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Dissipativity-preserving model reduction for linear systems using moment matching

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ABSTRACT

This article introduces a novel model reduction method that preserves dissipativity properties while achieving moment matching at user-specified interpolation points. Building on the time-domain moment matching framework, our approach leverages the flexibility of free parameters in reduced-order models to ensure the preservation of $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity, independent of the state dimension of the reduced model or the location of the interpolation points. By preserving dissipativity, this method enables more reliable and efficient reduced-order models for large-scale systems, while maintaining key physical and stability properties. Numerical examples are presented to illustrate the effectiveness and applicability of the proposed approach.

1. Introduction

Model reduction techniques play a crucial role in control theory and systems engineering, enabling the simplification of complex dynamical systems while preserving essential properties (Antoulas, 2005a). The goal of these techniques is to create reduced-order models that accurately represent the original system's behaviour, particularly in terms of stability, performance, and specific structural properties like dissipativity. As introduced by J.C. Willems in the seminal paper (Willems, 1972), dissipativity of dynamical systems is a fundamental property of storing, dissipating, and exchanging energy over time. The success of dissipativity is attributed to its ability to be preserved in the interconnection of systems, which simplifies the construction of energy-based controllers, and its strong connection with the Lyapunov stability theory (van der Schaft, 2000).

Several characterisations and extensions of the dissipativity property have been developed so far, see, *e.g.*, (Angeli, 2006; Forni & Sepulchre, 2018; Hill & Moylan, 1976; Moreschini, Bin, Astolfi, & Parisini, 2023, 2025; Pavlov & Marconi, 2008; van der Schaft, 2013, 2020; Simpson-Porco, 2018). Dissipativity naturally emerges when analysing the stability and performance of several classes of systems such as port-Hamiltonian, Lur'e-type systems, and in the construction of reduced-order models (Besselink, van de Wouw, & Nijmeijer, 2013; Moreschini & Astolfi, 2024; Moreschini, Simard, & Astolfi, 2024; Shakib et al., 2024; Shakib, Scarciotti, Pogromsky, Pavlov, & van de Wouw, 2023a). Port-Hamiltonian systems are a type of dissipative system arising from Dirac structures that integrates interconnection and energy dissipation properties (Moreschini, Monaco, & Normand-Cyrot, 2024; van der Schaft & Jeltsema, 2014).

Over the past decades, numerous methods for model reduction have been developed. Balanced truncation (Moore, 1981) has been widely used thanks to its simplicity and the ability to provide error bounds. This method aims to balance the controllability and the observability Gramians of a system, reducing the model by truncating less significant states. Alternatively, interpolation-based approaches (Antoulas, 2005a), also known as moment matching-based approaches, generate reduced-order models that retain specific behaviour at so-called interpolation points. These methods, closely related to the Nevalinna-Pick interpolation problem (Byrnes, Georgiou, & Lindquist, 2001), are favoured for their applicability to large-scale systems and their ability to preserve accuracy in targeted regions of the complex plane where system performance is most critical. While effective, these methods often overlook the specific structural properties of the system, such as passivity or dissipativity, which are critical in many applications, especially in physical systems.

In response to these limitations, more recent efforts have focused on dissipativity-preserving model reduction. Balanced truncation approaches have been extended to positive-real (Phillips, Daniel, & Silveira, 2002) and bounded-real (Opdenacker & Jonckheere, 1988) balancing, thereby preserving passivity and L_2 system norm, respectively. Furthermore, it was shown in Antoulas (2005b), Moreschini, Simard, and Astolfi (2024), Sorensen (2005) that interpolation-based reduction approaches preserve the passivity property and the underlying physical structure, such as the port-Hamiltonian structure, by a careful selection of the interpolation points. Recently, Cheng (2023) proposed an optimal projection framework for model reduction that preserves

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$(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity through a specific selection of the projection matrices.

In this article, we propose a novel approach to model reduction that not only preserves the dissipativity properties of the system but also achieves moment matching at user-defined interpolation points. The proposed approach relies on the so-called *time-domain moment matching* approach introduced in Astolfi (2010), which proposed a family of models that achieve moment matching and leave some parameters free. In Astolfi (2010), these free parameters were used to enforce passivity or a pre-defined L_2-gain, however, the more general concept of dissipativity was not studied. In the current article, the free parameters of this reduced-order model are used to ensure that the dissipativity property of the original system is preserved with the same $(\mathcal{Q},\mathcal{S},\mathcal{R})$. We furthermore show that this problem can be solved independently of the choice of the state dimension of the reduced-order model and of the location of the user-defined interpolation points. The applicability of the results of this article is demonstrated in numerical examples. In summary, the main contribution of this article is a model reduction by moment matching technique which preserves the $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity property of the original system to the reduced-order model.

Notation: The symbols $\mathbb{R},\mathbb{C},\mathbb{C}^0,\mathbb{C}^-$, $\mathbb{C}^+$, and $\mathbb{N}$ denote the set of real numbers, complex numbers, complex numbers with zero real parts, complex numbers with negative real parts, complex numbers with positive real parts, and non-negative integers, respectively. The symbol I_n denotes the $n\times n$ identity matrix, while $\mathbf{0}$ denotes the zero matrix of which the dimensions can be inferred from the context. For $A\in\mathbb{R}^{n\times n}$, $\sigma(A)$ denotes its spectrum and $A^\top$ denotes its transpose. A matrix $A\in\mathbb{R}^{n\times n}$ is Hurwitz if all its eigenvalues have a strictly negative real part. For a symmetric matrix $A\in\mathbb{R}^{n\times n}$, $A\succ 0$ ($A\prec 0$) denotes that A is positive (negative) definite. In symmetric block matrices, the element $\star$ is induced by transposition. The symbol j denotes the imaginary unit.

2. The dissipativity-preserving reduction problem

We first recall the time-domain moment matching approach proposed by Astolfi (2010) in Section 2.1. After that, we give preliminaries on $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity in Section 2.2. Finally, we pose the formal problem addressed in this article in Section 2.3.

2.1. A family of moment matching models

Consider a multiple-input, multiple-output, continuous-time, linear time-invariant (LTI) system described by the equations

$$\begin{aligned}\dot{x}(t)&=Ax(t)+Bu(t),\ \ x(0)=x_0\in\mathbb{R}^n,\\ y(t)&=Cx(t),\end{aligned}\tag{1}$$

with state $x(t)\in\mathbb{R}^n$, input $u(t)\in\mathbb{R}^m$, output $y(t)\in\mathbb{R}^p$, and real-valued matrices A, B, and C of appropriate dimensions. Let system (1) be minimal, i.e., (A,B) defines a reachable pair and (A,C) defines an observable pair. The notion of moment of an LTI system is defined as follows (Astolfi, 2010; Shakib, Scarciotti, Pogromsky, Pavlov, & van de Wouw, 2023b).

Definition 1. Consider the system (1) and an observable pair (S,L) with $S\in\mathbb{R}^{\nu\times\nu}$ and $L\in\mathbb{R}^{m\times\nu}$ such that $\sigma(S)\cap\sigma(A)=\emptyset$. The matrix $C\Pi$ is the moment of system (1) at (S,L), where $\Pi\in\mathbb{R}^{n\times\nu}$ is the unique solution of the Sylvester equation

$$A\Pi+BL=\Pi S.\tag{2}$$

As a replacement for the system (1), consider a reduced-order model described by the following equations

$$\begin{aligned}\dot{\xi}(t)&=F\xi(t)+Gu(t),\ \ \xi(0)=\xi_0\in\mathbb{R}^\nu,\\ \psi(t)&=H\xi(t),\end{aligned}\tag{3}$$

where $\xi(t)\in\mathbb{R}^\nu$ is the state, $u(t)\in\mathbb{R}^m$ is the input, $\psi(t)\in\mathbb{R}^p$ is the output, and F,G, and H are real-valued matrices of appropriate size. For an observable pair (S,L), the moment of (3) is denoted by HP, where $P\in\mathbb{R}^{\nu\times\nu}$ is the unique solution of the Sylvester equation

$$FP+GL=PS.\tag{4}$$

Let us define the notion of moment matching model.

Definition 2. Consider the matrix $C\Pi\in\mathbb{R}^{p\times\nu}$, which characterises the moment of the system (1) at (S,L). The model (3) is said to achieve moment matching at (S,L) if its moment HP at (S,L) equals $C\Pi$. Furthermore, if $\nu<n$, then (3) is called a reduced-order model of system (1).

Following (Astolfi, 2010; Shakib et al., 2023b) by taking $P=I_n$ in (4), the family of models (3) described by

$$F=S-GL,\quad H=C\Pi,\tag{5}$$

and parametrised by $G\in\mathbb{R}^{\nu\times m}$ such that $\sigma(S)\cap\sigma(S-GL)=\emptyset$, contains *all* the models of dimension ν that achieve moment matching (up to a similarity transformation), see Astolfi (2010), Shakib et al. (2024). A general family of *all* models of any (finite) dimension larger than ν was recently given in Simard, Moreschini, and Astolfi (2025).

In the parametrisation (5), the matrix G is considered a free variable, which has been used for various purposes, e.g., enforcing a port-Hamiltonian model structure (Ionescu & Astolfi, 2013) or minimising the $\mathcal{H}_\infty$ or $\mathcal{H}_2$-norm of the error system, see Shakib et al. (2021), and see Scarciotti and Astolfi (2024) for a recent overview. In this article, we are interested in using the freedom in G to ensure that the reduced-order model preserves the so-called $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity property of the original system.

Remark 1. The interpolation points are characterised through the set $\sigma(S)$, while L characterises the so-called tangential directions, see Shakib et al. (2023b) for details. When restricting to the imaginary axis, the interpolation points can often be chosen intuitively: for example, selecting the zero frequency to obtain a steady-state match for constant inputs, and choosing frequencies corresponding to resonance peaks to accurately capture the behaviour of the system close to such peaks. Alternatively, interpolation points and tangential directions that satisfy first-order necessary conditions for $\mathcal{H}_2$ optimality are given in Dooren, Gallivan, and Absil (2008). Such conditions depend on the unknown reduced-order model and are hence typically found through an iterative scheme, such as, e.g., (Gugercin, Antoulas, & Beattie, 2008).

2.2. Dissipativity theory

The theory of dissipativity was introduced in the seminal paper by Willems (1972) to characterise the energy behaviour of dynamical systems in relation to their inputs and outputs. Dissipativity relies on two key notions: the *storage function*, a scalar function of the state of the system representing the energy stored in the system at that state; and a *supply rate*, a scalar function of the input and the output of the system representing the rate at which energy is supplied to a system. Motivated by Willems (1972), D. Hill and P. Moylan in Hill and Moylan (1976) specified the dissipativity property using a general quadratic supply rate. Dissipativity with a quadratic supply rate is referred to as $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity, as formally defined next.

Definition 3 (*Kottenstette, McCourt, Xia, Gupta, and Antsaklis (2014, Definition 4)*)**.** The system (1) is *dissipative* with respect to the energy supply rate $s:\mathbb{R}^m\times\mathbb{R}^p\to\mathbb{R}:(u,y)\mapsto s(u,y)$, if there exists a non-negative storage function $V:\mathbb{R}^n\to\mathbb{R}$, such that for all inputs signal $u(\cdot)$, all trajectories $x(\cdot)$, and all $t_2\geq t_1$, the following inequality holds

$$V(x(t_2))\leq V(x(t_1))+\int_{t_1}^{t_2}s(u(t),y(t))\mathrm{d}t.\tag{6}$$

Additionally, the system is $(\mathcal{Q},\mathcal{S},\mathcal{R})$-*dissipative* if it is dissipative with respect to the quadratic supply rate

$$s(u,y)=y^\top \mathcal{Q} y+2u^\top \mathcal{S} y+u^\top \mathcal{R} u, \tag{7}$$

where $\mathcal{Q}\in\mathbb{R}^{p\times p}$, $\mathcal{S}\in\mathbb{R}^{m\times p}$, and $\mathcal{R}\in\mathbb{R}^{m\times m}$.

The notion of $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity encapsulates, amongst others, the notions of passivity and L_2-gain. In particular, if the inequality (6) with s in (7) holds for a square system, i.e., $m=p$, with $\mathcal{Q}=\mathbf{0}, \mathcal{S}=\frac{1}{2}I_m$, and $\mathcal{R}=\mathbf{0}$, then the system is *passive*. Furthermore, if it holds with $\mathcal{Q}=-I_p, \mathcal{S}=\mathbf{0}$, and $\mathcal{R}=\gamma^2 I_m$, then the system is L_2-stable from u to y with L_2-gain γ, where the L_2-gain γ is further connected to the $\mathcal{H}_\infty$-norm of the system (Scherer, 1990). Whether an LTI system is $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative can be checked by solving a linear matrix inequality problem defined next.

Lemma 2 (*Kottenstette et al.* (*2014*, *Lemma 2*)). *For the LTI system (1), a necessary and sufficient test for $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity is that there exists a matrix $\mathcal{X}\in\mathbb{R}^{n\times n}$ such that $\mathcal{X}=\mathcal{X}^\top\succ\mathbf{0}$ and the following linear matrix inequality is satisfied:*

$$\Gamma:=\begin{bmatrix}A^\top\mathcal{X}+\mathcal{X}A & \mathcal{X}B\\ \star & 0\end{bmatrix}-\begin{bmatrix}C^\top\mathcal{Q}C & C^\top\mathcal{S}\\ \star & \mathcal{R}\end{bmatrix}\preceq\mathbf{0}. \tag{8}$$

The matrix inequality (8) is linear in the matrix $\mathcal{X}$. The next section formulates the problem addressed in this article.

2.3. Formal problem statement

Based on the family of moment matching models and the notion of $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity, we introduce the $(\mathcal{Q},\mathcal{S},\mathcal{R})$-*dissipativity-preserving model reduction by moment matching problem*. We summarise all the assumptions required first.

Assumption 1. Given $\mathcal{Q},\mathcal{S}$, and $\mathcal{R}$, the system (1) is $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative and minimal. The pair of matrices (S,L) is observable with $S\in\mathbb{R}^{\nu\times\nu}$ and $L\in\mathbb{R}^{m\times\nu}$ such that $\sigma(S)\cap\sigma(A)=\emptyset$ and $\nu<n$.

We formulate the problem addressed in this article next.

Problem 3. Consider the system (1) and the matrices (S,L), and suppose Assumption 1 holds for a given $\mathcal{Q},\mathcal{S}$, and $\mathcal{R}$. The $(\mathcal{Q},\mathcal{S},\mathcal{R})$-*dissipativity-preserving model reduction by moment matching problem* consists in finding a matrix $G\in\mathbb{R}^{\nu\times m}$ such that the reduced-order model (3) with F and H as in (5), achieves moment matching at (S,L) and is $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative for the same $\mathcal{Q},\mathcal{S}$, and $\mathcal{R}$.

The considered problem aims jointly at achieving moment matching at (S,L) and preserving the $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity property of the original system. While moment matching is achieved for any G that satisfies $\sigma(S)\cap\sigma(S-GL)=\emptyset$, at this point it is unknown whether a G exists that preserves the dissipativity property and how such a G can be computed. In the next section, we show that, independent of dimension ν and the choice of (S,L) (as long as the conditions in Assumption 1 are satisfied), a G exists such that dissipativity is preserved.

3. Dissipativity-preserving moment matching

This section presents a selection of the matrix G that preserves $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity as the main result of this article in Section 3.1. After that, we show that this result has an interpretation as a Petrov–Galerkin projection in Section 3.2.

3.1. Main result

The main result presented next shows that at least one matrix $G\in\mathbb{R}^{\nu\times m}$ exists that preserves $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity. After the theorem, we show a constructive way of computing such a G.

Theorem 4. *Consider the system (1) and the matrices (S,L), and suppose Assumption 1 holds for given $\mathcal{Q},\mathcal{S}$, and $\mathcal{R}$. Then, model (3) with matrices F and H as in (5) and*

$$G=\left(\Pi^\top\mathcal{X}\Pi\right)^{-1}\Pi^\top\mathcal{X}B, \tag{9}$$

where $\mathcal{X}\succ\mathbf{0}$ is any solution to (8) and Π is the unique solution to (2), preserves the $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity property. Furthermore, if $\sigma(S)\cap\sigma(S-GL)=\emptyset$, then model (3) also achieves moment matching at (S,L).

Proof. The matrix inequality (8) written for the reduced-order model (3) reads as

$$\begin{bmatrix}F^\top\tilde{\mathcal{X}}+\tilde{\mathcal{X}}F & \tilde{\mathcal{X}}G\\ \star & 0\end{bmatrix}-\begin{bmatrix}H^\top\mathcal{Q}H & H^\top\mathcal{S}\\ \star & \mathcal{R}\end{bmatrix}\preceq\mathbf{0}, \tag{10}$$

for some symmetric matrix $\tilde{\mathcal{X}}\succ\mathbf{0}$. Let $\mathcal{X}\succ\mathbf{0}$ be such that (8) is satisfied for the original system (1) (such an $\mathcal{X}$ exists by Assumption 1). We show that the selection

$$\tilde{\mathcal{X}}=\Pi^\top\mathcal{X}\Pi \tag{11}$$

results in the satisfaction of the matrix inequality (10). Note that $\tilde{\mathcal{X}}$ is a positive-definite matrix since Π is a full column-rank solution of the Sylvester equation (2) by the observability of (S,L) and controllability of (A,B), see de Souza and Bhattacharyya (1981) for details.

Using $\tilde{\mathcal{X}}$, the matrix G in (9) can be written as

$$G=\tilde{\mathcal{X}}^{-1}\Pi^\top\mathcal{X}B. \tag{12}$$

By direct computation, we find the following identities:

$$\begin{aligned}\tilde{\mathcal{X}}F&=\tilde{\mathcal{X}}(S-GL) && (13a)\\ &=\tilde{\mathcal{X}}(S-\tilde{\mathcal{X}}^{-1}\Pi^\top\mathcal{X}BL) && (13b)\\ &=\Pi^\top\mathcal{X}\Pi S-\Pi^\top\mathcal{X}BL && (13c)\\ &=\Pi^\top\mathcal{X}A\Pi, && (13d)\end{aligned}$$

where to arrive at (13d), the Sylvester equation (2) has been used. Furthermore, the identities

$$\begin{aligned}\tilde{\mathcal{X}}G&=\Pi^\top\mathcal{X}B, && (14a)\\ H^\top\mathcal{S}&=\Pi^\top C^\top\mathcal{S}, && (14b)\\ H^\top\mathcal{Q}H&=\Pi^\top C^\top\mathcal{Q}C\Pi, && (14c)\end{aligned}$$

can be found by direct substitution of (12) for G and $H=C\Pi$ (as in (5)). Using the identities (13) and (14) in (10), the inequality (10) can be written as

$$\begin{bmatrix}\Pi^\top & 0\\ 0 & I_m\end{bmatrix}\underbrace{\Gamma}_{\preceq 0}\underbrace{\begin{bmatrix}\Pi & 0\\ 0 & I_m\end{bmatrix}}_{=:T}\preceq\mathbf{0}, \tag{15}$$

which holds true because T is a full column rank matrix, and where Γ is the matrix defined in (8) which is negative semi-definite by the $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity of the original system. We have thus shown that (10) is satisfied, which guarantees that the reduced-order model (3) is $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative. Finally, it follows from (Astolfi, 2010; Shakib et al., 2023b) that moment matching is achieved if $\sigma(S)\cap\sigma(S-GL)=\emptyset$ is satisfied, which completes this proof. ■

Note that the solution $\mathcal{X}$ to (8) is not unique (Kottenstette et al., 2014). Hence, Theorem 4 presents a well-defined family of models parametrised by $\mathcal{X}$. We will consider finding the model within this family that minimises some error criterion with respect to the full-order system, e.g., the $\mathcal{H}_2$-error criterion, as part of future work.

The result of Theorem 4 can be used to define a procedure to find a $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative reduced-order model. First, for the original system (1), solve (8) to obtain $\mathcal{X}$. Then, select F and H as in (5) and use the computed $\mathcal{X}$ to select G as in (9). This procedure ensures that the reduced-order model preserves the $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity property. If, in addition, the condition $\sigma(S)\cap\sigma(S-GL)=\emptyset$ is satisfied, then moment matching is also achieved at (S,L).

In some specific cases, the reduced-order model also preserves asymptotic stability. We formalise this result in the next corollary.

Corollary 5. *Consider the setting in Theorem 4. If A is Hurwitz and $\mathcal{Q}\preceq 0$, then F is Hurwitz.*

Proof. We start by noting that Π is a full column rank matrix and $\tilde{\mathcal{X}}=\Pi^\top\mathcal{X}\Pi$ is a positive-definite matrix, see the proof of Theorem 4. Then, if $\mathcal{Q}\preceq\mathbf{0}$ and given that Π is full column rank, one has that

$$A^\top\mathcal{X}+\mathcal{X}A\prec\mathbf{0}\implies \Pi^\top A^\top\mathcal{X}\Pi+\Pi^\top\mathcal{X}A\Pi\prec\mathbf{0}.$$

Note that the equality sign is excluded because $A^\top\mathcal{X}+\mathcal{X}A=\mathbf{0}$ can only be true when A is a skew-symmetric matrix, which it is not given that A is Hurwitz. Now, given that $\Pi^\top\mathcal{X}A\Pi=\tilde{\mathcal{X}}F$ (see (13)), we have that

$$\Pi^\top A^\top\mathcal{X}\Pi+\Pi^\top\mathcal{X}A\Pi=F^\top\tilde{\mathcal{X}}+\tilde{\mathcal{X}}F\prec\mathbf{0}.$$

Hence, since $\tilde{\mathcal{X}}=\Pi^\top\mathcal{X}\Pi$ is positive definite, we conclude that F is Hurwitz, which completes the proof. ■

The selection of G in (9) results in a Petrov–Galerkin projection. In the next section, we show this explicit relation.

Remark 6. The condition $\sigma(S)\cap\sigma(S-GL)=\emptyset$ is a mild condition that is typically satisfied. For example, in the setting of Corollary 5 in which asymptotic stability is preserved, selecting $\sigma(S)\subset\left(\mathbb{C}^0\cup\mathbb{C}^+\right)$ ensures that $\sigma(S)\cap\sigma(S-GL)=\emptyset$.

Remark 7. The result of Theorem 4 can also be used to preserve the port-Hamiltonian structure of a given system when reducing it to a reduced-order model. Whenever the given system (1) is $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative with $\mathcal{Q}=\mathbf{0},\mathcal{S}=\frac{1}{2}I_m$, and $\mathcal{R}=\mathbf{0}$ and represented as a port-Hamiltonian system, i.e.,

$$A:=(J-R)\mathcal{X},\qquad C:=B^\top\mathcal{X},$$

where $J=-J^\top$, $R=R^\top\succeq\mathbf{0}$, and with storage function $V(x):=\frac{1}{2}x^\top\mathcal{X}x$, the obtained reduced-order model (3) is again $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative with $\mathcal{Q}=\mathbf{0},\mathcal{S}=\frac{1}{2}I_m$, and $\mathcal{R}=\mathbf{0}$. Furthermore, such a reduced-order model can be equivalently rewritten as a reduced-order port-Hamiltonian system with matrices

$$F:=(\tilde{J}-\tilde{R})\tilde{\mathcal{X}}^{-1},\ H:=G^\top\tilde{\mathcal{X}}^{-1},\ G:=\Pi^\top\mathcal{X}B,$$

where

$$\tilde{J}:=\frac{1}{2}\Big((\tilde{\mathcal{X}}S-\Pi^\top\mathcal{X}BL)-(\tilde{\mathcal{X}}S-\Pi^\top\mathcal{X}BL)^\top\Big),$$
$$\tilde{R}:=-\frac{1}{2}\Big((\tilde{\mathcal{X}}S-\Pi^\top\mathcal{X}BL)+(\tilde{\mathcal{X}}S-\Pi^\top\mathcal{X}BL)^\top\Big),$$

and where $\tilde{\mathcal{X}}^{-1}$ is the matrix characterising the storage function associated with the reduced-order model, i.e. $\tilde{V}(\xi)=\frac{1}{2}\xi^\top\tilde{\mathcal{X}}^{-1}\xi$, with $\tilde{\mathcal{X}}$ as in (11).

Remark 8. The proposed reduced-order model with G as in (9) potentially achieves so-called *two-sided* moment matching. Following (Ionescu, 2015), if there exists a reachable pair $M\in\mathbb{R}^{\nu\times\nu}$ and $R\in\mathbb{R}^{\nu\times p}$ such that $M\Pi^\top\mathcal{X}=\Pi^\top\mathcal{X}A+RC$ is satisfied, then moment matching is achieved at (S,L) and (M,R) simultaneously.

Remark 9. As part of future work, we consider extending Theorem 4 to the case of nonlinear systems. In particular, we highlight the potential of formulating error bounds between the responses of a $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity nonlinear system and the responses of a reduced-order nonlinear model that preserves the same dissipativity property. For example, in the LTI case, if the system has a finite L_2-gain $\gamma\in[0,\infty)$, and the reduced-order model preserves this gain, then 2γ is a trivial bound for $\mathcal{H}_\infty$-norm of the error dynamics.

3.2. Connection to Petrov–Galerkin projection

The reduced-order model obtained Theorem 4 has a connection to Petrov–Galerkin projection. A Petrov–Galerkin projection is defined through the projection matrices $V\in\mathbb{R}^{n\times\nu}$ and $W\in\mathbb{R}^{n\times\nu}$ which satisfy $W^\top V=I_\nu$. After applying the projection, the matrices of the reduced-order model (3) read as

$$F=W^\top AV,\quad G=W^\top B,\quad H=CV.\tag{16}$$

Both the moment matching approach as well as the balanced truncation approach can be cast via appropriate choices of these projection matrices, see Antoulas (2005a) for details.

The method in Cheng (2023) proposes a $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative reduction approach based on a suitable choice of the Petrov–Galerkin projection matrices. In particular, it was shown in Cheng (2023) that $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity can be preserved by selecting

$$V=\mathcal{X},\quad W=(V^\top\mathcal{X}V)^{-1}V^\top\mathcal{X},\tag{17}$$

where $\mathcal{X}\succ\mathbf{0}$ is any solution to (8). Note that achieving moment matching was not an objective in Cheng (2023) and the resulting model does not achieve moment matching.

Let us show that the parametrisation (5), (9) we propose also corresponds to a specific selection of V and W. Substituting the expression for G in (9) in the expression for F in (5) results in:

$$\begin{aligned}F&=S-\left(\Pi^\top\mathcal{X}\Pi\right)^{-1}\Pi^\top\mathcal{X}BL\\&=\left(\Pi^\top\mathcal{X}\Pi\right)^{-1}\left(\Pi^\top\mathcal{X}\Pi S-\Pi^\top\mathcal{X}BL\right).\end{aligned}$$

We now use the Sylvester equation (2) to arrive at

$$F=\left(\Pi^\top\mathcal{X}\Pi\right)^{-1}\Pi^\top\mathcal{X}A\Pi.$$

From here, it is easy to see that

$$V:=\Pi,\quad W^\top:=\left(\Pi^\top\mathcal{X}\Pi\right)^{-1}\Pi^\top\mathcal{X}\tag{18}$$

are the projection matrices leading to the projection (16) and are consistent with $H=CV$ and $G=W^\top B$. Furthermore, the matrix W is a generalised inverse of V, i.e., V and W satisfy the condition $W^\top V=I$, since

$$W^\top V=\left(\Pi^\top\mathcal{X}\Pi\right)^{-1}\Pi^\top\mathcal{X}\Pi=I_\nu,\tag{19}$$

as $\left(\Pi^\top\mathcal{X}\Pi\right)$ is a positive-definite matrix. Thus, we have shown that a specific selection of V and W in a Petrov–Galerkin projection as in (16) results in a reduced-order model that jointly preserves $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity and achieves moment matching (if $\sigma(S)\cap\sigma(S-GL)=\emptyset$ is satisfied). We now illustrate these results using numerical examples.

4. Numerical examples

Two numerical examples are considered for special cases of $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity preservation. For the first example, we preserve the L_2-gain of the system, while reducing its order. For the second example, we start with a passive system and we preserve passivity for the reduced-order model. The Matlab code corresponding to these examples can be downloaded from https://github.com/FahimShakib.

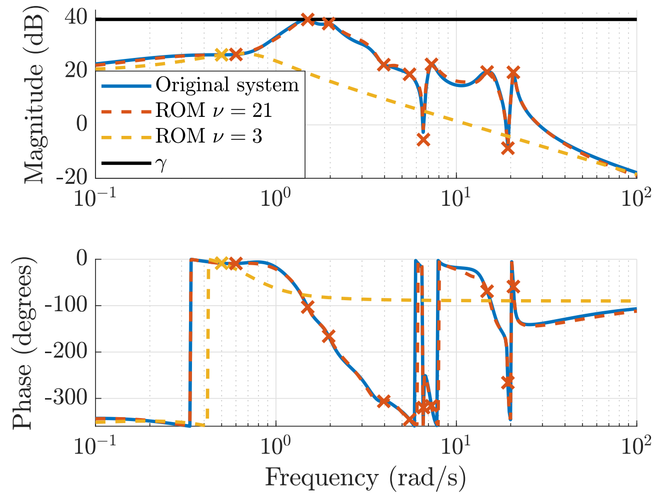

Fig. 1. The Bode plot of the original system and of the two reduced-order models for Example 1. The correspondingly-coloured crosses represent the interpolation points. The interpolation point at 0 is not visible in this plot.

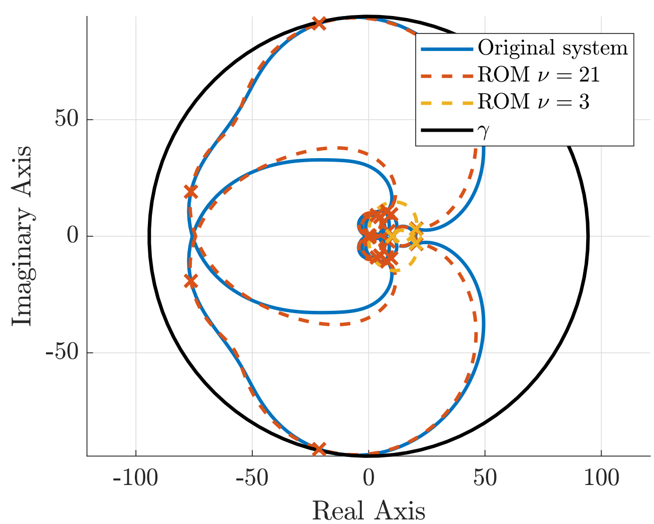

Fig. 2. The Nyquist plot of the original system and of the two reduced-order models for Example 1. The correspondingly-coloured crosses represent the interpolation points.

4.1. Example 1: L_2-gain-preserving model reduction

We consider an asymptotically stable LTI system of order $n = 128$ with its matrices randomly generated through the Matlab command `rss`. The L_2-gain of this system from u to y is 94.3. We compute the symmetric and positive-definite matrix $\mathcal{X}$ by solving the LMI (8) for $Q = -1$, $\mathcal{S} = \mathbf{0}$, $\mathcal{R} = \gamma^2$ with $\gamma := 94.3$.

We first show that the L_2-gain is preserved even when we reduce the state dimension drastically. We select $\nu = 3$ and

$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -0.5 \\ 0 & 0.5 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}, \tag{20}$$

which corresponds to the interpolation points $\{0, \pm 0.5j\}$ (corresponding to 0 and ± 0.5 rad/sec). The pair (S, L) is observable. The reduced-order model (ROM) matrices are then defined through (5), (9). Figs. 1 and 2 depict the Bode and Nyquist plots of the original system (blue curve) and this reduced-order model (yellow curve). Evidently, even though there is a large mismatch between the frequency response of the original system and the reduced-order model, the L_2-gain of the reduced-order model does not exceed the one of the original system. This can be inferred from the preservation of the peak gain in Fig. 1 and the peak radius in Fig. 2.

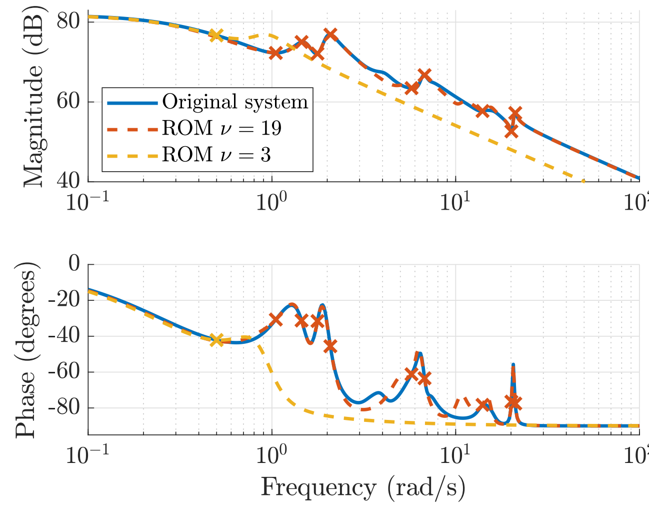

Fig. 3. The Bode plot of the original system and of the two reduced-order models for Example 2. The correspondingly-coloured crosses represent the interpolation points. The interpolation point at 0 is not visible in this plot.

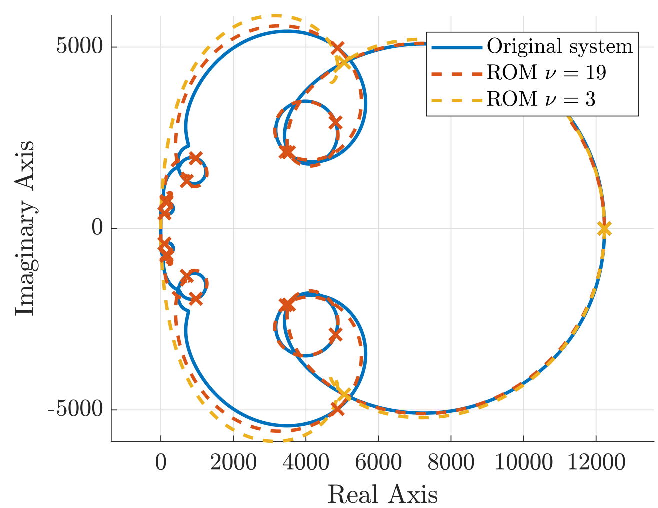

Fig. 4. The Bode plot of the original system and of the two reduced-order models for Example 2. The correspondingly-coloured crosses represent the interpolation points.

We now include additional interpolation points to improve the approximation. We select a total of 11 interpolation points (leading to the dimension $\nu = 21$), namely 0 and $\{\pm j\omega_k\}_{k=1}^{10}$, where ω_k are the frequencies marked by red crosses in Figs. 1 and 2. The matrices S and L are constructed as follows:

$$S = \mathrm{blkdiag}\left(0, \Psi(\omega_1), \ldots, \Psi(\omega_{10})\right), \tag{21a}$$

$$\Psi(\omega) = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}, \tag{21b}$$

$$L = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & \ldots & 1 & 0 \end{bmatrix} \in \mathbb{R}^{1\times\nu}. \tag{21c}$$

The pair (S, L) is observable by construction. Again, the reduced-order model matrices are defined through (5), (9). The red curve in Figs. 1 and 2 represents the frequency response of this reduced-order model. In addition to the L_2-gain of the original system being preserved, also the frequency responses now match accurately across all frequencies. In general, including more interpolation points results in a larger dimension for the reduced-order model and a closer fit between the frequency response of the reduced-order model and the full-order model.

4.2. Example 2: Passivity-preserving model reduction

We modify the system of Example 1 to make it passive. Hereto, we first compute the symmetric and positive-definite matrix $\mathcal{X}$ as the solution to $A^\top\mathcal{X}+\mathcal{X}A=-I_n$. Then, we select $C=2B^\top\mathcal{X}$ for the matrix C of system (1).

As in Example 1, we again construct a third-order reduced-order model, i.e., $\nu=3$, using the (S,L) given in (20). Figs. 3 and 4 show that even though there is a large mismatch between the frequency response of the original system (blue) and the reduced-order model (yellow), the passivity property of the system is preserved. The latter can be confirmed by noting that the phase never exceeds −90 degrees in Fig. 3 and lies in the right half plane in Fig. 4.

To improve the approximation accuracy, we include additional interpolation points. We select a total of 10 interpolation points (leading to $\nu=19$), namely 0 and $\{\pm j\omega_k\}_{k=1}^{9}$, where ω_k are the frequencies marked in red in Figs. 3 and 4. The matrices S and L are as in (21) and the reduced-order model is constructed in the same way as in Example 1. The red curve in Figs. 3 and 4 represents the frequency response of this reduced-order model. In addition to passivity preservation, also the accuracy between the frequency responses has improved significantly across all frequencies.

5. Conclusions

In this article, we introduced a novel model reduction approach that preserves the $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity properties of a linear time-invariant system while achieving moment matching at user-defined interpolation points. The proposed approach can reduce the order of the system to any arbitrarily small order. Although formalised in a time-domain moment matching framework, we show the relation of the approach with model reduction by Petrov–Galerkin projections. Future work aims at extending this approach to nonlinear systems and exploiting dissipativity properties to derive error bounds for nonlinear model reduction.

CRediT authorship contribution statement

Mohammad Fahim Shakib: Conceptualization, Investigation, Software, Formal analysis, Methodology, Validation, Visualization, Writing – original draft, Writing – review & editing. **Alessio Moreschini:** Conceptualization, Investigation, Formal analysis, Methodology, Validation, Visualization, Writing – original draft, Writing – review & editing. **Giordano Scarciotti:** Conceptualization, Investigation, Formal analysis, Methodology, Visualization, Project administration, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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