

Buckling of elliptical hollow section members under combined compression and uniaxial bending

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Abstract

Experimental and numerical investigations into the behaviour of elliptical hollow section beam-columns under axial compression and uniaxial bending have been performed and described in this paper. A large-scale experimental programme, comprising a total of 10 tensile coupon tests and 24 beam-column tests, was carried out. The beam-column tests included 6 pure compression tests, 3 buckling about the major axis and 3 about the minor axis, and 18 eccentric compression tests, 9 inducing compression plus bending about the major axis and 9 inducing compression plus bending about the minor axis. All tested elliptical hollow sections were EHS 150×75×5, and three member lengths of 1 m, 2 m and 3 m were considered. The test results have been supplemented by numerically generated results based on validated FE models to assess the influence of member slenderness and cross-sectional aspect ratio. On the basis of the experimental and numerical findings, design rules covering instabilities in hot-finished EHS beam-columns have been assessed and verified by statistical analysis. The limiting length concept has also been extended to EHS beam-columns.

Keywords: Beam-columns; elliptical hollow sections; in-plane behaviour; instability; laboratory testing; limiting length; numerical modelling; steel structures.

Symbols

A	Cross-sectional area
a	Half of the larger outer diameter of an EHS
b	Half of the smaller outer diameter of an EHS; average ratio of experimental to model resistance based on a least squares fit
b_{tc}	Necked width of tensile coupon
e_y	Eccentricity to the major (y-y) axis
e_z	Eccentricity to the minor (y-y) axis
E	Young's modulus
EHS	Elliptical hollow section
f_u	Ultimate tensile stress
f_y	Yield stress
G	Shear modulus
i	Radius of gyration
I	Second moment of area
I_t	Torsion constant
I_w	Warping constant
$k_{d,n}$	Design fractile factor
k_{ij}	Interaction factors
L	Length
$L_{c,N}$	Limiting length in the presence of axial load
M_{1st}	First-order elastic moment
$M_{2nd,el.}$	Second-order elastic moment
$M_{2nd,incl.}$	Second-order inelastic moment
$M_{b,Rd}$	Design lateral torsional buckling resistance
M_{cr}	Elastic buckling moment
$M_{cr,0}$	Elastic critical moment for pure bending
$M_{cr,N}$	Elastic buckling moment for pure bending in presence of axial load
$M_{el,Rd}$	Design elastic moment resistance
$M_{pl,Rd}$	Design plastic moment resistance
$M_{y,Ed}$	Design value of major axis moment
$M_{y,Rk}$	Characteristic value of major axis moment resistance

$M_{y,Rd}$	Design value of major axis moment resistance
$M_{z,Ed}$	Design value of minor axis moment
$M_{z,Rk}$	Characteristic value of minor axis moment resistance
$M_{z,Rd}$	Design value of minor axis moment resistance
n	Number of tests; axial load level = N_{Ed}/N_y
$N_{b,Rd}$	Design flexural buckling resistance
N_{cr}	Elastic buckling load
$N_{cr,T}$	Elastic torsional buckling load
N_{Ed}	Design value of compression force
N_{Rd}	Design value of compression resistance
N_{Rk}	Characteristic value of compression resistance
N_u	Ultimate applied load
N_y	Yield load
R_m	Mean value of the utilisation ratio
t	Thickness
U	Utilisation
V_r	Combined coefficient of variation incorporating both model and basic variable uncertainties
V_R	Coefficient of variation of resistance predictions
V_δ	Coefficient of variation of the tests relative to the resistance model
W_y	Major axis section modulus
γ	cross-sectional aspect ratio = a/b
γ_{M1}	Partial factor for member instability
ΔM_{Ed}	Moments due to the shift of the centroidal axis for Class 4 sections
ϵ_{f1}	Plastic strain at fracture based on elongation over the standard gauge length
ϵ_{f2}	Plastic strain at fracture based reduction of cross-sectional area
$\bar{\lambda}$	Non-dimensional flexural buckling slenderness
$\bar{\lambda}_{LT}$	Non-dimensional lateral torsional buckling slenderness
$\bar{\lambda}_{LT,0}$	Plateau length for lateral torsional buckling curves
$\bar{\lambda}_y$	Major axis non-dimensional flexural buckling slenderness

$\bar{\lambda}_z$	Minor axis non-dimensional flexural buckling slenderness
$\bar{\lambda}_{z,lim}$	Limiting non-dimensional slenderness
ω_g	Maximum global geometric imperfection
ω_u	Mid-span lateral deflection at ultimate load
χ_{LT}	Reduction factor for lateral torsional buckling
χ_y	Reduction factor for major axis flexural buckling
χ_z	Reduction factor for minor axis flexural buckling

1. Introduction

The addition of hot-finished elliptical hollow sections (EHS) to the tubular product standard EN 10210 [1] and to industry design guidance [2] has increased awareness among engineers and architects of their availability and led to more practical applications. Recent research into the cross-sectional response of EHS has included studies under isolated compression and bending [3-6] and combined compression plus uniaxial bending [7]. On the basis of experimental and numerical findings, proposals for the cross-section classification of EHS were made and, under combined loading, it was found that a fully plastic interaction formula [8] for Class 1 and 2 cross-sections and an elastic interaction formula for Class 3 and 4 cross-sections give safe predictions of capacity and are suitable for design purposes. Classification of EHS based on the determination of an equivalent circular section was proposed in [3-5] and adopted in [2], while an alternative approach whereby an equivalent rectangular hollow section is defined was proposed in [6, 9].

More recently, research focus has shifted to member instabilities. Column buckling was examined by Chan and Gardner [10], while lateral instability of EHS beams was investigated by Law and Gardner [11], and the applicability of the existing buckling curves adopted in EN 1993-1-1 (2005) [12] was assessed. Member instability of EHS beam-columns remains unexplored; hence, an experimental and numerical study of EHS members under combined compression plus uniaxial bending is the subject of the present paper.

The beam-column problem is complex since it involves the features of column buckling, uniaxial or biaxial beam bending and beam buckling (Figure 1). Since the late nineteenth century, substantial research on beam-column behaviour has been carried out; the development of the theory of beam-columns has been summarised by Massonnet [13] and Chen and Atsuta [14, 15]. Initial analysis of beam-column behaviour was confined to the elastic range, but with the development of powerful computational tools, inelastic behaviour of beam-columns has also been examined. The behaviour of beam-columns can be categorised into three types: in-plane behaviour, lateral torsional buckling and biaxial bending. In-plane behaviour refers to a beam-column which is bent about its major principal axis while restrained from deflecting laterally or is bent about its minor principal axis. When a beam-column which is bent about its stronger axis is not restrained laterally, it may buckle prematurely out of the plane by deflecting laterally and twisting and this action is regarded as lateral torsional buckling. Biaxial bending occurs when a beam-column is bent about both principal axes; this biaxial bending involves interactions of beam bending with beam and column buckling. The present paper focuses on the structural response of EHS beam-columns which are loaded eccentrically about one axis.

2. Experimental studies

A full-scale experimental programme on EHS member instability under combined compression and bending has been carried out in the Structures Laboratory at Imperial College London. The tested sections were all hot-finished from grade S355 steel and produced by Tata Steel Tubes. The test programme comprised 10 material tensile coupon tests and 24 beam-column tests - 6 pure compression tests; 3 buckling about the major axis and 3 about the minor axis and 18 eccentric compression tests, 9 inducing bending about the major axis and 9 about the minor axis. The tested EHS had an aspect ratio of two, overall outer cross-section dimensions of 150×75 mm and a thickness of 5 mm, which is the thinnest non-slender section of the range (based on a yield strength of 355 N/mm²). The primary aim of the member tests was to investigate the beam-column behaviour of EHS members with pinned end conditions and under eccentric compression (generating uniform moment along the member length).

2.1 Tensile coupon tests

Material tensile coupon tests were conducted in accordance with EN 10002-1 [16] to determine the basic engineering stress-strain response of the material of the tested sections. The specimens originated from 10 lengths of material, and one coupon was taken from each length for material testing. Full details of the tensile coupon tests have been described in Law and Gardner [11], while mean measured dimensions and the key results from the tensile coupon tests are reported in Table 1.

The reported material parameters are the necked width b_{tc} and thickness t of the coupons, Young's Modulus E , static yield stress f_y , static ultimate tensile stress f_u and the plastic strain at fracture based on elongation over the standard gauge length ϵ_{f1} and the reduction of cross-sectional area ϵ_{f2} of the coupons. A typical stress-strain curve is shown in Figure 2. The obtained material properties were subsequently employed to facilitate the analysis of the beam-column test results and were incorporated into the numerical models to replicate the structural response of the tested specimens.

2.2 Pure compression and uniaxial eccentric compression tests

Six pure compression tests and 18 eccentric compression tests were performed. Three different nominal column lengths of 1 m, 2 m and 3 m (see Figure 3), to which the dimensions of the end conditions were added, were tested to provide a range of member slendernesses $\bar{\lambda}$ ranging from 0.36 to 0.85 for the pin-ended beam-columns eccentrically loaded about the major axis and 0.63 to 1.51 about the minor axis. For the pure compression test specimens (BC-1- $e_y=0$ and BC-1- $e_z=0$), where the measured global imperfection was less than $L/1000$, an eccentricity of loading was applied such that the combined imperfection plus eccentricity was equal to $L/1000$. For other columns, the load was applied concentrically since the measured imperfections were greater than $L/1000$. For the eccentric compression tests, the load eccentricity was varied so that a range of proportions of axial load to bending could be achieved.

2.2.1 Experimental set-up

The general configuration of the experimental set-up is depicted in Figure 4. The specimens were loaded by a 2000 kN Instron hydraulic machine through hardened steel knife-edges at both ends to provide pinned end conditions about the axis of buckling and fixed conditions in the orthogonal direction. The knife-edges were carefully designed to allow for a maximum of 15 degrees of end rotation. For the pure compression tests, steel plates with slotted holes were utilised to clamp the specimen into position on 40 mm plates, which were attached to the knife edges at both ends. For the eccentric compression tests, end plates were welded onto the specimens, since bending moments were induced at the member ends.

Two draw wire transducers were located at the mid-height of the specimens to measure the lateral deflections in both principal directions. Inclinoimeters were positioned at each end of the members to measure the end rotations about the axis of buckling. Four linear electrical resistance strain gauges were affixed to the extreme tensile and compressive fibres of the section at a distance of 20 mm from the mid-height of the member to avoid contact with the draw wire transducers. Applied load and vertical displacement were obtained directly from the loading machine. Displacement control was employed to drive the hydraulic machine at a constant rate of 0.25 mm/min, 0.50 mm/min and 1.0 mm/min for 1 m, 2 m and 3 m specimens respectively. Load, strain, lateral and vertical displacements, end rotations and input voltage were all recorded using the data acquisition equipment DATASCAN and logged using the DSLOG computer package. All data were recorded at one second intervals.

The geometry of an elliptical hollow section is depicted in Figure 5, where $2a$ is the larger outer diameter, $2b$ is the smaller outer diameter, t is the thickness and e_y and e_z are the load eccentricities to the major and minor axes, respectively. Measurements of the cross-section geometry, member length L (measured between the pinned ends) and maximum global geometric imperfections in the direction of buckling ω_g (determined by means of a laser beam directed along the member length) were taken prior to testing and are reported in Table 2. Residual stresses were not measured in this study since the

level of residual stresses in hot-finished EHS members has been found to be very low in previous studies [11]. Local geometric imperfections were not measured in this study since the proportions of the cross-sections examined are relatively stocky and insensitive to local buckling. The geometric properties for the EHS specimens were calculated using the exact formulae adopted in previous studies of EHS [3-5].

2.2.2 Test results

The load eccentricities and key test results are summarised in Table 3, in which N_u is the ultimate applied load, M_{1st} is the first-order elastic moment, $M_{2nd,el.}$ is the second-order elastic moment and $M_{2nd,incl.}$ is the second-order inelastic moment given by Equations (1) to (3) respectively.

$$M_{1st} = N_u \times (e + \omega_g) \quad (1)$$

$$M_{2nd,el.} = M_{1st} \times \frac{1}{1 - \frac{N_u}{N_{cr}}} \quad (2)$$

$$M_{2nd,incl.} = N_u \times (e + \omega_g + \omega_u) \quad (3)$$

in which, ω_u is the mid-span lateral deflection of the specimen at ultimate load.

All the beam-columns failed by flexural buckling and in-plane bending with no sign of lateral torsional buckling, as shown by Figures 6 and 7. For the beam-columns loaded eccentrically about the minor axis, all specimens failed in the minor axis direction only (see Figure 6). For the beam-columns loaded eccentrically about the major axis, all specimens failed in the major axis direction only (see Figure 7) since the knife edges provided fixed end conditions about the minor axis and the buckling length in that direction was therefore halved, making major axis buckling critical. Graphs showing the applied axial load versus lateral deflection response of the 24 beam-columns are shown in Figures 8 to 13. Overall, a general reduction in ultimate test load was observed with

increasing eccentricities due to the higher bending effects. Plots of ultimate load versus corresponding first-order elastic, second-order elastic and second-order inelastic moments are depicted in Figures 14 to 19. It is shown that the second-order inelastic moment was still increasing after the ultimate load was reached due to the increasing mid-span lateral deflection. In addition, the second-order effect on the test specimens becomes more significant compared with the first-order elastic moment with increasing beam-column length. This is due to the larger mid-span lateral deflections that arose in the more slender specimens. It should be noted that the sudden change of slope in the later part of some load-lateral deflection curves (BC-1- $e_y=0$, BC-1- $e_z=15$, BC-1- $e_z=25$, BC-2- $e_z=0$ and BC-2- $e_z=25$) indicates the formation of a plastic hinge at the most heavily stressed cross-section, generally occurring at the mid-height of the specimens.

3. Numerical simulations

The behaviour of the tested beam-columns was initially replicated by finite element analysis using the measured dimensions of the test specimens and measured material stress-strain data. The finite element (FE) package ABAQUS [17] was used throughout the study. Four-noded, reduced integration shell elements with six degrees of freedom per node, designated as S4R in the ABAQUS element library, were chosen for the numerical models since they were employed successfully in the previous simulations of hot-finished EHS [3-5, 7, 9, 10, 17]. A suitable size of the uniform mesh was found to be 15×15 mm by performing a mesh convergence study; this mesh density is able to achieve accurate results with reasonable computational effort.

Initial global geometric imperfections were incorporated by applying a lateral point load to the specimens at mid-height. The true material stress-strain relationship was generated by converting a multi-linear stress-strain curve derived from the static engineering stress-strain curves obtained from the tensile coupon tests. Boundary conditions were modelled to simulate pinned conditions at the ends of the beam-columns in the axis of buckling. The modified Riks method [17], which enables the post-ultimate behaviour to be traced, was employed to solve the geometrically and materially nonlinear numerical models.

The ultimate loads obtained from the FE models are compared with those obtained from the tests in Table 4. Replication of test results was found to be satisfactory with a mean ratio of test to FE ultimate load of 1.05. Typical comparisons of failure mode and load versus lateral deflection curves are depicted in Figures 20 and 21 respectively. The numerical models were able to successfully capture the initial stiffness, ultimate capacity, general load-deformation response and failure patterns observed in the tests. It should be noted that the ultimate loads obtained from the numerical models were generally lower than the test results; this is attributed to the use of static material properties in the FE models; the static material properties were obtained by pausing the coupon tests at points along the yield plateau, though the member tests themselves were not paused.

Upon verification of the general ability of the FE models to replicate the beam-column behaviour of EHS members with a cross-sectional aspect ratio of two, a series of parametric studies were conducted, aiming to investigate the influence of different aspect ratios and member slendernesses on the beam-column behaviour. The obtained results are used to develop design guidance for EHS beam-columns. A piecewise linear material stress-strain model developed from an average of the stress-strain curves obtained from the tensile coupon tests, was employed in the parametric numerical models. Initial geometric imperfections with an amplitude ω_g of $L/500$, which is the tolerance stated in EN 10210-2 [1], were included in the nonlinear parametric analyses by applying a lateral point load to the specimen at mid-height. The outer height and thickness of the cross-section considered in the parametric studies were fixed to 150 mm and 5 mm respectively. A range of cross-sectional aspect ratios from 2 to 4 and member slenderness $\bar{\lambda}_z$ of 0.5, 1.0 and 1.5 was considered. The results are utilised for the validation of proposed beam-column interaction formulae for elliptical hollow sections, as discussed in the following section.

4. Development of design guidance

In the design of an eccentrically loaded beam-column, either the strength of the cross-section or the stability of the member will govern the load carrying capacity, depending on the length of the beam-column.

The cross-section strength criterion has been described briefly in Section 1. For the member stability criterion, which is the focus of the present investigation, extensive studies have been undertaken. During the development of the Eurocodes, some physical inconsistencies and over-conservatisms in the design rules for beam-columns given in ENV 1993-1-1 [19] were found [20]. Hence, two new sets of design formulae for assessing the member stability of beam-columns, named Method 1 and Method 2 in EN 1993-1-1 [12], were developed by the Technical Committee 8 – Structural Stability of the European Convention for Constructional Steelwork [21]. Method 1 was derived based on second-order in-plane elastic theory and further extended to spatial and elastic-plastic behaviour by a French-Belgian team [22]. Method 2 was developed on the basis of a semi-empirical approach utilising curve-fitting techniques to mainly numerical results by an Austrian-German team [23]. The applicability of the two different approaches to hot-finished elliptical hollow section beam-columns is assessed in this section.

4.1 Ultimate member capacity – Stability interaction

The design approach for treating member instability under combined compression and bending in EN 1993-1-1 (2005) [12] is based on the use of interaction formulae. In general, both Equations (4) and (5), i.e. Equations (6.61) and (6.62) in EN 1993-1-1 respectively, must be satisfied.

$$\frac{\frac{N_{Ed}}{\chi_y N_{Rk}}}{\gamma_{M1}} + k_{yy} \frac{\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} M_{y,Rk}}}{\gamma_{M1}} + k_{yz} \frac{\frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{z,Rk}}}{\gamma_{M1}} \leq 1 \quad (4)$$

$$\frac{\frac{N_{Ed}}{\chi_z N_{Rk}}}{\gamma_{M1}} + k_{zy} \frac{\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} M_{y,Rk}}}{\gamma_{M1}} + k_{zz} \frac{\frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{z,Rk}}}{\gamma_{M1}} \leq 1 \quad (5)$$

where N_{Ed} , $M_{y,Ed}$ and $M_{z,Ed}$ are the design values of the compression force and the maximum moments about the major and minor axes along the member length respectively, $\Delta M_{y,Ed}$ and $\Delta M_{z,Ed}$ are the moments due to the shift of the centroidal axis

for Class 4 sections, N_{Rk} , $M_{y,Rk}$ and $M_{z,Rk}$ are the characteristic compression resistance and moment resistances about the major and minor axes of the cross-section respectively, k_{yy} , k_{yz} , k_{zy} and k_{zz} are interaction factors, χ_y and χ_z are the reduction factors due to flexural buckling, χ_{LT} is the reduction factor due to lateral torsional buckling and γ_{M1} is the partial factor for member instability. Note that, since the recommended partial factor for member instability γ_{M1} is equal to unity in EN 1993-1-1 [12], the characteristic values N_{Rk} , $M_{y,Rk}$ and $M_{z,Rk}$ are equal to their design values N_{Rd} , $M_{y,Rd}$ and $M_{z,Rd}$.

Interaction factors can be found by either Method 1 or 2, which are described in Annex A and B of EN 1993-1-1 respectively. In general, the interaction formulae were both derived from the classical behaviour of an elastic beam-column based on in-plane second-order theory and then extended to spatial behaviour and modified to allow for plasticity. However, in Method 1, each constitutive factor is normally associated with a single physical effect, such as material and geometrical nonlinearities and interactions between loading components, which was derived theoretically or if a theoretical approach was not possible, numerical simulations were used to calibrate the coefficients. In contrast, Method 2 employs a reduced number of factors as a result of integration of several effects, which were calibrated based on extensive numerical simulations. Comparatively, Method 1 provides a wide range of applicability with a high level of accuracy and consistency, whereas simplicity prevails against transparency in Method 2. The background of the derivation of interaction factors for both methods was described by Boissonnade et al. [21].

4.2 Comparison of experimental and numerical results with existing design rules

Applicability of the EN 1993-1-1 [12] interaction formulae to EHS members is assessed in this section on the basis of the experimental and numerical results generated in this study. Two principal cases: compression with major axis moment and compression with minor axis moment, are considered herein and design recommendations for EHS beam-columns are proposed.

For the practical range of EHS column slendernesses considered ($\bar{\lambda}_z \leq 1.5$), the non-dimensional slenderness for lateral torsional buckling $\bar{\lambda}_{LT}$ is always less than 0.4 (see Figure 22), 0.4 being the slenderness below which LTB can be neglected [11]. Furthermore, for a cross-sectional aspect ratio $\gamma = a/b$ of 2, which represents current commercially available sections, $\bar{\lambda}_{LT}$ is less than 0.4 up until a column slenderness $\bar{\lambda}_z$ of 2.7. It is therefore assumed that lateral torsional buckling need not be considered in this study for practical EHS beam-columns. Figure 23 shows a plot of $\bar{\lambda}_{LT}$ versus $\bar{\lambda}_z$ for EHS members with various aspect ratios $\gamma = a/b$, in which the outer diameter and thickness of the section are taken as 150 mm and 5 mm respectively, $f_y = 355 \text{ N/mm}^2$ and $E = 210\,000 \text{ N/mm}^2$. In this section, applicability of the Method 1 and Method 2 design rules to EHS beam-columns is assessed.

4.2.1 In-plane buckling under $N + M_y$

For the studied EHS beam-columns loaded at an eccentricity to the major axis, all the tested specimen are classified as either Class 1 or 2 sections according to the classification guidance proposed by Gardner et al. [7]. For the stability interaction, Equation (4) is always critical and the design buckling formula simplifies to:

$$\frac{N_{Ed}}{N_{b,y,Rd}} + k_{yy} \frac{M_{y,Ed}}{M_{b,Rd}} \leq 1 \quad (6)$$

where $N_{b,y,Rd}$ is the major axis flexural buckling resistance and $M_{b,Rd}$ is the lateral torsional buckling resistance. Different interaction factors k_{yy} and hence design curves are obtained from Method 1 and Method 2. Figures 23 and 24 show the test results with their corresponding interaction curves obtained from Method 1 and Method 2 respectively. The non-dimensional slenderness $\bar{\lambda}_y$, the corresponding k_{yy} factors and the total utilisations U (i.e. test/design resistance) are presented in Table 5 for the two methods. Note that ultimate test loads and the corresponding first-order moments M_{1st} , as reported in Table 3, have been plotted, and that $M_{b,Rd} = M_{pl,y,Rd}$, where $M_{pl,z,Rd}$ is the major axis cross-section plastic bending resistance

The test results generally follow the trend of the interaction curves, whereby failure occurs earlier for more slender columns due to more significant secondary effects. For Method 1, four test data points (BC-1- $e_y=25$, BC-1- $e_y=150$, BC-2- $e_y=50$ and BC-2- $e_y=150$) lie marginally below their corresponding design curves, while for Method 2, there are two such data points (BC-2- $e_y=50$ and BC-2- $e_y=150$). The experimental results and resistance functions are subjected to reliability analyses in Section 5.3 to verify their applicability.

The test results are supplemented by the validated numerical findings in order to illustrate the effects of member slenderness $\bar{\lambda}_y$ and cross-sectional aspect ratio γ on the in-plane buckling behaviour of EHS beam-column members under axial compression and major axis bending. Numerical results from the parametric studies described in Section 3 have been plotted in Figures 25 to 27 for Method 1 and Figures 28 to 30 for Method 2, and compared with the respective design curves. The member slendernesses considered were 0.5, 1.0 and 1.5, while the cross-sectional aspect ratios considered were 2, 3 and 4. For Method 2, it should be noted that the interaction factor k_{yy} reaches its upper bound at $\bar{\lambda}_y = 1.0$ and therefore the design curves are the same for $\bar{\lambda}_y = 1.0$ and 1.5.

The design curves are similar for the different aspect ratios since the stability effects are accounted for by the χ factor in the axial compression term. Compared with the test and numerical results, the design curves obtained from Method 1 seem to provide marginally unconservative predictions, especially for stocky members. This may be due to inappropriate plasticity factors which were derived mainly on the basis of I-sections; further investigation is required in this area. Method 2 generally provides safe-side predictions for the buckling resistance of EHS beam-columns under compression and major axis bending.

4.2.2 In-plane buckling under $N + M_z$

For the studied EHS beam-columns loaded at an eccentricity to the minor axis, all the tested specimen are classified as Class 3 sections according to the classification

guidance proposed by Gardner et al. [7]. For the stability interaction, Equation (5) is always critical and the design buckling formula simplifies to:

$$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zz} \frac{M_{z,Ed}}{M_{el,z,Rd}} \leq 1 \quad (7)$$

where $N_{b,z,Rd}$ is the minor axis flexural buckling resistance and $M_{el,z,Rd}$ is the minor axis cross-section elastic bending resistance. Different interaction factors k_{zz} and hence design curves are obtained from Method 1 and Method 2; the applicability of the two methods to hot-finished EHS under axial compression and minor axis bending is discussed in this section. Figures 31 and 32 show the test results with their corresponding interaction curves obtained from Method 1 and Method 2 respectively. The non-dimensional slenderness $\bar{\lambda}_z$, the corresponding k_{zz} factors obtained and the total utilisations U are given in Table 6. Note that ultimate test loads and the corresponding first order moments M_{1st} , as reported in Table 3, have been plotted.

As for the case of axial load plus major axis bending, the test results generally follow the trend of the interaction curves for both methods, with failure occurring earlier for the more slender members due to more significant secondary effects. Note in Figure 31 that the normalised interaction curve for the 3 m members is marginally above that for the 2 m members according to Method 1, though the overall prediction capacities of the more slenderness remain lower due to the lower value of the pure compression buckling resistance $N_{b,Rd}$. For Method 2, the findings are comparable, with the same interaction curve applying to the 2 m and the 3 m beam-columns, since both have a non-dimensional slenderness greater than unity. The above design treatment is supported by the experimental results, which show the response of the 2 m and 3 m test specimens to be similar, as shown in Figure 32. The experimental results and resistance functions are subjected to reliability analyses in Section 4.3.

In order to illustrate the effects of member slenderness $\bar{\lambda}_z$ and cross-sectional aspect ratio γ on the in-plane buckling behaviour of EHS beam-column members under axial compression and minor axis bending, numerical results from the parametric studies described in Section 4 have been plotted in Figures 33 to 35 and 36 to 38 for Method 1

and Method 2 respectively. The member slendernesses considered herein are 0.5, 1.0 and 1.5, while three cross-sectional aspect ratios (2, 3 and 4) are compared. Similar to the in-plane buckling behaviour of EHS beam-column members under axial compression and major axis bending, for Method 2, the interaction factor (k_{zz}) reaches its upper bound at $\bar{\lambda}_z = 1.0$ and therefore the design curves are the same for $\bar{\lambda}_z = 1.0$ and 1.5.

The behaviour of EHS members under axial compression and minor axis bending is similar to that under axial compression and major axis bending. Generally, the design curves give satisfactory and safe-side predictions of the observed behaviour for a range of member slendernesses and cross-sectional aspect ratios. It can be concluded that, through Equation (7), the design formulae for the in-plane buckling under axial compression and minor axis bending from both Methods 1 and 2 are applicable to EHS members.

4.3 Reliability analysis

The behaviour of beam-columns depends on a number of factors, including member slendernesses, cross-section geometry, type of loading and support conditions. In the development of the Eurocode 3 beam-column design expressions [21], instead of performing standard statistical analysis based on Annex D of EN 1990 [24], a simplified approach was adopted by comparing the ratio R between the theoretical and experimental loadings. Adopting a similar approach herein, Table 7 shows the key statistical parameters from the EHS beam-column test data, in which R is the ratio between the total utilisation under the combined loading, incorporating the corresponding interaction factors and the full utilisation limit (i.e. unity), n is the number of tests, R_m is the mean value of the utilisation ratio and V_R is the coefficient of variation.

Comparing the statistical results above with those for other cross-sectional shapes obtained from Boissonnade et al. [21] shows that a similar level of reliability is achieved for EHS members; generally more test results with an R value less than unity

were reported when using Method 1. A standard statistical analysis based on Annex D of EN 1990 [24] is performed herein for the EHS beam-column tests to assess the reliability of the interaction formulae. The key statistical parameters are given in Tables 8 and 9 for Method 1 and Method 2, respectively. The resistance term in the statistical analysis is taken as the total utilisation from the compression and major axis or minor axis moment. The following symbols are used: $k_{d,n}$ = design (ultimate limit state) fractile factor for n tests, where n is the population of test data under consideration; b = average ratio of experimental to model resistance based on a least squares fit to the test data; V_δ = coefficient of variation of the tests relative to the resistance model; and V_r = combined coefficient of variation incorporating both model and basic variable uncertainties. The obtained partial factors are higher than 1.0, but offer similar reliability to other section types, which may be considered adequate. It is recommended herein that the scope of Annex B (Method 2) of EN 1993-1-1 (2005) [12] be extended to include EHS beam-columns, while Annex A may require modification for application to EHS.

4.4 Limiting member length in the presence of axial load

In Law and Gardner [11], the concept of a limiting length below which lateral torsional buckling need not be considered for EHS beams was discussed. This limiting length concept is extended to beam-columns herein.

For a member subjected to uniform major axis bending moment and axial compression, the theoretical value of the critical moment can be estimated by [25]:

$$M_{cr,N} = M_{cr,0} \sqrt{\left(1 - \frac{N_{Ed}}{N_{cr,z}}\right) \left(1 - \frac{N_{Ed}}{N_{cr,T}}\right)} \quad (8)$$

where $M_{cr,0}$ is the elastic critical moment for pure bending and can be obtained from:

$$M_{cr,0} = \sqrt{\left(\frac{\pi^2 EI_z}{L^2}\right) \left(GI_t + \frac{\pi^2 EI_w}{L^2}\right)} \quad (9)$$

$N_{cr,z}$ is the elastic buckling load about the minor axis:

$$N_{cr,z} = \frac{\pi^2 EI_z}{L^2} \quad (10)$$

and $N_{cr,T}$ is the elastic torsional buckling load:

$$N_{cr,T} = \frac{A}{I_y + I_z} (GI_t + \frac{\pi^2 EI_w}{L^2}) \quad (11)$$

For closed sections, it is assumed that the effects of warping are negligible ($I_w = 0$) and therefore, after rearranging the terms from these expressions, the limiting length in the presence of axial load can be obtained as:

$$L_{c,N} = \frac{\bar{\lambda}_{LT,0}^2 \pi \sqrt{EI_z GI_t}}{W_y f_y} \sqrt{(1 - \frac{N_{Ed}}{N_{cr,z}})(1 - \frac{N_{Ed}}{N_{cr,T}})} \quad (12)$$

Clearly, when the member is under pure bending, the above expression reduces to that described in [11]. The above relationship may be more conveniently presented in terms of axial load level $n = \frac{N_{Ed}}{N_y}$, where N_y is the yield load, and cross-sectional aspect ratio

$\gamma = a/b$, by utilising the simplified expressions for section properties, derived in [11]. Hence, in the presence of axial load, the limiting length above which lateral torsional buckling needs to be considered may be obtained as:

$$L_{c,N} = \sqrt{\frac{1 - \frac{nf_y(I_y + I_z)}{GI_t}}{\frac{W_y^2 f_y^2}{\bar{\lambda}_{LT,0}^4 \pi^2 EI_z GI_t} + \frac{nAf_y}{\pi^2 EI_z} [1 - \frac{nf_y(I_y + I_z)}{GI_t}]}} \quad (13)$$

Substituting the approximated plastic section properties [11] into the above $L_{c,N}$ formulation, for Class 1 and 2 sections gives:

$$\frac{L_{c,N,pl}}{2b} = \frac{\pi}{4} \sqrt{\frac{E}{f_y}} \sqrt{\frac{\xi}{\frac{1.21 f_y}{9 \bar{\lambda}_{LT,0}^4 \pi^2 G} \left[\frac{(19\gamma + 24)(\gamma + 2)^2}{(3\gamma + 1)(7 - \gamma)} \right] + 1.15 n \frac{\gamma + 1}{3\gamma + 1} \xi}} \quad (14)$$

where

$$\xi = 1 - \frac{n f_y (\gamma + 1)^3 (19\gamma + 24)}{64G \gamma^2 (7 - \gamma)} \quad (15)$$

As discussed in [11], the influence of the pre-buckling in-plane deflections may also be included through the parameter ψ to achieve improved efficiency, hence:

$$\frac{L_{c,N,pl}'}{2b} = \frac{\pi}{4\psi} \sqrt{\frac{E}{f_y}} \sqrt{\frac{\xi}{\frac{1.21 f_y}{9 \bar{\lambda}_{LT,0}^4 \pi^2 G} \left[\frac{(19\gamma + 24)(\gamma + 2)^2}{(3\gamma + 1)(7 - \gamma)} \right] + 1.15 n \frac{\gamma + 1}{3\gamma + 1} \xi}} \quad (16)$$

in which

$$\psi = \sqrt{\left[1 - 0.93 \frac{(3\gamma + 1)}{\gamma^2 (3 + \gamma)} \right] \left[1 - 0.93 \frac{32G(7 - \gamma)}{E(3 + \gamma)(19\gamma + 24)} \right]} \quad (17)$$

For Class 3 sections, the expression of $W_{pl,y}$ is replaced with $W_{el,y}$ in Equation (12) and the limiting length in the presence of axial load with the inclusion of the influence of pre-buckling in-plane deflections is found to be:

$$\frac{L_{c,N,el}'}{2b} = \frac{\pi}{\psi} \sqrt{\frac{E}{f_y}} \sqrt{\frac{\xi}{\frac{1.13 f_y}{16 \bar{\lambda}_{LT,0}^4 G} \left[\frac{(19\gamma + 24)(\gamma + 3)^2}{(3\gamma + 1)(7 - \gamma)} \right] + 1.15 \times 16 n \frac{\gamma + 1}{3\gamma + 1} \xi}} \quad (18)$$

Graphs showing the limiting length versus aspect ratio for the exact formula (Equation (12)) and the approximate formulae (Equations (16) and (18)) for different levels of axial loads are depicted in Figures 39 and 40. For the comparisons, the larger outer diameter and thickness of the EHS were fixed to 150 mm and 5 mm respectively. The

assumptions also included a plateau length $\bar{\lambda}_{LT,0} = 0.4$ and the nominal values of $E = 210000 \text{ N/mm}^2$ and $f_y = 355 \text{ N/mm}^2$. Good agreement between the exact and approximated formulations may be observed and it is concluded that the proposed expressions can be used to estimate the limiting length of a beam-column for various axial load levels and aspect ratios with high accuracy.

The limiting non-dimensional slenderness $\bar{\lambda}_{z,\text{lim}}$ can be obtained by dividing the limiting length expression by i_z and $\lambda_1 = \pi \sqrt{\frac{E}{f_y}}$. Therefore, for Class 1 and 2 sections,

$$\begin{aligned} \bar{\lambda}_{z,N,\text{lim,pl}} &= \frac{L_{c,N,\text{pl}}}{i_z \sqrt{\frac{E}{f_y}} \pi} \\ &\approx \frac{1}{0.94\psi} \sqrt{\frac{\gamma+1}{3\gamma+1} \frac{\xi}{\frac{1.21 f_y}{9\bar{\lambda}_{LT,0}^4 \pi^2 G} \left[\frac{(19\gamma+24)(\gamma+2)^2}{(3\gamma+1)(7-\gamma)} \right] + 1.15n \frac{\gamma+1}{3\gamma+1} \xi}} \end{aligned} \quad (19)$$

And for Class 3 sections,

$$\begin{aligned} \bar{\lambda}_{z,N,\text{lim,el}} &= \frac{L_{c,N,\text{el}}}{i_z \sqrt{\frac{E}{f_y}} \pi} \\ &\approx \frac{1}{0.23\psi} \sqrt{\frac{\gamma+1}{3\gamma+1} \frac{\xi}{\frac{1.13 f_y}{16\bar{\lambda}_{LT,0}^4 G} \left[\frac{(19\gamma+24)(\gamma+3)^2}{(3\gamma+1)(7-\gamma)} \right] + 1.15 \times 16n \frac{\gamma+1}{3\gamma+1} \xi}} \end{aligned} \quad (20)$$

The above limiting non-dimensional slenderness values simply offer an alternative presentation of the limiting lengths. Both offer the opportunity to assess whether or not lateral-torsional buckling needs to be considered in a particular design.

5. Conclusions

Member instability in hot-finished EHS beam-columns under axial compression and uniaxial bending has been studied in this paper. An experimental programme comprising 10 tensile coupon tests and 24 beam-column tests was carried out. The beam-column tests included 6 pure compression tests, 3 buckling about the major axis and 3 about the minor axis, 18 eccentric compression tests, 9 inducing bending about the major axis and 9 about the minor axis. All tested elliptical hollow sections had a constant cross-section aspect ratio of two and the non-dimensional column slenderness ranged from 0.36 to 0.85 in the major axis bending beam-column tests and from 0.63 to 1.51 in the minor axis bending beam-column tests. The key material properties, geometric measurements, and test results from the beam-column tests have been reported.

The test results were subsequently utilised to validate numerical models. Satisfactory agreement between the test results and those obtained from the numerical models was achieved. A series of parametric studies based on the validated numerical models considering a range of slendernesses and aspect ratios was then performed to investigate the influence of cross-sectional aspect ratios ranging from 2 to 4 and member slendernesses $\bar{\lambda}$ ranging from 0.5 to 1.5. On the basis of the experimental and numerical findings, design rules covering the instability of hot-finished EHS beam-columns have been assessed. Methods 1 and 2 of EN 1993-1-1 (2005) [12] were shown to provide similarly accurate predictions of the capacity of hot-finished EHS beam-columns as for other section shapes, with Method 2 yielding the more reliable results. It is recommended herein that the scope of Annex B of EN 1993-1-1 (2005) be extended to include elliptical hollow section beam columns. The limiting length concept, previously applied to EHS beams, has also been extended to EHS beam-columns and approximate formulations for the limiting length below which lateral torsional buckling in the presence of axial load can be ignored has been derived analytically, and presented in terms of axial load level and aspect ratio.

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Table 1: *Mean measured dimensions and key results from tensile coupon tests*

Specimen	Width	Thickness	E	f_y	f_u	ϵ_{f1}	ϵ_{f2}
	b_{tc}	t					
	mm	mm	N/mm ²	N/mm ²	N/mm ²	%	%
BC-TC1	20.09	5.01	212900	350	481	40	64
BC-TC2	20.12	4.97	218200	343	472	36	68
BC-TC3	20.00	4.94	217000	360	485	38	64
BC-TC4	20.01	4.83	214600	344	470	39	65
BC-TC5	20.06	4.93	222400	388	510	42	65
BC-TC6	20.07	4.95	219500	326	452	39	68
BC-TC7	20.02	5.03	220200	341	473	35	65
BC-TC8	20.05	4.93	221100	358	468	37	64
BC-TC9	20.08	4.92	219200	373	489	40	66
BC-TC10	20.05	5.02	212100	377	491	40	63

Table 2: Mean measured specimen dimensions and global imperfections for uniaxial eccentric compression tests

Specimen	Tensile coupon	Length L	2a	2b	t	ω_g
		m	mm	mm	mm	mm
BC-1- $e_y=0$	BC-TC8	1.33	150.35	75.74	4.93	1.00
BC-1- $e_y=25$	BC-TC4	1.31	150.42	75.33	5.01	0.25
BC-1- $e_y=50$	BC-TC3	1.31	150.50	75.58	5.02	0.20
BC-1- $e_y=150$	BC-TC5	1.31	150.33	75.87	4.98	0.53
BC-1- $e_z=0$	BC-TC8	1.33	150.46	75.56	4.97	1.00
BC-1- $e_z=15$	BC-TC6	1.31	150.27	75.80	4.97	0.26
BC-1- $e_z=25$	BC-TC6	1.31	150.36	75.81	5.08	0.52
BC-1- $e_z=100$	BC-TC6	1.31	150.20	75.70	4.98	0.50
BC-2- $e_y=0$	BC-TC10	2.33	150.32	75.70	4.98	2.50
BC-2- $e_y=25$	BC-TC4	2.31	150.00	75.88	4.93	1.03
BC-2- $e_y=50$	BC-TC5	2.31	150.28	75.73	4.95	2.48
BC-2- $e_y=150$	BC-TC9	2.31	150.36	75.33	4.99	2.00
BC-2- $e_z=0$	BC-TC10	2.33	150.33	75.41	4.98	3.00
BC-2- $e_z=25$	BC-TC4	2.31	150.03	75.84	5.06	3.05
BC-2- $e_z=50$	BC-TC4	2.31	150.00	75.57	5.03	3.01
BC-2- $e_z=150$	BC-TC9	2.31	150.16	75.86	4.98	2.51
BC-3- $e_y=0$	BC-TC6	3.19	150.16	75.64	4.84	3.02
BC-3- $e_y=25$	BC-TC7	3.17	150.16	75.50	4.92	2.47
BC-3- $e_y=50$	BC-TC7	3.17	150.03	75.95	4.90	1.52
BC-3- $e_y=150$	BC-TC7	3.17	150.13	75.62	4.96	2.03
BC-3- $e_z=0$	BC-TC5	3.18	150.18	75.68	4.82	5.05
BC-3- $e_z=50$	BC-TC6	3.17	150.17	75.40	4.93	5.00
BC-3- $e_z=100$	BC-TC8	3.17	150.30	75.52	4.95	2.96
BC-3- $e_z=200$	BC-TC8	3.17	149.95	75.54	4.96	3.46

Table 3: *Summary of uniaxial eccentric compression test results*

Specimen	Axis of Buckling	Eccentricity		N_u kN	M_{1st} kNm	$M_{2nd,el.}$ kNm	$M_{2nd,incl.}$ kNm
		e_y mm	e_z mm				
BC-1- $e_y=0$	Major	0	0	645.4	0.7	0.77	0.77
BC-1- $e_y=25$	Major	25	0	390.8	9.9	10.3	11.9
BC-1- $e_y=50$	Major	50	0	323.4	16.2	16.9	19.6
BC-1- $e_y=150$	Major	150	0	142.4	21.4	21.8	23.4
BC-1- $e_z=0$	Minor	0	0	636.7	0.6	0.8	1.1
BC-1- $e_z=15$	Minor	0	15	350.5	5.3	6.1	7.5
BC-1- $e_z=25$	Minor	0	25	271.5	6.9	7.7	9.4
BC-1- $e_z=100$	Minor	0	100	112.3	11.3	11.8	13.1
BC-2- $e_y=0$	Major	0	0	594.9	1.5	2.1	3.2
BC-2- $e_y=25$	Major	25	0	339.8	8.8	10.6	13.3
BC-2- $e_y=50$	Major	50	0	245.7	12.9	14.6	18.5
BC-2- $e_y=150$	Major	150	0	123.5	18.8	20.0	22.4
BC-2- $e_z=0$	Minor	0	0	430.3	1.3	3.6	5.8
BC-2- $e_z=25$	Minor	0	25	211.7	5.9	8.5	11.4
BC-2- $e_z=50$	Minor	0	50	146.2	7.7	9.8	12.5
BC-2- $e_z=150$	Minor	0	150	73.3	11.2	12.5	14.1
BC-3- $e_y=0$	Major	0	0	475.4	1.4	3.0	4.6
BC-3- $e_y=25$	Major	25	0	281.8	7.7	11.2	14.4
BC-3- $e_y=50$	Major	50	0	212.5	10.9	14.2	18.4
BC-3- $e_y=150$	Major	150	0	113.5	17.2	19.6	22.4
BC-3- $e_z=0$	Minor	0	0	226.0	1.1	4.7	9.3
BC-3- $e_z=50$	Minor	0	50	109.4	6.0	9.4	12.0
BC-3- $e_z=100$	Minor	0	100	80.2	8.3	11.2	13.6
BC-3- $e_z=200$	Minor	0	200	51.6	10.5	12.6	13.9

Table 4: *Comparison between test and FE ultimate loads*

Specimen	Test N_u / FE N_u
BC-1- $e_y=0$	1.08
BC-1- $e_y=25$	1.04
BC-1- $e_y=50$	1.07
BC-1- $e_y=150$	1.04
BC-1- $e_z=0$	1.09
BC-1- $e_z=15$	1.10
BC-1- $e_z=25$	1.08
BC-1- $e_z=100$	1.10
BC-2- $e_y=0$	1.00
BC-2- $e_y=25$	1.07
BC-2- $e_y=50$	1.02
BC-2- $e_y=150$	1.07
BC-2- $e_z=0$	1.00
BC-2- $e_z=25$	1.05
BC-2- $e_z=50$	1.01
BC-2- $e_z=150$	1.03
BC-3- $e_y=0$	1.04
BC-3- $e_y=25$	1.08
BC-3- $e_y=50$	1.04
BC-3- $e_y=150$	1.09
BC-3- $e_z=0$	0.95
BC-3- $e_z=50$	1.02
BC-3- $e_z=100$	1.03
BC-3- $e_z=200$	1.05
Mean	1.05
COV	0.04

Table 5: *Calculated k_{yy} factors from Method 1 and Method 2*

Specimen	$\bar{\lambda}_y$	Method 1		Method 2	
		k_{yy}	U	k_{yy}	U
BC-1- $e_y=0$	0.36	0.71	1.10	1.18	1.12
BC-1- $e_y=25$	0.36	0.78	0.99	1.11	1.11
BC-1- $e_y=50$	0.36	0.82	1.04	1.09	1.21
BC-1- $e_y=150$	0.37	0.92	0.92	1.04	1.02
BC-2- $e_y=0$	0.67	1.20	1.12	1.49	1.13
BC-2- $e_y=25$	0.63	1.05	1.03	1.28	1.12
BC-2- $e_y=50$	0.66	1.04	0.91	1.19	0.98
BC-2- $e_y=150$	0.65	1.01	0.93	1.10	0.99
BC-3- $e_y=0$	0.83	1.71	1.21	1.70	1.21
BC-3- $e_y=25$	0.85	1.30	1.04	1.41	1.08
BC-3- $e_y=50$	0.85	1.21	1.03	1.31	1.07
BC-3- $e_y=150$	0.85	1.10	1.03	1.16	1.07

Table 6: *Calculated k_{zz} factors from Method 1 and Method 2*

Specimen	$\bar{\lambda}_z$	Method 1		Method 2	
		k_{zz}	U	k_{zz}	U
BC-1- $e_z=0$	0.63	1.56	1.25	1.44	1.24
BC-1- $e_z=15$	0.60	1.24	1.30	1.25	1.30
BC-1- $e_z=25$	0.60	1.17	1.26	1.19	1.26
BC-1- $e_z=100$	0.60	1.07	1.32	1.08	1.34
BC-2- $e_z=0$	1.16	1.96	1.38	1.71	1.36
BC-2- $e_z=25$	1.09	1.33	1.26	1.35	1.26
BC-2- $e_z=50$	1.09	1.21	1.22	1.24	1.24
BC-2- $e_z=150$	1.12	1.09	1.17	1.12	1.19
BC-3- $e_z=0$	1.56	1.41	1.13	1.60	1.14
BC-3- $e_z=50$	1.45	1.19	1.16	1.30	1.21
BC-3- $e_z=100$	1.51	1.12	1.13	1.21	1.19
BC-3- $e_z=200$	1.51	1.08	1.17	1.14	1.23

Table 7: Results of statistical analysis of test data for EN 1990 (2002) based on the resistance function obtained from Method 1

Data set	n	Method 1		Method 2	
		R_m	V_R	R_m	V_R
N + M_y	12	1.030	0.086	1.092	0.069
N + M_z	12	1.228	0.066	1.248	0.050

Table 8: Results of statistical analysis of test data for EN 1990 (2002) based on the resistance function obtained from Method 1

Data set	No. of	Fractile	$R_{Test} /$	Model	Resistance	γ_{M1}
	tests	factor	R_{Code}	scatter	scatter	
	n	$k_{d,n}$	b	V_δ	V_r	
N + M_y	12	4.19	1.030	0.0850	0.1026	1.25
N + M_z	12	4.19	1.228	0.0655	0.0871	0.97
All tests	24	3.56	1.129	0.1176	0.1308	1.21

Table 9: Results of statistical analysis of test data for EN 1990 (2002) based on the resistance function obtained from Method 2

Data set	No. of	Fractile	$R_{Test} /$	Model	Resistance	γ_{M1}
	tests	factor	R_{Code}	scatter	scatter	
	n	$k_{d,n}$	b	V_δ	V_r	
N + M_y	12	4.19	1.092	0.0690	0.0898	1.11
N + M_z	12	4.19	1.248	0.0497	0.0759	0.91
All tests	24	3.56	1.170	0.0905	0.1072	1.07

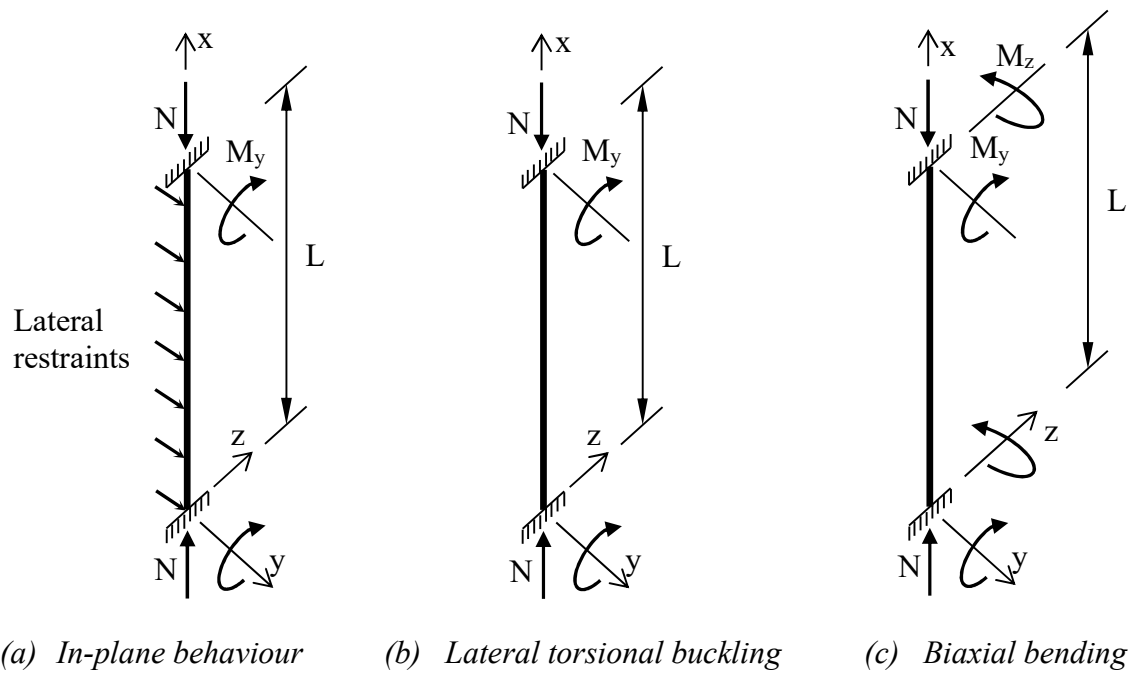


Figure 1: *Beam-column behaviour*

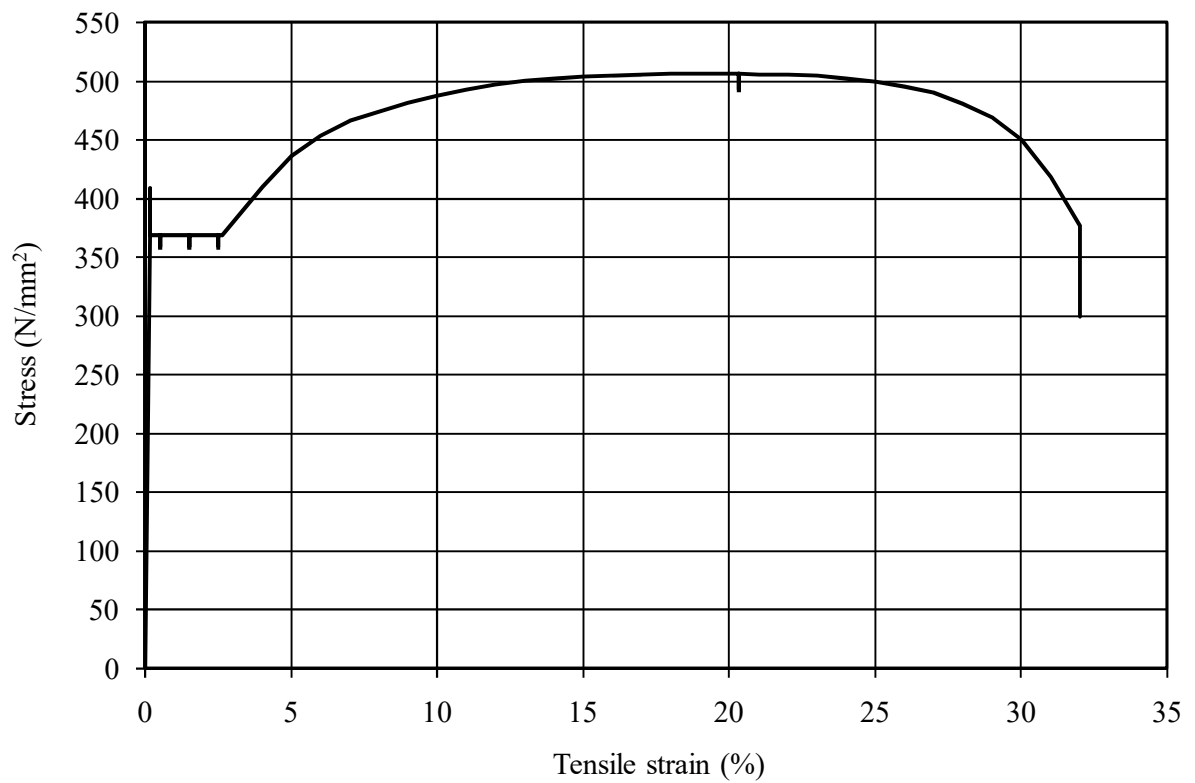


Figure 2: *Tensile material stress-strain curve for coupon BC-TC3*



Figure 3: Deformed specimens of the three lengths of the tested beam-columns

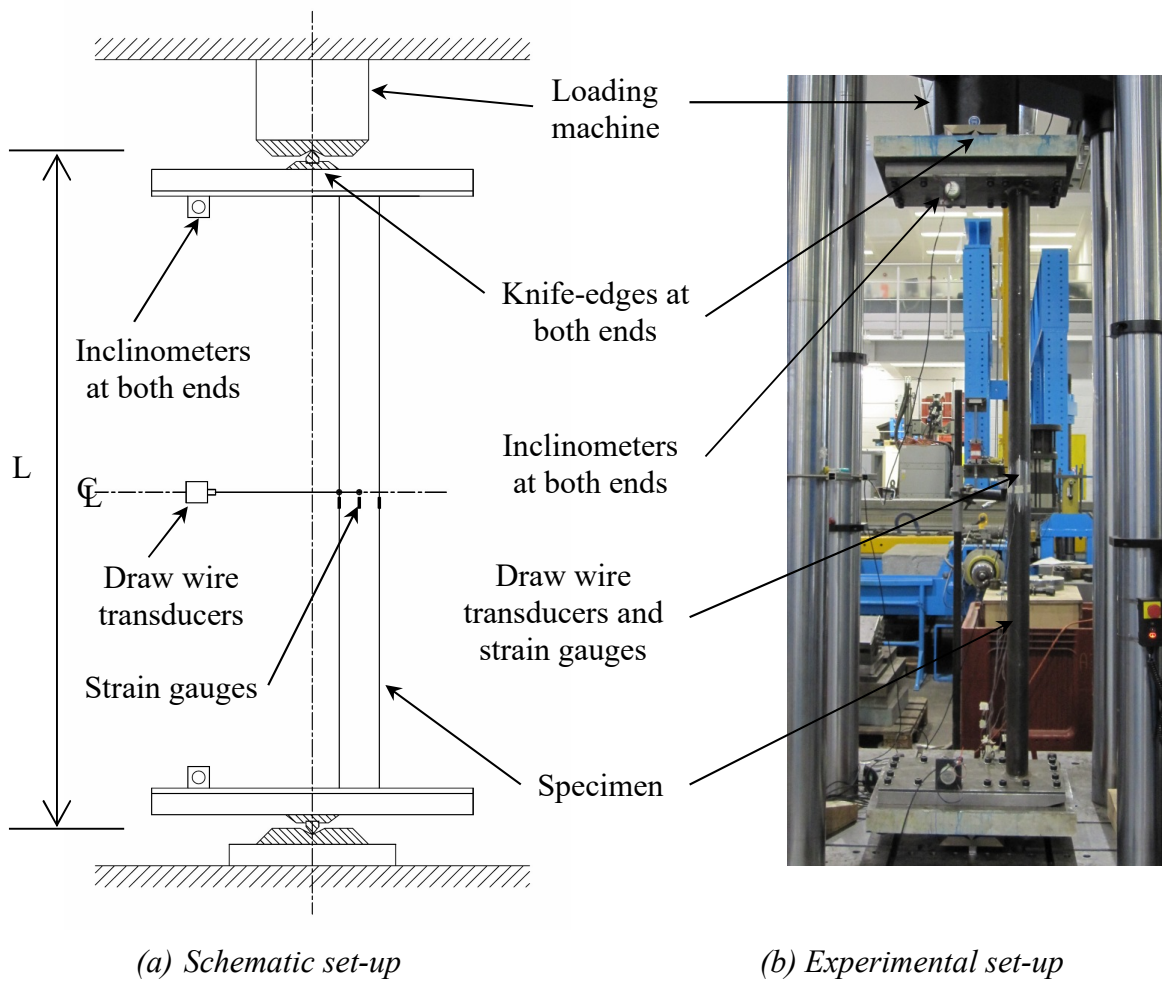


Figure 4: Pure and uniaxial eccentric compression test configurations

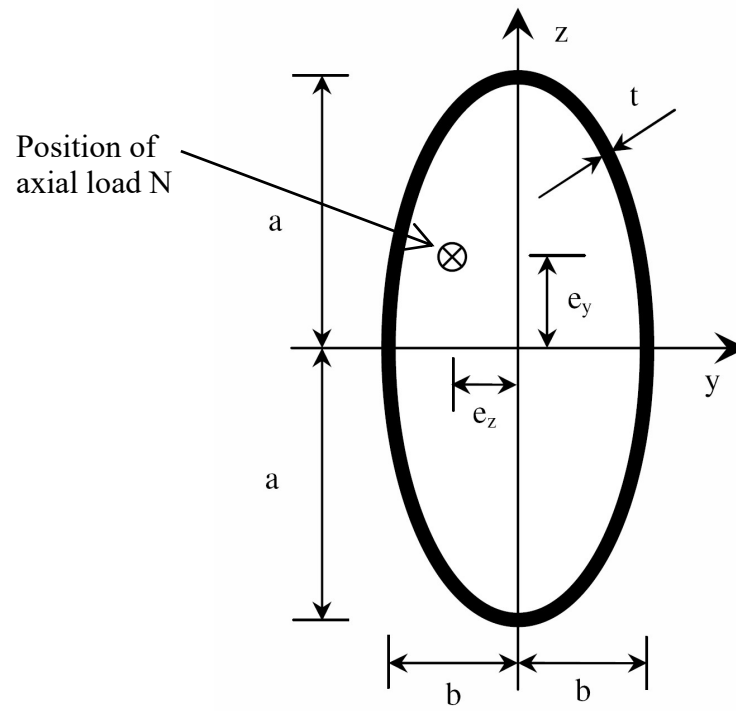


Figure 5: *Geometry of an elliptical hollow section*



Figure 6: *Deformed specimen of BC-2- $e_z=150$*



b) View from the major axis



a) View from the minor axis

Figure 7: Deformed specimen of BC-2- $e_y=150$

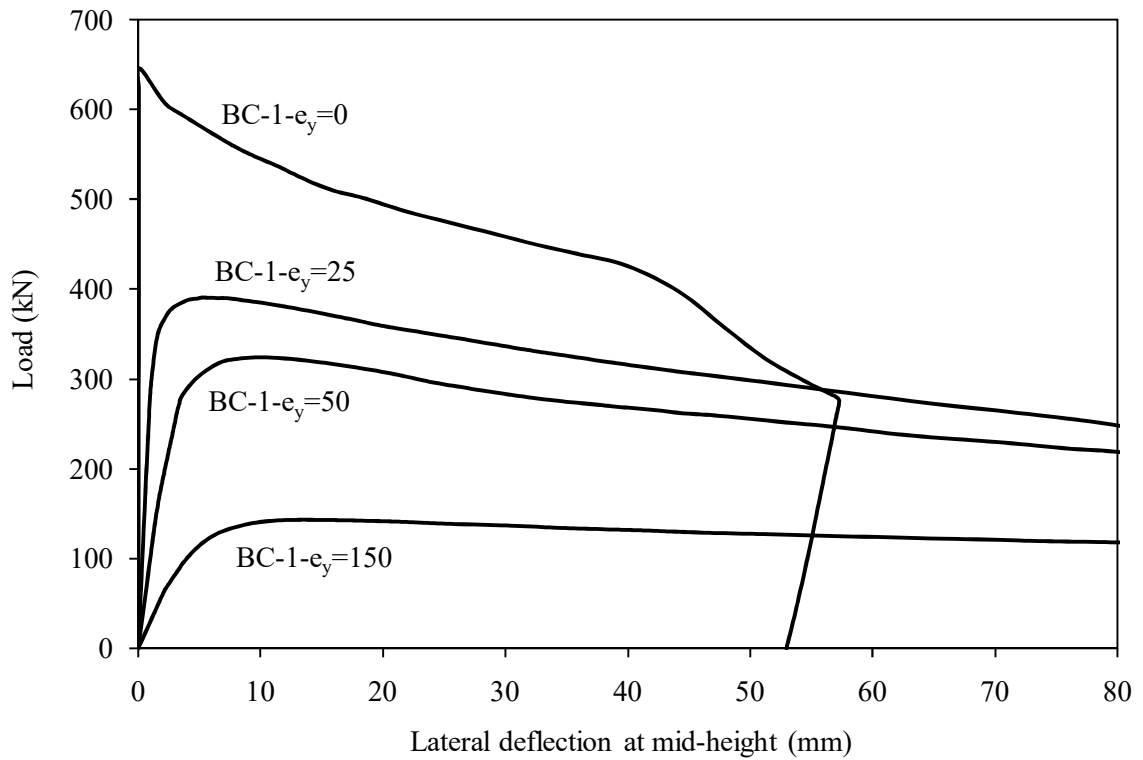


Figure 8: Load-lateral deflection curves for 1 m beam-columns eccentrically loaded about the major axis

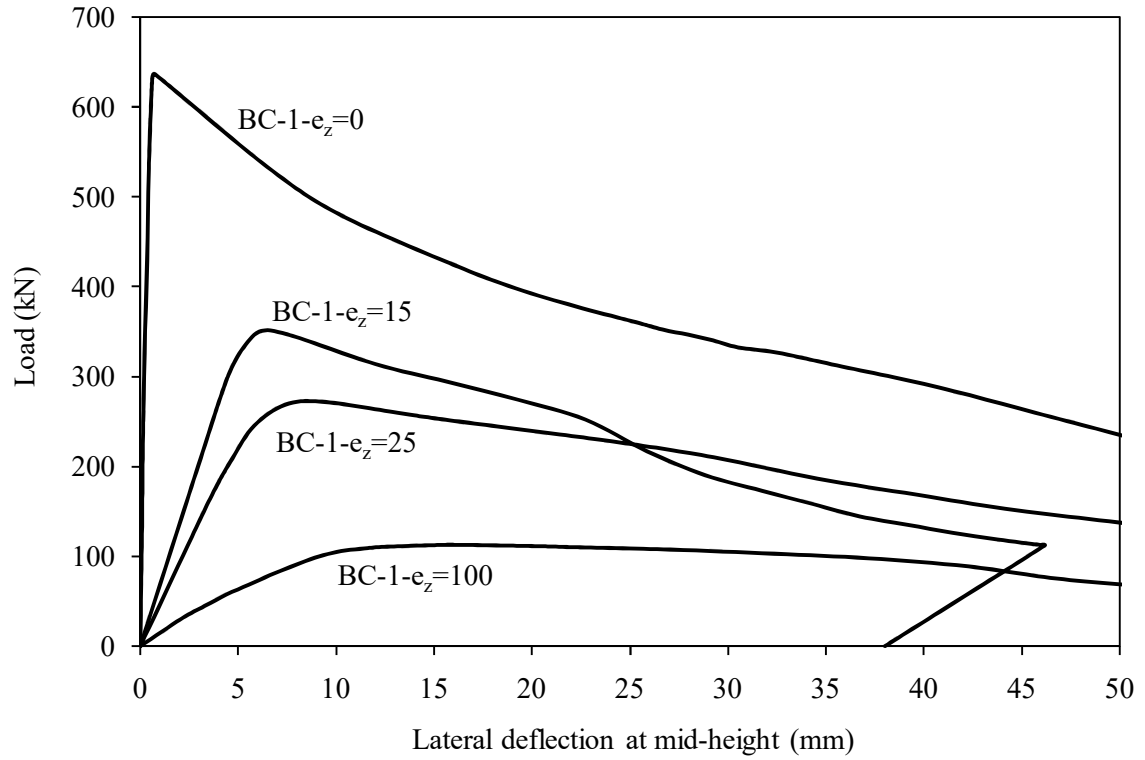


Figure 9: Load-lateral deflection curves for 1 m beam-columns eccentrically loaded about the minor axis

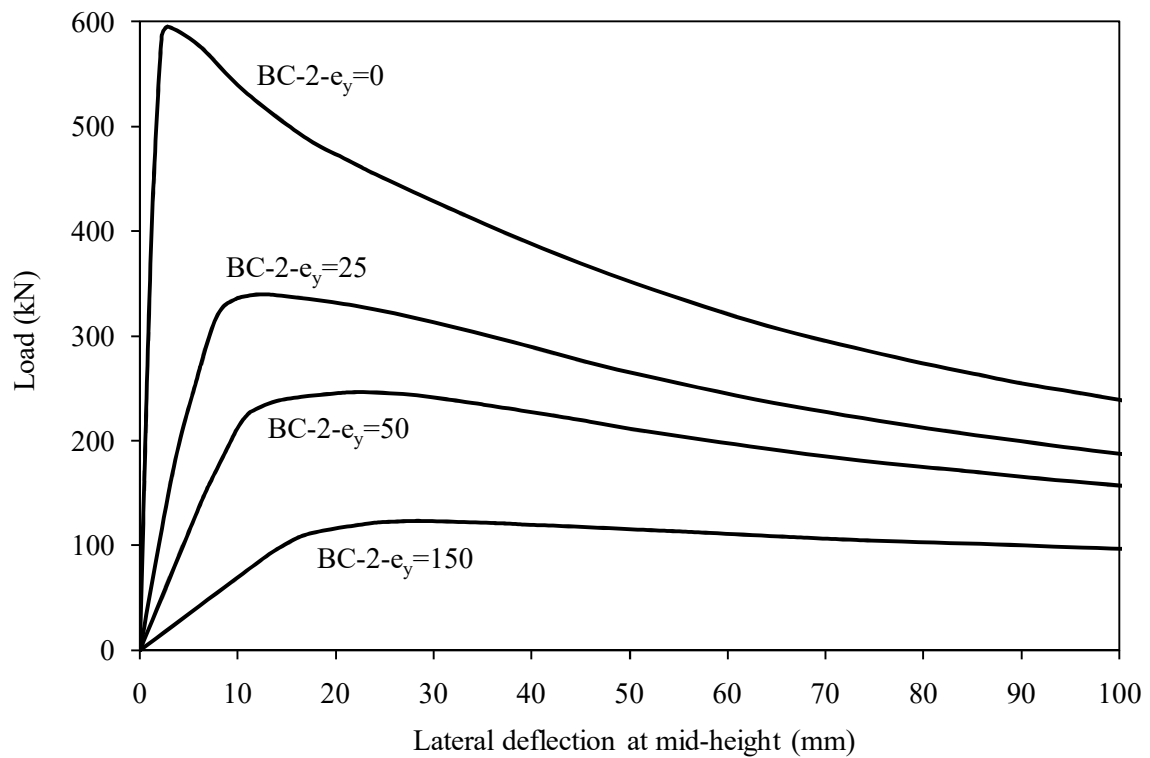


Figure 10: Load-lateral deflection curves for 2 m beam-columns eccentrically loaded about the major axis

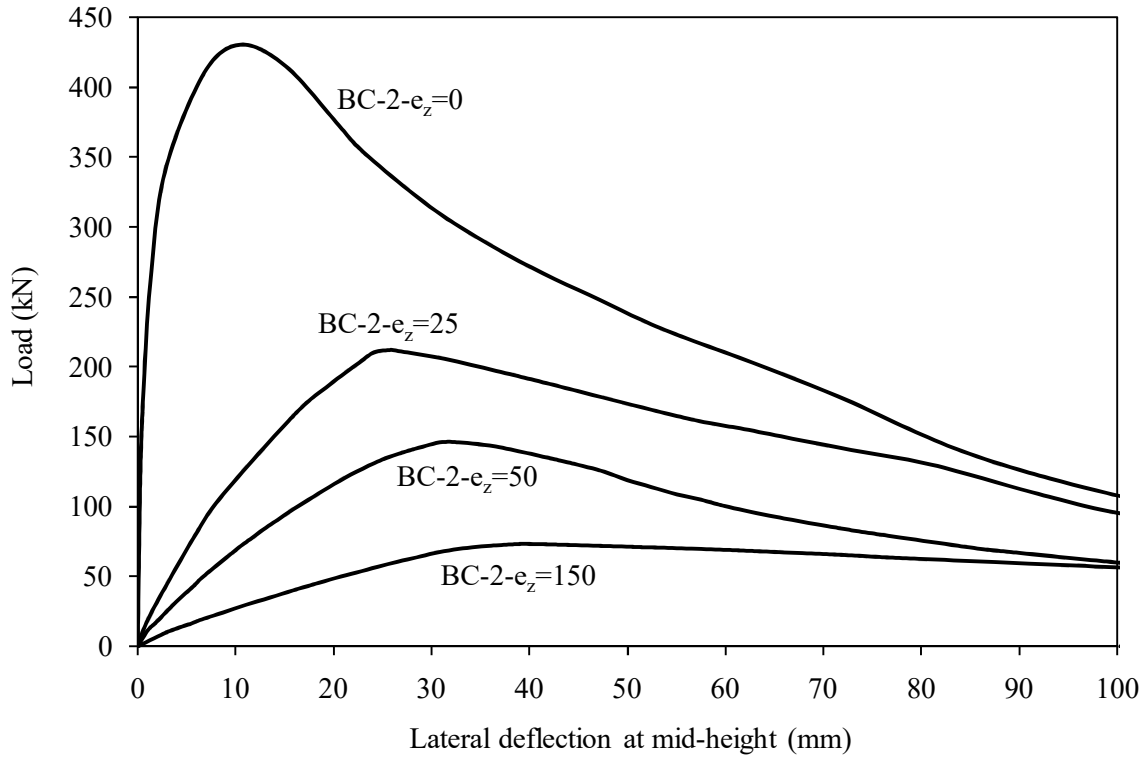


Figure 11: Load-lateral deflection curves for 2 m beam-columns eccentrically loaded about the minor axis

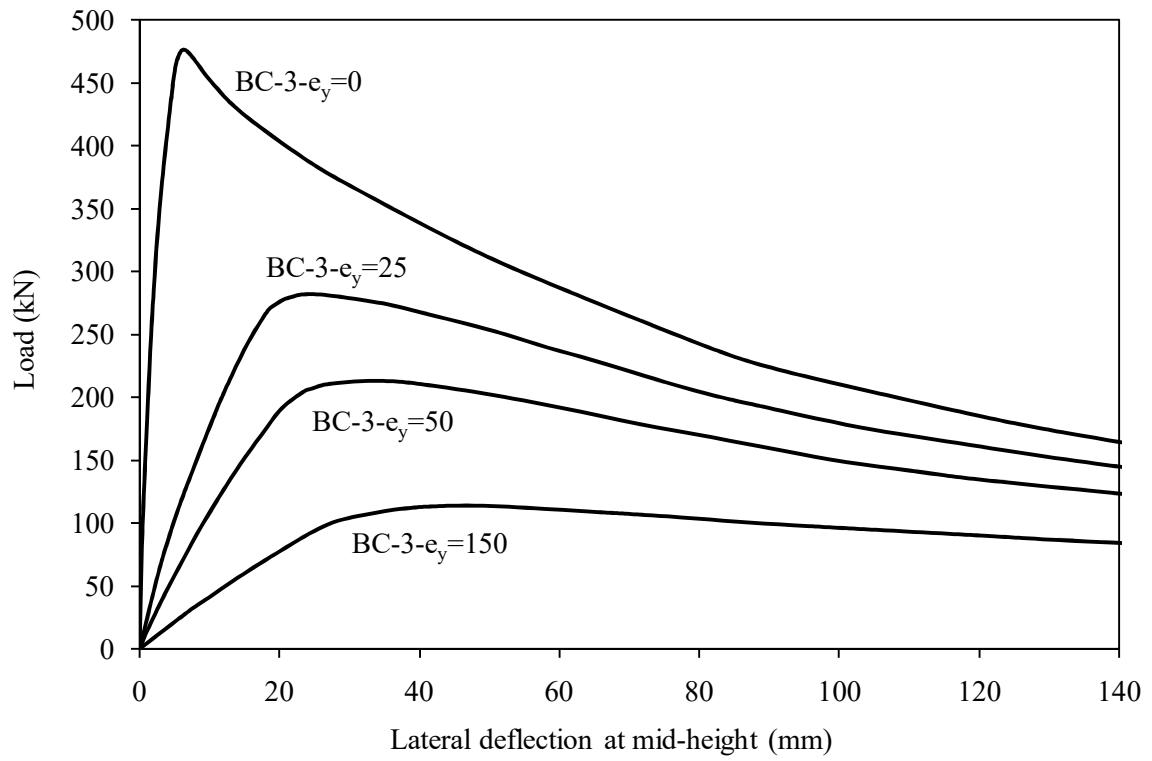


Figure 12: Load-lateral deflection curves for 3 m beam-columns eccentrically loaded about the major axis

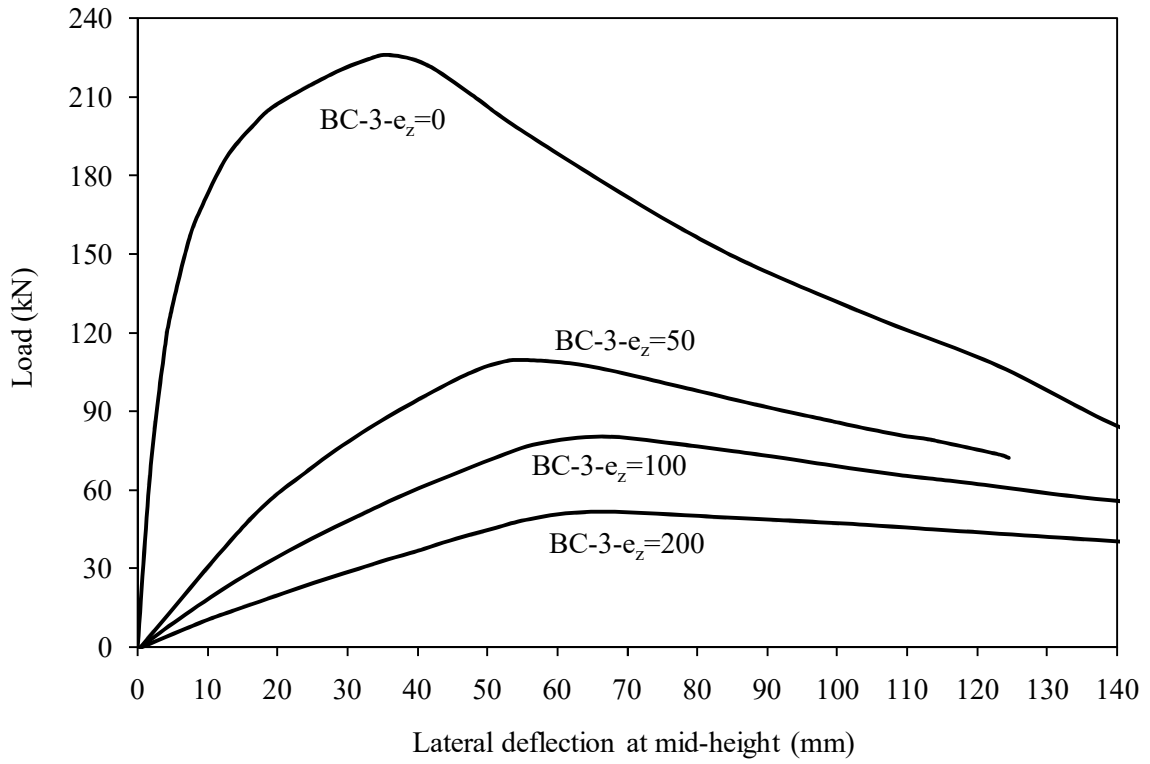


Figure 13: Load-lateral deflection curves for 3 m beam-columns eccentrically loaded about the minor axis

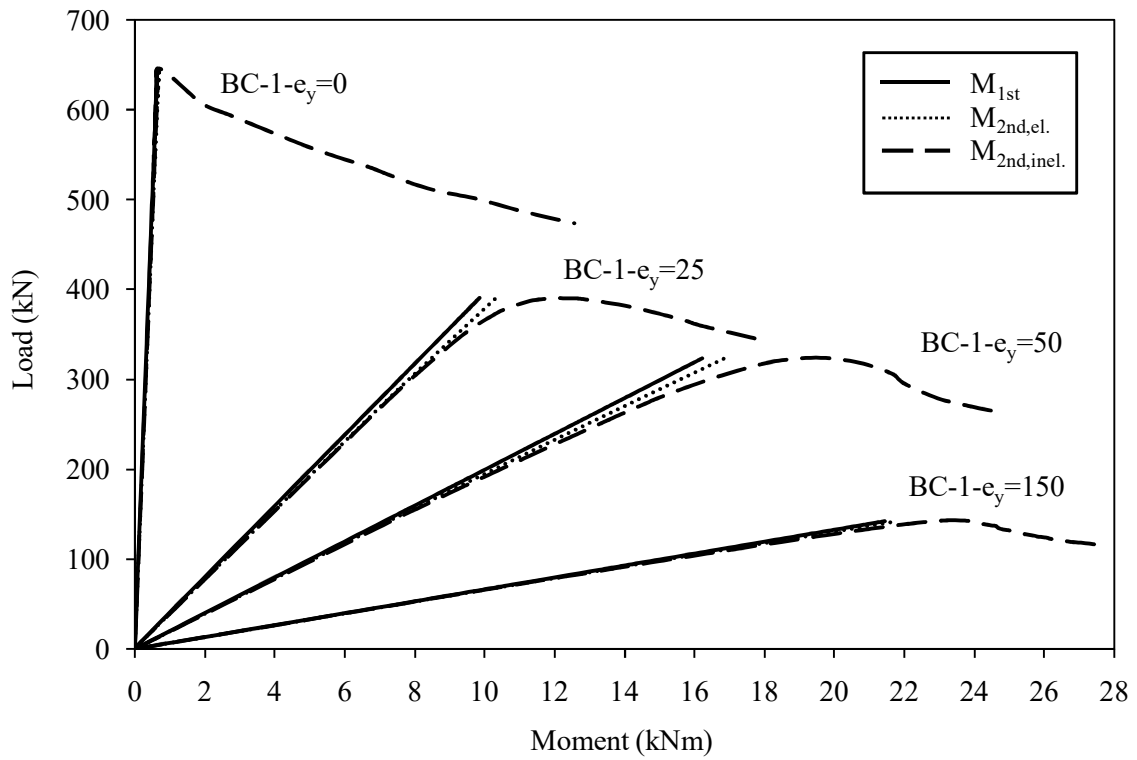


Figure 14: Load versus moment for 1 m beam-columns eccentrically loaded about the major axis

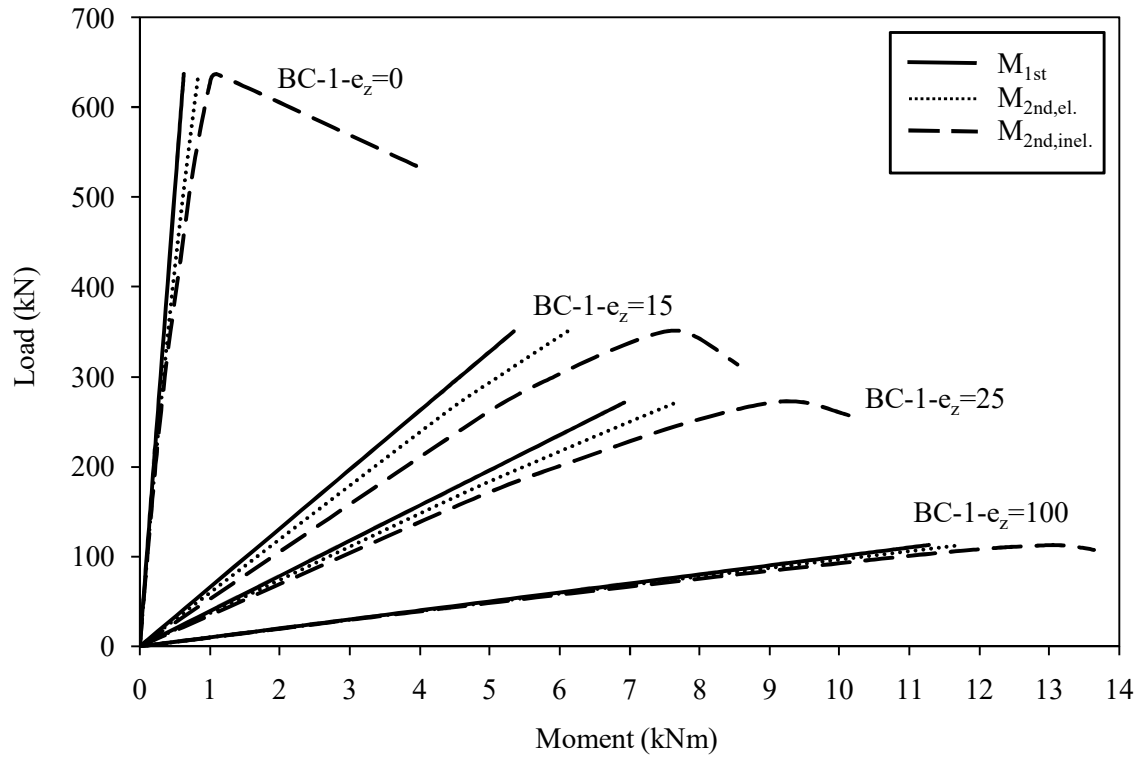


Figure 15: Load versus moment for 1 m beam-columns eccentrically loaded about the minor axis

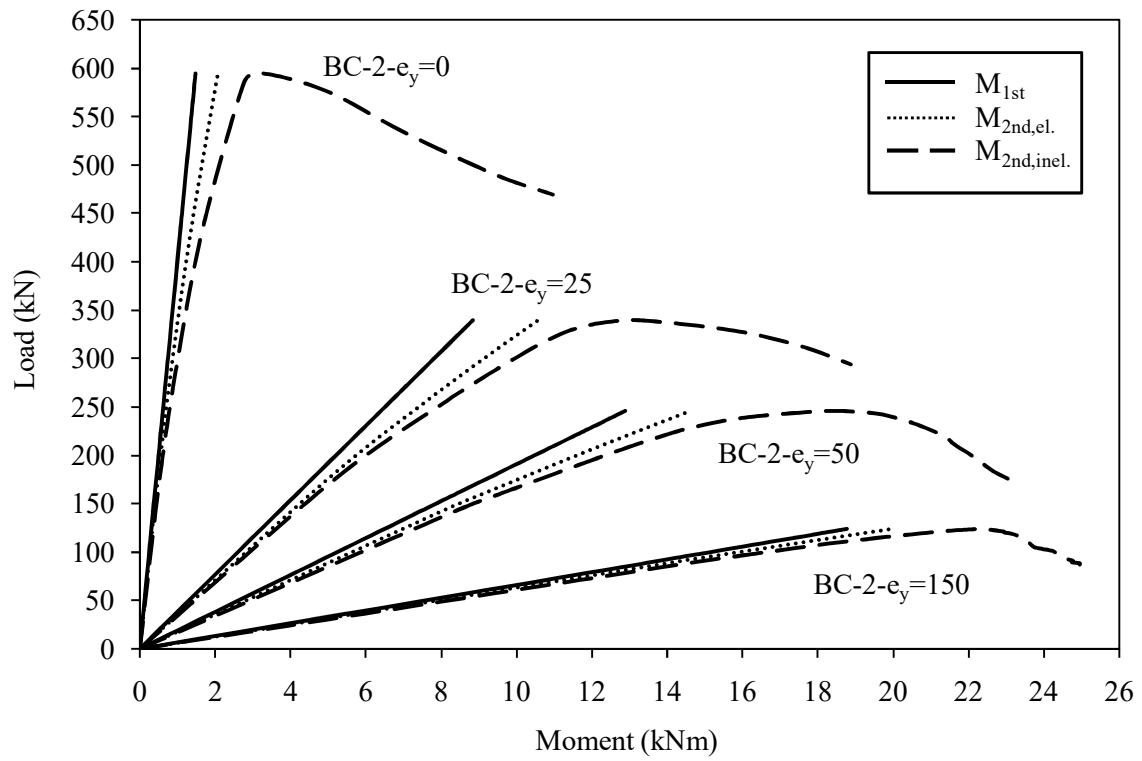


Figure 16: Load versus moment for 2 m beam-columns eccentrically loaded about the major axis

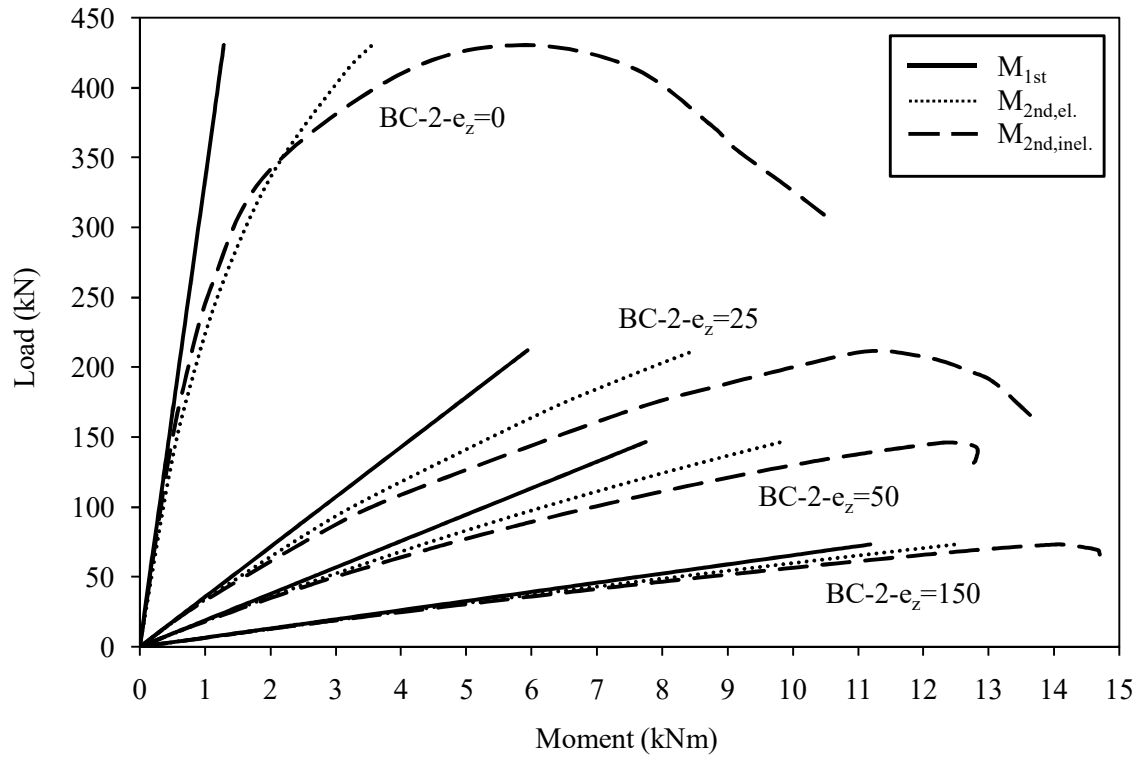


Figure 17: Load versus moment for 2 m beam-columns eccentrically loaded about the minor axis

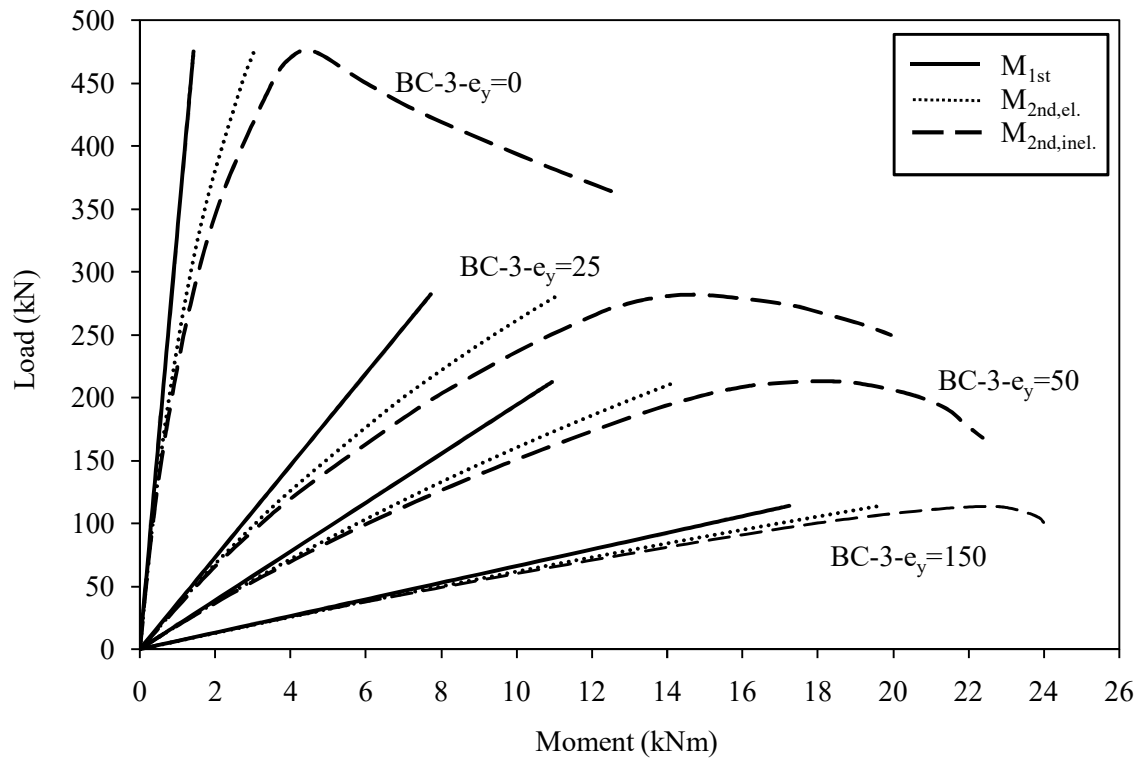


Figure 18: Load versus moment for 3 m beam-columns eccentrically loaded about the major axis

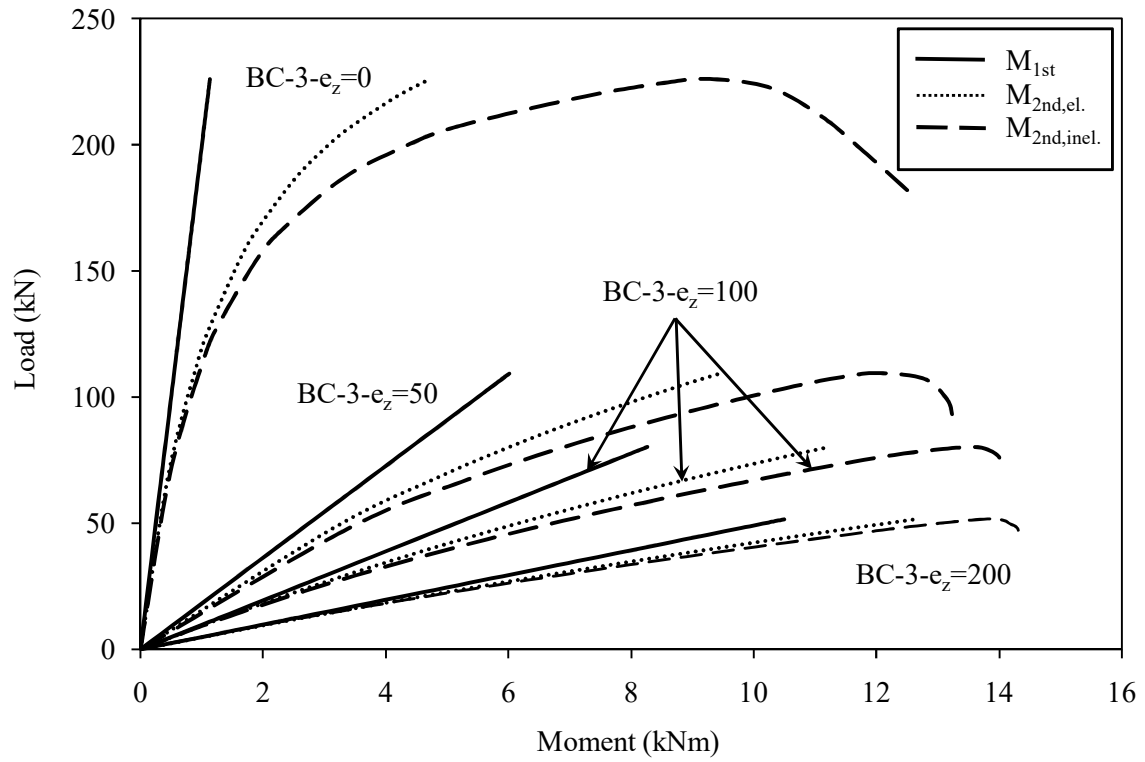


Figure 19: Load versus moment for 3 m beam-columns eccentrically loaded about the minor axis

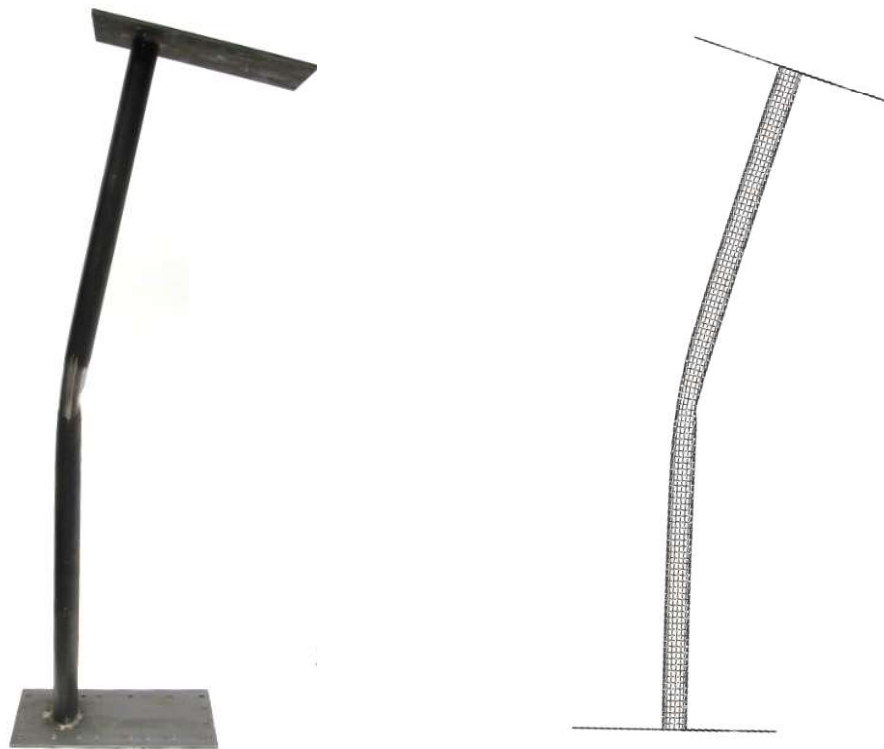


Figure 20: Comparison of deformed specimens for BC-2- $e_z=150$

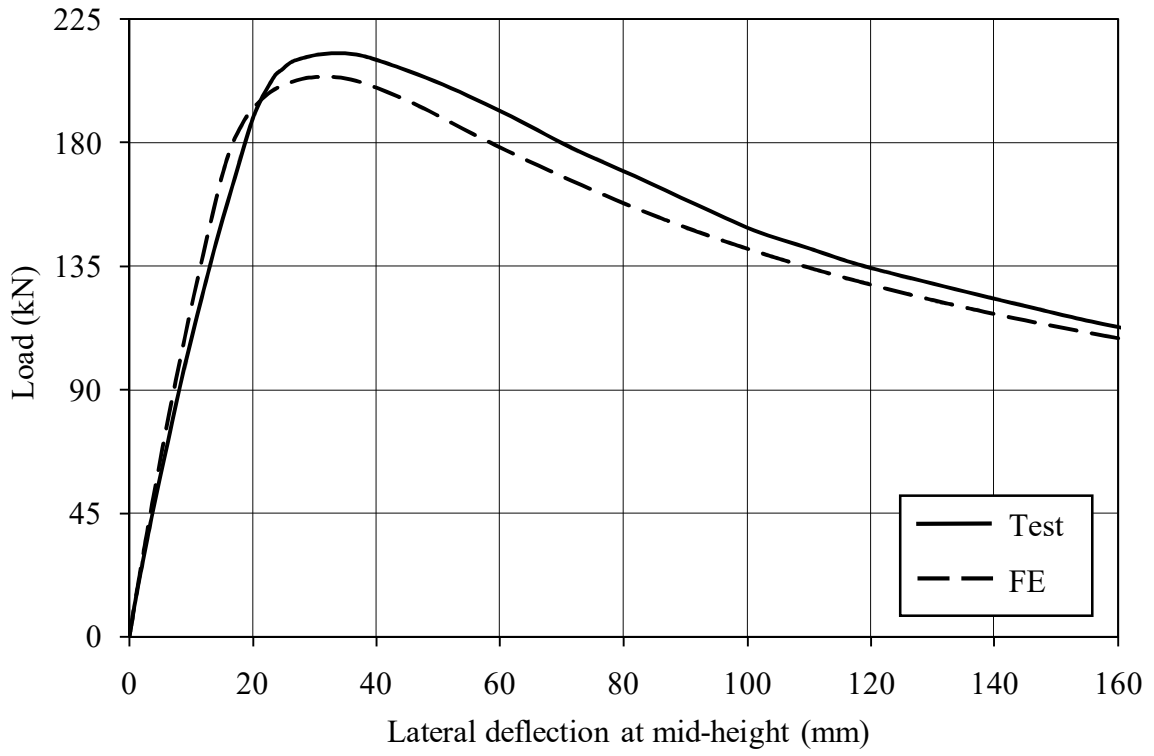


Figure 21: Comparison between test and numerical results for BC-3- $e_y=50$

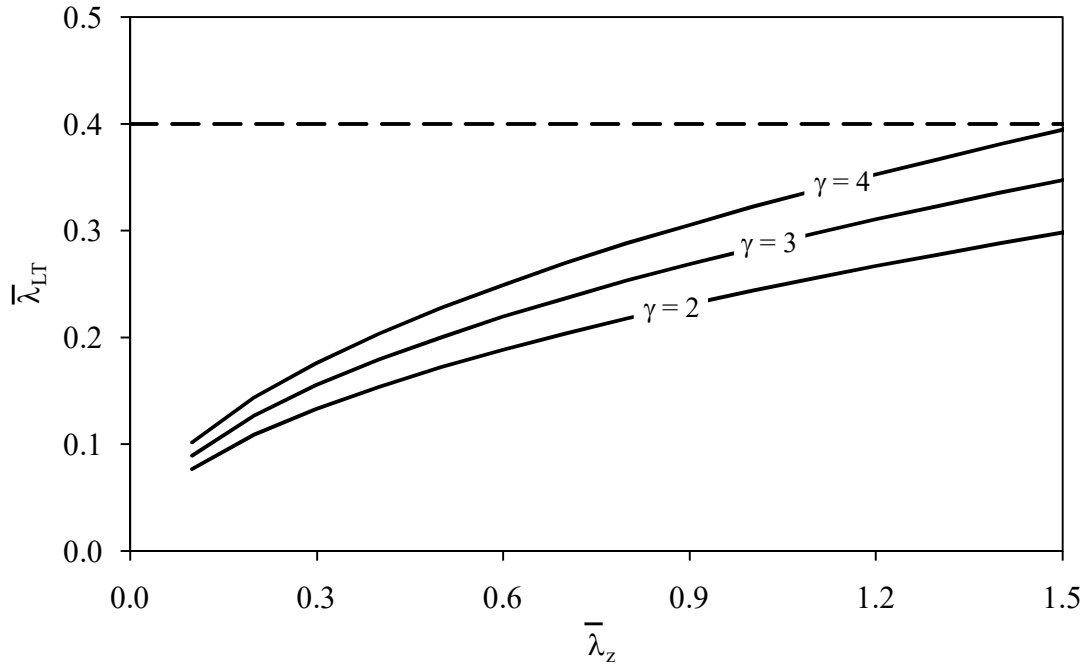


Figure 22: $\bar{\lambda}_{LT}$ versus $\bar{\lambda}_z$ for EHS members with various aspect ratios

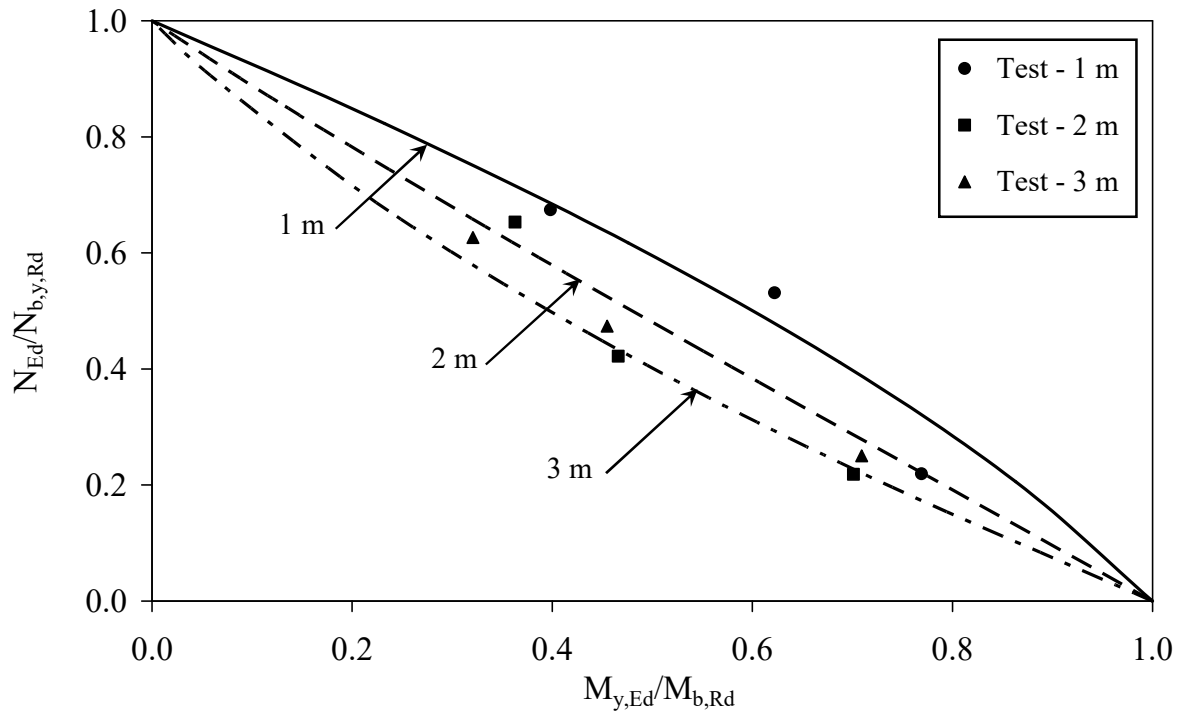


Figure 23: Interaction of axial load and major axis moment – tests and design curves obtained from Method 1 in EN 1993-1-1 (2005)

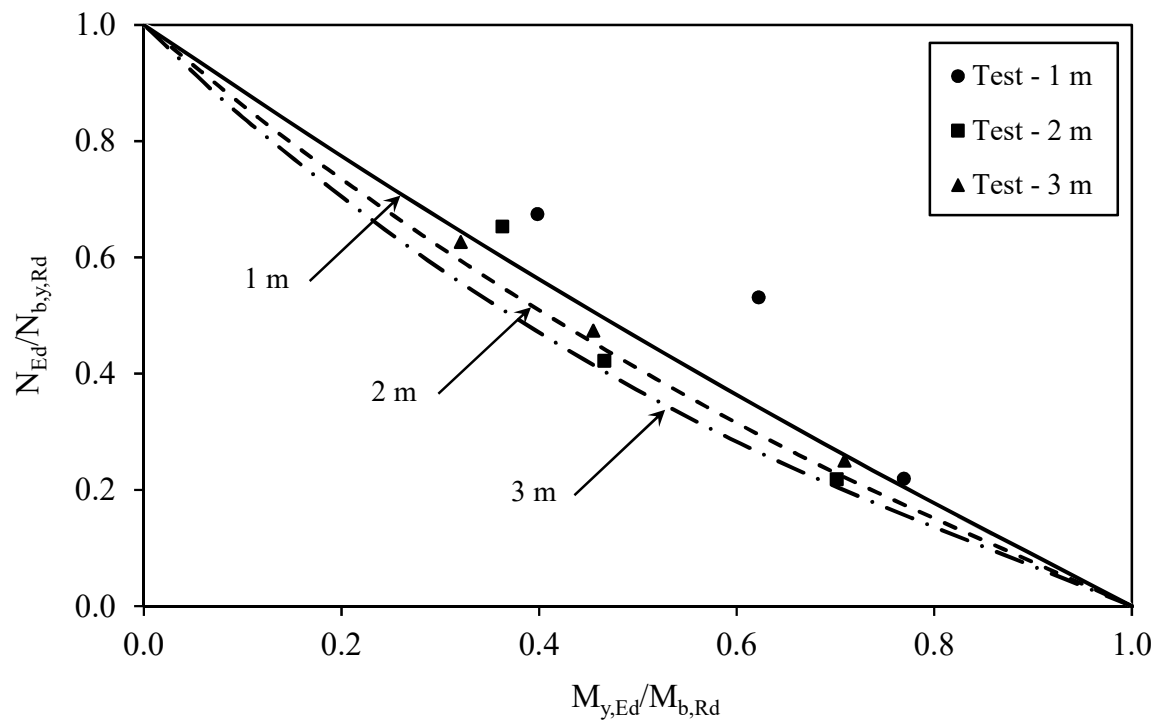


Figure 24: Interaction of axial load and major axis moment – tests and design curves obtained from Method 2 in EN 1993-1-1 (2005)

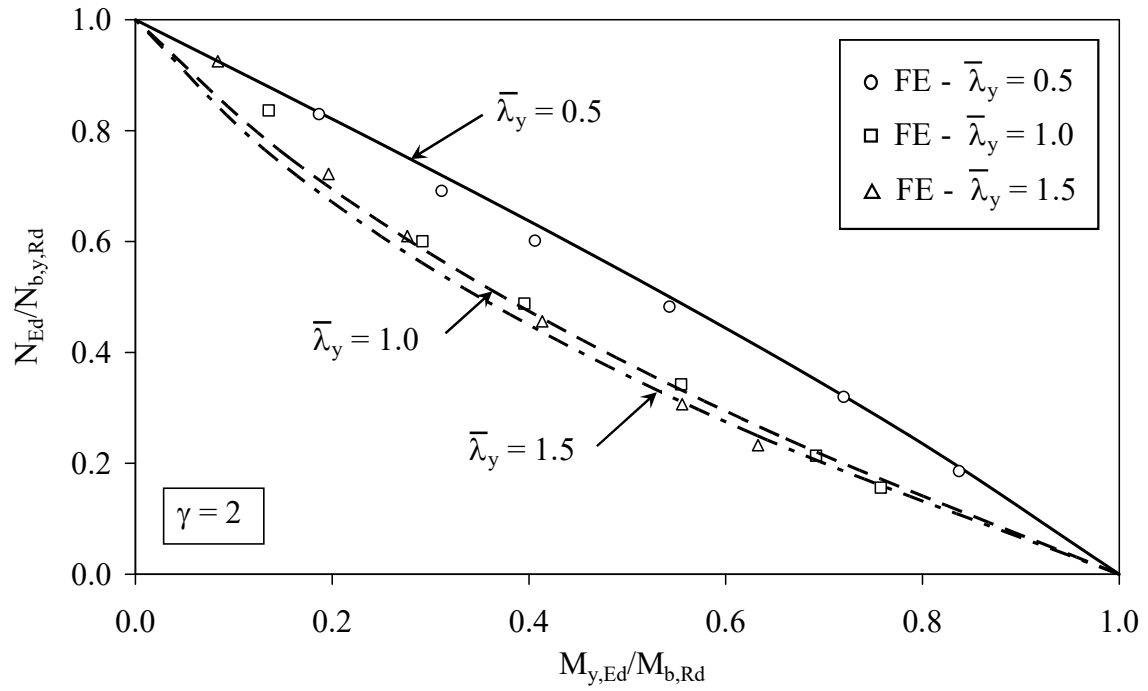


Figure 25: Interaction of axial load and major axis moment – FE results and design curves of members with $\gamma = 2$ (Method 1)

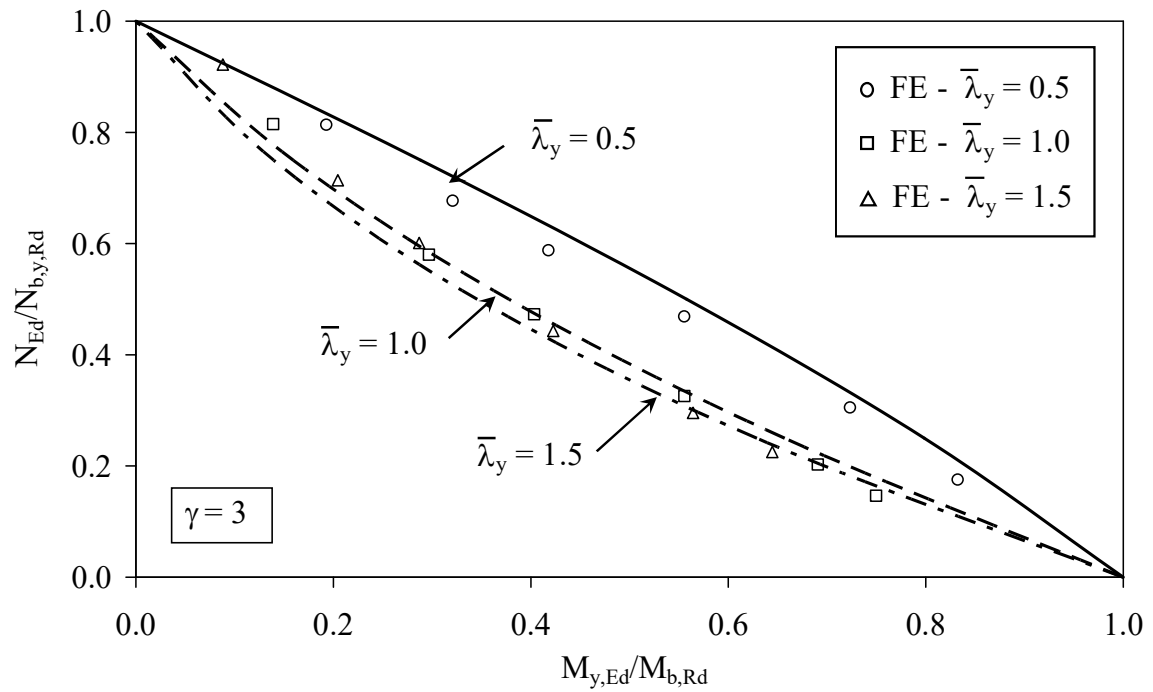


Figure 26: Interaction of axial load and major axis moment – FE results and design curves of members with $\gamma = 3$ (Method 1)

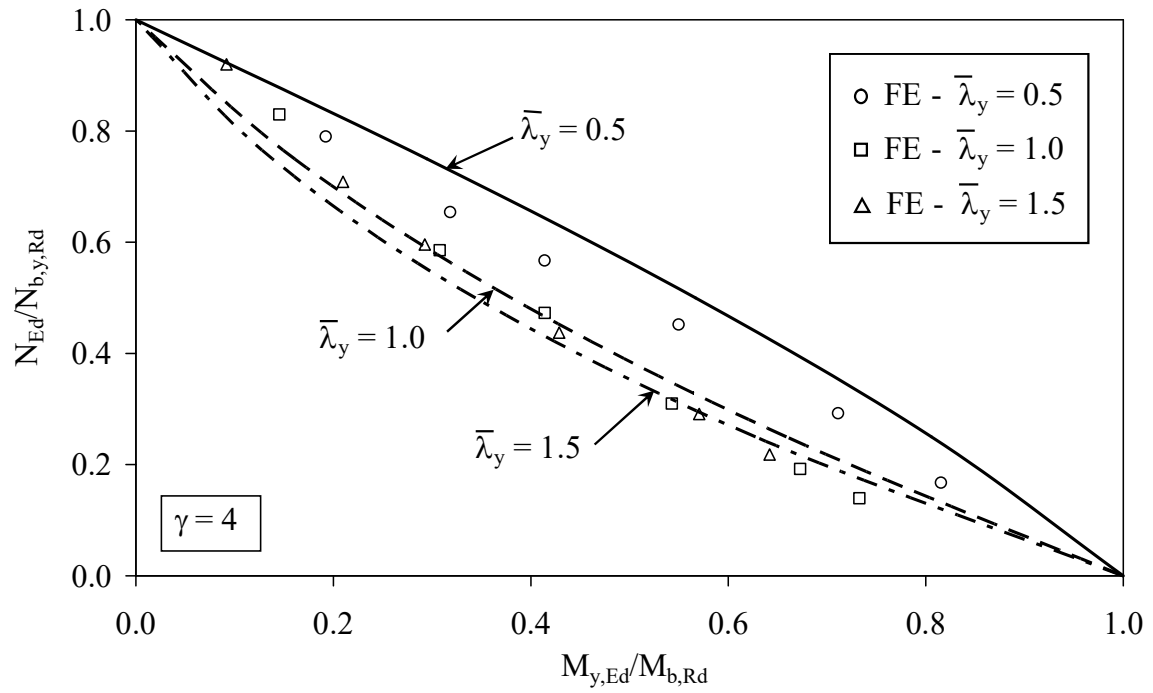


Figure 27: Interaction of axial load and major axis moment – FE results and design curves of members with $\gamma = 4$ (Method 1)

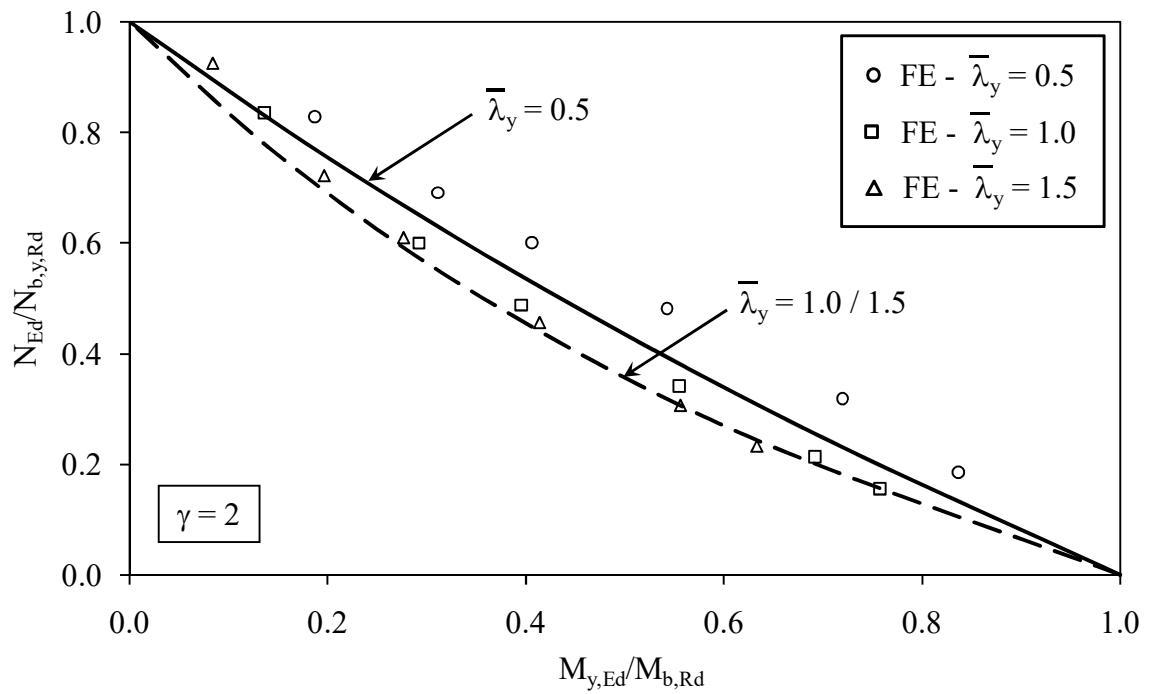


Figure 28: Interaction of axial load and major axis moment – FE results and design curves of members with $\gamma = 2$ (Method 2)

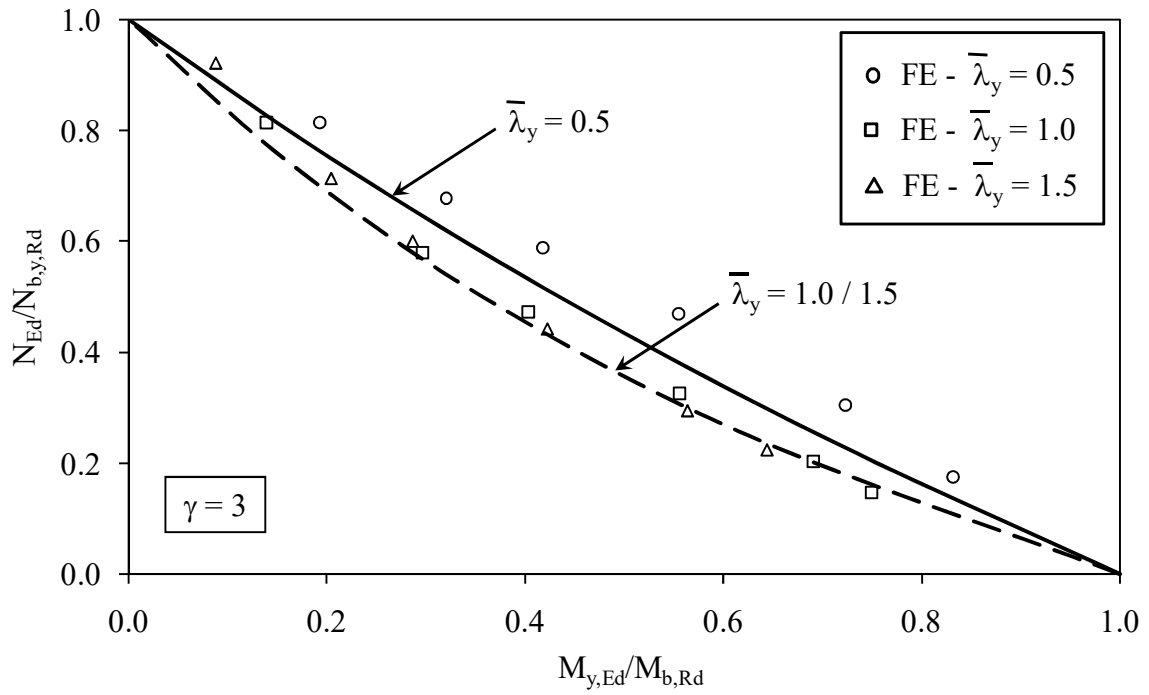


Figure 29: Interaction of axial load and major axis moment – FE results and design curves of members with $\gamma = 3$ (Method 2)

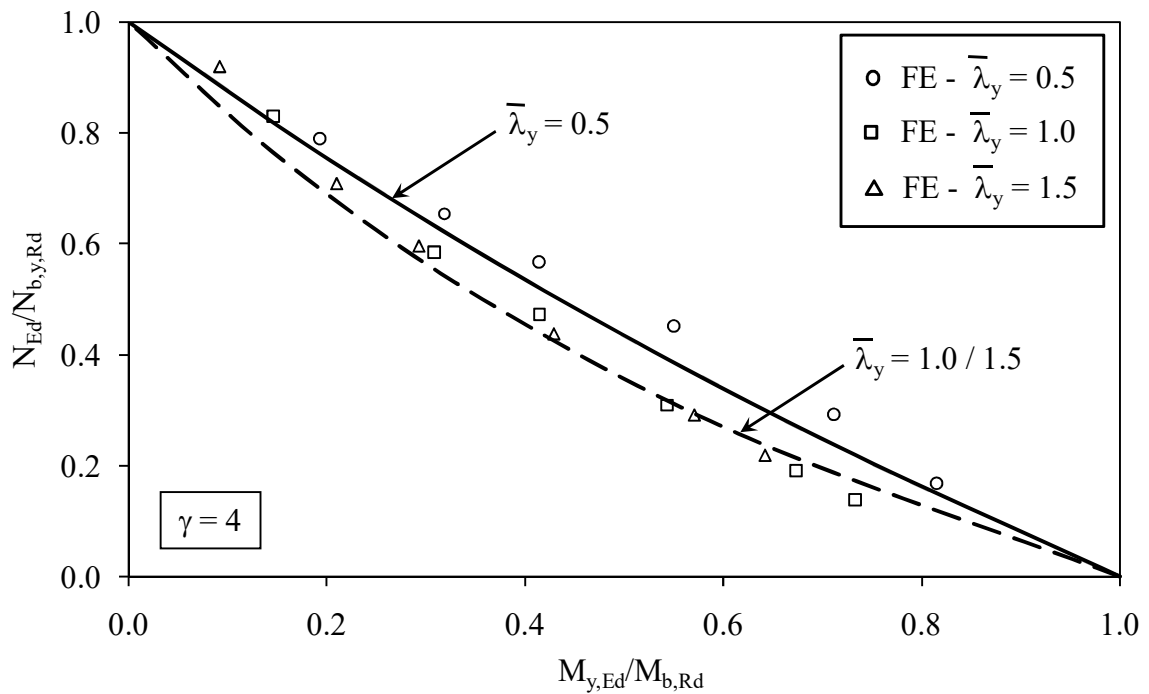


Figure 30: Interaction of axial load and major axis moment – FE results and design curves of members with $\gamma = 4$ (Method 2)

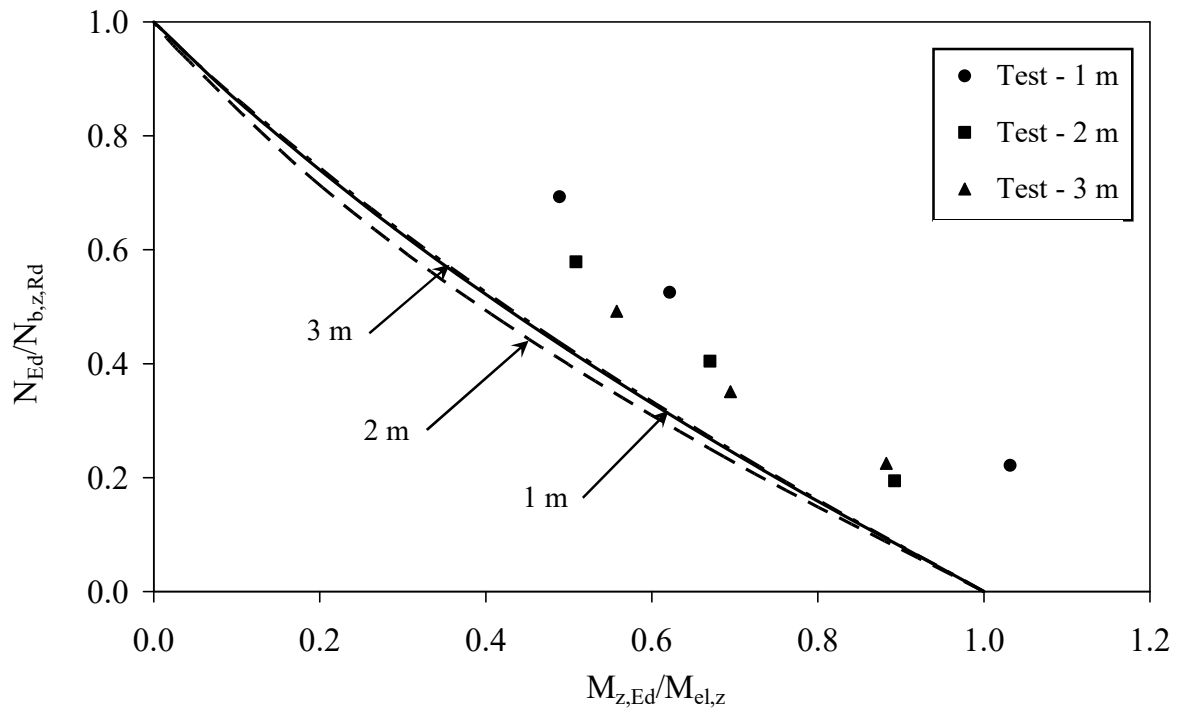


Figure 31: Interaction of axial load and minor axis moment – tests and design curves in *EN 1993-1-1 (2005) (Method 1)*

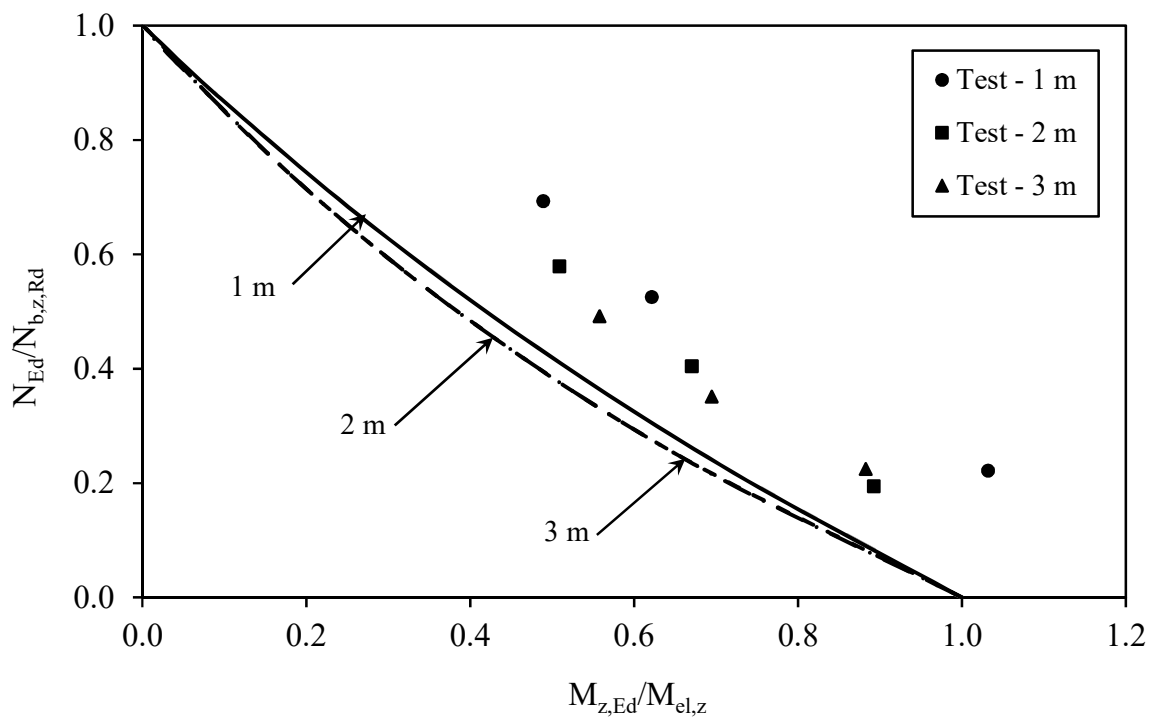


Figure 32: Interaction of axial load and minor axis moment – tests and design curves in *EN 1993-1-1 (2005) (Method 2)*

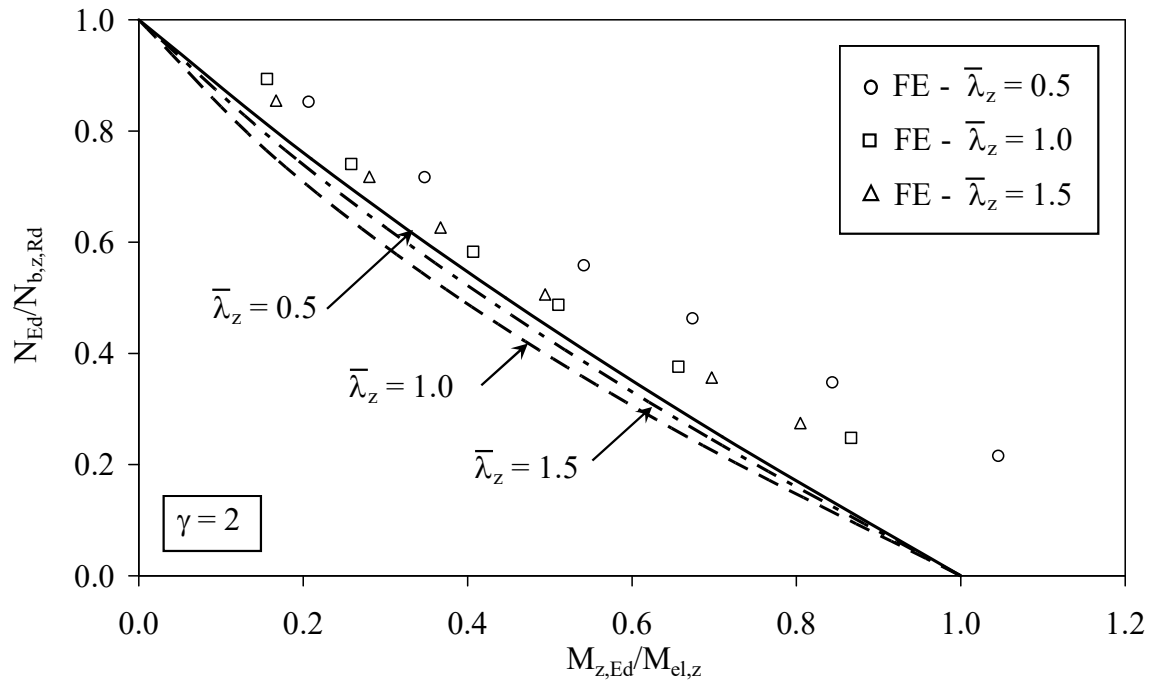


Figure 33: Interaction of axial load and minor axis moment – FE results and design curves of members with $\gamma = 2$ (Method 1)

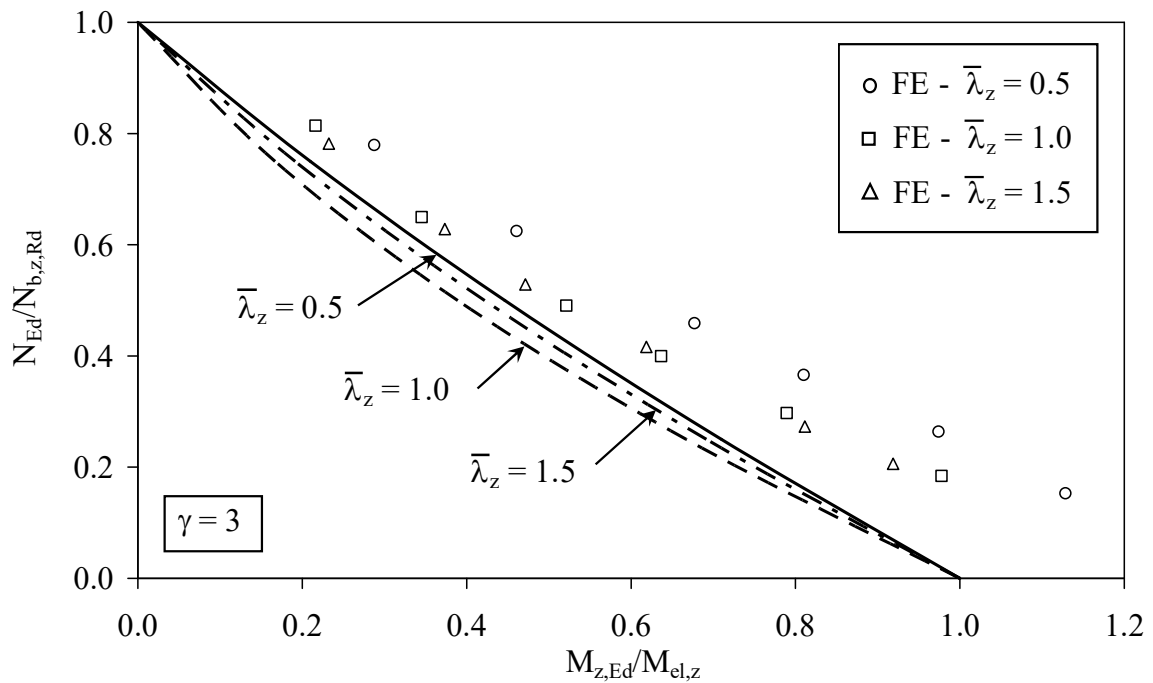


Figure 34: Interaction of axial load and minor axis moment – FE results and design curves of members with $\gamma = 3$ (Method 1)

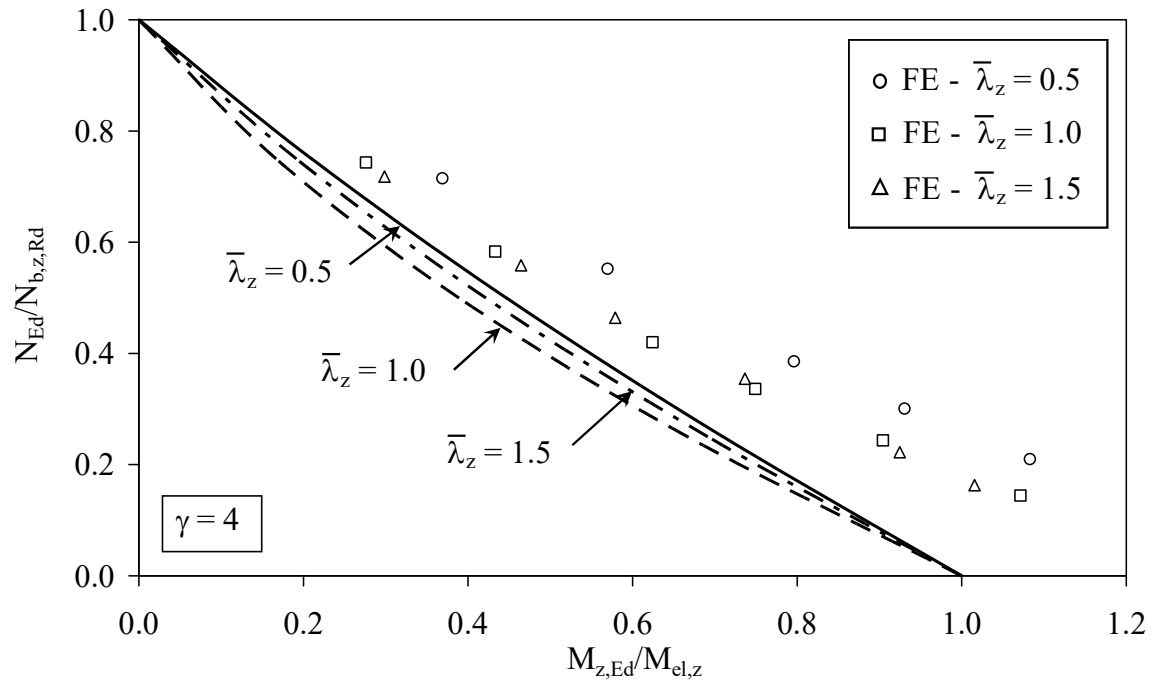


Figure 35: Interaction of axial load and minor axis moment – FE results and design curves of members with $\gamma = 4$ (Method 1)

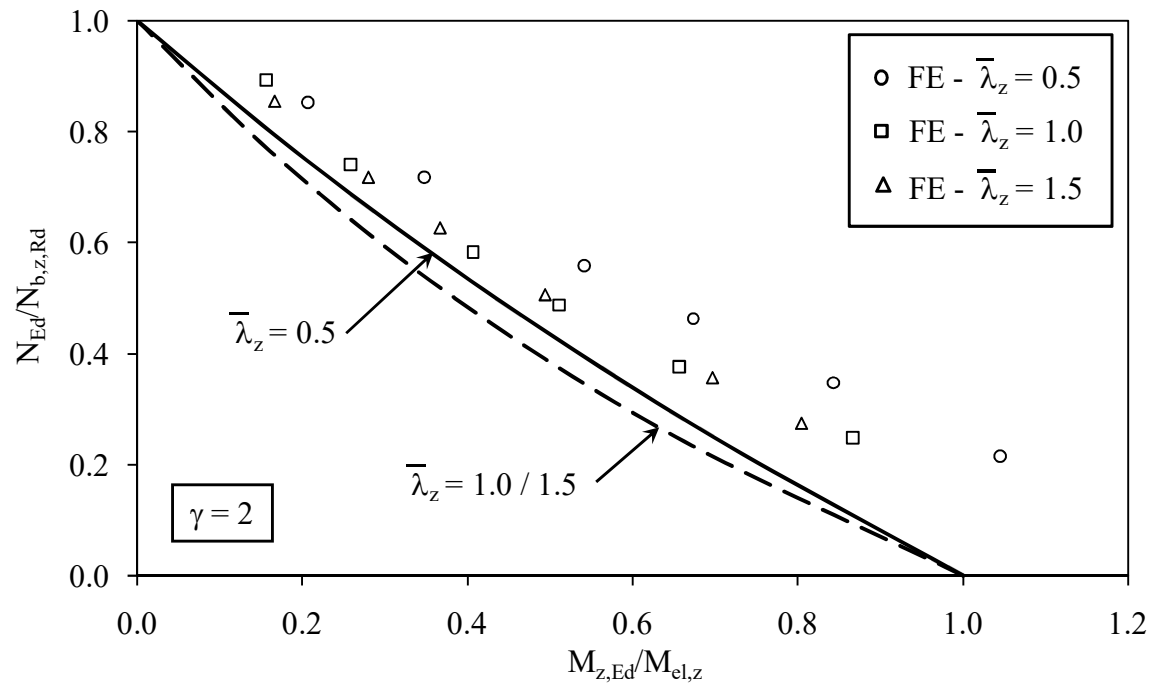


Figure 36: Interaction of axial load and minor axis moment – FE results and design curves of members with $\gamma = 2$ (Method 2)

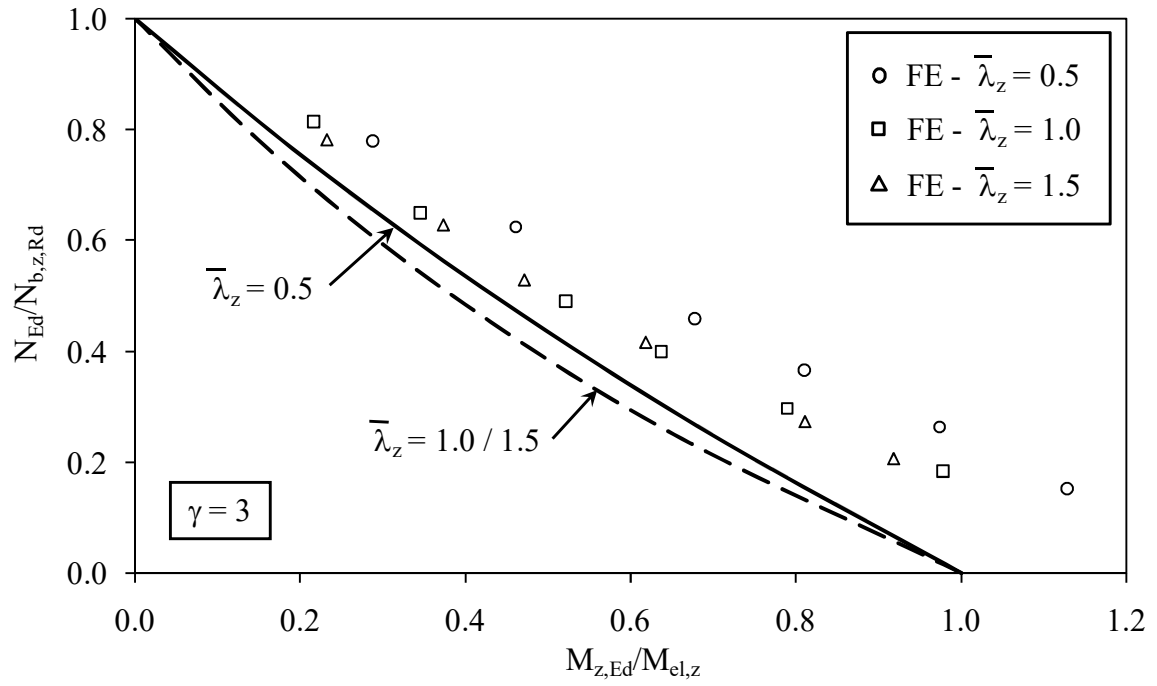


Figure 37: Interaction of axial load and minor axis moment – FE results and design curves of members with $\gamma = 3$ (Method 2)

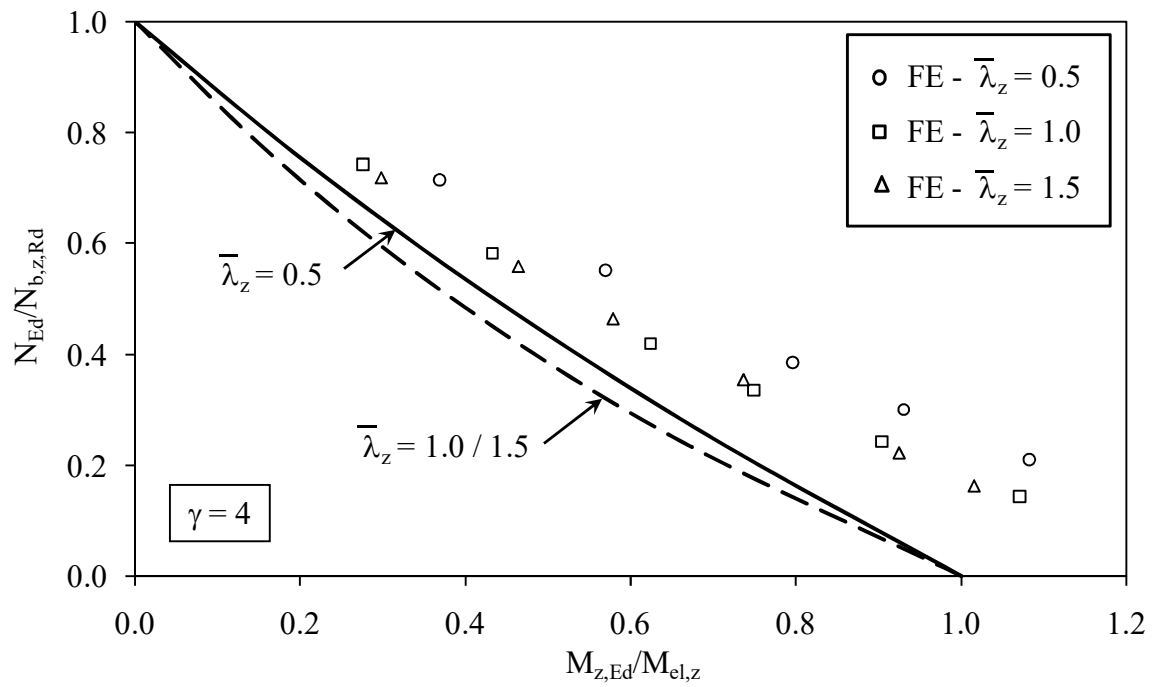


Figure 38: Interaction of axial load and minor axis moment – FE results and design curves of members with $\gamma = 4$ (Method 2)

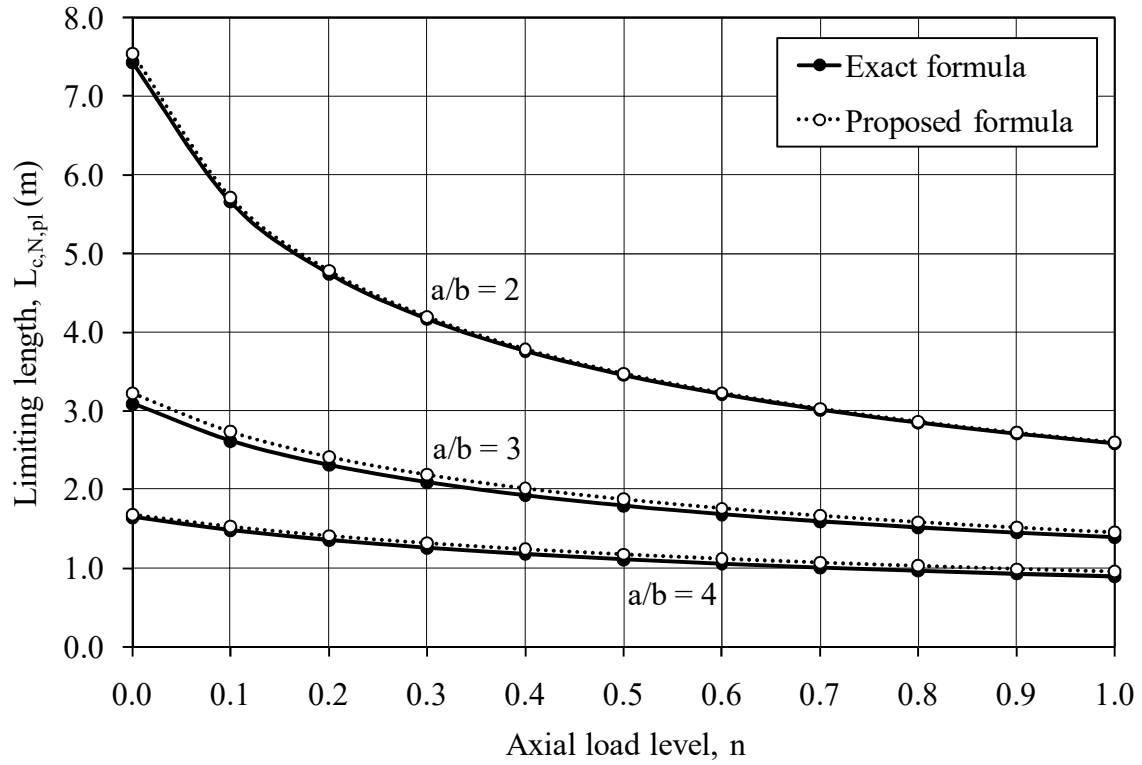


Figure 39: Limiting length versus axial load level with various aspect ratios for Class 1 and 2 cross-sections

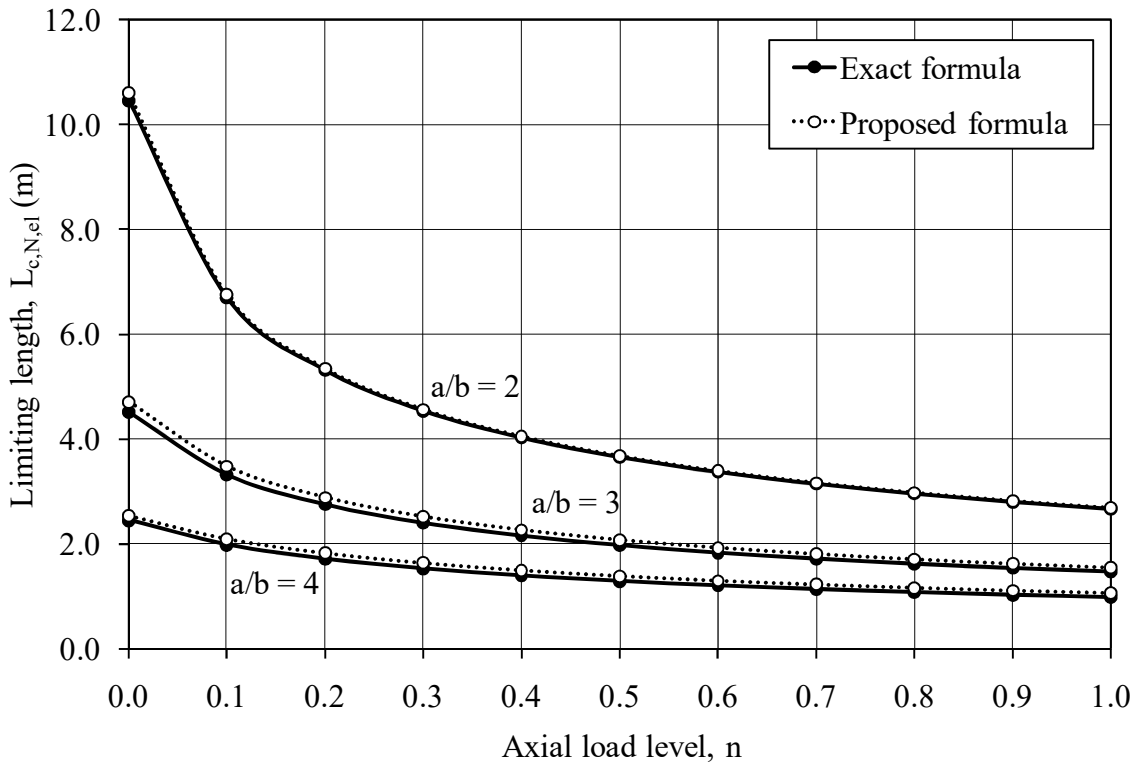


Figure 40: Limiting length versus axial load level with various aspect ratios for Class 3 cross-sections