BROKEN SYMMETRIES OF STRONG INTERACTION

by

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PREFACE

The work presented in this thesis was carried out in the Department of Theoretical Physics, Imperial College, London, between October 1963 and October 1966 under the supervision of Professors P.T. Matthews and A. Salam. The author wishes to thank them and Dr. R. Delbourgo for their continued guidance and encouragement.

Except where stated in the text, the work described here is original and has not been submitted in this or any other University for any other degree. The thesis is based principally on two papers by the author and one with collaborators. He is grateful to his collaborators Dr. D. Akyeampong and Mr. P. Mahanta for permission to include part of the joint work in this thesis. He is also grateful to the members of the Theoretical Physics Group for useful discussions.

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ABSTRACT

Chapters I and II contain a review of U(6,6) symmetry and its hierarchy of subgroups. The problem of proton - anti-proton annihilation at rest into two mesons is treated in chapter III in the framework of broken U(4,4). A detailed comparison with broken U(6,6) is drawn. In chapter VI we assume that the physical baryons belong to the mixed states of the 220^{+}, 364^{+} and 572^{+} irreducible representations of U(6,6), and find the consequences of this assumption on the weak and electromagnetic vertices. Finally, chapter V deals with the problem of higher boson mass spectra in the group $U(6) \otimes U(6) \otimes O(3)$.

INTRODUCTION

The discovery, at Brookhaven, of Ω^- in January 1964 was not only a landmark to unitary symmetry scheme of Gell-Mann and Ne'eman - the 'Eight-Foldway', but also a signpost to many a scholar who desired a clearer understanding of the structure of 'Elementary Particles' by symmetry principles. Following this, Gursey and Radicati[1] arrived, in the summer of 1964, at SU(6) symmetry scheme which is a non-trivial merging of space-spin and unitary-spin. Thus, for the first time in the history of particle physics, it became possible to classify the octet pseudoscalar and the nonet vector mesons into one single multiplet - the 35, and the octet and decuplet baryons into another - the 56, with a unique coupling for all baryon-baryon-meson vertices. One of the good consequences of this was the prediction of $-3/2$ for the magnetic moment ratio of proton and neutron.

Nevertheless, SU(6) had its limitations, one of which is that it could only be applied to situations at rest. A relativistic generalization of it was needed. And it did not take long before Fulton and Wess[2], Charap and Matthews, Ruhl, and many others, thinking along somewhat different lines, introduced the group SL(6,C) as the least non-trivial relativistic completion of SU(6). At nearly the same time, Salam, Delbourgo and Strathdee[3], Sakita and Wali[4], independently proposed the group U(6,6), which has most of the good features of SL(6,C) with the added advantage of accommodating more particles in its regular representation, and of providing a definite reasonable value for the proton magnetic moment. However, it may be true to say that the

explanation of the hitherto mysterious equality of the proton electric and magnetic form factors is the most striking prediction of U(6,6) theory. For, it was soon realised that U(6,6) is incompatible with the requirements of unitarity[5] of the S-matrix, and that at any rate, ignoring this very problem, calculations done on scattering processes with U(6,6) invariant amplitudes, in more cases than not, give results in violent disagreement with experiment.

The main aim of this work, therefore, is to emphasise some of the weak points of this theory and to illustrate using three specific examples how one can obtain better results, and possibly predictions, from a strong interaction symmetry theory of this type, by broken symmetry techniques.

Thus in chapter I we make a brief survey of the development of U(6,6) theory by means of auxiliary operators, following closely the systematic logical approach of Feldman and Matthews.[6] The limitations on the S-matrix elements, due to the requirement of pure U(6,6) index invariance, are discussed in detail in chapter II under "The need for symmetry breaking". This chapter closes with a discussion on the hierarchy of subgroups of U(6,6) produced by the insertion of symmetry breaking spurions.

We choose in chapter III, as our example, the problem of $p\bar{p}$ annihilation at rest into two mesons, in the dynamical spurion scheme of U(4,4) (in which the λ-quarks are separated from the non-λ quarks). The results[8] led to the conclusion that, among other factors, these dynamical spurions were necessary for better agreement with experiment.

Similar to what people do in current algebra, we assume, in chapter IV, that the physical baryons exist[9] as a mixed state of the

three low lying irreducible representations of U(6,6) - $(220,L=0)^+$, $(364,L=0)^+$ and $(572,L=0)^+$. This breaks the U(6,6) symmetry since the physical baryon tensor is no longer an irreducible representation of this group. The consequences of this assumption are studied with special reference to electromagnetic and weak vertices. Calculations of the G_A/G_V and D/F ratios led to the conclusion that such a mixing, at best, reproduces the old SU(6) results.

Finally, in chapter V, we choose as our example the two point function for mass spectra, and treat the problem of further classification of higher boson resonances, by deriving an approximated mass formula for the group $U(6)\otimes U(6)\otimes O(3)_\ell$, (for any given angular momentum ℓ) and exhibiting the[10] exact values of the spin-orbit and tensor mass splitting terms for $\ell = 1, 2, 3$. Equal spacing mass relations which give fitting predictions are obtained. Our mass formula contains terms which determine the mixing angles unambiguously.

More detailed summaries are given at the beginning of each of the last three chapters.

CHAPTER I

U(6,6) AS AUXILIARY GROUP

As mentioned earlier, the contents of the first two chapters are not new, but they are necessary for a logical introduction to, and a clear understanding of the motivations for symmetry breaking.

1. Homogeneous Lorentz Group as Auxiliary Operators.

The homogeneous, proper orthochronons$^+$ Lorentz group $L^{\uparrow}_{+}$, is the group of transformations

$$x_\mu \rightarrow \Lambda_\mu{}^\nu \, x_\nu \qquad \ldots\ldots 1.0$$

with

$$\det \Lambda = +1, \quad \Lambda^\circ_\circ \geq 1 \qquad \ldots\ldots 1.1$$

and which leaves invariant the quadratic form

$$g^{\mu\nu} \, x_\mu \, x_\nu \qquad \ldots\ldots 1.2$$

Since the conditions (1.1) exclude separate space or time reflections, the group elements are continuously connected with the identity. The infinitesimal Lorentz transformation

$$\delta x_\mu = \eta_{\mu\nu} \, x^\nu \qquad \ldots\ldots 1.3$$

are expressed in terms of six real antisymmetric parameters $\eta_{\mu\nu}$ and are generated by six infinitesimal antisymmetric operators $J_{\mu\nu}$ satisfying the commutation relations:

$$(J_{\mu\nu}, J_{\lambda\rho}) = -\, i(g_{\mu\lambda} J_{\nu\rho} + g_{\nu\rho} J_{\mu\lambda} - g_{\mu\rho} J_{\nu\lambda} - g_{\nu\lambda} J_{\mu\rho}) \qquad \ldots\ldots 1.4$$

Let us add to this the displacement in space-time

$$x_\mu \rightarrow x_\mu + \ell_\mu \qquad \ldots\ldots 1.5$$

which form a four parameter Abelian group, with infinitesimal generators P_μ satisfying

$$(P_\mu, P_\nu) = 0 \qquad \ldots\ldots 1.6$$

The Poincare group is the semi-direct product of this Abelian group with the proper homogeneous Lorentz group. It is defined with the additional commutation relation

$$(P_\lambda, J_{\mu\nu}) = i(g_{\lambda\mu}P_\nu - g_{\lambda\nu}P_\mu) \qquad \ldots\ldots 1.7$$

which is the requirement that under Lorentz transformation P_μ transforms like a 4-vector.

Thus the general Poincare transformation, parametrised by a displacement ℓ^μ and a Lorentz transformation $\Lambda^{\mu\nu}$ ($\det \Lambda = 1, \Lambda^{\circ\circ} = 1$) is

$$U(\ell, \Lambda) = \exp\{i(P_\mu \ell^\mu + \tfrac{1}{2}\eta^{\mu\nu} J_{\mu\nu}) \qquad \ldots\ldots 1.8$$

In (1.8) P_μ is interpreted physically as the total energy and momentum operator. J_{ij} (or equivalently η^{ij}) are interpreted as angular momentum. J_{oj} (or η_{oj}) generate pure Lorentz transformation and are related to the velocity of the transformation parallel to j-axis.

The Poincare group generated by $J_{\mu\nu}$, P_λ is non-compact so that all its unitary representations are infinite. However we require that the physical states be normalizable and hence form unitary representations of the Poincare group. These states are specified by the eigen-values of the operators of a complete commuting set constructed from these group generators. Thus an irreducible representation is partially specified by giving the value of the Casimir operator

$$P^2 \,|\, m^2\rangle = m^2 \,|\, m^2\rangle \qquad \ldots\ldots 1.9$$

and for any state in this representation we can further specify the eigenvalue of $\underline{P}$.

$$\underline{P}\,|m^2,\underline{p}\rangle = \underline{p}\,|m^2,\underline{p}\rangle \qquad \ldots\ldots 1.10$$

where for physical states,

$$m^2 \geq 0,\ p_o = +\,(\underline{p}^2 + m^2)^{\frac{1}{2}} \qquad \ldots\ldots 1.11$$

and in particular we restrict ourselves to

$$m^2 > 0 \qquad \ldots\ldots 1.12$$

Clearly, physically we have selected those states corresponding to a given mass. The manifold of states satisfying (1.9) to (1.12) is infinite, and it is called an orbit. A particular point of interest on this orbit, corresponding to a particular component of this infinite component multiplet, is given by the state of zero momentum,

$$p_\mu = m_\mu = (m,\underline{0}) \qquad \ldots\ldots 1.13$$

Now, by definition, the Little Group of the Poincare group corresponding to a state of four momentum p_μ is the subgroup of the homogeneous Lorentz group which leaves p_μ invariant. Since the little group is the same for all points on an orbit, it is convenient to specify this subgroup - the little group - with respect to the special point defined by equation (1.13) which corresponds to the rest frame $p_i = 0$. The little group therefore is generated by J_{ij} - the angular momentum operators. Thus we may further specify J^2 and J_{12} for a state in this representation. For one particle state these reduce to the spins, and spin components s_3. This, apart from parity, completely determines the representation of the one particle states, as far as their space-time properties are concerned. We denote these one particle states by

$$|m^2,s;\underline{p},s_3\rangle = |p,s\rangle \qquad \ldots\ldots 1.14$$

where m^2, s determine an irreducible representation and p and s_3 a state in this representation.

To obtain an explicit representation for the general Poincare transformations, equation (1.8), on the one particle states, we shall for convenience introduce the boost operator which takes the rest state $|m,s\rangle$ to a moving state $|p,s\rangle$.

$$|p,s\rangle = N \exp(-i \underline{\epsilon}(p) \cdot \underline{K}) | m,s\rangle \qquad \ldots\ldots 1.15$$

where

$$K_j = J_{oj} \qquad \ldots\ldots 1.16a$$

and $\underline{\epsilon}(p)$ is parallel to p and satisfies

$$\cosh|\underline{\epsilon}| = p_o/m \ , \ \sinh|\underline{\epsilon}| = |\underline{p}|/m \qquad \ldots\ldots 1.16b$$

and N is a normalization factor chosen conveniently.

Under a pure Lorentz transformation η, a general state $|p,s\rangle$ is transformed to

$$U(\eta)|p,s\rangle = e^{-i\eta\cdot\underline{K}} |p,s\rangle \qquad \ldots\ldots 1.17$$

$$= e^{-i\underline{\epsilon}'\cdot\underline{K}}\left[e^{i\underline{\epsilon}\cdot\underline{K}} e^{-i\eta\cdot\underline{K}} e^{-i\underline{\epsilon}\cdot\underline{K}}\right]|m,s\rangle \qquad \ldots 1.18$$

where $\underline{\epsilon}' \equiv \underline{\epsilon}(p')$, $p' = \Lambda^{-1} p$ $\qquad \ldots\ldots 1.19$

and Λ is the Lorentz transformation.

From equation (1.15) the effect of the three exponential factors in the square brackets in equation (1.18) is successively to take $m \rightarrow p$, $p \rightarrow p'$, $p' \rightarrow m$. Thus they take a rest frame state to a rest frame state. In other words, they are just the little group spin rotation. Therefore, if we define the Wigner rotation as

$$D(p) = e^{i\underline{\epsilon}'\cdot\underline{K}} e^{-i\eta\cdot\underline{K}} e^{-i\underline{\epsilon}\cdot\underline{K}} \qquad \ldots\ldots 1.20$$

Then

$$D(p)|m,s\rangle = |m,s'\rangle\langle m,s'|D(p)|m,s\rangle \qquad \ldots\ldots 1.21$$

and equation 1.18 may be written

$$U(\eta)|p,s\rangle = |p',s'\rangle\langle m,s'|D(p)|m,s\rangle \qquad \ldots\ldots 1.22$$

and under translations

$$U(\ell)|p,s\rangle = e^{iP_\mu \ell^\mu}|p,s\rangle = e^{ip_\mu \ell^\mu}|p,s\rangle \qquad \ldots\ldots 1.23$$

Let us now express these transformation properties in terms of ordinary Fock annihilation and creation operations on a non-degenerate vacuum state $|\rangle$,

$$a^+(p,s)|\rangle \equiv |p,s\rangle \qquad \ldots\ldots 1.24$$

where with covariant normalization

$$\left[a(p,s), a^+(p',s')\right]_\pm \; 2\pi\theta(p_0)\delta(p^2-m^2) = (2\pi)^4\delta^4(p-p')\delta ss' \qquad \ldots\ldots 1.24a$$

Then by (1.22) - (1.24a)

$$U(\eta)\; a(p,s)\; U^{-1}(\eta) = \langle m,s|D^{-1}(p)|m,s'\rangle a(p's') \qquad \ldots 1.25$$

and

$$U(\ell)\; a(p,s)\; U^{-1}(\ell) = e^{-ip_\mu \ell^\mu}\; a(p,s) \qquad \ldots\ldots 1.26$$

From these we can see that the transformation properties of these operators under Lorentz transformations are complicated because the Wigner rotation, D, depends not only on η, the parameter of pure Lorentz transformation, but also on the momentum of the particular state being transformed. We therefore want to construct operators with simple transformation properties. To do so, we must first develop explicit representations for the three factors appearing in D. This is done by introducing an auxiliary group which must contain the Lorentz boosts $\underline{K}$ and the spin operators J_{ij} as subgroups. The simplest group

to take is the homogeneous Lorentz group, and we may use any representation which includes the spin s in the decomposition. Again, it is simplest to use finite (but non-unitary) representations $|\alpha\rangle$.

We can write

$$\langle m,s|D(p)|m,s'\rangle = \langle m,s|\alpha\rangle\langle\alpha|e^{i\underline{\xi}^{0}\cdot\underline{K}}|\beta\rangle\langle\beta|e^{-i\underline{\eta}\cdot\underline{K}}|\delta\rangle$$

$$\times\ \langle\delta|e^{-i\underline{\xi}\cdot\underline{K}}|\gamma\rangle\langle\gamma|m,s'\rangle \qquad \ldots.1.27$$

and define the auxiliary operator

$$A\alpha(p) \equiv \langle\alpha|e^{-i\underline{\xi}\cdot\underline{K}}|\beta\rangle\langle\beta|m,s\rangle\, a(p,s)$$

$$\equiv U_{\alpha}(p)^{s}\, a(p,s) \qquad \ldots.1.28$$

Then by (1.25) and (1.27)

$$\begin{aligned} U(\eta)A_{\alpha}(p)U^{-1}(\eta) &= \langle\alpha|e^{-i\underline{\xi}\cdot\underline{K}}|\beta\rangle\langle\beta|m,s\rangle U(\eta)a(p,s)U^{-1}(\eta) \\ &= \langle\alpha|e^{i\underline{\eta}\cdot\underline{K}}|\beta\rangle A_{\beta}(p^{0}) \\ &= S_{\alpha}{}^{\beta}A_{\beta}(p^{0}) \qquad \ldots.1.29 \end{aligned}$$

where we have made use of the relation

$$\langle\beta|m,s\rangle\langle m,s|D|m,s'\rangle = \langle\beta|D|\rho\rangle\langle\rho|m,s'\rangle \qquad \ldots.1.30$$

which is true since D is an element of the rotation group, which is a subgroup of the auxiliary group and the representation $|\alpha\rangle$ includes the spin $|s\rangle$.

In equation (1.29) the spin label s in $a(p,s)$ is replaced by the index α in $A_{\alpha}(p)$, and this has the consequence that the spinor matrix $S_{\alpha}{}^{\beta}$ is a pure index transformation, parametrised only by η and decoupled from the momentum p. In $A_{\alpha}(p)$, p is replaced by $p^{0} = \Lambda^{-1}p$. Therefore $A_{\alpha}(p)$ transforms as a density representation of the homogeneous Lorentz group. The finite non-unitary states $|\alpha\rangle$ have no direct physical meaning.

The link between the physical states $|m,s\rangle$ and the auxiliary states $|\alpha\rangle$ is the spinor $\langle\alpha|m,s\rangle$ appearing in equation (1.28).

To construct $A\alpha(p)$ it is necessary that the representation $|\alpha\rangle$ contains the spin $|s\rangle$. If $|\alpha\rangle$ contains several spins, then $A\alpha(p)$ may be constructed to include particles of different spin. If α runs over more values than the label s, then there must be restrictions which reduce the number of independent auxiliary operators $A_\alpha(p)$ to the number of independent $a(p,s)$. It is in this respect that the equations of motion play a vital role. Let us consider for example the case of spin $\frac{1}{2}$. Here s runs over two values. We may choose for $|\alpha\rangle$ the representation $(\frac{1}{2},0) \equiv |a\rangle$ or $(0,\frac{1}{2}) \equiv |\dot{a}\rangle$, and given explicitly as

$$\begin{aligned} \langle a|\underline{K}|b\rangle &= -\,{}^{i}/_{2}\,\sigma_a^{\ b} \qquad (a,b = 1,2) \\ \langle \dot{a}|\underline{K}|\dot{b}\rangle &= \ {}^{i}/_{2}\,\sigma^{\dot{a}}_{\ \dot{b}} \qquad (\dot{a},\dot{b} = 1,2) \end{aligned} \qquad \ldots\ldots 1.31$$

where $\sigma_a^{\ b}$, $\sigma^{\dot{a}}_{\ \dot{\beta}}$ are the conventional Pauli matrices. In either choice, α runs over two values and there will be no constraints. But if we choose for $|\alpha\rangle$ the Dirac representation $(\frac{1}{2},0) + (0,\frac{1}{2})$ for which

$$\langle\alpha|K_i|\beta\rangle = \tfrac{1}{2}(\sigma_{oi})_\alpha^{\ \beta} \qquad \alpha,\beta = 1\ldots4 \qquad \ldots\ldots 1.32$$

where

$$\sigma_{\mu\nu} = {}^{i}/_{2}[\gamma_\mu, \gamma_\nu]$$

and $\qquad \ldots\ldots 1.33$

$$\{\gamma_\mu, \gamma_\nu\} = 2\, g_{\mu\nu},$$

then α runs over four values, and we may impose the conditions

$$(\gamma_o)_\alpha^{\ \beta}\langle\beta|m,s\rangle = +\,\langle\alpha|m,s\rangle \qquad \ldots\ldots 1.34$$

From (1.32)

$$\langle\alpha|e^{-i\underline{\xi}\cdot\underline{K}}|\beta\rangle = \left[\exp(-i\xi_i\sigma_{oi}/2)\right]_\alpha^{\ \beta} \qquad \ldots\ldots 1.35$$

Using (1.16b), (1.35) in (1.34), we obtain

$$(\gamma_o)\exp(i \xi_i \sigma_{oi}) = \frac{\gamma_\mu p_\mu}{m} \quad \ldots$$

from which we derive, using (1.28), (1.34) and (1.35),

$$(\gamma_\mu p_\mu - m)_\alpha^\beta \, A_\beta(p) = 0 \quad \ldots\ldots 1.36$$

where, we use the representation in which γ_o is hermitian, γ_i is antihermitian. If we now define Parity Operator R which satisfies the commutation relations,

$$\{R, P_i\} = [R, P_o] = \{R, J_{oi}\} = [R, J_{ij}] = 0 \quad \ldots\ldots 1.37$$

we can see that in the Dirac representation,

$$J_{\mu\nu} = \frac{i}{2}\sigma_{\mu\nu} \quad \ldots\ldots 1.38$$

and γ_o satisfies (1.37) and may serve as a representation of R. Therefore equation (1.34) may be interpreted as a specification of the particle parity. Thus equation (1.36) shows that for a particle of spin $\frac{1}{2}$ and definite parity, the $U_\alpha(p)^s$ defined in (1.28) is the conventional Dirac spinor. This expression for the spinor in terms of the auxiliary group matrix elements of the boost operator is the crucial link between the group theoretic and field theoretic approaches to relativistic particle theories. The role of the Dirac equation in reducing, in a convariant manner, the number of independent $A_\alpha(p)$ to the number of independent $a(p,s)$ is typical of all such 'equations of motion', other than the Klein-Gordan equation.

2. Construction of Poincare scalar densities with Auxiliary Operators.

The auxiliary operator dual to $A_\alpha(p)$ is

$$A^\alpha(p) \equiv a^+(p,s)\langle m,s|\beta\rangle\langle\beta|e^{+i\underline{\xi}\cdot\underline{K}}|\alpha\rangle \qquad \ldots.1.39$$

so that under Lorentz transformation

$$U(\eta)A^\alpha(p)U^{-1}(\eta) = A^\beta(p')\langle\beta|e^{-i\underline{\eta}\cdot\underline{K}}|\alpha\rangle \qquad \ldots.$$

$$= A^\beta(p')(S^{-1})_\beta{}^\alpha \qquad \ldots.1.40$$

This ensures that the quantity

$$A^\alpha(p)A_\alpha(p) \qquad \ldots.1.41$$

is a scalar density. Since the matrix element $\langle\alpha|K|\beta\rangle$ is antihermitian, $A^\alpha(p)$ is not the hermitian conjugate of $A_\alpha(p)$. The hermitian conjugate of $A_\alpha(p)$ is to be constructed for each particular case considered. For example, for the Dirac representation of the spin $\frac{1}{2}$ particles,

$$(A_\alpha(p))^+ = A^\beta(p)(\gamma_o)_\beta{}^\alpha \quad (+ \text{ means hermitian conjugate}) \qquad \ldots.1.41a$$

Under translations

$$A_\alpha(p) \to e^{-ip_\mu\ell^\mu} A_\alpha(p) \qquad \ldots.1.42$$

$$A^\alpha(p) \to e^{+ip_\mu\ell^\mu} A^\alpha(p) \qquad \ldots.1.43$$

Thus with these auxiliary operators one can construct Poincare scalar densities. To do so, we notice that under Lorentz transformation $p_\mu A_\alpha(p)$ transforms effectively as a 4-vector. The vector representation in the Lorentz group is the $(\frac{1}{2},\frac{1}{2})$ representation, so one may introduce the matrices

$$(\sigma_\mu)_{a\dot{b}} \equiv (1,\underline{\sigma})_{a\dot{b}} \qquad \ldots.1.44$$

$$(\tilde{\sigma}_\mu)^{\dot{a}b} \equiv (1,-\underline{\sigma})^{\dot{a}b} \qquad \ldots.1.45$$

As is suggested by the notation, σ_μ and $\tilde{\sigma}_\mu$ transform as Lorentz 4-vectors.

Therefore given any 4-vector p_μ we can construct Lorentz spinors by saturating the indices μ:

$$p_{a\dot{b}} = p^\mu (\sigma_\mu)_{a\dot{b}} \qquad \ldots\ldots 1.46$$

$$\tilde{p}^{\dot{a}b} = p^\mu (\sigma_\mu)^{\dot{a}b} \qquad \ldots\ldots 1.47$$

The spinor $p_{a\dot{b}}$, $\tilde{p}^{\dot{a}b}$ transforms as indicated by their indices, so that for example under Lorentz transformation $(p \to p')$

$$A^\alpha(p) p_{a\dot{b}} A^{\dot{b}}(q) \to A^c(p') S^{-1}{}_c{}^a \, p_{a\dot{b}} \, S^{\dot{b}}{}_{\dot{d}} \, A^{\dot{d}}(q')$$

$$= A^a(p') p'_{a\dot{b}} A^{\dot{b}}(q') \qquad \ldots\ldots 1.48$$

is a true scalar. So that to construct Poincare invariant densities, all one has to do is to saturate indices in the product of four momentum spinors, and multiply by a four momentum δ-function. A typical example is

$$A^{a\ldots}_{b\ldots}(p) \, A^{\dot{c}\ldots}_{\dot{d}\ldots}(q) \, p_{a\dot{c}\ldots} \, A^{bd\ldots}_{\ldots\ldots}(r) \, \delta^4(p + q - r) \qquad \ldots\ldots 1.49$$

where the form of the δ-function argument implies that A(p) and A(q) are creation operators, and A(r) annihilation operators, or vice versa.

3. Fields, Statistics, CTP and Crossing.

We have been able, in the previous section, to construct Poincare scalar densities of type (1.49) with the auxiliary operators, without introducing the notion of antiparticles, causality, statistics, crossing and CTP. Weinberg[11], Feldman and Matthews[6] have, by introducing a local field $\psi_\alpha(x)$, defined as some linear combination of auxiliary operators

which transforms under homogeneous Lorentz transformation as indicated by the space-time variable x and the suffix α, and satisfies causality in the form of the local commutation relations:

$$[\psi_\alpha(x),\psi^+_\beta(y)]_\mp = 0 \qquad (x-y)^2 < 0 \qquad \ldots\ldots 1.50$$

shown that

(i) $\psi_\alpha(x)$ must be a combination of annihilation and creation operators, and that the auxiliary operator associated with a creation operator can be made to have the same transformation properties as that associated with an annihilation operator.

(ii) If for particles of spin j we[11] use the representation (j,o) or (o,j) for the α, the usual condition between spin and statistics** is obtained:

$$[\psi_\alpha(x),\psi^+_\beta(y)]_- = 0, \qquad (x-y)^2 < 0 \qquad \ldots\ldots 1.51$$

only if j is an integer and $|\lambda| = |\mu|$, and that

$$[\psi_\alpha(x),\psi^+_\beta(y)]_+ = 0 \quad \text{for} \quad (x-y)^2 < 0 \qquad \ldots\ldots 1.52$$

only if j is one half an odd integer and $|\lambda| = |\mu|$ and these causality conditions lead to crossing symmetry and CTP.

(iii) If for particles[6] of spin $\frac{1}{2}$, we use the Dirac $(\frac{1}{2},0) + (0,\frac{1}{2})$ representation, then if we assume the parity doubling theory i.e. there are as many s as there are α, then our fields can satisfy causality, but will not satisfy any equation of motion except the Klein-Gordan type. However, if we put in a definite parity on our particle state $|m,s>$, and put in opposite parity on the antiparticle state $|m,\bar{s}>$, then for our field operator $\psi(x)$ to satisfy causality it will in addition satisfy, in configuration space, the equation

of motion

$$(i\gamma_\mu \partial_\mu - m)\psi(x) = 0 \qquad \ldots.1.53$$

which is the Dirac equation of motion for 4 component spinor $\psi(x)$.

4. Extension of Auxiliary Group to U(2,2).

We prescribed earlier that any $A_\alpha(p)$ which contains the pure Lorentz transformation K_i as infinitesimal generators, and which includes the required spin s in its decomposition can serve as an auxiliary group. Let us now exploit the freedom in this prescription, and consider the extension of the auxiliary group to U(2,2), which is the group of transformations whose infinitesimal generators are F_R (R = 1,2....16). In the basic (Dirac) 4 x 4 representation $|\alpha\rangle$,

$$\langle\alpha|F_R|\beta\rangle = \tfrac{1}{2}(\gamma_R)_\alpha{}^\beta \qquad \ldots.1.54$$

where

$$\gamma_R = 1, \gamma_5, \gamma_\mu, i\gamma_\mu\gamma_5; \quad \sigma_{\mu\nu} = \tfrac{i}{2}[\gamma_\mu, \gamma_\nu] \qquad \ldots.1.55a$$

$$\gamma_5 = \gamma_o\gamma_1\gamma_2\gamma_3 \qquad \ldots.1.55b$$

$$\gamma_o(\gamma_R)^+\gamma_o = \gamma_R \qquad \ldots.1.55c$$

We define an auxiliary operator

$$A_\alpha(p) = \langle\alpha|e^{-i\underline{\xi}\cdot\underline{K}}|\beta\rangle\langle\beta|m,s\rangle a(p,s) \qquad \ldots.1.56$$

This has exactly the same form as (1.28), but now $|\alpha\rangle$ is to be interpreted as a representation of U(2,2) and will transform like some reducible representation of the Lorentz Group. Thus under pure Lorentz transformation

$$U(\eta)A_\alpha(p)U^{-1}(\eta) = \langle\alpha|e^{i\underline{\eta}\cdot\underline{K}}|\beta\rangle A_\beta(p')$$

$$= S_\alpha{}^\beta A_\beta(p') \qquad \ldots.1.57$$

which is exactly the same as equation (1.29). However we now have the possibility of performing U(2,2) index transformation on the auxiliary operators

$$A_\alpha(p) \rightarrow \langle\alpha|e^{i\eta_R F_R}|\beta\rangle A_\beta(p)$$

$$\equiv T_\alpha^{\ \beta} A_\beta(p) \qquad \ldots\ldots 1.58$$

The auxiliary operator dual to $A_\alpha(p)$ is $A^\alpha(p)$ where

$$A^\alpha(p) = a^+(p,s)\langle m,s|\beta\rangle\langle\beta|e^{i\underline{\xi}\cdot\underline{K}}|\alpha\rangle \qquad \ldots\ldots 1.59$$

and under U(2,2) index transformation

$$A^\alpha(p) \rightarrow A^\beta(p)(T^{-1})_\beta^{\ \alpha} \qquad \ldots\ldots 1.60$$

It is worth pointing out at this stage that under the U(2,2) index transformations of type (1.58) and (1.60) there is no effective transformation on p. Thus if we were to insist on invariance under these index transformations we would be led to a restricted class of Poincare scalar densities. This distinction is important, especially in chapter II where we show how restriction on U(6,6) index invariants leads to a restricted class of Poincare invariant S-matrix elements.

The condition (1.55c) ensures that the relationship between hermitian conjugate operators and dual for U(2,2) is the same as for the homogeneous Lorentz group. Thus for the basic 4 x 4 representation

$$(A_\alpha(p))^+ = A^\beta(p)\gamma_{o\beta}^{\ \ \alpha}$$

and this ensures that $(A^*(p)\gamma_o)^\alpha$ transforms like $A^\alpha(p)$.

We can now build auxiliary operators of the form $A^{\gamma\ldots}_{\alpha\beta\ldots}(p)$ out of many index tensors, and these will represent particles of higher spins. We may then define corresponding fields $\psi^{\gamma\ldots}_{\alpha\beta\ldots}(p)$ which transforms under U(2,2) index transformation as

$$\psi^{\gamma\cdots}_{\alpha\beta\cdots}(p) \rightarrow T_{\alpha}^{\alpha'} T_{\beta}^{\beta'} \cdots \psi_{\alpha'\beta'\cdots}^{\gamma'\cdots}(p)(T^{-1})^{\gamma}_{\gamma'} \cdots \qquad \ldots\ldots 1.61$$

and

$$(\psi^{\gamma\cdots}_{\alpha\beta\cdots}(p))^{+}$$

transforms like

$$(\gamma_o)^{\gamma'}_{\gamma} \cdots \psi(p)^{\alpha'\beta'\cdots}_{\gamma'\cdots} (\gamma_o)^{\alpha}_{\alpha'} (\gamma_o)^{\beta}_{\beta'} \cdots$$

Feldman and Matthews[6] have carried out the construction of fields $\varphi_{\alpha}{}^{\beta}$ composed of auxiliary operators of the form $A_{\alpha}{}^{\beta}(p)$ $(\alpha,\beta = 1\ldots4)$ which transforms as an irreducible representation of U(2,2) and which describes particles of both spin 0 and 1. They have shown that if one assumes parity doubling and that the creation and annihilation operators are bose operators, then the fields $\varphi_{\alpha}{}^{\beta}$ will satisfy causality, and there will be no constraint on the indices (i.e. no equations of motion except the K.G. type). Whereas if one assigns <u>negative</u> parity to both the particle and the antiparticles states then $\varphi_{\alpha}{}^{\beta}$ will satisfy causality in addition to the constraint, the equation of motion

$$(i\gamma\partial)_{\alpha}{}^{\pi}\, \varphi_{\pi}{}^{\rho}(x)(i\gamma\partial)_{\rho}{}^{\beta} = -m^2\, \varphi_{\alpha}{}^{\beta}(x) \qquad \ldots\ldots 1.62$$

The above parity assumption implies that $\varphi_{\alpha}{}^{\beta}(x)$ is a self conjugate field consisting of two singlets and two triplets, all of negative parity.

However, if one imposes further restrictions by specifying separately the parities of the quark and antiquark components, so that for instance the quark state $|m,s\rangle$ has a positive quark parity and the state $|m,\bar{s}\rangle$ has negative quark parity, then $\varphi_{\alpha}{}^{\beta}(x)$ will satisfy causality together with the equations

$$(i\gamma\partial - m)_{\alpha}{}^{\rho}\, \varphi_{\rho}{}^{\beta}(x) = 0 \qquad \ldots\ldots 1.63$$

and

$$\varphi_\alpha{}^\lambda(x)\ (i\gamma\partial + m)_\lambda{}^\beta \quad = \quad 0 \qquad \ldots\ldots 1.64$$

which are Bargman-Wigner (B.W.) equations first introduced into U(6,6) theory by Salam et al.[3] The fields in (1.63) and (1.64) cannot be made self conjugate because the quark parities of $|m,\bar{s}\rangle$ are opposite to those of $|m,s\rangle$. This is a direct[6] consequence of causality. This has physical significance only if the interactions are such that the quark parities are observable.

The problem of constructing fields $\psi_{\alpha\beta\gamma}(p)$ from auxiliary operators of type $A_{\alpha\beta\gamma}(p)$ runs along similar lines.[12] By assigning positive and negative quark parities to the states $|m,s\rangle$ and $|m,\bar{s}\rangle$ respectively, one obtains the B.W. equation

$$(i\gamma\partial - m)_\alpha{}^{\alpha'} \psi_{\alpha'\beta\gamma}(x) \quad = 0 \qquad \ldots\ldots 1.65$$

in configuration space.

5. Extension to U(6,6).

We have extended the auxiliary group to include U(2,2). We shall now discuss briefly the inclusion of the well-known internal symmetry group SU(3). The rest states are now defined as

$$|m,s,N,I;s_3 Y I_3\rangle \qquad \ldots\ldots 1.66$$

where N is an SU(3) irreducible representation, I is the total isospin, Y and I_3 the hypercharge and third component of isospin respectively. If, in addition to U(2,2) invariance, we assume that the single particle

state (1.66) transforms as a representation of SU(3), then we can take our auxiliary group to be U(2,2)⊗SU(3), and the corresponding (spinor) field in the basic representation is

$$\psi_{\alpha,r} \qquad (\alpha = 1\ldots4),\ (r = 1\ldots3) \qquad \ldots.(1.67)$$

where α specifies a representation of U(2,2) and r, of SU(3). The auxiliary group transformations are index transformations carried out independently on these two indices and U(2,2) and SU(3) invariants are constructed by taking products of these fields and saturating independently, the two types of indices. The little group of this auxiliary group is[++] U(2)⊗U(2)⊗SU(3).

Now we can extend the auxiliary group to U(6,6) by requiring the transformation, in the basic representation, on the corresponding twelve component field (spinor)

$$\psi_A = \psi_{\alpha,r} \qquad (A = 1\ldots.12) \qquad \ldots.1.68$$

to include transformations which couple U(2,2) indices, α to SU(3) indices, r. Like in the previous sections our fields here satisfy causality, and if we restrict them to satisfy the requirement that our particle and anti-particle states have opposite quark parity, they will in addition satisfy the B.W. equations. This will then correspond to the theory of Salam, Delbourgo and Strathdee.[3]

Our ability to extend to higher groups, the auxiliary operators out of which we construct local fields, stems from the fact that we restrict ourselves to <u>pure index</u> transformations on these fields. In the latter part of the next chapter, we shall discuss the difficulties one encounters in trying to construct Poincare invariant S-matrix elements out of these local fields.

CHAPTER II.

U(6,6) ALGEBRA AND THE NEED FOR SYMMETRY BREAKING

1. Algebra of U(2,2) and its subgroups.

We shall first review the algebra of U(2,2) and its subgroups. The extension to U(6,6) becomes trivial. The non-compact U(2,2) algebra is defined in terms of sixteen 4 x 4 Dirac matrices γ_R (R = 1....16) defined in equation (1.55):

$$\gamma_R = 1, \gamma_\mu, i\gamma_\mu\gamma_5, \quad \sigma_{\mu\nu} = \tfrac{1}{2}[\gamma_\mu, \gamma_\nu], \quad \gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3 \qquad \ldots.2.0$$

$$\{\gamma_\mu, \gamma_\nu\} = 2\, g_{\mu\nu}$$

Here we adopt a representation in which γ_0 is hermitian and γ_i (i = 1,2,3) is antihermitian, so that the set of γ-matrices obeys the defining property

$$\gamma_0\gamma_R\gamma_0 = \gamma_R^{+} \qquad \ldots.2.1$$

of U(2,2) algebra. Of the total sixteen matrices, eight of them

$$1, \vec{\sigma}, \gamma_0, \gamma_0\vec{\sigma} = i\gamma_i\gamma_5 \qquad \ldots.2.2$$

are hermitian, and eight

$$\gamma_5, \gamma_5\vec{\sigma} = i\vec{\sigma}_o, i\gamma_0\gamma_5, i\gamma_0\gamma_5\vec{\sigma} = \gamma_i \qquad \ldots.2.3$$

anti-hermitian, where $\vec{\sigma} = (\sigma_{23}, \sigma_{31}, \sigma_{12})$, $\vec{\sigma}_o = (\sigma_{o1}, \sigma_{o2}, \sigma_{o3})$. In Pauli representation,

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \quad i\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \vec{\sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \qquad \ldots.2.4$$

The hermitian and anti-hermitian matrices appear as block diagonal and block off-diagonal respectively. This provides a mnemonic for

remembering them.

The γ_R so defined obey the multiplication rules

$$\begin{aligned}
\gamma_\mu\gamma_\nu &= -i\sigma_{\mu\nu} + g_{\mu\nu} \\
\gamma_\lambda\gamma_{\mu\nu} &= i(g_{\lambda\nu}\gamma_\mu - g_{\lambda\mu}\gamma_\nu) + \epsilon_{K\lambda\mu\nu}\, i\gamma_K\gamma_5 \\
\gamma_{\mu i}\gamma_\nu\gamma_5 &= ig_{\mu\nu}\gamma_5 + \tfrac{1}{2}\epsilon_{\mu\nu K\lambda}\,\sigma_{K\lambda} \\
\sigma_{K\lambda}\sigma_{\mu\nu} &= i(g_{K\nu}\sigma_{\lambda\mu} + g_{\lambda\mu}\sigma_{K\nu} - g_{K\mu}\sigma_{\lambda\nu} - g_{\lambda\nu}\sigma_{K\mu}) \\
&\quad + (g_{K\mu}g_{\lambda\nu} - g_{\lambda\mu}g_{K\nu} - \epsilon_{K\lambda\mu\nu}\gamma_5) \qquad \ldots.2.5 \\
\sigma_{K\lambda}\, i\gamma_\mu\gamma_5 &= i(g_{\lambda\mu}\, i\gamma_K\gamma_5 - g_{K\mu}\, i\gamma_\lambda\gamma_5) - \epsilon_{K\lambda\mu\nu}\gamma_\nu \\
\sigma_{K\lambda}\gamma_5 &= \tfrac{1}{2}\epsilon_{K\lambda\mu\nu}\sigma_{\mu\nu} \\
i\gamma_\mu\gamma_5 i\gamma_\nu\gamma_5 &= i\sigma_{\mu\nu} - g_{\mu\nu}
\end{aligned}$$

Use will be made of these rules in the computation without explicit reference to them.

The U(2,2) algebra is realized in general by a set of matrices J_R which satisfy commutation rules analogous to those of the γ_R. These commutations rules are much simplied by defining alternative bases,

$$J_\alpha{}^\beta = \tfrac{1}{2}(\gamma_R)_\alpha{}^\beta\, J_R \qquad \alpha, \beta = 1,2,3,4 \qquad \ldots.2.6$$

It takesthe form,

$$\left[J_\alpha{}^\beta, J_\gamma{}^\delta\right] = \delta_\alpha{}^\delta\, J_\gamma{}^\beta - \delta_\gamma{}^\beta\, J_\alpha{}^\delta \qquad \ldots.2.7$$

In this new basis, the hermiticity condition (2.1) becomes

$$(J_\alpha{}^\beta)^+ = (\gamma_o)_\beta{}^\gamma\, J_\gamma{}^\delta (\gamma_o)_\delta{}^\alpha \qquad \ldots.2.8$$

The relations(2.6) and (2.7) are now easily generalized to U(P,P) with

$$\gamma_o = {}^{PxP}\left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & -1 \end{array}\right)_{PxP}$$

Now, there exists local isomorphism between the U(2,2) group and the O(4,2) group - the real orthogonal group in six dimensions with metric

$$g_{MN} = (+ \ldots +) \; ; \qquad MN = 0,1,2,3,5,6$$

We will write the γ-matrices in the form γ_{MN} where

$$\gamma_{\mu\nu} = \sigma_{\mu\nu}, \quad \gamma_{\mu 5} = i\gamma_\mu\gamma_5, \quad \gamma_{\mu 6} = \gamma_\mu, \quad \gamma_{56} = \gamma_5 \qquad \ldots.2.9$$

Then equations (2.5) and (2.7) will take the simple form

$$\gamma_{KL}\gamma_{MN} = i(g_{KN}\gamma_{LM} + g_{LM}\gamma_{KN} - g_{KM}\gamma_{LN} - g_{LN}\gamma_{KM}) + (g_{KM\,LN} - g_{KN}g_{LM} + \tfrac{1}{2}\epsilon_{KLMNIJ}\gamma_{IJ}) \qquad \ldots.2.10$$

and

$$[J_{KL}, J_{MN}] = i(g_{KN}J_{KM} + g_{LM}J_{KN} - g_{KM}g_{LN} - g_{LN}J_{KM}) \qquad \ldots.2.11$$

with

$$J_{MN} = \tfrac{1}{2}(\gamma_{MN})_\alpha^{\ \beta}\, J_\beta^{\ \alpha} \qquad \ldots.2.12$$

Using this local isomorphism one can systematically break down U(2,2) into all its important subgroups. A comprehensive account of this is contained in reference (13). We shall here discuss only those, the direct application of which we need in this work.

The maximal compact subgroup O(4) locally isomorphic to U(2)⊗U(2) is composed of the generators,

$$1, \vec{\sigma}, \gamma_0, \gamma_0\vec{\sigma} = \tfrac{1}{2}(1 \pm \gamma_0), \tfrac{1}{2}(1 \pm \gamma_0)\vec{\sigma} \qquad \ldots.2.13$$

which commute with γ_0. It is therefore the little group of U(2,2). If we select from the set (2.13) those generators which commute with σ_{12}, we have the set

$$1, \sigma_{12}, \gamma_0\sigma_{23}, \gamma_0\sigma_{31} \qquad \ldots.2.14$$

which generate the W-spin group of Lipkin,[14] which is relativistic for all processes whose kinematics restrict the free momenta to the z-direction. If we further restrict the set (2.14) to

$$1, \vec{\sigma} \qquad \ldots.2.15$$

we have the ordinary spin group (SU(2)).

The most important non-compact subgroup of U(2,2) for our purpose is the (already introduced) homogeneous Lorentz group $\mathcal{L}_4$ locally isomorphic to O(3,1) locally isomorphic to SL(2,C), with generators $\sigma_{\mu\nu}$. Within the algebra of $\mathcal{L}_4$, as indeed within the algebra of SO(3,2) composed of the generators $(\sigma_{\mu\nu}, \gamma_\mu)$, there exists an antisymmetric tensor $C_{\alpha\beta}$ which has the property that

$$\begin{aligned} (C^{-1})^{\alpha\beta}\ C_{\beta\gamma} &= \delta_\gamma{}^\alpha \\ (C^{-1})^{\alpha\beta}\ (\sigma_{\mu\nu})_\beta{}^\gamma\ C_{\gamma\delta} &= -\ (\sigma_{\mu\nu})_\delta{}^\alpha \\ (C^{-1})^{\alpha\beta}\ (\gamma_\mu)_\beta{}^\gamma\ C_{\gamma\delta} &= -\ (\gamma_\mu)_\delta{}^\alpha \end{aligned} \qquad \ldots.2.16$$

In fact one can show that[3] the 16 Dirac matrices of U(2,2) fall into two distinct classes. The matrices

$$(\gamma_\mu\ C)_\alpha{}^\beta,\ (\sigma_{\mu\nu})_{\alpha\beta}$$

are symmetric, and

$$C_{\alpha\beta},\ (\gamma_5 C)_{\alpha\beta},\ (i\gamma_\mu\gamma_5 C)_{\alpha\beta}$$

are antisymmetric. The tensor $C_{\alpha\beta}$ is identified with charge conjugation operator. The transformation properties of the fields under C will be defined later.

2. Extension to U(6,6).

We shall now pass smoothly on to the algebra of U(6,6), which we define as the algebra of the 144 matrices

$$F_R{}^i = \gamma_R\ T^i \qquad (R = 1 \ldots 16),\ (i = 0 \ldots 8) \qquad \ldots.2.17$$

where γ_R are as defined in (2.0) and T^i are the unitary spin matrices which we conventionally define as

$$T^i = \tfrac{1}{2}\lambda_i, \quad T^o = 1/\sqrt{6}\,1 \qquad (i = 1 \ldots 8) \qquad \ldots.2.18$$

where λ_i are the Gell-Mann[15] matrices. The T^i satisfies the ~~commutation~~ relations

$$T^iT^j = \tfrac{1}{2}(d^{ijk} + if^{ijk})T^k \,; \quad i,j,k = 0 \ldots 8 \qquad \ldots.2.19$$

where d^{ijk}, f^{ijk} $(i,j,k = 1 \ldots 8)$ are completely symmetric and completely antisymmetric tensors respectively. They are all given by Gell-Mann, and

$$f^{ojk} = o\,, \quad d^{ojk} = \delta^{jk}\sqrt{\frac{2}{3}} \qquad \ldots.2.20$$

$$\therefore\ T_r(T^iT^j) = \tfrac{1}{2}\delta^{ij} \qquad \ldots.2.21$$

In the same way as in U(2,2), the generators of U(6,6) algebra denoted by J^A_B, $(A,B = 1\ldots.12)$ will satisfy the commutation relation

$$\left[J^A_B, J^C_D\right] = \delta^C_B J^A_D - \delta^A_D J^C_B \qquad \ldots.2.22$$

and the hermiticity condition

$$(J^A_B)^+ = (\gamma_o J \gamma_o)^B_A \qquad \ldots.2.23$$

analogously to (2.7) and (2.8).

If we now define

$$J_R{}^i = (\gamma_R)_\alpha{}^\beta\ (T^i)^s_r\ J^{\alpha r}_{\beta s} \qquad \ldots.2.24$$

where $A = \alpha, r$, $B = \beta, s$; $\alpha,\beta = 1\ldots.4$, $r,s = 1,2,3$,

the relation (2.22) will give:

$$\begin{aligned}
&\left[J^i, J^j\right] = -\left[J_5{}^i, J_5{}^j\right] = if^{ijk}J^k \\
&\left[J^i, J_5{}^j\right] = if^{ijk}J_5{}^k \\
&\left[J^i, J^j_{\mu\nu}\right] = if^{ijk}J^k_{\mu\nu} \\
&\left[J_5{}^i, J^j_{\mu\nu}\right] = \tfrac{1}{2}\,if^{ijk}\epsilon_{\mu\nu\rho\lambda}\,J^k_{\rho\lambda} \qquad \ldots.2.25 \\
&\left[J^i_{\rho\lambda}, J^j_{\mu\nu}\right] = id^{ijk}(g_{\rho\nu}\,J^k_{\lambda\mu} + g_{\lambda\mu}\,J^k_{\rho\nu} - g_{\rho\mu}\,J^k_{\lambda\nu} - g_{\lambda\nu}\,J^k_{\rho\mu}) \\
&\qquad + if^{ijk}\left\{(g_{\rho\mu}\,g_{\lambda\nu} - g_{\lambda\mu}\,g_{\rho\nu})J^k - \epsilon_{\rho\lambda\mu\nu}\,J_5{}^k\right\}
\end{aligned}$$

$$
\begin{aligned}
[J_\mu^i, J_\nu^j] &= if^{ijk}\, g_{\mu\nu}\, J^k - id^{ijk}\, J_{\mu\nu}^k \\
[J_\mu^i, J_{\nu 5}^j] &= id^{ijk}\, g_{\mu\nu}\, J_5^k + \tfrac{1}{2} if^{ijk}\, \epsilon_{\mu\nu\rho\lambda} J_{\rho\lambda}^k \\
[J^i, J_\mu^j] &= if^{ijk}\, g_\mu\, J_\mu^k \\
[J^i, J_{\mu 5}^j] &= if^{ijk}\, J_{\mu 5}^k \\
[J_5^i, J_\mu^j] &= id^{ijk}\, J_{\mu 5}^k \\
[J_5^i, J_{\mu 5}^j] &= id^{ijk} J_\mu^k \\
[J_\lambda^i, J_{\mu\nu}^j] &= id^{ijk}(g_{\lambda\mu}\, J_\nu^k - g_{\lambda\nu}\, J_\mu^k) - if^{ijk}\, \epsilon_{\lambda\mu\nu\rho}\, J_{\rho 5}^k \\
[J_{\lambda 5}^i, J_{\mu\nu}^j] &= id^{ijk}(g_{\lambda\mu}\, J_{\nu 5}^k - g_{\lambda\nu}\, J_{\mu 5}^k) - if^{ijk}\, \epsilon_{\lambda\mu\nu\rho}\, J^k
\end{aligned}
\qquad \ldots.2.25 \text{ (continued)}
$$

and if we use (J^i, J_{MN}^i) as introduced in (2.9), in place of J_R^i, equation (2.25) collapses into

$$
\begin{aligned}
[J^i, J^j] &= if^{ijk}\, J^k \\
[J^i, J_{MN}^j] &= if^{ijk}\, J_{MN}^k \qquad \ldots.2.26 \\
[J_{KL}^i, J_{MN}^j] &= if^{ijk}\left[(g_{KM}\, g_{LN} - g_{KN}\, g_{ML}) J^k + \tfrac{1}{2}\epsilon_{KLMNIJ}\, J_{IJ}^k\right] \\
&\quad + id^{ijk}\left[g_{KN}\, J_{LM}^k + g_{LM}\, J_{KN}^k - g_{KM}\, J_{LN}^k - g_{LN}\, J_{KM}^k\right]
\end{aligned}
$$

which summarizes the algebra of U(6,6).

3. Multispinor Representation of U(6,6).

In the fundamental (quark) representation of U(6,6), under an infinitesimal transformation, the 12-component spinor ψ_A (A = 1....12) transforms similarly to equation 1.58,

$$\psi_A(p) \rightarrow S_A{}^B \psi_B(p) \qquad ....2.27$$

where now

$$S = e^{i\epsilon^i_R J_R{}^i} = e^{i\epsilon_B{}^A J_A{}^B} \qquad (\epsilon, \text{ real}) \qquad ....2.28$$

and the adjoint (anti-quark) spinor which is defined by

$$\bar{\psi}^A = \bar{\psi}^{\alpha r} = (\psi_{\beta r})^+ (\gamma_o)^\alpha_\beta \qquad ....2.29$$

transforms

$$\bar{\psi}^A \rightarrow (S^{-1})^A_B \bar{\psi}^B \qquad ....2.30$$

so that the quantity $\bar{\psi}^A \psi_A$ is an invariant. As indicated in (1.61) multispinors representing particles of higher quantum numbers can be built up from these basic spinors, and they transform according to the general rule,

$$\psi^{CD....}_{AB....} \rightarrow S_A{}^{A'} S_B{}^{B'} ....(S^-)^C_{C'} (S^{-1})^D_{D'} .... \psi^{C'D'....}_{A'B'....} \qquad ....2.31$$

The representations of interest in this work are the regular representation $\varphi_A{}^B(143)$, the completely symmetric representation, $\psi_{\{ABC\}}{}^{(364)}$, the mixed symmetry representation, $\psi_{[AB]C}{}^{(572)}$, and the totally antisymmetric representation $\psi_{[ABC]}{}^{(220)}$. The full decomposition of these into their spin and unitary spin contents, after the application of B.W. equations are:

$$\varphi_A{}^B = \varphi_{\alpha r}^{\beta s} = \frac{1}{2\mu}\left[(\not{p} + \mu)(\gamma_\mu\, \varphi_{\mu r}^{\ s} + \gamma_5\, \varphi_{5r}^{s})\right]_\alpha^\beta \qquad \ldots.2.32$$

$$\Psi_{\{ABC\}} = \Psi_{\{\alpha\beta\gamma\}\{pqr\}} = \frac{1}{2m}\sqrt{\frac{3}{2}}\left[(\not{p} + m)\gamma_\mu\, C\right]_{\alpha\beta} D_{\mu\gamma\{pqr\}}$$

$$+ \frac{1}{2m}\sqrt{\frac{1}{6}}\left\{\left[(\not{p} + m)\gamma_5\, C\right]_{\alpha\beta} \epsilon_{pqs}\, N_{\gamma r}^{s} + \text{cy.}(A,B,C)\right\} \qquad \ldots.2.33$$

and

$$(\gamma_\mu)_\alpha^\beta\, D_{\mu\beta\{pqr\}} = 0$$

$$\Psi_{[AB]C} = \frac{1}{4m}\left[(\not{p} + m)\gamma_\mu\, C\right]_{\alpha\beta} \epsilon_{pqs} \mathcal{N}_{\mu\gamma,r}^{\ s}$$

$$+ \frac{1}{2m}\sqrt{\frac{1}{6}}\left\{\epsilon_{qrs}\left[(\not{p} + m)\gamma_5\, C\right]_{\beta\gamma} \mathcal{N}_{\alpha p}^{s} - \epsilon_{rps}\left[(\not{p} + m)\ {}_5C\right]_{\gamma\alpha} \mathcal{N}_{\beta,q}^{\ s}\right\}$$

$$+ \frac{1}{2m}\sqrt{\frac{1}{6}}\ \epsilon_{pqr}\left\{\left[(\not{p} + m)\gamma_5 C\right]_{\alpha\gamma} \mathcal{J}_\beta + \left[(\not{p} + m)\gamma_5 C\right]_{\beta\gamma} \mathcal{J}_\alpha\right\}$$

$$+ \frac{1}{2m}\sqrt{\frac{1}{2}}\left[(\not{p} + m)\gamma_5\, C\right]_{\alpha\beta} \mathcal{D}_\gamma\, \{pqr\} \qquad \ldots.2.34$$

and

$$(\gamma_\mu)_\alpha^\beta\, \mathcal{N}_{\mu\beta,r}^{\ s} = 0$$

$$\Psi_{[ABC]} = \frac{1}{2m}\sqrt{\frac{1}{12}}\, \epsilon_{pqr}\left[(\not{p} + m)\gamma_\mu\, C\right]_{\alpha\beta}\, \Delta_{\mu\gamma}$$

$$+ \frac{1}{6m}\sqrt{\frac{1}{6}}\left\{\left[(\not{p} + m)\gamma_5\, C\right]_{\alpha\beta}(\epsilon_{rps}\, \nu_{\gamma q}^{\ s} - \epsilon_{prs}\, \nu_{\gamma p}^{s}) + \text{cy.}A,B,C\right\} \qquad \ldots.2.35$$

with

$$(\gamma_\mu)_\alpha^\beta\, \Delta_{\mu\beta} = 0$$

Here, $\{\ \ \}$ means symmetry, $[\ \ \ ]$, antisymmetric.

These decompositions have been done according to the reduction

$$U(6,6) \longrightarrow U(2,2) \otimes SU(3) \qquad \ldots.2.36$$

and the spin and unitary spin of the particles are as indicated by the indices α,β,γ and p,q,r respectively.

We can generalize[12] the charge conjugation operator introduced in (2.16) such that under C,

$$\begin{aligned} \psi_A(p) &\longrightarrow (\psi_A(p))^C = C_{AB}\bar{\psi}^B(-p) \\ \bar{\psi}^A(p) &\longrightarrow (\bar{\psi}^A(p))^C = (C^{-1})^{AB}\psi_B(-p) \end{aligned} \qquad \ldots.2.37$$

and for the multispinors, we have, for example,

$$\begin{aligned} \psi_{ABC}(p) &\longrightarrow C_{AD}\, C_{BE}\, C_{CF}\bar{\psi}^{DEF}(-p) \\ \varphi_B^A(q) &\longrightarrow (C^{-1})^{AD}\, \varphi_D^E(+q)\, C_{EB} \end{aligned} \qquad \ldots.2.38$$

The generalized C will now have the properties,

$$\begin{aligned} C_{AB}(C^{-1})^{BD} &= \delta_A^D \\ (C^{-1})^{AB}\,(\gamma_\mu\, T^i)_B^{\ C}\,(C)_{CD} &= -\,(\gamma_\mu (T^i)^T)_D^A \end{aligned} \qquad \ldots.2.39$$

The relations (2.38) and (2.39) will be extensively used in our calculations.

4. The need for Symmetry breaking.

We showed in Chapter I how one can construct Poincare scalar densities by use of auxiliary groups which we extended to include $U(6,6)$. Let us for the moment suppress the internal symmetry indices and consider only the auxiliary groups which belong to $U(2,2)$ symmetry. In this case we are now dealing with physical particle local fields such as

$$\psi_{\{\alpha\beta\gamma\}},\ \psi_{[\alpha\beta]\gamma},\ \psi_{[\alpha\beta\gamma]},\ \varphi_\alpha^\beta$$

built up from the fundamental fields $\Psi_\alpha(p)$, and its dual $\Psi^\alpha(p)$. Now the most natural requirement for theories constructed from such fields is that the effective interaction Lagrangian, or S-matrix elements, are limited to be invariant under the index transformations of U(2,2) of type, equations (1.58) and (1.60). This restriction to index U(2,2) invariance leads to a subset of Poincare invariants. Thus a term like

$$\Psi^\alpha(q)\ (\gamma_\mu)^\beta_\alpha \Psi_\beta(t)\ p_\mu \qquad \ldots.2.40$$

which is Poincare invariant is excluded by U(2,2) invariance. Even terms of the form

$$\Psi^\alpha(\gamma_\mu)^\beta_\alpha \Psi_\beta\ \chi^\epsilon\ (\gamma_\mu)^\delta_\epsilon\ \chi_\delta \qquad \ldots.2.41$$

which are index Lorentz invariant are not index U(2,2) invariant because the γ_μ's are only a subset of F_R's which would have, if inserted into (2.41), made it full U(2,2) invariant.

Thus we see that any form which was index invariant under auxiliary group transformation was automatically Poincare invariant, but certainly not vice-versa. Now it has been pointed out[16] that even if the interaction Lagrangian is restricted by requiring index invariance under auxiliary group U(2,2), it is <u>not possible</u> to construct a theory (that is a specification of an interaction Lagrangian[6] and free particle propagators from which S-matrix elements can be calculated to any order in the coupling strength) which leads in general to S-matrix elements which show this U(2,2) index invariance. This is because any such theory based on local fields leads to S-matrix elements which contain terms of the form $p_\mu\gamma_\mu$, $\gamma_\mu\gamma_\mu$ which arise from propagators and summation over spins. These terms certainly destroy the invariance of the S-matrix element under index

transformation of the auxiliary group. Thus we are left with S-matrix which is invariant only under Poincare group, P. Thus, with U(6,6) as auxiliary group, the surviving invariance of our theory is P⊗SU(3).

On the other hand if we force in a <u>presciption</u> that our S-matrix be U(6,6) invariant, then we come into direct conflict with the principles[5] of unitarity. This can easily be seen as follows: the unitary condition is

$$\mathrm{Im}\,\langle f|T|i\rangle = (2\pi)^4 \sum_n \langle f|T^+|n\rangle\langle n|T|i\rangle\,\delta^{(4)}(Pn - Pi) \qquad \ldots.(2.42)$$

where, the summation is over positive energy intermediate states n. Now, the projection operator

$$\sum_n |n\rangle\langle n| = \not{p} + m \qquad (\not{p} = \gamma_\mu p_\mu) \qquad \ldots.2.43$$

introduces terms $\gamma_\mu p_\mu$, which are not U(6,6) index invariant. Thus any S-matrix built on such a prescription, cannot in general be unitary. It is perhaps worth remarking in passing at this stage that even SU(3) is compatible with unitarity only if we neglect the physical mass differences produced by symmetry-breaking effects in the S-matrix elements. It is therefore not surprising that SU(3) predictions for scattering cross-sections are not, in general, good compared with experiment. This point will be stretched further in chapter III.

If one decidedly ignores the difficulties with unitarity mentioned above, and makes numerical calculations with a U(6,6) index invariant amplitude, one does obtain results which are, to be fair, in remarkable agreement with experiments for <u>two</u> and <u>three</u> point functions. Notably, its prediction of the exact magnetic moment of the proton and the proton-

neutron magnetic moment[3] ratios. However when applied to four point functions for reactions it gives some results which are in violent disagreement with experiment. A few of these include:

(a) Zero polarization[5] for various meson baryon scattering processes, in particular

$$K^- + p \longrightarrow \Xi^- + K^+ \qquad \ldots.2.44$$

Experimentally[17] at K^- lab. momentum of 1.95 Bev/c the average Ξ^- polarization is 0.77 ± 0.24.

(b) Equality of the differential[5,7] cross-section for

$$K^- + p \longrightarrow \pi^+ + \Sigma^- \quad \text{and} \quad K^- + p \longrightarrow K^\circ + \Xi^\circ \qquad \ldots.2.45$$

At low energies the left hand side is about one hundred times greater than the right hand side, but at higher energies both tend to zero very rapidly. This result is also true for SU(3) alone.

(c) The ratios of cross-sections of various two meson proton-antiproton annihilation processes are also bad compared with[18] experiment, e.g.

$$\sigma(K^+K^{*-}) : \sigma(K^\circ\bar{K}^{\circ *}) = 16 : 1 \qquad \ldots.2.46$$

Experimentally the ratio is 0.92 : 1.2. This particular problem is treated in great detail in chapter III.

Since the requirements of U(6,6) index invariance have led to S-matrix elements which are in conflict with unitarity, and which predict some results in conflict with experiment, it certainly needs some modification. We were led to U(6,6) index invariance and not true invariance because we restricted our space time dimension to four.[19] As suggested by many authors,[2] for example in the case of U(2,2) we can extend the space-time to 16, so that we have 16 energy-momentum operators P_R which obey the

commutation relations

$$\left[P_R, P_S\right] = 0 \qquad \ldots.2.47$$

$$\left[F_R, P_S\right] = i f_{RS}^{T} P_T \qquad \ldots.2.48$$

The last relation shows that P_R transforms like a 16-vector under U(2,2) transformation. We can then form the spinor $(P_R F_R)_\alpha^{\ \beta}$ which is a generalization of (1.46) and (1.47) and our auxiliary operators will be of the form

$$A_\alpha(p) = \left\langle \alpha \middle| e^{-i\xi_R F^R} \middle| \beta \right\rangle \left\langle \beta \middle| m, s \right\rangle a(p, s) \qquad \ldots.2.49$$

where now p runs over sixteen values and under IU(2,2) transformation

$$A_\alpha(p) \longrightarrow \left\langle \alpha \middle| e^{i\eta_R F_R} \middle| \beta \right\rangle A_\beta(p') \qquad \ldots.2.50$$

which is a <u>true</u>, not an index, transformation. We make contact with the physical world by requiring that the p runs over the real four values μ. The little group is still $U(2) \otimes U(2)$. We would then proceed as in the case of U(2,2). The only difference now is that with IU(2,2) transformation, the <u>derivative</u> or <u>irregular couplings</u> are no longer excluded from the interaction Lagrangian as they were in pure U(2,2) index transformation. In the same way IU(6,6) admits the irregular couplings into the Lagrangian and is therefore less restrictive than U(6,6).

On the other hand we can look at the problem from a slightly different view point. We take U(6,6) index invariance as a working hypothesis, first order approximation in the scattering amplitude, and then, as need arises, supplement this with irregular couplings. These irregular couplings will then <u>break</u> the U(6,6) symmetry. The inclusion of irregular terms into U(6,6) invariant matrix elements may be looked upon as <u>intrinsic</u> symmetry breaking, since the requirement that the physical particle fields, constructed from U(6,6) auxiliary operators,[3] obey Bargman Wigner equations

of motion, introduces these irregular terms $\gamma_\mu p_\mu$ intrinsically into the theory.

Equation (2.43), together with the remark we passed on SU(3) and unitarity, reveals that the inclusion of these 'intrinsic' irregular terms will not be sufficient to restore unitarity, since the phsyical multiplets of U(6,6) binds together particles of different spins, and therefore largely different masses. This suggests then[20] the inclusion of spin-dependent mass splitting terms which[21] may be called 'unitarity corrections'. These terms are of the form $\gamma_R \otimes \gamma_R$. Again, these corrections may give only an order of approximation to unitarity, and cannot restore it completely. In this way we can write the S-matrix elements as a sum of three leading terms,

$$S = S_o + S_D + S_u \qquad \ldots.2.51$$

where S_o has full homogeneous U(6,6) symmetry, S_D are the derivative terms which break 'intrinsically' the U(6,6) symmetry, and S_u are the spin-dependent 'unitary corrections'.

5. The hierarchy of U(6,6) subgroups.

An interesting equation now arises: with these symmetry breaking terms, can any vestige of our symmetry still possibly survive?

We shall answer this question first by considering the derivative terms S_D only. The derivative coupling is of the form $\gamma_\mu p_\mu = \not{p}$. Thus if in any process it is not possible to find a Lorentz frame where any of the components of p_μ is zero, then the term $\not{p}$ breaks down U(6,6)

symmetry to P⊗SU(3), and there will be no new prediction except SU(3) predictions. Let us consider the following more interesting cases:

(i) Some particular processes may be such that the S-matrix depends on only one 4-momentum (this includes mass matrices for two point functions). In this case we can go to the rest frame where $p_1 = p_2 = p_3 = 0$, and the surviving symmetry is that whose generators commute with γ_o - the non-chiral U(6)⊗U(6) composed of the generators

$$\tfrac{1}{2}(1 \pm \gamma_o)T^i,\ \tfrac{1}{2}(1 \pm \gamma_o)\vec{\sigma}T^i \qquad \ldots\ldots 2.52$$

This is the little group of U(6,6).

(ii) For some processes involving two independent 4-momenta p_μ, q_μ, we may go to a frame

$$p_\mu = (p_o,0,0,0) \text{ and } q_\mu = (0,0,0,q)$$

and the surviving symmetry will be the collinear $U(6)_W$ of Lipkin and Meshkov[14] or the lesser group of Matthews, Charap and Streater with generators

$$(1,\ \sigma_{12},\ \gamma_o\sigma_{23},\ \gamma_o\sigma_{31})\,.T^i \qquad \ldots\ldots 2.53$$

Such processes include vertex functions, all forward and backward scattering and $p\bar{p}$ annihilation at rest into two mesons. $U(6)_W$ is compatible with unitarity provided all its intermediate states are collinar.

(iii) If three momenta p_μ, q_μ, k_μ are independent, we can set

$$p_\mu = (p_o,o,o,o),\ q_\mu = (o,o,o,q),\ k_\mu = (0,0,k,0)$$

and the symmetry that remains will be generated by all matrices which commute with γ_o, γ_3, γ_2,

$$(1,\ \gamma_o\sigma_{23})\ T^i \qquad \ldots\ldots 2.54$$

This is the coplanar or the least group $U(3)\otimes U(3)$.

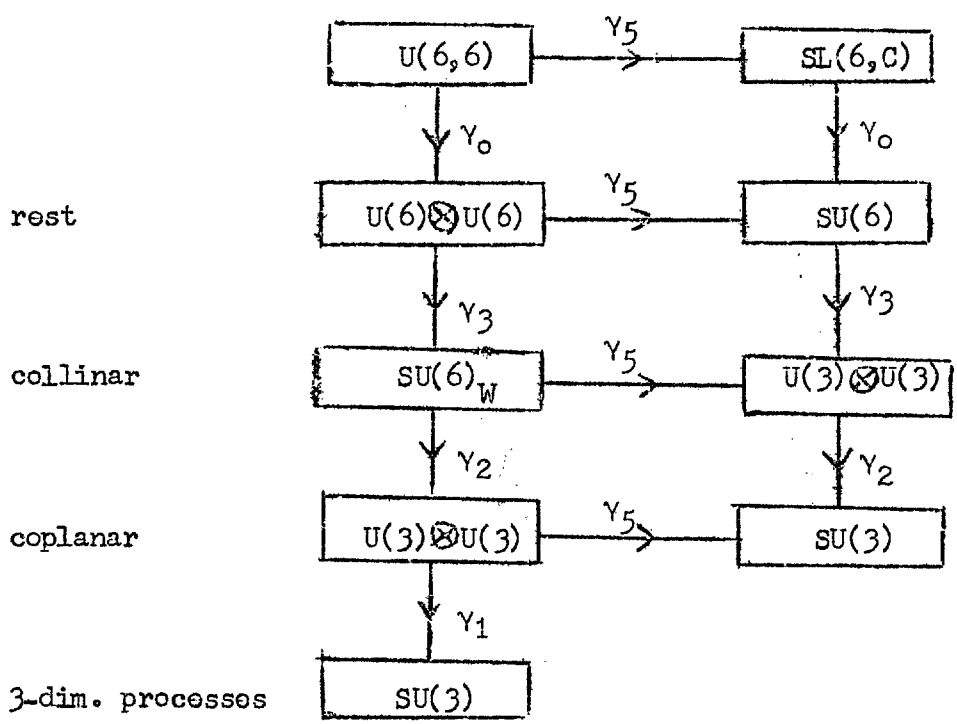
We consider next the effects of insertion of such non-momentum dependent spurions of the form

$$\gamma_5\otimes\gamma_5,\ \gamma_\mu\otimes\gamma_\mu,\ i\gamma_5\gamma_\mu\otimes i\gamma_5\gamma_\mu,\ \sigma_{\mu\nu}\otimes\sigma_{\mu\nu} \qquad \ldots\ldots 2.55$$

into U(6,6) invariant amplitude.

As in the case of derivative coupling, the insertion of any but $\gamma_5\otimes\gamma_5$ reduces the U(6,6) symmetry down to $P\otimes SU(3)$. With the insertion of $\gamma_5\otimes\gamma_5$, the residual symmetry is generated by all matrices which commute with γ_5. An excellent diagrammatic[23] way of representing the hierarchy of subgroups is as shown in fig. 1.

Fig. 1: Hybrids of U(6,6)



Note: In this figure, A $\xrightarrow{\gamma}$ B means the generators of A that commute with γ generate B.

Other interesting subgroups of U(6,6) which are not obtained directly by insertion of dynamical spurions include:

(a) $U_{SI}(4) \otimes U_{SY}(2)$ with the generators

$$1,\ \vec{\sigma},\ \vec{\tau},\ \vec{\sigma}\vec{\tau},\ Y,\ Y\vec{\sigma} \qquad \ldots.2.56$$

where

$$\vec{\tau} = (T^1, T^2, T^3),\quad Y = \sqrt{\frac{2}{3}}\,(T^0 - \sqrt{2}\,T^8) \qquad \ldots.2.57$$

and S, I, Y stand for space spin, isospin and hypercharge respectively. We recognise $U_{SI}(4)$ as the original supermultiplet group of Wigner[24] and $U_{SY}(2)$ as the strange quark spin introduced by Lipkin.[25] The application of the relativistic extension of this group forms the basis of our discussion in the next chapter.

(b) The chiral $U(6) \otimes U(6)$ with generators

$$\tfrac{1}{2}(1 \pm i\gamma_5)\,T^i,\quad \tfrac{1}{2}(1 \pm \gamma_5)\,\vec{\sigma}\,T^i$$

obtained from SL(6,C) by performing Weyl's unitary trick.[26]

(c) $U(2,2) \otimes SU(3)$, the generators of which we have already displayed.

CHAPTER III.

BROKEN U(4,4) AND $p\bar{p}$ ANNIHILATION AT REST INTO TWO MESONS

1. Introduction.

Since the advent of U(6,6) several authors have investigated its predictions on $p\bar{p}$ annihilation at rest into two mesons with or without dynamical spurions. It has been shown[18,27] that the full U(6,6) invariant term S_0 (in equation 2.51) vanishes in annihilation amplitude while, under certain conditions S_D, the irregular momentum dependent term, gives a unique coupling which predicts results in violent disagreement with experiments. Attempts[28] at including the momentum non-dependent spurions S_u does improve the results on some events but not on others. It becomes therefore apparent that the trouble with U(6,6) symmetry does not only lie in its dynamical structure but also in its internal symmetry structure. Recalling that even at SU(3) level some of the bad predictions of U(6,6), for example

$$d\sigma(K^-p \rightarrow K^\circ \Xi^\circ) = d\sigma(K^-p \rightarrow \Sigma^-\pi^+) \qquad \ldots.3.0$$

still survive, and the fact that Lipkin's λ-quark selection rule[29] does fairly well agree with experiments we are tempted to attribute, to some degree, the failures of U(6,6), for reactions,[30] to the merging of strange and non-strange quarks.

Following Macfarlane, O'Raifeartaigh and Sudarshan,[31] therefore, we propose and test, within the spurion scheme, a covariant U(4,4) generalization of the Wigner supermultiplet SU(4). This group, U(4,4)

has been shown[31] to reproduce the same predictions as U(6,6) on magnetic moment of the proton and on the proton-neutron magnetic moment ratio. We shall here treat in great detail the application[8] of this group to the problem of $p\bar{p}$ annihilation at rest into two mesons.

2. Particle classification in U(4,4).

Under the reduction

$$U(6,6) \longrightarrow U(4,4)_{SI} \otimes U(2,2)_{SY} \qquad \ldots.3.1$$

a fundamental twelve component (Dirac) quark goes into

$$\Psi_A \longrightarrow \Psi_{\alpha'}(p+3) \qquad \ldots.3.2$$

so that a U(6,6) meson tensor φ_A^B will reduce according to

$$\varphi_A^B \longrightarrow \varphi_{\alpha(p+3)}^{\beta(p+3)}$$

$$\equiv \varphi_{1,\alpha p}^{\beta q} + \varphi_{2,\alpha 3}^{\beta q} + \varphi_{\bar{2}\alpha p}^{\beta 3'} + \varphi_{3,\alpha 3}^{\beta 3'} \qquad \ldots.3.3$$

where, S, I, Y denote space-spin, iso-spin, and hypercharge respectively, α runs from 1 to 4, p from 1 to 2, and similarly for all small greek and latin indices.

The generators of the algebra of U(4,4) are $\gamma^R \otimes T^i$ where the γ^R's, R running from 1 to 16, are the Dirac matrices, the same as for U(6,6) but now the T^i, i running from 0 to 3, with $T^\circ = 1$, are the Pauli spin matrices.

The physical particle content of the φ_i's are realised in the maximal compact subgroup of U(4,4), the non-chiral $U(4) \otimes U(4)$ composed of the generators

$$\tfrac{1}{2}(1 \pm \gamma_0)T^i, \quad \tfrac{1}{2}(1 \pm \gamma_0)\vec{\sigma}\, T^i \qquad \ldots.3.4$$

Thus under the reduction (3.1) and (3.3) the 36 low-lying mesons are classified as follows:

$$\varphi_1 \supset (\rho, \pi, \omega, \eta)_{\lambda=0}, \qquad \varphi_2 \supset (K, K^*)_{\lambda=1}$$
$$\varphi_{\bar{2}} \supset (\bar{K}, \bar{K}^*)_{\lambda=1}, \qquad \varphi_3 \supset (\varphi, \chi)_{\lambda=2} \qquad \ldots.3.5$$

where here, $\lambda = n$, means the total number of λ-quark and antiquark content of each particle in the multiplet.

In terms of this classification, ω, φ, χ, η are physical in U(4) space. This can easily be seen. The classification here is a complete separation of particles according to their λ-quark and antiquark contents. No two particles with different λ-quark contents can mix. Therefore, ω cannot mix with φ and η cannot mix with χ. Again, although ω and η now belong to the same multiplet, they are distinguished by their space spin contents and cannot mix. Similarly φ and χ cannot mix. The expression of these phsyical classifications in terms of the unphysical SU(6) classification of these singlets determines, unambiguously, the mixing angles in SU(6) space.

We now apply Bargman Wigner equations to each φ, so that, for instance (equation 2.32),

$$\mu\, \varphi_{1\ \alpha q}^{\beta p}(k) = [(\mu + \not{k})(\gamma_5 P_5 {}^{p}_{q} + \gamma_\mu V_\mu {}^{p}_{q})]^{\beta}_{\alpha} \qquad \ldots.3.6$$

with subsidiary condition

$$k_\mu V_\mu = 0$$

The baryons are similarly treated. Under (3.1)

$$\Psi_{ABC} \rightarrow \Psi_{\alpha(p+3)\beta(q+3)\gamma(r+3)}$$

$$\equiv \Psi_{1,\alpha\beta\gamma,pqr} + \Psi_{2,\alpha\beta\gamma,(pq3 + cy.p,q,r)}$$

$$+ \Psi_{3,\alpha\beta\gamma,(p33+cy.p,q,r)} + \Psi_{4,\alpha\beta\gamma,333} \quad \ldots.3.7$$

where

$$\Psi_1 \supset (N,N^*)_{\lambda=0}, \quad \Psi_2 \supset (\Sigma,\Lambda,Y^*)_{\lambda=1},$$

$$\Psi_3 \supset (\Xi,\Xi^*)_{\lambda=2}, \quad \Psi_4 \supset (\Omega\Sigma)_{\lambda=3} \quad \ldots.3.8$$

and each Ψ satisfies the B.W. equations of motion. In this particular problem we shall be concerned with only the Ψ_1. The spin $\frac{1}{2}$ beat of Ψ_1 is abstracted from equation (2.33),

$$2\sqrt{6}\,\Psi_{1,\alpha\beta\gamma,p,q,r} = e_{pq3}\, N^3_{\alpha\beta\gamma,r} + cyc.p,q,r;\alpha,\beta,\gamma \quad \ldots.3.9$$

where

$$mN_{\alpha\beta\gamma,r} = [(m + \not{p})\,\gamma_5\, C]_{\alpha\beta}\, N_{\gamma,r} \quad \ldots.3.10$$

3. Construction of U(4,4) Invariant Amplitudes.

Since U(6,6) contains in its regular representation all the different multiplets of U(4,4), to construct U(4,4) invariant amplitudes it is more convenient to construct U(6,6) invariant amplitudes and then use the reduction (3.1) to express them in terms of U(4,4) amplitudes.

To begin with let us assume that all the mesons have equal mass μ (in the actual computation, physical masses are used where these make a substantial difference), and let the proton, anti-proton mass be m. We specify the rest condition by going to a frame where

$$p = (m,\underline{0}) = \text{momentum of proton}$$
$$= \text{momentum of anti-proton} \quad \ldots.3.10a$$

$q_1 = (\epsilon, \underline{q}')$ = momentum of meson 1

$q_2 = (\epsilon, \underline{q}')$ = momentum of meson 2 ....3.10a contd.

Energy momentum conservation gives

$$q_1 + q_2 = 2p \Longrightarrow \epsilon = m \qquad \ldots.3.11$$

We define

$$q = q_1 - q_2 = (0, \underline{q}), \text{ where } \underline{q} = 2\underline{q}' \qquad \ldots.3.12$$

so that we have only two independent momenta q and p. Thus the irreducible insertion of $\not{q}$ and, or $\not{p}$ into U(6,6) [U(4,4)] invariant amplitude will leave us with residual symmetry $U(6)_W$ [$U(4)_W$] (see fig. 1).

The reductions

$$143 \otimes 143 \rightarrow \underline{1} \oplus \underline{143}_F \oplus \underline{143}_D \oplus \underline{5940} \oplus 4212 \oplus 5005 \oplus 5005^*$$

and

$$364 \otimes 364^* \rightarrow \underline{1} \oplus \underline{143} \oplus \underline{5940} \oplus 126412$$

indicate that there are four U(6,6) invariant amplitudes,

$$\underline{1},\ \underline{143}_F,\ \underline{143}_D,\ \underline{5940}$$

which can be constructed for $p\bar{p}$ annihilation into two mesons. These are given by

$$\begin{aligned} S_o = {} & \bar{\Psi}^{ABC}(-p)\Psi_{ABD}(p)\ \varphi_E{}^D(q_1)\ \varphi_D{}^E(q_2) \\ & + \bar{\Psi}^{ABC}(-p)\Psi_{ABD}(p)\left\{\varphi_C{}^E(q_1)\ \varphi_E{}^D(q_2) \pm \varphi_E{}^D(q_1)\ \varphi_C{}^E(q_2)\right\} \\ & + \Psi^{ABC}(-p)\Psi_{ADE}(p)\ \varphi_B{}^D(q_1)\ \varphi_C{}^E(q_2) \qquad \ldots.3.16 \end{aligned}$$

where $\pm$ indicate F or D couplings respectively, and $\Psi(-p)$ represents incoming antiparticle with momentum p.

It can be shown[18,27] that

$$S_o \equiv 0 \qquad \ldots.3.17$$

because

$$\bar{\Psi}^{ABC}(-p)\,\Psi_{ADE}(p) = 0 \qquad \text{for } p = (m,\underline{0}) \qquad \ldots.3.18$$

i.e. the left hand side of (3.17) vanishes whenever there is a direct contraction between $\bar{\Psi}(-p)$ and $\Psi(p)$. This property arises purely from the Lorentz indices and has nothing to do with the internal symmetry indices. Thus the 'leading' term, the U(6,6) invariant term in the S-matrix element, vanishes identically. This result is however in reasonable agreement with experiment for two meson production compared with higher meson production rates. Except the $\rho\pi$ production, the two meson production is considerably damped.

We are now left with the terms S_D and S_u. Let us treat S_D first. These are terms obtained by the insertion of the two independent spurions $\not{p}$ and $\not{q}$ in the U(6,6) amplitude in such a way as to obtain all possible irreducible independent terms. On account of the 'rest conditions' and equation (3.18) the number of independent terms one gets in this particular problem is very much limited. This is more so because whenever $\not{p}$ acts on either Ψ or $\bar{\Psi}$, the vertex is reducible because of the B.W. equations. Similarly, whenever $\not{p}$ or $\not{q}$ acts between φ_1 and φ_2, or $\varphi_{1,2}$ and Ψ or $\varphi_{1,2}$ and $\bar{\Psi}$, the vertex is reducible. So that the only irreducible terms are:

$$\begin{aligned} S_D = {} & a_1 \bar{\Psi}^{ABC}(-p)\,(\not{q})^D_A\,(\not{q})^E_B\,(\not{q})^F_C \Psi_{DEF}(p)\,\varphi^G_H(q_1)\,\varphi^H_G(q_2) \\ & + a_2^D \bar{\Psi}^{ABC}(-p)\,(\not{q})^D_A\,(\not{q})^E_B \Psi_{DEF}(p)\left\{\varphi^G_C(q_1)\varphi^F_G(q_2) + \varphi^F_G(q_1)\varphi^G_C(q_2)\right\} \\ & + a_2^F \bar{\Psi}^{ABC}(-p)\,(\not{q})^D_A\,(\not{q})^E_B \Psi_{DEF}(p)\left\{\varphi^G_C(q_1)\varphi^F_G(q_2) - \varphi^F_G(q_1)\varphi^G_C(q_2)\right\} \end{aligned}$$

$$+ a_3 \bar{\Psi}^{ABC}(-p)(\not{q})^D_A (\not{q})^E_B \Psi_{DEF}(p)\ \varphi^F_C(q_1)(\not{q})^G_H \varphi^H_G(q_2)$$

$$+ a_4 \bar{\Psi}^{ABC}(-p)(\not{q})^D_A (\not{q})^E_B \Psi_{DEF}(p)\ \varphi^F_C(q_2)(\not{q})^G_H \varphi^H_G(q_1)$$

$$+ a_5 \bar{\Psi}^{ABC}(-p)(\not{q})^D_A \Psi_{DEF}(p)\ \varphi^E_B(q_1)\ \varphi^F_C(q_2) \qquad \ldots.3.19$$

where the a's are in general invariant functions of (s,t,u). However, the rest conditions reduce the a's to functions of the energy only. We shall now assume that strong interaction is invariant under charge conjugation C defined in (2.37) - (2.39). To test (3.19) on C, we first symmetrize and antisymmetrize Bose and Fermi Operators respectively. For example, consider the last term a_5.

$$\left[\bar{\Psi}^{ABC}(p_1)(\not{q})^D_A \Psi_{DEF}(p_2) - \Psi_{DEF}(p_2)(\not{q})^D_A \bar{\Psi}^{ABC}(p_1)\right] \times$$

$$\times \left[\varphi^E_B(q_1)\ \varphi^F_C(q_2) + \varphi^F_C(q_2)\ \varphi^E_B(q_1)\right] \qquad \ldots.3.20$$

Under C, the first term of (3.20) goes into

$$(C^{-1})^{AA'}\ (C^{-1})^{BB'}\ (C^{-1})^{CC'} \Psi_{A'B'C'}(-p_1)(\not{q})^D_A\ C_{DD'} C_{EE'} C_{FF'} \bar{\Psi}^{D'E'F'}(-p_2)$$

$$(C^{-1})^{EG}\ \varphi^H_G(q_1)\ C_{HB}\ (C^{-1})^{FI}\ \varphi^J_I(q_2)\ C_{JC}$$

$$= \Psi_{ABC}(p_1)(\not{q})^A_D \bar{\Psi}^{DEF}(-p_2)\ \varphi^B_E(q_1)\ \varphi^C_F(q_2)$$

here we have used (2.38) and (2.39). Treating all the other terms in a similar way, we obtain for a_5:

$$\bar{\Psi}^{ABC}(-p_1)(\not{q})^D_A \Psi_{DEF}(p_2)\ \varphi^E_B(q_1)\ \varphi^F_C(q_2) \xrightarrow{C}$$

$$- \bar{\Psi}^{ABC}(-p_2)(\not{q})^D_A \Psi_{DEF}(-p_1)\ \varphi^E_B(q_1)\ \varphi^F_C(q_2) =$$

$$= -\bar{\Psi}^{ABC}(-p_1)(\not{\phi})^D_A \Psi_{DEF}(p_2)\, \varphi^E_B(q_1)\, \varphi^F_C(q_2) \qquad \ldots.3.21$$

obtained by making the replacement $p_1 \longrightarrow + p_2$, $q_{1,2} \longrightarrow q_{1,2}$ which is consistent with energy momentum conservation equation (3.11).

$$\therefore \quad a_5 = 0$$

by charge conjugation invariance. Applying C to the other terms we find that the only non-vanishing amplitude is

$$S_D = g_D \bar{\Psi}^{ABC}(-p)(\not{\phi})^D_A\, (\not{\phi})^E_B \Psi_{DEF}(p) \left\{ \varphi^G_C(q_1)\varphi^F_G(q_2) + \varphi^F_G(q_1)\varphi^G_C(q_2) \right\} \qquad \ldots.3.22$$

We next consider the terms S_u arising from the insertion of non-momentum dependent spurions of the form,[20,28]

$$\gamma_5 \otimes \gamma_5, \quad i\gamma_\mu\gamma_5 \otimes i\gamma_\mu\gamma_5, \quad \gamma_\mu \otimes \gamma_\mu, \quad \sigma_{\mu\nu} \otimes \sigma_{\mu\nu} \qquad \ldots.3.23$$

These terms when inserted into baryon and meson mass terms split their masses as follows:

$\gamma_5 \otimes \gamma_5$, $i\gamma_\mu\gamma_5 \otimes i\gamma_\mu\gamma_5$ splits 0^- singlet from 0^- octet

$\sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$, $\gamma_\mu \otimes \gamma_\mu$ splits 1^- singlet from 1^- octet

$\sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$, $i\gamma_\mu\gamma_5 \otimes i\gamma_\mu\gamma_5$ splits 0^- nonet from 1^- nonet

$\sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$, $i\gamma_\mu\gamma_5 \otimes i\gamma_\mu\gamma_5$ splits $\frac{1}{2}^+$ octet from $3/2$ decuplet

$\ldots.3.24$

The insertion of the $\not{\phi}$'s break U(6,6) symmetry down to $U(6)_W$. The additional liberal insertion of (3.23) will break the symmetry down to SU(3). To prevent this, therefore, we shall choose from (3.24), one term that splits the masses of the 0^- nonet from the 1^- nonet. Moreover, since our ultimate goal is U(4,4) where ω and φ are physical and belong to different multiplets, it will be desirable also to split their masses. The only term that accomplishes these two requirements is

$$\sigma_{\mu\nu} \otimes \sigma_{\mu\nu} \qquad \ldots.3.25$$

which we choose. (By this economy of choice the only term we really miss is $i\gamma_\mu\gamma_5 \otimes i\gamma_\mu\gamma_5$, or $\gamma_5 \otimes \gamma_5$.) In spite of all this, we realize that the term (3,25), if contracted in all possible ways, still breaks down our symmetry to SU(3). To prevent this we have to impose the extra condition that our annihilation amplitude is dominated by a single 35 plet meson pole. This is analogous to the octet dominance assumption in SU(3), and will correspond in SU(4) to the 15 plet dominance. With this restriction, and with charge conjugation invariance, (3.25) will give the unique term

$$S_u = g_u \Psi^{ABC}(-p)(\sigma_{\mu\nu})^D_A\,(\sigma_{\mu\nu})^E_B \Psi_{DEF}(p)\left\{\varphi^G_C(q_1)\varphi^F_G(q_2) + \varphi^F_G(q_1)\varphi^G_C(q_2)\right\} \qquad \ldots\ldots 3.26$$

so that S will now be

$$S = S_D + S_u \qquad \ldots\ldots 3.27$$

Now we have to express every U(6,6) amplitude in terms of the corresponding U(4,4) amplitude. Using (3.1) and (3.3) we find that on the basis of the above prescriptions, notably 15 plet dominance, a typical U(6,6) amplitude, say $\bar\Psi\Psi\varphi\varphi$, will now break into two U(4,4) amplitudes:

$$\bar\Psi\Psi\varphi\varphi \rightarrow \bar\Psi_1\Psi_1\left[\alpha_1\,\varphi_1\,\varphi_1 + \alpha_2\,\varphi_2\,\varphi_2\right] \qquad \ldots\ldots 3.28$$

Thus, we have for the general U(4,4) amplitude,

$$\begin{aligned} A = {} & \bar\Psi_1^{ABC}(-p)(\not{q})^D_A(\not{q})^E_B \Psi_{1,DEF}(p)\left\{\alpha_1\left[\varphi^G_{1,C}(q_1)\varphi^F_{1,G}(q_2) + \varphi^F_{1,G}(q_1)\varphi^G_{1,C}(q_2)\right]\right. \\ & \left. + \alpha_2\left[\varphi^G_{2,C}(q_1)\varphi^F_{2,G}(q_2) + \varphi^F_{2,G}(q_1)\varphi^G_{2,C}(q_2)\right]\right\} \\ & + \bar\Psi_1^{ABC}(-p)(\sigma_{\mu\nu})^D_A(\sigma_{\mu\nu})^E_B \Psi_{1,DEF}(p)\left\{\beta_1\left[\varphi^G_{1,C}(q_1)\varphi^F_{1,G}(q_2) + \varphi^F_{1,G}(q_1)\varphi^G_{1,C}(q_2)\right]\right. \\ & \left. + \beta_2\left[\varphi^G_{2,C}(q_1)\varphi^F_{2,G}(q_2) + \varphi^F_{2,G}(q_1)\varphi^G_{2,C}(q_2)\right]\right\} \end{aligned} \qquad \ldots\ldots 3.29$$

Here, A, B, C are now U(4,4) indices with the appropriate decomposition for Ψ_1 and $\varphi_{1,2}$ in terms of the α's and the p's as indicated in equations (3.3) and (3.7), and $\alpha_1, \alpha_2, \beta_1, \beta_2$ are coupling constants.

4. Evaluation of the amplitudes.

Before we demonstrate how to evaluate (3.29), we shall first analyse the state of our initial and final systems. The $p\bar{p}$ system at rest is in the $^3S_1$ and ^1S_o states. The parity and charge parity of these states are summarised in Table 1.

Table 1: Show parity and charge parity of $(p\bar{p})$ system.

	P	C
^1S_o	-	+
$^3S_1$	-	-

Thus while the single and triplet states have same space parity, they have opposite charge parity. We shall assume here, as in the case[32] of Gell-Mann's SU(3), that for pseudoscalar mesons

$$\begin{aligned} C|\pi^{\pm}_{o}\rangle &= +|\pi^{\mp}_{o}\rangle \\ C|K^{\pm}\rangle &= +|K^{\mp}\rangle \\ C|K^{o}\rangle &= +|\bar{K}^{o}\rangle \end{aligned} \qquad \ldots\ldots 3.30$$

and for vector mesons

$$C|\rho^{\pm}_{o}\rangle = -|\rho^{\mp}_{o}\rangle$$

$$C|K^{*\pm}\rangle = -|K^{*\mp}\rangle$$
$$C|K^{o*}\rangle = -|K^{*o}\rangle \qquad \ldots\ldots 3.31$$

Since the initial state is eigenstate of P and C with definite eigenvalue we must construct final states which are also eigenstates of P and C with same eigenvalues. Consider for instance the production of two pseudoscalar mesons. The ^1S_o mode is forbidden by space parity, since the conservation of J implies $\ell = 0$. But the ^3S_o mode is allowed, since $\ell = 1$, and

$$\begin{aligned} C|\pi^+\pi^-\rangle &= |\pi^-\pi^+\rangle \\ &= (-)^{\ell}\,|\pi^+\pi^-\rangle \\ &= -|\pi^+\pi^-\rangle \end{aligned} \qquad \ldots\ldots 3.32$$

Therefore C invariance will thus forbid singlet particle pseudoscalar (PP) production. Therefore a C invariant amplitude will be of the F type, i.e. $|\pi^+\pi^- - \pi^-\pi^+\rangle$, since

$$^3S_o,\ C|\pi^+\pi^- - \pi^-\pi^+\rangle = -|\pi^+\pi^- - \pi^-\pi^+\rangle \qquad \text{F coupling} \qquad \ldots\ldots 3.33$$

In the same way, we can construct for C invariant amplitude for other modes:

$$^1S_o\ (PV):\ C|K^+K^{*-} - K^-K^{*+}\rangle = +|K^+K^{*-} - K^-K^{*+}\rangle \quad \text{F coupling}$$
$$^3S_o\ (PV):\ C|K^+K^{*-} + K^-K^{*+}\rangle = -|K^+K^{*-} + K^-K^{*+}\rangle \quad \text{D coupling}$$
$$^1S_o\ (VV):\ C|K^{*+}K^{*-} + K^{*-}K^{*+}\rangle = +|K^{*+}K^{*-} + K^{*-}K^{*+}\rangle \quad \text{D coupling}$$
$$^3S_o\ (VV):\ C|K^{*+}K^{*-} - K^{*-}K^{*+}\rangle = -|K^{*+}K^{*-} - K^{*-}K^{*+}\rangle \quad \text{F coupling}$$

Here V stands for vector meson, P for pseudoscalar meson.

Equations (3.33) and (3.34) will act as rough guides in our computations.

Let us now indicate how one computes specific contributions from (3.29):

$$\bar{\Psi}_1^{ABC}(-p)(\not{A})_A^D(\not{A})_B^E \Psi'_{DEF}(p) = \bar{\Psi}_1^{\alpha\beta\gamma,pqr}(-p)(\not{A})_\alpha^\delta \delta_p^t (\not{A})_\beta^\epsilon \delta_q^u \Psi_{\delta\epsilon\lambda,tus}(p)$$

$$= \Psi_1^{\alpha\beta\gamma,pqr}(-p)(\not{A})_\alpha^\delta \delta_p^t (\not{A})_\beta^\epsilon \delta_q^u \Psi_{1,\delta\epsilon\lambda,tus}(p)$$

$$= \Psi_1^{\alpha\beta\gamma,pqr}(-p)(\not{A})_\alpha^\delta (\not{A})_\beta^\epsilon \Psi_{1,\delta\epsilon\lambda,pqs}(p)$$

$$= \frac{1}{24}\left\{\left[\epsilon^{pq3} N_3^{[\alpha\beta]\gamma,r} + \epsilon^{qr3} \bar{N}_3^{[\beta\gamma]\alpha,p} + \epsilon^{rp3} \bar{N}_3^{[\gamma\alpha]\beta,q}\right]\right\}$$

$$\times \left\{\left[\epsilon_{pq3} N^3_{[\delta\epsilon]\lambda,s} + \epsilon_{qs3} N^3_{[\epsilon\lambda]\delta,p} + \epsilon_{sp3} N^3_{[\lambda\delta]\epsilon,q}\right]\right\}$$

$$\times (\not{A})_\alpha^\delta (\not{A})_\beta^\epsilon$$

(the last equation comes from using 3.9)

$$= \frac{1}{12}\left[2\bar{N}_3^{[\alpha\beta]\gamma,r} N^3_{[\delta\epsilon]\lambda,s} + \bar{N}_3^{[\beta\gamma]\alpha,r} N^3_{[\lambda\delta]\epsilon,s}\right.$$

$$\left. + (\bar{N}_3^{[\beta\gamma]\alpha} N^3_{[\epsilon\lambda]\delta} - \bar{N}_3^{[\beta\gamma]\alpha} N^3_{[\lambda\delta]\epsilon})\delta_s^r\right] (\not{A})_\alpha^\delta (\not{A})_\beta^\epsilon$$

....3.35

Here we have used

$$N_{[\alpha\beta]\gamma} + N_{[\beta\alpha]\gamma} = 0$$

$$N_{[\alpha\beta]\gamma} + N_{[\beta\gamma]\alpha} + N_{[\gamma\alpha]\beta} = 0 \qquad \text{....3.36}$$

$$\epsilon^{pq3}\epsilon_{st3} = \delta_s^p \delta_t^q - \delta_t^p \delta_s^q$$

A similar result is obtained for the other term with $(\sigma_{\mu\nu})_\alpha^\delta (\sigma_{\mu\nu})_\beta^\epsilon$ replacing $(\not{A})_\alpha^\delta (\not{A})_\beta^\epsilon$.

For the meson part, consider for example the term $\varphi_1\varphi_1$, using equation (3.6)

$$\varphi_1\varphi_1 \equiv \varphi^{G}_{1G}(q)\varphi^{F}_{1G}(q) + q_1 \leftrightarrow q_2$$

$$= \left\{(1 + \frac{\not{q}_1}{\mu})\ \gamma_5\ \varphi^i_5(q_1) + (1 + \frac{\not{q}_1}{\mu})\ \gamma_\mu\ \varphi^i_\mu(q_1)\right\}^{k}_{\gamma}\ (T^i)^{t}_{r}$$

$$\times \left\{(1 + \frac{\not{q}_2}{\mu})\ \gamma_5\ \varphi^j_5(q_2) + (1 + \frac{\not{q}_2}{\mu})\ \gamma_\nu\ \varphi^j_\nu(q_2)\right\}^{\lambda}_{k}\ (T^j)^{s}_{t}$$

$$+ q_1 \longleftrightarrow q_2 \qquad (i,j = 1,2,3) \qquad \ldots\ldots 3.37$$

so that, using the kinematical relations

$$(\not{q}_1 + \mu)\ \gamma_5 (\not{q}_2 + \mu)\ \gamma_5 = \not{q}(\not{p} - \mu)$$

$$(\not{q}_2 + \mu)\ \gamma_5 (\not{q}_1 + \mu)\ \gamma_5 = -\not{q}(\not{p} - \mu)$$

where

$$q_1 = (p + q/2)\ ;\ q_2 = (p - q/2)$$

we obtain

$$\varphi_1\varphi_1 = \frac{1}{\mu}\left[\not{q}(\not{p} - \mu)\right]^{\lambda}_{\gamma}\ \varphi^i_5(q_1)\ \varphi^j_5(q_2)\left[T^iT^j - T^jT^i\right]^{s}_{r} \qquad \ldots\ldots 3.38$$

The same is true for $\varphi_2\varphi_2$. We can now couple (3.38) to (3.35) and apply the B.W. equation given in (3.10) to the N's to obtain

$$\frac{1}{\mu}(m - \mu)\ q^2\ \bar{N}\ \not{q}\ N\left\{(\bar{N}\ N)^{r}_{s}\ \left[T^iT^j - T^jT^i\right]^{s}_{r}\right\} \qquad \ldots\ldots 3.39$$

Notice the F combination as expected from (3.33). All other terms are similarly treated. Finally, by defining the following quantities:

$$G = \frac{q^2}{3\mu^2}\ (m - \mu)\ \bar{u}\ \not{q}\ u$$

$$Q = \frac{q^2}{3\mu^2}\ (m - \mu)\ \bar{u}\left[\not{q}\ \gamma_\mu\gamma_\nu - (q_\mu\gamma_\nu - q_\nu\gamma_\mu) - g_{\mu\nu}\not{q}\right]u$$

$$H = \frac{q^2}{3\mu^2}\ (m - \mu)\ \bar{u}\left[\not{q}\ \gamma_\nu\gamma_5 + \gamma_\nu\gamma_5\ \not{q}\right]u$$

$$I = \frac{q^2}{3\mu^2}\ (m^2 - \mu^2)\ \bar{u}\left[\gamma_\nu\gamma_5 + \frac{1}{2(m+\mu)}\ q_\nu\gamma_5\right]u$$

$$J = \frac{q^2}{3\mu^2}\ \bar{u}\Big[-2(m^2 - \mu^2)\ \gamma_\mu\gamma_\nu + (m - \mu)(q_\mu\gamma_\nu + q_\nu\gamma_\mu - q_{\mu\nu}\not{q}) - 2\ p_\nu\gamma_\mu\ \not{q}\Big]u$$

$$L = \frac{q^2}{3\mu^2}\,\bar{u}\left\{4\,p_\nu\gamma_\mu + \left[2(m-\mu)\,g_{\mu\nu} + \frac{1}{(m+\mu)}\,q_\mu q_\nu\right]\not{A} + (q_\mu\gamma_\nu\not{A} - q_\nu\gamma_\mu\not{A})\right\}u$$

$$M = \frac{q^2}{3\mu^2}\,\bar{u}\left[\frac{4}{m}\,p_\mu p_\nu - \frac{1}{m}(p_\mu q_\nu - p_\nu q_\mu) - \frac{1}{m+\mu}\,q_\mu q_\nu - 2(m-\mu)\,g_{\mu\nu}\right]\not{A}\,u$$

$$R = \frac{q^2}{3\mu^2}\,\bar{u}\left\{\frac{1}{m}(m-\mu)(m+2\mu)(q_\nu\gamma_\mu + q_\mu\gamma_\nu) + (m-\mu)\,g_{\mu\nu}\not{A} - \left[2m^{-1}\,p_\mu p_\nu\right]\not{A}\right\}u \qquad \ldots.3.40$$

we have the results:

For $p\bar{p} \longrightarrow PP$ via $^3S_1$,

$$A = G\left\{(\bar{N}N)^r_s\left[(3\alpha_1 - 13\beta_1)(P^tP_t)^s_{-r} + (3\alpha_2 - 13\beta_2)(P^3P_3)^s_{-r}\right] + (\bar{N}N)^r_r\left[(3\alpha_1 - \beta_1)(P^tP_t)^s_{-s} + (3\alpha_2 - \beta_2)(P^3P_3)^s_{-s}\right]\right\} \qquad \ldots.3.41$$

For $p\bar{p} \longrightarrow PV$ via $^1S_0$,

$$A = (\bar{N}N)^r_s\left\{\left[(10I + J)\,\alpha_1 - 60I\beta_1\right](P^tV_t)^s_{-r} + \left[(10I + J)\,\alpha_2 - 60I\beta_2\right](P^3V_3)^s_{-r}\right\} + (\bar{N}N)^r_r\left\{\left[-(2I + J)\,\alpha_1 + 12I\beta_1\right](P^tV_t)^s_{-s} + \left[-(2I + J)\,\alpha_2 + 12I\beta_2\right](P^3V_3)^s_{-s}\right\} \qquad \ldots.3.42$$

For $p\bar{p} \longrightarrow PV$ via $^3S_1$,

$$A = H\left\{(\bar{N}N)^r_s\left[(\tfrac{5}{2}\alpha_1 - 13\beta_1)(P^tV_t)^s_{+r} + (\tfrac{5}{2}\alpha_2 - 13\beta_2)(P^3V_3)^s_{+r}\right] + (\bar{N}N)^r_r\left[(-\tfrac{1}{2}\alpha_1 + \beta_1)(P^tV_t)^s_{+s} + (-\tfrac{1}{2}\alpha_2 + \beta_2)(P^3V_3)^s_{+s}\right]\right\} \qquad \ldots.3.43$$

For $p\bar{p} \longrightarrow VV$ via $^1S_0$,

$$A = Q\left\{(\bar{N}N)^r_s\left[(-5\alpha_1 + 30\beta_1)(V^tV_t)^s_{+r} + (-5\alpha_2 + 30\beta_2)(V^3V_3)^s_{+r}\right] + (\bar{N}N)^r_r\left[(\alpha_1 - 6\beta_1)(V^tV_t)^s_{+s} + (\alpha_2 - 6\beta_2)(V^3V_3)^s_{+s}\right]\right\} \qquad \ldots.3.44$$

For $p\bar{p} \longrightarrow VV$ via $^3S_1$,

$$A = (\bar{N}N)^r_s \{ [(5K + L)\alpha_1 - 26K\beta_1](V^t V_t)^s_{-r} + [(5K + L)\alpha_1 - 26K\beta_2](V^3 V_3)^s_{-r} \}$$
$$+ (\bar{N}\bar{N})^r_r \{ [(-K - L + M)\alpha_1 + (K + 4R)\beta_1](V^t V_t)^s_{-s} + [(-K - L + M)\alpha_2$$
$$+ (K + 4R)\beta_2](V^3 V_3)^s_{-s} \} \quad \ldots\ldots 3.45$$

where

$$(\bar{N}N)^r_s = \bar{N}^r_3 N^3_s, \quad (P^t P_t)^s_{\pm r} = (P^t_r P^s_t \pm P^s_t P^t_r)$$

5. Results and Conclusions.

We shall first discuss those results which are directly obtained from equations (3.41) to (3.45) making no use of experimental input. We shall classify them according to their modes.

From equation (3.41) we deduce

$$^3S \; ; \; A(\chi^\circ\chi^\circ) = A(\pi^\circ\pi^\circ) = A(\pi^\circ\chi) = 0 \quad \ldots\ldots 3.46$$

This result is in fact a consequence of charge conjugation invariance. It was also obtained[18] in U(6,6) theory.

From equation (3.43),

$$^3S \; ; \; A(\rho\pi) = A(\omega\eta) \quad \ldots\ldots 3.47$$

$$^3S \; ; \; A(\rho\,) = A(\omega\pi) \quad \ldots\ldots 3.48$$

$$^3S \; ; \; A(\varphi\eta) = A(\varphi\pi) = A(\varphi\chi) = A(\rho\chi) = A(\omega\chi) = 0 \quad \ldots\ldots 3.49$$

Equation (3.39) is satisfied experimentally in so far as these events have not been seen. It could easily have been deduced from (3.29). It also agrees with U(6,6) results. Equations (3.47) and (3.48) are hardly easy to justify since there are no reliable experiments on $(\omega\eta)$ and $(\omega\pi)$ events, while data on $(\rho\pi)$ and $(\rho\eta)$ are pretty sure.[33] Physical mass insertions enhance $A(\rho\pi)$ desirably but depress $A(\rho\eta)$ undesirably. U(6,6) predicts[12]

$$^3S;\ A(\rho\pi) : A(\omega\pi) : A(\omega\eta) : A(\rho\eta) : A(\rho\chi) : A(\omega\chi)$$
$$= \quad 3 : \quad 5 : \quad \sqrt{3} : \quad 5/\sqrt{3} : \quad 10/\sqrt{6} : \quad \sqrt{6}$$

Thus, experimentally, the predictions (3.47), (3.48), (3.49) appear more tolerable than their U(6,6) equivalent. Equation (3.44) gives

$$^1S\ ;\ A(\omega\omega) : A(\rho\rho) : A(\rho\omega) = 3 : 3 : 5 \qquad \ldots\ldots 3.50$$

$$^1S\ ;\ A(K^{*+}K^{*-}) : A(K^{*\circ}\bar{K}^{*\circ}) = 4 : -1 \qquad \ldots\ldots 3.51$$

$$^1S\ ;\ A(\varphi\rho) = A(\rho\varphi) = A(\omega\varphi) = 0 \qquad \ldots\ldots 3.52$$

Equation (3.52) agrees with experiment and with U(6,6) predictions. The square of the ratio $A(\rho\rho) : A(\rho\omega)$ in (3.50) is in fair agreement with experiment rate $\sigma(\rho\rho) : \sigma(\rho\omega) = 3.8 : 7.0$. But there are no data on $(\omega\omega)$. Equation (3.51) is conspicuously bad compared with experimental rate $\sigma(K^{*+}K^{*-}) : \sigma(K^{*\circ}\bar{K}^{*\circ}) = 1.3 : 2.9$. It was also obtained in U(6,6), and is the single disturbing aspect of U(6,6) which survives a broken U(4,4) theory in this problem. However, it arises from ^1S mode which accounts for at most 35% of most two-meson annihilation[33] processes.

Using equations (3.42) and (3.45):

$$^1S\ ;\ A(K^+K^{*-}) - A(K^\circ\bar{K}^{\circ *}) = \{(10I + J)\,\alpha_2 - 60I\beta_2\}$$
$$^1S\ ;\ A(\rho^+\pi^-) \qquad\qquad = [(10I + J)\,\alpha_1 - 60I\beta_1] \qquad \ldots\ldots 3.53$$

and

$$^3S\ ;\ A(^{*+}K^{*-}) - A(K^{*\circ}\bar{K}^{*\circ}) = \{(5K + L)\,\alpha_2 - 26K\beta_2\}$$
$$^3S\ ;\ A(\rho^+\rho^-) \qquad\qquad = [(5K + L)\,\alpha_1 - 26K\beta_1\} \qquad \ldots\ldots 3.54$$

Thus we see that none of the U(6,6) sum rules can be simply derived in U(4,4). The U(6,6) sum rules are not well satisfied[28] by experiment.

It is worth mentioning that we can derive all U(6,6) results by going to the U(6,6) limit $\alpha_1 \to \alpha_2$, $\beta_1 \to \beta_2$.

Let us now turn to a more exciting aspect of our problem. Let us assume that the annihilation amplitude for PP and VP productions are dominated by the ^3S mode - a very reasonable assumption.[33,34] Then from equation (3.41), by putting

$$^3S \; ; \; A(K^+K^-) \; : \; A(K^\circ\bar{K}^\circ) \; = \; 7 \; : \; 5 \qquad \ldots\ldots 3.55$$

we obtain the ratio

$$\beta_2 \; : \; \alpha_2 \; = \; 1 \; : \; 7 \qquad \ldots\ldots 3.56$$

and equation (3.43) gives

$$^3S \; ; \; A(K^+K^{*-}) \; : \; A(K^\circ\bar{K}^{\circ *}) \; = \; 4 \; : \; -5 \qquad \ldots\ldots 3.57$$

the square of which compares favourably with experimental cross section

$$\sigma(K^+K^{*-}) \; : \; \sigma(K^\circ\bar{K}^{\circ *}) \; = \; 0.92 \; : \; 1.2 \qquad \ldots\ldots 3.58$$

This result is rather satisfying since the ratio is insensitive to mass factors. In the same way, using the results

$$^3S \; ; \; A(\pi^+\pi^-) \; : \; A(K^+K^-) \; = \; 9 \; : \; 5 \qquad \ldots\ldots 3.59$$

$$^3S \; ; \; A(\rho^\circ\pi^\circ) \; : \; A(\rho^\circ\eta^\circ) \; = \; 5 \; : \; 2 \qquad \ldots\ldots 3.60$$

we obtain the ratios

$$\beta_1 \; : \; \alpha_2 \; = \; 1 \; : \; 11 \qquad \ldots\ldots 3.61$$

$$\alpha_1 \; : \; \alpha_2 \; = \; 9 \; : \; 50 \qquad \ldots\ldots 3.62$$

and equations (3.43) and (3.44) give

$$^3S \; ; \; A(\rho^{\pm}_{\circ}\,\pi^{\mp}_{\circ}) \; : \; A(K^{*+}K^{*-}) \; = \; 3 \; : \; 1 \qquad \ldots\ldots 3.63$$

$$^1S \; ; \; A(\rho^{+}_{\circ}\,\rho^{-}_{\circ}) \; : \; A(K^{*+}K^{*-}) \; = \; 2 \; : \; 1 \qquad \ldots\ldots 3.64$$

which are also good compared with experiment. However, equation (3.64)

cannot be taken too seriously in view of our assumption of the ^3S mode dominance. If this is so the possibility of the existance of a singlet resonance in the effective mass region of $(K^{*+}K^{*-})$ cannot be ruled out.

In our calculations we have used physical masses of the form $(m-\mu)/\mu^2$ where this will make significant difference. But we have not taken phase space corrections into account. The fact that the results obtained in this way are consistently good indicate that phase space effects tend to cancel out in the ratios. In fact, as pointed out by[35] Fraser, phase space corrections change the results only slightly, and in the right direction. In table II we exhibit these results, comparing them with $U(6,6)$ results and with experiment.

We can make one more interesting deduction from this problem. The ratios in equations (3.56) and (3.61)

$$\beta_2 : \alpha_2 = 1 : 7; \quad \beta_1 : \alpha_1 = 1 : 11 \qquad \ldots.3.65$$

indicate that the coupling strengths of the derivative term S_D are an order of magnitude greater than those of the unitarity terms S_u. This is only in keeping with the expression

$$S = S_o + S_D + S_u + \ldots. \qquad \ldots.3.66$$

as a rythmic perturbative expansion in the coupling strengths. This result is however contingent on the kinematics of this particular problem and the assumptions made in deriving (3.65). Here, S_o, the leading term, vanishes identically.

In conclusion, this problem has served three useful purposes.

First, it has demonstrated how one can obtain better results from higher symmetry groups by symmetry breaking techniques using momentum

dependent and momentum non-dependent spurions.

Next, that the bad effects of SU(3) for reactions are to a good extent due to the lumping together of the non-strange and the strange quarks. These bad effects are obviously amplified at U(6,6) level, because of its high predictive power. U(4,4) similar to U(6,6) in all respects except that the λ-quarks are separated from the non-λ-quarks is, therefore, a better approximate symmetry for reactions than U(6,6).

Finally, subject to the assumptions made here, and to the kinematics of this problem, there is sense and rythmn in the expansion (3.66) as a perturbation expansion in the coupling strengths.

Table II: The predictions of U(4,4) in this table are subject to the input data (underlined) which fix the ratios $\beta_1 : \alpha_1$, $\beta_2 : \alpha_2$ and $\alpha_1 : \alpha_2$ to 1 : 11, 1 : 7 and 9 : 50 respectively.

Ratios of amplitudes	Modes	U(6,6) predictions	U(4,4) predictions	Ratios of experimental[a] ratios $\sim \sigma$
PP $A(\pi^+\pi^-) : A(K^+K^-)$	^3S	1 : 2	<u>9 : 5</u>	3.2 : 1.1
PP $A(K^+K^-) : A(K^\circ\bar{K}^\circ)$	^3S	2 : 1	<u>7 : 5</u>	1.1 : 0.56
PV $A(\rho^\circ\pi^\circ) : A(\rho^\circ\eta^\circ)$	^3S	$3 : 5/\sqrt{3}$	<u>5 : 2</u>	14 : 2.2
PV $A(K^+K^{*-}) : A(K^\circ\bar{K}^{\circ *})$	^3S	4 : -1	4 : -5	0.92 : 1.2
PV $A(\rho^{\pm}\pi^{\mp}) : A(K^+\bar{K}^{\circ *})$	^3S	3 : 4	3 : 1	$(14:0.92)_0, (29:0.92)^{\pm}$
VV $A(\rho^{\pm}\rho^{\mp}) : A(K^{*+}K^{*-})$	^1S	3 : 4	2 : 1	3.8 : 1.3
VV $A(K^{*+}K^{*-}) : A(K^{*\circ}\bar{K}^{*\circ})$	^1S	4 : -1	4 : -1	1.3 : 2.9
VV $A(\rho\rho) : A(\rho\omega)$	^1S	3 : 5	3 : 5	3.8 : 7.0

(a) Most of the experimental data are taken from Table I of ref. (33).

CHAPTER IV

REPRESENTATION MIXING IN U(6,6)

1. Introduction.

Applications of the algebra of integrated current components[36] to weak and electromagnetic phenomena had, about a year ago, indicated that although the assumed commutation relations between the integrated currents seem to be correct, the assumption that the obtained sum rules are saturated by very few intermediate states is, in most cases, inadequate. In particular, it is realised that the saturation of the Adler-Weisberger sum rule,

$$1 - \left(G_V/G_A\right)^2 = \frac{2}{\pi}\left(\frac{M}{g_{\pi NN}}\right)\int_{\mu}^{\infty}\frac{q\,d\nu}{\nu^2}\left[\sigma_\nu{}^{tot}(\pi^+p) - \sigma_\nu{}^{tot}(\pi^-p)\right] \qquad \ldots\ldots 4.0$$

(here, G_V/G_A is ratio of vector to axial vector coupling constants, $\sigma^{tot}(\pi^+p)$ is the total cross-section for elastic π^+p scattering, ν is the poin lab energy and q is the magnitude of the three-momentum of the poin in the lab system. M and μ are the nucleon and pion masses respectively,) with the spin $\frac{1}{2}^+$ octet and $3/2^+$ decuplet alone gives results for G_V/G_A, and G^* (the strength of the axial vector transition between the nucleon and $N^*(1238)$ which are bad compared with experiment. On the other hand it is observed that in (4.0) the contribution of states like $N^*(1512,\ J^P = 3/2^-)$, $Y_o^*(1405,\ J^P = \frac{1}{2}^-)$ and $Y_o^*(1520, J^P = 3^-/2)$ gives terms of the same order of magnitude as the decuplet.[37] Again it is to be noted that none of the low lying irreducible representations of U(6,6), alone, reproduces the experimental values of both G_A/G_V and D/F ratios. Thus if we still believe that the sum rules are saturated by a few states,

these states must then belong to more than one irreducible representation of our symmetry group. This idea has led many authors[36,37,38] to the concept of the physical baryons belonging to more than one irreducible representation of $U(3) \otimes U(3)$ and $SU(6)_W$. Investigation of the consequences of this assumption have been successfully carried out in the context of algebra of current[37,38] components.

We shall here consider assumptions of similar nature within the context[9] of $U(6,6)$ symmetry. We shall assume specifically that the physical baryons belong to some mixed states of

(i) $(220, L = 0)^+$ and $(364, L = 0)^+$

and

(ii) $(220, L = 0)^+$, $(364, L = 0)^+$ and $(572, L = 0)^+$.

These assumptions are motivated by our belief that the recent discovery of the resonances [39] like $N^*(1450, \frac{1}{2}^+)$ and $Y_0^*(1663, \frac{1}{2}^+)$ serve as an incentive to the consideration of the 220^+ and the 572^+ multiplets.

It is worthwhile to remark that these assumptions break the $U(6,6)$ symmetry, since the tensor for a baryon single particle state is no longer irreducible in $U(6,6)$.

From the decomposition equations (2.33) - (2.35), it is seen that only the 364 contains the decuplet $\frac{3}{2}^+$. Therefore we shall focus attention on the $\frac{1}{2}^+$ octet parts.*

2. (i) The $(220, L = 0)^+$ and $(364, L = 0)^+$ mixing.

We assume that the physical baryon wave function can be written as

$$|B\rangle = \alpha |364\rangle + \beta |220\rangle \qquad \ldots\ldots 4.1$$

where α and β are mixing parameters with constraint

$$\alpha^2 + \beta^2 = 1 \qquad \ldots\ldots 4.2$$

We compute the expectation values of the 143 vectors

$$(F_R{}^i)^A_B = (\gamma_R T^i)^B_A$$

between the states $|B\rangle$, namely

$$J_R{}^i = \alpha^2 \langle 364|F_R{}^i|364\rangle + (1-\alpha^2)\langle 220|F_R{}^i|220\rangle$$

$$= \alpha^2 \overline{\Psi}^{\{ABC\}} (\gamma_R T^i)^D_C \Psi_{\{ABD\}}$$

$$+ (1-\alpha^2) \overline{\Psi}^{[ABC]} (\gamma_R T^i)^D_C \Psi_{[ABD]} \qquad \ldots\ldots 4.3$$

where we are only interested in the spin $\frac{1}{2}^+$ octets.

$$\Psi_{\{ABC\}} = \frac{1}{2m}\sqrt{\frac{1}{6}} \left\{ [(\not{p} + m)\gamma_5 C]_{\alpha\beta} \epsilon_{pqs} N^s_{\gamma r} + \text{cy.A,B,C} \right\} \qquad \ldots\ldots 4.4$$

$$\Psi_{[ABC]} = \frac{1}{6m}\sqrt{\frac{1}{6}} \left\{ [(\not{p} + m)\gamma_5 C]_{\alpha\beta} (\epsilon_{rps} N^s_{\gamma q} - \epsilon_{qrs} N^s_{\gamma p}) + \text{cy.A, B, C} \right\} \qquad \ldots\ldots 4.5$$

Using the functions,

$$q = p' - p, \; P = p' + p, \; r_\mu = \epsilon_{\mu\nu k\lambda} P_\nu q_k \gamma_\lambda \gamma_5 \qquad \ldots\ldots 4.6$$

and the identity

$$\overline{N}\gamma_\mu \left(1 + \frac{p\cdot p'}{m^2}\right) N = \overline{N}\left(\frac{P_\mu}{m} + \frac{r_\mu}{2m^2}\right) N \qquad \ldots\ldots 4.7$$

We ontain for the first terms of (4.3)

$$J_5{}^i = \frac{p^2}{12m^2} (\overline{N}\gamma_5 N)_{3D+2F} \qquad \ldots\ldots 4.8$$

$$J_{\mu 5}{}^i = \frac{p^2}{12m^2} (i\,\overline{N}\gamma_\mu\gamma_5 N)_{3D+2F} \qquad \ldots\ldots 4.9$$

$$J_\mu{}^i = \frac{P_\mu}{2m} (\overline{N}N)_F + \frac{1}{3}\left(\overline{N}\frac{r_\mu}{4m^2} N\right)_{3D+2F} \qquad \ldots\ldots 4.10$$

$$J_{\mu\nu}^{\ i} = \frac{P^2}{12m^2}(\bar{N}\,\sigma_{\mu\nu}\,N)_{3D+2F} + \frac{i}{12m^2}(P_\mu q_\nu - P_\nu q_\mu)(\bar{N}N)_{F-3D} \qquad \ldots\ldots 4.11$$

and for the second term,

$$J_5^{\ i} = -\frac{P^2}{12m^2}(\bar{N}\,\gamma_5\,M)_D \qquad \ldots\ldots 4.12$$

$$J_{\mu 5}^{\ i} = -\frac{P^2}{12m^2}(\bar{N}\,\gamma_\mu\gamma_5\,N)_D \qquad \ldots\ldots 4.13$$

$$J_\mu^{\ i} = -\frac{1}{12m^2}\left[(\bar{N}\,r_\mu\,N)_D - 2m\,P_\mu\,(\bar{N}N)_F\right] \qquad \ldots\ldots 4.14$$

$$J_{\mu\nu}^{\ i} = \frac{P^2}{12m^2}(\bar{N}\,\sigma_{\mu\nu}\,N)_D + \frac{i}{12m^2}(P_\mu q_\nu - P_\nu q_\mu)(\bar{N}N)_{D+F} \qquad \ldots\ldots 4.15$$

(we do not need the scalar current).

Using the free field equations (B.W.)

$$\mu\varphi_5^{\ i} = i\,q_\mu\,\varphi_{\mu 5}^{\ i}, \qquad \mu\varphi_\nu = i\,q_\mu\,\varphi_{\nu\mu} \qquad \ldots\ldots 4.16$$

we can define the total pseudoscalar and vector currents as

$$j_5^{i(tot)} = j_5^{\ i} + iq_\mu j_{\mu 5}^{\ i}/\mu \qquad \ldots\ldots 4.17$$

$$j_\mu^{i(tot)} = j_\mu^{\ i} + iq_\nu j_{\nu\mu}^{\ i}/\mu \qquad \ldots\ldots 4.18$$

and have respectively for the set (4.8) - (4.11) and (4.12) - (4.15),

$$j_5^{i(tot)} = \left(1 + \frac{2m}{\mu}\right)\frac{P^2}{12m^2}(\bar{N}\,\gamma_5\,N)_{3D+2F} \qquad \ldots\ldots 4.18$$

$$j_\mu^{i(tot)} = \frac{P_\mu}{2m}\left(1 + \frac{q^2}{2m\mu}\right)(\bar{N}N)_F + \frac{1}{3}\left(1 + \frac{2m}{\mu}\right)(\bar{N}\,\frac{r_\mu}{4m^2}\,N)_{3D+2F} \qquad \ldots\ldots 4.19$$

and

$$j_5^{i(tot)} = -\frac{P^2}{12m^2}\left(1 + \frac{2m}{\mu}\right)(\bar{N}\,\gamma_5\,N)_D \qquad \ldots\ldots 4.20$$

$$j_\mu^{i(tot)} = \frac{1}{3}\frac{P_\mu}{2m}\left(1 + \frac{q^2}{2m\mu}\right)(\bar{N}N)_F - \frac{1}{3}\left(1 + \frac{2m}{\mu}\right)(\bar{N}\,\frac{r_\mu}{4m^2}\,N)_D \qquad \ldots\ldots 4.21$$

From equations (4.9), (4.13) and (4.3) we have

$$\frac{D}{F} = \frac{4\alpha^2 - 1}{2\alpha^2} \qquad \ldots\ldots 4.22$$

The common assumption about weak interactions is that they proceed via the coupling $G\, W_{\mu,\mu 5}\ (\gamma_\mu + i\gamma_\mu\gamma_5)$ which in the static limit gives,

$$\left|\frac{G_A}{G_V}\right| = \frac{6\alpha^2 - 1}{2\alpha^2 + 1} \qquad \ldots\ldots 4.23$$

From (4.22) and (4.23) we obtain

$$\left|G_A/G_V\right| = \frac{D + F}{3D - F} \qquad \ldots\ldots 4.24$$

in agreement with the $U(3)\otimes U(3)$ results obtained by Cabibbo and Ruegg[38] from a consideration of the mixing of the $(56,L=0)^+$ and the $(20,L=0)^-$ and the $(56,L=0)^+$ and $(70,L=0)^-$ in the context of current algebra. If in (4.24) we put $D/F = 1.75$, we get $\left|G_A/G_V\right| = 2.2$. Alternatively, if $\left|G_A/G_V\right| = 1.2$, $D/F = 1.18$. These results are bad compared with the experimental result of $\left|G_A/G_V\right| = 1.2$ and $D/F = 1.75$.

3. (ii) <u>The $(220,L=0)^+$, $(364,L=0)^+$ and $(572,L=0)^+$ mixing.</u>

As in the case of (i), we assume that the physical baryon state can be written as

$$|B\rangle = \alpha_1 |364\rangle + \alpha_2 |220\rangle + \alpha_3 |572\rangle \qquad \ldots\ldots 4.25$$

with $\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = 1$ $\qquad \ldots\ldots 4.26$

Then

$$\begin{aligned} j_R^i = {} & \alpha_1^2 \langle 364|F_R^i|364\rangle + \alpha_2^2 \langle 220|F_R^i|220\rangle \\ & + \alpha_3^2 \langle 572|F_R^i|572\rangle_I + g_1\alpha_3^2 \langle 572|F_R^i|572\rangle_{II} \\ & + g_2\alpha_1\alpha_3 \langle 572|F_R^i|364\rangle + g_3\alpha_2\alpha_3 \langle 572|F_R^i|220\rangle \end{aligned} \qquad \ldots\ldots 4.27$$

where g_1, g_2, g_3 are constants. The first two terms have been worked out in (i). To work out the rest we need the $\frac{1}{2}^+$ octet part of the 572 representation shown in equation (2.34).

$$\Psi_{[AB]C} = \frac{1}{2m}\sqrt{\frac{1}{6}}\left\{\epsilon_{qrs}\left[(\not{p} + m)\ \gamma_5\ C\right]_{\beta\gamma}\ N^s_{\alpha p} - \epsilon_{rps}\left[(\not{p} + m)\ \gamma_5\ C\right]_{\gamma\alpha}\ N^5_{\beta q}\right\} \qquad \ldots.4.28$$

Using (4.4), (4.5) and (4.28) we work out the respective F/D ratios due to the relevant components of current in the right hand side of (4.27). These, together with their corresponding G_A/G_V ratios are displayed in Table III.

From this table we see that the current $\langle 572|F_R^i|572\rangle_I$ has the same F/D ratio as the current $\langle 220|F_R^i|220\rangle$ and is therefore not independent.

The usual practice in the application of group theory to physics is to assume that the interaction is invariant under a group of transformations, and to classify our physical particle states into definite irreducible representations of our group so that the interaction will take a given irreducible representation into itself. However, if the interaction is approximately invariant under the group, it will induce some transition between states belonging to different irreducible representations of the group. It can then be argued that such transitions are small compared with the transitions between states of same irreducible representation. They break the group symmetry. Now we can consider the case for approximate invariance in another light. We assume, as in this problem, that our physical particle state is a mixed state of the irreducible representations of the group. Here we build in the symmetry breaking. We may then

reasonably require that our interaction takes an irreducible representation of the group into itself. This eliminates the cross terms $\langle 364|F_R^i|572\rangle$ and $\langle 220|F_R^i|572\rangle$.

This heuristic argument is supported by the fact that these cross terms give non-zero electric charge to the neutron. This can be demonstrated as follows:

We can express all the currents in (4.27) in terms of the total pseudoscalar and vector currents from which we can read off the magnetic moments and electric charges.

For $\langle 364|F_R^i|364\rangle$ and $\langle 220|F_R^i|220\rangle$ see equations (4.18), (4.19) and (4.20), (4.21).

For $\langle 220|F_R^i|572\rangle$

$$J_5^{i(tot)} = \frac{P^2}{24m^2}\left(1+\frac{2m}{\mu}\right)(\bar{N}\gamma_5 N)_{D-F} \qquad \ldots.4.29$$

$$J_\mu^{i(tot)} = \frac{1}{6}\left(1+\frac{2m}{\mu}\right)\left(\bar{N}\frac{r_\mu}{4m^2}N\right)_{D-F} + \frac{P_\mu}{6m}\left(1+\frac{q^2}{2m\mu}\right)(\bar{N}N)_{3D+F} \qquad \ldots.4.30$$

For $\langle 364|F_R^i|572\rangle$

$$J_5^{i(tot)} = -\frac{P^2}{24m^2}\left(1+\frac{2m}{\mu}\right)(\bar{N}\gamma_5 N)_{3D+5F} \qquad \ldots.4.31$$

$$J_\mu^{i(tot)} = -\frac{1}{6}\left(1+\frac{2m}{\mu}\right)\left(\bar{N}\frac{r_\mu}{4m^2}N\right)_{3D+5F} + \frac{P_\mu}{6m}\left(1+\frac{q^2}{2m\mu}\right)(\bar{N}N)_{D+\frac{1}{3}F} \qquad \ldots.4.32$$

For $\langle 572|F_R^i|572\rangle_{II}$

$$J_5^{i(tot)} = \frac{P^2}{12m^2}\left(1+\frac{2m}{\mu}\right)(\bar{N}\gamma_5 N)_{\frac{1}{2}(D+F)} \qquad \ldots.4.33$$

$$J_\mu^{i(tot)} = \frac{1}{3}\left(1+\frac{2m}{\mu}\right)\left(\bar{N}\frac{r_\mu}{4m^2}N\right)_{\frac{1}{2}(D+F)} + \frac{P_\mu}{6m}\left(1+\frac{q^2}{2m\mu}\right)(\bar{N}N)_F \qquad \ldots.4.34$$

We now assume that the electromagnetic interactions proceed via ρ, φ intermediate states, which in the Lagrangian model[13] corresponds to introducing couplings of the form $(e\mu^2/g)A_\mu\varphi_\mu$ in the U-spin scalar projection $(\varphi^3 + \varphi^8/\sqrt{3})$. Then to lowest order in e, we have for Sach's electric form factor[40]

$$(1 - \frac{q^2}{4m^2})^{-1} G_E(q^2) = \left[(1 + \frac{q^2}{2m_\mu}) \mu^2 F(q^2)\right]/(\mu^2 - q^2) \qquad \ldots\ldots 4.35$$

where $F(q^2)$ is an ad hoc form factor introduced into U(6,6) vertex. According to Oehme,[41] on account of pole dominance, at $q^2 = \mu^2$ we can replace the factor $\left[(1 + \frac{q^2}{2m_\mu}) \mu^2 F(q^2)\right]$ by the residue at μ^2. Thus (4.35) becomes

$$(1 - \frac{q^2}{4m^2})^{-1} G_E(q^2) = \left[(1 + \frac{q^2}{2m_\mu}) \mu^2\right]/(\mu^2 - q^2) \qquad \ldots\ldots 4.36$$

from which we can read directly the electric charge of the octet baryons using the D and F parts of the coefficients of P_μ in equations (4.19), (4.21) (4.30), (4.32) and (4.34). (These ratios are also shown in Table III under P_μ) The cross terms are not pure F and therefore give non-zero charge to the neutron.

Eliminating these, (4.27) becomes

$$j_R^i = \langle 364|F_R^i|364\rangle + \alpha_1 \langle 220|F_R^i|220\rangle + \alpha_2 \langle 572|F_R^i|572\rangle_{II} \qquad \ldots\ldots 4.37$$

where we have made use of the constraint (4.26).

4. Results and Conclusions.

From (4.37) and Table III we obtain

$$D/F = \frac{3 - \alpha_2 + \frac{1}{2}\alpha_3}{2 + \frac{1}{2}\alpha_3} \qquad \ldots\ldots 4.38$$

$$\left|\frac{G_A}{G_V}\right| = \frac{5 + \alpha_3 - \alpha_2}{3 + \alpha_2 + \alpha_3} \qquad \ldots.4.39$$

from which we obtain

$$\left|\frac{G_A}{G_V}\right| = \frac{F + D}{3F - D} \qquad \ldots.4.40$$

which is the same as we obtained in the case (i) equation (4.24). As we have commented, this result is bad compared with experiment, and with the result obtained by Gatto et al[38] and Harari,[37] whose consideration of $(56, L=0)^+$ and $(20, L=1)^-$ in U(3)⊗U(3) algebra of currents yielded

$$\left|\frac{G_A}{G_V}\right| = \frac{D + F}{3(D - F)} \qquad \ldots.4.41$$

which is in excellent agreement with experimental values of 1.2 for G_A/G_V and 1.75 for D/F. This good result (4.41) has been obtained by Akyeampong[42] in U(6,6) symmetry scheme by mixing $(364, L=0)^+$ and $(220, L=1)^-$. However, if in (4.38)

$$D/F = 3/2 \qquad \ldots.4.42$$

which corresponds to

$$\mu(p)/\mu(n) = -3/2 \qquad \ldots.4.43$$

a good experimental result, then (4.40) gives

$$\left|\frac{G_A}{G_V}\right| = 5/3 \quad , \qquad \ldots.4.44$$

the old SU(6) result.

Thus, in conclusion, if we assume that the physical baryons belong to mixed states of the <u>three</u> low-lying irreducible positive parity

representations of U(6,6), $(220,L=0)^+$, $(364,L=0)^+$ and $(572,L=0)^+$, then the requirements that

(a) $u_3(n) = 0$

for all currents, and that

(b) $u(p)/u(n) = -3/2$

automatically fix the ratio $|G_A/G_V|$ to 5/3. No more, no less. So far, it appears that only the mixture of the $(56,L=0)^+$ with the $(20,L=1)^-$ can really depress 5/3 to the experimental value 1.2.

It is useful to remark[9] that under mixing assumptions (i) and (ii) Johnson Treiman relations are reproduced if the parameters are chosen in such a way as to eliminate the cross terms.

Current for each I.R.	F : D for Mag. moment part of vector current (r_μ)	F : D for charge part of vector current (P_μ)	F : D for pseudo-scalar and axial vector current $\gamma_5, i\gamma_\mu\gamma_5$	$G_A : G_V$
$\langle 364 \vert F_R^i \vert 364 \rangle$	3D + F	3F	3D + 2F	5/3
$\langle 220 \vert F_R^i \vert 220 \rangle$	- D	F	- D	- 1
$\langle 572 \vert F_R^i \vert 572 \rangle_I$	- D	F	- D	- 1
$\langle 572 \vert F_R^i \vert 572 \rangle_{II}$	$\frac{1}{2}$(D + F)	F	$\frac{1}{2}$(D + F)	1
$\langle 572 \vert F_R^i \vert 364 \rangle$	- (3D + 5F)	D + $\frac{1}{3}$F	- (3D + 5F)	-
$\langle 572 \vert F_R^i \vert 220 \rangle$	D - F	- (3D + F)	D - F	-

Table III: Shows the different F/D and G_A/G_V ratios for the irreducible representations considered. The calculations are based on the expression for the multiplets given by Delbourgo et al.[13] It is worth pointing out that the magnetic moment part of the vector current has the same F/D ratio as the pseudoscalar and axial vector currents.

CHAPTER V.

ℓ-EXCITATION IN U(6)⊗U(6).

1. Introduction.

As remarked by Professor A. Salam, at the September (1965) Oxford International Conference[21] on Elementary Particles, the year 1965 was a year of rare achievement in a phenomenological correlation of the resonances. A greater portion of this achievement stems from the concept that higher resonances could be thought of as angular momentum excitation of the lower ones. We shall here restrict ourselves to the bosons. The rest symmetry subgroup U(6)⊗U(6), of U(6,6), with its magic multiplet of 36 provides the most convenient basis for this orbital excitation. Several authors, notably, Gatto et al,[43] Mahanthappa and Sudarshan,[44] and Sinanoglu[45] have considered, with some measure of success, the possible classification of higher bosons in U(6)⊗U(6)⊗O(3)$_\ell$ with certain definite approximate assumptions regarding the spin-orbit and tensor symmetry breaking terms, and the mixing angles. So far, it is not known for sure, for example, the exact value of the tensor symmetry breaking terms, and the general effect of these spin-breaking terms on the mixing angles between the T = Y = 0 octet-singlet members of the nonets.

It is therefore the purpose of this report[10] to conduct a systematic study of the above problems, deriving an approximated mass formula for the group U(6)⊗U(6)⊗O(3)$_\ell$ (for any given ℓ) by dropping certain well-defined mass tensors, and exhibiting the exact forms (values) of the symmetry breaking terms. Our approximated mass formula contains terms which determine the mixing angles.

2. Mass tensors and Mass Formula.

Throughout this work we shall assume the quantum number assignments of mesons consistent with the model of bound quark-antiquark ($q\bar{q}$) system in relative ℓ-wave. So that

$$J = \ell + s, \quad C = (-)^{\ell+s}, \quad P = (-)^{\ell+1} \qquad \ldots.5.0$$

where ℓ is the angular momentum, s the intrinsic spin, P is space parity, and C charge parity of the uncharged mesons. And we extend the concept of charge parity to the kaon by using the definition that if $C|\pi^\circ\rangle = |\pi^\circ\rangle$, then $C|K\rangle = |\bar{K}\rangle$ where K and π belong to the same multiplet. This helps to distinguish, for instance, the $^1P_1$ $K^*(1^{+-})$ and the $^3P_1$ $K^*(1^{++})$.

The physical meson state in $U(6)\otimes U(6)\otimes O(3)_\ell$ is represented by $(6,\bar{6};\ell)$. To construct the mass operator which we will require to transform like an $I = Y = J = 0$ tensor in the product space of $U(6)\otimes U(6)\otimes O(3)_\ell$, we shall use the chain reduction

$$\begin{aligned} U(6)\otimes U(6)\otimes O(3)_\ell &\rightarrow SU(6)\otimes O(3)_\ell \\ &\rightarrow SU(3)\otimes SU(2)_S\otimes SU(2)_\ell \end{aligned} \qquad \ldots.5.1$$

Consider then the reduction processes

$$(6,\bar{6};\ell)\otimes(6,6;\ell) \rightarrow 1 \oplus 35,\ 35 \oplus 1;\ 2\ell \oplus 2\ell - 1 \oplus \ldots. \oplus 0 \qquad \ldots.5.2$$

and

$$35\otimes 35 \rightarrow 1 \oplus 35_F \oplus 35_D \oplus 189 \oplus 280 \oplus \overline{280} \oplus 405 \qquad \ldots.5.3$$

$$\begin{aligned} (35,1) &\rightarrow 35 \\ (1,35) &\rightarrow 35 \\ (1,1) &\rightarrow 1 \end{aligned} \qquad \ldots.5.4$$

Charge conjugation invariance applied to (5.3) eliminates the 35_F and the 280, $\overline{280}$, and groups together the (35,1) and (1.35). Since an $SU(3)\otimes SU(2)_S$ reduction of 35,189 and 405 of SU(6):

$$35 \rightarrow (8,3) \oplus (1,3) \oplus (8,1) \qquad \ldots.5.5$$

$$189 \rightarrow (8,5) \oplus (1,5) \oplus 2(8,3) \oplus (27,1) \oplus (8,1) \oplus (1,1) \oplus \ldots. \qquad \ldots.5.6$$

$$405 \rightarrow (27,5) \oplus (8,5) \oplus (1,5) \oplus (27,3) \oplus 2(8,3) \oplus (27,1) \oplus (8,1) \oplus (1,1) \oplus \ldots. \qquad \ldots.5.7$$

gives a maximum spin of 2, to construct a scalar in total spin space, the maximum value of the irreducible representation of the orbital spin in the right hand side of (5.2) must be 2. This maximum value is already reached in the case $\ell = 1$. Thus the maximum number of mass tensors is the same for all $\ell \geq 1$. Moreover, exactly the same mass tensors operate for all $\ell \geq 1$.

The coupled state of our system in the product space is defined in terms of the uncoupled[46] state by

$$|\mu_\gamma \nu j j_3; 6,\bar{6}, \ell_s\rangle = \sum_{\substack{\nu_1,\nu_2 \\ s_3,\ell_3}} c\begin{pmatrix} 6 & 6 & u_\gamma \\ \nu_1 & \nu_2 & \nu \end{pmatrix} c\begin{pmatrix} s & \ell & j \\ s_3 & 3 & j_3 \end{pmatrix} |6\nu_1\rangle\,|\bar{6}\nu_2\rangle\,|\ell,\ell_3\rangle \qquad \ldots.5.8$$

and the mass tensor is similarly defined, to wit:

$$M^{\delta(\rho,S)}_{[\alpha,\beta;L]} = \sum_{\substack{\lambda_1\lambda_2 \\ S_3L_3}} c\begin{pmatrix} \alpha & \beta & \delta \\ \lambda_1 & \lambda_2 & \lambda \end{pmatrix} M^{\alpha}_{\lambda_1} M^{\beta}_{\lambda_2} M^{L}_{L_3}\, \delta_{SL}\, \delta_{S_3L_3} \qquad \ldots.5.9$$

where δ corresponds to some SU(6) irreducible representation in the decomposition of $\alpha\otimes\beta$, and (ρ,S) are (SU(3), SU(2)) decompositions of δ. L is some irreducible representation of $\ell\otimes\ell$ in $SU(2)_\ell$ space. Using (5.8) and (5.9) we write the transition element as

$$\langle \psi' | M | \psi \rangle = \sum C_1 \begin{pmatrix} \sigma & \bar{\sigma} & \mu_\nu \\ \nu_1 & \nu_2 & \nu \end{pmatrix} C_2 \begin{pmatrix} \sigma & \bar{\sigma} & \mu_\nu \\ \nu_1' & \nu_2' & \nu \end{pmatrix} C_3 \begin{pmatrix} \sigma & \alpha & \sigma \\ \nu_1 & \lambda_1 & \nu_1' \end{pmatrix}$$

$$\times \; C_4 \begin{pmatrix} \bar{\sigma} & \beta & \bar{\sigma} \\ \nu_2 & \lambda_2 & \nu_2' \end{pmatrix} C_5 \begin{pmatrix} \alpha & \beta & \delta \\ \lambda_1 & \lambda_2 & \lambda \end{pmatrix} C_6 \begin{pmatrix} s & \ell & j \\ s_3 & \ell_3 & j_3 \end{pmatrix}$$

$$\times \; C_7 \begin{pmatrix} s & \ell & j \\ s_3' & \ell_3 & j_3' \end{pmatrix} C_8 \begin{pmatrix} \ell & L & \ell \\ \ell_3 & L_3 & \ell_3' \end{pmatrix} \delta_{SL} \, \delta_{S_3 L_3}$$

$$\times \langle \sigma | M^\alpha | \sigma \rangle \langle \bar{\sigma} | M^\beta | \bar{\sigma} \rangle \langle \ell | M^L | \ell \rangle, \quad \ldots.5.10$$

the summation being taken over all possible magnetic quantum members.

Equation (5.10) reveals some interesting features. The coefficients C_6, C_7, C_8 are independent of the internal symmetry quantum numbers and are directly responsible for spin-splitting. This splitting falls into three categories according as L takes the values 0,1,2. For L = 0, there is no spin splitting in total spin space and all particles come out with degenerate mass in this space. The most interesting case is L = 1. In this case the spin-splitting term, denoted by ζ in Table IV shown below, is given by

$$\zeta = \underline{\ell} \cdot \underline{s} \; / \sqrt{4\ell(\ell+1)} \qquad \ldots.5.11$$

and is proportional, for a given ℓ, to the anticipated spin-orbit splitting (reference (44) and (45), and Dalitz (39)). For L = 2, we have the spin-splitting terms denoted by ξ in table IV. This surely corresponds to some tensor forces and its exact values are computed for ℓ = 1,2,3. It cannot be simply expressed in terms of J, L, S. This is not surprising since the supposed tensor forces

$$T = \frac{3(\underline{\sigma}_1 \cdot \underline{r})(\underline{\sigma}_2 \cdot \underline{r})}{r^2} - \underline{\sigma}_1 \cdot \underline{\sigma}_2 \qquad \ldots.5.12$$

unlike $\underline{L}.\underline{S}$, does not commute with J^2, L^2 and S^2 and so its eigen-values cannot be simply expressed in terms of these quantities.

The coefficients C_1, C_2, C_3, C_4, C_5 contribute to the splitting within the $U(6)\otimes U(6)$ multiplets. In working out these coefficients it is reasonable to neglect[47] contributions from the $189^{27,8}$ and $405^{27,8}$, drawing our morals from the case of SU(6).

Equation (5.10) gives[48] then, using conventional mass squared for mesons,[49] the approximated mass formula

$$M^2 = A_o + (A_1 + B_1\zeta)p + (A_2 + B_2\xi)q + (A_3 + B_3\xi)r + A_4 v + B_5\zeta + (A_6 + B_6\zeta)t \qquad \ldots.5.13$$

for any given ℓ.

Here A_o, (A_1,B_1), (A_2,B_2), (A_3,B_3), A_4, B_5, (A_6,B_6) are respectively the reduced matrix elements corresponding to

$$M^{1(1,1)}_{[1,1:1]}\ ,\quad M^{35(8,1)}_{[(35,1)+(1,35);1]}\ ,\ M^{35(8,3)}_{[(35,1)+(1,35);3]}\ ,$$

$$M^{189(1,1)}_{[35,35;1]}\ ,\ M^{189(1,5)}_{[35,35;5]}\ ,\quad M^{405(1,1)}_{[35,35;1]}\ ,\ M^{405(1,5)}_{[35,35;5]}\ ,$$

$$M^{1(1,1)}_{[35,35;1]}\ ,\ M^{35(1,3)}_{[(35,1)+(1,35);3]}\ ,\quad M^{35(8,1)}_{[35,35;1]}\ ,\ M^{35(8,3)}_{[35,35;3]} \qquad \ldots.5.14$$

and p, q, r, v, t are products of $C_1 \ldots C_5$ in (5.10) and are shown below in Table V.

3. Results and Conclusions.

From Tables IV and V, we observe[50] that if we set $A_6 = B_6 = 0$ we come out with equidistance relations,

$$^3S_1\ [K^*(1^{--}) - \rho(1^{--})] = {}^1S_o\ [K(0^{-+}) - \pi(0^{-+})]$$
$$= {}^1P_1\ [K^*(1^{+-}) - \rho(1^{+-})] \quad ....5.15$$

and

$$^3P_o\ [K^*(0^{++}) - \rho(0^{++})] = {}^3P_1\ [K^*(1^{++}) - \rho(1^{++})]$$
$$= {}^3P_2\ [K^*(2^{++}) - \rho(2^{++})] \quad ....5.16$$

The first part of (5.15) is well known and is satisfied by experiment.[51] For the second part, fitting in the masses of 891 for $^3S_1$ $K^*(1^{--})$, 769 for $^3S_1$ $\rho(1^{--})$ and 1220 for $^1P_1$ $\rho(1^{+-}) \sim B$, we come out with a satisfactory prediction of 1225 MeV for $^1P_1$ $K^*(1^{+-})$. This can be identified with the C(1215±15) resonance.

From relation (5.16), feeding in the masses 1410 for $^3P_2$ $K^*(2^{++})$, 1324 for $^3P_2$ $\rho(2^{++}) \sim A_2$, 1072 for $^3P_1$ $\rho(1^{++}) \sim A_1$ and 725 for ^3P_o $K^*(0^{++}) \sim k$, we come out with 1175 MeV for $^3P_1$ $K^*(1^{++})$. This fits exactly with the $K\pi\pi(1175)$ resonance. Also from (5.16) we predict 540 MeV for ^3P_o $\rho(0^{++})$, which falls neatly into pattern, but has not yet been seen.

The relations (5.15) and (5.16) are expected to repeat for corresponding particles as we go up the ladder. Thus for $\ell = 2$, we expect to have

$$^1D_2\ [K^*(2^{-+}) - \rho(2^{-+})] = {}^3S_1\ [K^*(1^{--}) - \rho(1^{--})]$$
$$= {}^1S_o\ [K(0^{-+}) - \pi(0^{-+})] \quad ...5.17$$

and

$$^3D_2\ [K^*(1^{--}) - \rho(1^{--})] = {}^3D_2\ [K^*(2^{--}) - \rho(2^{--})]$$
$$= {}^3D_3\ [K^*(3^{--}) - \rho(3^{--})] \quad ...5.18$$

The validity of the first part of (5.17) and of (5.18) will be a matter

for future experimental test. Some of these equal spacing relations were also obtained by Gatto, Miani and Preparata,[43] who considered only the case $\ell = 1$ in $U(6) \otimes U(6) \otimes O(3)_\ell$ "moving states" of besons.

Now from tables IV and V, it is clear that for any given ℓ, the mixing between the I = Y = 0 octet-singlet within each nonet depends not only on p, q, r, v, t, but also on ζ and ξ and on the masses of the particles belonging to the multiplets. Thus we do not expect in general to reproduce the $\eta - \chi$ or $\omega - \omega'$ type of mixing. With explicit knowledge of nine masses for any given ℓ, we can calculate the seven other masses and the four mixing angles, using for each mixing angle the formula[52]

$$\sin^2\theta = (f' - f_8) / (f' - f) \qquad \ldots.5.19$$

$$f + f' = f_1 + f_8, \qquad ff' = (f_1 \leftrightarrow f_8)^2 - f_1 f_8 \qquad \ldots.5.20$$

where f_1, f_8 represent the singlet and the octet I = Y = 0 particles of each nonet and f, f' are their corresponding physical states, $(f_1 \leftrightarrow f_8)$ given in table V is the singlet-octet transition element.

In deriving these results we have neglected the effects of possible mixing between, for example, $^1P_1$ $K^*(1^{+-})$ and $^3P_1$ $K^*(1^{++})$. The fact that our predictions fall into nice pattern with experiment justifies this neglect a posteriori, and indicates that such a mixing is expected to be small. It is interesting to notice that for $\ell = 0$, our approximated mass formula agrees with $U(6) \otimes U(6)$ mass formula of reference (50). Although our method here is fairly unwieldy and tends to make the true nature of the symmetry breaking forces less transparent, it has the sole advantage of giving the exact values of these symmetry breaking

terms, notably, the so-called tensor forces, and that, for any given ℓ. Moreover, if we believe in orbital excitation, it contains in it the ingredients that will supply most of the information needed about higher boson mass spectra, once as little as nine masses are known, for any given value of the orbital excitation.

ℓ	s	J^{PC}	Spect.Not^{n}	L = 0	L = 1, ζ	L = 2, ξ
0	0	0^{-+}	$^1S_0$	1	0	0
	1	1^{--}	$^3S_1$	1	0	0
1	0	1^{+-}	$^1P_1$	1	0	0
	1	0^{++}	$^3P_0$	1	-1	1
		1^{++}	$^3P_1$	1	$-\frac{1}{2}$	$-\frac{1}{2}$
		2^{++}	$^3P_2$	1	$\frac{1}{2}$	$1/10$
2	0	2^{-+}	$^1D_2$	1	0	0
	1	1^{--}	$^3D_1$	1	$-3/2$	$21/10$
		2^{--}	$^3D_2$	1	$-\frac{1}{2}$	$-21/10$
		3^{--}	$^3D_3$	1	1	$3/5$
3	0	3^{+-}	$^1F_3$	1	0	0
	1	2^{++}	$^3F_2$	1	-2	6
		3^{++}	$^3F_3$	1	$-\frac{1}{2}$	$-21/2$
		4^{++}	$^3F_4$	1	$3/2$	$9/2$

TABLE IV: Shows values of the spin splitting terms for $\ell = 1,2,3$. ζ and ξ are written up to factors of $1/\sqrt{4\ell(\ell+1)}$ and $1/\sqrt{4(2\ell-1)\,\ell(\ell+1)(2\ell+3)}$ respectively. These factors are however irrelevant for considerations within a given ℓ.

Particle Symbol	s	I	p	q	r	v	t
K^*	1	$\frac{1}{2}$	-1	-2	10	-1	0
ρ		1	2	-11	1	-1	3
ω^u		0	0	-32	-32	-1	0
φ^u		0	-2	-2	10	-1	0
$\omega \leftrightarrow \varphi$		0	$2\sqrt{2}$	0	0	0	0
K	0	$\frac{1}{2}$	-1	18	-18	-1	4
π		1	2	9	-27	-1	-5
χ^u		0	0	0	0	5	0
η^u		0	-2	18	-18	-1	8
$\eta \leftrightarrow \chi$		0	$2\sqrt{2}$	0	0	0	0

TABLE V.

APPENDIX

In this appendix we shall compute all possible baryon-baryon-meson vertices related to the processes

$$B' \longrightarrow B + M \qquad \text{....A.1}$$

where M is a member of the 143 irreducible representation of U(6,6) and B', B are members of the 220, 364 and 572 irreducible representations of U(6,6). First, such a computation will illustrate the procedure outlined in chapter IV. Next, if U(6,6) invariance is approximately true, the large widths of some of the newly discovered[53] resonances, notably the $N^*_{\frac{1}{2}}$ and Y^*_0, strongly suggests that they may well belong to the three low-lying irreducible representations of U(6,6), parity discrepancies notwithstanding. It is therefore worthwhile, in the manner of R. Delgourgo and M.A. Rashid,[54] to compute the coupling strenghts for such transitions as (A.1).

From the product decompositions,

$$\begin{aligned}
220 \otimes \overline{220} &\longrightarrow 1 \oplus 143 \oplus 4212 \oplus \ldots\ldots \\
364 \otimes \overline{364} &\longrightarrow 1 \oplus 143 \oplus 5940 \oplus \ldots\ldots \\
572 \otimes \overline{572} &\longrightarrow 1 \oplus 143_1 \oplus 143_2 \oplus 4212 \oplus \ldots\ldots \qquad \text{....A.2} \\
220 \otimes \overline{572} &\longrightarrow \quad 143 \oplus 4212 \oplus \ldots\ldots \\
364 \otimes \overline{572} &\longrightarrow \quad 143 \oplus 5005 \oplus \ldots\ldots
\end{aligned}$$

we see that there are unique invariant couplings for all but the process $572 \longrightarrow 572 + 143$ which has two distinct invariant couplings. These invariants are

$$\overline{\Psi}^{[ABC]} \Psi_{[ABC']} \varphi^{C'}_{C} \;, \qquad \overline{\Psi}^{\{ABC\}} \Psi_{\{ABC'\}} \varphi^{C'}_{C} \;,$$

$$\bar{\Psi}^{[AB]C}\Psi_{[AB]C'}\ \varphi_C^{C'}\ ,\quad \bar{\Psi}^{[AB]C}\Psi_{[AB']C}\ \varphi_B^{B'}\ ,$$
$$\bar{\Psi}^{[ABC]}\Psi_{[AB]C'}\ \varphi_C^{C'}\ ,\quad \Psi^{\{ABC\}}\Psi_{[AB']C}\ \varphi_B^{B'}\ . \qquad \ldots\ldots A.3$$

where

$$\Psi_{[AB]C} + \Psi_{[CA]B} + \Psi_{[BC]A} = 0 \qquad \ldots\ldots A.4$$
$$\Psi_{[AB]C} + \Psi_{[BA]C} = 0$$

The $SU(3)\otimes SU(4)$ reduction of $\underline{220}$, $\underline{364}$ and $\underline{572}$ are given by:

$$\underline{220} = (8,20') \oplus (10,4^*) \oplus (1,20)$$
$$\underline{364} = (10,20) \oplus (8,20') \oplus (1,4^*) \qquad \ldots\ldots A.5$$
$$\underline{572} = (10,20') \oplus (8,20) \oplus (8,20') \oplus (1,20) \oplus (8,4^*)$$

The decomposition in terms of the wave functions are given in equations (2.33) to (2.35).

As an example we shall compute the first term of (A.3) Writing

$$\varphi_C^{C'} = (\gamma_R)_\gamma^{\gamma'}(T^i)_r^{r'}\ \varphi_R^i \qquad \ldots\ldots A.6$$

we compute, using equation (2,35), the form

$$\bar{\Psi}^{[ABC]}\ (\gamma_R T^i)_C^{C'}\Psi_{[ABC']} =$$
$$\frac{1}{48}\Big\{\epsilon^{pqr}\Big[C^{-1}\gamma_\mu\frac{(\not{r}' + m)}{m}\Big]^{\alpha\beta}\bar{\Delta}_\mu^{\gamma} + \sqrt{\frac{2}{3}}\Big[C^{-1}\gamma_5\frac{(\not{p}' + m)}{m}\Big]^{\alpha\beta}$$
$$\times\ (\epsilon^{rps}\bar{\nu}_s^{\gamma,q} + \epsilon^{rqs}\bar{\nu}_s^{\gamma,p}) + \text{cy.A,B,C}\Big\}$$
$$\times\ (\gamma_R)_\gamma^{\gamma'}(T^i)_r^{r'}\Big\{\epsilon_{pqr'}\Big[\frac{(\not{p} + m)}{m}\gamma_\mu C\Big]_{\alpha\beta}\Delta_\mu\gamma'$$
$$+ \sqrt{\frac{2}{3}}\Big[\frac{(\not{p} + m)}{m}\gamma_5 C\Big]_{\alpha\beta}(\epsilon_{r'pt}\nu^t_{\gamma',q} + \epsilon_{r'qt}\nu^t_{\gamma',p}) + \text{cy.A,B,C}\Big\}$$
$$\ldots\ldots A.7$$

where Δ_μ is a spin $\frac{3}{2}^+$ singlet and ν a spin $\frac{1}{2}^+$ octet. The SU(3) indices are trivially contracted using

$$\epsilon^{pqr}\epsilon_{stu} = (\delta^p_s\,\delta^q_t\,\delta^r_u + \delta^p_t\,\delta^q_u\,\delta^r_s + \delta^p_u\,\delta^q_s\,\delta^r_t) - (\delta^p_s\,\delta^q_u\,\delta^r_t + \delta^p_t\,\delta^q_s\,\delta^r_u + \delta^p_u\,\delta^q_t\,\delta^r_s) \qquad \ldots\ldots A.8$$

$$\epsilon^{pqr}\epsilon_{ptu} = \delta^q_t\,\delta^r_u - \delta^q_u\,\delta^r_t \qquad \ldots\ldots A.9$$

$$\epsilon^{pqr}\epsilon_{pqu} = 2\,\delta^r_u \qquad \ldots\ldots A.10$$

We consider in detail the contraction of the U(4) indices. For $\Delta_\mu \rightarrow \Delta_\mu$, $\Delta_\mu \rightarrow \nu$ transitions, we have,

$$\bar{\Delta}^\gamma_\mu\left[C^{-1}\,\gamma_\mu(\not{p}' + m')\right]^{\alpha\beta}\,(\gamma_R)^{\gamma'}_{\gamma}\left\{\left[(\not{p} + m)\,\gamma_\nu\,C\right]_{\alpha\beta}\,\Delta_{\nu,\gamma'} + \left[(\not{p} + m)\,\gamma_5\,C\right]_{\beta\gamma'}\,\nu_\alpha\right\}$$

$$= \Delta_\mu\,\gamma_R\,\Delta_\nu\,S_p\left\{(\not{p}' + m')(\not{p} + m)\,\gamma_\nu\gamma_\mu\right\} + \Delta_\mu\gamma_R(\not{p} + m)\gamma_5\gamma_\mu(\not{p}' + m)\nu \qquad \ldots\ldots A.11$$

and for $\nu \longrightarrow \nu$ transitions,

$$\bar{\nu}^\gamma\left[C^{-1}\,\gamma_5(\not{p}' + m')\right]^{\alpha\beta}\,(\gamma_R)^{\gamma'}_{\gamma}\left\{\left[(\not{p} + m)\,\gamma_5\,C\right]_{\alpha\beta}\,\nu_{\gamma'} + \left[(\not{p} + m)\,\gamma_5\,C\right]_{\beta\gamma'}\,\nu_\alpha\right\}$$

$$= \bar{\nu}\,\gamma_R\,\nu\,S_p\left\{(\not{p} + m)(\not{p}' + m')\right\} - \bar{\nu}\,\gamma_R\,(\not{p} + m)(\not{p}' + m')\,\nu \qquad \ldots\ldots A.12$$

and

$$\bar{\nu}^{\alpha} \left[C^{-1} \gamma_5 (\not{p}' + m')\right]^{\beta\gamma} (\gamma_R)_{\gamma}^{\gamma'} \left[(\not{p} + m) \gamma_5 C\right]_{\beta\gamma'} \nu_{\alpha}$$

$$= \bar{\nu} \nu S_p \left\{(\not{p}' + m') \gamma_R (\not{p} + m)\right\} \qquad \ldots\ldots A.13$$

where p', p are the momenta of outgoing and incoming baryons respectively.

Taking into account the B.W. equations for the mesons, we can define the analogue of the total current equations (4.17) and (4.18) in terms of (γ_R),

$$\gamma_5^{tot} = \frac{1}{\mu} (\not{q} + \mu) \gamma_5 \qquad \ldots\ldots A.14$$

$$\gamma_\mu^{tot} = \frac{1}{\mu}(\not{q} + \mu)\,\gamma_\mu \qquad \ldots\ldots A.15$$

where μ is the mass of the meson. Plugging (A.14) and (A.15) into (A.11), (A.12) and (A.13), and using the relations

$$q = p' - p, \quad P = p' + p,$$

$$r_\mu = \epsilon_{\mu\nu k\lambda}\, P_\nu\, q_k\, \gamma_\lambda\, \gamma_5 \qquad \ldots\ldots A.16$$

$$p_\mu\, \Delta_\mu = q_\mu\, \varphi_\mu = 0 \qquad \ldots\ldots A.17$$

and the identity

$$\bar{u}(p')\left[\frac{p'_\mu}{m'} + \frac{p_\mu}{m} - \left(1 + \frac{p\cdot p'}{mm'}\right)\gamma_\mu\right] u(p) = -\,\bar{u}(p')\,\frac{r_\mu}{2mm'}\,u(p) \qquad \ldots\ldots A.18$$

we obtain, for $220 \to 220 + 143$,

$$\begin{aligned} g^{-1}\, J_5^i = &-\frac{1}{12m^2\nu}\, P^2\left(1 + \frac{2m}{\mu}\right)(\bar{\nu}\,\gamma_5\,\nu)_D - T_r(T^i)\,\nu\,\gamma_5\,\nu \\ &- \frac{1}{3\sqrt{2}\,mm'}\left[\left(1 + \frac{m+m'}{\mu}\right) m\, q_\mu\, \bar{\nu}\, T^i\, \Delta_\mu + h.c\right] \\ &+ \frac{1}{12m^2}\left(1 + \frac{2m}{\mu}\right)\left[P^2\, \Delta_\mu\, \gamma_5\, \Delta_\mu + (P_\mu q_\nu - q_\mu P_\nu)\,\bar{\Delta}_\mu\, \gamma_5\, \Delta_\nu\right] T_r(T) \end{aligned} \qquad \ldots\ldots A.19$$

$$\begin{aligned} g^{-1}\, J_\lambda^i = &-\frac{1}{3}\left(1 + \frac{2m}{\mu}\right)\left[\left(\bar{\nu}\,\frac{r_\lambda}{4m^2}\,\nu\right)_D - T_r(T^i)\left(\nu\,\frac{r_\lambda}{4m^2}\,\nu\right)\right] \\ &+ \frac{1}{3}\,\frac{P_\lambda}{2m}\left(1 + \frac{q^2}{2m\mu}\right)\left[(\bar{\nu}\nu)_F + T_r(T^i)\,\bar{\nu}\nu\right] \\ &+ \frac{1}{3}\sqrt{2}\left(1 + \frac{m+m'}{\mu}\right)\left[\left(1 + \frac{p\cdot p'}{mm'}\right) g_{\mu\lambda}\,\bar{\nu}\, T^i\, \gamma_5\, \Delta_\mu\right. \\ &\qquad - \frac{1}{2mm'}\, P_\lambda q_\mu \bar{\nu} T^i \gamma_5 \Delta_\mu + \frac{1}{m'}\, q_\mu \bar{\nu} T^i \gamma_5 \gamma_\lambda \Delta_\mu \\ &\qquad \left. + h.c\right] \\ &+ \frac{1}{12m^2}\left[\left(1 + \frac{2m}{\mu}\right)\left\{P^2\,\bar{\Delta}_\mu\,\gamma_\lambda\,\Delta_\mu + (P_\mu q_\nu - q_\mu P_\nu)\,\bar{\Delta}_\mu\,\gamma_\lambda\,\Delta_\nu\right\}\right. \\ &\left. - \frac{P_\lambda}{\mu}\left\{P^2\,\bar{\Delta}_\mu\,\Delta_\mu + (P_\mu q_\nu - q_\mu P_\nu)\,\bar{\Delta}_\mu\,\Delta_\nu\right\}\right]\, T_r(T^i) \end{aligned} \qquad \ldots\ldots A.20$$

For $572 \longrightarrow 220 + 143$:

$$g^{\circ} J^{i}_{5} = \frac{1}{12}\left(1 + \frac{m+m^{\circ}}{\mu}\right)\Big\{\left(1 + \frac{p\cdot p^{\circ}}{mm^{\circ}}\right)\Big[(\bar{N}\gamma_5\,\nu)_{D-F} - 2T_r(T^i)\,\bar{N}\gamma_5\,\nu$$
$$+\frac{2}{\sqrt{6}}\,\bar{y}T^i\,\gamma_5\,\nu - 2\bar{\mathcal{D}}T^i\,\gamma_5\,\nu + \frac{6}{\sqrt{2}}\,\bar{N}_\mu\,(T^i)\,\gamma_5\,\Delta_\mu\Big]$$
$$+\frac{2\sqrt{3}}{m^{\circ}}\,q_\mu\,(\bar{N}T^i\,\Delta_\mu + \frac{1}{\sqrt{6}}\,\bar{y}\Delta_\mu\,Tr(T^i) + \sqrt{3}\,\bar{N}_\mu\,T^i\,\nu)$$
$$x\,\frac{\sqrt{3}}{mm^{\circ}}\,(P_\mu q_\nu - q_\mu P_\nu)\,\bar{N}_\mu\,T^i\,\gamma_5\,\Delta_\nu\Big\} \qquad \ldots\ldots A.21$$

$$g^{-1} J^{i}_{\lambda} = \frac{1}{12}\left(1 + \frac{m+m^{\circ}}{\mu}\right)\Big\{\left(1 + \frac{p\cdot p^{\circ}}{mm^{\circ}}\right)\Big[\frac{4}{\sqrt{6}}\frac{P_\lambda}{\mu}\,\bar{y}T^i\,\nu + \frac{P_\lambda}{2\mu}\,\bar{\mathcal{D}}T^i\,\nu + 2\sqrt{6}\,\bar{N}_\lambda\,\gamma_5\,T^i\,\nu$$
$$+\sqrt{3}\,\bar{N}_\mu\,T^i\,\gamma_\lambda\,\Delta_\mu - \frac{2}{\sqrt{3}}\,\bar{y}\gamma_5\,\Delta_\lambda\,T_r(T^i) + 2\sqrt{2}\,\bar{N}\,T^i\,\gamma_5\,\Delta_\lambda$$
$$+\left(\frac{p^{\circ}_\lambda}{m^{\circ}} + \frac{P_\lambda}{m}\right)\Big[(\bar{N}\nu)_{3D+F} - 2T_r(T^i)\,\bar{N}\,\nu - \frac{4}{\sqrt{6}}\,\bar{y}T^i\,\nu - \tfrac{1}{2}\bar{\mathcal{D}}\,T^i\,\nu\Big]$$
$$+\frac{r_\lambda}{2mm^{\circ}}\Big[(\bar{N}N)_{D-F} - T_r(T^i)\,\bar{N}\nu + \frac{1}{3}\,\bar{y}T^i\,\nu - \tfrac{1}{2}\bar{\mathcal{D}}T^i\,\nu\Big]$$
$$\left(\frac{2q_\mu\gamma_\lambda\gamma_5}{m} - \frac{P_\lambda q_\mu}{2m^{\circ}m}\,\gamma_5\right)\Big[\,\bar{N}_\mu\,T^i\,\nu + \frac{1}{\sqrt{3}}\,\bar{y}\Delta_\mu\,T_r(T^i) - \sqrt{2}\,\bar{N}T^i\,\Delta_\mu\Big]$$
$$+\frac{1}{3\sqrt{3}mm^{\circ}}\,(P_\mu q_\nu - q_\mu P_\nu)\,\bar{N}_\mu\,T^i\,\gamma_\lambda\,\Delta_\nu$$
$$-\frac{1}{12}\frac{P_\lambda}{\mu}\,(1 + \frac{p\cdot p^{\circ}}{mm^{\circ}})\Big[(\bar{N}N)_{3D+F} - 2T_r(T^i)\,\bar{N}\,\nu + \sqrt{3}\,\bar{N}_\mu\,T^i\,\Delta_\mu\Big]$$
$$-\frac{1}{8\sqrt{3}mm^{\circ}\mu}\,P_\lambda(P_\mu q_\nu - q_\mu P_\nu)\,\bar{N}_\mu\,T^i\,\Delta_\nu \qquad \ldots\ldots A.22$$

For $572 \longrightarrow 572 + 143$, the first coupling,

$$g^{-1}_{1} J^{i}_{5} = \frac{1}{12}\left(1 + \frac{m+m^{\circ}}{\mu}\right)\Big\{\left(1 + \frac{p\cdot p^{\circ}}{mm^{\circ}}\right) - (\bar{N}N)_D + T_r(T^i)\,\bar{N}N + \frac{1}{\sqrt{6}}\bar{N}T^i\,\gamma_5 y$$
$$+\frac{1}{\sqrt{2}}\bar{N}T^i\,\gamma_5\mathcal{D}\Big] + \frac{\sqrt{6}}{m^{\circ}}\,q_\mu\,\bar{N}_\mu\,T^i N + \frac{1}{2m}\,\bar{N}_\mu\,T^i\,y$$

$$+\frac{2}{3mm^{\circ}}(P^2\delta_{\mu\nu}+P_\mu q_\nu-P_\nu q_\mu)\bar{N}_\mu T^i\gamma_5 N_\nu\Big\}+h.c$$

....A.23

$$g_1^{-1}J_\lambda^i=\frac{1}{3}(1+\frac{m+m^{\circ}}{\mu})\Big\{(1+\frac{p.p^{\circ}}{mm^{\circ}})\Big[\bar{N}_\lambda T^i\gamma_5 Y-\frac{3}{\sqrt{6}}\bar{N}_\lambda T^i\gamma_5 N$$

$$\Big(-\frac{P_\lambda}{\mu}+\frac{p_\lambda^{\circ}}{m^{\circ}}+\frac{p_\lambda}{m}\Big)\Big[-\frac{\sqrt{6}}{4}\bar{N}T^i Y+\frac{1}{2\sqrt{2}}\bar{N}T^i\Theta\Big]$$

$$\Big(-\frac{P_\lambda q_\mu\gamma_5}{2mm^{\circ}}+\frac{q_\mu}{m^{\circ}}\gamma_\mu\gamma_5\Big)\Big[\sqrt{\frac{3}{6}}\bar{N}_\mu T^i N+\bar{N}_\mu T^i Y\Big]$$

$$+\frac{r_\lambda}{4mm^{\circ}}\Big[T_r(T^i)\bar{N}N-(\bar{N}N)_D+\frac{1}{\sqrt{6}}\bar{N}T^i Y+\sqrt{\frac{1}{2}}\bar{N}T^i\Theta$$

$$+\frac{3}{4mm^{\circ}}(P^2\delta_{\mu\nu}+P_\mu q_\nu-P_\nu q_\mu)\gamma_\lambda\bar{N}_\mu T^i N_\nu\Big\}$$

$$-\frac{P_\lambda}{4mm^{\circ}\mu}(P^2\delta_{\mu\nu}+P_\mu q_\nu-P_\nu q_\mu)\bar{N}_\mu T^i N_\nu$$

$$+\frac{P_\lambda}{6m}(1+\frac{q^2}{2m\mu})\Big\{T_r(T^i)\bar{N}N+(\bar{N}N)_F\Big\}+h.c \qquad ....A.24$$

For the second coupling of $572\longrightarrow 572+143$,

$$g_2^{-1}J_5^i=\frac{1}{12}(1+\frac{m+m^{\circ}}{\mu})\Big\{(1+\frac{p.p^{\circ}}{mm^{\circ}})\Big[(\bar{N}\gamma_5 N)_{D+F}+\sqrt{\frac{5}{6}}\bar{N}T^i\gamma_5 Y$$

$$+\sqrt{\frac{1}{2}}\bar{N}T^i\gamma_5\Theta\Big]$$

$$+\frac{q_\mu}{m}\Big[\bar{N}_\mu T^i Y-3\sqrt{2}\bar{N}_\mu T^i\Theta+\sqrt{6}\,T_r(T^i)\bar{N}_\mu N-(\bar{N}_\mu N)_D^i\Big]$$

$$+\frac{1}{4mm^{\circ}}\epsilon_{\nu\mu\rho\sigma}\Big[p_\rho p_\sigma'+\frac{m^{\circ}}{\mu}(p_\rho p_\sigma^{\circ}-p_\rho^{\circ}p_\sigma)\Big]\Big\{T_r(T^i)\bar{N}_\mu N_\nu-\bar{N}_\mu N_\nu T^i\Big\}$$

....A.25

$$g_2^{-1}J_\lambda^i=\frac{1}{12}(1+\frac{m+m^{\circ}}{\mu})\Big\{(1+\frac{p.p^{\circ}}{mm^{\circ}})\gamma_5\Big[\sqrt{6}\,T_r(T^i)\bar{N}_\mu N-\sqrt{6}(\bar{N}_\mu N)_D$$

$$+N_\mu T^i Y\Big]$$

$$(-\frac{1}{m^{\circ}}q_\mu\gamma_\lambda\gamma_5+\frac{p_\lambda^{\circ}q_\mu\gamma_5}{mm^{\circ}}-\frac{p_\lambda^{\circ}}{m^{\circ}\mu})\Big[\sqrt{6}\,T_r(T^i)\bar{N}_\mu N-\sqrt{6}(\bar{N}_\mu N)_D$$

$$+ \bar{N}_\mu T^i \mathcal{Y} \Big]$$

$$+ \frac{r_\lambda}{2mm^\circ}\Big[(\bar{N}N)_{D+F} + \frac{5}{\sqrt{6}}\bar{N}T^i\mathcal{Y} + \frac{1}{\sqrt{2}}\bar{N}T^i\Phi\Big]$$

$$+ \frac{1}{\sqrt{2}}\Big(\frac{p^\circ_\lambda}{m^\circ} + \frac{p_\lambda}{m}\Big)\Big[\sqrt{3}\bar{N}T^i\mathcal{Y} - \bar{N}T^i\Phi\Big]\Big\}$$

$$+ \frac{1}{6m}\Big(1 + \frac{q^2}{2m\mu}\Big)P_\lambda\Big\{T_r(T^i)\bar{N}N + (\bar{N}N)_F\Big\}$$

$$\frac{1}{4m^\circ}\Big[\Big(1 + \frac{m^\circ}{\mu}\Big)(q_\mu\,\delta_{\lambda\nu} - q_\nu\,\delta_{\lambda\nu}) + P_\lambda\,\delta_{\nu\mu}\Big]\Big\{T_r(T^i)\bar{N}_\mu N_\nu - \bar{N}_\mu N_\nu T^i\Big\}$$

$$- \frac{1}{2\sqrt{2}mm^\circ}\epsilon_{\lambda\mu\rho\sigma}\Big\{\Big(1 + \frac{m^\circ}{\mu}\Big)p_\rho p^\circ_\sigma - \frac{m}{\mu}p_\sigma p^\circ_\rho\Big\}\bar{N}_\mu T^i\Phi$$

$$+ \Big(1 + \frac{p\cdot p^\circ}{mm}\Big)\frac{P_\lambda}{4\mu}\Big[-\frac{1}{\sqrt{6}}\bar{N}T^i\mathcal{Y} + \frac{1}{3\sqrt{2}}\bar{N}T^i\Phi\Big] \qquad \ldots\ldots A.26$$

where m, m° are respectively the masses of the incoming and outgoing baryons at each vertex. We have not included here the results for $572 \to 364 + 143$ and $364 \to 364 + 143$ as these have been published in references (54) and (3) respectively. The presence of the characteristic factor $\big[1 + (m + m^\circ)/\mu\big]$ which enters through the equations of motion, and the appearance of the same F/D ratio in the magnetic moment part of the vector current as well as in the pseudoscalar current, are noteworthy.

Here, the particle symbols $\mathcal{D}$, $\mathcal{N}_\mu$, $\mathcal{N}$, $\mathcal{Y}$ are respectively decuplet of spin $\frac{1}{2}^*$, octet of spin $\frac{3}{2}^*$, octet of spin $\frac{1}{2}$ and singlet of spin $\frac{1}{2}$, all belonging to the 572 irreducible representation of U(6,6).

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+ Unless otherwise stated, throughout this discussion we use the metric (1 - 1 - 1 - 1). All Greek indices of type μ, ν run from 0 to 3

and all Latin indices of type i, j run from 1 to 3. Summation is implied for repeated indices.

++ See chapter II on the classification of the subgroups of U(2,2).

* For detailed calculation, see Appendix.

** λ, μ are the constant coefficients of the auxiliary operators associated with creation and annihilation operators respectively, which appear in $\psi_\alpha(x)$.