



Vibrating-Wire Viscometry

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Abstract

The theory and application of the vibrating-wire technique for the measurement of viscosity, as well as both viscosity and density, are reviewed. Theory is presented in the form of practical working equations and well-established limitations on their ranges of validity. The cases of both transient and steady-state excitation of the vibrating wire are considered in detail. For the steady-state mode, we describe a variant of the method in which the density is also measured. Practical details including wire materials, magnet systems and instrumentation are discussed, and several design examples from the literature are reviewed. Relative uncertainties in vibrating-wire viscometry vary from, at best, 0.2 % to about 2 % at 95 % confidence. In an appropriately designed instrument, density can be measured simultaneously with a relative uncertainty of about 0.2 %.

Keywords High pressure · Vibrating wire · Viscometer · Viscosity

1 Introduction

In essence, a vibrating-wire viscometer comprises a thin wire or slender rod immersed in the fluid of interest and caused to execute transverse oscillations. The damping of these oscillations, under either steady-state or transient excitation, is determined largely by the viscosity and density of the surrounding fluid and, with knowledge of the latter, the former can be deduced precisely from experimental measurements. A key feature of the method is that there exists a rigorous theoretical connection between the measured damping and the viscosity of the fluid. The technique is especially well suited to measurements of viscosity at high pressure and over extended ranges of temperature in electrically-insulating fluids. Advantages of the technique include no macroscopically moving parts, a lack of bulk fluid flow, no requirements to measure either flow rate or differential pressure, and the possibility

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to operate in any orientation. Vibrating-wire viscometers provide an electrical output signal which is advantageous for automation and/or on-line monitoring applications. Direct contact between the wire and the fluid results in some complications in the case of a highly corrosive and/or electrically conductive fluid. Vibrating-wire viscometers have not been commercialized but a variety of bespoke designs have been described in the literature, some of which are relatively easy to implement. A properly designed and operated instrument is capable of measuring viscosity with a relative uncertainty on the order of 1 % at 95 % confidence. A variant of the instrument permits simultaneous measurement of density and viscosity.

This article is intended as a guide to best practice in the design and operation of vibrating-wire instruments. To start, we briefly review the relevant theory of the instrument, including the variant in which the fluid density is measured simultaneously with the same sensor. Next, we discuss practical implementation in detail including analysis of the achievable experimental uncertainty. Finally, some example designs from the literature are reviewed.

2 Theory and Design of Vibrating-Wire Instruments

The theory of operation can be traced back to efforts to understand the oscillatory motion of pendulums. Stokes [1], recognizing the important influence of both the viscosity and density of the surrounding fluid, derived a theory for the damping of an infinitely-long cylinder executing transverse oscillations in a viscous medium of infinite extent. This theory was subsequently exploited by Tough et al. [2, 3] in the first modern implementation of the technique. The theory of vibrating-wire instruments for density and viscosity measurements was revisited by Retsina et al. in two seminal papers published in the 1980s [4, 5]. Returning to first principles, these authors distinguished between the steady-state and transient modes of operation, derived the corresponding working equations and established the limits of their validity.

The theory assumes a long straight cylindrical wire or rod of radius R and length $L \gg R$ fixed at both ends and surrounded by an infinite homogeneous fluid of density ρ and viscosity η . The wire itself is a homogeneous material of density ρ_w subject to a tension $T \geq 0$. When subjected to a small transverse displacement, restoring forces arise from a combination of the tension and the flexure stiffness of the wire or rod. In the case of ‘thin’ wires, tension provides the dominant restoring force, whereas for ‘thick’ wires or rods, the flexure stiffness dominates, and an imposed tension might not be required.

Although not an essential feature of the theory, practically, a metallic wire is used, and a magnetic field is imposed perpendicular to its axis. Then, when a current is passed through the wire, it will experience a force perpendicular to both the magnetic field and the axis of the wire; this provides the means of exciting wire motion. At the same time, oscillation of the wire within the magnetic field will induce a potential difference φ between the ends of the wire; this provides the means of detecting the wire motion.

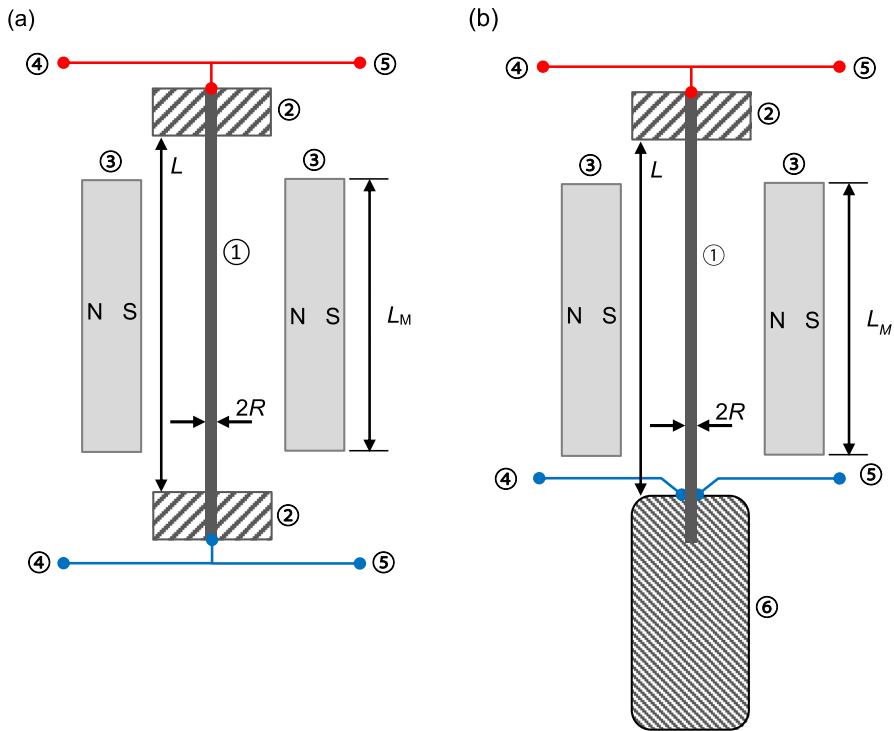


Fig. 1 Geometry of the vibrating-wire instrument: **a** wire clamped in fixed supports at both ends; **b** wire clamped to a fixed support at the top and to a freely suspended sinker at the bottom. 1, wire or rod; 2, fixed end supports; 3, –magnets; 4, current leads; 5, potential leads; 6, freely suspended sinker

Figure 1 illustrates the geometry of the vibrating-wire instrument, including symmetrically placed magnets of length L_M , and separate electrical leads for passing drive current and measuring the voltage across the wire. In Fig. 1a, the wire is clamped between two fixed supports, whereas in Fig. 1b, the top support is fixed, but the bottom of the wire is clamped to a freely suspended sinker which serves to determine the tension. The practical working equations for both transient and steady-state operation [4–6] are summarized in Table 1 and discussed further below in the context of the different modes of operation.¹

Common to both modes of operation are the definitions of β as a dimensionless added mass and β' as a dimensionless added damping of the wire arising from the presence of the surrounding fluid. These quantities are, in turn, related through A to the dimensionless frequency $\Omega = \omega R^2 \rho / \eta$ wherein the fluid viscosity appears. Note, however, that the detailed expressions for β' are slightly different for the transient and steady-state modes. The associated quantities, $k = (\rho_w / \rho) \beta$ and $k' = (\rho_w / \rho) \beta'$, which are independent of the wire density, are also defined and, in the steady-state

¹ Equation (48) in reference [5] for A in the transient mode is incorrect; the correct expression is given by Wilhelm et al. [6].

Table 1 Working equations of the vibrating-wire viscometer for steady-state and transient operation^a

Transient	Steady state
$\varphi(t) = \varphi_0 \sin(\omega t + \theta) \exp(-\Delta \omega t)$ (1)	$\varphi(\omega) = IZ + \varphi_1(\omega)$ (7)
$\Delta = \frac{\beta' + 2\Delta_0}{2(1 + \beta)}$ (2)	$\varphi_1(\omega) = \frac{i\Lambda\omega}{[\omega_0^2 - \omega^2(1 + \beta)] + i\omega^2(\beta' + 2\Delta_0)}$ (8)
$\beta = (\rho/\rho_w)[2\text{Im}(A) - 1]$ (3)	$\beta = (\rho/\rho_w)[2\text{Im}(A) - 1]$ (9)
$\beta' = 2(\rho/\rho_w)[\text{Re}(A) + \Delta\text{Im}(A)]$ (4)	$\beta' = 2(\rho/\rho_w)\text{Re}(A)$ (10)
$A = i(1 + i\Delta) \left[1 + \frac{2K_1(\sqrt{i\Omega'})}{\sqrt{i\Omega'}K_0(\sqrt{i\Omega'})} \right]$ (5)	$A = i \left[1 + \frac{2K_1(\sqrt{i\Omega})}{\sqrt{i\Omega}K_0(\sqrt{i\Omega})} \right]$ (11)
$\Omega' = (1 + i\Delta)(\omega R^2 \rho / \eta) = (1 + i\Delta)\Omega$ (6)	$\Omega = (\omega R^2 \rho / \eta)$ (12)

^a K_0 and K_1 are the modified Bessel functions of order zero and one, respectively. Other quantities are defined in the text

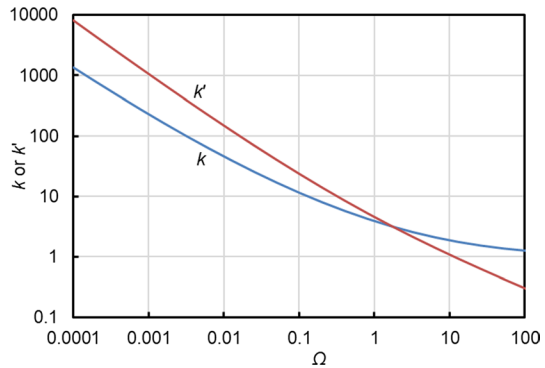
mode, depend only on Ω . Figure 2 shows k and k' for the steady-state mode over the range $0.0001 \leq \Omega \leq 100$ within which practical operation usually falls. For larger values of Ω , k and k' have the asymptotic behavior $k \rightarrow 1$ and $k' \rightarrow 4/\sqrt{2\Omega}$. In the transient mode, k and k' are similar functions of Ω but dependent parametrically upon Δ .

2.1 The Transient Mode

When plucked by some means, the wire will exhibit transient damped simple harmonic motion in one or more of its normal modes of oscillation. Initiated in an appropriate way, the fundamental transverse mode of oscillation is dominant, and the induced voltage $\varphi(t)$ across the wire follows Eq. 1. Here, φ_0 is a voltage amplitude, ω is the angular natural frequency of the fundamental mode, t is time measured from the cessation of the excitation signal, θ is a phase angle and Δ is the logarithmic decrement. The values of φ_0 and θ are determined by the means of excitation. The essence of the method is to first determine both ω and Δ from measurements of the decaying alternating voltage across the wire and then to determine the viscosity from those two quantities making use of Eqs. 2-6. It is necessary to know the densities ρ of the fluid and ρ_w of the wire material, as well as the radius R of the wire and the logarithmic decrement Δ_0 measured in a vacuum. Starting from an initial estimate of η , Eq. 6 yields an estimate of Ω' , and Eqs. 2-5 yield an estimate of Δ . A standard iterative scheme can be used to refine these estimates by manipulating η until the calculated and experimental values of Δ coincide.

The wire might be set in motion by passing a short current pulse through it. Unfortunately, this would have the effect of exciting many modes of oscillation simultaneously, which is not the desired situation. As an improvement, one can apply two opposing current pulses separated in time by one half of the anticipated period of oscillation. However, it is preferable to excite the motion with a sinusoidal

Fig. 2 Functions $k = 2\text{Im}(A) - 1$ and $k' = 2\text{Re}(A)$ plotted against dimensionless frequency $\Omega = \omega R^2 \rho / \eta$



tone burst at a frequency close to the natural frequency of the fundamental mode. This tone burst should contain an integral number of cycles N ; if $N \gg 1/(2\pi\Delta)$, then the wire motion rings up to a steady-state amplitude. The fundamental mode may be favored further by optimizing the length of the magnets relative to the length of the wire. If $L_M \geq L$, then, when a current flows, the wire experiences a uniform transverse force along its length, which suppresses all even-order modes of oscillation.² In this situation, the fundamental mode is excited efficiently, and higher-order odd modes are excited with rapidly diminishing amplitude. Wilhelm et al. [6] showed that making $L/L_M = 1.5$ also eliminates the third mode of oscillation and this choice, together with tone-burst excitation, is recommended. Operated in this way, the initial condition at $t=0$ is a nearly pure fundamental mode of oscillation so that the measured signal conforms closely to Eq. 1 at $t>0$. Even when excited with opposing current pulses, the unwanted higher modes decay very quickly relative to the fundamental [7, 8].

2.2 The Steady-State Mode

In the steady-state or forced mode of oscillation, the wire is driven continuously at an imposed angular frequency ω , and the complex voltage $\varphi \cdot \exp(i\omega t)$ across the wire is measured. The complex notation used here represents the magnitude $|\varphi|$, phase $\arg(\varphi)$ and time dependence $\exp(i\omega t)$ of the voltage when current $I \cdot \exp(i\omega t)$ is passed. Henceforth, the time-dependent factor $\exp(i\omega t)$ is implicit and dropped from subsequent steady-state equations.

As shown by Eq. 7, φ comprises two terms. The first, IZ , is simply the product of the drive current I and the electrical impedance Z of the stationary wire, which may be measured as a function of frequency in the absence of the magnetic field. The second term, given by Eq. 8, is the emf induced as a consequence of the oscillation of the wire. This term describes a resonance curve for the driven wire with angular resonance frequency.

² Even-order modes are anti-symmetric with respect to the perpendicular plane at the mid-point of the wire.

$$\omega_{\text{res}} = \omega_0 / \sqrt{1 + \beta}, \quad (13)$$

at which the imaginary part of φ vanishes, and a resonance value is given by:

$$\varphi_{\text{res}} = \frac{\Lambda}{\omega_{\text{res}} (\beta' + 2\Delta_0)}. \quad (14)$$

Here, ω_0 may be identified as the angular resonance frequency of the wire in a vacuum. Note that Eqs. 13 and 14 are implicit because β and β' are themselves functions of frequency. The quantity Λ is an amplitude proportional to the product of the current I and the square of the magnetic field strength B as follows [9]:

$$\left. \begin{aligned} \Lambda &= \left(\frac{4IL_M B^2}{\pi^2 \rho_w R^2} \right) \sin \left(\frac{\pi L_M}{2L} \right) & L > L_M \\ \Lambda &= \left(\frac{4IL B^2}{\pi^2 \rho_w R^2} \right) & L \leq L_M \end{aligned} \right\}. \quad (15)$$

The SI units of Λ are $\text{V}\cdot\text{s}^{-1}$. Equation (15) indicates the important role of the magnetic field strength in determining the magnitude of the resonance signal. This equation may be useful for estimating expected signals when designing an instrument; however, Λ itself is usually treated as an adjustable parameter in the analysis of experimental data.

The resonance curve does not generally conform to a simple Lorentzian function although, in cases where the damping is small ($\beta' + 2\Delta_0 \ll 1$), the resonance is nearly symmetrical with a quality factor $Q \approx (1 + \beta)/(\beta' + 2\Delta_0)$.

2.3 Constraints on the Working Equations

The working equations detailed in Table 1 were obtained by solving the Navier–Stokes equations for the motion of the fluid around a cylinder subject to simple harmonic transverse displacement of small amplitude. Several assumptions were made in obtaining the solution as follows:

$$R \ll L \quad (16)$$

$$R \ll R_c \quad (17)$$

$$\varepsilon \Omega \ll 1 \quad (18)$$

$$\omega \varepsilon R \ll c \quad (19)$$

Here, R_c is the radius of a cylindrical container within which the wire is coaxially aligned, εR is the displacement amplitude, and c is the speed of sound in the fluid. Criterion (16) ensures that end effects are negligible; criterion (17) ensures that the

influence of the outer boundary is negligible; criterion (18) is that the Reynolds number is small so that non-linear terms in the Navier–Stokes equations are negligible; and criterion (19) ensures that the Mach number is small so that compressibility effects are negligible. Further details can be found in the paper by Retsina et al. [4, 5].

A steady-state solution of the problem for the case in which R/R_c is *not* very small has also been derived by Chen [10] and adapted to the transient case by Retsina et al. [5] and Assael et al. [8], yielding alternative expressions for the function A to replace Eqs. 5 and (11). These can be used as a basis for determining the error in a viscosity measurement arising from a finite ratio R/R_c . Retsina et al. [5] showed that the relative error in viscosity is less than 0.4 % when $R_c/R > 33$ and $\Omega < 1000$. In principle, the full expression for A , including the outer boundary, might be used as the working equation, but this approach has not usually been followed in practice.

2.4 Resonance Frequencies

It is also worth noting that the constraints on the validity of the theory do *not* extend to the precise nature of the boundary conditions at the end of the wire. All that is required is that the wire or rod is fixed at each end in a way that does not cause too much damping of the oscillation so that $\Delta_0 \ll \beta'/2$. However, the boundary conditions, including the imposed tension and constraints on the curvature of the wire or rod, do influence the fundamental resonance frequency. Equation (13) shows that the added mass of the fluid around the wire, expressed by the parameter β , also influences the resonance frequency, and so, in principle, the density might be determined simultaneously with the viscosity making use of experimental measurements of both the resonance frequency and the damping of the wire. However, in practice, it is very difficult to design fixed wire supports that maintain repeatable tension and end conditions in the face of temperature and pressure variations. Therefore, when using an instrument in which the wire is fixed to rigid supports at each end, the fluid density should be an independent input.

It is necessary to recognize that, in a system having perfect symmetry about the axis of the wire, the fundamental mode of oscillation is a degenerate pair. These two modes oscillate in mutually perpendicular planes. In a real system, the symmetry is usually broken to some extent, for example, by the method of clamping the wire at each end and/or by a slightly non-circular wire cross-section. This results in a splitting of the otherwise degenerate modes, and it is necessary to tune the system such that just one of these modes is excited. This can be achieved by rotation of the magnetic field until only a single resonance mode is excited.

3 Simultaneous Measurements of Viscosity and Density

An alternative to the arrangement shown in Fig. 1a is to fix the lower end of the wire to a sinker of mass m_s and volume V_s as shown in Fig. 1b. The weight of the sinker imposes a tension T on the wire, which is modified by the buoyancy

effect: $T = (m_s - \rho V_s)g$. Therefore, the resonance frequency of the wire acquires an additional dependence upon the fluid density, which is generally significantly larger than the effect of β ; at the same time, tensioning by means of a freely suspended sinker imposes a much more consistent end condition such that the resonance frequency can be used to determine the density of the fluid with a small uncertainty.

A thin wire behaves much like a flexible string with a resonance frequency in a vacuum of.

$$\omega_{0,\text{vac}} = \frac{\pi}{L} \sqrt{\frac{m_s g}{\rho_w \pi R^2}}. \quad (20)$$

The quantity ω_0 appearing in Eq. 8 now pertains to the hypothetical situation in which the wire is in a vacuum but the sinker is in the fluid so the tension is modified by the buoyancy effect. In that case, the fundamental transverse resonance frequency of the wire treated as a flexible string would be.

$$\omega_0 = \sqrt{\omega_{0,\text{vac}}^2 - \frac{\pi g \rho V_s}{\rho_w R^2 L^2}}. \quad (21)$$

Nevertheless, a wire also possesses stiffness, and the resonance frequency then depends not only upon tension but also upon Young's modulus E for the wire material and the end conditions [11]. Typically, these end conditions approximate clamped boundary conditions. The vacuum resonance frequency of a stiff tensioned string subject to such boundary conditions has been derived by Morse and Ingard [12]. When the tension is the dominant restoring force and the buoyancy effect is taken into account, ω_0 is given by [11]:

$$\omega_0 = \frac{\pi}{L} \left[\alpha \sqrt{\frac{(m_s - \rho V_s)g}{\rho_w \pi R^2}} + \sqrt{\frac{ER^2}{\rho_w L^2}} + \left(4 + \frac{\pi^2}{2}\right) \cdot \sqrt{\frac{E^2 \pi R^6}{16(m_s - \rho V_s)g \rho_w L^4}} \right], \quad (22)$$

where, theoretically, $\alpha = 1$. However, a complication is that the real end conditions may not conform to the clamped-clamped assumption; therefore, Ciotta and Trusler [11] proposed incorporating the empirical multiplier α and determining its value by calibration in a vacuum. This is discussed further in connection with practical realizations. Note that several authors [13, 14] have employed the simpler Eq. 21 with $\omega_{0,\text{vac}}$ determined in a vacuum calibration measurement. This approach largely compensates for stiffness and unknown boundary conditions, but Eq. 22 is recommended as the preferred approach.

To make sensitive fluid density measurements, the sinker should be made from a low-density solid such as aluminum, which can be anodized to obtain a relatively inert surface.

The working equations of the steady-state vibrating-wire viscometer-densimeter are those given in Table 1, Eqs. 7-12, supplemented by Eq. 22, and the constraints on their validity remain those detailed in Sect. 2.3. The analysis of

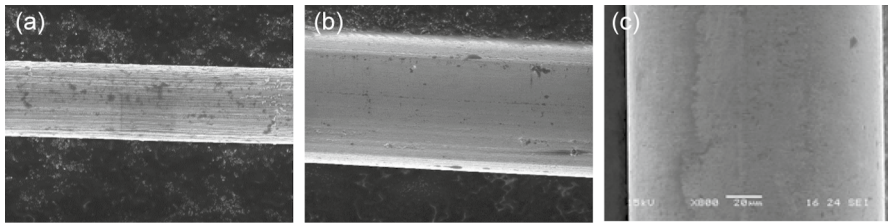


Fig. 3 Scanning electron microscope (SEM) images of commercial wire samples: **a** 7 μm diameter tungsten wire; **b** 25 μm diameter Chromel wire; **c** 150 μm diameter centerless-ground tungsten wire [15]

experimental data then requires refinement of trial values of both η and ρ until, for example, the sum of the squared differences between the calculated and experimental resonance curves, $\varphi(\omega)$, is minimized.

4 Practical Implementation

Practical instruments usually employ a wire of between (10 and 100) mm in length tensioned such that its fundamental resonance frequency is between a few hundred and a few thousand Hertz. The wire assembly is housed within a fluid-containment vessel with the magnets either within or without. Of course, for precise measurements, the temperature of the instrument and the pressure of the fluid that fills it must both be controlled. The instrument design involves a number of key aspects that are discussed in this section.

4.1 Wire Selection and Mounting

The wire material should be a non-magnetic elastic conductor with low mechanical and electrical dissipation. To conform with the theoretical model of the instrument, it must also have good cylindrical geometry, i.e., circular cross-section, uniform diameter, and a smooth surface. Adequate strength and creep resistance are also required to allow sufficient tension to obtain a convenient resonance frequency. Equations (3), (4), (9) and (10) demonstrate that the wire density ρ_w is also a significant parameter controlling the extent to which the fluid properties are coupled to the resonance properties of the wire. Finally, the wire must be chemically inert to the fluid that it contacts.

Practically, tungsten satisfies most of these criteria. As well as having high density, high strength, low electrical resistivity and low mechanical damping, it is available commercially in a wide range of diameters from 7 μm upwards. As shown by the scanning electron microscope images in Fig. 3, drawn wire tends to have a rough surface. However, centerless-ground tungsten wire, which has a smooth surface and uniform circular cross-section, is available in diameters ≥ 0.1 mm, and, for many applications, such material is ideal. Thinner tungsten wires can also be

electropolished to obtain a smooth surface, although not necessarily a perfect circular cross-section.

For measurements on gases, which generally require thin wires with diameters $\leq 25 \mu\text{m}$, the thermocouple alloy Chromel (90 mass % Ni + 10 mass % Cr), annealed in vacuum, has been recommended for its smooth surface and good mechanical properties [6], although its electrical resistivity is higher and its density is lower than tungsten. Platinum-iridium alloy (90 mass % Pt + 10 mass % Ir) has also been recommended for aqueous systems on grounds of chemical inertness [16]. Tungsten wire can be gold-plated to obtain a chemically inert surface.

Generally, one can recommend centerless-ground tungsten wires for measurements on insulating liquids, and vacuum-annealed Chromel wires for measurements on gases. Having chosen the material, the diameter is the next quantity to consider, and the main associated criterion is that the logarithmic decrement Δ should be much greater than Δ_0 but small enough that oscillations do not decay too rapidly in the transient mode or that the resonance quality factor Q is not too low in steady state mode. A suggested range is $0.01 \leq \Delta \leq 0.1$ [5], corresponding approximately to $5 \leq Q \leq 50$, although good results may also be obtained for smaller damping (higher Q) provided that Δ_0 is sufficiently small. The equations in Table 1 can be used to calculate the expected values of Δ for various wire diameters over a suitable range of fluid properties with an assumed fundamental resonance frequency. Once a nominal radius is selected, a length can be chosen, and the tension adjusted to obtain the desired resonance frequency. The actual radius of the wire must be obtained by means of either direct measurement or calibration as discussed below.

Thick wires can be gripped in a non-magnetic collet, which provides a secure and well-defined end condition. This method of gripping the wire does not create preferred planes of oscillation and it will be necessary to rotate the wire or magnet assembly until a pure mode of oscillation is achieved. A common alternative is to grip the wire between a pair of plane-parallel jaws. This typically does create preferred planes of oscillation such that, when the clamping planes at both ends are either parallel or perpendicular to the magnetic field, a pure mode is obtained. The jaws may be either harder or softer than the wire material. If softer, the wire will tend to make an indentation in the jaws, whereas hard jaws tend to slightly compress the wire into a locally oval cross-section. Either arrangement is acceptable.

To fix the wire under tension, it is usual to first clamp one end of an over-long wire. Then, with the assembly in the vertical orientation, the wire is passed through the lower clamping device, and a suitable weight is attached to the free end. Finally, the lower clamp is tightened, and excess wire below it can be cut off. For wire that has been coiled, it may be helpful to leave the assembly for some time to relax with the weight hanging and the lower clamp loose. For tungsten, it is recommended to choose a tensile stress of between 20 and 50 % of the yield stress.

4.2 Magnet System

Permanent magnet systems comprise two magnetic blocks linked by a soft-iron or mild-steel yoke so as to create a magnetic circuit. This arrangement, illustrated

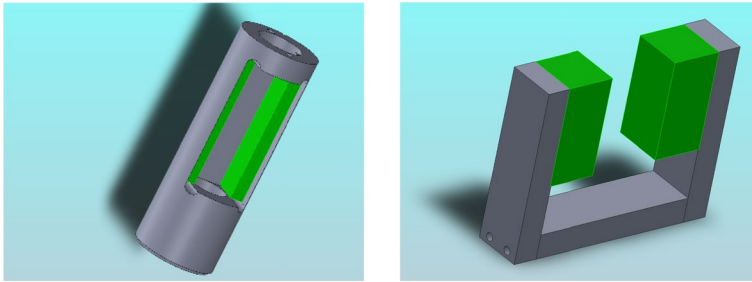


Fig. 4 Magnet assemblies comprising rare-earth magnet blocks (green) linked by mild steel or iron yokes (grey): **a** internal design with magnets of 4 mm×10 mm×40 mm and a gap of 12.2 mm; **b** external design with magnets of 50 mm×50 mm×100 mm and a gap of 71 mm [18]

in Fig. 4, ensures a nearly uniform magnetic field in the gap where the wire is to be located. A field strength of a few tenths of a Tesla is achievable and sufficient. Whether or not to place the magnets within the pressure vessel is partly determined by the wall thickness of the latter. For high-pressure operation, it may be more convenient to locate the magnet assembly within the vessel. The magnets are then closer to the wire, and they do not have to be very large. For example, rectangular cross-section blocks of 10 mm wide×4 mm thick×52 mm long have been used with a gap of 8 mm for a wire of 0.1 mm diameter and 70 mm length [9]. If the magnets are external to the vessel, then much larger dimensions are needed. For example, Peleties et al. [17] used 50.8 mm×50.8 mm×50.8 mm NdFeB magnet blocks with a mild-steel yoke and a gap of 50 mm, within which the field was 0.38 T.

High-performance magnetic materials are typically slightly porous and should be plated or encapsulated to resist fluid ingress when used within a pressure vessel. Nickel is a common plating material, although gold may be preferable on grounds of chemical inertness. Encapsulation within a sealed non-magnetic casing may also be used.

4.3 Instrumentation: Steady-State Operation

For steady-state operation, one needs to pass a constant sinusoidal current through the wire and simultaneously measure the complex voltage developed across it. Constant excitation current can be obtained most easily by using a sine-wave function generator operating in constant voltage mode and a resistance R_s in series with the wire, where R_s is much greater than the magnitude of the electrical impedance of the wire at resonance. In that case, $I = \varphi_0/R_s$, where φ_0 is the amplitude of the excitation voltage. Alternative circuits are illustrated in Fig. 5. The potentiometric circuit shown in Fig. 5a is often used. However, the bridge circuit shown in Fig. 5b may be preferable to partially balance out the background voltage IZ in Eq. 7. This may be achieved with $R_1 \approx R_s$ and $R_2 \approx R_w$, where R_w is the DC resistance of the wire.

The voltage response $\varphi_A - \varphi_B$ should be measured with a high-input impedance differential detector. A dual-phase lock-in amplifier is probably the best instrument as these lock on to the measurement frequency, providing narrow-band low-noise

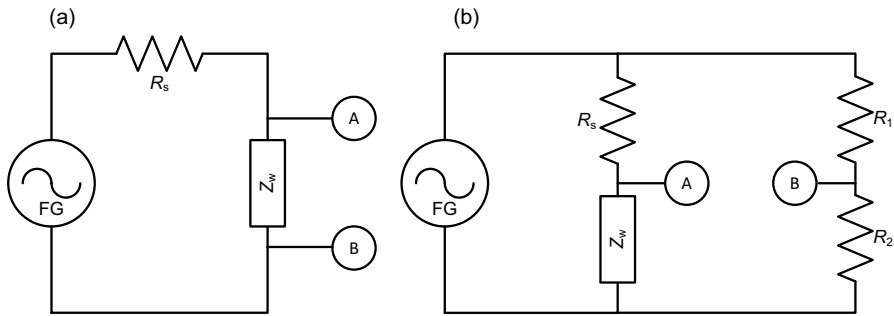


Fig. 5 Circuits for steady-state and transient measurements: Z_w , wire impedance; R_s , R_1 , R_2 , resistances; FG, function generator; Ⓐ, Ⓑ, connection points for differential voltage measurement. **a** potentiometric circuit; **b** bridge circuit

measurements. Many such instruments also incorporate an adequate sine-wave function generator. Vector impedance analyzers can be used in a similar way. The current required to excite the wire oscillations (without over-driving them into a non-linear regime) should be determined by trial. As a rule of thumb, one can aim for $|q_1|$ to reach a value of (10 to 300) μV at resonance.

The circuit diagrams shown in Fig. 5 are best implemented as a four-wire configuration in which separate current and potential leads are attached to each end of the wire. In that case, the measured differential voltage in Fig. 5a corresponds to that developed by the constant alternating current flowing through the impedance of the vibrating wire alone. The impedance of the stationary wire (Z in Eq. 7) is then $R_w + i\omega L_w$, where L_w is the inductance. In some situations, it might be difficult to run a pair of leads exactly to each end of the wire, in which case the measured differential voltage will not correspond exactly to that across the wire alone. This problem has been analyzed by Padua et al. [19], who developed a detailed electromechanical model for the steady-state mode, which can be used in such situations.

4.4 Instrumentation: Transient Operation

The circuit used for transient measurements can be as shown in Fig. 5a. The function generator is then required to emit a sinusoidal tone burst at a frequency close to the fundamental resonance frequency of the wire. The differential voltage $\varphi_A - \varphi_B$ should be measured with a high-speed analog-to-digital converter (DAC) equipped with a suitable wide-bandwidth pre-amplifier such that the signal passed to the DAC reaches almost the full-scale amplitude at the end of the initiation tone burst. The DAC should log data at a rate much greater than the period of oscillation and for a sufficient number of cycles for the oscillations to decay to a small fraction of their initial value. A single acquisition is likely to be too noisy, but the signal-to-noise ratio can be improved by averaging many synchronous repetitions, each separated by sufficient time for the previous oscillation to have decayed completely. The input impedance of the pre-amplifier must be sufficient to avoid loading the oscillating wire, as discussed by Vogel et al. [6].

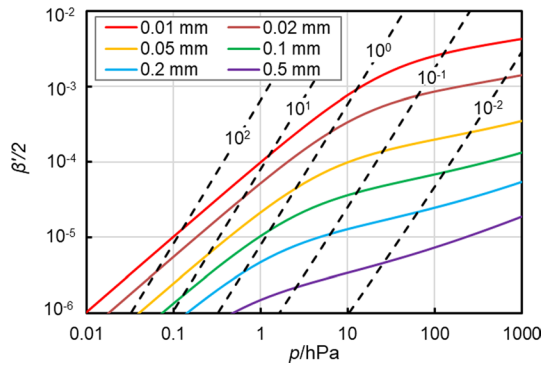


Fig. 6 Residual-gas damping of vibrating wires, expressed as $\beta'/2$, as a function of pressure p in the continuum, transition and free-molecule regimes for different wire diameters in nitrogen at $T=298.15$ K. The wire material is tungsten, and the natural frequency of oscillation is 1 kHz. The effective viscosity is approximated as $\eta = \eta_0 / (1 + 2K_n)$, where η_0 is the viscosity in the continuum regime, $K_n = \lambda/R$ is the Knudsen number and λ is the mean free path [22]. Solid lines: constant wire diameter (see legend). Dashed lines: constant Knudsen number (labeled)

4.5 Calibration

The key physical parameter for a vibrating-wire viscometer is the wire radius. If centerless-ground or other smooth uniformly cylindrical wire of sufficient diameter is used then the radius may be measured directly (e.g., with a laser micrometer) at a number of points along the intended length, and at several angles around the axis, leading to a robust mean squared value (note the presence of R^2 in Eqs. 6 and 12). In this case, an absolute measurement of the viscosity becomes possible.

Alternatively, the mean squared radius may be obtained by calibration in a fluid of known viscosity and density, leading to relative measurements. This calibration approach is also required when, e.g., the wire diameter is too small for direct measurements with adequate precision. As noted above, drawn wires tend to have a rough finish such that direct dimensional measurements are not meaningful; in these cases, calibration in fluid returns an effective mean squared radius. While this is a step back from fulfilling all the requirements of the theory, the evidence is that errors cancel between calibration and measurement.

In either approach, one requires knowledge of the density of the wire material. Furthermore, when calibration is carried out at one state point, but subsequent measurements are made at different temperatures and pressures, both the wire radius and density must be corrected for thermal expansivity and compressibility.

A second parameter to determine by calibration is the vacuum decrement Δ_0 . In principle, this is straight forward but attention should be paid to achieving an adequate vacuum as this may be challenging for small-diameter wires installed in a high-pressure system connected to the vacuum pump via long small-bore tubes. The problem is illustrated in Fig. 6 where we plot the principal contribution to the logarithmic decrement arising from residual gas, given by $\beta'/2$, using an approximate model for the viscosity of the rarefied gas. This contribution may be compared

with typical Δ_0 values [6, 20, 21] which roughly correlate with wire diameter D as $\Delta_0 = 4 \times 10^{-4} D/\text{mm}$. One can see that quite high vacuum is required for small-diameter wires. In the case of larger diameter wires (e.g., ≥ 0.1 mm) it may be preferable to make a calibration measurement in, for example, nitrogen gas at a low pressure that can be measured precisely (e.g., 1000 hPa) and to deduce Δ_0 from the total logarithmic decrement Δ taking account of the viscous damping provided by the gas.

In the case of the vibrating-wire viscometer-densimeter, there are additional parameters to determine by measurement and/or calibration. These are the mass m_s and volume V_s of the sinker, the length L of the wire and a parameter to determine the resonance frequency in a vacuum. The mass and volume of the sinker can be determined by weighing the sinker in air and when immersed in a fluid of known density. The length of the wire can be measured after assembly of the wire-sinker unit, for example by means of a cathetometer and calibrated scale or a precision internal micrometer. Finally, if Eq. 22 is used for the vacuum resonance frequency, then the parameter α may be adjusted to fit the vacuum resonance frequency. Alternatively, one might set $\alpha = 1$ and determine L not by direct measurement but to fit the measured vacuum resonance frequency.

Typically, R , L and V_s are determined at a single temperature and pressure. Values at other temperatures and pressures are then calculated based on the thermal expansivity and isothermal compressibility of the relevant materials. The properties of tungsten have been reviewed and correlated by Mohammed et al. [23]. In the vibrating-wire viscometer-densimeter, it is important that the vacuum resonance frequency is accurately represented, and so α should be measured over the working temperature range of the instrument.

4.6 Uncertainty

The rigorous assessment of uncertainty by means of conventional Gaussian error propagation is somewhat hampered by the complexity and non-linearity of the working equations. As a result, most results have been reported based on estimated uncertainties of the measured and calibrated quantities combined with an approximate sensitivity analysis. The relative standard uncertainty $u_r(\eta)$ in viscosity may be expressed in the following form:

$$u_r^2(\eta) = \sum_i \left[\left(\partial \ln \eta / \partial \ln z_i \right)_{z_j \neq i} u_r(z_i) \right]^2. \quad (23)$$

Here, z_i ($i = 1, 2, 3 \dots$) are the set of experimental quantities required to determine η . For example, in the absolute transient mode of operation, the required inputs are Δ , ω , R^2 , ρ , ρ_w , and Δ_0 . The differential coefficients can be determined numerically. In the cases of a relative measurement, one must consider, first, the calibration step in which the mean value of R^2 is determined and, second, the actual measurement. In the first step, the inputs are Δ_{cal} , ω_{cal} , η_{cal} , ρ_{cal} , ρ_w , and Δ_0 , where subscript 'cal' denotes either quantities measured in or properties of the calibration fluid. Typically, $u_r(R^2) \approx u_r(\eta)$.

In the steady-state mode, one does not usually calculate ‘intermediate’ quantities equivalent to Δ and ω in the transient method. Instead, the parameters η , Λ , ω_0 are regressed against the entire resonance curve. Nevertheless, Mohammed et al. argue that the process of regressing η is equivalent to fitting the value of Ω at the experimental resonance frequency [23]. On that basis, they report analytical expressions for the differential coefficients in Eq. 23. Their analysis extends also to the vibrating-wire viscometer-densimeter and includes a detailed analysis of the uncertainties in the properties of both the wire and the sinker.

The lowest uncertainties have been reported for relative viscosity measurements on gases where the density was known independently with negligible uncertainty. In these cases, expanded relative uncertainties as low as 0.2 % (at 95 % confidence) have been reported [24]. In the absolute mode, slightly larger relative uncertainties of, e.g., 0.5 % [7] and 0.9 % [25] (also at 95 % confidence) have been reported. With the combined vibrating-wire viscometer-densimeter, expanded relative uncertainties of 2 % for viscosity and 0.2 % for density are typical [23, 26, 27].

The problem of estimating uncertainties might be better resolved by application of the Monte Carlo method [28]. Such an analysis would utilize the raw data (either $\varphi(t)$ or $\varphi(\omega)$) and simultaneously consider calibration and measurement steps, as well as the additional parameters and their uncertainties required to compute the influence of temperature and pressure upon the properties of the wire and, where relevant, the sinker.

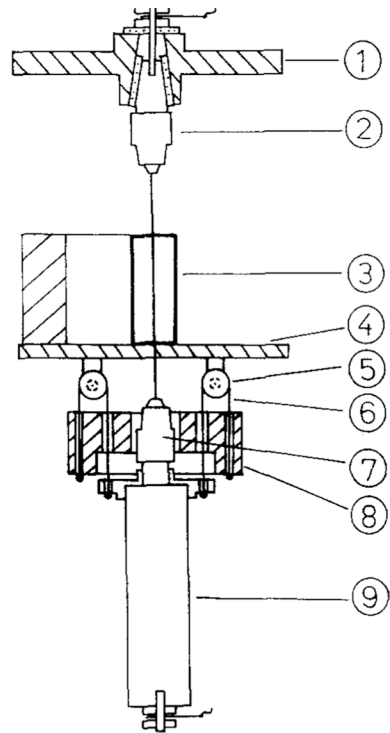
5 Examples from the Literature

Over the past three decades, much work has been done on improving vibrating-wire viscometry, and many data sets for pure substances and mixtures, in both liquid and gaseous states, have been published. Kandil et al. [29] presented a brief literature review as of 2005, including a differentiation of transient and steady-state instruments. Here, we present a few selected practical approaches from the literature, chosen to give an impression of how the measuring principle can be realized. For information about measurement uncertainties, we refer to the respective papers because consistent information across all methods is not possible without an equal uncertainty analysis.

5.1 Transient Instruments

Assael et al. [8] established a vibrating-wire viscometer dedicated to the operation in the liquid phase up to high pressures (up to a few hundred MPa) and over a moderate temperature range from 270 K to 370 K. A schematic of this transient instrument is shown in Fig. 7. The vibrating wire was made of tungsten and was clamped between two chucks where the bottom chuck carried a central weight (also made of tungsten) providing the main tension to the wire. A second weight, made of stainless steel, was connected to the central weight through a special balance mechanism. A tungsten wire passed through a small hole at the top of the central weight. It was then

Fig. 7 Schematic overview of the vibrating-wire viscometer cell of Assael et al. (1992). 1, support plate (which is connected to the not shown upper closure of the high-pressure vessel); 2, upper chuck; 3, samarium-cobalt magnets; 4, middle plate; 5, pulley; 6, tungsten wire; 7, bottom chuck; 8, second weight; 9, central weight. 1 is insulated from 2 by a PTFE washer and a ceramic cone. The vibrating wire is made of 100 μm diameter and 56 mm length tungsten wire supported between 2 and 7 leading to a vacuum resonant frequency of 1 kHz. Reprinted with permission from Assael et al. [8], copyright 1991 Springer Nature



held in place by a crimped tube and passed over a pulley. The appropriate choice of the volumes and masses of the two weights ensured that buoyancy forces exerted on the inner weight were compensated by an equal and opposite force on the outer weight while maintaining the lower attachment point of the vibrating wire as a node.

In 1998, Wilhelm et al. [6] presented a transient instrument for measurements in dilute and dense gases. Gas phase measurements were possible at temperatures from 298 K to 523 K with pressures up to 40 MPa. Improvements to this instrument were, for instance, published in 2000 by Wilhelm and Vogel [24] within the scope of accurate measurements of argon, krypton, and propane. In one instrument, two wires with different radii were used to cover wide ranges of density and viscosity (Fig. 8). A 25 μm wire was used to study gases at low pressures, while a 50 μm wire worked for supercritical pressures. Simultaneous measurements in the intermediate pressure range verified the instrument's operation. The wires were tightly clamped between glass blocks at the top and brass-stubbed cones at the bottom to prevent relative motion and to avoid torsion. The weights at the lower ends were 1.1 g and 4.5 g, resulting in an angular vacuum frequency of 1800 s^{-1} . The authors report that they chose Chromel as the wire material instead of tungsten as conventionally used in measurements on liquids. They did this because the rough surface of drawn tungsten wires was found to give rise to systematic errors in measurements on gases.

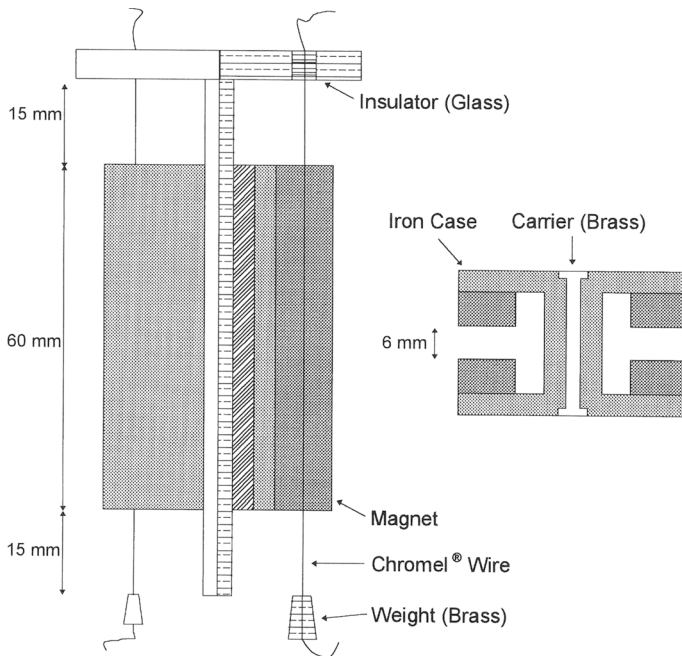


Fig. 8 Schematic illustration of the vibrating-wire viscometer cell of Wilhelm et al. (1998) showing the side view (left) with a cut-out in the right half and the top view (right in the picture). The magnets are made of samarium-cobalt. Reprinted with permission from Wilhelm et al. [6], copyright 1998 Springer Nature

5.2 Steady-State Instruments

Kandil et al. [29] described the development of a vibrating-wire viscometer using an electrically insulating wire-tensioning mechanism. Goodwin and Marsh [25] further developed this instrument to operate as an ‘absolute’ technique to measure the viscosity of water under ambient conditions. The vibrating wire, as shown in Fig. 9, was a cold-drawn tungsten wire approximately 40 mm long with a nominal diameter of either 50 μm or 150 μm . Clamps made from austenitic stainless-steel grade 316 were used to restrict the wire’s motion to one plane perpendicular to the magnetic field. The wire was clamped within a slot to confine its motion, and the magnetic field was aligned to vibrate the wire in a plane perpendicular to the slot. One end of the wire was clamped between the two plates using stainless steel screws, while the other end was attached to a mass for approximately 24 h before the clamps were tightened and the mass removed. The tensioning mass was selected on the basis of the desired fundamental resonance frequency, neglecting the wire stiffness. The two clamps were separated from each other by a tube made of Macor machinable ceramic.

In parallel to the work of Kandil et al. [29] published in 2005, Lundstrum et al. [30] introduced a different design of the vibrating-wire holder as shown in panel (a) of Fig. 10. Again, the wire was held at each end using a two-part assembly: a flat

Fig. 9 Simplified cross-section of the vibrating-wire viscometer showing the holder made from Macor, the SmCo magnets, and the non-magnetic wire clamps of Kandil et al. (2005). Reprinted with permission from Kandil et al. [29], copyright 2005 American Chemical Society

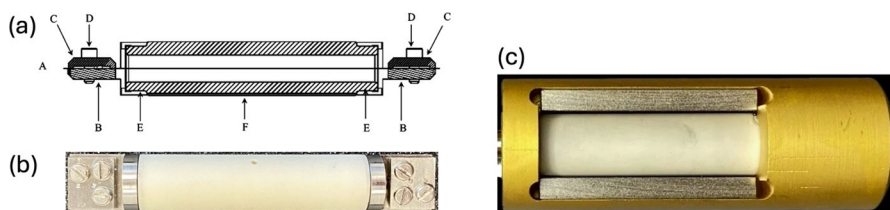
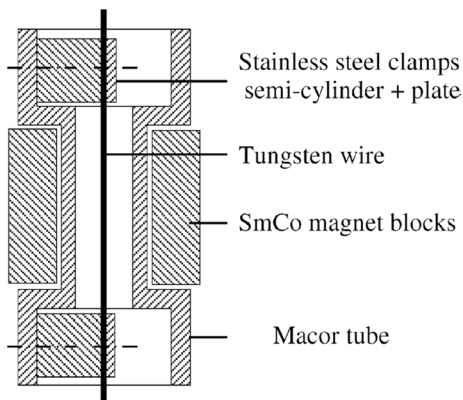


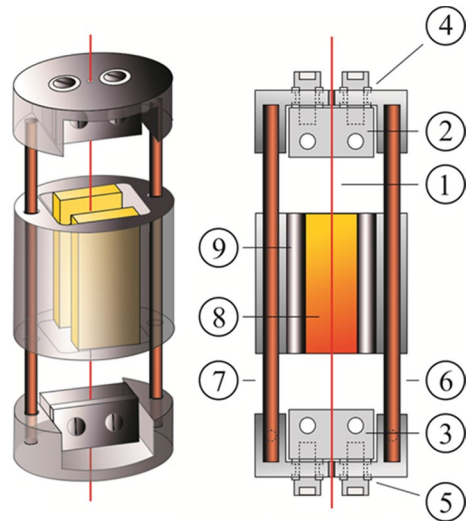
Fig. 10 **a** Cross-section through the vibrating-wire holder reported by Lundstrum et al. [30]. The wire A is clamped at each end between two Monel parts, B and C, which are held together with screws D; the neck plate of B attaches to a spacer F fabricated from Shapal-M (machinable aluminum nitride ceramic) at an interlocking step E. **b** Photograph of a similar assembly with all components except the spacer F made of austenitic stainless-steel grade 316. **c** Photograph of the vibrating-wire holder placed inside a yoke (made of carbon steel, gold-plated) that holds the two rectangular samarium-cobalt magnets. Not visible: two PEEK rings as spacers between the holder and the yoke. Panel **a** reprinted with permission from Lundstrum et al. [30], copyright 2005 American Chemical Society

plate with a circular boss that slid onto a separating tube, and a flat clamping plate. These two pieces were held together with two M2 austenitic stainless-steel screws. A pin made from Monel 400 was used to maintain the wire's axial symmetry and to align the two plates. The clamps were both mechanically supported and electrically isolated from each other with a ceramic tube made from Shapal-M (aluminum nitride) machinable ceramic. This material was chosen to reduce the effects of differential thermal expansion on the wire tension.

To provide tension, one end of the wire was clamped between plates B and C (see Fig. 10a), while the other end of the wire was passed through the second open clamp and attached to a mass for about 24 h. After this time, the lower clamp was carefully tightened and the mass removed. The tensioning mass was selected to determine the fundamental resonance frequency and to maintain the tension between 20 % and 50 % of the tensile yield stress for tungsten (approximately 720 MPa).

The design of the vibrating-wire viscometer shown in panels (b) and (c) of Fig. 10 has been used by the authors' research groups for many studies on liquids and gases

Fig. 11 Schematic illustration of the vibrating-wire viscometer of Assael and Mylona [31]. 1, vibrating wire (300 μm diameter tungsten wire); 2, 3, wire clamps; 4, 5, ceramic washers; 6, 7, 3 mm diameter tungsten rods; 8, magnets; 9, magnet support. Reprinted with permission from Assael and Mylona [31], copyright 2013 American Chemical Society

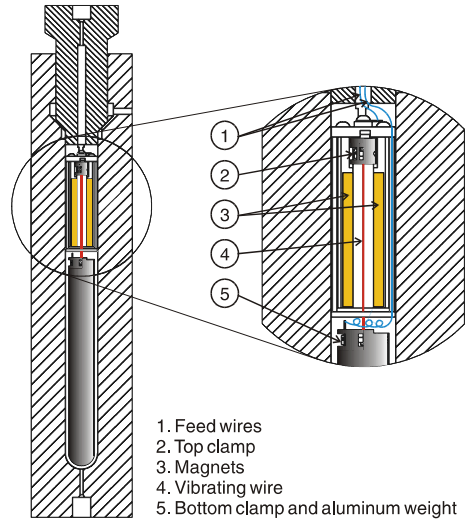


over large temperature, pressure and viscosity ranges. Recently, such a viscometer was used for measurements on cryogenic liquid mixtures such as liquefied natural gas at temperatures as low as 117 K with pressures up to 8.5 MPa (with the results to be published soon by the present authors). In 2011, Peleties and Trusler [17] used this instrument design with a 300 μm diameter tungsten wire to investigate high-viscosity liquids such as oils, a case for which Assael and Mylona [31] report a different construction as shown in Fig. 11. Furthermore, the instrument developed by Kandil et al. [29] was modified by Locke et al. [32] to enable gas-mixture viscosity measurements. However, it was operational over the entire fluid range (gas, liquid, supercritical) as shown, for example, by Yang et al. [33] and Xiao et al. [34] for refrigerant mixtures.

5.3 Viscometer-Densimeters

The simultaneous measurement of viscosity and density is particularly useful from both a modeling perspective and because the fluid density is part of the working equations (see Table 1). Furthermore, commercially available densimeters are restricted to pressures below approximately 140 MPa while the vibrating-wire approach may be extended to much higher pressures. Instruments in which both viscosity and density can be measured together are more complicated to use. Nevertheless, a number of different instruments have been reported to date. A design for operation at moderately-high pressures (up to 100 MPa) and temperatures between 298 K to 348 K was established by Pádua et al. [13] in 1994 and further developed by Audonnet and Pádua [20]. Caudwell et al. [14, 26] described a similar instrument, shown in Fig. 12, with an extended working range permitting pressures up to 200 MPa at temperatures between 298 K and 473 K. This was subsequently used by Ciotta et al. [35] and Mohammed et al. [23] to study hydrocarbon liquids with dissolved gases.

Fig. 12 Vibrating-wire apparatus of Caudwell et al. [14] where the vibrating wire and magnetic assembly were housed in a commercially available pressure vessel. Reprinted with permission from Caudwell et al. [14], copyright 2004 Springer Nature



In some situations, density is well known from independent sources. If this is not the case, then it is necessary to measure it. A unique alternative to obtaining the density from the vibrating-wire sensor itself was described by Seibt et al. [36]. Their instrument combined the vibrating-wire viscometer of Wilhelm et al. [6] and Wilhelm and Vogel [24] with an absolute magnetic-suspension densimeter, reported internationally for the first time by Wagner et al. [37]. This approach was designed for gas-phase measurements over the temperature range from 253 K to 423 K at pressures up to 30 MPa. Another common alternative dedicated to high-pressure, high-density measurements involves combining a vibrating-wire viscometer with a commercial vibrating-tube densimeter. This approach was followed by McBride-Wright et al. [16] for the measurement of aqueous solutions of carbon dioxide at temperatures from 274 K to 449 K and at pressures up to 100 MPa. Compared to a combined vibrating-wire viscometer-densimeter, this is less complex and more robust.

6 Conclusions

This review demonstrates that the vibrating-wire viscometer is a well-established instrument with a rigorous working equation and the potential for achieving reliable measurements with comparatively low relative uncertainties. It may be operated in either an absolute or a relative manner and with either transient or steady-state excitation. The relative mode has been employed most often as it avoids the problem of determining the wire radius directly. There is little to choose between the transient and steady-state modes and both have been widely used. Although not available commercially, the information and references provided in this article should help guide the design, construction and use of vibrating-wire instruments for viscosity or combined viscosity and density measurement.

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Author Contribution MR reviewed relevant literature and prepared part of the original draft. JPMT reviewed relevant literature and prepared part of the original draft. Both authors reviewed and revised the manuscript.

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Data Availability No datasets were generated or analysed during the current study.

Declarations

Competing interests The authors declare no competing interests.

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References

1. G.G. Stokes, J. Larmor, J.W.S.B. Rayleigh, *Mathematical and Physical Papers* (University Press, 1901)
2. J.T. Tough, W.D. McCormick, J.G. Dash, *Phys. Rev.* **132**, 2373 (1963)
3. J.T. Tough, W.D. McCormick, J.G. Dash, *Rev. Sci. Instrum.* **35**, 1345 (1964)
4. T. Retsina, S.M. Richardson, W.A. Wakeham, *Appl. Sci. Res.* **43**, 127 (1986)
5. T. Retsina, S.M. Richardson, W.A. Wakeham, *Appl. Sci. Res.* **43**, 325 (1987)
6. J. Wilhelm, E. Vogel, J.K. Lehmann, W.A. Wakeham, *Int. J. Thermophys.* **19**, 391 (1998)
7. M.J. Assael, M. Papadaki, M. Dix, S.M. Richardson, W.A. Wakeham, *Int. J. Thermophys.* **12**, 231 (1991)
8. M.J. Assael, C.P. Oliveira, M. Papadaki, W.A. Wakeham, *Int. J. Thermophys.* **13**, 593 (1992)
9. D.R. Caudwell, PhD Thesis, University of London, 2004
10. S.S. Chen, M.W. Wambsganss, J.A. Jendrzejczyk, *J. Appl. Mech.* **43**, 325 (1976)
11. F. Ciotta, J.P.M. Trusler, *J. Chem. Eng. Data* **55**, 2195 (2010)
12. P.M.C. Morse, K.U. Ingard, *Theoretical Acoustics* (McGraw-Hill, 1968)
13. A.A.H. Padua, J.M.N.A. Fareleira, J.C.G. Calado, W.A. Wakeham, *Int. J. Thermophys.* **15**, 229 (1994)
14. D.R. Caudwell, J.P.M. Trusler, V. Vesovic, W.A. Wakeham, *Int. J. Thermophys.* **25**, 1339 (2004)
15. F. Ciotta, *PhD Thesis* (Imperial College London, London, 2010)
16. M. McBride-Wright, G.C. Maitland, J.P.M. Trusler, *J. Chem. Eng. Data* **60**, 171 (2015)
17. F. Peleties, J.P.M. Trusler, *J. Chem. Eng. Data* **56**, 2236 (2011)
18. M. McBride-Wright, *PhD Thesis* (Imperial College London, London, 2013)
19. A.A.H. Pádua, J.M.N.A. Fareleira, J.C.G. Calado, W.A. Wakeham, *Rev. Sci. Instrum.* **69**, 2392 (1998)
20. F. Audonnet, A.A.H. Padua, *Fluid Phase Equilib.* **181**, 147 (2001)

21. F. Peleties, *PhD Thesis* (Imperial College London, London, 2007)
22. V.K. Michalis, A.N. Kalarakis, E.D. Skouras, V.N. Burganos, *Microfluid. Nanofluid. Nanofluid.* **9**, 847 (2010)
23. M. Mohammed, F. Ciotta, J.P.M. Trusler, *J. Chem. Eng. Data* **62**, 422 (2017)
24. J. Wilhelm, E. Vogel, *Int. J. Thermophys. Thermophys.* **21**, 301 (2000)
25. A.R.H. Goodwin, K.N. Marsh, *J. Chem. Eng. Data* **56**, 167 (2011)
26. D.R. Caudwell, J.P.M. Trusler, V. Vesovic, W.A. Wakeham, *J. Chem. Eng. Data* **54**, 359 (2009)
27. D.R. Caudwell, J.P.M. Trusler, V. Vesovic, W.A. Wakeham, *J. Chem. Eng. Data* **55**, 5396 (2010)
28. C.E. Papadopoulos, H. Yeung, *Flow Meas. Instrum. Instrum.* **12**, 291 (2001)
29. M.E. Kandil, K.N. Marsh, A.R.H. Goodwin, *J. Chem. Eng. Data* **50**, 647 (2005)
30. R. Lundstrum, A.R.H. Goodwin, K. Hsu, M. Frels, D.R. Caudwell, J.P.M. Trusler, K.N. Marsh, *J. Chem. Eng. Data* **50**, 1377 (2005)
31. M.J. Assael, S.K. Mylona, *J. Chem. Eng. Data* **58**, 993 (2013)
32. C.R. Locke, P.L. Stanwix, T.J. Hughes, A. Kisselev, A.R.H. Goodwin, K.N. Marsh, E.F. May, *J. Chem. Eng. Data* **59**, 1619 (2014)
33. X. Yang, A. Arami-Niya, X. Xiao, D. Kim, S.Z.S. Al Ghafri, T. Tsuji, Y. Tanaka, Y. Seiki, E.F. May, *J. Chem. Eng. Data* **65**, 4252 (2020)
34. X. Xiao, D. Kim, F. Jiao, X. Yang, S. Al Ghafri, A. Siahvashi, T. Tsuji, A. Yukumoto, Y. Seiki, P.L. Stanwix, E.F. May, *Int. J. Refrig. Refrig.* **152**, 331 (2023)
35. F. Ciotta, G. Maitland, M. Smietana, J.P.M. Trusler, V. Vesovic, *J. Chem. Eng. Data* **54**, 2436 (2009)
36. D. Seibt, S. Herrmann, E. Vogel, E. Bich, E. Hassel, *J. Chem. Eng. Data* **54**, 2626 (2009)
37. W. Wagner, K. Brachthäuser, R. Kleinrahm, H.W. Lösch, *Int. J. Thermophys. Thermophys.* **16**, 399 (1995)

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