

TRACTION INDUCED COMPACTION OF MAIZE POWDER

by

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TO MY PARENTS AND MY FIANCE'

Preface

This dissertation is a description of the work carried out in the Department of Chemical Engineering and Chemical Technology, Imperial College, University of London, between October 1983 and October 1986. Except where acknowledged, the material presented is the original work of the author and includes nothing which is the outcome of work done in collaboration, and no part of it has been submitted for a degree at any other university.

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Abstract

The thesis reports the results of an experimental and analytical study of the compaction of maize powder in the feed zone of a typical single screw extruder. The thesis is in three interconnected parts. A direct experimental study of the compaction zone is described for a particular extruder over a range of operating conditions. The main measurements include the extent of the compaction zone and the wall stress generated. The major part of the thesis, however, deals with a series of small scale simulations which attempt to reproduce the essential and controlling compaction variables. The central argument is that the feed zone behaves as a traction induced compaction process. The relative movement of the screw and barrel is thought of as providing a net normally acting compaction stress in the direction of the extruder channel. The problem is amenable to analysis using established but extended compaction theories. The necessary data to test these models has been obtained from a series of friction and uniaxial compaction data. These data have been studied in detail as a rather separate exercise and physically significant and usefully predictive models are described. These studies have also considered the effects of moisture and temperature particularly upon the magnitude of the wall friction.

Incorporating the compaction and friction data into the traction induced compaction model provides a quantitative prediction of the feed zone characteristics which have been compared with the extruder feed zone measurements. The

predictions provided by the model closely follow the observed trends although it has not been possible to explore fully the range of variables included in the traction induced compaction model. In some cases, the predicted pressure profiles are remarkably close to the experimentally observed profile. The study does however indicate that a series of relatively simple measurements of friction and uniaxial compaction may be used to provide a first order quantitative prediction of the extent of the compaction zone and wall pressure profile within a single screw extruder.

NOTATION

Chapter 1

- F - Volumetric flow rate of solids
- V_{pl} - Velocity of the solid plug
- D - Outside diameter of the screw
- D_b - Diameter of the extruder barrel
- D_s - Diameter of the extruder screw
- H - Channel depth
- e - Flight width
- p - No. of flights
- $\bar{\theta}$ - Mean helix angle
- θ_b - Helix angle at the barrel
- θ_s - Helix angle at the screw
- ϕ - Solids conveying angle
- V_b - Velocity of the barrel
- μ_s - Coefficient of friction at the screw
- μ_b - Coefficient of friction at the barrel
- K_1, K_2 - Anisotropic pressure constants at barrel, flight edge and
and K_3 screw root
- W - Channel width
- Z_b - Barrel length
- P - Pressure
- K_w - Stress transmission ratio

Chapter 2

- F - Friction force

τ - Interfacial shear strength
 $A_{app.}$ - Apparent area of contact
 A_{real} - Real area of contact
 W - Normal load
 P_0 - Flow stress
 μ - Coefficient of friction
 k - Constant
 n - Load index
 R_s - Radius of sphere
 ν_1, ν_2 - Poisson's ratios for substrates 1 and 2
 E_1, E_2 - Young's moduli for substrates 1 and 2
 m - Contact area scalar
 ω_i - Load per particle
 N - Number of asperities
 E^1 - Reduced Young's modulus
 P_0 - Local flow stress
 σ - Mean deviation of asperity heights
 β - Radius of the tips of asperities
 ψ - Plasticity index
 τ_0 - Intrinsic shear stress (eq. 2.5)
 α - Pressure coefficient (eq. 2.5)
 P - Contact pressure
 τ_0' - Intrinsic shear stress (eq.2.7)
 α' - Pressure coefficient (eq. 2.7)
 τ'' - Constant (eq. 2.10)
 Q - Activation energy
 R - Universal gas constant
 T - Absolute temperature

Chapter 3

τ - Shear stress

σ_n - Normal stress

σ_0 - Pre-consolidation stress

C - Cohesion

ϕ_i - Angle of internal friction

ϕ_w - Angle of wall friction

ϕ_E - Effective angle of internal friction

T_i - Tensile strength

n - Shear index

σ_{xx}, σ_{yy} - Normal stresses in x and y directions

τ_{xy}, τ_{yx} - Shearing stresses

τ_w - Shearing stress at the wall

θ - Inclination of the major principal stress with the x axis

θ_{wall} - Inclination of the major principal stress with the x axis
at the wall

X, Y, Z - Body forces per unit volume

K_A, K_P - Coefficients of active and passive pressures

P - Mean of the principal stresses

K_{WA}, K_{WP} - Coefficients of active and passive pressures at the
wall

R_w - Interior normal reaction

γ - Weight density

σ_w - Wall stress

h - Height

α - Inclination of the slip plane

D - Diameter

μ_{ω} - Coefficient of friction at the wall
 $\sigma_{yy_{\omega}}$ - Vertical stress at the wall
 $\overline{\sigma_{yy}}$ - Mean vertical stress
 $\mathcal{D}$ - Distribution factor
 $\sigma_{xx_{\omega}}$ - Horizontal stress at the wall

Chapter 4

σ_0 - Pre-consolidation stress
 h - Gap separation in cone and plate geometry
 a - Radius of cone
 φ - Tilt
 χ - Off-centre
 H - Gap set
 S - Local dilatancy paramter
 τ_0 - Intrinsic shear stress (eq. 2.5)
 α - Pressure coefficient (eq. 2.5)
 τ_0' - Intrinsic shear stress (eq. 2.7)
 α' - Pressure coefficient (eq. 2.7)
 P_{app} - Normal pressure based on the apparent area of contact
 R_c - Radius of an asperity
 μ_{ω} - Coefficient of friction at the wall

Chapter 5

A_r - Real contact area
 E - Young's modulus
 E_p - Particle modulus

τ - Interface shear stress
 τ_R - Rupture stress
 F - Friction force
 W - Normal load
 n' - Contact density
 R_p - Radius of particle
 R_s - Radius of compact hemisphere
 n - load index
 m - Contact area scalar
 P_o - flow stress
 μ - coefficient of friction
 T - Interface temperature
 P - Normal contact pressure
 α' - Pressure coefficient (eq. 2.7)
 τ_o' - Intrinsic shear stress (eq. 2.7)

Chapter 6

Q - Surcharge
 σ_{yy} - Vertical stress
 $\overline{\sigma_{yy}}$ - Mean vertical stress
 σ_{yy_w} - Vertical stress at the wall
 σ_{yy_w} - Horizontal stress at the wall
 ξ - Ratio of real to apparent contact area
 τ_o' - Intrinsic shear stress
 α' - Pressure coefficient
 D - Diameter
 H - Height

$\mathcal{D}$ - Distribution factor
 K_{WA}, K_{WP} - Active and pasive pressure coefficients at the wall
 A_r - Real area of contact
 A_c - Area of a single contact
 A_{app} - Apparent area of contact
 R_c - Radius of a single asperity
 V_c, V_s - Poisson's ratio of an asperity and surface repectively
 n_c - Total no. of contacts
 W_T - Total normal load
 P_{app} - Contact pressure based on the apparent area of contact
 n' - No. of contacts per unit area of A_{app}
 n - Load index
 μ_ω - Coefficient of wall friction
 μ_i - Coefficient of internal friction
 ϕ_E - Effective angle of internal friction
 ϕ_ω - Angle of wall friction
 F_o, F_t - Force before decay and decaying force after time t
 E_A - Asymptotic modulus
 K_ω - Ratio of horizontal to vertical stresses

Chapter 7

Tr - Torque
 α - Cone angle
 R - Radius of the plate
 τ - Shear stress.

φ - Tilt
 χ - Off-centre
 H - Gap set
 S - Local dilatancy parameter
 τ_o - Intrinsic shear stress (eq. 2.5)
 α - Pressure coefficient (eq. 2.5)
 τ_o' - Intrinsic shear stress (eq. 2.7)
 α' - Pressure coefficient (eq.2.7)
 P_{app} - Normal Pressure passed on the apparent area of contact
 R_c - Radius of an asperity
 μ_w - Coefficient of friction at the wall

Chapter 8

μ_b - Coefficient of friction at the barrel
 μ_s - Coefficient of friction at the screw
 K_w - Stress transmission ratio
 ϕ - Conveying angle
 T_i - Oil bath set temperature
 T_o - Discharge temperature of the oil
 T_s - Screw surface temperature
 $\dot{Q}_{conv}$ - Rate of heat flow by convection
 $\dot{Q}_{cond}$ - Rate of heat flow by conduction
 $\dot{Q}_{total}$ - Total rate of heat flow
 h_i - Inside film heat transfer coefficient
 r_{is} - Inside screw radius
 r_{os} - Outside screw radius
 L - Length of screw

K_m - Thermal conductivity of the metal
 K_{oil} - Thermal conductivity of the oil
 d_{is} - Inside screw diameter
 ρ - Density of oil
 μ - Viscosity of oil
 C_p - Specific heat capacity of the oil
 v - Velocity of oil
 $\mathcal{D}$ - Distribution factor
 ξ - Ratio of real to apparent contact area
 K_1 - Fraction of the mean axial pressure acting on the barrel surface
 K_2 - Fraction of the mean axial pressure acting on the flight surface
 K_3 - Fraction of the mean axial pressure acting on the screw surface
 P_i - Inlet pressure to feed zone
 h - Channel depth
 σ_{xx_b} - Radial stress at the barrel wall
 τ_o' - Intrinsic shear stress (eq. 2.7)
 α' - Pressure coefficient (eq. 2.7)
 $\overline{\sigma_{yy}}$ - Mean axial stress
 r_s - Mean screw radius
 r_b - Mean barrel radius
 P_{max} - Maximum pressure developed in the melt zone
 P_o - Exit axial from the transition zone
 μ_m - Melt viscosity
 v - Velocity of extruder
 H_o - Channel depth at the beginning of the extruder

H_1 - Channel depth at the end of the extruder

L_f - Length of feed zone

P_D - Die Pressure

ϕ_i - Angle of internal friction.

Chapter 1

Introduction

This thesis describes a study of the feed zone of a single screw extruder which is regarded as a problem in traction induced dynamic compaction. A simple analytical model has been developed from an acquired understanding of the friction and compaction characteristics of a model powder and the predictions of this model have been tested by experimental studies on a feed zone. This chapter introduces the relevant precedents in food extrusion followed by a discussion of appropriate work published to-date on solids flow. The final section outlines the methods adopted in this study and in the modelling of the extruder feed zone.

1.1. *Extrusion of foods*

Extrusion is a process whereby a highly deformable solid is forced or pushed through an orifice. The process is widely used commercially and conveys the idea of high pressures and invariably involves consolidation and shear induced mixing. Several extruder designs are commercially available, but the central theme of this thesis centres around the modelling of parts of a single screw extruder. Single screw extruders were first used for extrusion of plastics in the early 1930's and their use in food processing dates back to 1935 when a single screw extruder was first applied for the continuous extrusion of pasta products⁽¹⁾. These

extruders basically consist of a flighted screw rotating within a barrel. The "pushing" action of the flights conveys the material forward and thereby creates the necessary compaction pressure , which forces or extrudes the material through a suitable die.

Originally, for example in the processing of synthetic polymers, the material was "softened" within the extruder by selectively heating the barrel to facilitate plastic flow within the channels. The material transformations in this case are essentially reversible although highly oriented materials are often produced. In the processing of food these thermal gradients provide a means of cooking or chemically modifying the ingredients. It is this feature which is attractive commercially as continuous mixing, consolidation, cooking and forming can be achieved within one unit. Clearly, the introduction of these additional features makes the control of this equipment more difficult and the rapid expansion of the use of single screw extruders in the food processing industry has shown the need for a better basic understanding of extrusion cooking processes. During cooking, extrusion heating is generally introduced through jackets surrounding the barrel and/or by the mechanical energy dissipated as a result of the shearing motion of the material. The discharge temperatures may exceed 200°C and the residence time varies between 60 and 270 seconds for a typical starch processing unit. This high temperature (HT) processing capability of extruders combined with short residence times (ST) has been termed HTST processing. At present this HTST process is utilized in a myriad of operations such as the preparation of snacks, cereals, pasta foods, confectionary products, pet foods, meat analogues and beverage powders. Cooking extruders come in a variety of sizes

and shapes and normally have the capability to vary the screw, barrel and die configurations to tailor the machine to a specific product's processing requirements⁽²⁾.

Single screw extruders are commonly thought of as having three functional zones (Fig. 1.1). The solid particulate material enters the "feed zone" through a hopper. The flow of the solid particles in the hopper is usually induced by gravitational forces, but at times forced feeding with an auxiliary screw arrangement is used. In this section, mixing and compaction of the feed occurs and in some cases heat is introduced from the surroundings. The conveying mechanism in the feed zone is governed by the interfacial tractions generated between the feed material and the confining metal surfaces. The frictional and mechanical work produces heating which, combined with the external heat and the local pressures generated, transforms the powder into a compacted solid with some molten regions. The feed section is considered to end where the solids begin to undergo gross melting. Complete melting is achieved in the "transition region" where the cooking and final mixing processes occur. The length of this zone is a function not only of the screw geometry, but also the operating conditions and the physical properties of the solid⁽³⁾. The conveying is now governed largely by viscous forces. The mechanism which causes conveying in the transition section was first investigated qualitatively by Street⁽⁴⁾ and a theoretical model describing the process was developed later by the same author. In the early years, the theoretical modelling of this section was largely neglected partly due to the difficulties associated with analysing the flow behaviour of a two phase system. In recent years this section has received a great deal of

attention⁽³⁾⁽⁵⁾⁽⁶⁾⁽⁷⁾⁽⁸⁾ and this effort has been directed principally at predicting the length of screw required to ensure the completion of the melting process⁽⁹⁾. The fully molten melt is then conveyed by the "metering section" to the extruder die and this final zone controls, in part, the pressure distribution along the extruder and in particular in the feed zone. The level of the understanding of the metering zone has now become highly sophisticated. The modelling of this section is based on the application of established fluid mechanics albeit in a complicated environment. However, these models are restricted in various degrees by their respective simplifying assumptions, the most important of which are the assumptions of steady state, the neglect of body forces, a developed incompressible laminar type of flow and the assumption of zero interface slip⁽³⁾. With these assumptions, the complex geometry of the confining solid surfaces, that is the barrel and screw, is amenable to simplification and flow may be considered as that between two parallel plates with one surface moving relative to the other. The basic problem in describing the physical processes in the transition and the metering section is that the flow is often non-Newtonian and non-isothermal. Therefore a complete solution can only be arrived at by solving the equation of motion as well as the equation of energy. The solution to these equations is feasible if the cited assumptions and simplifications are adopted and successful modelling of the flow behaviour of the metering zone has been demonstrated⁽¹⁰⁾⁽¹¹⁾⁽¹²⁾.

It is now clear that the forward motion of the material in the "feed", "transition" and "metering" sections of an extruder is governed by different physical processes. A complete mathematical

treatment of extrusion can only be arrived at by coupling the functionalities describing these processes, together with suitable models for die and hopper flow. This is a difficult task since the interaction between the various physical processes must be incorporated into the modelling. A recent publication by Zavadsky⁽¹³⁾ discusses this approach. His computer program EXTR3 contains several subroutines which enable the computation of the parameters describing the various processes. Further, he considers the solids conveying zone to end when the solid bed temperature becomes higher than the melting temperature of the solid and the transition zone to end when the solid bed width is less than 10% of the channel width. However in dealing with food materials there are added complexities. The non-homogeneous nature of the food makes it a difficult task to characterize its rheology. Further, the heat and shear environment in the extruder screws causes chemical alterations such as gelatinization, denaturization of proteins and the restructuring of these food constituents. During this process, the granular or amorphous food takes on a "dough" or plasticized state. To model these events in the case of food extrusion requires an extensive knowledge of the reaction kinetics of the food constituents and how those changes are manifest in the changes in the rheology of the system⁽²⁾. Nevertheless, in dealing with food extrusion problems the methods developed for plasticating extrusion have been successfully used⁽¹⁴⁾⁽¹⁵⁾⁽¹⁶⁾.

A detailed study of the extrusion process must therefore begin with the solids conveying mechanisms and in particular the way in which the governing parameters in this process are affected by changes in extrusion variables. In the past this section has

received the least amount of attention by the extruder analysts largely because the contribution from this section to the overall extrusion process have been estimated as small. The melting mechanism was regarded as being the dominant controlling process in the extruder. However, for the melting mechanism to operate efficiently, the material must be delivered to the transition and metering sections at the correct pressure, temperature and flow rate and this requires a knowledge of the solids conveying mechanism.

1.2 Review of Previous work on solids flow

The first reported work found by the current author on the solids conveying processes in single screw extruders was given by Decker⁽¹⁷⁾. His conclusions are in broad agreement with behaviour found in practice. However, the formulae derived by Decker do not provide an accurate quantitative description of the generally observed actual behaviour. He does however discuss the variables which control solid transport such as the screw geometry, the surface temperatures, the back pressure, and the coefficient of friction between the solids and the screw and the barrel surfaces. His conclusions are that the coefficient of friction between the material and the barrel wall must be greater than that between the material and screw in order to facilitate flow. This is apparently achieved in practical cases of extrusion by polishing the screw surface, using a grooved barrel wall and, more critically, by having different temperatures on the two surfaces (screw and barrel). For materials with a temperature dependent

coefficient of friction, which is generally the case for foods, this is the most obvious means of control. The geometry of the screw must also be chosen to maximise axial movement of the solid plug. Later, Weber⁽¹⁸⁾ treated the feed as a solid plug but he neglected some of the important forces in the system. A better approach to the problem of solid conveying is that due to Pawlowski⁽¹⁹⁾. He neglected internal shear and essentially considered the balance of forces on the screw. The first really significant attack on the problem of solids flow was made by Maillefer⁽²⁰⁾ who recognized that the problem is essentially one of the frictional force differentials. However, in deriving the equation to compute the conveying angle an important force contribution was overlooked. The definitive work on solids transport is that by Darnell and Mol⁽²¹⁾ which has now become the classic analysis in this field. The equations derived by these authors are based on the following assumptions:

1. The solid in the screw channel behaves as a plug, and contacts all sides of the channel, i.e. barrel wall, screw root and both the pushing and trailing sides of the flight and no internal shear takes place.
2. The pressure developed in the feed zone is a function of the channel length.
3. The coefficient of friction between the solid and the metal surfaces is independent of pressure.
4. Leakage flow in the radial clearance of the flights δ is neglected.
5. The coefficient of friction μ between the material and the screw and barrel surfaces is assumed to be the same.

6. The body forces such as gravitational and centrifugal forces are neglected.
7. The velocity of the plug is constant with time i.e. steady state conditions hold.
8. The width of flights is neglected.

Some of these assumptions have been relaxed by subsequent workers. For example, Schneider⁽²²⁾ allows the pressure developed to be different at the barrel, screw and flight walls. The flight width is taken into consideration in expressing the solids flow rate equation by Tadmor and Klein⁽³⁾. These latter authors further allow for different coefficients of friction at the barrel and screw wall. Lovegrove⁽²³⁾ has computed the relevance of body forces in the conveying mechanism; generally they may be neglected.

The various forces acting on the solid plug are shown in Fig. (1.2). The flow rate or solids conveying rate F is obtained by multiplying the solid plug velocity V_{pl} by the cross-sectional area of the plug.

$$F = V_{pl} \left[\frac{\pi}{4} (D_b^2 - D_s^2) - \frac{peH}{\sin \bar{\theta}} \right] \quad (1.1)$$

where D_b and D_s are barrel and screw diameters, p the number of flights in parallel, e is the flight width, H the channel depth and $\bar{\theta}$ the mean helix angle. Before eq. (1.1) can be used, the plug velocity must be known and this is obtained by considering the velocity vector diagram (Fig. 1.3).

$$V_{pl} = \frac{\tan\phi \tan\theta_b}{\tan\phi + \tan\theta_b} V_b$$

where V_b is the barrel velocity and ϕ the conveying angle. It is usual to establish the conveying angle via a force and torque balance on the solid plug under the constraint of the listed assumptions given previously. The conveying angle ϕ is given by⁽²³⁾

$$\begin{aligned} \cos\phi = & K\sin\phi + \frac{\mu_s}{\mu_b} \frac{K_2}{K_1} \frac{2HE}{(W - e)} (K\tan\bar{\theta} + E) \\ & + \frac{\mu_s}{\mu_b} \frac{K_3}{K_1} C \cos\theta_s (K\tan\theta_s + C) \\ & + \frac{1}{K_1} \frac{HE\cos\bar{\theta}\sin\bar{\theta}}{Z_b\mu_b} (K\tan\bar{\theta} + E) \ln \frac{P_2}{P_1} \end{aligned} \quad (1.2)$$

where H is the channel depth, W is the channel width, μ_s and μ_b the coefficients of friction at the screw and barrel surface, $\bar{\theta}$ and θ_s the mean and screw helix angles, K_1 , K_2 and K_3 are constants describing pressure anisotropy at barrel surface, flight edge and the screw root, Z_b the barrel length and P_2 and P_1 are the exit and entry pressures to the feed zone. The constants K , C and E are defined as follows;

$$K = \frac{E\{\tan\bar{\theta} + \mu_s\}}{\{1 - \mu_s \tan\bar{\theta}\}}$$

$$E = \frac{D - h}{D}$$

$$C = \frac{D - 2h}{D}$$

D is the outside diameter of screw.

The analysis of eq. (1.2) indicates that the angle of movement ϕ is a function of screw geometry, the friction coefficients μ_s and μ_b and the pressure profile in the conveying zone. Therefore, for a given screw geometry, the variation of the coefficient of friction with the process variables and its influence on the pressure profile must be known *a priori* in order to compute the throughput from eq. (1.1).

The pressure profile in the feed zone has been shown experimentally to be exponential⁽²³⁾⁽³⁾⁽²⁴⁾ in form and it is now appropriate to review the work to-date on the theoretical and the experimental studies on the form of the pressure profile. The combination of the torque and force balance equations yields a one-dimensional differential equation describing the rate of change of pressure P in the major axis of the channel direction Z. The integration of this equation, subject to the obvious boundary conditions, results in

$$P_2 = P_1 \exp \left\{ \frac{AZ}{D} \right\} \quad (1.3)$$

where A is a function of the screw geometry and is given by

$$A = \frac{K_2 \mu_b (\cos \phi - K \sin \phi) D}{h \cos \bar{\theta} (\tan \bar{\theta} + E)} - \frac{K_1 2 \mu_s}{(W - e) \cos \bar{\theta}} - \frac{K_3 \mu_s C \cos \theta_s D (K \tan \theta_s + C)}{h E \cos \bar{\theta} \{K \tan \bar{\theta} + E\}}$$

The constants K_1 , K_2 and K_3 in these equations are functions of the screw geometry and are dependent upon a material stress

transmission property K_w which is the ratio of the lateral to axial stress in a compression test in an enclosed container⁽²²⁾. Eq. (1.3) predicts the pressure profile to be proportional to the initial pressure P_1 . It must therefore follow that the pressure development would be dependent on the "head" of material in the hopper. However, experiments conducted by Lovegrove⁽²⁴⁾ have indicated that the pressure profile in the feed zone is independent of the head of material in the hopper. The inference here is that some non-frictional mechanism may be responsible for pressure initiation in this case. Lovegrove and Williams⁽²⁵⁾ incorporated gravitational and centrifugal forces to account for the pressure initiation. The inclusion of gravity forces produces an essentially exponential pressure rise with a sinusoidal ripple superimposed. These body forces are relatively small compared to the pressures generated at the exit of the solids conveying zone. However they are very significant at the beginning of the feed zone where the pressures are low, nominally zero in the limit. The exponential form of the pressure profile was experimentally confirmed by Jacobi⁽³⁾ in a cooled and long conveying zone, and some simulated pressure profiles are given by Tadmor and Klein⁽³⁾. It is interesting to note that the pressure profile equation (1.3) has no terms explicitly involving screw speed so if the coefficients of friction are independent of velocity the pressure profile should also be unaffected by the velocity. Schenkel⁽²⁶⁾ has published the results of some experimental work on the pressure build-up in the feed zone and demonstrated that significant pressures may be built up in the solids conveying process. More recently, Lovegrove⁽²⁴⁾ has reported experiments on the output/pressure build-up characteristic of the feed zone. He

worked on a piece of apparatus similar to the feed section of a single screw extruder with a device for applying back pressure attached to the end. The application of back pressure was thought of as simulating what the feed zone would experience from the rest of a conventional extruder. The validity of the plug flow assumption was investigated and it was concluded that for a flood-fed extruder the deviation from plug flow was very small. Further the effect of screw speed and geometry on the output rate was investigated. Measurements were taken at the fully "smeared" condition or steady state which took approximately between 1/2 - 1 hour to generate. It has also been suggested that the smaller the particle size compared with the size of the channel the more valid the continuum assumption.

In dealing with food extrusion, the methods developed in the field of plastic extrusion are often used. Jasberg et al⁽¹⁴⁾ have demonstrated the applicability of the Darnell and Mol⁽²¹⁾ equation in modelling low temperature extrusion of defatted soy flakes. These authors found that at low temperatures (30 - 50°C) the back pressure from the die was unaffected by an increase in the flow rate. The major difficulty encountered in describing the flow patterns during the extrusion of biopolymers is the lack of reliable rheological properties relevant to the extrusion conditions. The objective of this thesis is to predict the pressure profile and the length of the compaction zone from an understanding of the frictional and stress transmission characteristics of the powder, in the current work a maize flour has been used.

1.3. *The Approach to the Problem*

The present solids conveying theories (eq. 1.1) require a knowledge of the absolute values of the friction coefficients between the material and the barrel and screw surfaces μ_b and μ_s and the stress transmission characteristics (ratio of radial stress to vertical stress, K_w), in order to predict the generated pressure profile. A qualitative and preferably a quantitative knowledge of the magnitude of the friction and the K ratio and its variation with the significant variables, interface temperature, moisture and velocity are therefore required to enable a predictively accurate model to be constructed. Direct measurements inside the appropriate machines are not practical currently and the alternative of a simple simulation has obvious attraction. The theme of the present study is that the necessary information may be obtained from a range of small scale simulations by addressing the processes occurring in the extruder. The loose powder is forced or drawn into the compaction zones by the traction stress at the interface. The relative tractions partly define the compaction rate and at the same time the extent of compaction defines the mechanical properties of the system and hence the local normal stress particularly at the walls. The local normal stress scales the traction. The study of the feed or the solids conveying zone of a single screw extruder therefore may be regarded as a problem in traction induced compaction where the material is progressively changing its properties due to the chemical reaction (cooking) and densifications which take place.

This study has investigated the feeding processes in a single screw extruder operating on a maize powder. Chapter 2 reviews the

available work to-date on polymer friction. In particular the adhesion model of friction and the various contact mechanical treatments available to compute the real area of contact A_r for elastic and plastic deformation. The history of the interfacial shear stress τ measurement is also discussed. Finally the available work on the coefficient of friction of polymeric powders, obtained in order to simulate extruder feed zone, is presented.

Chapter 3 reviews the work on the compaction of powders and the relevant theories used to evaluate the stress transmission factors and failure or flow properties of powders. It further discusses the advantages and disadvantages of the methods used for the prediction of wall stresses.

Chapter 4 describes the experimental methods employed in this study. Chapter 5 presents data and analysis of the traction of compacted maize hemi-spheres on flat glass surfaces as a function of the fabrication temperature and pressure, the interface temperature, the moisture content and the effect of normal pressure. The data obtained in thin film shear experiments, carried out in order to characterize the interfacial rheology, are also reported.

Chapter 6 presents data on the compaction of maize powder and discusses the relevance of the Janssen-Walker (J-W) analysis for the prediction of wall stresses in the present experiments. A contact mechanical model appropriate to hemi-spherical particles on a flat wall is also developed and the existing Janssen-Walker analysis is modified to account for the variation of the coefficient of wall friction μ_w with the local normal stress. A comparison of the data obtained from the original Janssen-Walker

(J-W) and the modified Janssen-Walker (M.J.W.) analyses with experimental measurements are also presented.

Chapter 7 presents the data on the analysis on the simulation of the low stress range of the extruder feed zone. The effects of particle dilatancy and its inclusion in the prediction of wall frictional coefficients are also discussed. Chapter 8 presents experimental data obtained in a single screw maize extruder and the applicability of the traction induced compaction model derived in this thesis is discussed. In the chapters to follow, compressive stresses, and angles measured in an anti-clockwise direction are considered as positive.

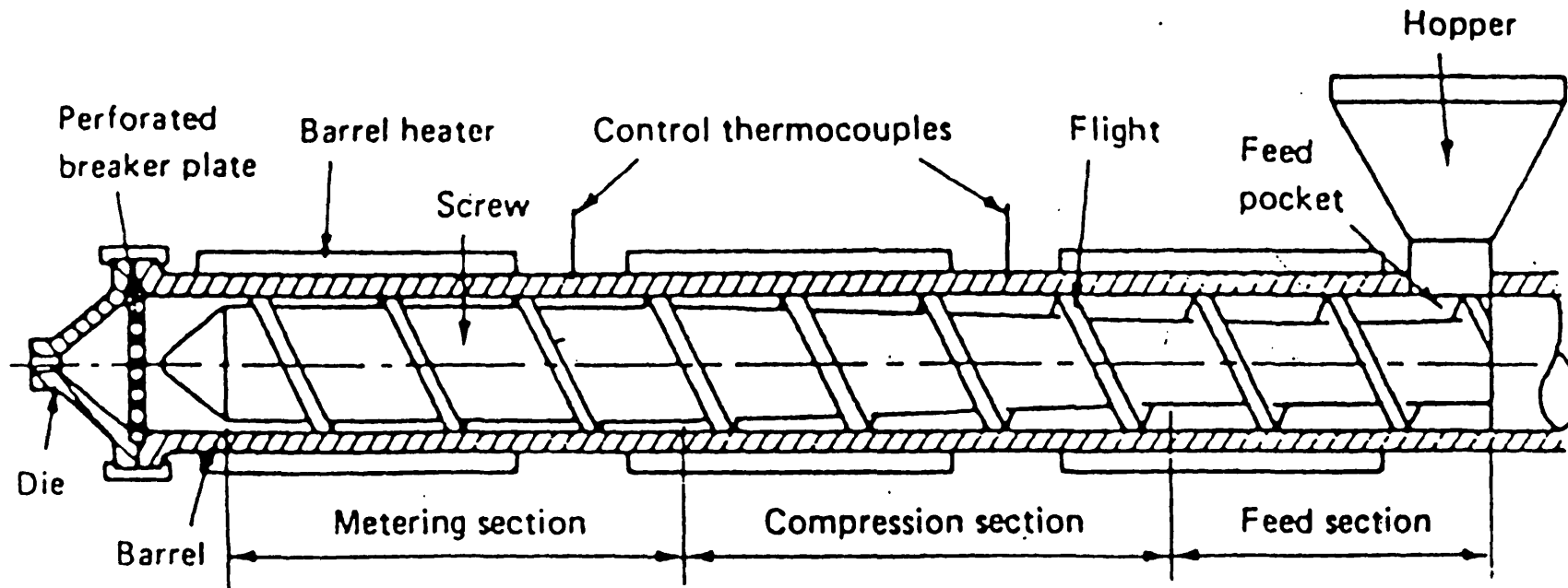


Fig.1.1 A schematic diagram of a single screw extruder.[3]

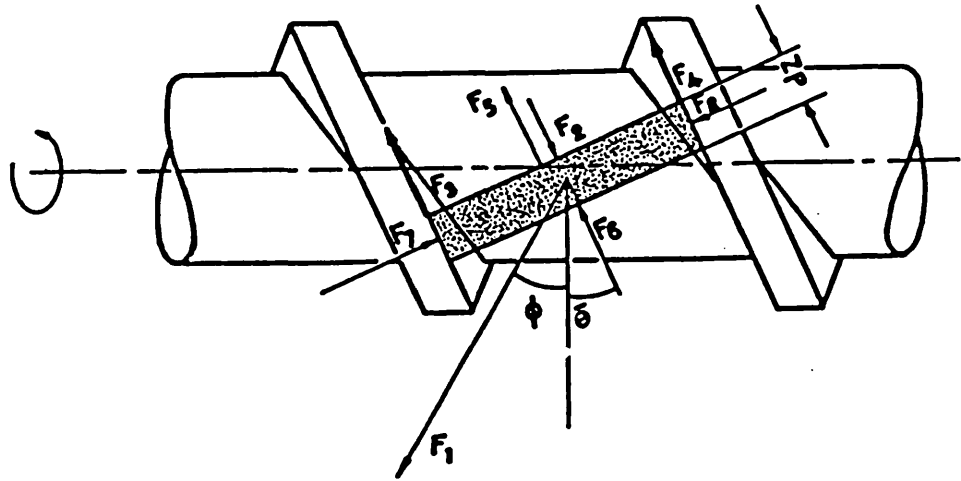


Fig.1.2 The forces acting on the solid plug in the screw channel.[3]

F_1 = friction force between the barrel surface and the plug.

F_2 = Normal force

F_3, F_4, F_5 = Friction forces on the two flights and on the root of the screw.

F_6 = Normal force

F_7, F_8 = Forces exerted by the plug on the flights.

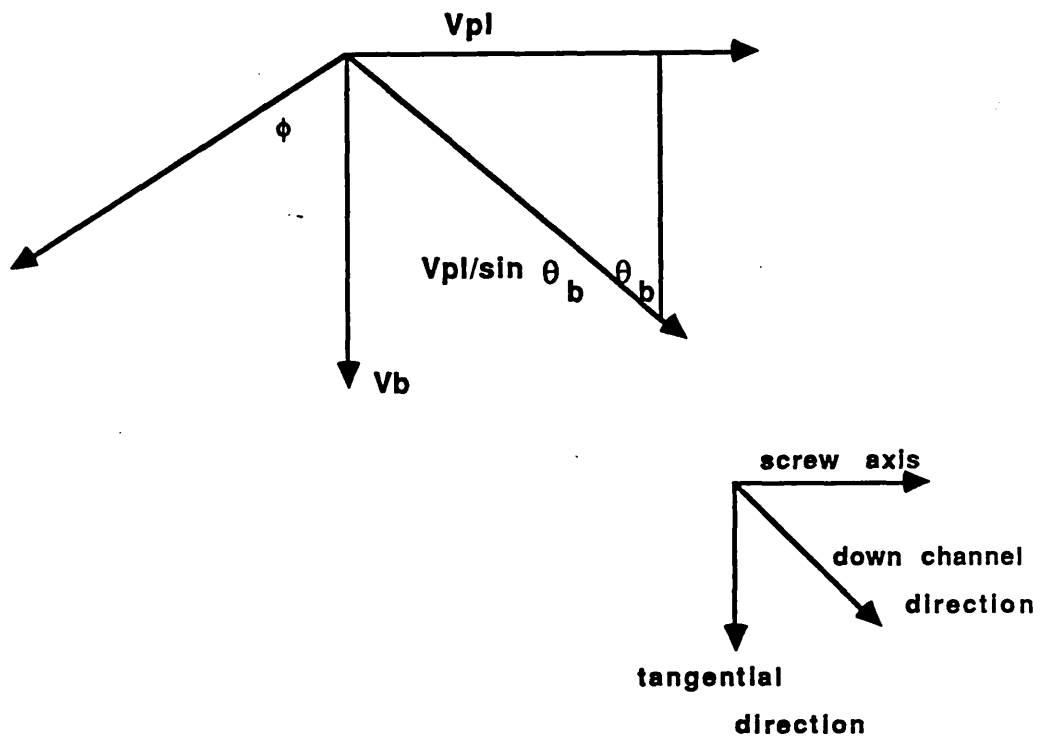


Fig.1.3 Velocity vector diagram

Chapter 2

Friction of Polymers and the Adhesion Model of Friction

2.1. Introduction

. In the middle of the fifteenth century Leonardo da Vinci⁽²⁷⁾ deduced the basic "laws of friction". However, the first systematic work was due to Amontons⁽²⁸⁾ in the period 1663-1705. In addition to the validation of the basic laws of friction, Amontons recognised that most surfaces are rough on a molecular scale and offered asperity engagement as the origin of friction. Today this mechanism is termed Coulombic friction. It was John Theophilus Desaguliers⁽²⁹⁾ who first attributed friction to adhesion between surfaces although it is believed to be an insignificant source of energy dissipation. The many subtle subdivisions which exist today in the mechanism of polymer friction lie in the early work on metallic friction. In addition to the adhesion component of friction which is currently regarded as generally important, a second component of energy dissipation arises from the ploughing of one surface by asperities of the other. This component is often called the deformation friction. The adhesion work is thought of as being dissipated in a narrow region close to the interface and is equivalent to the energy required to shear adhesive junctions formed at points of real contact between the surfaces. The ploughing term involves

deformation of a greater volume, which is comparable with overall asperity dimensions. In this case the stress is transmitted purely by geometric constraint and surface forces are considered to be absent. This aspect of energy dissipation has been extensively studied both experimentally and theoretically and is well documented⁽³⁰⁾⁽³¹⁾. However, this process is not the major energy dissipative mechanism in the context of the present work and it is not appropriate to consider it any further.

This chapter therefore discusses the available work to date on bulk friction and the interpretation of this data in terms of the "Adhesion Model of Boundary Friction". Section 2.3 introduces this model and describes how the parameters needed to compute the friction coefficient may be estimated. Finally, Section 2.4 reviews the published work on polymeric powders, in particular how friction is effected by certain extrusion variables, such as temperature, pressure, moisture and relative velocity. It will be shown that food materials, which are the subject of the current thesis, can be usefully analysed using the same basic mechanisms established for the interpretation of the data of polymer friction.

2.2. *Adhesion Model of Friction*

2.2.1. *Introduction*

The notions of the interaction of surface forces and the subsequent rupture of the junctions they produce at interfaces form the basis of what is now known as the adhesion model of boundary friction. To amplify this point further, when two bodies are brought into molecular contact a series of discrete adhesive junctions are formed. On the application of a tangential stress these junctions are sheared and sliding commences. Energy is dissipated in a region close to the interface and the frictional force in this interfacial zone is given by:

$$F = \tau A_{\text{real}} \quad (2.1)$$

where τ is the energy dissipated per unit area, per unit sliding distance or the interfacial shear strength and A_{real} is the real area of contact. The real area of contact is given by the sum of the areas of the individual contact points, and is usually a factor of 10^{-3} less than the apparent area of contact with the exception of molecularly smooth surfaces such as mica and some rubbers. The analysis of surface tractions *in situ* presents a problem in view of the difficulty associated with defining quantitatively the real area of contact. The measurement or the accurate prediction of the real area of contact produced by the contact of two rough bodies remains a difficult problem. However,

realistic first order estimates can be obtained by established contact mechanical models for elastic and plastic interfacial deformations. These models take into account the surface topography, contact geometry and the mechanical properties of the contacting solids. In principle, and indeed often in practice, the contact area is a function of the interface shear stress; the junction increases in area as the tangential force is applied. Tabor (1972)⁽³²⁾ has reviewed this phenomenon in the context of plastically deforming metallic contacts. The extent of junction growth is a strong function of the quantity τ and can be estimated from a simple model.

Many authors have contributed to the formulation of the two term non-interacting dissipation models of friction but it was the monographs of Bowden and Tabor⁽³³⁾ that established its validity as a predictive tool.

2.2.2. Area of Real Contact between Solid Surfaces

The estimation of contact area has posed an intractable problem. Many theories have been advanced, based on plastic and elastic flow. An estimation of the contact area in plastically deforming contacts is given by $A = \frac{W}{P_0}$ where W is the load and P_0 is a flow stress of the softer solid. This relation was shown to hold well for metallic contacts by Bowden and Tabor⁽³³⁾. For plastic deformation of the asperities in contact, the frictional force is $F = \mu W$ where μ is the coefficient of friction. In the case of polymers, however, it has been shown that the relationship

can often be of the form $F = kW^n$, where n may take any value between $2/3$ and unity for the contact of rough surfaces deforming elastically. The origins of these various possibilities are now considered and the current literature on the prediction of contact area is reviewed.

For the ideal elastic case of a smooth contact of a hemi-sphere on a flat, the contact area is given by the Hertz solution:

$$A_r = A_{app} = \pi[DWR_s]^{2/3} \quad (2.2)$$

where R_s is the radius of the sphere and D a quantity which characterises the mechanical properties of the contacting substrates.

$$D = \frac{3}{4} \left[\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right] \quad (2.3)$$

where ν_1 , ν_2 , E_1 and E_2 are the Poisson's ratio and moduli of the contacting solids. This idealized case is depicted in Fig. (2.1) where the contact area scalar m is $2/3$.

$$A_r = kW^m \quad (2.4)$$

Most engineering surfaces contain many protuberances or valleys which make the real area of contact A_r orders of magnitude less than the A_{app} predicted from the Hertz equation. In general, if N asperities are involved in the frictional process, the real

contact area is given by the sum of the individual contact points⁽³⁴⁾.

$$F = \sum_{i=1}^N k_i w_i^{2/3}$$

where k_i is a constant and w_i the load per particle. This relation, however, assumes all the asperities to be of the same shape and size and obey the same deformation law and to be uniformly distributed. In addition the load dependence of the shear stress is omitted. The load index n was predicted to be 8/9 by Lincoln⁽³⁴⁾ for the friction between two cylindrical fibres, possessing longitudinal fluted asperities of uniform cylindrical cross-section and being uniformly distributed. He explains this departure from the Hertzian power law of 2/3 as indication of the deformation of asperities being partly plastic. However, as has been shown later, this supposed departure from elastic behaviour is not necessary in order to explain this result. Archard⁽³⁵⁾ in 1953 considered the load dependency on the contact and rubbing of flat surfaces, and obtained an index of 0.8 for multiple areas of elastic contact. Howell and Lodge⁽³⁶⁾ have considered the appropriate case for a spherical surface of radius R_s covered with spherical asperities of r_s loaded onto a smooth flat. The value of m was found to be 0.89. Further, in 1957 Archard⁽³⁷⁾ shows for a sphere on a flat geometry the index m may progress from 2/3 to unity depending on the number of layers of protuberances superimposed on the first layer. The various cases of the multiple asperity model of Archard are shown in Fig. (2.2). It is

useful at this point to emphasize the physical concepts which are inherent in these treatments and to indicate some general considerations which are independent of the particular model used.

In the contact of two bodies two extreme types of deformation are possible, purely plastic and purely elastic. It was shown that for purely plastic deformation the contact area scalar was unity. For elastic deformation with increasing normal load, two distinct cases are possible:

- 1) The number of asperities may remain constant and each contact will deform in a Hertzian manner in which case the total contact area will be given by the sum of these contacts yielding a value of $2/3$ for the contact area scalar m .
- 2) The increasing load will increase the number of contacts while the average area of the existing contacts remains constant. In this case the real area of contact is proportional to W .

The more realistic case however is the simultaneous occurrence of both these phenomena, the increased normal load increasing both the number and the size of the existing contact points. Clearly the contact area scalar m will take any value between $2/3$ and unity. Archard⁽³⁷⁾ considers that if one set of protuberances were to be imposed on the first layer an index of 0.89 will be obtained for spheres on a flat geometry. In contrast, for nominally flat surfaces which may have widely separated regions of contact, m will be 0.8.

Experimentally it is difficult to distinguish between these two cases. Adams⁽³⁸⁾ in 1963 has shown the insensitivity of

asperity height variation in the predicted values of n for nylon. Table (2.1) lists the expressions based on Lodge and Howell's and Archard's theory. The latter caters for asperity height variation and the contribution from a load dependent shear stress, while the former only considers the shear stress contribution. Adams⁽³⁸⁾ also measured, using optical interference for nylon on a glass flat, the variation of friction force and contact area with load during sliding. He wrote $F = K'W^n$ and found n and m to be 0.78 and 0.71 respectively. It was demonstrated that a pressure dependent shear stress was necessary to explain the difference in the indices n and m . Another criterion for this difference was postulated as the asperity shape. This contribution has been discussed by Adams, considering a symmetrical asperity profile and shows that m will be unity for a conical asperity.

A more thorough analysis is given by Greenwood and Williamson⁽³⁹⁾. They consider the contact of nominally flat surfaces and define them as those in which the area of apparent contact is large, so that the individual contacts are dispersed and the forces acting through neighbouring contacts do not influence each other. However, when curved surfaces touch, the apparent contact area is limited by their gross geometry: the individual contact areas are then tightly clustered and it becomes necessary to take into account the interaction between contacts and how this may influence the contact area. They argue that the contact area is a function of the standard deviation of the height distribution, and for the sake of simplicity, they consider the behaviour of surfaces for which the asperity heights follow an

exponential distribution. In this case, irrespective of whether the deformation is elastic or plastic, the area of contact is directly proportional to the load. For many real surfaces, however, the height distribution has been shown to be very nearly Gaussian, and the deformation, elastic or plastic, depends on the surface topography. The contributions for the two extreme modes of deformation are described by the ratio σ/β , where σ is the mean deviation of the asperity heights and β the mean radius of the tips of the asperities and the deformation properties of the material, as represented by the ratio of the elastic to the plastic properties of the material. This is represented as $\frac{E'}{P_0}$ where E' is the reduced Young's modulus and P_0 the local flow stress of the material. The product of these is termed the plasticity index ψ

$$\psi = \left[\frac{E'}{P_0} \right] \sqrt{(\sigma/\beta)}$$

Greenwood shows that if $\psi < 0.6$ the deformation will be elastic over a large range of loads.

A wide range of values for the load index n has been experimentally obtained. Bowden and Young⁽⁴⁰⁾ obtained $n = 0.8$ for diamond, Lincoln⁽³⁴⁾ 0.67 for nylon, Howell and Mazur⁽⁴¹⁾ 0.96 for cellulose acetate and 0.91 for viscous rayon, 0.90 and 0.80 for undrawn and drawn nylon respectively.

In general a typical contact produced between rough surfaces will comprise multiple asperities, some of which will deform elastically and others plastically. In addition, the deformation

of polymeric matrices will also be sensitive to time, history and strain. Therefore a more realistic estimation of the contact area can only be achieved from a contact mechanical model which incorporates the contribution from these parameters.

2.2.3. *Interfacial Shear Strength τ*

The interface shear strength of contacting surfaces is defined as the work done per unit sliding distance. It is necessary to measure the quantity τ directly and in the past a number of experiments have been devised to facilitate this operation. The first of these techniques is that due to Bridgman⁽⁴²⁾. An organic layer was interposed between two anvils, which were loaded and the torque required to rotate one anvil relative to the other was measured and subsequently converted to shear stress and pressure respectively. Rabinowicz and Tabor⁽⁴³⁾ successfully used a second technique which involves sliding of plastically deforming substrates over each other and a normal load, sufficient to induce permanent deformation, is applied.

A third method which involves the shearing of a film between elastically deforming substrates has been extensively used⁽⁴⁴⁾⁽⁴⁵⁾. Several methods of film deposition have been employed and these will now be outlined. The first of these is the Langmuir-Blodgett⁽⁴⁶⁾ method for depositing monolayers on solid substrates. This technique facilitates the deposition of layers of well defined composition, structure and thickness. A multilayer substrate can also be built up from a single monolayer

of the material. However, this technique is restricted to compounds of low molecular weight, which contain strongly adsorbing functional groups⁽⁴⁶⁾.

Many polymers possess transferring properties and this technique of deposition has been used to deposit films of approximately 10 nm in thickness, which are highly drawn in the direction of rubbing⁽⁴⁷⁾. Two types of transferred film have been identified; thin films of 10-100 nm in thickness and thick films about one micron in thickness. At 25°C polypropylene (PP) and low density polyethylene (LDPE) represent good examples of the latter type while polytetrafluoroethylene (PTFE) and high density polyethylene (HDPE) at 25°C and at low sliding speeds are examples of the former type of film formation. The third technique involves the deposition of the film from a volatile solvent in which the material is not highly soluble. The original material should be highly purified to prevent selective deposition of less soluble or more volatile minor constituents⁽⁴⁸⁾.

The deformation characteristics of polymeric matrices have been shown to be sensitive to a range of contact parameters: temperature, hydrostatic stress, strain, strain rate⁽⁴⁹⁾, history, contact time and to some extent on the chemistry of the environment. Experimental data are presented later which indicate in a quantitative way how the interfacial shear strength τ varies with some of these variables.

Towle⁽⁵⁰⁾ used the rotating anvil technique over a confined pressure range and obtained the dependence of shear strength on pressure. Prior to this, Bowers and Zisman⁽⁵¹⁾ fitted the data of

Bridgman⁽⁴²⁾ on paraffins to an equation of the form

$$\tau = KP^N$$

This functionality is inadequate since it implies a zero interface shear strength at zero contact pressure which is not a plausible solution. Later, experiments of Briscoe *et al* ⁽⁵²⁾ and Amuzu⁽⁵³⁾ demonstrated the pressure dependency to be more precisely described by an equation of the form

$$\tau = \tau_0 + \alpha P \quad (2.5)$$

at constant sliding velocity, contact time and temperature. This functionality was obtained over a wider pressure range and the data of Amuzu⁽⁵³⁾ for various polymeric films at 25°C and 0.20 mm sec⁻¹ sliding velocity is shown in Fig. (2.3). The values of τ_0 and α , which are the intrinsic shear stress and the pressure coefficient respectively for these polymers, are given in Table (2.2). The pressure dependency for most of these terms is adequately described by equation (2.5). Nevertheless, certain exceptions have been observed. PTFE shows a discontinuity in α and τ_0 at a contact pressure of 5×10^8 Pa at 20°C. This anomaly has been attributed to a pressure induced phase change⁽⁵⁰⁾⁽⁵⁴⁾. The coefficient of friction in this contact geometry is obtained by dividing equation (2.5) by the mean normal contact stress .

$$\mu = \frac{\tau_0}{P} + \alpha \quad (2.6)$$

The coefficient of friction μ is expected to decrease with increasing pressure tending to α at high contact pressures. The α and τ_0 values are affected by the type of casting technique used and these have been discussed in detail by Briscoe and Smith⁽⁵⁵⁾. The application of eq. (2.5) to some shear strength data reveals negative values of τ_0 which is physically unacceptable. Isotactic polystyrene cast from toluene at room temperature is an example of this behaviour. A more general empirical equation which adequately describes the shear-strength - pressure functionality for solvated films was established by Briscoe and Smith⁽⁵⁶⁾.

$$\tau = \tau_0' \exp \alpha' P \quad (2.7)$$

τ_0' and α' are constants for a given temperature and strain rate. These values together with τ_0 and α values for a range of polymers is given in Table (2.2). The coefficient of friction μ is now given by

$$\mu = \frac{\tau_0' \exp (\alpha' P)}{P} \quad (2.8)$$

The exponential term can be approximated as $e^{\alpha' P} \approx 1 + \alpha' P$ resulting in

$$\begin{aligned} \mu &= \frac{\tau_0'}{P} \{1 + \alpha' P\} \\ &= \frac{\tau_0'}{P} + \alpha' \tau_0' \end{aligned} \quad (2.9)$$

As the pressure tends to infinity the limiting value of the coefficient of friction is given by $\alpha'\tau_0'$.

The sliding of bulk polymers on elastic substrates has been investigated by many authors. Pooley⁽⁵⁷⁾, Eldredge and Tabor⁽⁵⁸⁾ and Amuzu⁽⁵³⁾ studied extensively the effect of the contact parameters on the coefficient of friction of bulk polymers sliding on glass surfaces and Smith⁽⁵⁹⁾ presents data on the coefficient of friction during interface shear and further makes a comparison between these values and the data obtained during bulk sliding.

The temperature dependence of τ , for polymers was investigated by Briscoe and Tabor⁽⁶⁰⁾ and Smith⁽⁵⁹⁾. The shear strength of all the materials investigated has indicated three types of behaviour: those materials which exhibit a shear strength independent of temperature, those in which there is a marked decrease as the temperature increases and finally a combination of both these types of responses. Fig. (2.4) depicts the data of Smith⁽⁵⁹⁾ for a number of polymers. It has been noted that polymers which exhibit a glass transition temperature show a temperature independent response below this temperature and a marked decrease in τ as the temperature is increased. The data for these materials were found to correlate well with an empirical expression

$$\tau = \tau'' \exp \{Q/RT\} \quad (2.10)$$

where Q is the activation energy, R the universal gas constant, T the absolute temperature and τ'' a constant for a given pressure

and sliding velocity. The observed values of Q are listed in Table (2.3). These indicate the value of Q to lie between 8 - 32 kJ mole⁻¹ or is zero for materials which demonstrate a temperature independent response. High density polyethylene films show a peculiar transition in their behaviour with temperature⁽⁶¹⁾. Initially the value of τ decreases with temperature until a critical temperature is reached. At this point the friction increases and then becomes essentially independent of temperature. This transition behaviour, which is highly reproducible and reversible, is a function of contact pressure and sliding velocity. Increasing contact pressure and decreasing sliding velocity shifts this transition to lower temperatures. The shear strength functionality of thin HDPE films was rationalised in terms of a ductile-brittle transition and results were cited by Amuzu et al⁽⁶¹⁾ from bulk deformation studies to substantiate this belief.

A brief description of the effect of sliding velocity on friction is now appropriate. Data available on this parameters influence is limited. Amuzu⁽⁵³⁾, Briscoe and Smith⁽⁴⁹⁾, Evans⁽⁶²⁾ and Smith⁽⁵⁹⁾ have established in general terms the influence of sliding speed on the shear strength. Fig. (2.5) depicts data of Amuzu⁽⁵³⁾ on the effect of strain rate on the shear-strength. For polymeric thin films a general analysis of the effect of sliding velocity has been put forward by Briscoe and Tabor⁽⁶³⁾⁽⁶⁴⁾. These authors argue that the effective strain rate in the film is given by the velocity V divided by h , the film thickness, and therefore the effective strain rate and the velocity would increase

simultaneously. A more detailed analysis of the velocity dependence is given by Smith⁽⁵⁹⁾.

2.3. *Coefficient of Friction of Polymeric Powders*

A number of authors⁽⁶⁵⁾⁽⁶⁶⁾ have also investigated the effect of temperature, pressure, moisture, surface roughness and speed on the coefficient of friction of powders in conditions which are thought to be comparable with the feed zone of an extruder. This parameter plays a decisive role in the solids conveying mechanism and therefore a brief review of the available literature on the coefficient of friction and its variation with the cited variables is now appropriate. It will however be realised that it may be calculated from equations such as [2.9].

Numerous techniques have been employed to measure the coefficient of friction μ in conditions which simulate the extruder feed zone. The classical method is to slide a box filled with the polymer along a metal plate. This approach has been favoured by Shooter⁽⁵⁷⁾, Grant⁽⁶⁸⁾, Chung⁽⁶⁹⁾, Gregory⁽⁷⁰⁾ and Schneider⁽⁷¹⁾ but is necessarily limited in providing velocities comparable with extruders. Spencer *et al*⁽⁷²⁾ used a pressurised container while Chang and Daane⁽⁷³⁾, Evans⁽⁷⁴⁾, Klein⁽⁷⁵⁾ and Briscoe *et al*⁽⁷⁶⁾ preferred rotating cylinders or annuli. Emmanuel and Schmidt⁽⁷⁷⁾ use a specially designed fixture with a RMS - 7200 Rheometrics, mechanical spectrometer and Huxtable *et al*⁽⁶⁵⁾, an apparatus based on an instrumented annular shear cell. The value of the coefficient of friction for a low density

polyethylene at temperatures between 20°C and 50°C was found to be 0.46 ± 0.04 by Cogswell and Huxtable⁽⁷⁸⁾, which compares with Chung's⁽⁶⁹⁾ figure of 0.44. This indicates that the coefficient of friction is, for these cases at least, a material property rather than a property of the particular instrument on which it is measured. However, surface conditions in different instruments may alter the value of the coefficient of friction.

A large number of published values depict the variation of the coefficient of friction with experimental parameter⁽⁷⁷⁾⁽⁷⁸⁾. Many workers have observed a reduction in the coefficient of friction with increasing normal force or pressure. The work of Chang and Daane⁽⁷³⁾ working with polyvinylchloride powder, Schmidt and Emmanuel⁽⁷⁷⁾ on polycarbonate, low density polyethylene, ABS, high density polyethylene and Schneider on nylon-6 are some of the examples depicting this behaviour. It was shown previously that the contact area of elastically deforming substrates and the shear stress were described by equation (2.4) and (2.5). Incorporating these into the adhesion model of friction (eq. 2.1) yields

$$F = \left[\tau_0 + \frac{\alpha W}{kW^m} \right] kW^m$$

$$\mu = \frac{F}{W} = \frac{\tau_0 k}{W^{1-m}} + \alpha \quad (2.11)$$

Therefore, the coefficient of friction would be expected sometimes to decrease with increasing pressure for a system undergoing interfacial elastic deformation.

Temperature has been established as the single most important

technically significant factor in determining the friction behaviour of polymers. The observed trends in the values of the coefficient of friction for a wide range of polymers has shown an increasing, decreasing and temperature independent behaviour. Fig. (2.6) exemplifies Chaang and Daane⁽⁷³⁾ data on PVC powder, PVC crystal and HDPE disc. All these polymers show a minimum and then a rapid increase in the coefficient of friction with interface temperature. This pattern of behaviour was also observed by Huxtable *et al*⁽⁶⁵⁾ with LDPE and he further noted a maximum at a temperature of 140°C. The work of Schmidt⁽⁷⁷⁾ unfortunately did not extend to this higher temperature range. Polypropylene also shows a temperature maximum approximately at 100°C as shown both by Schneider⁽²²⁾ and Huxtable⁽⁶⁵⁾. It is interesting to note that both these authors obtained the same temperature response of polypropylene working with two different types of apparatus. High density polyethylene is an example of a temperature independent polymer, while nylon shows an increase in the coefficient of friction with temperature. The temperature response of these polymers can be rationalised in terms of this parameters influence on the shear strength and the effective contact area. In general, the real area of contact increases with temperature due to a reduction in the modulus or the flow stress. However, the shear strength may decrease or remain constant with increasing temperature. Therefore, the coefficient of friction may increase or decrease depending on whichever of the two factors happens to be predominant. A third situation may arise if the relative magnitude of the changes in contact area and

shear-strength are equal and opposite. In this latter case the coefficient of friction will be independent of temperature; this is most often the case over limited temperature ranges.

The coefficient of friction can be influenced in various ways by speed. Tabor and McLaren⁽⁷⁹⁾ observed a small increase in μ for a number of polymers. A maximum was observed at approximately 100 cm sec^{-1} followed by a fall in the frictional coefficient. This maximum was attributed to melting occurring at the interface due to surface heat generated. Schneider⁽²²⁾ also investigated this dependence with nylon-6 granules but at speeds considerably lower than the speed at which melting would be expected to commence. The frictional coefficient was found to be independent of speed at lower temperatures. However, a slight increase was found at elevated temperatures. Since the shear-strength and contact area are both affected by the rate and duration of the applied load and the frictional heat generated, the frictional coefficient would be expected to change accordingly. Nevertheless, for a large number of materials the coefficient of friction was found to be independent of speed over a wide temperature range.

Schneider⁽²²⁾ further tested the effect of surface roughness with experiments on 12 plates with different roughness coefficients. Six of these plates were machined longitudinally and the remaining six perpendicularly to the direction of rubbing. These results were then compared with data on a freshly machined plate. The frictional coefficients increased with surface roughness and the values for the longitudinally grooved plates

were found to be close to the smooth plate. However, the data of Schmidt and Emmanuel⁽⁷⁷⁾ seem to infer that significant changes in the coefficient of friction with roughness can only be expected for materials with particle sizes greater than the scale of the roughness peaks. Their data on the coefficient of friction of polycarbonate powder was found to be insensitive to surface roughness. In contrast, substantial changes were observed with polycarbonate pellets. Similar experiments of PVC powder by Chaang and Daane⁽⁷³⁾ on a "Xaloy" 101 and stainless steel surface with identical root mean square roughness indicated no change in the coefficient of friction. Therefore it appears that it is the roughness of the surface that is critical and not the actual material of construction of the surface.

The amount of moisture also plays a crucial role. For most systems moisture reduces the shear strength and increases the contact area. The data of Schneider on nylon confirmed those previously obtained by Cohen and Tabor⁽⁸⁰⁾ with saturated granules having moist surfaces. The frictional coefficient was 10% lower than those for dry granules and with saturated granules having dry surfaces they were 25% higher.

In extruders, surface "smearing" or transfer is observed with transferring polymers. This situation presents the greatest technical interest since this certainly would be the case during steady state operation. The available data on powders indicated an increase in the coefficient of friction with the onset of "smearing".

It has been suggested that the powder surface is similar in many ways to the topography of a rough solid and a series of experiments were conducted by Briscoe *et al*⁽⁷⁶⁾ to substantiate this belief. By and large it was shown that the interfacial friction of powders is amenable to analysis based on the adhesion model of friction.

Table 2.1.

Theoretical equations for the prediction of the load index $(n)^{(38)}$

	n Lodge and Howell	n Archard
Rough hemisphere on plane	$m + (1 - m)[m' + h(1 - m')]$	$\left[\frac{g + h(1/m - 1) + 3}{g + 1/m + 2} \right]$
Smooth hemisphere	$m + h(1 - m)$	

m and m' are the area scalars for the specimen and asperity respectively.

h is the index of the shear strength variation with pressure $\tau = \tau_i p^h$ and g

is the frequency of the asperity frequency/height relation $N = N_1 x^g$.

Table 2.2.

 τ_0 and α values of Briscoe and Smith⁽⁵⁶⁾

	τ_0 (Pa)	τ_0'	α	α'
PTFE	1.0×10^6		0.08	
HDPE	2.5×10^6		0.10	
LDPE	6.0×10^6		0.14	
PP	6.0×10^6		0.17	
PMMA	1.0×10^7		0.36	
PS	4.0×10^6	2.3×10^7	0.45	1.2×10^{-8}
PC (80°C)		0.7×10^7		2.1×10^{-8}
PET		0.7×10^7		1.9×10^{-8}
PVAL		0.6×10^7		1.1×10^{-8}

Table 2.3.

Q value (kJ mole⁻¹) for various polymers(Smith⁽⁵⁹⁾)

Polytetrafluoroethylene (PTFE)	=	20
High density polyethylene (HDPE)	=	16
Polypropylene (PP)	=	17
Low density polyethylene (LDPE)	=	9-30

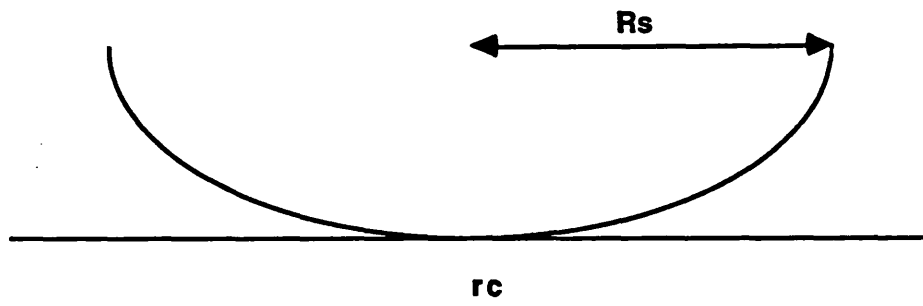


Fig.2.1 Idealized case. Hertzian contact. Real area of contact= apparant area of contact.

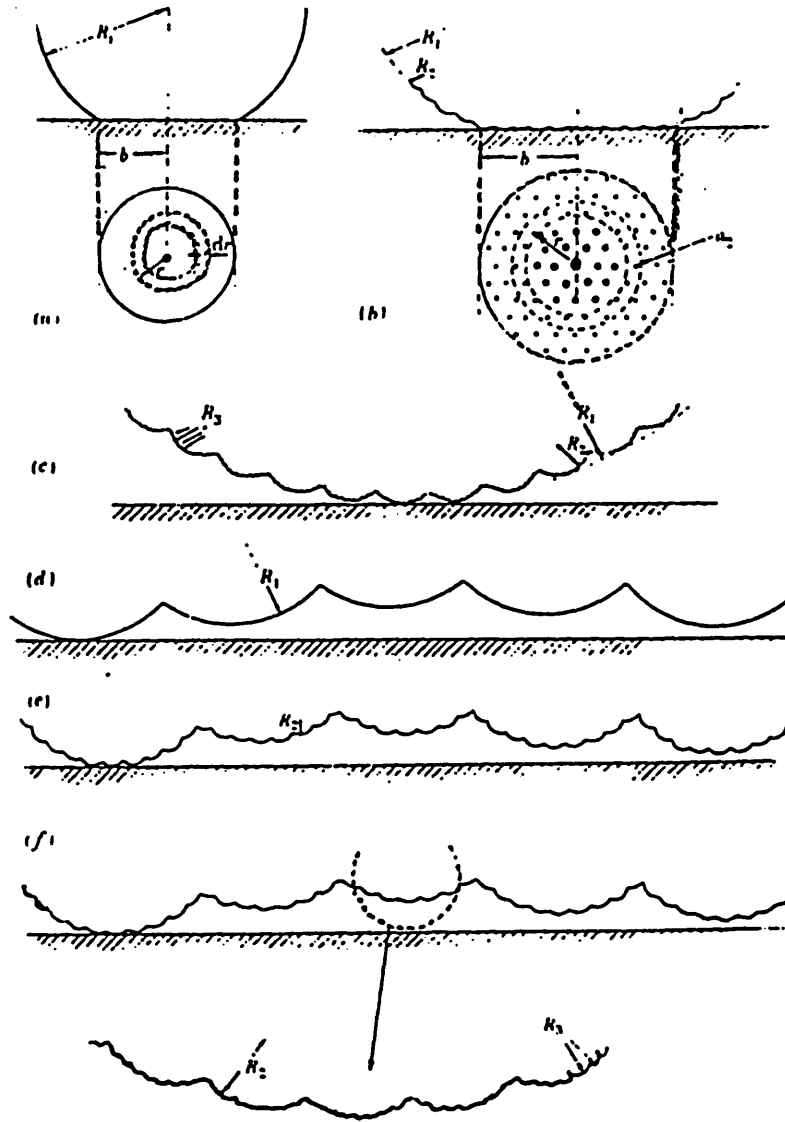


Fig.2.2 The multiple asperity contact mechanical models. (Archard²⁷). The deduced relations between A and W are;
 a. $A \propto W^{2/3}$, b. $A \propto W^{8/9}$, c. $A \propto W^{26/27}$, d. $A \propto W^{4/5}$,
 e. $A \propto W^{14/15}$, f. $A \propto W^{44/45}$.

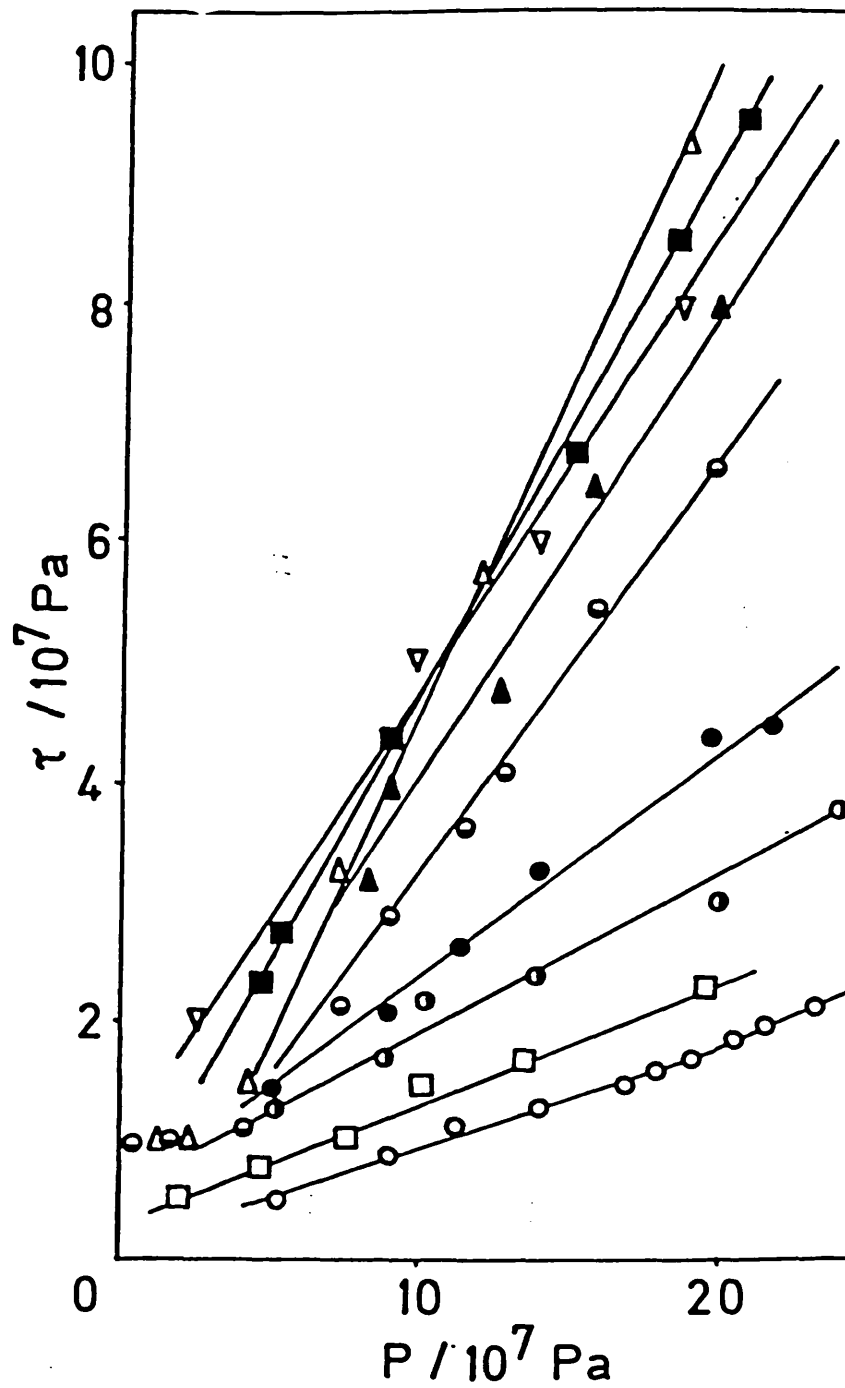


Fig.2.3 The shear strength τ , of polymeric films against mean contact pressure, P , at 25°C and 0.20mmsec^{-1} (Amuzu⁵³) on glass substrates.

- | | |
|-----------------|--------------------|
| Δ HDPE | $\bigcirc$ NYLON 6 |
| $\bullet$ LDPE | Δ PET |
| $\bigcirc$ PTFE | ∇ PMMA |
| $\bullet$ PP | $\blacksquare$ PS |
| $\bigcirc$ PVC | |

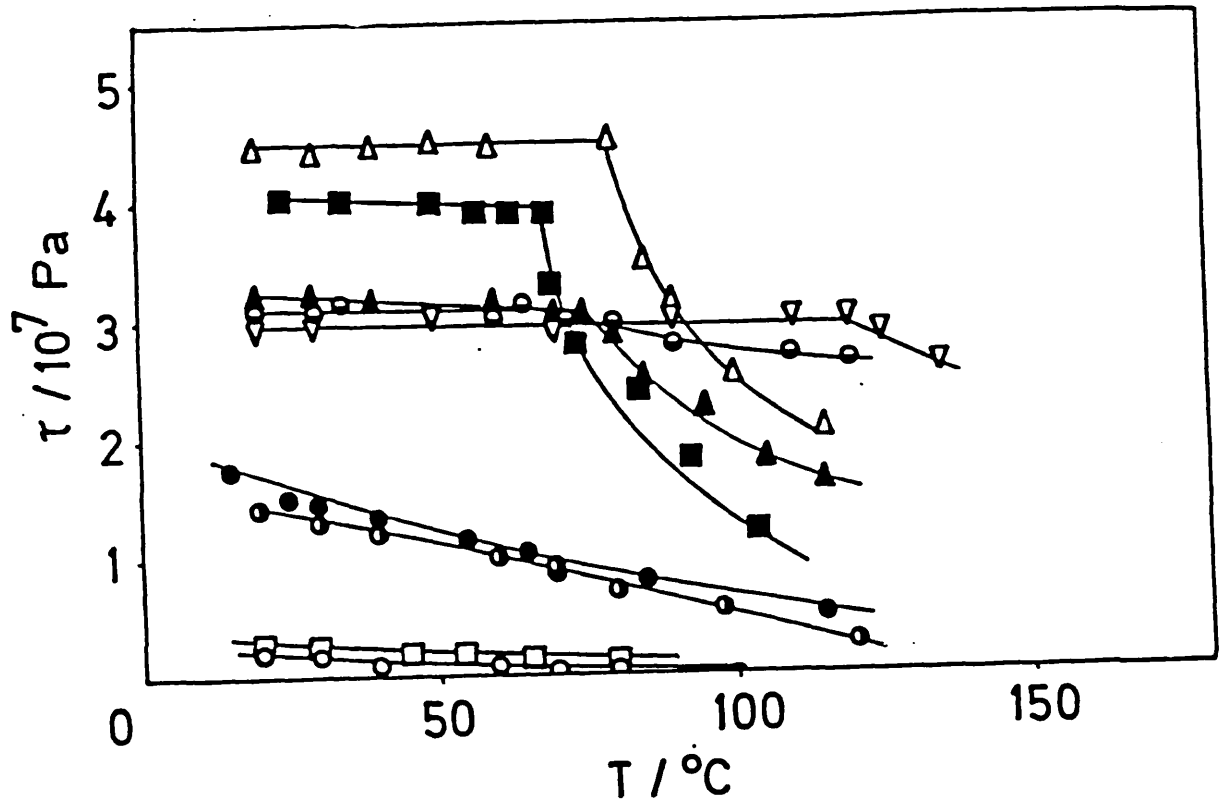


Fig.2.4 Shear strength, τ , against temperature, T , for films of polymers on glass substrates. Contact pressure 7.2×10^7 Pa and sliding speed 0.2 mm sec^{-1} (Amuzu⁽³⁾).

- | | |
|-----------|--------|
| □ HDPE | ● LDPE |
| ● PTFE | ● PP |
| ● NYLON 6 | ▲ PET |
| ▽ PMMA | ■ PS |
| △ PVC | |

Fig.2.5 Log_{10} (shear strength, τ) plotted against log_{10} (strain rate $\dot{\epsilon}$) for films sheared between glass substrates (Amuzu⁽³⁾).

○PTFE Contact pressure, $P = 6.7 \times 10^7$ Pa; Temperature, $T = 25^\circ\text{C}$.

●PP $T = 25^\circ\text{C}$ $P = 7.2 \times 10^7$ Pa

□HDPE $T = 25^\circ\text{C}$

■PS $T = 25^\circ\text{C}$

▽PMMA $T = 30^\circ\text{C}$

△PMMA $T = 200^\circ\text{C}$

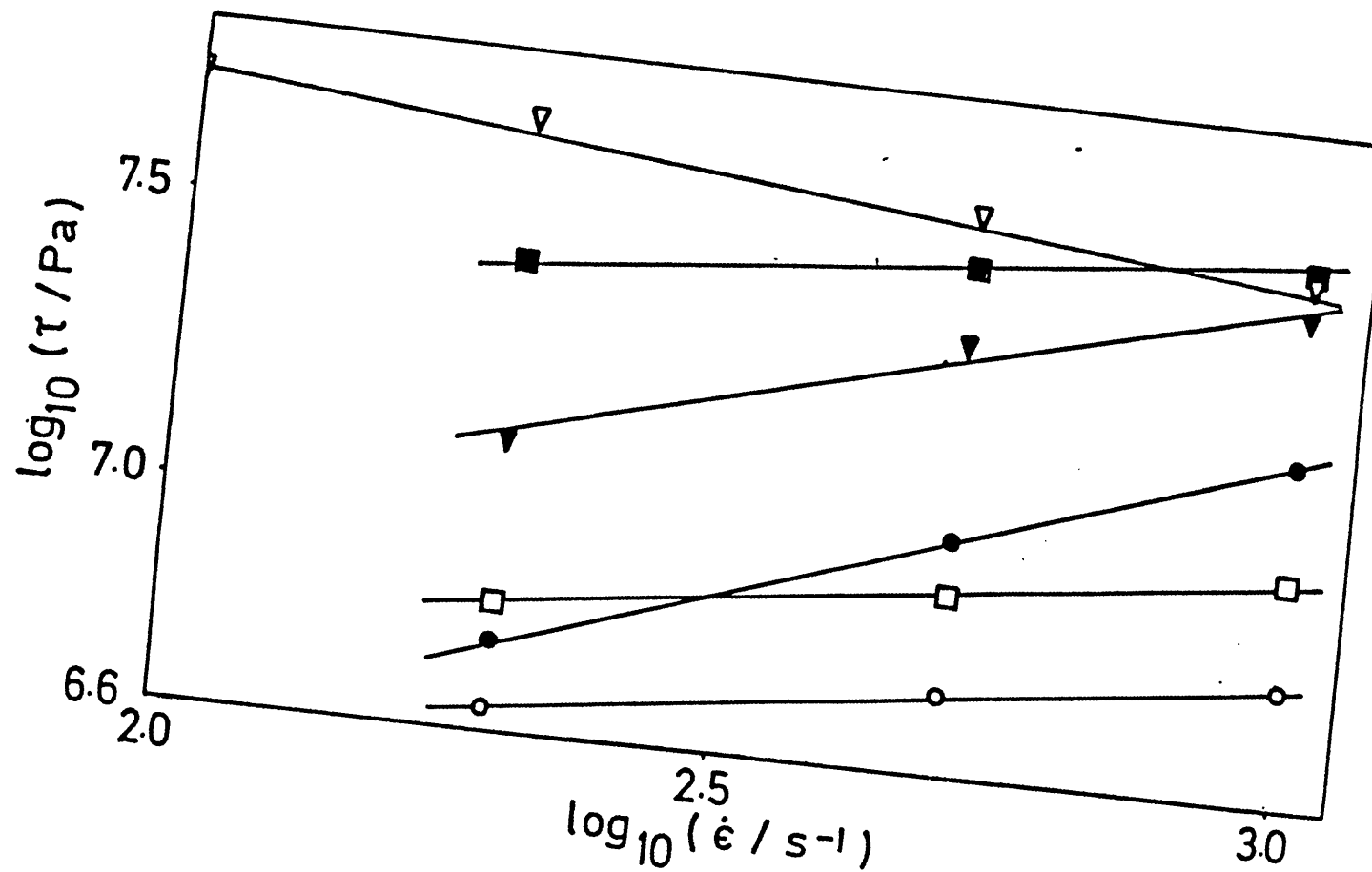
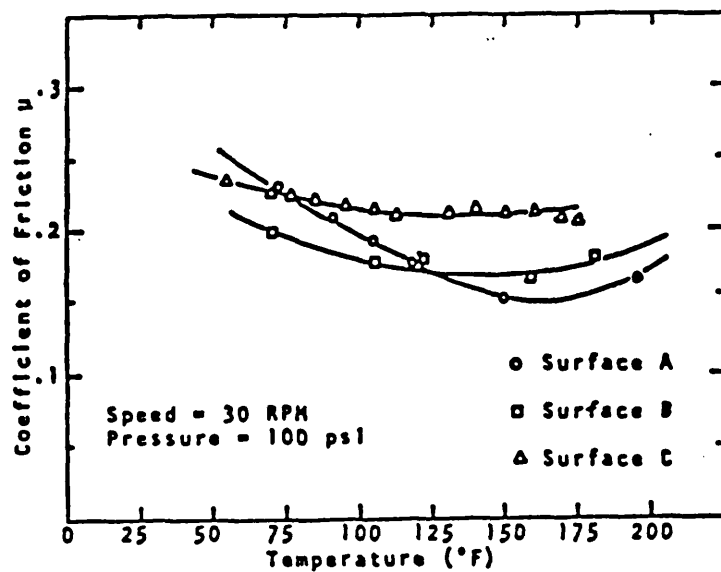
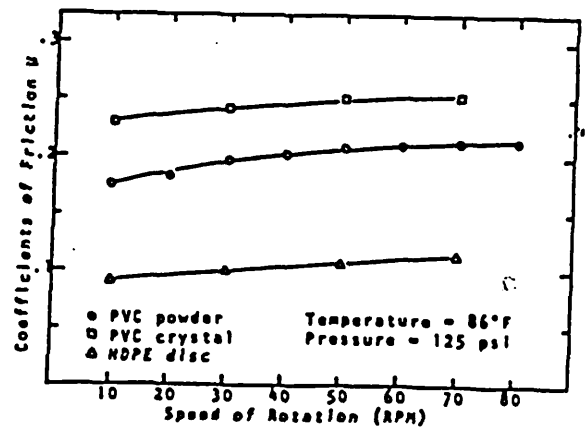
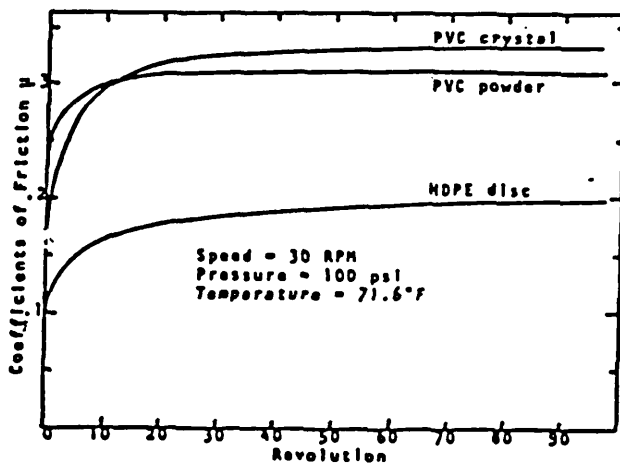
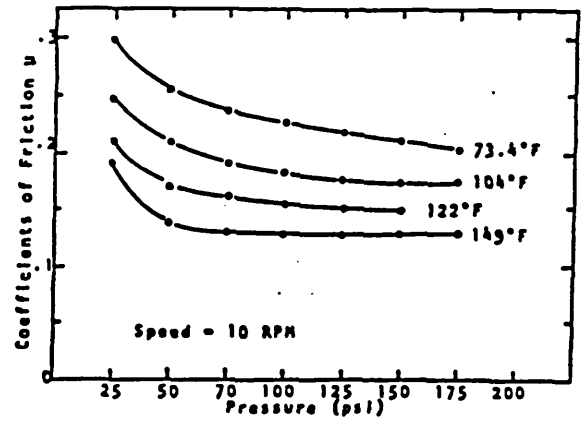
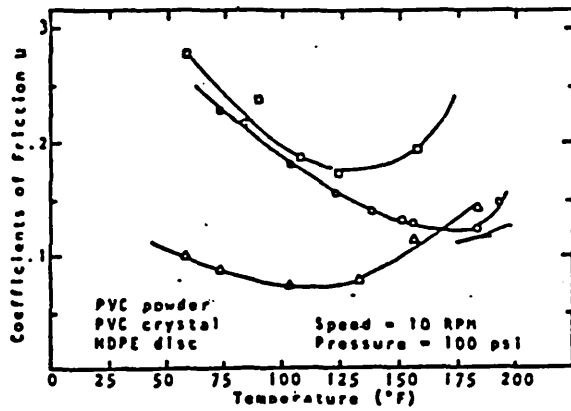


Fig 2.6 The friction data of Chang and Daane⁴³().

- a. Effects of temperature on the co-efficient of friction for various polymers against metal surface A. Speed 10 RPM, Pressure=100Psi.
 ● PVC powder □ PVC crystal △ HDPE disc.
 - b. Effects of pressure on co-efficients of friction for PVC powder against metal surface A. Speed 10 RPM.
 - c. Co-efficient of friction in transient state for various polymers on metal surface C. Speed 30 RPM, Pressure 100Psi, Temperature 71.6°F.
 - d. Effects of rotating speed on co-efficient of friction for various polymers against surface A. Pressure 125Psi, Temperature 86°F.
 ● PVC powder □ PVC crystal △ HDPE disc
 - e. Effects of surface roughness on co-efficients of friction for PVC powder at various temperatures. Speed 30 RPM, Pressure 100Psi.
- | | RMS Roughness |
|---------------------------------|--------------------|
| Surface A Xaloy 101 | -10 to 20 inches. |
| Surface B Stainless Steel | - 10 to 20 inches. |
| Surface C Chromium plated steel | - 20 to 30 inches |



Chapter 3

Compaction of Powders

3.1. *Introduction*

In the transportation of particulate solids the efficiency of conveying is governed by the state of stress in the material. This is dependent on the strength and failure properties of the particular material being processed. In processing machinery such as extruders, the locally induced stresses govern the rate of deformation. This deformation may be elastic, plastic or destructive, depending on the rate and magnitude of the applied stress and on the physical properties of the material. If deformation is elastic and the modulus of the system is low the system recovers completely on removal of the load. If the modulus of the system is high there is negligible movement and the system remains rigid. Once the elastic limit is exceeded, plastic deformation follows with consolidation of the material. If the rate of application of the compacting stress exceeds a critical threshold value, the material is incapable of deforming plastically at a rate sufficient enough to accommodate the induced strain so that it fractures⁽⁸¹⁾.

This chapter reviews the available literature on the failure

properties of powders and the methods available for the prediction of wall stresses.

3.2. *Failure properties of Powders*

3.2.1. *Introduction*

In the processing of powders flow difficulties and "caking" are common problems. Several powders such as powdered fruits and vegetables are renowned for their tendency to cause flow problems and "cakes" while other powders like flour and starch under normal conditions are known to be relatively free flowing⁽⁸²⁾. Therefore, it is necessary to identify those factors which enhance flow problems and to evaluate how these are affected by the process variables. High moisture content, compaction pressure, small particle size and increased temperatures are some of the factors which promote these problems. Recent methods for identifying and quantifying these effects are based on estimating the strength and failure properties of powders⁽⁸³⁾. The failure properties of food powders and the parameters available for predicting these properties have been discussed by several authors⁽⁸²⁾⁽⁸⁴⁾. The most general technique involves the compaction of a powder specimen under a normal stress, followed by a shear operation in the type of shear cell developed by Jenike in 1967⁽⁸⁵⁾.

The "shear test" has been widely used to study the failure properties of powders⁽⁸⁶⁾ and the response curve is a plot of shear stress versus time or displacement. The shape of the curve depends on the initial density, the normal stress and the physical properties of the material to be tested⁽⁸¹⁾. A typical shear stress - shear strain plot from a Jenike shear cell is shown in Fig. (3.1). In general, when the specimen is sheared the sample volume will change until it reaches a critical value. If the material is initially highly compacted it will dilate in shear. A response curve of type II (Fig. 3.1) is obtained. If the initial density of the powder is less than the critical density, the bed will compact in shear and a response curve of type I is obtained. Once the critical density has been reached, all the shear work is done in overcoming the internal friction and the powder undergoes no change in volume with increasing normal load but is sheared on planes where the effective applied stress is greater than the inherent shear strength of the material.

3.2.2. Yield locus

The strength of a powder prepared to a specific degree of consolidation can be represented on a shear - compressive stress diagram and is termed the yield locus (YL). This represents the limiting shear stresses (τ) for a range of compressive stresses (σ_n) corresponding to a specific initial degree of consolidation σ_0 (Fig. 3.2). A family of yield loci can be obtained by proceeding from a different degree of initial consolidation and is

depicted in Fig. (3.3). These individual yield loci each terminate at a specific stress condition corresponding to the initial consolidation stress σ_0 . If the powder were to be loaded to a stress beyond the pre-consolidating stress it would fail at a state different from that obtained during ready sliding. The intersection of the yield locus on the τ axis corresponds to the cohesion, c , of the material. For the hypothetical case of a free flowing powder, c will be approximately zero. The inclination of the yield locus with the normal stress axis is termed the angle of internal friction, ϕ_i , which is a measure of resistance to flow. Jenike found the end points of the individual yield loci to lie on a straight line through the origin and termed this the effective yield locus (EYL) and the corresponding angle, the effective angle of internal friction ϕ_E . The position of the individual yield loci are functions of the history of the material as shown in Fig. (3.3). Increasing the pre-consolidating stress (σ_0) increases the cohesion and alters the position of the YL.

However, for non-cohesive Coulomb materials the YL and EYL will be coincident and the final stress state will be independent of the consolidation history of the powder. This was shown experimentally by Mannheim and Passy⁽⁸²⁾ working on a range of starches which are known for their non-cohesivity. These authors found the increase in consolidation pressure to have little effect on the bulk density, cohesion and the angle of internal friction. Lovegrove⁽²⁴⁾ concludes this to be the case but fails to recognise that this only is applicable to non-cohesive powders. He obtained his data on YL working with polystyrene beads which are

non-cohesive. Brown and Richards⁽⁸⁷⁾ working on granular powders also found that the value of YL was independent of consolidating stress, σ_0 .

For a rigid-plastic Coulomb solid the YL is linear and is represented by the equation:

$$\tau = \tan\phi_i \sigma_n + c \quad (3.1)$$

This is known as the Coulomb failure criterion. Failure occurs on planes where the shear stress is greater than the shear strength of the material.

Ashton *et al*⁽⁸³⁾ working on a range of powders found the YL to be a power law type and to conform to the equation:

$$\left(\frac{\tau}{c}\right)^n = \frac{\sigma_n}{T_i} + 1 \quad (3.2)$$

where T_i is the tensile strength and n is the shear index. A material obeying this equation is termed a Warren-Spring material. If n in equation (3.2) is equal to 1, we obtain the Coulomb equation (3.1). A detailed description of the derivation of equation (3.2) is given in reference (83). The values of n were found to lie between 1 and 2, characterising a perfectly free-flowing powder and, at the other end of the spectrum, a highly cohesive powder respectively.

Williams and Birks⁽⁸⁸⁾ have demonstrated a means of reducing a family of YL to a single straight line by expressing the shear-normal stresses as reduced stresses, thereby reducing the

number of experiments required to obtain a YL.

The strength and failure properties of powders and their variation with the process parameters provides a useful insight and assists in the minimising of potential flow problems in practice.

3.3. *Stress State in a Powder*

3.3.1. *Mohr Stress Circle*

The stresses at any point in a powder at equilibrium can be represented by three mutually perpendicular principle stresses. The largest and the smallest of these stresses will prescribe whether the powder will fail and flow. The Mohr circle represents all possible states of normal and shear stresses on any plane through a stressed point in the material and was developed by the German engineer Otto Mohr⁽⁸⁹⁾.

Fig. 3.4 shows a two-dimensional representation of the Mohr Circle. The stress state at any point in a powder can be evaluated by resolving the forces on a triangular element of powder Fig. (3.4). The element of material is subjected to direct stresses σ_{xx} and σ_{yy} in the x and y directions together with shear stresses τ_{xy} and τ_{yx} tending respectively to cause anti-clockwise and clockwise rotation of the element. The triangular element is maintained in equilibrium by a direct stress σ and a shear stress τ acting in an anti-clockwise sense. The element is considered to

be of unit thickness and is in equilibrium under the action of the various forces. Three equations of equilibrium can be obtained.

1. The sum of moments in the plane of the element must be zero.

If moments are taken through a point at the middle of face AB the only stresses producing moments are τ_{xy} and τ_{yx} which are complementary shear stresses since they act on planes separated by 90° .

$$\tau_{xy} AB \cos\theta \cdot 1 \cdot \frac{AB}{2} \sin\theta = \tau_{yx} AB \sin\theta \cdot 1 \cdot \frac{AB}{2} \cos\theta$$

$$\tau_{xy} = \tau_{yx} = \tau' \quad (3.3)$$

2. Resolving forces perpendicular to AB

$$\sigma = \left[\frac{\sigma_{xx} + \sigma_{yy}}{2} \right] + \left[\frac{\sigma_{xx} - \sigma_{yy}}{2} \right] \cos 2\theta + \tau' \sin 2\theta \quad (3.4)$$

3. Resolving forces parallel to AB and equating the sum to zero

$$\tau = - \left\{ \frac{\sigma_{xx} - \sigma_{yy}}{2} \right\} \sin 2\theta + \tau' \cos 2\theta \quad (3.5)$$

Combining and re-arranging equations (3.4) and (3.5)

$$\left[\sigma - \left[\frac{\sigma_{xx} + \sigma_{yy}}{2} \right] \right]^2 + \tau^2 = \left[\frac{\sigma_{xx} - \sigma_{yy}}{2} \right]^2 + \tau'^2 \quad (3.6)$$

This is the equation of a circle of radius:

$$R = \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2} \right)^2 + \tau'^2 \right]^{1/2} = \tau_{\max}$$

and centre at: $\left\{ \frac{\sigma_{xx} + \sigma_{yy}}{2}, 0 \right\}$.

In Fig. (3.4) σ_1 and σ_2 represents principle stresses which act on orthogonal planes and the shear stresses on them are zero.

For a powder in the state of plastic equilibrium on the point of plastic flow the Mohr circle representing the stress conditions is a tangent to the yield locus, Fig. (3.5). If the stress condition in a powder is represented by a Mohr circle lying below the yield locus the powder will remain at rest unless it is adjacent to a wall in which case it may slip along the wall. Points A and B in Fig. (3.5) represent the wall plane. There cannot be anywhere in a powder an equilibrium stress state for which the Mohr circle intersects the yield locus; plastic flow would set in before such a state would develop.

3.3.2. Equations of Equilibrium

The stresses acting on a cube of element are shown in Fig. (3.7). A three dimensional cartesian co-ordinate system is considered. A detailed derivation of the equations of equilibrium is given by Sokolovskii⁽⁹⁰⁾.

A force balance on the x,y and z planes yields the following three equations.

$$\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \frac{\partial \sigma_{xx}}{\partial x} = X \quad (3.7)$$

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = Y \quad (3.8)$$

$$\frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{zx}}{\partial x} = Z \quad (3.9)$$

where X,Y and Z are body forces per unit volume.

These equations hold everywhere in the powder where the stresses vary continuously.

3.3.3. *Active and Passive Stress States*

The active and passive Rankine⁽⁹¹⁾ stress states constitute two limiting states of plastic equilibrium when the material is on the point of shearing (Terzaghi⁽⁹²⁾, Jenike and Johanson⁽⁹³⁾). On filling a container the stress state is near the active state of stress and on initiation of a discharge there is a switch to the passive stress state giving the dynamic condition. A large lateral stress termed the switch stress exists on the transition from the active to a passive stress state. In the active stress state the material will tend to expand horizontally and contract vertically. The major principle stress direction will be near the vertical axis. In contrast, in the dynamic stress state the material will contract radially and expand vertically, the

corresponding principle stress direction will be near the horizontal axis. The wall stresses generated in the dynamic condition will be greater than the static condition. In these states the co-efficients of active and passive pressures K_A and K_P are defined as

$$K_A = \frac{\sigma_{xx}}{\sigma_{yy}}$$

$$K_P = \frac{1}{K_A}$$

The powder may assume any equilibrium state between the limits defined by K_A and K_P depending on the method of filling and on the prior history of the material.

An analytical expression for the active and passive pressure ratios can be obtained by considering the geometry of the Mohr stress circles as shown in Fig. (3.9). For active failure (Fig. 3.9a)

$$\sigma_{xx} = P - [P \sin\phi_i \cos 2\theta]$$

Angle 2θ is measured in the anti-clockwise direction and is positive in the adopted sign convention

$$\sigma_{yy} = P + [P \sin\phi_i \cos 2\theta]$$

$$K_A = \frac{\sigma_{xx}}{\sigma_{yy}} = \frac{P[1 - \sin\phi_i \cos 2\theta]}{P[1 + \sin\phi_i \cos 2\theta]}$$

$$K_A = \frac{1 - \sin\phi_i \cos 2\theta}{1 + \sin\phi_i \cos 2\theta} \quad (3.10)$$

Similarly for passive failure (Fig. 3.9b)

$$K_P = \frac{P[1 + \sin\phi_i \cos 2\theta]}{P[1 - \sin\phi_i \cos 2\theta]} \quad (3.11)$$

The angle of wall friction ϕ_w will be different from ϕ_i , and the ratios K_A and K_P at the wall will therefore assume different values.

K_{wA} = K ratio for active failure at the wall Fig. (3.9a)

$$K_{wA} = \frac{[1 - \sin\phi_i \cos(\omega - \phi_w)]}{[1 + \sin\phi_i \cos(\omega - \phi_w)]} \quad (3.12)$$

$$\sin\omega = \frac{\sin\phi_w}{\sin\phi_i}$$

$$K_{wP} = \frac{[1 + \sin\phi_i \cos(\omega + \phi_w)]}{[1 - \sin\phi_i \cos(\omega + \phi_w)]} \quad (3.13)$$

This analysis for the K_w ratios at the wall was originally presented by Walker⁽⁹⁴⁾.

3.4. Prediction of Wall Stresses

3.4.1. Introduction

The methods available for the prediction of the wall stresses are reviewed in this section and their relative merits and drawbacks are discussed. These techniques are invaluable in the design of process machinery such as extruders and storage vessels. The available methods are:

1. Coulomb's Method of Wedges (Coulomb⁽⁹⁵⁾).
2. Differential Slice Technique (Janssen⁽⁹⁶⁾, Walker⁽⁹⁴⁾, Walters⁽⁹⁷⁾, Nedderman⁽⁹⁸⁾).
3. Method of Characteristics (Sokolovskii⁽⁹⁰⁾, Nedderman⁽⁹⁸⁾).

Coulomb's method of wedges and the differential slice technique are both approximate solutions requiring some assumption about the stress distribution on a control face while the method of characteristics yields an exact solution. It is appropriate at this point to list the assumptions common to all three techniques.

1. The material is assumed to behave as a continuum and is thought of as being isotropic.
2. The material is everywhere in a state of incipient failure obeying the Mohr-Coulomb failure criterion. In principle, any failure criterion could be used but in practice this is not done.
3. Time dependency is neglected and flow is assumed to be steady.

3.4.2. Coulomb's Method of Wedges

Coulomb⁽⁹⁵⁾ in 1776 was the first to analyse forces on retaining walls. A detailed description of his work is given in Terzaghi⁽⁹²⁾, Schofield and Wroth⁽⁹⁹⁾. In his analysis Coulomb assumes the material to fail as a rigid block sliding on a plane where the shear stress is greater than the shear strength of the material. Fig. (3.7) represents the relevant forces on a single retaining wall. The slip surfaces are assumed to be planar. However, the real surfaces of sliding are curved but the error involved due to curvature is less than 5%. Nedderman⁽⁹⁸⁾ has discussed this in his recent review and concludes that, except for the case of a fully rough wall, Coulomb's method gives an excellent prediction of stresses on a single wall in active failure of a cohesionless material. This method is not to be recommended for cohesive materials and for a material in passive failure since the slip surface will in this latter case be bent through a larger angle and the resulting error is much larger than in the former case.

Resolving forces perpendicular to the wall reaction P (Fig. 3.7), gives:

$$R \cos \phi_w = \frac{\gamma y^2}{2} \frac{\cos^2 \phi_i}{[1 + \sqrt{(1 + k) \sin \phi_i}]^2}$$

where γ is the weight density and

$$k = \frac{\mu_{\omega}}{\mu_i} = \frac{\tan\phi_{\omega}}{\tan\phi_i}$$

and the wall stress is given by

$$\sigma_{\omega} = \frac{\gamma h \cos^2 \phi_i}{[1 + \sqrt{(1+k)} \sin \phi_i]^2} \quad (3.14)$$

The twin wall analysis and the relevant equations for the computation of wall stresses are given in a review by Nedderman⁽⁹⁸⁾.

In conclusion, Coulomb's method is essentially two dimensional and is appropriate for bunkers of large aspect ratio. It is unsuitable for bunkers of square cross-section and circular cross-section. The wall stresses predicted increase linearly down to a depth $d \tan \alpha_c$, where α_c is given by

$$\alpha = \tan \phi_i + \frac{\sec \phi_i}{\sqrt{(1+k)}}$$

and d is the separation of the walls. Beyond this point there is a lesser decrease in wall stress and the extent of this decrease is dependent on the surcharge⁽⁹⁸⁾.

3.4.3. Differential Slice Technique

This was originally proposed by Janssen⁽⁹⁶⁾ in 1895. His original analysis was for a bin of circular cross-section, however it can be modified to apply to any other geometry. The solutions

obtained from Janssen type techniques require some assumption about the stress distribution, in addition to the assumptions listed in the introduction of section 3.3. The analysis is subjected to the following further assumptions.

1. The failure is specified in one plane. Strain in the third direction is neglected. The analysis is essentially two dimensional.
2. The material cohesion is neglected.
3. The material has undergone sufficient strain and is at the critical state at which failure can occur at constant volume and constant yield stress.
4. The vertical and horizontal stresses are only functions of the y direction.

Some of these assumptions are relaxed by subsequent workers⁽¹⁰⁰⁾⁽¹⁰¹⁾.

A two dimensional formulation of the Janssen analysis is considered subject to the given assumptions. The forces acting on an element of material in a cylindrical container is shown in Fig. (3.8).

Force balance on an element of thickness dy

$$\frac{\pi D^2}{4} [\sigma_{yy} + d\sigma_{yy} - \sigma_{yy} + \tau dy] = - \tau_w \pi D dy \quad (3.15)$$

Janssen further assumed σ_{xx} and σ_{yy} to be principle stresses and therefore $2\theta = 180^\circ$, and equation (3.10) and (3.11) reduces to

$$K_A = \frac{1 - \sin \phi_i}{1 + \sin \phi_i} = \frac{1}{K_P} \quad (3.16)$$

The shearing stress at the wall $\tau_\omega = \mu_\omega \sigma_{xx}$ substitution of $\mu_\omega \sigma_{xx}$ for τ_ω and $\sigma_{yy} K_A$ for σ_{xx} in equation (3.15) yields:

$$d\sigma_{yy} = \left[-\frac{4}{D} \mu_\omega K_A \sigma_{yy} + \tau \right] dy \quad (3.17)$$

intergration subject to boundary conditions: $\sigma_{yy} = Q$ at $y = 0$ and $\sigma_{yy} = \sigma_{yy}'$ at $y = H$ yields:

$$\sigma_{yy} = \frac{\tau D}{4\mu_\omega K_A} \left[1 - \exp - \left[\frac{4\mu_\omega K_A H}{D} \right] \right] + Q \exp \left[- \frac{4\mu_\omega K_A H}{d} \right] \quad (3.18)$$

As $y \rightarrow \infty$ the exponential terms dissappear and

$$\sigma_{yy} \rightarrow \frac{\tau D}{4\mu_\omega K_A}, \quad \sigma_{xx} \rightarrow \frac{\tau D}{4\mu_\omega} \quad \text{and} \quad \tau_\omega \rightarrow \frac{1}{4} \tau D.$$

These expressions give the asymptotic values of σ_{yy} , σ_{xx} and τ_ω at great depth.

For bunkers of other than circular cross-section the groups $\frac{\pi D^2}{4}$ and πD can be suitably altered.

A similar analysis exists for passive failure, where K_A is replaced by K_P .

For passive failure neglecting weight density

$$\sigma_{yy} = Q \exp \left[-K_P \frac{4\mu_\omega H}{D} \right] \quad (3.19)$$

Equations (3.19) and (3.18) prescribe the lower and upper limit of the range of possible vertical stresses generated. These vertical stresses can be converted to the appropriate wall normal stress σ_{xx} by multiplying eq. (3.18) by K_A and eq. (3.19) by K_P . These wall stresses generated in active failure will be less than those for the case of passive failure.

To emphasize the limiting values of the wall stresses, Sundaram and Cowin⁽¹⁰¹⁾ expressed their equations as a set of inequalities. These authors also discuss the various assumptions which are not necessary to obtain the essential feature of Janssen's result. They argue that it is not necessary to assume uniform vertical and horizontal stresses but that they could be averaged over the cross-section and perimeter respectively. Cowin⁽¹⁰⁰⁾ has also extended the Janssen formula to account for material consolidation and the dependency of the K ratio on the vertical stress. A major drawback of the original Janssen formula is the assumption that σ_{xx} is a principle stress. This is clearly unsatisfactory since if this were to be the case the associated shearing stress at the wall is zero and the analysis breaks down.

A better analysis of stresses near the wall is given by Walker (Fig. 3.9) and the respective K_{WA} and K_{WP} are given by eq. (3.12) and (3.13). Walker allows for the variation of the vertical stress and distinguishes between an average value $\overline{\sigma_{yy}}$ and $\sigma_{yy\omega}$. He defines the distribution factor $\mathcal{D}$ as the ratio of these vertical stresses.

$$\mathcal{D} = \frac{\sigma_{yy\omega}}{\sigma_{yy}}$$

The original Janssen force balance eq. (3.15) now becomes

$$\frac{\pi D^2}{4} [\overline{\sigma_{yy}} + d\overline{\sigma_{yy}} - \overline{\sigma_y} + \gamma dy] = \mu_\omega K_{WA} \mathcal{D} \overline{\sigma_{yy}} \pi D dy \quad (3.20)$$

The distribution factor $\mathcal{D}$ is an unknown function of depth and in order to solve eq. (3.20) this functionality must be known. However, Walker⁽⁹⁴⁾ derives an expression for $\mathcal{D}$ at great depth .

$$\mathcal{D}_{\text{at great depth}} = \frac{\cos\phi \{1 + \sin^2\phi_i\} \pm 2\{\sin^2\phi_i - \sin^2\phi_\omega\}^{1/2}}{\cos\phi_i \{(1 + \sin^2\phi_i) \pm 2Z\sin\phi_i\}}$$

$$Z = \frac{2}{3c} [1 - (1 - c)^{3/2}]$$

and

$$c = \left\{ \frac{\tan\phi_\omega}{\tan\phi_i} \right\}^2$$

The upper sign refers to static conditions and the lower sign to the dynamic condition. The distribution factor $\mathcal{D}$ is therefore only a function of ϕ_i and ϕ_ω . Walters⁽⁹⁷⁾ has computed values of $\mathcal{D}$ at great depth for a range of ϕ_i and ϕ_ω combinations.

Walters and Nedderman⁽¹⁰²⁾ point out that in a cylindrical system σ_{xx} is not constant because of the influence of the circumferential stress but the resulting error in $\mathcal{D}$ for active failure is small. Neglecting weight density the solution to the

Janssen-Walker equation (3.20) for active failure is:

$$\overline{\sigma_{yy}} = Q \exp \left[- 4\mu_{\omega} K_{WA} \mathcal{D} \frac{H}{D} \right] \quad (3.21)$$

$$\sigma_{yy_{\omega}} = \mathcal{D} Q \exp \left[- 4\mu_{\omega} K_{WA} \mathcal{D} \frac{H}{D} \right] \quad (3.22)$$

$$\sigma_{xx_{\omega}} = K_{WA} \mathcal{D} Q \exp \left[- 4\mu_{\omega} K_{WA} \mathcal{D} \frac{H}{D} \right] \quad (3.23)$$

and for passive failure:

$$\frac{\sigma_{xx_{\omega}}}{\sigma_{yy_{\omega}}} = K_{WP} \quad (3.24)$$

The resulting equations are:

$$\overline{\sigma_{yy}} = Q \exp \left[\frac{4\mu_{\omega} \mathcal{D} K_{WP}}{D} \right] \quad (3.25)$$

$$\sigma_{yy_{\omega}} = \mathcal{D} Q \exp \left[\frac{4\mu_{\omega} \mathcal{D} K_{WP}}{D} \right] \quad (3.26)$$

$$\sigma_{xx_{\omega}} = K_{WP} \mathcal{D} Q \exp \left[\frac{4\mu_{\omega} \mathcal{D} K_{WP}}{D} \right] \quad (3.27)$$

The Janssen-Walker analysis is completely rigorous apart from the assumption of the value at great depth for the distribution factor $\mathcal{D}$.

3.4.4. *Method of Characteristics*

This method follows closely the work of Sokolovski⁽⁹⁰⁾ and involves the solution of the basic differential equations. No assumption about the stress distribution is required and the method is exact for a Coulomb material in the critical state. At every point in the material, the stress state is completely specified by the value of the mean of the principle stress P and the orientation of the principle stress to a horizontal x -axis, θ .

$$\frac{\sigma_{xx} + \sigma_{yy}}{2} = P$$

The equations of equilibrium for a two dimensional system are

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = \gamma y$$

where γ is the weight density of the material.

These equations are statically indeterminate and in order to solve them some relation between the stresses and the major principle stress direction θ is required. The Mohr-Coulomb failure criterion provides this third relation and converts the system to a deterministic form. The appropriate Mohr-Coulomb failure equations are:

$$\sigma_{xx} = P[1 - \sin\phi_i \cos 2\theta]$$

$$\sigma_{yy} = P[1 + \sin\phi_i \cos 2\theta]$$

$$\tau_{xy} = -\tau_{yx} = P \sin\phi_i \sin 2\theta$$

Substitution of these into the equations of equilibrium yields a pair of partial differential equations which can be shown to be hyperbolic in form and reduce to ordinary differential equations in two directions α and β through each point.

The directions are given by

$$\frac{dy}{dx} = \tan\{\theta \pm \epsilon\} \text{ where } 2\epsilon = \frac{\pi}{2} - \phi_i \quad (3.28)$$

The corresponding ordinary differential equations are

$$dP \pm 2P \tan\phi_i d\theta = dy \pm \tan\phi_i dx \quad (3.29)$$

The upper sign refers to a β characteristic and the lower sign to an α characteristic. Equations (3.28) and (3.29) provide two equations but there are four unknowns at each point, x, y, P and θ . However, equations (3.28) and (3.29) can be expressed in finite difference form to provide the required four relations⁽¹⁰³⁾.

$$y - y_1 = (x - x_1) \tan(\theta_1^* - \epsilon)$$

$$y - y_2 = (x - x_2) \tan(\theta_2^* - \epsilon)$$

$$P - P_1 - 2P_1^*(\theta - \theta_1)\tan\phi_i = [y - y_1] - (x - x_1)\tan\phi_i]$$

$$P - P_2 + 2P_2^*(\theta - \theta_2)\tan\phi_i = [y - y_2] + (x - x_2)\tan\phi_i]$$

Since θ is a variable, the characteristics are not linear and the equations are solved iteratively by replacing the quantities denoted by $*$ by the average value over the interval. If boundary conditions are known this method will give stress parameters at a mesh of points of any desired spacing⁽¹⁰³⁾.

For axi-symmetric situations as in the case of a cylindrical container it is only necessary to perform computations on one side of the bunker as the state of stress is symmetrical about the y axis. A characteristic mesh for the problem is shown in Fig. (3.10). The wedge-shaped region OAB beneath the top surface is an active Rankine zone where the characteristics are linear and an analytical solution for the stress distribution is available. In the Rankine zone

$$y = x \tan\left\{\frac{\pi}{4} + \frac{\phi_i}{2}\right\}, \quad \theta = \theta_{\text{top}} = \frac{\pi}{2} \text{ for active failure and} \\ = \pi \text{ for passive failure.}$$

$$\sigma_{yy} = Q = P[1 \pm \sin\phi_i],$$

the upper and lower signs refer to active and passive failure respectively .

The region BAC is a zone of "radial shear". The change in slopes of the characteristic lines reflect the change in value of

P and θ due to stress discontinuity at A. At A, equation (3.29) can be integrated since at this point dx and dy are zero.

$$dP = \pm 2P \tan\phi_i d\theta$$

$$\sigma_{yy} \int_Q \frac{dP}{2P} = \pm \tan\frac{\pi}{2} \int d\theta$$

$$\sigma_{yy} = P(1 + \sin\phi_i)$$

$$P = \frac{Q}{1 + \sin\phi_i} \exp[(\pi - 2\theta)\tan\phi_i] \quad (3.30)$$

The remaining boundary conditions required for calculation of data at all points in the grid are the values of θ along the centre line and the wall. Symmetry provides θ along the centre line $\theta = \frac{\pi}{2}$ and θ_ω is determined simply from a Mohr circle of stress at the wall:

$$\theta_\omega = \frac{\pi}{2} \pm \frac{1}{2} \left[\frac{\sin\phi_\omega}{\sin\phi} - \phi_\omega \right]$$

for active and passive failure.

In the characteristic mesh shown in Fig. (3.10) it is clear that the value of θ at the discontinuity A is multivalued and is given by θ_{top} and θ_{wall} and the resulting plots reveal a blank region between the α type characteristic emanating from the top right hand corner of the wall and the Rankine zone boundary. In order to facilitate the continuation of the computation of stresses a "fictitious grid" of fan characteristics is

generated⁽¹⁰³⁾. This involves assuming values of θ in the range

$$\theta_{\text{top}} \leq \theta \leq \theta_{\text{wall}}$$

and obtaining the corresponding P values to build up the fan region to enable the calculations to proceed to the wall.

The method of characteristics will generate stress parameters at a mesh of points by solving the respective α and β type characteristic equations provided there are sufficient boundary conditions for the solution to proceed.

3.5. *Concluding Remarks*

The methods for the prediction of wall stresses differ principally in the assumption about the distribution of stresses. At great depths the stresses tend to limiting values and are independent of the stress distribution and all the methods give essentially the same result. This is confirmed by Hancock and Nedderman⁽⁶³⁾ for a range of ϕ_i and ϕ_w values. Further, the use of the method of characteristics to solve problems in axi-symmetry as in a cylindrical container poses computational difficulties⁽¹⁰³⁾.

It can be concluded that at great depth the method of characteristics and Janssen-Walker analysis becomes identical and the only difference which remains is that the former technique is numerical while the latter is analytical. The differential slice technique gives results in good agreement with the exact solution

with typical deviations of less than 5%, which is less than the errors associated in stress measuring equipment. This is particularly true for active failure. In passive failure the influence of the circumferential stress on the $\mathcal{D}$ ratio should be included. In view of the difficulties encountered, theoretical predictions based on Janssen's method are probably accurate enough for most practical purposes⁽⁹⁸⁾.

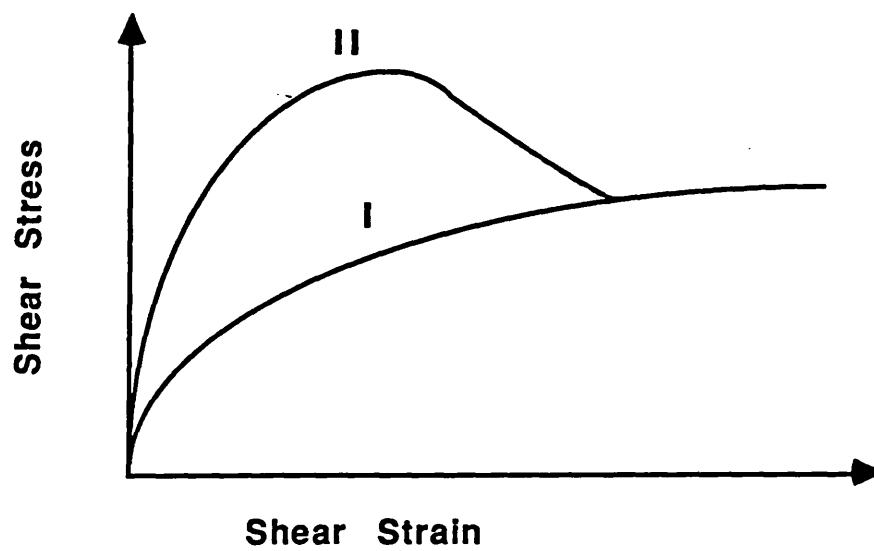


Fig.3.1 A typical plot from a "shear test" . I, initial density of the powder less than the critical density. II a highly consolidated powder showing dilatancy during shear.

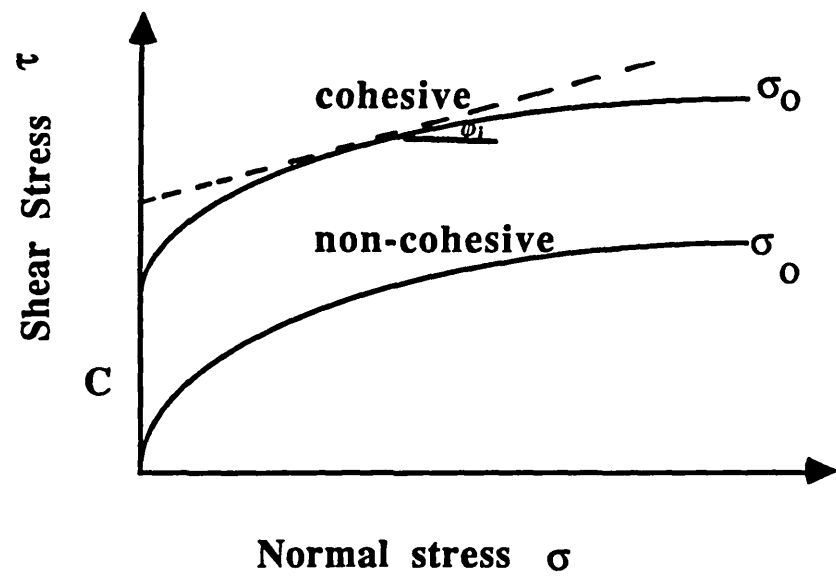


Fig.3.2 The yield loci obtained in a shear test for a cohesive and non-cohesive powder. σ_0 is the pre-consolidation stress and ϕ_i is the angle of internal friction.

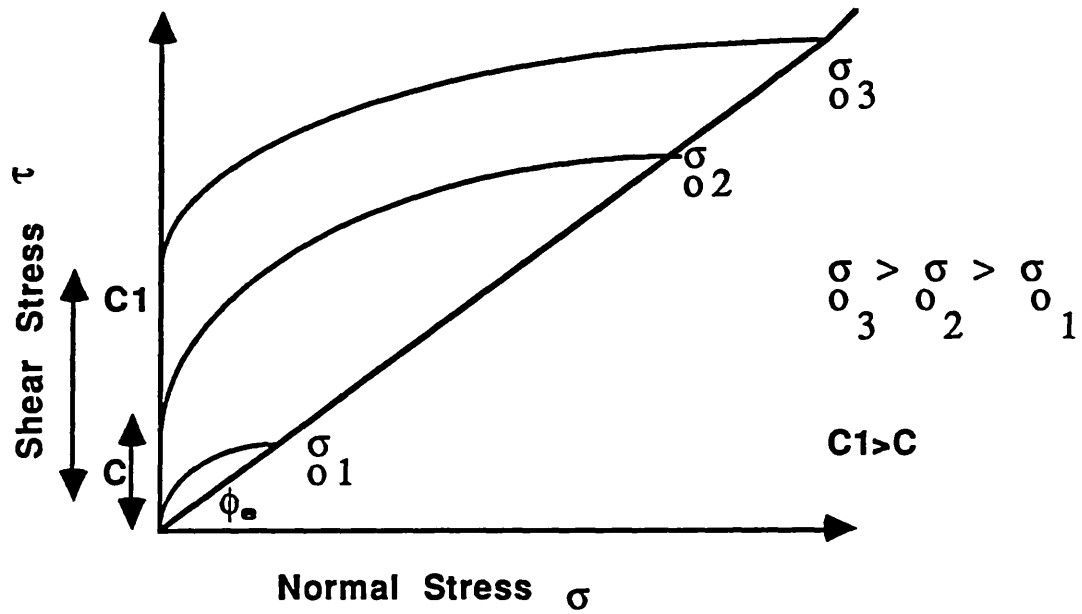


Fig.3.3 The variation of the yield locus with consolidation pressure for a cohesive powder. C is the cohesion and ϕ_e the effective angle of internal friction.

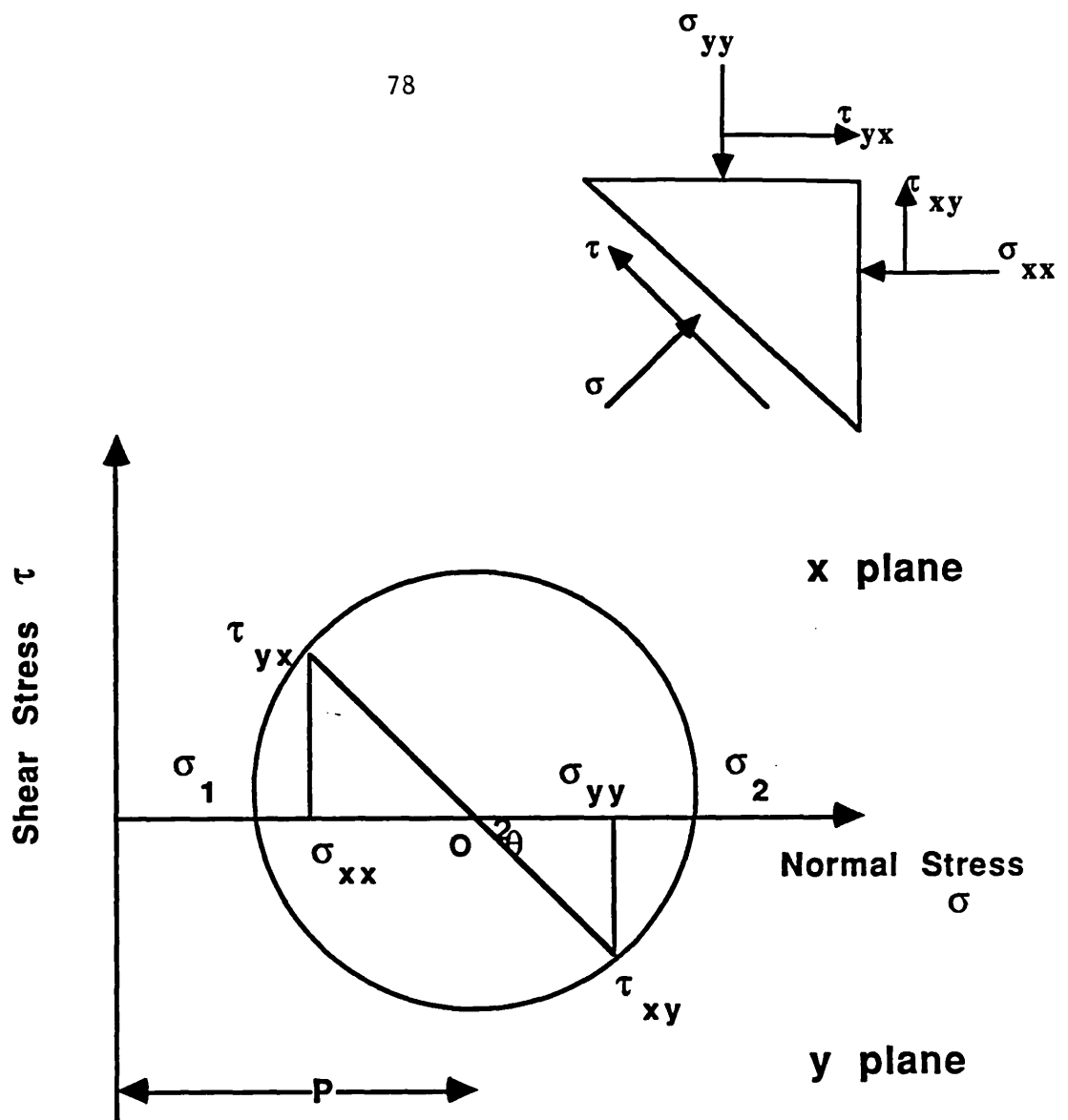


Fig 3.4 The Mohr stress circle for an element in plastic equilibrium. σ_1, σ_2 are principle stresses, σ_{xx}, σ_{yy} are horizontal and vertical stresses. P is the mean of the principle stresses and θ the inclination of the major principle stress direction with the x-axis.

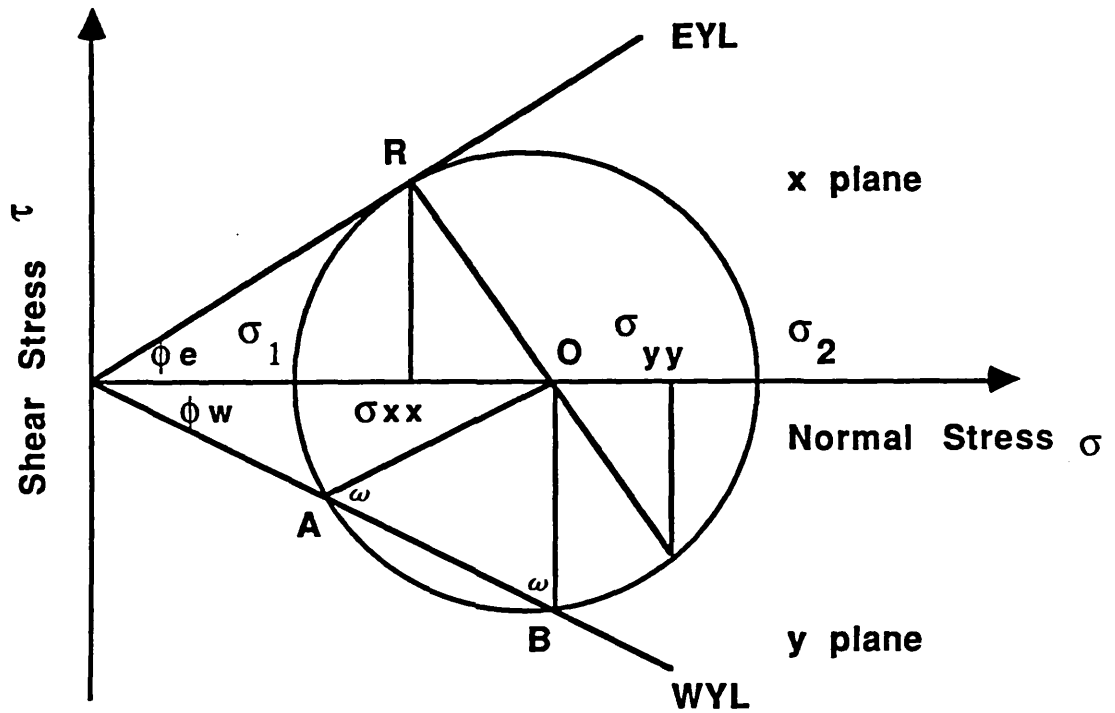


Fig.3.5. The Mohr stress circle for a powder in plastic equilibrium at the point of plastic flow. $\sin \omega$ is the ratio of the sines of the angles of wall ϕ_w and internal friction ϕ_e

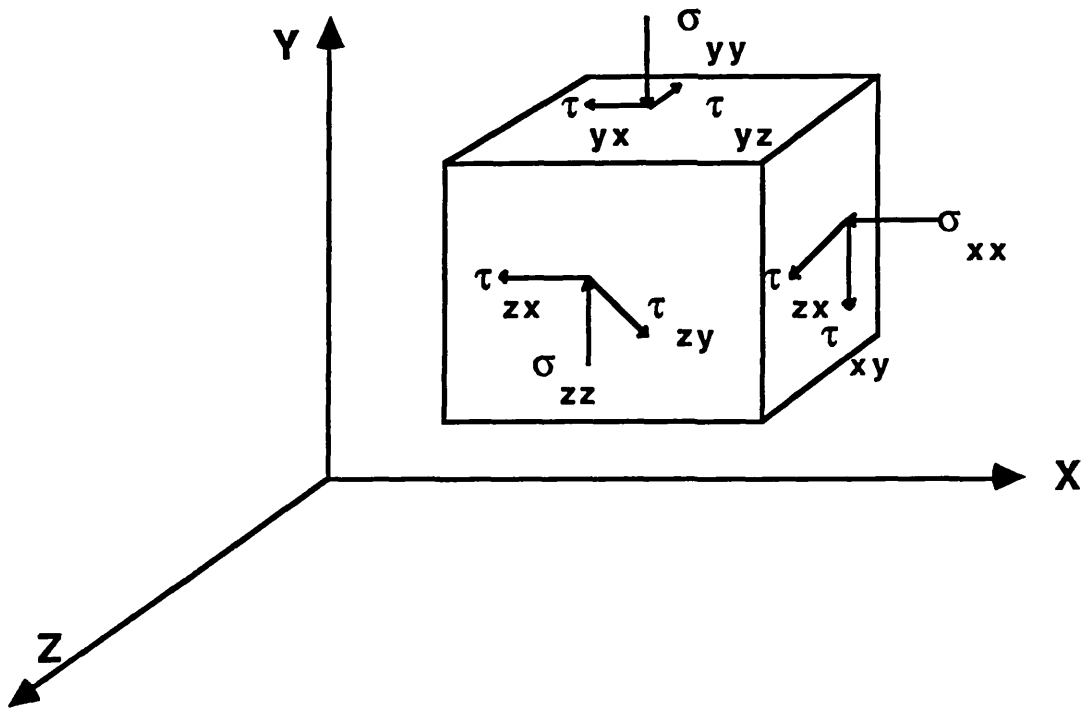


Fig 3.6 A three dimensional representation of the stresses acting on a cube of element. σ, τ are normal and shear stresses respectively.

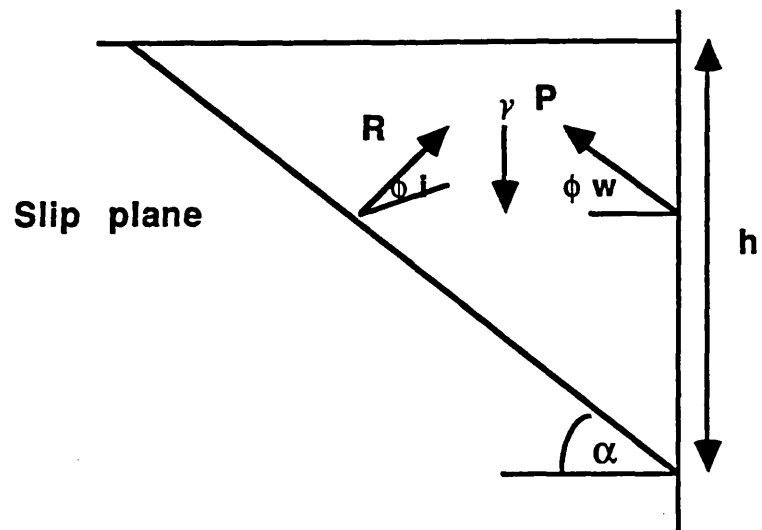


Fig.3.7 Forces on a single retaining wall. P is the wall reaction, α the inclination of the slip plane with the x axis. ϕ_i , ϕ_w are angles of wall and internal friction respectively.

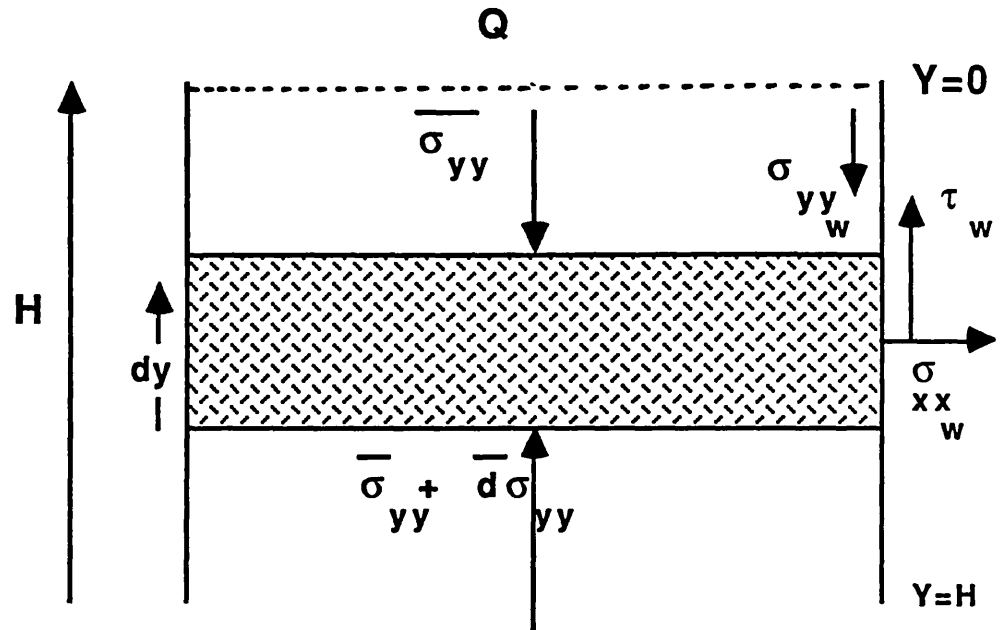
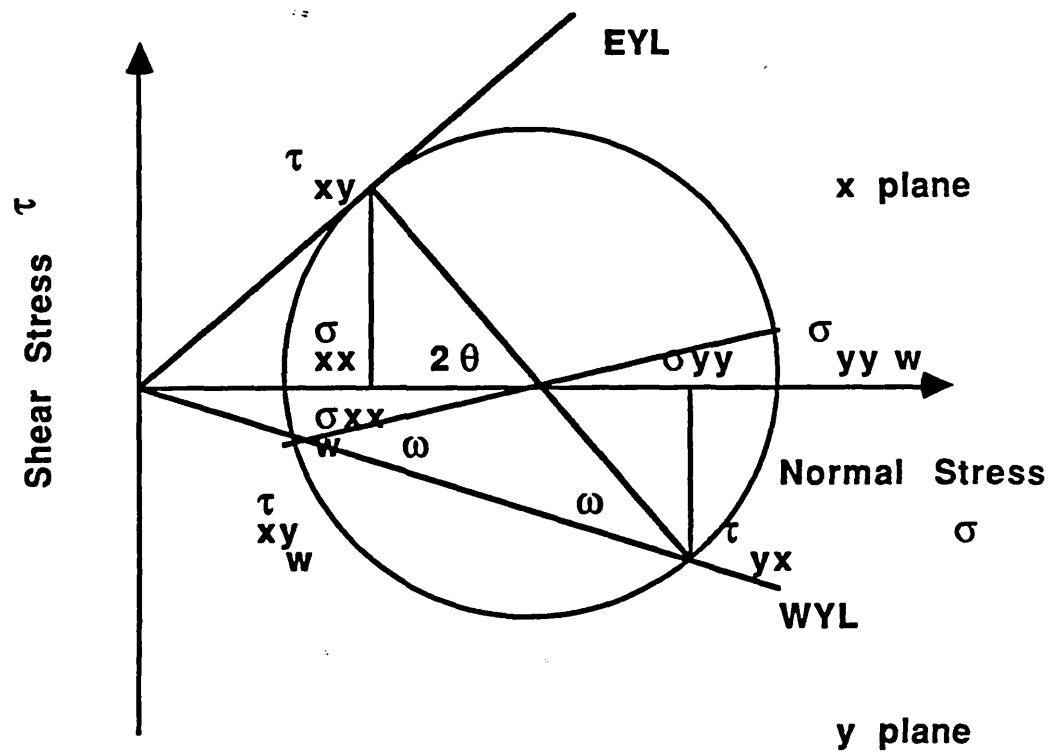


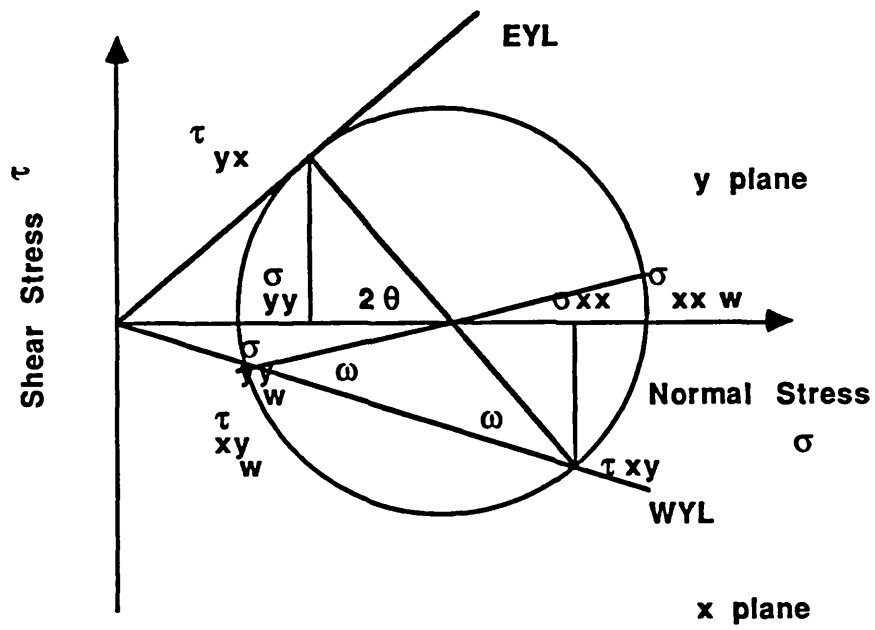
Fig.3.8 A two dimensional representation of stresses acting on an element in a cylindrical container.

Fig.3.9 The Mohr stress circles for active and passive failure. K_{wa} and K_{wp} are the co-efficients of active and passive pressures at the wall respectively.



$$\text{Active failure } K_{wa} = \frac{\sigma_{xx w}}{\sigma_{yy w}}$$

Fig.3.9a



Passive failure $K_{wp} = \frac{\sigma_{xx}}{\sigma_{yy}}$

Fig.3.9b

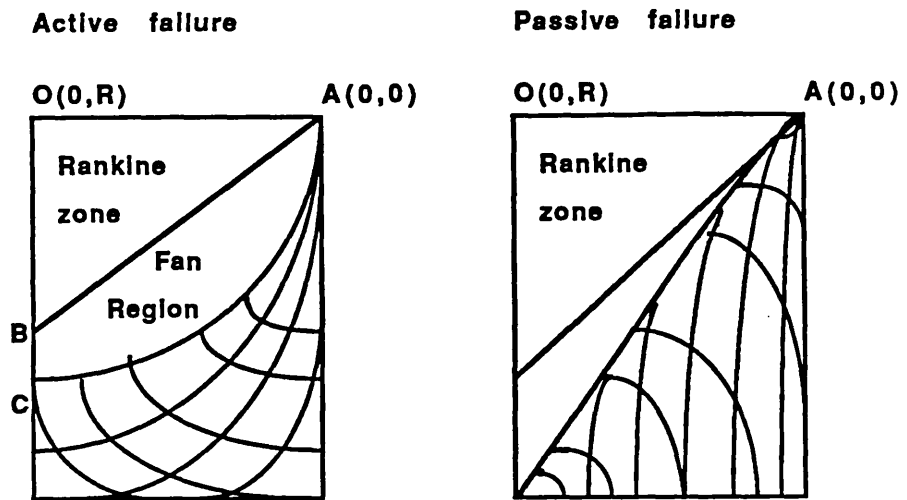


Fig.3.10 A characteristic mesh of the variation of stresses depicting active and passive failures.

Chapter 4

Experimental Methods

4.1. *Introduction*

This chapter describes the experimental techniques adopted in this study to simulate and investigate the role of wall traction in the continuous extrusion of maize. The first part will deal with the friction of compacted maize powder as a means of simulating wall traction in both extrusion and uniaxial compaction. Later sections deal with normal force generation, uniaxial compaction and extrusion *per se*.

4.2. *Material*

The powder used in this study was a maize flour (fine polenta, F. Smiths Mills, Workshop, U.K.). Its dry matter composition was obtained by chemical analysis as 80.5% starch, 13% protein, 1.7% dietary fibre, 1.4% fat, 0.7% free sugar and 0.2% ash. The moisture content of the raw material was 13%.

4.3. Friction Measurements of Compacts

4.3.1. Introduction

The basic experimental and analytical techniques used in this study have been extensively applied to the investigation of the frictional properties of synthetic polymers (Briscoe *et al* 1973⁽⁵²⁾, Briscoe and Smith 1980⁽⁴⁹⁾). Samples of maize flour were compression moulded to produce hemi-spherical surfaces at a number of fabrication temperatures with the addition of various amounts of water. A schematic view of the compacted surface is shown in Fig. (4.1). Two sliding configurations were used.

1. A compact hemi-sphere sliding over a smooth glass substrate.
2. A flamed glass substrate loaded against a thin film of the material deposited on a smooth glass flat.

4.3.2. The Fabrication of Compact Hemi-spherical Indentors

The compacted samples were fabricated using a plunger mould compressed between the heated plates in an Instron testing machine. The mould consisted of a highly polished aluminium base plate which had been indented using a steel ball bearing 10mm in diameter. A brass collar was attached to the base plate and the material compressed into a hemi-spherical compact using the plunger. In most of the current experiments the indenter was 6mm in radius and the maize powder compressed to a pressure of 3×10^7 Pa at a number of temperatures up to 200°C. The effect of

primary extrusion variables - interface temperature, normal pressure and moisture content - on friction were investigated by attaching these compacts to the load head of a sensitive friction machine (Fig. 4.2) and sliding these on smooth glass counterfaces. The friction force was also recorded as a function of the compact radius and moisture. Scanning Electron Micrography was used to study the surfaces of deformed and undeformed samples (Fig. 4.3).

4.3.3. *Friction Machine*

A detailed description of the friction apparatus is given by Scruton⁽¹⁰⁴⁾ who was responsible for the initial design of this machine. It was built around a chemical balance which allows the application of small and accurately known loads to the contact. The upper contact is spring loaded, such that its deflection changes the inductance of the displacement transducers (Type T₆/UNF, Sangamo Controls Ltd.) mounted on the beam assembly. These transducers constitute two arms of a carrier wave frequency bridge circuit, the output of which was electrically filtered and fed to a fast response chart recorder {Philips PM 8222}. The signal was of the form of a square wave of amplitude equivalent to twice the frictional force; the indenter travelled over the substrate in a reciprocating manner. The position of the contact between the upper and lower substrates, controlled by a x-y microscopic traverse, was attached to the lower stage. The bottom substrate was clamped to a heating stage which was driven in a reciprocating motion of 2mm amplitude by a hydraulic link acting

through two bellows from a constant speed electric motor. Repeated traversals of the lower surface were made at a sliding speed of 0.15mm sec^{-1} until an equilibrium friction force was recorded. The recorder trace was calibrated prior to testing, by imposing known horizontal loads via a slider-pulley system. The temperature of the glass flat was varied from ambient to 200°C by means of a heating block. The output of the block was controlled by a "Eurotherm" temperature controller and the surface temperature of the glass flat was monitored by a Chromel-Alumel thermocouple fixed to the glass with high conductivity silver paint.

4.3.4. *Thin Film Shear*

Thin films were prepared by transfer deposition, which involves rubbing a compact over the surface of a clean glass plate. This technique of film deposition is known to produce thin layers of material of approximately 10nm to 100nm in thickness which are often highly drawn in the direction of rubbing (Pooley and Tabor⁽⁴⁷⁾). It was not considered necessary during the present investigation to characterize these films in terms of uniformity and thickness.

The upper surface was made by mounting a tear drop shaped piece of silica glass in a brass collar using an adhesive. After flame cleaning and optical microscopy examination for scratches and debris, the collar was screwed to the load head of the friction machine and the beam balanced. Friction force was

recorded as a function of the slider radius, normal load, moisture content and interface temperature. The temperature of the interface was varied between ambient and 200°C and the normal load range between 0.05N to 1.00N.

Prior to testing, the glass substrates were cleaned by soaking in a methanolic potassium hydroxide solution followed by multiple rinsing in double-distilled water and dried overnight in an air circulating oven. The substrates were then allowed to cool to room temperature before use.

4.3.5. *Mechanical Properties of Compacted Maize Samples*

A commercial Spec-ac press was used to fabricate maize cylinders of 13mm in diameter and to a height of 5mm. The Spec-ac press was enclosed in a heating chamber which allowed the fabrication temperature to be raised to a maximum of 250°C. The Spec-ac mould containing the powder was placed in an Instron testing machine and compressed to a load of approximately 3kN. Samples were also made at 50°, 100°, 150° and 200°C and with 16% w/w added water. These samples were then removed from the mould and compressed in an Instron testing machine until fracture occurred at a cross-head speed of 5mm min⁻¹. The force displacement trace obtained during pressing was used to compute the apparent stress and strain values to rupture. The data obtained were then fitted to obtain the stress-strain correlations and the apparent Young's Moduli in compression at a strain of 20%.

The respective rupture stress and strain were also recorded for each sample.

4.4 Pressure Propagation in Uni-axial Compaction

4.4.1. Introduction

In an experimental extruder, the compaction of the material induces traction forces and it is the traction which creates the compaction. The traction scales the normal force along the barrel wall. These wall stresses can be simulated in a uni-axial compaction experiment.

4.4.2. Test Equipment

A schematic diagram of the specially constructed compaction rig is shown in Fig. (4.4). It consisted of a hollow cylinder with a fixed base plate and a moving piston. The cylindrical die was 51mm in diameter and 110mm in height. It was machined to a normal smooth finish of 1.6 μ m CLA. The transmitted and radial pressures generated during compaction were monitored by the application of a range of surcharge stresses to the top surface. The compression was performed in an Instron testing machine with a full scale deflection of 5kN. The maximum compaction load reached in the present work was 4.5kN. The applied and transmitted loads were measured by two pre-calibrated load cells attached to the

upper plate and the fixed lower base plate. These loads were converted to pressures by dividing by the area of the cylinder normal to the loading direction. The radial pressure generated at the cylinder wall was also measured using a Dynisco pressure transducer (Model PT 420F). The sensing end of the transducer consists of a one piece machined diaphragm on which pressure acts. The resulting displacement of the diaphragm was transmitted through a mercury filled capillary tube to strain gauges from which an electrical signal was obtained. The signal from the pressure transducer and the load cells were both fed into a bridge amplifier prior to recording. A x-y plotter continuously recorded the output from the load cells while an ultra-violet recorder monitored the output from the pressure transducer.

4.4.3. *Pressure Measurement in Uni-axial Compaction*

Compaction data were obtained at three cross-head speeds 12.5, 25.4, 50.8mm min⁻¹ and three moisture levels %, 8% and 16% added moisture respectively. In all these experiments the test temperature was kept constant at ambient; ca 20°C. Unfortunately it was not possible to vary the test temperature due to serious malfunctioning of the pressure transducer at elevated temperatures.

The compaction rig was also used to determine stress relaxation curves of maize powder at 0%, 8% and 16% w/w added moisture respectively. The powder was consolidated to a load of 4kN and held constant at this value. The decay in the transmitted

pressure to a constant steady value was monitored. It was not possible to monitor the decay of the radial pressure transducer since this transducer was ruptured during testing and a replacement was not available.

The pressure transducer was calibrated before use with a Budenberg dead weight pressure gauge tester and was energised by a 5V DC external power supply.

4.5. *Shear Cell*

Bulk solids, unlike liquids, can support static shear stresses and the shear-normal stress characteristics of a solid can be measured using a Jenike shear cell⁽⁸⁵⁾ developed in 1967. The technique involves the modest compaction of a powder specimen under a normal stress followed by a shear operation. In its simple form the shear cell consists of a box which is split horizontally. One half of the box is displaced at a constant rate and the resultant force on the other half of the box is monitored by a proving ring.

The shear cell used in the present work is similar to that of Jenike, and is shown in Fig. (4.6). Material was packed in the base and moulded in several layers and slightly tapped with a metal flat to attain some uniformity in packing. A twisting top was now rested on the free surface of the material and a compressive stress σ_0 of $0.77 \times 10^6 \text{Pa}$ applied for a period of 2min. After this time the load was removed and the top lid

twisted by a number of revolutions which was determined previously by repeated preliminary runs. This number of twists corresponds to the case when the resulting displacement time curve attains and maintains a steady value. If the number of twists is too large, the material will be over-consolidated and the resultant displacement - time curve passes through a maximum. If the number is too small, the curve maintains a positive slope throughout the test. A compressive load less than the initial consolidation load was then applied to the top surface and the material sheared until a steady shear force was obtained. This procedure was repeated for a range of normal loads between 0.1kN - 3.2kN, and at moisture levels of 8% and 16% respectively. The proving ring had a calibration constant of 2.1N/division and the shear cell was displaced at a rate of 0.581mm min^{-1} . The steady state shear force was plotted for each normal load in order to determine the yield locus of the material. Ideally, this process should be repeated starting with a different initial consolidation load in order to establish the EYL. However, this process was only repeated for a single pre-consolidation stress and the EYL was found by connecting the origin with the terminal point of the yield locus. A displacement - time plot obtained in the shear - test at a pre-consolidation load of 0.4kN is shown in Fig.(4.5).

4.6. *Normal and Shear Force Measurements*

4.6.1. *Introduction*

In most of the experiments described, a pre-determined normal load was applied. In contrast, a normal force will be generated at the extruder barrel due to traction during extended flow. Therefore it was decided to investigate the normal and shear forces generated during flow of maize powder. A Weissenberg Rheogoniometer offered this possibility since it is able to measure normal as well as shear components of the stress tensor. In the rheogoniometer, the test sample is contained between two instrument members, one of which is driven at a prescribed angular velocity while the other is stationary. Facilities were available to determine the torque on the stationary member and the normal force on the rotary member.

4.6.2. *Weissenberg Rheogoniometer*

The Weissenberg Rheogoniometer used in the present work was model R16 {Sangamo Controls Ltd., Bognor Regis}. It basically consists of a torsion head, normal force measuring unit and the plates. A brief description of each of these units is now appropriate.

The torsion head mounted on the column of the Weissenberg Rheogoniometer carries the top plate. A wide range of torsion bars are available, the selection of which depends on the torque

generated. In all the experiments, the torsion bar used had a calibration constant of 3.51×10^4 dyne cm per 0.001 inch of deflection. The air bearing in the torsion head is supplied with air at a pressure of 80p.s.i. Air passes through a filter unit mounted at the back of the instrument and is regulated at the top and bottom. The top regulator is used to select the working pressure of 20p.s.i. The lower end of the torsion bar carries the 10cm radius arm to which is attached the armature of the torsion head transducer {F.51/T/M}. The displacement of this transducer, read from the appropriate transducer meter {Boulton Paul EP.597} gives, with the calibration constant of the appropriate torsion bar, a direct reading of the torque and hence the tangential stress in the specimen. On the left hand side of the carriage there is an adjustable knurled screw which bears on the gap setting transducer mounted on the side of the column. This monitors the clearance between the plates.

The normal force spring is 1.25 cm in width with a spring constant of 1.5383×10^5 dynes per 0.001 inch of deflection. At the rear end it is held rigidly as a cantilever in a rigid block which is positioned accurately in the base of the column. The free end of the cantilever spring rests on a ball-bearing providing a frictionless joint for positioning the free end by the micrometer. During measurements of normal force it was found to be important to keep the gap between the two plates constant. This can be detected by the servo transducer which bears on the underside of the normal force pivot bearing and its deflection seen on the appropriate transducer meter. A correction for gap

separation can be made by rotating the micrometer until the reading is zero on the servo transducer meter. The reading of the normal force transducer meter {EP.597} with the spring constant gives a direct reading of the normal force.

A cone and plate geometry was used in all the experiments and the plates were machined from a high temperature resistant grade of steel (EN41A) to minimise thermal expansion, thus reducing its contribution to the normal force. The plates were 7.5cm in diameter and the cone angle was 1.5° . Another pair of plates were machined with the same dimensions but with a circular groove of 1mm depth and 4mm width to establish the effect of grooving on normal and shear force generation.

The effect of rotational speed on normal force and torque generation was established in the speed range 0.3 to 30r.p.m. Rotational speeds above 30r.p.m. could not be used due to the effect of centrifugal force becoming dominant at higher speeds, resulting in the sample being thrown out of the plates. A series of experiments were carried with mis-aligning the plates since this corresponds in some degree to the effective extruder geometry. It was found to be difficult to obtain an accurate measurement of the tilt and off-centre of the plates but an approximate measurement was made with the aid of a depth gauge and a vernier. The plate temperature was varied between ambient and 200°C . The plates were surrounded by an aluminium block which carried the electrical heater assembly. The input to the heaters was controlled by a Stanton Redcroft Universal Temperature Programmer while the sample temperature was monitored using a

Chromel-Alumel thermocouple. Finally, these experiments were repeated with the addition of 16% w/w moisture to the powder.

4.6.3. *Possible sources of error*

The basic equations adopted for the determination of the shear stress from the torque measurements were based on the assumption of the existence of a simple type of flow. An important consideration therefore was the extent to which these assumptions were valid in practical situations and the limitations, if any, they impose on the operating conditions. This section briefly considers these assumptions in relation to the operation of the Weissenberg Rheogoniometer and how they influence the interpretation of experimental results.

The constant shear rate assumption in a cone and plate geometry has been considered by Walters and Waters⁽¹⁰⁵⁾. They have shown this to be clearly dependent on the gap angle but for gap angles less than 4° , the constant shear rate assumption is reasonable. Table (4.1) quotes the associated errors involved for gap angles up to 10° from Adams and Lodge⁽¹⁰⁶⁾.

In the steady simple shearing flow proposed for a cone and plate geometry, the governing equations are applicable when inertial effects are negligible. However it has been shown that even in the absence of inertial effects there is a need to invoke secondary flows in order to satisfy compatibility conditions. The associated error is yet again large for large cone angles.

Further edge and end effect errors arise from the fact that

the instrument members have finite dimensions, while theory is based on infinite cone and plate dimensions. The flow field is distorted near the cone rim and the extent of this distortion is a function of $\frac{h}{a}$ where h is the separation between the cone and plate at the rim and a , the radius of the cone. Griffiths and Walters⁽¹⁰⁷⁾ have estimated percentage differences and for the particular $\frac{h}{a}$ ratio in the present experiments, the error involved was negligible. However, this error estimate should only be used for liquids where surface tension effects play a dominant role at the edges.

Other sources of error are those due to instrument imperfection and non-homogeneity of the test sample.

4.7. *Extrusion*

4.7.1. *Introduction*

The final part of the experimental programme involved the monitoring of the pressure propagation along the extruder and to establish how this could be enhanced by altering the extrusion variables such as the temperature, the moisture, the rotational speed and the geometry. These extrusion data provide a means of evaluating the predictive capacity of the traction induced compaction model which is developed in chapter 6.

4.7.2. *Extruder Instrumentation*

The extruder used in this study was a 48mm Almex single screw extruder with a L/D ratio of 12 : 1 with longitudinal grooves of 4 x 1mm. A standard square pitched screw with a compression ratio of 2 : 1 was used in most of the experimental runs. The extruder was equipped with a circular die of 4mm, and was instrumented for the measurement of the extruder variables which were logged by a microcomputer. The temperatures and pressures were measured at points along the barrel by five pre-calibrated Gentran pressure transducers and Eurotherm thermocouples. Provisions were available for heating by means of electrical elements in the barrel and cooling with a mixture of freon and water through a chiller unit. The temperature was set in four different zones enabling a wide choice of temperature profiles. An iron constantan thermocouple and a Gentran transducer monitored the temperature and pressure at the die. The extruder screw was heated by pumping hot oil through the bore of the screw from a Grant hot oil bath which contains a built-in over temperature protection device. The bath temperature could be altered from ambient to 250°C. It was not possible to position pressure transducers or thermocouples along the extruder screw. A conventional heat balance analysis was adopted in order to determine the screw surface temperature and this temperature was assumed constant along the screw. The raw material was fed into the extruder using a K-tron 7100 loss-in-weight feeder-controller which utilizes microprocessor control. The screw speed generated

was measured by a tachometer and logged to the microprocessor, via the Almex data acquisition system.

4.7.3. *Mode of Operation*

The zone temperatures and the oil bath temperatures were set and left to equilibrate for one hour. After this time the pressure probes were re-calibrated since the calibration altered at elevated temperatures and the process controls were switched on at this stage. As with any other process, start-up played a crucial role in the running of the machine. A practice was developed where for one minute the extruder was run with moistened maize grits since these were easier to extrude than the dry powder. At this stage the extruder was generally running at the maximum possible speed of 190r.p.m. The maize powder was then introduced from the K-tron feeder at a rate of 5kg/hr. During some trial runs it was established that the screw speed and feed rate would have to be altered together in order to maintain flood feeding conditions. It was therefore decided that a screw speed of 133.4r.p.m. was best suited for the running of the extruder. Therefore the screw speed was gradually reduced in steps of 10r.p.m. to the required screw rotational speed of 133.4r.p.m. The extruder was allowed to run at this screw speed for a period of 5 minutes and the feed rate was then increased in steps of 2kg/hr to the maximum possible level available within the power rating of the machine. After each step change in feed rate, the extruder was allowed to run at that particular flow rate for a

period of ten minutes. The machine was run at the full production rate for about 40 minutes (time taken to attain steady state running conditions) and then the machine was stopped and the screw extracted to measure the length of the compaction zone. This procedure was repeated for different screw and barrel set temperatures the details of which are given in Table (4.2). It was not possible to run the extruder with the addition of 16% w/w moisture since this increased the slip at the contacting surfaces which eventually stalled the machine due to material "build-up" at the feed port. A similar problem, but to a lesser extent, was encountered in all the screw heating runs and the 50/50/50/50°C barrel heating run. These runs were performed with a new batch of maize powder which contained a little more moisture than the old batch, which caused a rather low pressure profile to be generated. The geometry design of the extruder screw is shown in Table (4.3)

TABLE 4.1

The percentage error involved in assuming a
constant shear rate [Adams and Lodge]⁽¹⁰⁶⁾

Gap angle (degrees)	Variation of shear rate across gap (%)	Error in using formula to compute shear rate (%)
1	0.03	0.02
2	0.21	0.08
3	0.28	0.18
4	0.49	0.32
5	0.77	0.50
7	1.5	0.98
10	3.1	2.00

TABLE 4.2

Extrusion runs, set temperatures on the barrel and screw.

All runs were at a screw speed of 133.4 r.p.m.

Barrel heating zones ($^{\circ}\text{C}$)	zone 1	zone 2	zone 3	zone 4
Screw heating ($^{\circ}\text{C}$)				
Ambient	20	20	20	20
Ambient	50	50	50	50
Ambient	80	80	80	80
Ambient	120	120	120	120
50	50	50	50	50
80	50	50	50	50
120	50	50	50	50
150	50	50	50	50

TABLE 4.3

Geometrical design data of extruder screw (mm)

Barrel diameter $D_b = 48.3\text{mm}$

Die diameter = 4mm

Screw comp. ratio	1 : 2
Screw root diameter D_s	39
Screw lead L_s	45
Channel depth	4.6
$H = \frac{\{D_b - D_s\}}{2}$	
Av. diameter $\bar{D}$	43.7
$D_b - H_c$	
Helix angle θ_b	16.52°
Helix angle θ_s	20.17°
Helix angle $\bar{\theta}$	18.14°
Flight width e	0.19
Channel width:	
$W_b = L \cos \theta_b - e$	1.51
$W_s = L \cos \theta_s - e$	1.49
$\bar{W} = L \cos \bar{\theta} - e$	1.47

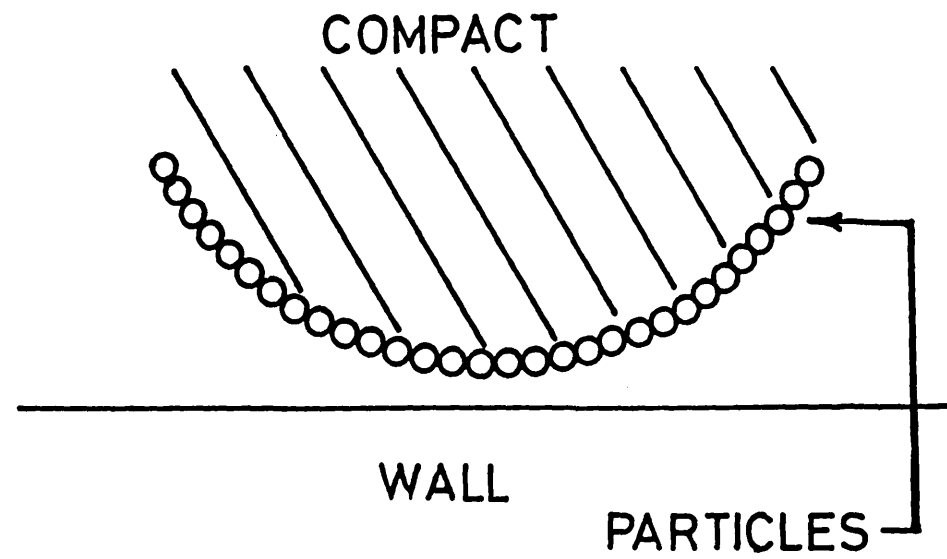


Fig.4.1 A schematic view of the compacted surface.

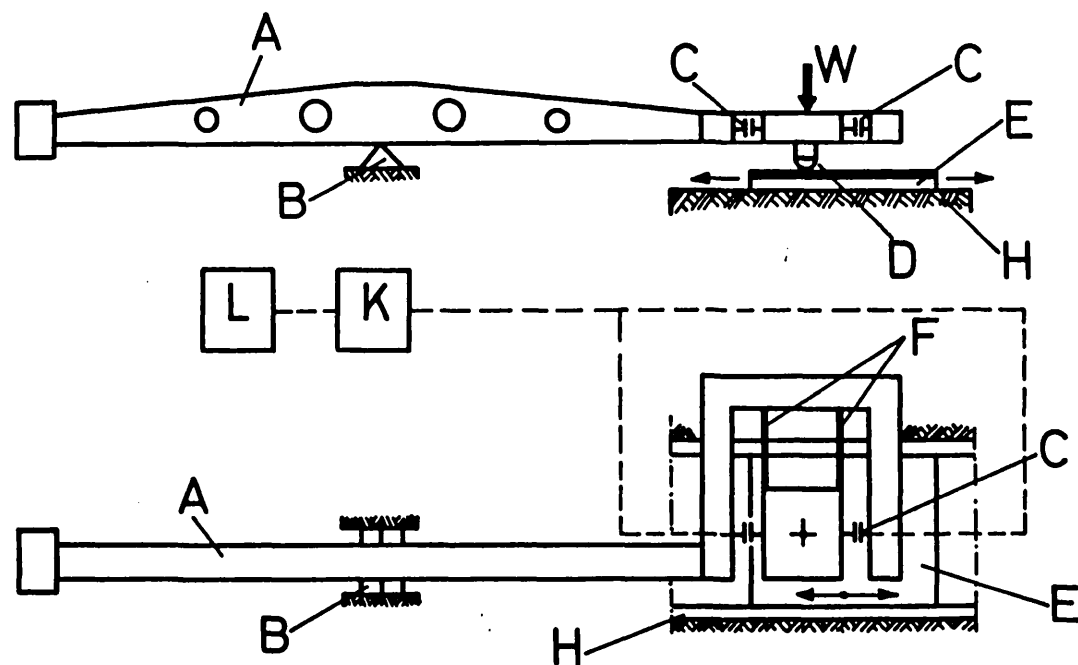
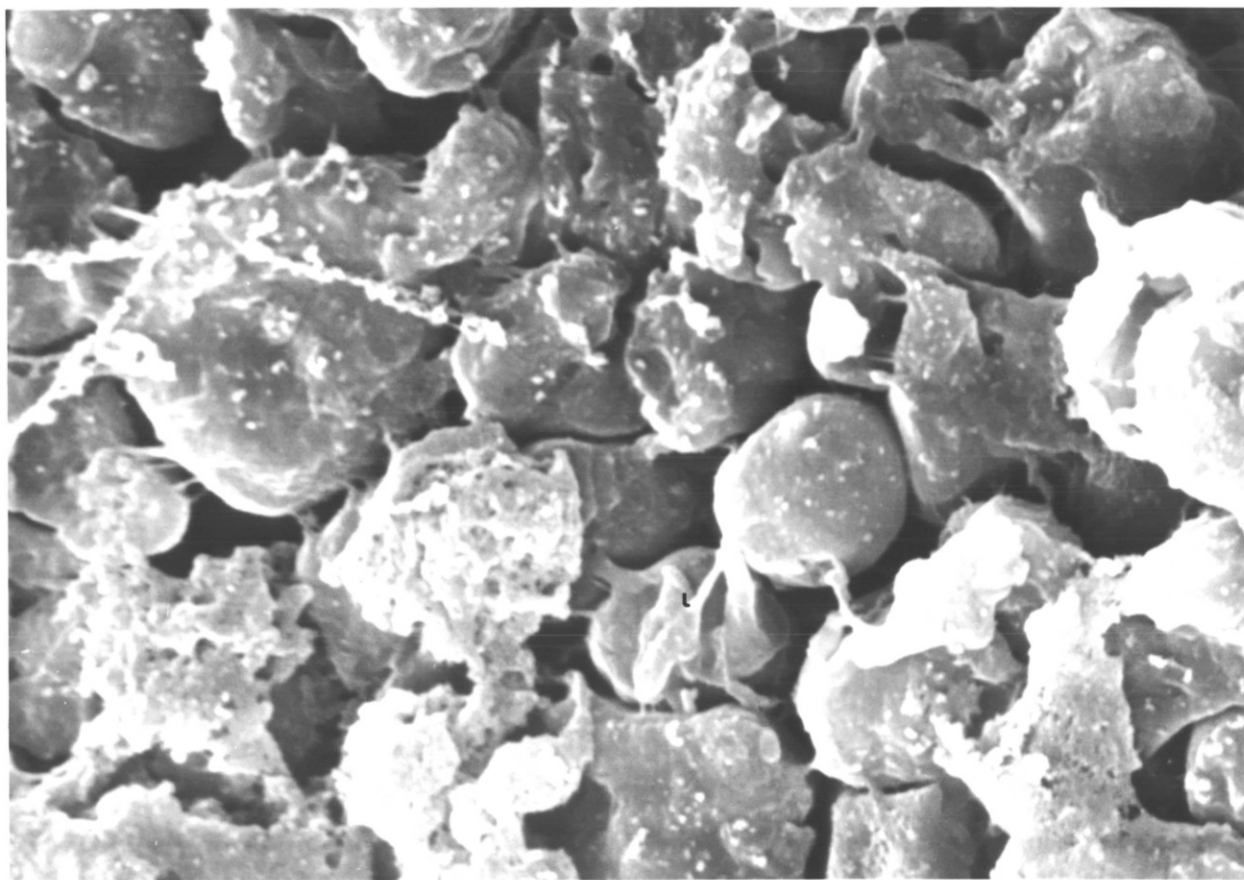
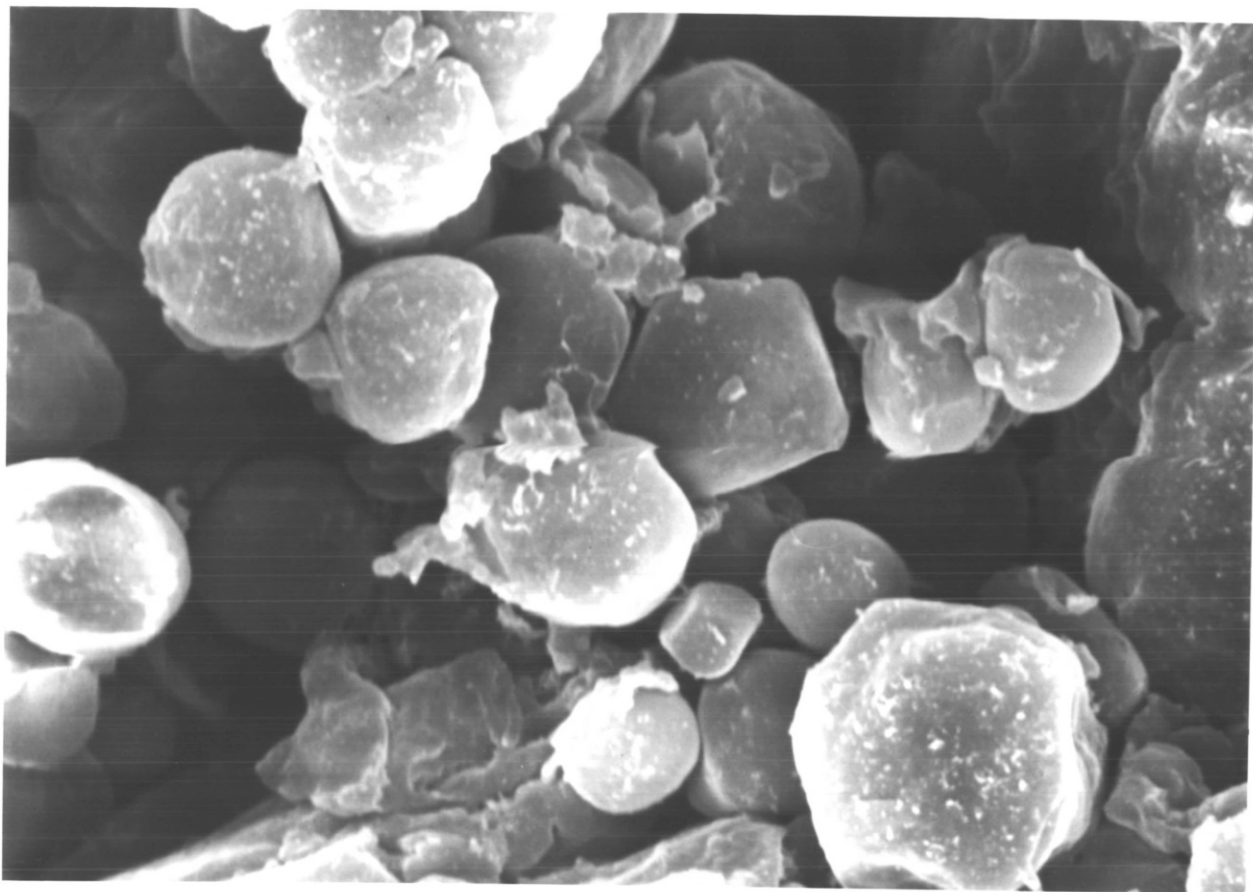


Fig.4.2 The friction machine. A; Beam of chemical balance. B; Agate fulcrum. C; Displacement transducers. D; Malze hemisphere. E; Float glass substrate. F; Leaf springs. H; Heating block. K; Carrier wave frequency bridge; L; Ultraviolet recorder; W; applied load.



160 μm

Fig.4.3 Scanning electron micrographs of undeformed and deformed surfaces. Mag $\times 3640$

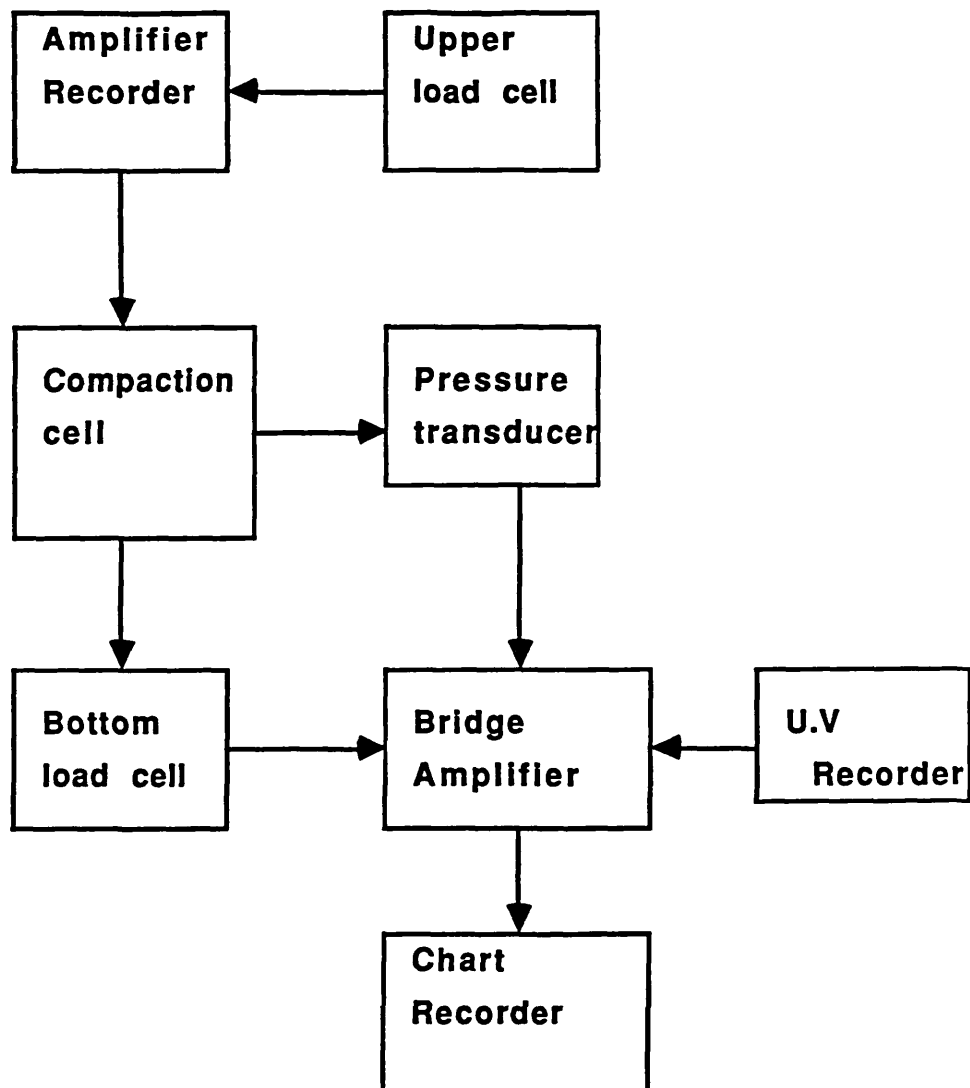


Fig.4.4 A schematic representation of the compaction rig.

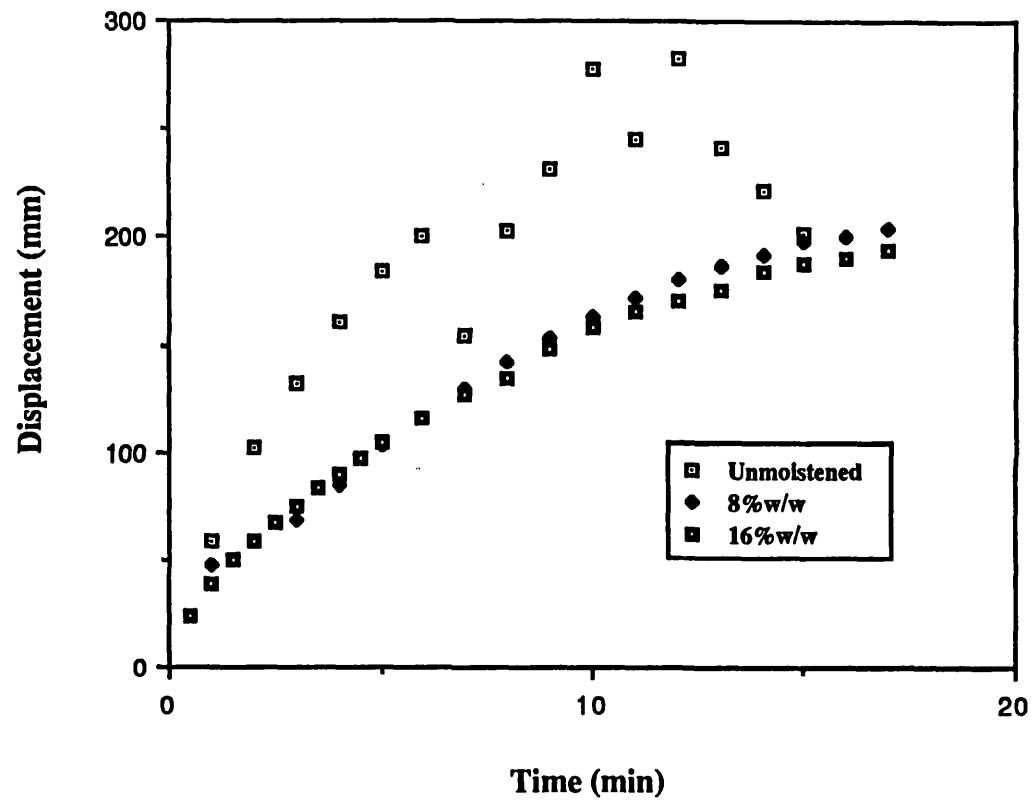


Fig.4.5 The displacement time plot at a consolidation load of 0.4kN from the shear test at different levels of added moisture.

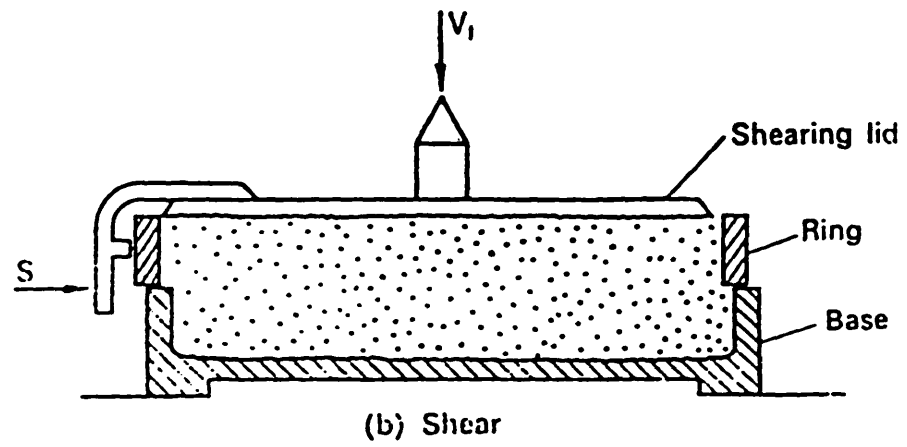


FIG.4.6 . Jenike shear cell⁽⁸⁵⁾

Chapter 5

Friction Data and Analysis

5.1. Introduction

The deformation characteristics of particulate solid masses critically depend upon the tractions produced at the interfaces between the particles and the container. This process is also recognised as being the controlling factor in the initial stages of the extrusion of particulate solids. A knowledge of the magnitude and variation of the interface friction with the process parameters is necessary to evaluate any model of these types of processes. The direct measurement of tractions *in situ* is possible in principle but currently impracticable. Therefore, the alternative of simple simulation has obvious attraction. This chapter presents data obtained to elucidate the role of interface friction in the continuous processing of a maize powder.

The simulation involves producing spherically shaped compacts in conditions which are thought to approximate to the dynamic compacting conditions which exist in the extruder and the subsequent measurement of the friction forces as a function of certain contact parameters. The choice of contact geometry: spherically ended compacts sliding on a smooth glass counterface were preferred because the evaluation of the real contact area A_r is made possible in this configuration based on the contact

mechanical models presented in chapter 2. The data obtained during sliding is then analysed using a combination of topographical models and the adhesion model of friction. This approach assumes that it is realistic to arrest the compaction process and study the traction periodically and that shear mixing and sliding at the interface do not greatly modify the nature of the interface topography and rheology.

Data are also presented for thin film interface shear experiments and the mechanical properties of maize cylinders. The interfacial shear stress data obtained from the thin film shear experiments described in chapter 2, together with the contact area computed from a suitable topographical model, provides the necessary predictive capacity of the friction force generated under a given set of circumstances by the compacts and ultimately at the extruder walls. This then allows a direct comparison to be made between the experimental measured forces and predicted values of friction.

5.2. *Mechanical Properties of Compacted Maize Cylinders*

In order to use eq.(5.1) for the prediction of contact area the moduli E of the system is required. Therefore it was decided to obtain the Young's moduli E of these systems for maize cylinders since the evaluation of E for hemi-spherical compacts proved to be problematical. Table (5.1) lists the apparent Young's moduli E and rupture strengths τ_R obtained in compressive

testing at a strain of ca 20% for maize cylinders fabricated at different temperatures. The apparent modulus is defined as σ (20%)/ ϵ (20%) where σ is the stress at 20% strain and ϵ the strain. The form of the stress-strain curves obtained was non-linear and the compact moduli were found to be a function of the effective strain. The modulus was observed to increase with increase in strain. The moduli-strain data conforms to a general geometrical relationship of the form $E \approx a.(\text{strain})^b$, where a and b are constants for a given system. These parameters for the moistened and unmoistened systems are listed in Table (5.2). For the dry powders a , is of the order 5×10^8 Pa and b ranges from ca 1.5 to ca 1.0. The moistened systems have values two orders of magnitude less and the parameter b is also greatly reduced.

The observed increase in rupture stress τ_R (Table 5.1) with compaction temperature is notable. This increase was found to be very much greater for the moistened systems; for the unmoistened cylinders this increase was only by a factor of 1.4 compared with a factor of 10 for the moistened systems.

In the presence of excess moisture at elevated compaction temperatures, the powder is chemically modified as a result of complex reactions, which may explain this marked change (108). The addition of moisture has weakened the matrix as indicated by the reduced rupture strengths τ_R and moduli E for this system. Increasing fabrication temperature increases the modulus of the unhydrated system by a factor of 2 approximately. However, no appreciable change was observed in the moduli for the hydrated system.

The mechanical properties of maize cylinders obtained in compressive testing has demonstrated that the modulus E is a non-linear function of the strain. The addition of moisture reduces the modulus, while there is an increase in E with fabrication temperature. These data suffice for a first order evaluation of the Young's modulus E of the compacts processed at different fabrication temperatures. However, as the fabrication temperature is increased the evaluation of the modulus based on 20% strain to be incorporated into the Archardian contact mechanical treatment (eq. 5.1) underestimates the value of the real area of contact. Therefore the value of E was optimised to get the best fit over the experimental load range as will be shown in Section 5.5.

5.3. *Friction of Hemi-spherical Compacts*

The friction force F , as a function of the normal load W for compacts prepared at various temperatures sliding on smooth glass counterfaces, is shown in Figs. (5.1) and (5.2) for unmoistened and moistened systems respectively. Both sets of data clearly demonstrate that the frictional force increases slowly with load. This trend is in line with those observed by other workers for a variety of organic polymers⁽⁵³⁾⁽¹⁰⁹⁾. The data for the moistened maize is generally greater than those for the dry compacts. The influence of the heat treatment temperature for both cases shows a number of peculiar features, but the general trend is for the

friction to decrease with increasing processing temperature. The data for the dry compacts formed at 200°C and for moistened compacts at 150°C are anomalous in their respective sets. This may be due to a shift of the gelatinisation temperature with moisture (Takakashi et al 1982⁽¹⁰⁸⁾). The sudden change in the modulus E for the dry system is consistent with this behaviour. Fig. (5.3) depicts clearly the variation of friction with fabrication temperature for four normal loads, when the compact is slid over a clean glass plate at ca 20°C. The average coefficient of friction at ambient was 0.09 and 0.38 for the unhydrated and hydrated systems respectively; an increase of a factor of 4.2 with the addition of moisture. These data can be rationalised in terms of the effect of water on the shear stress τ and contact area A_r . The absorption of water into the matrix swells the individual particles, but reduces the shear stress of the matrix. The observed increase in the frictional coefficient can be attributed to the contact area A_r increasing more rapidly than the shear stress τ , thereby producing a nett increase in the coefficient of friction. This type of response in the coefficient of friction for saturated granules having dry surfaces was observed by Cohen⁽¹¹⁰⁾; values of friction coefficients of 0.40 and 0.57 respectively were found for dry nylon and saturated nylon sliding on a dry glass surface.

In order to interpret these data quantitatively a number of variable changes which occur during the fabrication procedure must be considered. These include the interfacial shear strength τ , the particle contact density n' and the mechanical properties of

both the granular structure and the supporting matrix E_p and E . These parameters are required in the prediction of contact area in eq. (5.1).

The matrix suffers extensive plastic flow during the fabrication process. It is, however, shown later that these contacts behave as though they are elastic bodies. An insight into the mode of deformation can be established by evaluating the magnitude of the load index n . The data of Figs. (5.1) and (5.2) are plotted on logarithmic axis in Figs. (5.4) and (5.5) and the gradients n are listed in Table (5.3). The values of n for the unhydrated maize fall in the range 0.87 to 1.0. In chapter 2 it was demonstrated that for a rough elastic system, where both matrix and asperities deform in an elastic manner, the expected contact area scalar m is 0.88. This case, with an array of elastic spheres of radius R_p residing on a sphere of radius R_s which is also deforming in an elastic manner, has been solved explicitly by Archard⁽³⁷⁾. The result for the total contact area is:

$$A_{\text{real}} = K^1 \frac{1}{E_p^{2/3}} \frac{1}{E^{2/9}} R_p^{2/3} R_s^{2/9} n'^{1/3} W^{8/9} \quad (5.1)$$

where K^1 is a constant, n' is the contact density, E_p and E are particle and compact moduli respectively.

For the case where the interfacial shear stress τ is not a strong function of the load, the load index n and the contact area scalar m will be nearly identical. Therefore, for the present system, if it is assumed that τ is very nearly independent of load, the values of load indices are expected to be near 0.88 if the Archard model is appropriate. The experimental values

obtained for the dry compacts are all close to the Archard's value of 0.88 except for one system. Data are later presented to substantiate the belief that the interface shear stress τ is not a strong function of the applied load. The load indices for the moistened systems are all near unity. The following points remain to be discussed at this stage; the apparent applicability of an elastic model to a contact which has suffered plastic flow during its creation and the reason why n takes a value of 1.0 or approximately 0.88 for a given system. On the first point the original Archard idea of the progressive attainment of elastic deformation may be invoked. Clearly this will induce changes in certain contact parameters such as R , but these are seemingly second order effects. Archard⁽³⁹⁾ was probably the first to point out that asperities cannot deform plastically in an indefinite way and load is supported by elastic strains. The analysis of the Lodge and Howell⁽³⁶⁾ provides an expression for the load index given in chapter 2, Table (2.1). A value of unity for m' which is the asperity deformation index, results in a load index n , equivalent to the contact area scalar m of the specimen as a whole. This clearly indicates that even if the individual asperities were to flow initially, it is possible for these contacts subsequently to produce elastically deforming junctions. The sub-matrix also undergoes gross flow⁽³⁷⁾ but the same argument may be applied. This topic is referred to again in the context of the stress transmission process operating in compaction.

The interesting point is that although the whole system is at elastic equilibrium and not in plastic flow it is often sufficient

to equate the mean asperity contact stress with a plastic flow stress. Bowden and Tabor⁽³³⁾ adopted this approach (chapter 2) and if we take the flow stress as P then $A = \frac{W}{P_0}$, then μ is just $\frac{\tau}{P_0}$, a constant. This was indeed the point which Archard sought to make; the asperities are deforming in an elastic manner while at the same time the mean contact stress is a constant. The latter condition arises when the asperities are not all at the same height and the mean contact area for the asperity population remains constant. The mean pressure is often close to P_0 , the flow stress. Some of the current compacts demonstrate in a neat way the Howell and Lodge⁽³⁶⁾ or the so-called Archard⁽³⁷⁾ pre-cursor case; $n \approx 0.88$. Others approach the more common case of the higher Archard models which include height distributions and n is unity; now the model is less explicit and if $A_r \approx \frac{W}{P_0}$ then $\mu = \frac{\tau A}{W} = \frac{\tau(P_0)}{P_0}$; $n \approx 1$.

This behaviour was also observed for PEEK contacts by Briscoe and Stolarski⁽¹¹¹⁾. A more recent analysis for the load index has been offered by Adams *et al*⁽¹¹²⁾ but remains unpublished. In this analysis it is suggested that the load index n should be a function of the slider radius, the interfacial rheology terms, α and τ_0 , and the elastic properties of the substrate; the modulus and the Poisson's ratio, E and ν . Briscoe and Stolarski present sufficient data for a range of organic polymers (Table (5.4) and (5.5)) to elucidate the role of compact radius on the load index n . By and large the observed values of the indices n decrease with increasing compact radius. The data for the present system also follows the same trend. Table (5.6) and (5.7) depicts data

for the unhydrated and hydrated systems at various fabrication temperatures. At higher temperatures the index was also insensitive to changes in compact radius. The presence of moisture increases the load index and this can be attributed to the matrix being weaker and therefore allowing more deformation to take place during sliding. These data can be rationalised in terms of the effect of contact pressure or interface compressive strain on friction. Small radii produce an effectively larger contact pressure and more interface deformation. It is therefore to be expected that the smaller sliders would tend to produce an index tending towards unity as the higher orders of the Archard model were more likely. The coefficient of friction was found to be independent of compact radius at elevated temperatures, but a finite change was observed at a forming temperature of ca 23°C, (Fig. (5.6)).

In addition to the variables already discussed, the extent of compaction also influences the magnitude of the frictional coefficient. Data are presented in Fig. (5.7) to demonstrate the effect of consolidation load on the coefficient of friction for maize compacts at a normal load of 0.2 N. Higher consolidation loads increase the modulus of the system thereby reducing the contact area A_r and the coefficient of friction $\mu = \frac{\tau A_r}{W}$ is reduced.

The experiments described so far involved sliding compacts over glass surfaces at ca 20°C. A series of experiments have been carried out where the counterface is heated. The change in friction was expected to be nominally a reflection of the change

in τ . The friction force was found to decrease markedly between 20°C and 60°C and there was considerable scatter above 60°C. These experiments are somewhat closer to the extrusion process where there is usually a difference in the traction forces induced by different temperatures on the screw and barrel. The coefficient of friction μ as a function of counterface temperature for a moistened and unmoistened compact processed at ambient is shown in Figs. (5.8) and (5.9). The frictional coefficient reduces with interface temperature for both systems and an increase in μ was observed with the addition of moisture system. These data can be rationalised in terms of the effect of temperature on the shear stress τ and contact area A_r ; increase in interface temperature reduces both the shear stress τ and the contact area, thereby reducing the friction coefficient.

It has been demonstrated that the friction of hemi-spherical maize compacts reduces with temperature but contrary to what was expected, the major part of the reduction in observed friction is caused by a decrease in contact area, induced by changes in mechanical properties of the compact surface.

5.4. *Interface Rheology*

The interface shear strength τ is shown as a function of the mean contact pressure in Fig. (5.10) for maize films at a sliding velocity of 0.15 mm sec⁻¹. The contact geometry in this configuration is well defined, since the contact is between two

very smooth surfaces. This allows the application of Hertzian contact mechanics to compute the real area of contact.

The shear strength - contact pressure data for both the solvated and unsolvated films fits the exponential relation given in eq. (2.7). The α' and τ_0' values for these films show no particular trend but in general, the films, transfer deposited from a hydrated compact, exhibit τ_0' and α' values less than the unhydrated case. It will be shown later that the parameters τ_0' and α' which characterize the interface rheology are necessary for the quantitative analysis of the friction data (Section (5.5)). It is not surprising that the exponential equation applies both to the hydrated and unhydrated systems, although this behaviour was only characteristic for solvated films in the work of Amuzu et al⁽⁶¹⁾. This discrepancy is attributed to the ability of maize granules to absorb water from their surroundings and therefore possess characteristics of a solvated film, even though it was transferred from an unmoistened compact. The α' and τ_0' values for these films are given in Table (5.8). The changes associated with this interfacial shear process have been interpreted in terms of a molecular process for polymeric films⁽⁴⁹⁾⁽⁵⁹⁾. The low stress limit of the Eyring molecular theory was shown to be more appropriate for the exponential pressure dependence of polymer films⁽⁴⁹⁾.

The temperature dependency of τ for maize films is similar in nature to the response of HDPE films. These data are depicted in Fig. (5.11). The linear part of the shear-stress - temperature graph yields a value of 16 K J mol^{-2} for the activation energy

term Q . This transition shifts to lower temperatures with increasing pressure. The shear-strength decreases with temperature to approximately ca 60°C and thereafter increases and then remains constant. The coefficient of friction in thin film shear also decreases with interface temperature and this trend is analogous to the variation in the frictional coefficient observed in the bulk friction work. Table (5.9) lists data on the frictional coefficient obtained in bulk and interface shear experiments, together with those predicted from eq. (2.9). The agreement between predictions from eq. (2.9) with experiment is within a factor of 1.5 for the unhydrated system. However, for the hydrated systems the comparison between the predictions from eq. (2.9) and the experiment values is less exact, especially at elevated temperatures. This large error could be due to the inaccurate estimation of the flow stress as $\frac{1E}{10}$.

Irrespective of these differences, the apparent agreement between the two types of experiments, bulk shear and thin film shear, vindicates the use of the adhesion model of friction to account for energy dissipation in these systems.

5.5. *The Prediction of Friction Force F*

This section presents a theoretical model which provides a good account of the simulated data. The contact mechanical treatment appropriate to curved rough surfaces for the case where a second layer of asperities reside on the first layer were solved

explicitly by Archard⁽³⁷⁾. The result for the total contact area for elastic deformation of asperities is given by equation (5.1). It was also demonstrated that the interfacial shear stress varied as an exponential function of the contact pressure (eq. 2.7) and its variation was relatively insensitive to changes in contact pressure over the experimental load range. Therefore we may calculate the absolute value of the frictional force from a knowledge of $\tau(P)$ (eq. 2.7) and $A_r(P)$ (eq. 5.1) and substituting these in the adhesion model of friction (eq. 2.1). The resulting expression for the frictional force is given by:

$$F_{\text{theor}} = \tau e^{\alpha P_r} K^1 \frac{1}{E_p^{2/3}} \frac{1}{E_c^{2/9}} R_p^{2/3} R_s^{2/9} n^{1/3} W^{8/9} \quad (5.2)$$

In this equation K^1 is a numerical constant whose value is estimated as 1.81 for the current system. The parameter E was optimized for the best fit since the magnitude of the strain in these configurations as a function of temperature was unknown. It has also not been possible to distinguish between E_c and E . E is a function of both strain and temperature and a common value E has been evaluated. In evaluating the constant K^1 , a Poisson's ratio for maize of 0.35 was used in the absence of more specific values (Muller (113)).

Theory and experiment are in excellent accord for compacts fabricated at ambient temperature. The quality of the predictions are less exact at the higher fabrication temperatures. This may be due to the significant uncertainties in E and R_p , but equation (5.2) suggests that the uncertainties in E would pre-dominate

since it is the changes in real area of contact which govern the friction. Fig.(5.12) also shows the results of the optimization of the modulus E to give the best agreement between theory and experiment of the friction data. There are many reasons why the direct estimate of the moduli is in error; the extent of the compaction and the strain rate in a cylindrical specimen tested in compression will most certainly be different to those pertaining in a compacted hemi-spherical specimen. The overall predictions for the absolute value of friction with the optimized value of E for the unhydrated system is shown in Tables (5.10) and (5.11). The average ratio of the predicted values to the experimental values is ca 0.70, but many of the individual values are close to unity, particularly at a forming temperature of 20°C.

An estimate of friction in the "plastic" region may be obtained using

$$F = \tau'_0 \exp^{\alpha' P_0 \frac{W}{P_0}} \quad (5.3)$$

where P_0 is the flow stress Tables (5.11) and (5.10.5) give a comparison of prediction and experiment with this model for the hydrated cases and for the 200°C unhydrated case. The flow stress was assumed to be $\frac{1}{10}E$.

The predictions at ambient from eq.(5.3) is within a factor of 1.5, however there are large differences in the predictions at higher temperatures. This is probably due to the errors involved in the estimation of P_0 and the relevance of the estimated parameter deduced in compaction to the sliding contact.

5.6. *Conclusions*

The friction behaviour of maize compacts can be effectively modelled using a precursor of the classical multiple asperity model, for the calculation of real areas of elastic contact in conjunction with the adhesion model of boundary friction. In this system the interface shear stress is remarkably insensitive to contact pressure and may be regarded as constant in this simulation. It is the real area of contact which controls the magnitude of the frictional force. Data were presented to elucidate the role of the major extrusion parameters, pressure, temperature and moisture on friction. However, the apparatus was unable to simulate velocities comparable to those present in an extruder and therefore the influence of this parameter on the friction force was not investigated. Data are presented in chapter 7 to account for the variation of friction force with test speed. In using eq. (5.2) and (5.3) for the prediction of friction, it is necessary to take into account the change in mechanical properties with the fabrication and interface temperatures. The major uncertainties arise in the estimation of the appropriate moduli and hence the flow stress P_0 . It is however gratifying that this first order prediction provides a good account of the experimental data, for compacted maize hemi-spheres sliding against glass counterfaces. The major point of interest in extruder modelling is the temperature dependence of the friction.

Table 5.1.

Young's Modulus E (Nm^{-2}) measured in compression at a strain rate of 0.42 s^{-1} and rupture stress τ_R (Nm^{-2})

Water content	None		16% w/w	
Temp. $^{\circ}\text{C}$	E	τ_R	E	τ_R
Unheated	5.00×10^7	2.63×10^6	0.345×10^7	0.09×10^6
50	5.21×10^7	2.95×10^6	0.345×10^7	0.21×10^6
100	5.47×10^7	3.35×10^6	0.452×10^7	0.67×10^6
150	12.73×10^7	3.75×10^6	0.344×10^7	0.74×10^6
200	10.92×10^7	5.66×10^6	0.402×10^7	1.04×10^6

Table 5.2.

The parameters a and b obtained in compression of maize cylinders at a strain rate of 0.42 sec^{-1} . $E = a (\text{strain})^b$

Fabrication Temp. °C	Unmoistened		Moistened	
	a(Pa)	b	a(Pa)	b
Ambient } 50 }	5.24×10^8	1.46	0.68×10^7	0.46
100	6.32×10^8	1.52	0.57×10^7	0.31
150	5.78×10^8	0.93	0.59×10^7	0.31
200	5.14×10^8	0.96	0.57×10^7	0.28

Table 5.3

The load index $\{F = K W^n\}$

Added Water	None	16% w/w
Temp. °C		
Unheated	0.87 ± 0.02	1.02 ± 0.03
50	0.87 ± 0.02	1.01 ± 0.03
100	0.87 ± 0.03	1.00 ± 0.05
150	0.90 ± 0.05	0.99 ± 0.03
200	1.00 ± 0.05	0.96 ± 0.04

Table 5.4.

Experimental values of load indices n as a function of compact radius for a range of organic polymers (Briscoe and Storlaski 1985)

Radius/mm	1.59	3.18	6.35	12.7	19.1	50	100	125
Ps	0.96	0.92	0.88	0.86	0.84	0.79	0.81	0.80
PTFE	0.87	0.75	0.68	-	-	-	-	-
PC	0.91	0.89	0.88	0.85	0.78	-	-	-
PP	0.89	0.80	-	0.79	0.77	-	-	-

Table 5.5

Load indices n for nylon 6:6 as a function of relative humidity.

Radius of slider = 1.59 mm

Relative Humidity %	Load index n
58	0.75
84	0.82
100	0.88

Table 5.6

Experimental values of load indices as a function of compact radius
for dry maize powder at various fabrication temperatures

Radius/mm	2.5	3.5	4.0	6.5
Fabrication Temp. °C				
25	0.97	–	0.86	0.86
50	1.03	–	1.03	1.00
100	1.03	1.10	0.99	0.99
150	1.03	1.04	0.89	0.89
200	1.03	1.05	1.00	0.99

Table 5.7

Experimental values of load indices as a function of compact radius
for hydrated maize powder at various fabrication temperatures

Radius/mm	5.0	4.0	3.5
Fabrication Temp. °C			
25	0.98	1.07	1.03
50	1.00	1.30	1.30
100	0.91	1.04	1.24
150	-	1.15	1.13

Table 5.8

Interfacial Rheology data for thin films
 transfer deposited from compact maize hemispheres
 $(\tau = \tau_0' e^{\alpha' P})$

Temp. °C	unmoistened		moistened	
	α'	τ_0'	α'	τ_0'
unheated	1.27×10^{-8}	1.4×10^6	1.76×10^{-8}	0.15×10^6
30	3.2×10^{-8}	0.9×10^6		
40	3.5×10^{-8}	0.7×10^6		
50	2.27×10^{-8}	1.5×10^6	2.70×10^{-8}	0.91×10^6
60	1.58×10^{-8}	1.92×10^6		
70	1.60×10^{-8}	2.1×10^6		
80	2.3×10^{-8}	1.2×10^6		
90	2.3×10^{-8}	1.2×10^6		
100	2.3×10^{-8}	1.2×10^6	3.38×10^{-8}	0.59×10^6
110	2.3×10^{-8}	1.2×10^6		
120	1.3×10^{-8}	2.0×10^6		
130	2.3×10^{-8}	1.2×10^6		
140	2.3×10^{-8}	1.2×10^6		
150	2.3×10^{-8}	1.2×10^6	1.73×10^{-8}	0.71×10^6
200	2.3×10^{-8}	1.2×10^6	1.73×10^{-8}	0.71×10^6

Table 5.9

Coefficient of friction during thin film and bulk shear at a normal load of 0.2 N together with predictions

based on $\mu_T = \frac{\tau_0 e^{\alpha' Pr}}{Pr}$ where Pr is the contact pressure based on an Archardian contact area

unmoistened

Interface Temp. °C	μ_T	μ_{thin}	μ_{bulk}
25	0.11	0.16	0.10
50	0.14	0.12	0.09
100	0.11	0.08	0.09
150	0.08	0.06	0.07
200	0.08	0.09	0.08

moistened $\mu_T = \tau_0 e^{\alpha P_0/P_0}$

Interface Temp. °C	μ_T	μ_{thin}	μ_{bulk}
25	0.44	0.09	0.44
50	2.68	0.07	0.42
100	1.31	0.09	0.12
150	1.77	0.03	0.27
200	1.77	0.05	0.06

Table 5.10

Predicted and experimental values of friction data together
with predicted contact area from equation (5.1) (dry compacts)

Table 5.10.1

Compact processing temp. = Unheated

Load (N)	Contact Area predicted m^2	Friction F_1 predicted (N)	Friction F_2 experimental (N)	Ratio of F_1/F_2
0.049	3.74×10^{-9}	0.005	0.003	1.70
0.098	6.90×10^{-9}	0.014	0.014	1.00
0.196	1.28×10^{-8}	0.018	0.020	0.91
0.491	2.89×10^{-8}	0.040	0.042	0.95
0.981	5.36×10^{-8}	0.074	0.077	0.97

Table 5.10.2

Compact processing temp. = 50°C

Load (N)	Contact Area predicted m ²	Friction F ₁ predicted (N)	Friction F ₂ experimental (N)	Ratio of F ₁ /F ₂
0.049	3.6 x 10 ⁻⁹	0.006	0.007	0.87
0.098	6.7 x 10 ⁻⁹	0.012	0.010	1.20
0.196	1.24 x 10 ⁻⁸	0.022	0.017	1.30
0.491	2.79 x 10 ⁻⁸	0.049	0.034	1.46
0.981	5.17 x 10 ⁻⁸	0.092	0.073	1.26

Table 5.10.3

Compact processing temp. = 100°C

Load (N)	Contact Area predicted m ²	Friction F ₁ predicted (N)	Friction F ₂ experimental (N)	Ratio of F ₁ /F ₂
0.049	3.45 x 10 ⁻⁹	0.001	0.003	0.30
0.098	6.39 x 10 ⁻⁹	0.003	0.010	0.30
0.196	1.18 x 10 ⁻⁸	0.005	0.017	0.30
0.491	2.67 x 10 ⁻⁸	0.011	0.028	0.39
0.981	4.9 x 10 ⁻⁸	0.021	0.065	0.32

Table 5.10.4

Compact processing temp. = 150°C

Load (N)	Contact Area predicted m ²	Friction F ₁ predicted (N)	Friction F ₂ experimental (N)	Ratio of F ₁ /F ₂
0.049	1.63 x 10 ⁻⁹	0.001	0.003	0.34
0.098	3.0 x 10 ⁻⁹	0.004	0.006	1.17
0.196	5.58 x 10 ⁻⁹	0.002	0.013	0.15
0.491	1.26 x 10 ⁻⁸	0.005	0.040	0.12
0.981	2.33 x 10 ⁻⁸	0.010	0.044	0.23

Table 5.10.5

Compact processing temp. = 200°C

Flow stress $\approx P_0 = 0.1E = 10.92 \times 10^6$ Pa

Load (N)	Contact Area predicted m^2	Friction F_1 predicted (N)	Friction F_2 experimental (N)	Ration of F_1/F_2
0.049	4.49×10^{-9}	0.001	0.002	1.00
0.098	8.97×10^{-9}	0.002	0.005	0.40
0.196	1.79×10^{-8}	0.004	0.016	0.25
0.491	4.50×10^{-8}	0.010	0.040	0.25
0.981	8.98×10^{-8}	0.020	0.044	0.45

Table 5.11

Predicted and experimental values of friction data together

with predicted contact area from equation $A = \frac{W}{P_0}$, $P_0 \approx 0.1E$

Table 5.11.1

Compact processing temp. = Unheated

Flow Stress $\approx P_0 = 0.34 \times 10^6$ Pa

Load (N)	Contact Area predicted m^2	Friction F_1 predicted (N)	Friction F_2 experimental (N)	Ratio of F_1/F_2
0.049	1.44×10^{-7}	0.022	0.015	1.47
0.098	2.88×10^{-7}	0.044	0.053	0.83
0.196	5.76×10^{-7}	0.089	0.086	1.03
0.491	1.44×10^{-6}	0.216	0.160	1.35
0.981	2.88×10^{-6}	0.432	0.293	1.47

Table 5.11.2

Compact processing temp. = 50°C
 Flow stress $\simeq P_0 = 0.34 \times 10^6$ Pa

Load (N)	Contact Area predicted m ²	Friction F ₁ predicted (N)	Friction F ₂ experimental (N)	Ratio of F ₁ /F ₂
0.049	1.44×10^{-7}	0.13	0.013	10.00
0.098	2.88×10^{-7}	0.26	0.050	5.20
0.196	5.76×10^{-7}	0.52	0.082	6.34
0.491	1.44×10^{-6}	1.31	0.153	8.56
0.981	2.88×10^{-6}	2.62	0.293	8.90

Table 5.11.3

Compact processing temp. = 100°C

Flow stress $P_0 = 0.45 \times 10^6$ Pa

Load (N)	Contact Area predicted m^2	Friction F_1 predicted (N)	Friction F_2 experimental (N)	Ratio of F_1/F_2
0.049	1.09×10^{-7}	0.064	0.006	10.67
0.098	2.18×10^{-7}	0.128	0.010	12.80
0.196	4.35×10^{-7}	0.256	0.024	10.67
0.491	1.09×10^{-6}	0.643	0.065	9.90
0.981	2.20×10^{-6}	1.298	0.100	12.98

Table 5.11.4

Compact processing temp. = 150°C

Flow stress $\simeq P_0 = 0.34 \times 10^6$ Pa

Load (N)	Contact Area predicted m ²	Friction F ₁ predicted (N)	Friction F ₂ experimental (N)	Ratio of F ₁ /F ₂
0.049	1.44×10^{-7}	0.100	0.020	5.00
0.098	2.88×10^{-7}	0.204	0.047	4.34
0.196	5.76×10^{-7}	0.408	0.054	7.55
0.491	1.44×10^{-6}	1.027	0.065	15.72
0.981	2.88×10^{-6}	2.045	0.073	28.00

Table 5.11.5

Compact processing temp. = 200°C

Rupture stress $\tau_R = P_0 = 0.4 \times 10^6$ Pa

Shear stress, = 7.1×10^5 Pa

Load (N)	Contact Area predicted m^2	Friction F_1 predicted (N)	Friction F_2 experimental (N)	Ratio of F_1/F_2
0.049	1.22×10^{-7}	0.087	0.003	29.00
0.098	2.45×10^{-7}	0.173	0.010	17.30
0.196	4.90×10^{-7}	0.348	0.024	36.40
0.491	1.23×10^{-6}	0.873	0.054	32.22
0.981	2.45×10^{-6}	1.740	0.044	39.53

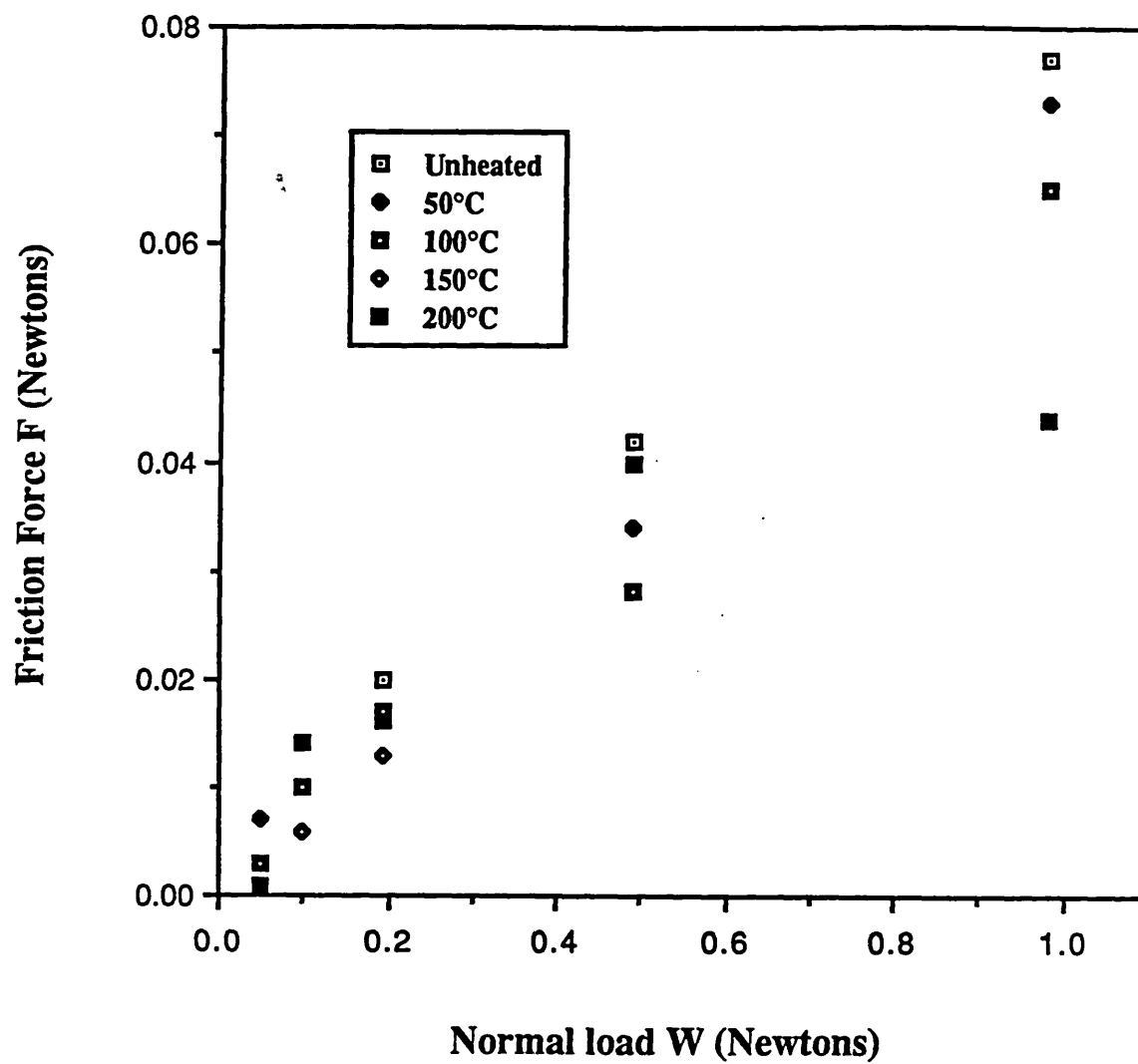


Fig.5.1 The frictional force F as a function of the normal load W for compacted maize hemi-spheres fabricated at different temperatures slid over glass counterfaces.

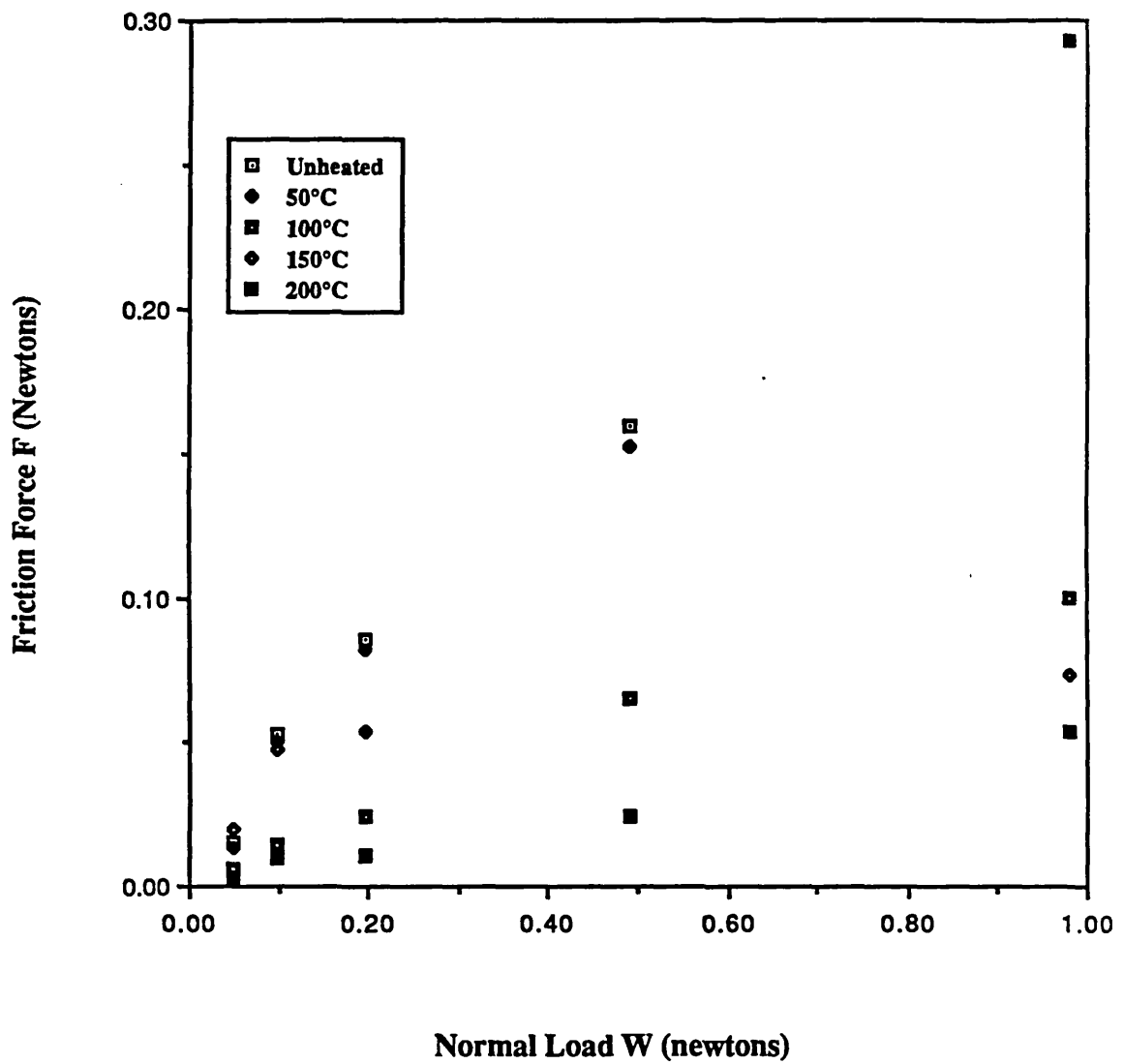


Fig.5.2 The frictional force F as a function of the normal load W for compacts fabricated from powder containing 16% w/w water slid over glass counterfaces.

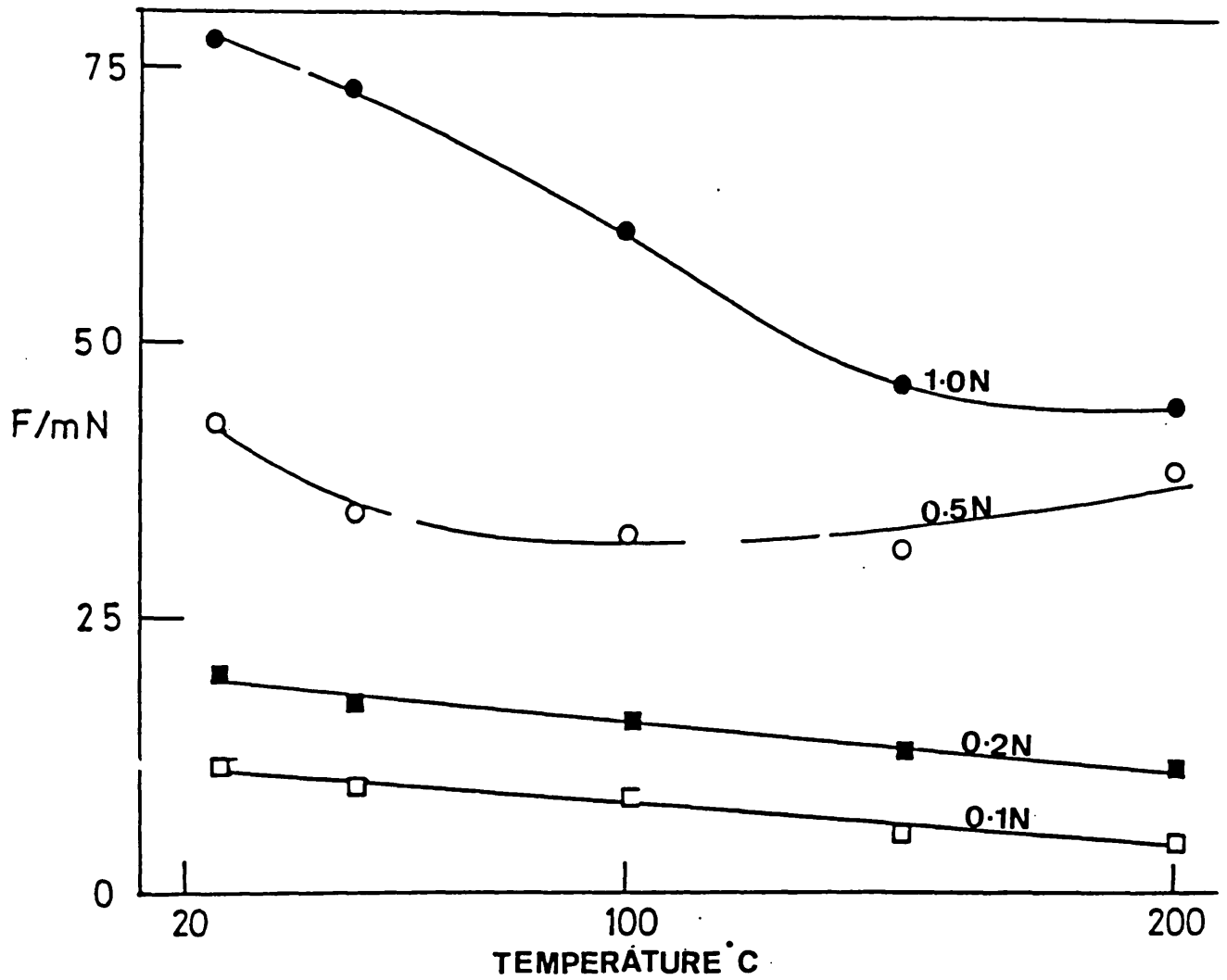


Fig.5.3 The friction force F for compacts produced at different compaction temperatures: sliding at 20°C on smooth glass counterfaces.

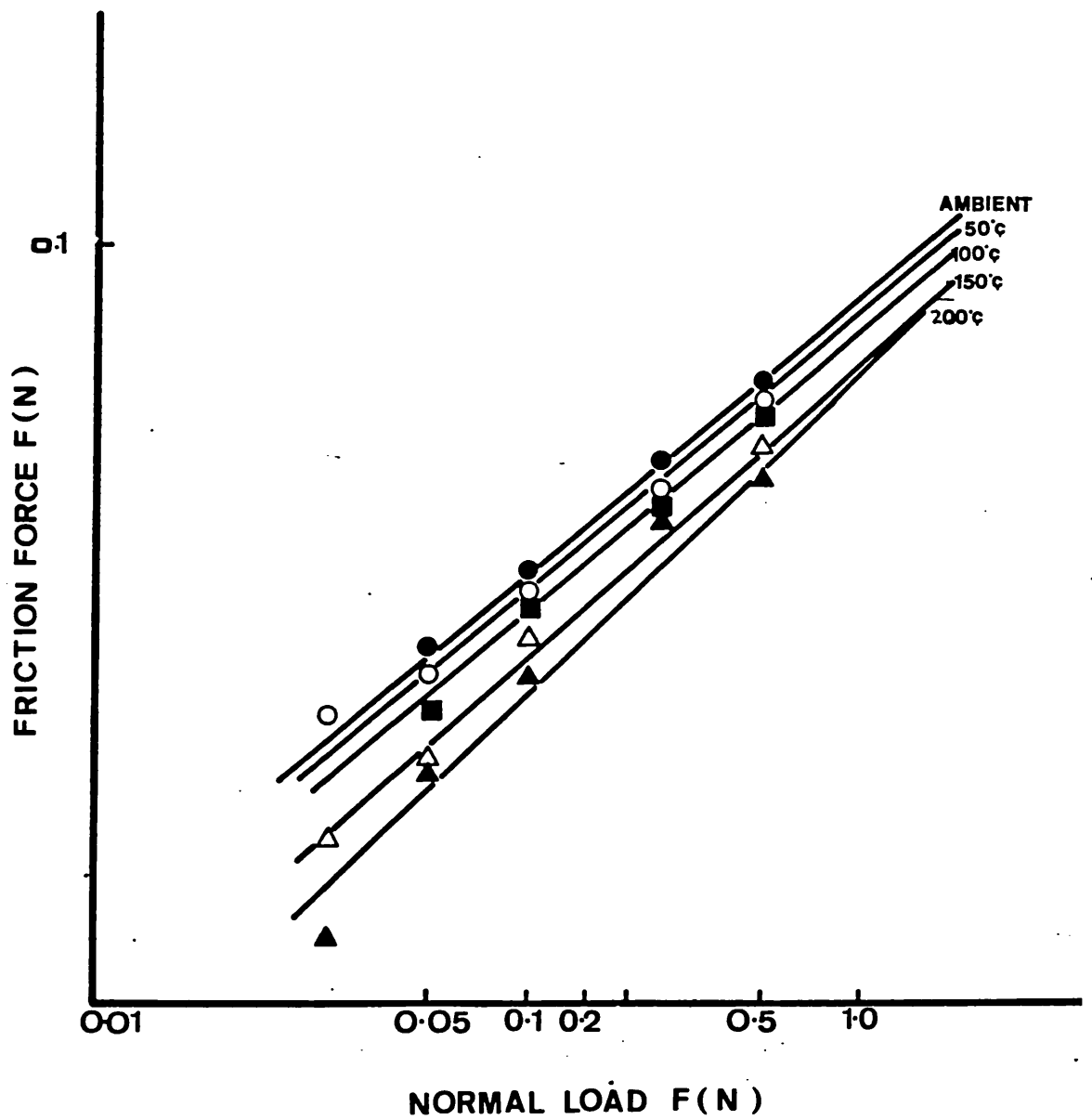


Fig.5.4 A logarithmic plot of the frictional force F as a function of the normal load W for a compact prepared from unmoistened maize powder.

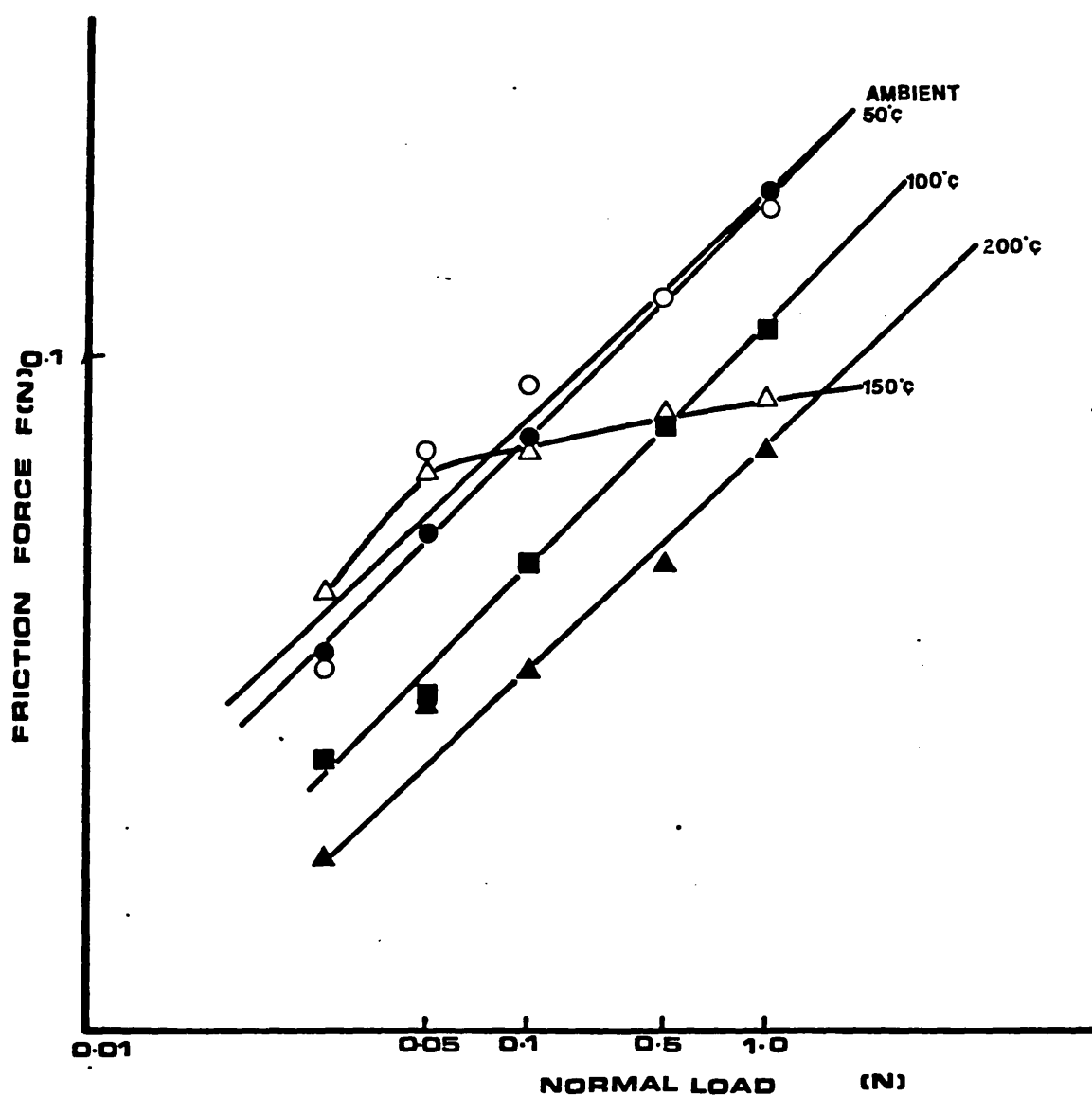


Fig.5.5 A logarithmic plot of the friction force F as a function of the normal load W for a compact prepared from moistened maize powder (level of added moisture 16% w/w.).

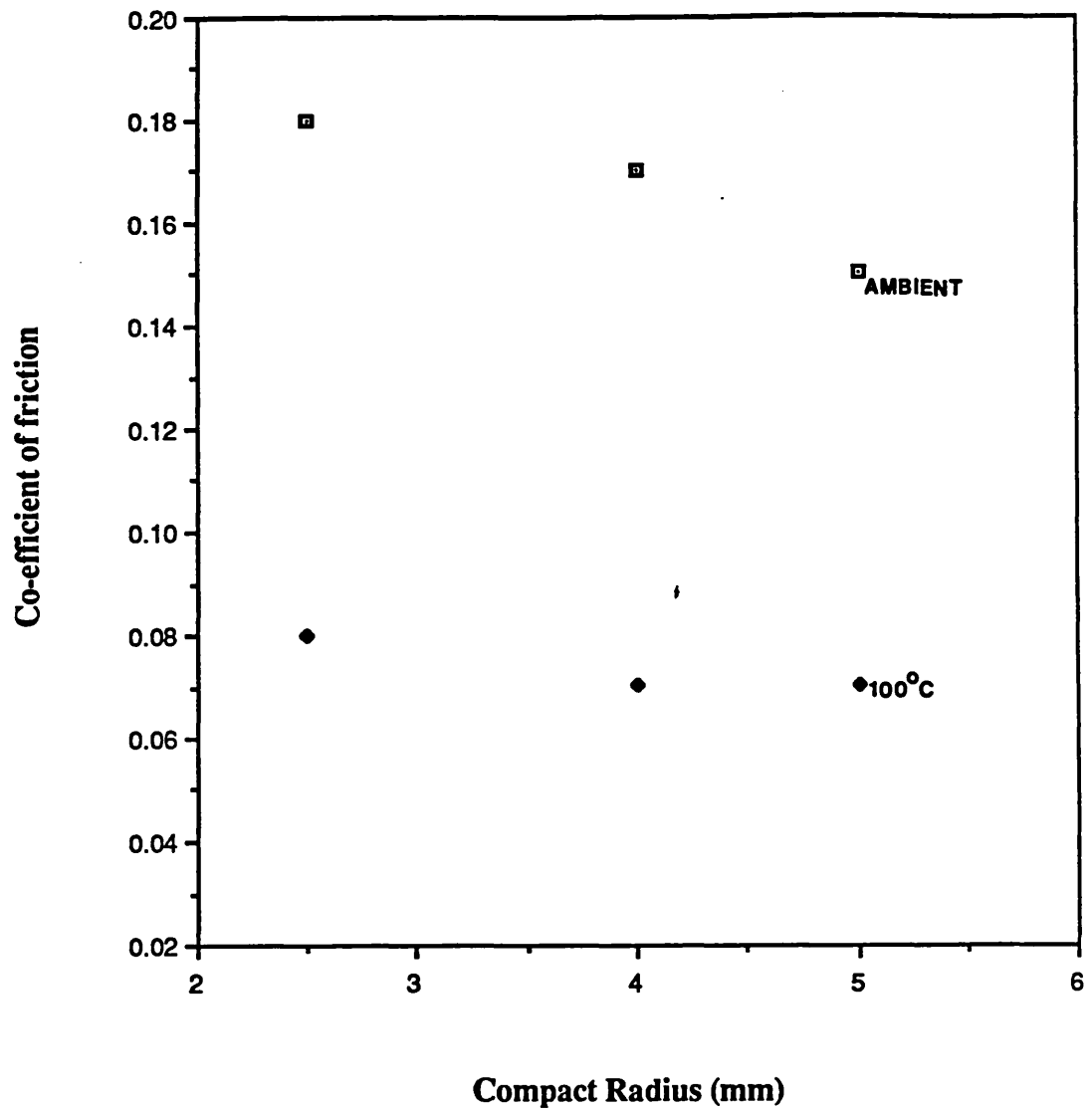


Fig.5.6 The effect of compact radius on the co-efficient of friction for a maize compact fabricated from unmoistened maize powder at ambient and 100°C.

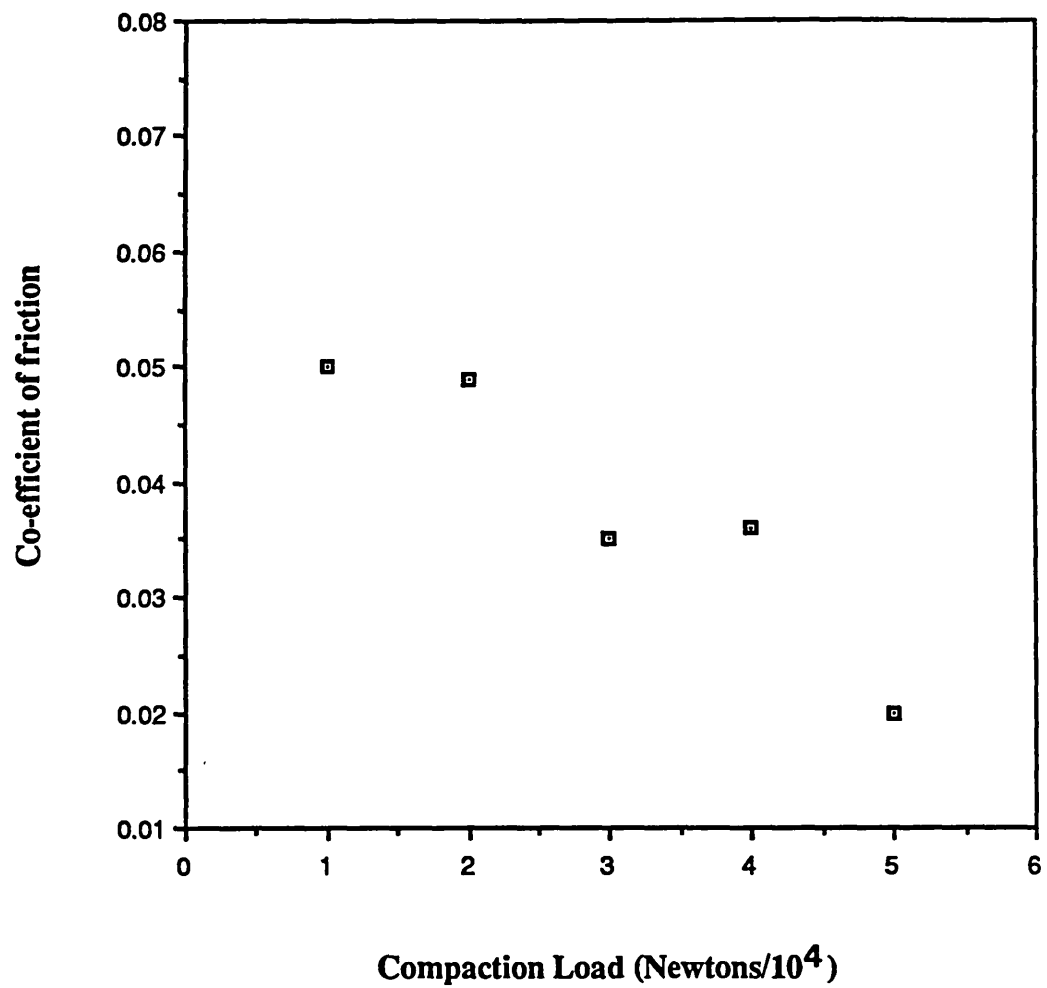


Fig.5.7 The influence of pre-consolidation load on the co-efficient of friction. Data at a normal load of 0.2N.

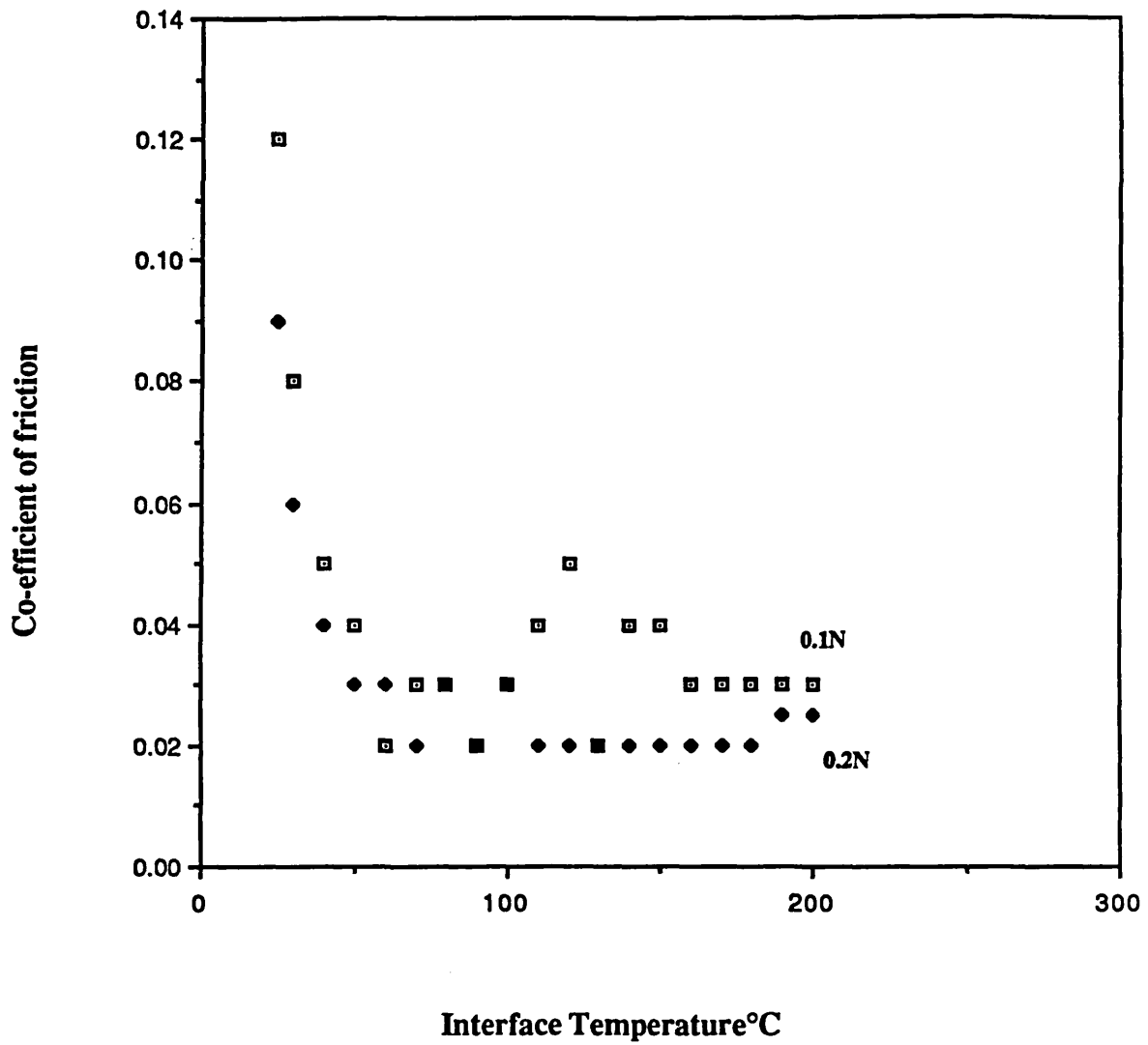


Fig.5.8 The influence of interface temperature on the co-efficient of friction for a compact fabricated from unmoistened maize powder at ambient *Ca.20°C*.

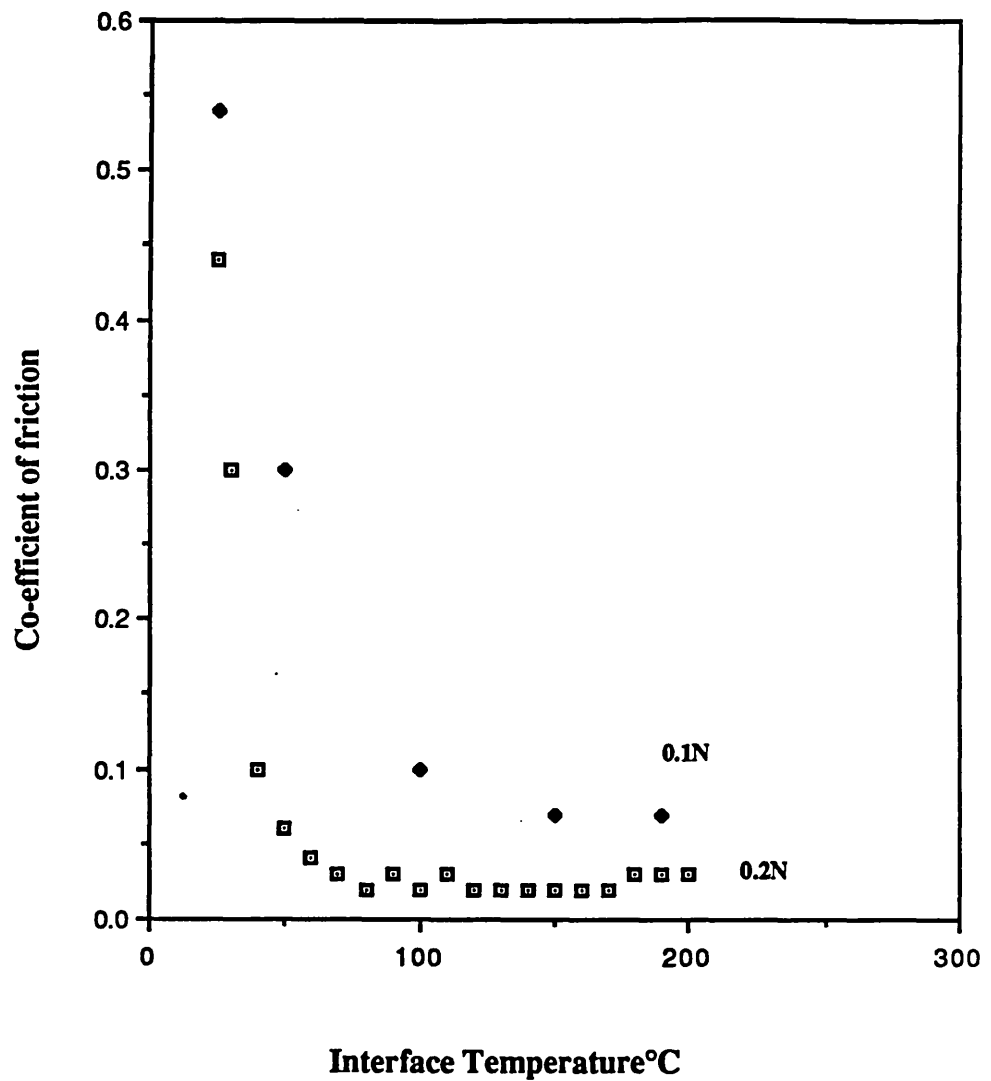


Fig.5.9 The influence of interface temperature on the co-efficient of friction for a compact fabricated from moistened maize powder at ambient *Ca* 20°C.

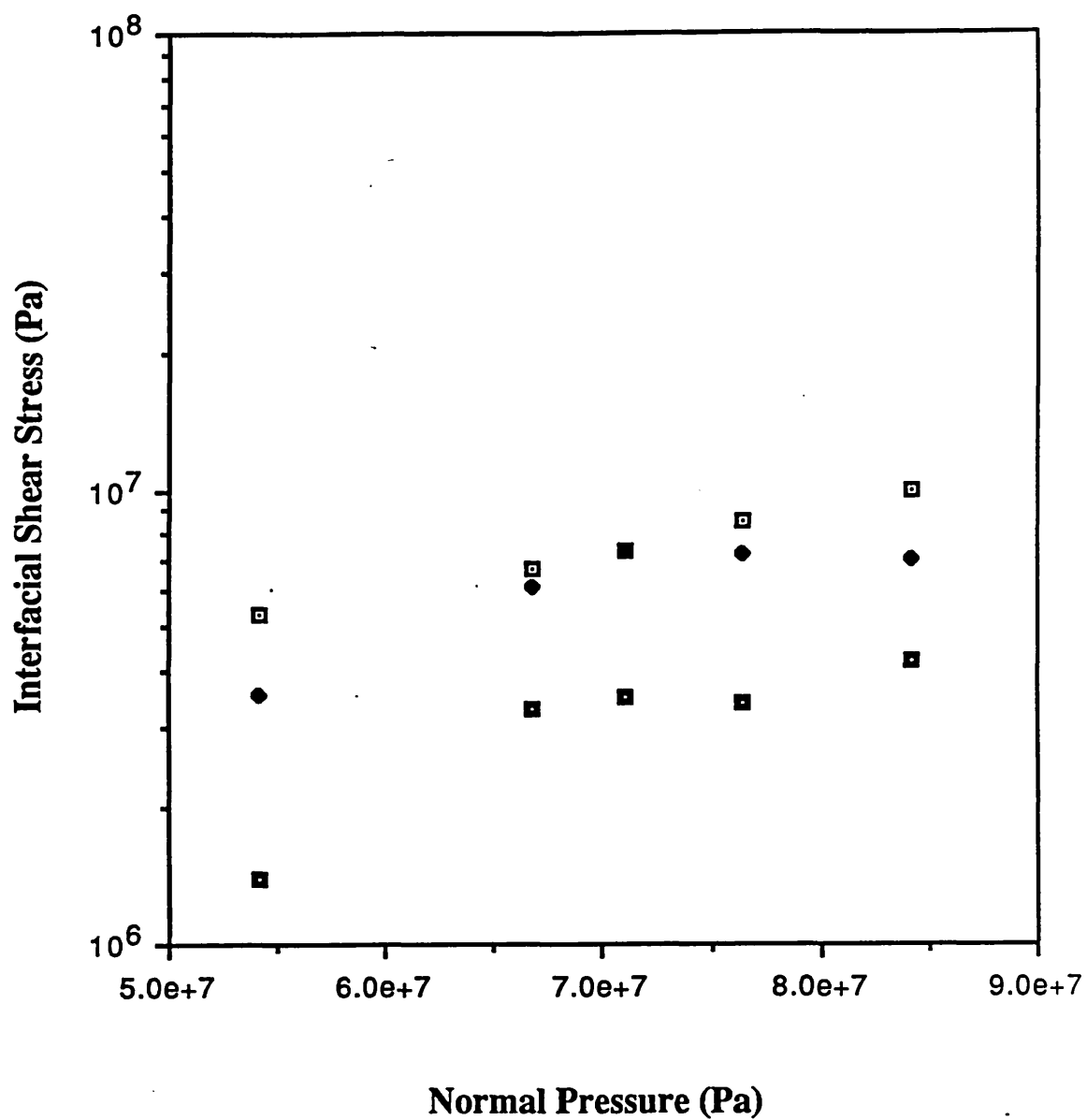


Fig.5.10 The logarithm of the interfacial shear strength τ as a function of the mean contact pressure P for maize films at a sliding velocity of 0.15mm/sec for various temperatures($^{\circ}\text{C}$) ■ unheated, □ 50 $^{\circ}\text{C}$, ◆ 100 $^{\circ}\text{C}$, ◆ 150 $^{\circ}\text{C}$, ◆ 200 $^{\circ}\text{C}$.

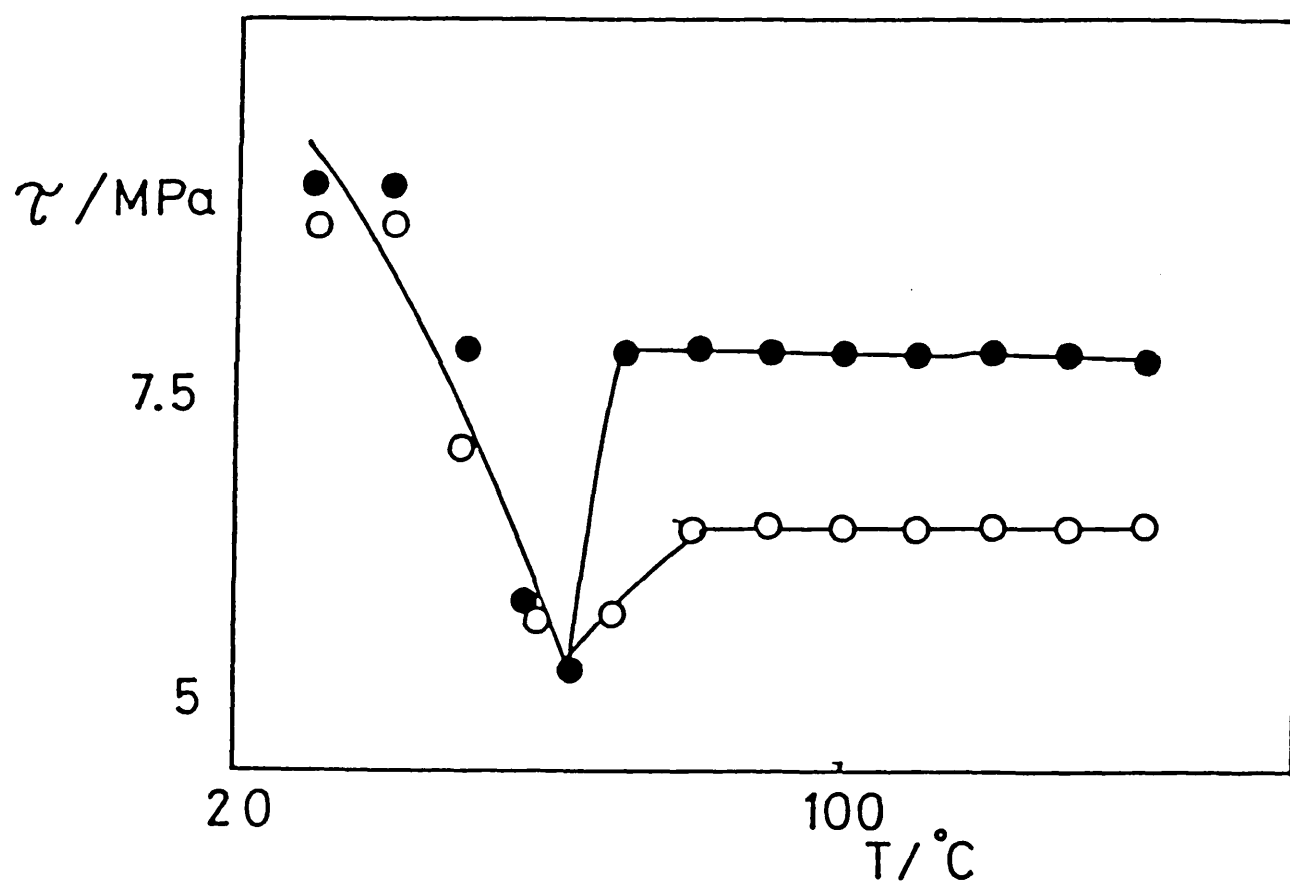


Fig.5.11 The interfacial shear stress τ as a function of contact temperature for maize films (Ca.100nm in thickness) deposited at Ca.20°C. Contact pressures ● 7.1×10^7 Pa, ○ 6.7×10^7 Pa.

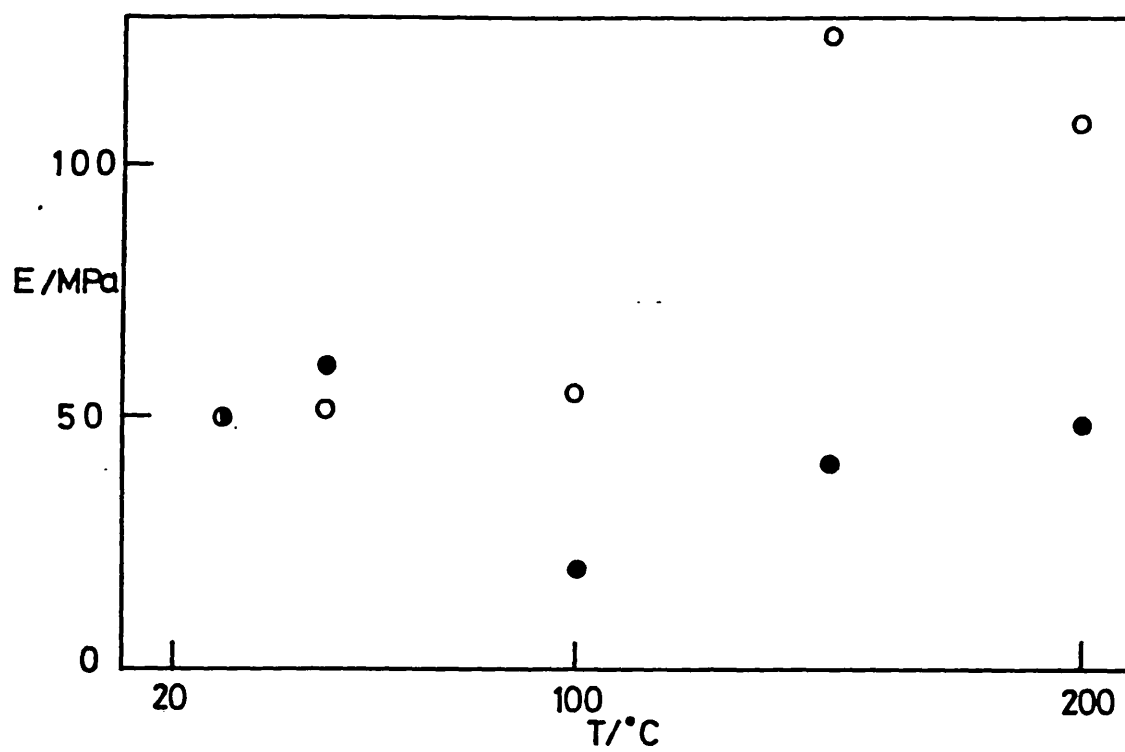


Fig.5.12 The influence of fabrication temperature on the modulus E . Open symbols are for measured values of E and closed symbols are for optimized values of E .

Chapter 6

Compaction Data and Analysis

6.1. *Introduction*

In an earlier chapter the notion was introduced that the uni-axial compaction process is similar in many respects to the compaction process which exists in the feed zone of an extruder. The similarity of the piston-cylinder compaction geometry to that which exists in the extruder feed zone will become clearer if one were to unravel the extruder screw and approximate the flow in the screw channel to be that between two parallel plates. There are, however, several important differences between the processes in the extruder feed zone and those which occur in a uni-axial compaction process: Apart from the simplified geometry in the latter type of experiment, the stress state here is near the active or the lower limit of the possible stresses. In contrast, the flowing material in an extruder feed zone would be expected to generate stresses near the passive stress state. In spite of these differences a first order prediction of wall stresses in an extruder feed zone can be obtained through an analysis based on a uni-axial compaction process; this is described in chapter 4. The available literature on the prediction of wall stresses was reviewed in chapter 3, and having considered these, it was decided

that a Janssen-Walker type of analysis would be best suited for the current problem.

Several workers⁽¹¹⁴⁾⁽¹¹⁵⁾ have studied powder compaction in cylindrical dies and derived relationships between the applied stress Q , commonly known as the surcharge, and the vertical transmitted stress, σ_{yy} at a given depth. In deriving these functionalities the dependency of friction on the process variables is neglected and a constant friction coefficient is assumed. However, experimental data on the friction properties of materials reveals that the coefficient of friction may be significantly modified by the locally induced normal stress, interface temperature and relative interfacial movement⁽⁴⁹⁾. In the early years of compaction studies, Unckel⁽¹¹⁶⁾, Seelig and Wulff⁽¹¹⁷⁾ and Bal'shin⁽¹¹⁸⁾ demonstrated the types of density distributions produced in a powder as a result of friction at the die-wall interface. More recently Train and Lewis⁽¹¹⁹⁾, Isherwood⁽¹¹⁵⁾, Bransby *et al*⁽¹²⁰⁾ have probed further into this mechanism of the complex stress and density distributions caused by wall friction.

This chapter extends the Janssen-Walker analysis described in chapter 3 to cater for the variation of the coefficient of friction μ_w with normal stress and interface temperature. The predictions from these extended analyses are then compared with the original Janssen-Walker analysis and with data obtained in uni-axial compaction experiments.

6.2. Extended Janssen-Walker Analysis

The formulae of Janssen yield the static and the dynamic stresses exerted on bin walls by the compacting mass. The analysis to be presented is subjected to the assumptions made in the derivation of the original Janssen-Walker analysis.

In considering the elemental force balance (eq. 3.15) appropriate to the cylindrical geometry, the shearing stress at the wall can be presented as a function of the wall normal stress $\sigma_{xx\omega}$. For maize powder this was shown to be exponential and was given by $\tau_{\omega} = \tau_o' e^{-\frac{\alpha' \sigma_{xx\omega}}{\bar{\xi}}}$ in chapter 5, where $\bar{\xi}$ is the ratio between the real area of contact and the apparent area of contact and σ_{xx} the wall normal stress based on the apparent area of contact. This exponential relation can be substituted for τ_{ω} in the force balance

$$\frac{\pi D^2}{4} [d\bar{\sigma}_{yy} + \bar{\sigma}_{yy} + \bar{\sigma}_{yy}] = - \bar{\xi} \tau_o' \exp \frac{\alpha' \sigma_{xx\omega}}{\bar{\xi}} \pi D dy \quad (6.1)$$

$\bar{\xi}$ is the average value of the area ratio over the pressure range.

For active failure,

$$\sigma_{xx\omega} = K_{WA}^{-} \sigma_{yy\omega} \quad \text{and} \quad \frac{\sigma_{yy\omega}}{\bar{\sigma}_{yy}} = \mathcal{D}.$$

Substitution of these in equation (6.1) and approximating the

exponential term as

$$\exp\left[\frac{\alpha' \sigma_{xx\omega}}{\bar{\xi}}\right] = 1 + \frac{\alpha \sigma_{xx\omega}}{\bar{\xi}}$$

yields:

$$\frac{\pi D^2}{4} [d\bar{\sigma}_{yy}] = -\bar{\xi} \tau_o' \left[1 + \frac{\alpha K_{\omega}^{-} \mathcal{D}}{\bar{\xi}} \bar{\sigma}_{yy}\right] \pi D dy \quad (6.2)$$

Simplification leads to

$$\bar{\sigma}_{yy} \int_Q \frac{d\bar{\sigma}_{yy}}{1 + A\bar{\sigma}_{yy}} = -\frac{4\bar{\xi}\tau_o' H}{D} \int dy$$

where

$$A = \frac{\alpha' K_{\omega}^{-} \mathcal{D}}{\bar{\xi}}.$$

Solution to the integral equation is

$$\bar{\sigma}_{yy} = \frac{1}{A} \left[(1 + AQ) e^{-4A\bar{\xi}\tau_o' H/D} - 1 \right] \quad (6.3)$$

This expression can now be converted to $\sigma_{yy\omega}$ and $\sigma_{xx\omega}$ respectively.

$$\sigma_{yy\omega} = \frac{\mathcal{D}}{A} \left[(1 + AQ) e^{-4A\bar{\xi}\tau_o' H/D} - 1 \right] \quad (6.4)$$

$$\sigma_{xx\omega} = K_{\omega} \frac{\mathcal{D}}{A} \left[(1 + AQ) e^{-4A\bar{\xi}\tau_o' H/D} - 1 \right] \quad (6.5)$$

Similarly for passive failure

$$\bar{\sigma}_{yy} = \frac{1}{A'} \left[(1 + A'Q) e^{-4A'\bar{\xi}\tau_o'H/D} - 1 \right] \quad (6.6)$$

$$\sigma_{yy_\omega} = \frac{\mathcal{D}}{A'} \left[(1 + A'Q) e^{-4A'\bar{\xi}\tau_o'H/D} - 1 \right] \quad (6.7)$$

$$\sigma_{xx_\omega} = \frac{K_{\omega P} \mathcal{D}}{A'} \left[(1 + A'Q) e^{-4A'\bar{\xi}\tau_o'H/D} - 1 \right] \quad (6.8)$$

where $A' = \frac{\alpha' K_{\omega P} \mathcal{D}}{\bar{\xi}}$. In these equations τ_o' and α' can be expressed in terms of temperature and the appropriate value of $\bar{\xi}$ is evaluated from a contact mechanical model appropriate for the conditions near the wall. A schematic presentation of the powder wall interface is shown in Fig. (6.1).

If n' is the number of contacts per unit area of A_{app} along the length of the cylinder and W_T the total normal load acting on the wall, then the total no. of contacts = $n' \times A_{app} = n_c$. Hence, the load acting on a single contact = $\frac{W_T}{n'A_{app}} = \omega_c$.

Area of a single contact is given by the Hertz equation

$$A_c = \pi \left[D_c \frac{W_T}{n'A_{app}} R_c \right]^{2/3} \quad (6.9)$$

where R_c is the radius of a single asperity and

$$D_c = 0.75 \left[\frac{1 - \nu_c^2}{E_c} + \frac{1 - \nu_s^2}{E_s} \right] \quad (6.10)$$

ν_c , ν_s , E_c and E_s are Poisson's ratio and Young's Moduli for the asperity and the surface respectively. The real contact area, $A_r \simeq \text{no. of contacts} \times \text{area of a contact} = n_c \times A_c$. Substitution of eq. (6.9) for A_c and $n \times A_{app}$ for n_c and division by A_{app} yield

$$\xi = \frac{A_r}{A_{app}} = n^{1/3} \pi \left[\frac{D_c W_T R_c}{A_{app}} \right]^{2/3}$$

but

$$\frac{W_T}{A_{app}} = P_{app}$$

$$\xi = n^{1/3} \pi \left[D_c R_c \right]^{2/3} P_{app}^{2/3} \quad (6.11)$$

The analysis can then be extended to obtain an analytical expression for the coefficient of wall friction μ_w , as follows.

The adhesion model of boundary friction described in chapter 2 for the powder-wall contact in the cylinder is

$$\begin{aligned} F &= \tau \times A_r \\ &= \tau_0 \left[1 + \frac{\alpha P_{app}}{\xi} \right] \xi A_{app} \end{aligned}$$

Dividing by $P_{app} \times A_{app}$

$$\mu_{\omega} = \frac{\tau_o' \xi}{P_{app}} + \tau_o' \alpha'$$

Eq. (6.11) is substituted for ξ

$$\mu_{\omega} = \frac{\tau_o' n'^{1/3} \pi (D_c R_c)^{2/3}}{P_{app}^{1/3}} + \tau_o' \alpha' \quad (6.12)$$

As $P_{app} \rightarrow \infty$, $\mu_{\omega} \rightarrow \tau_o' \alpha'$, this gives the limiting value of μ_{ω} for a particular medium.

It has been shown that ξ is a function of the normal stress σ_{xx} , but the equations derived for the prediction of wall stresses neglect this dependency. A more appropriate expression can be derived by substituting eq.(6.11) in (6.2) and the respective expressions for active and passive failure, which accounts for the change in ξ with normal stress, are

$$\int_Q^{\bar{\sigma}_{yy}} \frac{d\bar{\sigma}_{yy}}{\sigma_{yy}^{1-n} (1 + A' \sigma_{yy}^{1-n})} = - \frac{4HB\tau_o}{D} (K_{\omega_A} \mathcal{D})^n \quad (6.13)$$

$$\int_Q^{\bar{\sigma}_{yy}} \frac{d\bar{\sigma}_{yy}}{\sigma_{yy}^{1-n} (1 + A' \sigma_{yy}^{1-n})} = - \frac{4HB\tau_o}{D} \left[\frac{\mathcal{D}}{K_{\omega_P}} \right]^n \quad (6.14)$$

where $n \simeq 0.67$, $B = n'^{1/3} \pi (D_c R_c)^{2/3}$ and $A' = \frac{\alpha}{B} (K_{\omega_A} \mathcal{D})^{1-n}$.

The above numerical integrations can be solved by a fourth order Runge-Kutta technique developed by Gill and Miller⁽¹²¹⁾. At each temperature the appropriate values of α' , τ_o' , K_ω and D_c are incorporated in these equations to predict the stresses generated.

These equations therefore provide a means of estimating the stress distribution in a compacting medium provided its frictional properties τ_o' , α' , μ_i , μ_ω and mechanical properties of the contacting substrates E_s , E_c , v_s , v_c , and R_c are available. The only unknown quantity, as with the original Janssen analysis is the distribution factor $\mathcal{D}$. However for containers of large aspect ratio the great depth value of $\mathcal{D}$ at great depth can be used without serious loss of accuracy.

6.3. *Failure Properties and Compaction Data and Analyses of Maize Powder*

6.3.1. *Yield Loci of Maize Powder*

The failure properties of maize powder obtained in a shear test, are presented in this section. The powder was approximately at its critical volume since volume change during shear was negligible. This state is particularly important as it represents a condition in which all the work done by the shear force is used in overcoming internal friction. The shear strength of the material in this state, plotted against the normal load for maize powder at 0%, 8% and 16% added moisture is shown in Figs. (6.2 -

6.4). The cohesion c and the angle of internal friction ϕ_i were calculated by linear regression analysis of the data and listed in Table (6.1). For the present system the cohesion c increases with moisture, while the angle of internal friction ϕ_i decreases. Mannheim and Passy⁽⁸²⁾ also observed a decrease in the angle of internal friction with moisture and attributed this to the existence of a liquid film on the particle surfaces which is acting as a lubricant. The data obtained on the failure properties of starches by these authors are listed in Table (6.2) for comparison with data on the failure properties of maize powder (Table 6.1). These indicate the cohesion and the angle of internal friction are little affected by experimental variations for starch and flour, both of which are known to be relatively non-cohesive powders. It was only possible to establish the yield loci for maize powder for a single pre-consolidation load within the available time constraint and the effect of pre-consolidation history on the yield locus was not investigated. Nevertheless, the EYL was shown in chapter 3 to pass through the origin and the terminal points of all the yield loci. Therefore, by connecting the origin and the single terminal point of the yield locus an estimate for the effective angle of internal friction ϕ_E was found. The YL for maize powder was found to deviate from a straight line at low values of the normal stress. This was observed by Ashton et al⁽⁸³⁾ working on a range of powders. His data for lithophane is shown in Fig. (6.5).

In the available literature, there is relatively little reported on the effect of temperature on the failure properties of

powders. This is somewhat surprising since temperature is the single most important parameter in many processes. The work of Gale⁽¹²²⁾ on the temperature dependency of ϕ_i for both PVC and polyethylene powder shows a decrease in ϕ_i with increasing temperature. Jenike *et al*⁽¹²³⁾ however, observes an increase in ϕ_i with temperature for adipic acid. For the present system it was not possible to establish the temperature dependency but some indication of this trend can be deduced from the rheogoniometer work to be discussed in chapter 7.

The major drawback of YL studies is the uncertainty as to whether the failure plane is co-incident with the plane of shear. If this is not the situation in a shear cell, the data for ϕ_i are subject to an unknown uncertainty.

A further point worthy of mention is the scatter in shear strength data obtained for the unmoistened system. This was also observed by Schramali⁽¹²⁴⁾ working on cement and Golder⁽¹²⁵⁾ working on sand and suggests that a powder can become locked as a result of the initial condition and can only be unlocked if allowed to expand momentarily. This dilatant behaviour gives rise to an erratic variation of the shear strength.

In conclusion, it appears that the point of incipient failure of a powder depends not only on the forces between particles and the way they adhere together but also on the elastic deformation and residual elastic strains in the matrix of which for the latter very little is known at present.

6.3.2. Computation of Residual Elastic Strains

It has recently been shown that the compressive stress relaxation curves of powders can be normalized and linearized in the form⁽¹²⁶⁾

$$\frac{F_0 t}{F_0 - F_t} = k_1 + k_2 t$$

where F_0 is the force before decay, F_t the decaying force after time t and k_1 and k_2 are constants characteristic of the actual shape of the recorded curve. The value of $\frac{1}{k_1}$ depicts the initial relaxation rate and $\frac{1}{k_2}$ is an asymptotic value of $\frac{(F_0 - F_t)}{F_0}$. This allows the calculation of an asymptotic residual modulus E_A from $E_A = \frac{F}{A\epsilon} \left\{ 1 - \frac{1}{k_2} \right\}$, where A is the cross-section area and ϵ the strain. The magnitude of E_A indicates the degree of solidity, since it represents those components of stress that do not dissipate through flow or structural re-orientation⁽¹²⁶⁾.

Fig. (6.6) shows a plot of the relaxation behaviour of maize powder at three moisture levels. The "wet" powder was found to be more deformable than the dry powder and the time taken to reach a steady value of the transmitted stress was longer, thus allowing more stress relaxation to take place during deformation. Table (6.3) lists the data obtained on food powders by Peleg et al⁽¹²⁶⁾ together with the data for the current maize powder. The addition of moisture drastically reduces the asymptotic modulus, E_A . Although part of this decrease can be attributed to differences in the density of the compacts, it is still mainly a reflection of

stress dissipation through flow of liquid or weak interparticle bridges⁽¹²⁷⁾.

Since these stress relaxation data can be obtained with relative ease with the widely available universal testing machines, they have provided useful complementary information in assessing the deformation properties of the powder.

6.3.3. *Compaction Data Analysis*

In order to use the Janssen-Walker analysis and the extended Janssen-Walker analysis for the prediction of wall stresses, the quantities $\bar{\xi}$, $K_{\omega A}$, μ_{ω} , μ_E and $\mathcal{D}$, D_c must be known *a priori*. The effective angle of internal friction, μ_E was established from the shear test experiments and the angle of wall friction μ_{ω} was computed from eq (6.12) and $\bar{\xi}$ to be incorporated into this equation from the derived contact mechanical model (eq. 6.11). The only available accurate solution for $\mathcal{D}$ uses the method of characteristics. However, as was pointed out by Nedderman⁽⁸⁹⁾, difficulties are encountered with this technique for axi-symmetric cases. Horne⁽¹²⁸⁾ was forced to abandon his computation of wall stresses in a cylindrical container with this method and for this reason it was decided not to use this technique in the current work. Therefore, in the absence of a more suitable technique for predicting $\mathcal{D}$, the value at great depth was used in the present work. The great depth value would be expected to be an overestimation of the true stress variation since the aspect ratio of the cylinder for the present system was small. The vertical

stress variation at small depths is greater than at large depths, but this variation is also a function of the difference in the internal and wall angles of friction ϕ_E and ϕ_w . The greater the difference between ϕ_E and ϕ_w , the nearer will the appropriate value of $\mathcal{D}$ at small depth approach the great depth value. This was found to be the case for the current problem, where a great depth value of unity for $\mathcal{D}$ gave excellent agreement between theory and experiment. It was discussed in chapter 3 that exact agreement between theory and experiment should not be expected as theory would only predict the limits of the possible range of stresses in plastic equilibrium. Nevertheless, the experimental values accord well with theoretical predictions, especially for the low moisture systems. These predictions are shown in Tables (6.4 - 6.8). It is also interesting to note that the value of K_{ω_A} for the low moisture systems predicted from a model based on elastic equilibrium yields a value of 0.54, as opposed to a value of 0.2 predicted from a model based on plastic equilibrium derived by Long⁽¹²⁵⁾. The analysis for K_{ω} in elastic equilibrium is given by $\frac{v}{1-v}$, where v is the Poisson's ratio, the value of which for maize powder is 0.35. For a highly elastic material like a rubber with a Poisson's ratio of 0.5 the K_{ω} ratio will be 1.00 the stress transmission is hydrostatic. This seems to indicate that the powder is in plastic flow and the predictions based on a plastic model are less appropriate for the high moisture systems, since these systems are capable of retaining large amounts of residual elastic strains. The stress relaxation data supports this view where it was shown that the asymptotic

residual modulus for the 16% w/w system was 40 kg/cm^2 compared to a value of 88 kg/cm^2 for the unmoistened powder. Nevertheless, the material has undergone plastic flow in all cases, but stress transmission may be governed by residual elastic strains in the moistened systems.

The computed value of ξ increases with increasing surcharge and cross-head speed (Fig. 6.7). In these conditions the deformation of the existing contact points are increased and the ratio between the real and apparent contact area tends to unity. The same is true for the moistened systems where moisture absorption swells the particles resulting in an increase in ξ . The relevant ξ values were calculated by eq. (6.11) and some preliminary computations using the extended Janssen-Walker analysis which allows for the variation of ξ with normal pressure gave predictions 3% less than the extended analysis which uses an average value of $\bar{\xi}$. In view of the simplicity in the use of the exact solution, added to the fact that stress measuring instruments are unable to monitor stresses to an error limit less than 5%, it was decided that the analysis based on the average $\bar{\xi}$ value would be sufficiently accurate for the current series of experiments.

The predictions from the original Janssen-Walker and the extended analysis yields predictions which are similar, irrespective of the fact that the extended analysis caters for the variation in the coefficient of wall friction μ_w with the local normal stress. This is a result of the high normal pressures generated which yields the coefficient of wall friction μ_w , to be

rather independent of normal stress in the present experimental stress range and therefore the original Janssen-Walker and the extended analysis basically predict the same result. However, for low surcharge stresses, the corresponding wall stresses generated would be lower and a significant difference can be expected in the predicted values of the wall stresses between the two analyses. It is envisaged that this would be the case in the extruder where the normal pressures generated in the feed zone are known to be much lower. Similar effects would be expected in very high aspect ratio dies in uni-axial compaction. Fig. (6.8 - 6.11) demonstrates the effect of moisture, surcharge and cross-head speed on the transmitted vertical stress and the normal wall stress. The transmitted stress is virtually unaffected by experimental variations. However, a marked increase in the normal wall stress was observed with moisture and cross-head speeds.

7.4. Concluding Remarks

The major uncertainty in the Janssen-Walker types of analysis is created by the use of the great depth value of $\mathcal{D}$. Nevertheless, considering the large difference between the effective angle of internal friction ϕ_E and the wall angle of friction ϕ_w , the great depth value of unity for $\mathcal{D}$ provides a good prediction of the wall stresses in the uni-axial compaction of maize powder.

An added drawback of this quasi-static analysis is the use of contact mechanics based on a particulate medium, to be incorporated into a theoretical model derived from continuum mechanics. A more appropriate analysis can only be derived by a better understanding of the internal micro-mechanics of which relatively little is known at present. Thornton's⁽¹³⁰⁾ computer simulation work goes some way in remedying this situation but the current major weakness here is the neglect of boundary friction. In the real situation the internal and the wall processes cannot be isolated since the behaviour of the interior to a large extent will be governed by what takes place next to the wall.

In spite of these inaccuracies it has been demonstrated that a first order prediction of wall stresses could be obtained by a Janssen-Walker type of analysis for the current problem.

Table 6.1.

Failure Properties of Maize Powder

	0% moisture	8% moisture	16% moisture
Angle of internal friction ϕ_i	42°	41°	42°
Effective angle of friction ϕ_E	35°	31°	26°
Angle of wall friction in compaction, ϕ_ω computed from $\tan \phi_\omega = \frac{\tau_{\theta} e^{\alpha' \sigma_{xx\omega} / \xi}}{\sigma_{xx\omega}}$	2.4°	1.7°	0.2°

Table 6.2

Failure Properties of Starches, obtained by Pelag *et al*⁽⁸⁴⁾

Food Powder	Moisture %	Cohesion c (g/cm ²)	Angle of internal friction ϕ_i
Milk powder	1.05	7.0	39.4°
Milk powder	4.4	9.6	38.4°
Red beets	3.3	8.6	42.8°
Sodium chloride	0.32	2.0	36.4°
Gelatine	-	0.8	39.4°
Soy flour	-	0.9	34.3°

Table 6.3

Stress relaxation characteristics of powders obtained
by Pelag *et al*⁽⁸⁴⁾, together with data on maize powder

Powder	Moisture (%)	Irrecoverable work in composition (% of total)	Asymptotic residual modulus (kg/cm ²)
Glass beads (175 micron)	dry	65	60 - 72
	1.0	>99	1.5 - 2.8
Powdered salt (-100 + 200 mesh)	dry	75	89
	0.6	98	0.01
Coffee creamer (commercial)	dry	92	21
	7.0	99	13
Maize powder	14	78	88
	22	50	48
	30	42	40

Table 6.4

Janssen-Walker predictions.

Cross-head speed 12.7 mm min^{-1} , $\bar{\xi}_{\text{av}} = 0.044$, $\phi_E = 35^\circ$, $\phi_\omega = 2.4^\circ$,
 $K_{\omega_A} = 0.27$, $\mathcal{D}_{\text{at great depth}} = 1.0$, Unmoistened system

$Q/10^7 \text{ Pa}$ surcharge	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (experimental)	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (Janssen-Walker)	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (extended)
0.23	0.05	0.06	0.05
0.46	0.14	0.12	0.11
0.69	0.17	0.17	0.17
0.92	0.20	0.23	0.22
1.15	0.31	0.30	0.30
1.38	0.38	0.35	0.36
1.61	0.43	0.41	0.40
1.84	0.48	0.47	0.47
2.07	0.55	0.51	0.51
2.30	0.59	0.59	0.59

Table 6.5

Janssen-Walker predictions.

Cross-head speed 25.4 mm min^{-1} , $\bar{f}_{av} = 0.05$, $\phi_E = 35^\circ$, $\phi_\omega = 2^\circ$,
 $K_{\omega A} = 0.27$, $\eta_{\text{at great depth}} = 1.0$, Unmoistened system

$Q/10^7 \text{ Pa}$ surcharge	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (experimental)	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (Janssen-Walker)	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (extended)
0.23	0.07	0.06	0.06
0.46	0.14	0.12	0.12
0.69	0.19	0.18	0.18
0.92	0.25	0.24	0.24
1.15	0.30	0.30	0.30
1.38	0.40	0.36	0.36
1.61	0.48	0.42	0.42
1.84	0.55	0.48	0.48
2.07	0.59	0.54	0.54
2.30	0.64	0.60	0.60

Table 6.6

Janssen-Walker predictions.

Cross-head speed 51.0 mm min^{-1} , $\bar{\xi}_{av} = 0.06$, $\phi_E = 35^\circ$, $\phi_\omega = 2^\circ$,
 $K_{\omega A} = 0.27$, $\mathcal{D}_{\text{at great depth}} = 1.0$, Unmoistened system

$Q/10^7 \text{ Pa}$ surcharge	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (experimental)	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (Janssen-Walker)	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (extended)
0.23	0.06	0.06	0.06
0.46	0.14	0.12	0.16
0.69	0.21	0.18	0.18
0.92	0.38	0.24	0.24
1.15	0.52	0.30	0.30
1.38	0.60	0.36	0.36
1.61	0.74	0.43	0.43
1.84	0.93	0.48	0.48
2.07	0.96	0.55	0.54
2.30	1.00	0.61	0.61

Table 6.7

Janssen-Walker predictions.

Cross-head speed 12.7 mm min^{-1} , $\bar{\xi}_{av} = 0.06$, $\phi_E = 31^\circ$, $\phi_\omega = 1.7^\circ$,
 $K_{\omega A} = 0.32$, $\vartheta_{\text{at great depth}} = 1.0$, Moisture = 8% added

$Q/10^7 \text{ Pa}$ surcharge	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (experimental)	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (Janssen-Walker)	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (extended)
0.23	0.07	0.07	0.07
0.46	0.14	0.14	0.14
0.69	0.20	0.21	0.21
0.92	0.27	0.28	0.28
1.15	0.39	0.35	0.35
1.38	0.50	0.42	0.42
1.61	0.74	0.49	0.49
1.84	1.00	0.56	0.56
2.07	1.15	0.64	0.63
2.30	1.31	0.71	0.71

Table 6.8

Janssen-Walker predictions.

Cross-head speed 12.7 mm min^{-1} , $\bar{\xi}_{av} = 0.1$, $\phi_E = 26^\circ$, $\phi_\omega = 0.2^\circ$,
 $K_{\omega A} = 0.39$, $\eta_{\text{at great depth}} = 1.0$, Moisture added 16% w/w

$Q/10^7 \text{ Pa}$ surcharge	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (experimental)	$\sigma_{xx_\omega}/10^7 \text{ Pa}$ (Janssen-Walker)	$\sigma_{xx_\omega} 10^7 \text{ Pa}$ (extended)
1.15	0.5	0.44	0.44
1.38	0.73	0.53	0.53
1.61	0.98	0.62	0.62
1.84	1.15	0.71	0.71
2.07	1.27	0.80	0.80
2.30	1.50	0.89	0.89

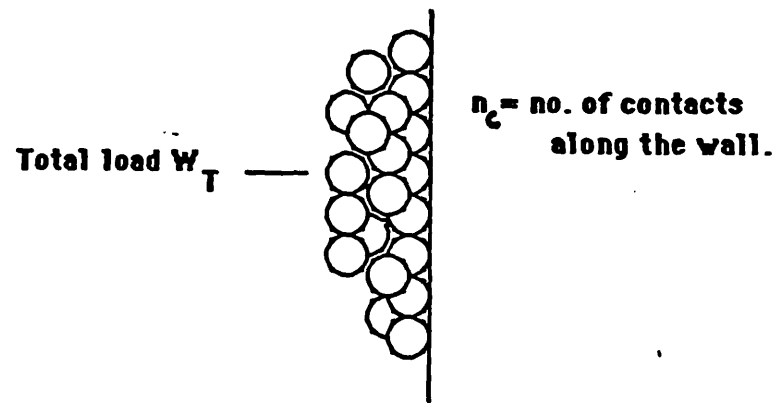


Fig.6.1 A representation of the powder/wall interface.

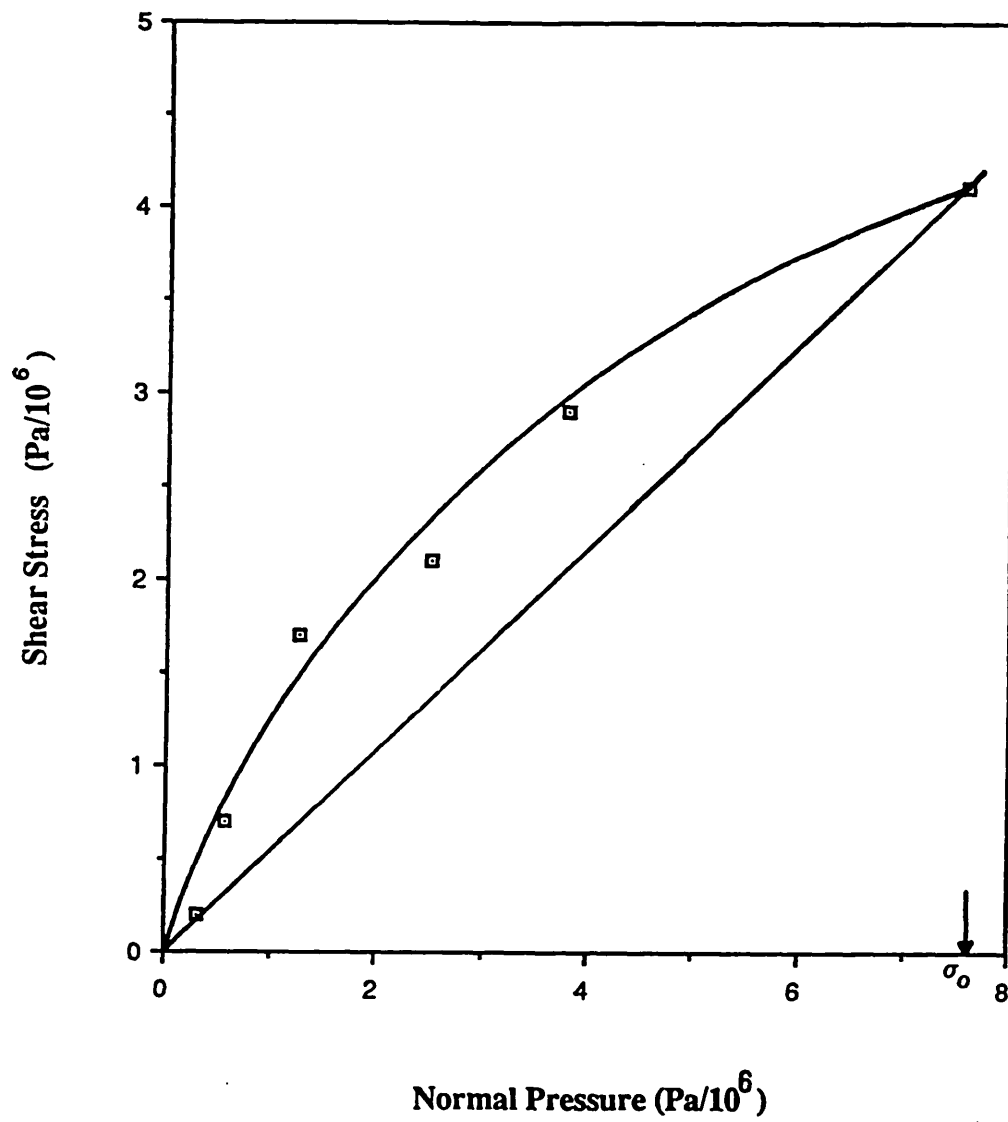


Fig.6.2 The yield locus for unmoistened maize powder at a pre-consolidation stress σ_0 of 0.77×10^6 and a sliding velocity of 0.581 mm/min. The effective angle of internal friction $\phi_e = 35^\circ$.

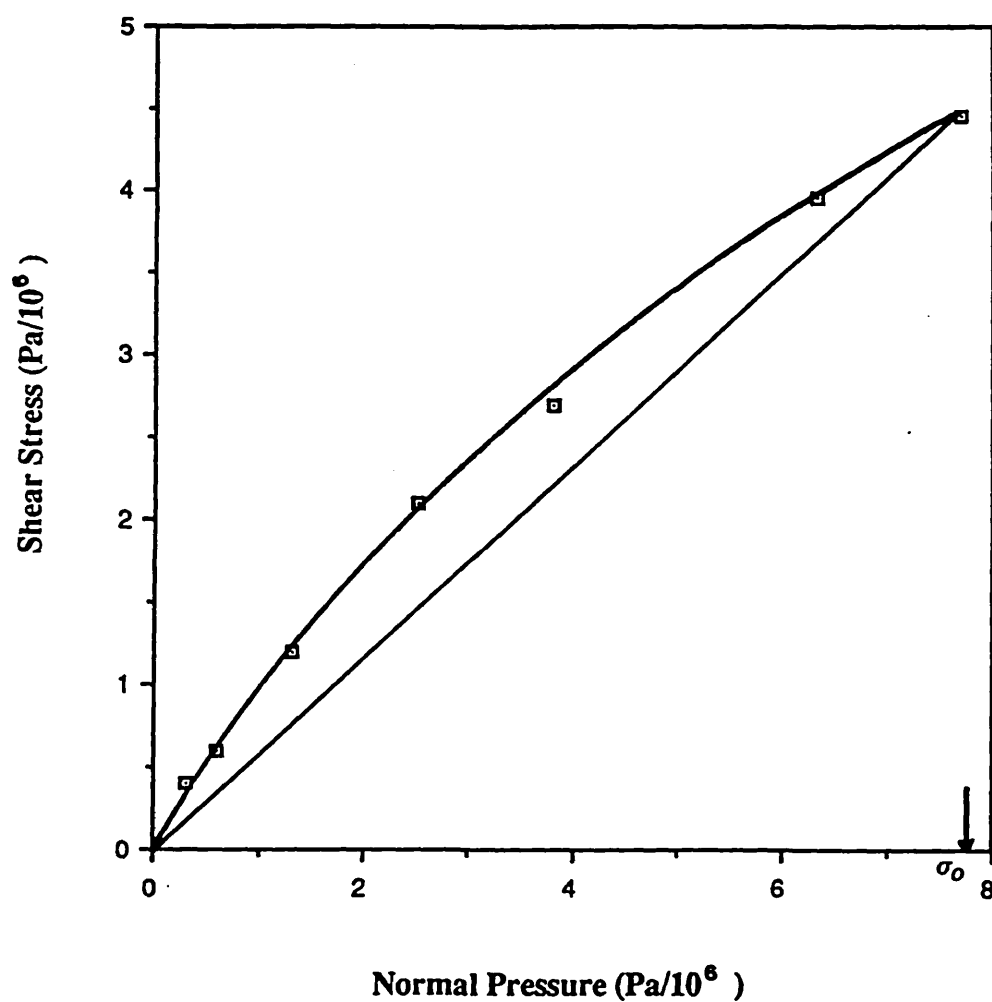


Fig.6.3 The yield locus for moistened maize powder at a pre-consolidation stress σ_0 of 0.77×10^6 Pa and a sliding velocity of 0.581mm/min. The level of added moisture 8% w/w. The effective angle of internal friction $\phi_c \approx 31^\circ$.

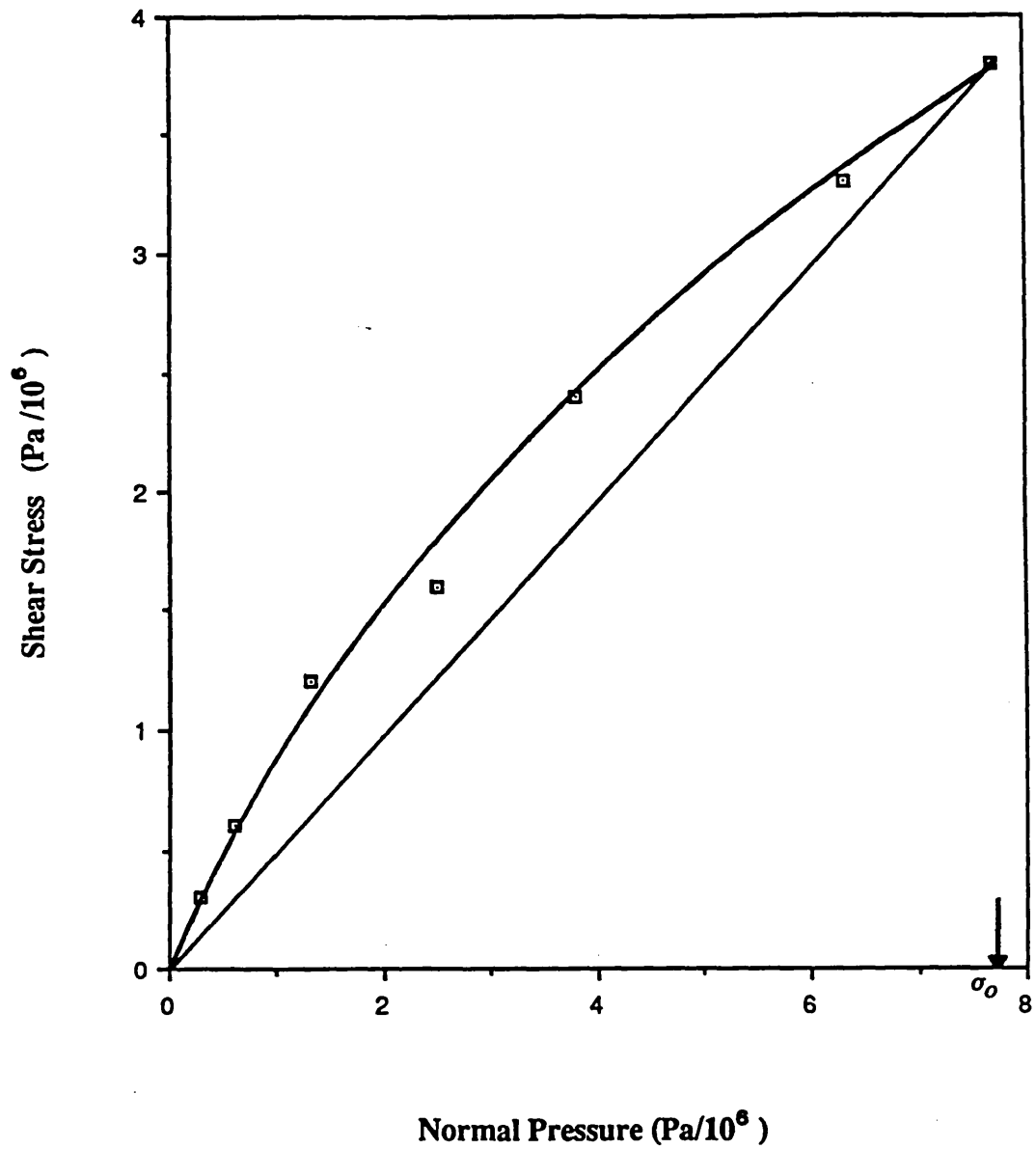


Fig.6.4 The yield locus for moistened maize powder at a pre-consolidation stress σ_0 of 0.77×10^6 Pa and a sliding velocity of 0.581 mm/min. The level of added moisture 16w/w. The effective angle of internal friction $\phi_e = 26^\circ$.

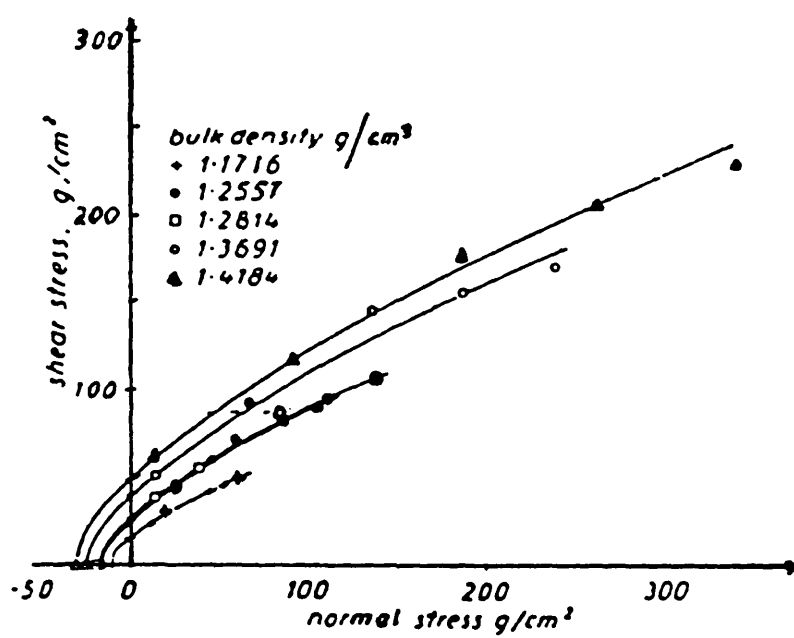


Fig.6.5 Yield loci of lithophane.(83)

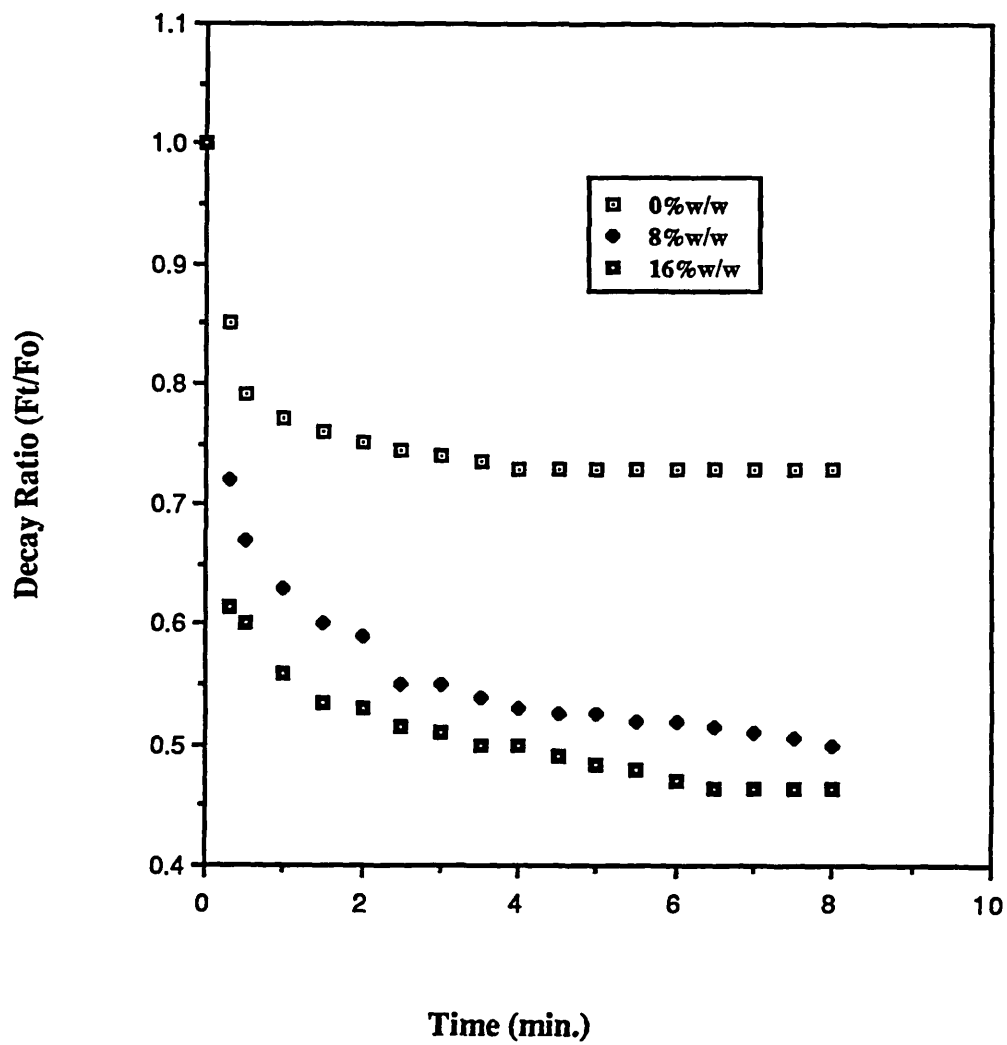
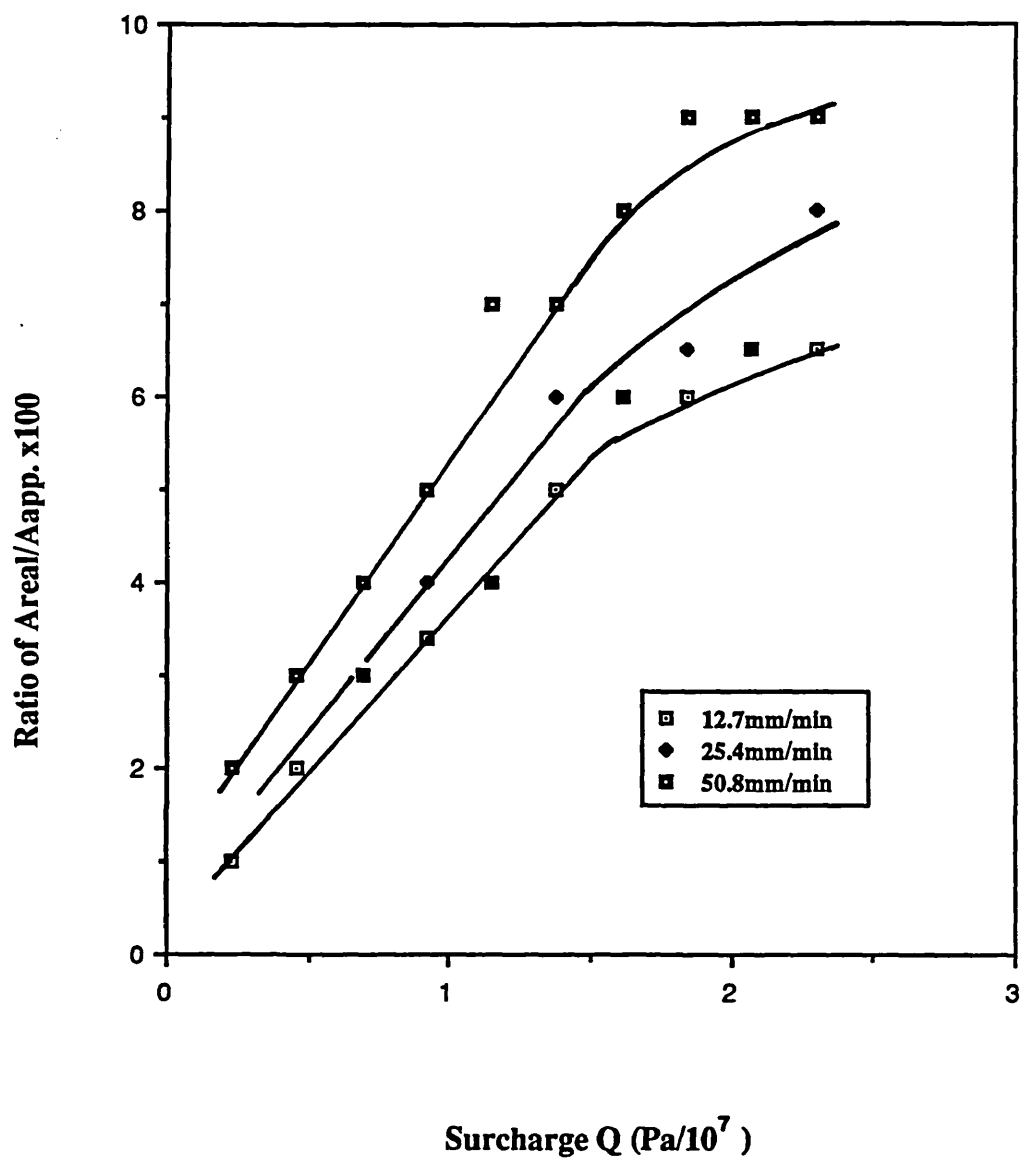
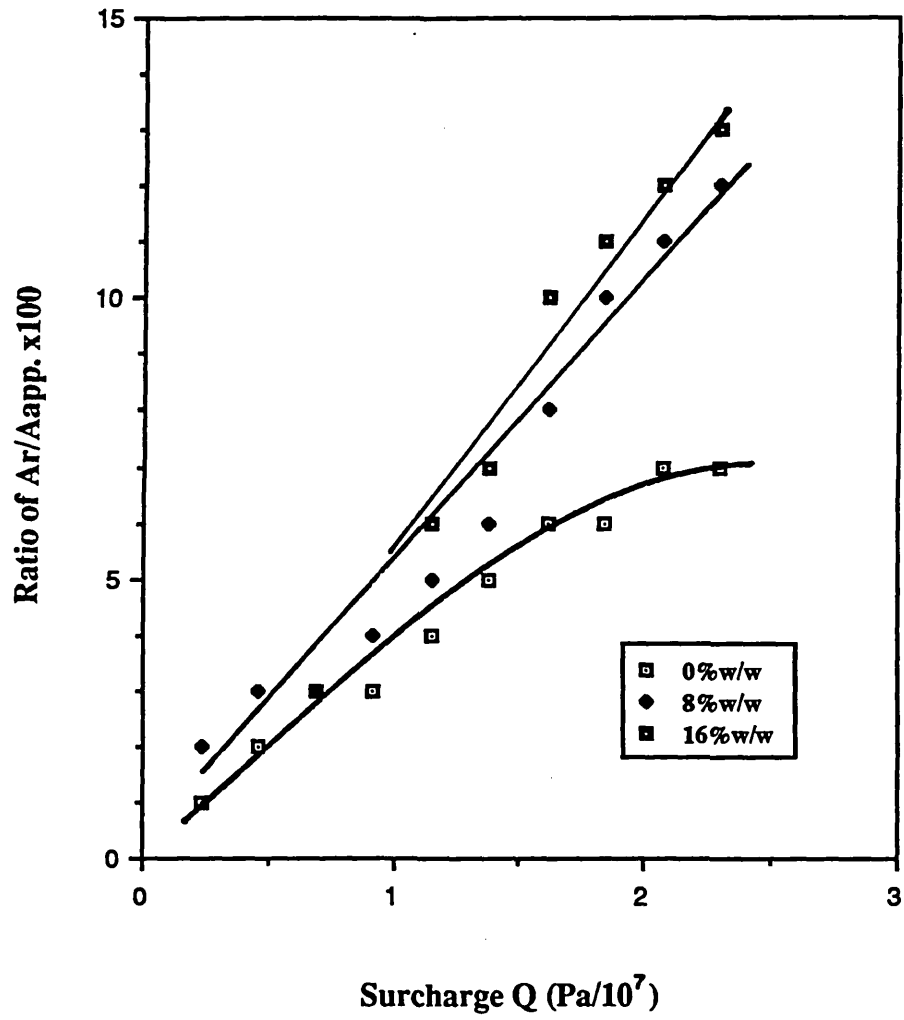


Fig.6.6 Stress relaxation data obtained for maize powder containing different levels of inherent moisture.

Fig.6.7 The ratio of real to apparant contact area as a function of the surcharge stress Q . (a). The effect of cross-head speed on ζ . (b). The effect of moisture on ζ .





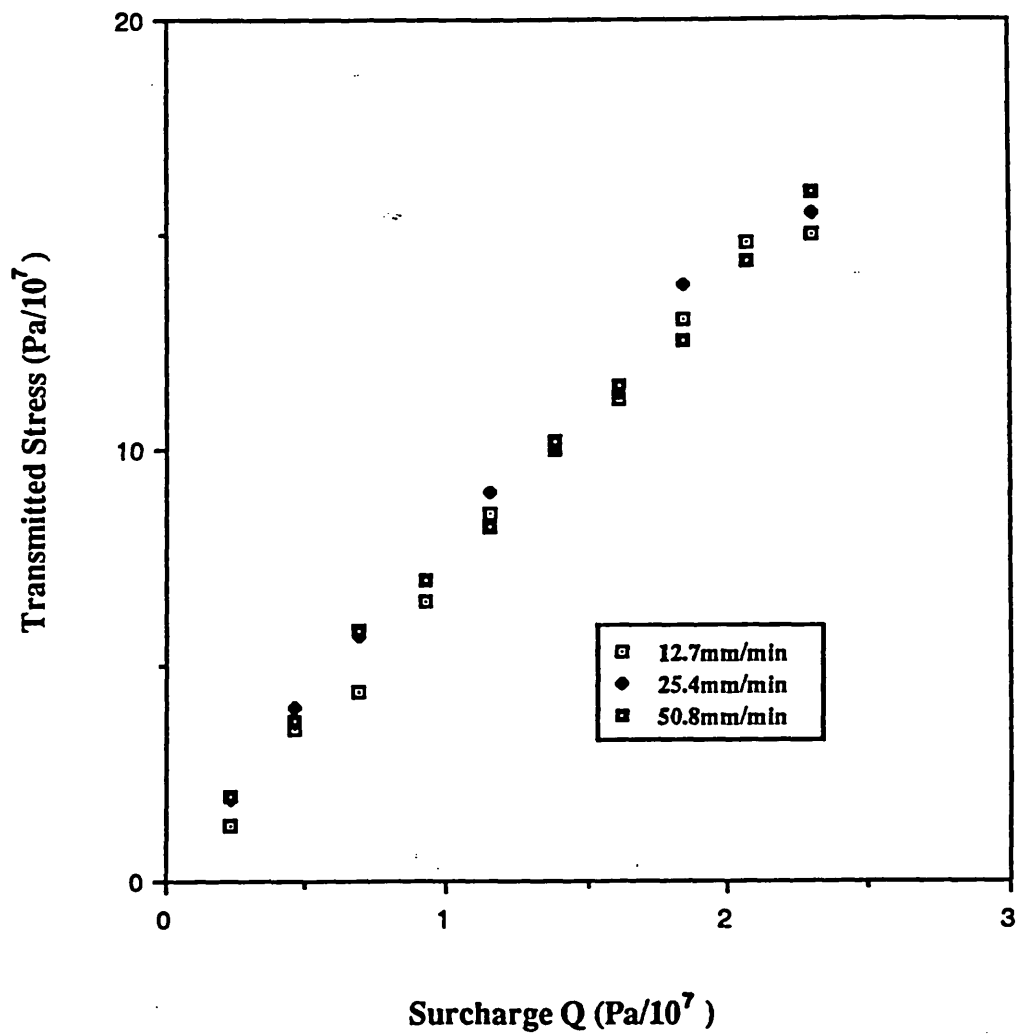


Fig.6.8 The effect of cross-head speed on the transmitted stress σ_{yy} for the unmoistened powder.

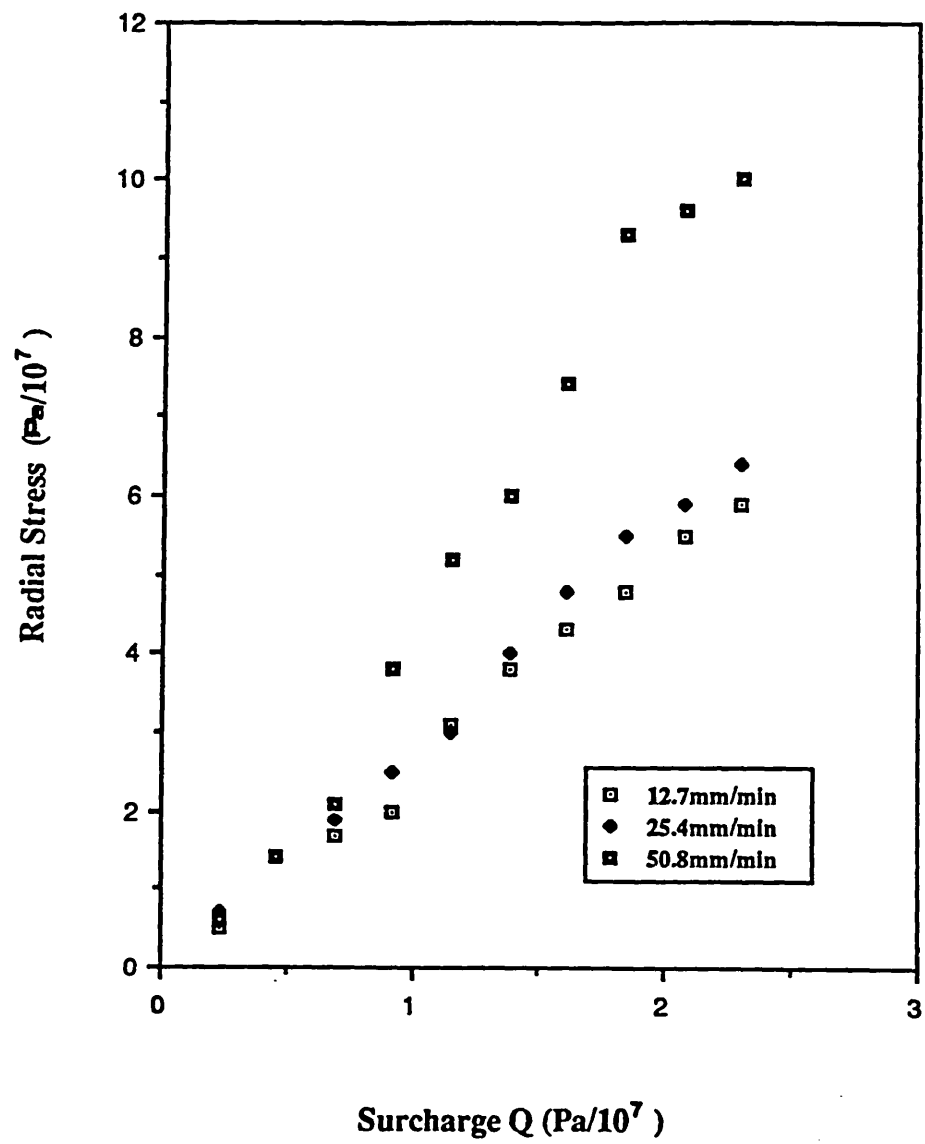


Fig.6.9 The effect of cross-head speed on the radial stress σ_{xxw} for the unmoistened system.

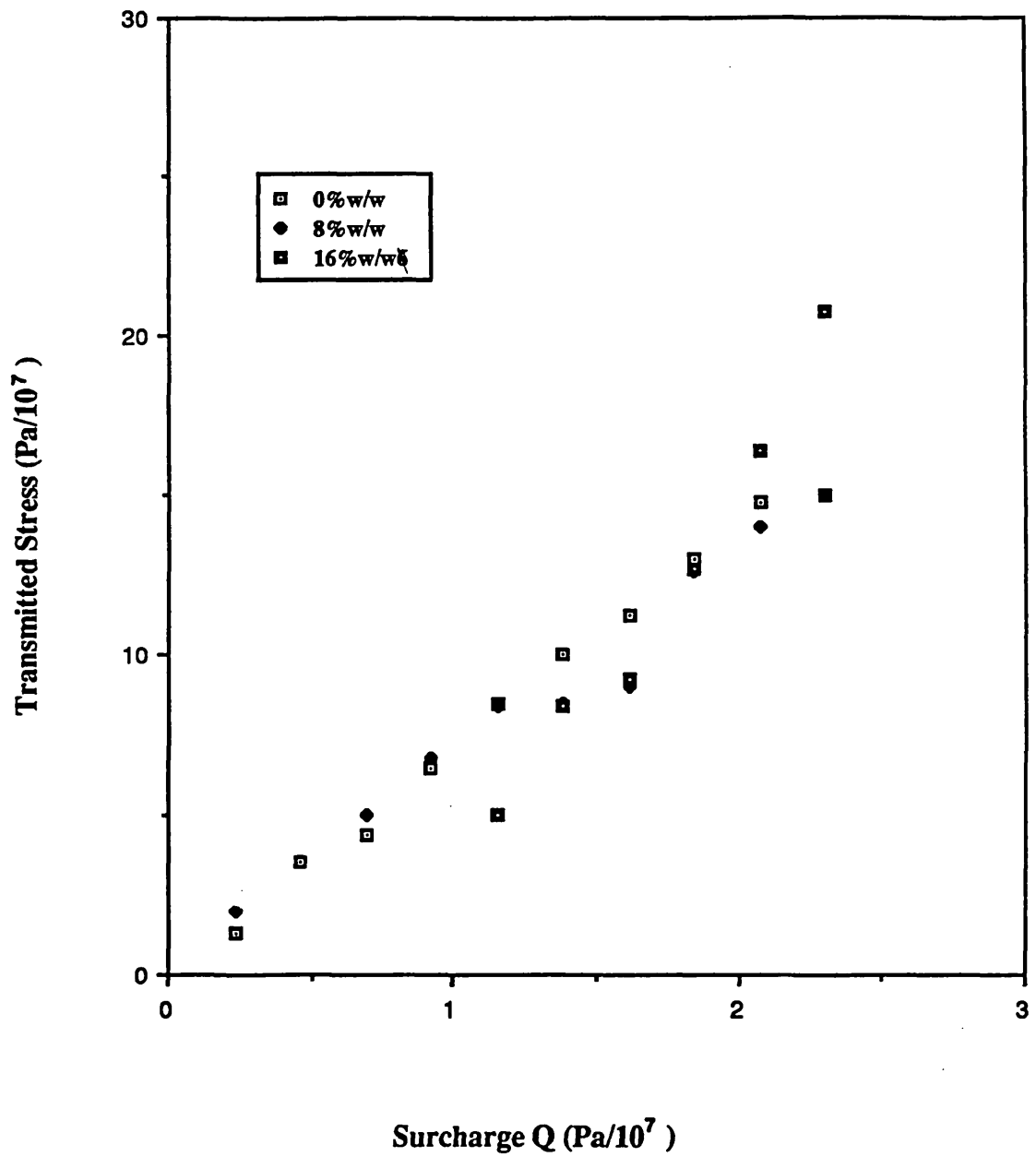


Fig.6.10 The influence of moisture on the transmitted stress σ_{yy} .

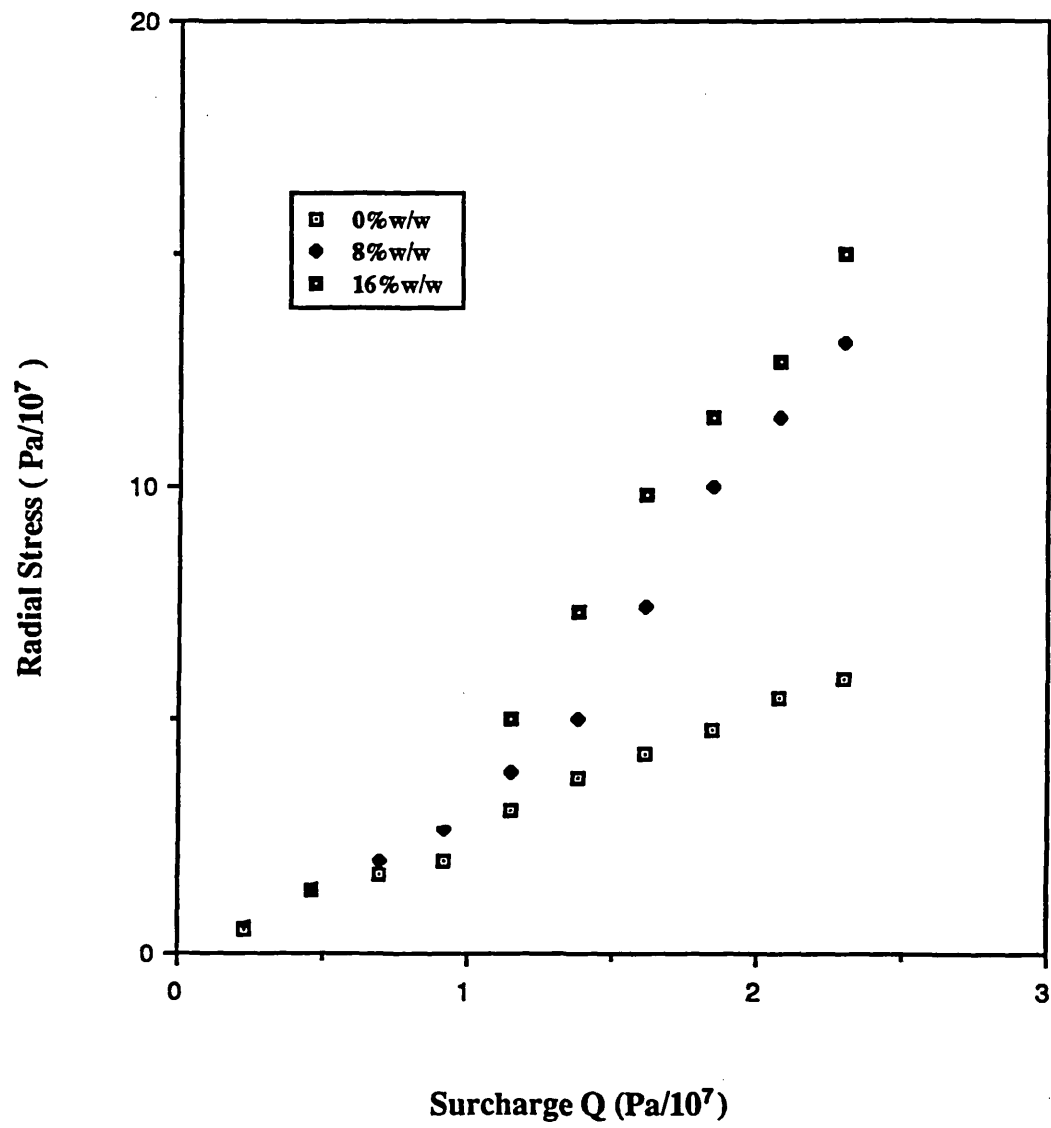


Fig.6.11 The influence of moisture on the radial stress

σ_{xxw} .

Chapter 7

Pressure Generation During Shear Flow in a Weissenberg Rheogoniometer

7.1. Introduction

The importance of interfacial frictional forces in controlling solids conveying has been discussed in earlier parts of this theses. The variation of the friction with the process parameters was reviewed in chapter 2, in the context of the friction of compacts. In these experiments a pre-determined normal load was applied to a pre-formed solid and the friction force generated was measured when the compact was slid against smooth counterfaces. In a conventional food processing extruder a shearing force is continuously exerted on the solid plug in the screw channel and a normal stress develops naturally at the walls. The initial section of the feed zone can be regarded as a "flow zone" where the particle assembly is relatively less compacted compared with the "compacted zone" simulated by compact friction studies described in chapter 5. The resulting normal forces generated in the "flow zone" are much lower and the effects of particle dilatancy may need to be considered. The comprehensive evaluation of the single screw feed zone mechanism therefore requires an additional study of the "flow zone" and in particular how the induced normal stress varies with the primary extrusion

variables, such as interface temperature, moisture and relative velocity. The experimental extruder itself is not suitable for this investigation as it would only enable the overall torque to be measured while the shearing forces in the individual sections would be unknown. Therefore the "flow zone" was simulated by using a Weissenberg Rheogoniometer which measures simultaneously the normal and shear forces generated during the shear flow of the maize granules. The sample was constrained in a cone and plate geometrical configuration and a range of normal pressures were generated by the introduction of mis-alignment into the cone and plate geometry. These data are then amenable to analysis using a combination of the adhesion model of boundary friction and the contact mechanical model appropriate to particle-wall contact developed in chapter 6.

This chapter presents data on the normal and shearing forces developed as a function of interface temperature, moisture, velocity and the mis-alignment of the shear plates. The appropriate frictional coefficients are then predicted via equation (6.12) and from an equation which will be developed which incorporates particle dilatancy effects during sustained flow. These predictions are then compared with the experimental data. The chapter further presents relevant theories on the effects of the geometrical factors, such as tilt and off-centre, on the torque measurements. The influence of grooves on the shear plates, on the generated forces is also described. The grooved plate systems correspond more closely to the experimental extruder which contains a grooved barrel.

7.2. *Normal force and torque measurements*

The normal force and torque generated for the unmoistened and moistened systems are shown in Figs. (7.1 - 7.8). The generated stresses were much larger for the grooved plate than for the corresponding case of the smooth plate. This is also true for the mis-aligned systems. The inference here is that the mis-aligned geometry favours material compaction to a greater degree than the aligned systems resulting in higher torques and normal forces. The presence of extensive wall slip in the smooth plate cases reduces the material compaction, thereby reducing the measured forces. Many workers⁽¹³¹⁾⁽¹³²⁾ have used serrated cones and plates in an attempt to eliminate wall slip, and these surface features have generally led to the generation of higher stresses. The effects of off-centre of the shear plates on the generated forces are more significant than those due to tilting. This is a result of the more complex flow geometry created in the gap when the axis of the cone is displaced. More compaction ensues and as the material is compacted a greater force is required to shear the powder or the powder wall interface. Therefore the apparent increase in torque with normal force is to be expected. Increasing the interface temperature increases the torque and normal force but this increase is more marked for the smooth plates. This may be a consequence of the different dissipation processes in the two cases. Smooth plates may promote metal-powder contacts and the powder-powder contacts may be favoured with the grooved plates. The rate of normal force and

torque generation increases rapidly for the smooth plates system at a temperature of approximately 120°C . In this temperature range starch is known to undergo a series of complex chemical reactions⁽¹⁰⁸⁾ including protein denaturation and gelatinisation. The more efficient rate of heat transfer between a powder-metal contact as opposed to a powder-powder contact will increase the rate of these reactions thereby accelerating the generation of normal force and torque in the smooth plate. The moistened system also shows similar trends to those observed when the temperature is increased but there is an anomaly in the response of the aligned plate. The normal force generated at low temperatures in the moistened smooth plate is less than the corresponding unmoistened system. At these temperatures the inherent water may act as a surface lubricant, thus reducing the normal force generated. There was no change however in the torque and normal force in the grooved plate cases with the addition of water.

Figs. (7.9) and (7.10) present data on the dependence of shear rate on the measured forces at ca 20°C . In these experiments it was not possible to simulate the range of velocities involved in extrusion since at higher rotational speeds the domineering effect of the centrifugal force field made it difficult to contain the sample within the cone and plate geometry. In this low shear rate regime the torque and normal force were independent of rotational speed. This was also found to be the case at elevated temperatures.

7.3 Basic theory for cone and plate arrangement

The available analysis of the torque - speed relationship which allows the computation of shear stresses and shear rates is based on the solution of the Navier-Stokes equations. A detailed description of the solution of these equations with appropriate boundary conditions for a cone and plate geometry has been given by Bird et al⁽¹³³⁾. In the solution of these equations the flow is assumed to be steady, laminar and incompressible. Further the contribution from body forces are neglected. The torque Tr , transmitted by the material to the cone is given by

$$Tr = \frac{2\pi R^3 \sin^2 \alpha_c \tau}{3} \quad (7.1)$$

where α is the cone angle, R the radius of the plate and τ the associated shear stress. For small cone angles $\sin^2 \alpha_c$ will be very nearly unity and the shearing stress τ is equivalent to:

$$\tau = \frac{3}{2} \frac{Tr}{\pi R^3} \quad (7.2)$$

The computed shear stress τ is therefore independent of the radial position and does not involve any parameter describing the nature of the test sample. By using a more rigorous analysis Cheng⁽¹³⁴⁾ has shown that equation (7.2) is valid for most practical purposes when cone angles of less than 4° are used. Equation (7.2) has been used to compare τ for the present data in

spite of the fact that some of the relevant assumptions contained in its derivation are not appropriate for the current species.

7.4. *The effect of mis-alignment on the torque generated*

The solution of the Navier-Stokes equations in deriving an expression for the torque generated assumes perfect alignment of the cone and plate. However there is appreciable scope for generating mis-alignment in the setting up of the cone and plate geometry. It is therefore appropriate to elude to the effects of tilt, concentricity and gap setting on the torque measurements.

If the cone axis is tilted at an angle φ to the plate axis, the torque is expected to multiply by a factor:

$$\left[1 - \frac{\varphi^2}{\alpha^2}\right]^{1/2} \quad (7.3)$$

and if the cone axis though normal to the plate is displaced from the axis of the plate by a distance χ from the plate axis the torque is multiplied by a factor:

$$\left\{1 + \frac{7\chi^2}{3R^2}\right\} \quad (7.4)$$

A third type of error often occurs in incorrect gap setting. If gap setting error $\delta = \frac{H}{R\alpha}$, the generated torque is multiplied by a

factor:

$$\left[1 - \frac{3}{2} \delta \right] \quad (7.5)$$

The theoretical derivations of the above equations have been given by Markovitz⁽¹³⁵⁾ and they are based on Newtonian flow.

Further, the normal forces may also be affected by mis-alignment of the cone and plate⁽¹³⁶⁾ and by secondary flows⁽¹³⁷⁾⁽¹³⁸⁾⁽¹³⁹⁾ induced by the combined effects of the centrifugal-force field and elastic forces. In addition, the normal force measurements are directly affected by the imperfections and improper positioning of the transducers. However, no analytical solution is available in order to assess the magnitude of these errors. Equations (7.3), (7.4) and (7.5) were used to compute the increase in torque in the mis-aligned experiments. The measured increments in the torque measurements do not correspond with those predicted from the given equations. The observed increases were greater.

7.5. Data Analysis

In order to interpret these data in terms of the adhesion model of boundary friction and the contact mechanical model presented in chapter 6, the measured forces must be converted to shear stresses and normal pressure respectively. The torque was converted to shear stress by means of eq. (7.2) and the normal

force was divided by the area of the shear plates to convert it to normal pressure. These derived data are plotted in Figs. (7.11 - 7.14) for the grooved plate. A small increase in normal pressure and shear stress with temperature was observed with the exception of the aligned moistened system. In this latter system the observed normal pressure and shear stress increased by a factor of 8 and 2 respectively. This may be a result of starch gelatinisation⁽¹⁰⁸⁾ in the presence of excess moisture. However, this trend is not observed with the mis-aligned experiments. The response of the normal pressure and shear stress with increasing temperature in the smooth plate is greater than the corresponding grooved plates for both the unmoistened and moistened systems, (Figs. 7.15, 7.16) respectively. The improved rate of heat transfer for the metal-powder contact cited earlier, increase the rate of gelatinisation reactions and may thereby increase the degree of compaction and hence the measured stresses.

The response of the coefficient of friction (computed from the ratio of shear stress and normal pressure) to temperature is a reflection of the relative changes associated with the normal pressure and shear stress. In the grooved plate systems (Figs. 7.17 - 7.18), the change in normal pressure and shear stress with increasing temperature was small and therefore no significant change in the coefficient of friction was noted. There is one exception however for the aligned geometry where a decrease in the coefficient of friction was observed.

In both the unmoistened and moistened systems the coefficient of friction was found to be largest for the aligned geometry. The

normal pressures generated in this geometry are relatively small and therefore higher frictional coefficients are to be expected (eq. 6.12) for elastically deforming asperities. No significant change in the coefficient of friction was observed with the addition of water for the grooved plates but the mean coefficient of friction was found to increase with moisture content for the smooth configuration plate. Figs. (7.19 - 7.20) presents a comparison of data on the smooth plate, grooved plate and the compact friction data reported in chapter 5. These changes follow the trends expected with the change in mean pressure. The experiments conducted with the grooved plates are in some respects analogues to the "shear test" experiments reported in chapter 6. However, the effective coefficient of friction μ_E is much larger than the coefficient of friction for the grooved plate experiments. This may be a result of the higher shear rates associated with the "shear test" experiments and the fact that some wall slip occurs for the grooved plate geometry.

The derived experimental data obtained have been compared with the predictions from equation (6.12). These predictions together with the experimental data are plotted in Figs. (7.21 - 7.30) for the different systems studied. In order to use equation (6.12) to compute the coefficient of friction a knowledge of the particle moduli E_p is required. This was computed from the functionalities listed in Table 5.2. In order to use these functionalities the mean strain was estimated as the deflection of the normal force spring divided by the initial gap separation. The agreement between theory and experiment was found to be within

a factor of 2 for the unmoistened system. This difference in the experimental and the predicted values of the frictional coefficient can be attributed to the inaccuracies associated with the estimation of the modulus E or the neglect of potential dilatancy effects experienced by particles in a dynamic stress field. If the material is closely packed so that voidage effects are negligible, the material response can be predicted from a model based on continuum mechanics (eq. 6.12). However, during sustained flow of a less dense phase, particle dilatancy can no longer be neglected. Any analytical model describing the behaviour of particles in this low stress field must take into consideration dilatancy and its influence on the resulting stress field. These effects are considered in detail in a recent publication by Tuzun *et al*⁽¹⁴⁰⁾. These authors consider an assembly of closed packed spheres in contact with a flat wall. The resulting expression for the coefficient of wall friction for materials with a linear interfacial shear-stress - normal pressure functionality described by equation (2.5) is

$$\mu_w = \pi \tau_o \left[\frac{D_c R_c}{S} \right]^{2/3} \frac{1}{p_{app}^{1/3}} + \alpha \quad (7.6)$$

The corresponding expression for the present system with an exponential correlation between shear stress and normal pressure

(eq. (2.7)) is

$$\mu_{\omega} = \pi \tau_0' \left[\frac{D_c R_c}{S} \right]^{2/3} \frac{1}{p_{app}^{1/3}} + \alpha' \tau_0' \quad (7.7)$$

In these equations the local voidage parameter S is defined as the distance between two point contacts at the wall and is given by

$$S = (2 + \beta)R_c, \quad \text{where } 0 \leq \beta \leq 2.$$

In the most dilated state the ratio $\frac{S}{R_c}$ is equivalent to $3^{(140)}$, and this corresponds to a local bed porosity of 0.45. Equations (6.12) and (7.7) predict that the coefficient of friction is a function of the normal pressure for elastically deforming hemi-spheres on a flat smooth surface. The predictions from equation (7.7) are in better agreement with experiment for the unmoistened systems (Figs. 7.21 - 7.25). In these predictions the value of S was equated to $3R_c$. Further, in these analyses the suitability of a mean modulus extrapolated from the compressive testing of pre-formed maize cylinders for the prediction of friction coefficients in a less dense system is most questionable. In this low density regime the appropriate moduli would be the particle modulus E_p , which would nominally be higher than those computed from the functionalities listed in Table (5.12). As there seems to be no way of accurately establishing the magnitude of the particle modulus E_p an optimised value of the modulus E was used to give the best fit of the experimental data in the normal

pressure range. The theoretical predictions from eq. (7.7) with the optimised moduli for the unmoistened systems are also shown in Figs. (7.21 - 7.25). These indicate the sensitivity of the modulus E_p to the theoretical predictions. For the moisture system the errors involved in the estimation of moduli from compressive testing are greater. The predictions from eq. (7.7) were a factor of 10 greater than those measured experimentally. Therefore an optimised modulus was used to predict μ_w from eq. (6.12) and (7.7). These predictions together with experimental measurements for the moistened systems are indicated in Figs. (7.26 - 7.30). The improvements shown by including particle dilatancy (eq. 7.7) is less marked in this latter system. The addition of moisture swells the particle thereby reducing the separation between two neighbouring contacts. Therefore a theoretical prediction based on continuum mechanics is more appropriate for the moistened systems. The relative change in the experimental value of the coefficient of friction with normal pressure is typical of an elastically deforming contact. This change in μ with contact pressure for the wet powder was less, especially at elevated interface temperatures. This is in agreement with the findings obtained from the compact friction data (chapter 5) where the addition of moisture resulted in a load index n closer to unity as the higher order cases of the multiple asperity models were approached. In the experiments reported here it was difficult to establish ^{the} load index n but an average value near to $2/3$ was observed for the unmoistened system whilst a value nearer to unity was found for the wet powder. Figs. (7.31) and

(7.32) plot the average experimental values of the coefficient of friction as a function of the reciprocal of the mean pressure. At high normal pressures the compact friction data also falls on the same curve.

The mean shear stress - contact pressure data (Fig. 7.33 and 7.34) is adequately described by the exponential functionality (eq. 2.7) established during thin film interface shear experiments reported in chapter 5, with an average coefficient of correlation of 0.99. However the τ_0' and α' values in the thin film experiments were 4 orders of magnitude greater; this is possibly a result of the different interface rheology associated with the prolonged sliding in these experiments as opposed to a film deposited from a compact specimen where little sliding was introduced.

7.6. *Concluding Remarks*

In these experiments a range of normal pressures were developed by the mis-alignment of the flow geometry. It has been possible to simulate the "flow zone" in extrusion within the low stress field generated in this work. The coefficient of friction was found to be a strong function of the normal pressure and this vindicates the use of the "adhesion model of friction" and the "Hertzian contact mechanics" appropriate to elastically deforming hemi-spheres on a flat surface to analyse the data obtained. The influence of particle dilatancy on the theoretical predictions has

been discussed. However it is not possible to establish if the inclusion of dilatancy has improved the predictions since the uncertainties associated with the estimation of E_p and R_p undermine the value of this exercise. A sensitivity analysis of eq. (7.12) indicates that if the optimised particle moduli E_p is increased by a factor of 3, without the inclusion of the local voidage parameter S , the predictions would give an equally good description of the experimental data. Therefore it follows that the governing parameter for the accurate prediction of frictional coefficients is the modulus E and the errors involved in its estimation are significant. However, during sustained flow particles do experience wall dilatancy and thus the inclusion of this effect is appropriate.

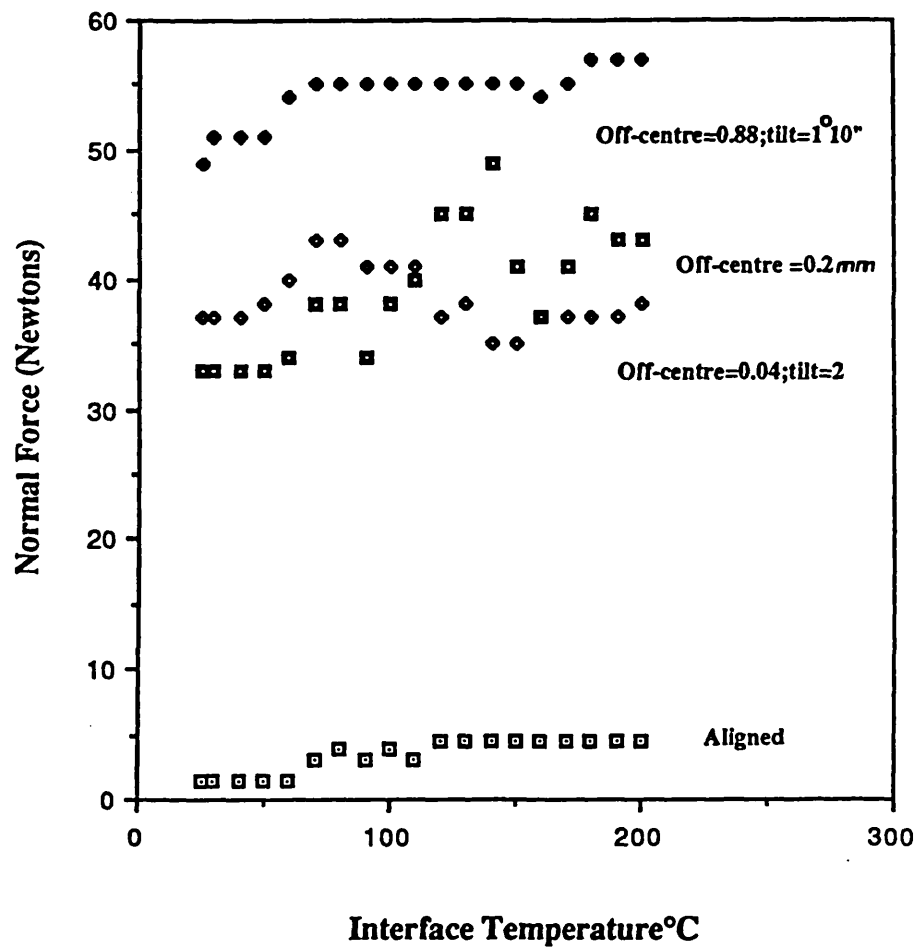


Fig.7.1 The influence of interface temperature on the generation of normal force for the grooved plate system. The powder does not contain any added moisture.

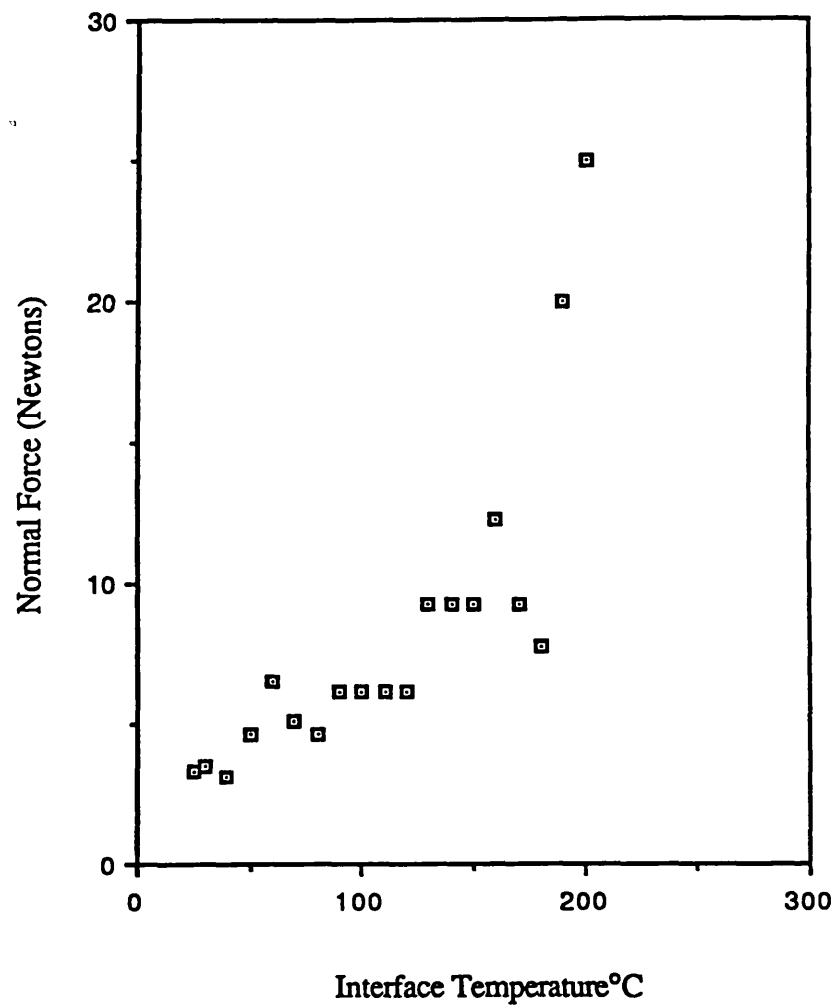


Fig.7.2 The influence of interface temperature on normal force generation for the smooth plate. The powder does not contain any additional moisture.

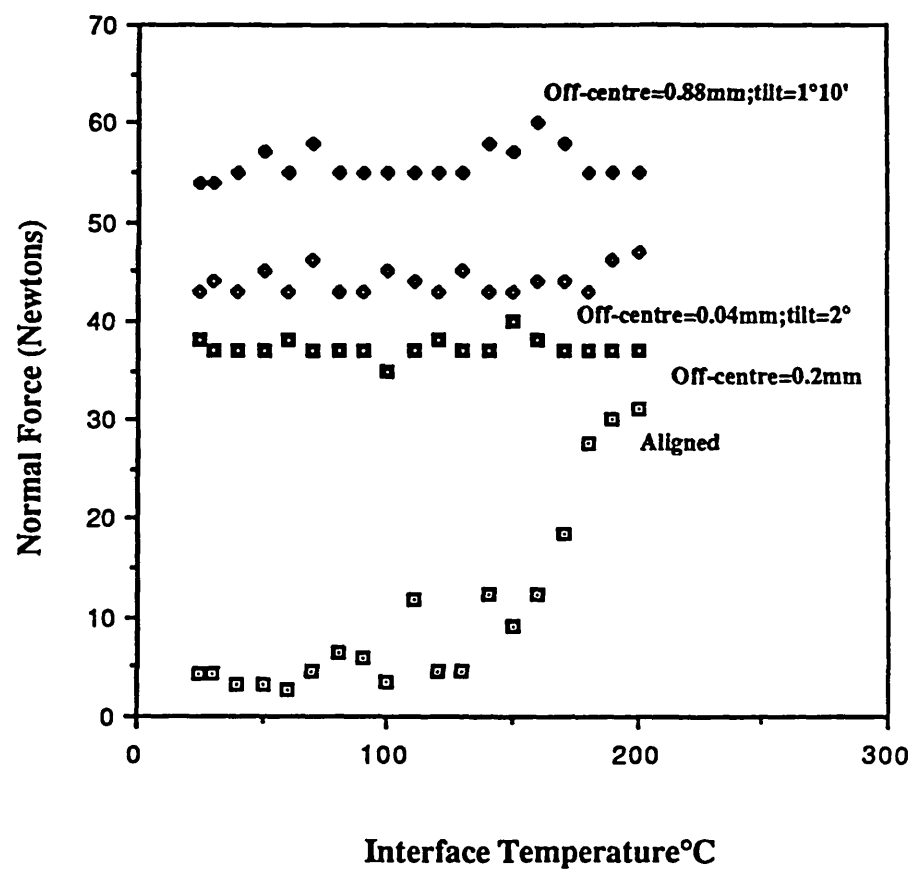


Fig.7.3 The influence of interface temperature on normal force generation for the grooved plate. The powder contains 16% w/w additional moisture.

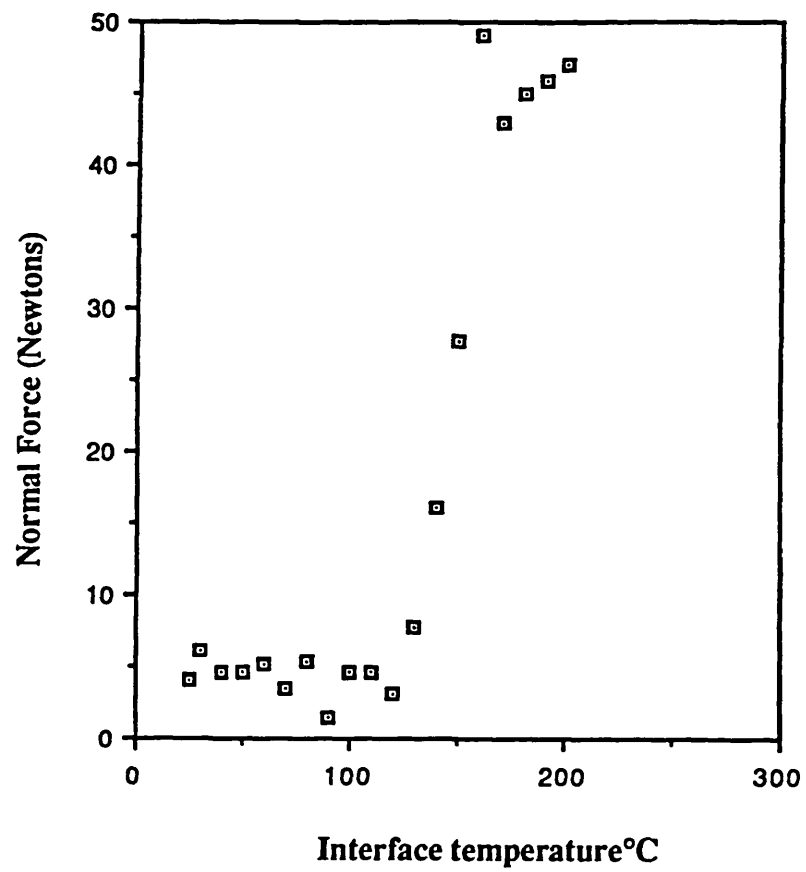


Fig.7.4 The influence of interface temperature on normal force generation for the smooth plate. The powder contains 16% w/w additional moisture.

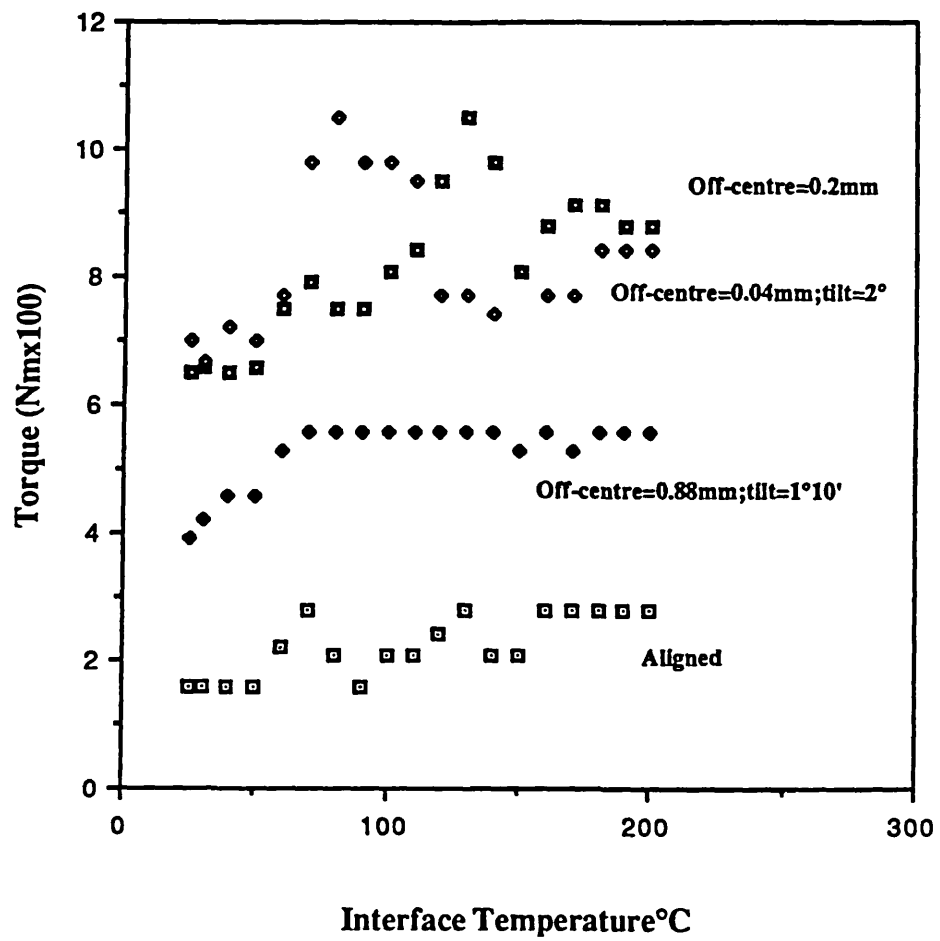


Fig.7.5 The influence of Interface temperature on torque generation for the grooved plate. The powder does not contain any additional moisture.

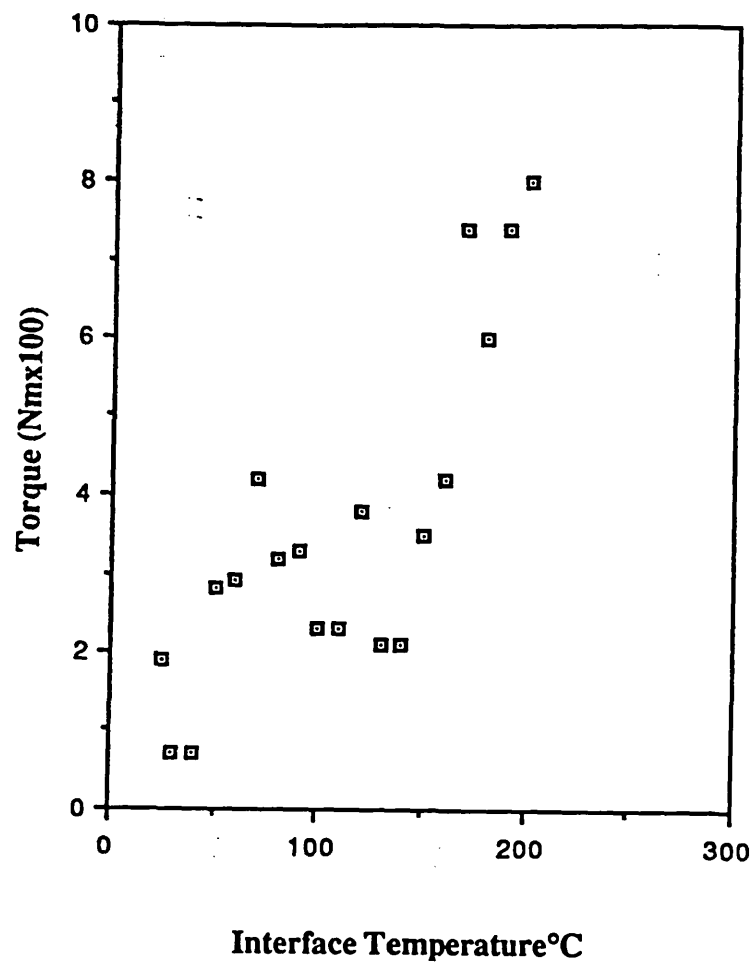


Fig.7.6 The influence of interface temperature on torque generation for the smooth plate. The powder does not contain any additional moisture.

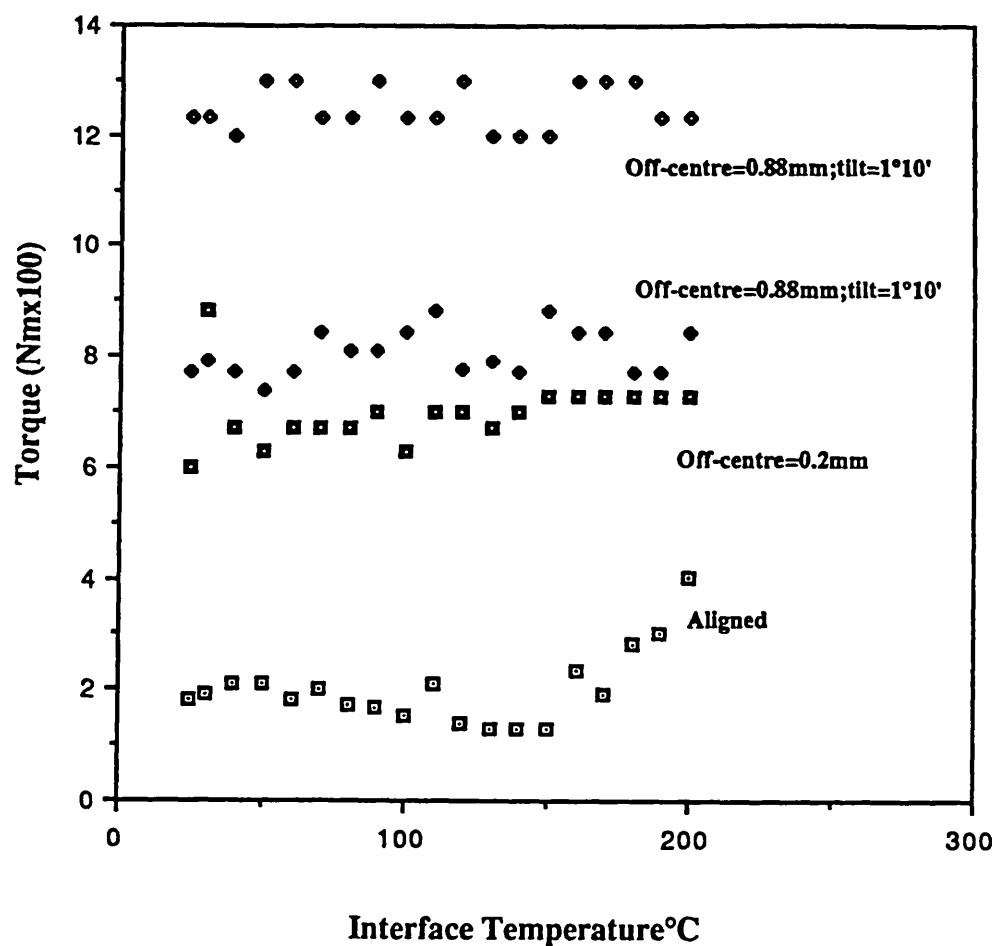


Fig.7.7 The influence of interface temperature on torque generation for the grooved plate. The powder contains 16% w/w additional moisture.

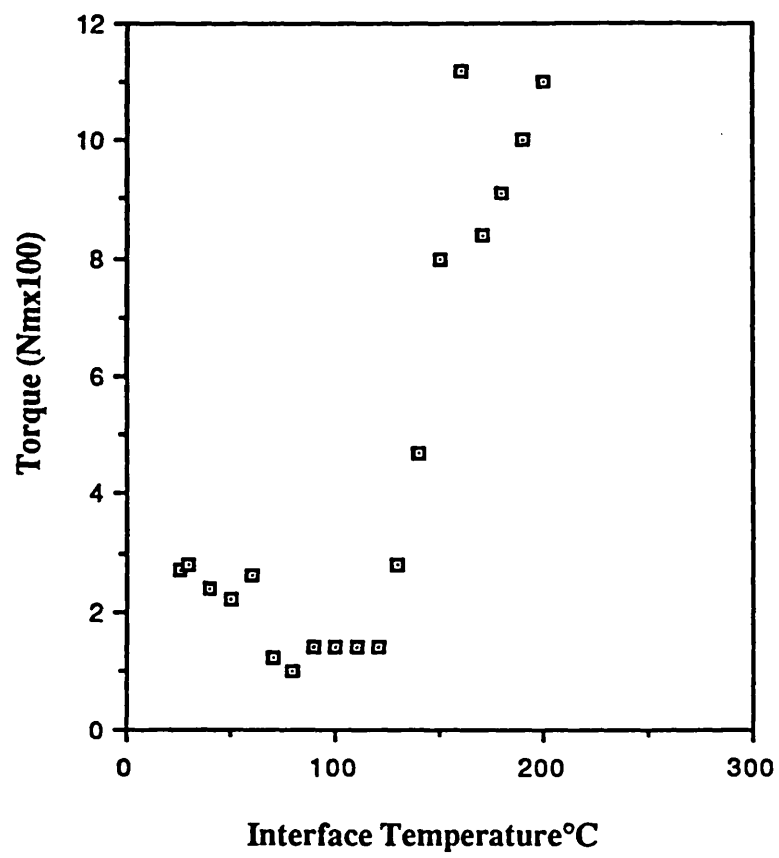


Fig.7.8 The influence of interface temperature on torque generation for the smooth plate. The powder contains 16% w/w additional moisture.

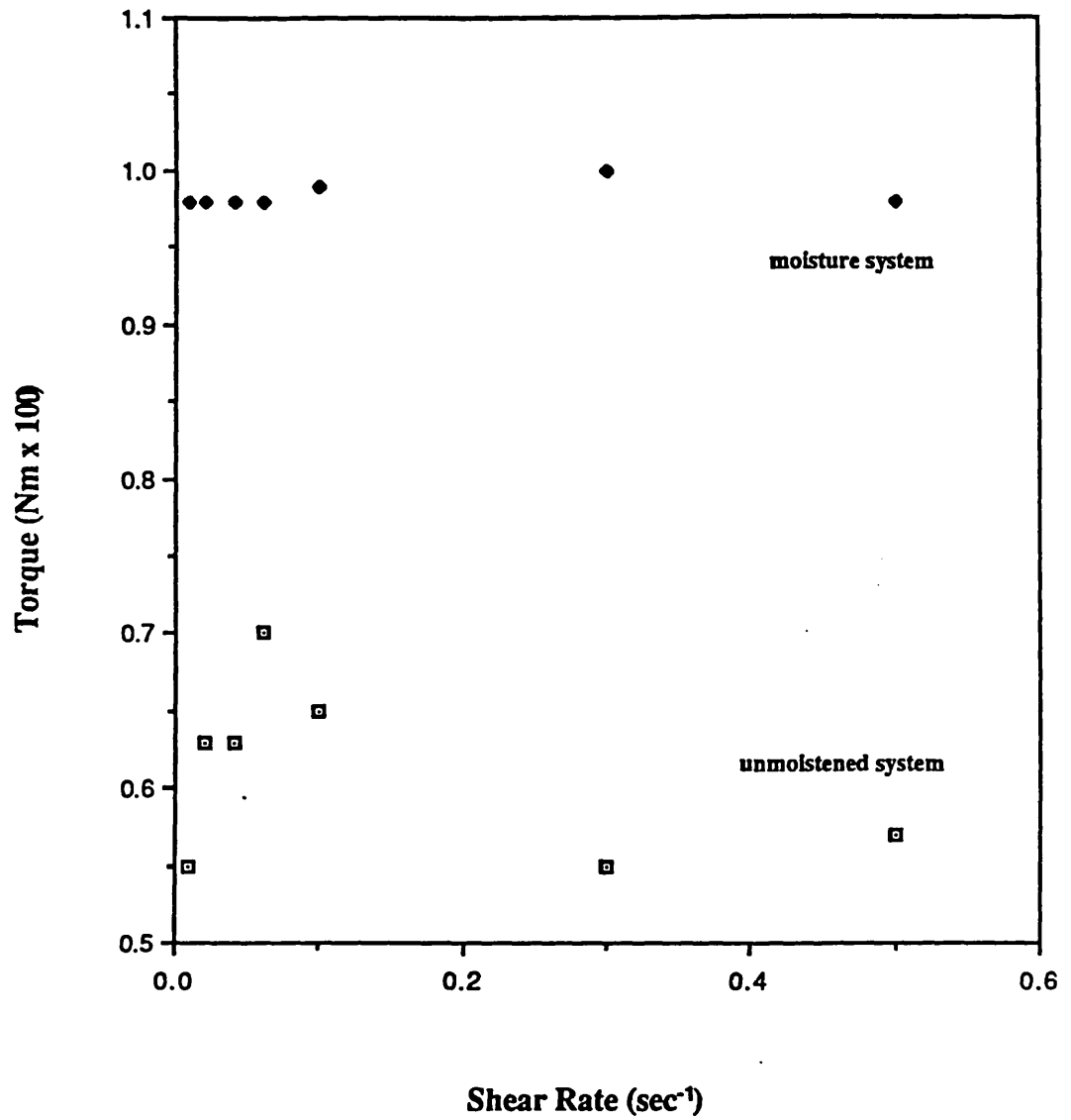


Fig.7.9 The torque generated during shear flow of maize granules as a function of shear rate.

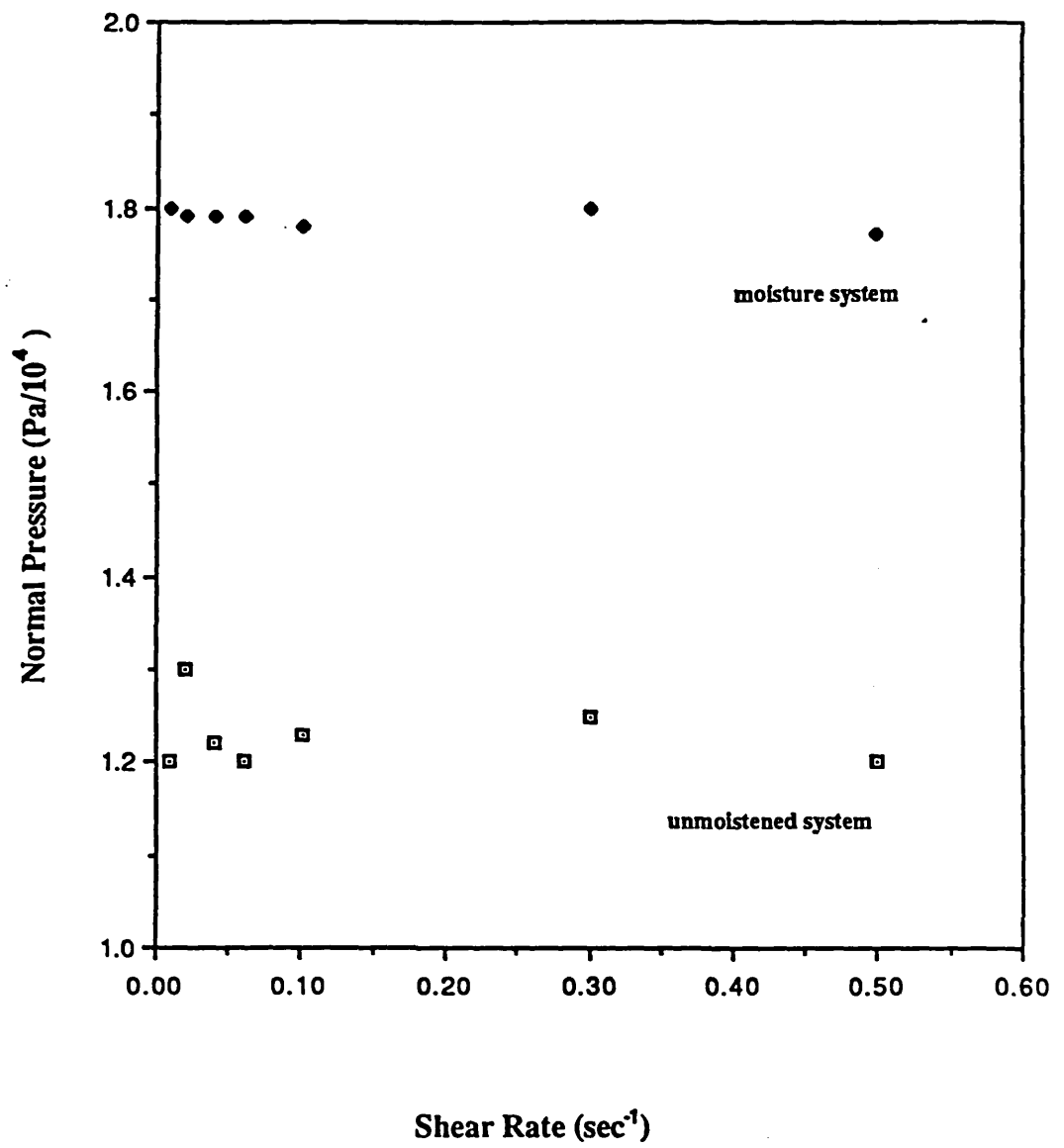


Fig.7.10 The normal pressure generated during shear flow of maize granules as a function of shear rate.

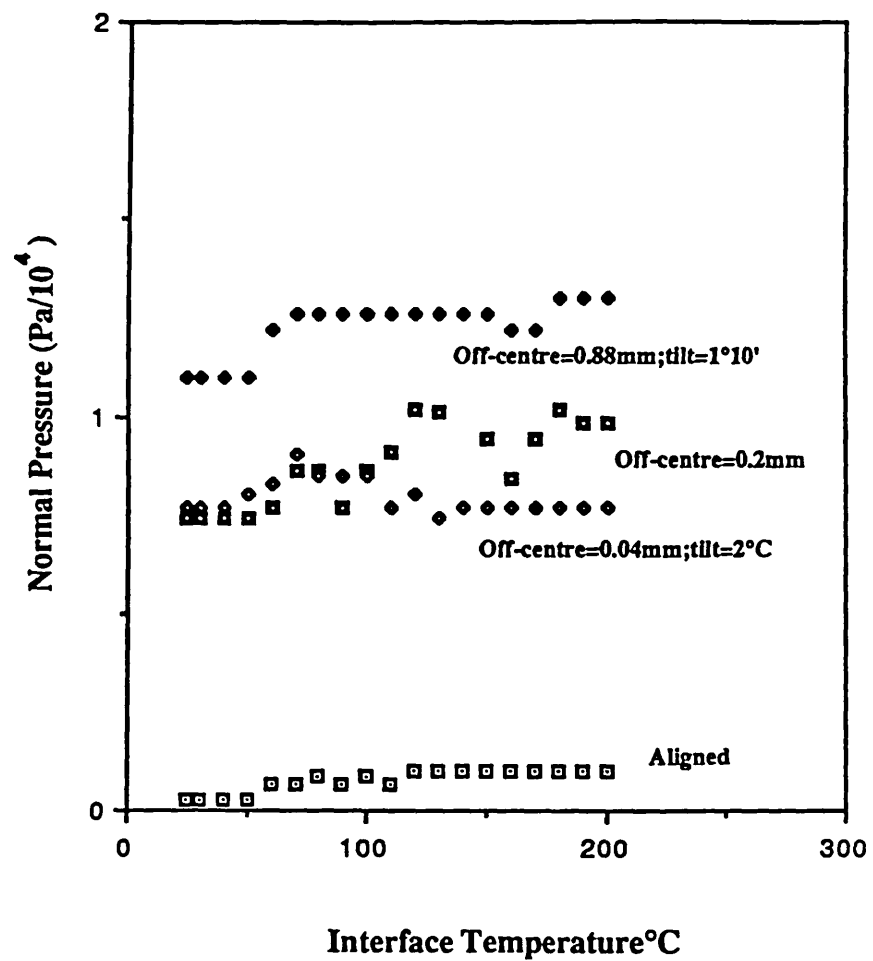


Fig.7.11 The effect of interface temperature on the normal pressure generation for the grooved plate. The powder does not contain any additional moisture.

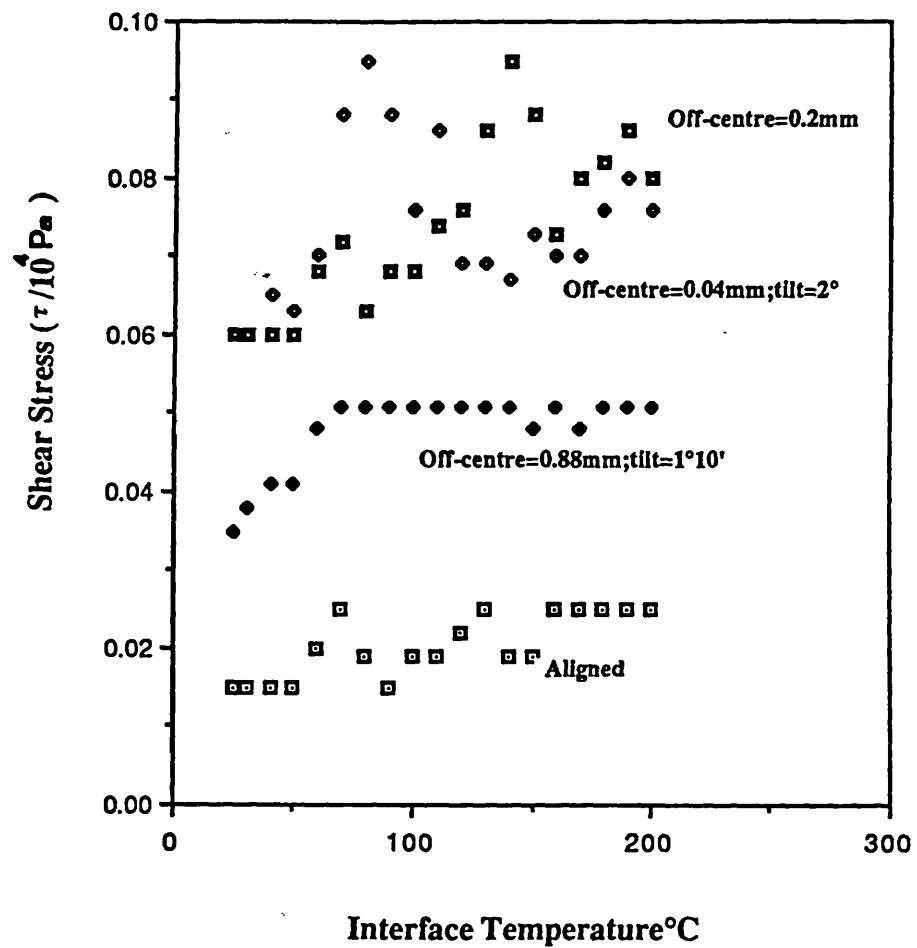


Fig.7.12 The effect of interface temperature on the shear stress for the grooved plate. The powder ^{does not} contain 16% w/w additional moisture.

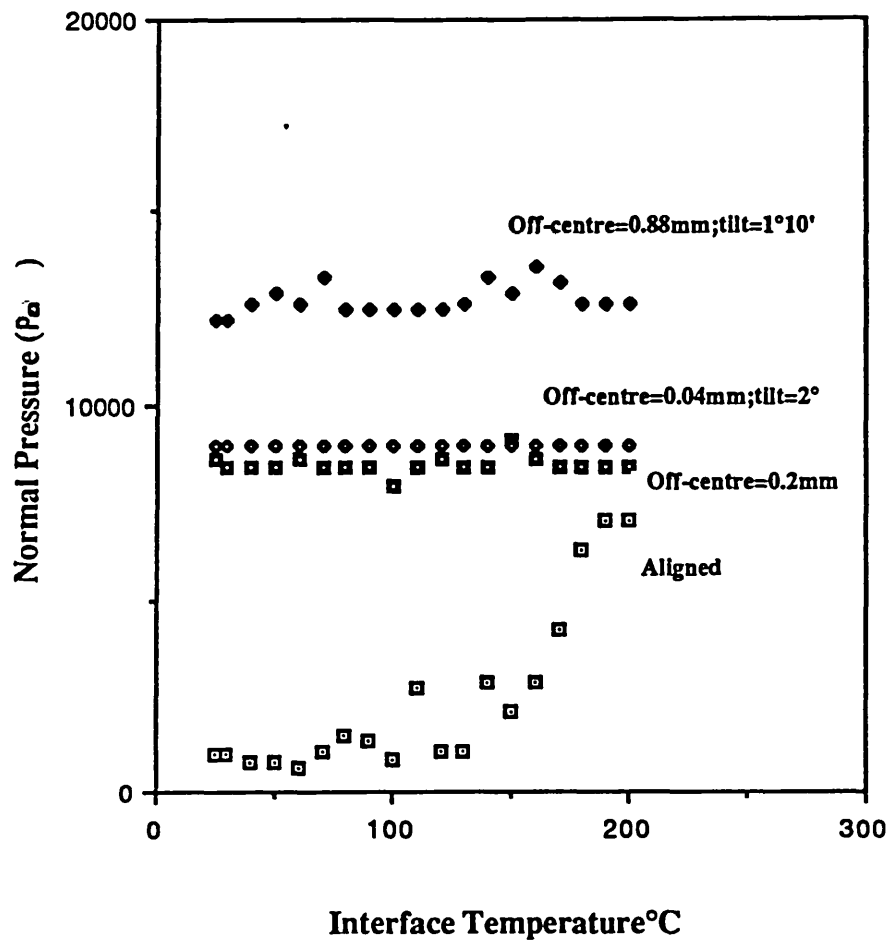


Fig.7.13 The influence of interface temperature on the normal pressure generation for the grooved plate. The powder contains 16% w/w additional moisture.

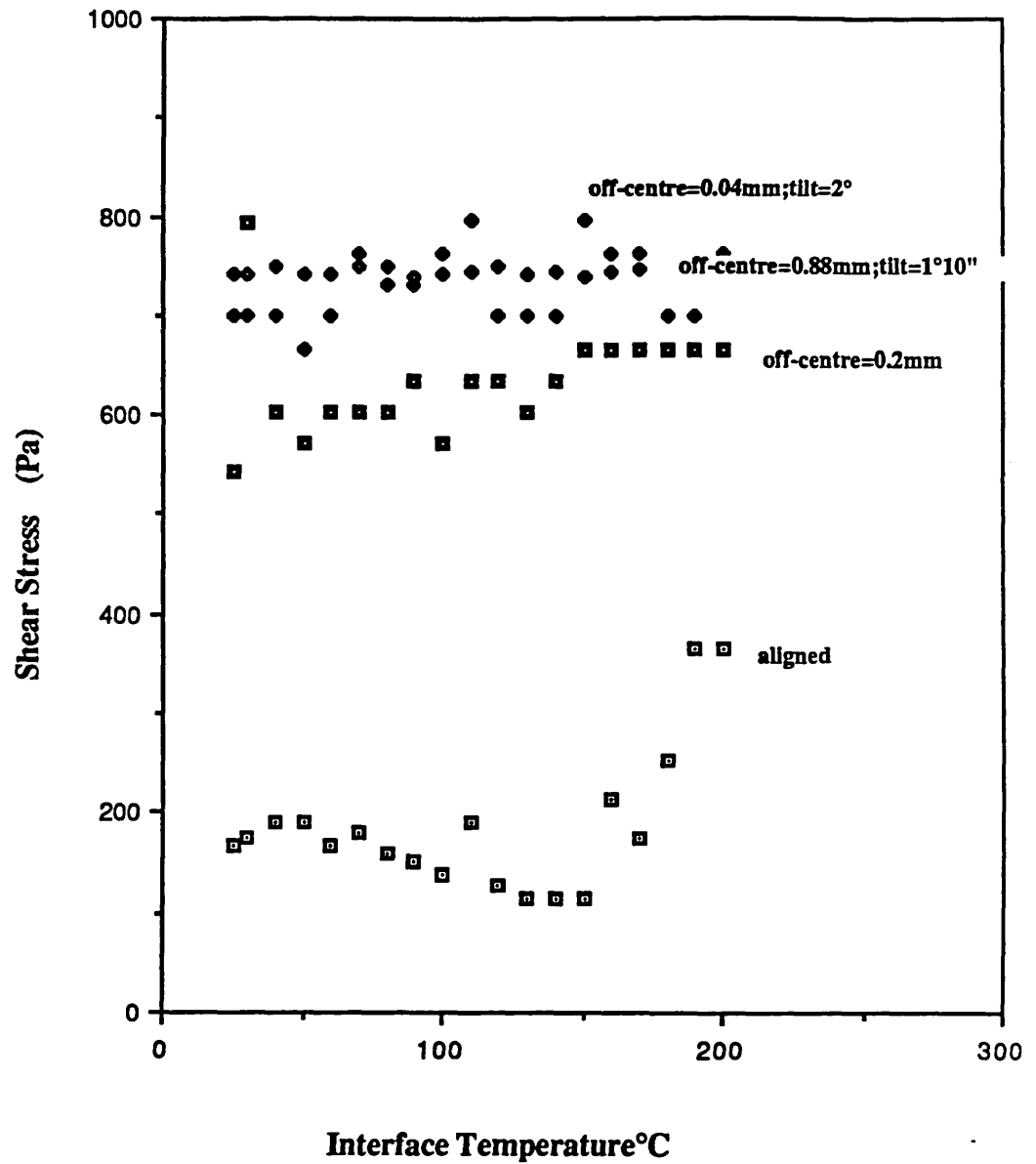


Fig.7.14 The influence of interface temperature on the shear stress generation for the grooved plate. The powder contain 16% w/w additional moisture.

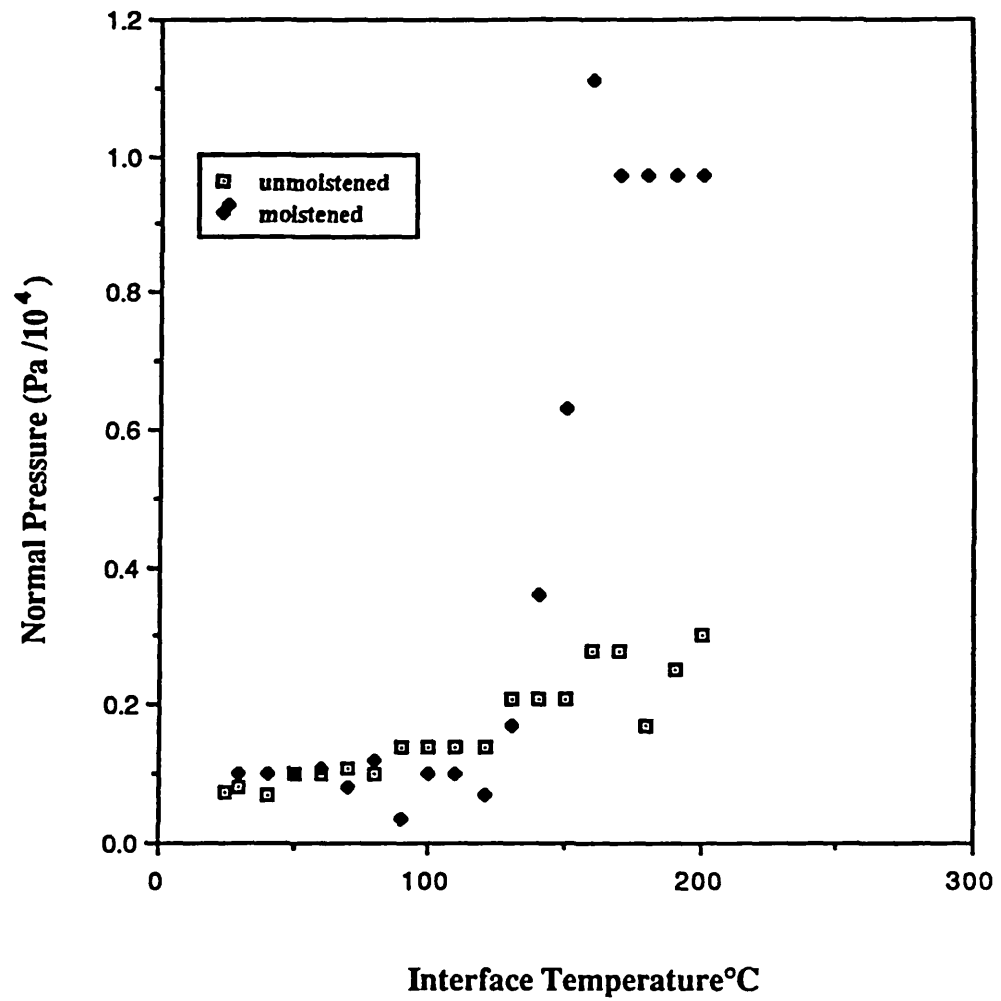


Fig.7.15 The Influence of interface temperature on the normal pressure generation for the smooth plate.

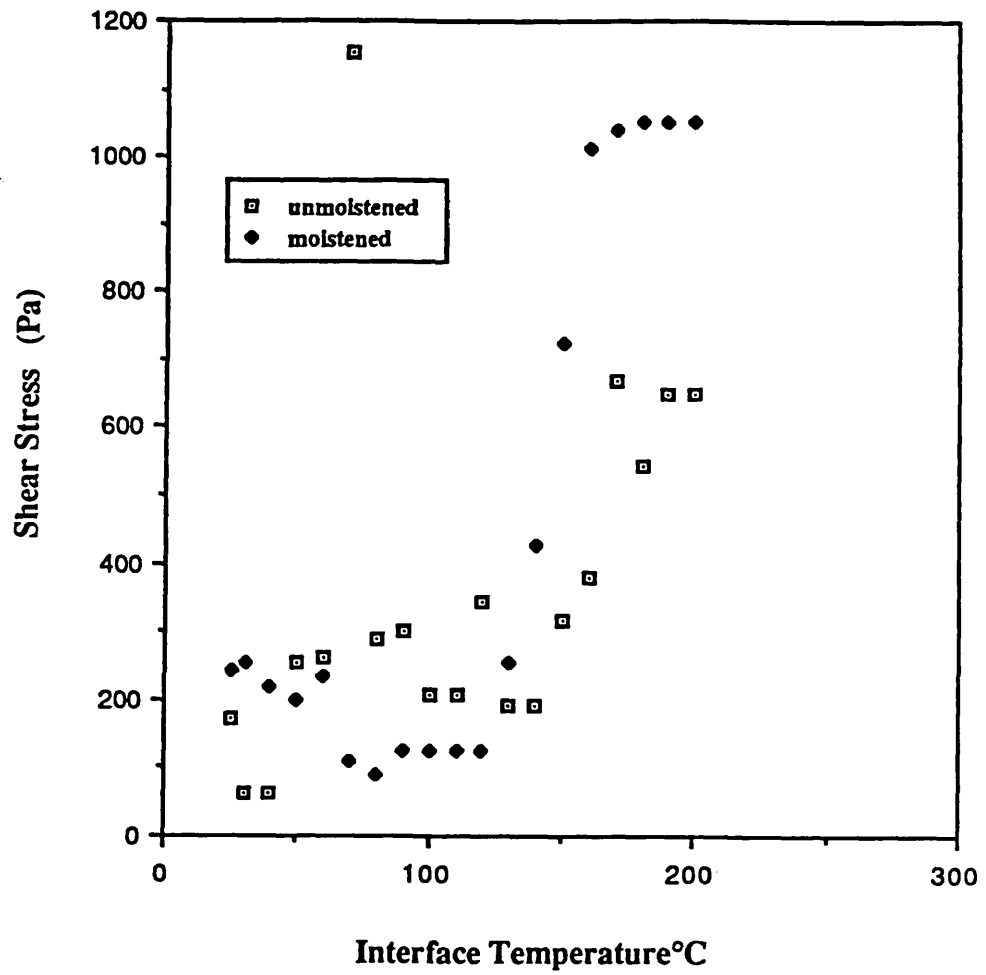


Fig.7.16 The influence of interface temperature on the shear stress for the smooth plate.

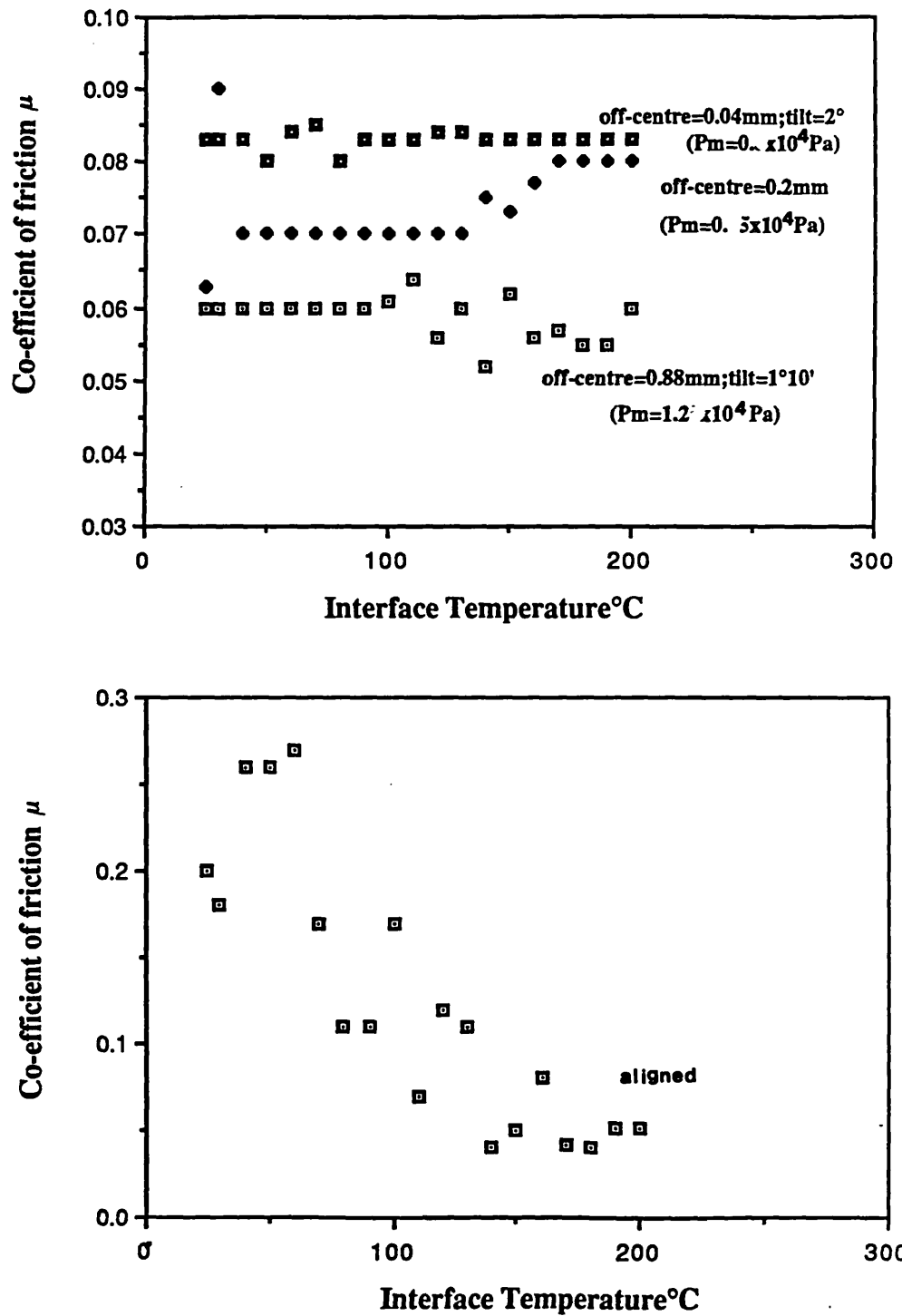


Fig.7.17 The influence of interface temperature on the co-efficient of friction. The powder does not contain any additional moisture (grooved plate)

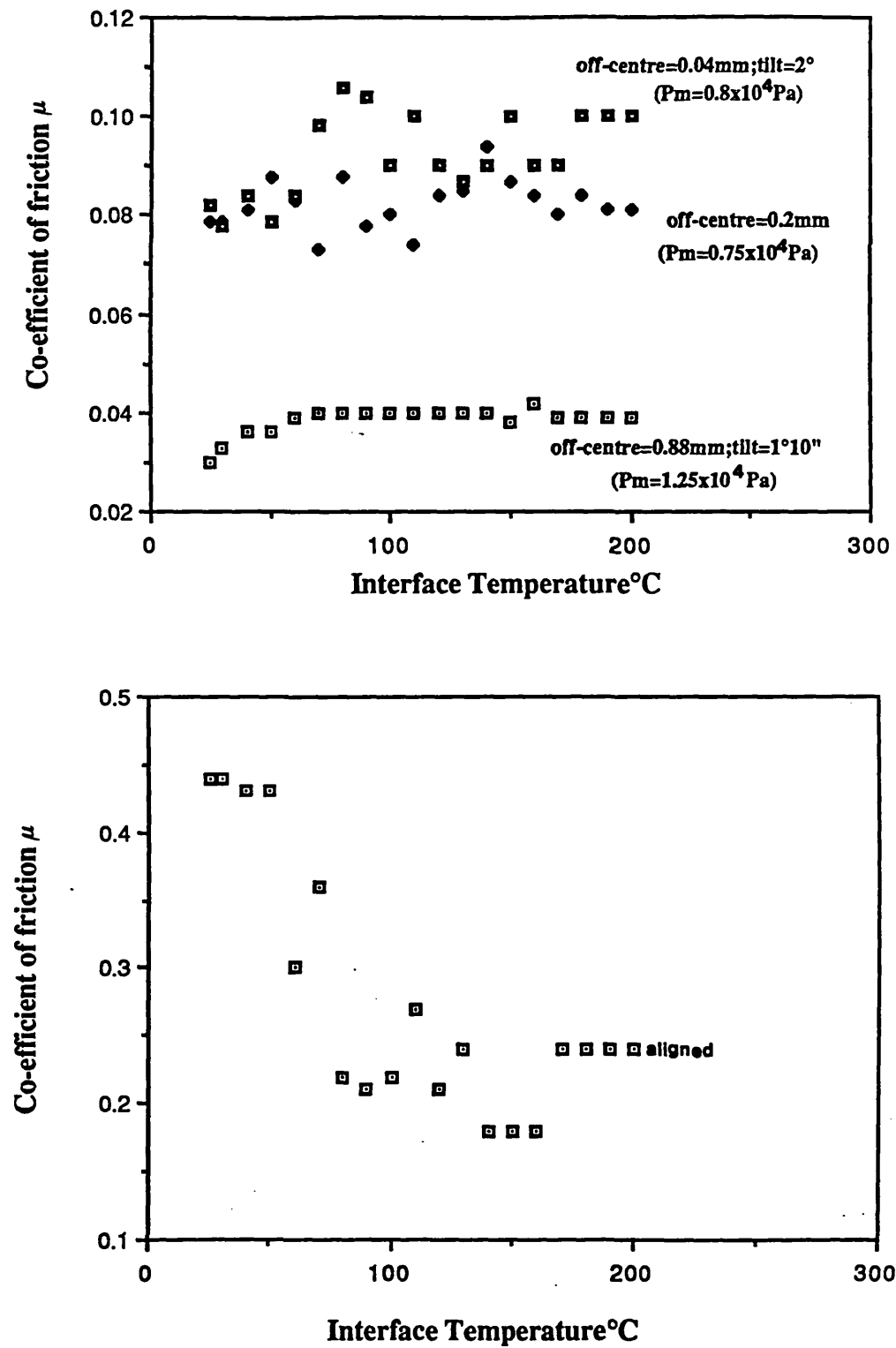


Fig.7.18 The influence of interface temperature on the co-efficient of friction. The level of additional moisture 16% w/w. (grooved plate)

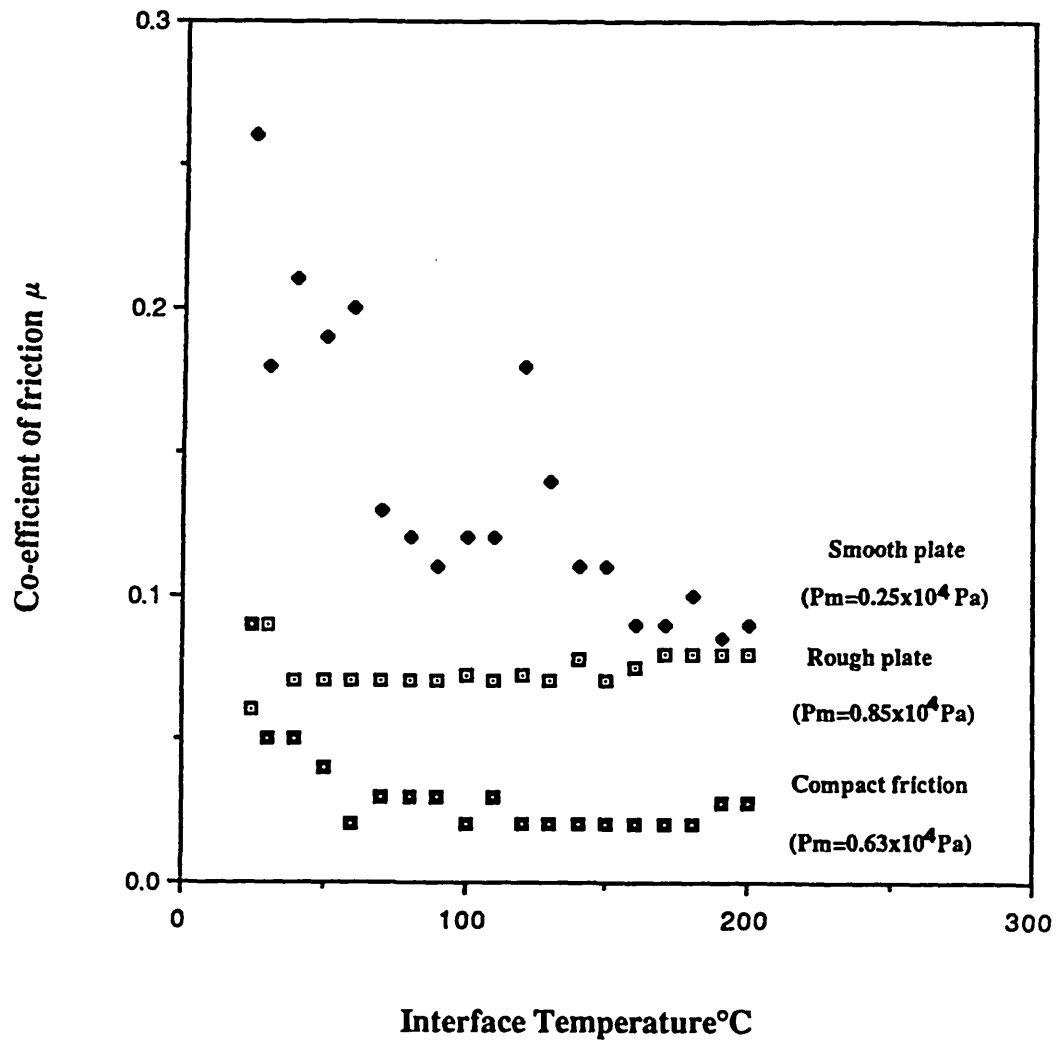


Fig.7.19 A comparison of the co-efficient of friction for smooth and rough plates to-gether with data on compact friction as a function of interface temperature. The powder does not contain any additional moisture.

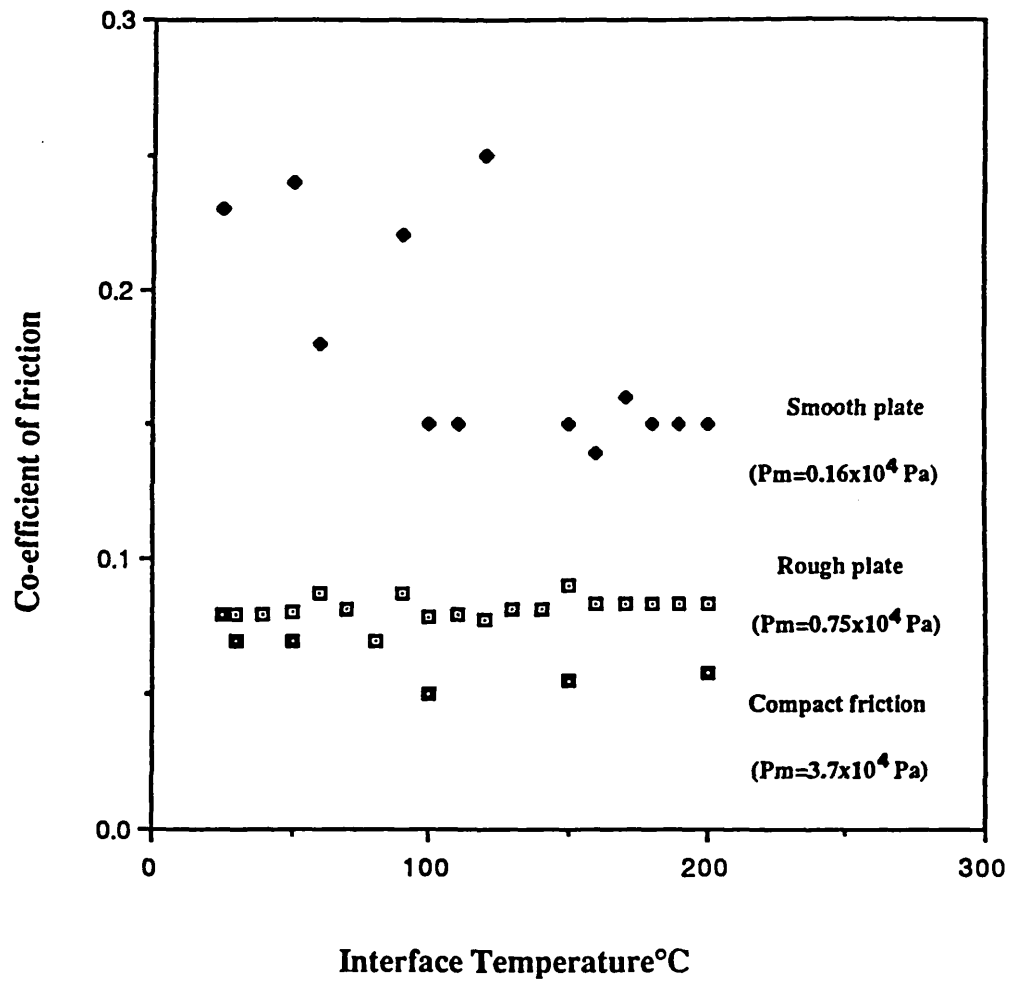


Fig.7.20 A comparison of the co-efficient of friction for smooth and rough plates together with data on compact friction as a function of interface temperature. The powder contains 16% w/w additional moisture.

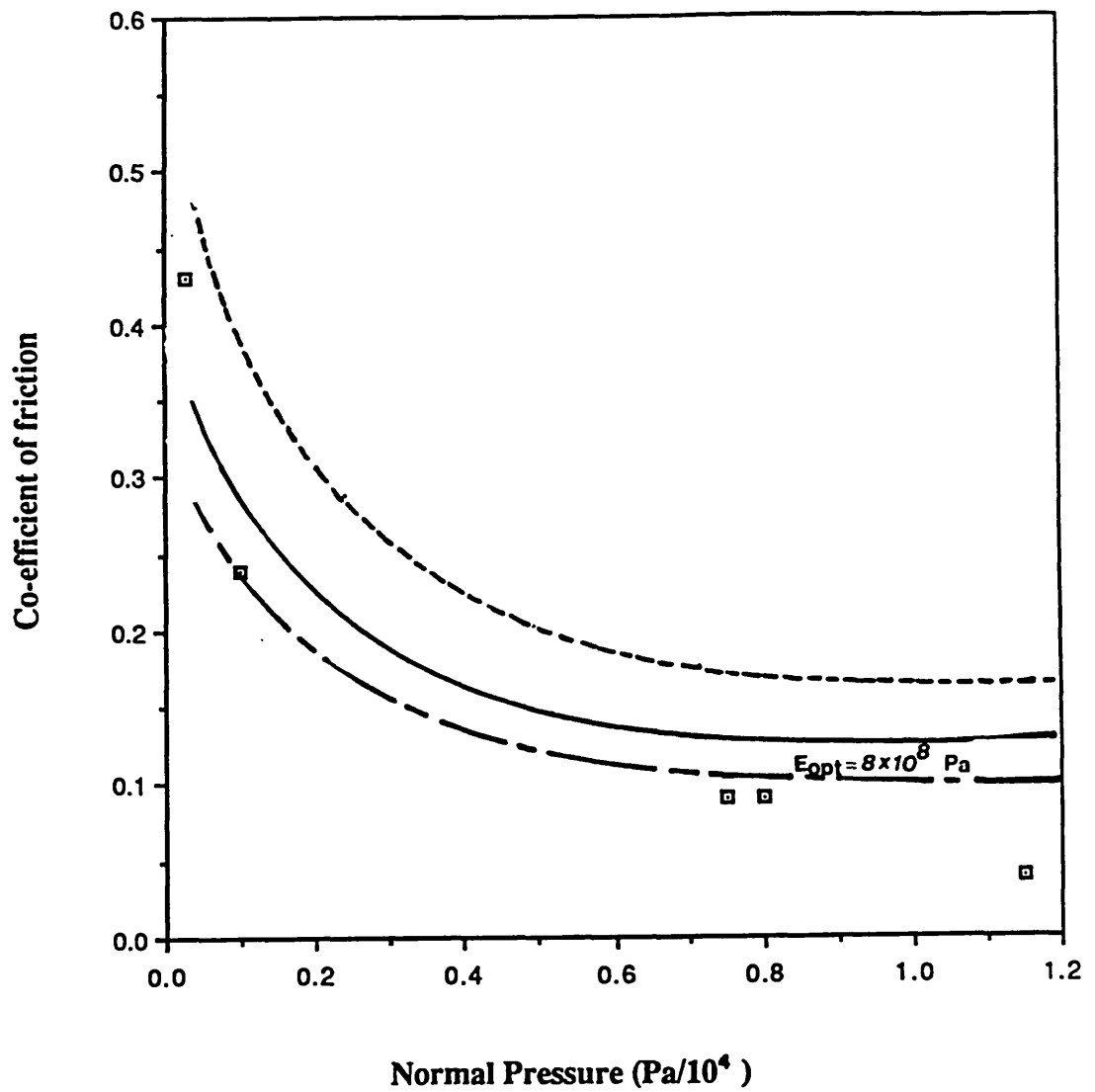


Fig.7.21 A comparison of the experimental and theoretical predictions of the co-efficient of friction. — predictions from eq.6.12. — predictions from eq.7.7. Unmoistened powder at ambient.

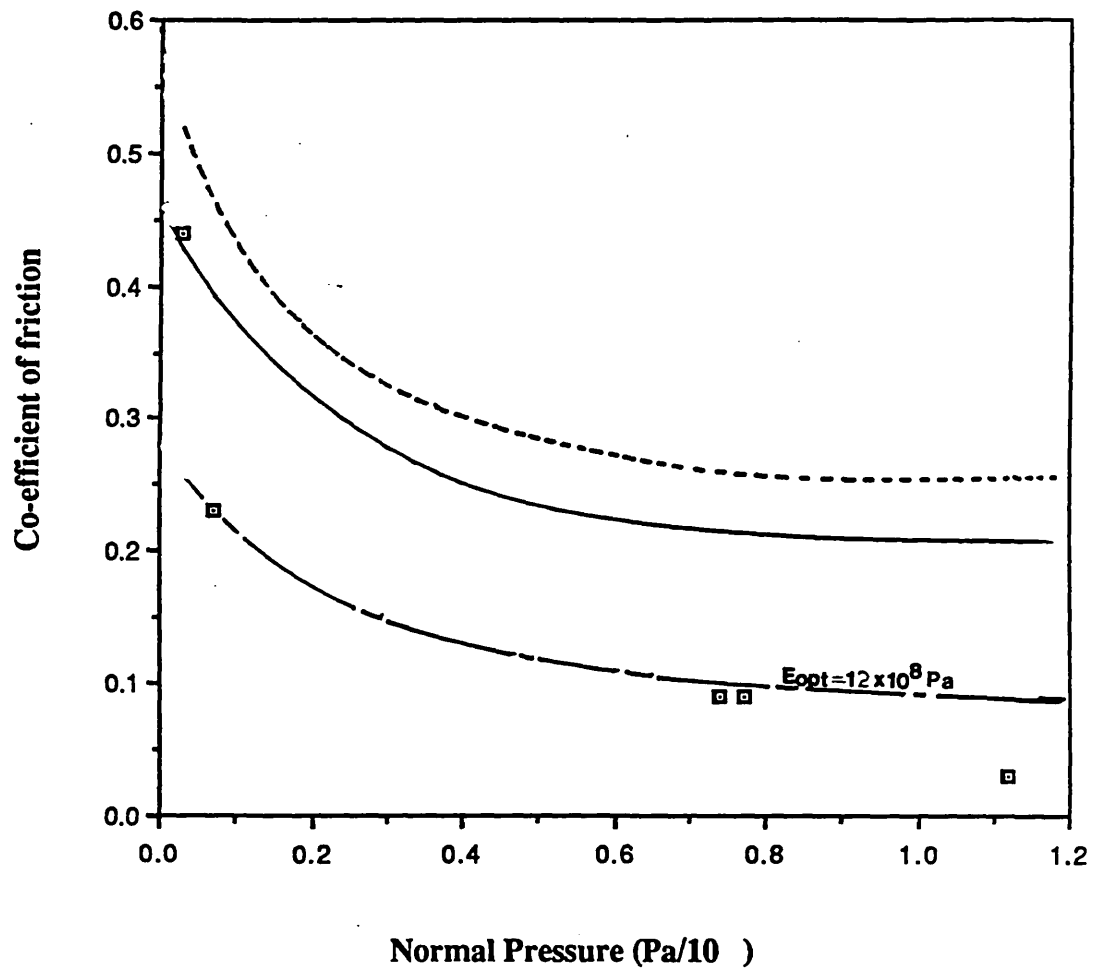


Fig.7.22 A comparison of the experimental and theoretical predictions of the co-efficient of friction. — predictions from eq.6.12. ____ predictions from eq.7.7. unmoistened powder at 50°C.

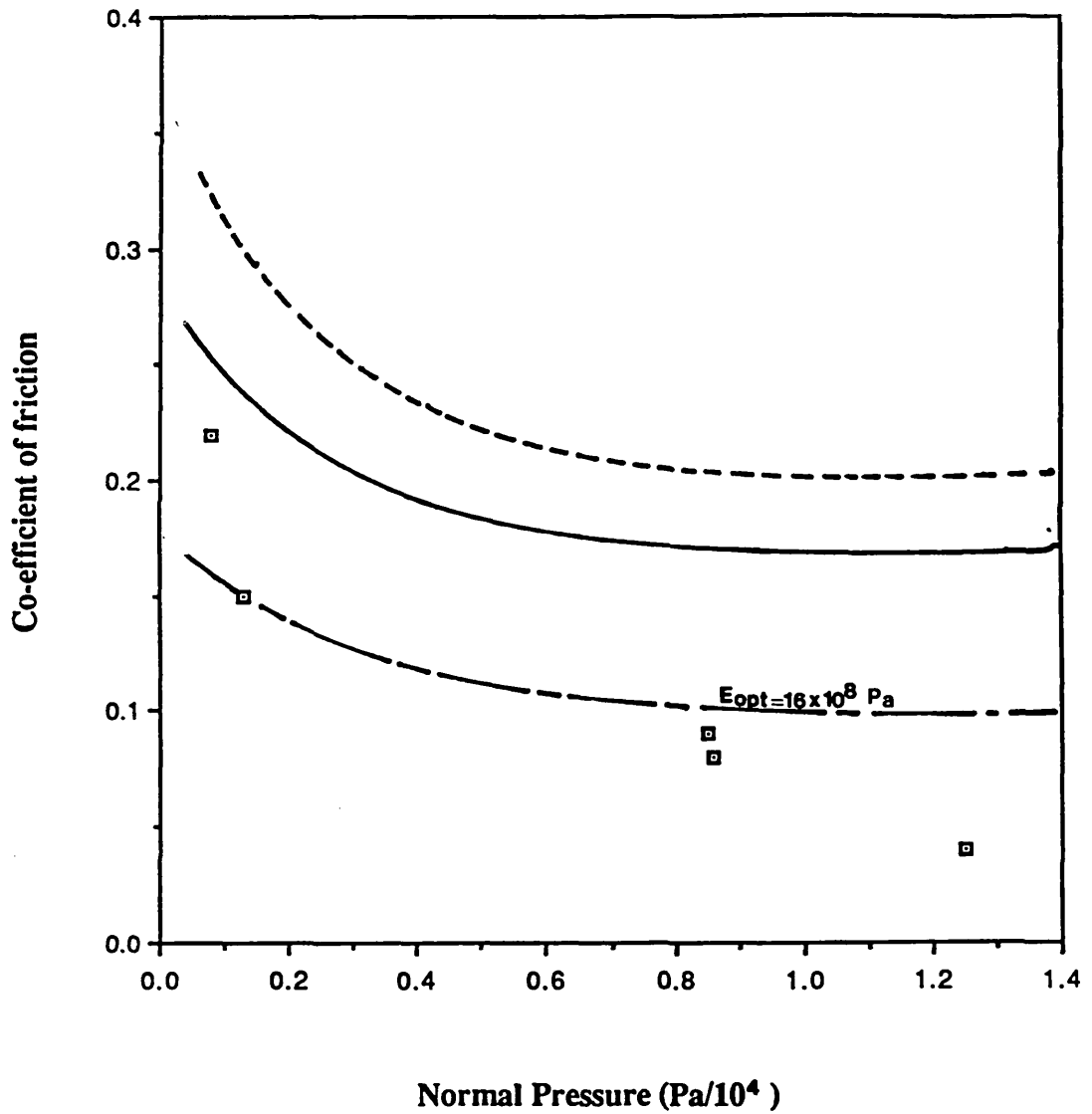


Fig.7.23 A comparison of the experimental and theoretical predictions of the co-efficient of friction. — predictions from eq.6.12. ____ predictions from eq.7.7. Unmoistened powder at 100°C.

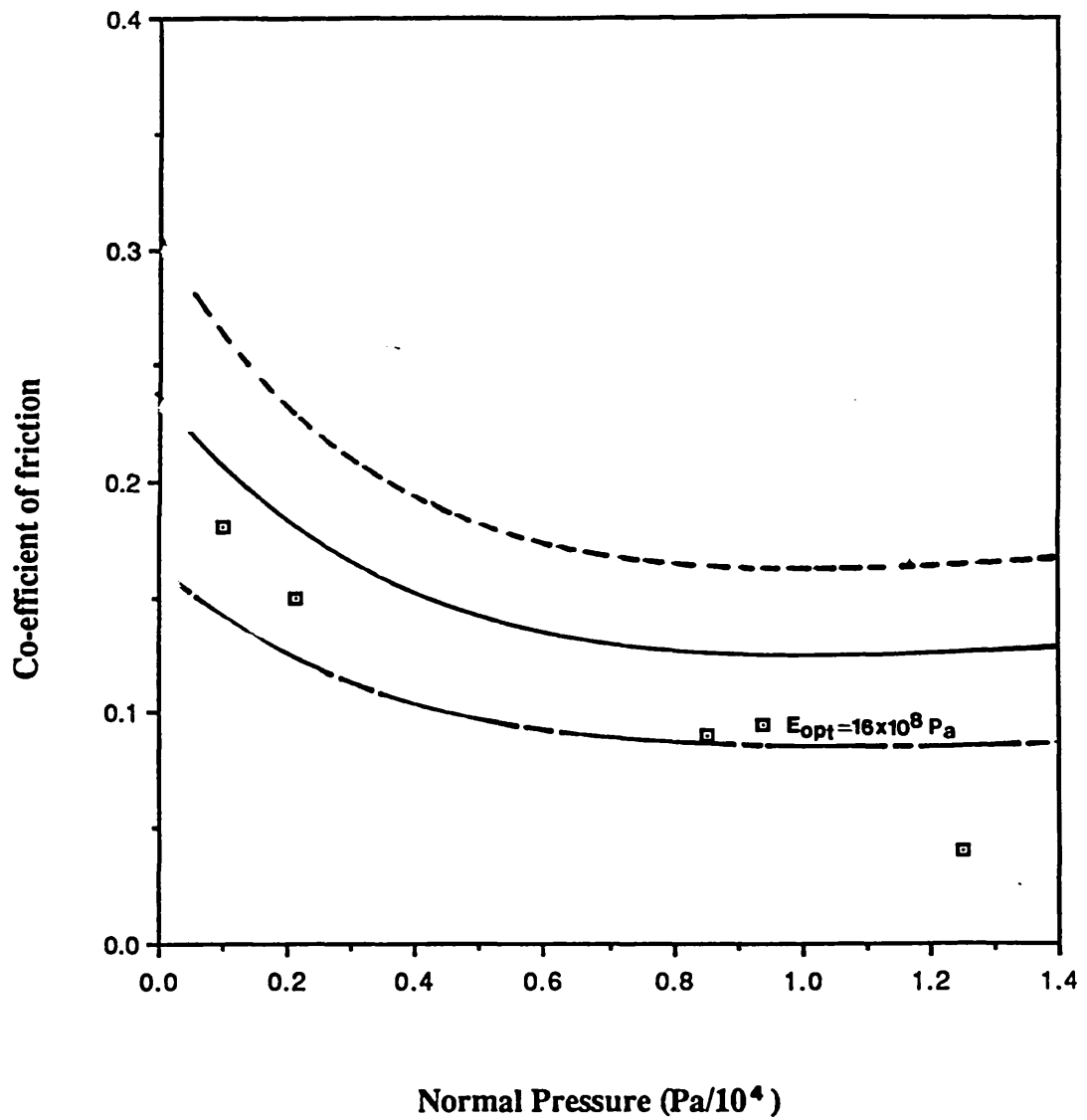


Fig.7.24 A comparison of the experimental and theoretical predictions of the co-efficient of friction. — predictions from eq.6.12. — predictions from eq. 7.7. Unmistened powder at 150°C .

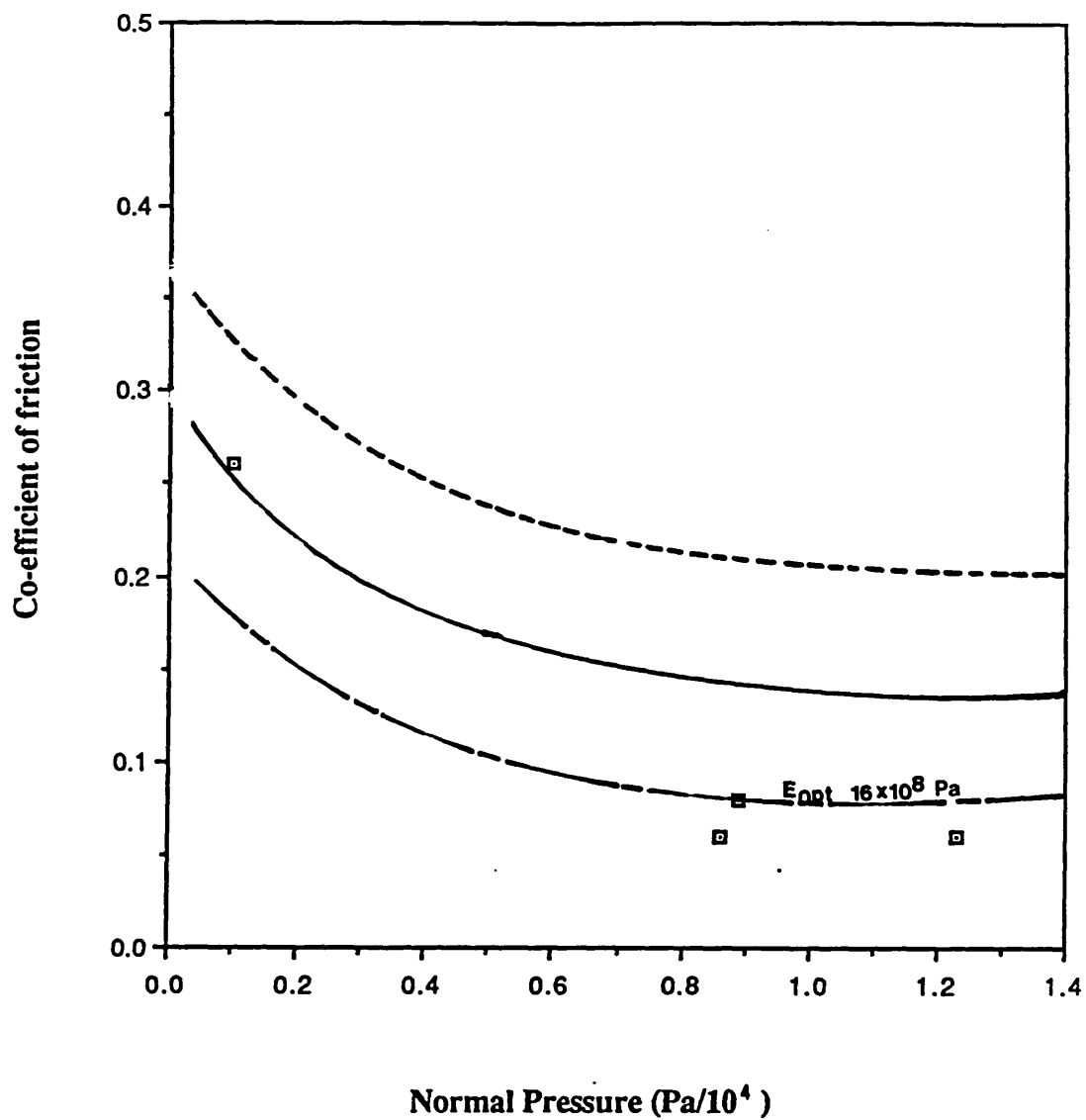


Fig.7.25 A comparison of the experimental and theoretical predictions of the co-efficient of friction. — predictions from eq.6.12.
 — predictions from eq.7.7. Unmoistened powder at 200°C.

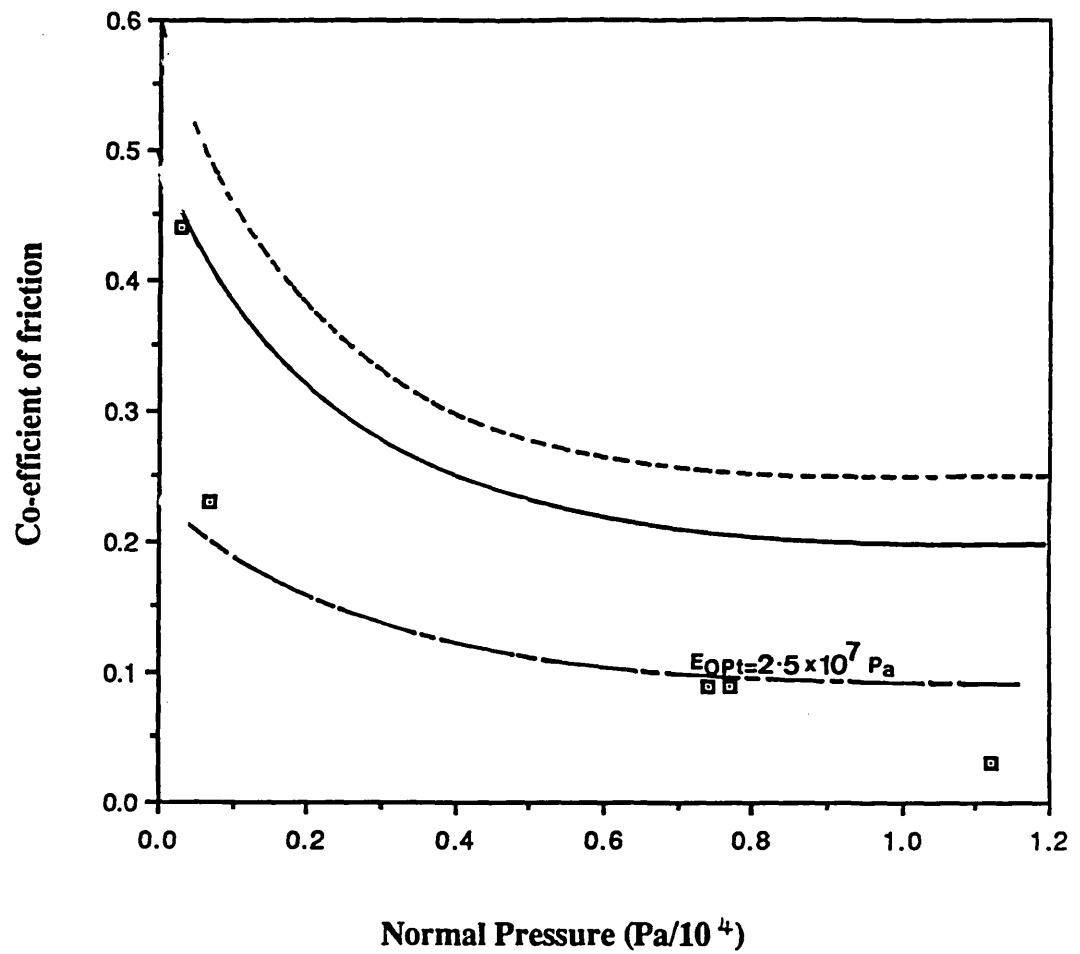


Fig.7.26 A comparison of the experimental and theoretical predictions of the co-efficient of friction. — predictions from eq.6.12. — predictions from eq.7.7. Moistened powder at ambient.

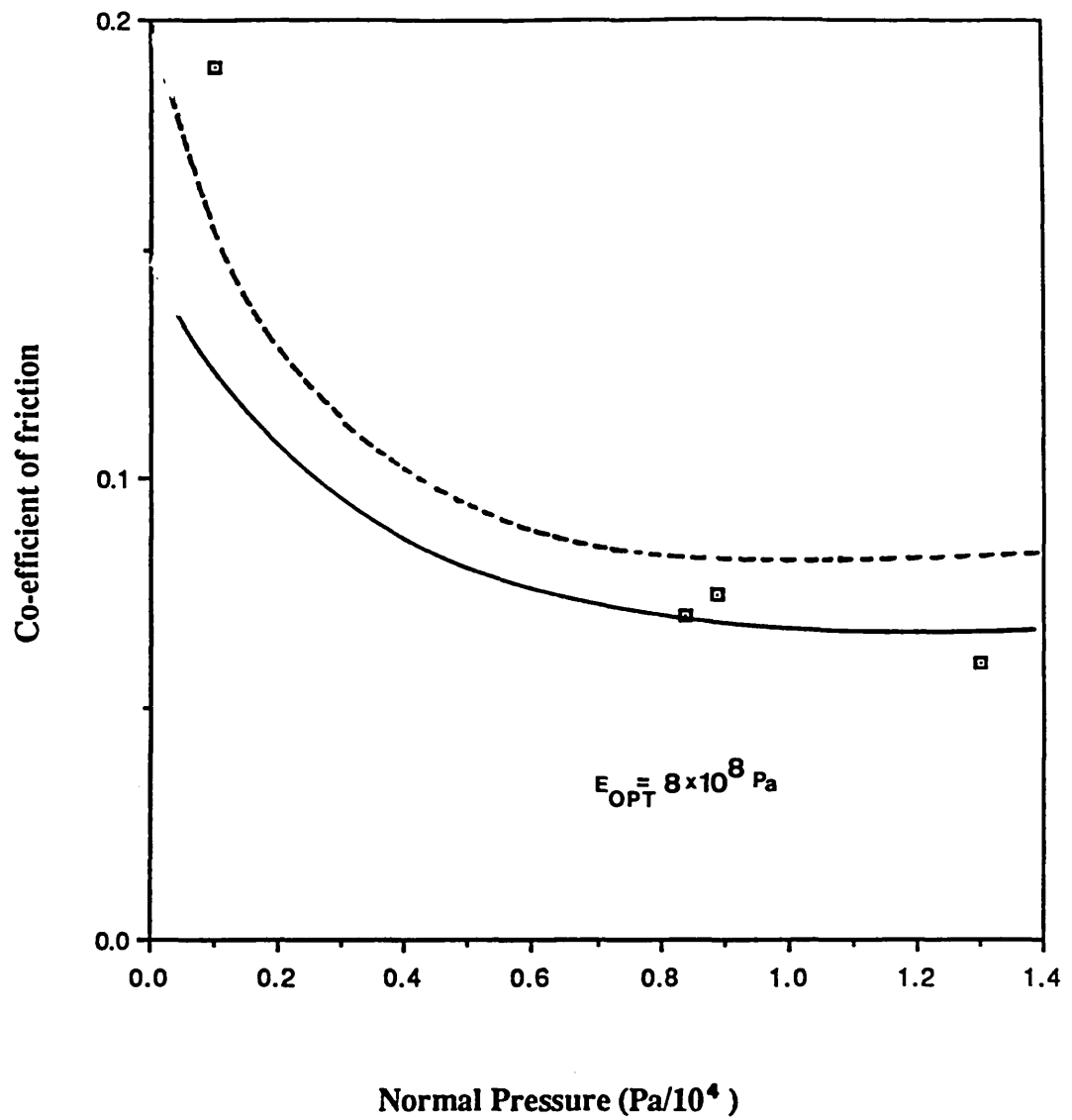


Fig.7.27 A comparison of the experimental and theoretical predictions of the co-efficient of friction. — predictions from eq.6.12. — predictions from eq.7.7. Moistened powder at 50°C.

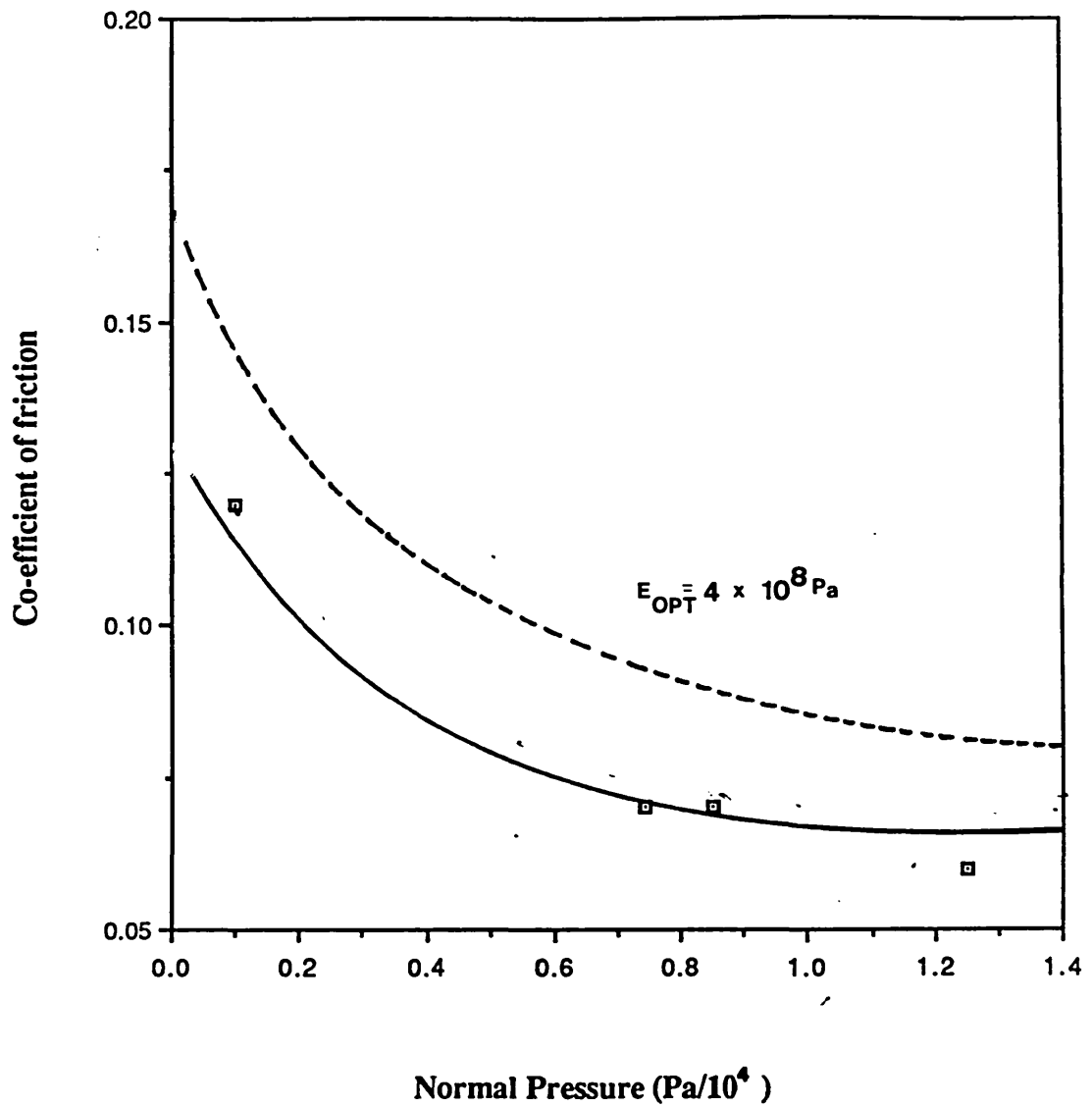


Fig.7.28 A comparison of the experimental and theoretical predictions of the co-efficient of friction. — predictions from eq.6.12. ____ predictions from eq.7.7. Moistened powder at 100°C .

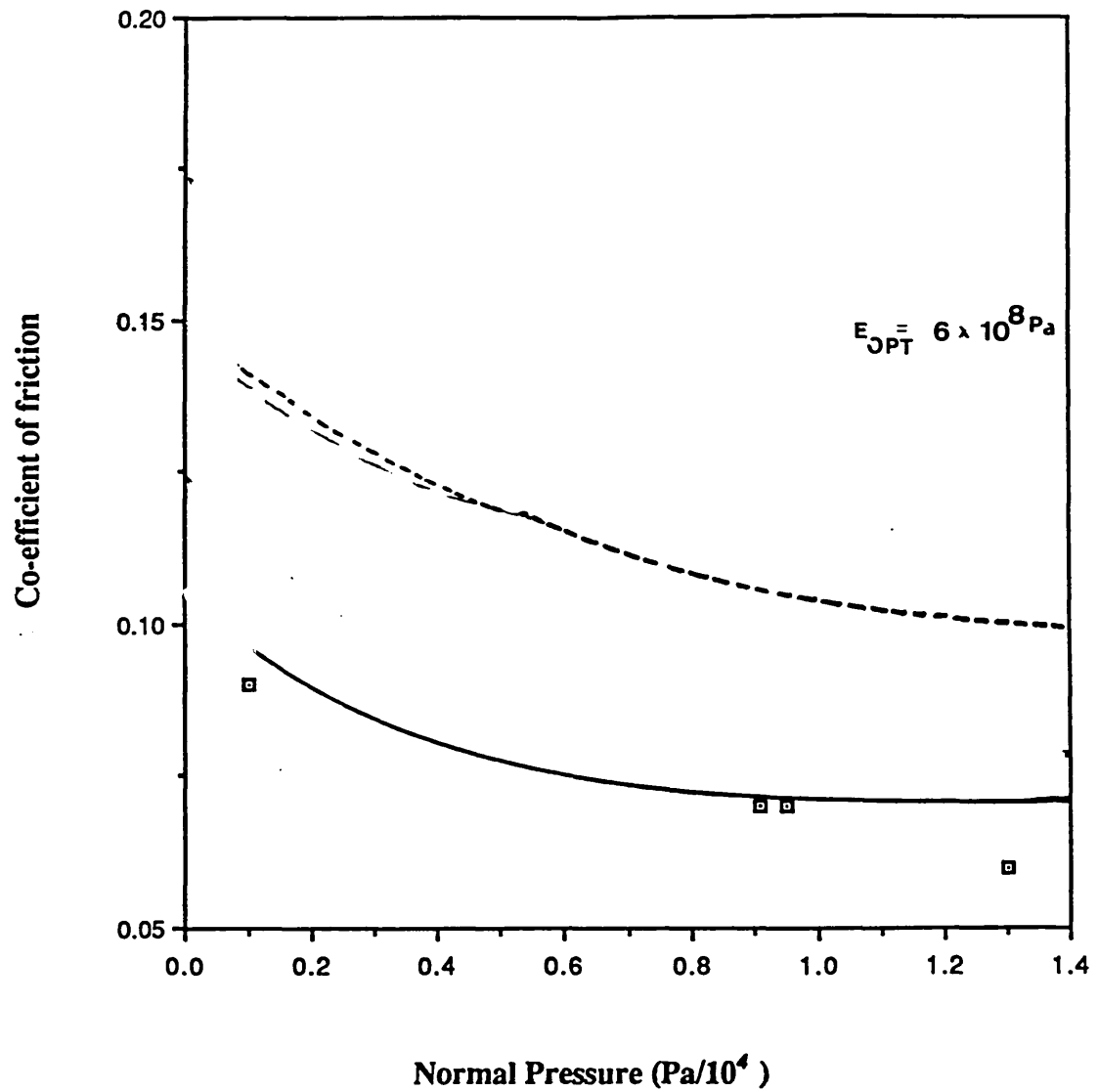


Fig.7.29 A comparison of the experimental and theoretical predictions of the co-efficient of friction. — predictions from eq.6.12. ____ predictions from eq.7.7. Moistened powder at 150°C .

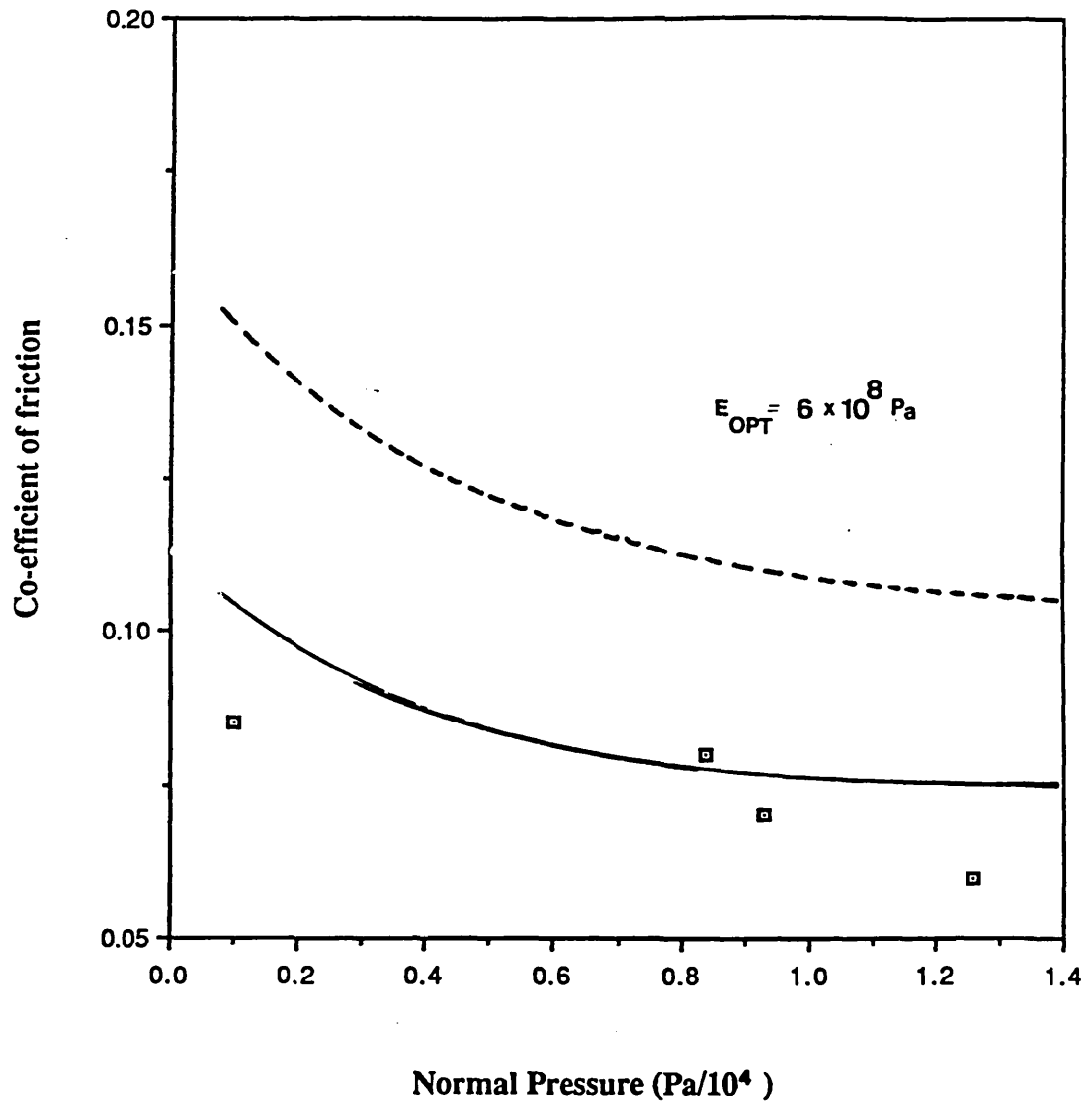


Fig.7.30 A comparison of the experimental and theoretical predictions of the co-efficient of friction. — predictions from eq.6.12. ____ predictions from eq.7.7. Moistened powder at 200°C.

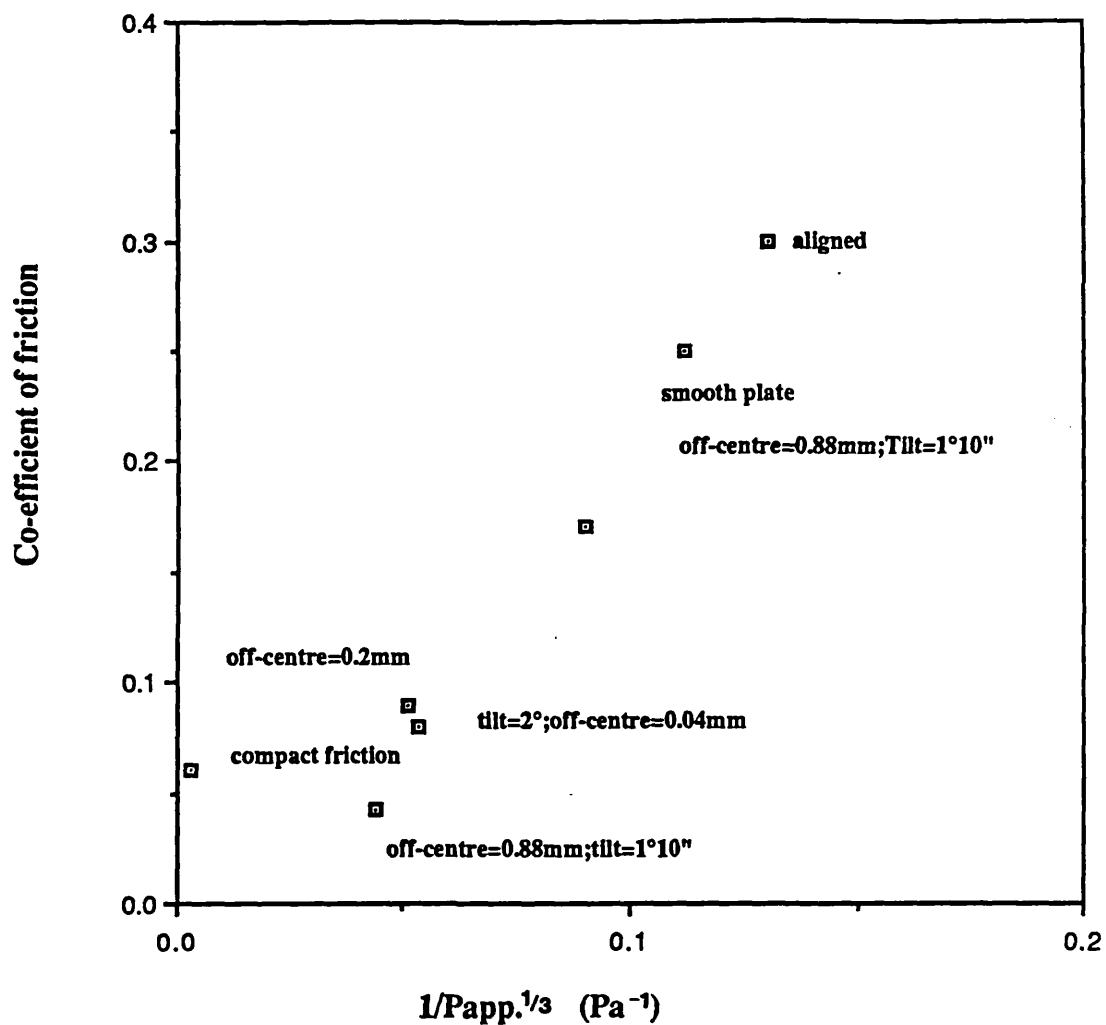


Fig.7.31 The mean co-efficient of friction as a function of the inverse normal pressure. Temperature range ambient-200°C. The powder does not contain any additional moisture.

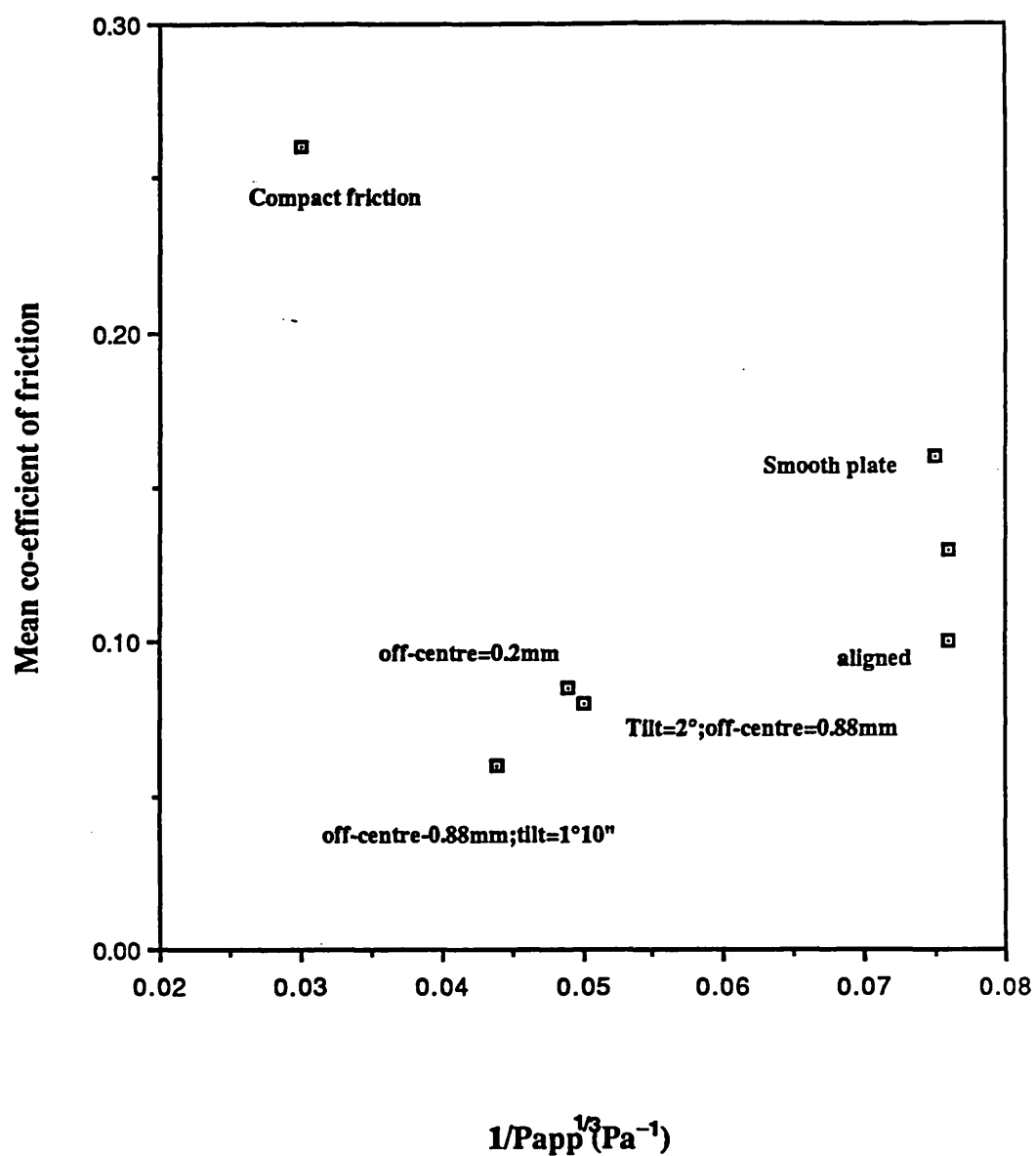


Fig.7.32 The mean co-efficient of friction as a function of the inverse normal pressure. Temperature range ambient -200°C. The powder contains 16% w/w additional moisture.

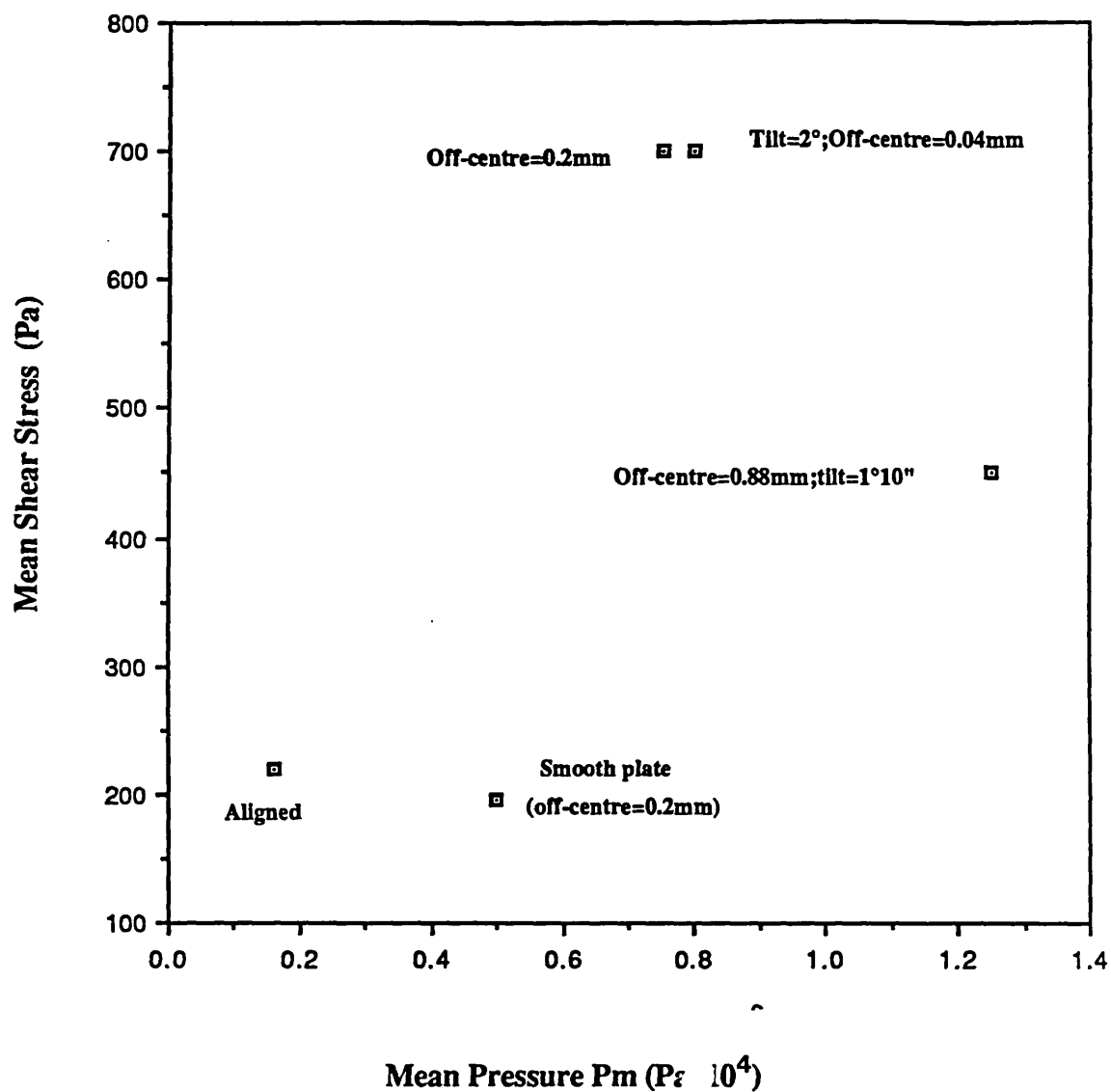


Fig.7.33 The mean shear stress as a function of the mean pressure for the unmoistened powder. Temperature range ambient to 200°C.

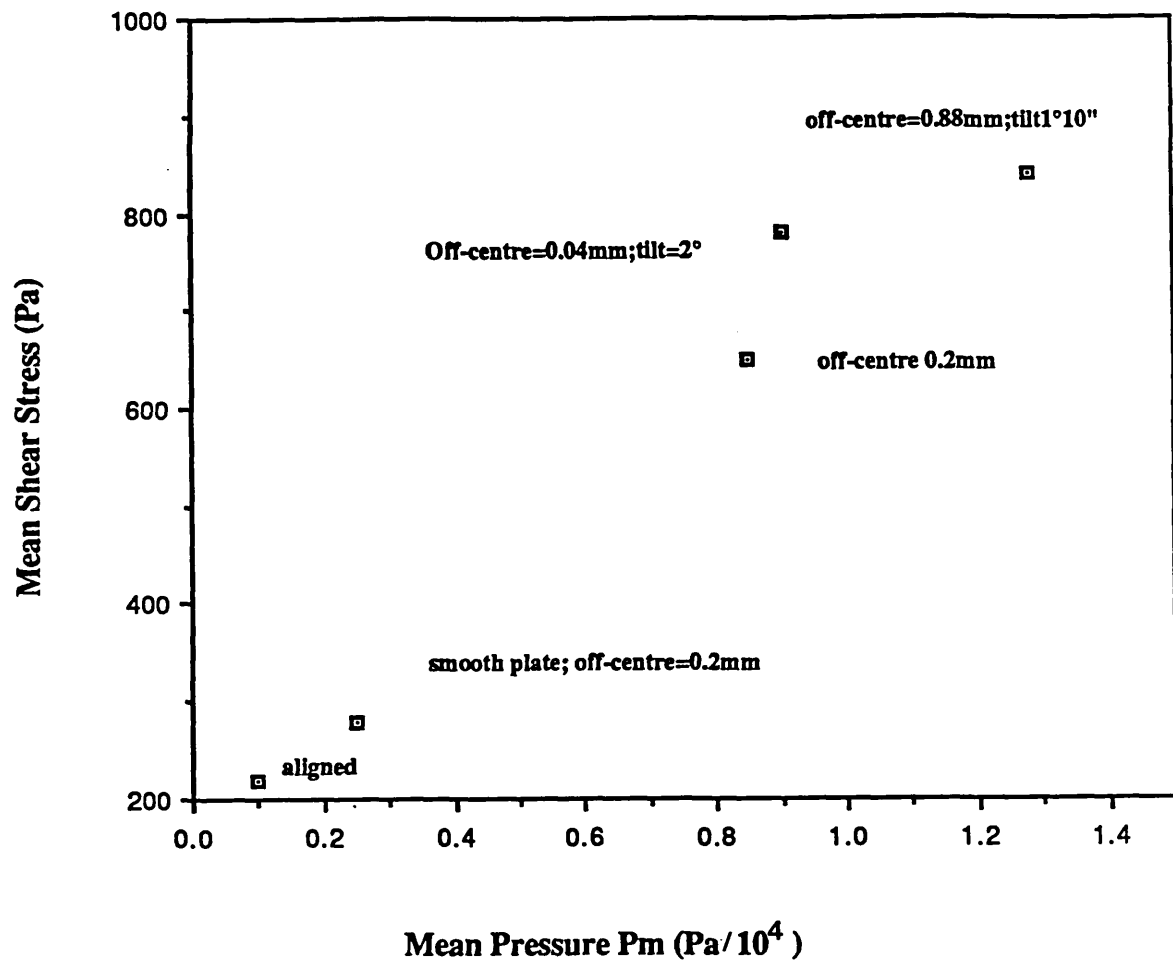


Fig.7.34 The mean shear stress as a function of the mean pressure for the moistened powder. Temperature range ambient to 200°C.

Chapter 8

Extrusion Data and Analyses

8.1. Introduction

The Darnell and Mol⁽²¹⁾ analysis for the prediction of solids flow in the feed section of a single screw extruder was presented in chapter 1. The relevant variables which govern the feeding mechanism were established as: the coefficients of friction between the barrel and screw surfaces μ_b , μ_s ; the stress transmission ratio K_w and the screw geometry. The prediction of the conveying angle ϕ (eq. 1.2) and hence the throughput requires a knowledge of the pressure profile in a single screw extruder. The main objective of the current work has been the prediction of the form of this pressure profile and hence the effective length of the feed zone of a typical single screw maize extruder. For a fixed screw geometry, the major variables governing the profile are the coefficients of friction, μ_s , μ_b and the stress transmission ratio K_w . The magnitude of these parameters for the present system and their variation with the extrusion variables were presented in chapters 5 and 6 and these studies provided explicit relationships for the friction coefficient and the transmission factor K_w , which are necessary in order to predict the stress generation from the traction induced compaction model

developed in chapter 6. Further, this model also allows for the variation of the coefficient of friction with the induced normal pressure. This chapter presents data obtained during single screw extrusion of maize powder. These data are then compared with the analysis cited above for the wall traction induced compaction.

8.2. *Extrusion Data*

The pressure profiles in these experiments were found to be exponential in form (Figs. 8.1 - 8.3). This form for the pressure generation along the extruder was also observed by Lovegrove⁽²⁴⁾ for PVC powders, although he was generally only able to measure pressures at the delivery end (Fig. 8.4). Tadmor and Klein⁽³⁾ have also presented simulated and experimental pressure profiles for low density polyethylene (LDPE), polypropylene (PP) and high density polyethylene (HDPE) which also indicate an exponential pressure rise in the feed zone for these powders. In the present study the pressure profiles and the measured values of the length of the feed zone were not strong functions of the barrel heating temperature. A series of experiments were also conducted to investigate the influence of screw heating on the pressure profile for maize powder and in these experiments the barrel was maintained at a temperature of 50°C. The maize powder used in these screw heating experiments was of a different chemical composition to that which was used in earlier experiments and it was only possible to measure significant pressures towards the

delivery end of the extruder, with the exception of the 120°C and 150°C screw heating experiments. The pressure profiles obtained in these screw heating experiments are depicted in Figs. (8.5 - 8.6). The temperature profiles measured by the thermocouples along the barrel wall, for both the barrel heating and screw heating experiments, are shown in Figs. (8.7 and 8.8). In order to analyse quantitatively the data obtained during the screw heating experiments, the magnitude of the screw surface temperature is required. This allows the frictional properties of the powder to be evaluated at the appropriate screw surface temperature and hence the coefficient of screw friction μ_s , may be computed. Therefore the screw surface temperature was estimated from a conventional heat balance, the details of which are presented in section 8.3.

8.3. *Computation of Screw Surface Temperature*

The screw was heated prior to the commencement of the experiment by pumping hot oil from an overhead oil bath, which was set at a temperature T_i . On the discharge end of the screw the temperature is T_o such that $T_o < T_i$. The screw surface temperature will also be influenced by heat flow from the barrel to the screw surface. However, since the entrapped air layer between the barrel and the screw surfaces will act as an effective insulator, the influence of heat flow from the barrel on the screw surface temperature T_s , will be negligible. Therefore it is only

necessary to consider the convective heat transfer $\dot{Q}_{\text{conv.}}$ from the oil to the inside screw surface and heat flow by conduction $\dot{Q}_{\text{cond.}}$ to the outside surface of the screw.

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv.}} + \dot{Q}_{\text{cond.}}$$

The combination of the appropriate equations for the computation of the forced convective term $\dot{Q}_{\text{conv.}}$ and conduction $\dot{Q}_{\text{cond.}}$ and assuming steady state yields:

$$Q \left[\frac{1}{h_i 2\pi r_{is} L} + \frac{1}{2\pi k_m L} \ln \frac{r_{os}}{r_{is}} \right] = T_i - T_s \quad (8.1)$$

where h_i is the convective heat transfer coefficient, r_{is} and r_{os} the inside and outside radius of the screw, L the length of the screw and T_s the outside screw surface temperature and k_m the thermal conductivity of the metal. The computed Reynolds numbers Re for the flow of oil in these screw heating experiments were within the laminar flow region; i.e. $Re < 2100$. Therefore the appropriate convective film coefficient h_i was computed from⁽¹⁴¹⁾:

$$\frac{h_i d_{is}}{K_{\text{oil}}} = \frac{3.66 + 0.0688 \left\{ \frac{d_{is}}{L} \right\} Re Pr}{1 + 0.04 \left[\frac{d_{is}}{L} Re Pr \right]^{2/3}} \quad (8.2)$$

Re and Pr are Reynolds and Prandtl numbers respectively given

as

$$\text{Re} = \frac{\rho v d_{is}}{\mu}; \quad \text{Pr} = \frac{C_p \mu}{K_{oil}}$$

where ρ , μ , C_p , K_{oil} are the density, viscosity, specific heat capacity and thermal conductivity of the oil. The rate of heat flow is equivalent to the heat lost by the oil and is given by:

$$\dot{Q} = m C_p (T_i - T_o) \quad (8.3)$$

The screw surface temperature T_s is found by combining and solving eq. (8.1), (8.2) and (8.3). The computed values of T_s are given in Table (8.1). In eq. (8.1) the contribution to the heat flow from the presence of extended surfaces was neglected. Further, during the experiment the frictional heat generated may increase the surface temperatures and the convective heat transfer is that of natural convection. The estimated screw temperatures were assumed to be constant along the length of the screw.

8.4. Extrusion Data Analyses

In the barrel heating experiment the pressure build up was found to be improved at ambient; ca 20°C. In this experiment both the screw surface and barrel surface were at the same temperature. Therefore the coefficient of friction between the solids and the barrel and screw surfaces would be expected to be similar if the normal pressure acting on both the surfaces is the same. However,

as Schneider⁽²²⁾ has demonstrated the pressure in the extruder channel is anisotropic and the fraction of the axial pressure which acts on the barrel wall is larger than that on the screw surface. It therefore follows that if the stress transmission is governed by elastic properties the coefficient of friction will decrease with increase in normal pressure (eq. 6.12). In the extruder the material is flowing plastically. However, the maize powder used in this study is capable of sustaining a considerable amount of residual elastic strains. This is demonstrated by the stress relaxation data reported in chapter 6. Following these arguments it is to be expected that the coefficient of friction μ_b at the barrel wall will be less than that at the screw surface. The same arguments apply to the 80°C and 120°C barrel heating experiments. This is an interesting practical consequence of the normal stress dependence of the wall friction.

After each experiment the screw was extracted and the length of the feed zone was measured. These data are shown in Table (8.2). The length of the feed is greater at ambient than those at 80°C and 120°C. This is probably a result of the die temperature being larger at these temperatures than at ca 20°C resulting in a longer melt zone and therefore a shorter solids conveying zone. In the screw heating experiments the low coefficients of friction at the screw μ_s at elevated temperatures results in an improved pressure profile relative to the experiment conducted at ambient. This indicates that the relative tractions at the two surfaces, screw and barrel, enhance the pressure build up. This was also pointed out by Maillefer⁽²⁰⁾ who recognized that the conveying

mechanism is essentially a problem in the frictional force differential between the barrel and the screw surfaces.

In order to analyse these data quantitatively using the Janssen-Walker (J-W) (eq. 3.23) and Modified Jansen-Walker (M.J.W.) (eq. 6.5) equations, the ratio of real to apparent contact area ξ , the coefficient of friction at the barrel wall interface μ_b , the distribution ratio $\mathcal{D}$ and K_w ratio are required. These were computed from the functionalities presented in chapters 2 and 6. The distribution factor $\mathcal{D}$ is near to unity for the particular combination of the wall angle ϕ_w and the internal angle ϕ_i of friction. The computed values of μ_s and μ_b for the barrel heating experiments are shown in Table (8.3 and 8.4). These data demonstrate that the coefficient of friction reduces with increasing normal pressure as would be expected for a system where deformations are governed by residual elastic strains which accompany the plastic flow.

The direction of the shear-stress in the extruder channel is opposite to that in an uni-axial compaction experiment. Therefore the appropriate J-W and M.J.W. equations for the active analysis in the extruder are:

$$\sigma_{xx_{w_b}} = \mathcal{D} K_1 P_i e^{4\mu_w K_1 L/h} \quad (8.4)$$

$$\sigma_{xx_{w_b}} = \frac{\mathcal{D} K_1}{A} \left[(1 + A P_i) e^{4\bar{\xi} \tau_o' A L/h} - 1 \right] \quad (8.5)$$

where L/h is the ratio of length to channel depth and K_1 a

constant which characterizes the pressure anisotropy at the barrel. The value of K_1 is derived from a consideration of the radial equilibrium equation in polar co-ordinates as shown by Schneider⁽²²⁾. This also requires a knowledge of the stress transmission ratio K_w and the resulting equations are:

$$\frac{\sigma_{xx_b}}{\sigma_{yy}} = \frac{1 - (r_s/r_b)}{1 - (r_s/r_b)^{1/K_w}} = K_1 \quad (8.6)$$

$$\frac{\sigma_{xx_s}}{\sigma_{yy}} = \frac{r_b/r_s - 1}{(r_b/r_s)^{1/K_w} - 1} = K_3 \quad (8.7)$$

where r_s and r_b are mean screw and barrel radii and σ_{xx_s} , σ_{xx_b} are normal stresses at the screw and barrel wall respectively. The ratio K_2 is the fraction of the mean pressure acting on the flights and is equivalent to K_w . In the derivation of the J-W and the M.J.W. functionalities appropriate to extrusion, the contribution from the screw traction to the pressure profile has been neglected. However, as has been demonstrated by other workers⁽³⁾⁽²⁰⁾ and by the 150°C screw heating experiment for the maize powder in this study, the pressure profile is improved by the magnitude of the relative friction at the barrel and screw surfaces. A complete quantitative analysis should therefore encompass the contribution from screw traction. However, as pointed out by Tadmor and Klein⁽³⁾ it is easier to understand the conveying mechanism if it assumed that the screw is stationary and

the barrel rotates. Therefore the driving force for the movement of the plug is clearly the friction force between the barrel and solid while that between screw and solid would tend to retard axial motion of the plug. Based on these ideas it is reasonable to translate the uni-axial compaction modelling to describing quantitatively the extruder feeding mechanisms by only considering barrel traction. Equations (8.4) and (8.5) were used to predict the pressure profile along the barrel wall and these predictions together with experimental measurements are shown in Tables (8.6 and 8.7). The initial pressure P_i was equated to the reading from the P_2 transducer and the computed values of K_1 , K_2 and K_3 were 0.33, 0.27 and 0.23 respectively.

It was argued in chapter 6 that the flowing material in the extruder would generate stresses near the upper limit or the dynamic stress condition. However, the experimental measurements were very much closer to the active or the lower limit of the possible stresses. This is not surprising since the discharge rates from an extruder feed zone are much lower than a vertically discharging hopper which generates stresses near the dynamic stress condition. The theoretical predictions however should be lower than the experimental measurements as the active analysis yields the lower limit of the possible stresses and this was confirmed by experiment. The predictions from the modified Janssen-Walker analysis (M.J.W.) which caters for the variation of the coefficient of friction with the local normal pressure are in better agreement with experiment than those from the original Janssen-Walker (J-W) analysis based on an average value of the

coefficient of friction.

The observed difference in the prediction from the M.J.W and the J-W analyses is a consequence of the relative change in the coefficient of friction along the barrel of the extruder induced by the changes in the normal pressure. This large difference between the two analyses was not observed in modelling the uni-axial compaction data. This is a result of the normal pressures being much higher with uni-axial compact experiments, and thus the resulting coefficient of frictions are relatively independent of normal pressure, in these compaction experiments. However, in the extruder feed zone where the pressures generated are lower over an appreciable length of the zone, the coefficient of friction is a strong function of the local normal pressure and the difference in the predictions between the two techniques (J-W and M.J.W) are now significant. In order to predict the length of the solids conveying zone, the maximum pressure developed in the melting mechanism was coupled with the stress transmission equation (8.5) in the feed zone. Costas and Gogos⁽¹⁴²⁾ have demonstrated that the maximum pressure $P_{\max}$ developed in the melting zone for non-parallel plate flow, as is the case in the extruder, is given by:

$$P_{\max} = P_0 + \frac{6\mu LV_0}{H_0 H_1} \quad (8.8)$$

where P_0 is the exit axial pressure from the transition zone, μ the viscosity of the melt, L the length of the extruder, V_0 the velocity and H_0 , H_1 are the channel depths at the beginning and end of the extruder. In deriving eq. (8.8) it has been assumed

that the melt flow is steady, isothermal and involves an incompressible Newtonian liquid. However, it has been demonstrated⁽²⁾ that most food materials exhibit non-Newtonian properties. Nevertheless in the present experiments the residence times were small and the melt may be considered as Newtonian to a good approximation. Further, if the percentage of melt is small in relation to the solid in the transition zone, the behaviour in this section of the machine can be predicted from an extension of the feed zone theory. In the present experiments the P_6 transducer was in this transition region and the predictions based on the traction induced compaction model are within a factor of 2. Therefore it is reasonable to equate the entrance pressure to the melt zone P_0 as the exit pressure from the solids conveying zone, and a combination of eq. (8.8) with the mean axial pressure from the feed zone (eq. 8.5) result in:

$$P_{\max} - \frac{6\mu LV_0}{H_0 H_1} = \frac{1}{A} \left[(1 + A P_i) e^{\frac{4\bar{\tau}_0' AL_f}{h}} - 1 \right] \quad (8.9)$$

If $P_{\max}$ is equated to the die pressure P_D , the length of the feed zone L_f can be computed from eq. (8.9). The viscosity data at different temperatures for maize powder (fine Polenta F) was taken from Siodlak⁽¹⁴³⁾. The length of the feed zone can also be predicted by replacing the M.J.W. equation in eq. (8.9) by the J-W equation:

$$P_{\max} - \frac{6\mu LV_0}{H_0 H_1} = P_i \exp^{\frac{4\bar{\mu}_w K_1 L_f}{h}} \quad (8.10)$$

These predictions together with experimental measurements are shown in Table (8.8). The M.J.W. analysis gives a better prediction of the feed zone length than the J-W analysis. However, the trends shown with temperature between experiment and theory are in opposite directions. In these predictions the variations of the K_w ratio with temperature were neglected. The magnitude of K_w will tend to unity with increasing temperature as the hydrostatic condition is approached. An evaluation of equations (8.9) and (8.10) with increased K_1 ratios with temperature indicates that the direction of the predictions with temperature would be similar to the trend shown by experiment.

The pressure profile and the length of the solids conveying zone has been predicted from an understanding of the traction and compaction characteristics of the powder. In order to use the developed traction induced compaction model (M.J.W.) to predict throughput an equation similar to equation (1.1) which caters for the variation in the coefficient friction with normal pressure needs to be developed. However, eq. (1.1) and (1.2) can be used for the computation of the ratio $\frac{\mu_s}{\mu_b}$ for the present experiment where the throughput was $0.028 \text{ m}^3/\text{hr}$. Therefore by computing an average value of $\overline{\mu_s}$ (Table 8.4) the average $\overline{\mu_b}$ may be computed from the ratio $\frac{\mu_s}{\mu_b}$. This value is then compared with the average $\overline{\mu_b}$ from Table (8.3). This indicates that the coefficient of friction in the experimental extruder is larger than that predicted from theory by a factor of 3.

8.5. Concluding Remarks

The pressure generation process which occurs during the single screw extrusion of maize powder is amenable to analysis based on a model derived from a uni-axial compaction theory. The experimental measurements were compared with theoretical predictions from the Janssen-Walker (J-W) analysis and a modified Janssen-Walker (M.J.W.) analysis which caters for the variation in the coefficient of friction with the induced normal pressure.

The theoretical predictions yield the lower limit of the possible range of stresses in plastic equilibrium. The necessary parameters μ , μ_b , ϕ , and K_w for use in eq 8-5 are calc. (6.11), (6.12), (3.20) and (3.12).

The MJW analysis gives a better description of the experimental data. It was the low stress limit (Active) of the possible range of stresses in plastic equilibrium, which was appropriate and not the passive analysis as thought previously. A sensitivity analysis of eq. (8.5) indicates that if the modulus E is reduced by a factor of 10 the predicted value of σ_{xx_w} will increase by a factor of 30 and if the product $\alpha\tau_0$ is increased by a factor of 10, the predictions increase by a factor of 9. These indicate the errors involved in the estimation of E would predominate. In the analysis described in this chapter, it was the optimised modulus E_{opt} from the rheogoniometer data modelling which was used. The pressures generated during shear flow in the rheogoniometer were lower than those developed in extrusion. Therefore it follows that the appropriate value of the modulus for extrusion will be lower than the E_{opt} used. Further, particle dilatancy effects which were described in chapter 7 were not considered in the extrusion analysis. Unfortunately it was not possible to measure any pressure from the P_1 transducer. Had this

been possible the effects of particle dilatancy could have been explored and the quality of the simulation of the low pressure flow zone described in chapter 7 could have been tested directly. The variation of the K_w ratio with temperature has been neglected. However, if the interparticle friction follows similar trends to that of interface friction (chapter 5) the angle of internal friction ϕ_i will reduce with increasing temperature. This would result in the K_w ratio approaching unity with increasing temperature. On this basis, the differences in the trends shown by experiment and theory in predicting the length of the feed zone can be rationalised. Irrespective of these discrepancies a good prediction of the pressure profile and the length of the feed zone in a single screw maize extruder has been achieved from a theory based on the interface traction developed for a relatively smooth surface.

Table 8.1

Screw surface temperatures T_s , computed from eq. (8.1), (8.2) and (8.3) for the screw heating experiments.

Oil bath set temp. °C	Thermocouple temp. during sampling T_i °C	Computed screw surface temp. R_s °C
20	35	32
40	45	40
60	66	59
70	77	66
80	93	79
120	130	117
150	140	130

Table 8.2

The experimental values of the length of the feed zone L_f
in (m) for barrel heating experiments

Barrel set temp. °C	Length of feed zone (m)
20	0.5
80	0.4
120	0.4

Table 8.3

The coefficient of barrel friction μ_b computed from eq. (6.12) as a function of normal pressure for barrel heating experiments.

Pressure in (Pa)

	20°C		80°C		120°C	
	σ_{xx_b}	μ_b	σ_{xx_b}	μ_b	σ_{xx_b}	μ_b
P ₂	9.7 x 10 ⁴	0.060	9.7 x 10 ⁴	0.05	-	-
P ₃	8.25 x 10 ⁵	0.045	2.4 x 10 ⁵	0.05	1.65 x 10 ⁵	0.05
P ₄	10.08 x 10 ⁵	0.044	26.7 x 10 ⁵	0.04	11.25 x 10 ⁵	0.04
P ₅	42 x 10 ⁵	0.040	64 x 10 ⁵	0.03	88.7 x 10 ⁵	0.03
P _D	171 x 10 ⁵	-	154 x 10 ⁵	-	175 x 10 ⁵	0.03

Table 8.4

The coefficient of screw friction μ_s computed from eq. (6.12) as a function of normal pressure for barrel heating experiments.

Pressure in (Pa)

	20°C		80°C		120°C	
	σ_{xx_b}	μ_s	σ_{xx_b}	μ_b	σ_{xx_b}	μ_s
P ₂	6.8 x 10 ⁴	0.064	6.8 x 10 ⁴	0.064	—	—
P ₃	5.8 x 10 ⁵	0.046	1.6 x 10 ⁵	0.056	1.15 x 10 ⁵	0.06
P ₄	7.02 x 10 ⁵	0.046	18.6 x 10 ⁵	0.041	7.84 x 10 ⁵	0.044
P ₅	29.3 x 10 ⁵	0.040	44.6 x 10 ⁵	0.040	61.8 x 10 ⁵	0.04

Table 8.5

The variation of the coefficients of friction μ_b and μ_s with normal pressure for the 150°C screw heating experiments.

Pressure in (Pa)

	σ_{xx_b}	μ_b	σ_{xx_s}	μ_s
P ₂	-	-	-	-
P ₃	2.3 x 10 ⁵	0.07	1.6 x 10 ⁵	0.06
P ₄	11.6 x 10 ⁵	0.06	8.1 x 10 ⁵	0.05
P ₅	56.8 x 10 ⁵	0.05	39.6 x 10 ⁵	0.04

Table 8.6

Predictions from Janssen-Walker (J-W) and Modified Janssen- Walker (M.J.W.)
together with experimental measurements for barrel heating experiments. σ_{xx_b} (Pa)

Post. of Transd. distance from feed pocket [m]	Exp.	20°C		Exp.	80°C		Exp.	120°C	
		J-W	M.J.W.		J-W	M.J.W.		J-W	M.J.W.
0.35	9.7×10^4	-	-	9.7×10^4	-	-	-	-	-
0.41	8.3×10^5	0.7×10^5	4.4×10^5	2.4×10^5	0.6×10^5	5.7×10^5	1.6×10^5		
0.47	10.1×10^5	1.4×10^5	7.7×10^5	26.7×10^5	1.31×10^5	10.0×10^5	11.2×10^5	1.2×10^5	11×10^5
0.56	42×10^5	3.5×10^5	14.16×10^5	64×10^5	3.15×10^5	15.3×10^5	88.7×10^5	2.6×10^5	19×10^5

Table 8.7

The experimental and theoretical predictions from eq. (8.4) and (8.5)
of the pressure generation for the 150°C screw heating experiment.

Pressures in (Pa)

Pos'n. Of Transducers Dist. from feed pocket(m)		Exp.	J-W	M.J.W.
0.35	P ₂	-	-	-
0.41	P ₃	2.3 x 10 ⁵	-	-
0.47	P ₅	11.6 x 10 ⁵	3 x 10 ⁵	7 x 10 ⁵
0.56	P _D	56.5 x 10 ⁵	9 x 10 ⁵	35 x 10 ⁵

Table 8.8

The experimental and theoretical predictions from eq. (8.9) and (8.10)
for the length of the feed zone L_f(m)

Barrel set temp. °C	Exp.	J-W	M.J.W.
20	0.5	0.6	0.4
80	0.4	0.6	0.4
120	0.4	0.7	0.5

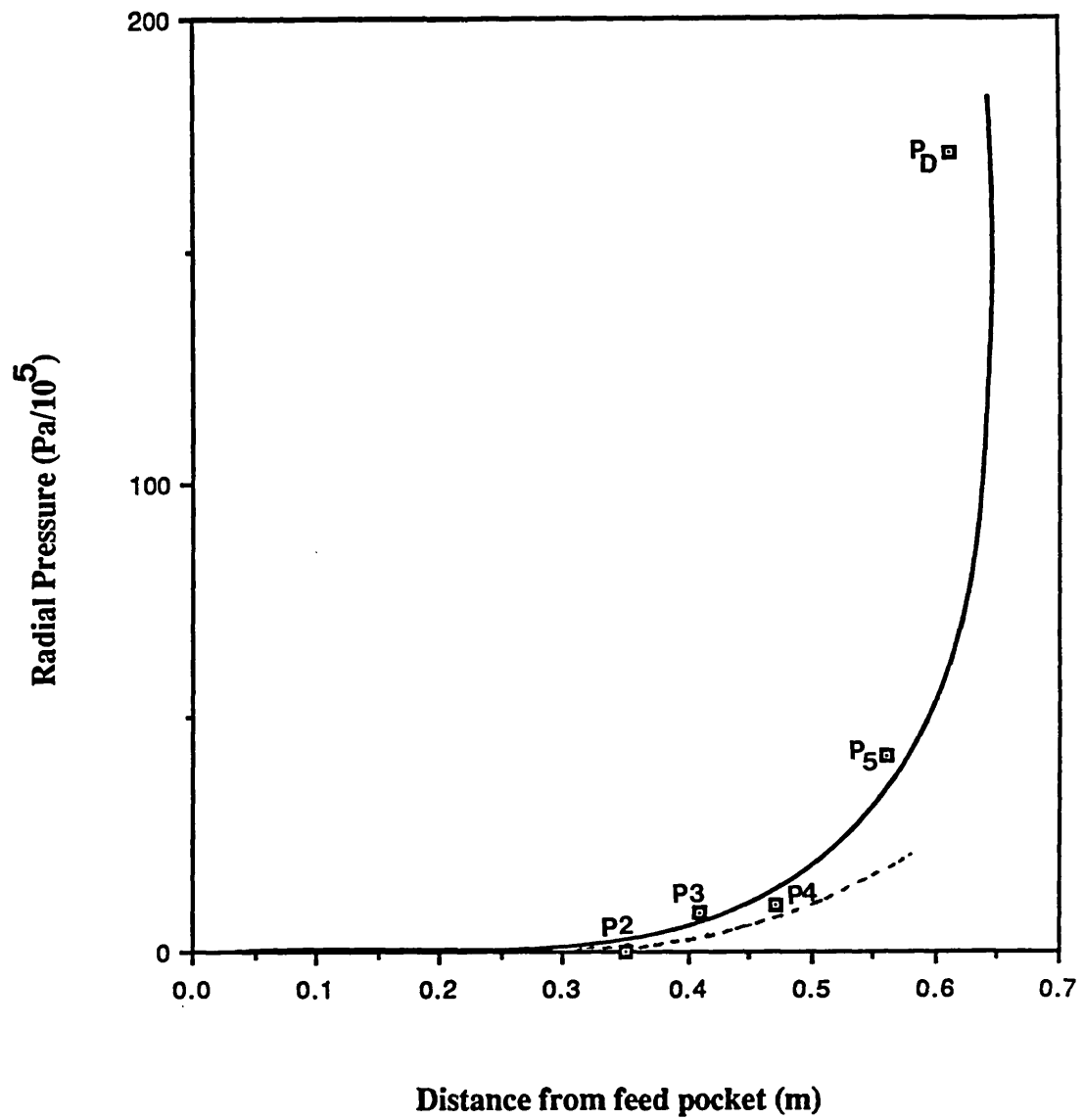


Fig.8.1 The measured barrel pressure along the extruder. Both the barrel and screw surfaces are at ambient. -----Theoretical lower limit (MJW)

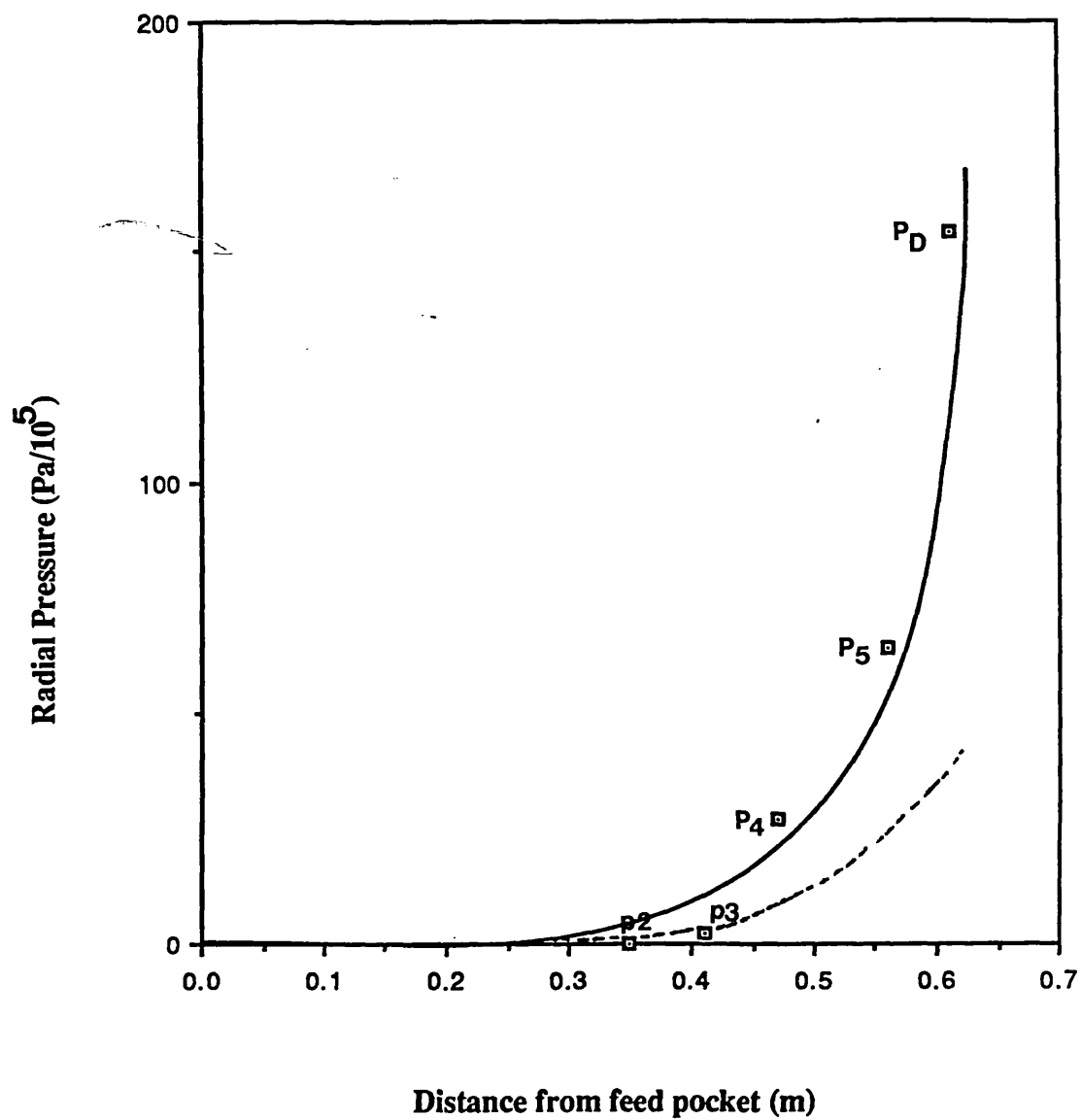


Fig.8.2 The measured barrel pressure along the extruder. Barrel heated to 80°C. Screw surface at ambient. - - - - -Theoretical lower limit[MJW]

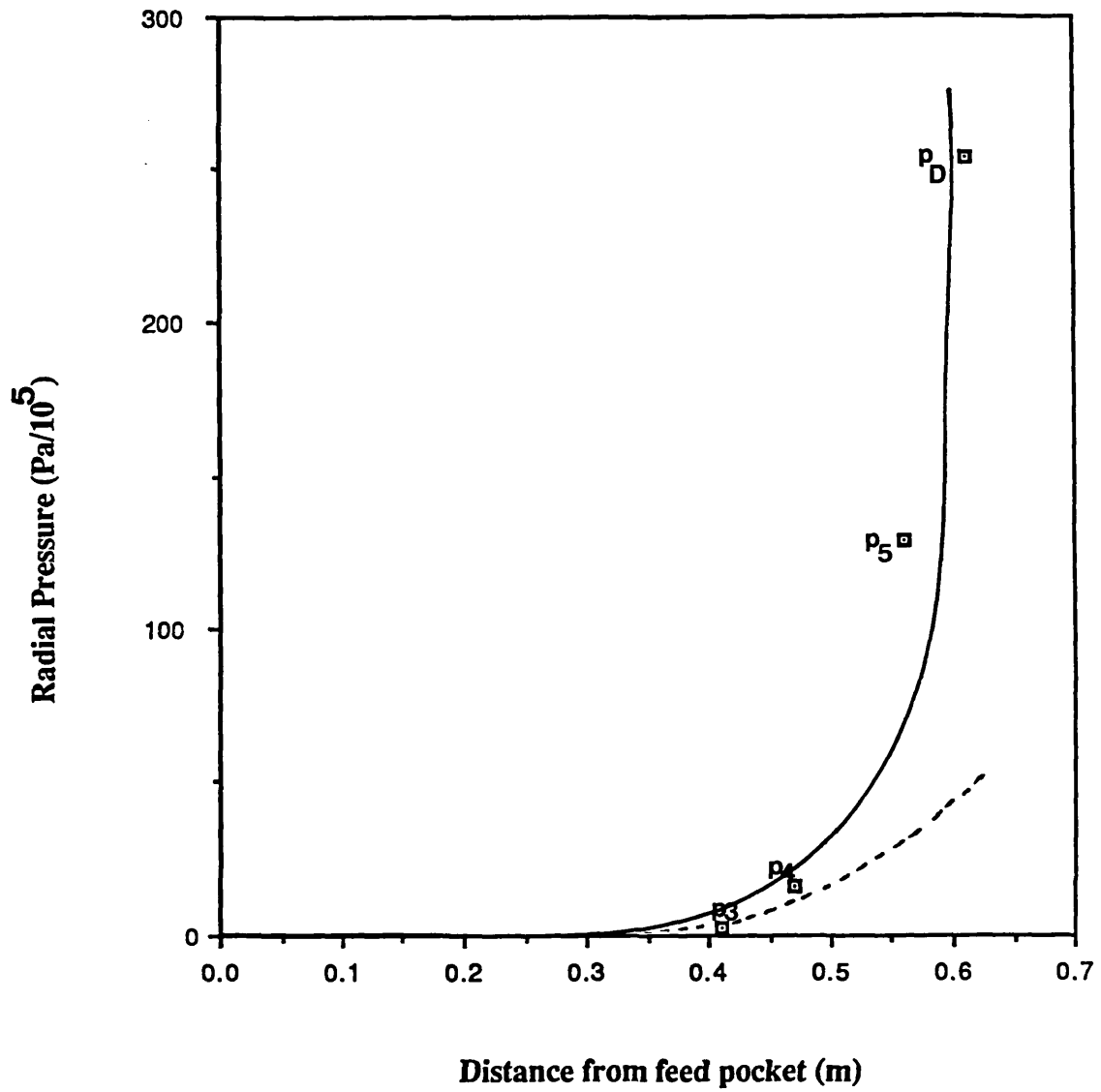


Fig.8.3 The measured barrel pressure along the extruder. Barrel heated to 120°C. Screw surface at ambient. ----- Theoretical lower limit [MJW]

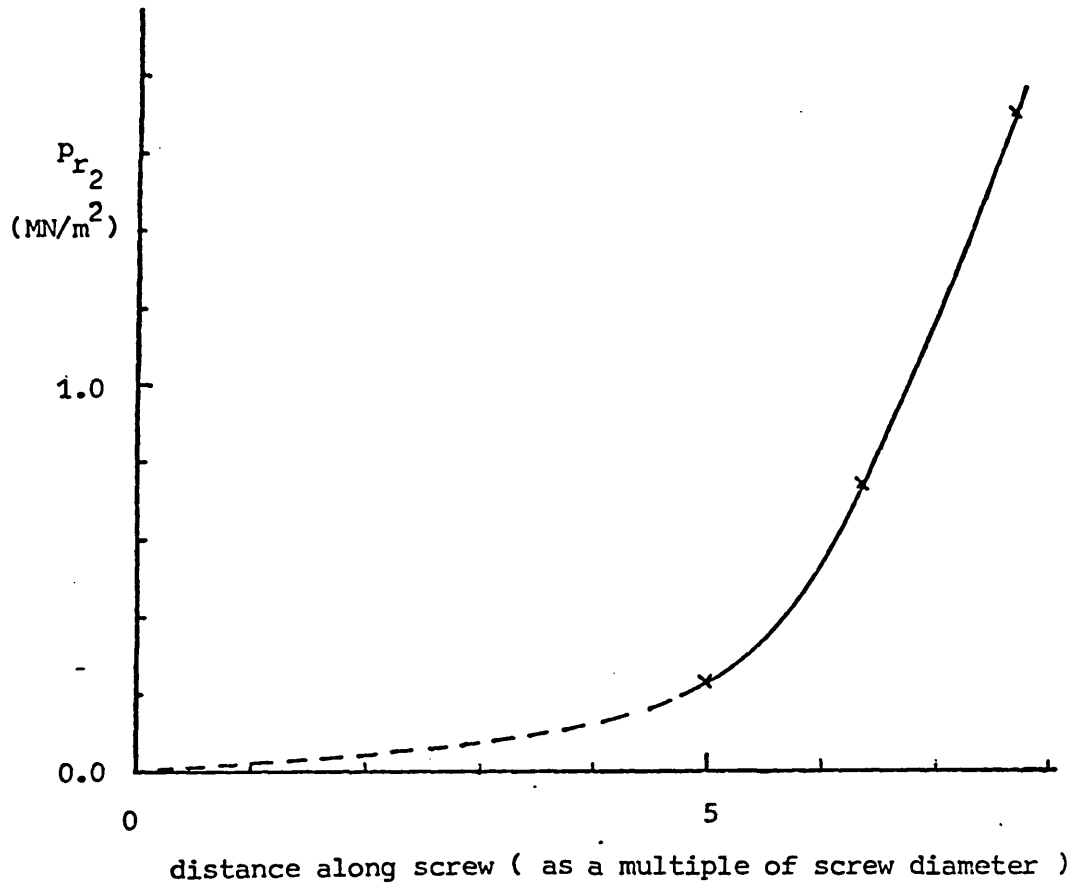


Fig.8.4 The measured barrel pressure along the extruder by Lovegrove ⁽²⁴⁾.

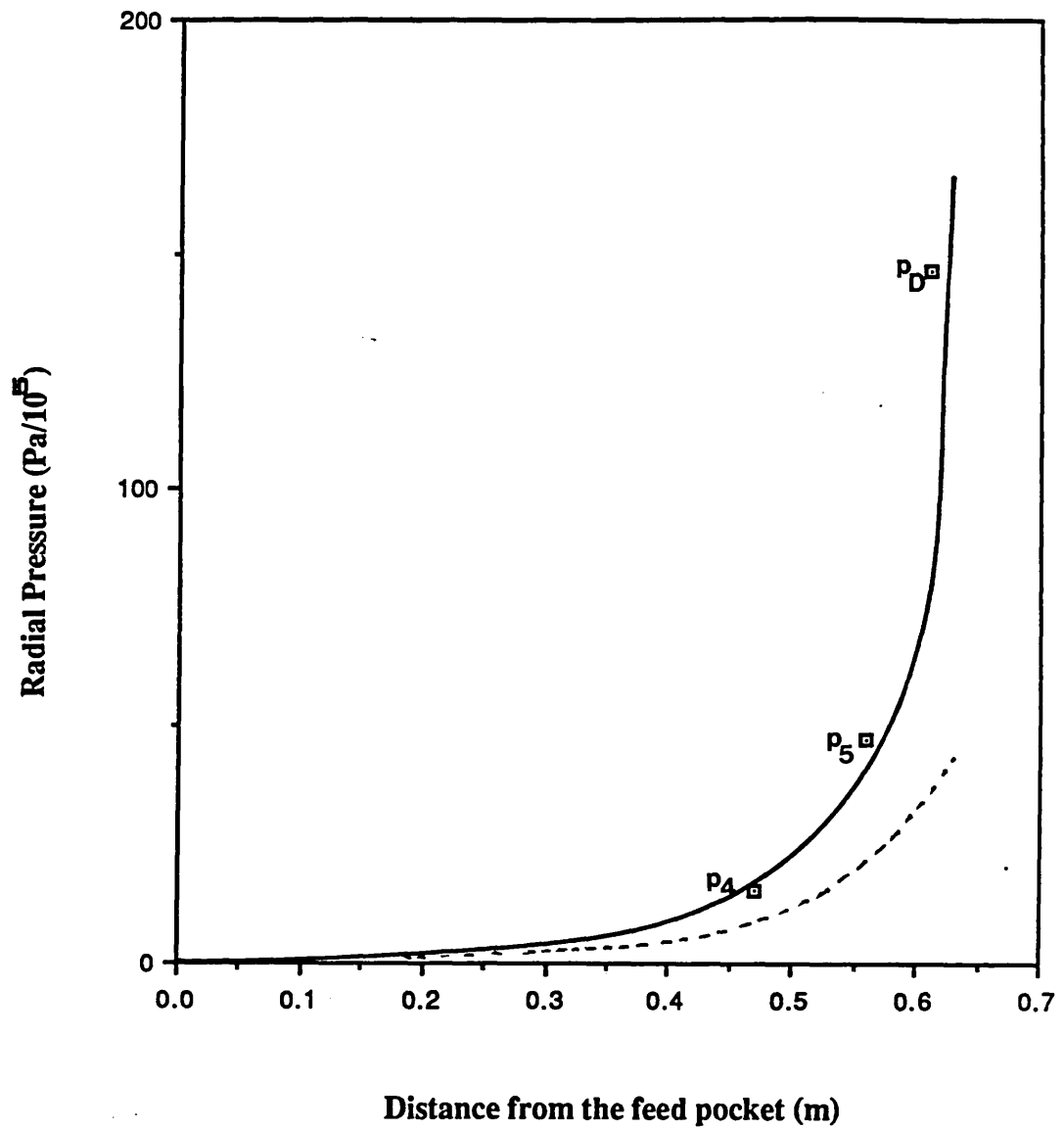


Fig.8.5 The measured barrel pressure along the extruder. Barrel heated to 50°C. Screw heated to 120°C. - - - - Theoretical lower limit(MJW)

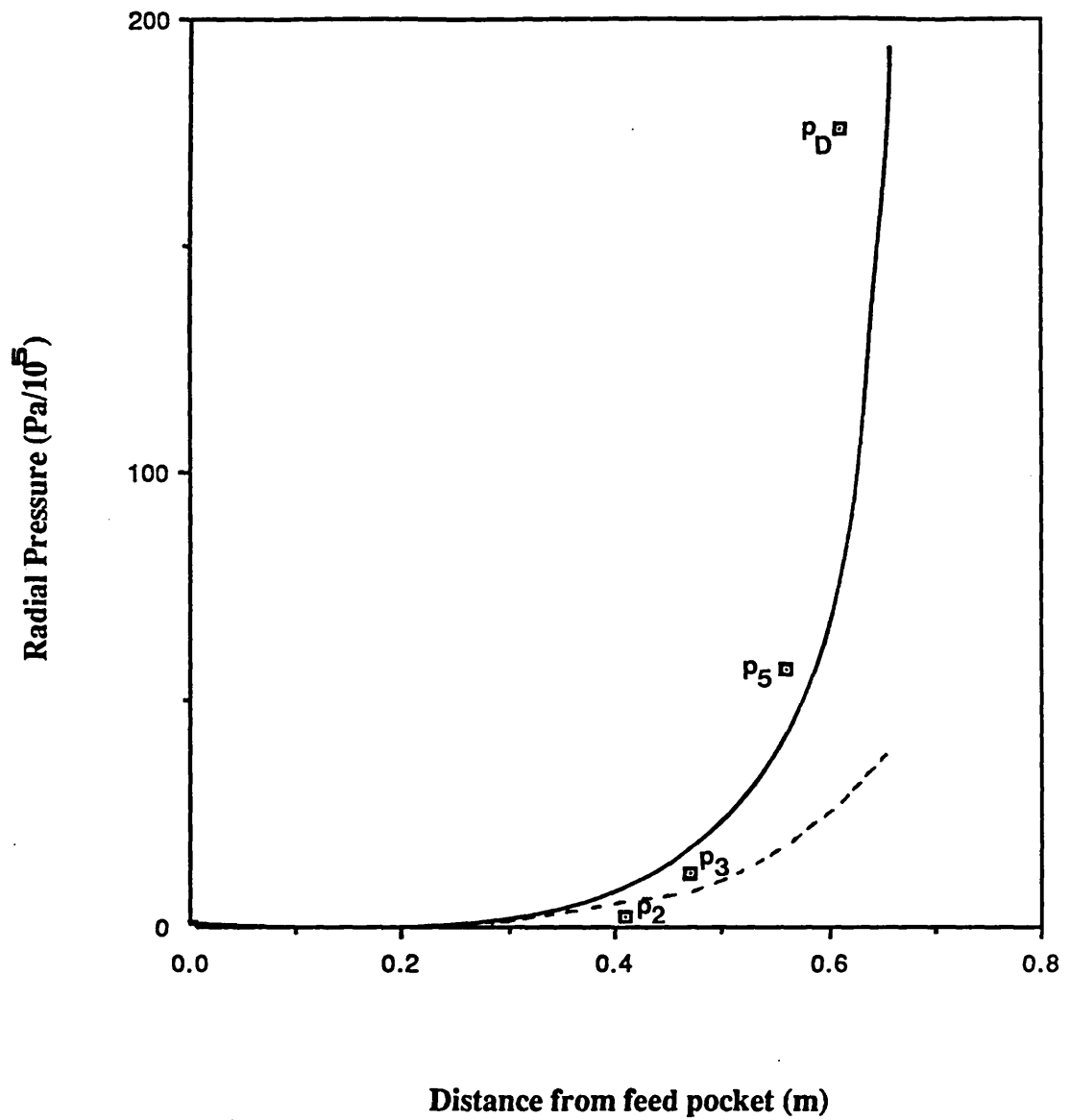


Fig.8.6 The measured barrel pressure along the extruder. Barrel heated to 50°C. Screw heated to 150°C. -----Theoretical lower limit(MJW)

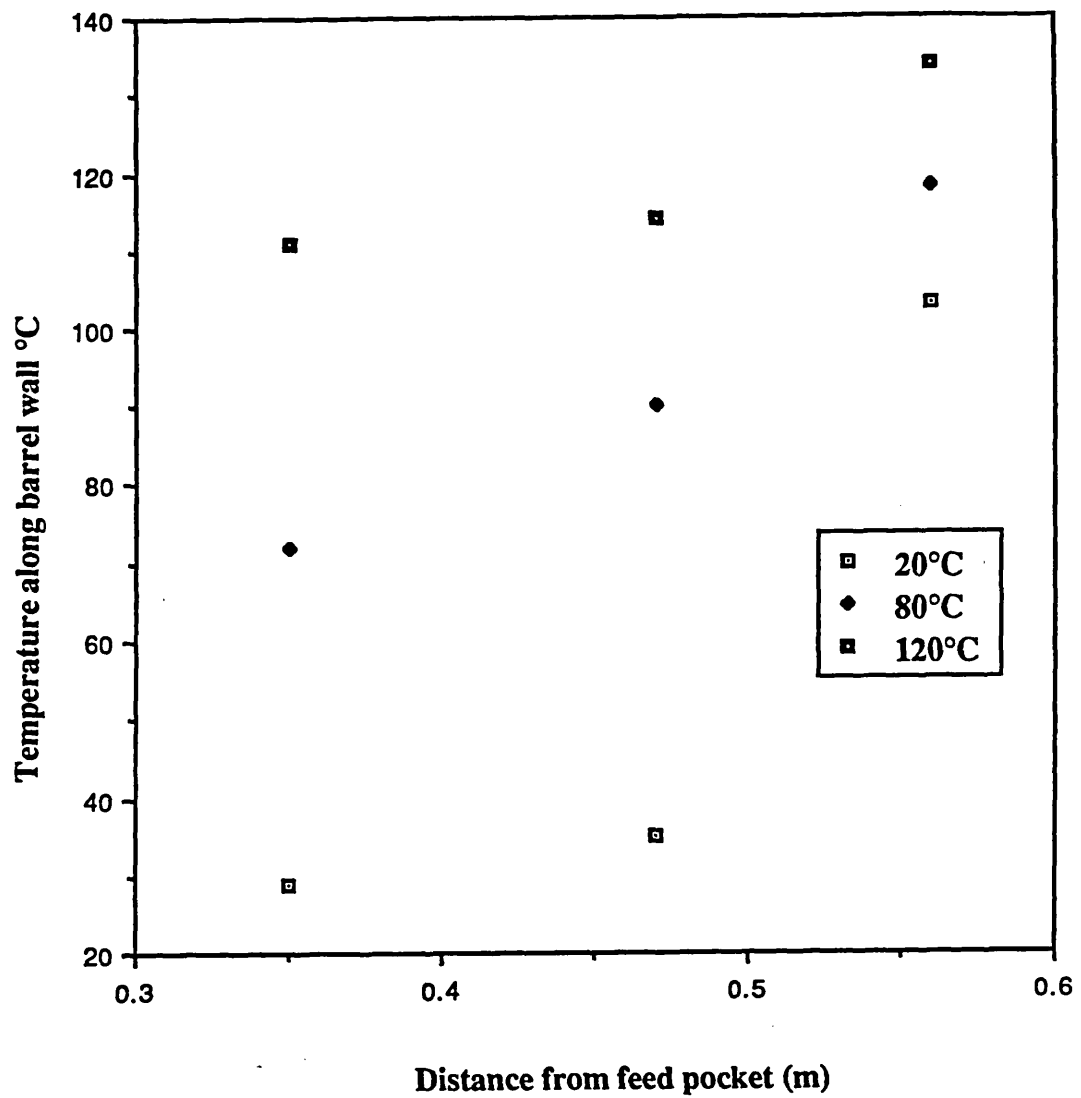


Fig.8.7 The measured temperature profile along the extruder for barrel heating experiments.

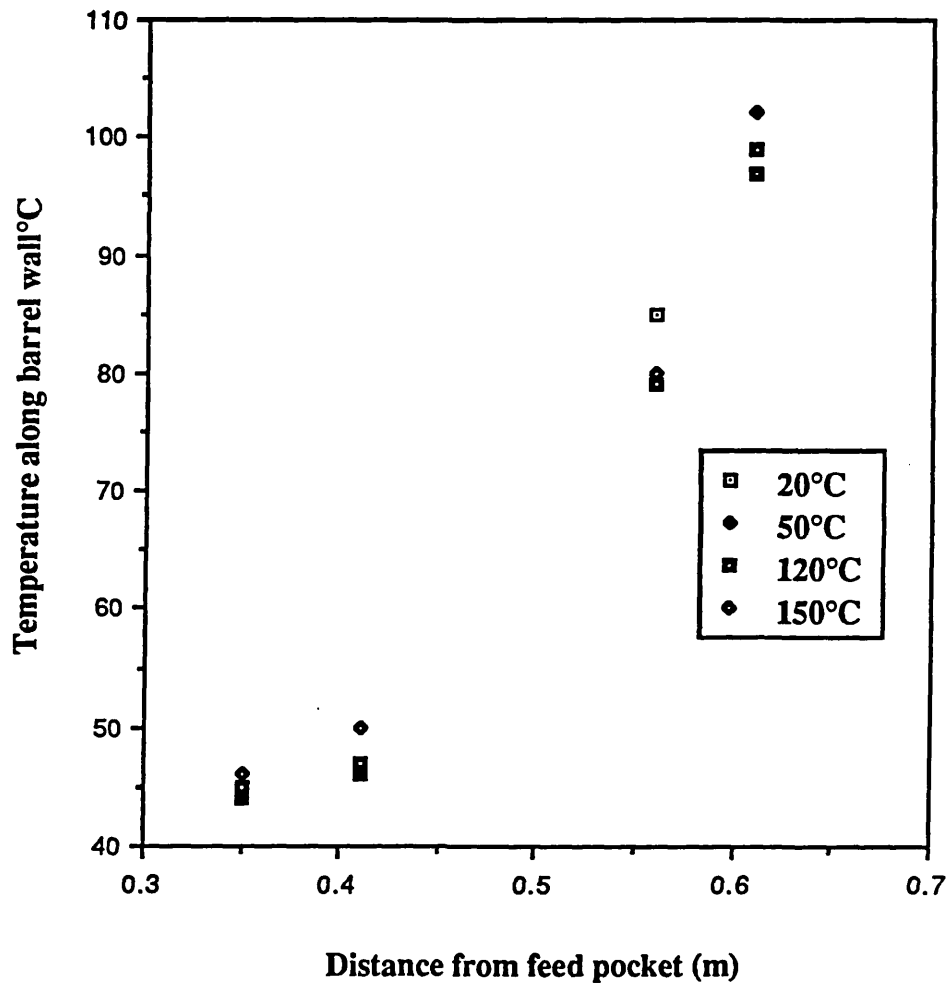


Fig.8.8 The measured temperature profile along the extruder for screw heating experiments.

Chapter 9

Conclusions

This thesis has sought to establish if it is realistic and predictively accurate to assume that the feed zone of a single screw extruder induces a particle deformation field which is very similar to that created in simple uni-axial compaction. This has involved the development and evaluation of models to describe uni-axial compaction processes for a starch based powder and the application and extension of these models to the feed zone. The necessary interfacial and material properties have been obtained by a range of simulations. The major conclusion of the thesis is that this approach provides a reasonably self consistent and predictively accurate account of the major features of the particle flow in the feed zone. A number of subsidiary conclusions may be drawn;

- (i) The wall traction processes which are generated in either uni-axial compaction and the feed zone may be effectively simulated. For high normal stresses hemi-spherical compacts are a convenient form of simulation but for the low normal stress range a semi-constrained flow is more suitable. The measured tractions are functions of many variables including the normal stress σ_n , the temperature T , and the moisture content.
- (ii) The friction data are reasonably well modelled using

established elastic contact mechanical models for rough surfaces and The Adhesion Model of Friction. A more interesting result is observation of the Archard precursor load index of $8/9$ for elastic asperities in flow. The effects of water and temperature are adequately described. The higher cases of the Archard contact mechanical models are approached with the addition of water. In the low normal stress range particle dilatancy may exist and the coefficient of friction in this dilute phase is a notable function of the induced normal stress.

- (iii) The effective bulk stress transmission parameter, K_w , has been measured directly in uni-axial compaction and computed from shear test and wall friction data. Further, the computed great depth value for the distribution factor $\mathcal{D}$ is unity for the particular combination of the internal and wall angles of friction ϕ_i and ϕ_w appropriate for the present systems.
- (iv) The bulk and interfacial parameters have enabled the testing of the Janssen-Walker uni-axial compaction model for active failure. These predictions are based on an average value of the coefficient of friction, and may be compared with predictions from an extended Janssen-Walker analysis which effectively caters for the dependency of the coefficient of friction on normal pressure. The other established methods for the prediction of wall stresses, the method of characteristics and Coulomb's method of wedges, were concluded to be less appropriate for the

treatment of the current problem.

- (v) The combination of an extended Janssen-Walker model with the appropriate wall and bulk parameters included in a Darnell and Mol extruder model produces explicit relationships for the pressure profile and feed rates. These analyses consider the conveying to be largely governed by the barrel tractions. The predictions are in good agreement with experiment and further indicate the trends associated with changes in the processing variables. The variation of the stress transmission ratio, K_w , with temperature is apparently an important consideration in the feed zone length predictions.

The sought after conclusion of identifying the similarity of uni-axial compaction and screw induced traction induced compaction model is not proven. However, it is found that the equations appropriate to describe uni-axial compaction after some modification also adequately describe the traction induced compaction process in the feed zone. The main implication of the conclusions is that it is viable to predict *a priori* the pressure profile, feed zone length and throughputs from a single screw extruder based upon a series of simple simulations and a modified Janssen-Walker compaction theory. These studies also elucidate some of the important tribological factors present in the processing of soft, highly deformable solid powders.

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