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Applications of the Capital Asset Pricing Model to planning in a decentralized department store

by

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TO MY PARENTS

ABSTRACT

A town or village attracts people from its hinterland to purchase goods by virtue of the amount and variety of goods on offer. Similarly, a department store attracts customers away from competitors by means of displays, staff expertise, stock holdings and numerous other parameters. The store is generally free to manipulate these parameters subject to certain restrictions; the manipulations will have a direct effect on the ultimate profitability of the store. This thesis presents a systematic approach to the issue of resource allocation in a decentralized department store chain operating under uncertainty. We link the corporate planning of the department store group with the performance of each of its branches, itself a function of the socio-economic data describing the population attracted by the branch. Using the Capital Asset Pricing Model to explain how capital costs and risk are related, and portfolio theory to describe the inter-relationships between product lines within each store and between stores within the group, we suggest various models of the corporate planning process. Specifically, we demonstrate how typical operating decisions in a department store affect the market value of the individual store and the group as a whole, and how different market conditions call for varying degrees of intervention by the corporate centre in the planning within its branches. Special emphasis is put on the analysis of changes in the corporate cost structure and their impact on the capital budgeting and resource allocation decisions.

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TABLE OF CONTENTS

CHAPTER 1		INTRODUCTION	10 - 12
CHAPTER 2		MODELS OF RESOURCE ALLOCATION IN A DECENTRALIZED FIRM	13 - 28
	2.1	Models of resource allocation	14
	2.2	Classical economic models	15 - 18
	2.3	The budgeting approach	19 - 20
	2.4	Behavioural models	21 - 23
	2.5	Resource allocation in the retail trade	24 - 25
	2.6	Allocation of financial resources	26 - 28
CHAPTER 3		THE POTENTIAL SALES OF A DEPARTMENT STORE	29 - 62
	3.1	Introduction	30 - 32
	3.2	Defining the trading area	33 - 38
	3.2.1	The Gravitational approach	33 - 37
	3.2.2	Empirical methods	38
	3.3	Assessing potential demand within a trading area	39 - 47
	3.3.1	Estimating a shopping centre's potential sales	39 - 43
	3.3.2	Estimating the potential demand for a store	44 - 47
	3.4	Regional demand functions	48 - 60
	3.4.1	Consumer preferences	48 - 51
	3.4.2	Income effects	51 - 56
	3.4.3	Price effects	56 - 60
	3.5	Summary	61 - 62
CHAPTER 4		MODELLING A DEPARTMENT STORE	63 - 96
	4.1	The structure of a department store	64 - 67
	4.1.1.	The store manager's problem	64 - 65
	4.1.2	The scarce resources	65 - 67
	4.2	A heuristic	68 - 77
	4.3	The store layout	77 - 89

	4.3.1	The store layout problem	77 - 79
	4.3.2	Formulation of the problem	79 - 81
	4.3.3	Methods of solution and results	81 - 89
	4.4	Optimizing models	90 - 94
	4.4.1	A deterministic model	90 - 92
	4.4.2	A stochastic model	92 - 94
	4.5	Summary	95 - 96
CHAPTER 5		CORPORATE CAPITAL BUDGETING WITH THE CAPM	97 - 162
	5.1	Introduction	98 - 101
	5.2	Capital budgeting with the CAPM	102 - 108
	5.3	A critical analysis of the CAPM	109 - 114
	5.4	The capital market mechanism	115 - 124
	5.5	Capital budgeting in a perfect competition	125 - 137
	5.5.1	Expanding the mean-variance theorem	125 - 134
	5.5.2	Expanding the uncertainty of operating efficiency	135 - 137
	5.6	Capital budgeting with economies of scale	138 - 150
	5.6.1	Economies of scale in decentralized firms	138 - 144
	5.6.2	The CAPM and economies of scale	145 - 150
	5.7	A numerical example	151 - 160
	5.8	Summary and conclusions	161 - 162
CHAPTER 6		RESOURCE ALLOCATION IN A DEPARTMENT STORE OPERATING UNDER UNCERTAINTY	163 - 212
	6.1	Introduction	164 - 166
	6.2	A department store's portfolio	167 - 169
	6.3	A department store's risk	170 - 192
	6.3.1	Investment proportions	170 - 185
	6.3.2	A product line's risk	185 - 188
	6.3.3	The correlation coefficient	188 - 192
	6.4	Resource allocation in a department store	193 - 210
	6.5	Summary	211 - 212

CHAPTER 7		RESOURCE ALLOCATION IN A DECENTRALIZED DEPARTMENT STORE UNDER UNCERTAINTY	213 - 251
	7.1	Introduction	214 - 216
	7.2	Decentralized resource allocation procedures	217 - 229
	7.2.1	The perfect market case	217 - 226
	7.2.2	The case of a changing cost structure	226 - 229
	7.3	Interventionist methods of resource allocation	230 - 243
	7.3.1	Behavioural patterns in decentralized firms	230 - 231
	7.3.2	The economics of intervention	231 - 236
	7.3.3	Interventionist models	236 - 243
	7.4	Behavioural externalities	244 - 248
	7.5	Summary	249 - 251
CHAPTER 8		SUMMARY AND CONCLUSIONS	252 - 258
		REFERENCES	259 - 277
		APPENDIX I	278 i - viii
		APPENDIX II	ix - xv

LIST OF FIGURES

		PAGE
Fig. (3.1)	Planning process flowchart	32
Fig. (3.2)	Trade distribution	35
Fig. (4.1)	A department store model	69
Fig. (5.1)	Market Price of Risk criterion	105
Fig. (5.2)	Standard price increase mechanism	117
Fig. (5.3)	Capital market equilibrium	121
Fig. (5.4)	Investment project ranking	123
Fig. (5.5.a)	A demand curve in perfect competition	131
Fig. (5.5.b)	Revenue curves in perfect competition	131
Fig. (5.6)	Cost curves in a perfectly competitive market	133
Fig. (5.7.a)	Marginal and average cost curves in economies of scale	143
Fig. (5.7.b)	Long range total cost curve in economies of scale	143
Fig. (5.8)	The expanded market price of risk criterion	147
Fig. (5.9)	Project ranking	160
Fig. (6.1)	Department store structure	165
Fig. (6.2)	Production technology curve	171
Fig. (6.3)	Effects of technology change	178
Fig. (6.4)	Wealth increase and normality of wealth and security	180
Fig. (6.5.a)	Financial product transformation curve	183
Fig. (6.5.b)	- " -	183
Fig. (6.5.c)	- " -	184
Fig. (6.5.d)	- " -	184
Fig. (7.1)	Organizational structure of department store chain	215
Fig. (7.2)	Decentralized approach flow chart	220
Fig. (7.3)	Resource expansion effect	237
Fig. (7.4)	The effects of resource commitment	243

LIST OF TABLES PAGE

Table (3.1)	Centre probability matrix	42
Table (3.2)	Trading area potential demand matrix	43
Table (3.3)	Regional budget elasticity of demand matrix	53
Table (3.4)	Own price elasticity	57
Table (3.5)	Cross price elasticities	58
Table (3.6)	Price elasticity matrix for SEG k	59
Table (3.7)	Regional price elasticity of demand matrix	60
Table (4.1)	Iteration by smallest profit index	73
Table (4.2)	Iteration by smallest marginal value	74
Table (4.3)	Iteration by smallest percentage fall	75
Table (4.4)	Results of heuristic	76
Table (4.5)	Three layout options	78
Table (4.6)	Site cost matrix	84
Table (4.7)	Substitution matrix	85
Table (4.8)	Distance matrix	86
Table (4.9)	Constructive section	87
Table (4.10)	Improving with SDPI	88
Table (4.11)	Improving with SDTI	89
Table (5.1)	Capital market changes	120
Table (6.1.a) - (6.1.b)	Cross-elasticities : examples	190

LIST OF ABBREVIATIONS

CAPM	-	Capital Asset Pricing Model
CML	-	Capital Market Line
ESC	-	Efficient Set Curve
FPTC	-	Financial Product Transformation Curve
LP	-	Linear Programme
LRAC	-	Long Range Average Cost
LRMC	-	Long Range Marginal Cost
LRTC	-	Long Range Total Cost
NBED	-	Net Budget Elasticity of Demand
MP	-	Mathematical Programming
MPR	-	Market Price of Risk
MPRX	-	Expanded Market Price of Risk
PEM	-	Price Elasticity Matrix
RBD	-	Regional Budget Elasticity of Demand
RPED	-	Regional Price Elasticity of Demand
RWBD	-	Regional Weighted Budget Elasticity of Demand
SDPI	-	Steepest Descent Pair-wise Interchange
SDTI	-	Steepest Descent Three-way Interchange
SML	-	Security Market Line
TR	-	Total Revenue
WACC	-	Weighted Average Cost of Capital

C H A P T E R 1.

INTRODUCTION

1. INTRODUCTION

Because they handle a large number of products and because of the multi-plant nature of their operations, decentralized organizations in general (and chain retail corporations in particular) face special problems in pricing and profit maximization. Ten or twenty years ago, it was not too difficult for a retailer to assess the potential of a new site and develop it. The pattern of local trading was well established and the opening of a new store required a relatively small capital investment. At present, with the existence of larger stores offering a wider range of merchandise to customers from a very much wider area, the complexity of the problem has substantially increased. Not only does the opening of a new store nowadays require a considerably increased capital outlay, the retailer's knowledge of its potential has simultaneously diminished. If the store is a failure, there are slim chances that the retailer will not suffer an appreciable loss; thus, if an existing store does not live up to the retailer's expectations, it is crucial for him to understand why. This holds even if the reasons indicated are beyond the retailer's own control, understanding them should at least prevent him from repeating the same mistake.

Clearly, these trends have increased the retailer's search for some guidance in his management decisions. Not surprisingly, more attention is now being devoted to the contribution which market research can make to the location, development and appraisal of stores, and on a higher organizational level, to the profit maximization in the chain as a whole. The primary purpose of this thesis is to offer a model which can assist management with the selection of investment projects and the allocation of resources between stores and products under uncertainty. The model also makes it possible to deal with locating product lines within a store, considering alternative marketing policies and assessing the impact of centralized procurement of goods and capital.

The structure of this work is best understood if one accepts Simmons' (138) approach to the role of market research in planning and optimizing investment in stores. Specifically, he identifies two broad types: studies prior to the opening of a new store and studies post-opening.

Studies prior to the opening of a new store can be classified into those aimed at estimating the likely turnover, and those designed to formulate the promotional strategy for the new store. On the other hand, post-opening studies, appraising the performance of existing stores, can be classified into those designed to estimate share of trade, and those designed to establish the "image" of the store.

This work attempts to produce a model which is not limited in its scope to either one of the planning stages, and consequently it develops gradually from a model designed to estimate an individual store's likely turnover and promotion strategy (defined in terms of layout and resource allocation) into a more ambitious effort directed towards appraising the performance of a store as a component of the corporate stucture.

As a starting point for our work, we examined a case study. Specifically, we studied the operations of a branch of Debenham Ltd., located in Romford, Essex. Through a series of discussions with both local managers and the corporate headquarters in London we gathered information on the centre-branch relationships, the characteristics of a modern shopping centre and the local management's decision making process. The results of this investigation into a decentralized retail chain were subsequently incorporated throughout our discussion, thus, hopefully, adding to it realism.

Broadly, we can divide this work into two main parts. First, we develop a heuristic planning model of a store; in chapter 3, we examine the external environment in which the store operates, the factors which influence consumers' attraction to a particular shopping centre, and to a particular store

within it. On the basis of this information, we produce a model used to derive the potential sales the store can hope to attract. In chapter 4, we suggest a heuristic for allocating resources between product lines in view of the demand patterns derived in the preceding chapter.

In chapter 5, we turn to analyse the capital market mechanism, and the share price equilibrium in particular. We then examine, in chapter 6, the planning implications of our findings on the share price maximizing firm. In chapter 7, we suggest various approaches to the problem of resource allocation under uncertainty in a decentralized firm, and we summarize our main conclusions in chapter 8.

To conclude, the main contribution of this thesis lies in the development of a heuristic for resource allocation in an individual department store, the analysis of a department store's portfolio and investment proportions and a study of resource allocation in a decentralized retail chain. These contributions are summarized in chapter 8.

We start, however, with a survey of the main modelling efforts in the field in chapter 2. In view of the fact that most sections of this work have implications which are not confined to the retail trade, i.e. they can be applied to any firm with a decentralized structure, we open our survey with the most important models of resource allocation in such organizations. Following this, we present a survey of work done on allocating resources in the retail trade, and the allocation of financial resources in particular.

C H A P T E R 2

MODELS OF RESOURCE ALLOCATION IN A DECENTRALIZED FIRM

2.1 MODELS OF RESOURCE ALLOCATION

Numerous models of resource allocation in decentralized organizations appear in the literature. These models range from verbalizations of flow diagrams of decision-making processes, such as Mesarovic et al (107) and Young (156), to massive simulations, such as Forrester (52), or analytic programming models.

We direct our attention upon the latter group, namely analytic programming models of the planning process. Following Ruefli (130), we will distinguish two sets of analytic programming models, which we shall call "classical" and "behavioural" models respectively.

The "classical" models are mainly based on what can be called a classical economic interpretation of systems management. The concept underlying this approach is the assumption that the ideal solution for a system is precisely that solution which would be obtained if the system functions as a centralized monolithic decision entity.

The fundamental difference between classical and behavioural models is that, in the latter, the solution reached by the system is a function of its organizational structure. In other words, given a resource allocation problem in an organization with structure A, we assume that a behavioural model achieves a solution denoted Z_A. If we now alter, by defining a new set of sub-units, the structure to one denoted B, and apply the behavioural type of model to the resource allocation problem, we will produce a solution denoted Z_B. While in similar circumstances, if the classical model were applied, Z_A will always equal Z_B, the very nature of the behavioural approach means we cannot assume $Z_A = Z_B$. Consequently, behavioural models often yield solutions that would be considered suboptimal in a classical framework.

2.2 CLASSICAL ECONOMIC MODELS

Despite the many differences between them, all the models discussed in this section share a common approach to the structure of the organization for planning purposes. Specifically, the organizational framework is assumed to be a strictly pyramidal hierarchy, composed of subordinate units. In addition to the similar structure, those models assume that the organization has a global objective (usually profit maximization) that is perceived by the superordinate unit, and the problem consists of co-ordinating the subordinate units in a way so as to maximize the organization's attainment of the global objective. In their analysis of hierarchical multi-level systems, Mesarovic et al (107) provide a general conceptual framework for the organization resource allocation process as well as a critique of some of the models to be discussed.

Several articles had appeared on the subject of decentralized decision-making prior to the development of the simplex method for decomposing a linear programme. Among the most notable and early statements in the economics management science literature are those by Lange and Taylor (91) and Lerner (92).

Their approach has a superordinate unit allocating a multi-period budget to sub-unit projects, by setting shadow prices on each period's budget. These shadow prices are then used by the sub-units to establish resource requirements for their investment proposals. Given these data, the superordinate unit then compares these expenditures against the appropriate budgets, and if they are not optimal, new shadow prices are generated. This process is repeated until an optimum is reached.

The methodology described above appears in various forms as the framework for other efforts that we will consider. A static model was suggested by Koopmans (89a), who looked

at an activity analysis formulation of the problem. Koopmans' model concentrates on establishing the conditions required for maintaining an "efficient" point once it has been achieved. On the basis of this formulation, Charnes and Cooper (31) offered a linear programming statement of Koopmans' problem and discussed the critical concepts of "Koopmans' efficiency" to the case of functional efficiency. They then developed a computational method for generating the problems and prices required in the planning process.

In 1960, Arrow and Hurwicz (10) discussed the non-linear formulation of the decentralization problem. In a subsequent work, Arrow (11) studied the general linear model, which does not present a computation scheme for generating the prices used to achieve decentralization, but has the strength of providing a conceptual approach which proved to be an anticipation of later developments. On a more general basis, works by Malinvaud (97), Hurwicz (80), Marschak (102), Katsenelinboigen et al (84, 85 and 86), Albegov (2), Moeseke and de Ghellinck (108), and Weil (150), provide theoretical structuring of the environment for decentralized resource allocation.

A rather different approach proceeds via decomposition methods and algorithms. The original decomposition method for linear programming was developed by Dantzig and Wolfe (38) and Dantzig (39). The decomposable linear programming problem is a model type with a characteristic structural formulation. This model type and solution process can be interpreted as a description of the economic activity of a decentralized organization.

The solution mechanism proposed by Dantzig and Wolfe consists of an iterative process in which the central unit generates shadow prices which are used by the subunits to generate proposed levels of activity for their operations. These proposals are then sent

to the central unit which performs an evaluation and generates another set of shadow prices. This process is repeated until an equilibrium is reached, and a global optimum is achieved. At this stage, the centre informs each sub-unit of the optimal activity levels for that unit, i.e. following the conclusion of the iterative procedure, a set of instructions is sent, and therefore this is not a pure price-planning system per se.

The importance of the Dantzig-Wolfe model can be summarized as follows:

(i) its use as a computation aid in solving large linear programmes.

(ii) its position as the first model to incorporate a finite adjustment process for establishing efficient prices.

Since later researchers modified and extended the basic assumptions underlying the above model, it is useful to list them briefly:

(i) all relations in the mathematical model are linear;

(ii) co-ordination of the organization's activities by the central unit is achieved through the mechanism of prices it generates;

(iii) the whole organization's goal is the optimization of a single goal level;

(iv) information concerning the characteristics of the projects and the goal levels are known a priori; and

(v) the information and activities of the organization are partitioned a priori into global (inter-divisional) and local (intra-divisional) categories.

Whinston (153) and Baumol and Fabian (14) relaxed the assumptions that all of the relationships are linear and that divisional constraints are entirely local in nature. Hass (67) contributed a major extension by relaxing the assumption of a linear objective function. He developed an algorithm that uses the demand and supply curves generated by the central unit as the basis for an iterative process that results in a global optimum for a quadratic objective function.

A different approach to the nature of the information used in the iterative process was suggested by Balas (12), namely, he developed a modified algorithm where the divisions optimize their segment of the global objective function and send their planned activities to the central unit. The divisions continue to generate revised plans in response to the prices generated by the central unit until a set of these divisional plans is feasible with respect to the global constraints. At this point, a feasible global optimum has been reached but, as required in the Dantzig-Wolfe model, the centre may need to direct the divisions to combine one or more of their proposed plans in order to yield the final plan of activities.

The underlying concept, common to the models mentioned so far, is that they all use prices originated by the central unit as the co-ordinating mechanism. As an alternative, a number of important models use a different mechanism, which we shall name the "budgeting" approach.

2.3 THE BUDGETING APPROACH

The budgeting approach is the dual of the pricing approach. Generally, the central unit allocates resources (or goals) to each of the divisions. These, in turn, send shadow prices, or marginal values, back to the central unit, who then adjusts the resource allocations until an acceptable allocation is determined. The underlying economic principle is the concept that in equilibrium all resources yield equal marginal return, and consequently adjustments should continue to be made until such an equilibrium is reached.

Two Hungarian economists, Kornai and Liptak (90), are the pioneers of the budgeting approach. Using the Kornai-Liptak method, the central unit may, by using the appropriate resource allocation, force the division to choose any point in its original feasible region as optional, since that allocation will reduce the feasible region, and any interior point of the original feasible region could be made a corner point of a revised feasible region by imposing the appropriate constraints. This is the source of what possibly is the main advantage of the resource allocation decomposition procedures. Each division is free to determine its own optimal decision independently, once the optimal allocation of corporate resources has been established. This clearly is an improvement over the Dantzig-Wolfe algorithm, which requires a corporate optimal solution prior to the divisional optimal decision.

The implications of this difference can be of considerable importance when one takes into account organization control. In the Kornai-Liptak approach, each division may be allowed to maximize its own objective funtion, knowing that the result would simultaneously maximize the corporate objective function. On the other hand, the fact that the Dantzig-Wolfe algorithm requires some type of quota system with

changes for either positive or negative deviations, may cancel some of the advantages offered by "pure" decentralized management.

Several authors have extended the basic model of Kornai and Liptak. Heal (69), modified the model to include non-convex technologies, intermediate products and joint products. Aoki (5) developed an iterative method that combines both resource directive and price directive measures. In another paper, Heal (70) presents a mixed price and resource directive system that operates even under increasing returns to scale. Combining a price and quota system, Heal's procedure consists of a two stage iterative process, with the subordinate first informing the superordinate unit of their current programmes (outputs). This information generates a response by the superordinate unit which attaches a price on each subordinate unit's output. Following this, the subordinate units respond to the prices with a shadow price on each global resource, and the superordinate unit uses these shadow prices to reallocate the global resources between the subordinates. This process is repeated until a global optimum is reached.

Other algorithms have been developed for the budgeting approach. Sharing the concept of divisional programmes, they differ in the way adjustment of common resources is made. Examples of such cases are Geoffrion (57), Sanders (132), Schleichers(133), Silverman (137) Abadie and Sakarowitch (1) and Zchau (157). In section 2.5, we discuss the issue of implementation of these results.

2.4 BEHAVIOURAL MODELS

In the previous section we discussed models which, based on the assumption that the organization modelled is searching for an optimum, seeked to construct means whereby a decomposed model could generate the same solutions as an undecomposed model. In that approach, sub-optimization is ruled out as a possibility, the optimal solution is unaffected by whether the problem is solved as a centralized problem or as a decomposed one. Given such assumptions, it follows that the organizational dimension is only relevant to the mechanics of deriving a solution and the potential effects of organizational structure on the solution are ignored.

In this section, however, we will discuss some of the more important models of resource allocation which attempt to achieve some agreement with behavioural theories that recognise that the structure of an organization can affect its performance. Various methods have been suggested to make organization structure an implicit parameter; mainly, they involve replacing the classical optimization along a single dimension with a satisficing or a multiple goal approach.

Cassidy et al (29) presented a satisficing objective function which is a significant variation on the Kornai and Liptak formulation. Using as an example a federal government allocating resources to states, who in turn allocate them to cities, the objective of the federal unit is formulated so as to minimize differences in regret among the states. Given this objective, the solution to the problem can be shown to be sensitive to the organization structure. This is primarily due to the nature of the objective function which is sensitive to the number of states and the way in which projects are distributed among states.

Cassidy et al (29) developed a centralized solution algorithm to the above problem. However, while their algorithm could be implemented on a decentralized basis, the information requirements would be substantial. In a two-level hierarchical model, Collomb (33) developed a formulation in which the central unit and the subordinate units all have satisficing objectives on the attainment of multiple goals.

Kornai and Liptak's model was generalized to an n-level hierarchy with multiple goals by Ruefli (126,128,129 and 130). Assuming linear relations, the process consists of the centre setting initial goal levels for the subordinate management units which, at any state, use the current set of goals to generate shadow prices to the central unit and to its subordinate operating units. The operating units use these shadow prices to generate new proposals which are then sent to their respective superordinate management units. The central unit uses the shadow prices to generate new sets of goals which are transmitted to the management units where they and the alternative proposals are used to generate new sets of shadow prices. This process is repeated until the deviations from the management unit goals are at a minimum and no readjustment of goal levels on the part of the management units or modifications of proposals on the part of the operating units will yield a net decrease in the deviations from the goal levels for the organization as a whole. If the goal levels, alternatives and shadow prices are generated using the rules of the simplex procedure, the process will terminate in a finite number of steps.

Ruefli has shown that this model can incorporate externalities, either technological or behavioural. Specifically, three types of technological externalities were considered: those involving inter-dependencies among

projects subordinate to the same management unit, cases when the goal levels of a management unit are dependent on those of some other management unit, and when project characteristics of projects generated by different management units are related.

Freeland & Baker (56) have shown that, for Ruefli's algorithm to work, all units subordinated to a given unit must solve their resource allocation problem simultaneously. This shortcoming can be overcome by using a partitioning algorithm, as suggested by Benders (17). Using Freeland and Baker's algorithm, Hurwitz (80) developed two stochastic models; one examined the case of uncertainty in the subordinate units about the reliability of the sub-goals set by the central unit and the other, the case of uncertainty in the subordinate units about the reliability of their constraints.

2.5 RESOURCE ALLOCATION IN THE RETAIL TRADE

Applications of analytic programming models to decision making in the management of retail stores seem to be few and far between. Although economic studies of the retail trade have been available for quite some time, E.L. Salkin's (131) paper, published in 1964, is probably one of the earliest examples of mathematical programming being used as a managerial tool in retailing. In his discussion, Salkin demonstrates how linear programming can assist management of a retail store in its merchandizing decisions and provides an example of a two-product problem.

A notable analysis of the economics of retailing was published in the early sixties by Preston (118), who examined pricing policies and decisions within the individual store and reconciled economic theory with data on prices to explain individual store decisions. On a similar basis, Pfouts (116) attempted to incorporate the unusual nature of the profit maximizing problems of the chain store manager and the role of incomplete information and search and information costs in the pricing decision in a chain's central office.

The behavioural assumptions made by Pfouts are primarily concerned with the pricing policies and practices. Specifically, the retail chain is assumed to follow the policy of establishing prices at their headquarters. Usually, the pricing process is viewed as that of choosing a markup to be added to the unit cost which will cover overhead and provide a profit, while keeping the final price sufficiently low for the product to sell.

A second major assumption concerns the degree of freedom the store manager is given by the central unit. Basically, the store manager is assumed to be motivated by the need to operate his store as profitably as possible, and in the process he is limited by the storage and display space

available in the store. An additional constraint arises because it is not unusual for a store manager to be required by the central office to stock certain items even though it might be more profitable to use the space for other products.

On the basis of these behavioural assumptions, Pfouts suggests two alternative methods of solving the profit maximization problem facing the store manager. The first approach uses inventory control methods to establish optimum order sizes for each type of good. The sum of procurement costs and carrying costs is viewed as a constant for each good since it is assumed the same quantity is ordered each time. The results of this model require that the gross profit per unit of a good equal the shadow value of space occupied by a unit of the good, plus the value to the firm of the good over alternative goods.

A second and more intuitive model is used in order to make the method more attractive to the store managers. The underlying concept is that different types of space are independent of each other so far as sales and profit are concerned, and consequently the model can be applied to each type of space. Basically, the goods showing the largest gross profit per unit of space are stocked by the model, and this process is repeated until all space is in use. The most profitable goods are to be offered in quantities which meet demand, and thus profit is maximized.

2.6 ALLOCATION OF FINANCIAL RESOURCES

Another extension of the general resource allocation problem in a decentralized firm model has focused on the firm's financial resources. Special attention has been devoted in an effort to produce an optimizing model for the allocation of corporate funds in a decentralized firm. The reason for such treatment probably lies in the strategic importance corporate managers tend to associate with capital funds. In view of this approach, they are usually reluctant to decentralize totally in this area, and consequently adopt direct capital rationing procedures as administrative safeguards.

The main difficulties in providing a complete framework of capital rationing in a decentralized firm are related to the complexity of the problem. Linear programming models of specialized decision problems such as working capital management (115) and short term financing (122), are deficient in that they may lead to decisions which are suboptimal with respect to the firm as a whole. This is directly related to the fact that each of the above models focuses on a single problem, thereby neglecting its interaction with the other activities of the firm. On the other hand, if one were to incorporate all these interdependencies into a single model, it would tend to become excessively large and inefficient to use.

Weingartner (151) first provided a complete framework within which project selection under capital rationing could be made for a deterministic case. On the basis of this work, extensions have been made which eventually have lead to the application of the decomposition principle to financial decision models.

Carleton, Kendall and Tandon (27) produced the first comprehensive attempt to join the techniques of mathemat-

ical models for capital budgeting and decomposition for decentralized decision-making. Prior to their effort, very little work had been done to synthesize the two, which they explained by the fact that a satisfactory construction for administrative capital rationing at the corporation level was not available. Their approach was to define overall investment opportunities at the firm - not divisional - level. By doing so, interdivisional externalities can be captured without the divisions themselves needing to know of opportunities other than specific projects or of fund sources outside the corporate pool.

The basic concept underlying their algorithm is to allow each division to calculate an optimal division capital budget for itself, without reference to the corporate capital constraints. The divisions then inform the central unit only of their net cash returns or requirements. On the basis of this information, the centre then determines the aggregate effect of the divisional flows on the corporate constraints, and subsequently defines a net set of capital prices for each division to use. These are calculated by adding or subtracting from the previous prices each division's marginal net contribution or burden upon the other divisions, and following this, new divisional plans based on the new prices are generated. This iterative process is repeated until a corporate optimum is reached within a finite number of steps.

While Carleton et al concentrated more upon the decentralization of capital budgeting decisions between corporate divisions, Morris (109) applied the decomposition principle to decentralization according to functional areas: Investment, Financing and Production. The interaction among decisions made in different decision centres is reflected in a set of prices for resources which they share. The primary contribution of Morris' paper was the analysis of the mechanism of internal pricing of financial resources, providing considerable insight into the inter-

action among dividends, equity financing and capital budgeting. It is shown that, under conditions of capital rationing, capital budgeting decisions should be made independently of financing and production decisions only if the "conventional" cost of capital discount factor is modified by an internally generated price for funds which reflects the internal supply and demand for funds.

Despite their major differences, both these models share a common feature, namely: certainty. Relaxing the assumption of certainty is the primary objective of this work; we suggest a method of allocating resources in a decentralized firm operating under uncertainty, based upon portfolio theory, rather than on the decomposition principle. We start, however, with a heuristic approach to the resource allocation problem in a decentralized department store.

C H A P T E R 3

THE POTENTIAL SALES OF

A DEPARTMENT STORE

3.1 INTRODUCTION

In theory, because a multi-branch department store is a highly complex system of interactions, both within itself and with the environment, any attempt to model the department store would result in a very large and complicated problem. Since such a model is usually of little practical application, what tends to happen in practice is that the modelling process is broken down into a set of interlocked subproblems.

The model developed in this chapter and the subsequent one follows the practical approach, i.e., the large problem is split into manageable subunits, and each is analysed in its turn. The characteristics of the department store problem lend themselves to a broad division into three major subproblems, each of which in its turn is composed of several minor subproblems. Specifically, the modelling process of a department store consists of:

a) The External Environment

   i) Defining the trading area boundaries.

   ii) Estimating the potential demand in the trading area.

   iii) Estimating the potential store sales.

   iv) Analysing the regional demand functions.

b) The Internal Environment

   i) Defining the scarce resources.

   ii) Determining the relationship between both environments.

c) Formulating and Solving the Store Manager's Problem

In this chapter we concentrate on all the aspects related to the store's external environment, while in Chapter 4

we direct our attention to the internal mechanism of the store, formulating and solving the store manager's problem.

Our approach to the task of modelling the external environment is based on the concept of the store's market being a function of a combination of parameters within the firm's control, and others beyond it. The first type of parameters consists of the allocation of resources between the store's various lines and its pricing policy, while the exogenous type is reflected by the surrounding population's demand functions, their elasticities, and by the consumers' budgets. It is therefore a combination of economy wide, socio-economic, and internal parameters which determine the external environment within which the store operates. This concept is graphically illustrated in Fig. (3.1).

The structure of this chapter reflects the above approach. Section (3.2) examines the importance of socio-economic factors, Section (3.3) concentrates on the relative importance of internal variables, while Section (3.4) provides an analysis of the effects of changes in consumers' budget and the firm's pricing policy on the regional demand function. Finally, Section (3.5) summarizes the main points and conclusion of our discussion.

Several parts of this and the following chapter have been presented at the III Symposium über Operations Research, Universität Mannheim, West Germany, September 1978, and subsequently published by Flavell, Penn and Salkin (51). The published version of this discussion is included in the form of an appendix at the end of this volume.

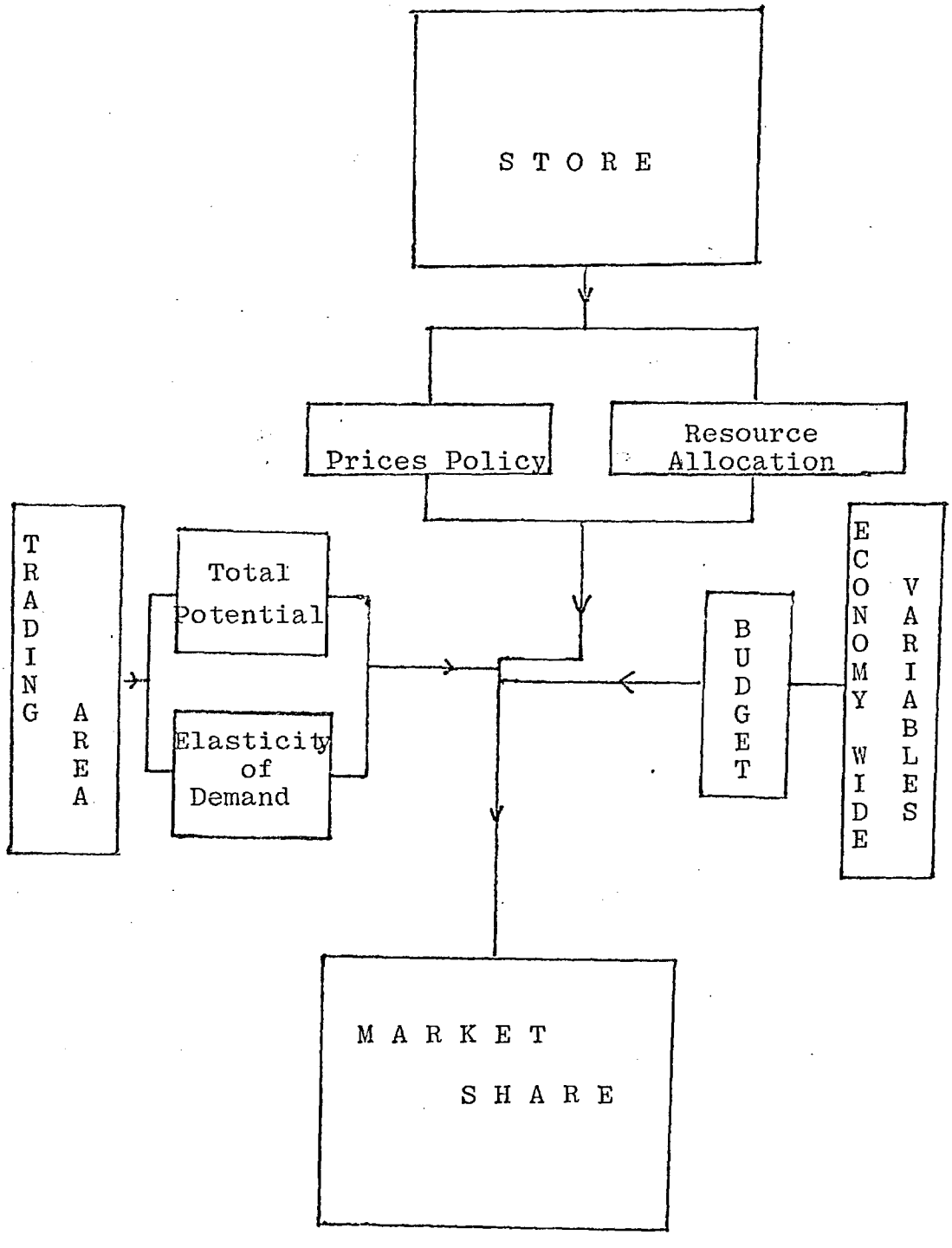


Fig: (3.1) :- Planning Process Flowchart.

3.2 DEFINING THE TRADING AREA

Defining the trading (or catchment) area of a shopping area is one of the classical problems in marketing research. The problems involved in this task are both theoretical and practical and the approaches vary from the intuitive, such as Applebaum's (6) to the highly sophisticated and computerised by Huff and Batsell (79). Mercer (105) identifies two major approaches to management in the operation definition of a trading area. They are:-

a) the gravitational approach;

b) the empirical approach.

3.2.1 The Gravitational Approach

The major advantage of the gravitational method is that, assuming a satisfactory formula can be found, it is relatively cheap and quick when compared with expensive empirical studies. Reilly (119) can be credited with being the pioneer of defining the catchment area in mathematical form. In his book, "The Laws of Retail Gravitation", Reilly puts forward the idea that two cities can attract trade from an intermediate town approximately in direct proportion to the population of the two cities, and in inverse proportion to the squares of the distances from the two cities to the intermediate town. This is analogous to the Newtonian law of gravitation has become widely known in Classical Physics, and hence, Reilly's law of retail gravitation.

Mathematically, this can be put as:

$$\frac{B_a}{B_b} = \frac{P_a}{P_b}\left(\frac{D_b}{D_a}\right)^2 \qquad (3.1)$$

where:

B_a - the proportion of trade which town a would attract from an intermediate town.

B_b - the proportion of trade which town b would attract from an intermediate town.

P_a - the population of town a.

P_b - the population of town b.

D_a - the distance from town a to the intermediate town.

D_b - the distance from town b to the intermediate town.

This theory was widely accepted as an improved approach to the understanding of retail penetration, and attempts to refine it, thus making it more suitable for practical purposes, have been numerous and varied. Converse (34) extended this formulation to indicate the breaking point for trade gravitation, suggesting that:

$$D_{br} = \frac{D_{ab}}{1 + \sqrt{\frac{P_a}{P_b}}} \qquad (3.2)$$

where:

D_{ab} - the distance between towns a and b.

D_{br} - the breaking point for trade between a and b measured in miles from b.

Empirical evidence derived from the Census of Distribution by McClelland(104), and illustrated in Fig. (3.2) suggests that such generalizations as implied by the gravity model are not always valid. Namely, that they vary throughout the whole range of town sites, and according to the product or service under consideration.

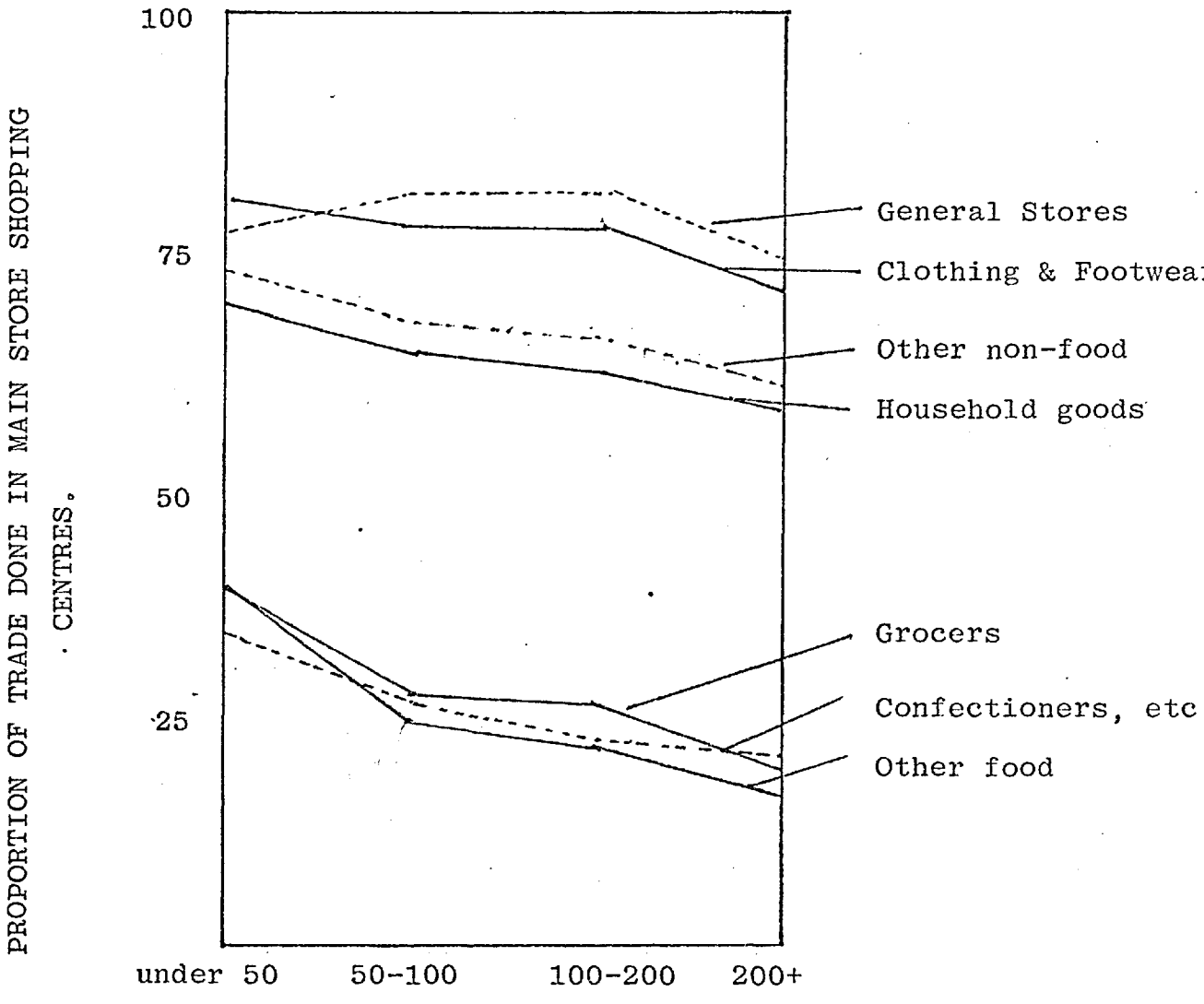


Fig: (3.2): Trade distribution
Source: McCLELLAND, (104) p.215
from the 1961 Census of Distribution.

One trend of research, of which Carroll (28) was a pioneer, followed by Stanton (142) and Dixon (41), developed a "circle of influence" model. This approach defined the catchment area more realistically as a three dimensional shape of attractive force fields exerted by competing centres. Consequently, the equilibrium line between the centres of attraction is a circle surrounding the smaller circle. The smaller circle's radius R_b can be found using

$$R_b = \frac{m\ D_{ab}}{m^2 - 1} \qquad (3.3.a)$$

where

a - represents the largest retail centre;

b - represents any other retail centre;

D_{ab} - the distance from the centre of the circle to the large retail centre.,

where, if S_a and S_b represent the sizes of centres a and b respectively, we get:

$$m = (S_a / S_b)^{1/\lambda} \qquad (3.3.b)$$

where λ, the exponent, is a constant.

Reilly (120), suggested that λ was equal to 2.0 for any product or service; Huff (78) however produced evidence that seems to contradict such a claim. Using a probabilistic model rather than a deterministic one, Huff incorporated approximations of consumer preferences into his model. This last model suggests that a contour map should be drawn for customers' probability of shopping in any particular group of centres, and that the inter-

locking nature of the hierarchy of centres will be demonstrated. The probability that any particular centre will obtain trade in any particular product or service group is formally expressed by Huff in the following model:

$$P_{ij} = \frac{S_j / T_{ij}^{\lambda}}{\sum_{j=1}^{m} S_j / T_{ij}^{\lambda}} \qquad (3.4.a.)$$

where:

P_{ij} - represents the probability of customer at point of origin i travelling to a centre j;

S_j - represents the size of shopping centre j (in square feet of sales area devoted to the product, or service group in question);

T_{ij} - represents the travelling time involved in getting a customer from his origin at i to the centre at j.

λ - represents a parameter estimated empirically for each product, or service, group to reflect the relative perception of travel time arising from the product, or service, group to be purchased.

As a result, the number of consumers from point i, shopping at centre j, E_{ij}, will be:

$$E_{ij} = P_{ij} c_i \qquad (3.4.b)$$

where c_i represents the total number of consumers at point i. As previously mentioned, Huff does not fix λ at 2.0 as Reilly did; for example, he estimated λ to be 2.723 for furniture and 3.191 for clothing. The larger the magnitude of λ, the smaller the time the customer will be willing to spend travelling for a given probability P_{ij}.

3.2.2. Empirical Methods

The set of empirical approaches to the definition of a trading area can be broadly classified into two groups:

i) external to the centre,

and ii) internal to the centre.

Basically, external methods rely on gathering data through normal consumer surveys to determine where potential customers come from. Because it is labour-intensive, this type of survey is prohibitively expensive.

Internal Account Analysis, as suggested by Plotkin (117), attempts to examine the store's customers in a less costly method by using existing data processing systems within the store. Despite its obvious appeal to management, this method has a major disadvantage in that it monitors only existing trade, rather than the full potential which could be available.

The second empirical internal method for defining the trading area of a store also relies on consumer survey. In this case, however, customers are interviewed not just in a particular store, but in the whole shopping centre.

In our case studies, we were able to use Internal Account Analysis due to the existence of extensive delivery and credit files and the discussions we had with management at the store. This however probably did not provide us with the boundaries of the potential trading area, but rather with those of the existing one. We felt, however, that our talks with local management, and their knowledge of the region, compensated for any shortcomings of the method.

3.3 ASSESSING POTENTIAL DEMAND WITHIN A TRADING AREA

Having defined the trading area boundaries, we can proceed to estimate the potential demand for a product or service provided by the store. This process was executed in two stages:

i) estimating the total potential demand in the trading area;

ii) estimating the potential sales of a store.

3.3.1 Estimating a Shopping Centre's Potential Sales

The first step in estimating the total potential of the trading area was to assess the attractiveness of each of the competing shopping centres operating within it. The attractiveness was measured in terms of the probability of a person living in any one of the wards within the catchment area being attracted to any one of the centres. In order to estimate the probability, we first had to establish which are the variables that influence a shopper's decision to prefer one centre to another.

Several studies have looked into the question of establishing the parameters which affect a potential customer's decision in selecting a shopping centre. In 1966, Black (20) attempted to define the attractiveness function as the amalgamation of two variables - the intrinsic attractive power of a centre, and the accessibility of the centre to a given zone. A function of the following form was suggested:

$$A_{ij} = S_i^{ae} \; ^{-bd}{}_{ij} \qquad (3.5)$$

where:

A_{ij} - attractive power of centre i;
S_i - the sales volume;
d_{ij} - the distance between i and j;
a,b - constants.

This work was supported by Wilson (154) and Eilon, Tilley and Fowkes (43) who analysed this model.

A different approach, using the relationship between retail turnover and the total number of shops in a centre, was developed by Muller (112). His study of a sample of shopping centres in the North-Midlands area established a correlation coefficient of 0.995 between retail turnover and the number of shops in a centre. This result was supported both by the findings of Haydock's (68) study, and by Dalrymple (37), who tested this factor hypothetically and obtained a 98% fitness of its variations with the profitability levels. The pulling power of large stores derived from these findings can be formally expressed:

$$A_i = \text{Total number of shops} + 2V + 3D + C + M \qquad (3.6)$$

where:
A_i - attractiveness of centre i;
V - the number of variety stores;
D - the number of deparment stores;
C - the number of chain stores;
M - the number of markets.

This attraction function has both advantages and disadvantages when compared with the simple measure of sales.

While sales figures are difficult to obtain, they offer possibly a more plausible measure of attractiveness. The main criticism that can be directed at the index in Eq. (3.6) is that it does not take into consideration the floorspace of each of the stores and that the categorization of stores may sometimes be questionable.

The importance of this last point is supported by the findings of Kilsby, Tulip and Bristow (88) who examined the importance of forty-three different variables related to the attractiveness of a shop. These variables can be classified into the following groups:

i) those associated with shopping provisions and opportunities in the centre as a whole;

ii) those which described the provision of non-shopping amenities;

iii) those concerned with shopkeepers' policies and the qualities of the individual shops themselves;

iv) those concerned with local customers in a centre, and

v) those related to the design of the shopping centres.

Using principal components analysis, they concluded that a simple measure such as the total gross floorspace of shops is an adequate index of attractiveness in a gravity model.

Information about the identity, precise location and total trading floorspace of shopping centres was collected from a research carried out by the Greater London Council (61) on the Census of Distribution. Distances between the population and the shopping centres was measured between

each of the wards, after their 1971 boundaries were supplied by local planning officers, and the centres in terms of main road distance. It was felt that, as the entire area consisted of relatively wealthy people, most of them travelled by private car rather than by public transport. Hence, using Huff's probabilistic model, i.e. Eq. (3.4.a) with λ set at 2, we generated a matrix containing the probability of consumers living in ward s being attracted to a shopping centre t within the trading area:

shopping centres

wards

$$\begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1t} \\ P_{21} & & & \vdots \\ \vdots & & & \vdots \\ P_{s1} & \cdots & & P_{st} \end{bmatrix}$$

Table 3.1: Centre probability matrix

The above analysis does ignore the possibility of consumers buying goods near their place of work; it was felt that such purchases tended to be mainly for foodstuffs, which a department store does not generally sell, and that hopefully the total discrepancy was small because we had no way of checking on these sales.

With this probability matrix in hand, we proceeded to estimate the potential demand for each item offered by the store for the whole trading area. As it would have been impossible to consider every single item sold by the store, we used 18 categories of similarly grouped items, corresponding

to the internal product groups as defined by the store; indeed, throughout this work, the words "item" and "groups of similar items" are synonymous. There were, as one might imagine, considerable difficulties in combining together statistics from different sources and of different ages. We attempted to reconcile figures wherever we could; where we could not, a qualified pragmatism was used.

The 1971 Census of Population (30) identified pari passus, the number of households in each ward, whilst two reports by the Greater London Council defined the socio-economic grouping (SEG) of the population within each ward (60, 61). Finally, a recent Family Expenditure Survey (49) provided the income and expenditure distribution, coded by occupation of the head of the household, who may be identified with a particular SEG. Thus, from combining these three sets of statistics, we arrived at D_{iqs}, the demand for item i by SEG q in ward s. Hence:

$$D_i^* = \sum_s P_{st} \sum_q D_{iqs} \qquad (3.7)$$

the potential demand for item i in centre t. This data can be presented in form of a matrix:

item

ward

$$\begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & & & \vdots \\ \vdots & & & \vdots \\ d_{s1} & \cdots & & d_{sn} \end{bmatrix}$$

Table (3.2): Trading area potential demand matrix.

With this information in hand, we moved from the shopping centre to the individual store.

3.3.2 Estimating Potential Demand for a Store

Having calculated the probability of a customer being attracted to a centre, for all centres within the trading area, we directed our attention to that centre in which the store we were interested in is located. Several methods have been suggested for assessing potential demand of a store in a trading area, of which, perhaps, the best known is the grid method developed by Ritland (121) and Applebaum (7).

This approach superimposes a grid on a set of contour lines representing either shopping probabilities at the various centres shown, after Huff's method, or drawing power as measured through an empirical approach. Within each grid-square the investigation will:

i) estimate the population;

ii) determine how great its current relevent expenditure is with the particular store or centre under investigation;

iii) estimate the total spending of the population on the relevent product-service group.

From these data, one can calculate the extent to which potential trade is being obtained on a comparative basis. The major disadvantage of this approach is the fact that it does not incorporate any of the store manager's decision variables. In other words, by determining the total potential demand in each grid for a product and comparing the store's performance with it, one is ignoring the level of competition, and therefore at best merely stating the store's share.

Beckman (16) suggested that, for each range of homogeneous goods, the potential buyer is influenced by the store's

attractiveness in respect of the competition and by the distance he has to travel to each one. Hence, for two equal ranges of products, the sales ratio of any two competitors can be formally expressed as:

$$\frac{S_i}{S_j} = f(d_j - d_i + a_{ij}) \qquad (3.8)$$

where:

S_i - the sales volume of store i;

d_i - the distance between the store and the potential buyer;

a_{ij} - the difference in the attractiveness of stores i and j,

the main problem in applying this model being the ambiguity of the difference between the attractiveness of two stores.

If a model is to be of practical use, a manager must be able to determine how changes in his decision variables affect the store's performance. It is with this thought in mind that we approached the problem. First it was necessary to compile a list of the factors influencing a consumer's decision to buy in one shop rather than another. However, a general consensus among store managers in the group was that there were three principal factors, namely floorspace, stock and labour, that would influence demand. Strangely, price was not regarded as such an important factor; on the whole, there was little price distinction between shops in the centre, unless there was a deliberate attempt to sell off a line.

Having established the factors which determine market share, we turned to examine their inter-relationship, for it is their combination which attracts customers. It is firstly suggested that the demand function for a particular item is expressed in terms of relative allocations, i.e. the

market share λ_i is given by:

$$\lambda_i = \frac{D_i}{D_i^*} = f_i \quad (\beta_{ki}/C_{ki}) \qquad \forall k \qquad (3.9)$$

where D_i is the demand for item i at store D, and β_{ki} and C_{ki} the scarce resources of type k allocated to item i in D and in the entire centre respectively. A more precise statement of the form of this function requires a further assumption and we consider two alternative ones.

Let us consider the case of a multiplicative demand function. This is based upon the assumption that, if the allocation of any scarce resource is zero, then the demand is also zero, and furthermore that there are decreasing returns to scale, i.e.

$$\lambda_i = \prod_k r_{ki}^{a_{ki}} \qquad 0 \leq a_{ki} \leq 1 \quad \forall k,i$$

where $r_{ki} \equiv (\beta_{ki}/C_{ki})$. It is felt that the manager would be able to give an indication of the "relative importance" of each scarce resource in influencing demand; for a multiplicative function, we interpret this statement as an estimate of the relative elasticities of demand. Assuming that resource 1 is the base resource, the relative importance is defined by:

$$\gamma_{ki} = \left(\frac{\partial \lambda_i}{\partial r_{ki}} \cdot \frac{r_{ki}}{\lambda_i}\right) / \left(\frac{\partial \lambda_i}{\partial r_{1i}} \cdot \frac{r_{1i}}{\lambda_i}\right) = \left(\frac{r_{ki}}{r_{1i}}\right)\left(\frac{\partial \lambda / \partial \lambda_{ki}}{\partial \lambda / \partial r_{1i}}\right)$$

or $$\gamma_{ki} = a_{ki}/a_{1i} \qquad \forall k,i$$

The interpretation is not entirely fortuitous as it does result in an expression that is independent of r_{ki}. The demand function may now be written as:

$$\lambda_i = \left(\prod_k r_{ki}^{\gamma_{ki}}\right)^{a_{1i}} \qquad \forall i \qquad (3.10)$$

Given the status quo (λ_i^0 , r_{ki}^0), we can compute

$$a_{1i} = \ln\lambda^0 \,/ \sum_k \gamma_{ki} \ln r_{ki}^0 \qquad \forall i \qquad (3.11)$$

Let us now consider the case of a separable demand function. The underlying assumption here is that demand is generated by each scarce resource in isolation, and that the total demand is computed from the addition of the individual demands, i.e.,

$$\lambda_i = \sum_k f_{ki}(r_{ki}) \qquad \forall i$$

Notice that this assumption makes the clear distinction between demand and sales, for obviously there can be no sales unless a positive quantity of all the scarce resources is allocated. To produce a logical interpretation of "relative importance" in this situation is, however, difficult, and perhaps the easiest direction is to assume that, in practice, there would be little movement away from the status quo, so that a linear demand function would not be unrealistic, i.e.,

$$\lambda_i = \lambda_i^0 + \sum_k d_{ki}(r_{ki} - r_{ki}^0) \qquad \forall i$$

Now, using the same definition of relative importance yields:

$$\lambda_i = \lambda_i^0 + d_{1i}\, r_{1i} \sum_k \gamma_{ki}\left(1 - \frac{r_{ki}^0}{r_{ki}}\right) \qquad \forall i \qquad (3.12)$$

But there is no simple means of calculating d_{1i} from the status quo; the only way is, when the allocation has altered from the original status quo, to make a direct substitution.

3.4 REGIONAL DEMAND FUNCTIONS

So far we have outlined various methods of deriving the demand function facing a store and discussed each method's relative merits. Before we can proceed any further, we need to direct our attention to one of the major aspects of demand functions, namely, their response to changes in one or more variables which determine their value. In other words, perhaps the most important feature of demand functions for planning purposes is their elasticities to changes in parameters within the store's control, and to a lesser extent, to changes in economy wide variables. Consequently, there follows a brief discussion of some of the topics in consumer theory which are relevant in determining the elasticities of regional demand functions.

3.4.1 Consumer Preferences

A basic prerequisite to any analysis of demand functions is a definition of the assumptions that govern consumers' behaviour. The subject of consumers' choice is a bundle of goods by which we mean a collection of different goods, the quantity of each being given. Let us follow Hadar's (65) notation whereby there are n goods denoted by $x^1, x^2, \ldots, x^n$, then a bundle is represented by the vector $x = (x^1, x^2, \ldots, x^n)$. Bundles are identified by subscripts such as $x_0 = (x_0^1, x_0^2, \ldots, x_0^n)$, and by the nature of the problem contain only non-negative quantities. For the purpose of comparing bundles, the following symbols will be used:

Vectors		Numbers	
$x_0 = x_1$	means	$x_0^i = x_1^i$	$\forall i$
$x_0 \geq x_1$	means	$x_0^i \geq x_1^i$	$\forall i$
$x_0 \geq x_1$	means	$x_0^i \geq x_1^i$	and
		$x_0^i > x_1^i$	for at least one i
$x_0 > x_1$	means	$x_0^i \geq x_1^i$	$\forall i$

and similarly the phrase "is at least as good" will be denoted by the symbol ζ.

We now can proceed to state formally the assumptions about consumers' preference:

A.3.1. Completeness. Given any set of bundles, then for every two bundles x_0 and x_1 in that set, either $x_0 \zeta x_1$, or $x_1 \zeta x_0$,

The purpose of this assumption is to rule out any possbility of indecision on the part of the consumer. By this, we mean that if shown any two bundles, the consumer will either prefer one to the other or be indifferent, but never will there be a case of indecision.

A.3.2. Transitivity. For any three bundles x_0, x_1 and x_2 if $x_0 \zeta x_1$, and $x_1 \zeta x_2$, then $x_0 \zeta x_2$.

This assumption expresses an intuitively reasonable consistency restriction on the consumer's preferences. In fact, assumptions A.3.1 and A.3.2 impose a ranking on the bundles which make it possible to order them in accordance with the consumer's preferences, and any set whose elements satisfy such assumptions is said to be completely

preordered. Sometimes a preordering such as the above is also referred to as "weak" because of the inclusion of indifference.

A.3.3. Greediness. For any two bundles x_0 and x_1, if $x_0 \geq x_1$; then $x_0 \zeta x_1$.

This assumption represents the property that of any pair of bundles the consumer always prefers the "larger of the two". By larger, we mean that the bundle has more of at least one good , and no less of any good than the other bundle.

These are the major assumptions we make about the rational consumer's behaviour. Essentially, we have assumed that the consumer knows his preferences, that in some sense these are consistent, and finally, that the consumer always prefers to have "more" than "less".

The analysis of consumer behaviour is greatly simplified if a numbering scheme is introduced. By that, we mean a system where, for each possible bundle, a matching number is attached in such a way that for any two bundles, it would be true that the consumer always chooses that bundle which has the largest number. In other words, a numbering scheme is tantamount to a functional relationship, whose domain is the set of all possible bundles and whose range is the set of real numbers. If such a function exists, then we can say that the system of preferences is representable, and the function itself referred to as a utility function. The last basic assumption is, therefore:

A.3.4. Representability. The consumer's system of preferences is representable.

Having highlighted the rationale underlying a consumer's behaviour, we need not specify the properties of a utility

function. Suffice to say that the utility function, by its very nature, must be capable of reflecting all the properties of the consumer's preference structure.[1]

Let us adopt Hadar's definition of a demand function of a single consumer as a relationship between, on the one hand, the amount of commodity he is able and willing to buy, and on the other hand, his budget and the prices of all commodities. Such a function may be written as:

$$x^j = h^j(p^1, p^2, \ldots, p^n, m) \quad j = 1,2,\ldots,n \qquad (3.13)$$

This expression says that the dependent variable x^j is a function of all the independent variables $p^1, p^2, \ldots, p^n, m$ that appear in the brackets, where p^n denotes the price of item n, m the consumer's budget, and x^j the demand for item j.

3.4.2. Income Effects

Although, when operating in a perfect competition, the firm is assumed to have no influence on economy wide variables such as consumers' income, this does not imply that, when planning ahead, they should be ignored. Clearly, a firm's performance is related to the level of the consumers' budget and therefore the impact of changes in it need to be analysed.

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1. Two further assumptions, of a technical nature, concerning utility functions:

   i) Continuity - in order to guarantee the existence of a utility function, a certain continuity condition has to be imposed.

   ii) Convexity - we assume indifference curves exist and slope towards the right, i.e. the indifference curves are convex to the origin.

In order to predict whether an increase in a consumer's income will cause an increase in his consumption of a particular product or service, we need to investigate the derivatives of the demand function. Using Cramer's rule, Hadar defines:

$$h_m^j = \partial h^j / \partial m = \frac{D_{n+1,j}}{D} \qquad (3.14)$$

where the derivative h_m^j indicates the rate of change in the proportion of the budget spent on j, with all the prices given, as there is an infinitesimal increase in the budget. D is the determinant of the coefficient matrix of the objective function and the budget constraint, and D_{n+1} is a typical cofactor. While the sign of D given n is known, the sign of $D_{n+1,j}$ is indeterminate.

There are three possible signs the income effect can have, and each implies a customary class of goods, namely that:

If $h_m^j > 0$ then j is said to be a superior good ;
If $h_m^j = 0$ then j is said to be a neutral good ;
If $h_m^j < 0$ then j is said to be an inferior good .[2]

There are two points worth mentioning; first, the budget elasticity of demand, h_m^j, is a function of p^i and m, so that the values of h_m^j can change as price-income combinations change. The second point is that, due to the greediness assumption A.3.3, which implies that a consumer will always spend all his budget, it is impossible for all products to be neutral or inferior, consequently at least one must be superior, i.e. have a positive h_m^j.

---

2) This is the Hicksian definition of an inferior goods as opposed to one which depends on the Giffen paradox.

It is against this background that we can turn to analyse the special characteristics of the demand functions facing the store managers. For a meaningful assessment of the impact of changes in consumers' income on the firm, the demand functions developed in sections 3.2 and 3.3 have to be analysed in greater detail. To the existing data, budget elasticities for each product and service have to be added, and this has to be repeated for each SEG since we assume (and empirical data supports it) that SEGs respond differently to changes in their income.

This data can be clearly presented using matrix notation, hence:

$$\begin{bmatrix} h^1_{m_{0,1}} & h^2_{m_{0,1}} & \cdots\cdots\cdots & h^n_{m_{0,1}} \\ h^1_{m_{0,2}} & & & \vdots \\ \vdots & & & \vdots \\ h^1_{m_{0,k}} & \cdots\cdots\cdots & & h^n_{m_{0,k}} \end{bmatrix}$$

Table (3.3): RBD Matrix

Where the Regional Budget Elasticity of Demand (RBD) table elements are defined as follows, $h^n_{m_{0,k}}$ is the budget elasticity of demand of SEG k's demand function, at the point defined by budget m_0, for good n.

The above table, however, does not provide the full data requirements of the store planners. In addition,

the firm would need to know the distribution of budget elasticities of demand in order to be in a position to evaluate the net aggregate effect of the exogenous change in income level, a consequence of the catchment area not being equally divided between SEGs. For this purpose, the data related to the socio-economic distribution of the population living in the trading area can be used to weigh the relative importance of the different budget elasticities. Let us formally represent the socio-economic distribution of households in the catchment area by a diagonal matrix, where each of the non-null elements s_k, represents the number of households belonging to SEG k, and the trace of which is the total number of households living within the boundaries of the catchment area. The product of using this latter matrix as a premultiplier on the RBD matrix is the Regional Weighted Budget Elasticity of Demand (RWBD). This can be formally expressed by:

$$\begin{bmatrix} s_1 & 0 & 0 & \cdots & 0 \\ 0 & s_2 & \cdots & & 0 \\ \vdots & & & & \\ 0 & \cdots & & & s_k \end{bmatrix} \begin{bmatrix} h^1_{m_{0,1}} & h^2_{m_{0,1}} & \cdots & h^n_{m_{0,1}} \\ h^1_{m_{0,2}} & \cdots & \cdots & h^n_{m_{0,2}} \\ \vdots & & & \\ h^1_{m_{0,k}} & \cdots & \cdots & h^n_{m_{0,k}} \end{bmatrix} = \begin{bmatrix} b^1_{m_{0,1}} & \cdots & b^n_{m_{0,1}} \\ b^1_{m_{0,2}} & \cdots & b^n_{m_{0,2}} \\ \vdots & & \\ b^1_{m_{0,k}} & \cdots & b^n_{m_{0,k}} \end{bmatrix}$$

where $b^n_{m_{0,k}}$ represents the budget elasticity of demand of SEG k, for product n, at the point defined by m_0, and weighted by the number of households belonging to SEG k.

A more meaningful measure of the effect of a change in the consumers' income is the value of the change in monetary units, specifically the value of the incremental sales. As the data in previous sections points out, SEGs do not differ solely in their expenditure pattern; there is also a difference between the SEGs' average annual income. As a result, the incoporation of income into the RWBD matrix introduces a second weighting, this time according to the average SEG income. Let us formally represent the distribution of SEG income by a diagonal matrix, where the non-null elements, I_k, represent the annual income of SEG k. We use this income matrix as a premultiplier on the RWBD matrix, hence:

$$\begin{bmatrix} I_1 & 0 & 0 & 0 & 0 \\ 0 & I_2 & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & I_k \end{bmatrix} \begin{bmatrix} b^1_{m_{0,1}} & \cdot & \cdot & \cdot & b^n_{m_{0,1}} \\ b^1_{m_{0,2}} & \cdot & \cdot & \cdot & b^n_{m_{0,2}} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ b^1_{m_{0,k}} & \cdot & \cdot & \cdot & b^n_{m_{0,k}} \end{bmatrix} = \begin{bmatrix} c^1_{m_{0,1}} & c^2_{m_{0,1}} & \cdot & \cdot & c^n_{m_{0,1}} \\ c^1_{m_{0,2}} & \cdot & \cdot & \cdot & c^n_{m_{0,2}} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ c^1_{m_{0,k}} & \cdot & \cdot & \cdot & c^n_{m_{0,k}} \end{bmatrix}$$

where $c^n_{m_{0,k}}$ is the budget elasticity of demand of SEG k for product n defined at point m_0 and weighted both by the size of SEG k and its average annual income. The product of the multiplication is the Net Budget Elasticity of Demand (NBED).

The net regional effect of a change in income on the demand for product i can now be expressed as a summation of column i. For each summation, the three possible outcomes are:

if $\sum_{z=1}^{k} c^{i}_{m_{0,z}} > 0$ consumption of good i will increase with income.

if $\sum_{z=1}^{k} c^{i}_{m_{0,z}} = 0$ consumption of good i will remain unchanged with changes in income.

if $\sum_{z=1}^{k} c^{i}_{m_{0,z}} < 0$ consumption of good i will decline as income increased.

Clearly the overall regional effect for all products and services must equal the changes in regional income, hence:

$$\sum_{i=1}^{n}\sum_{z=1}^{k} c^{i}_{m_{0,z}} = \sum_{z=1}^{k} s_z \Delta I_z,$$

where ΔI_z is the income increment in SEG z's income. There is no possibility of determining the net regional effect a priori to the incorporation of the budget elasticities of demand of all SEGs, the sizes of the SEGs and their annual average income.

3.4.3 Price Effects

We now turn to examine the effects of a price change. This, being a managerial decision parameter, is probably a more practical topic from the firm's point of view. In a manner similar to the treatment of income effects, i.e., using Cramer's rule, Hadar defines:

$$h^{j}_{i} = \partial h^{j}/\partial p^{i} = -\lambda \frac{D_{ij}}{D} - x^{-i} \frac{D_{n+1,j}}{D} \qquad (3.15)$$

and similarly the sign of h^{j}_{i} is indeterminate, since the right hand side of Eq. (3.15) involves cofactors of D, whose signs are not restricted by the conditions of utility

maximization. Two points need to be borne in mind; first, that a change in the price of one commodity can have an effect on both the quantity consumed of the product itself and on the quantity consumed of other items. Secondly, these changes can be split into a substitution effect and an income effect. This characteristic of the price change is due to the fact that price variations have a direct effect on the consumer's purchasing power as well as presenting him with a new relative prices structure. Since these two effects can operate on the demand function in opposite directions and the store manager is interested in the net effect, we will examine the sum of the two effects.

A formal illustration of the separation of the total price effect into its two components is the Slutsky equation, which, using Hadar's notation, can be written:

$$h_i^j = S_i^j - \bar{x}^i h_m^j \qquad (3.16)$$

where the two terms on the right hand side stand for the substitution and income effect respectively. Using Eq. (3.16), we can examine both the own-price elasticity of product i, and the cross-elasticity of the demand for project j following a change in p^i. The case where i = j, i.e. the own-price elasticity of item i, is represented by Table (3.5). Hence:

(3)

S_i^j	h_m^j	$-\bar{x}^i h_m^j$	S_i^j	$-\bar{x}_i h_m^j$	h_i^j	Type
<0	≥ 0	≤ 0	$\gtreqless$		≤ 0	Normal
	<0	>0	$\leq$			
			<0		>0	Giffen

Table (3.4): Own price elasticity

and for $i \neq j$;

(4)

S_i^j	h_m^j	$-\bar{x}_i h_m^j$	S_m^j	$-\bar{x}_i h_m^j$	h_i^j	Type
≥ 0	≥ 0	≤ 0	$<$		<0	j is GC for i
			$>$		≥ 0	j is WGS for i
	<0	>0	$\gtrless$			
<0	≥ 0	≤ 0			<0	j is GC for i
	<0	>0	$>$			
			$\leq$		≥ 0	j is WGS for i

Table (3.5): Cross price elasticities

---

3) That $S_i^j < 0$ for any $i=j$ is the Fundamental Theorem of Consumption Theory, known also as the Law of Demand, which states that a compensated change in the price of the good (the compensation being equal to the loss in real income) always causes a decrease in the quantity demanded of that good . This result is valid only for compensations through nominal income changes. If real income is kept constant by suitable changes in prices, the theorem does not necessarily hold.

4) The S_i^j values in Table (3.6) are known as Hicksian substitutes which represent a symmetrical relationship, contrary to that reflected by gross substitues. Clearly, since $S_i^j > 0$ for all $i=j$ at least one S_i^j must be positive where $i \neq j$.

where GC and WGS stand for gross complement and weak gross substitute respectively.

Returning to the problem facing the decision makers in a department store, clearly the store planners would require both own price and cross price elasticities before formulating a pricing policy. The data required by management for such purposes can be illustrated in a matrix form, hence:

$$\begin{bmatrix} h_1^1 & \cdots & h_n^1 \\ h_1^2 & \cdots & h_n^2 \\ \vdots & & \vdots \\ h_1^n & \cdots & h_{n-1}^n \; h_n^n \end{bmatrix}$$

Table (3.6): Price Elasticity Matrix for SEG k

where the element h_{n-1}^n in the Price Elasticity Matrix (PEM) is the price elasticity of the demand for product n as a result of a change in p^{n-1}. Obviously, the elements on the main diagonal represent the own price elasticities, i.e. h_i^j for i=j, and all other elements the cross price elasticities, i.e. where $i \neq j$.

Two goods can be substitutes for one type of consumer and complements for another. Since the firm is interested in the overall price effect within its catchment area, an aggregation of cross price elasticities over all SEGs is necessary. As in the previous case of

income elasticities, a weighting by expenditure and number of households of SEG k is to be incorporated in the model. Hence, by definition:

$$\sum_{z=1}^{k} \delta_z = \sum_{z=1}^{k} (s_z E_z^i / \sum_{z=1}^{k} s_z E_z^i) = 1 \quad i = 1,2,..,n \quad (3.17)$$

where s_z is the number of households in the trading area belonging to SEG z and E_z^i their expenditure on product i. By doing a scalar multiplication, i.e. δ_z on PEM_z, and a matrix addition for SEGs, we produce the Regional Price Elasticity of Demand (RPED) matrix, which can be illustrated as follows:

$$\begin{bmatrix} \delta_1^1 & \cdots & \delta_n^1 \\ \delta_1^2 & \cdots & \delta_n^2 \\ \vdots & & \vdots \\ \delta_1^n & \cdots & \delta_{n-1}^n \; \delta_n^n \end{bmatrix}$$

Table (3.7) : RPED Matrix

Where δ_{n-1}^n is the price elasticity of the demand function for item n in respect to changes in the price of product n-1, weighted by the number of households and average expenditure of the z SEGs, i.e.

$$\delta_{n-1}^n \equiv \sum_{z=1}^{k} (h_{n-1}^n \delta_z).$$

3.5 SUMMARY

Studies designed to assess the trade available in the area surrounding a store, and to establish the share likely to be secured by it, can provide guidance to retailers on their management decisions. This chapter discusses various methods of estimating these values, and the factors which determine them.

We divided the process by which a potential customer is attracted to a store into two stages. First, we examined the stage in which a shopper selects the shopping centre he will frequent among those operating within the catchment area of the store. The measure of attractiveness was reduced to two parameters, the competing centres' floorspace, and an inverse relationship with the square of the travelling distance between the customer and the centres.

Having established the probability of customers from different points in the trading area being attracted to the centre in which the store under examination was located, we analysed the second stage in the process, i.e. that of the customer selecting a store within the centre. We established, through talks with store managers, that three factors affect the potential buyers' second decision, specifically floorspace, stock levels and manning. From this, and a combination of socio-economic data, we produced the store's market share, defined in terms of sales.

If this information is to enable the retailer to develop an effective strategy for his store, namely what to promote by allocating more resources, some idea of the response of demand to such changes is vital. Consequently, we discussed some of the topics related to regional demand functions, such as the definition of a rational consumer behaviour, the effects of changes in demand as a result of budget increments, and the various impacts of price

movements both on the product whose price changes, and on its complements and substitutes. This was done taking into account the relative buying power of the various SEGs in the particular trading area.

A successful application of this model depends on the store manager possessing an equally good understanding of the way the internal environment, i.e. the store itself, functions, and of the interaction between the two environments. These, and their managerial implications, are the subject of the chapter which follows.

It is worth noting that any manager who requires the data required for computing a store's potential need not go through the long and tedious process of collecting all the data in order to use our model. These data are available from commercial firms, whose business it is to calculate the potential demand in any area defined by its reference in the U.K. grid. Such an example is SITE (140), which uses a combination of public and private data to provide this kind of service.

C H A P T E R 4

MODELLING A DEPARTMENT STORE

4.1 THE STRUCTURE OF A DEPARTMENT STORE

4.1.1 The Store Manager's Problem

Certainly in the group which we were studying, and generally in many such groups, the store manager has considerable autonomy provided that he (or she) is reasonably successful. The manipulation of certain decision parameters is under his own control; hence the title of this section.

Superficially, his problem is easily stated: for each item in the store, and this may run into thousands, he has to match supply with demand in such a way as to optimize some objective(s). Supply is very much under his control, and as such is a primary decision parameter; unfortunately, demand is not fixed and independent of a number of other parameters under the manager's control.

As we have discussed in great depth in the previous chapter, it is possible (in theory, at any rate) to compute the potential demand that the consumer will place on any particular shopping centre. A department store manager in such a centre can do little to influence this potential demand, assuming, of course, that the size of the store is small in relation to the overall size of the centre. The assumption is likely; department stores are usually (invariably) placed in large shopping centres.

Once the consumer has arrived in the shopping centre, the influence that the manager has is considerably increased. He is competing using the resources at his disposal with the other shops and stores in the centre for the consumers' attention. If he increases the resources allocated to one particular item, ceteris paribus, he must increase the demand in his store for the item.

But, the manager's problem has not finished. The resources he is allocating are strictly finite, and hence the various items are in competition with one another. For example, short of rapidly expanding the store, the total selling floor space is fixed; if more space is allocated to one item, less space has to be allocated to another. The demand for the former may increase, but for the latter will decrease, which could result in a net detrimental effect on the manager's objective. Thus, the manager has to decide upon the optimal allocation of these resources, a typical mathematical programming problem, although of course highly complex. The next sections will discuss the construction of the various relationships necessary to solve the manager's problem optimally.

4.1.2. The Scarce Resources

In the previous chapter, we briefly mentioned the fact that when asked, store managers in the group reached a consensus about the existence of three major factors, namely: floorspace, stock, and labour, that would influence demand. In this section, we discuss some of the characteristics of these inputs.

a) Floorspace is perhaps the most obvious scarce resource. It is obviously limited by the physical size of the store and also, ceteris paribus, an increase in the space allocated to a goods should increase its relative demand. However, in practice it is not as simple as that: floorspace is not homogeneous in its appeal. If we adopt as an index profit/sq.ft., then the index decreases as the distance from the ground floor increases. Also, the space on each floor may be subdivided; prime space is opposite the main entrance, good space

is opposite the side entrances and the mouths of escalators and lifts, whilst the rest is simply standard space. To increase the space allocated to a good and simultaneously move it to a worse kind of space, need not necessarily increase the demand for that good at all.

This lack of homogeneity presents a major problem. What we would like to do is construct a model that would ultimately compute the amount of floorspace that should be allocated to a particular item and also where the floorspace should be located. This would require such data as the change in sales as a goods is moved around the store: these data are just not available, nor are any records kept.

b) Stock Generally, department stores have little storage space, their total stock being usually on display to encourage the customer. The manager faces the classical stock control problem (except that here it is exacerbated by the fact that demand is a function of the stock level) examplified by two situations that may occur:

i) the level of stock needed to generate a certain level of demand is greater than that level of demand, which implies that there will always be unsold items on display. This may create problems of stock turnover and cause low profitability returns per stock item.

ii) the level of stock needed to generate a certain level of demand is less than that level of demand, which implies that customers will have an unsatisfied demand, and sales will be lost. This situation

is generally an anathema to managers who primarily see themselves as traders rather than businessmen.

There are further complications, namely that the stock level must be restricted by the floor space available and also by the working capital funds. The store can only afford to hold limited stocks.

c) Labour: In the front line of any store are the sales staff. Such staff are extremely important, as they directly influence the customer. As the level of staffing drops, the waiting time of customers increases and ultimately demand will decrease. But staff are a direct cost to the manager, and as such he has to decide upon a suitable level for each item. His task is complicated because many of the staff are specialists, and thus are not interchangeable.

4.2 A HEURISTIC

So far, we have outlined the various scarce resources that a manager must allocate and discussed the restrictions and interdependencies that impinge on each. Before we can proceed further, we need to define the relationship between the store's internal and external environments, and identify the objective that the manager is trying to optimize.

Basically, we can assume that, irrespective of the resource allocation policy adopted by the store manager, the store cannot attract a demand which exceeds the total potential of the shopping centre in which the particular store operates. Even the case of the individual store attracting the full potential demand of the centre seems highly improbable in view of the assumptions made earlier concerning the relative size and influence of a department store. In other words, the potential demand generated in a centre can be treated as an upper limit on the sales a store can hope to achieve. In terms of the constraints limiting the options open to the store manager, we can view the information sent by the external environment surrounding the store as a fourth constraint to be added to the constraints generated by the finite amounts of floorspace, labour and stock. This concept is graphically illustrated in Fig.(4.1) where the inter-relationship between the store and its environment and the intra-relationships within the store are reflected.

As for the objective the manager is trying to optimize, the evidence for the identification tends to be contradictory. While most managers in isolation are primarily concerned with turnover, they are principally traders, and their status increases with turnover; furthermore, in the group the pecking order of stores was based upon turnover. On the other hand, taking the group view, store contribution (an approximation to profit) is a much more useful

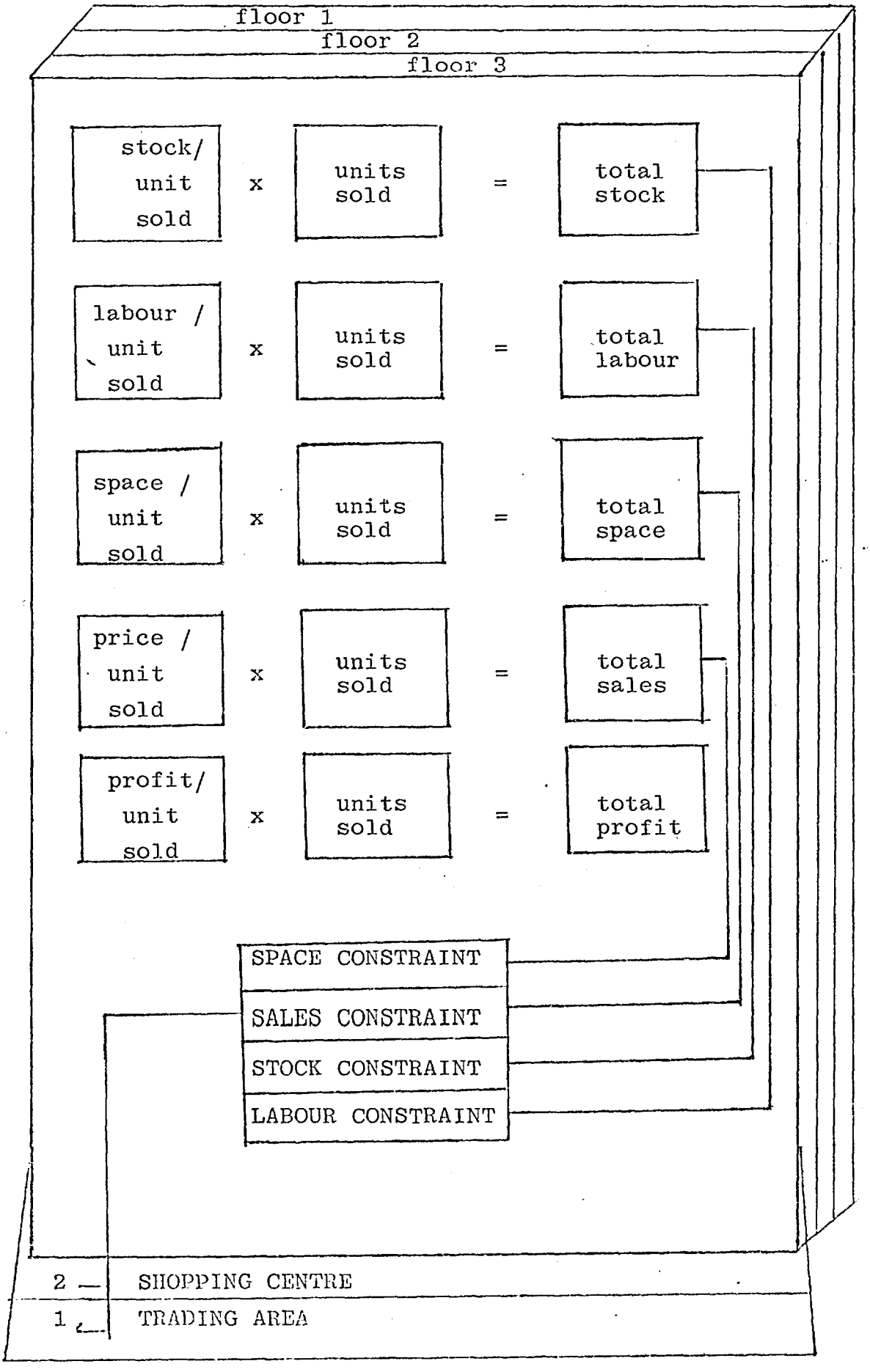


Fig. (4.1): A Department Store Model

objective and the one that the head office is principally concerned with. We adopted a pragmatic approach: as we had been briefed by the head office, we used a profitability objective.

As we have seen, it is possible to formulate this problem as a mathematical programme. The profitability is easily calculated as the prices of goods are fixed; buying costs, costs of floorspace (rent, rates and depreciation) and labour (various hierarchies of sales staff) are all easily obtainable. The different types of scarce resources, their limitations and inter-relationships with each other and with the demand function, have been described. So one could solve the allocation problem as described relatively easily, although the model might be extremely large. Section 4.4 describes simple models and discusses appropriate solution methods.

However, our first approach for the management of the group was not to use an MP, but to develop a heuristic. The heuristic made certain gross assumptions, but it had the virtue of being easily and quickly understood. It started by assuming that, just as the division of demand between centres was on the basis of relative floorspace, the potential demand in i could be divided up in proportion to the relative floorspace in C - i.e.,

$$D_i = \left(\frac{\beta_{2i}}{C_{2i}}\right) \quad D_i^*$$

where the subscript 2 refers to floorspace.

For each item, or rather group of items (i), various (p) discrete sales levels were identified, S_{ip}, p = 1, 2, ... With each level were associated four figures: the profit generated by that level of sales (P_{ip}), the space required for those sales (SP_{ip}), the stock required for the sales

(ST_{ip}) and finally the level of staffing required to achieve the sales ($\dot{L}_{ip}$). Rather than considering every possible combination of scarce resources as the MP would do, the heuristic looks only at certain a priori values. One detailed concession was made, namely to divide the floorspace up into different types. In a three-floor store, we examined, we identified seven grades of space depending on the floor one was on, and whereabouts on that floor. This involved (in theory, at any rate) producing the above figures for each type of space as well (let the subscript t denote the type of space). Calculate as a profitability index, the profit/sq.ft.

$$\bar{P}_{ipt} = P_{ipt}/SP_{ipt} \quad \forall\ i,p,t.$$

For each scarce resource, we assume certain known limitations, which are easily ascertainable from the manager.

The detailed heuristic is as follows:

STEP 1: Scan the matrix $\bar{P}$ for the highest figure; let this be $\bar{P}_{xyz}$. As decreasing returns to scale were assumed, the greatest profitability was generally at the lowest level of sales.

STEP 2: Calculate the running totals:

i) add SP_{xyz} to total floor space of type t so far consumed.

ii) add ST_{xyz} to total value of stock held so far.

iii) add L_{xyz} to total staff employed so far.

STEP 3: Compare the current usage of the scarce resources with the limitations. If any constraint is violated, go to 4. Otherwise go to 5.

STEP 4: Identify the accepted item with the lowest profitability. Delete its current level from further consideration and adjust the running totals accordingly. Go to 3.

STEP 5: Delete from further consideration, This ensures that the same item will not be spread out on different types of floor.

STEP 6: If there exists spare capacity and some items still have to be allocated, go to 1. Otherwise calculate store-wide figures and stop.

In STEP 4, we used three different decision rules to identify the item to be deleted:

i) by cancelling the smallest value in the initial solution;

ii) by cancelling the item with the smallest marginal fall in profit;

iii) by cancelling the item with the smallest percentage fall in profit.

Such an heuristic is not optimal, but it does enable the treatment of, e.g. the discreteness of labour, non-convex profit functions, discontinuities in floorspace, as well as being extremely rapid to compute. Data collection is a major disadvantage and, whilst we have demonstrated its usefulness to the management, the application to an entire store is still in the future. We now present some of the results of a numerical example run on partly arbitrary data, consisting of 18 product groups, 7 types of space on 3 floors and 5 levels of performance for each product.

TABLE (4.1): Iteration by smallest profit index

```
   THE INITIAL SOLUTION IS
    130     96    127     99      0    130     95    125     91     42     50     75    130     45     88    123    139    110

THE MARGINAL PROFITABILITY VECTOR IS
 NOT USED IN THIS VERSION .

   SPACE REQ.      8952      STOCK REQ.      1363      LABOUR REQ.      4210

   SPACE CON.      2000      STOCK CON.      2000      LABOUR CON.      6000

MINIMAL VALUE        9  LOCATION        5

   THE TRANSFORMED MATRIX IS
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45
    78     47     30      0     45     39     35     23      8     35     36     15      0     45

   THE INITIAL SOLUTION IS
    130     96    127     99      3    130     95    125     91     42     50     75    130     45     88    123    139    110

THE MARGINAL PROFITABILITY VECTOR IS
 NOT USED IN THIS VERSION

   SPACE REQ.      8917      STOCK REQ.      1353      LABOUR REQ.      4774

   SPACE CON.      2000      STOCK CON.      2000      LABOUR CON.      6000

MINIMAL VALUE        3  LOCATION        5

   THE TRANSFORMED MATRIX IS
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42
    75     47     30      3     42     36     32     20      5     32     33     12      3     42

   THE INITIAL SOLUTION IS
    130     96    127     99      3    130     95    125     91     42     50     75    130     45     88    123    139    110

THE MARGINAL PROFITABILITY VECTOR IS
 NOT USED IN THIS VERSION

   SPACE REQ.      8843      STOCK REQ.      1303      LABOUR REQ.      4507

   SPACE CON.      2000      STOCK CON.      2000      LABOUR CON.      6000

MINIMAL VALUE       42  LOCATION       11
```

```
THE INITIAL SOLUTION IS
   130    99   127    99    13   130    95   125    91    40    50    75   130    45    88   123   139   110

THE MARGINAL PROFITABILITY VECTOR IS
    30    17    12    11    13    20    12    25    21     5    10    15    31     5    21    22    16    22

 SPACE REQ.     6959    STOCK REQ.     1374    LABOUR REQ.     4923

 SPACE CON.     2000    STOCK CON.     2000    LABOUR CON.     6000

MINIMAL VALUE         5  LOCATION       14

 THE TRANSFORMED MATRIX IS
    71    47    83     3    -3    45    39    35    23    25     8    35    30    -3    38    15    80    40
    73    55    91    75    -1    70    48    70    33    30    17    40    48    15    45    33    95    55
    90    70    93    80     3    80    70    95    48    35    25    47    57    30    59    75    99    77
   101    73   119    88     3   110    83   100    70    40    40    60    99    40    67   101   123    88
   130    90   127    99    13   130    95   125    91     0    50    75   130     0    88   123   139   110

 THE INITIAL SOLUTION IS
   130    90   127    99    13   130    95   125    91    40    50    75   130    40    88   123   139   110

THE MARGINAL PROFITABILITY VECTOR IS
    30    17    12    11    13    20    12    25    21     5    10    15    31    10    21    22    16    22

 SPACE REQ.     6626    STOCK REQ.     1384    LABOUR REQ.     4904

 SPACE CON.     2000    STOCK CON.     2000    LABOUR CON.     6000

MINIMAL VALUE         5  LOCATION       10

 THE TRANSFORMED MATRIX IS
    73    47    83     3    -3    45    39    35    23    25     8    35    36    -3    38    15    80    40
    75    55    91    75    -1    70    48    70    33    30    17    40    48    15    45    33    95    55
    90    70    93    80     3    80    70    95    48    35    25    47    57    30    59    75    99    77
   101    73   113    88     3   110    83   100    70     0    40    60    99    40    67   101   123    88
   130    90   127    99    13   130    95   125    91     0    50    75   130     0    88   123   139   110

 THE INITIAL SOLUTION IS
   130    90   127    99    13   130    95   125    91    35    50    75   130    40    88   123   139   110

THE MARGINAL PROFITABILITY VECTOR IS
    30    17    12    11    13    20    12    25    21     5    10    15    31    10    21    22    16    22

 SPACE REQ.     6455    STOCK REQ.     1342    LABOUR REQ.     4757

 SPACE CON.     2000    STOCK CON.     2000    LABOUR CON.     6000
```

TABLE (4.2): Iteration by smallest marginal value

TABLE (4.3): Iteration by smallest percentage fall

```
 130   90   127   99   13   130   95   125   91   0   50   75   130   45   88   123   139   110

 THE INITIAL SOLUTION IS
  130   90   127   99   13   130   95   125   91   40   50   75   130   45   88   123   139   110

THE MARGINAL PROFITABILITY VECTOR IS
 .2813 .1889 .0945 .1111 .5263 .1538 .1263 .2000 .2308 .1250 .2000 .2000 .2385 .1111 .2386 .1789 .1151 .2000

 SPACE REQ.    6959    STOCK REQ.    1374    LABOUR REQ.    4323

 SPACE CON.    2000    STOCK CON.    2000    LABOUR CON.    6000

MINIMAL VALUE  .0945  LOCATION      3

 THE TRANSFORMED MATRIX IS
  75   47   60   74   13   45   39   35   23   25   8   35   38   1   8   43   33   44
  75   55   70   74   13   75   65   70   38   30   17   45   63   15   8   55   55   55
  90   70   95   74   13   105   65   95   38   35   25   45   57   30   8   55   77   77
 100   73   110   99   13   110   65   110   70   40   40   60   99   45   8   67   88   88
 130   90   127   99   13   130   95   125   91   0   50   75   130   45   88   123   139   110

 THE INITIAL SOLUTION IS
  130   90   115   99   13   130   95   125   91   40   50   75   130   45   88   123   139   110

THE MARGINAL PROFITABILITY VECTOR IS
 .2308 .1889 .1391 .1111 .5263 .1538 .1263 .2000 .2308 .1250 .2000 .2000 .2385 .1111 .2386 .1789 .1151 .2000

 SPACE REQ.    6670    STOCK REQ.    1395    LABOUR REQ.    4746

 SPACE CON.    2000    STOCK CON.    2000    LABOUR CON.    6000

MINIMAL VALUE  .1111  LOCATION      14

 THE TRANSFORMED MATRIX IS
  75   47   60   74   13   45   39   35   23   25   8   35   38   1   8   43   33   44
  75   55   70   74   13   75   65   70   38   30   17   45   63   15   8   55   55   55
  90   70   95   74   13   105   65   95   38   35   25   45   57   30   8   55   77   77
 100   73   110   99   13   110   65   110   70   40   40   60   99   45   8   67   88   88
 130   90   115   99   13   130   95   125   91   0   50   75   130   40   88   123   139   110

 THE INITIAL SOLUTION IS
  130   90   115   99   13   130   95   125   91   40   50   75   130   40   88   123   139   110

THE MARGINAL PROFITABILITY VECTOR IS
 .2313 .1889 .1391 .1111 .5263 .1538 .1263 .2000 .2308 .1250 .2000 .2000 .2500 .2386 .1789 .1151 .2000
```

TABLE (4.4): Results of heuristic

```
THE PROFIT FROM    1  IS     36428  SPACE USED IS      234  AT     156  PER SQUARE FOOT   THE SALES TOTAL     99
THE PROFIT FROM    2  IS      6761  SPACE USED IS      123  AT      47  PER SQUARE FOOT   THE SALES TOTAL     78
                      ITEM    3  WILL NOT BE SOLD ON SPACE TYPE   1   ON FLOOR    1
THE PROFIT FROM    4  IS     24375  SPACE USED IS      333  AT      75  PER SQUARE FOOT   THE SALES TOTAL     66
THE PROFIT FROM    5  IS      -321  SPACE USED IS      321  AT      -1  PER SQUARE FOOT   THE SALES TOTAL     78
THE PROFIT FROM    6  IS     24150  SPACE USED IS      345  AT      70  PER SQUARE FOOT   THE SALES TOTAL     69
                      ITEM    7  WILL NOT BE SOLD ON SPACE TYPE   1   ON FLOOR    1
THE PROFIT FROM    8  IS     24220  SPACE USED IS      340  AT      70  PER SQUARE FOOT   THE SALES TOTAL     55
                      ITEM    9  WILL NOT BE SOLD ON SPACE TYPE   1   ON FLOOR    1
                      ITEM   10  WILL NOT BE SOLD ON SPACE TYPE   1   ON FLOOR    1
                      ITEM   11  WILL NOT BE SOLD ON SPACE TYPE   1   ON FLOOR    1
                      ITEM   12  WILL NOT BE SOLD ON SPACE TYPE   1   ON FLOOR    1
THE PROFIT FROM   13  IS     59280  SPACE USED IS      486  AT     156  PER SQUARE FOOT   THE SALES TOTAL     77
                      ITEM   14  WILL NOT BE SOLD ON SPACE TYPE   1   ON FLOOR    1
THE PROFIT FROM   15  IS      4674  SPACE USED IS      123  AT      38  PER SQUARE FOOT   THE SALES TOTAL     96
THE PROFIT FROM   16  IS     19275  SPACE USED IS      257  AT      75  PER SQUARE FOOT   THE SALES TOTAL     99
THE PROFIT FROM   17  IS     10800  SPACE USED IS      135  AT      80  PER SQUARE FOOT   THE SALES TOTAL     90
                      ITEM   18  WILL NOT BE SOLD ON SPACE TYPE   1   ON FLOOR    1
THE TOTAL PROFIT GENERATED BY THIS SPACE TYPE    203254
THE SPACE USED IS   1992  OUT OF    2000  AVAILABLE.
THE REDUCED MATRIX IS
   0    0   60    0    0    0   39    0   25   25    8   35    0   -3    0    0    0   41
   0    0   91    0    0    0   49    0   38   30   17   40    0   15    0    0    0   55
   0    0   99    0    0    0   73    0   43   55   25   47    0   30    0    0    0   77
   0    0  115    0    0    0   83    0   70   40   40   60    0   40    0    0    0   88
   0    0  127    0    0    0   95    0   81   42   50   75    0   45    0    0    0  110
   0    0   70    0    0    0   23    0   13   15   -5   25    0   -9    0    0    0   41
   0    0   75    0    0    0   44    0   20   20    0   33    0    7    0    0    0   45
   0    0   86    0    0    0   57    0   33   32   21   44    0   19    0    0    0   67
   0    0   99    0    0    0   74    0   65   34   46   55    0   33    0    0    0   89
   0    0   56    0    0    0  101    0   78   67   55   79    0   66    0    0    0   55
   0    0   -6    0    0    0   -4    0   -2   -1   -2   -3    0   -4    0    0    0   -9
   0    0    5    0    0    0    9    0    7    6    5    4    0    2    0    0    0    1
   0    0   45    0    0    0   22    0   33   44   55   66    0   22    0    0    0   26
   0    0   46    0    0    0   53    0   50   57   56   59    0   62    0    0    0   67
   0    0   76    0    0    0   72    0   81   82   83   84    0   86    0    0    0   90
   0    0   60    0    0    0   39    0   23   25    8   35    0   -3    0    0    0   41
   0    0   91    0    0    0   49    0   38   30   17   40    0   15    0    0    0   55
```

4.3 THE STORE LAYOUT

4.3.1 The Store Layout Problem

The problem of deciding upon the physical layout of goods in a department store is a complex and important issue for its management. The importance of product display area in retail stores has caused one author, Cairns (25) to define the retailer as a merchant of product display area. Consequently, as Anderson (4) points out, the determination of the shares of display area has become one of the most significant short-run decisions of retail management.

The traditional theoretical approach, as represented by Cairns (26), Buzzel, Salmon and Vancil (24), Hubbard (76) and Curhan (36), assumes a direct relationship exists between the amount of shelf space used by a product or a brand and its sales. Brown and Tucker (23) suggest that this relationship varies for different items; that for some items the quantity sold associated with various quantities display exposure may be regarded as a discontinuous increasing concave function that eventually reaches a maximum for some finite quantity of shelf space, while others are completely insensitive to quantity of display space.

However, while the importance of the layout of goods is recognised by most, the variation between layout formulae used in practice is quite impressive. In Fig. (4.5) we provide a graphical representation of a debate concerning the physical layout of goods in a department store that exists within a UK retail group. This example illustrates the existing differences of opinion which are the result of the use of a blend of experience and intuition by management when deciding upon the exact location of a particular item within a store.

LIST OF NOTATIONS FREQUENTLY USED IN CHAPTERS 5, 6 AND 7

bs_i - the systematic risk in the return of store i.

b_m - the systematic risk in the return of product line m.

CS_i - budget constraint of store i.

cs_i - expected cash flow of store i.

c_k - unit cost of resource k.

C_j - budget constraint on firm j.

CL_m - product line m's budget.

c_M - expected cash flow of the capital market.

$E(RS_i)$ - expected rate of return of store i.

$E(R_j)$ - expected rate of return of firm j.

$E(R_M)$ - expected rate of return on the market portfolio of risky securities.

$E(RL_m)$ - expected rate of return of product line m.

H - the price of risk.

L^* - target debt ratio

ps_i - average unit price of store i.

P_j - the price of the j'th security.

p_m - unit price of product line m.

QS_i - sales of store i.

Q_m - sales of product line m.

R_F - risk-free rate of return.

S_M - the total market value of all stocks.

vs_i - average unit variable cost of store i.

v_k^* - a dual variable.

v_m - unit variable cost of product line m.

W_0 - wealth of portfolio holder at the beginning of the period.

W_1 - wealth of portfolio holder at the end of the period.

W_k^* - a dual variable.

α_i - the proportion of the total resources allocated to store i.

$\hat{\alpha}_i$ - the amount of resources allocated to store i.

α_{mk} - the proportion of resource k allocated to product line m.

α^*_{ik} - the proportion of resource k allocated to store i.

αf_{jk} - the amount of resource k available to firm j.

ζ_1 - a lagrangian multiplier.

ζ^*_m - an integer decision variable.

λ - the market price of risk.

μ - the mean of the return of a portfolio.

μ_{im} - the complementarity effect of having product lines i and m in the portfolio.

Π_k - a dual variable.

ρ_{ij} - the correlation between assets i and j.

ρ_{ij}, - the partial correlation between assets i and j.

σ_i^2 - the variance on the returns of store i.

σ_i - the standard variation of the return of store i.

$\sigma_{i.}$ - asset i's partial standard variation.

τ - the tax rate.

τ^X_{j0} - after tax return before investment is undertaken.

τ^X_{j1} - after tax return after investment is undertaken.

ϕ_k - a dual variable

Ω^*_i - a constant in corporate centre's objective function.

Ω_m - a constant in store manager's objective function.

TABLE (4.5): Three layout options

While it cannot be totally dismissed, this method suffers from the disadvantage of relying on subjective assessments which tend to differ considerably, even within the same firm, and it is virtually impossible to determine, a priori, one layout's supremacy over another. The inevitable conclusion of all this is that there is some place for a more systematic approach to the issue of store layout, an approach which would take into account the relationship between various products and services within the store as perceived through customers' behaviour.

4.3.2 Formulation of the Problem

The store layout problem has certain characteristics which are not unique to the retail trade. In fact, it can be defined as a special case of the facility layout problem, or more generally as a quadratic assignment problem. The quadratic assignment is a more advanced method of looking at the facility layout problem. The formulation of this problem is similar to that of the linear assignment problem, which is a special case of the well known transportation problem.

The quadratic assignment problem can be formulated as follows:

given n^4 cost coefficients C_{ijpq} $(i,j,p,q = 1,2,\ldots,n)$

determine n x n solution matrix $X = ||X_{ij}||$ so as to

minimize $\sum_j \sum_i \sum_p \sum_q C_{ijpq} X_{ij} X_{pq}$

Subject to the following constraints:

$$\sum_j X_{ij} = 1 \qquad (i = 1,2,\ldots,n)$$

$$\sum_i X_{ij} = 1 \qquad (j = 1,2,\ldots,n)$$

$$X_{ij} = \begin{cases} 1, & \text{if } i \text{ is assigned to } j \\ 0, & \text{otherwise} \end{cases}$$

Koopmans and Beckman (89) formulated a less general form of the quadratic assignment problem. Their own formulation is as follows:

> Let it be required to assign n facilities to n locations in such a way that the total cost of interfacility transportation is minimized. Two n x n matrices, $D = (d_{ij})$ and $W = (w_{pq})$ are given where:
>
> d_{jq} - cost of transporting one unit of commodity from location j to location q.
>
> w_{ip} - number of units of commodity to be transported from facility i to facility p.
>
> Each mapping of facilities to locations is represented by the n x n matrix $X = (X_{ik})$, where
>
> $$X_{ik} = \begin{cases} 1, & \text{if facility i is assigned to location k} \\ 0, & \text{otherwise} \end{cases}$$

In other words, the Koopmans-Beckmann problem is to minimize the dot product of D and T with respect to a symmetric permutation of the rows and columns of one of the matrices. That is to minimize $T(XDX^T)$ where X^T is the transposed matrix of X. It can easily be seen that the Koopman-Beckmann formulation is a special case of the general formulation with $C_{ijpq} = d_{jq}\ w_{ip}$. The coefficient C_{ijpq} represents the cost of transportation from plant i at location j to plant p at location q.

In the objective function of the Koopmans-Beckmann formulation, an additional linear cost function may be added. This may be of form AX. In practice, the cost matrix A may represent the cost of assigning a particular facility to a particular location. In the Koopman-Beckmann formulation and using a permutation notation, the problem

is now to

$$\text{minimize} \quad \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij}\, d_{\rho(i)\,\rho(j)} + \sum_{i=j}^{n} A_{i\rho(i)}$$

$$\text{subject to} \quad \rho(i) \epsilon S_i \quad \text{for } i = 1,2,\ldots,n$$

where the notation $\rho(i)$ is the position in which facility i is located, and S_i is the set of locations whose characteristics (size, proximity to service areas, etc...) are compatible with the requirements for facility i.

4.3.3 Methods of Solution and Results

The literature contains several algorithms for solving the quadratic assignment problem, a comprehensive survey of their relative merits and the programmes used later can be found in Alli's (3) study of plant layout. In the context of our problem, it is sufficient to state that two suboptimal algorithms were tried when solving the store layout problem. These were based on the works of Hillier (72) and Armour and Buffa (9), and were a combination of "constructive" and "improvement" techniques. While the first method starts with a null solution, the latter improves non-null ones. As for the "improvement" methodology, we tried two heuristic techniques:

Steepest Decent Pairwise Interchange (SDPI),
and Steepest Descent Three-Way Interchange (SDTI).

Using the notation similar to Graves and Whinston (59) and Loss (96), the following matrices are defined:

$A = (a_{ij})$ n x n matrix.

a_{ij} - site cost of assigning facility i to location j.

$Q = (q_{ij})$ n x n matrix.

q_{ij} - interaction between the facilities i and j

$C = (c_{ij})$ n x n matrix.

c_{ij} - cost per interaction unit between the locations i and j.

$\rho(i)$ - location assigned to facility i.

$\delta(k)$ - facility assigned to location k,

i.e. $\delta(k) = \rho^{-1}(k)$.

For any particular layout, the total cost of assigning facility i to location $\rho(i)$ and facility j to location $\rho(j)$ all over the possible values of i and j is given by:

$$TC(P) = \sum_{i=1}^{n} a_{i\rho(i)} + \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} c_{\rho(i)\rho(j)}$$

where TC(P) is the total cost of the layout denoted by P.

In the context of the problem of the layout of goods in a department store, the following interpretations can be given to the above matrices:

A the elements of this two-dimensional array represent the cost of assigning a particular product to a particular location in the store, eg. the cost of developing the site accommodating the new department.

Q the elements of this two-dimensional array represent the flow between any pair of products, i.e. the degree of substitutability or complementarity

between the two goods, defined in Hicksian (symmetrical) terms.

C the elements of this two-dimensional array represent the distance between any two locations in the store.

We now present some of the results of a numerical example run on arbitrary data. The size of the problem is 12 x 12, we start with a null solution, construct an initial solution, and then improve on it using SDPI and SDTI respectively. We assumed there was no different in the cost of developing sites between the products, hence the null site cost matrix. Both methods improved the initial solution and ended with the same final sub-optimal layout. In view of the short time it took to run the two methods, no meaningful comparison as to their relative efficiency can be made on the basis of our example.

SITE COST MATRIX

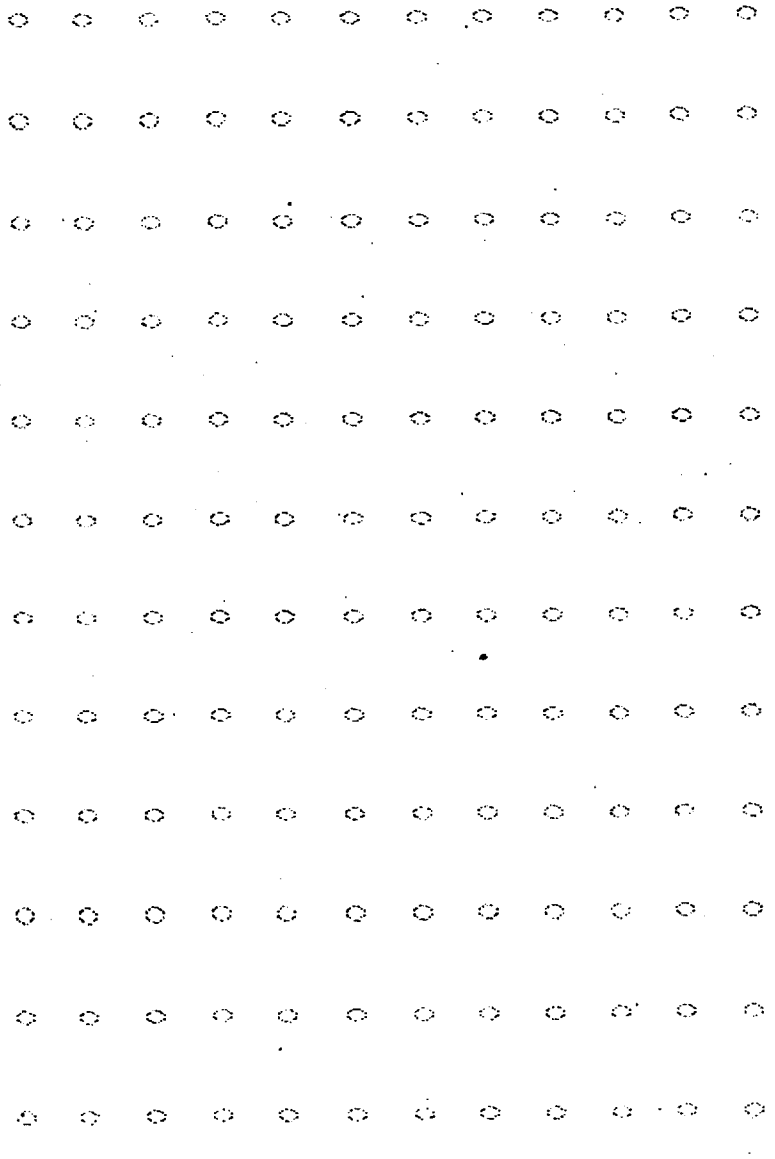


TABLE (4.6): Site Cost Matrix

FLOW MATRIX

0	-1.0	.5	2.0	1.5	-1.0	2.0	1.5	-1.0	1.0	1.0	1.0	0
-1.0	0	1.0	3.0	2.0	-1.0	3.0	2.0	-1.0	1.0	1.0	1.0	0
.5	1.0	0	2.5	1.5	.5	2.5	1.5	-1.0	1.0	1.0	1.0	0
1.5	2.0	1.5	0	1.0	1.5	2.0	1.0	-1.0	1.0	1.0	1.0	0
1.0	3.0	2.5	2.0	0	2.5	3.0	2.0	-1.0	1.0	1.0	1.0	0
1.0	2.0	1.5	1.0	0	0	2.0	1.0	-1.0	1.0	1.0	1.0	0
1.0	3.0	2.5	2.0	1.0	1.0	0	2.0	-1.0	1.0	1.0	1.0	0
1.0	4.0	3.5	3.0	2.0	2.0	1.0	0	-1.0	1.0	1.0	1.0	0
1.0	2.0	1.5	1.0	0	0	2.0	1.0	0	1.0	1.0	1.0	0
1.0	3.0	2.5	2.0	1.0	1.0	0	2.0	-1.0	0	1.0	1.0	0
1.0	4.0	3.5	3.0	2.0	2.0	1.0	0	-1.0	1.0	0	1.0	0
1.0	3.0	2.5	2.0	1.0	1.0	0	2.0	-1.0	1.0	1.0	0	0
1.0	4.0	3.5	3.0	2.0	2.0	1.0	0	-1.0	1.0	1.0	1.0	0

TABLE (4.7): Substitution Matrix

DISTANCE MATRIX

0	1.0	2.0	1.4	2.2	2.0	1.4	1.0	2.0	2.2	2.0	3.2	3.2	3.4
1.0	0	1.0	1.0	1.4	2.0	1.0	1.4	1.4	2.0	2.2	3.0	3.2	3.2
2.0	1.0	0	1.4	1.0	2.2	1.4	2.2	2.2	2.0	2.2	3.0	3.0	3.0
1.4	1.0	1.4	0	1.0	2.0	1.4	2.2	2.0	2.2	2.2	3.2	3.2	3.2
2.2	1.4	1.0	1.0	0	1.4	1.4	2.2	2.2	2.2	2.2	3.2	3.2	3.2
2.0	2.0	2.2	2.0	1.4	0	2.2	2.0	2.2	2.2	2.2	3.2	3.2	3.2
1.4	1.0	1.4	1.4	2.2	2.2	0	1.4	1.4	2.2	2.2	3.2	3.2	3.2
1.0	1.4	2.2	1.4	2.0	2.0	1.4	0	1.4	2.2	2.2	3.2	3.2	3.2
2.0	1.4	2.2	1.4	2.2	2.2	1.4	1.4	0	2.2	2.2	3.2	3.2	3.2
2.2	2.0	2.0	2.2	2.2	2.2	2.2	2.2	2.2	0	1.4	2.2	2.2	2.2
2.0	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	1.4	0	2.2	2.2	2.2
3.2	3.0	3.0	3.2	3.2	3.2	3.2	3.2	3.2	2.2	2.2	0	1.4	2.2
3.2	3.2	3.0	3.2	3.2	3.2	3.2	3.2	3.2	2.2	2.2	1.4	0	1.4
3.4	3.2	3.0	3.2	3.2	3.2	3.2	3.2	3.2	2.2	2.2	2.2	1.4	0

TABLE (4.8): Distance Matrix

INITIAL FACILITY ASSIGNMENT

		1	2	3	4	5	6	7	8	9	10	11	12	TOTAL COST
0	SITE NUMBER													
0	SITUATION AFTER A NEW ASSIGNMENT	0	0	0	0	5	0	0	7	0	0	0	0	6.0
0	NO IMPROVEMENT IS POSSIBLE	0	0	0	0	5	0	0	7	0	0	0	0	6.0
0	SITUATION AFTER A NEW ASSIGNMENT	0	0	0	0	5	0	0	7	10	0	0	0	10.8
0	NO IMPROVEMENT IS POSSIBLE	0	0	0	0	5	0	0	7	10	0	0	0	10.8
0	SITUATION AFTER A NEW ASSIGNMENT	0	0	0	3	5	0	0	7	10	0	0	0	25.8
0	NO IMPROVEMENT IS POSSIBLE	0	0	0	3	5	0	0	7	10	0	0	0	25.8
0	SITUATION AFTER A NEW ASSIGNMENT	0	0	0	3	5	0	6	7	10	0	0	0	47.4
0	NO IMPROVEMENT IS POSSIBLE	0	0	0	3	5	0	6	7	10	0	0	0	47.4
0	SITUATION AFTER A NEW ASSIGNMENT	0	0	0	3	5	2	6	7	10	0	0	0	71.6
0	NO IMPROVEMENT IS POSSIBLE	0	0	0	3	5	2	6	7	10	0	0	0	71.6
0	SITUATION AFTER A NEW ASSIGNMENT	0	0	0	3	5	2	6	7	10	0	11	0	92.0
0	NO IMPROVEMENT IS POSSIBLE	0	0	0	3	5	2	6	7	10	0	11	0	92.0
0	SITUATION AFTER A NEW ASSIGNMENT	0	4	0	3	5	2	6	7	10	0	11	0	130.0
0	IMPROVEMENT DUE TO AN INTERCHANGE	0	10	0	3	5	2	6	7	4	0	11	0	129.4
0	SITUATION AFTER A NEW ASSIGNMENT	0	10	0	3	5	2	6	7	4	0	11	12	160.0
0	NO IMPROVEMENT IS POSSIBLE	0	10	0	3	5	2	6	7	4	0	11	12	160.0
0	SITUATION AFTER A NEW ASSIGNMENT	0	10	0	3	5	2	6	7	4	8	11	12	214.0
0	IMPROVEMENT DUE TO AN INTERCHANGE	0	10	0	3	5	2	6	7	8	4	11	12	204.6
0	SITUATION AFTER A NEW ASSIGNMENT	0	10	9	3	5	2	6	7	8	4	11	12	249.4
0	NO IMPROVEMENT IS POSSIBLE	0	10	9	3	5	2	6	7	8	4	11	12	249.4
0	SITUATION AFTER A NEW ASSIGNMENT	1	10	9	3	5	2	6	7	8	4	11	12	276.6
0	IMPROVEMENT DUE TO AN INTERCHANGE	4	10	9	3	5	2	6	7	8	1	11	12	268.4

TABLE (4.9): Constructive Section

```
TWO-WAY INTERCHANGE BEGINS
--------------------------

SITE NUMBER          1   2   3   4   5   6   7   8   9  10  11  12
INITIAL LAYOUT       4  10   6   3   5  12   3   7   6   1  11  12
CURRENT LAYOUT       4   2   6   3   5  10   6   7   7   1  11  12

SUB-OPTIMAL SOLUTION FOR TWO-WAY INTERCHANGE REACHED
***************************************************

COST OF THIS ASSIGNMENT=      244.8

COST OF INITIAL LAYOUT=       268.4

SUB-OPTIMAL ASSIGNMENT IS :
***************************

SITE NUMBER          1   2   3   4   5   6   7   8   9  10  11  12
FACILITY             4   2   6   3   5  10   6   7   7   1  11  12
```

TABLE (4.10): Improving with SDPI

THREE-WAY INTERCHANGE SEQENS

SITE NUMBER	1	2	3	4	5	6	7	8	9	1	11	12	TOTAL COST
INITIAL LAYOUT	4	12	9	3	5	2	6	7	8	1	11	12	236.4
CURRENT LAYOUT	4	12	9	3	5	10	6	7	8	1	11	12	244.4

SUB-OPTIMAL SOLUTION FOR THREE-WAY INTERCHANGE REACHED

COST OF THIS ASSIGNMENT= 246.8

COST OF INITIAL LAYOUT= 244.4

SUB-OPTIMAL ASSIGNMENT IS **

SITE NUMBER	1	2	3	4	5	6	7	8	9	10	11	12	TOTAL COST
FACILITY	4	2	9	3	5	10	6	7	8	1	11	12	246.8

TABLE (4.11): Improving with SDTI

4.4 OPTIMIZING MODELS

We now return to the optimizing model as this is probably of longer-term interest. One fundamental assumption throughout the work above is that of determinism. Yet, when all analyses are performed, and the manager has allocated his resources, the final demand is still very much unknown. The sources of this uncertainty are manifold; errors in the potential demand computation, the form of the demand function, a shift in buying patterns, and so on. But any worthwhile model must attempt to take it into account. However, before we suggest such a model, we present a deterministic model which, despite its simplicity, highlights all the points made up till now.

4.4.1 A Deterministic Model

This model is a very simple one. It does not distinguish between different types of floor space, nor different grades of sales staff. It also assumes that all the items within a group are homogeneous.

1. Demand

$$D_i = D_i^* \left\{ \prod_{k=1}^{3} \left(\frac{\beta_{ki}}{c_{ki}}\right)^{\gamma_{ki}} \right\} a_i \qquad \forall\ i$$

2. Supply

$$S_i \leq \beta_{1i}$$

$$S_i \leq D_i$$

$$\eta_i \beta_{1i} \leq \beta_{2i}$$

$$S_i \leq \theta_i \beta_{3i}$$

where η is the floorspace consumed per item and θ_i the selling capacity per member of staff. It is implied that fractional staff may be theoretically considered equivalent either to staff working in more than one department of the store, or the employment of part-time staff.

3. Overall

$$\sum_i c^i \beta_{1i} \leq M \qquad \text{current asset constraint}$$

$$\sum_i \beta_{2i} \leq N \qquad \text{total floor space}$$

$$\sum_i \beta_{3i} \leq \Omega \qquad \text{total level of staff}$$

4. Contribution to Profit

$$\text{Income} \qquad \sum_i p_i S_i$$

$$\text{Outgoings} \qquad \sum_i c_i S_i + r \sum_i c_i (\beta_{1i} - S_i) + d\sum_i \beta_{2i} + e\sum_i \beta_{3i}$$

where p_i and c_i are the prices and cost of the i^{th} item respectively, d the charge per unit of floorspace, e the labour rate and r the interest charge on stock holding.

Other than the demand function, this model is a LP. It is possible to approximate the demand function by only allowing small variations around the current status quo, i.e. :

$$D_i \leq D_i^0 \{1 + \sum_k (\frac{\beta_{ki}}{\beta^0_{ki}} - 1) \gamma_{ki} a_i\} \qquad \forall\ i$$

using a first order approximation of a Taylor series together with the fact that: $\dfrac{\partial^2 D_i}{\partial \beta_{ki}^2} \leq 0.$

If this approximation is insufficient, then because D_i is monotonic increasing in all β_{ki}, it is highly possible that a simple application of an iterative linear approximation technique (see (74) for further details) might suffice (14).

4.4.2 A Stochastic Model

How a stochastic demand is to be included is still very largely a matter of discussion. However, for the want of any better information, we made the gross assumption that the demand is distributed with an expected value given by the demand function (D_i) and a fixed exogenous variance (σ_i^2). This implies no increase in the uncertainty as the market share increases. It is supposed that an estimate of this variance may be provided from the historical records of the store, together with the experience of the manager.

In the vernacular, the desired stochastic model is an "here and now" one (149) because the resource allocations have to be performed before the actual demand can be observed. The class of models may be subdivided further into indirect corresponding to the active approach of stochastic linear programming (135, 145), and direct. We will concentrate on this latter set, which contains both chance-constrained models and recourse models.

If the deterministic model is defined as

$$\{\max.\ p(\underline{x})\ \text{s.t.}\ \underline{f}\ (\underline{x}) \leq \underline{\tilde{D}}\ ;\ \underline{x} \in \psi\}$$

where p(.) is the profit function, $\underline{x}$ the set of decision variables, ψ the feasible space defined by the intersection of all the other constraints, f(.) a vector (of dimension equal to the number of items) representing [sales + expected demand] and finally $\underline{\tilde{D}}$ a vector of

random variables with zero mean and standard variation $\underline{\sigma}$. Before we can proceed much further, we have to discuss the effect of this uncertainty on the manager. It implies that there are times when he will have allocated resources to account for a particular level of demand, and that the demand has fallen below this level, thus reducing his profits. The chance-constrained formulation comes to hand immediately; the uncertainty in the model being catered for by $\Pr\{\underline{f}(\underline{x}) < \tilde{\underline{D}}\} > \underline{\alpha}$. This model restricts the violation of the demand constraint to, at the most, $(1 - \alpha_i)$ proportion of the time, for each item. The certainty equivalence of these constraints is simply:

$$f_i(\underline{x}) \leq F_i^{-1}(1 - \alpha_i) \qquad \forall\, i$$

where $F_i^{-1}(.)$ is the inverse cumulative density function of $\tilde{D}_i$. The uncertainty here is measured by $\underline{\alpha}$ and it is quite feasible to generate a surface $\{p^*(\underline{x}), \underline{\alpha}\}$ as naturally $\underline{\alpha}$ and p^* are inversely related.

However, such a model does not really get to the heart of the manager's problem. His objective is profit and, at the end of the day, trading risk must be measured in terms of lost profits.

Writing the model as

$$\{\max.\ p(\underline{x}) - g(\underline{y}^+) - h(\underline{y}^-) \text{ s.t. } \underline{f}(\underline{x}) + \underline{y}^+ - \underline{y}^- = \tilde{\underline{D}}\ ;\ \underline{y}^+\ ,\ \underline{y}^- \geq 0\ ;\ \underline{x} \in \psi\}$$

where $g(\underline{y}^+)$ is a measure of the profits foregone as demand was greater than allowed for, and $h(\underline{y}^-)$ a measure of profits lost due to the overallocation of resources. Such a model can be simplified by assuming, a priori, that the manager would find acceptable a certain minimum profit level (p^*) or a range of market shares, say $\{\underline{\lambda}_L, \underline{\lambda}_u\}$.

This would remove the profit function from the objective, which now becomes {min. $g(\underline{y}^+)$ + $h(\underline{y}^-)$}. This model is an example of a programme with simple recourse and as such has been studied in some detail (152, 158).

Unfortunately, it is not easy to solve, unless some further simplifying assumptions can be made. Let us assume that the penalty functions are both linear, e.g. $h(\underline{y}^-) = \sum_i h_i y_i^-$ and that we are concerned about the expected violation of the demand constraint, so that $g(\underline{y}^+) = 0$, then the objective may be written as:

$$\text{max.} \quad p(\underline{x}) - \sum_i h_i \int \{f_i(\underline{x}) - \tilde{D}_i\} \quad dF(\tilde{D}_i)$$

integrating over the range

$$f_i(\underline{x}) \geq \tilde{D}_i$$

For an appropriate distribution, e.g. uniform, the objective becomes:

$$\text{max.} \quad p(\underline{x}) - 1/4 \sum_i \left(\frac{h_i}{\hat{D}_i}\right) \left[f_i(\underline{x}) + \hat{D}_i\right]^2$$

where $\hat{D}_i \sim R\{-\hat{D}_i, +\hat{D}_i\}$. This may be easily solved using either a quadratic algorithm (if the LP approximation for $f(\underline{x})$ is suitably valid) or some non-linear algorithm.

Another approach, because the integral is in the objective function, and because the integral is monotonic in $f_i(x)$, we could use again a version of linear approximation programming.

4.5 SUMMARY

In this chapter we have attempted to describe and model how an individual department store operates, what are the constraints under which it must act, and the various decision parameters under its control. In the process of our analysis we have had several discussions with managers in the retail trade which have lead us to the conclusion that, in practice, the manager of a department store operates within a set of constraints which are generated by three different sources.

The first source which limits the store manager's freedom is the environment in which his store is located. This external constraint requires the manager to tailor the types of goods he stocks to the tastes of the population attracted to the shopping centre in which he operates. In spite of this need being obvious, the store manager may find it difficult to accommodate his potential customers' needs, since it often conflicts with the other two sources of restrictions, namely the limitation on working capital and the finite amounts of local resources.

The second source of constraints is the corporate centre, which not only sets targets,but also allocates the total corporate resources between the various activities of the firm, i.e. determines the budget available to each store. A third and less obvious restriction imposed by the firm's headquarters is on the supply of goods. In practice, the manager often finds that the purchasing requirements of his store conflict with the overall procurement interests of the firm. Such a case may arise where the centre is offered a quantity rebate on a certain product and consequently applies pressure on local managers to request the discounted item for the centre's order to reach the minimum size

needed. Pressures of this kind tend to create a certain discrepancy between the type and quantities of products actually on display in a store and the range of goods the local manager would have liked to see.

The third and last type of restriction originates from the store level. Obviously, local factors such as the quality and quantity of manpower available, and the total amount of floor space cannot be changed in the short term, not even by some corporate decision.

In Chapter 7, we concentrate mainly on the second and most interesting source of constraint on store managers, i.e. the conflicts between local and corporate objectives, and how these can be resolved. In the following two-chapters, we develop the theoretical framework necessary for the relationship between a holding company and its subsidiaries under uncertainty.

C H A P T E R 5

CORPORATE CAPITAL BUDGETING WITH THE CAPM

5.1 INTRODUCTION

The importance of determining the appropriate cost of capital for an investment project considered by the firm can hardly be over-stressed. The cost of capital is used extensively as a cut-off rate for new projects and as a result, its estimation has become a fundamental part of many financial decisions.

Traditionally, methods were used in which the firm's cost of capital was perceived to be the criterion by which its capital investments were evaluated. Consequently, hurdle rates for new projects were assumed to depend on financial parameters of the individual firm.

These methods suffered from both theoretical and practical flaws, and as a result were generally invalid. For example, the "Weighted Average Cost of Capital" (WACC) criterion, one of the better known techniques of that kind, not only fails to consider marginal values, it also totally ignores the risk of firms and projects. The practical drawback of using such methods is the obvious need to calculate an appropriate WACC for each and every firm because of firms differing in their sources of capital, investment strategy, and capital structure.

The next step in asset expansion theory was to foresake the methods using the firm's cost of capital for those which are based on market data. Keane (87) presents a clear analysis of the major points in favour of the use of market data. The two main arguments against using the firm's cost of capital are, first, that in spite of the firm's ability, through its financial policies to affect the probability distribution of investment's cash flows, it is beyond its power to influence the discount rate

required by the capital market to any particular probability distribution. In other words, the cost of capital appropriate to the project is independent of the firm which undertakes it.

The second problem when using the firm's cost of capital, is due to the fact that it is an average rate, and therefore useless when evaluating investments of varying maturities and risks. Clearly, it is necessary to compare the performance of a project with that of a publicly traded asset of similar maturity and risk.

However, despite the clear case for the use of market data, this approach is not homogeneous and controversy exists as to the correct method for measuring the firm's cost of capital. Furthermore, market data required in these methods do not exist for non-traded firms, nor is it clearly defined for a division of a firm. The lack of the latter becoming particularly problematic in cases of highly decentralized firms. Using the firm's cost of capital under such circumstances involves error, since it is analogous to using an economy-wide cost of capital for a single firm, a point discussed thoroughly by Turnball (147). These difficulties not withstanding, the use of techniques which rely on market data for evaluating a new project's discount rate has become dominant in financial theory.

Among the market data approaches, clearly the most influential (if one is to judge by the amount of literature it generated, and still continues to do) is the well known Markowitz Sharpe Capital Asset Pricing Model (CAPM). Basically developed in the sixties by Sharpe (136), Lintner (95), Mossin (110) and Fama (47) to explain the behaviour of a security market, the CAPM has been extended to investment evaluation models. Mean-variance capital budgeting models have been proposed by Hamada (66), Bierman and Hass (18), Rubinstein (125),

Mossin (111) and Bogue and Roll (21).

Hamada (66) related an investment project's cost of capital with its systematic risk rather than that of the firm, which may or may not be the same. However, measuring the systematic risk of a project, i.e. the covariance between some measure of return on the investment and the return on all assets, using the CAPM is very difficult and not practical in many situations. Gordon and Halpern (58) suggested a model for calculating the cost of capital for both a non-traded firm and a division of a traded or non-traded firm. Their model computes the over-all cost of capital for a company then, on the basis of the allocation of the firm's joint costs to the various divisions, generates estimates of divisional costs of capital. However, they assumed that the break-down of costs would be an unsurmountable task. Jarrett (81) takes this idea one step further by using estimation theory to solve the allocation of joint costs problem. Although estimated costs of capital are produced for the divisions, they are highly dependent on accurate forecasts of accounting income numbers and underestimation of the cash flows results in an error in estimating the cost of capital. Although these models have all contributed to highlight different aspects of the capital budgeting problems of the firm, they can, as claimed by Senbet and Thompson (134) be reduced to common equivalence.

The most promising model, in the context of a decentralized firm, mentioned amongst the above, is the one suggested by Rubinstein. The main advantage of Rubinstein's approach is his expansion of the firm's risk element to include variables related to the production level, a point that will be clarified in the following chapter, thus enabling us to analyse the connection between various levels within the firm. As a result, it is Rubinstein's model which we

use as a basis for our study of the capital budgeting problems in a decentralized firm. In section 2, we start with a brief presentation of the CAPM and Rubinstein's model. In section 3, we highlight the main points of criticism the CAPM has drawn, following which, in section 4, we discuss the capital market equilibrium in view of the firm's ability to alter its riskiness, and suggest an appropriate asset expansion decision rule. In section 5, we prove, and demonstrate the shortcomings of, Rubinstein's expansion of the mean-variance theorem. In section 6, we present the case of economies of scale in general, and in department stores in particular, and suggest how these can be treated within the CAPM framework. In section 7 we provide a simple numerical example to illustrate our findings, and section 8 is a summary of the main points made in this chapter.

5.2 CAPITAL BUDGETING WITH THE CAPM

The CAPM provides, under certain assumptions, a theoretical relationship between the expected rate of return and the risk of an asset under conditions of market equilibrium. The underlying concept is the assumption that there is a market rate of interest which reflects the market's time value of money, and a market price of risk which determines the premium or discount on the market rate of interest necessary to produce the return from any individual risk asset.

Furthermore, the model states that the expected return on an asset is linearly related to its systematic risk, which is a measure of the responsiveness of the asset's return with the return on the market portfolio, which is composed of all risky assets. This can be formally expressed:

$$E(\bar{R}_j) = R_F + \lambda Cov(\bar{R}_j, R_M) \qquad (5.1)$$

where:

$\bar{R}_j$ - rate of return on security j (random variable),

R_F - rate of return on riskless security,

R_M - rate of return on market portfolio of risky securities (random variable),

λ - a positive constant equal to $\{E(R_M) - R_F\}/Var\ R_M$:

which is the familiar mean-variance security valuation theorem.

This theorem holds, in equilibrium, for any security j under a set of fairly stringent assumptions; these assumptions can be summarized as:

i) All investors are single-period expected utility of terminal wealth maximizers who choose among alternative portfolios on the basis of mean and variance (or standard deviation) of returns.

ii) All investors can borrow or lend an unlimited amount at an exogenously determined risk-free rate of interest, R_F, and there are no restrictions on short sales of any asset.

iii) All investors have identical subjective estimates of the means, variances and covariances of return among all assets.

iv) All assets are perfectly divisible and liquid and there are no transaction costs.

v) There are no taxes, all investors are price takers and the quantities of all assets are given. (1)

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(1) Recent work has focused on relaxing some of the assumptions underlying the model by introducing various market imperfections. Fama (48) has demonstrated that, given an investor has a concave multi-period utility function, he will behave in the first period as if he possesses some concave single-period utility function. Rubinstein(124) demonstrates that provided at least two risky securities exist, the existence of a risk-free security is not vital. Brennan (22) has concentrated on the effects of personal taxation; Black (19) on the effects of investing in only risky securities; Levy (93) on the existence of a constraint on the number of securities in a portfolio; Chen and Boness (32) on the effects of uncertain inflation; and Mayers (103) on the effects of non-marketable assets.

Footnote (1) continued..

The mean-variance security valuation theorem can be used as a cut-off rate for new projects. If j refers to firm j and R_j represents the rate of return on its equity, an investment project should be accepted only if:

$$E(\bar{R}_j^0) > R_F + \lambda Cov(\bar{R}_j^0, R_M) \qquad (5.2)$$

where $E(\bar{R}_j^0)$ is the expected return on the project.

Since λ may be interpreted as the risk standardized cost of capital, then if $Cov(\bar{R}_j^0, R_M) > 0$ a firm should accept a project only if:

$$\frac{E(\bar{R}_j^0) - R_F}{Cov(\bar{R}_j^0, R_M)} > \lambda \qquad (5.3)$$

where Eq.(5.3) is the "Market Price of Risk" (MPR) asset expansion criterion suggested by Rubinstein. The MPR decision rule advocates acceptance of a project only if its expected rate of return, $E(\bar{R}_j^0)$, exceeds the appropriate risk-adjusted discount rate for the project, $R_F + \lambda\, Cov(\bar{R}_j^0, R_M)$, i.e. the expected rate of return on a security with the same risk as the project. The MPR is graphically illustrated in Fig. (5.1) where only projects A and B whose risk-return ordered pairs fall above the Securities Market Line (SML) are to be accepted by the firm.

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(1) Continued...
Turnball (148) has attempted to synthesize some of these imperfections and analyse their combined effect on the structure of asset returns, introducing in the process a second risk variable.

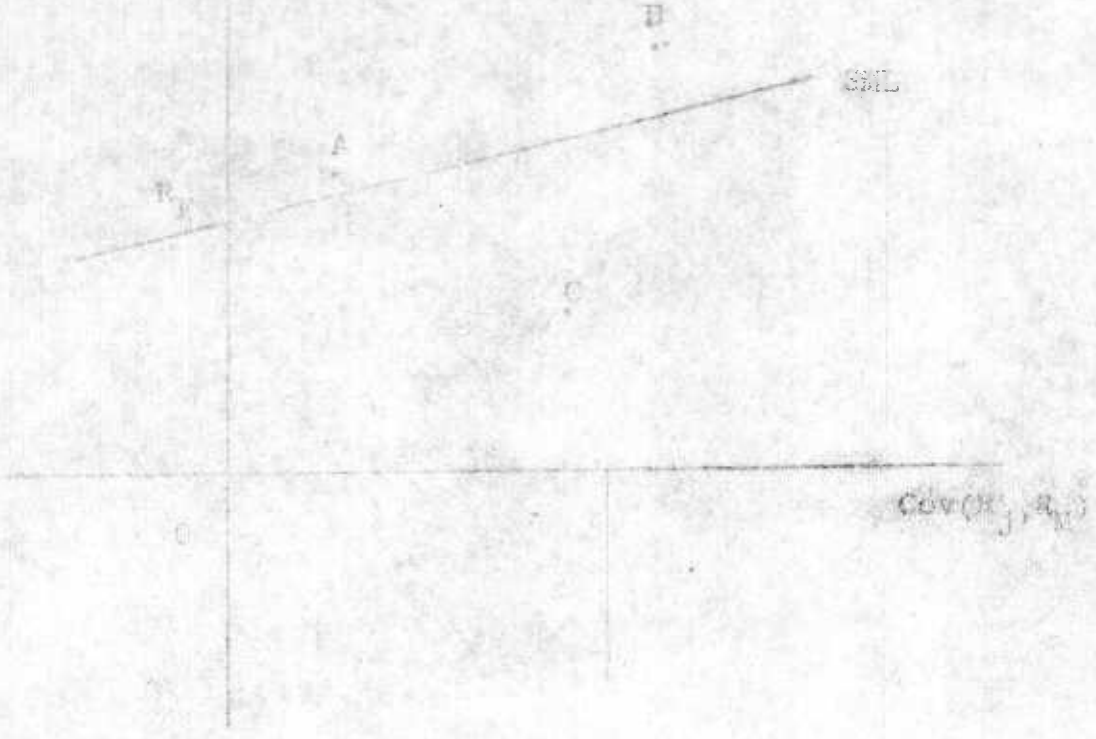


Fig. (5.1): Market Price of Risk Criterion

The MPR decision rule has several properties which on one hand make it a useful tool, and on the other hand are the reasons why the MPR requires modifications if it is to be used in the context of a decentralized firm. The main advantage of the MPR is the fact that it is an economy wide decision rule appropriate to all firms and all projects. Furthermore, the MPR is independent from the firm's variance of equity rate of return, i.e. notwithstanding cases of synergy, if $\bar{R}_j^0 = R_j$, the rate of return if the project were itself a firm, diversification, i.e. reduction of Var R_j, can be ignored in budgeting decisions and each project is judged on the basis of its own performance, regardless of the firm's other investments. This is explained by Mossin's (110) suggestion that individuals can, by their own diversification, costlessly eliminate any diversifiable risk present in a firm's investment portfolio, so that the firm need not diversify for individuals, a concept known as Mossin's "homemade diversification" theorem[2].

Even in the case of a perfect market, as defined by the CAPM, using the capitalization rate as a criterion for accepting or rejecting investment projects can be misleading hence resulting in non-optimal decisions. Myers and Turnball (114) point out that the growth rate of expected cash flows, the project life and other variables, which are not generally used when assessing risk, do

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(2) This observation holds in a capital market, as defined by the CAPM, and in a similar form, as demonstrated by Myers (113), for a state preference model under complete, i.e. Arrow-Debreu, markets for securities, and under complete, i.e. generalized, markets with the existence of security classes.

influence the observed systematic risk. However, it is mainly because of the issues of synergy and the "home-made diversification" that the MPR requires certain modifications if it is to be used as an asset expansion decision rule in a decentralized firm.

The non-synergy assumption, about the independence of a firm's new investments from its existing ones, is a fairly restrictive one. Even if we do accept the possibility of such cases existing, it is not difficult to imagine a realistic business environment in which the firm undertakes projects which in fact do affect, positively or not as the case might be, other parts of the firm's operations. A simple example of the firm wide impact generated by a new investment is the possible effect the opening of a new distribution outlet could have on the production divisions in a vertically integrated firm with excess production capacity. If, in fact, this cross-firm effect does materialize we would be justified in expecting a subsequent change in the firm's variance. This point is particularly relevent where the firm has potential economies of scale, a topic which will be covered in the following chapters.

The reservation concerning the "homemade diversification" theorem follows directly from the comment made about the issue of synergy. The firm's variance can be broken down to:

$$\sqrt{\text{Var } \bar{R}_j} = \rho(\bar{R}_j, R_M)\sqrt{\text{ Var } \bar{R}_j} + \{1 - \rho\ (\bar{R}_j, R_M)\sqrt{\text{ Var } \bar{R}_j}\} \tag{5.4}$$

i.e. the firm's total risk, when its security is held alone and not in a portfolio, is the sum of its systematic risk, $\rho(\bar{R}_j, R_M)\sqrt{\text{ Var } \bar{R}_j}$, and its diversifiable risk, $\{1-\rho(\bar{R}_j, R_M)\sqrt{\text{Var } \bar{R}_j}\}$. Clearly, if the firm, through its expansion policy, can change its variance this will have an effect on both components of its total risk, which is tantamount

to stating that the firm can diversify part of what is from the investors' viewpoint, undiversifiable. This leads to the conclusion that there exists a certain part of the firm's total risk which can be reduced solely by the firm's diversification, i.e. new investments, and which the investors in the corporate bonds market view as undiversifiable.

It is crucial to recognize that the comments mentioned above do not invalidate the MPR decision rule, or the "homemade diversification" theorem for as long as the CAPM assumptions hold. Both remain valid for any portfolio in a perfect market. However, our underlying assumption is that the firm, which is viewed as a holder of a portfolio of investments in various business activities, can, by restructuring its investments, redefine its portfolio, thus creating a new one. In other words, the "homemade diversification" and MPR asset expansion decision are indeed valid for a firm which does not rearrange its investments in a way which creates a new portfolio - by which we mean investments which are not proportional increases of all assets. We do not, therefore, contradict the generally accepted concept that, without synergism, firm diversification generates no benefit to shareholders that they themselves cannot achieve through their own portfolios. However, we do consider it necessary to incorporate in our model the overall effects an investment can have on the firm's performance, i.e. allow for synergy when restructuring portfolios, if the special characteristics of decentralized firms are to be reflected in our model.

5.3 A CRITICAL ANALYSIS OF THE CAPM

The CAPM is based on two sets of assumptions, the first is derived from portfolio analysis and can be summarized as follows:

i) The rate of return from an investment adequately reflects the investment's outcome, and investors assess the various rates of return in form of probability distributions of rates of return.

ii) The risk estimates of the investors are proportional to the variability of return as they evaluate it.

iii) Investors make their decisions on the basis of two parameters: the probability distributions of returns, i.e. the expected rate of return, and the variance.

iv) For any level of risk, the investors prefer a higher to a lower rate of return. Alternatively, they prefer a less risky investment, rather than a riskier one, from the set of investments with an identical rate of return.

Given that investors conform to the preceding assumptions, the following set of assumptions forms the basis of the capital market theory:

i) All investors are Markowitz efficient (conform to the above assumptions) and attempt to attain the efficient frontier.

ii) Any amount of money can be borrowed or lent at the risk-free rate of interest.

iii) All investors hold the same beliefs as to the probability distributions of the future

rates of returns.

iv) A single-period horizon.

v) All investments are indefinitely divisible.

vi) There exist no taxes and transactions costs.

vii) Inflation and changes in the rate of interest do not exist - or are fully anticipated.

viii) The capital markets are in equilibrium.

Although our main concern is with the second group of assumptions, it is vital to consider the important question of the exact relationship between the mean-variance approach and the expected - utility approach to portfolio selection.

The question which arises is: in what conditions will an expected utility maximizer be able to neglet all features of the expected returns on his portfolio other than the mean and variance of their probability distributions?

One or both of the following conditions must be satisfied if the mean-variance and expected utility approaches are to be reconciled. First, the probability distributions of expected returns must be 'normal', regardless of the form of the utility-of-wealth function. Secondly, the expected utility maximizer will be indifferent between any two investment projects with equal means and equal variances (regardless of the type of probability distribution) if, and only if, his utility of wealth function is quadratic.

Both these assumptions have generated heated debates. Most writers on the subject question the use of the normal distribution, even as an approximate pattern, for the returns on a typical portfolio. As to the second assumption, Green (63) points at two implications of the combination of risk-aversion together with a quadratic utility function which to his opinion make the latter quite unacceptable. The first point is that the range of quantities of wealth over which the quadratic utility function may be used is restricted if the

marginal utility of wealth is to be positive and diminishing (the latter is due to the risk-aversion). The second point is that even within the range of wealth where the function may be used it is possible that with a riskless asset and a risky asset the latter is an inferior good.

Francis and Archer (54) argue that although all the above assumptions underlying portfolio analysis have been shown in the literature to be inaccurate to varying degrees, this does not invalidate the theory as a whole. They claim that, as long as the analysis rationalizes complex behaviour which is observed and consequently yields valid predictions, it can be valuable in spite of fallacious assumptions, and state that even if the above assumptions are slight oversimplifications, they are no cause for alarm.

Accepting these portfolio assumptions leads us to the second assumption of the CAPM, namely, the existence of one risk-free interest rate for both borrowers and lenders. A realistic approach to capital markets reveals that investors with poor credit ratings are required to pay a higher borrowing rate than those with good credit ratings. Furthermore, the borrowing rate in most capital markets is higher than the lending rate. The implications of the relaxation of the above assumption would be a curved CML, becoming more curved as the difference between the lending and the various borrowing rates increases.

The third assumption, concerning the homogeneity of beliefs of investors, basically means that given any preselected information set, e.g. the past price observations, investors share a common ex ante distribution that they perceive generates returns. Clearly, casual observation suggests that investors do not show the homogeneity of beliefs which characterize the theory. Quite the opposite, it is the differences in beliefs and changes in beliefs which have been suggested as the causes for trade. Models of such differences suffer from the fact that they imply a persistent irrationality on the part of the investors. Ross (123) suggests that it would not be difficult

to incorporate some random cross sectional heterogenity in investor beliefs into the CAPM, however, since a single ex post distribution of returns is observed by all investors, systematic and persistent differences would probably not be observed. If, however, an attempt to relax this assumption was made, it would imply drawing an opportunity locus and CML composed of "fuzzy" curves and lines. The more investors' expectations differed, the "fuzzier" all lines and curves would become. As a result the analysis would be determinate only within limits and only major disequilibriums corrected.

Since the CAPM is a static one period model, considerable effort has been directed at intertemporal extensions. Among the numerous attempts to relax this assumption the most successful are based on Merton's (106) lognormal diffusion model for capital gains and assume that returns follow log-normal diffusions and instantaneous portfolio choice is identical to that of the CAPM. There are a number of discrete time models but most simply exogenously assume that the market price of risk remains unchanging over time. The important difficulties associated with intertemporal models are identification of the factors which cause return anticipations to change, the understanding of how changes in information sets over time affect equilibrium prices and how will these prices and their movements feed back into individual information sets. Finally, the question of how these observations are to be modelled in a manner which allows tests to be made has to be answered.

The fifth assumption, when relaxed, would cause the SML to transform to a dotted line where each dot represents an opportunity attainable with an integral number of shares.

The existence of taxes and transaction costs is a well known characteristic of modern capital markets. However, the recognition of the existence of different tax rates on ordinary income and capital gains would confuse the picture defined by the CAPM. This is a direct consequence of the fact that each investor would see a slightly different CML

and SML depending on his tax bracket. As a result, a static equilibrium could never emerge under existing tax laws even if the remaining assumptions were not violated. The analysis of the impact of differential taxation of capital gains and dividends on the efficient set mean-variance analysis has been developed by Brennan (22).

The relaxation of the seventh assumption concerning the existence of inflation, a fact only too obviously clear, derives from the fact that while returns are measured in a nominal accounting unit preference is over real goods. It therefore becomes necessary to deflate returns by a price index to transform them into real returns. This causes the CAPM to hold identically as before with the need to measure all returns in real terms and the CAPM is thus equivalent to the real efficiency of the market portfolio. As to a change in the riskless rate of interest, this implies that a true riskless asset does not really exist and even default-free securities will experience systematic variability of return with the effect of obscuring the CML and SML.

The eighth and final item on our list of assumptions is the existence of an equilibrium in the capital market. Since in reality information reaches various investors at different times it is unrealistic to expect investors' expectations to change simultaneously. Equally unrealistic is therefore the belief that prices adjust so that all investors are satisfied with their holdings at any point in time. In fact, in view of the excess demands for certain or all assets, assets' prices are in a dynamic continuous disequilibrium - hopefully, a minor one.

Before we summarize this part of our analysis, it is necessary to specify the main obstacles which lie in the way of any attempt to make an empirical test of the CAPM. Namely:

i) Variability of return measures are only risk surrogates which, although widely used, could

probably be replaced with better risk surrogates.

ii) The bulk of empirical tests rely on explicit proxies for the market portfolio in view of the difficulty in observing the market portfolio in practice. Thus, using the equal weighted portfolio may or may not be efficient, but in any case, would have no implications on the acceptance or rejection of the CAPM.

iii) In the event that the market portfolio, or whatever proxy being used, is not ex ante efficient, there exists a set of inefficient portfolios for which statistical tests of the linear regression usually used will falsely accept the efficiency of the proxy.

In view of the defective empirical tests made up till now, the considerable difficulties in testing the theory and the undeterminate effects on the theory of relaxing simultaneously the underlying assumptions, the CAPM has proved to be somewhat less robust than might have been hoped for. This, however, does not make the CAPM redundant; the theory still rationalizes much observed behaviour, explains previously unexplained practices such as diversification, and offers realistic suggestions about the restoration of equilibrium.

5.4 CAPITAL MARKET MECHANISM

In order to understand the rationale underlying the capital budgeting decisions of a value maximizing firm, it is useful to first proceed by analysing the mechanism which governs share price behaviour. A modified version of the standard formulation of the relationship between a firm's financial performance, i.e. its risk and expected return, and its share price in a single period horizon is:[3]

$$P_j = \frac{1}{R_F}\left[E(c_j) - H\ Cov(c_j, c_M)\right] \qquad (5.5.a)$$

or

$$P_j = \frac{E(c_j)}{R_F + H\ Cov(R_j, c_M)} \qquad (5.5.b)$$

where:

P_j - firm j's share price,

c_j - firm j's expected cash flow,

$cov(c_j, c_M)$ - the covariance between firm j's expected cash flow and the expected cash flow of the market portfolio of risky securities,

$R_j = c_j/P_j$ - firm j's expected rate of return,

H - the price of risk

$$H = \left[\sum_i \left[- \frac{E\left[u''(w_i)\right]}{E\left[u'(w_i)\right]} \right]^{-1} \right]^{-1} \qquad (5.6)$$

where $u(w_i)$ is the utility of wealth function for investor i.

These expressions are suggested by Fowler and Rorke (53) in their analysis of capital budgeting with the CAPM. In their analysis Fowler and Rorke consider the case of a scale changing investment, i.e. an expansion of existing facilities to supply more of current product lines for the same markets,

footnote 3 : Throughout our analysis of both the independent department store and the decentralized retail chain we assume a single period horizon.

and prove that under a non-stochastic expansion there is a change in the firm's risk. In other words they show that $Cov(R_j + \Delta R_j, c_M + \Delta c_j)$ is not equal to $Cov(R_j, c_M)$ under most imaginable circumstances. Furthermore, they suggest that despite conflicting forces, on balance an increase in scale will generate a higher rate, i.e. $Cov(R_j + \Delta R_j, c_M + \Delta c_j) > Cov(R_j, c_M)$.

In view of these conclusions it seems that an incremental cash flow will generate two conflicting forces on the share price: the increase in the expected return will operate on the firm's share price while, if one adopts Fowler and Rorke's approach, the higher discount rate will reduce the latter. These findings are consistent with our suggestion that an investment project can affect not only the firm's expected return but its risk as well, and in this sense we depart from conventional CAPM theory.

In the following chapter we set to examine the accuracy of the statement that a non stochastic scale increase generates an increase in the firm's covariance for the case of a department store. At this stage we merely state that under reasonably realistic assumptions an increase in the scale of the firm's activities can leave its covariance unchanged or even reduced.

The effect of an investment project on the firm's value when accepted on the basis of the MPR asset expansion rule will be to cause a temporary disequilibrium due to the firm's rate of return being greater than that required by the market. An upward revision of the firm's share price will restore the equilibrium, i.e. the firm's risk-return ordered pair will return to the original point on the SML, since by bidding up the share price the firm's investors will lower its expected return. This case is illustrated graphically in Fig. (5.2) where the firm is seen to move from its origin A to B and back.

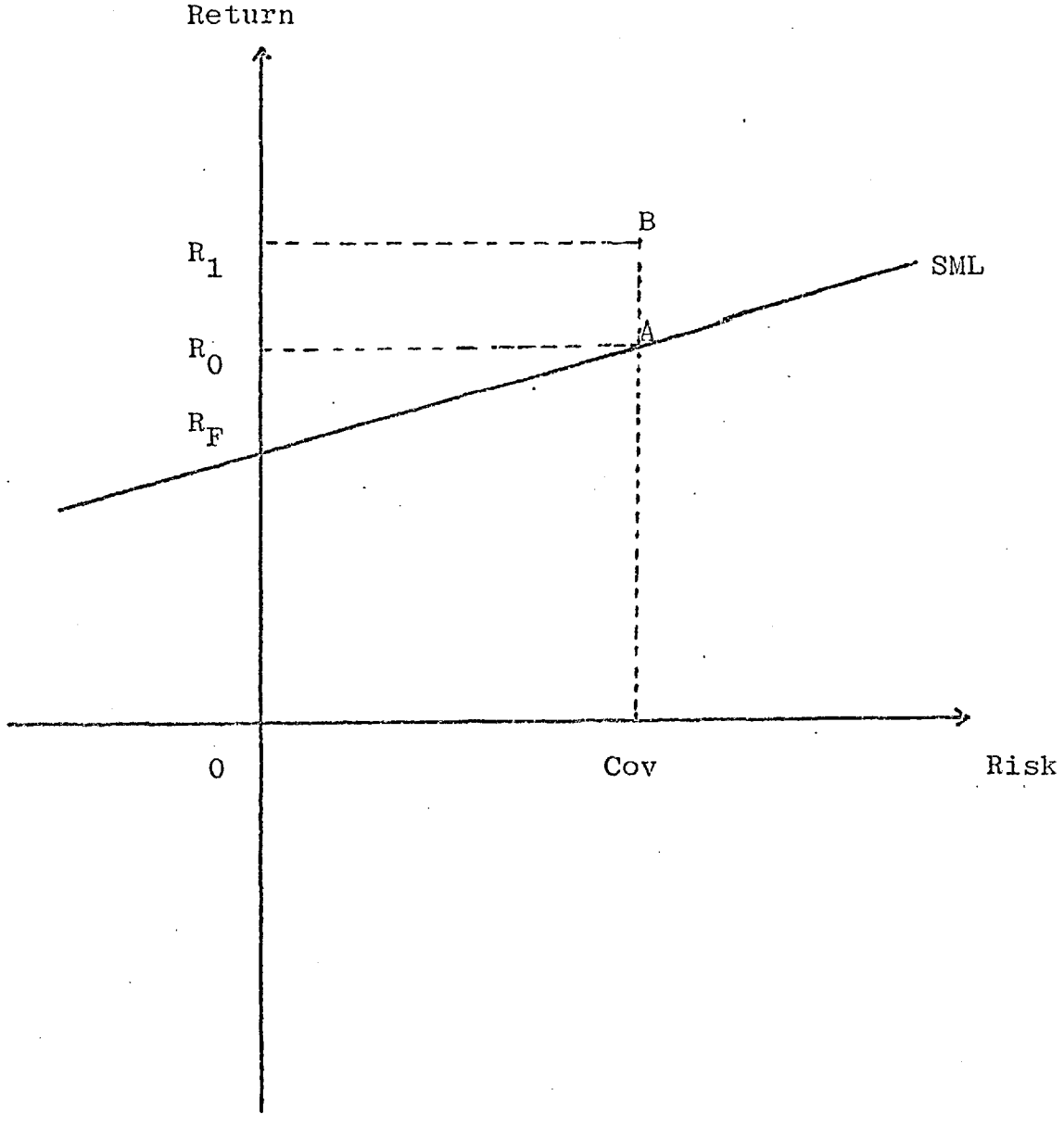


Fig. (5.2): Standard Price Increase Mechanism

In fact, when we examine a portfolio whose assets are economically dependent it is not surprising that a decision regarding the level of investment in one would have some effect on at least part of the remaining assets. Fowler and Rorke offer a general formulation of the two effects of an investment project, i.e. the direct contribution to the firm's return and the effect on the existing cash flows. They express the change in the firm's share price, i.e. the value of the project, as :

$$\Delta P_j = P_j^* - P_j = \frac{E(c_j + \Delta c_j)}{k^*} - \frac{E(c_j)}{k}$$

$$= \frac{E(\Delta c_j)}{k^*} - \frac{E(c_j)(k^* - k)}{kk^*} \qquad (5.7)$$

where:

$k = R_F + \lambda cov(R_j, c_M)$

$k^* = R_F + \lambda cov(R_j + \Delta R_j, c_M + \Delta c_j)$

The terms of the right hand side of Eq. (5.8) represent the direct and indirect effects of the new project respectively.

The fact that a project can alter the existing cash flows of a firm requires a revision of the analysis of both the investment acceptance rule and the capital market equilibrium. Clearly, in a situation where both the firm's expected return and risk are liable to be affected by the acceptance of a new project it would be misleading to use decision rules, such as the MPR, which advise acceptance or rejection solely on the basis of the project's return contribution. A modified criterion, one which considers both the direct and indirect effects, is required. In terms of Eq. (5.8) we can advise investing in a project which generates an upwards revision of the firm's share price, i.e. $\Delta P_j > 0$. In other words, only a project which has a combined positive effect on the firm's value is to be accepted; this decision rules explains cases where firms reject intuitively attractive investments, i.e.

projects which are profitable yet have a negative dominant effect on the existing cash flows, and cases where a project which does not generate a particularly high level of return becomes attractive due to its positive impact on other investments.

In view of the potentially conflicting forces generated by an investment we need to reassess our analysis of the possible disequilibriums the firm can undergo. In our previous discussion we adopted the conventional approach, consistent with CAPM theory, that a firm's ordered pair can be shifted either upwards or downwards. We now extend our analysis to include cases where the firm's risk is altered, resulting in sideways movements. We can summarize all possible outcomes following an investment in the form of a table. Specifically, Table (5.1) lists all possible combinations of the direct and indirect effects of a project and the matching graphical implications, defined in terms of Fig. (5.3). For the sake of a simple presentation, let us define:

$$d = \frac{E(\Delta c_j)}{k^*} \quad \text{and} \quad d' = \frac{E(c_j)(k^*-k)}{kk^*} \qquad (5.8)$$

hence:

CONDITIONS	LOCATION
A. if $\Delta P_j > 0$ and	
$d > 0;\ d' > 0$	I
$d > 0;\ d' = 0$	AY
$d > 0;\ d' < 0$	VI
$d = 0;\ d' < 0$	AZ
$d < 0;\ d' < 0$	V
B. if $\Delta P_j < 0$ and	
$d > 0;\ d' < 0$	II
$d = 0;\ d' < 0$	AZ'
$d < 0;\ d' < 0$	III
$d < 0;\ d' = 0$	AY'
$d < 0;\ d' > 0$	IV
C. if $\Delta P_j = 0$ and	
$d < 0;\ d' < 0$	SML Left
$d > 0;\ d' > 0$	SML Right
$d = 0;\ d' = 0$	A

TABLE (5.1): Capital Market Changes

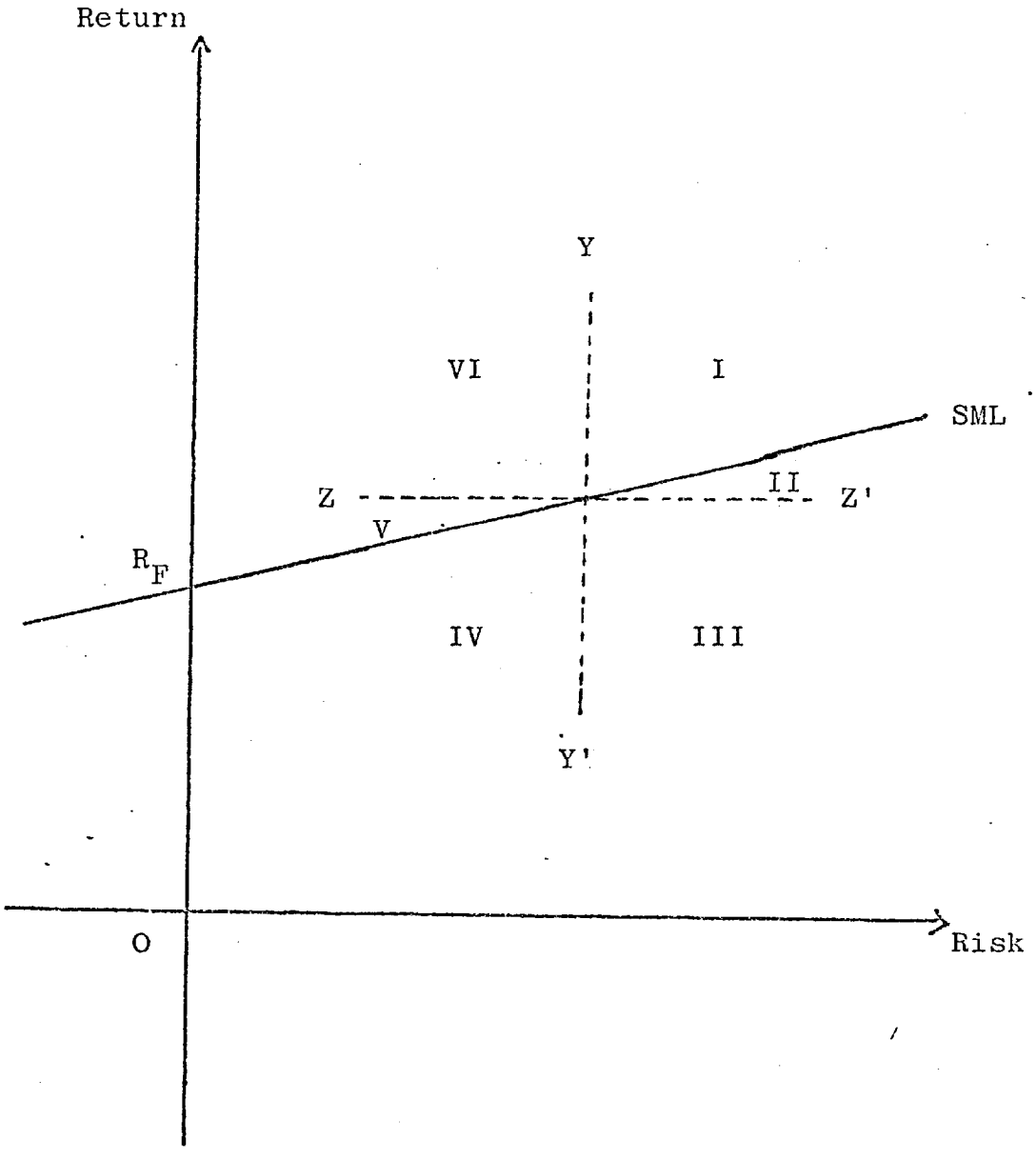


Fig. (5.3): Capital Market Equilibrium

The fact that a firm can embark on a policy which alters both its return and its risk means that two forces operate on its share price. As a result of the combined changes in risk and return it is clear that the MPR is an inadequate decision rule; we therefore turn to examine the aggregate effect of a project on the firm's share price. We can best make our point with the following example: let us consider the case of a firm which faces the need to select the best from two mutually exclusive projects. These hypothetical investments can be graphically illustrated so as to reflect not simply the total effect but also the breakdown to direct and indirect effects. Suppose projects B and C contribute to the firm's performance, both by their return and by their impact on the existingcash flows discount rate, yet to different extents. In Fig. (5.4) two such projects are reflected, starting from an initial point of equilibrium A, the firm is required to choose between projects B and C. Project B is seen to yield a greater excess rate of return than C, hence $d_B > d_C$ (where d_B is equal to project B's contribution d as defined earlier), yet project C reduces the firm's existing cash flows' discount rate by more than B, hence $d'_B < d'_C$ (in a similar way to d, d'_B is related to our earlier definition of d'). From Fig. (5.4) we can determine that project B offers a superior risk-return package than C, as reflected by the greater increase in share price it generates, and it is therefore project B that the firm should accept.

So far we have pointed out the general mechanism which governs the process which takes place when the firm alters its risk as well as its return. If, however, this is to become of any practical use in the planning and resource allocation within the firm, a more detailed model is required. Specifically, one would need to establish a connection between the operating units which form the firm, and the risk-return it generates. Such a link would enable the corporate planners to assess the impact of their operating decisions on the firm's value in the securities market, thus providing them with an objective criterion

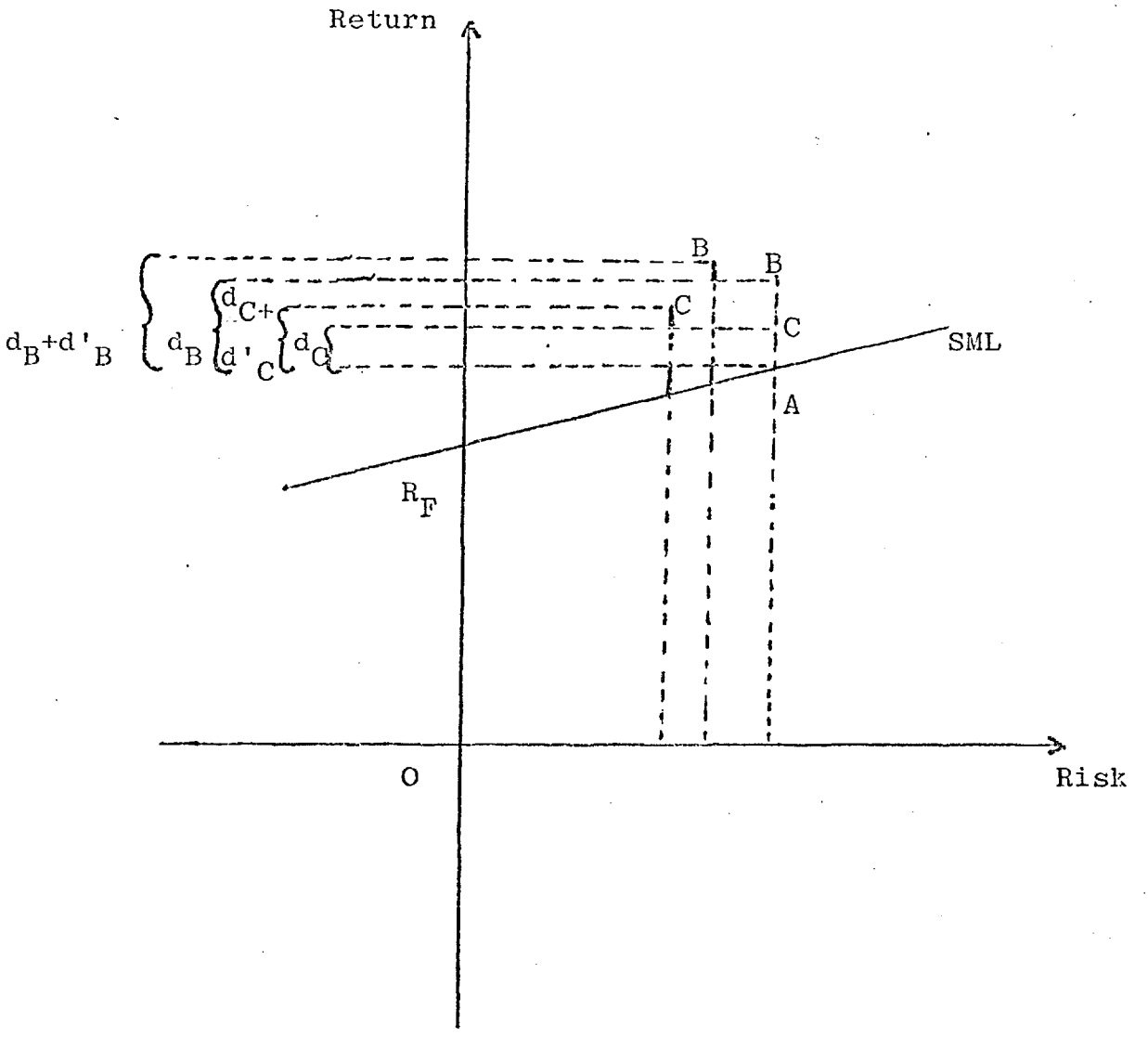


Fig. (5.4): Investment Project Ranking

for allocating the limited resources at their disposal between the economic opportunities available. We therefore turn our attention to an analysis of the firm within the CAPM framework, starting with a CAPM perfect world, and following with the case of economies of scale.

5.5 CAPITAL BUDGETING IN PERFECT COMPETITION

5.5.1 Expanding the Mean-Variance Theorem

We start with the mean-variance theorem, Eq. (5.1), which Rubinstein (125) expands to produce:

$$E(\tilde{R}_j) = R_F + \lambda^* \rho(\tilde{R}_j, R_M) \sqrt{\text{Var}\tilde{R}_j} \qquad (5.9)$$

where $\rho(\tilde{R}_j, R_M)$ is the correlation coefficient between the return on firm j and that on the market portfolio of risky securities, and $\lambda^* = \lambda\sqrt{\text{Var } R_M}$.

For a firm with debt Eq. (5.9) becomes

$$E(\tilde{R}_j) = R_F + \lambda^* \rho(\tilde{R}^*_j, R_M) \sqrt{\text{Var}\tilde{R}^*_j} \quad (1 + B_j/S_j) \qquad (5.10)$$

where:

B_j - present value of debt.

S_j - present value of equity

$V_j = B_j + S_j$, present total value of the firm's securities,

$*$ - superscript for a firm with zero debt, i.e., $B^*_j = 0$

We can rewrite Eq. (5.10) to separate the total risk of firm j:

$$E(\tilde{R}_j) = R_F + \lambda^* \rho(\tilde{R}^*_j, R_M) \sqrt{\text{Var } \tilde{R}^*_j} + \lambda^* \rho(\tilde{R}^*_j, R_M)\sqrt{\text{Var } \tilde{R}^*_j} \ (B_j/S_j)$$

(5.11)

re:

$$\rho(\bar{R}_j^*, R_M)\sqrt{\text{Var } \bar{R}_j^*} \quad - \quad \text{operating risk, and}$$

$$\rho(\bar{R}_j^*, R_M)\sqrt{\text{Var } \bar{R}_j^*}\ (B_j/S_j) \quad \text{financial risk}$$

hence, the sum of the operating and financial risk being the firm's total systematic risk.

We now proceed to prove Rubinstein's expansion of the operating risk. Let us take firm j with m products for which:

Q_m - the output in units, random variable;

v_m - variable cost per unit,

p_m - sales price per unit,

F_m - the fixed costs,

and α_m - the proportion of assets, i.e., of V_j, allocated to product m.

STEP 1: Suppose all fixed costs of the firm can be allocated, $X_j = \sum_m (Q_m p_m - Q_m v_m - F_m)$, where X_j, a random variable, is the value of the total net operating income. Since $\rho(X,Y) = \text{Cov}(X,Y)/\sigma_X\sigma_Y$ we can write:

$$\rho(\bar{R}_j^*, R_M)\sqrt{\text{Var } \bar{R}_j^*} = \text{Cov}(\bar{R}_j^*, R_M)/\sqrt{\text{Var } R_M} \qquad (5.12)$$

STEP 2: Since $R_j \equiv X_j/V_j$[3] we can rewrite the covariance operator in the right hand side of Eq.(5.12)to

3. Notice the change in the definition of the firm's return when compared with that on page 115. In our discussion of the capital market mechanism we referred to the economic definition of rate of return, namely $R_j = c_j/P_j$, as suggested by Fowler and Rorke (53). However, for the purpose of our analysis of the capital budgeting process in a decentralized firm it is preferable to adopt the accounting definition of the corporate rate of return, $R_j = X_j/V_j = \sum_m (Q_m p_m - Q_m v_m - F_m)$, which provides the basis for an expansion of the mean-variance theorem and the linkage of product line variables with the corporate performance.

produce:

$$\mathrm{Cov}(\bar{R}_j^*, R_M) = \mathrm{Cov}(X_j/V_j, R_M)$$

$$= 1/V_j \ \mathrm{Cov}(X_j, R_M)$$

STEP 3: We can substitute X_j to produce

$$\mathrm{Cov}(\bar{R}_j^*, R_M) = 1/V_j \ \mathrm{Cov}(\sum_m (Q_m(p_m - v_m) - F_m), R_M)$$

STEP 4: Which, since F_m is constant, can be rewritten

$$\mathrm{Cov}(X_j, R_M) = \mathrm{Cov} \sum_m (Q_m(p_m - v_m), R_M)$$

STEP 5: Since $\mathrm{Cov}(\sum_i X_i, Y) = \sum_i \mathrm{Cov}(X_i, Y)$ we can write

$$\mathrm{Cov}(X_j, R_M) = \sum_m \mathrm{Cov}(Q_m(p_m - v_m), R_M) \qquad (5.13)$$

STEP 6: and since $(p_m - v_m)$ is assumed fixed, we can rewrite Eq.(5.13)

$$\mathrm{Cov}(X_j, R_M) = \sum_m ((p_m - v_m)\mathrm{Cov}(Q_m, R_M))$$

STEP 7: hence the operating risk becomes

$$\rho(\bar{R}_j^*, R_M)\sqrt{\mathrm{Var}\ \bar{R}_j^*} = \sum_m ((p_m - v_m)\mathrm{Cov}(Q_m, R_M))/V_j\sqrt{\mathrm{Var}\ R_M}$$

(5.14)

STEP 8: which can be rewritten as:

$$\rho(\bar{R}^*_j, R_M)\sqrt{\text{Var } \bar{R}^*_j} =$$

$$\sum_m \{((p_m - v_m)\rho(Q_m, R_M)\sqrt{\text{Var } Q_m}\sqrt{\text{Var } R_M})\}/V_j\sqrt{\text{Var } R_M}$$

STEP 9: which, if we cancel Var R_M and multiply by α_m/α_m, becomes

$$\rho(\bar{R}^*_j, R_M)\sqrt{\text{Var } \bar{R}^*_j} =$$

$$\sum_m \{(\alpha_m(p_m - v_m)\rho(Q_m, R_M)\sqrt{\text{Var } Q_m/\alpha_m V_j}\}$$

In this form the elements of the operating risk can be interpreted as follows:

α_m - measures the relative importance of each of the firm's activities (assuming all assets of the firm can be allocated between them).

$(p_m - v_m)$ - reflects operating leverage.

$\rho(Q_m, R_M)$ - the pure influence of economy wide events on output, and

$\sqrt{\text{Var } Q_m/\alpha_m V_j}$ - the uncertainty of output per unit of asset, which indicates the uncertainty of operating efficiency.

We can now reformulate the mean-variance theorem, i.e., Eq.(5.1)

$$E(\bar{R}_j) = R_F + \lambda^* \sum_m \{\alpha_m(p_m - v_m)\rho(Q_m, R_M)\sqrt{\text{Var } Q_m/\alpha_m v_j}\}$$

$$+ \lambda^* \sum_m \{(\alpha_m(p_m - v_m)\rho(Q_m, R_M)\sqrt{\text{Var } Q_m/\alpha_m V_j})\} (B_j/S_j)$$

(5.15)

In this latest form, the mean-variance theorem is a powerful tool. It links a firm's expected rate of return with variables related to its product lines and, by doing so, establishes a vital connection between the firm's major parameters - financial leverage, allocation of resources between activities and output level, and its financial performance. This concept is both fundamental and far-reaching in its consequences; however, before we proceed to examine its implications, a clarification of the assumptions underlying it is required.

Let us begin by looking at STEP 3 where we freely extracted V_j from the covariance operator. The statistical implication of this move is the independence of V_j from X_j, and the economic one, the independence of the firm's total net operating income from either its present value of equity, assuming the firm is debtless, or its present value of securities, if a positive debt does exist. In other words, for STEP 3 to be legitimate, the following assumption has to be made concerning the relationship between the above parameters, specifically, that changes in the value of the firm's securities are not affected by fluctuations in its total net operating income, and vice versa. Clearly, this is a fairly restrictive assumption, since it is hard to imagine a realistic situation where investors in the stock market fail to adjust share prices to changes in a firm's performance, or alternatively, where a reduction in the firm's equity would not

affect indirectly, i.e., through the working capital, the firm's operating income.

Similarly restrictive is the assumption necessary to support STEP 6, where we extracted the unit profit, $(p_m - v_m)$, from the covariance operator. As in the previous case, we have made an implicit assumption about the relationship between parameters within the operator, namely that the sales of product m, Q_m, are statistically independent of its unit profit, and vice versa. The economic implications of such a relationship are far reaching, and can best be examined by looking separately at the connection between a product's sales, with its unit price, p_m, and with its unit variable cost, v_m.

The economic meaning of sales being independent of price can only be one, namely, that the firm has no influence on the market price of the products it supplies. In economic theory terminology, such a case is termed as the firm operating in a perfectly competitive market for its output, where it faces a unique market price determined by an infinitely elastic demand curve, such as the one graphically illustrated in Fig.(5.5.a.). Under such conditions, a firm can increase its sales rate without shading its price to get the extra business; a small decrease in price would result in an indefinitely large increase in the quantity it could sell, and a very small increase in price would result in its selling nothing. Moreover, since price remains constant, each additional unit sold increases total revenue by the amount of the price, the consequence being that price and marginal revenue are equal, as illustrated in Fig.(5.5.b).

Let us now return to the analysis of the economic implications of the independence between the sales of product m and the unit variable cost. When making such an assumption, we are in fact implying two complementary things;

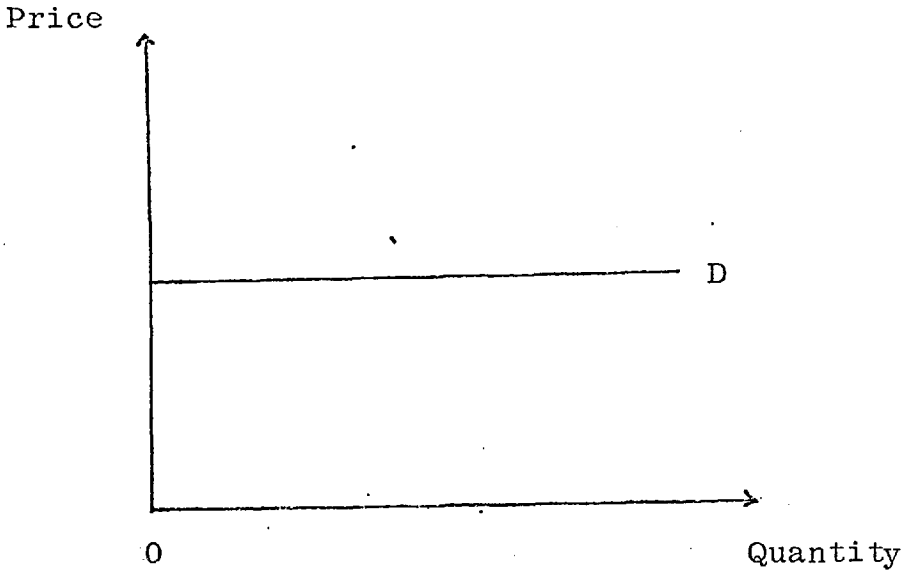


Fig.(5.5.a): A demand curve in perfect competition

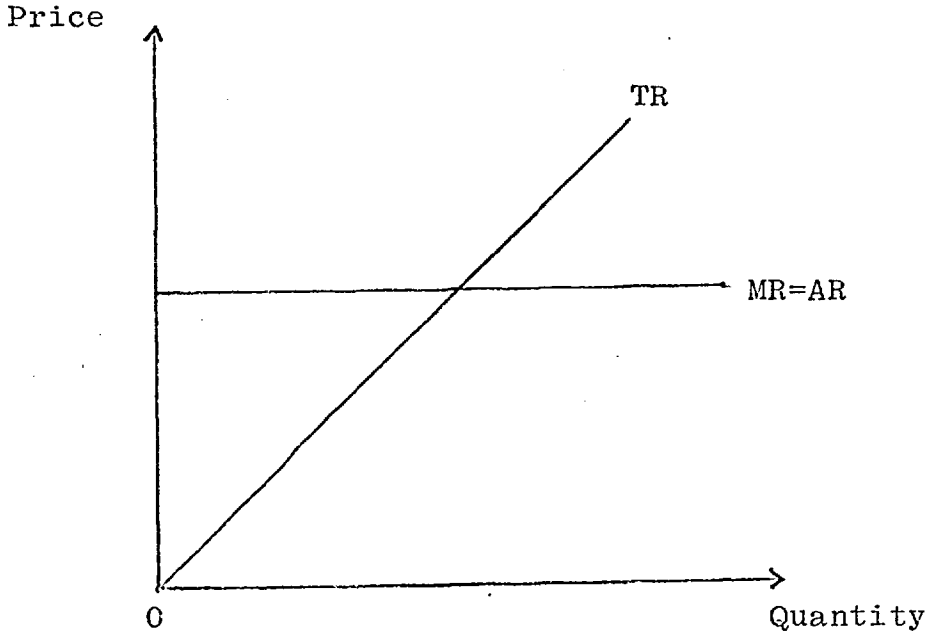


Fig.(5.5.b): Revenue curves in perfect competion

first, that an increase in sales does not alter the variable unit cost, and secondly, that variable unit costs are identical for all input quantities purchased by the firm. This distinction is hardly semantic; while the former suggests that the firm's production function has constant returns to scale, the latter implies the firm purchases its inputs in a perfectly competitive market. Both these conditions are required if the firm's unit variable cost is to remain fixed for all levels of sales. If the firm was to have increasing or decreasing returns to scale, combined with fixed unit input costs, its unit variable costs would be decreasing or increasing respectively. If, on the other hand, the firm was to produce with constant returns to scale, yet face increasing or decreasing input unit costs, the variable unit cost would fluctuate accordingly. Consequently, the firm which has a unit variable cost independent of the output level, faces costs curves as represented in Fig.(5.6).

The conclusion of this economic analysis of the statistical independence is that, for STEP 6 to be legitimate, the market in which the firm operates has to be a perfectly competitive one, where limitless quantities of inputs can be purchased at fixed prices, and infinite amounts of products and services can be sold at market prices. In other words, the firm is certain that, no matter what its production level is, it is guaranteed access to production resources at guaranteed prices, and similarly a market and fixed prices are certain to exist for its output.

This set of assumptions leads to one obvious question: if both demand and supply are riskless, why is the firm considered to be operating under uncertainty? - i.e. the return on the firm's securities is defined in terms of an expected value and a variance. Indeed, the two concepts,

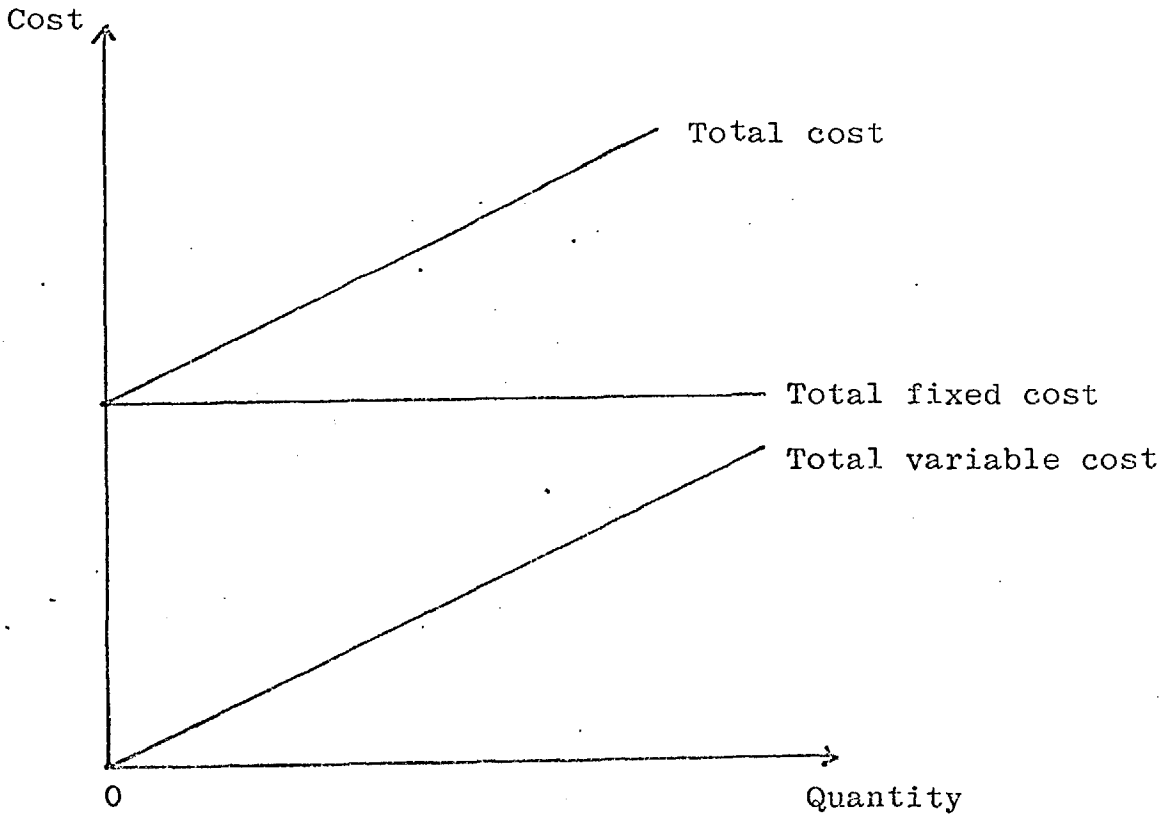


Fig.(5.6): Cost curves in perfectly competitive market

uncertainty and perfect competition conflict unless some other source of uncertainty exists in the firm's operations. Consequently, STEP 6 requires an assumption about a source of uncertainty in the firm which exists exogenously to the firm's production and marketing activities.

Another step which requires a rather restrictive assumption, if it is to be legitimate, is STEP 9, specifically the incorporation of α_m, the proportion of assets allocated to product m, into the equation. For this move to be acceptable, either α_m is statistically independent of all her parameters in the expanded mean-variance theorem, or it is a fixed value. If we accept the latter proposition, the mean-variance theorem in its expanded form becomes of little analytical use in a dynamic context, i.e. it can only be of use in an equilibrium analysis. On the other hand, if we assume statistical independence we are implying that a product's performance, as measured by its sales, Q_m, is unsensitive to changes in the proportion of total resources available to the store manager allocated to it. Clearly, a highly unrealistic suggestion.

To recapitulate, the rather stringent set of assumptions necessary to transform Rubinstein's expansion of the mean-variance theorem into a useful tool in a dynamic context, consists of:

i) the independence of present value of equity and total net operating income;

ii) the existence of uncertainty in a perfectly competitive market, and

iii) the independence of the sales of product m and proportion of resources allocated to it.

Obviously, any analysis of the firm's risk, which is to be of managerial use, will require a relaxation of these assumptions. In the subsequent section, i.e. 5.6, we provide an expansion of the mean-variance theorem which does not require any of the above conditions. However, prior to that, we produce one example of how the mean-variance theorem can be further expanded under the above set of assumptions.

5.5.2 Expanding the Uncertainty of Operating Efficiency

One of the potentially more useful components of the expanded mean-variance theorem is the uncertainty of operating efficiency,

$$\sqrt{\text{Var } Q_m / \sum_k \alpha_{mk} c_k}$$

In this element, sales and resources are linked, thus providing a measure of the risk involved in allocating additional amounts of resources to the various products. In other words, as a product's uncertainty of operating efficiency increases, it becomes riskier to increase the amount of resources to that particular product. Clearly, this kind of information is potentially of use in the resource allocation process, since a product's contribution to the corporate performance is measured in terms of risk as well as return.

Greater insight can be generated by the uncertainty of operating efficiency if the aggregate amount of resources are separated into the individual types of resources used by the products. Such a breakdown would provide management with a specific measure of the riskiness of each type of resource when allocated to a particular product or service. We therefore proceed to expand the aggregate expression of the uncertainty of operating

efficiency, starting with a simplified example illustrating the method used.

For any a,b independent of a random variable X, we can write:

$$\mathrm{Var}\left(\frac{X}{a+b}\right) = \left(\frac{X}{a+b}\right)^2 \mathrm{Var}\ X \qquad (5.16)$$

which, if multiplied by a^2/a^2, becomes:

$$\mathrm{Var}\left(\frac{X}{a+b}\right) = \left(\frac{a}{a+b}\right)^2 \mathrm{Var}\left(\frac{X}{a}\right) \qquad (5.17)$$

and similarly, if Eq. (5.17) is multiplied by b^2/b^2, it becomes:

$$\mathrm{Var}\left(\frac{X}{a+b}\right) = \left(\frac{b}{a+b}\right)^2 \mathrm{Var}\left(\frac{X}{b}\right) \qquad (5.18)$$

Clearly, Eq.(5.17) and Eq. (5.18) are equal, hence we can sum them and divide them to produce:

$$\mathrm{Var}\left(\frac{X}{a+b}\right) = \frac{1}{2(a+b)^2}\left\{a^2\mathrm{Var}\left(\frac{X}{a}\right) + b^2\mathrm{Var}\left(\frac{X}{b}\right)\right\}$$

Naturally, this procedure can be repeated for any number of elements, thereby enabling us to write:

$$\sqrt{\mathrm{Var}\ Q_m/\Sigma_k \alpha_{mk} c_k} = \sqrt{\Sigma_k\{(\alpha_{mk} c_k)^2\ \mathrm{Var}(Q_m/\alpha_{mk} c_k)\}/K(\Sigma_k \alpha_{mk} c_k)^2}$$

(5.19)

where Var $(Q_m/\alpha_{mk}\, c_k)$ is the uncertainty of a monetary unit of resource of type k allocated to product m, and $K = \sum_k 1$.

This expression of the uncertainty of operating efficiency has the advantage of establishing a linkage between a product's performance and the relative riskiness of the various types of resources it uses. A similar relationship, between the riskiness of resource when allocated to a particular product and the corporate equilibrium rate of return required by the capital market, can be established by substituting Eq.(5.19) into Eq.(5.15) Hence:

$$E(R_j) = R_F + \lambda^* \sum_m \{\alpha_m (p_m - v_m)\rho(Q_m, R_M)\} \Big/ \sqrt{\sum_k \{(\alpha_{mk}\, c_k)^2\, \mathrm{Var}(Q_m/\alpha_{mk}\, c_k)\}} \Big/ \sqrt{K(\sum_k \alpha_{mk}\, c_k)^2\}} (1 + B_j/S_j) \qquad (5.20)$$

We have therefore formulated the mean-variance theorem in a way which enables us to measure the impact of changes in the firm's resource allocation between activities on the riskiness of the firm, and thereby on its expected equity rate of return. We now proceed to examine the implications of the existence of economies of scale on the mean-variance theorem, relaxing in the process the assumptions made in this section.

5.6 CAPITAL BUDGETING WITH ECONOMIES OF SCALE

5.6.1 Economies of scale in decentralized firms

Recent empirical research by Corey (35) into the organisation of the procurement function in major Amercian corpporations identifies the existence of significant restructuring since the early 1970's.

Specifically, the trend is towards greater centralization of procurement in decentralized companies at the expense of the pure concepts of decentralization and profit centre management. The main factors prompting the development of centralized purchasing are the wish to secure future growth through assuring long-term availability of supply and the tremendous potential for improving profit margins. To illustrate this last point, Corey cites a study made at General Motors of the purchase of one supply item, which revealed that ten GM divisions were paying seven different prices for the same item, and that one supplier alone, supplying five divisions, quoted different prices for the same item to each of the divisions. A subsequent contract negotiated by GM's centre meant estimated savings of more than 12% over prices paid the previous year. With the costs of parts, materials and other purchased products and services accounting, in GM's case, for 28 billion dollars in 1977, the motivation to maximize purchasing power by procurement centralization to obtain favourable terms is obvious. Corey's work, however, was restricted to the examination of the potential economies of scale a powerful firm could achieve by centralizing the supply of goods and services to its divisions, ignoring the potential of a similar centralization of activities in the capital market, an area which could prove equally beneficial for large decentralized firms.

Some idea of the impact of a firm's size on the capital market is provided by a study by Sullivan (143). In his research, Sullivan examines the relationship between capital costs and the market power of firms, using the CAPM to explain how capital costs and risk are related. The results of this analysis point out that the systematic risk, hence the capital costs, in general appears to be lower for firms that are large and for firms that sell highly in concentrated markets.

Archer & Faerber (8) provide one explanation for this observation, namely that the powerful firm would seek to raise, ceteris paribus, larger amounts of capital than a small firm, and therefore the floatation costs per unit raised by the large firm would be smaller. To that, Sullivan adds another probable explanation: that the securities of a powerful firm, after being issued, would be subject to less non-diversifiable price volatility as the secondary market fluctuates in response to changes in broad systematic factors. This results in suppliers of capital requiring a lower expected dividend and price appreciation return, i.e. lower cost of capital, to invest in the powerful firm in comparison with the non-powerful one. In other words, the powerful firms are less risky because they are less susceptible to non-diversifiable periodic revaluation in the secondary market in light of new systematic information.

Further support for these findings can be found in Davis and Yeomans' (40) study of the influence of firm size, method of issue, and market volatility on the market discount on new issues of equity. Their conclusion about the importance of the firm's size was that: it appears that, at least in periods of market stability corresponding loosely to market equilibrium, small firms utilising offers for sale (the most popular method of issue) incur market discounts.

In addition to these general findings concerning large firms, there exist several suggestions that certain types of economies of scale are characteristic to the retail business in particular. Jeffereys (82), in his massive study of the evolution of retail trade in Britain, concludes that department stores have various forms of economies of scale. The first source he identifies is the advantages large firms have in buying, namely the fact that they buy in bulk, bypass the wholesaler, and have a specialization of the function within the organization. This specialization is not confined purely to buying, a second source of economies of scale is derived from the store's skilled selling of a wide range of goods under one roof.

More generally, de Somogyi (141) identifies three types of economies of scale in retail:

i) Vertical

ii) Horizontal

iii) Central

Specifically, the vertical advantages are due to the operation of large stores, the horizontal ones to the operation of numerous stores, and finally the central economies of scale due to the existence of one head office for all stores within the chain which provides centralized (i.e. at reduced cost per store) services.

Having examined the costs and competion in retailing, McClelland (104) finds that the department store can be said to represent expansion by diversification. Consequently, he identifies the economies of the multiple mainly on the supply side, i.e. those of the department store in relation to the customer, e.g. single building, single credit facility.

Size can also create opportunities for economies of scale in a firm's marketing activities. An established brand name, common to all parts of the firm, and the impact of publicity are both more cost effective, within a certain range, given a greater number of outlets. Having analysed data for retail firms, Douglas (42) concludes that the large firm is capable to use resources in distinctive ways to build a corporate image, and that they may be economies of scale in this phase of retailing. These economies manifest themselves, not in the relative level of costs, but in the relative level of the cost-revenue relationship.

Kaldor (83) offers two possible explanations for the fact that, over at least some range of output levels, there are increasing returns to advertising expenditures. First, as the number of messages produced by the firm increases (as a result of an expansion) the cost per message decreases in a way similar to that described earlier regarding discounts on other goods and services. The second reason suggests that it is plausible that, over at least a limited range, there are increasing sales returns to advertising messages, though there is likely to be a saturation point.

Another possible explanation is based on the public awareness of the firm and its products generated by the promotion efforts. These will be more beneficial as the number of outlets selling the goods increases. It seems that part of the increased awareness generated by an advertising campaign is wasted if potential customers cannot complete the buying process initiated by it. The probability of the purchasing process being successfully completed depends on the availability of the goods which, in turn, is determined, inter alia, by the number of outlets in the chain.

Regardless of the precise explanation for these observa-

tions, all these studies point towards the same direction, namely, that powerful decentralized firms can, within a certain range, increase their purchasing power in the products, services and capital markets by operating in them through strong procurement bodies, with the effect of reducing costs and securing supply.

The observations mentioned up till now, regarding the behaviour of large firms, are consistent with basic economic theory. The theory of the firm has long ago concluded that, faced with a totally elastic demand curve, i.e. the firm can at a given price sell any level of output, and economies of scale, the firm can reduce its long range average costs by increasing its output. This increase in scale will generate greater profits since as long as the market is perfectly competitive and long range costs decline, average revenue, i.e. unit price, does not change, while at the same time average costs fall. It follows that, in a long-run equilibrium, a firm must have no unexploited economies of scale within its reach.

Empirical data gathered by Douglas (42) provides some evidence that the short-run cost curves and long-run envelope curves described by economic theory are appropriate for the retail trade. Analysing data from the Inland Revenue Service and U.S. Treasury sources on corporate income tax returns of nine types of retail, Douglas concludes that retailing appears to be an industry of decreasing costs up to a point; however, due to lack of sufficient data, a mathematical description of the above curves was impossible to compute.

The classic cost curves suggested by economists are graphically illustrated in Fig.(5.7.a) which shows the demand curve D, faced by a firm in perfect competion, the Long Range Marginal Costs (LRMC) curve, and the Long Range

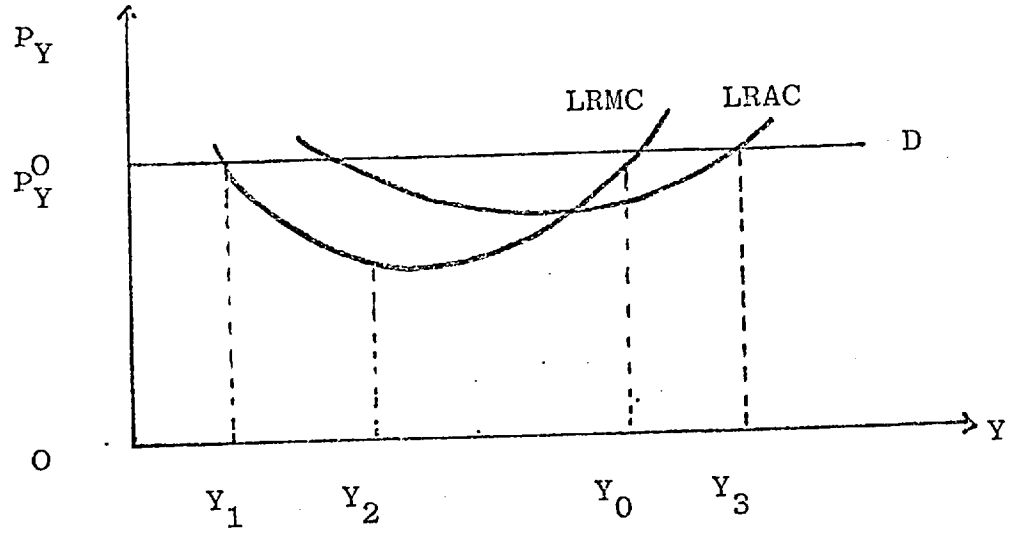


Figure (5.7.a)

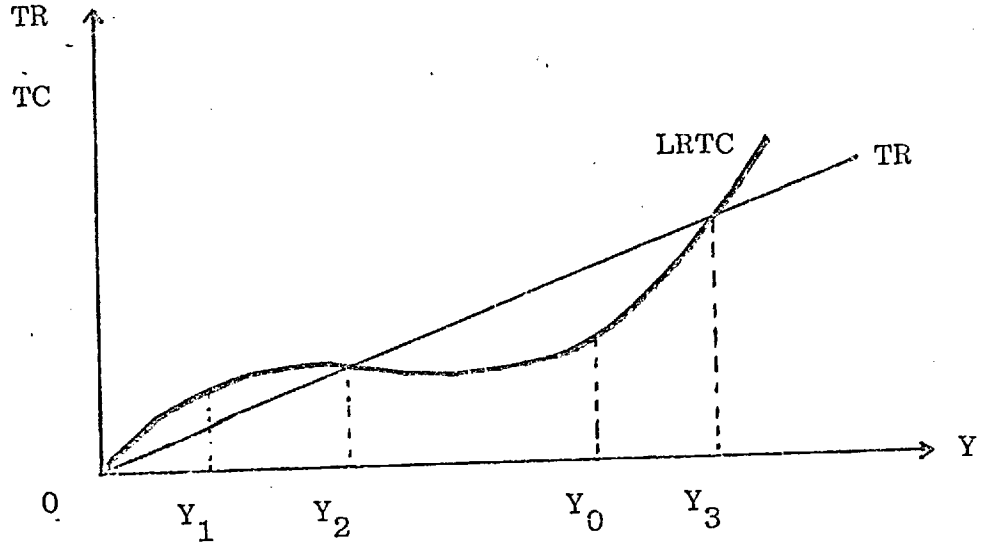


Figure (5.7.b)

Average Costs (LRAC) curve for a firm with varying returns to scale. In Fig. (5.7.b), the impact of a change in output, under the above conditions, can be analysed as follows: the revenue can be represented by a straight line, starting from the origin, Total Revenue (TR), with a slope equal to the product's price, P_Y (hence the higher the price the steeper the slope), the Long Run Total Costs (LRTC) curve reflects first economies of scale then increasing average costs, and the profit is the difference between the TR and LRTC.

A level of output lower than Y_2 means the firm will suffer losses; levels between Y_2 and Y_3 will generate profits, which will be maximized at that output level where the difference between TR and LRTC is greatest and their slopes identical, i.e. at Y_0. This can be explained by examining the options open to the firm at point Y_0; a reduction of output by one unit will reduce revenue by a greater amount than costs, since LRTC's slope is greater than TR's, while an increase of output by one unit will increase revenue by less than the increase in costs, hence both options have a net detrimental effect on the profit orientated firm. It follows that, if a firm is in a position to shift itself towards Y_0, it will aim to do precisely that. If, as in the hypothetical case we are discussing, the firm can take advantage of economies, this implies that it is at Y_0's left, and therefore expansion will move it to the right and closer to Y_0, its long-range equilibrium.

We can sum up the main points up to here as follows: decentralized firms, above a certain range, can reduce part of their unit costs by expanding the scale of their activities, thereby exploiting falling costs and increasing sales, which smaller firms, or those beyond the relevant range, do not have. We now proceed to incorporate this concept into the CAPM and analyse its implications on capital budgeting decision rules.

5.6.2. The CAPM and Economies of Scale

In order to incorporate economies of scale into the CAPM, let us return to the expanded mean-variance theorem, Eq.(5.11), and expand it under assumptions different to those made in section (5.5), i.e. we acknowledge the relationship between sales, variable cost per unit and sales price per unit.

Let us return to Eq.(5.13) and proceed from there, hence

$$\rho(\bar{R}^*_j, R_M)\sqrt{\text{Var } \bar{R}^*_j} = \{ \sum_m \text{Cov}\{(Q_m(p_m - v_m)/V_j), R_M\}/\sqrt{\text{Var } R_M}\}$$

which, if we cancel $\sqrt{\text{Var } R_M}$, becomes:

$$= \sum_m \{\rho\{(Q_m(p_m - v_m)/V_j, R_M\}\sqrt{\text{Var}\{Q_m(p_m - v_m)/V_j\}} \qquad (5.21)$$

and can be rewritten:

$$= \sum_m \{\rho\{Q_m(p_m - v_m)/ \sum_m \sum_k \alpha_{mk} c_k), R_M\}\sqrt{\text{Var}\{Q_m(p_m - v_m)/ \sum_m \sum_k \alpha_{mk} c_k\}\}} \qquad (5.22)$$

and substituted into Eq.(5.9) to produce:

$$E(R_j) = R_F + \lambda^* \sum_m \{\rho\{\{Q_m(p_m - v_m)/\sum_m \sum_k \alpha_{mk} c_k\}, R_M\}\sqrt{\text{Var}\{Q_m(p_m - v_m)/ \sum_m \sum_k \alpha_{mk} c_k\}\}}(1 + B_j/S_j) \qquad (5.23)$$

This formulation of the mean-variance theorem does not require any of the assumptions made in the previous section, and therefore is perfectly capable of handling the case of a firm with economies of scale. However, before we proceed to discuss the various aspects of this concept, let us pause to point out that the MPR asset expansion rule can be redefined on the basis of Eq.(5.22), so as to provide greater insight, and thus becomes:

$$\frac{E(\bar{R}_j) - R_F}{\sqrt{\text{Var } R_M \sum_m \{\rho\{Q_m(p_m - v_m)/\sum_m \sum_k \alpha_{mk} c_k, R_M\} \sqrt{\text{Var}\{Q_m(p_m - v_m)/\sum_m \sum_k \alpha_{mk} c_k\}}\}}} > \lambda$$

for a debtless firm, and assuming

$$\sqrt{\text{Var } R_M \sum_m \{\rho\{Q_m(p_m - v_m)/\sum_m \sum_k \alpha_{mk} c_k, R_M\} \sqrt{\text{Var}\{Q_m(p_m - v_m)/\sum_m \sum_k \alpha_{mk} c_k\}}\}} > 0$$

Clearly, the expanded MPR criterion (MPRX) uses the same reasoning as the MPR iteself since both advise acceptance of a project only if its expected internal rate of return, $E(\bar{R}_j)$, exceeds the appropriate risk-adjusted discount rate for the project. The advantage of this formulation is in the linkage it offers between the decision rule and individual product line variables. The MPRX decision rule is graphically illustrated in Fig.(5.8), where the firm should accept a project only if the project's risk-return ordered pair falls above the market line, e.g. projects A and B.

The expanded version of the mean-variance theorem, Eq.(5.23) has the structure necessary for the analysis of a firm with potential economies of scale. Suppose firm j has non-

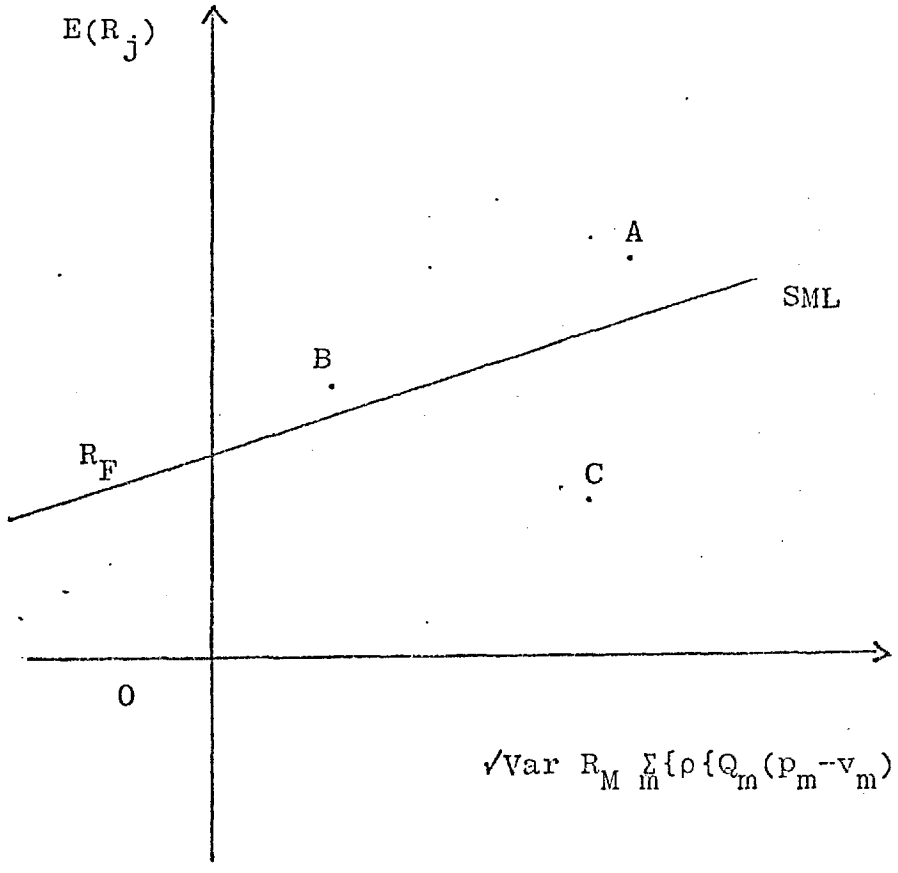


Fig.(5.8): The MPRX criterion

exploited decreasing unit costs, i.e. it is on the left of Y_0 in Fig.(5.7.a) and Fig.(5.7.b), and that at that volume of production the firm faces the following set of unit prices of resources it purchases $(c_1, c_2, \ldots, c_k)$. If the firm takes advantage of the declining LRAC curve and increases its output by establishing a new division, it can expect part or all of the unit prices of resources to fall accordingly. As a result of this move to the right and along the LRAC the firm faces a new set of unit prices for its resources, specifically $(\hat{c}_1, \hat{c}_2, \ldots, \hat{c}_k)$, where $\hat{c}_k$ is the new unit resource price, hence $(c_k \geq \hat{c}_k \mid k \epsilon K)$.

We can now proceed to incorporate into the mean-variance theorem the effects of expansion with economies of scale. First let us examine the impact of the reduction of procurement costs; the results of expansion will be reflected in cuts in the variable unit cost, v_m, and the unit cost of resource, c_k. Both these values will decrease as the firm moves towards Y_0, i.e. as long as the LRMC is smaller than the marginal revenue. The second type of advantages, i.e. those available in the firm's marketing activities, will generate a fall in the firm's unit cost of advertising, hence in its variable unit cost, v_m, and possibly an increase in sales of certain products, i.e. Q_m.

Naturally, the firm's main concern, when considering the expansion projects, is the net effect increased output will have on its risk-return performance. Under our set of assumptions, the acceptance of an investment project can lead not only to changes in the firm's return, but in its risk as well. In other words, in face of considerable empirical evidence, it is only reasonable to assume that the increased power the firm has in the various markets as a result of its increased output level will generate benefits which, to some extent, will be shared by numerous branches of the organization. In terms of a department

ore chain, such a case can be illustrated by the effect
opening of a new branch has on the reduction of unit
t of a product stocked by existing stores as a result
the increase in the quantities purchased by the firm
llowing the expansion. Basically, we are implying that
several branches hold homogeneous products and the open-
g of a new one cuts some of their unit costs, as well as
ducing the unit fixed costs through the central purchas-
g, this benefit will be reaped by all branches concerned
not limited to the latest one which generated the savings.

can now turn to analyse the process of expansion with
onomies of scale within the CAPM framework. In section 5.3
referred to Fowler and Rorke's (53) assessment of the
anges in the firm's risk-return performance following an
crease in scale, we are now in a position to provide greater
onomic insight into this discussion. Let us consider the
se of a firm which operates m product lines and which is
nsidering adding one, denoted m+1. Assuming the firm is,
remains, debtless throughout the analysis, we can denote
s value by using the expanded mean-variance theorem,
. (5.23), hence:

$$= \frac{E(\sum_{m}\alpha_m R_M)}{R_F + \lambda\sqrt{\sum_{m}\{\rho\{\{Q_m(\rho_m - v_m)/\sum\sum\alpha_{mk}c_k\}, R_M\}\sqrt{Var\{Q_m(\alpha_m - v_m)/\sum\sum\alpha_{mk}c_k\}}\}}} \qquad (5.24)$$

llowing the firm's expansion, the value of the firm becomes:

$$= \frac{E(\sum_{m+1}\alpha_{m+1}R_{m+1})}{R_F + \lambda^{*}\sum_{m=m+1}\{\rho\{\{\hat{Q}_{m*}(\alpha_{m*} - \hat{v}_{m*})/\sum_{m*}\sum\hat{\alpha}_{mk*}\hat{c}_k, R_M\}}{\sqrt{Var\{\hat{Q}_m(\alpha_{m*} - \hat{v}_{m*})/\sum_{m*}\sum\hat{\alpha}_{mk}\hat{c}_k\}}} \qquad (5.25)$$

can now express the change in the value of the firm as the
of the new line's contribution to the corporate return

and its effect on the established lines. Using the previous definitions of d and d', we can write:

$$d = \frac{E(R_{m+1})}{R_F+\lambda^*\sum_{m^*}\{\rho\{\{\hat{Q}_{m^*}(p_{m^*}-\hat{v}_{m^*})/\sum_{m^*}\sum_k\hat{\alpha}_{mk^*}\hat{c}_k\},\ R_M\}\sqrt{Var\{\hat{Q}_{m^*}(p_{m^*}-\hat{v}_{m^*})/\sum_{m^*}\sum_k\hat{\alpha}_{mk^*}\hat{c}_k\}}}$$

and

$$d' = \frac{(\sum\alpha_m R_m)}{R_F+\lambda^*\sum_{m^*}\{\rho\{\hat{Q}_{m^*}(p_{m^*}-\hat{v}_{m^*})/\sum_{m^*}\sum_{m^*}\hat{\alpha}_{mk^*}\hat{c}_k\},\ R_M\}\sqrt{Var\{\hat{Q}_{m^*}(p_{m^*}-\hat{v}_{m^*})/\sum_{m^*}\sum_k\hat{\alpha}_{mk^*}\hat{c}_k\}}}$$

We can define the condition for a decrease in the old investments' discount rate as follows:

$$\sum_m\{\rho\{Q_m(p_m-v_m)/\sum\sum\alpha_{mk}c_k, R_M\ \ Var\ Q_m(p_m-v_m)/\sum\sum\alpha_{mk}c_k\}\} \ >$$

$$\sum_{m^*=1}^{m+1}\{\rho\{\hat{Q}_{m^*}(p_{m^*}-\hat{v}_{m^*})/\sum_{m^*}\sum_k\hat{\alpha}_{mk^*}\hat{c}_k, R_M\}\sqrt{Var\{\hat{Q}_{m^*}(p_{m^*}-\hat{v}_{m^*})/\sum_{m^*}\sum_k\hat{\alpha}_{mk^*}\hat{c}_k\}}\}$$

(5.26)

In other words, if and only if the condition expressed in Eq. (5.26) holds will the project's indirect effect be positive, i.e. contribute to the increase of the firm's share price. If, however, the above inequality does not hold, as Fowler and Rorke suggest, the investment's indirect effect would be to lower the market value of the firm's old investments, and the aggregate impact would depend on the relative magnitudes of the two contradicting effects.

In the following chapter we turn our attention to discuss the structure of a department store's portfolio in general and its risk in particular. We will concentrate on demonstrating the conditions which generate a risk reduction and consequently a lowering of the firm's discount rate. Before we proceed to this topic, we first illustrate the points raised in our discussion so far with a numerical example.

5.7 A NUMERICAL EXAMPLE

Let us take the case of a firm which operates 5 divisions and is considering opening a sixth. The problem the firm's planners face is to select the best proposal out of three available to it, or alternatively to reject the idea of increasing its operations. In order to keep the presentation of this example within a reasonable size, we assume throughout it that the firm is originally, and remains, debtless. If the firm's existing investments are left intact we assume the firm will generate a rate of return of 12.0%, while the market portfolio's performance, on the basis of a 5 period forecast, is estimated as:

YEAR	1	2	3	4	5
RETURN	0.05	0.05	0.06	0.055	0.06

Assuming the riskless rate of return is $R_F = 0.045$, we can calculate the market price of risk:

$$\lambda = \frac{0.055-0.045}{0.25}$$

$$\lambda = 0.04$$

and

$$\lambda^* = 0.04\ 0.5 = 0.02$$

Given that the firm was charged, prior to the expansion, the following unit prices for the three types of inputs it uses:

$$c_1 = 5 \text{ m.u.} \ ; \ c_2 = 6 \text{ m.u.} \ ; \text{ and } c_3 = 7 \text{ m.u.}$$

We can calculate the original corporate risk on the basis of the following data:

p_i	v_i	$\sum_m \alpha_{1m}$	$\sum_m \alpha_{2m}$	$\sum_m \alpha_{3m}$	Q_1	Q_2	Q_3	Q_4	Q_5	ρ_i	$\sqrt{Var_i}$
15	13	60	100	100	100	225	240	270	320	0.7188	0.2980
17	14	35	60	95	110	230	230	240	250	0.6074	0.1440
16	12	35	60	95	115	235	250	270	290	0.6923	0.2248
17	15	190	190	160	130	250	260	280	295	0.6676	0.0523
17	13	60	50	30	160	265	270	275	280	0.6173	0.2573

where ρ_i and $\sqrt{Var_i}$ are store i's correlation coefficient and standard variation respectively.

From which we can calculate the firm's required rate of return:

$$\begin{aligned} E(R_j) &= 0.045 + 0.02\ (0.7188\ 0.2980 + 0.6074\ 0.1440 \\ &\quad + 0.6923\ 0.2248 + 0.6676\ 0.0523 + 0.6173\ 0.2573) \\ &= 0.045 + 0.02\ 0.6180 \\ &= \underline{0.0574} \end{aligned}$$

In other words, the rate of return required by the investors in the capital market in view of the firm's risk premium is 5.74%. We now turn to investigate the three projects available to the firm.

PROJECT 1

We assume this project generates economies of scale which reduce the unit cost of inputs 1 and 3; hence

$$\hat{c}_1 = 5 \text{ m.u.} \ ; \ \hat{c}_2 = 6 \text{ m.u} \ ; \text{ and } \hat{c}_3 = 5 \text{ m.u.}$$

and that all five, possibly six, divisions of the firm offer the same five lines, although in varying quantities. Let us assume that the first project's estimated performance is summarized in the following table:

p_m	v_m	α_{1m}	α_{2m}	α_{3m}	Q_1	Q_2	Q_3	Q_4	Q_5	ρ_m	$\sqrt{Var_m}$
18	13	30	15	12	35	40	45	45	45	0.8385	0.0595
17	15	4	5	50	15	20	20	35	25	0.3297	0.0508
15	14	60	30	24	18	25	30	40	50	0.7340	0.0302
14	13	32	15	30	25	30	40	40	50	0.9129	0.0195
14	12	32	15	30	25	35	35	35	35	0.5590	0.0179

from which we can calculate the project's required rate of return given its risk level. Hence,

$$
\begin{aligned}
E(R_1) &= 0.045 + 0.02(0.8385\ 0.0595 + 0.3297\ 0.0508 \\
&\quad + 0.7340\ 0.0302\ 0.9129\ 0.0195 + 0.5590\ 0.0179) \\
&= 0.045 + 0.02\ 0.1166 \\
&= \underline{0.0473}
\end{aligned}
$$

This however is the excess rate of return if the project were to be evaluated solely on the basis of its own merits. Given that it generates indirect effects on the rest of the firm, the discount rate used to assess the project's excess rate of return depends on the total effect, we therefore turn to examine the effect on the rest of the firm as summarized in the following table:

p_i	$\hat{v}_i$	$\sum_m \hat{\alpha}_{1m}$	$\sum_m \hat{\alpha}_{2m}$	$\sum_m \hat{\alpha}_{3m}$	$\hat{Q}_1$	$\hat{Q}_2$	$\hat{Q}_3$	$\hat{Q}_4$	$\hat{Q}_5$	ρ_i	$\sqrt{Var_i}$
15	13	60	100	100	115	235	250	270	290	0.6923	0.0980
17	14	35	60	95	150	235	235	260	255	0.5587	0.1336
16	12	35	60	95	130	250	260	280	295	0.6676	0.2595
17	15	35	190	160	160	265	270	275	280	0.6173	0.0479
17	13	190	50	30	170	290	280	280	280	0.4951	0.1060

from which we can calculate the existing investments' new risk level and the resulting equilibrium rate of return for those divisions:

$$E(R_{j1}) = 0.045 + 0.02\ (0.6923\ 0.00980 + 0.5587\ 0.1336 + 0.6676\ 0.2595 + 0.6173\ 0.0479 + 0.4951\ 0.1060)$$
$$= 0.045 + 0.02\ 0.3977$$
$$= \underline{0.0530}$$

However, the firm's risk given that project 1 is accepted is calculated as follows: under the assumption that the new project requires 24% of the firm's resources:

$$Cov_{j1} = 0.24\ 0.1166 + 0.76\ 0.3977$$
$$= \underline{0.3303}$$

We can now calculate the contribution of project 1 on the corporate risk:

$$\Delta Cov_{j1} = 0.6180 - 0.3303$$
$$= \underline{0.2877}$$

In other words, the acceptance of project 1 reduces the firm's risk by 0.2877, which in view of the market price of risk implies that the project's indirect contribution to the corporate excess rate of return is

$$d'_1 = \Delta R_{j1} = 0.02\ 0.2877$$
$$= \underline{0.0058}$$

while the project's direct contribution, calculated on the basis of the assumption that project 1 generates a rate of return of 8.75%, is:

$$d_1 = \Delta R_1 = 0.0875 - (0.045 + 0.02\ 0.3303)$$
$$= 0.0875 - 0.0516$$
$$= \underline{0.0359}$$

We can now calculate the net total contribution of Project 1 hence:

$$D_1 = d_1 + d'_1 = 0.0359\ 0.24 + 0.0058\ 0.76$$
$$= \underline{0.0130} = \underline{1.30\%}$$

PROJECT 2

In a similar manner we can examine the second alternative facing the firm, which, as its predecessor, requires 24.0% of the firm's resources, and generates the following economies of scale:

$$\hat{c}_1 = 4.5 \text{ m.u.} \; ; \; \hat{c}_2 = 5 \text{ m.u.} \; ; \text{ and } \hat{c}_3 = 6 \text{ m.u.}$$

with the following results for the new division:

p_m	v_m	α_{1m}	α_{2m}	α_{3m}	Q_1	Q_2	Q_3	Q_4	Q_5	ρ_m	$\sqrt{Var_m}$
18	12	30	35	15	55	40	40	40	60	0.1382	0.1462
17	13	55	15	20	60	40	55	60	65	0.5199	0.0962
17	13	35	20	18	25	40	45	55	50	0.6516	0.1358
16	13	40	20	25	30	40	40	40	40	0.5590	0.0419
15	11	40	25	30	10	40	45	40	45	0.6781	0.1281

from which we can calculate the required rate of return were the project economically independent from the rest of the firm's investments, hence:

$$\begin{aligned} E(R_2) &= 0.045 + 0.02(0.1282\ 0.1462 + 0.5199\ 0.0962 + \\ &\quad 0.6516\ 0.1358 + 0.5590\ 0.0419 + 0.6781\ 0.1281) \\ &= 0.045 + 0.02\ 0.2675 \\ &= \underline{0.0504} \end{aligned}$$

The indirect effect of the acceptance of Project 2 is summarized in the following table:

p_i	$\hat{v}_i$	$\sum_m \hat{\alpha}_{1m}$	$\sum_m \hat{\alpha}_{2m}$	$\sum_m \hat{\alpha}_{3m}$	$\hat{Q}_1$	$\hat{Q}_2$	$\hat{Q}_3$	$\hat{Q}_4$	$\hat{Q}_5$	ρ_i	$\sqrt{Var_i}$
15	13	60	100	100	120	240	255	280	300	0.6926	0.1023
17	13	35	60	95	155	245	245	270	280	0.6323	0.1977
17	13	190	190	160	170	275	275	295	310	0.6351	0.0772
16	10	35	60	95	160	265	270	275	280	0.6173	0.3022
17	12	60	50	30	160	230	250	255	260	0.7265	0.2808

from which we can calculate the existing investments' new risk level and the resulting equilibrium rate of return for those divisions:

$$E(R_{j2}) = 0.045 + 0.02(0.6962\ 0.1028 + 0.6323\ 0.1977 + 0.6351\ 0.0772 + 0.6173\ 0.3022 + 0.7265\ 0.2808)$$
$$= 0.045 + 0.02\ 0.6361$$
$$= \underline{0.0577}$$

However, the firm's risk given that Project 2 is accepted can be calculated as follows:

$$Cov_{j2} = 0.24\ 0.2675 + 0.76\ 0.6361$$
$$= \underline{0.5476}$$

We can now calculate the contribution of Project 2 to the corporate risk:

$$\Delta Cov_{j2} = 0.6180 - 0.5476$$
$$= \underline{0.0704}$$

Therefore, the indirect contribution of Project 2 can be measured as follows:

$$d'_2 = \Delta R_{j2} = 0.02\ 0.0704$$
$$= \underline{0.0014}$$

while the project's direct contribution, calculated on the assumption that Project 2 generates a rate of return of 6.25% is:

$$d_2 = \Delta R_2 = 0.0625 - (0.045 + 0.02\ 0.5476)$$
$$= \underline{0.0065}$$

We can now calculate the net total contribution of Project 2, hence:

$$D_2 = d_2 + d'_2 = 0.0065\ 0.24 + 0.0014\ 0.76$$
$$= \underline{0.0027} = \underline{0.27\%}$$

PROJECT 3

Let us assume the third alternative facing the firm generates the following set of unit costs:

$$\hat{c}_1 = 5 \text{ m.u} \; ; \; \hat{c}_2 = 4 \text{ m.u.} \; ; \text{ and } \hat{c}_3 = 5 \text{ m.u.}$$

and the new division's estimated performance:

p_m	v_m	α_{1m}	α_{2m}	α_{3m}	Q_1	Q_2	Q_3	Q_4	Q_5	ρ_m	$\sqrt{Var_m}$
18	14	30	90	70	25	70	77	89	97	0.7037	0.1319
16	15	70	50	65	38	35	49	77	85	0.6666	0.0253
15	14	90	70	35	41	80	93	93	93	0.7217	0.0237
17	16	90	20	30	50	95	96	100	111	0.6607	0.0345
17	11	90	85	65	60	85	87	79	80	0.5135	0.0568

from which we can calculate the required rate of return were the project economically independent from the rest of the firm:

$$\begin{aligned} E(R_3) &= 0.045 + 0.02(0.7037\ 0.1319 + 0.6666\ 0.0253 \\ &\quad + 0.7217\ 0.0237 + 0.6607\ 0.0345 + 0.5135\ 0.0568) \\ &= 0.045 + 0.02\ 0.1788 \\ &= \underline{0.0486} \end{aligned}$$

The indirect effect of Project 3 is summarized in the following table:

p_i	$\hat{v}_i$	$\sum_m \hat{\alpha}_{1m}$	$\sum_m \hat{\alpha}_{2m}$	$\sum_m \hat{\alpha}_{3m}$	$\hat{Q}_1$	$\hat{Q}_2$	$\hat{Q}_3$	$\hat{Q}_4$	$\hat{Q}_5$	ρ_i	$\sqrt{Var_i}$
15	14	60	100	100	160	265	289	315	365	0.7526	0.0634
17	16	35	60	95	180	245	260	275	310	0.7571	0.0527
16	14	35	60	95	185	270	295	315	365	0.7722	0.1430
17	16	190	190	160	190	250	260	295	315	0.7026	0.0183
17	14	60	50	30	215	270	315	335	360	0.8278	0.2655

from which we can calculate the existing investments' new risk level and the resulting equilibrium rate of return for those divisions:

$$\begin{aligned} E(R_{j3}) &= 0.045 + 0.02(0.7526\ 0.0634 + 0.7571\ 0.0527 + \\ &\quad 0.7722\ 0.1430 + 0.7026\ 0.0183 + 0.8278\ 0.2655) \\ &= 0.045 + 0.02\ 0.4307 \\ &= \underline{0.0536} \end{aligned}$$

However, the firm's risk given that Project 3 is accepted can be calculated as follows:

$$\begin{aligned} Cov_{j3} &= 0.24\ 0.1788 + 0.76\ 0.4307 \\ &= \underline{0.3702} \end{aligned}$$

We can now calculate the contribution of Project 3 to the corporate risk:

$$\begin{aligned} \Delta Cov_{j3} &= 0.6180 - 0.3702 \\ &= \underline{0.2478} \end{aligned}$$

Therefore, the indirect contribution of Project 3 can be measured as follows:

$$\begin{aligned} d'_3 &= R_{j3} = 0.02\ 0.2478 \\ &= \underline{0.0050} \end{aligned}$$

and its direct contribution, given the assumption that it generates a rate of return of 7.30%, is:

$$\begin{aligned} d_3 &= R_3 = 0.0730 - (0.045 + 0.02\ 0.3702) \\ &= \underline{0.0656} \end{aligned}$$

Hence, Project 3's net total contribution is equal to:

$$\begin{aligned} D_3 &= d_3 + d'_3 = 0.0656\ 0.24 + 0.0050\ 0.76 \\ &= \underline{0.0185} = \underline{1.95\%} \end{aligned}$$

In view of the above calculations we are now set to assess the excess rate of return of each project, and of the non-expansion alternative. Starting with the latter:

$$XR_j = 0.1200 - (0.045 + 0.02\ 0.6180)$$
$$= \underline{0.0626} = \underline{6.26\%}$$

Consequently, we can calculate the following:

$$XR_{j1} = XR_j + D_1 = 0.0626 + 0.0130$$
$$= \underline{0.0756} = \underline{7.56\%}$$

and

$$XR_{j2} = XR_j + D_2 = 0.0626 + 0.0027$$
$$= \underline{0.0653} = \underline{6.53\%}$$

and

$$XR_{j3} = XR_j + D_3 = 0.0626 + 0.0195$$
$$= \underline{0.0821} = \underline{8.21\%}$$

The conclusion from our calculations is that the ranking of the four alternatives facing the firm is as follows:

1.	PROJECT	3	excess rate of return	8.21%
2.	PROJECT	1	" " " "	7.56%
3.	PROJECT	2	" " " "	6.53%
4.	No expansion		" " " "	6.26%

i.e. clearly the best choice is Project 3. These results are graphically illustrated in Fig. (5.9)

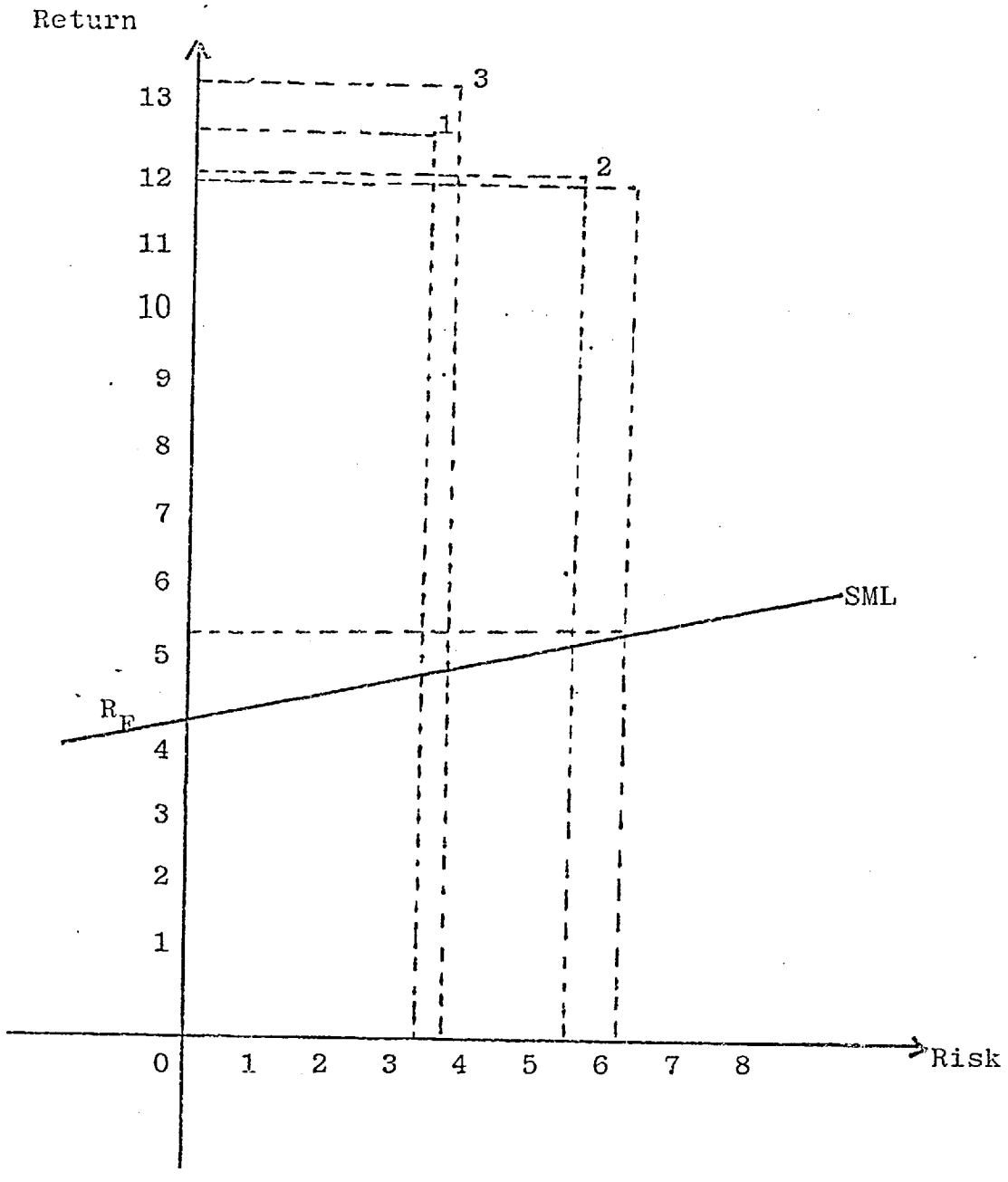


Fig. (5.9): Project Ranking

5.8 SUMMARY AND CONCLUSIONS

In this chapter we have presented the theoretical framework necessary for a model of a decentralized firm operating under uncertainty. Based on the CAPM, our discussion started with a brief review of existing asset expansion decision rules, and the criticism the CAPM has drawn over the years. We then examined the capital market mechanism and stressed the case for a departure from conventional CAPM theory by acknowledging the fact that in a realistic business environment there exist circumstances where a new investment undertaken by the firm does affect its existing projects. Having made this point, and sustained it with recent evidence, we set out to understand the mechanism by which an expansion of the firm's operations alters both its risk and return. For this purpose we turned to Rubinstein's expansion of the mean-variance theorem, and in the process of proving it we pointed out the stringent set of assumptions which govern the economic environment within which the firm operates, and its shortcomings. Consequently, we modified the mean-variance theorem in a way which allowed for economies of scale to be incorporated, and which enabled us to shed some light on the economic processes which take place when a decentralized firm expands its activities in a way which alters its risk.

The main contribution of this chapter is in the way it treats a decentralized firm and in the linkage it offers between the individual product lines and the firm's market value, both in a CAPM perfect world and in one where economies of scale exist. Furthermore, on the basis of the above, we reformulated the capital budgeting decision rules used in earlier sections of this chapter, and despite the fact that they are based on identical reasoning as their predecessors they provide the considerable advantage of enabling the corporate planners to evaluate the impact of changes at product line level on their investment criterions.

With the vital link between the firm's operating decisions at the production level and its market value in hand, we are now able to take our analysis one step further by focusing on the implications of the firm's managerial decisions on its market value. In the following chapter we concentrate on the problem of resource allocation within a department store using as a theoretical basis some of the concepts introduced in this chapter. Specifically, we discuss in greater detail the structure of the corporate risk and the forces which operate on it to either increase or reduce its value following an expansion of the firm's activities, or a change in the internal resource allocation.

C H A P T E R 6

RESOURCE ALLOCATION IN A DEPARTMENT STORE OPERATING UNDER UNCERTAINTY

LIST OF NOTATIONS FREQUENTLY USED IN CHAPTERS 6 AND 7

b_i - the systematic risk in the return of store i.

b_m - the systematic risk in the return of product line m.

C_i - budget constraint of store i.

c_i - expected cash flow of store i

c_{ik} - store i's unit cost of resource k allocated

C_j - budget constraint on firm j.

C_m - product line m's budget.

c_m - expected cash flow of the capital market.

$E(R_i)$ - expected rate of return of store i.

$E(R_j)$ - expected rate of return of firm j.

$E(R_M)$ - expected rate of return on the market portfolio of risky securities.

$E(R_m)$ - expected rate of return of product line m.

H - the price of risk.

L* - target debt ratio

p_i - average unit price of store i.

P_j - the price of the j'th security.

p_m - unit price of product line m.

Q_i - sales of store i.

Q_m - sales of product line m.

R_F - risk-free rate of return.

S_M - the total market value of all stocks.

v_i - average unit variable cost of store i.

v_k - a dual variable.

v_m - unit variable cost of product line m.

W_0 - wealth of portfolio holder at the beginning of the period.

W_1 - wealth of portfolio holder at the end of the period.

W_k - a dual variable.

α_i - the proportion of the total resources allocated to store i.

$\hat{\alpha}_i$ - the amount of resources allocated to store i.

α_{mk} - the proportion of resource k allocated to product line m.

α_{ik} - the proportion of resource k allocated to store i.

α_{jk} - the amount of resource k available to firm j.

ζ_1 - a lagrangian multiplier.

ζ_m - an integer decision variable.

λ - the market price of risk.

μ - the mean of the return of a portfolio.

μ_{im} - the complementarity effect of having product lines i and m in the portfolio.

Π_k - a dual variable.

ρ_{ij} - the correlation between assets i and j.

$\rho_{ij.}$ - the partial correlation between assets i and j.

σ_i^2 - the variance on the returns of store i.

σ_i - the standard variation of the return of store i.

$\sigma_{i.}$ - asset i's partial standard variation.

τ - the tax rate.

τX_{j0} - after tax return before investment is undertaken.

τX_{j1} - after tax return after investment is undertaken.

ϕ_k - a dual variable

Ω_i - a constant in corporate centre's objective function.

Ω_m - a constant in store manager's objective function.

6.1 INTRODUCTION

In chapter 5 we suggested that a multi-product firm can be viewed as a holder of a portfolio consisting of both financial and physical assets. In this chapter, we turn our attention to the implementation of this concept to the problem of resource allocation under uncertainty in a department store.

The basic claim of modern portfolio theory as developed by Markowitz (100, 101) and Tobin (146) is that, the risk of a portfolio of capital assets is not solely a function of the variance of the returns of individual assets in the portfolio but also of their correlation (or covariance). An examination of the structure of a department store, whose typical structure is graphically illustrated in Fig. (6.1), reveals that a portfolio whose asset base consists of the product lines carried by the store can be associated with such a firm.

Several properties of the department store make it a particularly suitable candidate for the application of portfolio theory. As pointed out in the previous chapter, for a firm to have a portfolio problem the economic opportunities (projects) considered by the firm must be correlated. Such a statement is valid for numerous types of industry and the nature of the retail trade would certainly include it in such a list. The fact that a department store is a combination of correlated product lines is what attracts its customers and therefore its raison d'être. Specifically, one can view the existence of operationally autonomous product lines within a department store, each with its own physical location and management, as the store portfolio's asset base while the relationships between the various lines of products and services, i.e. substitutability and complementarity, are the measures of correlation between them.

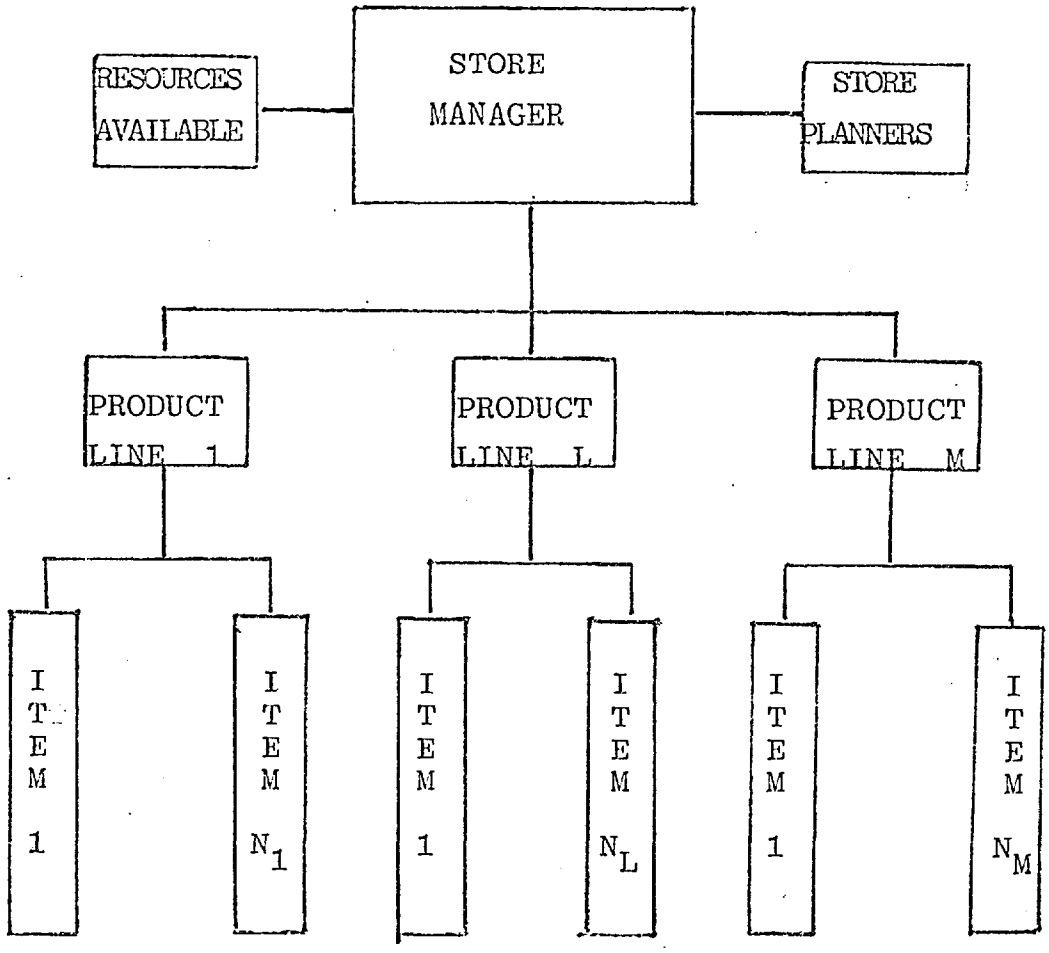


Fig. (6.1): Organizational Structure of a Department Store

In terms of the managerial decision making, such an approach implies that planners in a department store face a problem analoguous to one facing an investor in the capital market. Specifically, both are required to determine the optimal resource allocation between the available investments, each of which is defined in terms of a mean-variance outcome. While the investor in the capital market has to divide his budget between the various types of financial products offered by the market, i.e. risk-free and risky securities, the store manager is required to allocate the limited amounts at his disposal between the various business activities available to him.

Keeping these concepts in mind we focus our attention on the development of a model for the allocation of scarce resources in a department store. We start with a formal presentation of the theoretical framework necessary for such an analysis, in section 6.2, following which we discuss in section 6.3 the structure of corporate risk and its economic implications. Finally, in section 6.4, we suggest several formulations of the store manager's problem.

6.2 A DEPARTMENT STORE'S PORTFOLIO

The expected value of a portfolio at the end of a single period horizon is given by:

$$E(W_1) = \sum_{i=1}^{m} \hat{\alpha}_i(1+E(R_i) + (W_0 - \sum_{i=1}^{m} \hat{\alpha}_i)(1 + R_F)$$

$$= W_0 + \sum_{i=1}^{m} \hat{\alpha}_i E(R_i) + (W_0 - \sum_{i=1}^{m} \hat{\alpha}_i)R_F \qquad (6.1)$$

and the variance of the portfolios' return:

$$\sigma^2(W_1) = \sum_{i=1}^{m} \hat{\alpha}_i^2 \sigma_i^2 + 2 \sum_{\substack{i=1 \\ j>i}}^{m} \hat{\alpha}_i \hat{\alpha}_j \sigma_{ij} \qquad (6.2)$$

where:

W_0 - the store manager's wealth at the beginning of the period which is assumed to be constant.

W_1 - the terminal value of the store manager's wealth which is a stochastic variable.

$\hat{\alpha}_i$ - the amount invested in the i-th product line.

$E(R_i)$ - the expected rate of return on the i-th product line.

σ_i^2 - the variance on the rate of return of the i-th product line.

σ_{ij} - the covariance of the rate of return on product lines i and j.

R_F - the risk-free rate of interest.

We can rewrite Eq. (6.2) in terms of the correlation rather than in terms of variances and covariances, i.e. :

$$\sigma^2(W_1) = \sum_{i=1}^{m} \alpha_i^2 \rho_{ii} \sigma_i^2 + 2 \sum_{\substack{i=1 \\ j>i}}^{m} \hat{\alpha}_i \hat{\alpha}_j \rho_{ij} \sigma_i \sigma_j \qquad (6.3.a)$$

where ρ_{ij} is the correlation of the returns on product lines i and j and hence $\rho_{ii}=1$. Since the amounts invested in the i-th line can be substituted for the proportion of the store manager's wealth at the beginning of the period allocated to line i, where $\alpha_i \equiv \hat{\alpha}_i / \sum_{i=1}^{m} \hat{\alpha}_i$, Eq. (6.3.a) can be rewritten:

$$\sigma^2(W_1) = \sum_{i=1}^{m} \alpha_i^2 \sigma_i^2 + 2 \sum_{\substack{i=1 \\ j>i}}^{m} \alpha_i \alpha_j \rho_{ij} \sigma_i \sigma_j \qquad (6.3.b)$$

From our expansion of the mean-variance theorem in chapter 5, i.e. Eq. (5.23), we know that:

$$\sigma_i = \sigma \left(QS_i (ps_i - vs_i) / \sum_k \alpha^*_{ik} c_k \right) \forall i$$

which if substituted in Eq. (6.1) and (6.3.b) respectively gives us:

$$E(W_1) = \sum_{i=1}^{m} \alpha_i \left(1 + QS_i (ps_i - vs_i) / \Sigma \alpha^*_{ik} c_k\right) + \left(W_0 - \sum_{i=1}^{m} \alpha_i\right)(1+R_F)$$

$$= W_0 + \sum_{i=1}^{m} \alpha_i (QS_i (ps_i - vs_i) / \Sigma \alpha^*_{ik} c_k) + \left(W_0 - \sum_{i=1}^{m} \alpha_i\right) R_F \qquad (6.4.a)$$

and

$$\sigma^2(W_1) = \sum_{i=1}^{m} \alpha_i^2 \sigma (QS_i (ps_i - vs_i) / \Sigma \alpha^*_{ik} c_k)^2 + 2 \sum_{\substack{i=1 \\ j \equiv i}}^{m} \alpha_i \alpha_j \rho_{ij}$$

$$\sigma (QS_i (ps_i - vs_i) / \Sigma \alpha^*_{ik} c_k) \sigma (QS_i (ps_i - vs_i) / \Sigma \alpha^*_{ij} c_k) \qquad (6.4.b)$$

In Eq. (6.4.b) we have the vital link between the department store's risk, as measured by the variance of its return, and the individual product lines it operates. Such a model is a prerequisite for any attempt to assess the effect of managerial decisions made in a department store operating under uncertainty.

Specifically, our model of a store is defined in terms of two parameters - risk and return, and consequently is suitable for handling the analysis of a change in any one of the two, or both, as a result of a fluctuation in the firm's operating environment.

Studies of parameters which determine a firm's rate of return and the effects of changes in their values are available in the literature. Recent examples of such studies are provided by Eilon and Cosmetatos (45) who have examined the effects on the unit cost, the revenue and profit, of combined changes imposed on the firm by managerial decisions or external forces. Specifically, their analysis focuses on how actions within a firm designed to enhance a certain performance criteria may be found to have an adverse effect on other criteria. In view of the existence of such studies and the fact that they are restricted to the deterministic case, we now proceed to discuss the impact of changes in the system on the firm's level of risk. We concentrate on the structure of a department store's risk and assess the effect of changes in the system on each of its components and as a whole.

6.3 A DEPARTMENT STORE'S RISK

From the equations developed in the previous section it is clear that virtually any decision made at various organizational levels within the firm will have a potential influence on its risk-return parameters. Consequently, the basic concept underlying our approach to the analysis of the firm's risk is to direct our dicussion to the economic interpretation of the parameters which form it. The structure of our analysis will obviously reflect this attitude and as a result each subsection examines the economic insight provided by the individual components of the corporate portfolio's risk.

6.3.1 Investment Proportions

The first term in the expression of a portfolio's variance is the proportion, or alternatively amount, of the investor's budget invested in a given asset. In the case of a portfolio of securities these proportions merely reflect the investor's demand for each of the risky assets offered to him by the capital market. In the context of a corporate portfolio the interpretation and analysis of the investment proportions provides considerable economic insight into the complex corporate mechanism.

Let us start by examining one of the fundamental determinants of a firm's behaviour - the range of production possibilities available to it. Specifically, the problem facing any multi-product firm is choosing the output combination which maximizes some objective(s) function. This problem can be presented in a portfolio context by introducing into our model the concept of an "Efficient Set Curve" (ESC). Known also as an "Efficiency Boundary" and "Envelope Curve", the ESC as illustrated in Fig. (6.2) reflects all the risk-return performances which the firm is capable of generating by combining product lines and services

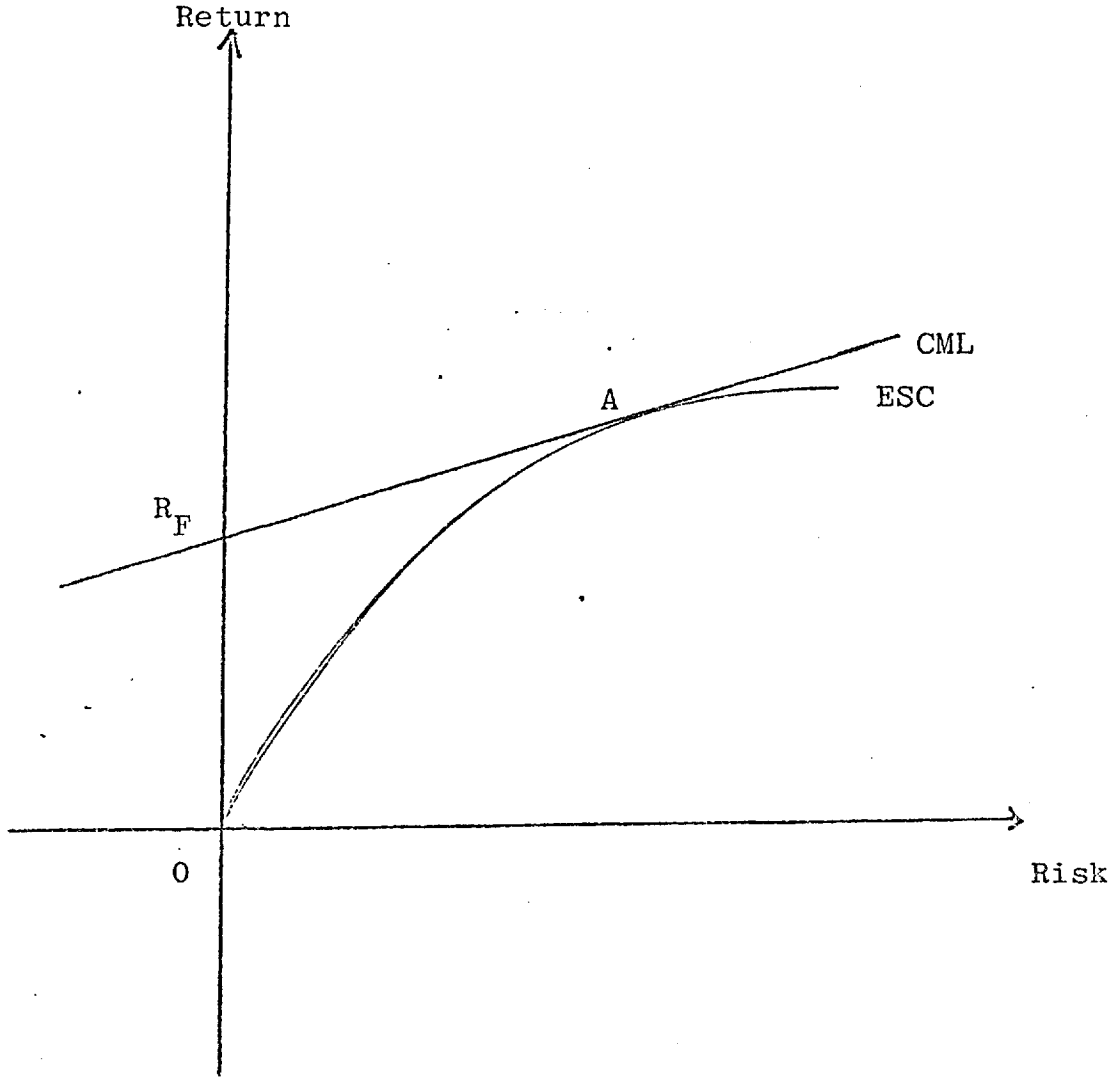


Fig. (6.2): Production Technology Curve

and altering the proportion of resources invested in each. In the case of a corporate portfolio the set of feasible risk-return combinations which lies under the ESC represents all those output combinations that are dominated from the firm's point of view since for any such point, say B, it can produce with the existing technology a higher expected return for any given risk.

This concept can be better understood when one views the firm as an offeror of financial products to investors through the mechanism of the capital market. In the CAPM firms offer two such financial products, expected return and risk, where risk is reflected by the covariance between the firm's rate of return and the return on the market portfolio of risky securities. The firm can, by allocating its resources in different ways, vary the risk-return combinations it sends to the market.

Greenberg, Marshall and Yawitz (64) name the set of attainable risk-return combinations-the firm's "technology" for the production of risk and return. They denote the efficient set of combinations by:

$$f(\mu, \text{Cov}) = 0$$

depicted by a convex curve to reflect the idea that increases in Cov yield successively smaller increases in the rate of return.[1] Following this they proceed to find the conditions which maximize the unconstrained firm's market value and

---

1. In the case of the firm the curve may not be continuous due to the indivisibility of investment projects. Fama (49) provides a discussion of the geometry of the efficient set under various assumptions concerning the availability of borrowing and lending.

prove that in order to achieve maximum value the following condition must hold:

$$\frac{\partial\rho/\partial Cov}{\partial\rho/\partial\mu} = \lambda$$

i.e. maximum will be achieved at that risk-return combination where the firm's price of risk and that of the capital market are identical. Graphically, this is equivalent to the point where the firm's ESC is tangent to the CML, i.e. point A. In other words, the multi-product firm must attempt to select the investment proportions which yield a risk-return result which is not only internally efficient but externally as well, i.e. where the slopes of the CML and ESC are equal.

Greenberg et al's formulation of the condition for the maximization of the unconstrained firm's value can be substituted with an alternative expression which can provide greater opportunity for economic analysis of the above condition.

This substitution is based on Fowler & Rorke's (53) work which provides us with the following definitions:

$$\Delta Cov_i = Cov(c_i, c_M) - Cov(c_i + \Delta c_i, c_M + \Delta c_i)$$

and

$$\Delta E(c_i) = E(c_i) - E(c_i + \Delta c_i)$$

hence the value maximizing condition can be rewritten as follows:

$$\frac{Cov(c_i, c_M) - Cov(c_i + \Delta c_i, c_M + \Delta c_i)}{E(c_i) - E(c_i + \Delta c_i)} = \lambda$$

We now turn to discuss how changes in the performance of a particular product line's performance will affect the store manager's selection of the optimal portfolio.

Basically, the problem seems simple enough and the answer straightforward: how does a store manager who has already selected his optimal portfolio, i.e. determined among other his optimal investment proportions, and who then faces a change in the information regarding one of the assets included in his portfolio, react? It is only natural that in view of additional information there might be a change in the relative attractiveness of the product lines carried by the store. Intuitively, a product line whose results prove to be better than predicted, i.e. a higher profit without a change in risk, will cause management to shift additional resources to that line at the expense of others. Similarly, if a product line generates a lower level of risk than expected, combined with an unchanged level of return, this too intuitively suggests that the product line in question would enjoy an increased level of investment which would be matched with a simultaneous reduction in the level of investment in the remaining lines. However, a careful analysis reveals that these are not necessarily the case.

The complexity of the matter is mathematically presented by Levy (94) in his study of the demand for assets under risk. Using a notation similar to ours, Levy writes the demand for asset i as:

$$\alpha_i = \zeta_1 \left\{ \frac{E(R_i) - R_F}{\sigma_{i.}^2} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{E(R_j) - R_F}{\sigma_{i.}\,\sigma_{j.}}\, \rho_{ij.} \right\} \qquad (6.5)$$

where:

$E(R_i)$ - the expected rate of return on the i-th asset (in our case a product line)

ζ_1 - a Lagrange multiplier derived from the return constraint when solving for the vector X which minimizes the portfolio's variance.

$\sigma_{i.}$ - asset i's partial standard deviation, i.e. that part of the variance which remains after removing that variance due to other possible assets in the portfolio.

$\rho_{ij.}$ - the partial correlation between assets i and j, i.e. after the removal of any part of the variation due to the influence of the other n-2 variables.

From Eq. (6.5) Levy proceeds to differentiate the demand for asset i in respect to changes in its return and risk respectively. Hence:

$$\frac{\partial \alpha_i}{\partial E(R_i)} = \frac{\partial E(W_1)}{\partial E(R_i)} \frac{\partial \zeta_1}{\partial E(W_1)} Y_i + \zeta_1 \frac{1}{\sigma_{i.}^2} \qquad (6.6.a)$$

and

$$\frac{\partial \alpha_i}{\partial \sigma_{i.}} = \frac{\partial \sigma(W_1)}{\partial \sigma_{i.}} \frac{\partial \zeta_1}{\partial \sigma(W_1)} Y_i - \frac{\zeta_1}{\sigma_{i.}^2} \left\{ \frac{E(R_i) - R_F}{\sigma_{i.}} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{E(R_j) - R_F}{\sigma_{j.}} \rho_{ij.} \right\} - \frac{\zeta_1}{\sigma_{i.}} \frac{E(R_i) - R_F}{\sigma_{i.}^2} \qquad (6.6.b)$$

where

$$Y_i \equiv \left\{ \frac{E(R_i) - R_F}{\sigma_{i.}^2} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{E(R_j) - R_F}{\sigma_{i.} \sigma_{j.}} \rho_{ij.} \right\}$$

Following this formulation of the changes in the demand for assets under risk it is clear why our previous statements were made with certain reservations. Both the reaction of demand to changes in asset i's rate of return, Eq. (6.6.a), and to changes in its risk, Eq. (6.6.b), are seen to be functions of elements whose signs are undetermined.

The response of demand to changes in return is dependent on the two parts: the "substitution effect", i.e. the change in α_i as a result of the change in $E(R_i)$ when the expected wealth is held constant, and the "wealth effect", or "income effect" i.e. the change in α_i due to change in the expected wealth. While the substitution effect is positive the income effect might be either positive or negative. This latter point is due to the fact that the income effect can be positive, in which case $\partial E(W_1)/\partial E(R_i) > 0$, or not, i.e. $\partial E(W_1)/\partial E(R_i) \leq 0$.

Similarly, the response change in the investment proportion of an asset whose risk has deviated from the planners' estimates is dependent on whether the income effect is positive, i.e. $\partial E(W_1)/\partial \sigma_i > 0$, in which case the investor will allocate more of his resources to risky assets, or alternatively he might opt to consume more security, i.e. $\partial \sigma(W_1)/\partial \sigma_i < 0$.[2]

These non-intuitive conclusions are of substantial interest to our analysis of the financial impacts of operating decisions in a department store. Past studies of the theory of the firm, such as classic micro-economics and recent profitability evaluations (45), have highlighted the fact that given the firm's dependency on exogenous market parameters, policies which attempt to achieve a greater rate of return by promoting one of the firm's products can not only fail to achieve their narrow objective but can also backfire in the wider sense, i.e. while enhancing one objective they can act adversely on others, ending with a net detriment effect on the firm as a whole.

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2. Two assumptions made throughout Levy's analysis and worth noting are:

   i) the variance-covariance matrix is regular, i.e. two assets with perfect correlation do not exist;

and ii) the investor does not sell short, i.e. $\alpha_i \geq 0$ and hence $Y_i \geq 0$.

In an uncertain world, where the firm is represented by a portfolio of physical and financial assets, a new element is added to the already complex situation. Not only can a policy designed to generate an overall positive contribution to the firm's risk-return performance fail, moreover, even if it succeeds to do so it is not at all clear whether as a result the investors' i.e. managers and/or shareholders, wealth will be altered and if the answer is positive in which direction. In other words it is theoretically conceivable that a policy designed to reduce a product line's riskiness achieves its target without any negative cross-effects yet fails to reduce the overall corporate risk because of the investors response of shifting resources to riskier lines. Such a situation would arise in the case where security is not a normal good. Similarly, a promotion effort which manages to increase a line's rate of return without altering any other parameter can cause the investors to form a new portfolio with a lower expected wealth.

We can clarify the issue of the choices facing the management of a department store following a change in one of the product line's risk-return performance with a simple example. Let us consider the case of an increment in a line's expected rate of return, the effect of which on the store's technology is graphically illustrated in Fig. (6.3) by the outwards shift of ESC^1_i to ESC^2_i. In view of this new set of production possibilities the store manager can react in three different ways:

- by moving from his original location on ESC^1_i, point A, to point A', thus generating more wealth without altering his risk level.
- by generating more wealth and at the same time increasing the store's risk, i.e. moving to point B.
- by reducing the store's wealth and simultaneously increasing its security, i.e. moving to point C.

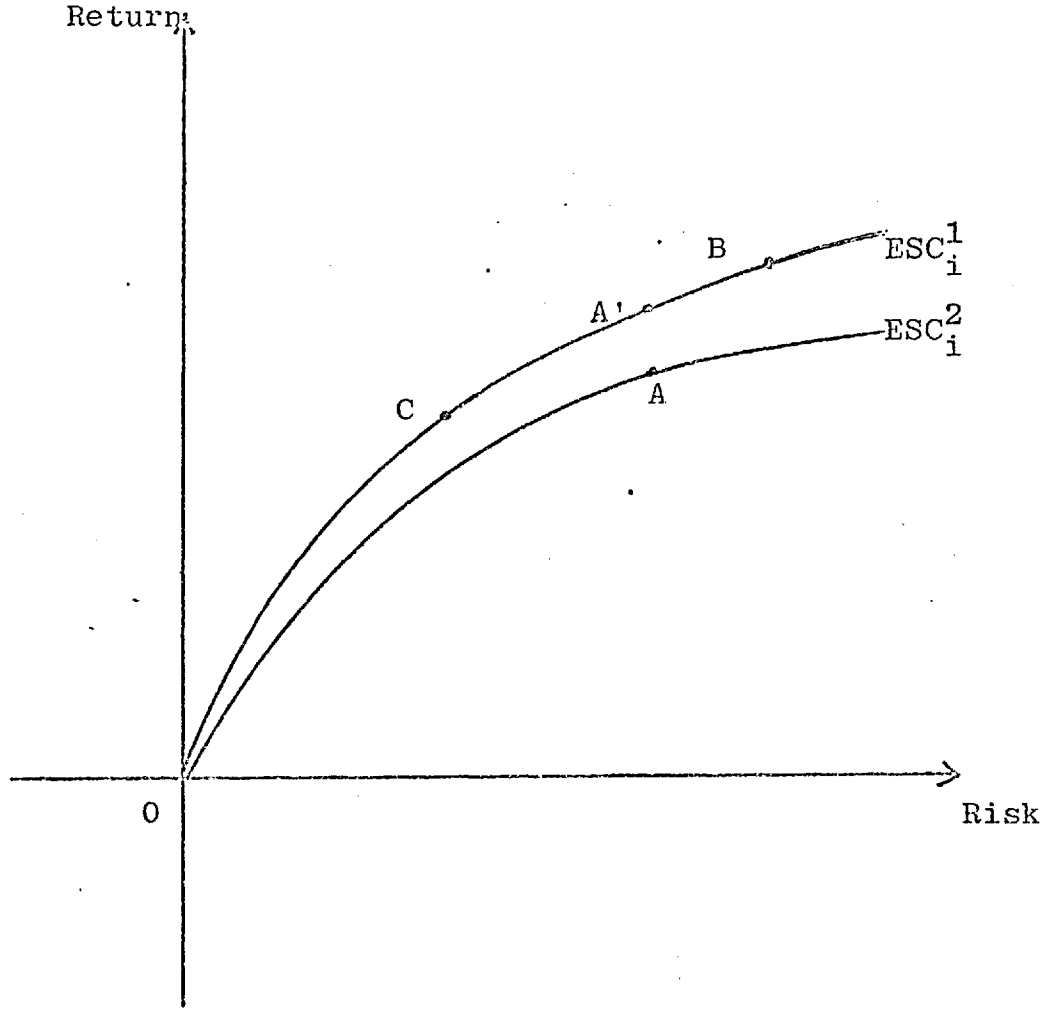


Fig. (6.3): Effects of technology change.

The fact that in a mean-variance framework security and expected wealth might be either normal or inferior goods has behavioural implications both on the department store's management and on those responsible for the individual product lines. Particularly interesting is the effect of the normality or inferiority of expected wealth and security on the allocation of additional resources in the event that these become available. For example, if new sources of funds become available to the store planners after they have selected their optimal resource allocation the additional income will be divided between both expected wealth and security if both are normal goods, ceteris paribus. In other words, given that the two are normal goods, the new portfolio will reflect a higher expected wealth and lower level of risk than the old one. Such a case is graphically illustrated in Fig. (6.4) by the movement from point A to point B, the old and new portfolios respectively.

The implications of the analysis of the demand for assets under risk on the behaviour of the product line manager can be quite unconventional. Specifically, the knowledge of the shareholders' attitude towards wealth and security can lead under certain circumstances to the product line manager adopting what initially seems to be a negative approach. For example, if the line manager perceives that the store management's response to a possible increase in the return of his line is such that they will not increase the amount of resources allocated to it, i.e. the store management has a negative income effect greater than the positive substitution effect, this might reduce the line manager's desire to excell by generating results which surpass the estimates set by the store management. Such an attitude by the store management may lead a line manager to keep his line's performance below expected targets which should, ceteris paribus, generate a shifting of resources from other lines to his. Although this latter case seems unlikely the first and less extreme possibility is sufficient to create problems in motivating lower management in a department store who expect the size of their

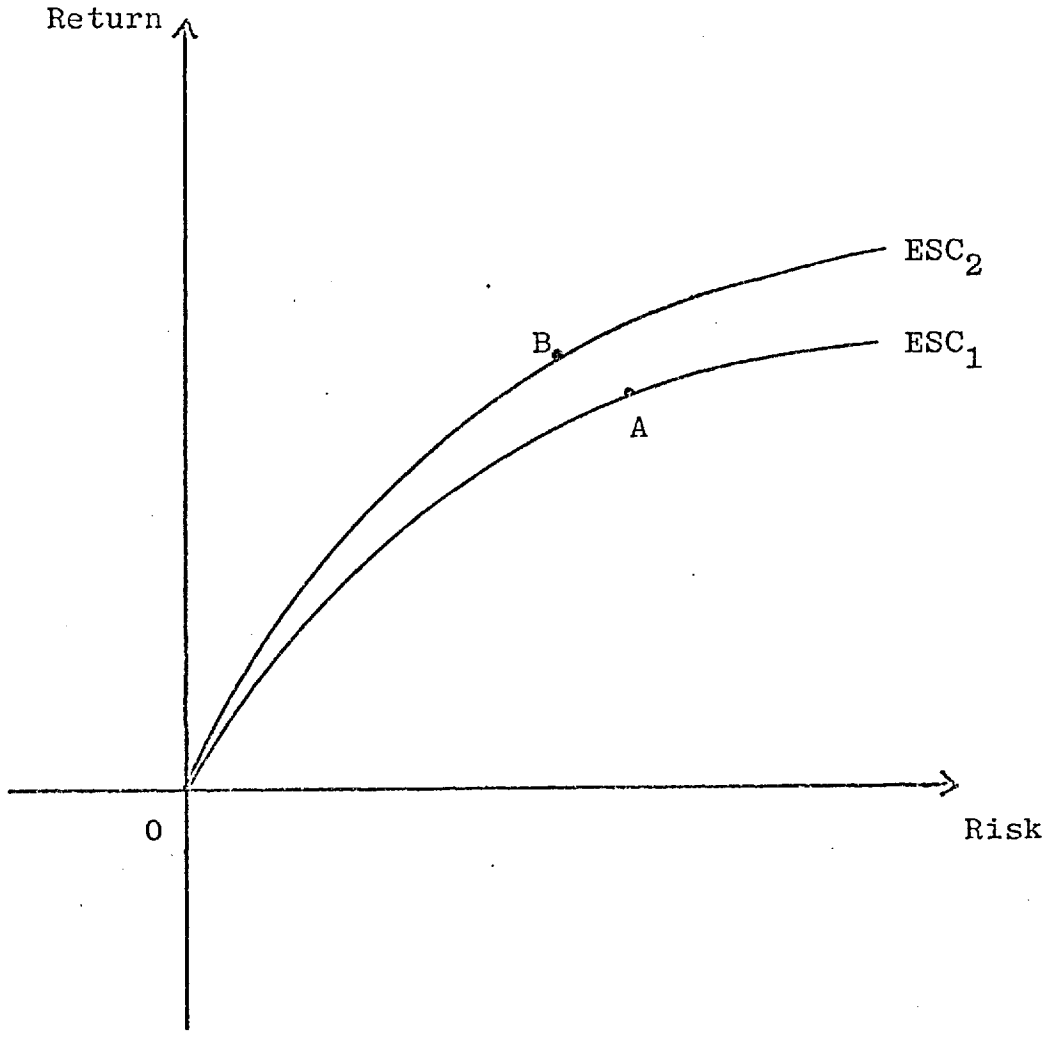


Fig: (6.4): Wealth increase and normality of wealth and security

lines to be a reflection of their performance. Unless a growth in return is matched with an increase in floorspace, stock, manpower etc. . . a certain element of the positive internal competition will probably be lost.

The problem facing a department store's management when confronted with a change in the estimated expected return and/or risk can be described in more familiar terms borrowed from both consumer theory and the theory of the firm. We can illustrate this point with a simple example. Consider the case of a single product firm, whose resources can be invested either in its product or in a riskless asset. If we adopt Greenberg et al's (64) approach, i.e. that increases in risk yield successively smaller increases in return, we can represent the firm's "production possibilities" with a conventional concave product transofrmation curve. We graphically illustrate this concept in Fig. (6.5.a - d), where the intersection of the financial product transformation curve (FPTC) with the wealth axis represents the maximum attainable level of wealth, which if we assume positive diminishing marginal returns will be achieved when all corporate resources are allocated to the product line. There is however no intersection with the security (measured by some negative value of the line's variance or standard deviation) axis since even a risk free policy, i.e. all resources are allocated to the riskless asset, generates a certain amount of wealth, W_1 and W_2 are the wealth generated when all resources are invested in the risky asset before and after a change respectively.

Using this framework we can describe, in relatively simple terms, the possible responses a department store's management can adopt in face of a change in expected return. An increase in the expected return will be graphically reflected by an outwards shift of the firm's FPTC, from $FPTC_1$ to $FPTC_2$, without affecting the maximum security point's location.[3] Management can respond

3. The only possible parallel movement of the FPTC would have to be inwards, and would reflect a disturbance in the system which introduces risk into what previously was a risk-free asset.

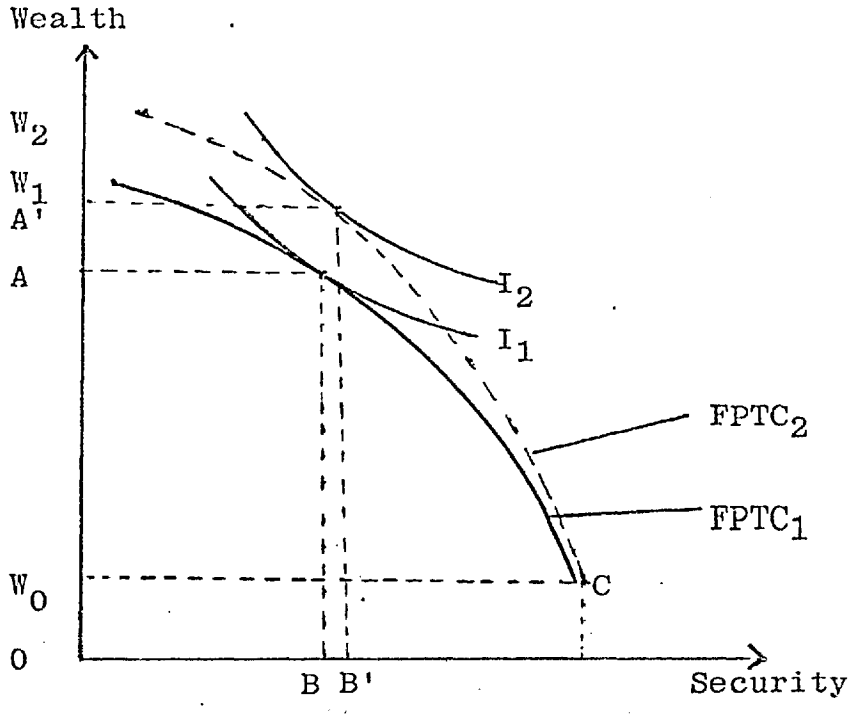


Fig. (6.5.a)

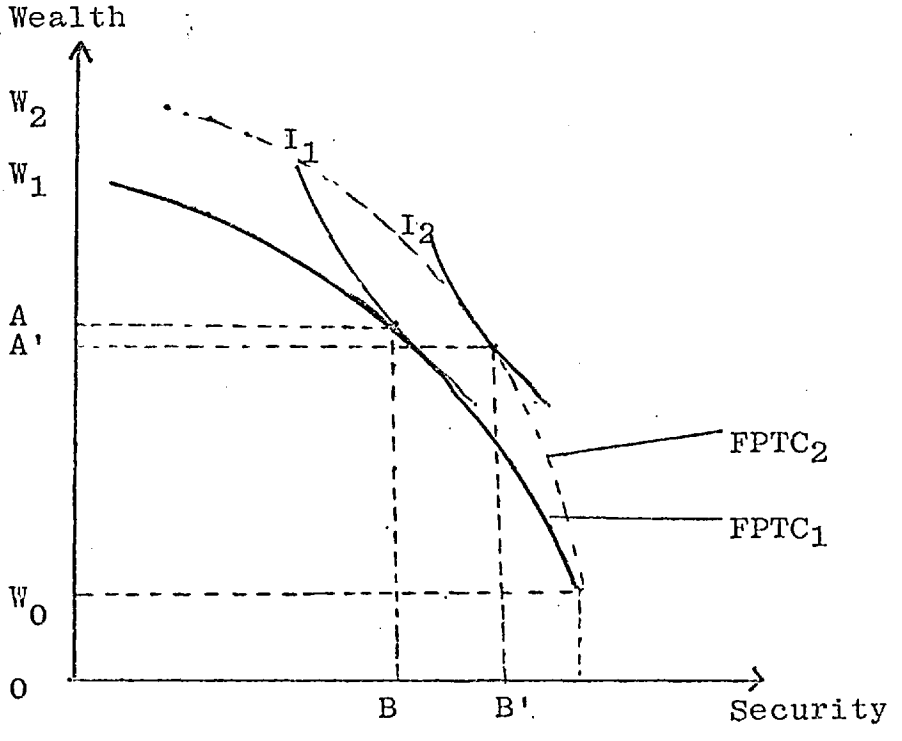


Fig. (6.5.b)

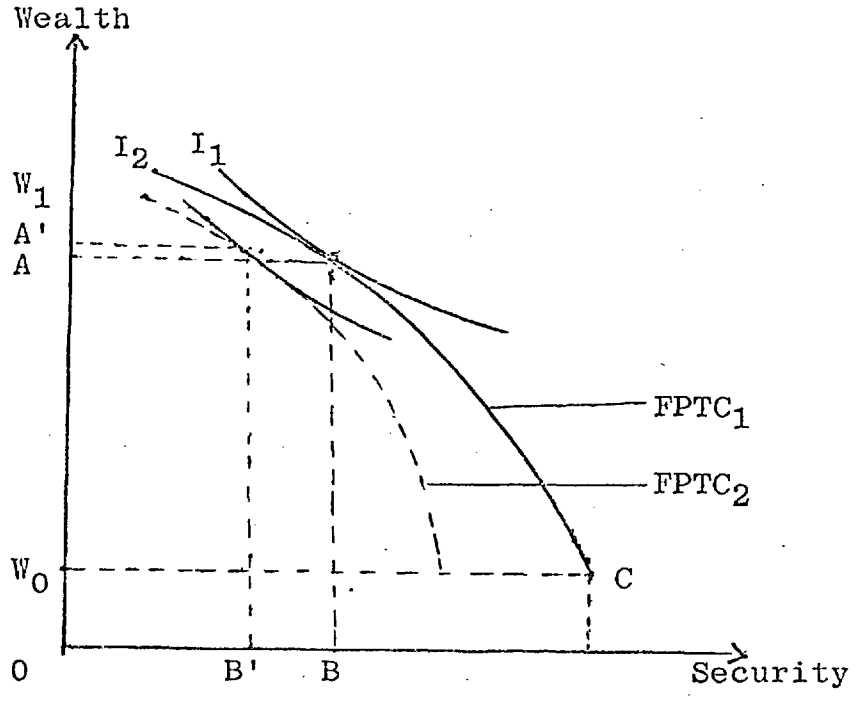


Fig. (6.5.c)

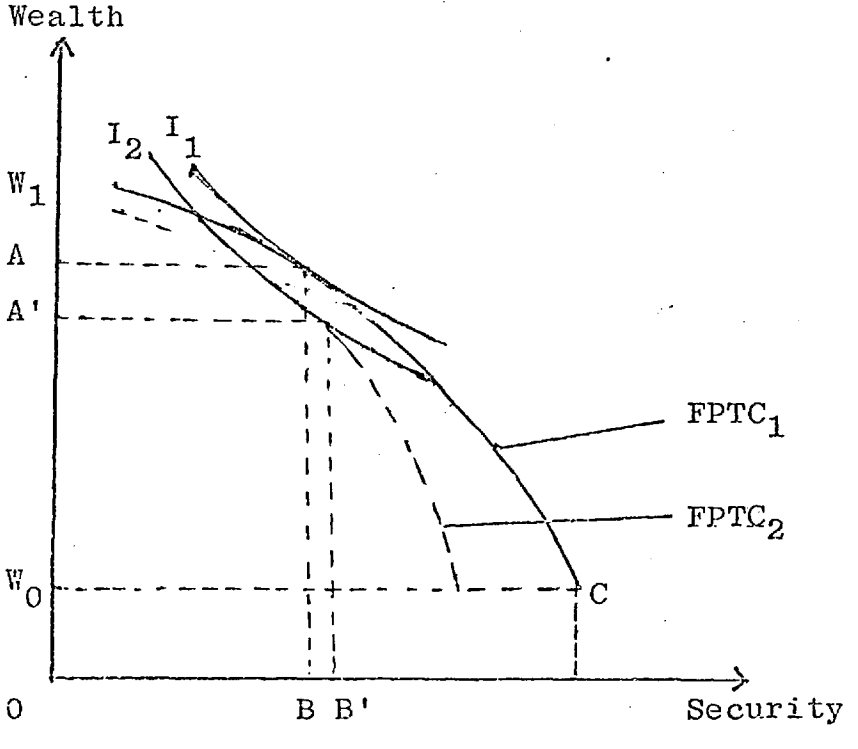


Fig. (6.5.d)

either by increasing its holding of the product line, which is the intuitive reaction, or alternatively shift resources from the product line to the riskless asset, Fig. (6.5.a) and Fig. (6.5.b) respectively.

Similarly, one can explain in these terms the potential effects on the resource allocation in a department store of a change in a line's risk. In Fig. (6.5.c) the firm's FPTC becomes discontinuous since the movement from $FPTC_1$ to $FPTC_2$ leaves the risk free option, denoted C, unaltered. Despite the fact that $FPTC_2$ does not intersect the risk free option it remains a feasible alternative for the firm. It does imply however that, the operation of the line at even a minimal level entails an immediate loss of security. While Fig. (6.5.c) reflects a policy which increases the holding of the line, Fig. (6.5.d) implies a decrease in the amount of resources allocated to the line.

The fact that one can, within the portfolio framework, deal with situations where an asset's rate of return or risk is subject to reassessment is an important contribution to the modelling of a firm operating under uncertainty. An example of a managerial problem which faces practically all firms, regardless of size and product range, and which one can handle on the basis of our previous discussion, is the impact on the firm of a change in consumers' tastes. A shift in the pattern of the consumers' expenditure, regardless of its source, i.e. a change in fashion or in real income, is bound to alter the distribution of the store's trade. One of the direct results of such circumstances would be to change the profitability, i.e. the rate of return, contribution of the various product lines, making some more attractive than when management constructed the original store portfolio and vice versa. In terms of a corporate portfolio one would normally expect such changes in the system to trigger a reassessment of the investment proportions in an effort to shape the asset base in a way which generates the best expected wealth and risk combination. In practical terms this would mean shifting

resources between lines in an effort to shape the product range offered by the store in order to meet changing demand. Such a process could, however, lead to a conflict between top and lower levels of management in view of the possible discrepancy in their attitudes towards the new levels of return. Let us summarize this point by stating that the number of operating decisions which can be handled by using the above concepts is as large as the number of changes in a department store's environment which generate changes in the performance of one or more assets. New levels of technology, introduction of new product lines and greater efficiency are but three of such issues.

Up till now we have concentrated mainly on the influence changes in risk and return can have on the resource allocation within a corporate portfolio, briefly discussing ways changes in a product line's expected rate of return can arise. We now turn our attention to the issue of a product line's risk as measured by its variance of return.

6.3.2 A product line's risk

The analysis of a product line's variance of return and the impact of operating decisions on it involves an examination of the structure of the second element in the expression of the corporate portfolio's risk, Eq.(6.4.b).

A particularly revealing method of studying the above term involves replacing the measure of the rate of return with the familiar ratio:[4]

$$\frac{Q_m(p_m - v_m)}{\sum_k \alpha_{mk} c_k} = \frac{\text{Profit}}{\text{Capital Employed}} = \frac{P}{CE}$$

4. Since our analysis is confined to the single-period context, which we equate with the short term, fixed costs do not appear in our measure of an asset's variance of return since we assume that there are no marginal fixed costs.

which can be expanded to

$$\frac{P}{CE} + \frac{\text{(Sales Volume. Selling Price - Discounts) -}}{\text{Current Assets +}}$$

$$\frac{\text{(Production Costs + Selling Costs + Administration Costs)}}{\text{Fixed Assets}}$$

$$= \frac{(SV.SP - D) - (PC + SC + AC)}{CA + FA}$$

In this form we can write the variance of return as follows:

$$\sigma^2\left(\frac{Q_m(p_m - v_m)}{\sum_k \alpha_{mk}\, c_k}\right) = \sigma^2\left(\frac{(SV.SP - D) - (PC + SC + AC)}{CA + FA}\right)$$

which can be expanded:

$$\sigma^2\left(\frac{Q_m(p_m - v_m)}{\sum_k \alpha_{mk}\, c_k}\right) = \sigma^2\left(\frac{(SV.SP - D}{CA + FA}\right) + \sigma^2\left(\frac{PC + SC + AC}{CA + FA}\right) + 2\rho\left(\frac{SV.SP - D}{CA + FA}, \frac{PC + SC + AC}{CA + FA}\right) \sigma\left(\frac{SV.SP - D}{CA + FA}\right)\sigma\left(\frac{PC + SC + AC}{CA + FA}\right) \quad (6.7)$$

A study of Eq. (6.7) provides some insight into the complexity of the problem facing the store planners when they come to assess the effects of a certain operating decision on the riskiness of a particular product line. The right hand side of Eq. (6.7) consists of three elements: the variance of gross return, the variance of costs and the weighted correlation between the two, respectively. This formulation of a product line's variance provides detailed information about the complexity of the problem raised earlier concerning the existence of contradicting forces

which operate on a line's variance when the former is expanded. If we concentrate on the case which is of particular interest to our discussion, i.e. expansion with economies of scale, it is clear that while the first two terms are positive the weighted correlation between gross return and costs can be negative. Such a situation would occur when unit costs fall as output increases and therefore an expansion which generates economies of scale would create conflicting pressures: on the one hand an increase in the positive values of the first two terms and on the other hand and increase (or possibly decrease) in the last term's value, thus making it impossible to make any general a priori statement as to the expansion's net effect on the line's risk.

The fact that an expansion can yield inconclusive effects on a product line's variance obviously has implications on its aggregate effect on the lines' risk. A study of the forces operating on the corporate risk in such an event reveals that they too, not surprisingly, are capable of moving in opposite directions. Specifically, one would expect the reduction in the unit cost, v_m, assuming unit price remains unchanged, to increase the value of the reflection of the operating leverage $(p_m - v_m)$, which in its turn increases the value of the operating efficiency

$$\text{Var}\ \frac{Q_m(p_m - v_m)}{\sum_k \alpha_{mk} c_k}$$

the latter being further increased through the drop in the unit cost of resource, c_k, and the increase in sales, Q_m. On the other hand however, there exists the possibility that an increase in sales and operating leverage can generate a fall in the value of the correlation coefficient between the firm's return and the return on the market portfolio of risky securities, as illustrated in the numerical example in the previous chapter. The expansion of operations will yield a reduction of the firm's

systematic risk if the following condition holds:

$$\sum_m \left\{ \rho \left\{ Q_m(p_m - v_m) / \sum_k \alpha_{mk} c_k, R_M \right\} \sqrt{\text{Var}\ Q_m(p_m - v_m) / \sum_k \alpha_{mk} c_k} \right\} >$$

$$\sum_{m^*=m+1} \left\{ \rho \left\{ \hat{Q}_{m^*}(p_{m^*} - \hat{v}_{m^*}) / \sum_k \hat{\alpha}_{mk^*} \hat{c}_k, R_M \right\} \sqrt{\text{Var}\ \hat{Q}_{m^*}(p_{m^*} - \hat{v}_{m^*}) / \sum_k \hat{\alpha}_{mk^*} \hat{c}_k} \right\}$$

Whether in fact such a condition does hold depends on the magnitude of the cost reductions and the sales increases as well as on the response of other product lines to changes both in the unit costs of inputs and the sales of other lines. It is the analysis of these kind of relationships between lines which is the subject in the following sub-section.

6.3.3 The Correlation Coefficient

The third and last term in the formal expression of the store's variance of return is ρ_{ij}, the correlation coefficient of the returns on asset i and j. Essential to establishing optimal investment proportions in any type of portfolio, this term is the basis for a concept of considerable importance in the case where the portfolio in question is a firm and the assets - product lines, namely the concept of complementarity and substitution.

In chapter 3, within the context of consumer theory, we discussed complementarity - substitution relationships between goods in a riskless world. Very similar relationships have been shown to exist between risky assets in a portfolio and have been the subject of various studies. Most interesting from our point of view is the fact that some of the complementarity - substitution characteristics established in the portfolio context are very close to those in the standard consumer theory, and consequently are of practical meaning to planners in a department store.

The conventional definitions of relationships between two assets in a portfolio are based on the response of the demand for one asset as a result of a change in the return of the other, given that the wealth effect is neglected. Specifically, two assets are defined as complements if a rise in the return on one asset increases the demand for the other, and the two are substitutes if a rise in the return of one asset decreases the demand for the other.

Levy (94) provides a mathematical formulation of these concepts in his analysis of the demand for assets under risk. Specifically, he examines the second term of the following expression:

$$\frac{\partial \alpha_i}{\partial E(R_j)} = \frac{\partial \zeta_1}{\partial E(R_j)} Y_i + \zeta_1 \frac{\partial Y_i}{\partial E(R_j)}$$

and concludes that the substitution - complementarity relationship is determined by the term:

$$\left.\frac{\partial \alpha_i}{\partial E(R_j)}\right|_{E(W_1) \text{ const.}} = \zeta_1 \frac{\partial Y_i}{\partial E(R_j)} = - \zeta_1 \frac{\rho_{ij.}}{\sigma_{j.} \sigma_{i.}}$$

which since $\zeta_1/\sigma_{i.}\sigma_{j.}$ is positive, implies that the crucial element is the partial correlation $\rho_{ij.}$[5] Specifically, his analysis concludes the following results:

i and j are complements if $\rho_{ij.} < 0$

i and j are substitutes if $\rho_{ij.} > 0$

i and j are independent if $\rho_{ij.} = 0$

and that these relationships are symmetrical and that all assets cannot complement each other.

---

5. Levy offers a comprehensive discussion of the relationship between the total and partial correlations between two assets, and why the use of the former for determining the substitution-complementarity relationship might mislead.

If we now assume that the partial correlation can be approximated by the determinant of complementarity in consumer theory, i.e. the cross-elasticity between products, then in the context of a department store these relationships are particularly useful in the planning process since the assets in question are goods and services whose relationships have, to a large extent been the subject of numerous econometric studies. Tables (6.1.a) and (6.1.b) illustrate the results of such studies with various cross-elasticities of demand for selected commodities;

COMMODITY	CROSS-ELASTICITY WITH RESPECT TO PRICE OF:	CROSS-ELASTICITY
Beef	Pork	+ 0.28
Butter	Margarine	+ 0.67
Margarine	Butter	+ 0.81
Pork	Beef	+ 0.14

Source: H. Wold, Demand Analysis, New York, John Wiley & Sons, 1953

Table (6.1.a)

COMMODITY	CROSS-ELASTICITY WITH RESPECT TO PRICE OF:	CROSS-ELASTICITY
Beef (Home)	Mutton/Lamb (imported)	+ 0.50
Pork	Mutton/Lamb (imported)	+ 0.85
Butter	Flour	- 0.21
Oranges	Dried Fruit	+ 0.63
Margarine	Butter	+ 1.01
Apples	Other fresh fruit	+ 2.77
Tea	Coffee	+ 0.14

Source: Stone, R., The Measurement of Consumers' Expenditure and Behaviour in the U.K., 1920-38 Vol. 1, Cambridge University Press, London, 1953.

Table (6.1.b)

The combination of such studies with the experience gained by management can provide the department store's decision makers with information about the relationships between product lines within the store. This kind of data, which we assume available, are clearly of vital importance for determining the exact composition of the range of products and services offered by the store and therefore, have an economic and not merely statistic role in computing the optimal store portfolio.

If we are to allow for changes in consumers' tastes to be incorporated in our model of the firm we must acknowledge the possibility of such a change being reflected by a change in the correlation coefficient between two product lines. Such an alteration of the value of the cross-elasticity between two goods can be a consequence of a change in the consumers' expenditure or income pattern or possibly a result of the introduction of a new product into the market.

Levy (94) provides the mathematical framework for the analysis of the case of a change in the correlation between assets i and K without any change in the other parameters. Basically, the effect on the demand for asset i as a result of a change in the cross-elasticity is measured by the quantity $\partial\alpha_i/\partial\rho_{iK.}$. By differentiating the demand for i with respect to $\rho_{iK.}$ he gets:

$$\frac{\partial\alpha_i}{\partial\rho_{iK.}} = \frac{\partial\zeta_1}{\partial\rho_{iK.}} Y_i + \zeta_1 \frac{\partial Y_i}{\partial\rho_{iK.}}$$

Hence,

$$\frac{\partial\alpha_i}{\partial\rho_{iK.}} = \frac{\partial\sigma(W_1)}{\partial\rho_{iK.}} \frac{\partial\zeta_1}{\partial\sigma(W_1)} Y_i - \zeta_1 \frac{E(R_K) - R_F}{\rho_{K.}\rho_{i.}} \qquad (6.8)$$

Since the second term in Eq. (6.8)'s right hand is negative, i.e. a negative substitution effect, a reduction of $\rho_{iK.}$ without a change in the store manager's income would increase

the amount of resources allocated to product line i. However, since $\partial\sigma(W_1)/\partial\rho_{iK}$ can be either positive or negative, the income effect is undetermined.

Finally, let us point out that there is evidence that the returns from capital projects are not perfectly correlated in most types of industries. This is due to the fact that projects are differently affected by factors such as differences in the products and industries and their vulnerability, geographical location, sensitivity to various social and governmental factors, and their competitive environment. As a result, the exploration of the economic forces underlying the correlation coefficient of a corporate asset under risk can be of interest in industries other than the retail trade.

6.4 RESOURCE ALLOCATION IN A DEPARTMENT STORE

The concept we have adopted, i.e. treating a department store as a portfolio, raises three questions, typical to any type of portfolio, which face the store manager. Specifically, the following problems have to be solved when one constructs the store manager's portfolio:

i) which assets to include in the asset base;

ii) what are the optimal investment proportions;

iii) what is the total level of investment in the portfolio?

In this section we attempt to answer, within the realities of a department store, the first two questions, leaving the treatment of the last one for the following chapter.

Determining the asset base:

In the case of an investor in the capital market the answer to this question is straightforward: in a perfect market, where borrowers and lenders face the same riskless rate, the risk portfolio of all investors will include all securities independent of their taste. This statement, known as the separation theorem, follows from the fact that if a particular security is not included in the market portfolio of risky securities, and no investor holds it, its price must fall, thereby raising its return until it becomes sufficiently attractive to be included in the market portfolio of risky securities.

In the context of a department store however, one cannot apply the same approach without further study. Product lines are not necessarily always profitable, i.e. a line might generate a level of return lower than that required by the firm's riskiness, and the concept of a pricing mechanism which adjusts

the actual return to an equilibrium level is not at all clear. Furthermore, the department store manager, unlike the investors in the capital market, does not have access to endless amounts of resources.

As a result, the store manager's first task is to identify and select those product lines whose rate of return justifies inclusion in the asset base. Specifically, this implies that the store manager should consider allocating a proportion of the scarce resources at his disposal only to lines whose expected rate of return exceeds the appropriate risk adjusted discount rate. In the previous section we formulated the asset expansion rule necessary for the selection of attractive lines.

Determining the investment proportions:

In this section we suggest methods for dividing the total assets at the store manager's disposal between the elements of the set of investments which have successfully passed the hurdle defined above. We tackle the store manager's problem from two behaviourally different angles; first, we formulate the case where the manager is in search for an overall optimum, and second, we abandon the attempt to maximize the store's value and instead assume the manager tries to attain target standards of performance.

A). OPTIMAL INVESTMENT PROPORTIONS:

There are numerous ways of determining the optimal resource allocation for a firm which seeks to maximize its value. In the following discussion we suggest two alternative formulations of the department store manager's problem given the above assumption about his target.

The first of such formulations is a direct extention of our earlier analysis of the concept of the excess

rate of return. Using the asset expansion criterion developed earlier one can view the combinations of projects which have passed the profitability hurdle and satisfy the rationing constraints (yet to be defined) as mutually exclusive bundles. Consequently, the firm should accept the bundle with the highest excess expected rate of return weighted by its cost. Hence, for a store operating m product lines, the problem is to accept the bundle which:

$$\text{Max. } Z = \sum_m CL_m \left[E(R_m) - R_F - \lambda \text{Cov}(R_m, R_M)\right] \qquad (6.9.a)$$

$$\text{S.T. } \sum_m CL_m \leq CS_i \qquad (6.9.b)$$

where:

CL_m - is the investment in product line m,

CS_i - is the total capital available to store i.

and Eq. (6.9.b) is the capital rationing constraint.

This formulation's main attraction is its relative simplicity and the fact that it highlights the type of problem in a very general manner. This advantage however, is also its very weakness, there is no mention of the internal mechanism of the store, making it quite impractical. Clearly there is a need for a formulation of the problem in a manner which links the corporate performance with that of the indivi-ual product lines.

A suitable approach to the problem of the mathemat-ical programming of interrelated projects has been developed by Thompson (144) on the basis of Hamada's (66) capital budgeting model. Basically, Hamada's

hurdle rate for a new investment is defined as:

$$\frac{dE({}_{\tau}X_j)}{dI} \geq R_F\,(1 - \tau L^*) + \frac{\lambda}{S_M}\,\frac{dCov({}_{\tau}X_j, X_M)}{dI}$$

where:

${}_{\tau}X_j$ - is firm j's after tax returns,

$dI = \Delta I$ - the investment project,

$-{}_{\tau}L^*R_F$ - the tax subsidy for debt,

L^* - the target debt ratio,

S_M - the total market value of all stocks including firm j.

In the case of interrelated product lines within a department store, the effect of a reallocation of resources on the store's risk as measured by the covariance of its returns with those of the market can be estimated as:

$$\frac{\lambda}{S_M}\,\frac{d\ Cov\ ({}^{\tau}X_j, {}^{\tau}X_M)}{dI} = \frac{\lambda}{S_M}\,\frac{Cov\left({}_{\tau}X_{j0} + \sum_{i=1}^{n} {}_{\tau}X_{ji}, {}^{\tau}X_M + \sum_{i=1}^{n} {}^{\tau}X_{ji}\right)}{\Delta I}$$

$$- \frac{Cov({}_{\tau}X_{j0}, {}^{\tau}X_M)}{\Delta I}$$

where:

$_{\tau}X_{j0}$ - is the after tax return before the projects are undertaken,

$_{\tau}X_{ji}$, i-1,..,n - the after-tax returns for the individual projects,

$\Delta I = \sum_{i=1}^{n} \Delta I_i$ - is the sum of the reallocations in the individual product lines.

which can be manipulated as follows:

$$\frac{\lambda}{S_M} \frac{d\ Cov(_{\tau}X_j, _{\tau}X_M)}{dI} = \frac{\lambda}{S_T \Delta I} \left\{ \sum_{i=1}^{n} Cov\left(_{\tau}X_{ji}, _{\tau}X_M\right) + \sum_{i=1}^{n} Cov\left(_{\tau}X_{j0}, _{\tau}X_{ji}\right) + Cov\left(\sum_{i=1}^{n} {}_{\tau}X_{ji}, \sum_{i=1}^{n} {}_{\tau}X_{ji}\right)\right\}$$

$$= \frac{\lambda}{\Delta I} \sum_{i=1}^{n} Cov\left(_{\tau}X_{ji}, R_M\right) + \frac{\lambda}{S_T \Delta I} \left\{ \sum_{i=1}^{n} Cov\left(_{\tau}X_{jo}, _{\tau}X_{ji}\right) + Cov\left(\sum_{i=1}^{n} {}_{\tau}X_{ji}, \sum_{i=1}^{n} {}_{\tau}X_{ji}\right)\right\}$$

which after further manipulation becomes:

$$\frac{\lambda}{S_M} \frac{d\ Cov\ (_{\tau}X_j, _{\tau}X_M)}{dI} = \frac{\lambda}{\Delta\ I} \sum_{i=1}^{n} Cov(_{\tau}X_{ji}, R_M)$$

(6.10)

If we now assume that the systematic risk in the rate of return of an individual product line can be explained by a linear relationship with R_M we can write:

$$\frac{{}_{\tau}X_{ji}}{\Delta I_i} = a_i + b_i R_M + \varepsilon_i \qquad (6.11)$$

we can incorporate Eq. (6.10) and Eq. (6.11) to write:

$$\frac{\lambda}{S_M} \quad \frac{d\ Cov({}_{\tau}X_j, {}_{\tau}X_M)}{dI} = \frac{\left[E(R_M) - R_F\right]}{\Delta I} \sum_{i=1}^{n} b_i\ \Delta I_i \qquad (6.12)$$

since $Cov\ ({}_{\tau}X_{ji}, R_M) = \Delta I_i\ Cov({}_{\tau}X_{ji}/\Delta I_i, R_M) = \Delta\ I_i b_i \sigma^2\ (R_M)$.

On the basis of this framework Thompson develops a formulation of the investment selection problem which, after modification, can be used to formulate the store manager's problem.

I) <u>A Linear Programming approach:</u> We can write the store manager's problem as follows:

$$\text{Max.}\quad Z = \sum_m \left\{ \frac{E(X_m) - \left\{R_F + b_m \left[E(R_M) - R_F\right]\right\} \sum_k \alpha_{mk} c_k}{R_F} \right\} \qquad (6.13.a)$$

$$\text{S.T.} \qquad \sum_m \alpha_{mk} \leq \alpha^*_{ik} \quad \forall k \qquad (6.13.b)$$

$$\alpha_{mk} \geq 0 \quad \forall m,k \qquad (6.13.c)$$

where:

X_m - the returns generated by product line m; and

α^*_{ik} - the amount of resource k available to the manager of store i.

Since our objective in the following chapter is to develop a resource allocation procedure in a decentralized department store chain, we now proceed to formulate the dual of the problem of store i's manager. Let us start, however, by simplifying the notation so that we can rewrite the primal problem as follows:

$$\text{Max.} \quad Z = \sum_m \sum_k \Omega_m c_k \alpha_{mk} \qquad (6.14.a)$$

$$\text{S.T.} \quad \sum_m \alpha_{mk} \leq \alpha^*_{ik} \quad \forall k \qquad (6.14.b)$$

$$\alpha_{mk} \geq 0 \quad \forall m,k \qquad (6.14.c)$$

where $\Omega_m = -\left\{R_F + b_m \left[E(R_M) - R_F\right]\right\}$ and the true solution is $\sum_m \frac{E(X_M)}{R_F}$ more than this.

We can write the above problem's dual in the following manner:

$$\text{Min.} \quad Z = \sum_k \alpha^*_{ik} W_k \qquad (6.15.a)$$

$$\text{S.T.} \quad W_k \geq 0 \quad \forall k \qquad (6.15.b)$$

$$W_k \geq \Omega_m c_k \quad \forall m,k \qquad (6.15.c)$$

where the dual variables W_k can be interpreted as the marginal price of resource k when used by department store i. The importance of this marginal price in the process of resource allocation in a department store is in its use as a bidding price by the management of the store when requesting its share of the total amount of resource k available to the

central unit. This point will be further elaborated in the following chapter where we develop an iterative procedure for resource allocation for a decentralized department store chain, using the concept of marginal prices as bidding prices for resources.

Using as a framework the LP defined above, we can now turn to examine several topics which face a department store manager in the running of his branch.

i) Increasing the investment in a particular product Line:

One of the questions which preoccupy department store managers to a large extent is the issue of expanding a particular line at the expense of some other(s). Clearly, no sensible decision of this type can be made unless the store manager has a good idea as to the effect such a change will have on the value of his objective function. Any increase in the investment in a product line will be done voluntarily by the store manager only if his performance consequently improves. The kind of information which the manager requires can be generated by looking at the following formulation of the store manager's problem:

$$\text{Max. } Z = \sum_m \sum_k \Omega_m c_k \alpha_{mk} \qquad (6.16.a)$$

$$\text{S.T. } \sum_k \alpha_{mk} \leq \alpha l_{im} \quad \forall m \qquad (6.16.b)$$

$$\alpha_{mk} \geq 0 \qquad (6.16.c)$$

where αl_{im} is the limit on the amount of resources allocated by the manager of store i to produce line m.

Let us turn to the above problem's dual which can be written as follows:

$$\text{Min. } Z = \sum_m \alpha l_{im} \phi_m \qquad (6.17.a)$$

$$\text{S.T. } \phi_m \geq 0 \quad \forall m \qquad (6.17.b)$$

$$\phi_m \geq \Omega_m c_k \quad \forall k \qquad (6.17.c)$$

where ϕ_m, the dual variables, can be economically interpreted as the marginal contribution to the store manager's objective function of an increase of the investment in product line m. In other words, the store manager can, by examining the above dual variables, assess how the store portfolio's performance will be affected by increasing the amount of resource allocated to a particular product line. Specifically, the higher a product line's dual variable, the greater its marginal contribution to the store's performance in the event of an increase in the amount of resources allocated to it.

(ii) Normality and Inferiority under Uncertainty:

Additional insight into the properties of a product line can be achieved by examining the change in the store manager's "consumption" of the line in question as a result of a change in the amount of resources at his disposal. Specifically, the answer to the above question, i.e. how does the demand for a product line change as a result of a change in the store manager's budget, is analogous to the concepts of inferiority and normality in the deterministic world of consumer theory. Generally, one can state that, given that the investment in a particular product line increases (decreases) as a result of an increase in the amount of resources at the store manager's disposal - the line is normal (inferior). Since the lines in a department store are groups of goods and as such are the subjects of consumer theory, it is particularly interesting to investigate the case of a department store, where the results can be subsequently compared with those derived under deterministic conditions. In other words, we could then examine the question: does an item maintain its normality (inferiority) from a deterministic world as one moves to a mean-variance one?

Although no a priori answer can be given for any specific item, we can proceed to examine the set of information required for such an answer. Let us define $B = (a_{ij})$, whose dimensions are (k x k) and which is the basis matrix formed from linearly independent columns of the coefficient matrix

of the constraints of the store manager's problem in its LP formulation. By definition:

$$B^{-1} = \frac{1}{|B|} \text{ adj } B$$

where $|B|$ is the determinant of B, and adj B the adjoint matrix.

The question we are examining can be defined as follows: how does the store manager's investment in product line m, i.e. α_{mk}, change as a result of a change in α_{ik}? Let us assume that in view of the marginality of the change in the store manager's budget there will be no change in the basic solution, and that the constraints are strict, otherwise any marginal change in α_{ik} will leave all α_{mk} unchanged. Using standard notation, we can write:

$$x^* = B^{-1} b$$

where x^* is the optimal solution of the original problem, i.e. prior to the change in the constraint α_{ik}, and

$$b = \begin{pmatrix} \alpha_{i1} \\ \alpha_{i2} \\ \cdot \\ \cdot \\ \cdot \\ \alpha_{ik} \end{pmatrix}$$

If the dual simplex method has been used to solve the original problem, B^{-1} will be known. We now distinguish between the case where the perturbation in the constraint generates a basic solution which is feasible and that where it is infeasible. The simpler case is the former, if b is perturbed to $b + \Delta b$, and the new basic solution

$$\hat{x} = B^{-1}(b + \Delta b)$$

is feasible, then the change in α_{mk} will be known immediately.

If on the other hand the new basic solution is infeasible, then we can calculate the marginal change in x* as follows:

by definition $BB^{-1} = I$

where I is the identity matrix, and by the product differentiation rule

$$\frac{\partial B}{\partial b} B^{-1} + B \frac{\partial B^{-1}}{\partial b} = 0$$

which, rearranged, gives us:

$$B^{-1} \frac{\partial B}{\partial b} B^{-1} = - \frac{\partial B^{-1}}{\partial b}$$

since, by well known theory, $x^* = B^{-1}b$, we can write:

$$\frac{\partial x^*}{\partial b} = \frac{\partial B^{-1}}{\partial b} b + B^{-1}$$

$$= - B^{-1} \frac{\partial B}{\partial b} B^{-1}b + B^{-1}$$

$$= -B^{-1} \frac{\partial B}{\partial b} x^* + B^{-1}$$

$$= B^{-1} \left[I - \frac{\partial B}{\partial b} x^* \right]$$

However, if we are interested in determining the normality or inferiority of a particular product line say m, in the case of the store manager whose problem we are examining, a look at the m-th entry of the vector $\partial x^*/\partial b$ will tell us the marginal change in a_{mk} and answer our question. Let us point out that, in view of the fact that any solution is determined by the product line's risk, among other parameters, it follows that

any change in the risk characteristics of a product line's risk can result in a change in the line's classification, i.e. making a normal good inferior or vice versa.

iii) Department Store Efficiency

A different insight into the store manager's behaviour under uncertainty can be derived from an examination of the store manager's portfolio performance as a result of a change in the efficiency with which his store is run. Let us assume that the store manager's problem is formulated as follows:

$$\text{Max. } Z = \sum_m \sum_k \Omega_m c_k \alpha_{mk}$$

$$\text{S.T.} \quad \sum_m \alpha_{mk} c_k \leq C_{ik}$$

$$\alpha_{mk} \leq 0 \quad \forall m,k$$

where C_{ik} is the budget available for the purchase of resource k. If we now assume that the unit cost of resource k, i.e. c_k, is a function of a parameter t, where t could reflect some efficiency in staff training when k is manpower, then the change in the store's excess rate of return as a result of an improvement in training can be defined as follows:

$$\frac{dZ}{dt} = \sum_m \sum_k \frac{\partial Z}{\partial C_{ik}} \frac{\partial Z}{\partial \Omega_m c_k} \frac{\partial \alpha_{mk}}{\partial t}$$

which as Saaty[6] proves is equal to:

$$\frac{dZ}{dt} = \sum_m \sum_k \frac{\partial Z}{\partial \alpha_{mk}} \frac{d\alpha_{km}}{dt} + \sum_m \frac{\partial Z}{\partial \Omega_m c_k} \frac{d\Omega_m c_k}{dt} + \sum_k \frac{\partial Z}{\partial C_{ik}} \frac{dC_{ik}}{dt}$$

which we can rewrite:

$$\frac{dZ}{dt} = \sum_m \sum_k \Pi^*_k x^*_m \frac{dc_k}{dt} + \sum_m x^*_m \frac{d\Omega_m c_k}{dt} + \sum_k \Pi^*_i \frac{dC_{ik}}{dt}$$

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6. Saaty, T.L., (1959) Coefficient Perturbation of a Constrainted Extremum. Operations Research, Vol. 7, pp. 294 - 302.

where the dual variables and solution at optimum, Π^*_m and x^*_k respectively, can be derived as we have demonstrated earlier, and therefore are assumed to be known, which enables us to substitute them into the above equation and measure the change in the store manager's performance as a result of a change in parameter t. If in fact t does denote some reflection of managerial efficiency, the above information could be used by the store manager to establish the size of the reward, if any, measured in terms of the change in the value of the objective function, which results from the above effort. If a cost could be associated with the achievement of the new level of efficiency, the store manager would be in a position to determine whether the effort is worthwhile, i.e. if the incremental excess rate of return is greater than the cost of attaining it.

II) A Non-Linear Programming Approach: In our LP formulation of the store manager's problem we made an implicit assumption as to the effect of the existence of complementarity-substitution relationships in the store. Specifically, we incorporated these relationships through the store's covariance, i.e. the inclusion of a particular product line in the store portfolio affected the store's risk in a manner which obviously depended on the inclusion, or non-inclusion, of other product lines in the store portfolio. As a result, the store portfolio's risk was directly affected by the relationships which exist between the product lines operated by the store.

There exists however an alternative method of incorporating the concept of substitution-complementarity in the formulation of the store manager's problem: through a specific complementarity effect on the store's return. This concept is based on Hillier's (73) method of letting the competitive and complementary effects be deterministic and additive and is written by Thompson (144) as:

$$E({}_{\tau}X_{ji}) = E({}_{\tau}X'_{ji}) + \sum_{\substack{m=1 \\ m\neq i}}^{n} \mu_{im}\zeta^*_m \qquad (6.18)$$

where $E({}_{\tau}X'_{jm})$ is the direct effect of product line m, and μ_{im} is one half of the substitution or complementarity effect of having both i and m in the solution, i.e. the incremental return of the store manager's portfolio of both product lines i and m are carried

by the store, and ζ_m^* is a decision variable where $\zeta_m^* = 0(1)$ following m's rejection (acceptance).

Let us replace the integer decision variable, ζ_m^*, with a continuous ratio so that the return effect of complementarity and substitution is a function of the proportion of the resources allocated by the store manager to the product line in question. We therefore rewrite Eq. (6.16) in the following manner:

$$E({}_{\tau}X_{ji}) = E({}_{\tau}X'_{ji}) + \sum_{\substack{m=1 \\ m\neq h}}^{n} \mu_{mh} \frac{\Sigma_k \alpha_{mk} c_k}{\Sigma_k \alpha^*_{ik} c_k}$$

where $\Sigma_k \alpha^*_{ik} c_k$ is the total amount of resources at store i's manager's disposal (a constant at any particular iteration).

We can formulate the store manager's problem in a manner which incorporates the return effect of complementarity-substitution as follows:

$$\text{Max. } Z = \sum_m \left\{ \frac{E(X'_m) - \left\{ R_F + b_m \left[E(R_M) - R_F \right] \right\} \Sigma_k \alpha_{mk} c_k}{R_F} \right\}$$

$$+ \frac{1}{R_F} \sum_{h=1}^{m} \sum_{j=1}^{m} \mu_{hj} \frac{\Sigma_k \alpha_{hk} c_k}{\Sigma_k \alpha^*_{ik} c_k} \frac{\Sigma_k \alpha_{jk} c_k}{\Sigma_k \alpha^*_{ik} c_k} \qquad (6.19.a)$$

$$\text{S.T.} \quad \Sigma_m \alpha_{mk} \leq \alpha^*_{ik} \quad \forall\, k \qquad (6.19.b)$$

$$\alpha_{mk} \geq 0 \quad \forall m,k \qquad (6.19.c)$$

Thompson points out the fact that the solution to the above type of problem is not straightforward. He suggests spliting the problem into independent subproblems, which would be possible in the event of not all the μ_{ij}'s being non-zero and the inter-relationships between projects allow such a split, which could be solved using Hillier's branch-and-bound method for quadratic objective functions with linear constraints.

One can however simplify the problem which faces the store manager as defined above by examining its dual. Let us denote the first part of the objective function $L(\alpha)$, i.e. $L(\alpha)$ is a vector function with elements α_{km} $\forall k,m$, and denote:

$$f_1 \equiv \frac{\sum_k \alpha_{hk} c_k}{\sum_k \alpha^*_{ik} c_k} \quad ; \quad \text{and} \quad f_2 \equiv \frac{\sum_k \alpha_{jk} c_k}{\sum_k \alpha^*_{ik} c_k}$$

and Π_k be the dual variables. Hence we can write the dual of the problem defined by Eq. (6.17a) - (6.17.c) as follows:[7]

$$\text{Min.} \quad L(\alpha) + \sum_{hj} \mu_{hj} f_1(\alpha) f_2(\alpha) + \sum_k \Pi_k \left(\sum_m \alpha_{mk} - \alpha^*_{ik}\right) \tag{6.20.a}$$

$$\text{S.T.} \quad \frac{\partial L}{\partial \alpha} + \sum_{hj} \mu_{hj} \left\{ \frac{\partial f_1(\alpha)}{\partial \alpha} f_2(\alpha) + \frac{\partial f_2(\alpha)}{\partial \alpha} f_1(\alpha) \right\} + \sum_k \Pi_k = 0 \tag{6.20.b}$$

$$\Pi_k, \alpha_{mk} \geq 0 \tag{6.20.c}$$

We can write the above dual in a lagrangian form, using α, the primal variable, as below:

$$\text{Min.} \quad L(\alpha) + \sum_{hj} \mu_{hj} f_1(\alpha) f_2(\alpha) + \sum_k \Pi_k \left[\alpha^*_{ik} - \sum_m \alpha_{mk}\right] - \left[\sum_k \Pi_k C - \frac{\partial L}{\partial \alpha} - \sum_{hj} \mu_{hj} \left\{ \frac{\partial f_1(\alpha)}{\partial \alpha} f_2(\alpha) + \frac{\partial f_2(\alpha)}{\partial \alpha} f_1(\alpha) \right\}\right] \alpha \tag{6.21.a}$$

$$\text{S.T.} \quad \sum_k \Pi_k C \geq \frac{\partial L}{\partial \alpha} + \sum_{hj} \mu_{hj} \left\{ \frac{\partial f_1(\alpha)}{\partial \alpha} f_\alpha(\alpha) + \frac{\partial f_\alpha(\alpha)}{\partial \alpha} f_1(\alpha) \right\} \tag{6.21.b}$$

$$\Pi_k, \alpha_{mk} \geq 0 \quad \forall k,m \tag{6.21.c}$$

where $\frac{\partial \alpha_{mk}}{\partial \alpha} = C$, where C is a constant.

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7. See Mangasarian (98) and Balinsky and Baumol (13).

which can be rearranged as follows:

$$\text{Min} \left[L(\alpha) + \frac{\partial L}{\partial \alpha}\alpha\right] + \sum_{hj} \mu_{hj}\left[f_1(\alpha)f_2(\alpha) + \left\{\frac{\partial f_1(\alpha)}{\partial \alpha} f_2(\alpha) + \frac{\partial f_2(\alpha)}{\partial \alpha} f_1(\alpha)\right\}\alpha\right] + \left[\sum_k \Pi_k\left[\alpha^*_{ik} - \sum_m \alpha_{mk}\right] - \alpha \sum_k \Pi_k C\right] \quad (6.22.a)$$

$$\text{S.T.} \quad \sum_k \Pi_k C \geq \frac{\partial L}{\partial \alpha} + \sum_{hj} \mu_{hj}\left\{\frac{\partial f_1(\alpha)}{\partial \alpha} f_2(\alpha) + \frac{\partial f_2(\alpha)}{\partial \alpha} f_1(\alpha)\right\} \quad (6.22.b)$$

$$\Pi_k, \alpha_{mk} \geq 0 \qquad \forall k,m \qquad (6.22.c)$$

Given the fact that

$$f_1(\alpha)f_2(\alpha) \simeq f_1(0)\ f_2(0) + \alpha \frac{\partial (f_1 f_2)}{\partial \alpha}$$

and also that

$$f_1(0) \simeq f_2(0) \simeq 0$$

we can rearrange the objective function, thus simplifying it, by approximating it as follows:

$$\text{Min.} \left[L(\alpha) + \frac{\partial L}{\partial \alpha}\alpha\right] + \sum_{hj} \mu_{hj}\left[\alpha \frac{\partial (f_1 f_2)}{\partial \alpha} + \left\{\frac{\partial f_1(\alpha)}{\partial \alpha} f_2(\alpha) + \frac{\partial f_2(\alpha)}{\partial \alpha} f_1(\alpha)\right\}\alpha\right] + \left[\sum_k \Pi_k\left[\alpha^*_{ik} - \sum_m \alpha_{mk}\right] - \alpha \sum_k \Pi_k C\right]$$

Given the product differentiation rule we can write:

$$\frac{\partial f_1(\alpha)}{\partial \alpha} f_2(\alpha) + \frac{\partial f_2(\alpha)}{\partial \alpha} f_1(\alpha) = \frac{\partial (f_1 f_2)}{\partial \alpha}$$

and given Taylor's series we can write:

$$L(\alpha) \simeq L(0) + \alpha \frac{\partial L}{\partial \alpha}$$

Hence $$\alpha \frac{\partial L}{\partial \alpha} \simeq L(\alpha) - L(0)$$

and since $L(0) \simeq 0$, we can approximate the dual problem and simplify it as follows:

$$\text{Min. } 2L(\alpha) + \sum_{hj} \mu_{hj} 2\alpha \frac{\partial (f_1 f_2)}{\partial \alpha} + \sum_k \Pi_k \left[\alpha^*_{ik} - \sum_m \alpha_{mk} \right]$$

$$- \alpha \sum_k \Pi_k C \qquad (6.23.a)$$

$$\text{S.T.} \quad \sum_k \Pi_k C \geq \frac{\partial L}{\partial \alpha} + \sum_{hj} \mu_{hj} \frac{\partial (f_1 f_2)}{\partial \alpha} \qquad (6.23.b)$$

$$\Pi_k, \alpha_{mk} \geq 0 \quad \forall k,m \qquad (6.23.c)$$

B) The risk-minimizing approach: It has been suggested by Simon (139) and others that the firm "satisfices" rather than maximizes some objective(s). That is, business firms aim at "satisfactory" rates of profit rather than at a maximum figure. In fact, Eilon (44) claims that while the optimizing philosophy is the dominant one in the literature, satisficing prevails in practice. The implications of such an attitude on the planning of the firm's resource allocation is that the objectives are defined in terms of the need to meet levels of performance, such as capital expenditure limits, and attaining target of sales, market penetration and so on.

In view of the considerable data requirements necessary for a successful application of the optimizing approach in

the context of a department store, it seems reasonable that a store manager would be satisfied with achieving some degree of improvement in the present period over the previous period's performance. Hence, a formulation of the store manager's problem under such circumstances could be written as follows:[8]

$$\text{Min. } Z = \sum_{v=1}^{m} \sum_{w=1}^{m} \delta_v C_v \sigma_v \delta_{vw} \sigma_w C_w \delta_w \Big/ \left(\sum_m C_m \delta_m\right)^2 \qquad (6.24.a)$$

$$\text{S.T.} \quad \sum_m E(X'_m)\delta_m + \sum_{\substack{s=1 \\ n \neq s}}^{m} \mu_{sn} \delta_s \delta_n \Big/ \sum_m C_m \delta_m \geq e \qquad (6.24.b)$$

$$\sum_m C_m \delta_m \leq C_i \qquad (6.24.c)$$

$$\delta_m = 0,1 \qquad (6.24.d)$$

where each product line has a cost C_m and an associated expected internal return, which when divided by the total invesment, as in Eq. (6.24.b), yields a pure rate of return measure. The objective function, Eq. (6.24.a), is simply an expression for the store portfolio's variance, where δ_{vw} are the cross-elasticities between product lines v and w. The target rate of return required by the store manager is denoted by e, and it can bear some kind of relationship with the previous period's performance, e.g. $e_t = e_{t-1} + g$, where e_{t-1} represents the previous period's performance and g the growth rate the store manager hopes to achieve.

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8. The basic structure of this kind of formulation and a discussion of its methods of solution can be found in Findlay et al's (50a) analysis of the applications of the mean-variance theory to indivisible assets.

6.5 SUMMARY

In chapter 6 we have analysed the structure of a department store portfolio consisting of the products and services offered by the store. We have demonstrated how the main features of a department store can be incorporated into a portfolio model and then used as a framework in which the outcomes of operating decisions can be assessed.

In view of the fact that we assume the store is operating under uncertainty, and the existence of various studies of the corporate return's structure, our main emphasis was put on the identification and subsequent analysis of the economic forces which form the risk of an individual product line and that of the department store as a whole. We used the expanded mean-variance theorem to break down the individual line's risk to:

i) the investment proportion associated with that particular product line;

ii) the product line's variance of the rate of return; and

iii) the cross-elasticity of demand for the line in question and the remaining lines carried by the store.

An analysis of each of the above components of the store's risk revealed the factors which determine the possible reactions of a store manager to changes in the estimated performance, i.e. rate of return, risk and cross-elasticity, of a particular product line, where the response is measured by the amount of resources allocated to a line. We discussed the repercussions of various types of responses of the store manager on the attitude of the product line managers.

We then showed the potential impacts of an expansion policy with a changing cost structure can have on the variance of

the rate of return on an individual line. Depending on the magnitude of the increase in sales and cost reductions, we demonstrated that an increase in scale which generates a reduction in some or all unit costs can reduce as well as increase the variance, and possibly risk, associated with a particular product line.

The analysis of the last term in the risk expression of a product line's risk, the correlation coefficient, enabled us to incorporate into our model of the department store the concept of substitution and complementarity. Using Levy's study of the demand for assets under risk, we discussed the effects of a change in a line's correlation with another line carried by the store on the amount of resources allocated by the store manager to the line in question.

In the final section of this chapter we examined how portfolio theory could be applied to the store manager's problem, i.e. to the allocation of resources between product lines. We distinguished between two approaches: first, we formulated the problem for the case where the store manager seeks to find the optimal investment proportions, secondly, the problem was defined for the less ambitious case, namely, where the store manager sets himself a target and any solution which attains it is satisfactory, despite the fact that it may not be the optimum.

In this chapter we have avoided the third question which determines the store manager's portfolio, specifically, the size of the investment in the store. Since the ultimate objective of our analysis is to model the resource allocation process in a decentralized department store chain, we turn in chapter 7 to the analysis of the corporate portfolio, in the process of which we will show how the amount of resources invested by the corporate centre in each store is determined.

C H A P T E R 7

RESOURCE ALLOCATION IN A DECENTRALIZED DEPARTMENT STORE UNDER UNCERTAINTY

7.1 INTRODUCTION

In this chapter we take the concept of a corporate portfolio one step further by applying it to a two level organization, i.e. a decentralized firm. An examination of an example of such firms, in our case a decentralized department store chain whose organizational structure is graphically illustrated in Fig. (7.1), suggests that one can associate three different portfolios with it, namely:

i) the firm's securities are one of the assets which form the market portfolio of risky securities;

ii) the stores which form the company chain are the assets which constitute the risky element of the corporate portfolio; and

iii) the product lines operated by each store form the divisional portfolio.

The basic concept underlying the above classification is that the analogy drawn in chapter 6 between the investor in the capital market and the department store manager can be equally applied at the corporate level. In other words, in a department store chain the corporate planners and the store managers face a problem similar to that of the investor in the corporate securities market, specifically, that of dividing the limited resources at their disposal between the economic opportunities available to them. In the previous chapter we discussed in detail the issue of a store portfolio and we now turn our attention to the upper level of the organization.

Various properties of the department store chain, and the decentralized firm in general, make it a suitable candidate for the application of portfolio theory. The corporate centre can be seen as an investor faced with a set of mean-variance combinations, generated by the stores which form the organization,

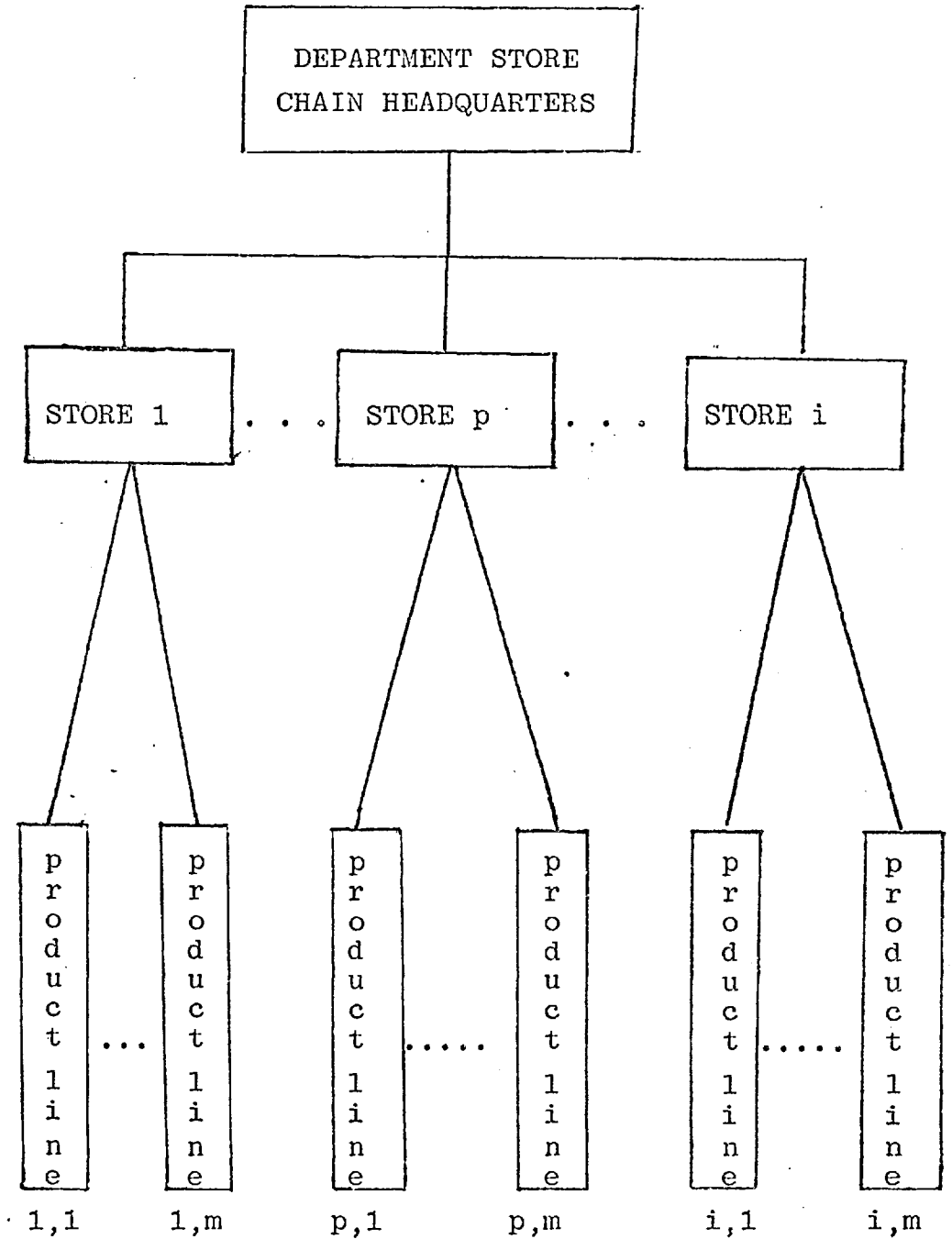


Fig. (7.1): Organizational structure of department store chain

and the need to decide upon the optimal portfolio's structure. The concept underlying our analysis is that the corporate centre's role is similar to that of a holding company, owner of several subsidiaries, whose planning requirements include determining the level of investment in each of the latter. The clear divisibility of a chain retail organization combined with the economic relationships between branches make the idea of a corporate portfolio a meaningful one.

Keeping this concept in mind, we focus our attention in this chapter on the planning of the resource allocation in a department store chain operating under uncertainty. We start by examining the process which takes place when the store managers are granted a large degree of autonomy in determining their portfolio, in section 7.2. Following this we discuss the case where the corporate centre reduces the freedom of the store managers and gets involved in the selection of the stores' policies, in section 7.3. In section 7.4 we discuss the case of behavioural externalities.

7.2 DECENTRALIZED RESOURCE ALLOCATION PROCEDURES

7.2.1 The Perfect Market Case

In a CAPM efficient market we can define the following relationships for firm j with i divisions each operating m product lines:

$$E(R_j) = R_F + \lambda^* \sum_i \left[\alpha_i (p_i - v_i)\rho(Q_i, R_M)\sqrt{\text{Var } Q_i / \sum\sum_{km} \alpha_{ikm} c_k}\right](1 + B_j/S_j) \qquad (7.1.a)$$

and

$$E(RS_i) = R_F + \lambda^* \sum_m \left[\alpha_{im}(p_{im} - v_{im})\rho(Q_{im}/R_j)\sqrt{\text{Var } Q_{im} / \sum_k \alpha_{ikm} c_k}\right] \qquad (7.1.b)$$

where:

α_i - proportion of firm j's total assets allocated to division i;
α_{im} - proportion of division i's total assets allocated to product line m;
p_i - average weighted unit price of the m lines carried by division i;
p_{im} - unit price of product line m in division i;
v_i - average weighted variable unit cost in division i;
v_{im} - unit variable cost of product line m in division i;
$\rho(Q_i, R_M)$ - correlation between the return on the market portfolio of risky securities and division i's sales;
$\rho(Q_{im}, R_M)$ - correlation between the return on the market portfolio of risky securities and the sales of product line m in division i;
α_{ikm} - number of units of resource type k allocated to product line m operating in division i;
c_k - unit price of resource k;
B_j - debt of firm j;
S_j - equity of firm j.

The assumptions underlying Eq. (7.1.a) and Eq. (7.1.b) are those discussed in detail in chapter 5, namely that the firm is incapable of any risk

reducing diversification which the investors in the capital market are not in a position to costlessly eliminate. Furthermore, the economic implications of the existence of a perfect market on a decentralized firm is that there are no divisional externalities that can be internalized in a central structure, hence there is no gain in the linkage of independent activities. In the following section we will show how the respective roles of the corporate centre and store manager are greatly modified as one moves from a perfect world to an imperfect one.

In section 6.4 we defined the three problems that face the department store manager and suggested that he was free to establish the asset base and optimal investment proportions, while leaving the question of the level of investment in the store unanswered. In this section we tackle the third stage of the store manager's problem and suggest that in a perfect market one can view the corporate centre's role as one which is confined to the final stage. In other words, we adopt the approach that, as long as the assumptions of a perfectly efficient market hold, the intervention of the centre in the divisional planning and resource allocation is limited to defining the amounts of the various resources which will be available to each division in the relevant period.

This last point has an obvious effect on the definition of the mathematical problem facing the corporate centre: If we assume all inputs used at the divisional level are either physically supplied or financed by the central unit, it clearly follows that the problems of determining the size of a store manager's portfolio and that of establishing the corporate holding of that particular store in its portfolio are none but one and the same. Consequently, we can treat the problem of resource allocation in a decentralized firm as that of determining the optimal investment proportions within the corporate portfolio.

However, before we turn our attention to the second and third elements of the corporate planner's problem a few words of clarification concerning the first one, i.e. the establishment

of the corporate asset base. In a manner similar to an investor in the securities market and the department store manager, the first problem facing the corporate planners is that of determining the assets which, when combined, will form the corporate portfolio. The economic interpretation of this problem, in the context of a department store, is basically, which stores to include in the firm during the planned period. Since the alternative to inclusion in the corporate chain is either closure or change of ownership, both of which can be interpreted as an exit of the firm from the industry, it is doubtful whether the option of not including one of the firm's stores does not contradict the concept of the short term. As a result, we can reasonably assume that the assumption of the short run implies that while the store manager can delete product lines if unprofitable, corporate planners cannot alter the number of stores operated by the firm. Consequently, the first problem when defining a portfolio of any kind, i.e. determining the asset base, does not exist at the corporate level in a single period planning horizon.

The resource allocation processes which we are about to discuss share the common feature, which Carleton, Kendall and Tandon (27) claim is typical to planning in decentralized firms, namely that the divisions, in our case - stores, are the originators of projects and their associated performance. The first resource allocation process is distinctive in that it allows the store managers a large degree of autonomy at the central unit's expense and as a result we name it the "decentralized" approach. We now proceed to analyse the decentralized approach whose structure is graphically illustrated in Fig. (7.2).

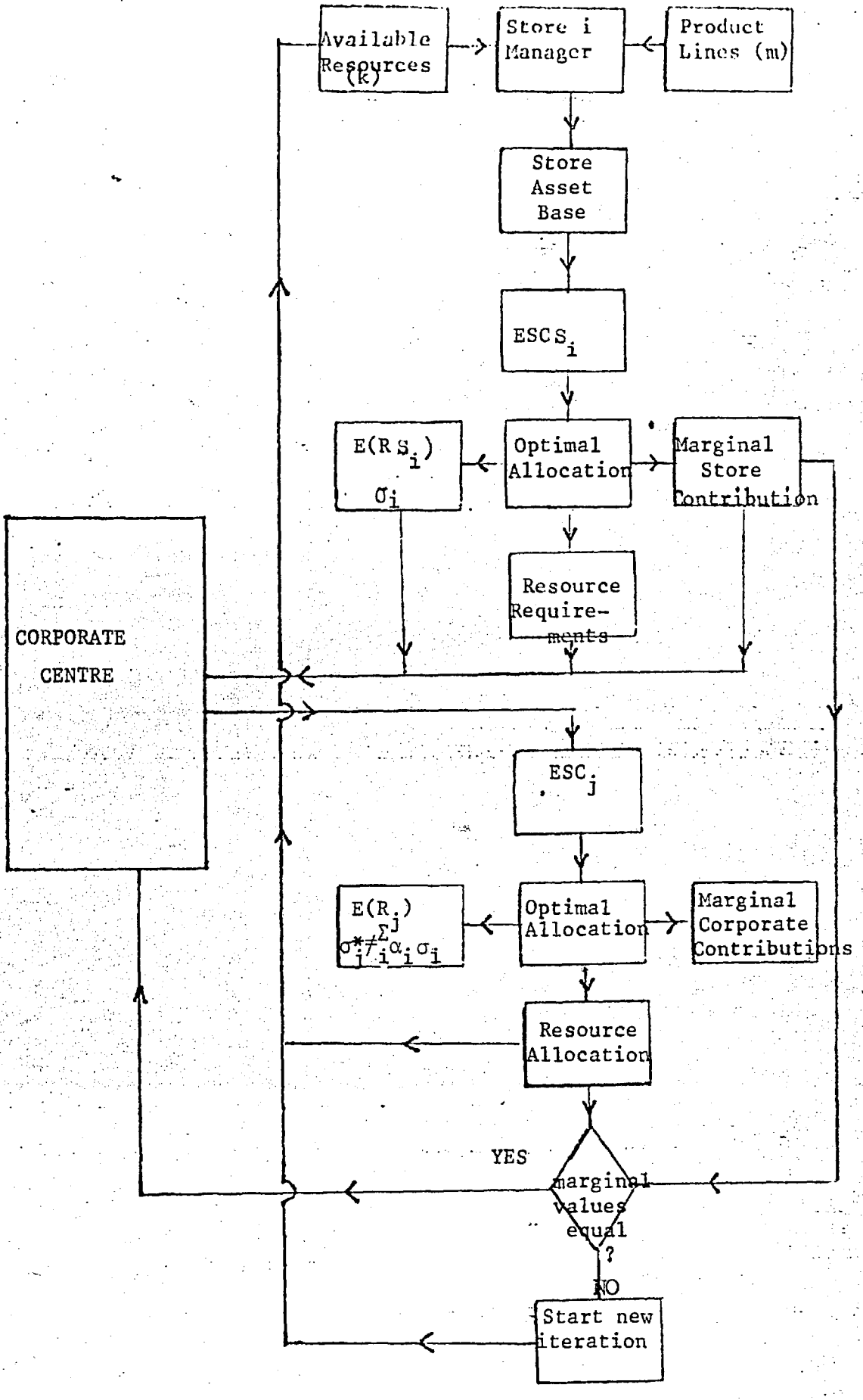


Fig. (7.2): Decentralized approach flow-chart

The process is initiated by the central unit which allocates in an arbitrary manner the total amount of resources at its disposal between the stores in the corporate chain. The store managers, upon receival of the amount of resources which at that stage are at their disposal, set out to construct their optimal portfolios. The first step in the store managers' problem is, as discussed in chapter 6, the determination of the appropriate asset bases. Once the store manager has selected those product lines which he judges worthy of consideration he can proceed, on the basis of past and present performance, to evaluate the expected performance of the product lines included in the store portfolio's asset base. Having determined the expected results of each product line, the store manager can generate his ESC_i and calculate the optimal resource allocation for his store by solving the familiar problem:

$$\text{Max. } Z = \sum_m \left[\frac{E(X_m) - \left\{ R_F + b_m \left[E(R_M) - R_F \right] \right\} \sum_k \alpha_{mk} \, c_k}{R_F} \right] \qquad (7.2.a)$$

$$\text{S.T.} \quad \sum_m \alpha_{mk} \leq \alpha^*_{ik} \quad \forall k \qquad (7.2.b)$$

$$\alpha_{mk} \geq 0 \quad \forall k, m \qquad (7.2.c)$$

and its dual:

$$\text{Min. } Z = \sum_k \alpha^*_{ik} \, W_k \qquad (7.3.a)$$

$$\text{S.T.} \quad W_k \geq 0 \quad \forall k \qquad (7.3.b)$$

$$W_k \geq \Omega_m c_k \qquad (7.3.c)$$

At this stage of the process the store managers inform the central planning unit of their portfolio's risk-return performance and the value of the dual variables. These dual variables can be economically interpreted as the stores' shadow prices and therefore can be used as bid prices for the global resources. The purpose of this information being to inform the central planning unit how store i's objective function value would change if the amount of resources allocated to it changed.

The central unit can then use the bidding prices sent by the stores to do the following marginal adjustments: the allocation of resources is increased for stores with higher bid prices, and simultaneously reduced for those stores with lower bid prices. The rationale underlying this system of reallocation of resources, i.e. using shadow prices as bidding prices, is the economists' desire to divide up a resource among competing usages - in this case department stores - so that a resource yields equal marginal return in all uses.

In other words, given the stores' bidding prices, the central planning unit needs to solve the following problem:

$$\text{Max. } Z = \sum_i \sum_k \Omega_i^* c_{ik} \alpha_{ik}^* \qquad (7.4.a)$$

$$\text{S.T.} \quad \sum_i \alpha_{ik}^* \leq \alpha f_{jk} \quad \forall k \qquad (7.4.b)$$

$$\alpha_{ik}^* \geq 0 \quad \forall i,k \qquad (7.4.c)$$

where $\Omega_i^* = -\left\{R_F + bs_i\left[E(R_M) + R_F\right]\right\}$, αf_{jk} is amount of resource k available to the planners of firm j, and the unit of resource prices c_{ik}'s are none other than the dual variables W_{ik} derived from the solution of the store managers' dual problem. Let us point out that the true solution to the problem defined by Eq. (7.4.a) - (7.4.c) is $\sum_i E(X_i)/R_F$ more than this due to our notation.

The dual of the centre's problem can be written as follows:

$$\text{Min.} \quad Z = \sum_k \alpha f_{jk} W_k \qquad (7.5.a)$$

$$\text{S.T.} \quad W_k \geq 0 \quad \forall k \qquad (7.5.b)$$

$$\sum_k \alpha^*_{ik} W_k \geq \sum_k \Omega^*_i c_{ik} \quad \forall i \qquad (7.5.c)$$

where αf_{jk} is the constraint on the amount of resource k available to the central unit, and the α^*_{ik}'s generated by solving the central unit's problem are none other than the α^*_{ik}'s used as constraints in the store managers' problem.

The values of the dual variables of the central unit's problem can be used as guidelines in the event of the firm considering the increase of the amounts of resources at its disposal, i.e., if faced with the problem of selecting which type of resource to acquire in the case of additional income becoming available the central unit's planners could use the marginal return of each resource (as reflected by the corresponding dual variable) to indicate the best form of expansion.

This iterative process can be continued until all bid prices are equalized and therefore there is no gain to be achieved by shifting resources from one store to another. We can summarize the iterative procedure described above as follows:

STEP 1: The central unit allocates the global resources between the stores in an arbitrary manner, i.e., it sends to each store manager his associated α^*_{ik}, and an opening set of unit costs of resources, c_{ik}'s, whose purpose is purely to initiate the resource allocation process.

STEP 2: Each of the i department store managers responds to the α^*_{ik} and c_k's sent to him by the chain's centre by solving his internal resource allocation problem subject to the above data.

STEP 3: The store managers then inform the corporate centre of their store's performance as defined by its risk-return pair and, more important for the purpose of the resource allocation procedure, the bidding prices as defined by the W_k's generated by each division.

STEP 4: The corporate centre proceeds then to substitute the initial unit costs of each of the k resources with the stores' bid prices and then solves its resource allocation problem. The amount of resource k allocated to each store is sent to the corresponding store manager.

STEP 5: In view of the amount of resources allocated to him in the previous step, and using the dual variables' values as unit costs for each type of resource, the store managers resolve their respective internal resource allocation problems. At the end of this step they each inform the corporate centre of their latest set of dual variables and risk-return performance.

STEP 6: The corporate centre examines the sets of dual variables generated by the store: if there exists any potential gain to be achieved by shifting resources from one store to another, i.e. a difference between marginal returns, STEP 4 is reinitiated; if, on the other hand, no potential gains are available, i.e. marginal returns are equalized, optimum has been reached and the process is brought to a halt.

This iterative procedure for resource allocation under uncertainty in a department store chain has characteristics which, according to Freeland and Moore (55), classify it as a resource-directive procedure. The major advantages of resource-directive methods are two: first, their economically appealing adjustment process, i.e. at each iteration the corporate headquarters resources are reallocated to the advantage of the high bidders until equality of bids across all stores exists, and, secondly, the fact that at each iteration's end the existing solution does not violate any constraint and consequently can be implemented. In other words, if for any reason, for example - prohibitive costs, the corporate planners do not pursue the iterative procedure until an optimal solution is found, there is no theoretical obstacle to the current solution being carried out.

Despite these attractive properties, resource-directive procedures may prove to be problematic and consequently fail to equalize bid prices. Specifically, Freeland and Moore point out that, the main danger when using a resource-directive algorithm lies in the danger of a degenerate solution. Such a situation can arise when the optimal bid price vector is not unique, i.e. multiple bid prices exist, and can create problems in the convergence of the iterative process. In view of the fact that multiple bid prices, and consequently degeneracy, are not scarce, Freeland and Moore's study focused on the circumstances under which unique bid prices, i.e. non-degeneracy, exist in resource-directive procedures. The results of the above study reveal that, given the properties of resource directive methods, it is necessary to adopt the assumption that only those cases where the solutions to the store manager's problem are non-degenerate are considered. Failing to do that would require devising more sophisticated rules for the corporate centre, such as those developed by Geoffrion (57), with the penalty of the increase in the process' complexity and data

requirements, resulting in a greater problem in implementation.

Let us point out that, in terms of the portfolio vernacular, the process which takes place at each iteration of the resource-directive procedure described above can be defined as the store managers' performance being the basis for construction of the corporate production possibilities curve, denoted ESC_j. In other words, each store's risk-return combination is an asset in the corporate asset base and consequently, the set of dominating options available to the centre can be defined by its efficient set curve. In the following sections of this chapter we shall consider cases where each store informs the central unit about more than one risk-return combination, i.e. more than one policy is considered by the corporate planners.

7.2.2 The case of a changing cost structure

In our analysis in subsection 7.2.1 we made the assumption of a perfectly competitive market. One of the implications of the above assumption was that the firm had no influence on the price of the inputs it purchases. In this subsection we turn our attention to the case where this assumption is relaxed to allow the impact of bulk buying by the corporate centre to be incorporated into our model.

The assumption of the existence of a dependency between the quantities purchased by the corporate centre and the unit costs it pays, a realistic one - as discussed in chapter 5, can be written as follows:

$$\alpha^*_{ik}\, c_{ik} \equiv f_{ik}(\alpha^*_{ik})$$

and when incorporated into the formulation of the corporate

centre's problem gives the following:

$$\text{Max.} \quad Z = \sum_i \sum_k \Omega_i^* f_{ik}(\alpha_{ik}^*) \qquad (7.6.a)$$

$$\text{S.T.} \quad \sum_i \alpha_{ik}^* \leq \alpha f_{jk} \quad \forall k \qquad (7.6.b)$$

$$\alpha_{ik}^* \geq 0 \quad \forall i,k \qquad (7.6.c)$$

where Ω_i^* is defined as in the previous subsection and, similarly, the true solution to the problem is $\sum_i E(X_i)/R_F$ greater.

Let us now simplify the notation by denoting the objective function, i.e. Eq. (7.6.a), $g(\alpha)$ and, using Balinsky and Baumol's (13) method, derive the dual of the above problem, hence:

$$\text{Min.} \quad h(\alpha,v) = g(\alpha) + \sum_k v_k^* \left[\alpha f_{jk} - \sum_i \alpha_{ik}^*\right] - \sum_i \sum_k \left[v_k^* \frac{\partial \sum_i \alpha_{ik}^*}{\partial \alpha_{ik}^*} - \frac{\partial g}{\partial \alpha_{ik}^*}\right] \qquad (7.7.a)$$

$$\text{S.T.} \quad v_k^* \geq \partial g/\partial \alpha_{ik}^* \quad \forall k \qquad (7.7.b)$$

$$v_k^* \geq 0 \qquad (7.7.c)$$

If we now incorporate the concepts of substitution-complementarity relationships and changing cost structure, we can write the corporate centre's problem as follows:

$$\text{Max.} \quad Z = \sum_i \sum_k \Omega_i^* f_{ik}(\alpha_{ik}^*) + \sum_{hl} \mu_{hl} e_1(\alpha) e_2(\alpha) \qquad (7.8.a)$$

$$\text{S.T.} \quad \sum_i \alpha_{ik}^* \leq \alpha f_{jk} \quad \forall k \qquad (7.8.b)$$

$$\alpha_{ik}^* \geq 0 \quad \forall i,k \qquad (7.8.c)$$

where

$$e_1 = \frac{\sum_k \alpha_{hk}\, c_k}{\sum_k \alpha f_{jk}\, c_k} \quad \text{and} \quad e_2 = \frac{\sum_k \alpha_{ek}\, c_k}{\sum_k \alpha f_{jk}\, c_k}$$

which, if we denote the first part of the objective function, i.e. Eq. (7.8.a), $G(\alpha)$, has the following dual problem:

$$\text{Min.} \quad G(\alpha) + \sum_{hl} \mu_{hl}\, e_1(\alpha)\, e_2(\alpha) + \sum_k v_k \left[\alpha f_{jk} - \sum_i \alpha^*_{ik}\right]$$

$$- \left[\sum_k v^*_k C - \frac{\partial G}{\partial \alpha} - \sum_{hl} \mu_{hl} \left[\frac{\partial e_1(\alpha)}{\partial \alpha} e_2(\alpha) + \frac{\partial e_2(\alpha)}{\partial \alpha} e_1(\alpha)\right]\right] \alpha \qquad (7.9.a)$$

$$\text{S.T.} \quad \sum_k v^*_k C \geq \frac{\partial G}{\partial \alpha} + \sum_{hl} \mu_{hl} \left[\frac{\partial e_1(\alpha)}{\partial \alpha} e_2(\alpha) + \frac{\partial e_2(\alpha)}{\partial \alpha} e_1(\alpha)\right] \qquad (7.9.b)$$

$$v^*_k,\ \alpha^*_{ik} \geq 0 \qquad \forall i,k \qquad (7.9.c)$$

As in the previous case, in chapter 6, we can apply the product differentiation rule, hence:

$$\frac{\partial e_1(\alpha)}{\partial \alpha} e_2(\alpha) + \frac{\partial e_2(\alpha)}{\partial \alpha} e_1(\alpha) = \frac{\partial (e_1 e_2)}{\partial \alpha}$$

and Taylor's series:

$$G(\alpha) \simeq G(0) + \alpha \frac{\partial G}{\partial \alpha}$$

hence:

$$\alpha \frac{\partial G}{\partial \alpha} \simeq G(\alpha) - G(0)$$

However, in this case, as opposed to that analysed in chapter 6, one cannot safely assume that $G(0) \simeq 0$ due to the non-linearity of the function in G. As a result, the best we can do to simplify the dual problem facing the corporate centre is:

$$\text{Min.} \quad \left[2\,G(\alpha) - G(0) \right] + \sum_{hl} \mu_{hl}\, 2\alpha \frac{\partial(e_1 e_2)}{\partial \alpha} + \left[\sum_k v_k^* \left[\alpha f_{jk} - \sum_i \alpha_{ik}^* \right] - \alpha \sum_k v_k^* C \right] \qquad (7.10.a)$$

$$\text{S.T.} \quad \sum_k v_k^* C \geq \frac{\partial G}{\partial \alpha} + \sum_{hl} \mu_{hl} \frac{\partial(e_1 e_2)}{\partial \alpha} \qquad (7.10.b)$$

$$V_k^*, \alpha_{ik}^* \geq 0 \qquad \forall i,k \qquad (7.10.c)$$

7.3 INTERVENTIONIST METHODS OF RESOURCE ALLOCATION

One of the main limitations of the resource-directive method discussed in the previous section lies in its fundamental property, namely, that it leaves all decisions as to the optimal resource allocation within a department store within the local manager's hands. Despite the appeal of such a system to decentralization purists, it is doubtful how realistic such an attitude is given the modern business environment. In this section, we present the case for a model of the resource allocation in a decentralized department store chain which allows for a greater involvement by the centre in the construction of the store managers' portfolios.

7.3.1 Behavioural patterns in decentralized firms

Studies which have examined the behavioural practices typical to decentralized firms in general, and chain retail stores in particular, tend to support the suggestion of a far more extensive involvement of the corporate centre in divisional planning than incorporated in our previous models.

For example, Holdern (75) claims that pricing is generally done at the corporate centre. Specifically, he suggests that retail chains usually follow the pattern of establishing prices at their headquarters and permitting the managers of individual stores to decide the particular amounts of each good they will stock given the above set of prices.

Pfouts (116) takes this last point further and argues that it is simplistic to assume that the store manager is totally free to use the limited space at his disposal as he wishes. In fact it is typical for a store manager to be required by the corporate centre to stock certain items though it might be more profitable from his point of view, to use the space for other products. As a result a certain amount of space will be pre-

empted by these required goods. Pfouts provides examples of such intervention in chain retail stores: chain drugstores are usually required to stock items such as hydrogen peroxide and epsom salts while a supermarket must carry salt and baking powder.

Carleton, Kendall and Tandon (27) note the conflict which emerges when capital funds are the scarce resource to be allocated, specifically, between the behavioural and informational benefits of decentralization, on the one hand, and the advantages of maintaining some corporate control over such a strategic activity, on the other. In other words, given the strategic importance of capital investments, corporate managers are reluctant to decentralize totally in this area. Consequently, they typically adopt direct capital rationing procedures as administrative safeguards.

7.3.2 The economics of intervention

In view of the fact that the mere existence of typical behavioural patterns in a firm are hardly a justification for their perpetuation, despite both their importance and tendency to become permanent, we now turn to examine the conditions which make an intervention by the corporate centre in the divisional planning economically sound.

Fowler and Rorke (53) provide, in their analysis of projects which involve external effects, and philanthropic investments in particular, a framework which we can use to stress our point. Using their model of the effects of a firm devoting money to charity, we can denote the value of store i, which is instructed by the corporate planners to carry a line which generates no direct contributions to its cash flows, yet increases the income of the remaining branches of the corporate chain, as follows:

$$P^*_i = \frac{1}{R_F}\left[E(c_i) - H\ \mathrm{Cov}(c_i, c_M + \Delta c_M)\right] \qquad (7.11)$$

where:

c_i - the cash flow generated by store i both prior to and after the acceptance of the new product line in the store manager's portfolio,

c_M - the cash flow generated by the market portfolio of risky securities prior to the inclusion of the new line in store i; and

Δc_M - the incremental cash flows generated as a result of store i including the new product line in its range of products.

The resulting change in the store's price is:

$$\Delta P_i = -\frac{H \, Cov(c_i, \Delta c_M)}{R_F} \qquad (7.12)$$

Even if we assume that Cov $(c_i, c_M) = 0$, and therefore $\Delta P_i = 0$, this does not necessarily imply that the centre's intervention was pointless since the price of the other stores becomes:

$$\Delta P_j = \frac{1}{R_F}\left[E(\Delta c_j) + H \, Cov(c_j, \Delta c_M) - H \, Cov(\Delta c_j, c_M) - H \, Cov(\Delta c_j, c_M)\right] \qquad (7.13)$$

within this framework based on Fowler and Rorke's analysis it is clearly seen why the individual store manager, having nothing to gain by including a particular line in his store, will not do so unless the centre intervened and forced him, one way or the other, to comply. The corporate centre is motivated by the desire to maximize the net aggregate change in the value of all the corporate stores, hence it will have an economical justification for forcing a store manager to commit a proportion of his store's resources to a new line, if the resulting increase in the value of the chain's assets is greater than the required investment. Hence the decision for the corporate centre whether to intervene

or not can be defined as:

$$\Delta Q = \sum_{\substack{j=1 \\ j \neq i}}^{n} \Delta P_j - I > 0 \qquad (7.14)$$

where:

ΔQ - is the change in the value of all the stores which form the corporate portfolio; and

I - the cost of the lost opportunity in store i due to the resources committed to the new line.

We can therefore advise central intervention in a store's resource allocation if as a result $\Delta Q > 0$.

We can further analyse the case for the centre's intervention in the stores' planning within the framework of the expanded mean-variance theorem. Let us consider, for example, the case where the centre is considering intervention in an environment of economies of scale. This intervention can be done either directly by instructing the relevant store manager to include product line m' among the m lines it carries, or alternatively, indirectly by sending him, in a price-directive resource allocation method, the discounted unit cost of a unit of input rather than the full price. By having the store manager use the discounted unit price the corporate centre reduces the probability of a conflict with the store manager since, when the latter uses the discounted unit costs when calculating a product line's performance it probably will use a lower discount rate and therefore increase the chance of the store manager including the product line in question on his portfolio and the corporate centre thus achieving the minimum order necessary for a quantity discount, without any need to resort to direct intervention which can result in conflict.

We can express formally the change in the value of the store's excess rate of return, denoted ΔXR_i, as a result of replacing

the full unit costs, which would be used either if the store was not part of a change or if it operated in a perfectly competitive market, by the reduced unit costs, as follows:

$$\Delta XR_i = \Big[R_i - R_F - \lambda * \sum_m \Big[\rho(Q'_m(p_m - v_m)/\sum\sum \alpha'_{mk} c'_k R_M)$$

$$\sqrt{\text{Var } Q'_m(p_m - v'_m)/\sum\sum \alpha'_{mk} c'_k}\Big]\Big] - \Big[R_i - R_F -$$

$$\lambda * \sum_m \Big[\rho(\alpha_m(p_m - v_m)\rho(Q_m, R_M)\sqrt{\text{Var } Q_m/\sum\sum \alpha_{mk} c_k}\Big]\Big]$$

$$= \lambda * \Big[\sum_m \Big[\rho(Q'_m(p_m - v'_m)/ \sum\sum \alpha'_{mk} c'_k, R_M)\sqrt{\text{Var } Q'_m(P_m - v'_m)}$$

$$/ \sum\sum \alpha'_{mk} c'_k\Big] - \sum_m \Big[(\alpha_m(p_m - v_m)\rho(Q_m, R_M)$$

$$\sqrt{\text{Var } Q_m / \sum\sum \alpha_{mk} c_k}\Big]\Big] \qquad (7.15)$$

Thus, if we assume that the aggregate effect on the store's risk of the change in cost structure is such that the former is reduced, the divisional discount rate will be consequently reduced and the store manager will have a commercially justifiable reason for including the line required by the corporate centre in his portfolio.

It is conceivable that the centre will impose its will on a particular store manager who judges it not worthwhile stocking the goods suggested by the centre. Such a situation can occur when, despite the use of discounted unit costs, a certain product line's performance falls below required by the store manager for it to be included in his portfolio. If the centre wishes to assess the option of ignoring the above decision and

instructing the store manager to carry the line in question, it must be certain that the contribution of the economies of scale throughout the rest of the firm is large enough to cover any instances of local losses. We can define the condition where such an intervention by the centre is economically sound as follows:

$$\Delta XR_j = \left[R_j - R_F - \lambda^* \sum_i \sum_m \left[\rho(Q'_m(p_m - v'_m)/\Sigma\Sigma\alpha'_{mk} c'_k, R_M)\right.\right.$$

$$\left.\left.\sqrt{\text{Var } Q'_m(p_m - v'_m)/ \sum_m \sum_k \alpha'_{mk} c'_k}\right]\right] - \left[R_j - R_F - \lambda^* \sum_i \sum_m\right.$$

$$\left.\left[(\alpha_m(p_m - v_m)\rho(Q_m, R_M) \sqrt{\text{Var } Q_m / \sum_k \sum_m \alpha_{mk} c_k}\right]\right]$$

$$= \lambda^* \left[\left[\sum_i \sum_m (\rho(Q'_m(p_m - v'_m)/ \sum_m \sum_k \alpha'_{mk} c'_k) \sqrt{\text{Var } Q'_m}\right.\right.$$

$$\left.\left.\overline{(p_m - v'_m)/ \sum_m \sum_k \alpha'_{mk} c'_k}\right] - \sum_i \sum_m \left[(\alpha_m(p_m - v_m)\rho(Q_m, R_M)\right.\right.$$

$$\left.\left.\sqrt{\text{Var } Q_m / \sum_k \sum_m \alpha_{mk} c_k}\right]\right] \qquad (7.16)$$

Therefore, if $\Delta XR_j > 0$, i.e. the net aggregate effect on firm j of the economies of scale is positive, this implies that the poistive contributions of the economies of scale is sufficiently large to offset any negative ones that may exist in certain stores, where otherwise consumers' demand does not justify inclusion of the particular product line in quantities which generate savings.

Finally, let us point out that the concept of required goods is not limited to an environment of changing cost structures.

Prerequisite goods may be necessary conditions to profit maximization in any type of cost environment because a store which does not carry certain traditional goods may fail to attract customers in the same manner as one which does.

7.3.3 Interventionist models

Before we attempt to formulate the problem facing the corporate centre which is involved in the stores' planning it is useful to highlight one of the implications of the models discussed in section 7.2. Specifically, in view of the fact that the determination of the store manager's portfolio was mainly in the local managers' hands, and the fact that the corporate centre's sole role was that of determining the amount of resources allocated to each store, the corporate centre was in no position to select the range of goods and services offered by the stores - the only direct influence the centre had was on the size of the investment. As a result, any movement of the firm along its ESC reflects a change in the level of the corporate investment in the stores - and their respective policies. If the increased level of investment in a store does not alter the proportion of its resources allocated to each of the product lines, i.e. the store manager opts for a proportionate change in the investment in all product lines, this implies that the store's policy, as defined by the range of goods it carries, has not altered as a result of the change in the level of the aggregate investment by the corporate headquarters in the store. Seen from the corporate centre's point of view, it has merely changed its holding of the old policy adopted by the store manager. We can graphically illustrate this point in Fig. (7.3) where an increased investment by the centre enables the store manager to move from his original production possibilities, as reflected by ESC_{i1}, to a higher mean-variance combination, represented by ESC_{i2}, thus leaving the store's policy unchanged, a fact reflected by the store's movement along the "policy line", denoted AA.

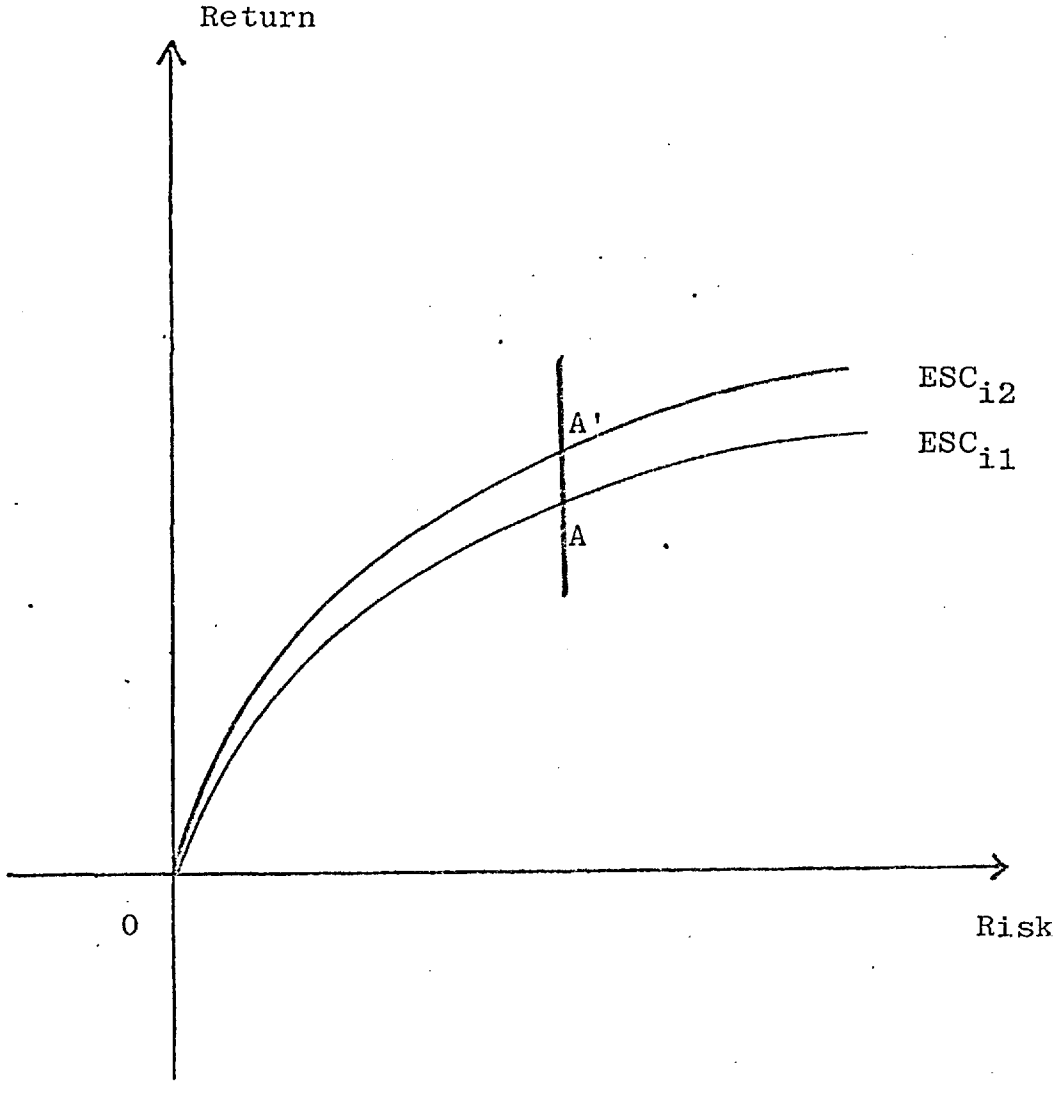


Fig. (7.3): Resource Expansion Effect

Having presented the empirical evidence as to the typical behaviour in decentralized firms and the economic justification for corporate involvement under certain circumstances, we now turn our attention to two models of the resource allocation process in a decentralized department store chain, each of which allows for a different degree of central involvement in local planning.

i) The "mixed" approach: We start with a model in which the store manager is left with a considerable degree of autonomy. Specifically, he is free to determine the optimal investment proportions at each stage in the iterative procedure but not to the same extent as in our previous model: we now increase the set of data that each store manager is required to provide the centre with at the end of each iteration from one mean-variance combination to several ones. The rationale underlying such an approach is that the corporate centre may well find it worthwhile to instruct the store managers to consider not merely the question of which policy is the optimal one for each store but other types of policies as well. Such an action could be justified on the basis that the corporate centre may desire to present a consistent, and therefore coherent, image throughout all its chain of stores. In other words, the centre, wanting to aim at the same market segment(s) throughout the chain, may wish to instruct the store managers to assess which is the optimal policy for capturing the alternate segments. This approach would imply that in practice the manager of store i would have to generate s_i, where s is the number of alternative marketing targets, mutually exclusive policies and associated outcomes. These would be sent to the corporate centre which, on the basis of the resulting ESC_j, would select the best overall policy, i.e. would select the one marketing target which if common to all the chain's stores promises the best results.

For the purpose of formulating the corporate problem we can use the framework developed by Findlay et al (50a) for the

selection of indivisible projects. Let us define the corporate objective function as one which minimizes the corporate risk for a target rate of return. we can formulate the problem facing the corporate planners as follows:

$$\text{Min. } Z = \sum_i \sum_j \sum_s \zeta_i^s \, C_i^s \rho_{ij}^s \, C_j^s \zeta_j^s \, / \, (\sum_{j=1}^{n} C_j^s \, \zeta_j^s)^2 \qquad (7.17.a)$$

$$\text{S.T.} \quad \sum_i \sum_s \zeta_i^s C_i^s \sum_i X_i^s / \sum_i \sum_s C_i^s \zeta_i^s \geq e \qquad (7.17.b)$$

$$\sum_i \sum_s \zeta_i^s C_i^s \geq C_j \qquad (7.17.c)$$

$$\zeta_i^s = 0,1 \qquad \forall i,s \qquad (7.17.d)$$

$$\sum_s \zeta_i^s = 1 \ \forall i \qquad (7.17.e)$$

$$s_i = s_j \qquad \forall i,j \qquad (7.17.f)$$

where: each store generates s alternative policies (which obviously are mutually exclusive), $\zeta_i^s = 0(1)$ represents a decision variable denoting rejection (acceptance) of policy s at store i, Eq. (7.17.b) represents the target rate of return, X_i^s is the return generated by policy s when implemented at store i, Eq. (7.17.c) the centre's resource constraints, Eq. (7.17.d) and Eq. (7.17.e) guarantee that only one policy will be chosen at any given store, and finally, Eq. (7.17.f) implies that the same policy must be implemented at each and every store.

The main advantage of the above formulation of the corporate planners' problem is that it enables the store managers to continue selecting optimal resource allocations on the basis of purely local information flows, by that we imply that, given a marketing target the store managers are free to select the method of achieving it, and hence, while increasing the

corporate planners' influence on the choice of the stores' policies, does leave a considerable degree of autonomy in the store managers' hands.

ii. Direct intervention: If we attempt to assess the impact of the centre's intervention in terms of the stages of construction of the store manager's portfolio, the following picture emerges: In the totally decentralized resource-directive procedure analysed in section 7.2 the store manager was totally free in the first two stages of the selection of his portfolio. Specifically, there was no intervention by the centre in the:

i) determination of the store's asset base; and

ii) selection of the optimal investment policy.

and the sole influence the centre was guaranteed to have was over the level of investment in the store as a whole.

When we moved to discuss the mixed approach to resource allocation in a decentralized department store chain we were implicitly reducing the store manager's autonomy down to the first stage of the portfolio selection. In other words, in fact the centre's intervention expanded to include both the second and third stages of the local planning.

If we now proceed to discuss the case where the centre adopts a direct and full intervention, the implications in terms of the construction of the store manager's portfolio should be clear: we now consider those cases where the centre is involved in all three stages of the store manager's portfolio selection, from the selection of the asset base up to the determination of the total investment in the store.

Let us consider the case where the centre instructs the store manager to carry certain lines by requiring that certain amounts of resources be allocated in advance to the lines in question. Such an instruction can be incorporated into our model by replacing the non-negativity constraints in the store manager's problem with specific requirements which represent the amounts dictated by the corporate centre. The store manager's problem therefore becomes:

$$\text{Max. } Z = \sum_m \left\{ \frac{E(X_m) - \left\{R_F + b_m \left[E(R_M) - R_F\right]\right\} \sum_k \alpha_{mk} c_k}{R_F} \right\} \tag{7.18.a}$$

$$\text{S.T.} \quad \sum_m \alpha_{mk} \leq \alpha^*_{ik} \quad \forall k \tag{7.18.b}$$

$$\alpha_{mk} \geq \hat{\alpha}_{mk} \tag{7.18.c}$$

where $\hat{\alpha}_{mk}$ is the amount of resource type k that the central unit wants the manager of store i to allocate to product line m.

If we now return to the simplified form of notation of the store manager's problem presented in the previous chapter, and in addition define:

$$\beta_{mk} = \alpha_{mk} - \hat{\alpha}_{mk}$$

we can rewrite the store manager's problem as follows:

$$\text{Max. } Z = \sum_m \sum_k \Omega_m c_k (\beta_{mk} + \hat{\alpha}_{mk}) \tag{7.19.a}$$

$$\text{S.T.} \quad \sum_m (\beta_{mk} + \hat{\alpha}_{mk}) \leq \alpha^*_{ik} \quad \forall k \tag{7.19.b}$$

$$\beta_{mk} \geq 0 \quad \forall m,k \tag{7.19.c}$$

Since there is no fundamental change in the nature of the store manager's problem following the incorporation of lower bounds on the resource allocation, and no change in the corporate centre's problem, one can use a resource-directive resource allocation procedure similar to the one suggested in the previous section.

Put in terms of the construction of the store's production possibilities curve, $ESCS_i$, the shape of the store's technology will differ from that available prior to the change in the store manager's liberty to allocate freely the resources at his disposal. Strictly speaking, the above comment excludes the remote possibility that given the freedom of choice the store manager would allocate the store's resources in the manner identical to that which is imposed upon him, in which case the centre's intervention becomes superfluous. In Fig. (7.4) we graphically illustrate the kind of impact an intervention of the kind described above can have on the production possibilities of a store: the intervention-free set of efficient risk-return combinations, denoted $ESCS_{i1}$, represents higher rates of returns for the same levels of risk when compared with the post-intervention efficient set, denoted $ESCS_{i2}$.

In appendix II of this work we present a numerical example of the effects of an intervention by the centre in the store's resource allocation at the lowest level. Specifically, we calculate the cost, measured in terms of the drop in the store's performance, as the central planning unit imposes a set of constraints which specify the precise amounts of resources the store manager is required to allocate to a particular product line (under the assumptions that these differ from the amounts of resources allocated to the line in question when the store manager was granted total freedom to allocate resources within his store).

Before we turn to the last section of this chapter, which deals with the treatment of behavioural externalities in a mean-variance world, let us summarize the main points of this and the previous sections.

Generally speaking, the objective of the two sections was to examine, under different types of environment, some of the managerial problems in a retail chain. The dominant problem throughout our analysis was that of resource allocation in a decentralized department store chain. We first suggested an algorithm for a resource-directive iterative procedure for resource allocation in the case of a perfect market.

Subsequently, we examined the case of a changing cost structure and incorporated into it the concept of substitution-complementarity relationships. However, since we recognized the unrealistically decentralized nature of the assumptions underlying our first model, we turned to examine the same problem under two different sets of assumptions, both of which share the common trait of a greater degree of involvement by the centre in the individual stores' planning.

Having established, through our discussions with the managers of the retail chain with which we cooperated and evidence published in the literature, the typical centre-store relationships in the retail industry, we analysed the economic rationale for such an intervention. Two slightly different theoretical frameworks were used to determine the conditions necessary for an economically justifiable intervention. The first model, drawn from a paper by Fowler and Rorke (53), examined

the acceptance-rejection decision rules concerning a project which involves external effects. By examining the change in the individual stores' prices as a result of the acceptance of the new project and the aggregate change in the chain the decision rule reached was that an intervention which yield a positive aggregate change in the stores' prices is economically sound in spite of the existence of certain local, i.e. at the store level, instances of a drop in the store price as a result of the acceptance of the new project.

The alternative method we suggested analyses the same problem not through the price mechanism but through the excess rate of return generated by the stores and the chain as a whole. As one would expect, the decision rule for the acceptance-rejection of a project with external effects depends on the aggregate change in the chain's excess rate of return following the acceptance of the new project; a positive net change in the chain's excess rate of return implies that the project should be accepted on economic grounds and vice versa.

The first interventionist model limits the centre's intervention to the second and third stages of the local planning, i.e. the selection of the optimal investment policy and the determination of the total investment in the store. The third, and to our opinion the most realistic model, allowed for an intervention by the corporate centre in the stores' selection of their asset bases, i.e. in all three stages of the construction of a store's portfolio. On the basis of this interventionist model we presented, in form of an appendix, a numerical example which served to highlight the cost at the store level of this

new type of involvement by the centre in the store planning process.

Let us recapitulate at this point : the discussion presented in chapter 7 up to this stage gradually develops a serie of three types of models of the resource allocation process in a decentralized retail chain. moving from a purely decentralized system to one which leaves relatively little autonomy in the store managers' hands. The combination of the models developed in chapters 5, 6 and 7 provides a framework in which a large number of managerial problems typical to the retail chain environment can be examined. Specifically, we discussed the problem of resource allocation in an individual department store, resource allocation in a decentralized retail chain, the effect of shifting resources from one product line in a store to another in the same store, the question of normality-inferiority of a particular product line under uncertainty as opposed to its properties in a deterministic environment, the topic of a change in the efficiency of the use of a particular type of resource in a store, and the concept of complementarity-substitution.

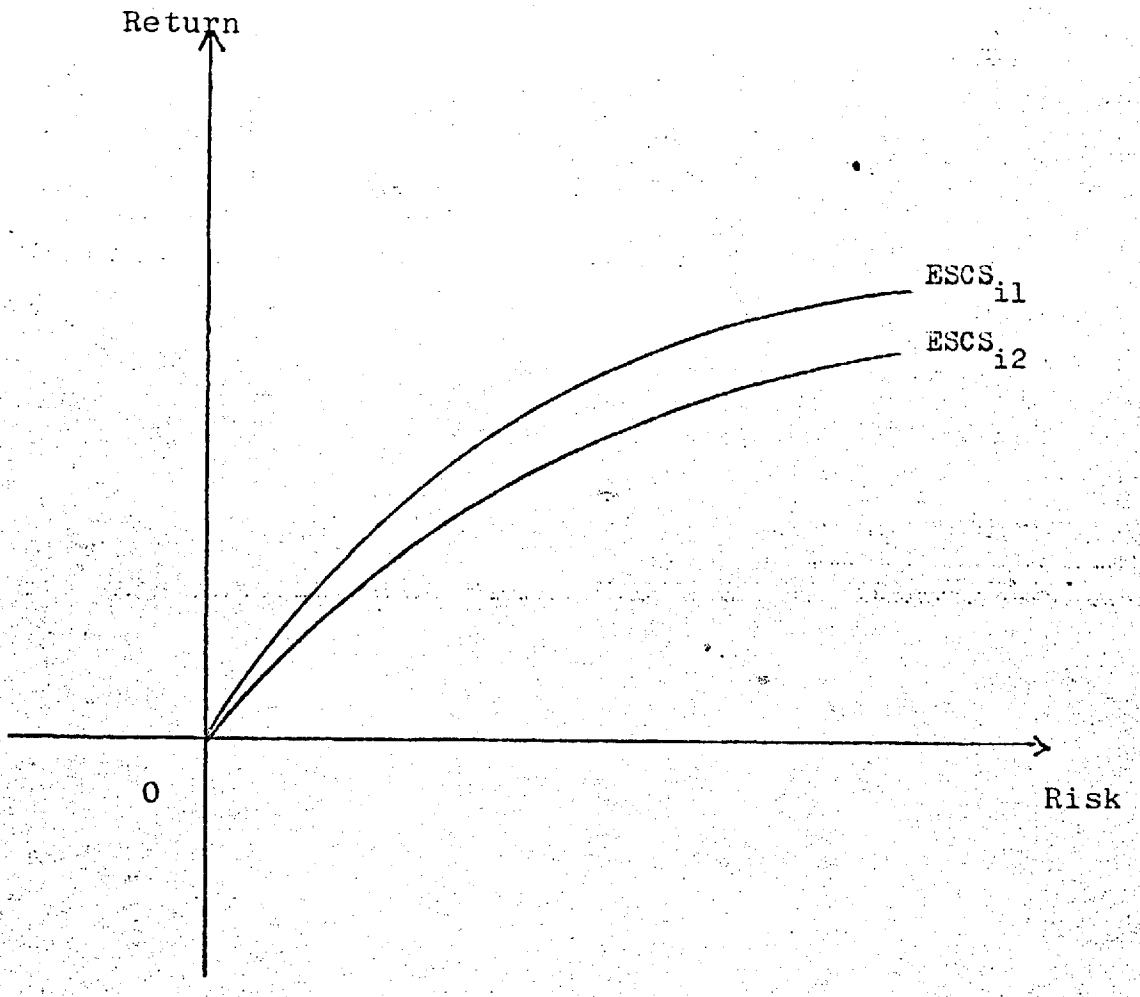


Fig. (7.4): The effects of resource commitment

7.4 BEHAVIOURAL EXTERNALITIES

The effects of economic externalities in decentralized organizations have been discussed throughout the various chapters of this work. In this section we turn our attention to the concept of behavioural externality and its impact on the decision making process in a decentralized organization in general, and in department store chains in particular.

Ruefli (127) defines a behavioural externality in a decentralized organization as the case when the goal seeking behaviour of one management unit, say unit k, is affected by the goal seeking behaviour of another management unit, say unit 1. This concept is expressed formally by incorporating parameters related to unit 1's performance in the objective function for management unit k, hence:

$$\text{Min.} \quad Z_k \left(Y_k^{\pm} , Y_k^{-}, Y_1^{+}, Y_1^{-} \right)$$

Where:

Y_k^{+} - is an $(m_k \times 1)$ vector of positive deviations from unit k's goal levels,

Y_k^{-} - is an $(m_k \times 1)$ vector of negative deviations from unit k's goal levels,

and the same applies for unit 1's deviations from its goals, Y_1^{+} and Y_1^{-} respectively.

Similar types of behavioural imperfections can be dealt within the framework of a corporate portfolio. Let us take the case of a department store manager aiming to surpass the performance of some other store within the corporate chain, perhaps in search for promotion or in hope of diverting certain resources to his store. Since our analysis is two-dimensional,

the target in question can either be the rival unit's risk level, or its rate of return, and equally possible - both. We now proceed to define formally the above ideas.

i) Aiming at a return goal: Consider the case of the manager of store i attempting to excell over and above store p's rate of return whatever the price in terms of risk. In such a case we can write the problem facing the store manager of unit i as follows:

$$\text{Min. } X(e_p) = \sum_m \left[b_m \sigma^2 (R_M) \sum_k \alpha_{mk} c_k \right] / \left(\sum_m \sum_k \alpha_{mk} c_k \right)^2 \qquad (7.20.a)$$

$$\text{S.T.} \quad \sum_m \left[E(X_m) - \left\{ R_F + b_m \left[E(R_M) - R_F \right] \right\} \sum_k \alpha_{mk} c_k \right] /$$

$$\sum_m \sum_k \alpha_{mk} c_k \geq e_p \qquad (7.20.c)$$

$$\sum_m \alpha_{mk} \leq \alpha^*_{ik} \quad \forall k \qquad (7.20.c)$$

$$\alpha_{mk} > 0 \quad \forall k,m \qquad (7.20.d)$$

where e_p is store p's money unit return. In the case described above, at every iteration of the resource allocation process of the kind described earlier, the dependent unit receives the independent unit's return and then proceeds to solve the problem defined above which assures that its own return does not lag behind that of store p (although there is no certainty that such a feasible solution exists in every case).

(ii) Aiming at a target risk level: In a similar manner the dependent store's manager may attempt to attract resources to his store by offering a "safer", i.e. less risky, performance than the competing store. Under such circumstances the dependent unit's store manager's problem can be formulated as follows:

$$\text{Max.} \quad Z(e'_p) = \sum_m \left[E(X_m) - \left\{ R_F + b_m \left[E(R_M) - R_F \right] \right\} \sum_k \alpha_{mk} c_k \right] \Big/ \sum_m \sum_k \alpha_{mk} c_k \qquad (7.21.a)$$

$$\text{S.T.} \quad \sum_m \left[b_m \sigma^2 (R_M) \sum_k \alpha_{mk} c_k \right] \Big/ \left(\sum_m \sum_k \alpha_{mk} c_k \right)^2 \leq e'_p \qquad (7.21.b)$$

$$\sum_m \alpha_{mk} \leq \alpha^*_{ik} \quad \forall k \qquad (7.21.c)$$

$$\alpha_{mk} \geq 0 \quad \forall k, m \qquad (7.21.d)$$

where e'_p is store p's money unit measure of risk. Our earlier comments, regarding the availability of the independent unit's performance to the dependent store's manager during the resource allocation process, are equally valid in this context.

(iii) Combined risk-return targets: A more general situation arises when the dependent unit is not satisfied with equalizing the independent unit's performance merely in one respect, either risk or return, and sets itself a target which if attained means that it surpasses, or at least equalizes, the independent unit in both its risk and return. Clearly, there is greater sense, in a mean-variance world, for the dependent unit to aim at both parameters of the independent store's performance, since, by concentrating on only one of the two, the dependent store is not certain that by achieving the target

in question it is necessarily in a position to offer the corporate centre a more attractive financial package than that of the rival store. On the other hand, by setting itself a combined goal, the dependent store's manager is assured that, given that the goal is attained, his store generates an excess rate of return which is as good as, or possibly superior, than the independent one.

We can formulate the dependent store manager's problem as follows:

$$\text{Max. } Z = \sum_m \left[\frac{E(X_m) - \left\{ R_F + b_m \left[E(R_M) - R_F \right] \right\} \sum_k \alpha_{mk} c_k}{R_F} \right] \tag{7.22.a}$$

$$\text{S.T.} \quad \sum_m \left[E(X_m) - \left\{ R_F + b_m \left[E(R_M) - R_F \right] \right\} \sum_k \alpha_{mk} c_k \right] /$$

$$\sum_m \sum_k \alpha_{mk} c_k \geq e_p \tag{7.22.b}$$

$$\sum_m \left[b_m \sigma^2 (R_M) \sum_k \alpha_{mk} c_k \right] / \left(\sum_m \sum_k \alpha_{mk} c_k \right)^2 \leq e'_p \tag{7.22.c}$$

$$\sum_m \alpha_{mk} \leq \alpha^*_{ik} \quad \forall k \tag{7.22.d}$$

$$\alpha_{mk} \geq 0 \quad \forall k, m \tag{7.22.e}$$

Let us point out that these behavioural externalities are not necessarily limited to the uni-directional cases defined above, i.e. cases where there exists a clear dist inction between and independent and dependent unit. As suggested by Ruefli (127), it is conceivable that a feedback process takes place

in both directions, and that the goal dependencies are mutual. Such a case would naturally complicate the solution procedure considerably since it is not clear whether a feasible solution exists for all stores involved.

7.5 SUMMARY

In chapter 7 we have shown how the problem of resource allocation in a decentralized firm can be dealt within a mean-variance framework. Our analysis covered a wide range of assumptions: on one hand a model which minimizes the corporate centre's involvement in the department stores' internal planning, and on the other, a model which incorporates central intervention in every step of the local planning.

Basically, our approach to the modelling of the resource allocation process reflects the fact that the construction of any portfolio can be divided into the following familiar stages:

i) determining the asset base,

ii) finding the optimal investment proportions; and

iii) establishing the level of investment in the portfolio

and we therefore focused our attention on an examination of the conditions which justify the corporate centre's intervention in stages i) and ii) of the store manager's portfolio problem, as well as stage iii) in which it is involved even in the most decentralized of our models.

We started with a model which assumed the store managers enjoyed total freedom both in selecting the product lines they want their stores to carry and in deciding how to allocate the resources at their disposal between them. This was equivalent to confining the corporate centre's involvement to the third stage in the construction of the store managers' portfolio. In order to establish the optimal allocation of the corporate resources between the various stores which form

the corporate chain a resource-directive procedure was developed using the values of the store manager problem's dual variables as his bid prices for the corporate resources. We pointed out the potential problems that have been shown to exist with resource-directive methods, and consequently made the assumption that only non-degenerate solutions submitted by the store managers would be considered by the corporate centre. We then suggested two additional forms of the corporate centre's problem, given a restricted involvement in department store affairs, namely, the case where the complementarity-substitution had a deterministic and additive effect on the centre's return and that of a changing cost structure, and demonstrated how the above problems can be simplified by examining their duals.

Having discussed these decentralized models, we turned our attention to the cases where the central headquarters are involved to a greater degree in the stores' planning - a situation which, in face of the empirical evidence, seems to typify real practices in such firms. Within this category, we first looked at the case where store managers did retain some of their autonomy, although admittedly to a lesser degree, namely in the selection of the products and services their stores were to carry. The centre, under such circumstances, did not allow the store managers to remain the sole selectors of the policy adopted by their respective store. Following this, we focused on the most extreme case of corporate centre involvement, i.e. the case where it was involved in all three stages of the store managers' portfolio construction, where the centre instructed the store managers to allocate certain amounts of resources to given lines in order to guarantee that these will be carried throughout the chain - a practice common in the retail industry.

Finally, we examined behavioural externalities in a mean-variance

framework and pointed out the fact that any store manager who wanted his store to surpass a rival store's performance would have to attempt to attain both the latter's risk and return performance. This is a direct consequence of the fact that, in the two-dimensional framework of uncertainty, achieving merely one of the targets would not be sufficient.

C H A P T E R 8

SUMMARY AND CONCLUSIONS

8. SUMMARY AND CONCLUSIONS

This work has studied the problems of planning and resource allocation in a decentralized department store operating under uncertainty. Broadly speaking, our discussion is divided into three main parts according to the following lines:

i) A Heuristic for resource allocation in an individual department store,

ii) An analysis of a department store's portfolio and optimal investment proportions; and

iii) An analysis of a decentralized department store chain's portfolio and optimal investment proportions.

The Heuristic: In chapter 3, following a review of the literature on resource allocation in decentralized firms in general, with particular reference to the retail trade and financial resources, we proceeded to examine the complex issue of assessing the potential sales of a department store located in a shopping centre. In view of the empirical evidence suggesting that different parameters influence a potential customer's choice of a shopping centre and that of a particular store within it, we decided to break the problem into two sub-problems:

i) Assessing the attractiveness of a shopping centre in terms of the potential sales it attracts; and

ii) Assessing the potential sales of a particular store located within the centre.

Using the familiar gravitational model to assess the former we turned to the second sub-problem. In an effort to keep our model as close to reality as possible, we adopted the approach recommended to us by store managers with which we were in contact, namely that within a modern competitive shopping centre price differentiation between stores does not play a dominant role in determining the level of sales attracted by a particular store. The three main parameters which affect the attractiveness of a store, and which are manipulated by the store manager, were found to be: floorspace, manning and stock. By combining the probability of a customer then being attracted to the store in question, and the socio-economic data related to the income and expenditure patterns of the population living in the store's trading area, we produced a model for evaluating the potential sales of a particular item by the department store. We then turned, in chapter 4, to examine how changes made by the store manager in the three variables mentioned earlier affect the store's sales, resource requirements and profitability. Having established a link between the parameters under the store manager's control, the socio-economic data and the store's performance we developed a profit maximizing heuristic for resource allocation in a department store. We then computerized the heuristic and ran a numerical example using three variations of the reallocation decision rule.

A topic whose importance in the retail trade is such that it merited a separate discussion is the store layout problem. We examined this highly controversial issue and suggested a method for incorporating it in the planning process which takes place in a department store. Specifically, we suggested assigning different levels of profit/square foot according to the quality of the floorspace in which the line is situated. We then used a modified version of the plant layout problem to computerize the store layout problem in an effort to assess the optimal layout for a particular store given the external and internal environments in which it operated.

A department store's portfolio: In chapter 5 we developed the theoretical framework necessary for the analysis of the operating decisions of a department store within the context of portfolio theory. We started by discussing the capital budgeting procedures in a department store, first under assumptions of a perfect market and subsequently in an environment of changing cost structures. Using the Capital Asset Pricing Model to explain the relationship between capital costs and corporate risk, we first proved the standard expansion of the mean-variance theorem and discussed the stringent set of assumptions which underlie it. Following this, we suggested a different expansion, one which enabled us to incorporate the firmwide effects of an increase in the firm's operations in an environment of economies of scale. Having linked the corporate performance with that of the individual lines which form its portfolio we proceeded to investigate the potential effects the acceptance of a new investment project can have on the value of the firm's existing investments. We demonstrated that the aggregate contribution of a new project could be separated into its direct contribution, i.e. its risk-return performance, and the indirect effect it has by altering the risk level of the old investments. We suggested an appropriate decision rule for the acceptance of a project which generates economies of scale and we demonstrated the application of the above rule for the case of ranking mutually exclusive investments.

Having established the above framework in chapter 5, we then proceeded to analyse the structure and potential applications of a department store's portfolio. Our main analysis concentrated on the components of the department store's risk and their interactions. Specifically, we considered the economic pressures which operate on the individual terms of the store's risk expression, and their combined effect on the firm's discount rate following an expansion which generates

economies of scale. The final section of chapter 6 dealt with the issue of resource alllocation between product lines in an independent department store. Using portfolio theory terminology, we equated the store manager's problem with that facing any investor contemplating a portfolio of assets, i.e. determining the optimal investment proportions.

A decentralized chain's portfolio: In chapter 7 we extended our analogy, borrowed from the capital market, to cover the problem facing the corporate planners in a decentralized department store chain. We examined the two-level portfolio problem under three main sets of assumptions related to the economic and behavioural environment in which the stores and the central unit operate. The first model, called the decentralized approach, allowed for perfectly competitive markets and practically no intervention in the store planning process by the central unit. The store manager was free to formulate the store's policy on the basis of purely local information flows and the sole direct involvement by the centre was in determining the total level of investment in the store.

An alternative model, the mixed approach, relaxes some of the behavioural assumptions made in the earlier model and allows for a greater degree of intervention by the centre in the individual store's resource allocation process. Specifically, we enabled the centre to be selector of the store's policy rather than be faced with the store manager's choice as in the previous case. In other words, we acknowledged that it is highly improbable that the centre would allow the store manager to be the sole influence determining the range of goods carried by the store.

The third, and probably most realistic model of the three, is the interventionist one. In this model we allow not only for imperfections in the capital and goods market, but for an extensive involvement of the central unit in the individual

stores' planning process. In the interventionist model the centre is in fact part of every single stage of the decision making process which takes place in the store from the determination of the level of investment down to the range of product lines offered by the store and possibly even their precise location within the store.

FUTURE RESEARCH: It is evident that many research avenues remain to be explored. In the process of this work we were faced with two particular topics which could potentially make a contribution to the planning process in a decentralized department store chain operating under uncertainty. Specifically, they are:

i) the refinement of the heuristic; and

ii) the development of a multi-period model.

i) Refining the heuristic: While formulating the heuristic for resource allocation under uncertainty in a department store we encountered certain difficulties in obtaining the full set of data necessary for conducting a satisfactory test of its accuracy. Specifically, serious gaps in past and present information records of the firm with which we were in contact, and the lack of full co-operation of both public and private sources as a result of the confidential nature of various "commercial secrets" made it quite impossible to collect all data requirements.

In view of the practical, rather than theoretical, nature of these obstacles, it seems that they may not prove to be unsurmountable. Future efforts, which would concentrate on limited aspects of the heuristic, may perhaps be more successful in their attempt to gather all the data necessary for comprehensive evaluation of the heuristic. A first and recent step in this direction was taken by Yeo (155) who used the heuristic developed in this work to assess the resource allocation in a newly established store in the Blackburn

area. Facing similar difficulties in obtaining data on the relationships between inputs, sales and profits, Yeo resorted to a linear programming technique which enables a sensitivity analysis to be made, and the effects of alternative resource allocations to be assessed.

ii) A Multi-Period Model: The attempt to provide a multi-period resource allocation model for a decentralized department store chain operating under uncertainty faces the considerable tasks that need to be overcome for an intertemporal capital model to be produced. Described in chapter 5, these consist of: establishing the factors which determine the change in return anticipations, the changes in equilibrium prices and their feedback effects into individual information sets. Only after these problems have been satisfactorily solved can one seriously attempt to provide, within the Capital Asset Pricing Model framework, a multi-period model of the resource allocation in a decentralized firm in general, and of a department store chain in particular.

An alternative approach altogether to the problem of ranking the options facing the store manager and the corporate centre is one which uses stochastic dominance. One could use tests for stochastic dominance in an effort to rank investment projects in order of preference, thus reducing the size of the efficient set of ventures.

To conclude, let us stress the fact that the latter parts of this thesis, i.e. those which follow the heuristic developed in chapter 3 and 4, are potentially applicable to multi-product and multi-branch profit orientated organizations in general. In spite of the fact that the behavioural assumptions underlying our models, e.g. the commitment of a proportion of a department store's resources to certain products following corporate instructions, are typical to the retail trade, the framework developed in our discussion can be modified to accommodate the behavioural characteristics of other industries. In view of the potential flexibility of our models, it is conceivable that they could be implemented in various industries where limited amounts of resources need to be allocated in a rational manner between a number of divisions, subsidiaries or branches under conditions of uncertainty.

R E F E R E N C E S

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APPENDIX I

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The Planning of a Department Store

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A town or village attracts people from its hinterland to purchase goods by virtue of the amount and variety of goods on offer. Similarly, a department store attracts customers away from competitors by means of displays, staff expertise, stock holdings and numerous other parameters. The store is generally free to manipulate these parameters subject to certain restrictions; the manipulation will have a direct affect on the ultimate profitability of the store. This paper describes some current research that is being carried out in conjunction with a department store group. The ultimate aim is to link the corporate planning of the group and each store to detailed socio-economic data describing the population. Some aspects of the problem have been investigated in depth, some are currently being discussed and some have not been considered at all as yet. The latest position will be reported and hopefully some feedback will be engendered.

INTRODUCTION

This paper is the initial report of some work undertaken in the UK on behalf of a large department store group. The scope of the research overall is to develop a methodology for the future capital expenditure of the group, and in particular to identify where new stores could be opened, to suggest the size of such stores, to decide whether or not existing stores should be expanded, and so on. At the present time, such expenditure lacks a global view and is thus undertaken piecemeal; the aim of the project is ultimately to present a group-wide view, with all the various interactions this would involve.

However this paper has a far less ambitious objective. It is obviously necessary in this work to understand how an individual department store operates, what are the constraints under which it must act, and what are the various decision parameters under its control. This paper describes the work that has been carried out to develop a planning model of a store; some of the work has been completed, but much is still in the formative stage and it is hoped that this presentation will generate comments and feedback to us.

The paper is divided into a number of parts. Firstly the general problems of controlling a store will be discussed in some detail. This discussion will identify a number of separate aspects whose derivation and measurement will be analysed. Finally the threads will be drawn together with the presentation of various models.

THE STORE MANAGER'S PROBLEM

Certainly in this group, and generally in many such groups, the store manager has considerable autonomy provided that he (or she) is reasonably successful. The manipulation of certain decision parameters is under his *own* control; hence the title of this section.

Superficially his problem is easily stated: For each item in the store, and this may run into

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thousands, he has to match supply with demand in such a way as to optimise some objective(s). Supply is very much under his control and as such is a primary decision parameter; unfortunately demand is not fixed and independent of supply nor is it independent of a number of other parameters under the manager's control.

The derivation of a demand function is highly complex. Moving away from the store for the moment, picture a country homogeneously scattered with consumers. Now place in this country a limited number of independent and separate shopping centres; it is not impossible to imaging the consumers gravitating towards the centres as a direct function of their 'attractiveness' and an inverse function of their 'distance'. Briefly completing the analogy, which will be discussed in far greater depth in the next section, it would therefore be possible, in theory at any rate, to compute the potential demand that the consumers will place upon any particular shopping centre. A department store manager in such a centre can do little to influence this potential demand, assuming of course that the size of the store is small in relation to the overall size of the centre. The assumption is likely; department stores are usually (invariably) placed in large shopping centres.

Once the consumer has arrived in the shopping centre, the influence that the manager now has is considerably increased. He is competing using the resources at his disposal with the other shops and stores in the centre for the consumer's attention. If he increases the resources allocated to one particular item, *ceteris paribus*, he must increase the demand in his store for the item. A more detailed discussion of the various resources he can allocate, and the nature of the demand function, is left to a later section.

But the manager's problem has not yet finished. The resources he is allocating are strictly finite and hence the various items are in competition with one another. For example, short of rapidly expanding the store, the total selling floor space is fixed: if more space is allocated to one item, less space has to be allocated to another. The demand for the former may increase, but for the latter will decrease, which could result in a net detrimental effect on the manager's objective. Thus the manager has to decide upon the optimal allocation of these resources, a typical mathematical programming problem, although of course highly complex. The next sections will discuss the construction of the various relationships necessary to solve the manager's problem optimally.

THE DERIVATION POTENTIAL DEMAND

In this section we will examine the measurement of the potential demand for an item (or collection of items) in terms of the success of a shopping centre in attracting consumers from within the surrounding area. For confidentiality, we will identify the particular department store under consideration as D, situated in shopping centre C.

The underlying basis for work in this area is due to Reilly [9], who postulated that competing shopping centres would extract custom from the intervening hinterland in direct proportion to the 'attractiveness' of the centres and in inverse proportion to the square of the physical distance from each centre to the hinterland. Mathematically, one may define

$$p_{st} = (a_t/d_{st}^2) \sum_{\forall r} (a_r/d_{sr}^2) \qquad (1)$$

where p_{st} is the proportion of customers from area s going to shopping centre t, a_r the 'attractiveness' of centre r and d_{sr} the distance between s and r. The statement is termed the law of retail gravitation after the obvious analogy with the Newtonian law of gravitation. Since Reilly in 1929, there have been a number of suggestions and improvement to this basic model. Perhaps the most recent version is that, after statistically examining some 43 measures, 'attractiveness' has been shown to be highly correlated with the total available floorspace in a shopping centre [8]. Another important modification is to replace the physical distance with a measure of economic distance, i.e. how much would it cost a consumer to travel from s to r, or travelling time.

The actual use made of equation (1) in practice was as follows. First we examined D's delivery and credit files to identify the boundaries of the catchment area. Using the 1971 Census of Population [2], this area was subdivided into 74 wards. This was necessary because a lot of the data we were able to collect was

only by ward. The next step was to identify the major shopping centres competing over this catchment area. As centre C was situated to the north-east of London, it was felt necessary to subdivide London into identifiable shopping centres, e.g. Oxford Street, Brent Cross. The location and total trading floor-space of each centre was available from a 1971 Census of Distribution, carried out by the Greater London Council [4]. Finally, after identification by planning offices of the precise boundaries of each ward as in 1971, the distance between each ward and each centre was measured in terms of 'main road distance'. It was felt that, as the entire area is relatively wealthy, most people travelled by private car rather than public transport. Hence, a vector containing the proportion of consumers living in ward i shopping in centre C was calculated. The above analysis does ignore the possibility of consumers buying goods near their place of work; it was felt that such purchases tended to be mainly for foodstuffs (which a department store does not generally sell) and that hopefully the total discrepancy was small because we had no way of checking on these sales.

We now have to move from the proportion to the potential demand for each item (or group of items). The 1971 Census of Population [2] identified *pari passu*, the number of households in each ward, whilst two reports by the GLC defined the socio-economic grouping (SEG) of the population within each ward [5, 6]. Finally a recent Family Expenditure Survey [3] describes the income and expenditure distribution, coded by occupation of the head of the household, who may be identified with a particular SEG. Thus, from combining these three sets of statistics, we arrived at D_{iqs}, the demand for item i by SEG q in ward s.

Hence,

$$D_i^t = \sum_s p_{st} \sum_q D_{iqs}, \quad \text{where } t \equiv C \tag{2}$$

the potential demand for item i in centre C.

As we had no intention of considering each item sold by D individually, we used instead 18 groups of similar items, corresponding to the internal product groups as defined by the store; indeed, throughout this paper, the words 'item' and 'groups of similar items' are synonymous. There were, as one might imagine, considerable difficulties in combining together statistics from different sources and of different ages. We attempted to reconcile figures where ever we could; where we could not, a qualified pragmatism was used.

THE SCARCE RESOURCES

We have so far described how the potential demand for a particular item in a particular shopping centre may be calculated. We will now turn to the next problem, namely a discussion of the scarce resources under the control of the manager and the relationship between these resources and the actual demand in the store.

One could have drawn up a very long list of factors that would obviously influence a consumer into buying in one shop as opposed to another. Such a list would not be very practical; we had discussions with a number of store managers of this particular group and the general consensus was that there were three principal factors, namely floorspace, stock and labour, that would influence demand. Strangely, price was not regarded as such an important factor; on the whole there was little price distinction between shops in the centre unless there was a deliberate attempt to sell off a line.

(a) *Floorspace*

Floorspace is perhaps the most obvious scarce resource. It is obviously limited by the physical size of the store and also, *ceteris paribus*, an increase in the space allocated to a good should increase its relative demand. However in practice it is not quite as simple as that: floorspace is not homogeneous in its appeal. If we adopt as an index profit/sq. ft., then the index decreases as the distance from the ground floor increases. Also the space on each floor may be subdivided; prime space is opposite the main entrance, good space is opposite the side entrances and the mouths of the escalators and lifts, whilst the rest is simply standard space. To increase the space allocated to a good and simultaneously move it to a worse kind of space, need not necessarily increase the demand at all.

This lack of homogeneity presents a major problem. What we would like to do is construct a model that would ultimately compute

the amount of floorspace that should be allocated to a particular item and also where the floorspace should be located. This would require such data as the change in sales as a good is moved around the store: these data are just not available nor are any records kept.

(b) *Stock*

Generally department stores have little storage space, their total stock is usually on display. This of course does encourage the customer if a large, and varied stock is available. But for the manager it does create a problem. As demand is a function of the stock level then one of two situations may occur:

(i) the level of stock needed to generate a certain level of demand is greater than that level of demand, which implies that there will always be unsold items on display. This may create problems of stock turnover and cause low profitability returns per stock item.

(ii) the level of stock needed to generate a certain level of demand is less than that level of demand, which implies that customers will have an unsatisfied demand and sales will be lost. This situation is generally an anathema to managers who primarily see themselves as traders rather than businessmen.

There are further complications, namely that the stock level must be restricted by the floor space available and also by the working capital funds. The store can only afford to hold limited stocks.

(c) *Labour*

In the front-line of any store are the sales staff. Such staff are extremely important as they directly influence the customer. As the level of staffing drops, the waiting time of customers increases and ultimately demand will decrease. But staff is a direct cost to the manager and as such he has to decide upon a suitable level for each item. His task is complicated because many of the staff are specialists and thus not interchangeable.

THE CONSTRUCTION OF A DEMAND FUNCTION

In discussing the key parameters under the control of the manager, we must note that not only do they all directly influence demand, but also that they are all inter-related. One must be careful here to distinguish between demand and sales, for demand may exist independent of sales. It is firstly suggested that the demand function for a particular item is expressed in terms of relative allocations, i.e. the market share λ_i is given by

$$\lambda_i = \frac{D_i}{D_i^*} = f_i\left(\frac{\beta_{ki}}{c_{ki}}\right)$$

where D_i is the demand for item i at D, and β_{ki} and c_{ki} the scarce resources of type k allocated in D and in the entire centre C respectively to item i.

A more precise statement of the form of this function requires a further assumption and we consider two alternative ones.

(a) *Multiplicative demand function*

This is based upon the assumption that if the allocation of any scarce resource is zero, then the resulting demand is also zero, and furthermore that there are decreasing returns to sale, i.e.

$$\lambda_i = \prod_k r_{ki}^{a_{ki}} \quad 0 \leqslant a_{ki} \leqslant 1$$

where $r_{ki} \equiv (\beta_{ki}/c_{ki})$. It is felt that the manager would be able to give an indication of the 'relative importance' of each scarce resource in influencing demand; for a multiplicative function, we interpret this statement as an estimate of the relative elasticities of demand. Assuming that resource 1 is the base resource, the relative importance is defined by

$$\gamma_{ki} = \left(\frac{\partial\lambda_i}{\partial r_{ki}} \cdot \frac{r_{ki}}{\lambda_i}\right) \Big/ \left(\frac{\partial\lambda_i}{\partial r_{1i}} \cdot \frac{r_{1i}}{\lambda_i}\right) = \left(\frac{r_{ki}}{r_{1i}}\right)\left(\frac{\partial\lambda/\partial r_{ki}}{\partial\lambda/\partial r_{1i}}\right)$$

or

$$\gamma_{ki} = (a_{ki}/a_{1i})$$

The interpretation is not entirely fortuitous as it does result in an expression that is independent of r_{ki}. The demand function may now be written as

$$\lambda_i = \left(\prod_k r_{ki}^{\gamma_{ki}}\right)^{a_{1i}} \tag{3}$$

Given the status quo (λ_i^0, r_{ki}^0) we can compute

$$a_{1i} = \ln \lambda^0 / \sum_k \gamma_{ki} \ln r_{ki}^0 \tag{4}$$

(b) *Separable demand function*

The underlying assumption here is that demand is generated by each scarce resource

in isolation, and that the total demand is computed from the addition of the individual demands, i.e.

$$\lambda_i = \sum_k f_{ki}(r_{ki})$$

Notice that this assumption makes the clear distinction between demand and sales, for obviously there can be no sales unless a positive quantity of all the scarce resources is allocated. To produce a logical interpretation of 'relative importance' in this situation is however difficult, and perhaps the easiest direction is to assume that in practice there would be little movement away from the *status quo*, so that a linear demand function would not be unrealistic, i.e.

$$\lambda_i = \lambda_i^0 + \sum_k d_{ki}(r_{ki} - r_{ki}^0)$$

Now, using the same definition of relative importance yields

$$\lambda_i = \lambda_i^0 + d_{ii} r_{ii} \sum_k \gamma_{ki}\left(1 - \frac{r_{ki}^0}{r_{ki}}\right) \qquad (5)$$

But there is no simple means of calculating d_{ii} from the *status quo*; the only way is, when the allocation has altered from the original *status quo*, to make a direct substitution.

ALLOCATION OF SCARCE RESOURCES

So far we have outlined the various scarce resources that a manager must allocate and discussed the restrictions and interdependencies that impinge on each one of them. We have also described the construction of a demand function related to the scarce resources and also the potential demand of the entire shopping centre. Before we can proceed further, we need to identify the objective that the manager is attempting to optimise. The evidence for the identification tends to be contradictory: most managers in isolation are primarily concerned with turnover. As stated above, they are principally traders and their general status increases with turnover: furthermore, in the group the pecking order of stores was based upon turnover. On the other hand, taking the group view, store contribution (an approximation to profit) is a much more useful objective and one that the head office is principally concerned with. We adopted the pragmatic approach: as we had been briefed by the head office we used a profitability objective.

As we have seen, it is feasible to formulate this problem as a mathematical program. The profitability is easily calculated as the prices of the goods are fixed, buying costs, costs of floorspace (rent, rates and depreciation) and labour (various hierarchies of sales staff) are all easily obtainable. The different types of scarce resources, their limitations and interrelationships with each other and with the demand function, have been described. So one could solve the allocation problem as described relatively easily although the model might be extremely large. Appendix 1 describes a simple model and discusses appropriate solution methods.

However our first approach for the management of the group was not to use a MP but to develop an heuristic. The heuristic made certain gross assumptions but it had the virtue of being easily and quickly understood. It started by assuming that, just as the division of demand between centres was on the basis of relative floorspace, the potential demand in C could be divided up in proportion to the relative floorspace in C, i.e.

$$D_i = \left(\frac{\beta_{2i}}{\epsilon_{2i}}\right) D_i^*$$

where the subscript 2 refers to floorspace. For each item, or rather group of items, various discrete sales levels were identified. S_{ip}, $p = 1, 2, \ldots$. With each sales level was associated four figures: the profit generated by that level of sales (P_{ip}), the space required for those sales (SP_{ip}), the stock required for the sales (ST_{ip}) and finally the level of staffing required to achieve the sales (L_{ip}). Rather than considering every possible combination of scarce resources as the MP would do, the heuristic only looks at certain *a priori* values. One detailed concession was made, namely to divide the floorspace up into different types; in D we identified seven grades of space depending upon the floor one was on and one's whereabouts on that floor. This involved, in theory at any rate, producing the above figures for each type of space as well (let the subscript t denote the type of space). Calculate as a profitability index, the profit/sq.ft. $\bar{P}_{ipt} = P_{ipt}/SP_{ipt}$ ($\forall i, p, t$). For each scarce resource, we assume certain known limitations, which are easily

ascertainable from the manager. The detailed heuristic is as follows:

1. Scan the matrix $\bar{P}$ for the highest figures; let this be $\bar{P}_{rst}$. As decreasing returns to scale were assumed, the greatest profitability was generally at the lowest level of sales.

2. Calculate running totals. (i) add SP_{rst} to total floor space of type t so far consumed; (ii) add ST_{rst} to total value of stock held so far; (iii) add L_{rst} to total staff employed so far.

3. Compare the current usage of the scarce resources with the limitations. If any constraint is violated, go to 4. Otherwise go to 5.

4. Identify the accepted item with the lowest profitability. Delete its current level from further consideration and adjust the running totals accordingly. Go to 3.

5. Delete from any further consideration, items (rsu) $\forall u \neq t$. This ensures that the same item will not be spread out on different types of floor.

6. If there exists spare capacity and some items still have to be allocated, go to 1. Otherwise calculate store-wide figures and stop.

Such an heuristic is not optimal, but it does enable the treatment of, e.g. the discreteness of labour, non-convex profit functions, discontinuities in floor space, as well as being extremely rapid to compute. Data collection is a major disadvantage and whilst we have demonstrated its usefulness to the management, the application to an entire store is still in the future.

STOCHASTIC DEMAND MODELS

We now return to the optimising model as this is probably of longer-term interest. One fundamental assumption throughout the work above is that of determinism. Yet, when all the analyses are performed, and the manager has allocated his resources, the final demand is still very much unknown. The sources of this uncertainty are manifold, errors in the potential demand computation, the form of the demand function, a shift in buying patterns, and so on. But any worthwhile model must attempt to take it into account.

How a stochastic demand is to be included is still very largely a matter of discussion. However, for the want of any better information, we made the gross assumption that the demand is distributed with an expected value given by the demand function (D_i) and a fixed exogenous variance (σ_i^2). This implies no increase in the uncertainty as the market share increases. It is supposed that an estimate of this variance may be provided from the historical records of the store, together with the experience of the manager.

In the vernacular, the desired stochastic model is a "here-and-now" one [12] because the resource allocations have to be performed *before* the actual demand can be observed. The class of models may be subdivided further into indirect, corresponding to the active approach of stochastic linear programming [11, 10], and direct. We will concentrate on this latter set, which contains both chance-constrained models and recourse models.

Let us write the deterministic model as

$$\{\max P(x) \text{ s.t. } f(x) \leqslant \tilde{D}; x \in \psi\} \qquad (6)$$

where $p(.)$ is the profit function, x the set of decision variables, ψ the feasible space defined by the intersection of all the other constraints, f(.) a vector (of dimension equal to the number of items) representing [sales + expected demand] and finally $\tilde{D}$ a vector of random variables with zero mean and standard deviation σ. Before we can proceed much further, we have to discuss the effect of this uncertainty on the manager. It implies that there are times when he will have allocated resources to account for a particular level of demand, and that the demand has fallen below this level, thus reducing his profits. The chance-constrained formulation comes to hand immediately; the uncertainty in the model being catered for by $\Pr\{f(x) < \tilde{D}\} > \alpha$. This model restricts the violation of the demand constraint to, at the most, $(1 - \alpha_i)$ proportion of the time, for each item. The certainty equivalence of these constraints is simply

$$f_i(x) \leqslant F_i^{-1}(1 - \alpha_i)$$

where $F_i^{-1}(.)$ is the inverse cumulative density function of $\tilde{D}_i$. The uncertainty here is measured by α and it is quite feasible to gener-

ate a surface $\{p^*(x), \alpha\}$ as naturally α and p^* are inversely related.

However, such a model does not really get to the heart of the manager's problem. His objective is profit and, at the end of the day, trading risk must be measured in terms of lost profits. Writing the model as

$$\{\max. p(x) - g(y^+) - h(y^-) \text{ s.t. } f(x) + y^+ - y^- = \bar{D}; y^+, y^- \geq 0; x \in \psi\} \quad (7)$$

where $g(y^+)$ is a measure of the profits foregone as demand was greater than allowed for, and $h(y^-)$ a measure of profits lost due to the overallocation of resources. Such a model can be simplified by assuming *a priori*, that the manager would find acceptable a certain minimum profit level (p^*) or a range of market shares, say $[\lambda_L, \lambda_u]$. This would remove the profit function from the objective, which now becomes $\{\min. g(y^+) + h(y^-)\}$. The model is an example of a programme with simple recourse and as such has been studied in some detail [13, 14]. Unfortunately it is not easy to solve unless some further simplifying assumptions can be made. Appendix 2 contains the details of a simple model and also discusses the implications of other assumptions.

SUMMARY

It is very common, in the UK at any rate, to find that managers of department stores do not regard themselves as businessmen at all but more as traders. This has obviously got an historical basis. In this paper, we have discussed some of the problems of running a store and have attempted to display a scientific approach to these problems. Like any business, at the heart of a store is the economic relationship between supply and demand. We have constructed a demand function for the various items in the store and examined how this demand will vary as the manager allocates the scarce resources under his control. We have outlined various models that may be used as the basis of any structural planning process and suggested algorithms that may be implemented. Uncertainty plays a large part in the manager's problems and we have proposed methods of manipulating it so as to reflect its true role.

We have not attempted to answer the fundamental question: what makes a department store different from a collection of individual shops? We could put forward suggestions related to synergy, ambience and so on, but ultimately it is a question we are unable to answer. There are other facets that have not been discussed. In retailing there is the theory of complementarity—namely, that goods that tend to be sold together or to the same group of customers are put in close physical proximity. This can have a major impact on the layout of a store. One technique we think might be applicable here is graph theory, with weights on the arcs denoting the degree of complementarity and trying to identify spanning trees or restricted degree. This would certainly be a very difficult problem to handle using programming techniques.

The demand pattern for goods may be highly seasonal, e.g. toys, garden tools, and yet all of the models so far presented do not possess a time dimension. One answer might be to produce a multi-period model, but perhaps a more realistic one is to run the static model at, say, monthly intervals with appropriate (and largely subjective) modifications to the demand function. Data are just not available to build up monthly demand functions in the manner suggested in the main text. Finally, to reiterate the limitations we laid down at the beginning of the paper, there is deliberately no discussion concerning capital expenditure and the more major *group* task of assessing and controlling a large number of stores. These topics will be the subject of a future paper.

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Appendix 1: A deterministic model

The aim of this appendix is to demonstrate how a deterministic mathematical programming model of a store may be constructed, to build a simple model and to suggest various appropriate solution strategies.

Concentrating firstly on an individual item, say the ith one, the supply and demand constraints must be constructed. The sales of this item, S_i, are limited by:

the stock of goods, β_{1i}
the floorspace allocated, β_{2i}
the labour force available, β_{3i} and, of course, the demand, D_i.

The precise nature of the limitations depends upon the required detail, as discussed in the main text, but the simplest assumptions of homogeneous floor space and sales staff might produce constraints of the form

$$S_i \leq \beta_{1i}$$
$$y_i\beta_{1i} \leq \beta_{2i}$$
$$S_i \leq \theta\beta_{3i}$$
$$S_i \leq D_i$$

where y_i is the floorspace consumed per item and θ_i the selling capacity per member of staff. Fractional staff are deliberately implied, equivalent perhaps to staff working in more than one section of the store or the employment of part-time staff.

The demand, being a function of the various β's, could be as described either in equations (3) and (4) or as in (5), i.e. multiplicative or linear. If the multiplicative function is adopted, then an obvious strategy is to employ geometric programming; this in addition would be able to handle easily conditional demands between items. Alternatively, because D_i is monotonic in all the β's, it is possible that a simple application of an iterative linear approximation technique (see [7] for further details) might suffice [11]. Finally, if only small variations in the demand around the status quo are considered to be likely or permissible, use of a first order approximation of a Taylor series, together with the fact that the second differentials are all non-negative would result in a constraint of the form

$$D_i \leq D_i^0\left[1 + \sum_k \left(\frac{\beta_{ki}}{\beta_{ki}^0} - 1\right)\pi_{ki}a_i\right]$$

The profit contribution of this item consists of the gross operating profit less various charges for excess stock holding, labour and, possibly, the use of floorspace and other facilities. In the simple model this may be written as:

$$(p_i - c_i)S_i - rc_i(\beta_{1i} - S_i) - d\beta_{3i} - e\beta_{2i}$$

where p_i and c_i are the price and cost of the item, r the interest charge on stock holding, d the labour rate and e the charge per unit of floorspace.

Looking at the overall picture, there are restrictions on the various decision parameters relating to, for example, the physical size of the store, the size of the labour force and the available working capital. Simple descriptions of these constraints would be:

$$\sum_i c_i\beta_{1i} \leq M \quad \text{—current asset constraint}$$
$$\sum_i \beta_{2i} \leq N \quad \text{—total floorspace}$$
$$\sum_i \beta_{3i} \leq \Xi \quad \text{—total level of staff}$$

but of course they could be far more complex. Examination of the shadow prices attached to these constraints would be of considerable interest, both to the manager and the head office, as they might form the basis for inter-store efficiency measures.

Appendix 2: A stochastic demand model

We demonstrated in the main text that one approach to handling a stochastic demand might result in the following model:

$$\{\max.\ p(\mathbf{x}) - g(\mathbf{y}^+) - h(\mathbf{y}^-) \text{ s.t. } \mathbf{f}(\mathbf{x}) + \mathbf{y}^+ - \mathbf{y}^- = \tilde{\mathbf{D}};$$
$$\mathbf{y}^+, \mathbf{y}^- \geq \mathbf{0}; \mathbf{x} \in \psi\}. \quad (7)$$

If we make the assumption that the penalty functions are both linear, e.g. $h(\mathbf{y}^-) = \sum_i h_i y_i^-$ *and* that we are concerned about the *expected* violation of the demand constraint, so that $g(\mathbf{y}^+) = 0$, then the objective may be written as:

$$\max.\ p(\mathbf{x}) - \sum_i h_i \int [f_i(\mathbf{x}) - \tilde{D}_i]\, dF(\tilde{D}_i)$$

integrating over the range

$$f_i(\mathbf{x}) \geq \tilde{D}_i$$

For an appropriate distribution, e.g. uniform, the objective becomes

$$\max.\ p(\mathbf{x}) - \tfrac{1}{4}\sum_i \left(\frac{h_i}{\tilde{D}_i}\right)[f_i(\mathbf{x}) + \tilde{D}_i]^2$$

where $\tilde{D}_i \sim R[-\tilde{D}_i, +\tilde{D}_i]$. This may be easily solved using either a quadratic algorithm (if the L^2 approximation for $f(\mathbf{x})$ is suitably valid) or some non-linear algorithm.

Another approach, because the integral is in the objective function, and because the integral is monotonic in $f_i(\mathbf{x})$, we could use again a version of linear approximation programming.

APPENDIX II

A NUMERICAL ILLUSTRATION OF CENTRAL INTERVENTION

We can illustrate the effect of the centre's intervention into the store planning with the help of a simple numerical example. Specifically, we examine the case of two-level organisation where the centre imposes a detriment on the the divisional manager in form of a pre-emption of space.

Let us define the divsional manager's problem for a department store i where two lines (activities) are operated using two resources (inputs). The formulation of the primal problem, along the lines discussed in Chapter 6, is :

$$\text{Max.} \quad Z = \Omega_1 (\alpha_{11}c_1 + \alpha_{12}c_2) + \Omega_2(\alpha_{21}c_1 + \alpha_{22}c_2)$$

$$\text{S.T.} \quad \alpha_{11} + \alpha_{21} \leq RL_{i1}$$

$$\alpha_{12} + \alpha_{22} \leq RL_{i2}$$

$$\alpha_{11}, \alpha_{21}, \alpha_{12}, \alpha_{22} \geq 0$$

where :

α_{11} - the amount of resource type 1 allocated to product line 1.

α_{21} - the amount of resource type 1 allocated to product line 2.

α_{12} - the amount of resource type 2 allocated to product line 1.

α_{22} - the amount of resource type 2 allocated to product line 2.

c_1 - the unit cost of resource type 1.

c_2 - the unit cost of resource type 2.

RL_{i1} - the constraint on the amount of resource type 1 allocated to store i.

RL_{i2} - the constraint on the amount of resource type 2 allocated to store i.

Let us now assume the following values :

$\Omega_1 = 2$ $\quad c_1 = 3$ $\quad RL_{i1} = 20$

$\Omega_2 = 8$ $\quad c_2 = 64$ $\quad RL_{i2} = 276$

which we can insert into the above formulation to produce :

$$\text{Max. } Z_1 = 2(3\alpha_{11} + 64\alpha_{12}) + 8(3\alpha_{21} + 64\alpha_{22})$$

$$\text{S.T. } \alpha_{11} + \alpha_{21} \leq 20$$

$$\alpha_{12} + \alpha_{22} \leq 276$$

$$\alpha_{11}, \alpha_{21}, \alpha_{12}, \alpha_{22} \geq 0$$

We can rewrite the objective function

$$Z_1 = 6\alpha_{11} + 128\alpha_{12} + 24\alpha_{21} + 512\alpha_{22}$$

and solve the problem, which gives us the following values :

$\alpha_{21} = 20$ $\quad \alpha_{11} = 0$

$\alpha_{22} = 276$ $\quad \alpha_{12} = 0$

and

$$Z_1 = 24 \cdot 20 + 512 \cdot 276$$
$$= 480 + 141312$$
$$= \underline{141792}$$

The above problem's dual can be written as follows :

$$\text{Min.} \quad Z = \alpha_{i1} w_1 + \alpha_{i2} w_2$$
$$\text{S.T.} \quad w_1, w_2 \geq 0$$
$$w_1 + w_2 \geq \Omega_1 c_1 + \Omega_1 c_2$$

which, given the values of our numerical example, becomes

$$\text{Min.} \quad Z = 20 w_1 + 276 w_2$$
$$\text{S.T.} \quad w_1, w_2 \geq 0$$
$$w_1 \geq 6$$
$$w_1 \geq 24$$
$$w_2 \geq 128$$
$$w_2 \geq 512$$

and the optimal solution is :

$$w_1 = 24 \quad \text{and} \quad w_2 = 512$$

and the slack variables (s.v.) are :

$\alpha_{11} = 0$ $\quad w_1 \geq 6$ $\quad$ s.v. $\neq 0$

$\alpha_{12} = 0$ $\quad w_2 \geq 128$ $\quad$ s.v. $\neq 0$

$\alpha_{21} \neq 0$ $\quad w_1 \geq 24$ $\quad$ s.v. $= 0$

$\alpha_{22} \neq 0$ $\quad w_2 \geq 512$ $\quad$ s.v. $= 0$

since

$$w_1 + t_1 = 6$$

$$w_2 + t_2 = 128$$

$$w_1 + t_3 = 24$$

$$w_2 + t_4 = 512$$

Let us now turn to the store manager's problem given that the centre instructs the store planners to allocate specific quantities of both types of resources to product line no. 1, say :

$$\alpha_{11} = 10 \quad \text{and} \quad \alpha_{12} = 15$$

The new primal is :

$$\text{Max.} \quad Z_2 = 6\alpha_{11} + 128\alpha_{12} + 24\alpha_{21} + 512\alpha_{22}$$

$$\text{S.T.} \quad \alpha_{11} + \alpha_{21} \leq 20$$

$$\alpha_{12} + \alpha_{22} \leq 276$$

$$\alpha_{11} = 10$$

$$\alpha_{12} = 15$$

$$\alpha_{21}, \alpha_{22} \geq 0$$

and the optimal solution is :

$$\alpha_{11} = 10 \qquad \alpha_{12} = 15$$

$$\alpha_{21} = 10 \qquad \alpha_{22} = 261$$

hence the value of the objective function can be computed to be :

$$Z_2 = 6 \cdot 10 + 128 \cdot 15 + 24 \cdot 10 + 512 \cdot 261$$

$$= \underline{135852}$$

The new dual is :

$$\text{Min. } 20\, w_1 + 276\, w_2 + 10\, w_3 + 15\, w_4$$

$$\text{S.T. } w_1, w_2 \geq 0$$

$$w_1 + w_3 \geq 6$$

$$w_2 + w_4 \geq 128$$

$$w_1 \geq 24$$

$$w_2 \geq 512$$

and the optimal solution is

$$w_1 = 24 \qquad w_3 = -18$$

$$w_2 = 512 \qquad w_4 = -384$$

and the objective function of the dual is :

$$= 20 \cdot 24 + 276 \cdot 512 - 10 \cdot 18 - 15 \cdot 384$$

$$= \underline{135852}$$

hence all the dual slack variables are zero.

These calculations provide us with the data necessary for measuring the cost of the detriment put by the centre on the store manager. This is done by calculating the difference in the original store manager problem's solution and that derived following the incorporation of the additional, i.e. pre-emption, constraints. The net change is

$$Z = 141792 - 135852$$

$$= \underline{5940}$$

As could be expected, the interference of the centre in the store manager's problem has a cost attached to it since it prevents the store manager from optimizing his resource allocation in face of purely local indormation flows. The fact that such a potential penalty exists means that prior to any interference by the centre a study should be made in order to establish whether the profit to the organization as a whole as a result of the imposition of central interests outwheighs the local instances of losses (assuming such exist). Hence our analysis of the economics of intervention, see section 7.3.2.