

Numerical Modelling of Wave Interactions with a Vertical Surface-piercing Column

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Declaration of originality

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Abstract

This thesis concerns the numerical modelling of the wave interactions with a vertical, surface-piercing, column. The study focuses on the unexpected high-frequency wave scattering and the nonlinear loading of columns located in the inertia regime. The test cases include both regular and irregular incident waves. With the wave-column interactions involving highly nonlinear free-surface effects, the two-phase Volume of Fluid (VOF) model adopted. This choice was based upon preliminary calculations in which the model was shown to be capable of simulating appropriate extreme wave conditions. These calculations involved a wide range of incident wave conditions including as standing, regular, irregular and breaking waves. In all cases, comparisons with existing theory and lab data confirmed that the waves were successfully simulated.

The numerical method was first employed in the simulation of *Type-1* and *Type-2* scattered waves due to the interaction between incident regular waves and a vertical column. Comparisons to the laboratory data presented in Swan & Sheikh (2015) allowed a rigorous validation of the proposed method. Further investigation to determine the dependency of the scattered waves on the size of the column and the incident wave steepness was undertaken. For wave-structure interactions involving irregular focused waves, an additional wave scattering mechanism was also identified. The physical origins of this new mechanism were carefully investigated and shown to be closely linked to the increased steepness of the wave cases. The nonlinear wave-wave interactions between the incident and scattered wave fields, for both regular and irregular waves, were examined and their practical implications discussed.

The nonlinear loading, in particular, the *secondary loading cycle* was successfully predicted using the numerical model. Frequency analysis and comparisons with available analytical force models were carried out to highlight important changes in the higher order force components, particularly in respect of increasing wave steepness and varying column diameter. A clear linkage between the *secondary loading cycle* and high-frequency wave scattering has been established and the physical origins of the former studied and explained.

Finally, the importance of the *secondary loading cycle* in provoking a dynamic excitation was investigated using a simple single degree of freedom model. The results reveal a large dynamic excitation in both regular and irregular waves in the presence of the *secondary loading cycle*. This has important practical implications, not least because the load models commonly applied in design are not able to predict the *secondary loading cycle*. As such, they will undoubtedly under-estimate both the magnitude and occurrence of dynamic excitation. Key insights into this important problem are provided.

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1

Introduction

1.1 Overview

Worldwide energy demand is rising at an alarming rate. According to the International Energy Outlook (EIA, 2013), the total world energy consumption will grow by 56 % in the next three decades. Much of this increase is expected to occur in developing countries such as China and India due to strong economic growth and expanding populations. Given this increase, political and industrial groups have expressed their concern over the shortage of energy supply in the coming years. This has added to the challenges for energy companies to keep up with ever-increasing demand. With the depletion of current hydrocarbon deposits, energy companies are being forced to broaden their portfolio and secure new energy reserves. Large multinational oil companies have turned their attention to abundant supplies of natural gas in deep water as a potential energy reserve. This has forced the industry to embark upon exploration and production in deeper waters that are potentially subject to more severe storm conditions.

As production moves to deeper water, offshore structures are exposed to greater risk from the environmental conditions such as wind, waves, and currents. Of these, waves are probably the most important and significant environmental concern for offshore structures due to the fact that they exert the largest and the most damaging

forces. Therefore, careful consideration of the wave forces is important, especially in the planning and design stages. In an offshore design context, the structure needs to be designed to withstand and/or avoid loading from the waves. For substructures, such as the legs of a steel jacket platform and/or a Gravity Based Structure (GBS), wave loading is unavoidable. In order to ensure that the substructures do not fail when subjected to large forces from the wave, their ultimate limit strength needs to be larger than the applied load. In traditional design methodologies where the understanding of the wave process was not well advanced, a large safety factor was used to take into account the uncertainty in the wave forcing. This can lead to an over designed structure which is not only expensive, but also excessively heavy, requiring a much more sophisticated design of the foundations. On the other hand, the design of the topside structure principally concerns the choice of a suitable deck elevation to avoid wave-in-deck loading. If the deck is placed too low, it is susceptible to green water effects and impact from the undersides. In contrast, a higher deck placement increases the construction costs and exposes the structure to large overturning moments from the wind.

In recent years, with the increase in mean sea level and storm severity, the risk associated with the environmental conditions has increased tremendously. For example, in 2005, during hurricanes Katrina and Rita in the Gulf of Mexico, 113 oil platforms were destroyed (Guidotti, 2006) while many others suffered extensive local damage. This caused significant financial loss to the oil and gas industry with a huge economic impact that was felt around the globe. Indeed, the failure statistics in these storms has raised questions regarding the adequacy of current design standards. Therefore, in order to provide reliable and safe future design, there is a need to increase our understanding of the physical mechanism that governs extreme waves and their interactions with offshore structures.

1.2 Research Motivation

The motivation of this research arose from a series of studies to explain the damage sustained at the Brent Bravo GBS (Figure 1.1), in a relatively mild storm in January 1995. During this event, an unexpected wave impact occurred on the underside of the deck causing temporary disruption and significant local damage. This was inconsis-



Figure 1.1: The Brent oil platforms. From left to right; Alpha, Bravo, Charlie and Delta. Image taken from Shell (2016).

tent with observations made in the same storm at an adjacent platform, Brent Alpha, where no damage was recorded. Observations by Swan et al. (1997), indicated that the difference between the waves observed by Brent Bravo and Brent Alpha were due to their structural configuration. Brent Alpha is a typical steel jacket structure, while Brent Bravo is a GBS with three legs of relatively large diameter, located close to each other. Although each individual leg lies outside the linear diffraction regime (the latter defined by $D/\lambda > 0.2$ (Havelock, 1940), where D is the column diameter and λ the wavelength), the interaction between the incident waves and the structure produced unexpected high-frequency wave scattering. Subsequent interactions between the incident waves and the scattered waves produced substantially increased water surface elevations, causing damage to the platform. Studies by Ohl et al. (2001b,a) confirmed the observations by Swan et al. (1997) and further demonstrated that wave nonlinearity plays a critical role in the occurrence of larger crest elevations when compared to the linear diffraction theory.

In order to provide a much better explanation of the physical process involved, Sheikh (2004) conducted a scaled model study involving a single leg of the Brent Bravo GBS. The study was further extended in Swan & Sheikh (2015). Taken to-

gether, these studies reported the generation of two undocumented scattered wave modes radiating outwards from the column. The interaction between the incident and scattered waves further justified the observations made by Swan et al. (1997); that significant amplifications of the water surface elevation will arise in steep waves. The occurrence of wave breaking was also observed in the form of vertical jetting and wave over-turning. While the former has serious implications for the provision of a suitable deck elevation, the latter will give rise to uncertainty in the force prediction, particularly the occurrence of wave slamming.

Accurate force predictions for each structural member due to the action of waves is essential in the design of any offshore structure. For a slender structure, the commonly adopted method of load calculation is based on Morison's equation, or an improved slender body approximation. On the other hand, a linear or higher-order diffraction solution is used for large volume structures. Recently, Masterton (2006), carried out an extensive laboratory study of both the scattered wave field and the applied loading on columns of varying diameter. These results suggested the existence of high-frequency loading components in some of the wave cases considered. Further study revealed that none of the existing force models can accurately predict the existence of these highly nonlinear loading components. Given the nonlinear and complex interactions arising in realistic design sea states, there will be many examples in which these commonly adopted force models will be inadequate, failing to describe both the magnitude and the frequency of the applied loads.

In the design of many offshore structures, particularly where they involve a degree of novelty, detailed design work is often supported by data from scaled laboratory tests. Laboratory or model testing has long been considered to be the best source of reliable information regarding the realistic behaviour of waves and their interaction with large volume structures. Realistic sea states can be simulated in a laboratory environment involving highly nonlinear, random and directionally spread sea states. Furthermore, extreme wave conditions involving wave breaking and slamming gives the best possible description of the nonlinear wave-wave and wave-structure interactions arising in a given sea state. However, model testing is expensive and problem specific; critically dependent upon both the structural configuration and the relative size of the structural members, D/λ . Furthermore, model testing does not always represent an accurate description of the actual conditions owing to scale effects.

With the advancement of computational procedures and the increase in computational power, there has been a shift from expensive laboratory experiments towards the utilisation of nonlinear numerical methods for solving wave-structure interaction problems. Indeed, the present state-of-the-art is one in which nonlinear numerical models can provide an alternative methodology to further understand the complex phenomenon of wave-structure interaction.

1.3 Aim and Objectives

The aim of the present study is to explain the complex phenomena of high-frequency wave scattering and the nonlinear loading of a vertical surface-piercing column using nonlinear numerical methods. The objectives of the study are to:

1. Numerically predict the high-frequency wave scattering observed by Swan & Sheikh (2015).
2. Provide a theoretical explanation for the origin of the high-frequency wave scattering in both regular and irregular waves.
3. Investigate the importance of wave scattering, particularly its relevance for the prediction of an effective air-gap; the latter defined as the distance between the underside of the deck structure and the maximum incident crest elevation.
4. Confirm the existence of the so-called *secondary loading cycle* in both regular and irregular waves, providing a physical explanation for its occurrence.
5. Identify and explain the connection, if any, between the nonlinear wave scattering and the *secondary loading cycle*.
6. Determine whether the *secondary loading cycle* can provoke a structural excitation leading to a *ringing*-type response; the latter occurring at frequencies well above the incident wave frequency.

1.4 Thesis Outline

This thesis is divided into six main chapters. An overview is given at the beginning of each chapter. This serves to set the specific work contained within the chapter in

the wider context of the thesis, as well as to introduce any additional background information that may be relevant.

Chapter 2 : This provides background information on the existing analytical wave and wave-structure interaction models. In addition, important state-of-the-art numerical methods used in the study of wave-structure interactions are presented.

Chapter 3 : This provides an introduction to the numerical code, OpenFOAM, used in the present study and the solution strategies to be employed.

Chapter 4 : This presents a detailed numerical validation of the various incident wave cases used through-out the study, with particular attention paid to the occurrence of wave breaking.

Chapter 5 : This presents the numerical simulation of regular waves interacting with a vertical surface-piercing column. The results are first validated against the experimental observation by Swan & Sheikh (2015). Once complete, a parametric investigation involving different incident wave conditions and varying column diameters is presented; the purpose being to provide detailed physical insights into the occurrence of the unexpected modes of wave scattering.

Chapter 6 : This concerns numerical simulations of the interactions between a focused wave and a surface-piercing column; the aim being to identify whether the observed effects are similar to those recorded in regular waves. This represents a first step in determining the practical importance of the nonlinear scattering since focused waves are representative of the largest waves arising in a realistic irregular sea state.

Chapter 7 : This considers the nonlinear potential flow loading for a vertical surface-piercing column. The presence of a *secondary loading cycle*, believed to be responsible for the onset of structural *ringing*, is of primary interest; the results showing that the physics responsible for these loads is closely connected to the nonlinear scattering discussed earlier.

Chapter 8 : This chapter draws the conclusion of the present study and provide several suggestions for future work.

2

Literature Review

2.1 Introduction

This chapter presents the relevant background information relating to the modelling of the wave and wave-structure interactions. The chapter starts with the classification of the flow regime in the context of engineering design. This is followed by a review of the available analytical wave models and existing diffraction solutions; the former providing possible inputs to the numerical calculations and the latter solutions against which the numerical calculations can be compared. The final section of this chapter summarises the different numerical models that can be (have been) used in the study of wave and wave-structure interaction problems; the purpose being to identify the relative advantages of the different approaches.

2.2 Classification of Flow Regime

In the design of most offshore structures, the flow regime to which the structure is exposed can be determined by comparing the relative size of the structure to the incident wavelength. If the diameter of the structure, D , is small when compared to

the incident wavelength, λ , the structure is assumed to be slender. This is given by

$$\frac{D}{\lambda} < 0.2. \quad (2.1)$$

In this case, it is assumed that the body causes no disturbance to the incident flow, except for the development of a downstream wake and the loads are specified in terms of the incident flow conditions. This describes the so-called drag-inertia regime which is dependent upon the Keulegan-Carpenter number ($KC = UT/D$), where U is a characteristic velocity, T the wave period, and D the column diameter. In effect, KC describes the magnitude of the wave-induced orbital displacement (UT) to the column diameter. If $KC > 20$ the relative fluid displacement is large, the problem is then characterised by flow separation and wake formation, and the flow regime is drag dominated. Alternatively, if $KC < 5$ the relative fluid displacement is small, limited flow separation occurs, and the flow regime is inertia dominated. The intermediate regime where both the drag and the inertia elements are non-trivial occur between the range of $5 < KC < 20$. In the slender body regime, design calculations to determine, for example, wave overtopping or the extent to which waves pass over a structure, the effective air-gap and wave loading are based on the description of the incident waves alone. It therefore follows that an accurate description of the incident waves represents a critical input into the design process. Although this seems a trivial comment, the description of a realistic incident wave continues to represent a major challenge and is still an active area of research. The explanation for this lies in the fact that realistic design waves are highly nonlinear, unsteady and directional. A brief review of existing wave models is given in Section 2.3.

In contrast, if the ratio between the diameter of the structure and the incident wave-length is large,

$$\frac{D}{\lambda} > 0.2, \quad (2.2)$$

a significant disturbance of the flow field due to the presence of the structure is expected (Havelock, 1940). This disturbance arises in the form of diffracted or radiated waves; the former occurring when the structure remains static and the latter driven by the motion of the body. If both diffraction and radiation are present, then this is commonly referred to as wave scattering. For the conditions outlined in Equation (2.2) the flow is said to be in the linear diffraction regime, assuming a low steepness incident

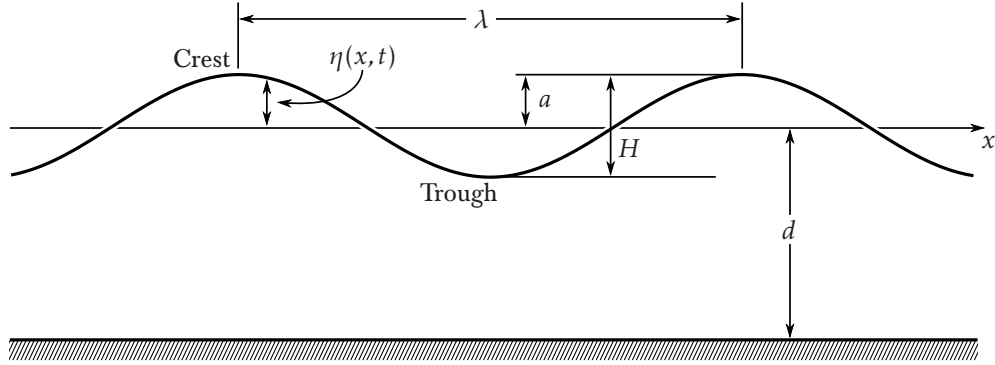


Figure 2.1: Wave characteristics

wave. This classification is entirely dependent upon the expected linear loads and does not therefore account (or relate) to the occurrence of any scattered wave field. For design purposes, any calculation should include the effect of both the incident wave and the disturbance caused by the wave-structure interaction. Much work has been carried out in this field; the available diffraction solutions being presented in Section 2.4.

2.3 Analytic Wave Models

A schematic of a two-dimensional regular wave propagating in the positive x direction is given in Figure 2.1. This represents the simplest wave form, with the surface elevation, $\eta(x, t)$, assumed to be sinusoidal in space (x) and time (t). Individual waves are commonly described by their wave height, H , wave period, T , and water depth, d . In addition, the wave amplitude, a , defines the crest elevation above the SWL ($z = 0$) and the phase speed or the wave celerity is defined by $c = \lambda/T$ or ω/k ; where k is the wave number ($2\pi/\lambda$) and ω the circular wave frequency ($2\pi/T$).

The physical behaviour of real gravity waves in nature is inherently complex. However, based on simplified theoretical assumptions, the wave system can be represented by mathematical equations in the form of an ordinary differential equation (ODE) or a partial differential equation (PDE). The ODE or PDE can be solved analytically subject to appropriate boundary conditions and the essential characteristics of the wave system can be extracted from this closed-form solution. Analytical solutions are extremely powerful as they provide both spatial and temporal information over the entire domain, enabling a full description of a specific wave system. Although

many wave systems can be formulated theoretically, solving them analytically is a non-trivial task. In the following section, analytic wave models commonly used in the design of offshore structures will be presented.

2.3.1 Governing Equations and Boundary Conditions

In deriving an analytical wave model, assumptions have to be made to reduce the complexity of the problem. For a gravity wave, the wave-induced fluid flow is assumed to be homogeneous and incompressible. Assuming that viscosity is only important within the bottom boundary layer, the majority of the fluid is assumed as inviscid. To further simplify the description of the fluid flow, it is assumed that the fluid remains irrotational, unless mechanisms such as wave breaking, flow separation, aeration and wake formation are present. Moreover, for a surface gravity wave, the length scales involved (and particularly the surface curvature) are such that surface tension can be neglected; the only restoring force being gravity.

Under these assumptions, the wave can be modelled using a potential flow theory. In this case, the velocity potential $\phi(x, y, z, t)$ is defined such that

$$\mathbf{u} = \nabla \phi, \quad (2.3)$$

where $\mathbf{u} = (u, v, w)$ is the velocity vector describing the wave-induced fluid velocities, $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$ is the gradient operator and (x, y, z) represent the Cartesian coordinate system with z measured vertically upward from still water level (Figure 2.1). With the introduction of ϕ , mass continuity is given by the Laplace equation such that

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0. \quad (2.4)$$

This defines the governing field equation. For the wave field to be uniquely defined, this equation must be subject to a number of boundary conditions. At the free surface ($z = \eta(x, y, z, t)$), two boundary conditions apply. The first is given by the Kinematic Free Surface Boundary Condition (KFSBC). This requires the water surface to be a streamline and, hence, the water particles on the free surface remain there. This

condition is defined by

$$\frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial \eta}{\partial y} - \frac{\partial \phi}{\partial z} = 0 \Big|_{z=\eta}. \quad (2.5)$$

The second boundary condition requires the pressure, p , to be constant on the free surface. This is referred to as the Dynamic Free Surface Boundary Condition (DFSBC). Adopting Bernoulli's equation, it can be expressed as

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 + \left(\frac{\partial \phi}{\partial z} \right)^2 \right] + g\eta = \text{constant} \Big|_{z=\eta}, \quad (2.6)$$

where g is the gravitational acceleration.

Assuming the sea bed, defined by $z = -d$, is both horizontal and impermeable, a no flow boundary condition must be applied such that

$$w = \frac{\partial \phi}{\partial z} = 0 \Big|_{z=-d}. \quad (2.7)$$

The real challenge in obtaining an analytical solution for a water wave problem is usually associated with the free surface boundary conditions, Equation (2.5) and Equation (2.6). The difficulty arises because the free surface is part of the solution to be obtained, and cannot be known a priori. Simultaneously solving the nonlinear PDE on an unknown surface is notoriously difficult. To overcome this problem, the equations are commonly linearised; the result either being used as an appropriate approximation or as an initial estimate from which the nonlinear terms can be progressively estimated.

2.3.2 Regular Wave

Small Amplitude Wave Theory

The simplest form of wave model is given by the so-called Small Amplitude Wave Theory (SAWT). This provides the description of a regular wave train and was first derived by Airy (1841). The difficulty associated with the nonlinear boundary conditions is alleviated by expanding the free surface, $\eta(x, t)$ about the SWL ($z = 0$) by means of a Taylor's series expansion. The expansion is expressed in term of wave

steepness, ak , where the wave amplitude, a , is assumed to be small in comparison to both the wavelength, λ , and water depth, d . Adopting this approach, the KFSBC and DFSBC can be linearised allowing the solution for ϕ and η to be defined by

$$\phi(x, z, t) = \frac{a\omega}{k} \frac{\cosh(k(z+d))}{\sinh(kd)} \sin(kx - \omega t) \quad (2.8)$$

and

$$\eta(x, t) = a \cos(kx - \omega t). \quad (2.9)$$

In deriving this solution, the DFSBC defines the relationship between the wave-number, k and angular frequency, ω . This relationship is commonly referred to as the dispersion equation, and defined by

$$\omega^2 = gk \tanh(kd). \quad (2.10)$$

Stokes Waves

To overcome the limitations of this linear approximation, which restricts the applicability of the solution to waves of small amplitude, Stokes (1847) developed a second-order model in an attempt to improve the fit to the nonlinear free surface boundary condition. This was achieved by including second-order terms in the perturbation expansion, effectively producing a series solution for η and ϕ . Based upon this approach, a higher order solution can be derived by incorporating more terms in the series expansion. The additional terms arising from this series expansion are interpreted as additional wave components that are needed to satisfy the nonlinear free surface boundary condition. These waves do not satisfy the dispersion equation and are bound to the linearly propagating wave component. The presence of these bound waves is responsible for the crest-trough asymmetry observed in regular waves. More recently, analytical solutions up to fifth-order of wave steepness has been derived by, for example Fenton (1985). The fifth-order Stokes solution for ϕ and η are given by

$$\phi(x, z) = -\bar{u}x + C_o \left(\frac{g}{k^3} \right)^{\frac{1}{2}} \sum_{i=1}^5 \epsilon^i \sum_{j=1}^i A_{ij} \cosh(jkz) \sin(jkx) \quad (2.11)$$

and

$$\begin{aligned}
k\eta(x, t) = & \epsilon \cos(kx) \\
& + \epsilon^2 B_{22} \cos(2kx) \\
& + \epsilon^3 B_{31} [\cos(kx) - \cos(3kx)] \\
& + \epsilon^4 [B_{42} \cos(2kx) + B_{44} \cos(4kx)] \\
& + \epsilon^5 [-(B_{53} + B_{55}) \cos(kx) + B_{53} \cos(3kx) + B_{55} \cos(5kx)],
\end{aligned} \tag{2.12}$$

where $\bar{u}$ is a depth uniform current, ϵ is the wave steepness, ak and the constant coefficients A_{ij} and B_{ij} are given in Fenton (1985). Stokes'-type wave solutions have been widely accepted in the design of intermediate and deep-water offshore structure owing to their ease of use and relatively accurate prediction of the water surface elevation and water particle kinematics associated with large regular or steady waves. However, it is widely acknowledged that the largest waves in the real sea are neither regular nor steady.

2.3.3 Irregular Wave

Linear Random Wave Theory

While Stokes' waves take the nonlinearity into consideration, the solution is derived based on a unidirectional, single frequency wave in a steady frame of reference. Therefore, Stokes' waves do not provide a physically realistic description of waves in a real sea state. A real sea state is characterised by multiple wave components, each with a different frequency, amplitude, and direction. By assuming the wave components to be linear, the superposition of all wave components produce a linear approximation of the sea state (Figure 2.2). This approach is known as Linear Random Wave Theory (LRWT). A LRWT solution, for a uni-directional wave form is given in term of η and ϕ as

$$\eta(x, t) = \sum_{n=1}^{Nc} a_n \cos(k_n x - \omega_n t + \psi_n) \tag{2.13}$$

and

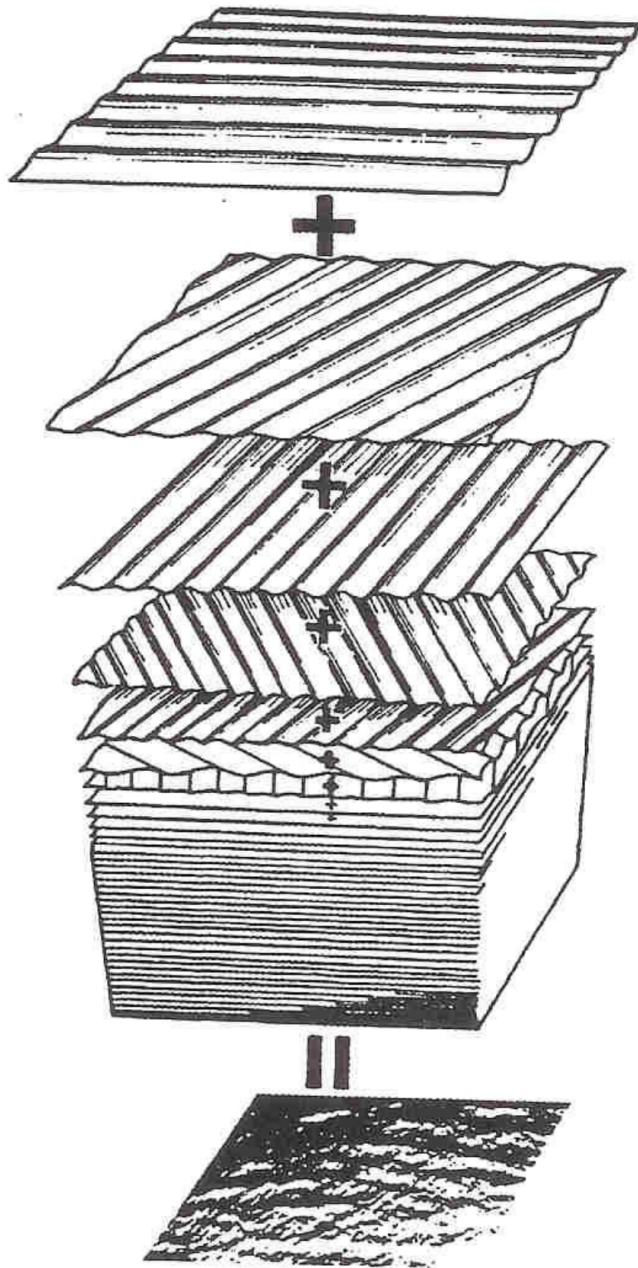


Figure 2.2: Superposition of regular waves with different frequency, amplitude and direction. Figure after Pierson et al. (1971).

$$\phi(x, z, t) = \sum_{n=1}^{N_c} \frac{a_n \omega_n}{k_n} \frac{\cosh(k_n(z+d))}{\sinh(k_n d)} \sin(k_n x - \omega_n t + \psi_n), \quad (2.14)$$

where N_c is the number of wave components, a_n , ω_n and ψ_n are amplitude, circular (wave) frequency and phase angle of individual wave component, respectively. While LRWT provides an improved physical representation to the real sea state, the solution is inaccurate due to the applied linearisation. In effect, this ignores the interaction between the wave components. In term of the kinematics predictions, the extrapolation of all the wave component about SWL, rather than incorporating the fact that individual wave component ride over one another, leads to high-frequency contamination (Sobey, 1990) and gross over-prediction of the velocities high in the wave crest.

Second Order Random Wave Theory

To overcome the very significant shortcomings of the LRWT, especially in respect of the near-surface properties, a higher order random wave theory is needed. Longuet-Higgins & Stewart (1960) provide the mathematical basis for such a model. Their paper describes a perturbation expansion, similar to the Stokes (1847) solution, for the interactions of two freely propagating waves up to a second-order of wave steepness. Using this solution, they identified two sources for the nonlinearity. First, there are the Stokes' term or the self-interaction terms for each of the wave components. These are consistent with those outlined in Equation (2.11) and (2.12). Second, there are the cross-interactions terms, or frequency-sum and frequency-difference terms. Building on the work by Longuet-Higgins & Stewart (1960), Sharma & Dean (1981) applied the solution to a random wave field by summing up the interactions for all possible pairs of wave components. This solution is the basis for what is commonly referred to as second-order random wave theory. Although this theory provides a much better description of the irregular sea-states when compared to LRWT, important nonlinear terms are still missing from the derivation. For example, Gibson & Swan (2007) showed that for a highly nonlinear problem, representative of a steep sea state, there are rapid changes in the underlying wave spectrum during the evolution of an extreme wave event. These changes occur at third-order and above. Experimental investigation by Baldock et al. (1996) reported that under extreme wave conditions,

the maximum water surface elevation and the horizontal fluid velocity high in the wave crest may be under-estimated by 20 % and 30 % respectively by a second-order random wave solution. In effect, a second-order random wave theory only provides a first approximation to the nonlinearity and is therefore only applicable to weakly nonlinear waves.

Higher Order Random Wave Solution

Higher order random wave solutions (above second order) can be based upon the Zakharov equation (Zakharov, 1968). Applying this equation Krasitskii (1994) provides an explicit numerical implementation, valid to a fourth-order of wave steepness. At second-order, the solution is equivalent to Longuet-Higgins & Stewart (1960) and Sharma & Dean (1981) in that only bound wave terms are present; the latter corresponding to the frequency-sum and frequency-difference terms discussed earlier. However, at third-order and above, additional terms commonly referred as the *resonant* interactions are present. These allow the transfer of energy to additional wave components that satisfy (or nearly satisfy) the dispersion equation (Equation (2.10)). This process changes the underlying wave spectrum, particularly in the vicinity of a highly nonlinear extreme wave event. The practical importance of this result is that the wave spectrum can no longer be assumed constant. Another class of highly nonlinear random wave solutions are the spectral wave models, typically based on a Fourier series representation of the free surface. These Fourier-based methods have been successfully applied in Bateman et al. (2001, 2003) to investigate the evolution of steep, near-limiting wave groups. Although they provide a computationally efficient process, enabling realistic fully nonlinear wave calculations, they do not provide a physical understanding of the nonlinear evolution.

Despite the recent advances in wave modelling and the ability to provide an accurate description of highly nonlinear wave problems, the adaptation of a Fourier series representation ensures that the water surface, $\eta(x, y, t)$ is restricted to a single value function. As such, the solution can only be applied to non-breaking waves.

2.4 Diffraction Solution

In the previous section, solution procedures appropriate to the modelling of progressive water waves were briefly introduced. Despite the complexity and importance of this subject, the majority of the problems encountered in the context of offshore structures are related to the interaction between the incident waves and the structure. In this section, models for the wave-structure interaction will be presented. The present review will concentrate on a simple structural form consisting of a single surface piercing vertical column.

2.4.1 Linear Diffraction

An exact representation of the physical problem involving the interaction between steep incident waves and structures is extremely complex. This complexity can be reduced by making appropriate assumptions, some of which have been discussed. In the case of a large-volume structure for which $D/\lambda > 0.2$, where D is the diameter of the body and λ the incident wavelength, the effect of viscosity can be neglected and the potential flow assumption can be employed. Havelock (1940) was the first to provide a linear diffraction solution for a flow around a vertical column in an infinite water depth. This was then extended by MacCamy & Fuchs (1954) to finite water depth.

For linear diffraction to remain valid, the incident wave steepness, ak needs to be small. The total velocity potential can then be expressed as,

$$\phi = \phi_I + \phi_S, \quad (2.15)$$

where subscript ϕ_I and ϕ_S correspond to the velocity potentials describing the incident and scattered waves respectively. The governing equation for linear diffraction is given by the Laplace's equation presented in the previous section. Once again, appropriate boundary conditions are needed to solve the problem. These are given by linearised free surface boundary conditions, no flux boundary conditions, at both the bed and on the surface of the column, and a radiation boundary condition. Adopting cylindrical co-ordinates (r, θ, z) , where z -axis is aligned with the centerline of the

column, the linear diffraction solution for ϕ and η are given by

$$\phi(r, \theta, z, t) = -i \frac{gH}{2\omega} \frac{\cosh(k(z+d))}{\cosh(kd)} \sum_{m=0}^{\infty} \epsilon_m i^m \left[J_m(kr) - \frac{J'_m(\frac{1}{2}kD)}{H_m^1(\frac{1}{2}kD)} H_m^1(kr) \right] \cos(m\theta) e^{-i\omega t} \quad (2.16)$$

and

$$\eta(r, \theta, z, t) = -\frac{1}{g} \left(\frac{\partial \phi}{\partial t} \right), \quad (2.17)$$

where i is the imaginary number, $J_m(kr)$ and $H_m^1(kr)$ are the Bessel and Hankel functions of the first kind, of order m and argument kr , where k is again the wave number $2\pi/\lambda$. The prime indicates the first derivative with respect to this argument, $\partial/\partial(kr)$, with the gradient evaluated at the surface of the column, $r = D/2$.

Although linear diffraction can describe both the loading on the structure and the wave scattering around the structure, it is based on small amplitude incident waves. Given the comments in Section 2.3, linearly predicted loads are unlikely to be sufficiently accurate for design calculations which will necessarily involve large, steep waves.

2.4.2 Higher-Order Diffraction

Linear diffraction provides, at best, a first-order approximation of the wave loads. With an increased incident wave steepness, analytical diffraction solutions are again notoriously difficult to obtain. This is principally related to the difficulty of satisfying the inhomogeneous free surface boundary condition and the radiation condition for the higher-order scattered wave components. Given this difficulty, nonlinear diffraction solution are only available up to a third-order of wave steepness. Furthermore, these solutions are limited to regular or steady incident waves.

At second-order, diffraction solutions can be divided into two approaches. The first approach is called the *indirect* method. In this method, the second-order diffraction forces are evaluated without explicitly calculating the second-order potential. This is achieved by using Green's theorem to express the second-order forces in term of the first-order potentials only. In deep water, the second-order forces are given by

Lighthill (1979), while Molin (1979) and Eatock Taylor & Hung (1987) provide the corresponding expressions in finite water depth. Whilst the indirect methods are convenient for the determination of forces, the calculation of pressures, the water particle velocities and the water surface elevations requires the evaluation of the second-order potential. The second approach, referred to as the *direct* method involves this evaluation. An example of this latter approach is given by, amongst other, Chau & Eatock Taylor (1992) who present a semi-analytical second-order solution for a single, bottom-mounted, circular cylinder. Extending the solution further, Malenica & Molin (1995) gives a third-order semi-analytical diffraction model for the same problem.

Although diffraction solutions provide the description of the scattered wave field, they do not take into consideration the nonlinear interaction between the incoming incident waves and the out-going scattered waves. As a result, they typically provide an inadequate prediction of wave-structure interaction problems involving highly nonlinear incident waves.

2.5 Numerical Models for Waves and Wave-Structure Interactions

It has already been noted that obtaining an analytical solution for the water wave problem is extremely challenging. Although potential flow assumptions are commonly made, reducing the problem to one which is homogeneous, incompressible, inviscid and irrotational, the solution to this problem remains difficult owing to the nonlinear boundary conditions that need to be imposed on the unknown free surface. Furthermore, with the introduction of a structure, and hence additional boundary conditions, the complexity of the problem increases considerably. While potential flow theory remains valid for a simple structure of relatively large size, existing solutions are restricted to incident waves that are, at most, weakly nonlinear. On the other hand, if the potential flow assumptions cease to be valid, due, for example, to the occurrence of wave breaking, viscous and rotational flow effects will become increasingly important. In this case, no analytical solution exist. Indeed, the only viable option is to approximate the solution using numerical methods.

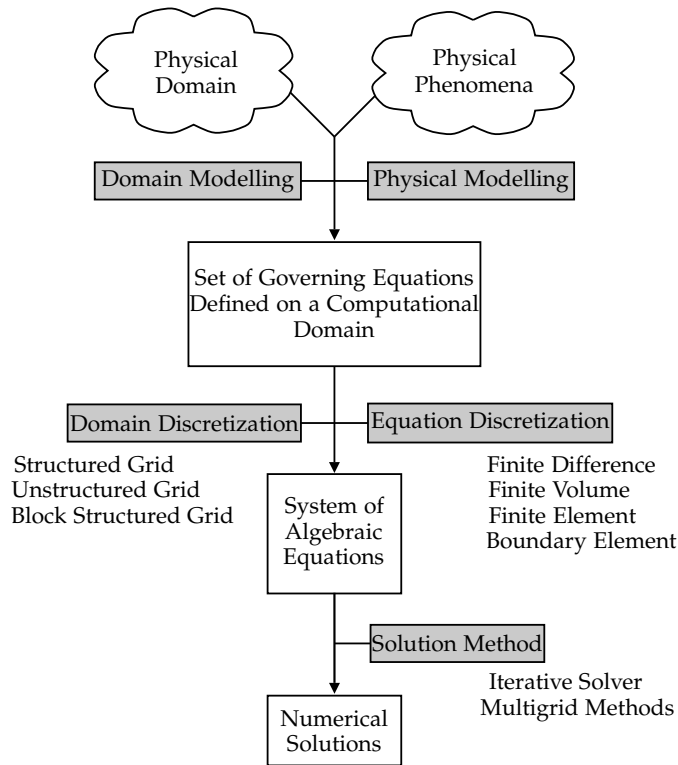


Figure 2.3: Numerical solutions, an overview of the modelling process (Moukalled et al., 2015).

2.5.1 Numerical Discretization

A numerical method is best described as the mathematical representation of a physical problem using numerical techniques to approximate and hence solve the governing equation. Unlike analytical solutions, numerical methods provide a discrete solution to the problem. The numerical modelling of water waves may be sub-divided into two groups according to the representation of the physical problem. If the potential flow assumption is made, the problem is governed by the Laplace equation. On the other hand, if the flow is considered viscous and rotational, the Navier Stokes equations are used to represent the problem. Based on these assumptions, the governing equations can then be discretized using various numerical methods.

The solution procedure applied in the commonly adopted numerical methods follows the general methodology outlined in Figure 2.3. First, the domain and physical phenomena of the problem need to be defined; an example being a wave propagating in an experimental wave basin. The physical domain then needs to be represented by a computational domain using sets of coordinates, both line and plane, defining the domain boundary appropriately. Next, the physical phenomena must be repre-

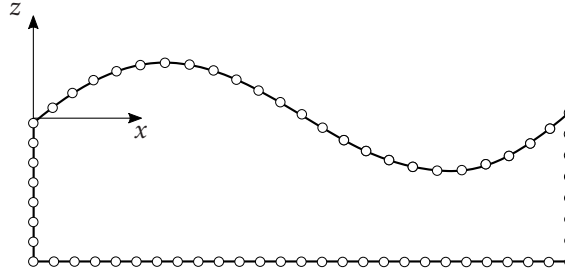


Figure 2.4: Boundary discretization in BEM.

sented by a suitable governing equation. In fluid problems, this typically represents the conservation of mass and momentum, applied throughout the computational domain. This domain then needs to be discretized into a computational mesh using sets of nodes and elements; the elements covering the whole domain, with no overlapping allowed. The domain can be discretized using structured, unstructured or block-structured grids. The governing equation must then be discretized using an established method such as the Finite Difference Method (FDM), Finite Element Method (FEM), Finite Volume Method (FVM) or Boundary Element Method (BEM). Discretization converts the governing equation into discrete algebraic equations, defined over the discrete computational nodes. These can then be solved using various linear algebra solvers which can be either direct or iterative.

In the sections that follow, the commonly adopted numerical methods used in the study of wave and wave-structure interactions problem will be presented and, where appropriate, the advantages and short-comings of the methods will be discussed.

2.5.2 Boundary Element Method (BEM)

BEM is one of the most widely used numerical methods in the design of offshore structures. Unlike other numerical methods where the discrete numerical solution is sought over the entire domain, BEM solves the Boundary Integral Equation (BIE) on the boundary of the domain (Figure 2.4). This achieves a dimensional reduction with important implications for the computational efficiency. Using the BIE, the velocity potential, ϕ can be related to its normal derivatives along the boundary of the domain. This allows the complete solution to be obtained if either ϕ or its derivatives are known on the boundary.

The first application of BEM to model free-surface waves was undertaken by

Longuet-Higgins & Cokelet (1976). Following on from this success, many other researchers have extended and developed this modelling approach. Currently, the method can be divided into two categories. The first is based on the conformal mapping of the Cauchy integral formula, with the free surface mapped onto a complex plane. However, this method needs a periodic boundary condition and is typically restricted to a constant water depth. More importantly, it is only relevant to two-dimensional fluid flow problems.

The second category involves the solution of the BIE in physical space using Green's second identity. Using this method, there is no need for conformal mapping, allowing the use of non-periodic boundary conditions and non-uniform water depths. Furthermore, within this category, the method can easily be extended to a three-dimensional problem. While the latter is undoubtedly an improvement over the former, at least in term of applicability, it suffers from the so-called corner problem since the use of Green's second identity assumes a continuous, smoothly varying, boundary. Geometrical discontinuities occur at the corners (2D) and the edges (3D) of a computational domain and, particularly, at the intersection between the free surface and a body. Multiple solutions have been proposed to mitigate the corner problem. These include the double node and discontinuous element approaches. Brebbia & Dominguez (1994) introduced the multi-flux approach to tackle the corner problem in generic BEM applications. Indeed, they argued that it facilitates a more accurate representation of the behaviour at corners/edges when compared to the double node and discontinuous element approaches. This approach has been specifically applied to the study of water waves by Hague & Swan (2009). More recently, Christou (2009) and Archibald (2011) extended this solution to three dimensional by introducing a more efficient iterative solver and a parallel implementation of the code.

In the context of wave modelling, BEM has been used extensively to model highly nonlinear waves. Longuet-Higgins & Cokelet (1976) were the first to apply BEM to the simulation of a 2D, steep, over-turning, wave in an infinite water depth. The extension to finite water depths was provided by Vinje & Brevig (1981), New et al. (1985) and Grilli et al. (1989). In much of this earlier work, wave breaking was induced by applying a non-uniform pressure on the free surface of a regular wave train. In contrast, the simulation of 2D over-turning waves in deep water driven by frequency dispersion was first carried out by Skyner (1996) with good agreement between the

BEM simulation and experimental observation. Similarly, Christou (2009) managed to model an over-turning wave in deep water, based upon a realistic frequency spectrum using a multiple-flux BEM. Modelling of 3D extreme waves using BEM has also been carried out, although limited, primarily due to the computationally demanding nature of BEM for large problems. These works were performed by Xue et al. (2001), Fochesato & Dias (2006), Fochesato et al. (2007) and Archibald (2011) amongst others.

BEM has also been utilised in the study of the wave-structure interaction problem for the past three decades. Early implementations of a fully nonlinear BEM describing a surface piercing vertical column has been carried out by Isaacson (1982), Yang & Ertekin (1992), Ferrant et al. (1995), Boo & Kim (1996) and Hong et al. (2000). In addition, a high-order BEM was used by Liu et al. (2001) to study the higher-harmonic loading on a vertically truncated cylinder. In this study, regular waves (Stokes' wave) with a wave steepness, $Hk/2$, lying in the range of 0.13 to 0.2 were incident to a single cylinder with a diameter to wave-length ratio, D/λ lying between 0.6 to 0.12. For the case with $D/\lambda = 0.12$ and $Hk/2 = 0.2$, a maximum run-up in excess of 80% of the incident wave amplitude was reported. This is in marked contrast to comparisons made with a linear diffraction solution where the maximum diffracted amplitude is underestimated by a factor of more than two. Furthermore, comparison between the BEM predicted and experimentally observed wave forces showed good agreement. This includes the higher-order wave forces arising with increasing incident wave steepness. Interestingly, they also reported an effort to model a higher incident wave steepness ($Hk/2 \approx 0.25$). However, wave breaking was observed and the code stopped before the steady state solution was reached.

Despite the ability to model the nonlinear water wave problem, the computational effort associated with 3D BEM calculation has limited its application in practical design work. Notwithstanding this shortcoming, BEM has been used in conjunction with higher-order diffraction solutions since most are not entirely analytical. To solve the diffraction problem, the solution is truncated at a certain order, leading to a numerical integration that needs to be approximated by BEM. This allows for a faster computation and reducing overall computational cost when compared to fully a nonlinear BEM solution.

In the offshore industry, BEM is commonly referred to as the panel method. A commercial BEM code (WAMIT) solving the diffraction problem (up to second-order),

has been developed at the Massachusetts Institute of Technology and is currently the leading industry standard code for modelling of wave-structure interaction problem (Lee & Newman, 2005). The code has been used by, amongst others by Stansberg & Kristiansen (2005) in the study of wave-structure interactions involving both a single and multiple vertical columns. In a similar frame, Oxford University has developed a second-order BEM called DIFFRACT (Eatock Taylor & Kernot, 1999). Sheikh (2004) used DIFFRACT and compared the results against his experimental observations of wave scattering from a surface-piercing column. In both Stansberg & Kristiansen (2005) and Sheikh (2004) the second-order prediction provide a much better description than linear theory. However, important discrepancies still remain when predictions are compared to experimental observation. Whilst these discrepancies may not be large when compared to the magnitude of the linear predictions, the frequency at which they occur may be important for dynamic excitation. Finally it is relevant to note that an open-source linear diffraction code based on BEM has recently been made available by L'École Centrale De Nantes under code named NEMOH (Babarit & Delhommeau, 2015). This confirms the continued interest in this method.

Whilst BEM is undoubtedly attractive not least because the dimensional reduction substantially reduces the time needed to prepare a complex mesh and results in a much smaller matrix that needs to be solved, the method has several important shortcomings. First, the use of potential theory to represent the system restricts the modelling of rotational and viscous flow. Second, while BEM can be used to model overturning waves, the method usually terminates before the plunging jet touches down. Following on, BEM is unable to provide any information on post-breaking behaviour which is characterised by a surface break-up and vortex formation. Third, BEM is also computationally expensive when applied to fully nonlinear three-dimensional problems. This is because the discretization of the BIE results in a large, dense matrix, the solution of which requires large computation and storage costs at each time step.

2.5.3 Finite Element Method (FEM)

An alternative to BEM for solving water wave problems is provided by FEM. In FEM, a discretized solution to the problem is given by sets of elements, covering the entire domain (Figure 2.5). Across each element, the solution is approximated using a so-

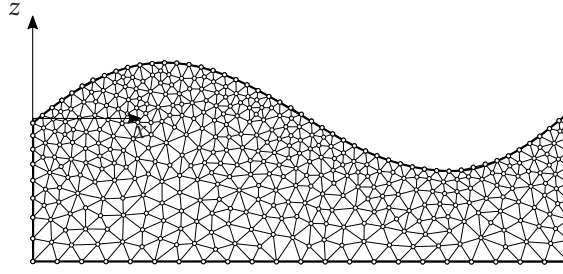


Figure 2.5: Domain discretization in FEM.

called shape function. This describes the variation of the solution between the nodes using either a linear, quadratic or higher-order polynomial function. By substituting the discretized variables (in term of the shape function) into Laplace equation and applying a weighted residual method (frequently based on the Galerkin approach), a set of linear algebraic equations can be formed. Appropriate boundary conditions can then be implemented and the evolution of the free surface can be obtained by time marching the solution.

Although FEM requires the generation of a mesh over the entire domain, the solution method is relatively cheap in a computational sense. This is due to the fact that a node is only influenced by those nodes directly linked to it and restricted to those defined by the connectivity matrix. As a result, the matrix generated by FEM is banded (for a structured mesh) or sparse (for a unstructured mesh) leading to only a small non-zero part. This allows the FEM matrix to be stored using limited storage. This is in marked contrast to a BEM where a fully populated matrix is expected. Furthermore, a sparse matrix can be solved very efficiently and can easily be parallelized to achieve significant additional gains. Early applications of FEM in the study of water wave problems were provided by Wu & Eatock Taylor (1994). In addition, comparisons between FEM and BEM have been carried out by Wu & Eatock Taylor (1995). These showed that FEM was superior in terms of calculation time and storage requirements. Following on, Wu & Eatock Taylor (2003) developed an effective coupling between BEM and FEM, taking into account the advantages of each method.

For wave-structure interaction problems, FEM has been used by Ma et al. (2001a,b). This considered the flow around a single vertical column and a pair of vertical columns in steep regular and irregular waves. The results clearly showed the presence of high-frequency wave scattering from a single column and the enhanced interaction

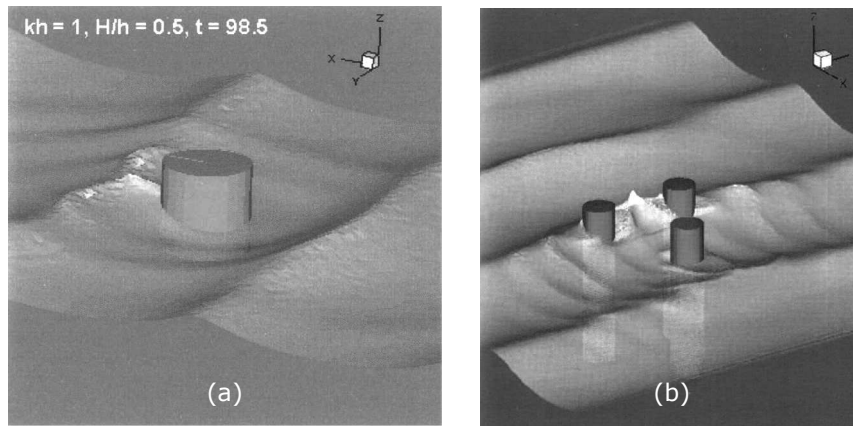


Figure 2.6: Snapshot of surface elevation around (a) single cylinder (b) multiple columns from work by Kim et al. (2006)

for the two column cases. Whilst the scattering pattern was not discussed in any detail, this was perhaps the first of a small number of numerical studies to provide snapshots of high-frequency wave scattering from a vertical surface piercing column. Similarly, FEM has been used by Kim et al. (2006) to study the interaction between steep Stokes waves and both single and multiple columns in varying water depths. In this study, they provide evidence of high-frequency wave scattering from the column face as incident waves interact with the column. They also observed that as the water depth decreases, the incident wave becomes nonlinear, leading to larger wave run-up. Indeed, this can be as high as three times the incident wave amplitude in the steepest wave cases. Interestingly, the Type-2 wave scattering pattern reported in Sheikh & Swan (2005) was also observed in one of the cases presented in Kim et al. (2006); the data reproduced in Figure 2.6 (a). Furthermore, for steep Stokes wave interacting with a multiple column arrangement (similar to work by Swan et al. (1997)), high frequency wave scattering was again observed. Moreover, qualitative comparison between these results and independent experimental observations (Swan et al., 1997) show good agreement; the multiple column arrangement capable of projecting the scattered waves to higher elevations as shown in Figure 2.6 (b).

Although FEM has distinct advantages over BEM in term of computational performance, the method also has several disadvantages. Since FEM requires the whole domain to be meshed, any deformation to the boundary of the domain, especially if it is locally large, can lead to a severely distorted element. This, in turn, leads to a loss in element quality and in the worse case, an overlapping element. To avoid this

problem, remeshing over the entire domain is required. Unfortunately, remeshing is undesirable not only because it is computationally expensive, but frequent remeshing may also cause a loss of energy. Furthermore, interpolation is required after each remeshing step to update information for the newly generated nodes. This will lead to decrease in solution accuracy. Recently, Ma & Yan (2006) developed a quasi arbitrary Lagrangian-Eulerian FEM (QALE-FEM) to avoid the need to remesh the domain at each time step. The arbitrary Lagrangian-Eulerian (ALE) concept was borrowed from the numerical modelling of the Navier Stokes equation (NSE) where a moving (lagrangian) mesh was used instead of a fixed mesh. In, Ma & Yan (2006) a spring analogy method was used to redistribute the internal nodes each time the domain boundary deformed. Despite the many improvements associated with this method, it is still restricted to the modelling of non-breaking waves.

2.5.4 Finite Volume Method (FVM)

Another popular method used in the study of water waves is based on the FVM. This is a mesh based method where discretization is again required over the whole domain. An unstructured mesh can be used with FVM, allowing irregular domains to be modelled with ease. Using the divergence theorem, volume integrals of partial differential equations are converted to surface integrals, allowing the fluxes to be evaluated at the surface of the control volume. Assuming that the size of the control volume is fixed and completely full of fluid (no free surface), the flux entering and leaving a control volume will be equal. In this case FVM is considered as a conservative scheme. This makes FVM highly relevant to systems, such as the NSE, which are based upon the conservation of mass and momentum. In the context of surface water wave, solutions based on BEM and FEM are commonly used with a potential flow assumption. In contrast, FVM is often used to solve the NSE. Unlike the potential flow theory, which assumes that the fluid is inviscid and irrotational, the NSE describes the real behaviour of a fluid, allowing for viscous effects, flow separation and the formation of vortices in the flow. As such the NSE has been used to simulate difficult wave problems such as wave breaking and wave-structure interactions.

For most water wave problems, the fluid is considered incompressible. This assumption leads to an incompressible version of NSE in which the density is assumed

to be constant. The difficulty in solving the incompressible NSE is associated with the nonlinearity of the equation and the pressure variable. Unlike the velocity, the pressure is not linked to any constitutive equations. As such, the pressure adjusts itself instantaneously based on the velocity field. Several approaches have been proposed to solve the NSE, amongst them are, the artificial compressibility method, the fractional step method and the pressure correction method. In the pressure correction method (PCM), the momentum equations are substituted into the continuity equation, resulting in a pressure Poisson equation (PPE). In the PCM, the PPE is solved iteratively until a divergence free velocity is satisfied. Various algorithms have been developed based on PCM such as SIMPLE (Semi-Implicit Method for Pressure Linked Equations), SIMPLER (SIMPLE Revised), SIMPLEC (SIMPLE Consistent) and PISO (Pressure Implicit with Splitting Operators); the benefits of different approaches discussed in Ferziger & Peric (2012).

In the study of surface water waves, the NSE must be solved together with a description of the free surface. The free surface can be modelled using either an interface tracking method or an interface capturing method. In the interface tracking method, the moving free surface is explicitly described by the computational mesh. Due to the movement of the free surface, and hence the surface nodes, the position of the inner nodes has to be adjusted. While the movement of the free surface interface is governed by the velocity field, the adjustment of the inner nodes is usually arbitrary. For an accurate solution, the adjustment of the inner nodes has to preserve adequate mesh quality. If the mesh is represented in the physical space, techniques such as ALE are usually used to redistribute the internal mesh to avoid highly distorted elements. Alternatively, suitable transformations can be applied and the corresponding NSE solved in a fixed domain. In most cases, the interface tracking method will provide an accurate description of free surface; the solution limited to mild interface distortion.

In contrast to the interface tracking method, the interface capturing methods usually employ a fixed grid/mesh system. The interface defining the free surface is subsequently described by an artificial scalar field. The method is suitable for interfaces subjected to severe motions and can even model wave breaking. However, the method is more computationally expensive due to the increased number of cells. Furthermore, it is not as accurate as the interface tracking method when predicting the position of the free surface.

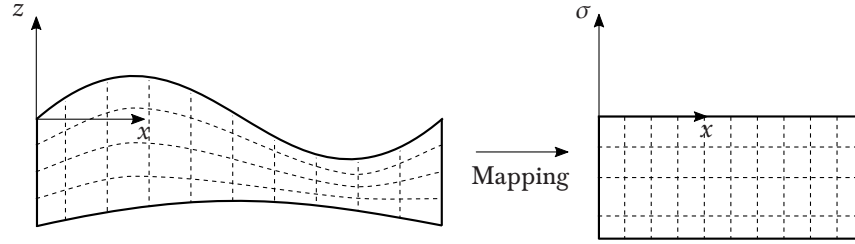


Figure 2.7: Mapping from the physical domain to σ -transformed coordinates.

Hydrostatic and Non-hydrostatic Formulation

For a single-phase free surface flow, the NSE can be solved using either a hydrostatic or a non-hydrostatic formulation. In the hydrostatic formulation, the assumed pressure distribution is appropriate for fluid flows in which the vertical acceleration is small. This is valid for flows such as long waves, tides, bores and ocean circulations. The hydrostatic assumption effectively reduces the vertical momentum equation into a single hydrostatic pressure assumption. In making this assumption it is also assumed that the free surface slope is mild and a single-valued function. Substituting the hydrostatic pressure assumption into the horizontal momentum equation eliminates the pressure term. This simplifies the solution of the NSE considerably not least because an iterative solution to the pressure is no longer required. In this case, the velocity field can be defined by forward marching in time and the surface elevation determined using the integrated continuity equation in the vertical direction. Whilst the hydrostatic formulation is not relevant to deep water waves, it has been used extensively in the simulation of near-shore flows where the water depth is much reduced. Indeed, there exist quite a number of hydrostatic wave models, among them are the Princeton Ocean Model (POM) (Blumberg & Mellor, 1987) and the Coupled Hydrodynamical Ecological Model for Regional Shelf Seas (COHERENS) (Obino, 2002).

When the fluid acceleration becomes important, (short gravity waves in deep water and flow over abrupt changes in bathymetry) the hydrostatic pressure assumption becomes invalid. In such cases, the contribution due to the non-hydrostatic pressure has to be included. The non-hydrostatic formulation simply solves the NSE with the inclusion of pressure terms. Many of the non-hydrostatic formulations are constructed in σ -coordinates which map the physical space between the free surface and the sea bed to a regular grid (Figure 2.7). The transformation to σ -coordinates mod-

ifies the original NSE into a slightly more complicated form, but allow a much easier discretization. The non-hydrostatic formulation is usually solved either using FDM or FVM. The use of a σ transformation allows propagation of waves in deep water to be modelled with a relatively small number of vertical layers. This makes the non-hydrostatic formulation computationally cheap to run. Several open-source non-hydrostatic codes are available. Amongst these, SWASH (Simulating WAVes till SHore) developed at Delft University (Zijlema et al., 2011) and NHWAVE (Non-Hydrostatic WAVE) developed in the University of Delaware (Ma et al., 2012) are perhaps the most well known. Despite the cheap computational cost due to a relatively small number of vertical layers, the non-hydrostatic model cannot be used to model over-turning waves due to the requirement that the surface elevation is single-valued. This limits its use in simulating the most violent wave events. The non-hydrostatic model has been coupled with other methods such as Smoothed Particles Hydrodynamics (SPH) to allow the modelling of breaking waves (Altomare et al., 2015).

Level-set (LS) Method

The LS method developed by Osher & Sethian (1988) is a multiphase fluid formulation that falls under the interface capturing technique. In the LS formulation, a distance function ϕ is introduced to represent the interface. At the interface, ϕ is designed to have a zero value, changing smoothly across the interface with $\phi > 0$ for water and $\phi < 0$ for air. With the level set function known, the interface can be advected using the local velocity field (obtained from solving the NSE) and the scalar advection equation. In the case of interface break up, the LS method allows for reconnection and disconnection of the interface in a natural way, allowing the method to describe a highly distorted interface.

The LS method has been used to study 2D free surface flows by Iafrati et al. (2001). In this work, LS has been applied in the simulation of oscillating flow in a tank and in flow over a raised section of the sea bed and a submerged hydrofoil. The study highlights the capability of LS to model highly complex free surface distortion and surface break-up, including satisfactory agreement with experimental observations. In Iafrati (2009), this work was extended to investigate unsteady wave breaking generated by periodic wave-trains of varying amplitude. Within this latter study, they provided a detailed analysis of the role played by the breaking wave strength, consid-

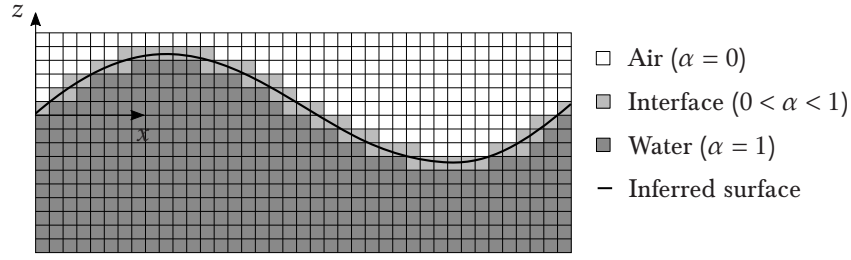


Figure 2.8: Volume fraction representation in the VOF method.

ering the geometric, kinematic and dynamic aspects of the flow. Following on, Chen & Yu (2009) applied LS with a large eddy simulation (LES) approach to investigate free surface flows in the vicinity of a fixed and moving structure. The study highlights the capability of LS to accurately predict wave-structure interactions involving violent free surface motions.

Although LS has provided some interesting insights into the simulation of extreme waves, the method suffers from poor mass conservation caused by reinitialization of the free surface. This leads to significant numerical dissipation. To overcome this issue, additional effort is undoubtedly required; an obvious next step being the introduction of higher order schemes and further grid refinement.

Volume of Fluid (VOF) Method

This method belongs to a family of multiphase interface capturing methods for tracking and locating the free surface. It was originally developed by Hirt & Nichols (1981) and is typically used to solve the NSE on a fixed grid using FVM. Unlike the LS method, which uses a distance function to represent the interface, VOF uses a volume fraction, α , to identify a different fluid phase. For a two phase system of water and air, water, is usually denoted by $\alpha = 1$ and air by $\alpha = 0$. The free surface interface is then defined as an intermediate α value such that $0 < \alpha < 1$. The volume fraction is updated at each time-step using a scalar advection equation based on the underlying flow field. An example of a volume fraction representation in VOF is given in Figure 2.8. VOF is capable of handling wave breaking and the subsequent merging of the interface. Moreover, it is relatively easy to implement and has better mass conserving properties when compared to the LS method. However, a fine mesh is required to get a detailed representation of the free surface.

Much of the research in VOF is devoted to achieving better description of the

interface for the $\alpha = 1$ field. In general terms, the method can be divided into two categories: the first described as geometrical VOF and the second algebraic VOF. In the geometrical VOF method, the interface is described by a set of polygons reconstructed from the volume fraction field. Methods such as the Simple Line Interface Calculation (SLIC) and the Piecewise Linear Interface Calculation (PLIC) have been proposed by Noh & Woodward (1976) and Youngs (1982) respectively. Although geometrical VOF allows a distinct interface to be defined, it is computationally expensive because the reconstruction algorithm must be applied at each time-step. On the other hand, algebraic VOF only considers the solution to the scalar advection equation of the volume fraction. Although solving the scalar advection equation is relatively cheap (when compared to the reconstruction of the free surface) solving a pure scalar advection equation is notoriously hard due to the absence of any diffusion term. This introduces problems with the boundedness of the solution and the smearing of the interface over several cells. To mitigate these issues, a higher-order bounded scheme has been introduced while smearing of the interface can be suppressed by introducing a compressive term in the scalar advection equation.

VOF has gained much attention in recent years and has been used in the study of water wave problems. For example, the method has been used to study the breaking of regular waves in deep water by Chen et al. (1999) and Xie (2012). In both cases, the simulations commenced with a steep Stokes wave ($ak = 0.55$) evolving to create a plunging breaker. Periodic boundary conditions were used at both the inlet and outlet, allowing a short computational domain. Although it is simple and computationally cheap to generate breaking wave using this method, validation against equivalent experimental setup is difficult. A more realistic representation of breaking waves in deep water can be achieved using a focused wave event, an example of this approach provided by Huang & Lin (2012).

Due to its ability to model both viscous processes and extreme wave events, VOF has been used in the study of wave-structure interactions. Mo et al. (2007) studied waves interacting with a slender pile using experimental observation and VOF calculations. In the slender body regime, no disturbance to the flow field is expected. However, with steep incident wave conditions, both the observed and predicted surface elevations showed the presence of high-frequency wave scattering. This is consistent with the finding of Swan et al. (1997) and highlights the capability of VOF

to model highly nonlinear waves. More recently, Chen et al. (2014) investigated the wave-structure interaction in regular and focused waves using the open-source code OpenFOAM. Comparison between the numerical results and laboratory observation performed at the Danish Hydraulic Institute (DHI) showed good agreement. Evidence of high-frequency scattering again observed in the simulation. Similarly, the nonlinear wave forcing produced by a steep regular waves on a vertical column in a wide range of wave conditions has been considered by Paulsen et al. (2014b). In this case, the *secondary loading cycle* has been reproduced in the simulation and good agreement with experimental work from the Danish ‘Wave Loads’ project obtained.

While VOF has been successfully used in modelling wave-structure interactions, the method is computationally demanding due to the need to solve the full NSE and additional gridding required to model the air phase. In order to reduce the computational cost of VOF simulation, hybrid methods have been proposed by several authors. Kim et al. (2012) investigated the ringing loads on a vertical column using the Euler Overlay Method (EOM). This approach combines an Euler equation solver in the far-field with a VOF solver in the near field. Using EOM, they were able to simulate *ringing* loads from a non-breaking focused wave and achieved good agreement with experimental data. They also undertook a simulation of a breaking wave case, showing that the nonlinearity and dynamic amplification of the *ringing* load increases significantly if the incident wave breaks on the column. An efficient domain decomposition strategy was also proposed by Paulsen et al. (2014a) to study wave loading on a vertical column in intermediate water depths. A 3D one-way coupling between a fully nonlinear potential flow model (OceanWave3D) developed by Engsig-Karup et al. (2009) and VOF based on NSE was used. This coupling allows both longer run times and, hence, irregular waves to be modelled efficiently. Using these results, the *secondary loading cycle* was successfully modelled with good agreement between the numerical simulations and experimental observations.

2.5.5 Smoothed Particle Hydrodynamics

In recent years, rapid advancement has been made in the field of meshless or mesh-free methods. Unlike conventional numerical methods that require a grid or mesh (FDM, FVM and FEM), meshfree methods rely on sets of nodes in the computational

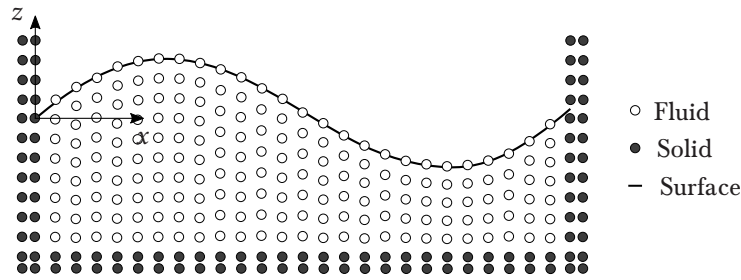


Figure 2.9: Lagrangian particles discretization in SPH.

domain, with no mesh connecting the data points (Figure 2.9). Instead, the solution to the governing equations is provided through interaction between particles, the latter defined using appropriate kernels. These particles carry the material properties of the fluid and are allowed to move freely and interact with each other. One widely used meshfree method for fluid flow problems is smoothed particle hydrodynamics (SPH). SPH was first developed in the context of astrophysics by Gingold & Monaghan (1977), before being adopted for free surface flow problems by Monaghan (1994). Unlike a typical CFD solver which uses the Eulerian description of the flow, in SPH the NSE are solved on a discretized computational domain consisting of Lagrangian particles. Without the need for a mesh, SPH can easily trace material interfaces and moving boundaries with large deformation. Furthermore, for the water wave problem, there is no special treatment needed for the free surface. These unique properties of SPH allow it to model surface breaking, merging, and splashing.

SPH has been used extensively in the study of wave breaking due to its capability to model large surface distortion. The method has been used to study dam break problems by, for example, Monaghan (1994), Dalrymple & Rogers (2006) and Gomez-Gesteira et al. (2010). Whilst these results serve as a useful benchmark case to test a CFD method, its application to real engineering application is fairly limited. In coastal engineering, SPH is widely used to model wave propagation on a sloping beach. As the wave propagates up the sloping bed, the wave-length decreases and the amplitude increases; as a result, the wave steepens and eventually breaks. Numerical simulations of this type have been carried out by, amongst other, Monaghan & Kos (1999), Dalrymple & Rogers (2006), and Khayyer et al. (2008). Recently, Dao et al. (2011) introduced a two-phase SPH model to capture the fine details of deep water wave breaking such as the formation and collapses of air pockets and splash-up. The breaking wave was generated by frequency dispersion using a moving wave paddle.

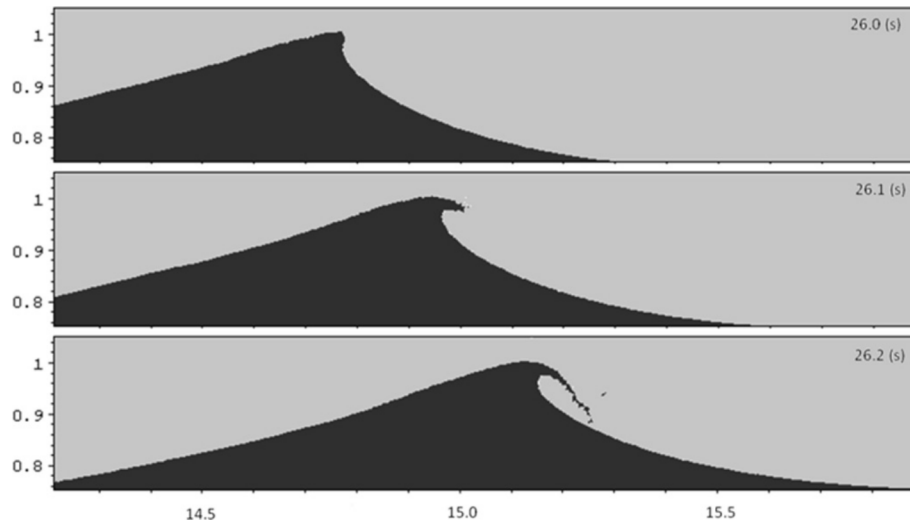


Figure 2.10: Plunging breaker from a focused wave using SPH from Dao et al. (2011).

Comparisons with the experimental observations of Kway et al. (1998) shows good agreement with the wave generation process. Figure 2.10 gives the results of a typical simulation showing the capability of SPH to simulate the breaking of an extreme wave. SPH has also been used to study the wave-structure interaction problem. For example, Gómez-Gesteira et al. (2005) employed SPH to study wave over-topping a flat deck and showed a good comparison with experimental observation. More recently, SPH has been used by Rudman & Cleary (2016) to investigate the effects of the impact of a large wave on a floating, moored structure.

Due to increased interest in SPH, an open-source SPH code, SPHysics (Gesteira et al., 2008) has been developed by collaborators from University of Manchester (UK), University of Vigo (Spain) and Johns Hopkins University (USA). The serial code was made public in 2007 and extended to include a parallel implementation in 2011; the latter called parallelSPHysics and GPUSPH (based on CUDA).

Despite its apparent advantages in modelling extreme surface flows, SPH has a number of drawbacks. First, problems arise in the definition of a solid boundary. In SPH, a solid boundary is represented by boundary particles which exert repulsive forces on the fluid particles to avoid penetration. However, near the free surface, where the contribution from the hydrostatic forces is weak, the repulsive forces can lead to the peeling of the fluid particles from the solid boundary. This problem can lead to uncertainty and inaccuracy in the prediction of near-boundary phenomena such as wave run-up. A second problem concerns the enforcement of the incom-

compressibility condition. Within an SPH calculation, a local density gradient is needed to drive the fluid particles. Thus, it is necessary to consider the fluid to be slightly compressible. If the exact value for the speed of sound in the relevant medium is used, the time step required to keep the solution stable needs to be very small. Consequently, an artificial speed of sound is introduced to keep the time-step fairly large, whilst maintaining negligible density fluctuations. This introduces an arbitrary constant that needs to be tweaked and adjusted to get a good solution. Whilst SPH certainly has the potential to model extreme wave conditions, the method is excessively dissipative for wave propagation over long periods of time and distance. For example, a reduction in wave amplitude of up to 40% can be observed for wave propagation over a distance of $27d$, where d is the water depth (Vaughan et al., 2006). With no way of imposing a rigorous energy conservation condition, this non-physical and unrealistic dissipation will limit the practical application of SPH modelling.

3

Numerical Model

This chapter describes the numerical model used in the present study. The first section introduces the open-source CFD code used. The second provides the description of the multi-phase solver and the governing equations with the free-surface treatments being discussed in detail. The third presents the discretization of the computational domain and the governing equation. This is followed by details of the solution procedure and linear algebra solver. The final part discusses the general boundary conditions, wave generation and the absorbing boundary conditions.

3.1 Introduction to OpenFOAM

An open-source Computational Fluid Dynamics (CFD) software, OpenFOAM was used in this study. OpenFOAM was originally developed at Imperial College London in the 1980s to provide a more robust and flexible simulation platform when compared to the more applied programming language of that time, Fortran. The adoption of C++ as the programming language in OpenFOAM results in more object oriented programming. The codes have been rewritten several times, exploiting advanced features of C++. This allows the adaptation of higher-order programming, enabling a much quicker code development environment. The code, previously called FOAM was sold by Nabla Ltd and was made public in 2004 under GNU license. Currently,

the code is maintained by OpenCFD Ltd.

OpenFOAM is essentially a collection of C++ codes for solving continuum mechanics problems. While OpenFOAM is mainly used for CFD simulations, it is also applicable to problems such as solid mechanics, electromagnetics, combustion and many others. The applicability of OpenFOAM to a wide range of physical modelling is achieved, in some part, through the use of equation mimicking. This allows the complex differential equations to be written (in the source code) conveniently by retaining their original mathematical form. Furthermore, unlike commercial software, which is essentially a ‘black-box’, the open-source nature of OpenFOAM allows the user to view, modify or even create and extend the functionality of the available source code. This is an attractive feature which allows users to tailor the code to their specified needs. While investing in commercial software for a specific task is usually expensive, writing code from scratch is cheaper but time-consuming. An open-source code provides a middle ground between the two by providing code that is readily available, well maintained and that can be modified for extended functionality. With this flexibility, OpenFOAM can be used not only to model complex multi-physics problems using its expanding collection of solvers, it can be coupled with other software to provide solutions for various types of physical problems.

Another advantage of OpenFOAM is the support for parallelism. Massive parallelism in OpenFOAM is done through domain decomposition using Message Parsing Interface (MPI). Unlike some commercial software which limits the number of cores that can be used for each licence, OpenFOAM has no such limitation. This means that OpenFOAM provides cost effective solutions for a problem that requires extensive parallel computation.

At its core, OpenFOAM uses a FVM discretization based on three-dimensional collocated polyhedral meshes. The use of FVM means that OpenFOAM is highly suitable for the modelling of a conservative problem such as the solution of NSE. Various discretization schemes in space and time are available. This suggests that, robust and highly accurate solutions can be obtained based on the careful selection of a discretization scheme tailored to a specific type of problem that needs to be solved. On the other hand, the adaptation of polyhedral meshes is more advantageous when compared to the commonly adopted meshes, particularly tetrahedral and hexahedral. Specifically, they allow easier mesh generation with higher solution accuracy.

Furthermore, OpenFOAM also provides dynamic mesh handling and adaptive mesh refinement. This increases the software applicability for problems involving complex moving meshes. Collocated variable arrangements in OpenFOAM means that all variables are stored at the cell centre, reducing the complexity associated with a staggered grid arrangement and allow much easier post-processing to be applied. For post-processing, OpenFOAM relies on a third party program called Paraview: an open-source, multi-platform data analysis and visualisation application.

3.1.1 OpenFOAM in Water Wave Problems

Although OpenFOAM has gained popularity with an increasingly active community covering a wide range of fields, the application of OpenFOAM in water wave problems is relatively new. Indeed, the lack of efficient and flexible wave generation and absorption utilities are some of the reasons why OpenFOAM has not been widely applied in the study of coastal and offshore engineering problems. Early attempts at the development of wave generation in OpenFOAM are by Morgan et al. (2011) and Amini Afshar (2010). Although they successfully modelled propagating water waves in OpenFOAM, their implementation suffers from inaccuracy due to the lack of an effective outlet boundary condition and the need for highly refined mesh around the water surface. Jacobsen et al. (2012) have addressed and solved these limitations by developing a wave generation toolbox in OpenFOAM called *wave2Foam*. This combines a generic wave generation ability and an effective wave absorption technique. The toolbox has been shown to accurately model long period simulations (over 100 times the wave period) as well as the propagation and breaking of individual water waves. More recently, Higuera et al. (2013) developed an alternative wave generation and absorption toolbox called *ihFoam*. While *wave2Foam* uses a passive wave absorption method, *ihFoam* uses active wave absorption; the latter technique being similar to that used in a physical wave tank. In applying *ihFoam*, Higuera et al. (2013) managed to simulate a broad range of coastal processes including solitary waves interaction with a vertical structure, a transient wave group, 3D solitary wave breaking, a rip current on a barred beach and, run up on a conical island.

3.2 Two-phase Flow Model

Based on the literature review presented in the previous chapter and taking into consideration the violent interaction between an incident wave and a surface piercing-column, a two-phase flow model was considered as the best method for the present study. Specifically, this model allows the modelling of extreme wave forms, including surface break-up. Within OpenFOAM, a two phase flow can be modelled using the *interFoam* solver. Within this, the two phases are represented by viscous, incompressible fluids, solved on a fixed mesh using VOF method. The present calculations were made with OpenFOAM version 2.2.x, with the wave generation toolbox, *waves2Foam* used to generate the incident waves at the inlet and absorb the waves at the outlet boundary. A description of VOF method used in the *interFoam* solver is given next, while the wave generation and absorption part will be discussed in the boundary condition section that follows.

3.2.1 Governing Equations

The *interFoam* solver, is based upon the Reynolds averaged NSE; the mass and momentum conservation equations being defined as follows:

$$\nabla \cdot \mathbf{u} = 0, \quad (3.1)$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \rho \mathbf{u} \mathbf{u} = -\nabla p^* - \rho \mathbf{g} \cdot \mathbf{x} \nabla \rho + \nabla \cdot [\mu \nabla \mathbf{u} + \rho \boldsymbol{\tau}] + \sigma_T \kappa_\alpha \nabla \alpha, \quad (3.2)$$

where $\mathbf{u} = (u, v, w)$ is the velocity vector in Cartesian coordinates, ρ is the density, $\mathbf{g}$ is the acceleration due to gravity, μ is the molecular viscosity, κ_α is the curvature of the interface, σ_T is the surface tension constant, and $\boldsymbol{\tau}$ is the Reynolds stress tensor. These equations are valid throughout the entire flow field and are applicable to each of the regions containing water or air; the solutions obtained in each region explicitly at each time step. Within these equations the gradient operator is given by

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right), \quad (3.3)$$

and the excess pressure given by

$$p^* = p - \rho \mathbf{g} \cdot \mathbf{x}, \quad (3.4)$$

where p is the pressure and $\mathbf{x} = (x, y, z)$ the coordinate vector. Within the present study, the surface tension is neglected, and the turbulence contribution is deliberately omitted. This reduces Equation (3.2) to

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \rho \mathbf{u} \mathbf{u} = -\nabla p^* - \rho \mathbf{g} \cdot \mathbf{x} \nabla \rho + \nabla \cdot [\mu \nabla \mathbf{u}]. \quad (3.5)$$

Within the *interFoam* solver, the free surface was described using a VOF two-phase model. In VOF, the different phases of the flow are represented by a scalar quantity, α , called the volume fraction. This accounts for the amount of fluid contained within a particular cell. For a two phase flow, such as air and water, $\alpha = 1$ if the cell is fully filled with water, while $\alpha = 0$ if the cell is filled with air. The free surface, or the interface between water and the air, is represented by an α value between 0 and 1. The evolution of the free surface is then tracked by advecting the volume fraction using the velocity field calculated from the NSE. The scalar transport equation for the volume fraction is given as

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot \mathbf{u} \alpha = 0. \quad (3.6)$$

As discussed in the previous chapter, much of research in VOF has been devoted to achieving a better description of the interface for the α field. In the *interFoam* solver, algebraic VOF is used to track the evolution of the free surface. The difficulty associated with solving equation Equation (3.6) is ensuring boundedness and preventing the smearing of the interface. The smearing of the interface is limited by the introduction of an artificial interface compression term such that

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot \mathbf{u} \alpha + \underbrace{\nabla \cdot \mathbf{u}_r \alpha (1 - \alpha)}_{\text{Interface compression}} = 0, \quad (3.7)$$

where $\mathbf{u}_r$ is the compression velocity given by $\mathbf{u}_r = \min[c_\alpha |\mathbf{u}|, \max |\mathbf{u}|]$. The factor c_α is used to control the compression of the interface with a default value of 1. The

compression of the interface can be enhanced by setting c_α to value larger than 1. If c_α is set equal to 0, no interface compression is applied, and equation Equation (3.6) is solved instead. The introduction of this compression term helps to limit the smearing of the interface and keeping the interface sharp. It should be noted that the interface is only compressed in the normal direction; from the air to the water phase. Furthermore, the artificial compression term is only active in the interface region ($0 < \alpha < 1$). Outside the interface region, the solution to the scalar advection equation is not affected by the artificial compression term.

In order to obtain a physical result, the solution to equation Equation (3.7) must be conservative and bounded between 0 and 1. The solution of a pure advection equation is difficult due to the absence of any diffusion term. In *interFoam*, the boundedness of the solution is achieved by using a solver called Multidimensional Universal Limiter for Explicit Solution (MULES). This is based on the flux-corrected transport (FCT) technique of Rudman (1997). In effect, it introduces a limiting factor applied to the fluxes of the discretized divergence term. This ensures that any under-shoot and overshoot of the solution is prevented and that the conservation of mass is always fulfilled.

Once the distribution of α is obtained, the local fluid properties in an individual cell can be calculated such that the effective density, ρ , and effective viscosity, μ , are defined by:

$$\rho = \alpha \rho_w + (1 - \alpha) \rho_a, \quad (3.8)$$

$$\mu = \alpha \mu_w + (1 - \alpha) \mu_a, \quad (3.9)$$

where subscripts w and a represent water and air respectively.

3.3 Finite Volume Discretization

Within the OpenFOAM framework, the solution to the differential equations are sought using the Finite Volume Method (FVM). Within this section, a brief description of the adopted geometric and equation discretization is provided.

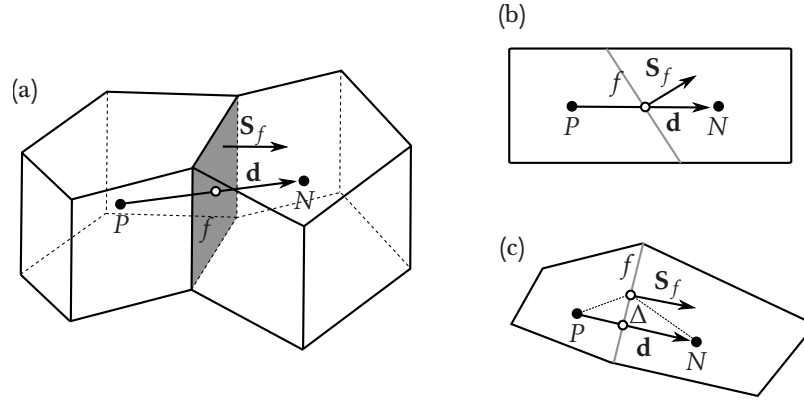


Figure 3.1: (a) Unstructured FV discretization. Examples of mesh (b) orthogonality, and (c) skewness.

3.3.1 Geometrical Discretization

Most numerical methods require geometrical discretization of a computational domain; the latter being a representation of the physical domain in which the governing equations are solved. The computational domain can be constructed to represent the exact physical domain or can be truncated to exploit the symmetry or periodicity of the solution. Geometrical discretization is achieved by dividing the computational domain into discrete non-overlapping meshes. The meshes need to completely fill the whole domain and conform as accurately as possible to the boundary of the domain. The meshes can be structured or unstructured and can have any shape; tetrahedras, pyramids, prisms, hexahedrals and polyhedrals being commonly adopted. During the mesh generation, topological information describing the mesh is generated along with basic geometrical information such as points, faces, and cells.

In FVM, the discretized meshes (commonly called control volumes or cells) are represented by points and faces that make up each cell. An example of three-dimensional unstructured cells for a FVM discretization is given in Figure 3.1(a). The cells are bounded by a set of flat faces with each face, f shared by two cells. In OpenFOAM, a collocated FVM approach is used where dependent variables and other properties are stored at the centroid of the cell, P . A vector connecting P and the centroid of a neighbouring cell, N , is given by $\mathbf{d}$. The face of each cell can either be an internal face or a boundary face. For the former, the face is shared between two cells, while the latter coincide with the boundaries of the domain. The face area vector, $\mathbf{S}_f$ is a normal vector that points outward from the cell and the magnitude is equal to the face area.

For simulations involving the NSE, the convergence and accuracy of the solution is highly dependent on the quality of the mesh used. Furthermore, the mesh resolution needs to be fine enough to capture all the relevant flow features. To date, there is no consensus, or standard metric, that can effectively assess the quality of the generated mesh. However, the usual criteria to check include:

- **Mesh orthogonality** is a measure of angular deviation of vector S_f from vector d as depicted in Figure 3.1(b). If this angle is high, the calculation of a gradient at the face centre, f , will be inaccurate. In this case a non-orthogonal correction should be applied. For the simulations undertaken in the present study, the non-orthogonality of the mesh is less than 60° . This ensured that only a small number of non-orthogonal corrections were performed to reduce the computational cost.
- **Mesh skewness** is the deviation, Δ , of the vector d from the face center, f , as shown in Figure 3.1(c). A large value of Δ will result in an inaccurate interpolation of the cell center quantities to the face centre, f . Since a structured block mesh was used in all cases described herein, a small value of the mesh skewness is ensured throughout. Nevertheless, skewness correction was performed automatically in every simulation.
- **Mesh aspect ratio** is the ratio of the longest side of the cell to the shortest side. If a large mesh aspect ratio is used, large gradients could be smeared. In practice, a large mesh aspect ratio can be employed provided the gradient of the solution in the long direction is small. However, in the present study, the maximum aspect ratio of 1 : 2 was maintained throughout. This limited any problems associated with smearing.

Within OpenFOAM, mesh generation can be carried out using (amongst others) the mesh generation utilities *blockMesh* and *snappyHexMesh*. For simple geometries, *blockMesh* can be used to generate multi-block meshes, while *snappyHexMesh* is often used for more complex geometries. In addition, OpenFOAM also provides support for mesh conversion to and from other mesh generation software. Within the present study, multi-block mesh for simple geometries were generated using *blockMesh*. For complex geometries (specifically those involving a column), an open-source mesh

generator called *Gmsh* was used. While *snappyHexMesh* was able to provide automated mesh generation for these geometries, there was little control over the quality of the generated mesh. In these cases, *Gmsh* was found to be preferable producing a higher quality mesh.

In undertaking the present calculations, the preliminary meshes were purely hexahedral. However, to achieve greater (finer) resolution, appropriate cells were converted to polyhedral meshes by internally splitting the original mesh. In this way, localised mesh refinement was achieved without disturbing the original mesh configuration. Moreover, this resulted in a lower number of cells when compared to meshes generated using other methods. It is important to note, that as far as the computations are concerned, local mesh refinement and the mixing between hexahedral and polyhedral meshes does not affect the solution accuracy, as long as the overall mesh density is sufficiently fine to resolve the physics of the problem.

3.3.2 Equations Discretization

Equation discretization is needed to convert the governing equations into a system of algebraic equations. Within this section, the FVM discretization of a generic transport equation is presented. This generic transport equation is used to represent any conservation law, including the NSE and the scalar transport equation of the volume fraction. Using a general scalar variable ϕ , the generic transport equation can be expressed as

$$\underbrace{\frac{\partial \rho \phi}{\partial t}}_{\text{time derivatives}} + \underbrace{\nabla \cdot (\rho \mathbf{u} \phi)}_{\text{convection term}} = \underbrace{\nabla \cdot (\rho \Gamma_{\phi} \nabla \phi)}_{\text{diffusion term}} + \underbrace{S_{\phi}(\phi)}_{\text{source term}}, \quad (3.10)$$

where ρ is the density, Γ the diffusivity and S_{ϕ} a source term.

In FVM discretization, Equation (3.10) is integrated over time, t , and volume, V_P , to yield

$$\begin{aligned} \int_t^{t+\Delta t} \left[\int_{V_P} \frac{\partial \rho \phi}{\partial t} dV + \int_{V_P} \nabla \cdot (\rho \mathbf{u} \phi) dV \right] dt \\ = \int_t^{t+\Delta t} \left[\int_{V_P} \nabla \cdot (\rho \Gamma_{\phi} \nabla \phi) dV + \int_{V_P} S_{\phi}(\phi) dV \right] dt. \end{aligned} \quad (3.11)$$

Using a generalized form of the Gauss theorem

$$\int_{V_p} \nabla \cdot \phi = \int_S \phi \cdot d\mathbf{S}, \quad (3.12)$$

where $d\mathbf{S}$ is the outward pointing differential of the surface area vector, the spatial derivatives in Equation (3.11) are converted from volume integrals to surface integrals.

The resulting equation is given by

$$\begin{aligned} \int_t^{t+\Delta t} \left[\int_{V_p} \frac{\partial \rho \phi}{\partial t} dV + \int_S (\rho \mathbf{u} \phi) \cdot d\mathbf{S} \right] dt \\ = \int_t^{t+\Delta t} \left[\int_S (\rho \Gamma_\phi \nabla \phi) \cdot d\mathbf{S} + \int_{V_p} S_\phi(\phi) dV \right] dt. \end{aligned} \quad (3.13)$$

At this point, no approximations have been applied in Equation (3.13) and the integration is exact. Beyond this point, appropriate approximation schemes need to be introduced to each discretized term.

Having converted the volume integral into a surface integral, the **convective term** can be discretized as follows:

$$\int_S (\rho \mathbf{u} \phi) \cdot d\mathbf{S} \approx \sum_f \mathbf{S} \cdot (\rho \mathbf{u})_f \phi_f. \quad (3.14)$$

The value of ϕ_f on the control volume surface, f , can be interpolated using values from the cell centre and the neighbouring cells using various face interpolation schemes. These include schemes such as central differencing (CD), upwind differencing (UD) and blended differencing (BD). In comparing these schemes, CD is second-order accurate, even on non-uniform meshes, but does not guarantee boundedness. On the other hand, UD guarantees boundedness, but is only first order accurate. In contrast, BD combines both CD and UD in one scheme, using a blending factor, in an attempt to provide a bounded solution with higher-order accuracy. In adopting this latter approach, the blending factor controls the relative contribution of CD and UD in the combined scheme.

Discretization of the **diffusion terms** is performed in a similar way. Having first integrated the term over the control volume and converted the volume integral to a

surface integral, the approximate discretization is defined by

$$\int_S (\rho \Gamma \nabla \phi) \cdot d\mathbf{S} \approx \sum_f \rho \Gamma_f (\mathbf{S} \cdot \nabla_f \phi). \quad (3.15)$$

The gradient term, $\nabla_f \phi$ on the face, f , also needs to be approximated. This can be achieved using schemes such as Gauss integration and face normal gradient.

For the **source terms**, first linearisation needs to be applied before discretization followed by integration over the control volume, yielding the following terms

$$\int_{V_P} S_\phi(\phi) dV = S_I V_P \phi_P + S_E V_P, \quad (3.16)$$

where S_I and S_E are implicit and explicit source terms, respectively.

Retaining the time derivative term, the semi-discretized generic transport equation is given by

$$\begin{aligned} \int_t^{t+\Delta t} \left[\int_{V_P} \frac{\partial \rho \phi}{\partial t} dV + \sum_f \mathbf{S} \cdot (\rho \mathbf{u})_f \phi_f \right] dt \\ = \int_t^{t+\Delta t} \left[S_I V_P \phi_P + S_E V_P + \sum_f \rho \Gamma_f (\mathbf{S} \cdot \nabla_f \phi) \right] dt. \end{aligned} \quad (3.17)$$

This method of discretization is called the Method of Lines in which the spatial discretization is performed first. Using this approach, the numerical approximation (or accuracy) can be selected to be different in the spatial and temporal terms. This flexibility offers a wide selection of approximation schemes. Throughout the present study, the discretization was achieved using an Implicit Euler scheme. After completing the volume integration and a finite difference like approximation for the time derivatives, the final discretization form of the generic transport equation is given by

$$\underbrace{\rho_P \frac{\phi_P^n - \phi_P^o}{\Delta t} V_P}_{\text{time derivatives}} + \underbrace{\sum_f \mathbf{S} \cdot (\rho \mathbf{u})_f \phi_f^n}_{\text{convection term}} = \underbrace{S_I V_P \phi_P^n + S_E V_P}_{\text{source term}} + \underbrace{\sum_f \Gamma_f (\mathbf{S} \cdot \nabla_f \phi^n)}_{\text{diffusion term}}, \quad (3.18)$$

where the superscripts n and o represent value at the current (new) and previous (old) time step respectively. It should be noted that the implicit Euler scheme is first-order

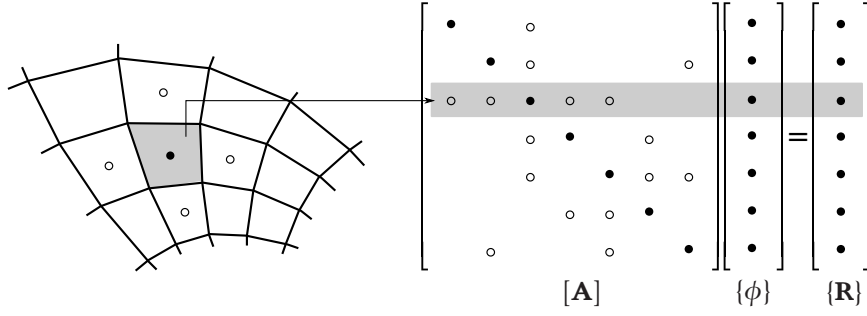


Figure 3.2: Sparse system of linear algebraic equation from FVM discretization of a two-dimensional mesh.

accurate. However, provided the entire spatial-temporal domain is discretized up to at least second-order, the overall accuracy of the solution will also be second-order.

3.4 System of Linear Algebraic Equation

The discretization of a generic transport equation yields a linear algebraic equation for each control volume. Based on the approximation and interpolation used for the convective and diffusion term, Equation (3.18) can be re-written in a generic form as

$$a_P \phi_P + \sum_N a_N \phi_N^n = R_P. \quad (3.19)$$

For each control volume, this linear algebraic equation links the variables, ϕ_P with its neighbouring value, ϕ_N . During the local mesh assembly, topological information from the geometrical discretization is used to find the neighbour of each control volume. After the local algebraic equation is calculated for each control volume, they can be collected and assembled into a system of linear algebraic equation. This process is referred to as a global assembly and leads to the matrix and vector form defined as follows

$$[\mathbf{A}]\{\phi\} = \{\mathbf{R}\}, \quad (3.20)$$

where $[\mathbf{A}]$ is a sparse square matrix containing the coefficients for the unknown variables in the column vector, $\{\phi\}$ and $\{\mathbf{R}\}$ is the column vector of the source terms. A sparse matrix is one in which most matrix coefficients are equal to zero. An example of the system of linear algebraic equation (presented in a matrix and vector form) for a two-dimensional mesh is given in Figure 3.2. The shaded region represents an

algebraic equation for a local control volume. The solid circles in the matrix $[\mathbf{A}]$ are the cell centred coefficients, a_P , while the opened circles are the coefficients for the neighbouring control volumes, a_N . After the imposition of appropriate boundary conditions, the system of linear algebraic equations can be solved for the unknown ϕ using direct or iterative methods.

Direct methods, such as Gauss elimination, can be used if the resulting system is small and provide an accurate solution limited only by machine precision. However, in CFD, the solution of the algebraic system does not need to be extremely accurate. Indeed, provided the accuracy is higher than the discretization scheme, this is usually sufficient. In terms of the computational effort, the use of a direct method can be prohibitive as the effort scales with $O(n^3)$, where n is the number of unknown in the system. Furthermore, a direct method does not necessarily preserve the sparseness of the system. As a result, memory requirements may become an issue when solving very large systems.

More typically, solving the system of algebraic equations is achieved using an **iterative method**. This is primarily due to the low storage requirements and the ability to preserve the sparseness of the resulting matrix system. For most of the simulations carried out in the present study, an iterative solver based upon the Preconditioned Conjugate Gradient (PCG) method was used. This is appropriate to solve a matrix that is both symmetric and positive definite. The convergence rate of the solver can be improved through the use of preconditioning. In the present study, a Diagonal Incomplete Cholesky (DIC) preconditioner was used with the PCG solver. Alternatively, in those cases when the matrix was asymmetric, a Preconditioned Bi-Conjugate Gradient (PBiCG) solver with Diagonal Incomplete Lower Upper (DILU) preconditioning was used.

3.5 Solution Procedure

Solutions to the multi-phase flow outlined earlier are obtained by solving the NSE and a scalar transport equation of the volume fraction. Although the general discretization technique presented in the previous section can be used to discretize the NSE, the solution of the NSE needs a special formulation that can deal with both **nonlinearity** and **pressure-velocity coupling**. Nonlinearity in the NSE comes from the convective

terms in the momentum equation. To obtain a solution for the nonlinear problem, a nonlinear iteration scheme such as the Picard or Newton-Raphson method could be used. However, introducing a nonlinear iteration in an unsteady NSE is computationally expensive. Alternatively, the convective terms can be linearised using the velocity field from the previous time-step. If the time-step is sufficiently small, the changes between consecutive time-steps is small and the changes due to nonlinearity insignificant. In effect, the linearised convective term can be treated explicitly and added to the source term.

3.5.1 Pressure-velocity Coupling

The main challenge in solving the NSE is associated with the pressure-velocity coupling. While the velocity can be calculated from the momentum equations, the pressure term is missing and cannot be determined from the continuity equation. As a result, an iterative method cannot be used to solve the velocity-pressure coupling. The solution to this problem involves using a segregate approach to effectively split the velocity and pressure term. A widely used segregate approach appropriate to the steady state solution of NSE pressure-velocity coupling is the Semi Implicit Pressure Linked Equation (SIMPLE) first proposed by Patankar & Spalding (1972). For a transient NSE, a related method called Pressure Implicit with Splitting of Operator (PISO), proposed by Issa (1986), is widely used.

The **PISO** algorithm is used in the current *interFoam* solver. Following the algorithm presented in Rusche (2003), the semi discretized momentum equation can be written as

$$a_P \mathbf{u}_P = \mathbf{H}(\mathbf{u}) - \nabla p^* - \mathbf{g} \cdot \mathbf{x} \nabla \rho, \quad (3.21)$$

where $\mathbf{H}(\mathbf{u})$ comes from the source term and the coefficient for the neighbouring cells multiplied by the corresponding velocity. Using Equation (3.21), the velocity $\mathbf{u}$ at the cell centre can be expressed as

$$\mathbf{u}_P = [a_P]^{-1} [\mathbf{H}(\mathbf{u}) - \nabla p^* - \mathbf{g} \cdot \mathbf{x} \nabla \rho]. \quad (3.22)$$

Substituting, Equation (3.22) into Equation (3.1), a Pressure Poisson Equation (PPE)

can be derived as follows

$$\nabla \cdot [a_P]^{-1} \nabla p_f^* = \nabla \cdot [a_P]^{-1} [H(\mathbf{u}) - \mathbf{g} \cdot \mathbf{x} \nabla \rho]. \quad (3.23)$$

In the PISO algorithm, the momentum equation is first solved using the pressure gradient from a previous time step to give an approximation of the new velocity field. This step is called the **momentum predictor** step. Using the predicted velocities, the new pressure field is calculated from the formulated pressure equation. This is called **pressure solution** step. Using the new pressure distribution, the velocity field is then corrected in an explicit manner. This last step is called the **explicit velocity correction** step.

The use of explicit velocity correction assumed that the correction is due to the pressure gradient only and neglects the influence from the neighbouring velocities. To improve the accuracy of the solution, the value of neighbouring velocities are corrected and a new pressure equation is formulated. This step is carried out until a pre-determined tolerance is reached. At each time step, an additional equation, such as the scalar transport equation is solved together with the PISO algorithm. A summary of the whole solution procedure for PISO algorithm is given as follows:

1. Set up the initial condition for all variables
2. Advance to a new time step
3. Solve the scalar transport equation in a temporal sub-cycling loop
4. Assemble and solve the momentum predictor equation using variables from previous time step
5. Enter PISO loop. Solve for the pressure and correct velocity. Iterate until tolerance is reached.
6. Return to step 2 if final time is not reached

3.5.2 Adaptive Time Stepping and Temporal Sub-cycling

Adaptive time stepping was used for all simulations undertaken in the present study. This involves the calculation of the appropriate time step at the beginning of each

time iteration based upon a constant Courant number; the latter defined by

$$C_o = \frac{\mathbf{u}_f \cdot \mathbf{S}_f}{d \cdot \mathbf{S}_f} \Delta t, \quad (3.24)$$

where Δt is the time step and $\mathbf{u}_f$ is the velocity vector. In adopting an adaptive time stepping scheme, the solution becomes more stable. Furthermore, it also allows much larger time steps to be used, provided that the global calculated velocity is smaller. This results in a much more efficient computation when compared to a calculation based on a fixed time-step.

Although the use of adaptive time stepping can stabilize a solution, both the convergence and the stability of the VOF solver are very sensitive to the scalar transport equation of the volume fraction. Whilst higher-order bounded schemes can be used to ensure a bounded solution, instabilities can still occur for a highly nonlinear problem. Given our interest in highly nonlinear (near breaking and breaking) waves, the stability of the whole solution can be enhanced further by using a temporal sub-cycling for the volume fraction. In this temporal sub-cycling, the time step for the sub-cycles is calculated using

$$\Delta t_{sc} = \frac{\Delta t}{n_{sc}}, \quad (3.25)$$

where Δt_{sc} is the time step for the sub-cycles and n_{sc} is the number of sub-cycles. Clearly, the use of temporal sub-cycling introduces additional computations in each time step. However, the solution of the volume fraction is much quicker when compared to the solution of the pressure and velocity in the PISO loop. Furthermore, the introduction of temporal sub-cycling provides a more accurate solution to the volume fraction equation. This enables the use of a larger global time step and therefore a considerable speed-up of the solution procedure. In most of the simulations carried out in this study, the maximum Courant number for both NSE and volume fraction was set to 0.3, while the number of temporal sub-cycling was set to $n_{sc} = 5$. As a result, at each sub-time step, the effective Courant number for the volume fraction is 0.06. Using these values, no stability issue is encountered in the present study.

3.6 Boundary Conditions

In the previous section, the numerical discretization of the multi-phase flow model has been presented but the discretized model was of a general form. For a specific problem to be uniquely defined, both the initial and boundary conditions are needed. The initial conditions are necessary at the start of the simulation to time march the solution; while boundary conditions are needed to uniquely define the problem spatially. The boundary conditions are specified on the cell faces that coincide with the boundaries of the physical domain. Imposing the correct boundary condition is essential to obtain accurate physical results.

In a general sense, boundary conditions can be classified into numerical and physical. In a numerical sense, they can either be Dirichlet or Neumann conditions; the former involving prescribed values of the variables on the boundary, and the latter the gradient of the variables normal to the boundary.

The implementation of a **physical boundary** in the numerical model can be based on either Dirichlet, Neumann or a combination of both. For a two-phase incompressible flow, the variables involved are velocity ($\mathbf{u}$), pressure (p) and volume fraction (α). Within OpenFOAM, the following physical boundary conditions are readily available:

- Basic boundary conditions
 - **A fixed value.** This is a Dirichlet type boundary condition where a fixed value is prescribed at the boundary face. At the inlet, this boundary condition is used to impose the velocity incident to the domain.
 - **A zero gradient.** This is a Neumann type boundary condition that extrapolates a zero gradient value to the face boundary from cell center value.
- Derived boundary conditions
 - **A free slip condition :** This ensures that the velocity normal to the surface is zero, while the pressure and volume fraction are defined as having a zero gradient.

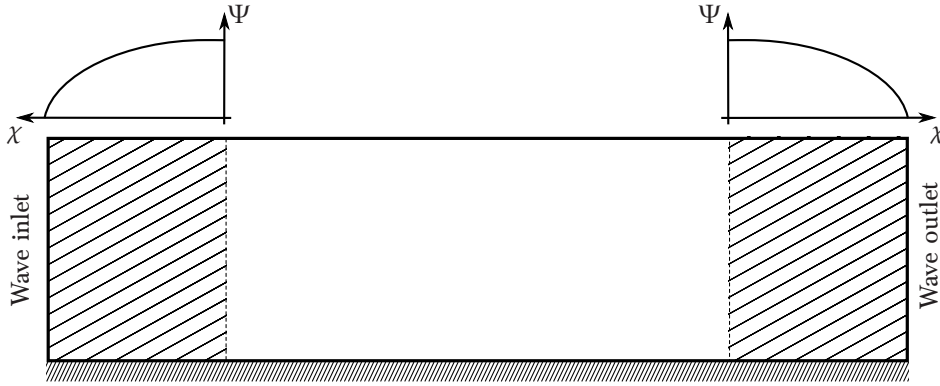


Figure 3.3: Sketch of the relaxation zone (hatch zone) and the variation of the blending function, Ψ for the inlet and outlet boundaries.

- **A plane of symmetry.** The value of gradient normal to the boundary is set to zero, while components parallel to the boundary are projected to the boundary from the cell centre.
- **A cyclic or periodic boundary.** In this case, the value on the first boundary is mapped onto the second boundary, creating a periodic solution.

3.6.1 Wave Generation and Absorption

The official version of the *interFoam* solver does not come with the capability to generate waves on an inlet boundary or to absorb waves on an outlet boundary. In the present study, the wave generation and absorption capability were introduced using a third-party toolbox called *wave2Foam* (Jacobsen et al., 2012). This was developed based on *interFoam* solver, and will be rigorously validated, for a broad range of wave events, in Chapter 4.

Using *wave2Foam*, **wave generation** is achieved by specifying the velocity, $\mathbf{u}$, the surface elevation, η and the normal derivatives of the pressure, $\partial p / \partial n$ on the inlet boundary. All the values are calculated from readily available analytical wave theories such as Stokes', stream-function, cnoidal, solitary and irregular (or random) waves. The value of the surface elevation was prescribed through the volume fraction, α . If the cell is partially submerged, α is calculated by the ratio of the wetted area to the total area of the boundary face. On the other hand, $\alpha = 0$ for dry face and $\alpha = 1$ for wetted or fully submerged face.

The **wave absorption** capability in *wave2Foam* is introduced through the use of a relaxation zone. This serves two purposes. First, it is used to prevent or reduce wave reflection from the outlet boundary. Second, if the wave is being reflected back from the outlet or some object placed inside the domain, the relaxation zone can help by reducing the interference between the reflected waves and the waves generated at the inlet boundary. Application of the relaxation zone is flexible and can be defined at any part of the computational domain to prevent wave reflection. In the present study, a rectangular shaped relaxation zone was used at the inlet and outlet boundary as outlined in Figure 3.3. Within each relaxation zone, a known solution at the boundary, ϕ_{known} , is blended with a computed solution, ϕ_{computed} , based on the following equation

$$\phi = (1 - \Psi) \phi_{\text{known}} + \Psi \phi_{\text{computed}}, \quad (3.26)$$

where the ϕ values represent either u or α . The exponential blending function, Ψ is given by

$$\Psi = 1 - \frac{\exp(\chi^\beta) - 1}{\exp(1) - 1}, \quad (3.27)$$

where β was taken as 3.5 through-out and χ represents the normalized distance from the internal side of the relaxation zone to the boundary. The sketch of the relaxation zone and the blending function is given in Figure 3.3. At the boundary, $\chi = 1$, giving $\Psi = 0$, and ϕ equal to the known value imposed on the boundary.

4

Numerical Validation

4.1 Introduction

This chapter presents the simulation of 2D unidirectional waves using OpenFOAM. The purpose of the simulations is to explore in detail, the capability of OpenFOAM to accurately model surface wave propagation. Simulations involve standing waves, regular waves, irregular waves, and breaking waves. These validations are critically important not only because they justify the suitability and accuracy of the method, but because an accurate representation of the incident wave field is essential for the subsequent work involving the interaction of the waves and a surface piercing column. In this sense, the present chapter presents preliminary calculations that are fundamental to the success of the overall study.

4.2 Standing Wave Simulation

In this section, the results of a standing wave simulation are presented. The purpose of this simulation is to assess the stability and accuracy of the *interFoam* solver. A standing wave simulation is selected for two reasons. First, the simulation allows a simple boundary condition to be used, eliminating the need for both wave generation and absorption to be addressed. Second, the simulation allows a straightforward

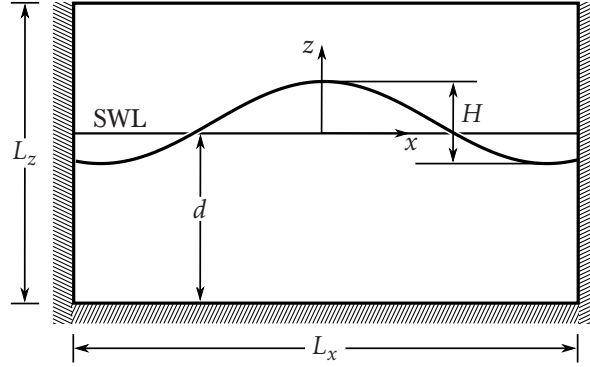


Figure 4.1: Computational domain for a standing wave.

calculation of the total energy in the system, allowing an assessment of the numerical stability.

The free surface, $\eta(x)$, corresponding to a linear standing wave with wave-height, H and wave-length, λ , was specified by

$$\eta(x) = \frac{H}{2} \cos(kx), \quad (4.1)$$

where $k = \frac{2\pi}{\lambda}$ is the wave-number. If $H = 0.02$ m and $\lambda = 2$ m, the corresponding wave-period defined by linear dispersion is $T = 1.134$ s. With the water depth set to $d = 1$ m ($kd = \pi$), a deep water wave condition was maintained.

The length of the computational domain, L_x , was set equal to the wave-length, λ , and the domain height was defined by $L_z = 1.2d$. A sketch of the computational domain is given in Figure 4.1. Initially, the domain was discretized using 200 cells in the x direction and 120 cells in the z direction. Local mesh refinement was then applied within the vicinity of SWL to increase the resolution of the mesh so that an accurate representation of the free surface could be achieved. After applying this refinement, a total of 81 060 hexahedra and 1470 polyhedra cells were generated with approximately 15 cells spread over the wave-height, H . A subsequent mesh convergence study (Section 4.3.1) has shown that this number of cells was adequate to fully resolve the wave propagation.

In this simulation, turbulence modelling was turned off and the flow assumed laminar. A slip boundary condition was applied at all boundaries except the top upper surface or boundary. At the top boundary, a special OpenFOAM boundary condition was prescribed such that the air phase is allowed to move in and out of

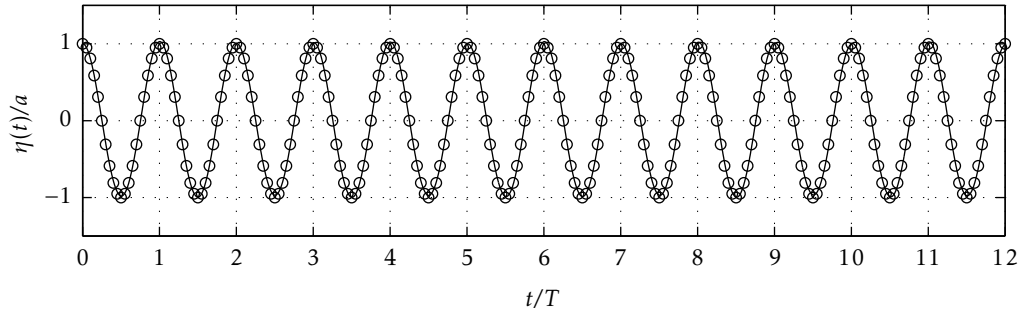


Figure 4.2: Time history of surface elevation at the centre of the domain, $\eta(t)$ at $x = 0$; comparisons between numerical prediction (—) and the analytical solution ($\circ$) for a standing wave.

the computational domain to prevent build up of pressure. For the sake of simplicity, this boundary condition will be called the *atmospheric* boundary. The slip boundary condition, indicated by the shading on Figure 4.1 is considered to avoid the need for increased resolution of the flow at these boundaries. This helps to reduce the number of cells and hence the computational effort. Despite the use of a slip boundary condition, *interFoam* is a viscous fluid solver and the properties of the fluid phases are required. These were set as follows: the viscosity and density of the water phase were $\mu_w = 1 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ and $\rho_w = 1000 \text{ kg m}^{-3}$, and for the air phase $\mu_a = 1.48 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ and $\rho_a = 1 \text{ kg m}^{-3}$. These values were considered appropriate and applied throughout the present study.

At the start of the simulation, the wave profile was prescribed using Equation (4.1), with the velocity and excess pressure set to zero over the entire domain. Adaptive time stepping was adopted with the Courant number set to 0.3 for both the velocity and the volume fraction field. The simulation was carried out for 11 s, or approximately 10 wave periods. The surface elevation was evaluated from the volume fraction, α , field as follows

$$\eta(x, t) = \int_{z=-d}^{z=\hat{z}} \alpha(x, z, t) dz - d, \quad (4.2)$$

where $\hat{z}$ is the coordinate of the top or *atmospheric* boundary.

Figure 4.2 shows the time history of surface elevation, η , measured at the centre of the domain. The time has been normalized by the wave period and the surface elevation normalized by the linear wave amplitude, $a = H/2$. It is clear from this figure, that excellent agreement is observed between the numerical prediction and

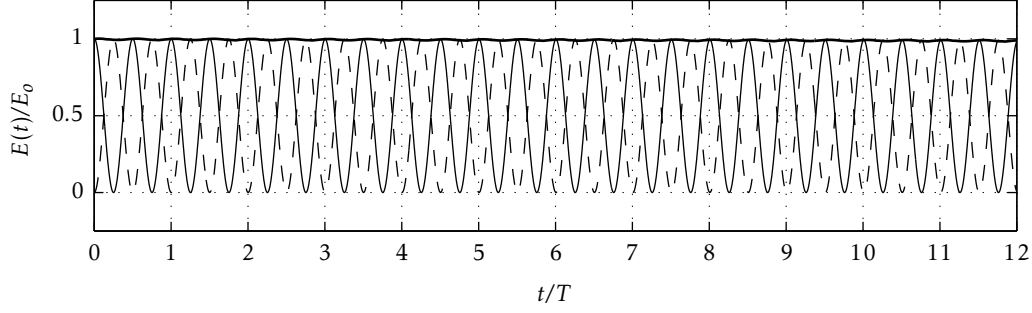


Figure 4.3: Time-histories of the potential energy, E_p (—), kinetic energy, E_k (- - -), and total energy, $E = E_p + E_k$ (— · —), for a standing wave described in Figure 4.1 and Figure 4.2. E_0 correspond to initial energy.

the analytical solution. Although the viscosity of the fluid was non-zero, there is no evidence of wave damping in the solution; the reduction in the wave height over the period being less than 2%. This suggests that the contribution from viscosity is weak, and the use of slip boundary condition is appropriate. Furthermore, the phasing of the solution is correctly predicted, even after 10 wave periods. These results confirm the ability of the numerical model to accurately describe a deep water standing wave.

The stability of the numerical solver can be determined by calculating the total energy in the system. For a numerical solver to be deemed stable, the total energy in the system must remain bounded, especially for long periods of simulation. The total energy, E , for the whole domain can be calculated by summing up the contribution from the kinetic energy, E_k , and the potential energy, E_p , at each time step. These are given by:

$$E_k(t) = \frac{1}{2} \rho_w \int_{-d}^{\eta} \int_{-\lambda/2}^{\lambda/2} u(x, z)^2 + w(x, z)^2 dx dz, \quad (4.3)$$

$$E_p(t) = \frac{1}{2} \rho_w g \int_{-\lambda/2}^{\lambda/2} (\eta(x) + d)^2 dx. \quad (4.4)$$

Time-histories of the normalized potential, kinetic and total energy from the numerical simulation are given in Figure 4.3. These results confirm that the total energy is conserved. Although there is a very small fluctuation in the total energy, the value stays bounded, with no apparent change in the total energy with time. Based on the result presented so far, it can be argued that the *interFoam* solver is numerically stable.

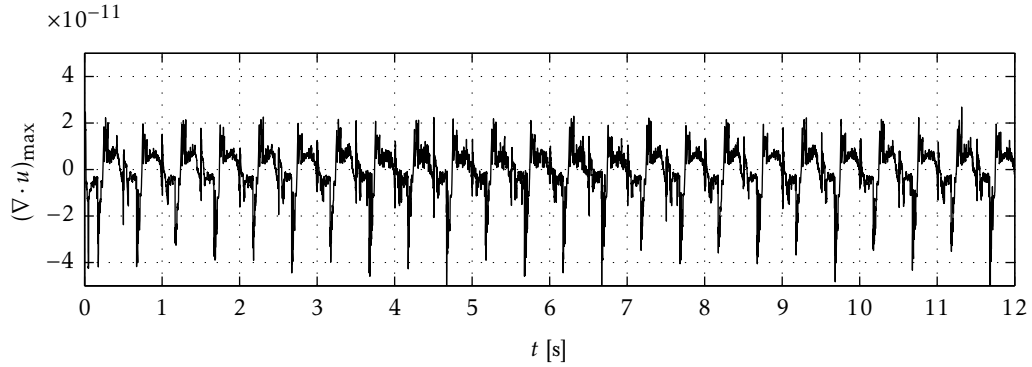


Figure 4.4: Global mass conservation in a standing wave simulation.

While energy conservation is important as an indication of the stability of the numerical scheme, it is of interest to see if other parameters particularly mass are also conserved. Figure 4.4 addresses this by providing a time-history of the maximum divergence, $(\nabla \cdot \mathbf{u})_{\max}$ for the whole domain. It is clear from the figure that over the whole period of the simulation, mass is well conserved and stays bounded. This is in agreement with the literature review where mass is well conserved for the VOF scheme.

4.3 Regular Wave Simulation

Having accurately modelled a standing wave, our attention now turns to the numerical simulation of a deep water regular wave. This section is divided into two parts. The first concerns a convergence study for regular waves generated using periodic conditions, while the second deals with validation work for the wave generation and absorption using the *wave2Foam* toolbox. In the second part, two regular wave conditions are considered: (a) waves propagating into still water, and (b) waves with an initially prescribed flow field.

4.3.1 Mesh Convergence Study

This section presents numerical results relating to the convergence of a two dimensional regular wave propagating in deep water under cyclic (or periodic) boundary conditions. The properties of the regular wave are tabulated in Table 4.1. Having adopted a linear representation of the wave-number, the wave steepness is given by $Hk/2 = 0.1$. Based upon this result, the wave can be considered weakly nonlinear.

Table 4.1: Regular wave parameters

Parameter	Value
Water depth, d	1.0 m
Wave period, T	1.0 s
Wave height, H	0.05 m
Wave-number, k	4.03 rad m^{-1}

Table 4.2: Test cases for mesh convergence

Mesh	Δx [mm]	Δz [mm]	$\lambda/\Delta x$	$H/\Delta z$
1	10	10	156	5
2	5	5	312	10
3	2.5	2.5	624	20

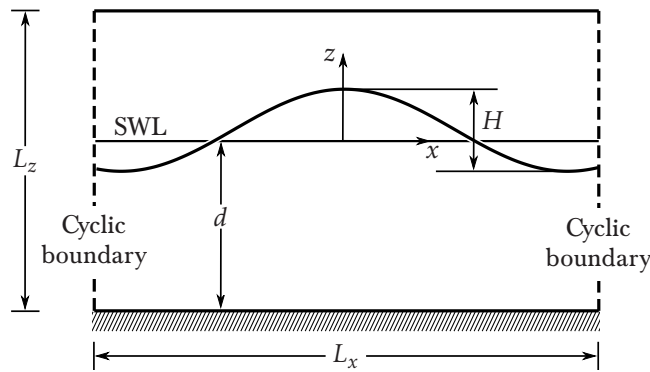


Figure 4.5: Computational domain for convergence study of a regular wave.

To avoid problems associated with spurious bound waves, the regular wave was initialized using a higher-order Stokes wave theory. With $kd = 4.03$ the wave is clearly propagating in deep water.

Adopting these wave properties, a convergence study was carried out using meshes with increasing spatial discretization. Three mesh of different densities were employed; their parameters given in Table 4.2. In all cases, the individual cells have an aspect ratio of unity.

Figure 4.5 illustrates the computational domain used in this convergence study. The domain length, L_x was defined to be equal to the wave-length, λ , and the domain height was set to $L_z = 1.2d$, where d is the water depth. The computational domain was discretized using the *blockMesh* utility based on the cell sizes tabulated in Table 4.2, where Δx and Δz are the horizontal and vertical cell size respectively.

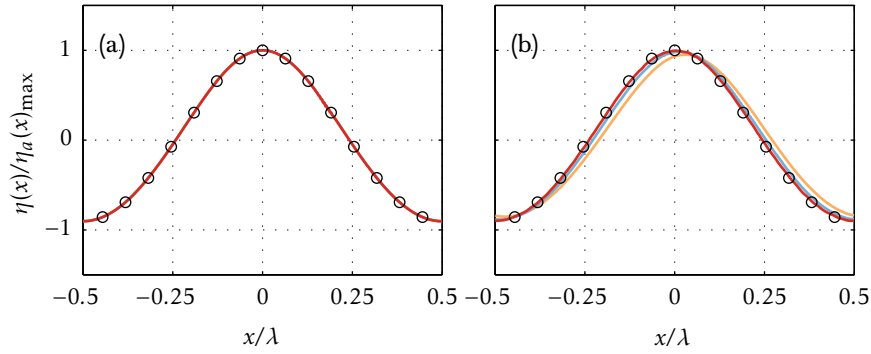


Figure 4.6: Spatial profile of the water surface elevation, $\eta(x)$, with comparisons between the numerical simulations and an analytical solution ($\circ$) at (a) $t/T = 0$ and (b) $t/T = 10$ for a regular wave. Legends: — mesh 1, — mesh 2, and — mesh 3.

A slip boundary condition was applied on the bottom boundary or bed and an *atmospheric* boundary condition was used for the top boundary; the cyclic behaviour being based on the periodic nature of a regular wave train. By adopting a cyclic boundary, the domain can be made relatively short (in this case, one wavelength), keeping the computational effort to a minimum. Furthermore, the use of cyclic boundary conditions eliminates the need for wave generation and absorption, thereby removing any possible errors associated with them. In effect, it isolates the role of the mesh density, with other waves being dealt with later.

Adaptive time stepping was adopted in this study, with Courant numbers of 0.3 and 0.1 used for each case. The simulation was carried out for a duration of 10 wave periods. At the start of the simulation, the initial surface elevation and the velocity field were prescribed through out the computational domain using a Stokes 5th-order solution (Fenton, 1985). During the simulation, the surface elevation, $\eta(x, t)$ was ‘measured’ by a set of numerical wave gauges, distributed uniformly along the domain with a spacing of 0.01 m. After the simulation was completed, the amplitude and phase error in the middle of the domain were measured and compared against the analytical solution. The maximum crest elevation and phase for every wave cycle were determined using a zero up-crossing analysis. The percentage error was calculated as

$$\text{error}_{\chi} = \frac{\chi_{\text{computed}} - \chi_{\text{analytical}}}{\chi_{\text{analytical}}} \times 100, \quad (4.5)$$

where χ is either the crest elevation or the phase.

Figure 4.6 shows the spatial profile of the water surface elevation at two different

time steps for a mesh of varying density with a Courant number of 0.3. The analytical solution from a Stokes 5th-order solution is overlaid on top for comparison. In Figure 4.6(a), corresponding to $t/T = 0$, the surface elevations shows perfect agreement with each other. This agreement is important, not only for the validation of the surface elevation but also because it highlights the reliability of the numerical wave gauges used to accurately evaluate the position of the free surface from the volume fraction field. After 10 wave periods, the data presented in Figure 4.6(b) shows good agreement for all cases, except with the coarsest mesh. In this case the maximum amplitude was under-predicted with a slight discrepancy of the wave phasing.

The magnitude of the errors in amplitude and phasing are further considered in Figure 4.7. Wave simulations corresponding to a Courant number of 0.3 are provided in the left hand column of Figure 4.7(a) and 4.7(c). In all cases the error is plotted against normalized time, t/T . At the start of the simulation, the amplitude error, given by Figure 4.7(a), is essentially zero for all cases. However, as time progresses, there is an increase in the error especially for the coarsest mesh (mesh 1). At the end of the simulation period ($t = 10T$), the amplitude error for mesh 1 is approximately 5%. Mesh 2 also shows an increased amplitude error, but at a much lower rate; the maximum error being approximately 1.6% at the end of the simulation. Not surprisingly, mesh 3 exhibits the best convergence behaviour, with no significant amplitude error for $t < 8T$ and a maximum error of 0.5% at $t = 10T$. The error in the wave phasing is given by Figure 4.7(c). This exhibits a similar trend; the errors reducing as mesh density increases. Overall, these result confirm the convergence of the numerical solution.

To further investigate the convergence of the numerical solution, the equivalent simulation results for a Courant number of 0.1 are presented in Figure 4.7(b) and 4.7(d). From the figures, it can be seen that a reduction in the Courant number decreases the errors in both the wave amplitude and the phasing for all cases. Comparing Figure 4.7(a) and 4.7(b), the error in the wave amplitude reduces by half for mesh 1 and mesh 2. In contrast, mesh 3 exhibits almost no error as compared to the small error with a Courant number of 0.3. Interestingly, for a Courant number of 0.1, the error in the wave phasing exhibits no increases in the error with time; the value fixed at about 0.5% through-out the simulation period. This error is equal to a difference of about 0.005s of the actual period. When compared to the exact wave

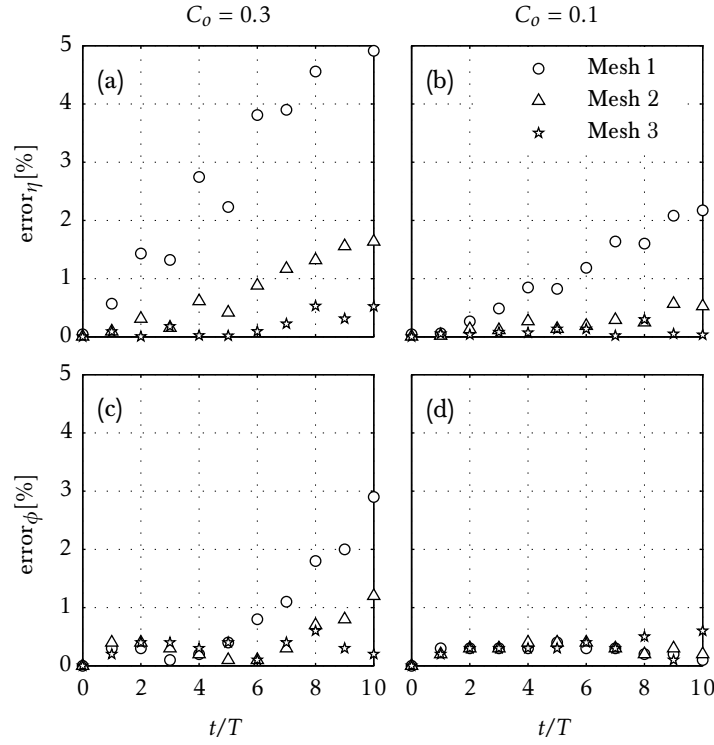


Figure 4.7: The time evolution of the percentage errors in the wave amplitude, (a) and (b), and the wave phase, (c) and (d), for a regular wave; the left hand column corresponding to $C_o = 0.3$ while the right hand column corresponds to $C_o = 0.1$.

period of 1 s, this error is considered very small and may be disregarded.

The data presented on Figure 4.7 shows good convergence as the cell size is reduced, but better convergence with the use of a smaller Courant number. This implies that although grid convergence is necessary, time discretization plays a rather important factor in achieving accurate results. However, the use of a smaller Courant number has important practical implications. This is primarily due to the fact that the Courant number is related to the time step; a decrease in Courant number implying a corresponding decrease in the maximum allowable time step. This results in a corresponding increase in the computational time. For example, the simulation time for the finest mesh case with a Courant number of 0.3 is around 30 min on a 12 cores Intel Xeon E5-1650 processor with 3.2GHz. However, this value increases to almost 100min when a Courant number of 0.1 was used. For the current 2D wave case, this difference seems small. However, when considering 3D wave cases, the increase in the total number of cells can render the simulation prohibitively long. In an attempt to achieve good accuracy within a reasonable computational

Table 4.3: Regular wave cases

Test case	T [s]	$Hk/2$	Description
6	1.0	0.1	Generated from SWL
7	0.8	0.2	Prescribed initial input
8	1.0	0.2	
10	1.4	0.2	

time, a Courant number of 0.3 was selected and used for the rest of the simulations presented hereafter. Nevertheless, the results presented in this section have shown the successful validation and convergence of a regular wave simulation with periodic boundary conditions applied.

4.3.2 Regular Waves, with Wave Generation and Absorption

Although the simulation of regular waves with periodic boundary conditions is computationally efficient, the practical application of this method is limited. For example, it cannot be used to model wave-structure interactions due to the absences of an outlet boundary to absorb any internally generated waves. Moreover, with a structure placed within the domain, the periodicity of the wave field will be lost. As a result, wave generation and absorption boundaries are required. In the present study, these were provided through the *wave2Foam* toolbox.

Within this section, the wave generation and absorption capability of *wave2Foam* are considered using regular wave cases. The regular wave parameters are presented in Table 4.3. All waves are generated in a water depth of $d = 1.0\text{m}$. In addition to validating the wave generation and absorption capability, these test cases serve to highlight the flexibility of *wave2Foam* in terms of wave modelling using two different methodologies. The first (corresponding to test case 6), generates or propagates the regular waves into quiescent or still water. This methodology is very similar to wave generation in a physical wave tank where waves are introduced at one end and absorbed at the other end, with the initial condition quiescent. The second method, uses a known solution to define the initial wave profile in space, throughout the computational domain. This is applied in test cases 7, 8 and 10. Although this second method appears quite similar to the methodology adopted in the cyclic or periodic boundary cases, there are important differences:

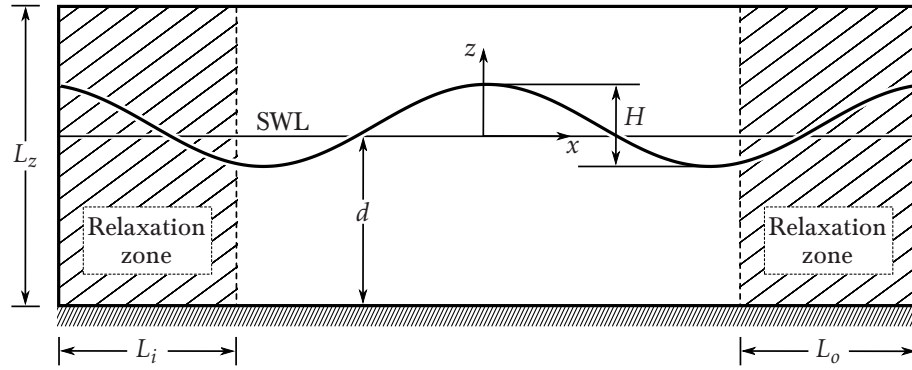


Figure 4.8: Computational domain for regular waves simulations.

- (i) the assumed solutions are not periodic,
- (ii) the wave generation at the input boundary continues beyond the initial spatial representation,
- (iii) wave absorption applies at the outlet boundary, and
- (iv) relaxation zones are used to achieve (ii) and (iii).

A schematic representation of the computational domain is presented in Figure 4.8. The total length of the computational domain is approximately 6.5λ and the wave height $1.2d$. The domain was discretized using the *blockMesh* utilities and mesh refinement was applied in the vicinity of the SWL. The refinement produces approximately 16 cells over the wave height, with a cell aspect ratio of 1:2. After the discretization, the total number of cells generated for each case ranges from around 80 000 to 100 000 cells.

In undertaking test case 6 (Table 4.3) an inlet boundary relaxation zone with length, $L_i = \lambda$ was applied whilst at the opposite end an outlet relaxation zone of length $L_o = 2\lambda$ was used. In terms of the initial conditions, the SWL was prescribed and the velocity and pressure set to zero value over the entire domain. A slip boundary condition was applied at the bottom (bed) boundary and an *atmospheric* boundary was used at the top. At the inlet boundary, an analytical solution based on a Stokes 5th-order theory was used to generate the desired wave form. At the outlet boundary, a SWL profile and zero velocity was applied. Finally, a set of numerical wave gauges was placed inside the computational domain from the inlet boundary to the outlet boundary with spacing of $\Delta x = 0.01$ m. At the start of the simulation

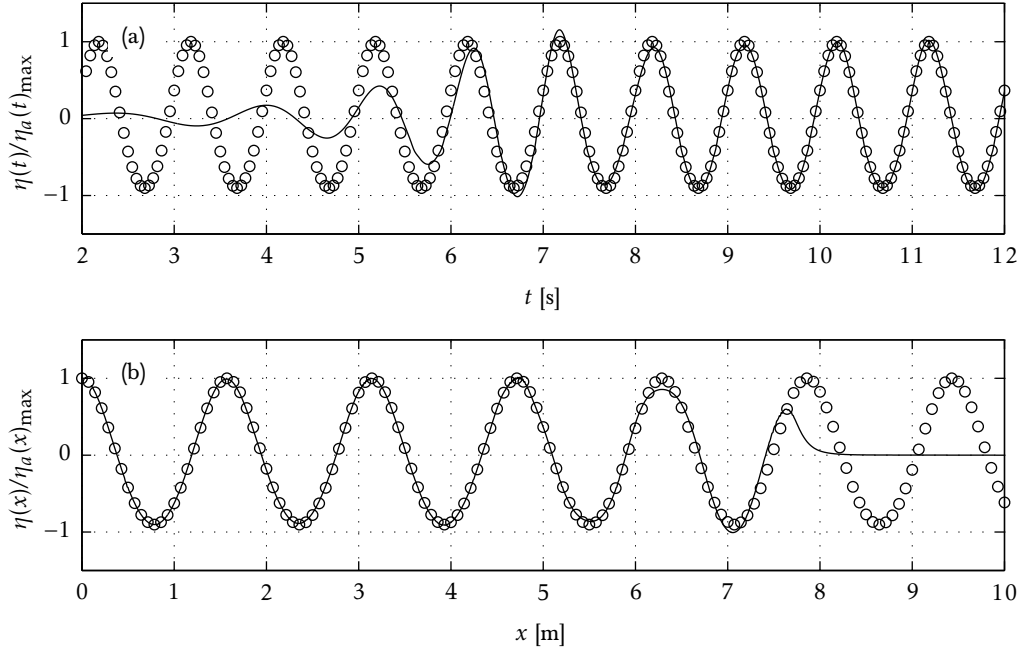


Figure 4.9: (a) Time history of surface elevation, $\eta(t)$, at the middle of the computational domain and (b) spatial profile of surface elevation, $\eta(x)$, at $t = 10$ s with comparison between numerical simulation (—) and Stokes 5th-order solution (○) for test case 6. Note: In Figure 4.9(b), the analytical Stokes solution has been extended (unchanged) into the relaxation zone to highlight the effects arising within this region.

a ramp-up time, $t_{\text{ramp}} = 1.0$ s was employed and the simulation was carried out over 12 wave periods.

Figure 4.9 presents the result of the surface elevation, (a) $\eta(t)$ at the middle of the computational domain and (b) $\eta(x)$ at $t = 10$ s predicted by the numerical simulation in test case 6. In both cases the results are compared with the analytical solution from a Stokes 5th-order wave theory. In terms of $\eta(t)$, Figure 4.9(a) shows that the solution progressively developed from the SWL reaching a steady state solution at around $t = 9.2$ s onward. Before this time, a steep transient wave event can be observed at $t = 7.2$ s. This is a consequence of the wave train propagating into still water, producing an evolving wave envelope at the front of the wave train. After a steady state has been reached, comparisons with the analytical solution show excellent agreement; both the amplitude of the crest and the trough being accurately predicted.

The spatial profile of the surface elevation, $\eta(x)$ at $t = 10$ s is shown in Figure 4.9(b). At this time, the solution has already reached a steady state. As such, comparisons with the analytical solution are excellent. Within the outlet relaxation

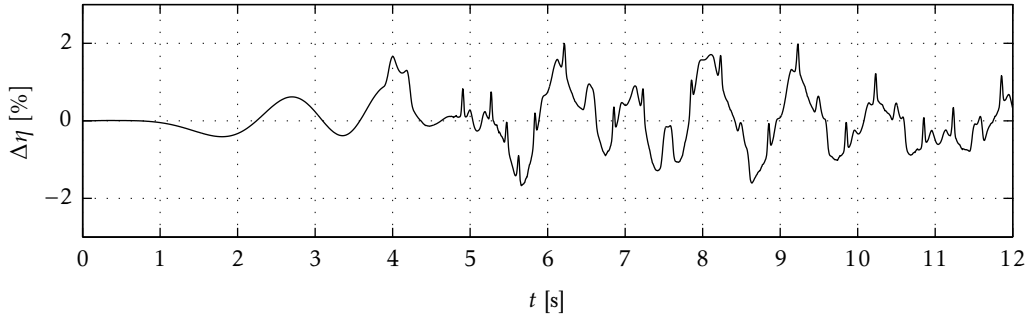


Figure 4.10: Percentage difference in the predicted water surface elevation, $\Delta\eta(t)$ between test cases 6 and 6c at $x = 3\lambda$.

zone, the solution from the inside of the domain is blended with the solution at the boundary. Since the latter is defined as zero, the surface elevation is progressively damped to zero as the waves propagate to the outlet boundary. This ensures that there is no wave energy to be reflected back into the computational domain thereby contaminating the solution.

To determine the reflection generated from the outlet boundary, a second simulation, test case 6c (where c stands for control case), similar to test case 6 but with a domain twice as long as the original, one was undertaken. By using a longer domain, the wave reflected from the outlet boundary does not have enough time to reach the measurement location. The reflection coefficient was then estimated by calculating the differences in the surface elevation between test cases 6 and 6c at $x = 3\lambda$. This was defined by

$$\Delta\eta[\%] = \frac{\eta_6(t) - \eta_{6c}(t)}{a} \times 100, \quad (4.6)$$

where $\Delta\eta$ is the non-dimensional difference in the surface elevation (expressed as %), $a = H/2$ is the linear amplitude, and η_6 and η_{6c} are surface elevation for test cases 6 and 6c respectively. This reflection coefficient (based on the surface elevation) is presented in Figure 4.10. This confirms that the reflection is quite small with a maximum coefficient of 2%. This is substantially better than that achieved in most laboratory facilities and confirms that the use of a relaxation zone at the outlet boundary can reduce wave reflections to an acceptable level.

Although a regular wave train can be generated from quiescent conditions, the propagation of a wave train into still water introduces a steep transient wave front that propagates and develops into the domain. Unfortunately, this requires a long

time to reach a steady state. Furthermore, with steeper incident waves, the transient wave front itself will become steeper and eventually break, introducing a disturbance in the wave field. The effect of this transient wave front can be reduced by using either a larger ramping time or a longer domain, or both. However, these methods will both increase the computational effort and for a 3D case, this could be prohibitive.

In order to address this problem, a second method of wave generation was explored. In this case, the same domain configuration and mesh as test 6 was adopted, but with inlet and outlet relaxation zone of lengths of $L_i = L_o = \lambda$. However, instead of starting the simulation from quiescent conditions, the surface elevation and velocity were specified across the domain using an analytical solution. At the outlet, instead of prescribing a zero velocity value, the known analytical solution was prescribed. During the simulation, the computed solution within the domain is blended with the analytical solution at the outlet boundary. As the difference between the computed solution and the analytical solution will be small, there will be a much smoother transition of the solution within the relaxation zone. This smoother transition was expected to produce a much smaller reflection coefficient when compared to the full absorption condition.

To validate the wave generation method, three regular wave cases were considered. The wave conditions are given in Table 4.3, noted as test cases 7, 8, and 10. These wave cases will subsequently be used in the wave-structure interaction studies presented in the next chapter. As a result, a careful validation is necessary at this stage. Figure 4.11 concerns the time history of surface elevation, $\eta(t)$, recorded at $x = 3\lambda$ and provides comparisons between the numerical calculations and the analytical solution for test cases 7, 8, and 10. Excellent agreement is shown in all test cases. The crest and trough height are accurately predicted, and there is no error in the phasing of the wave component. The root mean square (RMS) error for the predicted surface elevations can be calculated for each wave case using the following equation

$$\epsilon_{\eta}[\%] = \frac{1}{\eta_{\max}} \sqrt{\frac{\sum_{t=1}^n (\eta_s(t) - \eta_a(t))^2}{n}} \times 100, \quad (4.7)$$

where $\eta_s(t)$ and $\eta_a(t)$ are the surface elevations from the numerically simulated and the analytical solution respectively. The RMS error, given in percentage, is normalized by the analytical maximum surface elevation, $\eta_{\max}$. The RMS error for test case 7

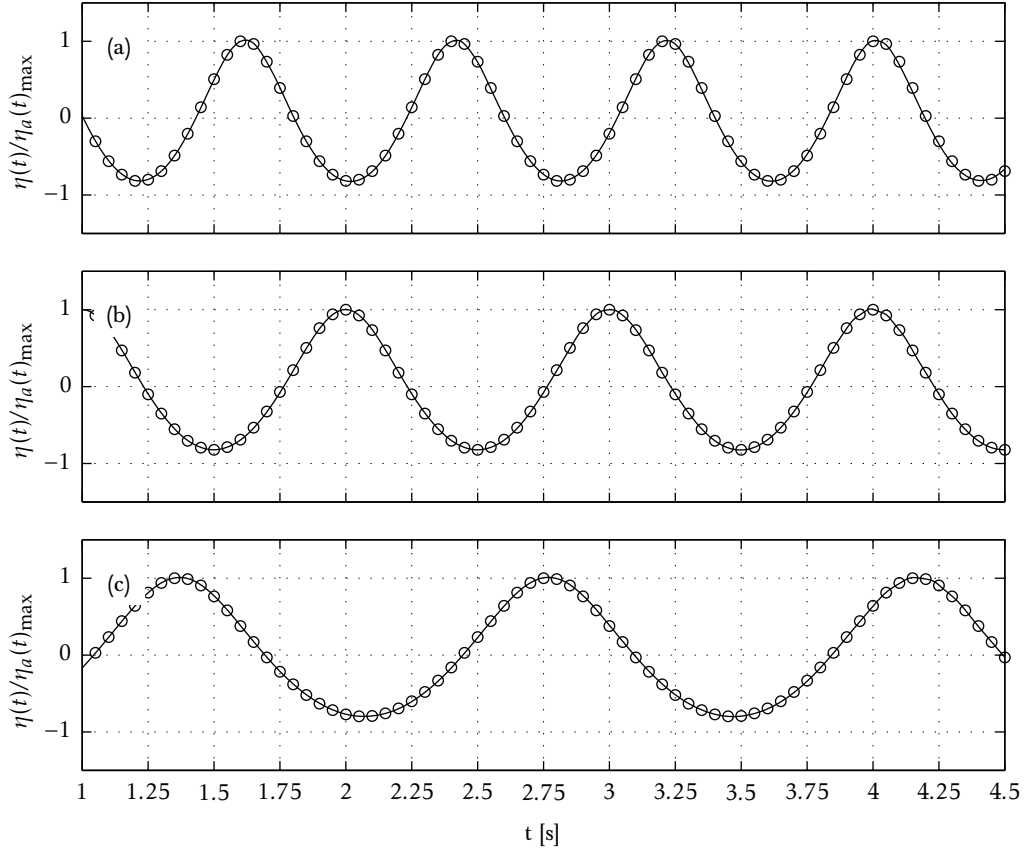


Figure 4.11: Time-history of the water surface elevation, $\eta(t)$, recorded at $x = 3\lambda$ with comparisons between numerical simulation (—) and a Stokes 5th-order solution ($\circ$) for (a) test case 7, (b) test case 8, and (c) test case 10.

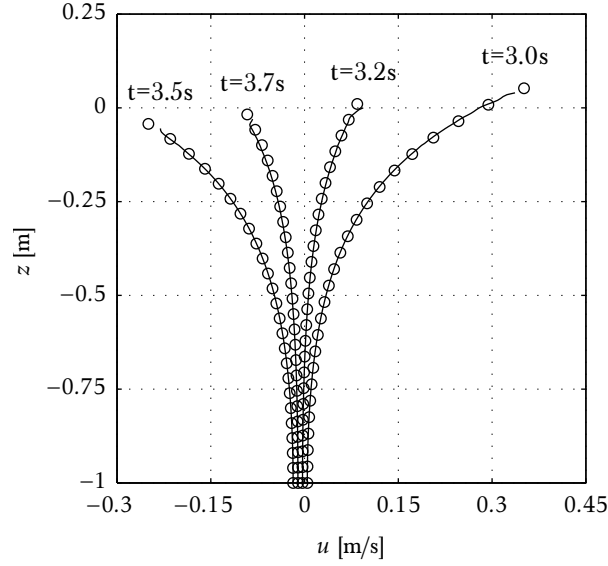


Figure 4.12: Horizontal velocity profile under a regular wave train with comparisons between numerical prediction (—) and analytical solution ($\circ$) for test case 8.

is 1.1 % while test cases 8 and 10 have RMS errors of 0.6 % and 0.5 % respectively. These results confirm that the simulations are both achievable and accurate, across all wave cases.

4.3.3 Kinematics Predictions Beneath a Regular Wave Train

The results presented thus far concern the predicted water surface elevations. To be of use in applied calculations, the numerical simulations must also reproduce the wave-induced water particle kinematics under a progressive wave train. Figure 4.12 concerns the horizontal water particle velocity in test case 8; with comparisons between the numerical simulation and an analytical solution based upon a Stokes 5th-order wave theory. The velocity profiles were predicted at a distance of 3λ from the inlet boundary, for four different time steps. At time $t = 3.0$ s a wave crest was located at $x = 3\lambda$, whilst $t = 3.5$ s is $T/2$ later and therefore corresponds to a wave trough. It is clear from Figure 4.12 that very good agreement is achieved at all times.

In comparing the kinematics predictions, it should be noted that the numerical results are plotted based on a volume fraction of $\alpha = 1$, corresponding to a water phase only. As a result, the predictions were truncated just below the free surface (interface) region. The reason for this is because at the instantaneous free surface, the blending between the velocities of the water and air phases produce an erroneous

result due to the contribution from high air velocity. This is an inherent problem in any VOF model in which the uncertainty in the predicted velocity is largest at the water surface. This is unfortunate since the interesting physics occurs close to the instantaneous free surface. To circumvent this problem, mesh refinement can be applied to reduce the size of the interface region, thereby reducing influence from the air phase. However, this has the disadvantage of increasing the computational cost. Alternatively, the velocity from the interior of the domain could be extrapolated up to the free surface. Since the numerically predicted free surface is accurate, this could prove to be an efficient and accurate calculation of velocity at the instantaneous free surface.

Overall, the present results show that the wave generation and absorption toolbox, *wave2Foam*, can be used to generate high quality regular wave and provide efficient wave absorption. Moreover, with the waves profile defined everywhere in the space at some initial time, the difficulties associated with a starting transient can be eliminated.

4.4 Focused Wave Simulation

Having established a capability to model regular waves, the simulation will be extended to two-dimensional irregular waves in this section. The purpose of this work is to consider the ability of the numerical scheme to accurately model the nonlinear wave-wave interactions. These interactions are critical, especially in the context of the wave-structure interaction problem. In this case, the nonlinear wave-wave interactions occur both as a component part of the incident waves and between the incident wave and the scattered wave field. Therefore, an accurate representation of these interactions is crucial for the success of the present study.

4.4.1 Underlying Wave Spectrum

Unlike the single freely propagating frequency component present in a regular wave train, irregular waves are characterised by multiple freely propagating wave components having different wave frequencies and amplitudes. In a numerical simulation in which computational resources are limited, the calculation of a long duration random wave record is usually prohibitive. As an alternative, a focused wave simulation is

readily achievable. This provides a nonlinear representation of the most probable shape of a large linear wave; the latter produced by focusing individual wave components at a single point in space and time. The focusing of these wave components will produce a highly nonlinear isolated wave event. It is widely believed that this focusing mechanism is responsible for the occurrence of *freak* or *rogue* waves in deep water; the nonlinear wave-wave interactions producing a maximum crest elevation that is larger than the linear sum of the wave components. As such an extreme wave event is characterised by a maximum crest elevation that is significantly higher than the underlying sea state. In this study, two-dimensional focused waves were simulated in a JONSWAP spectrum; the latter defined by

$$G_{\eta\eta}(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left(-\beta \left(\frac{\omega_p}{\omega}\right)^4\right) \gamma^{\exp\left(-\frac{1}{2} \left(\frac{\omega - \omega_p}{\sigma \omega_p}\right)^2\right)}, \quad (4.8)$$

where $G_{\eta\eta}(\omega)$ is the spectral density function, γ is the peak enhancement factor, ω and ω_p are the wave frequency and peak wave frequency respectively. The dimensionless constants are given by $\alpha = 0.0081$, $\beta = 1.25$ with $\sigma = 0.07$ for $\omega \leq \omega_p$ and $\sigma = 0.09$ for $\omega > \omega_p$. Following the work by Lindgren (1970) and Boccotti (1983) the most probable shape of a large linear wave is given by a focused wave event in which the amplitudes of individual wave components are proportional to the energy spectrum. Adopting this approach, the initial amplitude spectrum, $a(\omega)$, was calculated as

$$a(\omega) = A \frac{G_{\eta\eta}(\omega)}{\sum_{i=1}^{N_c} G_{\eta\eta}(\omega_i)}, \quad (4.9)$$

where A is the linear amplitude sum and N_c is the number of wave components used in the discretization of the spectrum.

4.4.2 Computational Domain and Boundary Conditions

The computational domain for the focused wave simulations is illustrated in Figure 4.13. The inlet boundary is located on the left side of the domain, with the right side being the outlet boundary. The height of the domain is given by $L_z = 1.2d$. In all cases, the water depth was set to $d = 1.0\text{m}$. The linear focused time and

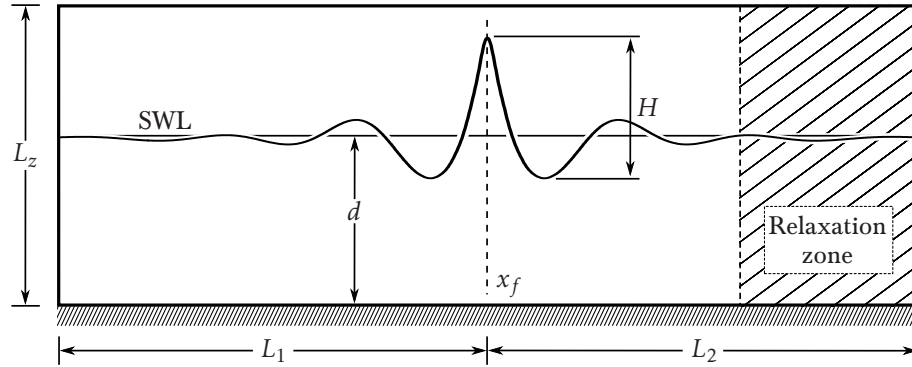


Figure 4.13: A sketch showing the focused wave domain.

position were set to $t_f = 0$ s and $x_f = 0$ m respectively. The distance between the inlet boundary and x_f is given by $L_1 = 5.0$ m; while the distance between x_f and the outlet boundary is given by $L_2 = 7.0$ m. The domain was initially discretized using a cell size of 0.02 m, both in the horizontal and vertical directions. The region near the SWL was then refined for each wave case; the final mesh having approximately 16 cells over the wave-height, $H = 2A$.

The input for the focused wave cases was prescribed using LRWT. Although this is a linear solution, it is preferable to a second-order solution due to the fact that it is computationally cheap to evaluate. Moreover, the linear solution will evolve fully nonlinearly as it propagates within the computational domain. To confirm that this is indeed the case, and that the final surface elevation predictions are independent of the input conditions, comparison of the results generated using linear and second-order inputs will be presented later in this section. At the outlet boundary, an outlet relaxation zone with a length of one wavelength was prescribed to absorb the outgoing waves. Unlike the regular wave simulations, no relaxation zone was used at the inlet boundary for the focused wave cases. Indeed, preliminary tests to assess the effectiveness of a relaxation zone at the inlet boundary in the case of a focused wave identified changes in the wave phasing as the waves propagate into the domain. As a consequence, the individual wave components were out of phase at the prescribed focused position; the relative phase changes becoming larger as the nonlinearity increases. As a result, no relaxation zone was applied at the inlet boundary for the rest of the focused wave simulation. However, given the efficiency of the relaxation zone employed at the outlet boundary, there was effectively no disturbance caused by unwanted reflected waves at the inlet boundary. Evidence to support this is given

Table 4.4: Input spectrum parameter

Parameter	Value
Water depth, d	1.0 m
Wave spectrum	JONSWAP (<i>NewWave</i>)
Number of wave components, N_c	103
Peak enhancement factor, γ	2.5
Spectral peak period, T_p	1.2 s
Peak wave number, k_p	2.815 rad m^{-1}

Table 4.5: Focused wave test cases

Test case	A [mm]	Ak_p	Comment
A05	5	0.02	Linear
A30	30	0.08	Weakly Nonlinear
A60	60	0.17	Nonlinear

in the reflection analysis presented in the section that follows.

4.4.3 Numerical Validation of Focused Wave Cases

To evaluate the accuracy of numerically simulated focused wave events, a JONSWAP spectrum with peak enhancement factor of $\gamma = 2.5$ and spectral peak period, $T_p = 1.2 \text{ s}$ was used. The spectrum was discretized into 103 frequency components generated within the range of $0.411 \text{ Hz} \leq f \leq 2.5 \text{ Hz}$. With $f_p = 1/T_p = 0.83 \text{ Hz}$, the high-frequency cut-off was applied at $3f_p$. This is consistent with best practise in terms of wave modelling. Furthermore, in the tail of the spectrum, the last 5 values were tapered to zero to reduce spectral leakage. A summary of the input spectrum used in the present cases is given in Table 4.4. Three focused wave cases with varying nonlinearity were considered; the latter achieved by changing the linear amplitude sum of the underlying wave components. These test cases vary from linear (test case A05), to weakly nonlinear (test case A30) and highly nonlinear (test case A60). The wave steepness for each of these cases is calculated by multiplying the linear amplitude sum, A , with the wavenumber of the spectral peak, k_p ; the relevant parameters for the focused wave cases being given in Table 4.5. The results from these simulations were compared with analytical solutions based upon linear and second-order random wave theory (Sharma & Dean, 1981), and the fully nonlinear multi-flux BEM

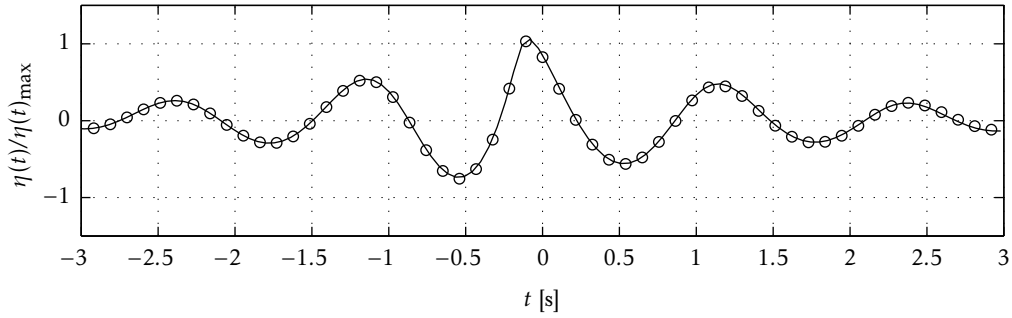


Figure 4.14: Time-history of normalized water surface elevation for a focused wave generated at $x = 0$ m between test cases A60 (—) and A60c ($\circ$).

of Hague (2006). In undertaking the BEM calculations, input conditions based on linear and second-order theory were employed. These are subsequently denoted by BEM_{LRWT} and $\text{BEM}_{\text{SORWT}}$ respectively.

In all cases, the simulations were commenced at $t_0 = -20$ s, for 40 s, or approximately 33 wave periods. A ramping time of $t_{\text{ramp}} = 2$ s was used at the start of the simulation to minimize the initial starting transient. The starting time was selected such that all the individual wave component has enough time to reach the linear focused position during focused event. Before validation work was carried out, preliminary calculations were undertaken to ensure that the results were not affected by reflection from the outlet boundary. To achieve this, a control case was established (referred to as test case A60c) with conditions similar to test case A60, but with double the distance from the linear focused position to the outlet boundary. The time-history of the water surface elevations, $\eta(t)$ at $x = 0$ m for both test cases are shown in Figure 4.14. It can be seen from these results that the water surface elevations are in almost perfect agreement. The RMS error of (of difference) being less than 0.5%. This result is extremely good, confirming that the relaxation zone at the outlet boundary effectively eliminated unwanted wave reflections.

With reflections eliminated as a potential issue, a full validation of the simulations were carried out. Figure 4.15 (a), (b) and (c) show the time-histories of the water surface elevation, $\eta(t)$, at the focused position for test cases A05, A30 and A60 respectively. In each figure, the results have been normalized with respect to linear amplitude sum, A , and shifted to the linear focused time, $t_f = 0$ s. A close up of the largest crest elevation is given by sub-plots (i) to (iii) at the bottom of Figure 4.15 (c). In Figure 4.15(a) and sub-plot (i), excellent agreement is observed between the

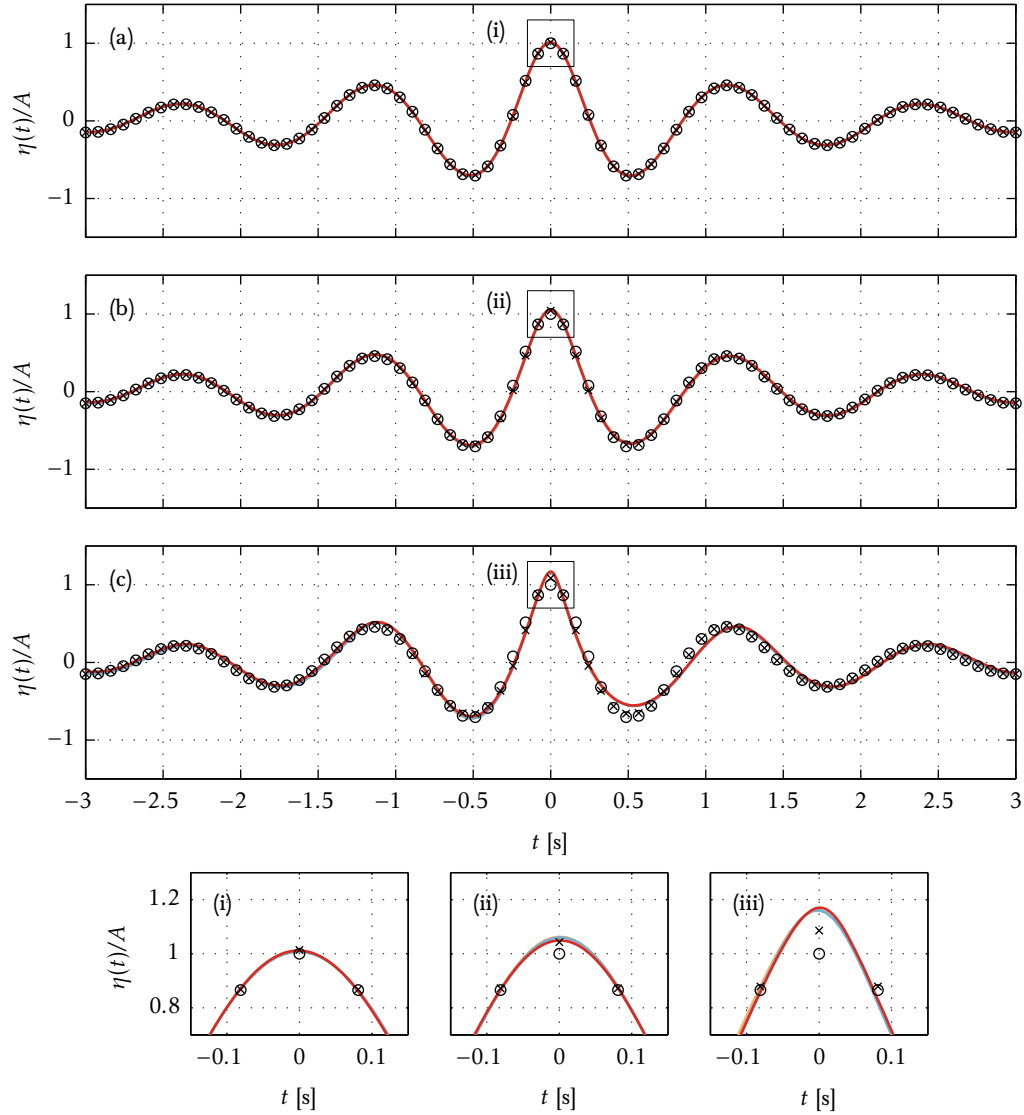


Figure 4.15: Time-history of normalized water surface elevation, $\eta(t)/A$, at the focused position for (a) test case A05 (b) test case A30 (c) test case A60, with comparisons between the available analytical solutions, BEM and present numerical simulation. In all cases, the results have been shifted to linear focused time, $t = 0$ s. Legends: — Present numerical, — BEM_{LRWT}, — BEM_{SORWT}, ○ LRWT, × SORWT.

analytical, BEM and numerical solutions. This is to be expected as case A05 is fundamentally linear, although even in this case, there is a small (second-order) increase in the maximum crest elevation. As the nonlinearity increases, the results shown in Figure 4.15(b) and sub-plot (ii) for case A30 show departures from the linear solution, particularly in term of the maximum crest elevation. However, solutions from both the BEM and the numerical simulation are in good agreement with the second-order solution. In this case, the difference in the maximum surface elevation, $\eta(t)_{\max}$, between the numerical predictions and the linear and second-order solution is 8 % and 1 % respectively. In contrast, the difference between the numerical and BEM solutions is less than 0.5 %. For the nonlinear case (case A60), the numerical solutions show a more pronounced departure from the analytical solutions. In this case, the difference between the numerical and analytical solutions grow to 17 % for the linear calculation and 7 % for second-order. However, the agreement between the numerical calculations and the BEM solutions remains very good, with differences of less than 1 %.

To further highlight accuracy of the present numerical simulations of focused wave events, the spatial profiles of surface elevations for cases A05, A30 and A60 are presented in Figure 4.16. In case A05, excellent agreement is observed between the numerical simulation and the analytical solutions. As the nonlinearity increases, agreement with analytical solutions reduces. However, in all cases, there is excellent agreement between the present numerical simulations and both BEM solutions. Indeed, it is clear from these results that the modelling of the largest wave cases requires a fully nonlinear model, incorporating effects beyond second-order. Furthermore, it is confirmed that the present numerical model can indeed provide a good prediction of a focused wave event and that the use of a linear solution at the inlet boundary is appropriate, even for the description of a nonlinear focused wave event.

4.4.4 Key Features of a Focused Wave Event

In addition to the nonlinear increase in the maximum crest elevation as the wave steepness increases, a key feature of the wave form observed on Figure 4.15 is the horizontal symmetry of the wave shape, $\eta(t)$. For the linear case, the surface elevation is symmetric about the linear focused time, $t_f = 0$ s. As the nonlinearity increases,

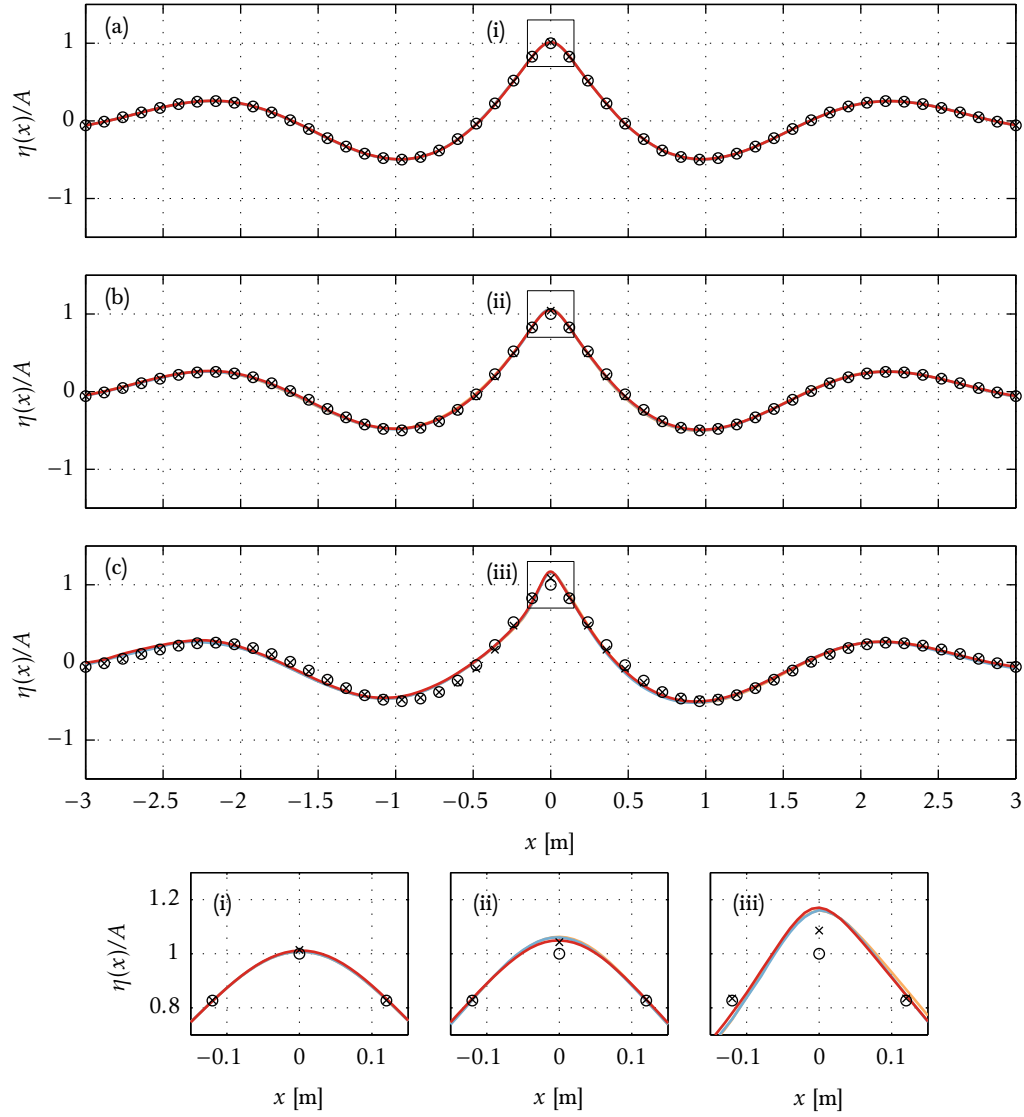


Figure 4.16: Spatial profile of normalized maximum water surface elevation, $\eta(x)/A$, at the focused time for (a) test case A05 (b) test case A30 (c) test case A60, with comparisons between the available analytical solutions, BEM and present numerical simulation. In all cases, the results have been shifted to linear focused position, $x = 0$ m. Legends: — Present numerical, — BEM_{LRWT}, — BEM_{SORWT}, ○ LRWT, × SORWT.

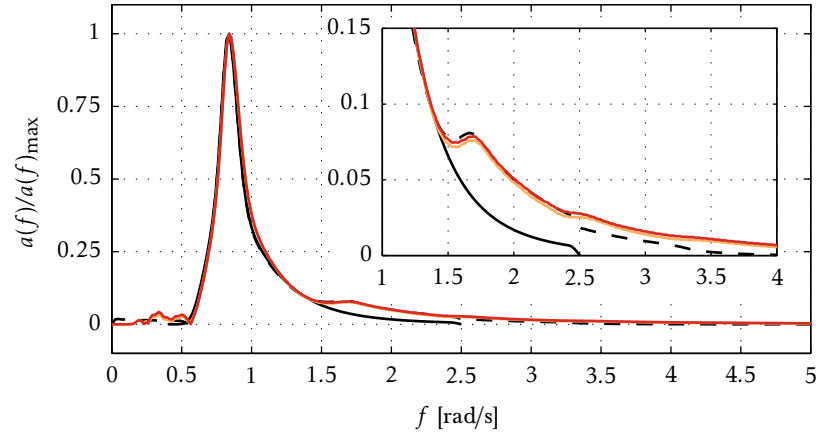


Figure 4.17: Normalized amplitude spectrum for the focused wave events shown on Figures 4.15 and 4.16. Legends: — Present numerical, — BEM, — LRWT, --- SORWT.

the symmetry of the wave shape breaks down; the depth of wave trough trailing and preceding the maximum crest elevation, $\eta_{\max}$ become unequal. This breakdown in the horizontal asymmetry cannot be predicted by either a linear and a second-order solution. This is because, at second-order, the wave interactions are bound to the freely propagating wave modes and are symmetric about the linearly predicted crest elevation. As such, they can contribute to the increase in the maximum crest elevation and the broadening of the wave trough (Sharma & Dean, 1981), but not the developing asymmetry. Above second-order, Gibson & Swan (2007) showed that the third-order resonant interactions lead to the generation of new freely propagating wave modes and a redistribution of wave energy to higher frequencies. Figure 4.17 shows the normalized amplitude spectrum for the present numerical simulations of the focused wave event described in Figures 4.15 and 4.16. The amplitude spectrum for the linear, second-order and a fully nonlinear BEM solution are also plotted for comparison. In all cases the amplitude spectra were normalised with respect to the amplitude of the input peak frequency, f_p . Comparisons between the present numerical results and the BEM calculations show excellent agreement. This confirms that the data describing the high-frequency energy is not due to some numerical error. In contrast, the lower frequency energy cannot be resolved due to the limited length of the predicted time-history. As the steepness of the wave form increases, the contribution from the second-order bound waves becomes increasingly important. For $f \leq 2.5$ Hz, the spectral form shows a notable contribution from the bound terms, with agreement between the second-order and numerically predicted results. However, the departure

from the second-order solution for $f > 2.5\text{ Hz}$ indicates that there is indeed an increase in the wave energy at higher frequencies. This movement of energy to the higher frequencies causes the changes of the underlying wave spectrum. As these new freely propagating wave modes travel at their own phase speed, the constructive and destructive interference of the wave components provides an explanation for the developing asymmetry with increasing nonlinearity.

A second feature of an extreme wave event is the drifting of the maximum crest location from the linearly predicted position. Figure 4.18 concerns the evolution of the surface profile, $\eta(x)$, for wave case A60, with comparisons between LRWT, a BEM solution and the present numerical solution; the data presented for $t = -1.0\text{ s}$ to $t = 1.0\text{ s}$, with a time-step of $\Delta t = 0.25\text{ s}$. The vertical grid lines represent the x coordinates, while the horizontal grid lines represents the SWL or $z = 0\text{ m}$. In these cases, the water surface elevation, $\eta(x)$, was plotted without any shifting of the data. These comparisons show very good agreement between the present numerical calculations and the BEM solution, indicating that the wave phases were correctly resolved by the two numerical methods. However, marked discrepancy was observed when compared to the linear solution. During the evolution of the nonlinear focused event, there is a clear downstream shifting of the wave event. This is in marked contrast to the equivalent calculations made for case A05 where the maximum crest elevation of the focused event occurs at exactly $x = 0\text{ m}$. Indeed, a careful analysis of the focal location reveals that the maximum crest elevation for case A30 and A60 (Table 4.5) occur at $x = 0.08\text{ m}$ and $x = 0.18\text{ m}$ respectively. This movement of the focused location can be attributed to the nonlinear changes in the underlying free wave spectrum arising due to the third-order resonance interactions.

The results presented in this section highlight the importance of the nonlinear wave-wave interactions in the description of a large wave event, and the ability to predict them using the present numerical solution. Indeed, the nonlinear interaction between the individual wave components produce a significant nonlinear increase in the crest elevation. Furthermore, as the wave steepness increases, the symmetric wave pattern no longer prevails, and the focus position moves ahead of the linearly predicted position. These changes are attributed to higher-order effects, including both nonlinear bound and nonlinear resonant interactions. The former are, in some part, responsible for the vertical asymmetry of the wave profile, making the crest

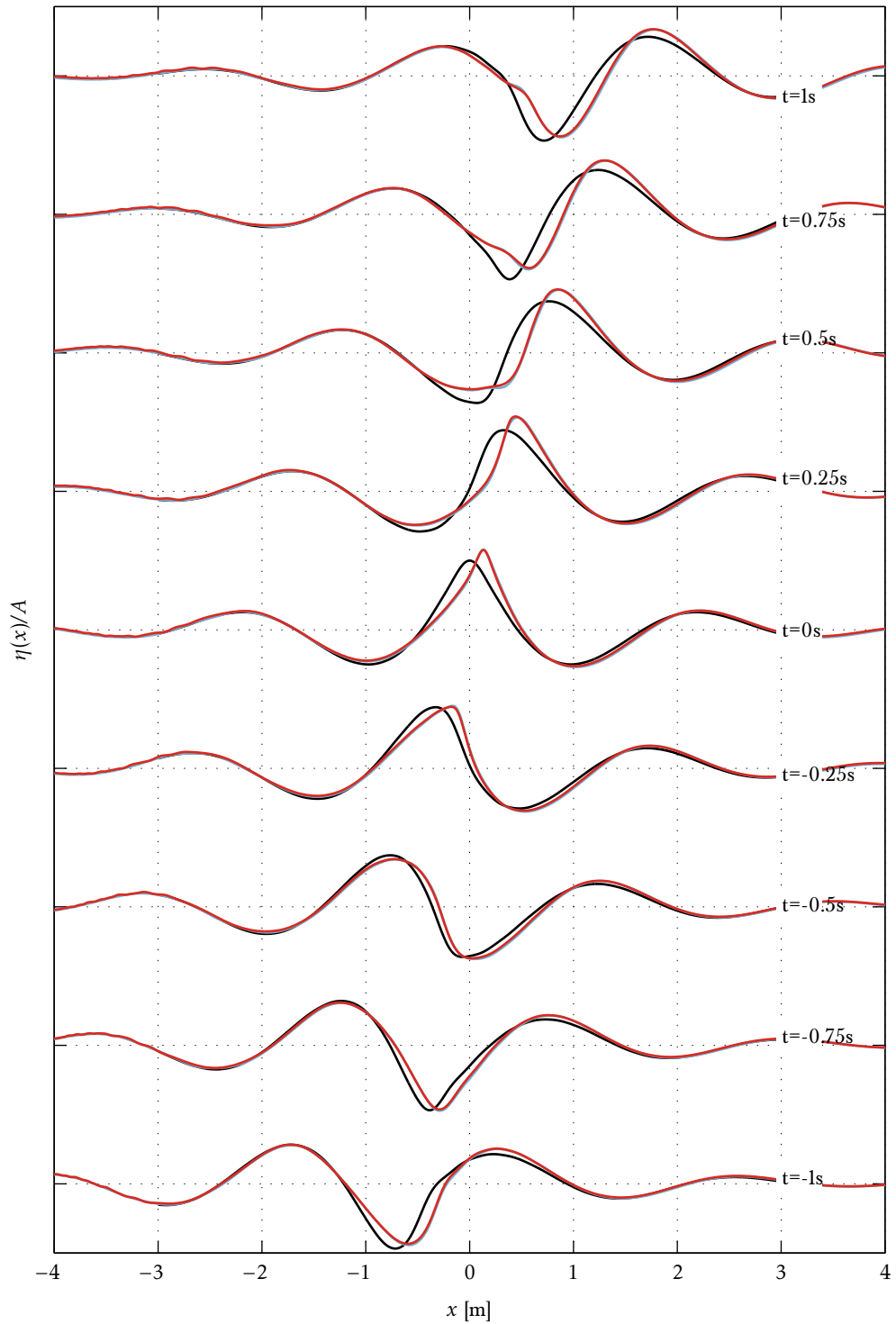


Figure 4.18: Evolution of the normalized water surface elevations, $\eta(x)/A$, with comparison between LRWT (—), BEM (—) and the present numerical simulations (—) for nonlinear focused wave case (test case A60). Note: in presenting these results no shifting has been applied to the numerical results.

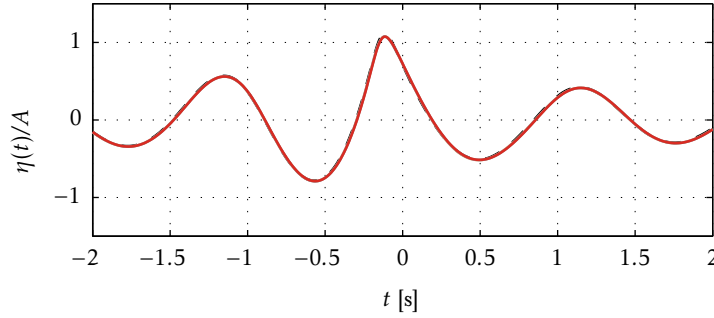


Figure 4.19: The time-history of the water surface elevation, $\eta(t)$, at $x = 0$ m for several different coupling of the present numerical scheme and the more efficient BEM solution. Legends: - - - BEM, — $t_i = 2.0$ s, — $t_i = 3.5$ s, and — $t_i = 5.0$ s.

steeper and the troughs broader. The latter, on the other hand, are responsible for the horizontal asymmetry of the wave profile and the movement of the maximum crest location due to changes in the underlying spectrum of the freely propagating components.

4.5 One Way Numerical Coupling

In calculating the earlier focused wave simulations, the only apparent drawback of the present numerical scheme concerns the required computational effort. Based on the earlier convergence study and repeated runs of specific test cases, achieving the same accuracy as the BEM solution requires the use of a smaller cell size and a smaller Courant number. The former is required to accurately resolve the free surface, while the latter is necessary to allow the accurate temporal evolution of the volume fraction field. Furthermore, the simulation of a focused wave requires an extended simulation period, to allow the evolution of the focused event, and a longer domain to ensure that reflections from the outlet boundary are minimised. This involves a significantly increased computational cost. Although direct comparison cannot be made between the present calculations and the BEM solution due to different discretization of the computational domain, the present calculations are at least seven times more expensive to run when compared to BEM. In an attempt to reduce this computational cost, a one-way coupling between the present numerical calculations and a BEM solution was initiated. This was achieved by starting the focused wave simulation using the quicker BEM solution up until several wave periods before the focused

event. At this point, both the velocity field and the free surface elevation from the BEM solution were evaluated and interpolated onto the required computational grid and used as the initial conditions for the present numerical calculations. In addition, the volume fraction field was calculated based on the cell submergence discussed in the previous section. With the present calculations using the pressure-velocity coupling algorithm, the initial pressure value was not needed, but calculated as part of the solution process. At the inlet boundary, the linear solution was matched to the BEM predictions. This avoided any discontinuities in the input conditions. Having prescribed the initial conditions, the present calculations were continued for several wave periods up to and beyond the focused event.

In order to test the proposed methodology, the numerical simulation was kick-started using initial conditions from different times, thereby evaluating the sensitivity of the solution to the initial conditions employed. Using wave case A60 as an appropriate test, Figure 4.19 contrasts the time-history of surface elevation, $\eta(t)$ at the linear focused position for start times of $t = -2.0$ s, -3.5 s and -5.0 s, where $t = 0$ s corresponds to the linearly predicted focus time. Excellent agreement is shown for all cases, including with the direct BEM calculations. This confirms that the adopted methodology is indeed independent of initial start time. Adopting this approach, the time required to run the present calculations was significantly reduced, and confirming the success of the coupling methodology. Given this success, the proposed methodology was used to kick-start the present numerical calculations, especially for several of the wave-structure interaction problems discussed in the subsequent chapters.

4.6 Simulation of a Limiting Wave Group

In the previous section, the capability of the present scheme to simulate a focused wave event was successfully demonstrated. In this section, the simulation of a limiting wave case is addressed. These cases are generated by increasing the linear amplitude sum, A , of the underlying spectrum until the maximum crest elevation reaches a threshold (or a limiting value) at which a further increase in A causes the wave to break. In a focused wave case, this limiting value is substantially lower than that predicted by a theoretical Stokes' breaking limit. For example, Christou (2009) inves-

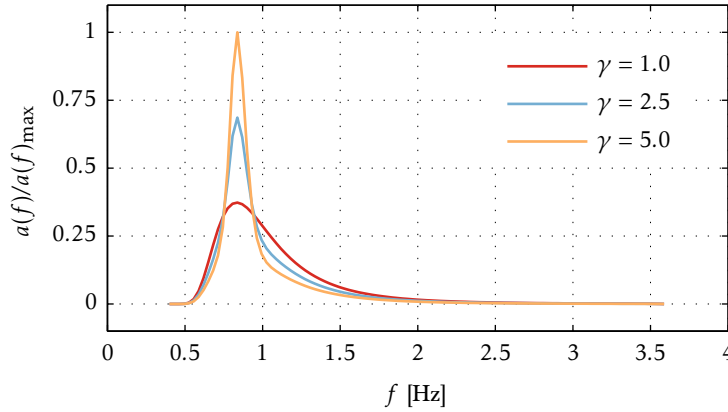


Figure 4.20: Normalized linear (input) amplitude spectrum, highlighting the variation with γ .

tigated the characteristics of a limiting wave using a BEM solution. In simulating the focused wave cases, a comparison between two frames of references, semi-Lagrangian (SL-BEM) and fully-Lagrangian (FL-BEM), were made. The former technique allows the movement of the computational nodes in the vertical direction only and, in doing so, limits the model to a single valued free surface. In contrast, the latter permits the free movement of the computational nodes, allowing over-turning waves to be modelled. The results of this study showed that a SL-BEM solution produces unrealistic results at the limiting condition. In contrast, more realistic results were obtained using a FL-BEM scheme; the limiting wave cases clearly observed to be related to the ratio of the maximum fluid velocity to the phase velocity. Overall, Christou (2009) clearly highlighted a problem with numerical methods based on a single-valued free surface when seeking to simulate a limiting wave case. Having shown good agreement between the present calculations and those obtained by a BEM solution, it is of interest to consider the success of the present approach when modelling a limiting focused wave case. Furthermore, unlike the BEM which is based on potential flow assumptions, the present VOF solution is free from either inviscid or irrotationality constraints. This is important since the limiting wave case is associated with the onset of wave breaking in which a small perturbation of the solution might induce breaking in the form of wave spilling. If this occurs, the potential flow assumption ceases to be valid and a computation based upon a BEM will break-down on the verge of doing so. In contrast, calculations based on a VOF method have the ability to model the break-up of the free surface without approaching any of the modelling constraints.

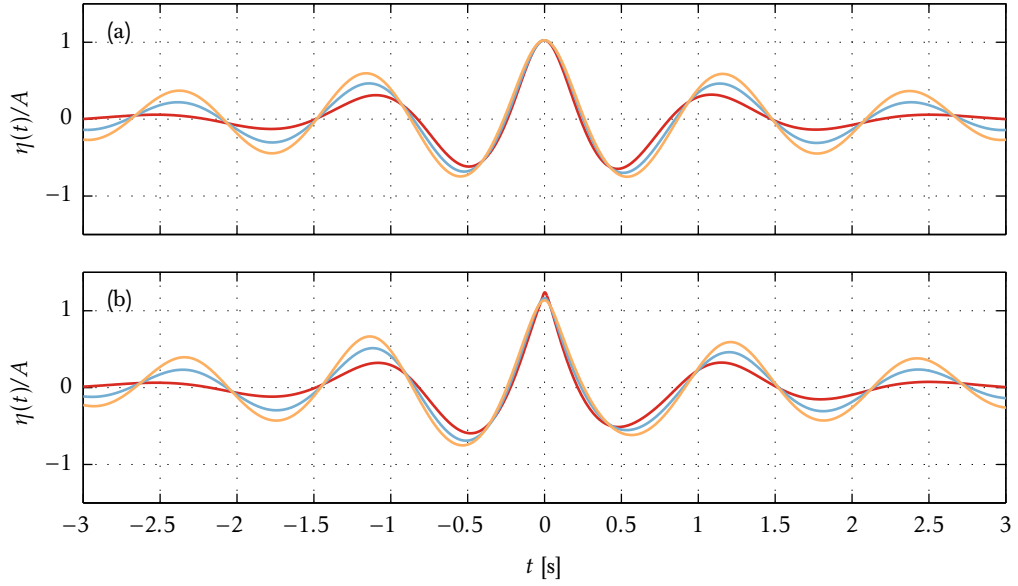


Figure 4.21: Time-histories of the normalized maximum surface elevation, $\eta(t)/A$, for varying γ and different linear amplitude sums, A ; (a) $A = 0.02$ m (b) $A = 0.06$ m. Note: the horizontal profiles have been shifted so that maximum crest elevation, $\eta(t)_{\max}$, occur at $x = 0$ m. Legends: — $\gamma = 1.0$, — $\gamma = 2.5$, and — $\gamma = 5.0$.

A set of focused wave cases was considered for the modelling of the limiting wave events. The underlying wave spectrum was again defined by a focused JONSWAP spectrum with peak enhancement factors given by $\gamma = 1.0$, $\gamma = 2.5$, and $\gamma = 5.0$. The normalized amplitude spectrum, $a(f)/a(f)_{\max}$, for each γ value are shown in Figure 4.20. In these comparisons, the γ value controls the peakiness of the linear (input) wave spectrum; a lower γ representing a broad-banded spectrum and a larger γ a more narrow-banded spectrum. For each γ value, the linear (input) amplitude sum was increased from $A = 0.02$ m to $A = 0.1$ m, with increments of $\Delta A = 0.01$ m used between $0.02 \text{ m} \leq A \leq 0.05 \text{ m}$ and a finer increment of $\Delta A = 0.002 \text{ m}$ between $0.05 \text{ m} \leq A \leq 0.1 \text{ m}$. If the steepness of the focused wave is defined as the product between the linear amplitude sum, A and the peak wave-number, k_p , the steepness for the present study lies within the range $0.08 \leq Ak_p \leq 0.42$. The input spectrum for each case was discretized into 112 wave components lying within the range of $0.4 \leq f \leq 3.6$. In all cases, the computational domain was as outlined in Section 4.4.

Before presenting the results of these limiting wave cases, it is interesting to investigate the changes in the shape of the focused wave for different γ values. Figure 4.21 presents the time-history of surface elevation, $\eta(t)$, for $A = 0.02$ m and $A = 0.06$ m; the results for the different γ values being superimposed. In each plot, the surface

elevation has been normalized by the corresponding linear amplitude sum. From Figure 4.21 (a), a distinct large crest is generated during the focusing process for $\gamma = 1.0$. This is characterized by a large isolated wave crest at the center of the group with preceding and trailing crests having substantially smaller elevations. In contrast, as γ increases, the relative variation in the crest elevation reduces; the wave pattern tending towards a regular wave profile as the spectral band-width reduces. As the nonlinearity of the wave event increases, a horizontal asymmetry develops. This is clearly shown in Figure 4.21(b), corresponding to almost 30% increase in the linear amplitude sum relative to the wave event shown on Figure 4.21(a). In the steeper wave case, the trough preceding the maximum crest is deeper than the trailing one; the developing asymmetry being caused by the higher-order resonant interactions. In terms of the crest elevation, the $\gamma = 1.0$ case has a larger maximum crest when compared to the other γ values. This suggests a higher nonlinearity for wave events with a more broad-banded spectrum. This is consistent with the steepness of the wave event, in both space and time; the increased bandwidth producing a reduction in the local wave length.

To investigate the changes in the underlying spectrum, the normalized amplitude spectrum for the maximum focused event are presented in Figure 4.22. This considers both the different γ values and the increasing linear amplitude sum. The left column of results concerns the $\gamma = 1.0$ cases, while the right column relates to $\gamma = 5.0$. In both cases, successive rows represent results with an increasing linear amplitude sum. In each plot, amplitude spectra from the present numerical simulation are compared with both the linear spectrum and the spectrum obtained from a second-order solution. As expected, for $A = 0.02\text{ m}$, the amplitude spectrum for both γ values is in close agreement with the second-order solution. This confirms that for this level of input amplitude sum, the wave is weakly nonlinear. Indeed, comparisons with the linear spectrum show that there is an increase in energy at the higher-frequencies due to the bound-wave interactions. For the $\gamma = 5.0$ case, a pronounced hump in the spectrum between $1\text{ Hz} \leq f \leq 2\text{ Hz}$ can be observed. This is due to the bound-interactions arising in the vicinity of the spectral peak. Indeed, in the limit of a really narrow banded spectrum, where the spectrum is characterized by a single freely propagating frequency, the hump will represent the second-order bound wave component arising in a regular Stokes waves.

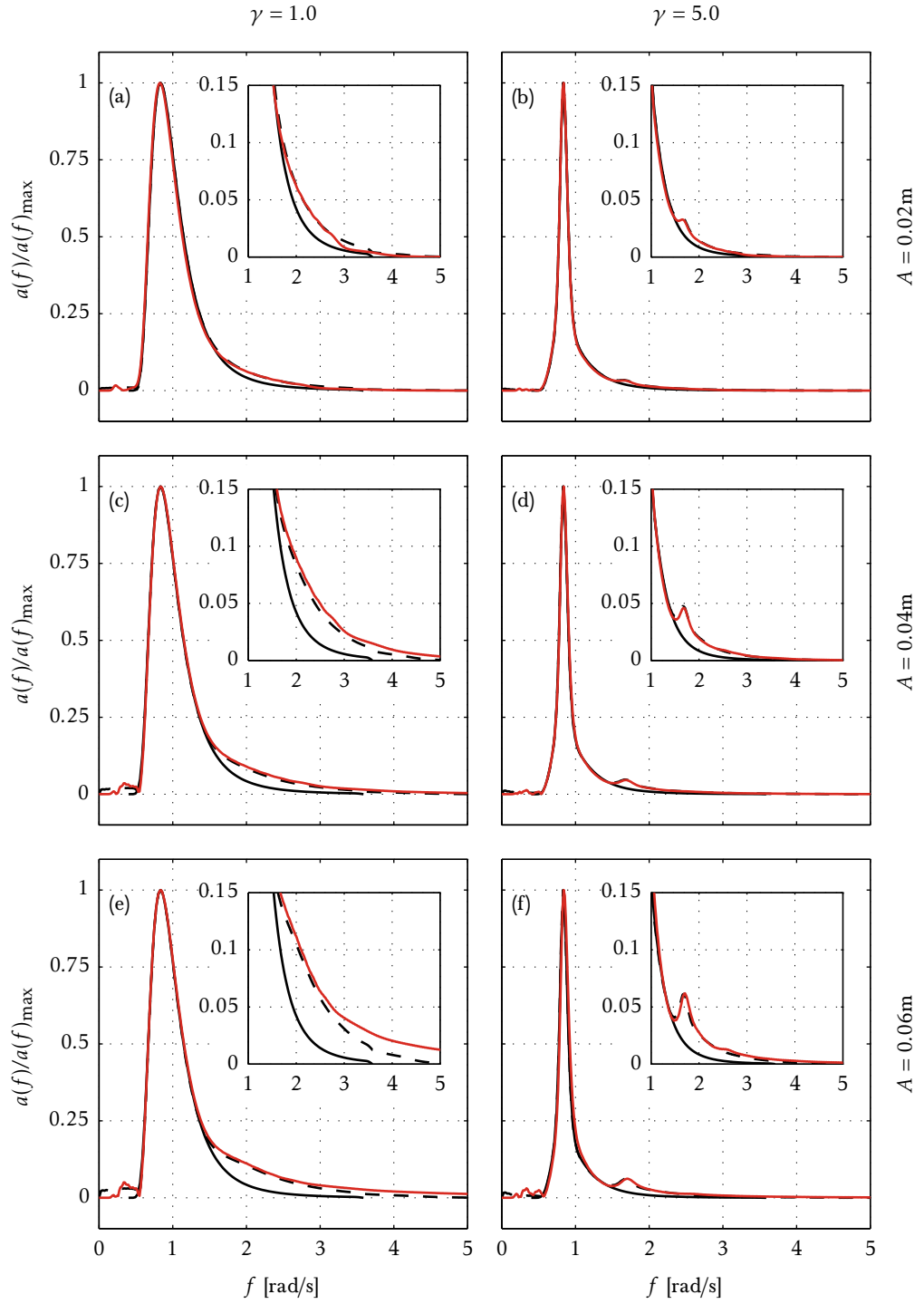


Figure 4.22: Normalized amplitude spectra for focused wave cases with increasing linear amplitude sum A for $\gamma = 1.0$ (left column) and $\gamma = 5.0$ (right column). Note: in each case, the small inset highlights the higher-frequency components. Legends: — Numerical, — LRWT, and - - - SORWT.

With an increase in the linear amplitude sum, the nonlinearity of the focused event is expected to increase. This is evident from the second and third rows of Figure 4.22; the energy in the higher frequency components increasing with the linear amplitude sum. Interestingly, for $\gamma = 1.0$, there is a rapid departure from SORWT for $A = 0.04\text{m}$, which increase further for and $A = 0.06\text{m}$. In contrast, the data for $\gamma = 5.0$ remains quite close to the SORWT spectrum for both cases. This confirms that waves arising in a broad-banded spectrum behave more nonlinearly than their narrow-banded counterpart. This nonlinearity is again attributed to *bound* and *resonant* interactions. However, the relative importance of these interactions is difficult to infer from the present results as there is no way of separating their respective contributions. Furthermore, the comparisons provided herein serve merely to investigate the nonlinear behaviour of waves with different spectral bandwidths.

The identification of the limiting wave cases is presented in Figure 4.23. Subplot (a) presents the normalized nonlinear increase in the crest elevation for different γ values, with comparisons between the present numerical calculations, LRWT, and SORWT. Open symbols describe the numerical results relating to non-breaking wave cases, while fully coloured symbols describe the conditions at the onset of wave breaking, or when the surface has just become multi-valued. The solid black line is based upon LRWT, while the coloured dashed lines are the results from SORWT. Figure 4.23(b) and (c) concerns the spatial and temporal location of the maximum crest elevation; the analytical values defined by $x_f = 0\text{m}$ and $t_f = 0\text{s}$ respectively. In Figure 4.23(a), it is shown that the second-order solution for $\gamma = 1.0$ describes a slightly larger increase in $\eta_{\max}$ as A increases, when compared to the $\gamma = 2.0$ and $\gamma = 5.0$ cases. This is consistent with the results presented earlier; the lower γ value corresponding to the broader-banded spectrum showing a higher nonlinear increase when compared to a narrower-banded spectrum. A similar trend is also exhibited by the numerical calculations. In Figure 4.23(a), for $0.02\text{m} \leq A \leq 0.03\text{m}$, the numerical calculations agree with SORWT for all γ values. However, for $A > 0.03\text{m}$, the present calculations start to diverge from SORWT. Furthermore, there is also a shift in both the spatial and temporal location of the maximum crest elevation, as shown in Figure 4.23(b) and (c). Interestingly, the rate at which $\eta_{\max}$ increases significantly higher for $\gamma = 1.0$ compared to the other γ values. This again highlights the link between spectral bandwidth and nonlinearity.

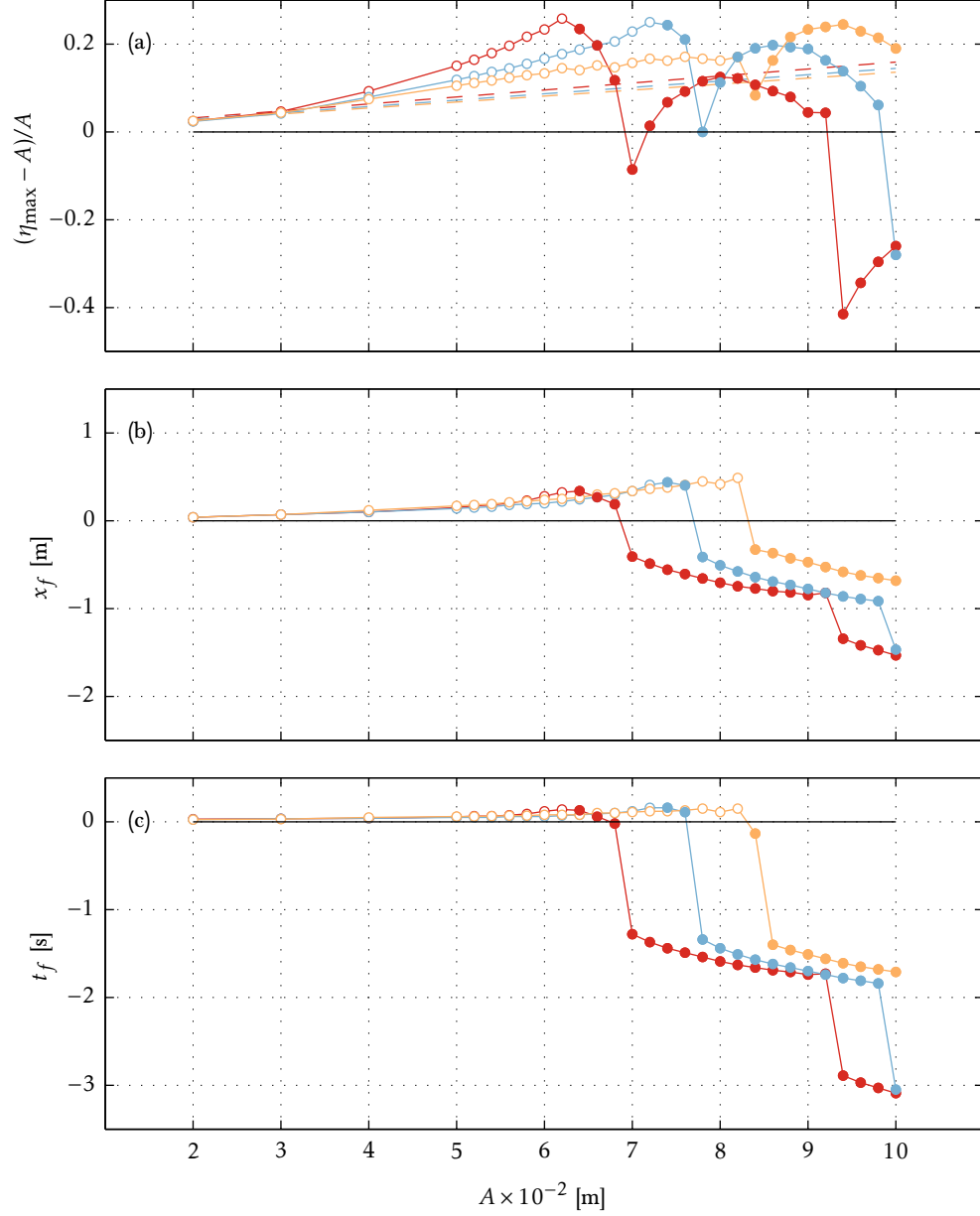


Figure 4.23: Identification of the limiting wave properties. (a) Normalized maximum crest elevations, (b) spatial location of maximum crest elevation, (c) temporal location of maximum crest elevation. Open symbols describe non-breaking wave cases, while fully coloured symbols relate to data taken just before the wave front becomes multi-valued. Legends: $\gamma = 1.0$ (present), $\gamma = 2.5$ (present), $\gamma = 5.0$ (present), — LRWT, and $\gamma = 1.0$ (SORWT), $\gamma = 2.5$ (SORWT), $\gamma = 5.0$ (SORWT).

With increased nonlinearity, the solution for $\gamma = 1.0$ reaches its limiting condition at a much lower amplitude sum, corresponding to $A = 0.062$ m; with the maximum crest elevation being significantly higher than those associated with the larger γ values. In a design context, if the limiting value is obtained from an analytical solution, the maximum crest elevation will be under-estimated by more than 20% if the spectrum is broad-banded. In contrast, solutions for the $\gamma = 2.5$ and $\gamma = 5.0$ wave cases reach limiting values at around $A = 0.072$ m and $A = 0.082$ m respectively. Beyond the limiting values, an increase in the linear amplitude sum leads to a reduced $\eta_{\max}$ prior to the occurrence of wave breaking. In considering these results, it is important to mention that the notation $\eta_{\max}$ beyond the limiting value, does not necessarily mean the maximum crest elevation across all time steps, but is a measure of the maximum crest just before the free surface becomes multi-valued; the latter signifying the onset of wave breaking. As A increases beyond the limiting value, there is a rapid shift in the time, t_f , and spatial location, x_f , in which $\eta_{\max}$ occurs. In all cases, this time, t_f , occur prior to the main focusing event. This change occurs much earlier for a broad-banded spectrum when compared to a narrow-banded spectrum. Further increase in the energy of the system, due to the increase in A , forces the wave to break prior to the focal event.

4.7 Breaking of Unidirectional Focused Waves

In order to further access the capability of the present numerical calculations to model deep water wave breaking, particularly involving unidirectional focused waves, this section presents a comparison between the numerical prediction and experimental observations of a large over-turning wave. The experimental data was obtained in work undertaken by Ma (2017) as part of his work to determine the wave-in-deck loading experienced by offshore structures under various incident wave conditions. In carrying out his experimental study, one of the wave cases involved a large over-turning wave in deep water. This extreme wave event was generated using a JONSWAP wave spectrum having a peak period of $T_p = 1.4$ s, a peak enhancement factor of $\gamma = 2.5$ and a linear amplitude sum of $A = 0.125$ m. The wave was propagating in a constant water depth of $d = 1.5$ m. Based on the peak period, the corresponding peak wave number was defined by $k_p = 2.06 \text{ rad m}^{-1}$. This gives an effective water

depth of $k_p d = 3.1$, confirming that the wave is propagating in deep water. Based upon these values the wave steepness is $Ak_p = 0.26$, far exceeding the limiting wave cases observed in the previous section. This steepness guarantees the occurrence of a large over-turning wave event.

This specific wave was generated in the Imperial College wave basin located in the basement of Civil and Environmental Engineering Department. This facility has an overall width and length of 20 m and 13 m respectively. To record the characteristics of this wave, seven surface-piecing wave gauges were placed 0.1 m apart in the central area of the wave basin. As the individual wave gauges record a time-history of the water surface elevation at one spatial location, they cannot be used to determine or differentiate the multi-valued free surface associated with an over-turning wave. To overcome this difficulty, the spatial evolution of the water surface elevation, $\eta(x)$, during wave over-turning, was recorded using a high speed video camera. During this recording, a transparent perspex sheet printed with a vertical and horizontal grid was placed on the glass side wall of the wave basin. This allowed the direct measurement of the surface elevation to be made without the need of complex image post-processing .

The numerical simulation of this breaking wave case was carried out using both the present numerical scheme and BEM; the latter providing a double check at least up to the point when the wave start to overturn. To provide a direct comparison with the experimental observations, the length of the computational domain was set to 13 m, where the last 3 m of the domain length was assigned as an outlet relaxation zone to reduce the occurrence of wave reflections. In undertaking the calculations, the spectral shape was identical to that employed in the laboratory tests and the overall spectrum discretized into 112 wave components. Furthermore, a set of numerical wave gauges, separated by $\Delta x = 0.01$ m apart were placed along the computational domain.

The first set of results describing this wave event are shown in Figure 4.24. This depicts the time-history of the water surface elevations recorded at the different wave gauge locations, with comparisons between the experimental observations and the numerical predictions. Wave gauges 1 is located nearest to the wave paddles or input boundary, while wave gauges 7 is located furthest downstream. It is clear from this figure, that the comparison between the experimental results and the numerical

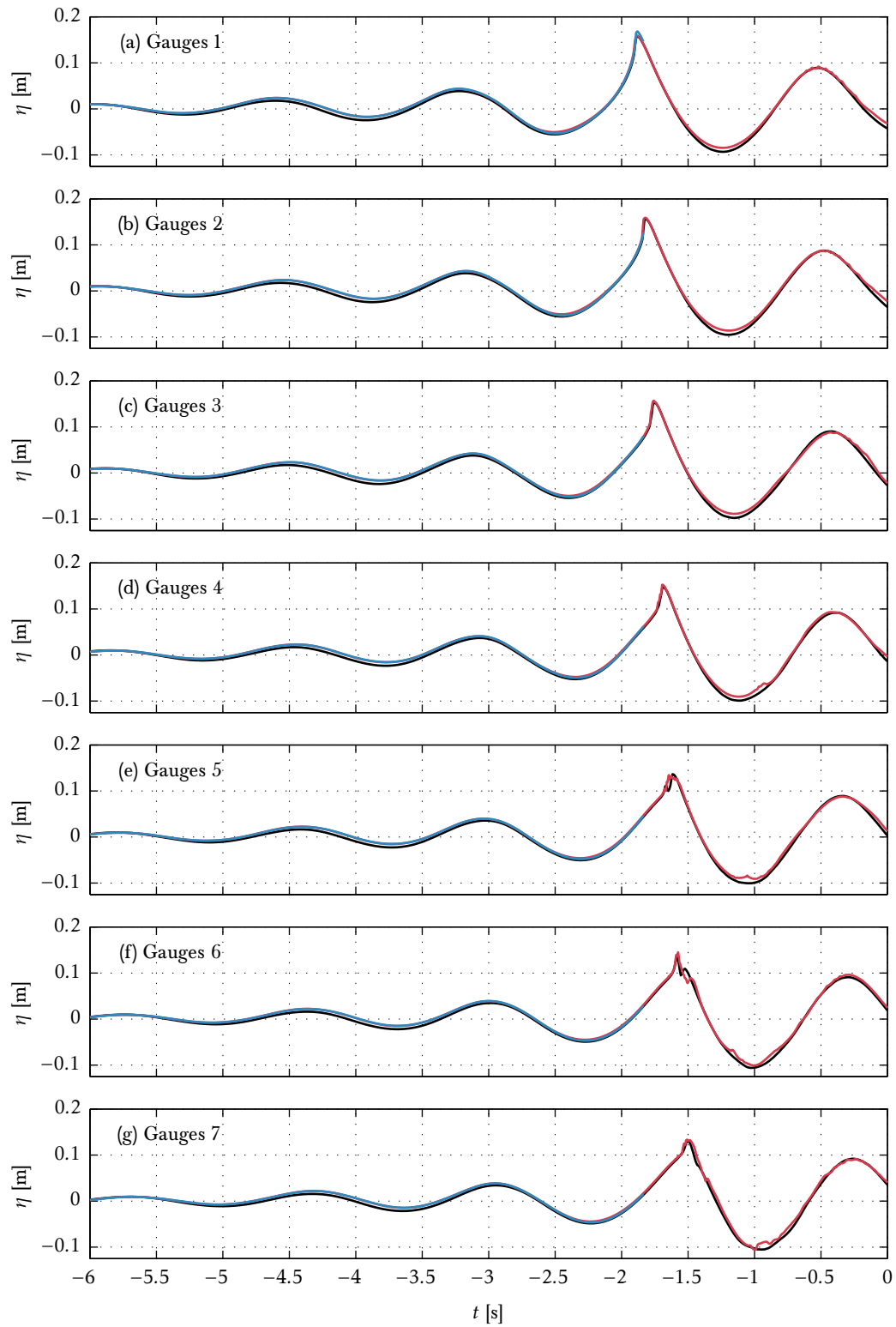


Figure 4.24: Time-history of the water surface elevation, $\eta(t)$, at various wave gauge locations for an over-turning wave case ($Tp = 1.4$ s, $\gamma = 2.5$ and $A = 0.125$ m). Comparison between experimental observation and numerical predictions. Legends: — Present numerical, — BEM, and — experiment.

predictions is extremely good. Although there is a slight mis-match at the trough of the wave record, the fact that the crest is predicted with a high degree of accuracy provides strong evidence of the ability of the present numerical calculations to model this extreme wave case. Interestingly, the maximum crest elevation at all wave gauges occur between $-2\text{ s} \leq t \leq -1\text{ s}$, where $t = 0\text{ s}$ is the time of the linear focused event. This is consistent with the results presented in previous section where the largest wave event consistently occurs prior to the prescribed linear focused event.

Figure 4.24 also shows the results of the BEM simulation, given by solid blue line. This is only partially successful due to the break down of the code during the over-turning of the wave event. Although the BEM code used herein is fully Lagrangian, it does not have a node redistribution capability. Therefore, during the process of wave over-turning, the nodes cluster together, resulting in the break down of the code. Even if a redistribution function were implemented, the code would still break down as the overturning wave front impinges on the free surface immediately in front. Indeed, this is a major drawback of a single phase numerical solver where the code breaks as soon as the free surface becomes heavily distorted or breaks apart. On the other hand, the robustness of two phase flow solver adopted in the present calculations is clearly demonstrated; there is no break down of the computation even in the event of significant free surface breaking.

Further comparisons of the water surface elevation are provided in Figure 4.25. This provides the images from the high speed video recording taken during the experiment with comparisons to the present numerical prediction (given by the black solid line). In addition, the surface elevations from both the experimental (red dot) and the numerical (black cross) wave gauges are also plotted on the same figure. In comparing these results, it should be noted that the time at which the experimental image was taken and the numerical result was predicted is slightly different. This is because the image was recorded using a high speed video recorder providing 240 fps, resulting in a time increment of $\Delta t = 0.0042\text{ s}$. In contrast, the numerical simulation was dumped at $\Delta t = 0.01\text{ s}$. In providing the comparisons on Figure 4.25, the smallest difference between the two time steps was used and plotted. Overall, the agreement between the observed and predicted water surface elevations is extremely good. However there seems to be a slight underestimation of the maximum crest elevation (Figure 4.25(a)) and a small delay in the formation of the over-turning wave

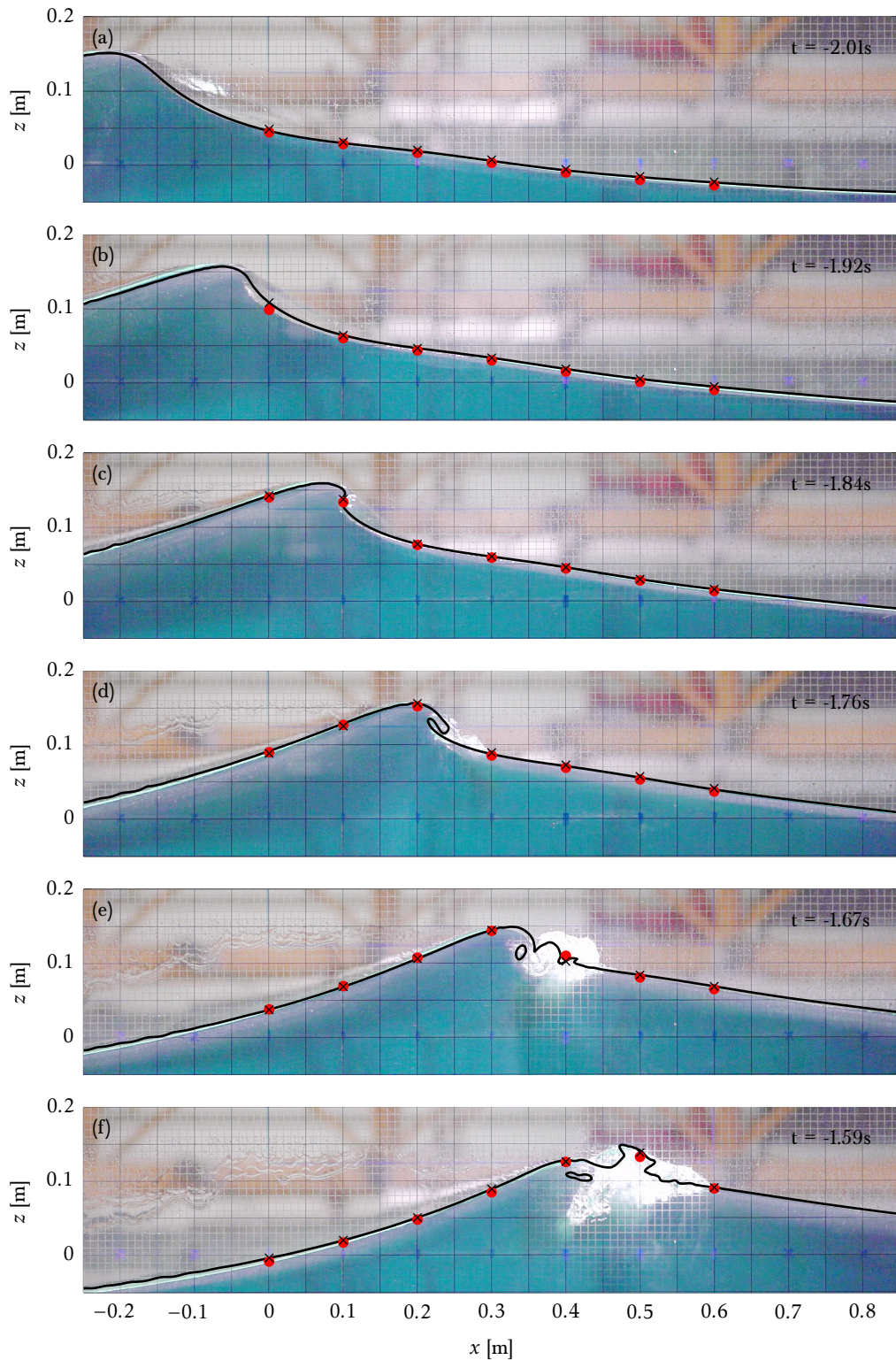


Figure 4.25: The spatial evolution, $\eta(x)$, of a plunging breaker arising in a unidirectional focused wave ($Tp = 1.4s$, $\gamma = 2.5$ and $A = 0.125m$) with comparisons between experimental observations and numerical predictions (solid red line). Note: the superimposed small grid size is 1 cm by 1 cm. Legends: — Surface elevation (numerical), • experimental gauges, and × numerical gauges.

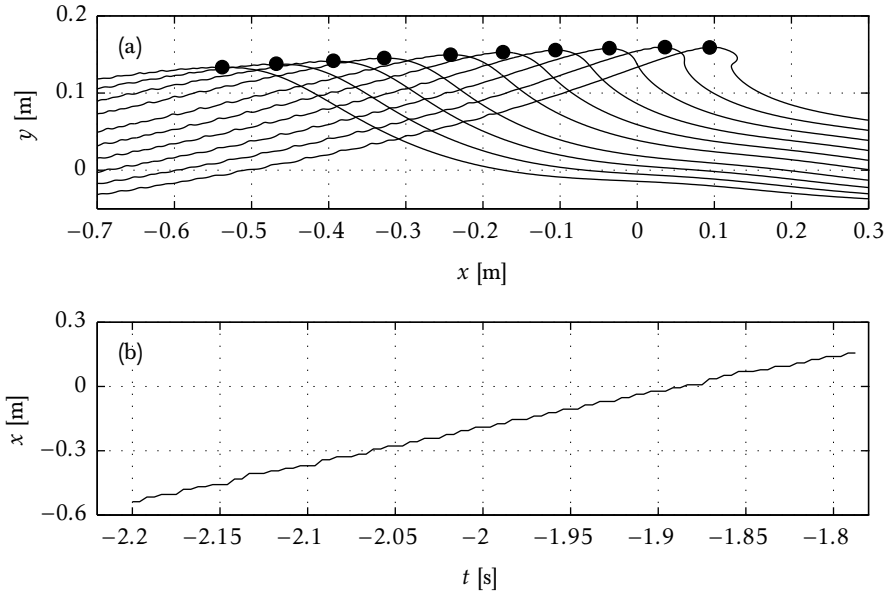


Figure 4.26: The evolution of an over-turning wave event: (a) The surface profile, $\eta(x)$, and maximum crest elevation, $\eta(x)_{\max}$ at various time steps. (b) A time-history defining the position of the maximum crest elevation. Legend: $\bullet$ $\eta(x)_{\max}$.

front ((Figure 4.25(b)-(d))). These small differences can be explained by the fact that the wave is generated, or introduced, in two totally different ways; the experiment using a moving wave paddle, and the numerical simulation a prescribed velocity profile (from LRWT) on a fixed inlet boundary.

With the impinging of the over-turning wave jet onto the free surface immediately in front, air entrainment in a form of white capping can be observed in Figure 4.25(e) and (f). This is accompanied by splash-up of the impinging wave front. The numerical method shows the formation of small air entrainment and subsequently smaller splash up. Whilst the subsequent splash-up is undoubtedly under-estimated by the present numerical method, the overall breaking process is captured surprisingly well. With the purpose of the present study being the provision of new physical insights into the nonlinear interaction of waves with a vertical surface-piercing column, the ability to model the generality of wave breaking is sufficient. The fact that the proposed model cannot model the details of the post-breaking two-phase flow is not of primary concern.

The evolution of the transient wave group is further considered in Figure 4.26. Figure 4.26(a) shows the surface profile for $-2.2 \text{ s} \leq t \leq -1.83 \text{ s}$, with $\Delta t \approx 0.04 \text{ s}$. At each time step, the maximum crest elevation is also plotted; the data represented by

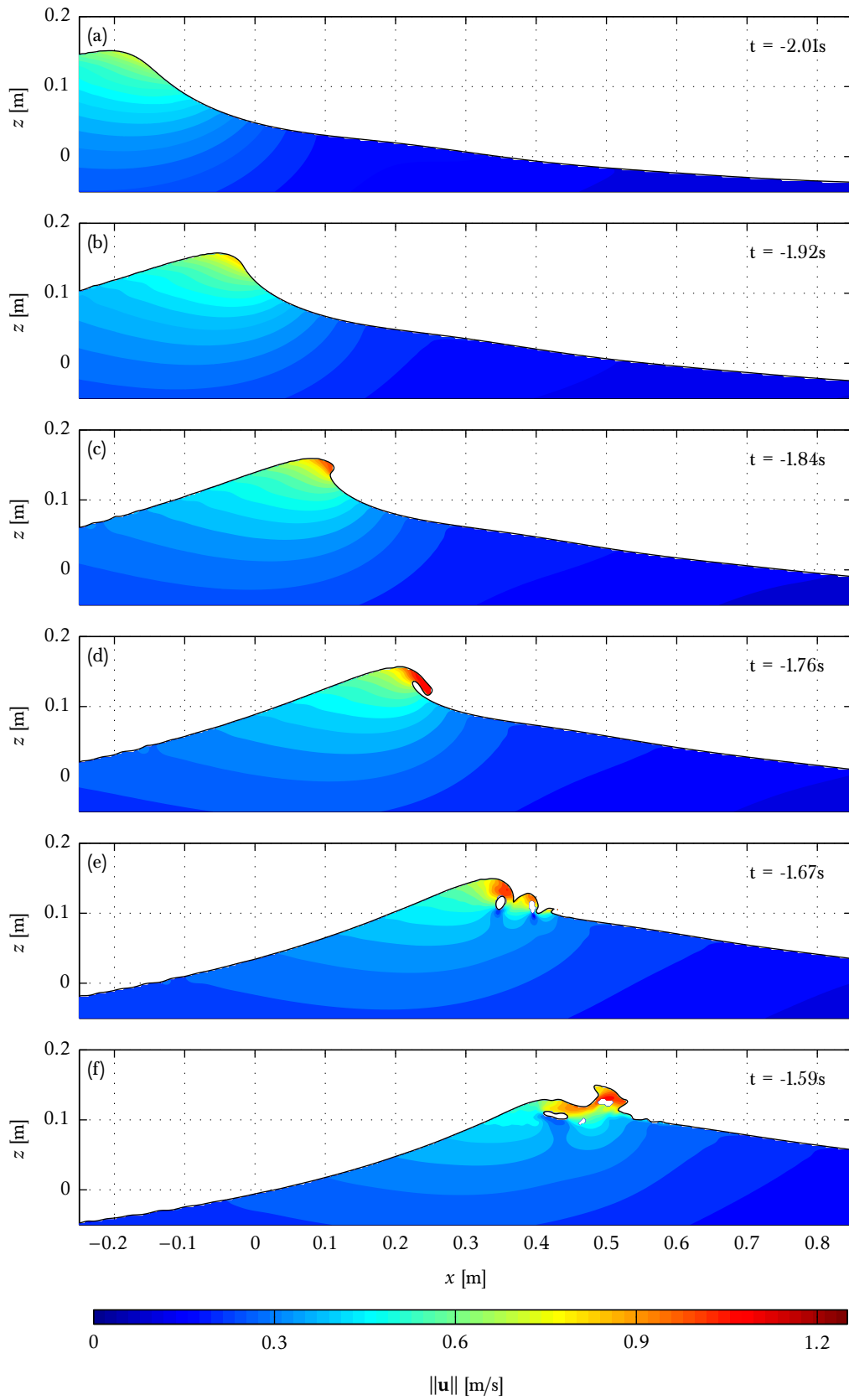


Figure 4.27: The contour plots associated with a large over-turning wave event produced by a focused wave ($T_p = 1.4s$, $\gamma = 2.5$ and $A = 0.125m$); numerical prediction confirming $u/c > 1$ at breaking.

the black dot. As the wave evolves, the overall shape of the wave starts to become horizontally asymmetric; the leading face becoming progressively steeper when compared to the back face. As this process continues, the wave front becomes so steep that it eventually over-turns to produce a plunging breaker. During the over-turning event, the crest reaches its maximum value. As soon as it breaks, much of the wave energy is dissipated and the subsequent crest (associated with the linearly predicted focused event) produces significantly reduced maximum elevations.

A plot showing the position of the maximum water surface elevation position at each time step, for $-2.2\text{ s} \leq t \leq -1.83\text{ s}$, is shown in Figure 4.26(b). Using this data, the average phase velocity of the wave up to over-turning can be determined by calculating the gradient of the line defining the arrival time. In this case, the calculated phase velocity is approximately 1.7 m s^{-1} .

It is well known that based on a kinematics argument, a wave will over-turn or break when the fluid velocity under the wave crest exceeds the phase velocity of the crest. To illustrate this, the velocity contour under the wave train is plotted in Figure 4.27. This sequence of plot is similar to the one presented in Figure 4.25, but includes colour coded velocity contour describing the magnitude of the fluid velocity, $\|\mathbf{u}\| = \sqrt{u^2 + w^2}$. It should also be noted that the value is based on the water phase only ($\alpha = 1$) to avoid erroneous prediction associated with velocity of the air. In some cases, this resulted in slight under-prediction of the maximum water velocity, since this will occur at the free surface. It is clear from the figure that as the wave evolves, the velocity at the crest becomes increasingly large. At $t = 1.84\text{ s}$ (Figure 4.25(c)), the velocity at the tip of the crest has exceeded the phase velocity, forcing the wave to overturn. This is an important physical result which the numerical model has successfully reproduced.

The results presented in this section confirm the capability of the adopted numerical method to accurately simulate the propagation of both non-breaking and breaking surface gravity waves. This validation is essential for the present study not least because an accurate description of the incident wave, including the occurrence of wave breaking is critically important to the simulation of subsequent wave-structure interactions.

5

Wave Scattering in Regular Waves

5.1 Introduction

This chapter presents the results from numerical simulations of regular waves interacting with a vertical surface-piercing column. The focus of the study lies in explaining the occurrence of high-frequency wave scattering. Relevant background information relating to the present study is given in Section 5.2. Section 5.3 outlines the numerical procedures. Validation of the numerical results using available laboratory observations is given in Section 5.4. Parametric studies involving a wide range of flow conditions, including various column diameters and varying wave steepnesses, are presented in Section 5.5 and Section 5.6 respectively.

5.2 Background

Many wave-structure interactions studies in the context of offshore engineering were carried out to explain the damage sustained by operational platforms (Kaplan et al., 1992; Swan et al., 1997; Bea et al., 1999; Stansberg et al., 2001). Some of this damage is associated with wave slamming on vertical members and wave impact on the underside of the platform deck. For the former, the damage is usually caused by steep incident waves undergoing over-turning, while the latter is associated with the loss of

an effective air-gap. An air-gap is defined as the vertical distance between the highest incident wave crest and the underside of the platform deck. A negative air-gap is undesirable, resulting in high-impact pressures and uplift forces, potentially causing damage and disruption to operations. An example of this is given by wave damage sustained at the Brent Bravo Gravity Based Structure (GBS) during a relatively mild storm in January 1995. During this event, a significant volume of water was projected upwards, hitting the underside of the platform deck causing local damage. With the underside of the deck structure located at a relatively high elevation of 25.4 m above still water level and the significant wave height and maximum crest elevation of the storm peaking at $H_s = 13$ m and $\eta_{\max} = 17$ m respectively, significant wave impact damage was unexpected. Indeed, this view is supported by the experience of the adjacent steel jacket platform Brent Alpha. This has a much lower deck elevation, but recorded no wave impacts.

A physical model study was carried out by Swan et al. (1997) to investigate and explain the damage sustained at the Brent Bravo GBS under various incident wave conditions. The structural configuration of Brent Bravo involves three closely spaced columns. Whilst the individual column diameter is relatively large, it is not sufficiently large that it lies within the linear diffraction regime ($D/\lambda > 0.2$). The experimental investigation undertaken by Swan et al. (1997) revealed that the interaction between the incident waves and the structure produces high-frequency wave scattering. Further observations showed that the subsequent interactions between these scattered waves and the next incident wave produces substantially increased water surface elevations. In steep incident wave cases, crest amplifications as much as three times the incident wave amplitude were observed. Indeed, they argued that the mechanism responsible for these large crest amplifications is in large part due to the classical short-wave long-wave interaction (Longuet-Higgins & Stewart, 1960); the high-frequency scattered waves riding on the much longer incident waves, and the consequent (fully nonlinear) interaction producing much larger crest elevations.

In another related study, Ohl et al. (2001b,a) investigated the interactions of regular and irregular waves with an array of bottom-mounted circular columns using theoretical, computational and experimental methods. Measurements made in the vicinity of the columns under various wave conditions reveal the existence of high-frequency wave scattering, confirming the earlier observation by Swan et al. (1997). For low

steepness incident waves, they showed good agreement between the linear diffraction theory and experimental observations. However, as the incident wave steepness was progressively increased, there was a large deviation between the second-order diffraction solution and the measured data; the semi-analytical solution under-predicting the maximum surface elevations. This again hints at the importance of wave steepness in the occurrence of the largest crest elevations.

Both Swan et al. (1997) and Ohl et al. (2001b,a) have highlighted the importance of high-frequency wave scattering when steep incident waves interact with multiple columns. Whilst this scattering is due to the interaction between the incident waves and the structure, concerns were also raised over the radiation of high-frequency wave modes trapped within this multiple-column arrangement. However, (Ohl et al., 2001a) showed that this only occurs at discrete incident wave frequencies. Furthermore, the effect of trapped wave modes can be eliminated by considering a single column arrangement. Whilst this has been the subject of numerous investigations, it is clear that the results are critically dependent upon both the diameter to incident wavelength ratio, D/λ , and the steepness of the incident waves. For example, Stansberg et al. (2001) and Stansberg & Kristiansen (2005) provide experimental data describing the interaction between a steep incident wave and a single vertical column. In both cases, high-frequency wave scattering was observed. Furthermore, the experimental results were compared with both the linear and second-order diffraction theory. For a moderately steep incident wave, linear diffraction under-predicts the crest amplification, while second-order diffraction provides much better agreement with experimental observations. However, in steep waves, the experimental data shows marked departures from both solutions.

Observation of new wave scattering mechanisms

In an attempt to provide a better understanding of the mechanisms responsible for the generation of this high-frequency wave scattering, Sheikh & Swan (2005) carried out an experimental study of a single column subjected to various regular and focused wave conditions. The diameter of the column was chosen to be equivalent to a single leg of the Brent Bravo GBS. Two new undocumented mechanisms responsible for the generation of high-frequency wave scattering were identified. Taken together, these mechanisms produce both concentric and non-concentric wave scatterings. The

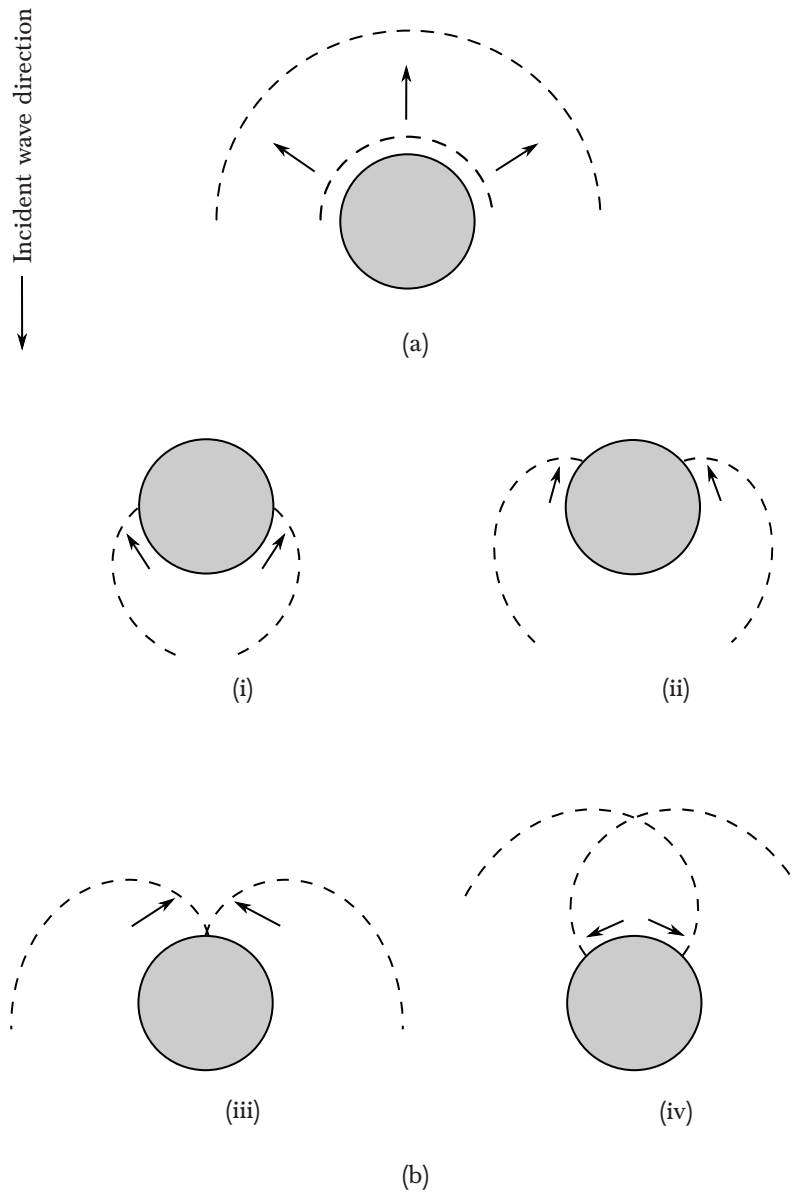


Figure 5.1: An idealized representation of upstream wave scattering for the (a) *Type-1* and (b) *Type-2* waves (Sheikh & Swan, 2005).

concentric waves were referred to as *Type-1* wave scattering and generated by run-up and wash down on the face of the column as the wave propagates past the column. In contrast, the non-concentric wave scattering was referred to as *Type-2* and associated with the movement of fluid around the column. These two modes of wave scattering are shown in Figure 5.1. A detailed extension of this work was given in Swan & Sheikh (2015) who further argued that the propagation of *Type-2* waves around the face of a column introduces an additional time-scale to the problem. This time-scale relates to the column diameter rather than the incident wave. As such, it is absent from a traditional harmonic-type analysis, explaining why diffraction solutions cannot describe these scattered wave components. Swan & Sheikh (2015) also showed that the nonlinear interactions between the scattered and the incident waves leads to a significant amplification of wave elevation, consistent with observations made by Swan et al. (1997). In the steepest wave cases, these nonlinear interactions lead to the scattered waves being washed backward, potentially creating wave slamming on the column face.

Masterton et al. (2007) extended the work undertaken by Sheikh & Swan (2005), by investigating the behaviour of the *Type-1* and *Type-2* scattered waves for a variety of column diameters subjected to steep regular waves. Based upon these results, it was concluded that the *Type-1* wave scattering was relatively insensitive to a change in the column diameter. However, the *Type-2* wave scattering, which corresponds to the motion of fluid around the column, was shown to be critically dependent upon the D/λ value. They attribute these changes to the time taken for the fluid disturbance to move around the column and its consequent phasing with the incident waves. In addition to the experimental observations, a comparison with industry standard, linear and second-order diffraction solutions was carried out. Unsurprisingly, the linear diffraction solution was unable to describe the scattered flow field, especially the maximum crest elevation and the high-frequency wave scattering. The second-order diffraction solution provided a slight improvement, especially for the larger diameter columns, but gave a poor description of the wave run up and *Type-2* wave scattering.

A number of extensive experimental investigations involving steep incident waves and a surface-piercing vertical column have been carried out. These provide some physical insights into the occurrence of unexpected high-frequency wave scatterings

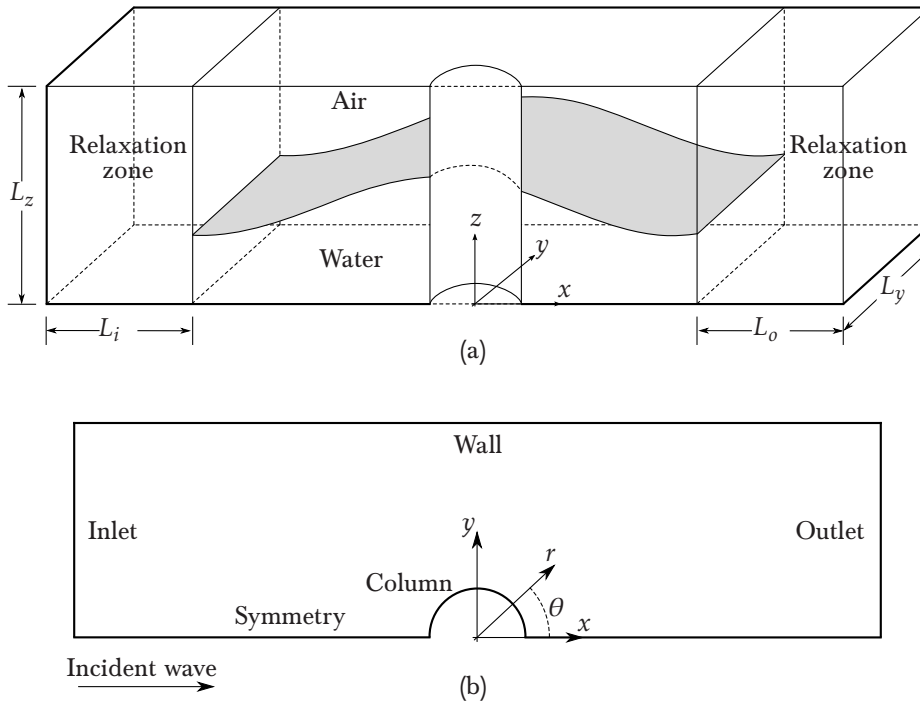


Figure 5.2: A schematic of the computational domain: (a) an orthogonal view with the column in place and (b) a plan-view showing the plane of symmetry.

and their importance in the context of offshore engineering. However, owing to the complex nature of these interactions, existing theoretical solutions are unable to provide a good description of the resulting flow field. Furthermore, with the interactions involving very steep breaking and near-breaking wave events, typical nonlinear numerical models cannot provide a full description of the process. In such cases, a more sophisticated model, capable of modelling the most extreme water surface elevations is required.

5.3 Numerical Procedure

The numerical procedure discussed earlier was used to numerically simulate the wave-structure interactions problem discussed above. This involved a single surface-piercing column; the wave generation and absorption capability provided through the use of the *wave2Foam* toolbox, as discussed in Chapter 4.

5.3.1 Computational Domain

In conducting the present study, two sets of numerical simulations were employed. The first involves the numerical simulations of regular waves in the absence of the column. This is referred to as the wave only solution. The second set of simulations were carried out with a vertical column located at the center of the domain. A sketch of the domain for this latter case is given in Figure 5.2; Figure 5.2(a) showing the full 3D domain and Figure 5.2(b) a plan view. The computational domain can be described by the height, L_z , the width, L_y , with the total length, L_x , the latter depending on the wavelength of the incident wave. The inlet and outlet boundaries were located on the left and right of the domain respectively. At both the inlet and outlet boundaries, a relaxation zone was used to absorb any reflected waves. The length of these zones again depended on the wavelength of the incident wave. The *atmospheric* boundary condition was applied at the top of the domain, allowing movement of the air phase both into and out of the domain. In order to reduce the computational cost, only half of the physical domain was simulated, a symmetric boundary condition being applied as shown in Figure 5.2. This is in line with laboratory observations reported by Swan & Sheikh (2015) in which they confirmed that both *Type-1* and *Type-2* waves were symmetric with respect to the column centreline. Furthermore, a slip boundary condition was applied on the sea bed, the back wall, and the column surface. The use of a slip boundary conditions was considered appropriate because the wave conditions adopted through out the present study lie within the inertia regime. In this case, viscosity plays an insignificant role and the effects due to various boundary layers can be neglected. Indeed, it will be shown in the subsequent section that the adopted boundary conditions are appropriate and generated results realistic.

The computational domain was discretized using the open-source mesh generator *Gmsh*; the adopted pattern involving sets of hexahedral meshes. Key parts of the computational mesh are shown in Figure 5.3. In the vicinity of the column ($D/2 \leq r \leq 4D$, where r is the radial distance from the centerline of the column and D is the column diameter), the mesh was discretized using a cylindrical polar grid. Adopting this method, the size of the mesh gradually decreases toward the surface of the column. Beyond the cylindrical polar grid, the mesh was discretized such that the aspect ratio of the vertical to horizontal cells was approximately 1 : 2. In order to obtain good approximation of the surface elevation, especially in the vicinity of the

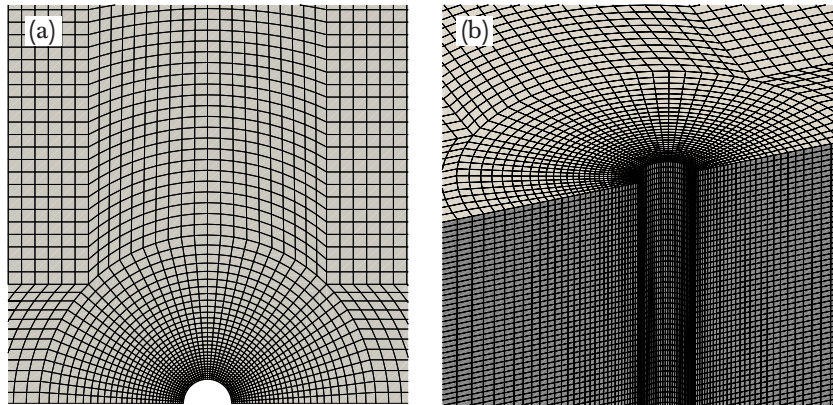


Figure 5.3: Computational mesh (a) plan view and (b) isometric view.

column, at least 20 vertical cells were required over the incident wave height, H .

5.3.2 Initialization Method

In the initial stages of the present study, the incident waves were generated from SWL. To enable the regular waves to reach a steady state, the simulations were carried out for a long time. This, in turn, requires a longer domain, to reduce possible reflections from the outlet boundary. Despite the use of a symmetric domain, effectively halving the domain size, the computational effort needed to advance this proved to be excessive. In order to avoid this difficulty, the simulations were initialized using a higher-order analytical wave solution, defined over the entire domain. By adopting this technique, a significant reduction in the computational effort was achieved. For the most computationally extensive cases, the simulation time was approximately 72h using a total of 32 processing cores on the Imperial College High Performance Computing (HPC) facility.

5.3.3 Surface Elevation Measurement

Most of the results presented in this chapter concern the predicted water surface elevations, with and without the column in place. The surface elevation in each numerical simulation was evaluated using two approaches. In the first, the surface elevation was calculated based on the iso-surface of the volume fraction evaluated at $\alpha = 0.5$. This allows the visualisation of multi-valued surfaces and the occurrence of surface break-up. The second adopts the use of numerical wave gauges to determine

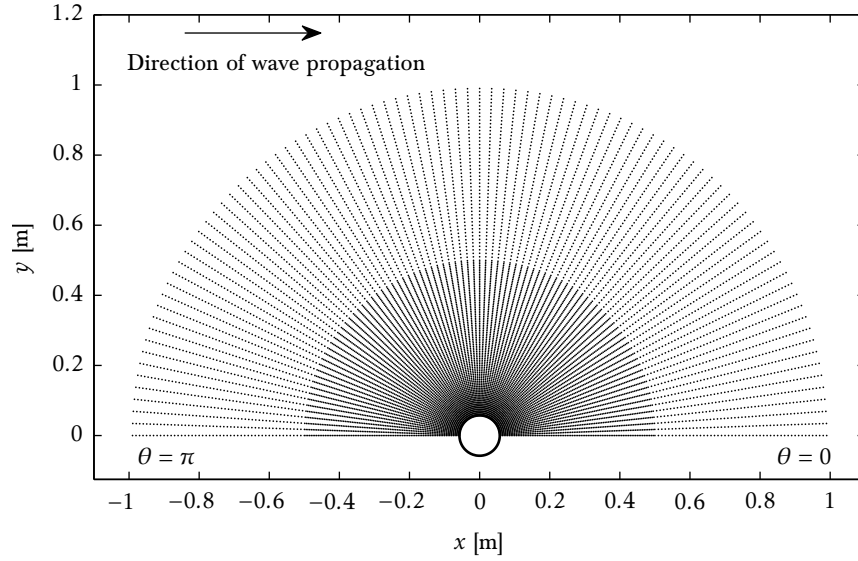


Figure 5.4: Typical layout of the numerical wave gauges.

the surface elevation. This produces a time series of surface elevation, $\eta(t)$, at each wave gauge location. In all cases the scattered wave field was determined using this second approach. If $\eta(t)_w$ and $\eta(t)_{wo}$ are the surface elevation predicted "with" and "without" the column in place, the scattered wave field is defined by $\Delta\eta(t) = \eta(t)_w - \eta(t)_{wo}$.

An extensive set of numerical wave gauges was used in each simulation. Adopting the usual polar coordinate system, centered about the column axes, an example of the numerical wave gauge arrangement is given in Figure 5.4; the incident wave direction being clearly noted in the figure. The upstream and downstream faces of the column are given by $\theta = \pi$ rad and $\theta = 0$ rad respectively. In this case, there are around 12 500 numerical wave gauges. With no need to worry about any possible disturbance to the flow, the number of wave gauges used herein is far greater than those reported in experiments of Swan & Sheikh (2015). This allows a much more detailed representation of scattered wave field. Furthermore, it should be noted that the results from the numerical wave gauges were calculated simultaneously at each time-step. This is in marked contrast with the experimental data outlined in Swan & Sheikh (2015) which was assembled from multiple runs of the same wave event using different wave gauge arrangements. In essence, the results obtained from the numerical wave gauges should prove to be more consistent and free from any unintentional averaging of the surface profile.

Table 5.1: Laboratory test cases from Swan & Sheikh (2015)

Test case	Wave period, T [s]	$Hk/2$	D/λ	KC
2	0.8	0.06	0.12	0.44
3	1.0	0.07	0.07	0.96
5	1.4	0.09	0.04	2.19
7	0.8	0.20	0.12	1.62
8	1.0	0.20	0.07	3.41
10	1.4	0.20	0.04	7.04
8a	1.0	0.28	0.07	4.75

5.4 Comparison with Experimental Results

A full description of the relevant laboratory data is given in Swan & Sheikh (2015). For the purpose of the present comparisons, all of the data relates to a single vertical surface-piercing column of diameter $D = 0.115$ m, located in a water depth of $d = 1.2$ m and subject to regular waves having a period lying in the range $0.8 \text{ s} \leq T \leq 1.4 \text{ s}$; the full details of the incident wave conditions are given in Table 5.1. All of these dimensions are given at laboratory-scale. Whilst the choice of the adopted scaling necessary to infer the equivalent full-scale values is arbitrary, an assumed length-scale of $l_s = 1 : 100$ is both appropriate to a problem dominated by potential flow forcing and provides guidance as to the relevant full-scale conditions. In this case, the application of Froude number similarity gives a time-scale of $t_s = \sqrt{l_s}$, such that the full-scale periods lie in the range $8 \text{ s} \leq T \leq 14 \text{ s}$. With a full-scale the column diameter of $D = 11.5$ m and a water depth of $d = 120$ m, these values are taken to be representative of a typical large volume GBS, located in the deeper end of the intermediate water depth regime. Indeed, the laboratory-scale test conditions are identical to those employed in an earlier physical model study of the Brent Bravo GBS (Swan et al., 1997).

Based upon the data given in Table 5.1, it is clear that some of the wave conditions (test cases 7, 8, 8a, 10) are steep or nonlinear ($Hk/2 \geq 0.2$); the (potential flow) inertia loading is expected to dominate ($KC \leq 7$), and the body lies outside the linear diffraction regime ($D/\lambda \leq 0.2$). This corresponds exactly to the troublesome loading regime identified in Section 5.2. Indeed, the wave loads experienced by the Brent Bravo GBS provided an important part of the motivation to investigate these

‘intermediate’ fluid-structure regimes.

In seeking to provide direct comparisons to the laboratory data, the first step involves the accurate representation of the incident wave conditions within a computational domain (Figures 5.2 and 5.3) that is large enough to avoid boundary effects and, at the same time, small enough to allow the discretization necessary to model any high-frequency effects. Figure 4.11 provides evidence of this. It addresses the three wave periods outlined in Table 5.1 and contrasts the laboratory measurements and the numerical predictions. Whilst comparisons of this type are of little value in their own right, they are fundamental to the comparisons that follow. The key point to note is that the incident wave conditions are very well described; the rms error in $\eta(t)$ is at most 1.1 % of the incident crest elevation. Moreover, the incident wave forms are genuinely long-crested; the rms variation in the transverse crest elevation, $\eta(y)$ being almost zero.

To achieve these results, the half-width of the computational domain was set to $L_y = 1$ m, the total height to $L_z = 1.5$ m (accommodating a mean water depth of 1.2 m) and the domain length, L_x varied with the wave period; the shortest waves ($T = 0.8$ s) being modelled in a domain of $L_x = 3$ m and the longest waves in a domain of $L_x = 6$ m. The input conditions were generated using a Stokes 5th order solution and the downstream boundary employed a relaxation zone of length $\lambda/2$, where λ is the wave length. Due to the way the cells are clustered near the free surface, the total number of cells varies from 2.5 million to 5.0 million. In all cases, a constant Courant number of $C_o = 0.3$ was maintained throughout the calculations.

The first quantitative evidence of an ability to model the flow around the vertical surface-piercing column, including the unexpected scattering of high-frequency wave components, is provided in Figure 5.5 and Figure 5.6. Each of these addresses the three incident wave periods ($T = 0.8$ s, $T = 1.0$ s and $T = 1.4$ s); Figure 5.5 considering the near-linear incident wave cases (Cases 2, 3 and 5 on Table 5.1) for which the steepness, $Hk/2 < 0.10$, and Figure 5.6 the nonlinear incident wave cases (test cases 7, 8 and 10 on Table 5.1) with a steepness of $Hk/2 = 0.20$. In each example, numerical data is provided describing the incident wave profile without the column in place, $\eta_{wo}(t)$, the wave profile with the column in place, $\eta_w(t)$ and the difference between the two, $\Delta\eta(t) = \eta_w(t) - \eta_{wo}(t)$; the latter describing the scattered wave field. In addition, the original laboratory data (from Swan & Sheikh (2015)) describing $\eta_w(t)$ and

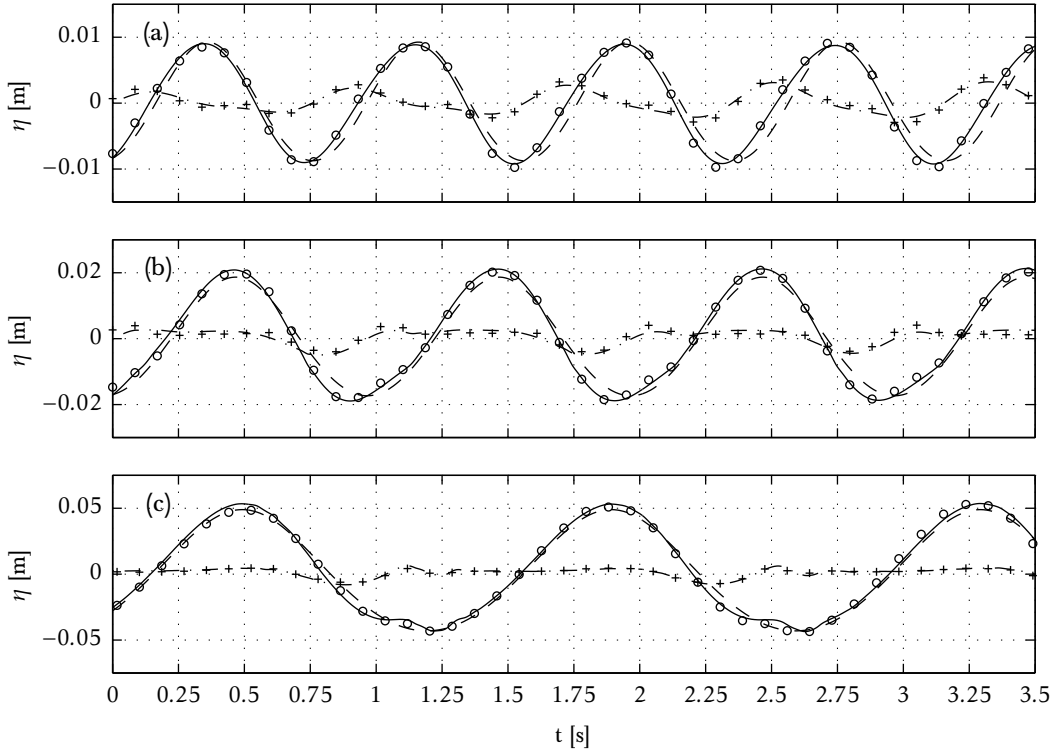


Figure 5.5: Time-histories of the water surface elevation, $\eta(t)$, with comparisons between numerical simulations and experimental observations for (a) test case 2, (b) test case 3, and (c) test case 5 recorded at $r = 0.1$ m and $\theta = \pi$ rad. Legends: --- η_{w0} , — η_w , - - - $\Delta\eta$, $\circ$ $\eta_w(\text{experiment})$, and + $\Delta\eta(\text{experiment})$.

$\Delta\eta(t)$ have been added as a direct comparison. In all six cases, the high-frequency scattered waves are clearly identified and good agreement between the experimental observations and the model predictions are established.

With $Hk/2 < 0.10$ the three wave cases on Figure 5.5 are (at most) weakly nonlinear. Nevertheless, the high-frequency wave scattering is clearly evident, particularly in test cases 2 and 3. This is important on two counts. First, given the D/λ ratios, linear diffraction will not be significant. Second, with diffraction solutions typically based upon a perturbation expansion involving the wave steepness, $Hk/2$, nonlinear diffraction occurring at integer multiples of the incident wave frequency should not occur in cases involving linear incident waves. Interestingly, the data presented in Figure 5.5 are not at odds with these assertions: there is no evidence of either linear diffraction or nonlinear scattering at integer multiples of the incident frequency. Nevertheless, high-frequency scattered waves are observed and their phasing, relative to the incident waves, appears to depend upon the relative column diameter, D/λ .

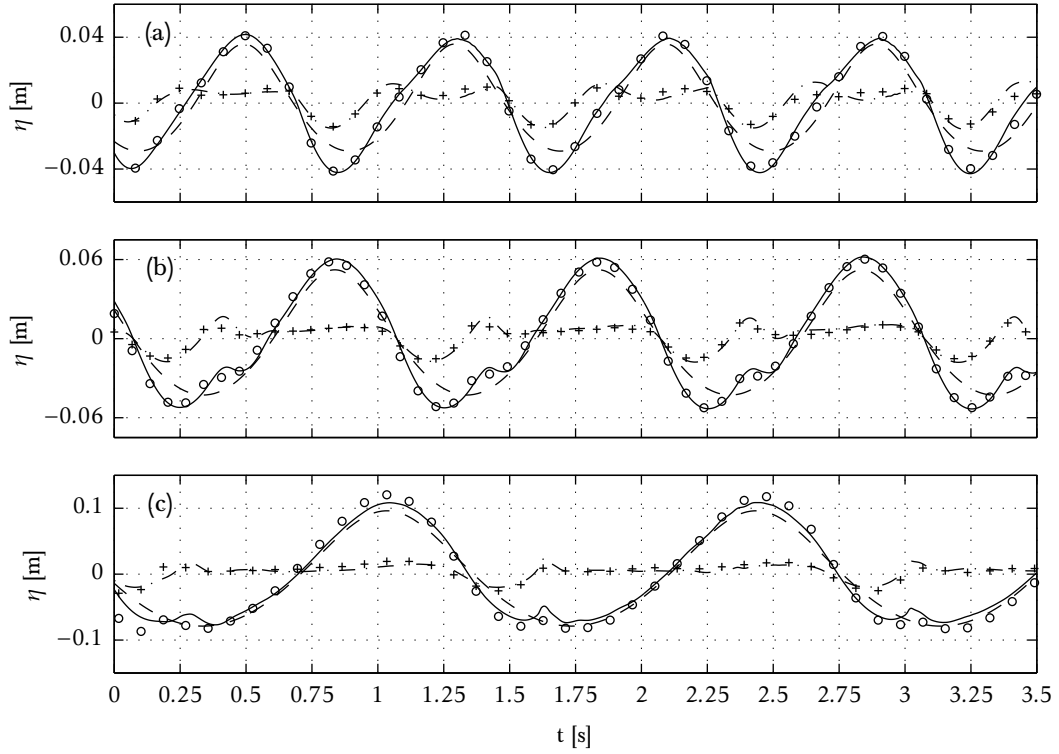


Figure 5.6: Time-histories of the water surface elevation, $\eta(t)$, with comparisons numerical simulations and experimental observations for (a) test case 7, (b) test case 8, and (c) test case 10 recorded at $r = 0.1$ m and $\theta = \pi$ rad. Legends: --- η_{w0} , — η_w , - - - $\Delta\eta$, $\circ$ $\eta_w(\text{experiment})$, and $+$ $\Delta\eta(\text{experiment})$.

This suggests that other mechanisms may be responsible for the high-frequency wave scattering.

Figure 5.6 provides a similar set of comparisons relating to three nonlinear incident wave cases. These again cover a range of effective column diameters (Table 5.1), whilst maintaining a constant incident wave steepness ($Hk/2 = 0.2$). Comparisons between the linear (Figure 5.5) and nonlinear (Figure 5.6) wave cases show that whilst the general pattern of high-frequency wave scattering remains broadly similar, the amplitude of the scattered waves is strongly dependent upon the incident waves. Evidence of this is provided by the fact that both the experimental observations and the numerical predictions show that the incident crest elevations (at $r \approx D$) are increased by some 20% due to the presence of the column. Furthermore, it is also clear that with a reduction in the effective diameter, D/λ , the high-frequency scattered wave appears earlier in the incident wave cycle.

For $T = 1.4$ s on Figure 5.6(c), the high-frequency scattered waves appear in the

trough of the longer incident wave. In this case there is some (small) discrepancy between the measured and numerically predicted form of the scattered waves. The explanation for this lies in the occurrence of the localised wave breaking on the column face. This adds to the complexity of interpreting the measured time-histories; the recorded data being unable to distinguish between multiple surface elevations at a single point and being sensitive to any air entrainment associated with the breaking process. Nevertheless, the measured and predicted data remains in good qualitative agreement, even in this most difficult of cases.

The occurrence of localised wave breaking was first observed in the context of flow around a vertical surface-piercing column by Chaplin et al. (1997) and Huseby & Grue (2000). These earlier studies addressed the higher harmonic forces acting on the column and did not consider the wave scattering. Nevertheless, there is striking similarity between the observed secondary loading cycle and the high-frequency wave scattering noted above. Specifically, neither can be predicted by classical (low-order) diffraction solutions and comparisons between the records suggests that their physical origins are closely related.

Further qualitative evidence of the success of the numerical calculations is provided in Figure 5.7. This provides a sequence of still images describing a plan view of the scattered wave field recorded in test case 8a outlined in Table 5.1. The left hand column of images are taken from the laboratory observations reported in Swan & Sheikh (2015). In each image, the incident waves are advancing from top to bottom, the column is clearly identified and the water surface seeded with aluminium dust to visualise the scattered wave field. Successive images, (a), (c), (e), (g), and (i) were taken with a time-lag of $\Delta t = 0.08$ s; the full sequence of images relating to approximately one half of a wave period in which the high-frequency wave scattering occurs in the upstream direction, opposing the incident wave direction. In contrast, the right hand column, images (b), (d), (f), (h), and (j) provide the corresponding numerical predictions; the pair of images given in each row representing the same phasing of the incident waves.

Taken as a whole, it is clear from Figure 5.7 that the observed and predicted wave scattering is in good general agreement. More specifically, both the *Type-1* concentric wave scattering and the *Type-2* non-concentric wave scattering, first reported by Swan & Sheikh (2015), is clearly predicted. Furthermore, the fact that the *Type-2* wave

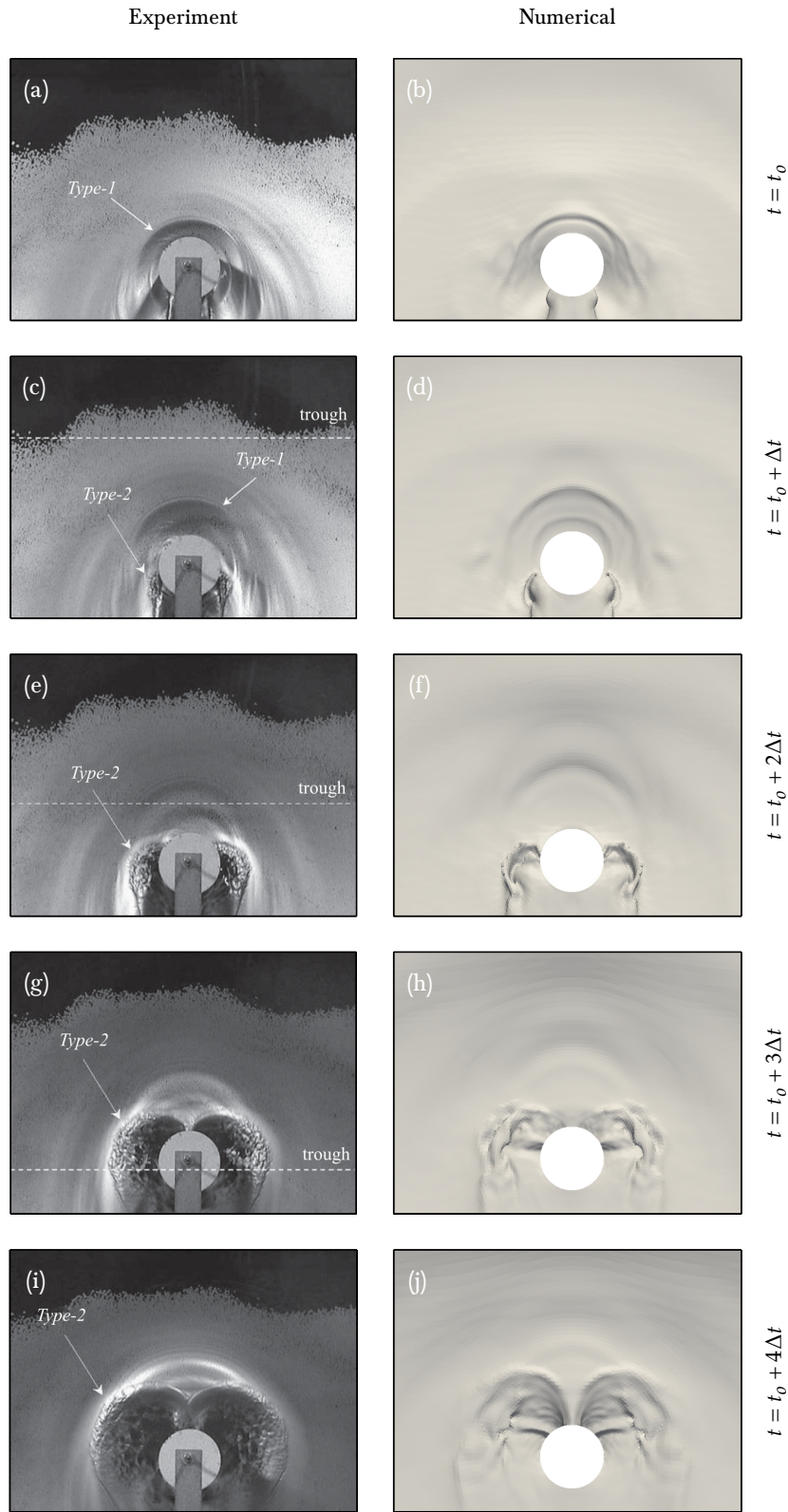


Figure 5.7: Plan view of wave-scattering from the upstream face of the column in test case 8a with comparisons between the experimental observations (left column) and the numerical simulations (right column). The time increment between each successive row is given by $\Delta t = 0.08$ s

scattering involves the movement of a disturbance (and hence fluid) moving around the circumference of the column is also clearly noted.

Quantitative descriptions of the *Type-1* and *Type-2* high-frequency wave scattering are provided in Figure 5.8. This again concerns wave case 8a and provides a sequence of spatial descriptions of the water surface elevation, $\eta(r)$, in the upstream direction ($\theta = \pi$); where (r, θ) are again polar co-ordinates whose origin lie on the centreline of the column with $\theta = 0$ defining the incident wave direction. In each sub-plot, data is provided describing the incident wave profile, $\eta_{wo}(r)$, the profile immediately upstream of the column, $\eta_w(r)$, and the scattered wave profile, $\Delta\eta(r) = \eta_w(r) - \eta_{wo}(r)$. In the case of η_w and $\Delta\eta$ comparative laboratory data (from Swan & Sheikh (2015)) is also provided. By contrasting the time-evolution of these profiles, both the *Type-1* and the *Type-2* scattered waves can be clearly identified. For example, with the arrival of an incident wave crest, the run-up on the front face of the column is clearly evident (Figure 5.8 (a) and (b)). As this runs back down, it creates the disturbance that propagates radially outwards as a concentric *Type-1* scattered wave (Figure 5.8 (c) and (d)). When this evolves in the vicinity of the incident wave trough, it is swiftly carried away from the column by the negative wave-induced fluid velocities (Figure 5.8 (e) and (f)).

With the disturbed fluid having moved around the column (Figure 5.7) it begins to build-up either side of the front face, driving the *Type-2* concentric wave scattering. The first evidence of this arises on Figure 5.8 (e), with the scattered wave (or waves) moving away from the column in Figure 5.8 (f) and (g). As the next incident wave crest approaches, the *Type-1* wave (located further upstream) is effectively brought to rest (Figure 5.8(g) and 5.8(h)) and the *Type-2* wave is sufficiently steep that it undergoes significant nonlinear amplification; the latter taking the classic form of a long-wave short-wave interaction (Longuet-Higgins & Cokelet, 1976) and being the most apparent in Figure 5.8 (h).

An alternative representation of the high-frequency wave scattering and the subsequent nonlinear wave-wave interactions is provided in Figure 5.9. This provides a contour plot of the incident wave, η_{wo} and the scattered wave field, $\Delta\eta$, in both space and the time (r, t) . This is assembled from time-histories of the water surface elevation recorded (or predicted) at multiple spatial locations. When presented in this form, a horizontal slice defines a time-history at one spatial location while a

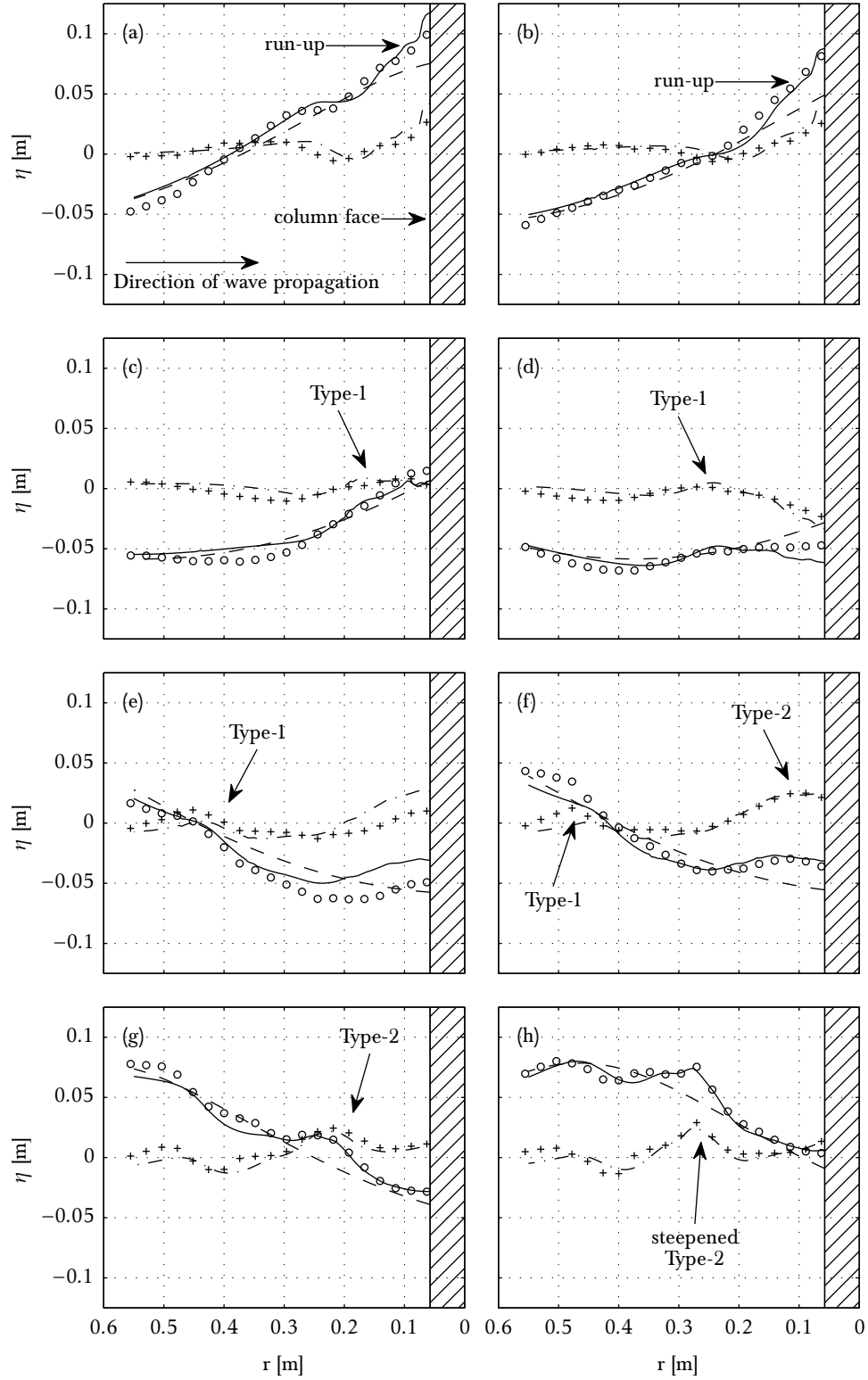


Figure 5.8: Spatial profiles of the water surface elevations showing the upstream scattering of *Type-1* and *Type-2* waves from the column for test case 8a with comparisons between numerical simulations and experimental observations. Phases of the incident wave cycle, Φ (radians) correspond to: (a) $\Phi = 0$, (b) $\Phi \approx 0.5$, (c) $\Phi \approx 1.1$, (d) $\Phi \approx 1.7$, (e) $\Phi = \pi$, (f) $\Phi \approx 3.3$, (g) $\Phi \approx 3.9$, (h) $\Phi \approx 4.5$. Legends: --- η_{w0} , — η_w , ···· $\Delta\eta$, $\circ$ $\eta_w(\text{experiment})$, and + $\Delta\eta(\text{experiment})$.

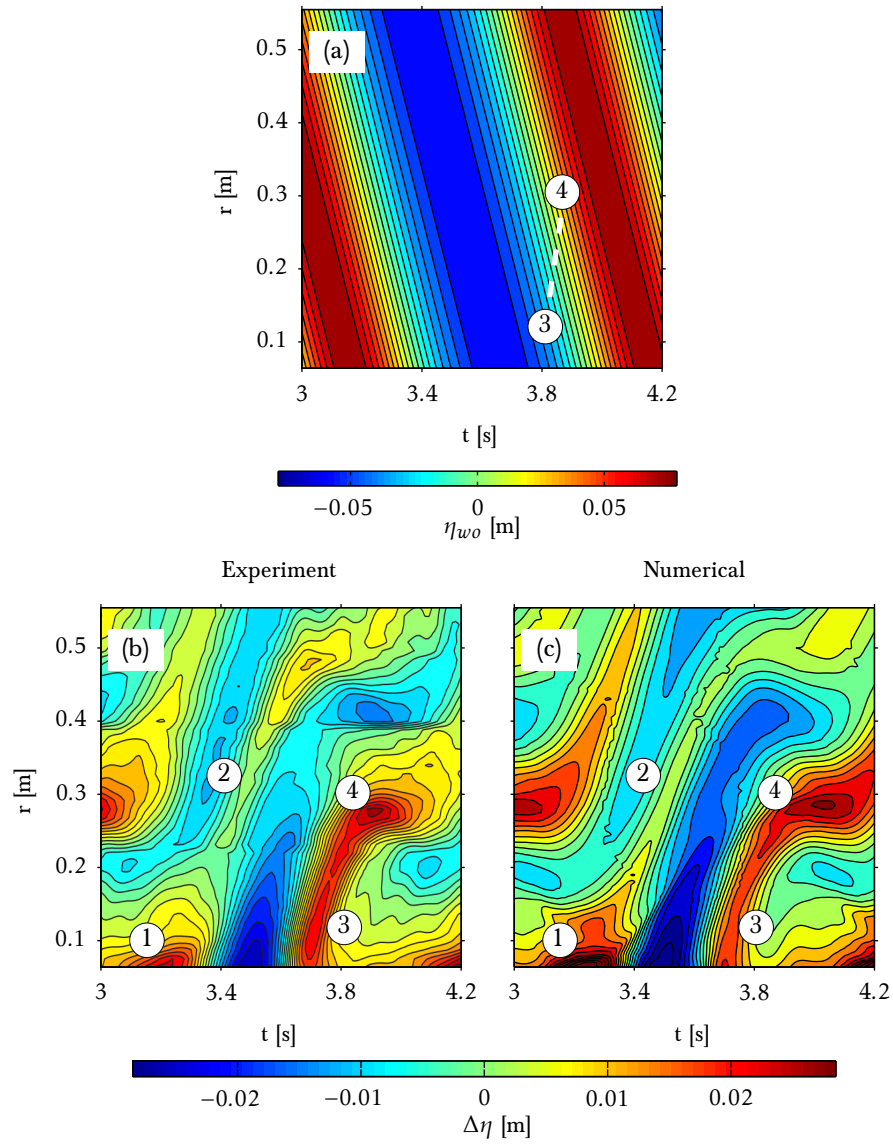


Figure 5.9: Contour plots showing spatial and temporal evolution of (a) the incident wave field, η_w and (b) & (c) the scattered wave field, $\Delta\eta$ on $\theta = \pi$ rad. All of the data relate to test case 8a, with (a) and (c) based on numerical simulations and (b) laboratory observations (Swan & Sheikh, 2015).

vertical slice defines spatial representation at one instant in time; the former related to the data presented in Figure 5.5 and Figure 5.6 and the latter the data given in Figure 5.8. Figure 5.9 provides three space-time plots: (a) the incident wave elevation from numerical simulations, (b) the scattered wave elevation based on the laboratory data given in Swan & Sheikh (2015) and, (c) the scattered wave elevation based upon the present numerical predictions. The similarity between plots (b) and (c) is obvious, as is the propagation of the *Type-1* and *Type-2* scattered waves; the former indicated by ① - ② and the latter ③ - ④.

In considering the scattered wave field, the orientation of r for $\theta = \pi$ is directly opposite to the direction of incident wave propagation; the contour describing the incident wave crest being defined by $\partial r / \partial t < 0$, as shown in Figure 5.9(a). The fact that both the *Type-1* and *Type-2* waves identified on Figure 5.9(b) and 5.9(c) have positive gradients, $\partial r / \partial t > 0$ confirms that they are propagating away from the column, in the opposite direction to the incident waves. Considering Figure 5.8(a), the large elevation noted at $r = D/2$ and $\theta = 0$ (or $t = 33.2$ s) corresponds to wave run-up on the front face of the column immediately following the passage of the incident wave crest. This corresponds to point ① on 5.9(b) and (c). The outwardly propagating wave that subsequent results denoted by ① - ② is a *Type-1* scattered wave. Approximately $T/2$ later (at $t = 33.7$ s), a second wave propagates outwards in the positive r direction. This is denoted by ③ - ④ on 5.9(b) and (c) and corresponds to a *Type-2* wave. This wave is larger and is brought to rest (momentarily) when it interacts nonlinearly with the next incident wave crest at $r \approx 0.28$ m and $t = 33.9$ s. This interaction occurs at point ④ and is further defined by the dashed line on 5.9(a). This produces the substantial increase in the local crest elevation first identified in Figure 5.8 (h). In the context of the present study, it is clear that the numerical predictions are able to reproduce the most important features of the wave-column interaction.

Given the importance of the *Type-2* scattered waves and their dependence on the circulation of fluid about the column, Figure 5.10 describes the water surface elevation on the surface of the column, $r = D/2$, and its variation with θ for different phases of the incident wave cycle. The upstream and downstream face of the column are given by $\theta/\pi = 1$ and $\theta/\pi = 0$ respectively. The incident wave profile, $\eta_{wo}(\theta)$, the wave profile around the column, $\eta_w(\theta)$, and the difference in the surface elevation,

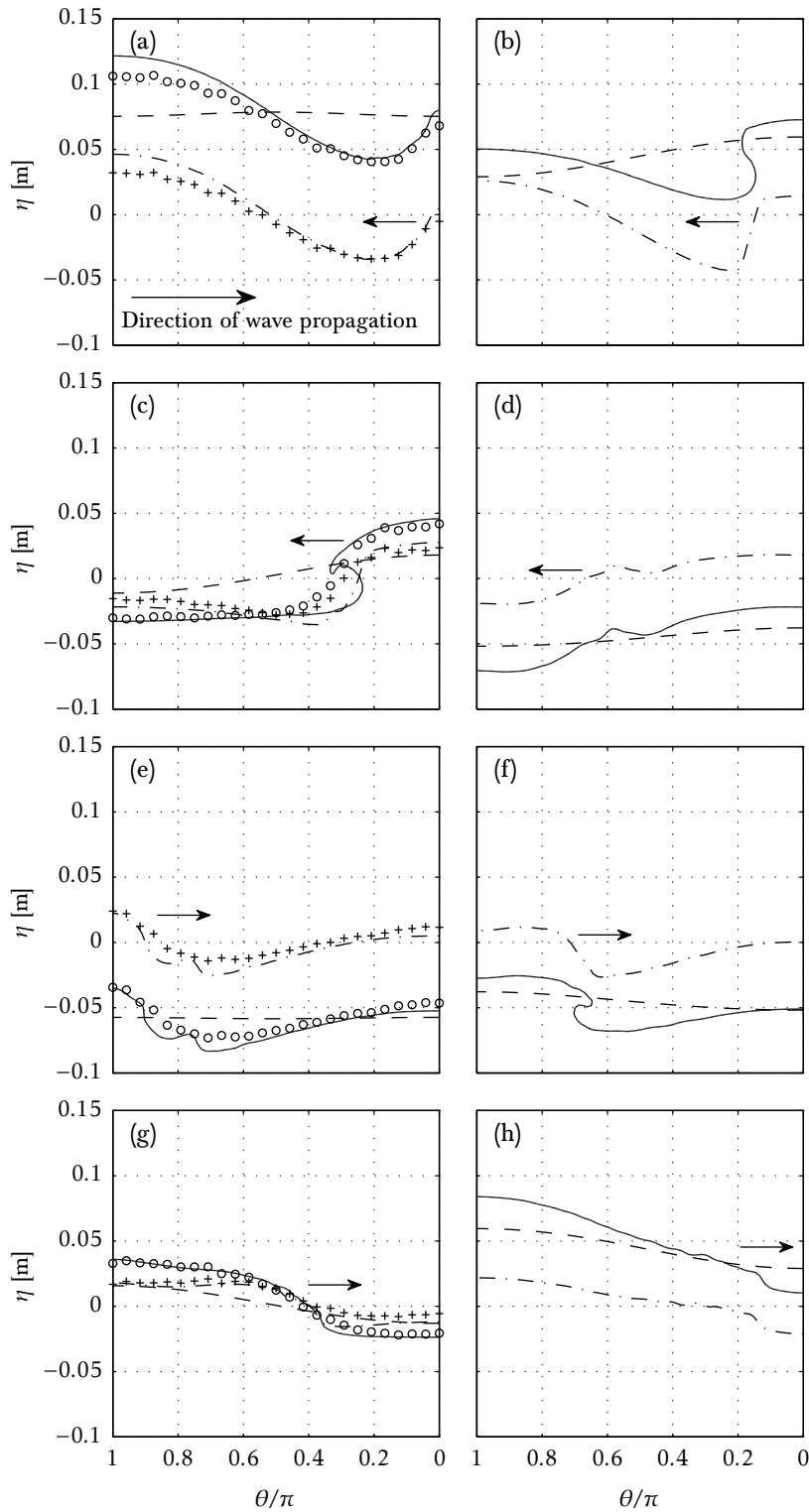


Figure 5.10: Spatial profiles of the water surface elevation on the column face, $r = D/2$, with comparisons between experimental observations and numerical simulations. Phases of the incident wave cycle, Φ (radians) correspond to: (a) $\Phi = 0$ (b) $\Phi \approx 0.8$ (c) $\Phi \approx 1.4$ (d) $\Phi \approx 2.4$ (e) $\Phi = \pi$ (f) $\Phi \approx 3.9$ (g) $\Phi \approx 4.9$ (h) $\Phi \approx 5.5$. Legends: --- η_{w0} , — η_w , - - - $\Delta\eta$, ○ $\eta_w(\text{experiment})$, and + $\Delta\eta(\text{experiment})$.

$\Delta\eta(\theta) = \eta_w(\theta) - \eta_{wo}(\theta)$ are provided in each subplot. Values for $\eta_w(\theta)$ is taken from an iso-surface of the volume fraction, $\alpha = 0.5$, allowing a detailed description of the wave pattern. Where available, experimental data (from Swan & Sheikh (2015)) is also provided for comparison. The small arrows placed on each plot shows the direction in which the fluid disturbance is moving.

With the arrival of the incident wave crest (Figure 5.10(a)), wave run-up occurs on the upstream face. In addition, a mound of water is also formed on the downstream face ($\theta/\pi = 0$). As this mound of water begins to move upstream (Figure 5.10(b)), the interaction with the positive wave-induced velocities steepen the surface profile. This forces the wave front to overturn and break (Figure 5.10(c)). The broken wave front continues to propagate upstream (Figure 5.10(d)), forming the origin of the non-concentric *Type-2* waves. Half a wave cycle later, with the arrival of the incident wave trough (Figure 5.10(e)), the run-up occurs on the back face and the mound of water on the front; essentially the reverse of that observed on Figure 5.10(a), but smaller in magnitude. At this phase of the wave cycle, the fluid disturbance begins to circulate, from the upstream face of the column to the downstream. As this disturbance moves downstream, the interaction with negative wave-induced velocities again steepens the profile, forcing the wave to break (Figure 5.10(f)). This broken wave continues to propagate round the column (Figure 5.10(g) and 5.10(h)) producing a pair of *Type-2* waves scattered in the incident wave direction.

It is clear from these results that the circulation of fluid around the column which forms the *Type-2* waves is greatly influenced by the incident waves. Specifically, the interactions with the positive wave-induced velocities (Figure 5.10(b) and 5.10(c)) are more pronounced when compared to the interactions with the negative wave-induced velocities (Figure 5.10(e) and 5.10(f)). The explanation for this lies in the fact that steep waves are vertically asymmetric, with larger fluid velocities in the crest as compared to the trough. Therefore, interactions arising close to the crest are larger than those in the trough. Setting these issues aside, the present results are in good agreement with the laboratory observations confirming that the interactions arising on the column face can be numerically predicted.

The importance of the nonlinear wave-wave interactions arising between the scattered high-frequency *Type-2* waves and the next incident wave crest, denoted by point ④ in Figure 5.9, is further addressed on Figure 5.11. Subplots (a) and (b) concern

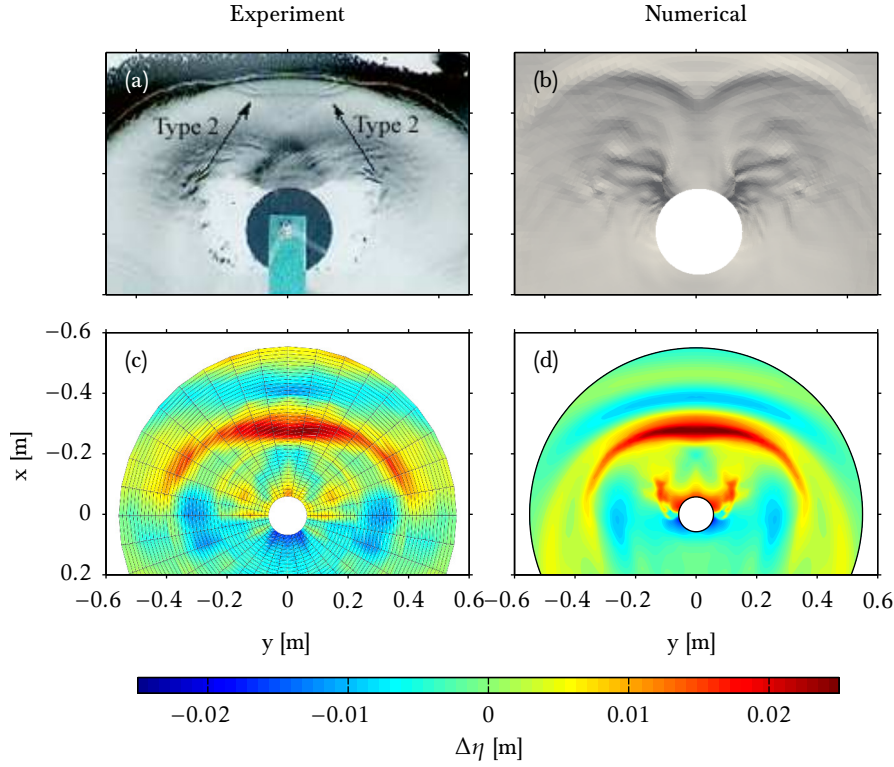


Figure 5.11: The nonlinear interaction between an incident wave and a Type-2 scattered wave upstream of the column. (a) Plan view from experimental observations, (b) plan view of numerical simulations, (c) contour plot of $\Delta\eta$ from experiment observations, and (d) contour plot of $\Delta\eta$ from numerical simulations.

the total water surface elevation, $\eta_w(x, y)$, recorded in the vicinity of the column; sub-plot (a) providing a still image taken from the earlier laboratory study (Swan & Sheikh (2015)) and sub-plot (b) the present numerical predictions at the same phase of the incident wave cycle. Adopting these results, sub-plots (c) and (d) provide contours of the change in the recorded water surface elevation $\Delta\eta = \eta_w - \eta_{wo}$, arising due to the scattered wave field and its subsequent interaction with the next incident wave crest; subplot (c) being based upon laboratory observations and subplot (d) numerical calculations. Comparing these results, two points merit particular attention. First, there is very close agreement between the experimental observations and the numerical predictions; this is true for both the largest crest elevation and the developing *Type-2* waves in the immediate vicinity of the column. Second, the magnitude of the increased crest elevations is such that $\Delta\eta/\eta_{wo} \approx 0.3$. This is very important from a practical perspective, particularly in respect of maintaining an effective air-gap, and all more significant because it arises outside the established diffraction regime

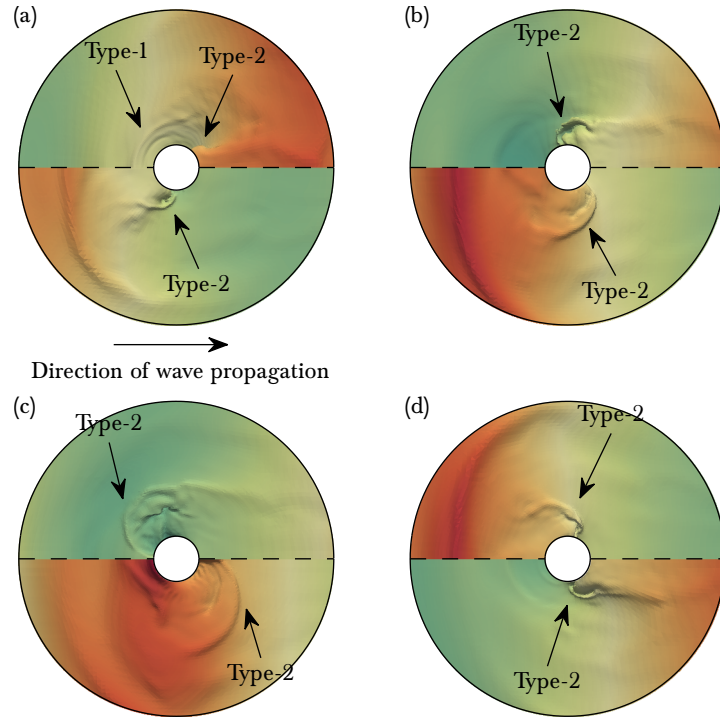


Figure 5.12: Plan view showing the upstream and down stream scattering based upon numerical simulations at different incident wave phases (a) $\Phi = 1.19$ rad, (b) $\Phi = 2.14$ rad, (c) $\Phi = 2.89$ rad, (c) $\Phi = 4.90$ rad. In each subplot, the top half corresponds to the phasing noted above and the bottom half is π rad (or half a wave cycle) later; the plotted area corresponding to $r = 5D$.

$(D/\lambda < 0.2)$.

Given the nature of the laboratory tests and the practical engineering application, the experimental data presented by Swan & Sheikh (2015) concentrated on the scattering of waves in the upstream directions. However, it is evident from the present calculations that wave scattering occurs both upstream and downstream; the pattern of *Type-1* and *Type-2* wave scattering occurring once, in each direction in each incident wave period. Evidence of this alternate scattering is provided in Figure 5.12. This again concerns the $D = 0.115$ m column subject to the wave case 8a ($D/\lambda = 0.07$) and presents four sub-plots relating to the different phases of the incident wave cycle. In each plot and all subsequent plots of these type, the colour represents the normalized water surface elevation. This lies within the range of $-1.3 \leq \eta_w(x, y)/\eta_{wo}(x, y)_{\max} \leq 1.3$, where the maximum and minimum values are represented by the red and green colours respectively. The shading is generated as a consequence of lighting applied at a certain angle from above during 3D rendering of the result. The figure also covers a radius of $5D$, where D is the column diameter.

Whilst this is sufficient to describe the relevant data, it represents only a small part of the total domain size. All of the data is numerically generated, with each sub-plot divided into an upper and lower half; the top relating to the stated phase (Φ) and the bottom π radians later.

In Figure 5.12 (a), $\Phi = 1.19$ rad so that in the top half of the figure, the wave crest has passed the column, the *Type-1* wave scattering is clearly visible in the upstream direction, and the circulation of fluid around the column (appropriate to the *Type-2* wave scattering) is also apparent. In the lower half of this figure, corresponding to $\Phi = 4.33$ rad, there is no evidence of *Type-1* wave scattering in the downstream direction due to the run-up of the negative fluid flow in the wave trough. The explanation for this again lies in the crest-trough asymmetry associated with the nonlinear incident wave, particularly, the fact that the magnitude of the positive velocities in the wave crest are larger than the negative velocities in the wave trough. However, the circulation of fluid around the column, associated with downstream propagation of *Type-2* wave scattering and driven by the advancing wave crest, is clearly visible in the lower half of this figure. Interestingly, this circulation is larger and has travelled further when compared to that in the top half of the figure. Again, this is due to wave asymmetry.

In Figure 5.12(b) and 5.12(c) corresponding to $\Phi = 2.14$ rad and $\Phi = 2.89$ rad, the progressive development of the circulating flow is clearly established; the direction of the circulation and the direction of the *Type-2* wave scattering being reversed every half wave period. In Figure 5.12(c), comparisons between the upper and lower values of the plots confirm that the downstream scattered *Type-2* waves are larger and occur a little earlier than the equivalent waves scattered in the upstream direction. Interestingly, Figure 5.12(d) corresponds to $\Phi = 4.90$ rad in the upper plot and $\Phi = 8.04$ rad or 1.76 rad in the lower plot. This is (approximately) half a wave cycle beyond Figure 5.12(b) and, as a result, the calculated data represents the mirror image of that presented in Figure 5.12(b). Taken together, Figure 5.12(a)-(d) show that the circulation of fluid around the column is a characteristic feature of the wave-column interactions, leading to the alternate scattering of *Type-2* waves in the upstream and downstream directions every half wave period.

Table 5.2: Test cases involving varying effective column diameters.

Test case	D [m]	$Hk/2$	D/λ	KC
D100	0.10	0.32	0.06	6.66
D150	0.15	0.32	0.10	4.44
D200	0.20	0.32	0.13	3.33
D250	0.25	0.32	0.16	2.66
D300	0.30	0.32	0.19	2.22

5.5 Effective Column Diameters

Having validated the numerical calculations and established the nature of the high frequency wave scattering, this section provides comparative calculations relating to five different column diameters subjected to the same incident wave conditions ($T = 1.0$ s and $H = 0.16$ m). Full details of the five cases are given in Table 5.2; the relative column diameters lying in the range $0.64 \leq D/\lambda \leq 0.192$, the incident wave steepness set at $Hk/2 = 0.32$, and deep water conditions maintained throughout, $kd \approx 4.02$. Figure 5.13 provides a sequence of still images showing a plan view of the total wave field, η_w , recorded in the presence of the column. These images are arranged in a block of three columns and six rows; the columns corresponding to a constant diameter of $D = 0.1$ m, 0.2 m and 0.3 m and the rows a constant incident phase angle, Φ . Taken together, the six rows cover a full incident wave cycle, $\Delta\Phi = 2\pi$, but the spacing between successive rows is variable allowing key stages in the scattering process to be identified.

With $\Phi = 0$ rad defined by an incident wave crest at $r = 0$, the first row of Figure 5.13 corresponds to $\Phi = -0.63$ rad. As such an incident wave crest is approaching the column. In Figure 5.13(a), corresponding to $D = 0.1$ m, the positive fluid velocities have produced wave run-up on the front face of the column ($\theta = \pi$) and have driven fluid (both clockwise, $\theta = \pi \rightarrow 0$, and anti-clockwise, $\theta = \pi \rightarrow 2\pi$) around the circumference of the column. In Figure 5.13(b) ($D = 0.2$ m) and 5.13(c) ($D = 0.3$ m), the run-up is more pronounced and the disturbance has propagated only part-way around the column; the latter being due to the reduced tangential velocities and the increased column circumference.

With the passage of the wave crest, the second row (Figure 5.13(d)-5.13(f)) correspond to $\Phi = 1.19$ rad. In all three cases, the *Type-1* scattered wave is immediately ap-

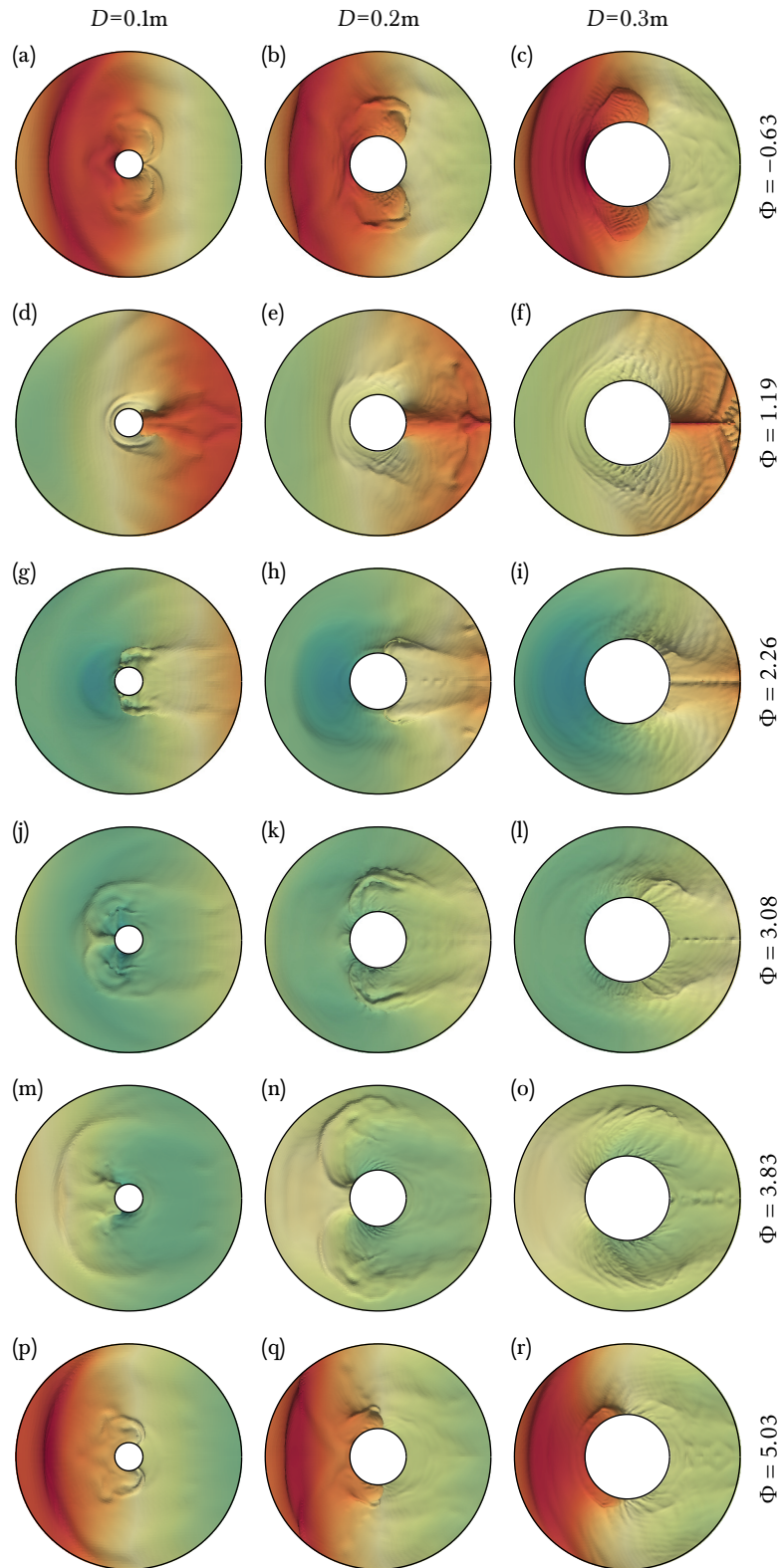


Figure 5.13: Numerical calculations showing plan views of the wave scattering for three different column diameters, ($D = 0.1\text{m}$ (first column), $D = 0.2\text{m}$ (second column), and $D = 0.3\text{m}$ (third column)) for the same incident wave conditions ($T = 1.0\text{s}$ and $H = 0.16\text{m}$) and different incident wave phases: (a-c) $\Phi = 0.63\text{rad}$, (d-f) $\Phi = 1.19\text{rad}$, (g-i) $\Phi = 2.26\text{rad}$, (j-l) $\Phi = 3.08\text{rad}$, (m-o) $\Phi = 3.83\text{rad}$, (p-r) $\Phi = 5.03\text{rad}$.

parent. In Figure 5.13(d) the negative velocities in the approaching trough encourage the continued propagation of the flow disturbance around the surface of the column. In Figure 5.13(e) evidence of this latter motion is less clear, whilst in Figure 5.13(f) it is absent. In Figure 5.13(g)-5.13(i) these differences become more apparent. Indeed, in Figure 5.13(i), there is little, if any, evidence of a returning circumferential flow.

In row 4, Figure 5.13(j)-5.13(l), the non-concentric disturbances have propagated around the smaller column ($D = 0.1$ m on Figure 5.13(j)) and the *Type-2* wave scattering in the upstream direction is apparent. In Figure 5.13(k), this process is established but delayed, while in Figure 5.13(l), the disturbance is moving radially rather tangentially. In row 5, the wave trough has passed and the next incident wave crest is approaching ($\Phi = 3.83$ rad). This causes a significant steepening of the *Type-2* scattered waves in Figure 5.13(m). This marks the beginning of the long-wave short-wave interactions, responsible for the large amplification of the maximum crest heights observed on Figure 5.11. Furthermore, with the approaching crest the first evidence of fluid being driven back around the circumference of the column is apparent. Similar observations can be made in respect of Figure 5.13(n), but in this case, the weaker *Type-2* waves did not reach the upstream stagnation point. As a result, the increased elevation on $\theta = \pi$ is much reduced. Likewise, in Figure 5.13(o) there is no evidence of increased elevations on $\theta = \pi$ due to the absence of *Type-2* wave scattering.

In the bottom row (row 6), corresponding to $\Phi = 5.03$ rad, the next incident wave crest is clearly apparent, and fluid is again driven around the column in all three cases (Figure 5.13(p), 5.13(q), 5.13(r)). Given the cyclic nature of the incident waves, the bottom row of figures will be followed by the top. In considering the overall motion, it is important to note that in the smallest column diameter ($D = 0.1$ m in Column 1), fluid is driven around the column circumference in both directions, each transit completing the full $\Delta\theta = \pi$ rad and each taking approximately half a wave period. This creates a near resonant condition in which fluid is permanently circulating about the column and the motion in one wave cycle re-inforces the next. In contrast, with an increase in the column diameter ($D = 0.2$ m in Column 2), the movement of the fluid from $\theta = \pi$ rad to $\theta = 2\pi$ rad is weaker and is not fully completed, leading to reduced scattering in the downstream direction. This undoubtedly arises due to the wave asymmetry; specifically the difference in the crest-column and trough-column interactions, including the fact that the wave-induced water particle velocities are of

Table 5.3: Test cases for varying wave steepness

Test case	$Hk/2$	D/λ	KC
A2	0.08	0.06	1.35
A4	0.16	0.06	2.88
A6	0.24	0.06	4.64
A8	0.32	0.06	6.66

larger magnitude in the crest than the trough. This difference becomes more apparent with the further increase in the column diameter ($D = 0.3$ m in Column 3). In this case, fluid is only driven around the column in one half of the wave cycle, leading to a very different, asymmetric pattern of wave scattering.

5.6 The Role of Wave Steepness

The effect of the incident wave steepness on the high-frequency wave scattering and particularly, the incident-scattered wave interactions are investigated in this section. All of the data relates to the smallest column diameter ($D = 0.1$ m) subject to an incident wave period, $T = 1.0$ s. This ensures that the relative column diameter for all cases is defined by $D/\lambda = 0.064$. The incident wave cases are defined in Table 5.3; the wave steepness lying in the range $0.08 \leq Hk/2 \leq 0.32$ with a deep water condition maintained throughout, by adopting a water depth of $d = 1.2$ m, the wave conditions vary from near-linear to near-breaking.

Figure 5.14 presents a sequence of still images showing the plan view of the wave field for each of the four wave cases. These images are arranged such that the columns correspond to cases involving varying wave steepness. In contrast, the rows represent different incident wave phases, Φ ; the values chosen such that they have an increment of $\Delta\Phi = \pi/2$ rad and cover the full incident wave cycle. The first row of Figure 5.14 corresponds to $\Phi = 0$ rad, defined such that the incident wave crest is at the centerline of the column. For the lowest steepness case (in the first column of Figure 5.14), the result shows almost no disturbance to the incident wave field. This is consistent with slender body approximation in which for value of $D/\lambda < 0.2$, any disturbance to the flow field due to the presence of the structure is negligible. However, as the wave steepness increases (moving from left to right

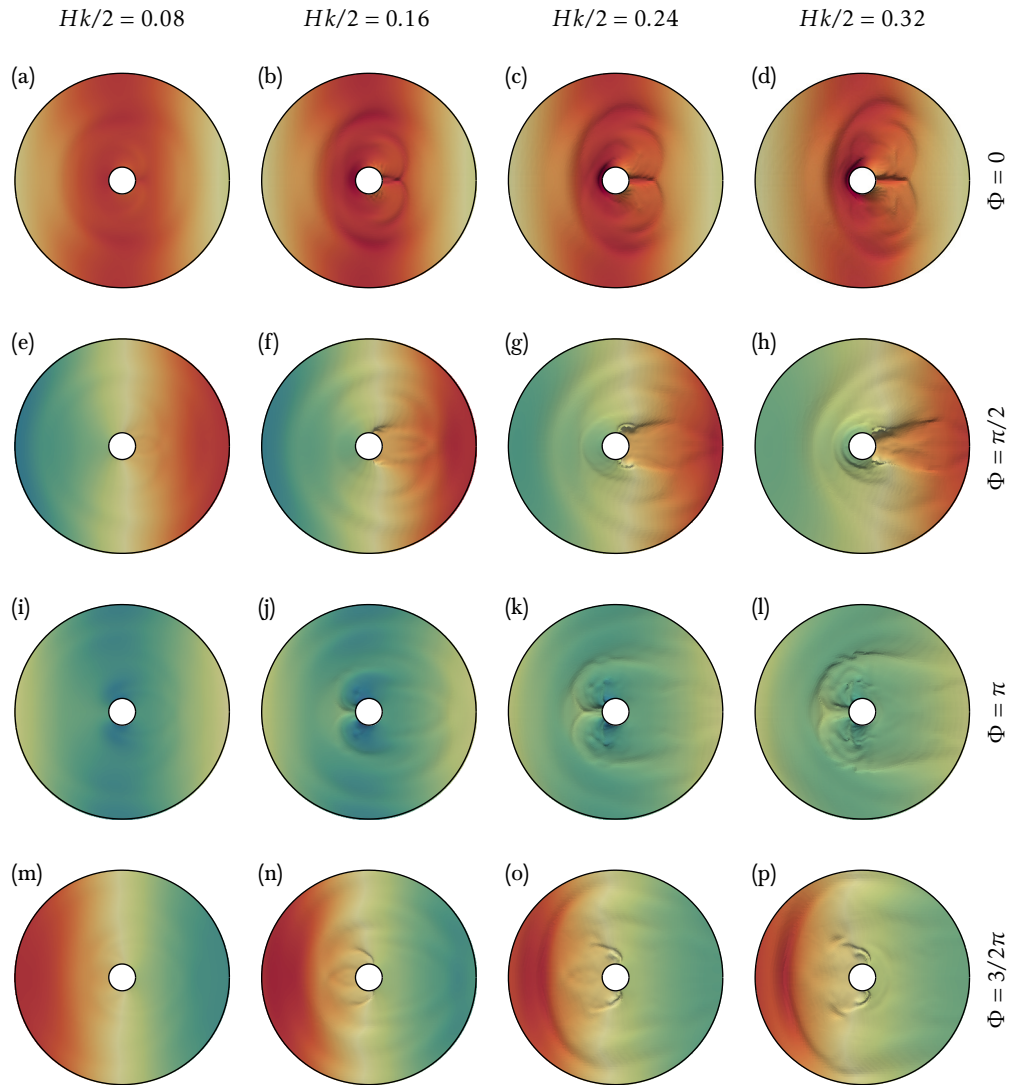


Figure 5.14: Numerical predictions of wave scattering for case A2 (first column), case A4 (second column), case A6 (third column), and case A8 (fourth column) at different incident wave phases. (a-d) $\Phi = 0$ rad, (e-h) $\Phi = \pi/2$ rad, (i-l) $\Phi = \pi$ rad, and (m-p) $\Phi = 3\pi/2$ rad.

across the first row of Figure 5.14), the disturbance in the immediate vicinity of the column becomes more apparent. Specifically, there is clear evidence of *Type-1* and *Type-2* waves being shed in the upstream ($\Phi = \pi$ rad) and downstream ($\Phi = 0$ rad) directions respectively. In the steepest wave case, the diameter of the non-concentric *Type-2* wave scattered in the downstream direction (Figure 5.14(d)) is appreciably larger than the equivalent *Type-2* wave scattered in the upstream direction (Figure 5.14(l)) half a wave period later. In addition, as the wave-wave interaction between the scattered *Type-2* waves and the incident wave takes place far upstream (Figure 5.14(n) - 5.14(p)), a pronounced increase in the surface elevation can be observed as the incident wave steepness increases. Given that this increase is dependent on a nonlinear wave-wave interaction, the increase with steepness is exactly as expected.

A quantitative description of the wave scattering is presented in Figure 5.15. This concerns the spatial contour of the scattered wave field, $\Delta\eta(x, y) = \eta_w(x, y) - \eta_{wo}(x, y)$; the data again being presented for varying incident wave steepness and different incident wave phases. Indeed, the arrangement of subplots is identical to that presented in Figure 5.14. Within Figure 5.15, the scattered wave field has been normalized by the maximum crest elevation of the incident wave, $\eta_{wo}(x, y)_{\max}$ and colour scale limited to lie within the range $-0.5 \leq \Delta\eta(x, y)/\Delta\eta(x, y)_{\max} \leq 0.5$ to highlight the scattered wave field in the extended vicinity of the column.

In contrast to Figure 5.14, Figure 5.15 indicates that the disturbance caused by the scattered wave field can be seen even in the lowest steepness wave case (presented in the first column). Taking each row in turn, the influence of the wave steepness is clearly highlighted. In the first row of figure Figure 5.15(a)-5.15(d) an increase in the wave run-up with the incident wave steepness is clearly noted. Moreover, when the run-up reverses (washes down), there is an increase in the depth of the depression immediately upstream of the column (Figure 5.15(e) - 5.15(h)). Also in the second row, the waves scattered in the downstream direction show a narrower angular spread with an increase in the incident wave steepness. This can be explained by the fact that a steeper incident wave produces a higher fluid velocity capable of washing the fluid disturbance further back. With the arrival of the wave trough in the third row (Figure 5.15(i)-5.15(l)), the *Type-2* wave scattering is observed in the upstream direction. In these plots, the scattered wave amplitude is clearly increasing with the incident wave steepness. From a practical perspective, the most important feature

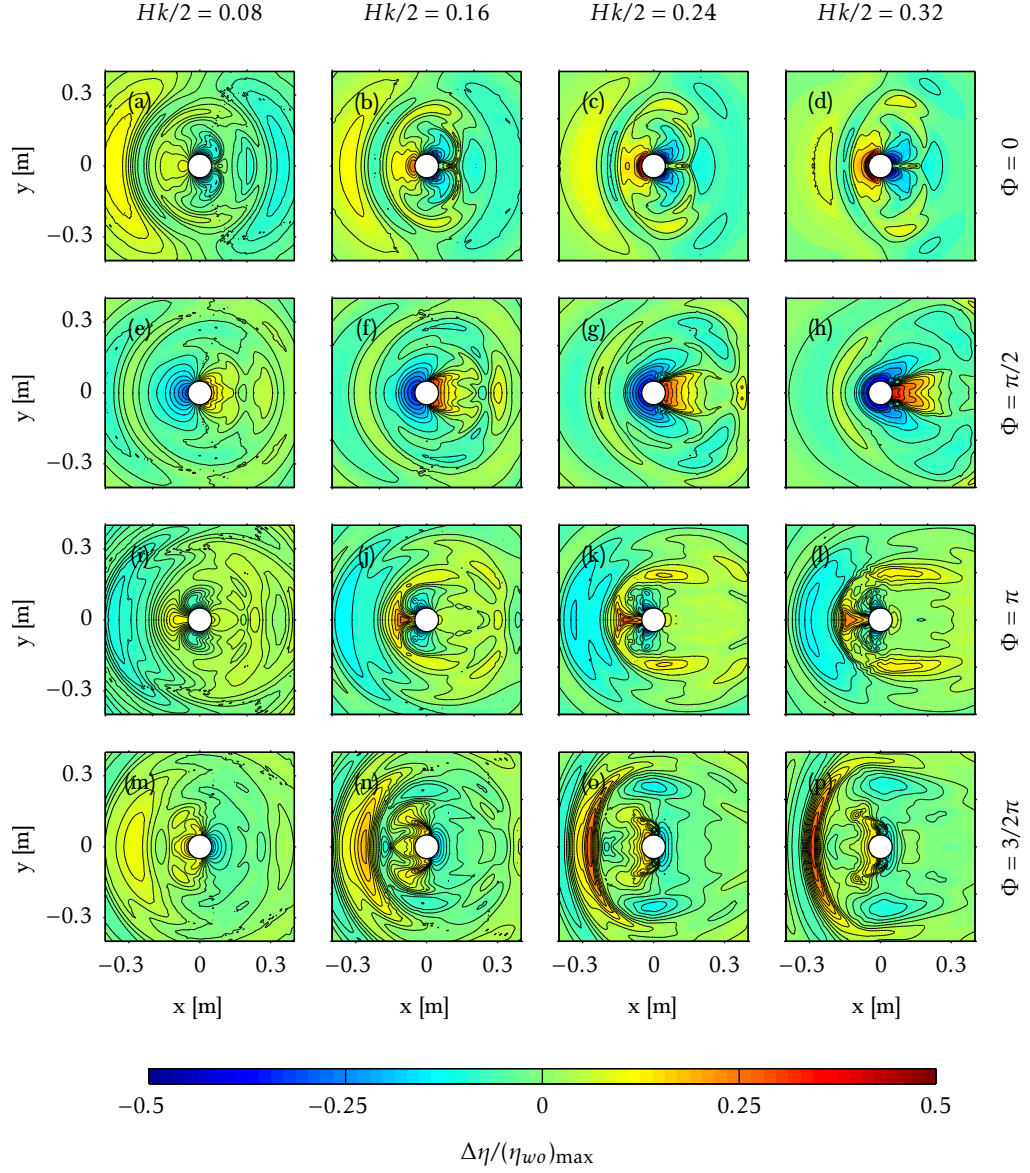


Figure 5.15: Contours of the normalized scattered wave field, $\Delta\eta/(\eta_{wo})_{\max}$ from the numerical simulation of wave case A2 (first column), wave case A4 (second column), wave case A6 (third column), and wave case A8 (fourth column) at different incident wave phases. (a-d) $\Phi = 0$ rad, (e-h) $\Phi = \pi/2$ rad, (i-l) $\Phi = \pi$ rad, (m-p) $\Phi = 3\pi/2$ rad.

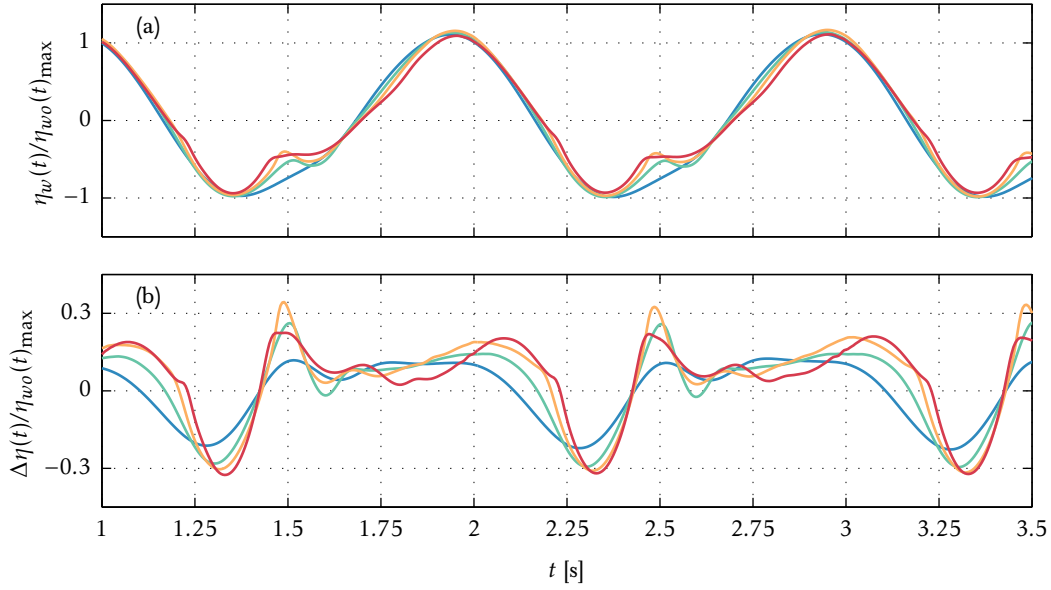


Figure 5.16: Time histories of normalized (a) surface elevation, $\eta_w/\max(\eta_{wo})$ and (b) scattered wave elevation, $\Delta\eta/\max(\eta_{wo})$, recorded at $r = 0.1$ m and $\Phi = \pi$ rad for varying incident wave steepness. Legends: — $Hk/2 = 0.08$, — $Hk/2 = 0.16$, — $Hk/2 = 0.24$, and — $Hk/2 = 0.32$.

of the scattered wave field is perhaps given by the last set of plots (Figure 5.15(m) - 5.15(p)). At this phase of the wave cycle, the interactions between the incident and the scattered waves produce a marked increase in the elevation of the scattered wave field. This increase is approximately 10% of the incident wave amplitude for the linear case and as high as 30% for the steepest case.

The role of the incident wave steepness is further investigated by comparing the time-history of the water surface elevation recorded one column diameter upstream of the column centerline (Figure 5.16). Figure 5.16(a) shows the total elevation in the presence of the column, $\eta_w(t)$ at $r = D$ while Figure 5.16(b) shows the scattered wave field, $\Delta\eta_w(t)$ at $r = D$. In both cases, the surface elevations have been normalized by the maximum incident crest elevation, $\eta_{wo}(t)_{\max}$. In Figure 5.16(a), high-frequency waves can be observed in the trough of the incident wave. This occurs at $t \approx 1.5$ s and again one period later at $t \approx 2.5$ s. With an increase in the incident wave steepness, it is perhaps expected that the amplitude of this high-frequency scattered wave will also increase. However, further examination of the scattered wave elevation (Figure 5.16(b)) shows otherwise. At the key times, $t = 1.5$ s and $t = 2.5$ s, the steepest wave case (A8) exhibits a lower (relative) scattered wave amplitude. This is at odds with comparable observations from the other wave cases; the clear trend being one

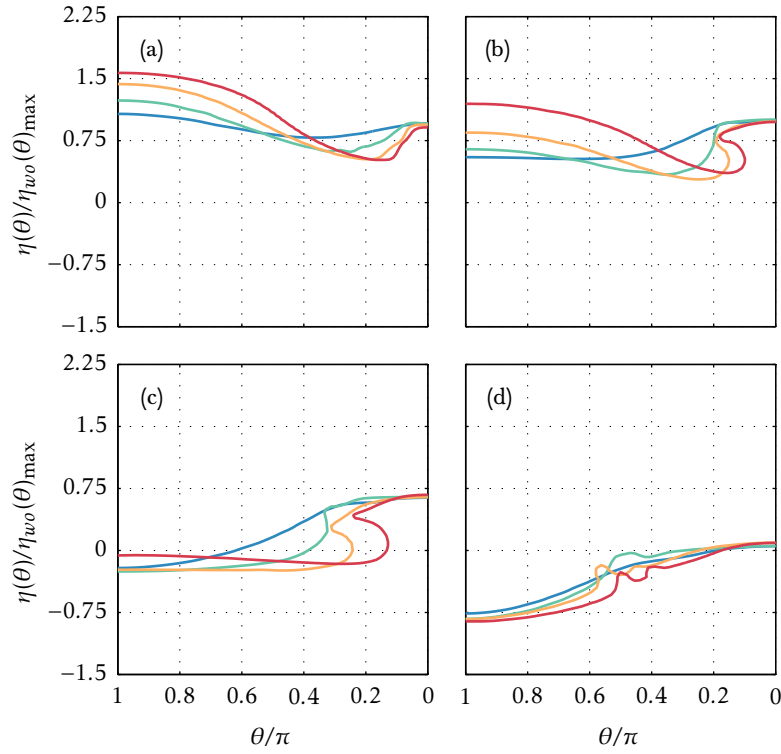


Figure 5.17: Spatial profile of the normalized surface elevation, $\eta_w(\theta)$, on the column face, $r = D/2$, for varying incident wave steepness at different incident wave phases, (a) $\Phi = 0$ rad (b) $\Phi = 0.63$ rad (c) $\Phi = 1.26$ rad (d) $\Phi = 1.88$ rad. Legends: — $Hk/2 = 0.08$, — $Hk/2 = 0.16$, — $Hk/2 = 0.24$, and — $Hk/2 = 0.32$.

in which the scattered wave amplitude increases with the incident wave steepness. To explain this effect it is first important to note that the scattered waves observed in Figure 5.16 are linked to *Type-2* waves. As such, the reduction in the scattered wave amplitude can perhaps be explained by examining the circulation of fluid around the column since this drives the formation of the *Type-2* waves.

The surface elevations, η_w , on the surface of the column at different phases of the incident wave cycle, Φ , for varying incident wave steepness are presented in Figure 5.17. The figure incorporate a few subplots, spaced $\Delta\Phi = 0.63$ rad apart, each providing data relating to the four wave cases (Table 5.3). In each case the surface elevation has been normalized by the maximum incident crest elevation. With the arrival of the incident wave crest (Figure 5.17(a)) the run-up on the upstream face ($\theta = \pi$ rad) shows a progressive increase with the incident wave steepness. In contrast, the elevations at the downstream side ($\theta = 0$ rad) are markedly different; the amplification of the surface elevation being approximately constant, albeit with

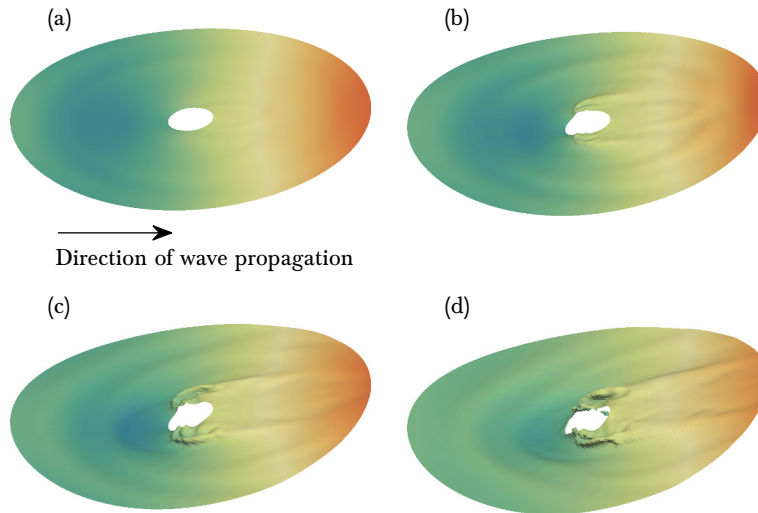


Figure 5.18: Images of the flow around the column for incident wave phase of $\Phi = 2.2$ rad and varying incident wave steepness, (a) $Hk/2 = 0.08$, (b) $Hk/2 = 0.16$, (c) $Hk/2 = 0.24$, and (d) $Hk/2 = 0.32$. The results have been truncated to show the surface elevation within $r = 5D$ from the column centerline.

a different shape. In Figure 5.17(b), the disturbed fluid begins to propagate around the column, interacting with positive wave-induced velocities associated with the incident wave. As a result, significant differences in the wave profile are observed for each of the four wave cases. Although, the linear case (A2) shows a steepening of the wave front, the interaction is not sufficiently strong to significantly distort the wave profile. However, as the incident wave steepness increases, the wave front becomes increasingly nonlinear; the wave interactions ensuring that the two steepest wave cases are on the verge of overturning. At the next incident wave phase, $\Phi = 1.26$ rad (Figure 5.17(c)), the nonlinear wave-wave interaction continues to develop; the motion of the wave fronts corresponding to the two steepest wave cases being delayed and well defined plunging breakers formed. As the wave front breaks, it is carried upstream by the negative wave induced velocity associated with the incident wave. This is part of the *Type-2* waves evolution. As this process continues, it is observed from Figure 5.17(d) that the surface elevation of the flow disturbance for the steepest case is much reduced as a consequence of the breaking process.

Figure 5.18 shows a set of images of the flow around the column for varying incident wave steepness. In all cases the phase of the incident wave cycle is defined by $\Phi = 2.2$ rad, corresponding to the arrival of the incident wave trough. With an increase in the incident wave steepness, a more defined and more vigorous wave breaking is observed. As the intensity of the wave breaking increases, the wave energy

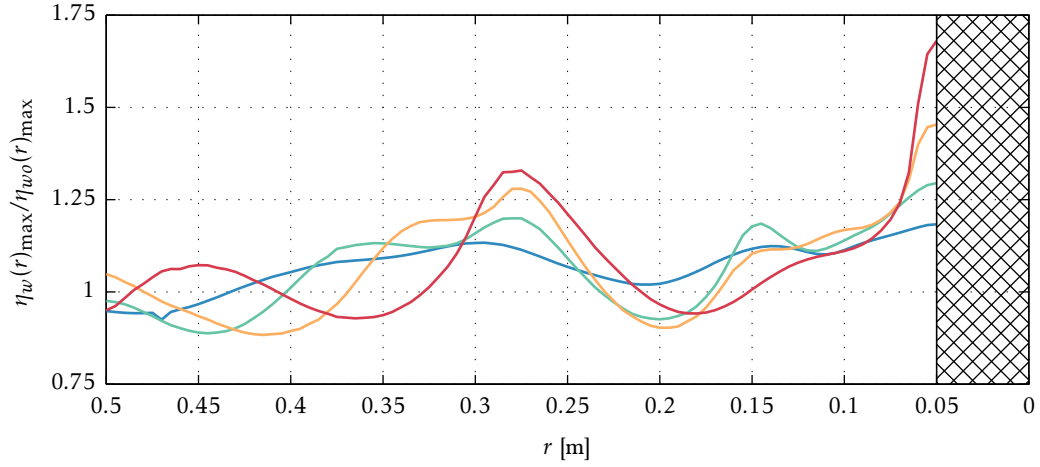


Figure 5.19: Spatial profile of normalized maximum surface elevation, $\eta(r)_{\max}$, in the upstream direction, $\theta = \pi$ rad. Legends: — $Hk/2 = 0.08$, — $Hk/2 = 0.16$, — $Hk/2 = 0.24$, and — $Hk/2 = 0.32$.

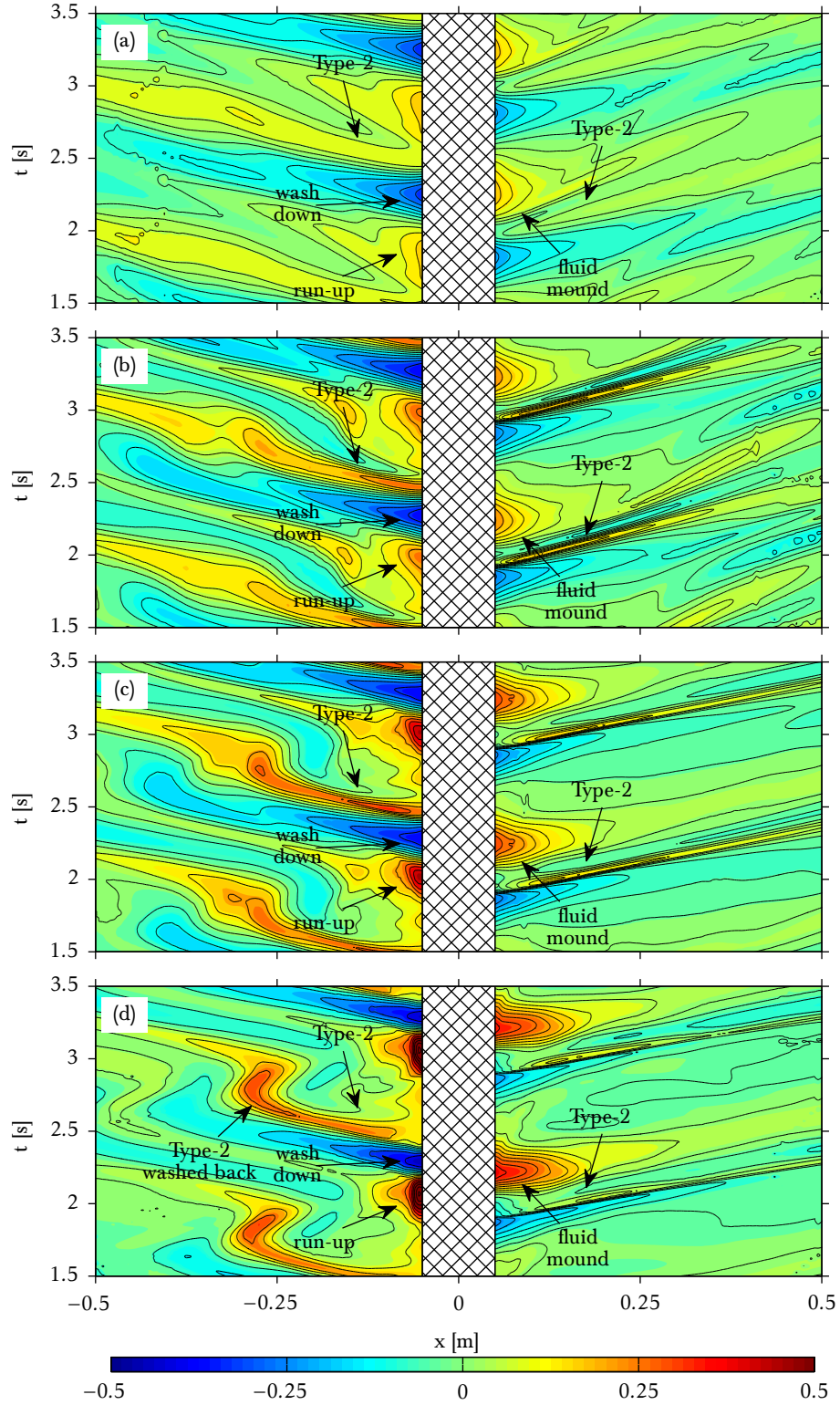
of the flow disturbance moving around the column will be reduced. This reduction translates into lower *Type-2* wave scattering from the upstream face of the column. This is consistent with the reduced scattered wave height observed in Figure 5.16(b). This result is important, not least because it shows the importance of wave breaking in limiting the growth of scattered waves for those cases of wave-structure interactions involving highly nonlinear incident waves.

Having considered the role of wave breaking in the wave scattering process, the nonlinear interactions between the incident and the scattered waves is further investigated. The maximum surface elevation upstream of the column, $\eta(r, t)$ for $\theta = \pi$ rad is considered in Figure 5.19. Within the figure, the maximum value of $\eta(t)$ was determined at each radial location, r . In each case, the surface elevation has been normalized by the maximum incident crest elevation to highlight the amplification of the maximum water surface elevation. It is clear from the figure that the largest amplification is associated with the wave run-up on the front face of the column with values as high as 70% for the steepest wave case. Moreover, an amplification of the scattered wave field also occurs further upstream, at around $r = 3D$. Interestingly, although it is shown that the *Type-2* wave amplitude for the steepest wave case is much reduced due to wave breaking, this case still produces the largest amplification. The explanation for this lies in the nonlinear wave-wave interactions. With a steep incident wave, the scattered waves are strongly modulated on the crest of the incident wave. This involves an increase in the local wave amplitude and a reduction in

the wavelength; the former producing an unexpectedly higher increase in the water surface elevation.

The extent of the nonlinear wave-wave interaction between the scattered waves and the next incident wave is further considered in Figure 5.20. This figure gives a related plot to that depicted in Figure 5.9(b) and 5.9(c). Essentially, it provides contour plots of the temporal and spatial evolution of the scattered wave field, $\Delta\eta$, for increasing incident wave steepness. However, in Figure 5.20, the plot has been rotated 90° counter-clockwise and extended to include both the upstream ($\Phi = \pi$ rad) and the downstream ($\Phi = 0$ rad) wave scattering. In this case, the incident waves, η_{wo} , now have a positive gradient $\partial t / \partial x > 0$; the radial distance, r , having been replaced by the horizontal distance, x . The hatched patch at the center of each plot represents the cross section of the column. In each case, the contours span over two wave periods, with $t = 1.0$ s, $t = 2.0$ s, and $t = 3.0$ s corresponding to the arrival of a wave at the column centerline, $x = 0$ m. Labels have been placed on the figure to indicate key points of the process of wave scattering.

With the arrival of an incident wave crest at $t = 2.0$ s, *Type-2* waves can be seen propagating away from the downstream side of the column. However, because the scattered wave is propagating in the same direction as the incident wave, the scattered wave is carried away from the column. In the upstream direction, the *Type-2* scattered wave is observed propagating away from the column approximately $T/2$ later at $t = 2.1$ s. In this case, the scattering occurs in the opposite direction to the incident waves producing a negative gradient of $\partial t / \partial x < 0$. As the wave-wave interaction takes place, the *Type-2* is essentially brought to rest at around $x = -0.27$ m. This effect becomes more pronounced with an increase in the incident wave steepness and is clearly apparent in Figure 5.20(b) and 5.20(c). However, in the steepest wave case (Figure 5.20(d)), the incident wave is so steep that the *Type-2* scattered wave is first brought to rest and then swept back in the incident wave direction. Evidence of this is provided by the fact that the wave propagation is reversed, given by $\partial t / \partial x > 0$. This observation has important practical implications suggesting that not only do the scattered wave produces a higher than expected increase in the maximum surface elevation, but with sufficiently steep incident waves, the scattered waves could be washed back, causing additional wave slamming on the column. This process may explain the higher than expected incidence of wave slamming that has been recorded



$$\Delta\eta/(\eta_{wo})_{\max}$$

Figure 5.20: Contour plots showing temporal and spatial evolution of the normalized scattered wave field, $\Delta\eta$, for (a) $Hk/2 = 0.08$, (b) $Hk/2 = 0.16$, (c) $Hk/2 = 0.24$, and (d) $Hk/2 = 0.32$. Note: the hatched area represent the column.

on some offshore installation, particularly those that are large but lie outside the linear diffraction regime.

5.7 Chapter Summary

This chapter has considered the numerical modelling of wave scattering produced by the interaction of regular waves and a vertical, surface-piercing, column. The present chapter has:

1. Demonstrated the ability of the current numerical method to predict the *Type-1* and *Type-2* scattered waves; the predicted behaviour shown to be in very good agreement with earlier laboratory observations.
2. Confirmed that the *Type-1* waves are generated by wave run-up and the subsequent wash-down, most notably on the front face of the column. In contrast, the *Type-2* waves are associated with the propagation of flow disturbance around the column.
3. Highlighted that the phasing of the *Type-2* waves is directly related to the time taken for the fluid to move around the column. This is, in large part, dependent upon the column diameter rather than the frequency of the incident waves.
4. Showed that the *Type-1* and *Type-2* waves scattered in the upstream and downstream direction are different due to the crest-trough asymmetry of the incident wave.
5. Demonstrated the suppression of *Type-2* waves in the upstream direction with increasing column diameter and showed that this relates to the increase in the time taken for the fluid disturbance to move around the column.
6. Established the importance of the steepness of the incident waves. With increasing steepness, the limiting factor for the amplitude of the scattered waves arises due to wave breaking around the column.
7. Identified the potential for a substantial nonlinear increase in the maximum water surface elevation in the immediate vicinity of the column. This arises

from the nonlinear wave-wave interaction between the high-frequency scattered waves and the next incident waves. With a large local increase in the wave steepness, this may also account for the increased occurrence of wave slamming on the column face.

6

Wave Scattering in Focused Waves

6.1 Introduction

The results presented in the previous chapter concern the interactions between regular waves and a surface-piercing column, highlighting the capability of the numerical method to accurately describe the high-frequency wave scattering. Moreover, the numerical calculations clarified aspects of the wave scattering that were not clear from earlier experimental observations. The high-frequency scattered waves appear as both concentric (*Type-1*) and non-concentric (*Type-2*) wave modes; the latter being the more important. The interactions between these scattered waves and the next incident wave provide a substantial increase in the water surface elevation in the immediate vicinity of the column. Despite the practical importance of this observation, especially in the context of engineering design, the results are limited in the sense that the adopted incident waves were regular. As such, they are characterised by a single, freely propagating, wave component and propagate with a fixed wave form. In contrast, a realistic sea-state is irregular and unsteady, having multiple freely propagating wave components of varying amplitude and frequency that propagate in different directions. Although several experimental studies have reported the presence of high-frequency wave scattering due to the interactions between irregular waves and a single surface piercing column (Sheikh, 2004; Stansberg & Kristiansen,

2005), a detailed description of the scattered wave field has not been provided.

The aim of the work presented in this chapter is to demonstrate the presence of wave scattering in focused irregular waves, providing a detailed description of the scattered wave field. This work also seeks to highlight the observation of a new wave scattering mechanism and assess the sensitivity of this scattered wave to parameters describing the incident wave. Finally, the increase in the maximum water surface elevation is examined and its practical implication discussed.

The layout of the chapter is described as follows. Details of the numerical procedure are outlined in Section 6.2. Section 6.3 presents and discusses the nature of the wave-structure interactions arising in incident focused waves. Finally, Section 6.4 concerns the amplification of the maximum water surface elevation, immediately upstream of the column due to incident-scattered wave interactions.

6.2 Numerical Procedure

6.2.1 Computational Domain and Boundary Conditions

In undertaking the present calculations, the computational domain was similar to that used in the previous chapter. This concerns domains both with and without the column in place; the scattered wave field being evaluated by taking the difference between the two. A column of diameter $D = 0.1$ m, located at $(x, y) = (0, 0)$, was used throughout these calculations. The computational domain was defined by a height and length of $L_z = 1.3d$ and $L_x \approx 8\lambda_p$ respectively, where d is the water depth and λ_p is the peak wave-length. The input boundary was located approximately $3.5\lambda_p$ from the column centerline. To reduce the computational effort half of the physical domain, with width of $L_y = 10D$, was modelled. This takes full advantage of the plane of symmetry described in Chapter 5. The applied boundary conditions and mesh discretization were exactly as described in Section 5.3.1. A relaxation zone with length, $L_o = \lambda_p$ was placed at the outlet boundary to reduce wave reflections. No relaxation zone was used at the inlet boundary, the reason been given in Chapter 4, Section 4.4.2. In all cases, adaptive time stepping was used with a Courant number of $C_o = 0.3$ maintained throughout.

Table 6.1: Test cases for focused waves interacting with a vertical column.

Test case	γ	T_p [s]	Ak_p	x_f [m]	D/λ_p	Note
11	2.5	1.0	0.12	0.0	0.064	Crest focused
12	2.5	1.0	0.16	0.0	0.064	Crest focused
13	2.5	1.0	0.18	0.0	0.064	Crest focused
14	2.5	1.0	0.20	0.0	0.064	Crest focused
15	2.5	1.0	0.24	0.0	0.064	Crest focused
11t	2.5	1.0	0.12	0.0	0.064	Trough focused
12t	2.5	1.0	0.16	0.0	0.064	Trough focused
13t	2.5	1.0	0.18	0.0	0.064	Trough focused
14t	2.5	1.0	0.20	0.0	0.064	Trough focused
15t	2.5	1.0	0.24	0.0	0.064	Trough focused
16	2.5	1.2	0.12	0.0	0.045	Crest focused
17	2.5	1.2	0.18	0.0	0.045	Crest focused
18	4.0	1.0	0.18	0.0	0.064	Crest focused
19	8.0	1.0	0.18	0.0	0.064	Crest focused
20	2.5	1.0	0.18	-0.30	0.064	Crest focused
21	2.5	1.0	0.18	-0.15	0.064	Crest focused
22	2.5	1.0	0.18	0.15	0.064	Crest focused
23	2.5	1.0	0.18	0.30	0.064	Crest focused

6.2.2 Wave Conditions

Within the present study, the incident waves were described by unidirectional focused waves, with the underlying wave components defined by a realistic JONSWAP spectrum combined with the theory of Lindgren (1970); the latter defining the most probable shape of a large linear wave (discussed in Section 4.4.1). The wave spectrum was discretized within a frequency range defined by $(1/3)f_p \leq f \leq 3f_p$, where f_p is the spectral peak frequency. A water depth of $d = 1.0$ m was maintained throughout, ensuring that the wave is effectively propagating in deep water. Details of the wave parameters used in this study are given in Table 6.1.

In most of the wave cases considered, the linear focused position was defined at the centre of the column, ($x_f = 0$ m). However, with an increase in the steepness of the incident wave, the focused position shifted downstream due to nonlinear wave-wave interactions. In these cases, iterative calculations were undertaken to ensure that the focused wave event occurs at the desired location, on the centreline of the

column. In all cases, the difference between the focused location and the centreline of the column was, at most, ± 0.01 m. For a peak period of $T_p = 1.0$ s, this corresponds to a spatial shift of less than 1 % of the peak wavelength, λ_p .

6.2.3 Simulation Methodology

In seeking to investigate the wave scattering produced by the interactions of focused waves and a vertical column, an accurate description of the incident wave is critically important. The capability of the numerical method to accurately describe the relevant focused wave events has been examined in Chapter 4. In the early stages of this study, the focused waves in the presence of the column were simulated from a quiescent condition; the total length of the simulations being approximately $30T_p$. This proved to be both computationally expensive and prohibitive in terms of the number of cases that could be run. Unlike the regular waves cases, for which higher-order analytical solutions exist and were used to initialize the simulation, the only higher-order analytical solution available for irregular waves is SORWT. Unfortunately, SORWT is inadequate when seeking to describe steep incident waves and, hence, cannot be used to initialize the simulation. Taking this limitation into consideration, the following methodology was adopted:

- (i) The unidirectional focused waves were simulated without the column in place for a time range of $-20T_p \leq t \leq 10T_p$. In these cases, only one cell was used in the transverse direction (y) to reduce the computational effort.
- (ii) Any shifting of the focused position from (i), due to nonlinear wave-wave interactions, was corrected by applying a phase difference

$$\Delta\Phi_n = k_n(\Delta x) - \omega_n(\Delta t), \quad (6.1)$$

where Δx , is the spatial shift in focused position, Δt , is the temporal shift in focused time, and k_n and ω_n are the wavenumber and angular frequency of the individual wave components respectively. Stages (i) and (ii) were then iterated until the desired focused wave event was achieved at the correct location and time.

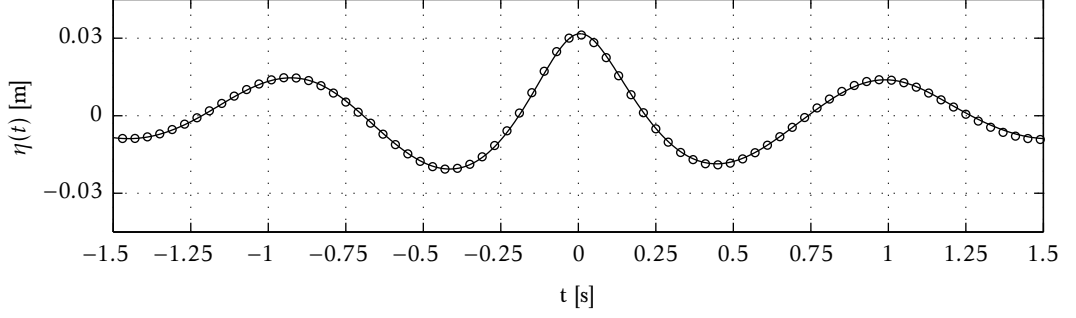


Figure 6.1: Time-histories of the incident surface elevation, η_{w0} ($\circ$), and water surface elevation, η_w (—), recorded at $r = 0.99\text{ m}$ and $\theta = \pi/2$ for test case 11 on Table 6.1.

- (iii) The solution arising from (ii), following the iterative procedure, was then used as the initial conditions for the numerical simulation with the column in place. In all cases the initial conditions were taken at approximately $t = -3T_p$ before the focused event.
- (iv) The simulation from (iii), with the column in place, was carried out for a total duration of approximately $6T_p$. $3T_p$ before and after the focused time.

In mapping the solution from (ii) in the presence of the column, (iii), it is important to note that the incident surface elevation, η_{w0} , around the column for $t < -3T_p$ is small. As such, there is little disturbance to the flow field. Using the method outlined above, a significant reduction in computational effort was achieved. On average, the simulations of the incident wave took around 3 hours to complete. With the column in place, the total number of cells was significantly increased. As a result, the numerical computation took around 70 hours to complete on the Imperial College HPC facility with 32 processing cores. In adopting this methodology, the reduction in focused wave duration had the added advantage of reducing wave reflections from the outlet boundary.

With the column in place, the mesh configuration (Figure 5.3) differs from the mesh used without the column. With the incident wave components having very different properties, there were concerns over the effect of the variation in the mesh density when modelling the incident waves. To ensure that near identical incident waves were generated in the different mesh configurations, comparison of the water surface elevations, η_{w0} and η_w , the latter generated on the side wall, inline with the column ($\Phi = \pi/2\text{ rad}$ and $r = 0.99\text{ m}$), was carried out. This provides a direct quan-

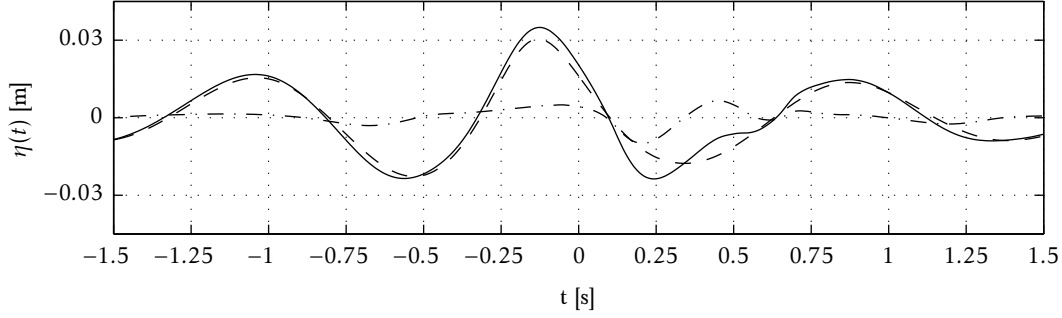


Figure 6.2: Time-histories of the water surface elevation, $\eta(t)$, recorded at $r = 0.1$ m and $\theta = \pi$ for test case 11 on Table 6.1. Legends: --- η_{w0} , — η_w , and $\Delta\eta$.

titative comparison of the generated incident wave for both domain configurations. It also serves the purpose of evaluating the extent of the transverse wave scattering and the outlet wave reflections. An example of this comparisons is provided in Figure 6.1 and shows that the recorded surface elevations are effectively identical. This confirms that any problems due to wave reflections, mesh size variation and the mapping of data from one mesh to another are minimal.

6.3 Results and Discussions

This section presents numerical data describing the interaction between unidirectional focused waves and a surface-piercing column. Evidence of wave scattering from a weakly nonlinear focused wave is first presented. This is followed by several nonlinear wave cases. With an increase in the incident wave steepness, a new wave scattering mechanism was observed. The origin of this new wave scattering mode and its sensitivity to the incident wave parameters are presented and discussed.

6.3.1 Evidence of Wave Scattering

Quantitative evidence of high-frequency wave scattering arising from the interaction of a focused wave event and a vertical surface-piercing column is provided in Figure 6.2. This figure concerns data recorded in test case 11 (Table 6.1) and contrasts the total wave elevation, $\eta_w(t)$, the incident wave elevation, $\eta_{w0}(t)$, and the scattered wave elevation, $\Delta\eta(t) = \eta_w(t) - \eta_{w0}(t)$, recorded one diameter upstream of the column. Based on the peak wave period, T_p , the effective column diameter is given by $D/\lambda_p \approx 0.064$. This is well outside the linear diffraction regime

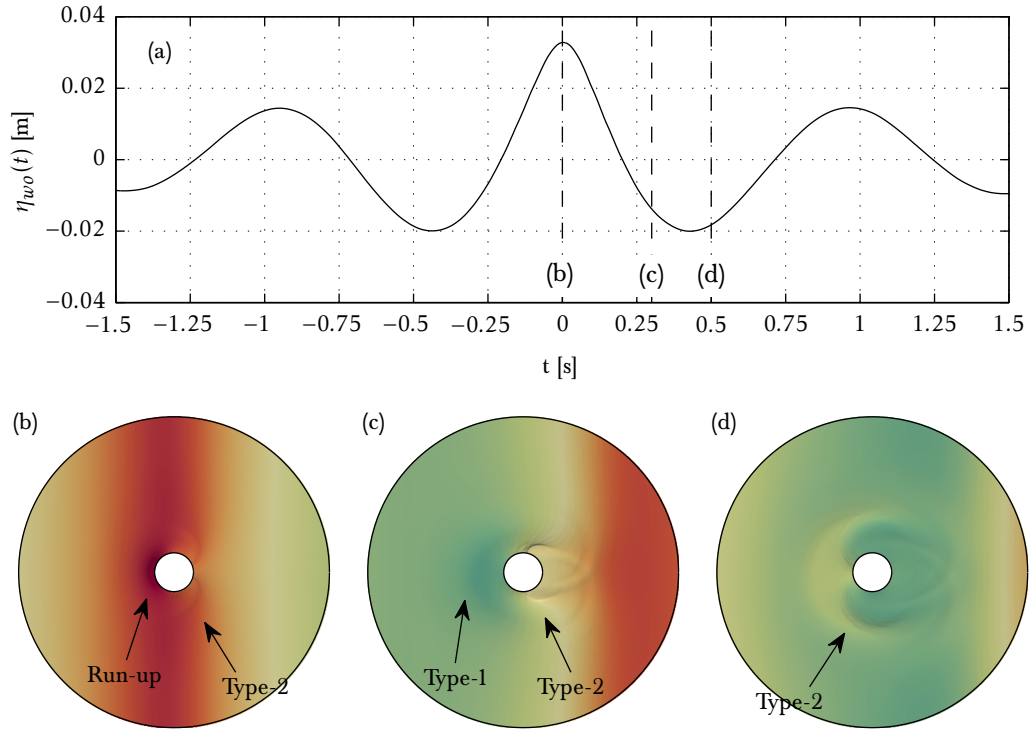


Figure 6.3: Evidence of high-frequency wave scattering in a focused wave event. (a) Time history of the water surface elevation without the column in place, $\eta_{w0}(t)$ and (b), (c) and (d) plan views of the surface elevation, $\eta_w(x, y)$, recorded in test case 11 at three different phases of the incident wave cycle (b), (c) and (d) marked on (a).

($D/\lambda_p > 0.2$) and, consequently, little or no wave disturbance was expected. Indeed, prior to the focused event ($t < 0$ s), there is little evidence of high-frequency wave scattering. However, with the arrival of the maximum incident crest, pronounced high-frequency wave scattering can be observed. Despite the fact that the incident wave is only weakly nonlinear ($Ak_p = 0.12$), there is clear evidence of high-frequency wave scattering in the trough of the incident wave (at around $t = 0.4$ s); the amplitude of the scattered wave being approximately 20% of the incident crest elevation, $\Delta\eta(t)/\eta_{w0}(t)_{\max} \approx 0.2$.

Having established the existence of high-frequency wave scattering for a weakly nonlinear focused wave, Figure 6.3 provides a sequence of still images describing a plan view of the scattered wave field, $\eta_w(x, y)$, for test case 11. Figure 6.3(a) presents the incident wave elevation, $\eta_{w0}(t)$, recorded at the column centerline with dashed lines overlaid at key phases of the incident wave cycle. Surface contour plots corresponding to these phases are given by Figure 6.3(b), (b) and (d). These show the details of the scattered wave fields. At each phase of the incident wave cycle, there is

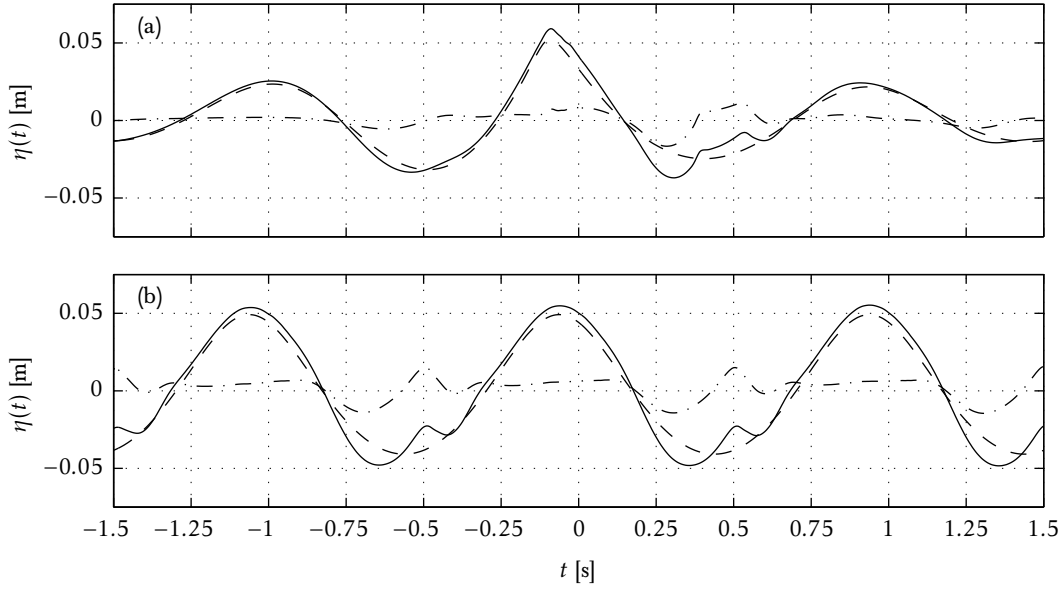


Figure 6.4: Time-histories of the water surface elevation, $\eta(t)$, recorded at $r = 100\text{mm}$ and $\theta = \pi$ allowing comparisons between (a) test case 13 (b) test case 13R. Legends: --- η_{w0} , — η_w , and -.- $\Delta\eta$.

evidence of both *Type-1* and *Type-2* wave scattering; the *Type-2* waves scattered alternately in the downstream and upstream directions. This is consistent with the results observed in the regular wave cases documented in Chapter 5 and proves that similar wave scattering occurs in a focused wave event.

6.3.2 Secondary Type-1 Wave Scattering

The data presented in Figures 6.2 and 6.3 highlight the generation of high frequency *Type-1* and *Type-2* wave scattering in focused waves. However, this case corresponds to a weakly nonlinear incident wave. In the present section, a fully nonlinear focused wave is considered. This case is defined by test case 13 in Table 6.1, having a steepness of $Ak_p = 0.18$.

The total wave elevation, $\eta_w(t)$, the incident wave elevation, $\eta_{w0}(t)$, and the scattered wave elevation, $\Delta\eta(t) = \eta_w(t) - \eta_{w0}(t)$, recorded one diameter upstream of the column in test case 13 is given in Figure 6.4(a). For comparison, a regular wave case having a wave period of $T = 1.0\text{s}$ and equivalent wave steepness, $Hk/2 = 0.18$, is given in Figure 6.4(b). To facilitate this comparison, the regular wave case has been labelled as test case 13R. With nonlinear incident waves, evidence of the high-frequency wave scattering is clearly observed at the trough of the incident wave immediately

following the arrival of the maximum crest elevation. In the regular wave case, 13R on Figure 6.4(b), the high-frequency wave scattering is observed in every wave cycle, consistent with periodic nature of a regular wave train. In the focused wave case on Figure 6.4(a), pronounced high-frequency wave scattering is only observed in the trough following the focused wave event, $0.25 \text{ s} \leq t \leq 0.75 \text{ s}$. During the focused wave event, the central crest elevation is a maximum, producing the steepest wave condition, and it is the event that produces the high-frequency scattering. The fact that significant scattering only occurs once, highlights the importance of incident wave steepness in the generation of high-frequency waves.

Although high-frequency waves were observed in both test case 13 and 13R, the high-frequency waves differs in some important respects. In test case 13R the high-frequency waves have a single peak at each incident wave cycle. In contrast, test case 13, exhibits two distinct scattered wave peaks immediately following the focused crest. These peaks occur roughly between $0.40 \text{ s} \leq t \leq 0.55 \text{ s}$ and are separated approximately $\Delta t \approx 0.15 \text{ s}$ apart. At this point, it is not clear what physical mechanism is responsible for the generation of this double-peak high-frequency wave.

Still images, or snapshots, of the wave field, $\eta_w(x, y)$, predicted in test case 13 are given in Figure 6.5. Eight subplots are provided corresponding to different phases of the focused event; Figure 6.5(a) corresponding to the arrival of the focused event at the center of the column. The subsequent images are given at a constant time interval of $\Delta t = 0.1 \text{ s}$. With the arrival of the maximum wave crest (Figure 6.5(a)), significant run-up is present on the upstream column face. This is followed by a subsequent wash-down (Figure 6.5(b)). The run-up and subsequent wash-down forces the generation of the *Type-1* wave as shown in Figure 6.5(c). At the next incident wave phase (Figure 6.5(d)), *Type-2* waves can be observed on either side of the column. At the same instant, a pronounced concentric disturbance can be observed immediately upstream of the column. This disturbance has a similar concentric pattern to a *Type-1* wave, but has a steeper wave front and occurs after the propagation of the *Type-1* wave. This disturbance has been labelled as a *Type-1** wave and is subsequently referred to as the *secondary Type-1* because of its close resemblance. With the arrival of the wave trough (Figure 6.5(e)), the *Type-1** wave is carried away from the upstream face of the column. As the pair of *Type-2* waves reach the upstream side of the column, a distinct pattern, formed by the intersection of *Type-1** and *Type-2* waves is

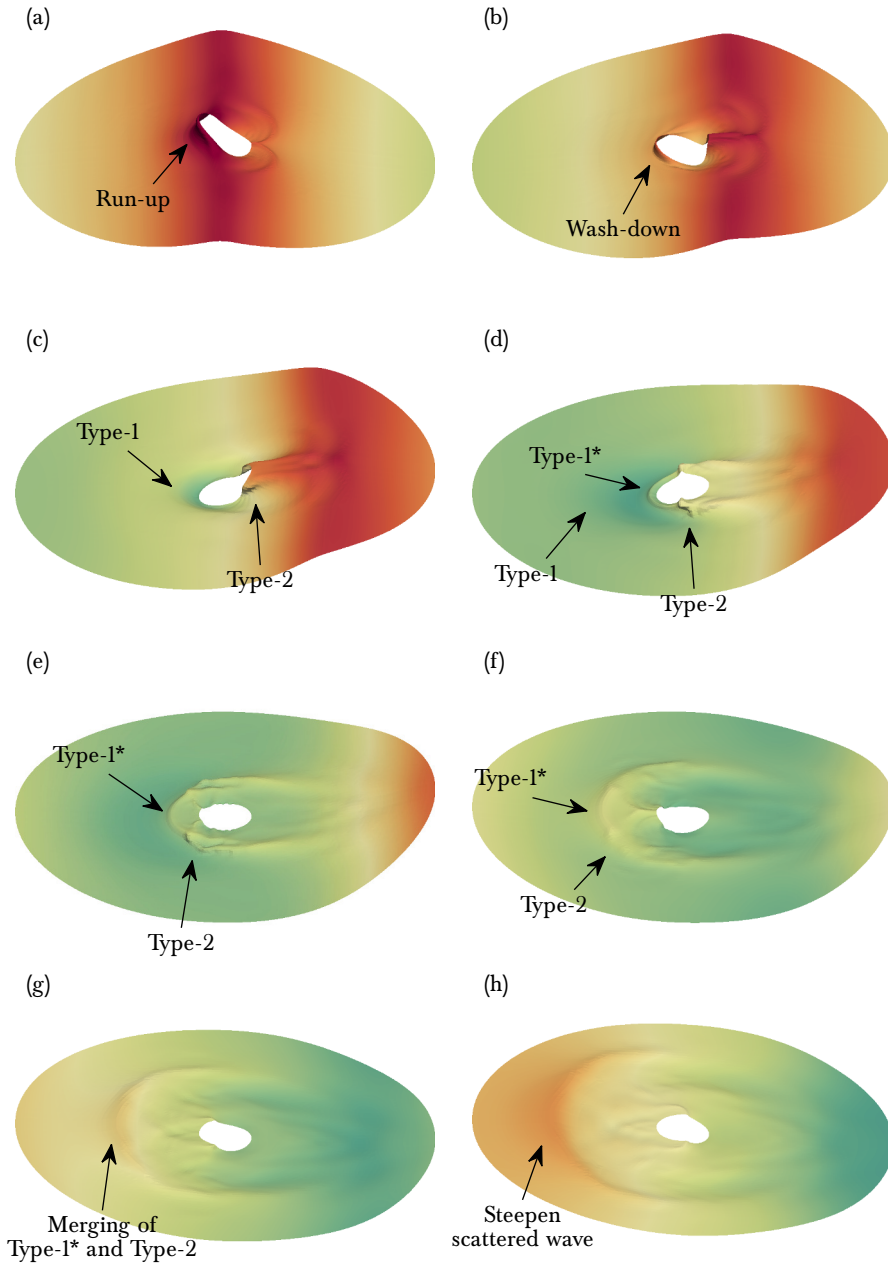


Figure 6.5: Still images (or snapshots) of the numerically predicted wave field showing the evolution of scattered waves, $\eta_w(x, y)$ in test case 13. The time interval between successive images is $\Delta t = 0.1$ s with (a) corresponding to the arrival of the to focused wave event at the column centerline.

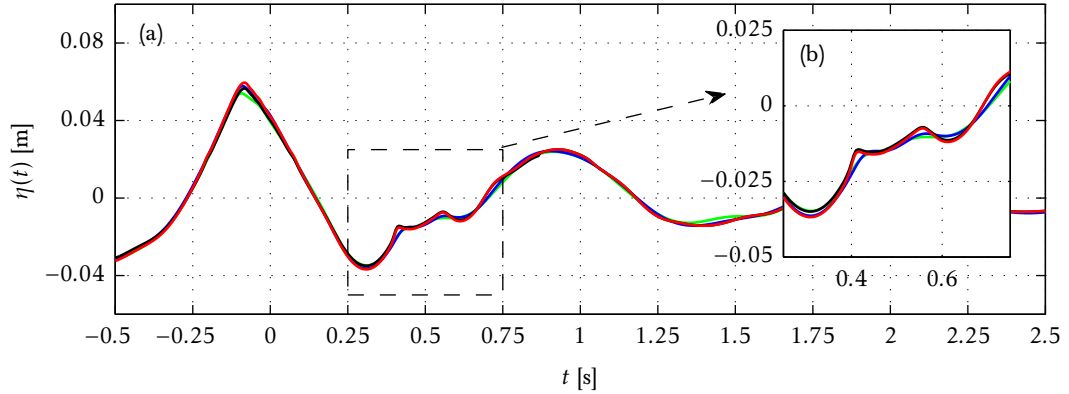


Figure 6.6: (a) Additional convergence tests showing time-histories of the water surface elevation, $\eta_w(t)$, relating to test case 13 recorded at $r = 0.1$ m and $\theta = \pi$ rad with comparison between different cell densities. (b) A close-up of sub-plot (a) in the region of *Type-1** scattering. — 1.5×10^6 cells, — 3.2×10^6 cells, — 5.2×10^6 cells (test case 13), — 9.8×10^6 cells.

evident (Figure 6.5(f)). The propagation of these interacting wave fronts explains the observed double peak in the high-frequency scattered wave in (Figure 6.4(a)). As the wave evolution continues, the *Type-2* and *Type-1** wave fronts can be seen merging on Figure 6.5(f) - 6.5(g). With the arrival of the next incident wave crest, the wave-wave interactions force the steepening of the combined scattered wave (Figure 6.5(h)), producing a large local increase in the crest elevation.

Before considering further the occurrence and properties of this secondary *Type-1* (*Type-1**) wave scattering, it is important to note that the waves have a substantially shorter wave length. This raises three additional concerns regarding

- (a) the convergence of the numerical scheme in term of these very short wave components,
- (b) the potential effects of surface tension, and
- (c) the possible effects of viscosity.

Numerical Convergence

In order to assess the sensitivity of the numerical results, in particular the formation of *Type-1** scattered waves, to the change in the cell density, three additional test cases were considered. These cases relate to test case 13, but with different cell densities; the total number of cells varying from 1.5×10^6 to 9.8×10^6 cells. Comparisons between surface elevation data predicted one diameter upstream of the column face

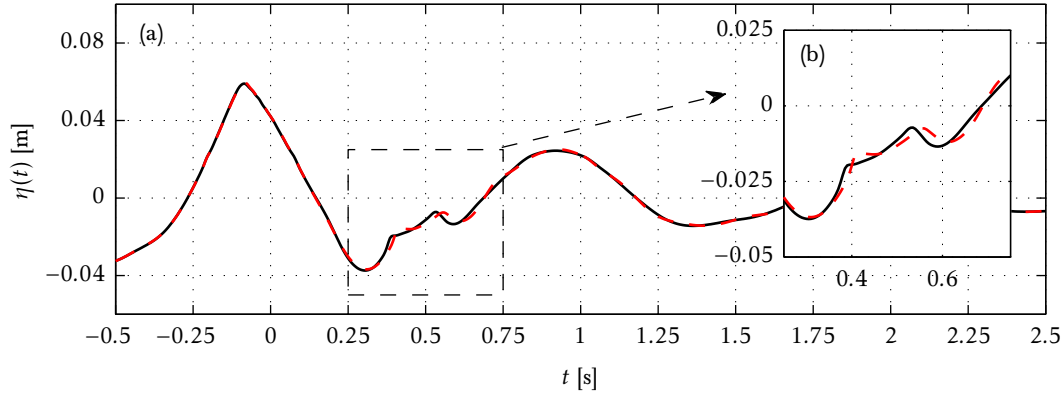


Figure 6.7: The role of surface tension. (a) Time-histories of the water surface elevation, $\eta_w(t)$, for test case 13 recorded at $r = 0.1$ m and $\theta = \pi$ rad with comparison between numerical results with (- - - -) and without (—) the inclusion of surface tension. (b) A close-up of sub-plot (a) in the region of *Type-1** scattering.

are presented in Figure 6.6. It is clear from this figure that with an increase in the cell density, the results become increasingly convergent. Specifically, the close-up of the *Type-1** wave scattering in sub-plot (b) shows agreement between the original test case 13 data (having 5.2×10^6 total number of cells) and the case with the highest cell density. Surprisingly, the test cases with the lower cell densities also show the evidence of *Type-1** wave, although with a slight under-prediction of the scattered wave amplitude and phasing. This data confirms the numerical convergence appropriate to the formation of the *Type-1** scattered waves.

Effect of Surface Tension

In considering the role of surface tension in relation to the *Type-1** scattered waves it is important to note that the surface tension (in Equation 3.2) was deliberately omitted by prescribing the surface tension constant, σ_T , as zero. To investigate the effect of surface tension, an identical simulation of test case 13 was carried out incorporating the non-zero surface tension constant. This was defined by $\sigma_T = 0.07 \text{ N m}^{-1}$ which represents a realistic surface tension value between water and air at 20°C . The results of this simulation are presented in Figure 6.7. This concerns the time-history of the surface elevation located one diameter upstream of the column face and compares data generated with and without the surface tension constant; all the data relating to test case 13. Once again, sub-plot (b) shows a close-up of the *Type-1** wave scattering. It is evident from this figure that although there is a small

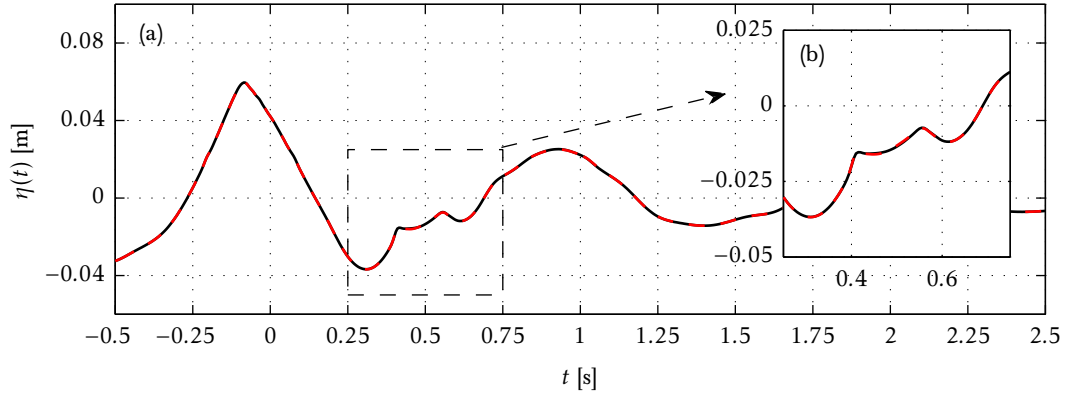


Figure 6.8: (a) Time-histories of the water surface elevation, $\eta_w(t)$, for test case 13 recorded at $r = 0.1$ m and $\theta = \pi$ rad with comparison between numerical results with different viscosity for the water phase. (b) A close-up of sub-plot (a) in the region of *Type-1** scattering. $\cdots$ $\mu_w = 1 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ and — $\mu_w = 1 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$.

difference between the two predictions, the fact that records are able to reproduce the double peak scattered waves means that surface tension is not the cause of the *Type-1** waves scattering.

Influence of Fluid Viscosity

The effect of viscosity is addressed by applying different fluid viscosities for the water phase. These are given by $\mu_w = 1 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ and $\mu_w = 1 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$ respectively, with both cases again applied to test case 13. By considering the surface elevation one diameter from the column upstream face, Figure 6.8 confirms that the results are identical, despite the fact that one of the cases has a fluid viscosity that is two order of magnitude larger. This evidence suggests that the *Type-1** wave scattering is not affected by changes in the fluid viscosity.

Having considered the numerical convergence and showed that neither surface tension nor fluid viscosity have significant influence on the formation of the *Type-1** scattered waves, Figure 6.9 further considers the occurrence and properties of these additional scattered waves but contrasting the data generated in test cases 13 and 13R. Within this figure, top row gives a plan view of the total wave fields, $\eta_w(x, y)$, while the bottom row is a similar view of the scattered wave fields, $\Delta\eta(x, y)$. Data relating to test case 13 is given in the left column, while the corresponding data for test case 13R is given on the right. In both cases, the plots are taken approximately $\Delta t = 0.33$ s

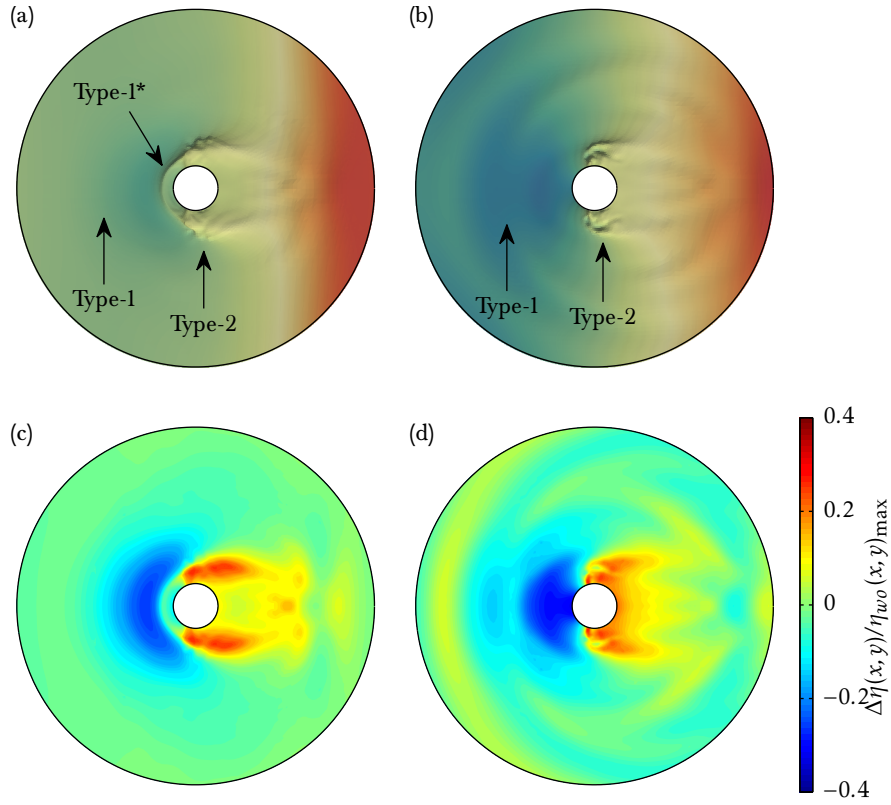


Figure 6.9: (a) & (b) Plan view of the wave fields, $\eta_w(x, y)$, and (c) & (d) contour of the scattered wave field, $\Delta\eta(x, y)$ for test case 13 (left column) and test case 13R (right column) at $\Delta t = 0.33$ s after passage of the maximum wave crest.

after the passage of the maximum crest elevation. By comparing the two cases, the presence of both a *Type-1* and the *Type-2* waves is again clearly evident. However, the *Type-1* wave is less pronounced in test case 13 (Figure 6.9(a) and (c)). Nevertheless, the characteristic concentric depression is observed upstream of the column (in both cases) suggesting that the concentric pattern associated with *Type-1* wave is indeed present. The striking difference between the two cases presented in Figure 6.9 lies in the occurrence of *Type-1** wave given in Figure 6.9(a). Indeed, the fact that *Type-1** wave has a similar concentric pattern and occurs immediately after the generation of the *Type-1* wave suggests that both these scattered waves might be generated by the same physical mechanism or at least are related to the same physical mechanism.

To further investigate the origin of the *Type-1** wave, the surface elevation, $\eta_w(t)$ recorded on the upstream face of the column ($r = D/2$ and $\theta = \pi$) is presented in Figure 6.10. This figure provides a comparison between the data recorded in test case 13 (focused) and test case 13R (regular). With the arrival of the maximum wave crest

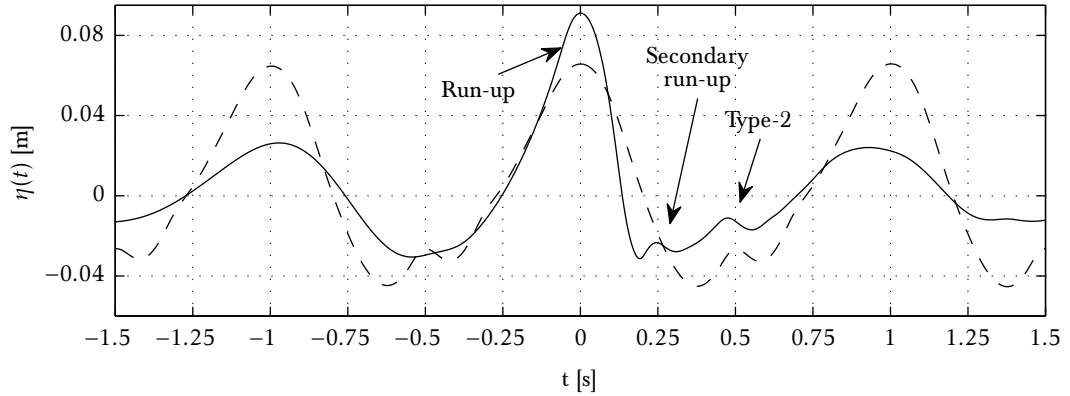


Figure 6.10: A comparisons of the time-histories of the water surface elevation, $\eta_w(t)$, recorded at $r = D/2$ and $\theta = \pi$, in test cases 13 (—) and 13R (- - -).

($t = 0$ s), a large increase in the surface elevation due to run-up is observed in both cases. For the incident focused wave, the run-up is projected to a higher elevation than the corresponding regular wave case. This can be explained by the fact that the focused wave case has a much steeper crest, with larger fluid velocities high in the wave crest. This gives a higher local energy density, allowing a higher projection of the fluid. With the passing of the maximum wave crest, the subsequent wash-down occurs for $t > 0$ s. In test case 13R there is an almost symmetric run-up and wash-down profile about $t = 0$ s. This means that the vertical velocities, $d\eta/dt$ associated with the run-up and wash down are almost identical; the only difference being the change in direction. However, the surface elevation associated with the run-up and wash down for test case 13 are markedly different; the velocity associated with the wash-down being much larger than the corresponding run-up. Evidence of this effect is given by the (steep) gradient of the water surface elevation, $d\eta/dt$.

In test case 13R, the end of the wash-down is followed by a peak in the trough. This occurs at approximately $t = 0.5$ s and is associated with the arrival of the *Type-2* waves at the upstream face of the column. A similar peak, again arising at $t = 0.5$ s, also occurs in test case 13. Once again, this corresponds to the arrival of *Type-2* waves; evidence to support this view being given in Figure 6.5(f). However, in this case there is an additional peak in the trough of the water surface elevation at $t \approx 0.25$ s. This has been labelled on Figure 6.10 as *secondary run-up*. Comparisons between observations of the occurrence of the *Type-1** wave scattering (Figure 6.5(d)-(f)) and the timing of *secondary run-up* suggests a link between these two physical mechanisms.

The spatial evolution of water surface elevations, $\eta(r)$, immediately upstream of

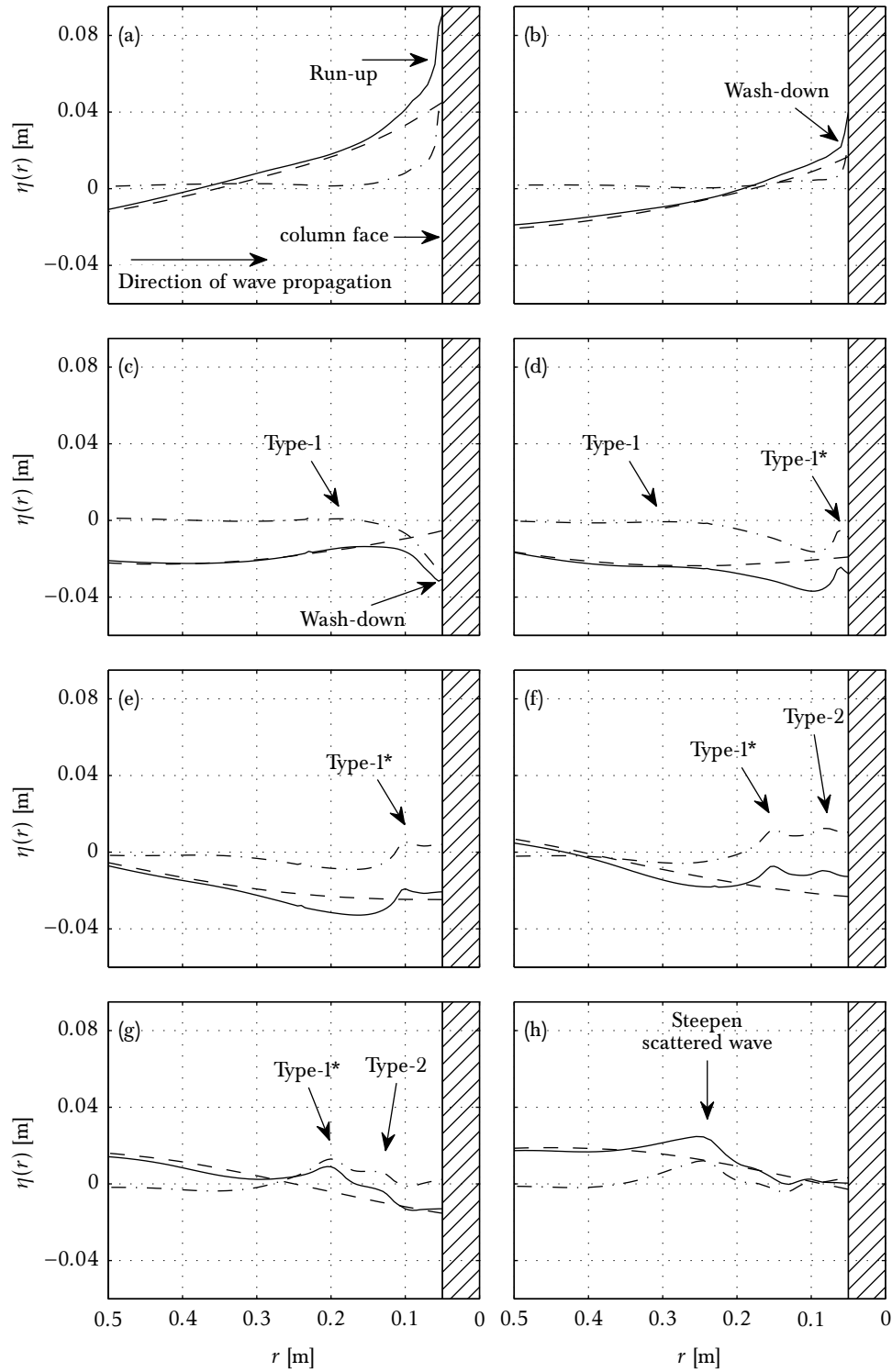


Figure 6.11: Spatial profiles of water surface elevations, $\eta(r)$, identifying the upstream scattering of *Type-1*, *Type-1** and *Type-2* waves in test case 13. Successive plots with the interval between each plot set at $\Delta t = 0.1$ s, with (a) corresponding to the arrival of the incident focused crest at the centerline of the column. Legends: --- η_{w0} , — η_w , and -.- $\Delta\eta$.

the column on $\theta = \pi$, is considered in Figure 6.11. This data related to test case 13; the time interval between successive sub-plots is $\Delta t = 0.1$ s and the arrangement of the sub-plots is similar to that given in Figure 6.5. In Figure 6.11, each subplot contrasts the total wave elevation, $\eta_w(r)$, the incident wave elevation, $\eta_{wo}(r)$, and the scattered wave elevation, $\Delta\eta(r) = \eta_w(r) - \eta_{wo}(r)$. The link between the *Type-1** wave scattering and the *secondary run-up* can be explained using data presented on Figure 6.11(b) - 6.11(e). After the passage of maximum crest elevation, the fluid washes-down the upstream face of the column (Figure 6.11(b)). Because the run-down velocities associated with the focused incident wave are significantly larger than might be anticipated, the fluid is driven down well below the incident wave profile (Figure 6.11(c)). This is immediately followed by a ‘bounce-back’ in which the volume of fluid on the column face undergoes a significant increase in elevation. Evidence of this is given in Figure 6.11(d). This forms what was previously termed the *secondary run-up*. As this *secondary run-up* occurs in the vicinity of the wave trough, it is then carried away from the column by the negative wave induced velocity to create a *Type-1** scattered wave (Figure 6.11(d) and (e)). With the arrival of incident wave trough, the *Type-1** scattered wave continues to propagate outwards, opposing the direction of the incident wave (Figure 6.11(e)). A couple of time-steps later, the *Type-2* wave is seen trailing the *Type-1** wave and continues to propagate in the same direction as shown in Figure 6.11(f). Eventually the *Type-2* wave catches up with *Type-1** wave (Figure 6.11(g) and (h)) and as these waves interact with the next incident wave crest, the wave-wave interaction steepens the wave profile as shown in the last plot (Figure 6.11(h)). Taken as a whole, it is clear from the results presented in Figure 6.10 and Figure 6.11 that the observed *Type-1** wave scattering is strongly dependent on the *secondary run-up* occurring on the upstream face of the column.

Although the *Type-1** wave scattering is present in test case 13, it is not yet clear whether the same mechanism will be observed in other focused wave cases. To investigate this, two additional focused wave cases were considered. These are defined as test cases 16 and 17 in Table 6.1. Both cases have spectral peak periods and peak enhancement factors of $T_p = 1.2$ s and $\gamma = 2.5$ respectively. The effective column diameter for both cases is $D/\lambda \approx 0.045$. For consistency, test case 16 and 17 have a wave steepness of $Ak_p = 0.12$ and $Ak_p = 0.18$ respectively. Figure 6.12(a) concerns the water surface elevations, $\eta(t)$, measured on the upstream face of the column

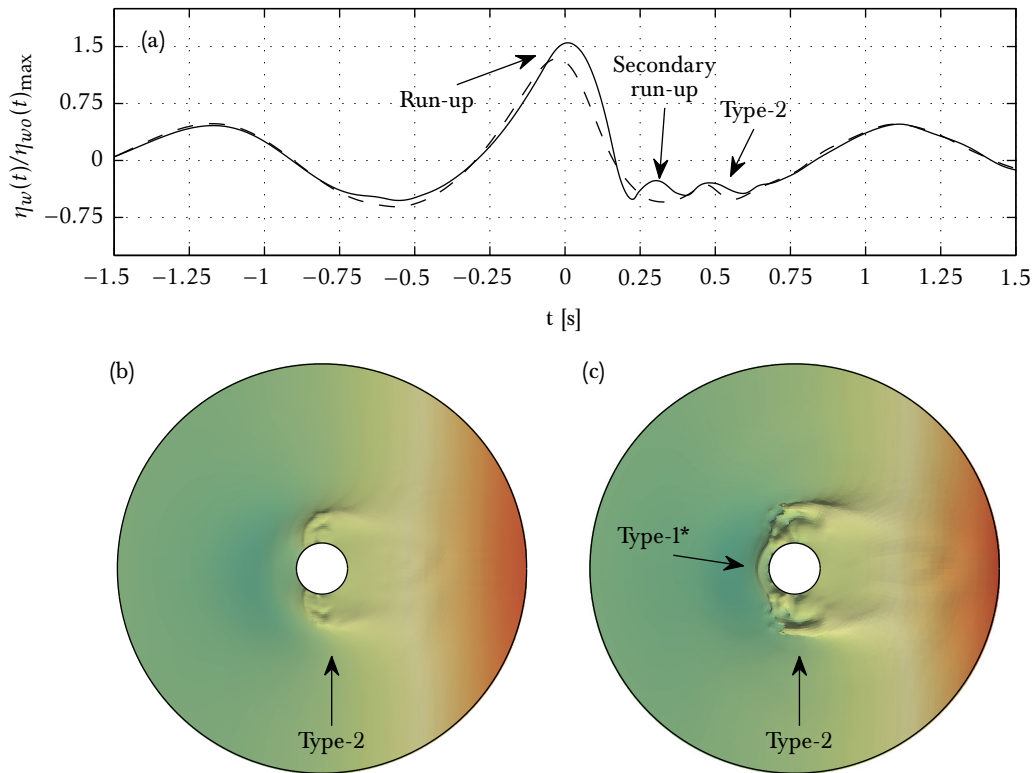


Figure 6.12: Wave run-up and wave-scattering in the focused wave cases of varying steepness. (a) Time-histories of the water surface elevation, $\eta(t)$, recorded at $r = D/2$ and $\theta = \pi$. Still images of the wave fields, $\eta_w(x, y)$, showing *Type-1** wave scattering at $\Delta t = 0.38$ s after passage of a focused wave crest for (b) test case 16 (---) (c) test case 17 (—).

($r = D/2$ and $\theta = \pi$); the data relating to both test case 16 and 17. Figure 6.12(b) and 6.12(c) provide the still images of the scattered wave fields, one for each case, taken approximately $\Delta t = 0.38$ s after the passage of maximum wave crest.

With test case 16 being weakly nonlinear, there is no *secondary run-up* and of *Type-1** wave scattering observed. In contrast, as the wave steepness increases, there is clear evidence of *secondary run-up* and *Type-1** wave scattering in test case 17. This is entirely consistent with the results obtained in test case 13. It is also relevant to note that with an increase in the peak period, the amplitude of the *secondary run-up* is slightly more pronounced. The reason for this again relates to the fluid velocities in the incident wave crest. With test case 17 having larger fluid velocities due to the increase in the linear amplitude sum, the maximum run-up is larger. This, in turn, produces a significant increase in the subsequent *secondary run-up* and consequently, the amplitude of the *Type-1* scattered wave. These observations show that the occurrence of *Type-1** wave scattering is not exclusive to one focused wave event. Indeed, a similar pattern of wave scattering can be observed provided that the incident wave is sufficiently steep to force the generation of the *secondary run-up* responsible for the formation of *Type-1** wave scattering.

Effect of Spectral Bandwidth

Having observed the generation of *Type-1** wave scattering from an incident focused wave, this section seeks to investigate the effect of the incident wave parameters, specifically the spectral bandwidth. In exploring this, three focused wave cases with different peak enhancement factors, γ , were considered. These are defined by test case 13, 18 and 19 (Table 6.1), having γ values in the range $2.5 \leq \gamma \leq 8.0$. The peak enhancement factor, γ , controls the shape of the JONSWAP spectrum; the larger the γ value, the more narrow-banded spectrum and hence the greater the proportion energy of that is concentrated around the spectral peak frequency. Each of the three wave cases considered in this section have a peak period of $T_p = 1.0$ s and a wave steepness of $Ak_p = 0.18$.

Time-histories of the water surface elevation, $\eta(t)$, recorded one diameter upstream of the column ($r = D$), for incident focused waves with varying γ values are provided in Figure 6.13. It is clear from this figure that an increase in γ suppresses the formation of the ‘double-peak’ in the high-frequency wave scattering occurring

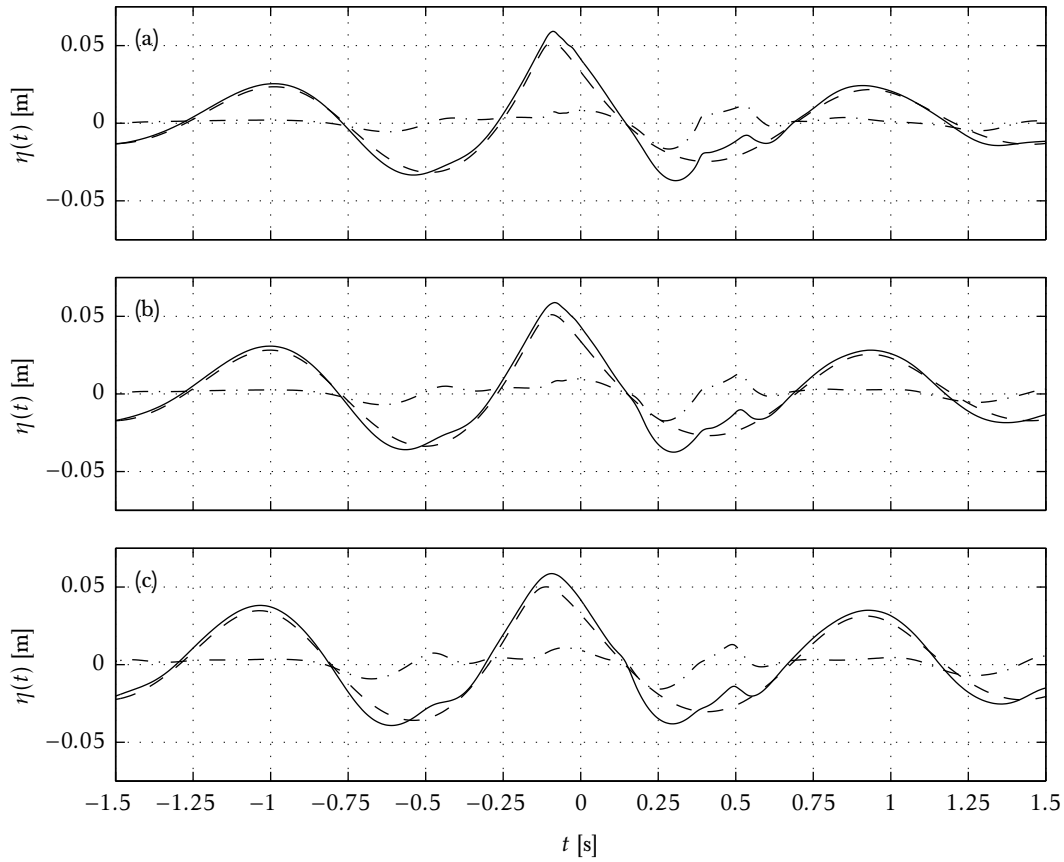


Figure 6.13: Time-histories of the water surface elevation, $\eta(t)$, recorded at $r = 100$ mm and $\theta = \pi$ for (a) test case 13, (b) test case 18, and (c) test case 19. Legends: --- η_{w0} , — η_w , and - · - $\Delta\eta$.

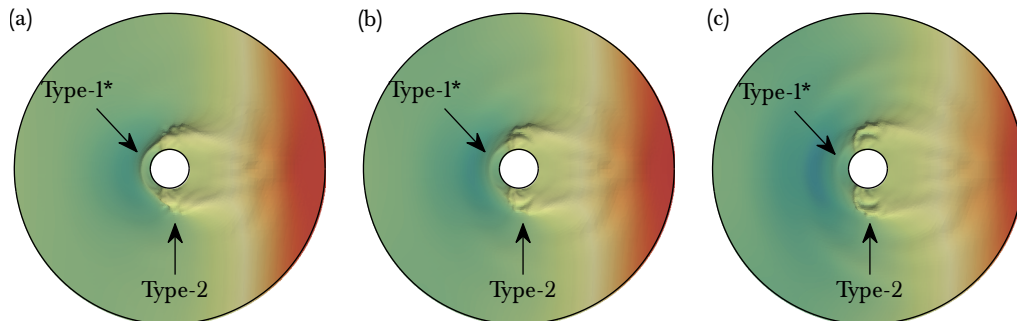


Figure 6.14: Snapshot of wave fields, $\eta_w(x, y)$ showing *Type-1** wave scattering at $\Delta t = 0.36$ s after passage of the wave maximum crest for (a) test case 13 (b) test case 18 and (c) test case 19.

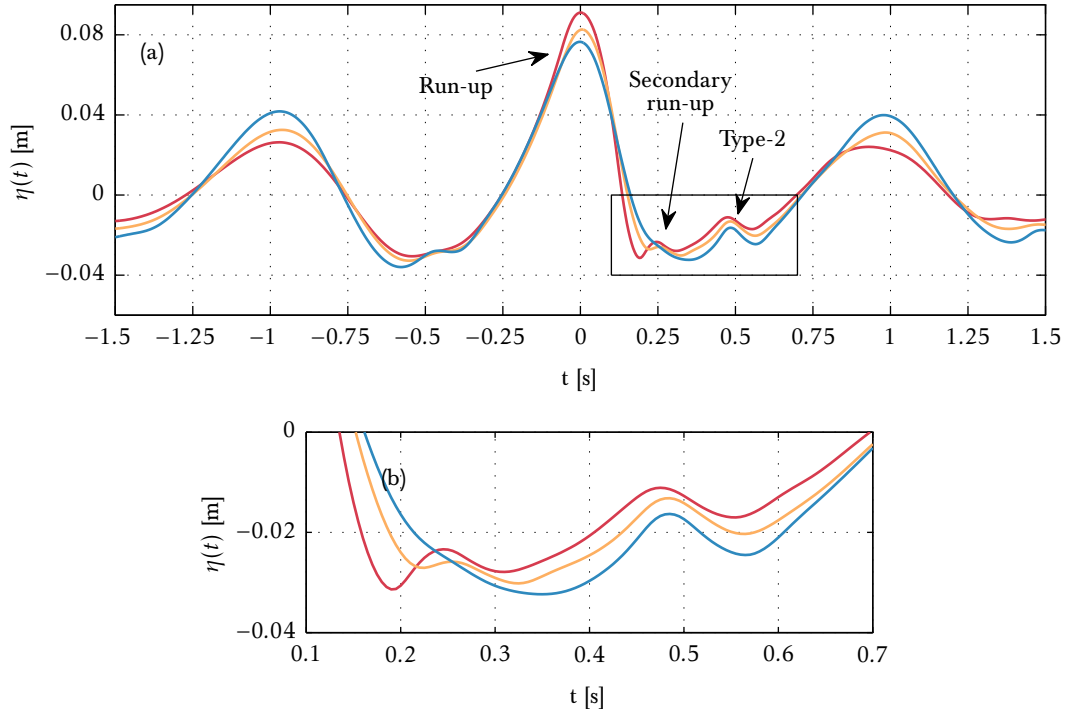


Figure 6.15: (a) Time-histories of the water surface elevation, $\eta(t)$, recorded on the upstream column face ($r = D/2$ and $\theta = \pi$) for focused waves with varying γ values. (b) Close-up of the wash-down and *secondary run-up*. Legends: — test case 13, — test case 18, and — test case 19.

immediately after the focused event. In this case, the first peak, associated with the *Type-1** wave scattering is much reduced with larger γ . In contrast, the amplitude of the second peak, associated with the *Type-2* waves scattering appears unaffected. The suppression of the *Type-1** wave scattering is further highlighted in Figure 6.14. This figure contrasts the scattered wave field, $\eta_w(x, y)$ in wave cases 13, 18 and 19 at $\Delta t \approx 0.36$ s after the focused event. Comparisons between these plots shows that with a reduction in the spectral bandwidth (increasing γ), the *Type-1** scattered wave front has a markedly reduced surface wave amplitude.

Although the incident wave steepness adopted for each wave cases is similar (based upon the relevant Ak_p), the fact that a broad-banded wave spectrum contains more high-frequency wave components merit particular attention. Section 4.6 provides evidence that large waves arising in broad-banded spectrum are inherently more nonlinear. With an increased presence of high-frequency waves, the wave-wave interactions produce steeper crests during the focusing process. This, in turn, produces subsequent increase in the wave run-up. This drives a larger wash-down, which then provokes the generation of *secondary run-up*, responsible for the formation of the

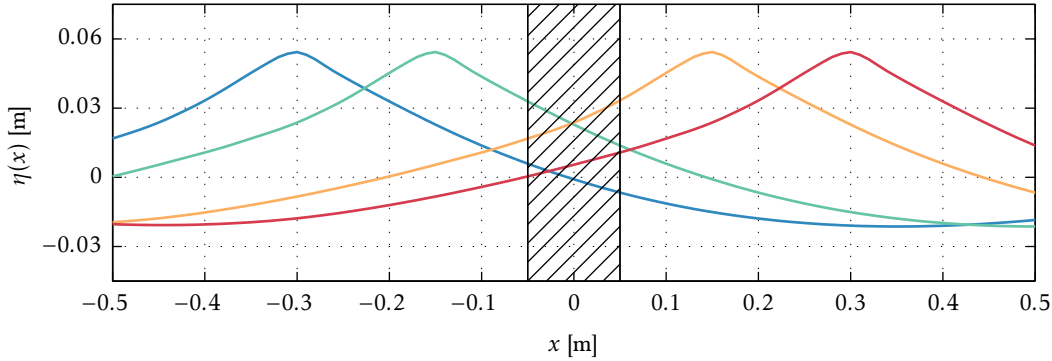


Figure 6.16: Spatial profile of the incident wave, $\eta_{wo}(x)$, at the focus time showing the distance between the location of the focused crest and the column; the latter represented by the shaded area. Legends: — test case 20, — test case 21, — test case 22, and — test case 23.

*Type-1** wave. Evidence of this is provided in Figure 6.15. This presents a comparison of the water surface elevation, $\eta(t)$, recorded on the front face of the column ($r = D$) in test cases 13, 18, and 19. It is clear from this figure that a broad-banded wave event produces significantly larger run-up when compared to the narrow-banded equivalent; with all other parameters held constant. Furthermore, with the subsequent wash down of fluid, a pronounced *secondary run-up* is generated. As this *secondary run-up* interacts with the negative wave-induced velocity, it is carried away from the column as a *Type-1** wave.

These results are consistent with the data presented in the previous section and confirm that the formation of the *secondary run-up* is directly linked to the generation of the *Type-1** scattered waves. Broad-banded wave spectra are associated with steeper waves that produce higher wave run-up. This, in turn, produces a greater wash-down, leading to larger *secondary run-up* and hence more pronounced *Type-1** wave scattering.

The effect of Focus Position

It has already been noted that whilst a regular wave is periodic and propagates with a fixed wave shape, this is not true of an irregular or focused wave. As such, the location of a column in a regular wave field has no effect on the wave-column interactions, producing a similar pattern of wave scattering. However, this is not the case in an irregular wave field. Indeed, the dispersive nature of deep water waves means that both the crest elevations and the wave steepness will vary spatially.

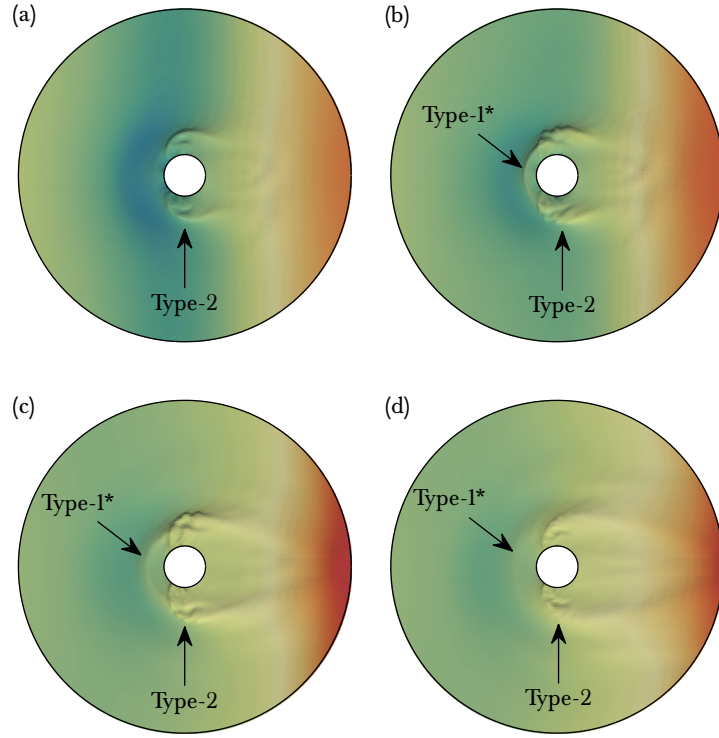


Figure 6.17: Still images or snapshot giving a plan view of the scattered wave field, $\eta_w(x, y)$ for (a) test case 20, (b) test case 21, (c) test case 22, and (d) test case 23. Note: In each case the time of the images (or the phasing of the wave components) have been chosen such that the *Type-2* scattered waves are at approximately the same spatial position.

Given the results presented thus far, this evolution is expected to have a significant effect on the scattered wave field. To explore this, four focused waves cases were considered. These are given as test cases 20, 21, 22, and 23 in Table 6.1. The focused positions for these cases are in the range of $-0.3 \text{ m} \leq x_f \leq 0.3 \text{ m}$, with $x_f = 0 \text{ m}$ corresponding to the column centerline. In each of these four cases the spectral peak period was held constant at $T_p = 1.0 \text{ s}$ and the peak wave steepness at $Ak_p = 0.18$. Figure 6.16 provides the spatial profile of the incident waves, $\eta_{wo}(x)$ at the focused time, highlighting the position relative to the column. In wave cases 21 and 22, the distance between the focused position and the column centerline is approximately $0.1\lambda_p$, where λ_p is the wavelength of the peak wave component. In wave cases 20 and 23, the distance is increased to $0.2\lambda_p$.

The scattered wave fields, $\eta_w(x, y)$, associated with wave cases 20, 21, 22, and 23 are presented in Figure 6.17. These views or still images are taken at different incident wave phases, the latter selected to show the extent of the scattered wave field. Specifically, the wave phases were selected such that the *Type-2* waves for all

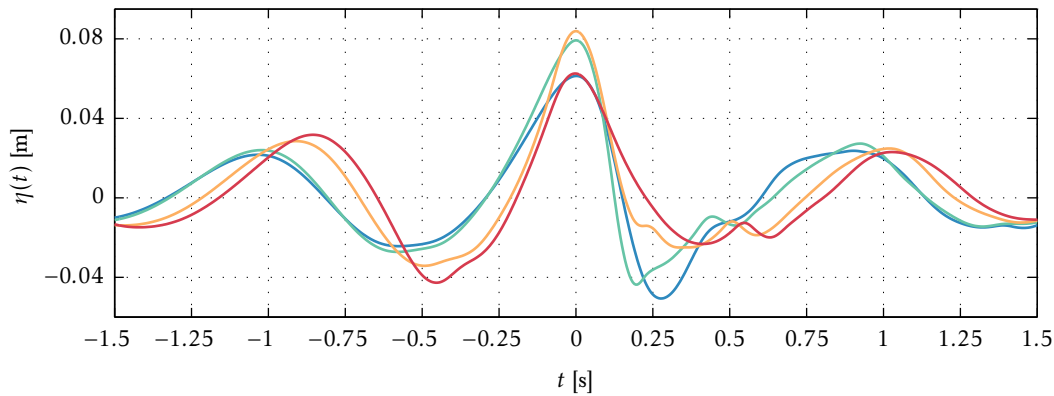


Figure 6.18: Time-histories of the water surface elevation, $\eta(t)$, recorded at $r = D/2$ and $\theta = \pi$ for incident wave cases having different focused locations. Legends: — case 20, — case 21, — case 22, and — case 23.

cases occur at the same spatial location. This allows an effective comparison between the cases. With varying focal positions, the *Type-1** wave patterns are clearly different. For incident waves with focus position close to the centreline of the column (test case 21 and 22), the formation of *Type-1** wave scattering is pronounced. As the distance between the focus position and the column centreline increases, this becomes less pronounced and in test case 20, it is not present at all.

The link between the steepness of the incident waves, the *secondary run-up* and the *Type-1** scattering is further explored in Figure 6.18. This figure shows the predicted water surface elevations, $\eta_w(t)$ on the upstream face of the column ($r = D/2$ and $\theta = \pi$ rad) for test cases 20, 21, 22 and 23. In each case the maximum run-up has been shifted to $t = 0$ s for ease of comparison. By contrasting Figure 6.17 with Figure 6.18, it is again evident that the presence of *Type-1** wave scattering is strongly dependent on the initial wave run-up and the subsequent wash-down. Since this is critically dependent on the incident wave steepness, it will inevitably vary with changes in the relative focus position. This has important practical implications when considering the occurrence of the different wave scattering modes in irregular seas and consequently, the increased crest elevations and the setting of appropriate deck elevations.

6.4 Far-field Amplifications in Focused Waves

Having presented and discussed the occurrence of high-frequency wave scattering in incident focused waves, this section seeks to investigate the increased water surface elevations arising due to the nonlinear wave-wave interactions between the incident and the scattered wave fields. The aim of this section is to establish whether this amplification is important from a practical perspective, especially in respect of maintaining an effective air gap and hence avoiding the occurrence of wave-in-deck loads.

In seeking to establish the importance of the increased water surface elevations for incident focused waves, the first step involves a qualitative assessment of the scattered wave field. This is provided in Figure 6.19. This concerns wave case 13 and 13R; the R denoting an equivalent regular wave. In both cases, the figure presents contour plots of the incident wave field, η_{wo} , and the scattered wave field, $\Delta\eta$, in space and time, (r, t) . In essence, this figure provides a condensed representation of the time-histories of water surface elevation recorded at multiple spatial locations, both upstream and downstream of the column. Figure 6.19 (a) and (c) concern the incident waves for test cases 13R and 13 respectively, whilst Figure 6.19(b) and (d) define the corresponding scattered wave fields. In all plots, the surface elevation has been normalized by the maximum incident crest elevation, η_{wo} .

In both cases 13R and 13, the maximum incident crest elevation at the column centreline occurs at $t = 0$ s. The next incident wave crest occurs one wave period later, at $t = 1$ s. In regular waves this second crest is of an identical height to the first (Figure 6.19(a)). However, in the focused wave case, it is substantially reduced due to the dispersive nature of the underlying sea state (Figure 6.19(c)).

With the arrival of an incident wave crest at $t = 0$ s, an increase in the surface elevation due to wave run-up occurs on both the upstream and downstream face of the column, the latter occurring later than the former. This occurs in both wave cases (13R and 13). Given the increased steepness of the focused wave (evidence of this being provided by the reduced spacing of the contour lines in Figure 6.19(c) when compared to Figure 6.19(a)), this case generates substantially larger run-up (Figure 6.19(d)). On the downstream face of the column, an increase in the surface elevation due to *Type-2* waves is clearly evident in both cases (Figure 6.19(c) and 6.19(d)); the scattered waves propagating in the same direction as the incident wave.

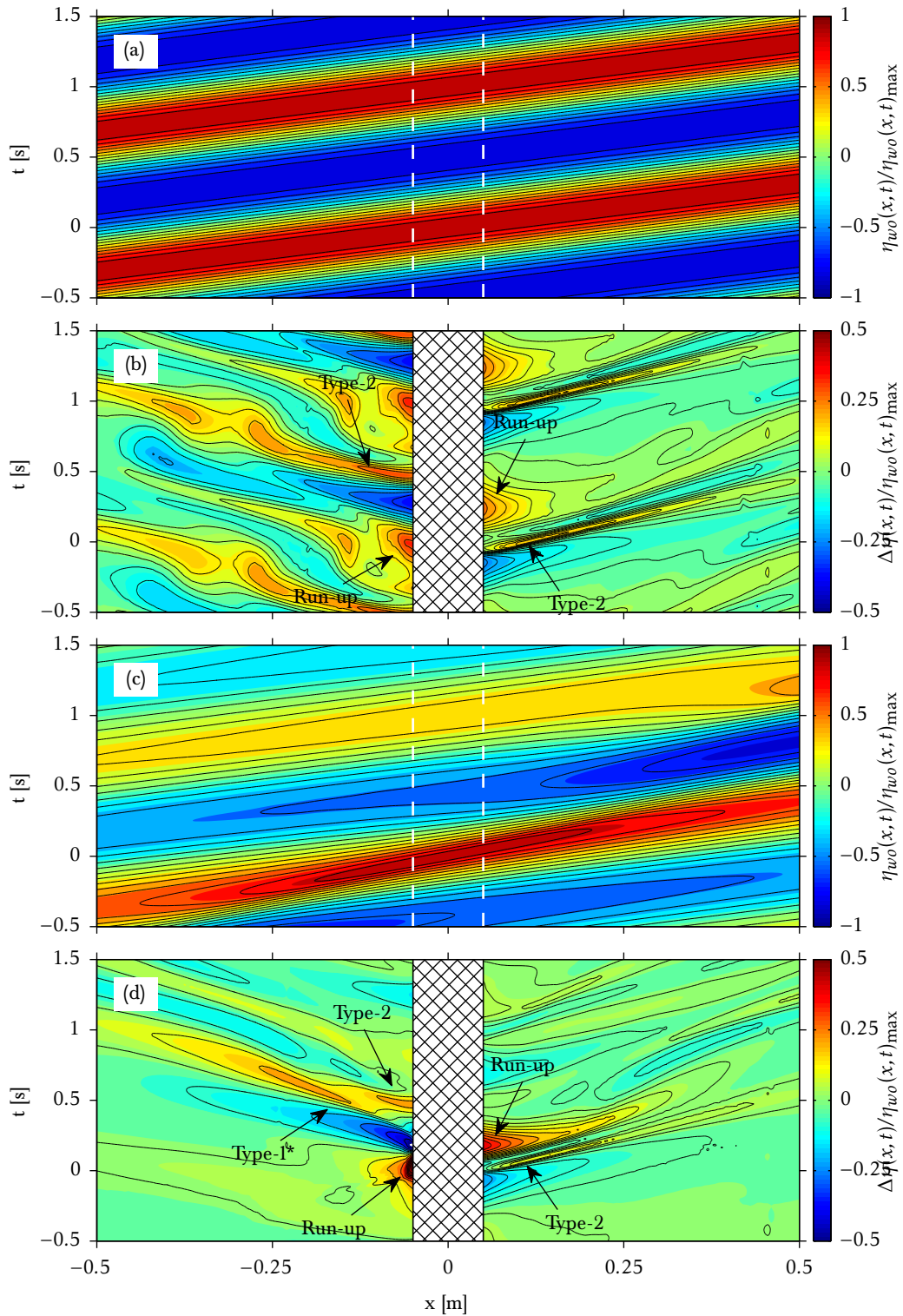


Figure 6.19: Contour plots showing the spatial and temporal evolution of the incident wave field, $\eta(x, t)$ and the scattered wave field, $\Delta\eta$, for wave cases 13R and 13, detailed of which are given on Table 6.1: (a),(b) η_{w0} and $\Delta\eta$ for case 13R, (c),(d) η_{w0} and $\Delta\eta$ for case 13.

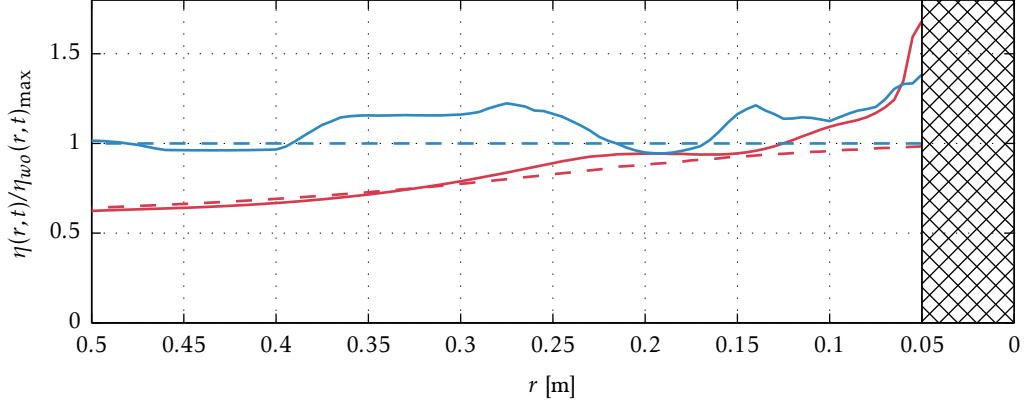


Figure 6.20: Spatial profile of maximum surface elevation, $\eta_w(r, t)$ and envelope of maximum incident elevation, $\eta_{w0}(r, t)_{\max}$ for test cases 13 and 13R. Legends: $---$ $\eta_{w0}(r, t)_{\max}$ (case 13R), $—$ $\eta_w(r, t)_{\max}$ (case 13R), $---$ $\eta_{w0}(r, t)_{\max}$ (case 13), and $—$ $\eta_w(r, t)_{\max}$ (case 13),

Confirmation of this is provided by the fact that the gradient of the contour lines on the rear side of the column is identical to that described by the incident waves (Figure 6.19(a) and 6.19(c)). Half a wave cycle later, at $t = 0.5$ s, the arrival of the incident wave trough ensures that upstream wave scattering also occurs. This time, the scattered waves are propagating in the opposite direction to the incident waves. In wave case 13R, much of the increase in the wave elevation is due to *Type-2* wave scattering. In contrast, in wave case 13, the increase in elevation is caused by the merging of the *Type-1** and *Type-2* waves. In both cases, the scattered wave elevation is a maximum at around $x = 0.27$ m, with an increase in the surface elevation of approximately $\Delta\eta/(\eta_{w0})_{\max} \approx 0.24$.

In considering the practical importance of the increase in the water surface elevation, it is essential to consider the corresponding total water surface elevation, η_w . For the regular wave case considered herein, the incident wave is relatively steep. In this case, the increased surface elevation, due to nonlinear wave-wave interactions, is larger than the maximum incident crest elevation. In contrast, the focused wave crest is characterised by a large single crest elevation, followed by waves that are substantially lower. As a result, although the magnitude of the scattered wave, $\Delta\eta$, is significant, because the next incident wave is smaller, the increase in water surface elevation is lower than the maximum incident elevation. This point is addressed in Figure 6.20. This describes the maximum incident crest elevation, $\eta_{w0}(r, t)$ and the maximum surface elevation, $\eta_w(r, t)$, recorded in the upstream of the column on both test cases 13 and 13R. The maximum value of $\eta_w(r, t)$ is calculated for $t > 0$ s

to reflect the increase in the total water surface elevation due to the scattered waves observed in Figure 6.19. It is clear from this figure that at some distance from the column ($r > 0.15$ m) the total water surface elevation in test case 13 is lower than the corresponding maximum incident wave elevation. This is in contrast with test case 13R which shows significantly larger maximum elevation for $r < 0.4$ m. At first sight this appears to suggest that the nonlinear (high-frequency) wave scattering and the subsequent nonlinear wave-wave interactions are less important in realistic focused waves. However, this does not necessarily follow. First, the substantial increase in the wave elevation due to run-up on the front face of the column in test case 13 (as given in Figure 6.19 and Figure 6.20) is undeniably important from a design perspective. Second, for $r < 0.15$ m, the data suggests that a large (20%) increase in the maximum incident crest elevation can occur. Moreover, the nature of these results brings into question whether a focused wave event, representing the most probable shape of the highest incident wave crest, is the correct wave event with which to examine the importance of the subsequent wave-wave interactions.

For the scattered waves to be important, two conditions are necessary. First, the incident wave needs to be sufficiently steep to promote the generation of high-frequency wave scattering. Second, in the presence of the scattered wave field, the next incident wave needs to be steep for a substantial nonlinear wave-wave interaction to occur; the latter producing a large increase in the incident wave elevation. Based upon these criteria, it would appear that a focused wave crest is not the most appropriate critical wave event.

To explore this further, two additional wave events were considered. The first relates to a focused wave event in a JONSWAP spectrum but with a peak enhancement factor of $\gamma = 8.0$ giving a substantially narrower spectral form. In this case, the reduction in the height of the second (or following) wave crest will be substantially reduced so that the wave appears more ‘regular’, local to the largest wave event. The second wave event concerns a trough focused wave. This is produced by adding a phase difference of $\Delta\Phi = \pi/2$ rad to all wave components; the end result being a deep trough formed between two relatively large crest. The interest in this case arises from the fact that the first crest produces the scattered waves and the second interacts nonlinearly with it. These additional cases are subsequently referred to as test case 19 and 13t respectively; full details of each being given in Table 6.1.

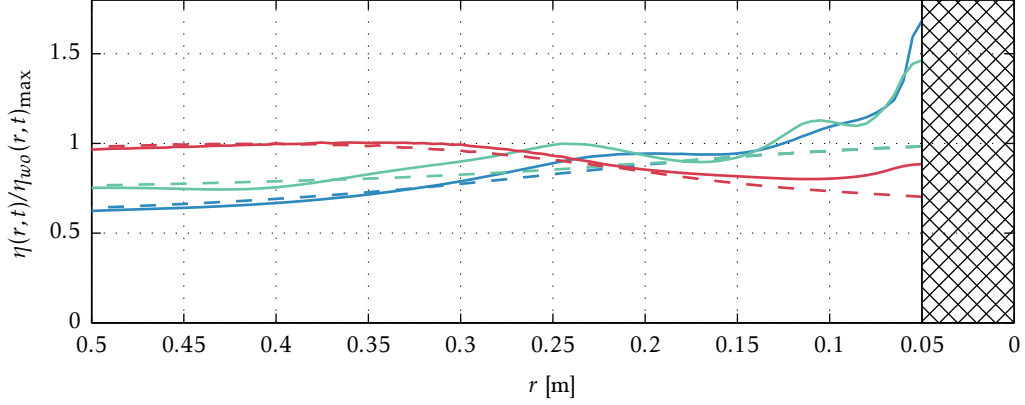


Figure 6.21: Spatial profile of maximum surface elevation, $\eta_w(r, t)$ and envelope of maximum incident elevation, $\eta_{w0}(r, t)_{\max}$ for test cases 13, 19 and 13t. Legends: $---$ $\eta_{w0}(r, t)_{\max}$ (case 13), $—$ $\eta_w(r, t)_{\max}$ (case 13), $---$ $\eta_{w0}(r, t)_{\max}$ (case 19), $—$ $\eta_w(r, t)_{\max}$ (case 19), $---$ $\eta_{w0}(r, t)_{\max}$ (case 13t), and $—$ $\eta_w(r, t)_{\max}$ (case 13t).

Figure 6.21 provides comparisons between test case 13, 19 and 13t, the format of the figure being identical to that given on Figure 6.20. In comparing these cases it is important to note that the incident waves have a similar steepness ($Ak_p = 0.18$) and that the maximum surface elevation in the upstream direction, $\eta_w(r, t)_{\max}$, is again normalised by the maximum incident crest, $\eta_{w0}(r, t)_{\max}$. Contrasting this data it is clear that for $r > 0.15$ m, none of the test cases produce upstream amplifications larger than their corresponding incident wave. However, the narrow banded spectrum of test case 19 produces a substantial increase above test case 13, emphasising the importance of the spectral bandwidth. Interestingly test case 13t produces the lowest amplification. This is due to the fact that the crests prior and after the focused trough are substantially smaller than the maximum incident crest elevation (which occur further upstream).

It is also interesting to note that with an increase in the linear amplitude sum for test cases 13, 19 and 13t, one might assume that the amplifications would be increased. However, this does not prove to be the case. The explanation for this lies in the occurrence of wave breaking which reduces the magnitude of the scattered waves considerably. As a result, the nonlinear wave interaction and consequently the maximum increase in the surface elevation are much reduced.

The present results clearly suggest that the nonlinear amplification of the maximum water surface elevations upstream of the column are substantially less in realistic focused waves when compared to regular waves. However, some care must be

exercised when drawing any wider conclusions because there may exist alternative incident wave forms that have not thus far been explored. Ideally, one would undertake long random simulations rather than employing individual deterministic wave events. However, the computational effort required to do this is prohibitively large.

6.5 Chapter Summary

This chapter has considered the numerical modelling of wave scattering produced by the interaction of focused waves with a vertical, surface piercing, column. The present chapter has:

1. Demonstrated the presence of *Type-1* and *Type-2* scattered waves in weakly non-linear focused wave cases.
2. Highlighted the presence of an additional wave scattering mechanism in the form of a *Type-1** scattered wave arising in steep focused wave events.
3. Provided an explanation for the origins of the *Type-1** scattered waves, highlighting its association with the *secondary run-up* occurring on the upstream face of the column.
4. Demonstrated the sensitivity of the *Type-1** waves with respect to the spectral bandwidth and the focal location of the incident waves.
5. Showed that the amplification of the crest elevations due to the nonlinear interaction between the incident and scattered waves is lower for focused wave cases when compared to those predicted using equivalent regular wave cases.

7

Nonlinear Wave Loading

7.1 Introduction

This chapter concerns the numerical descriptions of the nonlinear wave loading on a vertical surface-piercing column. Section 7.2 presents a review of existing work and the relevant background information. Section 7.3 outlines the numerical procedures used in the present study, while Section 7.4 presents a validation against available laboratory observations. Frequency analysis together with comparisons to the existing analytical force models are presented in Section 7.5. Further investigations of the dependence of the nonlinear loading on the column diameter are given in Section 7.6. This establishes a link between the *secondary loading cycle* and the high-frequency wave scattering around the column. This work is further extended in Section 7.7 which presents data describing the excitation of a single degree of freedom system subjected to the nonlinear wave loading. This, in turn, establishes a clear link between the high-frequency wave scattering and the onset of structural *ringing*.

7.2 Background

In the last couple of decades, much of the research into nonlinear wave loading, in the context of offshore engineering, has sought to provide an explanation for the onset of structural *ringing*; the latter used to describe the excitation of a structure at frequencies well above those associated with the incident waves. Structures with relatively large diameters and natural frequencies are known to be vulnerable to this type of response. These includes amongst others, several classes of Tension Leg Platforms (TLP) and Gravity Based Structures (GBS). *Ringling* responses were first observed on the Hutton platform and in the model tests of two production platforms: Heidrun and Troll (Natvig, 1994). Moreover, *ringing*-type responses were also observed during operations in storm conditions on the Gullfaks C platform (Langen et al., 1998).

Although *ringing* is often associated with impulsive loads arising due to wave slamming, the phenomena can also occur in steep non-breaking waves. As steep waves propagate past a structure, unexpected high-frequency loads are known to occur. These high-frequency loads induce a build-up of *ringing* vibrations at the natural frequency of the structure, and occur within a time interval of one wave period. Evidence of this was observed in the model tests of Draugen and Heidrun GBS structures. In these cases, the natural period of the structure was between three and five times shorter than the peak period of the incident wave. Previous studies (Faltinsen et al. (1995) and Tromans et al. (2006)) suggest that *ringing* usually occurs for slender vertical cylindrical structures when the incident waves are steep, and the diameter of the structure is comparable to the waveheight. Interestingly, this is similar to the flow regime where high-frequency wave scattering was also observed (Chapter 5 and 6).

Observations of structural *ringing* have also motivated several attempts at analysing higher harmonic forces using theoretical approaches. Faltinsen et al. (1995) provide an analytical solution for the third-order forcing using a perturbation solution. An extension to this theory (incorporating nonlinear diffraction) is provided by, amongst others Chau & Eatock Taylor (1992) and Malenica & Molin (1995). In both cases, the studies considered the loads acting in finite water depths. In addition, work on a higher-order slender body theory was undertaken by Rainey (1995). Although

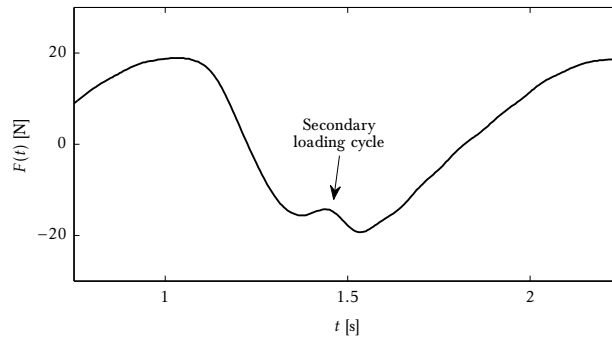


Figure 7.1: Approximate location of the *secondary loading cycle* in the force record.

the methodology under-pinning this work is fundamentally different, the results are strikingly similar to several of the solutions noted above.

7.2.1 Observation of a Secondary Loading Cycle

Early observations of *ringing* suggested that the phenomena might be governed by third-order, weakly nonlinear, terms. This argument is supported by the fact that observations show a burst like structural response that arises close to the third-harmonic of the wave frequency. While some believe that the third-order harmonics are responsible for the onset of structural *ringing*, an alternative view suggests that this phenomenon might be triggered by an unexpected *secondary loading cycle*; the latter first observed by Grue et al. (1993).

This *secondary loading cycle* corresponds to rapid force reversal that occurs just before the instant of minimum loading following the passage of a steep wave crest. Figure 7.1 gives a typical example, depicting the location of *secondary loading cycle* in the force record. Further study by Grue & Huseby (2002) found an association between the occurrence of *ringing* and the *secondary loading cycle*. Indeed, they suggested that the Froude number governs both the *ringing* event and the *secondary loading cycle*. Chaplin et al. (1997) on the other hand found an alternative correlation between the wave steepness and the magnitude of the *secondary loading cycle*. Following on, Rainey (2007) noted that the *secondary loading cycle* was associated with the violent motion of the water surface, often leading to local breaking around the column. In addition, Rainey (2007) argued that there was no connection between the third-harmonic of the wave frequency and the *ringing* phenomenon. Specifically, a change in the natural frequency of the structure (to a higher value) did not eliminate

the occurrence of *ringing*. This suggests that the *ringing* phenomenon might be both highly localised and highly nonlinear, occurring at a much higher frequency than previously predicted.

To date, the undocumented evidence suggests that the *secondary loading cycle* might hold the key to a physical understanding of the *ringing* phenomenon. In addition, recent studies by Masterton (2006), involving both single and multiple column arrangements in a wide range of wave conditions, reveal that the current industry standard theoretical force models are unable to predict the existence of this *secondary loading cycle*. This raises concerns over the accuracy of the current force models, especially for flow regimes in which the *secondary loading cycle* might become important. Interestingly, evidence from Masterton (2006) suggests that the *secondary loading cycle* and the nonlinear wave scattering might be closely linked; both events occurring in similar conditions. If this is true, a nonlinear model capable of predicting the nonlinear wave scattering might also provide insights into the *secondary loading cycle*, thus paving the way to a greater understanding of the *ringing* phenomenon.

7.2.2 Existing Force Models

If the effective column diameter is such that $D/\lambda < 0.2$, the column is considered slender and the effect of linear wave diffraction is neglected. In this case, the wave force or wave loading exerted on the column is typically calculated on the basis of the incident wave conditions. The most common method to calculate the wave forcing on a slender structure is given by Morison's equation

$$f = C_m \rho \frac{\pi D^2}{4} \frac{du}{dt} + C_d \frac{1}{2} \rho u^2 D, \quad (7.1)$$

where f is the wave force per unit length, D the column diameter, ρ the fluid density, u the wave-induced fluid velocity, C_m the inertia coefficient, and C_d the drag coefficient. The first and second terms on the right-hand side of Equation (7.1) correspond to the inertia and drag force respectively; the former due to irrotational flow effects and the latter associated with vortex formation due to viscous effects.

Using Equation (7.1), it is often argued that the first term in Morison's equation provides a linear approximation of the potential flow loading; the inertia force being linearly dependent on u . This suggests that any nonlinear wave loading is due to

the drag term alone. However, Lighthill (1979) argued that this view is incomplete. Indeed, he highlighted additional nonlinear force components that should be incorporated within the inertia term of Morison's equation. Taken together, Lighthill (1979) proposed the following potential flow force components

$$F_I = 2\rho g a \pi R^2 \cos(\omega t), \quad (7.2)$$

$$F_d = \left(\frac{1}{4}ak\right)\rho g a \pi R^2 \sin(\omega t), \quad (7.3)$$

$$F_w = (ak)\rho g a \pi R^2 \sin(\omega t), \quad (7.4)$$

$$F_q = \left[16\left(\log \frac{C}{kR}\right) - 3\right] \frac{1}{2}k^2 R^2 (ak)\rho g R \pi b^2 \sin(2\omega t), \quad (7.5)$$

where a is the wave amplitude, $R = D/2$ is the column radius, ω is the angular wave frequency, k is the wavenumber, and the constant C is taken as 0.28. In formulating this solution, F_I is the integral of unsteady pressure around the column surface up to SWL and corresponds to the inertia term in Morison's equation; F_d is the so-called dynamic force defined by the integral of dynamic pressure around the column surface up to SWL and F_w is the waterline force, defined as the integration of the hydrostatic pressure and the unsteady pressure around the column from the SWL up to the instantaneous water surface. Lastly, the quadratic potential, F_q , is the quadratic correction to the velocity potential due to the nonlinear terms in the free-surface boundary conditions. However, in the limit of small D/λ , F_q becomes very small, and the potential flow forcing is dominated by the first three terms.

Building upon the work by Lighthill (1979), Faltinsen et al. (1995) described the potential flow force acting on a vertical column subjected to regular incident wave having a relatively long wavelength, λ , but with wave amplitude, a , comparable to the column radius, $a = D/2$. This flow regime is characterised by $a \ll \lambda$, $d \ll \lambda$, but $a \approx D$. This is in contrast to conventional diffraction theories, based upon a perturbation expansion, where the wave amplitude is small as compared to other length-scales. In undertaking this analysis, Faltinsen et al. (1995), here after referred to as FNV, described the potential flow forces arising at increasing harmonics of the incident waves as

$$F_{x1} = 2\rho g a \pi R^2 \cos(\omega t), \quad (7.6)$$

$$F_{x2} = \frac{5}{4} \rho g k R^2 a^2 \sin(2\omega t), \quad (7.7)$$

$$F_{x3} = \pi \rho g k^2 R^2 a^3 [\cos(\omega t) - 2 \cos(3\omega t)]. \quad (7.8)$$

Within this solution, F_{x1} corresponds to the inertia force in Morison's equation, while F_{x2} is the summation of Lighthill (1979) dynamic force, F_d , and waterline force F_w . In contrast, the third force, F_{x3} is defined by the sum of a point load acting close to the water surface and a distributed force arising from a combination of the linear and quadratic potentials.

An alternative method for calculating the potential flow forces in the context of the slender body regime was proposed by Rainey (1995). Unlike Lighthill (1979) and Faltinsen et al. (1995), which are both based on a formal perturbation expansion, Rainey (1995) derived the potential flow forces from energy considerations. The total potential force is represented by the sum of three terms corresponding to the Morison's inertia force, F_I , the axial divergence force, F_{AD} and the free surface intersection force, F_{SI} . They are defined as follows

$$F_I^s = \int_{-d}^{\eta} 2\rho \pi R^2 \left(\frac{\partial u}{\partial t} + \frac{u \partial u}{\partial x} + \frac{w \partial u}{\partial z} \right) dz, \quad (7.9)$$

$$F_{AD}^s = \int_{-d}^{\eta} \rho \pi R^2 u \frac{\partial w}{\partial z}, \quad (7.10)$$

$$F_{SI}^s = -\frac{1}{2} \rho \pi R^2 \frac{\partial \eta}{\partial x} u^2, \quad (7.11)$$

where the superscript s denotes the slender body approximation, d is the water depth, η is the water surface elevation, and (u, w) are the velocity components. Rainey (1995) argued that the advantage of adopting energy considerations instead of a formal perturbation expansion is that the application is not restricted to the convergence of some series solution. Instead, the solution can be used to describe the wave forces for cases involving steep waves, where the surface elevation, η , and the wave-induced velocities, (u, w) can be approximated by some highly nonlinear wave theory. Whilst this is undoubtedly a significant advantage, the model provides little physical understanding of the origins of the various force components.

7.3 Numerical Calculations

The purpose of the present section concerns the numerical prediction of the nonlinear wave loads acting on a vertical surface piercing column subjected to regular and irregular incident waves. This builds upon the results of the previous two chapters. As such, both the computational domain and the boundary conditions are identical to those adopted previously. It should be noted that although the model used in this study is based on a two-phase (air and water) flow, the forces are calculated based on the volume fraction of the water phase only ($\alpha = 1$). In effect, it is assumed that the contribution from the air phase is small and can be neglected.

The total inline force, $F(t)$, and over-turning moment, $M(t)$, acting on the column are therefore calculated by integrating the pressure, p , over the wetted area of the column, such that

$$F(t) = \int_{z=-d}^{\eta} \int_{\theta=0}^{2\pi} p(t) R \cos(\theta) d\theta dz, \quad (7.12)$$

$$M(t) = \int_{z=-d}^{\eta} \int_{\theta=0}^{2\pi} (z+d) p(t) R \cos(\theta) d\theta dz, \quad (7.13)$$

where $R = D/2$ is the radius of the column. With the solution assumed to be symmetric about the $x-z$ plane, only half of the domain is modelled. This precludes any possibility of a lift force; the only force being the inline force, acting parallel to the direction of propagation of the incident waves.

7.4 Comparisons With Experimental Observations

To ensure that the predicted forces were correctly modelled, comparisons with earlier experimental observation were undertaken. In making these comparisons the domain size and mesh configuration were exactly as described in the previous chapters. The wave forces for both regular and irregular waves interacting with a surface-piercing column were compared with the experimental observations of Masterton (2006). In all cases considered herein, the diameter of the column was $D = 0.1$ m located in a water depth of $d = 1.2$ m. Similar domain and mesh configurations as previous two chapters were adopted. Due to the different wave generation mechanisms adopted in the numerical simulations and experimental observations, the initial numerically predicted water surface elevations were slightly different to those generated in the

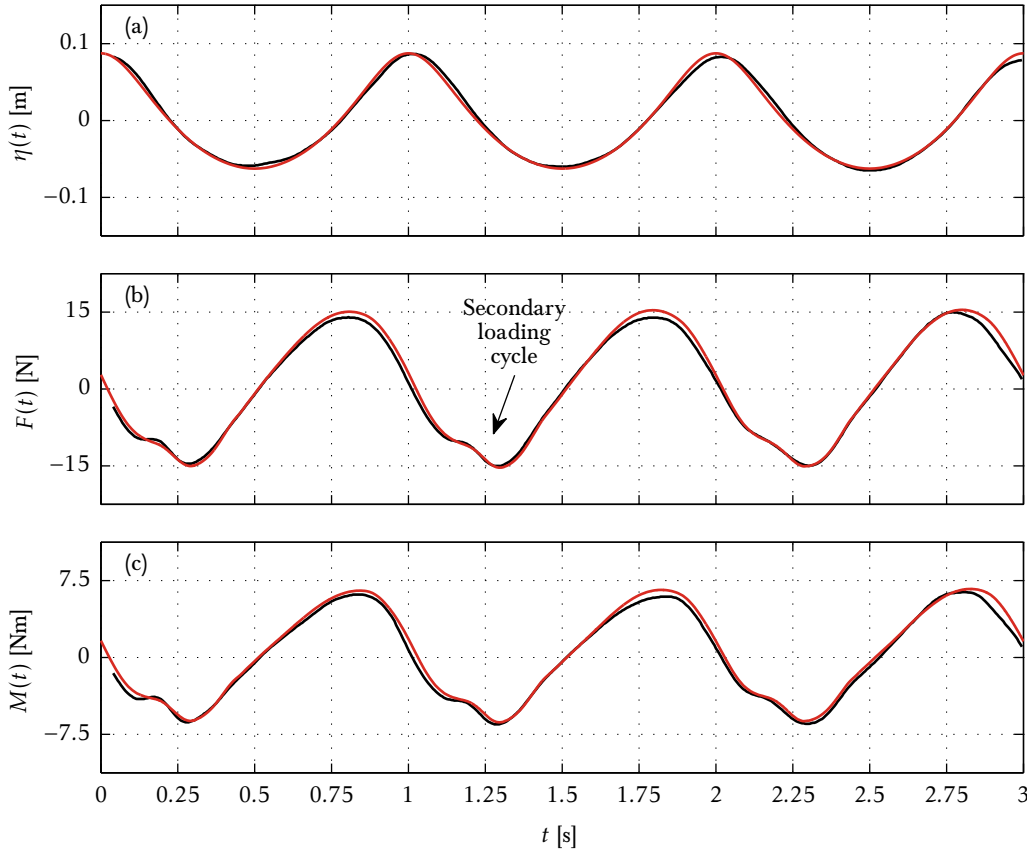


Figure 7.2: The load experienced by a surface-piercing column; time-histories of (a) the incident surface elevation, $\eta_{wo}(t)$, (b) the inline force, $F(t)$, and (c) the over-turning moment, $M(t)$ for steep incident waves ($T = 1.0$ s, $Hk/2 = 0.3$) with comparisons between experimental observations (—) and numerical predictions (—).

laboratory study. To minimize this difference, several iterations were carried out to ensure the best possible fit to the incident water surface elevation.

Figure 7.2 provides comparisons between the experimental observation and numerical simulation for a steep incident wave having a period of $T = 1.0$ s and a wave steepness of $Hk/2 = 0.3$. In this case, the effective column diameter is $D/\lambda \approx 0.07$. The incident water surface elevation, $\eta_{wo}(t)$, at the centre of the column (Figure 7.2(a)) shows good agreement with the experimental data. Based upon this result, good agreement is also observed between the numerically predicted and the experimentally observed inline force, $F(t)$ on Figure 7.2(b), and the overturning moment, $M(t)$ on Figure 7.2(c). Specifically, the occurrence of a high-frequency load, following the passage of maximum crest, is clearly observed. This is the *secondary loading cycle* and has been labelled in Figure 7.2(b).

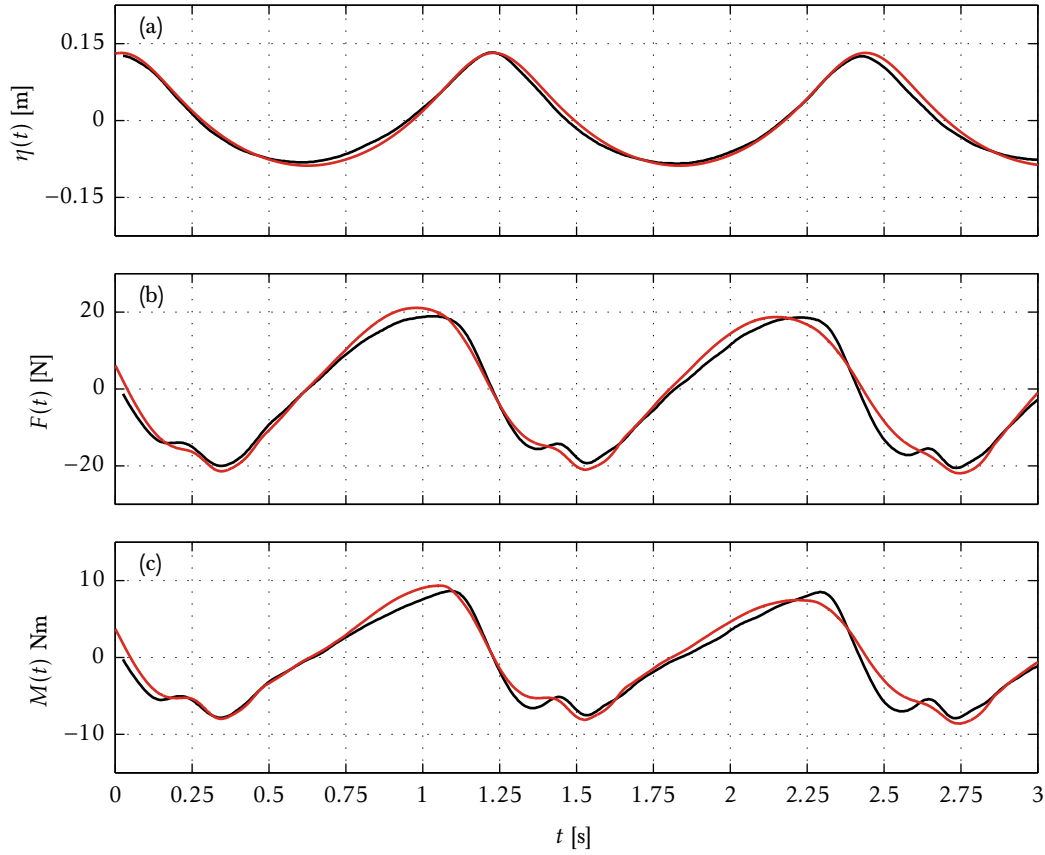


Figure 7.3: The load experienced by a surface-piercing column; time-histories of (a) the incident surface elevation, $\eta_{wo}(t)$, (b) the inline force, $F(t)$, and (c) the over-turning moment, $M(t)$ for steep incident waves ($T = 1.2$ s, $Hk/2 = 0.3$) with comparisons between experimental observations (—) and numerical predictions (—).

The capability of the numerical method to predict the nonlinear loading is further provided in Figure 7.3. In this case, the incident wave period is $T = 1.2$ s having a wave steepness of $Hk/2 = 0.3$ and an effective column diameter of $D/\lambda \approx 0.05$. Although the incident wave elevation, $\eta_{wo}(t)$, was shown to be in good overall agreement (Figure 7.3(a)), a slight discrepancy is observed in the description of the wave trough. Although several iterations were undertaken to reduce this discrepancy, the description of the incident wave elevation was not as good as that described in Figure 7.2(a). Nevertheless, the inline force and over-turning moment, given by Figure 7.3(b) and Figure 7.3(c) respectively, are shown to be in reasonable overall agreement. With an increase in the incident wave period, the *secondary loading cycle* is shown to be more pronounced. Interestingly, the over-turning moment also becomes more nonlinear; evidence of this being provided by the asymmetry of the loading

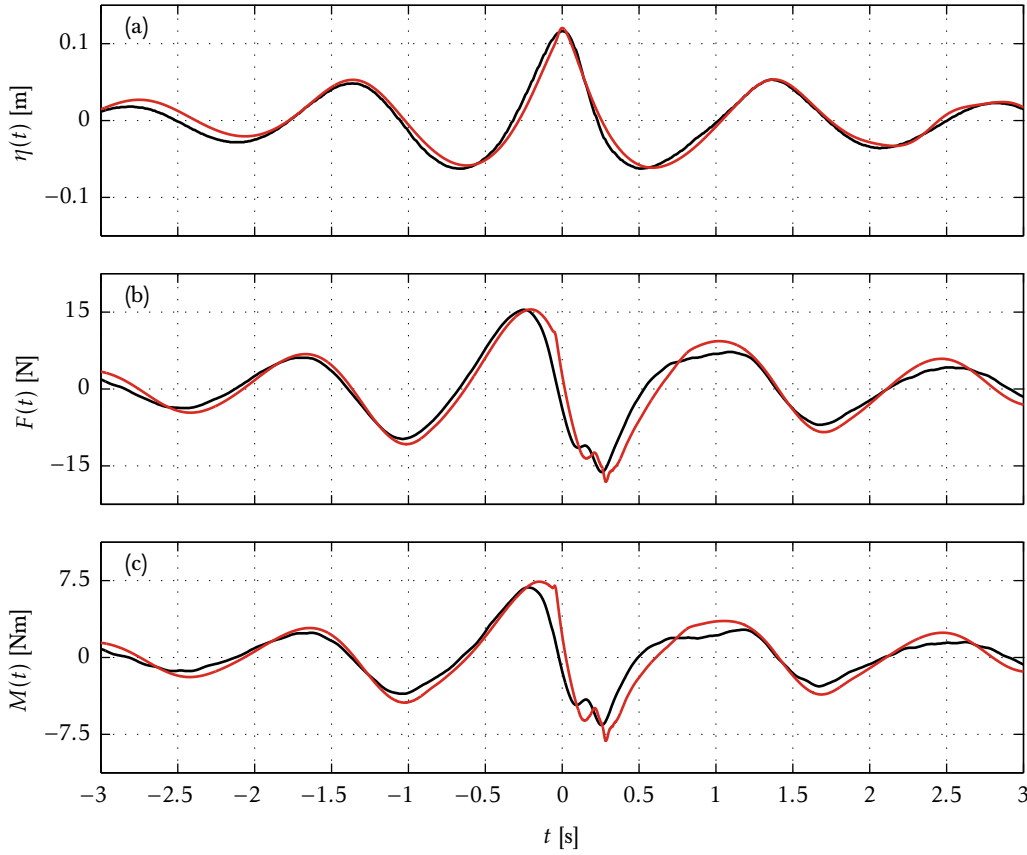


Figure 7.4: The load experienced by a surface-piercing column; time-histories of (a) the incident surface elevation, $\eta_{wo}(t)$, (b) the inline force, $F(t)$, and (c) the over-turning moment, $M(t)$ for steep incident focused wave ($T_p = 1.4$ s, $Ak_p = 0.2$) with comparisons between experimental observations (—) and numerical predictions (—).

record. With the moment related to both the applied forces and the lever arm, the fact that the moment shows significant nonlinearity hints at the importance of the forces acting at high elevations above the SWL.

Having demonstrated the capability of the numerical method to predict the non-linear loading in steep regular waves, Figure 7.4 provides a similar sequence of plots for an incident focused wave. This focused wave event was generated within a JONSWAP spectrum with $T_p = 1.4$ s and $\gamma = 2.5$; the wave steepness defined by $Ak_p = 0.21$, where A is the linear amplitude sum defined by $A = \sum a_n$. In the absence of a detailed description of the wave components used in the experimental work, an exact reproduction of the experimental data was not possible. Based on the available information, and after several iterations, the best fit to the experimentally observed incident surface elevation, η_{wo} , is presented in Figure 7.4(a). Using this

best fit to the incident water surface elevation, the numerically predicted inline force and over-turning moment (Figure 7.4(b) and Figure 7.4(c)) show good agreement with the experimental observations. In this case, The *secondary loading cycle* can again be observed at approximately 0.2 s after the passage of the maximum wave crest. Whilst the agreement is not exact, due to the mismatch in the incident wave elevation, both the predicted force and the overturning moment are in reasonable agreement. This highlights the capability of the numerical method to reliably predict the nonlinear loading experienced by a surface-piercing column subject to both regular and irregular incident waves.

7.5 Discussion of Results

Within this section, the nonlinear loading is investigated further. Specifically, data concerning the harmonic content of the nonlinear loading is provided together with comparisons to the available analytical force models.

7.5.1 Harmonic Reconstruction of the Nonlinear Loading

The *ringing* phenomenon discussed in Section 7.2 arises when the wave loading generated by the interaction between a steep incident wave and a surface-piercing vertical column produces a higher harmonic load at, or close to, the natural frequency of the structure. If this occurs, the structure will experience a (potentially) large structural response due to the resonance effect. While *ringing* is often associated with harmonic forces generated at third-order (Faltinsen et al. (1995) and Malenica & Molin (1995)), Grue & Huseby (2002) argued that *ringing* occurs in a non-breaking incident wave as a consequence of the observed *secondary loading cycle*. To explore this, a frequency analysis is carried out to investigate the harmonic order at which the *secondary loading cycle* starts to become significant.

The desired frequency analysis was undertaken by first decomposing the nonlinear load using a Fourier Transform (FT) and then applying a reconstruction by progressively summing the harmonic content. With the nonlinear loading periodic, the decomposition and reconstruction were done over a single wave period. The result of the harmonic reconstruction for the first six harmonics is presented in Figure 7.5. For ease of comparison, the original nonlinear loading (represented by the

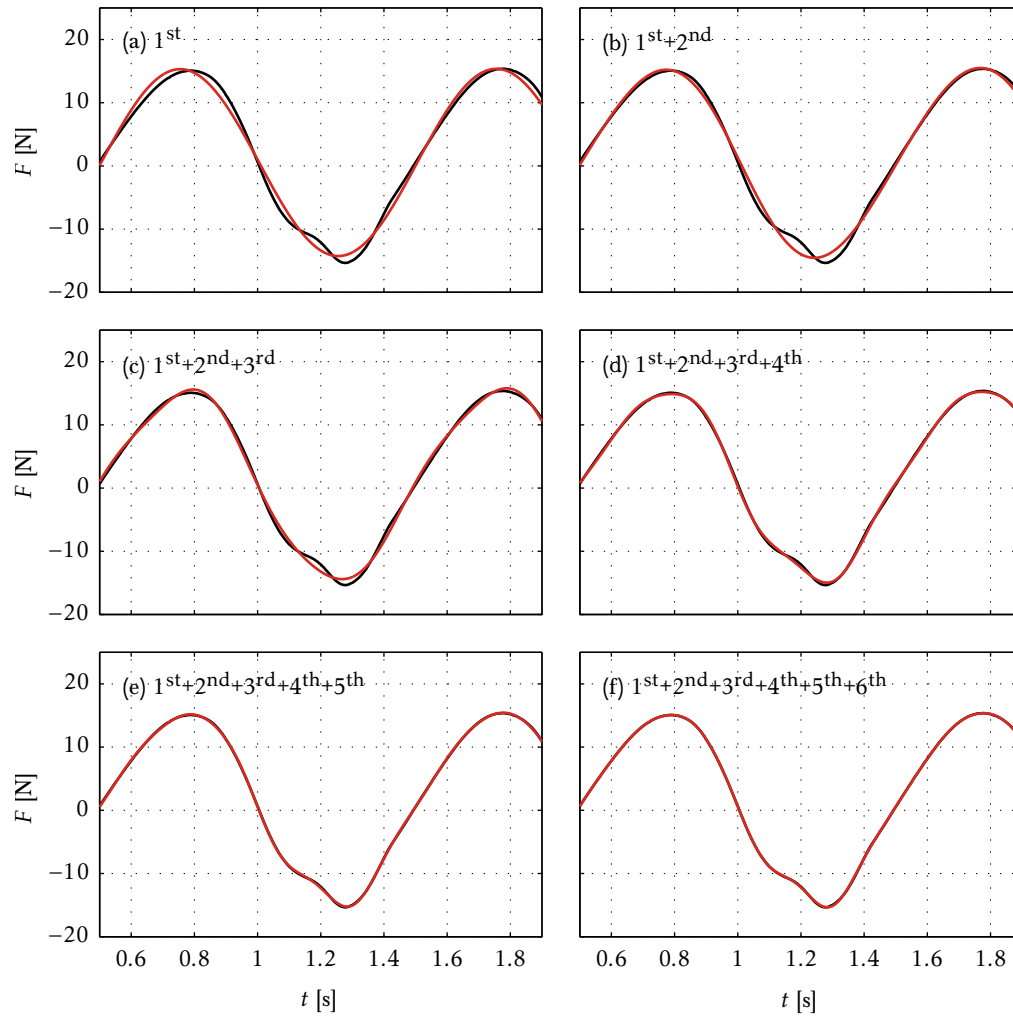


Figure 7.5: Harmonic reconstruction of the nonlinear loading in steep regular wave ($D = 0.115\text{m}$, $T = 1.0\text{s}$, $Hk/2 = 0.3$). Legends: — original and — reconstructed signals.

solid black line) is overlaid on each sub-plot. At the first harmonic, the overall result shows good agreement between original and reconstructed signal, especially with respect to the maximum and minimum values. However, with the inclusion of only the first harmonic, there is a slight phase shift of the re-constructed record and a complete absence of the *secondary loading cycle*. With the inclusion of the second and third harmonics, the overall phasing is correctly predicted but the *secondary loading cycle* is still absent. However, adding the fourth order of harmonic, the *secondary loading cycle* is clearly observed with a value comparable to the original signal. As far as the present result is concerned, it is clear that the *secondary loading cycle* is governed by harmonics generated at fourth order and above.

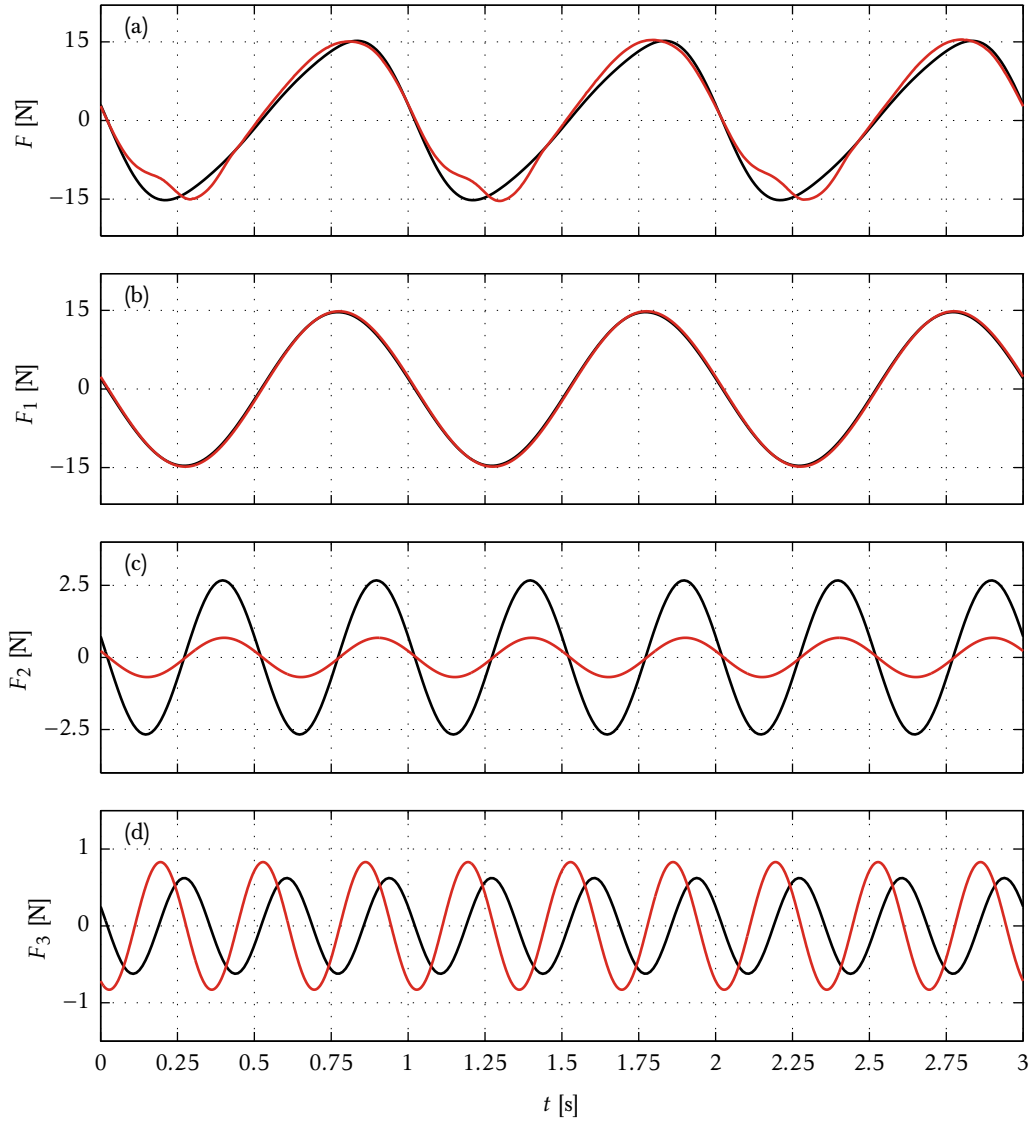


Figure 7.6: Nonlinear loads predictions for a steep regular wave ($D = 0.115\text{ m}$, $T = 1.0\text{ s}$, $Hk/2 = 0.3$), comparing the results of the numerical simulation (—) and the FNV force model (—). (a) Total inline force $F(t)$, (b) 1^{st} harmonic, (c) 2^{nd} harmonic, and (d) 3^{rd} harmonic.

7.5.2 Comparison With Analytical Force Model

Having shown that the *secondary loading cycle* is governed by harmonics larger than the third, this section seeks to compare the numerically predicted nonlinear loading with an existing analytical force model based on FNV. The FNV model is selected over Rainey's force model because the former allows a direct comparison of the underlying force components. Figure 7.6 provides the time-history of the nonlinear loading for a steep regular wave ($D = 0.115\text{ m}$, $T = 1.0\text{ s}$, $Hk/2 = 0.3$) with comparisons between the numerical prediction and the FNV force model. Figure 7.6(a) describes the total

inline force, while Figure 7.6(b)-(d) concerns the underlying force components, f_1 , f_2 and f_3 outlined in Equations (7.6) to (7.8); the subscript denoting the increasing harmonics.

It is clear from Figure 7.6(a) that comparisons between the numerical predictions and the FNV model for the total inline force shows good overall agreement. However, the analytical model fails to predict the presence of the *secondary loading cycle*. This is because the *secondary loading cycle* first occurs at the fourth harmonic. In contrast, the force model is only expanded to the third-order and, therefore, the third harmonic. Overall, the inline force is dominated by the first-order force component as shown in Figure 7.6(b). This is equivalent to the Morrison's inertia force. At second-order, the comparison between the numerical result and FNV force model starts to deviate; the numerical prediction describing a significantly smaller inline force. However, the phasing of the second-order force is correctly modelled. At third-order, the agreement is substantially improved, although the numerical method predicts a force of larger magnitude with a slight phase shift. Overall, although the FNV model provides a good description of the first harmonic inline force component, the model provide a poor description of the higher order components and, in particular the *secondary loading cycle*. With the analytical model derived on the basis of a formal perturbation expansion, it describes a monotonic reduction in the magnitude of the force components. However, this is not the case for the fully nonlinear numerical method; the third harmonic force component being larger than the second.

The numerically predicted higher-order force components extending from the second to the sixth are described in Figure 7.7. It is clear from this figure that there is indeed a break in the monotonic decrease in the magnitude of the higher harmonic force components. Indeed, the third-harmonic force is the largest, followed by the second and the fourth; the later two force components being of a similar magnitude. With the third-harmonic component the largest, this could explain why some believe that the third-order forcing is responsible for the phenomena of *ringing*. However, given the variation in the magnitude of the various harmonics, concerns must be raised concerning the convergence of a force model based on a formal perturbation expansion. The 'non-convergence' of higher order force component for the analytical solution is perhaps not surprising considering the steepness of the incident waves. Such waves will produce a significant nonlinear wave-structure

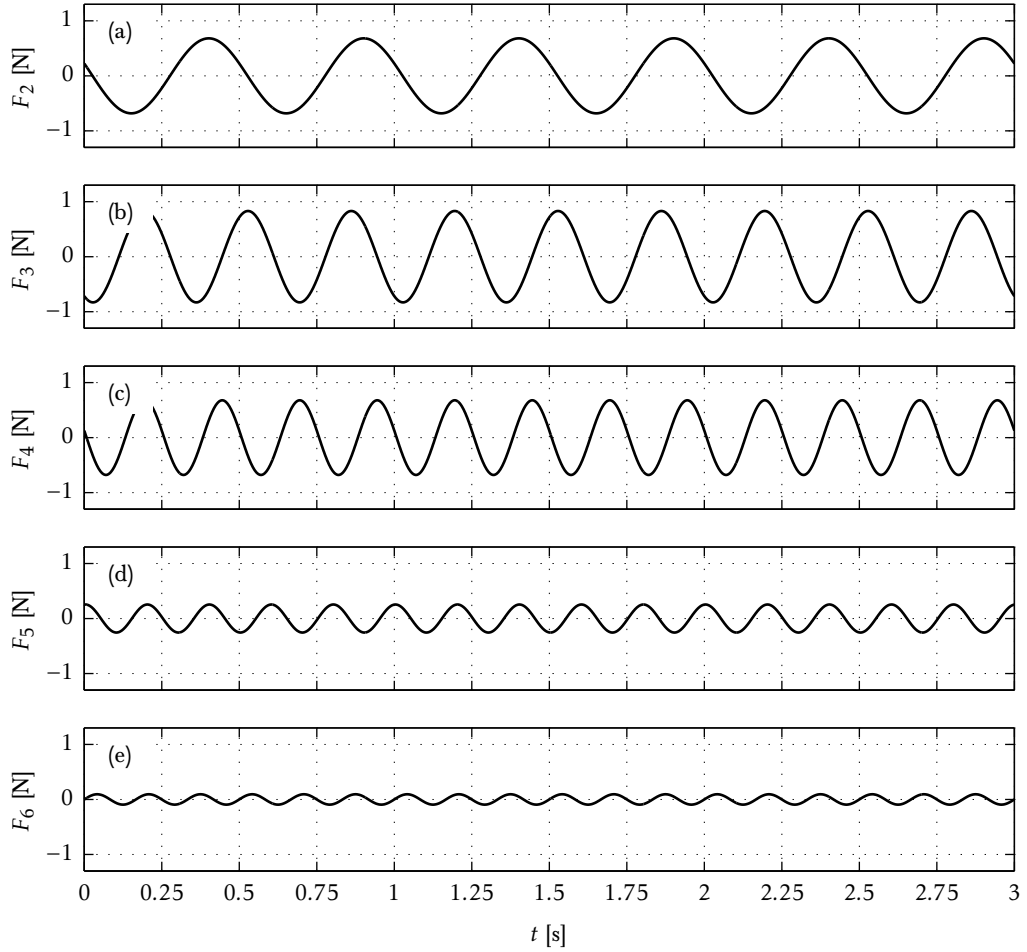


Figure 7.7: The sequence of higher harmonic force components arising from the numerical simulation of a steep regular wave ($D = 0.115\text{ m}$, $T = 1\text{ s}$, $Hk/2 = 0.3$): (a) 2^{nd} harmonic, (b) 3^{rd} harmonic, (c) 4^{th} harmonic, (d) 5^{th} harmonic, and (e) 6^{th} harmonic,

interaction and strong local surface effects, particularly relating to the high-frequency scattering. The relationship between the nonlinear loading and high-frequency wave scattering will be explored in the subsequent section.

7.6 Nonlinear Loading on Columns of Varying Diameter in Regular Waves

The variation in the nonlinear loading produced by regular waves acting on columns of varying diameters are considered in this section. A total of five column diameters lying in the range $0.1\text{ m} \leq D \leq 0.3\text{ m}$ were considered. For each column diameter, seven incident regular wave steepnesses ranging from $0.08 \leq Hk/2 \leq 0.32$ were

employed. This resulted in a total of 35 loading cases. In each case, the flow regime was well outside the linear diffraction regime ($D/\lambda < 0.2$) and the forces were inertia dominated; the latter defined by the relevant Keulegan-Carpenter number, $KC \leq 5$.

The time-history of the total inline loads, $F(t)$, produced by steep regular waves ($T = 1$ s, $Hk/2 = 0.32$) are presented in Figure 7.8. This provides five sub-plots relating to columns of varying diameter, D . The smallest column case (Figure 7.8(a)) correspond to a diameter of $D = 0.1$ m or $D/\lambda \approx 0.078$, while the largest case (Figure 7.8(e)) correspond to a column of diameter $D = 0.3$ m or $D/\lambda \approx 0.192$. In each subplot, the predicted Morison's inertia load is overlaid for comparison. Overall, it is clear from this figure that the simulated nonlinear loading is in reasonable agreement with Morison's inertia force; the magnitude of the maximum force increasing with the column size. For the smallest diameter column (Figure 7.8(a)), the nonlinear force associated with the *secondary loading cycle* can be clearly observed. Interestingly, with an increase in column size, the magnitude of the *secondary loading cycle* progressively reduces. This reduction is consistent with the suppression of the *Type-2* scattered waves observed in Chapter 5.

Using the data presented on Figure 7.8, the second and third-order harmonic loads were decomposed and presented on Figure 7.9. Within this figure, the left column represents the second harmonic load, while the right column gives the third harmonic load. Each row presents data relating to a different column size; the smallest column diameter considered in the top row and the largest in the bottom. In each subplot, the higher harmonic loading is represented as a percentage of the linear or first order harmonic loading.

While the first harmonic loading is undoubtedly dominant, and can be accurately predicted by Morison's inertia force, the 'convergence' of the higher harmonic loading is more problematic. For a series based solution or a perturbation expansion solution to be 'convergent', the magnitude of higher harmonic force components need to be monotonically decreasing. However, it is clear from Figure 7.9, that this is not the case. Apart from the largest column diameter (Figure 7.9(e)), the third-harmonic loading is larger than the second. Moreover, considering the second harmonic load, a decrease in column diameter reduces the magnitude of this load except in the smallest diameter case. In contrast, the magnitude of the third-harmonic loading increases with the reduction in column diameter. This suggests that in steep incident

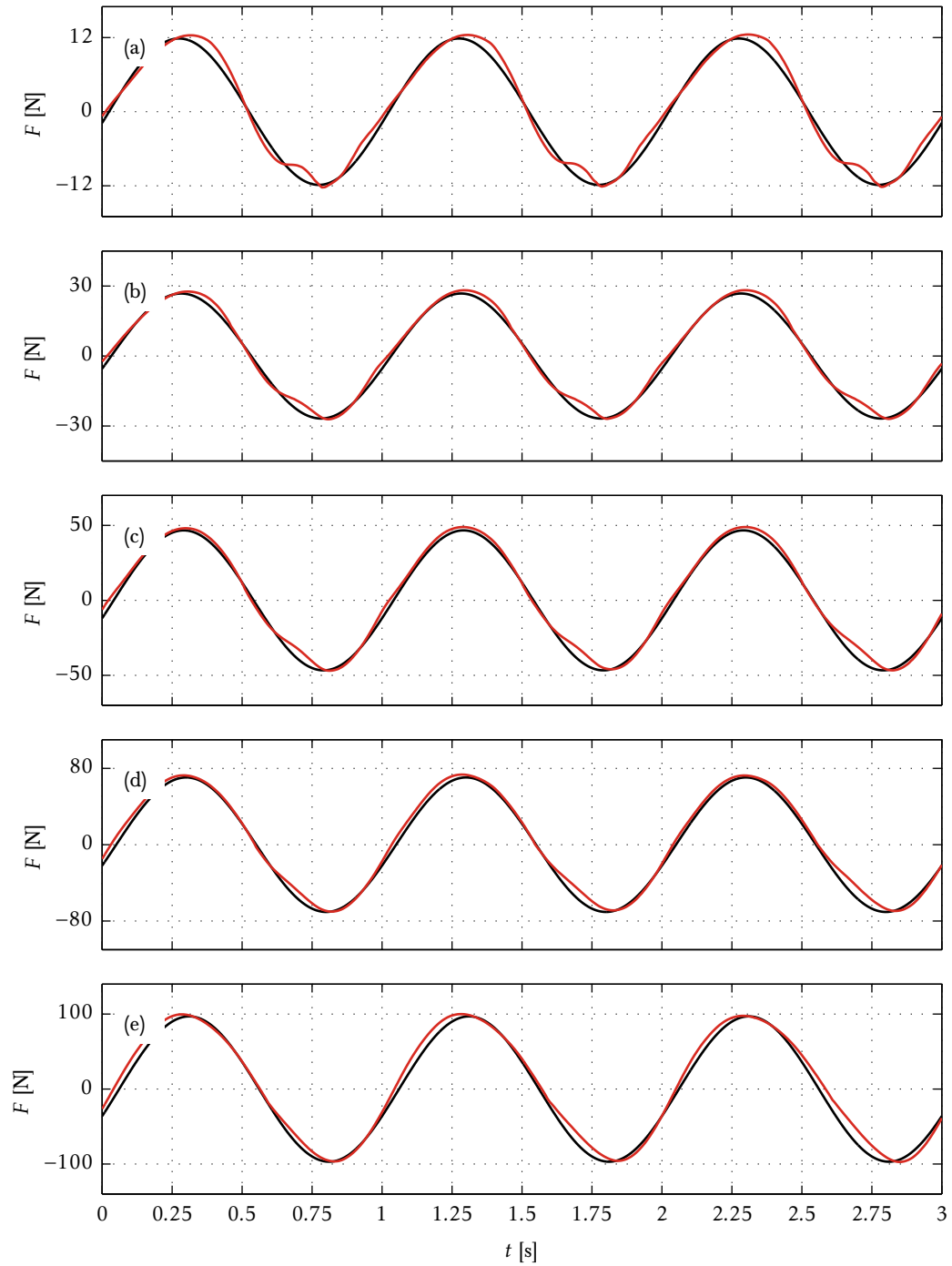


Figure 7.8: The role of varying column diameter; comparisons between the numerically predicted total inline load, F (—) and the first harmonic linear load corresponding to Morison's inertial component (—) in steep regular waves ($T = 1$ s, $Hk/2 = 0.32$) (a) $D/\lambda = 0.064$, (b) $D/\lambda = 0.096$, (c) $D/\lambda = 0.128$, (d) $D/\lambda = 0.160$, and (e) $D/\lambda = 0.192$.

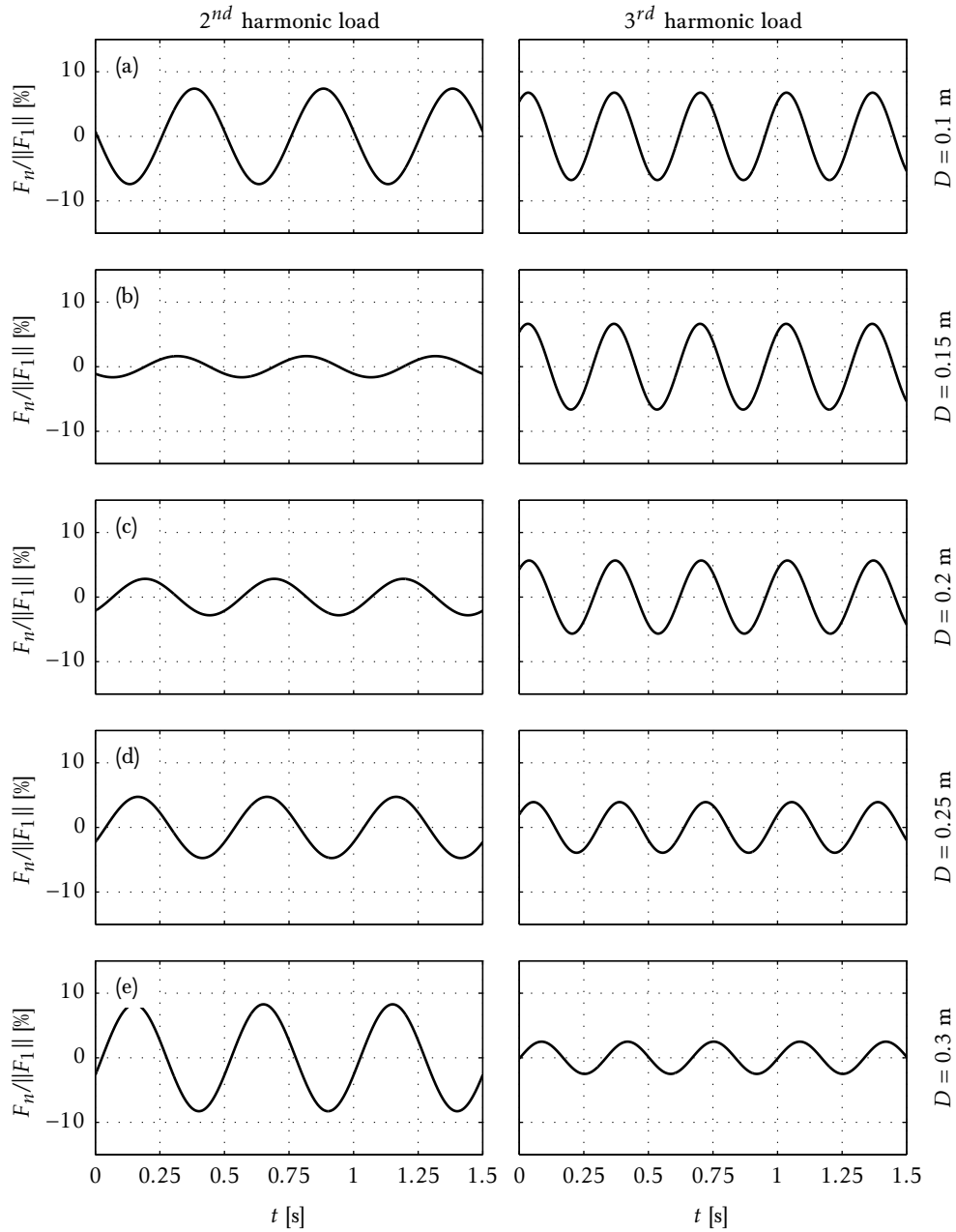


Figure 7.9: The relative magnitude of the second-harmonic (first column) and the third-harmonic (second column) loads, normalised by the first-harmonic, for column of varying diameter subject to steep regular waves ($T = 1$ s, $Hk/2 = 0.32$). (a) $D/\lambda = 0.064$, (b) $D/\lambda = 0.096$, (c) $D/\lambda = 0.128$, (d) $D/\lambda = 0.160$, and (e) $D/\lambda = 0.192$.

waves, the contribution from the third harmonic load becomes increasingly important and hence the solution ‘non-convergent’, as the column diameter reduces.

To further investigate the higher harmonic loading components, Figure 7.10 concerns a variety of column diameters (varying horizontally) and incident wave steepness (varying vertically). In each case histograms are provided showing the maximum amplitude of the second to fifth harmonic loads. The data is normalised with respect to the first harmonic load, F_1 , and the vertical scale fixed to 10%. Expressed in this way, the data allows the identification of ‘convergent’ or ‘non-convergent’ harmonics, assisting in determining the appropriate or valid flow regime for the application of the analytical force models.

For weakly nonlinear incident waves ($Hk/2 = 0.08$ and 0.12 in the lower most two rows), there is a progressive reduction in the magnitude of higher harmonic force components. However, with steeper incident waves, the order of the higher harmonics becomes increasingly ‘non-convergent’. The most problematic cases are perhaps given by $D/\lambda = 0.096$ and $D/\lambda = 0.128$. In these cases, the ‘non-convergence’ of the harmonic content becomes increasingly clear, especially in the steepest wave conditions. For example, in the case of $D/\lambda = 0.096$ and $Hk/2 = 0.32$, the magnitude of the third and fourth harmonics are respectively four times and two times larger than the second harmonic.

For the smallest diameter case ($D/\lambda = 0.064$ in the left hand column), the harmonics remain ‘ordered’ but with an increase in the incident wave steepness, the third and fourth harmonics become increasingly significant; their magnitude being similar to the second harmonic for the steepest incident wave. In this case, the contribution from each higher order harmonic (from second to fourth order) is significant, at around 8% of the amplitude of the linear load.

As the column size increases, especially for the largest column diameter ($D/\lambda = 0.192$ in the right hand column), the harmonics behave in a strongly ‘convergent’ manner for all incident wave conditions. Moreover, the second harmonic is dominant, increasing in amplitude with the incident wave steepness. This is perhaps unsurprising given that this case lies just outside the linear diffraction regime where the contribution from the higher-order diffraction effect (in particular the second-order) dominates with an increase in the wave steepness.

In considering the appropriate flow regime in which to apply the analytical force

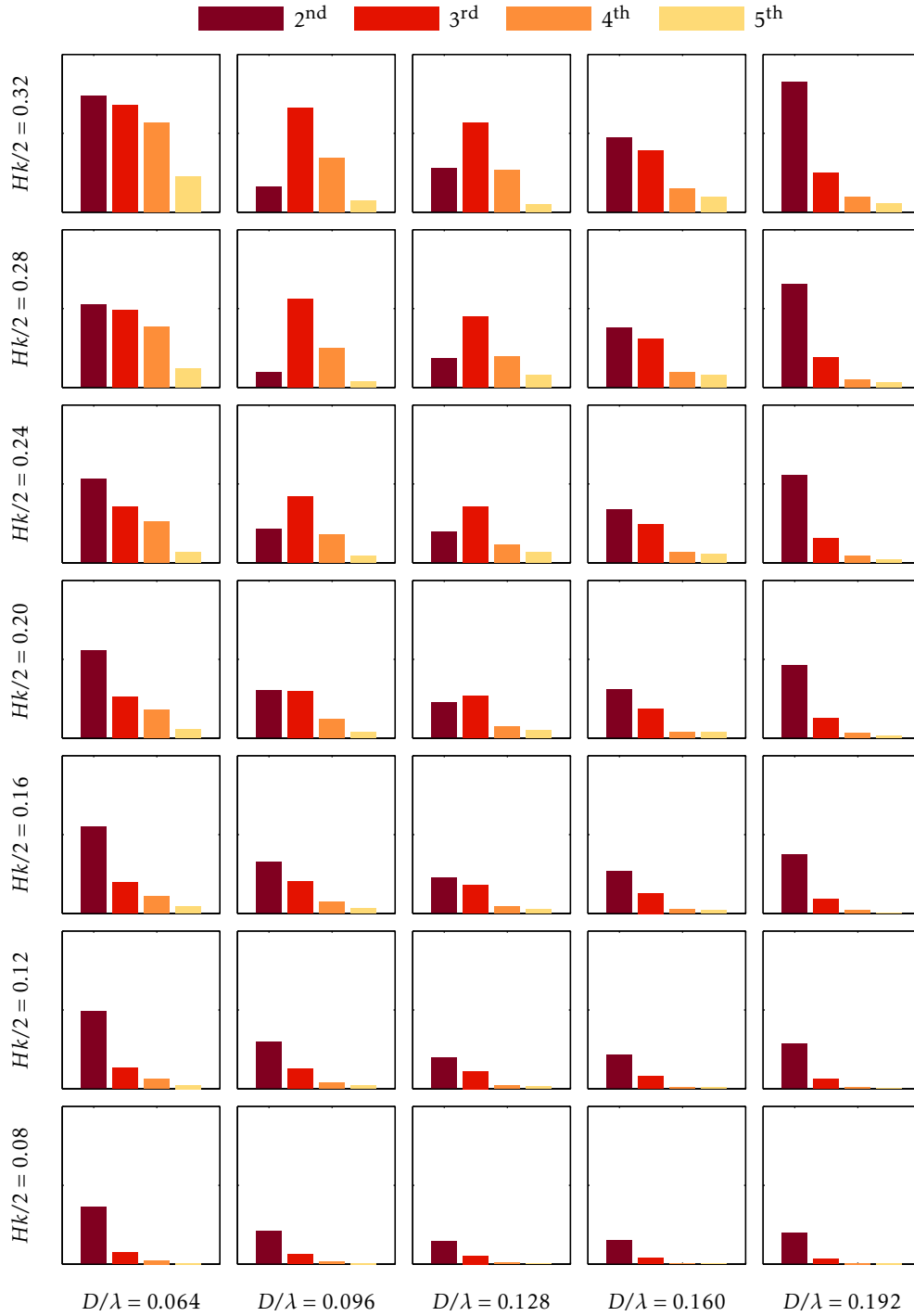


Figure 7.10: The amplitude of the higher harmonic loading expressed as a proportion of F_1 for variations in the wave steepness, $Hk/2$ and the effective column diameter, D/λ . Note: in each subplot the vertical scale has been fixed to maximum value of 10%.

models, the results presented here suggest that weakly nonlinear incident waves ($Hk/2 < 0.16$) will give ‘convergent’ high-order force harmonics. In this regime, the analytic FNV force model can be used with confidence. However, for steeper waves, while the first order force harmonic can perhaps be accurately described by the Morison’s inertia term, (as shown in Figure 7.6(b)), the fact that the higher harmonic components are increasingly ‘non-convergent’ means that the analytical force model will necessarily underestimate the nonlinearity of the applied loading.

7.6.1 Nonlinear Loading and Pressure Integration

Thus far, the total nonlinear loading is calculated by integrating the pressure distribution over the surface of the column, as indicated by Equation (7.12). By dividing this process into distinct vertical slices, further insights into the *secondary loading cycle* can be achieved. To achieve this, the pressure at each time step is integrated radially around the column on each slice of vertical elevation, Δz , where $\Delta z = 0.005$ m. The radially integrated pressure or force per unit length, $f(t, z)$ is presented as a contour plot in Figure 7.11; the $f(t, z)$ value normalised by the corresponding maximum value. The contour plots have been truncated to show data lying in the range $-0.25 \text{ m} \leq z \leq 0.25 \text{ m}$. The figure contrasts four different cases. Figure 7.11(a) corresponds to a small diameter column ($D = 0.1$ m) subject to a linear incident wave ($Hk/2 = 0.08$), while Figure 7.11(b) corresponds to the same column size in steeper incident waves ($Hk/2 = 0.32$). In contrast, Figure 7.11(c) and (d) correspond to the same steep incident waves ($Hk/2 = 0.32$) acting on columns with diameters of $D = 0.2$ m and $D = 0.3$ m respectively. In each subplot, a small inset represents the total load, $F(t)$, evaluated by integrating $f(t, z)$ over the vertical elevation, z . Dashed lines are overlaid on each subplot (and inset) to show the approximate location of the *secondary loading cycle*.

For linear incident waves and a small column diameter of $D = 0.1$ m (Figure 7.11(a)), there is no evidence of a *secondary loading cycle*; the contour of $f(t, z)$ being symmetric. With steeper incident waves, the presence of the *secondary loading cycle* is clearly identified in the inset of Figure 7.11(b). With the presence of the *secondary loading cycle*, the contour of $f(t, z)$ is distorted. Specifically the symmetry is lost and there is intrusion of positive $f(t, z)$ values after the passage of the wave

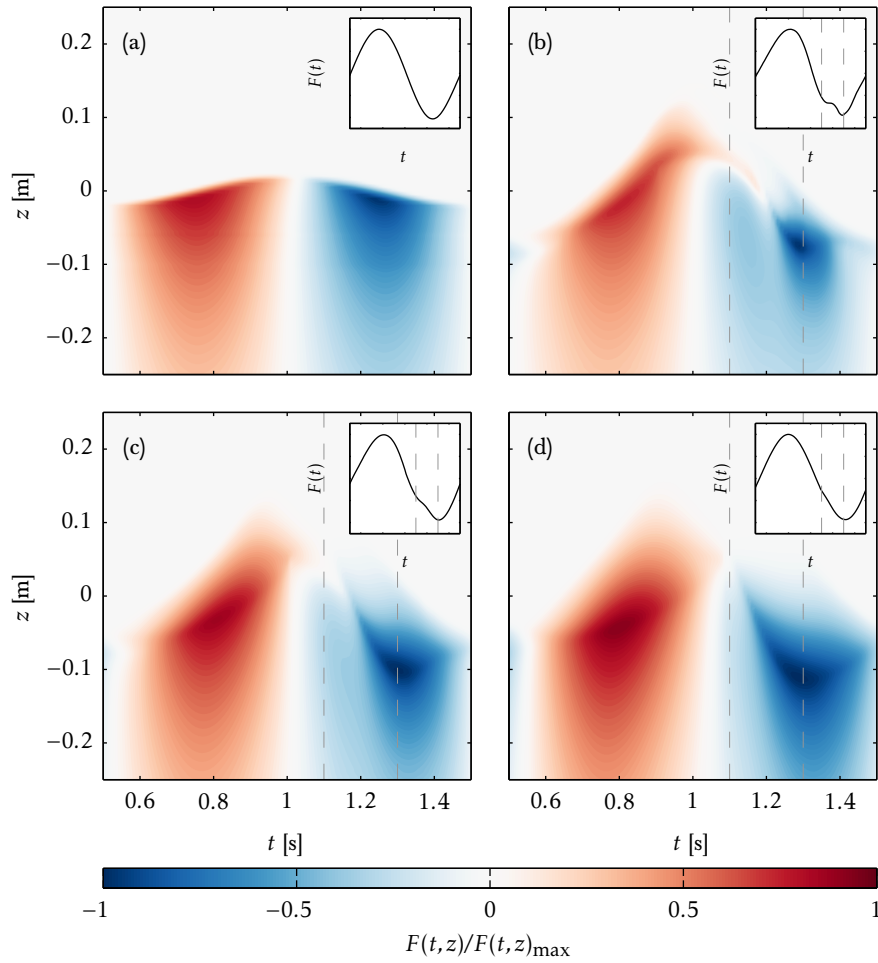


Figure 7.11: Contour plots of the normalized force per unit length, $f(t, z)/f(t, z)_{\max}$, for (a) $D = 0.1$ m and $Hk/2 = 0.08$, (b) $D = 0.1$ m and $Hk/2 = 0.32$, (c) $D = 0.2$ m and $Hk/2 = 0.32$, (d) $D = 0.3$ m and $Hk/2 = 0.32$. In each case, the small inset shows the total in-line force arising from a radial integration.

crest. This distortion occurs close to the SWL and again hints at the local disturbance of the free surface. With an increase in the size of the column and steep incident waves, the *secondary loading cycle* is less pronounced for $D = 0.2$ m (Figure 7.11(c)) and in the case of $D = 0.3$ m (Figure 7.11(d)) is absent. In both cases, the corresponding contours of $f(t, z)$ show a degree of asymmetry, but with markedly less intrusion of positive $f(t, z)$ values.

Figure 7.11 provides a qualitative assessment of the pressure field responsible for the formation of the *secondary loading cycle*. To further define the pressure contribution to the *secondary loading cycle*, Figure 7.12 presents the depth-integrated pressures over different sections of the column. In each of the three sub-plots (a-c) the left figure provides a sketch showing the integration limit, while the right figure shows the time-history of the corresponding inline load. For reference purposes, the total depth-integrated load is also overlaid on this figure.

The integration of the pressure from the SWL to the top of the column ($0 \text{ m} \leq z \leq \eta$) is given in Figure 7.12(a). This integration shows a localised loading occurring at around $t = 0.9$ s. This is associated with that part of the wave profile that exceeds the SWL. Interestingly, whilst there is evidence of a secondary loading peak, at $t = 1.2$ s, it is very small. A second integration covering the limit $-0.1 \text{ m} \leq z \leq 0 \text{ m}$ is given by Figure 7.12(b). Within this integration limit, the presence of the *secondary loading cycle* is clearly observed; the magnitude of this cycle being comparable to the fully depth-integrated value. Lastly, the integration between $-d \leq z \leq -0.1 \text{ m}$ on Figure 7.12(c) captures the underlying loading pattern, but exhibits little to no occurrence of a *secondary loading cycle*. It is clear from these results that the *secondary loading cycle* is governed by pressure close to the free surface. As the *secondary loading cycle* occurs after the arrival of the wave crest, the pressure distributions responsible for its formation lie within a small section of the column just below the SWL.

7.6.2 Nonlinear Loading and High-Frequency Wave Scattering

In the previous section, the integration of the hydrodynamic pressure acting on various part of the column suggests that the *secondary loading cycle* will be influenced by, or caused by, a fluid disturbance close to the free surface. This is further investigated

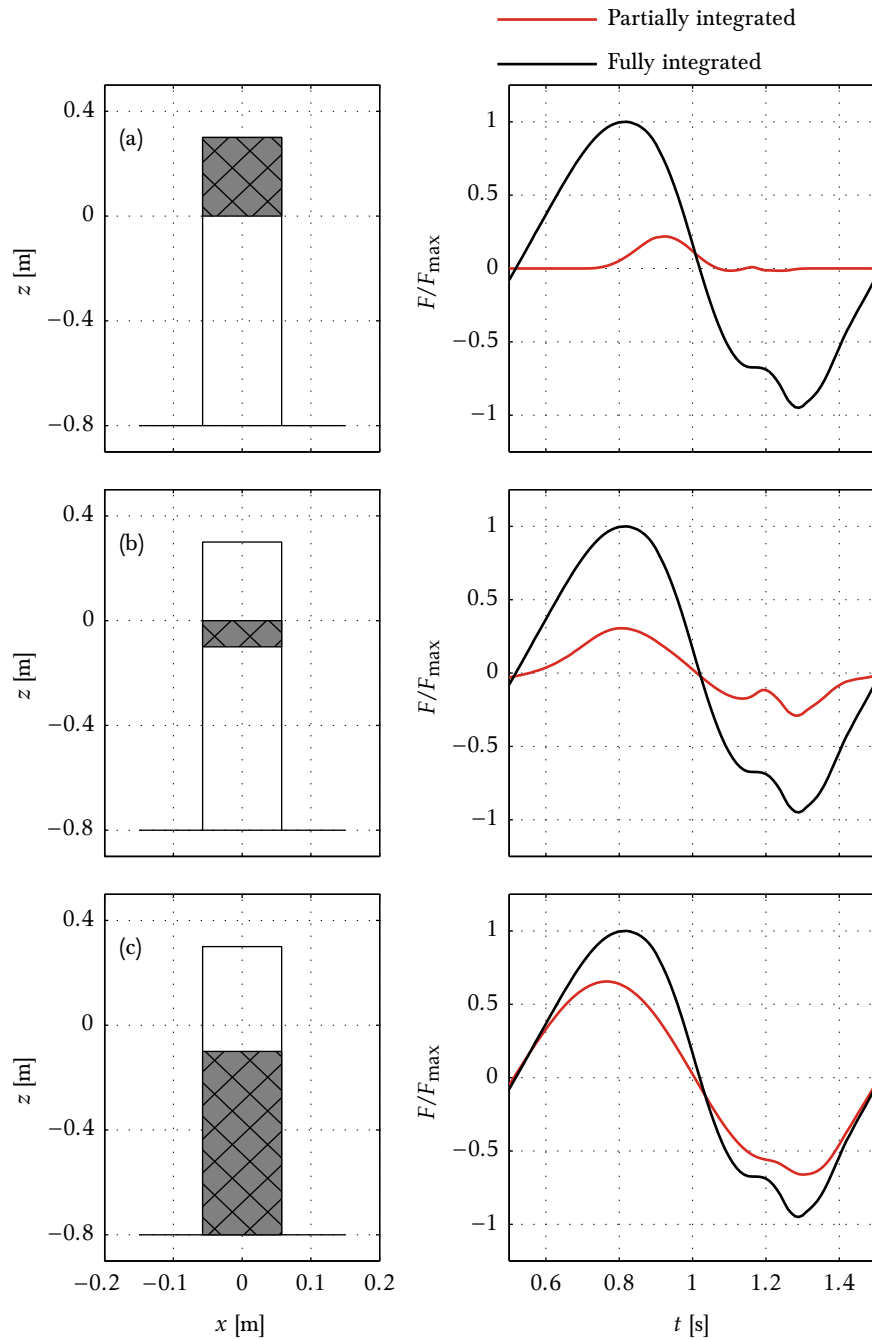


Figure 7.12: The nonlinear loads acting on a column of diameter $D = 0.1$ m in regular waves of steepness $Hk/2 = 0.32$; the time history of the loading derived from the integration of the numerically predicted pressures. Left column, a sketch of the integration limits. Right column, the load time-histories based upon integration limits defined by (a) $0 \leq z \leq z_{\max}$, (b) $-0.1 \leq z \leq 0$, and (c) $-d \leq z \leq -0.1$.

in this section, the aim being to seek a correlation between the *secondary loading cycle* and high-frequency wave scattering; the latter providing a possible physical explanation for the former. Once again, three columns of different diameters are considered, corresponding to 0.1 m, 0.2 m and 0.3 m.

Comparisons between nonlinear loading, in particular, the *secondary loading cycle* and the high-frequency wave scattering are given in Figures 7.13 to 7.15. In each figure, subplot (a) provides the time-history of nonlinear loading, $F(t)$, normalised with respect to its maximum magnitude, $F(t)_{\max}$. The normalised incident wave elevation, $\eta_{wo}(t)/\eta_{wo}(t)_{\max}$ recorded at the centre of the column is also provided on the same subplot. Following on, subplots (i)-(ix) provide snapshots of the total wave field, $\eta_w(x, y)$, given at different phases of the incident wave cycle. These snapshots are provided to allow the identification of important features of the scattered wave field that could be linked to the nonlinear loading.

From the previous discussions it has already been established that the *secondary loading cycle* originates from pressure arising at, or just below, the SWL (Figure 7.12). Moreover, the nature of the *secondary loading cycle* is such that it must involve the rapid reversal of pressure within this region. Contrasting the force time-histories given in Figure 7.13(a) ($D = 0.1$ m), Figure 7.14(a) ($D = 0.2$ m) and Figure 7.15(a) ($D = 0.3$ m) confirms that the magnitude of the *secondary loading cycle* is largest for $D = 0.1$ m, substantially reduced for $D = 0.2$ m and effectively absent for $D = 0.3$ m. Considering the associated snapshots of $\eta_w(x, y)$, particularly snapshots (iv), (v) and (vi) since these are centered about the *secondary loading cycle*, it is clear that the occurrence of this unexpected high-frequency loading is closely related to the evolution of the “*Type-2*” waves and the circulation of the fluid about the column. Specifically, the reduction in the magnitude of the *secondary loading cycle* is consistent with the suppression or delayed propagation of *Type-2* waves from the downstream side of the column.

For the $D = 0.1$ m case (Figure 7.13) it is clear that with the passage of the wave crest, the movement of the fluid around the column leads to a build-up of fluid on the back face of the column ($\theta = 0$). This begins in snapshot (ii), but is most pronounced between (iii) and (iv). With the build-up of fluid, positive pressures will be created at high elevations on the back face of the column. These will contribute to the rapidly reducing positive forcing which the column experiences between (ii) and

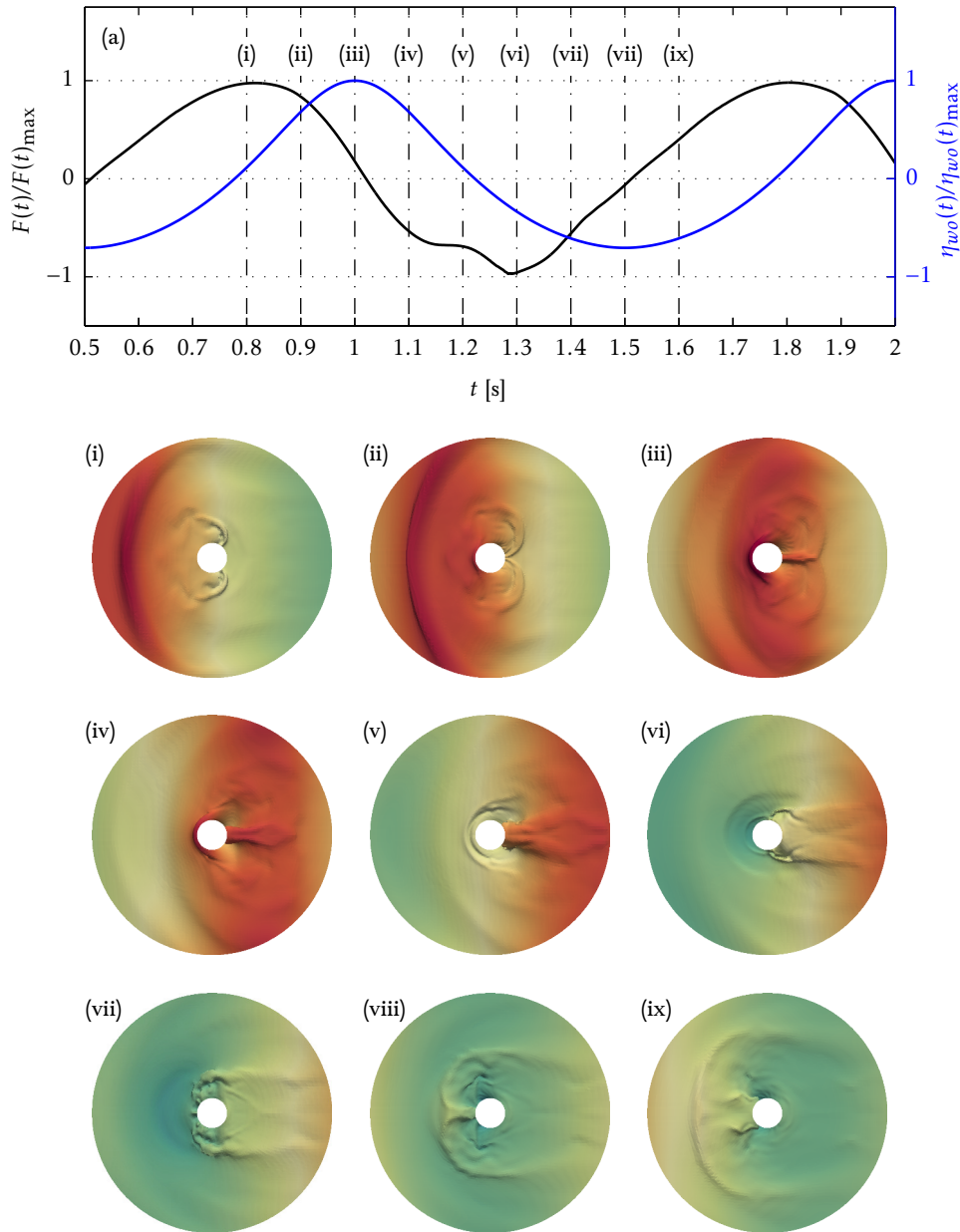


Figure 7.13: (a) Normalised nonlinear loading, $F(t)/F(t)_{\max}$ (—) and incident water surface elevation at the center of the column, $\eta_{wo}(t)/\eta_{wo}(t)_{\max}$ (—) in steep regular waves ($T = 1$ s, $Hk/2 = 0.32$) for a column of diameter $D = 0.1$ m. (i)-(ix) Snapshots of the water surface elevation, $\eta_w(x, y)$, at the time intervals indicated on part(a).

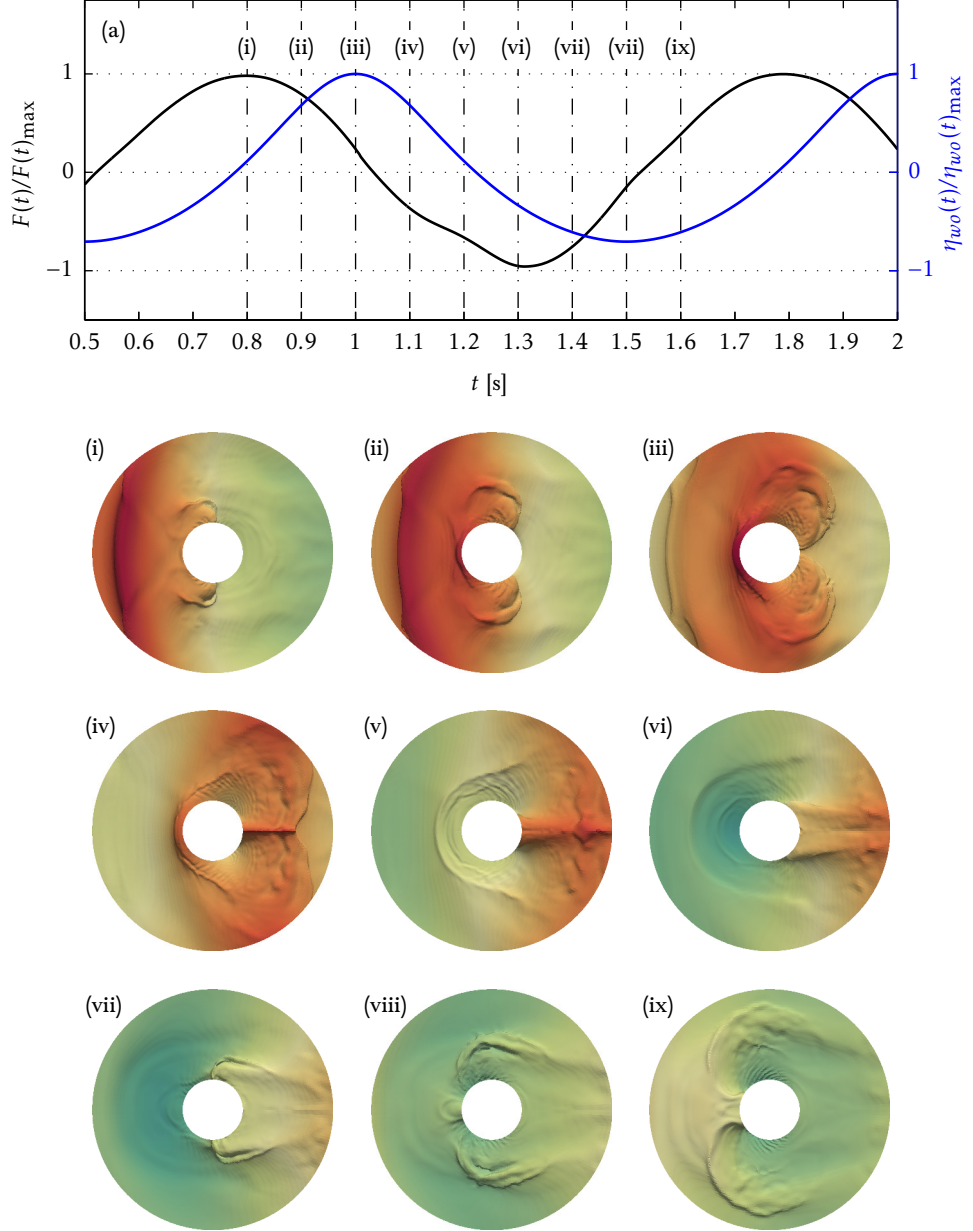


Figure 7.14: (a) Normalised nonlinear loading, $F(t)/F(t)_{\max}$ (—) and incident water surface elevation at the center of the column, $\eta_{wo}(t)/\eta_{wo}(t)_{\max}$ (—) in steep regular waves ($T = 1$ s, $Hk/2 = 0.32$) for a column of diameter $D = 0.2$ m. (i)-(ix) Snapshots of the water surface elevation, $\eta_w(x, y)$, at the time intervals indicated on part(a).

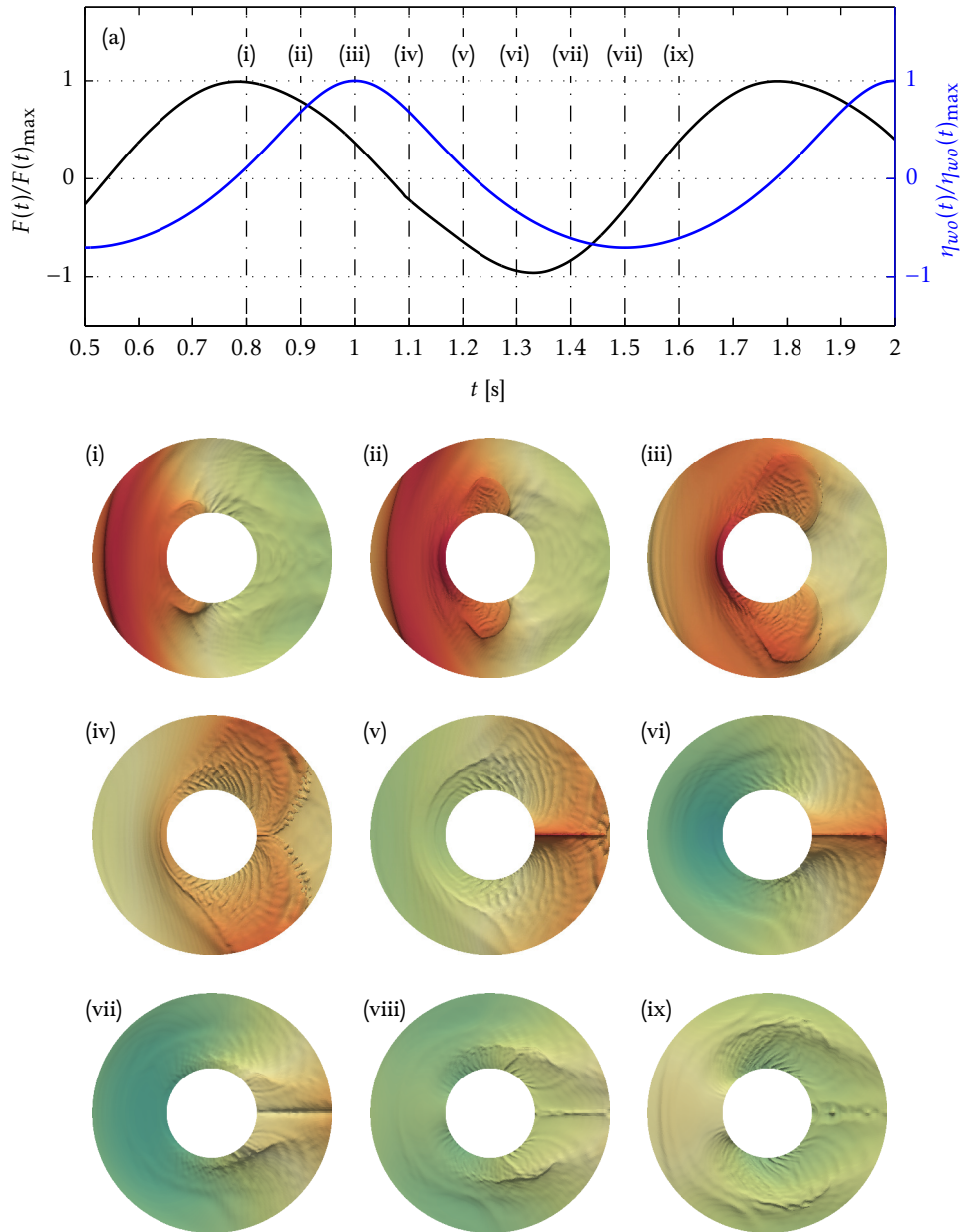


Figure 7.15: (a) Normalised nonlinear loading, $F(t)/F(t)_{\max}$ (—) and incident water surface elevation at the center of the column, $\eta_{wo}(t)/\eta_{wo}(t)_{\max}$ (—) in steep regular waves ($T = 1$ s, $Hk/2 = 0.32$) for a column of diameter $D = 0.3$ m. (i)-(ix) Snapshots of the water surface elevation, $\eta_w(x, y)$, at the time intervals indicated on part(a).

(iii), with the total loads becoming negative between (iii) and (iv). Whilst the loading time-history is undoubtedly dominated by the first-harmonic Morison's load, the fact that the loading time-history, $F(t)$, is highly asymmetric either side of the peak load (Figure 7.13(a)) is in part explained by the rear-surface effects. More importantly, when the *Type-2* waves begin to propagate back around the column, between snapshots (iv) and (v) on Figure 7.13, the build-up of water and the associated pressures are rapidly reduced. This rapid reduction in the negative rear-surface forcing provides the origins for the high-frequency *secondary loading cycle* observed between snapshots (iv) and (vi). Once the *Type-2* waves have advanced to the so-called shoulder position (at $\theta \pm \pi/2$) in snapshot (iv), the disturbance at the backface of the column has been dissipated and there is no further evidence of the *secondary loading cycle*.

With an increasing column diameter, $D = 0.2$ m on Figure 7.14 and (particularly) $D = 0.3$ m on Figure 7.15, the build-up of fluid on the back face is substantially reduced as is the magnitude of the *Type-2* waves that subsequently propagate around the column. Evidence of this is provided in the relevant snapshots, (iv), (v) and (vi), given in Figures 7.14 and 7.15 which show a marked contrast with the equivalent plots given on Figure 7.13. In these cases, the reduced importance of the *Type-2* waves and the circulation of fluid around the column ensures that the *secondary loading cycle* is also substantially reduced. Indeed, for $D = 0.3$ m on Figure 7.15, there is no evidence of these high-frequency loading components. Further evidence to support this view is given by the fact that for $D = 0.2$ m and $D = 0.3$ m, the loading time-history, $F(t)$ on Figure 7.14(a) and 7.15(a) are increasingly symmetric about their respective maximum.

To further explore the pressure reversals responsible for the *secondary loading cycle*, Figures 7.16 to 7.18 provides contour plots of $P(\theta, t)$ for the $D = 0.1$ m, $D = 0.2$ m and $D = 0.3$ m columns respectively. In each case, four sub-plots are provided corresponding to vertical elevations defined by $z = 0$ m, $z = 0.025$ m, $z = 0.05$ m, and $z = 0.075$ m; the pressures in each case being normalized by the maximum pressure occurring at that elevation. In Figure 7.16 ($D = 0.1$ m) the high pressures arising due to the run-up of fluid on the front face of the column ($\theta/\pi = 1$) are clearly apparent at $t \approx 0.95$ s. This arises at all four elevations and occurs just before point (iii) on Figure 7.13(a). Likewise, the movement of fluid around the column leading to increased elevations on the back face ($\theta/\pi = 0$) and hence higher pressures at $t = 1.05$ s (between points (iii) and (iv) on Figure 7.13(a)) is also well defined.

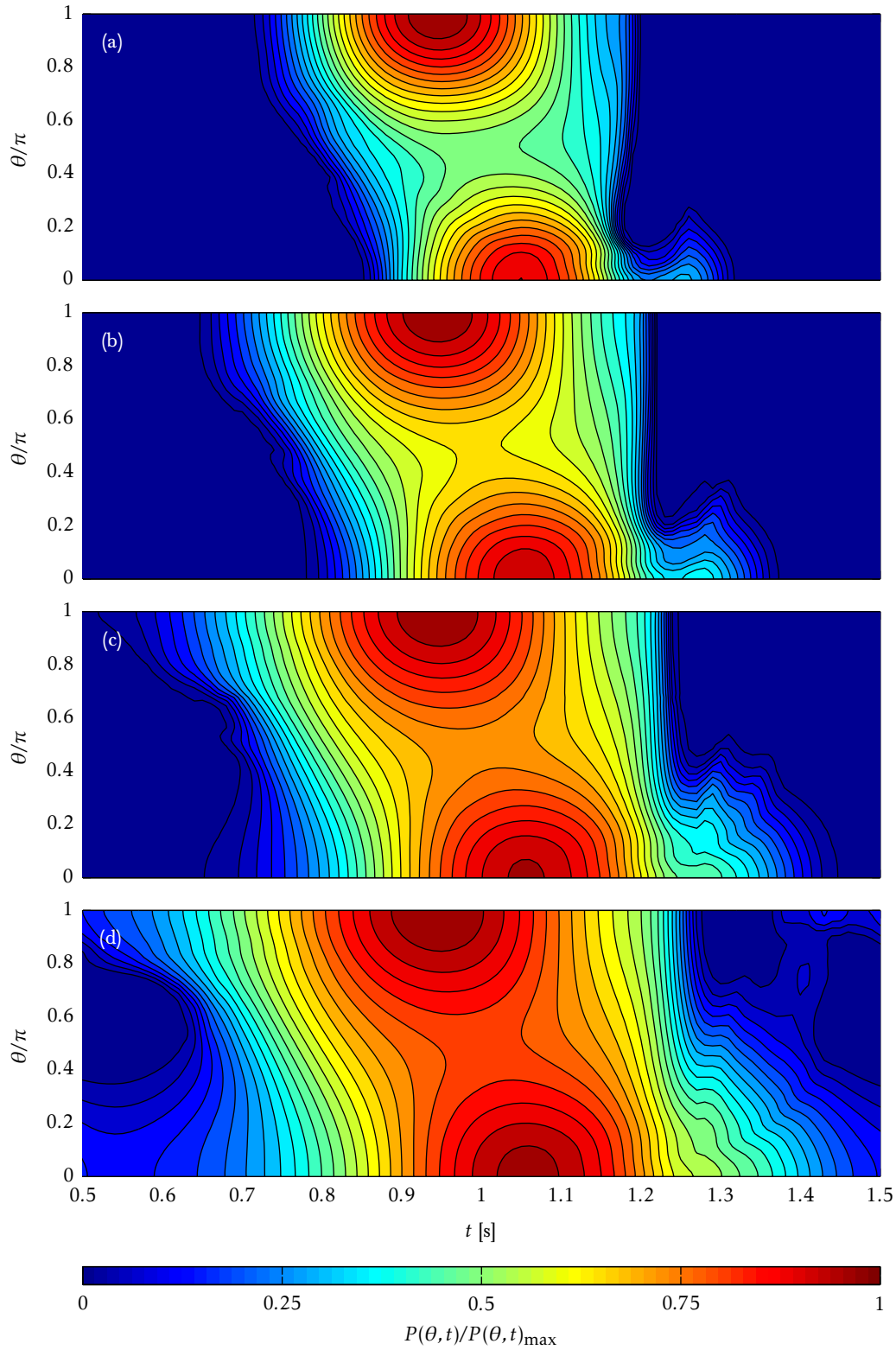


Figure 7.16: Contour plots of normalized pressure, $P(\theta, t)/P(\theta, t)_{\max}$, around the column ($D = 0.1$ m) in steep regular wave ($T = 1.0$ s, $Hk/2 = 0.32$) at various vertical elevations (a) $z = 0$ m, (b) $z = -0.025$ m, (c) $z = -0.05$ m, and (d) $z = -0.075$ m.

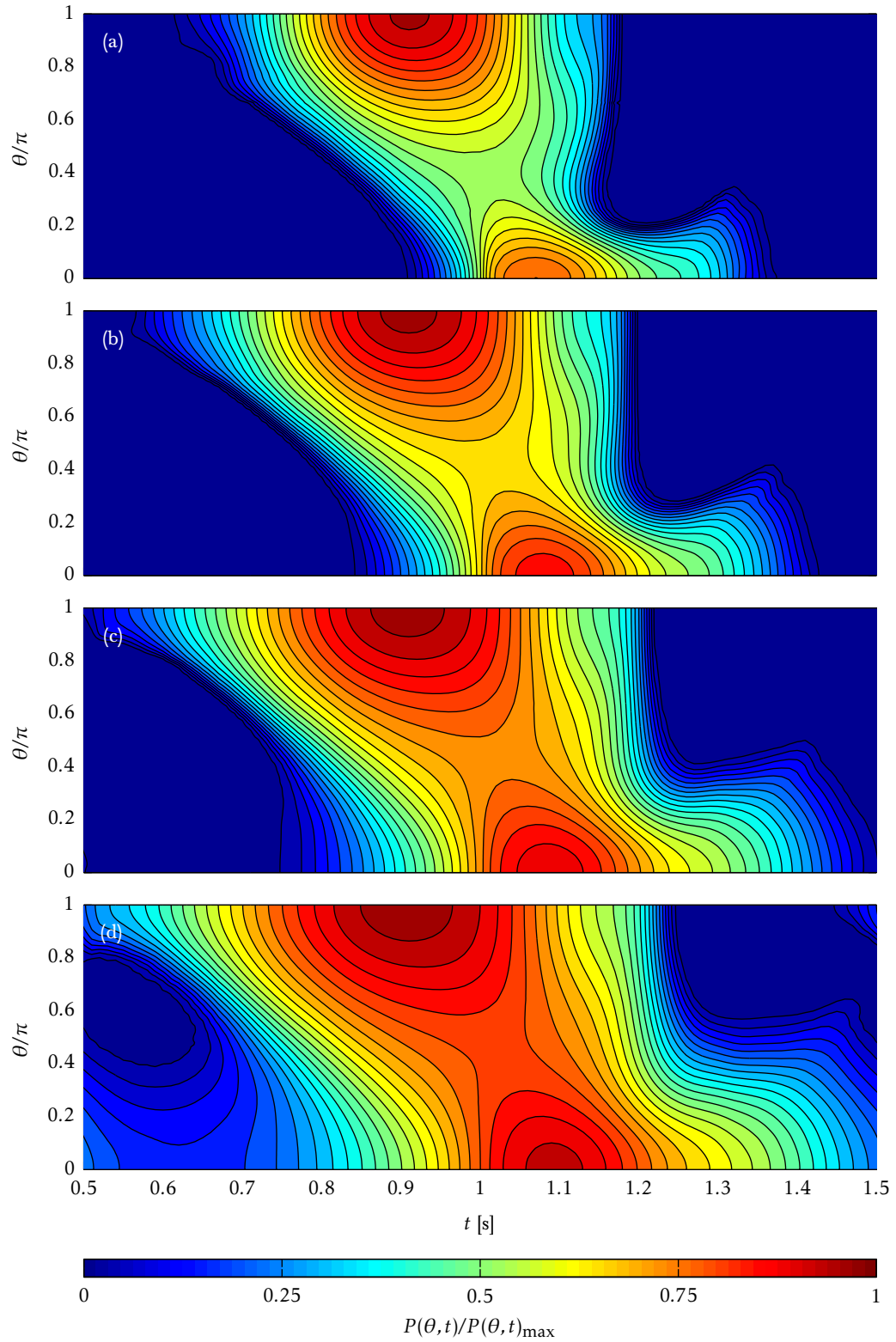


Figure 7.17: Contour plots of normalized pressure, $P(\theta, t)/P(\theta, t)_{\max}$, around the column ($D = 0.2$ m) in steep regular wave ($T = 1.0$ s, $Hk/2 = 0.32$) at various vertical elevations (a) $z = 0$ m, (b) $z = -0.025$ m, (c) $z = -0.05$ m, and (d) $z = -0.075$ m.

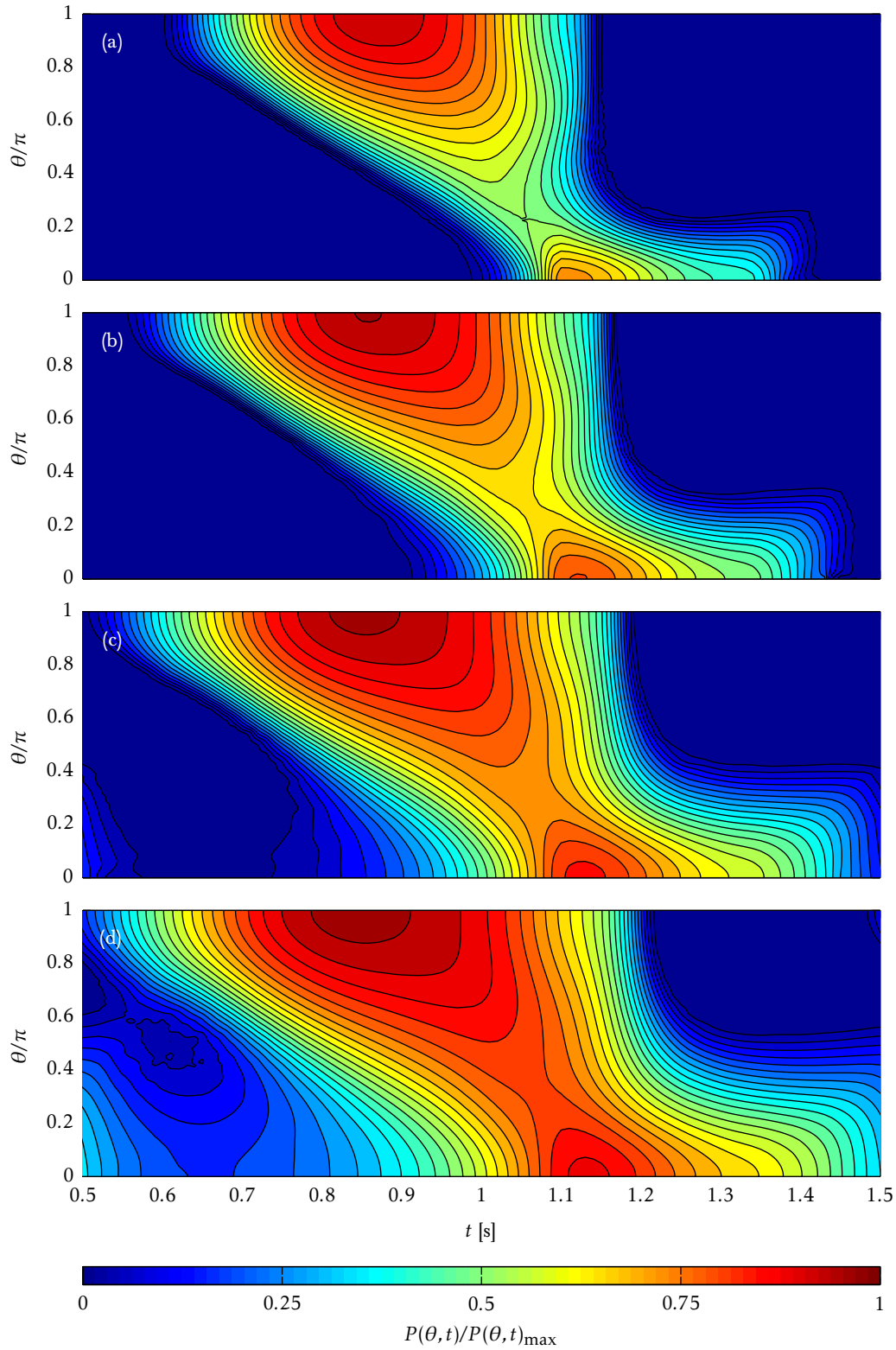


Figure 7.18: Contour plots of normalized pressure, $P(\theta, t)/P(\theta, t)_{\max}$, around the column ($D = 0.3$ m) in steep regular wave ($T = 1.0$ s, $Hk/2 = 0.32$) at various vertical elevations (a) $z = 0$ m, (b) $z = -0.025$ m, (c) $z = -0.05$ m, and (d) $z = -0.075$ m.

More importantly, as far as the *secondary loading cycle* is concerned, the increased elevations and hence increased pressures associated with the propagation of the *Type-2* wave around the column is also apparent. This arises at all four elevations and this is clearest in the region $0 \leq \theta/\pi \leq 0.4$ and occurs for $1.25\text{ s} \leq t \leq 1.35\text{ s}$, corresponding to point (v) and (vi) on Figure 7.13(a). In considering this pressure distribution, it is clear that there is a rapid pressure reversal, or pressure oscillation, on the back face of the column. This can be observed for θ values lying in the range $0 \leq \theta/\pi \leq 0.4$ and for z elevations defined by $0\text{ m} \leq z \leq -0.05\text{ m}$.

Interestingly, the equivalent data presented on Figure 7.17 and Figure 7.18 relating to $D = 0.2\text{ m}$ and $D = 0.3\text{ m}$ respectively again confirm the increased pressures associated with the run-up on the front and back faces, but the pressure reversals occurring immediately after are either reduced (for $D = 0.2\text{ m}$ on Figure 7.17) or eliminated (for $D = 0.3\text{ m}$ on Figure 7.18). In considering Figure 7.17, the pressure reversals are still apparent, but they are of substantially smaller magnitude and tend to occur at larger θ values. This latter point is important since it leads to a reduced contribution to the total inline force, $F(t)$. In contrast, Figure 7.18 exhibits no pressure reversals at any elevations. In this case ($D = 0.3\text{ m}$), the absence of any distinct *Type-2* wave scattering (Figure 7.15) ensures that there is no secondary pressure peak following the reduced run-up on the back face.

Explicit evidence of these pressure reversals is provided on Figures 7.19 to 7.22. These provide time-histories of the normalised pressure, $P(t)/P(t)_{\max}$, at elevations $z = 0\text{ m}$, $z = -0.025\text{ m}$, $z = -0.05\text{ m}$ and $z = -0.075\text{ m}$ respectively. In each case, data is provided at five radial locations corresponding to $\theta = 0\text{ rad}$, $\theta = 0.1\text{ rad}$, $\theta = 0.2\text{ rad}$, $\theta = 0.3\text{ rad}$ and $\theta = 0.4\text{ rad}$; with comparisons made between $D = 0.1\text{ m}$, $D = 0.2\text{ m}$ and $D = 0.3\text{ m}$ cases. It is clear from these figures that the pressure reversals during the *secondary loading cycle* are largest in the $D = 0.1\text{ m}$ case, are substantially reduced in the $D = 0.2\text{ m}$ case and do not occur in the $D = 0.3\text{ m}$ case. This is entirely consistent with the *Type-2* wave scattering or the associated movement of fluid around the columns indicated in Figures 7.13 to 7.15. Moreover, the fact that the pressure reversals decay rapidly with the depth is consistent with the origin of the applied loading (Figure 7.12) and physically realistic in the sense that *Type-2* waves evolving around the column have a very short wavelength. As a result, any motion or pressures associated with these disturbances will decay exponentially with depth.

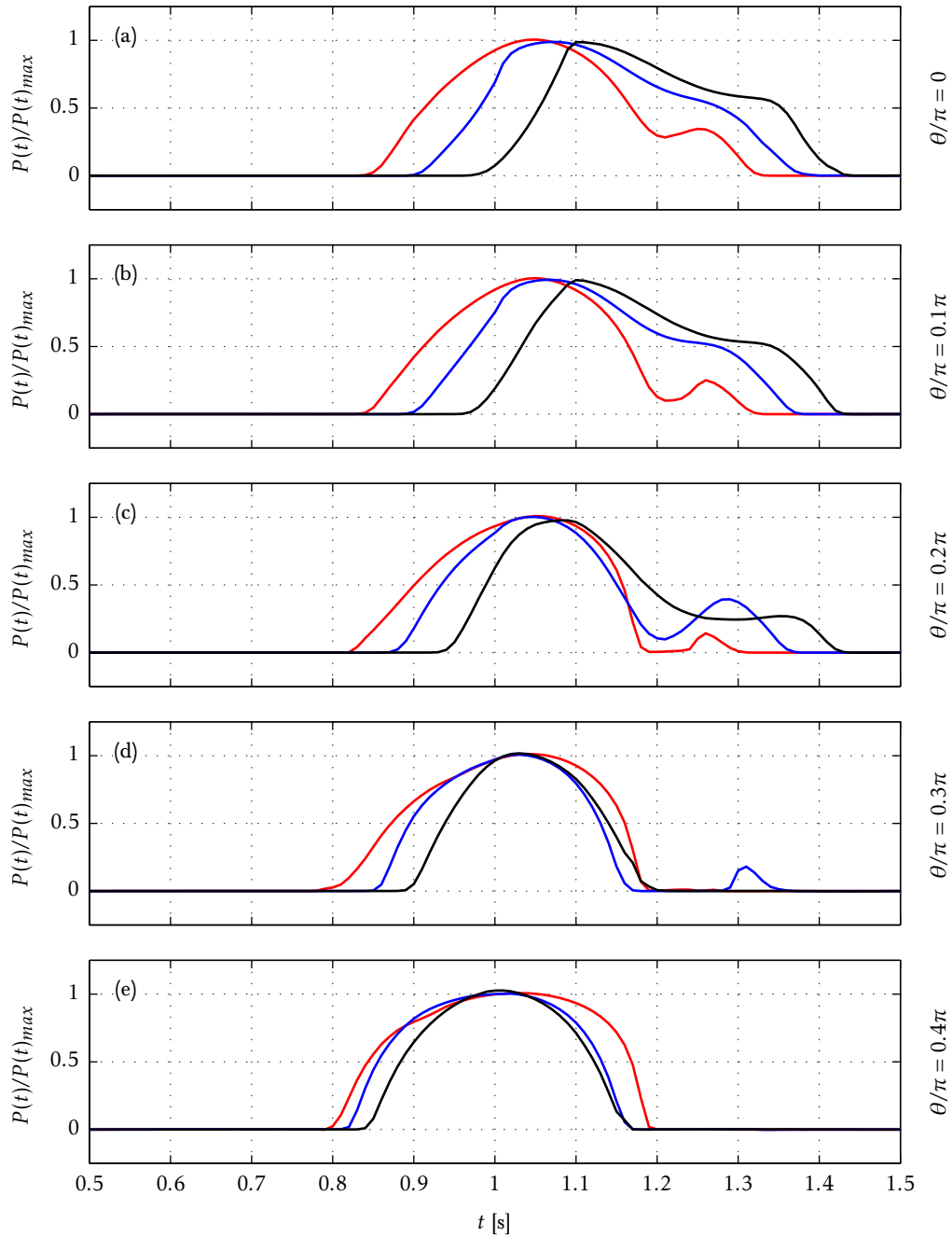


Figure 7.19: Normalised pressure time-histories, $P(t)/P(t)_{\max}$ on the surface of the column at $z = 0$ m for radial locations: (a) $\theta = \pi$ rad, (b) $\theta = 0.1\pi$ rad, (c) $\theta = 0.2\pi$ rad, (d) $\theta = 0.3\pi$ rad, and (e) $\theta = 0.4\pi$ rad. Legend: — $D = 0.1$ m, — $D = 0.2$ m, — $D = 0.3$ m.

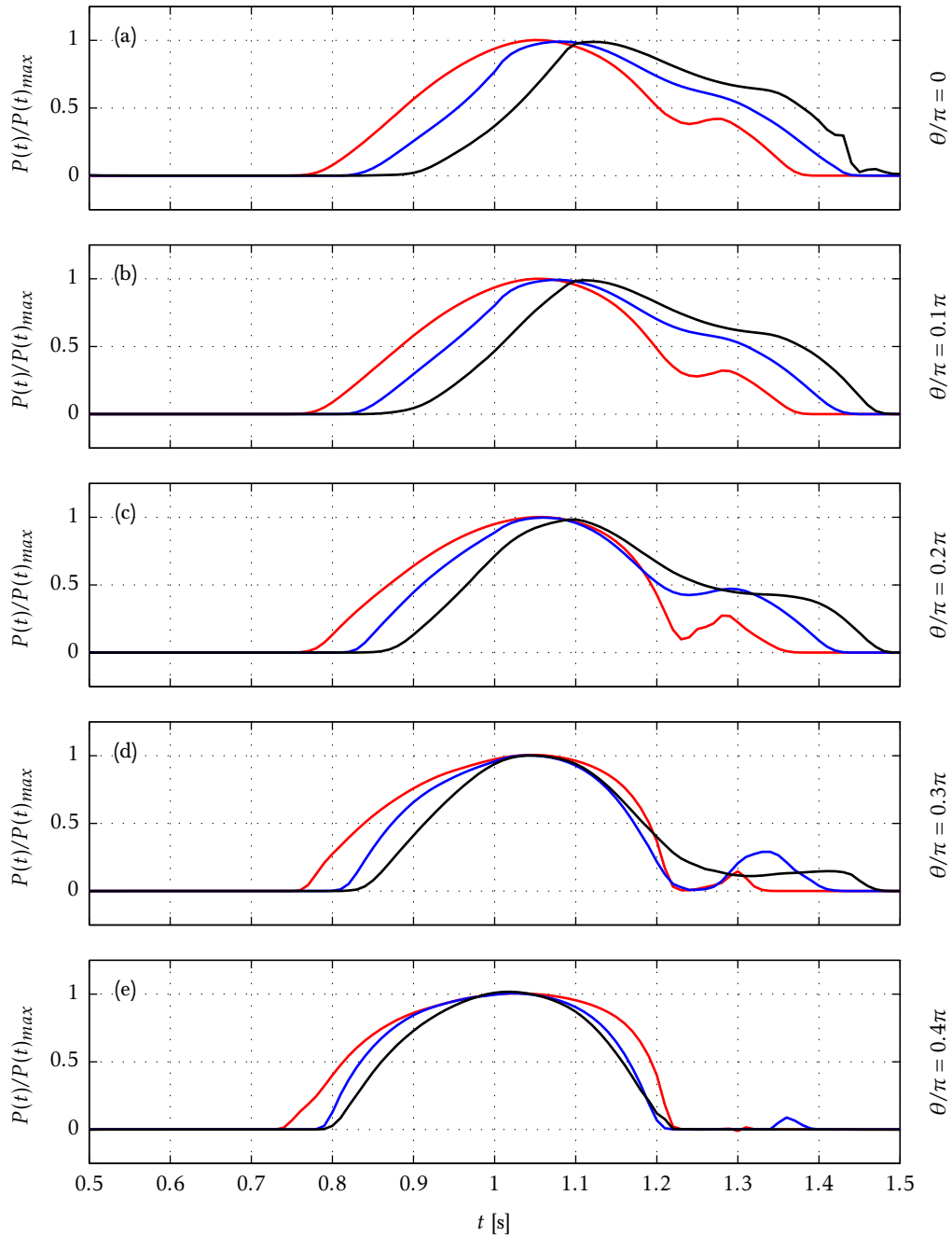


Figure 7.20: Normalised pressure time-histories, $P(t)/P(t)_{\max}$ on the surface of the column at $z = -0.025$ m for radial locations: (a) $\theta = \pi$ rad, (b) $\theta = 0.1\pi$ rad, (c) $\theta = 0.2\pi$ rad, (d) $\theta = 0.3\pi$ rad, and (e) $\theta = 0.4\pi$ rad. Legend: — $D = 0.1$ m, — $D = 0.2$ m, — $D = 0.3$ m.

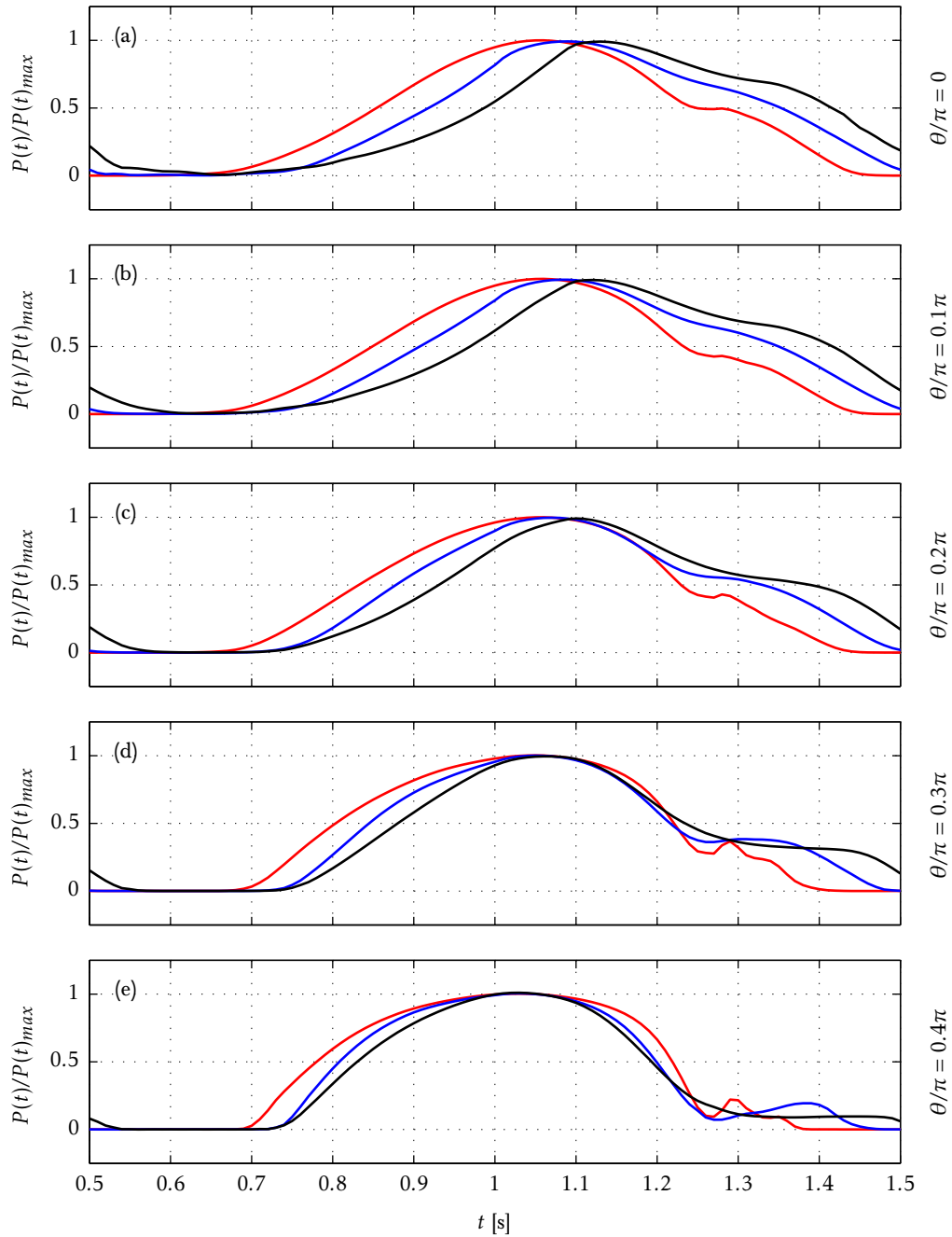


Figure 7.21: Normalised pressure time-histories, $P(t)/P(t)_{\max}$ on the surface of the column at $z = -0.05$ m for radial locations: (a) $\theta = \pi$ rad, (b) $\theta = 0.1\pi$ rad, (c) $\theta = 0.2\pi$ rad, (d) $\theta = 0.3\pi$ rad, and (e) $\theta = 0.4\pi$ rad. Legend: — $D = 0.1$ m, — $D = 0.2$ m, — $D = 0.3$ m.

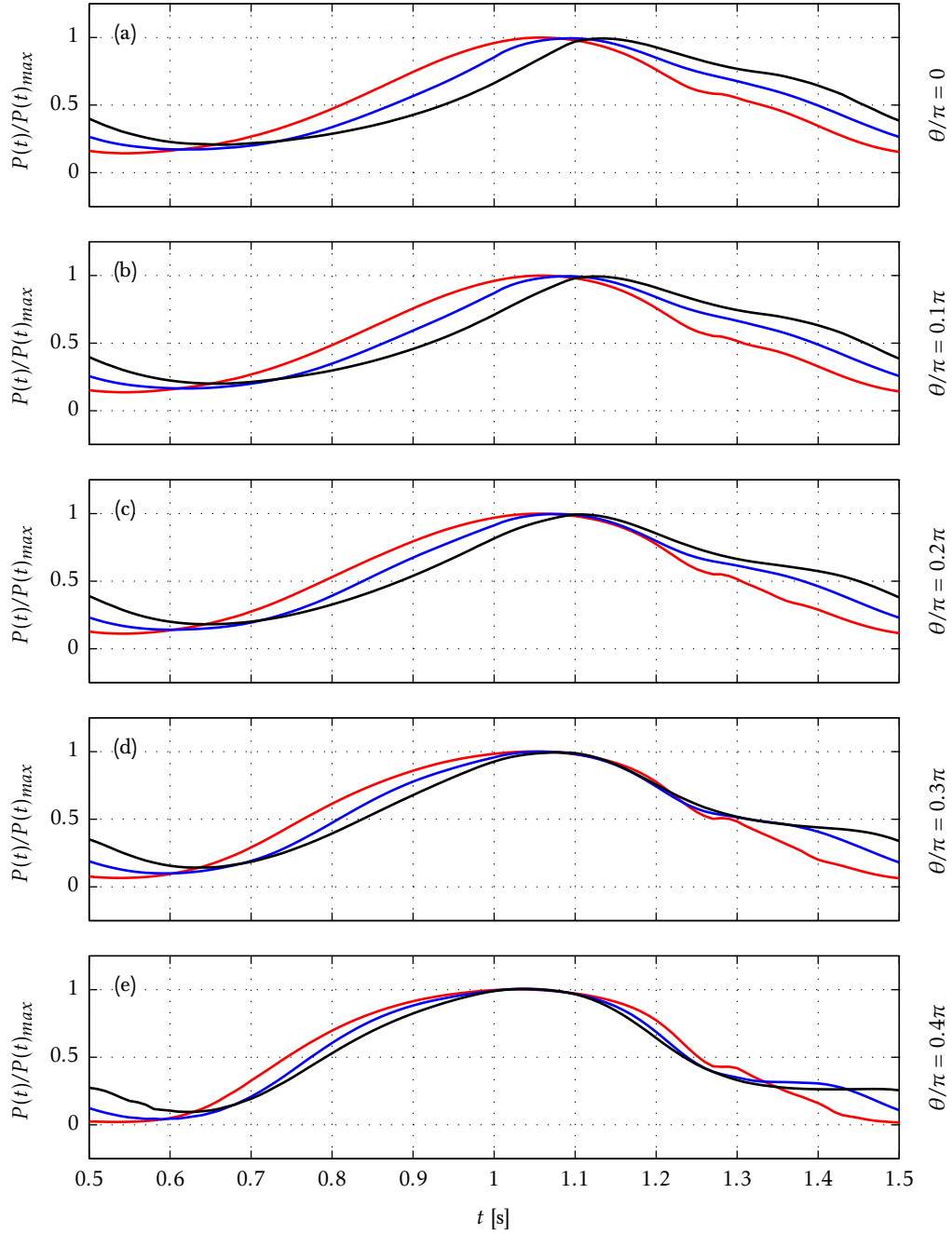


Figure 7.22: Normalised pressure time-histories, $P(t)/P(t)_{\max}$ on the surface of the column at $z = -0.075$ m for radial locations: (a) $\theta = \pi$ rad, (b) $\theta = 0.1\pi$ rad, (c) $\theta = 0.2\pi$ rad, (d) $\theta = 0.3\pi$ rad, and (e) $\theta = 0.4\pi$ rad. Legend: — $D = 0.1$ m, — $D = 0.2$ m, — $D = 0.3$ m.

7.6.3 The Possible Effect of *Type-1** Wave Scattering

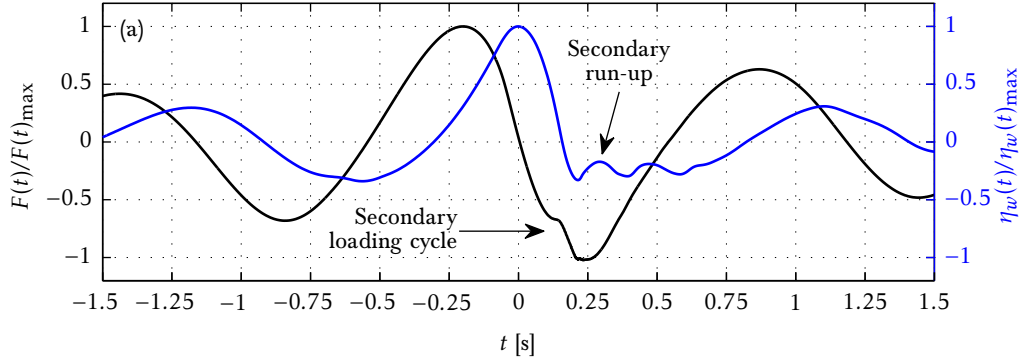


Figure 7.23: Time histories of the normalized nonlinear loading, $F(t)/F(t)_{\max}$ (—) and the water surface elevation, $\eta_w(t)/\eta_w(t)_{\max}$ (—) on the upstream face of the column ($r = D/2$ and $\theta = \pi$ rad) in steep focused wave; $T_p = 1.2$ s, $Ak_p = 0.18$ and $D = 0.1$ m.

Having established a clear linkage between the high-frequency *Type-2* wave scattering and the nonlinear wave loading, it is now important to consider whether there is any additional effect associated with the *Type-1** waves identified in Section 6.3.2. In order to do this, Figure 7.23 superimpose time-histories of the nonlinear loading, $F(t)$, and the water surface elevation, $\eta_w(t)$, on the upstream face of the column. Both the loading and the surface elevation have been normalized by their respective maximum values. From this figure, it is clear that the *secondary loading cycle* is most evident at $t = 0.15$ s after the focused wave event. In contrast, the *Type-1** wave scattering does not occur until $t = 0.3$ s when the *secondary run-up* produces a local maximum in the surface elevation. On close inspection, the timing between the *secondary loading cycle* and *secondary run-up* reveals that the latter occurs after the former ends. As such, the generation of *Type-1** wave scattering has no influence on the formation of the *secondary loading cycle*. Nevertheless, it is important to stress that it has the potential to be important in respect of the unexpected increase in the water surface elevation, especially in the immediate vicinity of the column.

7.7 Dynamic Response

In the previous section, it has been shown that the interaction between steep incident waves and a vertical surface-piercing column will produce significant nonlinear loading. In some cases, this is manifested in the form of a *secondary loading cycle*

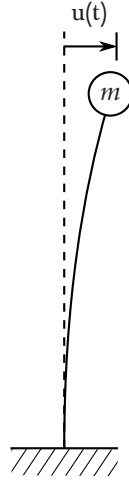


Figure 7.24: Single degree of freedom model of a mass, m , attached to a massless cantilever beam.

which exhibits a high degree of nonlinearity and is generated as a consequence of the pressure difference around the column due to high-frequency wave scattering associated with the *Type-2* wave. A frequency analysis of this *secondary loading cycle* indicates the importance of the higher harmonic components that may provoke the transient structural deflections associated with *ringing*. In an experimental context, the dynamic response of a vertical surface piercing column in steep incident waves has been extensively investigated by, amongst others, Chaplin et al. (1997). In this section, the numerically calculated nonlinear wave loading was used as the input to a single degree of freedom model to investigate the importance of this loading in term of a dynamic excitation.

7.7.1 Single Degree of Freedom Model

A vertical cantilever structure, such as a surface-piercing column, can be represented as a single degree of freedom system. This consists of mass, m , attached to a massless cantilever beam (Figure 7.24). Under the application of a forcing, $F(t)$, the linear equation of motion for the forced dynamics system is given by:

$$m \frac{\partial^2 s}{\partial t^2} + c \frac{\partial s}{\partial t} + k_s s = F(t), \quad (7.14)$$

where s is the structural displacement or response, m is the lumped mass, c is the damping coefficient, k_s is the stiffness of the beam and $F(t)$ is the applied force. By dividing Equation (7.14) by the mass, the equation can conveniently be written in

terms of the damping ratio, ζ , and the natural angular frequency, Ω_o as follows:

$$\frac{\partial^2 s}{\partial t^2} + 2\zeta\Omega_o \frac{\partial s}{\partial t} + \Omega_o^2 s = \hat{F}(t), \quad (7.15)$$

where

$$\hat{F}(t) = \frac{F(t)}{m}, \quad (7.16)$$

$$\zeta = \frac{c}{2\Omega_o m}, \quad (7.17)$$

$$\Omega_o = \sqrt{\frac{k_s}{m}}. \quad (7.18)$$

If, in the first instance, we assume that the damping coefficient is zero and the loading imposed on the structure harmonic, then

$$F(t) = F_o \sin(\Omega t), \quad (7.19)$$

where F_o is the amplitude of the applied loading and Ω is the angular frequency of the loading. In this case the analytical solution to Equation (7.14) is given by

$$s(t) = \frac{F_o}{k_s} \frac{1}{[1 - (\Omega/\Omega_o)^2]} \left(\sin(\Omega t) - \frac{\Omega}{\Omega_o} \sin(\Omega_o t) \right). \quad (7.20)$$

At this point, it should be noted that the solution given in Equation (7.20) is only valid for harmonic loading. In the case of nonlinear loading, the solution to the dynamic system can be obtained by first decomposing the applied loading into its Fourier components. Since Equation (7.20) is linear, it can be applied to each of the decomposed loading components and the solution to the nonlinear loading problem obtained from a linear summation. An alternative method is to solve the dynamic system using a numerical method. In the present study, Equation (7.14) is solved numerically using an explicit *Runge-Kutta* method. Using the MATLAB built-in solver ODE45, Equation (7.14) is solved using an adaptive time stepping algorithm. This ensures that the numerical solution is both accurate and stable. To confirm the results of this numerical solution, preliminary calculations were carried out to compare the results obtained from the numerical method and the analytical solution. In all cases, very good agreement was obtained; an example set of results being

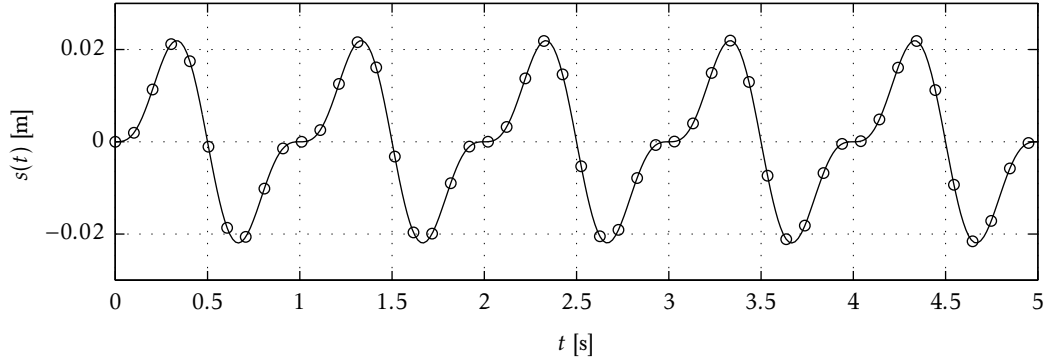


Figure 7.25: Dynamic response, $s(t)$, for a harmonic loading, $F(t)$, ($F_0 = 1 \text{ N}$, $\Omega = 1 \text{ rad s}^{-1}$, $T_n = 1 \text{ s}$, $m = 1 \text{ kg}$) with comparison between analytical solution ($\circ$) and numerical simulation ($—$).

provided in Figure 7.25.

7.7.2 Dynamic Response due to Nonlinear Loading

Within this section, data describing the response of a single degree of freedom system to nonlinear wave loading is presented; the purpose being to determine whether any structural excitation is due to the presence of the *secondary loading cycle*. Figure 7.26 concerns the response of a $D = 0.1 \text{ m}$ column subject to steep regular waves ($T = 1.0 \text{ s}$ and $Hk/2 = 0.32$); the four sub-plots corresponding to different natural period, T_n , such that T_n/T lies in the range $1/2 \leq T_n/T \leq 1/5$. In each plot, the dynamic response, based upon the solution of Equation (7.14), is compared to an ‘un-amplified response’ based on the statically applied load. This is subsequently referred to as the equivalent static (scaled) response and is defined by

$$s_{st}(t) = \frac{F(t)}{k_s}. \quad (7.21)$$

To compare these results, both are normalised with respect to the maximum ‘static response’; the difference between the two defining the dynamic amplification experienced by the structure. In all these initial cases, the damping ratio is set to zero. Although this is practically unrealistic, it serves to highlight the extent of the nonlinear response.

In considering the data presented on Figure 7.26, it is clear that the dynamic excitation of the structure increases with time. This is due to the absence of any damping. Moreover, it is also clear that the largest amplification occurs for $T/T_n =$

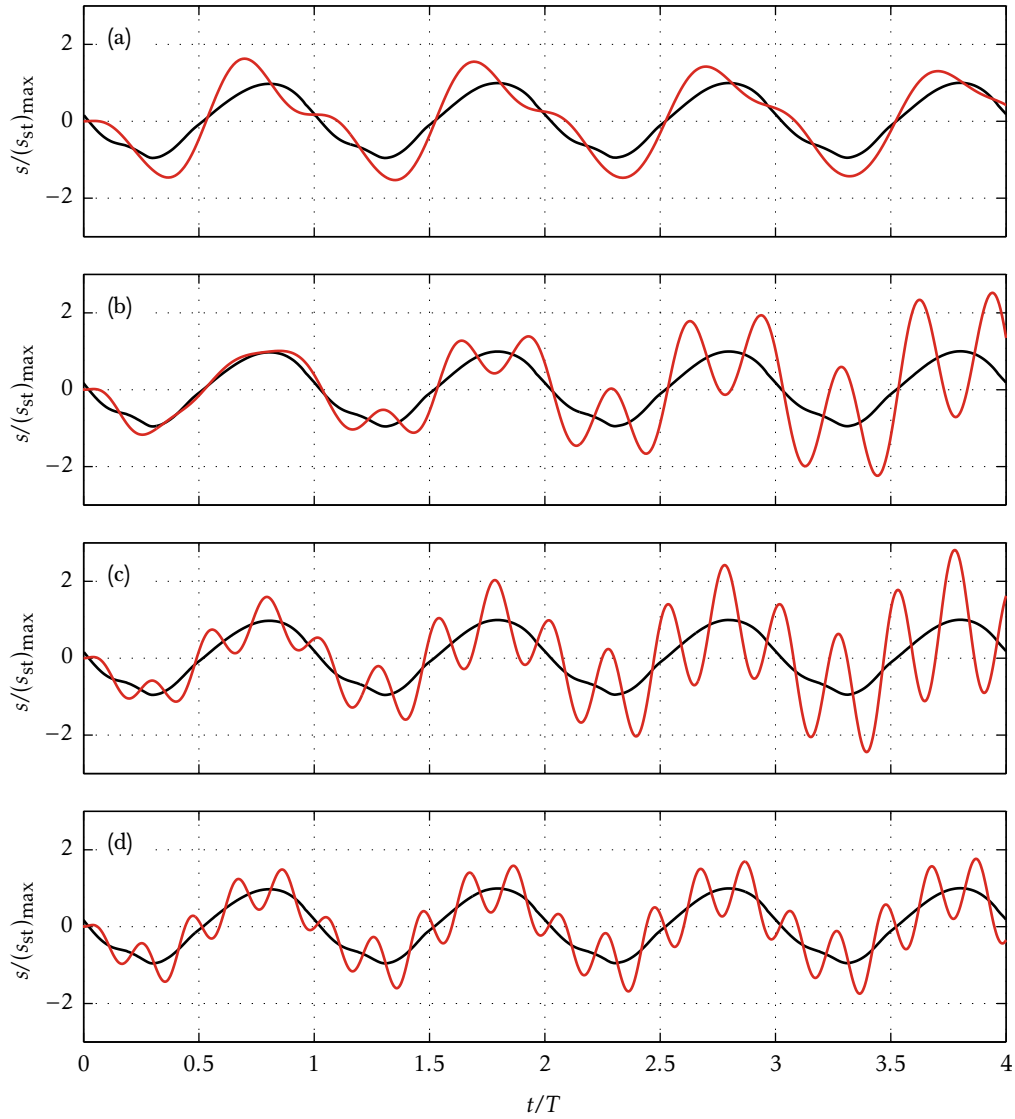


Figure 7.26: Dynamic response for a single vertical column in steep regular waves ($T = 1.0\text{ s}$, $D = 0.1\text{ m}$, $Hk/2 = 0.32$) with natural period, (a) $T_n/T = 1/2$ (b) $T_n/T = 1/3$ (c) $T_n/T = 1/4$ (d) $T_n/T = 1/5$.
 Legends: — 'static response', — dynamic response, both normalised by $(S_{st})_{max}$.

1/4 on Figure 7.26(c). This is perhaps unexpected, not least because an analysis of the nonlinear loads in exactly this case (given in the top left of Figure 7.10) shows that the second-harmonic load is the largest. However, the results of the harmonic reconstruction undertaken in Section 7.5.1 (see Figure 7.5), shows that the *secondary loading cycle* is associated with a significant fourth-harmonic component. The present results suggest that this fourth-harmonic component, acting high in the water column (Figure 7.12) drives the maximum dynamic amplification.

To further explore the effect of the *secondary loading cycle*, Figure 7.27 again concerns a $D = 0.1$ m diameter column in steep regular waves ($T = 1.0$ s and $Hk/2 = 0.32$) with a natural period defined by $T_n/T = 1/4$. In this case, five sub-plots are provided. These are based upon the reconstructed harmonic loading, each successive plot incorporating an additional harmonic component. As such, Figure 7.27(a) is based on the first harmonic of the numerically predicted load, Figure 7.27(b) the first and second harmonics, Figure 7.27(c) the first, second and third, and so on. With the inclusion of the fourth harmonic in Figure 7.27(d), the *secondary loading cycle* become apparent in the ‘static response’. Since this is close to the natural period of the structure, a large dynamic excitation is observed. Furthermore, the inclusion of the fifth harmonic in Figure 7.27(e) shows relatively little change in the dynamic amplification of the overall response. This result provides evidence of the role of the *secondary loading cycle* in the dynamic excitation of the structure. Moreover, although the fourth harmonic load is less than either the second or the third, this is more than compensated for by the fact that this loading component acts in the vicinity of the water surface and therefore has a very large effective moment arm.

Further evidence of the importance of the *secondary loading cycle* in provoking a dynamic excitation is provided in Figure 7.28. This again concerns a single column with natural period $T_n/T = 1/4$. In this case, four sub-plots are provided in which the column is subject to regular waves of increasing steepness, the four cases lying in the range of $0.08\text{ m} \leq Hk/2 \leq 0.32\text{ m}$. With an increase in the incident wave steepness, the *secondary loading cycle* becomes evident in Figure 7.28(c) and (d). With the presence of the *secondary loading cycle*, the dynamic excitation is substantially increased. Evidence of this is given in the two steepest wave cases. Comparison with the harmonic content of the applied forcing presented in the first column of Figure 7.10 shows that with an increase in the wave steepness, the higher harmonic

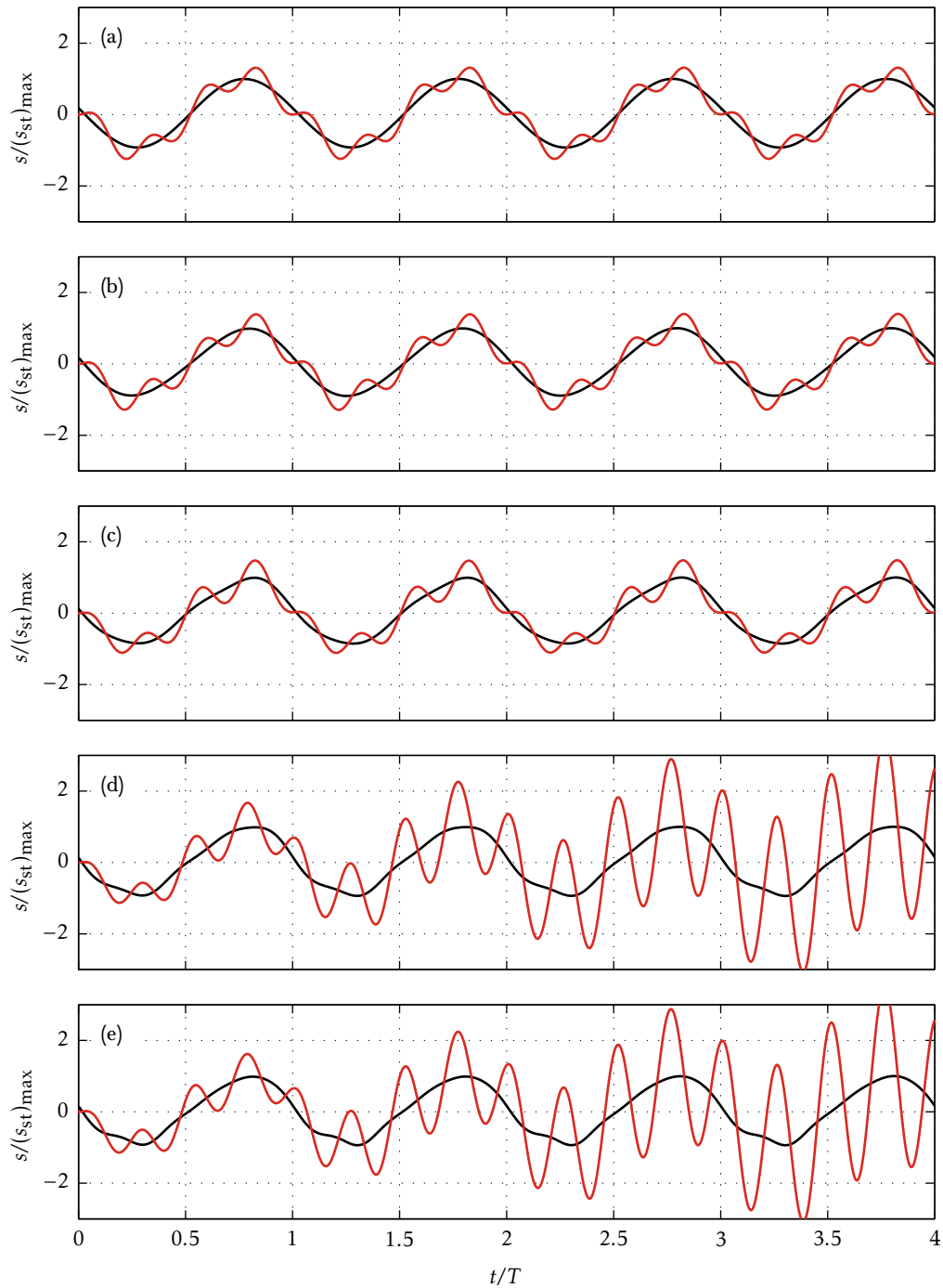


Figure 7.27: Dynamic excitation of a vertical column with natural period, $T_n/T = 1/4$, in a steep regular waves ($T = 1.0\text{ s}$, $D = 0.1\text{ m}$, $Hk/2 = 0.32$); calculations based upon a reconstructed force signal incorporating progressively higher harmonics (a) 1st harmonic, (b) 1st and 2nd harmonics, (c) 1st, 2nd and 3rd harmonics, (d) 1st, 2nd, 3rd and 4th harmonics (r) 1st, 2nd, 3rd, 4th and 5th harmonic. Legends: — ‘static response’, — dynamic response, both normalised by $(S_{st})_{\max}$.

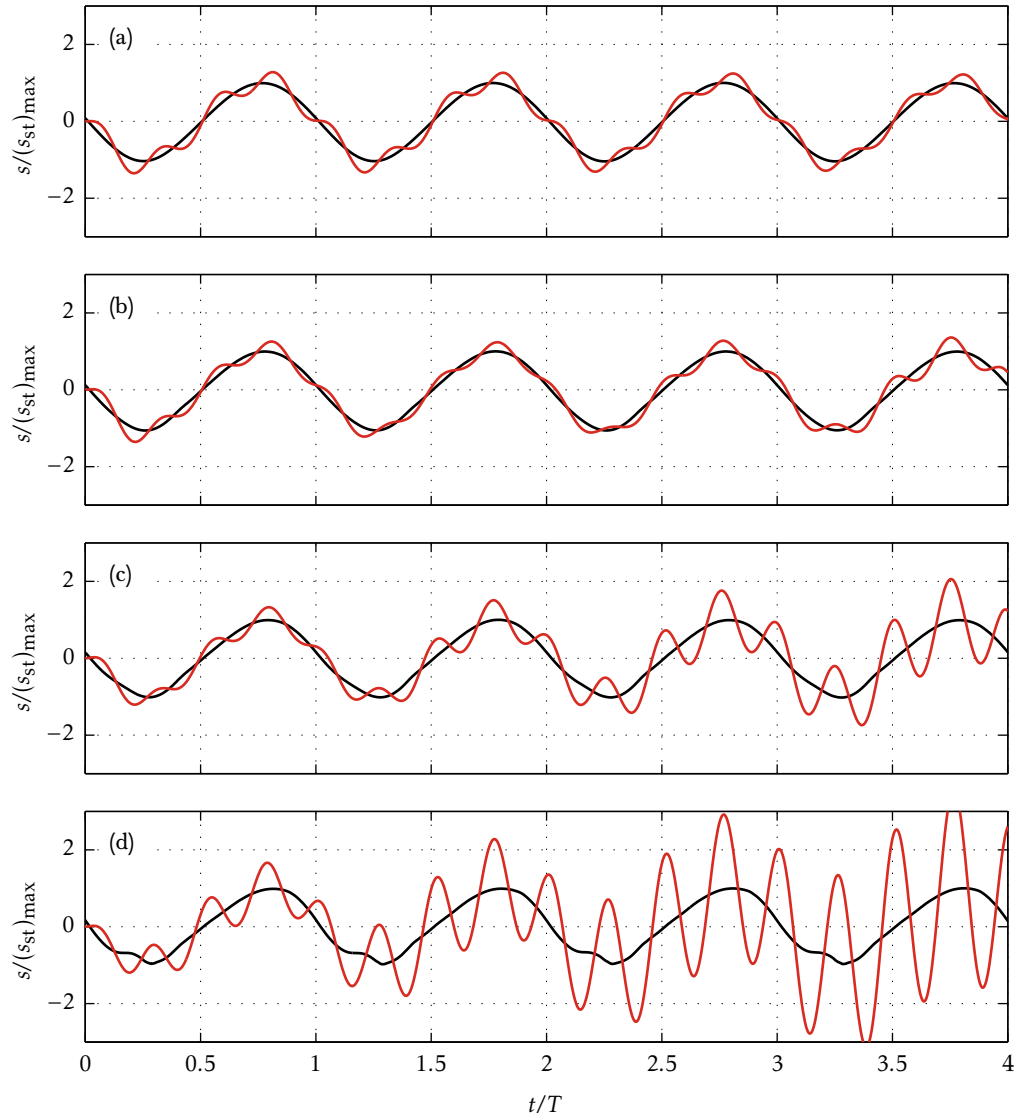


Figure 7.28: Dynamic excitation of a vertical column with natural period, $T_n/T = 1/4$, in a regular waves ($T = 1.0$ s, $D = 0.1$ m) of increasing incident wave steepness (a) $Hk/2 = 0.08$, (b) $Hk/2 = 0.18$, (c) $Hk/2 = 0.24$, and (d) $Hk/2 = 0.32$ with — ‘static response’ and — dynamic response, both normalised by $(s_{st})_{max}$.

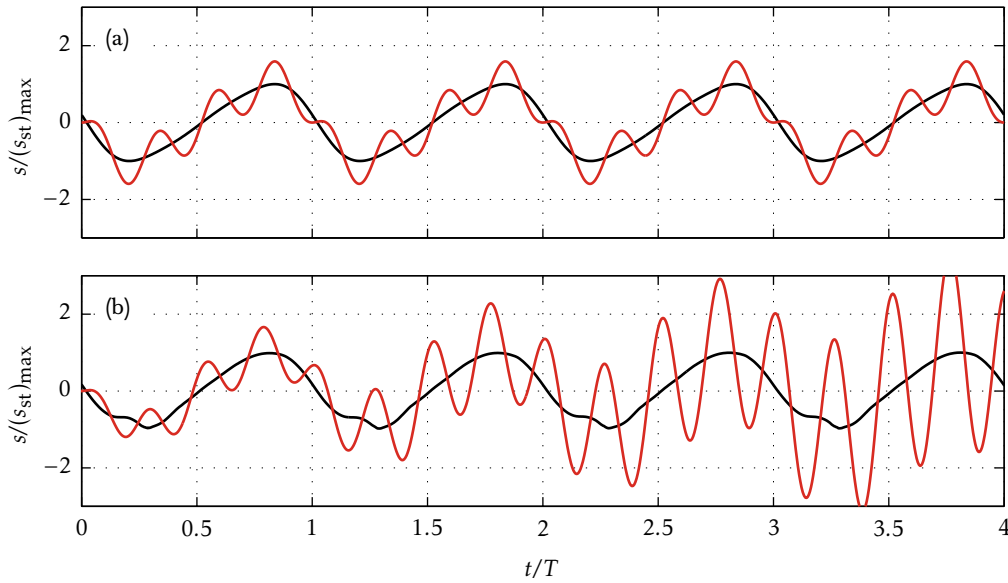


Figure 7.29: Dynamic excitation of a vertical column ($D = 0.1$ m) with natural period, $T_n/T = 1/4$, in a steep regular waves ($T = 1.0$ s, $Hk/2 = 0.32$), the force predictions based upon (a) the FNV force model and (b) the current numerical solution; with — ‘static response’ and — dynamic response, both normalised by $(S_{st})_{max}$.

forcing becomes increasingly significant. Whilst the fourth harmonic force is not dominant, its magnitude is comparable to the second and third order harmonic in the steepest wave case. The magnitude of this force, combined with the location at which it acts, ensures that the ‘*secondary loading cycle*’ plays a key role in defining the dynamic excitation.

The importance of the *secondary loading cycle* is further highlighted by comparing the predicted dynamic excitation based upon the FNV force model and the present numerical simulation. Once again, the vertical column of diameter $D = 0.1$ m with a natural period of $T_n/T = 1/4$ in steep regular waves ($T = 1.0$ s, $Hk/2 = 0.32$) is considered. The results of the comparison are presented in Figure 7.29. Although the FNV model undoubtedly provides an improvement over the Morison’s inertia load, the former only provides loading components up to the third harmonic of the incident waves. As such, the model under-predicts the nonlinear importance of the nonlinear loading components and cannot, therefore, model the dynamic excitation.

The results presented thus far concern the nonlinear loading and the dynamic excitation produced by regular incident waves. To show that the *Type-2* wave scattering and the ‘*secondary loading cycle*’ results are equally important for the excitation of a column subject to realistic focused waves, Figure 7.30 and Figure 7.31 presents

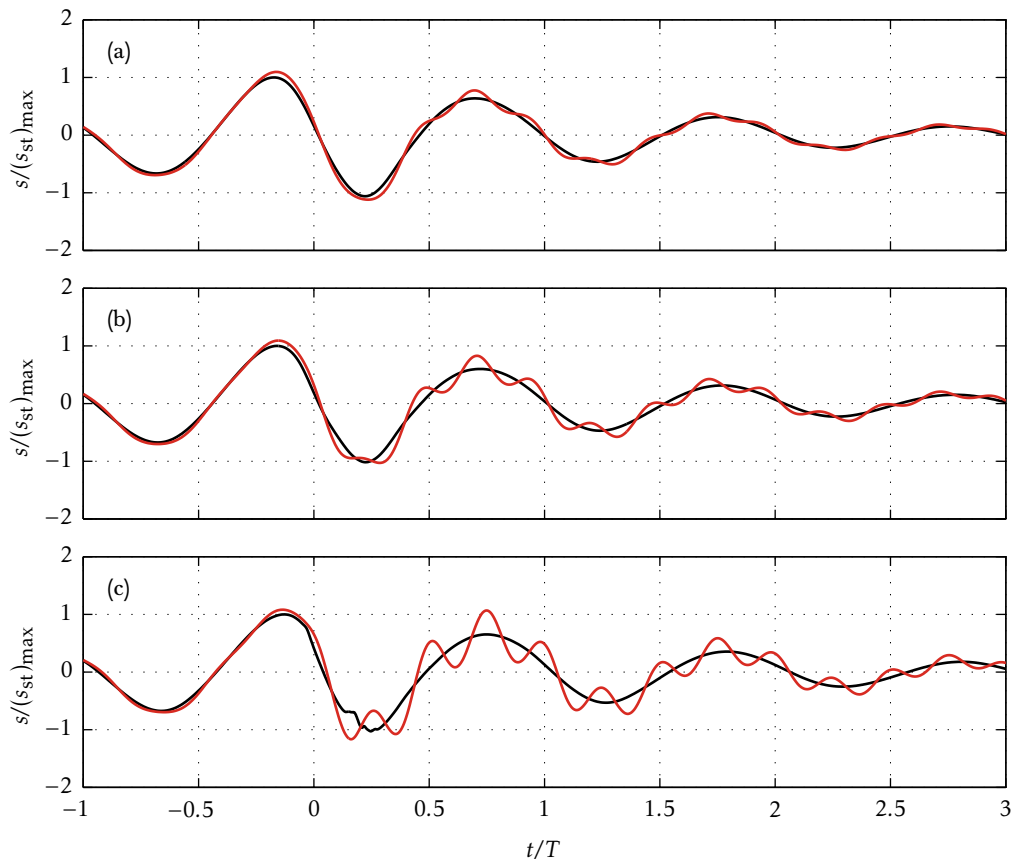


Figure 7.30: The dynamic excitation of a vertical column ($D = 0.1$ m) with a natural period of $T_n/T_p = 1/4$ subject to focused waves ($T_p = 1.0$ s, JONSWAP) of varying steepness: (a) $Ak_p = 0.12$, (b) $Ak_p = 0.16$ and (c) $Ak_p = 0.20$ with — static response and — dynamic response, both normalized by the maximum static response, $(s_t)_{\max}$

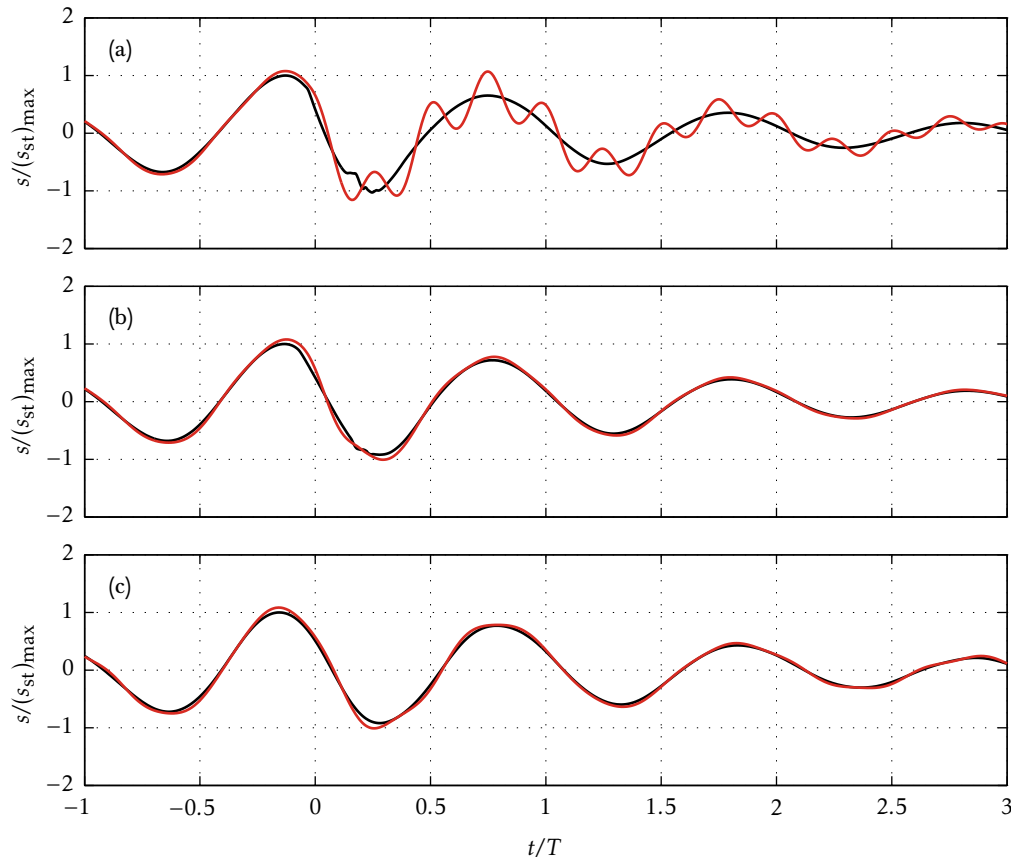


Figure 7.31: The dynamic excitation of vertical column with diameter (a) $D = 0.1$ m, (b) $D = 0.2$ m and (c) $D = 0.3$ m with a natural period of $T_n/T_p = 1/4$ subject to steep focused waves ($T_p = 1.0$ s, $Ak_p = 0.20$). Individual plots showing — static response and — dynamic response, both normalized by the maximum static response, $(s_t)_{\max}$

related data arising in different incident waves. The data presented in these figures is driven by large focused wave events arising in a JONSWAP spectrum with $T_p = 1.0$ s. In both cases, the natural period of the column is defined by $T_n/T_p = 1/4$, and the damping is assumed to be 2.5% of critical; the latter being typical value recorded in many types of offshore structure.

Figure 7.30(a)-(c) concern three focused wave events of increasing steepness such that $Ak_p = 0.12$, 0.16 and 0.2 . In each case, comparisons are made between the un-amplified or ‘static response’ which is simply proportional to the applied load, and the dynamic response; both being normalised by the maximum static response. It is clear from these figures that the only significant excitation arises in the steepest wave case. Moreover, it appears to be triggered by the onset of the *secondary loading cycle*, occurring approximately $t = 0.2$ s after the passage of the largest wave crest.

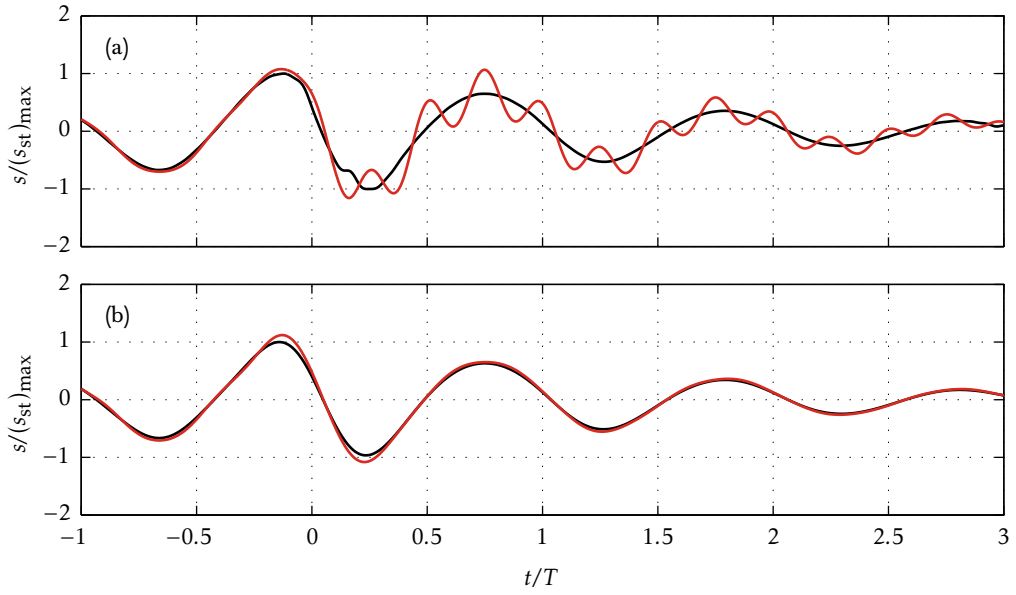


Figure 7.32: The dynamic excitation of a vertical column ($D = 0.1$ m) with a natural period of $T_n/T_p = 1/4$ subject to steep focused waves ($T_p = 1.0$ s, $Ak_p = 0.20$); (a) A close-up of the onset of dynamic excitation and (b) identical data, but with filtering applied to remove the *secondary loading cycle*. Individual plots showing — static response and — dynamic response, both normalized by the maximum static response, $(s_t)_{\max}$

Figure 7.31 reconsiders the steepest focused wave event ($T_p = 1.0$ s, $Ak_p = 0.20$) and contrasts the excitation of three different columns of varying diameter: (a) $D = 0.1$ m, (b) $D = 0.2$ m and (c) $D = 0.3$ m. In each case, comparisons are again made between the un-amplified ‘static response’ and the dynamic response, both normalised by the maximum static response. Comparing these results, it is clear that the only significant response occurs in Figure 7.31(a) corresponding to the smallest column case ($D = 0.1$ m) for which there is clear evidence of a *secondary loading cycle*. In making these comparisons, it is important to note that the structural properties have been held constant ($T_n/T_p = 1/4$ and 2.5% of critical damping); the only changes being the effective column diameter.

To further investigate the importance of the *secondary loading cycle*, Figure 7.32(a,b) provides a close-up of the excitation recorded in the $D = 0.1$ m diameter column subject to the steepest wave event. In Figure 7.32(a), the incident waves and the applied loading are exactly as described above. In Figure 7.32(b), the incident waves are identical, but the predicted forcing filtered using a windowed Fourier transform (FT) to remove the *secondary loading cycle*. It is clear from these comparisons that it is the *secondary loading cycle*, or more specifically the *Type-2* scattered waves, that drive the

dynamic excitation.

7.8 Chapter Summary

This chapter has considered the nonlinear loading arising from the interaction of regular and irregular waves with a vertical, surface piercing, column. The present chapter has:

1. Demonstrated the ability of the current numerical method to accurately predict the *secondary loading cycle* in both regular and irregular waves; the predicted behaviour shown to be in good agreement with earlier laboratory observations.
2. Established that, for smaller diameter column ($D/\lambda < 0.2$), the *secondary loading cycle* is associated with harmonic components arising at four times the incident wave frequency.
3. Confirmed the inadequacy of the FNV force model (Faltinsen et al., 1995) when seeking to predict the high-frequency force components.
4. Illustrated that in steep incident waves, the higher harmonic force components become ‘non-convergent’ for column diameters that lies outside the linear diffraction regime.
5. Provided evidence that the *secondary loading cycle* is associated with the pressure field close to the free surface.
6. Demonstrated that the *secondary loading cycle* and the *Type-2* waves are closely linked and that with an increase in the column diameter, both the *secondary loading cycle* and the *Type-2* wave scattering are suppressed.
7. Revealed that the *secondary loading cycle* is generated as a consequence of a pressure oscillation on the back of the column.
8. Confirmed that dynamic excitation associated with *ringing* occurs as a consequence of the *secondary loading cycle*. At present, this aspect of the work is (perhaps) limited by the classical linearisation in which the applied loads and the excitation are treated independently rather than as part of a coupled system. This limitation is addressed in the suggestions for further work.

8

Conclusions

A detailed understanding of wave-structure interactions is critical for both the design of new offshore structures and the re-assessment of existing structures. This thesis has addressed the interactions between deep water waves and a vertical, surface-piercing, column; the diameter of the latter being large, but not so large that it lies within the linear diffraction regime. The ultimate aim of this study was to provide an improved physical understanding of the resulting wave-structure interactions particularly in respect of high-frequency wave scattering and nonlinear loading.

8.1 Key Findings

8.1.1 Numerical Validation

The present study has presented a numerical investigation of regular and irregular waves interacting with a vertical, surface-piercing, column. The investigation has been carried out using open-source code, OpenFOAM. This provides a viscous, incompressible two-phase solver based on the VOF method. A preliminary study was first undertaken to test the capability of the adopted numerical method to describe various types of deep water waves. This is an essential part of the study since an accurate description of the incident wave is vital for the successful investigation of the wave-structure interactions that follow. Comparisons between the numerical method

and existing analytical solutions for both propagating regular waves and standing waves show excellent agreement. The method also displays good energy and mass conservation properties. The use of a relaxation zone to absorb wave energy at the outlet boundary was shown to be effective in reducing wave reflection. Moreover, a comparison of the water particle velocities beneath a steep regular wave train shows excellent agreement with existing analytical solutions.

In a nonlinear sense, the capability of the numerical method to model the relevant wave-wave interactions was addressed in the context of focused wave simulations. For linear and weakly nonlinear focused waves, the numerical method shows good agreement with the linear and second-order analytical solutions. More importantly, when considering fully nonlinear wave events, up to and beyond the point of wave breaking, comparisons with the fully nonlinear BEM solutions show excellent agreement. Specifically, important features of the nonlinear wave events such as the down-shifting of the focal location and the break down in the symmetry of the focused wave profile were all correctly modelled. Furthermore, an investigation of a number of limiting wave cases of varying spectral bandwidth revealed that a broad-banded spectrum was typically more nonlinear than a narrow-banded spectrum. The nonlinearity of the former is in some large associated with the shifting of wave energy to the higher frequencies. These results are inline with the earlier finding of Johannessen & Swan (2001), Bateman et al. (2001) and Gibson & Swan (2007). These energy transfer ensure that an extreme wave arising in a broad-banded spectrum reaches a limiting value much quicker than its corresponding narrow banded counterpart. Beyond this limiting value, further increases in the linear input amplitude sum, $\sum_{n=1}^N a_n$, lead to the onset of wave breaking. As part of the present study, unidirectional wave breaking has been successfully modelled; the results shown to be in good quantitative agreement with laboratory observations.

8.1.2 Wave Scattering in Regular Waves

Having confirmed the capability of the numerical method to accurately describe a wide range of incident wave conditions, the simulations were extended to include the interaction between regular waves and a vertical, surface-piercing, column. These were shown to be in good agreement with the laboratory observation by Swan &

Sheikh (2015). Specifically, the formation of both *Type-1* and *Type-2* scattered waves were clearly observed in steep incident wave conditions. The *Type-1* wave, exhibiting a concentric wave pattern, were observed to be generated by the run-up and wash-down on the surface of the column. In contrast, the *Type-2* waves, exhibiting a symmetric but non-concentric pattern are generated by the propagation of a fluid disturbance around the column. The phasing of the *Type-2* scattered waves suggests that they are not directly related to the frequency of the incident waves but rather to the diameter of the column and the time taken for the fluid to move around the column.

The subsequent interactions between the scattered waves and the next incident wave produce a very significant increase in the maximum surface elevation upstream of the column. This wave-wave interaction takes the form of a classical long-wave short-wave interaction and is thus caused by the modulation of the high-frequency scattered wave on the crest of the next incident wave. Further investigation revealed that the *Type-1* and *Type-2* waves scattered from the column in the upstream and downstream direction are slightly different. This difference is caused by the crest-trough asymmetry of the incident wave; the wave-induced velocity in the crest being larger than the trough.

An investigation of wave scattering for different column diameters was also undertaken. With an increase in the effective column diameter (D/λ), the formation of the *Type-2* waves was shown to be suppressed. This is caused by the time taken for the fluid disturbance, associated with *Type-2* waves, to move around the column. In the largest column diameter considered, the fluid was driven half the wave cycle before encountering the next incident wave, limiting the upstream scattering of *Type-2* waves. Lastly, the effect of the incident wave steepness on the high-frequency wave scattering was investigated. These results showed that although the steeper incident waves produce an increase in the scattered wave elevation, the increase was limited by the occurrence of wave breaking around the column. Nevertheless, in the steepest wave cases considered, the nonlinear wave-wave interactions between the incident and scattered waves still produce a very significant increase in the surface elevation. Indeed, in the most extreme cases, the wave-wave interactions were so strong that the scattered wave was effectively washed back into the column. This observation is important not least because the wash back has the potential to produce a much

higher incidence of wave slamming that might otherwise be predicted.

8.1.3 Wave Scattering in Focused Waves

The practical implications of this high-frequency wave scattering was further considered by investigating the wave-column interactions arising in a realistic focused wave. In the initial stage of this study, the numerical simulation of weakly nonlinear focused waves with a vertical, surface-piercing, column showed a pattern of wave scattering similar to that observed in regular waves. However, as the focused wave becomes more nonlinear, a new wave scattering mechanism referred to as *Type-1** was observed. This mechanism produces a concentric wave pattern similar to *Type-1* waves, but occurs between the propagation of *Type-1* and *Type-2* waves and only propagates in the upstream direction. The numerical calculations show that the *Type-1** waves are linked with a secondary run-up on the upstream face of the column; the latter generated by the occurrence of very large run-up and wash-down during a steep focused wave event. The magnitude of the *Type-1** wave was shown to reduce with the spectral bandwidth. With a narrow-banded spectrum, the initial wave run-up is significantly reduced, limiting the formation of the secondary run-up and the subsequent *Type-1** waves. The explanation for this lies in the reduced nonlinearity, particularly the transfer of energy in the higher frequencies. The generation of *Type-1** waves was also shown to be sensitive to the focal distance. This again emphasizes the importance of the local wave steepness in the vicinity of the column. Finally, the practical importance of the scattered waves was further considered by investigating the interaction between the scattered and the incident waves. These results show that the nonlinear wave-wave interaction associated with a focused wave produced a significantly smaller increase in total surface elevation when compared to the equivalent regular wave case. An explanation for this lies in the fact that a focused wave only incorporates one very large wave crest, the subsequent wave being substantially smaller. This is in marked contrast to regular waves. In an attempt to produce a significant increase in the maximum surface elevation, calculations were undertaken using a narrow-banded spectrum and a trough focused wave. Whilst significant increases in total water surface elevation were observed, they remained smaller than those predicted in regular waves.

8.1.4 Nonlinear Wave Loading

In respect of the total (or global) nonlinear wave loading, very good agreement is achieved between the present numerical calculations and laboratory observation by Masterton (2006). These address both regular and irregular incident waves; the *secondary loading cycle* clearly observed in both cases. For a small diameter column in steep regular waves, it was found that the *secondary loading cycle* was associated with harmonic components arising at four times the incident wave frequency. A comparison between the numerical force predictions and the FNV force model shows good agreement for the first harmonic force. This corresponds to Morison's inertia term integrated to the SWL. However, the model underestimates the amplitude and phasing of the additional high-frequency force components. In terms of the column size, the investigation has revealed that in steep incident waves the magnitude of the *secondary loading cycle* reduces with an increase in the effective column diameter. A detailed harmonic analysis was carried out for various incident wave steepness interacting with a column of varying diameter. These results show that linear incident waves produce a series of 'convergent' higher harmonic force components. However, with an increase in the incident wave steepness, the series becomes 'non-convergent', especially for smaller diameter columns. This suggests that analytical force models based on a formal perturbation expansion have to be used with care in this particular flow regime.

In explaining the origin of the *secondary loading cycle*, the link between the non-linear loading and pressure around the surface of the column was considered. Comparisons between the radially integrated pressures around the column shows that in the presence of *secondary loading cycle*, there is clear evidence of a distortion of the pressure field. Furthermore, the radially integrated pressures acting on different vertical sections of the column, confirm that the pressure field responsible for the formation of *secondary loading cycle* lies close to the SWL. This implies that changes in pressure, and subsequently, the development of the *secondary loading cycle* is associated with processes involving the disturbance of the flow close to the free surface. Indeed, comparisons between the high-frequency wave scattering and the nonlinear loading in steep regular waves for varying column diameters shows that there is a clear link between the two. Specifically, the results shows the suppression of both the *Type-2* wave scattering and the *secondary loading cycle* with an increase in the column

diameter. Further investigation reveals that the *secondary loading cycle* is generated as a consequence of pressure reversal (or oscillation) at the back of the column. This is most evident for the the smallest column diameter and again closely relates to the propagation of *Type-2* waves around the column. With an increase in the column diameter, the pressure reversal is suppressed, reducing the formation of the *secondary loading cycle*.

The final part of the thesis considers the response of a simple single degree of freedom (SDOF) system when subjected to the nonlinear loading arising from the wave-structure interactions. The aim was to demonstrate the importance of the *secondary loading cycle* in provoking large structural excitations, commonly referred to as *ringing*. If the natural period of the column is set to $1/4$ of the incident wave period, a not unrealistic situation, the nonlinear loading from a steep regular wave interacting with a small diameter column provokes a large dynamic excitation. Indeed, a large dynamic excitation was observed in all cases when the *secondary loading cycle* was present in the force records. More importantly, both the occurrence of the *secondary loading cycle* and the excitation that results is critically dependent upon both the steepness of the incident waves and the effective column diameter, D/λ . Furthermore, comparisons with the predicted excitation based upon the FNV load model confirm the inadequacy of the latter model. Finally, to confirm that the *secondary loading cycle* is relevant in realistic wave conditions, the nonlinear loads arising from steep focused waves were applied in the SDOF system. These results confirm the findings in the regular wave calculations, particularly the importance of wave steepness and the effective column diameter. Most importantly, when the *secondary loading cycle* is filtered out there is little evidence of significant dynamic excitation.

8.2 Suggestions for Future Works

The present study has demonstrated the ability of the adopted numerical method to predict the nonlinear wave-structure interaction arising from a surface-piercing column subject to steep incident waves. Whilst the study has provided some important physical insights into the nonlinear wave scattering and its link with the nonlinear loading, a number of important questions remain. The following points are highlighted as a suggestion for future works:

1. **Increasing computational efficiency.** Although the current numerical implementation is robust and accurate, a major drawback of the method lies in its computational cost. Specifically, a large number of computational cells are needed in the vicinity of the free surface to resolve the free surface interactions, the nonlinear wave scattering, the occurrence of wave breaking and the pressures defining the global loads. While an attempt was made to reduce the computational cost by coupling the present numerical method with an in-house BEM code, the coupling is only one way. A generic two-way coupling would allow the efficient modelling of both wave propagation in an outer domain and the extreme free surface conditions in the inner domain. Important steps in this direction have been made by Paulsen et al. (2014a).
2. **Directional incident waves.** Due to computational constraints, the current study was limited to unidirectional regular and irregular waves. Unfortunately, a realistic sea state will involve unsteady (irregular) waves with significant directional spread. The modelling of directional wave field would mean that the symmetric domain assumption used in the present study cannot be applied, further increasing the required computational effort.
3. **Long random wave simulations.** In a design context, key inputs involve the short-term (for a given sea state) distribution of wave loads and responses. This inevitably means undertaking long random wave simulations, including directional spreading (point (2) above). This presently lies well beyond the capability of the numerical methods. However, with the right coupling (point (1) above) and a sound physical understanding of the wave interactions involved, important progress can be made. Evidence of this is provided by the recent modelling of wave-in-deck loads within a Monte Carlo simulation by Ma (2017).
4. **Wave-structure interactions involving multiple columns.** The current study has only considered the wave-structure interactions appropriate for a single column case. From a practical perspective it is of interest to extend the current study to include multiple column arrangements. This would allow a clear understanding of the further amplification of the surface elevation due to the interaction of multiple scattered wave fields. This work should also include the corresponding nonlinear loading on multiple columns arrangements.

5. **Wave-structure interaction in intermediate and shallow water depth.** The current study was limited to deep water conditions. An extension of the work to intermediate and shallow water depths is essential considering that many existing structures, especially those with single column arrangements such as offshore wind turbines, are installed in shallower water depths. Furthermore, the wave scattering and subsequent wave-wave interactions are expected to be significant since waves propagating into intermediate and shallower water depth lead to a steepening of both the sea-state and hence the largest individual waves.
6. **Fully coupled wave-structure interactions involving a flexible structure.** While the purpose of the current single degree of freedom model was to investigate whether the nonlinear loading experienced by the structure was responsible for the large dynamic excitation, the model was undoubtedly oversimplified. A full three-dimensional structural model, fully coupled with the flow field, will provide a much more realistic representation of the excitation problem.

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