.uk

ABSTRACT

The effects of magnetic field intensity on the properties of the plasma discharge and on the underlying phenomena are studied for different propellants' ion mass. The plasma setup represents a 2D radial–azimuthal configuration with perpendicular electric and magnetic fields. The electric field is along the axial direction, and the magnetic field is along the radial direction. The magnetic field intensity is changed from 5 to 30 mT, with 5 mT increments. The studied propellant gases are xenon, krypton, and argon. The simulations are carried out using a reduced-order particle-in-cell code. It is shown that, for all the propellants, the change in the magnetic field intensity yields two distinct plasma regimes, where either the modified two-stream instability (MTSI) or the electron cyclotron drift instability (ECDI) are dominant. A third plasma regime is also observed for cases with moderate values of the magnetic field (15 and 20 mT), where the ECDI and the MTSI co-exist with comparable amplitudes. This described variation of plasma regime becomes clearly reflected in the radial distribution of the axial electron current density and the electron temperature anisotropy. At the relatively low-magnetic-field intensities (5 and 10 mT), the MTSI is mitigated. At relatively high magnitudes of the magnetic field (25 and 30 mT), the MTSI becomes strongly present, a long-wavelength wave mode develops, and the ECDI becomes suppressed. An exception to this latter observation was noticed for xenon, for which the ECDI was observed to be detectable with a notable strength up to the magnetic field value of 25 mT.

© 2024 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>). <https://doi.org/10.1063/5.0176586>

I. INTRODUCTION

$E \times B$ plasma discharges are of high applied relevance across the industries, from the manufacturing to the aerospace. These discharges present intricate physics that has gathered the attention of the scientific community for decades. The plasma in the $E \times B$ configuration is characterized by a vast array of underlying multiscale, multidimensional, and multiphysics processes.^{1,2} The resulting highly complex nature of the plasma has impeded achieving a comprehensive knowledge basis of the phenomena in the $E \times B$ plasmas despite years of effort and a large body of research.^{1–6} An enabling effort to achieve this full understanding consists of extensive parametric studies over wide ranges of plasma conditions and in controlled simulation environments. Such parametric studies allow one to isolate the effects of various physical factors and mechanisms on the underlying microscopic processes as well as on the global macroscopic behavior of the plasma.

The experimental works informed by the simulations are also of high importance to complement the parametric investigations since they enable the validation of the numerical observations and provide further insights from real-world devices and settings. The existing rich literature can indeed guide the joint numerical–experimental endeavors by highlighting the phenomena and/or the range of plasma parameters/configurations that require further in-depth analyses and more focused investigations.

The available high-fidelity plasma simulation codes, such as the ones based on the kinetic particle-in-cell (PIC) method,⁷ have so far proven to be highly useful to provide insights into the complicated physics of the $E \times B$ discharges. However, their enormous computational cost is not compatible with the extensive parametric investigations that are required to study the characteristics of the multiscale, multidimensional phenomena in detail over broad parameter spaces

and to investigate the involved dynamics over rather extended time frames. The reduced-order PIC scheme,^{8–12} developed and extensively verified at Imperial Plasma Propulsion Laboratory (IPPL), serves as a game-changer in this respect. This is because the reduced-order PIC allows us to perform high-fidelity simulations whose results closely resemble those of traditional PIC codes, and that also feature reduced computational costs by several factors to an order of magnitude.^{8,9,12}

Equipped with this cost-efficient numerical tool and following on the investigations presented in Paper I¹³ of this article, we focus in this Paper II on the effects that the intensity of the applied magnetic field in our adopted radial–azimuthal plasma configuration has on the development, the characteristics, and the impacts of the electron cyclotron drift instability (ECDI) and the modified two-stream instability (MTSI) modes. We analyze these aspects for the plasma discharges of xenon, krypton, and argon.

In the context of $E \times B$ plasma technologies, it is important to note that the magnetic field intensity represents a design parameter. Particularly in the case of Hall thrusters, which we have chosen as a proving ground for our analyses here, the topology of the applied magnetic field and its magnitudes within the domain remain invariant over time and through the operation of the device. Despite this temporal invariance, the magnetic field typically features notable spatial variations by design, with conventionally large gradients existing in the axial direction along the thrusters' discharge channel centerline.^{14,15}

The present effort aims to systematically assess in a radial–azimuthal Hall-thruster-representative setup the distinct influences that the magnetic field intensity has on the instability spectra and on the consequent variation(s) in the properties and the behavior of plasma discharges of multiple propellants. To the best of our knowledge, our effort is unique and un-attempted within the existing literature. Prior to this work, the effects of the magnetic field intensity in a similar context to that our paper have been numerically assessed for two settings:

- (1) In an axial–azimuthal Hall-thruster-representative configuration with xenon propellant.^{16,17} The research reported in Refs. 16 and 17 investigated the spatial variations in the characteristics of the ECDI and its transition dynamics to the ion acoustic instability (IAI)^{16,18} in the presence of various magnetic field magnitudes.
- (2) In cross-field configurations that exhibit large-scale spoke-like plasma structures.^{19–22} The authors in Refs. 21 and 22 demonstrated the formation of the “rotating spoke” structures from the gradient drift instability for various magnitudes of the magnetic field and explored the evolution of the gradient drift instability into an $m = 1$ rotating spoke mode. In Ref. 19, Powis *et al.* evaluated the parametric dependency of the rotating spoke frequency on several parameters, including the strength of the applied magnetic field. Boeuf and Takahashi in Ref. 20 showed that, depending on the magnetic field intensity, the rotating direction of the spoke mode can be either along the positive or the negative $E \times B$ direction.

From an experimental perspective, the authors are unaware of any effort with a similar scope to that of this article. However, there have been multiple publications in the context of the $E \times B$ plasma discharges that examined the influence of the magnetic field strength on the large-scale plasma instabilities such as the spoke mode,^{23–25} on the

electron species' properties,²⁶ and on the electrons' cross-field transport characteristics.^{27,28}

II. OVERVIEW OF THEORETICAL DESCRIPTIONS OF THE ECDI AND THE MTSI

In this section, we briefly discuss the 2D radial–azimuthal dispersion relations of the ECDI and the MTSI as derived from prior research in the literature.

Ducrocq *et al.*²⁹ formulated the dispersion relation of the ECDI in a plasma immersed in a constant electromagnetic field. They used kinetic description for the electrons and the fluid equations for cold, unmagnetized ions. The projection of their 3D dispersion relation onto a 2D plane perpendicular to the magnetic field is²⁹

$$k_{\perp}^2 \lambda_{De}^2 \left(1 - \frac{m_e \omega_{pe}^2}{m_i \omega^2} \right) + \left[1 - I_0(b) e^{-b} + \sum_{n=1}^{\infty} \frac{2(\omega - k_z V_{de})^2 I_n(b) e^{-b}}{(n\omega_{ce})^2 - (\omega - k_z V_{de})^2} \right] = 0. \quad (1)$$

In Eq. (1), $k_{\perp}$ is the wavevector component perpendicular to the magnetic field, k_z is the azimuthal wavenumber, ω_{pe} is the electron plasma frequency, ω_{ce} is the electron cyclotron frequency, λ_{De} is the Debye length, V_{de} is the electrons' $E \times B$ drift velocity, and m_e and m_i are the electron and ion mass, respectively. In addition, $b = k_z^2 V_{th}^2 / \omega_{ce}$, in which V_{th} is the electrons' thermal velocity. I_n denotes the modified Bessel function of order n .

According to the above dispersion relation, the unstable modes of the ECDI occur near certain azimuthal wavenumbers given by the following equation:^{29–31}

$$k_z = k_0 = N \frac{\omega_{ce}}{V_{de}} = N \frac{q B_x^2}{m_e E_y}; \quad N = 1, 2, \dots \quad (2)$$

In Eq. (2), q is the elementary charge. The transition of the ECDI modes to the ion acoustic instability (IAI) has been reported in Refs. 30 and 32 at sufficiently large amplitudes of the instability. The characteristics of the fastest growing mode of the ion acoustic instability are³³

$$k_{z,max} \approx \frac{1}{\sqrt{2} \lambda_{De}}, \quad \omega_{max} \approx \frac{\omega_{pi}}{\sqrt{3}}, \quad (3)$$

where ω_{pi} is the ion plasma frequency.

Regarding the MTSI, the author in Ref. 34 derived a dispersion relation based on the fluid treatment of the electrons and the ions in the radial–azimuthal plane. In this description, the electrons are assumed to be isothermal and isotropic, and the ions are considered to be unmagnetized and cold. Same relation can be obtained using kinetic treatment of the ions in the limit of $T_e \rightarrow 0$.³⁵ The resulting dispersion relation is given by Eq. (4) as

$$1 - \left(\frac{\omega_{pi}}{\omega} \right)^2 - \frac{m_i}{m_e} \frac{\omega_{pe}^2 k_x^2}{\left(\omega - k_z \frac{E_y}{B_x} \right)^2 k^2} - \frac{m_i}{m_e} \frac{\omega_{pe}^2 k_z^2}{\left(\left(\omega - k_z \frac{E_y}{B_x} \right)^2 - \omega_{ce}^2 \right) k^2} = 0. \quad (4)$$

The resonance condition of Eq. (4) establishes a relationship between the radial (k_x) and the azimuthal (k_z) wavenumbers of

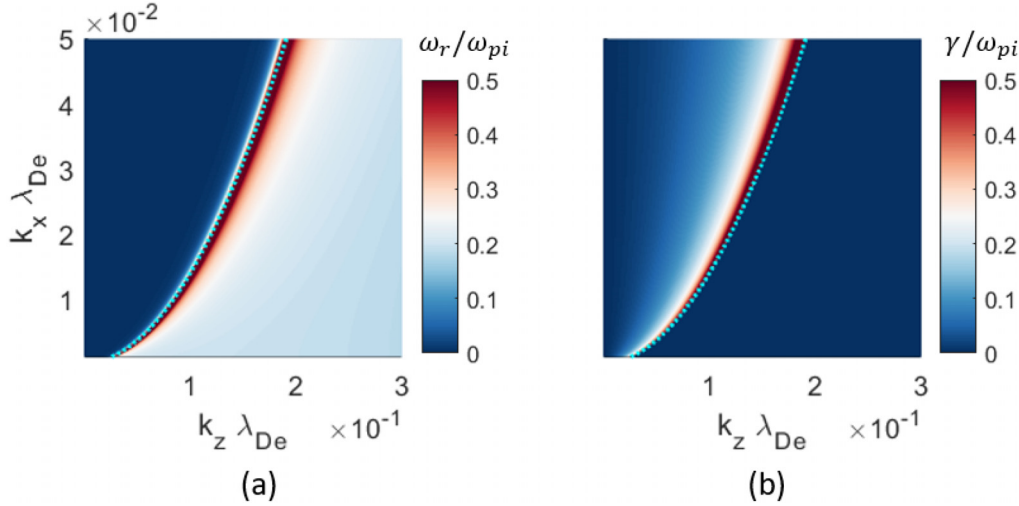


FIG. 1. Numerical solution of the MTSI's dispersion relation given by Eq. (4) with the plasma and the applied fields conditions corresponding to the baseline setup of the simulations in this work (see Sec. III), namely, xenon propellant, $n_i = 1.5 \times 10^{17} \text{ m}^{-3}$, $E_x = 10 \text{ kV m}^{-1}$, and $B_x = 20 \text{ mT}$; (a) normalized real frequency (ω_r/ω_{pi}), and (b) normalized growth rate (γ/ω_{pi}) of the MTSI. The dotted cyan curves show the loci of the k_x and k_z value pairs associated with the MTSI's fastest growing mode as determined by Eq. (5).

the fastest growing mode of the MTSI as the following equation:

$$k_z = \sqrt{\frac{q}{m_e} \frac{B_x^2}{E_y} k_x}. \quad (5)$$

To further illustrate the above statement concerning the resonance condition, we solved the MTSI's dispersion relation [Eq. (4)] for the plasma density and the field strength conditions of the baseline radial–azimuthal benchmark problem as an example.³⁶ Figure 1 shows the variations of the real frequency [plot (a)] and the growth rate [plot (b)] of the MTSI waves across the $k_x - k_z$ plane. Superimposed on the plots as cyan dotted lines are the loci of the (k_x, k_z) pairs that satisfy Eq. (5), hence, correspond to the MTSI's fastest growing mode. From Fig. 1(b), it is noticed that the MTSI is unstable only when $k_z \geq \sqrt{\frac{q}{m_e} \frac{B_x^2}{E_y} k_x}$. Accordingly, in line with the findings of Ref. 34, the relationship given by Eq. (5) can properly define the boundary between the stable and unstable regimes of the MTSI.

III. DESCRIPTION OF THE SIMULATIONS' SETUP

The simulations' domain is a 2D Cartesian plane representing the radial–azimuthal coordinates of a Hall thruster. This domain is schematically illustrated in Fig. 2. In the adopted coordinate system, x , y , and z indicate the radial, axial, and azimuthal directions, respectively.

The simulations' setup follows identically the one used in Paper I of the article.¹³ The only exception is that, for the simulations in this paper, the strength of the applied radial magnetic field (B_x) is varied across a range of values, which are summarized in Table I. The axial electric field remains constant at 10 kV m^{-1} for all the cases. This is the same electric field value as that adopted in the radial–azimuthal benchmark case in Ref. 36. The initial plasma properties and the computational parameters used for the simulations are provided in Table II.

Initially, electrons and ions are sampled from Maxwellian distributions at 10 and 0.5 eV, respectively. The particles are then uniformly

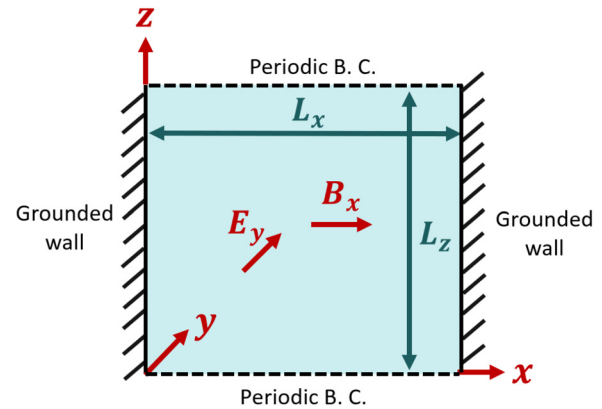


FIG. 2. Schematics of simulations' computational domain, the coordinate system, and the directions of the applied constant electric and magnetic fields.

TABLE I. List of the studied simulation cases, summarizing the values of the radial magnetic field (B_x) and the ion species in each case. B_0 refers to the magnetic field intensity in the baseline benchmark setup (20 mT).³⁶

Case no.	B_x (mT)	B_x/B_0	Propellant
1	5	0.25	Xe, Kr, Ar
2	10	0.5	Xe, Kr, Ar
3	15	0.75	Xe, Kr, Ar
4	20	1	Xe, Kr, Ar
5	25	1.25	Xe, Kr, Ar
6	30	1.5	Xe, Kr

TABLE II. Summary of the computational and the physical parameters used for the radial–azimuthal simulations.

Parameter	Value unit
Computational parameters	
Domain length ($L_x = L_z$)	1.28 cm
Virtual axial length (L_y)	1 cm
Cell size ($\Delta x = \Delta z$)	50 μm
Number of cells in each direction ($N_i = N_j$)	256
Time step (Δt)	1.5×10^{-11} s
Total simulated duration (t_{sim})	30 (μs)
Initial number of macroparticles per cell (N_{ppc})	100
Physical parameters	
Initial plasma density ($n_{i,0}$)	$5 \times 10^{16} \text{ m}^{-3}$
Initial electron temperature ($T_{e,0}$)	10 eV
Initial ion temperature ($T_{i,0}$)	0.5 eV
Axial electric field (E_y)	10 000 V m $^{-1}$
Potential at walls (ϕ_w)	0 V

loaded as pairs over the x – z simulation plane. To simulate the downstream convection of azimuthal waves and to control the particles' energy growth, a virtual axial extent ($L_y = 1$ cm) is considered. Along the axial (y) direction, Poisson's equation is not solved. The particles crossing the domain boundary in the y direction are resampled from a Maxwellian at their initial load temperatures (10 eV for electrons, 0.5 eV for ions). These resampled particles are then reinjected onto the simulation plane, preserving their azimuthal and radial positions.

The simulations are collisionless, and the establishment of a steady state is achieved though imposing an ionization source function $S(x, z)$. The profile of the ionization source is uniform along the azimuthal direction and follows a cosine function along the radial direction, spanning from $x = 0.09$ to 1.19 cm with the peak of $S_0 = 8.9 \times 10^{22} \text{ m}^{-3} \text{ s}^{-1}$. The detailed definition of the ionization source is provided in Ref. 36.

The potential boundary conditions consist of a zero-volt Dirichlet condition at the radial boundaries of the domain, which represent grounded walls. Along the azimuthal boundaries, a periodic condition is implemented.

The secondary electron emission (SEE) from the walls is not modeled, and all particles reaching the radial boundaries are, thus,

removed from the simulations. Additionally, to mimic the periodicity along the azimuthal direction, the particles leaving the azimuthal extent at one end of the domain are reintroduced from the opposite azimuthal boundary.

The simulations are performed using the IPPL-Q2D code. The domain decomposition associated with the reduced-order PIC scheme is applied along the radial and the azimuthal directions of the simulations using 50 regions.¹² In Sec. 1 of the Appendix, a summary of the results of the convergence study and the verifications of the Q2D simulations against the radial–azimuthal benchmark case³⁶ are provided. The results Sec. 1 of the Appendix demonstrate that, as the number of regions is increased, the predictions of the Q2D simulations approach the 2D results. In particular, simulations with 50 number-of-regions can reproduce the 2D results with sufficient accuracy, while offering a significant computational speedup (fivefold) over the conventional 2D simulations. With 50 regions, the Q2D approximation involves an error of about 12% in the ion number density and 5% in the electron temperature, with the characteristics of the resolved instabilities being almost identical to those captured by the full-2D simulation. Accordingly, 50 number-of-regions has been an appropriate choice for the parametric analyses and the study of the physics in the present effort.

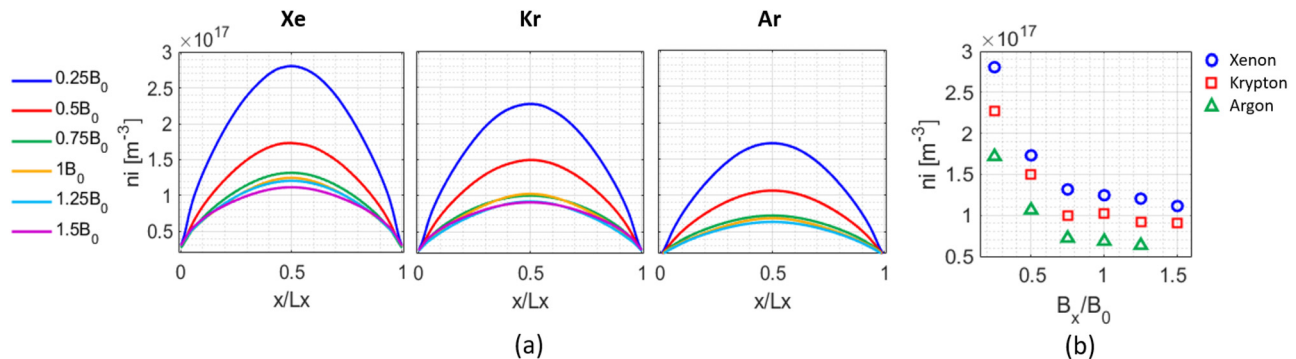
IV. RESULTS

In this section, we present the results of the simulations that we have performed to characterize the impact of the intensity of the radial magnetic field on the dynamics of the plasma discharge with various propellants (gases). Our focus is on studying the variations of the dominant instability modes with the B_x and the consequent effects on the plasma.

Following the same order of presentation as that pursued in Paper I,¹³ we begin by presenting the variations of the plasma properties' profiles. We then discuss how the magnitude of the B -field affects the instabilities' characteristics. The characteristics of the instabilities are deduced from the fast Fourier transform (FFT) analyses of the azimuthal fluctuations and are complemented by more comprehensive dynamic mode decomposition (DMD) analyses.

A. Variation in the plasma properties distribution

Figure 3(a) shows the time-averaged radial profiles of the ion number density from the different simulation cases, and Fig. 3(b)

**FIG. 3.** (a) Radial profiles of the ion number density (n_i) averaged over all azimuthal locations and over the time interval of 20–30 μs from the radial–azimuthal simulations with various radial magnetic field intensities and propellants. (b) Variation of the ion number density's peak value along the radial direction vs B_x/B_0 for the three studied propellants.

presents the variations of the peak values of the ion number density profiles with B_x/B_0 . In Sec. 2 of the Appendix, we have verified that the simulations had indeed arrived at a quasi-steady state. This has been achieved by assessing the time evolution plots of the spatially averaged plasma properties in addition to the time evolution plots of the amplitudes of the azimuthal fluctuations captured by the simulations.

From Fig. 3, we observe a consistent trend of decreasing density with the increasing magnetic field intensity, with xenon and argon consistently exhibiting the highest and the lowest densities, respectively. Furthermore, beyond $0.75 B_0$, the changes in the density become minor, and the evolution of the peak densities with B_x/B_0 appears to have reached an asymptote.

The radial distributions of the electron temperature (T_e) and the radial and the azimuthal electron temperature components (T_{ex} and T_{ez}) are shown in Fig. 4. Figure 4 also presents the variations of the average values of these properties over the radial extent of the domain vs B_x/B_0 .

From the plots in Fig. 4, we notice that T_e exhibits an increasing trend within the B_x range of $0.25 B_0$ to $0.75 B_0$ for all propellants. For xenon and krypton, the T_e remains nearly constant between $0.75 B_0$ and $1 B_0$ but decreases afterward toward higher B_x values. For argon, T_e consistently drops beyond $0.75 B_0$. The T_{ez} follows a similar variation with B_x/B_0 as that of the T_e for the cases of xenon and krypton propellants. For argon, however, the azimuthal electron temperature does not vary significantly for the magnetic field intensities of $0.25 B_0$ to $0.75 B_0$ before dropping in values toward higher B_x .

The T_{ex} for all the propellants increases with the magnetic field up to $B_x/B_0 = 0.75$, after which it remains almost constant. We can readily notice by comparing the T_{ex} vs B_x/B_0 plot in Fig. 4(d) against the n_i vs B_x/B_0 plot in Fig. 3(b) the inverse correlation that exists between the radial electron temperature, which determines the particles' thermal flux to the walls, and the plasma density. This inverse correlation arises due to the temporal constancy of the imposed particle injection source in the simulations as was elaborated on in Paper I.¹³ Nevertheless, it is emphasized that the extent to which the observed correlation may hold in more realistic scenarios needs to be investigated in a setup with self-consistent ionization process and with the axial fluxes of the particles also self-consistently resolved (hence, a 3D PIC simulation).

In Paper I,¹³ we exploited the knowledge of the different influences that the MTSI and the ECDI modes have on the electrons' heating and on the electrons' cross-field transport to distinguish the dominant instability between the ECDI and the MTSI across various simulation cases prior to analyzing the FFT/DMD spectra of the plasma fluctuations.

We introduced two parameters, namely, the average of the T_{ex}/T_{ez} ratio over the radial extent of the domain, and the radial profile of the normalized axial electron current density ($J_{ey}/J_{ey,mean}$), as two indicators for the above purpose. In this regard, as the ECDI contributes to the electrons' heating primarily along the azimuthal direction,^{37,38} whereas the MTSI heats up the electrons mostly in the radial direction,^{35,39,40} the ratio of the radial-to-azimuthal electron temperature can be an indirect indicator of the relative significance of the

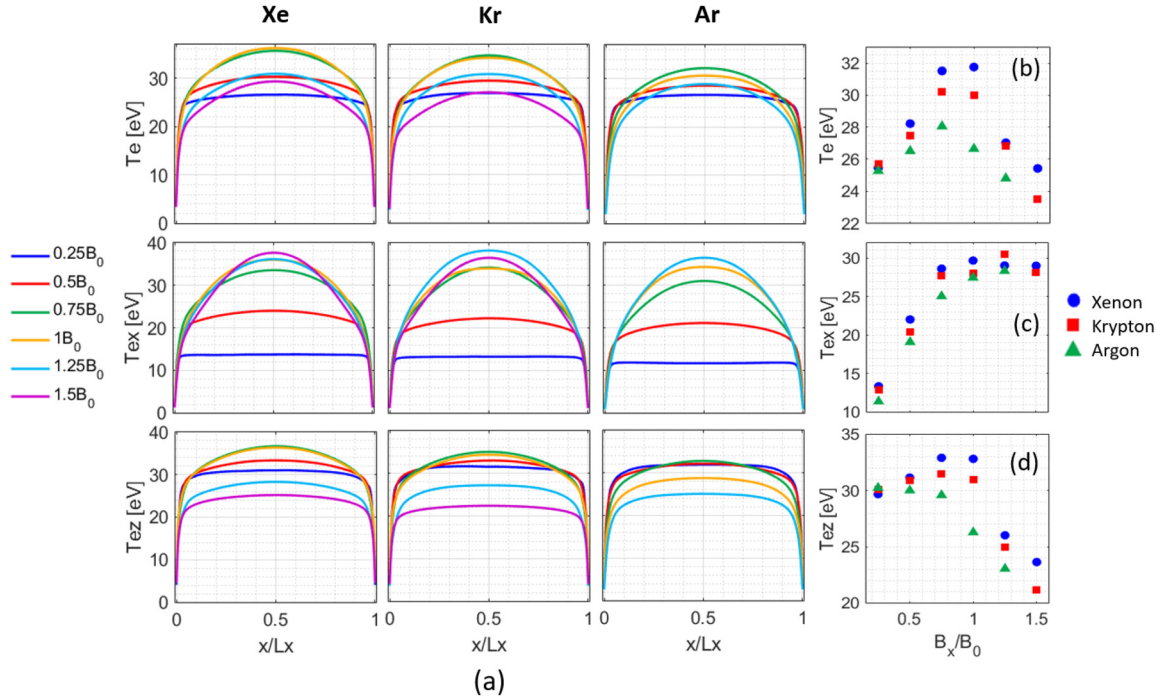


FIG. 4. (a) Radial profiles of electron temperature (T_e), radial electron temperature (T_{ex}), and azimuthal electron temperature (T_{ez}), averaged over all azimuthal locations, and over the time interval of 20–30 μ s from the radial-azimuthal simulations with various radial magnetic field intensities and propellants. Variation of the radially averaged T_e [plot (b)], T_{ex} [plot (c)], and T_{ez} [plot (d)] vs B_x/B_0 for the three studied propellants from the quasi-2D simulations.

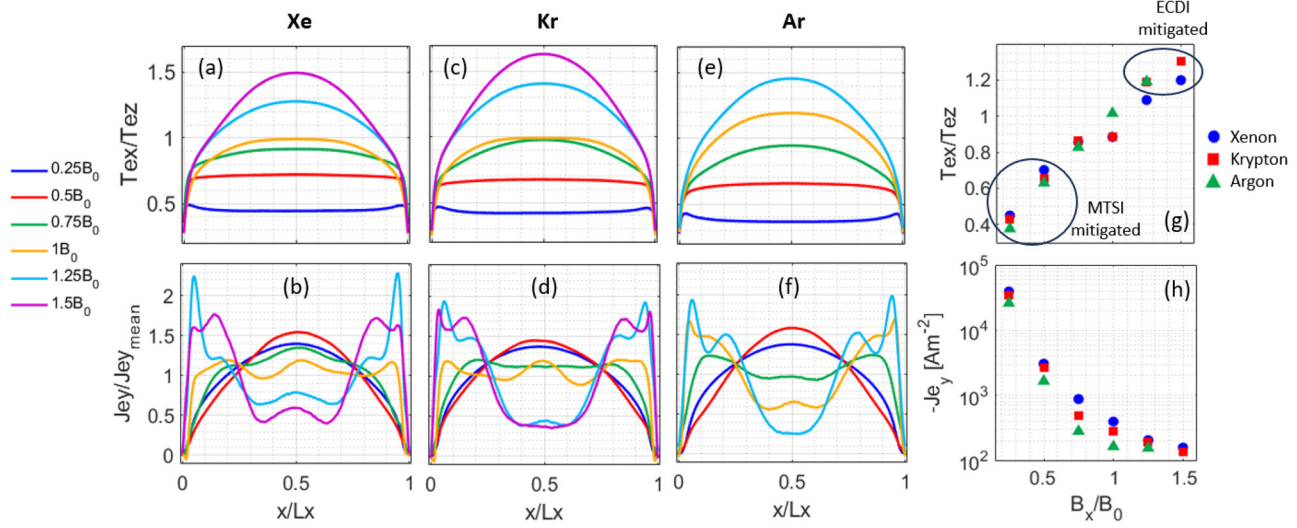


FIG. 5. (a)–(f) Radial profiles of the radial-to-azimuthal electron temperature ratio (T_{ex}/T_{ez}) (top row) and the normalized axial electron current density ($J_{ey}/J_{ey,mean}$) (bottom row) averaged over all azimuthal locations and over the time interval of 20–30 μs from the Q2D simulations with various axial magnetic fields and propellants. (g) Variation of the radially averaged values of T_{ex}/T_{ez} vs B_x/B_0 from the Q2D simulations with various propellants. (h) Variation of the radially averaged values of $-J_{ey}$ vs B_x/B_0 from the Q2D simulations with various propellants.

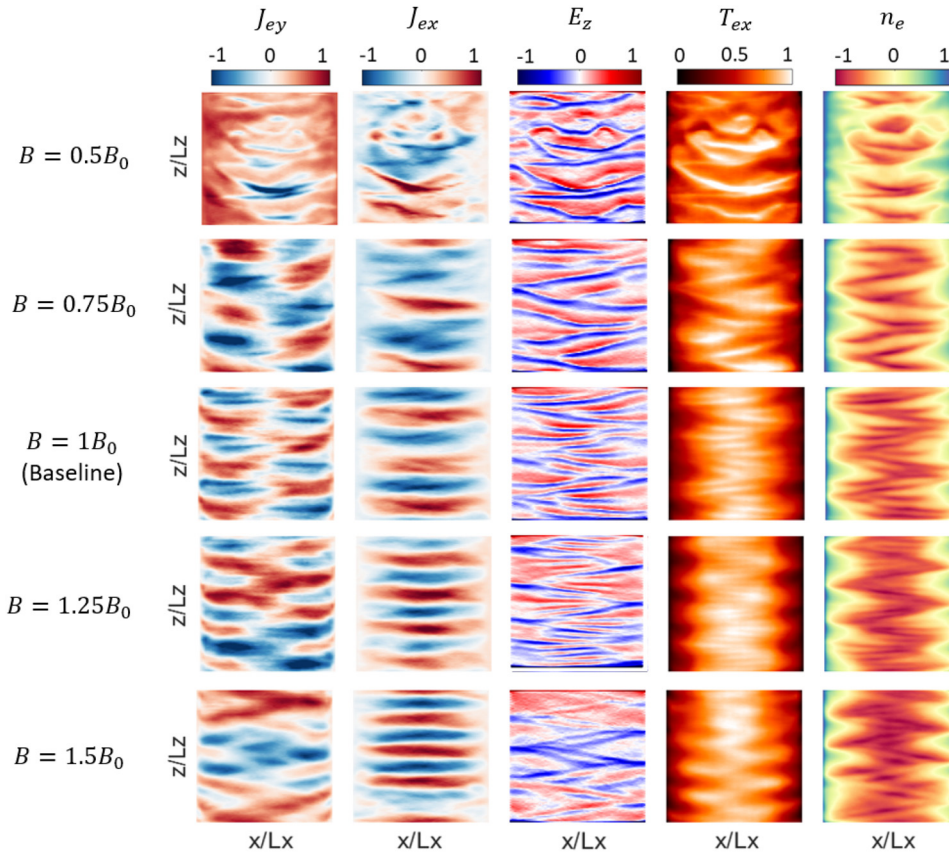


FIG. 6. Comparison of the 2D snapshots of the normalized plasma properties at a time of local maximum of the spatially averaged radial electron temperature for various radial B -field intensities and the xenon propellant. The columns, from left to right, represent the axial and radial electron current densities (J_{ey} and J_{ex}), the azimuthal electric field (E_z), the radial electron temperature (T_{ex}), and electron number density (n_e).

MTSI and the ECDI modes in a simulation. In addition, as was observed in Paper I¹³ and also in Ref. 41, the axial electron current induced by the MTSI is mainly concentrated near the walls as opposed to that carried by the ECDI, which is mostly focused in the central region of the domain.

Accordingly, looking at Fig. 5, we can see that, for the high values of the magnetic field ($1.25 B_0$ and $1.5 B_0$), $\frac{T_{ex}}{T_{ez}} > 1$, and the J_{ey} is mainly the largest near the walls for all the propellants. For the case of xenon with $B_x = 1.25 B_0$, the spikes in the J_{ey} near the walls are more pronounced than for the other two propellants. For these cases, we can, therefore, deduce that the MTSI shall be stronger than the ECDI. In Sec. IV B, we demonstrate from the spectral analyses that the ECDI is, in fact, highly mitigated at $B_x = 1.5 B_0$ for xenon and krypton (argon simulation not carried out) and at $B_x = 1.25 B_0$ for krypton and argon.

For low values of the magnetic field ($0.25 B_0$ and $0.5 B_0$), $\frac{T_{ex}}{T_{ez}} < 1$, and the J_{ey} is maximum at the centerline for all the propellants. This suggests that the ECDI must be the dominant mode under these magnetic field conditions. We will see in Secs. IV B and V A that the MTSI is mitigated in the low-magnetic-field intensity range.

At the intermediate magnetic field values ($0.75 B_0$ and $1 B_0$), particularly for xenon and krypton, $\frac{T_{ex}}{T_{ez}} \approx 1$, and the J_{ey} has a more uniform distribution across the radial extent of the domain. This implies that both the MTSI and the ECDI develop with comparable intensities in these cases. Nevertheless, for the argon simulation case with $B_x = 1 B_0$, the MTSI seems to be slightly stronger than the ECDI as the

ratio $\frac{T_{ex}}{T_{ez}}$ is moderately above unity, and the J_{ey} is larger toward the walls.

A sample set of 2D snapshots of various plasma properties are presented in Fig. 6. These snapshots are from the simulations with xenon and with the different magnetic field intensities. The snapshots for each B_x case correspond to an instance of time when the spatially averaged T_{ex} over the entire domain is at its local maximum. The snapshots from the $0.25 B_0$ case are not shown here, because they did not show distinguishable coherent structures as in the other cases. For the $0.5 B_0$ case, MTSI-like patterns are not observed, and only azimuthal waves are visible. The simulation cases with $0.75 B_0$ and $1 B_0$ feature both the ECDI-representative waves with short azimuthal wavelengths and the radial-azimuthal MTSI-like structures of longer azimuthal wavelengths. The ECDI waves become less distinct in the $1.25 B_0$ case and are not observed in the $1.5 B_0$ case. Furthermore, it is noteworthy that, at the high-magnetic-field-intensity limit ($1.25 B_0$ and $1.5 B_0$), a wave structure with a long azimuthal wavelength is seen to be developing as well.

In Fig. 7, we compare the 2D snapshots of the J_{ey} and the E_z from the simulations with the three propellants and the various studied magnetic field intensities. From these snapshots, we can qualitatively observe the overall similarity between the 2D spatial distributions of these two plasma properties among the different propellants for most of the magnetic field magnitudes. A disparity is noticed, however, for the $1.25 B_0$ case. In this case, the azimuthal ECDI waves are less

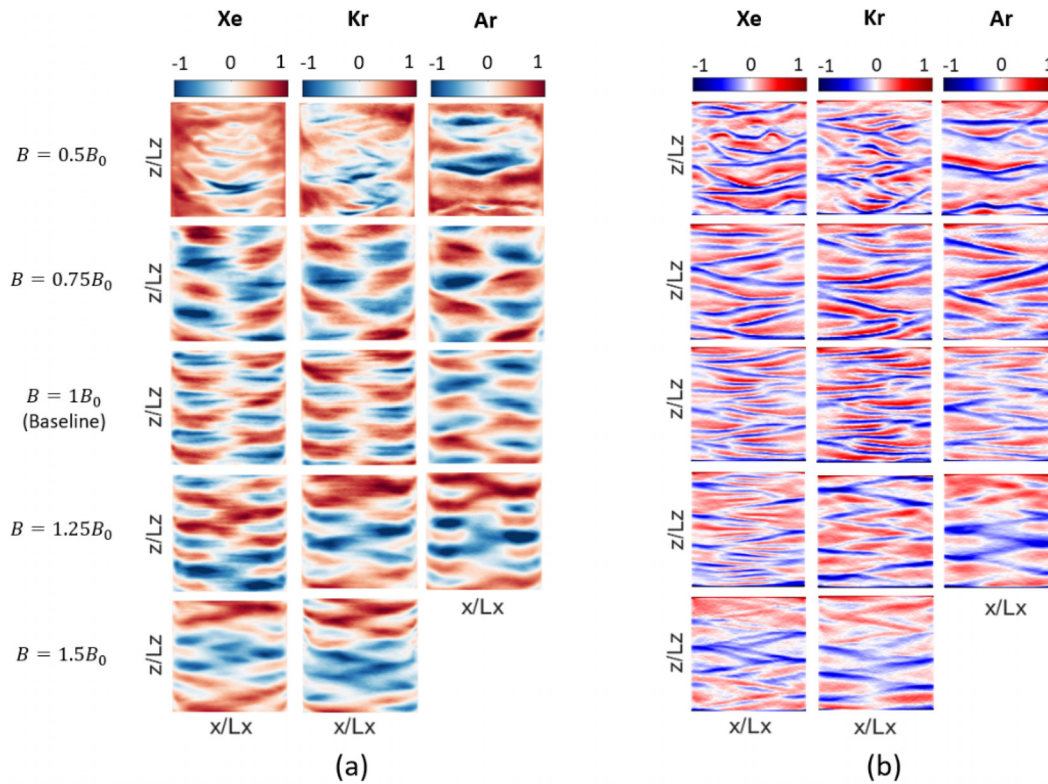


FIG. 7. Comparison of the 2D snapshots of (a) normalized radial electron current density (J_{ey}), and (b) the normalized azimuthal electric field (E_z) at a time of local maximum of the spatially averaged radial electron temperature for various radial B -field intensities and the three propellants.

pronounced for krypton than for xenon. Moreover, for argon, the ECIDI waves seem to be so weak as not to be noticeable.

With the above observations in mind, we present in Sec. IV B a comprehensive evaluation of the instabilities' characteristics by referring to the FFT and the DMD spectra.

B. Variation in the characteristics and the structure of the azimuthal instabilities

The averaged spatial FFT spectra of the azimuthal electric field (E_z) from the simulations with different B -field intensities and propellants are presented in Fig. 8. The wavenumbers are nondimensionalized with respect to the azimuthal wavenumber of the ECIDI's first harmonic (k_0) given by Eq. (2) (with $N = 1$, and using the values of the imposed E_y and B_x). We can see from the plots in Fig. 8 that, for the low-magnetic-field values ($0.25 B_0$ and $0.5 B_0$ cases) and for all the propellants, only the ECIDI mode is dominantly visible at $\frac{k_z}{k_0} \approx 1$.

For the simulation cases with the B -field intensity of $0.75 B_0$ and $1 B_0$, the theoretical range of the MTSI's azimuthal wavenumber from Eq. (5) is around $k_z \approx 0.1$ – 0.25 . This range is highlighted on the plots of Fig. 8 by gray-shaded areas. The theoretical k_z values are obtained from Eq. (5) by substituting the value of the radial wavelength (k_r) of the MTSI mode. The k_r of the MTSI from our simulations is about 245 rad/m because the captured MTSI waves have a radial wavelength twice the radial size of the domain. From the $0.75 B_0$ case, the FFT spectra for all the propellants start to show peaks near the normalized theoretical azimuthal wavenumber of the MTSI at $k_z/k_0 \approx 0.25$. For the $0.75 B_0$ case, the ECIDI mode's magnitude is comparatively lower in krypton than in xenon, and it reaches its lowest value for argon.

For the case $B_x = 1 B_0$, the FFTs for all the propellants show both the ECIDI and the MTSI peaks. Nevertheless, for the case of argon, the intensity of the ECIDI is nearly equal to that of the MTSI in contrast to the cases with xenon and krypton for which the ECIDI is stronger than the MTSI.

In the high magnetic field range ($1.25 B_0$ and $1.5 B_0$), the MTSI is clearly present with high amplitude. Another mode at a lower wavenumber and a comparable strength to the MTSI is also visible. We refer to this mode as the long- λ mode. This long- λ instability mode had been observed in our previous studies^{41,42} as well with a similar wavelength and a frequency of approximately 1 MHz. The long- λ mode was observed in Refs. 41 and 42 in the presence of strong radial magnetic field gradients, significant secondary electron emission (SEE), or at high number density conditions. The exact origin of this instability is still not fully understood; however, it is conjectured to be the result of an inverse energy cascade of shorter wavelength instabilities within the system, at least in certain conditions.⁴¹

At the highest magnetic field condition ($B_x = 1.5 B_0$), the ECIDI is not visible in the spatial FFT spectra of Fig. 8. In this regard, we refer to Sec. 3 of the Appendix, where we have compared the spatial and the temporal FFT spectra of the Q2D $1.5 B_0$ simulation with the xenon propellant against a corresponding full-2D simulation. In Sec. 3 of the Appendix, the k_z spectrum from the full-2D simulation exhibits a very small peak at the normalized wavenumber of the ECIDI's first harmonic. Accordingly, we conclude that the ECIDI is not entirely absent in the $1.5 B_0$ condition. Rather, the ECIDI mode is strongly suppressed to a very low amplitude. At such a low amplitude, the ECIDI is likely falling close to the noise level of the Q2D simulations, which makes it undetectable from the spatial FFT of the Q2D results.

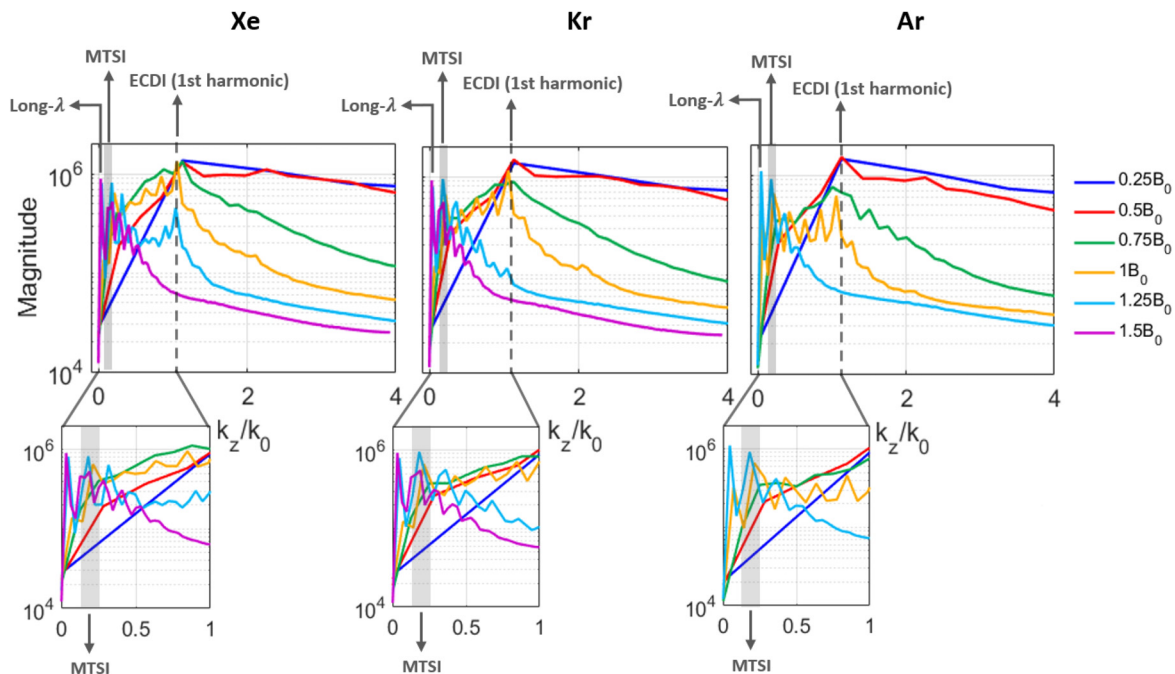


FIG. 8. 1D spatial FFT plots of the azimuthal electric field signal from the simulations with various values of B_x and the different propellants. The FFTs are averaged over all radial positions and over the time interval of 20–30 μ s.

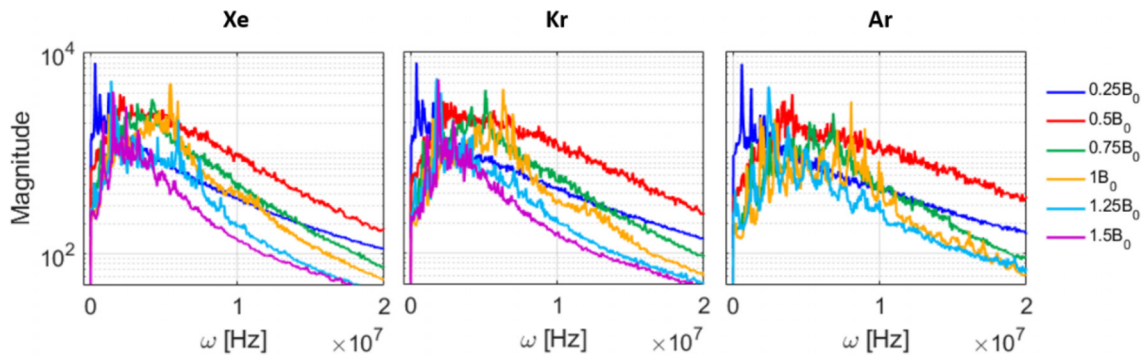


FIG. 9. 1D temporal FFT plots of the azimuthal electric field signal from quasi-2D simulations with various magnitudes of the radial magnetic field and different propellants. The FFTs are averaged over all radial and azimuthal positions.

To determine the frequency spectra of the instabilities, we look at the spatially averaged temporal FFT plots of the E_z signal as is shown in Fig. 9 for the various simulations. It should be noted from the FFT plots in Fig. 9, however, that, in some cases, there is more than one peak within the expected frequency range of the instability of interest. Thus, as was explained in Paper I,¹³ we have additionally used the dynamic mode decomposition (DMD) analysis⁴³ to distinguish the individual instability modes present in the system and to derive unambiguously their spatial structure together with their frequencies.

We would recall that the DMD is an interpretable data-driven method that finds a best-fit linear model describing the time evolution of *global* spatiotemporally coherent structures (dynamic modes) in data. A much more robust variant of DMD, called OPT-DMD, enables reliable identification of the dominant instability modes underlying plasma systems, even in the presence of high noise levels in the data.^{43–45}

The results of the OPT-DMD analysis for the various B -field simulations with xenon propellant are presented in Fig. 10. The results show the 2D spatial patterns of the dominant instability modes together with their associated frequencies. To identify among all the DMD modes the modes representing the MTSI and the ECDI, we have performed a cross-comparison between the wavenumber and the frequency of the DMD modes against the wavenumber and the frequency of the peaks in the FFT spectra. In case, a clear peak associated with an instability would have not been detectable, especially from the spatial FFT, the theoretical wavenumbers have been used. In Sec. 4 of the Appendix, we have additionally provided the OPT-DMD characterization results of the ECDI and the MTSI modes for the simulations with the different propellants.

In line with our conclusions from the spatial FFT analysis (Fig. 8), in the low-magnetic-field conditions ($0.25 B_0$ and $0.5 B_0$ cases), the MTSI is mitigated and is, thus, not detectable. This is because, in these two B_x cases, the DMD analysis does not show any mode with a radial–azimuthal structure resembling the MTSI. All the DMD-discovered modes in these two B_x cases amount to azimuthal fluctuations, which appear to be somewhat turbulent. In fact, the azimuthal fluctuations for the $0.25 B_0$ and $0.5 B_0$ cases are not as clean as the ECDI modes observed in the cases with the larger magnetic field values. Another interesting characteristic of the ECDI and other azimuthal modes in the $0.25 B_0$ and $0.5 B_0$ cases is that the amplitude of the fluctuations for each mode appears to be nearly uniform across the

domain (this is also the case for the $0.75 B_0$ case). On the contrary, for the higher B_x values ($1 B_0$ and $1.25 B_0$), the intensity of the ECDI modes is higher near the center of the domain along the radial direction.

At the highest simulated magnetic field, namely, the $1.5 B_0$ case, the DMD does not discover any mode with purely azimuthal oscillations that can be representative of the ECDI. This is again consistent with the absence of a clear peak near the first harmonic of the ECDI from the spatial FFT plots in Fig. 8.

For the $0.75 B_0$, $1 B_0$, and $1.25 B_0$ cases, DMD discovers both the ECDI-like and the MTSI-like modes. For these cases, also for the B_x of $1.5 B_0$, Fig. 10 shows several modes that feature fluctuations with both the radial and the azimuthal wavenumber components. As was pointed out in Paper I,¹³ these modes may represent the intermediate stages of discharge evolution during which the dominant instability modes are transitioning to one another. Finally, for the cases of $1.25 B_0$ and $1.5 B_0$, the long- λ mode is identified with a frequency of about 1.5 MHz.

V. DISCUSSIONS

In this section, we first compare the numerically observed characteristics of the resolved ECDI and MTSI modes for the different B -field intensities against the predictions of the corresponding theories for these two instability modes. Second, for the xenon propellant as an example, we discuss the contributions of the various momentum force terms to the electrons' cross-field transport, including the roles of the instabilities, the viscosity, and the convective inertia forces. Third, we analyze, for the different propellants, how the change in the instabilities' spectra with the magnetic field intensity impacts the plasma particles' velocity distribution functions along the radial and azimuthal velocity components.

A. Comparison of the simulated instabilities' characteristics with theory

The variations of the normalized characteristics of the dominant ECDI and MTSI modes vs B_x/B_0 are plotted in Fig. 11 for the three studied propellants. The instabilities' characteristics are obtained from the FFT and the DMD analyses of the Q2D simulations' results. The characteristics of interest are the nondimensional azimuthal wavenumber ($k_z \lambda_D$), the nondimensional frequency (ω/ω_{pi}), and the nondimensional azimuthal phase velocity (V_{ph}/C_s).

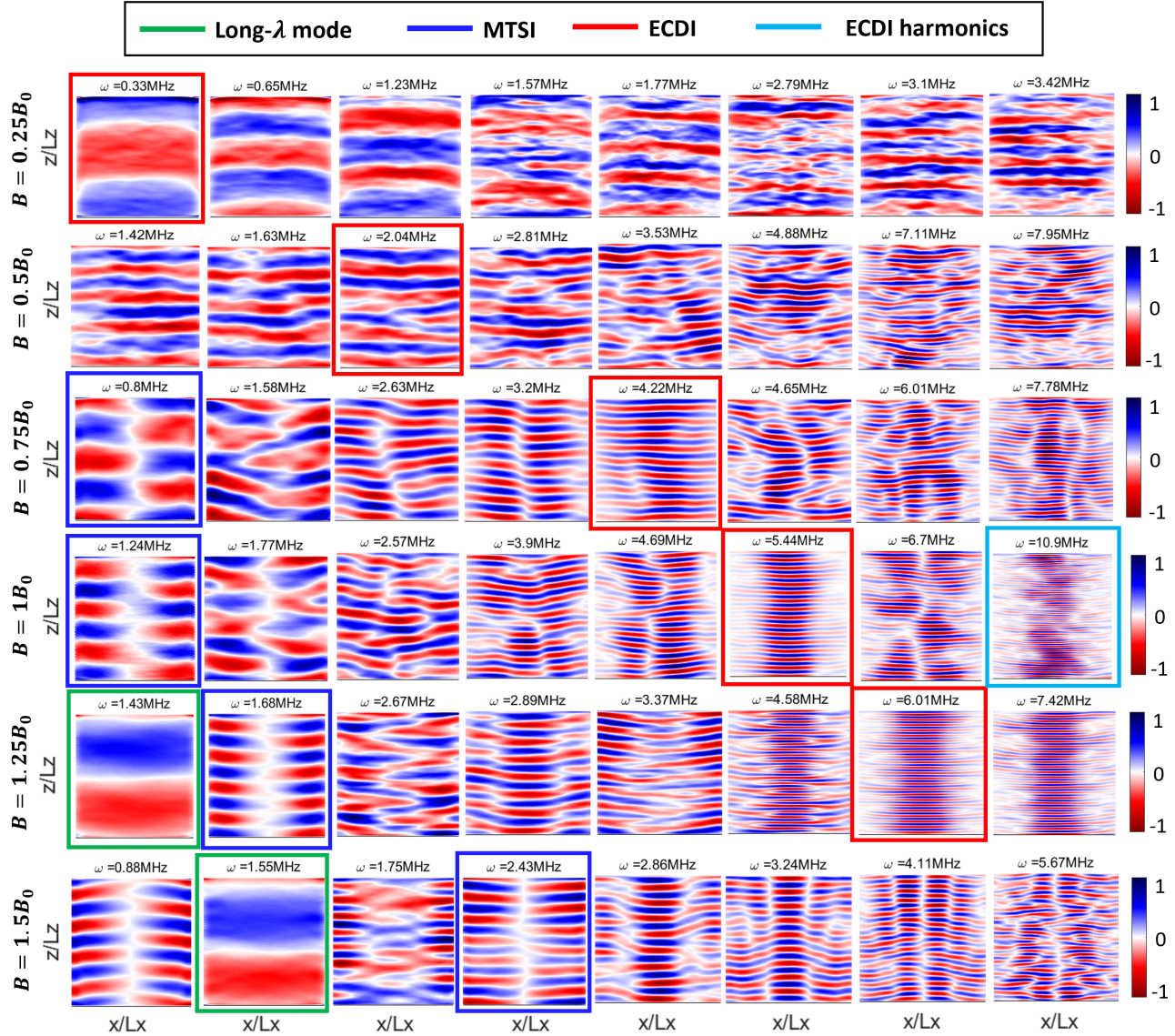


FIG. 10. Visualization of the several dominant DMD modes of the azimuthal electric field data from the radial–azimuthal simulations with different B_x values and the xenon propellant. The approach pursued to derive these modes is explained in detail in Ref. 43. The modes identified with a box correspond to those with wavelengths equal to the wavelengths of the dominant modes visible in the spatial FFT plot in Fig. 8.

To compare the instabilities' simulated characteristics against the theoretically predicted ones from the analytical relations reviewed in Sec. II, the plots in Fig. 11 also show the theoretical data points for the azimuthal wavenumber of the ECDI's first harmonic [Eq. (2)], as well as for the frequency, the wavenumber, and the phase velocity of the fastest growing MTSI mode from Eqs. (4) and (5). These data points are shown using hollow markers. On all the ECDI-related plots, the characteristics of the fastest growing ion acoustic instability mode from Eq. (3) are additionally superimposed as horizontal dotted black lines. It is also pointed out that, for the simulation cases in which the MTSI or the ECDI mode was not clearly detected from the FFT/DMD

analyses (the modes were strongly mitigated), the plots in Fig. 11 only present the theoretical data points.

To discuss the observations from Fig. 11, we first focus on the ECDI mode. We observe from the left-column plots in Fig. 11 that, as the magnetic field intensity increases, the normalized wavenumber of the dominant ECDI mode increases. Furthermore, the slight disparities in $k_z \lambda_D$ among the various propellants are also noticed to rather become more visible with the B_x/B_0 . The nondimensional frequency of the dominant ECDI mode shows an increasing trend vs B_x/B_0 within the field intensity range of $0.25 B_0$ to $1 B_0$. The ECDI's normalized frequency does not show a major variation across the different

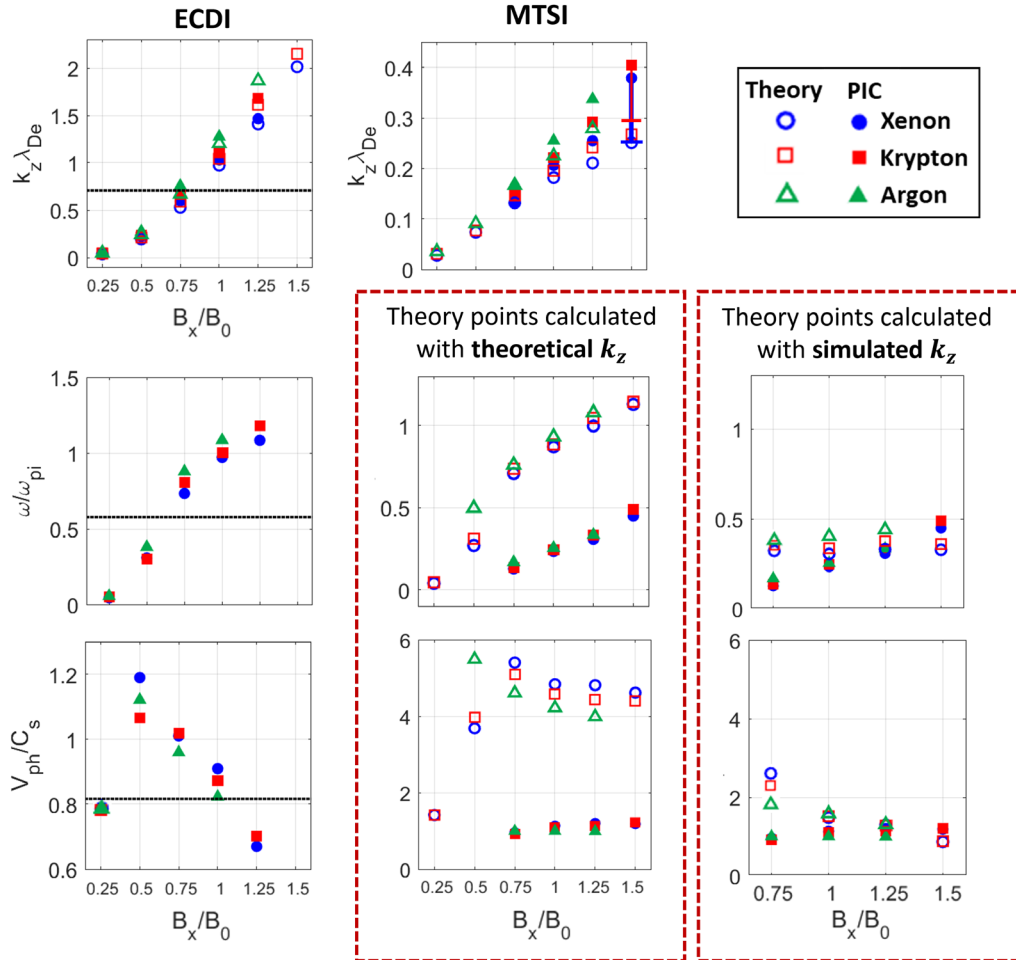


FIG. 11. Variation of the normalized characteristics of the dominant ECDI and the MTSI modes vs B_x/B_0 from the radial–azimuthal simulations with the different propellants. (Top row) normalized azimuthal wavenumber ($k_z \lambda_{De}$) compared against the theoretical data points from Eqs. (2) and (5) for the wavenumber of the ECDI's first harmonic mode and the MTSI's fastest growing mode, respectively; (middle row) normalized real frequency (ω/ω_{pi}); (bottom row) normalized azimuthal phase velocity (V_{ph}/C_s). Superimposed on the ECDI plots, the horizontal dotted black lines indicate the characteristics of the fastest growing mode of the ion acoustic instability from Eq. (3). The hollow markers on the MTSI frequency plot correspond to the solution to the instability's dispersion relation given by Eq. (4); the theoretical data points on the middle-column frequency plot are from solving Eq. (4) with the azimuthal wavenumbers from Eq. (5), whereas the theoretical points on the right-column frequency plot are from solving Eq. (4) using the simulated azimuthal wavenumbers. The same is true for the phase velocity (bottom row, middle, and right-most columns).

propellants. Finally, the normalized phase velocity exhibits an increasing-decreasing trend with the B_x/B_0 , with the peak value occurring at the magnetic field intensity of $0.5 B_0$. The ion mass is seen to minimally affect the V_{ph}/C_s variations of the ECDI mode with the B_x , with the most noticeable difference being observed among the three propellants at $B_x = 0.5 B_0$.

With regard to the theoretical comparisons, the B -field dependency of the ECDI's normalized wavenumber from the simulations [Fig. 11 (top row, left column)] is in very good agreement with the theory. From Eq. (2), the theoretical wavenumber of the ECDI's first harmonic has a quadratic relationship with the B_x , which is seen to be also quite well captured by the simulated wavenumbers as well across the B_x value.

Another comparison that we would like to make from the ECDI's characteristic plots is with respect to the theoretical characteristics of

the fastest growing IAI mode (the horizontal dotted black lines). In a similar manner to Paper I,¹³ we are interested to understand whether the ECDI modes show possible signs of transition to the ion acoustic instability from any of the various B -field simulations. From the plots in Fig. 11, and considering all the normalized wave properties, the simulated characteristics of the ECDI at the B_x/B_0 values of 0.75 and 1 are overall close to the characteristics of the IAI's fastest growing mode from Eq. (3).

To evaluate more closely possible similarities between the simulated ECDI's and the theoretical IAI's wave characteristics, we have shown the 2D dispersion plots of the azimuthal electric field fluctuations in Fig. 12 for the various B -field simulations and with the xenon propellant as an example. The theoretical dispersion relation of the IAI [Eq. (3) from Paper I¹³] is superimposed on all the plots in Fig. 12. It is indeed noticed that the cases with the $B_x/B_0 = 0.75$ and 1 show a

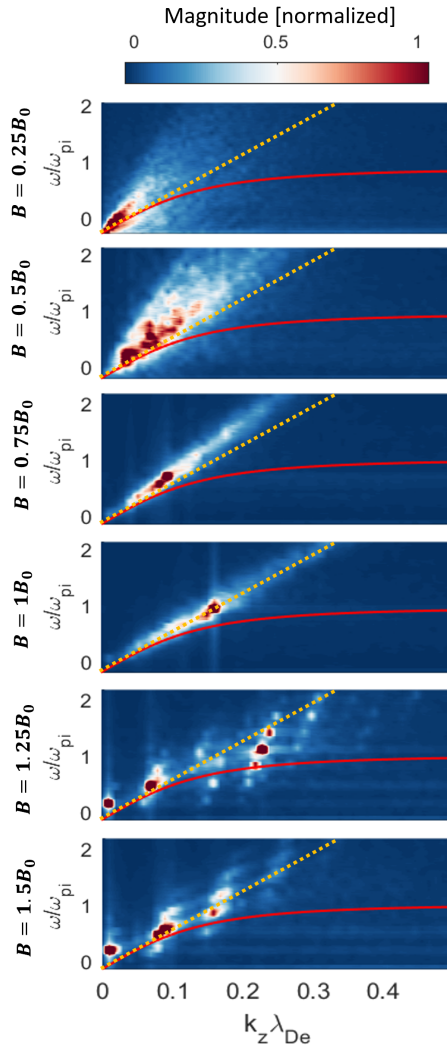


FIG. 12. Dispersion plots of the azimuthal electric field fluctuations from the simulations with various B_x values and the xenon propellant obtained from the 2D spatio-temporal FFT of the azimuthal electric field signal at the mid-radial location of the domain over $t = 20\text{--}30\ \mu\text{s}$. The superimposed red curves are the theoretical dispersion relation of the ion acoustic instability (see Paper I¹³) calculated using the time-averaged (within $20\text{--}30\ \mu\text{s}$ window) plasma properties. In each plot, the dashed yellow line is the tangent to the linear part of the ion acoustic dispersion relation.

resonance-broadening feature around the mode(s) with the largest magnitude. Such resonance broadening has been reported to occur when the ECDI modes transit to the IAI.³³ Nevertheless, the combined results from Figs. 11 and 12 are not fully conclusive so as to assert the ECDI modes' definitive transition to the IAI at the B_x/B_0 values of 0.75 and 1. In addition, the quite turbulent 2D dispersion plots of the fluctuations for the $0.25 B_0$ and the $0.5 B_0$ simulation cases are also noted.

Returning to Fig. 11, we now assess the variations in the MTSI mode's characteristics across the various B_x values. It is recalled that the MTSI was not observed as being distinctly present from the FFT and the DMD analyses of the azimuthal electric field from the simulations with the B -field intensities of $0.25 B_0$ and $0.5 B_0$. The MTSI was,

however, detected from $B_x/B_0 = 0.75$ onward, and the plots in Fig. 11 show that both its normalized wavenumber and frequency almost linearly increase with the magnetic field. As the B_x increases, the $k_z \lambda_D$ values for the different propellants become rather more disparate. However, the ω/ω_{pi} is noticed to be quite invariant with the ion mass across the B_x/B_0 values. The MTSI's V_{ph}/C_s characteristic is observed to be a rather weak function of the B_x and does not show any major variation among the different studied propellants.

With regard to the theoretical comparisons, the MTSI's simulated $k_z \lambda_D$ shows a good agreement with the theoretical relation [Eq. (5)] up to the B_x/B_0 of 1. At the higher B_x/B_0 values, the simulated wavenumbers show some deviation from the theory with a major discrepancy at $B_x = 1.5 B_0$.

With regard to the large discrepancy at $1.5 B_0$, it is first pointed out that we have taken the highest peak within the MTSI wavenumber range, as was highlighted in plots of the spatial FFT in Fig. 8, to represent the azimuthal wavenumber of the MTSI from the simulations. For the $1.5 B_0$ case, the spatial FFT spectra show a peak at $k_z/k_0 = 0.188$, which we selected as the simulated MTSI wavenumber. However, there also exists another nearby peak at $k_z/k_0 = 0.125$ with a slightly lower amplitude. This second, lower-amplitude peak coincides with the theoretical wavenumber of the MTSI from Eq. (5) for the $1.5 B_0$ case. Because it cannot be said with full certainty which one of the two peaks definitely corresponds to the MTSI in the simulation, we have added an error bar in the $k_z \lambda_D$ plot of Fig. 11 for the $1.5 B_0$ case. The error bar covers the wavenumber interval between the two identified peaks.

To ensure that this special double-peak feature of the FFT spectrum is not due to the Q2D approximation of the problem, we verified the Q2D simulation of the $1.5 B_0$ case with the xenon propellant against a full-2D simulation of the same case in terms of the spatial and temporal FFT spectra of the instabilities. The results are provided in Subsec. 3 of the Appendix. The 2D simulation also shows a broadening of the k_z/k_0 spectrum near the MTSI's azimuthal wavenumber (within $k_z/k_0 = 0.125\text{--}0.188$). The consistency between the Q2D and the 2D simulations bolsters our confidence in the Q2D results and in the derived insights.

As another point, we discussed at length in Paper I of the article¹³ how a slight variation in the theoretical value of the MTSI's azimuthal wavenumber can drastically change the theoretically predicted values of the frequency, and, in turn, the phase velocity, because Eq. (5) describes the MTSI's theoretical wavenumber at the resonance condition of the instability. In this regard, we observe from the results in Fig. 11 as well (middle- and right-most-column plots for the ω/ω_{pi} and V_{ph}/C_s) that if we use the MTSI's simulated wavenumber to solve the dispersion relation in Eq. (4) to obtain the theoretical frequency and subsequently the theoretical phase velocity, the simulations' predictions of these two characteristics are quite comparable with the theoretical predictions. However, if we use the theoretically predicted MTSI's wavenumber to solve Eq. (4), which is in any case rather similar to the simulated values at least in the range of $0.75 B_0$ to $1 B_0$, the theoretical frequencies and phase velocities become notably disparate compared to the simulations' predictions.

Finally, to explain why the MTSI is mitigated in the cases with low B -field intensities, i.e., for $B_x/B_0 < 0.75$, we have plotted in Fig. 13 the variation of the normalized MTSI's growth rate (γ/ω_{pi}) against the normalized azimuthal wavenumber ($k_z = k_z \lambda_D$) and the normalized

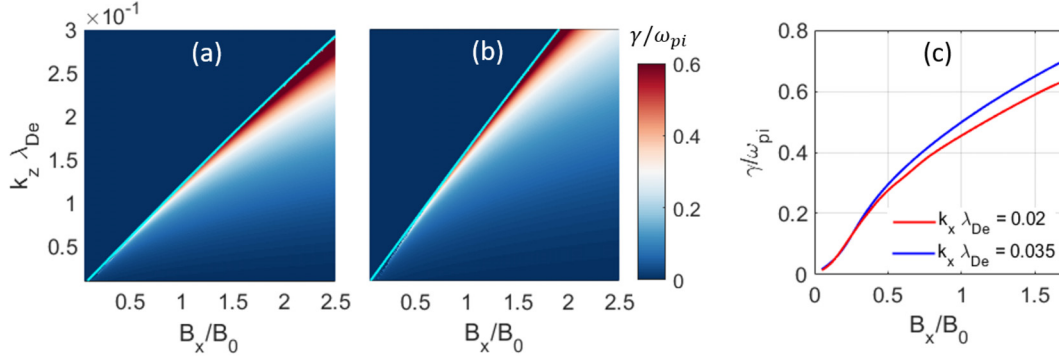


FIG. 13. Variation of the normalized MTSI's growth rate (γ/ω_{pi}) vs the normalized azimuthal wavenumber ($\tilde{k}_z = k_z \lambda_{De}$) and the B_x/B_0 obtained from the solution of the MTSI's dispersion relation in Eq. (4) for the xenon propellant. To solve the dispersion relation, we have used $n_i = 1.5 \times 10^{17} \text{ m}^{-3}$, $T_e = 30 \text{ eV}$, and $E_y = 10 \text{ kV m}^{-1}$. The dispersion relation is solved for two constant normalized radial wavenumber values ($\tilde{k}_x = k_x \lambda_{De}$) of (a) $\tilde{k}_x = 0.02$, and (b) $\tilde{k}_x = 0.035$. The cyan curves in plots (a) and (b) indicate the azimuthal wavenumber of the MTSI's fastest growing mode as is determined from Eq. (5). Plot (c) shows the variation of the normalized MTSI's growth rate vs B_x/B_0 with the two selected values of $\tilde{k}_x$.

radial magnetic field. The growth rate is obtained by solving the MTSI's dispersion relation as is given by Eq. (4) at two fixed values of the normalized radial wavenumber ($\tilde{k}_x = k_x \lambda_{De}$) of 0.02 and 0.035. These specific $\tilde{k}_x$ values are selected because they indicate the two bounds of $\tilde{k}_x$ for the MTSI modes that were developed in our simulations across the various magnetic field intensities. The MTSI's $\tilde{k}_z$ is varied from about 0.02 to 0.2 as the B_x increases from $0.25 B_0$ to $1.5 B_0$.

From the plots in Fig. 13, we observe that in the low-magnetic-field range, the growth rate of the MTSI's fastest growing mode is small, but it increases significantly as the magnetic field becomes stronger. This is consistent with our numerical observations that stronger MTSI modes occur at the higher magnetic field magnitudes.

B. Variation in the electrons' cross-field transport

In this section, we compare the significance of the roles played by various electron current carrying mechanisms as the magnitude of the B -field changes. This is done as an example for the xenon propellant. The mechanisms of interest are represented by the individual force terms in the electrons' azimuthal momentum equation^{13,18,46}

$$F_B = F_t + F_I + F_{\Pi} + F_E. \quad (6)$$

Equation (6) includes the temporal and the convective inertia forces (F_t and F_I , respectively), a force due to the non-diagonal pressure terms, i.e., the viscosity (F_{Π}), and the electric force term (F_E), which accounts for the momentum change in the electron species induced by the azimuthal instabilities. Additionally, the magnetic force term (F_B), which appears on the left-hand side of the azimuthal momentum equation, represents the aggregate effect of all the right-hand-side force terms mentioned above. The right-hand-side force terms are defined as

$$F_B = -qn_e v_{e,y} B_x, \quad (7)$$

$$F_t = \frac{\partial}{\partial t} (mn_e v_{e,z}), \quad (8)$$

$$F_I = \frac{\partial}{\partial x} (mn_e v_{e,x} v_{e,z}), \quad (9)$$

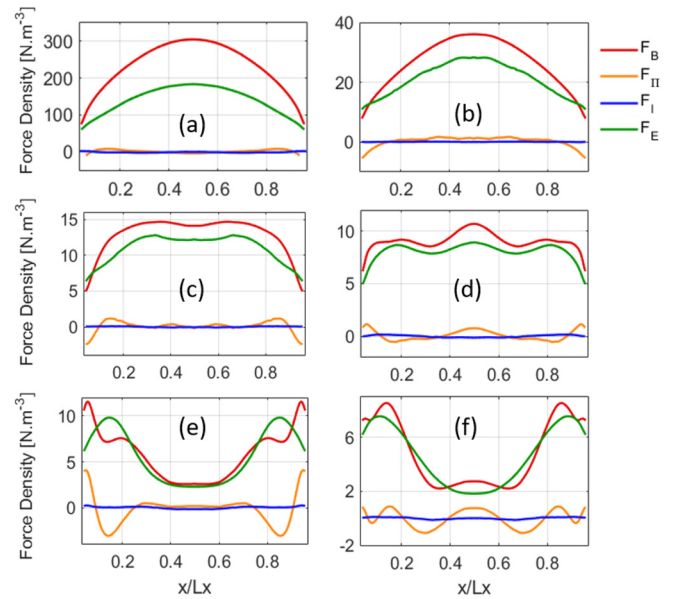


FIG. 14. Radial distribution of the force terms [Eqs. (7)–(11)] in the electrons' azimuthal momentum equation [Eq. (6)] for the xenon propellant and various values of the radial magnetic field: (a) $0.25 B_0$, (b) $0.5 B_0$, (c) $0.75 B_0$, (d) $1 B_0$, (e) $1.25 B_0$, and (f) $1.5 B_0$. The momentum terms are averaged over $10 \mu\text{s}$ of the simulations' time.

$$F_{\Pi} = \frac{\partial}{\partial x} (\Pi_{e,xz}), \quad (10)$$

$$F_E = -q\tilde{n}_e \tilde{E}_z. \quad (11)$$

In Eqs. (7)–(11), m is the electron mass, n_e is the electron number density, $v_{e,x}$ and $v_{e,z}$ are the electrons' radial and azimuthal drift velocities, and $\tilde{n}_e$ and $\tilde{E}_z$ are the number density and the azimuthal electric field fluctuations.

The time-averaged (over $10 \mu\text{s}$) radial profiles of all the above force terms and their average values over the radial extent of the

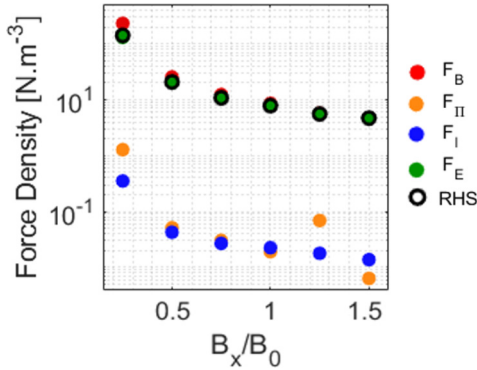


FIG. 15. Variation of the radially averaged value of each force term in the electrons' azimuthal momentum equation [Eq. (6)] vs B_x/B_0 from the radial-azimuthal simulations with xenon propellant. The hollow black dot represents the sum of all terms on the right-hand side of the azimuthal momentum equation.

domain are shown, respectively, in Figs. 14 and 15 for the various B -field simulations with xenon propellant. In these two figures, F_I is not shown due to its vanishingly small contribution.

From Figs. 14 and 15, it is evident that the F_E term is distinctly dominant for all the cases. In fact, the average value of the F_E term is more than two orders of magnitude larger than the value of the other terms across all the B_x intensities. The radial distributions of the F_B and the F_E terms for the $0.25 B_0$ and $0.5 B_0$ cases show a convex profile with the maximum occurring at the center. This is consistent with the transport profiles that we expect when the ECDI is the dominant instability in the radial-azimuthal configuration. For the $0.75 B_0$ and $1 B_0$ cases, the superposition of the contributions of the ECDI and the MTSI to the electron transport creates a more radially uniform F_E distribution. Particularly for the $1 B_0$ case, where the MTSI and the ECDI were observed in Sec. IV B to be of more comparable intensities, the F_E profile contains two peaks away from the center indicating the MTSI's transport and a peak at the center, which represents the transport carried mainly by the ECDI. Finally, the profiles of the F_E term for the $1.25 B_0$ and the $1.5 B_0$ conditions show peaks near the edges of plasma as a result of the strong MTSI contribution.

The role of the convective inertia (F_I) term is negligible for all the cases. The viscous term (F_{II}) is most significant in the $1.25 B_0$ and $1.5 B_0$ conditions. In this respect, although the radial average of the F_{II} term is still small compared to the F_E in these cases (Fig. 15), the local value of this term along the radius has a noticeable effect on the total momentum distribution captured by the F_B term in the high magnetic-field-intensity conditions [Figs. 14(e) and 14(f)]. In detail, in the $1.25 B_0$ case, the F_{II} accounts for nearly two-thirds of the total momentum (F_B) adjacent to the walls, and it becomes negative immediately after the near-wall region toward the center of the domain, where it counteracts the F_E term and, thus, reducing the F_B . The F_{II} term then assumes a nearly constant and slightly positive value in the central part of the domain. In the $1.5 B_0$ condition, the F_{II} demonstrates an oscillatory behavior across the domain reaching its peak relative significance at the centerline where it creates a bump in the F_B profile.

As one last point concerning the plot in Fig. 15, the discrepancy between the left-hand side (F_B) and the sum of the force terms on the

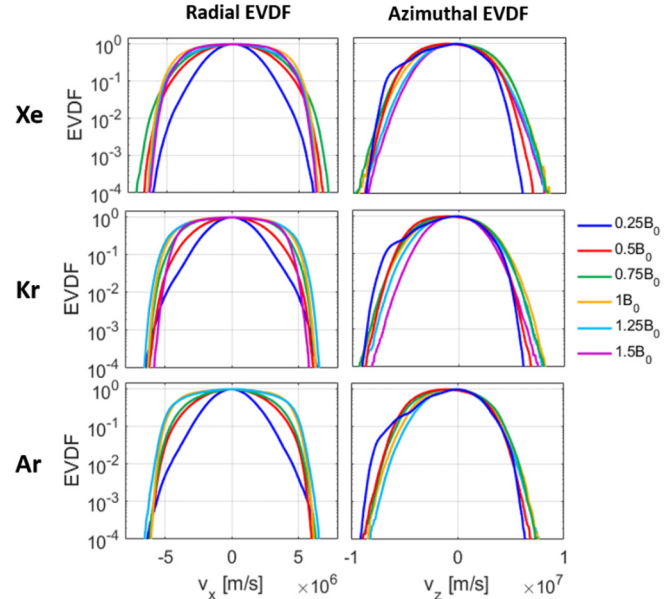


FIG. 16. Normalized radial and azimuthal velocity distribution functions of the electrons for various radial magnetic field values and the three studied propellants from the Q2D simulations.

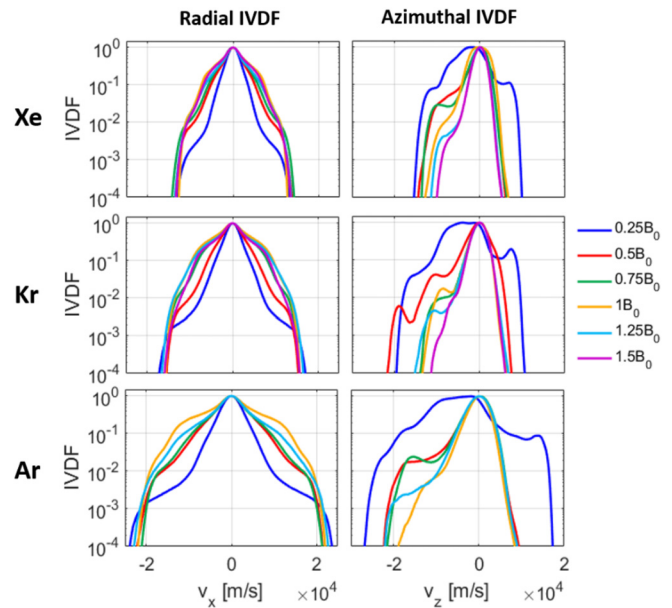


FIG. 17. Normalized radial and azimuthal velocity distribution functions of the ions for various radial magnetic field values and the three studied propellants from the Q2D simulations.

right-hand side (RHS) of the azimuthal momentum equation from the radial-azimuthal simulations at the low B_x intensities has been traced to be due to the momentum loss induced in the system by the introduction of an artificial boundary along the axial direction (Sec. III). Similar

to what was observed in Paper I of this paper,¹³ the imbalance between the right-hand side and the left-hand side is particularly pronounced for the cases that exhibit relatively larger axial electron currents ($B_x < 1B_0$) due to the higher particles' resampling rates in these cases.

C. Plasma species' distribution functions

To investigate how the changes in the dominant instability modes with the B_x intensity affect the plasma particles' velocities, we have provided, in Figs. 16 and 17, respectively, the radial and the azimuthal velocity distribution functions of the electrons and the ions at $t = 30 \mu\text{s}$. The VDFs are calculated for the plasma particles residing over the entire domain of the simulations.

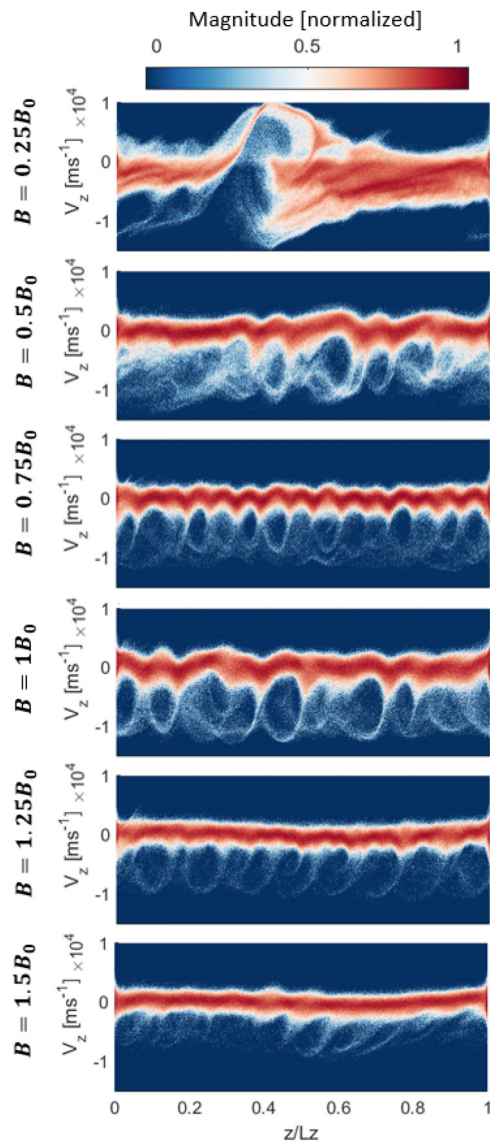


FIG. 18. Ions' azimuthal phase-space plots for various radial magnetic field values and the xenon propellant.

Looking at the electrons' radial VDFs (Fig. 16, left column), we notice that, at the lowest magnetic field intensity ($B_x = 0.25 B_0$), the distributions are closer to Maxwellian for all the propellants. However, for all the other cases, the high-velocity tail of the distribution functions is depleted, and most of the electrons have low- to mid-range velocities. The azimuthal EVDFs (Fig. 16, right column) become increasingly asymmetric with respect to the $v_z = 0$ axis as the B -field decreases. In particular, the azimuthal EVDF for the $0.25 B_0$ case shows a noticeable peak at the negative v_z values, which represents a large population of drifting electrons at relatively high azimuthal drift velocities compared to all the other cases.

Referring to the ions' velocity distribution functions in Fig. 17 (left column), we observe that, in the low B -field range ($0.25 B_0$ and $0.5 B_0$), the radial IVDFs have lost the mid-range velocity ions and retained the ions with high velocity in the tail. In the higher B -field intensity conditions, the mid-range velocity ions are seen to be replenished.

The azimuthal IVDFs in Fig. 17 (right column) show various degrees of broadening due to the ion-wave trapping phenomenon.⁴⁷ As a further evidence of the ion-wave trapping, the ions' azimuthal phase-space plots are provided in Fig. 18. The plots in Fig. 18 indicate that the ions' phase-space distribution is affected by the potential fluctuations of the azimuthal instability waves to different extents depending on the value of the radial magnetic field intensity.

Indeed, the azimuthal IVDF for the case with $B_x = 0.25 B_0$ features a strong broadening, which becomes increasingly more significant from xenon to argon. Unlike the other B_x conditions, and even unlike what was observed for the various E -field cases presented in Paper I,¹³ the azimuthal IVDF of the $0.25 B_0$ case broadens toward both the negative and the positive velocities. As it is seen from Fig. 18, the ions' phase-space plot shows a turbulent distortion of the ions toward both the positive and the negative velocities for the $0.25 B_0$ case. The turbulent behavior could be possibly because some of the azimuthal ECDI waves in this case may have positive phase velocities. For the other B -field intensities, the extent of the azimuthal IVDFs' broadening diminishes with the increasing B_x value, as is also evidenced from the decrease in the significance of the ion-wave trapping from the phase-space plots in Fig. 18.

VI. CONCLUSIONS

The studies in this paper complemented the discussions of Paper I¹³ by assessing in depth the effects that the radial magnetic field intensity have on the behavior of the plasma discharges of various gases—xenon, krypton, and argon. Similar to Paper I,¹³ we carried out the investigations in a 2D configuration representative of a Hall thruster's radial-azimuthal cross section. We also used the reduced-order quasi-2D PIC with a uniform 50-region domain decomposition approximation. The magnetic field value was varied over the range of 5 to 30 mT, with 5 mT increments. Having considered the B -field intensity of 20 mT as the baseline value (B_0), the studied B -field range corresponded to $0.25 B_0$ to $1.5 B_0$. We demonstrated that the changes in the strength of the magnetic field yield three plasma regimes for all the studied propellants.

In regime I associated with relatively low B -field values ($0.25 B_0$ to $0.5 B_0$), we observed the mitigation of the MTSI mode. Consequently, due to the dominance of the ECDI mode, the radially averaged electrons' radial-to-azimuthal temperature ratio (T_{ex}/T_{ez}) was seen to be less than one and the radial distribution of the axial

electron current density peaked around the center of the domain. We showed that the mitigation of the MTSI in this regime rooted in the mode's very small growth rates.

In regime II with moderate values of the magnetic field intensity ($0.75 B_0$ to $1 B_0$), the MTSI was seen to develop with a comparable strength with respect to the ECDI. The mean T_{ex}/T_{ez} ratio in this regime was accordingly observed to be close to 1, and the axial electron current density exhibited a rather uniform radial profile. An exception was observed for the argon propellant for which, at the B -field intensity of $1 B_0$, the MTSI was observed to be stronger than the ECDI. This resulted in the mean T_{ex}/T_{ez} ratio to be slightly larger than 1 and for the radial J_{ey} profile to feature a peak near the walls.

For relatively high values of the B -field ($1.25 B_0$ to $1.5 B_0$) corresponding to regime III, the ECDI was overall observed to be mitigated, and, hence, was not visible from the spectral analyses. The instabilities' spectra in this regime were dominated by the MTSI and a long-wavelength wave mode. For regime III, $\frac{T_{ex}}{T_{ez}}$ was greater than 1, and the radial profile of the J_{ey} had peaks in the vicinity of the walls. For the case of the xenon propellant, the ECDI mode was exceptionally observed to be still detectable from the FFT plots at $B_x = 1.25 B_0$.

The detailed FFT and DMD analyses of the azimuthal electric field fluctuations confirmed the observations summarized above concerning the nature of the dominant instability modes within each of the three plasma regimes—observations that had been inferred from the mean T_{ex}/T_{ez} ratio as well as the radial profile of the J_{ey} . The DMD analyses, in particular, casted light on the spatial structures of the dominant modes that were identified through the FFT plots. Indeed, the DMD method was shown to notably aid in disambiguating the results of the spatial and temporal FFT analyses.

Regarding the aggregated insights from the characterization of the instabilities, we first showed that the dependency of the azimuthal wavenumber of the ECDI's first harmonic on the B -field intensity from the simulations matches well the theoretical relationship. We also noticed that the simplified relation proposed in Ref. 34 for the azimuthal wavenumber of the MTSI's fastest growing mode overall captures the correct correlation with the magnetic field intensity, particularly for the intensity values corresponding to regime II. For plasma regime III, the capability of the relation from Ref. 34 to explain the numerically observed trend of the MTSI's azimuthal wavenumber vs the B -field intensity was debated. The frequencies of the ECDI and the MTSI were noticed to increase with the B_x . Regarding the phase velocity vs B_x for the ECDI and the MTSI, we noticed a non-monotonic increasing-then-decreasing trend for the case of the ECDI for all the propellants. For the MTSI, the trend was observed to be rather constant for all the propellants across the studied magnetic field intensities.

Concerning the changes in the significance of the force terms in the electrons' azimuthal momentum equation, we discussed, as an example for xenon, that the electric force term representing the effect of the instabilities is the dominant contributor to transport across the magnetic field values investigated. The radial distributions of the F_E term and the F_B term (the left-hand side of the momentum equation) were noticed to expectedly change with the B_x value in accordance with the observations reported from the time-averaged profiles of the axial electron current density. The viscous force term (F_{II}) was seen to

have a non-negligible contribution to the overall electrons' cross-field transport within regimes II and III.

Finally, we evaluated the impact that the increase in the B_x intensity has on the velocity distribution functions of the electrons and the ions. Regarding the electrons, we noted for all propellants that, with the rise in the B_x , the radial EVDFs depart from a nearly Maxwellian distribution at the lowest B -field intensity to the distributions for which the tail of the VDF is consistently depleted, whereas the azimuthal EVDFs became increasingly more symmetric with respect to $v_z = 0$ axis with the rise in the B_x value.

Regarding the ions, the extent of distortion and broadening of the radial and the azimuthal VDFs, respectively, were seen to decrease with the increase in the magnetic field strength. An observation of particular interest was that the ions' azimuthal VDF featured a broadening in both the positive and negative velocity directions at $B_x = 0.25 B_0$. This was the case for all the propellants albeit the extent of the broadening was different among the various gases.

ACKNOWLEDGMENTS

The present research is carried out within the framework of the project "Advanced Space Propulsion for Innovative Realization of space Exploration (ASPIRE)." ASPIRE has received funding from the European Union's Horizon 2020 Research and Innovation Programme under the Grant Agreement No. 101004366. The views expressed herein can in no way be taken as to reflect an official opinion of the Commission of the European Union.

The authors gratefully acknowledge the computational resources and support provided by the Imperial College Research Computing Service (<http://doi.org/10.14469/hpc/2232>).

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

M. Reza: Conceptualization (equal); Formal analysis (lead); Investigation (equal); Methodology (equal); Validation (lead); Writing – original draft (lead). **F. Faraji:** Conceptualization (equal); Formal analysis (supporting); Investigation (equal); Methodology (equal); Visualization (supporting); Writing – original draft (supporting); Writing – review & editing (equal). **A. Knoll:** Conceptualization (equal); Project administration (lead); Supervision (lead); Writing – review & editing (equal).

DATA AVAILABILITY

The simulation data that support the findings of this study are available from the corresponding author upon reasonable request.

APPENDIX: SUPPLEMENTARY ANALYSES

1. Verification results of the IPPL-Q2D code against the radial-azimuthal benchmark case

Figure 19 compares the predictions of the IPPL-Q2D with different number-of-region approximations against the full-2D results

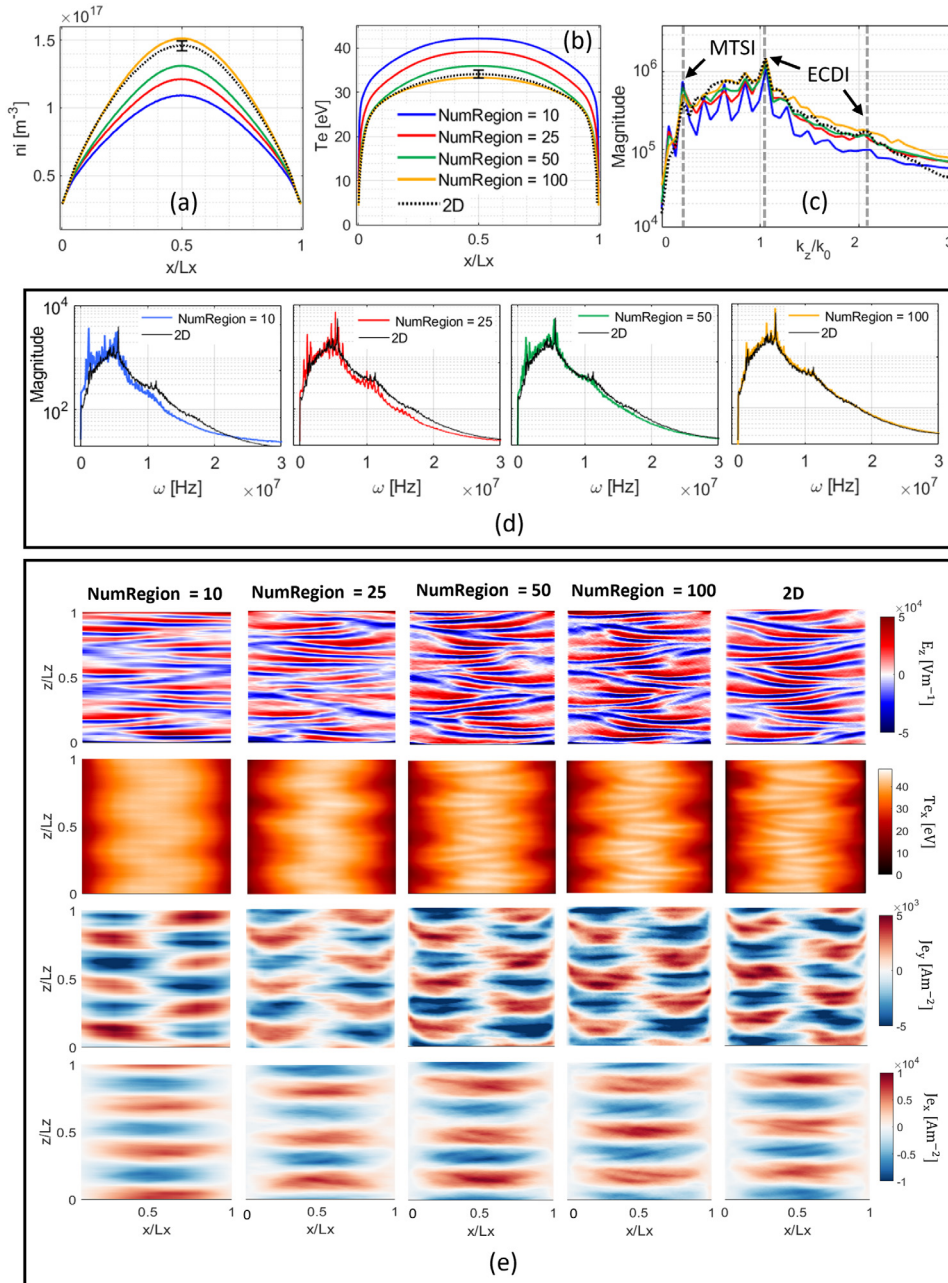


FIG. 19. Verification results of the Q2D simulations (using IPPL-Q2D code) with various number-of-regions against the full-2D simulation (using IPPL-2D¹²) of the radial–azimuthal benchmark case.³⁶ (a) and (b) Time-averaged (over 25–30 μ s) radial profiles of the ion number density and the electron temperature, respectively; (c) spatial FFTs, and (d) temporal FFTs of the azimuthal electric field fluctuations averaged over all radial locations and over 25–30 μ s duration from the various simulations; and (e) 2D snapshots of the plasma properties at the time of local maximum of the spatially averaged radial electron temperature [the rows, from top to bottom, represent the azimuthal electric field (E_z), the radial electron temperature (T_{ex}), and the axial and radial electron current densities (J_{ey} and J_{ex})]. The error bars in plots (a) and (b) indicate the variation bounds between the results from the different 2D codes that participated in the benchmarking activity. Adapted with permission from Faraji *et al.*, AIP Adv. **13**, 025315 (2023). Copyright 2023 Authors, licensed under a Creative Commons Attribution (CC BY) license.

obtained using IPPL-2D code in the radial–azimuthal benchmark setting.³⁶ For the Q2D simulations, the same number-of-regions are considered along the radial and the azimuthal directions. The comparison of the results is provided in terms of the time-averaged (over 25–30 μ s) radial profiles of the ion number density [Fig. 19(a)] and the electron temperature [Fig. 19(b)] as well as the wave characteristics as inferred from the spatial and the temporal FFTs of the azimuthal electric field fluctuations [Figs. 19(c) and 19(d), respectively]. Additionally, the 2D snapshots of several plasma properties are illustrated in Fig. 19(e). These snapshots are taken at a particular moment of the discharge

evolution corresponding to a local maximum of the globally averaged radial electron temperature.

2. Time evolution of the plasma properties and the waves' amplitude

The time evolution signals of the averaged (over the entire domain) ion number density and radial electron temperature are shown in Fig. 20 for the various B -field simulations with the xenon propellant. These plots indicate the establishment of a quasi-steady

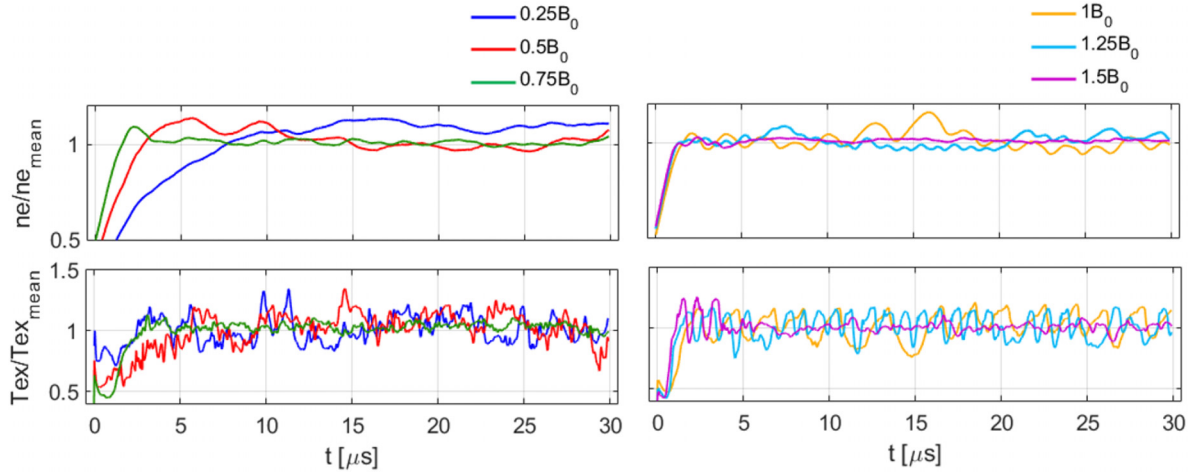


FIG. 20. Time evolution of spatially averaged ion number density (top row) and radial electron temperature (bottom row) from the simulations with various radial magnetic field values and the xenon propellant. Each trace is normalized with respect to the temporally averaged value of the corresponding signal.

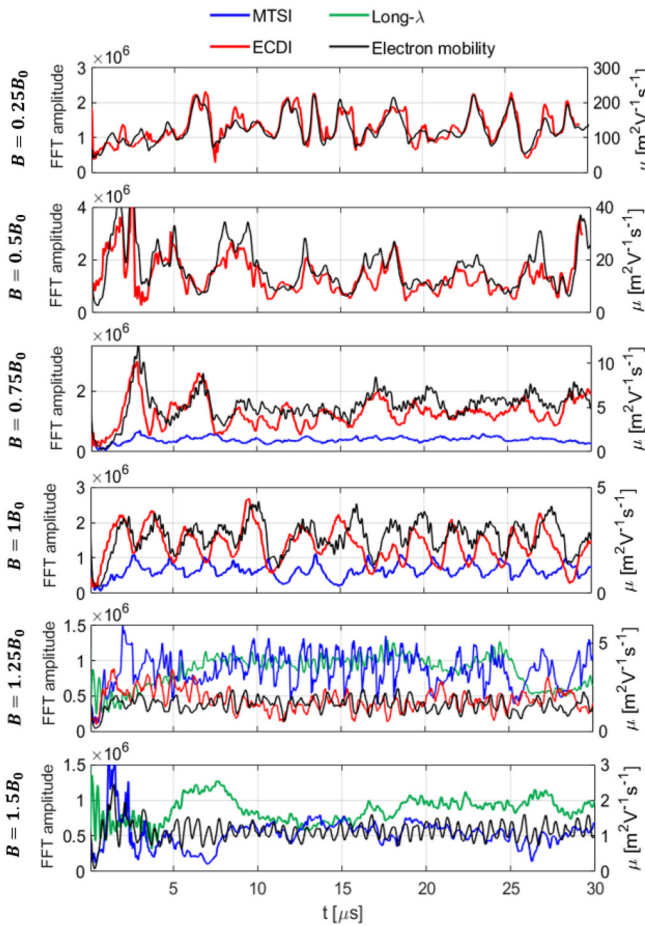


FIG. 21. Time evolution of the amplitude of the dominant azimuthal modes (left axes) and the average electrons' mobility (right axes) from the simulations corresponding to various radial magnetic field values and the xenon propellant.

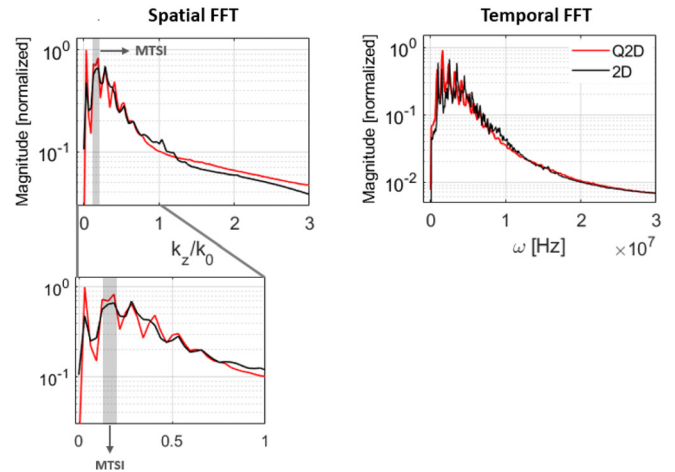


FIG. 22. Comparison of the averaged spatial and temporal FFT spectra of the azimuthal electric field fluctuations for the $B_x = 1.5B_0$ case with xenon propellant as obtained from the Q2D and full-2D simulations. The averaging is consistent with the averaging performed for the FFT analyses' results in Sec. IV B.

state in the system across all the simulated cases following an initial transient. This initial transient lasts for about 5 to 10 μs depending on the specific simulation case.

Furthermore, Fig. 21 presents the temporal evolution of the amplitude of the dominant azimuthal instability modes (discussed in Sec. IV B) within the simulations. The instability's amplitude at each time instant is determined by averaging the spatial FFT of the azimuthal electric field fluctuations across all radial positions. The resulting instantaneous amplitude corresponding to the wavenumber of each specific instability is then plotted over time. The plots also include the time evolution of electrons' cross-field mobility. The plots in Fig. 21 provide further evidence for the establishment of a quasi-steady state in all the simulations.

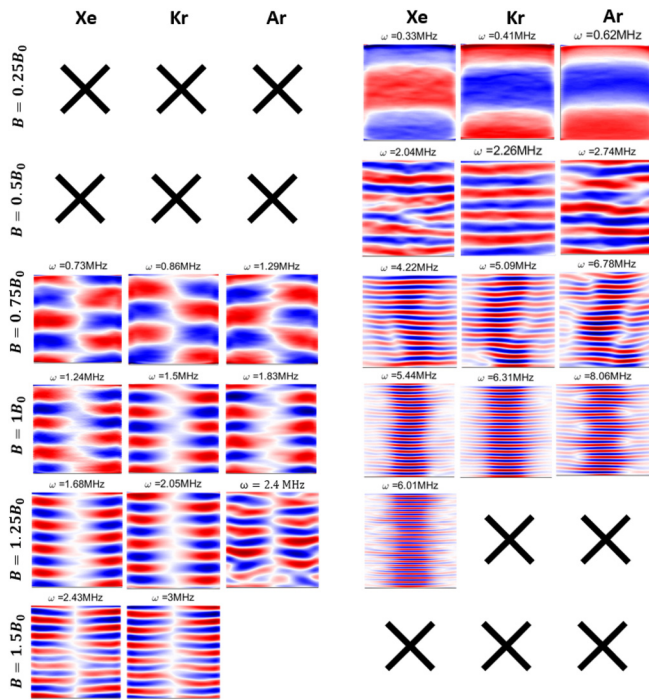


FIG. 23. Visualization of the dominant MTSI (left) and dominant ECDI (right) modes from the DMD of the azimuthal electric field data from the radial-azimuthal simulations with different B_x values and the three propellants. The "crosses" indicate that the mode was not dominantly detected from the simulations' data.

It is underlined that, from Fig. 21, we can infer the instability that has the dominant contribution to the electrons' mobility. This can be done by observing the correlation between the electrons' mobility oscillations and the oscillations in the instabilities' amplitude.

3. FFT spectra of the $B_x = 1.5 B_0$ case as obtained from the Q2D and 2D simulations

Figure 22 presents a comparison between the averaged spatial and temporal FFT spectra of the azimuthal electric field fluctuations as obtained from the Q2D simulation and a full-2D simulation of the extreme magnetic field intensity case of $1.5 B_0$.

In addition to the point inferred from the plots of Fig. 22 concerning the ECDI mode, the results additionally serve to verify the prediction of the Q2D simulation at $B_x = 1.5 B_0$ concerning the azimuthal wavenumber of the MTSI. It is recalled that, from Fig. 11, we saw some discrepancy between the simulated MTSI's azimuthal wavenumber and its theoretical value from Eq. (5). In this regard, from Fig. 22, we observe that the peaks associated with the MTSI mode from the spatial FFT spectra of the Q2D and the 2D simulations are coincident (at $\frac{k_z}{k_0} = 0.188$). However, as is illustrated in the figure, in both simulations, the FFT spectrum around this peak is broadened over a finite wavenumber interval of $\frac{k_z}{k_0} = 0.125\text{--}0.188$. As a result, we can expect the MTSI's azimuthal wavenumber to be somewhere within this interval.

4. Additional DMD results

Figure 23 compares the spatial structures of the ECDI and the MTSI modes as obtained from the OPT-DMD analyses of the various B -field simulations with xenon, krypton, and argon. At each B_x , the modes' shape is nearly identical among the different propellants; however, they are different in terms of the associated frequencies.

REFERENCES

- D. Kaganovich, A. Smolyakov, Y. Raitses, E. Ahedo, I. G. Mikellides, B. Jorns, F. Taccogna, R. Gueroult, S. Tsikata, A. Bourdon, J. P. Boeuf, M. Keidar, A. T. Powis, M. Merino, M. Cappelli, K. Hara, J. A. Carlsson, N. J. Fisch *et al.*, "Physics of $E \times B$ discharges relevant to plasma propulsion and similar technologies," *Phys. Plasmas* **27**, 120601 (2020).
- J. P. Boeuf and A. Smolyakov, "Physics and instabilities of low-temperature $E \times B$ plasmas for spacecraft propulsion and other applications," *Phys. Plasmas* **30**, 050901 (2023).
- F. Taccogna and L. Garriguez, "Latest progress in Hall thrusters plasma modeling," *Rev. Mod. Plasma Phys.* **3**(1), 12 (2019).
- J. P. Boeuf and A. Smolyakov, "Preface to special topic: Modern issues and applications of $E \times B$ plasmas," *Phys. Plasmas* **25**, 061001 (2018).
- Smolyakov A., "Towards predictive modeling of $E \times B$ plasma discharges," Report No. AFRL-AFOSR-VA-TR-2020-0087 (Air Force Research Laboratory, 2020).
- J. A. Carlsson, I. D. Kaganovich, A. V. Khrabrov, A. Smolyakov, D. Sydorenko, and Y. Raitses, "Multi-dimensional kinetic simulations of instabilities and transport in $E \times B$ devices," in *IEEE International Conference on Plasma Sciences (ICOPS)*, Antalya, Turkey (IEEE, 2015).
- C. K. Birdsall and A. B. Langdon, *Plasma Physics via Computer Simulation* (CRC Press, 1991).
- M. Reza, F. Faraji, and A. Knoll, "Concept of the generalized reduced-order particle-in-cell scheme and verification in an axial-azimuthal Hall thruster configuration," *J. Phys. D* **56**, 175201 (2023).
- M. Reza, F. Faraji, and A. Knoll, "Generalized reduced-order particle-in-cell scheme for Hall thruster modeling: Concept and in-depth verification in the axial-azimuthal configuration," *arXiv:2208.13106* (2022).
- F. Faraji, M. Reza, and A. Knoll, "Enhancing one-dimensional particle-in-cell simulations to self-consistently resolve instability-induced electron transport in Hall thrusters," *J. Appl. Phys.* **131**, 193302 (2022).
- M. Reza, F. Faraji, and A. Knoll, "Resolving multi-dimensional plasma phenomena in Hall thrusters using the reduced-order particle-in-cell scheme," *J. Electr. Propul.* **1**, 19 (2022).
- F. Faraji, M. Reza, and A. Knoll, "Verification of the generalized reduced-order particle-in-cell scheme in a radial-azimuthal $E \times B$ plasma configuration," *AIP Adv.* **13**, 025315 (2023).
- M. Reza, F. Faraji, and A. Knoll, "Effects of the applied fields' strength on the plasma behavior and processes in $E \times B$ plasma discharges of various propellants: I. Electric field," *Phys. Plasmas* **31**, 032120 (2024).
- M. Reza, E. Ferrato, F. Faraji, T. Andreussi, and M. Andrenucci, "Magnetic circuit optimization for Hall thrusters design," in 36th International Electric Propulsion Conference, Vienna, Austria, 15–20 September 2019.
- D. M. Goebel and I. Katz, *Fundamentals of Electric Propulsion: Ion and Hall Thrusters* (Wiley, 2008).
- T. Charoy, "Numerical study of electron transport in Hall thrusters," Doctoral thesis (Institut Polytechnique de Paris, 2020).
- T. Charoy, T. Lafleur, A. Tavant, P. Chabert, and A. Bourdon, "A comparison between kinetic theory and particle-in-cell simulations of anomalous electron transport in $E \times B$ plasma discharges," *Phys. Plasmas* **27**, 063510 (2020).
- T. Lafleur and P. Chabert, "The role of instability-enhanced friction on 'anomalous' electron and ion transport in Hall-effect thrusters," *Plasma Sources Sci. Technol.* **27**, 015003 (2018).
- A. T. Powis, J. A. Carlsson, I. D. Kaganovich, Y. Raitses, and A. Smolyakov, "Scaling of spoke rotation frequency within a Penning discharge," *Phys. Plasmas* **25**, 072110 (2018).

- ²⁰J. P. Boeuf and M. Takahashi, "New insights into the physics of rotating spokes in partially magnetized $E \times B$ plasmas," *Phys. Plasmas* **27**, 083520 (2020).
- ²¹L. Xu, D. Eremin, and R. P. Brinkmann, "Direct evidence of gradient drift instability being the origin of a rotating spoke in a crossed field plasma," *Plasma Sources Sci. Technol.* **30**, 075013 (2021).
- ²²L. Xu, D. Eremin, A. Smolyakov, D. Kruger, K. Kohn, and R. P. Brinkmann, "Evolution from the gradient drift instability to the formation of the spoke potential hump in a partially magnetized plasma," *Phys. Plasmas* **30**, 072112 (2023).
- ²³J. Hnilica, P. Klein, M. Šlapanská, M. Fekete, and P. Vašina, "Effect of magnetic field on spoke behaviour in HiPIMS plasma," *J. Phys. D* **51**, 095204 (2018).
- ²⁴Y. Raitses, E. Rodriguez, V. Skoutnev, A. Powis, B. Kraus, I. Kaganovich, and A. Smolyakov, "Characterization of the $E \times B$ penning discharge using electrostatic probes," in Proceedings of 36th International Electric Propulsion Conference, Vienna, Austria, 2019.
- ²⁵M. Cappelli, Y. Raitses, and I. Kaganovich, "Coherent structures in plasmas of conditions relevant to electric propulsion," Report No. AFRL-AFOSR-VA-TR-2016-0229 (2023).
- ²⁶P. J. Ryan, "Electron diagnostics of magnetron discharges," Doctoral dissertation (Department of Electrical Engineering and Electronics, University of Liverpool, 2019).
- ²⁷M. J. Brennan and A. M. Garvie, "An experimental investigation of electron transport in $E \times B$ discharges," *Aust. J. Phys.* **43**, 765 (1990).
- ²⁸E. C. Fossum and L. B. King, "Design and construction of an electron trap for studying cross-field mobility in Hall thrusters," AIAA Paper No. 2007-5207, 2007.
- ²⁹A. Ducrocq, J. C. Adam, A. Héron, and G. Laval, "High-frequency electron drift instability in the cross-field configuration of Hall thrusters," *Phys. Plasmas* **13**, 102111 (2006).
- ³⁰M. Lampe, W. M. Manheimer, J. B. McBride, J. H. Orens, K. Papadopoulos, R. Shanny, and R. N. Sudan, "Theory and simulation of the beam cyclotron instability," *Phys. Fluids* **15**, 662 (1972).
- ³¹T. Lafleur, S. D. Baalrud, and P. Chabert, "Theory for the anomalous electron transport in Hall effect thrusters. I. Insights from particle-in-cell simulations," *Phys. Plasmas* **23**, 053502 (2016).
- ³²M. Lampe, W. M. Manheimer, J. B. McBride, J. H. Orens, R. Shanny, and R. N. Sudan, "Nonlinear development of the beam-cyclotron instability," *Phys. Rev. Lett.* **26**, 1221 (1971).
- ³³J. P. Boeuf and L. Garrigues, " $E \times B$ electron drift instability in Hall thrusters: Particle-in-cell simulations vs. theory," *Phys. Plasmas* **25**, 061204 (2018).
- ³⁴F. Petronio, "Plasma instabilities in hall thrusters: A theoretical and numerical study," Ph.D. dissertation (Polytechnic Institute, NNT, Paris, 2023).
- ³⁵S. Janhunen, A. Smolyakov, D. Sydorenko, M. Jimenez, I. Kaganovich, and Y. Raitses, "Evolution of the electron cyclotron drift instability in two dimensions," *Phys. Plasmas* **25**, 082308 (2018).
- ³⁶W. Villafana, F. Petronio, A. C. Denig, M. J. Jimenez, D. Eremin, L. Garrigues, F. Taccogna, A. Alvarez-laguna, J. P. Boeuf, A. Bourdon, P. Chabert *et al.*, "2D radial-azimuthal particle-in-cell benchmark for $E \times B$ discharges," *Plasma Sources Sci. Technol.* **30**, 075002 (2021).
- ³⁷S. Janhunen, A. Smolyakov, O. Chapurin, D. Sydorenko, I. Kaganovich, and Y. Raitses, "Nonlinear structures and anomalous transport in partially magnetized $E \times B$ plasmas," *Phys. Plasmas* **25**, 011608 (2018).
- ³⁸J. C. Adam, A. Heron, and G. Laval, "Study of stationary plasma thrusters using two-dimensional fully kinetic simulations," *Phys. Plasmas* **11**, 295 (2004).
- ³⁹F. Taccogna, P. Minelli, Z. Asadi, and G. Bogopolsky, "Numerical studies of the $E \times B$ electron drift instability in Hall thrusters," *Plasma Sources Sci. Technol.* **28**, 064002 (2019).
- ⁴⁰J. B. McBride, "Theory and simulation of turbulent heating by the modified two-stream instability," *Phys. Fluids* **15**(12), 2367 (1972).
- ⁴¹M. Reza, F. Faraji, and A. Knoll, "Parametric investigation of azimuthal instabilities and electron transport in a radial-azimuthal $E \times B$ plasma configuration," *J. Appl. Phys.* **133**, 123301 (2023).
- ⁴²M. Reza, F. Faraji, A. Knoll, A. Piragino, T. Andreussi, and T. Misuri, "Reduced-order particle-in-cell simulations of a high-power magnetically shielded Hall thruster," *Plasma Sources Sci. Technol.* **32**, 065016 (2023).
- ⁴³F. Faraji, M. Reza, A. Knoll, and J. N. Kutz, "Dynamic mode decomposition for data-driven analysis and reduced-order modelling of $E \times B$ plasmas: I. Extraction of spatiotemporally coherent patterns," *J. Phys. D* **57**, 065201 (2024).
- ⁴⁴F. Faraji, M. Reza, A. Knoll, and J. N. Kutz, "Dynamic mode decomposition for data-driven analysis and reduced-order modelling of $E \times B$ plasmas: II. Dynamics forecasting," *J. Phys. D* **57**, 065202 (2024).
- ⁴⁵T. Askham and J. N. Kutz, "Variable projection methods for an optimized dynamic mode decomposition," *SIAM J. Appl. Dyn. Syst.* **17**(1), 380–416 (2018).
- ⁴⁶M. Reza, F. Faraji, and A. Knoll, "Influence of the magnetic field curvature on the radial-azimuthal dynamics of a Hall thruster plasma discharge with different propellants," *J. Appl. Phys.* **134**, 233303 (2023).
- ⁴⁷R. Dewar, "Saturation of kinetic instabilities by particle trapping," *Phys. Fluids* **16**, 431–435 (1973).