Modelling nonlinear structures using physics-guided, machine-learnt models.

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ABSTRACT

The constant drive to improve the performance of aeronautic structures is leading to new designs where nonlinearity is ubiquitous. Accurately predicting the dynamic behaviour of nonlinear systems is very challenging because they can exhibit a wide range of behaviours that have no linear equivalent and are very sensitive to parameter changes. In this work, we consider a physics-based model to capture the underlying linear behaviour of the system. This linear model is then augmented with a data-driven, machine-learnt model that captures the nonlinearities present in the system. Standard ML models have, however, several important shortcomings from an engineering point of view. They often require large training datasets, do not generalize well to unseen conditions, and can even be physically inconsistent. To overcome these limitations, we investigate the use of Lagrangian Neural Networks (LNNs) where a neural network is used to directly model the Lagrangian function of the system. To enforce physical consistency, the Euler-Lagrange equations of motion of the system are obtained by differentiating this neural network using automatic differentiation techniques. The potential of this modelling approach is numerically and experimentally shown on a range of systems with stiffness and damping nonlinearities.

Keywords: nonlinear systems, physics-guided machine learning, data-driven models

INTRODUCTION

Aerospace structures, rotating machines, turbines, pumps, and aero engines are complex dynamical systems that are known to suffer from vibration-related problems. Uncontrolled vibration leads to high cycle fatigue that might result in catastrophic failure [1]. Novel designs, strict regulations and composite materials in aerospace structures make predicting vibration responses of already complex structures even more challenging, as they inherently exhibit nonlinearity. Predicting the dynamic behaviour of nonlinear systems is difficult because of the wide range of behaviours that have no linear equivalent and are very sensitive to parameter changes. Identifying the source of nonlinearity and modelling requires domain-specific knowledge and experience, and to this day poses a significant challenge [2]–[4].

In recent years machine learning (ML) models started playing an important role in not only commercial applications but also in advancing scientific discovery in engineering [5]–[8]. Using state-of-the-art ML modelling techniques is tempting, especially in domains that are not fully understood, or where other well-established methods are computationally unfeasible. However, ML methods suffer from two main flaws: inefficient learning – as the models require large datasets, and lack of generalization beyond the training dataset – as the results often lack physical consistency [5], [9]. Therefore, in most cases, neither knowledge nor black-box approaches provide sufficient means to solve complex engineering problems. A recent research direction to address those issues is to implement the current understanding of physics in the data-driven or machine-learning model [7], [9]–[11], [11]–[15]. Such physics-guided data-driven machine learnt models provide an opportunity to capitalize on existing domain knowledge and fill in the lack of understanding with black-box ML approaches.

In this work, we introduce an approach that uses a linear model that is augmented with a modified Lagrangian Neural Network (LNN) proposed by Cranmer et al. [16]. We demonstrate how to expand the LNN formulation to an externally forced and dissipative system and show how to use this approach in a nonlinear structural dynamic context.

BACKGROUND

LNNs [16] provide a way to parametrize an arbitrary Lagrangian of a dynamical system. The well-established Lagrangian mechanics are based on the principle of least action [17] given by (1):

$$\mathcal{S} = \int_{t_0}^{t_1} \big(T(q_t, \dot{q}_t) - V(q_t)\big) dt = \int_{t_0}^{t_1} \mathcal{L}(q_t, \dot{q}_t) dt \tag{1}$$

where $\mathcal{S}$ is the scalar value of the action, $q, \dot{q}$ are generalized coordinates of the system, $\mathcal{L}$ is the Lagrangian defined as the difference between kinetic energy T and potential energy V. The Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial q} \tag{2}$$

can be expanded to the following equation of motion (EOM) in vector form

$$\ddot{q} = \left(\nabla_{\dot{q}} \nabla_q^T \mathcal{L}\right)^{-1} \left[\nabla_q \mathcal{L} - \left(\nabla_q \nabla_{\dot{q}}^T \mathcal{L}\right)\dot{q}\right], \tag{3}$$

as shown by Cranmer et al. [16].

The resulting EOM enables calculating $\ddot{q}_t$ for given a set of coordinates $q_t, \dot{q}_t$. In this work, we consider the case where $\mathcal{L}$ is an arbitrary Lagrangian function parametrized by a neural network. As integrating (3) yields the dynamics of the system, it is, therefore, possible to use it in a loss function that minimizes the discrepancy between model predictions, $\ddot{q}_t$, and experimental measurements, $\ddot{q}_t^{\,true}$. It is also worth noting that modern automatic differentiation frameworks have no problem with deriving the Jacobian, and hessian of $\mathcal{L}$ found in Eq. (3).

INTRODUCING NONCONSERVATIVE FORCES

To expand the use of LNN to an engineering setting let us consider a nonconservative system, with the presence of damping forces *D,* and other external excitation forces *F*. For such a system one can modify (2), and write the following expression

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} + \frac{\partial \mathrm{D}}{\partial \dot{q}} + \frac{\partial F}{\partial q} = 0. \tag{4}$$

Similarly, to how equation (3) was achieved, expanding (4) yields the following expressions

$$\ddot{q} = \left(\nabla_{\dot{q}} \nabla_q^T \mathcal{L}\right)^{-1} \left[\nabla_q \mathcal{L} + \nabla_q E - \left(\nabla_q \nabla_{\dot{q}}^T \mathcal{L} + \nabla_{\dot{q}} F\right)\dot{q}\right]. \tag{5}$$

Here both $\mathcal{L}$ and the dissipative forces *D* are parametrized by a neural network and the external forces $\nabla_q F$ are assumed to be measured directly from the experiment.

EXPERIMENTAL ANALYSIS

To test the application of the extended LNN, a set of numerical experiments was conducted. A damped non-linear mechanical system excited externally by a harmonic force F was simulated. The system comprised of a mass m, a linear *k* and a nonlinear cubic spring k_3 and a proportional damper *c*. The training data for the proposed presented method was simulated numerically using the following parameters m=0.64, k=1, k_3=0.5 and c=0.0064, and for a set of different excitation frequencies as well as forces. Solutions were obtained using shooting with pseudo arclength continuation methods [3] by fixing one parameter, the excitation frequency or forcing amplitude and using the other one as a continuation parameter. This provided frequency and S-shaped response curves [4] for the studied system. The training, testing and validation data set was built using families of periodic solutions, where each training point contained the state variables and the value of the excitation force.

Figures 1-2 show an excellent agreement between the model of the system and the trained LNN. The LNN can also be used to extract the stiffness and damping characteristics of the system, informing more detailed physics-based modelling of the system of interest.

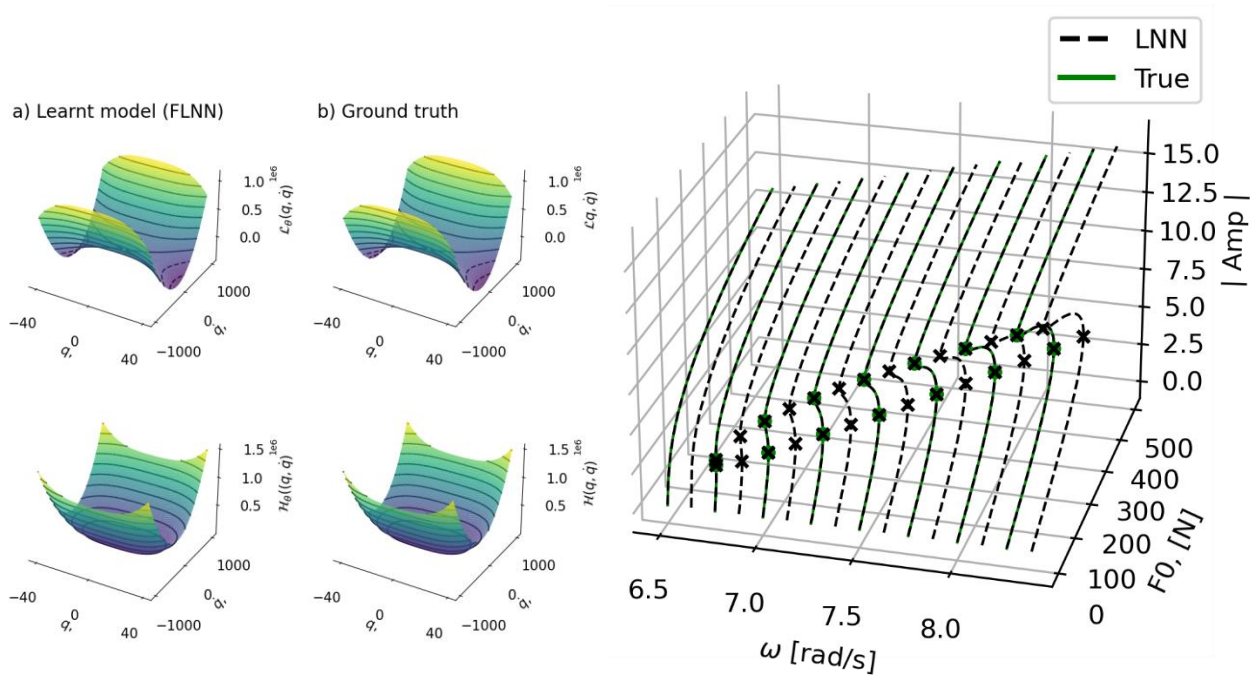


Figure 1. (Left) Lagrangian and Hamiltonian of the trained LNN and the analytical model. (Right) Comparison between the response obtained from the model (solid green) and the response obtained from the LNN (dashed black). For both models, shooting and pseudo-arclength continuation were used to obtain the response curves. Crosses highlight the location of the fold points.

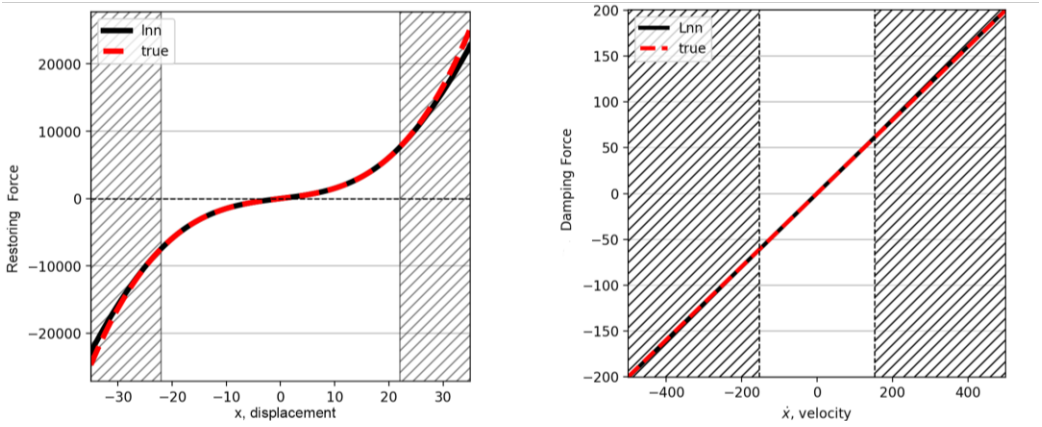


Figure 2. Comparison between the stiffness and damping forces obtained from the model (dashed-red) and derived from the LNN (solid black). The shaded area represents displacement/velocity amplitudes outside the training data set.

EXPLOITING KNOW PHYSICS

As demonstrated in the previous paragraph, LNNs provide an elegant way to model a non-linear dynamical system, without any assumptions about the underlying non-linearity. In structural dynamics, a linear model of the structure is often known in

the form of, for instance, a Finite Element Model. With LNNs, it is possible to exploit this prior knowledge about the system by simply adding the quadratic terms $\mathcal{L}_{lin}$ associated with the linear model to the LNN. The total Lagrangian of the system is thus given by

$$\mathcal{L}(q, \dot{q}) = \mathcal{L}_{lin}(q, \dot{q}) + \mathcal{L}_{NN}(q, \dot{q}), \tag{6}$$

where $\mathcal{L}_{NN}$ is an arbitrary function parametrized by a neural network that serves as a correction to the inaccuracies of the linear model and captures the nonlinear behaviour of the system. Our work further discusses how Eq (6) can be used to apply LNNs to more complex structures.

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