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RESEARCH ARTICLE

Optimization of passenger flow control and parallel bus bridging in urban rail transit based on intelligent transport infrastructure

Qingqing Zhao[1] | **Jinjin Tang**[1] | **Wen-Long Shang**[2,3] | **Chao Li**[1] | **Yifei Ren**[1] | **Mohammed Quddus**[2] | **Washington Ochieng**[2]

[1]School of Traffic and Transportation, Beijing Jiaotong University, Beijing, China

[2]Centre for Transport Studies, Imperial College London, London, UK

[3]School of Architecture and Cities, University of Westminster, London, UK

Correspondence
Wen-Long Shang, Centre for Transport Studies, Imperial College London, SW7 2AZ London, UK.
Email: wenlong.shang12@imperial.ac.uk

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Abstract

Passenger flow control and bus bridging are used widely in the operations and management of urban rail transit to relieve the pressure of urban rail transit passenger flow, especially in peak periods. This paper presents an optimization method based on time-varying running time in links. We first develop a mixed integer nonlinear programming model seeking to achieve the minimum total passenger travel time and operation cost. An optimization network and an algorithm are then designed to solve the model. We use the developed method to solve both a small-scale simulated case study and a real-world case study involving the Chengdu Metro. The results obtained by the designed algorithm are comparable with those obtained by the CPLEX solver but with a shorter calculation time. The results show that parallel bus bridging can effectively reduce the number of waiting passengers. A sensitivity analysis of weight suggests that the algorithm successfully balances passenger travel cost and operating cost while incorporating time-varying running times leads to more realistic and dynamic infrastructure planning. This work contributes to the development of intelligent urban rail and road infrastructure systems, promoting safer and more efficient public transport operations.

1 | INTRODUCTION

In many cities, however, and especially big cities, the capacity of urban rail transit is outpaced by the actual demand of passenger flow (Chen et al., 2020; Ma et al. 2024). Addressing the above issues requires both "demand side" and "supply side" interventions. Passenger flow control in urban rail transit and bus bridging now have become the major means to ease passenger flow pressure.

Many cities, especially those with large urban rail transit networks, have been at the forefront of implementing control measures to manage the imbalance between demand and supply of urban rail transit and ensure safe and reliable services (Yuan et al., 2020; Shang et al. 2023). For example, London, Beijing, Shanghai, Shenzhen, and Guangzhou (Zhao et al., 2024) have carried out passenger flow control. Despite the tentative efforts to explore passenger flow control, current passenger flow control measures largely

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depend on fixed plans based on human experience, with limited quantitative analysis involved. Network-level passenger flow control should coordinate passenger numbers at each station from a network-wide perspective to alleviate the passenger flow pressure. In practice, however, most passenger flow control interventions are only targeted at a single station level, without coordination with other stations, even though congestion appears in many stations (Zhao et al., 2024).

One effective strategy to ease overcrowding is to deploy shuttle buses. For example, Shenzhen (Transport Bureau of Shenzhen Municipality, 2023), Beijing (The People's Government of Beijing Municipality, 2024), and Toronto (Liu et al., 2020), to name a few, have launched urban rail transit bridging lines for large passenger flow and provided customized bus services to passengers. Many places have dedicated bus lanes and customizing bus routes according to passenger needs can reduce passenger travel time and improve passenger willingness to ride. Such measures are geared towards seamless transfer of passengers, relieving the pressure of metro passenger flow, and so on.

Considering these strategies in tandem, it is evident that, in scenarios of congested urban rail transit, optimizing the current passenger flow control alone cannot fully deal with congestion issues and is potentially disruptive in terms of passengers' travel needs. There is a need to facilitate the mobility of stranded urban rail transit passengers effectively by means of buses.

An intelligent transportation system platform requires facilities, equipment, vehicles, and advanced technology (Adeli & Karim, 2005; Karim & Adeli, 2003). To further implement the passenger flow control plan, Beijing Metro has built a real-time passenger flow monitoring platform and dispatchers can issue different levels of passenger flow early warning responses according to the status of passenger flow. For the coordinated development of multimode traffic, the Chengdu Traffic Operation Coordination Center's comprehensive analysis of passenger flow, traffic flow, and events helps the city's overall traffic operation monitoring, comprehensive resource scheduling, traffic policy adjustment, and public travel services. Thus, we develop a method for joint optimization of passenger flow control and parallel bus bridging in urban rail transit to support intelligent transportation systems.

The remainder of the paper is structured as follows: Section 2 presents a review of the relevant literature. Section 3 covers the notation and problem. In Section 4, a model of passenger flow control and bus bridging (PFCBB) is formulated, the complexity of the model is analyzed, and the model is decomposed. Section 5 develops a solution algorithm for the model. Section 6 reports the results of a small-scale example and a real-world case. Finally, Section 7 offers conclusions and provides recommendations for future research.

2 | LITERATURE REVIEW

2.1 | Passenger flow control in urban rail transit

There have been many studies on passenger flow control, and we review these here according to the control hierarchy, specifically, passenger flow control at the station level, line level, and network level.

During the past 10 years, much literature has become available on station-level passenger flow control. For instance, Seriani and Fernandez (2015a, 2015b) conducted a simulation analysis of the effects of passenger flow control and suggested a strategy for spatial planning at transfer stations in urban rail transit. With advances in computing technology, some literature adopts the heuristic method in addition to the simulation method. For example, Xu et al. (2016) proposed a station service evaluation model that incorporates demand uncertainty, using a genetic algorithm to find solutions. Following this, Zhang (2017) optimized a model for managing passenger flow in station corridors.

A further body of research focuses on passenger flow control on a line. The primary goal of the line-level passenger flow control strategy is to coordinate the organization of passenger flow along the entire line, which requires implementing control strategies at each station on the line. The literature about line-level passenger flow control pays attention to various indicators to realize the goal. A considerable amount of research has focused on passenger waiting times. For example, Xue et al. (2020) established an optimization model aimed at minimizing passengers' waiting time. Moreover, fairness has drawn attention from scholars examining line-level passenger flow management (Ma et al., 2017; Yin et al., 2021). Beyond traditional simulation and modeling approaches, machine learning has been explored by Jiang et al. (2018) to study passenger flow control in stations. Meanwhile, train operation optimization is the main work of railway line operation management (Barrena et al., 2014). A significant portion of the current literature on passenger flow control also focuses on train operation (Besinovic et al., 2022; Jiang et al., 2017; Li et al., 2017) to ease the contradiction between current passenger demand and transport capacity requires efforts on both the "demand side" and the "supply side."

As urban rail transit develops within cities, it often expands to become a complete network rather than just a single line. The above literature, however, only focuses

on one station, several stations, or one line, and lacks the capability to consider passenger flow control across the entire network, hindering the coordination of passenger flow. Ultimately, this implies that any passenger flow control strategy developed does not align with the actual patterns of passenger flow and proves ineffective when put into practice. Recent work, therefore, is increasingly shifting to consider the network level. This literature typically focuses on reducing passenger waiting times to improve satisfaction (Yuan et al., 2020). Since one of the key indicators is the number of passengers waiting on a platform, Shi et al. (2019) studied how to reduce passenger aggregation risk. How to match passenger flow demand with train capacity is another key to passenger flow control (Yao et al., 2015; Ye, 2016).

In general, the existing literature on passenger flow control primarily focuses on developing strategies for managing passenger flow. However, when the current capacity of urban rail transit is maximized, it becomes essential to explore alternative methods for alleviating passenger flow pressure.

2.2 | Bus bridging for urban rail transit

Research on the coordination between urban rail transit and bus services has been steadily advancing, in line with the growth of urban integrated transportation systems. In this context, bus bridging for urban rail transit passengers has emerged as a new solution to issues such as connecting urban rail transit with the first or last train (Kang et al., 2019, 2021), large passenger flow, disruption (Jin et al., 2014) and other related issues. Research about buses and urban rail transit is primarily categorized into macronetwork construction and microconnection schemes. At the macro-level, much of the research has focused on identifying and evaluating the bus and urban rail transit network (Mohaymany & Gholami, 2010). For example, Liang et al. (2019a) established a stochastic linear programming model to achieve a more balanced modal split between urban rail transit and bus systems. At present, the research at the micro level is mainly divided into two categories, aimed, respectively, at regular (Yu et al., 2017) and exceptional situations (Liang et al., 2019b, Bi et al., 2022). In the above literature, in addition to enterprise cost, time cost is a crucial metric for evaluating the quality of the connection between urban rail transit and bus services, and much of the available literature pays attention to reducing waiting time (Wang et al., 2014; Zhang & Lo, 2018) and traveling time (Yin et al., 2018). These studies highlight the need to improve service levels by reducing the time cost of passengers in addition to the operation cost.

Together, these studies indicate that joint optimization of urban rail transit and bus bridging is a significant component in facilitating efficient and safe passenger travel in the public transport system.

2.3 | Passenger flow control and parallel bus bridging in urban rail transit

Passenger pressure within urban rail transit can be relieved by bus bridging. This is most commonly manifested in the case of a parallel bus service stopping at the stations of a certain metro line (Y. Chen & An, 2021; Dou et al., 2019). "Parallel bus" refers to a bus service running parallel to the urban rail transit line. It is regarded as a specialized type of customized bus service (Tong et al., 2017) designed to provide direct and comfortable commuting services and only stop at the stations of this urban rail transit line (X. Li et al., 2024; Yang et al., 2017). In emergencies or during periods of high passenger flow, parallel buses are often deployed to reduce delays and alleviate passenger flow pressure, thereby enhancing the overall service level of urban rail transit.

Exactly how to use parallel buses to alleviate passenger flow pressure is therefore a popular topic. In order to get the scheme of passenger flow control and parallel bus connection, Yang et al. (2017) designed a two-stage model. Geng et al. (2024) developed a mixed integer linear programming model aimed at reducing passenger dissatisfaction and lowering system operating costs simultaneously. Li et al. (2024) proposed an integer nonlinear optimization model to minimize the weighted passenger waiting time and the operating costs of bus-bridging services. Many related studies, especially on joint optimization of passenger flow control and parallel bus bridging, only focus on line-level control and assume that buses follow fixed running times in links (Geng et al., 2024; Li et al., 2024). However, bus speeds (i.e., running times in links) are susceptible to road conditions at different times, and fixed bridging speeds, or running times in links, do not match the actual situations.

Aiming at filling these research gaps, the innovation of this paper includes two points: the joint optimization of network-level passenger flow control and parallel bus connection of urban rail transit is studied, and the time-varying running time is considered in bus connection optimization. For ease of comparison between our study and past research, the detailed features of some closely related literature are summarized in Table 1.

TABLE 1 Overview of the related literature.

Reference	Scope of passenger flow control in urban rail transit	Type of bus bridging	Running time in links	Solution method
Shi et al. (2019)	Network	×	×	Exact solution with the CPLEX solver
Yuan et al. (2020)	Network	×	×	Exact solution with the CPLEX solver
Zhao et al. (2024)	Network	×	×	A method for collaborative passenger flow control
Kang et al. (2019)	×	Bus bridging of the last train	×	Decomposition method
Kang et al. (2021)	×	Bus bridging of the first train	Fixed	A tailored algorithm
Y. Chen and An (2021)	×	Parallel bus bridging	Fixed	A tabu search-based heuristic with LP rounding
Yang et al. (2017)	Line	Parallel bus bridging	Fixed	Exact solution with the CPLEX solver
Geng et al. (2024)	Line	Parallel bus bridging	Fixed	Exact solution with the GUROBI solver
X. Li et al. (2024)	Line	Parallel bus bridging	Fixed	Exact solution with the GORUBI solver
This paper	Network	Parallel bus bridging	Time varying	CPLEX + simulated annealing algorithm

Note: × denotes not involved in the research.

2.4 | Focus of this study

To overcome the limitations, we propose a method for passenger flow control and parallel bus bridging of urban rail transit. The key contributions of this paper are outlined below:

1. We have constructed a joint optimization model of urban rail PFCBB that takes the variation in the running time of buses in links into account.
2. We propose the CPLEX + simulated annealing (SA) algorithm, built on an intelligent optimization network, to solve the joint optimization problem of passenger flow control and parallel bus bridging in urban rail transit.

3 | PROBLEM DESCRIPTION

3.1 | Notation

Table 2 presents the detailed definitions of the sets, parameters, and variables employed in this paper. The passenger flow waiting outside the station is a key factor affecting operational efficiency. Therefore, ς_t^s represents the number of passengers waiting outside the station at each timestamp, which helps manage station access and optimize the flow of passengers entering the station. Passenger flow control aims to avoid overcrowding inside the station, ensuring safety and operational smoothness. This variable, τ_t^s, represents the number of passengers inside the station who are waiting to board. x_t^s is a binary variable that indicates whether flow control measures are applied at a station at a given time. This variable is crucial for managing the application of flow control strategies within the model and allows for flexibility in adjusting flow control decisions based on system needs. For passengers to use a bus service, the bus must stop at a station. This variable determines whether a bus stops at a given station. It is essential for linking the bus network to the metro network. If a bus does not stop at a particular station, passengers cannot board, which restricts their options. y_k^s ensures that the bus service is appropriately integrated into the passenger flow optimization model. The departure time of buses is crucial for coordinating the timing between buses and trains. Proper scheduling ensures that passengers can make connections between the bus and metro services without excessive waiting. By

TABLE 2 Symbol definition.

Notations	Definition
Set	
L	Line set, $L = \{1, 2 \ldots |L|\}$, $l, l' \in L$
S^l	The urban rail transit station (bus stop) set on line l, $S^l = \{1, 2 \ldots |S^l|\}$, $s \in S^l$, $s' \in S^l$
T	Timestamp set, $t \in T$, $T = \{1, 2 \ldots |T|\}$
K^l	The bus set on line l, $k \in K^l$, $K = \{1, 2 \ldots |K^l|\}$
Parameter	
e_t^s	The number of passengers who arrive at station s at time t, which does not include the transfer passengers from the parallel bus alight to station s
σ	The maximum limit rate requirement of the station
α	The maximum capacity utilization factor of urban rail transit trains
δ^s	If station s is a transfer station, it is 1 or otherwise 0
$\beta^{s',s}$	The proportion of passengers boarding buses at station s' and transferring at station s
$\lambda^{s',s}$	The proportion of passengers entering at station s' and exiting at station s
$\lambda^{l',s',s,l}$	The proportion of passengers entering at station s' on line l'. and transferring to the line l at station s
c_1^l	The capacity of urban rail transit trains on line l
c_2^s	The maximum platform capacity of urban rail transit station s
c_3^s	The maximum inbound capacity of urban rail transit station s
c_4	Bus capacity
χ	Unit time length between adjacent timestamps
t^s	The stop time of the bus at the station s
$r_t^{s-1,s}$	At time t, the time it takes the bus to get to station s from station $s-1$
ε_1	Fixed cost of a bus
ε_2	Unit time cost of a passenger
ε_3	Unit variable cost of buses
w_1, w_2	Weight
Intermediate variable	
∂_t^s	At time t, the number of passengers getting off trains at station s
ℓ_t^s	At time t, the number of passengers boarding trains at station s
ψ_t^s	At time t, the number of passengers on trains leaving station s
i_t^s	The actual number of passengers entering station s at time t with passenger flow control
t_k^s	The arrival time of bus k at station s
ν_k^s	The number of passengers who get on bus k at station s
ξ_k^s	The number of passengers on bus k when it leaves station s
φ_t^s	At time t, the number of passengers on the bus who transfer outside station s
$\eta_{k,t}^s$	At time t, the number of passengers who take bus k at station s
ϕ_t^s	At time t, the number of passengers who transfer to the platform at station s in urban rail transit
$\mu_{k,t}^s$	At time t, if bus k stops at station s, it is 1, or otherwise it is 0
$t_k^{s-1,s}$	The running time of bus k from station $s-1$ to station s
Decision variables	
ς_t^s	At time t, the number of passengers waiting outside station s with passenger flow control
τ_t^s	At time t, the number of passengers waiting on the platform inside station s with passenger flow control
x_t^s	If passenger flow control is carried out in the station at time t, it is 1 or otherwise 0
h_k	If bus k is running it is 1 or otherwise it is 0
y_k^s	If bus k stops at station s it is 1 or otherwise 0.
ζ_k^s	The departure time of bus k at station s

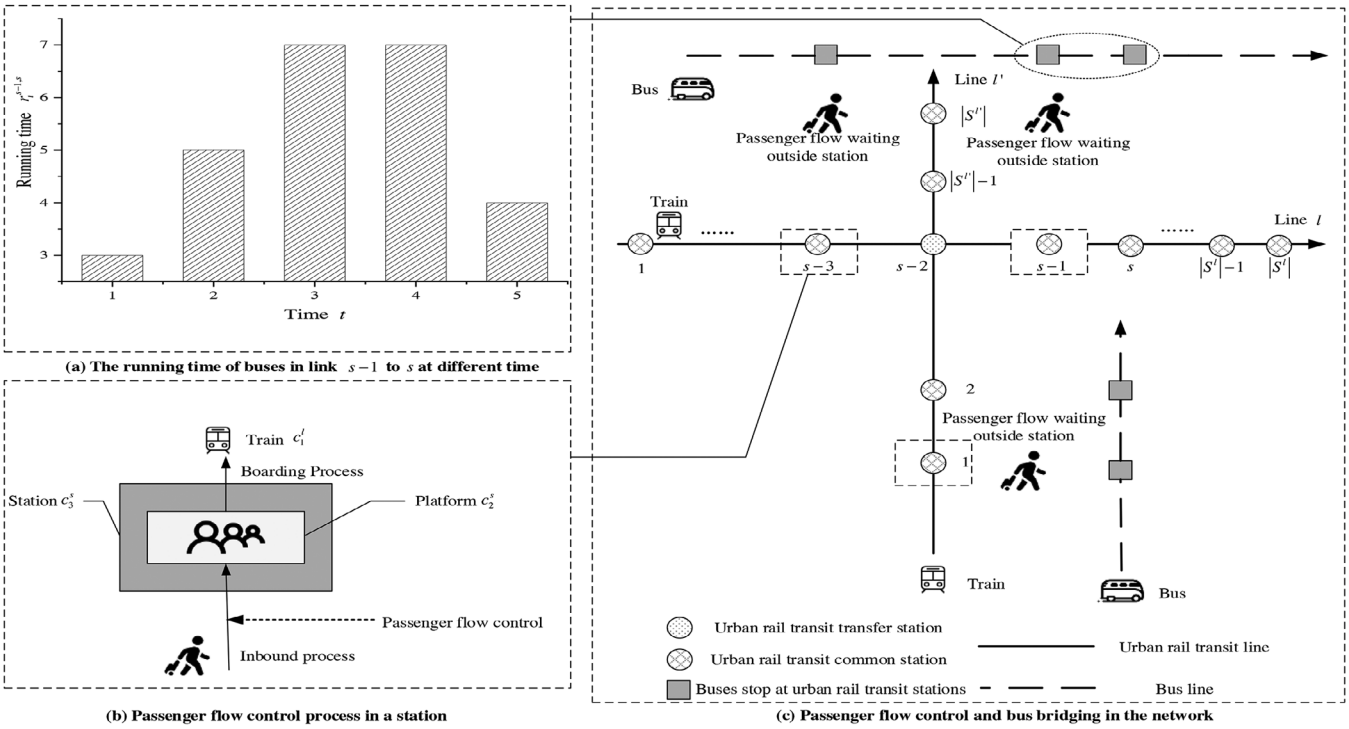


FIGURE 1 Coordination optimization of passenger flow control and parallel bus.

including ζ_k^s, the model can optimize bus departure times, improving the overall synchronization between the bus and metro systems and ensuring that passengers have access to timely and efficient transportation options.

3.2 | Problem statement

This paper investigates an urban rail transit network comprising multiple lines. As shown in Figure 1c, each line in the network $l, l' \in L$ contains the line code and train running direction. $S^l = \{1, 2 \ldots |S^l|\}$ represents a set of stations on line l, with an index of 1 to $|S^l|$ according to the direction of the train. Stations 1 and $|S^l|$ are the first station and terminal station of the line. Since the parallel bus service only stops at the stations of the urban rail transit line, it is sufficient to define the urban rail transit network to characterize the entire network structure.$T = \{1, 2 \ldots |T|\}$ represents a set of study timestamps. The duration of each study period is χ. The passenger flow data input in our model includes factors such as the origin–destination (OD) ratios and transfer proportions, which inherently account for two-way (up and down direction) travel demand.

Since failure to control the inbound passengers is likely to cause serious congestion or endanger operational safety, it is necessary to study strategies for passenger flow control. This paper aims to ease passenger congestion by implementing coordinated flow control across multiple stations. Train departure times are discretized to simplify the model construction and ensure the synchronization of train operations with passenger travel demand (Yang et al., 2017; Yin et al., 2021; Yuan et al., 2020). Figure 1b shows the process of passengers' movement in a station. Inbound capacity, platform capacity, and train capacity affect the passengers' distribution. When the inbound passenger flow reaches the inbound capacity threshold c_3^s, passengers need to wait outside the station. The limiting rate of the station should not exceed the threshold σ since an excessive limiting rate affecting inbound passengers may cause dissatisfaction among passengers. If passengers cannot board the train, they wait on the platform, and the number of waiting passengers cannot exceed the platform capacity limit c_2^s. The capacity of trains in urban rail transit is c_1^l, so the count of passengers on board cannot exceed c_1^l. The waiting passenger flow is divided into two parts, one is the waiting passenger flow τ_t^s, inside stations, and the other is the waiting passenger flow ς_t^s, outside stations. The concrete process affects constraints of the passenger flow control, as shown in Equations (1)–(12). Passenger flow control directly affects passenger flow waiting outside stations. To meet the requirements of the train, waiting passenger flow needs to be further determined to reduce the travel time of passengers.

There are $|K^l|$ buses running on line l, numbered $\{1, 2 \ldots |K^l|\}$ in turn. Therefore, the number of buses is certain, and this constraint needs to be considered. From the

point of view of passengers getting on and off, the bus stops at least two stations, and from the physical structure of the line, the train can stop at all stations on the whole line at most. Based on that, constraints (13)–(15) are proposed. As shown in Figure 1c, taking line l as an example, the inbound passenger flow at stations $s-3$ and $s-1$ is large, so passenger flow control is carried out at stations $s-3$ and $s-1$. Passengers can choose to use parallel buses to reach their destinations if they wait outside stations where parallel buses stop. Parallel buses can stop at stations $s-3$, $s-1$, and s. If there is spare capacity for the buses, passengers coming from station $s-3$ for destinations $s-1$ and s can choose to take the parallel buses, as can passengers coming from station $s-1$ for destination s. If stop station $s-2$ is added, passengers from $s-1$ can transfer to station $s-2$ to urban rail transit line l'. If the parallel buses stop only at stations $s-3$ and $s-1$, then only passengers with station $s-3$ as origin and station $s-1$ as the destination can choose to take the parallel buses, and passengers waiting outside other stations cannot choose the parallel buses. The above three cases, which explain the relationship between passengers getting on and off the bus and bus stops, are related to constraints (21)–(25). The running time $r_t^{s-1,s}$ of the buses in the link from station $s-1$ to station s is forecast and obtained from some intelligent transport infrastructure and collected by detection systems (vehicle detectors, cameras, automatic vehicle positioning systems, and so on). As shown in Figure 1a, $r_t^{s-1,s}$ varies at different periods, meaning that the running time $t_k^{s-1,s}$ of bus k from $s-1$ to s is different at $t=1$ compared to $t=3$. The running time $t_k^{s-1,s}$ directly affects the arrival and departure time. The count of passengers waiting outside station s varies according to the time of arrival and departure, and the count of passengers who can board bus k is also different, which constitutes constraints (16)–(20). If the running time $t_k^{s-1,s}$ is within a reasonable range and passengers are waiting outside the station as bus k passes station $s-1$, passengers will definitely choose to travel by bus. If the running time of the link is too long, passengers will keep waiting to enter the station. This means that the running time of the buses in the link is an important variable. Besides, $t_k^{s-1,s}$ is relevant to the arrival and departure time of bus k at station $s-1$. To reduce the passenger travel cost, it is necessary to plan a parallel bus stop scheme and the arrival and departure times at each station reasonably.

PFCBB are closely related in essence. In other words, with passenger flow control in place, the number of passengers seeking to enter the station dictates the count of passengers using the buses, while the count of passengers waiting outside the station is also influenced by the bus operation plan. Since both strategies aim to enhance the level of service, considering the strategies comprehensively can help optimize the entire public transport system service. This paper, therefore, examines a strategy for the collaborative optimization of urban rail transit PFCBB, aiming to minimize both passenger travel time and bus operation costs across the entire network, while aligning with train capacity.

To develop this collaborative optimization strategy, we assume the following:

1. Passenger travel demand is known. The main research content of this paper is to alleviate the passenger flow pressure of urban rail transit. Therefore, the passenger flow input in this paper is mainly the passenger flow of urban rail transit. This part of the passenger flow can be obtained by forecasting. Data used in this article such as passengers' origin stations, destination stations, and paths are historic and obtained by the clearing system of urban rail transit (Geng et al., 2024).
2. Passengers do not give up their desire to travel to their destination because of passenger flow control, nor do they change their travel destination (Shi et al., 2019; Yuan et al., 2020). This can make the input demand determined and conducive to the construction of mathematical models (Geng et al., 2024).
3. The bus running time between stations is predicted by intelligent transport infrastructure and collected by detection systems, and the urban rail transit trains run without line interruption (Dou et al., 2019).
4. We do not consider passengers transfer between different buses on one line, that is, passengers can only select buses that have stop plans covering both their origin and destination along the line (Geng et al., 2024). If the final destination of the passenger on the bus is on another line, the passenger needs to transfer to the urban rail transit at the urban rail transit interchange station.
5. The process of buses arriving from the depot to the station is not considered (Li et al., 2024; Yang et al., 2017).
6. Passengers waiting outside the station can choose parallel buses to reach their destination on the line, but those waiting inside the station can only choose urban rail transit (Li et al., 2024; Yang et al., 2017). This can make the choice of multimodal traffic passengers determined, conducive to the construction of mathematical models (Geng et al., 2024).

4 | MATHEMATICAL MODEL

4.1 | Constraints

Constraint (1) indicates that the actual number of passengers entering station s at time t with passenger flow control

does not exceed the passenger demand in the absence of passenger flow control. At a transfer station, it is crucial to consider passengers who exit the buses and proceed into the station.

$$i_t^s \le \begin{cases} e_t^s & \forall s \in S^l, l \in L, t = 1, \delta^s = 0 \\ e_t^s + \varsigma_{t-1}^s & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 0 \\ e_t^s + \varphi_t^s & \forall s \in S^l, l \in L, t = 1, \delta^s = 1 \\ e_t^s + \varphi_t^s + \varsigma_{t-1}^s & \forall s \in S^l, l \in L, t \in T, \delta^s = 1 \end{cases}. \quad (1)$$

A large number of passenger restrictions in a station are likely to cause passenger dissatisfaction and will cause passengers to gather, leading to insecurity. Constraint (2) ensures that the rate of passenger flow control does not surpass the maximum flow limit rate.

$$i_t^s \ge \begin{cases} (1-\sigma)e_t^s & \forall s \in S^l, l \in L, t = 1, \delta^s = 0 \\ (1-\sigma)(e_t^s + \varsigma_{t-1}^s) & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 0 \\ (1-\sigma)(e_t^s + \varphi_t^s) & \forall s \in S^l, l \in L, t = 1, \delta^s = 1 \\ (1-\sigma)(e_t^s + \varphi_t^s + \varsigma_{t-1}^s) & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 1 \end{cases}. \quad (2)$$

Constraint (3) indicates the relationship between the passenger flow waiting outside stations, the new arrival passenger flow, the passenger flow who take buses, the passengers who get off the bus and transfer to the station, along with the actual number of passengers entering the station under flow control. Security checks are of particular concern to urban rail transit operators for the enclosed systems and huge passenger flow in urban rail transit. Moreover, restricted by the station building structure, many stations have a limited number of entrances, which means some passengers may line up at station entrances for security checks. In particular, the insufficiency of security equipment at some urban rail transit stations tremendously delays the process of passengers entering the stations. Therefore, the number of passengers entering the station per unit time is limited. To account for these factors, inbound capacity should be regarded as a crucial element of passenger flow control. (Yuan et al., 2020). Constraint (4) indicates that inbound passenger flow does not exceed the inbound capacity.

$$\varsigma_t^s = \begin{cases} e_t^s - i_t^s - \sum_{k \in K^l} \eta_{k,t}^s & \forall s \in S^l, l \in L, t = 1, \delta^s = 0 \\ e_t^s + \varsigma_{t-1}^s - i_t^s - \sum_{k \in K^l} \eta_{k,t}^s & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 0 \\ e_t^s + \varphi_t^s - i_t^s - \sum_{k \in K^l} \eta_{k,t}^s & \forall s \in S^l, l \in L, t = 1, \delta^s = 1 \\ e_t^s + \varphi_t^s + \varsigma_{t-1}^s - i_t^s - \sum_{k \in K^l} \eta_{k,t}^s & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 1 \end{cases}, \quad (3)$$

$$i_t^s \le c_3^s \quad \forall s \in S^l, l \in L, t \in T. \quad (4)$$

For a station, the platform area is fixed, so the number of passengers waiting on the platform cannot exceed the capacity. Constraint (5) indicates that the number of passengers on the platform is less than the platform capacity. For a nontransfer station, the passengers on the platform include both passengers entering the station and those waiting inside the platform. For a transfer station, in addition to the above two parts, passengers on the platform also include transfer passengers.

$$c_2^s \ge \begin{cases} i_t^s & \forall s \in S^l, l \in L, t = 1, \delta^s = 0 \\ i_t^s + \phi_t^s & \forall s \in S^l, l \in L, t = 1, \delta^s = 1 \\ i_t^s + \tau_{t-1}^s & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 0 \\ i_t^s + \tau_{t-1}^s + \phi_t^s & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 1 \end{cases}. \quad (5)$$

The number of passengers on a train can be derived from the number of passengers boarding and alighting at each station. Constraint (6) indicates that if $s = 1$, the count of passengers on a train departing from station s equals the count of passengers who boarded the train at station s. If $s > 1$, when the train leaves station s, the count of passengers on the train is equal to that of passengers on the train leaving the previous station $s-1$, minus the count of passengers alighting at station s, plus the number of passengers boarding at station s.

$$\psi_t^s = \begin{cases} \ell_t^s & \forall l \in L, t \in T, s = 1 \\ \psi_t^{s-1} - \partial_t^s + \ell_t^s & \forall s \in S^l\backslash\{1\}, l \in L, t \in T \end{cases}. \quad (6)$$

The number of passengers on the train must remain within the limits set by the maximum load factor requirement, as specified in constraint (7). According to the proportion of passengers entering at station s' and exiting at station s, the number of passengers who get on at the previous $s-1$ stations and get off at station s can be calculated. Constraint (8) defines the number of passengers alighting at station s.

$$\psi_t^s \le \alpha * c_1^l \quad \forall s \in S^l, l \in L, t \in T, \quad (7)$$

$$\partial_t^s = \sum_{s' \in \{1,2,\ldots s-1\}} \lambda^{s',s} \ell_t^{s'} \quad \forall s \in S^l, l \in L, t \in T. \quad (8)$$

If passengers are unable to board a train, they must remain on the platform until the next available train arrives. Constraint (9) specifies that the number of passengers waiting inside the station is calculated as the total number of passengers on the platform minus those

boarding the train at that station.

$$\tau_t^s = \begin{cases} i_t^s - \ell_t^s & \forall s \in S^l, l \in L, t = 1, \delta^s = 0 \\ i_t^s + \phi_t^s - \ell_t^s & \forall s \in S^l, l \in L, t = 1, \delta^s = 1 \\ i_t^s + \tau_{t-1}^s - \ell_t^s & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 0 \\ i_t^s + \tau_{t-1}^s + \phi_t^s - \ell_t^s & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 1 \end{cases}. \tag{9}$$

Only the passengers on the platform can board the train. Therefore, the number of passengers at station s board the train is less than the number of passengers on the platform. Constraint (10) states that the count of passengers boarding the train cannot exceed the total number of passengers present on the platform.

$$\ell_t^s \le \begin{cases} i_t^s & \forall s \in S^l, l \in L, t = 1, \delta^s = 0 \\ i_t^s + \phi_t^s & \forall s \in S^l, l \in L, t = 1, \delta^s = 1 \\ i_t^s + \tau_{t-1}^s & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 0 \\ i_t^s + \tau_{t-1}^s + \phi_t^s & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 1 \end{cases}. \tag{10}$$

According to the proportion of transfer passengers, the count of passengers transferring to the urban rail transit is shown in constraint (11). The number of passengers transferring to the urban rail transit system is specified in constraint (11).

$$\phi_t^s = \sum_{l' \in L\backslash\{l\}} \sum_{s' \in S^{l'}} \lambda^{l',s',s,l} \ell_t^{s'} \quad \forall s \in S^l, l \in L, t \in T, \delta^s = 1. \tag{11}$$

Constraint (12) indicates that if the total passenger demand at station s at time t is greater than the number of passengers actually entering station s at time t with passenger flow control, then station s needs to carry out passenger flow control at time t with passenger flow control, then station s needs to carry out passenger flow control at time t, that is, $x_t^s = 1$ or otherwise, $x_t^s = 0$. In the case of a transfer station, passengers who disembark buses to transfer at station s need to be considered. If $t > 1$, the passenger flow ς_{t-1}^s waiting for outside stations in $t-1$ also should be considered.

$$\begin{cases} i_t^s - e_t^s + 1 \le M(1 - x_t^s) & \forall s \in S^l, l \in L, t = 1, \delta^s = 0 \\ e_t^s - i_t^s \le x_t^s M & \forall s \in S^l, l \in L, t = 1, \delta^s = 0 \\ i_t^s - e_t^s - \varphi_t^s + 1 \le M(1 - x_t^s) & \forall s \in S^l, l \in L, t = 1, \delta^s = 1 \\ e_t^s + \varphi_t^s - i_t^s \le x_t^s M & \forall s \in S^l, l \in L, t = 1, \delta^s = 1 \\ i_t^s - e_t^s - \varsigma_{t-1}^s + 1 \le M(1 - x_t^s) & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 0 \\ e_t^s + \varsigma_{t-1}^s - i_t^s \le x_t^s M & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 0 \\ i_t^s - e_t^s - \varphi_t^s - \varsigma_{t-1}^s + 1 \le M(1 - x_t^s) & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 1 \\ e_t^s + \varsigma_{t-1}^s + \varphi_t^s - i_t^s \le x_t^s M & \forall s \in S^l, l \in L, t \in T\backslash\{1\}, \delta^s = 1 \end{cases}. \tag{12}$$

The count of buses running on line l does not exceed the total number of buses configured on the line, as shown in constraint (13). Constraint (14) requires that when bus k does not run, subsequent trains, such as $k+1$ and $k+2$ also do not run. If bus k runs on line l, it will stop at no less than two stations and at most at $|S^l|$ stations. Therefore, the variable y_k^s, referring to whether the bus k stops at a station, and the variable h_k, referring to whether the bus k runs, are shown in constraint (15).

$$\sum_{k \in K^l} h_k \le \left|K^l\right| \forall l \in L, \tag{13}$$

$$h_k \ge h_{k+1} \forall k \in K^l\backslash\{\left|K^l\right|\}, l \in L, \tag{14}$$

$$2 * h_k \le \sum_{s \in S^l} y_k^s \le h_k \left|S^l\right| \quad \forall s \in S^l, l \in L, k \in K^l. \tag{15}$$

The arrival time of bus k at station s is calculated by adding the departure time of the bus at station $s-1$ to the running time from station $s-1$ to station s, as shown in constraint (16). Although the distance between the origin and destination does not vary, the running time is different at different periods. For example, buses spend 20 min if they depart at station $s-1$ at 8:00 am; however, they may spend 30 min if they depart at station $s-1$ at 9:00 am. Time-varying running time affects passengers' travel time on buses. Therefore, time-varying running time should be considered in a bus-bridging scheme to reflect a real-world situation. Because the running time of links varies with time, the running time of bus k in links is not fixed. The running time of bus k from station $s-1$ to station s depends on the departure time, as shown in constraint (17). This is a nonlinear constraint.

$$t_k^s = \zeta_k^{s-1} + t_k^{s-1,s} \quad \forall s \in S^l\backslash\{1\}, l \in L, k \in K^l, \tag{16}$$

$$t_k^{s-1,s} = r_t^{s-1,s} \quad \forall s \in S^l, l \in L, k \in K^l, t = \zeta_k^{s-1}. \tag{17}$$

The departure time of the bus at station s is shown in constraint (18). To simplify the calculation difficulty, this paper makes $t^s = \chi$. Also, the arrival and departure time of all buses should be within the research time range, as shown in constraint (19).

$$\zeta_k^s = t_k^s + y_k^s * t^s \quad \forall s \in S^l, l \in L, k \in K^l, \tag{18}$$

$$0 \le t_k^s, \zeta_k^s \le |T| \quad \forall s \in S^l, l \in L, k \in K^l. \tag{19}$$

If time t is between the arrival and departure time of the bus k at station s, $\mu_{k,t}^{s}=1$, or otherwise $\mu_{k,t}^{s}=0$.

$$\mu_{k,t}^{s}=\begin{cases}1 & \text{if } t_k^s<t\le\zeta_k^s\\ 0 & \text{else}\end{cases}\quad \forall s\in S^l, l\in L, t\in T, k\in K^l. \tag{20}$$

As shown in constraints (21) and (22), the count of passengers taking the kth bus at station s at time t is less than the total count of passengers waiting outside station s if the bus stops at their destination.

$$\eta_{k,t}^{s}\le\sum_{s'\in\{s+1,s+2,\dots|s|\}}\mu_{k,t}^{s}y_k^{s'}\lambda^{s,s'}\varsigma_t^s \quad \forall s\in S^l, l\in L, t\in T, k\in K^l, \tag{21}$$

$$\nu_k^s=\sum_{t\in T}\eta_{k,t}^{s}\quad \forall s\in S^l, l\in L, k\in K^l. \tag{22}$$

The load of buses is fixed, so the current number of passengers on a bus cannot exceed the bus capacity. Constraint (23) indicates that if $s=1$, the count of passengers on the bus at station s equals that of passengers boarding at station s. If $s\ge 2$, the number of passengers aboard the bus at station s is equal to that of passengers at the previous station $s-1$, minus those alighting at station, plus the passengers boarding at station. Constraint (24) means that the count of passengers on a bus must never exceed its maximum capacity.

$$\xi_k^s=\begin{cases}\nu_k^s & \forall l\in L, k\in K^l, s=1\\ \xi_k^{s-1}+\nu_k^s-\sum_{s'\in\{1,2,\dots s-1\}}\lambda^{s',s}\nu_k^{s'} & \forall s\in S^l\backslash\{1\}, l\in L, k\in K^l\end{cases}, \tag{23}$$

$$\xi_k^s\le c_4\quad \forall s\in S^l, l\in L, k\in K^l. \tag{24}$$

The number of passengers on buses who transfer to urban rail transit is shown in constraint (25).

$$\varphi_t^s=\sum_{s'\in\{1,2,\dots s-1\}}\sum_{k\in K^l}\mu_{k,t}^{s}\beta^{s',s}\nu_k^{s'}\quad \forall s\in S^l, l\in L, t\in T, \delta^s=1. \tag{25}$$

4.2 | Objective function

From the demand standpoint, the overall travel cost of passengers is a crucial metric. Passengers' travel cost mainly refers to the time cost passengers take to reach their destination. The travel time cost z_1 for all passengers during the study period consists of the travel time cost for those using urban rail transit (i.e., the sum of waiting time and time spent on trains) and the time cost z_2 for passengers traveling by bus.

$$z_1=\sum_{l\in L}\sum_{s\in S^l}\sum_{t\in T}\chi*(\varsigma_t^s+\tau_l^s)\varepsilon_2+\sum_{l\in L}\sum_{s\in S^l}\sum_{t\in T}\chi*\psi_t^s*\varepsilon_2. \tag{26}$$

$$z_2=\sum_{l\in L}\sum_{s\in S^l}\sum_{k\in K}(t^s+t_k^{s,s+1})\xi_k^s\varepsilon_2. \tag{27}$$

From the supply side, operational managers prioritize the operating cost of buses over passenger travel time. As shown in Equations (28) and (29), the whole operating cost of buses includes the fixed cost z_3 and the variable cost z_4 of buses. The variable cost (such as fuel consumption) is positively correlated with the running time in links and the number of passengers on buses.

$$z_3=\sum_{l\in L}\sum_{k\in K^l}h_k\varepsilon_1, \tag{28}$$

$$z_4=\sum_{l\in L}\sum_{s\in S^l}\sum_{k\in K}\xi_k^s t_k^{s,s+1}\varepsilon_3, \tag{29}$$

where w_1 is the weight of passenger travel cost and w_2 is the weight of bus operating cost. In summary, the joint optimization of PFCBB can be expressed as

$$\begin{cases}\min w_1(z_1+z_2)+w_2(z_3+z_4)\\ \text{s.t. Constraints (1)–(25)}\end{cases}. \tag{30}$$

4.3 | Model decomposition and complexity analysis

Model complexity depends on the number of variables and constraints. The PFCBB model has two kinds of constraints and variables. According to the features of the model, constraints (1)–(12) are passenger flow control constraints, all of which are linear constraints. Constraints (13)–(25) are bus-bridging constraints, whereas constraints (17), (20), (21), and (25) are nonlinear constraints.

If the bus bridging is set, the PFCBB model can be transformed into an accurate passenger flow control model (represented by PFC). As demonstrated below, the PFC model is an integer linear programming model.

$$\begin{cases}\min z_1\\ \text{s.t. Constraints (1)–(12)}\end{cases}. \tag{31}$$

The symbol "-" in Table 3 signifies that the associated variable or constraint is not included in the model. The size of the PFC and PFCBB models is solely dependent on the scale of the urban rail transit network, the number of buses, and the duration of the study period. An

TABLE 3 Model complexity.

Element	PFC	PFCBB
$\partial_t^s, \ell_t^s, \psi_t^s, \phi_t^s, i_t^s, x_t^s, \zeta_t^s, \tau_t^s$	$\sum_{l\in L} \lvert S^l\rvert \times \lvert T\rvert$	$\sum_{l\in L} \lvert S^l\rvert \times \lvert T\rvert$
Constraint (1–10)	$\sum_{l\in L} \lvert S^l\rvert \times \lvert T\rvert$	$\sum_{l\in L} \lvert S^l\rvert \times \lvert T\rvert$
Constraint (11)	$\sum_{l\in L}\sum_{s\in S^l} \delta^s \times \lvert T\rvert$	$\sum_{l\in L}\sum_{s\in S^l} \delta^s \times \lvert T\rvert$
Constraint (12)	$\sum_{l\in L} \lvert S^l\rvert \times \lvert T\rvert$	$\sum_{l\in L} \lvert S^l\rvert \times \lvert T\rvert$
h_k	–	$\sum_{l\in L} \lvert K^l\rvert$
φ_t^s	–	$\sum_{l\in L} \lvert S^l\rvert \times \lvert T\rvert$
$t_k^s, \nu_k^s, \xi_k^s, y_k^s, \zeta_k^s$	–	$\sum_{l\in L} \lvert S^l\rvert \times \lvert K^l\rvert$
$\eta_{k,t}^s, \mu_{k,t}^s$	–	$\sum_{l\in L} \lvert S^l\rvert \times \lvert K^l\rvert \times \lvert T\rvert$
$t_k^{s-1,s}$	–	$\sum_{l\in L} \lvert K^l\rvert * (\lvert S^l\rvert - 1)$
Constraint (13)	–	$\lvert L\rvert$
Constraint (14)	–	$\sum_{l\in L} (\lvert K^l\rvert - 1)$
Constraint (15–18), (24)	–	$\sum_{l\in L} \lvert S^l\rvert \times \lvert K^l\rvert$
Constraint (20–23)	–	$\sum_{l\in L} \lvert S^l\rvert \times \lvert K^l\rvert \times \lvert T\rvert$
Constraint (25)	–	$\sum_{l\in L}\sum_{s\in S^l} \delta^s \times \lvert K^l\rvert \times \lvert T\rvert$

Note: The symbol "-" signifies that the associated variable or constraint is not included in the model.

Abbreviations: PFC, passenger flow control; PFCBB, passenger flow control and bus bridging.

example is given to show the total count of decision variables and constraints for each of the models. We examine a scenario involving two urban rail transit lines, each with ten stations, a transfer station connecting the two lines, three buses operating on each line, and a planned time range of thirty periods. In this example, the PFC model has 4800 integer variables, of which 600 are binary variables; and 6660 constraints. There are 9360 integer variables, of which 246 are binary, and 14346 constraints in the PFCBB model. The PFCBB model is larger in scale compared to the PFC model. Moreover, the PFCBB model includes multiple nonlinear constraints, which increase the solution difficulty.

The bus-bridging problem contains many variables and nonlinear constraints, making it a challenging problem to solve. If the bus-bridging problem is treated as a subproblem (SP) (i.e., the bus-bridging scheme is the input data to the main problem (MP)), the MP is a PFC model that includes integer variables and constraints of modest scale, making it solvable efficiently with a CPLEX solver (Shi et al., 2018). The result of PFC can be input to the SP. Given that the dwell time is assumed to be fixed during the scheduling process, the primary decision variable in this problem is the arrival and departure time of each bus at every station. This design makes the problem easier to solve since it does not need to consider many binary variables in the bus-bridging part of the model. In this study, we intend to develop an optimization network and algorithm, utilizing an enhanced SA approach to address the bus-bridging problem, while employing the CPLEX solver to determine how to implement passenger flow control.

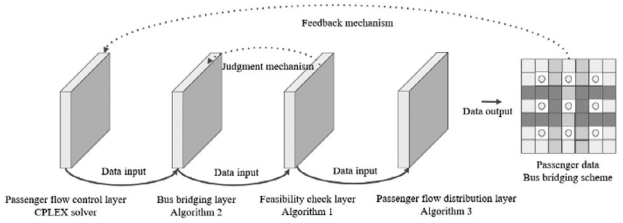


FIGURE 2 Intelligent optimization network of PFCBB.

5 | SOLUTION ALGORITHM

An intelligent optimization network is constructed according to the above processes as shown in Figure 2, which contains a passenger flow control layer, bus-bridging layer, feasibility check layer, and passenger flow distribution layer. Based on the optimization network, we design an algorithm to solve the joint optimization of passenger flow and bus bridging, including initial solution generation, new solution generation, passenger flow distribution, and search rules.

To facilitate the description, we introduce some important symbols used in the algorithm. m and i represent the temperature and the number of iterations, respectively.$t_k^s(1,1)$, $\zeta_k^s(1,1)$, and $y_k^s(1,1)$ are the initial solution. $z(1,1)$ represents the target value of the initial solution.$t_k^s(m,i)$, $\zeta_k^s(m,i)$, and $y_k^s(m,i)$ represent the new solution selected when the temperature is m at the ith iteration, and $z(m,i)$ is the target value corresponding to the current new solution.$t_k^s(*)$, $\zeta_k^s(*)$, and $y_k^s(*)$ are the best solution. $z(*)$ represents the current best target value. The maximum count of iterations is $i_{\max}$. $m_{\min}$, which stands for the minimum temperature. ϑ is the temperature change rate. z_{now} represents the optimal value now. gap stands for the gap value, that is, the algorithm stops when the gap of two iterations is smaller than this value.r_1, r_2, and r_3 are random numbers between 0 and 1.

5.1 | Initial solution generation

At the start of the local search process, an initial solution must be created to serve as the starting point for the first iteration. Typically, this initial solution can be generated

randomly. That is to say, we first randomly generate h_k, y_k^s, t_k^s, and ζ_k^s of buses at stations within a given time range. This allows an initial bus-bridging scheme to be generated. Then, it is necessary to test the feasibility of the scheme according to Algorithm A1 as presented in Appendix A. If this solution is valid, it will be selected as the initial solution. If this is not the case, the initial solution will be reinitialized and tested until a feasible one is achieved. For each feasible initial scheme, we use the CPLEX solver in the passenger flow control layer to determine the optimal strategy, with the resulting total passenger waiting time serving as an evaluation metric for the feasible schedule.

5.2 | New solution generation process

To obtain the new bus-bridging scheme, the original bus-bridging scheme will be randomly disturbed in the bus-bridging layer. The specific process is shown in Algorithm B1 (see Appendix B).

5.3 | Passenger flow distribution

The new passenger flow control strategy, that is, x_t^s, ς_t^s, and τ_t^s, is obtained using the CPLEX solver, and the bus-bridging scheme, that is, h_k, y_k^s, t_k^s, and ζ_k^s, is generated using Algorithm B1. In the actual situation, if the bus arrives at the station, the passengers will choose to board the bus if it has capacity remaining. Constraint (3) is used to update the number of passengers waiting outside the urban rail transit station. On this basis, Equations (26)–(29) are used to determine the objective function z at this time. The specific process of the passenger flow distribution layer is in Algorithm C1 (see Appendix C).

5.4 | Search rules

Note that since the traditional greedy algorithm chooses a current optimal solution every time it is updated, the solution obtained by the greedy algorithm easily falls into the local optimum. Although SA is also a greedy algorithm, it takes random factors into account in the search process. In the actual operation process, the SA algorithm can accept a new solution inferior to the current solution with a specific likelihood, which means that the SA can to some extent, escape local optimal solutions and explore the global optimal value. SA can solve engineering problems (Siddique & Adeli, 2016). It is also widely used in the transportation field (Kang et al., 2015), like bus bridging (Wang et al., 2021). This paper, therefore, utilizes the SA algorithm framework to address the intelligent optimization network for PFCBB. Based on Algorithms A1–C1, and the strong spatial and temporal coupling between the PFCBB scheme, the neighborhood search is carried out by disturbing the bus-bridging scheme, and the scheme of PFCBB is determined according to the optimization strategy of the SA algorithm. The algorithm terminates when the total target change value is within a certain range, or when a maximum iteration is reached. Otherwise, the bus-bridging scheme of SP is input into the MP to calculate the passenger flow control strategy, and the iteration continues, as shown in Algorithm 1.

6 | CASE STUDY

This section demonstrates the application of the proposed model and algorithm through a case study involving a small-scale scenario and a real-world example from the Chengdu Metro system. The proposed "CPLEX+SA" algorithm is programmed through Visual Studio 2019 on a Windows 10 computer with an Intel (R) Core (TM) i7-1195G7 @ 2.90 GHz processor and 16GB RAM. To explore the algorithm's effectiveness and make comparisons, we linearized the PFCBB model to obtain the PFCBB-L model (see Appendix D) and solved the model with a CPLEX solver.

6.1 | A small-scale case

6.1.1 | Parameter setting

The small-scale case study includes two lines, that is, $L = \{1, 2\}$. The running direction of the lines is known, with each line having three stations, $S^l = \{1, 2, 3\}$, including a transfer station, and the physical structure is shown in Figure 3. For ease of description, the stations are named A, B, C, D, E, and F, respectively. The running direction of line $l = 1$ is from station A to station C, and the running direction of line $l = 2$ is from station D to station F. In this case, if $l = 1$, there are two standby buses, and if $l = 2$, there are three standby buses. To establish the integer linear programming model, the unit time length is specified as 90 s. The research period encompasses a total of 30 timestamps (i.e., $T = \{1, 2, 3 \ldots 30\}$). The bus running time for every link in these settings is assumed as shown in Figure 4. The bus running time between A–B, B–C, D–E, and E–F is 4 at most and 1 at least. For every station, the maximum inbound capacity of the station is 120 passengers per time unit, the maximum capacity of the platform is 110 (unit:

ALGORITHM 1 The procedure of the "CPLEX+SA" algorithm.

Step 1: Initial maximum number of iterations $i_{\max}$, minimum temperature $m_{\min}$, temperature m, temperature change rate ϑ, and gap value gap, $z(*) = z_{now} = \infty$. A bus-bridging scheme is randomly generated, that is, $t_k^s(1,1)$, $\zeta_k^s(1,1)$, and $y_k^s(1,1)$, and Algorithm A1 is used to check the feasibility of the bus-bridging scheme. If possible, go to Step 2 or otherwise, repeat Step 1.

Step 2: Generate a passenger flow control strategy. According to the known bus-bridging scheme, combined with constraints (1)–(12), the objective function z_1 is minimized, and CPLEX is used to solve the passenger flow control strategy, that is, x_t^s, ς_t^s, and τ_t^s.

Step 3: Iterative optimization of bus-bridging scheme

While $m > m_{\min}$ do

While $i \le i_{\max}$ do

Step 3.1: Use Algorithm B1 to generate a new bus-bridging scheme $t_k^s(m,i)$, $\zeta_k^s(m,i)$, and $y_k^s(m,i)$.

Step 3.2: Use Algorithm A1 to check the feasibility of the bus-bridging scheme. If feasible, go to Step 3.3, or otherwise, go to Step 3.1.

Step 3.3: Use Algorithm C1 to generate the space-time distribution of passenger flow and calculate the target value of the current scheme.

Step 3.4: Determine the optimization result.

If $z(m,i) < z*$ then

$t_k^s(*) = t_k^s(m,i)$, $\zeta_k^s(*) = \zeta_k^s(m,i)$, $y_k^s(*) = y_k^s(m,i)$

$z(*) = z(m,i)$

Continue

End if

Calculate $p = \exp[-(-z(m,i) - z)/m]$ the acceptance of non-optimal values and generate random numbers 0–1.

If $r_3 < p$ then

$t_k^s(*) = t_k^s(m,i)$, $\zeta_k^s(*) = \zeta_k^s(m,i)$, $y_k^s(*) = y_k^s(m,i)$

$z(*) = z(m,i)$

End if

End While

Update Temperature $m = m * \vartheta$

End while

Step 4: Check the change value of the iteration value to determine whether the algorithm is finished.

If $\frac{|z_{now} - z(*)|}{z_{now}} < gap$ then

The algorithm ends. Go to Step 5

Else

$z_{now} = z(*)$, go to Step 2

End if

Step 5: Algorithm ends and output $t_k^s(*)$, $\zeta_k^s(*)$, and $y_k^s(*)$

passenger), the capacity of the urban rail transit train is 90 (unit: passenger), and the maximum load rate of the train is 120%. The maximum limit rate requirement of the station is 70%. The inbound volume of passengers in every period is shown in Figure 5. Inbound passenger demand reaches its peak during the period from 10 to 15. The costs ε_1, ε_2, and ε_3 are 100, 1, 5, respectively. The bus capacity is 10. w_1 and w_2 are set to 0.5. The transfer rate from A to B is 0.1, and the transfer rate from D to E is 0.15. The sufficiently large positive number M is 10,000. The CPLEX solver terminates when a solution is found within a duality gap of 0. $i_{\max}$ is set to 50. The gap value gap is 5%. m and $m_{\min}$ are set to 1000 and 0.01, respectively. The temperature change rate ϑ is 0.95.

6.1.2 | Result analysis

1. Passengers waiting outside and inside stations with bus bridging using CPLEX

The objective function value obtained by using CPLEX is 746,551. The calculation time is 50 s. As shown in Figure 6, at station A, the passengers wait outside the station during time 11–26 and 360 passengers wait outside the station at time 19. At station B, passengers begin to wait outside the station from time 11, and the passenger flow waiting outside the station peaks at time 19; passengers start waiting inside the station at time 14. Passenger flow waiting inside the station reaches its peak at time 26, and the passengers

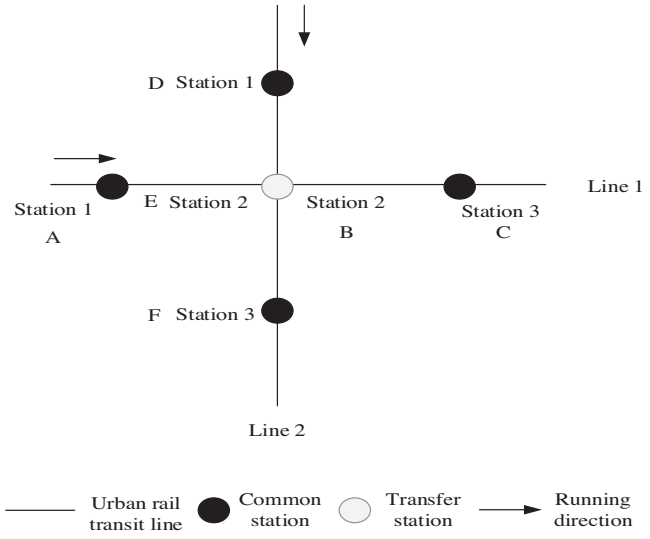


FIGURE 3 Physical structure of the small-scale case study.

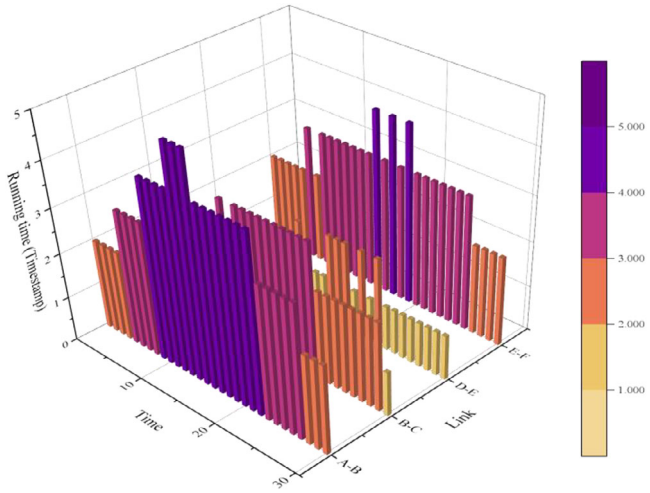


FIGURE 4 Running time in links between stations.

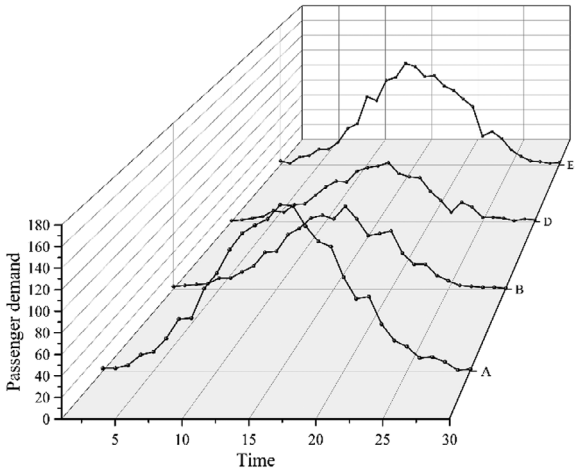


FIGURE 5 Inbound passenger flow in stations.

waiting inside and outside the station at time 27 are all evacuated. At station D, the passenger flow waiting outside the station reaches 228 at time 21, and the count of passengers waiting inside the station reaches a maximum of 30 at time 16. At station E, there are passengers waiting outside the station from time 10, and the passengers waiting outside the station are more than 200 from time 16 to 21. At time 27, all the passengers waiting inside and outside the station are evacuated. It is evident that the duration of control in the passenger flow control scheme is consistent with the demand for passengers waiting outside the station. Since evacuation takes time, the peak of passenger demand precedes that of waiting passengers.

2. Bus arrival and departure time and stopping plan using CPLEX

As shown in Figure 7a, bus 1 on Line 1 stops at stations B and C, while buses 2, 3, and 4 stop at stations A, B, and C, respectively. Bus 1 departs from station B at time 12 and arrives at station C at time 15. Bus 2 departs from station A at time 11 and takes 8 time units. Bus 3 takes 8 time units between A and C. Bus 4 takes 9 time units from A to C. Despite the same bus routes, the duration of buses 2, 3, and 4 is not identical due to time-varying running time. As shown in Figure 7b, buses 1 and 2 of Line 2 stop at stations D, E, and F. The two buses depart at time 11 and 12, respectively, at station D, with bus 1 taking 6 units of time and bus 2 taking 7 units of time. The bus stops at the station whenever there are passengers waiting outside. It reaches full capacity, with 10 passengers on board, when traveling along this route.

3. Passengers waiting outside and inside stations without bus bridging using CPLEX

When no buses are running, the total count of waiting passengers outside the station reaches 9174, while 1494 passengers are waiting inside the station. When there is bus delivery, the total passenger flow waiting outside the station is 8119, while 1296 passengers are waiting inside the station. The bus delivery effectively reduces the total waiting passenger flow, alleviating the pressure on the station.

4. Passengers waiting outside and inside stations with bus bridging using the CPLEX+SA algorithm

The objective value obtained by the CPLEX+SA algorithm is 757,108. The calculation time is 29 s. The passenger flow control strategy obtained using the CPLEX+SA algorithm is basically comparable with that obtained by CPLEX except for some minor differences. As depicted in Figure 8,

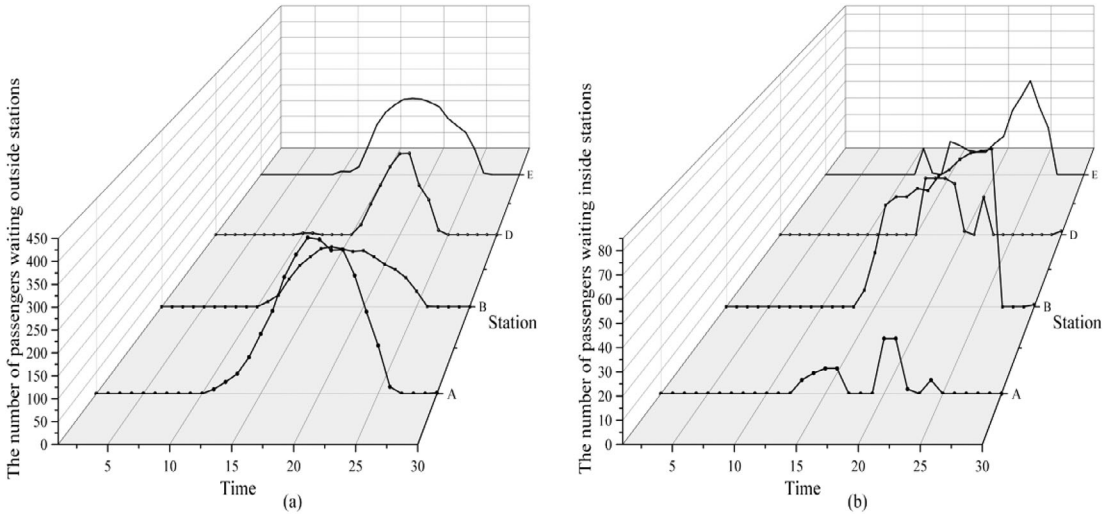


FIGURE 6 Waiting passenger flow with the CPLEX solver.

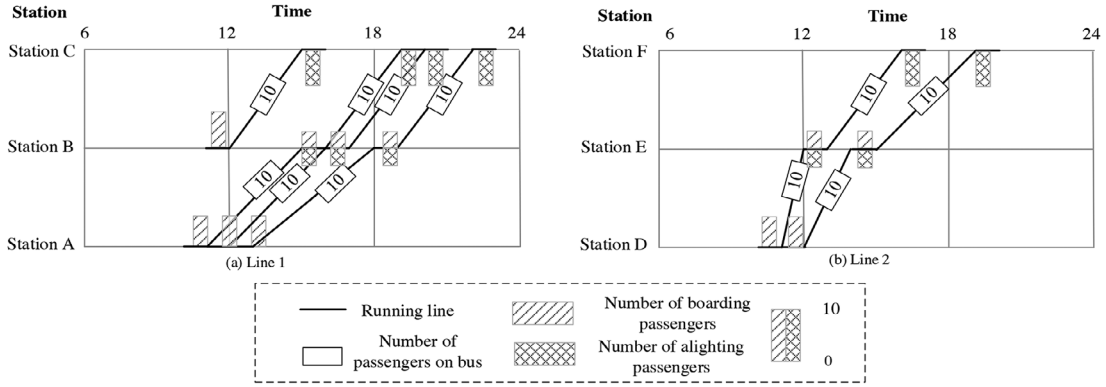


FIGURE 7 Bus-bridging scheme with the CPLEX solver.

there are 369 passengers waiting outside station A at time 19. The peak number of passengers waiting outside station B occurs at time 17, while the highest passenger flow inside the station is reached at time 25 and 26, and all the passengers waiting inside and outside the station at time 27 are evacuated. At station D, passengers should wait outside the station from time 15, and passenger flow control needs to be carried out at ten units of time. At time 21, the count of passengers waiting outside the station reaches 236, while the highest count of waiting passengers inside the station occurs at time 17 and 18, peaking at 58. Between time 15 and 21, the number of passengers waiting outside station E consistently exceeds 200.

5. Bus arrival and departure time and stopping plan using the CPLEX+SA algorithm

The bus-bridging scheme obtained using the CPLEX+SA algorithm differs from that obtained by CPLEX. As shown in Figure 9a, buses 1, 2, 3, and 4 on Line 1 stop at stations A, B, and C. Bus 1 departs from station A at time 14 and arrives at Station C at time 23, while bus 2 departs from station A at time 15 and takes nine time units to arrive at station C. Bus 3 takes ten time units from Station A to Station C. Bus 4 takes eight time units from A to C. Buses 3 and 4 run the same route at station B. Because of the time-varying running time, the duration of the bus may differ, even if the routes are identical. As shown in Figure 9b, buses 1 and 2 of Line 2 stop at stations D, E, and F. The two buses depart at time 16 and 17, respectively, at station D, with bus 1 taking eight units of time and bus 2 taking seven units of time. It can be seen that in the bus-bridging scheme obtained using the CPLEX+SA algorithm, buses park at stations where there are passengers waiting outside.

In summary, there is no significant difference between the objective function of the CPLEX solver and CPLEX+SA algorithm. Due to different bus schemes,

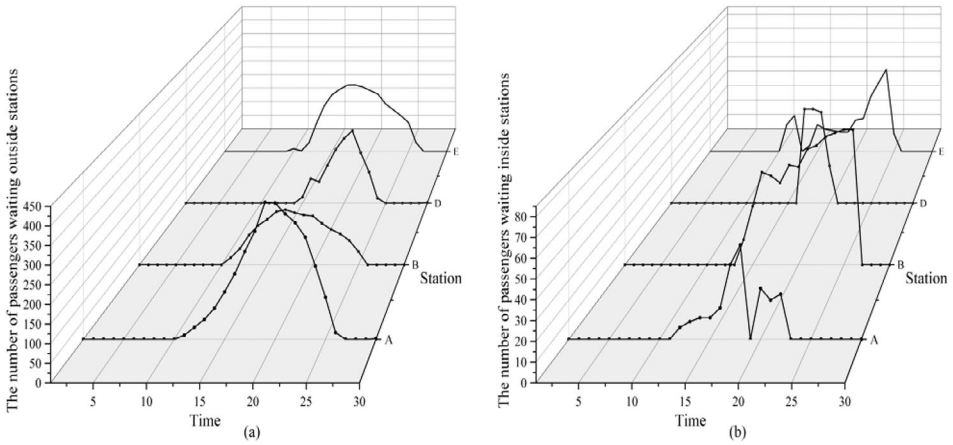


FIGURE 8 Waiting passenger flow with the CPLEX+SA algorithm.

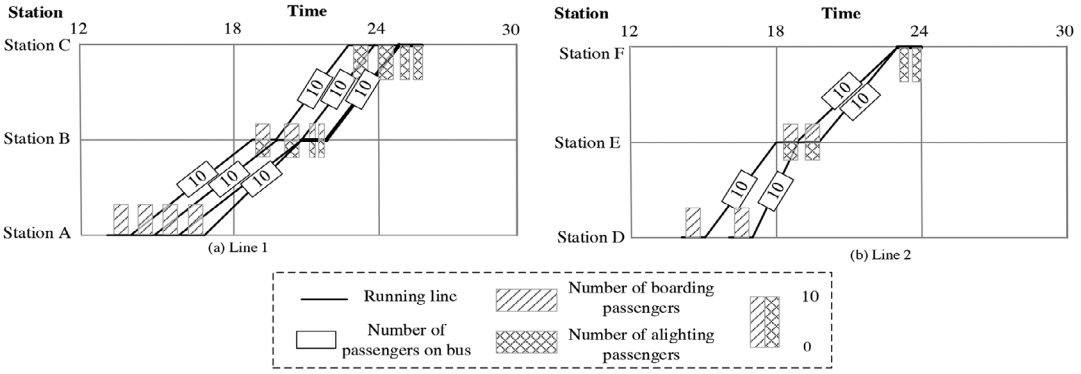


FIGURE 9 Bus-bridging scheme with the CPLEX+SA algorithm.

the passenger waiting time is also different. The overall cumulative count of waiting passengers outside the station across all periods is 8241, whereas the total number of passengers waiting inside the station reaches 1451. The cumulative number of waiting passengers is 3% greater than that obtained by CPLEX, but the difference between the two is not significant. The cumulative number of waiting passengers obtained by the CPLEX+SA algorithm is smaller than that without bus bridging. The CPLEX+SA algorithm can generate acceptable results in a shorter time.

6.2 | Numerical experiments on the Chengdu Metro

As of December 2021, the Chengdu Metro consisted of 12 lines, with a total of 333 stations in operation and 46 interchange stations, as shown in Figure 10. Its weekday passenger flow is normally more than six million, and urban rail transit accounts for more than 60% of public transport. The Chengdu Metro has significant network operation characteristics in terms of both construction scale and passenger flow scale. This paper, therefore, uses

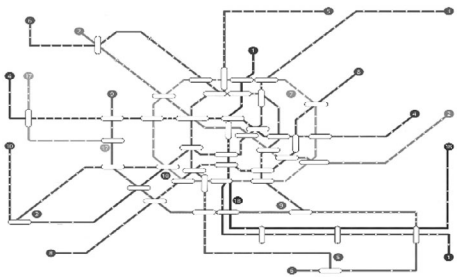


FIGURE 10 Network structure of the Chengdu Metro.

the proposed passenger flow control and bus connection model of urban rail transit for the Chengdu Metro.

6.2.1 | Parameter setting

Figure 11 shows the passenger flow of the Chengdu Metro on May 1, 2023. Inbound passenger flow shows obvious passenger flow peak characteristics, with its volume increasing first and decreasing later during the period 9:00 am to 11:00 am. Thus, 9:00 am to 11:00 am is taken as the study period in this paper.

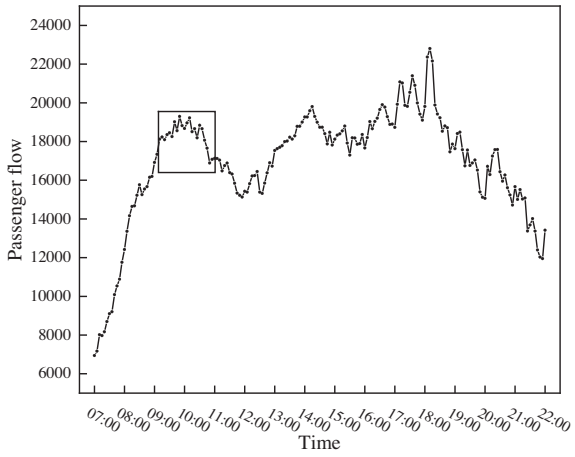


FIGURE 11 Inbound passenger flow of the Chengdu Metro. Source: Chengdu Metro.

TABLE 4 The correspondence between the actual line and the virtual line code in this research.

Line in reality	Line ID in this article		Capacity
	Downward direction	Upward direction	
Line 1	1	24	1468
Line 2	2	23	1468
Line 3	3	22	1460
Line 4	4	21	1468
Line 5	5	20	2414
Line 6	6	19	2480
Line 7	7	18	1828
Line 8	8	17	1804
Line 9	9	16	2456
Line 10	10	15	1504
Line 17	11	14	2096
Line 18	12	13	2958

This paper examines all 12 lines in the Chengdu Metro. To investigate both the passenger flow control scheme and the bus-bridging scheme, χ is set as 5 min in this case, and the period 9:00–11:00 is divided into 24 timestamps (i.e., $t \in \{1, 2 \ldots 24\}$). In every station, the maximum inbound capacity and the maximum capacity of the platform are obtained from the Chengdu Metro operator. We have set the capacity threshold to half of the provided value to account for the two-way nature of passenger demand. The maximum capacity of the urban rail transit train is shown in Table 4 (unit: passenger), the maximum train load factor of the train is 1.2, and the maximum capacity of the bus is 150 (unit: passengers). The value of both weight w_1 and w_2 is 0.5. The bus running time between stations in the links at different times is obtained from intelligent transport infrastructure API. The sufficiently large positive number M is 10,000. The costs ε_1, ε_2, and ε_3 are 200, 1, 8, respectively. The criteria for termination of the CPLEX solver is that a solution within the duality gap of 5% is found. The fixed cost of running a bus line is 1000. For a simpler description, the corresponding relation between the actual lines and the lines mentioned in this paper is shown in Table 4. It is worthwhile to note that the line IDs allocated here are the virtual numbers to serve the purpose of this paper. There are 2, 3, 2 buses on Lines 2, 4, 21. $i_{\max}$ is set to 50. The gap value gap is 5%. m and $m_{\min}$ are set to 1000 and 0.01, respectively. The temperature change rate ϑ is 0.95.

6.2.2 | Result analysis

1. Comparison of the results obtained by the CPLEX solver and the CPLEX+SA algorithm

The objective function value solved using the CPLEX solver is 3,711,097,204, and the calculation time is 72,312 s. The objective function value solved using the CPLEX+SA algorithm is 3,720,586,289, and the calculation time is 1,814 s. The waiting time outside stations calculated using the CPLEX solver is 173,474 time units, and the waiting time calculated using the CPLEX+SA algorithm is 193,440 time units. It is obvious that the scheme obtained using the CPLEX+SA algorithm is less effective in reducing the waiting time than the bus scheme calculated using the CPLEX solver. On the other hand, the calculation time of the CPLEX+SA algorithm is much shorter than that of the CPLEX solver. The CPLEX+SA algorithm is particularly valuable for short-term passenger flow prediction input (e.g., within a one-h timeframe), where rapid response is crucial.

With the passenger flow control, passenger flow waiting outside station 1 on Lines 2, 4, and 21 from time 11 to 20 is shown in Figure 12. The passenger flow control scheme in each line is closely related to the passenger flow trend. And the station with a large inbound volume has a large flow limit. In the two methods, the numbers of passenger flow on Lines 2 and 21 are close. However, the number of Line 4 obtained from the CPLEX+SA algorithm is smaller than that obtained from CPLEX. Since the bus-bridging scheme is related to passenger flow, when the passenger flow increases, the number of buses deployed needs to be increased. Therefore, the bus schemes under the two control scenarios also differ. The different bus schemes further affect the control plans. When no buses are deployed, the differences in the passenger flow control plans are more significant, and the model results tend to allow passengers to enter the stations. According to the result of the CPLEX solver in Table 5, one bus is running on Lines 2, 4, and 21. According to the result of the CPLEX+SA algo-

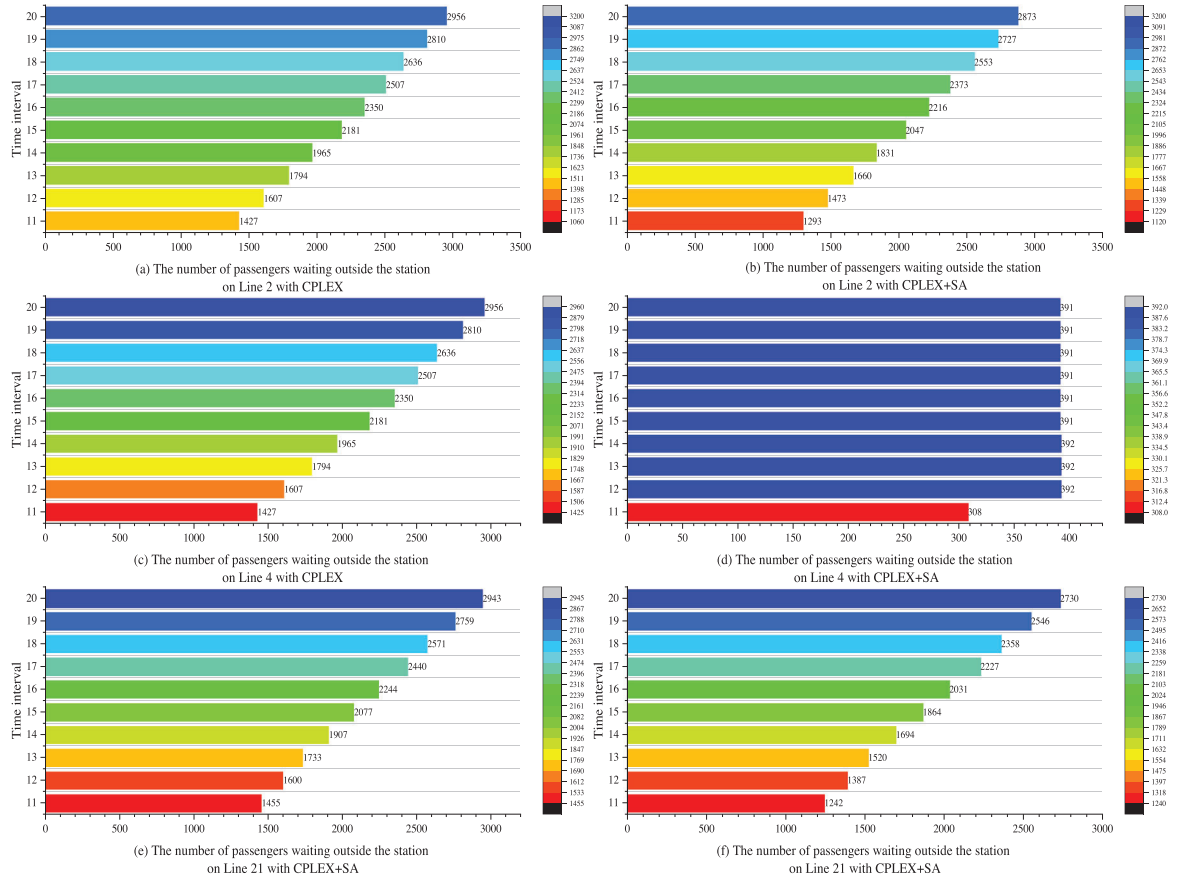


FIGURE 12 Passenger flow waiting outside stations.

rithm in Table 6, two buses are running on Line 2, two buses are running on Line 21, and no buses are running on Line 4. Every bus except bus 3 on Line 4 calculated using the CPLEX solver departs from station 1 since the first station of each line has passengers waiting outside the station. On Line 2, the two schemes basically have almost the same impact on the number of passengers waiting outside the station. On Line 4, three buses run in the scheme obtained using the CPLEX solver, so the waiting time of passengers outside the stations on Line 4 is 2,468, which is 7,087 in the result obtained from the CPLEX+SA algorithm. Whereas only one bus runs on Line 21 in the scheme obtained using the CPLEX solver, the scheme obtained using the CPLEX+SA algorithm runs two buses, which has a greater impact on the number of passengers waiting outside the station. According to the results obtained using CPLEX, there are waiting passengers at station 1 of Line 2 at time 5 but considering the long running time of buses at that time (659 s), the bus is selected to depart at time 11, with the running time at that time being 568 s, taking into account the running time of buses in each link. In the CPLEX+SA algorithm, the running time of buses from station 1 to station 2 on Line 2 at time 15 and 16 is also relatively short, so buses depart at time 15 and 16. Obviously, the arrival time and departure time of the buses take into account not only the number of passengers waiting outside the station but also the time-varying running time to reduce cost.

2. Comparison of the results in different scenarios

In this section, we compare the results with bus bridging, without bus bridging, and without both PFCBB. If there is only passenger flow control and no bus bridging, the waiting time of passengers inside and outside the station is 450,272. On the other hand, if there is bus bridging, the waiting time of passengers inside and outside the station is 443,742. If there is a bus bridging, the number of passengers waiting outside and inside the station can be effectively alleviated. If there is no PFCBB, the waiting time of passengers outside the stations is 744,510, and that of passengers inside the station is 265,447 according to the data from Chengdu Metro. In the scheme without PFCBB, the number of waiting passengers is higher than in the other two cases. To improve passengers' service level, both PFCBBs should be carried out.

TABLE 5 Bus arrival time and departure time with CPLEX.

Bus	Line	Station	Arrival time	Departure time
1	2	1	–	11
1	2	2	12	13
1	2	3	15	16
1	2	4	17	18
1	2	6	20	21
1	2	7	23	–
2	2	1	–	18
2	2	2	19	20
2	2	5	24	–
1	4	1	–	1
1	4	10	20	21
1	4	12	24	–
1	4	10	23	1
2	4	1	–	7
2	4	2	11	–
3	4	7	–	16
3	4	9	20	21
3	4	10	22	–
1	21	1	–	3
1	21	2	4	5
1	21	5	7	8
1	21	13	21	22
1	21	15	24	–

TABLE 6 Bus arrival time and departure time with the CPLEX+SA algorithm.

Bus	Line	Station	Arrival time	Departure time
1	2	1	–	15
1	2	2	16	17
1	2	3	18	19
1	2	4	20	21
1	2	5	22	–
2	2	1	–	16
2	2	2	17	18
2	2	3	19	20
2	2	4	21	–
1	21	1	–	17
1	21	2	18	19
1	21	3	20	21
1	21	4	22	–
2	21	1	–	18
2	21	2	19	20
2	21	3	21	22
2	21	4	23	–

6.2.3 | Sensitivity analysis

Since the weight involves the balance between the passenger service level and the operation cost, it is a subjective factor and has a greater impact on the model; hence, a sensitivity analysis of this factor is carried out in this paper. The time-varying running time of buses in links is also one of the main innovations of this paper, and thus, again, a sensitivity analysis is undertaken on this factor.

1. Weight analysis

As shown in Table 7, to study the influence of weight on the target value, four groups of different weight values are selected in this paper for experiments. As the value of w_1 increases, the value of w_2 decreases, and the value of the target value z continues to increase. According to the above experiments, it can be found that when w_1 grows, the number of buses first increases and then decreases, and the passenger travel time decreases initially and finally increases. When $w_2 > 0.4$, to reduce passengers' travel time, the number of buses running increases with the increase of the value of w_1; When $w_1 > 0.4$, the number of buses decreases to decrease passengers' travel time on buses and bus operation cost by carrying fewer passengers on buses. When $w_2 = 0.2$, the number of buses reaches the minimum. Therefore, $w_1 = 0.8$, $w_2 = 0.2$ is best for reducing bus operation cost. There are still buses running on Line 4; however, there is a gradual drop in the number of buses running on Line 21. In accordance with the result, the first of these experiments is more appropriate because the objective value is not too large and the number of buses in operation is relatively low. In the second experiment, passengers' travel time is shortest, so $w_1 = 0.4$ and $w_2 = 0.6$ is best for passengers. The weight change is mainly to change the strategy of bus connection because bus connection affects the operating cost but also affects the travel time of passengers. Through the experiments, it is found that when the weight changes, the bus-bridging scheme, and the urban rail transit traveling situation change accordingly. In the actual operation process, the urban rail transit operators need to determine the value of w_1 and w_2 according to the actual situation.

2. Analysis of running time in links

This paper changes the time-varying running time in links into a fixed value to analyze the influence of time-varying running time on the bus-bridging scheme. The bus running time of the links is fixed at 300, 600, 900, and 1200 s, respectively. As shown in Figure 13, when the running time of the links is 300 or 600 s, all buses will be on

TABLE 7 Objective function values and bus-bridging schemes under different weights.

w_1	w_2	$z_1 + z_2$	$z_3 + z_4$	z	Bus-bridging scheme
0.2	0.8	3,121,950,809	11,439	629,345,867	Line 2: 0,0 Line 4: 1,1,1 Line 21: 1,1
0.4	0.6	3,113,995,051	13,055	1,258,689,727	Line 2: 1, 1 Line 4: 1, 1, 1 Line 21: 1, 1
0.6	0.4	3,136,357,480	4,924	1,881,816,458	Line 2: 0, 0 Line 4: 1, 1, 1 Line 21: 1, 1
0.8	0.2	3,146,724,818	1,129	2,491,198,652	Line 2: 0, 0 Line 4: 1, 1, 1 Line 21: 0, 0

Note: 1 represents the bus is running, and 0 represents the bus is not running.

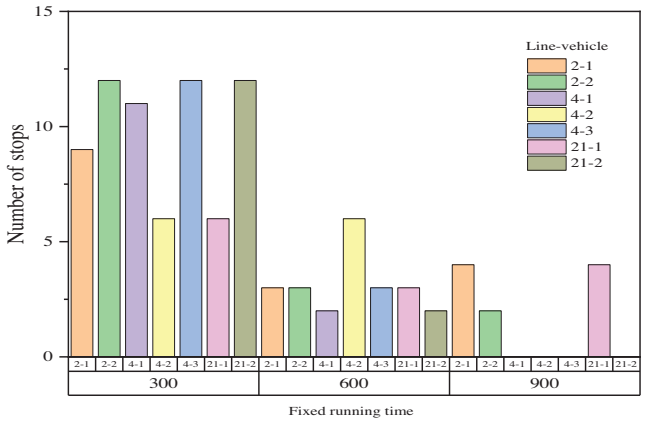


FIGURE 13 The number of bus stops under different fixed running time in links.

the road; however, when the running time of the links is 300 s, there will be a greater number of bus stops. When the running time of the links is short, there will be more bus stops. In the study time range, there will be more buses stop, and passengers will be more inclined to take buses. When the bus running time in links is increased to 900 s, only one bus runs on Line 4, and no buses run on Line 21. When the running time in links is fixed at 1200 s, no buses will run. The experiments reveal that with the increase in bus running time, the number of bus stops gradually decreases, and the number of running buses also gradually decreases. Whether it is higher or lower than the actual value, it will have a huge impact on the scheme. The results from the above four groups of tests differ from the schemes in Tables 5 and 6 because the fixed running time does not correspond to the actual situation, and overestimation or underestimation of the running time in links may lead to misjudgment of whether the bus is running or not and the stopping position. Operators should consider the time-varying running time when formulating the bus operation plan.

To compare the results related to fixed and time-varying running times, we employ the average value of the time-varying running time in a link as the fixed running time for that link. Only one bus runs on Line 21. Fixed running time leads to increased bus operating costs, so the number of buses is reduced. The waiting time of passengers outside stations is 196,167, which is lower than 202,586 in the scenario without bus bridging because bus bridging can reduce the number of passengers waiting outside stations. However, it is greater than that obtained using time-varying running time. Moreover, because fixed running time does not conform with reality, passengers prefer to choose urban rail transit and the number of waiting passengers increases. The objective value is 3,727,154,461, which is greater than the value found for a time-varying running time situation. Fixed travel time increases the objective function value. The model considering time-varying running time has more advantages in reducing the value of the objective function and waiting passengers. In addition, it shows that the time-varying running time is more realistic.

7 | CONCLUSION

With congestion growing increasingly serious in urban rail transit, PFCBB has become a common method of passenger flow organization. In recent years, artificial intelligence technology has developed rapidly, and it is increasingly easy to obtain the running time of the link through road intelligent transportation facilities. Accordingly, this paper developed the joint optimization of passenger flow control and parallel bus bridging in urban rail transit based on time-varying running time. An integer linear

programming model was constructed from the perspective of passenger travel time and enterprise operating cost. The proposed model was complex to solve since it contains many integer variables and constraints. With that in mind and considering that the model involves two distinct parts—PFCBB—the developed model was decomposed into MP and SP, and then an algorithm combining a CPLEX solver and SA was designed for the proposed model. In summary, the proposed method is effective in two experiments. The experimental results show that, compared with the strategy without bus bridging and passenger flow control, the proposed optimization model prevents the accumulation of waiting passengers inside and outside the station, and this reduces the total waiting time of passengers. This study will provide a basis for the joint optimization of passenger flow control and bus connection under large passenger flow conditions such as morning and evening peak hours.

Future research can explore several areas: (1) We only consider the parallel bus-bridging scheme. In practice, bus-bridging schemes between lines with nonfixed paths are a better fit for passengers' travel needs, and hence bus-bridging schemes between lines with nonfixed paths need further study. (2) The model construction in this paper is based on not considering the passenger's travel behavior. Although this reduces the difficulty of model construction, it is different from the actual situation. We will take the passenger's travel behavior into account to improve the accuracy of the model. (3) Although the solution speed of the algorithm proposed in this paper has been improved, it is still unable to give a real-time optimization scheme for the real-time situation of the subway. In the future, we will focus on the algorithm and try faster algorithms to solve the real-time problem.

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APPENDIX A
THE SOLUTION FEASIBILITY CHECK PROCEDURE

It is worth noting that during the random generation of the initial solution, a timetable for $|K^l|$ buses is produced, rendering the consideration of constraint (13) unnecessary in the feasibility assessment. During the bus-bridging process, both the arrival and departure

ALGORITHM A1 The solution feasibility check procedure.

Step 1: Calculate the running order of all buses.

For $k \in K^l$ do

Assess whether bus k satisfies constraint (14). If it does not, the scheme is considered infeasible, and the algorithm terminates.

If $k \neq |K^l|$ then

If $h_k > h_{k+1}$ then

The scheme is unfeasible and the algorithm terminates.

End if

End if

End for

Step 2: Verify if the bus satisfies the criteria outlined in constraint (15). If not affirmative, the scheme is deemed infeasible, prompting the algorithm to terminate.

Step 3: Examine whether both the arrival and departure times of the bus satisfy the requirements specified in constraint (19). If the criteria are not met, the scheme is deemed infeasible, prompting the algorithm to terminate.

Step 4: The scheme is determined to be feasible. Proceed to output the bus-bridging scheme.

times of all buses remain within the study time range. Given these conditions, the random generation of feasible solutions becomes straightforward. The procedure for verifying solution feasibility is outlined in Algorithm A1.

APPENDIX B
NEW SOLUTION GENERATION PROCESS

The roulette method is utilized for the random selection of the disturbance value. r_1 determines the departure time at the first station. Δt signifies that the disturbance quantity. The increment must be sufficiently diminutive to guarantee the adherence of the generated solution within the feasible range. This calculation process is illustrated in Equation (B1), where the disturbance quantity adheres to the probability distribution established by the disturbance function, and where r_2 dictates the decision of whether to stop at station s. In accordance with Equation (B2), if $r_2 < 0.5$, then $y_k^s = 0$; if $r_2 \geq 0.5$, then $y_k^s = 1$. During operational execution, buses are required to halt at stations with a considerable congregation of waiting passengers. Should the count of waiting passengers outside the station exceed the threshold $stopCriterion$ without any intervening stops from station 1 to station $s-1$, the inaugural bus halt necessitates a stop at the station with the highest density of external waiting passengers. This strategic maneuver optimizes bus capacity utilization, alleviates the waiting congestion in urban rail transit, and mitigates deadhead kilometers. Compliant with the constraints delineated in

ALGORITHM B1 New solution generation process.

Step 1: Initialize $h_k = 0$, $y_k^s = 0$, The index of the first stop $startIndex = 0$.

Step 2: Generate the binary variable indicating whether the bus makes stops at each station.

For $k \in K^l$ do

For $s \in S^l$ do

For $t \in T$ do

If $\varsigma_t^s > stopCriterion$ and $\sum_{s\in\{1,2..k-1\}} y_k^s < 1$ then

$y_k^s = 1$, $startIndex = s$

Break

End If

End For

If $y_k^s = 1$ then

Break

End if

End for

End for

For $k \in K^l$ do

For $s \in S^l$ do

If $y_k^s = 0$ and $s > startIndex$ then

Update y_k^s according to Equation (B2)

End if

End for

End for

Step 3: Generate h_k according to constraints (14) and (15).

For $k \in K^l$ do

If $\sum_{s\in S^l} y_k^s \geq 2$ then

$h_k = 1$

End if

End for

Step 4: Generate the departure time of the bus at the initial station s, where Δt represents the disturbance quantity.

For $k \in K^l$ do

If $h_k = 1$ then

Update Δt using Equation (B1). Update $\zeta_k^s(m,i) = \zeta_k^s + \Delta t$.

End if

End for

Step 5: Update the arrival and departure times of the bus at each station based on the departure time of the bus at the initial station and the binary variable indicating whether the bus stops at each station.

For $k \in K^l$ do

For $s \in S^l$ do

Update $t_k^s = \zeta_k^1 + \sum_{l\in L}\sum_{s\in S^l\setminus\{1\}} t_k^{s-1,s} + \sum_{l\in L}\sum_{s\in S^l\setminus\{1\}} y_k^{s-1} * t^{s-1}$

Update $\zeta_k^s = \zeta_k^1 + \sum_{l\in L}\sum_{s\in S^l\setminus\{1\}} t_k^{s-1,s} + \sum_{l\in L}\sum_{s\in S^l\setminus\{1\}} y_k^{s} * t^{s}$

End for

End for

Step 6: Verify the feasibility of the scheme following the guidelines outlined in Algorithm A1. If found infeasible, proceed with generating the bus-bridging scheme.

(16)–(18), the arrival and departure times of buses at each station are calculated.

$$\Delta t = \begin{cases} -\chi & 0 \le r_1 \le 0.3 \\ 0 & 0.3 < r_1 < 0.7 \\ \chi & 0.7 \le r_1 \le 1 \end{cases}, \tag{B1}$$

$$y_k^s = \begin{cases} 0 & 0 \le r_2 < 0.5 \\ 1 & 0.5 \le r_2 < 1 \end{cases}. \tag{B2}$$

APPENDIX C
PASSENGER FLOW DISTRIBUTION ANALYSIS PROCEDURE

The passenger flow distribution refers to the redistribution of passengers after the implementation of PFCBB. Based on the problem description in the paper, some passengers on the buses may disembark at transfer stations and reenter the stations, which introduces additional complexity in determining the final distribution of passengers. Integrated with the novel passenger flow control strategy, bus-bridging scheme, OD of passenger flow, and the bus timetable data, the passenger flow distribution in the network is computationally determined. Given the assumption in this paper regarding the bus's stopping interval $t^s = \chi$, it follows that $\sum_{t\in T} \eta_{k,t}^s = \eta_{k,t}^s$. Consequently, $\nu_k^s = \eta_{k,t}^s$.

ALGORITHM C1 Passenger flow distribution analysis procedure.

Step 1: Initialize the value of $\mu_{k,t}^s$ and y_k^s based on the arrival and departure times of the bus at each station.s

Step 2: Calculate ν_k^s, representing the number of passengers boarding the bus at each station, and φ_t^s, denoting the count of passengers transferring at the transfer station.

For $k \in K^l$ do
  For $s \in S^l$ do
    For $t \in T$ do
      If $t_k^s < t \le \zeta_k^s$ then

$$\nu_k^s = \min\{\sum_{s'\in\{s+1,s+2,\ldots|s|\}} \mu_{k,t}^s y_k^{s'} \lambda^{s,s'} \varsigma_t^s, c_4 - \xi_k^{s-1} + \sum_{s'\in\{1,2,\ldots s-1\}} \lambda^{s',s} \nu_k^{s'}\}$$

        If $\delta^s = 1$ then

$$\varphi_t^s = \sum_{s'\in\{1,2,\ldots s-1\}} \sum_{k\in K^l} \mu_{k,t}^s \beta^{s',s} \nu_k^{s'}$$

        End if
      End if
    End for
  End for
End for

Step 3: Based on the passenger counts $\eta_{k,t}^s$ and φ_t^s, in conjunction with constraint (3), adjust the tally of passengers awaiting transit outside each station ς_t^s.

Step 4: The objective function z is formulated based on τ_t^s, representing the count of passengers waiting inside the station, ς_t^s, the count of passengers awaiting transit outside the station, ψ_t^s, the count of passengers on urban rail transit train, $t_k^{s-1,s}$, the travel time of the bus along the link, and ξ_k^s, the count of passengers on the bus.

APPENDIX D
MODEL LINEARIZATION

As t is an integer, $\zeta_k^{s-1} \le t < \zeta_k^{s-1} + 1$ is equivalent to $t = \zeta_k^{s-1}$. To linearize constraint (17), binary variable $a_{k,t}^{s-1,s}$ is introduced. If $\zeta_k^{s-1} \le t < \zeta_k^{s-1} + 1$, $a_{k,t}^{s-1,s} = 1$ otherwise $a_{k,t}^{s-1,s} = 0$, as depicted in constraint (D1). Similarly, binary variable $b_{k,t}^{s-1,s}$ is introduced. if $t \ge \zeta_k^{s-1}$, $b_{k,t}^{s-1,s} = 1$, otherwise, $b_{k,t}^{s-1,s} = 0$, as depicted in constraint (D2). Binary variable $d_{k,t}^{s-1,s}$ is introduced. If $t < \zeta_k^{s-1} + 1$, $d_{k,t}^{s-1,s} = 1$, otherwise $d_{k,t}^{s-1,s} = 0$, as depicted in constraint (D3). Based on the logical relation, the relationship among $a_{k,t}^{s-1,s}$, $b_{k,t}^{s-1,s}$, and $d_{k,t}^{s-1,s}$ is depicted in constraint (D4). The linearization of constraints (D2)–(D4) is shown in constraints (D5)–(D11), and constraint (17) is transformed into constraint (D12).

$$a_{k,t}^{s-1,s} = \begin{cases} 1 & \text{if } \zeta_k^{s-1} \le t < \zeta_k^{s-1} + 1 \\ 0 & \text{else} \end{cases} \quad \forall s \in S^l\backslash\{1\}, l \in L, t \in T, k \in K^l, \tag{D1}$$

$$b_{k,t}^{s-1,s} = \begin{cases} 1 & \text{if } t \ge \zeta_k^{s-1} \\ 0 & \text{else} \end{cases} \quad \forall s \in S^l\backslash\{1\}, l \in L, t \in T, k \in K^l, \tag{D2}$$

$$d_{k,t}^{s-1,s} = \begin{cases} 1 & \text{if } t < \zeta_k^{s-1} + 1 \\ 0 & \text{else} \end{cases} \quad \forall s \in S^l\backslash\{1\}, l \in L, t \in T, k \in K^l, \tag{D3}$$

$$a_{k,t}^{s-1,s} = b_{k,t}^{s-1,s} d_{k,t}^{s-1,s} \quad \forall s \in S^l\backslash\{1\}, l \in L, t \in T, k \in K^l, \tag{D4}$$

$$\zeta_k^{s-1} - t \le M(1 - b_{k,t}^{s-1,s}) \quad \forall s \in S^l\backslash\{1\}, l \in L, t \in T, k \in K^l, \tag{D5}$$

$$t - \zeta_k^{s-1} + 1 \le M * b_{k,t}^{s-1,s} \quad \forall s \in S^l\backslash\{1\}, l \in L, t \in T, k \in K^l, \tag{D6}$$

$$t - \zeta_k^{s-1} \le M(1 - d_{k,t}^{s-1,s}) \quad \forall s \in S^l\backslash\{1\}, l \in L, t \in T, k \in K^l, \tag{D7}$$

$$\zeta_k^{s-1} - t + 1 \le M * d_{k,t}^{s-1,s} \quad \forall s \in S^l\backslash\{1\}, l \in L, t \in T, k \in K^l, \tag{D8}$$

$$a_{k,t}^{s-1,s} \le b_{k,t}^{s-1,s} \quad \forall s \in S^l\backslash\{1\}, l \in L, t \in T, k \in K^l, \tag{D9}$$

$$a_{k,t}^{s-1,s} \le d_{k,t}^{s-1,s} \quad \forall s \in S^l \backslash \{1\}, l \in L, t \in T, k \in K^l, \quad \text{(D10)}$$

$$a_{k,t}^{s-1,s} \ge b_{k,t}^{s-1,s} + d_{k,t}^{s-1,s} - 1 \quad \forall s \in S^l \backslash \{1\}, l \in L, t \in T, k \in K^l, \quad \text{(D11)}$$

$$t_k^{s-1,s} = \sum_{t \in T} a_{k,t}^{s-1,s} r_t^{s-1,s} \quad \forall s \in S^l \backslash \{1\}, l \in L, k \in K^l. \quad \text{(D12)}$$

Binary variable $f_{k,t}^s$ is introduced. If $t > t_k^s$, $f_{k,t}^s = 1$, otherwise $f_{k,t}^s = 0$, as depicted in (D13). Binary variable $g_{k,t}^s$ is introduced. If $t \le \zeta_k^s$, $g_{k,t}^s = 1$ otherwise $g_{k,t}^s = 0$, as shown in (D14). The relationship among $\mu_{k,t}^s$, $f_{k,t}^s$, and $g_{k,t}^s$ is depicted in constraint (D15). After linearization, constraints (D13)–(D15) are transformed into constraints (D16)–(D22), thereby converting constraint (20) into constraints (D16)–(D22).

$$f_{k,t}^s = \begin{cases} 1 & \text{if } t > t_k^s \\ 0 & \text{else} \end{cases} \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, \quad \text{(D13)}$$

$$g_{k,t}^s = \begin{cases} 1 & \text{if } t \le \zeta_k^s \\ 0 & \text{else} \end{cases} \quad \forall s \in S^l, l \in L, t \in T, k \in K^l \quad \text{(D14)}$$

$$\mu_{k,t}^s = f_{k,t}^s g_{k,t}^s \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, \quad \text{(D15)}$$

$$t_k^s - t + 1 \le M(1 - f_{k,t}^s) \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, \quad \text{(D16)}$$

$$t - t_k^s \le M * f_{k,t}^s \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, \quad \text{(D17)}$$

$$t - \zeta_k^s \le M(1 - g_{k,t}^s) \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, \quad \text{(D18)}$$

$$\zeta_k^s - t + 1 \le M * g_{k,t}^s \quad \forall s \in S^l, , l \in L, t \in T, k \in K^l, \quad \text{(D19)}$$

$$\mu_{k,t}^s \le f_{k,t}^s \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, \quad \text{(D20)}$$

$$\mu_{k,t}^s \le g_{k,t}^s \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, \quad \text{(D21)}$$

$$\mu_{k,t}^s \ge f_{k,t}^s + g_{k,t}^s - 1 \quad \forall s \in S^l, l \in L, t \in T, k \in K^l. \quad \text{(D22)}$$

By introducing variable $q_{k,t}^{s,s'} = \mu_{k,t}^s y_k^{s'}$, constraint (21) can be transformed into constraints (D23)–(D26). Let variable $o_{k,t}^{s,s'} = q_{k,t}^{s,s'} \zeta_t^s$ denote the most extreme scenario, where all passengers who entered the station before time t must wait for the bus, all buses are operational, and all passengers on the bus must disembark and transfer at station s. Accordingly, $\zeta_t^s \le \sum_{t \in \{1,2..t-1\}} e_t^s + c_4 * |K|$ and constraint (D23) is transformed into constraints (D27)–(D30).

$$\eta_{k,t}^s \le \sum_{s' \in \{s+1,s+2,\ldots|s|\}} q_{k,t}^{s,s'} \lambda_t^{s,s'} \zeta_t^s \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, \quad \text{(D23)}$$

$$q_{k,t}^{s,s'} \le \mu_{k,t}^s \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, \quad \text{(D24)}$$

$$q_{k,t}^{s,s'} \le y_k^{s'} \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, \; s' \in \{s+1, s+2, \ldots, |s|\}, \quad \text{(D25)}$$

$$q_{k,t}^{s,s'} \ge \mu_{k,t}^s + y_k^{s'} - 1 \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, s' \in \{s+1, s+2, \ldots, |s|\}, \quad \text{(D26)}$$

$$\eta_{k,t}^s \le \sum_{s' \in \{s+1,s+2,\ldots|s|\}} \lambda^{s,s'} o_{k,t}^{s,s'} \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, \quad \text{(D27)}$$

$$o_{k,t}^{s,s'} \le q_{k,t}^{s,s'} \Big(\sum_{t \in \{1,2..t-1\}} e_t^s + c_4 * |K| \Big) \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, s' \in \{s+1, s+2, \ldots, |s|\}, \quad \text{(D28)}$$

$$o_{k,t}^{s,s'} \le \zeta_t^s \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, s' \in \{s+1, s+2, \ldots, |s|\}, \quad \text{(D29)}$$

$$o_{k,t}^{s,s'} \ge \zeta_t^s - \Big(\sum_{t \in \{1,2..t-1\}} e_t^s + c_4 * |K| \Big)(1 - q_{k,t}^{s,s'}) \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, s' \in \{s+1, s+2, \ldots, |s|\}. \quad \text{(D30)}$$

Through the introduction of variable $u_{k,t}^{s',s} = \mu_{k,t}^s \beta^{s',s} \nu_k^s$, constraint (25) can be transformed into constraints (D31)–(D34).

$$\varphi_t^s = \sum_{s' \in \{1,2,\ldots,s-1\}} \sum_{k \in K} u_{k,t}^{s',s} \quad \forall s \in S^l, l \in L, t \in T, k \in K, t \in T, \delta_s = 1, \quad \text{(D31)}$$

$$u_{k,t}^{s',s} \le \beta^{s',s} \mu_{k,t}^s c_4 \quad \forall s \in S^l, l \in L, t \in T, k \in K, t \in T, s' \in \{1, 2, \ldots, s-1\}, \delta_s = 1, \quad \text{(D32)}$$

$$u_{k,t}^{s',s} \le \beta^{s',s} \nu_k^{s'} \quad \forall s \in S^l, l \in L, t \in T, k \in K, t \in T, s' \in \{1, 2, \ldots, s-1\}, \delta_s = 1, \quad \text{(D33)}$$

$$u_{k,t}^{s',s} \geq \beta^{s',s} v_k^{s'} - \beta^{s',s}(1-\mu_{k,t}^{s})c_4 \quad \forall s \in S^l, l \in L, t \in T, k \in K, t \in T, s' \in \{1,2,\dots,s-1\}, \delta_s = 1. \tag{D34}$$

If $m_{k,t}^{s,s+1} = \xi_k^s a_{k,t}^{s,s+1}$, $m_{k,t}^{s,s+1}$ satisfies constraints (D35)–(D37). The variable range is outlined in constraints (D38)–(D43).

$$m_{k,t}^{s,s+1} \leq c_4 * a_{k,t}^{s,s+1} \quad \forall s \in S^l \backslash \{\left|S^l\right|\}, l \in L, t \in T, k \in K, t \in T, \tag{D35}$$

$$m_{k,t}^{s,s+1} \leq \xi_k^s \quad \forall s \in S^l \backslash \{\left|S^l\right|\}, l \in L, t \in T, k \in K, t \in T, \tag{D36}$$

$$m_{k,t}^{s,s+1} \geq \xi_k^s - c_4 * (1 - a_{k,t}^{s,s+1}) \quad \forall s \in S^l \backslash \{\left|S^l\right|\}, l \in L, t \in T, k \in K, t \in T, \tag{D37}$$

$$u_{k,t}^{s',s}, v_k^{s',s}, \in z \quad \forall s \in S^l, l \in L, k \in K, s' \in \{1,2,\dots,s-1\}, \tag{D38}$$

$$q_{k,t}^{s,s'} = \{0,1\} \quad \forall s \in S^l, l \in L, t \in T, k \in K, s' \in \{s+1, s+2, \dots, |s|\}, \tag{D39}$$

$$o_{k,t}^{s,s'} \in z \quad \forall s \in S^l, l \in L, t \in T, k \in K, s' \in \{s+1, s+2, \dots, |s|\}, \tag{D40}$$

$$a_{k,t}^{s-1,s}, b_{k,t}^{s-1,s}, d_{k,t}^{s-1,s} = \{0,1\} \quad \forall s \in S^l \backslash \{1\}, l \in L, t \in T, k \in K^l, \tag{D41}$$

$$f_{k,t}^{s}, g_{k,t}^{s} = \{0,1\} \quad \forall s \in S^l, l \in L, t \in T, k \in K^l, \tag{D42}$$

$$m_{k,t}^{s,s+1} \in z \quad \forall s \in S^l \backslash \{\left|S^l\right|\}, l \in L, t \in T, k \in K. \tag{D43}$$

As both $t_k^{s,s+1}$ and ξ_k^s are variables, the objective function (27) is inherently nonlinear. However, from constraint (D12), it is evident that objective function (27) can be reformulated into objective function (D44), which subsequently transforms into objective function (D45). Similarly, objective function (29) can be converted into objective function (D46).

$$z_5 = \sum_{l \in L} \sum_{s \in S^l} \sum_{k \in K^l} t^s \xi_k^s \varepsilon_2 + \sum_{l \in L} \sum_{s \in S^l} \sum_{k \in K^l} \sum_{t \in T} \xi_k^s a_{k,t}^{s,s+1} r_t^{s,s+1} \varepsilon_2, \tag{D44}$$

$$z_6 = \sum_{l \in L} \sum_{s \in S^l} \sum_{k \in K^l} t^s \xi_k^s \varepsilon_2 + \sum_{l \in L} \sum_{s \in S^l} \sum_{k \in K^l} \sum_{t \in T} m_{k,t}^{s,s+1} r_t^{s,s+1} \varepsilon_2, \tag{D45}$$

$$z_7 = \sum_{l \in L} \sum_{s \in S^l} \sum_{k \in K^l} \sum_{t \in T} m_{k,t}^{s,s+1} r_t^{s,s+1} \varepsilon_3. \tag{D46}$$

To summarize, the linearized model for joint optimization of passenger flow control and parallel bus bridging (PFCBB-L) is represented as follows:

$$\begin{cases} \min w_1(z_1 + z_6) + w_2(z_3 + z_7) \\ \text{s.t.} \\ (1)\text{–}(16), (18)\text{–}(19), (22)\text{–}(24), \\ (D5)\text{–}(D12), (D16)\text{–}(D22), (D24)\text{–}(D43) \end{cases}. \tag{D47}$$