London, March, 6, 2022

p-adic Cohomology and Arithmetic Applications: Conference in Banff, 2017

This part of the present special issue of JNT is related to the conference *p-adic Cohomology and Arithmetic Applications*, BIRS, Banff, Canada, October 2-6, 2017. The subject of p-adic cohomology, a Weil cohomology theory for algebraic varieties defined over fields of positive characteristic p, has a long and distinguished history. It began with Dwork's proof of one of the most important results in arithmetic geometry, the rationality of zeta functions of algebraic varieties defined over finite fields, originally conjectured by Weil. This proof in fact predated that given by Artin and Grothendiek, and, very surprisingly at the the time, used p-adic analytic methods. Soon the cohomology theory that was hiding behind these methods was identified by Monsky and Washnitzer. At the same time, Grothendieck and Berthelot introduced crystalline cohomology, motivated by the desire to capture p-torsion phenomena in the cohomology of varieties in characteristic p. These two approaches were later unified, at least rationally, via Berthelot's theory of rigid cohomology. After a period of intensive work, due to the efforts of Kedlaya, Caro, and others, the theory reached a new level of sophistication with the proof of Grothendieck's six functors formalism, replicating those properties of ℓ-adic cohomology which occupied Grothendieck and his school for many years, and considered as a central achievement in arithmetic geometry.

Recent years have seen somewhat of a maturation of the subject; many basic foundational results are now settled, and interest has started to turn towards applications of these results to long standing arithmetic questions and conjectures, some of which were the original motivation for the development of p-adic cohomology. The conference brought together mathematicians working on p-adic cohomology to discuss the latest developments, and to explore possible new directions of research.

Contents. This volume opens with a paper by S. Agrawal where formal deformation theory of overconvergent isocrystals on U is studied with fixed local monodromy along Z, via a Hochschild cochain complex and its relation to the de Rham complex. Then a paper by V. Ertl follows, where she gives a new, more concise proof of a vanishing result due to Berthelot, Esnault, and Rülling, relating for a regular, proper, and flat scheme X over a discrete valuation ring of mixed characteristic $(0, p)$ the vanishing of the cohomology of the generic fibre of X with the vanishing of the Witt vector cohomology of its special fibre. Next the reader will find an article by O. Gregory where the author uses the crystal of relative displays, which can be thought of relative versions of Fontaine's notion of *strongly divisible lattice* from integral p-adic Hodge theory, to prove a Grothendieck-Messing type result for the deformation theory of Calabi-Yau threefolds. Among several other things, the analogue of the Simpson correspondence is studied for q-difference equations when q is a pth root of unity by M. Gros, B. Le Stum and A. Quirós. C. Huyghe, M. Strauch and T. Schmidt surveys sheaves of arithmetic differential operators with congruence level on formal schemes, presents first results about these sheaves and discuss some examples. The next paper, by K. Kedlaya, is a brief user's guide for the overconvergent F-isocrystals, which are analogous to lisse ℓ-adic sheaves in l-adic étale cohomology. The following paper by C. Lazda is a proof of the fact that proper surjective and faithfully flat maps are morphisms of effective descent for overconvergent isocrystals. Then he shows how to deduce the folklore theorem that for an arbitrary variety over a perfect field of characteristic p, the Frobenius pull-back functor is an equivalence on the overconvergent category. The collection closes with a study of the behaviour of discriminants in families of number fields arising from p-adic Galois representations, by J. Upton. Hist main theorem is analogous to the *genus-stability conjecture* of Daqing Wan. More about the conference (participants, speakers, organisers, schedule) can be found at the following URL: `https://www.birs.ca/events/2017/5-day-workshops/17w5118`. BIRS has put together a video playlist for each day of the conference which can be found following the previous link.

Acknowledgements. The organisers are thankful to the BIRS for the nice conditions under which the conference took place, and for the financial support of the conference.

On behalf of the editors of the present volume,