

STRATIFIED LIQUID-LIQUID FLOW

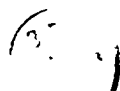
by

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To my mother

Did you think about your bills, your ex, your deadlines
Or when you think you're gonna die
Or did you long for the next distraction

A. Morissette

Abstract

Stratified flows of two immiscible liquids in horizontal pipes have been addressed in the experimental, modelling and numerical work presented in this thesis. Part of the experimental work performed involved the construction at Imperial College of TOWER, a new experimental flow test loop that has two 25 mm diameter test sections (one made of an acrylic material, commercially known as *transpalite*, and the other made of stainless steel). Data on liquid-liquid flow patterns, volume fractions and pressure drops were obtained in this facility for a system consisting of a light oil (viscosity of 1.6 mPa s and density of 800 kg/m³) and water. Data for a different oil-water system (oil viscosity and density of 45 mPa s and 865 kg/m³ respectively) were obtained at the WASP facility (another Imperial College multiphase flow rig that has a 77.9 mm diameter test section). The flow pattern maps drawn from these experiments were shown to be consistent with previous experimental work described in the literature survey.

A one-dimensional model was developed for the calculation of liquid-liquid stratified flow. The pressure drop calculated by this model was shown to agree with data from the TOWER stainless steel tube but to *overpredict* the data obtained in the *transpalite* test section when no inlet mixer was used. The model, though, *underpredicted* the pressure drop in the WASP experiments. A possible cause for this *underprediction* could be due to the effect that the higher oil viscosity might have on the interfacial structure. The interface level calculated by the model was also shown to be in good agreement with stratified flow interface level measurements that were performed in the TOWER facility using a high-frequency inductance probe. Two criteria for the *stratified/non-stratified* flow boundary for liquid-liquid flows were also presented, however their predictions

underestimated the area of occurrence of stratified flow observed in the experiments.

Two Computational Fluid Dynamics (CFD) methods were developed for the calculation of the velocity field of stratified flow. The first method solves the problem in bipolar coordinates and considers both fluids in laminar flow, but the second, implemented in body fitted coordinates (in the CFX-F3D flow solver) can treat both fluids in either laminar or turbulent flow. These two methods produced results in very good agreement with the analytical solution found for the special case of laminar stratified flow when the interface crosses the tube axis. Comparison between the numerical solutions and the one-dimensional model developed was also reported and showed that the results and trends produced by all methods were consistent among themselves and within an acceptable accuracy.

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Chapter 1

Introduction

Horizontal and near horizontal two-phase liquid-liquid pipeline flows are frequently encountered in the petroleum, petrochemical and chemical industries. In the petrochemical and chemical industries liquid-liquid mixtures may be transported through pipelines as part of reaction/separation processes. However, perhaps the most important occurrences of such flows are in the petroleum industry and it is these which have formed the impetus of the present study. Water is often produced simultaneously with oil as a result of the presence of natural (connate) water in oil reservoirs and also as a result of the injection of water into reservoirs for secondary recovery. The flow of oil-water mixtures therefore occurs over a range of conditions; of particular note is the operation of horizontal wells where the pressure may be kept above the bubble point and the flow is of a liquid-liquid (oil/water) mixture. Another example of liquid-liquid flows is where water is injected into a pipeline carrying highly viscous oil in order to reduce pressure drop and hence the pumping pressure and pumping power requirement.

The present work is focused on the study of the stratified flow of two immiscible liquids in horizontal pipes and comprises experimental and modelling work. An experimental test loop had been built to obtain data on flow characteristics such as flow patterns, volume fractions and pressure drops. These data were compared with one-dimensional phenomenological models and to two-dimensional

numerical solutions obtained both in bipolar coordinates and in body fitted coordinates. The bipolar coordinates code was written as part of the present study and the body-fitted coordinates solution was implemented in the commercial flow solver code CFX-F3D of AEA Technology plc. In the latter solution the interface was modelled as a moving wall across which there was continuity of shear stress and velocity but (in the case of the simulation for turbulent flow) no transfer of turbulent kinetic energy. The solution of these numerical codes were assessed against the analytical solution of the laminar stratified flow of two fluids whose interface crosses the tube axis. Figure 1.1 is a graphical representation of the velocity field for this system and provides an example of the kind of problems (and solutions) dealt with in this thesis.

In what follows in this chapter, a brief summary of the basic equations for two-phase flows is given in Section 1.1 (forming a foundation for the work described in the thesis) and (in Section 1.2), an overview is given of the contents of the succeeding chapters.

1.1- Basic equations of two-phase flows

Isothermal two-phase liquid-liquid flows can be described mathematically by the equations of conservation of mass and momentum for both phases. Based on the presentation by Ishii(1975), these equations are:

a) Continuity equations

$$\frac{\partial \rho_k}{\partial t} + \nabla \cdot (\rho_k \mathbf{v}_k) = 0, \quad k=1,2 \quad (1.1)$$

b) Momentum equations

$$\frac{\partial \rho_k \mathbf{v}_k}{\partial t} + \nabla \cdot (\rho_k \mathbf{v}_k \mathbf{v}_k) = -\nabla p_k + \nabla \cdot \tau_k + \rho_k \mathbf{g}, \quad k=1,2 \quad (1.2)$$

where the subscript k indicates each phase of the system.

These equations, expressed in vectorial notation, yield a set of eight, — two continuity and six momentum — equations when decomposed in an appropriate coordinate system. The number of equations can be reduced accordingly if for the sake of simplicity a problem is solved in one or two dimensions, instead of the full three-dimensional formulation.

Boundary conditions at the walls and jump conditions at the interface must be added to equations (1.1) and (1.2) for a fluid flow problem to be well posed (Ishii, 1975). These conditions can be expressed as:

a) No-slip condition at the wall

$$\mathbf{v}_k = 0 \quad k=1,2 \quad (1.3)$$

b) No-slip condition at the interface

$$v_{t_1} = v_{t_2} \quad k=1,2 \quad (1.4)$$

where v_t is the component of the vector velocity $\mathbf{v}$, tangent to the interface plane.

c) Momentum jump conditions at the interface

c1) Tangential direction

$$\sum_{k=1}^2 \tau_{t_k} = a^{\alpha\beta} \mathbf{t}_\beta \sigma_{,\alpha} \quad (1.5)$$

where τ_{ik} is the interfacial shear stress, σ is the interfacial tension and $a^{\alpha\beta}$ and $\mathbf{t}_\beta$ are the surface metric tensor and the hybrid interface tensor respectively, which are quantities related to the shape of the interface.

c.2) Normal direction

$$\sum_{k=1}^2 (\mathbf{n}_k p_k - \tau_{n_k}) = -2H_{21} \mathbf{n}_1 \sigma \quad (1.6)$$

where τ_n is the interfacial normal stress, $\mathbf{n}$ the unit normal vector and H_{21} the mean curvature of the interface.

The momentum jump conditions at the interface can be severely simplified if one considers the surface tension to be small when compared with the interface

pressure and tangential and normal stresses. Equations (1.5), (1.6) therefore reduce respectively to:

$$\sum_{k=1}^2 \tau_{t_k} = 0 \quad (1.7)$$

$$\sum_{k=1}^2 (\mathbf{n}_k p_k - \tau_{n_k}) = 0 \quad (1.8)$$

That is, Equation (1.7) expresses the continuity of tangential stress at the interface, and Equation (1.8) the force balance across it (moving interface). The case that the pressure of both sides of the interface is the same in equation (1.8) leads to a stationary interface, and also to normal stress continuity at this point.

Making certain simplifications, such as assuming steady and fully developed axial pipe flow, this system of equations reduces to that presented in Chapter 5 where the numerical simulation studies are described. Furthermore, if these equations are area averaged they reduce to the system studied in the one-dimensional model proposed in Chapter 4.

1.2 Description of this thesis

The remaining five chapters of this thesis contain the literature survey (Chapter 2); a description of the experimental set-up and procedures and results (Chapter 3); a discussion of one-dimensional modelling (Chapter 4); a description of the numerical studies (Chapter 5), and, finally, the conclusions and recommendations that can be drawn from this work (Chapter 6). A brief

description of the matters covered in each chapter of this dissertation is given below.

Chapter 2 -Literature Survey, comprises a review on liquid-liquid flow papers and publications. Previous works are described there and the importance of each one is highlighted. The papers covered in this survey are divided in two main sections. The first main section includes those works based on experiments and empirical models. This section itself is again divided according to the major interests that inspired each publication: either flow pattern recognition or pressure drop measurements. Regarding flow patterns, a brief description of gas-liquid flow patterns is presented to provide a comparative framework for standard classification of the liquid-liquid flow patterns. In this particular context the gas-liquid flow pattern classification presented in the book of Ishii (1975) and the flow pattern transition criteria of Taitel & Dukler (1976a) are used as a framework to introduce the liquid-liquid flow pattern standardisation proposed in this work.

The second main section of Chapter 2 refers to those works where an analytical, numerical or phenomenological approach is used to predict or explain the flow parameters in a liquid-liquid horizontal pipe flow system. Also in this section are included works that use energetic considerations in the form of the “*principle*”[†] of minimum viscous dissipation for the determination of the interface shape between the two liquids. In the range of phenomenological or mechanistic works special attention is given to the model presented by Brauner and Moalem Maron (1992a, b) whose importance for the future understanding of

[†] The word principle is quoted because this is not an actually proven principle but rather a hypothesis.

liquid-liquid flow pattern transition possibly parallels that which the work of Taitel and Dukler (1976a) represents for gas-liquid flows.

The Two-phase Oil and Water Experimental Rig — TOWER, and the Water, Air, Sand, and Petroleum — WASP systems are described in the first part of **Chapter 3 - Experimental Programme**. These rigs were those on which all the multiphase flow tests described in this work were performed. This section also details the special instrumentation used, such as pressure transducers, high speed video-recording, high frequency impedance probes, gamma ray densitometers, etc. Explanatory remarks on the construction, installation, working principles and operation of each of these devices, are included whenever it was felt that it could help to explain the physical meaning of the parameters being measured.

The second part of Chapter 3 is dedicated to report the data and results obtained during the experimental campaigns performed in both multiphase flow rigs. A total of six major experimental campaigns comprising a wide range of flow conditions were planned and performed. Three of the campaigns for flow pattern recognition and pressure drop measurements were performed on the TOWER system. These campaigns differed from each other either in the test section used (*transpalite* - a kind of acrylic material - or stainless steel tubes) or by the inclusion or absence of a mixer at the test section inlet.

Two experimental campaigns were performed on the WASP system. These two campaigns consisted mainly of flow pattern recognition and pressure drop measurement experiments. The difference between the two campaigns was that the former was conducted with a horizontally levelled test section while the latter was conducted with a slight upwardly (1°) tilted test section. One of the purposes of the tests on the inclined test section was to attempt to observe liquid-liquid

slug flow; this regime had not been found in any of the campaigns on horizontal sections. However, despite the usual increased probability of slugging in inclined pipes, no slug flow was observed in this case either. Gamma ray densitometry was also used in the horizontal flow experiments as an aid to flow pattern definition. This was particularly useful due to the fact that the oil used in the WASP system was opaque making it difficult to make simple visual observations of the flow structures.

The sixth and last experimental campaign was dedicated specifically for the observation of liquid-liquid stratified flow. In these experiments, run in the TOWER facility, besides the usual pressure drop measurements, the height of the interface separating the two liquids was measured by means of a high frequency inductance probe.

Graphs and maps summarising all the measurements and observations made are presented throughout Chapter 3. The experimental procedure followed in all campaigns is also detailed and the measurement and observation methods are described. The variables of interest are listed and difficulties encountered in the process of acquiring them are reported.

Chapter 4- One Dimensional Modelling, presents a simple phenomenological model to describe stratified flows and compares the results obtained with this model with the experimental measurements reported in Chapter 3. Also in this chapter, the boundary of the stratified flow pattern is investigated and two simple models that attempt to describe it are introduced. Comparison of the results of this flow pattern transition model with the experimental results are also included.

The first section of Chapter 4 is dedicated to present the model of Taitel and Dukler (1976a, b) and its extension to liquid-liquid flows. In the extended model a new set of useful dimensionless variables which may provide a clearer understanding of the phenomenon are introduced. In the following section of this Chapter the work of Andritsos and Hanratty (1987), for the interfacial shear stress in gas-liquid stratified flow is reviewed in the light of the one dimensional model. The next section discusses the correlation of Martinelli and Nelson (1948) and, again, presents an interpretation of this in the light of the one-dimensional model and dimensionless variables introduced previously.

The comparison of the pressure drop data and interface level measured in TOWER and pressure drop data measured in WASP with the predictions of the one dimensional model is shown next. Finally, the boundary between liquid-liquid stratified and non-stratified flows is modelled through an extension of Taitel and Dukler (1976a) methodology for gas-liquid flows and by an adaptation of Wicks and Fraser's (1975) model for the determination of the minimum flow velocity for the dispersion of a water layer in oil transportation lines. Results obtained with both of these models are compared with the flow pattern observation performed in the TOWER and WASP systems.

The numerical simulation of liquid-liquid stratified flows is dealt with in **Chapter 5 - Numerical Simulation**. This chapter starts with the presentation of an analytical solution of a special case of laminar stratified flow, that is when the interface of the two fluids corresponds to the diameter of the tube cross-sectional area. This solution is later used to assess the results from numerical methods. Then the two numerical methods used, the bipolar coordinates code and the commercial code CFX-F3D (which uses body-fitted coordinates) are described. The bipolar code was developed for this work and solves the laminar stratified

flow of two fluids at any given interface level. The implementation of a stratified flow of two fluids solution in CFX-F3D can solve both turbulent and stratified flow of both fluids. However it required that the interface were topologically modelled as a moving wall *permeable* to momentum fluxes (shear stresses) between the fluids.

The remaining part of Chapter 5 is used to compare the different solutions obtained: analytical in polar coordinates, numerical in bipolar coordinates and numerical in body-fitted coordinates with CFX-F3D. The solutions of the numerical codes were also integrated over the cross-sectional area of the tube and expressed in terms of parameters such as the Lockhart-Martinelli parameter (X) and the pressure drop multiplier (ϕ) so that a comparison between the numerical solutions and the one-dimensional model presented in Chapter 4 could also be performed.

Conclusions and recommendations arising from this study are presented in **Chapter 6 - Summary, Conclusions and Recommendations.**

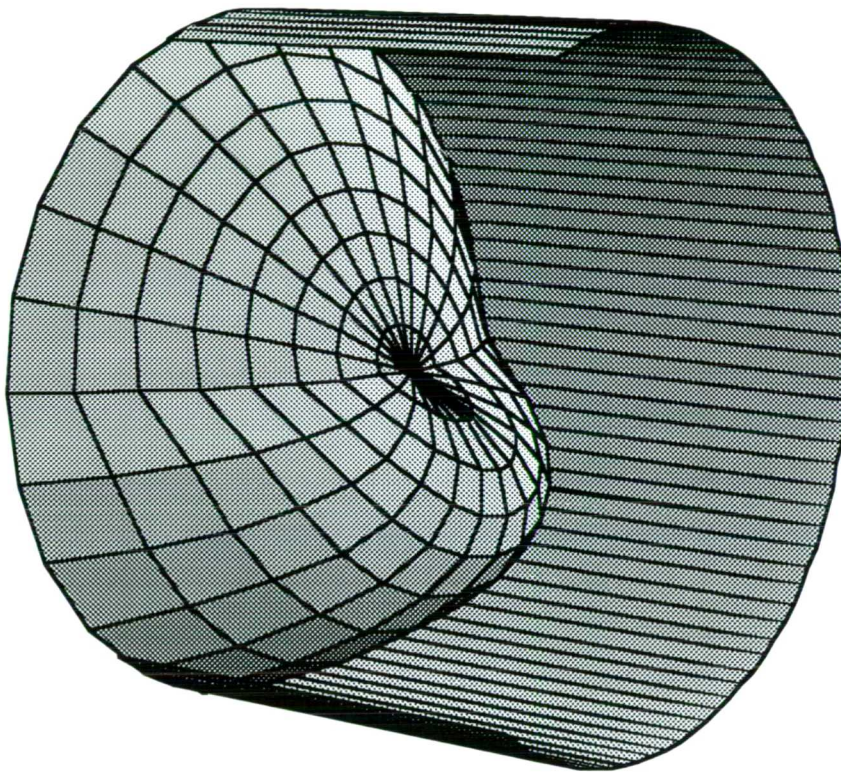


Figure 1.1: Pictorial of the analytical solution of the velocity field for a stratified liquid-liquid flow where the heavier (bottom) fluid is less viscous than the lighter (top) fluid, as in the case of oil-water flows[†].

[†] Courtesy of Dr C. J. Lawrence

Chapter 2

Literature Survey

Liquid-liquid flows occur in many chemical engineering applications, such as liquid-liquid extraction, chemical reactions and direct contact heat transfer operations. However, presently, it is in the transportation of oil with associated water from producing oil reservoirs that produces the main drive for the study of liquid-liquid hydrodynamics. As the petroleum industry ventures to exploit oil in harsher environments (the polar regions and deep sea waters, for instance) the deployment of separating equipment to these producing regions that would allow the produced fluids to be transported in individual streams of water, oil and gas has become very expensive, and in most of the cases is either economically or technically unfeasible. Thus, to overcome these problems, multiphase transportation lines for petroleum recovery systems are being designed to cover longer distances every year. Thus, the study of multiphase flow in pipes has become crucial for the development of new frontiers in the oil industry. This interest in multiphase flows has been manifested primarily by the increasing number of publications on two-phase gas-liquid flows during the last two decades, and as a consequence from these efforts, there is already a considerable amount of experimental data and predictive models for gas-liquid flow in pipes. On the other hand with liquid-liquid flow, despite a surge of research work and publications in the early 60's (Charles et al., 1961; amongst others), there is still a very limited availability of experimental data. Furthermore, models to describe liquid-liquid flows are rare, and when they exist, they represent extensions of gas-

liquid models rather than a methodology in their own right. As it will be shown in this Chapter the study of liquid-liquid flows is not only important in itself, due to the many applications as described above, but it is also important for the comprehension of the more complex multiphase phenomena such as three-phase flow systems (Stapelberg and Mewes, 1990, Hall and Hewitt, 1993 and Nädler and Mewes, 1995).

2.1- Experimental work and empirical models.

2.1.1- Flow patterns

2.1.1.1- Flow patterns in oil-water systems

Towards the end of the 50's and beginning of the 60's a group of researchers from the University of Alberta (Canada) published a series of papers about the simultaneous flow of two immiscible liquids in round pipes and square ducts (Russel et al., 1959; Russel and Charles, 1959, Charles et al., 1961, Charles and Redberger, 1962).

Russel et al. (1959) studied experimentally the horizontal flow of a white mineral oil ($\eta=18$ mPa s, $\rho=834$ kg/m³) and water in a smooth transparent 25.4 mm diameter pipe, for oil-water ratio values ranging from 0.1 to 10 and for water velocities ranging from 0.0350 to 1.08 m/s. They measured pressure drops and described the flow patterns observed. The flow patterns were obtained through photographs from the flow at different input rates of each phase and were generally classified as drop (the authors call it bubble in the paper), stratified and

mixed. They attempted to correlate these flow patterns to the Reynolds number of either phase, but it was shown that any of these patterns could exist in conditions of laminar, transitional or turbulent flows.

Charles et al. (1961) carried out a work similar to Russel et al.'s (1959), but added carbon tetrachloride to the oil so that its density would increase to 998 kg/m^3 , close to that of the water. Three oils were used in the experiments with different viscosities 6.29, 16.8 and 65.0 mPa s @ 25 °C and with oil-water interfacial tensions of 0.044, 0.045 and 0.030 N/m. The test section was a 26.4 mm internal diameter 7.3 m long acetate-butyrate flow tube. The flow patterns observed were oil drops in water, oil slugs in water, oil in water concentric (i.e. water annulus and oil core) and water drops in oil (dispersed). The flow pattern map derived from these observations is shown in Figure 2.2. Interestingly, the only form of annular flow found was the water-annulus/oil-core. Similarly, in the slug flow pattern, only slugs of oil in a continuous water phase were observed. Reciprocal situations (oil-annulus/water-core and water slugs in oil) were not recorded.

The flow pattern maps according to Russell et al. (1959) and Charles et al. (1961) are shown in Figures 2.1 and 2.2. As noted in the previous paragraphs, both the observations of Charles et al. (1961) and Russel et al. (1959) have found oil water two phase flows in which the phases were separated, dispersed or partly separated. Their main difference was that since there was no density contrast in Charles et al.'s (1961) flow experiments, the flow patterns observed in that work were axisymmetric. For this reason the stratified flow pattern is absent in Charles et al.'s (1961) experimental map.

Yu and Sparrow (1969) worked experimentally with two immiscible liquids flowing in the stratified pattern in a horizontal rectangular duct. They allowed the regime of each phase to be either laminar or turbulent. They observed that even though the interface was not completely planar, due to the formation of waves, these disturbances had almost no effect on the pressure gradients when the flow regimes in both phases were laminar.

Through empirical observation and using a mechanistic (phenomenological) approach, Hasson et al. (1970) studied the stability conditions for core-annular flow of a system of two immiscible liquids with similar densities: distilled water and a kerosene-perchloroethylene solution mixed to get a density of 1020 kg/m^3 . The viscosity and surface tension of these fluids at the temperature of 30°C , at which most of the flow experiments were conducted, were respectively 1.0 mPa s , and 0.067 N/m for the water phase and 0.82 mPa s and 0.0285 N/m for the organic phase. In the experiments with this peculiar set of fluids, a concentric flow, where the lighter and more viscous water phase would be at the core and the heavier and less viscous organic phase would be at the annulus region, was initially imposed at the inlet of a horizontal 12.6 mm internal diameter and 2.7 m long glass pipe. The break-up of this initial configuration was then observed, as also were the stable flow patterns that would follow it. The break-up mechanisms were observed to be of two types: either a wall film rupture, (a low speed phenomenon) or core collapse due to the growth of interfacial waves (a high speed phenomenon). The occurrence of the film rupture was attributed to be strongly related to the relative wall wetting properties of the two liquids, because it was found that the preferential glass wall wetting properties of water — the core liquid — exerted a potential disruptive influence on the wall film due to its tendency to cover the walls. To prove this influence, these researchers changed the wall wetting properties of the test glass pipe by treating it with a silicon

solution. As a result of this procedure, the pipe wall was turned hydrophobic, and this made it possible to obtain stable core-annular flows under conditions where, previously, film break-up had occurred near the pipe inlet. The second mechanism of core flow break-up was described to be the appearance of waves and ripples at the interface between fluids in the annulus and in the core. These waves were identified to be Rayleigh-type waves, which were originally observed in the studies of slow speed jet disintegration. Break-up of the annular flow through this mechanism occurred when the wave amplitude reached the core centre and this lead to the formation of slug flow.

Hasson et al. (1970) also determined the character of the more stable flow pattern that followed an annular flow break-up. As it was expected they found these patterns to be mainly determined by the break-up mechanism, the flow rates and the wetting properties of the pipe wall. The flow patterns observed were: water-in-organic and organic-in-water dispersions; water slugs in organic and organic slugs in water; stratified flow and stable annular flow. Flow pattern maps drawn for different wall wetting conditions showed that the area of occurrence of a stable annular flow was greater for a hydrophobic pipe wall than for a freshly-cleaned glass (hydrophilic) pipe wall, while the area of occurrence for all the other flow patterns remained more or less constant as illustrated in Figure 2.3. It should also be remarked that the stratified flow pattern did not occur in the hydrophobic pipe flow experiments and had a very limited occurrence in the hydrophilic flow pattern map. This can be attributed to the relatively small density difference between the two fluids used. However, in experiments conducted in a pipe that had not been freshly cleaned, there was a considerable region of occurrence of stratified flow. This, perhaps, can be explained by the fact that, in this case, the wall wetting properties could be considered neither hydrophilic nor hydrophobic, so the surface effects would not play an important role in the flow

characteristics. Therefore, for this case, the density difference between the two liquids (though small) would be the main factor determining the stable flow configuration.

Guzhov et al. (1973) determined the flow patterns and pressure drops for an oil-water system where a transformer oil with a viscosity of 21.7 mPa s and a density of 896 kg/m³ @ 20 °C was used as the non-aqueous phase. The flow pattern map shown in Figure 2.4 (as revised in the work of Oglesby, 1979) was drawn from data from this system. From this map it can be observed that all the flow pattern categories mentioned in Table 2.2 (separated, mixed and dispersed) seem to occupy significant zones in the map plotted in terms of oil and water superficial velocities.

Malinowsky (1975) constructed an experimental facility which had a 6.0 m long test section of tubes with internal diameter of 41.0 mm and used it to observe flow patterns, pressure gradients and in-situ phase fractions for two phase (oil-water) and three phase (air-oil-water) systems. The oil used had a viscosity of 4.7 mPa s and a density of 852 kg/m³, both at 21 °C. The oil-water flow pattern results presented in this work were somewhat limited and were only concerned with the boundary between the stratified and dispersed regions. Furthermore, no consideration was given to the kind of dispersion achieved (oil in water or water in oil) or to the transition flow patterns that the authors noted as the flow rates were gradually increased from those giving an initial stratified configuration.

Wicks and Fraser (1975), interested in avoiding corrosion in oil pipelines, carried out experiments aimed at determining the operating flow conditions that would force the entrainment of a stationary water layer into the oil mainstream. Laboratory experiments were conducted using kerosene and dyed water in a

25 mm diameter plastic pipe. If water was allowed to accumulate in a part of the pipe, for instance at the foot of a slope, it was shown that if the oil flow velocity were sufficiently high Kelvin-Helmholtz waves formed on the surface of the water layer. At higher velocities these waves became unstable and broke up into water droplets that were carried by the flowing oil. From the experimental data a simplified model was derived, based on the premise that the water droplets once entrained in the flowing oil would behave like solid particles.

Oglesby (1979) conducted a study on horizontal oil-water pipe flow for three oils with viscosities of 58.0, 84.0 and 115.0 mPa s @ 21 °C. The experimental facility used was the same one constructed and used by Malinowsky (1975). Flow pattern maps were drawn for each of the oils used from 422 data points generated in this work. Three basic flow patterns already found by other researchers (stratified, dispersed, and intermittent) were also observed in this work, however this author was the only one to report the extra flow configuration of an annular flow with a core of water rather than oil. The flow pattern map for the 84.0 mPa s viscosity oil is shown in Figure 2.5. It is interesting to note the long and narrow area in which intermittent flow occurs, which seems to indicate a unique water fraction at which this happens, and which is independent of the system total flow rate. Oglesby (1979) relates the occurrence of this intermittent flow with the inversion point at which a dispersion of, for example, water in oil changes to oil in water and vice-versa. As mentioned above, an aspect of this map, which had not been observed by any other researchers, is the existence of an annular flow in which the core fluid is the less viscous one, in this case, the water. This apparently would contradict the *minimum viscous dissipation principle* used in the works of MacLean (1973), Everage (1976), Joseph et al (1984) and Kurban (1989), for instance. This conflict could possibly be overcome if one could consider that the water core would be in turbulent flow, while the oil annulus would be in laminar

flow. By this means the apparent viscosity of the water would be greater than that of the oil and the *principle* would still hold. A similar explanation could also be extended to gas-liquid annular flows where the less viscous gas core is surrounded by a liquid film. The flow pattern maps for the other two oils had very similar trends, and their main difference was the shrinking of the area of occurrence for the *uncommon* more viscous annulus/less viscous core flow pattern when the system's oil viscosity was lowered, down to the point of its complete extinction for the 32 mPa s viscosity oil.

Ribeiro (1994) investigated the influence of the pipe wall material on the stability of oil-water core-annular flow (oil in the core and water in the annulus region), and showed that in flow conditions where core-annular flows are hydrodynamically stable (Joseph and Renardy, 1983), oil could foul the pipe wall. This was an effect of surface energy rather than of hydrodynamics, and had not been taken into consideration in the models used for the stability analysis. The stability of core-flows is very robust and even in the case of a thin oil layer fouling the walls, it is possible to observe a water annulus which lubricates an oil core (Bai, Chen and Joseph, 1992). However, on oleophilic pipe walls (such as carbon steel walls), progressive build-up of the oil may cause increasing pressure drops, up to the complete blockage of the pipe section by the oil. These observations seem to confirm those of Hasson et. al. (1970) who verified higher stability in core-annular flows where the pipe wall was preferentially wetted by the annulus phase rather than by the core phase. However the flows in those experiments were of different characteristics (an organic annulus and water core which are the reverse conditions of those performed by Ribeiro, 1994). It was also found by Ribeiro (1994) that there are preferential sites for oil fouling in a pipeline. Critical positions of the pipeline are usually sections located immediately downstream from pumping stations. Fouling is also heavier in the neighbourhood of line

irregularities such as curves, unions and flanges. Another blockage problem appears in the case of an unexpected shut-down in the line. In such cases, the oil and water stratify, causing the oil to stick to the pipe wall. The situation is worse in hilly terrain, where the oil can completely plug the cross section of the line, making it much harder to restart operation to resume core-annular flow. This adhesion problem is a function of the surface energies of the system wall-oil-water. Therefore, distinct oils will present this problem in different degrees. Ribeiro (1994) performed many adhesion and contact angle tests evaluating different wall materials for the lubricated transport of a specially problematic Venezuelan heavy crude. He concluded that the use of hydrophilic and oleophobic pipe walls (like, for instance, steel pipes lined with wet ceramics that can repel oil when proper hydrated) would reduce the problem. Indeed, pilot tests in pipelines using steel pipes lined with cement showed that this method was able to avoid oil fouling and reduced by six times the pressure levels needed to resume core-flow in a restart operation after a simulated shut-down in an inclined line (Ribeiro et al., 1995.)

Stapelberg and Mewes (1990) studied the two phase flow of oil and water as a limiting case for the analysis of three-phase flows of oil, water and air. The oil phase used in their experimental work was a white mineral oil of density of 850 kg/m^3 , dynamic viscosity of 30 mPa s and oil-water interfacial tension of 0.050 N/m . The experimental facilities built for the examination of two-phase oil-water and three-phase oil-water-air flows was equipped with an interchangeable test section where transparent tubes of 24 mm or 59 mm of internal diameter could be alternatively used. The flow pattern map of Figure 2.6 shows that only three flow regimes (stratified, oil in water dispersion and water in oil dispersions) were observed in these two-phase flow experiments. No transitional flow patterns were observed. The paper did not speculate as to the reasons for the absence of

such transitional flow patterns; however relatively large interfacial tension in the oil-water system used could be a cause for inhibiting the mixing of the phases until rather high velocities are reached. The predominant occurrence of stratified flow in most of the area of the flow pattern map depicted in Figure 2.6 corroborates this interpretation.

Nädler and Mewes (1995) performed liquid liquid two phase flow experiments with the same motivation as Stapelberg and Mewes (1990), that is a limiting case with which a three phase flow obtained by the introduction of a third (gas) phase into the system should be compared. Their experiments were conducted in a test section made of Perspex® (acrylic resin) tubes with 59 mm internal diameter. The oil used was a commercial mineral oil with a density of 841 kg/m^3 and a dynamic viscosity of 20 mPa s . The flow pattern map obtained from their experimental campaigns is shown in Figure 2.7, where seven flow patterns are identified: (1)Stratified (2)stratified with mixing at the interface (3)unstable water-in-oil emulsions (4)layers of water-in-oil dispersion and water (5)layers of dispersions and water (6)Oil-in-water dispersion above a water layer (7)unstable oil-in water dispersion.

2.1.1.2- Flow patterns in polymer systems

Another area of science and technology that has directed some research into the multiphase flow of two different liquids (not necessarily Newtonian ones) is the area of polymer engineering. The work in this area is worthy of mention because it gives more attention to the influence of the flow parameters on the interface shape between the two fluids. On the other hand, most of the published work in this area is experimental, and the mathematical treatment given to justify the

modelled phenomena is not conclusive. Moreover, the flow of melt polymers is normally vertical (Minigawa and White, 1975) and, therefore, there is no density contribution to the final shape of the interface.

Southern and Ballman (1973) were the first researchers to observe and quantify the distortion of an initially plane interface, when two immiscible liquids are forced to flow in the same pipe. In their work they used two polystyrenes whose apparent viscosities were shear rate dependent as pictured in Figure 2.8. At low shear rates the apparent viscosity of the fluid labelled as *B* in Figure 2.8 is greater than that of the fluid labelled as *A*, however, at high shear rates, the situation is inverted, and fluid *A* possesses a greater apparent viscosity than fluid *B*. It is clear that there should be a given shear rate where the apparent viscosity of both fluids is coincident. The experiment conducted by Southern and Ballman (1973) consisted of forcing these two fluids to flow under different shear rates in a vertical capillary tube and collecting the extruded material for further analysis. According to this procedure it was observed that at low shear rates, polystyrene *A*, then with lower apparent viscosity than polystyrene *B*, tended to wet the walls and encapsulate polystyrene *B*. At high shear rates this situation is inverted and polystyrene *B*, then possessing higher apparent viscosity than polystyrene *A*, tends to wet the wall and encapsulate polystyrene *A*. At the singular shear rate at which both polystyrenes possess the same apparent viscosity the plane interface initially imposed at the entrance of the capillary tube is kept practically unchanged. Figure 2.9 summarises these observations.

2.1.1.3- Correlations and semi-empirical models for flow patterns

It is seen from the literature reviewed above that, as with gas-liquid two-phase flows, when two liquids flow simultaneously in a pipe they can assume different configurations. The review of some concepts of gas-liquid multiphase flow, therefore, can provide an initial understanding of multiphase flows of two liquids, since many of the phenomena that govern each of these multiphase flow systems are similar.

In gas-liquid multiphase flow the so-called mechanistic or phenomenological models are widely accepted today. In this approach, the basic principles that govern each pattern of flow are described using simple formulations. By this means, even though the physics of the problem is modelled, the resulting equations are not much more complex than those which are purely empirically derived.

Ishii (1975) (after Ishii, 1971 and Kocamustafaogullari, 1971) presents a classification of multiphase flows based on the interfacial structures and topographical distributions of each phase. He classifies two phase flows in three major categories: *separated flows*, *mixed or transition flows* and *dispersed flows*. Considering the flow direction to be horizontal, the two-phase flow patterns observed in the literature would be divided into each category as shown in Table 2.1.

The category of *separated flows* is characterised by a total segregation of the two phases in the tube. According to the type of interface these separated flows can be subdivided in *stratified flow*, where a planar interface prevails, and *annular flows*, where the phase distribution is predominantly axisymmetric.

Drop and bubble flows make up the category of *dispersed flows* in Table 2.1, which is subdivided by the continuous phase rather than by the type of interface, for example spherical and elliptical drops or bubbles.

As the flow patterns do not abruptly change from separated structures to dispersed ones, a third class of mixed or transition flows is also shown in Table 2.1. In this case, Ishii (1975) found it more convenient to subdivide this flow pattern category according to the distribution of the dispersed phase. Therefore, for horizontal pipe flow, the slug flow and annular flow with entrainment regimes can be identified.

Taitel and Dukler (1976a) developed a model for predicting the flow pattern transitions in horizontal and near horizontal gas-liquid flow. This work is based in fundamental physical phenomena but uses a number of heuristic assumptions in the determination of the flow pattern boundaries. The analysis done by these researchers considered the conditions for transition between five basic flow patterns: smooth stratified, wavy stratified, intermittent, annular and dispersed bubble.

The core of the Taitel and Dukler's model for the determination of flow pattern transitions is the stratified-to-slug/annular flow boundary. The locus of this boundary is, in turn, dependent on the position of the interface in a hypothetical stratified flow between the two phases. According to the position of this interface and the velocity of the gas above, stratified flow may either exist or not.

Referring to Figure 2.10, the phenomena and equations descriptive of each flow-pattern transition assumed by Taitel and Dukler can be summarised as follows.

The stratified to intermittent or annular flow pattern boundary is determined by the Kelvin- Helmholtz instabilities; i.e. by a balance between the suction force resulting from the venturi effect caused by the reduction of gas flow area due to the appearance of an infinitesimal disturbance on the liquid surface and the gravity forces acting to smooth this disturbance. The critical velocities for the growth of the disturbance (and hence the formation of slugs) was given by Taitel and Dukler as:

$$u_G = C_2 \sqrt{\frac{(\rho_L - \rho_G) g \cos \alpha A_G}{\rho_G \frac{dA_G}{dh_L}}} \quad (2.1)$$

where ρ_L and ρ_G are the liquid and gas densities, g the acceleration due to gravity, α the angle of inclination of the channel to the horizontal and A_G and A_L the gas and liquid cross sections. C_2 is an arbitrary function used by Taitel and Dukler to allow matching of the experimental data.

The intermittent to annular flow pattern boundary, in turn, is only determined by the geometric consideration that if the liquid equilibrium level of a hypothetical stratified flow were above the centreline of the pipe a symmetrical liquid wave would eventually bridge the whole pipe and develop a liquid slug whereas, if it were below this value, annular flow would be obtained. Equation (2.2) below expresses this condition:

$$\frac{h_L}{D} = 0.5 \quad (2.2)$$

Based on the assumption that waves are generated when pressure and shear work can overcome the viscous dissipation in the wave, the smooth stratified to wavy flow pattern boundary could be written as:

$$u_G = \sqrt{\frac{4\nu_L(\rho_L - \rho_G)g \cos \alpha}{s\rho_G u_L}} \quad (2.3)$$

where s is a sheltering coefficient to which Taitel and Dukler assign the value of 0.01.

Finally, the transition boundary from intermittent to dispersed bubble flow patterns was obtained through a balance between the turbulent fluctuations in the liquid flow and the buoyancy forces that tend to move the gas to the top of the pipe:

$$u_L = \sqrt{\frac{4A_G}{P_i} \frac{g \cos \alpha}{f_L} \left(1 - \frac{\rho_G}{\rho_L}\right)} \quad (2.4)$$

A comparison of the flow pattern map for an air-water system calculated by the methodology of Taitel and Dukler (1976a) with the experimental map of Mandhane et al. (1974) is represented in Figure 2.11.

Using the categorisations of Ishii (1975) and the experimental observations reported by Oglesby (1979) and Arirachakaran et al. (1989) a general classification for liquid-liquid flows is obtained, and this is shown in Table 2.2. For the sake of clarity one phase is referred as water and the other as oil.

However it must be understood that these patterns should be valid for any two-phase pipe flow of two immiscible liquids.

According to Table 2.2, the category of *separated flows* for horizontal flow comprises the *stratified flow* and the *core-annular flow* configurations. Identically analogous to the gas-liquid classification these two flow pattern differ by the shape of the interface between the phases; in the *stratified flow* the interface is planar and in the *core-annular flow* the interface is circular and predominantly axisymmetric.

Water in oil dispersions and oil in water dispersions belong to the same category of dispersed flows; the subdivision criteria here being the continuous phase rather than the drop shape or size of the dispersion.

The category of *mixed or transition flows* includes: *intermittent or slug flow*; *stratified with dispersion flow*, and *annular with dispersion flow*. This category provides the link between the separated and dispersed categories as the fluid velocities are increased from the former to the latter. It is observed that the subdivision of this category was performed according to the distribution of the phase of dispersion, as suggested by Ishii (1975) and they are characterised as follows; *intermittent or slug flow* is that flow pattern where each of the phases successively predominates in the pipe either as a free phase or as a dispersion, and *stratified with dispersion flow and annular with dispersion flow* are similar to the *stratified and core-annular flow* configurations, where, however, one of the two liquids occupies a determined region of the pipe as a free phase and the other occupies the rest of the pipe as the continuous phase of a dispersion.

Table 2.3 summarises the works discussed in this section and presents a correlation for the nomenclature used amongst the different authors with that standardised in Table 2.2, which will be used throughout this work.

2.1.2- Pressure drop measurements

The motivation for the study of pressure drop in oil water systems is basically twofold. Firstly, there has been an interest from as far back as the beginning of the twentieth century (water lubricated lines have been tested periodically since 1906), in predicting the reduction in pumping power resulting from the injection of water into lines carrying highly viscous oils. Secondly, there is an increasing need to evaluate the pressure losses in naturally occurring multiphase oil-water flows originating from the production well streams; this second area is becoming more important as offshore installations penetrate harsher regions where separation is simply not possible.

Russel et al. (1959) represented pressure drop data measured in their horizontal oil-water experiments by means of a modified Fanning friction factor based on the water superficial velocity and developed from the analytical study of the laminar flow of two immiscible fluids between two parallel plates. This way of representing the pressure-drop was particularly useful in grouping the experimental data using the oil-water ratio as the correlation parameter in order to compare them with water single-phase flow friction factors.

Malinowsky (1975) compared the pressure gradients predicted by two simple oil-water flow models with his own experimental data and that of Russel et al. (1959). The two flow models studied were the homogeneous and no-slip stratified

models respectively. The no-slip stratified flow model assumes that the input volume fractions prevail along the whole length of the pipe (i.e. constant hold up) and that the wall shear stresses in each phase would be the same as the one that would be obtained if that phase were flowing alone in the pipe. Equation (2.5) summarises this model:

$$\left(\frac{dp}{dz}\right)_{TP} = \frac{P_o}{P} \left(\frac{dp}{dz}\right)_{fo} + \frac{P_w}{P} \left(\frac{dp}{dz}\right)_{fw} \quad (2.5)$$

where $(dp/dz)_{TP}$ is the two-phase flow pressure gradient, P the total tube periphery ($=\pi D$, for a round tube of diameter D) and P_o and P_w are the tube peripheries in contact with the oil and water respectively. $(dp/dz)_{fo}$ and $(dp/dz)_{fw}$ are the frictional pressure gradients for the average velocity in the pipe (oil plus water flow rate divided by the cross-sectional area) for a fluid having the properties of the oil and water respectively.

The dispersed or homogeneous model used by Malinowsky (1975) is even simpler, consisting of calculating the single phase flow pressure drop of a pseudo-fluid whose density and viscosity are evaluated as the input flow rate weighted average of the respective properties of each of the phases:

$$\eta_{TP} = \frac{\dot{V}_o \eta_o + \dot{V}_w \eta_w}{\dot{V}_o + \dot{V}_w} \quad (2.6)$$

and



$$\rho_{TP} = \frac{\dot{V}_o \rho_o + \dot{V}_w \rho_w}{\dot{V}_o + \dot{V}_w} \quad (2.7)$$

Additionally, dispersed flows were also investigated to determine if regions existed in which the pressure gradient might be controlled by only one of the liquid viscosities. Testing all his own experimental data against the flow models, Malinowsky (1975) noticed regions where one or the other method was more accurate in calculating the pressure drop. In an attempt to delineate these regions, the calculation method that most closely represented the pressure gradient was plotted on a map with coordinates of superficial oil and water velocities, and then by interpolation the region boundaries could be determined.

Arirachakaran (1983) studied the pressure drop and (continuous phase) inversion phenomenon in two dispersed oil-water systems (pure oil viscosities of 237 mPa s and 2116 mPa s at 21 °C) flowing in a 25.4 mm inside diameter pipe. In this work, he noticed that the pressure drop due to friction increased significantly when the flowing oil-water mixture reached a point where the continuous or “external” phase in a dispersed flow configuration changed from water to oil. It was also noticed that the water fraction necessary for this inversion to occur would be lower the higher the viscosity of the oil in the system. The pressure drop data was used to evaluate an apparent “bulk” viscosity for the whole dispersed system. Arirachakaran et al. (1989) had extended the pressure drop data base to include the data of Malinowsky (1975) and Oglesby (1979) encompassing four more different oil-water systems (oil viscosities of 4.7, 58, 84 and 115 mPa s at 21 °C) and including also stratified flow conditions.

Wu et al. (1986) performed a series of experiments with the objective of verifying the practical feasibility of stable annular-core flow as a means of reducing the

drag in the transportation of extremely viscous crude oils. The experimental set-up consisted of a 5.08 mm diameter flow loop with an effective test section length of 12 m. Three oil-water systems were used in the experiments. In the first system, the oil phase was fuel oil with a viscosity of 2000 mPa s and density of 975 kg/m³. In the second and third systems, crude oil was used at two different temperatures, giving oil viscosities of 2200 mPa s at 70 °C and 10000 mPa s at 40 °C. Stable operation in the core-flow mode was possible for all systems tested and the optimum water fraction for reducing the pressure drop was found to be around 0.10 for all oil flow rates imposed

Stapelberg and Mewes (1990) also performed oil-water flow pressure drop measurements in addition to the flow pattern studies mentioned in Section 2.1.1 above. They observed that as the rate of water injected in the oil stream was increased two minima in pressure drop occurred. The first minimum was the limit of the pressure-drop reduction that having stratified oil-water flow can promote relative to the pressure drop for single-phase flow of oil. Further increases of the water flowrate beyond this minimum have the effect of increasing the momentum exchange at the interface, which offsets and exceeds the advantage, delivered by the stratified flow, of having a larger fraction of tube wall wetted by water. As the water rate is increased even further a second minimum of the pressure drop is observed as a result of the breaking up of the stratified flow regime into oil-in-water dispersed flow. The experimental pressure drop data collected in this work was shown to be well represented by Equation (2.8) which follows the classical form (developed by Lockhart and Martinelli, 1949) of relating a pressure drop multiplier (ϕ_o) to a Martinelli parameter X (Charles and Lilleth, 1966). Thus:

$$\phi_o = \frac{X^{1.5579} + X^{0.5579} + 1.1}{X^{1.5579} + 1.1X} \quad (2.8)$$

where

$$X = \sqrt{\frac{\left(\frac{dp}{dz}\right)_o}{\left(\frac{dp}{dz}\right)_w}} \quad (2.9)$$

and

$$\phi_o = \sqrt{\frac{\left(\frac{dp}{dz}\right)_{TP}}{\left(\frac{dp}{dz}\right)_o}} \quad (2.10)$$

where $(dp/dz)_o$ and $(dp/dz)_w$ are the pressure gradients for the oil and water flows respectively flowing alone in the tube and $(dp/dz)_{TP}$ is the pressure gradient of the two-phase (oil-water) flow.

Figure 2.12 compares the experimental data of Stapelberg and Mewes (1990) with the predictions according Equation (2.8) and with the dispersed and stratified models of Malinowsky(1975) (Equations 2.5, 2.6 and 2.7).

2.2- Analytical, numerical and phenomenological models

Russel and Charles (1959) presented an analysis for the laminar one-dimensional flow of two fluids in rectangular and polar coordinates. For the rectangular

coordinates case, the flow between two plates is modelled and the fluids are assumed to be in a stratified configuration, i.e. one fluid on the top of the other. For the case of polar coordinates, the flow in a circular tube is modelled, and the fluids are assumed to be in a concentric annular configuration. Thus the velocity varies only in the direction perpendicular to the plane of the boundary wall (flat plates) in the flow in the rectangular coordinates case and only in the radial direction in the polar coordinates case. It is shown that the addition of a second less viscous phase to a more viscous primary stream can, in fact, reduce the pressure drop, in spite of the increase in the total flow rate.

Charles and Redberger (1962) solved numerically the case of a plane interface stratified flow of two immiscible liquids in a circular pipe. In this numerical procedure the momentum conservation equation for laminar flow of the two fluids was written in its Cartesian form and discretised by a finite difference technique. The calculation domain was, thus, Cartesian and the round pipe wall had to be approximated by a rectangular grid. Therefore, the representation of the pipe wall was as imprecise as the coarseness of the grid. This treatment of the pipe wall was acceptable at the time the paper was published, when scientific digital computing was still in its infancy. This study analysed five hypothetical oils with viscosities ranging from 1 to 500 mPa s. The computational procedure performed was to prescribe the pressure gradient and the position of the interface, and then calculate the velocity profiles for each of the phases. These velocity profiles were finally integrated numerically to provide the respective volumetric flow rates. From the results obtained in this way it could be observed that the pressure gradient reduction factor for a determined oil flow rate, i.e. the ratio of the pressure gradient for the single phase flow of the oil to the pressure gradient for the same oil throughput in the presence of water, was less than those computed

by Russel and Charles (1959) for stratified flow between parallel plates and concentric flow in a circular tube.

In the work of Gemmel and Epstein (1962), the case of a plane interface stratified configuration of two fluids in laminar flow in a circular tube was solved by a methodology almost identical to that of Charles and Redberger (1962). The pipe cross section was similarly covered by a rectangular grid so that the circular boundary was only approximated by a jagged line. Charles and Redberger (1962) had prescribed the standard zero velocity boundary condition to the points at the discretised pipe wall. In order to attempt to provide more accuracy to the calculations Gemmel and Epstein (1962) assigned negative values to the grid points which fell just outside of the curved boundary, so that the interpolated value at the pipe wall would be zero. Three viscosities ratios (10, 100 and 1000) were analysed at eight interface positions. The results of these calculations (which were performed on a desktop calculator, without the aid of digital computers!) were compared to the results of Russel et al. (1959). Good agreement was found between the data of these two works, despite the fact that the numerical calculations of Gemmel and Epstein (1962) considered laminar flow for both phases, while in the experimental work of Russel et al. (1959) most of the flow rate conditions were in the transitional and turbulent regions.

Achutaramayya and Sleicher (1969) performed a numerical analysis of the plane interface stratified flow of two immiscible fluids in circular and non-circular pipes. As in the work of Charles and Redberger (1962) and Gemmel and Epstein (1962), laminar flow was considered for both phases. The momentum conservation equation governing the flow of this system of fluids was discretised and solved numerically for five different pipe configurations. It was finally shown that the stratified flow of oil and water in tubes that had their lower part flattened gave a

slightly higher throughput of oil for a prescribed pressure gradient than did a perfectly round pipe. This increase, however, was not enough to recommend the use of flat bottomed tubes as a worthwhile alternative in practical applications.

Bentwich (Bentwich, 1964, 1976 and Bentwich et al., 1970) carried out extensive studies of the flow of two immiscible liquids in a pipe. Bentwich (1964), derived analytical solutions for the velocity distributions for the laminar flow (constant pressure gradient) of both phases with any viscosity ratio and with any prescribed eccentric circular interface, or in this work's own words for any annular, lunar or stratified flow configuration. To obtain these expressions he mapped the pipe geometry using a bipolar coordinate system and showed that the solutions were harmonic and, therefore, could be well represented by appropriate series and Fourier integrals whose coefficients, in turn, could be determined by the boundary conditions assumed, i.e. no slip at the walls and velocity and shear stress continuity at the interface. The laminar velocity profiles calculated in this manner were later integrated in Bentwich *et al.* (1970), to yield the volumetric flow rates in a tube of two immiscible liquids with an eccentric circular interface. Sample calculations for typical cases showing the flow rates for each phase as well as the pressure drop reduction (flow enhancement) and power reduction factors are also presented. Much later, Bentwich (1976) determined the static interface between two fluids using a standard calculus of variations technique to minimise the potential energy functional, taking into account their interface tension, capillary forces and densities, in a horizontal pipe cross-section. The interfacial shapes obtained by this methodology are those shown in Figure 2.13. The axial velocity profile for the two-phase laminar flow with this given interface was then solved by writing a functional whose minimisation, through the use again of variational calculus techniques, was equivalent to solving the momentum conservation partial

differential equations, with the respective boundary and interfacial conditions for this same problem.

Hasson and Nir (1970) presented a model for the ascending trajectory that a core of a less dense liquid follows as two liquids pass along a pipe in an annular configuration. The physical model used in this work assumed a smooth interface between the fluids, and considered that the buoyancy force acting on the less dense liquid core would be balanced by a vertical interfacial drag force. This drag force, in turn, was assumed to result from the wall drainage of the peripheral liquid, which was approximated by a local Couette flow in the space between the core-annulus interface and the pipe wall. This model predicts that the annulus thickness at the top of the pipe decreased as the fluids progressed along the pipe and the predictions were in good agreement with experimentally measured values obtained for flow conditions in cases where a stable non-wavy interface existed.

MacLean (1973) attempted to explain the results of some experiments performed by Southern and Ballman (1973) by means of energetic considerations. That is, MacLean (1973) endeavoured to determine the stable interface shape between two liquids using the so-called *variational principle of minimum viscous dissipation* (or *minimum entropy production*) which had already been used previously for the determination of velocity profiles of fully developed single phase flows. In his work, MacLean studied the two-phase flow between parallel plates under two geometrical configurations. In the first, the more viscous fluid occupies the centre of the channel while the less viscous fluid occupies the areas next to both plates. In the second configuration MacLean let the more viscous fluid to occupy half of the channel wetting one of the flat plates and the less viscous fluid to occupy the other half of the channel wetting the other plate. Finally, he proved, by allowing the same flow areas for each of the fluids in both

geometries, that the former, the sheath-core configuration (more viscous core and less viscous periphery), was energetically more favourable (i.e. provides higher flow rates for the same prescribed pressure gradient or a smaller pressure gradient for the prescribed flow rates) than the latter with a single flat interface. A similar analysis was also outlined, but not performed, for a cylindrical tube case where a core annular flow configuration would be compared to that one of stratified flow with a planar interface.

Everage (1973) did similar work to MacLean (1973) considering the flow in cylindrical tubes. In his work, for the case of fluids with equal density, it is shown that the concentric core-annular configuration with the more viscous fluid at the centre is energetically preferential to other configurations having eccentric circular interfaces, including the planar interface which is a particular case of this family of interfaces.

Williams (1975) performed the same evaluations as Everage (1973) but in a more direct and simple manner. He also proved by *energetic principles* (i.e. *principle of minimum viscous dissipation*) why the preferential configuration in a concentric two-phase flow should have the more viscous fluid in its core and the less viscous in the annular region.

More recently, Ooms et al. (1984) applied the lubrication theory to determine the pressure gradients in horizontal pipes where a very viscous oil core is encapsulated by a water annulus. Ooms et al. (1984) suggest an explanation of how the buoyancy force acting on the core, resulting from any density difference between the oil and the water, could be counterbalanced. Assuming the simplification that the very viscous core would behave as a solid shaft, it was shown that the moving ripples at the interface of the fluids could generate

pressure variations in the annular space, which, in fact, would result in a force acting perpendicularly on the core so as to compensate for the buoyancy effect. This model was compared with their own experimental results and those published later by Oliemans et al. (1987). In the latter paper, the original model was extended by the incorporation of turbulence in the water annulus surrounding the oil core. Furthermore, this group of Dutch researchers developed with their Venezuelan partners an extensive experimental programme to validate their results (Wu et al., 1986; Zubillaga, et al., 1985).

Joseph et al. (1984) confirmed the conclusions obtained by MacLean (1973), Everage (1973) and Williams (1975) with regards to the preferential configuration of two flowing fluids in a circular pipe, according to the *principle of minimum viscous dissipation* (or *principle of minimum entropy generation* — Bejan, 1982). However, Joseph et al. also carried out a linear stability analysis of the core-annular flow of two immiscible fluids of different viscosities and equal density. The numerical solutions from the linear stability analysis showed that whereas the *principle of minimum viscous dissipation* predicts that the fluids should always be arranged concentrically with the more viscous at the core, the stability analysis predicts that this configuration is only stable when the more viscous fluid occupies most of the pipe cross-sectional area. In conclusion, this paper states that the dissipation principle does not always hold, and that the *in-situ* volume fractions of the fluids should also be considered.

Kurban (1989) and Kurban and Bannwart (1990) extended the work of Joseph et al (1984) in order to study the simultaneous influence of different viscosities and different densities in the developed laminar flow of two liquids in a horizontal pipe. Kurban (1989), by means of the variational *principles* of minimum viscous dissipation and of minimum potential energy, derived the

following: the velocity field of a single phase in developed laminar flow; the interface shape of two fluids at rest (i.e. gravity stratification in a hydrostatic problem); the velocity fields and the interface shape in the flow of two immiscible liquids of same density but different viscosity in a horizontal pipe (i.e. pure viscosity stratification) and, finally, the velocity fields and the interface shape for the general case where the two fluids could have any arbitrary viscosities and densities, which was the final aim of that work. The results predicted by this theory were shown to be in agreement with the experimental observation, as, for instance, those of Southern and Ballman (1973), which indicate that when two immiscible liquids flow together in the same duct there is a tendency for the less viscous fluid to encapsulate the more viscous one either totally or, at least, in part. This work showed that the interface of the two liquids would always be an arc of circumference with varying radius of curvature and eccentricity depending on the viscosity and density ratios of the liquids under consideration. The plane stratified (eccentricity equal to infinity) and core-annular (zero eccentricity) patterns would be particular extreme cases for liquids of same viscosity but different densities and same density but different viscosities, respectively.

Kurban (1989) and Kurban and Bannwart (1990) calculated the laminar flow (velocity) fields of these prescribed interfaces by a numerical procedure where the linear momentum conservation equation in the axial direction of the pipe was discretised by the finite-volume method (Patankar, 1980). The problem was set in polar coordinates which presented some difficulties in matching the grid to the prescribed interface. To overcome these difficulties a *hybrid* procedure was devised for those cells which were crossed by the interface between the two phases. The viscosity in these *hybrid cells* was taken as a harmonic average of the values for the two fluids, weighted by the radial length of the cell occupied by each one of them. This procedure, despite being artificial, gave very good results

as it managed to conserve the actual momentum transfer (shear stress) that should occur through the prescribed interface. Finally, with the aid of the interfacial shape obtained by the variational *principle* of minimum viscous dissipation together with the results of the numerical simulations for several flows with varying interface eccentricity and radius of curvature, a graphical methodology was presented for the complete determination of the flow parameters (velocity fields and interface shape) for any given set of two fluids and flowing at any prescribed flow rates. An outline for a computational implementation of this methodology was also presented.

Issa (1987) developed a numerical procedure implemented in a computer program for the calculation of fully developed, turbulent stratified two-phase flow in horizontal and inclined pipes of circular cross section. Using the finite-volume approach, as described in Patankar (1980), the equations of momentum transfer were discretised, and by describing the flow domain with a bi-polar coordinate system the circular pipe cross section and the straight interface between the two phases could be fitted together. The implementation of this procedure includes the calculation of turbulent flows through two variants of the k - ϵ (turbulence kinetic energy-turbulence dissipation rate) model: one for low and the other for high Reynolds numbers. The discretised linear momentum conservation equation together with the transport equations for the conservation of turbulence kinetic energy k and of the turbulence dissipation rate ϵ formed a system of three equations which were solved sequentially in an external loop of the iterative procedure utilising the conjugate gradient method.

Hall and Hewitt (1993) produced a one-dimensional analytical model for the stratified flow of oil and water in a round pipe. Following a methodology similar

to that used by Taitel and Dukler (1976a) in modelling stratified gas-liquid flows, Hall and Hewitt's analysis lead to the following equations:

$$X_{ow}^2 \left[\left(\tilde{u}_w \tilde{D}_w \right)^{-n} \tilde{u}_w^2 \left(\frac{\tilde{P}_w}{\tilde{A}_w} + \frac{\tilde{P}_i}{\tilde{A}_w} + \frac{\tilde{P}_i}{\tilde{A}_o} \right) \right] - \left[\left(\tilde{u}_o \tilde{D}_o \right)^{-l} \tilde{u}_o^2 \frac{\tilde{P}_o}{\tilde{A}_o} \right] = 0 \quad (2.11)$$

where

$$X_{ow}^2 = \frac{\frac{4C_w}{D} \left(\frac{U_w D \rho_w}{\eta_w} \right)^{-n} \frac{\rho_w U_w^2}{2}}{\frac{4C_o}{D} \left(\frac{U_o D \rho_o}{\eta_o} \right)^{-l} \frac{\rho_o U_o^2}{2}} \quad (2.12)$$

and all the dimensionless parameters are the same as those defined by Taitel and Dukler (1976a, b) (only changing the gas phase references to oil) except for the effective diameters which, taking into consideration the higher viscosity of the oil in respect to that of the water, the oil is considered to be in an open surface flow while the water is assumed to be in a closed conduit flow.

For the sake of comparison with the above model and with the numerical solution of the flow field discussed later, Hall and Hewitt (1993) also developed a solution for a hypothetical oil-water stratified laminar flow between two flat plates of infinite width. Applying the same assumptions as in the Taitel and Dukler (1976a) pipe flow model and having in mind that due to the simpler geometry of the flow between flat plates and the linear relationship between pressure drop and flow rate for laminar flows, the Martinelli parameter X reduces to:

$$X_{ow}^2 = \frac{1}{\tilde{V}\tilde{\eta}} \quad (2.13)$$

where, in turn:

$$\tilde{V} = \frac{\dot{V}_w}{\dot{V}_o} \quad (2.14)$$

and

$$\tilde{\eta} = \frac{\eta_w}{\eta_o} \quad (2.15)$$

the following equation is derived:

$$2X_{ow}(1-\tilde{h})^2(2-\tilde{h})-\tilde{h}^3=0 \quad (2.16)$$

An *exact* solution of the oil-water flow obtained from the differential momentum conservation equations for developed flows can be represented in terms of $\tilde{h}$ and X_{ow} :

$$\begin{aligned} h^4 \left[(1-\tilde{\eta}) \left(\frac{1}{X_{ow}} + 1 \right) \right] + 2\tilde{h}^3 \left(\frac{1}{X_{ow}} + 3\tilde{\eta} - 2 \right) - \\ 3\tilde{h}^2 \left(\frac{1}{X_{ow}} + 3\tilde{\eta} - 2 \right) + 4\tilde{h}(\tilde{\eta} - 1) + 1 = 0 \end{aligned} \quad (2.17)$$

Comparing equations (2.16) and (2.17) it will be seen that Taitel and Dukler assumptions lead to a simple dependence of the equilibrium height h on the Martinelli parameter only, whereas, in reality there is also an influence of the viscosity ratio.

In this work, Hall and Hewitt also presented a numerical solution for developed stratified oil-water flows. This numerical model predicted the two dimensional flow field in a circular pipe by solving the mass and momentum conservation equations for each fluid through a finite difference technique and a bipolar coordinate transformation similar to that used by Issa (1987), which has the advantage of matching both the pipe walls and the interface in the stratified flow geometry.

Comparing the Taitel and Dukler type solutions with the exact solution in the flat plate flow case and with the numerical solution in the pipe flow case, Hall and Hewitt conclude that in spite of the existence a good agreement in the calculation of the volume fractions, via the equilibrium interface height, for high oil-water viscosity ratios, a significant deviation would exist in cases where this viscosity ratio was low (Figures 2.14 and 2.15).

Brauner and Moalem Maron (1992a) performed a stability analysis of stratified liquid-liquid flows. Starting with a hypothetical steady stratified flow configuration (Figure 2.16), they applied to it a harmonic disturbance in space at a specified time ($t=0$), and used linear stability analysis to establish whether the disturbance grew or collapsed.

The flow depicted in Figure 2.16 was area averaged in order to represent it by a two-fluid, one-dimensional model whose governing mass and linear momentum conservation equations are:

$$\frac{\partial}{\partial t}(\rho_b A_b) + \frac{\partial}{\partial x}(\rho_b A_b u_b) = 0 \quad (2.18)$$

$$\frac{\partial}{\partial t}(\rho_a A_a) + \frac{\partial}{\partial x}(\rho_a A_a u_a) = 0 \quad (2.19)$$

and

$$\begin{aligned} \frac{\partial}{\partial t}(\rho_b A_b u_b) + \frac{\partial}{\partial x}(\rho_b A_b \gamma u_b^2) = & \quad (2.20) \\ -\tau_b P_b \pm \tau_i P_i + \rho_b A_b g \sin \beta - \frac{\partial}{\partial x}(A_b p_b) + p_{ib} \frac{\partial A_b}{\partial x} \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial t}(\rho_a A_a u_a) + \frac{\partial}{\partial x}(\rho_a A_a \gamma u_a^2) = & \quad (2.21) \\ -\tau_a P_a \mp \tau_i P_i + \rho_a A_a g \sin \beta - \frac{\partial}{\partial x}(A_a p_a) + p_{ia} \frac{\partial A_a}{\partial x} \end{aligned}$$

where the subscripts a and b apply to the respective phases.

It should be noted that the above equations are written in their complete one-dimensional form (i.e. including the time dependent terms) so as to allow space and time perturbation functions to be applied to the main variables (phase velocities u_a and u_b , and interface height, h , which is implicitly present in Equations (2.18) through (2.21) through the flow areas A_a and A_b). The upper sign before the interfacial drag term ($\tau_i P_i$) in these equations should be used when the phase a is faster than phase b , and the lower sign when the opposite is true. The parameter γ is a shape factor in order to take account of the different velocity profiles encountered in the flow areas for each of the phases. The pressure for each phase includes the static pressure term and therefore varies due to gravity, hence the average pressures p_a and p_b differ from the corresponding values at the

interface of the fluids p_{1a} and p_{1b} , which, in turn, are not equal due to the existence of interfacial tension.

Through the appropriate analytical work of Equations (2.18) to (2.21) and by the inclusion of the necessary closure relationships regarding the system geometry and the interfacial tension formulation, the resulting system of equations was then linearised around a hypothetical smooth fully-developed stratified flow with superficial velocities U_a and U_b for each of the phases. The equations for the real neutral stable wave velocity C_{rn} and the neutral wave number k_n resulting from this stability analysis were of the form:

$$C_{rn} = f(\text{fully developed flow parameters}) \quad (2.22)$$

and

$$aC_{rn}^2 + bC_{rn} + (c + dk_n^2) = 0 \quad (2.23)$$

Equations (2.22) and (2.23) are only generic representations of the complex equations developed by Brauner and Moalem Maron (1992a), the original paper should be consulted for their detailed forms and derivation. For the sake of the present review this simplified representation should suffice, but it is important to remark that the parameters a, b, c and d of the quadratic Equation (2.23) are functions only of the geometry of the hypothetical stratified solution and of the two fluids properties.

Besides the stability analysis, Brauner and Moalem Maron (1992a) also derived the conditions for the well posedness of the initial value equations set formed by

the continuity and momentum Equations (2.18) to (2.21). This set of equations is said to be well posed if it possesses real characteristics. Therefore a test for real characteristics was carried out around the same initial conditions of a hypothetical fully developed stratified flow as in the linear stability analysis. After some analytical work, the characteristic equation for the set formed by Equations (2.18) to (2.21) was shown to be identical to Equation (2.23), already obtained from the stability analysis, so that the values of C_m determined that way would represent the desired characteristics. Thus, the condition for real characteristics was obtained by avoiding any complex root for (2.23), so:

$$b^2 - 4a(c + dk_{rc}^2) = 0 \quad (2.24)$$

where k_{rc} is the minimum wave number value for which real characteristics exist for the initial value problem under consideration.

In searching for all combinations of the initial stratified flow superficial velocities U_a and U_b (over which the stability analysis was applied) for which Equations (2.22) and (2.23) hold (in other words the pairs of U_a and U_b for which k_n vanishes), Brauner and Moalem Maron (1992b) could determine the zero neutral stability line (ZNS) for a given two phase system. By performing a similar search for combinations of U_a and U_b which satisfy the real characteristics condition expressed by Equation (2.21), the zero real characteristic boundary (ZRC) could also be constructed. Figure 2.17 is a plot of the ZNS and ZRC boundaries in terms of each of the phases superficial velocities calculated by Brauner and Moalem Maron for an oil water system ($\rho_a/\rho_b=0.834$; $\eta_a=18$ mPa s; $\eta_b=0.9$ mPa s; $\gamma_a=\gamma_b=1$) flowing in pipe with 20.5 mm internal diameter. Brauner and Moalem Maron (1992a, 1992b) associate the ZNS line with the boundary for the stable

smooth stratified flow pattern zone, as it is well understood from the linear stability analysis performed. The area between the ZNS and ZRC lines, is, in turn, identified with a buffer zone between the smooth stratified and the non-stratified flow pattern areas. Thus, Brauner and Moalem Maron (1992b) propose that this buffer zone could as well be associated with the area of occurrence of the wavy stratified flow pattern. These authors attempt to justify their proposal by reasoning that the complex characteristics found for operational conditions beyond the ZRC boundary imply that the equations for the stratified flow pattern at these flow rate ranges can no more accommodate the time and space variations associated with a certain range of amplified wave modes giving definite evidence that a flow pattern other than stratified should prevail therein. On the other hand, for operational points comprised between the ZNS and ZRC boundaries, i.e. inside the buffer zone, the stratified flow pattern is still well posed, though interfacial waves growth is still predicted to exist, indicating, perhaps, that wavy stratified flows may be found in this area.

After these considerations for the occurrence of smooth and wavy stratified flow pattern regions Brauner and Moalem Maron (1992b) tried to establish criteria for other flow pattern boundaries, with the aim of generating a general flow pattern transition methodology valid for all liquid-liquid systems flowing in horizontal pipes. The other boundaries defined in their work encompass annular, slug, stratified-dispersed and fully dispersed flow patterns in addition to the wavy and smooth stratified flow patterns defined by the ZNS and ZRC lines respectively.

Brauner and Moalem Maron (1992b) suggested that flow conditions outside the ZRC lines lead to either annular or slug flows. The annular/slug flow pattern transition criterion was based on geometrical considerations performed for a hypothetical annular flow configuration, whose flow parameters (e.g. interfacial

radius and pressure gradient for given input flow rates of the two fluids or vice versa) are determined according to the model presented by Brauner (1990). The transition line from annular to slug flow was established by searching the superficial velocities U_a and U_b for which the annular model equations fulfil the condition that either the interface diameter (D_c) is equal to half of the pipe diameter (D) or that the area occupied by the core fluid (A_c) is half of the total cross sectional pipe area (A), that is:

$$D_c = \frac{D}{2} \quad (2.25)$$

or

$$A_c = \frac{A}{2} \quad (2.26)$$

The criterion for the existence of stratified-dispersed flow was proposed by requiring that for given input superficial velocities, U_a , U_b , the available upper cross-sectional area of a hypothetical smooth stratified flow calculated for these parameters, is greater than that of a critical drop size diameter determined by a balance between surface tension and buoyancy forces. This criterion is justified by assuming that if the upper cross-sectional area is smaller than this critical diameter then the discrete drops will be forced to agglomerate forming a continuous phase layer, and, as a consequence, smooth or wavy stratified flow will be maintained. This boundary transition criterion is expressed mathematically by:

$$A_a = \frac{\pi d_{cr}^2}{4} \quad (2.27)$$

where

$$A_a = A_a(U_a, U_b, \rho_a, \rho_b, \eta_a, \eta_b) \quad (2.28)$$

and

$$d_{cr} = \sqrt{\frac{6\sigma}{(\rho_b - \rho_a)}} \quad (2.29)$$

The criterion above refers to the possibility of dispersed phase a in phase b for relatively low U_a . Brauner and Moalem Maron suggested that analogous reasoning can be applied for the case of dispersing the heavier phase b in the lighter phase a , when U_a is significantly smaller than U_b .

The methodology proposed by Brauner and Moalem Maron is not conclusive about the boundaries of the *fully* dispersed flow pattern. Basically they utilise Hinze's (1955) model for liquid-liquid dispersion, by which the maximum drop diameter of the dispersed phase is related to the flow turbulence, by:

$$\frac{d_m}{d_{cr}} = C_1 We_m^{0.1} Fr_m^{-0.5} Re_m^{0.08} \quad (2.30)$$

where d_{cr} is given by Equation (2.29) and We , Fr and Re are respectively Weber, Froude and Reynolds numbers, defined by:

$$\text{We}_m = \frac{\sigma}{D\rho_b U_m^2} \quad (2.31)$$

$$\text{Fr}_m = \frac{\rho_b U_m^2}{\Delta\rho g D} \quad (2.32)$$

$$\text{Re}_m = \frac{DU_m}{\nu_b} \quad (2.33)$$

where, in turn, U_m is the average flow velocity:

$$U_m = U_a + U_b \quad (2.34)$$

A typical flow pattern map presented in Brauner and Moalem Maron (1992b) is shown in Figure 2.18.

As it has been shown, the literature surveyed covers many diverse aspects of liquid-liquid flows. It is noteworthy that there has been a lack of connection between the analytical and experimental studies, possibly reflecting the limited number of the latter.

In the present work an attempt is made to carry out *both* experimental and analytical studies which complement the earlier studies whilst drawing on them where appropriate.

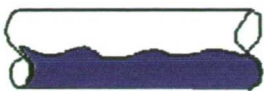
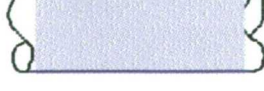
Category	Regime in Pipe Flow	Configuration	Sketch
Separated	Stratified	liquid layer in gas	
	Annular	gas core and liquid film	
Mixed or Transition	Slug	gas pocket in liquid	
	Annular with entrainment	gas core with droplets and liquid film with gas bubbles	
Dispersed	Bubbly	gas bubbles on liquids	
	Droplet	liquid droplets in gas	

Table 2.1: Classification of gas-liquid two-phase flow

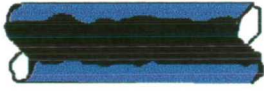
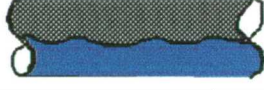
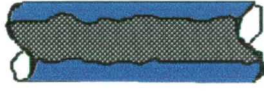
Category	Regime in Pipe Flow	Configuration	Sketch
Separated	Stratified	water layer on oil	
	Core - Annular	oil core and water film	
Mixed or Transition	Intermittent	phases alternately occupying the pipe as a free phase or as a dispersion.	
	Stratified with dispersion	layers of a dispersion with a free phase	
	Annular with dispersion	oil core with droplets and water film with oil droplets	
Dispersed	Water in oil	water droplets in oil.	
	Oil in water	Oil droplets in water	

Table 2.2: Classification of liquid-liquid two-phase flow

Category	Nomenclature adopted	Russel et al. (1959)	Charles et al. (1961)	Hasson et al. (1970)	Arrachakaran et al. (1989)	Stapelberg and Mewes (1990)	Nädler and Mewes (1995)
Separated	Stratified	Stratified		Stratified	Segregated	Stratified	Stratified
	Core-annular			Annular	Annular		
Mixed or transition	Intermittent		Slug	Slugs	Intermittent (oil continuous) Intermittent (water continuous)		
	Stratified with water in oil dispersion				Mixed (dispersion over a water layer)		Stratified with mixing at the interface Layers of water in oil dispersion and water Layers of dispersions and water Oil in water dispersion above a water layer
	Stratified with oil in water dispersion				Mixed (oil layer over a dispersion)		
	Annular with dispersion			Mixed			
Dispersed	Water in oil dispersion		Mist	Dispersion	Dispersed (oil continuous)	Water in oil dispersion	Unstable water in oil emulsions
	Oil in water dispersion	Bubble	Bubble	Dispersion	Dispersed (water continuous)	Oil in water dispersion	
		Mixed phases	Froth				

Table 2.3: Liquid-liquid flow pattern nomenclature

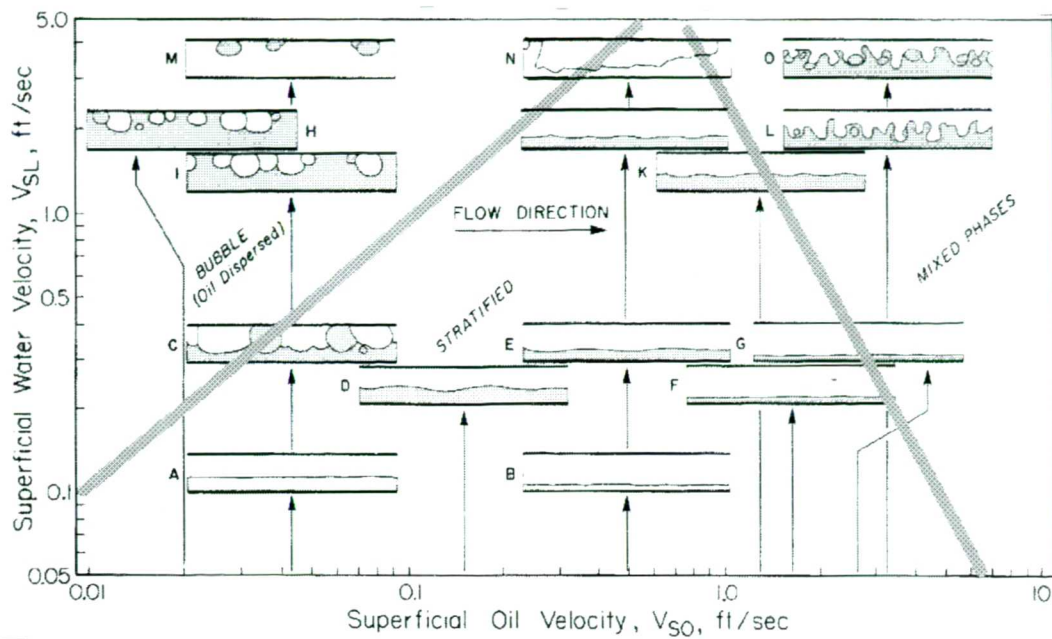


Figure 2.1: Flow patterns for a 18.0 mPa s, 834 kg/m³ oil and water system in a 0.025 m ID pipe based on observations of Russel et al. (1959). Map as published by Govier and Aziz (1972)

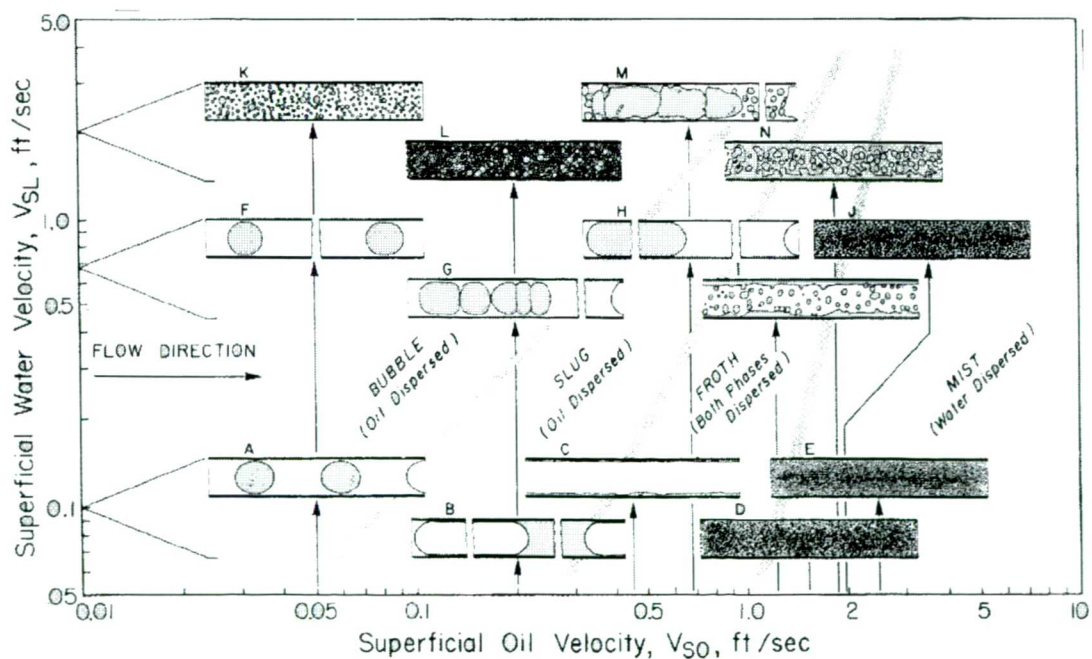


Figure 2.2: Flow patterns for a 16.8 mPa s, 998 kg/m³ oil and water system in a 0.026 m ID pipe based on observations of Charles et al. (1961). Map as published by Govier and Aziz (1972)

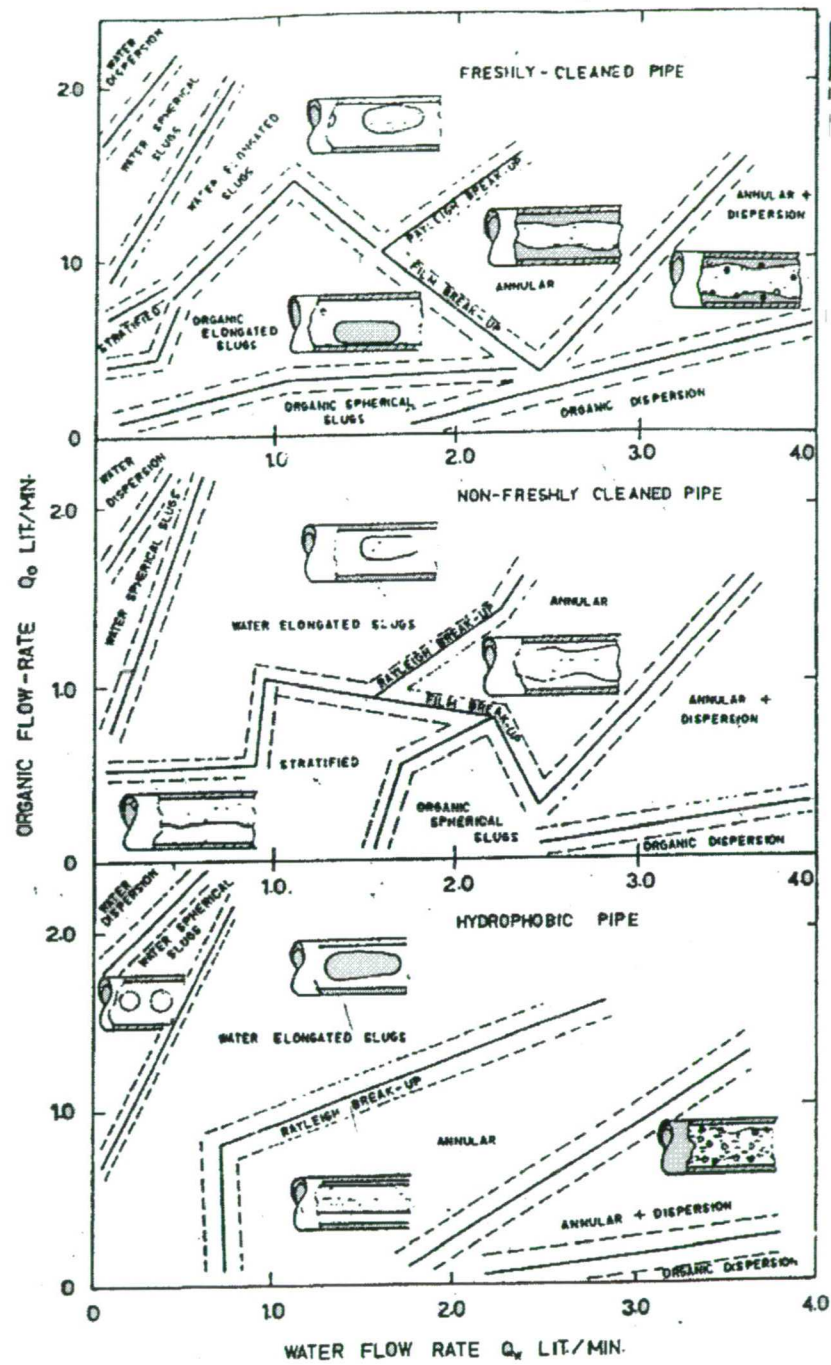


Figure 2.3: Flow patterns for a 0.82 mPa s, 1020 kg/m³ kerosene-perchloroethylene solution and water system in a 1.26 cm ID glass tube based on observations of Hasson et al. (1970).

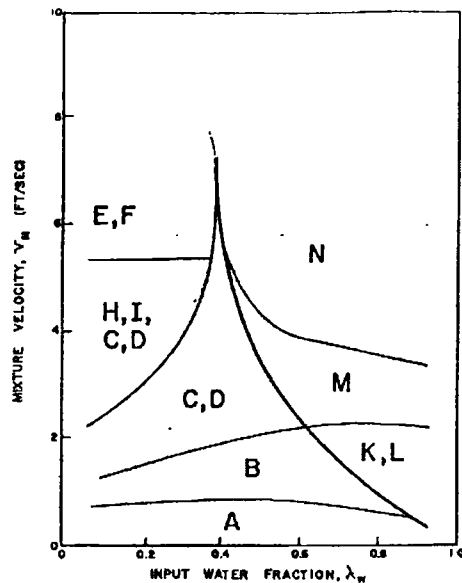


Figure 2.4: Flow patterns for a 21.7 mPas, 896 kg/m³ oil and water system in a 3.94 cm pipe based on observations of Guzhov *et al.* (1973), as revised and presented by Oglesby *et al.* (1979)

Flow patterns defined by Oglesby (1979):

A- Segregated: (no mixing at the interface)

B- Semi-segregated: (some mixing at the interface)

Oil dominant

C- Semi-mixed: Layers of free oil and oil in water dispersion. The layer of the dispersion occupies less than 50% of the pipe diameter.

D- Mixed: Layers of free oil and oil in water dispersion. The layer of the dispersion occupies more than 50% of the pipe diameter.

E- Semi-dispersed: Non-homogeneous water in oil dispersion.

F- Dispersed: Homogeneous fully dispersed flow.

G- Annular: Water core and oil film

H- Slug: Water slugs in oil

Water dominant

I- Slug: Oil slugs in water

J- Annular: Oil core and water film

K- Semi-mixed: Layers of free water and water in oil dispersion. The layer of the dispersion occupies less than 50% of the pipe diameter.

L- Mixed: Layers of free water and water in oil dispersion. The layer of the dispersion occupies more than 50% of the pipe diameter.

M- Semi-dispersed: Non-homogeneous water in oil dispersion.

N- Dispersed: Homogeneous fully dispersed flow.

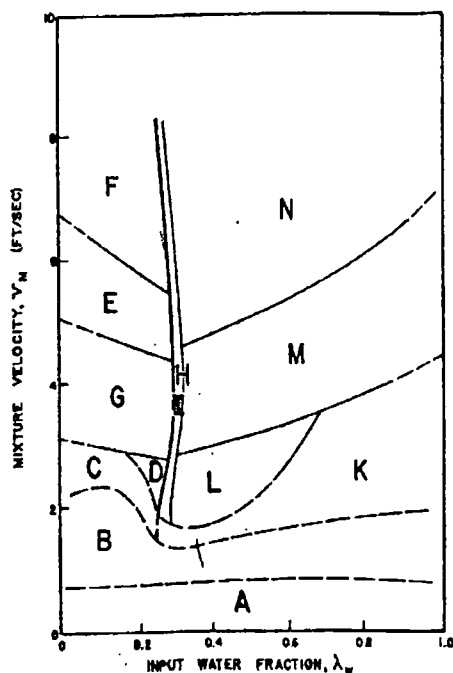


Figure 2.5: Flow patterns for a 84.0 mPa s oil and water system in a 41.0 mm pipe based on observations of Oglesby *et al.* (1979)

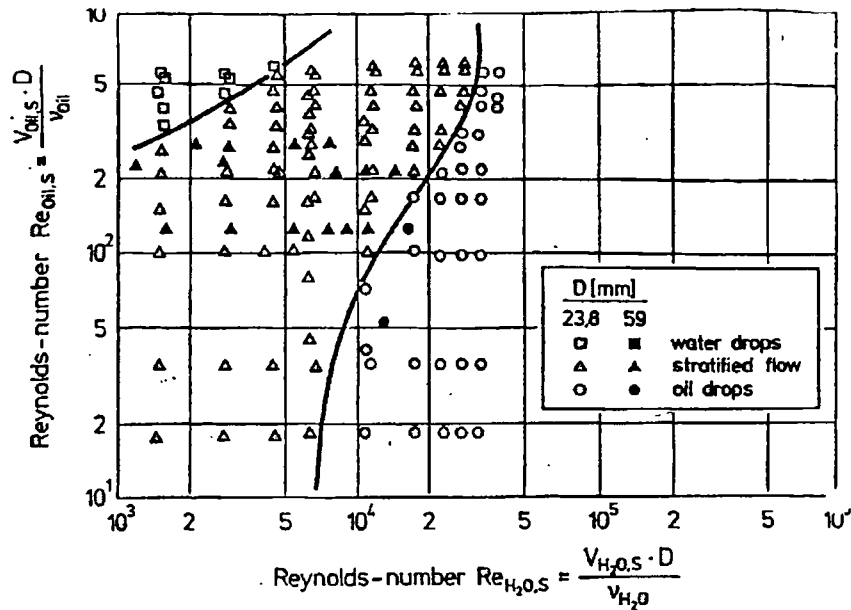


Figure 2.6: Flow patterns for a 30.0 mPas, 850 kg/m³ white mineral oil and water system in 2.38 and 5.90 cm perspex tubes based on observations of Stapelberg and Mewes (1990)

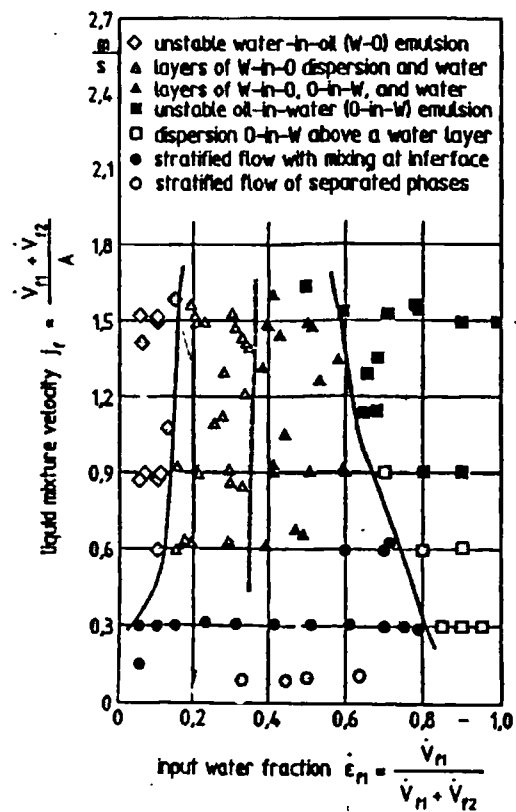


Figure 2.7: Flow patterns for a 20.0 mPas, 841 kg/m³ oil and water system in a 5.90 cm perspex tube based on observations of Nädler and Mewes (1995)

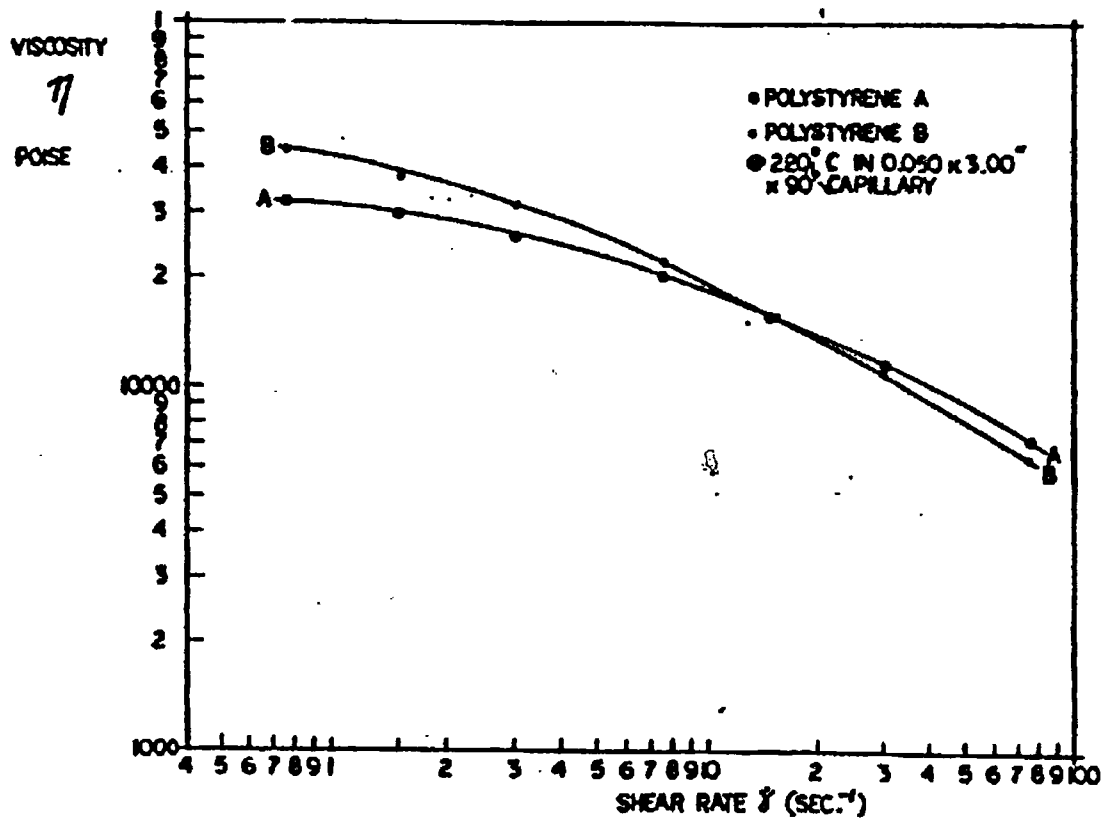


Figure 2.8: Viscosity vs shear rate crossover for the two polystyrenes used in the experiments of Southern and Ballman (1973)

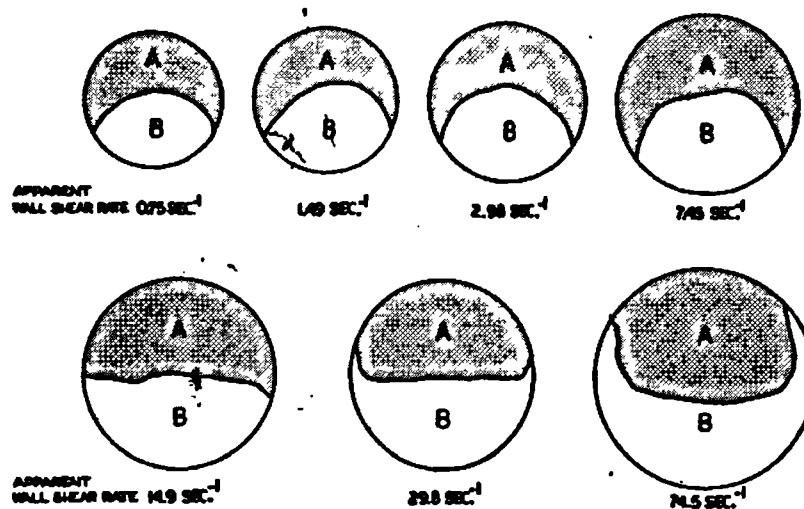


Figure 2.9: Interface shapes of two polystyrenes resulting from the simultaneous flow in a tube observed by Southern and Ballman (1973)

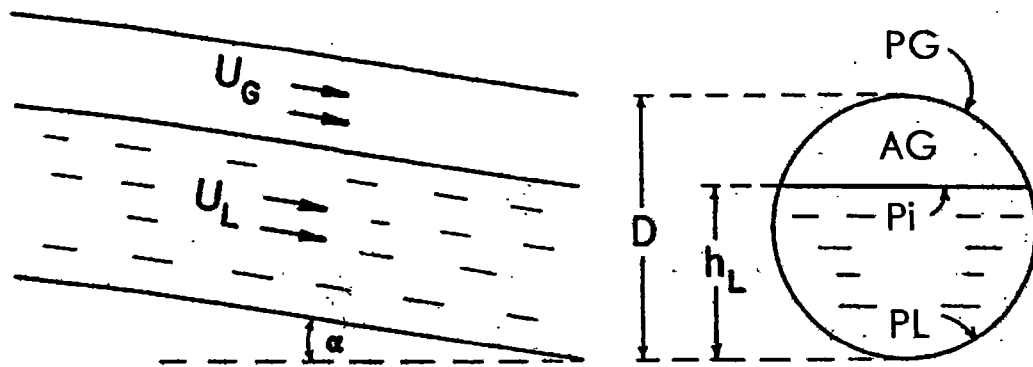


Figure 2.10: Equilibrium stratified flow for Taitel and Dukler (1976a, b) analysis

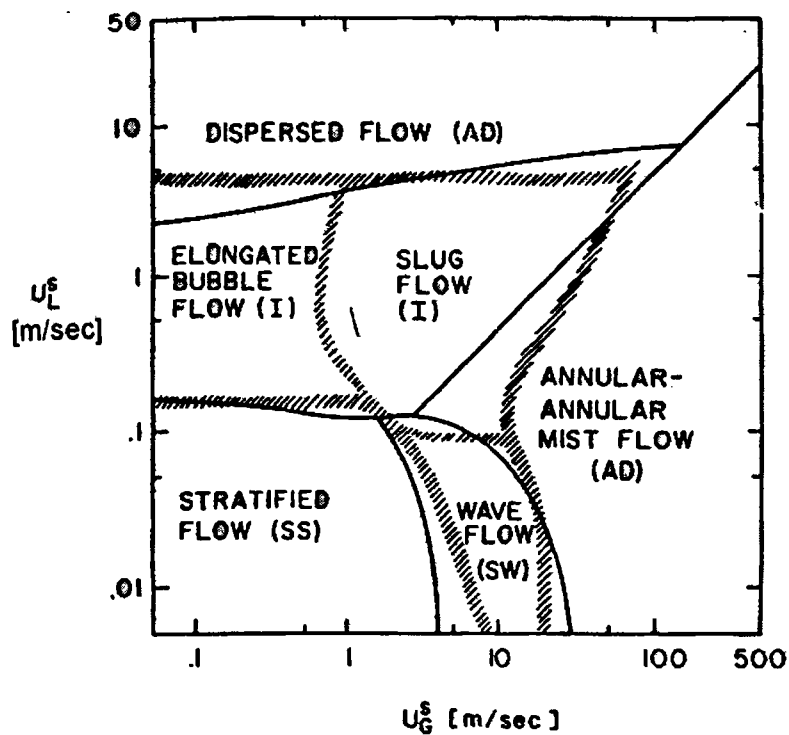


Figure 2.11: Comparison of flow pattern maps calculated by the methodology of Taitel and Dukler, 1976a (solid line) and the experimental flow pattern map of Mandhane, 1974 (hatched line).

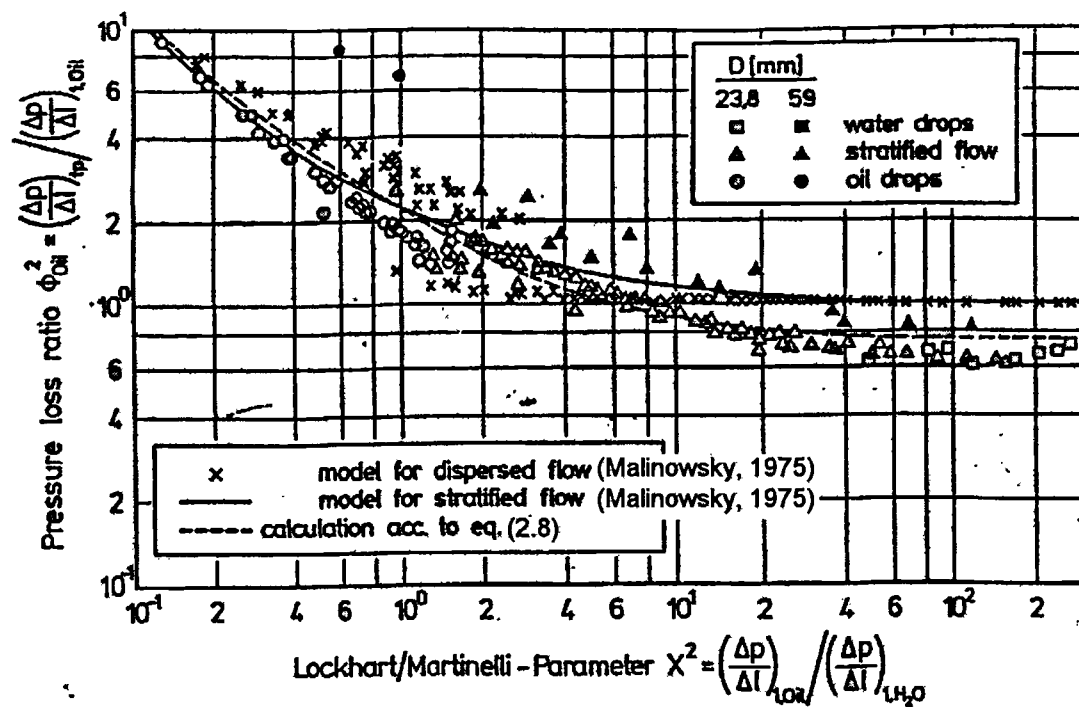


Figure 2.12: Comparison of the experimental data of Stapelberg and Mewes (1990) with the predictions of their own model— Equation (2.8) — and with those ones of the dispersed and stratified models of Malinowsky (1975) — Equations 2.5 through 2.7.

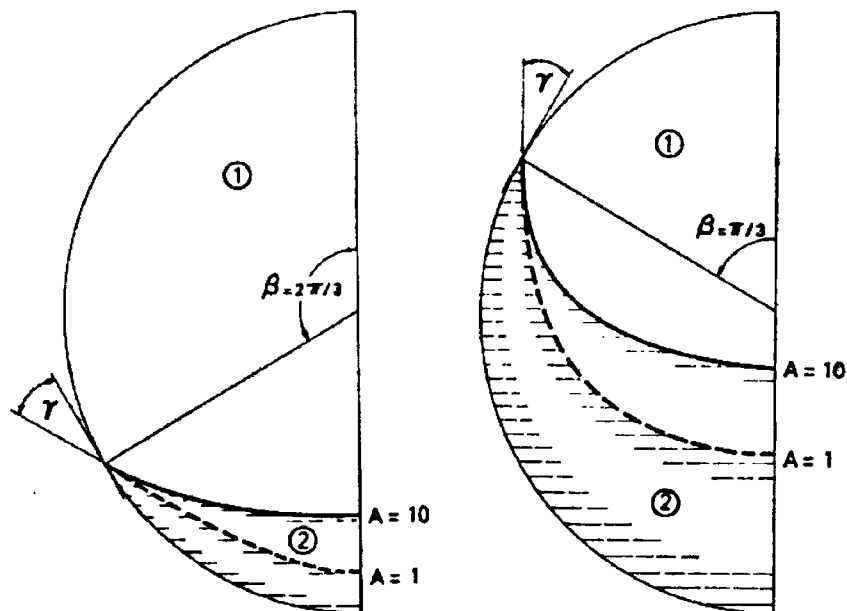


Figure 2.13: Interface shapes between two liquids in a circular pipe cross-section obtained analytically by Bentwich (1976) taking into account their surface tensions, capillary forces and densities. The parameter A is a non-dimensional stratification parameter, a function of the pipe radius, the interfacial tension, and the densities of the fluids and γ is the contact angle for the system.

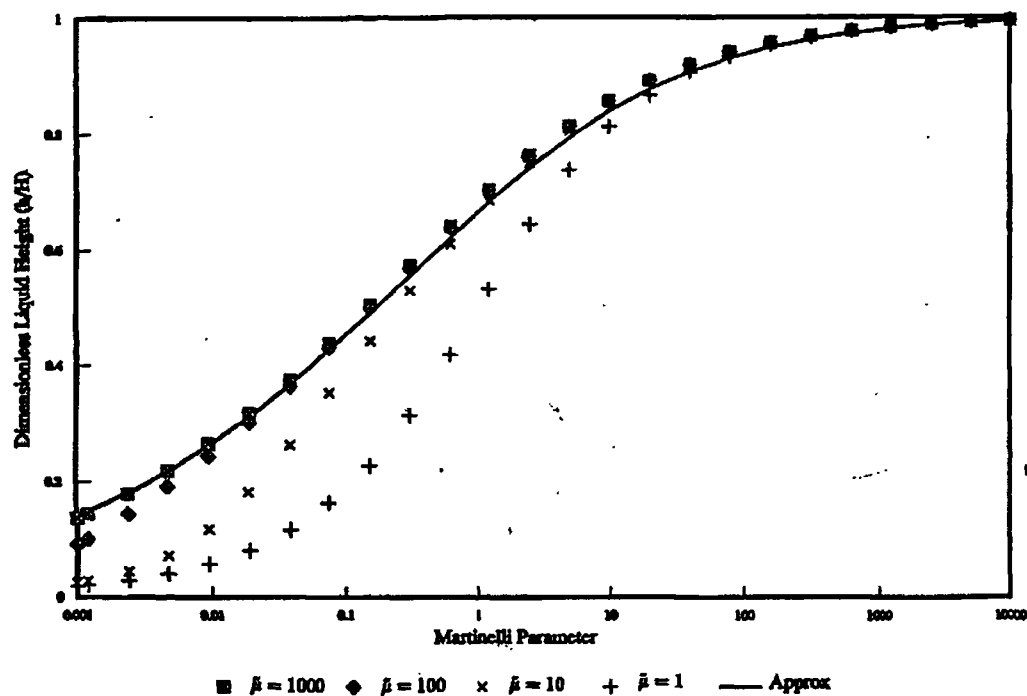


Figure 2.14: Comparison of the exact solution and the two-fluid model for the calculation of the equilibrium interface height in liquid-liquid flows between flat plates — Hall and Hewitt (1993)

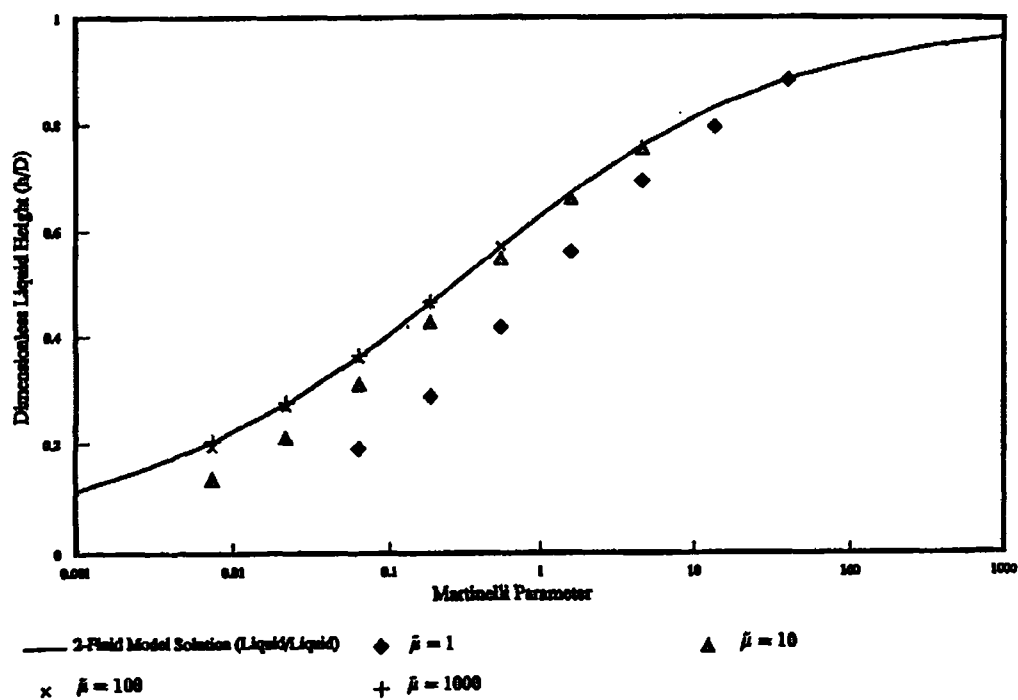


Figure 2.15: Comparison of the numerical solution and the two-fluid model for the calculation of the equilibrium interface height in liquid-liquid flows in a circular pipe — Hall and Hewitt (1993)

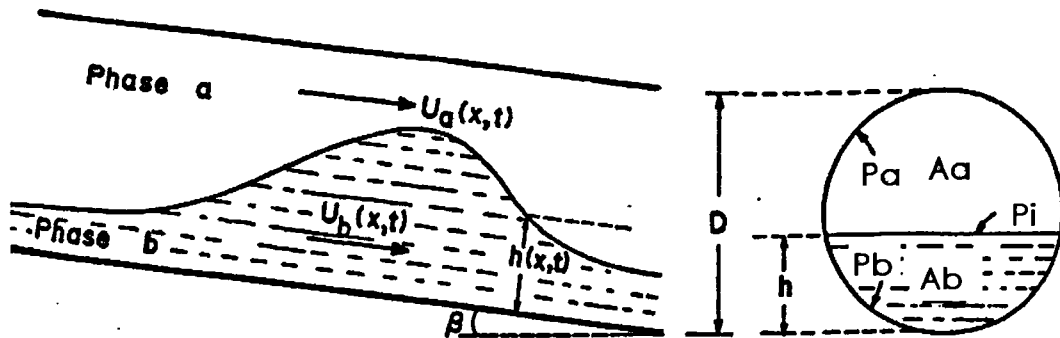


Figure 2.16: Schematic description of a stratified flow configuration used by Brauner and Moalem Maron (1992a)

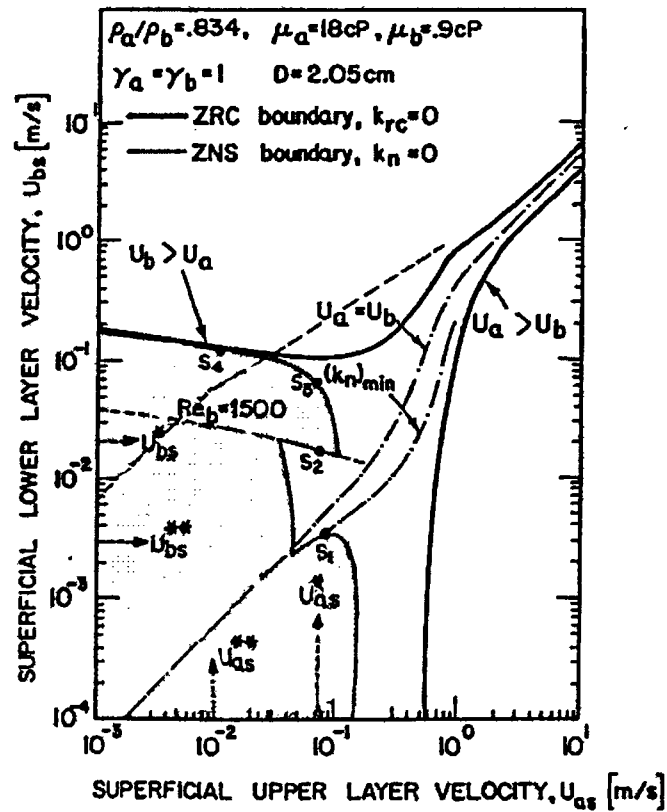


Figure 2.17: ZNS and ZRC boundaries for an oil-water system presented by Brauner and Moalem Maron (1992b), calculated according to their own methodology

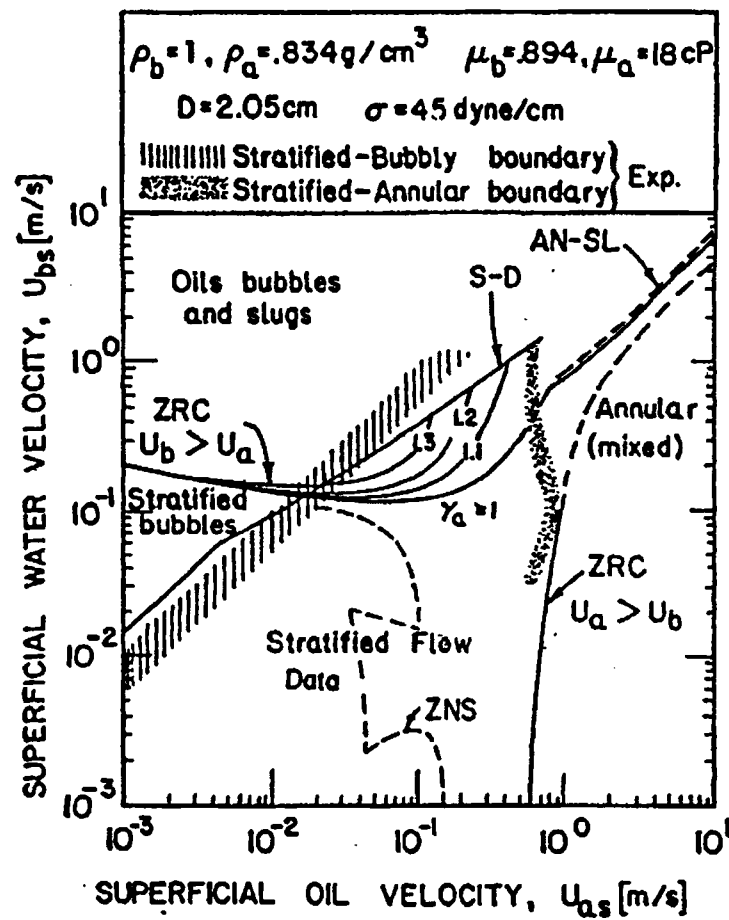


Figure 2.18: Analytical flow pattern map for an oil-water system presented by Brauner and Moalem Maron (1992b), calculated according to their own methodology

Chapter 3

Experimental Programme

The objective of this Chapter is to describe the experimental facilities used (TOWER and WASP) and the associated measurement methods and also to present the experimental results obtained. In Chapter 4 these results are used as a data base for the modelling studies; thus, no comparison with predictions are given in the present chapter.

In what follows, Section 3.1 presents a description of the experimental facilities and measurement methods, Section 3.2 presents the results from the main series of experiments in which the objectives were to produce data on flow pattern, pressure drop and (in the case of the WASP facilities) holdup (phase fractions). A special series of experiments were conducted in the TOWER facility in which the interface position was measured using a high frequency probe; these are described in section 3.3.

3.1- Description of experimental facilities

Two experimental flow-loops were used in this work for the generation of data, the TOWER and WASP rigs. The TOWER rig is a new facility designed during the course of this work. The WASP rig is a multiphase flow facility already existent at

Imperial College, which can run two, three and four phase flows. In this Section the TOWER and WASP rigs are described, as well as the instrumentation used.

3.1.1- The TOWER facility

The experimental facility on which most of the data presented in this work was obtained was the TOWER (Two-phase Oil and Water Experimental Rig) rig. This major new facility was designed as part of the project described in this thesis. The construction of the rig was financed by a grant from Norsk Hydro a.s.. The work was carried out in collaboration with Miss Panagiota Angeli, a fellow research student. Miss Angeli's work was concerned with fully dispersed liquid-liquid flows which was a natural corollary to the present work on separated liquid-liquid flows. The TOWER rig was designed to cover the full range of conditions required for the respective projects.

A schematic diagram of the TOWER rig is shown in Figure 3.1, it consists of the following sections:

- a) Tanks
- b) Pumps
- c) Mixer
- d) Test section
- e) Separator
- f) Instrumentation

These are described in detail in the following subsections.

3.1.1.1- Tanks

The two storage tanks are identical, are made of fibre glass reinforced plastic and can store as much as 600 litres of oil or water. They have two inlets: one from the separator, where the oil and water mixtures are separated, and the other from the pump recycling circuit, which controls the required flow rate through the system. There is also one outlet pipe which links the tanks to the fluid pumping section.

The water tank is filled directly from the mains through a rubber hose. The oil tank is filled by pumping oil from the supply drums through a special branch in the oil suction line of the oil pump; this pumps oil out of the drum and into the recycling line leading to the oil tank.

3.1.1.2- Pumps

Two identical centrifugal pumps supplied by Ingersoll-Dresser are used in the flow loop, but since the water and oil densities are different, their performance when pumping each of these liquids is different.

The flow rate of the liquids into the test section is regulated by means of two adjusting angle-seat valves; one fitted in the recycling line and the other in the flowmeter inlet line.

The flow rate is monitored by means of two variable area flowmeters (made by Fischer and Porter Ltd) and calibrated for the respective fluids with an accuracy of $\pm 1\%$ of the full scale reading. These devices have tapered tubes through which the fluid passes and the position of a float within the tapered tube gives an indication of the flowrate. This float position is governed by the balance between the gravitational

forces on the float and the drag forces exerted on the float by the fluid. Since the fluid drag forces, and consequently the float height, are dependent on the fluids properties, i.e. their densities and viscosities, the oil and water flowmeters have different scales and calibration.

Two different sets of flowmeters were used: one for high flow rates (up to $1.2 \times 10^{-3} \text{ m}^3/\text{s}$) and the other for low flow rates (up to $2 \times 10^{-4} \text{ m}^3/\text{s}$). Each of these flow meters can be independently interchanged, so that a wide range of combinations of oil and water flow rates can be achieved. When a change was needed in the flowmeters they were replaced (the PVC connectors made this easy); this was considered a better option than using a valve system from the point of view of avoiding leakages. Figure 3.2 shows the flowmeter assembly.

3.1.1.3- Mixers

In any multiphase flow experiment, the method by which the respective phases are introduced into the test section is of crucial importance. Entrance effects can persist for long distances; conversely, long distances may be required to reach equilibrium flow which is independent of entrance conditions. For the stratified flows which are the principal objective of the preceding study, the inlet effects are certainly less significant than in the parallel study of dispersed flow which has been carried out using the TOWER facilities. For this latter study, where the fluid velocities are much higher, it was felt important to control the drop size distribution at the inlet and, to achieve it, a five-element STATIFLO static mixer was used, supplied by Kenics. The numbers of elements used could be varied, giving some control of the initial drop size in dispersed flows. For the stratified flows studied in the present work, tests were carried out both with the full set of mixers elements in place and also without any mixers elements. The mixing section has a special distribution manifold and two oil

and water mixers, one for each flow line of the test section. The manifold consists of a set of three-way valves which can transfer the oil and water to each of the mixers. It was designed in such a way that the fluids can enter the mixer either axially or perpendicularly to the test section.

The static mixing elements were contained in a 20 cm long stainless steel 1" (2.54 cm) flanged tube. The static mixing elements, can be varied in number from zero to five. The mixing elements are 40 mm long pieces of a helicoid which make a once straight flow to continuously change its direction, causing turbulence, and mixing the two liquids. The mixing action increases with the number of elements in the tube spool. This process is illustrated in Figure 3.3.

3.1.1.4- Test sections

Two independent test sections were used to perform flow experiments, one made of stainless steel and the other of a clear acrylic plastic, commercially called *transpalite*.

The stainless steel test section is 9.7 m long and is composed of four 1" sch 80 (inside diameter equal to 24.3 mm) tubes (two tubes 1.0m long and the other two tubes 3.85 m long) linked together with flanged connections especially designed to give continuous and smooth inner walls.. Close to the outlet of the stainless steel tube there is, sandwiched between the flanges a monitoring spool, a hollow acrylic (*transpalite*) cylinder where the fluid flows can be visually observed. Alternatively, this spool can be replaced by the high-frequency probe assembly, which was used to determine the interface height level in the stratified flow experiments. Just upstream of the monitoring spools, two differential pressure transducer ports are located, 1.914 m apart with the first transducer being placed at 5.20 m from the injector. By this means the pressure gradients developed for each flow condition were also measured.

The clear acrylic (*transpalite*) test section is 9.5 m long and consists of five sections (two sections 1 m long and three sections 2.5 m long) of 1 inch nominal diameter tubes (inside diameter of 24 mm). This test section allowed the development of the flow patterns along the tube to be observed. However, the main reason for having both a stainless steel and a plastic test section was to allow the study of the effect of wall material on phase inversion in the study parallel to the present one on dispersed flow. Two differential pressure transducer ports (located 2.564 m apart and with the first port being placed at 3.80 m from the injector) allowed the pressure gradients developed for each flow condition to be measured in the acrylic test section.

The two test sections are also interconnected in order to allow more flexibility in future uses of this rig.

3.1.1.5- Separator

The separator is a 4 m³ horizontal vessel which is fitted internally with a 'knitmesh' coalescer to facilitate the segregation of the oil and water phases. The coalescer was supplied by KnitMesh, UK.

The 'knitmesh' is a mesh made up of two different materials with different wettabilities to oil and water. Drops of water in the oil-continuous region are collected and coalesced into larger water droplets on the hydrophillic elements of the mesh, and drops of oil in the water continuous region are collected and coalesced on the oil-wetted elements. The large drops leaving the coalescer are easily separated by gravity in the separator settling chamber downstream of the mesh zone.

3.1.1.6- Instrumentation

The instrumentation used in the TOWER facility in the present project comprised the differential pressure cell, the high-frequency impedance probe and the (high speed) video recording equipment.

The pressure gradients in each test section were measured with a Validyne model DP45 variable reluctance low-differential pressure transducer. This transducer was connected to the pressure ports in the test sections by means of high pressure nylon tubing, which could easily be moved from one test section to the other, depending on which test section was in use. The pressure ports (see Figure 3.4) were made of acrylic resin and sandwiched between flanges in the test-sections. This provides even more flexibility to the rig, as the length over which the pressure drop is to be measured can be varied by changing the selected flange positions. Since the test sections were carrying a multiphase oil and water mixture, a purge system was devised to keep the tubings from each chamber of the differential pressure transducer filled with water, in order to avoid errors due to a hydrostatic pressure unbalance. This purge was, therefore, performed before each run of the rig when pressure drop measurements were to be taken. Though this purge could be done continuously during the experiments this was avoided, for this would require perfect control of the flow rates being injected in each tubing in order to balance the pressure differential in the lines. Thus, accumulator capsules were installed near the pressure ports, so that in the unlikely event that the water in the conducting tubings were contaminated with oil, the effects of this contamination would be attenuated by the much larger diameter of the chamber in relation to the tubing. Figure 3.4 shows the pressure purge system and accumulator capsule assembly in more detail.

A high frequency impedance probe developed by Schlumberger® for measuring the local averaged volume fraction inside the pipe was used for determining the position

of the interface in the stratified flow experiments. The measurement is based on the difference between the probe impedance value when the probe tip is immersed in oil or water respectively. This type of meter has a very rapid response time, and due to the high frequency used is very sensitive to dielectric constant variations.

The tip of the probe consists of two electrodes separated by an insulator. When the tip is in water there is an effective short circuit at the tip, and, when it is in oil there is an open circuit, and a potential difference is measured. The impedance change caused by the passage of water and oil layers are converted electronically into a DC signal which is recorded in a data acquisition system. By changing the radial position of the probe tip in order to scan the whole pipe diameter the interface height in stratified flow is therefore determined when a discontinuity of the measured signal is observed. The high frequency probe and its mounting support are shown in Figure 3.5.

The identification of flow patterns could not be done by direct observation without the aid of a video recording device, because at some points, due to the mixing of the phases it was very difficult to decide if a layer of a pure fluid was still present, or if the two fluids were totally dispersed: in other words, if the flow regime was stratified with dispersion or dispersed. Thus a Hadland Photonics High Speed Videoscope was used which has a framing rate of 50 Hz, each frame being synchronised to powerful xenon strobe lights of 20 μ s duration. Thus the sharpness of each frame of the video is increased and good definition still pictures can be made of them. These still pictures provide an excellent aid for the definition of flow patterns in otherwise misleading conditions. Another point of importance was the illumination of the visualisation section. It was found that best contrast was obtained with top illumination and with the use of a black background — which was provided by a black screen placed at the back of the visualisation section in relation to the camera.

3.1.1.7- Fluids used and their maintenance

The fluids used in the TOWER system consisted of a commercial hydrocarbon oil, EXXSOL D-80 and tap water.

The EXXSOL D-80 is a very clear oil, that does not stain the walls of the transparent test section or the visualisation section. Even though the contrast of this oil to water was not ideal — in some cases it was not easy to visually distinguish between the oil and water — the above mentioned non-stain characteristic made it a good choice for most of the experiments performed. The properties of the oil are given in Table 3.1.

The tap water used, eventually, after a certain (large) number of runs became contaminated with some oil. This, apparently, caused some bacterial or mould growth in the water. The use of a biocide could prevent this occurrence, but this biocide addition was avoided because of the damaging effect of biocide agents on the system stability. Apparently the biocide tested acted as a surfactant, lowering the interfacial tension between the oil and water, and caused them to mix almost immediately they were brought in contact. The remedy employed for this problem of biological growth was a more frequent change of the water stored in the tanks. As a consequence of this procedure a very small amount of oil was lost from the system and after a very long time and numerous runs some oil had to be added to the system.

3.1.2- The WASP facility

As will be reported below, it was found impossible to obtain slug flow in the experiments in the TOWER rig. Slug flow in liquid-liquid flows was reported for limited ranges of conditions by Guzhov et al. (1973) and Arirachakaran et al. (1989).

Thus, it was decided to widen the data range covered by carrying out liquid-liquid flow experiments in the Imperial College WASP facility which used a more viscous oil (thus reducing the possibility of mixing before generation of slug flow) and the WASP test section was also inclinable and it is known that slug flow is more likely to occur in upwardly inclined pipes. The WASP rig is designed to operate with any combination of air, oil and water flows and is designed for a maximum working pressure of 50 bar; the rig operates on a “once through basis” as illustrated in the flow diagram shown in Figure 3.6. The main components of the rig are:

- a) a 42 m long tube with 77.90 mm internal diameter, that can be set at horizontal or near-horizontal positions;
- b) a visualisation section located at the end of the test section, where direct observation and video tape recording of the flows can be made;
- c) a slug-catcher which collects the fluids from the test section and separates the liquid phases from the air, discharging the liquids to the dump tank and the air, via a silencer, into the atmosphere;
- d) a dump tank which allows the oil and water to separate;
- e) a pump that drives the fluids from the dump tank to the system main supply tanks;
- f) two supply tanks, one for water and the other for oil, which feed the liquids to the test section using two pumps or, alternatively, that can be pressurised with air from the air supply systems, allowing the liquids to be fed into the test section without the use of the pumps;

- g) a magnetic (MAGFLO) flowmeter to measure the water flow rate;
- h) two orifice plates to measure the oil and air flow rates;
- i) either one of two air supply systems, one low pressure from the college utilities, and the other, high pressure, derived from large (40 m^3) tanks which are pre-pressurised using a compressor system;.
- j) a visualisation section for observation and filming of flow patterns and other flow characteristics (a PANASONIC MV-10 video camera is used to transmit the pictures from the visualisation section to a video recorder and a video monitor in the WASP control room).

For the present (liquid/liquid) tests, the rig was operated at low pressure and the fluids used in these experiments were tap water and oil (Shell Tellus 22) with density of 865 kg/m^3 @ 15°C and an absolute viscosity of 45 mPa s @ 25°C ,

The *in-situ* water fraction in the experiments was measured with a multi-energy gamma-ray densitometer which had been developed for the WASP facilities (Pan and Hewitt, 1993, 1995). This densitometer is designed to measure the phase fractions of air, water and oil respectively by measuring the absorption of two gamma beams with different energies (for instance, beam from caesium - Cs137; 661.6 keV and americium - Am241; 59.5 keV , or two beams at different energies from a barium - Ba133; 31 keV and 81 keV source). For the present experiments, only two phases (oil and water) were present and it was necessary only to use one gamma energy (namely Am241). The WASP densitometer is fitted with a motorised traversing mechanism which allows the beam to be traversed across the tube, measurement being taken at a series of chordal positions. In the present experiments, a vertical orientation for the beam was chosen and, by symmetry, only half the tube needs to be scanned. A total of

eight chordal positions were measured as shown in Figure 3.7. For each chordal position, the fraction of the chord length (l) which is in the water phase is given by

$$\frac{l_w}{l} = \frac{\ln I - \ln I_o}{\ln I_o - \ln I_w} \quad (3.1)$$

where I_o and I_w are the intensities of the beam at the given chordal positions when the tube is full of oil and water respectively. To determine the water fraction by volume in the pipe, an area-weighted mean value of l_w/l is obtained.

3.2- Flow pattern, holdup and pressure drop experiments

Flow pattern and pressure drop experiments were run on both TOWER and WASP rigs and are described in Sections 3.2.1 and 3.2.2 respectively. On the TOWER rig all the experiments were for horizontal flow, and an extensive range of oil and water flow rates — from 0.05×10^{-3} to 1.2×10^{-3} m³/s — was covered. On the WASP rig the experiments were run with the tube horizontal and also 1° upwards inclined. The flow rates range covered in each of these cases were not as extensive as those on the TOWER. On the WASP facility it was possible to measure the phase fractions using the scanning gamma densitometer system.

3.2.1- TOWER experiments

Experiments were carried out covering a range of mixture velocities (oil plus water flow rates divided by the pipe cross-sectional area) of 0.22 to 4.37 m/s and input

water fractions (inlet water volume flowrate divided by total of oil and water volume flowrate) ranging from 0.11 to 0.86. The oil used in the TOWER experiments had a density of 860 kg/m^3 and a viscosity of 1.6 mPa.s . Nearly 200 tests were performed with the main categories as follows: :

- a) Flow in the *transpalite* test section, without the mixer;
- b) Flow in the *transpalite* test section with the mixer;
- c) Flow in the **stainless steel** test section without the mixer.

The following sections describe each of these groups of experiments .

3.2.1.1- Experiments in the *transpalite* test section - no mixer

A total of 84 runs were done in the *transpalite* test-section without the mixer. In these experiments all the helicoidal elements were taken out of the static mixers, reducing them to simple flanged tube spools.

Three sets of experiments were performed. In the first set the oil and water flow rates were each varied from 0.2×10^{-3} to $1.2 \times 10^{-3} \text{ m}^3/\text{s}$ at increments of $0.2 \times 10^{-3} \text{ m}^3/\text{s}$. In the second set of experiments,- the oil and water flow rates varied independently from 0.05×10^{-3} to $0.25 \times 10^{-3} \text{ m}^3/\text{s}$ ($0.3 \times 10^{-3} \text{ m}^3/\text{s}$ for the oil) at increments of $0.05 \times 10^{-3} \text{ m}^3/\text{s}$. In the third set of experiments the water flow rate was varied from $0.05 \times 10^{-3} \text{ m}^3/\text{s}$ to $0.25 \times 10^{-3} \text{ m}^3/\text{s}$ at an increment of $0.05 \times 10^{-3} \text{ m}^3/\text{s}$, while the oil flow rate was varied from 0.2×10^{-3} to $1.2 \times 10^{-3} \text{ m}^3/\text{s}$. The purpose of this last set of experiments was to scan the area of occurrence of liquid-liquid slug flow as reported by Arirachakaran et al (1989) and Guzhov et al (1973). As it will be shown later the

slug flow pattern was not observed to occur in any of the systems of this work. Table 3.2 lists the results of the tests referred to above.

Four flow patterns were observed in the course of these experiments:

- a) *Stratified flow*, in which a layer of oil flows on top of a layer of water.
- b) *Stratified flow with water-in-oil dispersions*. Here, a layer of oil with water drops dispersed in it flows on top of a layer of water.
- c) *Stratified flow with oil-in-water dispersion*. Here, a layer of oil flows on top of a layer of water with oil drops dispersed in it.
- d) *Dispersed flow*, in which the two phases, oil and water, are totally entrained in each other forming a dispersion with either oil or water as the dispersed phase.

Figure 3.8 represents graphically the flow pattern map for all experiments run in the *transpalite* tube with no mixer. The coordinates of this map are given in terms of the total mixture velocity and the input water fraction (i.e. the water flow rate to the total flow rate ratio). This convenient set of coordinates allows comparison of this map with the maps of Arirachakaran et al. (1989) and Guzhov et al. (1973).

In the flow pattern map of Figure 3.8 it can be observed that the stratified flow patterns — i.e. (a), (b) or (c) — prevail below mixture velocities of approximately 1.0 m/s. Above this limit only dispersed flow occurs. However, it was not possible to determine visually which kind of dispersion (oil in water or water in oil) existed because of the similarities of colour of EXXSOL and water.

The boundaries between stratified and non-stratified (dispersed or slug) observed by Arirachakaran et al (1989) and Guzhov et al (1973) are also plotted in Figure 3.8 as are the locations on the map where these authors observed slug flow. The limits for stratified flow in the present experiments agree qualitatively with the limits observed in the earlier studies *but no sign of a slug type flow was detected*.

Photographs made from the video tapes of the runs are shown in Figures 3.9 to 3.12. The case number mentioned in each photo refers to the cases shown in Tables 3.2, 3.3 and 3.4.

The pressure drop data collected is represented in Figures 3.13 and 3.14. In these figures the pressure drop is presented as a function of the water flow rate with the oil flow rate as a parameter for the various curves shown. As expected the pressure drop increases with an increase in either oil or water flow rates. Also, since the viscosities of the two fluids are not very different and as the total Reynolds numbers based on the mixture velocity and on the average viscosity, i.e. the input volume fraction weighted mean viscosity:

$$\eta_{mixt} = \frac{\dot{V}_o \eta_o + \dot{V}_w \eta_w}{\dot{V}_o + \dot{V}_w} \quad (3.2)$$

were always high (from 10^4 to 10^5) all curves present a parabolic shape typical of single-phase turbulent flows.

3.2.1.2- Experiments in the *transpalite* test section - mixer

The second set of experiments (flow rates varying from $0.05 \times 10^{-3} \text{ m}^3/\text{s}$ to $0.25 \times 10^{-3} \text{ m}^3/\text{s}$ or $0.30 \times 10^{-3} \text{ m}^3/\text{s}$ for water and oil respectively) were also run in the

transpalite test section, but with the static mixer in place fitted with five helicoidal elements. Table 3.3 summarises the cases run and the measurements obtained.

The flow pattern map for this set of experiments is shown in Figure 3.15. Comparison of this figure with Figure 3.8 (cases with no mixing induced) shows that in the area where previously the stratified flow pattern prevailed (mixture velocity below approximately 1.0 m/s) the flow pattern observed was instead stratified with water-in-oil dispersions. This may indicate that mixing is only an inlet effect and that in these cases the flows could have returned to their original configuration, as observed in the experiments with no induced mixing, if the test section had been long enough.

The pressure drop data is presented in a similar way to the previous set of experiments in Figure 3.16 where, in a comparison with the experiments with no mixing action, an increase in the pressure gradients for the same set of flow rates can be noticed. The extra pressure drop may be a result of two different causes. The first one, the action of the mixer on each of the fluids which increases the turbulence in each of the phases of system and promotes a higher energy dissipation rate. The second cause is that the mixer will create a more finely dispersed mixture of the two fluids, which, as reported in Schramm (1992), may have a higher viscosity than either of the individual fluids. The effect of the mixer on the turbulence is likely to be less persistent down the channel than its effect on drop size and the second explanation of the effect is the more likely one.

3.2.1.3- Experiments in the stainless steel test section

The three sets of experiments performed in the *transpalite* section without the mixer were also performed in the stainless steel test section. Therefore the results of this group of experiments could be compared directly to those of the *transpalite* tube.

The 75 cases run on the stainless steel section are summarised in Table 3.4. The flow pattern map referring to these experiments is shown in Figure 3.17.

Comparison of Figures 3.17 and 3.8 shows that the dispersed flow region remained practically unaltered in both maps. The stratified flow region, on the other hand, showed a similar change to that resulting from the introduction of the mixer on the *transpalite* test section: that is, some tests that were observed to have a stratified flow pattern in the *transpalite* test section gave a stratified with water-in-oil dispersion pattern in the stainless steel tube. However, the extent of the change was not as great as with the introduction of the mixer. Generally speaking, the change occurred mostly at the upper limit of the stratified flow region. The cause could be rougher walls of the stainless steel pipe and the different wettability of the respective test-sections to oil and water. The stainless steel, being more likely to be wetted by water, would cause the oil layer to be less stable. In contrast to the case of the mixer, the regime changes promoted by the change from *transpalite* to stainless steel are not transient and persist over the whole channel length.

The pressure gradient data for the stainless-steel test-section experiments are shown in Figures 3.18 and 3.19. Comparison of these graphs with Figures 3.13 and 3.14 show an increase in the pressure drop for the same flow rate conditions. This is consistent with the higher wall roughness of the stainless steel section.

3.2.2- WASP experiments

The first set of WASP experiments were performed with the test section horizontal while on the second set the test section was tilted upwards 1° .

The flow patterns were determined by visual observation and video recording, and data were also collected for pressure drop and hold-up. The pressure drop data were obtained directly from a pressure transducer installed in the rig while the hold-up was determined using the gamma-ray densitometer. The latter information was also helpful in determining the flow pattern in the cases when the visual observations were unclear due to the opacity of the oil-water mixtures.

3.2.2.1- Horizontal oil and water flow experiments

Horizontal oil and water flow experiments were carried out with the main objective of determining the flowrate region where slug flow would occur in the WASP rig.

The oil and water flow rates set in the cases studied were those which, according to the maps of Guzhov et al. (1973) and Arirachakaran et al. (1989), would be in the slug flow region, i.e. approximately the area where the input water fraction was less than 25%. Due to operational limitations of the system and the short time available for the adjustment of the flow rates this water fraction limit was not strictly followed. Table 3.5 summarises the cases studied.

All the experiments were recorded on video tape, which was then used to determine the flow patterns for each set of conditions. From these results, also included in Table 3.5, it can be observed that the slug flow regime did not occur in any of the cases run. Most of the flow regimes observed were either stratified (a layer of oil on top of a layer of water) or stratified with oil-in-water dispersion. Only in the two highest flowrate cases was a totally dispersed flow regime observed.

Figure 3.20 shows the flow regimes observed in the experiments. In this the limits of the stratified and slug flow regions as established by Arirachakaran et al. (1989) and Guzhov et al. (1973) are also plotted.

There seems to be good agreement between the maps of Arirachakaran et al. (1989) and Guzhov et al. (1973) and the experimental data of this work. This fact is rather surprising since these two maps are purely empirical, based on systems different from the one in this study. This, added to the fact of the equally good agreement with the maps from the TOWER experiments reported in the previous section, may indicate that a general transition criteria might be determined to satisfy a wide range of oil-water flow systems.

In all the cases run, the pressure drop in the pipe was measured with a differential pressure transducer. These data were then recorded on a computer file. The video recordings also had displayed on them outputs from the computer indicating the flow rates of the respective fluids and the pressure drop. These "on-screen" data were particularly useful when reviewing the video-tapes so that they could be correlated with the computer files.. Figure 3.21 shows the pressure drop data. This data had to be plotted in a tridimensional graph as Figure 3.21 because, due to the way they were collected (from the video-tape and computer files), it was not possible to impose parameters of either constant oil or water flow rates. In this graph it can be seen that the pressure gradient seems to increase approximately linearly with the oil flow rate and to increase in a power function with the water flow rate. For the great majority of the experimental points plotted the regime was stratified with water-in-oil dispersions.

The in-situ water fraction in the experiments was measured with a multi-scan gamma-ray densitometry technique (Pan & Hewitt, 1993, 1995). In these measurements a half-pipe horizontal traversing method was used as described earlier and shown in

Figure 3.7; it is assumed that the horizontal flows being studied are symmetric about the vertical centreline of the pipe. As it was explained in the section describing the WASP facility, only one gamma-ray source was necessary for the present study whereas the gamma densitometer used was a dual energy one (as would be needed for a three phase flow). The results discussed below were obtained using an Americium source, the lower energy giving greater accuracy, (Pan & Hewitt, 1993, 1995).

Figure 3.22 show some of the results of the measurements obtained with the scanning gamma densitometer. Since the gamma densitometer gives an indication of the average distribution of the phases, it can in principle be used to help in the identification of flow patterns. Figure 3.23 and 3.24 show the kind of results which would be obtained using the scanning densitometer for ideal stratified and ideal homogeneous flow respectively. Both vertical and horizontal scans can be performed using the WASP scanning gamma densitometer but, unfortunately, only horizontal traverses (with a vertical gamma beam) were possible in the present experiments due to restrictions in rig availability (the WASP rig is scheduled intensively for a whole range of projects and, thus, the time allocated to a given project is necessarily short). Using the vertical beam the distinction between dispersed flow and stratified flow becomes less clear if the interface is not flat and the utility of the gamma scans in determining flow patterns is reduced. However, in the present experiments, *the flow pattern is known from visual observation*. Thus, the results for the gamma traverses may be used to infer some features of interfacial behaviour.

Consider the series of results from the gamma scans illustrated in Figure 3.22. They are from an approximately constant water superficial velocity (0.165 ± 0.015 m/s) but with oil superficial velocity increasing from 0.2 m/s (Figure 3.22-a) to 1.10 m/s (Figure 3.22-d). At the lowest velocities, a dip appears in the interface near the wall, indicating the beginning of mixing processes in that region with oil droplets being entrained in the water. This mixing of the oil and the water in the wall region

continues, leading to the upward displacement of the apparent interface (Figure 3.22-b) and to more complete dispersion of the water in the oil at the highest velocity (Figure 3.22-d). Scans with the beam in the horizontal position (not possible in the present work) would have been particularly helpful in elucidating the competition between the mixing and separation processes and are recommended for future work in the area.

3.2.2.2- Inclined oil and water flow WASP experiments

Continuing the search for a possible liquid-liquid slug flow pattern, further experiments were conducted in the WASP rig, which was inclined 1° upwards. The experimental data obtained were somewhat limited due to the short time of this campaign. The slug flow pattern, however, was not observed but the data collected were used in the context of the stratified flow studies.

Table 3.6, summarises the cases run and describes the flow pattern observed for each condition. Figure 3.25 is a graphical plot of Table 3.6 and represents the flow pattern map obtained from the experimental observations.

To have some insight of the oil-water flow pattern transition, the inclined flow experiments were plotted again in Figure 3.26, which also includes the resulting empirical flow pattern map generated. The transitions shown in this map seem to be consistent with what could be expected theoretically, i.e., that as the mixture velocity increases, a transition from smooth stratified flow to wavy stratified flow occurs. As the velocity increases further, inter-phase mixing begins giving oil-in-water dispersion at high water fractions and water-in-oil dispersions at high oil fraction.

In order to determine the effects of the pipe inclination Figures 3.20 and 3.25 may be compared. This comparison shows that the effects of the pipe inclination are not dramatic; in equivalent areas of both graphs the same flow patterns seem to prevail. However, due to the very limited data obtained, with the majority of data in the mixed-stratified flow pattern, this statement may not, and should not be taken neither as definite or conclusive. Further data covering the area of different flow patterns are necessary for more objective and clearer conclusions to be taken.

3.3- Liquid level measurement in stratified flow using high frequency probes

As will be seen from the above descriptions of the main series of experiments, measurements were made of the hold-up (or mean water level) in the WASP facility. However equivalent measurements were not possible in the TOWER facility. However, for pure stratified flow, an indication of the water level can be obtained using the high frequency probe system. Thus, a special series of measurements were made at low velocities (where stratified flow occurred) using this technique. Stratified flow of oil and water occurs when the superficial velocity of both of the fluids is low as can be seen in Figures 3.8, 3.15 and 3.17 which show flow pattern maps derived from experimental data obtained in the TOWER facility.

The interface level h was measured experimentally in the *transpalite* test-section of the rig using the high-frequency impedance probe as explained in the Section 3.1.1.6. It was assumed that the measurement of the interface level with a perpendicular scan passing through the tube axis would give a value characteristic of the whole two-dimensional planar interface. Since the interface in the actual experiments is not a perfect smooth horizontal plane, having some degree of waviness and dispersion

between the phases, the average interfacial value was determined by scanning the probe through the interfacial zone.

The superficial velocities of oil and water were limited to 0.44 m/s in order to maintain the stratified flow pattern. All cases run are listed in Table 3.7.

Figure 3.27 shows the measured interface level of the experiments. As expected it is seen that the average interface level increases with the water flow rate for a given oil flow rate, and that it decreases with the oil flow rate for a given water flow rate. Comparisons of the result with prediction are described in Chapter 4.

Product name	Exxsol D80
Flash Point	75 °C
Density	800 Kg/m ³
Viscosity	1.6 mPa s @ 25 °C
Interfacial tension air-oil	0.027 N/m @ 22 °C
Interfacial tension oil-water	0.017 N/m @ 22 °C

Table 3.1: Properties of the oil used (EXXSOL D80) in the TOWER experiments.

Case	Oil flow rt (l/s)	Water flow rt (l/s)	Water frac	Mix. veloc. (m/s)	Press grad. (mbar/m)	Flow pattern
1	0.20	0.20	0.50	0.87	3.71	Stratified
2	0.20	0.40	0.67	1.31	8.00	Stratified with water in oil dispersion
3	0.20	0.60	0.75	1.75	13.46	Stratified with water in oil dispersion
4	0.20	0.80	0.80	2.18	19.85	Dispersed
5	0.20	1.00	0.83	2.62	28.12	Dispersed
6	0.20	1.20	0.86	3.06	38.88	Dispersed
7	0.40	0.20	0.33	1.31	6.59	Stratified
8	0.40	0.40	0.50	1.75	11.90	Stratified with water in oil dispersion
9	0.40	0.60	0.60	2.18	18.68	Dispersed
10	0.40	0.80	0.67	2.62	25.62	Dispersed
11	0.40	1.00	0.71	3.06	36.15	Dispersed
12	0.40	1.20	0.75	3.49	48.17	Dispersed
13	0.60	0.20	0.25	1.75	10.02	Dispersed
14	0.60	0.40	0.40	2.18	17.12	Dispersed
15	0.60	0.60	0.50	2.62	25.08	Dispersed
16	0.60	0.80	0.57	3.06	34.67	Dispersed
17	0.60	1.00	0.63	3.49	45.59	Dispersed
18	0.60	1.20	0.67	3.93	59.79	Dispersed
19	0.80	0.20	0.20	2.18	13.14	Dispersed
20	0.80	0.40	0.33	2.62	21.02	Dispersed
21	0.80	0.60	0.43	3.06	32.88	Dispersed
22	0.80	0.80	0.50	3.49	45.05	Dispersed
23	0.80	1.00	0.56	3.93	58.62	Dispersed
24	0.80	1.20	0.60	4.37	73.05	Dispersed
25	1.00	0.20	0.17	2.62	20.63	Dispersed
26	1.00	0.40	0.29	3.06	25.08	Dispersed
27	1.00	0.60	0.38	3.49	45.36	Dispersed
28	1.00	0.80	0.44	3.93	62.52	Dispersed
29	1.00	1.00	0.50	4.37	75.78	Dispersed
31	1.20	0.20	0.14	3.06	27.03	Dispersed
32	1.20	0.40	0.25	3.49	30.54	Dispersed
33	1.20	0.60	0.33	3.93	41.07	Dispersed
34	1.20	0.80	0.40	4.37	79.68	Dispersed
37	0.05	0.05	0.50	0.22	0.35	Stratified
38	0.05	0.10	0.67	0.33	0.82	Stratified
39	0.05	0.15	0.75	0.44	1.60	Stratified
40	0.05	0.20	0.80	0.55	2.15	Stratified
41	0.05	0.25	0.83	0.65	2.85	Stratified
42	0.10	0.05	0.33	0.33	0.74	Stratified
43	0.10	0.10	0.50	0.44	1.29	Stratified
44	0.10	0.15	0.60	0.55	1.91	Stratified
45	0.10	0.20	0.67	0.65	2.61	Stratified

CONTINUES

Table 3.2: Summary of the horizontal oil-water flow experiments. TOWER, *transpalite* test-section, no mixer. The pressure gradient measurements are at least 0.1 mbar/m accurate and a reproducibility with a standard deviation of 0.85 %.

Case	Oil flow rt (l/s)	Water flow rt (l/s)	Water frac	Mix. veloc. (m/s)	Press grad. (mbar/m)	Flow pattern
46	0.10	0.25	0.71	0.76	3.39	Stratified
47	0.15	0.05	0.25	0.44	1.21	Stratified
48	0.15	0.10	0.40	0.55	1.83	Stratified
49	0.15	0.15	0.50	0.65	2.46	Stratified
50	0.15	0.20	0.57	0.76	3.24	Stratified
51	0.15	0.25	0.63	0.87	3.94	Stratified
52	0.20	0.05	0.20	0.55	1.91	Stratified
53	0.20	0.10	0.33	0.65	2.46	Stratified
54	0.20	0.15	0.43	0.76	3.08	Stratified
55	0.20	0.20	0.50	0.87	3.86	Stratified
56	0.20	0.25	0.56	0.98	4.56	Stratified
57	0.25	0.05	0.17	0.65	2.30	Stratified
58	0.25	0.10	0.29	0.76	3.00	Stratified
59	0.25	0.15	0.38	0.87	3.94	Stratified
60	0.25	0.20	0.44	0.98	4.33	Stratified
61	0.25	0.25	0.50	1.09	5.42	Stratified
62	0.30	0.05	0.14	0.76	3.00	Stratified
63	0.30	0.10	0.25	0.87	3.71	Stratified
64	0.30	0.15	0.33	0.98	4.56	Stratified
65	0.30	0.20	0.40	1.09	5.11	Stratified
66	0.30	0.25	0.45	1.20	6.20	Stratified
67	0.20	0.05	0.20	0.55	1.56	Stratified
68	0.30	0.05	0.14	0.76	3.04	Stratified
69	0.40	0.05	0.11	0.98	4.29	Stratified with oil in water dispersion
70	0.50	0.05	0.09	1.20	5.77	Stratified with oil in water dispersion
71	0.60	0.05	0.08	1.42	8.19	Dispersed
72	0.70	0.05	0.07	1.64	11.08	Dispersed
73	0.80	0.05	0.06	1.86	14.04	Dispersed
74	0.20	0.10	0.33	0.65	2.18	Stratified
75	0.30	0.10	0.25	0.87	3.43	Stratified
76	0.40	0.10	0.20	1.09	5.15	Stratified with oil in water dispersion
77	0.50	0.10	0.17	1.31	6.79	Dispersed
78	0.60	0.10	0.14	1.53	8.50	Dispersed
79	0.70	0.10	0.13	1.75	10.84	Dispersed
80	0.80	0.10	0.11	1.96	14.74	Dispersed
81	0.20	0.15	0.43	0.76	2.81	Stratified
82	0.30	0.15	0.33	0.98	4.13	Stratified
83	0.40	0.15	0.27	1.20	5.77	Stratified
84	0.50	0.15	0.23	1.42	7.72	Dispersed
85	0.60	0.15	0.20	1.64	9.67	Dispersed
86	0.70	0.15	0.18	1.86	11.70	Dispersed
87	0.80	0.15	0.16	2.07	14.27	Dispersed

Table 3.2: Summary of the horizontal oil-water flow experiments. TOWER, *transpalite* test-section, no mixer. The pressure gradient measurements are at least 0.1 mbar/m accurate and a reproducibility with a standard deviation of 0.85 %.

Case	Oil flow rt (l/s)	Water flow rt (l/s)	Water frac	Mix. veloc. (m/s)	Press grad. (mbar/m)	Flow pattern
37	0.05	0.05	0.50	0.22	0.51	Stratified
38	0.05	0.10	0.67	0.33	1.13	Stratified with water in oil dispersion
39	0.05	0.15	0.75	0.44	2.54	Stratified with water in oil dispersion
40	0.05	0.20	0.80	0.55	3.24	Stratified with water in oil dispersion
41	0.05	0.25	0.83	0.65	3.78	Stratified
42	0.10	0.05	0.33	0.33	1.13	Stratified with water in oil dispersion
43	0.10	0.10	0.50	0.44	2.54	Stratified with water in oil dispersion
44	0.10	0.15	0.60	0.55	4.02	Stratified with water in oil dispersion
45	0.10	0.20	0.67	0.65	4.72	Stratified with water in oil dispersion
46	0.10	0.25	0.71	0.76	5.42	Stratified with water in oil dispersion
47	0.15	0.05	0.25	0.44	1.60	Stratified
48	0.15	0.10	0.40	0.55	3.55	Stratified with water in oil dispersion
49	0.15	0.15	0.50	0.65	5.34	Stratified with water in oil dispersion
50	0.15	0.20	0.57	0.76	5.97	Stratified with water in oil dispersion
51	0.15	0.25	0.63	0.87	6.51	Stratified with water in oil dispersion
52	0.20	0.05	0.20	0.55	1.91	Stratified
53	0.20	0.10	0.33	0.65	3.63	Stratified with water in oil dispersion
54	0.20	0.15	0.43	0.76	6.59	Stratified with water in oil dispersion
55	0.20	0.20	0.50	0.87	7.45	Stratified with water in oil dispersion
56	0.20	0.25	0.56	0.98	7.61	Stratified with water in oil dispersion
57	0.25	0.05	0.17	0.65	2.15	Stratified
58	0.25	0.10	0.29	0.76	3.47	Stratified
59	0.25	0.15	0.38	0.87	6.90	Stratified with water in oil dispersion
60	0.25	0.20	0.44	0.98	8.62	Stratified with water in oil dispersion
61	0.25	0.25	0.50	1.09	8.93	Stratified with water in oil dispersion
62	0.30	0.05	0.14	0.76	2.85	Stratified
63	0.30	0.10	0.25	0.87	3.71	Stratified with water in oil dispersion
64	0.30	0.15	0.33	0.98	6.12	Stratified with water in oil dispersion
65	0.30	0.20	0.40	1.09	9.87	Stratified with water in oil dispersion
66	0.30	0.25	0.45	1.20	10.41	Stratified with water in oil dispersion

Table 3.3: Summary of the horizontal oil-water flow experiments. TOWER, *transpalite* test-section, mixer. The pressure gradient measurements are at least 0.1 mbar/m accurate and a reproducibility with a standard deviation of 0.85 %.

Case	Oil flow rt (l/s)	Water flow rt (l/s)	Water frac	Mix. veloc. (m/s)	Press grad. (mbar/m)	Flow pattern
1	0.20	0.20	0.50	0.87	6.01	Stratified with water in oil dispersion
2	0.20	0.40	0.67	1.31	10.19	Stratified with water in oil dispersion
3	0.20	0.60	0.75	1.75	15.73	Dispersed
4	0.20	0.80	0.80	2.18	24.92	Dispersed
5	0.20	1.00	0.83	2.62	36.62	Dispersed
6	0.20	1.20	0.86	3.06	50.94	Dispersed
7	0.40	0.20	0.33	1.31	8.52	Stratified with water in oil dispersion
8	0.40	0.40	0.50	1.75	17.29	Dispersed
9	0.40	0.60	0.60	2.18	24.19	Dispersed
10	0.40	0.80	0.67	2.62	35.16	Dispersed
11	0.40	1.00	0.71	3.06	49.58	Dispersed
12	0.40	1.20	0.75	3.49	63.17	Dispersed
13	0.60	0.20	0.25	1.75	12.49	Dispersed
14	0.60	0.40	0.40	2.18	28.16	Dispersed
15	0.60	0.60	0.50	2.62	36.42	Dispersed
16	0.60	0.80	0.57	3.06	49.37	Dispersed
17	0.60	1.00	0.63	3.49	65.67	Dispersed
19	0.80	0.20	0.20	2.18	20.43	Dispersed
20	0.80	0.40	0.33	2.62	26.70	Dispersed
21	0.80	0.60	0.43	3.06	50.42	Dispersed
22	0.80	0.80	0.50	3.49	65.26	Dispersed
23	0.80	1.00	0.56	3.93	66.30	Dispersed
25	1.00	0.20	0.17	2.62	26.18	Dispersed
26	1.00	0.40	0.29	3.06	32.65	Dispersed
27	1.00	0.60	0.38	3.49	66.72	Dispersed
28	1.00	0.80	0.44	3.93	84.80	Dispersed
31	1.20	0.20	0.14	3.06	36.21	Dispersed
32	1.20	0.40	0.25	3.49	42.16	Dispersed
33	1.20	0.60	0.33	3.93	55.64	Dispersed
37	0.05	0.05	0.50	0.22	0.89	Stratified
38	0.05	0.10	0.67	0.33	1.52	Stratified
39	0.05	0.15	0.75	0.44	2.14	Stratified
40	0.05	0.20	0.80	0.55	3.19	Stratified
41	0.05	0.25	0.83	0.65	4.34	Stratified with water in oil dispersion
42	0.10	0.05	0.33	0.33	1.20	Stratified
43	0.10	0.10	0.50	0.44	2.04	Stratified
44	0.10	0.15	0.60	0.55	3.19	Stratified with water in oil dispersion
45	0.10	0.20	0.67	0.65	4.34	Stratified with water in oil dispersion
46	0.10	0.25	0.71	0.76	5.80	Stratified with water in oil dispersion

CONTINUES

Table 3.4: Summary of the horizontal oil-water flow experiments. TOWER, stainless-steel test-section, no mixer. The pressure gradient measurements are at least 0.1 mbar/m accurate and a reproducibility with a standard deviation of 0.85 %.

Case	Oil flow rt (l/s)	Water flow rt (l/s)	Water frac	Mix. veloc. (m/s)	Press grad. (mbar/m)	Flow pattern
47	0.15	0.05	0.25	0.44	1.72	Stratified
48	0.15	0.10	0.40	0.55	2.66	Stratified
49	0.15	0.15	0.50	0.65	3.92	Stratified with water in oil dispersion
50	0.15	0.20	0.57	0.76	5.90	Stratified with water in oil dispersion
51	0.15	0.25	0.63	0.87	7.26	Stratified with water in oil dispersion
52	0.20	0.05	0.20	0.55	2.56	Stratified
53	0.20	0.10	0.33	0.65	3.29	Stratified
54	0.20	0.15	0.43	0.76	4.65	Stratified with water in oil dispersion
55	0.20	0.20	0.50	0.87	6.95	Stratified with water in oil dispersion
56	0.20	0.25	0.56	0.98	9.04	Stratified with water in oil dispersion
57	0.25	0.05	0.17	0.65	3.19	Stratified
58	0.25	0.10	0.29	0.76	4.13	Stratified
59	0.25	0.15	0.38	0.87	5.17	Stratified with water in oil dispersion
60	0.25	0.20	0.44	0.98	7.26	Stratified with water in oil dispersion
61	0.25	0.25	0.50	1.09	10.08	Stratified with water in oil dispersion
67	0.20	0.05	0.20	0.55	2.00	Stratified
68	0.30	0.05	0.14	0.76	3.35	Stratified
69	0.40	0.05	0.11	0.98	5.24	Stratified with oil in water dispersion
70	0.50	0.05	0.09	1.20	7.43	Stratified with oil in water dispersion
71	0.60	0.05	0.08	1.42	10.04	Dispersed
72	0.70	0.05	0.07	1.64	12.97	Dispersed
73	0.80	0.05	0.06	1.86	16.00	Dispersed
74	0.20	0.10	0.33	0.65	2.94	Stratified
75	0.30	0.10	0.25	0.87	4.19	Stratified
76	0.40	0.10	0.20	1.09	6.18	Stratified with oil in water dispersion
77	0.50	0.10	0.17	1.31	8.16	Dispersed
78	0.60	0.10	0.14	1.53	11.09	Dispersed
79	0.70	0.10	0.13	1.75	14.22	Dispersed
80	0.80	0.10	0.11	1.96	16.94	Dispersed
81	0.20	0.15	0.43	0.76	4.92	Stratified with water in oil dispersion
82	0.30	0.15	0.33	0.98	5.65	Stratified with water in oil dispersion
83	0.40	0.15	0.27	1.20	7.43	Stratified with water in oil dispersion
84	0.50	0.15	0.23	1.42	9.10	Dispersed
85	0.60	0.15	0.20	1.64	11.92	Dispersed
86	0.70	0.15	0.18	1.86	15.27	Dispersed
87	0.80	0.15	0.16	2.07	18.30	Dispersed

Table 3.4: Summary of the horizontal oil-water flow experiments. TOWER, stainless-steel test-section, no mixer.

Case	Oil sup. vel. (m/s)	Water sup. vel. (m/s)	DP measured (mbar/m)	Wat Frac. Am	Flow Pattern -
1	0.20	0.09	2.09	-	Stratified with oil in water dispersion
2	0.20	0.15	2.15	0.28	Stratified with oil in water dispersion
3	0.40	0.15	2.90	0.26	Stratified with oil in water dispersion
4	0.63	0.18	4.00	0.26	Stratified with oil in water dispersion
5	0.70	0.16	4.30	-	Stratified with oil in water dispersion
6	0.67	0.19	3.20	-	Stratified with oil in water dispersion
7	0.90	0.06	5.40	0.19	Stratified with oil in water dispersion
8	0.81	0.14	6.60	0.19	Stratified with oil in water dispersion
9	0.74	0.18	7.40	0.19	Stratified with oil in water dispersion
10	0.59	0.16	3.35	0.26	Stratified with oil in water dispersion
11	0.54	0.10	3.30	0.26	Stratified with oil in water dispersion
12	0.52	0.12	3.55	0.26	Stratified with oil in water dispersion
13	0.46	0.13	3.20	0.26	Stratified with oil in water dispersion
14	0.43	0.06	2.87	0.26	Stratified
15	0.37	0.10	3.80	0.21	Stratified with oil in water dispersion
16	0.38	0.09	3.25	0.21	Stratified with oil in water dispersion
17	0.38	0.19	3.13	0.27	Stratified with oil in water dispersion
18	0.39	0.16	2.70	0.27	Stratified with oil in water dispersion
19	1.25	0.20	8.20	0.34	Dispersed
20	1.10	0.18	-	0.34	Dispersed

Table 3.5: Summary of the horizontal oil-water flow experiments. WASP rig.

Run No.	Oil sup. vel. (m/s)	Water sup. vel. (m/s)	Mixt. vel. (m/s)	Input wat fr.	Flow pattern observed
I2	0.05	0.05	0.10	0.50	Stratified (smooth)
I4	0.07	0.16	0.23	0.70	Stratified (smooth)
I3	0.08	0.10	0.18	0.56	Stratified (smooth)
I35	0.10	0.10	0.20	0.50	Stratified (smooth)
I36	0.10	0.05	0.15	0.33	Stratified (smooth)
I6	0.11	0.09	0.20	0.45	Stratified (wavy)
I7	0.18	0.09	0.27	0.33	Stratified (wavy)
I8	0.20	0.09	0.29	0.31	Stratified (wavy)
I34	0.21	0.21	0.42	0.50	Stratified (wavy)
I9	0.23	0.10	0.33	0.30	Stratified (wavy)
I31	0.23	0.28	0.51	0.55	Stratified (wavy)
I26	0.24	0.41	0.65	0.63	Stratified (wavy)
I25	0.25	0.43	0.68	0.63	Stratified (wavy)
I16	0.27	0.19	0.46	0.41	Stratified (wavy)
I18	0.33	0.16	0.49	0.33	Stratified (wavy)
I19	0.34	0.15	0.49	0.31	Stratified (wavy)
I17	0.35	0.16	0.51	0.31	Stratified (wavy)
I20	0.38	0.20	0.58	0.34	Stratified (wavy)
I10	0.26	0.10	0.36	0.28	Stratified (wavy)
I11	0.31	0.09	0.40	0.23	Stratified (wavy)
I12	0.38	0.09	0.47	0.19	Stratified (wavy)
I13	0.37	0.09	0.46	0.20	Stratified (wavy)
I14	0.40	0.10	0.50	0.20	Stratified (wavy)
I15	0.44	0.10	0.54	0.19	Stratified (wavy)
I21	0.50	0.19	0.69	0.28	Stratified (wavy)
I23	0.52	0.21	0.73	0.29	Stratified (wavy)
I22	0.55	0.19	0.74	0.26	Stratified (wavy)
I27	0.69	0.37	1.06	0.35	Stratified (wavy)
I28	0.73	0.38	1.11	0.34	Stratified (wavy)
I30	0.21	0.52	0.73	0.71	Stratified with water in oil dispersion
I29	0.23	0.58	0.81	0.72	Stratified with water in oil dispersion
I33	0.30	0.30	0.60	0.50	Stratified with water in oil dispersion
I24	0.47	0.12	0.59	0.20	undefined
I1	0.50	0.05	0.55	0.09	undefined
I32	0.78	0.12	0.90	0.13	undefined

Table 3.6: Summary of the inclined (1° upwards) oil-water flow experiments. WASP rig.

Case stratified	Oil flow rt (l/s)	Water flow rt (l/s)	h/D
1	0.050	0.050	52.1%
2	0.100	0.100	54.2%
3	0.150	0.150	47.9%
4	0.200	0.200	47.9%
5	0.250	0.250	47.9%
6	0.050	0.100	66.7%
7	0.100	0.200	64.6%
8	0.075	0.150	66.7%
9	0.125	0.250	62.5%
10	0.100	0.050	47.9%
11	0.050	0.150	75.0%
12	0.050	0.200	77.1%
13	0.050	0.250	79.2%
14	0.100	0.150	64.6%
15	0.100	0.250	70.8%
16	0.150	0.050	45.8%
17	0.150	0.100	50.0%
18	0.150	0.200	56.3%
19	0.150	0.250	62.5%
20	0.200	0.050	37.5%
21	0.200	0.150	54.2%
22	0.200	0.250	60.4%

Table 3.7: Summary of the horizontal oil-water stratified flow experiments. TOWER, *transpalite* test-section, no mixer.

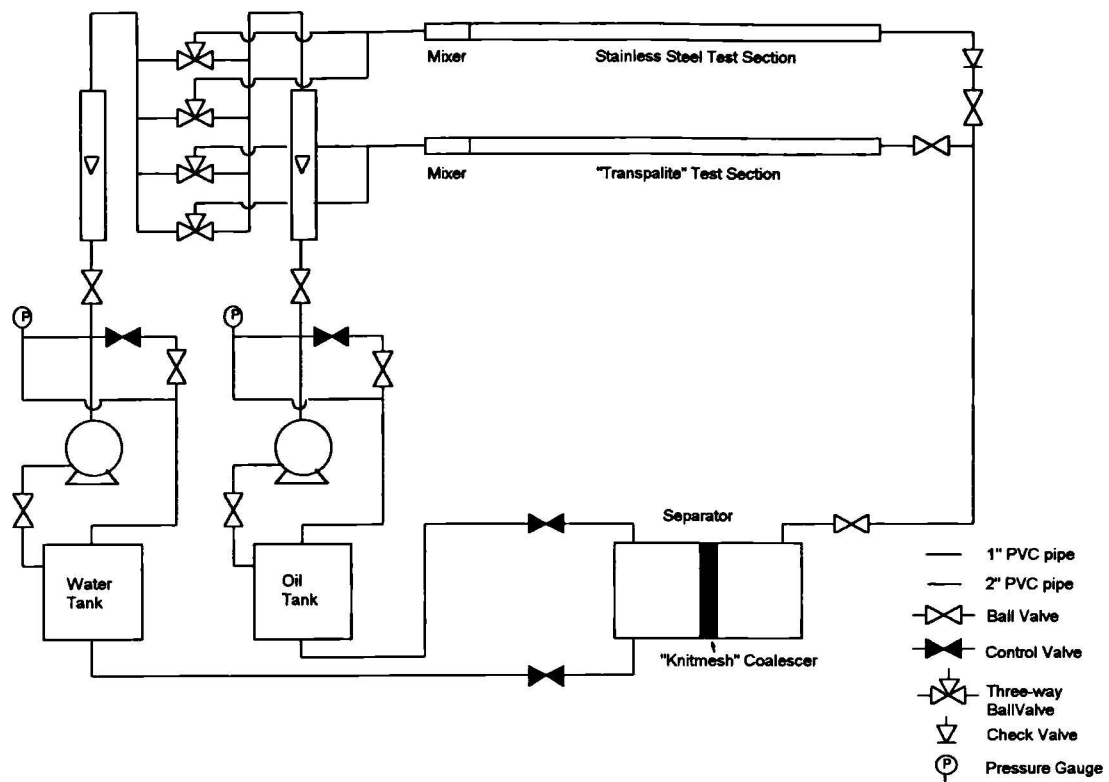


Figure 3.1: Diagram of the liquid-liquid flow facility - TOWER.

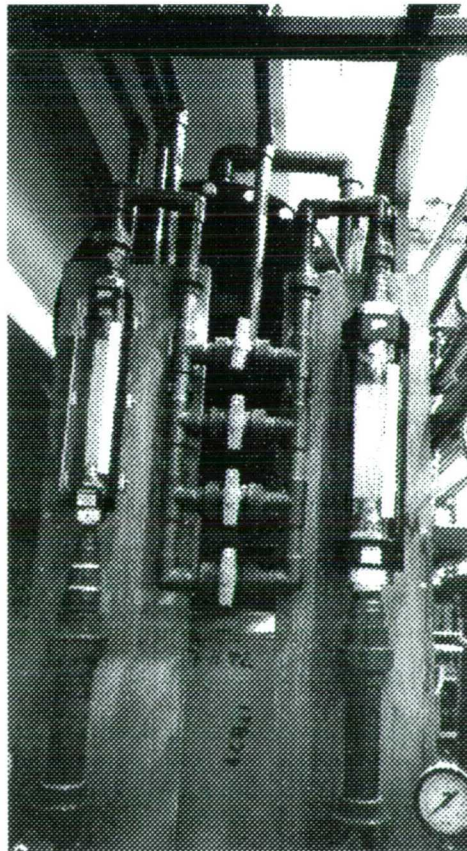


Figure 3.2: Flowmeters assembly (TOWER).

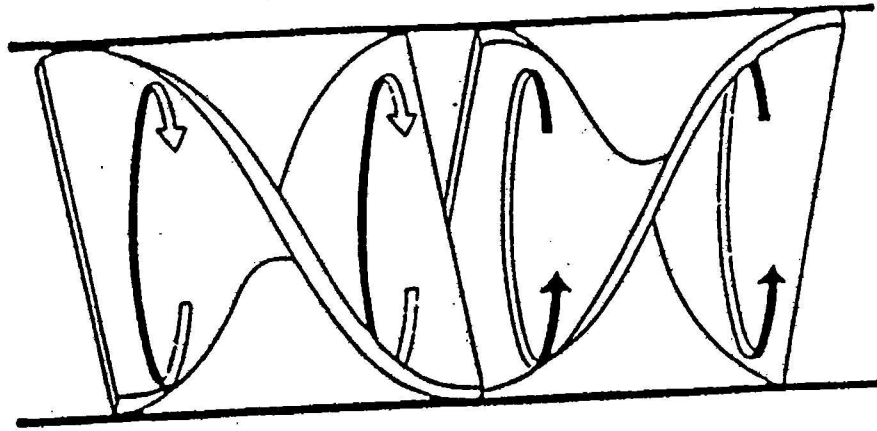


Figure 3.3: The in-line static mixer working principle.

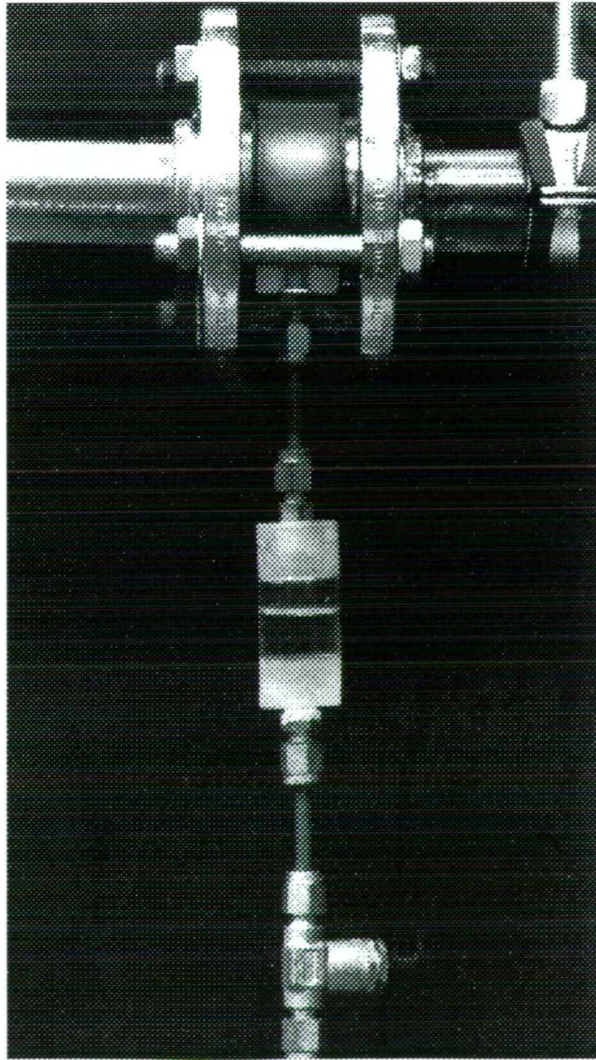


Figure 3.4: Pressure ports with accumulator chamber (TOWER).

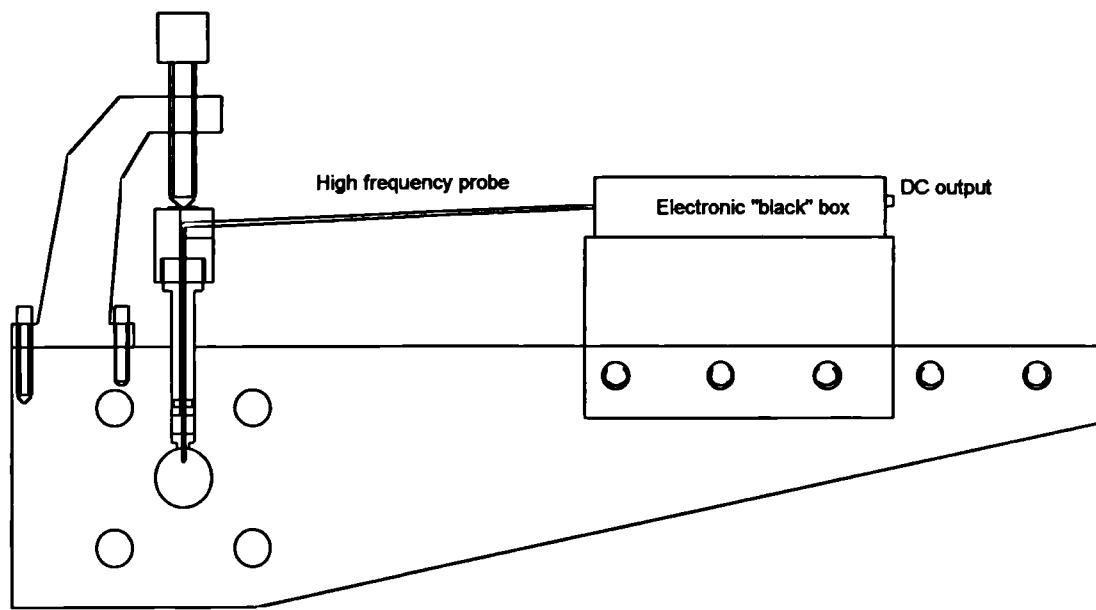


Figure 3.5: High frequency probe assembly

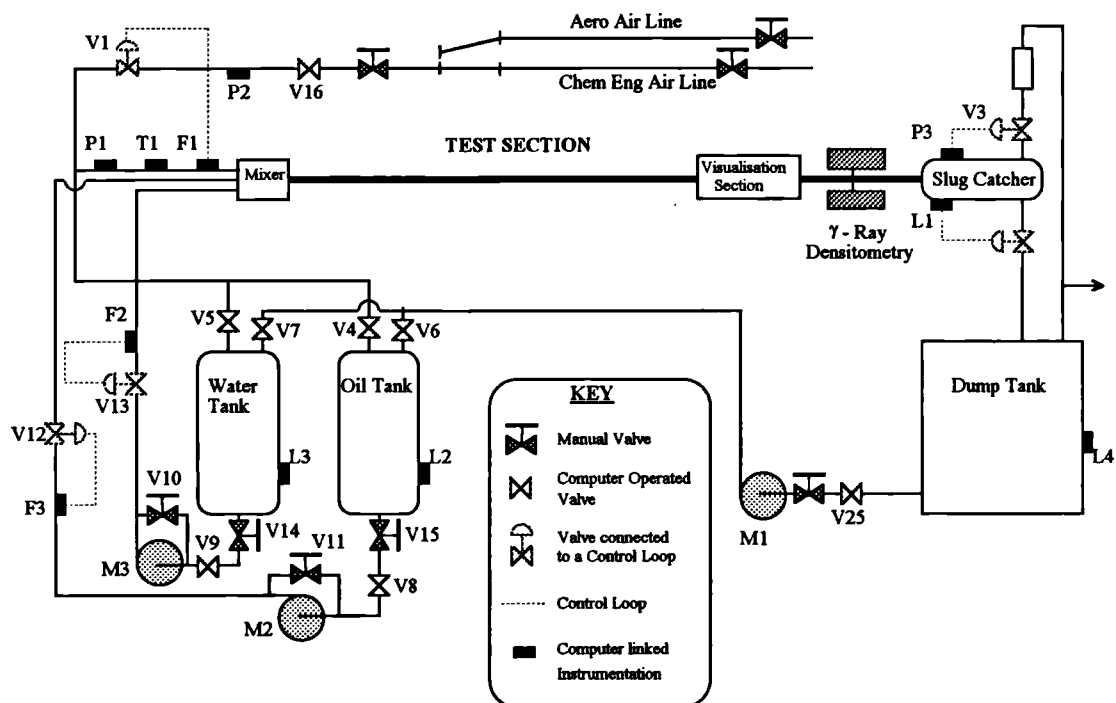


Figure 3.6: Schematic of the WASP facility.

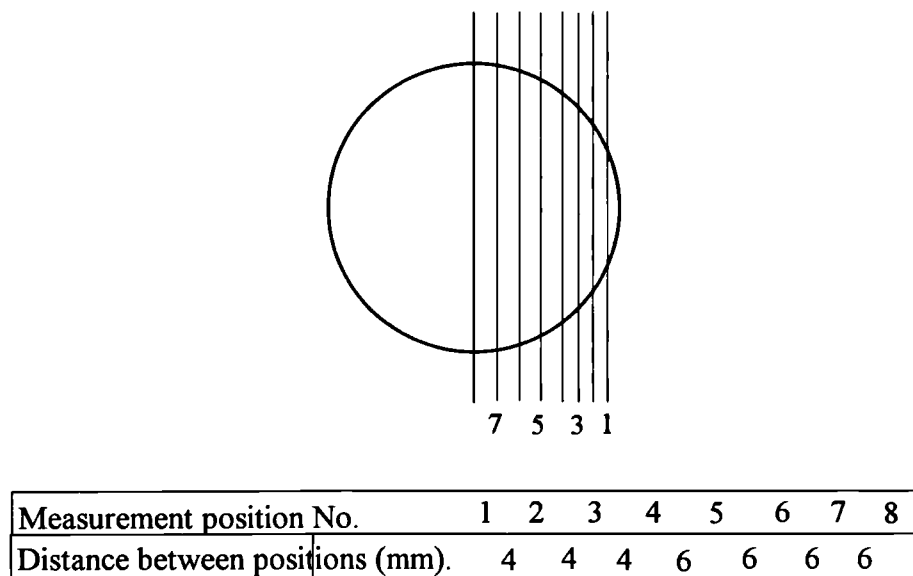


Figure 3.7: Position of measurements for the WASP gamma ray densitometer half pipe scan..

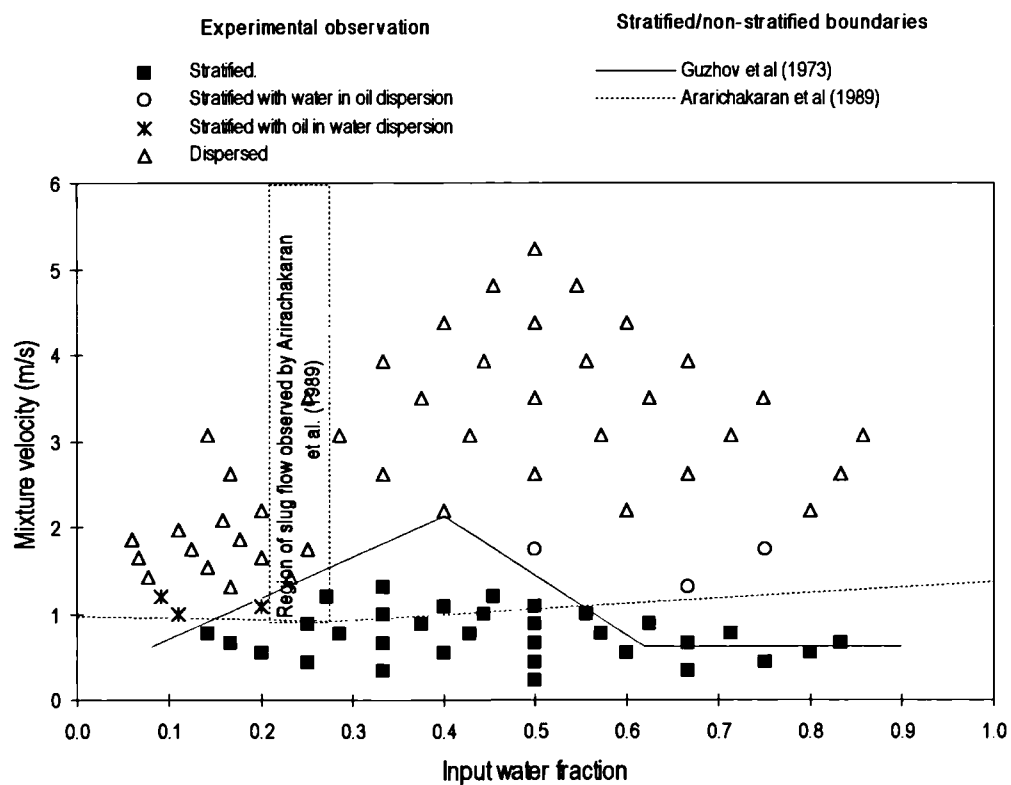


Figure 3.8: Experimental flow pattern map. TOWER *transpalite* test section - no mixer. Also plotted, the stratified/non-stratified boundaries according to Guzhov et al. (1973) and Arirachakaran et al. (1989) and the region of occurrence of slug flow reported by Arirachakaran et al. (1989).

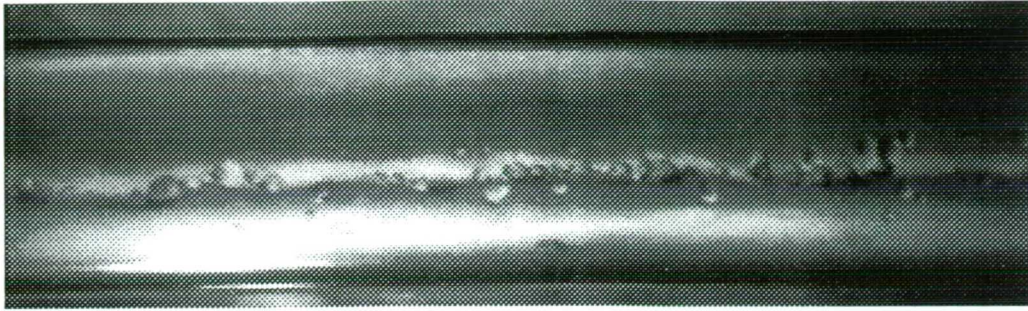


Figure 3.9: TOWER *transpalite* test section - no mixer (case 74). Flow pattern observation. $\dot{V}_o = 0.20$ l/s; $\dot{V}_w = 0.10$ l/s. Stratified flow.

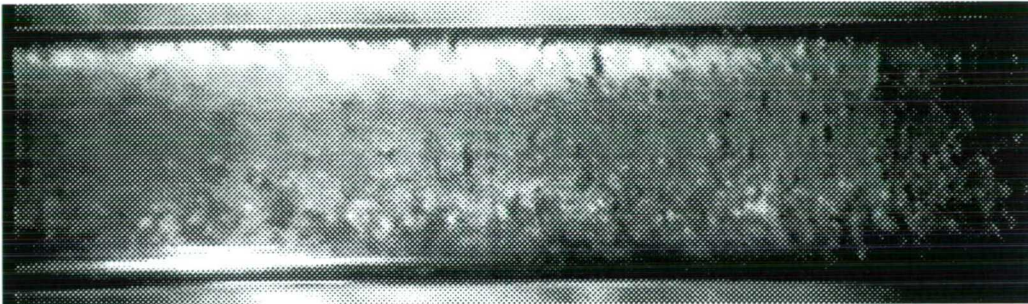


Figure 3.10: TOWER *transpalite* test section - no mixer (case 72). Flow pattern observation. $\dot{V}_o = 0.70$ l/s; $\dot{V}_w = 0.05$ l/s. Dispersed flow.



Figure 3.11: TOWER *transpalite* test section - mixer (case 40). Flow pattern observation. $\dot{V}_o = 0.05$ l/s; $\dot{V}_w = 0.20$ l/s. Stratified with water in oil dispersion.

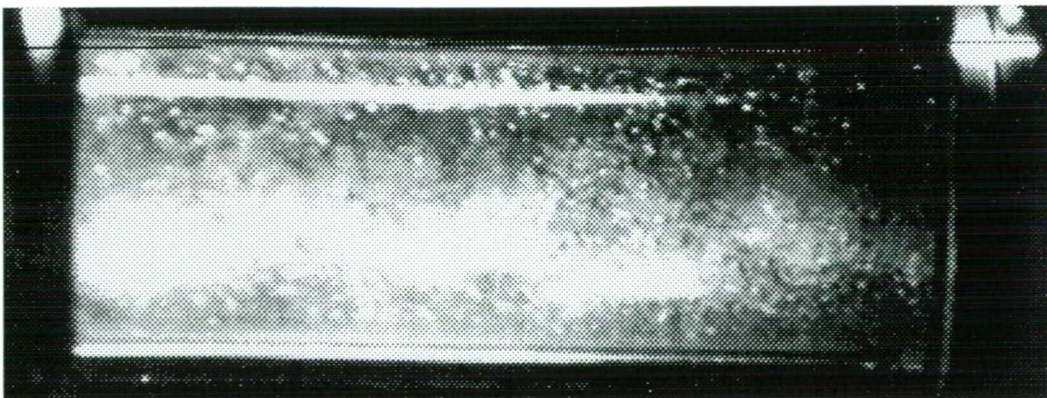


Figure 3.12: TOWER stainless steel test section (case 7) - Flow pattern observation. $\dot{V}_o = 0.40$ l/s; $\dot{V}_w = 0.20$ l/s. Stratified with water in oil dispersion.

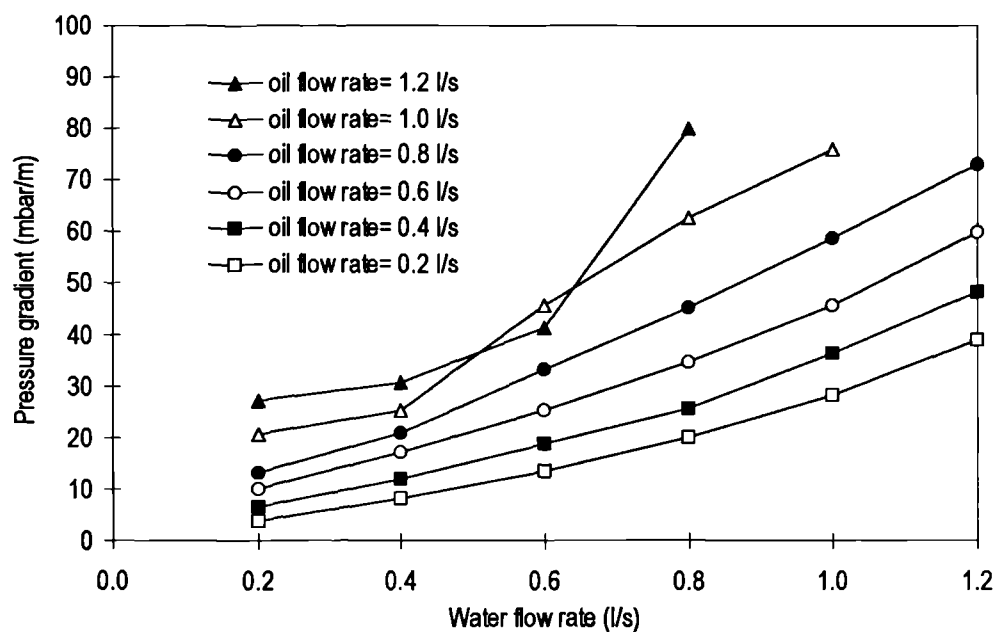


Figure 3.13: Pressure gradient. High flow rate experiments. TOWER *transpalite* test section - no mixer.

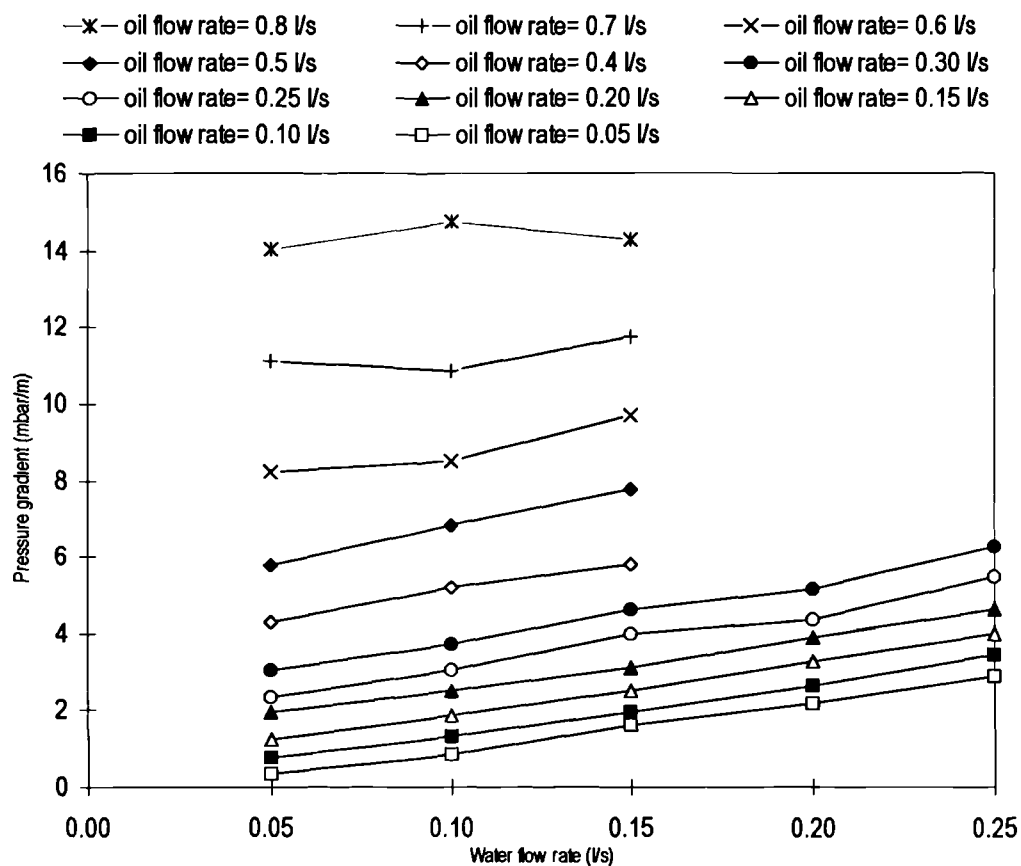


Figure 3.14: Pressure gradient. Low flow rate experiments. TOWER *transpalite* test section - no mixer.

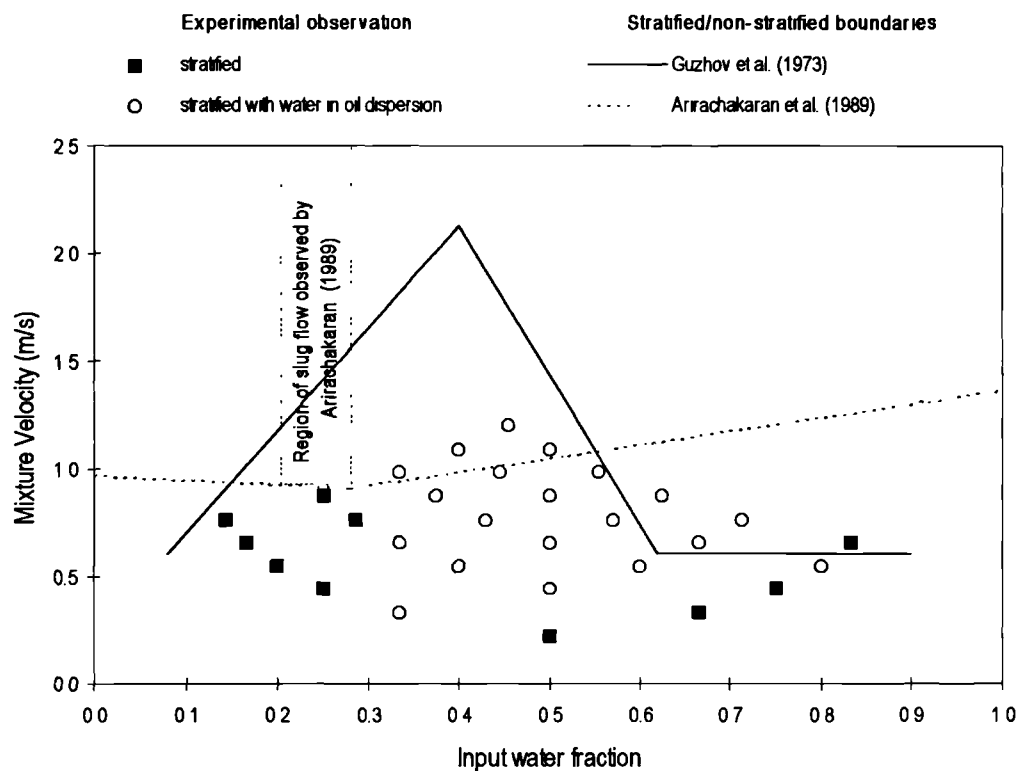


Figure 3.15: Experimental flow pattern map. TOWER *transpalite* test section - mixer. Also plotted, the stratified/non-stratified boundaries according to Guzhov et al. (1973) and Arirachakaran et al. (1989) and the region of occurrence of slug flow reported by Arirachakaran et al. (1989).

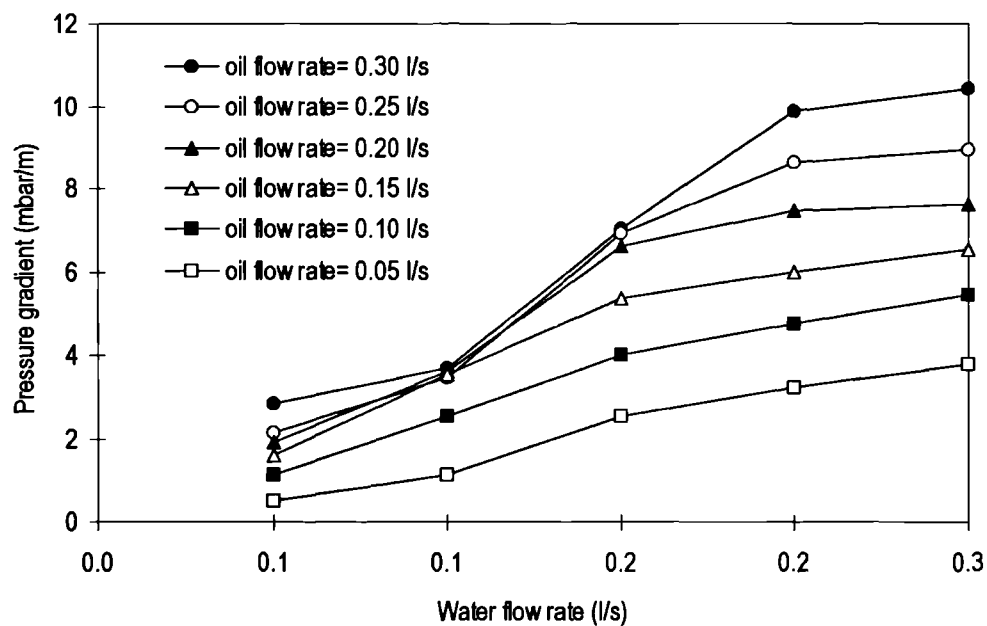


Figure 3.16: Pressure gradient. Low flow rate experiments TOWER *transpalite* test section - mixer.

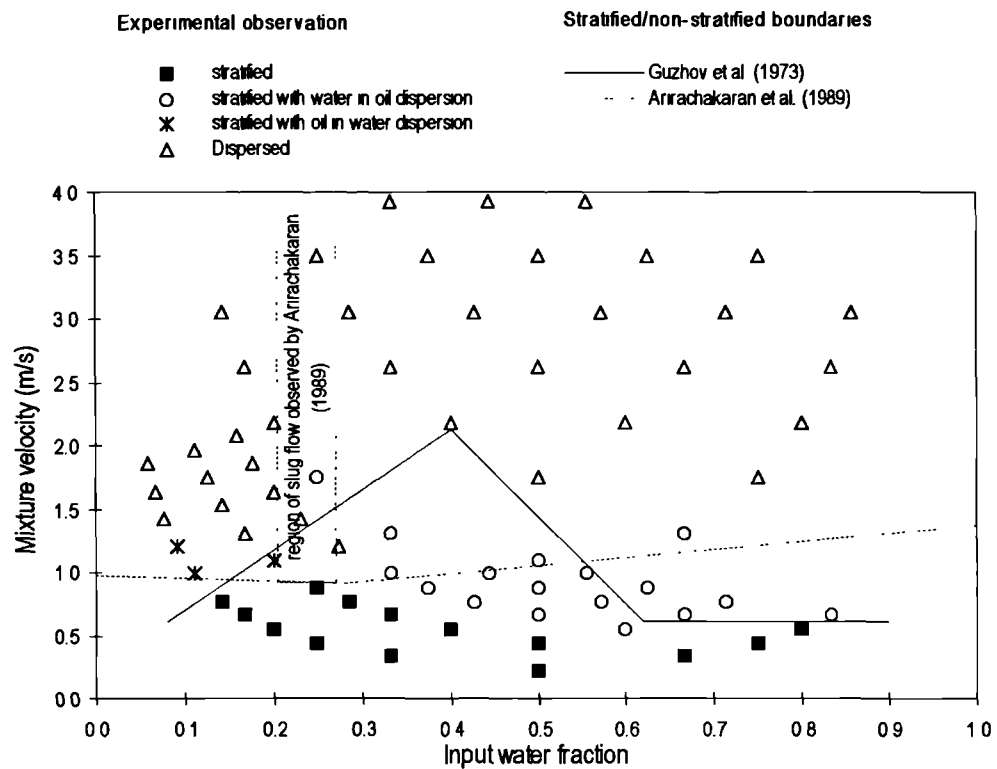


Figure 3.17: Experimental flow pattern map. TOWER stainless-steel test section.

Also plotted, the stratified/non-stratified boundaries according to Guzhov et al. (1973) and Arirachakaran et al. (1989) and the region of occurrence of slug flow reported by Arirachakaran et al. (1989).

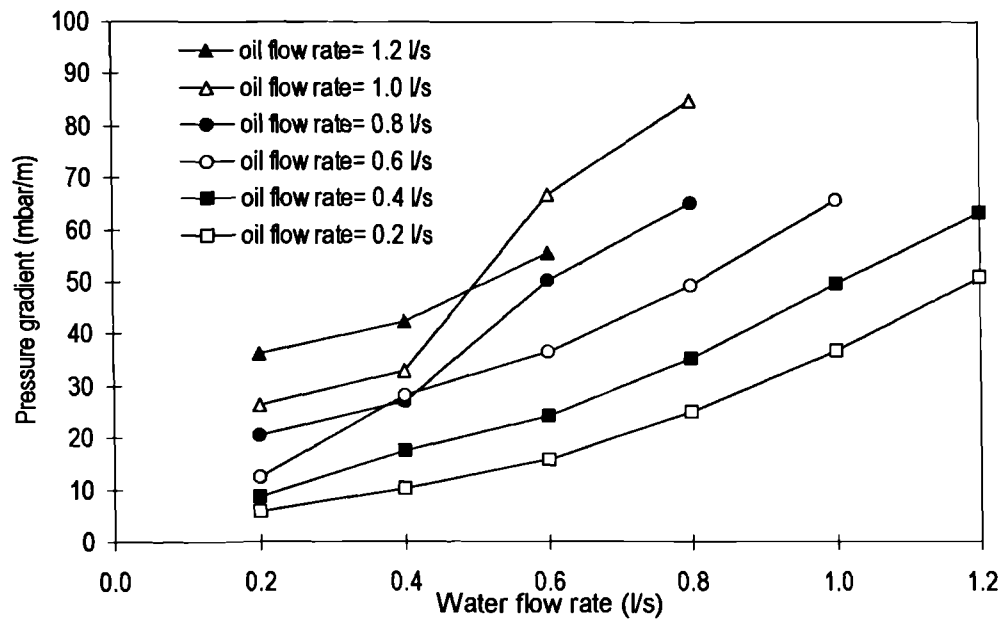


Figure 3.18: Pressure gradient. High flow rate experiments. TOWER stainless-steel test section.

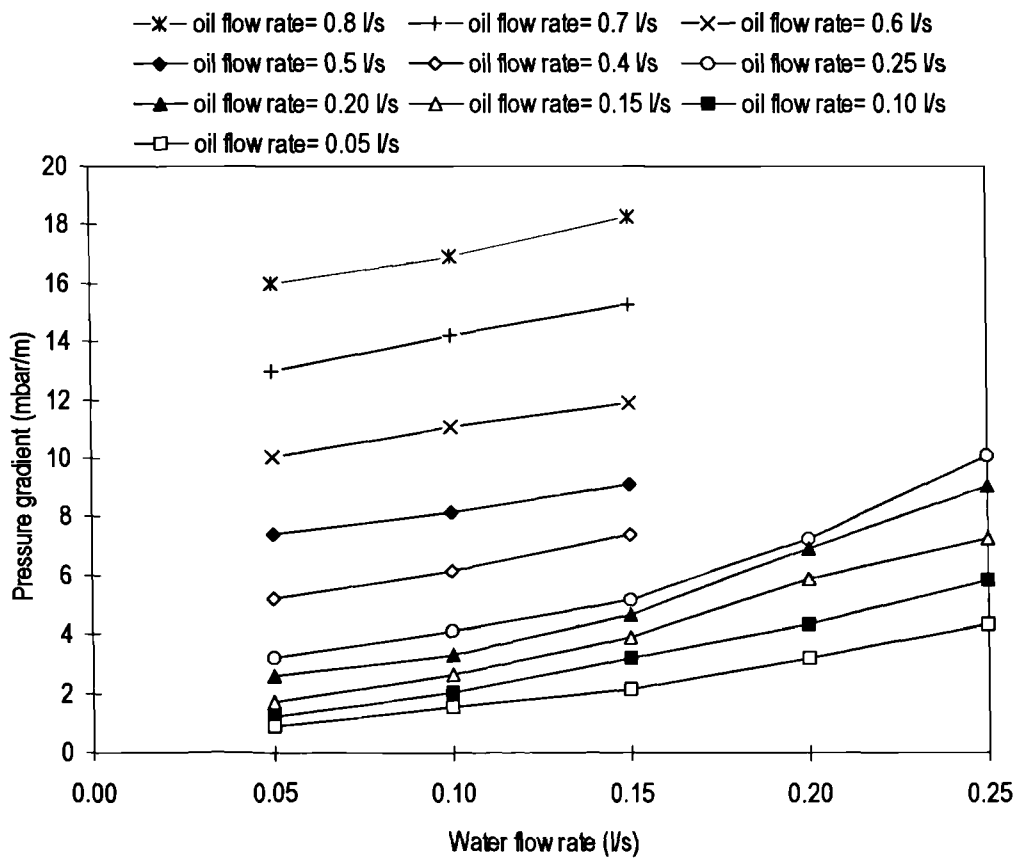


Figure 3.19: Pressure gradient. Low flow rate experiments. TOWER stainless-steel test section.

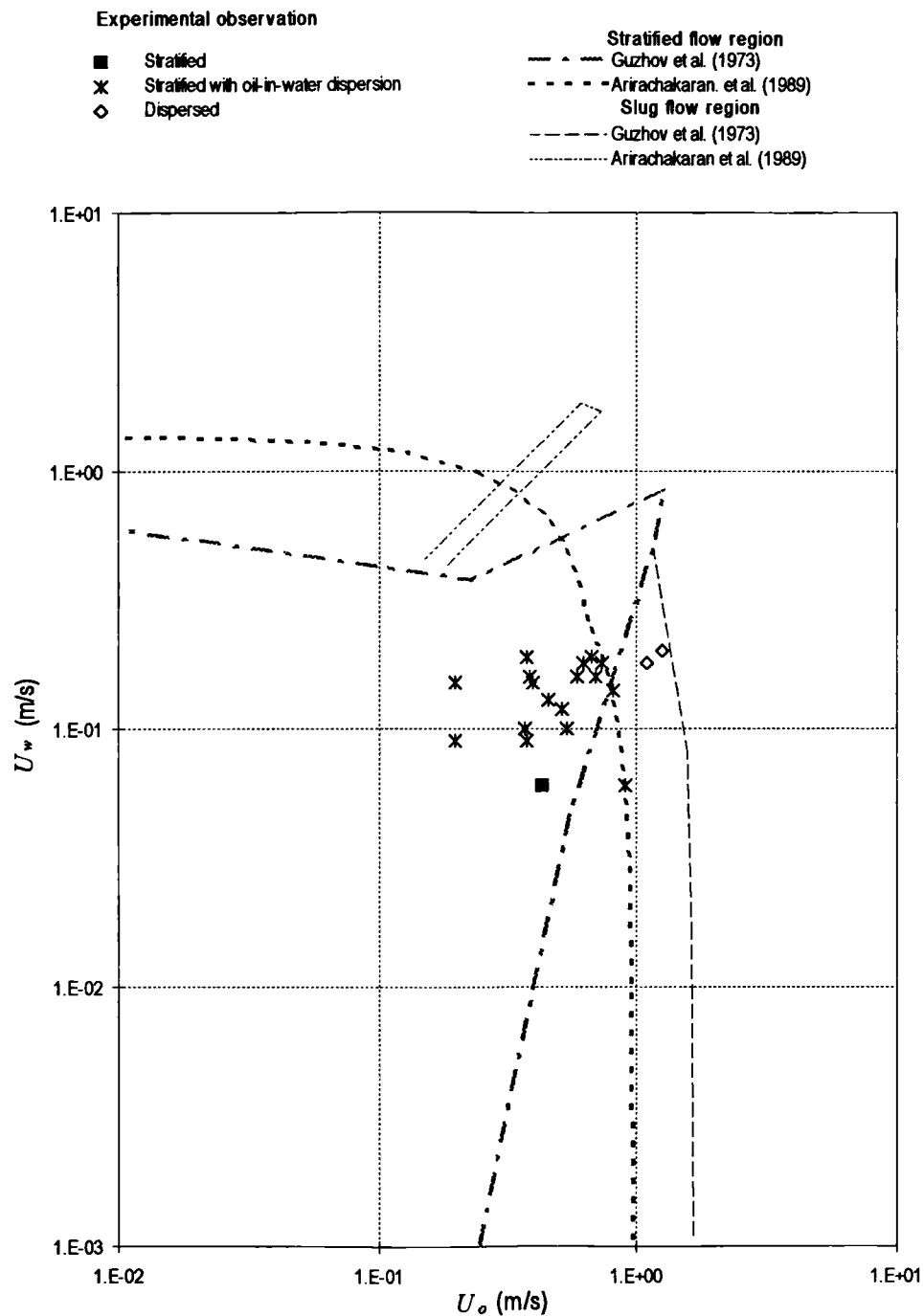


Figure 3.20: WASP horizontal flow experiments. Flow pattern map. Also plotted, the stratified flow and slug flow regions observed by Guzhov et al. (1973) and Arirachakaran et al. (1989).

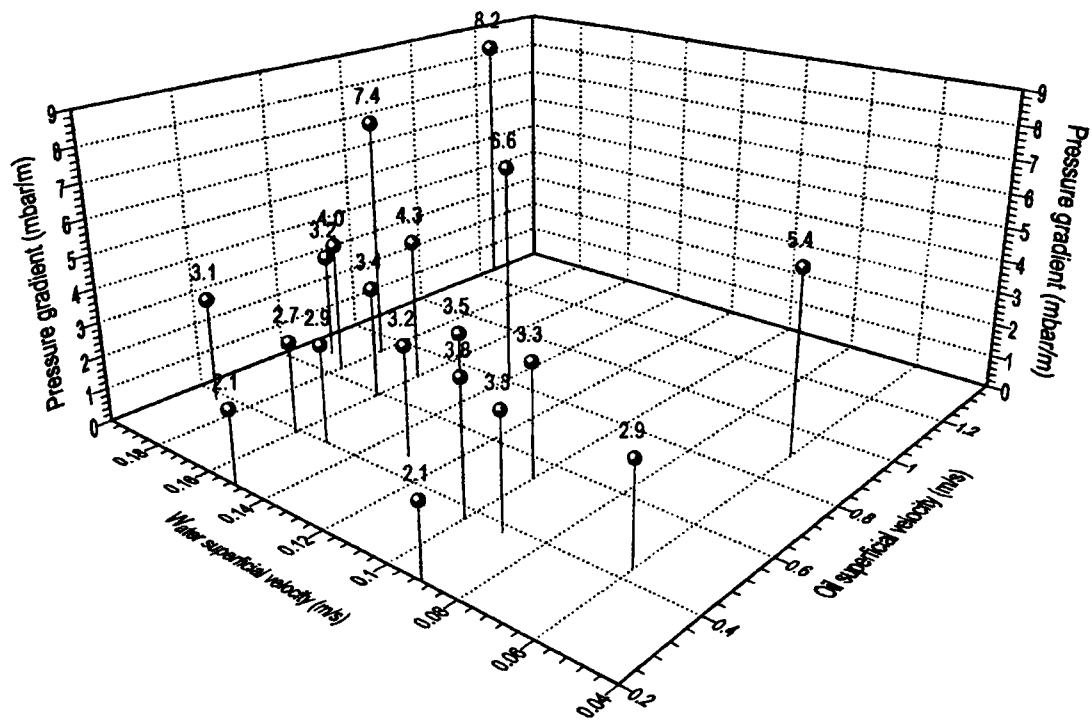


Figure 3.21: WASP horizontal flow experiments. Pressure gradients.

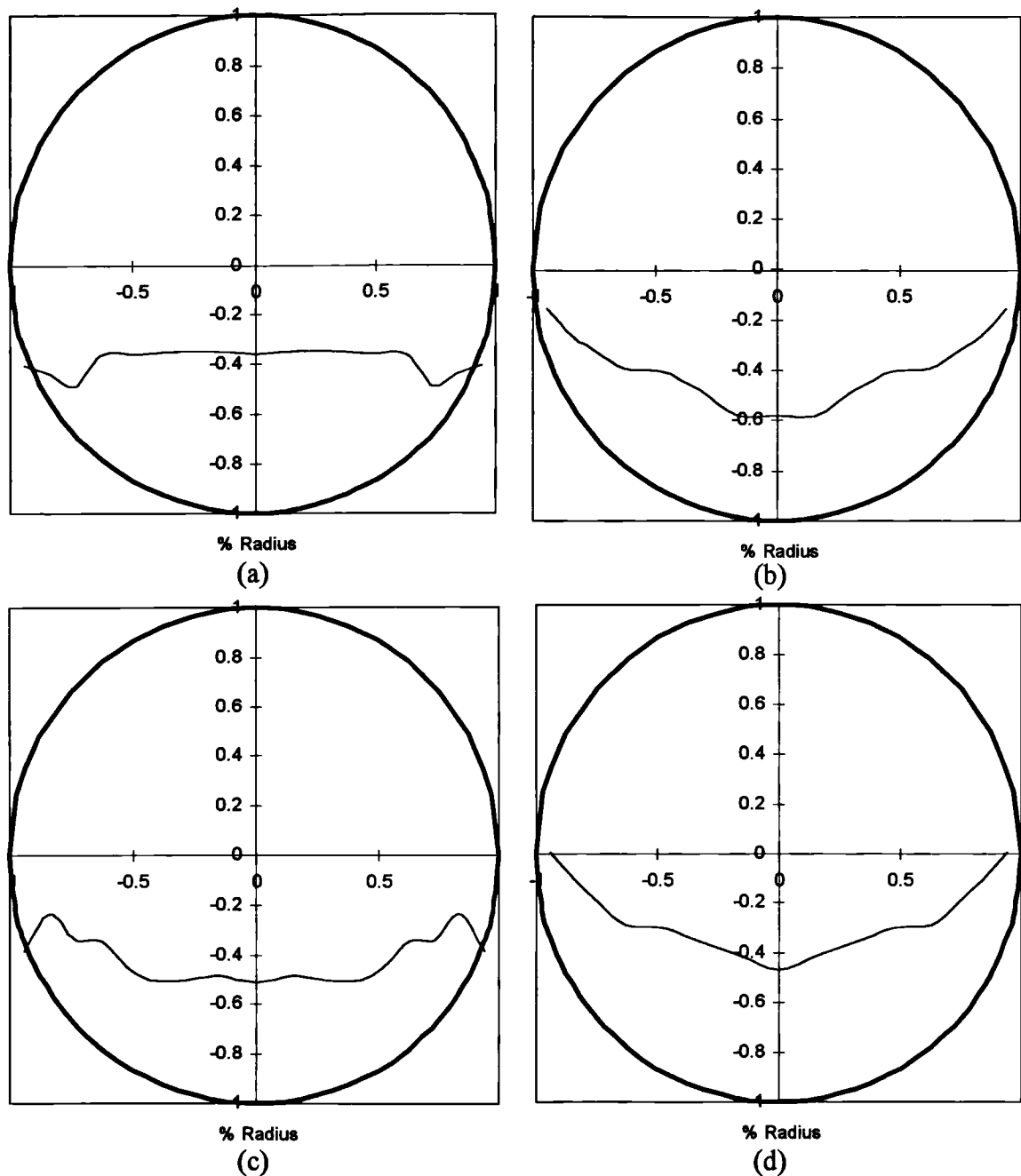
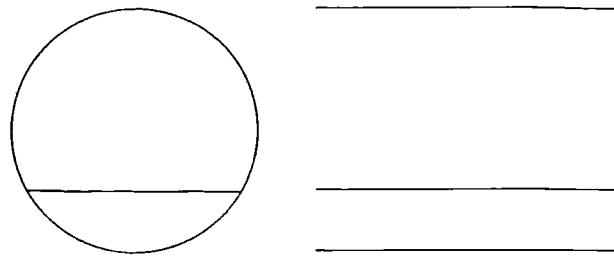
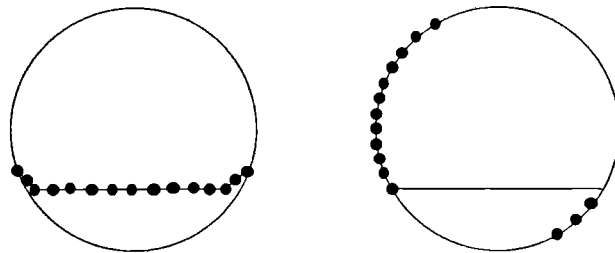


Figure 3.22: WASP horizontal flow experiments. Gamma ray scan with the Americium source. (a) Case 2, $U_o = 0.20$ m/s, $U_w = 0.15$ m/s, $\varepsilon_o = 0.28$, flow pattern observed: *stratified with oil in water dispersion*, flow pattern inferred: *stratified*; (b) Case 4, $U_o = 0.63$ m/s, $U_w = 0.18$ m/s, $\varepsilon_o = 0.26$, flow pattern observed: *stratified with oil in water dispersion*, flow pattern inferred: *dispersed*; (c) Case 10, $U_o = 0.59$ m/s, $U_w = 0.16$ m/s, $\varepsilon_o = 0.26$, flow pattern observed: *stratified with oil in water dispersion*, flow pattern inferred: *stratified*; (d) Case 20, $U_o = 1.10$ m/s, $U_w = 0.18$ m/s, $\varepsilon_o = 0.34$, flow pattern observed: *dispersed*, flow pattern inferred: *dispersed*.



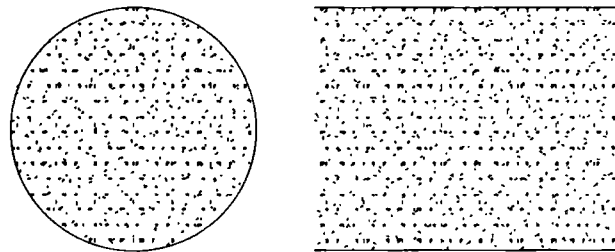
Ideal stratified flow



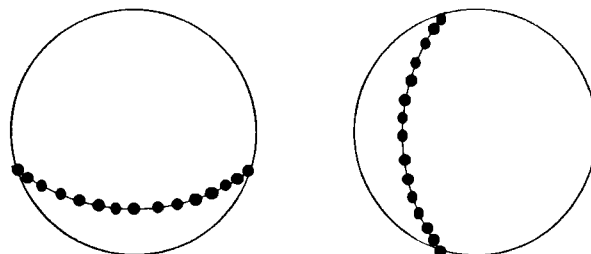
Horizontal traversing results

Vertical traversing results

Figure 3.23: Results given by the gamma ray densitometer for an ideal stratified flow (after Pan and Hewitt, 1993).



Ideal homogeneous (dispersed) flow



Horizontal traversing results

Vertical traversing results

Figure 3.24: Results given by the gamma ray densitometer for an ideal dispersed flow (after Pan and Hewitt, 1993).

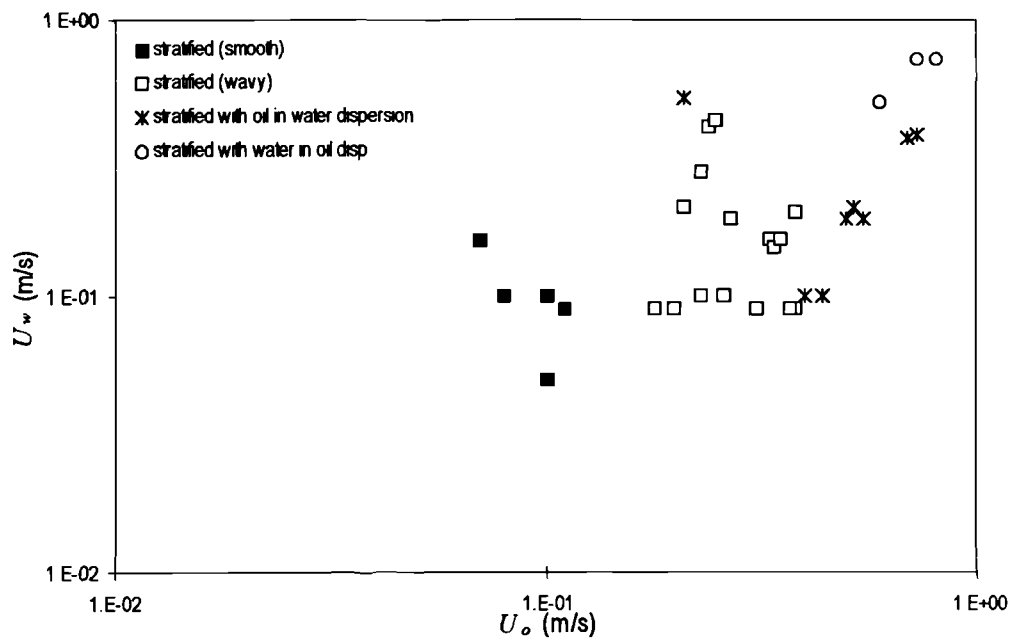


Figure 3.25: WASP inclined (1° upwards) flow experiments. Flow pattern map in terms of oil and water superficial velocities.

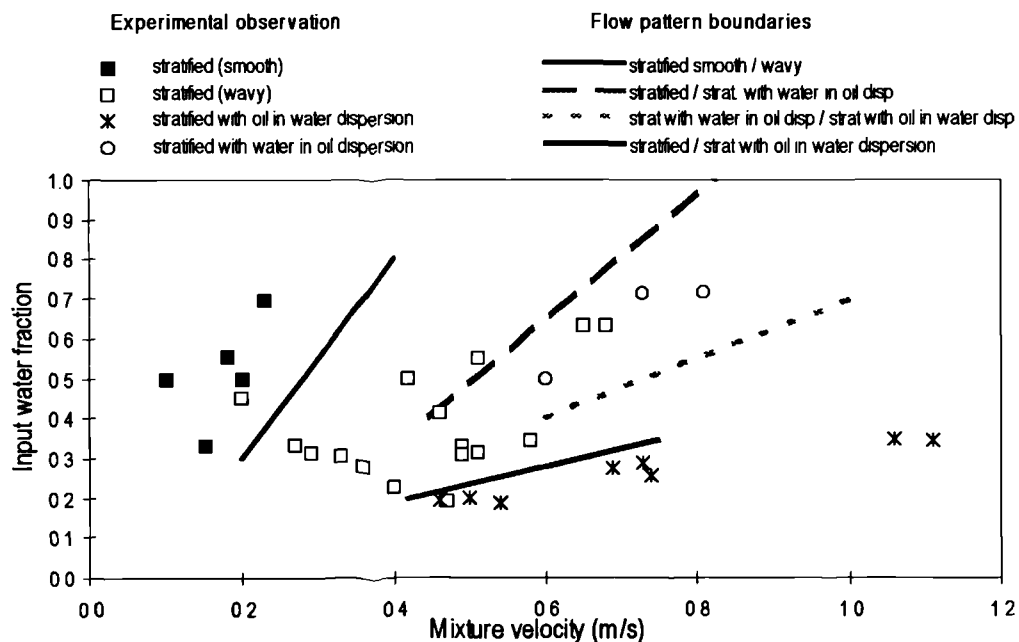


Figure 3.26: WASP inclined (1° upwards) flow experiments. Flow pattern map in terms of mixture velocity and water fraction.

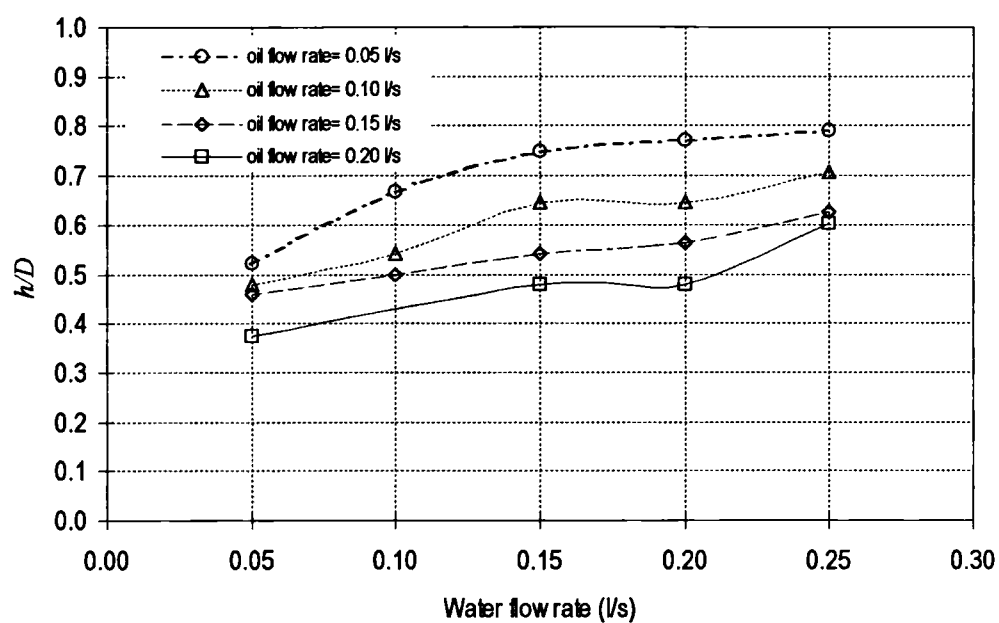


Figure 3.27: TOWER *transpalite* test-section. Interface level measured in stratified flow experiments.

Chapter 4

One Dimensional Modelling

In a *correlation* for a dependent variable in a system, an algebraic relationship is established between the variable and the independent variable based on experimental data. For example a *pressure drop correlation* or a *holdup correlation* for two-phase flow may relate these quantities to the channel geometry, the phase flowrates and the phase physical properties. Such correlations need take no account of the *physics* of the process occurring, though they are often most successful when expressed in terms of dimensionless terms (for example a relationship between a friction factor and a Reynolds number for single-phase flow); these dimensionless treatments sometimes may imply physical interpretations.

In a *phenomenological model*, on the other hand, the starting point is a phenomenological interpretation of the physics from which a consistent set of expressions is derived. Such models are often simplified greatly with respect to the real physics and there is no guarantee that, within a given data range, they perform better than correlation. However, it is normally argued that even simplified phenomenological models are more satisfactory in predicting conditions outside the available data base.

In this chapter, we deal with a specific example of a phenomenological model, namely the one dimensional model for stratified flow.

In the *phenomenological modelling* of the stratified flow the input is given by the geometry of the problem, Figure 4.1, which is a simplification of the more complex configuration of an actual flow. Then the area averaged momentum equations are applied over this geometry, resulting in a force balance where individual components can be identified and subsequently modelled, in general by correlations obtained from specially designed subsidiary experiments.

A phenomenological one-dimensional model of stratified flow was developed by Agrawal et al. (1973) and a similar model was developed later by Taitel and Dukler (1976a, b). The latter development is better known since it formed the basis of the widely used Taitel and Dukler model for two-phase flow patterns in horizontal gas liquid flows (Taitel and Dukler, 1976a)

In the following sections, the Taitel and Dukler gas-liquid stratified flow analysis is first presented. It is then followed by an extension of this analysis — applicable both to gas-liquid and liquid-liquid flows — where a new set of dimensionless variables is defined. Particular closure relationships (such as those of Andritsos and Hanratty, 1987) for stratified gas-liquid flow are finally applied within the framework of this extended model in the remaining sections.

4.1- Basic one-dimensional model for horizontal flow

4.1.1- Taitel and Dukler (1976a, b) one-dimensional momentum equations

The flow parameters used in Taitel and Dukler smooth stratified flow analysis are illustrated in Figure 4.1; u_G and u_L are the gas and liquid velocities, D the tube diameter, P_G and P_L the peripheries in contact with the gas and liquid, A_G and A_L the flow areas of the respective phases and h_L the height of the liquid layer as illustrated. The objective of the analysis was to calculate the dimensionless liquid level ($\tilde{h}_L = h_L/D$) and the pressure drop multiplier ($\phi_G^2 = (dp/dz)/(dp/dz)_G$) where $(dp/dz)_G$ is the frictional pressure drop for the gas flowing alone in the pipe) as a function of dimensionless flow parameters.

Taitel and Dukler assumed a uniform steady-state flow in which there were no hydraulic gradients (h_L constant) and in which there was no significant acceleration. For this case, the momentum equations for the liquid and gas phases respectively are as follows:

$$-A(1 - \varepsilon_G) \left[\left(\frac{dp}{dz} \right) + \rho_L g \sin \alpha \right] - \tau_{wL} P_L + \tau_i P_i = 0 \quad (4.1)$$

and

$$-A\varepsilon_G \left[\left(\frac{dp}{dz} \right) + \rho_G g \sin \alpha \right] - \tau_{wG} P_G - \tau_i P_i = 0 \quad (4.2)$$

where α is the angle of inclination of the channel, τ_{wL} and τ_{wG} are the wall shear stresses adjacent to the liquid and gas phases and τ_i is the interfacial shear stress.

Setting

$$\varepsilon_G A = A_G \quad (4.3)$$

and

$$(1 - \varepsilon_G) A = A_L \quad (4.4)$$

and eliminating the pressure gradient term $\left(\frac{dp}{dz} \right)$ between Equations (4.1) and (4.2) we have:

$$\tau_{wG} \frac{P_G}{A_G} - \tau_{wL} \frac{P_L}{A_L} + \tau_i P_i \left(\frac{1}{A_L} + \frac{1}{A_G} \right) - (\rho_L - \rho_G) g \sin \alpha = 0 \quad (4.5)$$

In order to calculate the shear stress terms, Taitel and Dukler invoked the following relationships involving friction factors:

$$\tau_{wL} = f_L \rho_L \frac{u_L^2}{2} \quad (4.6)$$

$$\tau_{wG} = f_G \rho_G \frac{u_G^2}{2} \quad (4.7)$$

and

$$\tau_i = f_i \rho_G \frac{(u_G - u_L)^2}{2} \quad (4.8)$$

Assuming a smooth interface and that u_G is much greater than u_L , Taitel and Dukler made the approximation:

$$\tau_i = \tau_{wG} \quad (4.9)$$

Taitel and Dukler related the friction factors to Reynolds numbers as follows:

$$f_L = C_L \left(\frac{u_L D_L \rho_L}{\eta_L} \right)^{-n} \quad (4.10)$$

and

$$f_G = C_G \left(\frac{u_G D_G \rho_G}{\eta_G} \right)^{-m} \quad (4.11)$$

where D_L and D_G are the hydraulic diameters and are given by:

$$D_L = \frac{4A_L}{P_L} \quad (4.12)$$

$$D_G = \frac{4A_G}{P_G + P_i} \quad (4.13)$$

For turbulent flow, Taitel and Dukler assumed that $C_G = C_L = 0.046$ and $n = m = 0.2$. Similarly for laminar flow, they assume that $C_G = C_L = 16$ and $n = m = 1$. In other words, Taitel and Dukler evaluate the friction factor using the Blasius equation for smooth pipes in turbulent flows or the Poiseuille† equation in laminar flows.

Taitel and Dukler proceeded to obtain a solution for equilibrium stratified flow by non-dimensionalising the various parameters in the above equations as follows:

$$\tilde{A}_L = \frac{A_L}{D^2} = \frac{1}{4} \left[\pi - \arccos(2\tilde{h}_L - 1) + (2\tilde{h}_L - 1) \sqrt{1 - (2\tilde{h}_L - 1)^2} \right] \quad (4.14)$$

$$\tilde{A}_G = \frac{A_G}{D^2} = \frac{1}{4} \left[\arccos(2\tilde{h}_L - 1) - (2\tilde{h}_L - 1) \sqrt{1 - (2\tilde{h}_L - 1)^2} \right] \quad (4.15)$$

† **Poiseuille, Jean-Louis-Marie**. (b. 22 April 1799, Paris, France--d. 26 Dec.1869, Paris), French physician and physiologist who formulated a mathematical expression for the flow rate for the laminar flow of fluids in circular tubes. Discovered independently by Gotthilf Hagen, a German hydraulic engineer, this relation is also known as the Hagen-Poiseuille equation:

$$\dot{V} = \left(\frac{\pi D^4}{128\eta} \right) \frac{dp}{dz}$$

Poiseuille received his medical degree in 1828 and established his practice in Paris. His interest in the circulation of the blood led him to conduct a series of experiments on the flow of liquids in narrow tubes, from which he determined the law that bears his name. This equation states that the flow rate is determined by the viscosity of the fluid, the drop in pressure along the tube, and the tube diameter. He also is believed to be the first to have used the mercury manometer to measure blood pressure.

$$\tilde{P}_i = \frac{P_i}{D} = \sqrt{1 - (2\tilde{h}_L - 1)^2} \quad (4.16)$$

$$\tilde{P}_L = \frac{P_L}{D} = \pi - \arccos(2\tilde{h}_L - 1) \quad (4.17)$$

$$\tilde{P}_G = \frac{P_G}{D} = \arccos(2\tilde{h}_L - 1) \quad (4.18)$$

$$\tilde{u}_L = \frac{u_L}{U_L} = \frac{\tilde{A}}{\tilde{A}_L} \quad (4.19)$$

$$\tilde{u}_G = \frac{u_G}{U_G} = \frac{\tilde{A}}{\tilde{A}_G} \quad (4.20)$$

$$\tilde{D}_G = \frac{4A_L}{P_L D} = \frac{4\tilde{A}_L}{\tilde{P}_L} \quad (4.21)$$

$$\tilde{D}_G = \frac{4A_G}{(P_G + P_i)D} = \frac{4\tilde{A}_G}{\tilde{P}_G + \tilde{P}_i} \quad (4.22)$$

It is observed that all the above parameters are uniquely related to $\tilde{h}_L$. Taitel and Dukler also introduced two further dimensionless parameters, namely X (the Martinelli parameter) and Y (an inclination parameter) defined as follows:

$$X = \sqrt{\frac{\left(\frac{dp}{dz}\right)_L}{\left(\frac{dp}{dz}\right)_G}} \quad (4.23)$$

and

$$Y = \sqrt{\frac{(\rho_L - \rho_G)g \sin \alpha}{\left(\frac{dp}{dz}\right)_G}} \quad (4.24)$$

where $(dp/dz)_L$ and $(dp/dz)_G$ are the pressure gradients for the liquid and gas phases respectively flowing alone in the pipe.

Substituting these non-dimensional quantities into Equation (4.5) leads, finally, to the following equation:

$$X^2 (\tilde{u}_L D_L)^{-n} \tilde{u}_L^2 \frac{\tilde{P}_L}{\tilde{A}_L} - (\tilde{u}_G \tilde{D}_G)^{-m} \tilde{u}_G^2 \left(\frac{\tilde{P}_G}{\tilde{A}_G} + \frac{\tilde{P}_i}{\tilde{A}_L} + \frac{\tilde{P}_i}{\tilde{A}_G} \right) - 4Y = 0 \quad (4.25)$$

Recognising that (with the exceptions of X and Y) all of the variables in Equation (4.25) are related uniquely to $\tilde{h}_L$, Equation (4.25) can be solved iteratively to give $\tilde{h}_L$ for any given values of X and Y. Nevertheless an easier process of obtaining the solution of Equation (4.21) is to give the values of $\tilde{h}_L$ and Y and then calculate X. Since, Equation (4.21) is implicit in $\tilde{h}_L$, but explicit in X, the iterative solution is, by these means, avoided. This solution presented in the form of Taitel and Dukler is shown in Figure 4.2.

The pressure drop multiplier ϕ_G^2 can also be written as a function of the dimensionless variables (Taitel and Dukler, 1976b):

$$\phi_G^2 = \frac{1}{4} \tilde{u}_G^2 \frac{(\tilde{u}_G \tilde{D}_G)^{-m}}{\tilde{A}_G} \left[\tilde{P}_G + \frac{f_i}{f_G} \tilde{P}_i \right] \quad (4.26)$$

For laminar liquid flows, Taitel and Dukler (1976b) report good agreement with gas-liquid pressure drop data by taking $f_i = f_G$.

Though the phenomenological analyses of the Taitel and Dukler type have provided a very useful approach to the understanding of smooth-stratified flows, the models are greatly oversimplified in their assumptions of the interfacial and wall friction terms. Even if the one-dimensional situation is retained the assumptions of the equations of the interfacial and wall friction factors for one of the phases and the neglect of interfacial velocity with respect to the velocities of that phase would appear to be of very doubtful generality, particularly for liquid-liquid flows. In order to address these deficiencies of the original Taitel and Dukler model, a modified one-dimensional analysis of stratified flow is presented in the next section. It should be remembered, of course, that the case of a one-dimensional analysis for what is (at least) a two-dimensional problem is in itself fundamentally flawed. This aspect will be explained in detail in Chapter 5.

4.1.2- Modified and extended 1-D analysis of stratified flow

Assuming the general horizontal stratified flow configuration and variables as shown in Figure 4.3[†], the momentum balance in the axial direction of the pipe can be expressed for each of the phases as:

$$-A_B \left(\frac{dp}{dz} \right) - \tau_{wB} P_B + \tau_i P_i = 0 \quad (4.27)$$

$$-A_T \left(\frac{dp}{dz} \right) - \tau_{wT} P_T - \tau_i P_i = 0 \quad (4.28)$$

where A_B and A_T are the cross sectional areas of the fluids B (Bottom: lower, heavier, phase in the pipe) and T (Top: upper, lighter, phase in the pipe), τ_{wB} and τ_{wT} the wall shear stresses for the respective phases, τ_i the interfacial shear stress and P_B , P_T and P_i the fluid B wall, fluid T wall and the interfacial peripheries respectively.

The above balances are valid for any two-phase system, irrespective if the nature of the phases (gas or liquid).

[†] For the sake of clarity and simplicity the phase names *fluid B* referring to the lower (Bottom) phase and *fluid T* for the upper (Top) phase are going to be used throughout the rest of this chapter. Therefore, when dealing with gas-liquid flows *fluid B* is the liquid phase and *fluid T* is the gas phase; and when dealing with oil-water flows *fluid B* is the water phase and *fluid T* is the oil phase. It is also assumed in gas-liquid flows that the absolute viscosity of the gas is always smaller than that of the liquid; and in liquid-liquid flows that the absolute viscosity of the upper phase (oil) is always greater than that on the lower phase (water).

Eliminating the pressure gradient terms in equations (4.27) and (4.28) by dividing them respectively by A_B and A_T and subtracting one equation from the other, the following relationship is obtained[†] :

$$\tau_{wB} \frac{P_B}{A_B} - \tau_{wT} \frac{P_T}{A_T} - \tau_i P_i \left(\frac{1}{A_B} + \frac{1}{A_T} \right) = 0 \quad (4.29)$$

Defining the following dimensionless parameters:

a) Dimensionless wall shear stresses:

$$\tilde{\tau}_{wB} = \frac{\tau_{wB}}{T_{wB}} \quad (4.30)$$

$$\tilde{\tau}_{wT} = \frac{\tau_{wT}}{T_{wT}} \quad (4.31)$$

where T_w is the superficial wall shear stress, i.e. the wall shear stress corresponding to the single flow of the phase at the respective superficial velocity.

At this point it is interesting to observe that if the friction factor concept in the form of Equations (4.10) and (4.11) is used to determine the values of the shear stresses, equations (4.30) and (4.31) reduce to:

[†] In this thesis, τ_i is defined as having a positive value if $u_T > u_B$ and a negative value if $u_T < u_B$.

$$\tilde{\tau}_{wB} = \frac{f_B \rho_B \frac{u_B^2}{2}}{F_B \rho_B \frac{U_B^2}{2}} = \tilde{f}_B \left(\frac{u_B}{U_B} \right)^2 = \frac{\tilde{f}_B}{\varepsilon_B^2} \quad (4.32)$$

and

$$\tilde{\tau}_{wT} = \frac{f_T \rho_T \frac{u_T^2}{2}}{F_T \rho_T \frac{U_T^2}{2}} = \tilde{f}_T \left(\frac{u_T}{U_T} \right)^2 = \frac{\tilde{f}_T}{\varepsilon_T^2} \quad (4.33)$$

where $\tilde{f}_B$ and $\tilde{f}_T$ are “dimensionless” friction factors, which are defined as the ratio between the friction factor for the actual flow configuration and the "superficial" friction factor for each phase (F_B and F_T), that is:

$$\tilde{f}_B = \frac{f_B}{F_B} \quad (4.34)$$

and

$$\tilde{f}_T = \frac{f_T}{F_T} \quad (4.35)$$

b) Dimensionless interfacial stress:

$$\tilde{\tau}_i = \frac{\tau_i}{T_{wT}} \quad (4.36)$$

c) Generalised Lockhart and Martinelli parameter:

$$X_{TB} = \sqrt{\frac{\left(\frac{dp}{dz}\right)_B}{\left(\frac{dp}{dz}\right)_T}} \quad (4.37)$$

Invoking single-phase flow relationships between the pressure gradient and the wall shear stress:

$$-\left(\frac{dp}{dz}\right)_B = \frac{4}{D} T_{wB} \quad (4.38)$$

and

$$-\left(\frac{dp}{dz}\right)_T = \frac{4}{D} T_{wT} \quad (4.39)$$

the generalised Martinelli parameter can be expressed alternatively by:

$$X_{TB} = \sqrt{\frac{T_{wB}}{T_{wT}}} \quad (4.40)$$

d) Dimensionless wetted perimeters:

$$\tilde{P}_B = \frac{P_B}{D} \quad (4.41)$$

$$\tilde{P}_T = \frac{P_T}{D} \quad (4.42)$$

e) Dimensionless interface length

$$\tilde{P}_i = \frac{P_i}{D} \quad (4.43)$$

f) Dimensionless flow areas:

$$\tilde{A}_B = \frac{A_B}{D^2} \quad (4.44)$$

$$\tilde{A}_T = \frac{A_T}{D^2} \quad (4.45)$$

Equation (4.29) can then be rewritten as:

$$X_{TB}^2 \tilde{\tau}_{wB} \frac{\tilde{P}_B}{\tilde{A}_B} - \tilde{\tau}_{wT} \frac{\tilde{P}_T}{\tilde{A}_T} - \tilde{\tau}_i \tilde{P}_i \left(\frac{1}{\tilde{A}_T} + \frac{1}{\tilde{A}_B} \right) = 0 \quad (4.46)$$

The expression given by Equation (4.46) — or (4.29) — is, in fact, an implicit solution for the height of the bottom fluid layer (h/D), since all geometric parameters are unique functions of it, while the shear-stresses are functions of the fluid properties and flow rates, which are supposedly the given input data for the problem.

In the light of equation (4.46) it is also convenient to derive a relation for the pressure gradient or its dimensionless equivalent ϕ_B , defined similarly to Lockhart and Martinelli (1949) as:

$$\phi_B = \sqrt{\frac{\left(\frac{dp}{dz}\right)}{\left(\frac{dp}{dz}\right)_B}} \quad (4.47)$$

Substituting Equation (4.47) in (4.27), gives:

$$\phi_B = \sqrt{\frac{\tau_{wB}P_B - \tau_iP_i}{A_B\left(-\frac{dp}{dz}\right)_B}} \quad (4.48)$$

and replacing the single-phase shear-stress-pressure gradient relationship of Equation (4.38) into Equation (4.48) gives:

$$\phi_B = \sqrt{\frac{D}{4A_B} \frac{\tau_{wB}P_B - \tau_iP_i}{T_{wB}}} \quad (4.49)$$

Using the definition of dimensionless shear and interfacial stresses, Equations (4.30) and (4.36), the expression for ϕ_B becomes:

$$\phi_B = \sqrt{\frac{D}{4A_B} \left(\tilde{\tau}_{wB}P_B - \tilde{\tau}_i \frac{T_{wI}}{T_{wB}} P_i \right)} \quad (4.50)$$

Recalling the definition of the generalised Martinelli parameter, as expressed by Equation (4.40), and the definition of the dimensionless wetted perimeter, interface length and flow area, Equations (4.41), (4.43) and (4.44), the final expression for ϕ_B is obtained:

$$\phi_B = \sqrt{\frac{1}{4\tilde{A}_B} \left(\tilde{\tau}_{wB} \tilde{P}_B - \frac{\tilde{\tau}_i}{X_{TB}^2} \tilde{P}_i \right)} \quad (4.51)$$

The expression given by Equation (4.51) provides a solution for the pressure drop multiplier ϕ_B as a function uniquely of the geometric configuration of the flow and the generalised Martinelli parameter. As X_{TB} and the geometric parameters are also inter-related, Equation (4.46), it can be said that Equation (4.51) represents a relationship between ϕ_B and X_{TB} . This relationship however needs to be solved in two successive steps, thus Equation (4.46) is first solved iteratively and Equation (4.51) is then solved using the shear stress and geometrical values that correspond to the interface height (h/D) determined.

In order to obtain results from Equations (4.46) and (4.51) the dimensionless shear stress terms present in these equations must be formulated. As this set of equations is normally preferred for the solution of gas-liquid flows, at this point in this study it was assumed that the interfacial shear stress was equal to be a multiple of the upper fluid wall shear stress (i.e. in this case the less viscous phase). The wall shear stresses were, in turn, modelled with a friction factor, so that the dimensionless wall shear stresses $\tilde{\tau}_{wB}$ and $\tilde{\tau}_{wT}$ are given by Equations (4.32) and (4.33). Furthermore it was assumed the friction factors present in these equations to have a Blasius-like form in a similar manner to Taitel and Dukler

(1976a, b). Therefore the dimensionless friction factors present in Equations (4.32) and (4.33) , and defined by Equations (4.34) and (4.35) were reduced to:

$$\tilde{f}_B = \frac{C \left(u_B D_B \rho_B / \eta_B \right)^{-m}}{C \left(U_B D \rho_B / \eta_B \right)^{-m}} = \left(\frac{\tilde{D}_B}{\varepsilon_B} \right)^{-m} \quad (4.52)$$

and similarly

$$\tilde{f}_T = \frac{C \left(u_T D_T \rho_T / \eta_T \right)^{-n}}{C \left(U_B D \rho_T / \eta_T \right)^{-n}} = \left(\frac{\tilde{D}_T}{\varepsilon_T} \right)^{-n} \quad (4.53)$$

where D_B and D_T are the hydraulic diameters for the lower and upper phases:

$$D_B = \frac{4A_B}{P_B + P_i \text{Max} \left(0, \frac{\eta_T - \eta_B}{|\eta_T - \eta_B|} \right)}^\dagger \quad (4.54)$$

and

[†] The introduction of the function "Max" in the denominator of Equations (4.52) and (4.53) was done in order to make them as general as possible. The use of this function here has the effect of adding the interface length S_i only to the perimeter of the less viscous phase. Therefore it is implied by these calculations of the hydraulic diameters that the less viscous fluid is assumed to be flowing as if it were in a close conduit, while the more viscous phase is assumed to be flowing as if it were in an open channel.

$$D_T = \frac{4A_T}{P_T + P_i \text{Max}\left(0, \frac{\eta_B - \eta_T}{|\eta_T - \eta_B|}\right)} \quad (4.55)$$

and $\tilde{D}_B$ and $\tilde{D}_T$ are the dimensionless hydraulic diameters for the indicated phases, defined as the ratio between the hydraulic diameter of each of these phases and the pipe diameter:

$$\tilde{D}_B = \frac{D_B}{D} \quad (4.56)$$

and

$$\tilde{D}_T = \frac{D_T}{D} \quad (4.57)$$

Figures 4.4 and 4.5 shows the influence of the flow regime, laminar or turbulent, in the solution of Equations (4.46) and (4.51) respectively. For the laminar solution it was assumed in Equations (4.52) and (4.53) that $n = m = 1.0$; while for the turbulent solutions it was assumed $n = m = 0.2$. Those curves correspond to the same solutions encountered by Taitel and Dukler (1976a). It is noticed that the laminar solution presents slightly lower interface levels at low values of X but slightly higher values of the interface level at high values of X in comparison with the turbulent solution.

Figures 4.6 and 4.7 show the turbulent solution of Equations (4.46) and (4.51) for the dimensionless interfacial shear stress to upper phase wall shear stress ratio

values $(\tilde{\tau}_i/\tilde{\tau}_{wT})$ of 1.0, 2.0, 10.0 and 100.0. It is observed from Figure 4.6 that the interface level can change very much according to the value of the dimensionless interfacial shear stress assumed. Taking, for instance, the flow conditions at which X is equal to 1.0, the interface level would be around 40 % of the pipe diameter if $(\tilde{\tau}_i/\tilde{\tau}_{wT})$ were equal to 1.0; or around 25% if $(\tilde{\tau}_i/\tilde{\tau}_{wT})$ were equal to 10.0. Similarly Figure 4.7 shows that the increase on $(\tilde{\tau}_i/\tilde{\tau}_{wT})$ also plays a role in the increase of the pressure gradient multiplier ϕ_B . For the same conditions as above, $X=1.0$, ϕ_B would be around 2.0 for $(\tilde{\tau}_i/\tilde{\tau}_{wT})$ equal to 1.0 and more than 2.5 for $\tilde{\tau}_i$ equal to 10.0. Figure 4.8 shows the variation of ϕ_B with $(\tilde{\tau}_i/\tilde{\tau}_{wT})$, where it can be seen that the effect of the variation $(\tilde{\tau}_i/\tilde{\tau}_{wT})$ in ϕ_B is more pronounced at lower values of X . That is not surprising because at high X values the pipe is mostly full with the lower phase (liquid in the gas-liquid flows) and under these conditions the total pressure gradient would be mostly controlled by the lower phase shear stress, due to the small values of the upper phase periphery and the interface length which would consequently cancel the effects of the interfacial shear stress and upper phase wall shear stress in Equation (4.27).

Equation (4.51), which gives the pressure multiplication factor in terms of the pressure gradient of the "superficial" (i.e. full pipe) flow of the lower phase ϕ_B , is normally applied in gas-liquid flows where the lower phase is also the more viscous. For liquid-liquid flows, like the simultaneous flow of oil and water, it is more interesting to have the pressure multiplication factor in terms of the upper phase, in this example the oil, which would normally be the more viscous liquid. In this case the pressure multiplication factor ϕ_T , defined by:

$$\phi_T = \sqrt{\frac{\left(\frac{dp}{dz}\right)}{\left(\frac{dp}{dz}\right)_T}} \quad (4.58)$$

A similar procedure to that one to determine ϕ_B is followed. Thus Equation (4.58) is substituted in Equation (4.28) and hence:

$$\phi_T = \sqrt{\frac{\tau_{wT}P_T + \tau_iP_i}{A_T\left(-\frac{dp}{dz}\right)_T}} \quad (4.59)$$

Subsequently, using the definition of dimensionless shear and interfacial stresses, Equations (4.30) and (4.36), and recalling once more the definition of the generalised Martinelli parameter, as expressed by Equation (4.40), and the definition of the dimensionless wetted perimeter, interface length and flow area, Equations (4.41), (4.43) and (4.44), the following expression for ϕ_T is obtained:

$$\phi_T = \sqrt{\frac{1}{4\tilde{A}_T}(\tilde{\tau}_{wT}\tilde{P}_T + \tilde{\tau}_i\tilde{P}_i)} \quad (4.60)$$

Although in Equation (4.60) the term X_{TB} does not appear explicitly it must be observed that it can still be considered a relation between ϕ_T and X_{TB} because the geometric terms present in Equation (4.60) are related to X_{TB} through Equation (4.46). One inconvenience still arises for the case of liquid-liquid flows where the upper phase is more viscous than the lower (as in oil-water flows), that is the definition of $\tilde{\tau}_i$ as the ratio of the interfacial stress and the top fluid superficial

wall shear stress, Equation (4.36). It would be better if $\tilde{\tau}_i$ were defined in terms of the lower viscosity fluid wall shear stress, as the interfacial shear stress is more likely to assume values of the same order of magnitude. Therefore a new dimensionless interfacial shear stress is defined for liquid-liquid flows:

$$\tilde{\tau}_{lli} = \frac{\tau_i}{T_{wB}} \quad (4.61)$$

With Equations (4.36), (4.40) and (4.61) the relation between $\tilde{\tau}_i$ and $\tilde{\tau}_{lli}$ can then be written as:

$$\tilde{\tau}_i = \tilde{\tau}_{lli} X_{TB}^2 \quad (4.62)$$

Substituting Equation (4.62) in Equations (4.46) and (4.60) a more practical set of equations for the solution of liquid-liquid (oil-water) problems is obtained:

$$\tilde{\tau}_{wB} \frac{\tilde{P}_B}{\tilde{A}_B} - \frac{1}{X_{TB}^2} \tilde{\tau}_{wT} \frac{\tilde{P}_T}{\tilde{A}_T} - \tilde{\tau}_{lli} \tilde{P}_i \left(\frac{1}{\tilde{A}_T} + \frac{1}{\tilde{A}_T} \right) = 0 \quad (4.63)$$

and

$$\phi_T = \sqrt{\frac{1}{4\tilde{A}_T} \left(\tilde{\tau}_{wT} \tilde{P}_T + X_{TB}^2 \tilde{\tau}_{lli} \tilde{P}_i \right)} \quad (4.64)$$

For the sake of completeness of the present work, Figures 4.9 to 4.13 are presented. These figures are the solutions of Equations (4.63) and (4.64) (liquid-liquid case) equivalent to those conditions of Figures 4.4 to 4.8 for the solution

of Equations and (4.46) and (4.51). All the remarks made for Figure 4.4 to Figure 4.8 are, in general, valid for Figures 4.9 to 4.13 if care is taken to observe that in the “*gas-liquid*” case high values of X_{TB} mean a high flow rate of the high viscosity (lower) phase, while in the liquid-liquid case high values of X_{TB} mean a high flow rate of the low viscosity (lower) phase.

4.2- Gas-liquid systems: Andritsos and Hanratty (1987)

Andritsos and Hanratty (1987) proposed a method for evaluating the interfacial shear stress by assuming that:

$$\tau_i = f_i \rho_G \frac{u_G^2}{2} \quad (4.65)$$

and by correlating the ratio of the interfacial friction factor to the gas friction factor on the wall (f_i/f_G). The correlation was based on the experimental data of air-water and air-oil flows, with liquid viscosity varying from 1 to 80 mPa s conducted in horizontal pipes with internal diameters of 25 and 95 mm. Andritsos and Hanratty observed the ratio (f_i/f_G) was almost unity when there were no waves at the interface but showed a sudden increase with the appearance of large-amplitude waves. Using the approximate theory of the inviscid, irrotational gas flow over an inviscid, irrotational stationary deep liquid, the following correlation was obtained:

$$\frac{f_i}{f_G} = 1 \text{ for } U_G \leq U_{G,t} \quad (4.66)$$

and

$$\frac{f_i}{f_G} = 1 + 15 \left(\frac{U_G}{U_{G,t}} - 1 \right) \sqrt{\frac{h_L}{D}} \text{ for } U_G > U_{G,t} \quad (4.67)$$

where $U_{G,t}$ is the gas superficial velocity at which large amplitude waves appear:

$$U_{G,t} = 5 \sqrt{\frac{\rho_{G0}}{\rho_G}} \text{ in m/s} \quad (4.68)$$

where ρ_{G0} is the gas density at atmospheric pressure and ρ_G the gas density under the conditions prevailing in the actual case.

Substitution of Equation (4.67) with (4.68) in Equations (4.36) and (4.46) provide a solution for the Andritsos and Hanratty model (substitution of Equation (4.66) instead of (4.67) in the general equations yields in an identical solution to that presented in section 4.1.2). Andritsos and Hanratty have also proposed that the gas wall shear stress should be calculated by an equation similar to that of Taitel and Dukler (1976a, b) but the liquid-wall shear stress to be calculated by a modified Cheremisinoff and Davies (1979) correlation. Jayanti (1991) suggests, however, the liquid wall shear stress be calculated by an explicit form of Colebrook-White's formula. Since, for smooth pipes the Colebrook-White's formula gives results close to Blasius Equation, and since it would be interesting to compare the effect of the interface shear stress calculated by the Andritsos and

Hanratty correlation to those with the arbitrary interfacial shear stresses already calculated, the dimensionless gas and liquid shear stresses were evaluated as indicated by Equations (4.52) to (4.57) of the previous section.

The dimensionless liquid level (h_L/D) and the pressure gradient multiplier ϕ_L , as calculated by the method of Andritsos and Hanratty, are shown in Figures 4.14 and 4.15. In both of these figures, the gas superficial velocity was used as the parameter to generate the curves illustrated because of the direct interdependency it holds with the interfacial friction factor — Equation (4.67). The variation of the interfacial and gas wall friction factors with the Martinelli parameter, X is shown in Figure 4.16 for various gas superficial velocities. It can be observed that for lower gas superficial velocities this friction factor ratio is almost independent of X , though different from unity, which is the case when no waves are present at the interface. At higher superficial gas velocities the mentioned friction factor ratio varies considerably with the intermediate values of X . At high and low X values this variation is not very pronounced as can be verified from Figure 4.14. In other words it could be said that for high superficial velocities, Figure 4.16, the curve of (f_i/f_G) seems to mimic the h_L/D versus X relationship (Figure 4.14).

4.3- Interpretation of the Martinelli and Nelson (1948) correlation

In 1948, Martinelli and Nelson observed that there were systematic deviations from the correlation of Lockhart and Martinelli as the pressure increased. The graph of these variations is shown in Figure 4.17. One limit represents the

correlation of Lockhart and Martinelli for a low pressure system and the other represents what would happen with the two fluids of identical (critical) properties (i.e. at the critical point). The curve for the critical point was obtained by the following argument.

Firstly, the two-phase pressure gradient was described by means of a pressure gradient multiplier as in Equation (4.46) giving:

$$\left(\frac{dp}{dz}\right)_l = \phi_L^2 \left(\frac{dp}{dz}\right)_{L,l} \quad (4.69)$$

where the subscript "l" in the pressure gradient terms denotes some point along an evaporator tube, which was fed at its beginning with a 100% liquid mass input $\dot{M}_{L,0}$.

We may define the quality of a liquid-vapour mixture as:

$$x = \frac{\dot{M}_G}{\dot{M}_G + \dot{M}_L} \quad (4.70)$$

where $\dot{M}_G$ and $\dot{M}_L$ are, respectively, the mass flow rate of the vapour (liquid that has evaporated) and mass flow rate of the liquid at any particular point of the evaporator tube.

It also follows that for any point along the tube, the pressure gradient for the liquid phase flowing alone in the channel can be related to the quality at this

particular point and the pressure gradient corresponding to the total (liquid) mass input at the entrance of the tube by:

$$\frac{\left(\frac{dp}{dz}\right)_{L,l}}{\left(\frac{dp}{dz}\right)_{L,0}} = \frac{C\left(\frac{\rho_L U_L D}{\eta_L}\right)^{-m} \frac{\rho_L U_L^2}{D 2}}{C\left(\frac{\rho_L u_{L,0} D}{\eta_L}\right)^{-m} \frac{\rho_L u_{L,0}^2}{D 2}} = \frac{U_L^{2-m}}{u_{L,0}^{2-m}} = (1-x)^{2-m} \quad (4.71)$$

or

$$\left(\frac{dp}{dz}\right)_{L,l} = \left(\frac{dp}{dz}\right)_{L,0} (1-x)^{2-m} \quad (4.72)$$

where $(dp/dz)_{L,0}$ is the pressure gradient at the entrance of the evaporator, when no liquid had yet evaporated.

At the critical point, however, since there is no distinction between phases, it can be written:

$$\left(\frac{dp}{dz}\right)_l = \left(\frac{dp}{dz}\right)_{L,0} \quad (4.73)$$

Combining Equations (4.69), (4.72) and (4.73), the value of the pressure gradient multiplier at the critical point is:

$$\phi_L = \frac{1}{(1-x)^{1-m/2}} \quad (4.74)$$

Moreover, the Martinelli parameter, Equation (4.23), at the critical point becomes

$$X^2 = \frac{\left(\frac{dp}{dz}\right)_{L,c}}{\left(\frac{dp}{dz}\right)_{G,c}} = \frac{C \left(\frac{\rho_c U_{L,c} D}{\eta_c}\right)^{-m} \frac{\rho_c}{D} \frac{U_{L,c}^2}{2}}{C \left(\frac{\rho_c U_{G,c} D}{\eta_c}\right)^{-m} \frac{\rho_c}{D} \frac{U_{G,c}^2}{2}} = \frac{U_{L,c}^{2-m}}{U_{G,c}^{2-m}} = \left(\frac{1}{x} - 1\right)^{2-m} \quad (4.75)$$

Combining Equations (4.74) and (4.75) gives the relation between ϕ_L and X at the critical point:

$$\phi_L = \frac{\left(X^{2/(2-m)} + 1\right)^{1-m/2}}{X} \quad (4.76)$$

It is interesting to compare the Martinelli and Nelson limit curve with that arising from the one-dimensional analysis given in section 4.1.2 above for the assumption that $\tilde{\tau}_i/\tilde{\tau}_G = 1$. The comparison is shown in Figure 4.18. The one-dimensional model applies to any ratio of physical properties but it contains the assumption of a shear-stress along the interface which is constant and equal to the wall shear stress of the less viscous phase, assuming the friction fraction ratio is unity. This would not give the correct limit for the case when the physical properties of the two phases are exactly identical. Consider for instance the case with an arbitrary plane interface as shown in Figure 4.19; if the two fluids had exactly identical properties and, the shear stress in the direction perpendicular to the interface would be given as:

$$\tau_{yz} = \tau_{rz} \cos \theta \quad (4.77)$$

however

$$\tau_{rz} = \frac{\tau_w}{R} r \quad (4.78)$$

and

$$r = \frac{(R-h)}{\cos \theta} \quad (4.79)$$

Therefore substituting Equations (4.78) and (4.79) in (4.77) the interfacial shear stress is evaluated as:

$$\tau_{yz} = \tau_w \frac{(R-h)}{R} \quad (4.80)$$

which is a constant value smaller than the wall shear stress and equal to τ_{rz} at the point at the middle of the interface line.

However, the basic assumption of the one dimensional model is that the shear stress has a constant value across the interface, given by the wall shear stress calculated for the specific zone. Thus, there is an important difference between the limiting case implicit from the one dimensional model and that calculated by the above analysis by Martinelli and Nelson. However, as it was seen in Figure 4.18, the differences when plotted in terms of the friction multiplier against the Martinelli parameter are relatively small (though significant).

It is interesting to consider, in terms of the present analysis, the implications of the Martinelli-Nelson correlation. Between the limiting (critical pressure) case represented by Equation (4.75) and the atmospheric pressure case represented by the Lockhart-Martinelli correlation, Martinelli and Nelson plotted (for steam-

water mixtures) a series of lines for given pressures and these are shown in Figure 4.17. Using the analysis reported above, these lines can be reinterpreted in term of the ratio τ_i/τ_G as shown in Figure 4.20. For the Lockhart-Martinelli (atmospheric condition) the ratio increases rapidly with X as shown. As the pressure increases, the ratio falls though all the curves approach a limiting line for the critical condition as X increases.

4.4- Comparison with experiments

4.4.1- Computer implementation of the model

The model presented in Section 4.1.2 was compared with the data obtained in the oil water experiments presented in Chapter 3.

In order to perform these comparisons a computer program was written. This program firstly transforms the actual flow and fluid properties (the input data) into the generalised Martinelli parameter X_{TB} and then solves Equation (4.63) for the geometrical configuration of the flow (basically to find h/D) and Equation (4.64) for ϕ_T . Finally, it transforms the dimensionless quantities calculated into their dimensional counterparts for comparison with the results from the experiments. It is important to remark that the solution of Equation (4.63), an implicit equation in the interface position (h/D), required an iterative process to be performed. For that purpose the secant method was used, which is an adaptation of Newton's method for finding the zero of functions in which the derivatives are approximated by the secant crossing the points of the present and previous iteration. This method is particularly useful when the derivative of the

function of which the zero is to be found cannot be differentiated or when its derivative is rather awkward.

The final closure relationship necessary for the complete solution of Equation (4.63) is a model for the interface shear stress. In the solutions presented in the following sections it was simply set to the same value as the water wall shear stress (the less viscous phase) in an analogous way of the approach of Taitel and Dukler (1976a, b) for gas-liquid flow. In turn, the wall shear stress was modelled using $m = n = 0.2$ (turbulent flow for both phases) in Equations (4.52) and (4.53), unless otherwise stated (Section 4.4.4).

4.4.2-TOWER results

The results from the TOWER experiment are presented in Table 4.1 and are compared with the one-dimensional model in Figure 4.21. The following main observations arise from these comparisons:

- (1) The data fell mainly within $\pm 25\%$ of the model, despite the simplifying assumptions made in the model.
- (2) The model tends to *overpredict* the data for the transpalite tube without mixer and gives better agreement with the stainless steel tube. The *overprediction* is surprising in view of the assumption (in the model) of a flow interface between the phases.
- (3) When a mixer is used at the tube inlet, the pressure gradient is significantly increased relative to the situation without a mixer. This may be a result of the flow taking some time to settle to its

equilibrium configuration following the initial disturbance. The presence of swirl within the flow after mixer may also be affecting the pressure drop measurement.

(4) At the higher values of flow rates significant departures may occur from idealised stratified flow as a result of partial inter-entrainment of the phases. Angeli (1996) has shown that, for a fully dispersed flow, the pressure gradient can, over a range of phase fractions, be lower than that for either phases flowing at the same velocity. The present results may be showing some manifestation of the same “*drag reduction*” effect.

There is a need for further investigation of the mixing and drag reduction effects and such work is currently being pursued at Imperial College.

4.4.3- WASP results

Comparisons of the pressure gradient data obtained in WASP horizontal flow experiments data with the one-dimensional model solution are illustrated in Table 4.2 and Figure 4.22. It is seen in Figure 4.22 that the model *underpredicts* the pressure gradients measured in the experiments by 50% or more. This trend is opposite to that observed in the TOWER facility. The principal difference between the WASP and TOWER experiments is the large difference in the viscosities of the oil phase (1.6 mPa s in TOWER and 45 mPa s in WASP, both at a temperature of 25°C) This implies that, for given *superficial* phase velocities, the holdup of the oil would be greater in WASP than in TOWER. Consequently the water would be travelling at a higher *actual* velocity and this would possibly

give rise to a more than commensurate increase in interfacial friction due to the creation of more intense wave activity at the interface. The fact that significant interface mixing is occurring in many of the tests could be a further manifestation of this effect. The viscosity of the mixed layer may also be actually higher than that of the oil phase (Schramm, 1992), further exacerbating these effects.

The one-dimensional model used for the pressure-drop predictions above was also used to predict the in-situ water-fraction and a comparison was made with the experimental results obtained by the gamma ray densitometer. These comparisons are shown in Table 4.4 and Figure 4.23. As will be seen, the model predicts water fractions which are, on average, *lower* than the measured values. This result is not inconsistent with the pressure drop data shown in Figure 4.22. Thus an increase in interfacial friction would lead to an increased pressure gradient and also to an increase in oil velocity, and a decrease in oil fraction, and by difference, to an increase in water fraction.

4.4.4- Interface level prediction

As was described in section 3.1.1.6, the interface level in the TOWER experiments was measured with the aid of the high frequency probe.

Figure 4.24 compares the dimensionless interface level ($\tilde{h} = h/D$) calculated from the theoretical model with the interface level measured in the experiments. The theoretical curves were calculated for *viscous viscous* and *turbulent turbulent* cases using Equations (4.62), (4.51) and (4.52) with $m = 1$ for the viscous cases and $m = 0.2$ for the turbulent cases. Good agreement is observed between the experimental points and the turbulent curve; the actual flows were in fact in the

turbulent region. However, the discrepancy in some points might be attributed to an improper interpretation, and subsequent interpolation, of the effect of the waviness of the actual interface in the measured signal. A better post-processing of this signal should be attempted in future measurements. Another possible explanation of these discrepancies could be related to the assumption in the model that sets the value of interface shear stress equal to that of the water wetted wall. Clearly, the presence of interfacial waves would lead to an increase in interfacial shear stress and this would influence the interfacial level. However, the effect is not as significant as it is in gas-liquid flow or as in the case of liquid-liquid flow with a high viscosity ratio.

4.5- Flow pattern modelling

4.5.1- Extension of Taitel and Dukler's (1976a) stratified/non-stratified flow transition to liquid-liquid flows

The model of stratified flow / non-stratified flow transition introduced by Taitel and Dukler (1976a) for gas-liquid flows has been extended to liquid-liquid flows.

The main alteration to the original methodology was the evaluation of the interfacial shear stress in the solution procedure used to determine the dimensionless equilibrium-height level in stratified flow. Similarly to the previous section (4.5), the interface shear stress was set equal to the less-viscous phase wall shear stress, which is consistent with what was done by Taitel and Dukler for gas-liquid systems. However, for gas-liquid systems the top phase (gas) is the less viscous one while the inverse is true for liquid-liquid (oil-water) systems. For the

present analysis Equations (4.63), (4.52) and (4.53) were solved coupled with the Taitel and Dukler (1976a) stratified flow stability criterion:

$$u_T \leq C_2 \sqrt{\frac{g(\rho_B - \rho_T)h_T}{\rho_T}} \quad (4.81)$$

where C_2 is a correction factor introduced by Taitel and Dukler, given by:

$$C_2 = 1 - \frac{h}{D} \quad (4.82)$$

The values of n and m were set to their turbulent flow value of 0.2.

It can be seen from Figures 4.25 to 4.28 that the stratified flow boundaries predicted by the modified Taitel and Dukler (1976a) model do not agree with the experimental results from any of the conditions investigated on either the WASP or TOWER facilities. This model seems to *underpredict* the area of occurrence of the stratified flow regime, which apparently exists at much higher velocities than those calculated by the model. However, it must be noticed that no provision is given in the model for the inclusion of the *stratified with dispersion* flow regimes, which by itself might require a description of a transition from the oil-water *stratified* flow pattern.

4.5.2- Adaptation of Wicks and Fraser (1975) model for the stratified flow boundary

Wicks and Fraser (1975) developed a model to determine a maximum critical flow velocity above which a water layer (i.e. stratified flow) may not exist.

This method compares the drop diameter and the velocity as determined by Hinze's critical Weber number, for given fluids —Equation (4.83)— with the maximum required velocity for a given pipe system for the existence of a water bed —Equations (4.84) to (4.88). The solution to this problem is that drop diameter and flow velocity that satisfy all these conditions.

a) Hinze's equation

$$U = \sqrt{\frac{22\sigma}{\rho_o d}} \quad (4.83)$$

where U is the total superficial velocity, σ the interfacial tension, ρ_o the oil density and d the maximum drop diameter.

b) Oil Pseudo-Reynolds number

$$\text{Re}_o = \frac{DU\rho_o}{\eta_o} \quad (4.84)$$

where D is the tube diameter and η_o the oil viscosity.

c) *Dimensionless parameter s (empirical parameter related to the condition of a zero thickness water layer)*

$$s = \text{Re}_o \left(\frac{d}{D} \right)^{\frac{2}{3}} \quad (4.85)$$

d) *The empirical correlations for the maximum velocity for which a layer of water may exist.*

d1) *If $s \leq 40$:*

$$U = \frac{Dd}{10\eta_o} (\rho_w - \rho_o) g \quad (4.86)$$

d2) *If $s \geq 400$:*

$$U = \sqrt[5]{\frac{10^4 (\rho_w - \rho_o)^2 g^2 \eta_o}{\rho_o^3}} \quad (4.87)$$

d3) *If $40 \leq s \leq 400$:*

$$U = \sqrt[4]{\frac{(\rho_w - \rho_o) g \eta_o^2 (52.5S + 4300)}{\rho_o^3 d}} \quad (4.88)$$

The flowchart of Figure 4.29 shows the Wicks and Fraser method as implemented in a computer program in the present work.

In Figures 4.25 to 4.28, which show the comparison between the experimental data and the stratified flow transition predicted by the modified Taitel and Dukler method, the Wicks and Fraser critical velocity line is also included. It is observed that there is a general agreement between the Taitel and Dukler and Wicks and Fraser modified methods, and consequently, as commented in section 4.5.1, both *underpredict* the actual transition boundary obtained from the experimental data.

Case -	Oil (l/s)	Water (l/s)	dp/dz(T) (mbar/m)	dp/dz(TMix) (mbar/m)	dp/dz(SS) (mbar/m)	dp/dz-Model (mbar/m)	Error - T	Error-TMix	Error - SS
1	0.20	0.20	3.71	—	6.01	4.246	14.60%	—	-29.33%
2	0.20	0.40	8.00	—	10.19	9.022	12.84%	—	-11.45%
3	0.20	0.60	13.46	—	15.73	15.47	14.97%	—	-1.63%
4	0.20	0.80	19.85	—	24.92	23.61	18.93%	—	-5.26%
5	0.20	1.00	28.12	—	36.62	33.43	18.88%	—	-8.72%
6	0.20	1.20	38.88	—	50.94	44.95	15.60%	—	-11.76%
7	0.40	0.20	6.59	—	8.52	8.713	32.19%	—	2.31%
8	0.40	0.40	11.90	—	17.29	15.21	27.86%	—	-12.05%
9	0.40	0.60	18.68	—	24.19	23.36	25.04%	—	-3.43%
10	0.40	0.80	25.62	—	35.16	33.12	29.25%	—	-5.81%
11	0.40	1.00	36.15	—	49.58	44.53	23.17%	—	-10.19%
12	0.40	1.20	48.17	—	63.17	57.61	19.60%	—	-8.80%
13	0.60	0.20	10.02	—	12.49	14.63	45.96%	—	17.16%
14	0.60	0.40	17.12	—	28.16	22.78	33.05%	—	-19.11%
15	0.60	0.60	25.08	—	36.42	32.63	30.11%	—	-10.40%
16	0.60	0.80	34.67	—	49.37	44.12	27.25%	—	-10.64%
17	0.60	1.00	45.59	—	65.67	57.19	25.44%	—	-12.92%
18	0.60	1.20	59.79	—	—	71.9	20.26%	—	—
19	0.80	0.20	13.14	—	20.43	21.99	67.31%	—	7.64%
20	0.80	0.40	21.02	—	26.70	31.75	51.03%	—	18.92%
21	0.80	0.60	32.88	—	50.42	43.25	31.55%	—	-14.22%
22	0.80	0.80	45.05	—	65.26	56.43	25.27%	—	-13.53%
23	0.80	1.00	58.62	—	66.30	71.25	21.55%	—	7.46%
24	0.80	1.20	73.05	—	—	87.66	20.00%	—	—
25	1.00	0.20	20.63	—	26.18	30.78	49.19%	—	17.59%
26	1.00	0.40	25.08	—	32.65	42.14	68.04%	—	29.05%
27	1.00	0.60	45.36	—	66.72	55.27	21.85%	—	-17.16%
28	1.00	0.80	62.52	—	84.80	70.11	12.14%	—	-17.32%
29	1.00	1.00	75.78	—	—	86.62	14.30%	—	—
30	1.00	1.20	—	—	—	104.8	—	—	—
31	1.20	0.20	27.03	—	36.21	41	51.69%	—	13.24%
32	1.20	0.40	30.54	—	42.16	53.94	76.63%	—	27.93%
33	1.20	0.60	41.07	—	55.64	68.7	67.28%	—	23.47%
34	1.20	0.80	79.68	—	—	85.18	6.90%	—	—
35	1.20	1.00	—	—	—	103.3	—	—	—
36	1.20	1.20	—	—	—	123.2	—	—	—
37	0.05	0.05	0.35	0.51	0.89	0.369	5.12%	-27.22%	-58.45%
38	0.05	0.10	0.82	1.13	1.52	0.745	-9.04%	-34.13%	-50.83%
39	0.05	0.15	1.60	2.54	2.14	1.24	-22.45%	-51.09%	-42.11%
40	0.05	0.20	2.15	3.24	3.19	1.84	-14.22%	-43.16%	-42.27%

Table 4.1: Comparison of horizontal oil-water flow experiments with the one-dimensional model .(T= *transpalite* ; T.w/Mix= *transpalite* with mixer, SS= *stainless steel*)

Case -	Oil (l/s)	Water (l/s)	dp/dz(T) (mbar/m)	dp/dz(TMix) (mbar/m)	dp/dz(SS) (mbar/m)	dp/dz-Model (mbar/m)	Error - T	Error-TMix	Error - SS
41	0.05	0.25	2.85	3.78	4.34	2.56	-10.08%	-32.33%	-40.97%
42	0.10	0.05	0.74	1.13	1.20	0.732	-1.22%	-35.28%	-39.08%
43	0.10	0.10	1.29	2.54	2.04	1.23	-4.43%	-51.48%	-39.64%
44	0.10	0.15	1.91	4.02	3.19	1.83	-4.24%	-54.45%	-42.58%
45	0.10	0.20	2.61	4.72	4.34	2.55	-2.41%	-45.97%	-41.20%
46	0.10	0.25	3.39	5.42	5.80	3.37	-0.68%	-37.84%	-41.89%
47	0.15	0.05	1.21	1.60	1.72	1.2	-0.75%	-24.96%	-30.40%
48	0.15	0.10	1.83	3.55	2.66	1.81	-1.26%	-49.00%	-32.07%
49	0.15	0.15	2.46	5.34	3.92	2.53	2.97%	-52.65%	-35.43%
50	0.15	0.20	3.24	5.97	5.90	3.35	3.49%	-43.86%	-43.26%
51	0.15	0.25	3.94	6.51	7.26	4.28	8.65%	-34.29%	-41.07%
52	0.20	0.05	1.91	1.91	2.56	1.77	-7.38%	-7.38%	-30.86%
53	0.20	0.10	2.46	3.63	3.29	2.48	0.93%	-31.63%	-24.66%
54	0.20	0.15	3.08	6.59	4.65	3.31	7.43%	-49.78%	-28.82%
55	0.20	0.20	3.86	7.45	6.95	4.25	10.07%	-42.95%	-38.84%
56	0.20	0.25	4.56	7.61	9.04	5.29	15.93%	-30.44%	-41.47%
57	0.25	0.05	2.30	2.15	3.19	2.43	5.60%	13.28%	-23.75%
58	0.25	0.10	3.00	3.47	4.13	3.25	8.22%	-6.37%	-21.26%
59	0.25	0.15	3.94	6.90	5.17	4.18	6.11%	-39.45%	-19.19%
60	0.25	0.20	4.33	8.62	7.26	5.23	20.81%	-39.32%	-27.98%
61	0.25	0.25	5.42	8.93	10.08	6.38	17.69%	-28.57%	-36.73%
62	0.30	0.05	3.00	2.85	—	3.2	6.56%	12.39%	—
63	0.30	0.10	3.71	3.71	—	4.11	10.93%	10.93%	—
64	0.30	0.15	4.56	6.12	—	5.15	12.86%	-15.89%	—
65	0.30	0.20	5.11	9.87	—	6.3	23.31%	-36.15%	—
66	0.30	0.25	6.20	10.41	—	7.56	21.91%	-27.40%	—
67	0.20	0.05	1.56	—	2.00	1.767	13.26%	—	-11.46%
68	0.30	0.05	3.04	—	3.35	3.198	5.12%	—	-4.66%
69	0.40	0.05	4.29	—	5.24	5.013	16.85%	—	-4.24%
70	0.50	0.05	5.77	—	7.43	7.205	24.82%	—	-3.02%
71	0.60	0.05	8.19	—	10.04	9.77	19.29%	—	-2.71%
72	0.70	0.05	11.08	—	12.97	12.7	14.66%	—	-2.06%
73	0.80	0.05	14.04	—	16.00	16	13.96%	—	0.01%
74	0.20	0.10	2.18	—	2.94	2.48	13.55%	—	-15.54%
75	0.30	0.10	3.43	—	4.19	4.112	19.81%	—	-1.87%
76	0.40	0.10	5.15	—	6.18	6.119	18.86%	—	-0.92%
77	0.50	0.10	6.79	—	8.16	8.498	25.22%	—	4.13%
78	0.60	0.10	8.50	—	11.09	11.24	32.20%	—	1.38%
79	0.70	0.10	10.84	—	14.22	14.36	32.44%	—	0.97%
80	0.80	0.10	14.74	—	16.94	17.83	20.94%	—	5.26%

Table 4.1: Comparison of horizontal oil-water flow experiments with the one-dimensional model .(T= *transpalite* ; T.w/Mix= *transpalite* with mixer, SS= *stainless steel*)

Case -	Oil (l/s)	Water (l/s)	dp/dz(T) (mbar/m)	dp/dz(TMix) (mbar/m)	dp/dz(SS) (mbar/m)	dp/dz-Model (mbar/m)	Error - T	Error-TMix	Error - SS
81	0.20	0.15	2.81	—	4.92	3.309	17.84%	—	-32.77%
82	0.30	0.15	4.13	—	5.65	5.149	24.55%	—	-8.92%
83	0.40	0.15	5.77	—	7.43	7.358	27.47%	—	-0.96%
84	0.50	0.15	7.72	—	9.10	9.934	28.64%	—	9.15%
85	0.60	0.15	9.67	—	11.92	12.87	33.06%	—	7.95%
86	0.70	0.15	11.70	—	15.27	16.18	38.29%	—	5.98%
87	0.80	0.15	14.27	—	18.30	19.84	38.99%	—	8.43%

Table 4.1: Comparison of horizontal oil-water flow experiments with the one-dimensional model .(T= *transpalite* ; T.w/Mix= *transpalite* with mixer, SS= *stainless steel*)

Case	Oil sup. vel. (m/s)	Water sup. vel. (m/s)	dp/dz measured (mbar/m)	Wat. Frac measured Americium	dp/dz calculated	Error dp/dz	Wat. frac. calculated	Error wat. frac.
1	0.20	0.09	2.1	—	0.319	-84.71%	0.27	—
2	0.20	0.15	2.2	0.283	0.406	-81.14%	0.36	25.80%
3	0.40	0.15	2.9	0.256	0.907	-68.73%	0.25	-0.82%
4	0.63	0.18	4.0	0.256	1.813	-54.68%	0.22	-13.13%
5	0.70	0.16	4.3	—	2.023	-52.95%	0.19	—
6	0.67	0.19	3.2	—	2.004	-37.38%	0.22	—
7	0.90	0.06	5.4	0.186	2.534	-53.07%	0.07	-62.32%
8	0.81	0.14	6.6	0.186	2.361	-64.23%	0.15	-21.94%
9	0.74	0.18	7.4	0.186	2.257	-69.50%	0.17	-10.00%
10	0.59	0.16	3.4	0.259	1.541	-54.00%	0.21	-18.65%
11	0.54	0.10	3.3	0.259	1.265	-61.67%	0.13	-48.57%
12	0.52	0.12	3.6	0.259	1.244	-64.96%	0.19	-25.14%
13	0.46	0.13	3.2	0.259	1.034	-67.69%	0.22	-16.06%
14	0.43	0.06	2.9	0.259	0.826	-71.17%	0.13	-50.62%
15	0.37	0.1	3.8	0.214	0.717	-81.14%	0.21	-2.20%
16	0.38	0.09	3.3	0.214	0.754	-76.81%	0.19	-9.35%
17	0.38	0.19	3.1	0.269	0.940	-69.98%	0.30	10.48%
18	0.39	0.16	2.7	0.269	0.901	-66.64%	0.27	-0.86%
19	1.25	0.2	8.2	0.337	4.942	-39.73%	0.14	-59.02%
20	1.1	0.18	—	0.337	3.975	—	0.16	-53.03%

Table 4.2: Comparison between horizontal WASP experiments and the one-dimensional model.

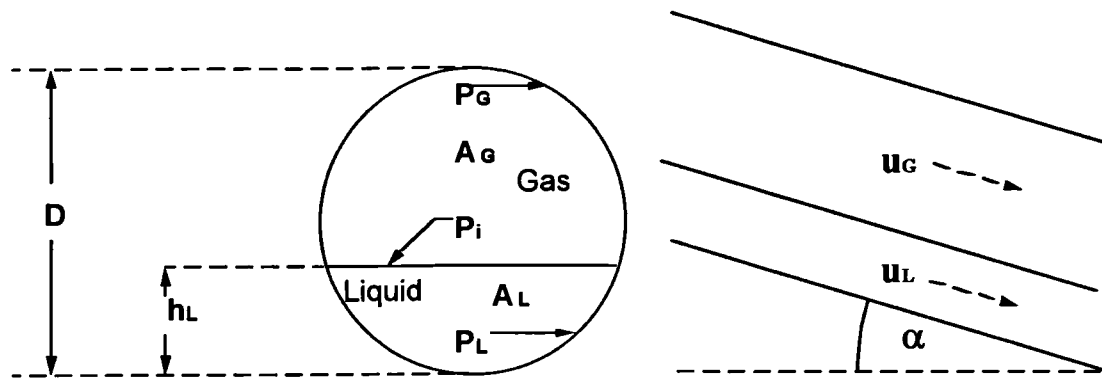


Figure 4.1: Equilibrium stratified flow

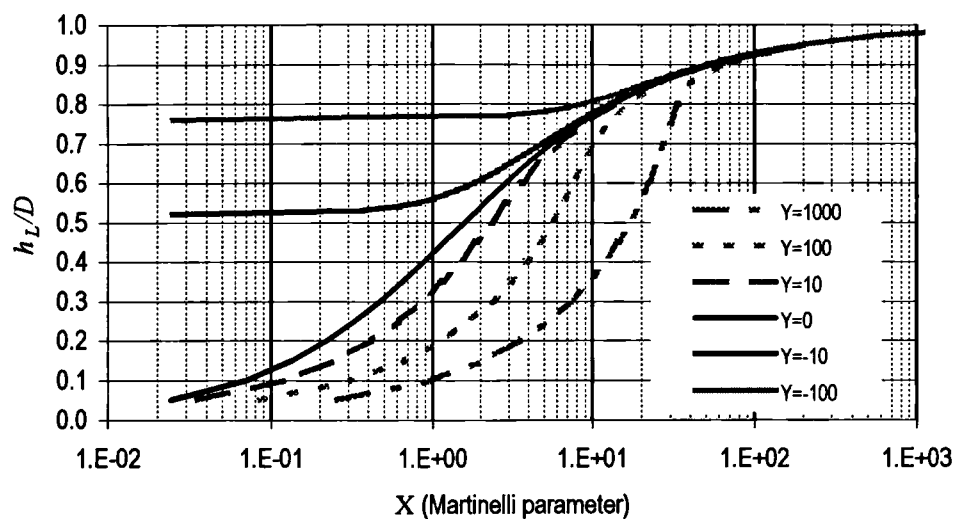


Figure 4.2: Taitel and Dukler model. Interface height (h_L/D) versus the Martinelli parameter (X) for different inclination parameters (Y)

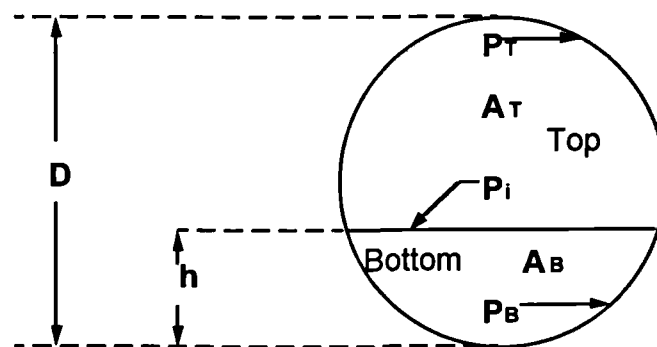


Figure 4.3: Horizontal stratified flow for two arbitrary fluids (Top, Bottom)

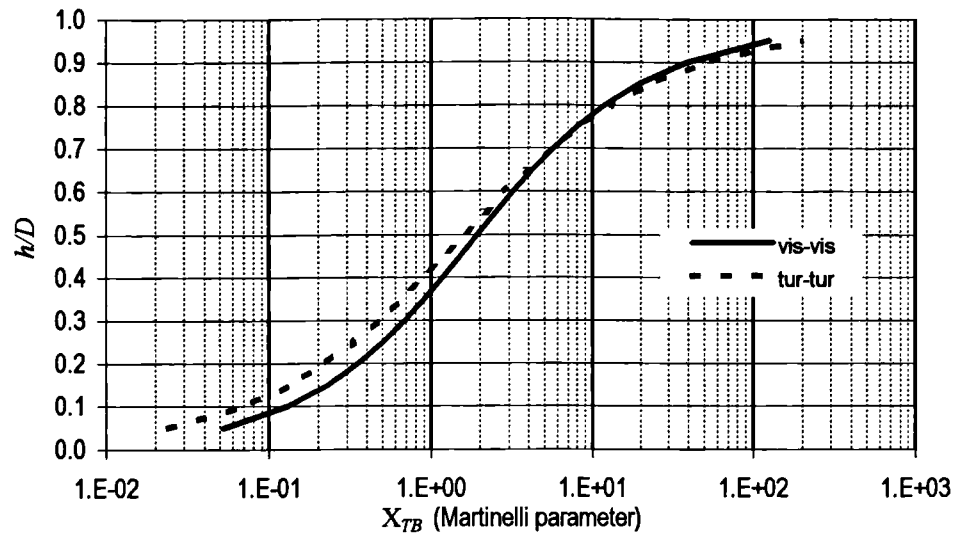


Figure 4.4: One-dimensional model. Gas-liquid case. Interface height (h_L/D) versus the generalised Martinelli parameter (X_{TB}) for laminar (vis-vis) and turbulent (tur-tur) flow of both phases.

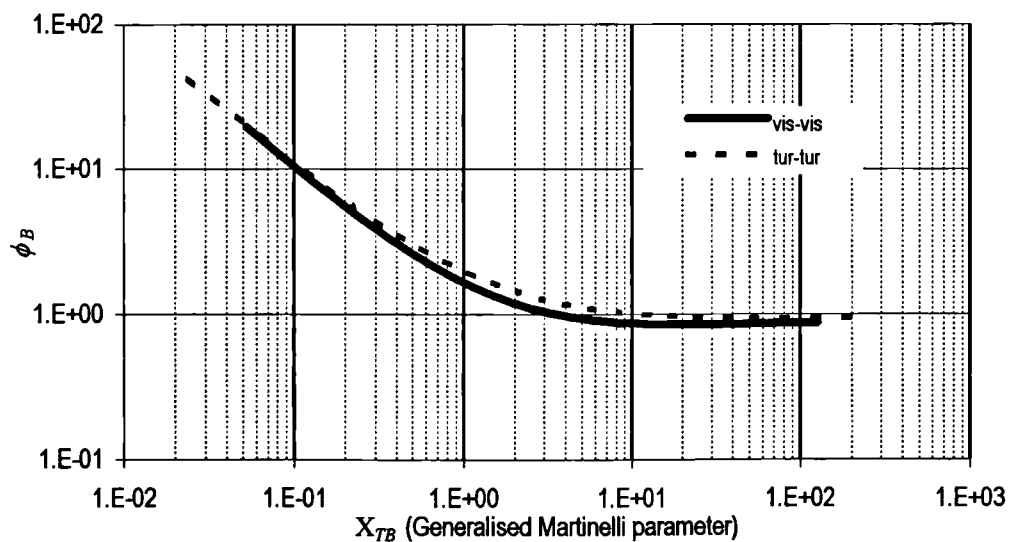


Figure 4.5: One-dimensional model. Gas-liquid case. Pressure multiplication factor (ϕ_B) versus the generalised Martinelli parameter (X_{TB}) for laminar (vis-vis) and turbulent (tur-tur) flow of both phases.

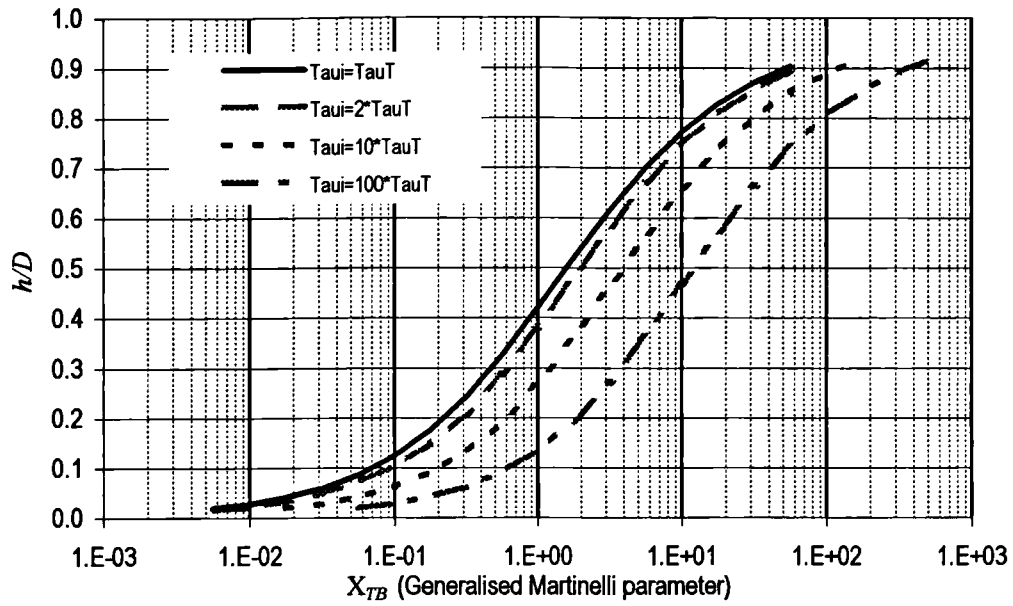


Figure 4.6: One-dimensional model. Gas-liquid case Interface height (h_L/D) versus the generalised Martinelli parameter (X_{TB}) for flow of both phases and different interfacial shear stress ($\tau_i = \tau_{wT}$; $\tau_i = 2\tau_{wT}$; $\tau_i = 10\tau_{wT}$; $\tau_i = 100\tau_{wT}$)

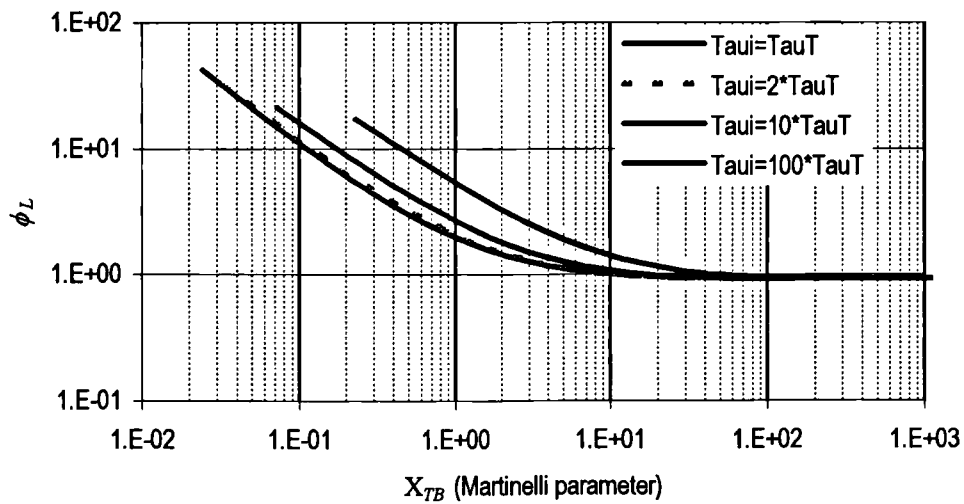


Figure 4.7: One-dimensional model. Gas-liquid case Pressure multiplication factor (ϕ_B) versus the generalised Martinelli parameter (X_{TB}) for flow of both phases and different interfacial shear stress ($\tau_i = \tau_{wT}$; $\tau_i = 2\tau_{wT}$; $\tau_i = 10\tau_{wT}$; $\tau_i = 100\tau_{wT}$)

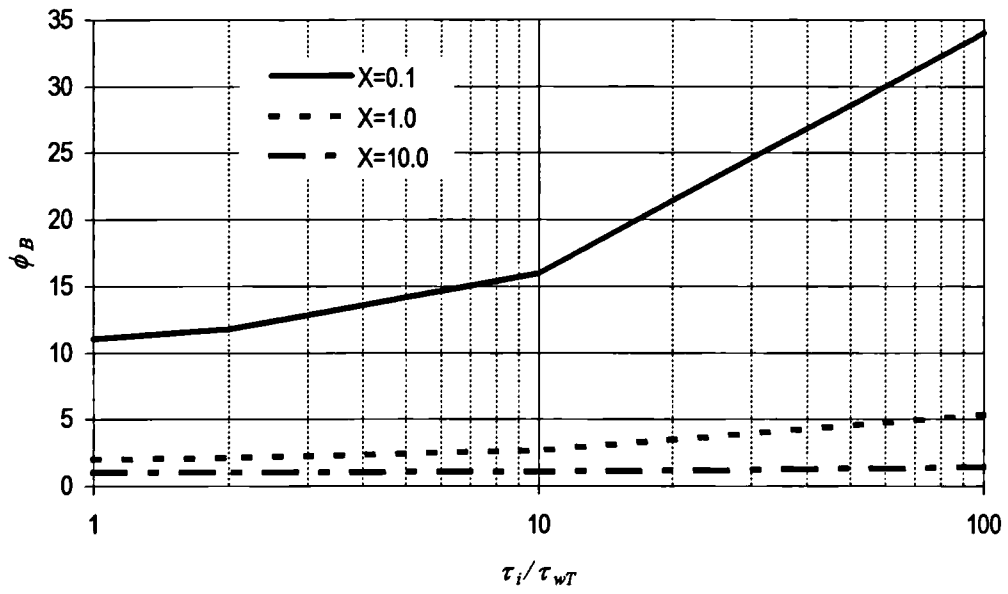


Figure 4.8: One-dimensional model. Gas-liquid case. Variation of the pressure multiplication factor (ϕ_B) with the ratio between the interfacial and wall shear stresses ($\tilde{\tau}_i / \tilde{\tau}_{wT}$).

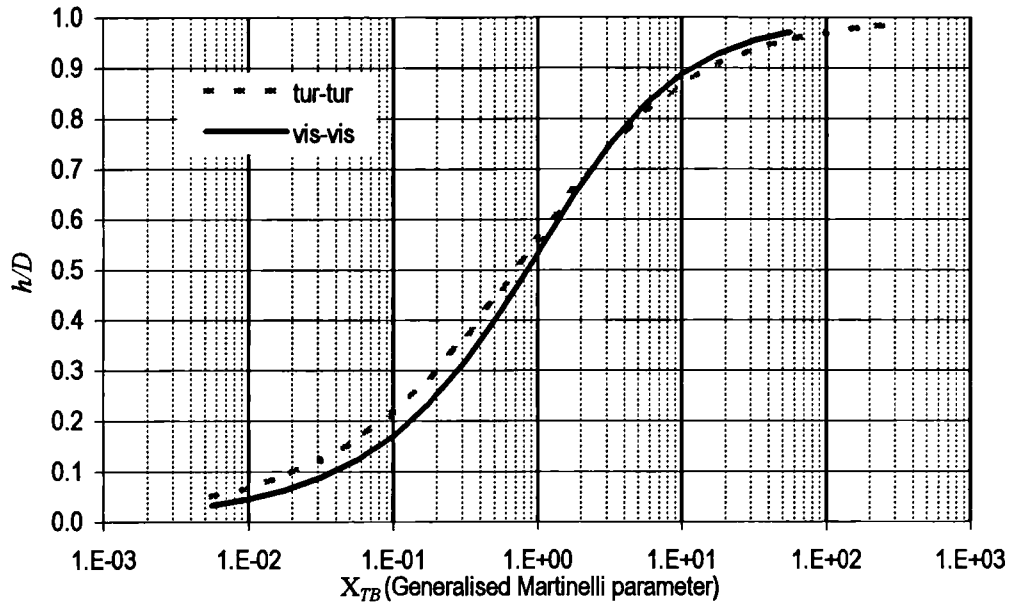


Figure 4.9: One-dimensional model. Liquid-liquid case. Interface height (h/D) versus the generalised Martinelli parameter (X_{TB}) for laminar (vis-vis) and turbulent (tur-tur) flow of both phases.

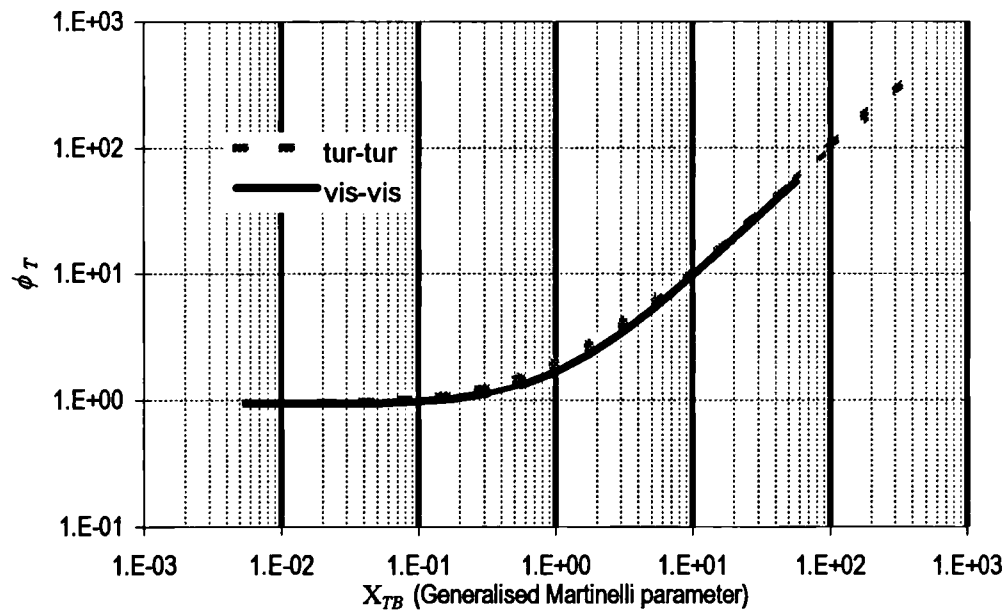


Figure 4.10: One-dimensional model. Liquid-liquid case. Pressure multiplication factor (ϕ_B) versus the generalised Martinelli parameter (X_{TB}) for laminar (vis-vis) and turbulent (tur-tur) flow of both phases.

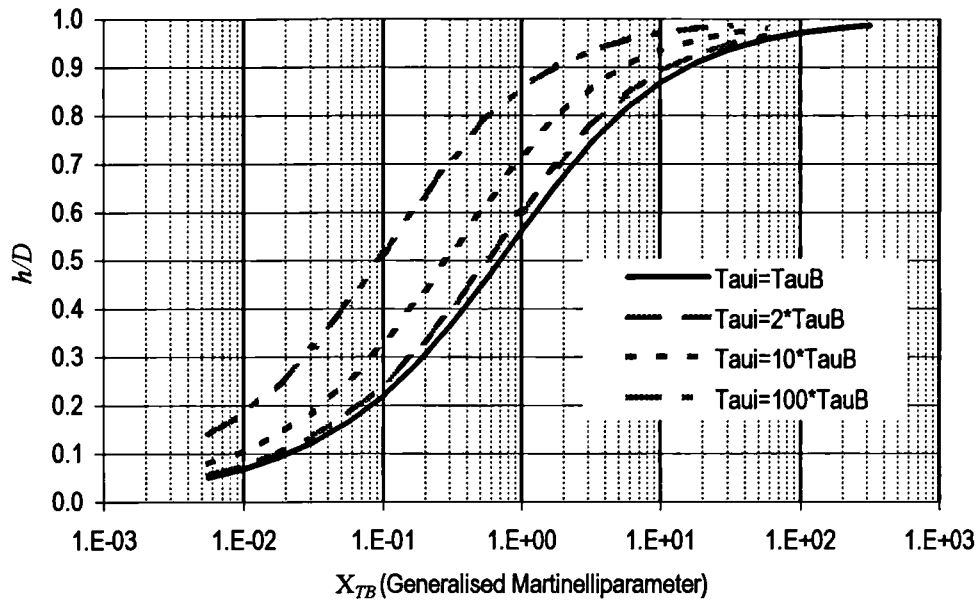


Figure 4.11: One-dimensional model. Liquid-liquid case Interface height (h/D) versus the generalised Martinelli parameter (X_{TB}) for flow of both phases and different interfacial shear stress ($\tau_i = \tau_{wB}$; $\tau_i = 2\tau_{wB}$; $\tau_i = 10\tau_{wB}$; $\tau_i = 100\tau_{wB}$)

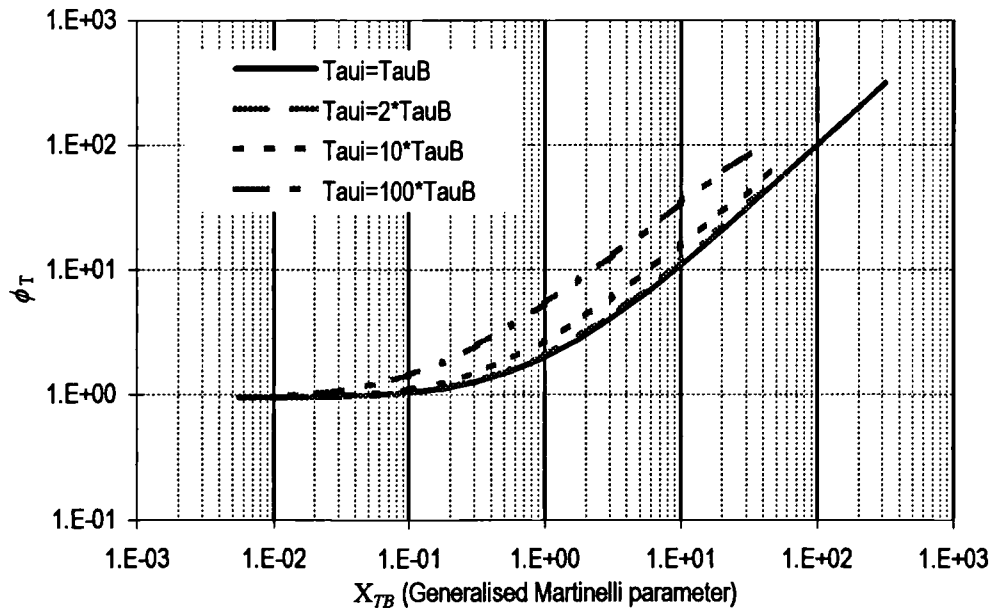


Figure 4.12: One-dimensional model. Liquid-liquid case Pressure multiplication factor (ϕ_T) versus the generalised Martinelli parameter (X_{TB}) for flow of both phases and different interfacial shear stress ($\tau_i = \tau_{wB}$; $\tau_i = 2\tau_{wB}$; $\tau_i = 10\tau_{wB}$; $\tau_i = 100\tau_{wB}$)

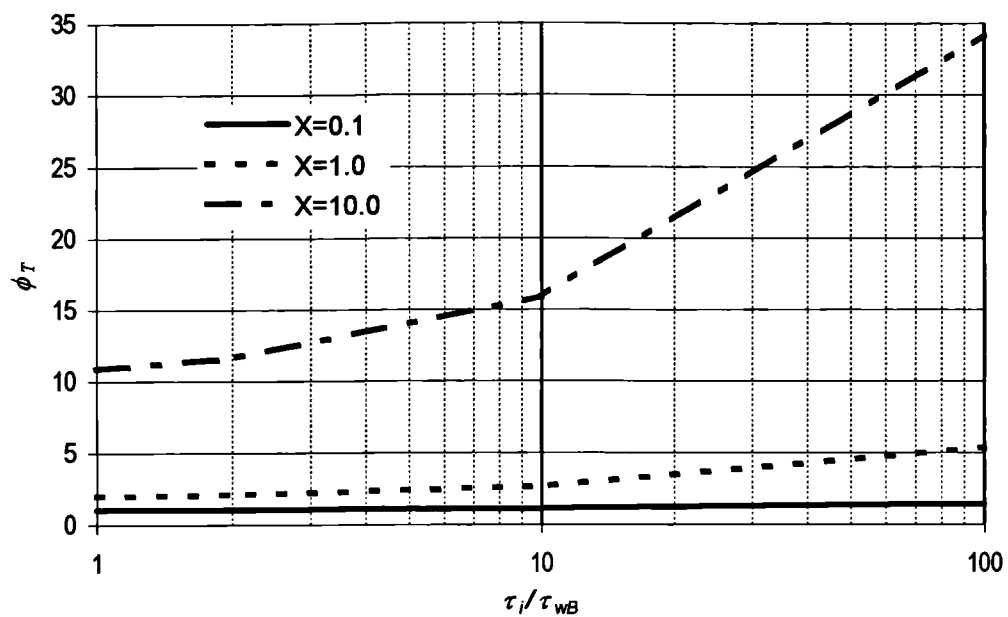


Figure 4.13: One-dimensional model. Liquid-liquid case. Variation of the pressure multiplication factor (ϕ_T) with the ratio between the interfacial and wall shear stresses ($\tilde{\tau}_i/\tilde{\tau}_{wB}$).

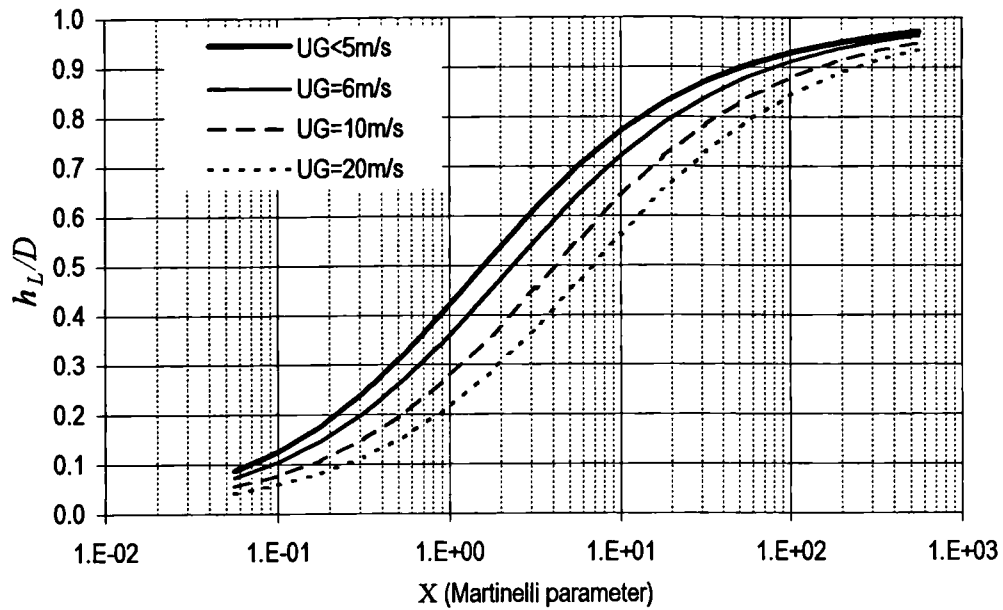


Figure 4.14: Andritsos and Hanratty (1987) model. Interface height (h_L/D) versus the Martinelli parameter (X) for different superficial gas velocities ($U_G < 5$ m/s; $U_G = 6$ m/s; $U_G = 10$ m/s; $U_G = 20$ m/s).

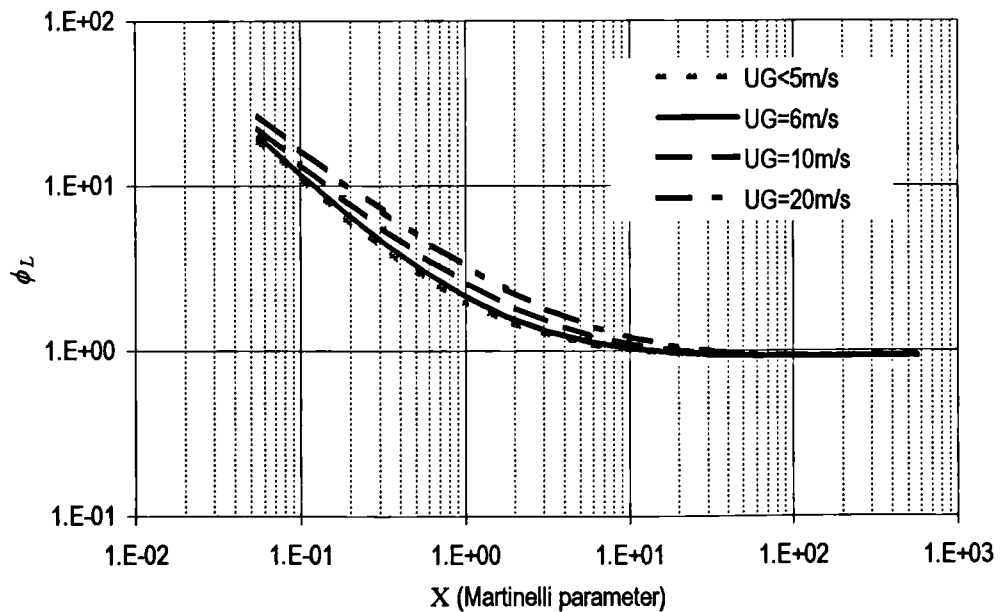


Figure 4.15: Andritsos and Hanratty (1987) model. Pressure multiplication factor (ϕ_L) versus the Martinelli parameter (X) for different superficial gas velocities ($U_G < 5$ m/s; $U_G = 6$ m/s; $U_G = 10$ m/s; $U_G = 20$ m/s).

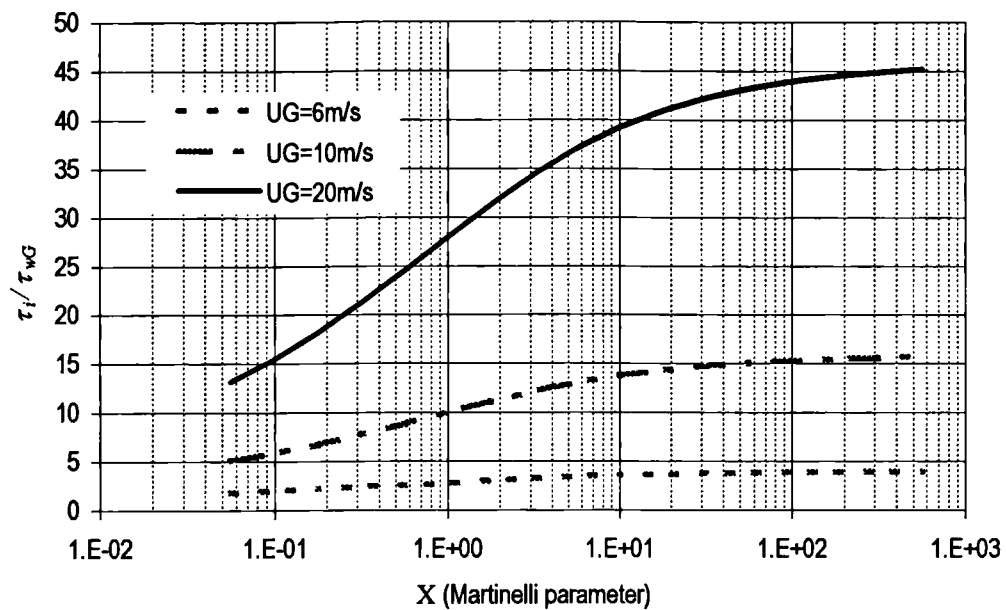


Figure 4.16: Andritsos and Hanratty (1987) model. Variation of the ratio between the interfacial and gas wall shear stresses (τ_i/τ_{wG}) with the Martinelli parameter (X).

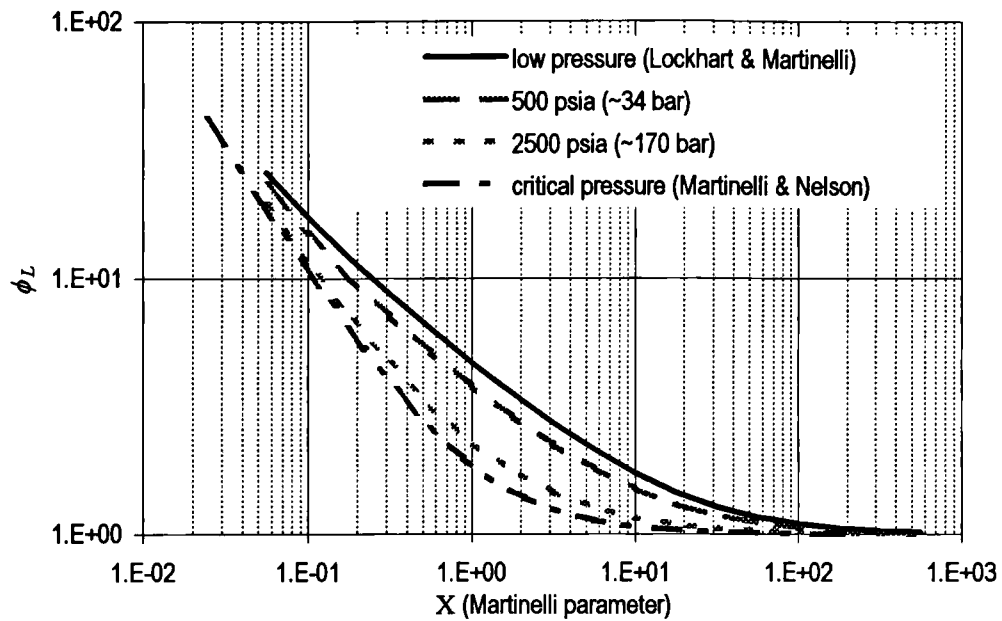


Figure 4.17: Martinelli and Nelson (1948). Pressure multiplication factor (ϕ_L) versus the Martinelli parameter (X) for various pressures for a water and steam system.

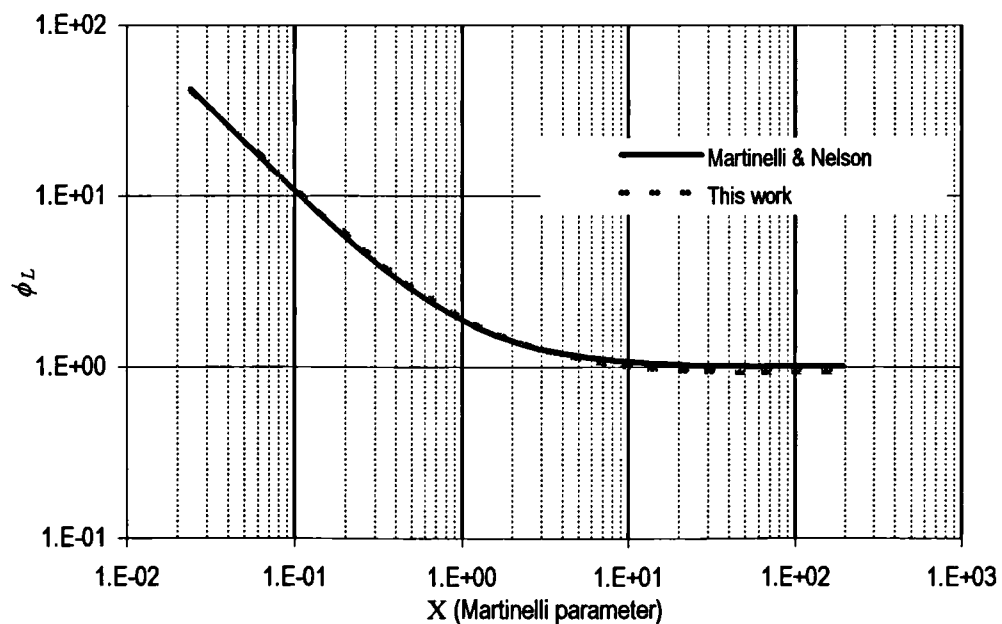


Figure 4.18: . Pressure multiplication factor (ϕ_L) versus the Martinelli parameter (X). Comparison between the critical pressure curve predicted by Martinelli and Nelson (1948) and the one-dimensional analysis developed in the present work.

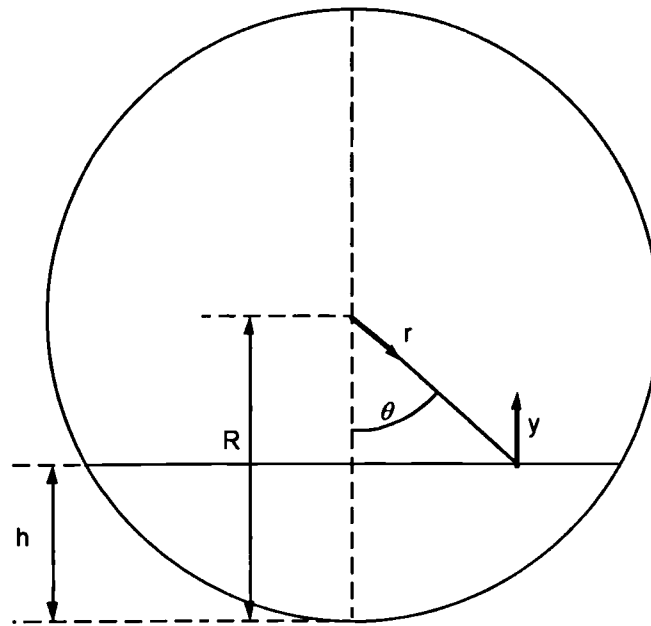


Figure 4.19: . Arbitrary horizontal “pseudo” interface

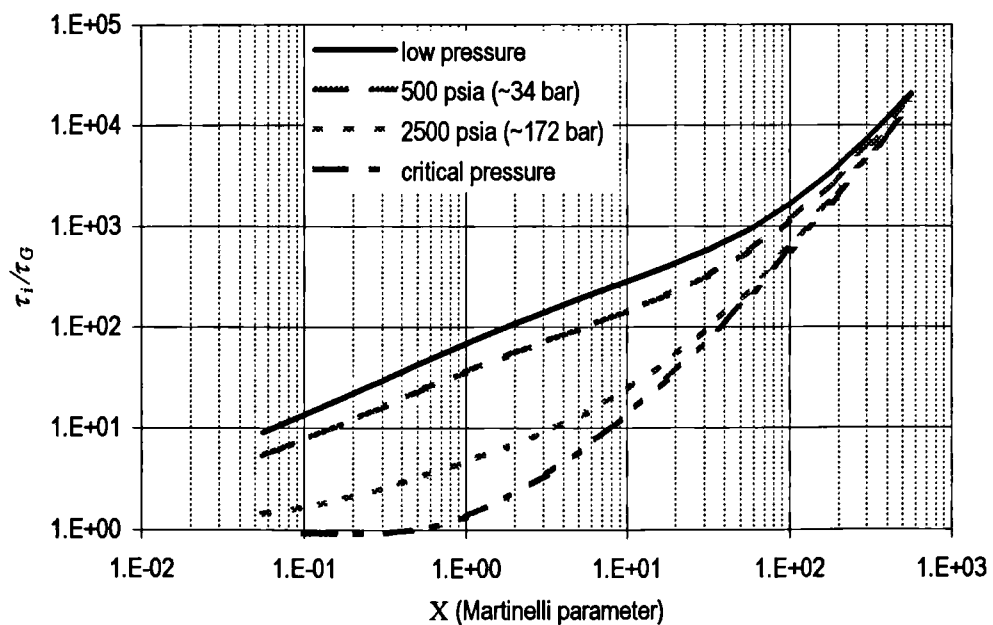


Figure 4.20: Martinelli and Nelson (1948). Variation of the ratio between the interfacial and gas (vapour) wall shear stresses (τ_i/τ_{wG}) with the Martinelli parameter (X) for various pressures for a water and steam system.

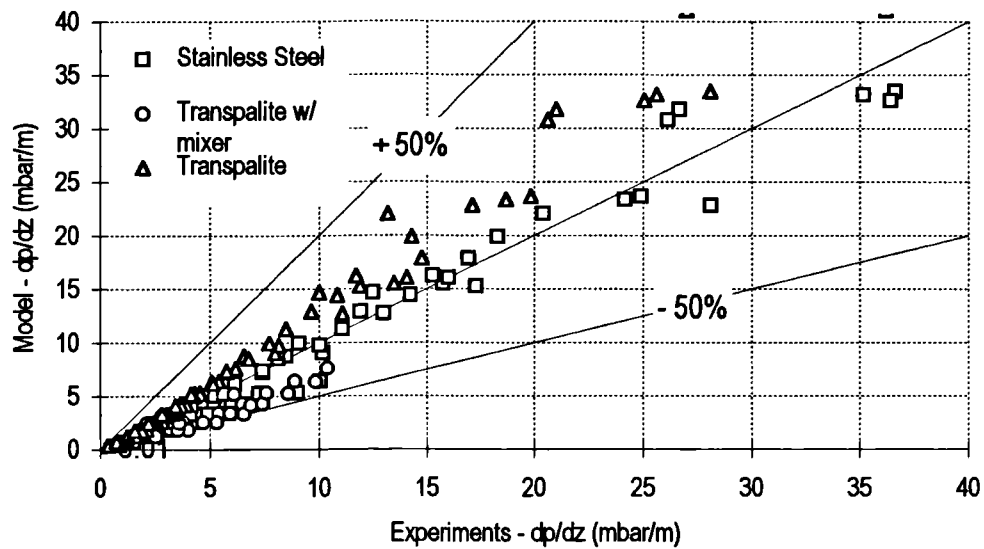


Figure 4.21: Comparison of the one dimensional model ($\tau_i = \tau_{wB}$) with the pressure gradient data obtained in TOWER oil-water flow experiments.

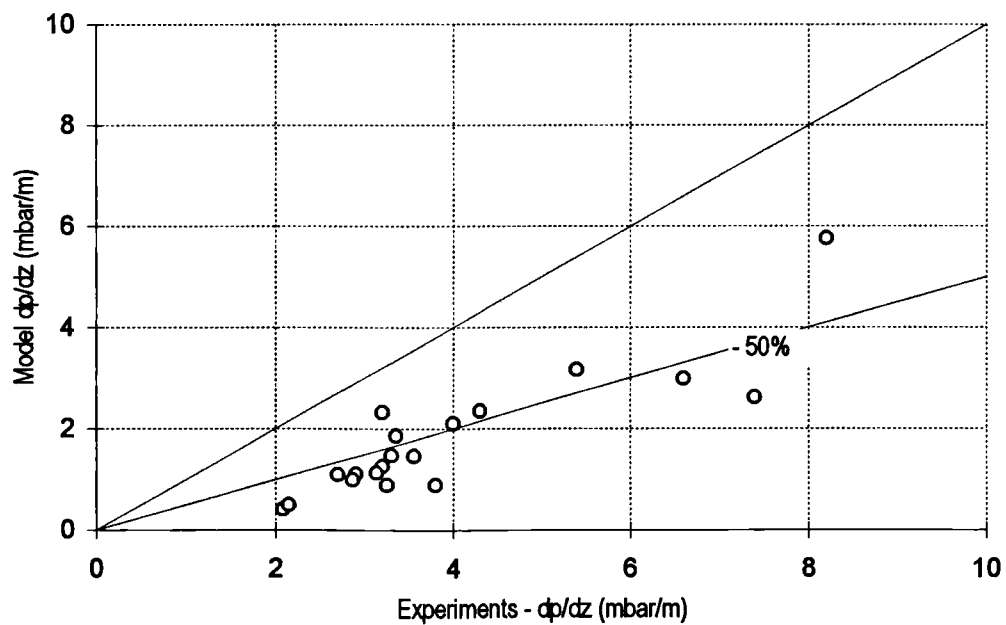


Figure 4.22: Comparison of the one dimensional model ($\tau_i = \tau_{wB}$) with the pressure gradient data obtained in WASP horizontal oil-water flow experiments.

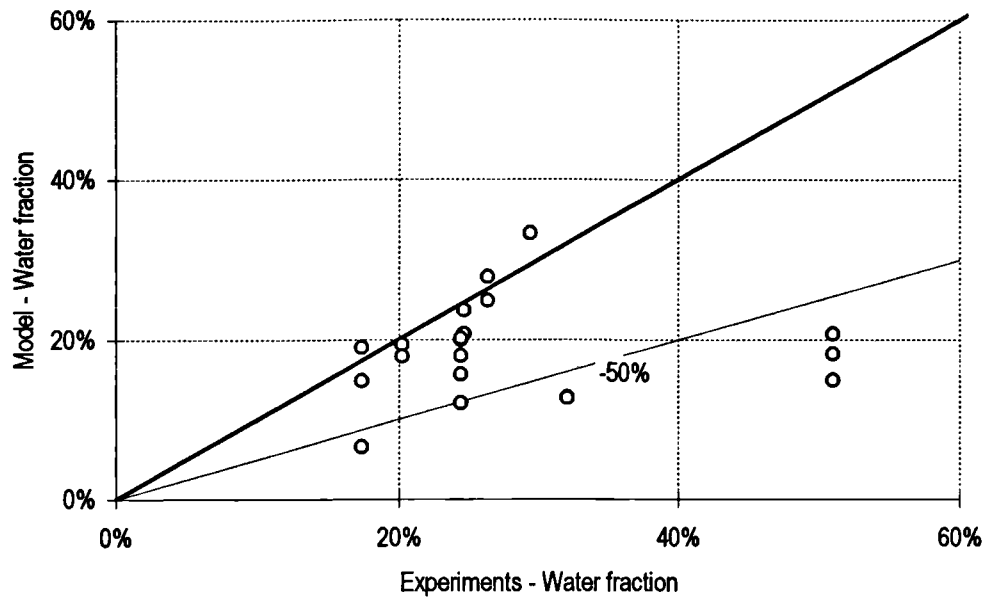


Figure 4.23: Comparison of the one dimensional model ($\tau_i = \tau_{wB}$) with the water fraction data obtained in WASP horizontal oil-water flow experiments.

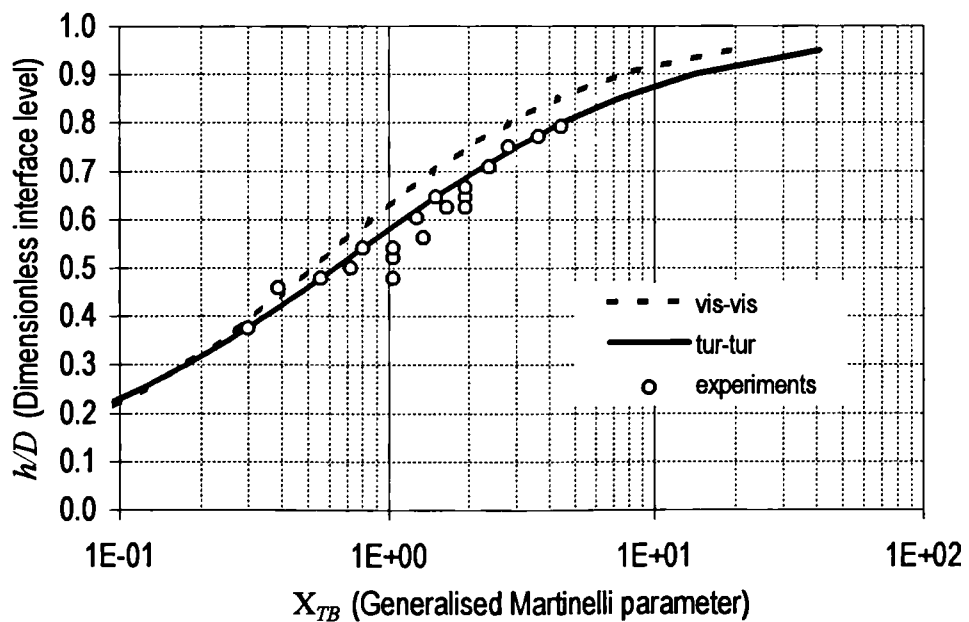


Figure 4.24: Comparison between the dimensionless interface level (h/D) calculated from the one-dimensional model for the laminar (vis-vis) and turbulent (tur-tur) flow of both phases with the results of interface level experiments performed on the TOWER.

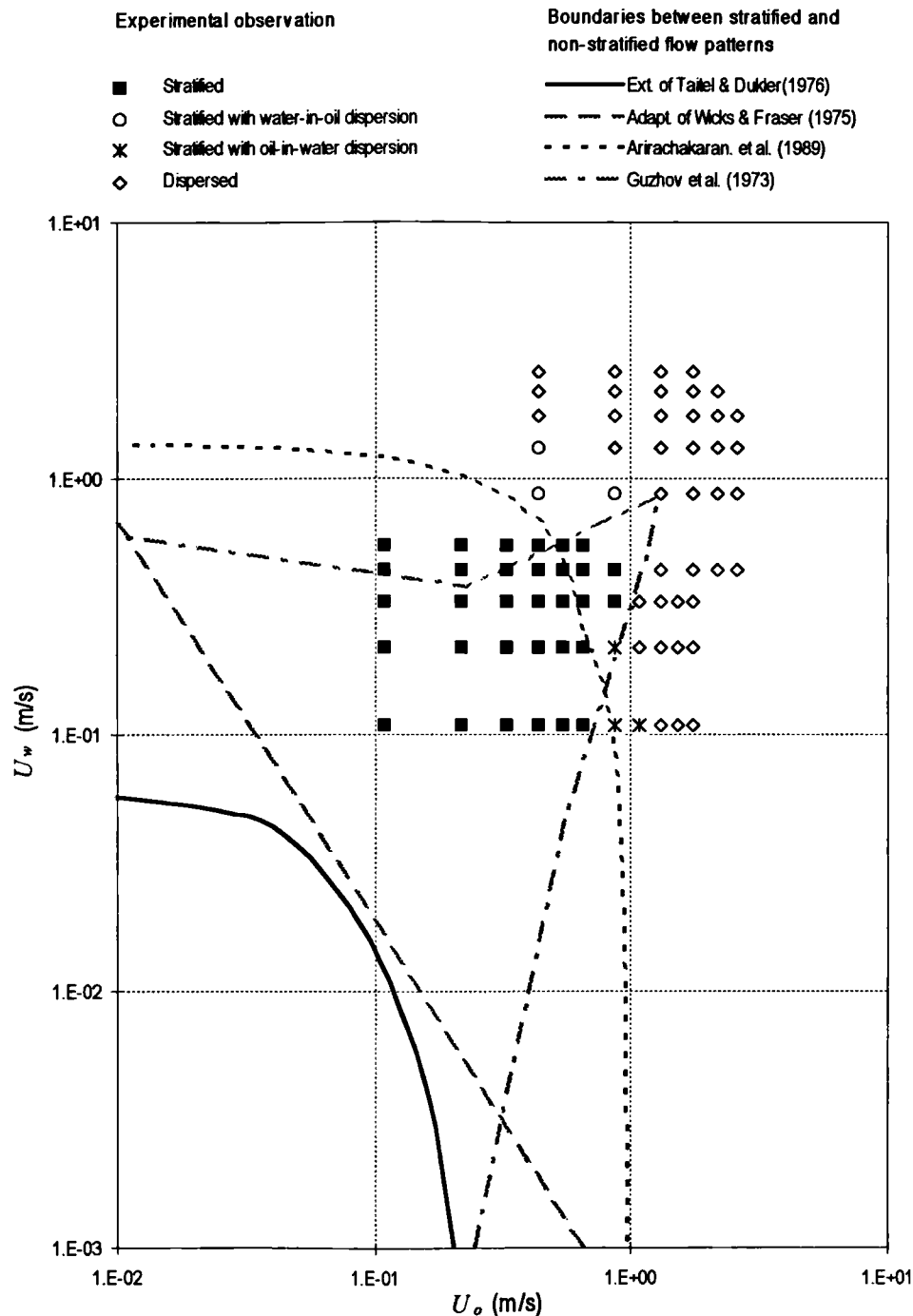


Figure 4.25: Flow patterns. Comparison of the extension of Taitel and Dukler (1976a) methodology and the adaptation of Wicks and Fraser (1975) model with the flow pattern data from the experiments on the TOWER *transpalite* test section (no mixer) and the stratified/non-stratified boundaries from the experimental flow pattern maps of Arirachakaran et al. (1989) and Guzhov et al. (1973).

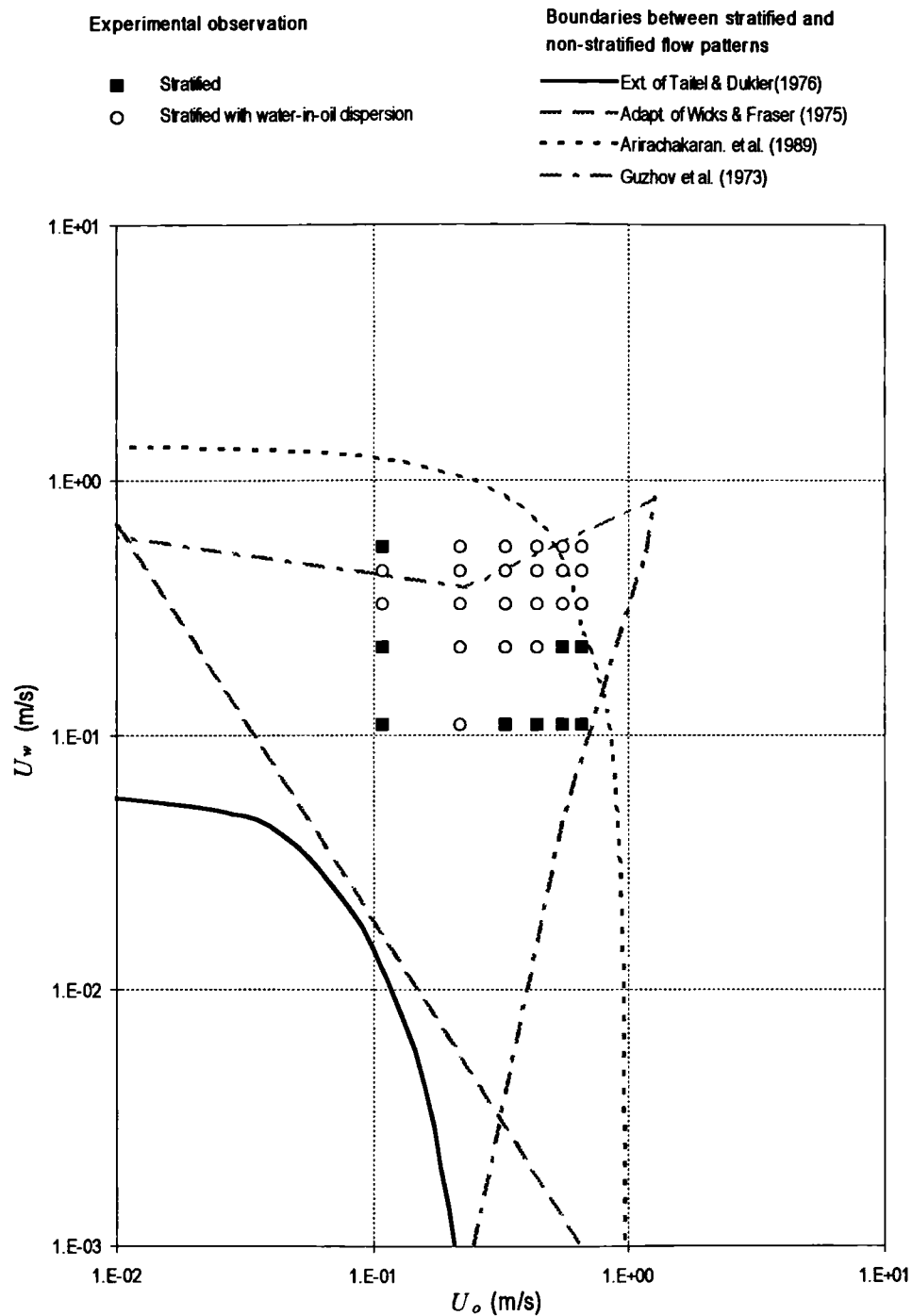


Figure 4.26: Flow patterns. Comparison of the extension of Taitel and Dukler (1976a) methodology and the adaptation of Wicks and Fraser (1975) model with the flow pattern data from the experiments on the TOWER *transpalite* test section (mixer) and the stratified/non-stratified boundaries from the experimental flow pattern maps of Arirachakaran et al. (1989) and Guzhov et al. (1973).

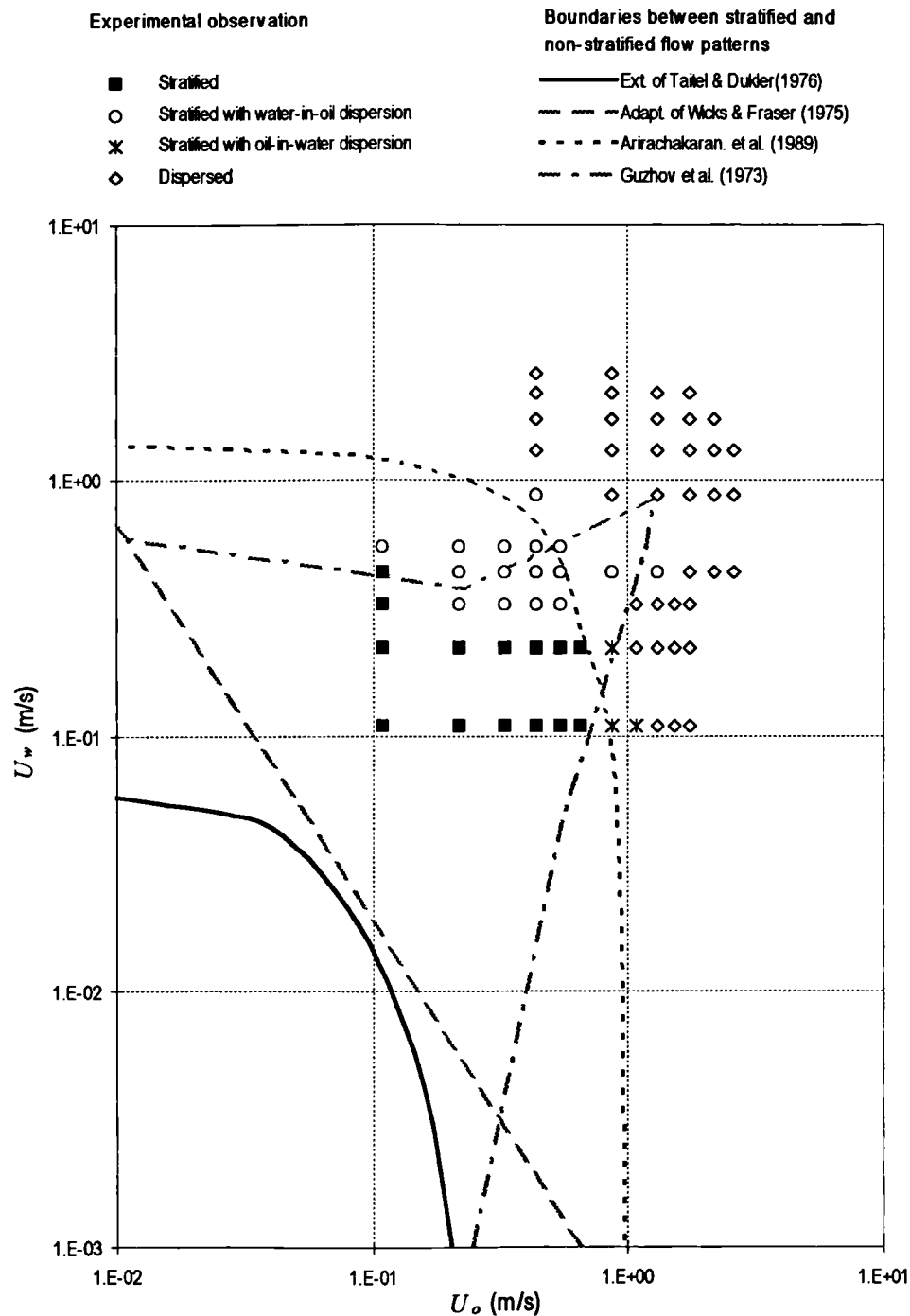


Figure 4.27: Flow patterns. Comparison of the extension of Taitel and Dukler (1976a) methodology and the adaptation of Wicks and Fraser (1975) model with the flow pattern data from the experiments on the TOWER stainless steel test section and the stratified/non-stratified boundaries from the experimental flow pattern maps of Arirachakaran et al. (1989) and Guzhov et al. (1973).

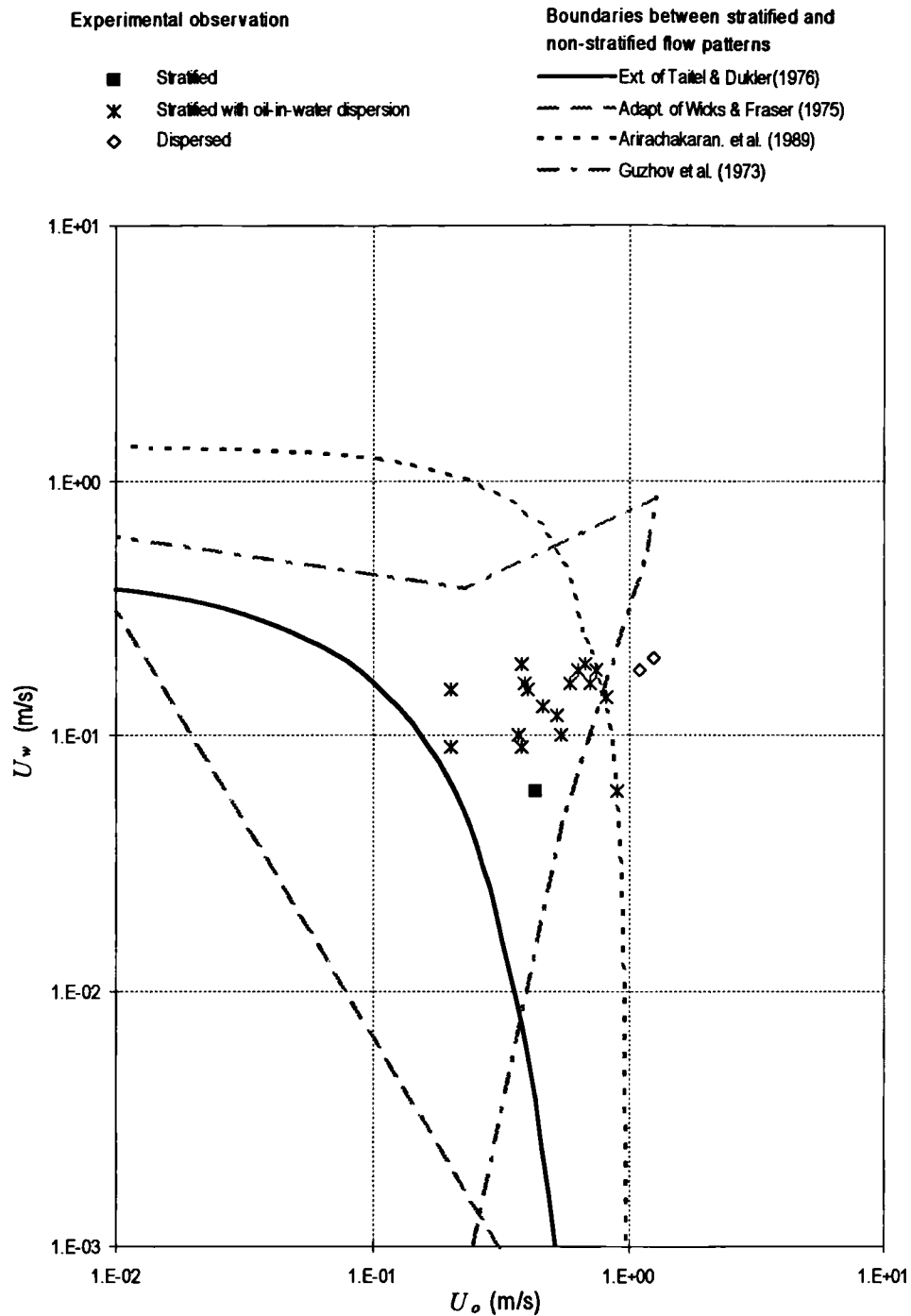


Figure 4.28: Flow patterns. Comparison of the extension of Taitel and Dukler (1976a) methodology and the adaptation of Wicks and Fraser (1975) model with the flow pattern data from the WASP horizontal oil-water experiments and the stratified/non-stratified boundaries from the experimental flow pattern maps of Arirachakaran et al. (1989) and Guzhov et al. (1973).

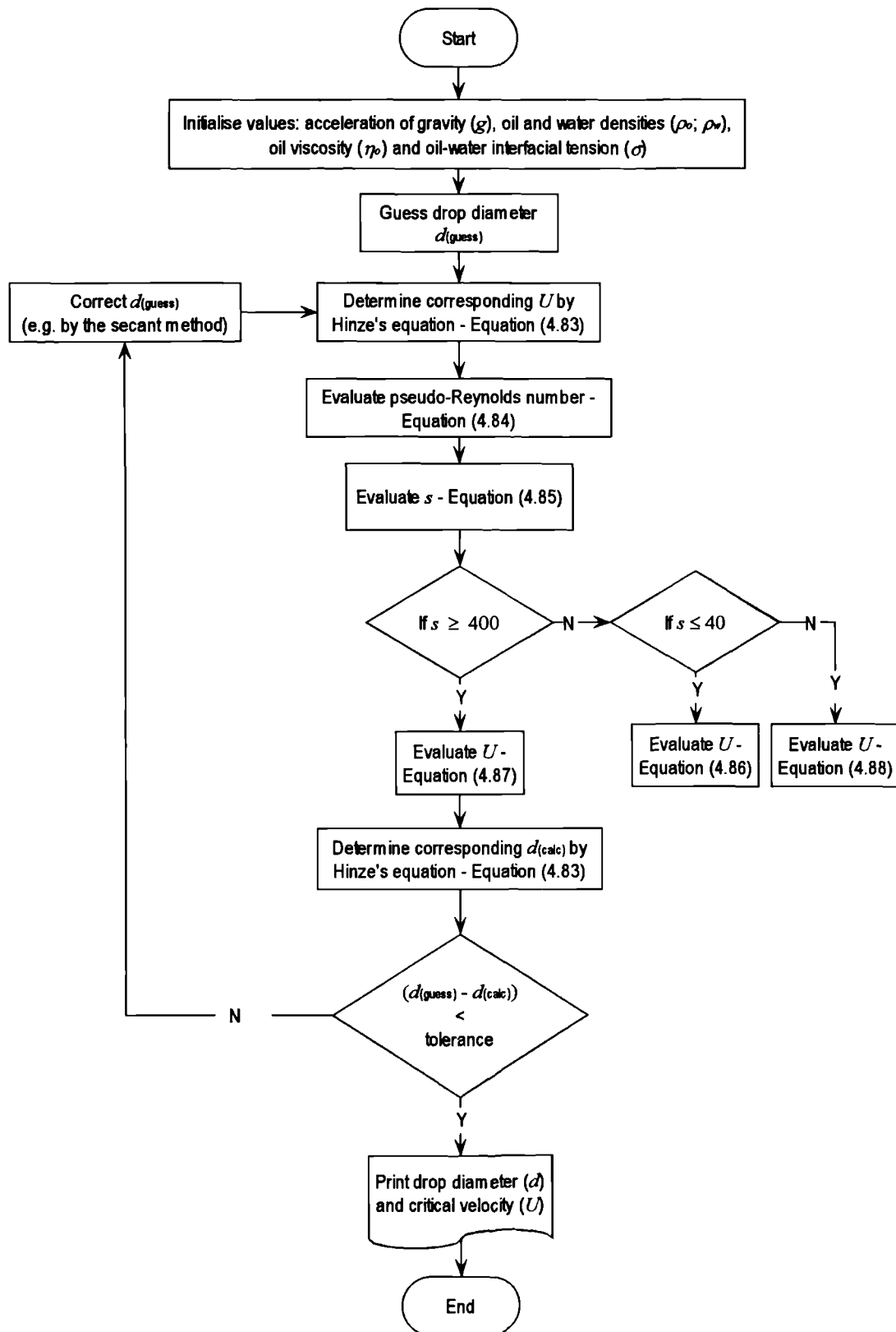


Figure 4.29: Adaptation of Wicks and Fraser (1975) method flowchart (as implemented in this work).

Chapter 5

Numerical Simulation

The one-dimensional model presented in Chapter 4 was developed using a phenomenological approach to describe the simultaneous flow of two liquids in a tube. In that model the velocities of each of the phases were averaged over cross sections of the tube occupied by the respective phase and these values used to calculate average values for the interface and wall shear stresses. Notwithstanding the obvious implications resulting from this averaging the phenomenological models have been used with relative success in many multiphase flow engineering design procedures, though they are clearly limited on the accuracy of the results they may produce. Similarly, it can be risky to extrapolate the one-dimensional models outside the application range or fluids for which the models were originally developed. Both of these problems arise from the fact that multiphase flow in tubes (particularly the stratified flow pattern, subject of this present study) is at least a one variable (the axial velocity w) two dimensional problem, even in the case of a smooth horizontal plane interface. However, if that interface should be described with more detail, the appearance of other components of the velocity vector in the flow are certain to require a full three-dimensional analysis for an accurate description of the phenomenon.

It has been shown in the work of Hall (1992) that the basic assumption of constant wall and interface shear stresses used in one-dimensional modelling does

not correspond to the true behaviour of a multiphase flow system. In stratified flow, the presence of a planar interface in a circular tube causes the flow field to be of much higher complexity than in the case of the purely axisymmetric single phase flow, for which most of the friction factor formulations used in one dimensional modelling were devised. Examples of numerical solutions showing the two-dimensional flow behaviour in gas-liquid multiphase flows were produced by Shoham and Taitel (1984) and Issa (1988). In both of these studies the problem was solved by the use of a bipolar coordinate system; in the study of Shoham and Taitel only the liquid phase flow was calculated numerically; the gas flow, described by a one dimensional model, was introduced as a boundary condition to the liquid flow problem. Though approximations like this might be valid for gas-liquid flow, where the liquid flow behaviour seems to be of little influence in the gas flow field, it is clearly not the case in liquid-liquid flows where both phases exert large influences on each other. In liquid-liquid systems, therefore, a full two-dimensional solution is required.

A multiphase flow model which, given the input flow rates of the fluids, would calculate both the flow fields and the configuration of the two fluids in a conduit is still in the realm of imagination. Kurban (1989), following the studies of Joseph et al. (1984), attempted to predict the interface shape in stratified and annular flows through the principle of minimum viscous dissipation, but his analysis only considered laminar and horizontal flows and his results are still awaiting experimental confirmation. The flow pattern transition criteria mostly used in engineering applications (e.g. Taitel and Dukler, 1976a; Moalem-Maron and Brauner, 1992a, b) are still based on macroscopic area averaged characteristics of the flows rather than on any principle that would tackle the basic physics of phase distribution in a conduit. This field of research, however, is potentially even more complex than studies of turbulence in single phase flows, which is still one of the

main topics of scientific investigation in fluid flow engineering today. Another alternative for the simulation of multiphase flows would be to observe the interfacial structure in specially designed experiments, on the lines of the method presented in section 3.3 (but much more detailed and sophisticated), and then input the interface shapes as given quantities in fluid flow solver algorithms. This seems an interesting path to follow, but it must be stressed that the interfaces are very rarely static, and a sound criteria to determine that interface condition that would best represent the average properties of the flowing system is yet to be devised. Besides, the computational facilities necessary to solve complex interface structures would still be formidable and most probably not currently accessible nowadays.

The numerical work developed in the present studies considered stratified flow configurations with simple planar interfaces. Firstly these flow structures, for different interface levels, were simulated in a bipolar coordinate system and later, through the aid of CFX-F3D, a commercial computer fluid dynamics code, in body fitted coordinates. The studies using the bipolar coordinate system were restricted to laminar flow for both phases, however in the body-fitted studies turbulence was taken in account by the use of the standard κ - ε formulation. For the case of laminar flow with the interface crossing the half diameter height of the circular cross section of the tube, an analytical solution was derived. These results were then used to assess the accuracy of the numerical schemes.

Section 5.1 includes a presentation of the analytical solution for the laminar flow of two fluids in a stratified flow configuration where each of the phases occupy half of the tube cross section, and describes the numerical methods used for solutions of cases which the interface does not necessarily pass through the tube axis, i.e. the bipolar algorithm for laminar flows and the commercial flow solver

CFX-F3D, used for both laminar and turbulent flows. Section 5.2 presents the results obtained by each method and compares them with each other and with the analytical solution wherever it is possible. Curves representing the flow fields and the behaviour of the interface and wall shear stresses are presented for various cases of fluids properties, interface position, and flow regimes. In this section the one dimensional model developed in Chapter 4 and its basic assumptions are also assessed against the numerical results obtained.

5.1- Description of the solution methods used.

5.1.1- Analytical solution

The particular case of laminar stratified flow in a circular tube half filled respectively by oil and water (or any other two immiscible fluids for this purpose) can be solved analytically as will be shown in this section. This solution is particularly important because it produces accurate flow field results which, therefore, can be used to assess the computational fluid dynamic methods described in sections 5.1.2 and 5.1.3. The author wishes to acknowledge the contribution of Dr. C. J. Lawrence of Imperial College in identifying this analytical solution.

A simultaneous pressure driven flow of two incompressible and totally immiscible fluids occurs in a horizontal circular tube; the flow is steady, fully developed and purely axial. The effects of interfacial tension are neglected. The pressure gradient is identical in the two phases and the interface position is assumed horizontal and to pass through the tube axis as shown in Figure 5.1.

The phenomenon is governed by the mass and linear momentum conservation (Navier-Stokes) equations for the fully developed axial flow in each phase present in the tube (Equations 5.1 to 5.5):

$$\frac{\partial w_T}{\partial z} = 0 \quad (5.1)$$

$$\frac{\partial w_B}{\partial z} = 0 \quad (5.2)$$

and

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r w_T) \right) + \frac{1}{r^2} \frac{\partial^2 w_T}{\partial \theta^2} = \frac{1}{\eta_T} \frac{\partial p}{\partial z} \quad (5.3)$$

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r w_B) \right) + \frac{1}{r^2} \frac{\partial^2 w_B}{\partial \theta^2} = \frac{1}{\eta_B} \frac{\partial p}{\partial z} \quad (5.4)$$

$$\frac{\partial p}{\partial r} = \frac{\partial p}{\partial \theta} = 0 \quad (5.5)$$

The boundary conditions to be applied are no slip at the tube walls and continuity of velocity and shear stress at the interface between the fluids:

on $r = R$:

$$w_T = 0 \quad (5.6)$$

$$w_B = 0 \quad (5.7)$$

and, at the interface, on $\theta = \pm \frac{\pi}{2}$

$$w_T = w_B \quad (5.8)$$

and

$$\tau_{iT} = \tau_{iB} \quad (5.9)$$

or

$$\eta_T \frac{\partial w_T}{\partial \theta} = \eta_B \frac{\partial w_B}{\partial \theta} \quad (5.10)$$

Solutions to this problem are better presented in terms of dimensionless variables:

$$\tilde{r} = \frac{r}{D} \quad (5.11)$$

and

$$\tilde{w} = w \left(\frac{\eta_B}{-\frac{\partial p}{\partial z} D^2} \right) \quad (5.12)$$

Then the governing equations (Equations 5.3 to 5.10) reduce to:

$$\frac{\partial \tilde{w}_T}{\partial \tilde{z}} = 0 \quad (5.14)$$

$$\frac{\partial \tilde{w}_B}{\partial \tilde{z}} = 0 \quad (5.15)$$

and

$$\frac{\partial}{\partial \tilde{r}} \left(\frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{w}_B) \right) + \frac{1}{\tilde{r}^2} \frac{\partial^2 \tilde{w}_B}{\partial \theta^2} = -1 \quad (5.16)$$

$$\frac{\partial}{\partial \tilde{r}} \left(\frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{w}_T) \right) + \frac{1}{\tilde{r}^2} \frac{\partial^2 \tilde{w}_T}{\partial \theta^2} = -\beta \quad (5.17)$$

where β , the inverse viscosity ratio, is defined by:

$$\beta = \frac{\eta_B}{\eta_T} \quad (5.18)$$

with the corresponding dimensionless boundary conditions:

on $\tilde{r} = 0.5$

$$\tilde{w}_T = 0 \quad (5.19)$$

$$\tilde{w}_B = 0 \quad (5.20)$$

and, at the interface, on $\theta = \pm \frac{\pi}{2}$

$$\tilde{w}_T = \tilde{w}_B \quad (5.21)$$

and

$$\frac{\partial \tilde{w}_T}{\partial \theta} = \beta \frac{\partial \tilde{w}_B}{\partial \theta} \quad (5.22)$$

The general solution for the Poisson equations represented by equations (5.16) and (5.17) can be written as

$$\tilde{w}_T = \tilde{w}_T^* + \tilde{w}_T^L \quad (5.23)$$

$$\tilde{w}_B = \tilde{w}_B^* + \tilde{w}_B^L \quad (5.24)$$

where the superscripts $*$ and L respectively represent a particular solution¹ and the solution of the Laplace equations obtained by neglecting the constant on the right hand side of equations (5.16) and (5.17).

The particular solution can conveniently be chosen to be of the form:

¹ The wall boundary conditions - Equations (5.19) and (5.20) - are relaxed for this particular solution.

$$\tilde{w}_B^* = A(\tilde{r} \sin \theta)^2 + B(\tilde{r} \cos \theta)^2 \quad (5.25)$$

and

$$\tilde{w}_T^* = C(\tilde{r} \sin \theta)^2 + D(\tilde{r} \cos \theta)^2 \quad (5.26)$$

which, in order for the interface conditions expressed by equations (5.23) and (5.24) to be satisfied one must have:

$$2(A + B) = -1 \quad (5.27)$$

$$2(C + D) = -\beta \quad (5.28)$$

and continuity of velocity requires that

$$A = C \quad (5.29)$$

The value for the constant A can be arbitrarily chosen as:

$$A = -\frac{1}{4} \quad (5.30)$$

and as consequence of (5.27), (5.28) and (5.29)

$$B = -\frac{1}{4} \quad (5.31)$$

$$C = -\frac{1}{4} \quad (5.32)$$

$$D = \frac{1}{4} - \frac{\beta}{2} \quad (5.33)$$

Equations (5.25) and (5.26) can then be re-written as:

$$\tilde{w}_B^* = -\frac{1}{4}\tilde{r}^2 \quad (5.34)$$

$$\tilde{w}_T^* = -\frac{\beta}{4}\tilde{r}^2 + \frac{1-\beta}{4}\tilde{r}^2 \cos(2\theta) \quad (5.35)$$

The Laplace equations are now:

$$\frac{\partial}{\partial \tilde{r}} \left(\frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{w}_T^L) \right) + \frac{1}{\tilde{r}^2} \frac{\partial^2 \tilde{w}_T^L}{\partial \tilde{r}^2} = 0 \quad (5.36)$$

$$\frac{\partial}{\partial \tilde{r}} \left(\frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{w}_B^L) \right) + \frac{1}{\tilde{r}^2} \frac{\partial^2 \tilde{w}_B^L}{\partial \tilde{r}^2} = 0 \quad (5.37)$$

and their boundary conditions, in order that the overall boundary conditions, (equations 5.19 to 5.22) be preserved, are:

on $\tilde{r} = 0.5$

$$\tilde{w}_B^L = \frac{1}{16} \quad (5.38)$$

$$\tilde{w}_T^L = \frac{\beta}{16} - \frac{1-\beta}{16} \cos(2\theta) \quad (5.39)$$

and on $\theta = \pm \frac{\pi}{2}$

$$\tilde{w}_B^L = \tilde{w}_T^L \quad (5.40)$$

$$\frac{\partial \tilde{w}_T^L}{\partial \theta} = \beta \frac{\partial \tilde{w}_B^L}{\partial \theta} \quad (5.41)$$

The Fourier series solution to Laplace's equation is given by

$$\tilde{w}_T^L = \sum_{n=0}^{\infty} \left[A_n \tilde{r}^n \cos(n\theta) + B_n \tilde{r}^n \sin(n\theta) + C_n \tilde{r}^{-n} \cos(n\theta) + D_n \tilde{r}^{-n} \sin(n\theta) \right] \quad (5.42)$$

$$\tilde{w}_B^L = \sum_{n=0}^{\infty} \left[a_n \tilde{r}^n \cos(n\theta) + b_n \tilde{r}^n \sin(n\theta) + c_n \tilde{r}^{-n} \cos(n\theta) + d_n \tilde{r}^{-n} \sin(n\theta) \right] \quad (5.43)$$

For the solution to be finite at $\tilde{r} = 0$, one has

$$c_n = C_n = d_n = D_n = 0 \quad (5.44)$$

For the solution to be symmetric about $\theta = 0$, one has:

$$b_n = B_n = 0 \quad (5.45)$$

Thus, Equations (5.42) and (5.43) simplify to:

$$\tilde{w}_T^L = \sum_{n=0}^{\infty} [A_n \tilde{r}^n \cos(n\theta)] \quad (5.46)$$

for $\frac{\pi}{2} < \theta < \pi$;

and

$$\tilde{w}_B^L = \sum_{n=0}^{\infty} [a_n \tilde{r}^n \cos(n\theta)] \quad (5.47)$$

for $0 < \theta < \frac{\pi}{2}$;

The value of the constants A_n and a_n in Equations (5.46) and (5.47) must be such that the boundary and interface conditions are satisfied. Thus for the equality of velocities at the interface (Equation 5.40) one has:

$$\sum_{n=0}^{\infty} (a_n - A_n) r^n \cos\left(\frac{n\pi}{2}\right) = 0 \quad (5.48)$$

which leads (if n is even) to:

$$A_n = a_n \quad (n = \text{even}) \quad (5.49)$$

Also, using the continuity of shear stresses at the interface (Equation 5.41) one obtains:

$$\sum_{n=0}^{\infty} (A_n - \beta a_n) r^n \sin\left(\frac{n\pi}{2}\right) = 0 \quad (5.50)$$

which in turn leads to, for odd values of n :

$$A_n = \beta a_n \quad (n = \text{odd}) \quad (5.51)$$

Now, applying the wall boundary conditions at $\tilde{r} = 0.5$

$$\tilde{w}_B^L = \frac{1}{16} = \sum_{n=0}^{\infty} \frac{a_n}{2^n} \cos(n\theta) \text{ for } 0 < \theta < \frac{\pi}{2} \quad (5.52)$$

and

$$\tilde{w}_T^L = \frac{\beta}{16} - \frac{1-\beta}{16} \cos(2\theta) = \sum_{n=0}^{\infty} \frac{A_n}{2^n} \cos(n\theta) \text{ for } \frac{\pi}{2} < \theta < \pi \quad (5.53)$$

Introducing $\phi = \pi - \theta$ in Equation (5.53) in order that its application range corresponds to that of Equation (5.52) leads to:

$$\tilde{w}_T^L = \frac{\beta}{16} - \frac{1-\beta}{16} \cos(2\phi) = \sum_{n=0}^{\infty} \frac{A_n}{2^n} \cos(n\phi) \text{ for } 0 < \phi < \frac{\pi}{2} \quad (5.54)$$

Now, Equation (5.54) can be subtracted from (5.52). Calling the generic angular variable of ψ leads to:

$$\sum_{n=0}^{\infty} \left[\frac{a_n - (-1)^n A_n}{2^n} \right] \cos(n\psi) = \frac{1-\beta}{16} (1 + \cos(2\psi)) \text{ for } 0 < \psi < \frac{\pi}{2} \quad (5.55)$$

recalling that $a_n = A_n$ when n is even and that $A_n = \beta a_n$ when n is odd, Equation (5.55) reduces to:

$$\sum_{n=\text{odd}} \frac{a_n}{2^n} \cos(n\psi) = \frac{1}{16} \frac{1-\beta}{1+\beta} (1 + \cos(2\psi)) \text{ for } 0 < \psi < \frac{\pi}{2} \quad (5.56)$$

Using ψ again, and adding Equation (5.52) multiplied by β to (5.54) it is similarly obtained:

$$\sum_{n=\text{even}} \frac{a_n}{2^n} \cos(n\psi) = \frac{1}{1+\beta} \left(\frac{\beta}{8} - \frac{1-\beta}{16} \cos(2\psi) \right) \text{ for } 0 < \psi < \frac{\pi}{2} \quad (5.57)$$

The coefficients a_n and A_n can be determined by the use of the properties of orthogonal functions. Therefore, for Equation (5.56) it can be set:

$$\int_0^{\pi/2} \left[\sum_{n=\text{odd}} \frac{a_n}{2^n} \cos(n\psi) \right] \cos(m\psi) d\psi = \quad (5.58)$$

$$\frac{a_m}{2^m} \frac{\pi}{4} = \frac{1}{16} \frac{1-\beta}{1+\beta} \int_0^{\pi/2} (1 + \cos(2\psi) \cos(m\psi)) d\psi \text{ for } m \text{ odd}$$

therefore

$$a_m = -\frac{2^m}{\pi} \frac{1-\beta}{1+\beta} (-1)^{\frac{m-1}{2}} \frac{1}{m} \left[\frac{1}{m^2-4} \right] \text{ for } m \text{ odd} \quad (5.59)$$

Similarly for Equation (5.57) it is set:

$$\int_0^\pi \left[\sum_{n=\text{even}} \frac{a_n}{2^n} \cos(n\psi) \right] \cos(m\psi) d\psi = \quad (5.60)$$

$$\frac{a_m}{2^m} \frac{\pi}{4} = \frac{1}{1+\beta} \int_0^\pi \left[\left(\frac{\beta}{8} - \frac{1-\beta}{16} \cos(2\psi) \right) \cos(m\psi) \right] d\psi \text{ for } m \text{ even}$$

and therefore

$$a_0 = -\frac{\beta}{8(1+\beta)} \quad (5.61)$$

$$a_2 = -\frac{1-\beta}{4(1+\beta)} \quad (5.62)$$

$$a_m = 0 \text{ for } m > 2 \text{ and even} \quad (5.63)$$

recalling Equations (5.23) and (5.24) the solution for the flow fields is finally obtained:

$$\begin{aligned} \tilde{w}_B = \frac{\beta}{8(1+\beta)} - \frac{\tilde{r}^2}{4} \left[1 + \frac{1-\beta}{(1+\beta)} \cos 2\theta \right] - \\ \frac{1}{\pi} \frac{1-\beta}{1+\beta} \sum_{n=odd} \left[(-1)^{\frac{n-1}{2}} \frac{2^n}{n} \left(\frac{1}{n^2-4} \right) \tilde{r}^n \cos(n\theta) \right] \end{aligned} \quad (5.64)$$

$$\begin{aligned} \tilde{w}_T = \frac{\beta}{8(1+\beta)} - \frac{\beta \tilde{r}^2}{4} \left[1 - \frac{1-\beta}{1+\beta} \cos(2\theta) \right] - \\ \frac{\beta}{\pi} \frac{1-\beta}{1+\beta} \sum_{n=odd} \left[(-1)^{\frac{n-1}{2}} \frac{2^n}{n} \left(\frac{1}{n^2-4} \right) \tilde{r}^n \cos(n\theta) \right] \end{aligned} \quad (5.65)$$

It should be noticed that when the viscosity ratio β is set to unity in the above equations both of them consistently reduce to:

$$\tilde{w} = \frac{1}{4} \left(\frac{1}{4} - \tilde{r}^2 \right) \quad (5.66)$$

which is the solution for the single phase flow of a fluid in a circular tube.

In order to find the velocity profile at the interface the value of θ is set to $\pi/2$, and from Equations (5.64) and (5.65) it is obtained:

$$\tilde{w}_B = \tilde{w}_T = \frac{1}{2} \left(\frac{\beta}{1+\beta} \right) \left(\frac{1}{4} - \tilde{r}^2 \right) \quad (5.67)$$

The expression for the shear rate on the interface is obtained by differentiating Equation (5.67) in relation to θ , therefore the expression for the shear stress at the interface is given as:

$$\tilde{\tau}_i = \frac{1}{r} \frac{\partial \tilde{w}_B}{\partial \theta} \Big|_{\theta=\frac{\pi}{2}} = -\frac{1}{\pi} \frac{1-\beta}{1+\beta} \sum_{n=odd} \left(\frac{2^n}{n^2-4} \right) r^{(n-1)} \quad (5.68)$$

Figure 5.2 shows the velocity profile on the symmetry plane, calculated by Equations (5.64) and (5.65) with θ set equal to 0 and π respectively; and Figures 5.3 and 5.4 show the velocity and shear stress profiles at the interface plane as predicted by Equations (5.67) and (5.68). In all these figures, solutions are presented for three values of β .

The shear stress on the wall for each phase is:

$$\begin{aligned} \tilde{\tau}_B = \frac{\partial \tilde{w}_B}{\partial \tilde{r}} \Big|_{r=1/2} = & -\frac{1}{4} \left(1 + \frac{1-\beta}{1+\beta} \cos(2\theta) \right) - \\ & \frac{2}{\pi} \left(\frac{1-\beta}{1+\beta} \right) \sum_{n=odd} \left[(-1)^{\frac{n-1}{2}} \frac{1}{n^2-4} \cos(n\theta) \right] \end{aligned} \quad (5.69)$$

and

$$\begin{aligned} \tilde{\tau}_T = \frac{1}{\beta} \frac{\partial \tilde{w}_T}{\partial \tilde{r}} \Big|_{r=1/2} = & -\frac{1}{4} \left(1 - \frac{1-\beta}{1+\beta} \cos(2\theta) \right) - \\ & \frac{2}{\pi} \left(\frac{1-\beta}{1+\beta} \right) \sum_{n=odd} \left[(-1)^{\frac{n-1}{2}} \frac{1}{n^2-4} \cos(n\theta) \right] \end{aligned} \quad (5.70)$$

Figure 5.5 show the behaviour of the wall shear stress along the perimeter of the tube for three values of β .

The flow rates of each phase are obtained by integrating their velocity profiles (Equations 5.64 and 5.65) over their respective flow areas:

$$\dot{V}_B = 2 \int_0^{\pi/2} \int_0^{1/2} \tilde{w}_B r dr d\theta = \frac{\pi}{64} \frac{\beta}{1+\beta} - \frac{\pi}{256} - \frac{1}{2\pi} \frac{1-\beta}{1+\beta} \sum_{n=odd} \frac{1}{n^2} \left(\frac{1}{n^2-4} \right) \frac{1}{n+2} \quad (5.71)$$

and

$$\dot{V}_T = 2 \int_0^{\pi/2} \int_0^{1/2} \tilde{w}_T r dr d\theta = \frac{\pi}{64} \frac{\beta}{1+\beta} - \frac{\pi\beta}{256} + \frac{\beta}{2\pi} \frac{1-\beta}{1+\beta} \sum_{n=odd} \frac{1}{n^2} \left(\frac{1}{n^2-4} \right) \frac{1}{n+2} \quad (5.72)$$

Figure 5.6 shows the dimensionless flow rates of each fluid as a function of the inverse viscosity ratio β .

It should be noticed again, as one should expect, that the Equations (5.71) and (5.72) above reduce to the single phase formulation when $\beta = 1$, that is:

$$\dot{V} = 2\dot{V}_T = 2\dot{V}_B = \int_0^{2\pi/2} \int_0^{1/2} \frac{1}{4} \left(\frac{1}{4} - r^2 \right) r dr d\theta = \frac{\pi}{128} \quad (5.73)$$

5.1.2- Numerical solution in bipolar coordinates

As it has been seen in the previous section the problem of the stratified flow of two immiscible fluids leads to the solution of two velocity fields governed by Poisson equations, separated by a plane interface. It was also seen that the analytical solution of these flow fields was rather complex, and that it was only obtained for the case where the interface would be coincident with the diameter of

the circular tube. A numerical solution on the other hand, although intrinsically inexact, allows basically that the physical problem may be tackled in its original formulation. The accuracy of the solution obtained represents a compromise between computational cost and the accuracy needed in practice. In fact, it is theoretically possible to get a numerical solution as accurate as one wishes, though this might not always be sensible to do.

The equations that govern flow field of the fluids in each of their sub-domains are differential Poisson equations of the type:

$$\nabla \cdot (\Gamma \nabla \varphi) - S = 0 \quad (5.74)$$

where Γ is a generic (not necessarily constant) diffusivity, φ a generic field variable and S a generic (not necessarily constant) source term.

Equation (5.74) in two dimensional cartesian coordinates takes the form:

$$\frac{\partial}{\partial x} \Gamma \left(\frac{\partial \varphi}{\partial x} \right) + \frac{\partial}{\partial y} \Gamma \left(\frac{\partial \varphi}{\partial y} \right) - S = 0 \quad (5.75)$$

Discretising Equation (5.75) using the control volume formulation presented by Patankar (1980) over the typical volume element described in Figure 5.7 yields an algebraic equation in the form of:

$$a_P w_P = a_E w_E + a_W w_W + a_N w_N + a_S w_S + b \quad (5.76)$$

where w_P , w_E , w_W , w_N , w_S are the velocities in the points indicated in Figure 5.7, and the coefficients a are defined by:

$$a_E = \frac{\Gamma_e \Delta y}{(\delta x)_e} \quad (5.77)$$

$$a_W = \frac{\Gamma_w \Delta y}{(\delta x)_w} \quad (5.78)$$

$$a_N = \frac{\Gamma_n \Delta x}{(\delta y)_n} \quad (5.79)$$

$$a_S = \frac{\Gamma_s \Delta x}{(\delta y)_s} \quad (5.80)$$

$$a_P = a_E + a_W + a_N + a_S \quad (5.81)$$

$$b = S \Delta x \Delta y \quad (5.82)$$

The flow geometry under consideration, shown in Figure 5.8, consists of a circular wall and a generic plane interface. As it is necessary to solve the flow fields for both phases, the numerical domain must have external boundaries at the pipe walls and a grid line coinciding with the interface. This clearly can be achieved neither in the cartesian coordinate system (Equation 5.75) nor in a cylindrical coordinate system as was done in Section 5.1.1 (Equation 5.3)². To get

² The only stratified case that can be fully described in cylindrical coordinates is that one presented in section 5.1.1, where the interface corresponds to the diameter of the tube, so that the interface is at a constant azimuthal coordinate value ($\pi/2$ in section 5.1.1)

around this apparent geometric heterogeneity one can map the flow domain of Figure 5.8 through a conformal transformation that will result in a bipolar coordinate system. The bipolar coordinates are the natural coordinate system to be used in dealing with field problems whose boundaries consist of intersecting a circular cylinder with a plane (Shoham and Taitel, 1984).

The bipolar coordinate system can be defined as the following transformation from the cartesian coordinate system:

$$x + iy = ic \cot\left(\frac{\mu + i\xi}{2}\right) \quad (5.83)$$

where x and y are the cartesian coordinates, ξ and μ the bipolar coordinates and c is the location of the pole on the x axis (Figure 5.8). Observe that $i = \sqrt{-1}$ in the above equation is used to represent the imaginary part of a complex number.

The relationship between the cartesian and the bipolar coordinates derived from the complex Equation (5.83) are:

$$x = \frac{-c \sinh \xi}{\cosh \xi - \cos \mu} \quad (5.84)$$

$$y = \frac{-c \sin \mu}{\cosh \xi - \cos \mu} \quad (5.85)$$

Figure 5.9 shows the bipolar coordinates in the real pipe domain for different interface levels (only half the pipes are represented since there is symmetry around the y axis) while Figure 5.8 also shows the relationship between the

conformal mapped and real domains. Observe that the pipe geometry in the transformed domain reduces to an infinite plane. Also the position of the interface will always be at $\mu = \pi$, while the wall wetted by the bottom fluid will be at $\mu = \pi + \gamma$ and the wall wetted by the top fluid will be at $\mu = \gamma$. The relationships among the pole coordinate (c), the interface height (h) and the angle γ , as can be derived from Figure 5.8, are:

$$\gamma = \arccos\left(1 - \frac{2h}{D}\right) \quad (5.86)$$

$$c = -\frac{D}{2} \sin \gamma \quad (5.87)$$

It should also be noticed that the symmetry line $x = 0$ in the pipe domain is mapped as the line $\xi = 0$ in the transformed domain.

The Equation for the velocity w in bipolar coordinates are written as:

$$\eta_T \left(\frac{\partial^2 w_T}{\partial \xi^2} + \frac{\partial^2 w_T}{\partial \mu^2} \right) = \frac{\partial p}{\partial z} \left(\frac{c}{\cosh \xi - \cos \mu} \right)^2 \quad (5.88)$$

and

$$\eta_B \left(\frac{\partial^2 w_B}{\partial \xi^2} + \frac{\partial^2 w_B}{\partial \mu^2} \right) = \frac{\partial p}{\partial z} \left(\frac{c}{\cosh \xi - \cos \mu} \right)^2 \quad (5.89)$$

Equations (5.88) and (5.89) become identical to Equation (5.75) provided that in the latter the generic variable φ is replaced by w ; the diffusivity Γ is replaced by either η_T or η_B and the source term S is replaced by the right-hand-side of Equations (5.88) or (5.89). Table 5.1 summarises these relationships. Therefore, the flow field solution can be obtained by solving Equations (5.88) and (5.89) with the same discretised equations (Equations 5.76 to 5.82). In fact, since in Equation (5.75) it is not implicit that neither Γ or S are constant Equations (5.88) and (5.89) can be reduced to a single equation where there is a discontinuity in the viscosity (diffusivity) across the interface (cell boundaries where $\mu = \pi$) and the source term varies continuously throughout the whole domain.

The solution of the flow field in bipolar coordinates was implemented using a FORTRAN code already developed for the solution of a system of algebraic equations (Equations 5.76 to 5.82) obtained from the discretisation of the general diffusion equation (Equation 5.75) by the *finite-volume* method (Altemani, 1988), as mentioned before and described in detail in Patankar (1980).

The computational domain was restricted to only half of the tube cross section due to the symmetry existent around the ordinate axes y or μ (see again Figures 5.8 and 5.9).

The computational domain, on the conformal mapped bipolar coordinate plane, was covered by a non-uniform grid, with the grid points centred in each *control volume*. The grid points were uniformly distributed along the μ direction and distributed along a geometric progression of growing rate of 1.3 along the ξ direction. This was done to compensate the effect of the bipolar coordinate grid: a uniform grid in the bipolar plane, produces in the real (x - y) pipe geometry an

over-refined grid near the pole c, while the bulk of the domain is left with a very coarse gridding.

The dimension of the computational grid was 20 x 20, including the grid points in the external boundaries (symmetry line and pipe walls). Therefore, a system of algebraic equations is formed by writing one equation of the form of (5.76) to every grid point. This system of equations was solved in the FORTRAN code (Altemani, 1988) by a iterative solution method called *line-by-line* and described in Patankar (1980). Basically this method consists of a combination of the *TriDiagonal-Matrix Algorithm* (TDMA) used for one-dimensional problems, and the Gauss-Seidel method. According to this method, each grid line is successively solved assuming that the values of the velocities in the grid points in the adjacent lines are known by the result of the last iteration performed. Thus, the tridiagonal matrix algorithm can be applied over the chosen line. This procedure is repeated sweeping all the lines alternately in the direct and reverse directions. In order to speed up convergence this method is also applied to every column in the computational grid.

The boundary and interface conditions are those described in section 5.1.1, that is:

a) *No slip at the pipe walls:*

On $\mu = \gamma$; $\mu = \pi + \gamma$ and $\xi = \infty$ ³

³ The domain mapped in bipolar coordinates goes to infinity at the ξ direction as it approaches the pole. Clearly this may not be implemented in the computational domain. For practical purposes, as in the computer solution developed, ξ was set to 10. The distance of this position from the pipe wall is in the order of 10^{-5} of the pipe radius. Therefore, setting a no-slip wall boundary condition at this point should introduce negligible error in the calculated velocity field.

$$w = 0 \quad (5.90)$$

b) *No slip at the interface:*

On $\mu = \pi$

$$w_T = w_B \quad (5.91)$$

c) *Continuity of shear stress at the interface:*

On $\mu = \pi$

$$\eta_T \frac{\partial w_T}{\partial \mu} = \eta_B \frac{\partial w_B}{\partial \mu} \quad (5.92)$$

The symmetry boundary condition to be added due to the fact that the computational domain maps only half the tube cross section is:

d) Symmetry around the μ axis:

On $\xi = 0$

$$\frac{\partial w}{\partial \xi} = 0 \quad (5.93)$$

The boundary conditions prescribed in (b) and (c) are internally satisfied by the numerical method used; on the other hand those prescribed by (a) and (d) must be imposed externally.

It is important to observe that, in Equations (5.90) to (5.93), the value of viscosities used are those prevailing at the *boundary* of each control volume.

In the case of a homogeneous medium the value of the viscosity at the boundary of two control volumes is the same as within each control volume. However, in the case that two adjacent control volumes possess different viscosities as in Figure 5.10, Patankar (1980) recommends that the weighted harmonic average of these viscosities should be used (Equation 5.94), so as to preserve the real shear stress (i.e. the *momentum flux*) between the two control volumes:

$$\eta_n = \frac{(\delta\mu)_n}{\frac{(\delta\mu)_{n+}}{\eta_N} + \frac{(\delta\mu)_{n-}}{\eta_P}} \quad (5.94)$$

In order to apply the above condition it was always guaranteed, by appropriate gridding, that the interface between the two phases coincided with the boundary between two control volumes.

The flow rate of the fluids were calculated by the numerical integration of the velocity field over the computational domain. This integration was proceeded considering that the velocity at each grid point prevails over the whole surrounding control-volume. Thus:

$$\dot{V}_T = \int_{A_T} w dA = \sum_{i=1}^n \sum_{j=1}^{p-1} (w(i,j) \Delta A(i,j)) \quad (5.95)$$

and

$$\dot{V}_B = \int_{A_B} w dA = \sum_{i=1}^n \sum_{j=p}^q (w(i,j) \Delta A(i,j)) \quad (5.96)$$

where i, j are respectively the grid point's ξ and μ coordinate indices; n and q are the grid dimensions in the ξ and μ directions and p is the index of the interface control-volume boundary. The area of the control-volume ΔA is the actual area it has in the original x - y plane (not in the conformal mapped ξ - μ plane). This area was obtained by calculating the x - y coordinate of each vertice of the control volume with the aid of Equations (5.84) and (5.85), and then approximating its area by the area of the trapezoid drawn from these points.

Figure 5.11 shows the dimensionless velocity profile for a single phase flow as calculated by the numerical bipolar code assuming a pseudo-interface at 50% of the pipe diameter. Figure 5.12 shows the perfect agreement between these results and those of analytical solution of single phase flow (Equation 5.66).

It is worth noting that, despite the fact that the computational domain in the bipolar ξ - μ plane has always the same rectangular gridding - with the interface always located at $\mu=\pi$; but with positions of the lower and upper walls varying according to the angle γ , as already mentioned in this section - its mapping in the original x - y plane is quite different according to the position of the interface, as can be seen in Figure 5.9. It is clear that the approximate integrals of the pseudo two phase flow (top and bottom fluids with same properties) performed by Equations (5.95) and (5.96) should yield different results. These errors in relation to the analytical single phase solution (Equation 5.73) of the flow rates calculated for five different interface positions are shown in Figure 5.13. It is seen then that the higher accuracy is achieved when the interface is at the 50% position, and that

the worst, but still very acceptable, results, occur when the interface is farthest from the diametrical line of the tube (values close to either 0 or 100%).

5.1.3- Numerical solution in body-fitted coordinates

The code presented in Section 5.1.2 was simple and compact and therefore did not allow for more elaborate studies to be performed. Besides, it was only a two-dimensional code, thus limiting its use for future applications where a full three-dimensional description of the interface may be required. Thus, further studies were conducted to implement a simulation of stratified flow in a commercial code, namely, the CFX-F3D produced by AEA Technology plc..

5.1.3.1- Description of CFX-F3D

The CFX-F3D flow solving software is a product of CFDS-AEA Technology, Harwell, UK for the prediction of laminar and turbulent flow, and heat transfer (Jones et al., 1985). This suite of programs was designed as a tool to aid the engineer in the design of processes and equipment in which fluid flow and heat transfer are of major concern. The first release of the code, originally named FLOW3D, was issued in 1985. In that time the code was limited to simple cartesian and cylindrical geometries only. CFX-F3D has been largely improved since then. Presently, CFX-F3D, release 4.1 has a vast range of features that allow it to simulate flow conditions in many complex systems. One of the most important characteristics of the current version of this code is its ability to generate and use body-fitted coordinates. This feature, implemented since Release 3, extends the geometric capabilities of the code by using the facility of multi-block, or block structured grids, which are grids that can be constructed by

glueing together an arbitrary number of topologically rectangular grids or blocks. Other features implemented through the years in the development of this code which make it more powerful are: different treatments for turbulence ($k-\varepsilon$ and low Reynolds number $k-\varepsilon$ turbulence models, Reynolds stress models, etc.); multiphase flow models, combustion models, porous media models, interactive interface front-end, adaptive time stepping for transient simulation, etc.

CFX-F3D is bundled in a CFDS suite of programs consisting of four main modules:

- (a) Pre-processing module or geometry and grid generators
- (b) CFX-F3D front-end module
- (c) CFX-F3D solution module
- (d) Post-processing or graphics modules

The pre-processing module can be used to create a computational grid based on the topology of the system. Simple geometries may be defined without the pre-processing module. However, complex geometries that require extensive use of body-fitted grids can be easily created using the pre-processing module. After the geometry is defined, the grid generator allows the user to subdivide each block created into the desired number of cells in each direction (x, y, z) and then, by propagating this subdivision information to every block in the geometry, the code calculates the overlaying grid and performs the necessary fitting to any other user-defined features.

The finite difference grid may be described in cartesian, cylindrical or body-fitted coordinates. For body-fitted coordinates, a curvilinear coordinate transformation is used to map a complex flow domain in physical space to a rectangular flow

domain in the computational space, in a procedure similar to that developed in the derivation of the bipolar grid presented in section 5.1.2. However it should be noticed that the bipolar solution of stratified flow is an analytical transformation, and that for more complex geometries no analytical transformation exists. Therefore, to overcome these limitations, the CFX-F3D code performs numerical transformations (body-fitted coordinates) which extend its capacity to handle the most complex geometries.

The *command file* that contains instructions which define the problem can be written either in the *frontend* module, or directly, as a programming language, once the user has gained confidence in his/her use of CFX-F3D. Parameters that can be defined in the *command file* language include: laminar flow or turbulence model to be used; single or multiphase flow; compressible or incompressible flow, steady or transient flow; solution of the energy equations or isothermal flow; solution of other user defined scalar transport equations.

Through the *command language* the user also possess a high level of control over the operation of the solver, including the choice of algorithm and convergence criteria. The treatment of the advective terms in the differencing schemes available in CFX-F3D are: upwind, hybrid (Wilkes and Thompson, 1983), higher-upwind (HUW) (Wilkes and Thompson, 1983) or quadratic upwind (QUICK) (Leonard, 1979). The upwind and hybrid schemes are formally first order accurate, whereas HUW is second order accurate and QUICK is third order accurate. Despite the fact that the accuracy is higher with HUW and QUICK the calculation time per iteration is increased and they are more difficult to converge than the hybrid differencing scheme. Again, there must be a compromise between accuracy and computational costs.

The most experienced and demanding users can also specify their problems with FORTRAN subroutines that can be linked with the CFX-F3D main code. These subroutines have names and framework predefined by CFDS-AEA, however they can be more-or-less freely modified by the user, allowing him/her to interact more directly with the main CFX-F3D code. Therefore the subroutines present in CFX-F3D *FORTRAN files*, are much more powerful than the instructions present in the *command files*.

A case study can be run in CFX-F3D after its command file, and optionally, its FORTRAN and geometry files have been prepared. The user can start this run either by using the interactive flow solver module or by a line command from any computer text terminal. When the program is running the general discretised equations are made to fit with the user specifications and the problem is solved in the computational space coordinates. All variables are evaluated in the centre of the cells or control-volumes in a non-staggered grid formulation. It must be noted here, that in order to avoid chequerboard oscillation in pressure and velocity inherent to the use of non-staggered grids for these variables, CFX-F3D uses the Rhie-Chow method, by which the velocities are consistently interpolated on the control-volume faces, in order to produce realistic velocity and pressure fields.

The results of CFX-F3D are presented in two files: an *output* file and a *dump* file. The output file is an ASCII file which comprises large tables that can be read by the user. It contains the input information of the problem, the values at each iteration of a monitoring grid point, which is very useful to check the convergence of the problem, and the solution of the velocity fields, temperature fields and other user scalar fields according to input files (command and FORTRAN files) submitted by the user. The dump file can be either formatted or unformatted. If unformatted it is written as a binary file, and can be read by the post processor

VIEW, one of the same CFDS suite of programs as CFX-F3D. In the case of a formatted dump file the value of the variables are written in an ASCII file whose format should be specified by the user in the command file. This formatted file can be post-processed by any data handling program of the user's preference. Also, through the FORTRAN files, the user can create as many auxiliary output files as desired, as in any standard FORTRAN program.

As mentioned above, the data stored in the dump file can be retrieved and handled by the post-processing or graphic module of the CFDS suite of programs. In the installation available at Imperial College the post-processor available was CFDS-VIEW. This program is a graphical package that can translate the dump file information into vector, contour and shade contour plots. There is a limited number of functions such as rotation, translation and expansion of the visualised geometry that permit the user to observe in more detail peculiarities of the solution obtained. This post-processor also has the ability to translate the final diagrams into post-script language files that can, for instance, be printed in any post-script compatible printer.

5.1.3.2- Simulation of liquid-liquid stratified flow with CFX-F3D

Liquid-liquid stratified flow can occur when either the two phases are laminar, turbulent or when one is laminar and the other turbulent. (Russel et al., 1959). This last case, common in gas-liquid flows applications, is rather rare in liquid-liquid flows where the difference of velocities between the fluids is not as great. Thus in the present work two general cases were implemented; i.e. those when the two phases are both laminar or both turbulent respectively.

Numerical codes like CFX-F3D have proved to give good results in the study of single-phase systems. Also, in multiphase systems where the geometry is such that the forces which determine the phase distribution are rather well defined (as in the case of the flow in vessels and separators) computational fluid dynamic codes have been shown to yield, at least, qualitatively acceptable results. In the cases of flow of multiphase systems in tubes, however, the forces that determine the phase configuration have only been established at a macroscopic and semi-empirical level.

At the level of present day scientific knowledge very little can be said of the chaotic processes that determine that fluids of certain properties should be at conditions of one or other flow pattern. This may seem odd, but even today there are aspects of single phase flow which have yet to rely on experimental data. For instance, except at low Reynolds numbers close to the onset of turbulence, models for single phase turbulence rely on the fitting of models to experimental observations to achieve closure.

A similar approach can be adopted in addressing multiphase flow in tubes. That is, the flow configuration can be set externally to the basic equations that govern the flow. In this way, then, the numerical code can be used to determine the velocity and pressure fields that satisfy the imposed boundary conditions. Unfortunately CFX-F3D does not allow the user to set up a flow pattern prior to solving the flow fields. Previous works, Srichai (1984) have solved the gas-liquid stratified flow by treating the gas and the liquid phases separately. The separate geometries and grids of each phase were generated with a symmetrically identical boundary at the interface. Then the flow of each phase was calculated in alternated runs of the numerical solver. The pressure gradient in both phases was assumed to be the same, and no-slip boundary conditions were applied to the pipe wall in both

geometries. The process started by first calculating the gas flow field. This produced values for the interfacial shear stress which were then used as a boundary condition for the liquid phase calculation. The calculation for the liquid zone, subject to the interfacial shear boundary conditions from the gas phase, would give new values of the interfacial velocity. These were then fed as a boundary condition to the gas phase, and the procedure was repeated. The basis of this iterative method lay in the assumption of the continuity of the interfacial velocity and the interfacial shear stress between the two phases. Hence, the calculation was stopped when the successive iterations gave substantially the same the shear stress and velocity for the two phases at the interface. This procedure, though, was not automated, and each run needed to be set and started manually. This process of copying the interface velocity and shear stress values and transferring manually from one run to the other was lengthy and meant that only a limited number of cases could be studied by this method. In the work presented in this present thesis, the interface conditions were included in the flow field solution process of CFX-F3D by a FORTRAN subroutine, which allowed the calculation process to be automated and several conditions tested, as it will be shown below.

The half-pipe geometry was divided in six blocks, as shown in Figure 5.14. The interface between blocks 2 and 4 and between blocks 3 and 5 was set topologically as a moving solid wall, and momentum was transferred from one side of the wall to the other, guaranteeing continuity of momentum and shear stress, through a FORTRAN subroutine linked to the main solver code. The interface conditions expressed by in the coordinate system shown in Figure 5.14 read:

a) *No slip at the interface:*

$$w_T = w_B \quad (5.97)$$

b) *Continuity of shear stress at the interface:*

$$\tau_{iT} = \tau_{iB} \quad (5.98)$$

Two situations of stratified flow between two liquids were implemented: one when the flow of both fluids is laminar; and the other when the flow of both fluids is turbulent. For laminar flow, Equation (5.98) implies:

$$\eta_T \frac{\partial w_T}{\partial y} = \eta_B \frac{\partial w_B}{\partial y} \quad (5.99)$$

Which, when discretised yields:

$$\eta_T \frac{(w_{T1} - w_i)}{(y_{T1} - y_i)} = \eta_B \frac{(w_i - w_{B1})}{(y_i - y_{B1})} \quad (5.100)$$

where w_i and y_i are the velocity and the y coordinate of the interface and w_{T1} , y_{T1} and w_{B1} , y_{B1} are the velocities and y coordinates prevailing at the nodes just above and below the interface.

Rearranging Equation (5.100), an explicit expression for w_i is obtained:

$$w_i = \frac{\frac{\eta_T}{(y_{T1} - y_i)} w_T + \frac{\eta_B}{(y_i - y_{B1})} w_B}{\frac{\eta_T}{(y_{T1} - y_i)} + \frac{\eta_B}{(y_i - y_{B1})}} \quad (5.101)$$

Therefore, by imposing equation (5.100) for every point of the interface one guarantees both the continuity of velocities and shear stress across it. Equation (5.100) was implemented in a subroutine FORTRAN that is called at each iteration by the CFX-F3D flow solver and sets the boundary conditions at the topological wall that is the interface line. This process is automatic and continuous and provides a simple way of calculating the flow fields of the two fluids at the same time that the equilibrium interface velocity (that will satisfy the interface conditions, Equations 5.98 and 5.99) is also found.

When the regime of the two fluids is turbulent, the k - ε model was used to describe the flow far from the walls (see a description of the k - ε in the appendix). However, near the walls it was assumed that the turbulence would be damped and that a log law wall velocity profile would exist. Therefore, the continuous shear stress boundary condition given by Equation (5.98) is now expressed by:

$$\rho_T w_T^{*2} = \rho_B w_B^{*2} \quad (5.102)$$

where w_T^* and w_B^* are the slip velocities at each “side” of the interface.

Writing the log law velocity profile for each “side” of the interface and solving it for the slip velocities, the following expressions are obtained:

$$w_T^* = \frac{\kappa(w_T - w_i)}{\log y_T^+ + B\kappa} \quad (5.103)$$

$$w_B^* = \frac{\kappa(w_B - w_i)}{\log y_B^+ + B\kappa} \quad (5.104)$$

where κ is the Von Karmann constant ($\kappa=0.4$), B is a log layer constant ($B=5.45$) and y_T^+ and y_B^+ are the local Reynolds number near the interface, defined by:

$$y_T^+ = \frac{\rho_T w_T^* y_T}{\eta_T} \quad (5.105)$$

$$y_B^+ = \frac{\rho_B w_B^* y_B}{\eta_B} \quad (5.106)$$

Now, substituting Equations (5.103) and (5.104) in Equation (5.102) and solving for w_i yields:

$$w_i = \frac{H_T w_T + H_B w_B}{H_T + H_B} \quad (5.107)$$

where

$$H_T = \frac{\sqrt{\rho_T}}{\log y_T^+ + B\kappa} \quad (5.108)$$

$$H_B = \frac{\sqrt{\rho_B}}{\log y_T^+ + B\kappa} \quad (5.109)$$

The procedure for calculating the velocity of the interface at each iteration implemented in the FORTRAN subroutine of the CFX-F3D flow solver was thus as follows.

First, calculate the values of y_T^+ and y_B^+ with Equations (5.105) and (5.106) using the w_T^* and w_B^* values of the previous iteration. Then evaluate H_T and H_B with Equations (5.108) and (5.109) and, finally, calculate w_i with Equation (5.107).

In the turbulent calculations, as in the laminar case, the value of the velocity of the interface is iterated together with the solution of the flow field, without the need of any inner iteration. This process was completely automated, and allowed each flow configuration (for given fluid properties and interface position) to be calculated with a single run of CFX-F3D, without any manual interventions as those necessary in the work described by Srichai (1994) and Wolf (1995). Thus a considerable number of combinations of different fluid properties and interface positions for the interface could be simulated. These results are shown in the following sections.

5.2- Results

Several stratified flow cases were simulated with the *bipolar code* and with CFX-F3D. The flows simulated in the bipolar code were all laminar, but with varying interface position. The cases run on the CFX-F3D flow solver were laminar for

both phases or turbulent for both phases. Three geometries representing different interface positions were also generated for the accomplishment of the CFX-F3D studies. These results are presented in the next sections. Furthermore, for the case where the interface passed through the tube axis, the code results are compared with the results of the analytical solution presented in Section 5.1.1. Finally the one-dimensional model presented in Chapter 4 is assessed against the numerical data obtained.

5.2.1- Simulations with the bipolar code

As mentioned in section 5.1.2, the *bipolar program* implemented can simulate stratified flow of two fluids in the laminar regime. One property of laminar flows is the direct proportionality between the flow field and the pressure gradient. These solutions, therefore, are better presented in dimensionless units, such as those introduced by Equations (5.11) and (5.12) in Section 5.1.1. The program was “forced” to give dimensionless solutions by inputting values of the pipe diameter, the pressure gradient and the bottom fluid viscosity fluid equal to one; and by entering the top fluid viscosity with the ratio of the top fluid and bottom fluid viscosities $\left(\tilde{D} = 1 ; \frac{\tilde{\mathcal{P}}}{\tilde{\mathcal{Z}}} = 1 ; \eta_B = 1 \text{ and } \eta_T = \frac{1}{\beta} \right)$ as can be observed from Equations (5.11), (5.12) and (5.16).

Three viscosity ratios ($\beta = 1/1.6$ corresponding to the TOWER system, $\beta = 1/45$ corresponding to the WASP system and $\beta = 1/16$ which is intermediate between the values for TOWER and WASP) and five interface positions were simulated, as shown in Table 5.2. The calculated flow rates, the generalised Martinelli

parameter and the pressure multiplication factor are also indicated in Table 5.2. The velocity profiles on the symmetry plane are shown in Figures 5.15 and the velocity profiles on the interface are shown in Figure 5.16. The shear stress on the walls is shown in Figure 5.17 and the shear stress on the interface in Figure 5.18. The plot of interface height and the pressure multiplication factor (ϕ_T) against the generalised Martinelli parameter (X_{TB}) are shown in Figure 5.19 and 5.20. It is noteworthy to observe that the generalised Martinelli parameter defined in Chapter 4, Equation (4.36), is equivalent to Equation (5.110) below:

$$X_{TB} = \sqrt{\frac{f_B \rho_B U_B^2}{f_T \rho_T U_T^2}} \quad (5.110)$$

where U_B and U_T are the superficial velocities of the two fluids, defined generically by:

$$U = \frac{\dot{V}}{A} \quad (5.111)$$

where A is the cross-sectional area of the tube.

However, for laminar flow the friction factor is given by:

$$f = \frac{16}{\text{Re}} = \frac{16\eta}{\rho U D} \quad (5.112)$$

Substituting Equation (5.111) into Equation (5.110), it can be shown that:

$$X_{TB} = \sqrt{\frac{U_B \eta_B}{U_T \eta_T}} \quad (5.113)$$

Then X_{TB}^2 , a parameter commonly used in one-dimensional analysis, is shown, for laminar flow, to be equal to the flow rate ratio between the two fluids multiplied by their viscosity ratio:

$$X_{TB}^2 = \frac{\dot{V}_B \eta_B}{\dot{V}_T \eta_T} \quad (5.114)$$

5.2.2- Simulations with CFX-F3D

The simulations using the commercial code CFX-F3D are summarised in Table 5.3 and 5.4. The grids for the cases where the interface was at 37.5%, 50% and 62.5% of the pipe diameter are shown in Figures 5.21, 5.22 and 5.23 respectively. The cases where the interface was set at 67.5% of the pipe diameter used the same 37.5 % grid, however with the fluids disposed inversely. Though this was clearly unphysical in the sense that the lighter fluid would always be at the top, this inversion of the fluids did not affect the results since the flow considered was purely axial and horizontal and the effects of gravity can be neglected. The laminar cases were simulated in dimensionless units in the same form as those used for the analytical solution (Section 5.1.1) and the bipolar code (Section 5.2.1). The turbulent cases, using the κ - ϵ model, on the other hand, can not be reduced to dimensionless equations due to the fact that in this model not only the linear momentum conservation equation should be made dimensionless but also the equations for the conservation of kinetic energy and kinetic energy

dissipation All these equations, despite having the same form of Equation (5.75), have extra variables playing an important role in the solution of the flow field (the densities of the fluids for example) which do not allow a consistent set of dimensionless variables to be devised for general solutions of the problem. Therefore, in these turbulent cases the geometry used was those of the TOWER rig, that is pipe diameter of 2.50 cm. The pressure gradient imposed were of 280 and 2800 Pa/m, which are characteristic values of the experiments performed (Chapter 3).

The solution of laminar cases (Table 5.3) are shown in Figure 5.24 (velocity profiles on the symmetry plane); Figure 5.25 (velocity profiles on the interface) and Figure 5.26 (shear stress on the interface). Identical plots for the turbulent cases (Table 5.4) are shown in Figures 5.27, 5.28 and 5.29. Plots of the interface height and the pressure multiplication factor (ϕ_T) against the generalised Martinelli parameter (X_{TB}), showing the results of the laminar and turbulent flow calculations are shown in Figures 5.30, 5.31, 5.32 and 5.33, respectively.

The whole flow field of laminar and turbulent cases where the viscosity ratio is equal to 1.6 are shown in three-dimensional shaded contour vector plots in Figures 5.34 to 5.39.

5.2.3- Comparison of the results obtained with different methods

Solutions for the stratified flow of two fluids were obtained by three methods: an analytical solution (for the case when the interface passed through the tube axis), a numerical solution using a conformal transformation (bipolar coordinates) and a

numerical solution using body-fitted coordinates with a commercial flow solver (CFX-F3D).

Comparison between the analytical solution and the results obtained numerically by the bipolar code and the moving wall method implemented on the CFX-F3D program and are shown in: Figures 5.40 (velocity profiles on the symmetry plane), Figures 5.41 (velocity profiles on the interface plane) and Figures 5.42 (shear stress on the interface plane) for the case where the interface traverses the tube axis. It can be seen that the bipolar numerical method reproduces well the exact solution obtained analytically. The solution obtained with the method implemented on the CFX-F3D program also has good overall agreement with the analytical solution. However, as seen in Figure 5.40, CFX-F3D underestimates the velocity field of the bottom (less viscous) phase. Regarding the properties of the fluids in the system, the performance of the bipolar code seems to be unaffected by the increase in the viscosity ratio of the top and bottom fluids, on the other hand, as shown in Figures 5.40, 5.41 and 5.42, the discrepancy between the results of the CFX-F3D code and the analytical solution grows with increasing viscosity ratios. Also, as a consequence of the underestimation of the velocity field the interface shear stress predicted by the CFX-F3D code is slightly smaller than the values obtained analytically.

Data from all the cases referred to in this Chapter (i.e. those for all interface positions) were plotted in two graphs, Figure 5.43 and 5.44. The first figure is a plot of the dimensionless interface height versus the generalised Martinelli parameter ($\tilde{h}$ vs X_{TB}) and the second one a plot of the pressure drop multiplier versus the generalised Martinelli parameter (ϕ_T vs X_{TB}). The solution of the one-dimensional model presented in Chapter 4. is also plotted in these graphs. It can

be seen that a good agreement exist among the numerical solutions and the one-dimensional model, especially in the laminar flow cases. In turbulent flow, the one-dimensional model predicts higher values of the dimensionless interface level and the pressure drop multiplier than those estimated from the numerical studies. It is worth observing that an increase in the interfacial shear stress in the one-dimensional model would lead to even larger discrepancies with the numerical solutions. The assumption of a constant shear stress on the interface in the one-dimensional model might play an important part in causing these discrepancies since, as may be seen in Figure 5.29, the numerical calculation indicates a large variation of shear stress along the interface.

In general, however, all the models produced results which were qualitatively consistent and gave the same overall behaviour with an acceptable margin of error as seen from Figures 5.43 and 5.44. Thus, the results obtained in this work, though limited, provide encouragement for further research with the numerical methods presented for modelling liquid-liquid stratified flows, especially the method implemented in the CFX-F3D program where the introduction of more complex interface geometries can be performed without many extra difficulties.

Equation (5.75)	Equations (5.88) and (5.89)
x	ξ
y	μ
φ	w
Γ	η
S	$\frac{\partial \mathcal{P}}{\partial z} \left(\frac{c}{\cosh \xi - \cos \mu} \right)^2$

Table 5.1: Term equivalence between the general diffusion equation (Equation 5.75) and the momentum conservation in bipolar coordinates (Equations 5.88 and 5.89)

η_T/η_B	$\tilde{h}$	$\tilde{V}_T$	$\tilde{V}_B$	X	ϕ_T
1.6	11.70%	0.0083876	0.0002345	0.132202	0.956259
	25.00%	0.0074734	0.001305	0.330361	1.013059
	50.00%	0.0043763	0.0052779	0.868191	1.323857
	75.00%	0.0010992	0.010055	2.391072	2.641536
	88.30%	0.0001889	0.0124496	6.418556	6.37257
16	11.70%	0.0009835	6.818E-05	0.065823	0.883074
	25.00%	0.0009679	0.0004555	0.171505	0.890189
	50.00%	0.0005901	0.0028445	0.548893	1.140083
	75.00%	0.0001412	0.0079379	1.874611	2.330843
	88.30%	2.28E-05	0.0115377	5.624114	5.800286
45	11.70%	0.0003582	3.977E-05	0.049672	0.872511
	25.00%	0.0003548	0.0003379	0.145482	0.876729
	50.00%	0.0002159	0.0025672	0.514000	1.123803
	75.00%	5.128E-05	0.0077299	1.830295	2.306158
	88.30%	8.229E-06	0.0114561	5.562047	5.756668

Table 5.2: Cases simulated with the *bipolar code*

η_T/η_B	$\tilde{h}$	$\tilde{V}_T$	$\tilde{V}_B$	X	ϕ_T
1.6	37.50%	0.006041	0.003005	0.557525	1.1267503
	50.00%	0.004317	0.005175	0.865573	1.3328629
	62.50%	0.0075	0.00254	1.359911	1.7389822
16	37.50%	0.000808	0.001204	0.305254	0.9743708
	50.00%	0.000568	0.002553	0.530165	1.1623801
	62.50%	0.00452	0.00032	0.935544	1.5420633
45	37.50%	0.000287	0.000872	0.259838	0.9747671
	50.00%	0.000198	0.001989	0.472783	1.1744531
	62.50%	0.0037	0.00011	0.863956	1.57359

Table 5.3: Laminar cases simulated with the CFX-F3D code.

η_T/η_B	$\frac{\partial p}{\partial z}$ Pa/m	$\tilde{h}$	$\dot{V}_T$ m ³ /s	$\dot{V}_B$ m ³ /s	X	ϕ_T
1.6	280	37.50%	9.972E-05	5.023E-05	0.56271	1.85091
		50.00%	7.010E-05	8.276E-05	1.29124	2.70959
		62.50%	1.210E-04	4.534E-05	2.59548	3.86941
16	2800	37.50%	2.006E-04	1.845E-04	0.65446	2.11070
		50.00%	1.330E-04	2.622E-04	1.10298	2.59239
		62.50%	4.469E-04	1.106E-04	1.74768	2.54246

Table 5.4: Turbulent cases simulated with the CFX-F3D code.

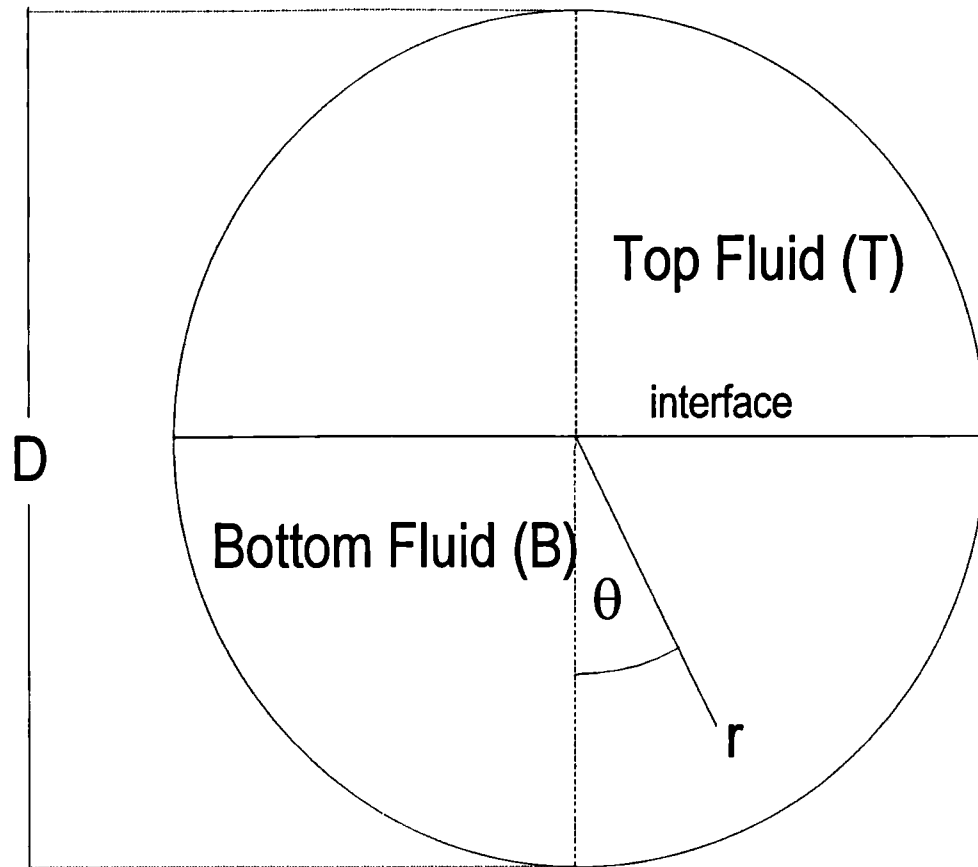


Figure 5.1: Analytical solution. Stratified flow geometry with diametrical interface

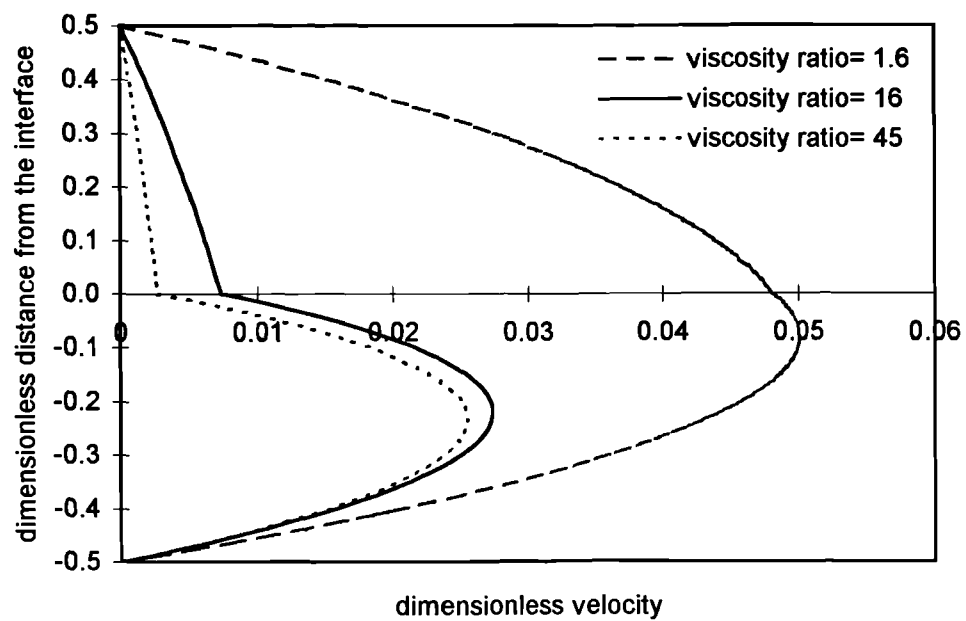


Figure 5.2: Analytical solution. Velocity profiles at the symmetry plane for three viscosity ratios.

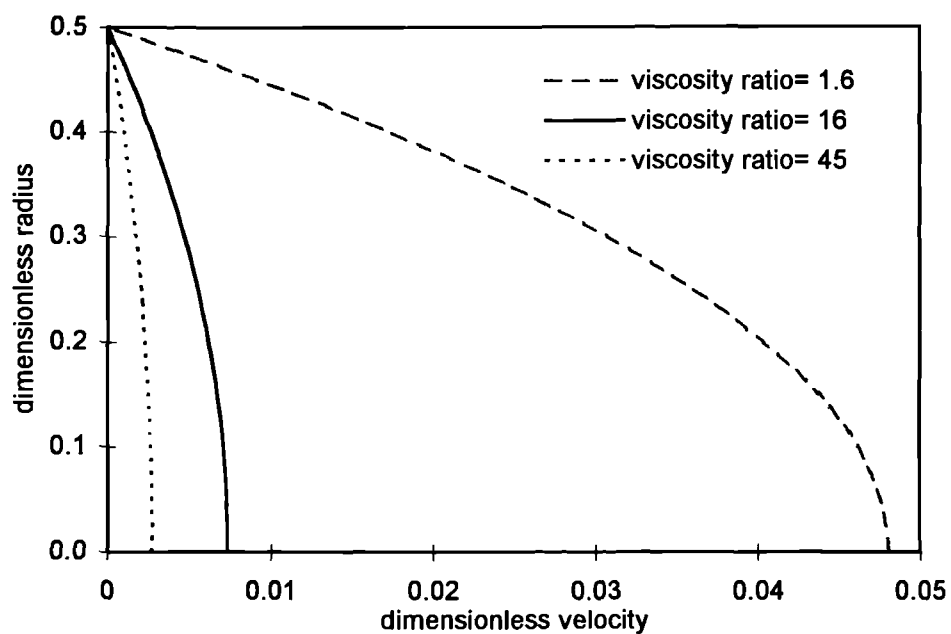


Figure 5.3: Analytical solution. Velocity profiles at the interface plane for three viscosity ratios.

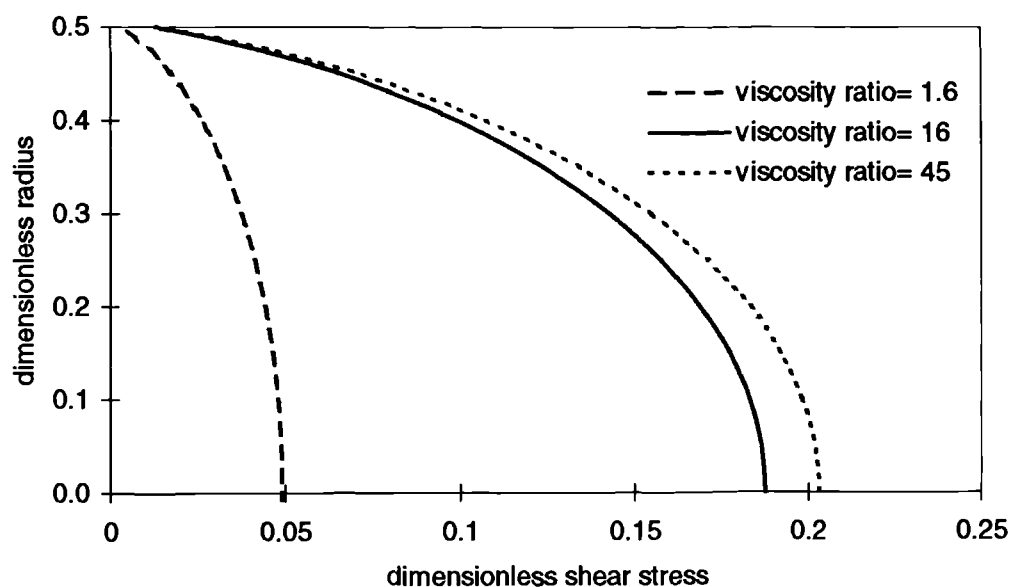


Figure 5.4: Analytical solution. Shear stress profiles on the interface plane for three viscosity ratios.

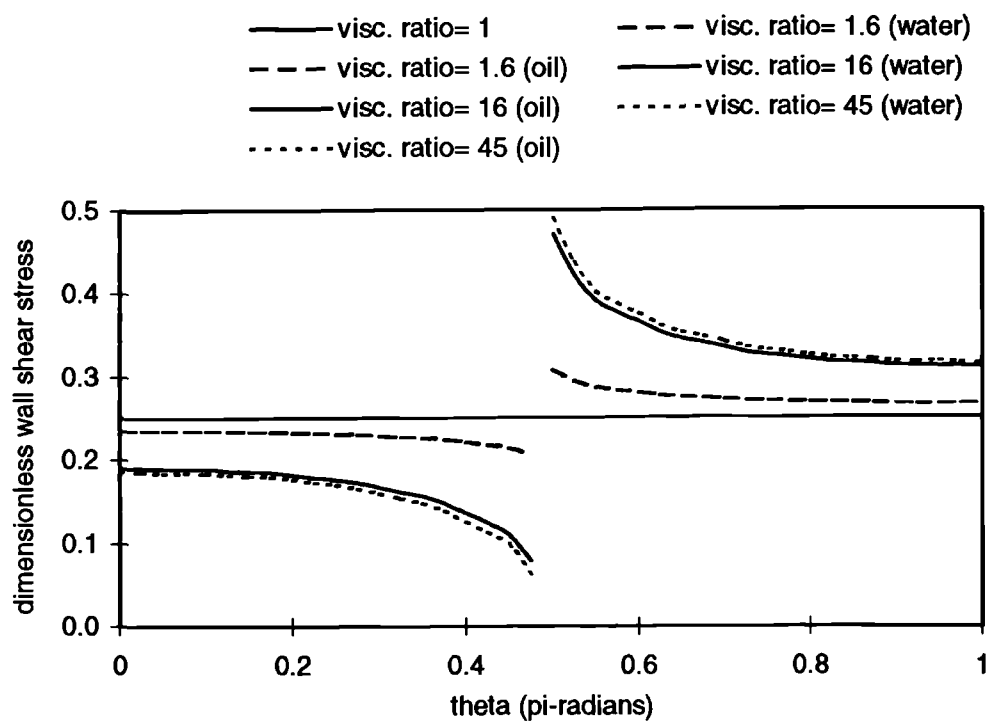


Figure 5.5: Analytical solution. Shear stress profiles on the tube wall for three viscosity ratios.

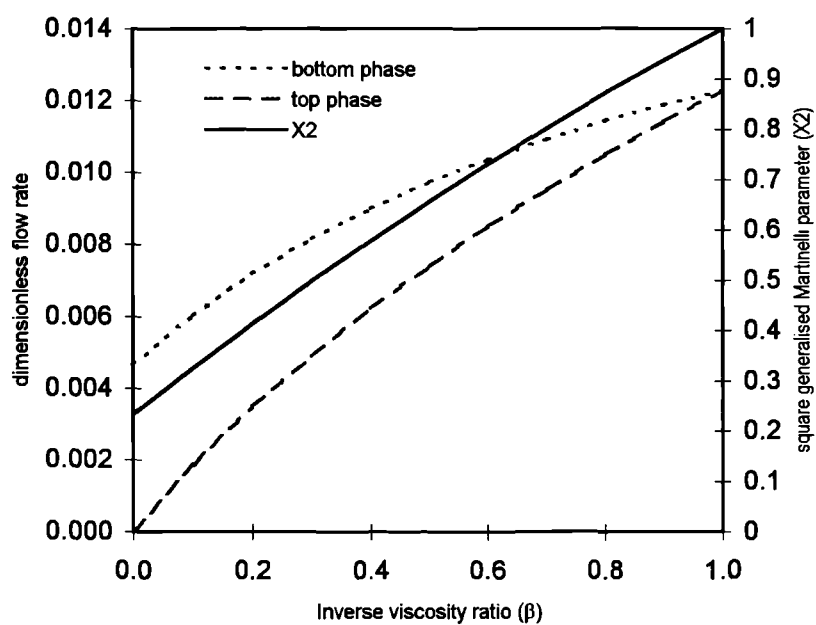


Figure 5.6: Analytical solution. Dimensionless flow rates and the generalised Martinelli parameter squared as functions of the inverse viscosity ratio (β).

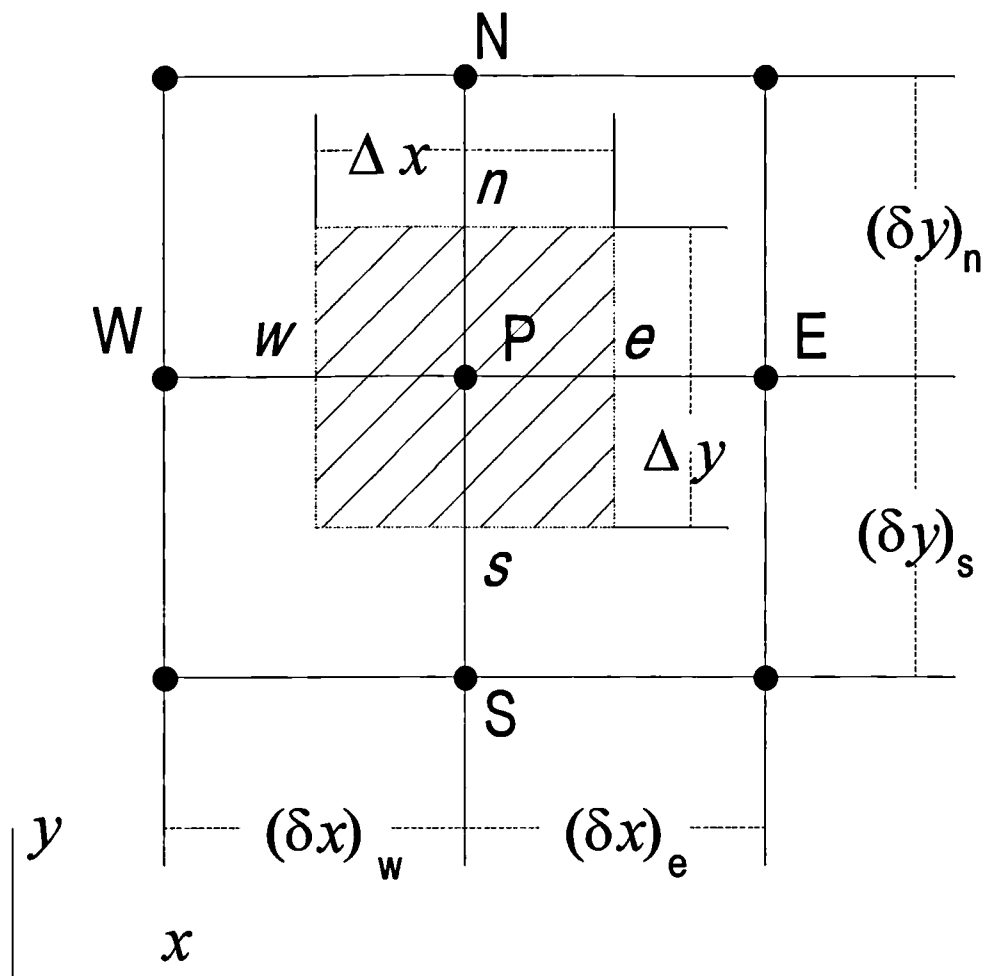


Figure 5.7: Control volume for a two dimensional domain

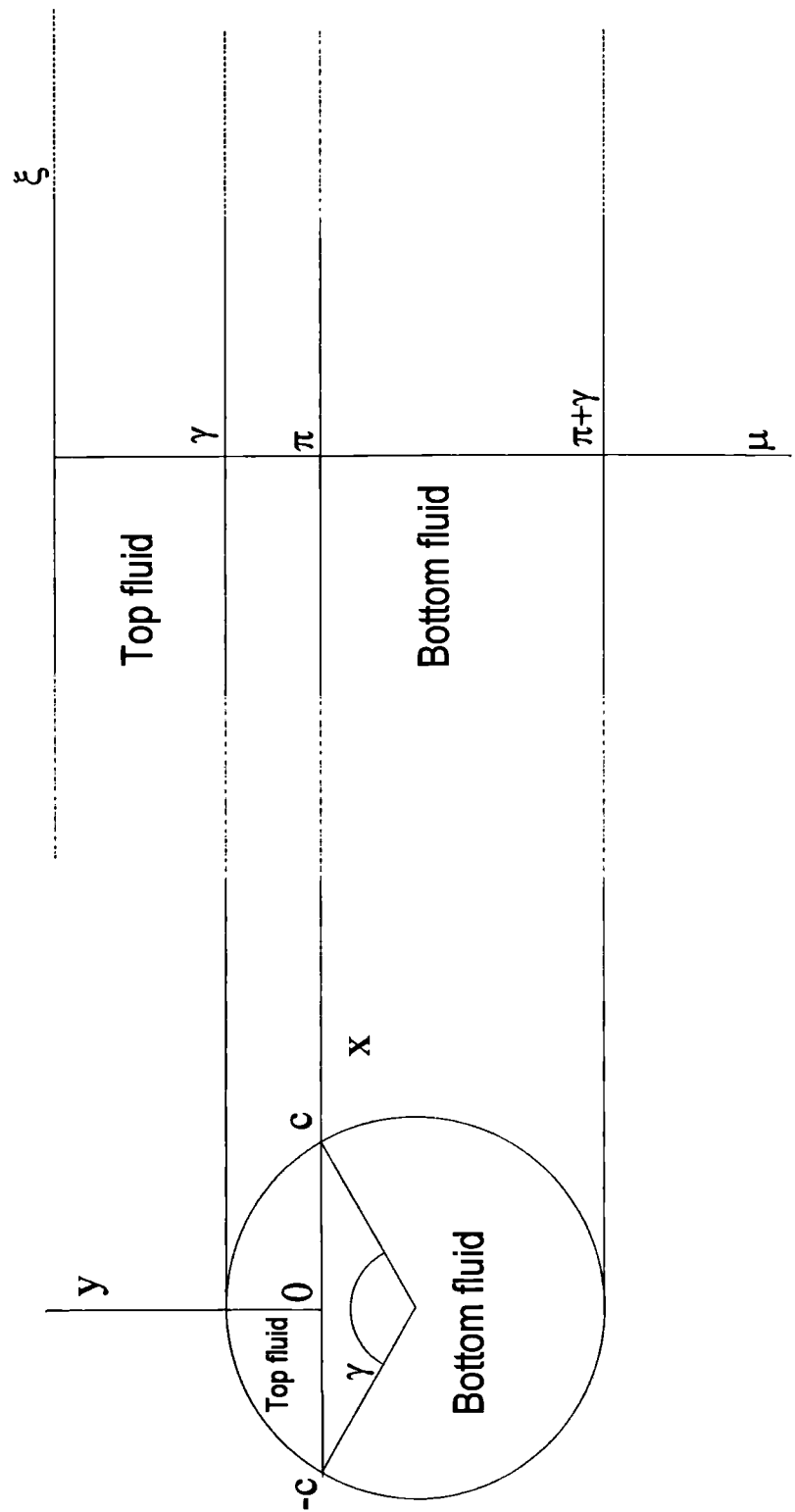


Figure 5.8: Stratified flow geometry and conformal mapping to bipolar coordinates

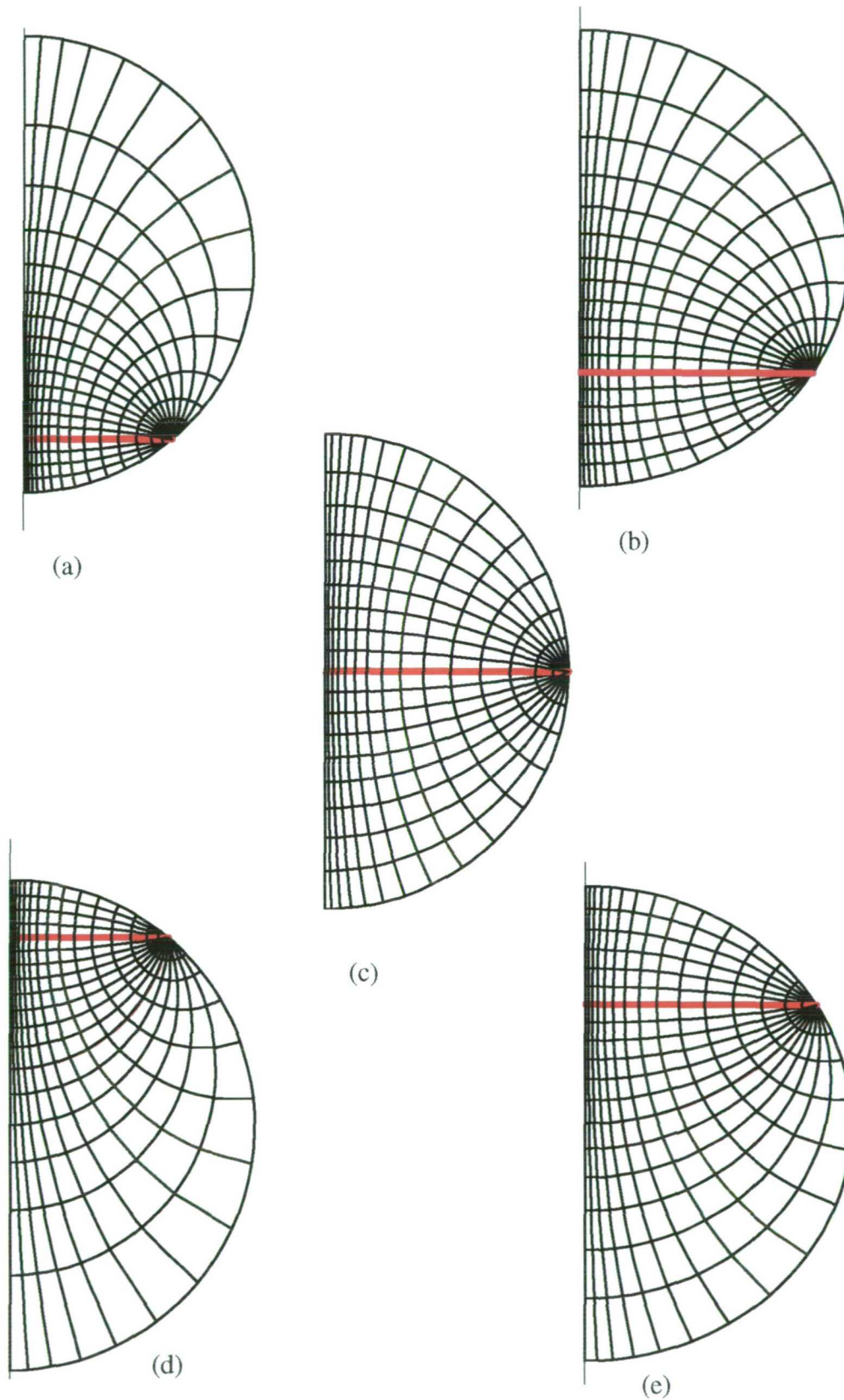


Figure 5.9: Bipolar grid for different interface positions:
(a) 12% ; (b) 25%; (c) 50%; (d) 75%; (e) 88%

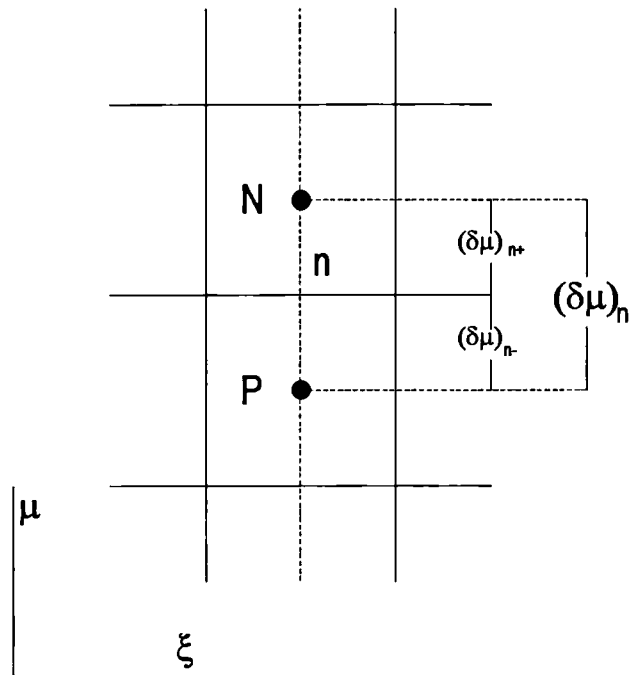


Figure 5.10: Geometry and distances associated to two neighbouring control-volumes of different viscosities.

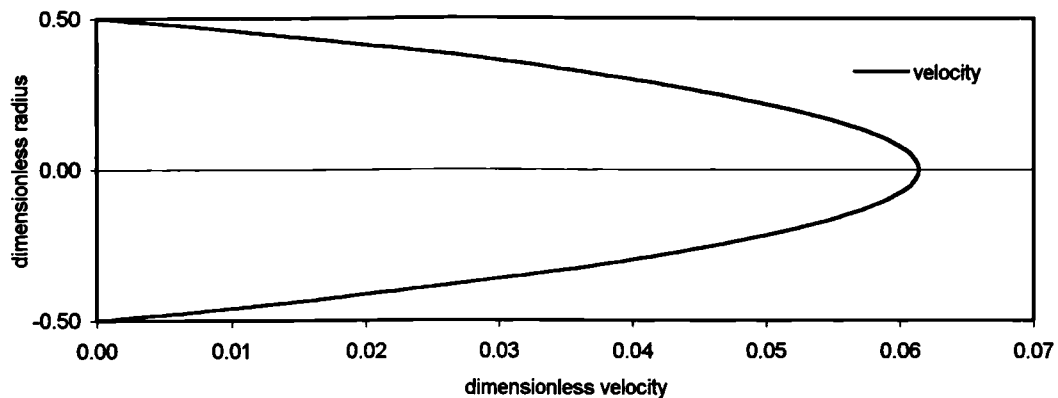


Figure 5.11: Velocity profile. Single phase flow calculated as pseudo-stratified flow (top and bottom fluid with same properties) using a bipolar grid with the interface located at 50% of the tube diameter.

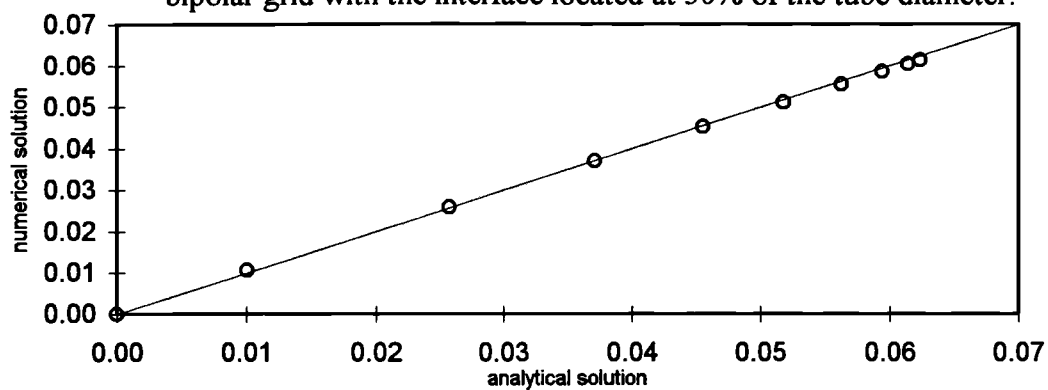


Figure 5.12: Assessment of the bipolar code. Comparison between the velocity fields calculated as a pseudo-two phase flow (top and bottom fluids with the same properties, pseudo interface at 50% of the tube diameter) and the analytical single phase flow solution.

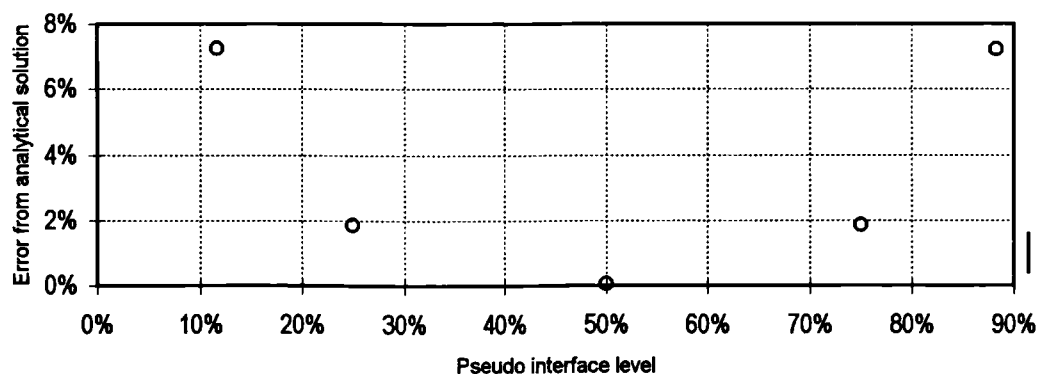


Figure 5.13: Assessment of the bipolar code. Error in relation to the analytical single-phase solution of the flow rates calculated as a pseudo-two phase flow (top and bottom fluids with the same properties) in function of the pseudo-interface location.

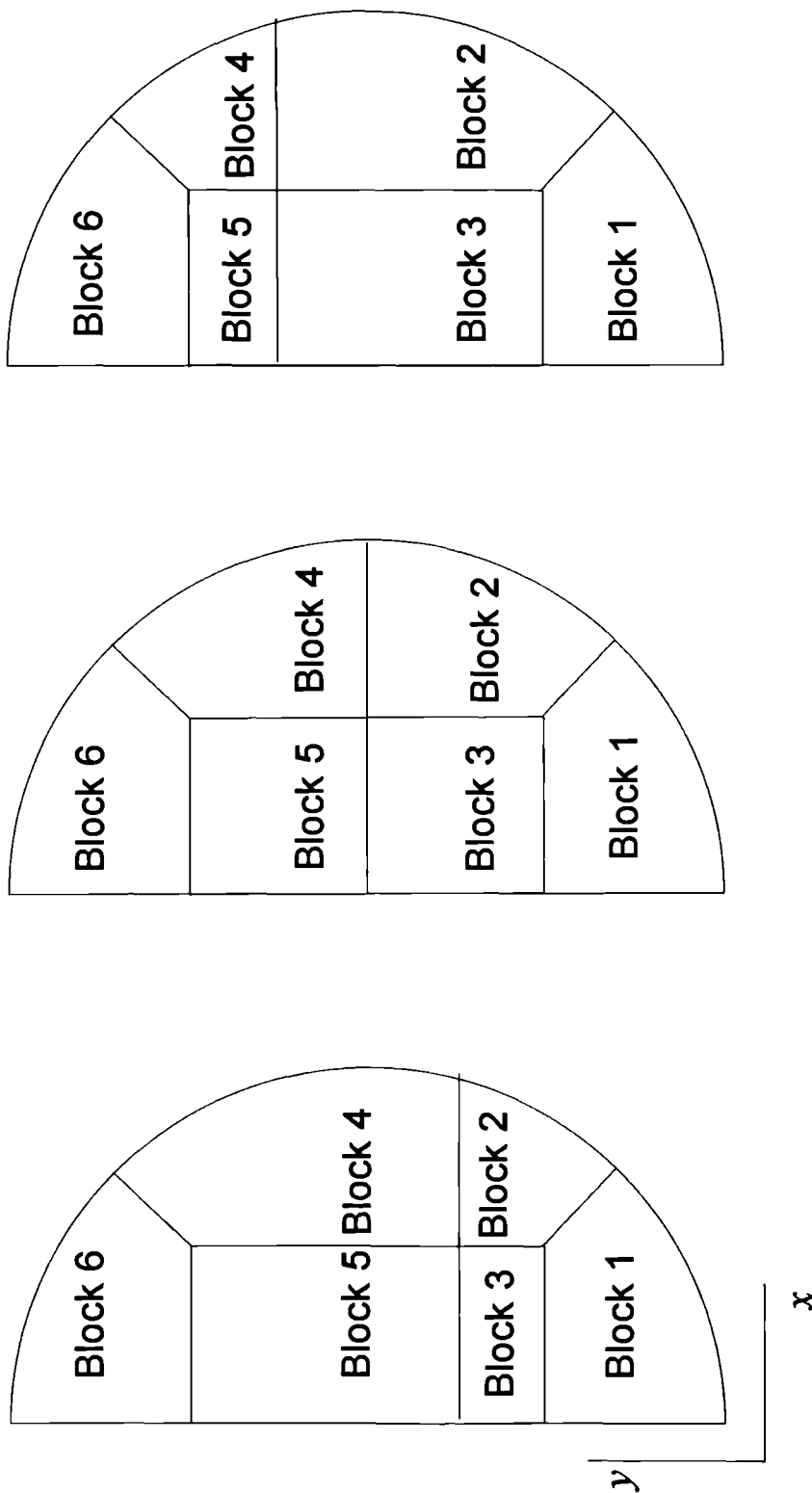


Figure 5.14: Calculations with the method implemented on the CFX-F3D code. Topology and block distribution of the computational domain for three interface positions: (a) 37.5%; (b) 50.0%; (c) 62.5%.

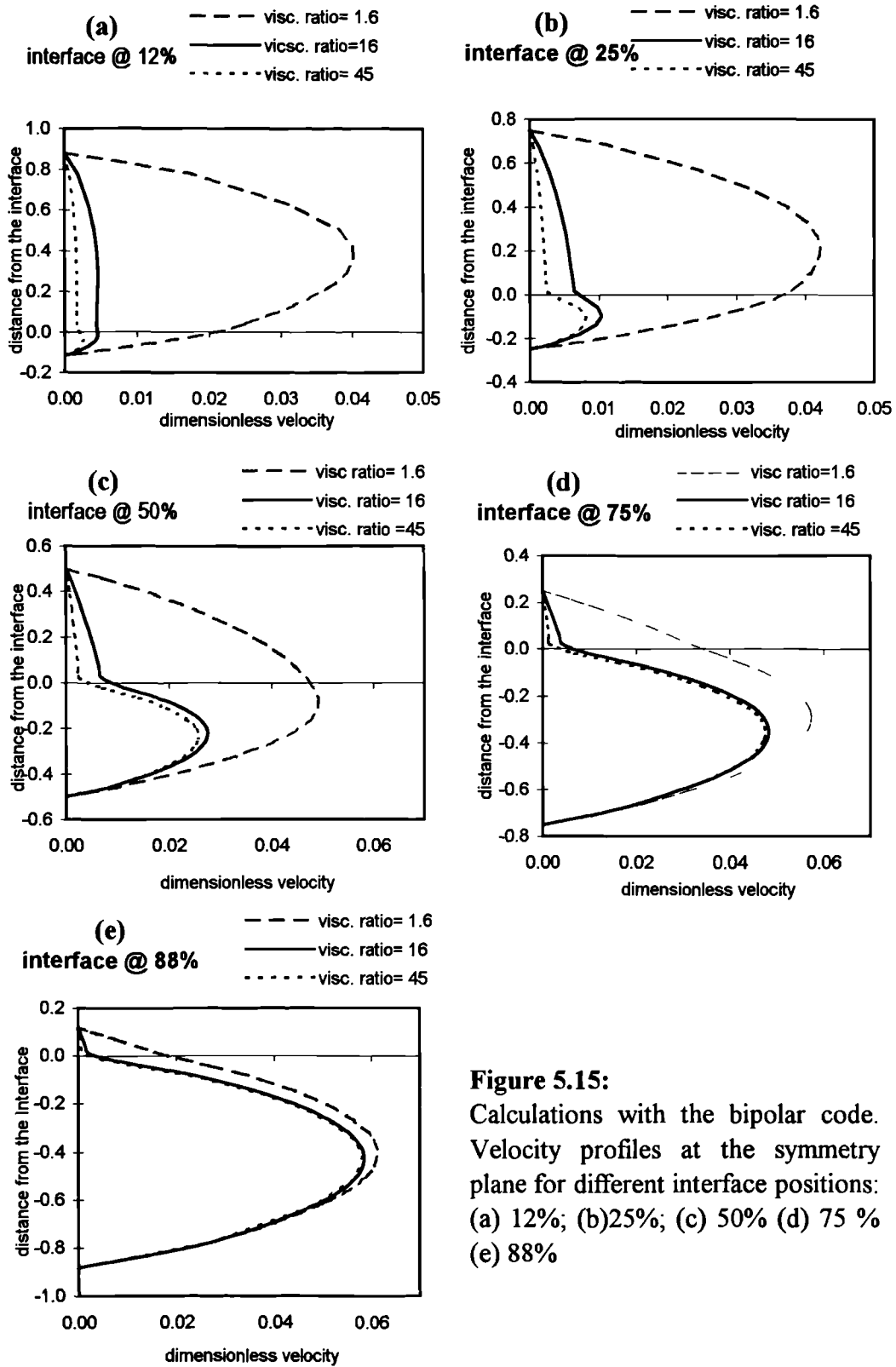
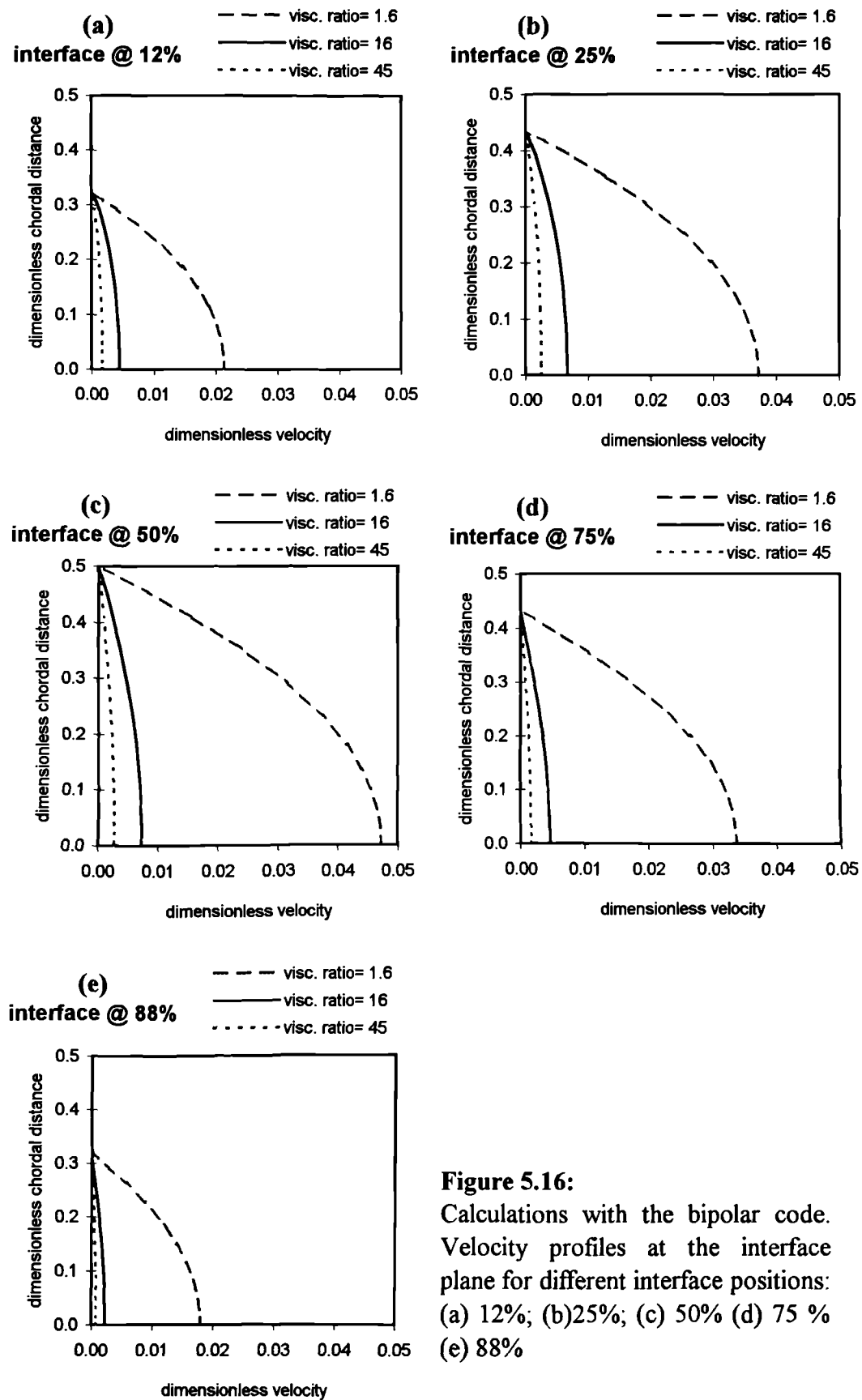


Figure 5.15:
Calculations with the bipolar code. Velocity profiles at the symmetry plane for different interface positions: (a) 12%; (b) 25%; (c) 50% (d) 75 % (e) 88%

**Figure 5.16:**

Calculations with the bipolar code. Velocity profiles at the interface plane for different interface positions: (a) 12%; (b) 25%; (c) 50% (d) 75 % (e) 88%

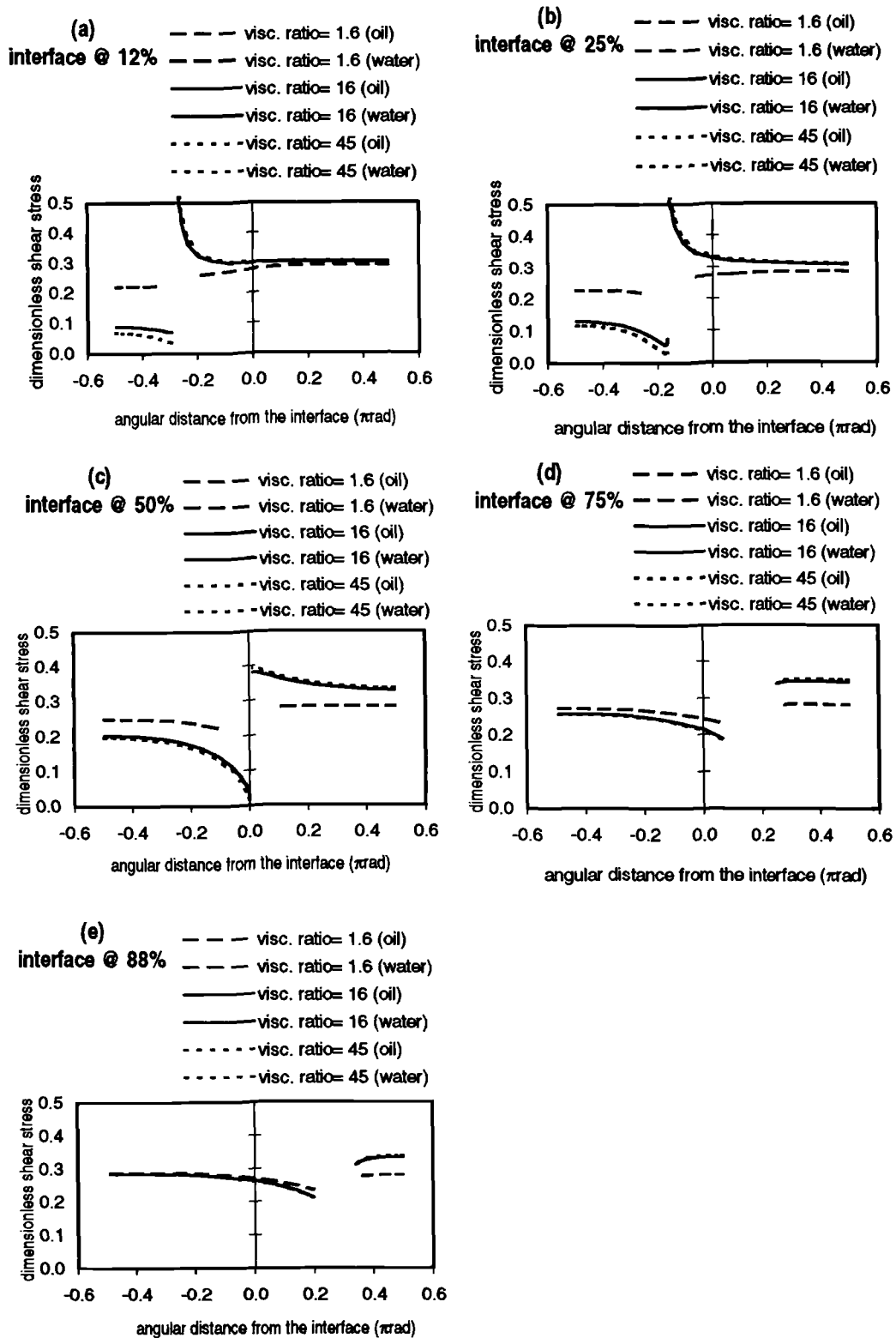
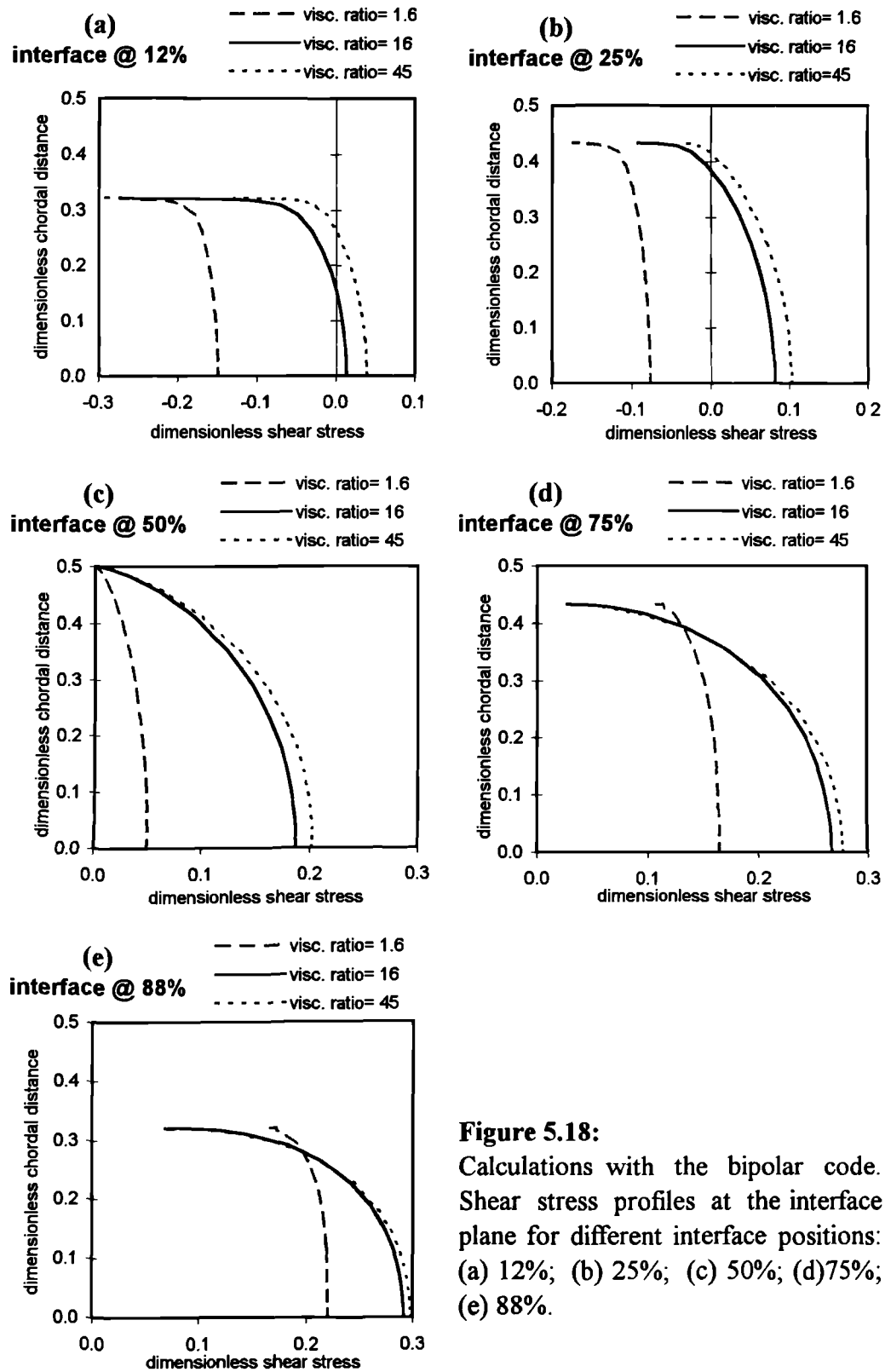


Figure 5.17: Calculations with the bipolar code. Shear stress profiles on the walls for different interface positions: (a) 12%; (b) 25%; (c) 50%; (d) 75%; (e) 88%.

**Figure 5.18:**

Calculations with the bipolar code. Shear stress profiles at the interface plane for different interface positions: (a) 12%; (b) 25%; (c) 50%; (d) 75%; (e) 88%.

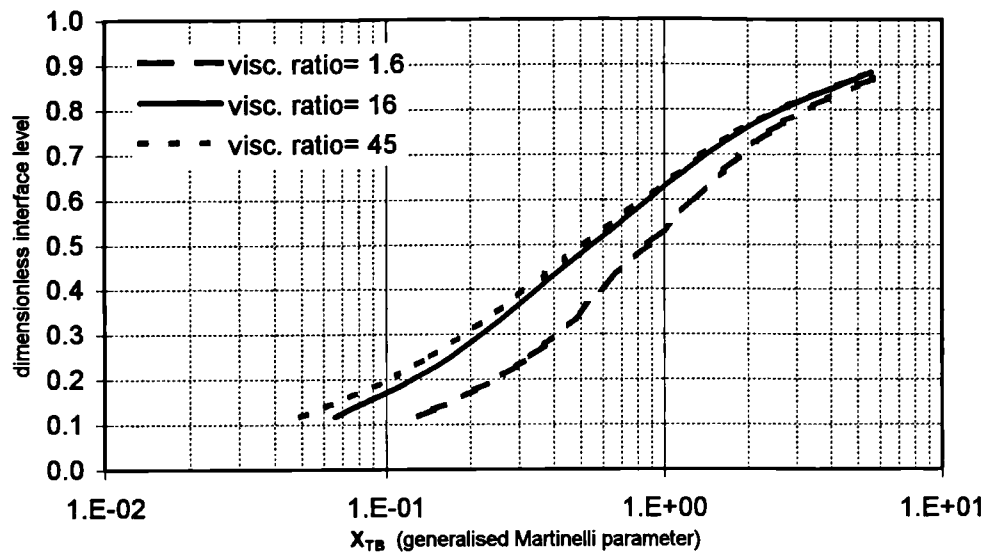


Figure 5.19: Calculations with the bipolar code. Dimensionless interface level versus the generalised Martinelli parameter (X_{TB})

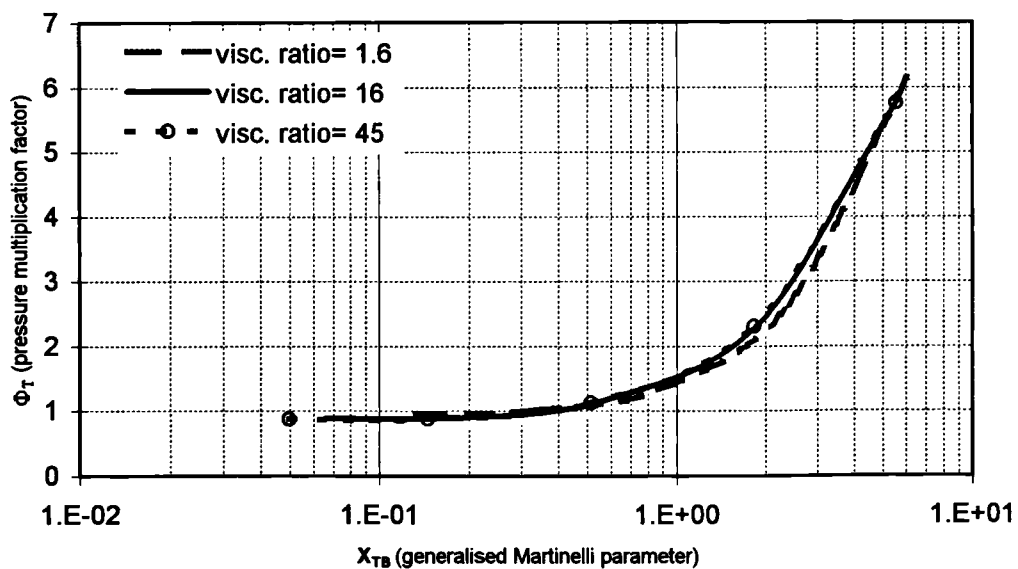


Figure 5.20: Calculations with the bipolar code. Pressure reduction factor (ϕ_T) versus the generalised Martinelli parameter (X_{TB})

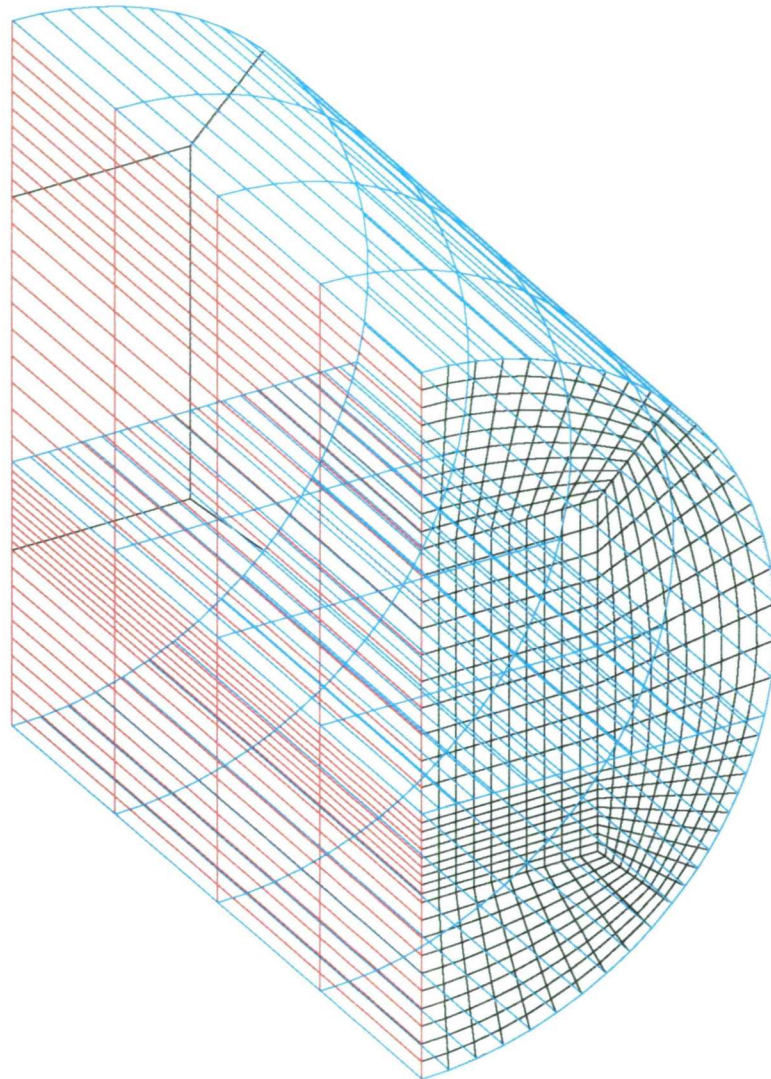


Figure 5.21 Calculations with the method implemented on the CFX-F3D code. Grid for the computational domain of the case when the interface is located at 37.5% of the tube diameter.

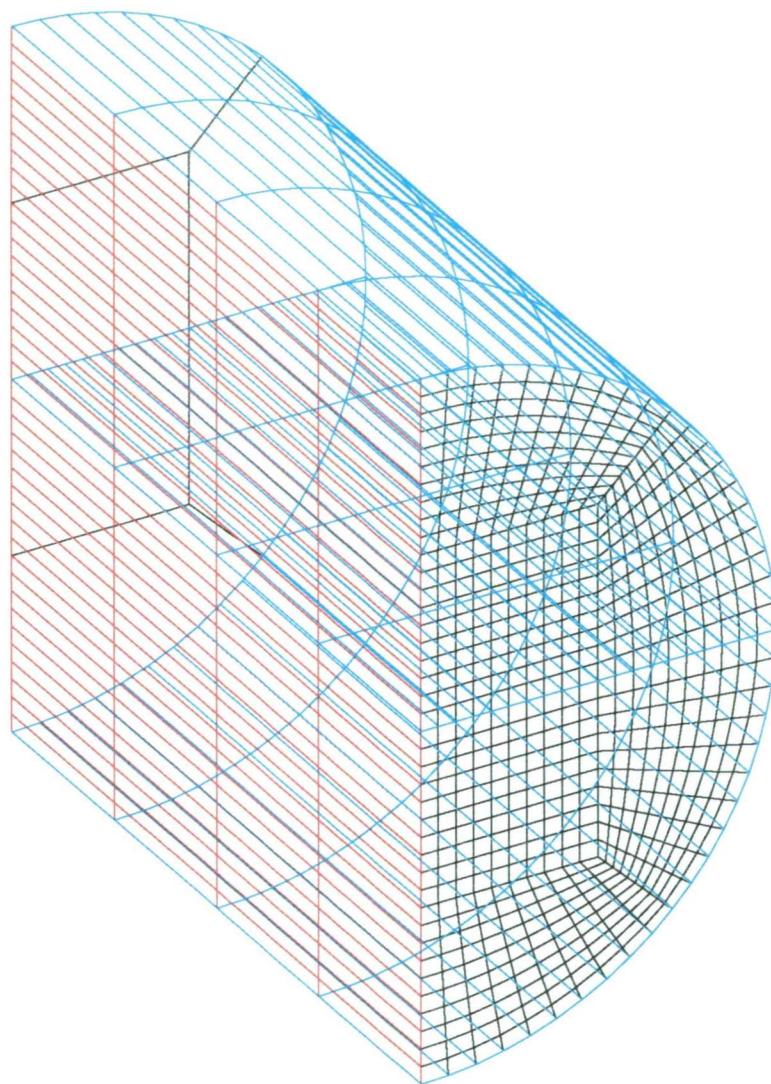


Figure 5.22 Calculations with the method implemented on the CFX-F3D code. Grid for the computational domain of the case when the interface is located at 50.0% of the tube diameter.

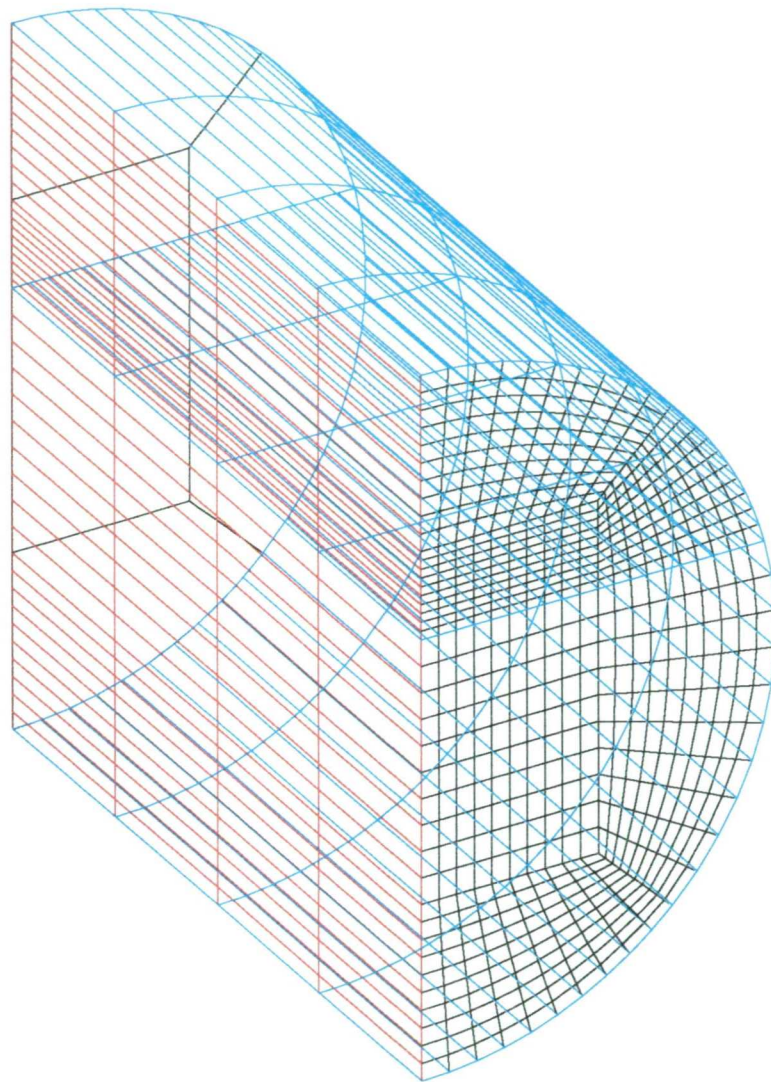


Figure 5.23 Calculations with the method implemented on the CFX-F3D code. Grid for the computational domain of the case when the interface is located at 62.5% of the tube diameter.

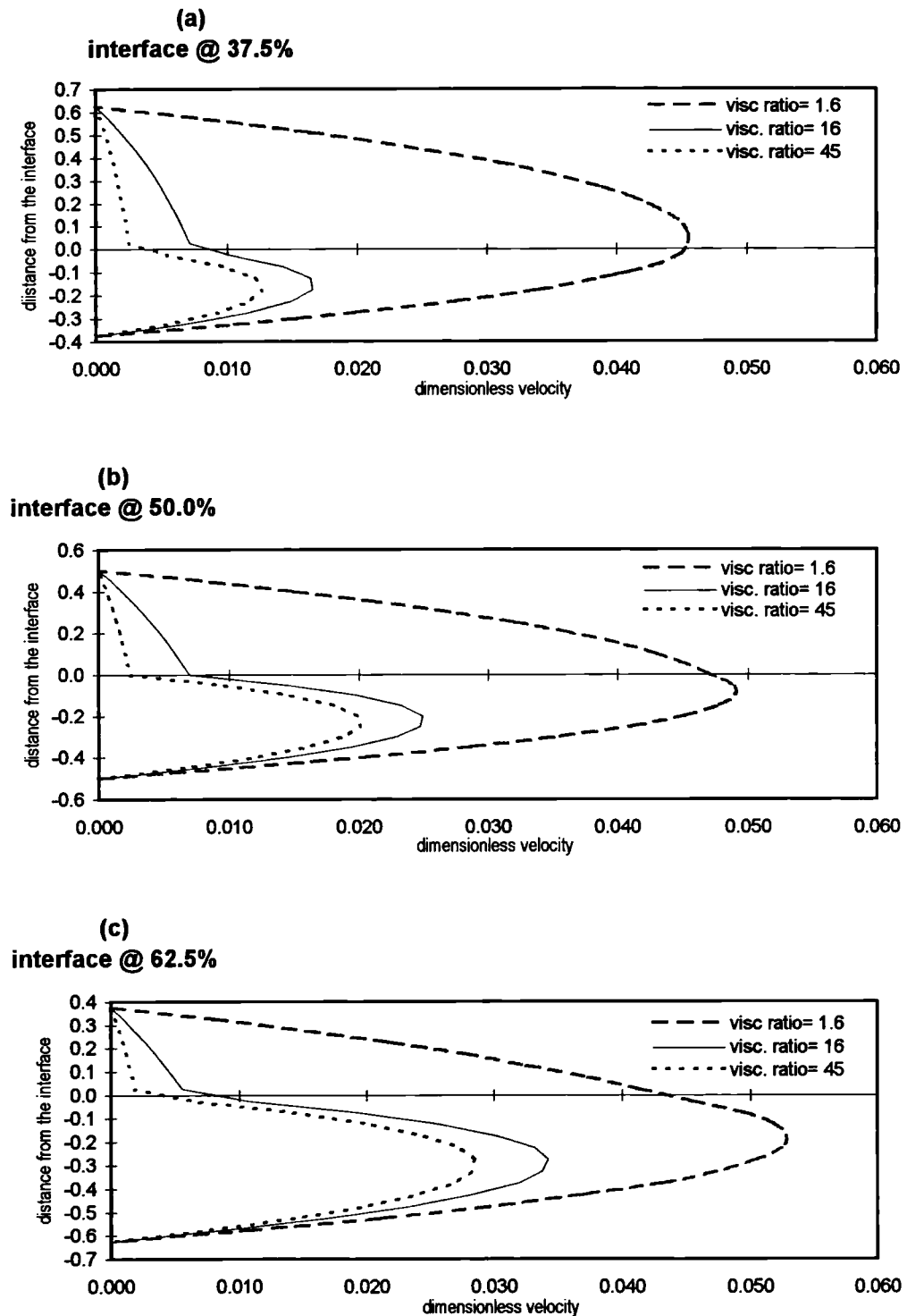


Figure 5.24: Laminar flow calculations with the method implemented on the CFX-F3D code. Velocity profiles at the symmetry plane for different interface positions: (a) 37.5%; (b) 50.0%; (c) 62.5%.

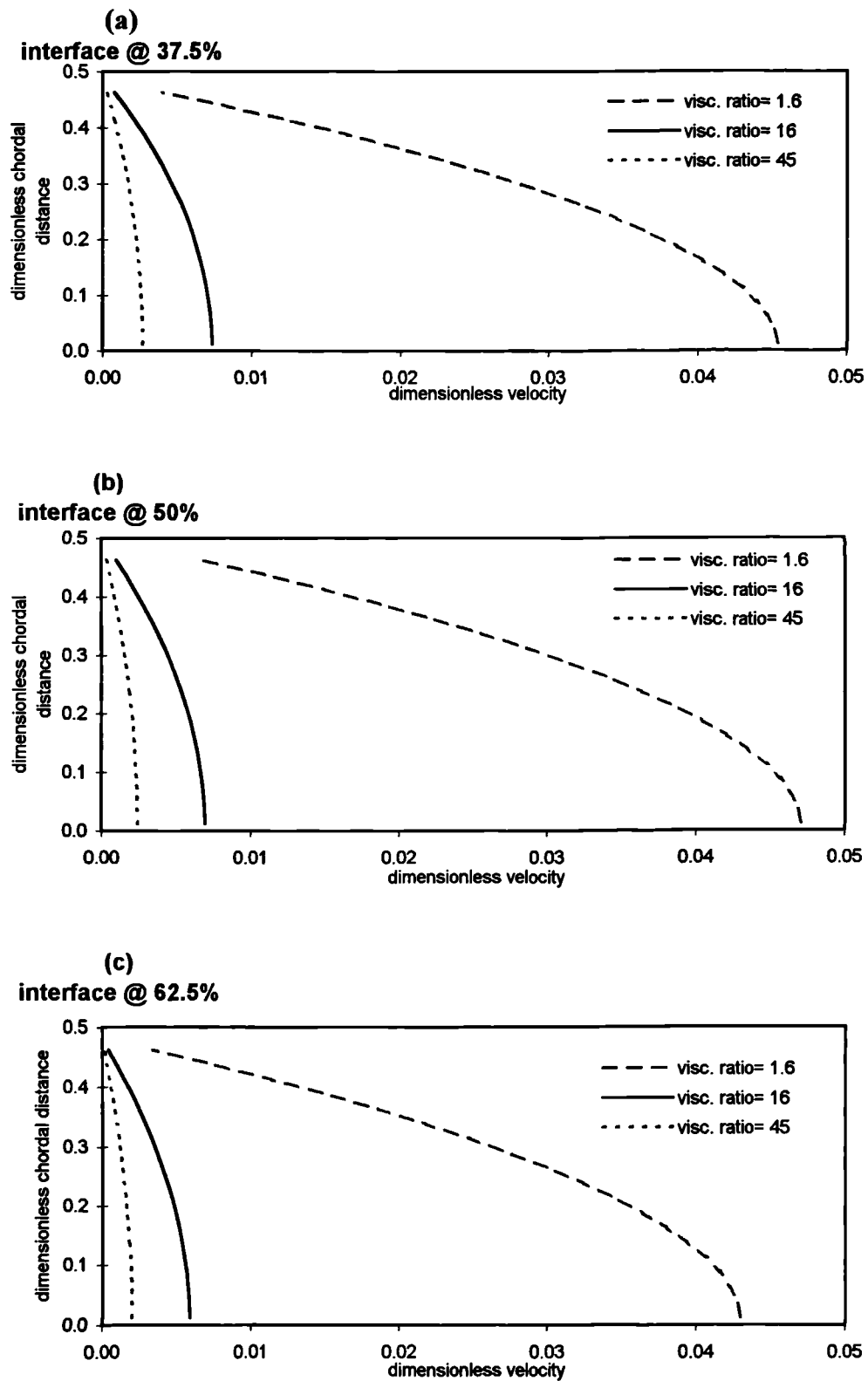


Figure 5.25: Laminar flow calculations with the method implemented on the CFX-F3D code. Velocity profiles at the interface plane for different interface positions: (a) 37.5%; (b) 50.0%; (c) 62.5%.

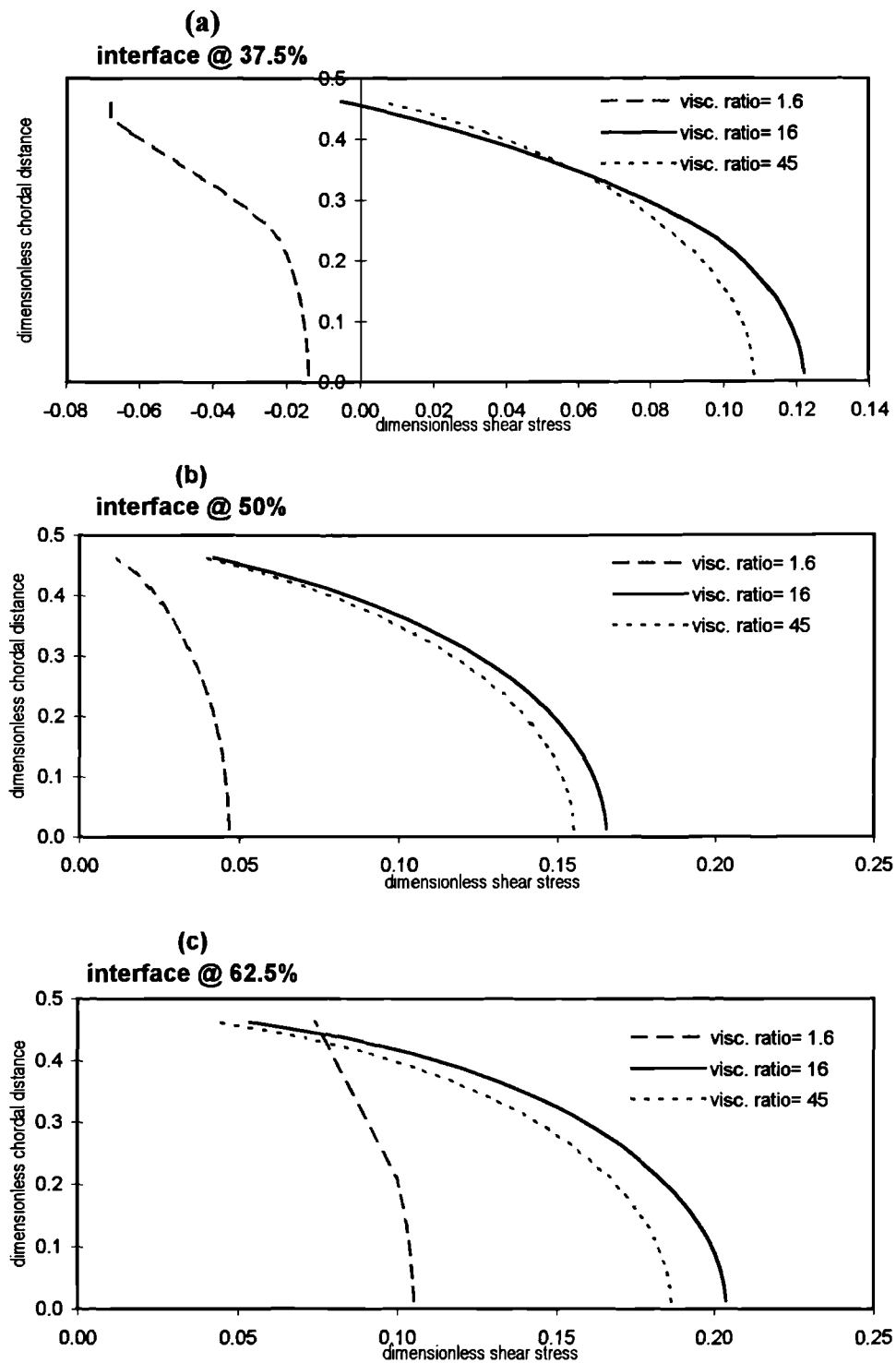
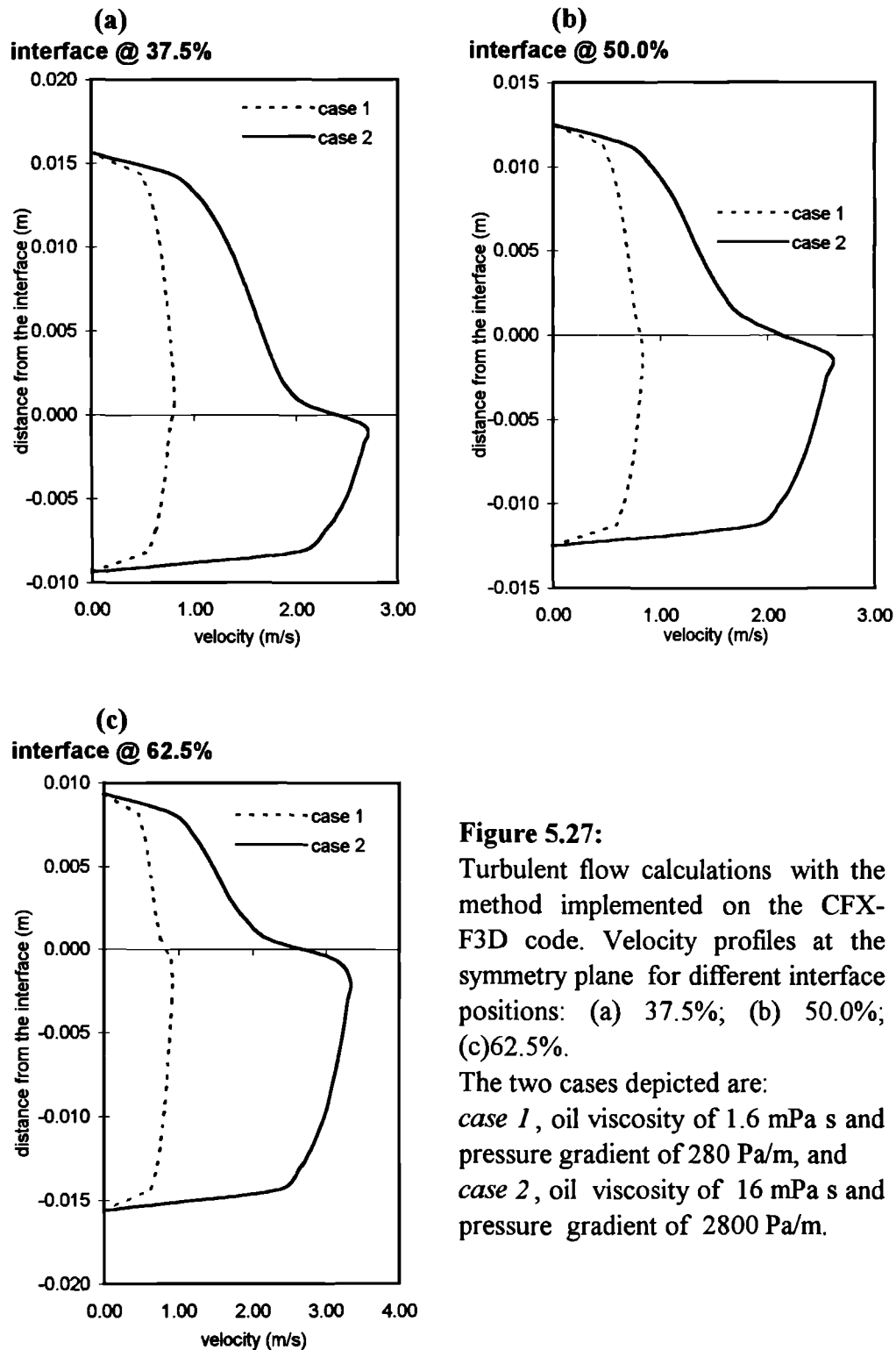


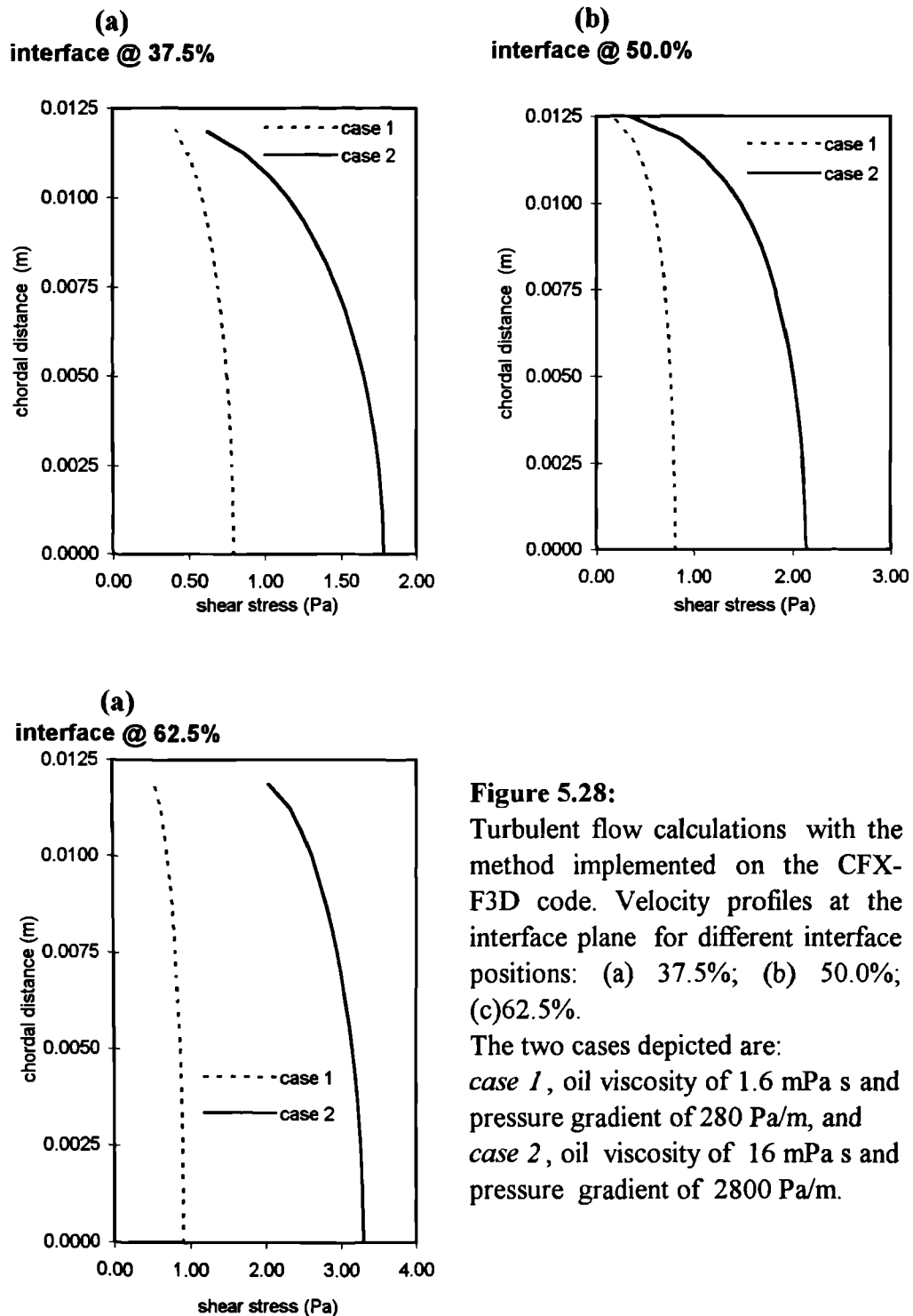
Figure 5.26: Laminar flow calculations with the method implemented on the CFX-F3D code. Shear stress profiles at the interface plane for different interface positions: (a) 37.5%; (b) 50.0%; (c) 62.5%.

**Figure 5.27:**

Turbulent flow calculations with the method implemented on the CFX-F3D code. Velocity profiles at the symmetry plane for different interface positions: (a) 37.5%; (b) 50.0%; (c) 62.5%.

The two cases depicted are:

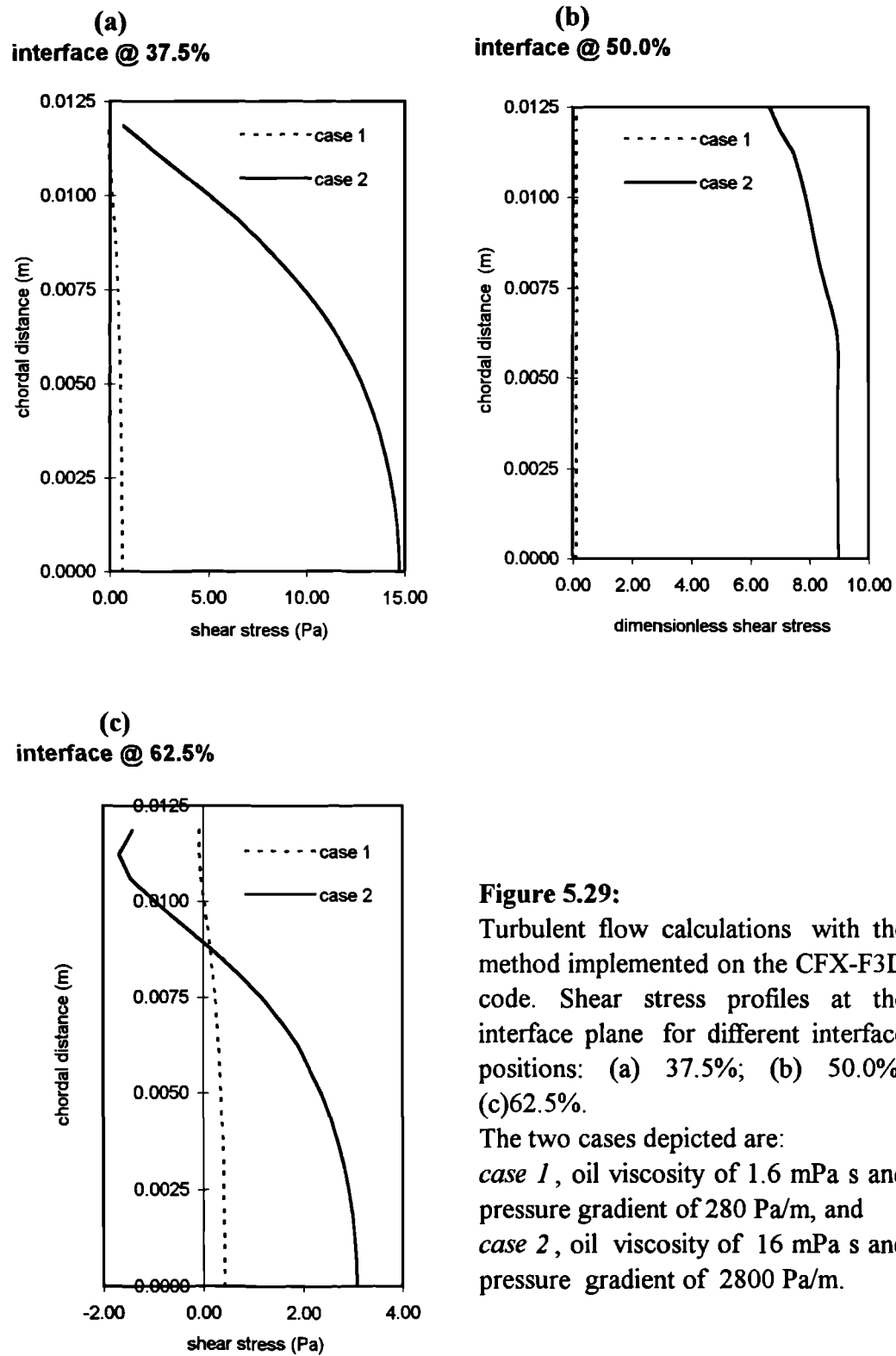
case 1, oil viscosity of 1.6 mPa s and pressure gradient of 280 Pa/m, and
case 2, oil viscosity of 16 mPa s and pressure gradient of 2800 Pa/m.

**Figure 5.28:**

Turbulent flow calculations with the method implemented on the CFX-F3D code. Velocity profiles at the interface plane for different interface positions: (a) 37.5%; (b) 50.0%; (c) 62.5%.

The two cases depicted are:

case 1, oil viscosity of 1.6 mPa s and pressure gradient of 280 Pa/m, and
case 2, oil viscosity of 16 mPa s and pressure gradient of 2800 Pa/m.

**Figure 5.29:**

Turbulent flow calculations with the method implemented on the CFX-F3D code. Shear stress profiles at the interface plane for different interface positions: (a) 37.5%; (b) 50.0%; (c) 62.5%.

The two cases depicted are:

case 1, oil viscosity of 1.6 mPa s and pressure gradient of 280 Pa/m, and
case 2, oil viscosity of 16 mPa s and pressure gradient of 2800 Pa/m.

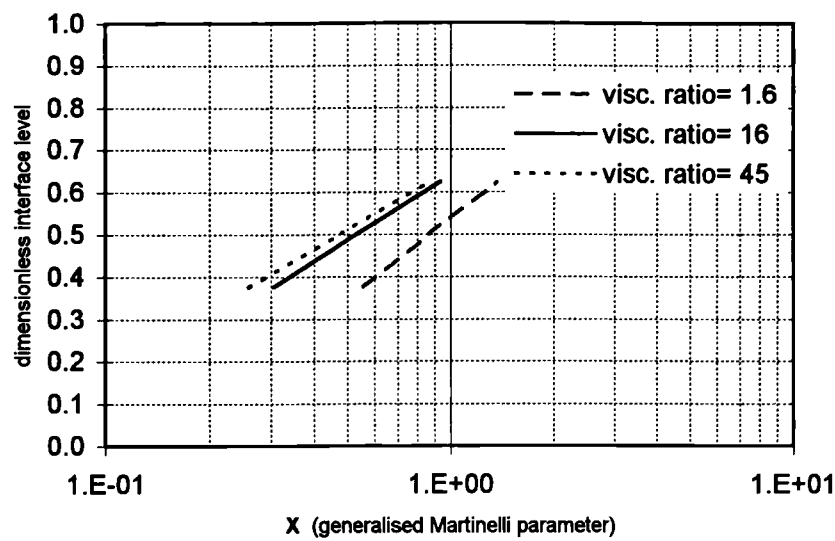


Figure 5.30: Laminar flow calculations with the method implemented on the CFX-F3D code. Dimensionless interface level versus the generalised Martinelli parameter (X_{TB})

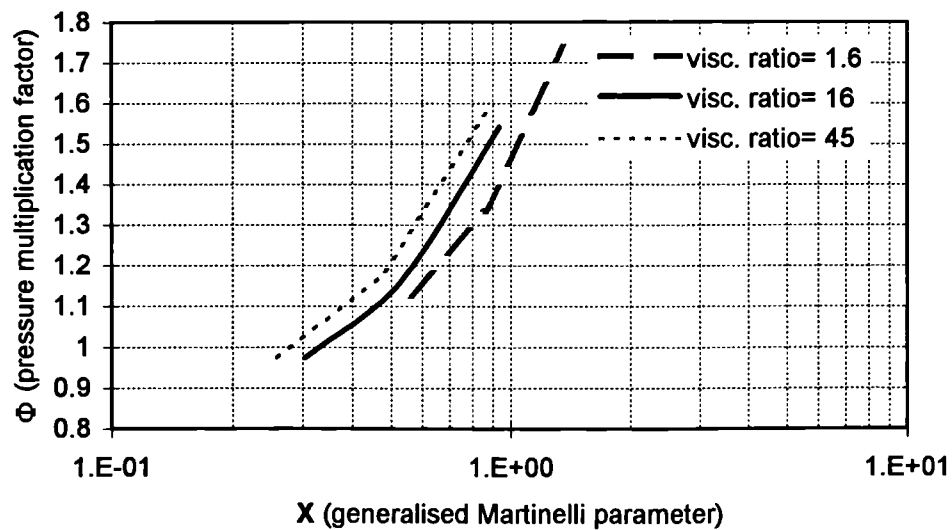


Figure 5.31: Laminar flow calculations with the method implemented on the CFX-F3D code. Pressure multiplication factor (ϕ_T) versus the generalised Martinelli parameter (X_{TB})

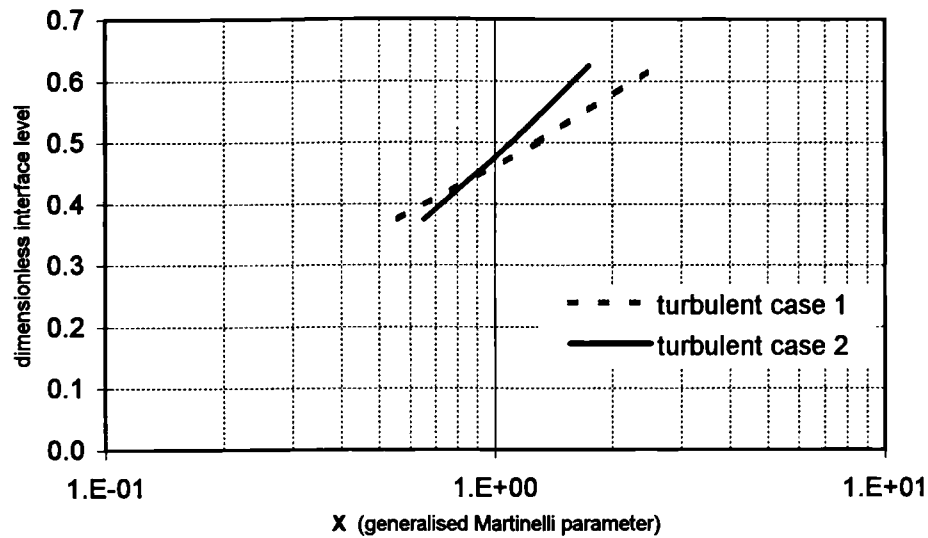


Figure 5.32: Turbulent flow calculations with the method implemented on the CFX-F3D code. Dimensionless interface factor versus the generalised Martinelli parameter (X_{TB}). The two cases depicted are: *case 1*, oil viscosity of 1.6 mPa s and pressure gradient of 280 Pa/m, and *case 2*, oil viscosity of 16 mPa s and pressure gradient of 2800 Pa/m.

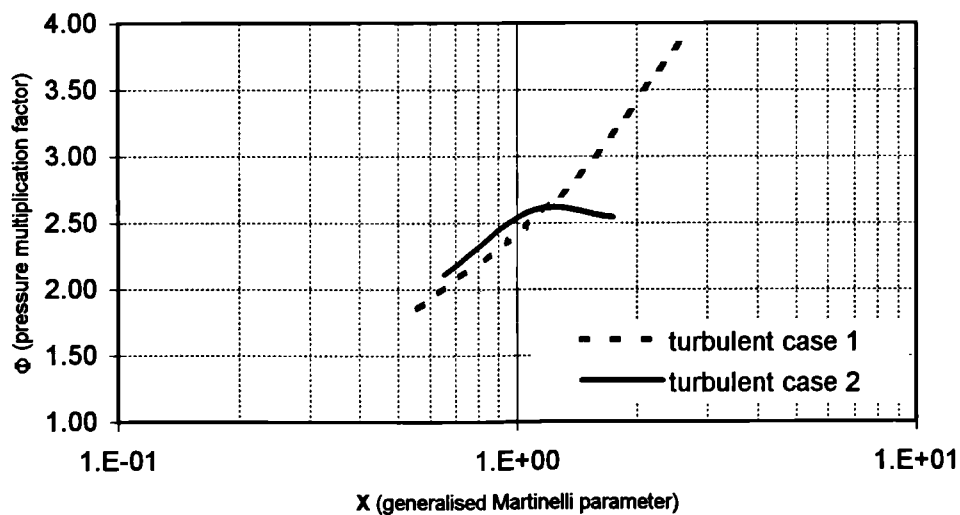


Figure 5.33: Turbulent flow calculations with the method implemented on the CFX-F3D code. Pressure multiplication factor (Φ_T) versus the generalised Martinelli parameter (X_{TB}). The two cases depicted are: *case 1*, oil viscosity of 1.6 mPa s and pressure gradient of 280 Pa/m, and *case 2*, oil viscosity of 16 mPa s and pressure gradient of 2800 Pa/m.

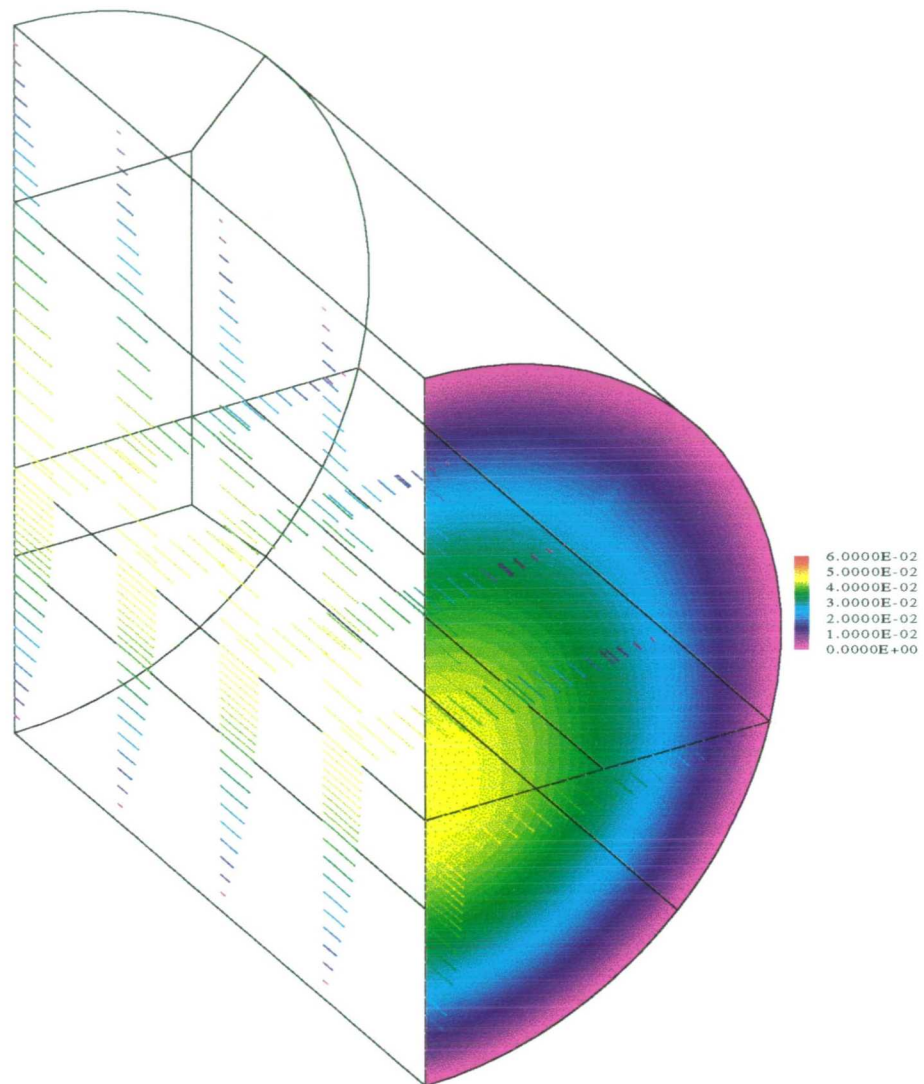


Figure 5.34 Laminar flow calculations with the method implemented on the CFX-F3D code. Cross sectional velocity field (shaded contours) and velocity profiles at the symmetry and interface planes. Interface position at 37.5% of the tube diameter

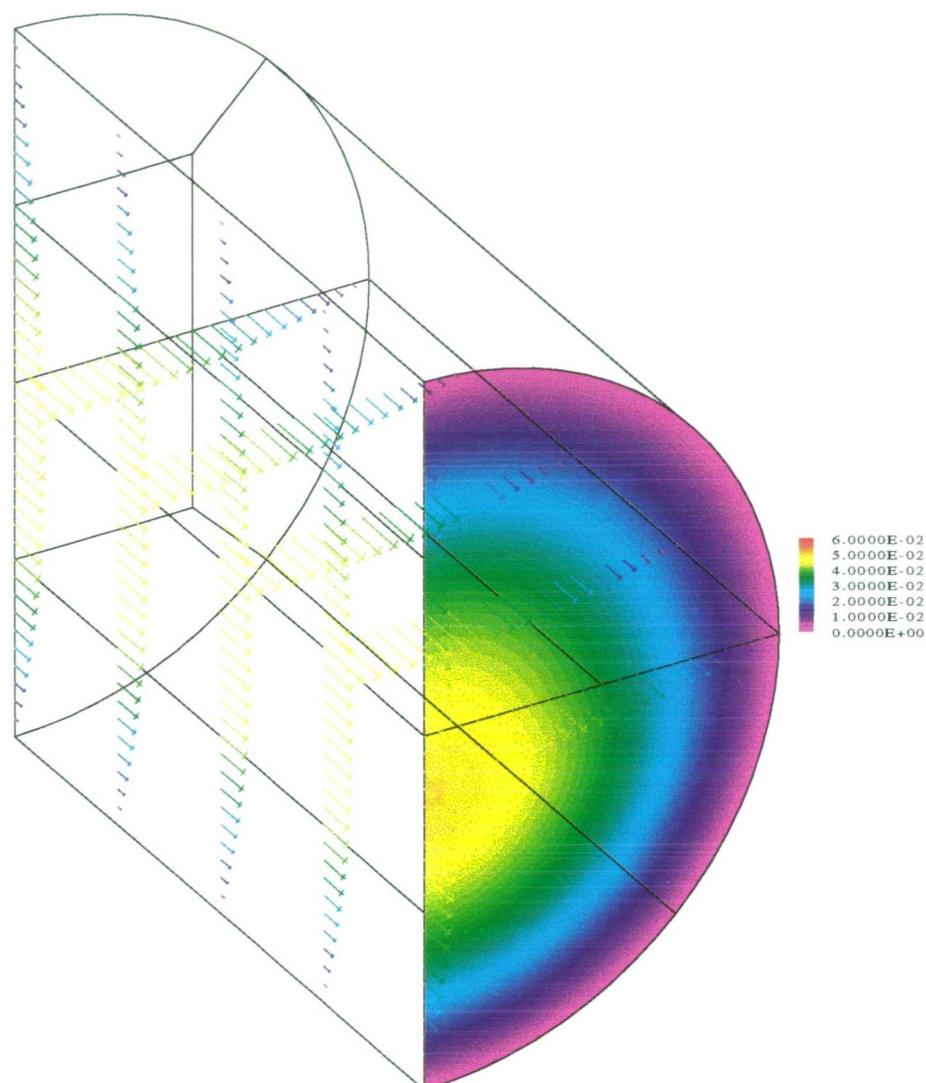


Figure 5.35 Laminar flow calculations with the method implemented on the CFX-F3D code. Cross sectional velocity field (shaded contours) and velocity profiles at the symmetry and interface planes. Interface position at 50.0 % of the tube diameter

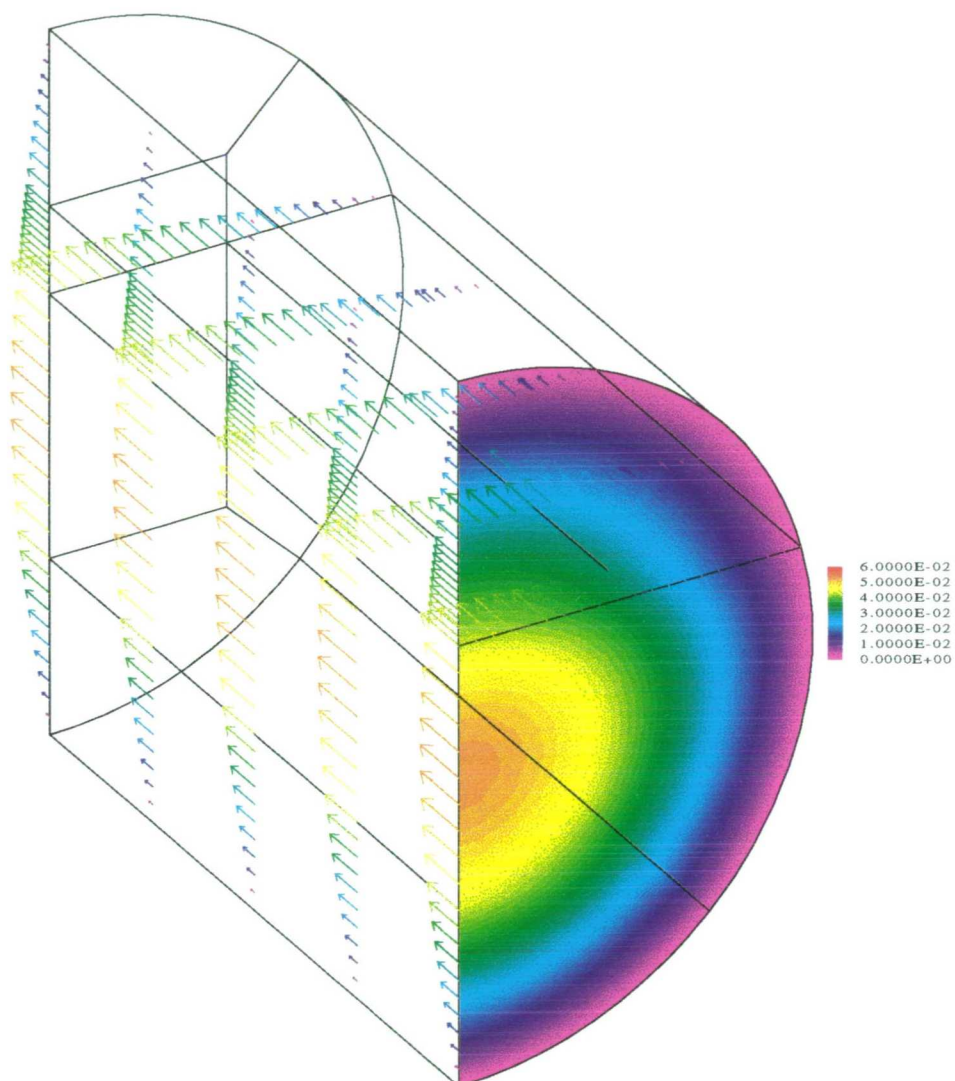


Figure 5.36 Laminar flow calculations with the method implemented on the CFX-F3D code. Cross sectional velocity field (shaded contours) and velocity profiles at the symmetry and interface planes. Interface position at 62.5% of the tube diameter

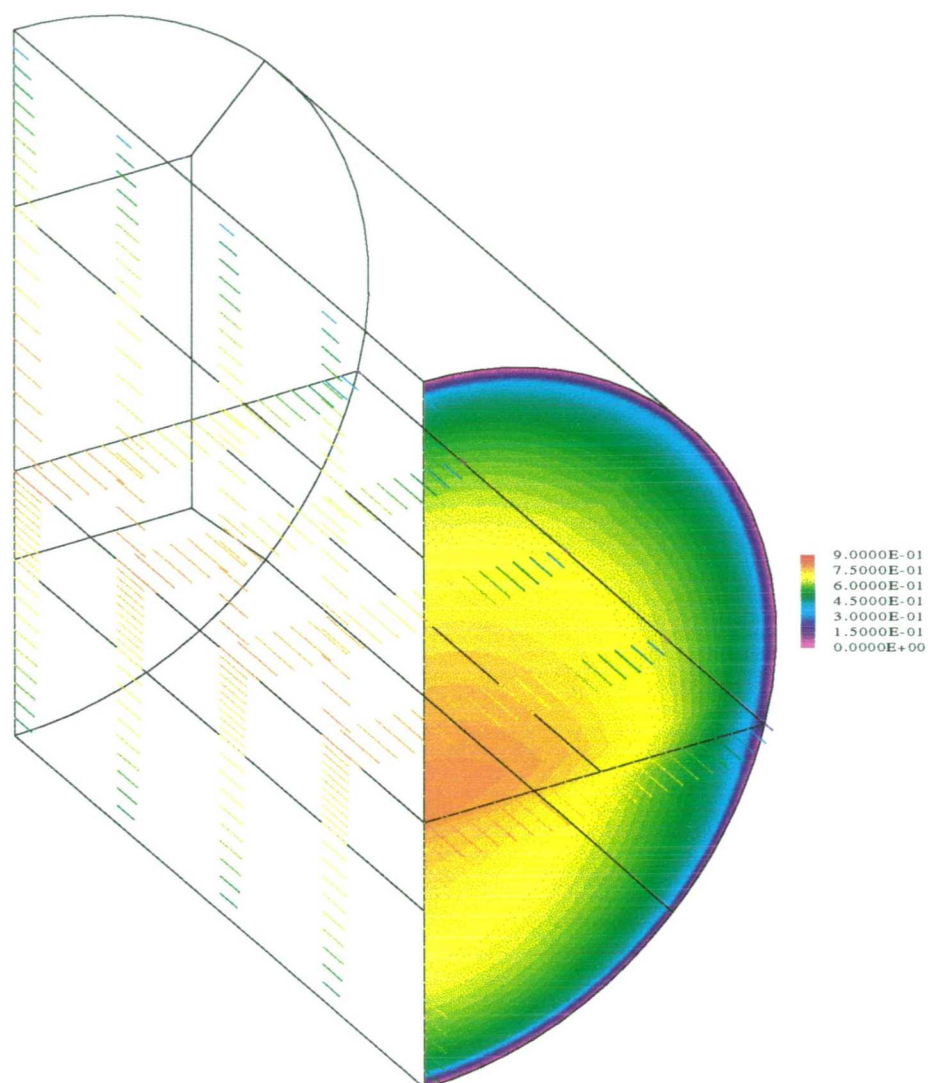


Figure 5.37: Turbulent flow calculations with the method implemented on the CFX-F3D code. Cross sectional velocity field (shaded contours) and velocity profiles at the symmetry and interface planes. Interface position at 37.5% of the tube diameter

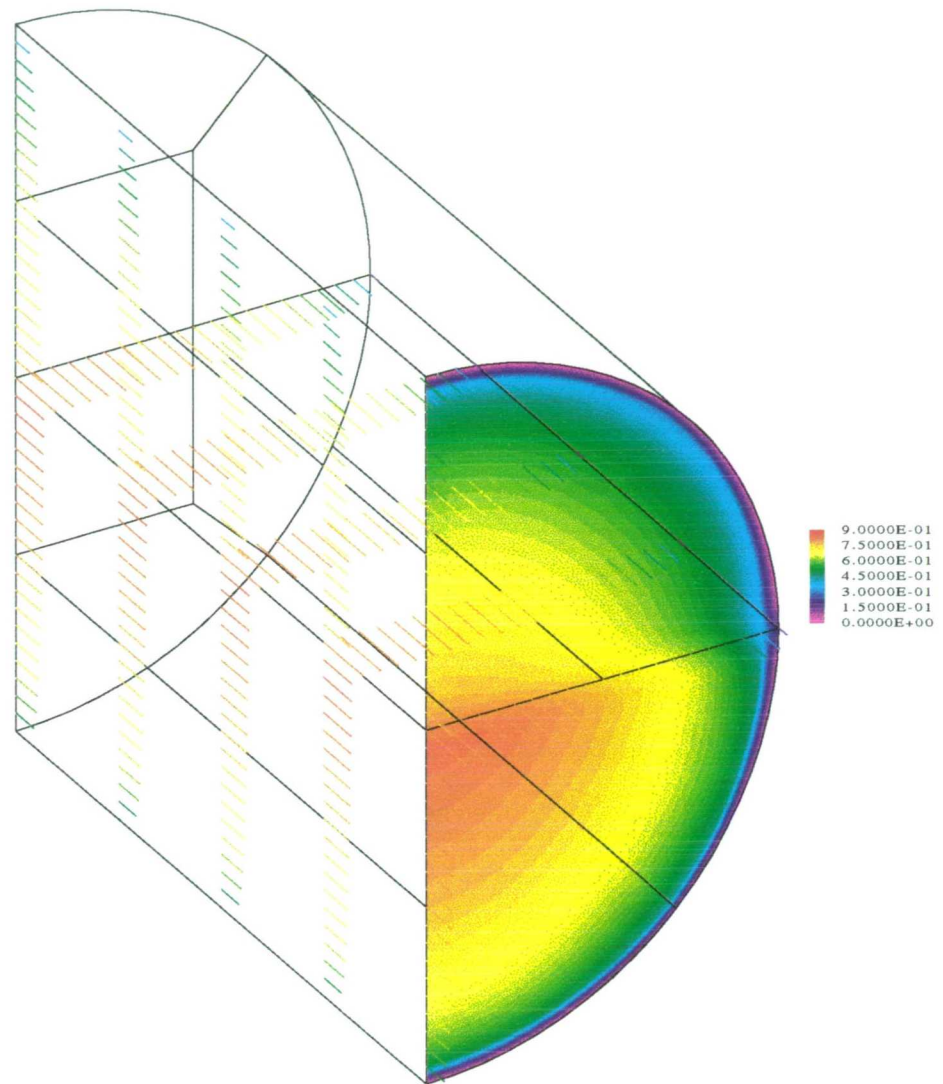


Figure 5.38: Turbulent flow calculations with the method implemented on the CFX-F3D code. Cross sectional velocity field (shaded contours) and velocity profiles at the symmetry and interface planes. Interface position at 50.0% of the tube diameter.

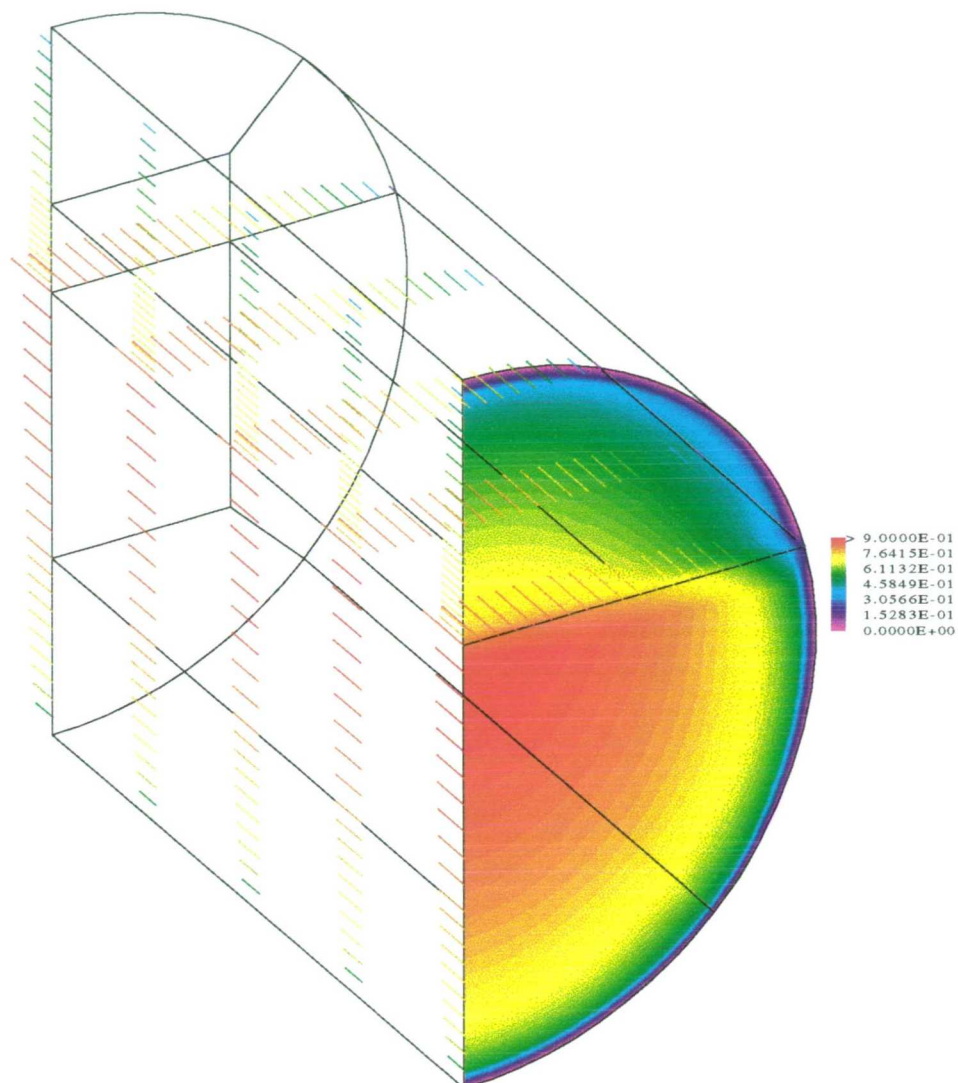


Figure 5.39: Turbulent flow calculations with the method implemented on the CFX-F3D code. Cross sectional velocity field (shaded contours) and velocity profiles at the symmetry and interface planes. Interface position at 62.5% of the tube diameter.

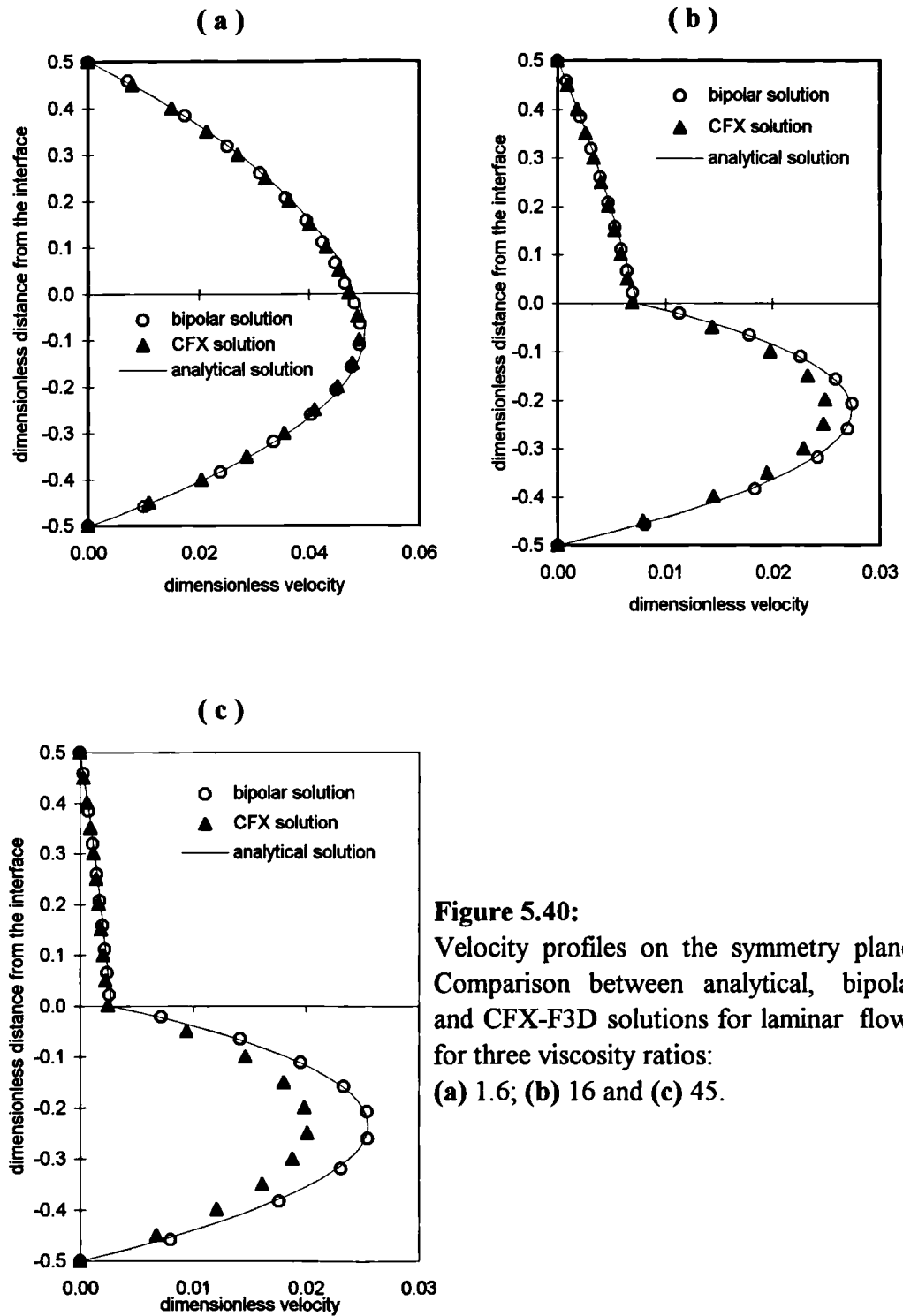


Figure 5.40:
Velocity profiles on the symmetry plane. Comparison between analytical, bipolar and CFX-F3D solutions for laminar flows for three viscosity ratios: (a) 1.6; (b) 16 and (c) 45.

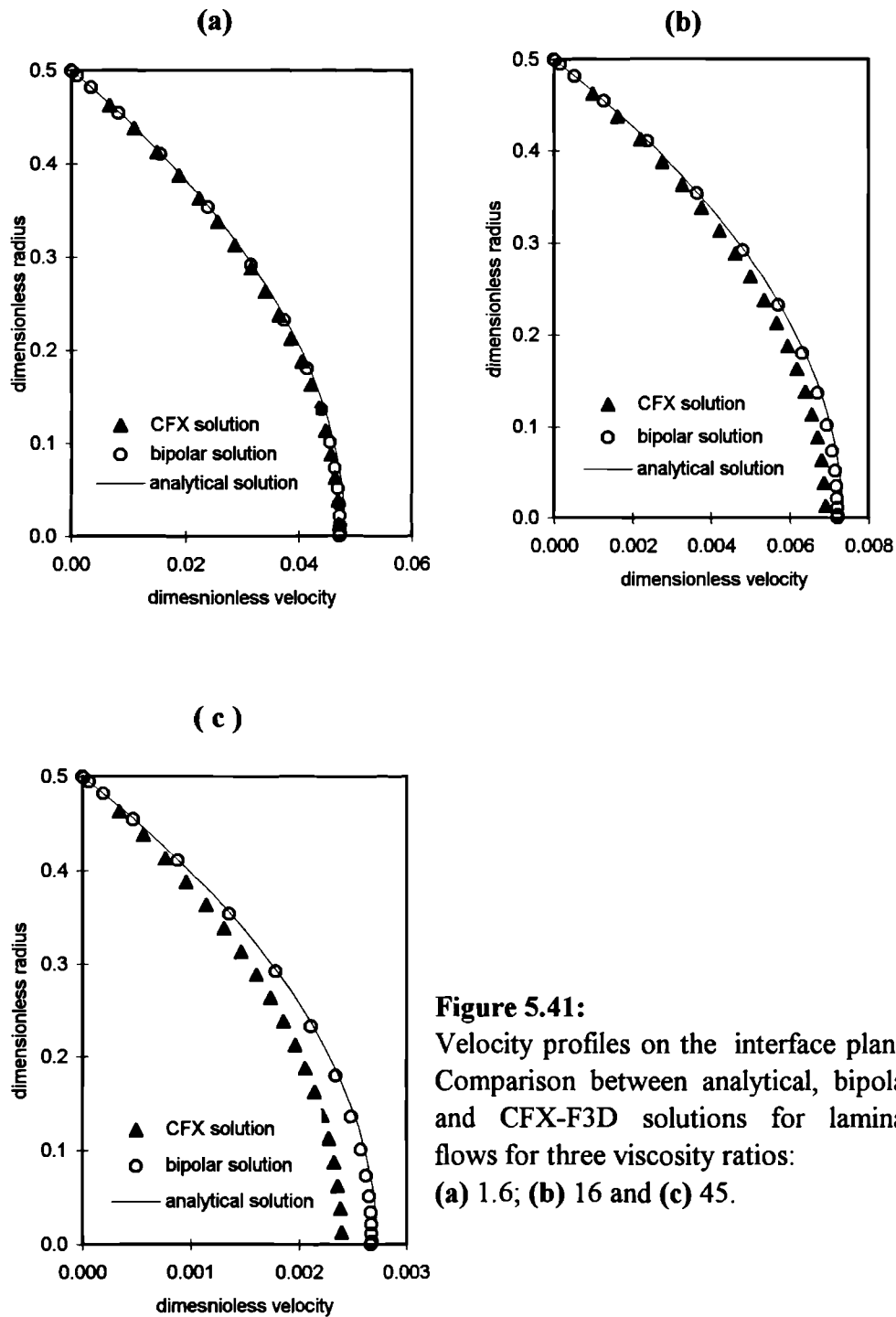
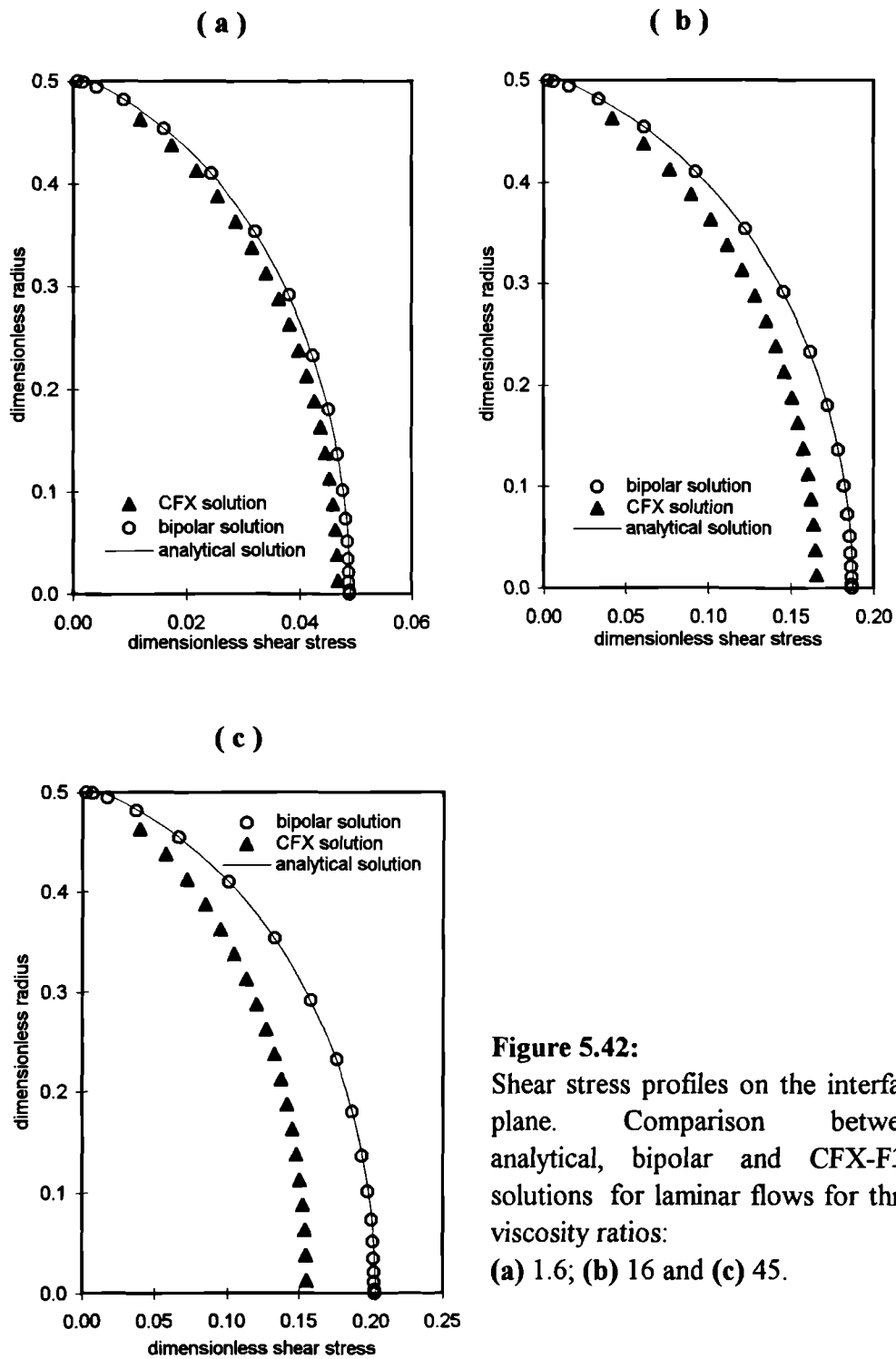


Figure 5.41:
Velocity profiles on the interface plane.
Comparison between analytical, bipolar
and CFX-F3D solutions for laminar
flows for three viscosity ratios:
(a) 1.6; (b) 16 and (c) 45.

**Figure 5.42:**

Shear stress profiles on the interface plane. Comparison between analytical, bipolar and CFX-F3D solutions for laminar flows for three viscosity ratios:

(a) 1.6; (b) 16 and (c) 45.

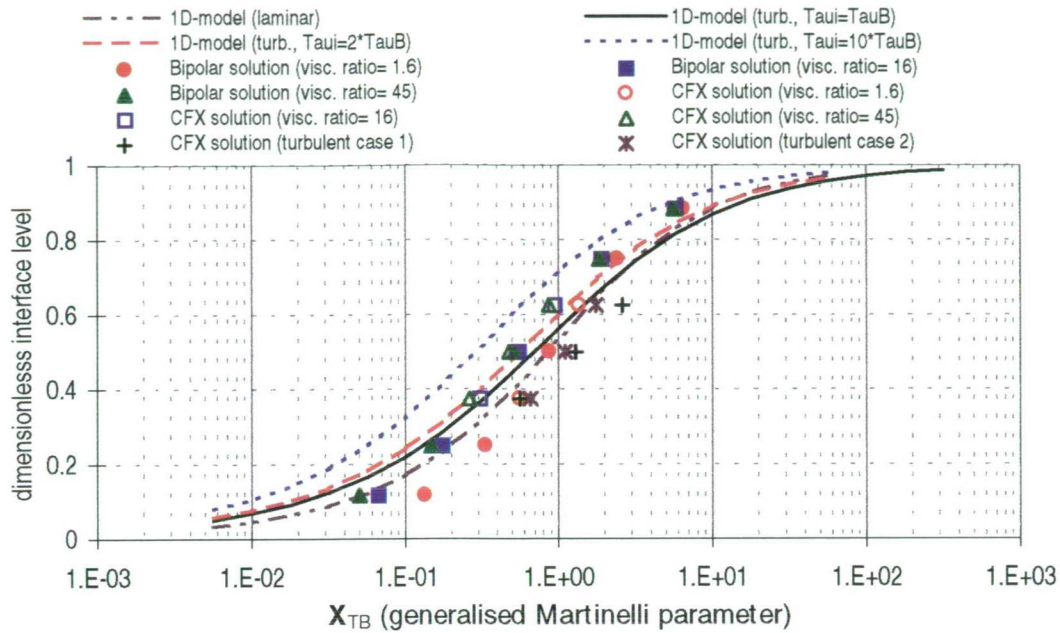


Figure 5.43: Comparison between the 1-D model and numerical solutions (bipolar, and CFX-F3D - laminar and turbulent). Dimensionless interface level versus the generalised Martinelli parameter (X_{TB}).

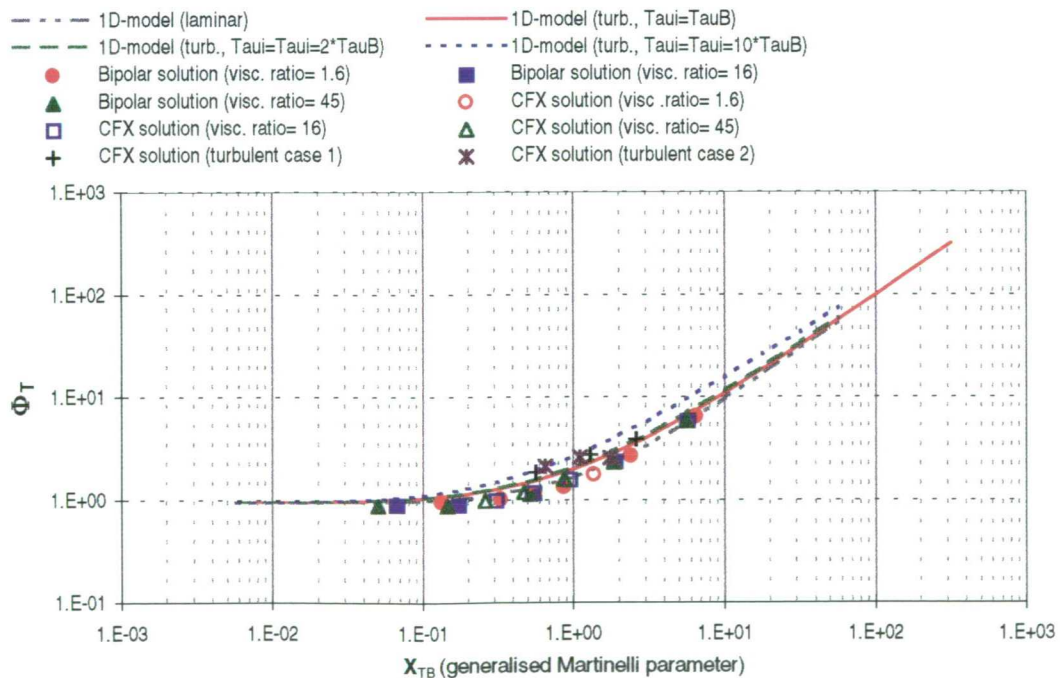


Figure 5.44: Comparison between the 1-D model and numerical solutions (bipolar, and CFX-F3D - laminar and turbulent). Pressure multiplication factor (ϕ_T) versus the generalised Martinelli parameter (X_{TB}).

Chapter 6

Summary, Conclusions and Recommendations

6.1- Summary and conclusions

This section presents a summary and the main conclusions from the work described in the thesis. It is convenient to divide the summary and conclusions into four subsections dealing with the findings arising from the literature survey (Section 6.1.1), the experimental work (Section 6.1.2), phenomenological modelling (Section 6.1.3) and numerical modelling (Section 6.1.7) aspects respectively.

6.1.1- Summary and conclusions arising from the literature studies

An extensive and rigorous literature survey on liquid-liquid flows was performed. Researchers, engineers and mathematicians have observed experimentally the simultaneous flow of two immiscible liquid phases in the most diverse technical situations and attempted to formulate and solve physically and mathematically this

phenomenon. It was noticed, though, that there was no unified work that linked the analytical and experimental studies.

- Based on the studies of the previous work a revised flow pattern nomenclature for liquid-liquid systems was proposed in this thesis based on the flow categorisation presented by Ishii (1975) and the experimental observation of Oglesby (1979) and Arirachakaran et al. (1989). The general liquid flow patterns were referred in three categories: (i) *separated flows*, including the *stratified* and *core-annular* flow patterns; (ii) *dispersed flows* including the oil-in-water and water-in-oil dispersions; and (iii) *transition flows*, including the *intermittent* or *slug* flow patterns, and the *stratified with dispersion* and the *annular with dispersion* flow patterns.

6.1.2- Summary and conclusions from experimental work

The design, construction, commissioning and operation of a new liquid-liquid flow facility was carried out in the course of this work at Imperial College in collaboration with Miss Panagiota Angeli and with financial support from Norsk Hydro a.s. The facility had 25 mm test sections and the oil used had a viscosity of 1.6 mPa s. A wide range of flow rates was tested in the TOWER facility with three test section configurations: transpalite test section without mixing at the inlet, transpalite test section with mixing at the inlet and stainless steel test section with no mixing at the inlet.

Two test campaigns were run in the WASP facility, which had 45 mPa s viscosity oil and used a 77.9 mm diameter test section. The first campaign was for a horizontal tube and the second for a 1° upwards inclined tube. Flow pattern observations were made and flow patterns were identified according to the new classification. The slug flow pattern did not occur in any flow conditions tested.

A sixth test campaign was run on TOWER and consisted of experiments dedicated to measure the interface level in oil-water stratified flows. These measurements were performed with a technique using a high frequency impedance probe. The results obtained showed the expected behaviour (higher interface level as the water input flow rate was increased) indicating the suitability of this measurement technique for the purpose intended.

The main conclusions from the experimental work carried out on the Imperial College WASP and TOWER facilities (and described in detail in Chapter 3) were as follows:

- The flow pattern maps for all three sets of experiments run on the TOWER facility, obtained by visual (high speed video recorded) observation, were similar and consistently showed a reduction in the area of occurrence of stratified flow as agitation was induced in the flow. That is the *transpalite* - no mixer tests produced a larger area of occurrence of stratified flow, the *transpalite* - mixer tests the most reduced area of occurrence while the tests on the stainless steel section, which has a rougher wall than the transpalite tubes (and therefore induces more turbulence), had intermediate values between the two other cases. In the cases where the stratified flow

region was reduced this flow pattern was replaced by a *stratified with dispersion* configuration.

- The pressure drop data obtained in the TOWER tests showed the same trends as the flow pattern observations, i.e. an increase in the pressure drop (probably related to an increase of turbulence) as the flow test section was changed from the *transpalite* - no mixer test section, to the stainless steel or *transpalite* - mixer test sections. Comparison between the pressure drop measured in the *transpalite* - mixer and stainless steel test sections did not show any significant difference between the two.
- The flow patterns in all four horizontal campaigns (three in TOWER and one in WASP) were consistent with the observations of Guzhov et al (1973) and those of Oglesby (1979) / Arirachakaran et al (1989). This was rather surprising since all these experimental data were generated using different oil-water systems. This fact may indicate that a general transition criteria might be found to produce good predictions for a wide range of oil and water systems
- The flow patterns in the inclined WASP experiments varied consistently with what theory and common sense would predict; the higher the fluid velocity the greater the propensity to dispersed type flow patterns. Comparison between the flow pattern occurrence in the WASP horizontal and inclined experiments did not show any significant discrepancy. However, the amount of data on inclined flows was limited and therefore more definite statement can not be

issued about the effects of pipe inclination on oil-water flow pattern occurrence.

6.1.3- Summary and conclusions from the phenomenological modelling studies

A one-dimensional phenomenological model was developed to describe stratified liquid-liquid flows. This model, an extension of the work of Taitel and Dukler (1976a) on gas-liquid flow, introduced a different treatment of the interface shear stress to account for the change to liquid-liquid (oil-water) systems. In this latter situation the lighter phase is also the more viscous and the modelling of the interface shear stress and hydraulic diameter needed to take account of this fact. Thus, the model (valid for both gas-liquid and liquid-liquid systems) provides means of always considering, independent of the position of the fluids in the pipe, that the more viscous phase is modelled in open-channel flow while the less viscous is modelled in closed conduit flow.

The following conclusions were drawn from the phenomenological modelling studies:

- A new set of dimensionless variables was introduced in the extended model, and quantities such as the *generalised Martinelli parameter*, the *superficial wall* and *interface shear stresses* and the *superficial friction factor* were defined. These new variables provide flexibility to the model to easily adapt to future developments.

-
- The framework of this extended model was applied to the analysis and interpretation of the works of Andritsos and Hanratty (1987) and Martinelli and Nelson (1948). It was shown that the assumption of a constant interface shear stress would be unrealistic and could induce some error in the results. For the limiting case of a laminar pseudo-stratified flow where the two fluids had the same properties it was shown that the interface shear stress was in fact constant, but that its ratio to the wall shear stress, rather than being unity, was equal to the ratio between the length of the perpendicular distance of the interface to the axis of the pipe and the pipe radius.
 - The results of the experiments run in TOWER and WASP were compared to the predictions of the model. The model *overpredicted* the data obtained in the *transpalite* test section of TOWER when no mixer was used, but gave better agreement with the stainless steel tube. The model *underpredicted* the pressure gradients measured in the WASP experiments by 50% or more, in an opposite trend to that observed in the TOWER experiments. The explanation may be that the effect of viscosity on interfacial structure would change the model predictions, even though the effect of the change of viscosity is included in the model *per se*.
 - The performance of the extended one-dimensional model in predicting the level of the interface in oil-water flows was assessed against the interface level data produced in the TOWER rig with the high frequency probe. Good agreement was observed between these data when the flow of the fluids in the model was considered turbulent. Some discrepancy existed at certain points, which could

be attributed to an inaccurate interpretation of the signal measured experimentally because of the inter-liquid dispersion that occurred at the interface.

- Two models that attempted to predict the *stratified/non-stratified* flow boundary for liquid-liquid flows were developed. The first of these models was an extension of Taitel and Dukler's (1976a) model for gas-liquid flows. The second was an adaptation of a model of Wicks and Fraser (1975) to predict the minimum oil velocity capable to take up to the flow mainstream a stationary water layer in a pipeline. The results of the two models were consistent with each other but predicted a smaller area of occurrence of stratified flow than what was really observed in the experiments. It must be noted though, that the models did not provide for the existence of a *stratified with dispersion* flow pattern, which by itself might require the definition of transition criteria to both *stratified* and *dispersed* flow.

6.1.4- Summary and conclusions from analytical and numerical (CFD) studies

The velocity field for the stratified flow configuration was solved analytically for the case where the interface crosses the pipe axis, numerically in bipolar coordinates for five interface levels, but for both fluids in laminar flow, and in body fitted coordinates (using the CFX-F3D code of AEA Technology plc) for three interface levels and for both fluids either in laminar or turbulent flow. Notwithstanding the fact that the numerical methods were applied for only a

limited number of interface levels, the methods developed are general and can be applied to any given interface position.

The following main findings arose from the analytical and numerical studies:

- All the solutions obtained by the three methods were physically realistic and showed good agreement amongst themselves. The program using bipolar coordinates, though, produced results closer to the analytical solution than the program that used body-fitted coordinates, which tended to underestimate the maximum velocity of the low viscosity phase.
- All the laminar solutions were presented in dimensionless coordinates, so they are general for a given viscosity ratio of the two fluids. On the other hand, the turbulent solutions can not be made dimensionless because this model, using the $k-\varepsilon$ model for turbulence, has to solve the equations of conservation of kinetic energy and turbulent energy dissipation in addition to the linear momentum conservation equation. Clearly, there is not a set of dimensionless variables that can generalise simultaneously the solution of these three coupled partial differential equations.
- The solutions obtained by the numerical methods were also compared to the one-dimensional model developed. This comparison was made in terms of the generalised Martinelli parameter (X_{TB}), the dimensionless interface level (h/D) and the pressure multiplier (ϕ_T). Good agreement existed between the numerical solutions of laminar stratified flows and the results

provided by the one dimensional model. In turbulent flow, however, the one dimensional model predicted higher interface levels and pressure drop multipliers than the numerical simulation. It was also shown that if the interface shear stress were modelled to be greater than the low viscosity fluid wall shear stress these turbulent flow discrepancies increased. It is speculated that the assumption of a constant value for the interfacial shear stress imposed by the one-dimensional model could be the cause of these discrepancies, because according to the numerical solutions obtained the shear stress varies considerably along the interfacial plane.

- In general the two numerical methods and the one-dimensional model provided results that were consistent among themselves and produced the same trends within an acceptable accuracy.

6.2- Recommendations for future work

Many further studies will be necessary to completely unveil the basic fundamentals of liquid-liquid flows. Following from the work described in the present thesis the following activities are proposed:

- (1) More flow pattern identification experiments with different liquid-liquid systems and tube diameters. The development of a general criterion for the transition between *stratified* and *non-stratified* flow patterns for oil-water systems would certainly be very interesting and technically desirable.

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- (2) Improvements to the experimental techniques for the measurement of the interface level in liquid-liquid flows. The high frequency impedance technique presented by Angeli (1996) in dispersed flow experiments could be used in an attempt to identify the dispersion gradients at the interface region in *stratified* flows and to characterise the *stratified with dispersion* flow pattern.
 - (3) More extensive pressure drop experiments in order to enlarge the data base against which one-dimensional and numerical models should be assessed.
 - (4) Improvement of the one-dimensional model in order to include a better description of the interface and wall shear stresses and of inter-dispersion phenomena. A revision of the definition of the phasic *hydraulic diameters* may be required.
 - (5) Inclusion of a turbulence model and, perhaps, a dispersion model in the bipolar coordinate numerical simulator.
 - (6) Inclusion of a dispersion model to the *moving wall* method implemented in the CFX-F3D code.
 - (7) Inclusion of the concept of equivalent roughness in the interface region in the bipolar coordinate program and in the CFX-F3D code.
 - (8) Inclusion of more realistic interface shapes (both in two dimensions at the pipe cross section plane and with a fully three-dimensional model accounting for the waviness along the pipe

axis) in the CFX-F3D code, taking advantage of its body-fitted capabilities

Nomenclature

Dimensions are given in terms of mass (M), length (L) and time (t). Boldface symbols are vectors or tensors.

Symbols	Description	Dimension
Latin Characters		
A	area	L^2
A	coefficient in Equation (5.25)	-
a	coefficient in the discretisation equation (Section 5.1.2)	varies
a	coefficient in Equations (2.23) and (2.24)	-
$\alpha^{\alpha\beta}$	surface metric tensor	-
B	coefficient in Equation (5.25)	-
B	log layer constant	-
b	coefficient in the discretisation equation (Section 5.1.2)	varies
b	coefficient in Equations (2.23) and (2.24)	-
C	constant in equation for the evaluation of the friction factor	-
C	coefficient in Equation (5.26)	-
C_2	constant used by Taitel and Dukler - Equations (2.1), (4.81) and (4.82)	-
C_m	neutral stable wave velocity	Lt^{-1}
c	coefficient in Equations (2.23) and (2.24)	-
c	pole coordinate given by Equation (5.87)	L
D	diameter (pipe diameter, hydraulic diameter)	L
D	coefficient in Equation (5.25)	-
d	drop diameter	L
d	coefficient in Equations (2.23) and (2.24)	-
F	superficial friction factor	-
Fr	Froude number, equation (2.32)	-
f	Fanning friction factor	-
g	acceleration of gravity	Lt^{-2}
H_{21}	mean curvature taken from phase 2 to phase 1	L^{-1}
h	interface height or level	L
I	intensity of gamma ray beam	Mt^{-3}
i	$= \sqrt{-1}$	-
k	turbulent kinetic energy	L^2t^{-2}
k_n	neutral wave number	-
k_{rc}	minimum wave number for real characteristics	-
l	chordal length	L

$\dot{M}$	mass flow rate	Mt^{-1}
m	exponent in equation for the evaluation of the friction factor	-
n	bipolar grid dimension in the ξ direction	-
$\mathbf{n}$	unit normal vector	-
P	perimeter, length	L
p	pressure	$\text{ML}^{-1}\text{t}^{-2}$
p	bipolar grid μ coordinate of the interface	-
q	bipolar grid dimension in the μ direction	-
R	pipe radius	L
Re	Reynolds number - equation (2.33)	-
r	radial coordinate	L
S	source term in Equation (5.74)	varies
s	sheltering coefficient used in Equation (2.3)	-
s	empirical parameter used in Equation (4.85)	-
t	time	t
$\mathbf{t}_\beta$	hybrid interface tensor	-
U	superficial velocity	Lt^{-1}
u	component of velocity in the x direction, generic component of velocity	Lt^{-1}
u^*	slip velocity near the wall	Lt^{-1}
$\dot{V}$	volumetric flowrate	L^3t^{-1}
v	velocity component in the y direction or generic component of velocity	Lt^{-1}
$\mathbf{v}$	velocity vector	Lt^{-1}
We	Weber number - Equation (2.31)	-
w	velocity component in the z direction or generic component of velocity	Lt^{-1}
w^*	slip velocity near the interface	Lt^{-1}
X	Lockhart and Martinelli parameter	-
x	cartesian coordinate	L
x	mass fraction (quality) of the gas in gas-liquid flow	-
Y	Taitel and Dukler inclination parameter	-
y	cartesian coordinate	L
y^+	log layer dimensionless distance to wall or interface (local Reynolds number)	L
z	cartesian or cylindrical coordinate	L
Greek characters		
α	angle of inclination	rad
β	angle of inclination	rad
β	inverse viscosity ratio	-
Γ	general diffusion coefficient - Equation (5.74)	varies
γ	shape factor - Equation (2.20), (2.21)	-

γ	angle relative to stratified interface position - Equation (5.86)	rad
$\dot{\gamma}$	shear rate	L^{-1}
ε	volume fraction	-
ε	energy dissipation rate	$L^2 t^{-3}$
H	group of variables defined by Equations (5.108) or (5.109)	$M^{1/2} L^{-3/2}$
η	dynamic viscosity	$ML^{-1} t^{-1}$
θ	angle or azimuth in cylindrical coordinates	rad
κ	von Karman's constant	-
μ	bipolar coordinate	-
ν	kinematic viscosity	$L^2 t^{-1}$
ξ	bipolar coordinate	-
ρ	density	ML^{-3}
σ	interfacial tension	Mt^{-2}
T	superficial shear stress	$ML^{-1} t^{-2}$
τ	shear stress	$ML^{-1} t^{-2}$
ϕ	Lockhart & Martinelli pressure drop multiplier,	-
ϕ	angle or azimuth coordinate	rad
φ	general dependent field variable	varies
ψ	generic angle coordinate	rad

Subscripts

a	phase a
B	bottom (lower) phase in stratified flow
b	phase b
c	core phase - Equations (2.25) and (2.26)
c	critical fluid - Equation (4.75)
cr	critical (maximum) - Equation (2.27)
G	gas
i	interface
k	generic phase
L	liquid
ll	liquid-liquid
m	mixture
n	normal direction - Equation (1.7)
o	oil
r	radial direction
T	top (upper) phase in stratified flow
TB	generalised stratified flow (top and bottom)
TP	two-phase
t	tangential direction - Equation (1.5)

t	appearance of large amplitude waves - Equation (4.68)
w	wall
w	water
y	y direction
z	z direction
0	atmospheric pressure
1	phase 1
2	phase 2

Superscripts

L	Laplace equation solution
$*$	particular solution

Overlines

$\sim$	dimensionless
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Appendix 1

Gamma Ray Scan in the WASP Experiments

Water fractions at eight chordal positions in the pipe cross section (Figure 3.7) were obtained with the gamma ray densitometry scan technique (Americium source) used in the horizontal flow WASP experiments described in Section 3.2.2.1. Some results of these scans were shown in Figure 3.22. For the sake of the completeness of this present work the remaining cases where results were obtained are shown in this Appendix (Figure A1.1).

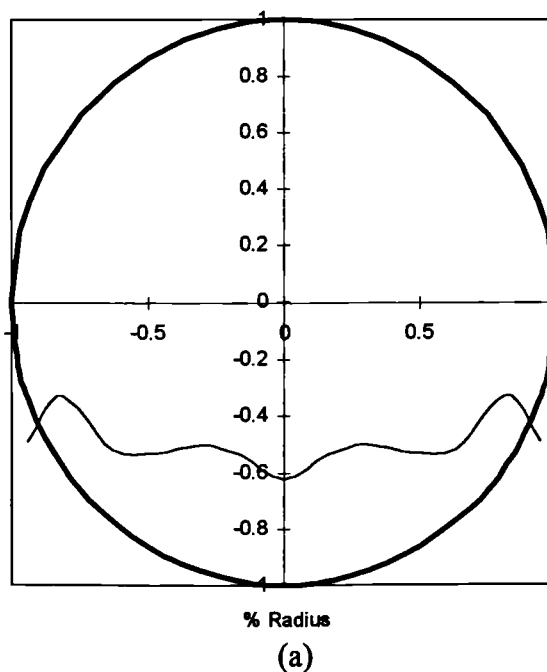


Figure A1.1: WASP horizontal flow experiments. Gamma ray scan with the Americium source. (a) Case 3, $U_o = 0.40$ m/s, $U_w = 0.15$ m/s, $\varepsilon_w = 0.26$, flow pattern observed: *stratified with oil in water dispersion*, flow pattern inferred: *stratified*; (continues on the next page)

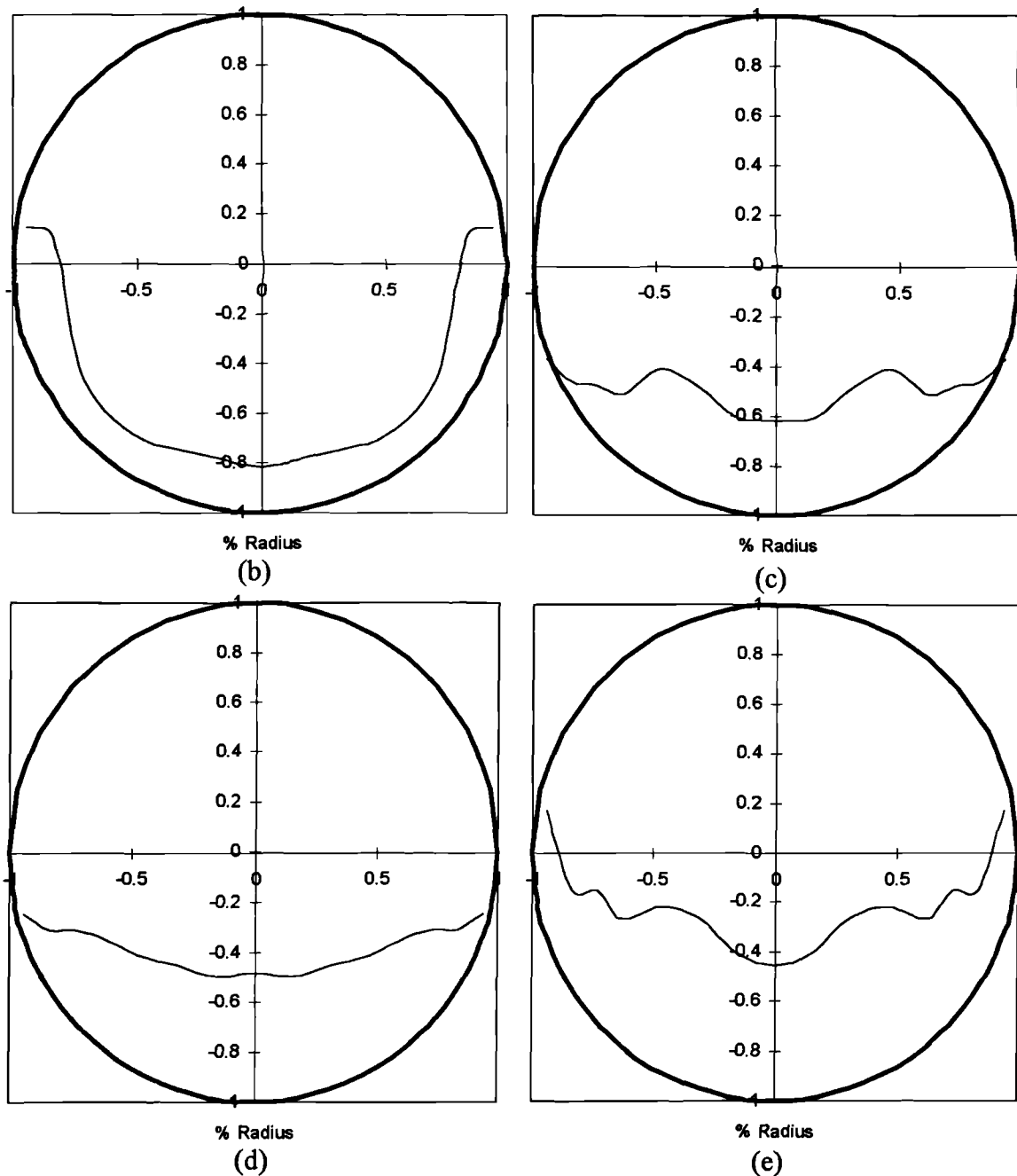


Figure A1.1: (continued from the previous page) WASP horizontal flow experiments. Gamma ray scan with the Americium source. (b) Case 8, $U_o = 0.81$ m/s, $U_w = 0.14$ m/s, $\varepsilon_o = 0.19$, flow pattern observed: *stratified with oil in water dispersion*, flow pattern inferred: *dispersed*; (c) Case 15, $U_o = 0.37$ m/s, $U_w = 0.10$ m/s, $\varepsilon_o = 0.21$, flow pattern observed: *stratified with oil in water dispersion*, flow pattern inferred: *stratified*; (d) Case 17, $U_o = 0.38$ m/s, $U_w = 0.19$ m/s, $\varepsilon_o = 0.27$, flow pattern observed: *stratified with oil in water dispersion*, flow pattern inferred: *stratified*; (e) Case 19, $U_o = 1.25$ m/s, $U_w = 0.20$ m/s, $\varepsilon_o = 0.34$, flow pattern observed: *dispersed*, flow pattern inferred: *dispersed*.

Appendix 2

The k - ε Turbulence Model

The k - ε model uses an eddy viscosity hypothesis for the turbulence. The continuity and linear momentum equations are then:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (\text{A2.1})$$

and

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla \mathbf{p} + \nabla \cdot (\eta_{eff} (\nabla \mathbf{v})^T) + \rho \mathbf{g} \quad (\text{A2.2})$$

where ρ and $\mathbf{v}$ are the mean fluid density and velocity, and η_{eff} is the effective viscosity defined by:

$$\eta_{eff} = \eta + \eta_T \quad (\text{A2.3})$$

where η is the molecular viscosity and η_T is the eddy or turbulent viscosity.

Kolmogorov and Prandtl suggested the calculation of the eddy viscosity by the following expression:

$$\eta_T = \rho C_\eta \frac{k^2}{\varepsilon} \quad (\text{A2.4})$$

where C_η is an empirical constant, k the local turbulence kinetic energy and ε the turbulent energy dissipation.

Jones & Launder (1972) derived differential equations for k and ε from the Navier-Stokes equations and these are stated as follows:

Turbulent kinetic energy

$$\begin{aligned} \frac{\partial(\rho k)}{\partial t} + \nabla \cdot (\rho \mathbf{v} k) = & \quad (\text{A2.5}) \\ \nabla \cdot \left[\left(\eta + \frac{\eta_T}{\sigma_k} \right) \nabla k \right] + (\eta + \eta_T) \nabla \mathbf{v} \cdot (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) - \rho \varepsilon \end{aligned}$$

Energy dissipation

$$\begin{aligned} \frac{\partial(\rho \varepsilon)}{\partial t} + \nabla \cdot (\rho \mathbf{v} \varepsilon) = & \quad (\text{A2.6}) \\ \nabla \cdot \left[\left(\eta + \frac{\eta_T}{\sigma_\varepsilon} \right) \nabla \varepsilon \right] + C_1 \frac{\varepsilon}{k} (\eta + \eta_T) \nabla \mathbf{v} \cdot (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) - C_2 \rho \frac{\varepsilon^2}{k} \end{aligned}$$

The model contains five empirical constants; i.e. C_η , σ_k , σ_ε , C_1 and C_2 . Their values are given below:

C_η	σ_k	σ_ε	C_1	C_2
0.09	1.0	1.22	1.44	1.92

The k - ε model as described above is valid only for turbulent flows with Reynolds numbers greater than 30000. It also fails to close to the wall where the turbulence is suppressed due to viscous effects (Rodi, 1982). A *wall function* is therefore imposed as a boundary condition for the region near the wall. Thus, the velocity components close to a smooth wall can be calculated from:

$$u^+ = y^+ \quad \text{for } y^+ < 5 \quad (\text{A2.7})$$

and

$$u^+ = \frac{1}{\kappa} \ln y^+ + B \quad \text{for } 5 < y^+ \leq 100 \quad (\text{A2.8})$$

where κ and B are constants having the values of 0.4 and 5.5 respectively and u^+ is the dimensionless velocity, defined by:

$$u^+ = \frac{u}{u_*} \quad (\text{A2.9})$$

and y^+ is the dimensionless distance from the wall (local Reynolds number):

$$y^+ = \frac{y u_* \rho}{\eta} \quad (\text{A2.10})$$

where, in turn, u^* is the slip velocity, defined by:

$$u^* = \sqrt{\frac{\tau_w}{\rho}} \quad (\text{A2.11})$$