

**STATISTICAL PROPERTIES OF LIGHT PROPAGATED  
THROUGH ATMOSPHERIC TURBULENCE AND  
SCATTERED BY DENSE RANDOM MEDIA**

by

**Dong Ning QU**

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## Publications

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2. D. N. Qu and J. C. Dainty, "Polarization dependence of dynamic light scattering by dense disordered media", Opt. Lett., **13**, p.1066(1988)
3. J. C. Dainty, D. N. Qu and S. Xu, "Statistical properties of enhanced backscattering produced by dense collections of latex spheres", Proc. SPIE, **976**, p.178(1988)
4. J. C. Dainty, D. N. Qu and M. J. Northcott, "Photon correlation of images through turbulence", Proc. SPIE, **976**, p.168(1988)
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## Abstract

In this thesis, the statistical properties of light propagated through atmospheric turbulence and scattered by dense random media are studied. Photon counting and correlation techniques were used in measuring these statistical properties.

In chapter 2 and 3, the principles of photon counting and correlation techniques are dealt with. The instrumentation aspects of the techniques, particularly the detector, are given. A performance study of a microchannel plate photomultiplier tube (MCP-PMT) for this purpose is presented in chapter 4. It is concluded that this MCP-PMT is not suitable for measuring space-time photon correlation functions due to the severe cross talk between the anodes and large spatial integration.

In chapter 5, speckle interferometry and several techniques of image processing and reconstruction from stellar speckle data are outlined. The temporal fluctuation of images viewed through atmospheric turbulence and its wavelength dependence are analyzed, and the importance of dealing with temporal properties is emphasised. Site measurements at La Palma Observatory, Canary Islands are reported for 11 nights during May 1988 and February 1989. It was found that the image fluctuation time scale (the  $1/e$  correlation time) there was on the order of  $5\text{ ms}$ . The wavelength dependence of the correlation time was consistent with the  $\lambda^{6/5}$  law. Measurements of the intensity correlation time for pairs of sub-apertures showed that their correlation times were on the same order as that for a full aperture image. The baseline dependence was found to be very weak when the baseline was larger than Fried's parameter  $r_o$ .

In chapter 6, the statistics of dynamic light scattering from dense random media are presented. The enhanced backscattering phenomenon is particularly dealt with. The polarisation and scattering angle dependence of scattered light around backscatter is studied and explained. It is found that the dynamic properties of

the scattered light have a strong polarisation dependence and it is related to enhanced backscattering. The measured correlation time shows completely different variation with angle in the co-polarised and cross-polarised cases. The normalised factorial moments of the detected photon counts (which equal to normalised intensity moments) are found independent of angle and detecting polarisations, and they are in agreement with those derived from gamma intensity distribution with the descriptive parameter  $u = 1.20$ .

In chapter 7, a summary of the measurements on the stellar image fluctuations is given and the photon localization effect related to enhanced backscattering is also discussed.

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# Chapter 1

## Introduction

### §1–1: General Introduction

Atmospheric turbulence has been a major problem for optical astronomers for a long time. The angular resolution of telescopes is greatly reduced by the inhomogeneity of the atmosphere. It is found that the “seeing” disc is always limited to an arcsecond or more no matter how large one increases the telescope aperture size. The desired diffraction-limited resolution cannot be achieved by conventional imaging in the presence of atmospheric turbulence.

In 1921, Michelson and Pease (1921) successfully measured the apparent diameter of Betelgeuse and of a few other bright red stars by using an interferometric technique. The basic theory is Young’s fringe theory. For two point sources separated by an angle  $\theta$ , with a two sub-aperture interferometer a combined interference pattern will be formed by summing at each point of the focal plane of the telescope the intensities in two interference patterns. These two patterns are mutually displaced and the fringes show periodic variations of distinctness or visibility. The first visibility minimum appears when  $d = \lambda/(2\theta)$  [Born and Wolf (1980)], where  $d$  is the baseline and  $\lambda$  is the incident wavelength. Since the visibility of the fringe pattern is related to the baseline, while  $d$  is inversely proportional to the angular diameter of the star, there is an exact relationship between fringe contrast and object structure. Therefore, by changing the baseline and detecting the visibility it is feasible to determine the angular diameter of the star. With a single telescope, the baseline is limited to the diameter of the aperture. To increase the resolving power beyond that of the telescope, Michelson used a long-baseline stellar interferometer. Because the sampling sub-apertures are smaller than the atmospheric turbulence characteristic size, the measurement of fringe visibility is not affected by the atmosphere, at least to first order (assuming wavefront tilts are removed). The whole fringe pattern is shifting because of the

turbulence effects but one can still follow the movement of the whole fringe pattern. Therefore, with the Michelson interferometer, the spread function consists of Young's fringes which can be analysed to extract diffraction-limited information. However the difficulties with Michelson interferometry lie in the fact that it is difficult to achieve the necessary mechanical precision in a large instrument and to make precise measurements in the presence of atmospheric scintillation.

In 1956, Hanbury-Brown and Twiss (1956) used a new type of interferometer, the intensity interferometer, which allowed the measurement of the correlation of the signals from two photomultiplier tubes (PMTs) that received light from a star. This correlation is proportional to the square of the modulus of the degree of spatial coherence of the starlight. Very good resolution may be obtained if the two PMTs are far enough apart. The angular diameter of a wide range of stars can be measured using intensity interferometer with precision through the atmospheric turbulence. The intensity interferometer does not have the stability problem of Michelson interferometry and it has a higher angular resolution. The main limitations are those of energy. In practice, the bandwidth is small and the available energy is greatly reduced, so that an integration time of several hours is required to give a reasonable signal-to-noise ratio. Because of the low sensitivity, to achieve a high signal-to-noise ratio, a very large light collector is required even for bright stars; a fairly long collecting time is also required. Another problem associated with the intensity interferometer is that the two detectors should have the same quantum efficiency.

Although interferometric techniques can yield diffraction-limited information they do not make full use of collected light and not give full diffraction-limited information, i.e. images of the objects. On the other hand, for a full aperture image the spread function consists of speckles which are the superposition of many randomly shifting fringe patterns of different frequencies. This makes it difficult to extract diffraction-limited information from a long exposure image. It is possible to get a slightly improved image by deconvolving the blurred images. The spread

function of such an image is, however, time-dependent and not completely known. Hence it is generally impractical to directly apply deconvolution procedures.

In 1970, Labeyrie (1970) suggested a technique called speckle interferometry in which the whole aperture of the telescope was used and many short exposure pictures were taken and processed. In contrast to the conventional observing procedures which ignore the interference structure, speckle interferometry takes advantage of the interference effects which result in a stellar speckle pattern. The diffraction-limited power spectrum or autocorrelation of the object can be extracted. Diffraction-limited images can also be reconstructed by using various high resolution imaging techniques to discover both modulus and phase of the object transform. A outline of these techniques is given in Chapter 5.

In speckle interferometry and related techniques, many short exposure data are taken in order to freeze the speckle patterns which contain diffraction-limited information. The exposure time is chosen so that it is short enough to freeze the speckle patterns and long enough to collect sufficient light and hence attain a reasonable signal-to-noise ratio. This exposure is related to the correlation time of the image intensity fluctuation due to atmospheric turbulence. Knowledge about the correlation time is clearly important in high resolution imaging techniques. At different observatories, the temporal “seeing” conditions are different. Several people have measured this time behaviour at different observatories, e.g. Parry *et al* (1977) at Hermonceux Observatory, U.K., Dainty *et al* (1980) at Belles Observatory, New York and O'Donnell *et al* (1982) at Mauna Kea Observatory, Hawaii. In this Thesis a report on the evaluation of temporal properties of image intensity fluctuations measured at La Palma Observatory, Canary Islands will be presented.

To achieve an adequate signal-to-noise ratio, we used a photon counting and correlation technique to analyze the statistical properties of image intensities. This technique is suitable for the analysis of statistical properties of light at low light level. In the technique, incoming individual photons are counted by a photo-

detector and related electronic circuit. Noise counts whose pulse heights are smaller than the signal pulse heights are eliminated by a reference threshold.

Using the same technique of photon correlation, the phenomenon of enhanced backscattering was studied. Enhanced backscattering has been of considerable interest recently. It is motivated by the possibility of observing Anderson localisation [Anderson (1958)] in the visible electro-magnetic spectrum. This strong type of localisation is due to the absence of diffusion in the random media caused by the interference of all the scattered waves. The enhanced backscattering is regarded as a precursor of strong localisation of light in very disordered media. This phenomenon is also related to the classical scattering problem and it is thought that the backscattered interference is the only interference effect presented in the dense random media.

It is established that the enhancement is due to a coherent backscattering effect [Akkermans *et al* (1986)] which is a general effect in the multiple scattering of waves by a disordered medium. In the classical approach to the scattering problem, it is assumed that the phases of scattered waves are uncorrelated and hence the scattered waves are incoherent. The scattering process is a process of transport of scattered energy. This is true for single scattering when the scattering medium is very dilute. Interference effects in the scattering process are neglected. However, when dealing with the scattering problem from highly concentrated media, the interference effects, particularly in the backscattering direction should be taken in account.

The effect of interference in the multiple scattering was first discussed by De Wolf (1971) in 1971 and Barabanenkov (1975) in 1975. In 1984, Kuga and Ishimaru (1984) demonstrated an enhancement of 1.15 in the backscattering direction of light scattered by a suspension of latex spheres for the co-polarised case. This was explained by Tsang and Ishimaru (1984) in terms of double scattering. Later Albada *et al* (1985) made measurements on enhanced backscattering and found the enhancement of 1.7 for the co-polarised case. The enhancement line shape

for the co-polarised case was then analysed by Akkermans *et al* (1986) in terms of classical interference effect. The dynamic effect of the scatterers due to Brownian motion on scattered light was studied by Maret *et al* (1987). This raised a great deal of interest and a deepening understanding the phenomenon. Many new features need to be fully studied and understood. They include the polarisation dependence of the scattered average intensity, the difference of enhancement factors for the co-polarised and cross-polarised cases due to vector characteristics of light wave, enhancement line shape and anisotropy of the enhancement cone and temporal fluctuations of the scattered intensity both inside and outside the enhancement cone.

## §1–2: Thesis Synopsis

In the thesis, we will present studies of atmospheric turbulence effects on imaging and dynamic light scattering due to dense media by means of photon counting and correlation.

In chapter 2 the principles of photon counting techniques are discussed. The advantages and disadvantages of these techniques are also examined. The photon counting statistics of various intensity distributions are analysed; so are the normalised factorial moments. In practical measurements, non-ideal effects such as dead time and temporal integration bias the real results. Such bias effects are also discussed in chapter 2. Finally the inverse problem of obtaining intensity distribution from the knowledge of photon counting statistics is considered. In chapter 3, we study the relationship between photon correlation and the relevant intensity correlation. It is established that the normalised photon correlation function is equal to the normalised intensity correlation function. Hence the intensity correlation functions can be analysed by means of the photon correlation technique with its advantage of high signal-to-noise ratio, particularly at low light level. The instrumentation aspects of photon correlation techniques are discussed and the possibility of using an avalanche photodiode as an alternative detector to

minimise the detecting system<sup>complexity</sup> is looked into. We also discuss various schemes to simplify digital photon correlation measuring devices by looking at clipped and scaled correlation techniques.

In chapter 4, we evaluate a micro-channel plate photomultiplier tube (MCP-PMT) for photon counting and correlation. A description of features of apparatus including the PMT, the MCP-PMT and their related electronic circuits is given. A performance study of the MCP-PMT is described in detail including quantum counting efficiency, dark count, afterpulsing, pulse height distribution, dead time and cross talk. Finally, measurements of photon correlation functions with the MCP-PMT for a Gaussian field are presented. The results are compared with known theoretical predictions and it is concluded that the cross talk and spatial integration severely limit the application of this type of tube in photon correlation measurements.

Chapter 5 summarises the correlation measurements of full aperture images and fringe patterns formed by two sub-apertures of a 1 m telescope at La Palma Observatory, Canary Islands. We first discuss the atmospheric turbulence effects on optical imaging and principles of speckle interferometry in which many short exposure data are taken in order to freeze the stellar speckle patterns and hence overcome the blurring effects caused by atmosphere. The optimum value of the exposure time is discussed and it is found that when the short exposure time is about twice the correlation time of the image intensity we can achieve a maximum signal-to-noise ratio. This is the motivation for measuring the correlation time at La Palma Observatory, in order to decide the optimum exposure time in high angular resolution stellar imaging for the data taken there. From our measurements it is found the correlation time for the full aperture images at La Palma Observatory is on the order of 5 ms and the correlation times for the fringe patterns are on the same order regardless of the baselines as long as they are longer than the Fried parameter  $r_0$ . This gives some indications for measurements using Michelson stellar interferometry.

In chapter 6, we discuss the enhanced backscattering phenomenon in detail. It is established that enhanced backscattering is due to multiple scattering. We first present an outline of the single scattering problem. The scattered intensities and their auto-correlation functions are summarised. Then a study of multiple scattering is presented. The measurements of enhanced intensities at backscatter for both co-polarised and cross-polarised detection are given. The principles of this phenomenon are explained. We discuss the spatial anisotropy of the enhancement line shape for co-polarised detection and the enhancement factor for both polarisation cases are discussed in terms of the vector characteristics of light waves. The polarisation degree of the backscattered light is also found to have an enhancement at backscatter. The intensity enhancement cone line shape for the co-polarised case is analysed. Chapter 6 also contains the measurements and analysis of dynamic properties of backscattered light. The correlation times for co-polarised and cross-polarised detection are found to have completely different behaviours. A physical explanation for this related to polarisation dependence of enhanced intensities is given. We also summarise the measurements and analysis of photon counting statistics. It is found that they are independent of angle and polarisation. The distribution obeys  $\chi^2$  Gamma distribution with the descriptive parameter  $u = 1.20$ . Finally a discussion of possible suppression of enhanced backscattering by the Faraday effect is presented.

In chapter 7, the conclusions about the temporal correlation time of stellar image intensity fluctuations at La Palma Observatory are given. The implementation aspect of high resolution speckle imaging is outlined. Finally, the possibility of observing Anderson localisation in the visible <sup>part</sup> of <sup>the</sup> electro-magnetic spectrum is discussed. We also speculate on potential developments in this field.

### §1-3: Original Contributions in the Thesis

The original work in this thesis consists principally of the experimental measurements described in chapters 4, 5 and 6. In order to place these measurements

in context, introductory and review material is given in chapters 2 and 3, and in the first parts of chapters 4, 5 and 6.

The measurements of the characteristics of the MCP–PMT detector, described in chapter 4, are of standard parameters but have not previously been reported for this detector. The measurements of the temporal correlation of stellar images at La Palma Observatory, described in chapter 5, involved the design and construction of an instrument mounted on the 1 *m* telescope, as well as the collection and interpretation of data. I was assisted at La Palma by J. C. Dainty and M. J. Northcott.

The measurements of the time dependence of dynamic light scattering from dense random media, described in chapter 6, were the first to compare the time behaviour of the co- and cross-polarised scattered beams.

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## Chapter 2

### Photon Counting Techniques

#### §2-1: Introduction

In order to detect a weak light intensity with reasonably high signal-to-noise ratio, several photodetection techniques have been designed. They are the lock-in amplifier [Alfano *et al* (1968), Moore (1962)], DC current [Malmstadt *et al* (1984)], noise voltage [Alfano *et al* (1968), Pao *et al* (1966)] and photon counting [Alfano *et al* (1968), Morton (1968)] techniques.

A lock-in amplifier is essentially a narrow-band, phase-sensitive amplifier followed by a low-pass filter. It selects a band of frequencies of bandwidth  $B_f$  about a reference frequency  $f_c$ , from a signal applied to its input, and converts the information to an equivalent bandwidth about zero frequency. The DC signal is amplified and passed on to a low pass filter which eliminates noise of high frequencies. To realize this principle, one often uses an optical chopper to add a frequency  $f_c$  at the input. The output signal then is modulated by this frequency and the detection system only selects the signals of this frequency; therefore the noise which has different frequencies can then be eliminated. The advantage of this technique is that it does not need very good environmental conditions, e.g. the background light with different frequencies can be filtered out easily. As long as the detected signal-to-noise ratio is not limited by shot noise\*, the lock-in amplifier technique is preferred.

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\* Shot noise is caused by two sources:

1. Photon noise, which is directly from the fluctuations of incident light field. By the virtue of fluctuation of incident field, the counting rate of photons fluctuates too.
2. Detecting noise, which is resulted from the statistical nature of secondary electron emission from the photocathode, dynodes or electron-multiply channels of the photo-detector.

The DC current technique is equivalent to the lock-in amplifier technique except that its reference frequency is at zero. In the DC current technique  $B_f$  refers to the bandwidth of the DC amplifier used. Hence, with equivalent bandwidth, the same sensitivity, in principle, is obtainable by the DC detection as by lock-in detection. However, DC detection is limited by drift due to ohmic and light leakage and by the fact that DC amplifiers are less stable and more difficult to construct and operate than AC amplifiers.

The noise-voltage detection technique recovers the signal by detecting the associated shot noise. The output current from a PMT irradiated by a photon flux consists of a DC component, which is usually referred to as signal, and an AC portion of unidirectional sharp pulses of variable amplitude, which is referred to as “noise”. For low incident power, there is more power in the AC portion than in the DC portion. Therefore by detecting the AC portion and then extracting the DC portion, the signal-to-noise ratio will be considerably enhanced. To fulfill this task, a circuit is made to amplify the AC portion. A low pass filter is then used to extract the DC portion of the amplified AC signal.

The photon counting technique directly amplifies pulses, discriminates against low height noise counts that did not originate from the photocathode and then counts output pulses. This is a more effective way to get rid of noise and indeed this is the principle of photon counting. The high signal-to-noise ratio comes from the discrimination of noise pulses which have a different pulse height from signal pulses.

It has been shown [Alfano *et al* (1968), Ingle *et al* (1972a,1972b), Nakamura *et al* (1968)] that when the photon counting technique is used to detect the weak light field, the output signal-to-noise ratio approaches theoretical shot noise limit. In fact, the photon counting technique is found to be superior to other methods with regards to both signal sensitivity and signal-to-noise ratio. It also has the advantage of system stability against drift. In addition, photoelectrons are by nature digital, therefore photon counting is easily implemented by digital data

processing. The disadvantage of the photon counting technique is that the background light cannot be eliminated in the detection process. The lock-in method is advantageous in this respect.

## §2-2: Photon Counting Statistics

Due to the inherent advantage of obtaining higher signal-to-noise ratio when shot noise limited, the photon counting technique has been widely used in detecting very low light level intensities. In applying the technique, a photo-detector such as a photomultiplier tube (PMT), is used and operated in photon counting mode. In this case, the output signal is digital in nature because of the shot noise photoelectric detection process. The output pulse numbers correspond to the photon number weighted by the quantum efficiency of the detecting system. The actual relationship between the statistics of the photoelectron count and that of the light falling on the detector is given by the following compound Poisson distribution [Mandel *et al* (1964)]—this is also called Mandel's formula:

$$\begin{aligned} P(n, T) &= \left\langle \frac{(\alpha I)^n}{n!} \exp(-\alpha I) \right\rangle_I \\ &= \int_0^\infty \frac{(\alpha I)^n}{n!} \exp(-\alpha I) P(I) dI \end{aligned} \quad (2.1)$$

where angular brackets represent an ensemble average and  $P(n, T)$  is the probability of counting  $n$  photoelectrons during the detecting time  $T$ ,  $\alpha$  is the detection system's quantum efficiency and  $P(I)$  is the probability density function (PDF) of the intensity  $I$ . Here we have assumed that the light field is stationary and ergodic, so that the statistics of the field are independent of time.

From the above formula, it can be seen that the fluctuations of the detected photon counts are dependent upon two causes:

- (1). The fluctuations of the light intensity itself.
- (2). The fluctuations in the detection process. This is because of the randomness of the secondary emission inside the detector. Even when there are no fluctuations in the light intensity, the detected photon counts will still result in a Poisson distribution.

The analysis of moments of the intensity probability density function has been widely accepted as a useful tool to study the statistics of the intensity. To implement this when photon counting and compare different statistics in different situations, it is necessary to choose *normalised* factorial moments as the descriptive parameters. The  $m$ -th factorial moment is defined as:

$$\mathcal{F}(m) = \left\langle \frac{n!}{(n-m)!} \right\rangle = \sum_{n=0}^{\infty} n(n-1)\cdots(n+m-1)P(n, T) \quad (2.2)$$

and the  $m$ -th *normalised* factorial moment is:

$$F(m) = \frac{\mathcal{F}(m)}{\langle n \rangle^m} \quad (2.3)$$

By using Mandel's formula it is found the  $m$ -th factorial moment of photon counts is proportional to the  $m$ -th moment of the intensity  $I$ ,

$$\mathcal{F}(m) = \left\langle \frac{n!}{(n-m)!} \right\rangle = \alpha^m \langle I^m \rangle \quad (2.4)$$

From Eq.(2.4) it is easy to find:  $\langle n \rangle = \alpha \langle I \rangle$

Hence we have:

$$F(m) = \frac{\alpha^m \langle I^m \rangle}{\langle n \rangle^m} = \frac{\langle I^m \rangle}{\langle I \rangle^m} \quad (2.5)$$

This relationship enable us to analyze the *normalised* moments of intensity by analyzing the *normalised* factorial moments of detected photon counts.

We shall here give a few examples of photon counting statistics  $P(n, T)$ , their factorial moments and normalised ones for several  $P(I)$ .

(1).  $P(I) = \delta(I - \langle I \rangle)$ , i.e. constant light intensity; by using Mandel's formula:

$$P(n, T) = \frac{\langle n \rangle^n}{n!} \exp(-\langle n \rangle) \quad (2.6)$$

which is the Poisson distribution. By using Eq.(2.4), the  $m$ -th factorial moment is:

$$\begin{aligned} \mathcal{F}(m) &= \alpha^m \langle I^m \rangle = \alpha^m \int_0^{\infty} I^m P(I) dI \\ &= \alpha^m \int_0^{\infty} I^m \delta(I - \langle I \rangle) dI \\ &= \alpha^m \langle I \rangle^m = \langle n \rangle^m \end{aligned} \quad (2.7)$$

and the normalised m-th factorial moment is:

$$F(m) = \frac{\mathcal{F}(m)}{\langle n \rangle^m} = 1 \quad (2.8)$$

(2).  $P(I) = \frac{1}{\langle I \rangle} \exp(-\frac{I}{\langle I \rangle})$ : This is the negative exponential distribution and describes the intensity of thermal light. If the photon counting sample time  $T$  is very large, the intensity fluctuations will be averaged out due to temporal averaging effects and the detected average photon counts during the sample time  $T$  will have Poisson statistics as shown for the constant intensity. If  $T$  is very small (very much less than the coherence time of the light), the following photon counting distribution will be obtained by Mandel's formula:

$$P(n, T) = \frac{\langle n \rangle^n}{(1 + \langle n \rangle)^{1+n}} \quad (2.9)$$

which is the Bose-Einstein distribution. The m-th factorial moment is:

$$\begin{aligned} \mathcal{F}(m) &= \alpha^m \langle I^m \rangle = \alpha^m \int_0^\infty I^m P(I) dI \\ &= \alpha^m \int_0^\infty I^m \frac{1}{\langle I \rangle} \exp(-\frac{I}{\langle I \rangle}) dI \\ &= \alpha^m \langle I \rangle^m \int_0^\infty x^m \exp(-x) dx \quad (x = \frac{I}{\langle I \rangle}) \\ &= \alpha^m \langle I \rangle^m \Gamma(m+1) \quad (m \text{ is an integer}) \\ &= m! \langle n \rangle^m \end{aligned} \quad (2.10)$$

and the normalised m-th factorial moment is:

$$F(m) = \frac{\mathcal{F}(m)}{\langle n \rangle^m} = m! \quad (2.11)$$

(3). For the Gamma intensity distribution:

$$P(I) = \frac{1}{\langle I \rangle} \left( \frac{I}{\langle I \rangle} \right)^{u-1} \frac{u^u}{\Gamma(u)} \exp(-\frac{uI}{\langle I \rangle})$$

where  $\langle I \rangle$  is the mean intensity and  $u$  is a constant; as  $u$  tends to 1, this distribution tends to a negative exponential distribution function. The photon counting distribution is calculated as:

$$P(n, T) = \frac{\langle n \rangle^n u^u}{(u + \langle n \rangle)^{u+n} n!} \frac{\Gamma(u+n)}{\Gamma(u)} \quad (2.12)$$

The m-th factorial moment is:

$$\begin{aligned}
\mathcal{F}(m) &= \alpha^m \langle I^m \rangle = \alpha^m \int_0^\infty I^m P(I) dI \\
&= \alpha^m \int_0^\infty I^m \frac{1}{\langle I \rangle} \left( \frac{I}{\langle I \rangle} \right)^{u-1} \frac{u^u}{\Gamma(u)} \exp\left(\frac{-uI}{\langle I \rangle}\right) dI \\
&= \frac{\alpha^m u^u}{\langle I \rangle^u \Gamma(u)} \int_0^\infty I^{u+m-1} \exp\left(\frac{-uI}{\langle I \rangle}\right) dI \\
&= \frac{\alpha^m \langle I \rangle^m}{u^m} \frac{\Gamma(u+m)}{\Gamma(u)} \\
&= \frac{\langle n \rangle^m}{u^m} \frac{\Gamma(u+m)}{\Gamma(u)} \tag{2.13}
\end{aligned}$$

so the m-th normalised factorial moment is:

$$\begin{aligned}
F(m) &= \frac{\mathcal{F}(m)}{\langle n \rangle^m} = \frac{1}{u^m} \frac{\Gamma(u+m)}{\Gamma(u)} \\
&= \frac{1}{u^m} (u+m-1)(u+m-2) \cdots (u+1)u \tag{2.14}
\end{aligned}$$

When  $u \rightarrow 1$ , it tends to  $m!$ , which corresponds to that of a Bose-Einstein distribution.

(4). The log-normal distribution function of the intensity is given by:

$$P(I) = \frac{1}{\sqrt{2\pi} \sigma I} \exp \left[ -\frac{(\ln I - \langle \ln I \rangle)^2}{2\sigma^2} \right] \tag{2.15}$$

where  $\sigma^2 = \langle (\ln I)^2 \rangle - \langle \ln I \rangle^2$  is the variance of the log-intensity. The photon counting distribution in this case cannot be given with an analytical form, but it can be evaluated by using a computer in terms of Mandel's formula.

However, the m-th moment of the intensity can be expressed as:

$$\begin{aligned}
\langle I^m \rangle &= \int_0^\infty I^m \frac{1}{\sqrt{2\pi} \sigma I} \exp \left[ -\frac{(\ln I - \langle \ln I \rangle)^2}{2\sigma^2} \right] dI \\
&\quad \left| \begin{array}{l} \text{Let } \ln I - \langle \ln I \rangle = \sqrt{2}\sigma x \\ \text{Let } x - \frac{1}{\sqrt{2}} m\sigma = y \end{array} \right. \\
&= \frac{\exp(m \langle \ln I \rangle)}{\sqrt{\pi}} \int_{-\infty}^\infty \exp[-(x^2 - \frac{1}{\sqrt{2}} m\sigma x)] dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{\exp(m \langle \ln I \rangle + \frac{1}{2}m^2\sigma^2)}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-y^2) dy \\
&= \exp(m \langle \ln I \rangle + \frac{1}{2}m^2\sigma^2) \operatorname{erf}(\infty) \\
&= \exp(m \langle \ln I \rangle + \frac{1}{2}m^2\sigma^2)
\end{aligned} \tag{2.16}$$

where  $\operatorname{erf}(t)$  is the error function and  $\operatorname{erf}(\infty) = 1$ .

From Eq.(2.16), it is then can be seen that:

$$\ln \langle I \rangle = \langle \ln I \rangle + \frac{\sigma^2}{2} \tag{2.17}$$

The  $m$ -th normalized factorial moment of photon counts which is equal to the  $m$ -th normalized intensity moment is given as follows:

$$\begin{aligned}
F(m) = M(m) &= \frac{\langle I^m \rangle}{\langle I \rangle^m} \\
&= \exp \left[ \frac{n(n-1)\sigma^2}{2} \right]
\end{aligned} \tag{2.18}$$

### §2-3: Non-ideal Effects

The above examples of photon counting distribution functions are derived under the assumption that the experimental conditions are ideal, i.e. the detector is perfect with no dead time and dark counts, there are no temporal and spatial integration effects, etc. In practice, all these factors affect the measured statistics. Let us now look at how these factors bias the measured statistics.

#### §2-3-1: Dead Time Effect

The measured photon counting distribution is affected by the dead time of the observing system. For a non-paralyzable detecting system, dead time  $\tau$  refers to the fact that after each registration of the arrival of a photon, there is a period during which no subsequent photon arrival can be detected. The dead time complicates the relationship between the measured photon counting distribution

$P(n, T, \tau)$  and intensity probability density  $P(I)$ . On the one hand, it is desirable to make a deduction of the form of measured (dead time affected) photon counting distribution  $P(n, T, \tau)$ . On the other hand, the requirement for the modification of measured photon counting distribution is needed to derive  $P(n, T, 0)$  from  $P(n, T, \tau)$  and thereafter to obtain  $P(I)$ .

The form of photon counting distribution with dead-time effects  $P(n, T, \tau)$  for the optical field with known statistical properties can be readily deduced by using Mandel's formula. For a random Poisson distribution of photons, i.e.  $P(I) = \delta(I - \bar{I})$ , the measured distribution to the second order of  $\tau/T$  has the form [Johnson et al (1966)] of:

$$\begin{aligned}
 P(n, T, \tau) = \frac{\bar{n}_o^n}{n!} \exp(-\bar{n}_o) \left\{ 1 + n(\bar{n}_o - n + 1) \frac{\tau}{T} \right. \\
 - \frac{1}{2} [(\bar{n}_o + 1)n - (\bar{n}_o^2 + 2\bar{n}_o + 3)n^2 \\
 \left. + (2\bar{n}_o + 3)n^3 - n^4] \left( \frac{\tau}{T} \right)^2 \right\} \quad (2.19)
 \end{aligned}$$

where  $\bar{n}_o$  is the mean numbers of photons during the sample time  $T$  in absence of dead time.

To the first order in  $\tau/T$ , a modified distribution function with experimental convenience, is defined as:

$$\begin{aligned}
 F(n) &= \frac{(n+1)P(n+1, T)}{P(n, T)} \\
 &= -2\bar{n}_o \frac{\tau}{T} n + \bar{n}_o (1 + \bar{n}_o \frac{\tau}{T}) \quad (2.20)
 \end{aligned}$$

As can be seen,  $F(n)$  is linear with the slope  $-2\bar{n}_o \tau/T$ . In fact, by plotting the measured data  $F(n)$  against  $n$  and measuring this negative slope (measured as  $-A$ ) and taking advantage of the fact that when  $n = 0$ ,  $F(0)$  is least affected by dead-time effects, we have:

$$\left. \begin{aligned} F(0) &= \bar{n}_o (1 + \bar{n}_o \frac{\tau}{T}) \\ -A &= -2\bar{n}_o \frac{\tau}{T} \end{aligned} \right\} \quad (2.21)$$

From the above equations, the mean counts and dead time of the detecting system can be deduced (see Chapter 4).

For any distribution measured with a sample time  $T$  much shorter than the inverse linewidth of the radiation, the measured photon counting distribution can be modified to form a compound Poisson process. Bearing in mind Eq.(2.19), to the second order, dead-time effects correction, by using Mandel's formula,  $P(n, T, \tau)$  is:

$$P(n, T, \tau) = \int_0^\infty \frac{(\alpha I)^n}{n!} \exp(-\alpha I) P(I) dI \left\{ 1 + n(\alpha I - n + 1) \frac{\tau}{T} - \frac{1}{2} [(\alpha I + 1)n - (\alpha^2 I^2 + 2\alpha I + 3)n^2 + (2\alpha I + 3)n^3 - n^4] \left(\frac{\tau}{T}\right)^2 \right\} \quad (2.22)$$

For Gaussian speckle which has the following intensity distribution:

$$P(I) = \frac{1}{\bar{I}_o} \exp\left(-\frac{I}{\bar{I}_o}\right) \quad (2.23)$$

where  $\bar{I}_o$  is the mean intensity in the absence of dead-time effects, the photon counting distribution with dead-time effects is given as follows:

$$P(n, T, \tau) = \frac{\bar{n}_o^n}{(1 + \bar{n}_o)^{n+1}} \left\{ 1 - n \left[ (n-1) - (n+1) \frac{\bar{n}_o}{1 + \bar{n}_o} \right] \frac{\tau}{T} + \frac{n}{2} \left[ (n-1)^3 - (2n^2 - 2n + 1)(n+1) \frac{\bar{n}_o}{1 + \bar{n}_o} + n(n+1)(n+2) \left( \frac{\bar{n}_o}{1 + \bar{n}_o} \right)^2 \right] \left(\frac{\tau}{T}\right)^2 \right\} \quad (2.24)$$

In the ideal case when  $\tau = 0$ , this gives the Bose-Einstein distribution.

As mentioned earlier, in practice, often the case is that one wishes to derive  $P(n, T, 0)$  from measured  $P(n, T, \tau)$  and therefore to get  $P(I)$ . This is referred to as the inverse problem. In general, as summarised by Mandel (1980), the relationship between dead-time affected photon counting probability and true photon

counting probability is as follows:

$$\left. \begin{aligned} P(n, T, \tau) &= P(n, T - n\tau, 0) \\ &+ \sum_{r=0}^{n-1} [P(r, T - n\tau, 0) - P(r, T - (n-1)\tau, 0)], \quad n \geq 1 \\ P(0, T, \tau) &= P(0, T, 0), \quad n = 0 \end{aligned} \right\} \quad (2.25)$$

From the above equation, it is possible to derive the form of  $P(n, T, 0)$  for light of various known photon counting statistics. In practice,  $P(n, T, 0)$  and the probability density  $P(I)$  are often unknown. However, as pointed out by Mandel (1980), the above equation can be inverted to get exact form of the probability  $P(n, T, 0)$  from measurements of  $P(n, T, \tau)$  by using several different sample times  $T$ .

In order to show this, let us assume the condition  $n\tau \ll T$  which is often the case in the experiments, i.e. the photon count that would be detected during a period of  $\tau$  is much less than 1. If we replace  $T$  by  $T + n\tau$  in Eq.(2.25), we can rewrite the equation as:

$$\left. \begin{aligned} P(n, T, 0) &= P(n, T + n\tau, \tau) \\ &+ \sum_{r=0}^{n-1} [P(r, T + \tau, 0) - P(r, T, 0)], \quad n \geq 1 \\ P(0, T, 0) &= P(0, T, \tau), \quad n = 0 \end{aligned} \right\} \quad (2.26)$$

Because when  $n = 0$ ,  $P(0, T, 0)$  is least affected by the dead time effect, therefore we take  $P(0, T + \tau, 0) = P(0, T + \tau, \tau)$  and it is immediately found:

$$\left. \begin{aligned} P(1, T, 0) &= P(1, T + \tau, \tau) \\ &+ P(0, T + \tau, \tau) - P(0, T, \tau) \end{aligned} \right\} \quad (2.27)$$

When  $n = 2$ , substitute Eq.(2.27) in Eq.(2.26); after manipulation of the formula, it yields:

$$\left. \begin{aligned} P(2, T, 0) &= P(2, T + 2\tau, \tau) \\ &+ \sum_{r=0}^1 [P(r, T + 2\tau, \tau) - P(r, T + \tau, \tau)] \end{aligned} \right\} \quad (2.28)$$

Similarly, by repeating above procedure, for any  $n$  under the condition of  $n\tau < T$ :

$$\left. \begin{aligned} P(n, T, 0) &= P(n, T + n\tau, \tau) \\ &+ \sum_{r=0}^{n-1} [P(r, T + n\tau, \tau) - P(r, T + (n-1)\tau, \tau)], \quad n \geq 1 \\ P(0, T, 0) &= P(0, T, \tau), \quad n = 0 \end{aligned} \right\} \quad (2.29)$$

For sufficiently small dead time  $\tau$ ,  $P(n, T + n\tau, \tau)$  may be an adequate first approximation to  $P(n, T, 0)$ .

Hence in principle, the true photon counting distribution can be deduced from the measured photon counting distribution to any desired accuracy. However, to do this, the measurements in several different sample times ranging from  $T$  to  $T + n_m\tau$  are required, where  $n_m$  is the largest value of  $n$  for which  $P(n, T, 0)$  is not negligible. Alternatively, if  $n_m\tau < T$ , which is normally the case, it may be sufficient to just measure  $P(n, T, \tau)$  and  $P(n, T + n_m\tau, \tau)$  plus a few others with sample times between  $T$  and  $T + n_m\tau$ , and the whole distribution function can then be obtained by an interpolation process.

Although in principle the true photon counting distribution can be derived as mentioned above, this requires tedious experimental measurements and a counter with very fine adjustable sample times. Chang *et al* (1969) derived approximate expressions in the form of a power series for the relation between factorial moments of  $P(n, T, 0)$  and  $P(n, T, \tau)$ . Although it is difficult to get the true photon counting distribution from the measured one, it is very useful in estimating the true general statistical properties from the dead time affected ones with this relationship.

For first order correction, if we denote  $a = \tau/T$ , the corrected factorial moments of photon counts (which are proportional to the moments of detected intensity) can be expressed [Chang *et al* (1969)] in terms of measured ones of two

different orders:

$$\left. \begin{aligned}
 M_5(1) &= f_{(5)} + a[20f_{(5)} + 5M_6(0)] \\
 M_4(1) &= f_{(4)} + a[12f_{(4)} + 4M_5(0)] \\
 M_3(1) &= f_{(3)} + a[6f_{(3)} + 3M_4(0)] \\
 M_2(1) &= f_{(2)} + a[2f_{(2)} + 2M_3(0)] \\
 M_1(1) &= f_{(1)} + aM_2(0)
 \end{aligned} \right\} \quad (2.30)$$

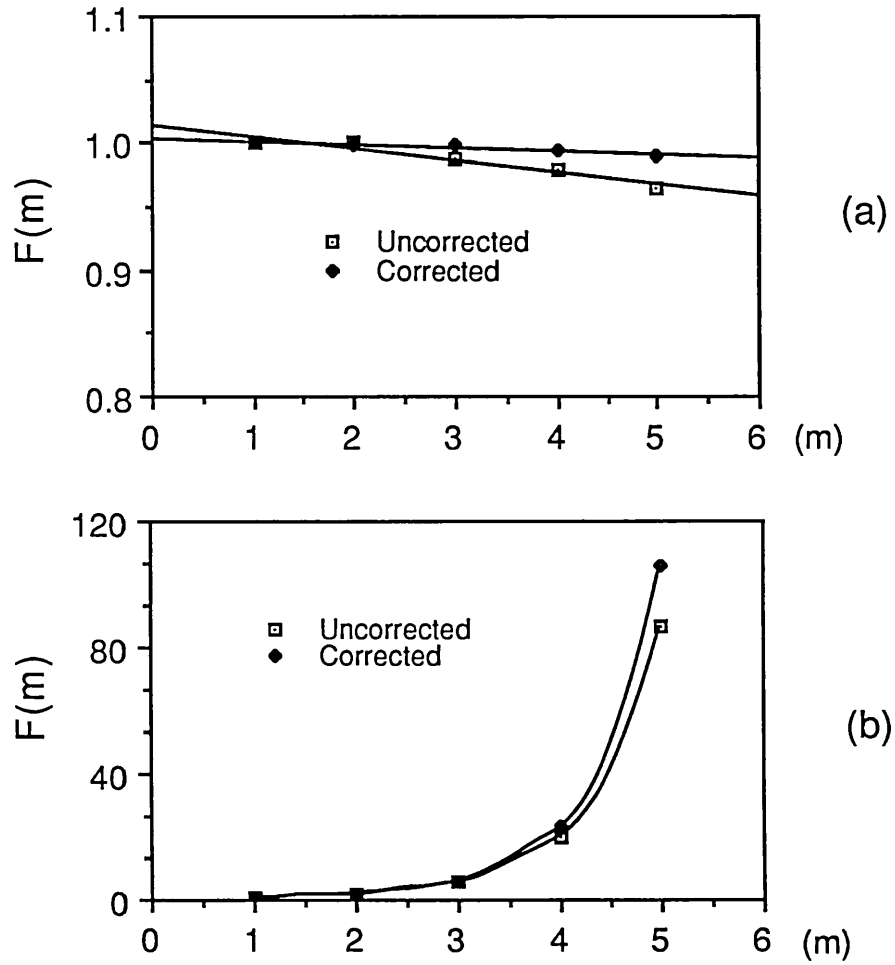


Fig.2.1. Dead time correction for the normalised factorial moments for (a). Poisson photon counting distribution and (b). Bose-Einstein photon counting distribution (Gaussian speckle). ( $\tau \approx 100 \text{ ns}$ ,  $T = 10 \mu\text{s}$ )

Eq.(2.30) can be generalised as follows:

$$M_n(1) = f_{(n)} + a[n(n-1)f_{(n)} + nM_{n+1}(0)], \quad n = 1, 2, \dots \quad (2.31)$$

where  $M_n(1)$  is the first order corrected  $n$ -th order factorial moment,  $f_{(n)}$  is measured  $n$ -th order factorial moment and  $M_{n+1}(0)$  is the “0-th order corrected”  $(n+1)$ -th order factorial moment; actually we take it as the measured  $(n+1)$ -th order factorial moment, i.e.  $M_{n+1}(0) = f_{(n+1)}$ . The above way of correcting factorial moments for dead-time effects can be readily carried out experimentally. Fig.2.1 shows two examples of this.

The second and third order correction of factorial moments of photon counts can be found in the paper by Chang *et al* (1969). As far as accuracy is concerned, dead-time effects on low order statistical quantities, such as mean and variance of a counting distribution are relatively insignificant and often negligible. But its effects on higher order statistical properties become more and more significant. The modification to the  $n$ th order statistics is roughly dependent upon the value of  $n\tau/T$ . Therefore when detailed features of the distribution involving higher order statistical properties are being investigated, it is essential to correct for dead-time effects.

## §2-3-2: Temporal Integration

The photon counting distribution is not only dependent upon the incident light intensity, but also the sample time  $T$ . To make the  $P(n, T)$  measurements reflect the statistics of the instantaneous light intensity rather than the integrated intensity, the sample time should be made much smaller than the coherence time of the intensity fluctuations, namely,  $\Gamma T \ll 1$ , where  $\Gamma$  is the half width at half maximum (HWHM) of the intensity fluctuation spectrum.

As an example, let us look at the effect of temporal integration on the measured photon counting distribution from Gaussian light. For Gaussian light, the true photon counting distribution is Bose-Einstein distribution which is for the

case when  $\gamma = \Gamma T = 0$ . As can be seen from the following graph, when the sample time  $T$  increases the curves are pushed towards larger  $n$ . The measured distribution functions are derivated more and more from the true ones. When  $T$  is very large, one will eventually get a Poisson distribution function.

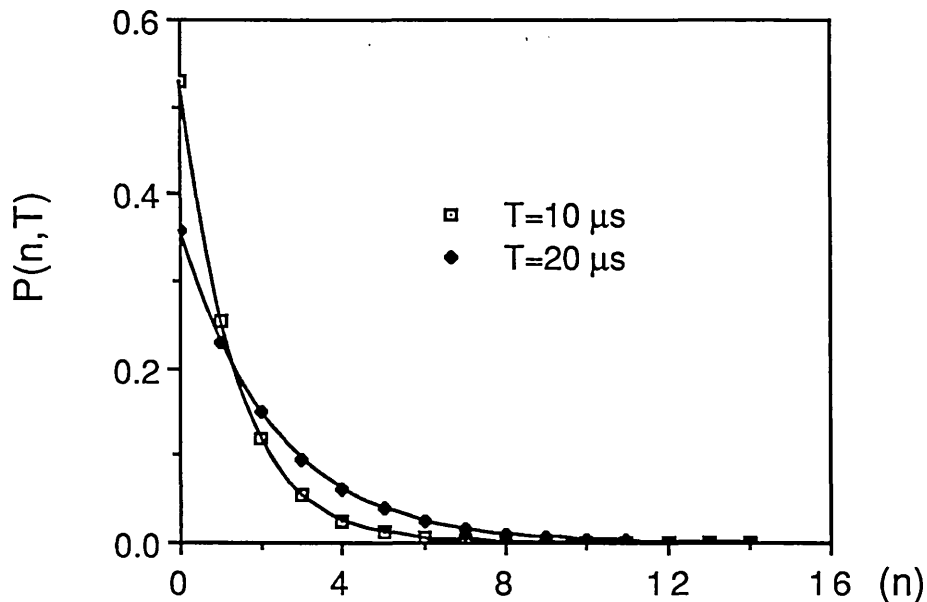


Fig.2.2. Curves of  $P(n, T)$  with sample times  $T$  set at  $10 \mu s$  and  $20 \mu s$  for Gaussian speckle. Due to temporal integration over the sample time, the curves show a decrease at the origin with the increase of the sample time.

The theoretical analysis [Jakeman *et al*, 1968(a),(b)] of temporal integration effects on the measured intensity distribution for thermal light which can be experimentally deduced from the measured photon counting distributon is as follows:

(1). For  $\gamma \ll 1$ :

$$P(I) \simeq \frac{1}{\langle I \rangle} \left\{ 1 + \left( 3 - \frac{2I}{\langle I \rangle} \right) \frac{\gamma}{4} \right\} \exp\left(-\frac{I}{\langle I \rangle}\right) \quad (2.32)$$

When  $\gamma \rightarrow 0$ :

$$P(I) = \frac{1}{\langle I \rangle} \exp\left(-\frac{I}{\langle I \rangle}\right) \quad (2.33)$$

which corresponds to Bose-Einstein photon counting distribution.

(2). For  $\gamma \gg 1$ :

$$P(I) \simeq \left( \frac{\gamma \langle I \rangle}{2\pi I} \right)^{1/2} \exp \left[ -\frac{\gamma \langle I \rangle}{2I} (I - \langle I \rangle)^2 \right] \quad (2.34)$$

When  $\gamma \rightarrow \infty$ :

$$P(I) = \delta(I - \langle I \rangle) \quad (2.35)$$

which corresponds to  $\hat{\alpha}$  Poisson photon counting distribution.

(3). For  $\gamma \sim 1$ :

$$P(I) = \frac{1}{\langle I \rangle} \exp \left( \gamma - \frac{\gamma I}{2 \langle I \rangle} \right) \times \sum_{n=1,2} (-1)^{n-1} \frac{2y_n^2}{y_n^2 + \gamma^2 + 2\gamma} \exp \left( -\frac{y_n^2 I}{2\gamma \langle I \rangle} \right) \quad (2.36)$$

where  $y_n$  are the solutions of the following equations:

$$\left. \begin{aligned} y_1 \tan\left(\frac{1}{2}y_1\right) &= \gamma, & 0 < y_1 < \pi \\ y_2 \cot\left(\frac{1}{2}y_2\right) &= -\gamma, & \pi < y_2 < 2\pi \end{aligned} \right\} \quad (2.37)$$

On the other hand, for Gaussian light with  $\hat{\alpha}$  Lorentzian spectrum from the measurements of  $P(n, T)$  with different sample times, one can decide the line width of  $\hat{\alpha}$  Lorentzian spectrum. For example, the variance or second moment and normalized second factorial moment can be measured and their theoretical expressions are given as follows:

$$\left. \begin{aligned} \alpha_T^2 &= \frac{\bar{I}^2}{2\gamma^2} (e^{-2\gamma} + 2\gamma - 1) \\ n_2 &= 1 + \frac{1}{2\gamma^2} (e^{-2\gamma} + 2\gamma - 1) \end{aligned} \right\} \quad (2.38)$$

By measuring  $\alpha_T^2$  or  $n_2$  and  $T$ , it is easy to get the HWHM of the intensity fluctuation spectrum  $\Gamma$ .

### §2-3-3: Spatial Integration

When the sample pinhole size is comparable with the speckle size, the measured statistics of the light intensity will show a spatial averaging effect. The

measured statistics do not reflect the true nature of the real statistics. As a guideline for measuring the statistical properties of the light intensity, when the sample pinhole size is one speckle size the contrast of the speckle pattern will be reduced by about 5%. Spatial averaging will mainly affect the measured speckle contrast, and have little effect on the temporal correlation.

The spatial and temporal integration effects are relatively easy to manipulate. One can make a trade off between integration effects and reasonable signal-to-noise ratio. Therefore, one can normally make the experimental conditions favourable to make the error caused by integration effects negligible. Dead-time effects are less manageable by the experimental conditions. Therefore one has to correct for these effects theoretically whenever possible.

#### §2-3-4: Dark Counts

Since dark counts originate from thermal emissions, they are usually observed as having a Poisson distribution. It is reasonable to imagine that dark counts are equivalent to a constant light intensity incident on the photocathode. Since dark counts are independent of the signal, the detected photon counting statistics which are biased and are therefore the product of unbiased photon counting statistics and a Poisson statistic due to dark counts. Thus the unbiased photon counting statistics can be easily deduced when dark counts are known. On the other hand, by cooling the detection system, dark counts are usually very small compared to the signal photon counts and the bias caused by the dark counts is therefore often negligible.

#### §2-3-5: Saturation Effect

Detection system saturation effects are caused by nonlinearity of the system when the incident intensity is above a certain value  $I_s$ . Their effects on detected moments are sometimes fairly large, particularly on higher order ones.

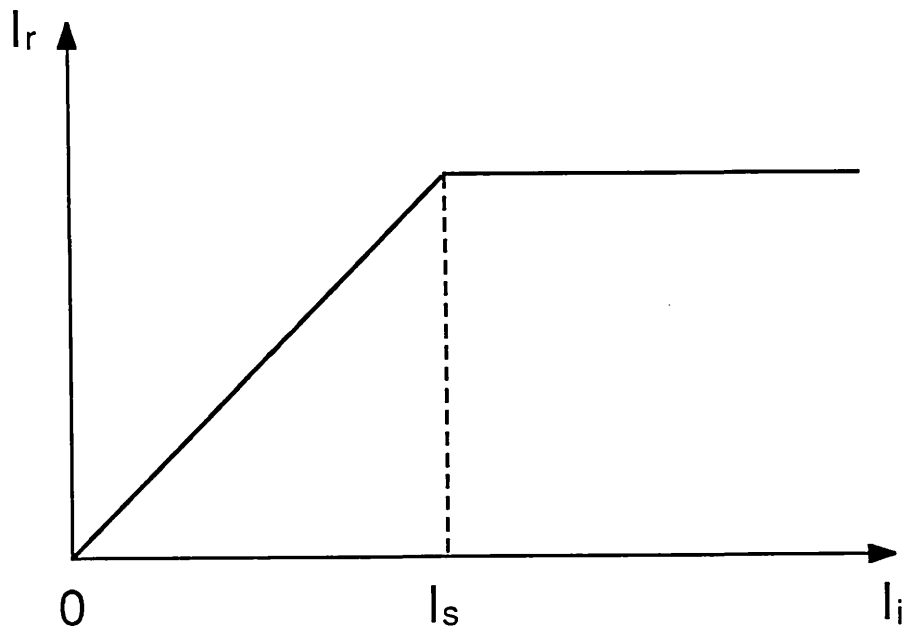


Fig.2.3. Detection system response against incident intensity.

Ideally, the measured moments in absence of saturation effects are defined as:

$$\langle I^n \rangle = \int_0^{\infty} I^n P(I) dI \quad (2.39)$$

with the saturation effects occurring at the threshold  $I_s$ , the actual measured moments are:

$$\langle I^n \rangle_d = \int_0^{I_s} I^n P(I) dI + I_s^n \int_{I_s}^{\infty} P(I) dI \quad (3.40)$$

which are smaller than they should be compared with ones in Eq.(3.39). The difference between the ideal and corrupted moments are therefore:

$$\begin{aligned} E_s^{(n)} &= \langle I^n \rangle - \langle I^n \rangle_d \\ &= \int_{I_s}^{\infty} I^n P(I) dI - I_s^n \int_{I_s}^{\infty} P(I) dI \\ &= \int_{I_s}^{\infty} (I^n - I_s^n) P(I) dI \end{aligned} \quad (3.41)$$

The system saturation effects can be described by the above parameter  $E_s^{(n)}$ .

The effects of detection system saturation on higher-order moments of intensity for a certain number of known statistics, such as log-normal and **K** distributions are investigated by Consortini *et al* (1984,1987).  $E_s^{(n)}$  is found to be dependent to be

upon the threshold and the orders of the measured moments. The effects become more significant for the measurement of high order moments. The measured moments are also dependent upon the threshold of the detection system when the detection system is saturated.

Hence, to avoid such effects the detection system's dynamic range must be large enough. However, in the cases where the detected light intensity is very weak, as in speckle imaging, the saturation effects are negligible.

### §2-3-6: Sample Size Influence (Finite Sample Size)

The finite sample size always causes error in estimating true statistical properties, such as moments. The higher moments measurements are strongly dependent upon the occasionally high intensity due to the fluctuations. Therefore a finite sample size could reduce the chance of detecting only high intensity and in turn the high moments are usually underestimated.

In practical measurement of statistical parameters, the sample size is always finite. When the total time available for a single experiment run is limited, the finite sample size influence becomes strong and non-negligible. For example, this happens in the case of measuring stellar scintillation or light propagating through atmospheric turbulence, where the nonstationarity of the medium restricts the duration of a single run. In such cases, the duration of stationarity is just a few minutes and as the correlation time is about  $10ms$ , the reasonable sample size in a real experiment is therefore a few thousand ( $\sim 10^4$ ) effectively uncorrelated samples.

Ben-Yosef *et al* (1988) and Goulder *et al* (1988) have evaluated the finite sample size effects on estimated moments of log-normal and K probability distribution functions (PDF). At the sample size of  $\sim 10^4$  which is a practically feasible size in scintillation measurements, the error caused by finite sample size makes the estimated high moments (e.g. 5-th) of log-normal and K PDFs indistinguishable.

The calculated maximum value of a higher order moment is found to be bounded for the given sample size, although the estimated moment will eventually converge to the theoretical value at a very large sample size. For different PDFs, the convergence rates are different. Typically, for PDFs with a long tail, the sample size required for estimated unbiased high moments is large and therefore the convergence rate is small, such as in the log-normal PDF case. For a PDF which goes to zero quickly at large intensities, the required sample size for correctly estimating unbiased higher order moments is relatively small, therefore the convergence rate is larger, such as in the  $K$  PDF case. Even though the log-normal PDF has a very long tail while  $K$  PDF has a very short one, it is found that we still need very large sample size to distinguish high order moments of the two extreme cases. In general, to distinguish the moments of two PDFs, a much larger sample size ( $\gg 10^4$ ) would be needed for the estimated high moment to reach its theoretical value. On the other hand, practical constraints make it impossible to obtain large sample sizes in such situations and the moments analysis used to decide uniquely the actual statistical properties, such as the PDF, is questionable. An alternative way to analyze high order moments might be needed.

## §2-4: Inverse Problem

Having dealt with photon counting statistics, we shall now look at how to find  $P(I)$  from the measured  $P(n, T)$ —this is referred to as the inverse problem.

In principle, it is possible to get the intensity distribution from the measurements of photon counting distribution  $P(n, T)$ .

$$P(n) = P(n, T) = \int_0^\infty \frac{(\alpha I)^n}{n!} \exp(-\alpha I) P(I) dI \quad (2.42)$$

Hence:

$$\begin{aligned} \left(\frac{ix}{\alpha}\right)^n P(n) &= \left(\frac{ix}{\alpha}\right)^n \int_0^\infty \frac{(\alpha I)^n}{n!} \exp(-\alpha I) P(I) dI \\ &= \int_0^\infty \frac{(ixI)^n}{n!} \exp(-\alpha I) P(I) dI \end{aligned} \quad (2.43)$$

Therefore:

$$\begin{aligned}
\sum_{n=0}^{\infty} \left(\frac{ix}{\alpha}\right)^n P(n) &= \sum_{n=0}^{\infty} \int_0^{\infty} \frac{(ixI)^n}{n!} \exp(-\alpha I) P(I) dI \\
&= \int_0^{\infty} \sum_{n=0}^{\infty} \frac{(ixI)^n}{n!} \exp(-\alpha I) P(I) dI \\
&= \int_0^{\infty} \exp(ixI) \exp(-\alpha I) P(I) dI
\end{aligned} \tag{2.44}$$

From above equation, if we denote:

$$\begin{aligned}
F(x) &= \sum_{n=0}^{\infty} \left(\frac{ix}{\alpha}\right)^n P(n) \\
&= \int_0^{\infty} \exp(ixI) \exp(-\alpha I) P(I) dI
\end{aligned} \tag{2.45}$$

by measuring  $P(n)$  we can calculate function  $F(x)$ . On the other hand, by noticing that  $F(x)$  and  $\exp(-\alpha I)P(I)$  are Fourier transform pairs, the inverse Fourier transform of  $F(x)$  leads to the intensity distribution  $P(I)$ :

$$P(I) = \frac{\exp(\alpha I)}{2\pi} \int_{-\infty}^{\infty} F(x) \exp(-ixI) dx \tag{2.46}$$

Thus, the intensity distribution function  $P(I)$  can be obtained from the knowledge of photon counting distribution  $P(n, T)$ .

In practice, this method is very sensitive to noise and often gives an erroneous estimate of the intensity distribution  $P(I)$ . This is because <sup>of</sup> the fact that experimentally it is very difficult to eliminate the errors in measuring photon counting distribution of *high* counts (i.e. large  $n$ ). Nevertheless, a very small error in estimating <sup>the</sup> photon counting distribution of *high* counts can cause a significant error in estimating  $F(x)$ . This can be seen from Eq.(2.45). In calculating  $F(x)$ , the  $P(n, T)$ s for *high* counts have large weighting factor because of the  $n$ -th power, while in experiments the  $P(n, T)$ s for very large  $n$  is very difficult to measure and often neglected. This gives rise to <sup>a</sup> large error in estimating  $F(x)$  and hence in turn causes a large error in calculating the intensity distribution function  $P(I)$ .

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## Chapter 3

### Correlation Techniques

#### §3–1: Introduction

For a long time, correlation techniques have been used to determine the similarity between events and distinguish signal from noise. During the last two decades, the techniques have been developed to make them applicable to many areas of scientific research. In the optical area, it has been found that light intensity correlation is of great importance because it can be practically measured and many major features of the light field can be deduced from the correlation function; for example, the power spectral density can be obtained by performing the inverse Fourier transform of the autocorrelation function through the Wiener–Khinchine theorem. The correlation function has also been found to be particularly useful in the area of speckle analysis and speckle interferometry. The study of the power spectral density of speckle patterns can be performed in the spatial domain by a study of the correlation function. Similarly, the spatial correlation can be measured and used to reconstruct the diffraction-limited correlation of an object from atmospherically degraded image in astronomy.

For an optical field represented by a complex analytical signal  $E(t)$  with corresponding instantaneous light intensity  $I(t) = E(t) E^*(t)$ , the normalised temporal correlation function of the light field is defined as:

$$C^{(1)}(\tau) = \frac{\langle E(t)E(t+\tau) \rangle}{\langle I \rangle} \quad (3.1)$$

and the normalised temporal correlation function of the intensity is defined as:

$$C^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau) \rangle}{\langle I \rangle^2} \quad (3.2)$$

For a Gaussian field, the relationship between the intensity correlation function and field correlation function is described by Siegert relation:

$$C^{(2)}(\tau) = 1 + |C^{(1)}(\tau)|^2 \quad (3.3)$$

This will enable us to deduce the modulus of the field correlation function from the measurements of the intensity correlation function which are readily measurable.

Broadly speaking, correlation techniques can be divided into two groups, i.e., homodyne (self-beat) and heterodyne techniques. In the homodyne technique, only the light of interest impinges on the detector, and the signal is in turn correlated. In heterodyne technique, a local oscillator (usually a small portion of reference beam with known frequency) is mixed with the light of interest, the combined light is then detected and correlated. As can be seen clearly, with the homodyne correlation technique, one gets the correlation of light of interest with itself. But for processes with several relaxation times such as in <sup>light scattering</sup> experiment with polydispersity, the analysis of homodyne data is quite difficult. From a spectrum point of view, there will be several different frequencies from which it is not easy to extract the required data. This fact can be seen from the following argument. For the Gaussian field, the intensity correlation is:

$$\langle I(t)I(t+\tau) \rangle = \langle I \rangle^2 + | \langle E(t)E(t+\tau) \rangle |^2 \quad (3.4)$$

Now for a process with several relaxation times:

$$| \langle E(t)E(t+\tau) \rangle |^2 = \sum_i a_i \exp\left(-\frac{\tau}{\tau_i}\right) \quad (3.5)$$

therefore:

$$\langle I(t)I(t+\tau) \rangle = \sum_{i,j} a_i a_j \left[ 1 + \exp -\tau \left( \frac{1}{\tau_i} + \frac{1}{\tau_j} \right) \right] \quad (3.6)$$

From the above formula, it is very difficult to get an analytical form (i.e. to decide  $a_i$  and  $a_j$ ) of intensity correlation function from the experimental data.

However, for the heterodyne technique, let  $E_l(t)$  represent the local oscillator electric field,  $E_s(t)$  represent signal electric field; thus the measured combined intensity autocorrelation function is:

$$\langle I(t)I(t+\tau) \rangle = \langle |E_l(t) + E_s(t)|^2 |E_l(t+\tau) + E_s(t+\tau)|^2 \rangle \quad (3.7)$$

We choose the experimental conditions properly so that the amplitude of the local oscillator may be made much greater than the amplitude of the signal electric field:

$$|E_l(t)| \gg |E_s(t)|$$

and the following approximations are made:

- (a) fluctuations of the local oscillator electric field are negligible;
- (b) the local oscillator electric field and signal field are statistically independent, so that  $\langle I_s I_l \rangle = \langle I_s \rangle \langle I_l \rangle$ .

Formula (3.7) is then expanded yielding 16 terms. From assumptions (a) and (b), 10 of them are zero, three are DC components and one is negligible (the term  $\langle |E_s(t)|^2 |E_s(t + \tau)|^2 \rangle$ ). Therefore by ignoring DC components, we have:

$$\langle I(t)I(t + \tau) \rangle \approx 2I_l \langle E_s(t)E_s(t + \tau) \rangle + I_l^2 \quad (3.8)$$

where  $I_l = \langle |E_l(t)|^2 \rangle$  is the intensity of the local oscillator signal. This shows that the desired signal field auto-correlation can be determined by the mixed intensity auto-correlation function. In addition, it is easier to get the analytical form of the correlation function from the experimental data in cases of processes which are sums of exponentials, as opposed to the homodyne technique.

As far as the correlation measurement itself is concerned, there are several ways to determine the intensity correlation. The intensity correlation can be used to determine characteristics of an object such as motion and shape. It has also been used in the diffusion coefficient measurements in light scattering experiments and in anemometry measurements. In addition, correlation measurements are extremely useful in describing the fluctuation properties of the radiation field. In the following sections several correlation techniques will be discussed in detail.

### §3-2: Photon Correlation

To determine the intensity correlation function at the very low light levels that are often encountered in astronomy, photon correlation techniques are often used.

Photon correlation techniques have also found many applications in light scattering studies, laser Doppler microscopy, polydispersities and microscopic particle sizing. The light field is detected by photon counting techniques and the correlation function can be produced by correlating the photon counts. The superiority of the photon counting method over the other methods for very low light fields is discussed in chapter 2.

### §3-2-1: Theoretical Analysis

*Jakeman (1974) first deduced the relationship between the photon correlation and the intensity correlation.* Let us first of all recall Mandel's formula (compound Poisson distribution) [see Eq.(2.1)]:

$$P(n, T) = \int_0^\infty \frac{(\alpha I)^n}{n!} \exp(-\alpha I) P(I) dI \quad (3.9)$$

where  $P(n, T)$  is the probability distribution of emission  $n$  photoelectrons during the detecting time  $T$ ,  $\alpha$  is the detection efficiency and  $P(I)$  is the probability distribution function of the intensity  $I$ . Here we have assumed that the light field is stationary and ergodic, so that the statistics of the field are independent of time  $t$ .

The joint distribution of counts at several detecting points  $(X_1, X_2, \dots, X_l)$  during the sample time  $T$  centered at  $(t_1, t_2, \dots, t_l)$  respectively can be obtained by generalizing formula (3.9):

$$\begin{aligned} & P[n(X_1, t_1), n(X_2, t_2), \dots, n(X_l, t_l); T] \\ &= P(n_1, n_2, \dots, n_l; T) \\ &= \int_0^\infty \frac{(\alpha I_1)^{n_1} (\alpha I_2)^{n_2} \dots (\alpha I_l)^{n_l}}{n_1! n_2! \dots n_l!} \exp[-\alpha(I_1 + I_2 + \dots + I_l)] \\ & \quad \times P(I_1, I_2, \dots, I_l) dI_1 dI_2 \dots dI_l \end{aligned} \quad (3.10)$$

where  $I_1, I_2, \dots, I_l$  are the intensity at points  $X_1, X_2, \dots, X_l$  and time  $t_1, t_2, \dots, t_l$  respectively,  $P(I_1, I_2, \dots, I_l)$  is the joint probability distribution function of multi-point intensities  $I_1, I_2, \dots, I_l$ . We have assumed that the detection efficiencies of the detector(s) at all points are the same, as  $\alpha$ .

We can give Eq.(3.10) an intuitive explanation. Suppose that at each point the intensity is not randomly fluctuating. The fluctuation of photon counts are purely due to the secondary emission process of the detector. Therefore each emission is a Poisson process and the Poisson distribution is as follows:

$$\begin{aligned} P'[n(X_i, t_i); T] &= P'(n_i, T) = \frac{\langle n \rangle_i^{n_i}}{n_i!} \exp(-\langle n \rangle_i) \\ &= \frac{(\alpha I_i)^{n_i}}{n_i!} \exp(-\alpha I_i) \end{aligned} \quad (3.11)$$

where  $I_i$  is the non-fluctuating intensity at a certain point, and  $\langle n \rangle_i = \alpha I_i$  is the average photon count. Under the above assumption, the counts at different points and/or in distinct time intervals are statistically independent. Hence, the joint distribution of counts at multi-points and times is shown as follows:

$$\begin{aligned} &P'[n(X_1, t_1), n(X_2, t_2), \dots, n(X_l, t_l); T] \\ &= P'(n_1, n_2, \dots, n_l; T) = \prod_{i=1}^l P'(n_i; T) \\ &= \frac{(\alpha I_1)^{n_1} (\alpha I_2)^{n_2} \dots (\alpha I_l)^{n_l}}{n_1! n_2! \dots n_l!} \exp[-\alpha(I_1 + I_2 + \dots + I_l)] \end{aligned} \quad (3.12)$$

In practice, the intensity at different points and time intervals are stochastic variables. Therefore on the basis of Eq.(3.12), we should take the average of it over the intensities at different points and time  $I_1, I_2, \dots, I_l$ . So the real probability distribution is:

$$\begin{aligned} &P[n(X_1, t_1), n(X_2, t_2), \dots, n(X_l, t_l); T] \\ &= \langle P'[n(X_1, t_1), n(X_2, t_2), \dots, n(X_l, t_l); T] \rangle_{I_1, I_2, \dots, I_l} \\ &= P(n_1, n_2, \dots, n_l; T) \end{aligned} \quad (3.13)$$

which is Eq.(3.10). With Eq.(3.10), the multi-point correlation function  $\langle n_1 n_2 \dots n_l \rangle$  can then be written as:

$$\langle n_1 n_2 \dots n_l \rangle = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \dots \sum_{n_l=0}^{\infty} n_1 n_2 \dots n_l P(n_1, n_2, \dots, n_l; T) \quad (3.14)$$

Finally, after manipulation of the Eq.(3.14) and Eq.(3.10), one finds that:

$$\frac{\langle n_1 n_2 \cdots n_l \rangle}{\langle n_1 \rangle \langle n_2 \rangle \cdots \langle n_l \rangle} = \frac{\langle I_1 I_2 \cdots I_l \rangle}{\langle I_1 \rangle \langle I_2 \rangle \cdots \langle I_l \rangle} \quad (3.15)$$

where  $n_i$  are detected at different points or times. From Eq.(15) a very important conclusion can be drawn: the normalised intensity correlation function, including autocorrelation, space-time correlation, triple correlation, etc., can be calculated by evaluating the normalised correlation function of the train of photoelectrons, which is proportional to incident photons. The photon correlation method can be used to get multi-channel or higher order correlation functions. In particular, for the intensity auto-correlation function (second order), Eq.(15) can be written as:

$$\frac{\langle n(t, T) n(t + \tau, T) \rangle}{\langle n \rangle^2} = \frac{\langle I(t, T) I(t + \tau, T) \rangle}{\langle I \rangle^2} \quad \tau \neq 0 \quad (3.16)$$

This provides the basis of digital photon correlation techniques.

The accuracy of the measured correlation function can be conveniently characterised by the relative variance:

$$\frac{Var(C^{(2)}(\tau))}{\langle C^{(2)}(\tau) \rangle^2} \quad .$$

For the stationary process

$$\begin{aligned} \langle C^{(2)}(\tau) \rangle &= \frac{1}{N} \sum_{i=1}^N n(iT) n(iT + \tau) \\ &= \langle n(0) n(\tau) \rangle \quad (N \rightarrow \infty) \end{aligned} \quad (3.17)$$

where  $N$  is the total samples.  $\langle C^{(2)}(\tau) \rangle$

is the measured correlation function. As pointed out by Davenport *et al* (1958), for a stationary process with no correlation,

$$Var(C^{(2)}(\tau)) = \frac{1}{N} Var(n(0) n(\tau)) \quad (3.18)$$

Separating the independent variables  $n(0)$  and  $n(\tau)$ , this gives:

$$\begin{aligned} Var(C^{(2)}(\tau)) &= \frac{1}{N} \left[ \langle n^2 \rangle^2 - \langle n \rangle^2 \right] \\ &= \frac{1}{N} (2 \langle n \rangle^3 + \langle n \rangle^4) \end{aligned} \quad (3.19)$$

for a Poisson distribution of  $n$ , since in this case,

$$\langle n^2 \rangle = \langle n \rangle^2 + \langle n \rangle, \quad \langle C^{(2)}(\tau) \rangle = \langle n \rangle^2$$

The relative variance is therefore given by

$$\frac{\text{Var}(C^{(2)}(\tau))}{\langle C^{(2)}(\tau) \rangle^2} = \frac{1}{N} \left( \frac{2}{\langle n \rangle} + \frac{1}{\langle n \rangle^2} \right) \quad (3.20)$$

When  $N \rightarrow \infty$ , the error can be ignored, i.e., one must average over many correlation times in order to obtain reasonably small error. This statistical error is always present in addition to the other noise effects. Such statistical error due to the finite sample size will be discussed later in this chapter.

There are other effects which influence the measurement of correlation functions. For example, the detector imperfections such as gain fluctuation, detector correlation which is caused by afterpulsing *etc.*

### §3-2-2: Instrumentation Aspects

As shown above, the number of pulses stimulated by photons can be used to compute the correlation function of intensity. This method is called photon correlation. The basic instrumentation setup is shown in Fig.3.1.

Incident light falling onto the photocathode of the detector causes the photocathode to emit photoelectrons which in turn are multiplied. At the output of the detector, a series of photoelectron pulses corresponding to the photon numbers incident onto the photocathode and noise pulses are obtained. The output pulses are amplified to a certain level by an amplifier and sent to a discriminator. The discriminator will accept the pulses higher than a certain threshold and suppress those whose heights are below the threshold. The threshold is chosen by taking advantage of the difference of the pulse height distributions between signal and noise counts. This threshold should be carefully chosen so that the noise pulses that do not originate from the photocathode and afterpulses are eliminated, while

the signal pulses and, inevitably, dark pulses which originate from the photocathode are counted. The output pulses from the discriminator are normalised logic pulses of standard shape and can be easily recognised by the correlator or counter. The correlator then performs the counting task and mathematical calculation to get the correlation function, and so on. It is worth mentioning that when the photoelectron overlap effects appear, it is still possible to extend the photon correlation method to this intermediate region between low photon counts and high intensity (when DC technique is used) by means of zero crossing method [Adrian *et al* (1988)]. In the zero crossing technique, instead of correlating photon counts directly, one correlates the pulse train obtained by a zero-crossing detector which produces one pulse for every zero of the output signals from a photodetector. This technique offers a chance to make use of the powerful photon correlation technique when there is a photoelectron overlap phenomenon.

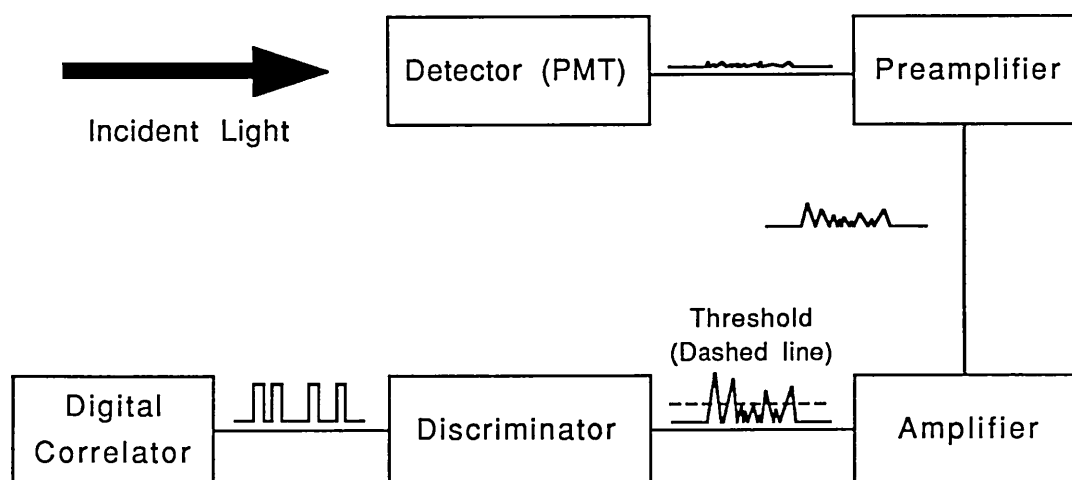


Fig.3.1. Schematic diagram of photon correlation setup.

The detection part of the photon correlation system is dominant in the whole arrangement. Usually, a photomultiplier tube is used to serve this purpose. The evaluation of parameters of this kind of detector will be discussed in chapter 4 and the emphasis will be put onto those parameters which are closely related to

the photon correlation method. In the following text, however, we will look into the avalanche photodiode (APD) as a possible alternative detector for photon correlation.

The APD is a solid state (and hence relatively inexpensive) photodetector with a relatively high quantum efficiency and a wavelength sensitivity from mid-visible through to  $\sim 1\ \mu\text{m}$ . It could be an alternative photon counting detector to replace the bulky photomultiplier tubes for photon correlation. An APD is usually operated in so called Geiger mode to achieve photon counting operation. In this mode, a reversed voltage beyond breakdown is applied across the APD. The incident light will cause an avalanche in charges and these will be detected by the following electronics. After each photodetection emission process, an avalanche quenching process is needed to get the APD recharged and ready for the next photon event. There are essentially two ways to quench the avalanche and recharge the APD, namely, passive and active quenching. In passive quenching, the APD is connected in series with ballast resistors after each photodetection process the APD gets recharged via the ballast resistors. Its dead time is the natural dead time associated with the time taken to get recharged. In active quenching mode, electronic circuitry is used to actively modify the voltage across the APD and minimize the recharging time. This technique can greatly reduce the detection system dead time.

Brown *et al* (1986,1987) have examined several parameters of APDs associated with photon correlation. They have shown that at room temperature the APD has a well defined peak for pulse height distribution with a quantum efficiency of  $\sim 7.5\%$  at  $632.8\ \text{nm}$ . The afterpulsing effects are found favourable compared with photomultiplier tubes. However, the dark counts are very high at room temperature. An acceptable dark count of a few hundreds per second can be achieved by cooling the APD down to around  $-20^\circ\text{C}$ . The output counting rates from the APD are found to be unstable with temperature and applied voltage. Another problem with passive quenching lies in the long dead time ( $\sim 2\ \mu\text{s}$ ) of

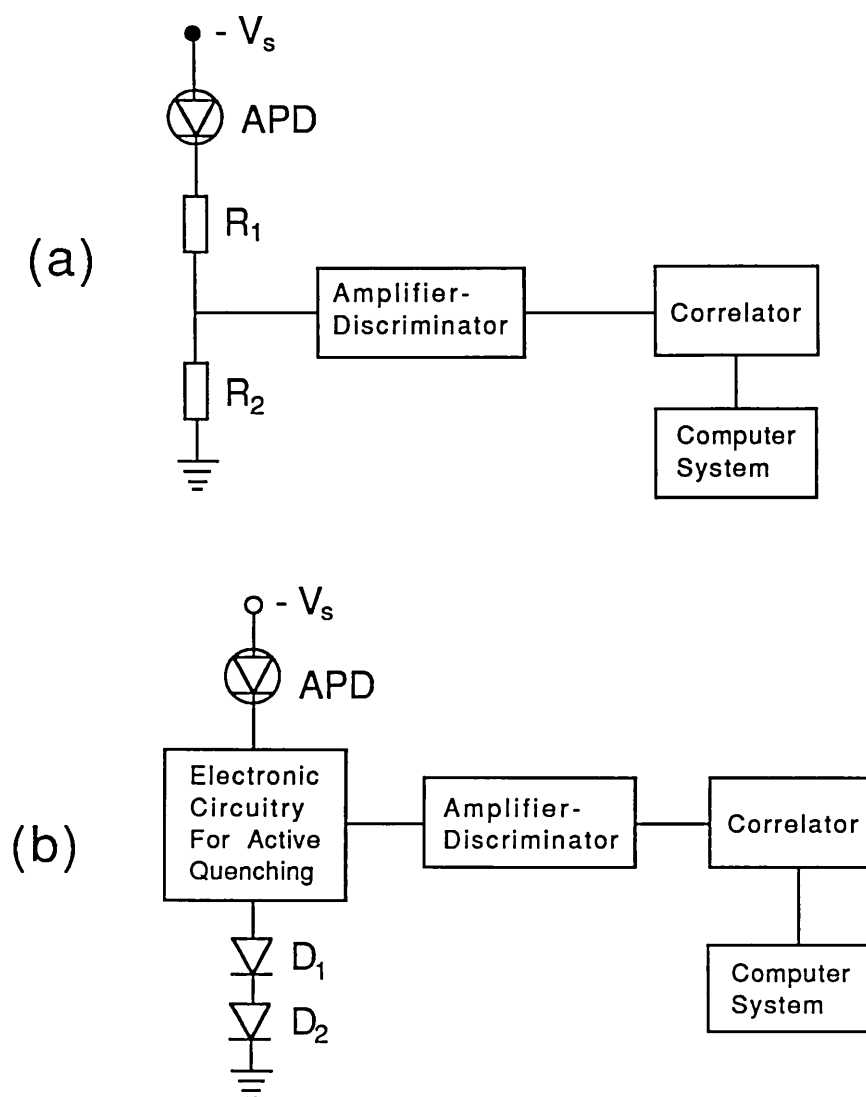


Fig.3.2. Schematic diagrams for (a). passive and (b). active quenching.

the system which severely limits the measurable correlation frequency (up to  $\sim 250\text{ KHz}$ ). The long dead time is basically determined by the time constant  $RC$  which governing the recharging process of the APD after each photodetection, where  $R$  is the ballast resistance and  $C$  is the APD capacitance. To reduce the dead time, it is desirable to reduce  $R$  and  $C$ . But to ensure the quenching avalanche upon each photodetection,  $R$  must be larger than a certain value decided by the APD characteristic parameters. Practically,  $R$  is in range of  $200\text{ K}\Omega$ . Therefore the only option is to reduce  $C$ , i.e., after each avalanche having been

quenched the APD should be recharged as soon as possible. This can be achieved [Brown *et al* (1987)] by an electronic circuit including fast switching Schottky diodes and a quenching transistor. This is so called active quenching. The active quenching technique can greatly reduce the system dead time to the order of a few tens of nanoseconds. This significantly extends the system measurable correlation frequency to well beyond one megahertz.

In conclusion, the APD is a favourable candidate to replace photomultiplier tubes as the detector for photon correlation at considerably smaller size and low cost. To overcome the long dead time of passive quenching, an active quenching scheme can be adopted. A very stable temperature and applied voltage is required to ensure the minimum counting rate fluctuation. On the other hand, because of higher dark counts, it is also necessary to cool the APD to a low temperature. A practical problem associated with this is that the APD entrance aperture might be frosted. To solve this problem, an integral optical fiber lead can be used to guide the light to the detector entrance.

### §3-3: Clipped and Scaled Correlation Techniques

#### §3-3-1: Clipped Correlation

To make a fast real-time digital correlator, the procedure of clipping of a signal is often used. The clipped laser speckle correlation is a representation of the original intensity correlation. The clipped intensity is defined as:

$$I_c = \begin{cases} 1, & I \geq I_T \\ 0, & \text{otherwise} \end{cases} \quad (3.21)$$

where  $I$  is the normal intensity and  $I_T$  is the clipping threshold.

In the clipping technique, instead of correlating two normal intensities, one correlates the clipped intensities. In the case known as single clipping, one of the intensities to be correlated is clipped, i.e.:

$$\langle I_{c1} I_2 \rangle = \int_0^\infty \int_0^\infty I_{c1} I_2 P(I_1, I_2) dI_1 dI_2$$

$$= \int_{I_T}^{\infty} dI_1 \int_0^{\infty} I_2 P(I_1, I_2) dI_2 \quad (3.22)$$

In the case of double clipping, both normal intensities are clipped and correlated, i.e.:

$$\begin{aligned} \langle I_{c1} I_{c2} \rangle &= \int_0^{\infty} \int_0^{\infty} I_{c1} I_{c2} P(I_1, I_2) dI_1 dI_2 \\ &= \int_{I_{T1}}^{\infty} dI_1 \int_{I_{T2}}^{\infty} P(I_1, I_2) dI_2 \end{aligned} \quad (3.23)$$

where  $I_{T1}$  and  $I_{T2}$  are clipping thresholds for  $I_{c1}$  and  $I_{c2}$  respectively.

The clipping technique offers the advantages of simpler and faster processing electronics than the conventional correlation technique. With the conventional technique, the speckle intensity is sampled and digitized by a multibit A/D and then correlated by multiplying the digital signals. Therefore the time scale of the technique is severely limited by the speed of A/D and multiplier required for correlation.

Electronically, the clipped signal can be easily obtained by using an analogue comparator, and these signals are then multiplied by the logical AND operation. The speed and simplicity are much more superior and a high-speed correlator permitting finer sample time and faster processing is possible.

Let us now look into what the clipping laser speckle correlations are when the speckle pattern is fully developed. For the fully developed speckle pattern, the joint probability density function is given as follows [Goodman (1975)]:

$$\begin{aligned} P(I_1, I_2) &= \frac{1}{\langle I \rangle^2 (1 - |u_{12}|^2)} \exp \left\{ -\frac{I_1 + I_2}{\langle I \rangle^2 (1 - |u_{12}|^2)} \right\} \\ &\times I_0 \left\{ \frac{2\sqrt{I_1 I_2} |u_{12}|}{\langle I \rangle^2 (1 - |u_{12}|^2)} \right\} \end{aligned} \quad (3.24)$$

where  $\langle I \rangle$  is average intensity,  $I_0(x)$  is the modified Bessel function of the zero order and  $|u_{12}|$  is the modulus of the normalized correlation function of the complex amplitude defined as:

$$|u_{12}| = \frac{|\langle E_1 E_2^* \rangle|}{\sqrt{\langle |E_1|^2 \rangle \langle |E_2|^2 \rangle}} \quad (3.25)$$

For the conventional normalized correlation, we have:

$$\frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} = 1 + |u_{12}|^2 \quad (3.26)$$

For single clipping, if we put Eq.(3.24) into Eq.(3.22), after manipulation of the integrals, it is found that the normalized clipping correlation function is given as follows:

$$\frac{\langle I_{c1} I_2 \rangle}{\langle I_{c1} \rangle \langle I_2 \rangle} = 1 + \frac{I_T}{\langle I \rangle} |u_{12}|^2 \quad (3.27)$$

It is easy to derive the conventional correlation function from the single clipping correlation. Moreover, if the clipping threshold is set equal to the average intensity, i.e.,  $I_T = \langle I \rangle$ , the single clipping correlation is exactly the same as nonclipping correlation.

For double clipping, if we put Eq.(3.24) into Eq.(3.23), after lengthy analysis, the double clipping correlation can be expressed in terms of the Laguerre polynomials:

$$\begin{cases} \langle I_{c1} I_{c2} \rangle = \sum_{n=0}^{\infty} a_n |u_{12}|^{2n} \\ a_n = \exp\left(-\frac{I_{T1}}{\langle I_1 \rangle}\right) L_n^{(-1)}\left(\frac{I_{T1}}{\langle I_1 \rangle}\right) \exp\left(-\frac{I_{T2}}{\langle I_2 \rangle}\right) L_n^{(-1)}\left(\frac{I_{T2}}{\langle I_2 \rangle}\right) \end{cases} \quad (3.28)$$

where  $L_n^{(\alpha)}(x)$  is the Laguerre polynomials:

$$L_n^{(\alpha)}(x) = \frac{1}{n!} \exp(x) x^{-\alpha} \frac{d^n}{dx^n} [\exp(-x) x^{n+\alpha}] \quad (3.29)$$

Therefore in general, under above clipping scheme, there is no closed form for the direct relationship between double clipping correlation and conventional correlation. They have different shapes of correlation function.

In spite of the fact that double clipping correlation has a different shape from that of the conventional correlation, it can still be used to determine the conventional correlation peak which is a very useful information in velocity and displacement measurement.

As the design of a double clipping digital correlator is even simpler than its single clipping counterpart, it is feasible to build such hardware for<sup>above</sup> mentioned special purpose.

The dynamic range of measured double clipping correlation is:

$$\begin{aligned} D &= \langle I_{c1} I_{c2} \rangle_{u=1} - \langle I_{c1} I_{c2} \rangle_{u=0} \\ &= \exp\left(-\frac{I_T}{\langle I \rangle}\right) - \exp\left(-\frac{2I_T}{\langle I \rangle}\right) \end{aligned} \quad (3.30)$$

In the equation we have assumed  $I_{T1} = I_{T2} = I_T$ . As shown above, the dynamic range is determined by the clipping threshold. If we differentiate Eq.(3.30):

$$\frac{dD}{dI_T} = \frac{1}{\langle I \rangle} \exp\left(-\frac{I_T}{\langle I \rangle}\right) + \frac{2}{\langle I \rangle} \exp\left(-\frac{2I_T}{\langle I \rangle}\right) \quad (3.31)$$

and set it to zero, we have:

$$\frac{dD}{dI_T} = 0, \quad I_T = \langle I \rangle \ln 2 \quad (3.32)$$

This implies that when the clipping threshold  $I_T$  is set to  $\langle I \rangle \ln 2$ , the measured double clipping correlation has maximal dynamic range and hence higher signal-to-noise ratio.

On the other hand, when we try to draw information from the measured double clipping correlation function, it is expected that the correlation function should have a broad width. As shown by Ohtsubo *et al* (1985,1988), the correlation width is also related to threshold setting and the optimal setting is at the signal's mean value. Thus it is concluded that one should make a trade-off in setting clipping threshold between correlation dynamic range and correlation width.

It is possible to determine approximately  $|u_{12}|$  from double clipping correlation. Marron *et al* (1985,1986) have plotted curves of  $\langle I_{c1} I_{c2} \rangle$  against  $|u_{12}|$  by using Eq.(3.28) and Eq.(3.29). It is found from their figure, the curves resembles a section of an ellipse as shown in Fig.3.3. Its extreme values are:

$$\begin{aligned} &\langle I_{c1} I_{c2} \rangle = \langle I_{c1} \rangle \langle I_{c2} \rangle \quad |u_{12}| = 0 \\ &\langle I_{c1} I_{c2} \rangle = \left\{ \begin{array}{l} \langle I_{c1} \rangle; \text{ if } \frac{I_{T1}}{\langle I_{c1} \rangle} \geq \frac{I_{T2}}{\langle I_{c2} \rangle} \\ \langle I_{c2} \rangle; \text{ if } \frac{I_{T2}}{\langle I_{c2} \rangle} \geq \frac{I_{T1}}{\langle I_{c1} \rangle} \end{array} \right. \quad |u_{12}| = 1 \end{aligned} \quad (3.33)$$

As can be seen from above, if we assume that  $(I_{T1} / \langle I_{c1} \rangle) \geq (I_{T2} / \langle I_{c2} \rangle)$ , the ellipse is therefore centered at  $(0, \langle I_{c1} \rangle)$  and the semimajor and semiminor axes are therefore 1 and  $(\langle I_{c1} \rangle - \langle I_{c1} \rangle \langle I_{c2} \rangle)$  respectively. Therefore the equation for the ellipse is then:

$$|u_{12}|^2 + \left( \frac{\langle I_{c1} I_{c2} \rangle - \langle I_{c1} \rangle}{\langle I_{c1} \rangle \langle I_{c2} \rangle - \langle I_{c1} \rangle} \right)^2 = 1 \quad (3.34)$$

Hence  $|u_{12}|$  is approximately:

$$|u_{12}| \approx \left[ 1 - \left( \frac{\langle I_{c1} I_{c2} \rangle - \langle I_{c1} \rangle}{\langle I_{c1} \rangle \langle I_{c2} \rangle - \langle I_{c1} \rangle} \right)^2 \right]^{\frac{1}{2}} \quad (3.35)$$

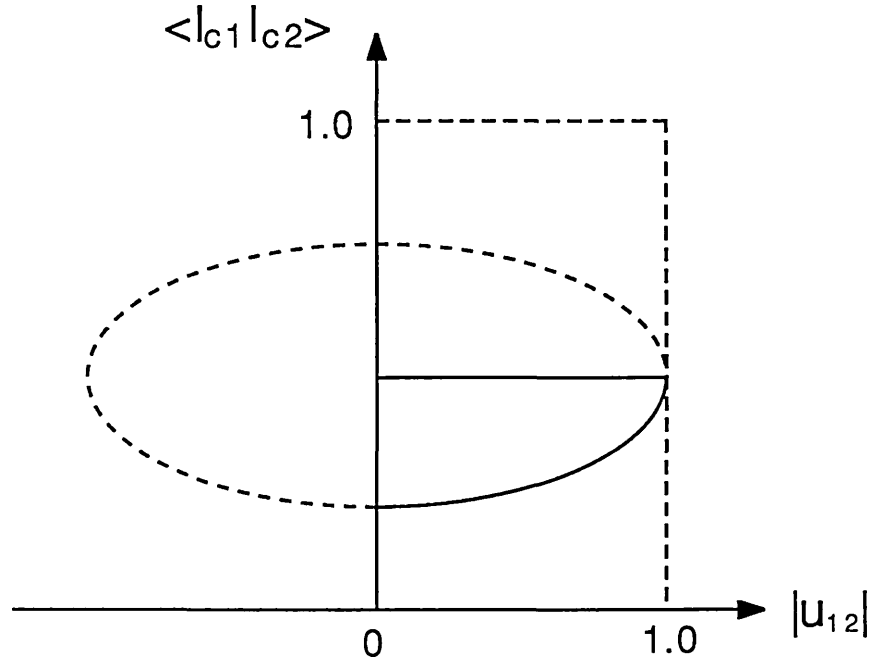


Fig.3.3. Ellipse showing an approximation of double clipping correlation function.

If we define the difference between Eq.(3.28) and the approximate expression Eq.(3.35) as the error, it is found that this approximation is most accurate when the clipping threshold holding the following formula:

$$\frac{I_{T1}}{\langle I_{c1} \rangle} = \frac{I_{T2}}{\langle I_{c2} \rangle} = \ln 2 \quad (3.36)$$

This claim is obtained by comparing curves plotted for errors against  $|u_{12}|$  for different clipping thresholds.

Churnside (1985) has also shown that double clipping correlation could be used to derive approximately non-clipping conventional correlation under the following clipping scheme:

$$I'_c = \begin{cases} 1, & I \geq I_T \\ -1, & \text{otherwise} \end{cases} \quad (3.37)$$

where  $I_T$  is the clipping threshold. Comparing this scheme with the one shown in Eq.(3.21), under this clipping scheme, the difference in implementation is to replace an OR gate for Eq.(3.21) by an XOR gate in the electronics so that no additional complexity is introduced. Under this scheme, if the clipping threshold is set to signal's mean value, the relationship between double clipping correlation and  $|u_{12}|$  can be approximately expressed as:

$$|u_{12}| = 1.123 \sin \frac{\pi \langle I'_{c1} I'_{c2} \rangle}{2} - 0.123 \quad (3.38)$$

As shown by Marron *et al* (1986) that two clipping schemes have linear relationship and therefore they are equivalent. The first method of approximation is most adequate when the clipping threshold is set to  $\langle I_c \rangle \cdot \ln 2$ , while for the second method of approximation is accurate only when the clipping threshold is equal to signal's mean value.

As with the clipped intensity technique, the photon counts clipping correlation technique is also a way to overcome difficulties in making fast real time correlator. The clipped photon counts are defined as:

$$\begin{cases} n_q(t) = 1, & n(t) \geq q \\ n_q(t) = 0, & \text{otherwise} \end{cases} \quad (3.39)$$

Like the intensity clipping technique, the photon counts clipping technique can be divided into single and double clipping techniques. For single clipping, the clipped correlation has the following relationship with the field correlation

function:

$$\frac{\langle n_q(0) n(t) \rangle}{\langle n_q \rangle \langle n \rangle} = 1 + \frac{1+q}{1+\langle n \rangle} |u_{12}|^2 \quad (3.40)$$

where  $\langle n \rangle$  is the mean counts, If the clipping threshold is set to the mean counts, then single clipping photon correlation is equal to nonclipping photon correlation.

For double clipping technique, one measures  $\langle n_{q1}(0) n_{q2}(t) \rangle$ . Generally speaking, double clipping correlation has a complicated relationship with the non-clipped photon correlation [Jakeman (1974)]. It is hard to derive information about  $|u_{12}|$ . However, similar to intensity clipping technique, the photon correlation peak can still be found from the measured double photon counts clipping correlation.

In a sense, the photon counts clipping is very much similar to intensity clipping technique. However, photon counts clipping technique is feasible when the detected light field is very weak, such as in stellar intensity analysis in which case the detecting procedure itself is photon counting.

### §3-3-2: Scaling-Clipping Correlation

Clipping methods, including intensity and photon counts clipping, have the disadvantage of being restricted to cases in which the Gaussian approximation applies. Only a few expressions for the relation between clipped and conventional correlation are derived for completely characterized field statistics, such as Gaussian statistics. In cases in which the Gaussian approximation breaks down, it is very difficult to analyze the unknown statistics from the measured clipping correlation which has a unknown relationship with conventional correlation. As suggested by Koppel *et al* (1973), the scaling-clipping method can be used here.

In the scaling method, one measures  $\langle n_o^{(s)}(0) n(t) \rangle$ , where  $n_o^{(s)}$  is the so called scaled-clipped photon counts, i.e., for every  $s$  input pulses there is one output pulse, the scaled photon counts are denoted as  $n^{(s)}$ , this scaled photon counts are then clipped again same as mentioned before to give scaled-clipped

photon counts.

As defined in photon counts clipping:

$$\begin{cases} n_q(t) = 1, & n(t) \geq q \\ n_q(t) = 0, & \text{otherwise} \end{cases} \quad (3.41)$$

It is obvious that:

$$\sum_{q=0}^{\infty} \langle n_q(0) n(t) \rangle = \langle n(0) n(t) \rangle \quad (3.42)$$

This implies that by averaging over a sufficiently large number of clipping levels, the conventional correlation function can be recovered.

It is easy to relate scaled-clipped correlation with clipping correlation by the following equation [Koppel *et al* (1973)]:

$$\langle n_o^{(s)}(0) n(t) \rangle = \sum_{q=0}^{s-1} \frac{\langle n_q(0) n(t) \rangle}{s} \quad (3.43)$$

If  $s \rightarrow \infty$ , bearing in mind Eq.(3.42), we have:

$$\langle n_o^{(s)}(0) n(t) \rangle = \frac{\langle n(0) n(t) \rangle}{s} \quad (3.44)$$

Thus, when the scaling scale is very large, the scaled-clipped correlation is proportional to the conventional correlation regardless of the field statistics. This technique has the advantages of easy interpretation of the measured scaling-clipping correlation to conventional correlation as opposed to the double clipping technique.

The problem with this technique is that the scaling scale must be very large both physically and mathematically to make Eq.(3.44) valid. On the other hand, the detected field particularly at low light level, such a high scaling scale would make the signal to noise ratio very poor. Therefore in practice, it is tricky to fulfill the scaling scheme.

### §3-4: Multichannel Optical Correlator

In real-time signal processing and optical computing, a fast multichannel correlation system is needed. The multichannel optical correlator is one of the

competitors among those including digital processing techniques to perform this task.

A multichannel optical correlator can be built utilizing acousto-optic [Berg *et al* (1983), Casasent (1985)], magneto-optic [Davis *et al* (1987)] devices and liquid crystal televisions [Yu *et al* (1988)]. They serve as signal bearing devices and convert time-varying signals into spatially distributed ones which are easily read out by a light beam. Several operations such as correlation and convolution then can be performed. The time delay and timing order can of course be manipulated by controlling the outside conditions. In the following example, we will briefly mention how an acousto-optic correlator works.

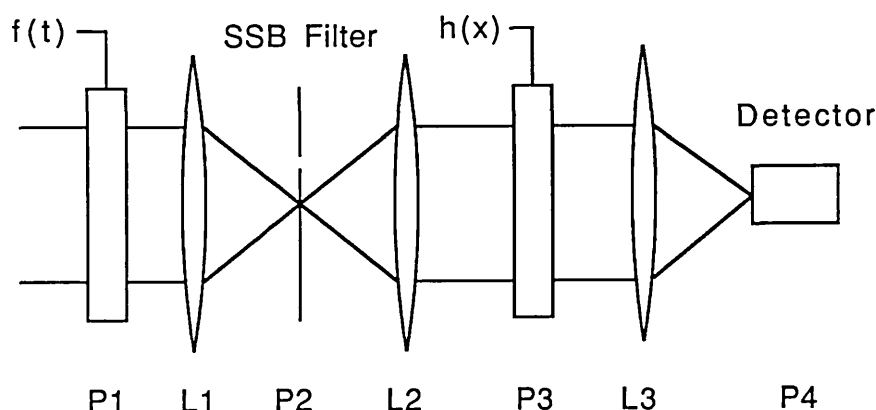


Fig.3.4. Schematic diagram of a multichannel acousto-optic correlator.

The principle of an acousto-optic correlator can be understood from Fig.3.4. The time-varying signal  $f(t)$  is fed into a signal bearing cell, such as an acousto-optic cell. This time dependent signal is then converted into a spatial one as the grating is formed when the signal travelling through the cell with time. This

spatially distributed signal can be written as  $f(vt + x)$ , where  $v$  is the velocity of sound. When a collimated beam passing through this cell, it will read out this spatial information and it will be imaged onto the second cell which stores the signal  $h[v(t + \tau) + x]$  being correlated with or in real time is being injected a signal. After the beam having past the second cell, it picked up the information of either correlation or convolution of the two signals depending on the order in which the second signal being injected and stored.

By replacing the reference channel of the above mentioned multichannel optical correlator with a magneto-optic spatial light modulator, we can take many advantages offered by this device. It has a number of favourable qualities for optical processing including a large number of parallel channels, infinite delay range and fast updating of the entire pattern when required. It is also programmable and does not have to be updated all the time. However, the problems with magneto-optic devices are the cross talk between channels and complicated electronic circuits needed to fulfil the task.

If liquid crystal televisions are used as the signal bearing cells, they offer several advantages such as being compact and cheap, addressable and easy to control. The disadvantages of using liquid crystal televisions are their low resolution (which limits the spatial carrier frequency) and slower response time.

In implementing the multichannel optical correlator, several practical constraints should be borne in mind: cross-talk, uniformity of laser beam, inherent gratings due to acousto-optic and magneto-optic devices themselves, *etc.* As can be seen, optical approaches are less accurate compared with their digital counterparts, but their parallel nature and great potential in real time signal processing make them the promising candidates in widespread applications where speed is the priority. For example, a multichannel optical correlator can be used to perform discrete Fourier transform [Goodman *et al* (1978), Huang *et al* (1988)], pattern recognition, *etc.*

### §3-5: Concluding Remarks

Photon correlation techniques provide a powerful way to measure the exact correlation function at very low light level. They also offer a way to reconstruct a less noisy correlation by certain algorithms in imaging through turbulence. To overcome difficulties in making a very fast correlator capable of measuring very high fluctuation frequencies, clipping techniques may be used. One can either clip the intensity or photon counts depending on the incident light level. However, only single clipping techniques can give the exact measurements of incident intensity correlation functions for Gaussian statistics at the expense of relatively complicated circuitry. Double clipping techniques can be used to estimate certain properties of detected light including the correlation peak but still the techniques are restricted to Gaussian statistics. As far as real-time signal processing is concerned, a fast multichannel correlator is of practical importance. Optical devices offer such possibilities but are still subject to certain practical constraints.

We used photon correlation techniques for measuring and analyzing statistical properties of light scattered by dense random media and stellar image fluctuations in which cases the light intensities are very weak. In the following chapter, we shall concentrate on the technique by evaluating a multi-anode micro-channel plate photomultiplier tube as the detector for the photon correlation technique.

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## Chapter 4

### Evaluation of a Micro-Channel Plate Photomultiplier Tube For Photon Counting and Photon Correlation

#### §4-1: Introduction

On the whole, the detector is the key part which has the greatest influence on a photon correlation system. Usually, a photomultiplier tube (PMT) is chosen as the photon event detector. A PMT is a device which can convert the incident electromagnetic field into pulses of charge at the anode. When the incident field is falling on the photocathode, the photoelectrons are emitted from the photocathode and they in turn release more electrons from the following secondary emission material (individual dynodes or continuous electron-multiply channel). The multiplied electrons are collected by the anode. At low light level, the output pulse overlap effect is insignificant and the output pulses are uniquely related to the incident photon events.

#### §4-1-1: General Parameters of a PMT

Because the PMT is a secondary emission device, its properties can be described by statistical parameters which can decide the effectiveness of use of the PMT. Foord *et al* (1969) have evaluated a number of PMTs for applications in experiments where the counting of individual photoelectrons is necessary or advantageous. Generally, the following parameters are important when a PMT is used for detecting photon events and in photon correlation measurements.

##### (1). Pulse Height Distribution

Physically, the gain of a PMT is defined as the number of output electrons stimulated by a single input event. The gain is not fixed quantity because the secondary emission process is a random process. The variation of the gain can be described by studying the charge distribution of the output pulses. We call this

charge distribution a pulse height distribution. It can be used to determine the optimum operating parameters of PMT for pulse counting measurements under different experimental conditions.

The pulse height distribution can be measured with a multi-channel analyzer [Stanford (1965)] consisting of a charge-sensitive preamplifier and a pulse-height analyzer, or alternatively, one can just measure the output counting rate against applied voltage (or gain) curve to choose the best operating parameters.

### (2). Quantum Counting Efficiency (QCE)

An ideal PMT would be expected to count photons with an efficiency equal to its photocathode quantum efficiency which depends upon the individual photocathode and incident wavelength. There are several reasons for the discrepancy between the total counting efficiency of the PMT and its photocathode quantum efficiency. These reasons are the loss in the multiplier chain, the loss in the input electron optics, dead time in the PMT or associated electronic circuits, etc. The basic way of measuring QCE is to measure the pulse count rate, multiply it by the photon energy and divide by incident power. A number of papers [Foord *et al* (1969), Poultney (1972), Lakes *et al* (1971)] have described the QCE and its measurements. Mathematically,

$$\text{QCE}(\alpha) = \frac{\text{Count rate} \times \text{Photon energy} (\hbar\nu)}{\text{Incident power}} \times 100\%$$

which can be obtained by measuring each variable in the formula.

### (3). Spurious Pulse Characteristics

The spurious pulses include dark count pulses, afterpulses and tube scintillation pulses. "Dark" pulses are those observed when the PMT is operated in total darkness. Afterpulses are spurious output pulses that are time correlated with the signal pulses. Afterpulses are generally of lesser amplitude than the signal pulses and occur right after the initiating pulses. Scintillation pulses are caused by light flashes that originate in the PMT structure, primarily in the faceplate and they

are usually produced by an interaction between luminescent centers within the glass and radiation of sufficient energy to produce a scintillation.

These spurious pulses can be partially eliminated by sending all the pulses through an amplifier–discriminator system which will reject the pulses whose height are lower than a certain threshold.

#### (4). Time Characteristics

In photon statistics experiments the time properties of a PMT are very important [Pike (1970)]. Leskovar *et al* (1975), Bebelaar (1986) have described the time property measurement in detail. The times taken by photoelectrons to travel from the photocathode to the anode (transit times) fluctuate. This gives rise to a transit time spread. The time resolution capabilities are essentially determined by the random deviations in the transit time of electrons, i.e. the transit time is spread over a period. Generally the value of the transit time spread depends on the PMT's geometric characteristics, its operating conditions, and the number of electrons emitted from the photocathode. Since in measuring the time spread function the measurements vary approximately inversely with the square root of the number of photoelectrons, the time behaviour information of single photoelectron is of particular importance when predicting the transit time spread for an arbitrary number of photoelectrons. Consequently, the single electron time performance is the best parameter for describing the characteristics of PMT electron transit time spread. There are a number of papers [Boutot *et al* (1978), Lo *et al* (1974,1981a) Artem'ev (1968), Hussain *et al* (1980), Poultney (1972), O'Connor *et al* (1984)] which give detailed experiments of single photoelectron timing performance measurement. Of course, in such measurements a fast light source is necessary. The light-emitting diode (LED) is a very inexpensive light source for this purpose.

#### §4-1-2: Micro-Channel Plate PMT (MCP-PMT)

With the advent of the micro-channel plate (MCP) photomultiplier tube (PMT), a lot of attention has been given to the applications and performance evaluation [e.g. Oba *et al* (1979)] of this device. The MCP-PMT offers the possibility of combining the advantages of both photographic and photoelectronic system. Namely, the MCP-PMT has abilities to gather spatial information with its multi-anode array and is sensitive and linear to the incident intensity. In particular, MCP-PMT has photon counting capability and a low dark count rate.

The MCP-PMT is essentially a large number of photomultipliers with partially independent signal channels arranged in close proximity in a single vacuum envelope. The tubes with a matrix anode design have the advantage of getting spatial information quite easily. Dhawan (1981) has given a very good introduction to MCP-PMTs. The MCP-PMT is different from the conventional PMT in the dynode structure for electron multiplication. In a conventional PMT, discrete dynode structures are adopted, while a continuous dynode structure (channel) is used in a MCP-PMT. As far as focusing photoelectrons from the photocathode to the front end of MCP is concerned, there are two ways to perform this. One is called proximity-focusing and the other electrostatic-focusing. The former kind focuses electrons only by putting photocathode and MCP close enough, whereas the latter uses a electron-optical lens to fulfill the focusing task; this offers better image quality and higher resolution but at the expense of being more bulky.

The MCP-PMT has several features resulting from the characteristics of the MCP. The detailed features of MCPs have been given by Manley *et al* (1969) and there have been some reports [Oba *et al* (1979), Lo *et al* (1977,1981b)] on the performance of MCP-PMTs. Here we summarise the main points:

(1). Advantages:

(a). It has the versatility of high gain, low noise, high speed.

(b). With an MCP-PMT, it is possible [Timothy *et al* (1975a,1975b,1979)] to get linear or two-dimensional information on <sup>the</sup> photon distribution falling on the photocathode.

(c). It has been reported [Lo *et al* (1977,1981b)] that MCP-PMT is immune to the magnetic field.

(d). The time resolution capability of MCP-PMT ( $\sim 3 - 5 \text{ nsec}$ ) is better than that of the conventional PMT. This has been reported by several authors [Bebelaar (1986), Oba *et al* (1981), Lo *et al* (1977,1981b)]. The transit time spread of an MCP-PMT is small and with this feature, an MCP-PMT can be used for time-correlated photon counting technique [O'Connor *et al* (1984)].

(2). Disadvantages:

(a). The life of the tube is fairly short.

(b). It has an exponential pulse height distribution and that will cause a disturbing effect of scintillations, particularly at very low light level. Therefore, high gain and operation in saturation mode is necessary to bring signal pulses of different heights to a certain level of similar height. Two MCPs in one tube can ensure high gain and saturation mode.

(c). Since an MCP-PMT is an assembly of channels fused together, variation in the channel's diameters and structure will introduce non-uniformity.

(d). MCP has a longer local dead time than the ordinary PMTs.

(e). Cross-talk between channels is rather serious.

An MCP-PMT can be operated in several modes and this has been summarised by Lakes *et al*. It will be used in photon counting mode in our experiments. When an MCP-PMT is operated in photon counting mode, as far as photon counting and correlation are concerned, the following parameters are of particular importance.

(1). Gain and Dark Count

The gain of an MCP-PMT is a function of the length to diameter ratio of

channels, the multiplier voltage, the penetration of the output electrode into the channels, the secondary emission coefficient of the channel walls, and the initial energy of secondary electrons. With the ready-made tube, the only factor we can control for the gain is the applied voltage. When the applied voltage is high, the gain is high and so is the output. However, the dark count is high too. If the applied voltage is low, although the dark count is very small, the output signal is sometimes too weak to be detected. So, there is a trade-off between the gain and dark count. The applied voltage should be carefully chosen to make a good compromise among the parameters. Also the signal-induced background should be considered. But this is normally negligible when the detector is operated in photon counting mode.

## (2). Dead Time

During the dead time interval, the tube will have no response to the incident events. With an MCP-PMT, the main reason for the dead time is that the MCP needs time to restore the charge to the used channels by means of the bias current.

The dead time can be measured by observing phenomena occurring after illuminating all the channels using a high intensity pulse sufficiently shorter than the dead time. Another way of measuring dead time is to measure the time interval between two successive pulses. In this case, the pulse rate should be in excess of maximum counting rate at that particular intensity. The above two methods are rather straightforward, but high precision apparatus is needed, for example, a very-short-width pulse generator and a high frequency observing oscilloscope. On the other hand, the dead time can be found by analyzing <sup>the</sup> detected photon counting distribution induced by an incident intensity which would produce a Poisson photon counting distribution if there were no dead time effects. This will be discussed in §4-2.

## (3). Spatial Resolution

The basic limitation to resolution obtainable from MCP-PMT is the channel <sub>an</sub>

and anode spacing. In our case the spatial resolution can be represented by the relationship between output from a row of anodes and position of a incident light scanning slit which is parallel to the row of anodes.

#### (4). Cross-Talk

Because of the imperfectness of the electron optical focusing system inside the tube and interference between the anodes, the output from the one anode will influence the output from an adjacent anode.

Apart from the parameters mentioned above, some parameters given in §4-1-1 are also of importance. The definitions and measuring methods for a MCP-PMT are the same or similar as the conventional PMT given before.

### §4-2: Evaluation of an MCP-PMT for Photon Correlation

The Hamamatsu R2519 photomultiplier tube is an MCP-PMT with a matrix of 100 anodes. It has an S-20 photocathode<sup>✓</sup> with a specified maximum sensitivity of 64mA/W at the wavelength of 420nm. <sup>of diameter 20mm,</sup> Its focusing method is electrostatic and the system magnification is 0.8. Each individual anode can detect the photon events incident on a small corresponding region of the photocathode. Therefore, by making full use of these anodes, both temporal and spatial information of photon events can be picked up simultaneously. In analyzing time-varying images such as speckle patterns and scintillation of the star light due to atmospheric turbulence, the space-time correlation can be measured and used to deduce certain features of both light sources and turbulent media [Dainty *et al* (1981)].

#### §4-2-1: MCP-PMT Evaluation

Among a hundred ( $10 \times 10$ ) anodes, forty-eight were connected with wires as shown in Fig.4.1(a) and they were ready for being connected to the output electronic circuits. As shown in the Fig.4.1(b), each anode was inserted into a spirally twisted wire and they were soldered together with a great deal of careful

work. Seven of forty-eight connected anodes were connected with BNC outputs and their spacings were arranged such that different spatial information could be obtained.

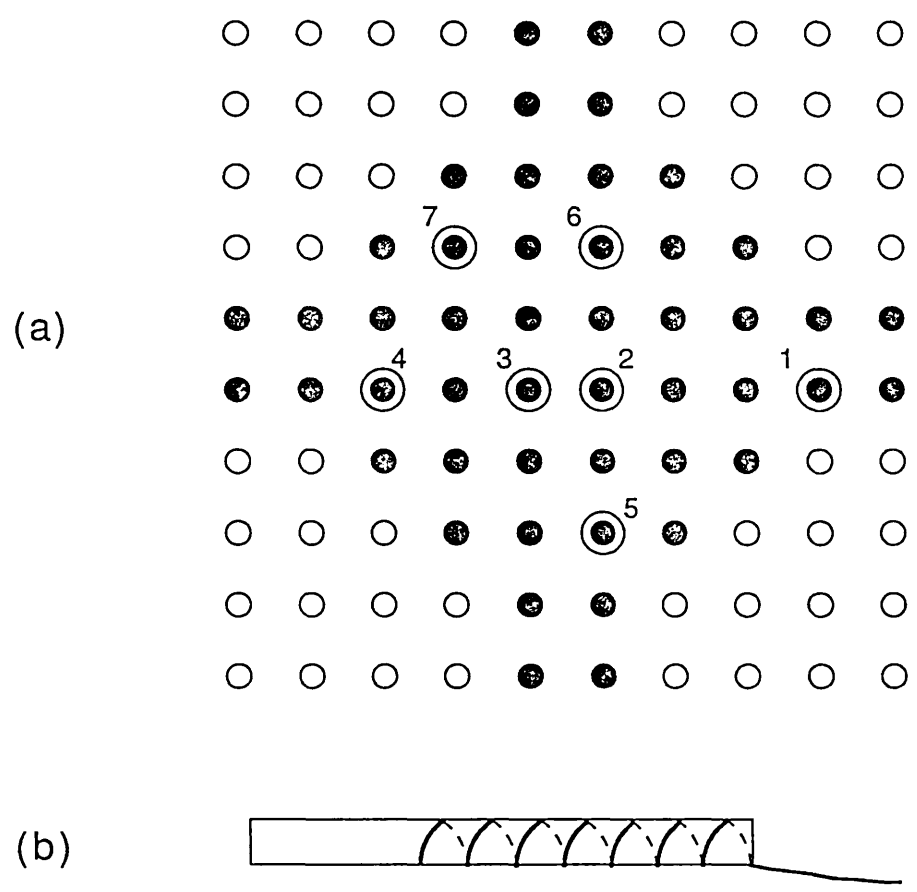


Fig.4.1. Schematic diagram of anode connections. (a). Forty-eight anodes were wired out and seven of them were connected to BNC connectors. (b). Each anode was soldered out with a spirally twisted wire.

Several parameters which are important for photon correlation measurements are evaluated. The operating performance of this MCP-PMT and measurements of the detected auto-correlation and space-time correlation of a randomly varying speckle pattern are also given below.

## (1). Quantum Counting Efficiency (QCE)

QCE is the combination of both Quantum Efficiency (QE) of photocathode and Collection Efficiency (CE) of both focusing system and MCP. The incident photons on the photocathode cause a certain number of photoelectrons to be emitted, which is governed by the QE of the photocathode. Due to inefficiencies of collection, a certain amount of the photoelectrons are lost within the tube and only a fraction of the electrons are finally collected by the anodes. In our case, the QCE is found to be substantially lower than the QE of the photocathode. As Fig.4.2. shows below, QCE was measured against applied voltage across the MCP for central and side located anodes when the incident light was at the wavelength of 632.8nm. At the typical operating point (2.1kV), the QCE is about 0.16%, while the QE of the photocathode is given by the manufacturer as about 4.8% at this wavelength. The error of the measurements was determined by the power meter and its calibration for the particular wavelength.

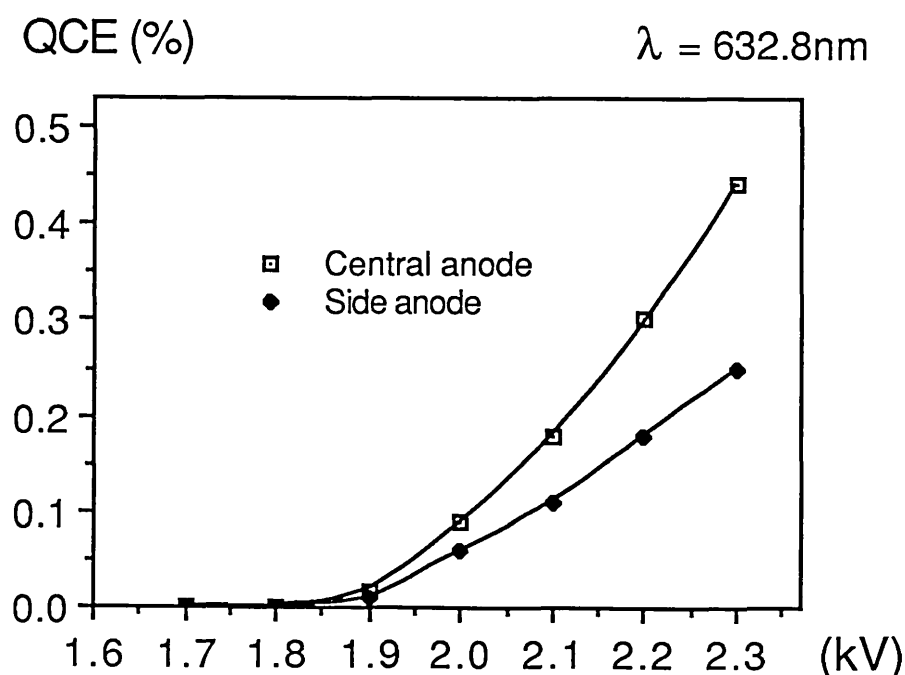


Fig.4.2. Measured QCE against voltage applied across the MCP at 20°C.

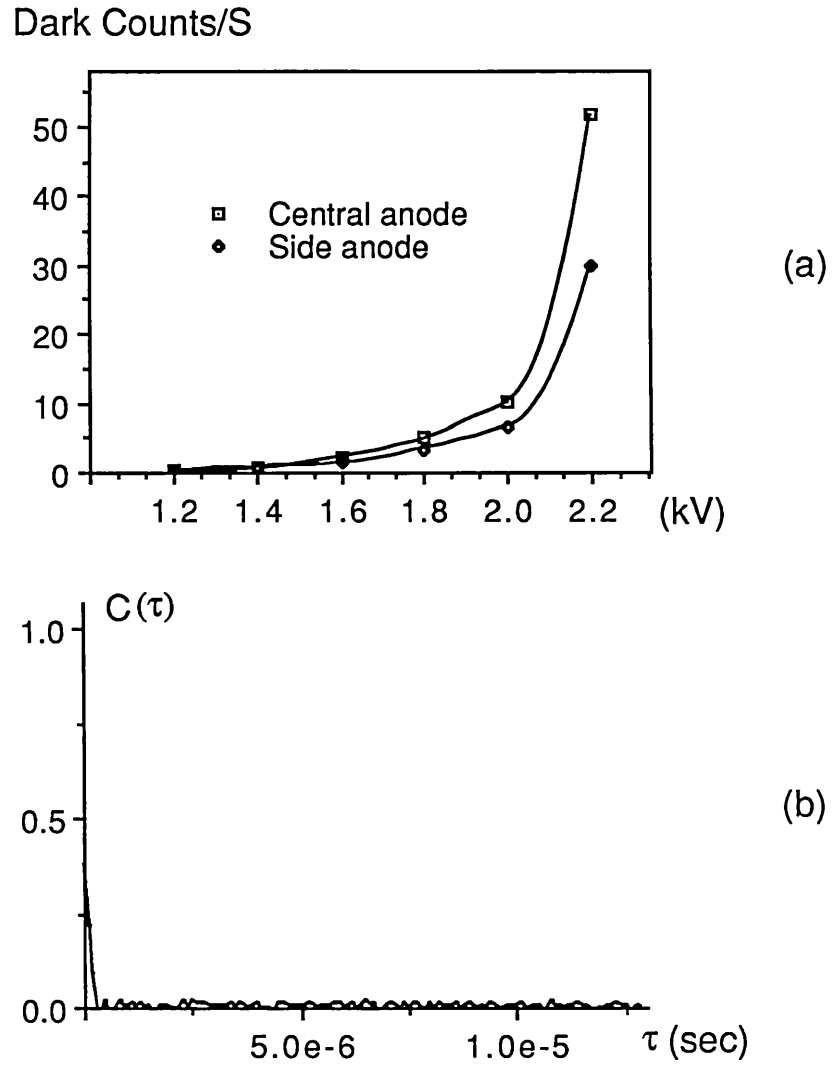


Fig.4.3. (a). Dark counts versus voltage applied across the MCP at 20°C with the threshold equal to zero.

(b). Correlation function showing afterpulsing effect. Sample time  $T = 100 \text{ ns}$ , peak value is  $C(0) = 0.38$ , average number of photon counts per sample time is  $\langle n \rangle = 1.1 \times 10^{-3}$ .

## (2). Dark Counts and Afterpulsing

Fig.4.3.(a) shows that the dark counts are quite low, and some of them can be removed by thresholding. For different anodes, the dark counts are slightly

different. The low dark count may be at the expense of low QCE and long dead time as shown later. Afterpulses are spurious output pulses that are time correlated with the signal pulses. Afterpulses are generally of lesser amplitude than the signal pulses and occur soon after the initiating pulses. A time auto-correlation peak can be observed in Fig.4.3.(b) caused by the afterpulsing effect. This correlation function was measured using a very short sample time of  $100\text{ ns}$  for an the incident light field with Poisson statistics, which implies that the light field has constant classical intensity and has normalised auto-correlation value of zero. The peak value of the correlation function (normalized in the sense of Eq.(4.3) below)  $C(0)$  is 0.38 and the average photon counts per sample time  $\langle n \rangle$  is  $1.1 \times 10^3$ . The correlation length is spread over the first three channels of delay time, corresponding to  $300\text{ ns}$ , or  $3.3\text{ MHz}$ . Therefore, to avoid the afterpulse effect on correlation measurements, this tube should be used to measure frequencies not more than  $3\text{ MHz}$ .

### (3). Pulse Height Distribution

For an MCP-PMT operating in linear gain mode, the output pulse height distribution is exponential. This causes difficulties in choosing the threshold voltage of the discriminator, for example to eliminate low height noise afterpulses. To overcome these difficulties, it is necessary to operate in gain-saturated mode so that the smaller signal pulses are amplified to the same level as large saturated signal pulses while a gap still remains between the heights of signal and noise pulses. Therefore all the signal pulses are expected to have similar pulse height at the output. By taking advantage of the difference between signal and noise pulses, a threshold voltage can be chosen to eliminate noise pulses and a higher signal-to-noise ratio can be obtained. To evaluate the pulse height distribution and obtain the best operating parameters, we measured the pulse counting rate versus applied voltage applied to the MCP for different threshold voltages of amplifier/discriminator, as shown in Fig.4.4.

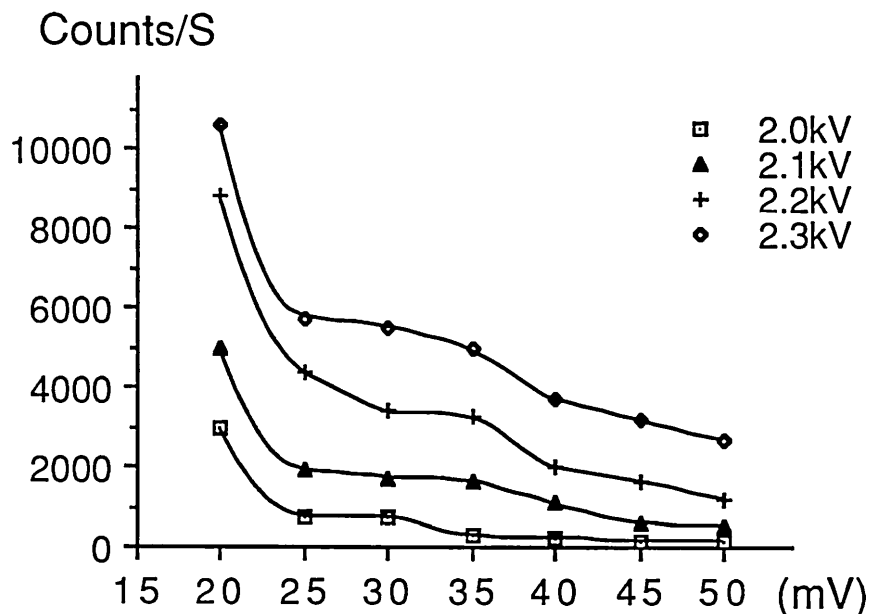


Fig.4.4. Pulse counting rate against discriminator threshold at various voltages applied across the MCP.

A fairly good pulse height distribution can be inferred from the fact that a large portion of each curve is plateau. According to Fig.4.4., the horizontal part should be taken as a reference for setting optimum operating parameters. In our case, an applied voltage of 2.1 *KV* for the MCP and a threshold voltage of 30 *mV* for the amplifier/discriminator are chosen as the best operating parameters.

#### (4). Maximum Counting Rate

When the tube is operated in photon counting mode, the incident light power should be kept low. The output counting rate decreases if the incident power is too high because of paralysable dead time effects. Therefore it is very important to keep the incident power to a certain low level so that the photon events can be counted correctly. Fig.4.5 shows the relationship between counting rate and incident intensity on the photocathode in cases of white and red ( $\lambda = 632.8nm$ ) light. In both cases, the maximum counting rate was found to be 45 *KHz*.

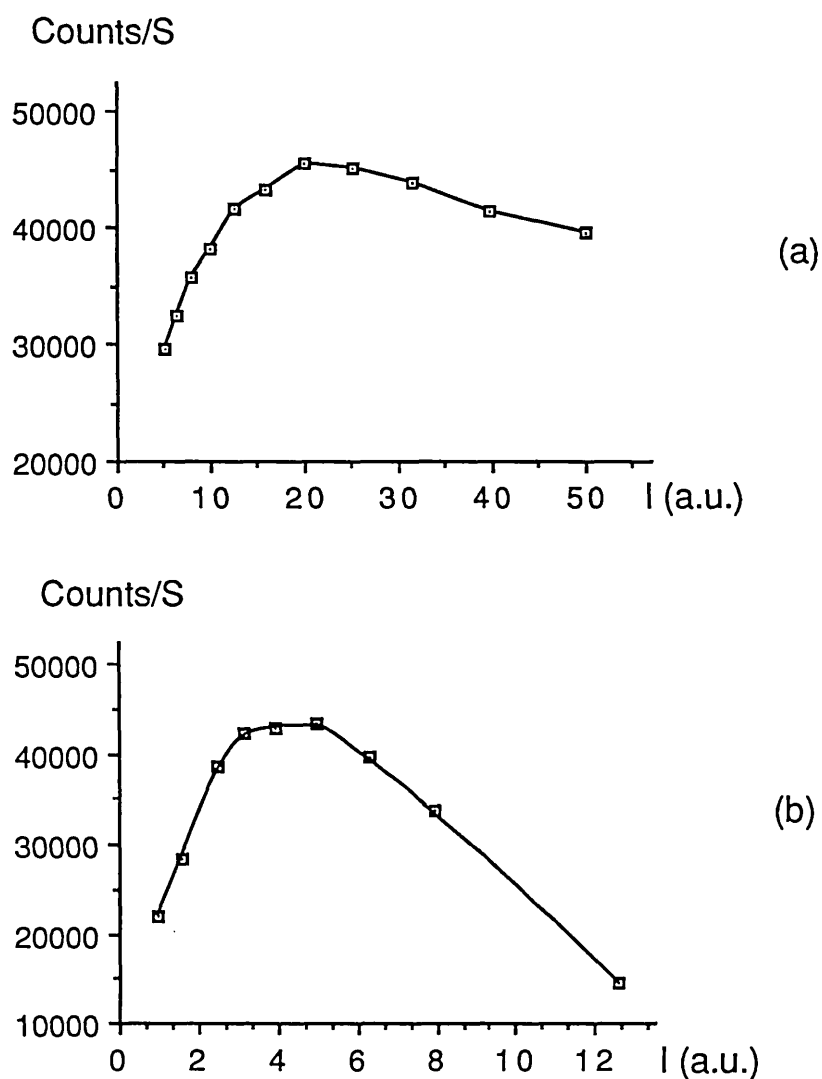


Fig.4.5. Counting rate versus incident intensity from a single anode for (a). white light and (b). light at  $\lambda = 632.8\text{nm}$ .

From these Figures, it is found that the counting rate has a linear relationship with the incident power up to a counting rate of approximately 30  $KHz$ . As the incident power increases, the counting rate reaches its maximum and decreases thereafter because of the paralysable dead time effects. A suitable incident power range can be indicated from the curves in order to keep the tube operating in photon counting mode. This range is rather small compared with that for the ordinary PMT.

## (5). Dead Time

We are here discussing the MCP-PMT's *local* dead time which is the time taken to recharge a *single* micro-channel. The dead time of this particular MCP-PMT is long due to the fact that a long time is needed to restore the charge to the used channels. The dead time was measured by measuring the photon counting distribution  $P(n, T)$ , which is the probability distribution of counting  $n$  photons during the sample time  $T$ . As defined in §2.2.20 :

$$F(n) = \frac{(n+1)P(n+1, T)}{P(n, T)} \quad (4.1)$$

$F(n)$  can then be calculated from the measurements of  $P(n, T)$ . It has been pointed out [Johnson *et al* (1966)] that for coherent incident light with Poisson statistics and with the sample time  $T$  much longer than dead time  $\tau_d$ ,

$$F(n) = -\frac{2\bar{n}_o\tau_d}{T}n + \bar{n}_o(1 + \frac{\bar{n}_o\tau_d}{T}) \quad (4.2)$$

where  $\langle n \rangle$  is the observed average number of counts per sample time  $T$ . The above relationship allows us to calculate dead time  $\tau_d$  if  $\langle n \rangle$  and  $T$  are known. The typical results are in Fig.4.6 with measured local dead time of about  $4.4 \mu s$ .

## (6). Cross Talk

When the photocathode is illuminated by light through a fine pinhole over a small area corresponding to one anode, the signal pulses are not only detected from this anode but also from the adjacent anodes. The cross talk is defined as the ratio of detected signals from adjacent anodes to that from the illuminated anode. There are two main causes responsible for the cross talk. One is the spreading of photoelectrons inside the tube, the other is the capacitance effect between the anodes. The measurement shown in Fig.4.7.(a) was made by scanning a pinhole image across the photocathode and the photon counts were recorded from one anode.

The spread over a certain area is mainly due to the focusing system and the spread inside the MCP. This also indicates that the anode is collecting signal

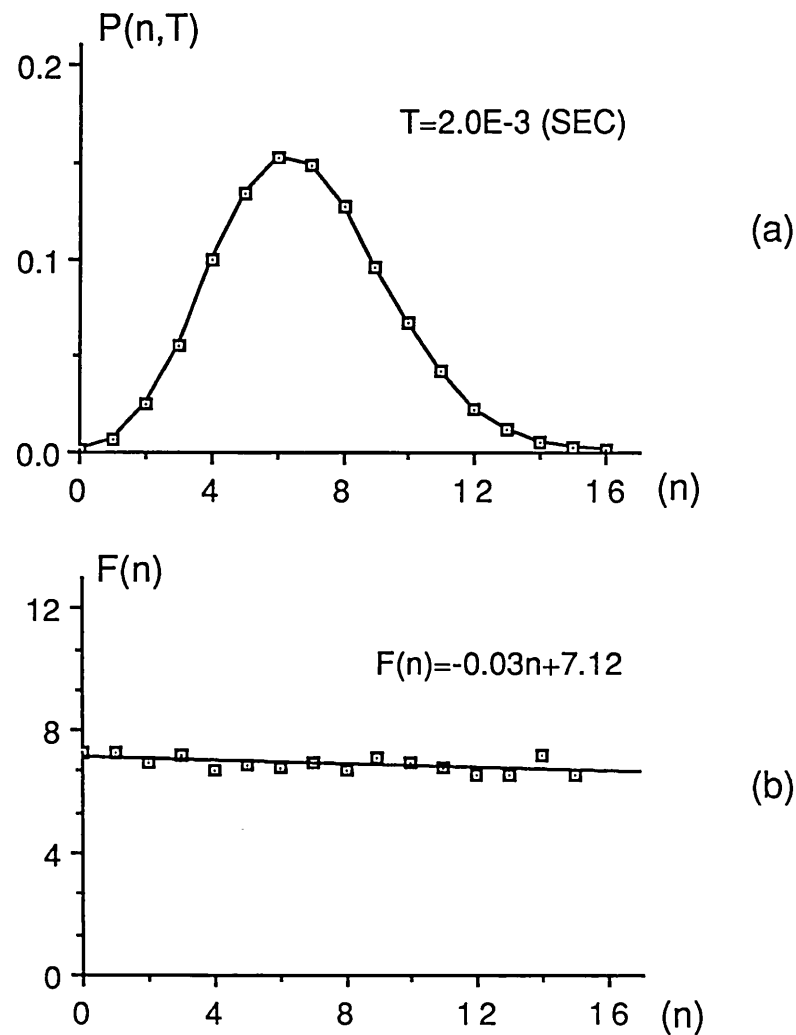


Fig.4.6. (a).  $P(n, T)$  measured with sample time  $T = 2.0 \times 10^{-3}(s)$  and average photon counts per sample time  $N = 6.8$ . (b).  $F(n)$  calculated from  $P(n, T)$  giving a best fit to the line  $F(n) = 0.03 n + 7.12$ , the implied dead time is  $\tau_d = 4.4 \mu s$ .

from an area of about  $2mm$  diameter circle on the photocathode. This spatial averaging effect, as discussed later, may severely degrade the contrast of measured correlation functions. In addition to the reason for cross talk mentioned above, the capacitance between the anodes also gives rise to cross talk among detecting channels. As Fig.4.7.(b) shows, the cross-correlation between two nearest anodes was measured at a very high sampling frequency ( $2 MHz$ ) when the incident light

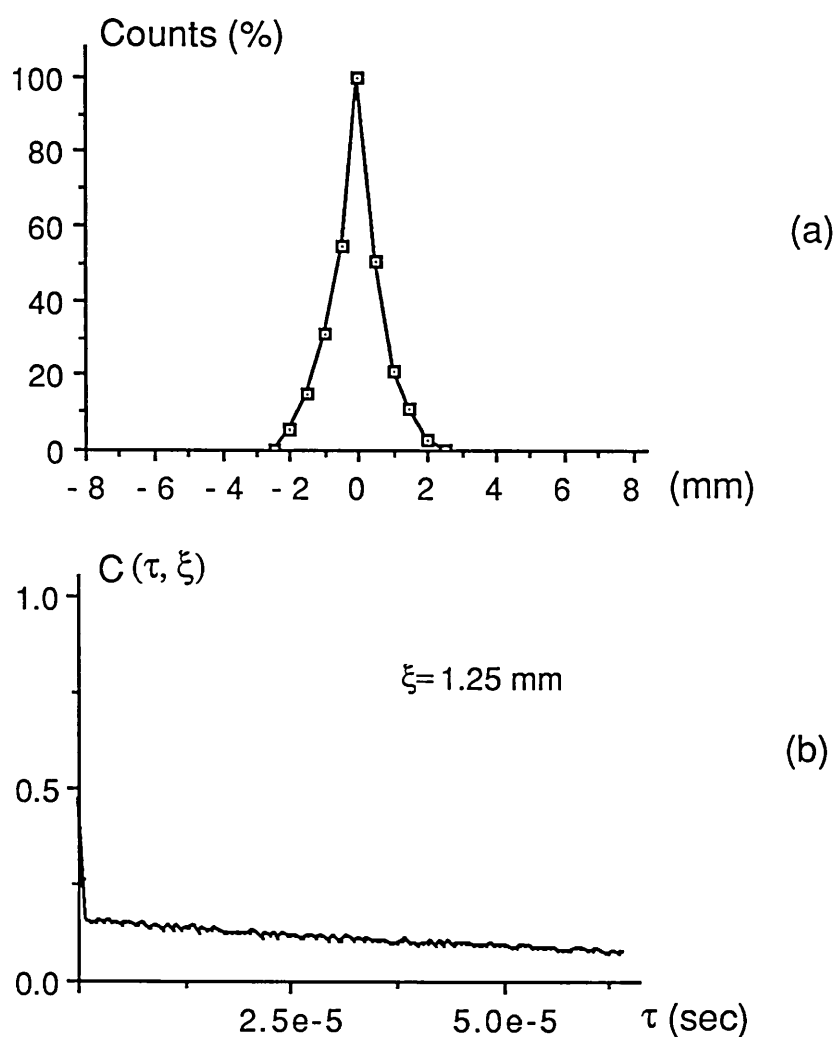


Fig.4.7. (a). Results of scanning a  $0.2\text{ mm}$  diameter pinhole image across photocathode; (b). Cross-correlation measured with a Poisson incident light beam showing a capacitive cross-talk.

was spatially uniform and had Poisson statistics. In this case, the light field did not introduce cross-correlation between two channels and the correlation value was caused by the correlation between the original signal from one anode and capacitance-induced signal from the other anode. There appears a sharp peak at very high frequency and a rather long tail following the peak. This peak covers the first two channels of delay time, this is corresponding to a frequency of  $1\text{ MHz}$ . Therefore it is clear that the capacitance between the anodes is quite large and

the capacitance-induced cross talk becomes dominant at very high frequency, i.e., above 1 *MHz*.

#### §4-2-2: Performance of the MCP-PMT for Photon Correlation

This MCP-PMT was used to measure the correlation function of the time-varying images. Let us recall the following formula stated in Chapter 3:

$$\frac{\langle n(t, T)n(t + \tau, T) \rangle}{\langle n \rangle^2} = \frac{\langle I(t, T)I(t + \tau, T) \rangle}{\langle I \rangle^2} \quad \tau \neq 0 \quad (4.3)$$

where  $n(t, T)$  is the photon counts at time  $t$  over a period of  $T$ ,  $I(t, T)$  is the intensity at time  $t$ , and  $\tau$  is the time delay. The angle brackets represent an ensemble average which for a stationary ergodic random process can be replaced by a time-average. The formula has a very important meaning: an unbiased estimate of the intensity correlation function can be obtained by means of correlating photon counts. In our case, the MCP-PMT is used to count photon events and these counts are then correlated by a digital correlator (Langley-Ford DC128). By making full use of an array of anodes, both autocorrelation and space-time correlation can be measured. The experimental setup is as in Fig.4.8.; the correlation function of the speckle pattern produced by a rotating disc is measured in the far field by the MCP-PMT system.

The results are shown in Fig.4.9. As mentioned earlier, the contrast was severely degraded by the finite detecting area. Fig.4.9.(b) shows that the contrast has been enhanced to unity by mounting a pinhole over one element as opposed to that in Fig.4.9.(a) without a pinhole. Figures 4.9.(c) and 4.9.(d) show the cross-correlation in  $x$  and  $y$  direction. The two-dimensional cross-correlation can be made quite easily by the tube. The speckle patterns produced in our experimental setup are expected to be Gaussian. The temporal and spatial correlation functions are Gaussian functions and the correlation measurements shown in the Fig.4.9 were fitted with Gaussian functions. This indicates that the temporal properties can be accurately measured by this MCP-PMT despite the fact that the speckle contrast is degraded in the measurements.

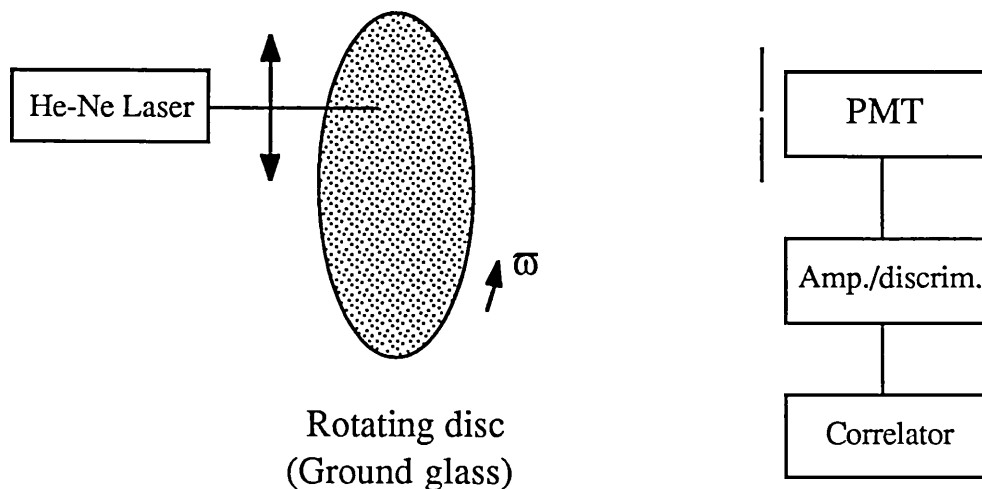


Fig.4.8. Experimental setup for measuring correlation function.

### §4–3: Concluding Remarks

The MCP–PMT has a good pulse height distribution and low dark count if it is operated optimally. It is a suitable candidate for detecting extremely low light intensity such as in analyzing single photoelectron time behaviour where the *global* dead time is a key parameter. The MCP–PMT has extremely small *global* dead time when it is used to detect very low level intensity. This is due to the fact that a huge number of micro–channels are built within a small area and a series of single photons can be detected and multiplied by *different* micro–channels within a very small time spread. The main drawbacks of this tube are due to the fundamental characteristics of MCP, such as long local dead time and low detection efficiency. Together with the long local dead time, both afterpulsing and cross talk limit the tube for measurement of frequencies up to  $1\text{ MHz}$ . On the other hand, the finite detection area can severely degrade the contrast of photon correlation function. Because of these drawbacks, we did not use the tube in our

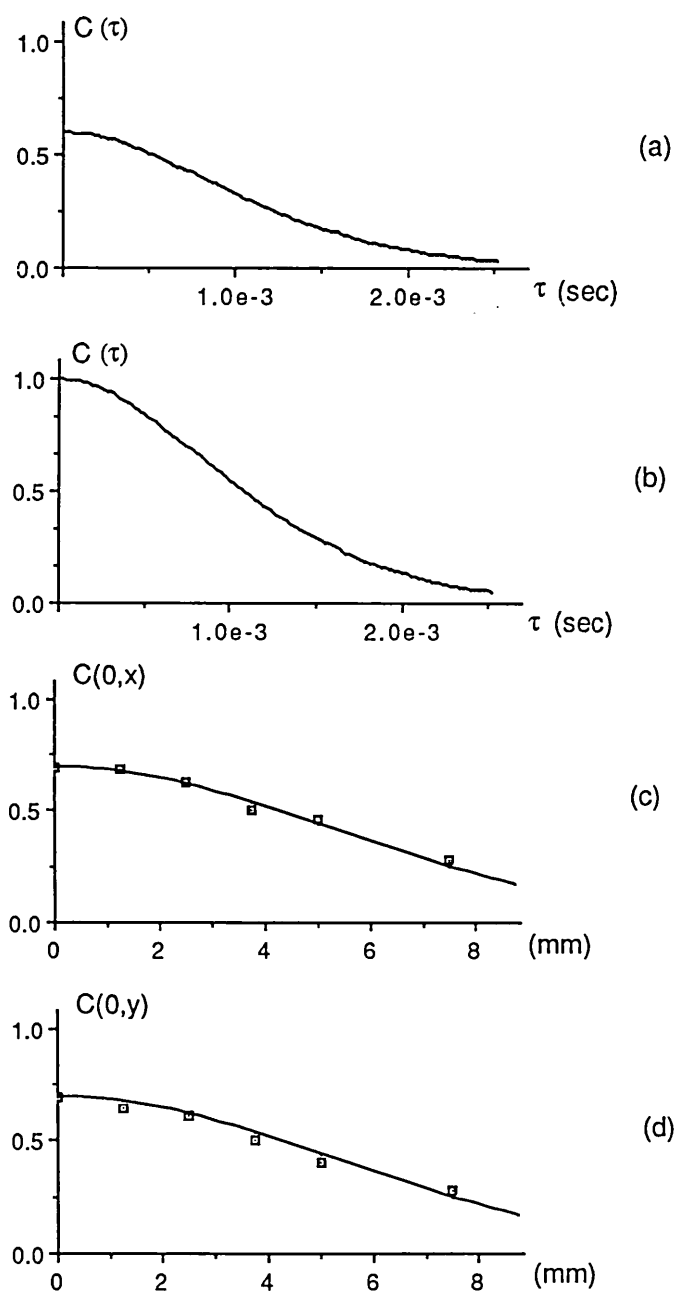


Fig.4.9. (a). Temporal correlation function measured using the MCP-PMT (spatial averaging has degraded the contrast  $C(0)$ ). (b). Temporal correlation determined by placing a small pinhole over the photocathode (the contrast  $C(0)$  increases to unity). (c). The spatial correlation in the  $x$ -direction. (d). The spatial correlation in the  $y$ -direction.

later observing measurements. If some drawbacks, particularly the cross-talk, can be overcome, it would be possible to use it in actual spatial-temporal correlation measurements.

Since <sup>the</sup> MCP-PMT has spatial imaging characteristics, the spatial-temporal correlation function and multi-channel correlation function can be obtained with a single tube. The MCP-PMT can possibly find applications in the following areas:

(1). Correlation measurement: with conventional PMTs, autocorrelation, cross-correlation, or multi-channel correlation measurements can be accomplished by using a single, twin or a tube array respectively. These measurements can be made by using only one MCP-PMT with a multi-anode array. The MCP-PMT photon correlation system may be particularly useful whenever a spatial information is needed. For example, in character recognition, it is necessary to measure the autocorrelations of every point in a multi-point array, in this case, when an MCP-PMT with sufficient anodes is used, possibly all the autocorrelations of the points can be obtained simultaneously. This will provide convenience for post-processing in which <sup>the</sup> autocorrelation process can be performed digitally.

(2). Space-time correlation: A two-dimensional photon counter is preferred to get a list of both coordinates of successively coming photons, particularly at low light level. The MCP-PMT photon detection system could be a suitable candidate for this purpose.

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## Chapter 5

### Statistical Properties of Images Through Atmospheric Turbulence

#### §5-1: Introduction

Stellar imaging techniques have been studied with considerable effort in order to determine the spatial structure of astronomical objects. With a single telescope, the possible angular resolving power should reach up to the diffraction limit, i.e.  $\lambda/D$  where  $\lambda$  is the wavelength and  $D$  is the diameter of the telescope. An ideal star image at the focal plane of a telescope is the diffraction pattern of the pupil of the telescope; that is, the light received by the telescope from the star must be a plane wave. However, in practice this does not happen even under the best observing conditions. Due to the presence of atmospheric turbulence the spatial resolution of images observed is severely degraded. The wave incident on the telescope is distorted with irregular sizes and random phases and the wave front changes very rapidly with time. The star image, therefore, has the appearance of a fuzzy spot that changes constantly and whose structure does not resemble the telescope diffraction pattern. The image is actually a mixture of  $\hat{a}$  star diffraction pattern and  $\hat{a}$  speckle pattern caused by atmospheric turbulence. For example the resolving power is degraded by a factor of about 30 in the visible for a 4-meter telescope. A review by Roddier (1981) gives a description of the effects of atmospheric turbulence in optical astronomy.

To overcome the atmospheric turbulence effects on stellar images, many techniques have been invented in order to enhance the spatial resolution of a large aperture telescope up to the diffraction limit. These include deconvolution techniques, adaptive optical systems and speckle interferometry. In this chapter, we shall briefly explain these techniques while putting our emphasis on the image temporal fluctuation properties, which are of particular use for speckle interferometry and high resolution image reconstruction.

## §5-2: Optical Atmospheric Turbulence Effects

### §5-2-1: Atmosphere Structure

In order to learn about the atmospheric turbulence effects in stellar optical imaging, let us now investigate the temperature structure of the atmosphere. In Fig.5.1. an atmosphere temperature structure is shown. The behaviour of the whole atmosphere is governed by the Sun. The temperatures of each part of the atmosphere are dependent upon their abilities to absorb the radiation energy. In the atmosphere up to 200 km from the sea level, there are four main divisions. The part of the atmosphere from the earth surface to  $7 \sim 17$  km, depending upon the latitude and season, is called troposphere; above troposphere up to about 50 km is stratosphere; from 50 km to about 75 km is mesosphere and above it is thermosphere. Right above troposphere there is a natural “lid” called tropopause. The atmosphere above this “lid” is also called upper atmosphere and below the “lid” lower atmosphere.

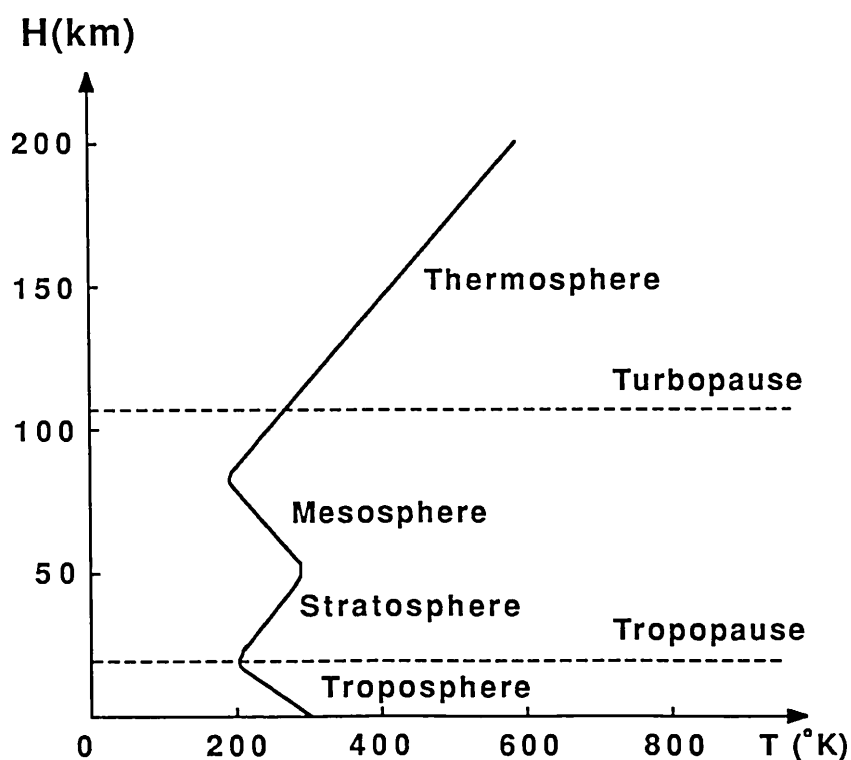


Fig.5.1. Temperature structure of atmosphere.

In the upper atmosphere, the temperature structure is completely determined by each region's ability to absorb the incident solar radiation. They can not absorb the re-radiated energy by the earth due to the long distances from the earth while in the lower atmosphere, the temperature is mainly determined by its ability to absorb the re-radiated infrared part of the spectrum by the earth. This infrared energy is absorbed by water vapour and carbon dioxide. Since there is a gradual decrease in the amount of water vapour and carbon dioxide with the height, the temperature in the lower atmosphere decreases with the altitude until it reaches the local minimum at the tropopause which acts as a natural "lid".

The lower atmosphere temperature and humidity fluctuations are subject to the influence of both solar radiation and the earth's re-radiation. These fluctuations in turn affect the fluctuations of optical refractive index. In the past years, people have realized that it is the lower atmosphere (troposphere) which has the most significant turbulent effects on optical wave propagations. In other words, the air flows in the low atmosphere in front of the telescope pupil have dominant turbulent effects.

The criterion for a flow to become turbulent is that its Reynolds number exceeds a certain critical value. The Reynolds number is defined as:

$$R_e = \frac{VL}{\nu} \quad (5.1)$$

where  $V$  is the characteristic velocity,  $L$  the characteristic length and  $\nu$  kinematic viscosity of the flow. The Reynolds number can be physically understood as the ratio of the power passing over from the initial flow to the fluctuation motion to the power dissipated as heat of the fluid. Therefore when the Reynolds number is greater than a critical value, i.e. the power passed onto the fluctuation motion is larger than the power consumed into heat, the flow will become unstable and turbulent.

Let us follow Roddier's (1981) analysis of the atmospheric turbulence. In a spectral analysis of the energy of the turbulence at scale  $L$ , for  $L_o^{-1} \ll k \ll l_o^{-1}$ , where  $L_o$  is the outer scale of the turbulence and  $l_o$  the inner scale where

viscous dissipation of the energy into heat occurs, the power spectrum of certain parameter is given by: and  $K$  is the modulus of the wave vector

$$E(k) \propto k^{-\frac{5}{3}} \quad (5.2)$$

where  $k$  is the wave number. This is called the Kolmogorov law [Tatarski (1961)] which describes the energy spectrum of a particular parameter due to the motion of the turbulence and/or fluctuations of the parameter. For example, the one dimensional power spectra of temperature and humidity fluctuations are given respectively by:

$$E_T^{(1)}(k) \propto k^{-\frac{5}{3}}, \quad E_H^{(1)}(k) \propto k^{-\frac{5}{3}} \quad (5.3)$$

In the isotropic case, the three dimensional temperature spectrum is given by integrating  $E_T^{(1)}(k)$  over all directions:

$$\begin{aligned} E_T(k) &= 4\pi k^2 E(k) \\ &= 0.033 C_T^2 k^{-\frac{11}{3}} \end{aligned} \quad (5.4)$$

where  $C_T^2$  is the structure constant of temperature fluctuations.

Due to the random fluctuations of temperature and humidity, the refractive index is also randomly fluctuating. The power spectrum of the refractive index fluctuations is similarly given by:

$$E_N(k) = 0.033 C_N^2 k^{-\frac{11}{3}} \quad (5.5)$$

where  $C_N^2$  is the index structure constant. The parameter  $C_N^2$  is a description of the contribution of the turbulence to optical propagation. The larger the  $C_N^2$  is, the larger the atmospheric turbulence degrades the stellar images. This will be clearly seen later from the relationship between  $C_N^2$  and Fried parameter  $r_o$ . Experimental results have shown [Roddier (1981)] that  $C_N^2$  decreases with the height. By experience it is often assumed that  $C_N^2$  decreases exponentially with the altitude. This is also the fact that the lower turbulence has larger degradation effects on stellar images. The random fluctuations of the refractive index cause the random distortion of the propagating wavefront. For a partially coherent

wave the distorted wavefront at the pupil plane of the telescope will give rise to a randomly fluctuating speckle pattern at the image plane. The detected intensity is therefore temporally and spatially fluctuating.

### §5-2-2: Long Exposure Images

In conventional astronomical imaging, the exposure time is of the order of a few minutes or even hours in order to gather enough light. The images taken are therefore patterns averaged over time rather than records of a random process. The properties of long exposure images have been investigated and reported in several papers [Hufnagel *et al* (1964), Fried (1965), Fried (1966)]. With a long exposure time, assuming the entire imaging system is linear and shift invariant, the following expression holds:

$$\langle I(x', y') \rangle = O(x', y') \otimes \langle P(x', y') \rangle \quad (5.6)$$

where  $I(x', y')$  is the observed intensity of the image,  $O(x', y')$  is the intensity of the object and  $P(x', y')$  is the instantaneous point spread function of the whole system, including both atmosphere and observing devices. Angular brackets denote the ensemble or time average—the two are equivalent in this case.

In Fourier space, Eq.(5.6) becomes:

$$\langle \tilde{I}(u, v) \rangle = \tilde{O}(u, v) \times \langle T(u, v) \rangle \quad (5.7)$$

where  $\langle T(u, v) \rangle$  is the time-averaged optical transfer function of the whole system. The quality of such images is basically decided by this transfer function.

Let us now analyse the long exposure transfer function. We shall assume here that the incident light is from a monochromatic point source. By diffraction theory, the complex amplitude at the focal plane of the telescope is given by the Fourier transform of the transmitted complex amplitude at the pupil of the telescope. By denoting the transmission function as  $p(x, y)$ , and the incident complex field as  $\psi(x, y)$ , the complex amplitude at the focal plane is given by:

$$A_f(x', y') \propto \mathcal{F}[\psi(x, y)p(x, y)] \quad (5.8)$$

where  $\mathcal{F}$  denotes the Fourier transform. The intensity point spread function is then given by:

$$S(x', y') = |A_f(x', y')|^2 \propto |\mathcal{F}[\psi(x, y)p(x, y)]|^2 \quad (5.9)$$

Since the optical transfer function is equal to the Fourier transform of the point spread function of the whole imaging system, the optical transfer function can be expressed as follows:

$$T(u, v) = \mathcal{F}[S(x', y')] \propto \mathcal{F}\left\{|\mathcal{F}[\psi(x, y)p(x, y)]|^2\right\} \quad (5.10)$$

According to the Wiener-Khintchine theorem the right hand side of the above equation is equal to the autocorrelation of  $\psi(x, y)p(x, y)$ , namely:

$$T(u, v) \propto \int \psi(x, y) \psi^*(x + u, y + v) p(x, y) p^*(x + u, y + v) dx dy \quad (5.11)$$

In the absence of atmospheric turbulence,  $\psi(x, y) = 1$ :

$$\begin{aligned} T(u, v) &\propto \int p(x, y) p^*(x + u, y + v) dx dy \\ &= \frac{1}{R} \int p(x, y) p^*(x + u, y + v) dx dy \\ &= T_o(u, v) \end{aligned} \quad (5.12)$$

where  $R$  is the area of the telescope pupil and  $T_o(u, v)$  is the optical transfer function due to the telescope alone. From Eq.(5.11) the long exposure optical transfer function is:

$$\langle T(u, v) \rangle = \frac{1}{R} \int \langle \psi(x, y) \psi^*(x + u, y + v) \rangle p(x, y) p^*(x + u, y + v) dx dy \quad (5.13)$$

In this expression if we put:

$$B(u, v) = \langle \psi(x, y) \psi^*(x + u, y + v) \rangle \quad (5.14)$$

which is the second order correlation of the complex random field and is also called coherence function. It is only dependent on  $u$  and  $v$ . Then we have:

$$\langle T(u, v) \rangle = B(u, v) T_o(u, v) \quad (5.15)$$

Eq.(5.15) shows that the optical transfer function of the whole system is the product of the transfer function of the telescope and the coherence function of the atmospherically disturbed complex field. The quality of long exposure stellar images are limited by this coherence function.

In a thin layer approximation, it is assumed that the thickness of the turbulent layer is small enough so that the diffraction effects are negligible. But the thickness should be larger than the correlation scale of the inhomogeneities. When a plane wave passing through a thin layer of altitude  $h$  and thickness  $\delta h$ , for an isoplanatic system in which case only one variable  $\xi$  determines coherence function, i.e.  $B(u, v) = B(\xi)$ , it has been shown [Roddier (1981)] that the coherence function of the complex random field  $\psi$  at the layer output is given by:

$$\begin{aligned} B_h(\xi) &= \langle \psi_h(x) \psi_h^*(x + \xi) \rangle \\ &= \exp\left[-\frac{1}{2}(2.91k^2 C_N^2(h) \delta h \xi^{\frac{5}{3}})\right] \end{aligned} \quad (5.16)$$

where  $k$  is the wave number and  $C_N^2(h)$  is the index structure constant and is height dependent.

Let us now assume that the whole atmospheric layer is made of several thin layers; for this case, the coherence function of the resultant output field at the ground is given by the product of the contributions from each turbulent layer:

$$\begin{aligned} B(\xi) &= \prod_i \exp\left[-\frac{1}{2}(2.91k^2 C_N^2(h_i) \delta h_i \xi^{\frac{5}{3}})\right] \\ &= \exp\left[-\frac{1}{2}(2.91k^2 \xi^{\frac{5}{3}}) \sum_i C_N^2(h_i) \delta h_i\right] \end{aligned} \quad (5.17)$$

If the turbulence is continuous the sum sign will become a integral extending throughout the atmosphere. For observing at an angle  $\gamma$  away from the zenith, to a good approximation the thickness  $\delta h$  of each layer is multiplied by  $(\cos \gamma)^{-1}$ , and the coherence function at the pupil plane of a ground-based telescope is therefore given by:

$$B(\xi) = \exp\left[-\frac{1}{2}(2.91k^2 (\cos \gamma)^{-1} \xi^{\frac{5}{3}}) \int C_N^2(h) dh\right] \quad (5.18)$$

Eq.(5.18) is of fundamental importance due to the fact that the qualities of long exposure images are completely determined by the coherence function  $B(\xi)$ .

In fact, the  $\underbrace{\text{resolving}}_{\text{angular}}$  power  $\mathcal{R}$  of the whole imaging system is described by the parameter:

$$\mathcal{R} = \int B(\xi) T_o(\xi) d\xi \quad (5.19)$$

For a very small diffraction-limited telescope of diameter  $D$ , turbulence effects are negligible and the  $\underbrace{\text{resolving}}_{\text{angular}}$  power is approximately given only by that of the telescope:

$$\mathcal{R} = \int T_o(\xi) d\xi = \frac{\pi D^2}{4\lambda^2} \quad (5.20)$$

For a very large telescope, the turbulence effects are dominating and the resolving power is approximately given by integrating Eq.(5.18):

$$\mathcal{R} = \int B(\xi) d\xi = \frac{6\pi}{5} A^{-\frac{6}{5}} \Gamma(6/5) \quad (5.21)$$

$$A = 1.45k^2 (\cos \gamma)^{-1} \int C_N^2(h) dh \quad (5.22)$$

Fried (1966) introduced a convenient parameter to describe the site observing condition. This is called Fried parameter and it is the critical diameter  $r_o$  of the telescope for which:

$$\int B(\xi) d\xi = \int T_o(\xi) d\xi \quad (5.23)$$

From the above definition of  $r_o$ , we can see that the Fried parameter is equal to the diameter of the diffraction-limited telescope which, in the absence of atmospheric turbulence, gives the same average image diameter as that observed with the turbulence in a large telescope. In other words,  $r_o$  is the average largest undisturbed aperture of the telescope pupil.

By definition in Eq.(5.23) putting  $D = r_o$  in Eq.(5.20) and equating it with Eq.(5.21), we have:

$$A = 3.44 \left( \frac{\lambda}{r_o} \right)^{\frac{5}{3}} \quad (5.24)$$

Hence by comparing Eq.(5.18), Eq.(5.22) and Eq.(5.24) the expression for  $B(\xi)$  can be written as:

$$B(\xi) = \exp \left[ -3.44 \left( \frac{\lambda \xi}{r_o} \right)^{\frac{5}{3}} \right] \quad (5.25)$$

From the above expression it is obvious that the larger the  $r_o$  is, the better the seeing condition is. The resolving power is limited by the telescope if the diameter of the telescope  $D$  is smaller than  $r_o$ . The limitation imposed by the atmospheric turbulence will play a major role when  $D$  is larger than  $r_o$ . On the other hand, for sampling apertures smaller than  $r_o$  which is the usual case in the classical stellar interferometry technique, the measuring results will not be affected much by the atmospheric turbulence. In the case of Michelson interferometry the resolution of the fringe pattern will not be disturbed by the atmospheric turbulence although the whole pattern is shifting randomly due to the phase differences at the two sub-apertures. Hence, with a long baseline, classical interferometry techniques often give turbulence-free results. However, these techniques only make use of a small portion of collected light and therefore are limited to measuring bright stars.

By equating Eq.(5.22) and Eq.(5.24),  $r_o$  can be expressed as:

$$r_o = \left[ 0.423 k^2 (\cos \gamma)^{-1} \int C_N^2(h) dh \right]^{-\frac{3}{5}} \quad (5.26)$$

From this equation we can see that  $r_o$  depends on the zenith angle and wavelength. The wavelength dependence is given by:

$$r_o \propto (\lambda^{-2})^{-\frac{3}{5}} = \lambda^{\frac{6}{5}} \quad (5.27)$$

It can be seen that a longer wavelength gives a larger  $r_o$ . In other words, the seeing condition improves for longer wavelengths. Intuitively, this can be understood in terms of the speckle phenomenon—a longer wavelength results in a larger speckle size which affects the undisturbed aperture and hence  $r_o$ . This is part of the reason that infrared observing has a better seeing condition than that in the visible although, of course this is at the expense of lower diffraction-limited resolution.

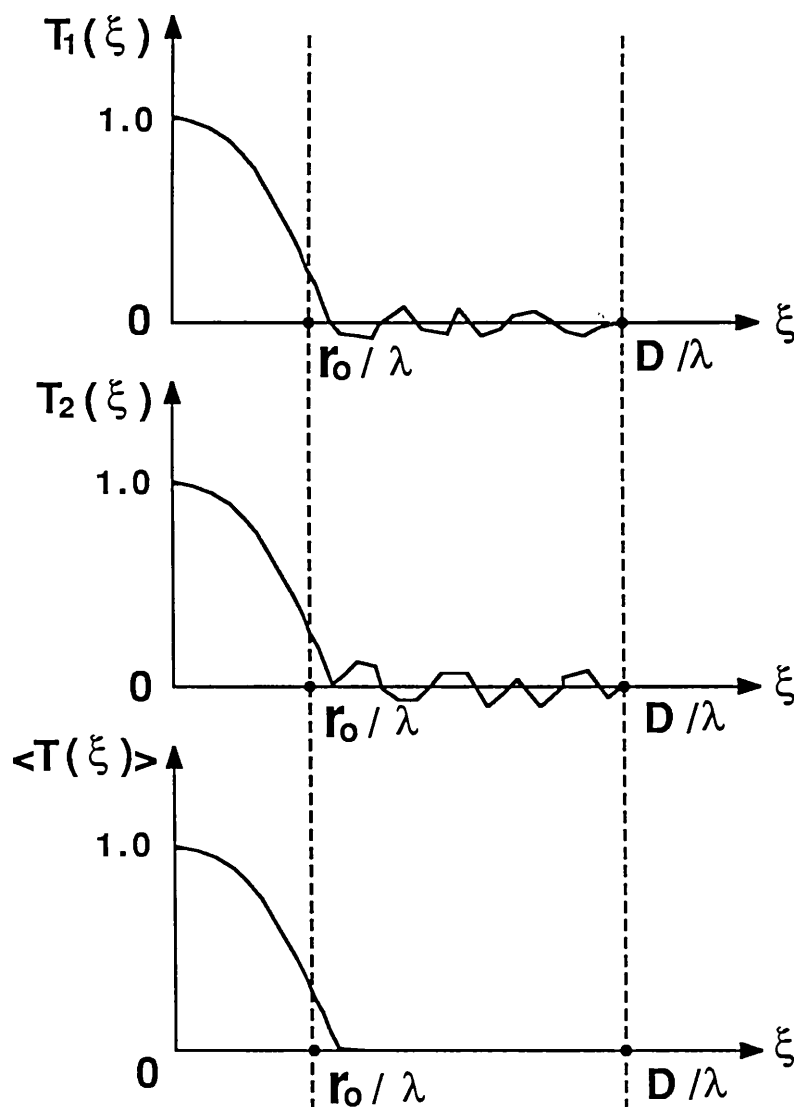


Fig.5.2. Schematic diagram showing how a long exposure optical transfer function  $\langle T(\xi) \rangle$  is added and recorded.  $T_1(\xi)$  and  $T_2(\xi)$  are two short exposure transfer functions;  $\langle T(\xi) \rangle$  is the average of many short exposure transfer functions.

Many experiments have shown that  $r_o$  is on the order of 10 cm at astronomical observing sites and from Eq.(5.25) one can see that  $B(\xi)$  tends to zero when the spatial frequencies are larger than  $r_o/\lambda$ . For a telescope of a diameter larger than  $r_o$ , from Eq.(5.15) it is found that  $\langle T(u, v) \rangle$  tends to zero at a spatial frequency well before it reaches the diffraction-limited spatial frequency.  $\langle T(u, v) \rangle$  can

be regarded as the sum of <sup>the</sup> transfer functions of many short exposure images. As can be seen in Fig.5.2, these transfer functions have values ranging from positive to negative at high frequencies. The ensemble average of these transfer functions will have value of zero at high frequencies. The high frequency information of the object is therefore lost in a long exposure picture and resolution of the image is seriously degraded.

There are two ways to measure the long exposure atmospheric transfer function  $B(\xi)$ . The first one is to measure the image profile of a point source;  $B(\xi)$  is then obtained by performing a two-dimensional Fourier transform of the image profile. For a rotationally symmetric system, this transform reduces to a one-dimensional Hankel transform. The accuracy of this method is easily affected by the telescope aberrations and focusing errors. To overcome this problem, in practice, the second method is often used in which  $B(\xi)$  is measured interferometrically. For a pair of very small sub-apertures with separation  $\lambda \xi_o$  sitting on the telescope pupil plane, the pupil transmission function is as:

$$P(u) = p(u) + p(u + \xi_o) \quad (5.28)$$

The optical transfer function of the telescope is then given by:

$$\begin{aligned} T_o(\xi) &= \int P(u) P^*(u + \xi) du \\ &= \delta(\xi) + \frac{1}{2}\delta(\xi + \xi_o) + \frac{1}{2}\delta(\xi - \xi_o) \end{aligned} \quad (5.29)$$

Hence the transform of the intensity of the fringe pattern due to the pair of sub-apertures is as follows:

$$\begin{aligned} \tilde{I}(\xi) &= \tilde{O}(\xi) T_o(\xi) B(\xi) \\ &= \tilde{O}(0) + \frac{1}{2}\tilde{O}(-\xi_o) B(-\xi_o) + \frac{1}{2}\tilde{O}(\xi_o) B(\xi_o) \\ &= \tilde{O}(0) + |\tilde{O}(\xi_o)| B(\xi_o) \cos(\alpha) \end{aligned} \quad (5.30)$$

where  $\alpha$  is the phase of product  $\tilde{O}(\xi_o) B(\xi_o)$ . We have assumed  $B(\xi_o)$  is real. From Eq.(5.30) it is found that the visibility of the fringe pattern is:

$$V = \frac{|\tilde{O}(\xi_o)| B(\xi_o)}{\tilde{O}(0)} \quad (5.31)$$

If we are observing a point source,  $|\tilde{O}(\xi_o)| = \tilde{O}(0) = 1$ , hence:

$$V = B(\xi_o) \quad (5.32)$$

This is a very important result. By observing a point source the atmospheric transfer function of a certain frequency is equal to the visibility of the fringe pattern produced a pair of very small sub-apertures at the pupil, their separation decides the measured spatial frequency.

### §5-2-3: Short Exposure Images

Short exposure imaging is a way to overcome this degradation and it is the principle of speckle interferometry. The basic idea was first realized by Labeyrie (1970). Since the astronomical images undergo random changes related to the motion of atmospheric inhomogeneities in front of the telescope, a short exposure image is in fact a speckle pattern. Speckle is inherently an interference phenomenon. When a coherent wave passes through a turbulent medium, or is reflected from a rough object, the output field is given some randomness in phase. Therefore the output sub-fields will interfere with each other and the resultant intensity distribution is a granular pattern called a speckle pattern. There have been a number of papers which describe the speckle pattern from different viewpoints, e.g. with statistical theory [Dainty (1976), Ohtsubo *et al* (1976), Dainty (1980)] and diffraction theory [George (1980)].

Stellar speckle patterns are time-varying. When the exposure time is very short the image can be frozen and the instantaneous speckle-affected images can be recorded. These images contain more information on smaller features than the long exposure images with a blurred speckle. Since the size of each speckle is of the order of the diffraction limit of the telescope, it is possible to extract high resolution information from such speckle patterns, while in conventional long exposure pictures the detailed speckle patterns have been averaged out. By this principle, Labeyrie invented a method in which a fairly large number of short exposure photographs of the star image are taken. These photographs are pro-

cessed by averaging the squared modulus of the Fourier transform of a series of pictures. By doing so, the atmospheric noise is suppressed and the modulus of the Fourier transform of the object can then be obtained up to the diffraction limit of the telescope. In image space, this type of operation corresponds to measuring the autocorrelation of the object. All these can be shown by the following mathematical analysis:

For each set of short exposure data:

$$I(\mathbf{s}) = O(\mathbf{s}) \otimes P(\mathbf{s}) \quad (5.33)$$

where  $I(\mathbf{s})$  and  $O(\mathbf{s})$  are the instantaneous image and object intensities respectively, and  $P(\mathbf{s})$  is the instantaneous point spread function of the whole observing system including atmosphere and detecting devices. According to diffraction theory,  $P(\mathbf{s})$  represents the speckle pattern initiated by a point source. Here  $\otimes$  denotes convolution and we have assumed that the observing system is isoplanatic within the imaging area.

The Fourier transform of Eq.(5.33) yields:

$$\tilde{I}(\mathbf{f}) = \tilde{O}(\mathbf{f}) \times T(\mathbf{f}) \quad (5.34)$$

where the tilde denotes Fourier transform. Instead of taking the average of the transform functions in Eq.(5.34) as done for long exposure imaging, we take the average of the squared modulus of Eq.(5.34) and since the object remains unchanged during the observing time, we have:

$$\langle |\tilde{I}(\mathbf{f})|^2 \rangle = |\tilde{O}(\mathbf{f})|^2 \times \langle |T(\mathbf{f})|^2 \rangle \quad (5.35)$$

Let us recall Eq.(5.11) for transfer function and rewrite it as:

$$T(\mathbf{f}) \propto \int \psi(\mathbf{s}) \psi^*(\mathbf{s} + \mathbf{f}) p(\mathbf{s}) p^*(\mathbf{s} + \mathbf{f}) d\mathbf{s} \quad (5.36)$$

where we have let the vector  $\mathbf{f}$  represent components  $u$  and  $v$  and the vector  $\mathbf{s}$  for components  $x$  and  $y$  in Eq.(5.11). The average squared modulus of  $T(\mathbf{f})$  is then

given by:

$$\langle |T(\mathbf{f})|^2 \rangle = C \int M(\mathbf{f}, \mathbf{f}') T_o(\mathbf{f}, \mathbf{f}') d\mathbf{f}' \quad (5.37)$$

$\langle |T(\mathbf{f})|^2 \rangle$  is also called speckle transfer function.  $C$  is a constant,  $M(\mathbf{f}, \mathbf{f}')$  and  $T_o(\mathbf{f}, \mathbf{f}')$  are the fourth order moments of atmospherically disturbed complex amplitude at the pupil plane of the telescope and the telescope pupil function respectively:

$$M(\mathbf{f}, \mathbf{f}') = \langle \psi(\mathbf{s}) \psi^*(\mathbf{s} + \mathbf{f}) \psi^*(\mathbf{s} + \mathbf{f}') \psi(\mathbf{s} + \mathbf{f} + \mathbf{f}') \rangle \quad (5.38)$$

$$T_o(\mathbf{f}, \mathbf{f}') = \int p(\mathbf{s}) p^*(\mathbf{s} + \mathbf{f}) p^*(\mathbf{s} + \mathbf{f}') p(\mathbf{s} + \mathbf{f} + \mathbf{f}') d\mathbf{s} \quad (5.39)$$

$T_o(\mathbf{f}, \mathbf{f}')$  describes the contribution from the telescope alone. Since  $M(\mathbf{f}, \mathbf{f}')$  is dependent on  $\mathbf{f}'$  so is  $T_o(\mathbf{f}, \mathbf{f}')$ , hence from Eq.(5.37) it can be seen that  $\langle |T(\mathbf{f})|^2 \rangle$  cannot be expressed as the product of a telescope function with an atmospheric function in contrast to the long exposure transfer function  $\langle T(\mathbf{f}) \rangle$  in Eq.(5.15).

When  $\psi(\mathbf{s})$  is assumed to follow circular Gaussian statistics, the fourth order moment is given by [Dainty (1975)]:

$$M(\mathbf{f}, \mathbf{f}') = B^2(\mathbf{f}) + B^2(\mathbf{f}') \quad (5.40)$$

where  $B(\mathbf{f}) = \langle \psi(\mathbf{s}) \psi^*(\mathbf{s} + \mathbf{f}) \rangle$ . Put Eq.(5.40) into Eq.(5.37), it becomes:

$$\langle |T(\mathbf{f})|^2 \rangle = B^2(\mathbf{f}) T_o^2(\mathbf{f}) + \int B^2(\mathbf{f}') T_o(\mathbf{f}, \mathbf{f}') d\mathbf{f}' \quad (5.41)$$

After the manipulation of Eq.(5.41), the speckle transfer function can be expressed as:

$$\langle |T(\mathbf{f})|^2 \rangle = \langle T(\mathbf{f}) \rangle^2 + \frac{1}{N_s} T_o(\mathbf{f}) \quad (5.42)$$

where  $\langle T(\mathbf{f}) \rangle$  is the long exposure transfer function,  $N_s = 2.3(D/r_o)^2$  is the average number of speckles in the image and  $T_o(\mathbf{f})$  is the diffraction-limited transfer function. The first term is the squared mean of long exposure transfer function and the second term is the diffraction-limited transfer function. Since the cut-off frequency for  $T(\mathbf{f})$  is  $r_o/\lambda$ ,  $\langle |T(\mathbf{f})|^2 \rangle$  can be represented by ~~the~~ long exposure transfer function at frequencies lower than  $r_o/\lambda$ ; at frequencies higher than  $r_o/\lambda$ ,

the second term shows that  $\langle |T(f)|^2 \rangle$  is attenuated by a factor of  $N_s$ . It has a long tail at high frequencies and this long tail goes up to <sup>the</sup> diffraction-limited cutoff frequency. In other words, the diffraction-limited power spectrum of the object can be extracted from the short exposure images by Eq.(5.35).

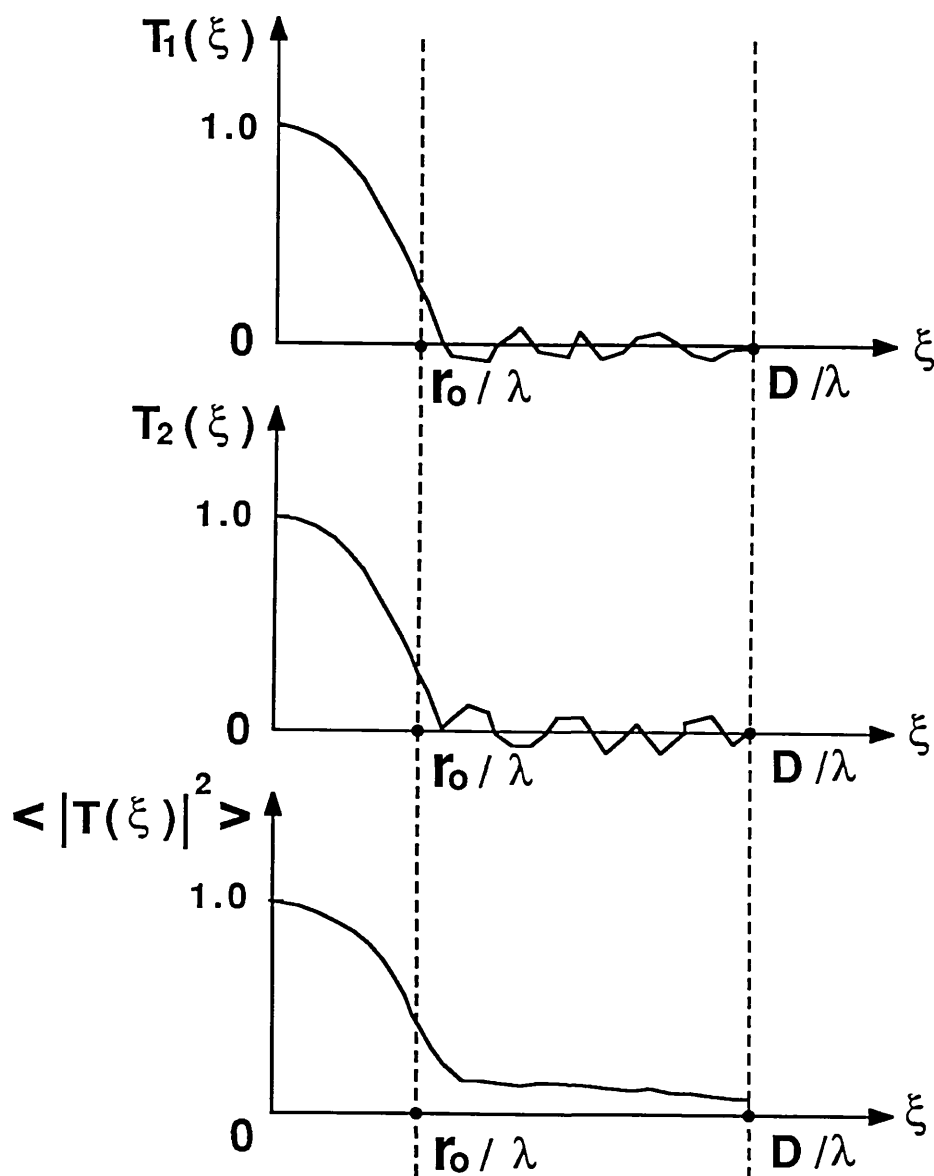


Fig.5.3.  $T_1(\xi)$  and  $T_2(\xi)$  are two short exposure transfer functions; speckle transfer function  $\langle |T(\xi)|^2 \rangle$  is the average of the squared modulus of many short exposure transfer functions.

Because the modulus of the short exposure transfer function is always positive, the speckle transfer function which is the sum of the power spectra of these short exposure images will have a positive value extending to the diffraction-limited cut-off frequency. Hence a diffraction-limited power spectrum of the object can be obtained. The modulus of the Fourier transform of the object  $|\tilde{O}(f)|$ , or equivalently the autocorrelation of the object can be reconstructed if  $\langle |\tilde{I}(f)|^2 \rangle$  and  $\langle |T(f)|^2 \rangle$  are known.  $\langle |\tilde{I}(f)|^2 \rangle$  can be readily measured, and  $\langle |T(f)|^2 \rangle$  is a well determined, simple function which is measurable for a given seeing conditions. Under stable atmospheric conditions,  $\langle |T(f)|^2 \rangle$  can be straight-forwardly measured from a point source. However, in most practical situations, the atmospheric conditions are not stable. Therefore it is practical to make simultaneous measurements of  $r_o$  and then use established theory [Rodier (1981)] to calculate  $\langle |T(f)|^2 \rangle$ , in which case the speckle transfer function is given by:

$$\langle |T(f)|^2 \rangle = |T_o(f)|^2 \exp \left\{ -6.88 \left( \frac{\lambda f}{r_o} \right)^{\frac{5}{3}} \left[ 1 - \left( \frac{\lambda f}{D} \right)^{\frac{1}{3}} \right] \right\} \quad (5.43)$$

The above formula was derived by separating the degradation effects due to the image motion from that due to the image blurring. The image blurring effects which cause the degradation in short exposure data remain in the formula.

The phase information of the Fourier transform of the object is lost in speckle interferometry. Only the diffraction-limited power spectrum or equivalently the autocorrelation of the object can be reconstructed. The real image cannot be uniquely reconstructed from its autocorrelation function except for centrosymmetric objects or in certain other special cases.

### §5-3: Image Reconstruction

Many efforts have been made to enhance the atmospherically degraded astronomical images. A few techniques have been invented. These include:

- (1). Simple deconvolution or inverse filtering, in which an enhancement filter – a high frequency boost – is used to compensate the long exposure MTF for

atmosphere which acts as a low pass filter.

(2). An adaptive optical system, in which a conjugate phase grating may be interposed at a suitable pupil of the telescope to cancel the phase variations of an incoming wavefront. [Hardy *et al* (1977), Buffington *et al* (1977), Grosso *et al* (1977)]

(3). Speckle interferometry with and without phase information.

A comparison of above three methods has been made by Richter (1977). We emphasise the third method.

As can be seen from the last section, to obtain diffraction-limited information, instead of studying the average intensities of images, one studies the power spectra or equivalently the autocorrelation of the image intensities. But the phase information is lost in the power spectra. To reconstruct the real image, both the modulus and phase of the Fourier transform are essential. So far several techniques have been developed to estimate both the modulus and the phase of the Fourier transform, to reconstruct real images from stellar speckle interferometry data. A review of this area has been given by Dainty(1975). In the following part of this section we shall outline three techniques.

### §5-3-1: Cross-Spectrum Technique

This technique is also called the Knox-Thompson technique [Knox *et al* (1974)]. It uses the same short-exposure pictures as in speckle interferometry to recover the phase of the object transform. In this technique, the phase information is extracted from the calculation of the autocorrelation of  $\tilde{I}(u)$ . Mathematically, from Eq.(5.34) this can be written as:

$$\langle \tilde{I}(u) \tilde{I}^*(u + \Delta u) \rangle = \tilde{O}^*(u) \tilde{O}(u + \Delta u) \langle T(u) T^*(u + \Delta u) \rangle \quad (5.44)$$

The shift  $\Delta u$  is made small compared with the correlation width of  $T(u)$  such that the autocorrelation of  $T(u)$  has a measurable value up to the diffraction

limit of the telescope. Eq.(5.44) contains the phase information and this phase information can be expressed as:

$$[\theta(u) - \theta(u + \Delta u)] = [\theta_o(u + \Delta u) - \theta_o(u)] + [\theta_T(u) - \theta_T(u + \Delta u)] \quad (5.45)$$

It has been shown [Knox (1976)] that the phase of the autocorrelation of  $T(u)$  is negligible and has little effect on the image reconstruction when  $\Delta u \ll (r_o/\lambda)$  and hence the last term in Eq.(5.45) can be neglected. The phase difference of the object transform can be well approximated by the measure of the phase difference of the  $\tilde{I}(u)$ .

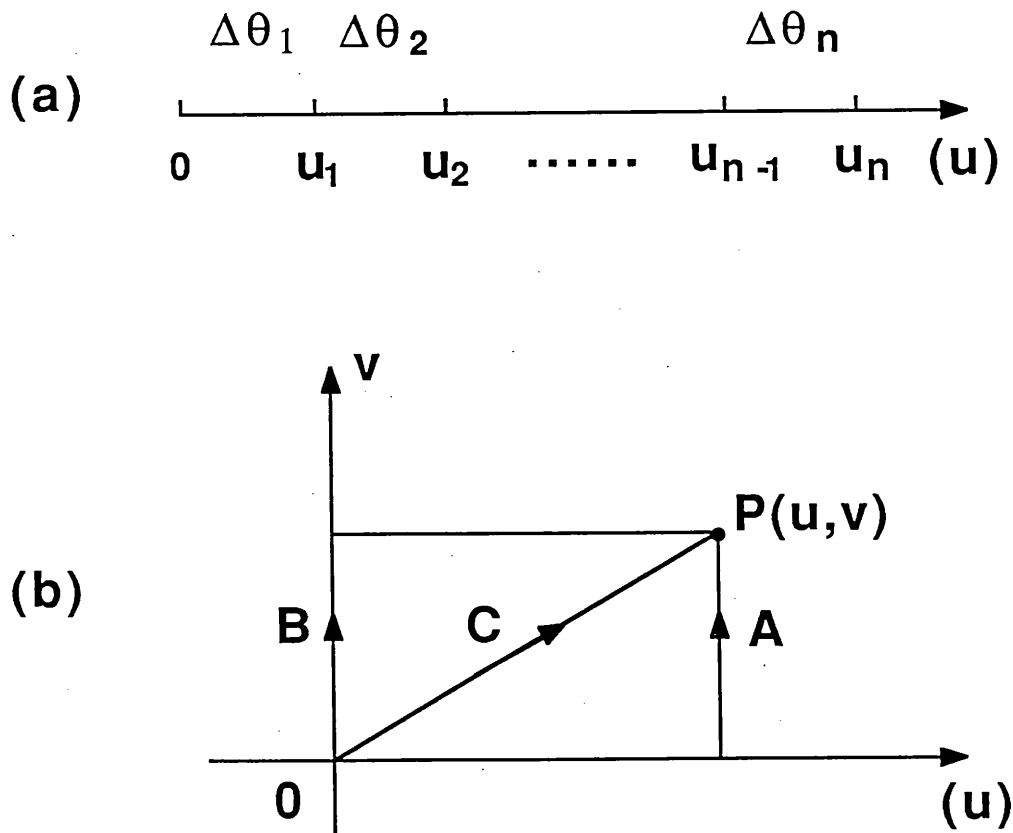


Fig.5.4. Extract phase by the Knox-Thompson technique.

(a). In one dimensional Fourier space;

(b). In two dimensional Fourier space.

Similarly one can find the phase difference  $\Delta\theta_i$  between any two points separated by a distance of  $\Delta u$  in Fourier space. For example in Fig.5.4(a), the phase gradients  $\Delta\theta_1, \Delta\theta_2, \dots, \Delta\theta_n$  between the corresponding points can be found in the manner mentioned above. It is obvious that the phase at point  $u_n$  is then the summation of all the phase differences starting from the origin where the phase is assumed to be zero, i.e.  $\theta(u_n) = \sum_{i=1}^n \Delta\theta_i$ .

This approach can be easily extended to two dimensional cases where the phase at a point  $P(u, v)$  can be obtained by summing up the phase differences along a certain path (A or B or C) shown in Fig.5.4(b). In principle, all the paths should give the same value for the phase at  $P(u, v)$ , while in practice due to the errors made in measuring the phase gradient and detection noise, there are variations for the values obtained from different paths. To reduce such errors, an average of the phase values obtained along more than one path is usually made to estimate the phase at  $P(u, v)$ .

This phase information is combined with the modulus of the object transfer to give a diffraction-limited image of the object. In principle, the Knox-Thompson technique can be used to reconstruct an object of any shape without requiring a reference star. However, this technique of phase retrieval seems to be sensitive to noise in the computed correlation [Dhawan (1981)].

### §5-3-2: Speckle Masking (Triple Correlation)

Analogous to the philosophy of many imaging techniques in which a reference point source is used, the speckle masking technique creates a mask which serves as an artificial reference star. This mask is produced by multiplying the speckle interferogram  $I(x)$  with the same but shifted speckle interferogram  $I(x + x')$ . The mask  $I(x)I(x + x')$  is then cross correlated with  $I(x)$  again. The product is a so-called triple correlation. The ensemble average of the triple correlation of speckle data can be used to yield diffraction-limited triple correlation of the object which contains information on both the amplitude and phase of the object transform.

Introductions and descriptions about triple correlation can be found in a number of papers [Lohmann *et al* (1983), Lohmann *et al* (1984), Wirnitzer (1985)]. The basic principle of the triple correlator is also given by Gamo (1963). For a one-dimensional real signal  $I(x)$ , the average triple correlation is expressed as follows:

$$\langle I(x, x') \rangle = \langle \int I(x'') I(x'' + x) I(x'' + x') dx'' \rangle \quad (5.46)$$

and its Fourier transform called the bispectrum is:

$$\tilde{I}^{(3)}(u, v) = \tilde{I}(u) \tilde{I}(v) \tilde{I}(-u - v) \quad (5.47)$$

For each short exposure data:

$$I(x) = O(x) \otimes P(x) \quad (5.48)$$

where  $O(x)$  is the intensity of the object and  $P(x)$  is the point spread function. From the above equation we have the following equation with regard to the bispectra in Fourier space:

$$\langle \tilde{I}^{(3)}(u, v) \rangle = \tilde{O}^{(3)}(u, v) \langle \tilde{P}^{(3)}(u, v) \rangle \quad (5.49)$$

where  $\tilde{I}^{(3)}(u, v)$ ,  $\tilde{O}^{(3)}(u, v)$  and  $\tilde{P}^{(3)}(u, v)$  are the bispectra of  $I(x)$ ,  $O(x)$  and  $P(x)$  respectively.  $\langle \tilde{P}^{(3)}(u, v) \rangle$  is called the bispectrum transfer function. It has been shown [Lohmann *et al* (1983)] that  $\langle \tilde{P}^{(3)}(u, v) \rangle$  is real and positive up to the diffraction limit of the telescope. Therefore the diffraction-limited information is contained in the bispectrum  $\tilde{O}^{(3)}(u, v)$  or equivalently the triple correlation of the object. The phase of the triple correlation of the object is equal to that of the triple correlation of the image data.  $\tilde{O}^{(3)}(u, v)$  can be evaluated from the average triple correlation  $\langle \tilde{I}^{(3)}(u, v) \rangle$  and  $\langle \tilde{P}^{(3)}(u, v) \rangle$  according to Eq.(5.49), where  $\langle \tilde{P}^{(3)}(u, v) \rangle$  can be evaluated from an astronomical point source. Another way to obtain  $\tilde{O}^{(3)}(u, v)$  is by means of so called Gaussian speckle-masking method [Lohmann *et al* (1983)] in which case a zero-mean Gaussian distribution for the real and imaginary part of the complex amplitude is assumed,  $\tilde{O}^{(3)}(u, v)$  can then be directly calculated from a set of values of average bispectrum  $\tilde{I}^{(3)}(u, v)$ .

Since  $\tilde{O}^{(3)}(u, v)$  contains diffraction-limited information on <sup>the</sup> phase of the object transfer, one can reconstruct diffraction-limited images from this information. Recall Eq.(5.47), for the bispectrum of object intensity, we have:

$$\tilde{O}^{(3)}(u, v) = \tilde{O}(u) \tilde{O}(v) \tilde{O}(-u - v) \quad (5.50)$$

and the phase information can be expressed as follows:

$$\phi(u, v) = \phi(u) + \phi(v) - \phi(u + v) \quad (5.51)$$

From Eq.(5.51) the phase of the object transfer  $\phi(u)$  can be evaluated. This can be seen from the following argument. If we set  $v = 1$ , we will have:

$$\phi(u + 1) = \phi(u) + \phi(1) - \phi(u, 1) \quad (5.52)$$

For  $u = 1, 2, \dots, n$ , we have the following iterative equations:

$$\left. \begin{aligned} \phi(2) &= \phi(1) + \phi(1) - \phi(1, 1) \\ \phi(3) &= \phi(2) + \phi(1) - \phi(2, 1) \\ \dots\dots\dots \\ \phi(n + 1) &= \phi(n) + \phi(1) - \phi(n, 1) \end{aligned} \right\} \quad (5.53)$$

The first phase value  $\phi(1)$  can be set arbitrarily since it only determines the position of the object. From Eq.(5.53), all phase values can be obtained and combined with the modulus of the object transfer obtained by speckle interferometry and the diffraction-limited image can be reconstructed.

Notice that for a one-dimensional signal the triple correlation or bispectrum is two-dimensional and for a two-dimensional signal it is four dimensional. This implies that triple correlation techniques require enormous amount of computing power and memory.

#### §5-4: Temporal Properties of Stellar Images

As discussed in previous sections, the “seeing” conditions give an indication of averaged blurring effects on the stellar images caused by the atmospheric turbulence. Equivalently, the Fried parameter  $r_o$  which is the spatial correlation

parameter is also used to describe the observing site condition. For conventional long exposure imaging, the Fried parameter  $r_o$  gives a good guide to image quality. However, for the various high angular resolution techniques both  $r_o$  and temporal properties of image intensity fluctuations are important parameters in determining the quality of the reconstructed information.

In all the high resolution stellar imaging techniques, short exposure data are taken in order to preserve the non-blurred stellar speckle patterns containing diffraction-limited information. Because the exposure time is finite the turbulence effects are not totally frozen. On the one hand, the exposure time should be short enough to freeze the speckle pattern; on the other hand, it is desirable to have long exposure time in order to gather as many photons as possible to achieve high signal-to-noise ratio. When the exposure time is too short there are not enough photons and the signal-to-noise ratio will be very poor. When the exposure time is too long, the high resolution information is suppressed and this also leads to a poor signal-to-noise ratio (SNR) at high frequencies. Hence in a photon-limited imaging case, there is a maximum exposure time with which an optimal SNR at high frequencies can be achieved [O'Donnell *et al* (1980)].

In actual experiments, for faint objects, because of photon noise the signal-to-noise ratio (SNR) of the power spectrum has been shown to be [Dainty (1975)]:

$$\text{SNR} = \sqrt{M} \bar{N} \Phi_m(f) \quad (5.54)$$

where  $M$  is the number of realisations of short exposure data,  $\bar{N}$  is average number of photons per exposure and  $\Phi_m(f)$  is the measured power spectrum of the image intensity normalised such that  $\Phi_m(0) = 1$ .

Because of the finite exposure time  $T$ , the actual measured intensity is the time average of the instantaneous intensity:

$$I_m(x) = \frac{1}{T} \int_0^T I(x, t) dt \quad (5.55)$$

It has been shown by O'Donnell *et al* (1980) that assuming at higher frequencies

all Fourier components decorrelate in the same manner, the related measured power spectrum can be expressed as:

$$\Phi_m(f) = \Phi(f) \frac{2}{T^2} \int_0^T (T - \tau) C(\tau) d\tau \quad (5.56)$$

where  $\Phi(f)$  is the ideal power spectrum when there is no time average;  $C(\tau)$  is the autocorrelation of the image intensity defined as:

$$C(\tau) = \langle I(x, t) I(x, t + \tau) \rangle \quad (5.57)$$

$C(\tau)$  is assumed to be even and it is the usual case in real experiments.

Hence putting Eq.(5.56) into Eq.(5.54) the signal-to-noise ratio can be expressed as:

$$\text{SNR} = \sqrt{M} \bar{N} \Phi(f) \frac{2}{T^2} \int_0^T (T - \tau) C(\tau) d\tau \quad (5.58)$$

For a Gaussian correlation function of  $1/e$  correlation time ( $T_\tau$ ), when the length of the entire experiment is finite, a numerical calculation of SNR against exposure time  $T$  calculated from Eq.(5.58) shows that the SNR has a maximum value when  $T$  is approximately at  $2T_\tau$ . In other words, the optimum exposure time is on the order of twice the correlation time. Clearly, a knowledge of the correlation time is essential if the maximum signal-to-noise ratio is to be achieved from the observational data. As a matter of fact, from an operational point of view, the correlation time is more important in short exposure data analysis than Fried parameter  $r_o$ , since with photon time-tagging detectors the exposure time can be optimised after data collection.

Measurements of the intensity temporal correlations of images and fringe patterns were carried out in the laboratory as well as at La Palma Observatory.

#### §5-4-1: Laboratory Simulations

To facilitate experiments at the observing site, the whole experimental arrangement was tested in the laboratory. The optical components were mounted

As shown in Fig. 5.6, on an optical breadboard, a star was simulated using a spatial filter; a He-Ne laser was focused onto a  $25\ \mu\text{m}$  pinhole and this pinhole image was then collimated as if the star was at an infinite distance. Another lens was used as the pupil lens of a telescope to form an  $f/15$  beam as would be encountered at the real site.

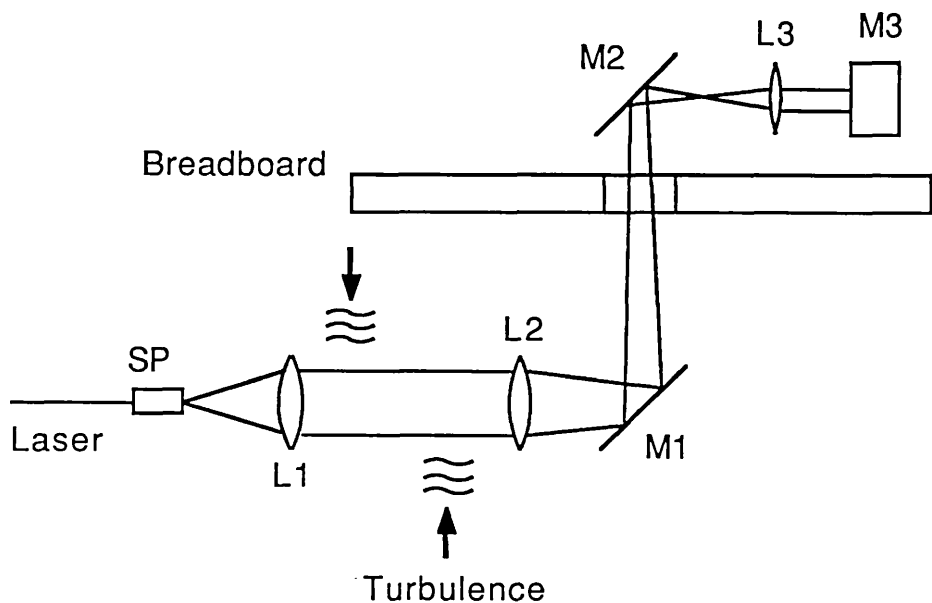


Fig. 5.5. Laboratory simulation setup.

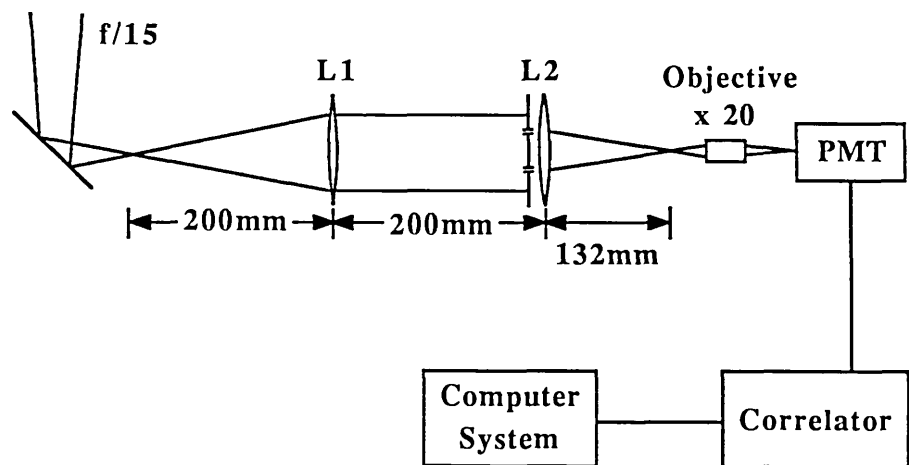


Fig. 5.6. Schematic diagram of instrumentation for measuring stellar temporal correlation.

In the laboratory experiments, two hairdryers were used to disturb the air and simulate atmospheric turbulence. The flow speed (gentle, medium and strong) and temperature (warm air and hot air) were adjustable so that one could vary the turbulence strength. When they were switched on we could clearly see the speckle moving randomly. In the case of fringe measurements, random shifting of the fringes was observed. Several correlation measurements were made on intensities of images and fringe patterns.

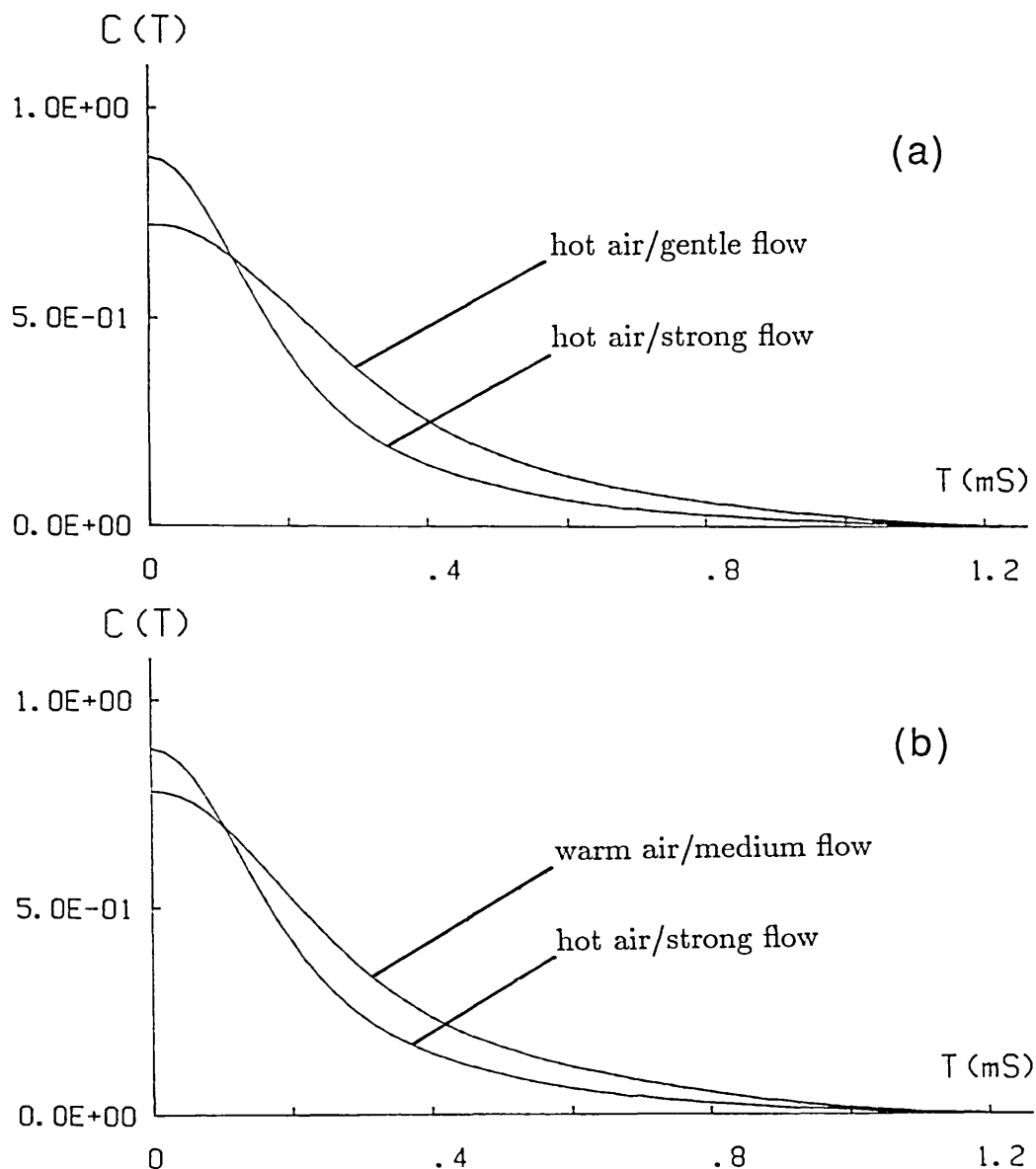


Fig.5.7. (a). Comparison of the correlation curves between created using hot air/gentle flow and hot air/strong flow. (b). between warm air/medium flow and hot air/strong flow.

In Fig.5.7 we compare the correlation curves of the image intensity obtained by using one hairdryer at different temperature and speed settings. As can be seen from the figures both temperature and flow speed influence the turbulence strength. Since hotter air will create larger temperature difference between the flow and ambient air, this causes a larger refractive index change. Therefore a hotter temperature gives rise to a more turbulent flow which gives a shorter correlation time. On the other hand, a faster flow speed will cause a faster phase change at the pupil plane and this of course causes a more turbulent flow and shorter correlation time.

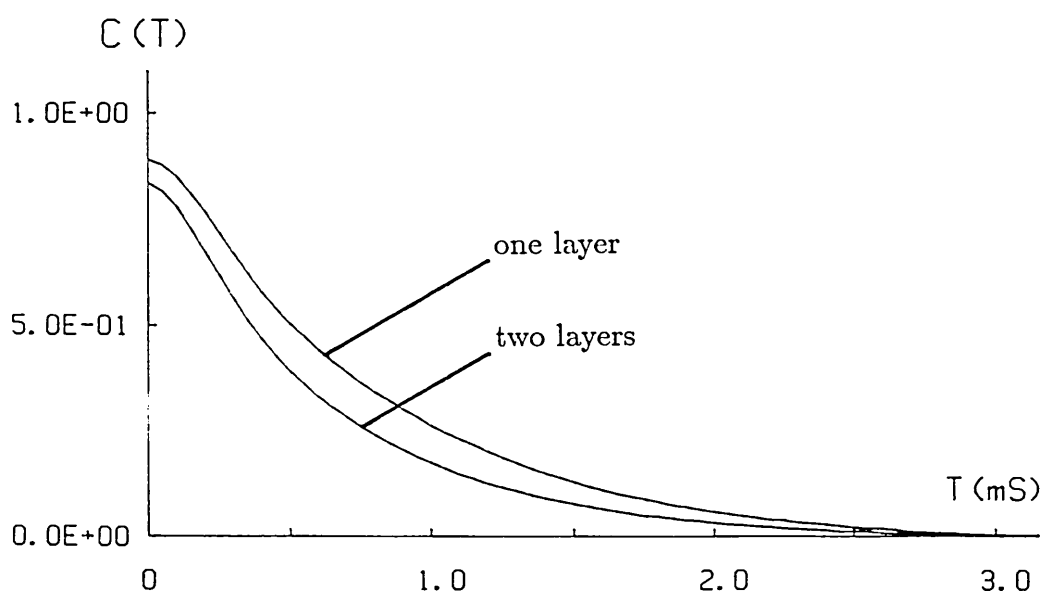


Fig.5.8. Comparison of the correlation curves between created using one turbulence layer and two turbulence layers.

In Fig.5.8, a comparison of the correlation curves of the image intensity obtained using one blowing hairdryer and two parallel blowing hairdryers. The blowing speed and temperature settings were the same for these hairdryers. It is obvious that two turbulence layers will cause faster phase changes and hence a shorter correlation time.

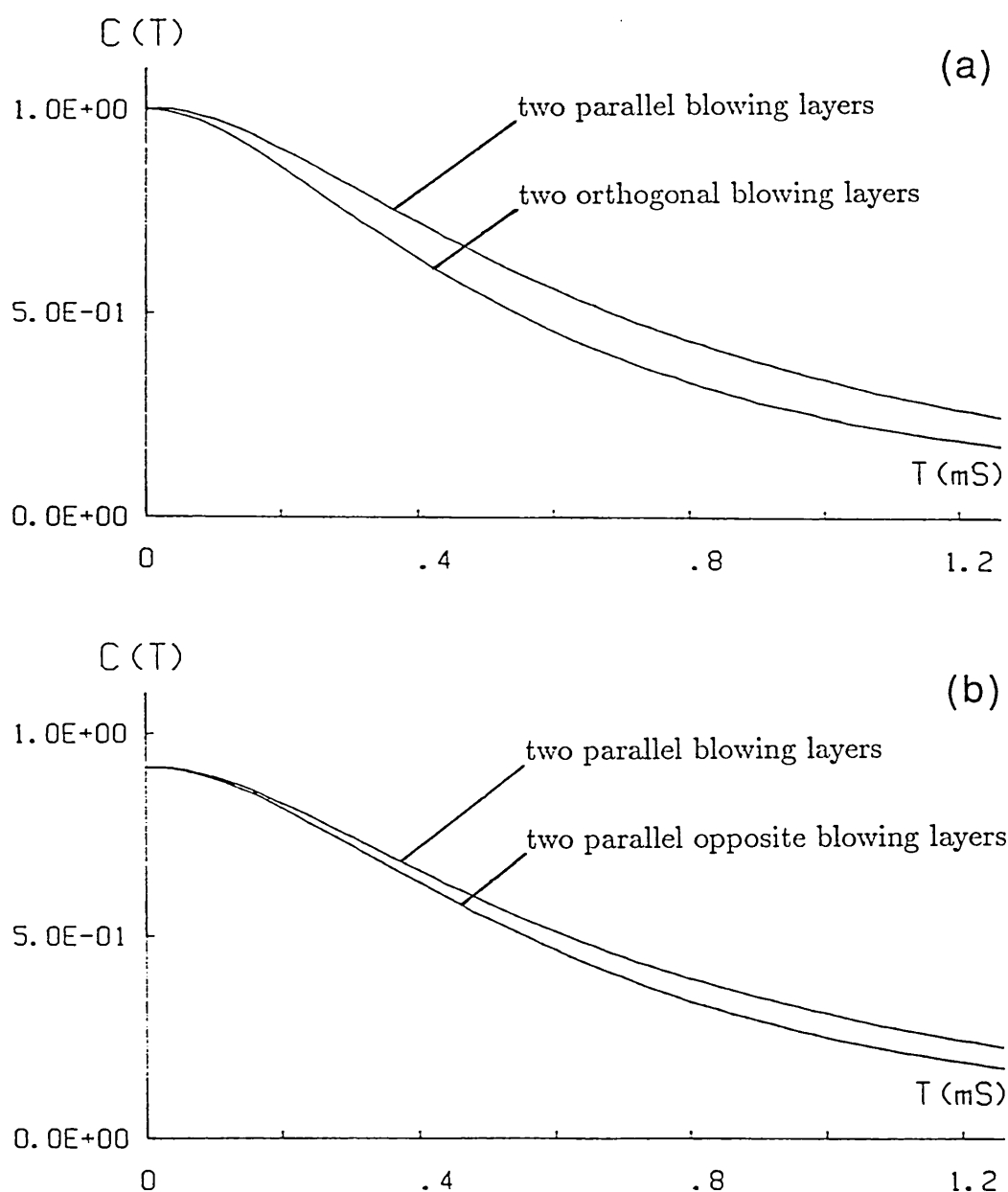


Fig.5.9. (a). Comparison of the correlation curves between created using two parallel blowing turbulences and two orthogonal blowing turbulences. (b). between two parallel blowing turbulences and two parallel opposite blowing turbulences.

In Fig.5.9, the correlation curves for the image intensity obtained using two parallel blowing hairdryers, orthogonally blowing hairdryers and anti-parallel blowing hairdryers. They were all set at the same blowing speed and temper-

ature. From Fig.5.9.(a), one would find that the orthogonal blowing turbulence layers gives a shorter correlation time than that caused by two parallel blowing turbulence layers. This is because for a multi-layer turbulence, the correlation time  $\tau_e$  is related to the standard deviation  $\sigma_v$  of the flow velocities of the layers [Roddier *et al* (1982)]:

$$\tau_e \approx 0.36 \frac{r_o}{\sigma_v} \quad (5.59)$$

The standard deviation of two orthogonal blowing layers is apparently larger than that of two parallel blowing layers when the other conditions are the same. From Eq.(5.59) a shorter correlation time is expected from the intensity correlation due to two orthogonal blowing turbulence layers. Similarly, the anti-parallel blowing turbulence layers causes a shorter correlation time than that caused by two parallel blowing turbulence layers.

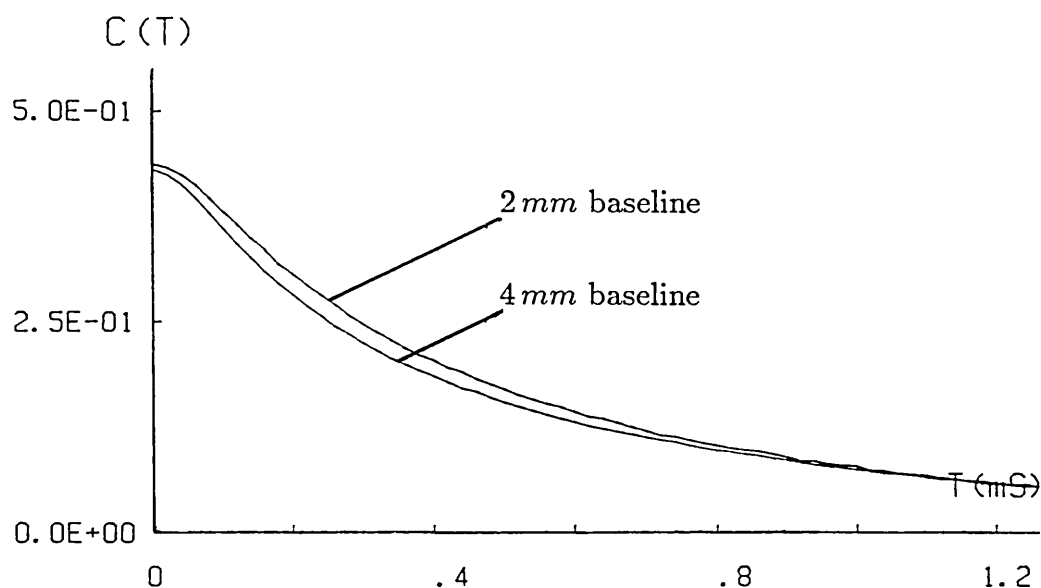


Fig.5.10. Comparison of the correlation curves of fringe intensity between created using 2 mm separation on the pupil and 4 mm separation on the pupil.

Correlation measurements of the fringe intensities were also made in order to look into the fringe shifting properties caused by the turbulence. The fringe

patterns were formed by putting a pair of sub-apertures of certain separations at the pupil plane. A comparison of fringe intensity correlation curves of  $2\text{ mm}$  and  $4\text{ mm}$  separations is presented in Fig.5.10. Basically, the correlation times for these two separations are found to be identical. This is further confirmed in the real site measurements at La Palma Observatory. The analysis of the results is given in the second part of the next section.

### §5-4-2: Site Measurements

#### (1). One aperture results

We carried out the measurements of the temporal correlation of the image intensity using a  $1\text{ m}$  telescope at La Palma Observatory, Canary Islands during the periods May 20 – 26, 1988 (nights 1 – 7) and February 15 – 22, 1989 (nights 8 – 15). No measurements were made on nights 6, 10, 11 and 12 due to bad weather. The bright stars observed were all unresolvable by a diffraction-limited  $1\text{ m}$  telescope.

The astronomical telescope used has a parabolic primary with a clear aperture of  $1.0\text{ m}$  and a focal length of  $4.596\text{ m}$  and a hyperbolic secondary constituting, with the primary a conventional  $f/15$  cassegrain system. The curved focal plane of the telescope is located  $760\text{ mm}$  behind the pole of the primary mirror, and  $370\text{ mm}$  behind the instrument mounting face giving a  $34.4\text{ arcmin}$  diameter field with a scale of  $13.8\text{ arcsec/mm}$ .

A schematic diagram of the instrumentation as used in the experiments is shown in Fig.5.6. The  $f/15$  Cassegrain beam is collimated by a doublet which produces an image of the telescope pupil. A second doublet forms an image again. By this arrangement, masks (e.g. sub-apertures) can be inserted in front of lens  $L2$ . The image formed by  $L2$  is then magnified by a microscope objective onto a sampling pinhole. To avoid spatial integration, we have to choose the magnification of the microscope objective and the size of a sampling pinhole carefully.

The size of the sampling pinhole has to be much smaller than the magnified speckle size. In the visible, the stellar speckle size at the original image plane of the telescope is on the order of the diffraction limit of the telescope, i.e.  $\sim 0.1 \text{ arcsec}$ . Once it is re-imaged by two doublets as shown in Fig.5.6 with a demagnification of 1.5. Similarly the telescope field at the original image plane is with a scale of  $13.8 \text{ arcsec/mm}$ , after re-imaging the scale is  $20.9 \text{ arcsec/mm}$ . Therefore, the pinhole size ( $H$ ) and magnification ( $M$ ) of the object have to be chosen such that:

$$\frac{20.9}{M} \times H \ll 0.1 \lambda \times M \quad (5.60)$$

In our experiments, we chose  $M = 20$  and  $H = 65 \mu\text{m}$  which satisfy the inequality (5.60).

The filters (wavelength/bandwidth) chosen in the experiments were blue (401/15 nm) and green (550/10 nm) and red (650/23 nm). In most practical cases, a larger bandwidth is desirable to improve the signal-to-noise. However, because speckle patterns are wavelength dependent, larger bandwidth will blur the speckle pattern and reduce the coherence and hence the resolution. Besides, the atmospheric refraction produces the atmospheric dispersion for the incident light. As a rule of thumb, the bandwidth ( $\Delta\lambda$ ) of the chosen filter has to satisfy:

$$\frac{\Delta\lambda}{\lambda} \ll \frac{r_o}{D} \quad (5.61)$$

such that the bandwidth has a negligible effect. For a 1 m telescope, taking  $r_o = 10 \text{ cm}$ , the bandwidths of the chosen filters have negligible effects.

A photomultiplier tube (EMI Type 9862), a preamplifier and a 128 channel digital correlator (Langley Ford DC128) were used to sample and process the data. Typical sample and experiment times were 0.5 ms and 20 – 60 s respectively. The average detected photon rate was 0.5 – 1.0 per sample time, or 1000 – 2000 per second. Typical dark counts of the detecting system were less than 10 counts per second and no correction for dark count or detector dead time was necessary. The data was stored and printed/plotted using a desktop computer linked to the correlator.

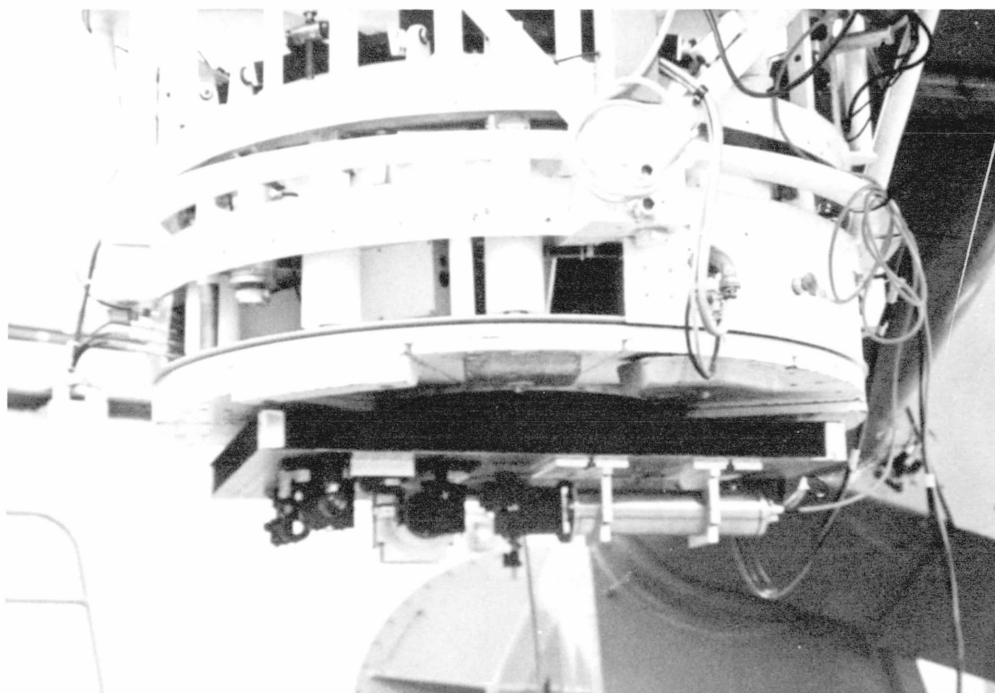


Fig.5.11. A photograph shows the whole layout of the experimental setup bolted onto the telescope.

All the optics and the PMT shown in Fig.5.6 were mounted onto a breadboard as shown in the above photograph. A  $40\text{ mm}$  hole was drilled in the middle of this breadboard to let the beam coming through. The breadboard was then mounted upside down to the mounting face of the telescope. A long cable was used to connect the output from the preamplifier to the correlator which, together with the computer system, was in the control room. In the experiments, one flip-in mirror was mounted between lens  $L2$  and the objective to divert the image onto an image intensifier to aid alignment of the image onto the sampling pinhole. Another flip-in mirror was also mounted between the objective and the PMT to divert the image to an eyepiece for the same purpose. For every run, because of the image wandering effects, we had to align the image onto the sampling pinhole. This was done with the help of these flip-in mirrors, the eyepiece and the image intensifier. Another very important aid for aligning the image was a pulse counter. The output from the preamplifier was connected to the pulse counter first to ensure

there was a satisfactory counts reading from the pulse counter. Once the counts were maximised, the output signal was then connected to the correlator and data accumulated.

In the data processing, all the correlation curves were corrected for image wander effects. The method used was the same used by Scaddan *et al* (1978). The image intensity fluctuations are due to the image boiling and the image wandering. The former are caused by the rapid variations of the phases at the pupil plane of the telescope and it is related to the optimum exposure time.

The image boiling is on a very fast time scale. The latter is caused by large scale phase change, for example, the tilting of the turbulence layers. It is on a relatively slower time scale. It is quite reasonable to assume that the two factors are independent of each other. This allows us to express the measured correlation function as the product of that due to boiling  $C_b(\tau)$  and wandering  $C_w(\tau)$ :

$$C(\tau) = C_b(\tau) C_w(\tau) \quad (5.62)$$

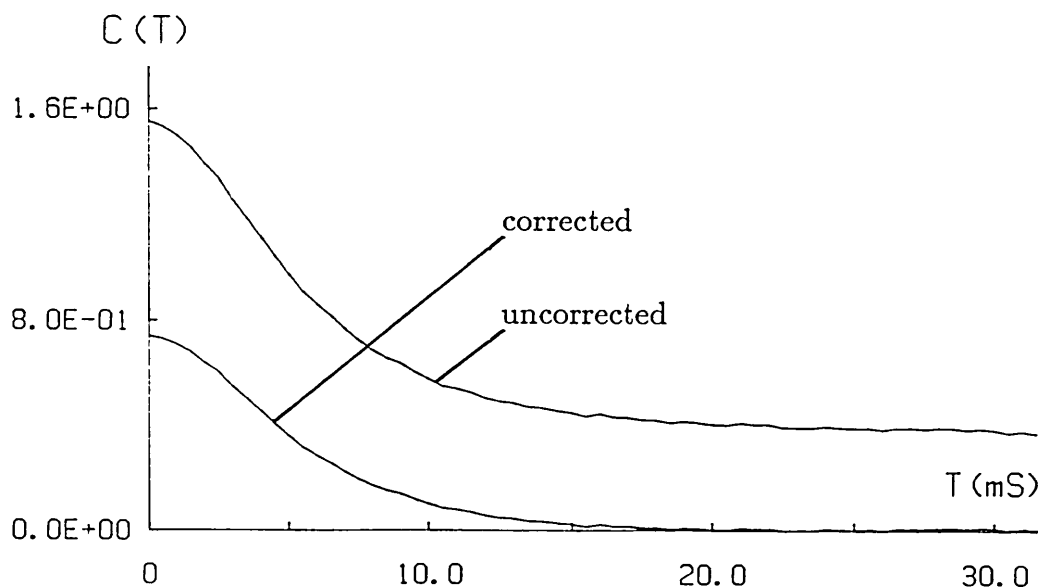


Fig.5.12. Correlation curves show the image wandering correction for measured temporal correlation.

Since the correlation functions due to the contributions from the image boiling and the image wandering have distinct time scales, more precisely the image wandering time scale is much slower than the image boiling time scale, the long tail in the measured correlation curve should be due to the image wandering effects. As shown in Fig.5.12, to a good approximation,  $C_w(\tau)$  can be obtained by extrapolating the roughly linear tail of  $C(\tau)$  back to the zero point. From Eq.(5.62), a corrected correlation function for image wandering can be obtained. The function  $C_b(\tau)$  is responsible for the image boiling effects.

A very large number of temporal correlation curves were measured over 15 nights. Fig.5.13 and 5.14 show typical measured temporal correlations of the image intensity fluctuations for nights 1 – 7 and 8 – 15 respectively. The  $1/e$  correlation times range from 3.2 to 8.9 *ms*, with an average value of 5.3 *ms*. Compared with the results (average 15 *ms* also with a large spread of values) obtained by O'Donnell *et al* (1982) at Mauna Kea Observatory, Hawaii, the image temporal intensity fluctuation times at La Palma Observatory seem to be relatively shorter. However, they were longer than the results obtained from Herstmonceux, U.K. [Parry *et al* (1979)] and Mees Observatory, New York [Dainty *et al* (1980)], where the values of 2 – 7 *ms* with an average of 4 *ms* and value of approximately 2 *ms* were obtained respectively.

In Fig.5.15 we show the correlation functions for the blue (415/15 *nm*), green (550/10 *nm*) and red (650/23 *nm*) wavelength on four different occasions. In general, the correlation time increases with wavelength. In the Taylor hypothesis [Roddier (1981)] in which it is assumed the intensity fluctuations are due to a single turbulence layer moving across the telescope pupil of diameter  $D$  at speed  $v$ , the correlation time  $\tau_e$  is given by:

$$\tau_e \approx 0.5 \frac{D}{v} \quad (5.63)$$

This formula has no wavelength dependence and predicts a correlation time much longer than those observed. For example, on one night the wind speed was measured at about 60 *km/h*; by Eq.(5.63) for the 1 *m* telescope we would have observed

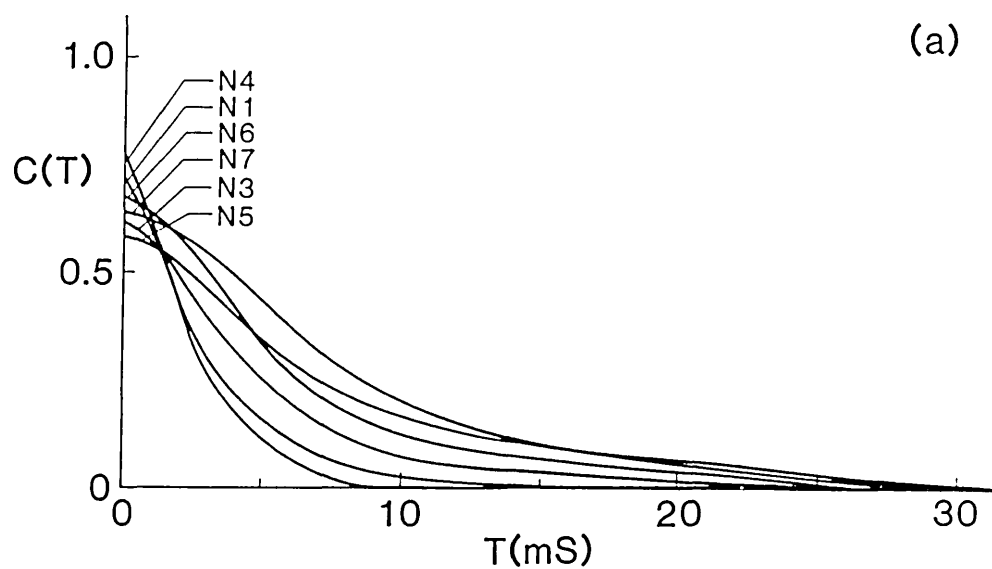


Fig.5.13. Measured temporal correlation curves at a wavelength of  $550\text{ nm}$  for nights 1 to 7. The curves are corrected for image wander effects.

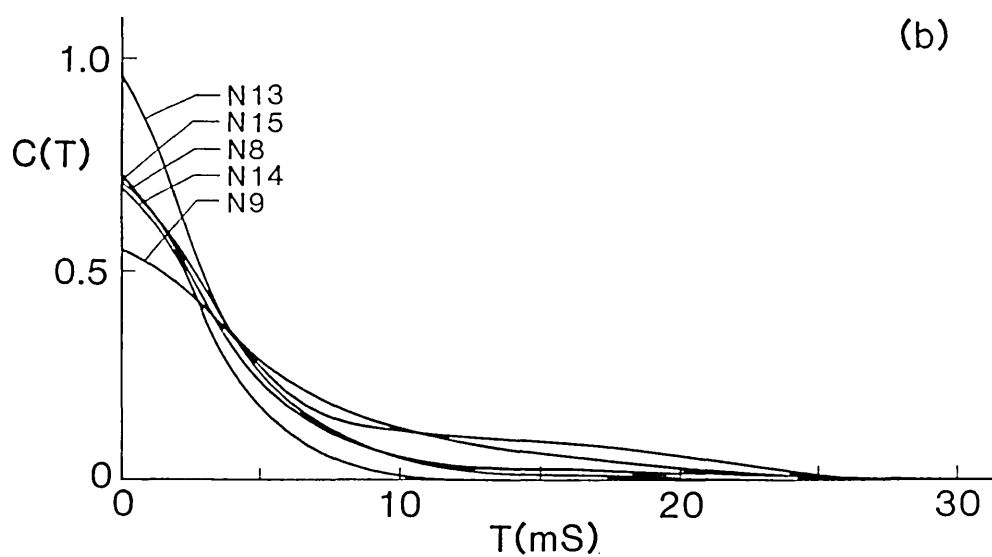


Fig.5.14. Measured temporal correlation curves at a wavelength of  $550\text{ nm}$  for nights 8 to 15. The curves are corrected for image wander effects.

a correlation time of about 30 *ms*. In fact the correlation times measured were only around 8 *ms* and they were wavelength dependent. On the other hand, if there are several turbulence layers moving at different velocities, then the correlation time is given by [Roddier *et al* (1982)]:

$$\tau_e \approx 0.36 \frac{r_o}{\sigma_v} \quad (5.64)$$

where  $\sigma_v$  is the standard deviation of the wind velocities of the turbulence layers. As mentioned before, the Fried parameter  $r_o$  has a wavelength dependence of  $\lambda^{6/5}$ ; hence from Eq.(5.64) the correlation time will also scale as  $\lambda^{6/5}$ . In Table 5.1, the ratios of the correlation times for blue:green:red for the curves shown in Fig.5.15 are listed and compared with the ratios predicted on the basis of a  $\lambda^{6/5}$  scaling. For this comparison, the ratios are taken relative to the correlation time for green (550/10 *nm*) light. The ratios in the first three cases are reasonably consistent with the theoretical predictions; in the fourth case [corresponding to Fig.5.15.(d)] there was very strong local gusting of the wind and it is likely that some combination of Eq.(5.63) and Eq.(5.64) is applicable. The wavelength dependence implies that correlation times increase significantly as the wavelength increases towards the infrared. In the infrared region, not only is the long exposure data less affected by the atmospheric turbulence because of larger  $r_o$ , but the exposure time of the short exposure data can be relatively longer thus providing more photons and a better signal-to-noise ratio.

On the basis of the above measurements and previous measurements at other observatories, the correlation time of the image intensity is highly variable and often shorter than 10 *ms*. Although these results were taken using a 1 *m* telescope, they are almost certainly representative for larger telescopes except when a single turbulence layer dominates the image intensity fluctuation. The results imply that the exposure times for speckle interferometry and related techniques should be shorter than 20 *ms* to achieve the optimum signal-to-noise ratio in the photon limited case. Similarly, for the photon time-tagging detectors, such as the IPD [Rees *et al* (1981)], PAPA [Papaliolios *et al* (1982)] and MAMA [Timothy *et al* (1986)] which allow the possibility of computing the correlation time from the data

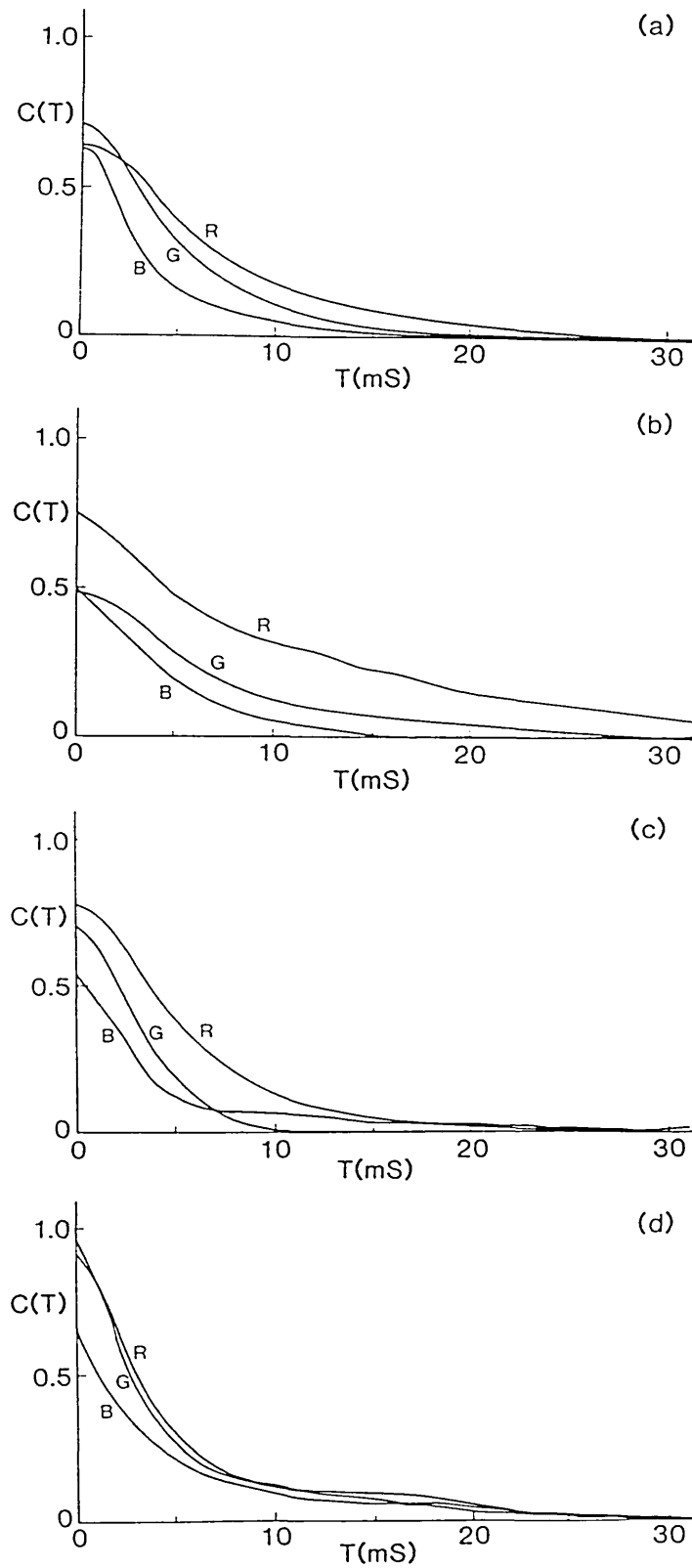


Fig.5.15. Four sets measurements of temporal correlation functions for blue (415/15 nm), green (550/10 nm) and red (650/23 nm).

| Figures | Ratio B : G : R    |
|---------|--------------------|
| (a)     | 0.62 : 1.00 : 1.38 |
| (b)     | 0.71 : 1.00 : 1.20 |
| (c)     | 0.86 : 1.00 : 1.60 |
| (d)     | 1.02 : 1.00 : 1.14 |
| Theory  | 0.68 : 1.00 : 1.23 |

Tab.5.1. Comparison of the ratio of the correlation times for the filters of blue (415/15 nm), green (550/10 nm) and red (650/23 nm), for the curves shown in Fig.5.15 and theoretical ratio according to the  $\lambda^{6/5}$  law.

and selecting the appropriate exposure time after data collection, the exposure time should be selected by taking the measured correlation time  $\sim 10$  ms as a reference.

(2). Two aperture results (Michelson interferometer)

The full aperture stellar images are random speckle patterns due to the random distortion of the wavefront at the pupil plane of the telescope. Equivalently, the degradation of the stellar images can also be understood as the result of addition of randomly shifting fringe patterns. Let us assume the telescope aperture is divided into  $N$  small sub-apertures and each sub-aperture is small enough so that the wavefront distortion due to the atmosphere over the sub-aperture portion is negligible. In other words, the size of the sub-apertures should be smaller than  $r_o$ . In the case of monochromatic light, the complex amplitude at the focal plane

of the telescope  $A$  can be obtained by summing up the contribution  $A_i$  of each sub-aperture  $i$ :

$$|A|^2 = \left| \sum_{i=1}^N A_i \right|^2 = \sum_{i=1}^N |A_i|^2 + \sum_{i \neq j} \sum_j A_i A_j^* \quad (5.65)$$

The ensemble average is then as follows:

$$\langle |A|^2 \rangle = \sum_{i=1}^N \langle |A_i|^2 \rangle + \sum_{i \neq j} \sum_j \langle A_i A_j^* \rangle \quad (5.66)$$

In the above equation, the first term is the sum of intensities due to all sub-apertures; it contains no high resolution information. The second term is the summation of cross products which are called mutual intensities.

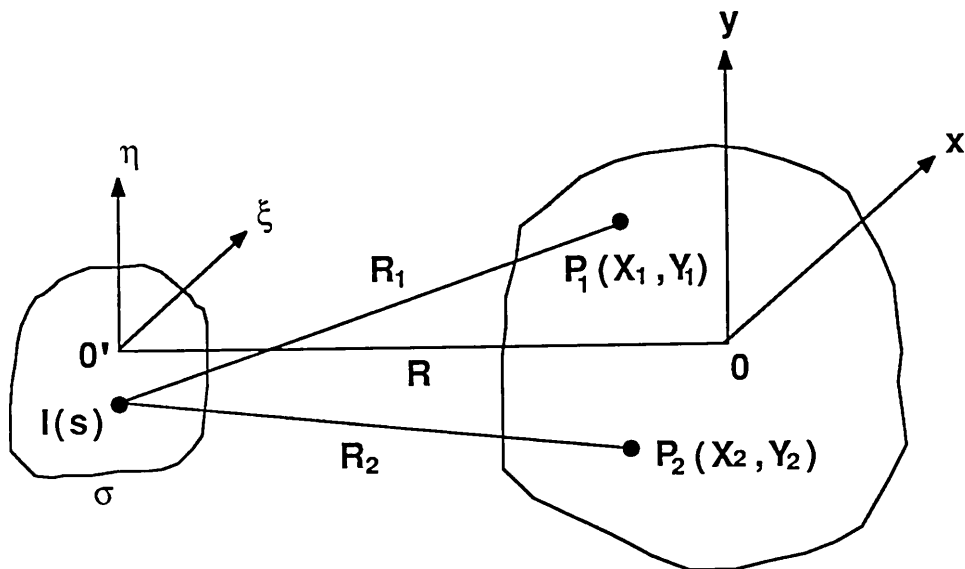


Fig.5.16. Schematic diagram showing the Van Cittert-Zernike theorem.

According to the Van Cittert-Zernike theorem, as shown in Fig.5.16, the mutual intensity due to two small sub-apertures is given by:

$$J_{ij} = \langle A_i A_j^* \rangle = \int_{\sigma} I(s) \frac{\exp[ik(R_1 - R_2)]}{R_1 R_2} ds \quad (5.67)$$

where  $I(s)$  is the intensity per unit area of the object;  $\sigma$  is the area of the object;  $R_1$  and  $R_2$  are the distances of the two sub-apertures from a particular point  $s$  on the object. When the distance between two sub-apertures is very small compared to the distance of these sub-apertures from the object, the mutual intensity is given by [Born and Wolf (1980)]:

$$\begin{aligned} J_{ij} &= \frac{\exp(i\varphi) \iint_{\sigma} I(\xi, \eta) \exp[-ik(p\xi + q\eta)] d\xi d\eta}{R^2} \\ &= \frac{\exp(i\varphi)}{R^2} \mathcal{F}_{ij}[I(\xi, \eta)] \end{aligned} \quad (5.68)$$

where  $R$  is the distance between 0 and 0',  $\mathcal{F}_{ij}$  denotes Fourier transform and the spatial frequency is decided by the separation sub-apertures  $i$  and  $j$ , the spatial information is contained in the variables  $p$  and  $q$  which are related to the coordinates of sub-apertures  $i$  and  $j$ :

$$\left. \begin{aligned} p &= \frac{X_1 - X_2}{R} \\ q &= \frac{Y_1 - Y_2}{R} \\ \varphi &= \frac{k[(X_1^2 + Y_1^2) - (X_2^2 + Y_2^2)]}{2R} \end{aligned} \right\} \quad (5.69)$$

Put Eq.(5.68) into Eq.(5.66) it becomes:

$$\langle |A|^2 \rangle = \sum_{i=1}^N \langle |A_i|^2 \rangle + \frac{1}{R^2} \sum_{i \neq j} \sum_j \exp(i\varphi) \mathcal{F}_{ij}[I(\xi, \eta)] \quad (5.70)$$

As can be seen from Eq.(5.70) each cross term is a Fourier component of a Fourier expansion. The image intensity distribution is actually the Fourier expansion of the object intensity. Physically, this means that the image intensity is the addition of fringes produced by all the possible combination of pairs of sub-apertures. The larger separation of the sub-apertures gives finer fringes which of course contain higher resolution information. Therefore preserving and recovering such information due to the longer baseline is one of the keys in high resolution imaging. This is also the basis of high resolution interferometric experiments.

Due to the atmospheric turbulence, the phases at  $P_1$  and  $P_2$  are randomly fluctuating, the fringes are therefore randomly shifting. Thus in a long exposure

image, the summation of these randomly shifting fringes will add up as a blurred image instead of a diffraction-limit image. In order to understand the implications of the fringe shifting at different baselines and their fluctuation properties, we carried out measurements on the temporal fluctuations of intensities of such fringe patterns and compared the results obtained by different baselines. Since the fringe intensity fluctuations were caused by the random shifting of the fringe pattern and this shifting was due to the random fluctuations of the phase difference between point  $P_1$  and  $P_2$ , we were in fact indirectly looking at the fluctuations of phase difference between two points at the pupil plane of the telescope.

The measurements were made using the same telescope as for the full aperture measurements. Two small sub-apertures were placed in the mask plane shown in Fig.5.6. The sub-apertures were carefully placed in order to avoid the central obscuration of the pupil imaged onto the mask plane with a diameter of 5.4 mm. The sub-apertures were made by drilling tiny holes in a few pieces of metal. The thickness of the sub-aperture edges were made very thin in order to avoid edge effects. This was done by drilling the pieces of metal with a pyramidal drill. The diameter of the sub-aperture on the metal is 1 mm, and separations were 2, 4, 7 and 10 mm. Since the demagnification from <sup>the</sup>telescope pupil to the masking plane was 75, the sub-aperture diameter corresponded to 7.5 cm in the telescope pupil and separations of 15, 30, 52.5 and 75 cm. For the above separations, at wavelength  $\lambda = 550 \text{ nm}$ , the corresponding fringe periods at the focal plane of lens "L2" were 36.3, 18.2, 10.4 and  $7.3 \mu\text{m}$  respectively. These fringes were then magnified by an  $\times 20$  objective, the magnified fringes with periods of 726, 364, 208 and  $146 \mu\text{m}$  were sampled by a slit of width  $30 \mu\text{m}$  parallel to the fringes. The slit was made by carefully aligning and gluing two pieces of <sup>razor</sup>blades under a microscope.

Interference fringes were clearly seen with an eyepiece and they were shifting all the time. The measurements are shown in Fig.5.17. These measurements were taken on night 13 using the 550 nm filter. Three graphs compare the temporal correlation of the image formed using 15 cm separation on the telescope pupil with

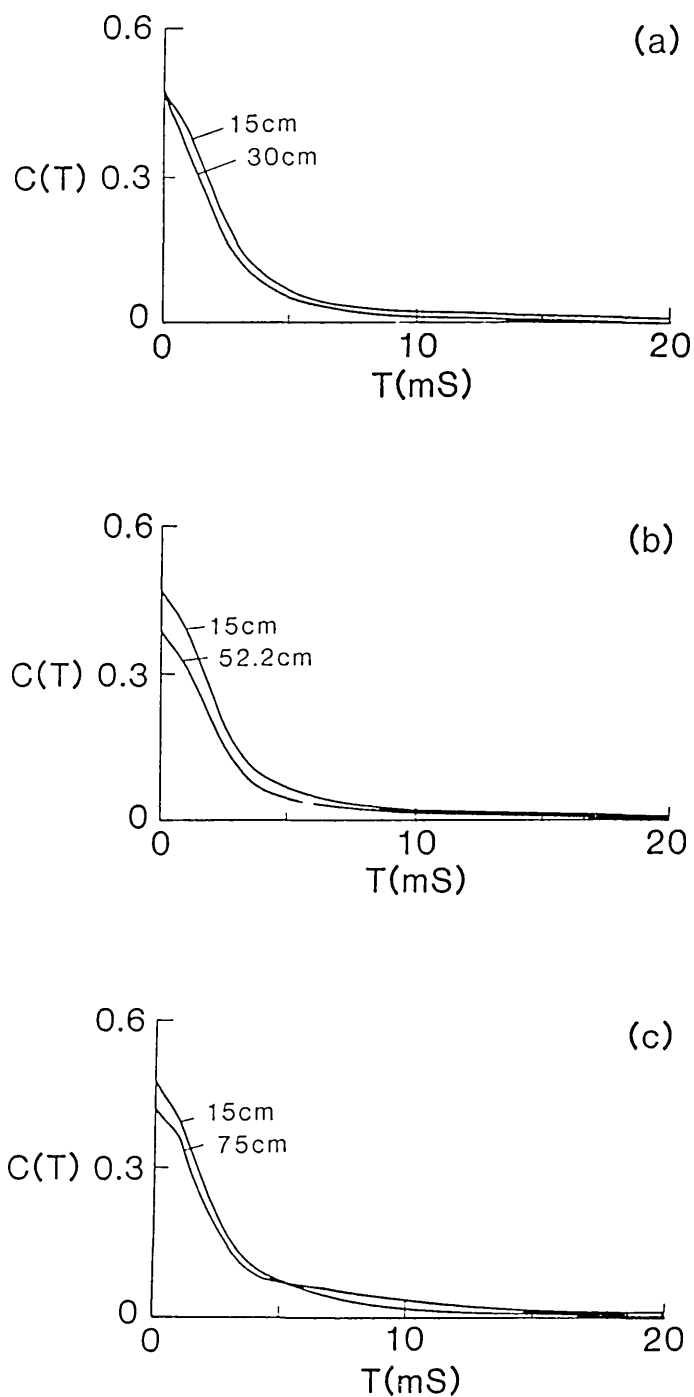


Fig.5.17. Temporal correlation curves of fringe intensity fluctuations. (a). Comparison between the results obtained from baselines 15 cm and 30 cm. (b). Comparison between the results obtained from baselines 15 cm and 52.5 cm. (c). Comparison between the results obtained from baselines 15 cm and 75 cm.

those obtained using sub-aperture separations of 30, 52.5 and 75 cm. As with the full aperture case, the long time scale image wander has been removed, only short time scale variations remaining. The scale values for larger baselines may be reduced due to mis-alignment of the slit relative the fringes and spatial averaging. This is because the larger baseline gives a finer fringe period. The two aperture results show that the fluctuation time scales of the fringe patterns for measured baselines 15cm, 30cm, 52.5cm and 75cm are very similar. They are on the order of 5ms similar to that of the full aperture image. This implies that the phase changes at two apertures of above mentioned baselines are random and uncorrelated. Even if the phases at two sub-apertures are influenced by one correlated factor, for example the translational movement of the turbulence, the *phase difference* at these two sub-apertures are still fluctuating at similar time scale for different baseline. However, for very small baseline ( $< r_o$ ) when the phase differences are also small ( $< 2\pi$ ), the fluctuation time scales for a shorter baseline should be longer than that for a longer baseline. This is true when two apertures are within the same homogeneous coherence patch. In our measurements, the baselines are larger than the dimension of this patch and therefore we observed similar fluctuation time scales for different baselines due to the uncorrelated phase changes at two apertures. This is also an indication that in our measurements the baselines were larger than the Fried parameter  $r_o$ . If the sub-aperture separation is smaller than  $r_o$ , one would expect the different correlation time scales for different baselines.

Another feature we can see from Fig.5.17 is that the relative fluctuation scales [standard deviation/mean intensity, i.e.  $C(0)$ ] are identical for all different baselines in our experiments. This implies that the average phase difference fluctuations are more than  $2\pi$ . If the phase difference fluctuations were smaller than  $2\pi$ , when the sampling slit is centred on a bright line the relative fluctuation scales would be different from that when the sampling slit is centred on a dark line. In our experiments, the sampling slit was not deliberately centred on any line yet the measurements showed the identical relative fluctuation scales for different measurements and baselines. For phase difference fluctuations larger than  $2\pi$  the

sampling slit would still sense the whole fluctuation since the fringes would still scan over the slit due to the sudden change of the phase difference. An implication for Michelson interferometry is that the fringe shifting scale and rates (time taken for a  $2\pi$  shifting) are the same for different baselines larger than the homogeneous patch.

## §5-5: Concluding Remarks

The temporal fluctuation properties of “seeing” at La Palma Observatory, Canary Islands were evaluated. This was done by evaluating the temporal correlation of stellar image intensities and fringe patterns. As far as high angular resolution imaging concerned, our measurements show that La Palma Observatory is a favourable observing site compared with Herstmonceux, U.K. and Mees Observatory, New York where shorter correlation times were found. The correlation time of the full aperture image intensity at La Palma is of the order of  $5\text{ ms}$ . This indicates that at La Palma the short exposure data should be taken or processed with an exposure time in the order of  $10\text{ ms}$  to achieve the optimal signal-to-noise ratio. In the measurements of intensity temporal correlation of fringe patterns, it is found from limited measurements that the values of temporal correlation time scale vary little for different baselines. The correlation times are in the same order of magnitude as the correlation time for images using the full aperture.

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## Chapter 6

### Statistical Properties of Light Scattered by Dense Random Media

#### §6-1: General Outline of Light Scattering

Light wave scattering has long been applied to solve many practical problems. It has been used to probe the structure of scattering molecules and measure certain properties of scattering media, such as the diffusion constant and particle sizes [e.g. Witkowski (1986)]. It is also used to estimate the fraction of spherical particles in a mixture [Rochon *et al* (1988)]. Because of the high intensities of laser sources and excellent monochromatic characteristics, laser light scattering is capable of measuring weakly scattered light and a very small frequency shift. Even at very low light levels, photon correlation techniques can be used to gain high signal to noise ratio in light scattering experiments [e.g. Moreno *et al* (1988)].

Physically light scattering can be explained in terms of interaction between the incident light wave and scattering medium. When light impinges on matter, the electric field of the light induces an oscillating polarisation of the electrons in the molecules. The molecules then serve as secondary sources of light and subsequently radiate (scatter) light. The frequency shifts, the angular distribution, the polarisation and the intensity of the scattered light are determined by the size, shape and molecular interactions in the scattering material. The energy exchanges between the light and medium will influence the translational, rotational, vibrational and electronic degrees of freedom of the molecules. They thereby suffer frequency shifts. Thus the frequency spectrum of the scattered light will exhibit resonances at the frequencies corresponding to these transitions. Rayleigh scattering deals with the characteristics of the light scattered from translational and rotational degrees of freedom, while Raman scattering, provides information about energy spectra of molecules. Apart from scattering theory which deals with the interaction between light waves and molecules, there is a fluctuation theory

of light scattering. The intensity of the light is obtained without considering the detailed molecular structure of the medium but the dynamic properties of the scatterers.

In this section, we will start with scattering by a single particle. Then the scattering by a collection of randomly distributed particles for single scattering in a dilute solution will be discussed. The multiple scattering process will also be briefly dealt with. In this section, we will follow Ishimaru's (1978) analysis.

### §6-1-1: Light Wave Scattered By A Single Particle

For the scattering geometry shown in Fig.6.1, if we denote the scattered field by the particle at point  $\vec{r}$  as  $\vec{E}_s(\vec{r})$ , then it is given by:

$$\vec{E}_s(\vec{r}) = \vec{f}(\vec{o}, \vec{e}) \frac{\exp(ikR)}{R} \quad (6.1)$$

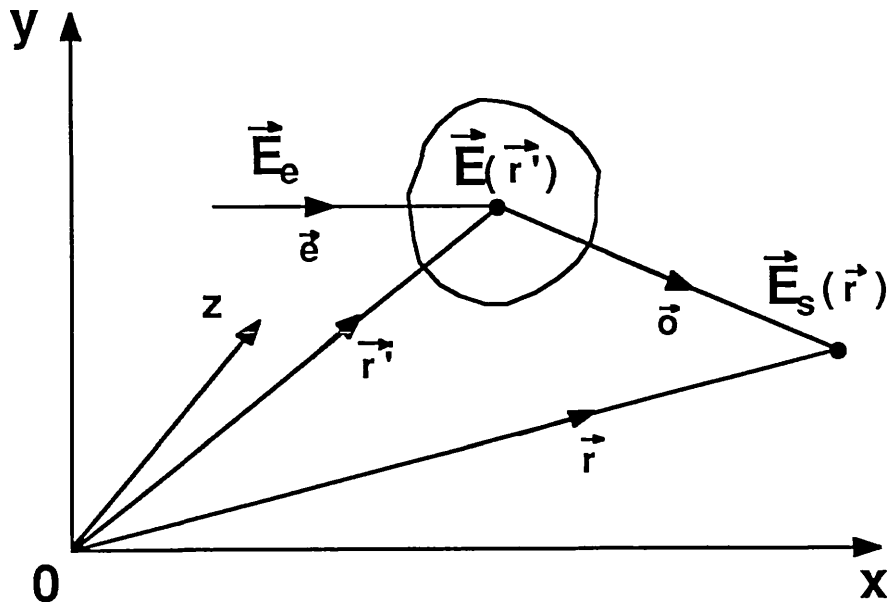


Fig.6.1. Scattering geometry by a single particle.

where  $k$  is the wave number,  $R$  is the distance between the detection point at  $\vec{r}$  and the particle,  $\vec{f}(\vec{o}, \vec{e})$  is the scattering amplitude given by:

$$\vec{f}(\vec{o}, \vec{e}) = \frac{k^2}{4\pi} \int_V \{[\vec{o} \times \vec{E}(\vec{r}')] \times \vec{o}\} (\varepsilon - 1) \exp(-ik\vec{r}' \cdot \vec{o}) dV \quad (6.2)$$

where  $\vec{e}$  and  $\vec{o}$  are unit vectors of incident and viewing directions respectively,  $\varepsilon$  is the relative dielectric constant of the particle and  $\vec{E}(\vec{r}')$  is the electric field inside the particle. The integration is over the whole volume of the particle. Therefore the question of scattering from a single particle becomes the question of how to find the electric field inside the particle in order to solve Eq.(6.2). The electric field *inside* the particle  $\vec{E}(\vec{r}')$  is generally unknown. However, in many practical situations, an approximation for  $\vec{E}(\vec{r}')$  can be made in order to obtain an approximate expression of  $\vec{f}(\vec{o}, \vec{e})$ . We will in the following context briefly mention a few examples, where we assume that all particles discussed below are spherical.

(1). Rayleigh scattering (for  $kd \ll 1$ ): It is assumed that the particle size  $d$  is sufficiently small compared with the wavelength and the particle is regarded as point-dipole oscillator. Thus the electric field inside the particle induced by an outside electric field  $\vec{E}_e$  is uniform and given by:

$$\vec{E}(\vec{r}') = \frac{3}{\varepsilon + 2} \vec{E}_e \quad (6.3)$$

Hence from Eq.(6.2):

$$\vec{f}(\vec{o}, \vec{e}) = \frac{k^2}{4\pi} \left[ \frac{3(\varepsilon - 1)}{\varepsilon + 2} \right] V \sin \theta \quad (6.4)$$

where  $\theta$  is the scattering angle. The scattering cross section  $\sigma_s$  is therefore given as:

$$\begin{aligned} \sigma_s &= \int_{4\pi} |\vec{f}(\vec{o}, \vec{e})|^2 d\omega \\ &= \frac{k^4}{16\pi} \left[ \frac{3(\varepsilon - 1)}{\varepsilon + 2} \right]^2 V^2 \sin^2 \theta \end{aligned} \quad (6.5)$$

The  $\lambda^{-4}$  dependence is a well-known result.

(2). Rayleigh-Debye scattering [ for  $(\varepsilon - 1)kd \ll 1$  ]: In this approximation, it is assumed that the particle's dielectric constant is close to 1 so that the field

inside the scatterer can be approximated by the incident field  $\vec{E}_e = \vec{e} \exp(ik\vec{r} \cdot \vec{e})$  (Born approximation). Note the particle size is not necessarily assumed small to make this approximation valid. The scattering amplitude therefore has the form:

$$\vec{f}(\vec{o}, \vec{e}) = \frac{3k^2(\epsilon - 1)V \sin \theta}{4\pi k_s^3 d^3} (\sin k_s d - k_s d \cos k_s d) \quad (6.6)$$

where  $k_s = 2k \sin(\theta/2)$ . The scattering amplitude shows a periodic pattern against the scattering angle.

(3). WKB\* scattering [ for  $(\epsilon - 1)kd \gg 1$ ,  $\epsilon - 1 < 1$  ]: The Born approximation mentioned above deals with particles whose dielectric constant is close to unity regardless the particle size as long as it is not too large. In contrast to the Born approximation, <sup>the</sup>WKB approximation is valid for the large particles whose dielectric constant is not close to 1.

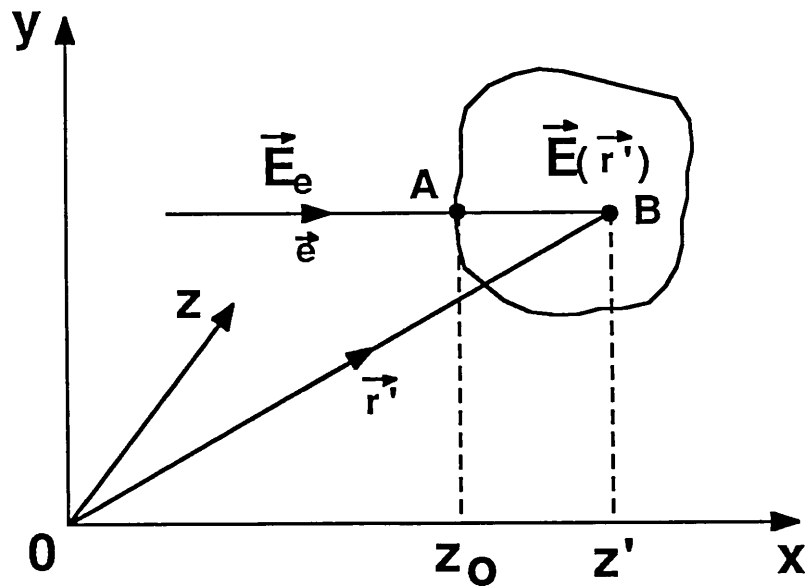


Fig.6.2. Particle diagram for WKB approximation.

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\* WKB stands for Wentzel, Kramers and Brillouin who developed this type of approximation.

Since it is also assumed  $\varepsilon - 1 < 1$ , this implies that inside the particle, the electric wave propagates approximately in the same direction of the incident wave. As shown in Fig.6.2, for an incident field  $\vec{E}_e = \vec{e} \exp(ik\vec{r} \cdot \vec{e})$ , the electric field at point  $B$  inside the particle is approximated by:

$$\vec{E}(\vec{r}') = T \exp[ikz_o + ikn(z' - z_o)]\vec{e} \quad (6.7)$$

where  $n = \sqrt{\varepsilon}$  is the refractive index of the particle,  $T$  is the transmission coefficient at the particle surface and it can be approximated by  $T = 2/(n + 1)$ . The scattering amplitude in this case can then be calculated as:

$$\left. \begin{aligned} \vec{f}(\vec{o}, \vec{e}) &= \frac{k^2}{4\pi} V \sin \theta X(\vec{o}, \vec{e}) \\ X(\vec{o}, \vec{e}) &= \frac{1}{V} \int_V 2(n - 1) \exp[ikz_o + ikn(z' - z_o) - ik\vec{r}' \cdot \vec{e}] dV \end{aligned} \right\} \quad (6.8)$$

(4). Mie scattering: This is the exact solution of the scattering of a plane wave by an isotropic, homogeneous sphere. In a spherical coordinate system, the scattered field can be represented by:

$$\left. \begin{aligned} E_\theta &= \frac{i \exp(ikr)}{kr} S_1(\theta) \cos \phi \\ E_\phi &= -\frac{i \exp(ikr)}{kr} S_2(\theta) \sin \phi \end{aligned} \right\} \quad (6.9)$$

where  $S_1(\theta)$  and  $S_2(\theta)$  are the series involving scattering angles. More details about them can be found in Ishimaru(1978).

### §6-1-2: Single Scattering

We have looked at the characteristics of the scattered wave by a single particle. In this subsection we shall study the scattered waves by a collection of randomly distributed particles. In other words, the scattering by a random medium. If the distribution of the particles is tenuous and the majority of the scattered waves have only undergone the scattering process once, we refer to this as single scattering. We will discuss the average detected scattered energy and the temporal auto-correlation related to the dynamic characteristics of the medium.

(1). Average detected power: Suppose the incident power density onto the scattering medium is  $S_i$  and the distance between the scattering medium and the detector is  $R$ , which is very large compared with the scattering medium size. Hence the scattered power density due to a single particle at the detector is given by:

$$S_{r1} = \frac{|\vec{f}(\vec{o}, \vec{e})|^2}{R^2} S_i \quad (6.10)$$

If we ignore the interference effect between scattered waves, the power scattered by a small volume  $dV$  is given by:

$$S_{r2} = \frac{|\vec{f}(\vec{o}, \vec{e})|^2}{R^2} S_i \times \rho dV \quad (6.11)$$

where  $\rho$  is the particle number density, i.e. the number of particles per unit volume. Therefore the total detected power by the detector with a receiving cross section  $A_d$  is as follows:

$$P_r = \int_V \frac{A_d |\vec{f}(\vec{o}, \vec{e})|^2}{R^2} S_i \times \rho dV \quad (6.12)$$

(2). Temporal auto-correlation: In the following treatment it is assumed the scattering medium is statistically stationary. The field temporal auto-correlation is, from its definition:

$$\begin{aligned} G^{(1)}(\tau) = & \left\langle \sum_{i=1}^N |\vec{f}(\vec{o}, \vec{e})| \exp[-i\vec{q} \cdot \vec{r}_i(0)] \right. \\ & \times \left. \sum_{j=1}^N |\vec{f}(\vec{o}, \vec{e})| \exp[i\vec{q} \cdot \vec{r}_j(\tau)] \right\rangle \exp(-i\omega_o \tau) \end{aligned} \quad (6.13)$$

where  $N$  is the total number of scatterers. For sufficiently dilute solutions the positions of different scatterers will be uncorrelated so that the cross terms vanish. If we also assume the scattering amplitudes from different particles are the same, the field temporal auto-correlation function can then be simplified as:

$$G^{(1)}(\tau) = N |\vec{f}(\vec{o}, \vec{e})|^2 \left\langle \exp \{i\vec{q} \cdot [\vec{r}(\tau) - \vec{r}(0)]\} \right\rangle \exp(-i\omega_o \tau) \quad (6.14)$$

By definition, the ensemble average in Eq.(6.14) can be replaced by:

$$G^{(1)}(\tau) = \int_V P(\vec{k}, \tau) \exp(i\vec{q}\vec{k}) dV \quad (6.15)$$

where  $P(\vec{k}, \tau)$  is the conditional probability of a particle located at the origin at  $t = 0$  moving to a position  $\vec{R}$  at  $t = \tau$ . For particles undergoing free isotropic diffusion,  $P(\vec{k}, \tau)$  obeys the diffusion equation:

$$\frac{\partial P}{\partial t} = D_T \Delta^2 P \quad (6.16)$$

where  $D_T$  is the diffusion constant of the scattering medium. Together with Eq.(6.13), we can solve for  $G^{(1)}(\tau)$ :

$$G^{(1)}(\tau) = N |\vec{f}(\vec{\sigma}, \vec{e})|^2 \exp(-q^2 D_T \tau) \exp(-i w_o \tau) \quad (6.17)$$

Hence the normalised field auto-correlation is given by:

$$C^{(1)}(\tau) = \exp(-q^2 D_T \tau) \quad (6.18)$$

By the Siegert relation, for a Gaussian field, the intensity auto-correlation is as follows:

$$C^{(2)}(\tau) = 1 + \exp(-2q^2 D_T \tau) \quad (6.19)$$

It is concluded that the intensity correlation of single scattered light from dilute solutions has the form of an exponential and its relaxation time is  $1/(2q^2 D_T)$ .

In Fig.6.3, a typical auto-correlation measurement of scattered intensity from dilute solution is plotted. The correlation scale is on a logarithmic scale. The experiments were done by using a very dilute suspension of polystyrene latex beads in water at room temperature. A 514.5 nm line Argon ion laser was used and the scattering angle was 2° off backscatter. As can be seen from the figure, the log-correlation shows a fairly straight line indicating the exponential correlation function. From the gradient of the logarithm curve, it is found the diffusion constant of this solution is  $D_T \approx 4.7 \times 10^{-9} (cm^2/s)$  at room temperature. In fact, this is a very important application of light scattering in industry.

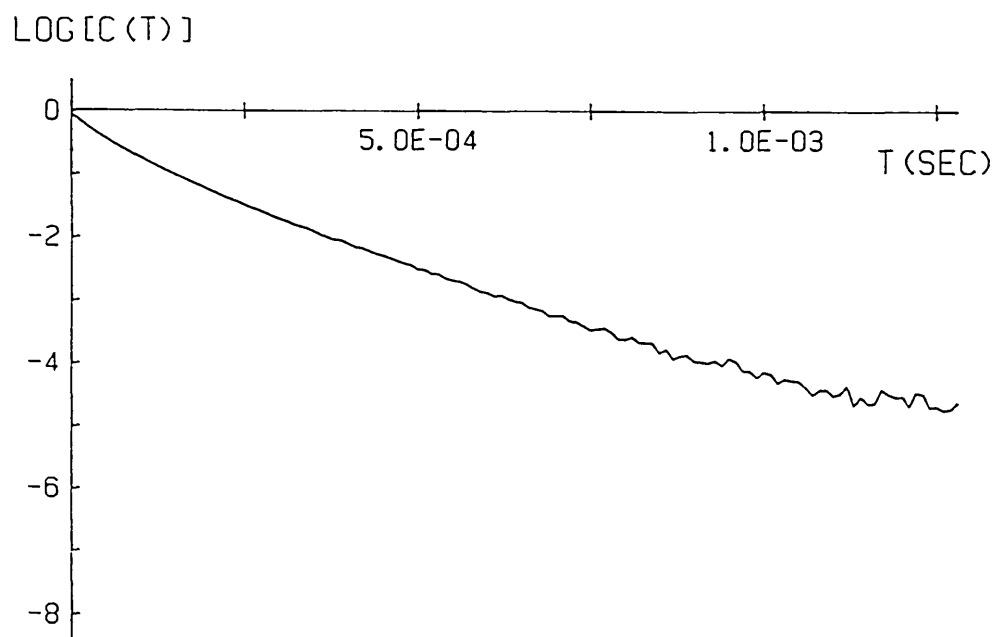


Fig.6.3. The logarithm of temporal correlation of singly scattered light intensity.

### §6-1-3: Multiple Scattering

When the density of the scattering medium is increased to a certain level, multiple scattering effects become dominant in determining the characteristics of the scattered waves. When this occurs, the single scattering approximation is no longer valid and multiple scattering theories must be used to cope with scattering problems. Historically, there are two distinct theories developed to solve multiple scattering problems. One is called analytical theory and the other transport theory.

In analytical theory, one starts with basic wave equations for scattered waves, takes into account the scattering and absorption characteristics of scattering particles and eventually gets appropriate differential or integral equations for the statistical properties such as correlation functions. Analytical theory is mathematically rigorous and in principle all the interference and diffraction effects can

be included in the theory. However, in practice it is very difficult to obtain the formulations which include all these effects. A variety of approximate theories have been developed to solve the problems for a specific range of parameters.

In transport theory, the propagation of intensities in a random medium is investigated by using the so-called transport equation. It emphasizes the transport of energy through a medium containing scattering particles. Transport theory ignores the details of the multiple scattering process, but the theory generally works well. However, because of the nature of the theory (i.e. only deals with the energy transport), it does not take account of diffraction or interference effects. Hence under certain circumstances, such as when dealing with the backscattered light intensity from dense random media, modifications are needed. The beauty of transport theory lies in the fact that it simplifies the multiple scattering process by only considering the intensity transport. For example this could be done by only considering the first and last scattering process and Green function which accounts for any intermediate scattering process. In this case, the Green function is the key factor to be studied for multiple scattering process. In section §6-3, we will use this feature to analyze the enhanced backscattering line shape. The details of multiple scattering theories (analytical and transport theories) are out of the scope of the thesis and they will not be discussed here. They can be found in the excellent textbook by Ishimaru(1978).

## **§6-2: Enhanced Backscattering Phenomenon**

There has been considerable interest in the phenomenon of enhanced backscattering exhibited by dense collections of particulate scatterers such as latex spheres and randomly rough surfaces. This phenomenon is of both basic physical importance and potential practical application. Because of the localisation of the photons, transport theory should be modified due to the modification of photon diffusion constant which is directly related to the localisation effect in certain situations. Coherent backscattering can also find application in analyzing certain

characteristics of the scattering media, in remote sensing, etc. A bibliography about the analysis of this phenomenon is listed in references 6.1–6.7, 6.10–6.14, 6.16, 6.18, 6.20–6.43 and 6.45–6.46.

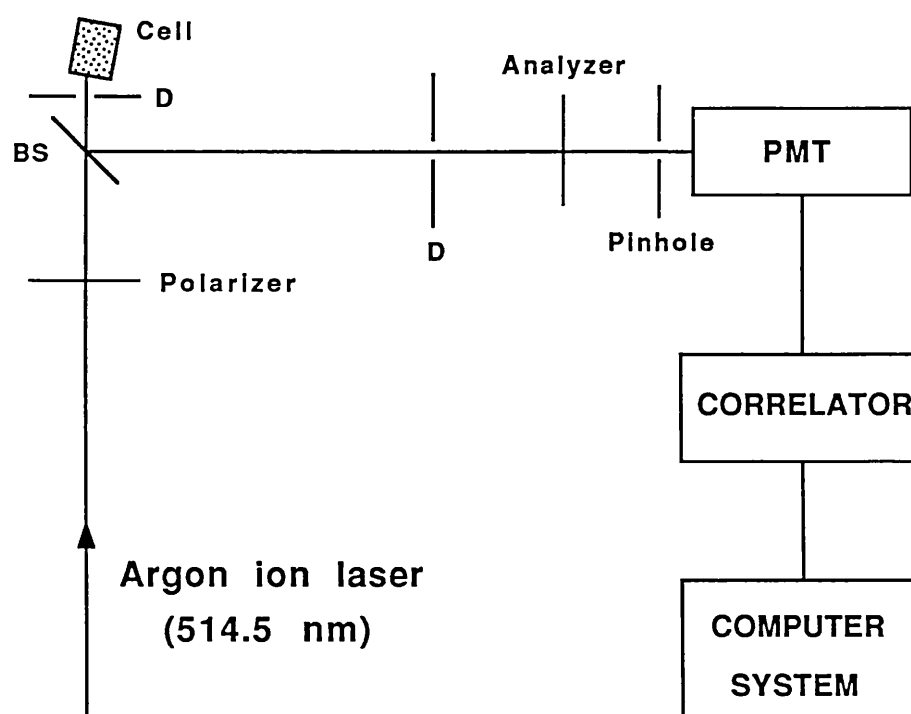


Fig.6.4. Experimental setup for measuring enhanced backscattering.

Our experimental setup is shown in Figure 6.4. In this Figure, polarised incident light illuminates a sample of polysyrene latex beads. The backscattered light passes through an analyzer and is reflected by a beamsplitter onto the detector. The detected signals are analysed by a digital correlator and its related computer system. Great care should be taken to block the stray light from various sources. The cell containing the sample was tilted off-axis in order to keep its surface reflection well away from the detector. Alignment for backscatter was done in the following way: put a tiny hole in front of the laser source and a flat mirror in the place where the sample cell should be. Then adjust both the flat mirror and the beamsplitter to let the reflected beam pass back through the tiny hole. Therefore both parts of the split beam represent the backscattering direction. The detector

was put in this direction and the backscattering direction was found and recorded.

Several measurements were made by using this experimental setup. The calibrations of the transmission and reflection of co-polarised and cross-polarised intensities by the beamsplitter were also made to obtain an absolute comparison between detected co-polarised and cross-polarised components. A typical measurement of coherent backscattering from a 10% by volume suspension of latex beads in water, with the average diameter  $0.46\ \mu m$  is plotted in Fig.6.5. As can be seen, in the backscatter direction, for illumination by linearly polarised light, the intensity of the linearly co-polarised scattered component is up to a factor of  $\sim 1.8$  greater than that away from backscatter (i.e.  $\approx 30\ mrad$  away) and the cross-polarised component also shows some enhancement, typically  $\approx 1.2$  for dense media. These peaks are due to multiple scattering and associated with weak localisation of photons. At and near backscatter, multiple scattering gives rise to constructive interference of the light travelling along the time reversed paths. For the co-polarised case, the two components of the time reversed paths are identical and therefore give higher enhancement. In the cross-polarised case, the two non-identical components still have correlated phases which give a smaller enhancement. The half-width at half-maximum (HWHM) of the enhanced backscatter peak for both co-polarised and cross-polarised detections is governed by diffraction and is on the order of  $\lambda/(2\pi l)$ , where  $l$  is the mean free path. It is also noted that at a large scattering angle, for example at  $\theta = 30\ mrad$  the detected light intensity is found to be almost totally depolarised.

Theoretically, for the co-polarised case, since forward and reverse amplitudes are identical and in phase at backscatter, an enhancement of up to a factor of two is possible. In practice, this enhancement factor is less than two due to several practical constraints affecting this behaviour. For example, the contributions from the multiple scattering whose first and last scattering are from the *same* scatterer are always in phase but are uniform with angle and this will bring down the total enhancement. Another example is the single scattering contributions which

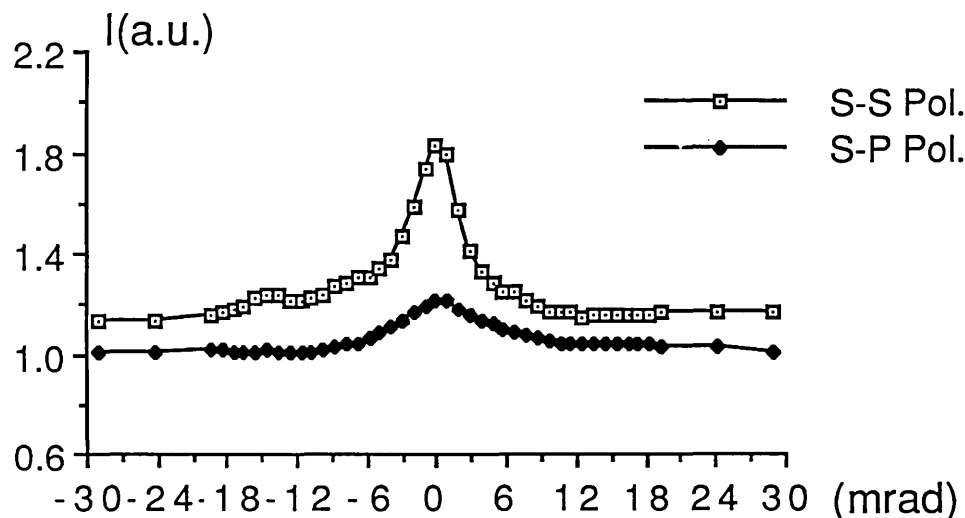


Fig.6.5. Measured scattered intensity versus angle showing enhanced backscattering for both co-polarised (s-s Pol.) and cross-polarised (s-p Pol.) detections.

do not contribute to the enhancement factor at all. It is possible to eliminate the single scattering contributions by shining the incident beam through a linear polariser followed by a quarter wave plate before the sample. The angle between the transmission axis of the linear polariser and the fast axis of the quarter wave plate is  $45^\circ$ . The beam incident onto the sample is then circularly polarised. This combination of a linear polariser and a quarter wave plate will not allow singly backscattered light passing back and therefore isolate the single scattering. However, by doing so other problems may occur, such as one cannot distinguish the co-polarised and cross-polarised light intensities, they are mixed up by the isolator. Besides, the isolator itself will introduce reflected and scattered light to make the measurements more noisy.

The measurements of backscattering intensity against angle for different concentrations were also made and plotted in Fig.6.6. The measurements were for

10%, 5% and 1% by volume respectively. The intensity was normalised to the large angle intensities of each one. This Figure therefore compares enhancement factor at different concentration densities. As can be seen from the Figure, the enhancement factor is decreased when the concentration becomes lower. This can be easily understood from the fact that the portion of single scattering contribution becomes large with the decrease of the concentration and single scattered light does not contribute to the enhanced backscattering. Another fact we can see from the Figure is that the enhancement cone is relatively broader for the higher density concentration. This is because the cone width is governed by diffraction and is the order of  $\lambda/(2\pi l)$ . The higher the concentration density, the smaller the effective mean free path length  $l$ , and of course the larger the enhancement cone width.

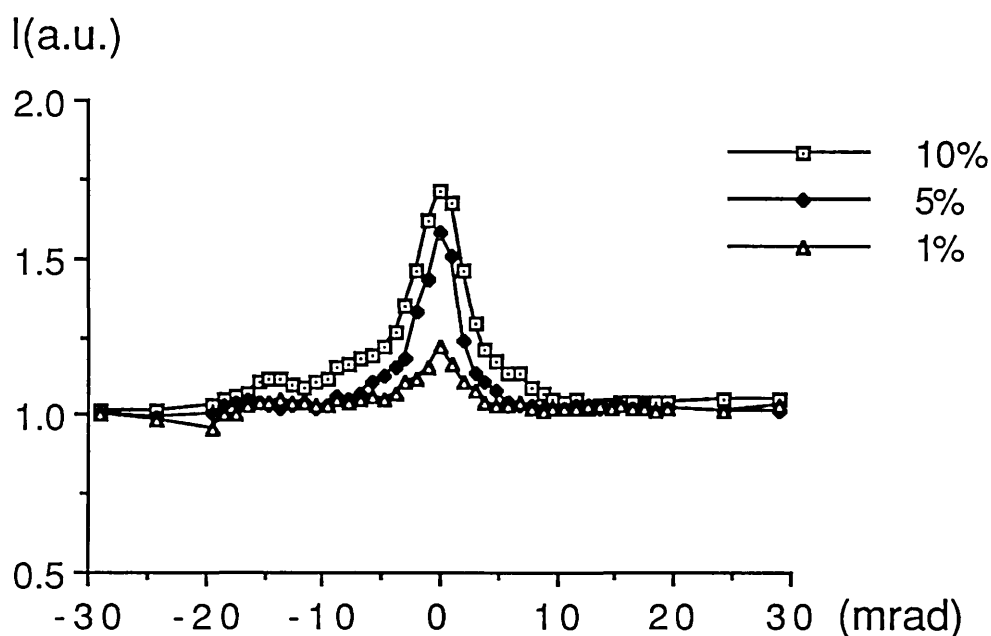


Fig.6.6. Measured backscattered light intensity versus angle for co-polarised detections for several concentrations, density in solids content percentage (%).

| Angles<br>(mrad)     | 0    | 5    | 10   | 15   | 20   | 25   | 30   |
|----------------------|------|------|------|------|------|------|------|
| C(0)<br>Copolarised  | 0.85 | 0.87 | 0.87 | 0.88 | 0.84 | 0.87 | 0.88 |
| C(0)<br>Cross-polar. | 0.83 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 |

Tab.6.1. Measured  $C(0)$  values at different angles.

Now let us discuss the standard deviation ( $\sigma$ ) of intensity fluctuations as a function of angle  $\theta$ . By definition, the standard deviation of intensity fluctuation is defined as:

$$\sigma = \sqrt{\langle (I - \bar{I})^2 \rangle} \quad (6.20)$$

For a stochastic variable it can be proven that:

$$\sigma^2 = \bar{I}^2 - \bar{I}^2 \quad (6.21)$$

Therefore:

$$\frac{\sigma^2}{\bar{I}^2} = \frac{\bar{I}^2}{\bar{I}^2} - 1 = C(\tau)_{\tau=0} = C(0) \quad (6.22)$$

Where  $C(\tau)$  is the measured intensity autocorrelation function defined as follows:

$$C(\tau) = \frac{\langle I(0)I(\tau) \rangle}{\bar{I}^2} - 1 \quad (6.23)$$

Hence we can see that:

$$\sigma = \sqrt{C(0)} \bar{I} \quad (6.24)$$

The relative intensity fluctuations at and around backscatter were studied against angle in the frame work of analysis of dynamic properties of scattered light presented in §6-5. The measurements were done by using a sample time of  $T = 2 \times 10^{-6} \text{ s}$ , small enough to avoid temporal averaging effects. The results are listed in Tab.6.1. As can be seen from the table, for both co-polarised and

cross-polarised detections the values of  $C(0)$  remain the same at different angles, therefore from Eq.(6.24) we can get  $\sigma \propto \bar{I}$  for different angles. In other words, the intensity fluctuation also shows an enhancement at backscatter, and its enhancement shape is identical to that of intensity enhancement for the co-polarised and cross-polarised cases respectively. A larger fluctuation will be observed at and around backscatter. Also from the Table, it is found that the relative intensity fluctuations  $C(0)$  for the co-polarised case are approximately the same as those for cross-polarised case.

### §6-3: Analysis of Enhancement Line Shape

The scattered light intensities have been intensively studied using transport theory. In the classical approach of analyzing the light transport, it is assumed that phases are uncorrelated on scales larger than the elastic mean free path  $l$ . This leads to an intensity transport equation of the Boltzmann type in which the phase relations between scattered waves are not taken into account because the scatterers are randomly distributed and hence any interference effects are neglected. When the scattering geometrical length scales are very large compared with  $l$ , in which case the attenuated energy can be neglected and diffused energy counts for the majority portion of observed energy, this equation reduces to a diffusion equation with a photon diffusion constant  $D = lc/3$ . The transport theory works well in many practical situations. However, it is necessary to make a modification around the backscattering direction in which two time reversed emergent waves interfere with each other causing enhanced backscattering.

#### §6-3-1: Enhancement Line Shape for a Scalar Light Wave

Figure 6.7. shows the scattering geometry of a semi-infinite medium occupying the half space  $z > 0$ . As can be seen from the Figure, the total phase difference between the two time reversed waves is  $\vec{q} \cdot (\vec{r} - \vec{r}')$ . This implies that the correction for interference due to multiple scattering can be obtained by weighting this phase difference factor  $\cos[\vec{q} \cdot (\vec{r} - \vec{r}')] to the corresponding incoherent intensity. This$

is done by putting an extra weighting factor in the calculation of the scattering albedo. The ratio of the emergent energy in direction  $\vec{s}$  per unit surface, unit time and unit solid angle  $d\Omega$  to the incident energy flux in direction  $\vec{s}_o$  is defined as albedo. For the multiple scattering part of albedo, the time dependent albedo is given as [Akkermans *et al* (1988)]:

$$\alpha(\vec{s}_o, \vec{s}; t) = \frac{c}{4\pi l^2} \int_0^\infty dz \int_0^\infty dz' \exp\left(-\frac{z}{\mu_o l}\right) \exp\left(-\frac{z'}{\mu l}\right) \times \int_s d^2\rho [1 + \cos \vec{q} \cdot (\vec{r} - \vec{r}')] G(\vec{\rho}, z, z'; t) \quad (6.25)$$

where  $\mu$  and  $\mu_o$  are projections of  $\vec{s}$  and  $\vec{s}_o$  on  $z$ -axis respectively,  $z$  and  $z'$  are projections of  $\vec{r}$  and  $\vec{r}'$  on  $z$ -axis respectively,  $\vec{q}$  is the transfer wave vector  $\vec{s} + \vec{s}_o$ .

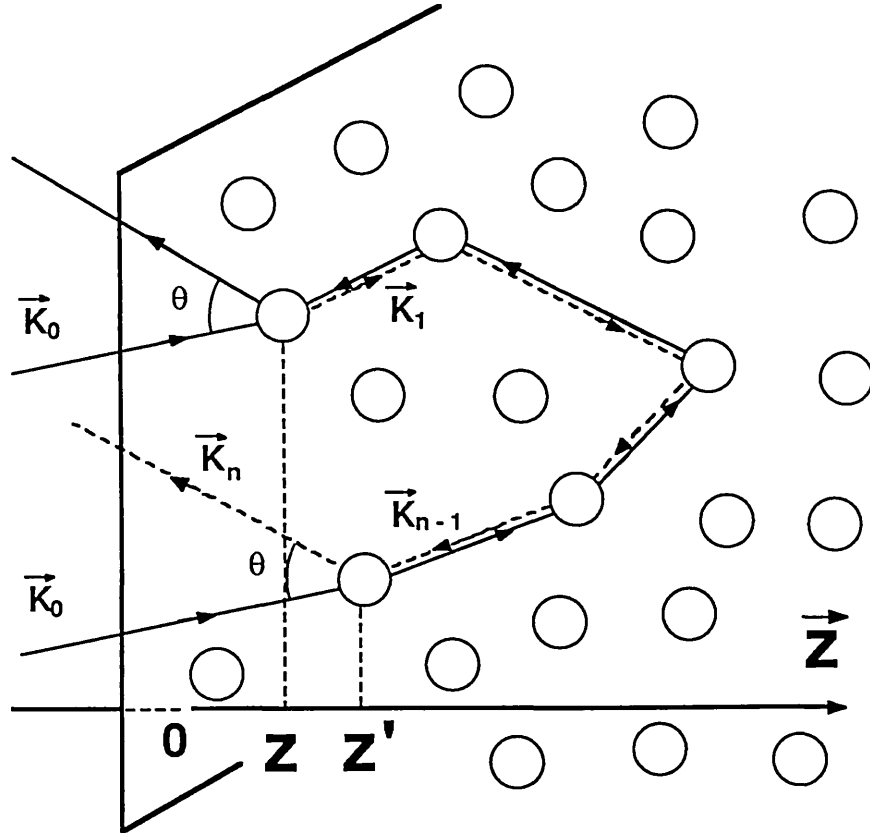


Fig.6.7. Scattering geometry used for calculation of albedo.

The stationary albedo is given by:

$$\begin{aligned}\alpha(\vec{s}_o, \vec{s}) &= \int_0^\infty \alpha(\vec{s}_o, \vec{s}; t) dt \\ &= \frac{c}{4\pi l^2} \int_0^\infty dz \int_0^\infty dz' \exp\left(-\frac{z}{\mu_o l}\right) \exp\left(-\frac{z'}{\mu l}\right) \\ &\quad \times \int_s d^2 \rho [1 + \cos \vec{q} \cdot (\vec{r} - \vec{r}')] G(\vec{\rho}, z, z')\end{aligned}\quad (6.26)$$

where  $G(\vec{\rho}, z, z')$  is the time integral of the Green function for the stationary process:

$$G(\vec{\rho}, z, z') = \int_0^\infty G(\vec{\rho}, z, z'; t) dt \quad (6.27)$$

This formula can be explained physically:  $dz d^2 \rho l^{-1} \exp[-z/(\mu_o l)]$  is the released energy density scattered from the elementary volume  $dz d^2 \rho$  where the incident energy is first scattered at point  $\vec{r}$  in the elementary volume.  $G(\vec{\rho}, z, z')$  is the Green function which describes the light transport from point  $\vec{r}$  to point  $\vec{r}'$  no matter how many scattering processes are involved between two points. The Green function is given by the ratio of the energy density at point  $\vec{r}'$  to the energy production rate at point  $\vec{r}$ .  $(c/4\pi) dz' l^{-1} \exp[-z'/(\mu l)]$  represents the last scattering process at point  $\vec{r}'$ . It is the fraction of emergent scattered energy density per solid angle at point  $\vec{r}'$  due to a incident energy density transported from Green function, while  $[1 + \cos \vec{q} \cdot (\vec{r} - \vec{r}')]$  accounts for the interference effects between the two emergent time reversed pairs.

As can be seen from the above equation, the calculation of albedo is then reduced to the calculation of the Green function which is a classical description of intensity transport. In transport theory, the transport of the light intensity in the medium is described by the Green function  $G(\vec{r}, \vec{r}'; t)$  defined by the response at  $\vec{r}'$  at the time  $t$  to a pulse at  $\vec{r}$  at time  $t = 0$ . For a large number of scatterings (or in the long time limit), Green function is well approximated by the solution of a time-dependent diffusion equation. As pointed by Akkermans *et al* (1988) the time dependent Green function  $G(\vec{\rho}, z, z'; t)$  is given as follows:

$$G(\vec{\rho}, z, z'; t) = \frac{1}{(4\pi Dt)^{3/2}} \exp\left(-\frac{\rho^2}{4Dt}\right)$$

$$\times \left[ \exp\left(-\frac{(z-z')^2}{4Dt}\right) - \exp\left(-\frac{(z+z'+2z_o)^2}{4Dt}\right) \right] \quad (6.28)$$

where  $D$  is the photon diffusion constant and  $z_o = (2/3)l$  within the diffusion approximation.

Therefore by integrating Eq.(6.26), denoting  $\theta$  as the scattering angle, the scattering angle dependent total albedo (coherent part and incoherent part) is:

$$\alpha_t(\theta) = \alpha_c(\theta) + \alpha_i(\theta) \quad (6.29)$$

where  $\alpha_c(\theta)$  is the coherent part of albedo:

$$\alpha_c(\theta) = \frac{3\mu\mu_o}{8\pi(1+\mu q_\perp l)(1+\mu_o q_\perp l)} \left( \frac{2\mu\mu_o}{\mu+\mu_o} + \frac{1-\exp(-2q_\perp z_o)}{q_\perp l} \right) \quad (6.30)$$

and  $\alpha_i(\theta)$  is the incoherent part of albedo:

$$\alpha_i(\theta) = \frac{3}{8\pi} \left( 1 + \frac{2z_o}{l} \right) \quad (6.31)$$

where  $q_\perp$  is the component of  $\vec{q}$  normal to the  $z$ -axis. Within the small angle limit,  $\mu \approx \mu_o$  and for normal incident  $\mu \approx \mu_o \approx 1$ , the total scattering albedo is therefore:

$$\alpha_t(\theta) = \frac{3}{8\pi} \left[ 1 + \frac{2z_o}{l} + \frac{1}{(1+q_\perp l)^2} \left( 1 + \frac{1-\exp(-2q_\perp z_o)}{q_\perp l} \right) \right] \quad (6.32)$$

Let us now take a look into the enhancement line shape. First, we analyse the scattering albedos in two extreme cases, namely when  $\theta = 0$  and  $\theta \rightarrow \infty$  where only incoherent part of albedo exists. For quasinormal incidence and emergence,  $q_\perp \approx 2\pi\theta/\lambda$ . Therefore when  $\theta \rightarrow 0$ ,  $q_\perp \rightarrow 0$  and  $\theta \rightarrow \infty$ ,  $q_\perp \rightarrow \infty$ , we have:

$$\left. \begin{aligned} \lim_{\theta \rightarrow 0} \alpha_t(\theta) &= \frac{6}{8\pi} \left( 1 + \frac{2z_o}{l} \right) \\ \lim_{\theta \rightarrow \infty} \alpha_t(\theta) &= \frac{3}{8\pi} \left( 1 + \frac{2z_o}{l} \right) \end{aligned} \right\} \quad (6.33)$$

It is easy to see that the enhancement at backscattering is theoretically a factor of two. Secondly the angular width of the albedo (FWHM) is found to be in the

order of  $\lambda/(2\pi l)$ . Thirdly, when  $\theta$  is very small, the albedo can be approximately expressed as:

$$\alpha_t(\theta) \approx \frac{6}{8\pi} \left( 1 + \frac{2z_o}{l} - \frac{2\pi z_o}{\lambda} |\theta| \right) \quad (6.34)$$

It can be seen from this formula that the albedo has a linear relationship with the scattering angle  $\theta$  near backscatter and the enhancement line shape is therefore triangular near backscatter.

The above analysis is only valid for isotropic scattering. For anisotropic scattering an approximate expression can be obtained by replacing the mean free path  $l$  by the transport mean free path  $l^*$ . For quasinormal incidence and emergence, a calculation of backscattering albedo can be made from Eq.(6.32). In Fig.6.8. a comparison between this theoretically calculated enhancement and experimental measurement is shown. The wavelength  $\lambda$  is at  $514.5 \text{ nm}$  and the transport mean free path  $l^*$  is  $\approx 19 \mu\text{m}$  in our experiments.

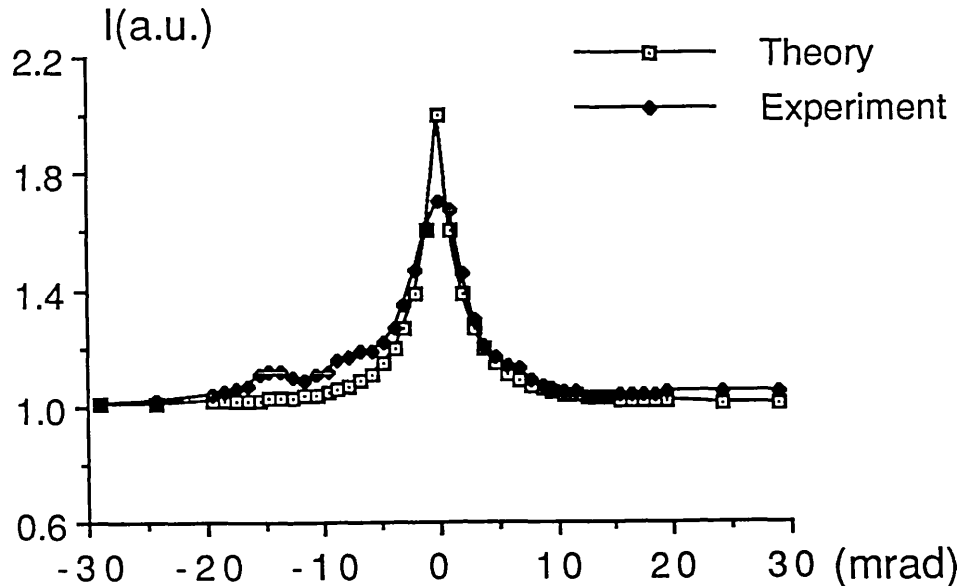


Fig.6.8. Theoretical curve compared with experimental curve.

From Fig.6.8, it is seen that the experimental enhancement peak is rounded from the theoretically predicted triangular shape.

Apart from the reasons mentioned in §6-2 (p.133), there are experimental conditions responsible for this as well. The first is due to angular averaging effects in the experiment which smooth the enhancement peak. In the experiment a laser beam of divergence of  $\approx 0.7 \text{ mrad}$  illuminated the sample. This divergence will give rise to an angular averaging effect. In addition, a  $65 \mu\text{m}$  pinhole was placed in front of the detector at a distance of  $2 \text{ m}$  away from the sample which gave rise to an averaging angle of  $0.32 \text{ mrad}$  for the detector. By taking into account the above mentioned two averaging factors the overall averaging angle is about  $1 \text{ mrad}$ . Therefore the detected value at backscatter is the average over  $1 \text{ mrad}$  around backscatter of the theoretical curve shown in Fig.6.8. The effect of this is small, except near the region of the peak; a simple calculation shows the value of the peak falls from 2.0 to approximately 1.9. This is not sufficient to explain the discrepancy between the experimental curve and theory.

The second reason is due to any effects which limit the maximum length of the scattering paths, such as absorption. Absorption will reduce the intensity contribution from both incoherent and coherent contributions. Longer coherent paths determine the magnitude and shape of the enhanced backscatter peak at the origin. Therefore, if these are diminished (e.g. by absorption), the magnitude of the peak is decreased and its shape changes. This is studied by Cwilich *et al* (1987) and they show that the enhancement peak is rounded and its shape is roughly Lorentzian rather than triangular in the presence of absorption. We cannot predict the magnitude of the effect as the absorption was not known in our experiments.

*see opposite page.*

### §6-3-2: Spatial Anisotropy of Enhancement Line Shape

The above analysis is for scalar waves and it is also valid for the co-polarised detection. For the cross-polarised detection the enhancement line shape is more complicated and it was studied by Stephen *et al* (1986b) with diffusion theory. Even for the co-polarised detection it is found [Albada *et al* (1987a, 1987b)] that the enhancement cone shows a spatial anisotropy, namely, its width (HWHM) is larger in the direction of the polarization vector of the incident beam than that in perpendicular direction.

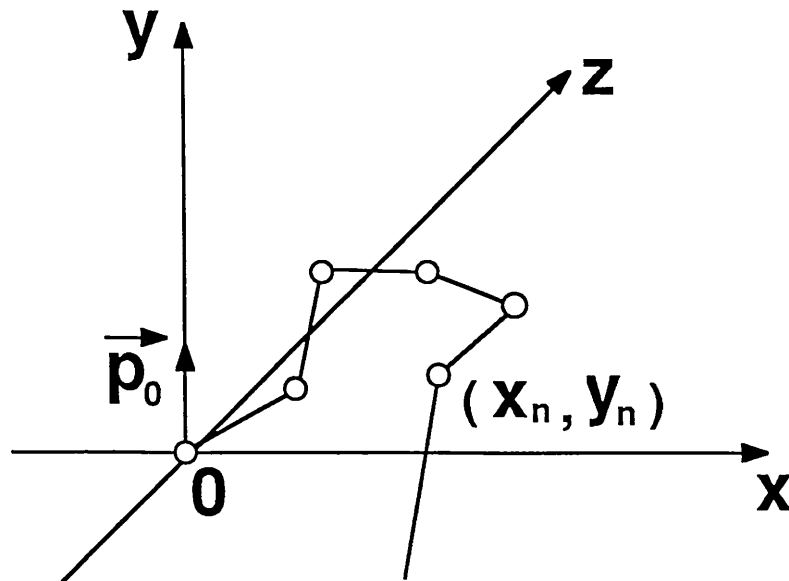


Fig.6.9. Scattering geometry showing the spatial anisotropy of the enhancement peak.

This spatial anisotropic characteristic can be understood from the vector characteristic of the light. In Fig.6.9, the  $z$ -axis is the incident direction and  $x$ - $y$  plane is perpendicular to the incident direction. The polarisation of the incident light is in the direction of the  $y$ -axis. Suppose there is a scatterer at the origin; the scattered amplitude by this scatterer in the  $x$ - $z$  plane is a maximum and in the  $y$ - $z$  plane is a minimum for the incident direction and polarisation direction. The amplitude scattered along the  $x$ -axis is maximal and along the  $y$ -axis is zero. For the other scatterers the scattered amplitudes should also have a preference to be in the  $x$ -direction rather than in the  $y$ -direction. Therefore on average the scattered light wave is more likely to transport in  $x$ -direction than in  $y$ -direction with the above given incident direction and polarisation direction. If we denote  $(x_n, y_n)$  as the coordinates of the last scatterer from which a co-polarised intensity can be detected, it can be concluded that under the same condition:

$$\bar{x}_n > \bar{y}_n \quad (6.35)$$

Hence on average the time reversed pairs which give rise to a constructive interference at and around backscatter have a larger separations in  $x$ -direction than in  $y$ -direction. The enhancement cone should therefore have a narrower width in  $x$ -direction which is the direction perpendicular to incident polarisation direction and a broader width in  $y$ -direction which is the direction parallel with the incident polarisation. It can also be intuitively imagined that a quantitative analysis of the enhancement spatial anisotropy should be related to the average of ratios of scattering phase function (the probability of scattered amplitudes in certain direction) from a particle in the  $y$ -axis to that in the  $x$ -axis.

#### §6-4: Polarization Dependence of Backscattered Light

The enhanced backscattering phenomenon can be understood in terms of classical interference between a wave multiply scattered along ~~a~~ certain path and a wave multiply scattered along its counterpath. Suppose the scatterers are small and point-like; a wave multiply scattered can be expressed by a product of a series

of matrices, each matrix representing the incident and viewing directions. For both forward and reversed waves, the matrices are related in such a way that at backscatter, for co-polarised detection, they have exactly the same element (amplitude) while for the cross-polarised case they have different amplitude although their phases are still correlated.

### §6-4-1: Analysis of Enhancement Factors

When a monochromatic beam is incident on a single molecule which has a polarisability specified by a polarisability tensor  $\eta_p$ :

$$\eta_p = \begin{pmatrix} \eta_{xx} & \eta_{xy} & \eta_{xz} \\ \eta_{yx} & \eta_{yy} & \eta_{yz} \\ \eta_{zx} & \eta_{zy} & \eta_{zz} \end{pmatrix} \quad (6.36)$$

the incident light induces a dipole moment:

$$\vec{D}_m(t) = \eta_p \cdot \vec{E}(t) = E(t) \eta_p \cdot \vec{P} \quad (6.37)$$

where  $\vec{E}(t)$  is the incident field which varies with time and  $\vec{P}$  represents the incident polarisation vector. According to classical radiation theory a time varying dipole emits electromagnetic radiation. The electric field  $\vec{E}_s$  of the radiated or scattered light at the detection point is proportional to a vector decided by the incident and viewing direction:

$$\vec{E}_s \propto (\vec{K} \times \vec{D}_m''(t)) \times \vec{K} \quad (6.38)$$

where  $\vec{K}$  is the viewing direction and  $\vec{D}_m''(t)$  is the second time derivative of  $\vec{D}_m(t)$ .

For a spherical molecule, the polarisability tensor is isotropic, namely:

$$\eta_p = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & \eta \end{pmatrix} \quad (6.39)$$

In this case the induced dipole moment is always parallel to the applied electric field. Therefore we have:

$$\vec{D}_m(t) = \eta \cdot \vec{E}(t) = E(t) \eta \cdot \vec{P} \quad (6.40)$$

$$\vec{E}_s = E_s(t) \quad \vec{P}_s \propto E''(t) \quad (\vec{K} \times \vec{P}) \times \vec{K} \quad (6.41)$$

and hence it is found that:

$$\vec{P}_s \propto (\vec{K} \times \vec{P}) \times \vec{K} \quad (6.42)$$

This implies that the scattered field amplitude (which is a vector) by an individual scatterer is proportional to a vector  $(\vec{K} \times \vec{P}) \times \vec{K}$ .

For vectors

$$\left. \begin{aligned} \vec{K} &= k_x \vec{i} + k_y \vec{j} + k_z \vec{k} \\ \vec{P} &= P_x \vec{i} + P_y \vec{j} + P_z \vec{k} \end{aligned} \right\} \quad (6.43)$$

It is found that

$$(\vec{K} \times \vec{P}) \times \vec{K} = M(k) \cdot \vec{P} \quad (6.44)$$

where  $M(k)$  is the representation of incident and viewing directions and it is a symmetric matrix:

$$M(k) = \begin{pmatrix} k_y^2 + k_z^2 & -k_x k_y & -k_x k_z \\ -k_y k_x & k_x^2 + k_z^2 & -k_y k_z \\ -k_z k_x & -k_z k_y & k_y^2 + k_x^2 \end{pmatrix} \quad (6.45)$$

Suppose the scatterers are point-like, in which case the scattering is Rayleigh scattering. For one scattering path the scattered field amplitudes are:

$$\left. \begin{aligned} \vec{P}_1 &= (\vec{K}_1 \times \vec{P}_0) \times \vec{K}_1 = M(K_1) \cdot \vec{P}_0 \\ \vec{P}_2 &= (\vec{K}_2 \times \vec{P}_1) \times \vec{K}_2 = M(K_2) \cdot \vec{P}_1 \\ &= M(K_2)M(K_1) \cdot \vec{P}_0 \\ &\dots\dots\dots \\ \vec{P}_n &= (\vec{K}_n \times \vec{P}_{n-1}) \times \vec{K}_n = M(K_n) \cdot \vec{P}_{n-1} \\ &= \dots = M(K_n)M(K_{n-1}) \dots M(K_1) \vec{P}_0 \end{aligned} \right\} \quad (6.46)$$

For the time reversed scattering path the scattered field amplitudes are:

$$\left. \begin{aligned} \vec{P}'_1 &= (\vec{K}_{n-1} \times \vec{P}_0) \times \vec{K}_{n-1} = M(K_{n-1}) \cdot \vec{P}_0 \\ \vec{P}'_2 &= (\vec{K}_{n-2} \times \vec{P}'_1) \times \vec{K}_{n-2} = M(K_{n-2}) \cdot \vec{P}'_1 \\ &= M(K_{n-2})M(K_{n-1}) \cdot \vec{P}_0 \\ &\dots\dots\dots \\ \vec{P}'_n &= (\vec{K}_n \times \vec{P}'_{n-1}) \times \vec{K}_n = M(K_n) \cdot \vec{P}'_{n-1} \\ &= \dots = M(K_n)M(K_1) \dots M(K_{n-1}) \vec{P}_0 \end{aligned} \right\} \quad (6.47)$$

Now we compare  $\vec{P}_n$  and  $\vec{P}'_n$ . Effectively, we are comparing the following products of a series matrices representing  $\underbrace{\text{certain scattering path and its time reversed counterpath}}_a$ :

$$\begin{cases} M(K_{n-1}) \dots M(K_1) \\ M(K_1) \dots M(K_{n-1}) \end{cases}$$

Mathematically, the transpose of a product of a series of matrices is equal to the product of these matrices in reverse order, namely:

$$[M(K_{n-1}) \dots M(K_1)]^T = [M(K_1)]^T \dots [M(K_{n-1})]^T \quad (6.48)$$

As pointed out already,  $M(K)$  is symmetric, therefore we have:

$$M(K_i) = [M(K_i)]^T \quad i = 1, 2, \dots, n-1 \quad (6.49)$$

hence:

$$[M(K_{n-1}) \dots M(K_1)]^T = M(K_1) \dots M(K_{n-1}) \quad (6.50)$$

and we set the above equation as  $M_p(K)$ . Therefore at backscatter, by setting:

$$M(K_n) = M(K_b) \quad (6.51)$$

For the time reversed pairs:

$$\begin{cases} \vec{P}_n = M(K_b) \cdot [M_p(K)]^T \cdot \vec{P}_0 \\ \vec{P}'_n = M(K_b) \cdot M_p(K) \cdot \vec{P}_0 \end{cases} \quad (6.52)$$

If we denote:

$$M_p(K) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad (6.53)$$

then we have:

$$[M_p(K)]^T = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \quad (6.54)$$

If the coordinate system is chosen in such a way so that the incident polarisation is:

$$\vec{P}_0 = \vec{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

therefore the backscattering (as well as incident) direction can be represented by:

$\vec{K}_b = k_{by}\vec{j} + k_{bz}\vec{k}$ . This is because  $\vec{P}_0$  and  $\vec{K}_b$  are perpendicular to each other.

Hence we have:

$$M(K_b) = \begin{pmatrix} k_{by}^2 + k_{bz}^2 & 0 & 0 \\ 0 & k_{bz}^2 & -k_{by}k_{bz} \\ 0 & -k_{by}k_{bz} & k_{by}^2 \end{pmatrix} \quad (6.55)$$

By substituting these known matrices into Eq.(6.52), it is found:

$$\vec{P}_n = \begin{pmatrix} a_{11}(k_{by}^2 + k_{bz}^2) \\ a_{12}k_{bz}^2 - a_{13}k_{by}k_{bz} \\ a_{13}k_{by}^2 - a_{12}k_{by}k_{bz} \end{pmatrix} \quad (6.56)$$

$$\vec{P}'_n = \begin{pmatrix} a_{11}(k_{by}^2 + k_{bz}^2) \\ a_{21}k_{bz}^2 - a_{31}k_{by}k_{bz} \\ a_{31}k_{by}^2 - a_{21}k_{by}k_{bz} \end{pmatrix} \quad (6.57)$$

If now we compare  $\vec{P}_n$  and  $\vec{P}'_n$ , it is found that they have the *same* components (same amplitude) in the  $\vec{i}$  direction, which is the incident polarisation direction shown above—this is the co-polarised case. In directions  $\vec{j}$  and  $\vec{k}$ , the cross-polarised cases, they have *different* amplitudes. From an interference point of view, therefore, at backscatter for co-polarised detection, we shall see an enhancement of up to a factor of 2, while for cross-polarised detection, a enhancement factor less than 2 should be observed, except for  $n = 2$  when we find that  $\vec{P}_2 = \vec{P}'_2$ .

From the above formula, it can be seen that:

1. For co-polarised detection, the enhancement factor of  $n$ -th order scattering is:

$$\begin{aligned}
 E_{co}^{(n)} &= \frac{\langle [a_{11}(k_{by}^2 + k_{bz}^2) + a_{11}(k_{by}^2 + k_{bz}^2)]^2 \rangle}{\langle [a_{11}(k_{by}^2 + k_{bz}^2)]^2 + [a_{11}(k_{by}^2 + k_{bz}^2)]^2 \rangle} \\
 &= \frac{4 \langle a_{11}^2 (k_{by}^2 + k_{bz}^2)^2 \rangle}{2 \langle a_{11}^2 (k_{by}^2 + k_{bz}^2)^2 \rangle} = 2
 \end{aligned} \tag{6.58}$$

Therefore the total enhancement factor contributed from all scattering orders and paths is 2.

2. For cross-polarised detection (see Appendix A), the enhancement factor of  $n$ -th order scattering is given by [Albada *et al* (1987a)]:

$$\begin{aligned}
 E_{cr}^{(n)} &= \frac{\langle [(a_{12}k_{bz}^2 - a_{13}k_{by}k_{bz}) + (a_{21}k_{bz}^2 - a_{31}k_{by}k_{bz})]^2 \rangle}{\langle (a_{12}k_{bz}^2 - a_{13}k_{by}k_{bz})^2 \rangle + \langle (a_{21}k_{bz}^2 - a_{31}k_{by}k_{bz})^2 \rangle} \\
 &= 1 + \frac{3(0.7^{n-1} - 0.5^{n-1})}{2(1 - 0.7^{n-1})}
 \end{aligned} \tag{6.59}$$

Summing up all the contributions from different scattering orders and paths, the total enhancement factor is found to be typically on the order of 1.2.

## §6-4-2: Polarisation Degree

Stokes' parameters are very important in analysing the polarization properties of the light field. The measurements of the Stokes' parameters of backscattered light at and around backscatter are presented below.

Stokes' parameters are defined as follows:

$$\left. \begin{aligned}
 S_0 &= I(0^\circ, 0) + I(90^\circ, 0) \\
 S_1 &= I(0^\circ, 0) - I(90^\circ, 0) \\
 S_2 &= I(45^\circ, 0) - I(135^\circ, 0) \\
 S_3 &= I(45^\circ, \frac{\pi}{2}) - I(135^\circ, \frac{\pi}{2})
 \end{aligned} \right\} \tag{6.60}$$

where  $I(\theta, \varepsilon)$  denotes the intensity of the light vibrations in the direction making an angle  $\theta$  with  $ox$  axis, when the  $y$ -component is subjected to a retardation  $\varepsilon$  with respect to the  $x$ -component.

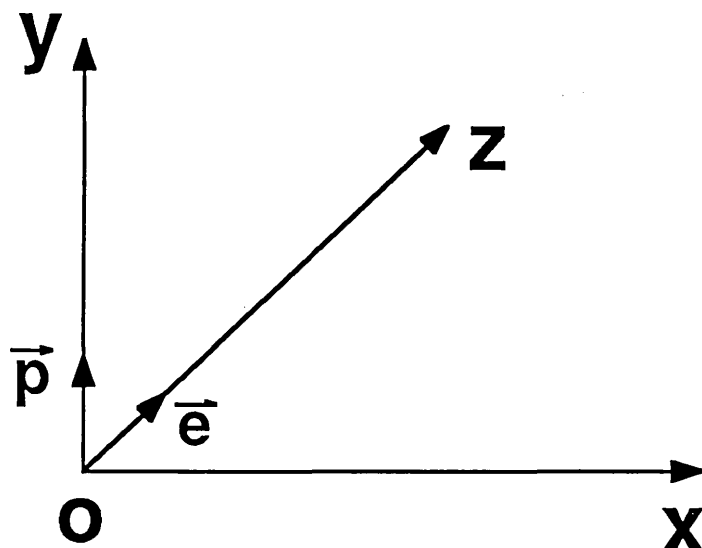


Fig.6.10. Geometry for calculating polarisation degree.

In our experiments, we choose the Cartesian coordinate system such that the incident polarisation direction is in the  $oy$  axis and the incidence direction is along the  $oz$  axis. Under such a scattering geometry, there is no preference for the output linear polarisation direction of scattered light in either  $45^\circ$  or  $135^\circ$  with respect to the  $ox$  axis or for left-handed or right-handed circular scattered light. In other words, we shall see that:

$$\begin{cases} S_2 = I(45^\circ, 0) - I(135^\circ, 0) = 0 \\ S_3 = I(45^\circ, \frac{\pi}{2}) - I(135^\circ, \frac{\pi}{2}) = 0 \end{cases} \quad (6.61)$$

Therefore the polarisation degree is:

$$P = \frac{S_0}{\sqrt{S_1^2 + S_2^2 + S_3^2}} = \frac{S_0}{S_1} \quad (6.62)$$

If we denote  $P_0$  as the polarisation degree at a large angle and the polarisation

( $\approx 30 \text{ mrad}$ )

degree around backscatter is  $P_b$ , then it is easy to write:

$$\begin{cases} P_b = \frac{E_{co}I_{co} - E_{cr}I_{cr}}{E_{co}I_{co} + E_{cr}I_{cr}} \\ P_0 = \frac{I_{co} - I_{cr}}{I_{co} + I_{cr}} \end{cases} \quad (6.63)$$

where  $E_{co}$  and  $E_{cr}$  are enhancement factors for co-polarised and cross-polarised detection respectively;  $I_{co}$  and  $I_{cr}$  are the intensities at a large angle for co-polarised and cross-polarised detection respectively. Solving the above equation we get:

$$P_b = \frac{(E_{co} - E_{cr}) + (E_{co} + E_{cr}) P_0}{(E_{co} + E_{cr}) + (E_{co} - E_{cr}) P_0} \quad (6.64)$$

As can be seen from Eq.(6.64) the polarisation degree also shows an enhancement at backscatter. Measurements listed in Tab.6.2 show and confirm an enhancement at backscatter.

| Angles<br>(mrad) | 0    | 3    | 6   | 9   | 12  | 18  | 23  |
|------------------|------|------|-----|-----|-----|-----|-----|
| P(%)             | 18.1 | 14.3 | 9.3 | 9.2 | 8.9 | 7.5 | 7.2 |

Tab.6.2. Measurements of polarisation degree against angle.

## §6-5: Dynamic Properties of Backscattered Light

The dynamic properties of scattered light are related to the Brownian motion of the scatterers in water. Because of the random motion of the scatterers the scattered light intensities are fluctuating randomly. In our experiments, the temporal intensity correlation functions and photon counting statistics are measured to describe the dynamic properties of backscattered light.

### §6-5-1: Temporal Correlation

Measurements of the temporal correlation of both co-polarised and cross-polarised returns around the enhanced backscattering cone of a dense collection of latex spheres are presented. The correlation time shows a completely different variation with angle in the two cases and an explanation of this is given. The photon statistics have also been measured for co-polarised and cross-polarised intensities. The normalised factorial moments of the photon counts (which equal the normalised intensity moments) are independent of angle (for angles within  $\approx 15\text{mrad}$  of backscatter) and detecting polarisations, and they are in agreement with those derived from Gamma intensity distribution with the descriptive parameter  $u = 1.20$ .

Much effort has been put into the study of this phenomenon. The statistical properties of such media, however, are very important in understanding the phenomenon, scattering process and the media itself. Maret *et al* (1987) have studied the temporal correlation of the intensity with respect to angle for the co-polarised return. Qu *et al* (1988) have reported temporal correlation studies with respect to the angle (at and around backscatter) for both co-polarised and cross-polarised detections. In this section, we shall first summarise the temporal correlation properties. The photon counting statistics at different angles for both cases will also be reported.

The experimental setup was the same as shown in Fig.6.4. The experiments were carried out by using the linearly polarised 514.5 nm line Argon ion laser. The sample was 10% by volume latex beads in water with an average diameter of 0.46  $\mu\text{m}$ . The scattered light was detected by using a photomultiplier tube with a pinhole much smaller than the backscattered speckle size. The signals were then analyzed by a digital correlator and its associated micro-computer system.

The temporal correlation function measured in our experiments is defined as:

$$C(\tau) = \frac{\langle I(0) I(\tau) \rangle}{\langle I \rangle^2} - 1 \quad (6.65)$$

where the angular brackets represent an ensemble average. Typical sets of data for both co-polarised and cross-polarised detections are plotted in Fig.6.11. It is found that the correlation curves are non-exponential due to multiple scattering. The resultant correlation curve is the superposition of the contributions from all different paths of different lengths.

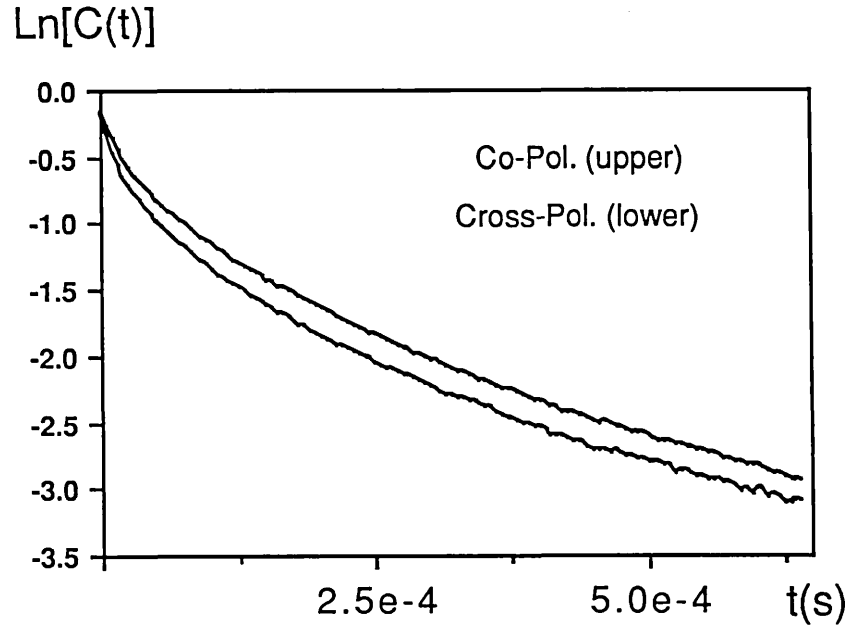


Fig.6.11. The logarithm of the temporal auto-correlations measured at backscatter for both co-polarised and cross-polarised cases, with a sampling time of  $5\mu s$ . The speckle contrast  $\frac{\sigma^2}{\langle I \rangle^2} \approx 0.85$ . The non-straight curves indicate the non-exponential decay of correlation functions (Upper curve is for the co-polarised and lower curve for cross-polarised cases).

For the co-polarised case, the correlation decay is slower than that for the cross-polarised case. This reveals that apart from the involvement of single scattering for co-polarised correlation curves, the shorter paths contribute more, while for cross-polarised detection the longer paths have a larger role to play. Therefore, as shown later, in general the correlation time is longer for the co-polarised case than that for the cross-polarised case.

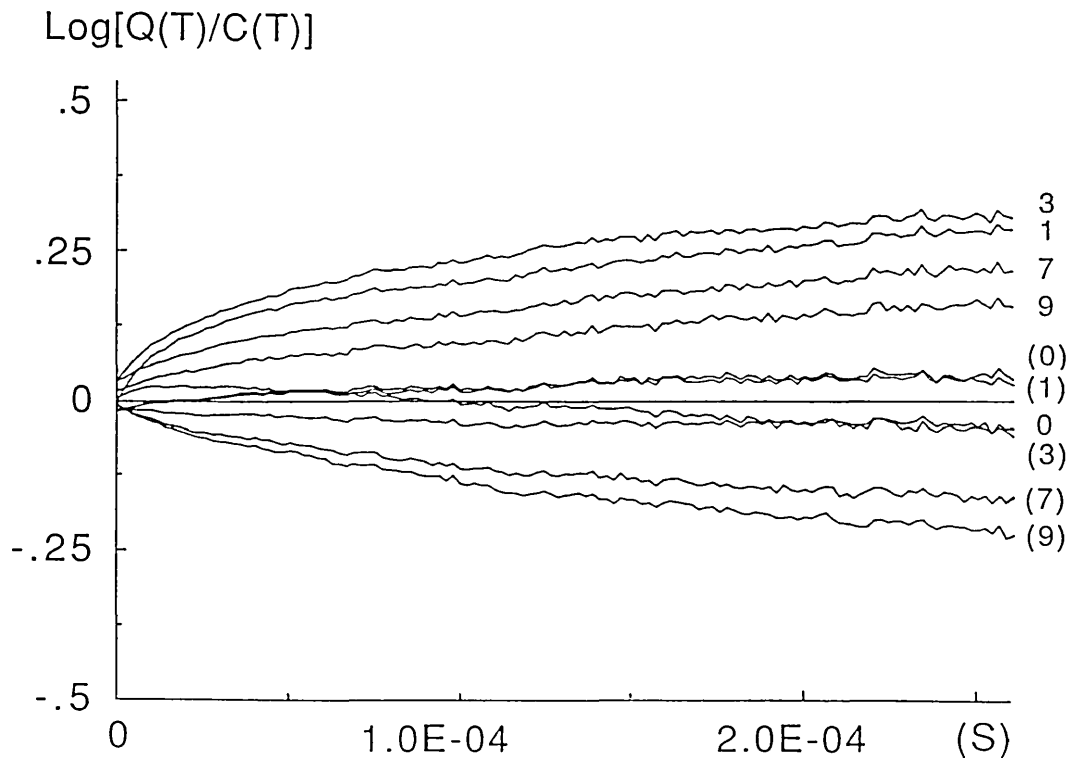


Fig.6.12. The correlation ratio as a function of correlation time for different scattering angles. The numbers at the right are the different measurement angles in milliradians (numbers without parentheses are for the co-polarised case; numbers with parentheses for the cross-polarised case).  $C(T)$  is measured at  $\theta = 40 \text{ mrad}$  in the co-polarised case.

Fig.6.12 summarises the measured correlation data for the co-polarised and cross-polarised cases at angles of 0,1,3,7 and 9 *mrad* from backscatter: the vertical axis is the logarithm of the **ratio** of the measured correlation function in each case to that of the co-polarised case at a large angle ( $\theta \approx 40 \text{ mrad}$ ). For the co-polarised case, this ratio starts at unity for  $\theta = 0$ , increases to a maximum at  $\theta \approx 3 \text{ mrad}$  and then falls to unity again at high angles. However, for the cross-polarised case, the ratio starts at unity at  $\theta = 0$  and decreases smoothly as the angle increases.

To describe the non-exponential fluctuation rate and compare the different fluctuating behaviour at and near backscatter for both co-polarised and cross-polarised cases, let us look at the time dependence effects of the backscattered intensity. There are three characteristic time scales which affect the dynamic properties of backscattered light. The first is transport elastic scattering time  $\tau_e$  which describes the time scale for the light to travel through the transport mean free paths and is in the order of  $10^{-13}$  s. The second is the time scale associated with the Brownian motion of the scatterers. It is given by:

$$\tau_B = \frac{\lambda^2}{D_B} \quad (6.66)$$

where  $D_B$  is the diffusion constant of the particles in suspension. Since the Brownian time scale can be effectively measured by the correlation time of scattered intensity, this time scale can be understood from the measurements of correlation time. The third time scale affecting the dynamic properties of backscattered light is the time reversal breaking down time scale ( $\tau_r$ ) within which the time reversed pairs still preserve the coherence and therefore can interfere with each other. This time scale is related to  $\tau_e$  and  $\tau_B$ . This can be seen from the following argument: For a  $N$ -th multiple scattering path of length  $L = Nl$ , the time taken for light to travel through this path is  $t = N\tau_e$ , during which, because of Brownian motion, each scatterer moves by diffusion over a distance  $\sqrt{\Delta r^2} \approx \sqrt{D_B t}$ . Therefore  $N$  scatterers move over the distance of  $N\sqrt{D_B t}$  and it must be less than a wavelength  $\lambda$  to preserve the coherence between the two time reversed pairs, i.e.  $t < \lambda^2/(ND_B)$ . Bearing in mind  $t = N\tau_e$  and the definition of  $\tau_B$  we have:  $t < \sqrt{\tau_e \tau_B}$ . Hence the time reversal break down time scale is  $\tau_r = \sqrt{\tau_e \tau_B}$  and the corresponding total scattering length is decided by:

$$\tau_r = N\tau_e = \sqrt{\tau_e \tau_B} \quad N = \sqrt{\frac{\tau_B}{\tau_e}} \quad (6.67)$$

this gives the maximal multiple scattering process from which the enhanced backscattering phenomenon can be observed.

To look at the dynamic scattering properties in detail, we shall look into the

behaviour of the correlation time. The correlation time is defined as follows:

$$\tau_c = \int_0^{+\infty} \frac{\langle \delta I(0) \delta I(\tau) \rangle}{\langle (\delta I)^2 \rangle} d\tau \quad (6.68)$$

while

$$\langle \delta I(0) \delta I(\tau) \rangle = \langle I(0) I(\tau) \rangle - \langle I \rangle^2 \quad (6.69)$$

$$\begin{aligned} \frac{\langle \delta I(0) \delta I(\tau) \rangle}{\langle \delta I^2 \rangle} &= \frac{\langle I(0) I(\tau) \rangle - \langle I \rangle^2}{\langle \delta I^2 \rangle} \\ &= \frac{C(\tau)}{C(0)} \end{aligned} \quad (6.70)$$

where

$$C(\tau) = \frac{\langle I(0) I(\tau) \rangle}{\langle I \rangle^2} - 1 \quad (6.71)$$

is our actual processed data measurement for the correlation from the digital correlator and therefore:

$$\tau_c = \int_0^{\infty} d\tau \frac{C(\tau)}{C(0)} \quad (6.72)$$

The measurements of  $\tau_c$  against angle for both co-polarised and cross-polarised detection have been made and are shown in Fig.6.13. As can be seen from the Figure, inside the enhancement cone, the temporal characteristics for co-polarised and cross-polarised cases have very different behaviour. For the co-polarised case, the curve has a double-peak structure while a single-peak similar to that of the intensity enhancement appears for cross-polarised detection.

Here we give a physical explanation for the observed phenomena:

The different behaviour of the two cases arises from the fact that the contributions from the diffraction of the scattering paths and from the coherence contribution are weighted in terms of the measuring angle.

The correlation function of scattered light consists of contributions from two parts, namely, one from the incoherent intensity and the other from the coherent intensity. The contributions from incoherent light are uniform against angle

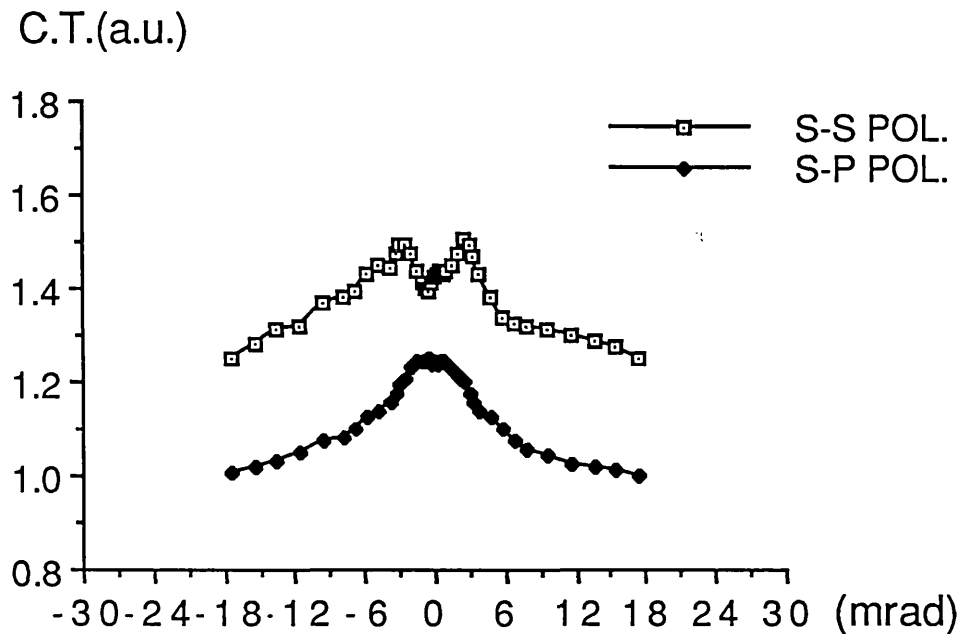


Fig.6.13. The measured correlation time against scattering angle for co-polarised and cross-polarised detections.

for both co-polarised and cross-polarised cases, while the coherent contributions result in different behaviours for co-polarised and cross-polarised detections.

Let us now concentrate on the coherent contributions to the correlation function. There are two fundamental factors among the coherent part contributions which have opposite effects on correlation time behaviour. One factor is from the contributions of the different light scattering paths of different lengths (path length contribution). The other factor accounts for the total coherent intensity contribution (enhancement factor contribution).

In the co-polarised case, the field correlation function is given by Maret *et al* (1987): (see Appendix B)

$$\langle E(0)E^*(\tau) \rangle = \int_0^{+\infty} I(L) \exp\left(\frac{-L\tau}{4l^*\tau_o}\right) dL \quad (6.73)$$

where  $I(L)$  is the intensity contribution of transport paths of length  $L$ ,  $l^*$  is transport mean free path and  $\tau_o$  is the single scattering relaxation time at backscatter.

It can be found from the formula that the contribution to the correlation from large  $L$  decays faster than that from shorter  $L$ , that is to say, the contribution from longer paths (larger  $L$ ) have shorter correlation time, and shorter paths have longer correlation time. The resultant correlation time due to path length contribution depends on how the contributions from different paths are weighted.

As pointed out by Akkermans *et al* (1986), when one moves away from the backscatter ( $\theta = 0$ ), but still inside the enhancement cone, only paths of length  $L < \lambda^2/(l\theta^2)$  contribute significantly to the intensity enhancement. Therefore when  $\theta$  becomes larger, the upper limit of  $L$  becomes smaller, shorter paths contribute more and the correlation time due to this contribution therefore becomes longer.

When one moves away from backscatter, the time-reversed paths become out of phase. This makes the coherent intensity decrease and subsequently reduce the coherent enhancement contribution to the correlation time.

It can be seen by combining the contributions to the correlation time from <sup>the</sup> two factors <sup>mentioned</sup> that at backscatter the longest paths count more and hence the correlation is shorter here. When  $\theta$  increases, as shorter and shorter paths contribute more, the correlation time becomes larger but there comes to a point where the coherent enhancement contribution has decreased to such a low amount that the correlation time goes down to the level of the incoherent correlation time.

In the cross-polarised case, the coherent enhancement is mainly due to lower-order multiple-scattering process and longer paths do not contribute significantly, even at backscatter. This can be justified from the fact that the enhancement factor for <sup>the</sup> cross-polarised case is very low (typically 1.2). The enhancement factor would be larger if longer paths contributed because longer paths would give rise to a sharper, narrower and higher enhancement peak centred at backscatter.

So the conclusion is <sup>that</sup> when one goes towards the backscatter right from the edge of the enhancement cone, slightly longer paths contribute more and more, and the

long path contribution saturates. Therefore the correlation time behaviour due to this contribution is as follows: it decreases when  $\theta$  decreases and levels off near backscatter because of the saturation effect. The enhancement contribution in cross-polarised case is the same as that in the co-polarised case. By superposing the above mentioned two factors, it is found that in cross-polarised detection the correlation time against angle appears to be a single peak structure in contrast of double-peak structure for co-polarised detection.

### §6-5-2: Photon Counting Statistics

The measurements of the photon counting distribution were made with a very small sampling pinhole ( $\approx 30 \mu rad$ ) and sampling time ( $= 10 \mu s$ ) to avoid spatial and temporal averaging effects. The normalised factorial moments were also calculated from the measured data. An experimental comparison between the probability distributions at backscatter ( $\theta = 0$ ) and away from backscatter ( $\theta \approx 12 mrad$ ) was made and the results are shown in Fig.6.14. The difference between the means of the two curves measured at backscatter and a larger angle respectively imply enhanced backscattering.

To compare and study photon statistics at the enhancement peak and wings for both co-polarised and cross-polarised cases, it is convenient to choose normalised factorial moments as the descriptive parameters. As defined in chapter 2 the  $m$ -th factorial moment is given as:

$$\mathcal{F}(m) = \left\langle \frac{n!}{(n-m)!} \right\rangle = \sum_{n=0}^{\infty} n(n-1)\cdots(n+m-1)P(n, T) \quad (6.74)$$

and the  $m$ -th normalised factorial moment is:

$$F(m) = \frac{\mathcal{F}(m)}{\langle n \rangle^m} \quad (6.75)$$

where angular brackets represent an ensemble average.

We have calculated the normalised factorial moments from the detected photon counts. Although the probability distributions at different angles in both

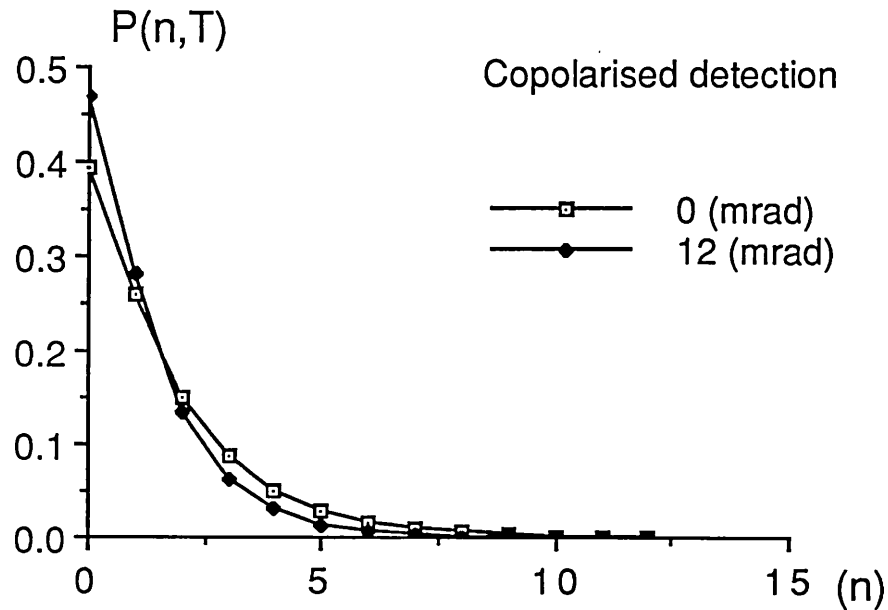


Fig.6.14. Photon counting distributions  $P(n,T)$  measured with sample time  $T = 10 \mu s$ , at  $\theta = 0$  and  $\theta = 12 mrad$ . The average photons per sample are  $\approx 1.27$  and  $\approx 0.89$  respectively.

co-polarised and cross-polarised cases are apparently different, the normalised factorial moments ( $F(m)$ ) at different angles are found to be similar for both detecting polarisations. The measurements of these are given in the following table. It is found, therefore, that the normalised factorial moments are independent of angle and detecting polarisation. The intensity probability distribution does not vary with the angles and polarisations except for the different mean counting rates.

To explain the measured statistics, a comparison of the measured photon statistics was made with Gaussian statistics for the field. For Gaussian speckle the intensity probability distribution has the form of negative exponential (Goodman):

$$P(I) = \frac{1}{\bar{I}} \exp\left(-\frac{I}{\bar{I}}\right) \quad (6.76)$$

which corresponds to a Bose-Einstein [Bertolotti (1974)] photon counting distri-

| F(m) | Co-polarised |          | Cross-polarised |          |
|------|--------------|----------|-----------------|----------|
|      | 0(mrad)      | 12(mrad) | 0(mrad)         | 12(mrad) |
| m=1  | 1.00         | 1.00     | 1.00            | 1.00     |
| m=2  | 1.86         | 1.85     | 1.82            | 1.81     |
| m=3  | 5.03         | 4.98     | 4.85            | 4.77     |
| m=4  | 17.53        | 17.19    | 16.25           | 16.08    |
| m=5  | 72.65        | 70.47    | 64.58           | 64.27    |

Tab.6.3. Comparisons of normalised factorial moments of co-polarised and cross-polarised cases, at backscatter and 12mrad, showing the independence of the normalised factorial moments upon the detecting polarisations and angles near backscatter.

bution: (see Eq.2.9)

$$P(n, T) = \frac{\bar{n}}{(1 + \bar{n})^{1+n}}, \quad \bar{n} = \alpha \bar{I}(T) \quad (6.77)$$

The comparison was done by comparing the measured normalised factorial moments with those of Bose-Einstein distribution  $[m!]$ .

Fig.6.15 shows that there is a slight departure between the measured statistics and those of Gaussian speckle. Kaveh *et al* (1986) have suggested that the intensity distribution in such cases can be described by a Gamma distribution function with its descriptive parameter  $u = 2.5$  (i.e. a significant departure from negative exponential). Wolf *et al* (1988) have also suggested the same distribution function but with  $u = 1.3$  for their case (i.e. very small departure from negative exponential). Here we try to describe the measured distribution function still with

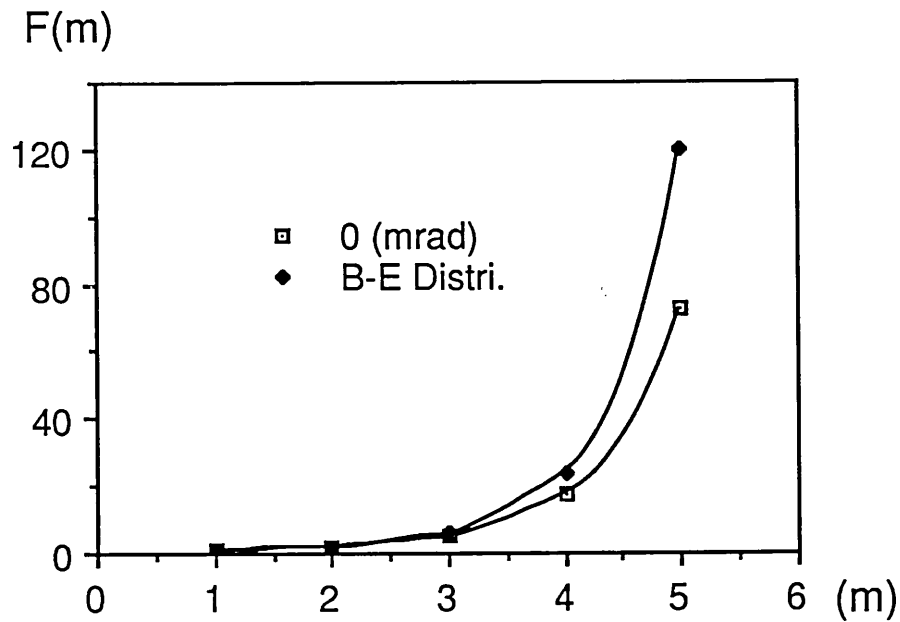


Fig.6.15. Measured normalised factorial moments at backscatter in co-polarised detection are compared with those of Bose-Einstein distribution.

Gamma distribution function, and try to get value for  $u$  by fitting the measured normalised factorial moments with those having a Gamma distribution function with the appropriate  $u$  value.

The Gamma intensity distribution is as follows:

$$P(I) = \frac{1}{\bar{I}} \left( \frac{I}{\bar{I}} \right)^{u-1} \frac{u^u}{\Gamma(u)} \exp\left(-\frac{uI}{\bar{I}}\right) \quad (6.78)$$

where  $\bar{I}$  is the mean intensity and  $u$  is a constant; as  $u$  tends to 1, this distribution tends to a negative exponential distribution function which is the intensity distribution shown by Gaussian speckle.

Let us recall the normalized factorial moments of Gamma distribution from Chapter 2. The  $m$ -th normalised factorial moment is:

$$\begin{aligned} F(m) &= \frac{\mathcal{F}(m)}{\bar{n}^m} = \frac{1}{u^m} \frac{\Gamma(u+m)}{\Gamma(u)} \\ &= \frac{1}{u^m} (u+m-1)(u+m-2) \cdots (u+1)u \end{aligned} \quad (6.79)$$

when  $u \rightarrow 1$ , it tends to  $m!$  which corresponds to that of Bose-Einstein distribution.

A comparison between the experimental data and the calculation values from Eq.(6.79) was made and shown in Fig.6.16. After the dead time correction it is found that the best fit is when  $u = 1.20$ . (see Eq. 2.30)

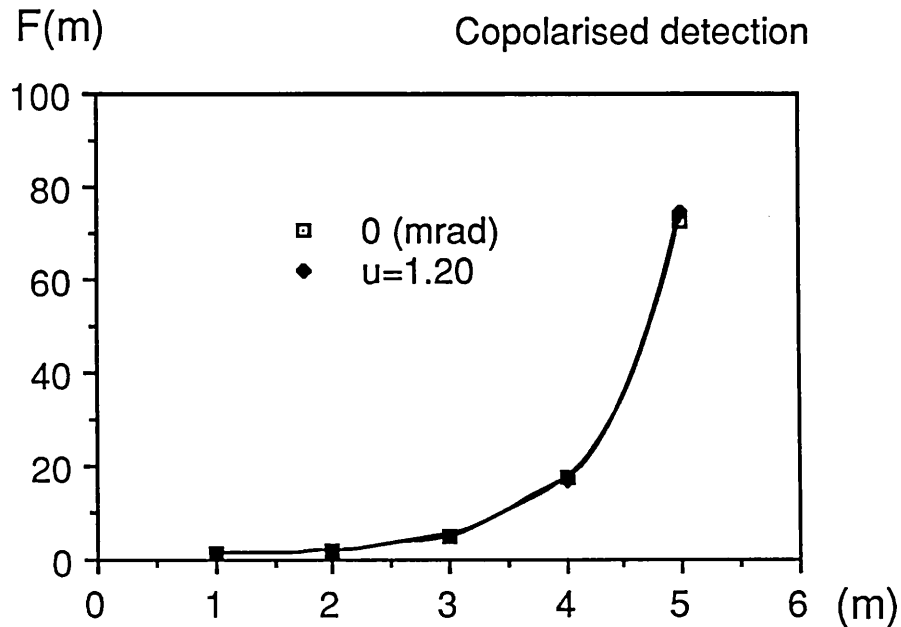


Fig.6.16. Measured normalised factorial moments are compared to those of a Gamma distribution with  $u = 1.20$ .

At present, we can only speculate as to the cause of the discrepancy between the measured fluctuation ( $u = 1.20$ ) and that associated with a Bose-Einstein photon counting distribution ( $u = 1$ ). The temporal and spatial integration was small and the difference is not due to this effect. On the other hand, the limited sample size used to measure the photon counting distribution, typically on the order of  $10^5$  independent samples, may explain the discrepancy [Ben-Yosef *et al* (1988), Goldner *et al* (1988)]. If the effect is real, it is possible that it may be accounted for by multiple scattering [Kelly (1973)] although this is unlikely. Another possibility to explain this discrepancy is as follows: In making the deduction [Goodman (1975)] of the statistics of Gaussian speckles which eventually

was found to have Bose-Einstein distribution, it is assumed that the amplitude and phase are statistically independent and phase is uniformly distributed. Now in our case, at and around backscatter, due to the presence of the coherent field, this assumption may not valid anymore and this could cause the discrepancy between the measured statistics of backscattered light and the Bose-Einstein distribution.

## §6-6: Suppression of Coherent Backscattering

The scattering by a nonstationary medium will often show total or partial suppression of the enhancement. There are several reasons responsible for this. One is the random motion of the scatterers. If the scatterers are moving with a speed of the order of the speed of the scattered wave, the time reversed scattering paths will be broken. This could happen in the case of acoustic scattering but not in the case of light scattering due to the very high speed of the light. To suppress the enhancement in the stationary media, the key is therefore to disturb the time reversed pairs. One way is to make the scatterers move faster by, for example, heating up the media. In the following, we will look at another method to suppress the coherent backscattering using the Faraday magneto-optic effect.

In the Faraday effect the polarisation direction of the light rotates when a strong magnetic field is applied in the light propagation direction. The angle of rotation is proportional to the applied static magnetic flux density and the length traversed. The proportionality coefficient, which is called the Verdet constant, is dependent upon the medium material. For example, the polarisation direction will rotate  $2^{\circ}11'$  for polarised light travelling 1cm in water in the moderately large magnetic field of  $10^4$  gauss. For a medium whose Verdet constant is positive, the Faraday effect is *I-rotatory* when the light propagates *parallel* to the applied magnetic field and *d-rotatory* when it moves *antiparallel* to the field. Therefore if the applied magnetic field is kept the same, the polarisation angle rotation (both direction and amount) of the light travelling from point *A* to point *B* will be the same as that of the light travelling from point *B* to point *A*. Notice also that the

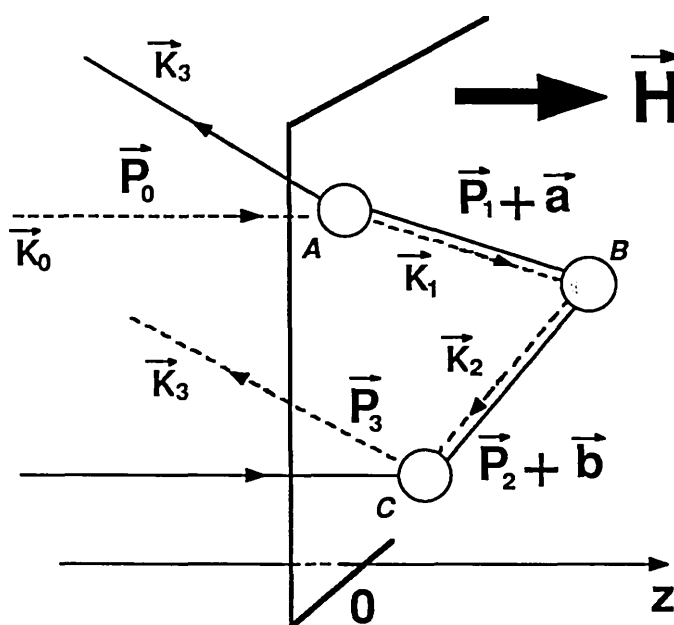


Fig.6.17. Faraday effect on amplitudes of two polarisations from time reversed pairs.

Faraday effect can be amplified by reflecting the light back and forth a few times through the sample.

The Faraday effect on the phases of time reversed pairs was discussed by Golubentsev (1984). Here we will concentrate on the Faraday effect on amplitudes of two output polarisations from time reversed scattering paths.

Let us recall the matrix solution for the co-polarised output amplitude after multiple scattering from the time reversed pairs in §6-3: To simplify the presentation we will only discuss a third order scattering process. In Fig.6.17,  $\vec{P}_0$  is the incident polarisation; the magnetic field is applied in the half space from  $z = 0$ . The vectors  $\vec{a}$  and  $\vec{b}$  are rotation vectors caused by <sup>the</sup> Faraday effect when light travels between  $A$  and  $B$  and between  $B$  and  $C$  respectively, while all the other polarisation vectors are the ones right after the light is scattered and before they are rotated by the Faraday effect.

- (1). For the forward scattering path, by the time the light is scattered to

scatterer  $B$  from point  $A$ , the polarisation vector has been rotated by  $\vec{a}$ . Hence the incident polarisation vector onto scatterer  $B$  will be  $\vec{P}_1 + \vec{a}$ . Similarly the incident polarisation vector onto scatterer  $C$  will be  $\vec{P}_2 + \vec{b}$ . Therefore the vector output amplitude after the third scattering process is given as:

$$\left. \begin{aligned} \vec{P}_1 &= (\vec{K}_1 \times \vec{P}_0) \times \vec{K}_1 = M(K_1) \cdot \vec{P}_0 \\ \vec{P}_2 &= [\vec{K}_2 \times (\vec{P}_1 + \vec{a})] \times \vec{K}_2 = M(K_2) \cdot (\vec{P}_1 + \vec{a}) \\ &= M(K_2)M(K_1) \cdot \vec{P}_0 + M(K_2) \cdot \vec{a} \\ \vec{P}_3 &= [\vec{K}_3 \times (\vec{P}_2 + \vec{b})] \times \vec{K}_3 = M(K_3) \cdot (\vec{P}_2 + \vec{b}) \\ &= M(K_3)M(K_2)M(K_1) \cdot \vec{P}_0 \\ &\quad + M(K_3)M(K_2) \cdot \vec{a} + M(K_3) \cdot \vec{b} \end{aligned} \right\} \quad (6.80)$$

(2). For the reverse scattering path, if we do the similar operation the vector output amplitude is given as:

$$\left. \begin{aligned} \vec{P}'_1 &= (\vec{K}_2 \times \vec{P}_0) \times \vec{K}_2 = M(k_2) \cdot \vec{P}_0 \\ \vec{P}'_2 &= [\vec{K}_1 \times (\vec{P}'_1 + \vec{b})] \times \vec{K}_1 = M(K_1) \cdot (\vec{P}'_1 + \vec{b}) \\ &= M(K_1)M(k_2) \cdot \vec{P}_0 + M(K_1) \cdot \vec{b} \\ \vec{P}'_3 &= [\vec{K}_3 \times (\vec{P}'_2 + \vec{a})] \times \vec{K}_3 = M(K_3) \cdot (\vec{P}'_2 + \vec{a}) \\ &= M(K_3)M(K_1)M(k_2) \cdot \vec{P}_0 \\ &\quad + M(K_3)M(K_1) \cdot \vec{b} + M(K_3) \cdot \vec{a} \end{aligned} \right\} \quad (6.81)$$

Now compare the two outputs  $\vec{P}_3$  and  $\vec{P}'_3$ . As shown in §6-3, at backscatter, the first terms in  $\vec{P}_3$  and  $\vec{P}'_3$  will have the same co-polarised components and therefore in <sup>the</sup> absence of the magnetic field, the enhancement would be a factor of two. However, due to the extra rotation vectors  $\vec{a}$  and  $\vec{b}$  representing the Faraday effect, there are extra second and third terms in  $\vec{P}_3$  and  $\vec{P}'_3$  and this makes the co-polarised components generally not equal for two time reversed outputs. Although approximately  $\vec{a}$  and  $\vec{b}$  are equal if the direction of the magnetic field is perpendicular to the line joining  $A$  and  $B$ , the difference between matrices  $M(K_1)$  and  $M(K_2)$  will still make the co-polarised components unequal. The enhancement will therefore be suppressed. This conclusion can obviously be extended to

a many-order scattering process. In summary, from the fact that the backscattered light has a polarization dependence, it is possible that if a magnetic field is imposed so that the polarization planes for the time reversed pairs ~~rotated~~<sup>are</sup>, they are no longer in the position to interfere with the same amplitude and in complete phase, therefore the enhancement factors can be suppressed. However, in practice a suitable medium with large ~~Verdet~~ <sub>$\alpha$</sub>  constant is required to show a large suppression of the enhancement factor.

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## Chapter 7

### Conclusions

The temporal correlation properties of stellar images and fringe patterns at La Palma Observatory were measured and analysed by means of the photon correlation technique. The correlation time at La Palma Observatory was on the order of  $5\text{ ms}$  during our observations. This implies that in high resolution imaging techniques such as speckle interferometry the exposure time should be in the order of  $10\text{ ms}$  in order to achieve an optimum signal-to-noise ratio in photon-limited situations. The correlation time of fringe intensity was also found to be on the same order of that for the full aperture images regardless of different baselines as long as they were larger than ~~the~~<sup>of</sup> Fried parameter  $r_o$ . Hence in the Michelson interferometer the fringe shifting rates for baselines longer than  $r_o$  are all identical.

To implement the high resolution speckle imaging technique a speckle camera is required. It consists of image guiding, data acquisition and image processing units. The image is guided onto the detecting unit and output data from the detector is transmitted into a host computer via a proper interface. The host computer performs the image processing task and also feeds back certain information to the image guiding unit to align the image.

As pointed out earlier, image wandering has a slower time scale ( $\sim 1\text{ sec}$ ) and therefore it is possible to compensate imaging wandering by electronically controlling a guiding mirror to center the image onto the detector. In the imaging guiding unit, a guiding mirror is controlled by a computer to which the magnitude of the image shifting is sent from the image processing unit. In the data acquisition unit, atmospheric dispersion effects are compensated by two rotary prisms orientated at an appropriate angle. A monochromatic filter is also used to select a narrow band of wavelengths. An image detector is the key part of the this unit. Its sampling pixels should be smaller than the size decided by the diffraction-limited resolution of the telescope. A variety of parameters should also be considered carefully, such as local dead time. Currently several types of

detector are suitable candidates for the purpose, for example, IPD, MAMA and PAPA as referred to in chapter 5. The image processing unit performs a high resolution algorithm. A variety of algorithms are available, each with its advantages and disadvantages; the triple correlation algorithm seems to yield good results. A speckle camera incorporating the above features is currently under construction at Imperial College.

Much progress has been made in high resolution imaging through atmospheric turbulence during the last decade. However, much work remains to be done to fully understand the atmospheric turbulence effects and further more to deal with them. For example, the relationship between speckle boiling and the structure of the atmosphere is not yet clear. The low frequency part of the spectrum of image motion is almost unknown. This raises <sup>the</sup> question of how to describe fully the atmospherically degraded images and subsequently what are the best standards to choose the observing sites.  
on which

In this Thesis, the enhanced backscattering phenomenon has also been studied in detail. Enhancements up to 1.8 for copolarised detection and 1.3 for cross-polarised detection are observed. The relative fluctuations of the backscattered intensities measured by their standard derivations are also found to have enhancements of the same order as for the intensities. Similarly the polarisation degree also has an enhancement at backscatter with a smaller enhancement factor than that for the intensities. The temporal correlation behaviour of light scattered by a dense collection of random scatterers is strongly dependent on the polarisation of the scattered intensity. The statistics of the scattered intensity show a small deviation from those expected of Gaussian speckle and this is probably due to experimental factors.

One of the most important motivations for the study of enhanced backscattering of light from dense random media is the possibility of observing Anderson localisation [Anderson (1958), Anderson (1985), Abrahams (1979)] of photons. When the localisation effect occurs, the propagating wave diffusion constant van-

ishes and there will be an anomalous rise in the energy absorption in the media even though the media is strongly scattering and weakly dissipative. Physically, the localisation of waves can be explained as a result of strong wave interference from different scattering events and the consequent renormalisation of the wave diffusion constant to zero. In such a case the medium serves as a trap to absorb the incident wave energy. The enhanced backscattering of the light wave is at the expense of reduced propagation of the light wave. This implies that the photon diffusion constant is reduced and hence this enhanced backscattering is also taken as a manifestation of weak localisation.

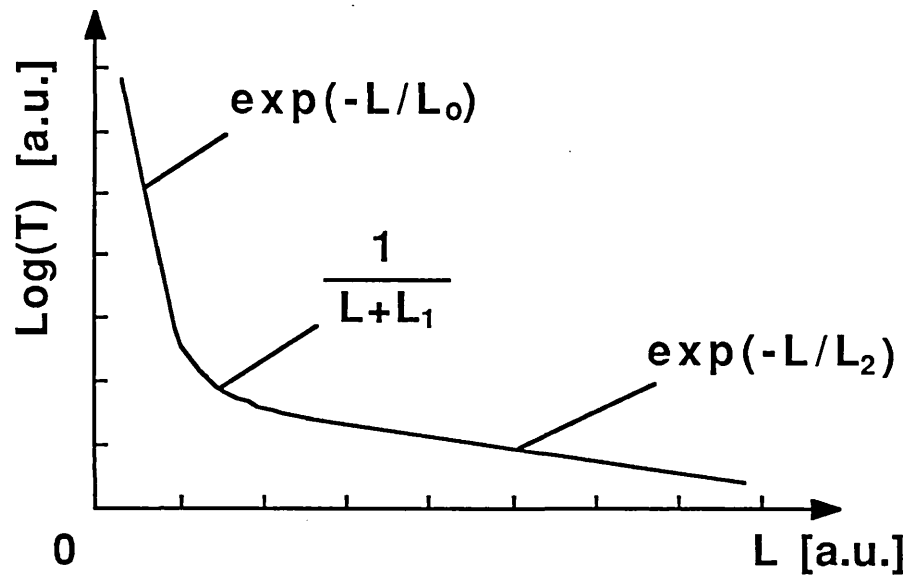


Fig.7.1. Schematic diagram of the transmission against slab thickness, where  $L_0$ ,  $L_1$  and  $L_2$  are the characteristic lengths for each region.

To observe strong photon localisation, the photon diffusion constant is the primary focus of the investigation. For a normal scattering process, we expect a light transmission behaviour as shown in Fig.7.1. This is a schematic diagram of the logarithm of the transmitted light intensity as a function of transmission length. The first region obeys the Lambert–Beer law in which the transmitted

light intensity decays exponentially with the transmission length. The second region of the curve should be described by a law in which the light transport is diffusive and the field satisfies the diffusion equation. The third region is associated with normal absorption after the light wave travels a long distance; the decay is also exponential but with a slower rate.

If there is strong localisation we will see totally different behaviour in the second region of the curve. In the limit, the diffusive part will be absent since the diffusion constant is expected to be zero. Hence by measuring the optical transmission in disordered media one can estimate the diffusion constant and judge the degree of photon localisation effects. Genack (1987) has reported such measurements by measuring very thin wedged samples with a computer controlled translation stage, although no strong localisation was observed.

Another way to observe the photon localisation is to investigate the fluctuation properties of backscattered intensity. Since only the long paths contribute at backscatter, the effects caused by localisation will show up at backscatter [Lagendijk *et al* (1986)]. It has also been shown [Maret *et al* (1987), Qu *et al* (1988)] that the correlation times around backscatter have a polarisation dependence and this is related to weak localisation of photons. Hence by judging the correlation time behaviour of the light intensity around backscatter it is possible to probe the localisation effects. It is expected an extremely short correlation time would be found at backscatter if localisation of photons were occurring. This is because when localisation occurs, there will be no diffusive scattering and the photons will be trapped in the medium travelling round and round for a very long way; eventually when they are detected at backscatter only very long paths contribute and this will give rise to a very short correlation time.

Although many attempts have been made to observe the localisation of light waves, it is still an open question whether it is possible to be able to observe such effects. The criterion for the occurrence of Anderson localisation is  $(\lambda/l^*) \gg 1$ , where  $\lambda$  is the probe wavelength and  $l^*$  is the effective transport mean free path

(i.e. the average distance over which the  $\vec{k}_i$  has become uncorrelated, where  $\vec{k}_i$  is the wave vector after the  $i$ -th scattering). Physically, this criterion suggests that the transport mean free path should be very much smaller than the probe wavelength to allow Anderson localisation to occur.

It is very difficult for  $l^*$  to be small enough for light waves, even though the concentrations of the scattering media are very high. There are two reasons for this. First of all,  $l^*$  is wavelength dependent [John (1984)] when the media concentrations are very high. Intuitively when  $l^*$  is about to be smaller than the probe wavelength when the concentrations become high, the wavelength cannot “see” the small  $l^*$ . In other words, the scattering cross section is limited by the wavelength and  $l^*$  is in turn limited by the scattering cross section. In such case several scatterers close enough may act as one scatterer for the wavelength. Secondly, by the definition of  $l^*$ , when the concentration is very high it still needs a fixed distance to get  $\vec{k}_i$  uncorrelated, even <sup>if</sup> there are more scatterers. In other words,  $l^*$  saturates when the concentration of the scatterers is very high. Hence it is expected that it would be easier to observe localisation effects with longer wavelengths.

The above points can be understood more if we now look at how the electron waves can satisfy the localisation condition. An electron is a very small particle with a diameter on the order of  $10^{-9}$  ( $\mu m$ ). For an electron subject to an electric voltage  $U$  in free space, the propagating electron wavelength is the de Broglie wavelength given by:

$$\lambda = \frac{h}{m_e v_e} = \frac{h}{\sqrt{-2em_e U}} \quad (7.1)$$

where  $h$  is the Planck’s constant,  $v_e$  is the electron propagating velocity,  $e$  is its electric charge and  $m_e$  is the mass of the electron. For an applied voltage of  $U = 1$   $kv$  the wavelength calculated from Eq.(7.1) is on the order of  $\lambda \sim 10^{-5}$  ( $\mu m$ ). The electron size [ $\sim 10^{-9}$  ( $\mu m$ )] is much smaller than its propagating wavelength. Since the diameter of an electron is much smaller than its wavelength, the electrons can still “see” and be scattered by scatterers whose transport mean free path  $l^*$

is much larger than the electron diameter while much smaller than the electron wavelength. Hence it is possible to have a situation when  $l^*$  is much smaller than the wavelength and observation of strong localisation of electron is feasible. But such a beautiful image cannot happen to light waves, since its wavelength is intrinsically considered as its size. Because of the connection between wavelength and transport mean free path it is basically impossible to have  $l^*$  much smaller than  $\lambda$ . However, it is still very interesting to investigate both transport and statistics of the light intensity at  $\lambda/l^*$  values approaching unity. Such values can be obtained by using spheres of large refractive index ( $\text{TiO}_2$  or diamond), or using a longer wavelength.

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## Appendix A

### Enhancement Factor Analysis

The enhancement factors have been analysed by several authors [e.g. Albada *et al* (1987a)]. In §6-4-1, we have derived the matrix representations of the output polarisation states for multiply scattered light. As can be seen from these matrices, the polarisation state for a forward scattering path can be written as  $\vec{P}_n = M^{(n)} \vec{P}_0$  and for the time reversed counterpath  $\vec{P}'_n = (M^{(n)})^T \vec{P}_0$ , where  $M^{(n)}$  is a  $3 \times 3$  matrix and  $(M^{(n)})^T$  is the tranpose of  $M^{(n)}$ . Hence the enhancement factors in the co-polarised and cross-polarised cases from a  $n$ -th scattering process are given respectively by:

$$E_{co}^{(n)} = \frac{\langle (2M_{11}^{(n)})^2 \rangle}{2 \langle (M_{11}^{(n)})^2 \rangle} = 2 \quad (A.1)$$

$$E_{cr}^{(n)} = 1 + \frac{\langle M_{12}^{(n)} M_{21}^{(n)} \rangle}{2 \langle (M_{12}^{(n)})^2 \rangle} \quad (A.2)$$

where  $M_{ij}^{(n)}$  is the corresponding  $i$ -th row and  $j$ -th column element in the related matrix. The co-polarised and cross-polarised fraction of the total backscattered intensity from such a  $n$ -th scattering process are given as:

$$F_{co}^{(n)} = \frac{\langle (M_{11}^{(n)})^2 \rangle}{\langle (M_{11}^{(n)})^2 \rangle + \langle (M_{12}^{(n)})^2 \rangle} \quad (A.3)$$

$$F_{cr}^{(n)} = \frac{\langle (M_{12}^{(n)})^2 \rangle}{\langle (M_{11}^{(n)})^2 \rangle + \langle (M_{12}^{(n)})^2 \rangle} \quad (A.4)$$

If we denote  $C_n$  as the fraction of the total backscattered intensity due to  $n$ -th order scattering processes, we then have the total enhancement factors for both co-polarised and cross-polarised cases:

$$E_{co}^{tot} = \frac{\sum_{n=1}^{\infty} C_n F_{co}^{(n)} E_{co}^{(n)}}{\sum_{n=1}^{\infty} C_n F_{co}^{(n)}} = 2 \quad (A.5)$$

$$E_{cr}^{tot} = \frac{\sum_{n=1}^{\infty} C_n F_{cr}^{(n)} E_{cr}^{(n)}}{\sum_{n=1}^{\infty} C_n F_{cr}^{(n)}} \quad (A.6)$$

As can be <sup>seen</sup> from the above analysis, the theoretical enhancement factor for the co-polarised case is a factor of 2. To obtain the answer for the enhancement

factor in the cross-polarised case, the key is therefore to solve for  $\langle (M_{11}^{(n)})^2 \rangle$ ,  $\langle (M_{12}^{(n)})^2 \rangle$  and  $\langle (M_{12}^{(n)})(M_{21}^{(n)}) \rangle$  from the matrix equations.

Let us recall Eq.(6.45) in §6-4-1, remember that:

$$k_{nx}^2 + k_{ny}^2 + k_{nz}^2 = 1 \quad (A.7)$$

We have:

$$M(k_n) = \begin{pmatrix} 1 - k_{nx}^2 & -k_{nx}k_{ny} & -k_{nx}k_{nz} \\ -k_{ny}k_{nx} & 1 - k_{ny}^2 & -k_{ny}k_{nz} \\ -k_{nz}k_{nx} & -k_{nz}k_{ny} & 1 - k_{nz}^2 \end{pmatrix} \quad (A.8)$$

Also we have:

$$M^{(n)} = M(k_n) M^{(n-1)} \quad (A.9)$$

From the above equation, the following iterative equations for the particular elements of the matrices can be deduced:

$$M_{11}^{(n)} = (1 - k_{nx}^2)M_{11}^{(n-1)} - k_{nx}k_{ny}M_{21}^{(n-1)} - k_{nx}k_{nz}M_{31}^{(n-1)} \quad (A.10)$$

$$M_{12}^{(n)} = (1 - k_{nx}^2)M_{12}^{(n-1)} - k_{nx}k_{ny}M_{22}^{(n-1)} - k_{nx}k_{nz}M_{32}^{(n-1)} \quad (A.11)$$

$$M_{21}^{(n)} = -k_{nx}k_{ny}M_{11}^{(n-1)} + (1 - k_{ny}^2)M_{21}^{(n-1)} - k_{ny}k_{nz}M_{31}^{(n-1)} \quad (A.12)$$

$$M_{22}^{(n)} = -k_{nx}k_{ny}M_{12}^{(n-1)} + (1 - k_{ny}^2)M_{22}^{(n-1)} - k_{ny}k_{nz}M_{32}^{(n-1)} \quad (A.13)$$

By multiplying Eq.(A.10) with Eq.(A.13) and averaging over all the scattering angles, the odd terms vanish because  $\oint$  randomness of the scattering directions and we get:

$$\langle M_{11}^{(n)} M_{22}^{(n)} \rangle = \eta^{(n)} = \langle (1 - k_{nx}^2)(1 - k_{ny}^2) \rangle \eta^{(n-1)} + \langle k_{nx}^2 k_{ny}^2 \rangle \xi^{(n-1)} \quad (A.14)$$

where  $\xi^{(n-1)} = \langle M_{12}^{(n-1)} M_{21}^{(n-1)} \rangle$ . Similarly by multiplying Eq.(A.11) and Eq.(A.12), we have:

$$\langle M_{12}^{(n)} M_{21}^{(n)} \rangle = \xi^{(n)} = \langle (1 - k_{nx}^2)(1 - k_{ny}^2) \rangle \xi^{(n-1)} + \langle k_{nx}^2 k_{ny}^2 \rangle \eta^{(n-1)} \quad (A.15)$$

In a spherical coordinate system ( $0 \leq \theta \leq \pi$ ,  $0 \leq \phi \leq 2\pi$ ):

$$k_{nx} = \sin \theta \cos \phi, \quad k_{ny} = \sin \theta \sin \phi, \quad k_{nz} = \cos \theta \quad (A.16)$$

Then we have:

$$\langle (1 - k_{nx}^2)(1 - k_{ny}^2) \rangle = \frac{6}{15}, \quad \langle k_{nx}^2 k_{ny}^2 \rangle = \frac{1}{15} \quad (A.17)$$

Hence we want to solve the following simultaneous equations:

$$\xi^{(n)} = \frac{6}{15}\xi^{(n-1)} + \frac{1}{15}\eta^{(n-1)} \quad (A.18)$$

$$\eta^{(n)} = \frac{1}{15}\xi^{(n-1)} + \frac{6}{15}\eta^{(n-1)} \quad (A.19)$$

Eq.(A.18)+Eq.(A.19):

$$\xi^{(n)} + \eta^{(n)} = \frac{7}{15}(\xi^{(n-1)} + \eta^{(n-1)}) \quad (A.20)$$

Eq.(A.18)-Eq.(A.19):

$$\xi^{(n)} - \eta^{(n)} = \frac{5}{15}(\xi^{(n-1)} - \eta^{(n-1)}) \quad (A.21)$$

Eq.(A.20)+Eq.(A.21):

$$\xi^{(n)} = \frac{1}{2} \left[ \frac{7}{15}(\xi^{(n-1)} + \eta^{(n-1)}) + \frac{5}{15}(\xi^{(n-1)} - \eta^{(n-1)}) \right] \quad (A.22)$$

By noticing the iterative nature of the Eq.(A.20) and Eq.(A.21) we have:

$$\xi^{(n)} = \frac{1}{2} \left[ \left(\frac{7}{15}\right)^{n-1}(\xi^{(1)} + \eta^{(1)}) + \left(\frac{5}{15}\right)^{n-1}(\xi^{(1)} - \eta^{(1)}) \right] \quad (A.23)$$

As it can be seen that:

$$\left. \begin{aligned} \xi^{(1)} &= \langle M_{12}^{(1)} M_{21}^{(1)} \rangle = \langle k_{1x}^2 k_{1y}^2 \rangle = \frac{1}{15} \\ \eta^{(1)} &= \langle M_{11}^{(1)} M_{22}^{(1)} \rangle = \langle (1 - k_{1x}^2)(1 - k_{1y}^2) \rangle = \frac{6}{15} \end{aligned} \right\} \quad (A.24)$$

Finally we have:

$$\xi^{(n)} = \frac{1}{2} \left[ \left(\frac{7}{15}\right)^n + \left(\frac{5}{15}\right)^n \right] \quad (A.25)$$

Thus at backscatter, if the observing direction is at  $-z$  direction, we have:

$$M(k_n) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (A.26)$$

Hence:

$$\langle M_{12}^{(n)} M_{21}^{(n)} \rangle = \xi^{(n-1)} = \frac{1}{2} \left[ \left( \frac{7}{15} \right)^{n-1} + \left( \frac{5}{15} \right)^{n-1} \right] \quad (A.27)$$

Similarly we can get:

$$\langle (M_{11}^{(n)})^2 \rangle = \frac{1}{3} \left[ \left( \frac{10}{15} \right)^{n-1} + 2 \left( \frac{7}{15} \right)^{n-1} \right] \quad (A.28)$$

and

$$\langle (M_{12}^{(n)})^2 \rangle = \frac{1}{3} \left[ \left( \frac{10}{15} \right)^{n-1} - \left( \frac{7}{15} \right)^{n-1} \right] \quad (A.29)$$

Therefore from Eq.(A.2) and Eq.(A.4):

$$E_{cr}^{(n)} = 1 + \frac{3(0.7^{n-1} - 0.5^{n-1})}{2(1 - 0.7^{n-1})} \quad (A.30)$$

and

$$F_{cr}^{(n)} = \frac{1 - 0.7^{n-1}}{2 + 0.7^{n-1}} \quad (A.31)$$

As pointed out by Albada *et al* (1987a), a computer simulation showed that  $C_n$  can be well approximated by:

$$C_n \approx 2 \left[ (n+3)^{-\frac{1}{2}} - (n+4)^{-\frac{1}{2}} \right] \quad (A.32)$$

Hence according to Eq.(A.6), from the above values of  $E_{cr}^{(n)}$ ,  $F_{cr}^{(n)}$  and  $C_n$  a computer calculation of the total enhancement factor can be made for the cross-polarised case and it is found to be  $\sim 1.2$ .

## Appendix B

### Analysis of Temporal Correlation From Dense Random Media

The approximate expression for the temporal correlation of the scattered field in enhanced backscattering phenomenon was analysed by Maret *et al* (1987). For a  $n$ -th scattering process, if the  $n$  scatterers are located at:

$$\vec{r}_1, \vec{r}_2, \dots, \vec{r}_n$$

Then the phase factors for each of these scatterers are:

$$\exp(i\vec{q}_1 \cdot \vec{r}_1), \exp(i\vec{q}_2 \cdot \vec{r}_2), \dots, \exp(i\vec{q}_n \cdot \vec{r}_n)$$

where  $\vec{q}_j = \vec{k}_j - \vec{k}_{j-1}$  are the transfer wavevectors after  $j$ -th scattering and  $\vec{k}_{j-1}$  and  $\vec{k}_j$  are incident and scattered wavevectors respectively. The temporal correlation of the resultant phase factor  $\phi_n$  after  $n$  scattering process is therefore given by:

$$\begin{aligned} \langle \phi_n(0) \phi_n^*(\tau) \rangle = & \langle \exp\{i\vec{q}_1 \cdot [\vec{r}_1(0) - \vec{r}_1(\tau)] + i\vec{q}_2 \cdot [\vec{r}_2(0) - \vec{r}_2(\tau)] \\ & + \dots + i\vec{q}_n \cdot [\vec{r}_n(0) - \vec{r}_n(\tau)]\} \rangle \end{aligned} \quad (B.1)$$

where  $\phi_n^*$  is the conjugate of  $\phi_n$ .

If we assume that  $n$  scatterers are not interacting and they are independent with each other, the ensemble average of the addition of these exponentials is equal to the addition of the ensemble average of each exponential for every single scatterer. For particles undergoing free isotropic diffusion, it can be seen (see §6-1-2) that:

$$\langle \exp\{i\vec{q}_j \cdot [\vec{r}_j(0) - \vec{r}_j(\tau)]\} \rangle = \exp(-D_T \tau q_j^2) \quad (B.2)$$

where  $D_T$  is the diffusion constant of the particles. Therefore we have:

$$\langle \phi_n(0) \phi_n^*(\tau) \rangle = \exp(-D_T \tau \sum_{j=1}^n q_j^2) \quad (B.3)$$

If we denote  $\langle q^2 \rangle$  as the ensemble average of all the  $q_j^2$ , then:

$$\langle \phi_n(0) \phi_n^*(\tau) \rangle = \exp(-D_T \tau n \langle q^2 \rangle) \quad (B.4)$$

The field temporal correlation for a given polarisation is then given by:

$$\begin{aligned} \langle E(0)E^*(\tau) \rangle &= \langle \left[ \sum_{n=0}^{\infty} A_n \phi_n(0) \right] \left[ \sum_{n=0}^{\infty} A_n^* \phi_n^*(\tau) \right] \rangle \\ &= \sum_{n=0}^{\infty} I_n \langle \phi_n(0) \phi_n^*(\tau) \rangle \\ &= \sum_{n=0}^{\infty} I_n \exp(-D_T \tau n \langle q^2 \rangle) \end{aligned} \quad (B.5)$$

where  $I_n = \langle A_n A_n^* \rangle$  is the ensemble time-averaged intensity of scattered by all sequences of order  $n$ . It has been assumed in the above equation that the amplitude is independent of the phase factor and the field contributions from different scattering orders are uncorrelated, and therefore the cross terms vanish.

If on average the total scattering path length is denoted as  $L$ , the mean free path as  $l$  and the effective transport mean free path as  $l^*$ , then  $n = L/l$  and the ensemble average  $\langle q^2 \rangle$  can be written as  $\langle q^2 \rangle = (2k_0)^2 l / (2l^*)$  by considering a random walk in  $\vec{q}$  space [Maret *et al* (1987)], where  $k_o$  is the modulus of the backscattering wavevector. Thus from Eq.(B.5) the temporal field correlation can be expressed in terms of scattering path length:

$$\langle E(0)E^*(\tau) \rangle = \int_0^{+\infty} I(L) \exp\left(\frac{-L\tau}{4l^*\tau_o}\right) dL \quad (B.6)$$

where  $\tau_o = 1/[2D_T(2k_o)^2]$  is the single relaxation time at backscatter. The contributions from different path lengths to the temporal correlation can therefore be evaluated by using the above equation.

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