

HORIZONTAL PENETRATION IN GRANULAR MEDIA: EFFECT OF INTRUDER SHAPE, DEPTH, ORIENTATION AND MATERIAL DENSITY ON PENETRATION FORCES

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ABSTRACT

Understanding the mechanics of horizontal penetration is fundamental for the development of new burrowing techniques for subsurface characterisation/monitoring, infrastructure construction, and the exploration of extreme environments. This contribution uses 3D discrete element simulations and 1-g physical model tests to study the effects of tip shape, intruder depth, soil density and tip orientation on the drag, lift, and lateral forces that develop during horizontal penetration. The intruder tips tested include the standard CPT tip and 3 tip morphologies optimised in a prior research study to reduce drag and/or lift forces .

The data generated reveal that using the optimised tips can reduce drag forces by up to 45% (compared to a standard cone penetration conical tip). The lift forces are depth-dependent, suggesting that a single intruder geometry cannot yield minimum lift for all depths. The penetration forces increase non-linearly with penetration depth, indicating a transition between shallow and deep failure mechanisms. The penetration forces increase with soil density due to the increase in the peak friction angle of the material. Tip rotation effectively changes the lateral/vertical forces during penetration. Still, continuous measurement and adaptation are needed to account for instrument compliance, soil variability and path deviations.

INTRODUCTION

Developing new underground exploration and characterisation methods is an active research topic in geotechnical engineering ([Zhang et al. 2023](#)). Such methods will benefit a wide range of applications, from infrastructure construction and monitoring to the exploration of extreme environments (including deep sea and extraterrestrial). There are also applications in soil science and agriculture ([Tao 2021](#)).

The cone penetration test (CPT), widely accepted as the industry-standard method for in-situ soil characterisation, consists of an instrumented conical probe that is vertically advanced into the ground from a surface-mounted testing rig. Recently, the industry has shifted towards construction and monitoring applications requiring continuous measurements over large distances or at considerable depths ([Broere and Van Tol 2003](#)). In response, researchers have conducted numerical ([Jiang et al. 2013](#)), laboratory ([Broere and Van Tol 2001](#)) and field ([Broere and Van Tol 2003](#)) studies to explore the use of horizontal and inclined CPT soundings for characterising soils around excavations and active tunneling operations. However, these studies primarily propose using horizontal CPT as a means to complement and verify findings from prior conventional soil characterisation, rather than as a tool for concurrent subsurface exploration and characterisation.

On the other hand, horizontal directional drilling (HDD) is the prevalent trenchless method for the installation of cables and pipelines with limited impacts on the surrounding structures. During HDD, a drilling rig, fixed at an angle between 8° and 16° from the horizontal, pushes a drill string

50 along a predefined path using pressurised drilling fluid to aid drilling, maintain borehole stability,
51 and transport drill cuttings back to the surface (Yan et al. 2018). High drilling fluid pressures can
52 result in hydrofracture (tensile failure), generalised shear failure of the soil around the borehole
53 (known as cavity blowout) (Xia and Moore 2006), and/or the formation of shear planes that may
54 reach the surface (Patino-Ramirez et al. 2021).

55 Burrowing techniques that do not require drilling fluid (or separation plants for slurry) avert
56 these failures, whilst potentially lowering the cost of drilling. In addition, these methods signifi-
57 cantly reduce soil disturbance around the burrowing probe, enabling the concurrent measurement
58 of soil properties during exploration. The reduced disturbance also allows the monitoring of ex-
59 isting structures (Pfändler 2023), and the measurement of mechanical, chemical or environmental
60 parameters. Moreover, the potential for untethered and autonomous operation of these burrowing
61 probes enables their use in extreme environments that require unmanned use. Such applications
62 include deep waters (e.g. offshore cable installation), disaster reconnaissance (e.g. contaminated
63 sites), extraterrestrial geotechnics, and agricultural applications.

64 The development of novel burrowing methods poses significant technical and technological
65 challenges. Technological challenges include real-time positioning, wireless communication from
66 the subsurface, miniaturisation of mechanical components and instrumentation, and reliable oper-
67 ation of the mechanical and electronic components in the hostile conditions of the subsurface. The
68 main (geo-) technical challenges are generating sufficient force to advance, the energy requirements
69 needed for burrowing and control of the exploration path (i.e. steerability) (Tao 2021).

70 To advance, the probe must overcome the drag force exerted by the surrounding soil; this
71 drag force acts in the direction opposite to the probe movement and is proportional to the rate of
72 energy exertion (work) during burrowing. Thus, minimising the drag forces reduces the thrust
73 force required and the energy consumption of the system. Autonomous burrowing systems cannot
74 rely on external rigs that generate thrust, instead, they must rely on anchoring systems responsible
75 for generating reaction forces for penetration, and may rely on internal batteries as a power source
76 during burrowing. Lower drag forces will ease the anchoring system requirements, while also

reducing energy consumption and hence maximising the exploration range of autonomous probes.

A number of strategies have been proposed to increase the available thrust force during horizontal burrowing. Constrained penetration can be achieved with rig-bound penetrometers, which hinder the autonomy of the methods, but leverage existing technologies used for CPT (Broere and Van Tol 2003; Wei et al. 2005), horizontal directional drilling, pipe jacking and/or micro-tunneling. Bio-inspired peristalsis (Borela et al. 2021; Das et al. 2023) and anchor-push mechanisms (Zhang et al. 2023; Tao et al. 2020; Naziri et al. 2024) alternate pressurisation/deformation in the axial and tangential directions of movement, mimicking the movement of earthworms and other organisms. Intruder/tip rotation or vibration (Jung et al. 2017; Tang et al. 2024) can reduce drag forces by inducing local disturbances ahead of the advancing intruder.

Steering forces are forces exerted by the soil orthogonal to the direction of movement and control the changes in the probe's orientation during burrowing. The vertical component of the steering force (parallel to the acceleration of gravity) is the granular lift force (granular buoyancy); previous studies (Potiguar and Ding 2013; Ding et al. 2011) have shown that symmetric intruders in granular media experience a net upwards force. Zhang and Fuentes (2022) demonstrated that granular lift develops due to the stress gradient induced by gravity, while Ding et al. (2011) and Patino-Ramirez and O'Sullivan (2023) showed that asymmetric intruder tips can neutralise and control lift forces. Lateral forces are the horizontal component of the steering force, orthogonal both to the direction of movement (horizontal in this study) and to the acceleration of gravity. Minimising lift and lateral forces enables the probe exploration course to be maintained during burrowing, while controlling their magnitude enables steering.

The prevalent steering method in HDD is based on the rotation of a drill string end bent at an angle of 0.5° , known as bent sub (Royal et al. 2010); with new methods like the EZ pilot system gaining popularity (Xinjun and Jing 2014). Additive manufacturing (3D printing) can be used to create a curved lining behind the tip of a soft intruder that can change the course of the intruder (Sadeghi et al. 2017). Mechanical systems to induce a force asymmetry around the intruder tip include a vibro-impact mechanism coupled with a rotating bevel tip head (Barenboim and Degani

2020) while [Naclerio et al. \(2021\)](#) used a slanted plate with a pressurised air jet to steer a soft robot.

This contribution studies the penetration forces (drag and steering) exerted on a cylindrical intruder during horizontal penetration in granular media at shallow depths. We use the discrete element model (DEM) and a 1-g physical model to study the influence of the intruder's tip shape, intruder depth and soil density on the penetration forces during horizontal burrowing. In addition, we explore the potential of a steering system based on the rotation of an asymmetric tip to control steering forces during exploration.

This study focuses on probe tip morphology and builds upon and complements many of these prior innovations. The paper firstly describes the 3D DEM model and simulation program which was designed to explore the effect of tip shape, and intruder depth. Then the experiments are outlined; the experimental plan was developed to explore the effect of tip shape and material density on the penetration forces. The numerical and experimental tests were carried out using similar intruder geometries, but different materials and testing conditions. The numerical and experimental studies are therefore combined to draw conclusions on the four main factors considered here, i.e. tip shape, intruder depth, material density and intruder tip orientation.

INTRUDER TIP SHAPES

The diameter of the intruders used in the numerical and experimental tests, $D_p = 40$ mm, is between that of the standard 10 cm^2 cone penetrometer test (CPT) cone (35.7 mm diameter), and the 15 cm^2 CPT cone, the second most popular CPT cone size (44 mm diameter).

The different intruder tips tested were proposed by [Patino-Ramirez and O'Sullivan \(2023\)](#) as the optimal shapes that minimise/balance penetration forces during horizontal penetration in a 2D virtual testbed comprising circular disks simulated using the discrete element method (DEM). The *MD* (minimum drag) tip minimises the drag force during penetration, the *ML* (minimum lift) tip minimises the granular lift force, and the *PO* (Pareto optimal) tip yields the best trade-off between drag and lift forces during penetration. In addition, A conical tip with the same apex angle (60°) as the standard cone penetrometer (CPT) tip is used as a baseline for comparison.

The optimal intruder tip shapes were proposed in 2D (XZ -plane in this study), and are described

by two super elliptical sections, one above the tip apex (Eq.1a) and one below the apex 1b.

$$\left| \frac{2x}{AR \cdot D_p} \right|^\eta + \left| \frac{z}{(1 - \zeta) \cdot D_p} \right|^\eta = 1 \quad \left(\frac{z}{D_p} > \zeta \right) \quad (1a)$$

$$\left| \frac{2x}{AR \cdot D_p} \right|^\eta + \left| \frac{z}{\zeta \cdot D_p} \right|^\eta = 1 \quad \left(\frac{z}{D_p} \leq \zeta \right) \quad (1b)$$

where ζ corresponds to the apex height, η is the tip curvature, and AR is the aspect ratio. The apex height ζ is the vertical distance (z -axis) from the bottom of the intruder to the tip apex, relative to D_p . The aspect ratio AR is the ratio between the tip length (x -axis) over D_p . The tip curvature η controls the curvature of the sides of the tip; $\eta = 1$ corresponds to straight sides, while $\eta < 1$ corresponds to concave tips, and $\eta > 1$ results in convex shapes. The *CPT* tip has shape parameters $AR = 0.87$, $\eta = 1$, $\zeta = 0.5$. The AR , η , and ζ parameters are schematically shown in 1a, and the values for the *MD*, *ML*, and *PO* morphologies obtained via the 2D simulation study (Patino-Ramirez and O'Sullivan 2023) are presented in Table 1.

Here, these 2D intruder geometries are extended to three-dimensional space, and attached to a cylindrical intruder body. Photographs of the side (XZ -plane) and top (XY -axis) intruder profiles are shown in Fig. 1b. The projection of the tip shape in the XY -plane is given as:

$$\left| \frac{2x}{AR \cdot D_p} \right|^\eta + \left| \frac{2y}{D_p} \right|^\eta = 1 \quad (2)$$

where AR and η are the same as in Eq. 1, and $\zeta = 0.5$ is fixed for a symmetrical profile. The shape of the cross-section of the tips in the YZ -plane is:

$$\left(\frac{2y}{Y(x)} \right)^2 + \left(\frac{2z}{Z(x)} \right)^2 = 1 \quad (0 \leq x \leq AR \cdot D_p) \quad (3)$$

where the curvature of the contour is set as $\eta = 2$ to create a smooth transition from the apex of the tip to the cylindrical body of the intruder, and $Y(x)$, $Z(x)$ are the bounds described by Eq. 2 and 1 (respectively).

NUMERICAL SIMULATIONS

The discrete element method (DEM) was used to simulate a horizontal probe (intruder) attached to a hollow cylinder penetrating a bed of granular material (see Fig. 2). These simulations examined the effect of tip morphology and penetration depth.

The model dimensions (see Table 2) were chosen to minimize boundary effects by reference to [Khosravi et al. \(2020\)](#) and recent studies on horizontal penetration using discrete elements ([Zhang and Fuentes 2022](#); [Patino-Ramirez and O’Sullivan 2023](#)). The length and width of the domain were set as $L_m = 18 \times D_p$ and $W_m = 13 \times D_p$ respectively.

Three intruder depths (H_p) measured from the free surface to the axis of the cylindrical intruder, were tested: $H_p = [5, 10, 15] \times D_p$. The distance from the bottom of the box to the centre of the intruder was set as $5.5 \times D_p$. During the simulations, the intruder moved horizontally in the positive x -direction at a constant speed $V_p = 25 \text{ mm s}^{-1}$ (consistent with standard CPT penetration rates) to a maximum penetration distance $X_p = 11 \times D_p$.

During penetration, the drag (x -axis) and lift (z -axis) forces acting on the tip (DF_{tip} and LF_{tip} respectively) and on the entire intruder (DF_{int} and LF_{int}) were recorded at a frequency of 500 Hz (every 0.05 mm of penetration). Particle and contact information were saved every $0.5 \times D_p$ (20 mm) of penetration. These forces are presented here normalised by the geostatic stress level at the depth of the intruder according to:

$$D_{tip} = \frac{DF_{tip}}{A_p \cdot \gamma \cdot H_p} \quad D_{int} = \frac{DF_{int}}{A_p \cdot \gamma \cdot H_p} \quad (4)$$

$$L_{tip} = \frac{LF_{tip}}{A_p \cdot \gamma \cdot H_p} \quad L_{int} = \frac{LF_{int}}{A_p \cdot \gamma \cdot H_p} \quad (5)$$

where $\gamma = 15.78 \text{ kN m}^{-3}$ is the unit weight of the material, $A_p = 1.26 \times 10^{-3} \text{ m}^2$ is the cross-sectional area of the intruder.

The DEM simulations were carried out using the granular package of the molecular dynamics software LAMMPS ([Thompson et al. 2022](#)). The interactions between the spherical particles were

calculated using the simplified Hertz-Mindlin (SHM) contact model (Itasca Consulting Group 2008). Material properties were selected according to the properties of quartz, with a shear modulus of $G = 29$ GPa, density $\rho_s = 2670 \text{ kg m}^{-3}$, and Poisson's ratio, $\nu = 0.2$ (Simmons and Brace 1965). A local damping coefficient $\eta = 0.1$ was used to reduce oscillations in the system.

The inter-particle friction coefficient ($\mu = 0.25$) and the (scaled) particle size distribution (PSD) of the particles follow those of Toyoura sand, a fine, rounded sand with mean particle size $d_{50} = 0.22$ mm, coefficient of curvature $C_c = 0.96$ and coefficient of uniformity $C_u = 1.39$ (Yang and Sze 2011). To reduce the computational cost, the PSD of Toyoura sand was scaled by multiplying the particle diameters by a factor of $k = 33.1$, to reach an intruder to particle size ratio $D_p/d_{50} = 5.5$ and a (scaled) mean particle diameter $d_{50} = 7.2$ mm, classified as a poorly graded gravel (GP) according to the USCS.

Scaling the PSD is a necessary measure to reduce the number of particles (N_p) in the system and achieve a tractable computational time. The number of particles in the system is proportional to the cube of the intruder to particle size ratio ($N_p \propto \left(\frac{D_p}{d_{50}}\right)^3$), therefore, increasing this ratio from 5.5 to 7 or 10, would increase N_p by 2 and 6 times (respectively).

The intruder to particle size ratio in this study is within the range of values reported by (Khosravi et al. 2020) for (vertical) CPT penetration studies using DEM. According to the guidance on particle size effects in soil-structure interaction proposed by Caicedo (2018), the ratio between intruder diameter and the maximum particle diameter in this study ($D_p/d_{max} = 4$), may result in a coefficient of variation on the stresses (vs. a continuum) lower than 20%. In addition, Arroyo et al. (2011) found that particle size effects lead to an increase in the observed fluctuations in forces during (vertical) CPT penetration; caused by the lower number of particles in contact with the probe. Still, the actual extent and magnitude of these particle size effects may be problem dependent, and is a limitation of this numerical study.

The critical timestep in the DEM simulations (Δt_{crit}) was calculated following (Otsubo et al. 2017) as:

$$\Delta t_{crit} = \min \left(2\sqrt{\frac{m_{min}}{8k_N + 16k_T}}, 2\sqrt{\frac{m_{min}}{40k_T}} \right) \quad (6)$$

where m_{min} is the minimum particle mass (from the minimum particle diameter $d_{min} = 2$ mm), and the normal (k_N) and tangential (k_T) contact stiffness values were calculated for the SHM model as $k_N = \frac{G\sqrt{d_{min}\delta}}{1-\nu}$, and $k_T = k_N \frac{2(1-\nu)}{2-\nu}$, assuming a maximum overlap $\delta = 5\% \times d_{min}$. Taking ($\Delta t_{crit} = 6.02 \times 10^{-7}$ s) the time step used was $\Delta t = 6 \times 10^{-7}$ s.

The domain boundaries were rigid and fixed, with a wall-particle friction coefficient ($\mu = 0.25$). Samples were prepared from a cloud of random particles that were allowed to settle under gravity until the total kinetic energy in the system was negligible ($\leq 1 \times 10^{-8}$ J). Table 3 lists the model height, Froude number (Fr) and number of particles for the penetration depths considered. The number of particles varied with H_p (Fig. 2, Table 2). After deposition, the samples had a void ratio $e = 0.435$ and a unit weight $\gamma = 15.78$ kN m⁻³.

The intruders were modelled as rigid bodies, formed by arranging spherical particles equidistantly along the surface of the intruder (intruder particles herein). The intruder particles had a diameter $d_I = 1.8$ mm, and a centre-to-centre distance $x_I = 0.5$ mm. At this level of overlap ($x_I/d_I = 0.27$), the geometrical texture of the intruders showed no influence on the penetration forces. A friction coefficient $\mu = 0.25$ was used for the contacts between intruder and soil particles. The intruder particles moved as a rigid body at a constant speed, with no interactions calculated between them, or between the intruder particles and the boundary walls. Therefore, the model walls appeared ‘invisible’ to the intruder as it entered the granular bed. The simulations were in the quasi-static regime, with Froude numbers $Fr \ll 1$. Fr were calculated from (Seguin 2022) as $Fr = V_p / \sqrt{g \cdot H_p}$, where V_p , g and H_p are the penetration speed, acceleration of gravity and the depth of the intruder (respectively). Each of the four tips considered was tested at depths $H_p = [5, 10, 15] \times D_p$. Four additional DEM simulations at an intruder depth $H_p = 5 \times D_p$ were performed by rotating the *PO* tip at $[45, 90, 135, 180]^\circ$ around the axis of the probe. Thus there were a total of 16 DEM simulations.

EXPERIMENTAL CAMPAIGN

In the 1-g physical model tests we measured the penetration forces during horizontal penetration at a fixed intruder depth (5 times the probe diameter) for 4 different tip shapes (CPT, *PO*, *MD*, *ML*), and two soil densities.

The experimental setup shown in Fig. 3 comprises three components: the testbed, the pluviation system, and the penetration frame. The testbed is a standard plastic pallet box with nominal dimensions $1\text{ m} \times 1.2\text{ m} \times 0.6\text{ m}$ (width, length, height). The internal length of the testbed is reduced with a barrier to reduce the mass of sand which required handling without affecting the experimental results. The dimensions of the sample (width W_E , length L_E and height H_E) are shown in Fig. 3 and summarised in Table 4.

Material characterisation and specimen preparation

Samples were prepared with fuse sand 80/20, a uniform, sub-angular and sub-rounded sand (SP according to the USCS); with size distribution parameters: median particle size $d_{50} = 0.54\text{ mm}$, coefficient of uniformity $C_u = 1.26$, coefficient of curvature $C_c = 0.99$, shape parameters: median sphericity 0.894, median convexity 0.955, and sub-angular classification according to [Altuhafi et al. \(2013\)](#). The size and shape parameters of the material and the size distribution in Fig. 4 were obtained with QICPIC, a laser diffraction system. Given the siliceous composition of the material, a specific gravity $G_s = 2.65$ was assumed for the calculations of void ratio. The hygroscopic water content of the material was measured before and after every test, with values $< 0.03\%$. The minimum density of the material $\rho_{min} = 1.46 \times 10^3\text{ kg m}^{-3}$ was calculated following method A from ASTM D4254-16 ([ASTM 2016](#)). The maximum density $\rho_{max} = 1.70 \times 10^3\text{ kg m}^{-3}$ was calculated using the air pluviation method proposed by [Lo Presti et al. \(1992\)](#).

The samples were prepared by air pluviation, using a frame similar to that proposed by [Gade et al. \(2017\)](#). Aluminium extrusions were used to build a prismatic frame that was fitted over the testbed. Rolling gantry plates were attached to the rails to allow the movement of a 3.2 litre funnel along the x and y directions (see Figure 3). Two telescopic perspex tubes were fitted to the funnel; the top perspex tube was directly fixed to the funnel, with internal and external diameters $ID = 30\text{ mm}$

and $OD = 38$ mm (respectively). The lower perspex tube ($ID = 42$ mm and $OD = 50$ mm) made it possible to adjust the height of fall during the pluviation. The height of fall (h_f) during pluviation was monitored with the help of a metal rod attached to the outside wall of the lower tube.

Five tight-fitting diffuser plates were placed inside the lower tube to guarantee a homogeneous distribution of the sand during pluviation. Each diffuser plate had 12 evenly distributed holes with a diameter of 8 mm. Diffusers were placed from the bottom of the lower tube and spaced every 25 mm (see Fig. 3). To calibrate the pluviation system a cylindrical container with a height 0.2 m, and a diameter of 0.36 m was filled with fuse sand 80/20 using the system. In this calibration process there were 4 specimens for each of 6 different heights of fall ($[0, 50, 100, 150, 200, 250]$ mm). Figure 5 shows the relation between height of fall h_f , and unit weight γ (5a), relative density DR (5b), and void ratio e (5c); dashed lines in Figure 5 show the quadratic relationships fitted using the calibration data.

During pluviation for the physical model tests, sand was fed into the funnel and deposited with continuous movements across the testbed. When needed, the height of the lower telescopic tube was adjusted to maintain the desired height of fall. Approximately 500 kg of sand and ~ 4 h were required to prepare each sample. Two packing densities were studied; ‘dense’ samples ($DR = 80.1\%$) are prepared with $h_f = 200$ mm, and medium-dense (referred to as ‘medium’ herein) samples ($DR = 52.0\%$) are prepared with $h_f = 60$ mm. During every test, two cylindrical containers (volume ~ 300 cm³) were placed at the same elevation of the intruder, and filled during pluviation. After testing, these containers were carefully removed and weighed, and the obtained density measurements are summarized as boxplots in Figure 5. The packing properties of the sand and tests are summarized in Table 5.

Penetration setup

A penetration system drove the cylindrical intruder horizontally into the testbed (in the x -direction, see Figure 3), at a penetration depth $H_p = 0.2$ m $= 5 \times D_p$. The penetration system was made from aluminium extrusions, and fitted with an electrical linear actuator with a stroke $X_p = 0.3$ m $= 7.5 \times D_p$, in line with an S-type load cell with a maximum capacity of 100 kg. The

actuator and external load cell were fixed to an aluminium tube with $OD = 40$ mm (matching the diameter of the intruder in Section 3) and total length 0.45 m, as the body of the probe. A multi-axis force sensor (Nano25 – ATI industrial automation) was fixed inside the free end of the aluminium tube. The intruder tips were 3D-printed following the geometries listed in Table 1.

Two linear bearings restricted the movement of the probe to the horizontal (x -axis) direction. A 3D-printed flanged bearing was fitted to the side of the testbed to guide the probe whilst preventing sand from escaping the testbed. Before pluviation (with an empty box), the intruder tip was fixed to the free end of the intruder, and the initial location of the tip of the intruder was set to one intruder diameter (40 mm) outside the inner wall of the testbed. During penetration, the actuator pushed the intruder at a constant speed of 4 mm s^{-1} (quasi-static regime $Fr \ll 1$), and the displacement (from the linear actuator), external load cell force, and multi-axis force sensor readings (axial, lateral and vertical) are recorded at a frequency of 20 Hz. Before and after each test, ‘empty-box’ tests were performed to verify the alignment of the setup and measure the baseline drag force generated by the bearings that guide the probe. Readings from the load cell were corrected by subtracting 20 N, a consistent average of the baseline drag forces.

Similar to the numerical study, four different tip orientations were tested experimentally, rotating the PO tip at $[0, 45, 90, 135]^\circ$ around the axis of the probe. The 180° orientation could not be tested due to logistic limitations in the testing program.

SYNTHESIS OF RESULTS

The combined experimental and numerical datasets generated here enabled the exploration of the influence of four different parameters on the resulting penetration forces exerted on an intruder during horizontal penetration in granular media.

Overview of data

DEM data

Figure 6 shows the tip (D_{tip}) and intruder (D_{int}) drag forces as a function of penetration distance X_p for the tip shapes and intruder depths tested. The initial portion of the response, $X_p < 2.5 \times D_p$, is affected by the presence of the container wall. Beyond this region, D_{tip} remains relatively

constant, while D_{int} increases linearly, proportional to the area of the intruder's body in contact with the media. Figure 7 shows the tip (L_{tip}) and intruder (L_{int}) lift forces as a function X_p for the tips and depths considered. The magnitude of the lift forces is significantly lower to the drag forces, and lift forces (L_{tip} and L_{int}) remaining relatively constant with no evident trend in the variation of either L_{tip} and L_{int} with X_p .

Experimental data

During the experiments the total intruder drag force (D_{tip}) was measured using the external load cell (corrected by baseline drag), the drag (D_{tip}) and lift (L_{tip}) forces at the tip were measured from the x and z force readings from the multi-axis force sensor (respectively), the drag force along the intruder shaft (D_{shaft}) was calculated as $D_{int} - D_{tip}$. Penetration forces are normalised by the stress level at the intruder's axis according to Eq. (5).

Figures 8 and 9 show the total drag (a), tip drag (b), shaft drag (c) and tip lift (d) forces, for the medium and dense samples (respectively). Data for the first 1.5 to $2 \times D_p$ of penetration distance were affected by the wall of the testbed, which induced a large increase in drag forces, and an increase followed by a sharp decrease in lift forces. The experimental penetration forces are significantly more steady than those in the discrete element simulations - attributed to the larger number of particles in contact with the intruder ($D_p/d_{50} = 74.1$). As a result, the shaded regions that show the variability of the penetration forces are nearly imperceptible.

To compare tip shapes, penetration forces are averaged over the range $X_p = [2.5, 11] \times D_p$. Figure 10 shows the average intruder drag force $\overline{D_{int}}$ (a) and tip lift force $\overline{L_{tip}}$ (b) for the different tips and densities tested. Values over each bar show the relative change in forces vs. the CPT intruder.

Tip Shape

The effect of tip shape was studied both numerically and experimentally. First, the performance of the different tip shapes was compared from the DEM simulations using the average intruder drag $\overline{D_{int}}$ and tip lift $\overline{L_{tip}}$ forces were calculated over the interval $5D_p < X_p < 10D_p$. Figure 11 shows the average value (height of bar) and indicates the forces for each of the new morphologies relative

to the CPT intruder values (percentage over each bar).

Considering the drag forces, the minimum drag (MD) intruder exhibits the best performance, reducing total drag force by between 8.6% and 12.1% (relative to the CPT). MD is followed by PO (5.6% to 9.1% reduction). The minimum lift (ML) intruder performs similarly to the CPT (2.9% reduction to 2.4% increase), with the CPT showing the highest drag forces overall.

The experimental data confirm that tip shape has a significant impact on the drag forces. The minimum drag (MD) tip yields the lowest drag force, which is 33.4% (medium-dense sand) to 45.3% (dense sand) lower than those of the CPT intruder. The Pareto Optimal tip (PO) is the second tip shape with the lowest drag forces, followed by ML. The CPT intruder had the highest overall drag force.

The drag forces at the tip account for between 89 and 94% of the total drag. Shaft drag (in Figures 8c and 9c) exhibits a linear increase with penetration distance, attributed to the increase in shaft area in contact with the soil. Still, the interface friction between the body of the intruder and the soil is affected by the intruder's tip shape, with the CPT and ML tips exhibiting the highest and lowest interface friction (respectively).

The fact that the optimised intruder tip shapes (MD and PO) reduce the drag forces during penetration, confirms the relevance of the 2D numerical results in [Patino-Ramirez and O'Sullivan \(2023\)](#) to 3D physically realistic systems.

The effect of tip shape is substantially more pronounced in the case of the lift forces, determining both the magnitude and the direction of the vertical forces. For the shallowest depth tested ($5 \times D_p$), the three optimal shapes significantly decrease the lift forces of the CPT, with PO (−88.1%) and ML (−75.7%) achieving the largest reductions. These results are consistent with the (2D) results from [Patino-Ramirez and O'Sullivan \(2023\)](#), as their tip shape optimisation was done for the same depth of penetration.

Similarly, in terms of lift force at a depth of $5 \times D_p$, the ML and PO tips display the lowest lift forces (in magnitude) in the numerical (reductions between 75.7% to 88.1% vs. CPT) and physical (reductions between 66.1% to 97.6% vs. CPT) tests. Interestingly, the PO tip yields lower lift

forces than the ML tip in some numerical ($5, 10 \times D_p$) and physical (medium dense sand) tests.

The experimental data confirm that the lift forces are significantly affected by tip shape. The ML and PO tips significantly reduce lift forces compared to the CPT, reducing their magnitude between 66.1% and 97.6% (see Figure 10b). As the lift forces for ML and PO are close to neutral, they oscillate between positive (upwards) and negative (downwards) during penetration, with PO showing mostly positive lift, albeit of relatively small magnitude. The high degree of asymmetry of MD leads to negative lift forces (i.e. in the downward direction) for all the depths tested.

Density

Material density plays an important role in the behaviour of granular media. Two different density levels were tested with the same experimental setup, the ‘medium’ ($DR = 52\%$) and ‘dense’ ($DR = 80.1\%$) samples as described above.

Drag forces significantly increase with soil density, with the average intruder drag (see Fig. 10a) in ‘dense’ samples being 38.5%, 35.5%, 43.0% and 15.6% higher than that of ‘medium’ samples for the CPT, PO, ML and MD (respectively). Interestingly, the smallest difference between different densities is observed for the MD tip which yields the lowest drag forces overall.

In terms of lift, increasing soil density results in a (positive) increase in lift forces. This difference is more pronounced for the CPT tip. A possible explanation for this behaviour is the higher unit weight of the soil in dense conditions, which increases the difference in geostatic stress between the top and the bottom of the intruder. As soil density ‘adds’ lift force, for tip shapes that yield negative lift forces (ML and MD), soil density decreases the magnitude of the lift force, whereas the same change in density increases the magnitude of the lift force for the CPT and PO tips (see Figure 10b). Similar to the drag force data, the CPT tip is the most sensitive to increases in density, while the tips with the best performance (ML and PO) are the least sensitive to the same increase.

The relationship between soil density and (vertical) cone penetration resistance is well-known (Jamiolkowski et al. 2003), dating back to Schmertmann (1976) who proposed the following correlation between cone resistance, relative density, and effective overburden stress for normally

consolidated, fine to medium, clean sands (such as the Fuse sand in this study): $q_c = C_0 \cdot (\sigma'_{vo})^{C_1} \cdot \exp(C_2 \cdot DR)$. Where q_c is the CPT tip resistance, C_0 , C_1 and C_2 are empirical correlation factors, and σ'_{vo} and DR , are the vertical effective overburden stress and relative density (respectively). Similar expressions have been proposed between the lateral stress index (from flat dilatometer tests) and relative density (Jamiolkowski et al. 2003; Garizio 1997).

More recent studies have proposed the use of the state parameter as a more meaningful parameter to capture changes in in-situ density and CPT penetration resistance (Robertson 2010; Jefferies and Been 2015). Lower state parameters (i.e. higher densities) lead to a direct increase in peak friction angle, and with it, an increase in cone penetration resistance (Bolton 1986; Schmertmann 1978). The results in this section are in agreement with these findings, showing that the increase in penetration forces with density is driven by the increase in the peak friction angle of the soil. Moreover, these results suggest that the effect of tip shape, density and depth in horizontal penetration forces cannot be decoupled, as they form a complex relationship that also depends on overburden stress, shear resistance of the soil, and changes in the failure mechanism.

Penetration depth

The influence of intruder depth was studied with DEM simulations, testing three intruder depths (H_p), measured from the free surface to the axis of the cylindrical intruder: $H_p = [5, 10, 15] \times D_p$. The drag forces increase with depth (see Fig. 12), suggesting an increased resistance from the soil, which may be related to a change in the failure mechanism around the probe, with a transition from shallow to deep conditions. Even though there are only three data points for each tip shape it is clear that the increase in drag force with depth is non-linear. Penetration forces are normalised by the stress at the level of penetration according to Eq. 5 and so the variation observed effect is not attributable to the increase in geostatic stress with depth. Similar behaviour has been observed during cavity expansion (e.g. during horizontal directional drilling), with a transition from shallow conditions controlled by shear planes extending from the cavity to the surface (Patino-Ramirez et al. 2021) to deep conditions, controlled by a concentric region of plastic deformation (Xia and Moore 2006). As the drag forces vs. penetration depth (from Fig. 12a) has not reached a plateau,

it can be inferred that the simulations in this study fall within the shallow-to-intermediate range, with full deep conditions not yet developed.

Lift forces develop due to the imbalance of forces between the vertical forces acting on the ‘top’ and ‘bottom’ of the intruder due to gravity (i.e. geostatic stress gradient) and intruder shape (Zhang and Fuentes 2022; Ding et al. 2011). In Fig. 12b normalised lift forces decrease with depth, changing their magnitude and direction. Tip shape affects the offset and (marginally) the rate of change of lift forces with depth. As a result, the ML and PO tips significantly reduce lift for an intruder depth of $5 \times D_p$ - depth used for the tip shape optimisation in Patino-Ramirez and O’Sullivan (2023). However, as lift forces continue to decrease with depth, the asymmetric optimal tips ‘overcorrect’ lift, resulting in a downward lift force during penetration. Similar to drag forces, the lift force is expected to stabilise in deep conditions - which has not yet been achieved within the depth range in this study ($H_p \leq 15 \times D_p$).

Effect of tip rotation

The effect of tip rotation was studied expanding on the experimental and numerical results published in (Patino-Ramirez et al. 2024). Five DEM simulations at an intruder depth $H_p = 5 \times D_p$ were performed rotating the PO tip at $[0, 45, 90, 135, 180]^\circ$ around the axis of the probe. Figure 13 shows the penetration forces at the tip, namely drag (D_{tip}), lift (L_{tip}) and lateral (H_{tip}) forces as a function of penetration distance X_p .

The drag forces increased monotonically with tip orientation, with the 180° orientation showing a 18% increase in D_{tip} compared to the 0° orientation. The lift forces (L_{tip}) also increased monotonically with orientation, from nearly neutral (0° orientation), to markedly positive as the apex of the tip moves towards the free surface with increasing tip rotations. Lateral forces (H_{tip}) oscillate from (nearly) neutral (0°) to reach a maximum at 90° orientation, before coming back down to nearly neutral at 180° when the symmetry around the XZ-plane is recovered. Beyond the zone of influence of the left boundary ($X_p = [0, 3] \times D_p$), steering forces remain relatively constant with X_p .

The average tip lift ($\overline{L_{tip}}$) and lateral ($\overline{H_{tip}}$) forces are calculated for the steady state range

$X_p = [5, 10] \times D_p$, and plotted against each other in Figure 14. The relationship between steering forces and tip orientation can be characterised by a circle, with the magnitude of the maximum lift force (achieved at 180° orientation) corresponding to about twice the magnitude of the maximum lateral force (achieved at 90° orientation).

Experimental results

The influence of tip orientation was explored both numerically and experimentally. Figure 15 shows the tip drag (D_{tip}), lift (L_{tip}) and lateral (H_{tip}) forces as a function of penetration distance X_p .

The experimental drag forces show a different trend than the numerical results, with the 45° and 90° orientations displaying lower drag forces than the 0° and 135° . Lift forces increase with tip orientation, with similar forces observed for the 0° and 45° orientations, and a large increase for the 135° angle. Lateral forces increase sharply with tip orientation, from (nearly) neutral to maximum for the 135° orientation.

The average tip lift ($\overline{L_{tip}}$) and lateral ($\overline{H_{tip}}$) forces are calculated for the range $X_p = [2.5, 7.5] \times D_p$, and plotted against each other in Figure 16. The circular relationship between steering forces and tip orientation between the experimental results and the DEM simulations is not reproduced in this plot of the experimental data - mostly due to the results at the 135° orientation, which show large values of $\overline{L_{tip}}$ and $\overline{H_{tip}}$. These results break the symmetry of the circular envelope obtained from the DEM simulations and suggest that variables like probe deformation and deviations from a purely horizontal path (not present in the numerical simulations) can significantly affect steering forces. Additional factors may include system compliance, intruder deformation, and spatial variability of the soils in-situ have a significant impact on the steering forces.

Reflection on combining numerical and experimental results

The physical experiments show significantly larger penetration forces than the DEM simulation results. We attribute this to the difference in shear strength between fuse sand and the spherical particles used in the DEM models. Fuse sand is sub-angular and sub-rounded (median sphericity 0.894, median convexity 0.955), whilst the DEM models used spherical particles with no rolling

resistance. Particle shape is recognised as a major contributor to the dilatancy and shear resistance of granular materials (Estrada et al. 2011; Bolton 1986). The difference between the DEM and physical materials amplified the effect of intruder tip shape in penetration forces, with optimal shapes (MD and PO) in physical experiments achieving significantly larger reductions in drag forces (vs. CPT). This finding highlights the potential of numerically-optimised intruders in physical applications.

This study (3-dimensional DEM and 1-g physical experiments) developed from an earlier 2-dimensional DEM study by Patino-Ramirez and O’Sullivan (2023) that proposed the intruder tips tested. 2D DEM models are computationally cheaper and the mechanisms can be more easily visualized in comparison with 3D models. However, 2D models cannot capture key physical phenomena, i.e. arching orthogonal to the probe and the related stress shielding effect which emerges as vertical stresses cannot re-distribute around the probe. It is therefore important to quantify the difference in performance between the 2D and 3D analyses to appreciate the applicability of 2D models to physical systems. Table 6 compares the drag ($\overline{D_{tip}}$) and lift ($\overline{L_{tip}}$) forces relative to the CPT for the two sets of analyses and experimental methods. As noted above, the overall trends are the same, i.e. the MD and ML topologies give the minimum drag and lift forces respectively in all cases considered. However the quantitative gains in performance differ. Generally, 2D simulations overestimate the drag performance relative to the 3D DEM; arguably because only the XZ-plane profile of the tips has been optimised, disregarding the out-of-plane stress distribution in the 3D domains, which significantly changes the failure mechanism, and the resulting drag force during burrowing. Lift performance (vs. CPT) is underestimated in 2D relative to the 3D DEM data.

This study also highlights the limitations of numerical and 1-g models in the development of geotechnical instruments. Numerical modelling allows to run a large number of tests, modify the testing setup, and increase the physical size of models with relative ease. However, numerical models are limited by computational cost, which may lead to particle size (and shape) and border effects, and hinder their ability to assess the impact of instrument compliance, and the spatial and intrinsic variability of soils in-situ.

On the other hand, 1-g experiments are limited by physical considerations such as time, weight

and physical space needed to manage large volumes of sand. Additionally, physical testing must consider the viability of the miniaturisation, manufacturing, instrumentation and operation of instruments, considerations that may be overlooked in numerical or theoretical analyses.

The differences between the numerical and experimental data highlight that potential steering systems need adaptive systems that can respond in real-time to changes in forces/direction.

CONCLUSIONS

This study has considered the influence of tip shape, intruder depth, soil density and intruder tip orientation on the penetration forces exerted on a cylindrical intruder penetrating horizontally in granular media. Discrete element models were used to study the effect of tip shape, penetration depth, and tip orientation, while 1-g physical experiments were used to study the influence of tip shape, soil density, and tip orientation. Data from the experimental and numerical studies were unified to develop the following main conclusions of the study:

- Penetration forces (drag, lift and lateral forces) show a complex relationship that depends on intruder shape/orientation, penetration depth and soil density; leading to changes in overburden stress, shear resistance of the soil and changes in failure mechanism.
- Drag and lift forces increase non-linearly with depth, indicating a transition from a shallow to a deep failure mechanism. For the testing conditions presented, full deep conditions are not yet developed at a depth of $15 \times D_p$ (D_p is the intruder diameter).
- The MD (minimum drag) and PO (Pareto optimal) tip shapes identified in 2D DEM simulations effectively reduce drag forces compared to the CPT conical tip. The relative reduction of drag forces is significantly higher in physical tests than in DEM simulations, with the MD and PO reducing drag by up to 45.3% and 25.3% in 1-g tests versus 12.1% and 9.1% in DEM simulations (respectively). In comparison with the 3D DEM dataset, the 2D DEM simulations overestimated the drag performance.
- Lift forces decrease with increasing penetration depth, with tip shape and material density controlling the ‘offset’ or initial value of lift. As a consequence, the ML and PO tips,

identified in the 2D DEM study, effectively reduce the lift forces at the target depth of $5 \times D_p$, but this reduction vanishes at different intruder depths.

- Two density levels ($DR = 52\%$ and $DR = 80.1\%$) were tested experimentally using a bespoke moving pluviometer. At the higher density, drag forces increase by 15.6% to 43.0% and lift forces increase (in the positive direction). The increase in penetration forces with density is driven by the reduction of the (initial) state parameter of the soil and the corresponding increase in the peak friction angle.
- Numerical and experimental tests at different tip orientations were carried out to evaluate the potential of a steering system based on adaptive tip rotation. Results demonstrate that tip orientation can change the steering (vertical and lateral) forces during penetration. Still, 1-g tests show that soil variability, instrument compliance and path deviations significantly affect the relationship between tip angle and steering forces.

In this study, numerical modelling and experimental testing are used to complement each other, circumventing the limitations of each method, and leveraging their advantages. We believe this framework combining computational optimisation, numerical modelling and physical testing can be used in other geotechnical (and non-geotechnical) applications.

Future directions of work include the study of the failure mechanism and transition between shallow and deep conditions during horizontal burrowing. From there, we envision to propose equations to estimate penetration forces. Moreover, future work may focus on the development of active steering systems that react in real-time to changes in forces and/or direction of moving probes during exploration.

Data Availability Statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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NOTATION

The following symbols are used in this paper:

D_p = Intruder diameter [m]

η = Tip curvature parameter [-]

ζ = Tip apex height [-]

AR = Tip aspect ratio [-]

L_m = Length of DEM model [m]

W_m = Width of DEM model [m]

H_m = Height of DEM model [m]

L_E = Length of 1-g testbed [m]

W_E = Width of 1-g testbed [m]

H_E = Height of 1-g testbed [m]

H_p = Intruder depth [m]

X_p = Penetration distance [m]

V_p = Penetration speed [mm/s]

Fr = Froude Number [-]

DF_{tip} = Tip drag force [N]

DF_{int} = Intruder drag force [N]

LF_{tip} = Tip lift force [N]

LF_{int}	=	Intruder lift force [N]
D_{tip}	=	Normalised tip drag force [-]
D_{int}	=	Normalised intruder drag force [-]
D_{shaft}	=	Normalised shaft drag force [-]
L_{tip}	=	Normalised tip lift force [-]
L_{int}	=	Normalised intruder lift force [-]
H_{tip}	=	Normalised lateral tip force [-]
DR	=	Relative density [%]
h_f	=	Pluviation height of fall [mm]
$\overline{D_{tip}}$	=	Average normalised tip drag [-]
$\overline{D_{int}}$	=	Average normalised intruder drag [-]
$\overline{L_{tip}}$	=	Average normalised tip lift [-]
$\overline{L_{int}}$	=	Average normalised intruder lift [-]
$\overline{H_{int}}$	=	Average normalised tip lateral force [-]

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Tip profile	ζ	η	AR
<i>CPT</i> (conical)	0.5	1	0.87
Minimum Drag (<i>MD</i>)	0.16	1.07	1.65
Minimum Lift (<i>ML</i>)	0.13	2.56	0.80
Pareto Optimal (<i>PO</i>)	0.23	1.18	1.29

TABLE 1. Shape parameters for the different tip profiles studied, ζ is the apex height, η is the curvature and AR is the aspect ratio.

Dimension	Symbol	Absolute [m]	Normalised by D_p [-]
Median particle size	d_{50}	7.2×10^{-3}	1/5
Model length	L_m	0.72	18
Model width	W_m	0.52	13
Model height	H_m	0.42, 0.62 or 0.82	10.5, 15.5 or 20.5
Intruder depth	H_p	0.2, 0.4 or 0.6	5, 10 or 15
Penetration distance	X_p	0.44	11

TABLE 2. Dimensions of the 3D DEM models.

Quantity	Symbol	$5 \times D_p$	$10 \times D_p$	$15 \times D_p$
Intruder depth	$H_p [m]$	0.2	0.4	0.6
Model height	$H_m [m]$	0.42	0.62	0.82
Number of particles	$N_p [-]$	863,212	1,243,925	1,658,336
Froude number	$Fr [-]$	17.8×10^{-3}	12.6×10^{-3}	10.3×10^{-3}

TABLE 3. Dimensions, number of particles and Froude number for the DEM models at different intruder depths.

Dimension	Symbol	Absolute [m]	Normalised by D_p [-]
Median particle size	d_{50}	0.54×10^{-3}	1/74.1
Sample width	W_E	0.97	24.25
Sample length	L_E	0.82	20.5
Sample height	H_E	0.42	10.5
Intruder depth	H_p	0.2	5
Penetration distance	X_p	0.3	7.5

TABLE 4. Dimensions of the 1-g experimental testbed.

Fuse sand packing	Density (ρ) [kg m^{-3}]	Relative Density (DR) [%]	Unit weight (γ) [kN m^{-3}]	Void ratio (e) [-]
Minimum density	(ρ_{min}) 1.46×10^3	0	14.33	(e_{max}) 0.81
Maximum density	(ρ_{max}) 1.70×10^3	100	16.71	(e_{min}) 0.56
Medium samples ($hf = 60mm$)	1.59×10^3	52.0	15.57	0.67
Dense samples ($hf = 200mm$)	1.65×10^3	80.1	16.24	0.60

TABLE 5. Packing density and void ratio for fuse sand 80/20. Average values across tests reported for the medium-dense and dense samples.

	Drag force ($\overline{D_{tip}}$)				Lift force ($\overline{L_{tip}}$)			
	2D DEM	3D DEM	Medium (Exp.)	Dense (Exp.)	2D DEM	3D DEM	Medium (Exp.)	Dense (Exp.)
PO	-9.8%	-2.3%	-25.3%	-25.0%	-12.0%	-98.1%	-16.6%	-62.1%
ML	-0.3%	+5.8%	-20.2%	-12.5%	-44.4%	-50.7%	-66.1%	-97.6%
MD	-12.5%	-5.4%	-33.4%	-45.3%	+44.7%	+19.4%	-85.4%	-75%

TABLE 6. Performance of the optimised tip shapes in terms of the average normalised tip drag ($\overline{D_{tip}}$) and lift ($\overline{L_{tip}}$) forces relative to CPT. Values shown for the 2D simulations performed by [Patino-Ramirez and O’Sullivan \(2023\)](#), and the 3D simulations and physical tests (in medium and dense samples) in this study. All values for an intruder depth $H_p = 5 \times D_p$.

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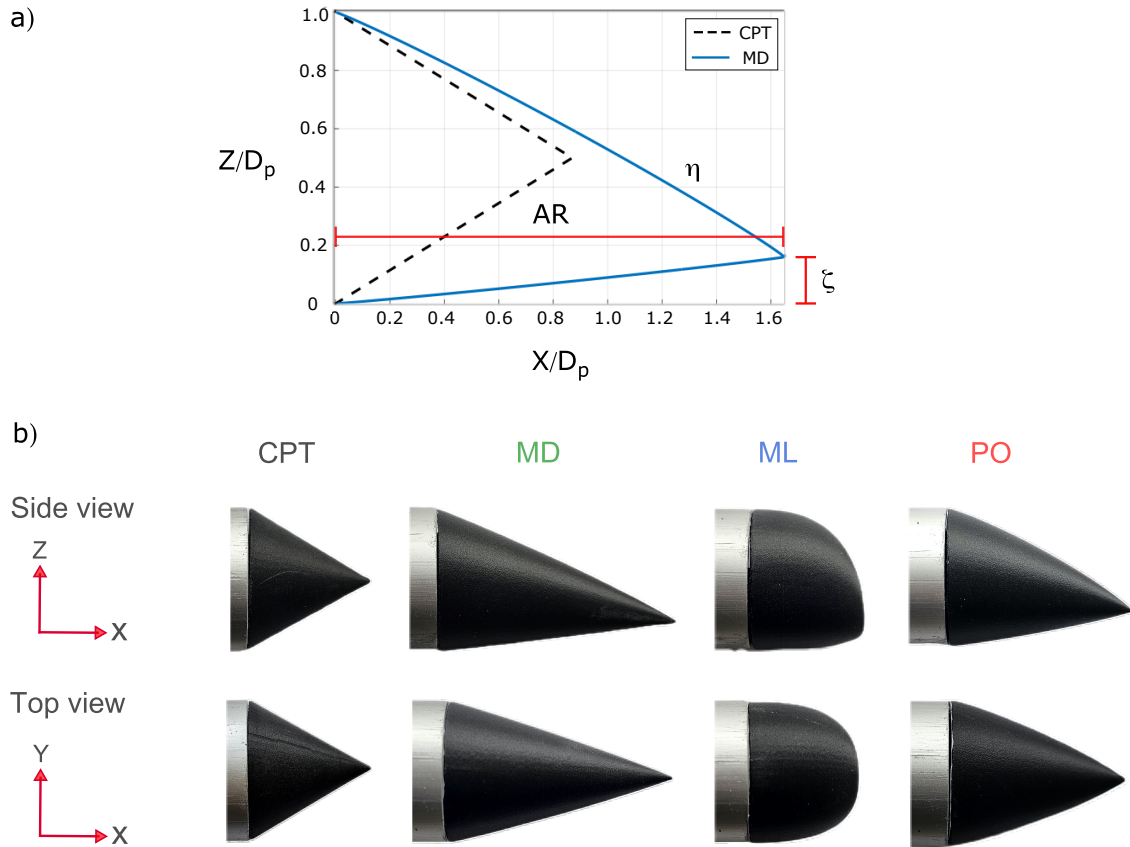


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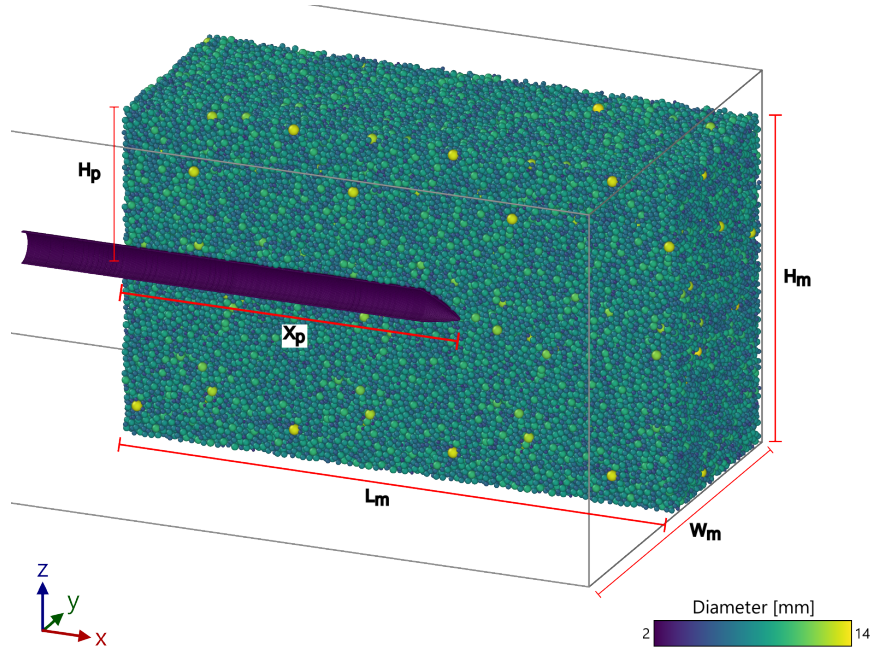


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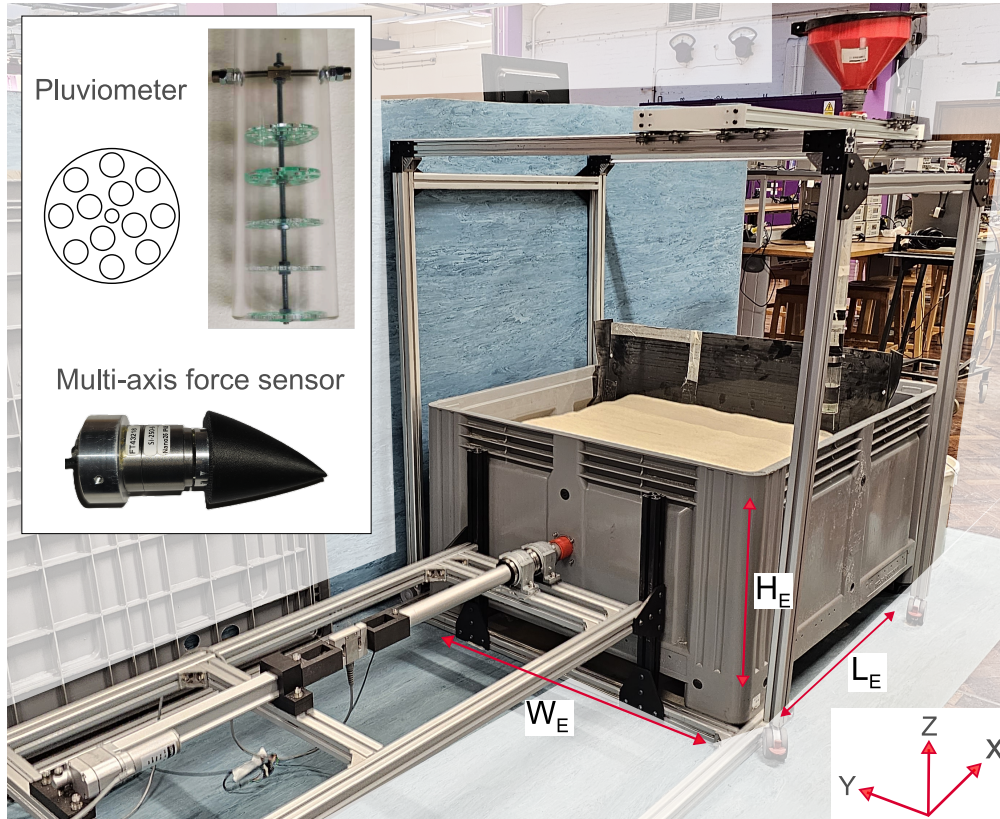


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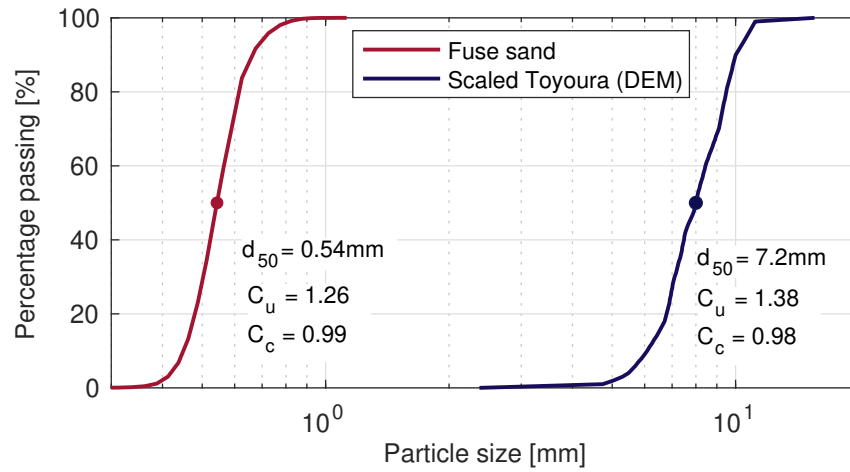


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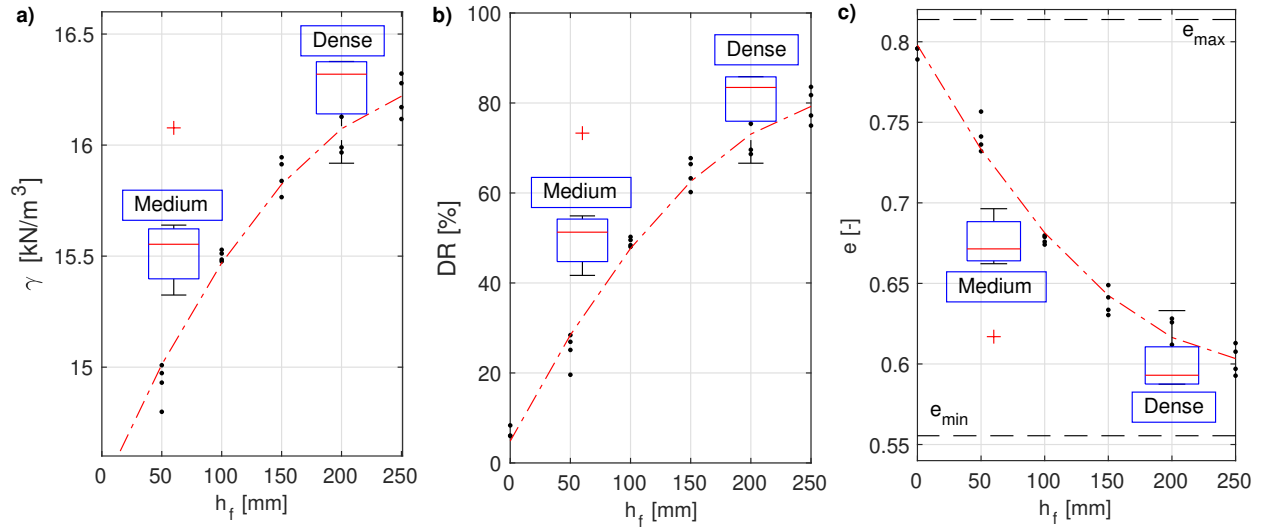


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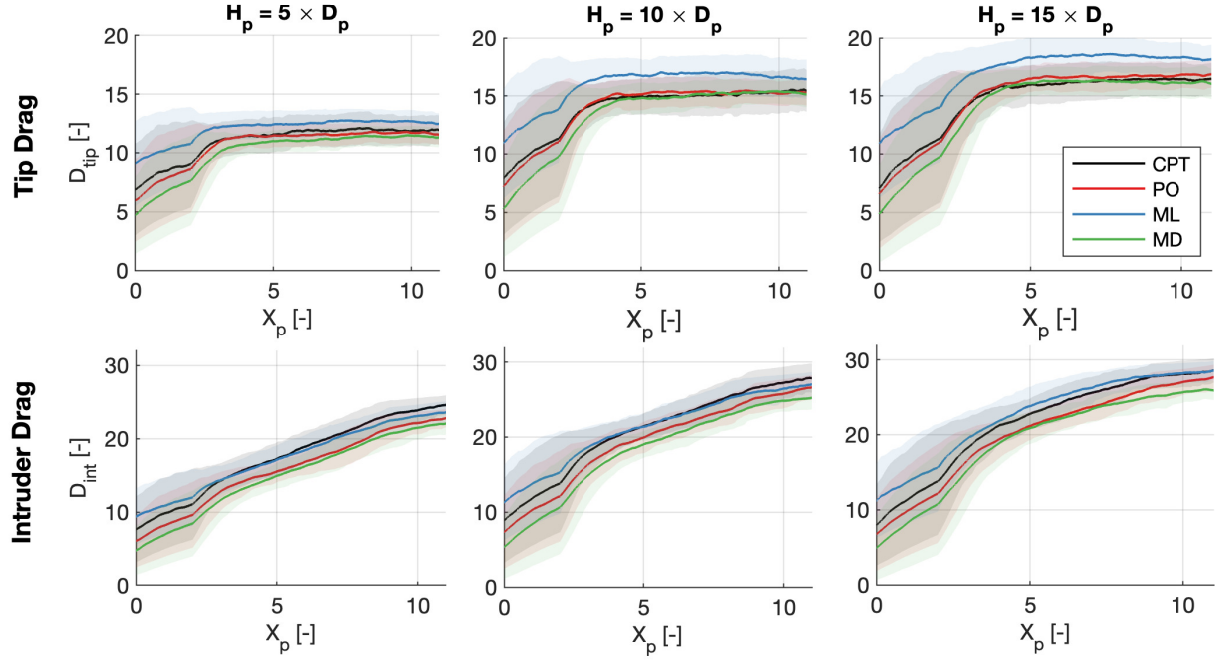


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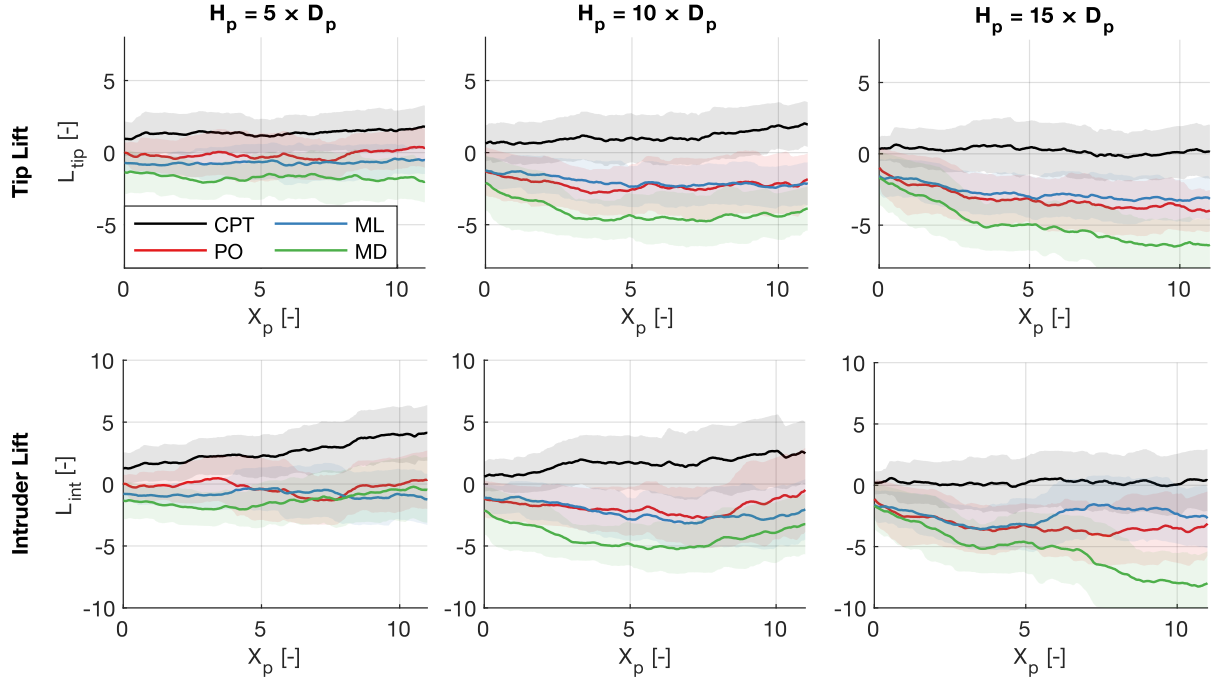


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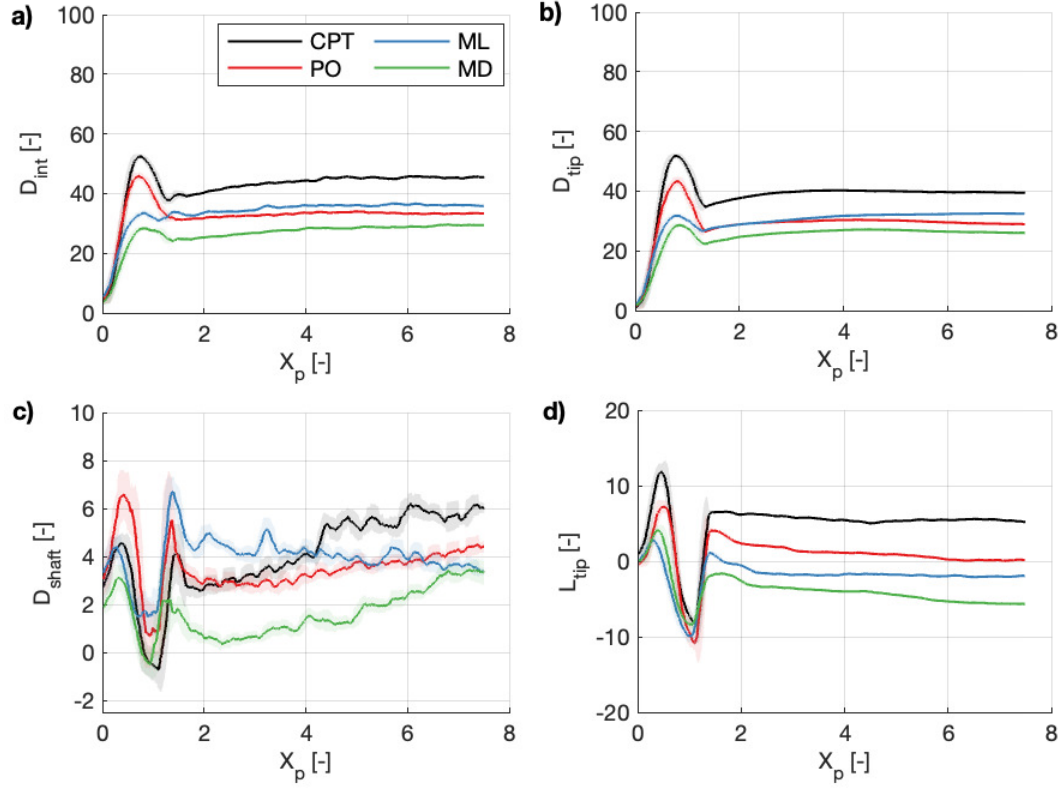


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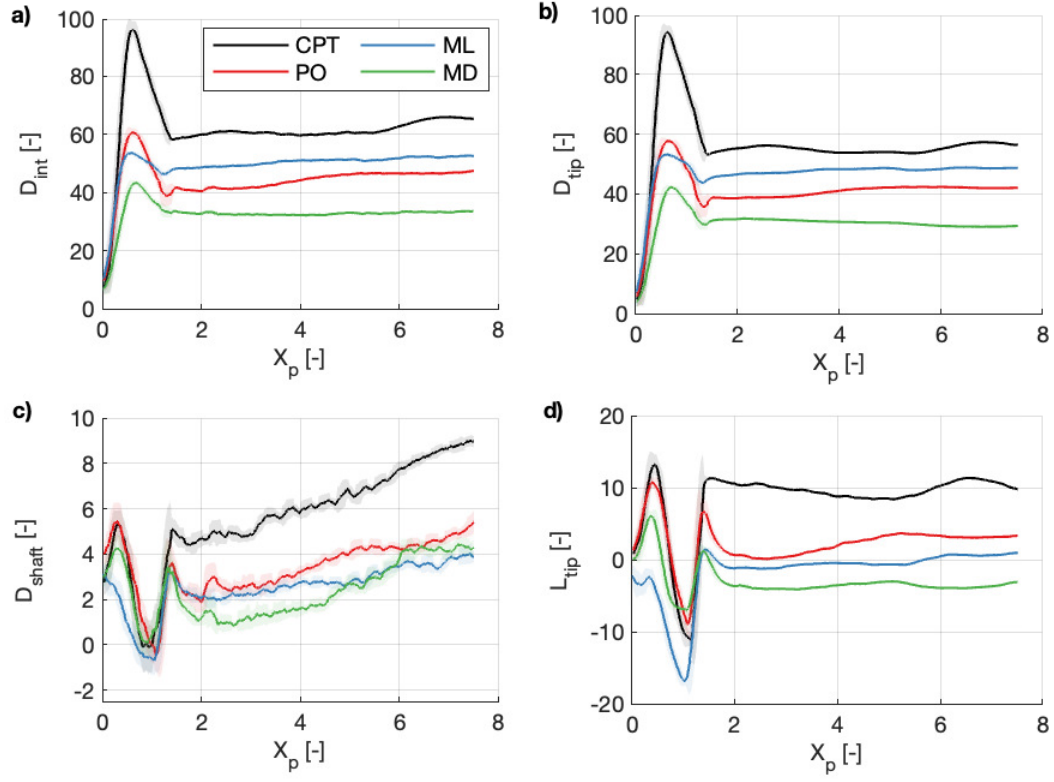


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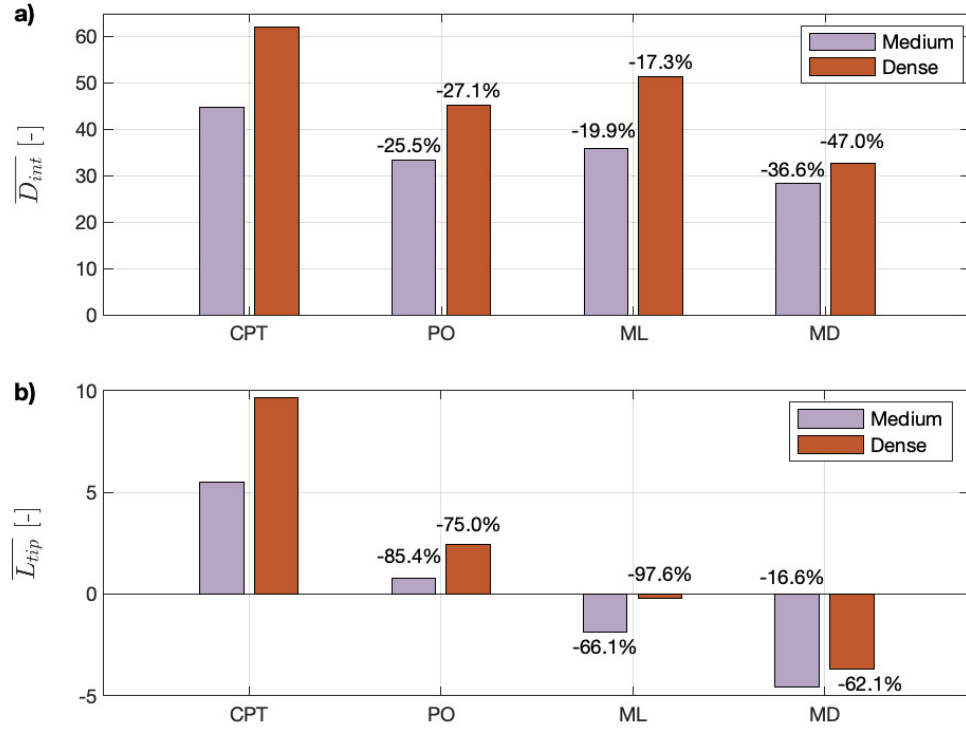


Fig. 10. Average penetration forces for the different intruder tip shapes and packing densities tested. The height of the bars shows the average intruder drag (a) and tip lift (b) over the range $2.5 \leq X_p \leq 11$. Percentages above each bar show the average increase/decrease in penetration forces versus the benchmark CPT tip shape.

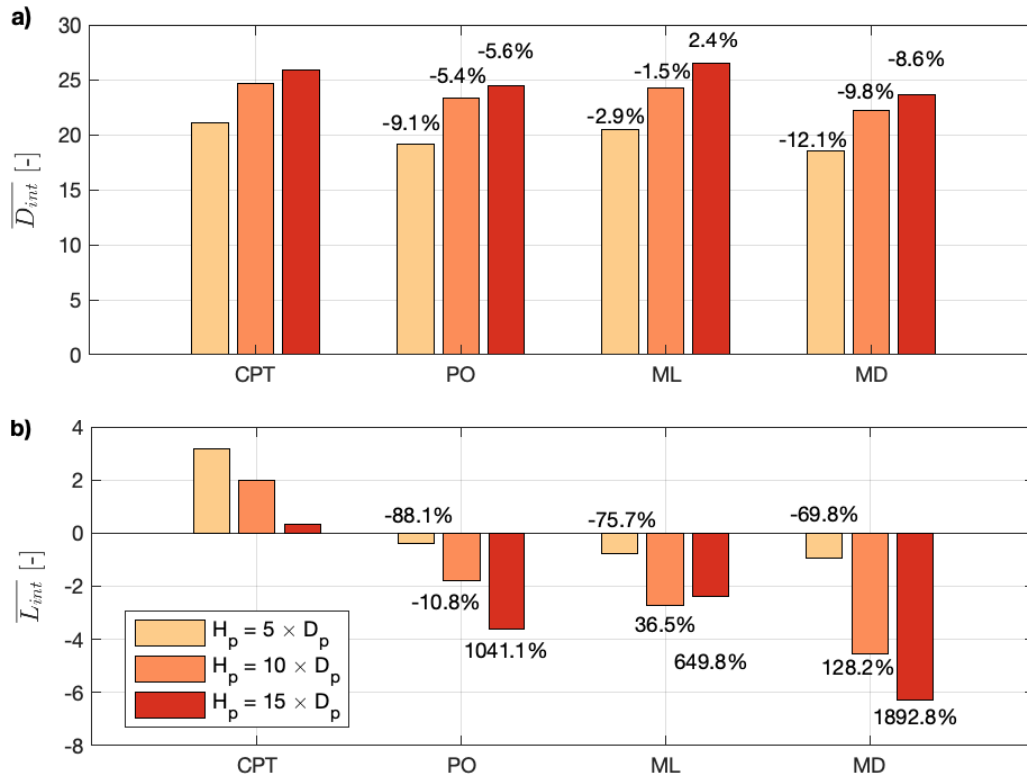


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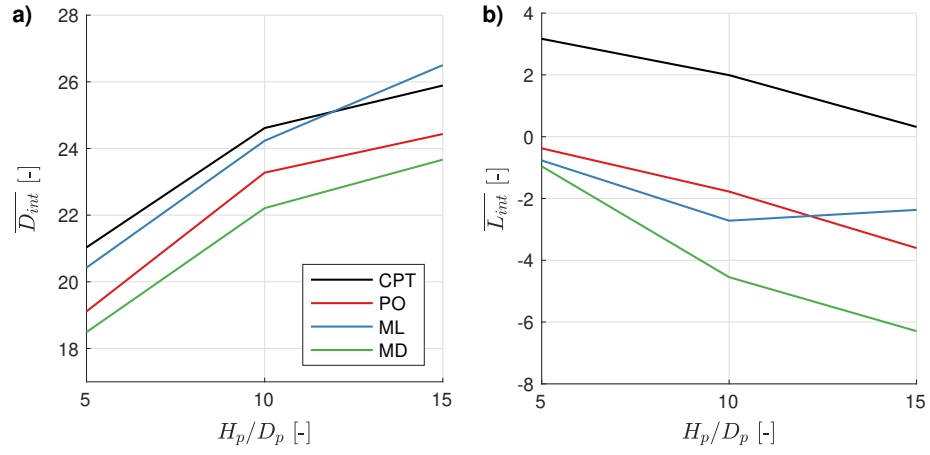


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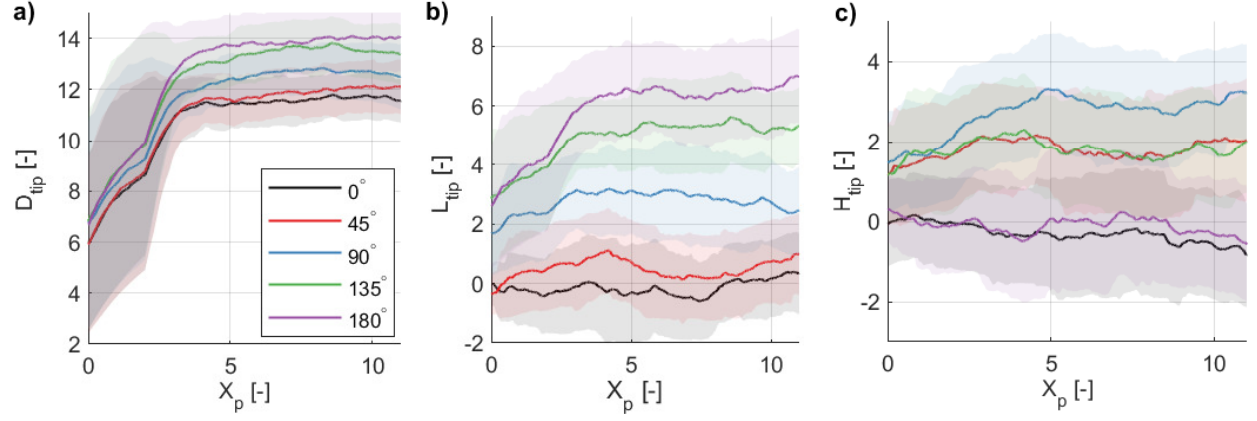


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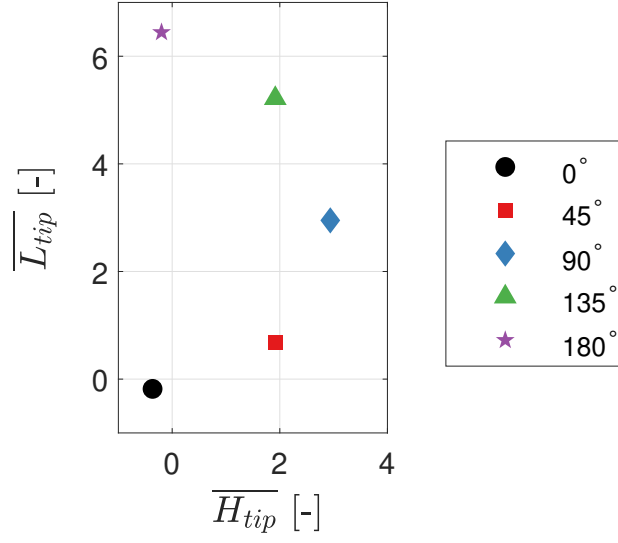


Fig. 14. Average tip lift ($\overline{L}_{tip}$) and lateral ($\overline{H}_{tip}$) forces for the different tip orientations tested. Results from DEM simulations at an intruder depth $H_p = 5 \times D_p$.

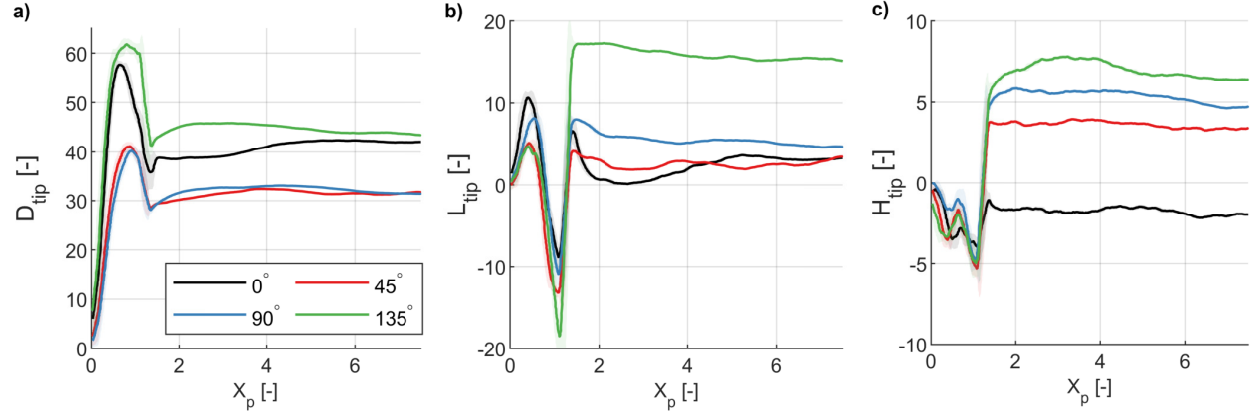


Fig. 15. Experimental penetration forces for different orientations of intruder tip. Results for PO tip at a depth $H_p = 5 \times D_p$. a) Tip drag D_{tip} , b) Tip lift L_{tip} , c) Tip lateral force H_{tip} . Values are shown as a function of penetration distance X_p . Solid lines and shaded regions show the moving average and the $\pm$ standard deviation over a window of $0.5D_p$. Note different ranges in the vertical axes.

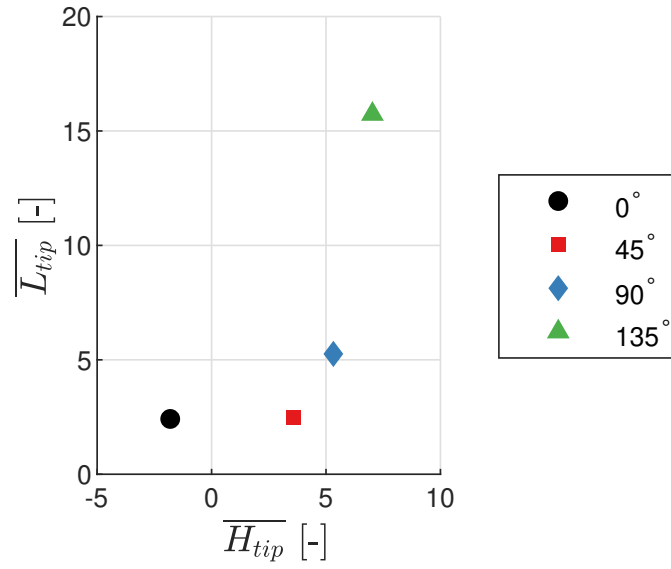


Fig. 16. Experimental comparison between average lift (vertical) vs. lateral forces. Values correspond to horizontal penetration experiments with PO tips at different orientations.