

THE INCORPORATION OF SUB-GRID SCALE CONVECTION

IN NUMERICAL WEATHER PREDICTION MODELS

A thesis submitted for the degree
of Doctor of Philosophy in the
University of London.

ALAN JOHN GADD

Department of Meteorology,
Imperial College.

April 1969

ABSTRACT

The ways in which static instability may develop, independently of surface heating, in the large-scale flow of the troposphere are investigated. Two destabilising processes, which operate mainly near fronts, are identified; and it is demonstrated that the model atmosphere described by Bushby and Timpson (1967) is capable of simulating these processes.

The deleterious effects of developments of static instability in the Bushby-Timpson model atmosphere are illustrated; thus showing the importance of incorporating a representation of the stabilising effects of sub-grid scale convection in such models. A number of such representations which have been used by previous workers are reviewed.

A representation of the vertical heat transports achieved by small-scale convection is formulated whereby a critical lapse rate of temperature is maintained whenever it tends to be exceeded as a result of the large-scale destabilising processes. This critical lapse rate is, unlike those previously used in such representations, a continuous function of the large-scale relative humidity, so that no unrealistically rapid convective heat transports

are implied as conditionally unstable air becomes saturated. Water vapour is also transported in the vertical, and convective rainfall may be produced; it is assumed that small-scale convection effects no mean transport of horizontal momentum.

The representation of sub-grid scale convection has been incorporated into the Bushby-Timpson model and is successful in preventing certain spurious developments. The results of predictions computed with the sub-grid scale convection included are described, and the meteorological properties of the representation are assessed.

CONTENTS

1.	INTRODUCTION	
1.1	Thermodynamic nomenclature	8
1.2	Convection in the atmosphere	9
1.3	Large-scale transports by small-scale convection	12
1.4	The general scheme of tropospheric motion	15
1.5	Numerical weather prediction	17
2.	THE DEVELOPMENT OF STATIC INSTABILITY	
2.1	Introduction	23
2.2	Differential advection	26
2.3	Vertical stretching	34
2.4	Differential diabatic heating	36
2.5	The saturation of conditionally unstable air	38
2.6	Destabilisation in the Bushby-Timpson model	41
2.7	Summary	43
3.	STATIC INSTABILITY AND SMALL-SCALE CONVECTION IN NUMERICAL WEATHER PREDICTION MODELS	
3.1	(a) Grid-scale convection	52
	(b) An example of the consequences of saturated instability	59
3.2	The meteorological importance of small-scale convection in numerical models	60

5.

3.3	A survey of representations of small-scale convection which have been used in numerical models of the atmosphere	62
3.3	(a) L.F. Richardson's model	62
	(b) Tropical cyclone models	63
	(c) General circulation models	64
3.4	Comment on the representations described above	65
4.	THE INCORPORATION OF SMALL-SCALE CONVECTION INTO THE BUSHBY-TIMPSON MODEL	
4.1	Introduction	74
4.2	An appropriate representation of small-scale convection	76
4.3	The neutral lapse-rate for small-scale convection	77
4.4	Water vapour transport and convective rainfall	81
4.5	Details of the convective adjustments scheme	
	(a) The test for instability	85
	(b) The solution for the adjusted parameters	87
4.6	The initial data	91
5.	RESULTS OF THE INCORPORATION OF THE CONVECTIVE ADJUSTMENTS	
5.1	Introduction	95
5.2	The case of 1 December 1961	98
5.3	The case of 21 August 1965	106
5.4	Summary of results	112

5.5 Assessment of the convective adjustments scheme	113
--	-----

DIAGRAMS: Figures 1-36 follow immediately after the
chapters to which they refer.

REFERENCES:	137
-------------	-----

'For the present purpose any motion which disappears on taking the average over our coordinate intervals has of necessity to be ignored. If it occurs in largenumbers - as cumulus eddies do for example - then its general effect can be satisfactorily represented by additional terms; although unfortunately this does not help us for example to say whether it will hail or not on Mr. X's field.'

L.F. Richardson, 1922.

'Weather prediction by numerical
process'

1. INTRODUCTION

1.1 Thermodynamic nomenclature

A number of thermodynamic parameters will be referred to in the following pages by means of the notation listed below.

Lapse rate of temperature $\gamma (= - \frac{\partial T}{\partial z})$.

Dry adiabatic lapse rate Γ_d .

Saturated pseudo-adiabatic lapse rate Γ_s .

Potential temperature θ . This is the temperature which a sample of air attains when brought to a pressure of 1000 mb by a dry adiabatic process.

Wet-bulb potential temperature θ_w . This is the temperature which a sample of air attains when lifted dry adiabatically to its condensation level, and then brought to a pressure of 1000 mb by a saturated pseudo-adiabatic process.

Saturated wet-bulb potential temperature θ_s . This is the value of θ_w which a sample of air would attain if saturated with water vapour at its present temperature and pressure.

In addition, reference will be made to the following three classes of static stability:

<u>Absolute instability</u>	$\gamma > \Gamma_d$	i.e. $\frac{\partial \theta}{\partial z} < 0$.
<u>Conditional instability</u>	$\Gamma_d > \gamma > \Gamma_s$	i.e. $\frac{\partial \theta}{\partial z} > 0$ $\frac{\partial \theta_s}{\partial z} < 0$.
<u>Absolute stability</u>	$\gamma < \Gamma_s$	i.e. $\frac{\partial \theta_s}{\partial z} > 0$.

1.2 Convection in the atmosphere

The motions of the earth's atmosphere are ultimately of a convective nature, in that they derive their kinetic energy from the potential energy associated with temperature gradients. These temperature gradients are in turn the result of differential heating by radiation, latent heat release, and conduction at the earth's surface.

Carlson and Ludlam (1965) have classified tropospheric convection according to scale, their categories being: small-scale, intermediate-scale, large-scale, and cumulonimbus convection. Of these, large- and small-scale convection may be regarded as primary, the two remaining kinds of convection being secondary developments on intermediate scales, both requiring the previous activity of small-scale convection.

Large-scale convection is a response to the horizontal equator-pole temperature gradient maintained by the net radiation balance of the earth and atmosphere, and grows

on horizontal scales in excess of 1000 km. The analysis of Eady (1949) indicates how, by virtue of this latitudinal temperature gradient, the troposphere is, in mid-latitudes, permanently unstable for perturbations of sufficiently long wave-lengths. This large-scale convection consists, in everyday terms, of cyclones and anti-cyclones. Fronts, which are regions of intensified horizontal temperature gradient accompanied by vertical motion, cloud, and precipitation, make up part of the structure of large-scale convection.

Small-scale convection is a response to statically unstable gradients of temperature in the vertical, and grows on horizontal scales of up to a few km. Perturbation analyses, such as that of Kuo (1963), indicate that, in dry atmospheres, convection with a sufficiently short wavelength can grow provided that θ decreases upwards; that is, provided that γ is greater than Γ_d .

In the real atmosphere the various effects of the water content modify this criterion so that small-scale convection can grow provided that γ exceeds some critical or neutral value γ_c . Apart from the virtual temperature effect (the reduction in the density of the air due to its water vapour content) which is of rather

minor importance in this context, γ_c lies between Γ_d and Γ_s , its value being determined by the distribution of saturated air in the small-scale convective circulations. In all cases, however, a decrease of θ_s with height is a necessary condition for the growth of small-scale convection.

Small-scale convection may be classified as follows according to its characteristic neutral lapse rate:

Dry convection, which occurs in entirely unsaturated air, can grow if γ exceeds Γ_d .

Cumulus convection, in which the ascending air only is saturated, is observed to have a neutral lapse rate which is about mid-way between Γ_d and Γ_s (Ludlam 1966).

Saturated convection, which occurs in air which is saturated on a scale larger than the dimensions of individual small-scale convective circulations, may be expected to have a neutral lapse rate close to Γ_s . This is because mixing between ascending and descending air produces no evaporative cooling, and also because the evaporation of liquid water in the descending air reduces its rate of warming to the saturated adiabatic.

There is no sharp distinction between cumulus and saturated convection. Intermediate forms occur where

the descending branches of the small scale circulations are only partly occupied by saturated air, as, for example, in the large convective cloud described by Malkus (1962).

1.3 Large-scale transports by small-scale convection

Whilst large- and small-scale convection may for some purposes be studied separately, their interaction is also important, because if small-scale convection produces fluxes of heat, water vapour, or momentum with divergences which do not vanish when meaned on the large-scale, the parameters of the large-scale flow are modified. The vertical fluxes associated with small-scale convection are more important than their horizontal counterparts, and therefore only modifications to the vertical distributions of the large-scale parameters are considered here.

Heat transport in the vertical always accompanies small-scale convection, because, in order that potential energy be released to drive the motion, warmer air must ascend and cooler air descend. This implies a net upward heat flux which reduces the large-scale temperature lapse rate.

Furthermore, this heat transport by small-scale convection operates with a very short time scale. Perturbation analysis indicates that, for static instability amounting to $0.1 \Gamma_d$, small-scale convection would double its amplitude in about 3 minutes. By comparison, large-scale convection systems have doubling times of at least 12 hours.

In reality, the rather rapid heat transport associated with small-scale convection quickly reduces the large-scale lapse rate to its neutral value. Indeed, from the point of view of the large-scale convection, the role of the small-scale convection may be seen as to effect vertical heat transports to maintain the neutral lapse rate γ_0 whenever this tends to be exceeded.

Water vapour transport by small-scale convection is generally thought to have an efficiency similar to that of heat transport. This view is supported by the observation that, in regions of dry small-scale convection, the neutral stratification of temperature ($\gamma = \Gamma_d$) is usually accompanied by a near-uniform humidity mixing ratio distribution (Ludlam 1966).

In saturated air vertical transport of water vapour is, of course, accompanied by condensation, so that the

vertical distribution of water vapour is uniquely related to the vertical temperature structure.

Momentum transport associated with small-scale convection is something of an unknown quantity. Scorer (1962) has argued qualitatively that small-scale convection must transport momentum less efficiently than heat. Even the direction of the momentum transport is not known in general, since the potential energy released in small-scale convection could, in some instances, be transformed into large-scale horizontal kinetic energy. This would produce an increase in vertical wind shear and therefore implies a transport of momentum up the gradient of velocity, contrary to the usual notion of 'mixing'.

Thus whereas the effect of the heat transport produced by small-scale convection must always be to reduce the lapse rate of temperature towards its neutral value, the nature of the corresponding momentum transport depends on the shape of the small-scale convective circulations, and even the sign of its effect cannot be specified unless specific models of the small-scale circulations are proposed.

1.4 The general scheme of tropospheric motion

Carlson and Ludlam (1965) have described the general scheme of motion in the mid-latitude troposphere in terms of a collaboration between small-scale convection, occupying a rather shallow layer near the surface, and large-scale convection systems, extending through the whole depth of the troposphere. Their schematic vertical projection illustrating this collaboration is reproduced here as figure 1.

The surface layer of small-scale convection, which results from static instability created by heating from the surface, consists of a cumulus layer above a layer of dry convection which together occupy a depth of up to a few km. The small-scale convection communicates heat and water vapour from the surface to air descending in the large-scale systems, which thereby acquires a high θ_w and eventually enters the large-scale ascent. θ_w is approximately conserved during the ascent, although most of the latent heat is converted to sensible heat by condensation, but falls during the rather long descent period because of radiative cooling. As the descending air re-enters the surface layer of small-scale convection θ_w is progressively restored, ready for the next large-scale ascent.

Small-scale convection near the surface thus plays a vital role in maintaining the general cycle of tropospheric motion, by restoring to the large-scale convection both the water vapour lost by precipitation and the energy lost by radiation.

This scheme of collaboration between large- and small-scale convection implies a certain characteristic stratification of θ_s throughout the mid-latitude troposphere. The surface layer of small-scale convection is characterised by the neutral lapse rates of temperature for dry and cumulus convection, respectively, below and above cloud base. There is therefore a decrease of θ_s with height throughout this layer. Above the surface layer the stratification is largely determined by the large-scale convective circulations, in which air is led from the surface layer at low latitudes polewards into the upper troposphere, where it arrives with values of θ_s greater than those prevailing directly beneath. Thus, from the top of the surface layer up to the tropopause, the mid-latitude atmosphere is characterised by an increase of θ_s with height.

This characteristic vertical distribution of θ_s is illustrated by figure 2 (after Carlson and Ludlam 1965) which is a mean vertical section, approximately along

the Greenwich meridian, for July 1959. The surface layer, governed by small-scale convection, is seen to occupy only the lowest 200 mb of the troposphere north of 45°N , although further south it extends to the 500 mb level.

1.5 Numerical weather prediction models

Numerical weather prediction models are usually systems of finite difference equations, approximating to the differential equations which describe atmospheric flow, which may be integrated numerically to simulate large-scale convection and thus predict the future state of the atmosphere from real initial data. As with any numerical model of fluid flow, motion systems on scales of time or space smaller than the finite difference intervals adopted are automatically excluded from explicit representation. Both the need for economy in computing time and the requirement, in most models, that the hydrostatic approximation be valid set lower limits on the grid-lengths used in numerical weather prediction models. These grid lengths are chosen to give adequate resolution of cyclones and, more rarely, frontal zones; thus small-scale convection inevitably falls into the category of sub-grid scale motion.

While the details of individual sub-grid scale circulations are of no concern to a numerical model, the overall effect of widespread sub-grid scale motion may well be important. In particular the neutralising of statically unstable lapse rates by small-scale convection is an important effect for the more sophisticated numerical weather prediction models, such as that described by Bushby and Timpson (1967). This is because meteorologically unrealistic motions, including the phenomenon known as grid-scale convection, develop in statically unstable regions of such model atmospheres.

In addition, more positive meteorological benefits, for example the possibility of forecasting showers, may accrue from the incorporation of the effects of small-scale convection in numerical weather prediction models. Certainly, the role of small-scale convection in the surface layer, as outlined in the previous section, must be represented in any model attempting simulations over long periods, though it will be comparatively unimportant for 1-2 day forecasts.

Small-scale convection can be represented only in those numerical weather prediction models which are capable of simulating realistically the changes in static stability which occur in the atmosphere. In this context

it is useful to note three features by means of which models of the atmosphere may be classified.

(a) Vertical resolution. Adequate definition of static stability is possible only in models with a rather fine spatial resolution in the vertical.

(b) Dynamics. Numerical weather predictions are computed by integrating either the quasi-geostrophic equations or the primitive equations (see for instance Thompson 1961). Only in primitive equation models is it possible in principle for ageostrophic wind components to modify the static stability through advective effects (see section 2.2 below).

(c) Latent heat release. Only in those models which compute water vapour content as an independent parameter can the important effect of latent heat release on the neutral lapse rate for small-scale convection be realistically represented.

It is thus necessary, in the context of a given numerical weather prediction model, to examine the possibilities for the development of static instability; to evaluate the significance of small-scale convection; and to incorporate into the model appropriate representations of the important effects of small-scale convection on the large-scale parameters. This is the

nature of the work described in this thesis; in this case the framework is provided by the 10-level wet primitive equation model described by Bushby and Timpson (1967).

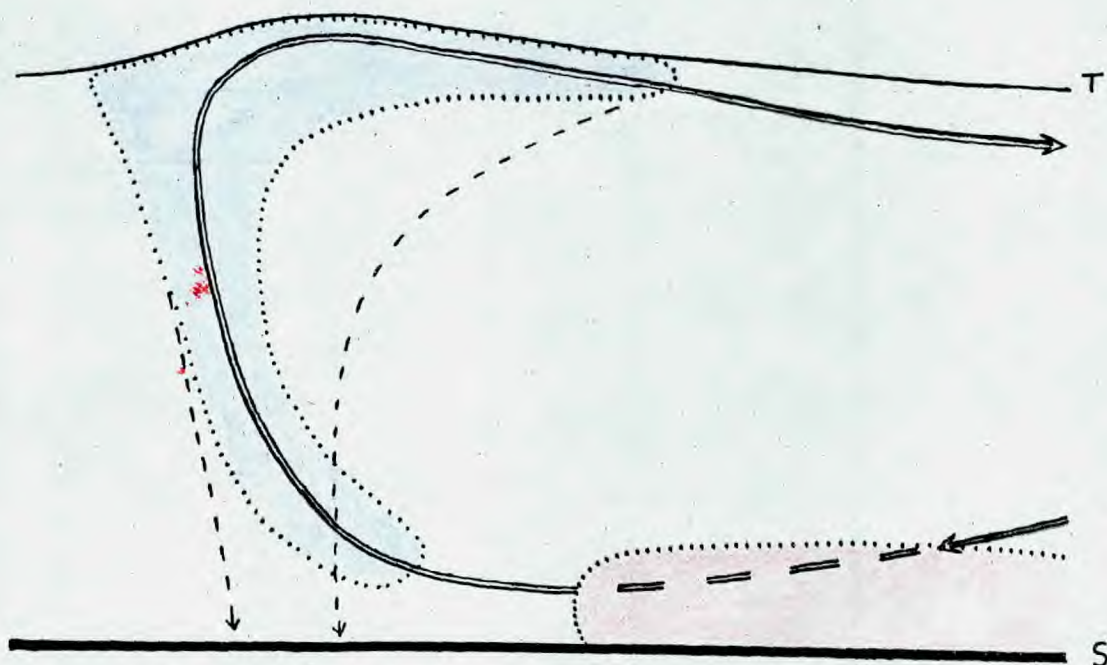


Figure 1 (after Carlson and Ludlam 1965) is a schematic vertical projection illustrating the general scheme of tropospheric convection. Flow in large-scale convection (double line) extends from the surface (S) to the tropopause (T) and in its ascending branch is characterised by condensation (blue shading) and precipitation (red asterisks). The descending branch, left open since it continues through several such systems, passes through the surface layer of small-scale convection (red shading) before re-entering the large-scale ascent.

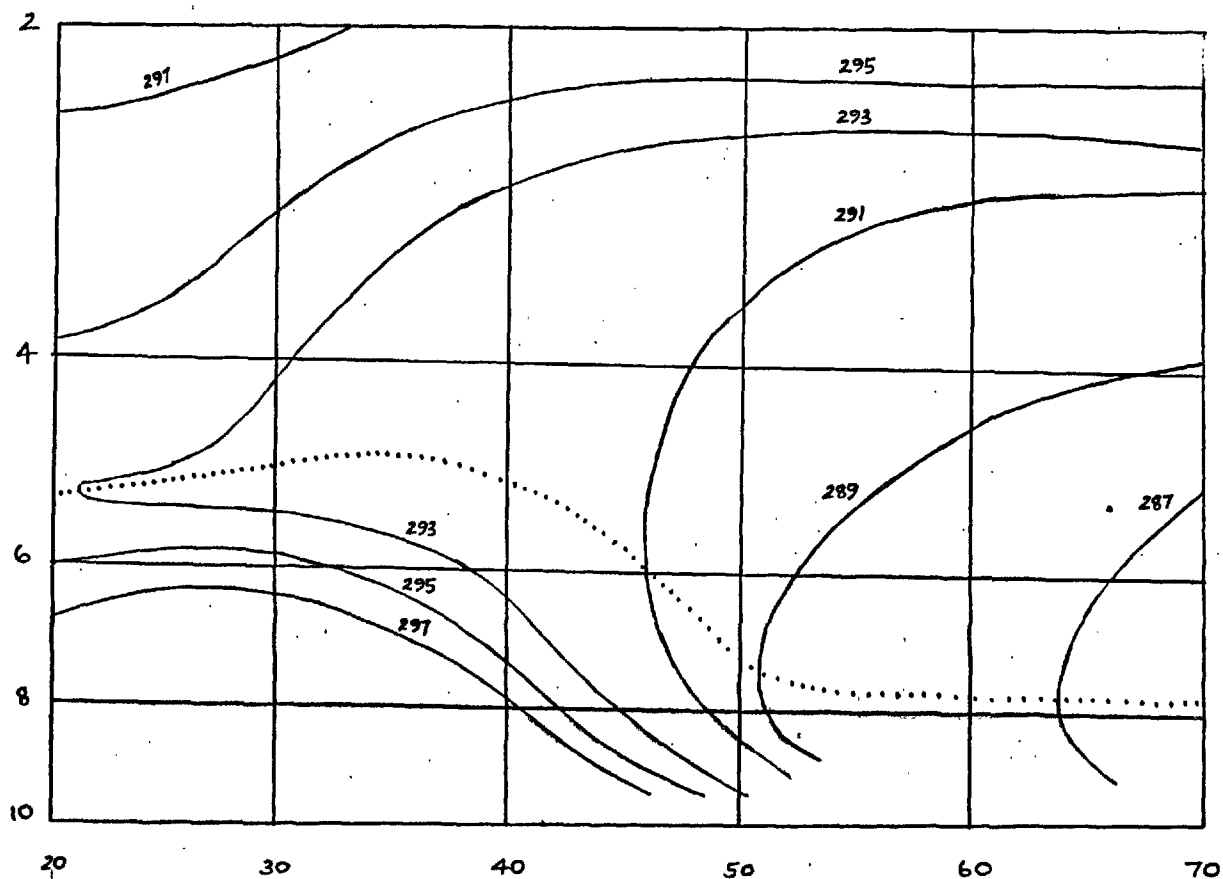


Figure 2 (after Carlson and Ludlam 1965) shows isonleths of Θ_s (full lines: labelled in degrees K.) on a vertical section, approximately along the Greenwich meridian, meaned for July 1959. The vertical axis is labelled in hundreds of mb and the horizontal in degrees of latitude north. The extent of the surface layer of small-scale convection, within which Θ_s decreases upwards, is shown by a dotted line.

2. THE DEVELOPMENT OF STATIC INSTABILITY

2.1 Introduction

The general scheme of tropospheric motion, as outlined in section 1.4 above, is such that, apart from a rather shallow surface layer, the mid-latitude troposphere is characterised by an increase of θ_s with height, and is therefore unfavourable for the development of small-scale convection. However, there are a number of physical and dynamical destabilising processes associated with large-scale convection which operate to off-set this general trend, and which may locally tend to produce static instability. Such developments of static instability (which are not directly related to surface heating) and the consequent growth of small-scale convection, are of computational and meteorological importance to numerical weather prediction models. In this chapter these destabilising processes are examined, and the possibility of their being simulated in various kinds of model atmosphere are noted.

Static instability is produced by the large scale flow when either γ is increased or γ_c is decreased until $\gamma > \gamma_c$.

Changes in γ are described by an equation which may be derived from the first law of thermodynamics. Using the static approximation to the pressure, this law may be written

$$\frac{DT}{Dt} + \Gamma_d w = \frac{Q}{C_p} \quad (1)$$

where $\frac{DT}{Dt}$ represents the rate of change of temperature following the motion, w the vertical velocity, C_p the specific heat of air at constant pressure, and Q the rate of diabatic heating. Among the various physical heating effects which may contribute to Q , the important role of latent heat release may be represented, to a close approximation, by replacing Γ_d by Γ_s in equation (1) whenever the air is saturated. The equation then becomes

$$\frac{DT}{Dt} + \Gamma w = \frac{Q}{C_p} \quad (1A)$$

where Γ is Γ_d or Γ_s as appropriate, and Q is the rate of heating by diabatic processes other than latent heat release. On differentiating (1A) with respect to the vertical co-ordinate z , the following equation for the rate of change of lapse rate following the motion is obtained.

$$\frac{D\gamma}{Dt} = \frac{\partial u}{\partial z} \frac{\partial T}{\partial x} + \frac{\partial v}{\partial z} \frac{\partial T}{\partial y} + \frac{\partial w}{\partial z} (\rho - \gamma) + w \frac{\partial \Gamma}{\partial z} - \frac{1}{C_p} \frac{\partial Q}{\partial z} \quad (2)$$

where u and v are the horizontal wind components in the co-ordinate directions of x and y respectively. Since Γ_d is a constant, whilst to a close approximation

$$\frac{D\Gamma_s}{Dt} = w \frac{\partial \Gamma_s}{\partial z}$$

equation (2) may be rearranged as follows.

$$\frac{D}{Dt}(\gamma - \rho) = \frac{\partial u}{\partial z} \frac{\partial T}{\partial x} + \frac{\partial v}{\partial z} \frac{\partial T}{\partial y} - \frac{\partial w}{\partial z}(\gamma - \rho) - \frac{1}{C_p} \frac{\partial Q}{\partial z} \quad (2A)$$

Changes in γ_c are associated with changes in the distribution of saturated air in the small-scale convective circulations. In particular, air becomes saturated on a large scale in the ascending branches of large-scale convective circulations so that its neutral lapse rate for small-scale convection is reduced to something close to the saturated adiabatic.

In the following four sections four destabilising processes which may operate to produce static instability in the large-scale flow are discussed. These comprise three processes represented by terms on the right of

equation (2A) and the destabilising change in γ_c which accompanies the large-scale saturation of previously unsaturated air.

2.2 Differential advection

The first part of equation (2A), namely,

$$\frac{D}{Dt} (\gamma - \gamma_c) = \frac{\partial u}{\partial z} \frac{\partial T}{\partial x} + \frac{\partial v}{\partial z} \frac{\partial T}{\partial y}$$

represents the rate at which the lapse rate is changed by the advection of temperature in a vertically sheared horizontal flow, a process which may be termed 'differential advection'.

If the flow is geostrophic, the thermal wind equations

$$\frac{\partial U}{\partial z} = - \frac{g}{fT} \frac{\partial T}{\partial y} \quad : \quad \frac{\partial V}{\partial z} = \frac{g}{fT} \frac{\partial T}{\partial x} \quad (3)$$

(where U and V are the components of the geostrophic wind, g is the acceleration due to apparent gravity, and f is the Coriolis parameter), ensure that the differential advection terms cancel one another exactly. The ability of differential advection to modify γ thus depends on the coincidence of ageostrophic wind shears

and horizontal temperature gradients, suggesting that this process is most likely to be important near fronts and jet streams.

In view of the complexity of the equations describing three-dimensional ageostrophic motion in the atmosphere, it is useful to exploit models of fronts and jet streams in which geostrophic balance is maintained in the component of the flow parallel to the front. The ageostrophic component of the flow is then two-dimensional in the vertical plane normal to the front. This approach has been developed by Sawyer (1956) and Eliassen (1962).

Consider a flow which consists of a non-divergent two-dimensional ageostrophic component (u,w) in the (x,z) plane, together with a horizontal geostrophic component (U,V) in the (x,y) plane.

A stream function ψ of the ageostrophic flow may be defined such that

$$u = - \frac{\partial \psi}{\partial z} \quad : \quad w = \frac{\partial \psi}{\partial x}$$

and the differential advection terms now reduce to

$$\frac{D}{Dt} (\gamma - \Gamma) = - \frac{\partial^2 \psi}{\partial z^2} \frac{\partial T}{\partial x} \quad .$$

For such a flow the y-momentum equation and the thermodynamic equation may be written

$$\frac{DV}{Dt} - f \frac{\partial \psi}{\partial z} = \frac{\partial}{\partial z} \frac{\tau}{\rho} \quad (4)$$

$$\frac{DT}{Dt} + \Gamma \frac{\partial \psi}{\partial x} = \frac{Q}{C_p} \quad (5)$$

(where τ is the y-component of the frictional stress and ρ is the air density). On differentiating (4) with respect to z and (5) with respect to x , and using equations (3), the following diagnostic differential equation for ψ may be obtained.

$$A \frac{\partial^2 \psi}{\partial z^2} + 2B \frac{\partial^2 \psi}{\partial z \partial x} + C \frac{\partial^2 \psi}{\partial x^2} = D \quad (6)$$

where A , B , C , and D are the following functions of the geostrophic flow.

$$A = f + \frac{\partial V}{\partial x} : B = - \frac{\partial V}{\partial z} : C = \frac{g}{fT} (\Gamma - \gamma)$$

$$D = \frac{\partial V}{\partial z} \left(\frac{\partial V}{\partial y} - \frac{\partial U}{\partial x} \right) + 2 \frac{\partial U}{\partial z} \frac{\partial V}{\partial x} + \frac{g}{fT} \frac{\partial}{\partial x} \frac{Q}{C_p} - \frac{\partial^2}{\partial z^2} \frac{\tau}{\rho} .$$

Equation (6) may be interpreted as follows. The effects of geostrophic advection, diabatic heating, and

friction are tending to destroy the geostrophic balance of the flow parallel to the front, D being the rate at which $\frac{g}{fT} \frac{\partial T}{\partial x} - \frac{\partial V}{\partial z}$ is increasing as a result of these processes. In order that the geostrophic balance be maintained, a vertical circulation, defined by ψ , in the normal plane is necessary. If D has the same sign as $\frac{\partial T}{\partial x}$, the frontal temperature gradient is being increased more rapidly than the vertical shear, and the required vertical circulation is in the thermodynamically direct sense, whilst if D and $\frac{\partial T}{\partial x}$ are of opposite sign, an indirect circulation is required.

The first term in D represents the effect of the deformation of the geostrophic wind in sharpening the frontal temperature gradient - the 'classical' frontogenesis process. This term is of the same sign as $\frac{\partial T}{\partial x}$ in active frontal zones.

The second term in D represents the effect of geostrophic advection parallel to the front when there is a temperature gradient along the front. For a front orientated N-S, and with temperature decreasing to the north, the effect is to sharpen the frontal temperature gradient of a cold front where the geostrophic component parallel to the front has cyclonic horizontal shear and of a warm front where there is anti-cyclonic shear.

The final two terms in D , involving τ and Q are comparatively insignificant (Eliassen 1966). Notice that the effect of latent heat release is accounted for in equation (6) by the definition of Γ (see section 2.1 above).

The lapse rate is increased by differential advection when $\frac{\partial^2 \psi}{\partial z^2}$ and $\frac{\partial T}{\partial x}$ have opposite signs, that is, in indirect circulations. Such circulations can only come about in active fronts where the second term of D is of opposite sign to $\frac{\partial T}{\partial x}$ and is not dominated by the first. Thus destabilisation by differential advection occurs when the parallel component of the geostrophic wind has a cyclonic shear in warm fronts, or an anti-cyclonic shear in cold fronts, if temperature decreases northwards along the fronts.

Figure 3 (after Palmen and Newton 1948) shows mean isotachs of the parallel geostrophic component in a vertical section normal to 12 cold fronts and their associated jet streams. Anti-cyclonic shear occurs extensively on the warm side of the frontal zone, and is intense (approaching $-f$) near the jet core. The fronts meant to obtain this diagram were orientated roughly E-W ; a similar geostrophic wind structure in a front with a significant parallel temperature gradient

would be characterised by rapid destabilisation in the regions of pronounced anti-cyclonic shear.

Such a front was considered by Todsén (1964), who solved an equation similar to (6) numerically for the cold front which was orientated almost N-S over Western Europe at 12 GMT on 8 April 1962. The terms A , B , C , and D were extracted from synoptic charts by mapping U , V , and T on a vertical section normal to the front. The terms in τ and Q were neglected.

Streamlines of the ageostrophic flow in the vertical plane normal to this front, as computed by Todsén, are shown in figure 4. Todsén used pressure as the vertical co-ordinate, so that the streamlines in this diagram are isopleths of ψ_p , defined by $u = \frac{\partial \psi_p}{\partial p}$, $\omega = -\frac{\partial \psi_p}{\partial x}$, where $\omega = \frac{Dp}{Dt}$. The computed ageostrophic flow consists of a major, thermodynamically direct circulation at and behind the surface cold front, together with a less extensive indirect circulation immediately ahead of the front. As may be expected from the above discussion, the indirect circulation occurs in the region of anti-cyclonic shear of the parallel geostrophic component. Over most of the region occupied by the indirect circulation the horizontal temperature gradient normal

to the front is rather small, so that the rate of destabilisation by differential advection is not very significant. In the vicinity of the point labelled 'A' in figure 4, however, the indirect circulation coincides with the frontal temperature gradient of about 1°C in 50 km, and a rate of destabilisation of about $0.1 \bar{\Gamma}_d$ per hour is implied.

Todsen also carried out a similar calculation in which the effect of latent heat release was neglected, that is, $\bar{\Gamma}$ was put equal to $\bar{\Gamma}_d$ everywhere in equation (6). The intensity of the indirect ageostrophic circulation was considerably reduced in this case, indicating that horizontally differential latent heat release may sometimes have an important effect on the destabilisation produced by differential advection near frontal zones.

In the case investigated by Todsen the implied destabilisation was confined to a region near the 500 mb level. However, the ageostrophic flow depicted in figure 4, which is rather like the reverse of that believed to characterise warm fronts, is typical of only some cold fronts (Miles 1962). Sawyer (1955) has commented on the deep convective clouds which were observed immediately ahead of a number of the cold fronts he studied. These

may perhaps be the result of the destabilisation of a deep layer of the troposphere by differential advection in the ageostrophic circulations associated with these fronts.

The region of intense anti-cyclonic shear of the parallel geostrophic component near the axes of jet streams (as, for example in figure 3) may on occasion be especially favourable for destabilisation by differential advection. Defant (1959) has noted near dry adiabatic lapse rates just south an E-W jet core.

Todsen's front was also untypical in that the associated ageostrophic flow was sufficiently two-dimensional to render the hypothesis underlying the derivation of equation (6) reasonably valid. However, there is no reason to suppose that destabilisation of a similar nature will not accompany more complicated flows. Indeed, the suggestion is that differential advection in ageostrophic circulations quite commonly tends to create static instability on the warm, anti-cyclonic sides of cold fronts and the associated jet streams. Similar effects may be expected in suitable regions near warm fronts.

With reference to numerical weather prediction models, it is clear that the destabilising effects of

differential advection cannot be simulated in quasi-geostrophic models, in which advection is entirely geostrophic. The effect may however be found in primitive equation models.

2.3 Vertical stretching

The third term of equation (2A) represents the rate at which γ is changed when columns of air are stretched in the vertical, namely,

$$\frac{D}{Dt} (\gamma - \Gamma) = - \frac{\partial w}{\partial z} (\gamma - \Gamma) \quad (7)$$

It may be seen from equation (7) that, in a convergent wind field ($\frac{\partial w}{\partial z} > 0$), the departure of the lapse rate from the appropriate adiabatic value is reduced by vertical stretching. However, static instability is never produced directly by this process since $\frac{D}{Dt} (\gamma - \Gamma) = 0$ when $\gamma = \Gamma$. Of course, static instability may be created if air which has been destabilised by this process while unsaturated subsequently becomes saturated.

Consider a simple model in which air is subjected to a constant stretching while ascending from level z_1 , where it has vertical velocity w_1 , to level z_2 , where it arrives with vertical velocity w_2 . If $s = \frac{w_2 - w_1}{z_2 - z_1}$,

equation (7) becomes

$$\frac{D}{Dt} (\gamma - \Gamma) = -s(\gamma - \Gamma) .$$

Thus if $\gamma = \gamma_1$ and $\Gamma = \Gamma_1$ at $z = z_1$, at $z = z_2$

$$(\gamma_2 - \Gamma_2) = (\gamma_1 - \Gamma_1) e^{-st}$$

where t is the time of ascent from z_1 to z_2 , given by

$$t = \int_{z_1}^{z_2} \frac{dz}{w} = \int_{z_1}^{z_2} \frac{dz}{w_1 + s(z - z_1)} = \frac{1}{s} \log \frac{w_2}{w_1}$$

so that

$$(\gamma_2 - \Gamma_2) = (\gamma_1 - \Gamma_1) \frac{w_1}{w_2}$$

Typically, with z_1 near 900 mb and z_2 near 700 mb, $w_2 \approx 2w_1$, so that the departure of the lapse rate from the appropriate adiabatic is about halved during such an ascent.

Static instability is most likely to be produced (indirectly) by vertical stretching if air first becomes saturated near the top of such an ascent. However, since the difference between Γ_d and Γ_s is also about halved in going from 900 mb to 700 mb, the significance of this

effect is inevitably rather limited.

2.4 Differential diabatic heating

The final term on the right of equation (2A) represents the rate at which the lapse rate is changed by vertically differential diabatic heating effects. These effects may be of three kinds in the atmosphere: heating by latent heat release, heating by radiation, and heating from the surface (essentially by conduction).

The most important effects of latent heat release have been catered for by the definition of Γ in equation (2A), and two aspects of these effects have been mentioned in sections 2.2 and 2.3 above. In addition, destabilisation may be produced by differential latent heat release at cloud edges (Ludlam and Scorer 1957) or when precipitation evaporates into unsaturated air.

Incoming solar radiation is mainly absorbed only at the earth's surface, so that the associated energy reaches the atmosphere in the guise of surface heating. Outgoing long-wave radiation has an overall destabilising effect in the troposphere which is off-set, in mid-latitudes, by the vertical heat transport achieved by large-scale convection. In shallow layers of cloud radiative

destabilising processes are accentuated, and static instability is often produced.

The destabilisation associated with heating from the earth's surface into the air above maintains the surface layer of small-scale convection discussed in section 1.4 above. Static instability is observed only in the lowest few hundred meters of the surface layer; elsewhere the appropriate neutral lapse rate for small-scale convection is rapidly established by the motion, though nevertheless the whole layer is characterised by a decrease of θ_s with height (that is, by conditional instability).

Several of the destabilising effects mentioned above influence only very shallow layers of air, and are therefore of little consequence to numerical weather prediction models. In general the extent to which these diabatic effects are represented in such models depends on the geographical location of the forecast region, and the range of prediction attempted. The most important effect of diabatic heating on the development of static instability is the indirect one, associated with the surface layer, which is discussed in the following section.

2.5 The saturation of conditionally unstable air

When air becomes saturated on a scale greater than the characteristic dimensions of small-scale convective circulations, the neutral lapse rate γ_c is reduced to something close Γ_s . Consequently, when conditionally unstable air, for which $\gamma > \Gamma_s$, becomes saturated in the large-scale flow, static instability is instantaneously created.

Thus static instability may occur when air which has been destabilised dynamically, that is, by differential advection or vertical stretching, until it is conditionally unstable, subsequently becomes saturated. A more common development of static instability by this process, however, is that implied by the general scheme of tropospheric motion as described in section 1.4 above.

Air in the descending branches of large-scale convective circulations becomes conditionally unstable in the lower troposphere under the influence of small-scale convection associated with surface heating. As described schematically in figure 1, this conditionally unstable air subsequently leaves the surface layer and enters the ascending branches of large-scale convection systems. Here it becomes saturated as a result of lifting, and is therefore statically unstable, with

the result that further small-scale convection is initiated. Green, Ludlam, and McIlveen (1966) have noted the effect of this phenomenon in relation to the construction of air trajectories.

Conditionally unstable air from the surface layer is continuously drawn into large-scale ascent during the life of a large-scale convection system, so that the lower regions of the ascending branches of such systems are generally characterised by the realisation of static instability, the onset of small-scale convection, and corresponding small-scale features in the precipitation pattern.

The development of static instability by the saturation of conditionally unstable air was discussed by Browning and Harrold (1969) in relation to a depression which crossed the British Isles on 16 October 1967. Figure 5, reproduced from their paper, shows isopleths of θ_w on a vertical section through the warm-sector of this depression at 12 GMT on that day. Conditionally unstable air is found at two levels, termed by the authors 'high- and low-level instability'. Conditionally unstable air was becoming saturated by the motion at both levels and initiating small-scale convection with consequences for the precipitation pattern which are discussed by the authors.

The low-level instability shown in figure 5 is a manifestation of the surface layer of small-scale convection, in which air has become conditionally unstable as a result of heating from the surface. In the region between the two fronts this conditionally unstable air is likely to become saturated in the large-scale ascent. The position of the high-level instability, in the warm air near 500 mb, is reminiscent of that of the ageostrophic destabilisation in the front studied by Todsén, although in this case the destabilisation is more closely associated with the warm front than with the cold. Thus there is a suggestion that this high-level instability is the result of differential advection in the ageostrophic flow associated with the depression. Again, where the air is ascending, saturation and the associated static instability are likely to be produced.

This particular destabilisation process may be simulated in those numerical weather prediction models which include latent heat release in the thermodynamic equation when the air is saturated and compute the water content of the air as an independent parameter.

2.6 Destabilisation in the Bushby-Tompson model

Bushby and Timpson (1967) have described a numerical model of the mid-latitude regions of the troposphere which has 10 levels in the vertical, uses the primitive equations, and retains the mixing ratio of water vapour as an independent parameter, but includes (in the version of the model discussed here) no representation of diabatic heating by radiation or from the surface.

Destabilisation may, in principle, take place as a result of both differential advection and the saturation of conditionally unstable air in the model atmosphere. That these processes are in fact simulated by the model is demonstrated in this section by means of an example of a prediction computed by a version of the model which included no representation of small-scale convection, so that the lapse rate was determined entirely by the initial data and the large-scale flow.

Figure 6 shows the surface analysis for 18 GMT 25 December 1960, when the warm sector of an intense depression lay over the British Isles. Figure 7 shows isopleths of θ_s on the vertical section A-B across this warm sector, as predicted for this time (from initial data 6 hours earlier) by the Bushby-Timpson model.

Conditional instability is found at two levels on this section, there being some general similarity with the observed section shown in figure 5. The low-level instability is here a manifestation of the surface layer as defined by the initial data 6 hours previously, so that, although the model includes no representation of surface heating, static instability will still develop according to the process described in the previous section, at least for relatively short predictions. The position of the high-level instability, just ahead of the cold front, is strongly suggestive of destabilisation by differential advection similar to that implied by Todsén's results.

Figure 8 shows the same vertical section 6 hours later. By this stage of the prediction some of the conditionally unstable air at both high- and low-levels has become saturated as a result of large-scale ascent and is therefore now statically unstable,

Figure 9 shows the evolution of the predicted θ structure, on this same vertical section, at 6 hour intervals from 00 GMT 26.12.60 onwards. From some stage between 00 GMT and 06 GMT 26.12.60 onwards, the region of high-level instability is characterised by absolute

instability. Among the destabilising processes which can be simulated by the Bushby-Timpson model, only differential advection is capable of increasing γ beyond $\bar{\gamma}_d$. The development of this high level instability must therefore be attributed to the effects of differential advection in thermodynamically indirect ageostrophic circulations.

Other aspects of the development of static instability in predictions computed with the Bushby-Timpson model are discussed in chapters 3 and 5 below.

2.7 Summary

Two destabilising processes, which may be simulated in suitable numerical weather prediction models, commonly operate in conjunction with large-scale convection and may tend to produce static instability in the large-scale flow.

(1) Differential advection of air in thermodynamically indirect ageostrophic circulations, associated with frontal regions of large-scale convection systems, produces destabilisation and may on occasion tend to produce absolute instability. This effect occurs, in particular, on the warm sides of frontal zones and the anti-cyclonic sides of jet streams.

(2) Conditionally unstable air, produced either by differential advection or as a result of heating from the surface, enters regions of large-scale ascent where it becomes saturated and therefore statically unstable.

In the real atmosphere both kinds of destabilisation may be expected to initiate small-scale convection which will rapidly transport heat in the vertical so as to maintain the appropriate neutral lapse rate. The consequences of such developments of static instability for numerical models of the atmosphere are discussed in the next chapter.

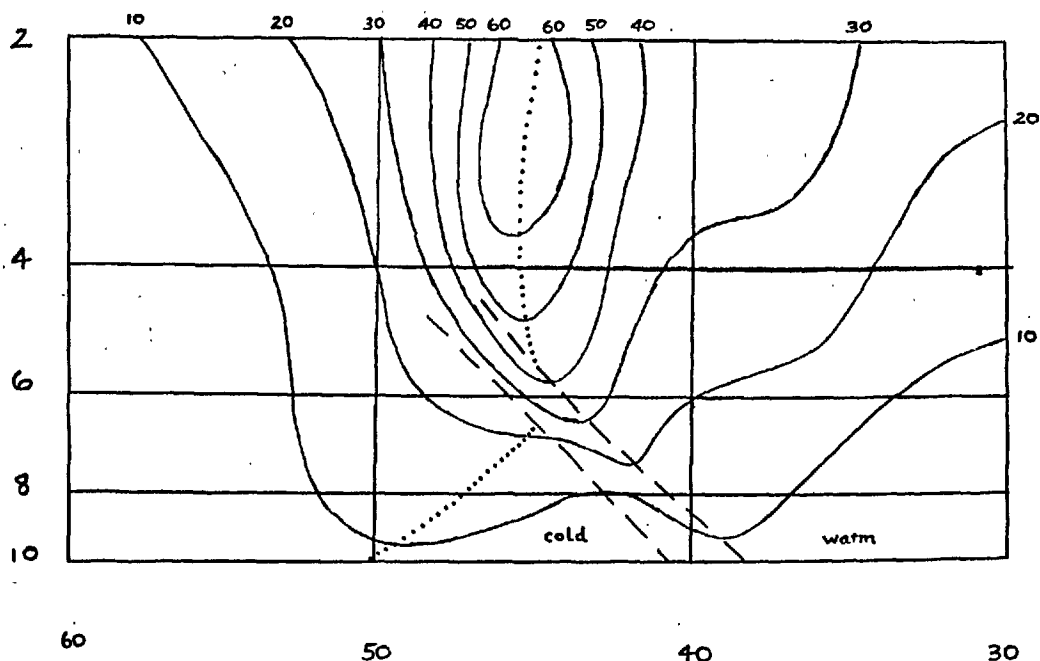


Figure 3 (after Palmen and Newton 1948) shows isotachs (labelled in m sec^{-1}) of the geostrophic component parallel to a cold front, meaned over 12 cases in December 1946. The vertical axis is labelled in hundreds of mb and the horizontal in degrees of latitude north along the 80°W meridian. Except within the frontal zone (pecked lines) the horizontal shear of this component is anti-cyclonic everywhere south of the dotted line.

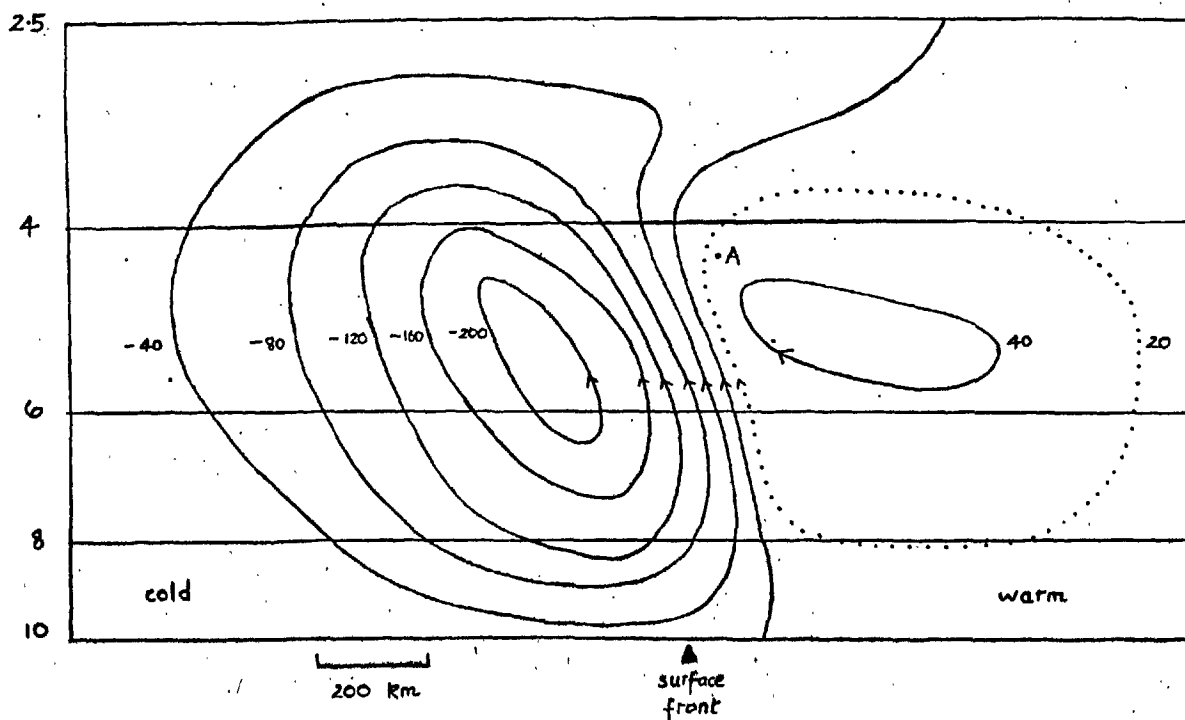


Figure 4 (after Todsén 1964) shows computed streamlines (isopleths of Ψ_p : labelled in units of 10^6 gm sec^{-2}) of the ageostrophic flow in a cold front at 12 GMT on 8 April 1962. In the neighbourhood of point 'A' the lapse rate is being increased by differential advection at a rate of about $0.1^\circ \text{C km}^{-1}$ per hour.

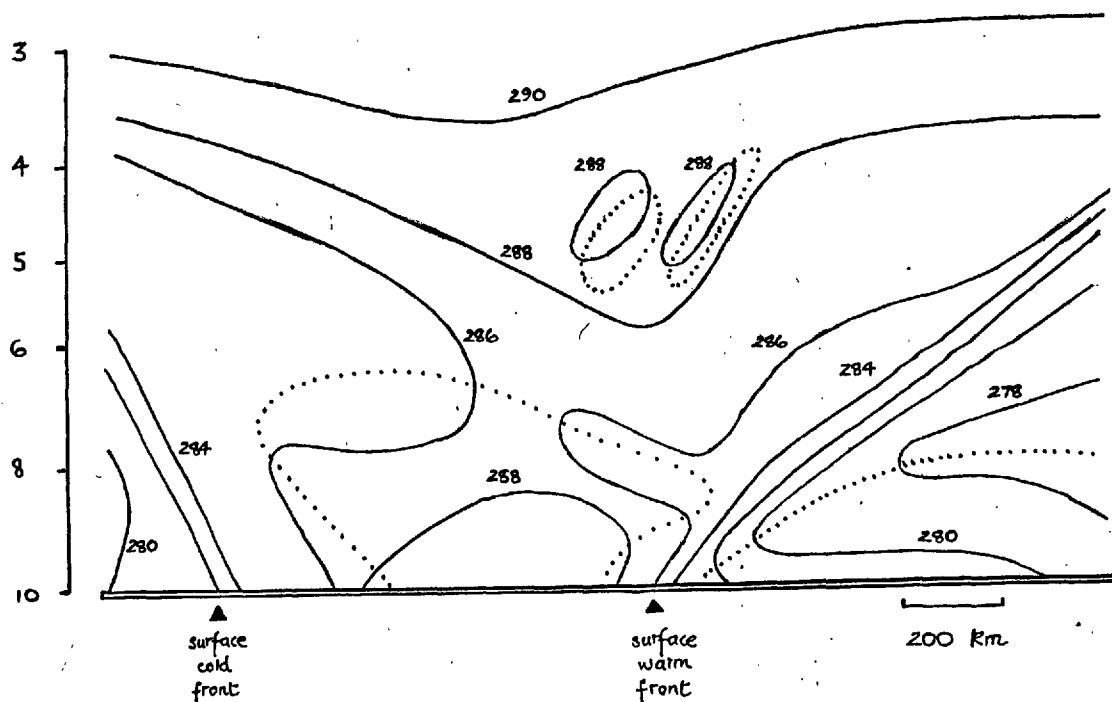


Figure 5 (after Browning and Harrold 1969) shows isonleths of Θ_w (labelled in degrees K) as observed on a vertical section across the warm sector of a depression at 12 GMT on 16 October 1967. The vertical axis is labelled in hundreds of mb. Regions of conditional instability (where Θ_w decreases upwards) are indicated by dotted lines.

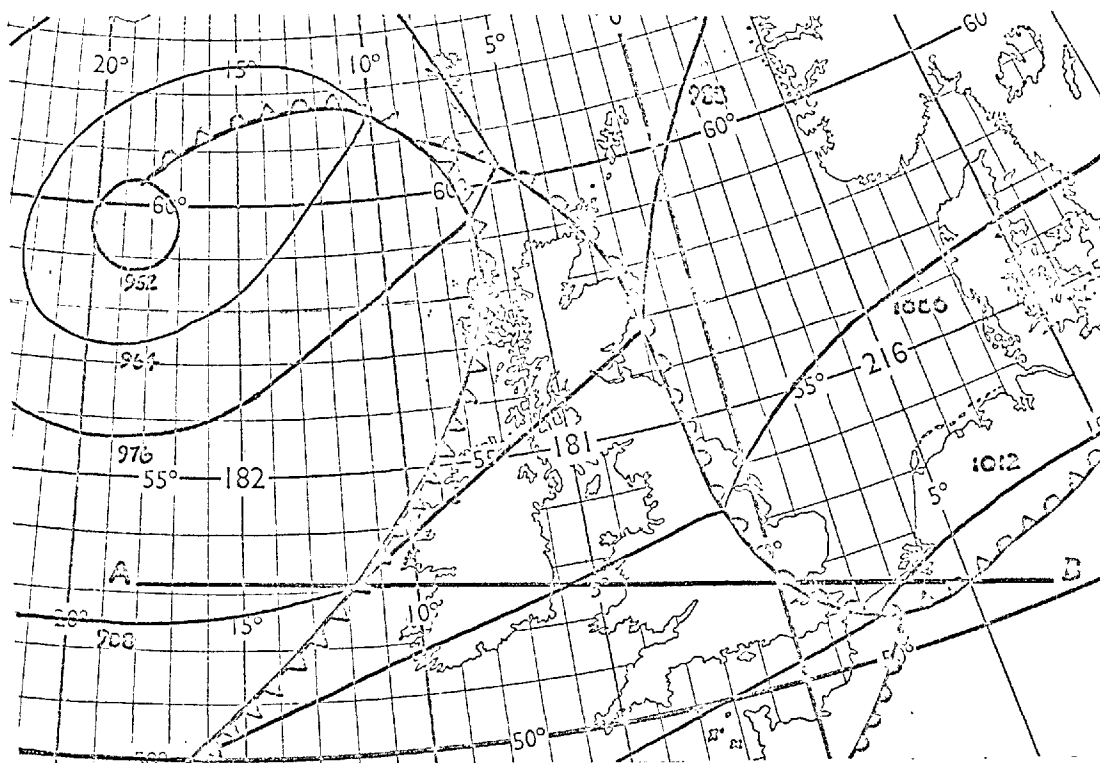


Figure 6 shows the position of the cross-section A-B (see figures 7, 8, and 9) in relation to the surface isobars and frontal analysis for 18 GMT 25.12.60.

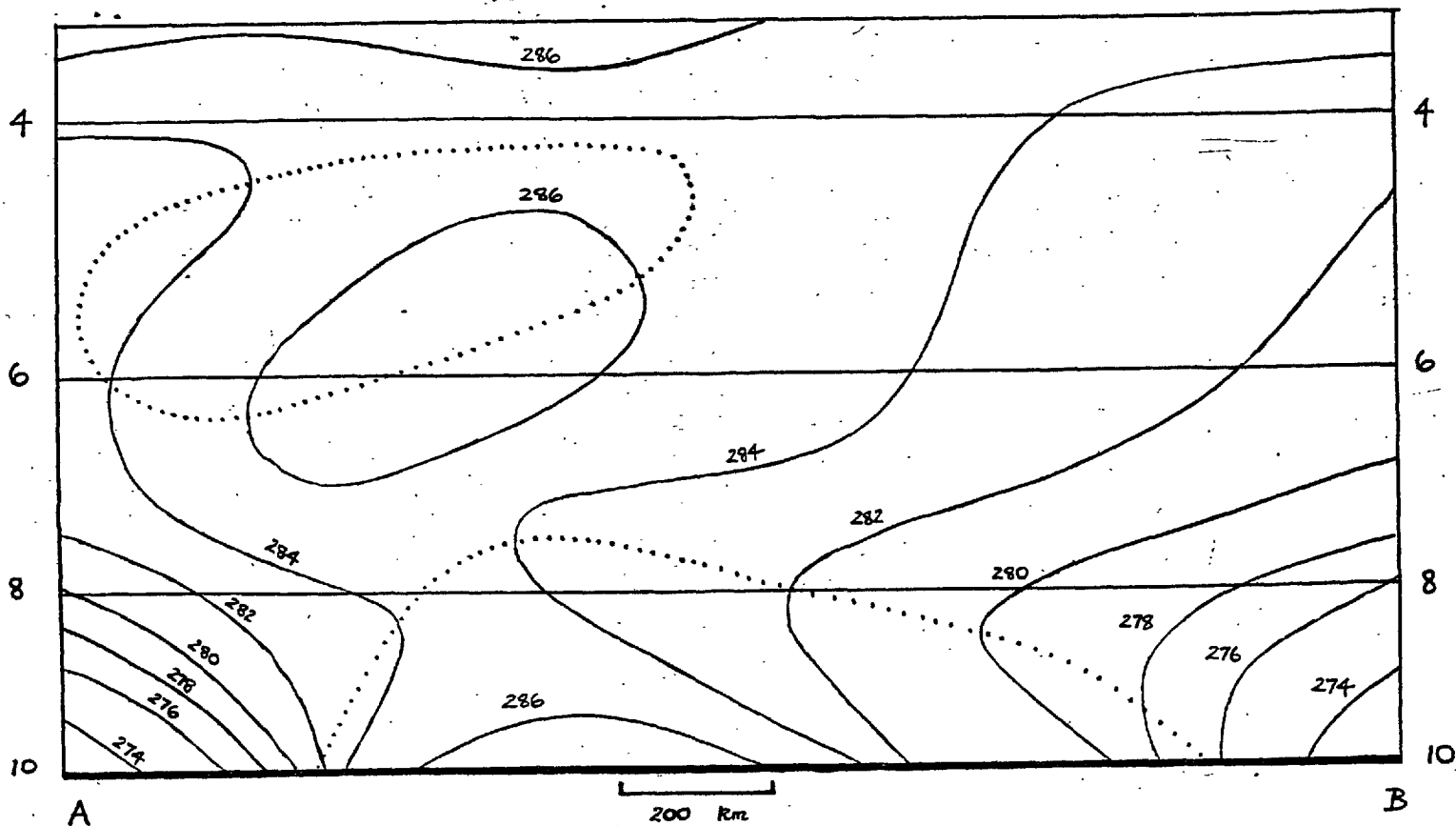


Figure 7 shows isonleths of θ_s (labelled in degrees K) on the vertical section A-B (see figure 6) as predicted by the Bushby-Timpson model for 18 GMT 25.12.60 (initial data 12 GMT 25.12.60). Regions of conditional instability are indicated by dotted lines.

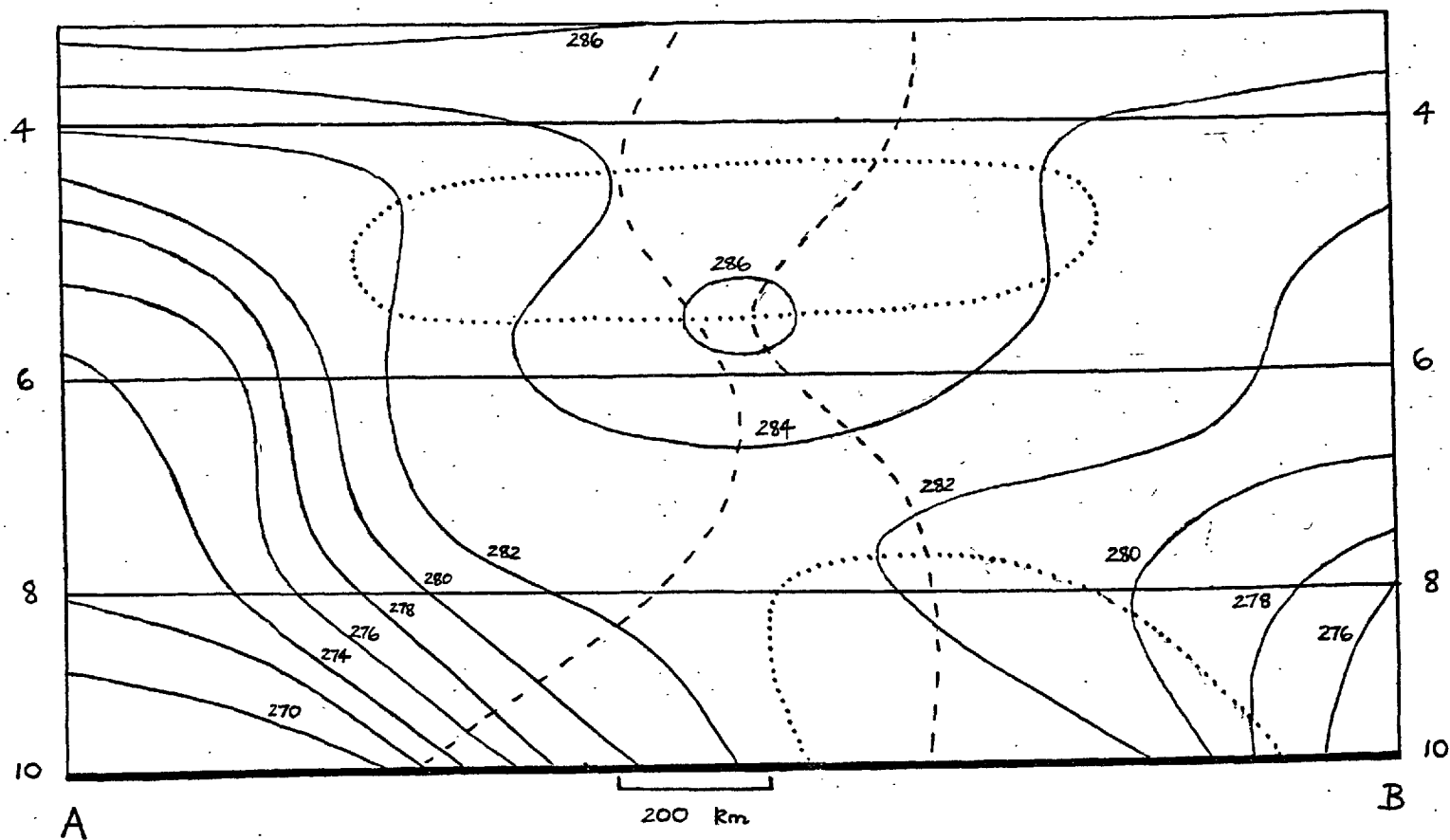


Figure 8 shows isopleths of θ_s (labelled in degrees K) on the vertical section A-B (see figure 6) as predicted by the Bushby-Timoson model for 00 GMT 26.12.60 (initial data 12 GMT 25.12.60). Regions of conditional instability are indicated by dotted lines and saturated regions by dashed lines.

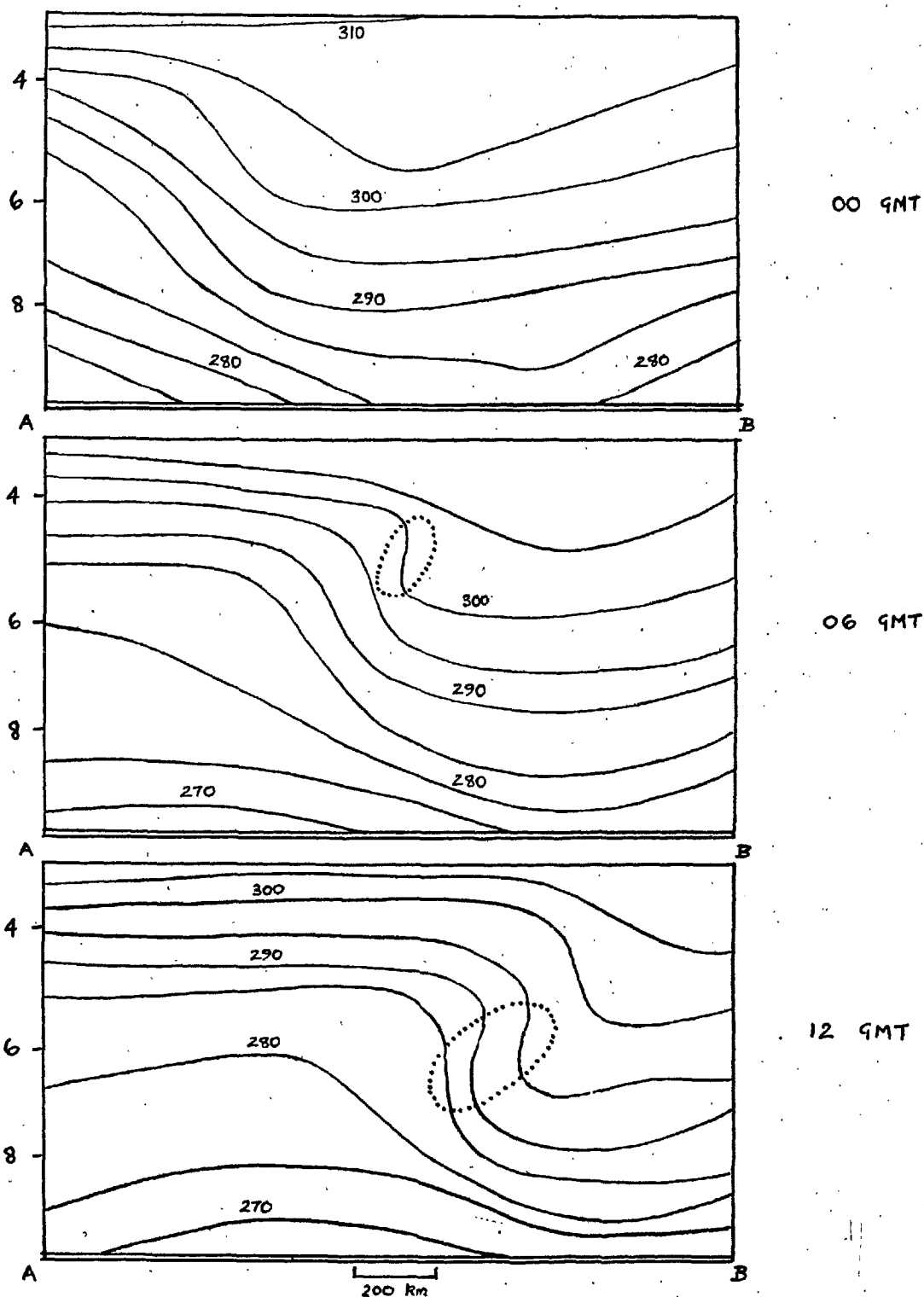


Figure 9 shows the evolution of the potential temperature structure (isentropes labelled in degrees K) on the vertical section A-B (see figure 6) predicted by the Bushby-Timpson model during the period 00 GMT to 12 GMT 26.12.60 (initial data 12 GMT 25.12.60). Regions of absolute instability are indicated by dotted lines.

3. STATIC INSTABILITY AND SMALL-SCALE CONVECTION IN NUMERICAL WEATHER PREDICTION MODELS

3.1 The consequences of the development of static instability in numerical models of the atmosphere

In chapter 2 it was shown that certain kinds of numerical weather prediction model, notably multi-level, wet, primitive equation models, are able to simulate certain tendencies for static instability to develop in the large-scale flow. In the real atmosphere such developments are accompanied by the growth of small-scale convection, which exercises a rather efficient control on the large-scale lapse rate of temperature. In model atmospheres, however, unless some representation of small-scale convection is included, the static instability may persist or become more intense under the influence of the large-scale flow. In such circumstances meteorologically unrealistic motions grow in the model atmosphere and these may spoil the computation.

(a) Grid-scale convection

As was pointed out in chapter 1, simple perturbation theory shows that small-scale convection will grow in statically unstable air, the fastest growth rate

corresponding to the shortest wavelengths. The shortest wavelength which can be resolved by a numerical model is equal to twice the grid length of the model, and consequently it is convection with such a wavelength, so-called grid-scale convection, which grows most rapidly in statically unstable regions of model atmosphere.

The orders of magnitude of the grid lengths which are chosen for numerical models of large-scale convection imply wavelengths for grid-scale convection which are much larger than those of small-scale convection in the real atmosphere. As a consequence grid-scale convection has a comparatively slow growth rate. Whereas, for typical small-scale convection in the real atmosphere, the doubling time corresponding to static instability amounting to $0.1 \tau_d$ is only about 3 minutes, grid-scale convection, in a numerical model with a grid length of 100 km, takes, for the same static instability, and for convection confined to a 2 km layer, roughly 1 hour to double its amplitude.

Numerical simulations of motion in conditionally unstable tropical atmospheres by Kasahara (1961) and Sasaki (1960) showed evidence of grid-scale convection in saturated regions of the model atmospheres, with

growth rates closely in accord with those obtained from linearised analysis. The grid lengths of these models were such, however, that the authors could attribute to these developments realistic meteorological origins.

In numerical models of large-scale convection, however, when grid lengths are of the order of 100 km, it is clear that grid-scale convection is a meteorologically quite unrealistic scale of motion. As such its growth is a very undesirable development in numerical weather prediction models. Manabe, Smagorinsky, and Strickler (1965) noted this effect, and took steps to eliminate it from their model.

Benwell and Timpson (1968) have described the difficulties encountered while attempting to produce a 24 hour forecast from initial data corresponding to 12 GMT 25 December 1960, using the Bushby-Timpson model. Figure 11 shows the central portion of the 600 mb vertical velocity field predicted for the 18 hour forecast stage from this initial data. As may be seen from figure 11 the prediction was dominated by short wave-length motions whose amplitudes had by that stage grown to the order of several hundreds of millibars per hour in some places.

The lapse rates of temperature implied by the height of the 10 isobaric surfaces at the initial time were

computed. Since temperature is not carried as an independent parameter of the Bushby-Timpson model, the lapse rates were derived at each of the 8 levels from 900 mb to 200 mb from the thicknesses of the nine 100 mb layers which make up the model atmosphere. Using the notation illustrated in figure 10, the static equation for the pressure, namely:

$$\frac{\partial p}{\partial z} = - \rho g = - \frac{p g}{R T}$$

may be integrated from pressure level n to pressure level $n + 1$ to give the following expression for the thickness of layer $n + 1/2$:

$$h'_{n+1/2} = \int_{p_{n+1}}^{p_n} dz = \frac{R}{g} \int_{p_{n+1}}^{p_{n+1}} T \, d \log p$$

A weighted mean temperature $\bar{T}_{n+1/2}$ may then be defined for this layer by:

$$\bar{T}_{n+1/2} = \int_{p_n}^{p_{n+1}} T \, d \log p \bigg/ \int_{p_n}^{p_{n+1}} d \log p = g h'_{n+1/2} \bigg/ R \log \frac{p_{n+1}}{p_n}$$

$\bar{T}_{n+1/2}$ is representative of the level $\bar{p}_{n+1/2}$ given by

$\bar{p}_{n+1/2}^2 = p_n p_{n+1}$, which is, approximately, the mid-point by distance of the layer. Thus the lapse rate of temperature implied at level n is:

$$\gamma_n = \frac{g}{R} \left\{ \frac{h'_{n+1/2}}{\log \frac{p_{n+1}}{p_n}} - \frac{h'_{n-1/2}}{\log \frac{p_n}{p_{n-1}}} \right\} \bigg/ \frac{1}{2}(h'_{n+1/2} + h'_{n-1/2})$$

Defining $c_{n+1/2} = \frac{2C_p}{R \log \frac{p_{n+1}}{p_n}}$, it is convenient to use

the dimensionless lapse rate N_n given by:

$$N_n = \frac{C_p}{g} \gamma_n = \frac{c_{n+1/2} h'_{n+1/2} - c_{n-1/2} h'_{n-1/2}}{h'_{n+1/2} + h'_{n-1/2}}$$

Since the air density inferred from the thickness of 100 mb layers in this derivation includes the effect of the water vapour content, N_n strictly represents the lapse rate of virtual temperature. This is a complication of rather minor significance which will not be considered here. Hence the model atmosphere is judged to be absolutely unstable at level n if $N_n > 1$.

The set of initial data mentioned above was found to be characterised by values of N greater than 1 over fairly extensive regions and at a number of levels.

Benwell and Timpson (1968) have discussed the way in which the data extraction procedures used for the model may inadvertently introduce unrealistic lapse rates, including super-dry-adiabatic lapses, into the initial data of the model. Notice, for example, that an error of only 10 m in the height of the 900 mb surface implies an error of about $0.5 \Gamma_d$ in the derived lapse rate at that level.

Benwell and Timpson went on to describe a scheme of vertical smoothing which, by introducing thickness changes of the order of 10m, which are virtually imperceptible when examined in horizontal fields, frees the initial data from absolutely unstable lapse rates. Figure 12 shows the 600 mb vertical velocity field comparable to that shown in figure 11 but predicted from initial data which had been smoothed in this way. The model atmosphere is now seen to be free from grid-scale features in this region, and the vertical motion now nowhere exceeds 25 mb/hr. The forecast was successfully computed to 24 hours. The intense spurious short wavelength motion depicted in figure 11 is thus seen to be a consequence of the spurious absolute instability in the initial data.

While it is predictable that convection will grow on the scale of the grid-length in statically unstable regions of model atmospheres, it is rather unexpected that the amplitude of the short wave-length components continues to increase in an uncontrolled way throughout the forecast. It might be thought that the effect of the grid-scale convection in modifying the vertical structure of the model atmosphere would be to remove the static instability and thus inhibit its own further growth. Before this could happen, however, considerable roughnesses will have been introduced into the horizontal fields of pressure and velocity of the model. Short wavelength components in these fields are known to give rise to non-linear computational instabilities in some numerical models (see for example Phillips 1959), and it may be that it is some such instability, initiated by the grid-length components associated with grid-scale convection, which leads to the final ruin of the computation.

That the initial data of a numerical weather prediction model should be absolutely unstable is, of course, meteorologically unrealistic. However, as was shown by the examples described in section 2.6, absolutely unstable lapse rates may develop, by differential advection, near fronts, even when the computation begins with realistic

initial data. Since destabilisation is achieved only rather gradually in this way, and since only limited regions of the model atmosphere are affected, such developments do not, in general, seriously detract from the quality of 24 hour predictions computed with the Bushby-Timpson model. In some cases however, especially when longer term forecasts are computed, serious deleterious effects, associated with the growth of grid-scale convection, must be expected unless some representation of the stabilising influence of small-scale convection is included in the model.

(b) An example of the consequences of saturated instability

Rather more common and extensive in model atmospheres such as the Bushby-Timpson model than regions of absolute instability are regions in which conditionally unstable air becomes saturated, a condition which may be described as 'saturated instability'.

Figure 13 shows the 1000 mb contour field predicted by the Bushby-Timpson model for 18 GMT 12 July 1965, from initial data corresponding to 00 GMT on that day. The contour pattern is seen to be rather distorted in the warm sector region, with a narrow (single grid length) trough running eastwards from the centre. Figure 14

shows isopleths of Θ_s and regions of saturated instability on the vertical section A-B along this trough. There is an extensive region of saturated unstable air near 700 mb.

Figure 15 shows the 1000 mb contour pattern predicted for the same time and from the same initial data, using a version of the model which differed from that which predicted the pattern illustrated by figure 13 only in that it included the scheme of convective adjustments described in chapter 4 below. Thus saturated instability was prevented from developing during this second prediction. The spurious trough referred to above is seen to be absent from figure 15, and it must be concluded that its development in the original simulation was a consequence of the persistent existence of regions of saturated instability in the model atmosphere.

3.2 The meteorological importance of small-scale convection in numerical weather prediction models

The undesirable effects described in the previous section make it imperative that some representation of the role of small-scale convection in inhibiting the development of static instability be included in multi-level wet primitive equation models of large-scale

convection. In addition, however, certain more positive meteorological benefits may be obtained with such an incorporation. These benefits vary according to the location of the modelled area and the time-scale of the simulation. In some cases the effects of the small-scale convection will have a direct and important bearing on the dynamics of the larger-scale motion under study. For instance, in simulations of tropical cyclones it is essential that the release of latent heat in small-scale convection be represented. Again, for long-term simulations of mid-latitude large-scale convection, the distribution of energy from the surface through the lower troposphere by small-scale convection is a fundamental process which must be taken into account.

In the context of 1-2 day mid-latitude predictions, however, such as those computed with the Bushby-Timpson model, the influence of small-scale convection on the large-scale flow is likely to be rather small. This is especially the case with the version of the Bushby-Timpson model used here, since no representation of heating from the surface is included.

When rainfall prediction is among the objectives of a numerical simulation, as is the case with the Bushby-Timpson model, small-scale convection, even in

the absence of surface heating, can be rather important. This aspect is illustrated by the results described in chapter 5.

3.3 A survey of representations of small-scale convection which have been used in numerical weather prediction models

A number of different representations of small-scale convection have been used in numerical models of atmospheric flow, and a brief description of them is given below.

(a) L.F. Richardson's model

Richardson (1922) designed the first numerical model of the atmosphere. In this model no distinction was drawn between thermally and mechanically driven sub-grid scale motions. The rate of change of the large-scale parameter A (horizontal velocity, potential temperature, or humidity mixing ratio) due to all sub-grid scale motions was represented by a term of the form

$$\frac{\partial}{\partial z} (K_A \frac{\partial A}{\partial z})$$

in the basic equations of the model. Richardson stressed that such an expression is merely a definition of K_A , the values of which must be tabulated, from such empirical

evidence as is available, for each large-scale parameter and for various large scale conditions.

(b) Tropical cyclone models

The important role which latent heat release in deep cumulus convection is thought to play in the development of tropical cyclones has led workers in this field to propose parameterisations of small-scale convection suitable for use in numerical simulations of such developments. Charney and Eliassen (1964) and Kuo (1965) have proposed essentially similar schemes in which a fraction α (determined by the low-level moisture convergence) of the area represented by a grid point is considered to be occupied by cumulus clouds whenever deep conditional instability and low-level convergence coincide. Air is assumed to ascend pseudo-adiabatically within the clouds and to mix instantaneously and completely with the environment. The rate of large-scale heating at a given level resulting from the small-scale convection is then given by

$$Q = \frac{\alpha C_p}{\Delta t} (T_s - T_e)$$

where T_s is the temperature on the pseudo-adiabat through the lifting condensation level of the surface

air, T_e is the environmental temperature, and Δt is an arbitrary time constant related to the rate at which the clouds are produced.

This parameterisation has also been used in mid-latitude regions by Krishnamurti (1967) in a diagnostic model.

(c) General circulation models

Vertical heat transports from the lower to the upper regions of the troposphere in low-latitudes by small-scale convection play a vital role in the general circulation of the atmosphere. Consequently a number of numerical models of the general circulation have incorporated representations of small-scale convection.

Mintz (1965), in a four layer model, used empirical time constants to describe the heating and stabilising effects of small-scale convection. Leith (1965), using a six level model atmosphere, suggested empirical relationships for the heat and moisture fluxes, cloud cover, and precipitation associated with small-scale convection.

Manabe, Smagorinsky, and Strickler (1965) have described a nine level wet primitive equation model of the general circulation. In this model the effects of small-scale convection are represented by making

adjustments, where necessary, to the vertical temperature structure of the model to ensure that γ never exceeds Γ_d in unsaturated air or Γ_s in saturated air. These adjustments are made at the end of each time step of integration. The total internal and potential energy of each column of the model atmosphere is conserved during the adjustments, and water vapour is transported or condensed as necessary to prevent supersaturation. In a later experiment, described by Smagorinsky and staff members (1965), this scheme was modified so that adjustments were made to ensure that γ never exceeded Γ_d for air less than 80% saturated, and never exceeded $0.2 \Gamma_d + 0.8 \Gamma_s$ for air more than 80% saturated.

3.4 Comment on the representations described above

No theory has been formulated which relates the statistical effects of small-scale convection to the large-scale winds, temperatures and humidities as defined by a numerical weather prediction model. Consequently all the representations of small-scale convection described above are essentially empirical in nature. Unfortunately observations are rarely available in the special detail required to test hypotheses as to the nature of small-scale convective transports, and in these circumstances

the simplest empirical method which represents all the important known effects recommends itself very strongly.

In some of the methods of representation described above, the empirical element is rather obscured by the fairly complicated details of the convective schemes. Ultimately, however, as with the simpler methods, these schemes are attempts to specify time scales for small-scale convective transports which agree with the behaviour of the real atmosphere in so far as it is known.

The representations of small-scale convection outlined in the previous section arise from one or the other of two basic approaches to the problem.

(a) Empirical flux methods. These methods seek, using either eddy transfer coefficients or simple models of convective clouds, to relate empirically the heat, moisture, and momentum fluxes due to small-scale convection to the parameters of the large-scale flow.

(b) Neutral lapse rate methods. These methods assume that small scale convection operates to maintain certain neutral lapse rates of temperature on the large-scale whenever these tend to be exceeded.

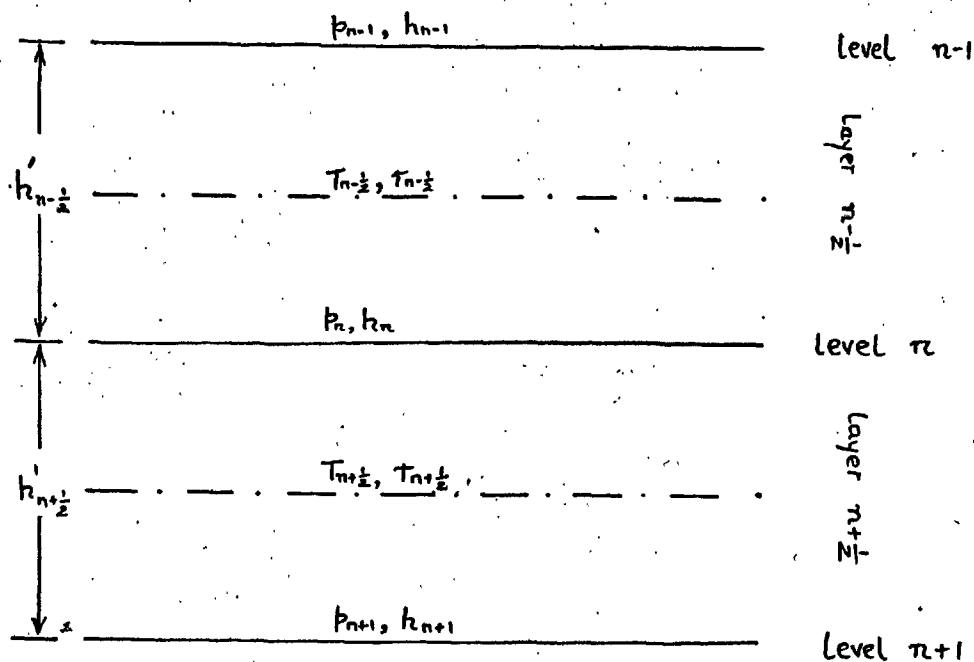
The second of these approaches seems to offer three advantages over the first.

(1) Its consequences are conceptually and computationally simpler, though they are not contrary to any observed feature of small-scale convection.

(2) It is in accord with observations of characteristic neutral lapse rates above and below cloud base in widespread cumulus convection described by Ludlam (1966).

(3) It allows for the fact that, in the real atmosphere, static instability is first manifest on scales smaller than the typical finite-difference intervals of numerical models, so that small-scale convective transports are initiated before static instability can develop on the grid-scale.

With the exception of Richardson's model, none of the representations described above includes any parameterisation of the vertical transport of horizontal momentum by small-scale convection. This reflects the uncertainty about the nature of this effect which was mentioned in section 1.3.



$$p_n = 100 \pi \quad \text{millibars}$$

Figure 10 illustrates the notation used in discussing the vertical structure of the Bushby-Timpson model.

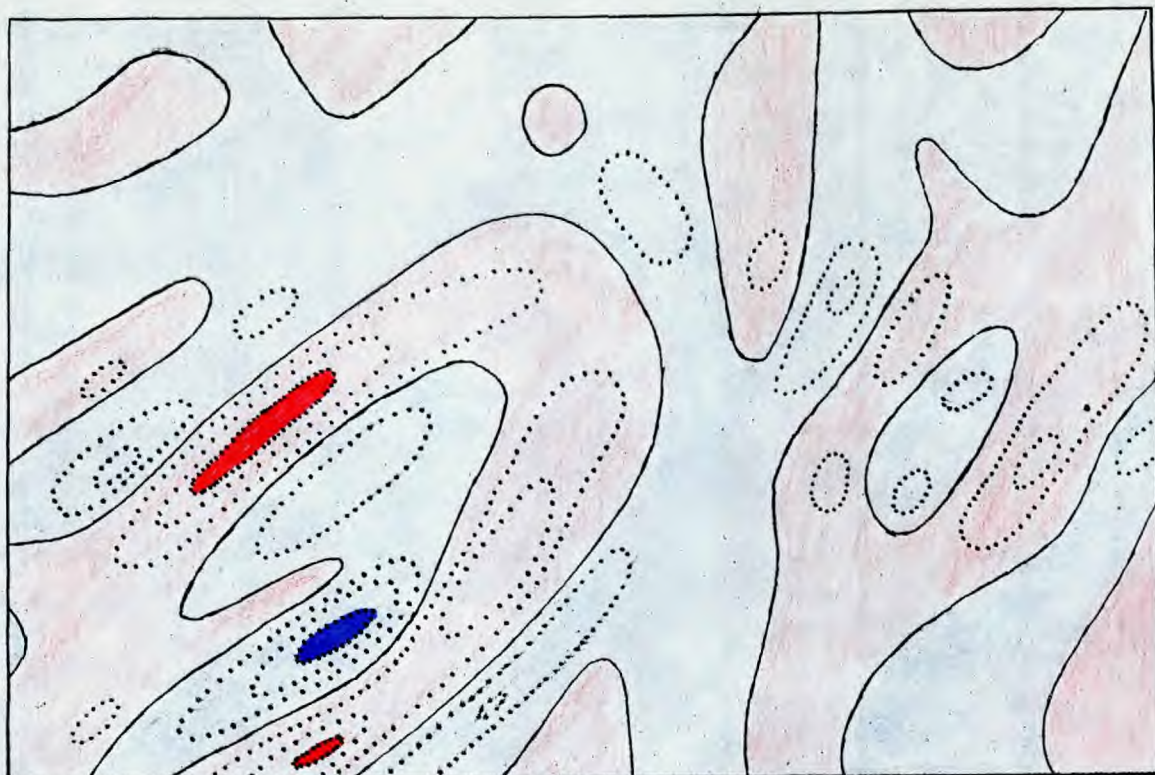


Figure 11 shows part of the 600 mb vertical velocity field computed for the 18 hour forecast stage from initial data (12 GMT 25.12.60) which was characterised by extensive regions of absolute instability. Regions of descent are indicated by blue shading; the isopleths are drawn at intervals of 200 mb hour.

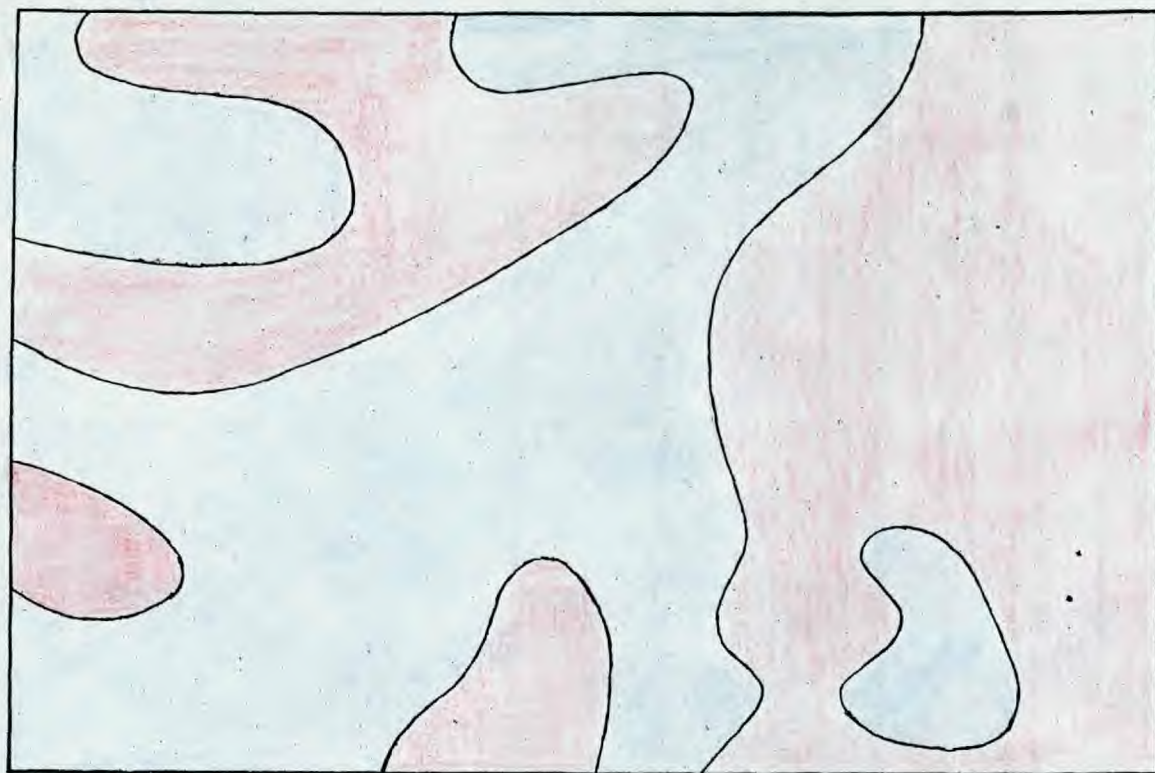


Figure 12 shows the 600 mb vertical velocity field which is comparable to that illustrated by figure 11 but which was computed from a vertically smoothed set of initial data. Regions of descent are indicated by blue shading; the vertical velocities nowhere have magnitudes in excess of 25 mb hour⁻¹.

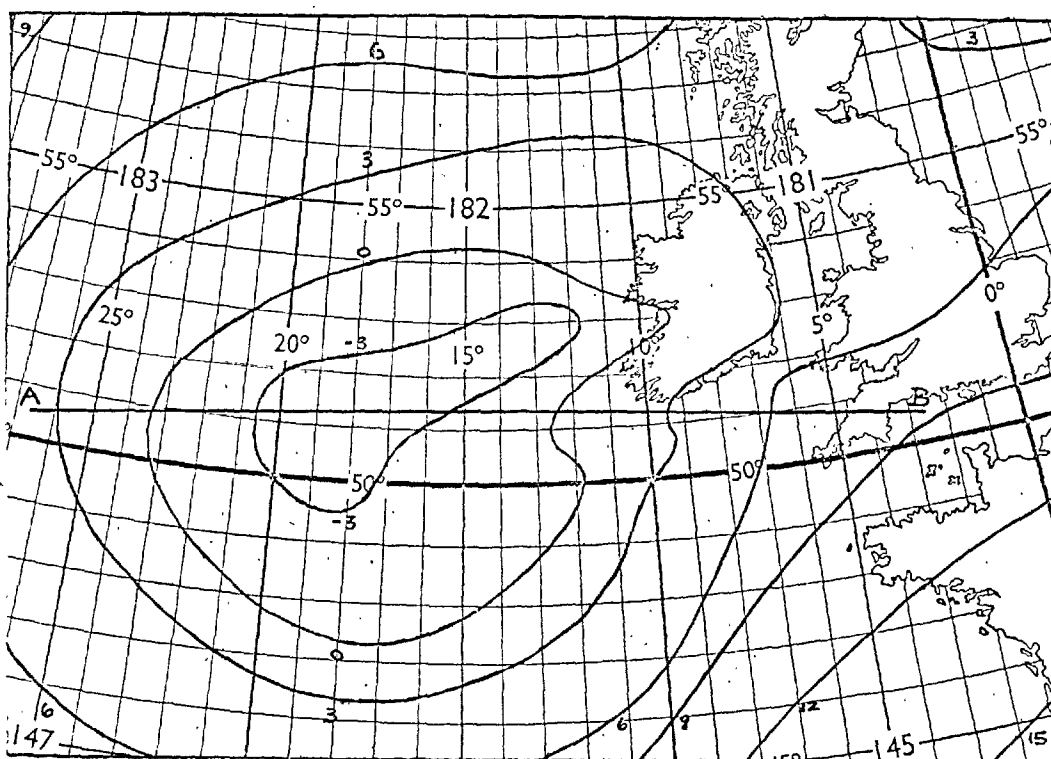


Figure 13 shows the forecast 1000 mb contours for 18 GMT 12.7.65 (initial data 00 GMT 12.7.65) and the position of the vertical section A-B. This forecast was computed using a version of the Bushby-Timpson model which does not include convective adjustments.

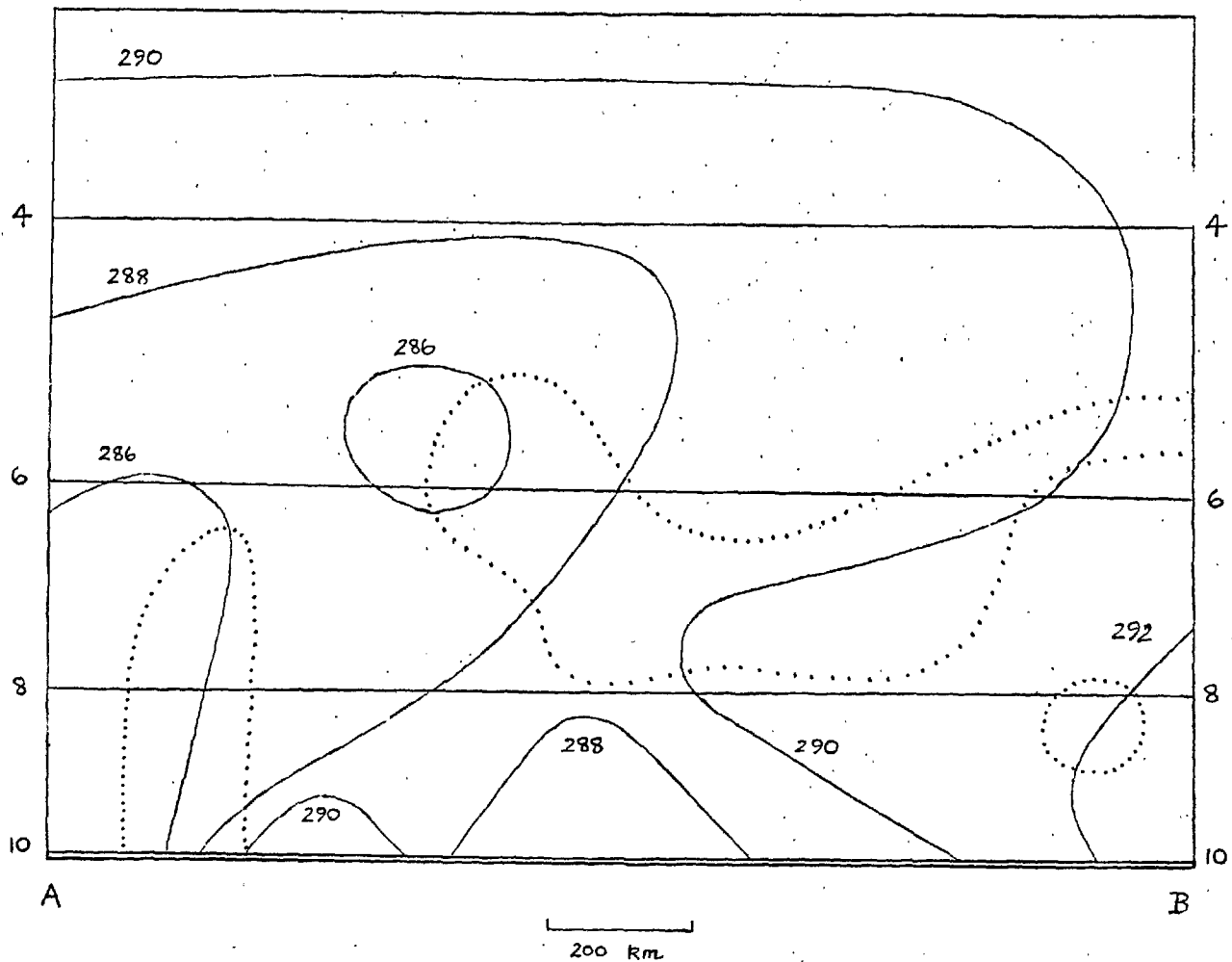


Figure 14 shows isopleths of Θ_s (labelled in degrees K) on the vertical section A-B of the depression shown in figure 13. Regions of saturated instability are indicated by dotted lines.

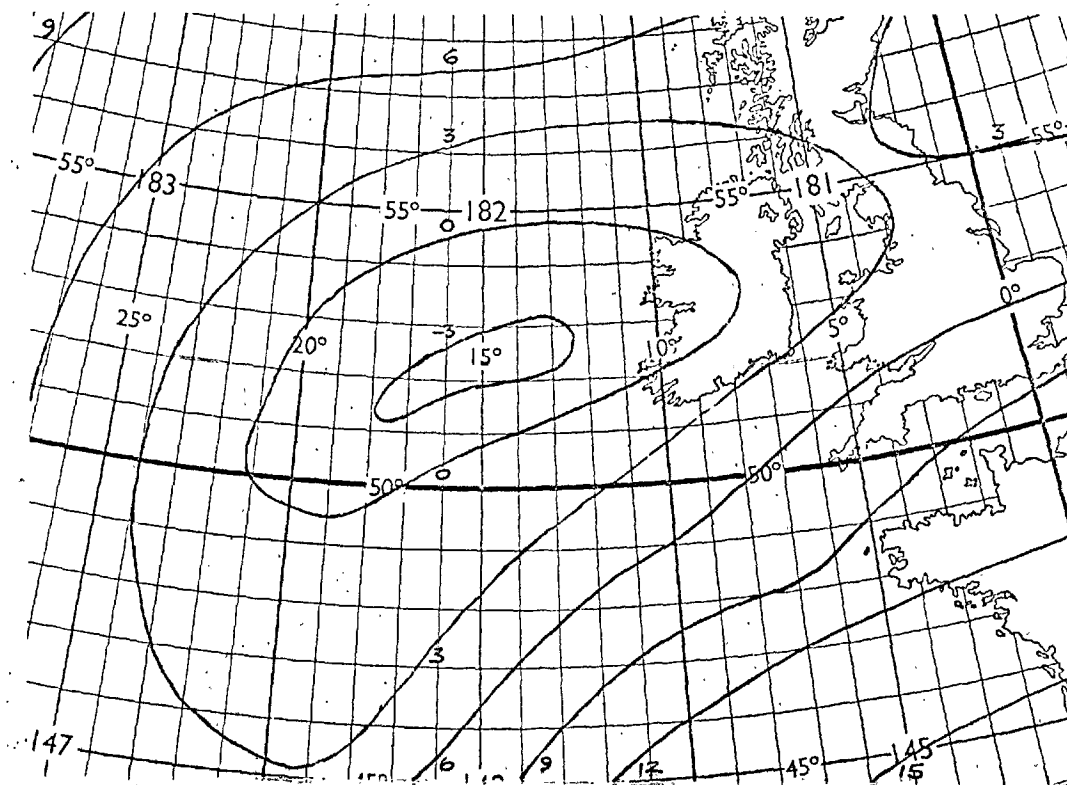


Figure 15 shows the field which is comparable to that shown in figure 13, but which was computed using a version of the Bushby-Timpson model which includes the convective adjustment scheme.

4. THE INCORPORATION OF SMALL-SCALE CONVECTION IN THE BUSHBY-TIMPSON MODEL

4.1 Introduction

As was pointed out in chapter 2, the 10-level, wet, primitive equation model described by Bushby and Timpson (1967) is capable of simulating certain tendencies of the large-scale flow in the troposphere towards producing statically unstable lapse rates. As was further indicated in chapter 3, the presence of static instability in the model atmosphere can have serious deleterious effects on the quality of the computed forecast. Hence it is important to incorporate into the Bushby-Timpson model some sufficient representation of the effects of small-scale convection, which in the real atmosphere operates rather quickly to restore neutral stability in such situations.

The Bushby-Timpson model is designed to simulate the development and movement of mid-latitude cyclones and fronts, and to predict the associated rainfall, over periods of 1-2 days. Showers are an important feature of the 'weather' associated with such motion systems, and the possibility of predicting such activity is a further reason for devising a representation of small-

scale convection for the model.

The finite-difference intervals of the model are 100 km (approx.) in the horizontal, 100 mb in the vertical, and about 3 minutes in time. Temperature is not retained as an independent parameter of the model, the thermodynamic equation being written in terms of the thicknesses of the nine 100 mb layers which make up the model atmosphere. The mixing ratio of water vapour is retained as an independent parameter at the midpoints of the 100 mb layers, except for the uppermost two layers, which are regarded as being completely dry.

In the version of the model which was used to compute the forecasts discussed in the next chapter, changes in thickness and mixing ratio appropriate to a time step of integration are computed for each layer at each grid point with no account taken of condensation or latent heat release. At the end of a time step each layer of the model atmosphere is tested for supersaturation, the required saturation mixing ratios being computed from the thicknesses of the layers by means of empirical quartic polynomials. In any layer which has become supersaturated, water vapour is condensed until the air is exactly saturated, the thickness of the layer being simultaneously augmented by the corresponding latent heat

release. The liquid water produced in this way falls immediately through the model atmosphere, but where it encounters an unsaturated layer during its descent it evaporates immediately until either it is exhausted or the layer becomes saturated. After the lowermost layer becomes saturated, liquid water may reach the surface as rainfall.

4.2 An appropriate representation of small-scale convection

In view of the discussions in sections 1.4 and 3.4 above, the important large-scale effects of small-scale convection ~~may~~ be represented in the Bushby-Timpson model as follows. After each time step of integration the temperature lapse rate is everywhere examined. Where some appropriate neutral lapse rate γ_c for small-scale convection has been exceeded by changes which have taken place during the time step, heat transports by small-scale convection to maintain the neutral lapse rate are inferred, so that the thicknesses of the layers concerned are adjusted to make $\gamma = \gamma_c$. Some corresponding vertical transport of water vapour is effected by adjusting the humidity mixing ratios (r) of the layers concerned, during which process liquid water may be produced by condensation and fall into

layers below as convective rainfall. It is assumed that the net vertical transport of horizontal momentum by the small-scale convection is zero.

During a convective adjustment involving the layers of the model between pressure level p_1 and pressure level p_2 the integral

$$\int_{p_2}^{p_1} (C_p T + Lr) dp$$

(where L is the latent heat of vapourisation of water) is conserved. The implicit assumption here is that the kinetic energy generated on the small-scale to achieve the heat and moisture transports is immediately dissipated as heat.

The principles and assumptions underlying this scheme of convective adjustments are, thus far, the same as those used by Manabe et al (1965). It remains to define the neutral lapse rate γ_c and to specify the water vapour transport effected during the convective adjustments.

4.3 The neutral lapse rate for small-scale convection

The most obvious definition of the neutral lapse rate γ_c for small-scale convection suitable for use in

a scheme of convective adjustments of this sort is as follows:

$$\begin{aligned}\gamma_c &= \Gamma_d & 0 \leq x < 1 \\ &= \Gamma_s & x = 1\end{aligned}$$

where x is the relative humidity. This was the definition of γ_c used by Manabe et al (1965). An experiment in which a scheme of convective adjustments using such a definition of γ_c was incorporated into the Bushby-Timpson model indicated that this form of neutral lapse rate is unsatisfactory, at least for this particular numerical weather prediction model. Unacceptably large values of vertical velocity were found to grow within a 6 hour forecast period in regions where convective adjustments had been made. This difficulty may be attributed to static instability arising from the saturation of conditionally unstable air. With the above definition of γ_c no small-scale convective transport of heat in the vertical is effected in conditionally unstable air until saturation is achieved. Consequently, if air with γ close to Γ_d is lifted in the large-scale motion and becomes saturated, the instantaneous adjustments to thicknesses which are

required at that stage to make γ equal to Γ_s may be as much as 10 m, implying unrealistically rapid convective transports. Such adjustments may produce rather sharp gradients of thickness in the horizontal near regions of small-scale convection, and these are reflected in large vertical velocities.

In the real atmosphere saturation is achieved locally in air which is being lifted long before volumes of air with the dimensions represented by a grid point in the model (i.e. $10^6 \text{ km}^2 \text{ mb}$) become saturated. As a result heat transports by small-scale convection begin to modify the large-scale lapse rate of conditionally unstable air while the air is, as yet, unsaturated on the large-scale. Thus the lapse rate is gradually reduced towards the saturated adiabatic and there are no sudden changes,

In order to represent this effect in the model γ_c is redefined as follows.

$$\begin{aligned}\gamma_c &= \Gamma_d & 0 \leq x \leq x_0 \\ &= \Gamma_d - \frac{x-x_0}{1-x_0} (\Gamma_d - \Gamma_s) & x_0 \leq x \leq 1.\end{aligned}$$

Here x_0 is an arbitrary parameter which defines the

lowest relative humidity at which convective transports can take place in conditionally unstable air. In the predictions discussed in the next chapter x_0 was taken to be 0.5, and figure 16 shows γ_c as a function of x in this case. Notice that γ_c as now defined is a continuous function of x , ensuring that no sudden and unrealistic thickness changes are required as air becomes saturated, and that γ_c becomes equal to Γ_s when $x = 1$.

The above definition of γ_c has no direct physical basis, although it clearly represents a reasonable and physically realistic neutral lapse rate in this context. It should be emphasised that the neutral lapse rates formulated for use in numerical weather prediction models cannot be obviously or directly derived from their counterparts in the real atmosphere. This is because the sub-grid scale distributions of relative humidity, which in the real atmosphere are important factors in the operation of small-scale convection, cannot be inferred from the grid-point parameters of numerical models. Thus both cumulus convection, and saturated convection operating over only part of a grid area, which in the real atmosphere effect vertical heat transports in air which is conditionally unstable but unsaturated on the large-scale, can only be represented

in a numerical model by some rather crude definition of γ_c such as that suggested above.

This modified definition of γ_c also opens the way to representing the surface layer of small-scale convection when surface heating is incorporated into the Bushby-Timpson model. Small-scale convection in the surface layer is observed to be characterised by neutral lapse rates which are intermediate between Γ_d and Γ_s , so that it could not be adequately represented in a scheme using the original definition of γ_c .

4.4 Water vapour transport and convective rainfall

The vertical heat transport effected by small-scale convection, which may be represented in the model atmosphere by supposing that the neutral lapse rate γ_c is never exceeded, is, in the real atmosphere, accompanied by vertical transport of water vapour. There is no neutral stratification of humidity mixing ratio comparable with the adiabatic lapse rates of temperature, so that there is no possibility of relating such convective transports of water vapour to the large-scale parameters in a manner analagous to that suggested above for heat transports.

When convective transports are taking place in air which is saturated on the large-scale, the distribution of water vapour is determined by the requirement that the air shall not become supersaturated. Thus the adjusted humidity mixing ratio of each layer of the model atmosphere involved in a convective adjustment is in this case the saturation mixing ratio corresponding to the adjusted thickness of the layer. Such an adjustment in saturated air inevitably leads to a net excess of water vapour, since a given mass of air with an unstable temperature lapse rate requires more water vapour to saturate it than the same air adiabatically transformed so that it has a neutral lapse rate. A proportion of this excess is considered to condense to liquid water, the appropriate latent heat being released, so that the layers concerned remain exactly saturated after the adjustment.

No such clear physical guidance is available concerning the large-scale effects of convective transports of water vapour in air which is unsaturated on the large-scale. Since the neutral temperature lapse rate as defined in the previous section is a function of relative humidity, it is clear that rather complex implicit relationships for the adjusted values of thicknesses and

mixing ratios could result from the adoption of some arbitrary principle governing the water vapour transport. It is very convenient, therefore, to assume that, subject to the overall requirement that total water content be conserved, the convective transport of water vapour is such as to conserve the relative humidity x of each layer of the model atmosphere involved in a convective adjustment, so that the neutral lapse rate γ_c does not change during convective adjustments. With such an assumption about the convective water vapour transport, the changes in humidity mixing ratio r during an adjustment are proportional to the changes in saturation mixing ratio r_s , which are in turn related to the temperature changes associated with the convective heat transports in such a way that water vapour is always transported upwards along with heat by the small-scale convection. This assumption that relative humidity is conserved by the small-scale convection thus leads to a physically realistic treatment of water vapour transport.

In order to conserve total water substance during the convective adjustments, either there is a net condensation to liquid water in the adjusted layers, or else the relative humidity of the uppermost of the

adjusted layers is reduced. This latter course of action leaves the uppermost level involved slightly stable compared with the neutral lapse rate γ_c .

Where a convective adjustment leads to a net condensation of water vapour in the layers concerned, the corresponding amount of liquid water falls into the layer immediately below these adjusted. During the descent of this liquid water to the surface, where it may eventually arrive as convective rainfall, it is subject to the same rather stringent evaporation model as is applied to large-scale rainfall in the Bushby-Timpson model, which was outlined in section 4.1. Some different treatment for convective rainfall might well be justified, but no experiments have yet been made in this direction.

The scheme of water vapour transport and convective rainfall production described above has the advantage that it makes possible predictions of convective rain from air which is conditionally unstable but unsaturated on the large-scale. This is clearly a useful feature in a numerical weather prediction model simulating motion systems which often include showery activity.

4.5 Details of the convective adjustment scheme

(a) The test for instability

After each time step of integration in the model each of the eight levels from 200 mb to 900 mb is tested for instability at every grid point.

Using the notation illustrated in figure 10, at each level n the neutral value of $T_{n-\frac{1}{2}}$, denoted by $T_{n-\frac{1}{2}}^c$, is related to $T_{n+\frac{1}{2}}$ by:

$$T_{n-\frac{1}{2}}^c = T_{n+\frac{1}{2}} - \frac{1}{2}h'_{n+\frac{1}{2}} \gamma_{n+\frac{1}{2}}^c - \frac{1}{2}h'_{n-\frac{1}{2}} \gamma_{n-\frac{1}{2}}^c$$

Level n is then convectively unstable if

$$T_{n-\frac{1}{2}}^c > T_{n-\frac{1}{2}}$$

This test is illustrated by figure 17.

As defined in section 4.3 above, the neutral lapse rate for small-scale convection is given by

$$\gamma_{n+\frac{1}{2}}^c = \int_d a_{n+\frac{1}{2}}$$

where

$$a_{n+\frac{1}{2}} = 1 \quad [0 \leq x_{n+\frac{1}{2}} \leq x_0]$$

$$1 - \frac{x_{n+\frac{1}{2}} - x_0}{1 - x_0} \left(1 - \frac{\int_{sn+\frac{1}{2}}}{\int_d}\right) \quad [x_0 \leq x_{n+\frac{1}{2}} \leq 1]$$

For the uppermost two (completely dry) layers of the model atmosphere $a_{n+\frac{1}{2}} = 1$. With the notation of section 3.1:

$$T_{n+\frac{1}{2}} = \frac{1}{2} \Gamma_d C_{n+\frac{1}{2}} h'_{n+\frac{1}{2}} \quad \text{where} \quad C_{n+\frac{1}{2}} = \frac{2Cp}{R \log_e \frac{n+1}{n}}$$

Thus level n is unstable if

$$C_{n+\frac{1}{2}} h'_{n+\frac{1}{2}} - a_{n+\frac{1}{2}} h'_{n+\frac{1}{2}} - a_{n-\frac{1}{2}} h'_{n-\frac{1}{2}} > C_{n-\frac{1}{2}} h'_{n-\frac{1}{2}}$$

that is if

$$\frac{C_{n+\frac{1}{2}} - a_{n+\frac{1}{2}}}{C_{n-\frac{1}{2}} + a_{n-\frac{1}{2}}} \frac{h'_{n+\frac{1}{2}}}{h'_{n-\frac{1}{2}}} > 1$$

The eight interior levels of the model atmosphere are tested in this way from the top downwards. If level k is the first level found to be unstable, and if level $k+r$ is the next stable level, then layers $k-\frac{1}{2}$, $k+\frac{1}{2}$,, $k+r-\frac{1}{2}$, are convectively adjusted in the way described below. After such an adjustment testing is resumed at the uppermost as yet untested level and continues until the 900 mb level has been tested.

(b) The solution for the adjusted parameters

Suppose that, at the end of a time step, levels k , $k+1$,, $k+r-1$ are found to have become unstable by the test described above. Then the thicknesses h' and mixing ratios r of layers $k-\frac{1}{2}$, $k+\frac{1}{2}$, $k+r-\frac{1}{2}$, must be adjusted so that each of these levels achieves neutral stability. Adjusted parameters are denoted below by starred symbols.

The requirement that levels k , $k+1$, $k+r-1$, be neutrally stable after adjustment gives r equations in the $r+1$ unknown adjusted thicknesses, viz:

$$h_{i+\frac{1}{2}}^{\star} = \frac{C_{i-\frac{1}{2}} + a_{i-\frac{1}{2}}}{C_{i+\frac{1}{2}} - a_{i+\frac{1}{2}}} h_{i-\frac{1}{2}}^{\star} \quad i = k \dots k+r-1 \quad (A)$$

(see previous section for working). The $a_{i+\frac{1}{2}}$ used in these equations may be calculated before the adjustment, since it is assumed that the relative humidities of the layers involved are, in general, conserved during the adjustment.

The set of simultaneous equations in the new thicknesses is completed by the energy equation

$$\int_{p_2}^{p_1} (C_p T^{\star} + Lr^{\star}) dp = \int_{p_2}^{p_1} (C_p T + Lr) dp$$

which, in finite difference terms, for layers $k-\frac{1}{2}$,
 $k+r-\frac{1}{2}$, may be rearranged in the form

$$\sum_{i=k}^{k+r} \frac{g_{p,i-\frac{1}{2}}}{2} (h_{i-\frac{1}{2}}^{\#} - h_{i-\frac{1}{2}}) = \begin{cases} L \sum_{i=k}^{k+r} (r_{i-\frac{1}{2}} - r_{i-\frac{1}{2}}^{\#}) & \text{(if there is net condensation)} \\ 0 & \text{(otherwise)} \end{cases}$$

A first solution is obtained by assuming that there is net condensation in the layers concerned during the convective adjustment. In this case

$$r_{i-\frac{1}{2}}^{\#} = x_{i-\frac{1}{2}} r_{s,i-\frac{1}{2}}^{\#} \quad i = k \dots k+r$$

In order to take account of the latent heat released during such condensation, the following linear approximation is used to relate the new saturation mixing ratio of a layer to the adjustment in the temperature of that layer.

$$r_{s,i-\frac{1}{2}}^{\#} = r_{s,i-\frac{1}{2}} + \left(\frac{\partial r_s}{\partial T} \right)_p (T_{i-\frac{1}{2}}^{\#} - T_{i-\frac{1}{2}})$$

From the Clausius-Clapeyron equation

$$\left(\frac{\partial r_s}{\partial T} \right)_p = \frac{\epsilon L r_s}{R T^2} \quad \text{where} \quad \epsilon = 0.622$$

so that

$$r_{s_{i-\frac{1}{2}}}^{\#} = r_{s_{i-\frac{1}{2}}} \left[1 + \frac{\varepsilon L}{RT_{i-\frac{1}{2}}^2} (T_{i-\frac{1}{2}}^{\#} - T_{i-\frac{1}{2}}) \right]$$

and

$$\begin{aligned} r_{i-\frac{1}{2}}^{\#} &= x_{i-\frac{1}{2}} r_{s_{i-\frac{1}{2}}}^{\#} = r_{i-\frac{1}{2}} \left[1 + \frac{\varepsilon L}{RT_{i-\frac{1}{2}}^2} (T_{i-\frac{1}{2}}^{\#} - T_{i-\frac{1}{2}}) \right] \\ &= r_{i-\frac{1}{2}} \left[1 + \frac{2C_p \varepsilon L}{gRc_{i-\frac{1}{2}} h_{i-\frac{1}{2}}'^2} (h_{i-\frac{1}{2}}'^{\#} - h_{i-\frac{1}{2}}') \right] \end{aligned}$$

Thus, if there is net condensation, the energy equation becomes

$$\sum_{i=k}^{k+r} \left(C_{i-\frac{1}{2}} + \frac{4C_p \varepsilon L^2}{Rg^2} \frac{r_{i-\frac{1}{2}}}{c_{i-\frac{1}{2}} h_{i-\frac{1}{2}}'^2} \right) (h_{i-\frac{1}{2}}'^{\#} - h_{i-\frac{1}{2}}') = 0 \quad (B)$$

Substituting successively from equations (A) into equation (B) a single linear equation in $h_{k-\frac{1}{2}}'^{\#}$ only may be obtained. After this equation has been solved equations (A) may be used again to obtain the other adjusted thicknesses. The values of the saturation mixing ratios $r_{s_{i-\frac{1}{2}}}^{\#}$ are then computed from these thicknesses using the quartic polynomial mentioned in section 4.1 above. The net mixing ratio of liquid

water produced in the layers during the convective adjustment is then, since all the layers have the same mass, given by

$$R = \sum_{i=k}^{k+r} (r_{i-\frac{1}{2}} - x_{i-\frac{1}{2}} r_{s_{i-\frac{1}{2}}}^{\#})$$

If R , computed in this way, turns out to be negative, then the assumption that net condensation would occur was false, and a second solution must be computed using equations (A) with the dry energy equation

$$\sum_{i=k}^{k+r} c_{i-\frac{1}{2}} (h_{i-\frac{1}{2}}^{\#} - h_{i-\frac{1}{2}}) = 0$$

where $h_{i-\frac{1}{2}}^{\#}$ are the original thicknesses. For this second solution

$$r_{i-\frac{1}{2}}^{\#} = x_{i-\frac{1}{2}} r_{s_{i-\frac{1}{2}}}^{\#} \quad i = k+1, \dots, k+r$$

as before, whilst, in order to conserve total water content

$$r_{k-\frac{1}{2}}^{\#} = r_{k-\frac{1}{2}} + \sum_{i=k+1}^{k+r} (r_{i-\frac{1}{2}} - x_{i-\frac{1}{2}} r_{s_{i-\frac{1}{2}}}^{\#})$$

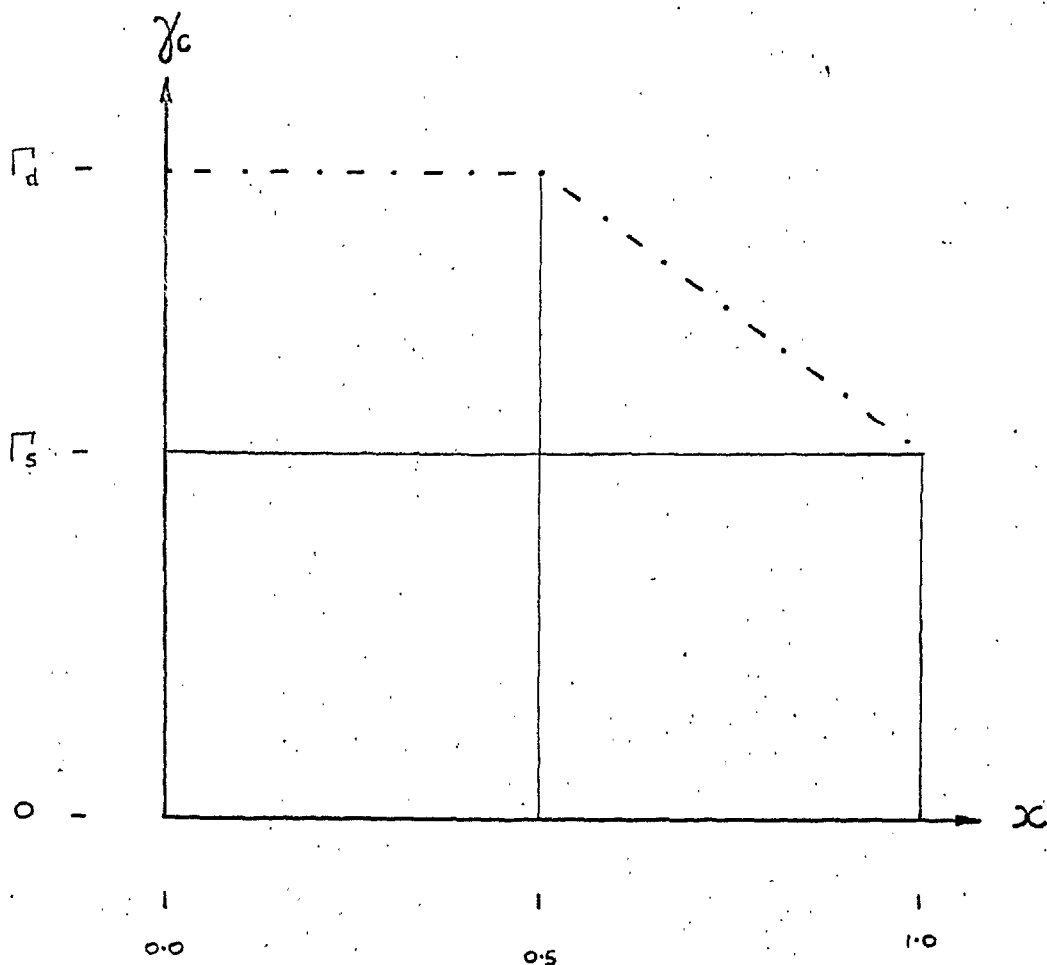
There is, of course no net production of liquid water in this case.

A convective adjustment made in the way described above to layers $k-\frac{1}{2}$, $k+\frac{1}{2}$, ..., $k+r-\frac{1}{2}$, may possibly create convective instability at the previously stable levels $k-1$ and/or $k+r$. The stability of these levels is therefore tested after such an adjustment, and if, for example, level $k+r$ has been rendered unstable by the adjustment, the process described above is repeated, this time making adjustments at levels k , $k+1$, $k+r-1$, and $k+r$. Thus by an iterative procedure it is ensured that, after the convective adjustments, each column of the model atmosphere is entirely free from static instability according to the stability criterion given in 4.5 (a).

4.6 The initial data

As was mentioned in section 3.1, certain procedures (described by Benwell and Timpson 1968) are carried out during the preparation of the initial data for the Bushby-Timpson model to remove vertical inconsistencies in the height fields which may inadvertently arise during data extraction.

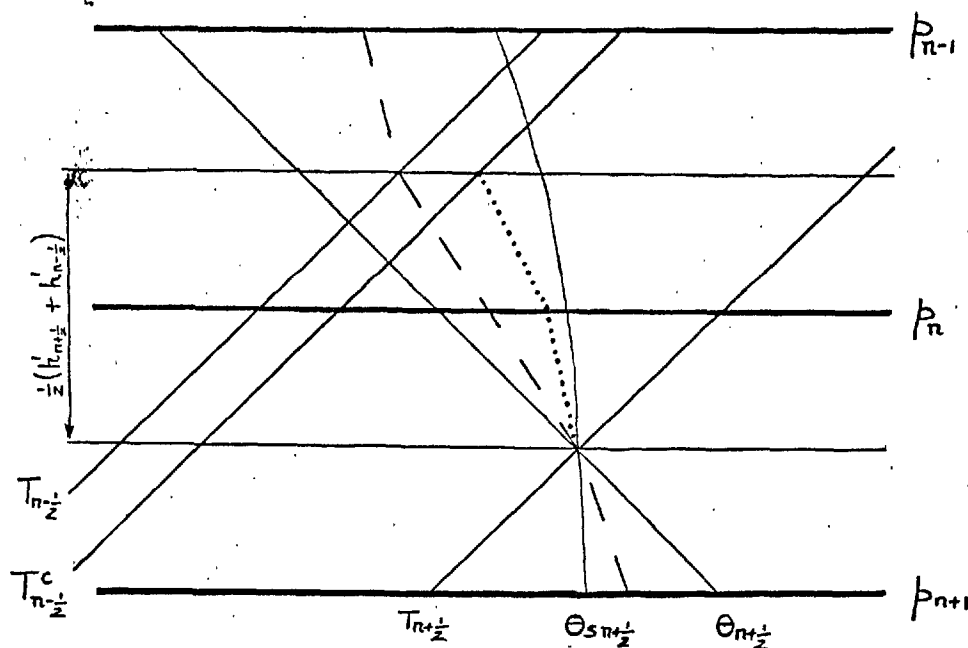
A further operation on the initial data becomes necessary with the incorporation of the scheme of convective adjustments described above. In order that sudden convective adjustments are not required at the end of the first time step of integration, the initial data must be free from instability according to the criterion of section 4.5 (a). Thus the convective adjustment procedure described above is applied to the initial height fields before the initial non-divergent wind field is calculated from the heights. During this initialising convective adjustment no latent heat release is permitted.



$$\gamma_c = \Gamma_d \dots\dots\dots 0 \leq x \leq x_0$$

$$\gamma_c = \Gamma_d \left(\frac{1-x}{1-x_0} \right) + \Gamma_s \left(\frac{x-x_0}{1-x_0} \right) \dots\dots\dots x_0 \leq x \leq 1$$

Figure 16 illustrates the neutral lapse rate γ_c for small-scale convection used in the Bushby-Timpson model as a function of the relative humidity x with $x_0 = 0.5$ (see section 4.3).



$$T_{n-\frac{1}{2}}^c = T_{n+\frac{1}{2}} - \left[\frac{1}{2} h'_{n+\frac{1}{2}} \gamma_{n+\frac{1}{2}}^c + \frac{1}{2} h'_{n-\frac{1}{2}} \gamma_{n-\frac{1}{2}}^c \right].$$

Figure 17 illustrates the test for instability (see section 4.5(a)) on a tephigram. The dashed line represents a temperature ascent in the model atmosphere as inferred from the thicknesses of the 100 mb layers. The dotted line illustrates the construction of the neutral temperature for layer $n-\frac{1}{2}$ from the temperature of layer $n+\frac{1}{2}$ using the neutral lapse rates $\gamma_{n+\frac{1}{2}}^c, \gamma_{n-\frac{1}{2}}^c$, appropriate to the two layers.

5. RESULTS OF THE INCORPORATION OF THE CONVECTIVE ADJUSTMENTS SCHEME INTO THE BUSHBY-TIMPSON MODEL

5.1 Introduction

As was pointed out in section 3.2, the primary objective of the incorporation of a representation of the effects of small-scale convection into the Bushby-Timpson model was to prevent the growth of spurious motions associated with developments of static instability in the large-scale flow. Since the particular representation of small-scale convection described in the previous chapter is such that at least neutral static stability is maintained everywhere during the integrations, this objective must, of necessity, be achieved. However, unless the scheme of convective adjustments is a realistic representation of the overall effects of small-scale convection on the scales of time and space implied by the finite difference intervals of the model, it is likely that, whilst one problem is solved, other equally undesirable spurious motions will be introduced into the model. This was indeed the case when the scheme used by Manabe et al (1965) was applied to the Bushby-Timpson model (see section 4.3 above).

Three pairs of 24 hour forecasts have been computed using two versions of the Bushby-Timpson model which are identical except that one version includes the convective adjustments scheme and the other does not. The results of one of these pairs of forecasts have already been referred to in section 3.1 (b) and are illustrated by figures 13 and 15. This is an example of a case in which spurious developments were prevented by the inclusion of the convective adjustments. A second pair of forecasts, that for 25 December 1960, is illustrated by figure 18, which compares 24 hour forecasts of vertical velocities in the vertical section A-B (see figure 6). This is an example where the incorporation of the convective adjustments scheme has considerably improved the organisation of the flow near fronts in the model atmosphere and considerably reduced the amplitudes of some short-wavelength features.

Other regions of the forecast vertical velocity fields in these three pairs of 24 hour predictions are, however, rather less flattering to the convective adjustments scheme. Short-wavelength components of the flow are in many places accentuated when the adjustments are included. This is probably due mainly to an error in the calculation of convective rainfall during these forecasts,

which resulted in an excessive release of latent heat during the convective adjustments.

A version of the Bushby-Timpson model has more recently been formulated which, as well as a correction to the error referred to above, includes lateral diffusion terms in the thermodynamic, humidity, and horizontal momentum equations. 24 hour forecasts have been computed from several sets of initial data using this improved version, and certainly the forecast fields are free from undesirable short-wavelength components. However, since such components are suppressed by the lateral diffusion terms, it is impossible to evaluate the genuine effects of the convective adjustments. It would appear though that the representation of horizontal sub-grid scale transfers by the lateral diffusion terms is a useful concomitance of the representation of vertical sub-grid scale transfers by the convective adjustments.

In summary then, the scheme of convective adjustments may be regarded as entirely successful in that it circumvents the problems associated with the development of static instability in the model atmosphere without introducing alternative deleterious effects. In addition the meteorological quality of the convective adjustments

scheme may be assessed from the forecast distributions of small-scale convection and convective rainfall. In the following sections forecasts computed for two days, one in December and the other in August, are described to this end.

5,2 The case of 1 December 1961

Figure 19 shows the 1000 mb and 500 mb analyses for 00 GMT 1 December 1961 over Europe and the North Atlantic. A 36 hour forecast has been computed from initial data corresponding to this time and the area shown, using the version of the Bushby-Timpson model which includes the convective adjustments scheme, with a grid-length of (approximately) 200 km. Two motion systems may be identified in figure 19 which turn out to be the most important features of the flow during the subsequent 36 hours:

- (1) a depression, and the associated fronts, which remained centered near Newfoundland during the forecast period;
- and (2) a wave-depression, situated, in nascent form, near ship J at the initial time, which deepened and moved rather rapidly eastwards during the forecast period.

Figures 20, 21, and 22 show the predicted distributions of large-scale rainfall and convective rainfall, in relation to the predicted 1000 mb contours, at the 12, 24, and 36 hour stages of the forecast respectively. The following points may be noted.

(1) The predicted positions of the convective rainfall are closely associated with the depression centres and frontal regions of the large-scale flow, thus reflecting the link between such regimes and the destabilising processes which initiate the small-scale convection. Unfortunately, since most of the areas concerned lie over the sea, it is difficult to check the accuracy of the forecast positions of the convective rainfall. However, such present weather reports as there are in these areas show that there were showers in or near the two belts where convective rainfall is predicted, near ships E and K respectively, at the 12 hour stage, and again in the belt near ship E at the 24 hour stage. These showers occurred in the regions to the south of the positions of the cold fronts on the actual charts for these times, which probably distinguishes their origin from that of the many showers reported throughout the forecast period in the region to the north of these fronts. The latter showers are almost certainly the

result of the rather intense sea surface heating which must be inferred in the northerly airstreams behind the fronts. Since no representation of surface heating is included in the version of the Bushby-Timpson model used here, these showers cannot be simulated.

(2) In the cold front region of the Newfoundland depression, and in a region near the centre of the wave-depression, convective rainfall is predicted throughout the forecast period, implying that the destabilising processes responsible for the small-scale convection in these regions are operative continuously during the 36 hours. Of the processes investigated in chapter 2, the dynamical destabilisation processes associated with differential advection and vertical stretching have this property. On the other hand, in the cold front region of the wave-depression, the convective rainfall which is dominant during the first half of the forecast period dies out completely by the 36 hour stage. This could, of course, be a genuine development in the situation, or, alternatively, it may indicate that the small-scale convection in this region is mainly the result of the saturation of conditionally unstable air from the surface layer, and that in the absence of surface heating, this layer disappears during the 36 hour period.

(3) There are extensive regions near the two cold fronts where the only rainfall predicted during the early part of the forecast period is of a convective origin. Subsequently, large-scale rainfall also develops in these regions. The present weather reports available indicate that this may be a genuine reflection of what actually occurred during the period, but the fact that the vertical velocities in the Bushby-Timpson model atmosphere are initially everywhere zero may also be relevant.

Figures 23 and 24 show the predicted distributions of the accumulative totals of large-scale and convective rainfalls for the 12 hour periods from 6 to 18 and from 18 to 30 hours respectively. The proportions of the predicted accumulative rainfalls which are of convective origin during these two periods, over the whole forecast area, and separately over the western and eastern halves of the area, are expressed as percentages in the table below.

It may be seen from this table that there is a significant difference between the convective contributions to the total rainfalls in the two halves of the forecast area. In the western half, which is, broadly speaking, the region occupied by the Newfoundland

	6-18 hours	18-30 hours
whole area	$8\frac{1}{2}$	5
western half	15	$8\frac{1}{2}$
eastern half	$2\frac{1}{2}$	1

depression and its associated fronts, convective rainfall is of the order of 10% of the total; whereas, in the eastern half, where the wave-depression is dominant, convective rainfall is only of the order of 1% of the total. As was mentioned previously, and as may be seen from figures 23 and 24, there are extensive regions where convective, but not large-scale, rainfall is predicted, so that care should be exercised in interpreting the figures in the above table in terms of the relative importance of convective and large-scale rainfall. Also, the release of latent heat corresponding to the convective rainfall increases the rate at which large-scale rainfall is produced by augmenting the large-scale vertical velocities. This effect must be very slight in the wave-depression area, but is probably not negligible in the western half of the area.

A comparison of the figures in the above table for the two 12 hour periods shows that there is a decrease in the relative size of the convective contribution during the 36 hours over the whole region. As was suggested previously, this is probably linked with the progressive removal of the surface layer in the absence of surface heating.

The regions near the two main centres of activity are now considered in more detail at the 12 hour forecast stage.

(1) The Newfoundland depression

The predicted distribution of small-scale convection in the region near the Newfoundland depression and its associated fronts at the 12 hour stage is shown in figure 25, together with the predicted 1000 mb contours. The vertical extent of the small-scale convection in each column of the model atmosphere is defined by the convective record, which records the uppermost and lowermost of the set of contiguous levels at which convective adjustments were required at the end of the previous time-step. (Where adjustments were required at more than one set of contiguous levels in a column, the extent of the lowermost set of levels is recorded).

Figure 26 shows the distribution of small-scale convection in a vertical section, orientated N-S across the region shown in figure 25, in relation to the isopleths of θ_s as implied by the predicted thicknesses of the 100 mb layers. It may be seen from figure 26 that, in this section, the convection, both near the cold front and near the depression centre, is taking place in the warm air, and it may reasonably be inferred from figure 25 that the belt of small-scale convection situated along the cold front and extending into the depression centre lies entirely in the warm air. The small-scale convection in the warm air near the cold front is a rather wide band of deep convection, extending from 900 mb up to 300 mb, and is situated about 200 km ahead of the surface front (as defined by the maximum horizontal θ_s gradient at the surface). Above 700 mb the convection is taking place in air which has become saturated on the large-scale as a result of the frontal ascent, but nearer the surface the convection is taking place in the conditionally unstable air of the surface layer, where the relative humidity is about 70%. The small-scale convection near the depression centre, which is taking place in saturated air, is confined to levels near 500 mb, and its position is similar to that of the

'high-level instability' observed by Browning and Harrold (1969), and illustrated by figure 5.

(2) The wave-depression

The predicted distribution of small-scale convection in the region of the wave-depression and its associated fronts at the 12 hour stage is shown in relation to the predicted distributions of convective and large-scale rainfall and the predicted surface isobars in figure 27. This diagram illustrated the results of a 24 hour forecast computed with the Bushby-Timpson model from initial data corresponding to 00 GMT, using a grid-length of (approximately) 100 km. (That is, half the grid-length used for the 36 hour prediction described above.) The reduction in grid-length produced no major changes in the predicted development of the wave-depression, or in the distribution of small-scale convection. However the shorter grid-length does make possible rather better resolution of the detail of the distribution. From figure 27 it may be seen that small-scale convection is predicted to occur in two distinct regions at this time:

(1) Near the centre of the wave-depression, above the surface warm front, small-scale convection is predicted to occur at levels between 800 mb and 500 mb. This position is again very similar to that of the

'high-level instability' in the case illustrated by figure 5.

(2) Parallel to the cold front rain belt small-scale convection is predicted to occur in three distinct bands, separated by 100 km or so. Ahead of this large-scale rain belt convection is predicted at 500 mb; coincident with it at levels between 500 mb and 900 mb; and behind it at the 800 mb and 900 mb levels. The convection at 500 mb ahead of the cold front is suggestive of destabilisation by differential advection similar to that illustrated by figure 9. The convection coincident with the large-scale rain belt is the result of conditionally unstable air becoming saturated in the large-scale ascent there, whilst the convection behind the rain belt is probably associated with large-scale ascent ahead of the trough of low pressure (see a similar example in the next section) which lies well behind the frontal rain belt.

5.3 The case of 21 August 1965

The surface analysis and present weather reports over the British Isles at 12 GMT 21 August 1965 are shown in figure 28. A depression was centred to the north of Scotland and the associated fronts brought rain to south-

east England, the Low Countries, and northern France. A rather pronounced trough of low pressure was situated over western districts of England and Wales and showers were widespread over most of the British Isles.

Figure 29 shows the surface isobars and the distributions of small-scale convection, convective and large-scale rainfall, predicted for the same time by the Bushby-Timpson model, using a grid-length of (approximately) 100 km, from initial data corresponding to 00 GMT on the same day. As may be seen from figure 29, the depression, frontal rain belts, and trough referred to above are fairly accurately simulated in the model atmosphere.

Small-scale convection is predicted to occur in two middle level regions associated with the large-scale rain belts; also in two low-level regions, one near the depression centre, and one on the forward side of the trough. (In addition, an extensive region of small-scale convection is predicted to occur at the 300 and 400 mb levels; for clarity this convection is considered separately below.) The low-level convection referred to above produces convective rainfall in northern Scotland, north-western England, Wales, south-western England, and the south-western approaches; these are

positions which correspond fairly well with those of reported showers shown in figure 28. However, no convective rainfall, or indeed small-scale convection, is predicted over Ireland or central Scotland, where showers were also reported at this time.

The predicted thermodynamic structure of the model atmosphere at this time on the vertical E-W section A-B across the British Isles is illustrated by figure 30, which shows isopleths of Θ_s implied by the thicknesses of the 100 mb layers. The region of small-scale convection is also marked. The mean values of Θ_s for the 100 mb layers have been extracted from the actual 12 GMT ascents at Valentia, Aughton, and Crawley, (all of which lie within a single grid-length of the section A-B). These values are plotted in the appropriate positions on figure 30, and they show that, except near 900 mb at Aughton, the predicted static stabilities do not deviate seriously from those of the real atmosphere at this time.

The dynamical influence of the trough of low pressure which is situated between Valentia and Aughton at this time is illustrated by figure 31, which compares the predicted vertical velocity profiles of the columns of section A-B which correspond to these two stations.

Ahead of the trough there is moderate large-scale ascent and behind it descent. Figures 30 and 31 together explain why small-scale convection is successfully forecast in some parts of the British Isles but not in others.

Ahead of the trough the conditionally unstable air near the surface is being lifted; its relative humidity is therefore increasing and it is becoming less stable according to the criterion of section 4.5. Behind the trough the air is subsiding and therefore becoming more stable.

Figures 32 and 33 show the actual and predicted situations respectively for the 24 hour stage of this forecast, i.e. 00 GMT 22 August 1965. The trough, which is now situated over south-western England, is no longer preceded by convective rainfall in either the real or the model atmospheres, although convection at low-levels is still predicted on its forward side. However the present weather reports still indicate showers in Ireland and south-western Scotland. The stationary position of this region of showers during the 12 hour period suggests that its origin is geographical rather than dynamical, whilst the persistence of the showers at night indicates that sea rather than land surface heating may be important.

The combined evidence of figures 28-33 indicates that the showers which occurred in the ascending air ahead of the trough were initiated by the saturation (on a small-scale) of conditionally unstable air, and that their onset and cessation were accurately predicted by the convective adjustments scheme included in the Bushby-Timpson model. The showers which occurred behind the trough were, it seems, initiated by surface heating and therefore could not be predicted by this version of the model.

Figure 34 shows the predicted distributions of accumulated convective and large-scale rainfall totals for the 06-18 hour period and emphasises the separation of the convective rainfall, associated with the trough, from the large-scale rainfall, associated with the fronts, during this period. Such a diagram would, except over Ireland, have provided a sound basis for a forecast of showers during the 12 hours. The convective rain totals predicted are small, the maximum value being 0.14 mm, but it must be remembered that such totals represent averages over the 10^4 km^2 areas represented by each grid-point in the model. Where showers occur over only a fraction of such an area, the inferred local rain total is proportionally enhanced.

As was mentioned above, small-scale convection was also predicted to occur throughout this prediction in an extensive region at 300 and 400 mb. Figure 35 shows the predicted 300 mb contour pattern for the 12 hour stage, together with the predicted distribution of small-scale convection at that level. The relationship of this small-scale convection to the predicted wind structure is illustrated by figure 36, which shows a vertical N-S section across the region shown in figure 35. The convection is seen to be occurring on the immediate anti-cyclonic side of the jet stream; the relative humidity in this region is less than 50%, so that the convection is operating to maintain the dry adiabatic lapse rate.

Taking the jet axis to be normal to the plane of figure 36, it may be seen from figure 35 that the flow normal to the jet axis is entirely ageostrophic at the 300 mb level over most of this section. Inspection shows that the sign of the shear of this ageostrophic component is such as to destabilise the air on the anti-cyclonic side of the jet stream, and spot checks of the magnitude of the differential advection terms appropriate to the 350 mb level indicate that the rate of destabilisation is about $0.01 \sqrt{\sigma}_d$ per hour. Although this destabilisation occurs rather slowly, the

process is obviously rather important to considerations of clear air turbulence.

5.4 Summary of results

During the forecasts described above, small-scale convection was predicted to occur mainly in regions related to features of the large-scale flow as follows:

- (A) on the warm sides of cold fronts, near the frontal cloud;
- (B) immediately ahead of troughs of low pressure;
- (C) in the warm air near centres of depressions;
- and (D) on the anti-cyclonic sides of jet streams.

These four regions are clearly related, all being situated in regions of large-scale ascent and on the warm sides of features with intensified horizontal temperature gradients.

Convection in regions A, B, and C generally produced convective rainfall. Convective rainfall in regions B was distinctive in that it was produced in air which was unsaturated on the large-scale.

Using the following code for the three destabilising processes identified in chapter 2:

Differential advection:	a
Vertical stretching:	s

Lifting of conditionally unstable air: 1

a number of facts about the predicted small-scale convection are conveniently summarised in the following table.

Region of convection	Illustrated by	Vertical extent (levels adjusted)	Typical relative humidity (%)	Probable destabilising factors	Maximum associated rate of convective rainfall (mm hr)
A.	fig. 26	9 - 3	100	a, s, and l.	0.37
B.	fig. 29	9 8	80	s and l	0.06
C.	fig. 27	5	100	a and s	0.04
D.	fig. 36	4 3	40	a	nil

5.5 Assessment of the convective adjustments scheme

The regions of the large-scale flow in which small-scale convection was predicted to occur during the forecasts described above correspond to regions of the real atmosphere where small-scale convection is observed. Such observations are described by, for example, Sawyer 1955 (deep convective cloud on the warm sides of cold fronts); Browning and Harrold 1969 (convection in a

warm sector near the depression centre); and Defant 1959 (near dry adiabatic lapse rates on the anti-cyclonic sides of jet streams).

This realism in the predicted occurrences of small-scale convection is, of course, a tribute to the simulation of certain destabilising processes by the Bushby-Timpson model rather than to the convective adjustments scheme. However, the prediction of convection ahead of the trough on 21 August 1965 supports the hypothesis implicit in the definition of γ_c used here, that small-scale convection is likely to be initiated in conditionally unstable air as it becomes more nearly saturated on the large-scale. On this occasion both the onset and cessation of convective rainfall associated with the trough, as simulated in the model, were in accord with events in the real atmosphere as revealed by the present weather reports of showers. The apparent ability of this scheme to deal with small-scale convection in conditionally unstable air which is unsaturated on the large-scale suggests that it may also be suitable for predicting small-scale convection associated with surface heating.

The scheme of convective adjustments used in these experiments may be compared with those described by Manabe et al (1965) and Smagorinsky et al (1965).

Manabe et al (1965) defined the neutral lapse rate γ_c for small-scale convection by:

$$\gamma_c = \begin{cases} \Gamma_d & 0 \leq x < 1 \\ \Gamma_s & x = 1 \end{cases}$$

This definition precludes the possibility of representing the effects of small-scale convection in conditionally unstable air which is unsaturated on the scale of a grid square. Smagorinsky et al (1965) attempted to remove this deficiency by redefining γ_c as:

$$\gamma_c = \begin{cases} \Gamma_d & 0 \leq x < 0.8 \\ 0.2\Gamma_d + 0.8\Gamma_s & 0.8 \leq x \leq 1 \end{cases}$$

Such a definition, however, has the disadvantage that Γ_s may be exceeded in air which is saturated on the large-scale, so that spurious developments, such as that described in section 3.1 (b), may not be prevented.

Both of the above definitions suffer from the further disadvantage that γ_c is somewhere discontinuous in x in the range $0 \leq x \leq 1$, so that when that value of x is achieved, unrealistically rapid convective heat transports are implied. This feature is clearly meteorologically unfortunate, and may also lead to

serious computational difficulties (see, for example, section 4.3 above).

The definition of γ_c used here, namely:

$$\gamma_c = \begin{cases} \Gamma_d & 0 \leq x \leq 0.5 \\ 2[(1-x)\Gamma_d + (x-\frac{1}{2})\Gamma_s] & 0.5 \leq x \leq 1 \end{cases}$$

overcomes each of these difficulties. Firstly, γ_c is a continuous function of x in the range $0 \leq x \leq 1$, so that large-scale transports associated with small-scale convection are always achieved gradually.

Secondly, the saturated adiabatic lapse rate is maintained in air which is saturated on the large-scale, so that spurious developments associated with saturated instability are avoided. Thirdly, small-scale convection can take place in conditionally unstable air which is sufficiently moist, but not necessarily saturated, on the large-scale.

In connection with small-scale convection in conditionally unstable air which is unsaturated on the large-scale, it should be pointed out that the scheme of convective adjustments cannot simulate the effects of the transformation of cumulus to cumulonimbus convection. Ludlam (1966) has suggested that this transformation

usually follows immediately upon the formation of showers in the cumulus. The sub-grid scale convection represented by this scheme is, in unsaturated air, confined to levels where $\gamma > \gamma_s$, as is characteristic of cumulus convection, whereas, after showers form, the cumulonimbus elements in the real atmosphere rise approximately pseudo-adiabatically through their environment. Thus, for instance, in the situation at Aughton at 12 GMT 21 August 1965 (illustrated by figure 30), it may be expected that in reality the convection eventually extended to about 550 mb following the shower formation, whereas the cumulus convection represented by the convective adjustments extended to only 750 mb. This factor will obviously lead to discrepancies between the predicted and actual duration and intensity of convective rainfall; since, however, the transformation to cumulonimbus occurs after the initial formation of showers, the accuracy of predictions of the time and position of the occurrence of such rainfall is not affected.

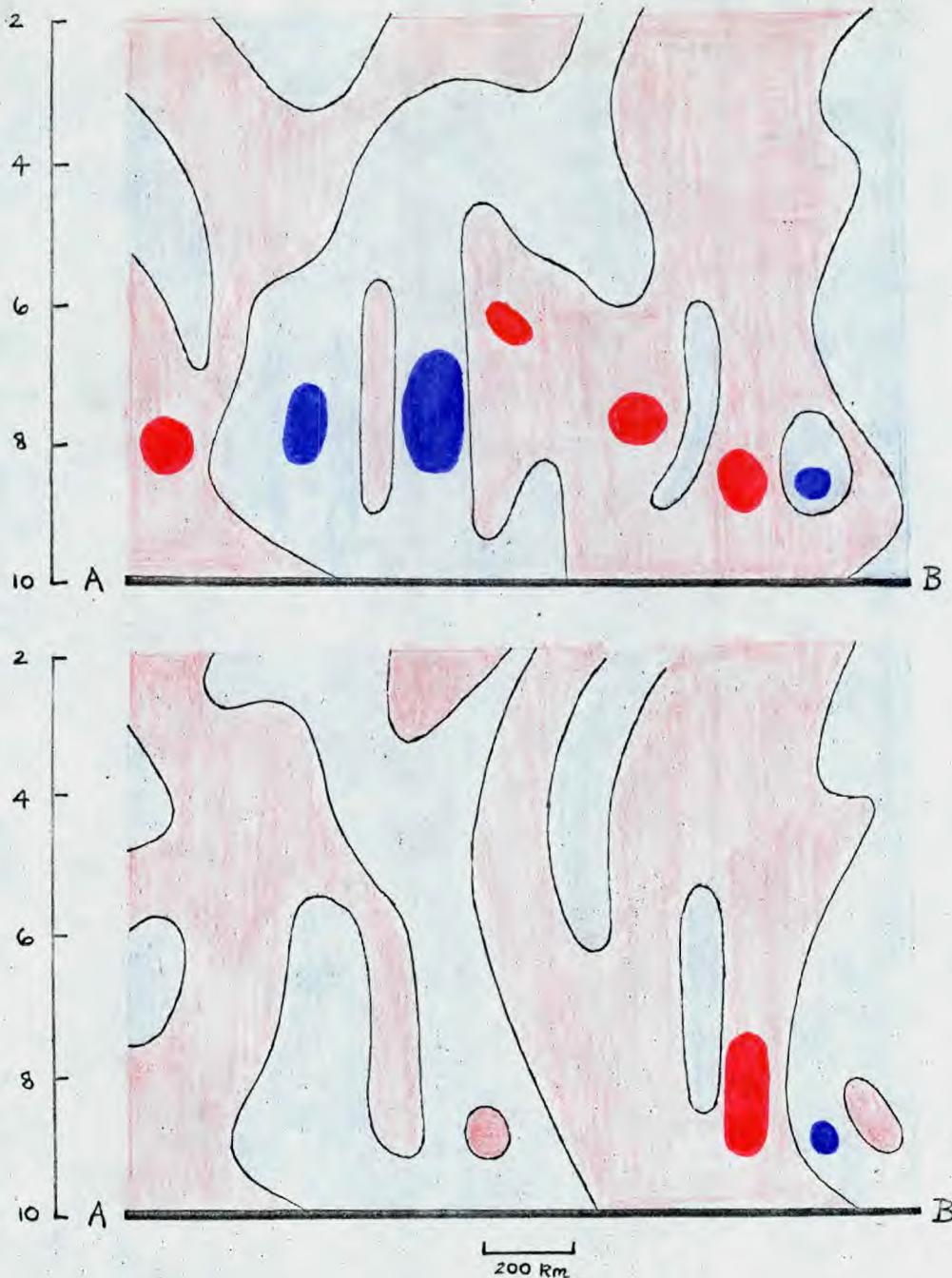


Figure 18 compares 24 hour forecast vertical velocities in the vertical section A-B (see figure 6), computed with (below) and without (above) convective adjustments by the Bushby-Timpson model, from initial data 12 GMT 25.12.60. Red shading indicates ascent; blue shading indicates descent; heavy shading indicates motion in excess of 20 mb hour^{-1} .

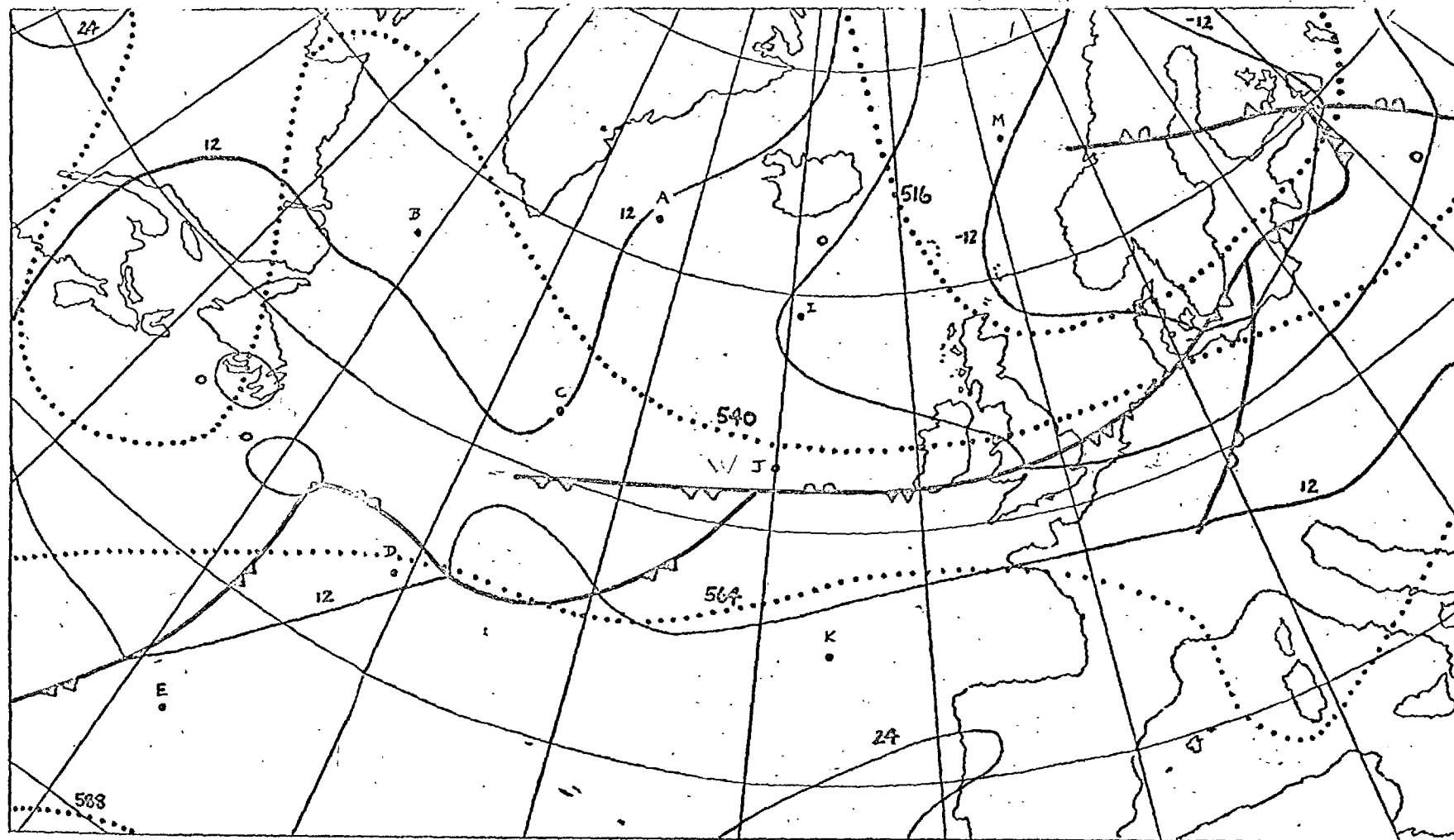


Figure 19 shows the actual 1000 mb (full lines) and 500 mb (dotted lines) contour analyses for 00 GMT 1.12.61 with heights labelled in dm. The positions of the Newfoundland depression (N) and the nascent wave-depression (W) may be noted.

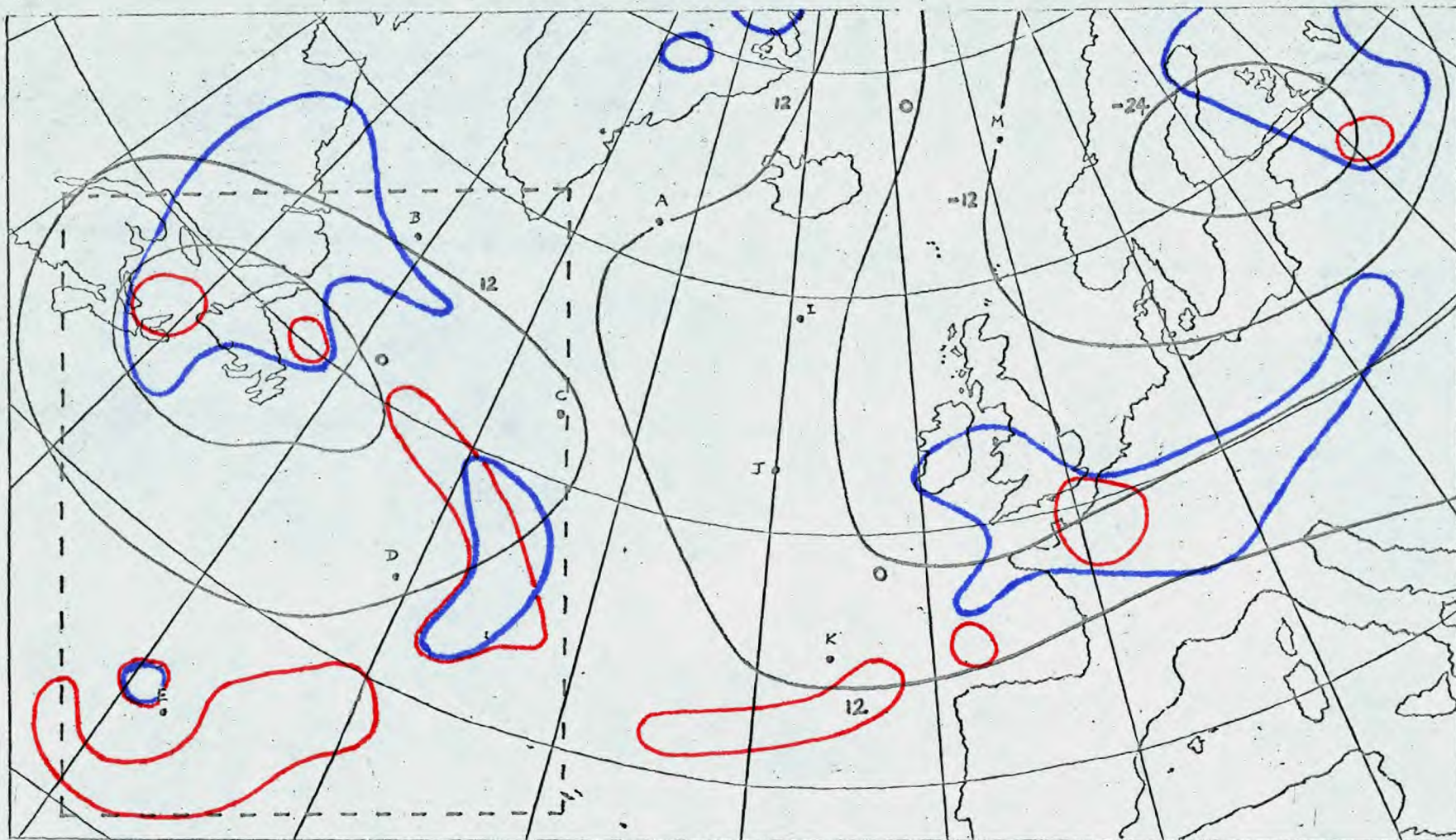


Figure 20 illustrates the 12 hour forecast for 1.12.61. The 1000 mb contours are labelled in dm, and the predicted distributions of convective (red) and large-scale (blue) rainfall are shown. The region enclosed by the pecked rectangle is illustrated in more detail in figure 25.

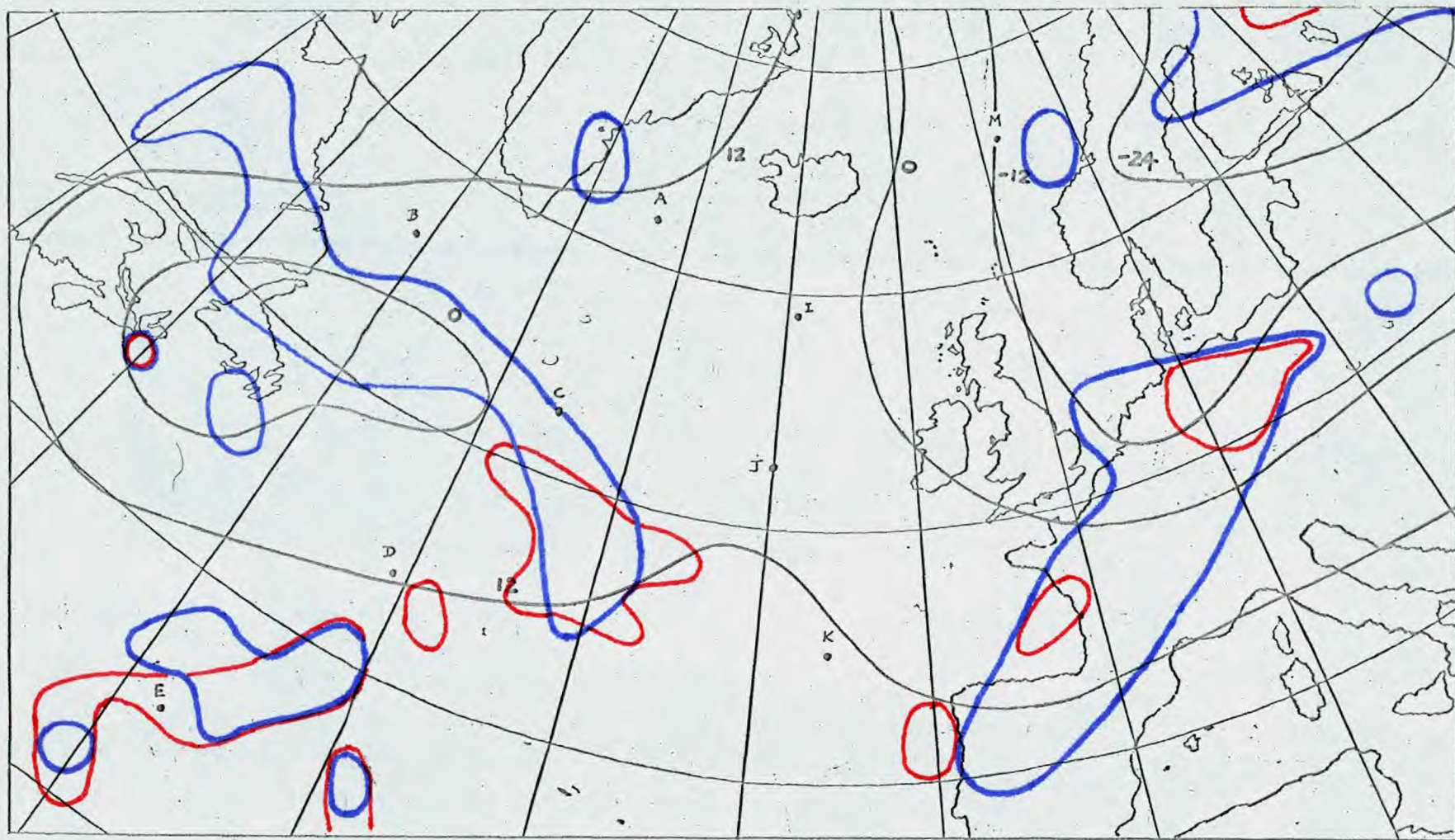


Figure 21 illustrates the 24 hour forecast for 1.12.61 with notation as in Figure 20.

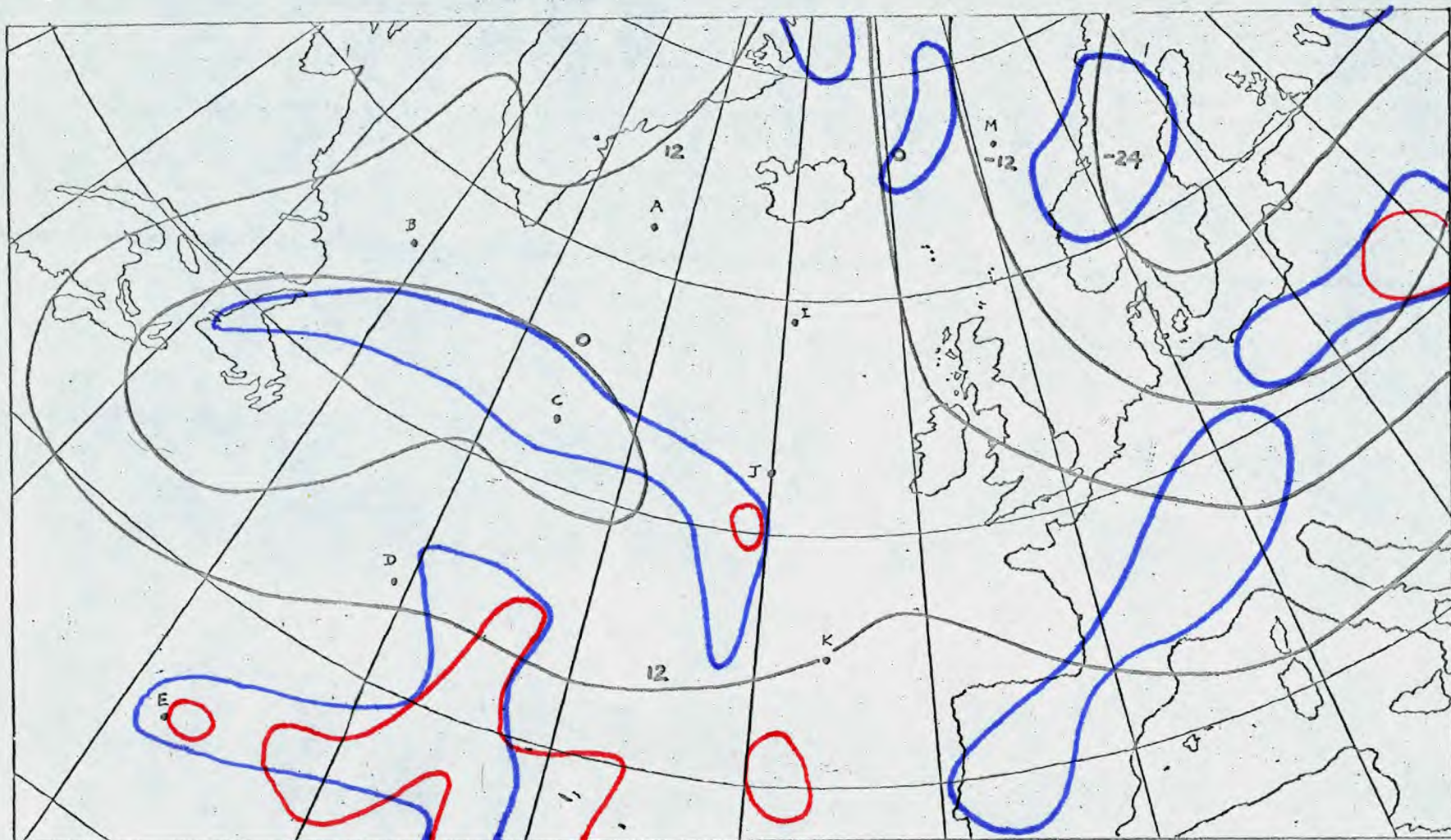


Figure 22 illustrates the 36 hour forecast for 1.12.61 with notation as in figure 20.

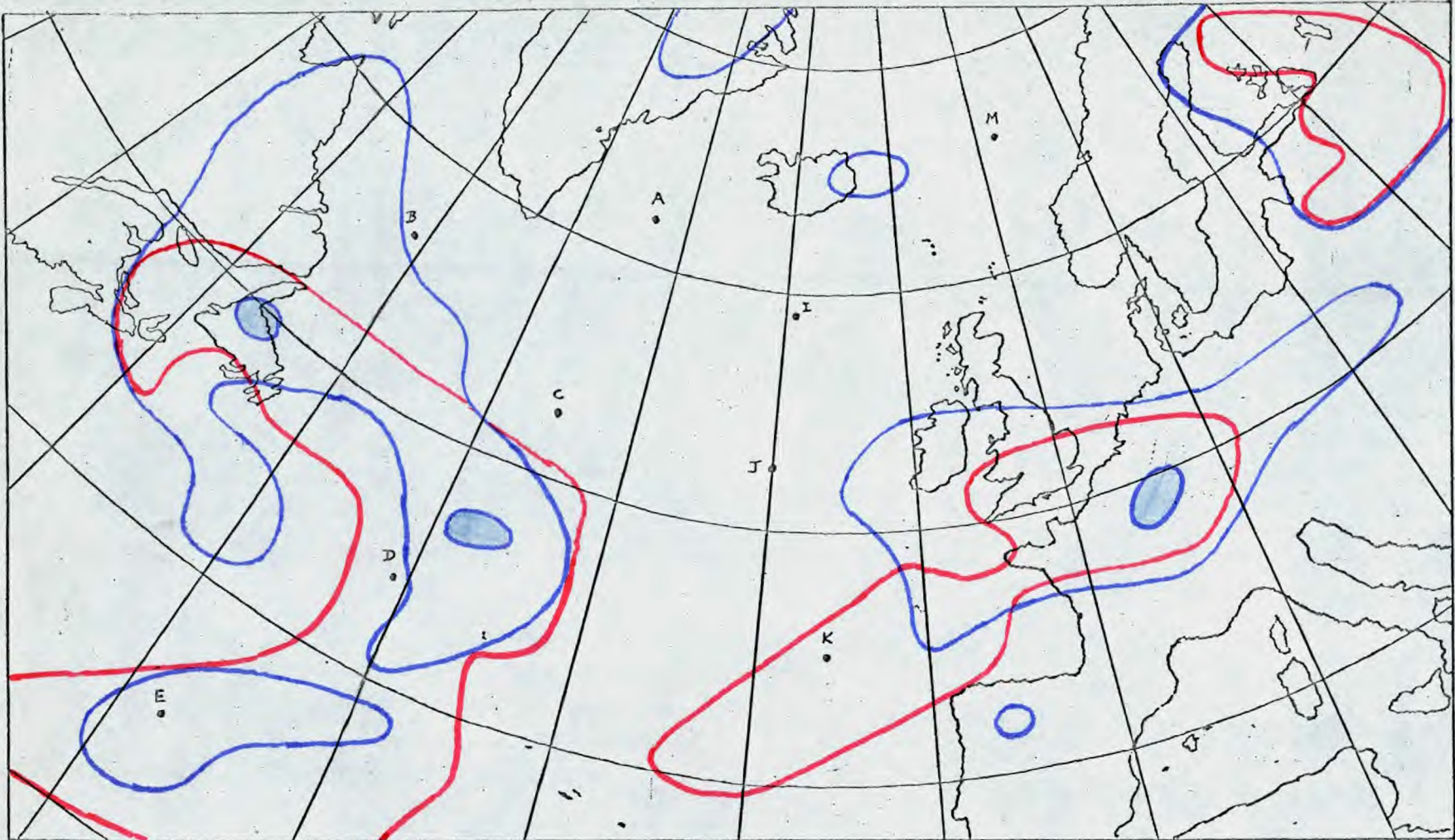


Figure 23 shows the predicted distributions of accumulated convective (red) and large-scale (blue) rain totals during the 6-18 hour forecast period on 1.12.61. Shaded regions indicate totals in excess of 8 mm.

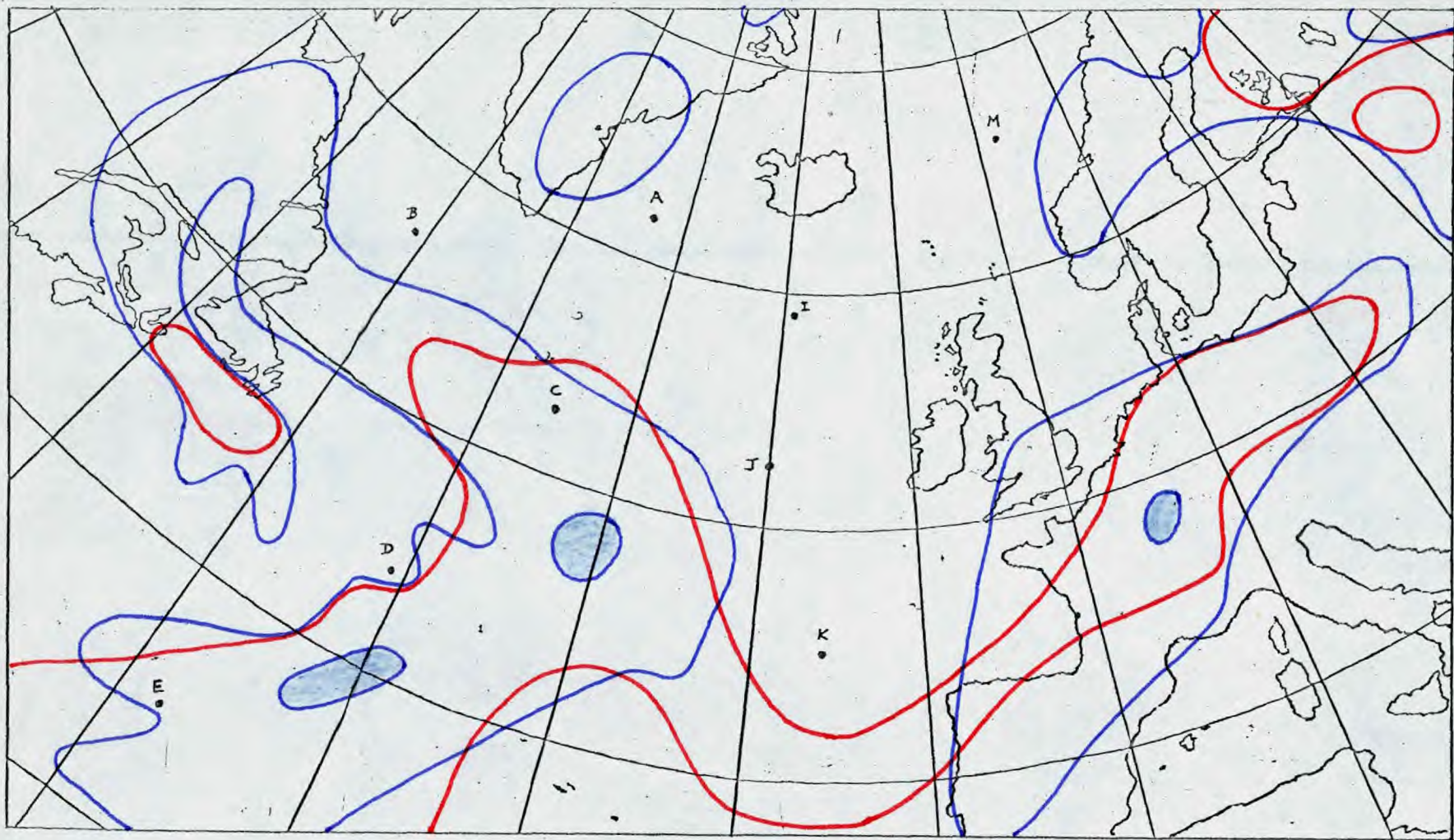


Figure 24 shows the predicted accumulated rainfalls for the 18-30 hour forecast period on 1.12.61 with notation as in figure 23.

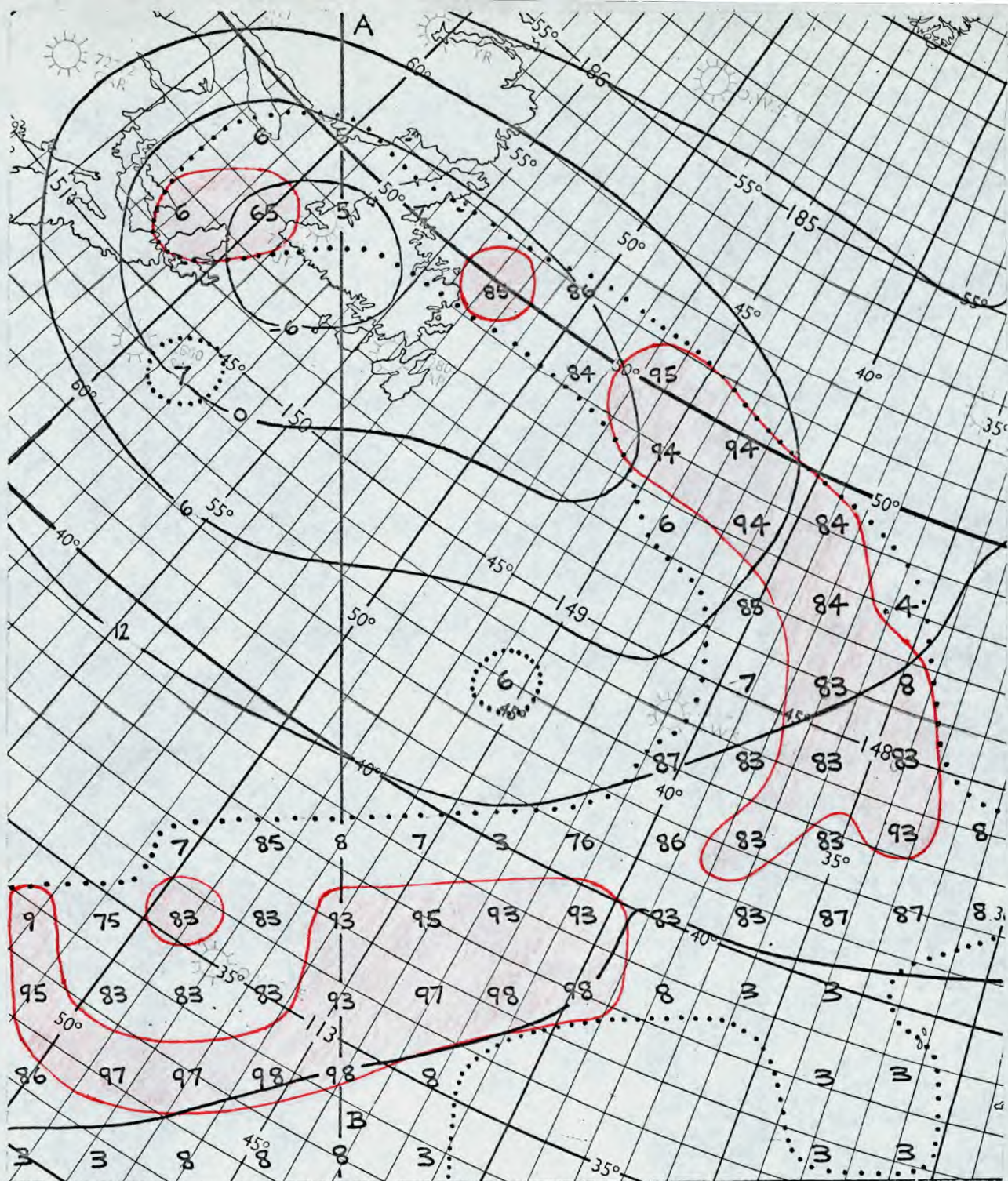


Figure 25 shows the convective record (black numerals : see page 103) and convective rainfall (red) in relation to the 1000 mb contours (full lines labelled in dm) as predicted for 12 GMT 1.12.61, over part of the area illustrated in figure 20. The position of vertical section A-B (see figure 26) is indicated.

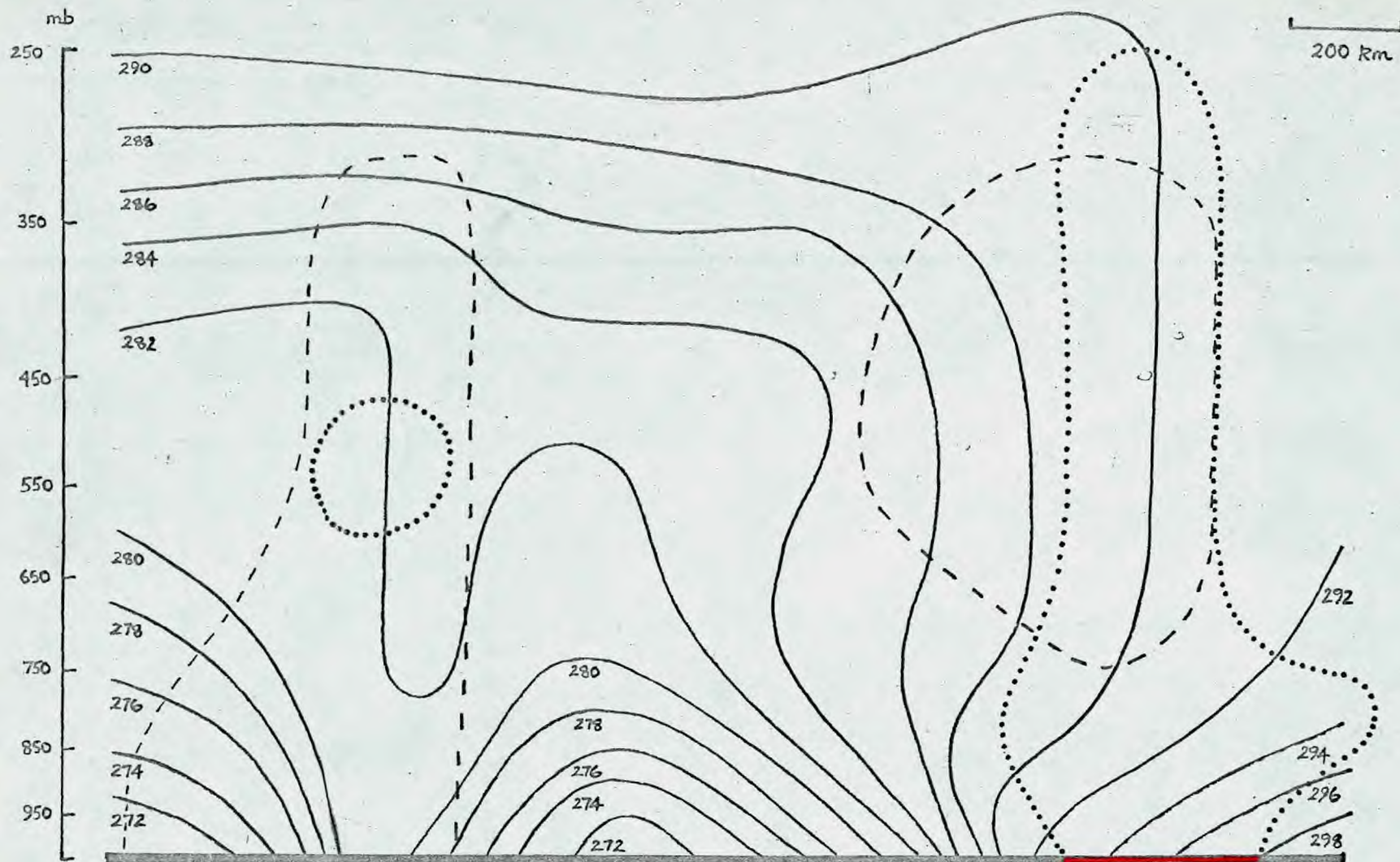


Figure 26 shows the predicted distribution of small-scale convection (dotted lines) and isopleths of (labelled in degrees K) in the vertical section A-B (see figure 25) at 12 GMT 1.12.61. Convective rainfall is indicated by a red line at the surface and regions of large-scale saturation by pecked lines.

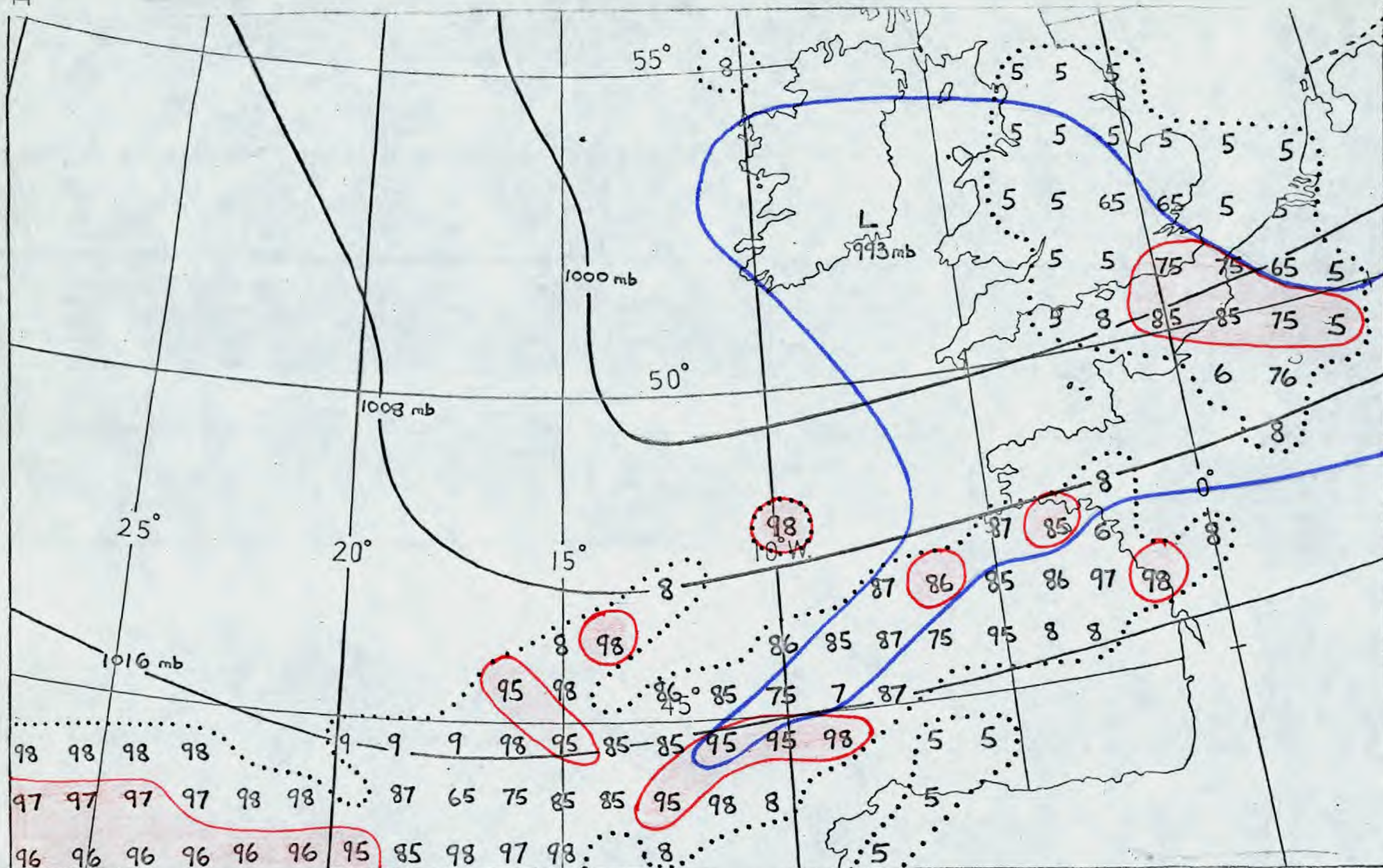


Figure 27 shows the convective record (black numerals: see page 103), convective rainfall (red), large-scale rainfall (blue), and surface isobars, predicted for 12 GMT 1.12.61 in the neighbourhood of the wave-depression.

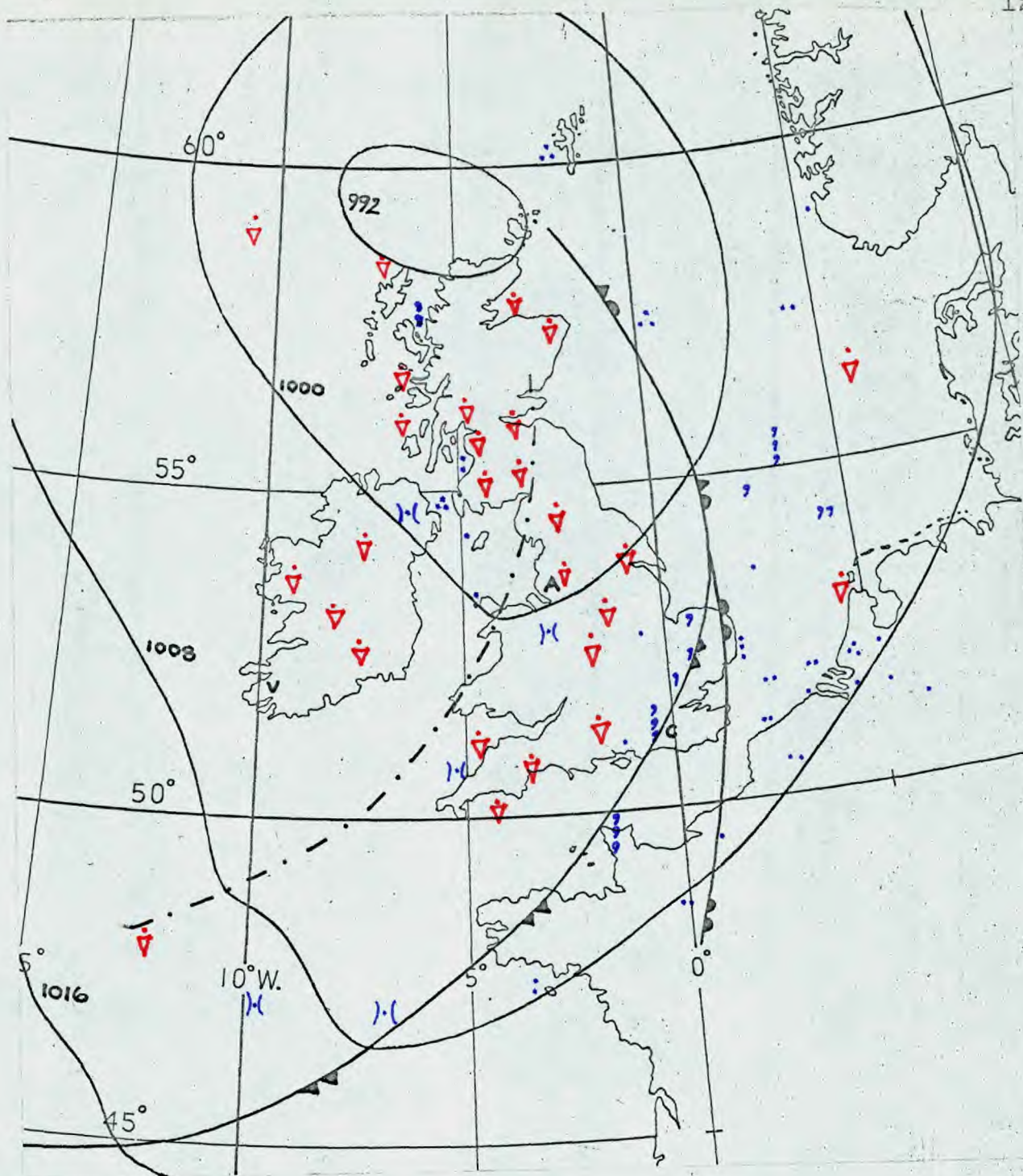


Figure 28 shows the surface pressure analysis (isobars labelled in mb) and present weather reports (see Handbook of Weather Messages, H.M.S.O., London) at 12 GMT 21.8.65.

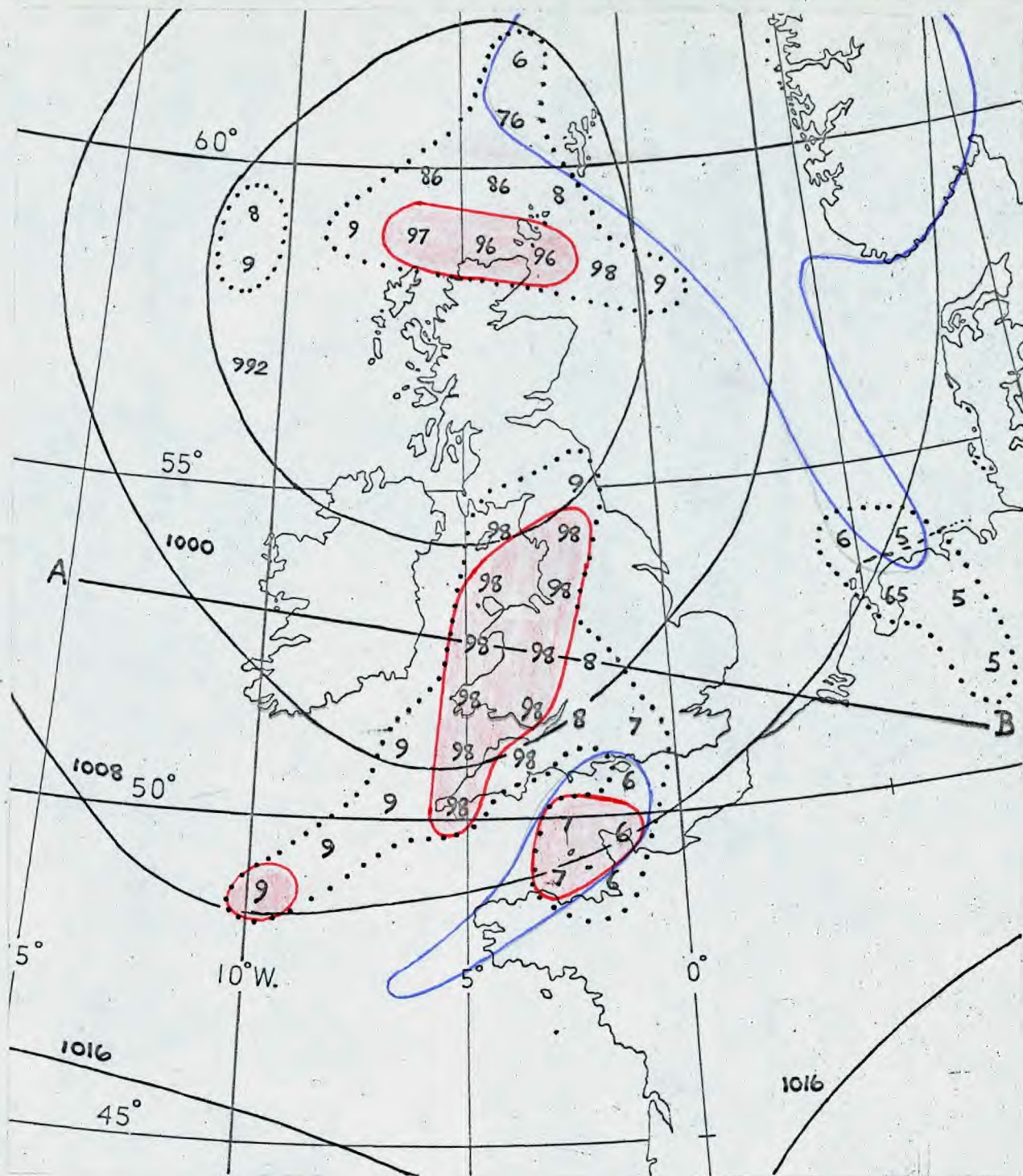


Figure 29 shows the convective record (black numerals: see page 103), convective rainfall (red), large-scale rainfall (blue), and surface isobars, predicted for 12 GMT 21.8.65 (initial data 00 GMT 21.8.65). The position of the vertical section A-B (see figure 30) is indicated.

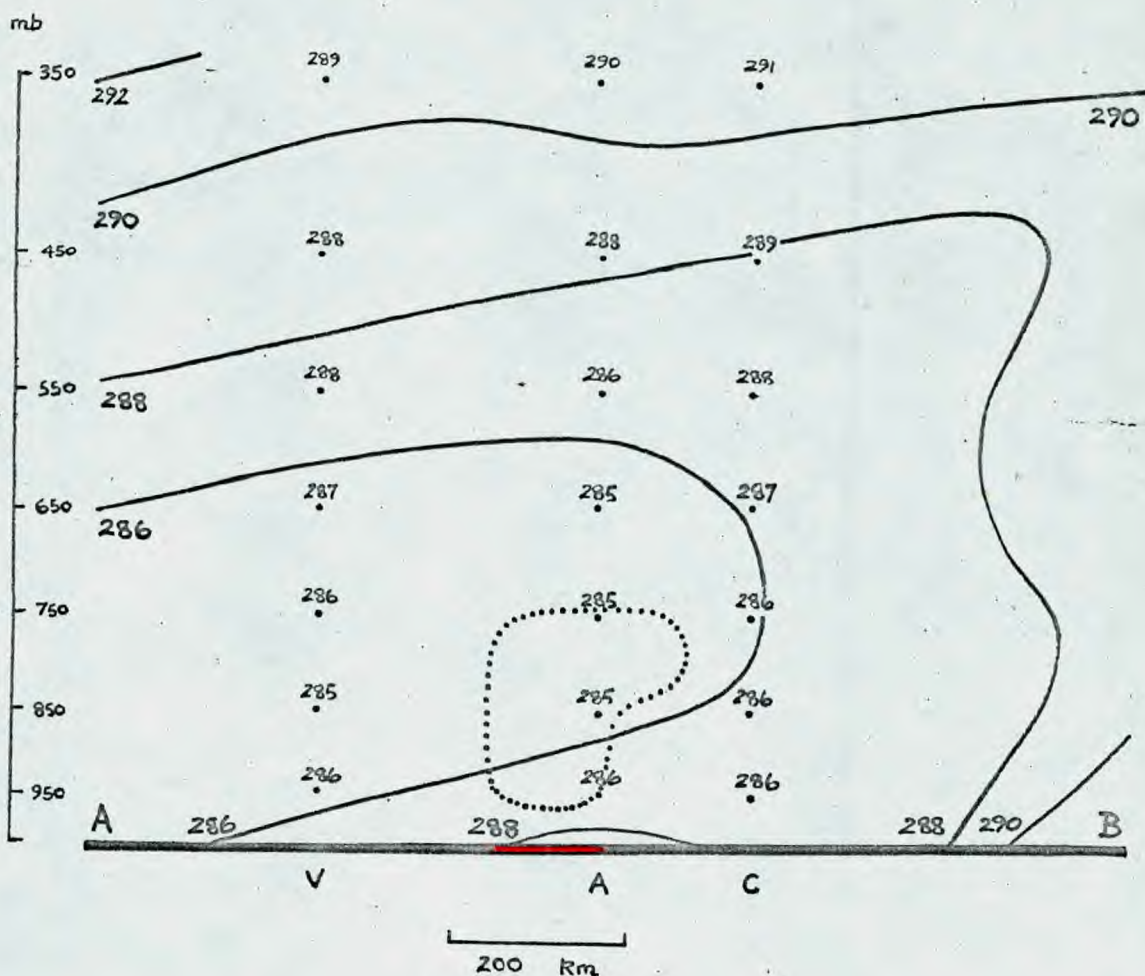


Figure 30 shows isopleths of Θ_s (labelled in degrees K) on the vertical section A-B (see figure 29) of the model atmosphere as predicted for 12 GMT 21.8.65. The dotted line indicates the region of small-scale convection and the red line at the surface shows the extent of convective rainfall. The mean values of Θ_s for 100 mb layers derived from the actual ascents at Aughton (A), Crawley (C), and Valentia (V), are also entered.

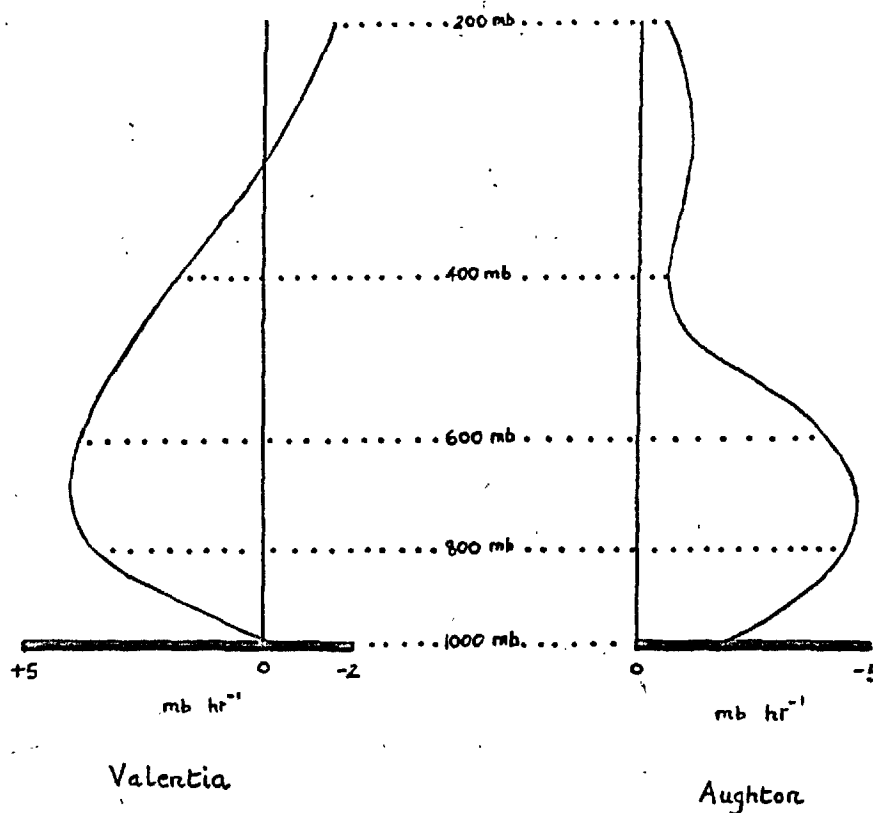


Figure 31 compares the predicted vertical velocity profiles of the columns of the cross-section illustrated by figure 30 which correspond to the positions of Aughton and Valentia.

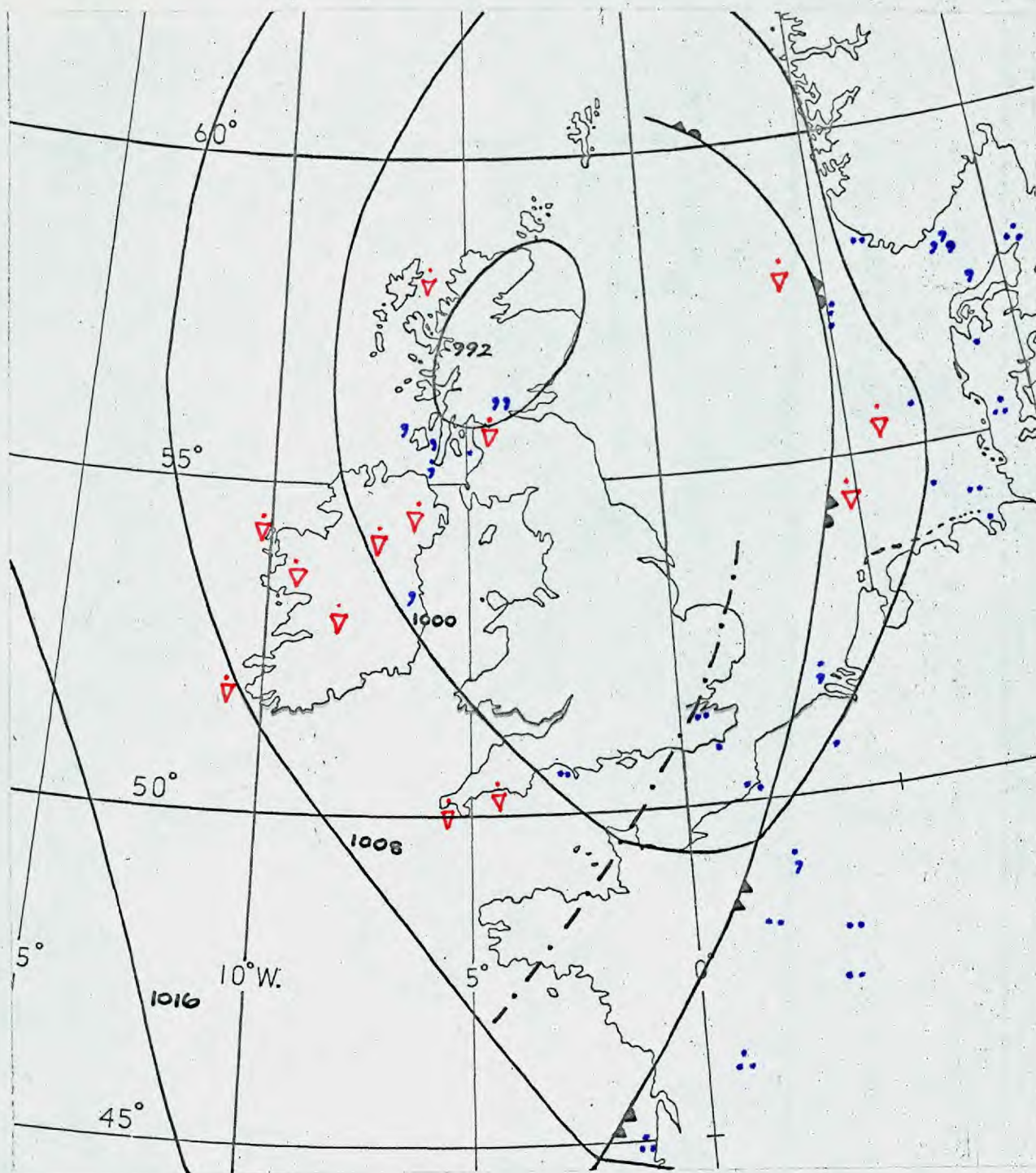


Figure 32 shows the surface pressure analysis (isobars labelled in mb) and present weather reports (see Handbook of Weather Messages, H.M.S.O., London) at 00 GMT 22.8.65.

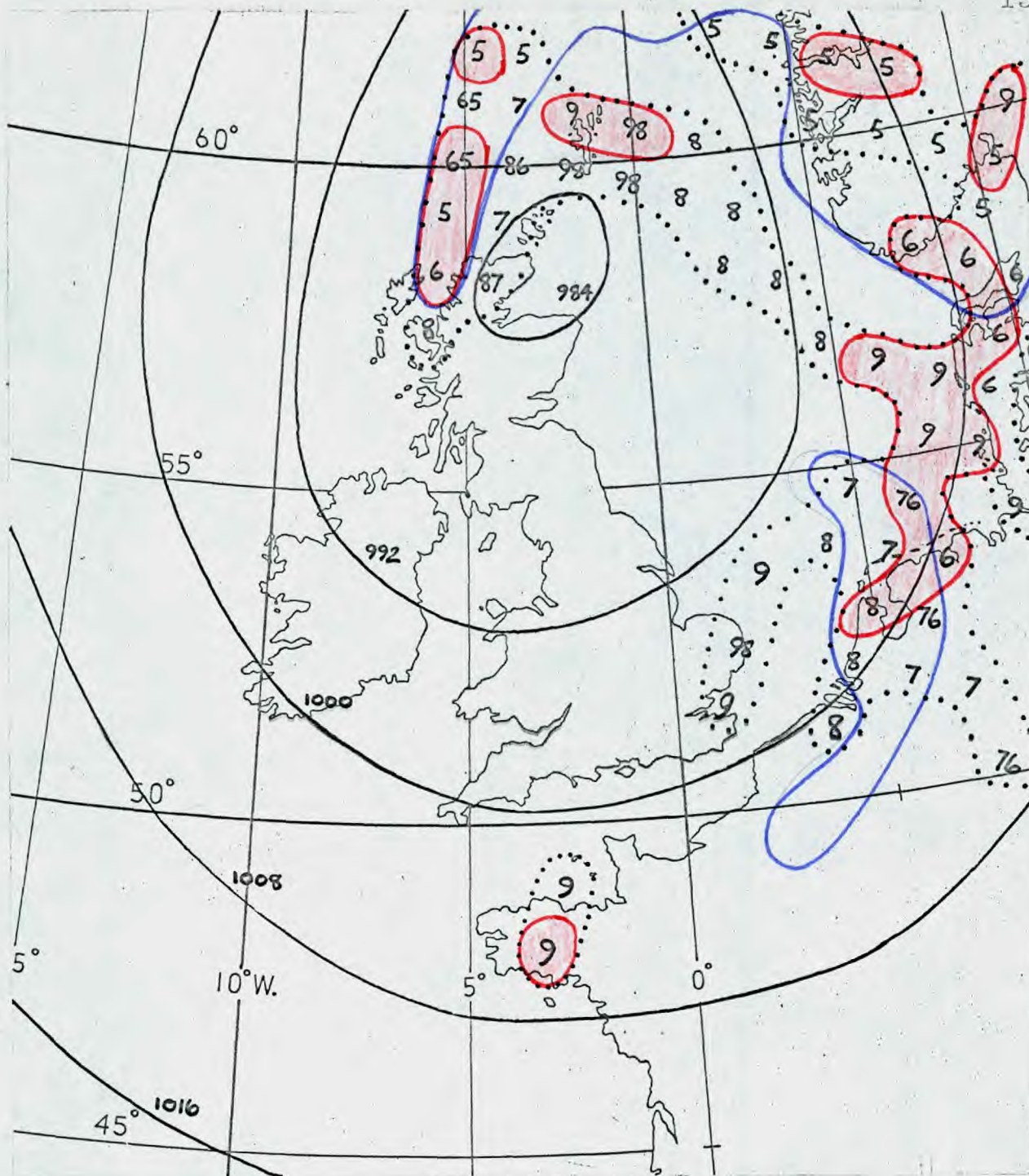


Figure 33 shows the convective record (black numerals; see page 103), convective rainfall (red), large-scale rainfall (blue), and surface isobars, predicted for 00 GMT 22.8.65 (initial data 00 GMT 21.8.65).

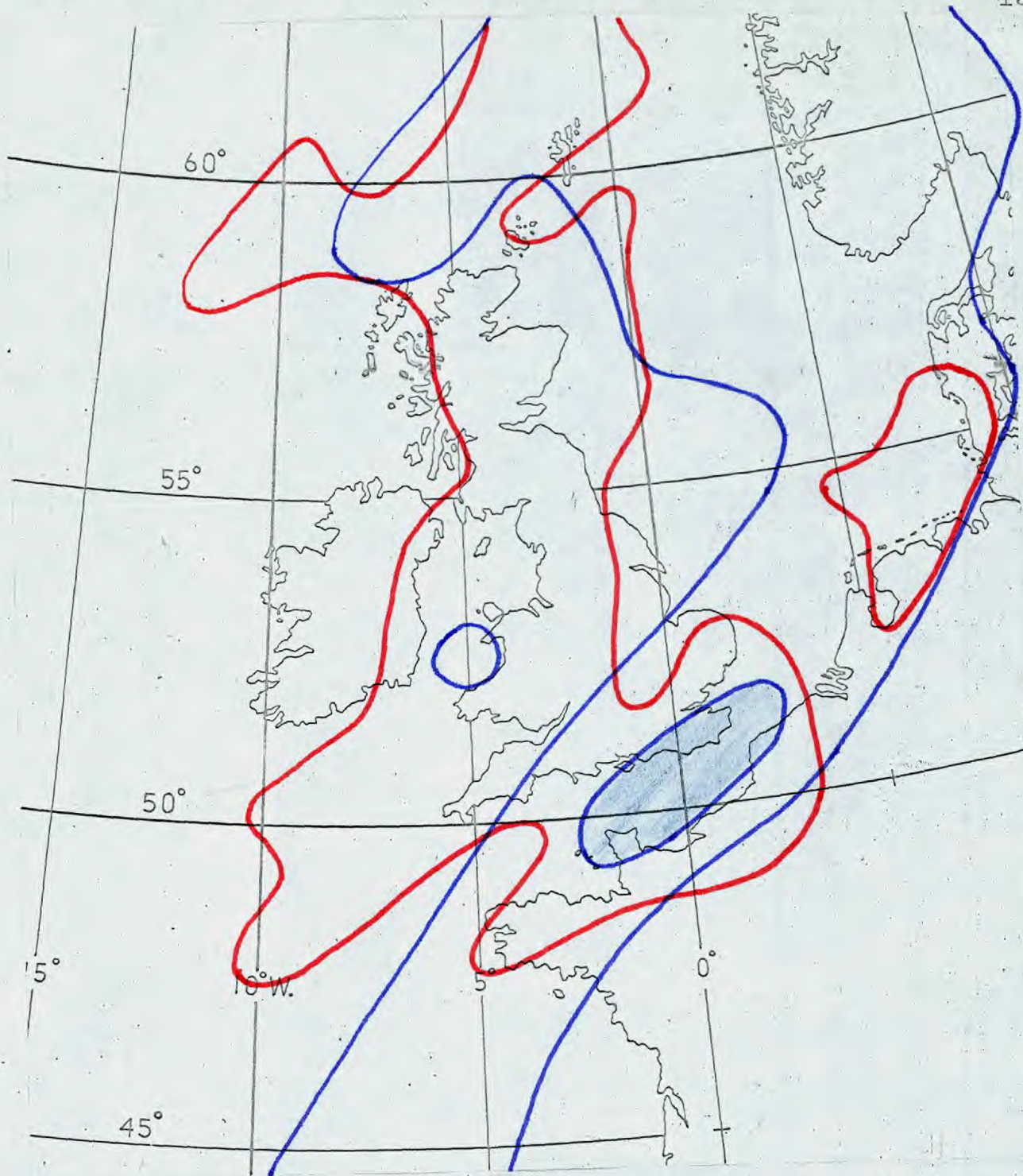


Figure 34 shows the predicted accumulated convective (red) and large-scale (blue: the shaded region denotes totals in excess of 4 mm) rain totals for the 6-18 hour forecast period on 21.8.65.

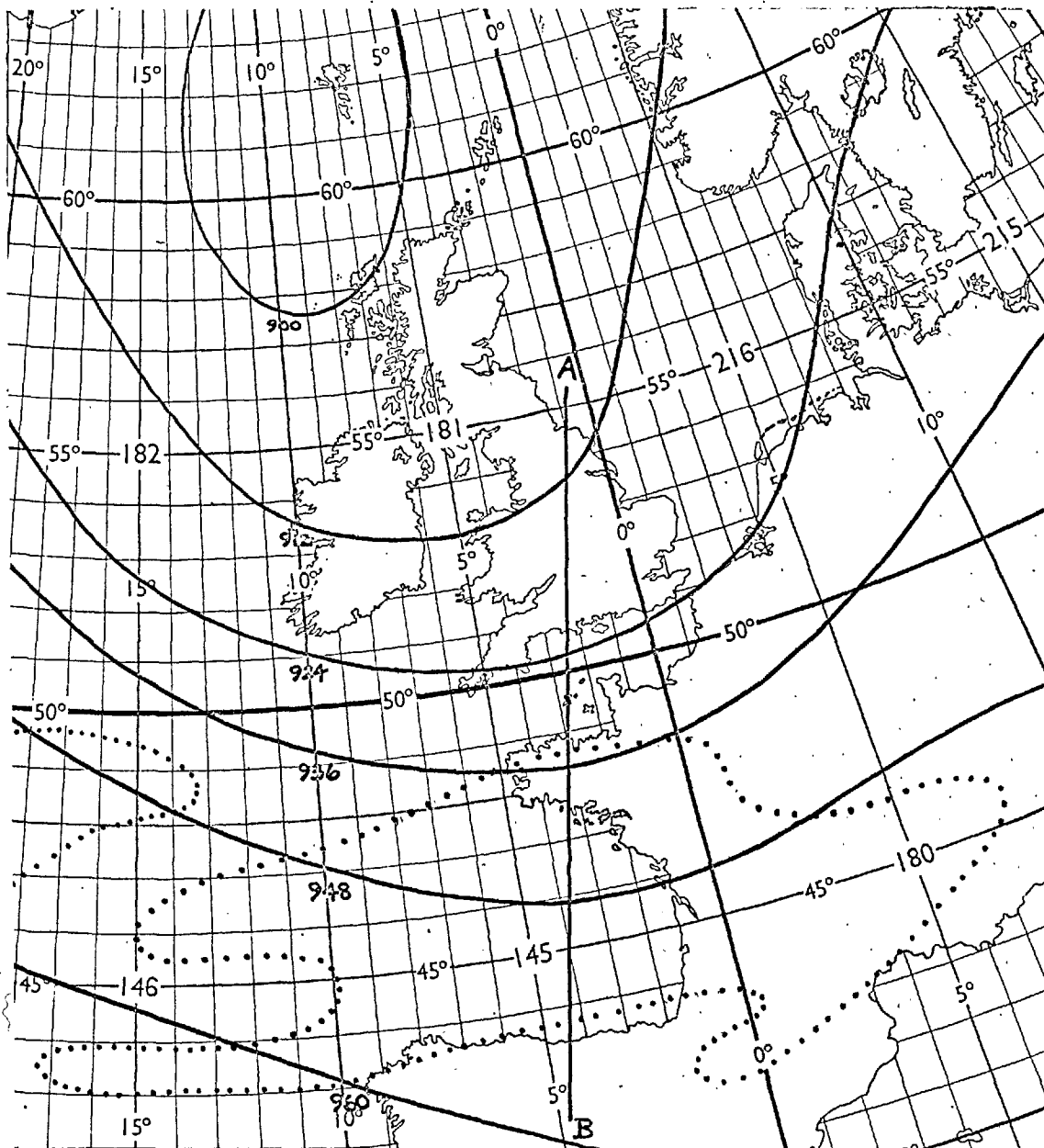


Figure 35 shows the extent of the region of small-scale convection (within dotted lines) at the 300 mb level and the 300 mb contours (labelled in dm) predicted for 12 GMT 21.8.65. The position of the vertical section A-B (see figure 36) is indicated.

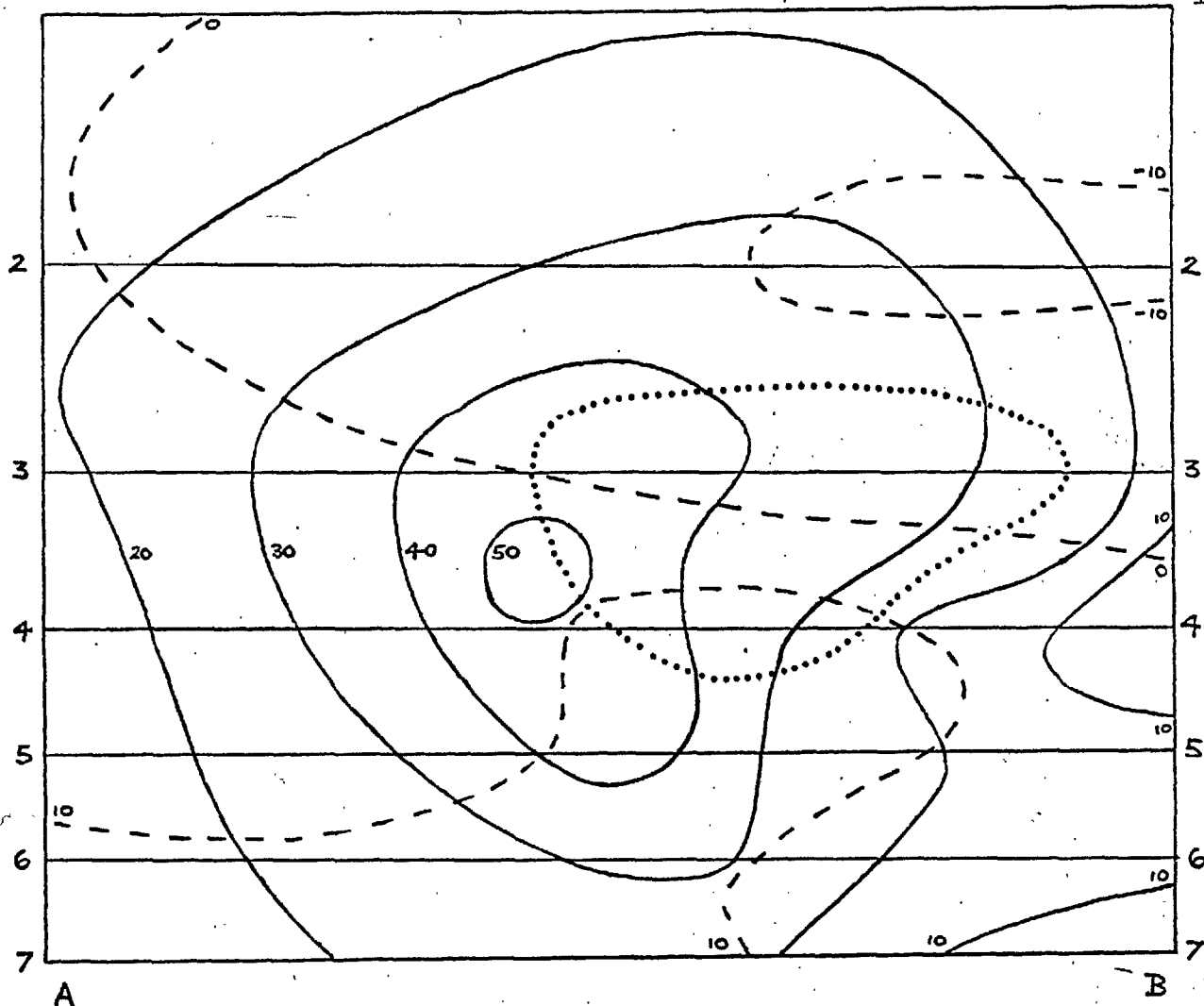


Figure 36 shows the vertical section A-B (see figure 35) across the jet stream in the prediction for 12 GMT 21.8.65. The full and the dashed lines are isotachs of the wind components normal and parallel to A-B respectively, and are labelled in metres per second. The vertical axis is labelled in hundreds of mb. The region within which small-scale convection was predicted to occur is indicated by a dotted line.

REFERENCES

- Benwell, G.R.R. and Timpson, M.S. 1968 Quart.J.R.Met. Soc. 94 p.12.
- Bushby, F.H. and Timpson, M.S. 1967 Quart.J.R.Met.Soc. 93 p.1.
- Browning, K. and Harrold, T. 1969 Quart.J.R.Met.Soc. 95
- Carlson, T.N. and Ludlam, F.H. 1965 Research on characteristics of severe storms. Annual summary report No.1, Grant AF EOAR 64-60, Imperial College Department of Meteorology, London.
- Charney, J.G. and Eliassen, A. 1964 J.Atmos.Sci. 21 p.68.
- Defant, F. 1959 The Atmosphere and the Sea in Motion (Rossby Memorial Volume), B.Bolin Ed., p.305.
- Eady, E.T. 1949 Tellus 1 p.33.
- Eliassen, A. 1962 Geof.Publ. 24 (The Vilhelm Bjerknes Centenary Volume) p.147.
- 1966 'The cloud forming circulations in frontal zones' - lecture given at N.A.T.O. advanced study institute 'The dynamics of clouds and cloud systems' Imperial College, July 1966.
- Green, J.S.A., Ludlam, F.H., and McIlveen, J.F.R. 1966 Quart.J.R.Met.Soc. 92 p.210.
- Kasahara, A. 1961 J.Meteorology 18 p.259.

- Krishnamurti, T.N. 1967 'Diagnostic studies of weather systems of low and high latitudes' Contract AF 19 (628)-477, Dept. of Met., U.C.L.A.
- Kuo, H.L. 1963 Physics of Fluids 6 p.195.
1965 J.Atmos.Sci. 22 p.140.
- Leith, C.E. 1965 Proceedings of the International Symposium on Dynamics of Large-Scale Processes in the Atmosphere, (IAMAP/WMO), Moscow, Ed. A.S. Monin p.134.
- Ludlam, F.H. 1966 Tellus 18 p.687.
- Ludlam, F.H. and Scorer, R.S. 1957 'Cloud Study', Murray, London.
- Manabe, S., Smagorinsky, J., and Strickler, R.F. 1965 Monthly Weather Review 93 p.769.
- Miles, M.K. 1962 Quart.J.R.Met.Soc. 88 p.286.
- Mintz, Y. 1965 Proceedings of the WMO-IUGG Symposium on Research and Development of Long-Range Forecasting, WMO No.162 T.P.79, Tech.Note No.66, Geneva, p.141.
- Palmen, E. and Newton, C.W. 1948 J.Meteorology 5 p.220.
- Phillips, N.A. 1959 'The Atmosphere and the Sea in Motion' (Rossby Memorial Volume), Ed. B. Bolin, p.501.
- Richardson, L.F. 1922 'Weather Prediction by Numerical Process', Cambridge University Press, London.

- Sasaki, Y. 1960 Proceedings of the International Symposium on Numerical Weather Prediction, Tokyo, Ed. S. Syone, Met.Soc. of Japan, p.477.
- Sawyer, J.S. 1955 Met.Office Geophysical Memoir No.96, London.
- 1956 Proc.Royal Soc. Series A 234 p.346.
- Scorer, R.S. 1962 Int.Journal Air Wat.Poll. 6 p.101.
- Smagorinsky, J. and Staff Members 1965 Proceedings of the International Symposium on Dynamics of Large-Scale Processes in the Atmosphere (IAMAP/WMO), Moscow, Ed. A.S. Monin, p.70.
- Thompson, P.D. 1961 'Numerical weather analysis and prediction' McMillan, New York.
- Todsen, M. 1964 'A computation of the vertical circulation in a frontal zone from the quasi-geostrophic equations', Tech.Note 4, Institute of Theoretical Meteorology, University of Oslo, Norway.