Incentive mechanism design for carbon-aware electric vehicle charging coordination problem

Tianyi Zhong and David Angeli, *Fellow, IEEE*

***Abstract*— In this paper, we study the problem of coordinating the charging pattern of strategic electric vehicle (EV) users by taking carbon intensity into consideration. We implement an incentive mechanism to address the misalignment between individual cost minimisation and system-wide carbon reduction. This mechanism leverages a modified VCG framework tailored to distributed computation, enabling the system to elicit flexible participation from EV users without compromising environmental goals. While the framework supports various optimisation algorithms, we specifically select a cutting plane-based distributed algorithm due to its computational efficiency.**

***Index Terms*— Mechanism design, VCG mechanism, EV charging coordination, Distributed algorithm**

I. Introduction

THE growing number of electric vehicles (EVs) has a significant impact on the carbon neutrality goal since it helps reduce the transportation sector's carbon emissions. However, large-scale EV charging increases electricity demand and can lead to higher system-level carbon emissions when charging peaks do not align with low-carbon generation periods. A commonly used modelling framework assumes that EV users are primarily motivated by lower charging costs and reduced battery degradation rather than environmental considerations [1], [2], [3]. Within this model, they respond mainly to electricity price signals and typically do not voluntarily account for carbon intensity in their charging behaviour. This raises one critical challenge on *how to incentivise EV users to provide their flexibility to contribute to decarbonisation*. In [4], [5], EVs are modelled collectively as an aggregated load, rather than representing each vehicle's charging behaviour individually. A fleet of EVs shifts its charging strategy across time in response to electricity price differences. To combat climate change, works in [6], [7] solve a scalarised multi-objective optimisation problem by introducing a carbon tax. Authors in [8] proposed a carbon-aware smart charging strategy model for EVs considering their responses to Carbon Intensity (CI) signals and provided methods to forecast CI signals based on historical data. These approaches generally assume that EV users act cooperatively to support global objectives when presented with appropriate incentives or signals.

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Tianyi Zhong and David Angeli are with the Department of Electrical and Electronic Engineering, Imperial College London, United Kingdom (e-mail: tianyi.zhong20@imperial.ac.uk; d.angeli@imperial.ac.uk).

David Angeli is also with the Department of Information Engineering of University of Florence, Italy.

Another line of research explicitly models EVs as non-cooperative and self-interested agents. In such frameworks, each agent seeks to minimise its own cost, potentially at the expense of others. Works including [9], [10] study non-cooperative frameworks of EV users with other agents, such as grid companies, EV aggregators and new energy power plants. However, these models often focus on strategic behaviour at the system level and do not fully capture individual EV user incentives. Authors in [1], [3] model interactions between strategic EV users as multi-agent non-cooperative games. For any finite number of agents, an approximate Nash equilibrium is computed, while the exact Nash one is reached only in the limiting case where the number of agents tends to infinity. The work in [3] provides the means to quantify the efficiency of Nash equilibria compared with the social optimum. In non-cooperative settings, incentive mechanisms [11] have shown potential to align societal and individual interests.

In contrast to both assumptions above, this work models EV users as self-interested yet cooperative agents. That is, while users are allowed to select strategies based on their own preferences and objectives (i.e., they are strategic and self-interested), they are also expected to comply with the protocol enforced by the system (i.e., they act cooperatively during execution of the optimisation algorithm). This modelling choice reflects situations where the central authority, which governs the distributed charging coordination process, expects regular cooperative behaviour from users and can monitor agent compliance through tools such as filters [12], [13]. If agents deviate from expected behaviour, the system can detect them and take corrective actions. As a result, self-interested agents may strategically shape their local objectives or decisions in a way that optimises personal benefit while still appearing to comply with the system protocol.

This work models the EV network as a hierarchical framework where there exists a central authority deciding the final outcome and payments. We consider that the central authority expects EV users to perform cooperatively while allowing them to be self-interested. To achieve this, we embed gradient filters within the central authority to check if the agents are regularly executing the protocol. Agents are self-interested in the sense that they can select an arbitrary convex cost function (not necessarily their true cost) before executing the protocol. To overcome the misalignment between individual cost minimisation and system-wide carbon reduction, we apply an incentive mechanism to encourage EV users to contribute to carbon reduction. The contributions of this work are three-fold: (1) we model a distributed, multi-agent EV charging

problem that explicitly incorporates carbon intensity signals into the agents' decision-making process; (2) we introduce a hierarchical coordination structure in which the mechanism is distributed and is compatible with any gradient-based distributed optimisation algorithm; (3) we design and analyse the incentive mechanism that aligns individual user incentives with carbon-aware charging behaviour, ultimately promoting socially and environmentally optimal outcomes.

II. Problem Formulation

A. Charging Model

Consider a population of N EVs charging over a common charging horizon of 24 hours from 24.00 on one day to 24.00 the next day, with a time interval of $\Delta T = 0.5$ h. Let the charging horizon be $\mathcal{T} = \{0, \Delta T, \cdots, T\}$ where $T = (n-1)\Delta T$ and $n = 48$. This interval aligns with the demand and carbon intensity measurement interval published by the National Grid (https://grid.iamkate.com/) and the National Energy System Operator (https://www.neso.energy/about-neso/our-progress-towards-net-zero/carbon-intensity-dashboard).

For each EV $i \in \mathcal{N} = \{1, 2, \ldots, N\}$, the objective function for a charging strategy $u_i = [u_{i,0}, u_{i,\Delta T}, \cdots, u_{i,T}]^T$ is:

$$w_i(u_i) = \sum_{t\in\mathcal{T}} v_i(u_{i,t}\Delta T) + \alpha_i(\sum_{t\in\mathcal{T}} u_{i,t}\Delta T - \Gamma_i)^2 \quad (1)$$

where $u_{i,t}\Delta T$ represents the charging energy supplied to EV i from the grid over the t-th time interval, $\alpha_i > 0$ is a given parameter and the convex function $v_i(\cdot)$ represents the battery degradation cost. This model incorporates a battery degradation cost because the performance of LiFePO4 lithium-ion batteries, which have been widely used in EVs, are influenced by charging behaviour over many cycles [1]. Also, the maximum energy that can be received is:

$$\Gamma_i = \Theta_i(\bar{x} - x_{i,0}) \quad (2)$$

where Θ_i is the battery capacity, $x_{i,t}$ is the battery state of charge (SoC) for EV i at time t and $\bar{x}$ is the maximum SoC. The dynamic between charging power and SoC is:

$$x_{i,t+1} = x_{i,t} + \frac{\delta}{\Theta_i} u_{i,t} \quad (3)$$

where δ is an EV-specific charging parameter used to convert charging power into the corresponding increase in SoC. The constraints on the charging strategy $u_{i,t}$ and the SoC $x_{i,t}$ are:

$$u_{i,t} \geq 0, \sum_{t\in\mathcal{T}} u_{i,t}\Delta T \leq \Gamma_i, \sum_{i\in\mathcal{N}} u_{i,t} \leq \bar{P} \quad (4a)$$

$$x_{i,0} \leq x_{i,t} \leq \bar{x} \leq 1 \quad (4b)$$

where $\bar{P}$ limits the total available charging power at time $t \in \mathcal{T}$. The local objective function (1) considers the tradeoff between battery degradation cost and the relative importance of charging the maximum energy.

Carbon intensity C_t (tCO2/GWh) reflects the grid-level carbon emission per energy unit, and it depends on the power grid's operational plan and the carbon intensity of each energy resource. Carbon intensity tends to be higher during the night and early morning, when solar energy availability is limited. Conversely, it typically decreases during the day, as renewable sources become the primary contributors to the power supply. The right plot in Fig. 1 shows the carbon intensity profile of the electricity system in the UK on a typical spring day.

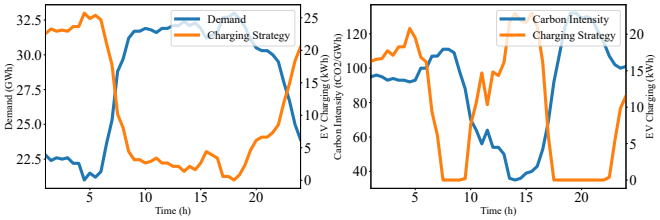


Fig. 1: Energy demand profile and charging strategy (left), carbon intensity profile and charging strategy (right) of carbon-aware EVs in a network of 100 EVs, where $\beta = 0.3$.

B. Social Model

The social model aims at deciding the charging power $u_{i,t}$ for vehicle i at time t based on the following social minimisation problem:

$$\min_{u} \sum_{t\in\mathcal{T}} \big(\lambda_t^p(rD_t + \sum_{i\in\mathcal{N}} u_{i,t}) + \beta\lambda_t^c C_t \sum_{i\in\mathcal{N}} u_{i,t}\big)\Delta T + \sum_{i\in\mathcal{N}} w_i(u_i) \quad (5)$$

s.t. (3) and (4) are satisfied,

where λ_t^p is an electricity price coefficient at time t, λ_t^c is a carbon price coefficient at time t. For each $t \in \mathcal{T}$, we denote D_t as the non-EV aggregate demand, r represents a scaling factor and β is a given parameter. The energy demand profile associated with the power demand $\boldsymbol{D} = (D_t), t \in \mathcal{T}$ is presented in the left plot in Fig. 1. The price of electricity λ_t^p depends linearly on the non-EV demand, $\lambda_t^p = \lambda D_t$. The quadratic terms in $w_i(u_i)$ show that the optimal charging strategy of problem (5) is unique. The social cost function (5) considers tradeoffs between total generation cost, carbon emission penalty, aggregate battery costs and the penalty for deviating from full charging. This problem can be solved by a distributed optimisation algorithm where agents transmit some of the local information to their neighbours.

As shown in Fig. 1, the carbon intensity profile tends to be inversely correlated with the demand profile. Carbon intensity is often higher during evening hours when fossil-fuelled generators are dispatched to meet demand, which coincides with lower electricity prices, as well as with the typical EV charging window after work hours. Despite this misalignment, individual users are primarily driven by cost, convenience, or operational constraints, rather than by environmental considerations. Hence, individuals have no motivation to alter their behaviour to reduce carbon emissions. Consequently, even though coordinated charging can help reduce system-level emissions by relieving grid stress during periods of high carbon intensity [14], individuals are unlikely to do so voluntarily. Incentive mechanisms are therefore essential to

bridge this gap and induce carbon-aware decision-making at the grid edge.

III. Carbon-Aware EV charging coordination under the DeVCG-G mechanism

According to (5) and $\lambda_t^p = \lambda D_t$, the local cost function of each cooperative EV $i \in \mathcal{N}$ is:

$$f_i(u_i) = \lambda \Delta T \sum_{t\in\mathcal{T}} D_t e_t^T u_i + \beta \Delta T \sum_{t\in\mathcal{T}} \lambda_t^c C_t e_t^T u_i + w_i(u_i) + \frac{\Delta T}{N} \sum_{t\in\mathcal{T}} \lambda r D_t^2 \tag{6}$$

where $e_t \in \mathbb{R}^n, t = 1, \ldots, n$ is the canonical basis and $\beta \in (0, 1)$. To capture the behaviour of strategic EV users who may not always act in accordance with the system-level cost, we allow each agent i to adopt a local evaluation function that may differ from (6) as shown in (7). In particular, each agent privately selects a local parameter γ_i, which defines the weight assigned to the carbon-related term.

$$f_i(u_i, \gamma_i) = \lambda \Delta T \sum_{t\in\mathcal{T}} D_t e_t^T u_i + \gamma_i \Delta T \sum_{t\in\mathcal{T}} \lambda_t^c C_t e_t^T u_i + w_i(u_i) + \frac{\Delta T}{N} \sum_{t\in\mathcal{T}} \lambda r D_t^2 \tag{7}$$

The parameter $\gamma_i \in [-1, 1]$ represents the agent's willingness to contribute to carbon emission reduction. The interval $[-1, 1]$ allows agents to either downplay or exaggerate the importance of the carbon-related term in a bounded manner, which is necessary to preserve a well-scaled convex cost function. The strategic behaviour $\gamma_i < \beta$ corresponds to under-weighting the environmental term, and $\gamma_i > \beta$ represents over-weighting of environmental concern. Here, we use $f_i(\cdot, \gamma_i)$ to denote the function (7) parameterised by γ_i. Notice that setting $\gamma_i = \beta$ is an operator-desired behaviour $f_i(u, \beta) = f_i(u)$. We remark that the social charging strategy is computed using the evaluation functions (7), and that agents' individual costs are also evaluated according to their chosen evaluation functions.

The bar plot in Fig. 2 (a) shows how individual agent's actual cost changes with respect to different γ_i. Notice that the coefficient γ_i is never really declared. So even if its true value is known, the designer cannot know if an agent is adopting a strategy compliant with the desired truthful parameter $\gamma_i = \beta$ or not. The charging strategy shown in the left plot in Fig. 1 is obtained when all agents choose $\gamma_i = 0$. Before proceeding, we note that the notation $f_i(\cdot, s_{ij})$ will be used in later sections to represent specific instances of $f_i(\cdot, \gamma_i)$ corresponding to particular parameter values.

A. Distributed VCG-Gradient (DiVCG-G) mechanism

In this subsection, we focus on using mechanism design approach as a tool to obtain desired solutions of the optimisation problem (5). We model EV users as players who make decisions according to the game rules and adopt the following assumption.

Assumption 1 (Strategic and Malicious Behaviour). We assume EV users act as strategic agents and assume that all agents are self-interested: they aim to minimise their game-theoretical cost (in this work, the incentivised cost as designed in (8)) without any regard for others. Meanwhile, we allow agents to be malicious, i.e. to increase others' game-theoretical costs when their own are not influenced.

The idea behind mechanism design is to construct the rules of the game so that the strategic behaviour of self-interested agents leads to a system-wide outcome that aligns with the optimal solution of problem (5). The game induced by the mechanism includes:

1) **Agents:** EVs in the network. Agents are given the option of not participating in the mechanism. However, a quitting EV cannot be charged, as its charging profile is fully determined by the decisions of the participating agents. We assume there is a set of $\mathcal{I} = \{1, \cdots, I\}$ agents that join the mechanism and the induced game. If an agent i decides not to join the mechanism, he can not enter the induced game after the game begins. The number of agents who join the mechanism is in range $I \in [0, N]$ and $\mathcal{I} = \emptyset$ if $I = 0$. Each EV $i \in \mathcal{I} \subseteq \mathcal{N}$ can only communicate with his neighbours. Assumption 2 shows the inter-agent communication topology.
2) **Sequences:** We denote the network involving $\mathcal{I}$ agents solving $\min_{u_1,\cdots,u_N} \sum_{i\in\mathcal{I}} f_i(u_i, s_{i0})$ as the social sequence and the network involving $\mathcal{I}\backslash\{j\}$ agents solving $\min_{u_1,\cdots,u_N} \sum_{i\in\mathcal{I}\backslash\{j\}} f_i(u_i, s_{ij})$ as sequence j.
3) **Strategies:** A strategy profile of an agent i is $s_i = (s_{i0}, (s_{ij})_{j\neq i}) \subset ([-1, 1] \cup \emptyset)^N$. The first element in the strategy profile s_{i0} is called social sequence strategy and s_{ij} is termed strategy for sequence j. The social sequence strategy represents agents' willingness to contribute to carbon reduction while sequence j's strategy represents agents' malicious manipulation to agent j.

Denote s_{-i0} as $(s_{j0})_{j\neq i}$ and denote the strategy profile in the social sequence as $\bar{s}_0 = (s_{i0}, s_{-i0})$ and that in sequence j as $\bar{s}_j = (s_{ij})_{i\neq j}$, all agents' incentivised cost function is:

$$\hat{f}_i(o^*, p_i, k_f) = f_i\big(o^*(\bar{s}_0, k_f), \beta\big) + p_i \tag{8}$$

The notation $o^*(\cdot, k_f)$ denotes the social outcome obtained by the distributed algorithm after k_f iterations. For non-participating agents $i \notin I$, the notation $o^*((\emptyset, s_{-i0}), k_f)$ is interpreted as agent i's strategy is $\emptyset$, and outcome is determined by the joining agents. The payment p_i and maximum iteration k_f are decided by the central authority of the mechanism.

The description of our mechanism Γ_{k_f} is divided into four main parts: the message space, the imposed filtering action, the outcome selection rule and the payment function.

1) The message space: The message space is two-fold:

$$M = \Pi_i\big((\Pi_k M_k^i) \times \hat{M}_i\big) \tag{9}$$

where M_k^i defines the set of possible messages of agent i at step k. Each $m_k^i \in M_k^i$ has the following form:

$$m_k^i = [(u_k^i, g_k^i), (u_k^{ij}, g_k^{ij})_{j\neq i}] \tag{10}$$

which contains I tuples with the first tuple corresponding to the transmitted messages in the social sequence and the other $I - 1$ tuples corresponding to that of sequences $j \neq i$.

Each tuple contains two vectors. The social sequence tuple includes: the local decision u_k^i, the gradient of cost function $g_k^i = \nabla f_i(u_k^i, s_{i0})$. Similarly, sequence j's tuple includes u_k^{ij}, $g_k^{ij} = \nabla f_i(u_k^{ij}, s_{ij})$. Each $\hat{m}_i \in \hat{M}_i$ is a vector that contains I scalars and has the following form:

$$\hat{m}_i = (\tau_i, (\tau_{ij})_{j\neq i}) \tag{11}$$

where the scalar τ_i is agent i's function value for the selected social outcome o^*, $\tau_i = f_i(o^*, s_{i0})$. Similarly, the scalar $\tau_{ij}, j \neq i$ is agent i's function value for the selected sequence j's outcome o_j, $\tau_{ij} = f_i(o_j, s_{ij})$.

2) Impose filters [12]: We employ I data filtering nodes within the central unit, denoted as $\mathcal{F}_i, i \in \mathcal{I}$. The central authority randomly decides steps $t_s < t_f < k_f$ at which $\mathcal{F}_i$ starts and ends filtering $\{g_k^i\}_{t_s\leq k\leq t_f}$, $\{g_k^{ij}\}_{t_s\leq k\leq t_f}$ and uses the filtered information to calculate agent i's payment. However, the filtered information will be kept private within the central authority. Adopting $0 < t_s < t_f < k_f$ is to reduce the computation and memory load, but ideally, the filter would be activated from step 0 to the final step. The central authority first causally combines the received gradient $\{g_t^i, (g_t^{ij})_{j\neq i}\}$ and state $\{u_t^i, (u_t^{ij})_{j\neq i}\}, t \leq k$ to obtain $\{\xi_t^i\}_{0\leq t\leq I(k+1)-1}$ and $\{\eta_t^i\}_{0\leq t\leq I(k+1)-1}$. Specifically, for any $t \in [0, k]$, $\xi_{It}^i = g_t^i, \xi_{It+j}^i = g_t^{ij}, \eta_{It}^i = u_t^i, \eta_{It+j}^i = u_t^{ij}, 1 \leq j \leq i-1$ and $\xi_{It+j}^i = g_t^{ij+1}, \eta_{It+j}^i = u_t^{ij+1}, i \leq j \leq I-1$. Then, $\mathcal{F}_i, \forall i \in \mathcal{I}$ filters the gradient data at $t \in [Ik, I(k+1)-1]$:

$$\begin{aligned}
&\tilde{\xi}_t^i = \arg\min_{\xi\in\mathbb{R}^n} \|\xi - \xi_t^i\|^2 \\
&\text{s.t. } (\tilde{\xi}_t^i)^T(\eta_\tau^i - \eta_t^i) + F_{\tau,m}^i(t-1) + (\tilde{\xi}_m^i)^T(\eta_t^i - \eta_m^i) \leq 0 \\
&\qquad \forall \tau, m \in \{1, 2, \ldots, t-1\}
\end{aligned} \tag{12}$$

where $F_{1,1}^i(0) := 0, \forall i \in \mathcal{I}$:

$$\begin{aligned}
F_{\tau,t}^i(t) &= \max_{m\in\{1,\ldots,t-1\}} F_{\tau,m}^i(t-1) + (\tilde{\xi}_i^m)^T(\eta_i^t - \eta_i^m) \\
F_{t,m}^i(t) &= \max_{\tau\in\{1,\ldots,t-1\}} (\tilde{\xi}_i^t)^T(\eta_i^\tau - \eta_i^t) + F_{\tau,m}^i(t-1) \\
F_{\tau,m}^i(t) &= \max\{F_{\tau,m}^i(t-1), F_{\tau,t}^i(t) + F_{t,m}^i(t)\}
\end{aligned}$$

The filtering process projects the agent's reported gradient across all sequences onto a set of gradients that are consistent with a single convex function. The filtered data are not directly used in the distributed algorithm and the computation of (12) does not influence the distributed optimisation process.

3) The outcome selection rule: After receiving the final step decisions from each agent, the social outcome o^* and sequence i's outcome o_i are calculated as the component-wise median of $\{u_{k_f}^i\}$ and $\{u_{k_f}^{ji}\}$:

$$\begin{aligned}
o^*(\bar{s}_0, k_f) &= Median(u_{k_f}^i) \\
o_i(\bar{s}_i, k_f) &= Median(u_{k_f}^{ji})
\end{aligned} \tag{13}$$

We remark that $\bar{s}_0 = (s_{i0}, s_{-i0})$ denotes all agents' strategies used in the social sequence (which decides the social outcome) and $\bar{s}_i = (s_{ji})_{j\neq i}$ represents strategies determined by all other agents $j \neq i$ in sequence i (which decides agent i's payment as shown in (14)).

4) The payment function: If all agents quit the game, the payment for each agent $i \in \mathcal{N}$ is $\bar{p}$, where $\bar{p} < +\infty$ is positive and sufficiently large. Otherwise, the payment for each agent $i \in \mathcal{N} \setminus \mathcal{I}$ is $p_i = 0$, and the payment for each agent $i \in \mathcal{I}$ is calculated as:

$$\begin{aligned}
p_i(s_i, \bar{s}_i, \bar{s}_0, k_f) &= \sum_{j\neq i} \tau_i - \sum_{j\neq i} \tau_{ji} + \pi_i(k_f) \\
&= \sum_{j\neq i} f_j(o^*, s_{j0}) - \sum_{j\neq i} f_j(o_i, s_{ji}) + \pi_i(k_f)
\end{aligned} \tag{14}$$

with

$$\pi_i(k_f) = \begin{cases} 0, & \text{if } \bar{e}_i = 0 \\ k_f \cdot \bar{e}_i + 1, & \text{otherwise} \end{cases} \tag{15a}$$

$$\bar{e}_i = \sum_{t=It_s}^{I(t_f+1)-1} \|\tilde{\xi}_t^i - \xi_t^i\|^2 \tag{15b}$$

where $\tilde{\xi}_t^i$ is calculated as in (12). We use the notation $p_i(s_i, \bar{s}_0, \bar{s}_i, k_f)$ to explicitly indicate the dependence of agent i's payment on his chosen strategy. Correspondingly, we denote agent i's incentivised cost (defined in (8)) by $\hat{f}_i(s_i, \bar{s}_i, \bar{s}_0, k_f)$. Notice that the factor $\bar{s}_i$ cannot be controlled by agent i, and it represents the maliciousness of other agents imposed on agent i. We remark that the inclusion of π_i is intended to mitigate malicious behaviour among agents, and the importance of this term is demonstrated in Fig. 2 (b).

B. Distributed calculation of outcomes

In general, any gradient-based distributed algorithm can be applied to calculate outcomes in each sequence. We base our distributed computation on the distributed cutting plane tracking algorithm [15]. The assumptions of the communication graph and the weights are as follows.

Assumption 2 (Connectivity and Communication): The undirected graphs $\mathcal{G}^\infty := (\mathcal{I}, \bigcup_{\tau=1}^\infty E(\tau))$ and $\mathcal{G}_q^\infty := (\mathcal{I} \setminus \{q\}, \bigcup_{\tau=1}^\infty E(\tau)), \forall q \in \mathcal{I}$ are fully connected. Moreover, there exists $T \geq 1$ such that $\bigcup_{\tau=k}^{k+T} E_q(\tau) = \mathcal{I}^2$ and $\bigcup_{\tau=k}^{k+T} E_q(\tau) = (\mathcal{I} \setminus \{q\})^2$ for all $k \in \mathbb{N}$, i.e. agent i receives information from a neighbouring agent j at least once every consecutive T iterations in all sequences.

Assumption 3 (Weight Coefficients): There exists $\zeta \in (0, 1)$ such that for all $i, j \in \mathcal{I}$ and all $k \geq 1$ $a_k^{ij} \geq \zeta$ if $(j, i) \in E(k)$, and $a_k^{ij} = 0$ otherwise. Likewise, for all $i, j \in \mathcal{I} \setminus \{q\}$ and all $k \geq 1$ $a_k^{ij}(q) \geq \zeta$ if $(j, i) \in E_q(k)$, and $a_k^{ij} = 0$ otherwise. Moreover, $\sum_{j\in\mathcal{I}} a_k^{ij} = 1$, for all $i \in \mathcal{I}$ and $\sum_{j\neq q} a_k^{ij}(q) = 1$, for all $i \neq q$; $\sum_{i\in\mathcal{I}} a_k^{ij} = 1$, for all $j \in \mathcal{I}$ and $\sum_{i\neq q} a_k^{ij}(q) = 1$, for all $j \neq q$.

During distributed computation, when the communication graphs $\mathcal{G}(k)$ and $\mathcal{G}_q(k), q \in \mathcal{I}$ are decided (either by the central unit or agents), each agent $i \in \mathcal{I}$ stores data from neighbour j, $(j, i) \in E(k) \cup_q E_q(k)$ including state $\{u_{k-1}^j, (u_{k-1}^{jq})_{q\neq i,j}\}$, gradient $\{g_{k-1}^j, (g_{k-1}^{jq})_{q\neq i,j}\}$ and function value $\{f_{k-1}^j, (f_{k-1}^{jq})_{q\neq i,j}\}$, where $f_{k-1}^j = f_j(u_{k-1}^j, s_{j0}), f_{k-1}^{jq} = f_j(u_{k-1}^{jq}, s_{jq})$. Let set $M_{k-1}^{ji} = \{m \leq k-1 : (j, i) \in E(m)\}$ be the set that includes steps (up to $k-1$) at which agent i receives data from j in the social

sequence and let set $M^{ji}_{q,k-1} = \{m \leq k-1 : (j,i) \in E_q(m)\}$ be the set that includes steps (up to $k-1$) at which agent i receives data from j in sequence q. Individual agent i locally builds $I-1$ estimation models $f^{i:j}_k(u)$ for agent $j \neq i$:

$$f^{i:j}_k(u) := \max\Big\{ \max_{m \in M^{ji}_{k-1}} f^j_m + (g^j_m)^T(u - u^j_m), \quad \max_{m_q \in M^{ji}_{q,k-1}} f^{jq}_m + (g^{jq}_m)^T(u - u^{jq}_m)\Big\}, \forall q \neq i \tag{16}$$

Agents initialises states in social sequence $y^i_0 \in \mathbb{R}^{In}$, $u^i_0 = \frac{1}{I}\sum_{i=1}^{I} y^i_0$ and initialise states in sequence, $q \neq i$ $y^{iq}_0 \in \mathbb{R}^{In}$, $u^{iq}_0 = \frac{1}{I-1}\sum_{i=1}^{I-1} y^{iq}_0$. At every step k, the updates are:

$$\begin{aligned}
y^i_k =& \arg\min_u \sum_{j \in \mathcal{I}} f^{i:j}_k(u) \quad \text{s.t.(3) and (4) are satisfied} \\
y^{iq}_k =& \arg\min_u \sum_{j \in \mathcal{I}\setminus\{q\}} f^{i:j}_k(u) \quad \text{s.t.(3) and (4) are satisfied} \\
u^i_k =& \sum_{j \in \mathcal{I}} a^{ij}_k u^j_{k-1} + y^i_k - y^i_{k-1} \\
u^{iq}_k =& \sum_{j \neq i,q} a^{ij}_k(q) u^j_{k-1} + y^{iq}_k - y^{iq}_{k-1}
\end{aligned}$$

where y^i_k and y^{iq}_k are lex-optimal solutions. The algorithm terminates at a pre-decided step k_f, and the central authority selects the outcome of each sequence according to agents' local decisions as in (13). The algorithm converges asymptotically (the proof is similar to [15]) and therefore:

$$\begin{aligned}
\lim_{k_f \to +\infty} u^i_{k_f} &= \arg\min_u \sum_{i \in \mathcal{I}} f_i(u, s_i) \\
\lim_{k_f \to +\infty} u^{iq}_{k_f} &= \arg\min_u \sum_{j \neq q} f_j(u, s_j)
\end{aligned} \tag{17}$$

A distinguishing feature of the distributed cutting plane algorithm is that the information transmitted across parallel optimisation problems contributes to building more accurate local models for each agent. The richer and more diverse information provided by parallel problems ensures that the algorithm refines local approximations of the global objective function more effectively and hence enhancing overall efficiency. Note that the algorithm used in this work is a primal version of the algorithm in [15] which can also be directly applied to problem (5) through solving its dual problem. The adaptation of DiVCG-G mechanism is to utilise the gradient of dual functions in (10) and (12).

C. Mechanism properties

A dominant strategy equilibrium (DSE) of agent i in the DiVCG-G game is a strategy profile $s^*_i = (s^*_{i0}, (s^*_{ij})_{j \neq i}) \in \mathbb{R}^N$ such that for all $s_{-i} \in (\mathbb{R}^N)^{N-1}$ and all $i \in \mathcal{N}$, $\hat{f}_i\big(s^*_i, \bar{s}_i, (s^*_{i0}, \bar{s}_{-i0}), k_f\big) \leq \hat{f}_i(s_i, \bar{s}_i, \bar{s}_0, k_f)$ holds, where $\bar{s}_i = (s_{ji})_{j \neq i}$ and $\bar{s}_0 = (s_{i0}, s_{-i0})$.

Lemma 1. The DSE of agent $i \in \mathcal{N}$ in the DiVCG-G game is $s^*_i = (s^*_{i0}, s^*_{ij})$ with $s^*_{i0} = \beta, s^*_{ij} = \beta, \forall j \neq i$ and the DSE is achieved asymptotically as $k_f \to +\infty$.

Lemma 1 characterises that agents' dominant strategy is to behave cooperatively in the algorithm. The social outcome is determined by the result of a distributed algorithm, terminating the process after a finite number of steps yields only an approximate solution. We employ the approximation notion that allows the dominant strategy to be satisfied up to a small ε, i.e., the agent is satisfied with the outcome where his incentivised cost is ε away from optimal.

Definition 1 (ε-Dominant Strategy Equilibrium). An ε-dominant strategy equilibrium (ε-DSE) of agent i in the DiVCG-G game for any given $\varepsilon \in (0,1)$ is defined as a strategy profile $s^*_i = (s^*_{i0}, (s^*_{ij})_{j \neq i}) \in \mathbb{R}^N$ such that for all $s_{-i} \in (\mathbb{R}^N)^{N-1}$ and all $i \in \mathcal{N}$, there exists a finite step k and the following holds for any $k_f \geq k$:

$$\hat{f}_i\big(s^*_i, \bar{s}_i, (s^*_{i0}, s_{-i0}), k_f\big) \leq \hat{f}_i(s_i, \bar{s}_i, \bar{s}_0, k_f) + \varepsilon \tag{18}$$

where $\bar{s}_i = (s_{ji})_{j \neq i}$ and $\bar{s}_0 = (s_{i0}, s_{-i0})$.

Definition 2 (Asymptotic Efficient). A sequence of distributed mechanism $\{\Gamma_k\}$ is asymptotic efficient if for every Γ_k, there exists $\varepsilon_k \in (0,1)$ such that an ε_k-DSE of its induced game $s^* = (s^*_i, s^*_{-i})$ with $s^*_i = (s^*_{i0}, s^*_{ij})_{j \neq i}$ and the corresponding social outcome $o^*((s^*_{i0}, s^*_{-i0}), k)$ fulfils:

$$\sum_{i \in \mathcal{N}} f_i(o^*((s^*_{i0}, s^*_{-i0}), k)) \leq \min_{u \text{ fulfils (3) and (4)}} \sum_{i \in \mathcal{N}} f_i(u) + \varepsilon_k$$

and $\lim_{k \to +\infty} \varepsilon_k = 0$.

Theorem 1. [16] Let all assumptions hold, given any tolerance $\varepsilon \in (0,1)$, there exists k_f such that the sequence of DiVCG-G mechanism $\{\Gamma_k\}_{k \geq k_f}$ is asymptotic efficient.

Theorem 1 establishes that as the iteration step of the distributed computation grows, the resulting social sequence outcome within the DiVCG mechanism framework converges to the socially optimal solution. It provides a theoretical guarantee that the performance gap between the implemented outcome and the social optimum can be made arbitrarily small by selecting a sufficiently large iteration step k_f.

IV. Numerical results

Consider a network with $N = 10, \alpha = 16, \beta = 0.3$ and the initial SoC values $x_{i,0}, i \in \mathcal{N}$ for each agent are randomly selected from $[0, 0.2]$. Also, we set $\lambda^c_t = 0.02$, $\Theta = 30$ kWh and $\bar{x} = 0.9$. The scaling factor is set as $r = 10^{-6}$, the electricity price is $\lambda^p_t = \lambda D_t$ with $\lambda = r$ and the battery degradation cost function is selected as $v_i(x) = 2x^2$. An agent i can be self-interested by deviating γ_i from its true value β in all sequences. We first consider that agents are using the same strategy in all sequences $s_i = s_{i0} = s_{ij}, \forall j \neq i$ and all agents joining the game $\mathcal{I} = \mathcal{N}$. Each agent $i \in \mathcal{N}$ chooses $s_i \in [-1, 1]$ before joining the game. The maximum iteration step is set as $k_f = 100$ to solve the optimisation problem in each sequence. Figure 2 (a) confirms that agents obtain the minimum incentivised cost when behaving truthfully, $s_i = \beta$.

We then examine a scenario where one agent i employs different manipulation strategies in different sequences and we illustrate why the term π_i is essential in the payment design. Specifically, we assume that agent i under-reports his carbon concerns by randomly selecting strategy s_{ij} from $s_{ij} \in [-\beta, 0]$ in sequence $j \neq i$ and other agents are playing truthfully. Figure 2 (b) shows how agents' incentivised cost (without π_i) changes, under the standard VCG mechanism

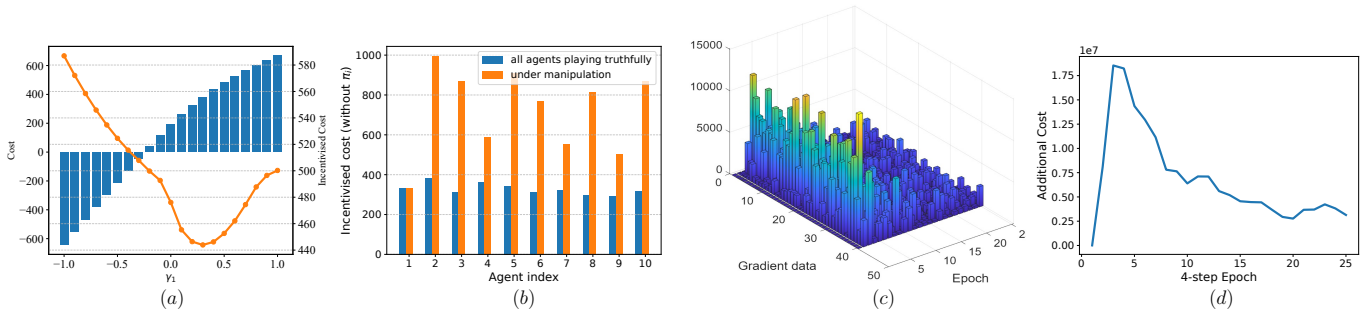


Fig. 2: (a) Agent 1's cost given by (7) (blue bars) and incentivised cost (8) (orange line) as γ_1 varies. (b) Agents are incentivised under standard VCG mechanism (without π_i term in (14)). Agent 1 plays maliciously by setting $\beta = s_1 \neq s_{1j}, \forall j \neq 1$. (c) The gradient violation error encountered by the strategic and malicious agent 1. The epoch length is 4 steps which includes 40 data samples provided by 10 sequences. The filter starting and ending steps of Epoch t are $t_s = 4(t-1)+1$ and $t_f = 4t$. (d) The additional cost obtained by agent 1 under the DiVCG-G mechanism (with π_i term in (14)).

framework (without π_i in (14)), as strategic agent $i = 1$ plays maliciously. Strategic and regular agents (agent 2-10) face an increased incentivised cost due to the malicious behaviour of agent 1. By carefully choosing s_{ij}, the malicious agent can increase others' incentivised cost to a considerably large value, hence discouraging others from joining the game. This result confirms that standard VCG framework (without π_i) is inefficient under manipulation. Figure 2 (c) shows how much the filter is violated in an epoch under the DiVCG-G mechanism framework. In this example, agent 1 encounters a significantly large additional cost (as presented in Fig. 2 (d)) caused by the gradient violation error. The proposed DiVCG-G mechanism aligns the cost of the malicious agent (agent 1) with its maliciousness. The greater the potential harm an agent can inflict on others, the greater the cost he will incur.

The complexity of deploying the DiVCG-G mechanism is primarily determined by the computational and communication requirements of running parallel distributed optimisation algorithms. The additional filtering process introduces negligible overhead. As shown in Fig. 2 (c)–(d), even a small number of filtering steps is sufficient to impose a substantial cost on malicious behaviour.

V. Conclusion

This work addresses the challenge of aligning self-interested EV user behaviour with system-wide carbon reduction goals in a carbon-aware charging coordination framework. We propose a hierarchical model in which a central unit selects outcomes based on distributed decisions and monitors agents' behaviour through a gradient filter. We introduce the concept of self-interested but cooperative agents who may strategically choose private costs but are expected to follow a prescribed protocol. To ensure that such strategic agents are incentivised to reveal useful information and act in ways that support decarbonisation, we apply a mechanism that is ε- efficient on finite-iteration distributed optimisation.

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