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Fast cooling of trapped ions using the dynamical Stark shift

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Abstract. A laser cooling scheme for trapped ions is presented which is based on the fast dynamical Stark shift gate, described in (Jonathan *et al* 2000 *Phys. Rev.* A **62** 042307). Since this cooling method does not contain an off resonant carrier transition, low final temperatures are achieved even in a traveling wave light field. The proposed method may operate in either pulsed or continuous mode and is also suitable for ion traps using microwave addressing in strong magnetic field gradients.

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1. Introduction

The ability to laser cool ions to within the vicinity of the motional ground state is a key factor in the realization of efficient quantum computation [1]. Various cooling schemes have been suggested and implemented, achieving lower and lower temperature and increased cooling rates. The schemes range from the simplest Doppler cooling [2, 3] to more sophisticated ion trap cooling methods, including sideband cooling for two level atoms [2, 4, 5], Raman side band cooling [6] which may be used for ions and for atoms [7, 8] and cooling schemes based on electromagnetically-induced transparency (EIT) [9]. The existing schemes may be divided into pulsed schemes [6] and continuous schemes.

The two-level and Raman sideband cooling schemes employ Rabi frequencies that are typically three orders of magnitude lower than the trap frequency. This is necessary to avoid off-resonant excitations which increase the average phonon number $\langle n \rangle$ and thus lead to heating. Hence, these off-resonant excitations increase the final temperature of the scheme. To overcome this limit, a scheme that employs EIT was introduced in [9] and demonstrated in [10]. This scheme uses interference to cancel the undesirable off-resonant carrier transition by suppression of absorption. This scheme can achieve low temperatures as well as high cooling rates, but the exact calibration of the laser intensity and the high intensities which are needed, makes the scheme challenging.

In this work, we present an alternative approach to this problem that relies on the fast Stark shift gate [11, 12]. In this scheme, off-resonant carrier transitions that would lead to heating are forbidden even when operating in the traveling wave regime at Rabi frequencies that are of the order of the trap frequency. The higher Rabi-frequency and the cancellation of the carrier transition promises increased cooling rates and lower final temperatures. This advantage applies both for the pulsed scheme and the continuous cooling schemes. It is important to note that unlike the original Stark shift gate which requires careful intensity stabilization to achieve high fidelity [11, 12] the Stark shift cooling method is only weakly affected by laser intensity fluctuations. Our scheme is applicable for trapped ions or atoms and is also suitable for an ion trap scheme employing a microwave addressing method in strong magnetic field gradients as proposed in [13] as the effective Hamiltonians in both settings are the same. The presented scheme satisfies the dark state condition [14], i.e. the ions are pumped into a zero-phonon state that no longer interacts with the applied laser fields.

This paper is organized as follows. We first describe the operation of the Stark shift gate and then explain the idea of using this gate for cooling both in pulsed and continuous operation. The main body of the paper will then be concerned with a detailed analysis of the continuous cooling scheme. We deduce the rate equation in the Lamb–Dicke limit and verify the predictions thus obtained with exact numerical simulations. In order to discuss the efficiency of the method we compare the proposed cooling scheme with previously proposed cooling schemes. We also demonstrate the applicability of the scheme to chains of ions, presenting numerical results for the simultaneous cooling of three ions and finish with a discussion and conclusion.

2. The Stark-shift gate

In the following, we briefly explain the basic mechanism underlying the Stark shift gate [11, 12] as it is crucial to our cooling scheme. The Hamilton operator of a single ion driven by a traveling-wave laser field of Rabi frequency Ω in the standard interaction picture with respect

to the free atom and phonons is given by

$$H = \Omega\{|\uparrow\rangle\langle\downarrow| \exp(\mathrm{i}\eta[a\mathrm{e}^{-\mathrm{i}\nu t} + a^{\dagger}\mathrm{e}^{\mathrm{i}\nu t}] - \mathrm{i}\delta t) + \mathrm{h.c.}\}, \tag{1}$$

where $\hbar = 1$, δ is the laser atom detuning, ν the trap frequency and η is the Lamb–Dicke parameter [15]. For $\delta = 0$ and employing $\eta \ll 1$, the exponent may be expanded to first order in η yielding $H \cong \Omega(\sigma_x + \eta\sigma_y(a\mathrm{e}^{-\mathrm{i}\nu t} + a^{\dagger}\mathrm{e}^{\mathrm{i}\nu t}))$. In a further interaction picture with respect to $H_0 = \Omega\sigma_x$, we obtain $H_{\mathrm{I}} = \mathrm{i}\eta\Omega'[\mathrm{e}^{\mathrm{i}(2\Omega-\nu)t}\sigma_+ a + \mathrm{e}^{\mathrm{i}(2\Omega+\nu)t}\sigma_+ a^{\dagger}] + \mathrm{h.c.}$, where $\sigma_+ = |+\rangle\langle -|(|\pm\rangle = |\uparrow\rangle \pm |\downarrow\rangle)$ and $\Omega' = \Omega\mathrm{e}^{-1/2\eta^2}$. Setting $\Omega = \frac{\nu}{2}$ (see also [11, 12, 16]) and neglecting off resonant terms we obtain

$$H_{\mathrm{ss}} = \frac{\mathrm{i}\eta\nu}{2}[\sigma_+ a - \sigma_- a^{\dagger}]. \tag{2}$$

Careful numerical investigations demonstrate that H_{ss} represents an excellent approximation to the exact dynamics [11, 12]. It is of particular importance to note that the Stark-shift gate does not have off-resonant carrier transitions, despite being operated in the traveling wave regime, making it a very accurate and fast quantum gate. As the carrier transition represents the dominant heating term in laser cooling, this cancellation of the carrier transition suggests the use of the Stark shift gate for fast laser cooling. The Stark shift gate requires stabilization of the Rabi frequency at $\Omega = \frac{\nu}{2}$. However, in laser cooling we are only interested in the preparation of a target state—the ground state. This suggests that the constraints on the stability of the Rabi frequency are much less severe in the application of the Stark shift gate to laser cooling than for its gate operation. The verification and exploitation of this idea is the purpose of the remainder of this work.

3. Stark-shift gate cooling

In the following, we will describe how the Stark shift gate may be used to implement both pulsed and continuous laser cooling schemes. We begin with the pulsed scheme as this clarifies the origin of the speed advantage in both schemes. We proceed in close analogy to the pulsed scheme based on the regular gate [6]. Assume that the system starts in state $|\downarrow\rangle|n\rangle$, where $|\uparrow\rangle$, $|\downarrow\rangle$ are internal states of the ion. In the Schroedinger picture, we apply a $\pi/2$ pulse implementing $|\downarrow\rangle|n\rangle \rightarrow |-\rangle|n\rangle$. Now we move to the interaction picture with respect to $\Omega\sigma_x$ and apply the Hamiltonian in equation (2), i.e. the Stark shift gate, to achieve $|-\rangle|n\rangle \rightarrow |+\rangle|n-1\rangle$. Now we return to the Schroedinger picture resulting in a global phase only. After some time spontaneous decay predominantly initializes the state to $|\downarrow\rangle|n-1\rangle$ and the process as a whole leads to the net loss of one phonon in the Schroedinger picture. This set of pulses is repeated until the final temperature is reached. This scheme yields lower final temperatures and is faster under ideal conditions than the usual sideband cooling. The bottleneck of the scheme is the $|-\rangle|n\rangle \rightarrow |+\rangle|n-1\rangle$, which lowers $\langle n\rangle$ and it is this process that is accelerated by the use of the Stark shift gate. The above scheme may also be implemented in the microwave regime [13] where instead of the cooling laser, a combination of a microwave and a steep magnetic field gradient is used and the coupling to the dissipative level is achieved by resonant lasers, which makes the effective Hamiltonian identical to the presented Hamiltonian.

The continuous scheme is most efficiently implemented in a three level configuration as shown in figure 1. The Hamiltonian of this system is given by

$$\begin{aligned} H = {} & \omega_1 |g_1\rangle\langle g_1| + \omega_2 |g_2\rangle\langle g_2| + \omega_3 |e\rangle\langle e| + \nu a^{\dagger}a \\ & -\Omega\left(|g_1\rangle\langle e|\,\mathrm{e}^{-\mathrm{i}(\omega_1+\Delta)t} + |g_2\rangle\langle e|\,\mathrm{e}^{-\mathrm{i}(\omega_2+\Delta)t} + \mathrm{h.c.}\right) - \Omega_{\mathrm{c}}\left(|g_1\rangle\langle g_2|\,\mathrm{e}^{\mathrm{i}(k_{\mathrm{c}}x-\omega_{\mathrm{c}}t)} + \mathrm{h.c.}\right). \end{aligned} \tag{3}$$

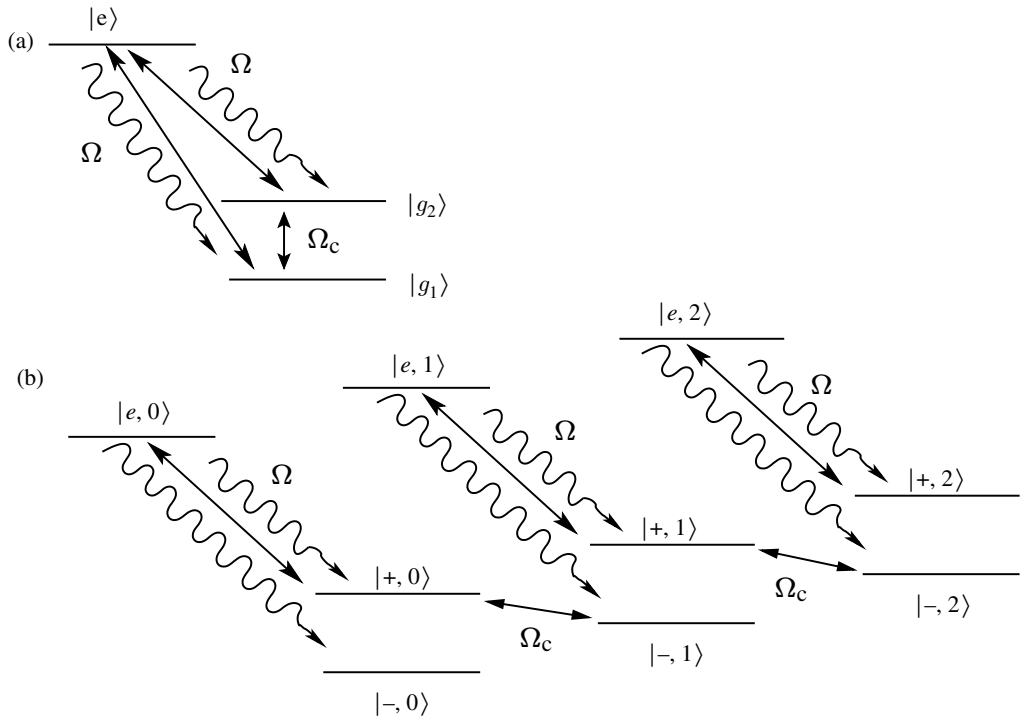


Figure 1. The required atomic level structure is given in the left upper corner (a). A cooling laser Ω_c manifests the Stark shift gate, creating a rotation $|-\rangle\,|n\rangle \leftrightarrow |+\rangle\,|n-1\rangle$. The Ω lasers couple only the $|+\rangle$ state to the dissipative level and leave the $|-\rangle$ state decoupled. The cooling process is schematically depicted in the lower right part of the picture (b). This scheme represents both the continuous and pulsed scheme the difference is in the way the coupling is done to the dissipative levels and the nature of lasers.

The laser on the narrow $|g_1\rangle \leftrightarrow |g_2\rangle(|\downarrow\rangle \leftrightarrow |\uparrow\rangle)$ transition is used to implement the Stark shift gate by tuning its Rabi frequency to the resonance condition $\Omega_c = \frac{\nu}{2}$. This implements the rotation $|-\rangle\,|n\rangle \rightarrow |+\rangle\,|n-1\rangle$. The two lasers on the $|g_i\rangle \leftrightarrow |e\rangle$ transition are chosen to have equal Rabi frequency and detuning and thus couple the $|+\rangle = \frac{1}{\sqrt{2}}(|g_1\rangle + |g_2\rangle)$ state to the dissipative level $|e\rangle$ with spontaneous decay rates $2\Gamma_1$ and $2\Gamma_2$ to the states $|g_1\rangle, |g_2\rangle$, respectively. The two lasers leave the state $|-\rangle$ decoupled from the dissipative level. It is important to note that the lasers on the $|g_i\rangle \leftrightarrow |e\rangle$ do not couple the internal and the phonon degrees of freedom, i.e their Lamb–Dicke parameter should be made as small as possible by adjusting the angle between the laser and the ion trap.

The basic cooling cycle in the continuous scheme works as follows. The laser with Rabi frequency Ω_c creates the rotation $|-\rangle\,|n\rangle \leftrightarrow |+\rangle\,|n-1\rangle$ based on the Stark shift gate. The lasers couple the $|+\rangle\,|n-1\rangle$ state to the dissipative level $|e\rangle|n-1\rangle$, which dissipates back to one of the $|g_i, n-1\rangle$'s which are superpositions of $|+\rangle|n-1\rangle$ and $|-\rangle|n-1\rangle$. If the decay is to the $|-\rangle|n-1\rangle$, one phonon has been lost and cooling is achieved, whereas if the decay is to the $|+\rangle|n-1\rangle$ state the cycle is repeated. Thus on an average phonons are dissipated out of the system. Note that this mechanism is independent of the sign of the detuning Δ.

3.1. Analytical results

Following the heuristic arguments presented so far we will now derive analytic expressions for the cooling rate and final temperature of our scheme based on a master equation approach. The dynamics and the final state can be obtained from the master equation,

$$\frac{\partial}{\partial t}\rho = \frac{1}{i}[H, \rho] + \mathcal{L}\rho, \tag{4}$$

where H is the Hamiltonian equation (3) and $\mathcal{L}$ is the Liouville operator describing the dissipative channels from level $|e\rangle$ to $|g_i\rangle$ including the effects of recoil due to photon emission. In the following, we use the techniques presented in [17, 18]. In this formalism, the equation of motion for the phonons is obtained by adiabatic elimination of the internal degrees of freedom of the ions. The final temperature and the cooling rate are obtained by expanding the Liouville operator and the density matrix in equation (4) in the Lamb–Dicke parameter $\mathcal{L} = \mathcal{L}_0 + \eta\mathcal{L}_1 + \eta^2\mathcal{L}_2 + \mathcal{O}(\eta^3)$, $\rho = \rho_0 + \eta\rho_1 + \eta^2\rho_2 + \mathcal{O}(\eta^3)$. Inserting this into equation (4) results in three equations, each for a different power of the Lamb–Dicke parameter as in [17]. From these three equations the final temperature and the cooling rate are calculated. The validity of the expansion in the Lamb–Dicke parameter at the point $\Omega_c = \frac{\nu}{2}$ is restricted by the three conditions $\Gamma\nu\eta$, $\nu^2\eta$, $\Delta\nu\eta \ll \Omega^2$ (which means that the internal dynamics has to be faster than the external one). The result of this approach is a rate equation for the phonon probability distribution $P(n)$ ($\Gamma = (\Gamma_1 + \Gamma_2)/2$) [19]

$$\frac{\mathrm{d}}{\mathrm{d}t}P(n) = \eta^2\Big(A_-[(n+1)P(n+1) - nP(n)] + A_+[nP(n-1) - (n+1)P(n)]\Big), \tag{5}$$

where

$$A_-(\nu) = \frac{2\Gamma\Omega^2\Omega_c^2}{\Gamma^2(\nu - 2\Omega_c)^2 + \big(2\Omega^2 + (\nu - 2\Omega_c)(\Delta - \nu + \Omega_c)\big)^2}, \tag{6}$$

and $A_+ = A_-(-\nu)$. From these expressions the final population and the rate are deduced to be

$$\langle n\rangle = \frac{A_+}{A_- - A_+} = \frac{\Gamma^2(\nu - 2\Omega_c)^2 + \big(2\Omega^2 + (\nu - 2\Omega_c)(\Delta - \nu + \Omega_c)\big)^2}{4\nu\left(2\Omega_c\Gamma^2 + (\Delta + 3\Omega_c)\left(\nu^2 + 2\left(\Omega_c(\Delta + \Omega_c) - \Omega^2\right)\right)\right)}, \tag{7}$$

$$W = \eta^2(A_- - A_+), \tag{8}$$

where W is the cooling rate. Numerics shown in figure 3 confirm that the $\langle n\rangle$ is minimized at the Stark shift gate point $\Omega_c \approx \frac{\nu}{2}$. At $\Omega_c = \frac{\nu}{2}$ we find $W = \frac{1}{8}\Gamma\eta^2\nu^2\Omega^2(\Omega^{-4} - 4[4\Gamma^2\nu^2 + (3\nu^2 + 2\Delta\nu - 2\Omega^2)^2]^{-1})$. The choice $\frac{\Gamma\nu}{\Omega^2}\eta = 1$ realizes a cooling scheme with a rate $W = \frac{1}{8}\eta\Omega_c$ which is close to the inverse of the gate time $T = \frac{\pi}{\eta\Omega_c}$. Note that this choice saturates the conditions of validity in the above derivation. If one of the other conditions is saturated as well the final population is smaller by the factor of 2, which means that the choice $\Delta = \Gamma$ is optimal. Nevertheless, numerical results shown in figure 2 corroborate that the cooling time is of the order of the time of a Rabi cycle of the Stark shift gate. Figure 3 shows numerically and analytically that for the final $\langle n\rangle$ as a function of Ω_c we obtain a parabola with a minimum around the Stark shift resonance. At this point we find

$$\langle n\rangle = \frac{\Omega^4}{\nu\left(\nu\Gamma^2 + (\Delta + (3\nu/2))\left(\nu^2 + 2\left((1/2)\,(\Delta + (\nu/2))\,\nu - \Omega^2\right)\right)\right)}. \tag{9}$$

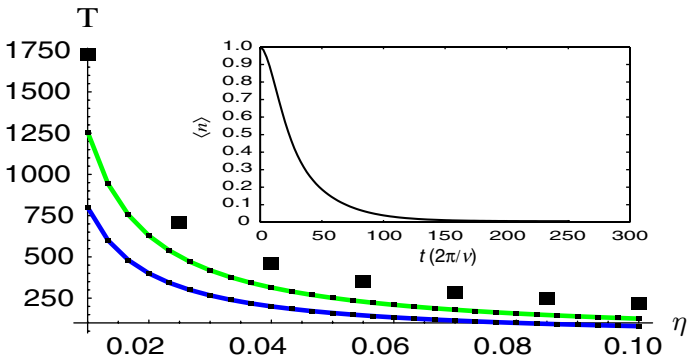


Figure 2. The cooling time in units of $2\pi/\nu$ at the optimal point is shown. The squares are the cooling time, defined as the time that takes to reduce the population from $\langle n\rangle = 1$ to $\langle n\rangle = 0.01$. The blue line corresponds to half a period of a Rabi cycle of the Stark shift gate and the green line to the cooling time calculated from the rate equations $W = \frac{1}{8}\eta\Omega_c$. The inset shows a numerical simulation of the time evolution of the mean phonon number $\langle n\rangle$ from the solution of the master equation. The parameters used in the inset are $\Delta = 0, \Gamma = 6, \nu = 1, \Omega_c = 1/2, \Omega = 1$ and $\eta = 1/10$. For these parameters the gate time is $\frac{\pi}{\eta\Omega} = 62.5$. Our numerics shows that the cooling time is of the same order of magnitude as the period of a Rabi cycle of the corresponding Stark shift gate.

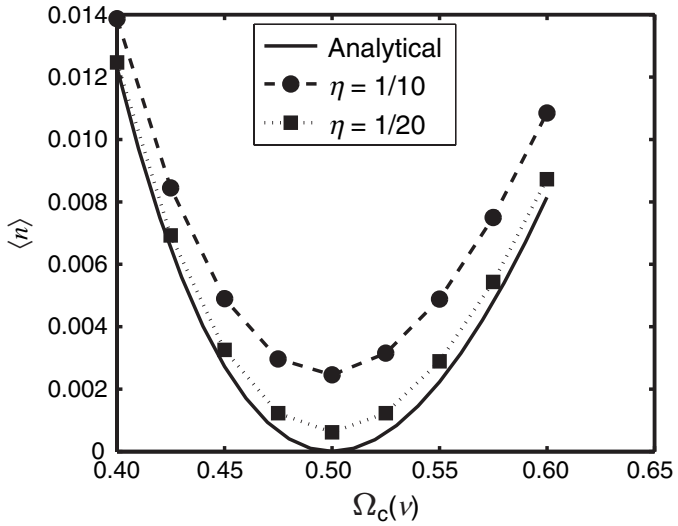


Figure 3. $\langle n\rangle$ as a function of Ω_c in units of ν. The minimum is at the Stark shift resonance. The continuous line is the rate equation result. The squares are calculated from a numerical solution of the master equation for $\eta = 1/20$ and the circles are for $\eta = 1/10$. In the limit $\eta \to 0$ the numerics coincide with the rate equations. The parameters are $\Gamma = 10, \Omega = 1/10, \nu = 1$ and $\Delta = 0$.

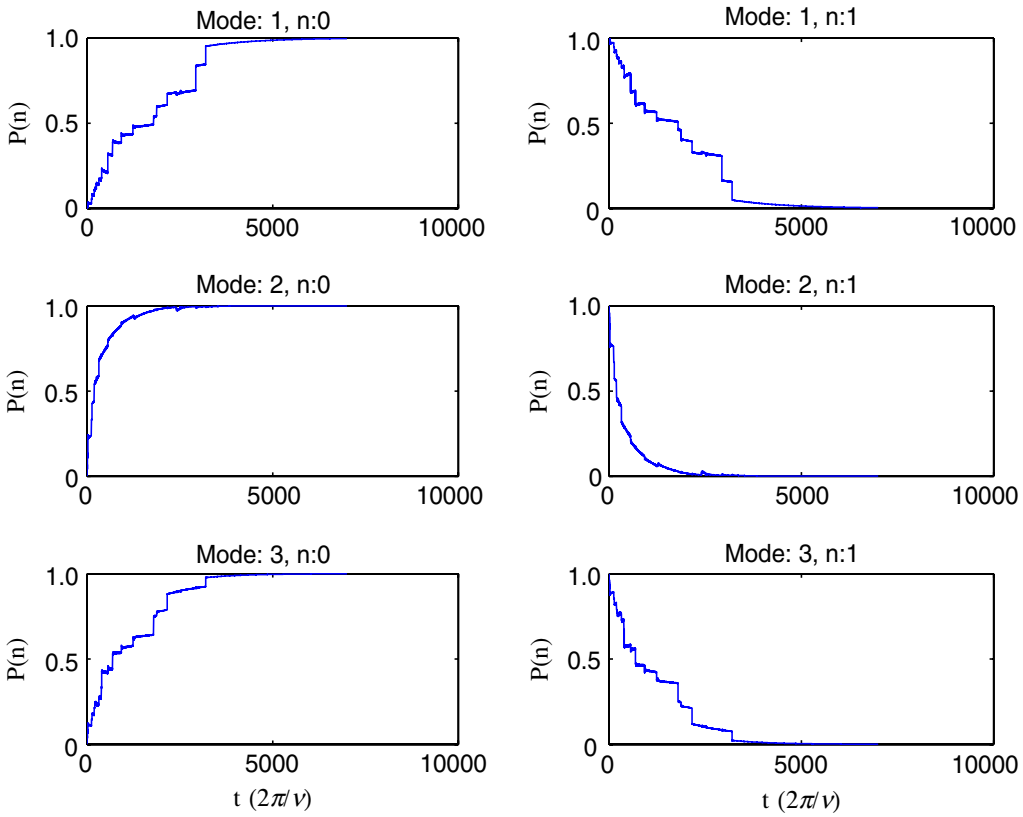


Figure 4. Monte-Carlo simulation for the cooling of three ions. The Rabi frequency is set to $\Omega_c = \frac{1}{2}\nu_2 + 0.15$. For three ions the frequencies are $\nu_2 = \sqrt{3}\nu_1$ and $\nu_3 = \sqrt{29/5}\nu_1$. The time units are $2\pi/\nu$. Modes 1, 2 and 3 are the three normal modes and n is the number of phonons which could be either 0 or 1.

Inserting the values of the optimum point, $\Gamma\nu\eta$, $\nu^2\eta$, and $\Delta\nu\eta = \Omega^2$, yields a temperature as low as $\eta^2/4$. Ω is set to $\sqrt{\Gamma\nu\eta}$ and we make the optimal choice $\Delta = \Gamma$. For $\Omega_c = \nu/2$ heating A_+ is weaker than cooling A_- whenever $\Omega^2 \leqslant \frac{\nu\left(4\Gamma^2+4\Delta^2+9\nu^2+12\Delta\nu\right)}{4(2\Delta+3\nu)}$. The cooling efficiency is not strongly affected by the fluctuations of the laser. For example, at the optimal point, at $\eta = 0.05$, for intensity fluctuations of 10%, the rate and the final temperature change by less than 50%.

3.2. Numerical results

The results deduced from the rate equations are correct in the limit $\eta \to 0$. In order to check the results for finite η and see the dynamics, we compare these results to a numerical solution of the master equation. In figure 3, a plot of the analytical final mean phonon number is compared to numerical results and as $\eta \to 0$ the rate equation results are reproduced. In order to compare the rates found for the final stages of cooling with the actual cooling time, we reproduce the dynamics by a numerical solution of the master equation for the cooling of one phonon. The inset in figure 2 shows the dynamics of the mean phonon number for the initial state of one phonon. The cooling scheme works for higher mean phonon numbers as well.

The mean phonon number and the rate in sideband cooling are $\langle n\rangle = (\alpha + \frac{1}{4})\,(\Gamma'/\nu)^2$ and $W \approx \eta^2\Gamma'$, where α is a geometric factor of order one [20, 21] and Γ' is the effective linewidth which can be controlled in three level systems [22]. In order for sideband cooling to operate at the same rate as the proposed cooling, Γ' has to be of the order of $\nu/2\eta$ which means that the sidebands are not resolved, hence, sideband cooling could not be applied. The cooling rates and temperatures found in our scheme could in principle be achieved by the EIT cooling method [9, 10] but in this method the laser power should be at least an order of magnitude stronger than in the cooling described in this work, due to the relation $\Omega = 2\sqrt{\omega\Delta_\sigma}$ [10], where Δ_σ is the detuning and ω is the vibrational frequency. Moreover, due to the high precision needed in the EIT cooling the final temperatures in the current proposal would be lower for given laser fluctuations.

3.3. Multi-mode cooling

The cancellation of the carrier transition suggests that our cooling method based on the Stark shift gate could be used to cool several modes simultaneously. Multi-mode cooling has been proposed by various set-ups, for example [23]. Figure 4 shows Monte-Carlo simulation [24] for the cooling of three modes in a three ion chain. Here, we chose the laser Rabi frequency near the second mode frequency. The cooling rate achieved in this way was of course slower than for a single ion, but still faster than side band cooling. In a different scheme individual ions could be addressed with lasers of different Rabi frequency each at the optimal working point for a different mode. Thus the whole chain could be cooled at times of the order of the Stark shift gate time.

4. Conclusions

We have introduced a method for the fast cooling of an ion trap which operates with rates of the fast Stark shift gate [11]. The final temperatures achieved are proportional to η^2 (recoil energy). The cooling method is efficient both in continuous and pulsed operation, achieving lower temperatures and faster rates than ordinary sideband cooling. The gate is also suitable for ion traps with microwave frequency addressing [13].

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