

NEUTRAL CURRENTS AND THE GAUGE
GROUP OF WEAK AND ELECTROMAGNETIC
INTERACTIONS

by

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A B S T R A C T

Up to the time when atomic physics experiments were first done and the period that followed thereafter, the only source of information regarding the properties of neutral currents was provided by the elastic and inelastic scattering of neutinos and antineutrinos of leptons and nucleons. This experimental data was described remarkably well by the Weinberg-Salam model of weak and electromagnetic interactions based on the gauge group $SU(2)_L \times U(1)$, everything being described in terms of one parameter, i.e. the ratio of the coupling constants $g_{SU(2)}$ and $g_{U(1)}$. The outcome from the analysis was that the neutral current was parity violating. However, the atomic physics experiments which employed the valency electrons interacting with the parent nucleus (Bismuth) seemed to indicate that the neutral current was parity conserving. This became more and more certain as the experimental accuracy improved. It is to accommodate this fact about neutral currents that one has to depart from the standard model of Weinberg and Salam. We extend the gauge group from $SU(2)_L \times U(1)$ to $SU(2)_L \times SU(2)_R \times U(1)$ which is already embedded in the Pati-Salam model of strong weak and electromagnetic interactions based on the group $SU(2)_L \times SU(2)_R \times SU(4)_{L+R}^C$. We only concentrate on the subgroup $SU(2)_L \times SU(2)_R \times U(1)$. The novel features of this new theory are:

1. The basic Lagrangian describing the universe is parity conserving and parity violation is attributed to spontaneous symmetry breaking.
2. All the neutrino neutral current data fits extremely well with experiments.
3. The parity violation parameter in Bismuth does not affect the neutrino neutral current data and can take any value lying between

positive infinity to about -150 in the limit of the standard $SU(2)_L \times U(1)$ theory.

4. The theory permits two massive neutral gauge bosons. If the parity violation parameter is positive, then one of the neutral gauge bosons is light and should be produced in the electron-positron colliding beams of PEP and PETRA in the near future.
5. Various asymmetries such as the forward-backward asymmetry and the longitudinal asymmetry associated with electron-positron going to muon-antimuon pairs are enhanced relative to the standard model. These tests should distinguish clearly between the standard model and the one based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. The gauge group $SU(2)_L \times SU(2)_R \times U(1)$ could well be the future group of weak and electromagnetic interactions.

This is what is highlighted in the thesis in greater details.

P R E F A C E

The work described in this thesis was carried out in the Department of Physics, Imperial College of Science and Technology, University of London, between October 1974 and September 1977 under the supervision of and in collaboration with Professors Jogesh Pati and Abdus Salam. Except when stated in the text, the material contained in this thesis is original and has not been previously presented for a degree in this or any other university. The results obtained have been published in the "Physical Review" and "Physics Letters".

The author wishes to thank Professor Pati, at the time visiting Professor from the University of Maryland, U.S.A., and Professor Salam for guidance, encouragement, interest and advice during the pursuit of this work. It was the arrival of Professor Pati that greatly facilitated the progress of this work.

While a postgraduate student at Imperial College the author wishes to thank members of staff of the Department of Theoretical Physics for help, many valuable discussions and suggestions, and also wishes to acknowledge^a grant from the Science Research Council. Financial aid towards the completion of the thesis from the Department is gratefully acknowledged.

It is seventeen years, this year, since my father died. I would like to dedicate this small piece of work to his memory.

C O N T E N T S

<u>Abstract</u>	<u>Page</u>
<u>Preface</u>	2
Part I: INTRODUCTION	4
A. The $SU(2)_L \times U(1)$ model; its adequacies and inadequacies regarding neutral current phenomenology	8
B. The model	9
C. Neutral currents of $SU(2)_L \times U(1)$	10
D. The role of Atoms in neutral currents	15
E. Modifications of and eventual departures from $SU(2)_L \times U(1)$	22
Part II: NATURAL LEFT-RIGHT SYMMETRIC THEORY AND ITS NEUTRAL CURRENT PHENOMENOLOGY	25
A. Introduction	28
B. Salient features of the basic theory; spontaneous symmetry breaking and gauge masses	35
C. General composition of the neutral gauge particles and their interactions within the left- right symmetric gauge structure G or $\bar{G}$	44
D. Two special cases	55
E. Phenomenology: Discussion of results	62
F. Footnotes and references	70
G. Figure Captions	84

Part III:	EFFECTS OF NEUTRAL WEAK CURRENTS IN ELECTRON-POSITRON ANNIHILATION	93
A.	Introduction	94
B.	Charge asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$	96
C.	Average helicity of μ^-	103
D.	Longitudinally Polarized beams	103
E.	Exclusive hadronic final states	105
F.	Discussion	107
G.	Footnotes	109
Part IV:	DILEPTON PRODUCTION IN PP AND $\bar{P}P$ COLLISIONS AS A PROBE TO THE NATURE OF THE NEUTRAL CURRENT INTERACTION	111
A.	Introduction	112
B.	Massive μ -pair production	113
C.	Energy asymmetry	115
D.	Forward-backward asymmetry	117
E.	Remarks	118
F.	Discussion	119
G.	Footnotes and references	121
H.	Figure Captions	123

PART I

INTRODUCTION

A. $SU(2)_L \times U(1)$ MODEL: ITS ADEQUACIES AND INADEQUACIES REGARDING

 NEUTRAL CURRENT PHENOMENOLOGY.

First we briefly present the $SU(2)_L \times U(1)$ model of Salam and Weinberg. Then we establish (in part) the properties of the neutral current by studying

- (a) the inelastic scattering of neutrinos and anti-neutrinos off isoscalar nucleons
- (b) the scattering of neutrinos and anti-neutrinos off protons
- (c) the elastic scattering of muon neutrinos and anti-neutrinos and electron anti-neutrinos off electrons

Finally we discuss the results of Atomic parity violation experiments on Bismuth and point out various ways in which the currents in the standard model can be modified so as to accommodate the findings of the experiment. Finding these modifications inadequate we depart from the standard theory of Salam and Weinberg and describe, in detail, the extended version of the $SU(2)_L \times U(1)$ theory based on the gauge-group $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ within the gauge model of Pati and Salam "Lepton number as the fourth colour".

B. THE MODEL

Without going into too much detail the salient features of the standard model can be derived as follows:

How does the gauge group $SU(2)_L \times U(1)$ come about?

Well, if one takes β -decay and writes out the matrix element in the current x current form of Fermi, the leptonic current suggests that the electron and its neutrino be put into a fermion doublet ψ where

$$\psi = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$$

and any fermion field f in its left and right handed chiral projections is represented by

$$f_L = \frac{1}{2} (1 + \gamma_5) f$$

$$f_R = \frac{1}{2} (1 - \gamma_5) f$$

The leptonic current can be written as

$$J^+ = \frac{1}{\sqrt{2}} \bar{\psi}_L \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \psi_L = \bar{\psi}_L T^+ \psi_L$$

Similarly, for its hermitian conjugate. If one now looks at the electromagnetic current of the electrons

$$\begin{aligned} J^{em} &= -\bar{e} \gamma_\mu e \\ &= -\bar{e}_L \gamma_\mu e_L - \bar{e}_R \gamma_\mu e_R \end{aligned}$$

one finds that the electromagnetic current can be written in a compact form

$$\begin{aligned} J^{em} &= \bar{\psi}_L \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \psi_L + \bar{\psi}_L \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \psi_L - \bar{e}_R \gamma_\mu e_R \\ &= \bar{\psi}_L T^0 \psi_L + \bar{\psi}_L U^0 \psi_L - \bar{e}_R \gamma_\mu e_R \end{aligned}$$

T_L^+ , T_L^- , T_L^0 , generate the weak isospin while U^0 which is just the unit matrix generates $U(1)$ and is identified with the weak hypercharge. The right-handed current is a singlet under the $SU(2)$ and the $U(1)$. So the input of leptonic weak current from β -decay and the leptonic electromagnetic current generate the gauge group $SU(2)_L \times U(1)$ as the unifying group of weak and electromagnetic interactions as output.

C. NEUTRAL CURRENTS OF $SU(2)_L \times U(1)$

The electric charge operator, from the discussion of the last section, is

$$Q^{em} = T_L^0 + U^0$$

The structure of the photon field which couples to the electromagnetic current can be written down immediately once ϕ^{em} is known,

$$\frac{A}{e} = \frac{W_L^0}{g} + \frac{U^0}{g'}$$

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}$$

$g \in SU(2)_L$; $g' \in U(1)$ and e is the electric charge. The other independent eigenvector, orthogonal to the photon, is the Z field and has the form,

$$\frac{Z}{e} = \frac{W_L^0}{g'} - \frac{U^0}{g}$$

The current coupling the photon we know already. Its properties such as the coupling constant (e) and its parity (conserving) are well established. So we will only concentrate on the Z field.

The current coupling the Z field can be written as

$$\frac{J_Z}{e} = \frac{g}{g'} J^{W^0} - \frac{g'}{g} J^{u^0}$$

or more compactly as

$$J_Z = \frac{g}{\cos \theta_W} (J^{W^0} - \sin^2 \theta_W J^{em})$$

$$g'/g = \tan \theta_W, \text{ the Weinberg angle.}$$

So far we have talked about the electron and its neutrino. We can treat the muon and its neutrino in exactly the same manner. From part A, (a) and (b) are absolutely necessary if we are to make predictions. (c) provides possible tests of the theory.

(α). The coupling constant of the neutral current is known. This is because it is related to the Fermi coupling constant G_F

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

All neutral current amplitudes have

$$\frac{G_N}{\sqrt{2}} = \frac{G_F}{\sqrt{2}} \left(\frac{M_W^2}{M_Z^2 \cos^2 \theta_W} \right)$$

as their coupling constant. These come from the usual tree diagrams in perturbation theory and M_W and M_Z are the masses of the mediating charged and neutral gauge bosons. Both theoretically and experimentally

$$\frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1$$

Having fixed the parameters of the neutral current we can make predictions for the following semi-leptonic processes.

- (i). $\nu_\mu + \text{Proton} \rightarrow \nu_\mu + \text{Proton}$
- (ii). $\nu_\mu + \text{Neutron} \rightarrow \mu^- + \text{Proton}$
- (iii). $\bar{\nu}_\mu + \text{Proton} \rightarrow \bar{\nu}_\mu + \text{Proton}$
- (iv). $\bar{\nu}_\mu + \text{Proton} \rightarrow \mu^+ + \text{Neutron}$

(i) and (iii) are elastic while (ii) and (iv) are quasi-elastic.

The theoretical predictions of the ratios

$$R_{el}^{\nu} = \frac{\sigma_{NC}^{\nu} (\nu_\mu + P \rightarrow \nu_\mu + P)}{\sigma_{CC}^{\nu} (\nu_\mu + N \rightarrow \mu^- + P)}$$

$$R_{el}^{\bar{\nu}} = \frac{\sigma_{NC}^{\bar{\nu}} (\bar{\nu}_\mu + P \rightarrow \bar{\nu}_\mu + P)}{\sigma_{CC}^{\bar{\nu}} (\bar{\nu}_\mu + P \rightarrow \mu^+ + N)}$$

$$R_{el} = \sigma_{NC}^{\bar{\nu}} / \sigma_{NC}^{\nu}$$

$P = \text{Proton}$ & $N = \text{Neutron}$.

lie within the range of the numbers quoted by the experimental groups, (we present all these numbers when we deal with the extended version of the model). Once again

$$R_{el} = \frac{\sigma_{\bar{\nu}}^{NC}}{\sigma_{\nu}^{NC}} \neq 1$$

Lastly, there are the elastic cross-sections for purely leptonic scattering

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$$

(β) $\sin^2 \theta_W$: Experimentally, the ratios of the cross-section of neutral current (NC) to charged currents (CC) for muon neutrinos and anti-neutrinos scattering inelastically off isoscalar targets have been measured with a great deal of accuracy by several groups all over the world. These ratios are defined as

$$R^\nu = \frac{\sigma_{NC}^\nu (\nu + N \rightarrow \nu + X)}{\sigma_{CC}^\nu (\nu + N \rightarrow \mu^- + X)}$$

$$R^{\bar{\nu}} = \frac{\sigma_{NC}^{\bar{\nu}} (\bar{\nu} + N \rightarrow \bar{\nu} + X)}{\sigma_{CC}^{\bar{\nu}} (\bar{\nu} + N \rightarrow \mu^+ + X)}$$

$$R = \sigma_{NC}^{\bar{\nu}} / \sigma_{NC}^\nu$$

For the standard theory, it is customary to plot a graph of R^ν against $R^{\bar{\nu}}$. Each point on the curve represents one value of $\sin^2 \theta_W$. Data from GGM, HPWF, CITF and CERN in particular, fix $\sin^2 \theta_W$ to lie between

$$0.24 \leq \sin^2 \theta_W \leq 0.30$$

[Ref. 40, 41, 42 on page 77 contain details about the above results.]

The prediction of the theory that the ratio

$$R = \sigma_{NC}^{\bar{\nu}} / \sigma_{NC}^\nu \neq 1.$$

is substantiated well by the data from the above four groups.

One parameterises the cross-sections in the following way

$$\sigma(\nu_\mu + e^- \rightarrow \nu_\mu + e^-) = C_{\nu_\mu e} \cdot 10^{-41} E^{\nu_\mu} \left(\frac{\text{cm}^2}{\text{GeV}} \right)$$

Here also one finds complete agreement between the standard $SU(2)_L \times U(1)$ theory and experiment. Also as has been the case previously

$$C_{\nu_\mu e} \neq C_{\bar{\nu}_\mu e}$$

i.e. the cross-sections for muon neutrinos and anti-neutrinos scattering off electrons elastically are not equal.

(γ) The outcome of all this is that the theoretical predictions of $SU(2)_L \times U(1)$ and the experimental data are in complete harmony with each other. Most important of all, the cross-sections, whether one considers elastic or inelastic scattering, for the neutrino neutral currents and the anti-neutrino neutral currents are not equal. This leads us to establish the nature of the neutral current. One concludes that the neutral current is Parity Violating!

Caution is needed in making such hasty decisions about this property of the neutral current for the following reason.

We employed neutrinos and anti-neutrinos in order to study the neutral current. The neutrinos and anti-neutrinos are two component objects. Due to their masslessness they come in definite handedness. Thus parity is naturally built in them through gauge invariance. This intrinsic property of the neutrinos and anti-neutrinos cannot be attributed to the neutral current.

It may well be the property of the neutral current but one has to make "intrinsic parity independent" measurements.

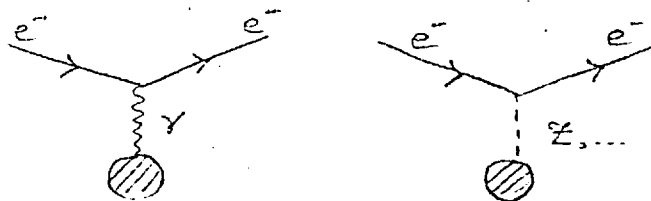
D. THE ROLE OF ATOMS IN NEUTRAL CURRENTS

If one wants to settle the parity of the neutral current, one must employ beams in which the constituents come with left as well as right handedness. The only other leptons left are the electrons and themuons which are indeed four component objects.

Electronic and Muonic atoms serve as natural models for studying neutral currents. Observations on muonic atoms are difficult because the muons decay too rapidly for reliable observations to be made. So we concentrate on electronic atoms and on Bismuth in particular since preliminary results are from experiments on Bismuth vapour.

Theory:

Consider an excited valency electron of the Bismuth atom.



INTERACTION OF ELECTRONS WITH THE NUCLEUS

It can interact with the parent nucleus through the exchange of a photon or through the exchange of a neutral intermediate boson. We will concentrate on the second diagram since the first is known to be parity conserving. The interaction hamiltonian is of the "current cross current" type.

$$H = \frac{G_F}{\sqrt{2}} \int d^3r \bar{e} \gamma_\mu (g_V^e + g_A^e \gamma_5) e \sum_{i=p,n} \bar{q}_i \gamma_\mu (g_V^i + g_A^i \gamma_5) q_i$$

where g_V^e , g_A^e , g_V^i , g_A^i , are known for any model and q^i is the quark of the i th type.

Only the cross terms in the interaction hamiltonian are parity violating.

In order to have some feel as to what the various parity violating terms mean, it is convenient to derive from the second quantised effective Hamiltonian, H , a non-relativistic electron nucleus potential in the following way.

In the non-relativistic approximation,

$$\bar{e} \gamma_\mu e \longrightarrow \text{velocity of the electron}$$

$$\bar{e} \gamma_\mu \gamma_5 e \longrightarrow \text{spin of the electron}$$

$$\sum_i \bar{q}^i \gamma_\mu q^i \sim \text{timelike} \rightarrow \text{charge density}$$

$$\sum_i \bar{q}^i \gamma_\mu \gamma_5 q^i \sim \text{spacelike} \rightarrow \text{spin of the nucleus}$$

Therefore,

$$H \equiv V_{(\Gamma)}^{\text{Parity violating}} \equiv V_{(\Gamma)}^{\text{PV}}$$

$$V_{(\Gamma)}^{\text{PV}} = -\frac{G_F}{\sqrt{2}} \left\{ \frac{\underline{S} \cdot \underline{P}}{m_e} Q_W + \left(\frac{\underline{P}}{m_e} - 2i \frac{\underline{S} \wedge \underline{P}}{m_e} \right) (\$P C_{AP} + \$N C_{AN}) \right\} S^3_{(\Gamma)}$$

+ hermitian conjugate

$\underline{r}$, $\underline{M}$, $\underline{S}$ and $\underline{P}$ are respectively the position, mass, spin $\underline{S} = \frac{1}{2} \underline{\sigma}$ and momentum operator of the electron. $\$P$ and $\$N$ are the total spin

operators of the protons and neutrons in the nucleus. C_{AP} and C_{AN} are

some linear combination of the gauge couplings $g_V^e, g_A^e, g_V^i, g_A^i$ introduced into the interaction Hamiltonian H . Q_W will be defined shortly.

Under the influence of the parity violating potential, parity mixing occurs between atomic wave functions. For heavy atoms the timelike $\sum_i \bar{q}_i \gamma_4 q_i$ dominates over the spacelike $\sum_i \bar{q}_i \gamma_\mu \gamma_5 q_i$ since the nuclear spin is of order unity. Therefore, in the parity violating potential, $(\sum p C_{AP} + \sum n C_{AN})$ term being proportional to nuclear spin is negligible and the electron-nucleus potential can be written as

$$V_{(r)}^{PV} = - \frac{G_F}{\sqrt{2}} \frac{\underline{S} \cdot \underline{P}}{m_e} \delta^3(r) Q_W.$$

So much for the physical insight into the various terms. The relativistic analogue for the above expression for H is the compact form

$$H = \frac{G_F}{\sqrt{2}} Q_W \int d^3r \bar{e} \gamma_\mu \gamma_5 e \rho(r)$$

where

$$Q_W = Z (2g_V^p + g_V^n) + (A - Z) (2g_V^n + g_V^p)$$

The electron sees Z protons and $(A - Z)$ neutrons through the neutral current and ρ is the nuclear charge density normalised to unity. Hence the form of Q_W .

In the dependent particle model of the atom, because of its zero-range nature, H will mix only $S_{\frac{1}{2}}$ and $P_{\frac{1}{2}}$ states. From ordinary perturbation theory, if S is the ground state, then it acquires some admixture of the $|n'P_{\frac{1}{2}}\rangle$ parity violating states of different principal quantum number

n' and the ground state is modified to

$$|n_{\frac{1}{2}}\rangle' = |nS_{\frac{1}{2}}\rangle + \sum_{n'} \frac{\langle n'P_{\frac{1}{2}}|H|nS_{\frac{1}{2}}\rangle}{E_{ns} - E_{n'p}} |n'P_{\frac{1}{2}}\rangle$$

The matrix element of H is given in terms of the radial function $R_{nl}(r)$ by

$$\langle nS_{\frac{1}{2}}|H|n'P_{\frac{1}{2}}\rangle \propto \frac{d}{dr} (R_{n1}(0) R_{n0}(0))$$

and simplifies to

$$\langle nS_{\frac{1}{2}}|H|n'P_{\frac{1}{2}}\rangle = \frac{\alpha^3 \left(\frac{\hbar}{c} \frac{G_F}{\sqrt{2}} \right) \cdot \{1 + S_0(E_n) + S_0(E_{n'})\} K_r}{2 m_e a_0^4 (2^n 2^{n'})^{\frac{3}{2}}}$$

where

$$a_0 = \text{Bohr radius}$$

$$S_0(E_n), S_1(E_{n'}) = \text{come from atomic wave functions}$$

$$E_{n,n'} = - \frac{1}{2^{n,n'}} = \text{binding energy in atomic units}$$

$$K_r = \text{Relativistic correcting factor}$$

(3 for Cesium, 9 for lead and increases as Z increases.)

The matrix element follow a Z^3 law. There is a simple physical interpretation

of the three powers of $\bar{Z}$.

1. One power of $\bar{Z}$ expresses the coherent effect of the nucleus due to the fact that the weak charge of the nucleus is the sum of the charges of the constituents.
2. The second power of $\bar{Z}$ comes from the fact that the electron feels the coulomb attraction of the nucleus which goes like $\bar{Z}$.
3. The third power follows from the proportionality of H to the velocity of the electron near the nucleus which grows like Z . It is clear that the matrix element is enhanced if we use heavy atoms with large $\bar{Z}$.

But why Bismuth? In the modified ground state, $|n'P_{1/2}\rangle$ the matrix element is weighted by the energy denominator $(\epsilon_{ns} - \epsilon_{n'p})$. The matrix element will be further enhanced if the levels are closely populated. Bismuth offers such a possibility. Hence the choice.

The first term in the modified ground state $|n'S_{1/2}\rangle$ is responsible for the normal magnetic dipole (M) transition and the second term is responsible for the parity violating electric dipole (E) transition.

Experiment:

In the actual experiment a plane polarised laser beam is divided into left and right circularly polarised light. It is then made to pass through a tube of Bismuth vapour and the intensities of the two beams emitted at the other end are separately measured. In the presence of H , the eigenstates corresponding to the left and right circularly polarised

light take the form

$$|ns_{\frac{1}{2}}\rangle_+ = |ns_{\frac{1}{2}}\rangle + \alpha |n'p_{\frac{1}{2}}\rangle$$

$$|ns_{\frac{1}{2}}\rangle_- = |ns_{\frac{1}{2}}\rangle - \alpha |n'p_{\frac{1}{2}}\rangle$$

The ratio of the intensities with and without the vapour is a measure of the angle of rotation of the plane of polarisation. This macroscopic effect manifests itself due to the fact that the refractive index of the medium is frequency dependent. The rotation of the plane of polarisation of light of wavelength λ transmitted through a length L of vapour having indices n_R and n_L for right and left circularly polarised light is given by

$$\angle\theta = -\frac{\pi}{\lambda} L (n_R - n_L)$$

An asymmetry parameter can be defined for the transmitted left and right circularly polarised light intensities (I_+, I_-)

$$a = (I_+ - I_-) / (I_+ + I_-)$$

$$= \frac{|M + \alpha E|^2 - |M - \alpha E|^2}{|M + \alpha E|^2 + |M - \alpha E|^2}$$

$$\sim 2\alpha E/M$$

It is also a measure of the angle of rotation between the left and right

circularly polarised light. α contains information about the parity violating Hamiltonian H . The asymmetry parameter, skipping the details, is usually expressed in the form

$$a = 10^{-8} \kappa Q_W$$

where $\kappa 10^{-8}$ comes from E/M and the absorption coefficient at the line peak assuming that the shape of the line is of the Breit-Wigner form. κ is of order (0.17). Experimentally, as measured by the Washington and Oxford groups, the value of $a/10^{-8}$ is

$$-0.7 \pm 3.2$$

Washington

$$+2.7 \pm 4.7$$

Oxford

For $\sin^2 \theta_W = 0.28$, Q_W for $SU(2) \times U(1)$ model is -138, way above the experimental limits (-138 in terms of $a/10^{-8}$ is -24). The experimental numbers suggest Q_W to be consistent with zero while the theoretical predictions are at least an order of magnitude great.

E. MODIFICATIONS OF AND EVENTUAL DEPARTURE FROM $SU(2)_L \times U(1)$

As we have seen, the standard model has been very successful in describing the neutrino neutral current data. It would be unjust if we were to advocate something new in place of the standard model without giving it a chance to explain the null or near null results of parity violation in Bismuth.

There are two ways one can accommodate these findings of the atomic physics experiments without departing from the $SU(2)_L \times U(1)$ theory.

(a) We could make the neutral current purely vector by introducing new quarks into the theory. The quarks would then go to make right handed and left handed doublets. There would be no singlets, e.g. in the vector-like theories

$$\begin{pmatrix} p \\ n \end{pmatrix}_L \quad \begin{pmatrix} c \\ \lambda \end{pmatrix}_L \quad \begin{pmatrix} p \\ b \end{pmatrix}_R \quad \begin{pmatrix} t \\ n \end{pmatrix}_R$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} M^0 \\ M^- \end{pmatrix}_R \quad \begin{pmatrix} E^0 \\ E^- \end{pmatrix}_R$$

Additional new leptons would have to be introduced to cancel the triangular anomalies. Such vector-like theories predict

$$\sigma_{Nc}^{\nu} (\nu + N \rightarrow \nu + X) = \sigma_{Nc}^{\bar{\nu}} (\bar{\nu} + N \rightarrow \bar{\nu} + X)$$

This, in fact, is a disaster since the result is in direct contradiction with experiments. These cross-sections have been measured with better accuracy than any other neutral current cross-sections.

(b) Since the dominant contribution to the parity violating hamiltonian was proportional to the axial vector electron current we could make the electron current pure vector by adding a right-handed weak doublet $(E^0, e^-)_R$. E^0 is some neutral heavy lepton. Anomaly cancellation requires an additional doublet of left-handed leptons. The same procedure for the muon current maintains μe symmetry through the addition of the $(M^0, \mu^-)_R$ doublet and another left-handed doublet for anomaly cancellation. The e^- is the charged lepton in one of the new doublets.

No change is made in the $SU(2)_L \times U(1)$ assignment of quarks. Only Higgs doublets are needed in the theory. The mixing angle between ν_e and E_L^0 and between ν_μ and M_L^0 are equal to m_e/m_{E^0} and m_μ/m_{M^0} . These are negligible. In the literature this modified version of the standard model has come to be known as the hybrid model. The salient features of the hybrid model are

(i) The masses of the charged W's and Z are the same as in the standard model.

(ii) There is no parity violation since the axial coupling of the electron does not exist.

(iii) The coupling of muon-neutrinos to electrons involves a pure electron current. This predicts $C_{\nu_\mu e} = C_{\nu_\mu e}$.

The last prediction seems to contradict experiment. But the data on lepton scattering is poor.

(c) The alternative solution that we propose is to depart from the gauge group of the standard model itself and replace it with some bigger unifying structure such as $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$.

Here we must stress that although the standard model predicts

observable parity-violating effects on Bismuth, the experiments are compatible with the null result but not with the prediction. The prediction due to the standard model lacks the theoretical precision with which the atomic calculations need to be performed. This, however, is our only reservation. In the event of $C_{\nu e} \neq C_{\bar{\nu} e}$ and for the lack of required precision in the atomic calculation the alternative solution might just be the answer.

In the next section we systematically develop the theory of weak and electromagnetic interactions based on the "left-right symmetric" group structure $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$. Since the main aim is to have a grand unified theory of all the basic forces eventually, as a first step in this direction we choose the Pati-Salam gauge group $SU(2)_L \times SU(2)_R \times SU(4)_{L+R}^C$ which already has $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ as its subgroup.

The most important result that originates out of our analysis is that neutral current processes initiated by left-handed neutrinos (or right-handed anti-neutrinos) are independent of the vacuum expectation values of those scalar multiplets which do not couple to the W_L 's, provided W_L and W_R mixing or equivalently quark-mass terms are ignored (W_L, W_R are the gauge bosons belonging to $SU(2)_L$ and $SU(2)_R$). This has the remarkable consequence that the theory can continuously reduce to the $SU(2) \times U(1)$ limit, and agree in all its predictions with those of that theory, except for the amount of atomic parity violations and the masses of the two neutral particles. The next part of the thesis is devoted to exhibiting the truth contained in the foregoing statement.

Our starting point is the Pati-Salam model itself.

P A R T I I

NATURAL LEFT-RIGHT SYMMETRIC THEORY
AND ITS NEUTRAL CURRENT PHENOMENOLOGY

The neutral-current weak interaction sector of the left-right symmetric unified theory of quarks and leptons based on the symmetry structure $SU(2)_L \times SU(2)_R \times SU(4)_{L+R}'$ and its subgroup $SU(2)_L \times SU(2)_R \times SU(3)_{L+R}' \times U(1)_{L+R}$ (suggested in earlier papers) is studied in detail here. The theory admits in general of two weak neutral gauge bosons N_1 and N_2 (and two sets of left and right charged W 's ($W_L^\pm, W_R^\pm$). As pointed out earlier, there are two distinct possibilities for the pattern of spontaneous symmetry breaking which lead to the mass ratios:

$$(i) \quad m_{N_1}^2 \sim m_{W_L^\pm}^2 \ll m_{N_2}^2$$

$$(ii) \quad m_{N_1}^2 \sim m_{N_2}^2 \sim m_{W_L^\pm}^2$$

Case (i) is identical to the familiar $SU(2) \times U(1)$ theory for all predictions. Case (ii), on the other hand, leads to departures from $SU(2) \times U(1)$. One of the most crucial results of our investigation is that such departures occur only for the electron-induced atomic parity experiments and not for neutrino-induced weak processes in the chiral γ_5 -invariant limit; in which W_L - W_R - mixing as well as fermion masses vanish. In this manner (for the chiral limit defined above), $SU(2) \times U(1)$ becomes an integral and stable ingredient of the left-right symmetric theory for both cases (i) and (ii) in so far as their predictions for neutrino-induced reactions are concerned; the differences between the two cases (in this chiral limit) lie solely in their predictions regarding atomic parity violation and, of course, the masses of the two neutral particles N_1 and N_2 . We study the chiral as well as

the non chiral cases and compare their predictions with experiments, observing that approximate or exact global chiral symmetry of the left-right symmetric theory is a point of good agreement between theory and all the present neutral-current experiments.

We exhibit the dependence of the atomic parity-violation-parameter Q_W on the masses of the two neutral particles N_1 and N_2 and remark in particular that (depending upon the sign and magnitude of Q_W), there is the exciting experimental possibility within the left-right symmetric theory that the mass of one of the neutral gauge particles is sensibly smaller than that of the charged $W_L^\pm$. This particular possibility does not materialise, if one insists on exact atomic parity conservation.

A. INTRODUCTION

The null or near null result of the present atomic parity violation experiments¹ (taken at its face value²) together with manifest ν - $\bar{\nu}$ -distinction in neutral current scattering experiments³ suggests that quite possibly a radical departure from the simple $SU(2) \times U(1)$ -group-structure is called for even at low energies. Neither the simple $SU(2) \times U(1)$ -theory⁴ nor the standard vector-like theory⁵ based on the $SU(2) \times U(1)$ -group can, it seems, account for the present set of experimental data.

The purpose of this chapter is to stress that such departures from the simple $SU(2) \times U(1)$ -theory are in general expected to occur within the left $\leftrightarrow$ right-symmetric unified theory of quarks and leptons proposed sometime ago⁶. The gauge structure is either $G = SU(2)_L \times SU(2)_R \times SU(4)_{L+R}'$, or its left $\leftrightarrow$ right symmetric subgroup⁷ $\bar{G} = SU(2)_L \times SU(2)_R \times U(1)_{L+R} \times SU(3)_{L+R}'$ both proposed in Ref. 6. There are two key features of this theory relevant for the present discussion.

(a) To begin with, the gauge structure $SU(2)_L \times SU(2)_R$ is left right-symmetric with left and right handed fermions transforming in a parallel fashion under two distinct $SU(2)$'s ($SU(2)_L$ and $SU(2)_R$). (This is unlike both the simple $SU(2)_L \times U(1)$ -theory, which treats the left and right handed fermions asymmetrically from the start, as well as the vector-like $SU(2) \times U(1)$ -theory for which the left and right handed fermions transform in a parallel manner, but under the same $SU(2)$ -group.) In contrast, for the symmetry-structure G , for every left handed (V-A)-current coupled to the gauge particles ($W_L^\pm$ or W_L^3), there exists a parallel right handed (V+A)-current coupled to a distinct set of gauge particles ($W_R^\pm$ or W_R^3).

The commuting group $SU(4)_{L+R}^1$ (as well as its subgroup $SU(3)_{L+R}^1 \times U(1)_{L+R}$) acting on the four color-indices generate just vector currents and thus also preserve the left $\leftrightarrow$ right symmetry.

As a result, (subject to⁸ the bare gauge coupling constants $g_L^{(0)}$ and $g_R^{(0)}$ associated with the gauge groups $SU(2)_L$ and $SU(2)_R$ being equal), the gauge lagrangian⁹ of the theory (prior to spontaneous symmetry breaking) is parity conserving. Parity violation is introduced into such a theory in the charged and independently in the neutral-current sectors, if, for example¹⁰, spontaneous symmetry breaking assigned asymmetric masses to W_L 's and W_R 's. The major motivation⁶ for the introduction of a left right symmetric gauge-structure was the fulfilment of the aesthetic desire that parity violation may be generated spontaneously within such a theory. (Additional motivations for the introduction of such a gauge structure are the elimination⁶ of abelian contributions to electric charge and the generation of a desirable milliweak theory of CP violation¹¹.)

(b) Second and equally important is the fact that this theory permits⁶ (depending upon the pattern and parameters of spontaneous symmetry breaking) the emergence of two relatively "light" neutral gauge particles¹² (rather than just one) in addition to the photon and the color-gluons. Each of these two can induce neutral-current interactions of order G_{Fermi} as stressed in Ref. 6. Likewise the theory permits two positively charged gauge particles to remain rather "light" (instead of the single W_L^+); one of them induces the familiar charged current interactions of strength G_F (by choice) and the other of strength $\leq G_F/10$ (this last restriction can be relaxed if we assume more than four quark flavours¹³). It may

be noted that the availability of this novel feature involving two relatively light neutral and likewise two rather light charged gauge particles (rather than one of each variety), is a very special property of the gauge structure $= SU(2)_L \times SU(2)_{R-} \times SU(4)_{L+R}$ (as well as of its left-right-symmetric subgroup⁷ and all its extensions to superunifying groups G); it is not available, by contrast within unification hypotheses such as the ones based on the group structures¹⁴ $SU(5)$ and E_7 . The availability of this new feature within the group structure was noted in Ref. 6, where the appropriate choice of Higgs-Kibble-multiplets permitting the emergence of the said solution was also given.

To be specific, it was pointed out in reference 6, that depending upon the choice of Higgs-Kibble multiplets and the pattern of their vacuum-expectation values, there are basically two extreme alternative modes by which the symmetry (or its superunifying extensions G) could descend spontaneously into the level of the symmetry (G_{obs}) of the known interactions. These are schematically presented in the chart on the next page.

$$G = [SU(4)]^4 \text{ or } SU(8) \times SU(8), \text{ etc.....}$$

(one coupling constant)

Heavy ∇ Associated Masses

$$G = SU(2)_L \times SU(2)_R \times SU(4)'_{L+R}$$

(2 effective constants: g, f)

mass $\nabla > 10^4 \text{ GeV}$

Case-I

$$G_0 = SU(2)_L \times U(1) \times SU(3)'_{L+R}$$

(g, g', f_s)

mass $\nabla \sim 100 \text{ GeV}$

$$G_{\text{obs}} = U(1)_f \times SU(3)'_{L+R}$$

(one "light" Neutral gauge particle: Z_0)

Case-II

$$\bar{G} = SU(2)_L \times SU(2)_R \times U(1)_{L+R} \times SU(3)'_{L+R}$$

(g, f_{15}, f_s)

mass $\nabla > 200 \text{ GeV}$

$$SU(2)_L \times U(1)_R \times U(1)_{L+R} \times SU(3)'_{L+R}$$

mass $\nabla \sim 100 \text{ GeV}$

$$G_{\text{obs}} = U(1)_f \times SU(3)'_{L+R}$$

(two "light" Neutral gauge particles: N_1 and N_2)

Here $U(1)_f$ denotes the abelian symmetry generated by the color-singlet part of the electromagnetic charge (which is the entire electromagnetic charge if quarks are fractionally charged). For the case of integer-charge quarks the symmetry $U(1)_f \times SU(3)_{L+R}^1$ breaks down further⁶ by a mass scale of order 1 Gev to the local symmetry generated by electric charge (which in this case becomes a sum of the color-singlet piece ($U(1)_f$) and an appropriate combination of color-octet-pieces). For the present discussions it is immaterial, however, we choose fractional or integer-charges for the quarks (see later).

In Ref. 6, only the first case (Case I) was exhibited in some detail for the sake of simplicity, although the basic ingredients needed to generate the second alternative as well as its general significance for neutral current-interactions due to the presence of two "light" neutral gauge particles (as opposed to one for the $SU(2) \times U(1)$ gauge-structure) were spelled out. In this part, we examine this second alternative in some detail and present the variety of new and experimentally interesting complexions, which could arise within this alternative by permitting the full generality of symmetry breaking, proposed in Ref. 6. We remark that atomic parity violation can be arbitrarily small within a class of solutions within the second alternative, while the distinction between neutrinos and antineutrinos even neutral-current scattering experiments can be large, this last feature produced only via parity violating charged current interactions and carry definite handedness (see later).

Before proceeding further, it needs to be noted that two sets of authors (Mohapatra and Sidhu¹⁶ and De Rujula, Georgi and Glashow¹⁷) have recently carried out investigations in this area. Our work and results partly overlap with the work of these authors to the extent that they are examining the consequences of the same group structure $\bar{G} = SU(2)_L \times SU(2)_R \times U(1) \times SU(3)_{L+R}$, which was proposed in Ref. 6. (De Rujula et al differ from Ref. 6. in their choice of fermion-representation and somewhat also in the scheme of spontaneous symmetry breaking. On the other hand, the fermion-representation with four flavors considered by Mohapatra and Sidhu (MS) is the one suggested in Ref. 6; also the schemes of spontaneous symmetry-breaking considered by MS is happily in one-one-correspondence¹⁸ with the one proposed in Ref. 6 (see later)). We however examine the problem in this note from the standpoint of the breaking of the bigger symmetry via the chain $G \rightarrow \bar{G} \rightarrow G_{\text{obs.}}$

Thus, we remark that the solutions of vanishing¹⁶ or small¹⁷ atomic parity violation presented in the above references are only special cases within a general class of solutions proposed in Ref. 6 for the breakdown of the symmetry structure . All solutions belonging to this class share the common feature of vanishing or small atomic parity violation, but they differ from each other notably in the predictions of the masses of the neutral particles N_1 and N_2 . As we shall show, within this class of solutions there exists the distinct possibility that one of the neutral masses can be sensibly lighter than the charged W^+ , (this particular feature is not obtainable for the special case of exact atomic parity-conservation considered by MS).

One of the most interesting (and somewhat unexpected) observations we make is to note that neutral current processes initiated by left-handed neutrinos (or right-handed anti-neutrinos) are independent of the expectation values of those scalar multiplets which do not couple to the W_L 's, provided W_L and W_R mixing or equivalently quark-mass terms are ignored. This has the remarkable consequence that the theory can continuously reduce to the $SU(2) \times U(1)$ limit, and agree in all its predictions with those of that theory, except for the amount of atomic parity violations and the masses of the two neutral particles.

The class of solutions permitting two relatively light neutral gauge particles is described in general in terms of three parameters (essentially two for most purposes, see sec. C), which represent the masses of the two relatively light neutral particles and one coupling constant to be contrasted with two parameters (in general) for the $SU(2) \times U(1)$ -theory.

We obtain a good description of the data involving at least eight independent neutral current parameters: R_ν , $R_{\bar{\nu}}$, $C_{\nu e}$, $C_{\bar{\nu} e}$, $C_{\nu e}$, Q_W (atomic) and two parameters describing elastic νp and $\bar{\nu} p$ -scatterings. The left $\leftrightarrow$ right-symmetric gauge-structure⁶ can thus be well tested once the body of neutral-current parameters settles experimentally.

B. SALIENT FEATURES OF THE BASIC MODEL; SPONTANEOUS
SYMMETRY BREAKING AND GAUGE MASSES

For the sake of completeness we wish to recall the following salient features of the basic model⁶:

(a) The model introduces a sixteen fold of basic four component fermions with four flavors (p, n, λ , c) and four colors (red, yellow, blue and lilac); the fourth color being identified with lepton number $L \equiv L_e + L_\mu$.

$$F_{L,R} = \begin{pmatrix} p_r & p_y & p_b & \nu_e \\ n_r & n_y & n_b & e^- \\ \lambda_r & \lambda_y & \lambda_b & \mu^- \\ c_r & c_y & c_b & \nu_\mu \end{pmatrix}_{L,R} \quad (1)$$

Note the necessity for the $L \leftrightarrow R$ -symmetric righthanded partners (ν_{eR} and $\nu_{\mu R}$) of the lefthanded neutrinos (ν_{eL} and $\nu_{\mu L}$).

(b) To implement left $\leftrightarrow$ right-discrete symmetry as well as quark-lepton-unification, the model assumes a minimal local symmetry

$$G = SU(2)_L \times SU(2)_R \times SU(4)_{L+R}' \quad (2)$$

where $\begin{pmatrix} p \\ n \end{pmatrix}_{L,R}$ and $\begin{pmatrix} c \\ \lambda \end{pmatrix}_{L,R}$ for each of the four colors (this includes the leptons) are treated as $SU(2)_{L,R}$ -doublets, while $(q_r, q_y, q_b, q_\lambda)_{L+R}$

for each of the four flavors are treated as $SU(4)'$ -antiquartets ($\bar{4}'$'s) (this puts quarks and leptons into one multiplet).

Following the suggestions in Ref. 6 and Ref. 8, we impose a discrete left $\leftrightarrow$ right-symmetry on the theory; thus the renormalised gauge-coupling constants g_L and g_R associated with the gauge groups $SU(2)_L$ and $SU(2)_R$ are equal up to finite order α' corrections ($(g_L/g_R) = 1 + O(\alpha')$). We will set throughout $g_L = g_R \equiv g$.

The gauge particles of the theory are conveniently represented by the matrices_

$$W_{L,R} = \frac{1}{2} \begin{pmatrix} \tau \cdot W & 0 \\ 0 & \tau_1 (\tau \cdot W) \tau_1 \end{pmatrix}_{L,R} \quad (3)$$

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} V(8) - \frac{S^0 \times 1}{\sqrt{12}} & X \\ \bar{X} & \sqrt{\frac{3}{4}} S^0 \end{pmatrix} \quad (4)$$

$V(8)$ denotes the $SU(3)'$ -octet of color-gluons, X is an exotic $SU(3)'$ -triplet ($B = -L = 1$) and S^0 is an $SU(3)'$ -singlet associated with the 15th generator of $SU(4)'$.

Defining $\nabla_\mu F = \partial_\mu F + ig W_\mu F - if V_\mu$, where f denotes the gauge coupling constant associated with the $SU(4)'$ -group¹⁹, the Fermi Lagrangian of the theory is given by

$$\mathcal{L}_{\text{Fermi}} = -\text{Tr } \bar{F}_L \not{V}_L F_L - \text{Tr } \bar{F}_R \not{V}_R F_R \quad (5)$$

where $F_{L,R} = \frac{1}{2} (1 \pm \gamma_5) F$. Note the manifest completely left $\leftrightarrow$ right-symmetric parity conserving nature of the gauge-interaction.

(c) Spontaneous Symmetry Breaking: The model⁶ introduces the following sets of Higgs-Kibble multiplets to induce a general pattern of spontaneous symmetry breaking (see section IV and Footnote 21 of Ref. 6):

$$\begin{aligned} A &= (2 + 2, 2 + 2, 1), \quad B = (1, 2 + 2, \bar{4}) \\ C &= (2 + 2, 1, \bar{4}), \quad D = (1, 1, 15) \end{aligned} \quad (6)$$

Note, the choice of the multiplets A, B, C, D is in accordance with the requirements of left $\leftrightarrow$ right-symmetry with B balancing C. If one wishes to assign a heavy mass to the charged $W_R^\pm$ without at the same time making the charged $W_L^\pm$ and the neutral components $W_{L,R}^3$ and S^0 heavy likewise, one must introduce⁶ an additional multiplet $E_R = (1, 3, 1)$ and therefore following L $\leftrightarrow$ R-symmetry a parallel multiplet⁸ $E_L = (3, 1, 1)$. (The introduction of E_L would be inconsequential operationally in the sense that we will assume for simplicity²⁰ that E_L is zero or small (perhaps $O(\alpha)$) compared to $\langle A \rangle$ and/or $\langle C \rangle$). Additional scalar multiplet may also be present to permit the emergence of fermion masses appropriately²¹; however we assume (as in Ref. 6) that such multiplets have small vacuum-expectation values and are inconsequential in so far as

gauge-masses are concerned.

Following Ref. 6 one may choose the parameters of the Higgs-potential (lying in a specified range) so as to obtain the following sets of vacuum-expectation-values consistent with minimisation of the potential.

$$A = \begin{pmatrix} a_1 & a_1 & a_1 & a_4 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 & b_4 \end{pmatrix}$$

$$C = \begin{pmatrix} c_1 & c_1 & c_1 & c_4 \end{pmatrix}$$

$$D = \begin{pmatrix} d & d & d & -3d \end{pmatrix}$$

$$E_R = \begin{pmatrix} 0 \\ \epsilon_R \\ 0 \end{pmatrix}$$

$$E_L = \begin{pmatrix} 0 \\ \epsilon_L \\ 0 \end{pmatrix} \sim 0 \quad (7)$$

We have not exhibited the Cabibbo rotated forms for the vacuum-expectation-matrices, since once again these are inconsequential for gauge-masses (see Ref. 6 for details). The above pattern can be preserved^{6,8}, consistent with the demands of natural left $\leftrightarrow$ right-symmetry (i.e. maintenance of the choice⁶ $g_L^{(0)} = g_R^{(0)}$).

Integer versus fractional quark-charges: Before proceeding further, it should be emphasised that the unification-hypothesis based on the symmetry structure G permits⁶ quark-charges to be either integral ($c_1 \neq 0$) or fractional ($c_1 = 0$).

$$\begin{array}{ll}
 c_1 = \text{finite} \sim 1 \text{ GeV} & (\text{quarks acquire integer-charges; only} \\
 & \text{photon remains massless; the octet of} \\
 & \text{color gluons acquire finite mass}) \\
 c_1 = 0 & (\text{quarks acquire fractional charges; photon} \\
 & \text{and the octet of color-gluons remain} \\
 & \text{massless}). \qquad (8)
 \end{array}$$

Whether c_1 is finite or zero thus makes a world of difference in general. However, as we would be interested in this note in studying neutral current phenomena below threshold for color-production and also because the parameter c_1 (even when it is non-vanishing) is still negligible compared to the other relevant mass scales in the problem, which are of order 100 GeV, it is fortunately immaterial for our present considerations, whether we set²² $c_1 = 0$, or $c_1 = \text{finite} \sim 1 \text{ GeV}$. Thus for simplicity of writing we henceforth set $c_1 = 0$ bearing in mind, however, that all our considerations in the present note regarding neutral and charged-current interactions below threshold for color production are equally valid for the cases of integer and fractionally charged quarks.

Setting $c_1 = 0$, we may now simply read off the gauge-mass-matrices from Eq. (17) and footnote 21 of Ref. 6. These are given below:

$$m_{X,\bar{X}}^2 = \frac{f_X^2}{2} (b^2 + c^2 + d^2)$$

$$m_{W_L, W_R}^2 = \frac{g^2}{4} \begin{pmatrix} W_L^+ & W_R^+ \\ a^2 + c^2 + \epsilon_L^2 & 2(a_1^2 + a_1 a_4) \\ 2(a_1^2 + a_1 a_4) & a^2 + b^2 + \epsilon_R^2 \end{pmatrix}$$

$$m_{\text{neutral}}^2 = \frac{1}{4} \begin{pmatrix} W_L^3 & W_R^3 & S^0 \\ g^2(a^2 + c^2) & -g^2 a^2 & \sqrt{\frac{3}{2}} f_{15} g c^2 \\ -g^2 a^2 & g^2(a^2 + b^2) & \sqrt{\frac{3}{2}} f_8 g b^2 \\ \sqrt{\frac{3}{2}} f_{15} g c^2 & \sqrt{\frac{3}{2}} f_8 g b^2 & \frac{3}{2} f_{15} (b^2 + c^2) \end{pmatrix}$$

where $a^2 \equiv 3 a_1^2 + a_4^2$, $b^2 \equiv b_4^2$ and $c^2 \equiv c_4^2$. We have inserted f_{15} and f_8 (rather than f) in the entries corresponding to the gauge particles $S_0 \equiv V_{15}$ and X respectively to denote the fact that the effective renormalised coupling constants associated with these gauge particles are in general expected to be quite different from that associated with the octet of color-gluons due to differing finite renormalisation effects, a feature, which was suggested in general terms in ref. 23. This has recently most strongly been emphasised by Elias and Shafi²⁴, who show that $f_8^2 \gg f_{15}^2$, (where f_8 denotes the effective quark-gluon coupling constant). We shall incorporate these results within our analysis in this note.

Note the important feature that the multiplet $D = (1, 1, 15)$ provides mass only to the exotic gauge particles X and $\bar{X}$, which are coupled to $\bar{q}_\mu \gamma_\mu l$ and $\bar{X}_\mu q$ -currents respectively but not to $V(8)$ and S^0 . Thus D splits $SU(4)'$ naturally to $SU(3)'_{L+R} \times U(1)_{L+R}$, where the $U(1)$ corresponds to the 15th generator of the $SU(4)'$ -group; this generator is associated with the gauge particle $S^0 \sim V_{15}$. Likewise the multiplets $E_R = (1, 3, 1)$ and $E_L = (3, 1, 1)$ provide masses only to the charged gauge particles $W_R^\pm$ and $W_L^\pm$ respectively but not to the neutral ones.

Thus note the interesting feature of the model⁶ that it is possible to destroy the left $\leftrightarrow$ right-symmetry altogether in the mass matrix of the charged gauge bosons and thereby in charged current-interactions (by choosing e.g. $\epsilon_R \gg \epsilon_L$, a, b, c) and yet preserve it approximately or exactly in the mass matrix of the neutral-gauge bosons and thereby in the neutral-current interactions by choosing $b \sim c$ or $b = c$ (see Eqs. (9.2) and (9.3)). Vice-versa is, however, not possible. The full implications of this remark for neutral current experiments will be apparent in a moment.

Phenomenologically, from the known degree of forbiddenness of $K_L \rightarrow \bar{\mu} e$, one may deduce that

$$m_X > 10^4 \text{ Gev} \quad (10)$$

for the basic model. Likewise from the known suppression of (V+A)-interactions in β -decay, one may deduce⁶ (within the four flavour-model²⁵) that

$$m_{W_R^\pm} > 3 \left(\frac{\cos \theta_R}{\cos \theta_L} \right)^{\frac{1}{2}} m_{W_L^\pm} \quad (11)$$

where θ_L and θ_R denotes the Cabibbo angles for the left and right handed fermions²⁶.

We are now ready to point out how in accordance with the above phenomenological constraints, the two cases I and II representing the two extreme alternative models for the breakdown of G to G_{obs} arise (these two alternative cases are described in section A).

$$\text{Case I: } \left(G \xrightarrow[\langle B \rangle]{10^4 \text{ GeV}} G_0 = SU(2)_L \times U(1) \times SU(3)_{L+R} \xrightarrow[\langle A \rangle, \langle C \rangle]{10^2 \text{ GeV}} G_{\text{obs}} \right)$$

This case arises if in the first place we choose $b \gg c$ and a .

Such choice, we note, destroys left $\leftrightarrow$ right-symmetry in the charged as well as in the neutral-current sectors, since with this choice all three right handed gauge-particles ($W_R^\pm, W_R^3$) acquire heavier masses than the corresponding left-handed ones ($W_L^\pm, W_L^3$) (see Eqs. (9.2) (9.3)).

As an extreme possibility, if we furthermore choose b to be the major contributor to the X-mass (i.e. $b > 10^4$ GeV), then left $\leftrightarrow$ right-symmetry as well as quark-lepton unification are lost in one stroke through $\langle B \rangle$; the symmetry G thereby reduces²⁷ to $G_0 = SU(2)_L \times U(1) \times SU(3)_{L+R}$. In turn, G_0 breaks through $\langle C \rangle$ and $\langle A \rangle \neq 0$ down to G_{obs} generating just one light neutral gauge-particle-the Z^0 . (Note that for this case the presence of the multiplets D , E_L and E_R is inessential for a gross view, since both X and W_R become heavy or superheavy through $\langle B \rangle$.)

$$\text{Case II: } \left(G \xrightarrow[\langle D \rangle]{10^4 \text{ GeV}} \bar{G} = SU(2)_L \times SU(2)_R \times U(1)_{L+R} \times SU(3)_{L+R} \right)$$

$$\xrightarrow[\langle A \rangle, \langle B \rangle, \langle C \rangle]{\sim 10^2 \text{ GeV}} G_{\text{obs}} \cdot)$$

This case arises⁶ provided we introduce the multiplets like $D = (1, 1, 15)$, $E_R = (1, 3, 1)$ and $E_L = (3, 1, 1)$ in addition to $\langle A \rangle$, $\langle B \rangle$ and $\langle C \rangle$ with the choice $d > 10^4$ GeV $\gg \epsilon_{R,L}$, a , b , c .

In this case, the theory loses quark-lepton-unification first (i.e. $SU(4)_{L+R} \rightarrow U(1)_{L+R} \times SU(3)_{L+R}$) through $\langle D \rangle$ being large, but

it still retains its intrinsic left $\leftrightarrow$ right-symmetric nature;
the symmetry $\overline{G}$ reducing to the left $\leftrightarrow$ rightsymmetric-structure

$$\overline{G} = SU(2)_L \times SU(2)_R \times U(1)_{L+R} \times SU(3)'_{L+R}.$$

The manner in which $\overline{G}$ reduces further down to $G_{\text{obs}} = U(1)_f \times SU(3)'_{L+R}$ depends very much on the relative magnitude of the parameters $\epsilon_{R,L}$ and a, b, c (for the neutral current-sector only the three parameters a, b, c are relevant).

If one allows $\epsilon_R \gg \epsilon_L, a, b, c$ (so that $m_{W_R}^+ > 3 m_{W_L}^+$), one loses left $\leftrightarrow$ right-symmetry in the charged current sector leading to predominance of V-A over V+A in this sector. One may still retain left $\leftrightarrow$ right-symmetry in the neutral current sector, however, as long $b = c$, which ensures that the mass parameters associated with W_L^3 and W_R^3 are equal (see Eq. (9.3)). Thus departures from the L $\leftrightarrow$ R-symmetry in the neutral current sector for the left $\leftrightarrow$ right-symmetric theory⁶ are strictly proportional to $(b-c)/(b+c)$.

Atomic parity violation would clearly be small if $b \sim c$; vanishing²⁸ in the limit $b = c$. In either case ($b \sim c$ or $b = c$), one expects⁶ to see two relatively light neutral gauge particles N_1 and N_2 . The case $b \sim c$ (rather than $b = c$) allows the interesting possibility that one of the neutralgauge particles can be sensibly lighter than the charged W_L^+ with the magnitude of atomic parity violation still remaining small (by about a factor ~ 2 to 4) compared to that of the $SU(2) \times U(1)$ -theory. We discuss this case as well as the case $b = c$ and their inter-relationship in some detail in the next two sections.

C. GENERAL COMPOSITIONS OF THE NEUTRAL GAUGE PARTICLES
AND THEIR INTERACTIONS WITHIN THE LEFT $\leftrightarrow$ RIGHT SYMMETRIC
GAUGE STRUCTURE G OR $\bar{G}$:

First we note the composition of the massless photon. This, for the two alternative cases of fractional and integer charge quarks ($c_1 = 0$ and $c_1 \neq 0$) is given by⁶:

$$\frac{A_\mu}{e} = \frac{W_L^2 + W_R^2}{g} - \sqrt{\frac{2}{3}} \frac{S^0}{f_{15}} \quad (\text{fractionally ch. quarks } c_1 = 0) \quad (12.a)$$

$$\frac{A_\mu}{e} = \frac{W_L^2 + W_R^2}{g} - \sqrt{\frac{2}{3}} \frac{S^0}{f_{15}} + \frac{2}{\sqrt{3}} \frac{U^0}{f_s} \quad (\text{Integer ch. quarks } c_1 \neq 0) \quad (12.b)$$

Here $U^0 \equiv (\frac{1}{2})(\sqrt{3}V_3 + V_8)$; f_s and f_{15} denote the effective renormalised gauge coupling constants associated with the color-gluon U^0 and the gauge particle $V_{15} \equiv S^0$ respectively; the electric charge for the two cases being simply related to these gauge coupling constants as follows:

$$\frac{1}{e^2} = \frac{2}{g^2} + \frac{2}{3} \frac{1}{f_{15}^2} \quad (13.a)$$

f.c.q.

$$\frac{1}{e^2} = \frac{2}{g^2} + \frac{2}{3} \frac{1}{f_{15}^2} + \frac{4}{3} \frac{1}{f_s^2} \quad (13.b)$$

i.c.q.

The abbreviations f.c.q. and i.c.q. stand for fractionally charged

and integer charged quarks respectively. Since we expect $f_s^2 \gg f_{15}^2 \sim g^2$ due to differing finite renormalisation effects^{23,24}, we see that either of the above two equations (i.e. (13.a) or (13.b)) leads essentially to the same value of g^2 for a given ratio (f_{15}^2/g^2). This is yet another reason why there is no difference worth noting between the cases of fractional and integer charge quarks in so far as neutral-current interactions below threshold for color-production are concerned.

From here on, therefore, we substitute, as mentioned in sec.II, $c_1 = 0$ for simplicity bearing in mind that our considerations in this part, involving neutral-current interactions below color-threshold, hold equally for both cases: $c_1 = 0$ (f.c.q.) and $c_1 \neq 0$ (i.c.q.).

The photon (Eq. (12.a)) and the two other eigenstates (N_1 and N_2) of the neutral mass-matrix (9.3) are then given by:

$$A_\mu = \frac{W_L^3 + W_R^3}{\sqrt{2}} \cos \alpha - S^0 \sin \alpha \quad (14.1)$$

$$N_1 = Y_1 \cos \beta + Y_2 \sin \beta \quad (14.2)$$

$$N_2 = Y_2 \cos \beta - Y_1 \sin \beta \quad (14.3)$$

where

$$Y_1 = (W_L^2 - W_R^2) / \sqrt{2} \quad (15)$$

$$Y_2 = \frac{W_L^2 + W_R^2}{\sqrt{2}} \sin \alpha + S^0 \cos \alpha \quad (16)$$

$$\cos \alpha = \frac{\sqrt{2} e}{g} \quad ; \quad \sin \alpha = \sqrt{\frac{2}{3}} \frac{e}{f_{15}} \quad (17)$$

Note that the normalisation condition (i.e. $\cos^2 \alpha + \sin^2 \alpha = 1$) is the same as Eq.(13.a) (corresponding to the case $c_1 = 0$). The mixing angle β determining the composition of the physical particles N_1 and N_2 as well as their masses may conveniently be obtained from (9.3) by first transforming the same into the orthonormal basis (A , Y_1 and Y_2). The photon, being an eigenstate, decouples from Y_1 and Y_2 ; the mass-matrix of the $(Y_1 - Y_2)$ complex, thus obtained, is given by:

$$m^2(Y_1, Y_2) = \frac{g^2}{8} \begin{pmatrix} Y_1 & Y_2 \\ 4a^2 + b^2 + c^2 & -\sqrt{3r+1}(b^2 - c^2) \\ -\sqrt{3r+1}(b^2 - c^2) & (3r+1)(b^2 + c^2) \end{pmatrix} \quad (18).$$

Note in the special limit $b = c$, Y_1 and Y_2 are the eigenstates of the mass-matrix, as in this limit they are decoupled from each other; their masses are given simply by the diagonal elements of the mass matrix (18). For the general situation, $b \neq c$, the physical particles are N_1 and N_2 (Eq. (14)). Diagonalising the matrix (18), we obtain:

$$\begin{pmatrix} m_{N_1}^2 \\ m_{N_2}^2 \end{pmatrix} = \frac{3}{16} \left(f_{15}^2 + \frac{2}{3} g^2 \right) (b^2 + c^2) + g^2 \frac{a^2}{4} \\ + \left[\left\{ \frac{3}{16} f_{15}^2 (b^2 + c^2) - g^2 \frac{a^2}{4} \right\}^2 + \frac{3}{64} g^2 \left(f_{15}^2 + \frac{g^2}{3} \right) (b^2 - c^2)^2 \right] \quad (19.1)$$

$$\tan 2\beta = \frac{\sqrt{3} g \left(f_{15}^2 + g^2 \frac{2}{3} \right)^{\frac{1}{2}} (b^2 - c^2)}{\frac{3}{2} f_{15}^2 (b^2 + c^2) - 2 g^2 a^2}. \quad (19.2)$$

The neutral-current-interaction induced by N_1 and N_2 is given by:

$$\mathcal{L}_{NC} = (J_\mu^{Y_1} \cos \beta + J_\mu^{Y_2} \sin \beta) N_{1\mu} \\ + (-J_\mu^{Y_1} \sin \beta + J_\mu^{Y_2} \cos \beta) N_{2\mu} \quad (20).$$

$$\begin{aligned}
J_\mu^{Y_1} = \frac{g}{2\sqrt{2}} & \left[\bar{\nu}_{eL} \gamma_\mu \nu_{eL} - \bar{\nu}_{eR} \gamma_\mu \nu_{eR} + \bar{\nu}_{\mu L} \gamma_\mu \nu_{\mu L} \right. \\
& - \bar{\nu}_{\mu R} \gamma_\mu \nu_{\mu R} - \bar{e} \gamma_\mu \gamma_5 e - \bar{\mu} \gamma_\mu \gamma_5 \mu \\
& \left. + \bar{p} \gamma_\mu \gamma_5 p + \bar{c} \gamma_\mu \gamma_5 c - \bar{n} \gamma_\mu \gamma_5 n - \bar{\lambda} \gamma_\mu \gamma_5 \lambda \right] \quad (21)
\end{aligned}$$

$$\begin{aligned}
J_\mu^{Y_2} = \frac{g}{2\sqrt{2} (3r+1)^{\frac{1}{2}}} & \left[(1-r) (\bar{p} \gamma_\mu p + \bar{c} \gamma_\mu c) \right. \\
& - (1+r) (\bar{n} \gamma_\mu n + \bar{\lambda} \gamma_\mu \lambda) \\
& + (3r+1) (\bar{\nu}_{eL} \gamma_\mu \nu_{eL} + \bar{\nu}_{\mu R} \gamma_\mu \nu_{\mu R} + \bar{\nu}_{eR} \gamma_\mu \nu_{eR} \\
& + \bar{\nu}_{\mu L} \gamma_\mu \nu_{\mu L}) \\
& \left. + (3r-1) (\bar{e} \gamma_\mu e + \bar{\mu} \gamma_\mu \mu) \right] \quad (22)
\end{aligned}$$

$$r = \frac{f_{15}^2}{g^2}$$

Note that J^{Y_1} is a pure axial-current for all four-component fields (this applies to neutrinos as well, if neutrinos are light four-component objects) and J^{Y_2} is a pure vector-current. Thus in the limit $b = c$ ($\tan 2\beta = 0$), for which N_1 and N_2 reduce Y_1 and Y_2 respectively, the neutral-current interaction is entirely parity conserving (for electrons, quarks and neutrinos as well) as

mentioned before. There would still be ν - $\bar{\nu}$ -distinction in neutral current scattering since the neutrinos and antineutrinos are produced by parity violating charged current interactions, which yield ν and $\bar{\nu}$'s with definite handedness (i.e. neutrinos with left helicities and antineutrinos with right helicities). Such neutrinos and antineutrinos contribute to neutral current scattering only through the term $\bar{\nu}_L \gamma_\mu \nu_L$ in the currents $J^{Y_{1,2}}$, but not through the term $\bar{\nu}_R \gamma_\mu \nu_R$ to the extent that the mass of the neutrino may be neglected. The current $\bar{\nu}_L \gamma_\mu \nu_L$ is a V-A for neutrinos, but V+A for antineutrinos, hence the distinction between ν and $\bar{\nu}$ in neutral-current-scattering even for the intrinsically parity conserving lagrangian ($b = c$). Note that unlike neutrinos, electrons and quarks appear with both helicities for processes of interest; thus in the limit $b = c$ atomic interactions will not show any parity violation.

Before plunging into phenomenology we must note the charged current-constraint; i.e.

$$\frac{g^2}{8m_{W_L}^2} = \frac{G_F}{\sqrt{2}} \quad (24)$$

For a given $r = (r_{15}^2 / g^2)$, we thus obtain by using Eqs.(13.a) and (24):

$$m_{W_L}^2 = \left(1 + \frac{1}{3r}\right) \frac{e^2}{\sqrt{8}} G_F^{-1} \approx \left(1 + \frac{1}{3r}\right) (53 \text{ GeV})^2 \quad (25)$$

Furthermore neglecting the small perturbation of $W_L^+ - W_R^+$ -mixing

(compared to the mass of W_R^+) as well as ²⁹ ϵ_L in (9.2), we also have:

$$m_{W_L^+}^2 = \frac{g^2}{4} (a^2 + c^2) \quad (26)$$

Combining Eqs. (24) and (25), we derive:

$$a^2 + c^2 = \frac{1}{\sqrt{2} G_F} = 10^{+5} \frac{m_N^2}{\sqrt{2}} \quad (27)$$

Thus, of the three parameters a^2 , b^2 and c^2 only two are independent.

Correspondingly we define two parameters³⁰

$$\zeta = \frac{c^2}{c^2 + a^2} ; \quad \eta = \frac{b^2}{c^2 + a^2} \quad (28)$$

Thus, $0 \leq \zeta \leq 1$

$$0 \leq \eta \leq \infty \quad (29)$$

It is convenient to express the masses of N_1 and N_2 , the mixing angle β and the amplitudes for the various neutral current-processes in terms of r (f_{15}^2 / g^2), ζ and η ; these we list below:

$$\begin{aligned} K \begin{pmatrix} 1 \\ 2 \end{pmatrix} &= \frac{m_{W_L^+}^2}{m_{N_1}^2} \\ &= \frac{4}{P_1 \mp P_2} \end{aligned} \quad (30)$$

$$\tan 2\beta = \frac{-(3r+1)^{\frac{1}{2}}(5-\eta)}{\frac{3r}{2}(5+\eta) - 2(1-5)} \quad (31)$$

$$P_1 = (3r+2)(5+\eta) + 4(1-5) \quad (32)$$

$$P_2 = \left[\{3r(5+\eta) - 4(1-5)\}^2 + 4(3r+1)(5-\eta)^2 \right]^{\frac{1}{2}}. \quad (33)$$

The Amplitudes

We define the relevant neutral-current amplitudes by the following effective lagrangian (here p and n denote quark-fields):

$$\begin{aligned} \mathcal{L}_{NC}^{eff} = & \frac{G_F}{\sqrt{2}} (\bar{\nu}_\mu \gamma_\mu (1+\gamma_5) \nu_\mu) \otimes \\ & \left[\bar{p} \gamma_\mu (g_V^p + g_A^p \gamma_5) p + \bar{n} \gamma_\mu (g_V^n + g_A^n \gamma_5) n \right. \\ & \left. + \bar{e} \gamma_\mu (M_V^e + M_A^e \gamma_5) e + \dots \dots \dots \right] \\ & + \frac{G_F}{\sqrt{2}} (\bar{\nu}_e \gamma_\mu (1+\gamma_5) \nu_{eL}) \bar{e} \gamma_\mu (\Xi_V^e + \Xi_A^e \gamma_5) e \\ & + \frac{G_F}{\sqrt{2}} (\bar{e} \gamma_\mu \gamma_5 e) (C_V^p \bar{p} \gamma_\mu p + C_V^n \bar{n} \gamma_\mu n \dots) \end{aligned} \quad (34)$$

Defining two special combinations

$$G_V = \frac{1}{2} \left[K_1 \sin^2 \beta + K_2 \cos^2 \beta + \frac{(K_1 - K_2) \sin \beta \cos \beta}{\sqrt{3\Gamma + 1}} \right] \quad (35)$$

$$G_A = \frac{1}{2} \left[K_1 \cos^2 \beta + K_2 \sin^2 \beta + \sqrt{3\Gamma + 1} (K_1 - K_2) \sin \beta \cos \beta \right] \quad (36)$$

which in terms of ζ and η acquire the simple forms:

$$G_V = \frac{\eta + 2(1 - \zeta)}{2(3\Gamma + 1)(\zeta - \zeta^2 + \eta)} \quad (37)$$

$$G_A = \frac{\eta}{2(\zeta - \zeta^2 + \eta)} \quad (38)$$

we now exhibit that the amplitudes of all neutrino-induced processes as defined by (34) are simply given in terms of either G_V or G_A , though not the electron-induced process.

Process

$$\left. \begin{aligned} g_V^p &= (r-1) G_V \\ g_A^p &= -G_A \end{aligned} \right\} \nu p \rightarrow \nu p$$

$$\left. \begin{aligned} g_V^n &= (r+1) G_V \\ g_A^n &= G_A \end{aligned} \right\} \nu n \rightarrow \nu n$$

$$\left. \begin{aligned} M_V^e &= -(3r-1) G_V \\ M_A^e &= G_A \end{aligned} \right\} \nu_e e \rightarrow \nu_e e$$

$$\left. \begin{aligned} E_V^e &= -1 + M_V^e \\ E_A^e &= -1 + M_A^e \end{aligned} \right\} \bar{\nu}_e e \rightarrow \bar{\nu}_e e$$

$$C_V^p = - \frac{(r-1)(K_1 - K_2)}{2(3r+1)^{\frac{1}{2}}} \cdot \sin 2\beta \quad ep \rightarrow ep$$

$$C_V^n = - \frac{(r+1)(K_1 - K_2)}{2(3r+1)^{\frac{1}{2}}} \cdot \sin 2\beta \quad en \rightarrow en$$

(39)

Here p,n denote quarks. Note that all neutrino-induced amplitudes are simply given by G_V or G_A .

Using the coupling parameters C_V^p and C_V^n , we may now obtain the effective charge Q_W characterizing atomic parity violation³¹. This is given by:

$$\begin{aligned} Q_W &= -2 \left\{ (2C_V^p + C_V^n) Z + (C_V^p + 2C_V^n)(A - Z) \right\} \\ &= \frac{(K_1 - K_2) \sin 2\beta [2Z - (3r+1)A]}{(3r+1)^{\frac{1}{2}}} \end{aligned}$$

(40)

$$Q_W = \frac{2Z - (3r+1)A}{(3r+1)} \left[\frac{\eta - 5}{5 - 5^2 + \eta} \right] \quad (40.1)$$

$$Q_W = Q_W(SU(2)_L \times U(1))_r \left[\frac{(\eta - 5)}{5 - 5^2 + \eta} \right] \quad (40.2)$$

where $Q_W(SU(2) \times U(1))_r = 2Z - (3r+1)A / (3r+1)$ is the value of Q_W obtained for the $SU(2) \times U(1)$ -theory (Ref. 4) with $\sin^2 \theta_W = 3r/(6r+2)$ (see Sec. IV).

D. THE TWO SPECIAL CASES

It is first important to notice the properties of two special cases:

Case A: The Case of Vanishing or small fermion-masses and small W_L - W_R -mixing $a^2/(c^2 + a^2) \rightarrow 0$, or $\zeta = c^2/(c^2 + a^2) \rightarrow 1$):

It turns out that one obtains good agreement with the entire body of present experimental values of neutral-current parameters for the case $\zeta \sim 1$ (i.e. $a^2 \ll c^2$). In order to understand the results for this case, which we discuss in detail later in this section, it is first helpful to note one (remarkable) general property of the special limit ($\zeta = 1$, i.e. $a^2 = 0$), a limit, in which W_L - W_R -mixing as well as fermion-masses vanish³². (Appropriately, this limit will be referred to as the global (γ_5 -invariant) chiral-symmetry limit).

A general property of the chiral symmetry-limit:

In the limit of exact global chiral symmetry ($a^2 \rightarrow 0$ or $\zeta \rightarrow 1$), even though the masses of the two neutral eigenstates N_1 and N_2 , (the mixing angle β), and the atomic parity violation-parameter Q_W vary sensitively with b^2 (i.e. with $\eta = b^2/(c^2 + a^2)$); the amplitudes of all neutrino-induced processes initiated by left-handed neutrinos (or right-handed antineutrinos) are totally independent of the vacuum-expectation values of "right-handed" scalar fields such as B . In the present case they are independent of b^2 , depending just on the single parameter $r = f_{15}^2 / g^2$. (By "right-handed scalar field", we simply mean a field which couples to W_R 's, but not to W_L 's, as for example $B = (1, 2 + 2, \bar{4})$).

The exact value of a^2 can not, of course, be zero, since fermions do have finite mass. However, as long as $a^2 \ll c^2$ (i.e. $\zeta \sim 1$), the amplitudes of neutrino-induced processes continue to remain insensitive to variations of b^2 (or η), varying just with r , on account of the result stated above. This is in fact borne out by our calculations (see Table III).

While the validity of the result stated above may be deduced on general grounds, it may be verified simply by noting the values acquired by the relevant parameters G_V , G_A , K_1 , K_2 and Q_W at $\zeta = 1$. These may be obtained from Eqs. (30), (37), (38) and (40) and are listed below:

$$G_V = [(\zeta = 1)] = \frac{1}{3r+1} \quad (41)$$

$$G_A (\zeta = 1) = 1 \quad (42)$$

$$\begin{aligned} K_{\binom{1}{2}} (\zeta = 1) &= \left[m_{W_L^+}^2 \div m_{N_{\binom{1}{2}}}^2 \right]_{\zeta=1} \\ &= 4 \left[(3r+2)(1+\eta) \mp \{9r^2(1+\eta)^2 + 4(3r+1)(1-\eta)^2\}^{\frac{1}{2}} \right]^{-1} \end{aligned} \quad (43)$$

$$Q_W (\zeta = 1) = \frac{2\zeta - (3r+1)A}{(3r+1)} \left(\frac{\eta - 1}{\eta} \right) \quad (44)$$

Note that G_V and G_A and therefore the amplitudes of all neutrino-

induced processes (see Eq. (39)) are independent of b^2 (or η) at $\zeta = 1$; they depend on the single parameter $r = (f_{15}^2 / g^2)$. On the other hand, the masses of the N_1 and N_2 (given by K_1 and K_2) as well as the atomic parity violation parameter³³ Q_W depends on r as well as b^2 (i.e. η) even at $\zeta = 1$ (the chiral-limit $a^2 = 0$).

Case B: The Case of $SU(2) \times U(1)$ -limit ($b^2 \gg (c^2 + a^2)$, or $\eta \gg 1$):

We present the results for the case $\eta \gg 1$, by assuming $r \neq 0$ (i.e. $\eta r \gg 1$). As we will show, the results in this case are independent of ζ (varying over its full range from 0 to 1). Except for the mass of N_2 which becomes superheavy depending upon the value of η , all relevant neutral-current parameters are independent of η as well. They depend once again (as in Case A) on the single parameter $r = f_{15}^2 / g^2$. Using Eqs.(14)-(19), we obtain in the limit $\eta \gg 1$ and $\eta r \gg 1$,

$$m_{N_1}^2 = m_{W_L^+}^2 \left(\frac{6r+2}{3r+2} \right) \left(1 + O\left(\frac{1}{\eta}\right) \right) \quad (45)$$

$$m_{N_2}^2 = \frac{m_{W_L^+}^2}{2} (3r+2) \eta \left(1 + O\left(\frac{1}{\eta}\right) \right) \quad (46)$$

$$\tan 2\beta = - \frac{2}{3} \frac{(3r+1)^{\frac{1}{2}}}{r} \quad (47)$$

$$A_\mu = W_L^2 \sin \Theta_W + B^0 \cos \Theta_W \quad (48)$$

$$N_1 = W_L^3 \cos \Theta_W - \sin \Theta_W B^0$$

(49)

$$N_2 = \frac{\sqrt{\frac{3r}{2}} V_{15} + W_R^3}{\left(\frac{3r}{2} + 1\right)^{\frac{1}{2}}}$$

(50)

where

$$B_0 = \frac{f_{15} W_{3R} - \sqrt{\frac{2}{3}} g V_{15}}{\left(f_{15}^2 + \frac{2}{3} g^2\right)^{\frac{1}{2}}}$$

(51)

$$\sin^2 \Theta_W = \frac{3r}{6r + 2}$$

(52)

Note that the expression (48) for the photon-field is the same as we had before (refer Eq. (14.1)). Thus we see that the theory reduces in the limit $\eta \gg 1$ to the $SU(2) \times U(1)$ -theory with the correspondence: B^0 is the $U(1)$ -gauge-field, N_1 is the Z^0 with a mass comparable to that of W_L^+ , while N_2 is superheavy. The familiar angle $\sin^2 \Theta_W$ is given by Eq.(52) with

$$m_Z^2 = m_{N_1}^2 = m_{W_L^+}^2 \left(\frac{6r+2}{3r+2} \right) \sim \frac{m_{W_L^+}^2}{\cos^2 \Theta_W}$$

Note in the bare limit of a unified theory, which incorporates

$G = SU(2)_L \times SU(2)_R \times SU(4)_{L+R}'$, one would obtain³⁴ $(\sin^2 \Theta_W)_{\text{bare}} = (3r)/(6r+2) = \frac{3}{8}$, since $r = f_{15}^2/g^2 = 1$ in this bare limit.

Finally, we see that the relevant parameters G_V , G_A and Q_W (in the limit $\eta \gg 1$) are given by:

$$G_V = \frac{\eta + 2(1-\zeta)}{(3r+1)(\zeta + \eta - \zeta^2)} \quad (53)$$

$$G_A = \left(\frac{\eta}{\zeta - \zeta^2 + \eta} \right) \xrightarrow[\zeta = 0 \text{ to } 1]{\eta \gg 1} \rightarrow 1 \quad (54)$$

$$Q_W = \frac{2Z - (3r+1)A}{(3r+1)} \left[\frac{\eta - \zeta}{\zeta - \zeta^2 + \eta} \right] \xrightarrow[\zeta = 0 \text{ to } 1]{\eta \gg 1} \frac{2Z - (3r+1)A}{3r+1} \quad (55)$$

The most amusing feature is thus that the expressions for G_V and G_A obtained in the $SU(2) \times U(1)$ -limit, are identical to those obtained in Case A (the chiral symmetry limit: $a^2 \sim 0$ or $\zeta = 1$); the two cases differ, in general (depending upon the value of η), only in their predictions of the masses of N_1 and N_2 as well as of the atomic parity violation-parameter Q_W . They agree entirely in all their predictions as regards amplitudes for neutrino-induced processes.

Case C: An Amusing special case ($r = (f_{15}^2/g^2) = \frac{1}{3}$)

The two cases A and B discussed above represent all the principal features of the left-right-symmetric gauge structure in so far as its predictions for neutral-current-phenomena are concerned. We note in passing a third amusing special case which may come under either Case A or Case B, or both (35) in view of the remarkably simple forms, which the basis-vectors (A, Y_1, Y_2), and consequently the eigenvectors (A, N_1 and N_2), acquire in this Case. This Case corresponds to the coupling constant-ratio:-

$$r = \frac{f_{15}^2}{g^2} = \frac{1}{3} \quad (51)$$

With $r = \frac{1}{3}$, the mixing angle α determining the composition of the photon and Y_2 (using Eqs. (13.a) and (14) - (18)) is given by:

$$\sin^2 \alpha = \frac{2}{3} \frac{e^2}{f_{15}^2} = \frac{1}{2} \quad (52)$$

Thus the angle α is simply 45° . Consequently,

$$\begin{aligned} A &= \frac{1}{\sqrt{2}} \left(\frac{W_L^3 + W_R^3}{\sqrt{2}} \right) - \frac{1}{\sqrt{2}} V_{15} \\ Y_1 &= \frac{1}{\sqrt{2}} (W_L^3 - W_R^3) \\ Y_2 &= \frac{1}{\sqrt{2}} \left(\frac{W_L^3 + W_R^3}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} V_{15} \end{aligned} \quad (53)$$

The $(Y_1 - Y_2)$ mass matrix reduces to:-

$$m^2(Y_1, Y_2) = \frac{g^2}{4} \begin{pmatrix} Y_1 & Y_2 \\ \frac{1}{2}(4a^2 + b^2 + c^2) & -\frac{1}{\sqrt{2}}(b^2 - c^2) \\ -\frac{1}{\sqrt{2}}(b^2 - c^2) & (b^2 + c^2) \end{pmatrix}$$

(54)

The point $r = (\frac{1}{3})$ is a point of good agreement between theory and experiments (as may be seen from Table I.). (Note, if in addition to $r = \frac{1}{3}$, one furthermore imposes $a^2 = (b^2 + c^2)/4$; the diagonal elements of the (Y_1, Y_2) -mass matrix become equal, so that the eigenstates N_1 and N_2 are given by the simple combinations $(Y_1 \pm Y_2)/\sqrt{2}$. A second interesting feature of this choice is that the vector coupling (M_V^e) for the process $\nu_e e \rightarrow \nu_e e$ and $\bar{\nu}_e e \rightarrow \bar{\nu}_e e$, both proceeding only through axial vector electron-current, have equal cross sections for the case $r = \frac{1}{3}$.

Case D: The Case of Exact Neutral current Parity Conservation ($b=c$)

If $b = c$, as noted in Secs. II and III, Y_1 and Y_2 are the eigenstates of the "neutral" mass matrix. In this case neutral current interactions (of electrons, quarks as well as of neutrinos) are exactly parity conserving, since Y_1 and Y_2 are coupled to axial vector and vector currents respectively. This is the case considered by MS. (Ref. 16). In this case, as noted before ν - $\bar{\nu}$ -distinction arises not because of intrinsic parity violating nature of the neutral

current - interactions, but because available neutrinos are only left-handed (and antineutrinos right-handed). Note this particular case can overlap with Case A (the chiral case); but it is exclusive of Case-B (the $SU(2) \times U(1)$ -limit), for which $b^2 \gg (c^2 + a^2)$. This particular case of $(b = c)$ of exact atomic parity conservation, it turns out it does permit the possibility of a rather light neutral gauge particle ($m_{N_1} < m_{W_L^+}$) (see discussions in the next section).

E. PHENOMENOLOGY: DISCUSSION OF RESULTS

With the amplitudes and K_1 , K_2 and Q_W listed in section C, it is straightforward to calculate the various neutral-current parameters³⁷ (R_ν , $R_{\bar{\nu}}$, R , $C_{\bar{\nu}e}$, $C_{\nu e}$, $C_{\bar{\nu}e}^{el}$, R_{ν}^{el} , $R_{\bar{\nu}}^{el}$ and R^{el}) using the quark-parton model for the semileptonic processes. For simplicity, we neglect the contributions of the antiquark-partons³⁸ in the sea. The results exhibited in Tables I, II and III as well as in Figures 1, 2, 3, and 4 represent the principal features³⁹ of our analysis. We draw attention to the following features of our results:

(1) We see first of all from Tables I and III that the point of exact or approximate chiral symmetry (i.e. $1 - 5 \equiv a^2/(c^2 + a^2) \sim 0$), together with the coupling constant ratio r being in the range $\sim .33$ to $.66$ (see below), is a point of good agreement between theory and experiments⁴⁰⁻⁴⁶ involving all present neutral current-processes. One obtains, in this case a fit⁴⁷ simultaneously for the neutrino-data as well as for the electron-atomic parity experiments, which presently seem to indicate^{1,2} a small value for Q_W .

The question arises: how far away from the chiral limit ($\xi = 1$ corresponding to $a^2 = 0$) may one depart and yet be in accord with the experiments. The answer, we find is interesting: one may choose $\xi = c^2/(c^2 + a^2)$ sensibly lower than unity (i.e. $a^2 \sim c^2$) if one wishes, and yet be in accord with all the neutrino-data provided, however, the special combination³⁰

$$\gamma = (1 - \xi)/\eta = \frac{a^2}{b^2}$$

(55)

is sufficiently small (~ 0.15). This may be inferred partly from table III and Figures 1, 2 and 3 which show that the values of R- and in particular R becomes lower⁴⁸ than those permitted by the present data, if γ exceeds nearly 0.15 with $\xi =$ either .9 or .8.

(It may be verified that the neutral-current cross-section ratio R is only a function of the coupling_g constant ratio r and the parameter γ , but it is independent of $x = (\eta - \xi)/\xi = (b^2 - c^2)/c^2$ and thus is an especially useful quantity for comparison of theory with experiment). The condition that γ be small ($< \frac{1}{6}$) can no doubt be satisfied even with arbitrarily small $\xi \rightarrow 0$ by choosing η to be sufficiently large, i.e.

$$\eta > 6(1 - \xi)$$

(56)

For example, if ξ is as small as 0.5, one would need $\eta > 3$; this is simply to explain the neutrino-data. The price one must pay in this case, however, (if it is a price²) is that the atomic parity

violation parameter Q_W would rapidly approach its $SU(2) \times U(1)$ -value for $\eta \gg 1$ (see Eq. (40.2) and our discussions in sec. IV) in apparent contradiction with the present data; simultaneously the mass of N_2 would become progressively heavier ($\gg m_{W_L^+}$) with N_1 remaining relatively light but still heavier than W_L^+ .

The precise constraint, which may be obtained on the chiral-symmetry-breaking parameter $(1 - \zeta)$, by the above considerations, can only be laid down after one knows the value of Q_W from experiments together with a reliable atomic physics calculation² for the relevant atom. However to demonstrate the point, let us assume for illustration only that the true value of Q_W is at least a factor of 3 lower in magnitude than its $SU(2) \times U(1)$ -value. This would imply, using eq.(40.2), that the ratio $Q_W/Q_W(SU(2) \times U(1)) = (\eta - \zeta)/(5 + \eta - \zeta^2)$ lies between $-\frac{1}{3}$ to $+\frac{1}{3}$; which in turn would imply (given either sign for Q_W) that $\eta < (25 - \zeta^2/2)$. Combining this with the constraint that the neutrino-data seem to require $6(1 - \zeta) < \eta$, as stated above, we would deduce

$$85 - 5\frac{1}{2} > 6$$

or,

$$1 - \zeta = \frac{a^2}{c^2 + a^2} < 0.2$$

We stress once again that this constraint should be taken at present as an illustration only, since the true value of Q_W is yet an unknown quantity.

We draw the following important conclusion from the above discussion: if Q_W indeed turns out to be rather small compared to its $SU(2) \times U(1)$ -value, and if the neutrino parameters retain their present values; one can well account for both these features

within the left-right symmetric theory; however, in this case the solution of the theory must lie near the chiral symmetric limit ($a^2 \ll c^2$); the limit $a^2 = 0$ corresponding to vanishing W_L - W_R -mixing as well as vanishing fermion-masses.

(2) We noted above that agreement between our predictions and the neutrino-data is obtained provided the coupling constant ratio $r = (f_{15}^2 / g^2)$ lies in a limited range. Examining table I, and Fig. 4, one may deduce (this holds for $\Sigma < 1$ as well) a guiding range:

$$0.33 < r = \left(\frac{f_{15}^2}{g^2} \right) < 0.66 \quad (57)$$

If the coupling constant ratio r is lower than 0.30, the neutrino-parameter R becomes too high, exceeding 0.30, which lies above the upper limits of Gargamelle and CITF-values. If, on the other hand, the coupling constant-ratio r exceeds 0.66, $R_{el} = \frac{\sigma_{el}^{\bar{\nu}p}}{\sigma_{el}^{\nu p}}$ becomes too high exceeding the upper limit ~ 0.6 of the present HPWF-result. (This last deduction should, however, be taken with caution, since the theoretical prediction of elastic-scatterings-parameters depend⁴⁷ upon $(\nu, \bar{\nu})$ -spectra and the mass characterising the axial form factor).

It is remarkable that the range of the coupling constant-ratio, exhibited above, deduced phenomenologically, lies well within the rather "limited" range of the same, deduced on the theoretical grounds²⁴ based on renormalisation group-considerations, which yield:

$$\frac{4}{3} e^2 \leq f_{15}^2 \leq 4 e^2$$

Together with Eq.(13.a), this implies

$$\frac{1}{3} \leq r = \frac{f_{12}}{g_1} \leq \frac{5}{3} \quad (59)$$

(3) The variation of Q_W as a function of³⁰ $x \equiv (\eta - \xi)/\xi = (b^2 - c^2)/c^2$ is exhibited in Fig. 5. As we see, it passes through zero at $x = 0$, approaching its $SU(2) \times U(1)$ -value in the limit $x \rightarrow \infty$ with $\xi \sim 1$ (i.e. $b^2 \gg c^2$, with $c^2 \neq 0$), while for negative x ($b^2 < c^2$), it rises rapidly acquiring unbounded positive values. We find that large negative x (i.e. $x < -0.6$, or $b^2 < 0.4c^2$) is probably excluded by the present atomic parity experiments, since such x implies $Q_W > +300$. On the other hand, small negative values of x ($-0.5 \leq x \equiv (\eta - \xi)/\xi \leq 0$, corresponding to $\frac{1}{2} \leq b^2/c^2 \leq 1$) is fully allowed by the data on Q_W (as well as by the neutrino-data).

(4) If $x > 0$ (i.e. if $b^2 > c^2$), then both N_1 and N_2 remain heavier than W_L^+ (see Fig. 6), with the mass of N_1 coinciding with that of W_L^+ for $b^2 = c^2$ (at $\xi = 1$, i.e. $a^2 = 0$). On the other hand, if $x < 0$ (i.e. if $b^2 < c^2$), N_1 is inevitably lighter than W_L^+ . For example, if the true value of the atomic parity violation-parameter Q_W is positive and as large as 100 (this is consistent with the present data, Ref. 1 and 2), then from Table II or Fig. 5, we may deduce⁴⁹ that the parameter $x = (\eta - \xi)/\xi$ should be ≈ -0.44 , and correspondingly (from Fig. 6.) with $r = 0.44$, we would obtain⁵⁰:

$$m_{N_1} \sim 53 \text{ GeV}, \quad m_{W_L^+} \sim 70 \text{ GeV}$$

(60)

By contrast, for the $SU(2) \times U(1)$ -theory with $r = 0.44$ (i.e. $\sin$

$\sin^2 \theta_W = (3r)/(6r+2) \sim 0.29$), we obtain:

$$m_{Z^0} \approx 83 \text{ GeV} ; m_{W_L^+} \sim 70 \text{ GeV}$$

(61)

Thus, as mentioned before, the left-right symmetric theory offers the distinct possibility of one of the neutral gauge particles being sensibly lighter than the Z^0 of the $SU(2) \times U(1)$ -theory as well as the charged W_L^+ , provided $b^2 < c^2$ (or equivalently provided Q_W is positive).

(5) In our analysis, we have neglected for simplicity the effect of $W_L - W_R$ mixing on the composition and masses of the charged eigenstates. (Compare the mass-matrix (9.2) with Eq.(26)). This is justifiable a posteriori within the four flavor-model⁶: First, because⁵¹ with only four flavors one must have $m_{W_R}^2 + > 10 m_{W_L}^2 +$ (following Eq. (11) and remark in Ref. (26)), to account for the observed predominance of V-A over V+A ($m_{W_L}^2 +$ and $m_{W_R}^2 +$ here refer to the diagonal elements of (9.2)). Second, because, as noted above, we found a posteriori that the solution with approximate chiral symmetry ($a^2 \ll c^2$) may even be favored by the data.

On the other hand, if we permit more than four flavors as in the mirror-model²⁵, there is no compelling need for $m_{W_R}^2 + \gg m_{W_L}^2 +$; the two masses can in this case be comparable to each other (the many Cabibbo-like angles arising in this case²⁵ together with heaviness of new fermions can account for the observed predominance of V-A over V+A). Thus with more than four flavors, there is the added distinct possibility within the left-right-symmetric theory

that there can be two sets of relatively light charged gauge particles (rather than just one), which would be fair mixtures⁵² of $W_L^+ - W_R^+$ -mixing. We will enter into a more detailed study of this possibility elsewhere.

To conclude, the left-right-symmetric theory⁶ accounts for all the present experimental neutral-current parameters including the phenomenon of possibly small atomic parity violation - in terms of three parameters in general ($r = f_{15}^2/g^2$, $\eta = b^2/(c^2 + a^2)$ and $\xi = c^2/(c^2 + a^2)$): The allowed range for the coupling constant-ratio r (~ 0.33 to 0.66) deduced phenomenologically falls within the theoretically deduced²⁴ rather limited range for the same. In addition to r , the combinations $(1 - \xi)/\eta = a^2/b^2$ and $(\eta - \xi)/\xi = (b^2 - c^2)/c^2$ also get constrained severely by the present data in spite of their large errors. Thus the left $\leftrightarrow$ right-symmetric theory can be well tested experimentally once the body of independent neutral-current measurements settles experimentally. There are at least eight such measurements involving scattering type experiments only. Including the masses of the two neutral particles, this number is ten.

The intricacies of the left-right-symmetric theory are remarkable. Depending upon the pattern and magnitudes of the parameter of spontaneous symmetry breaking⁶, it offers two interesting alternative limits: (i) the chiral symmetry-limit ($a^2 \ll c^2$) with $b^2 \sim c^2$ and (ii) the $SU(2) \times U(1)$ -limit ($b^2 \gg c^2 + a^2$). In either case, the neutrino-processes are described in terms of the single parameter r . The predictions of the two cases for these processes in fact coincide (in the extreme limit of $a^2 \rightarrow 0$ for case (i)); they differ from each other only in respect of electron-atomic parity experiment and the masses of the two neutral gauge

particles⁶ N_1 and N_2 , and subject to $b^2 < c^2$, it even offers the exciting possibility that one of the neutral gauge-particles can be considerably lighter than the charged W_L^+ . The second case generates the single relatively light familiar Z^0 (heavier than W_L^+). Which of these two limits of the left-right-symmetry theory⁶ is chosen by Nature, if any, appears to depend crucially upon a knowledge of the atomic parity violation parameter Q_W .

F. FOOTNOTES AND REFERENCES

1. P. Baird, M. Brimicombe, G. Roberts, P. Sandars, D. Soreide, E. Fortson, L. Lewis, E. Lindahl, D. Soreide; *Nature*, 264, 529 (1976).
2. As noted in Ref. 1, the uncertainty in the atomic physics calculations (on account of configuration-mixing) for the amount of parity violation to be expected in Bi^{209} (Ref. 1) is at present so large that no firm conclusion can be drawn from the present level of experimentation (see R.D. Cowan, S. Meshkov and S.P. Rosen (Preprint, 1977)). Thus one must wait for more refined theory and experiments on the problem before judging whether departures from the $\text{SU}(2) \times \text{U}(1)$ -theory as regards atomic parity violation are indeed showing themselves at low energies. For earlier calculations on atomic parity violation parameters, see M.A. Bouchiat and C.C. Bouchiat, *Phys. Lett.* 48B, 111 (1974), J. Bernabeu, T.E.O. Ericson and C. Jarlskog, *Phys. Lett.* 20, 315 (1974), I.B. Kriplovich, *Soviet Physics - JETP Lett.* 20, 315 (1974); E.M. Henley and L. Wilets, *Phys. Rev.* A19, 1911 (1976); M.W.S.M. Brimicombe, C.E. Loving and P.G.H. Sandars, *J. Phys.* B9, L237 (1976).
3. For measurements of $\sigma(\nu d \rightarrow \nu x)$ versus $\sigma(\bar{\nu} d \rightarrow \bar{\nu} x)$ see: A. Benvenuti et al, *Phys. Rev. Lett.* 37, 1039 (1976); J. Blietschau et al, Preprint CERN/EP/Phys. 76-55, B.C. Barish, Cal. Tech. Preprint CalT-68-544.
For measurements of $\sigma(\bar{\nu} p \rightarrow \bar{\nu} p)$ versus $\sigma(\nu p \rightarrow \nu p)$ see, D. Cline et al, *Phys. Rev. Lett.* 37, 252, 648 (1976); W. Lee et al, *Phys. Rev. Lett.* 37, 186 (1976).

4. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967), Abdus Salam in Elementary Particle Physics, N. Svartholm ed. (Almqvist & Wiksell, Stockholm p 367, 1968) and references therein to the work with J.C. Ward and of S.L. Glashow;
S.L. Glashow, J. Illiopoulos and L. Maiani, Phys. Rev. D2, 1285 (1970).
5. See for example R.M. Barnett, Review talk, Brookhaven APS meeting, SLAC-PUB 1850 for a list of references on standard vector-like theories based on $SU(2) \times U(1)$ -group. It should be noted that it is possible to build a notable class of vector-like $SU(2) \times U(1)$ -theories, as for example the one extracted from E, F (Gürsey and P. Sikivie, Phys. Rev. Lett. 36, 775 (1976), P. Ramond, Nucl. Phys. B110, 214 (1976); P. Sikivie and F. Gürsey, Preprint (1977)), in which atomic parity violation can be vanishingly small, and yet there can exist v - $\bar{v}$ -distinction in neutral-current scattering experiments (this is unlike the standard vector-like theories). This class of models, however, appears to suffer from one disadvantage: the strangeness changing neutral current-processes are not "naturally" suppressed at least within the $SU(2) \times U(1)$ -extractions of these theories. (S.L. Glashow and S. Weinberg, Harvard Preprint, HUTP-75/A158; see however remarks in P. Sikivie and F. Gürsey, Preprint, 1977.)
6. J.C. Pati and Abdus Salam, Phys. Rev. D10, 275 (1974).
7. The abelian-factor $U(1)_{L+R}$ within $\bar{G}=SU(2)_L \times SU(2)_R \times U(1) \times SU(3)'_{L+R}$ may be identified with the 15th generator F'_{15} of $SU(4)'_{L+R}$, as stressed in Ref. 6 (see Footnote 11).
8. This choice $g_L^{(0)} = g_R^{(0)}$ was proposed in Ref. 6, in order that the origin of parity violation should lie in spontaneous symmetry breaking only. See text p276 and Footnote 34, where it was stated that, provided $g_L^0 = g_R^0$, the "ratio of the renormalized coupling constant g_L/g_R would differ from unity by calculable

finite radiative corrections of order $(\alpha \log m_{W_R}^2 / m_{W_L}^2)$ to the extent that spontaneous symmetry breaking introduces different masses for W_L and W_R (for example)". The scalar (spontaneously symmetry breaking) multiplets introduced in Ref. 6(A, B, C) were left-right symmetric as indeed were the quarks and the leptons. With these choices of multiplets and the gauge structure, the proof that the radiative corrections to g_L^0 and g_R^0 can indeed be finite, through an appropriate choice of Higgs parameters, was later carried through in the papers of R.N. Mohapatra and J.C. Pati, Phys. Rev. D11, 566, 2558 (1975) and G. Senjanovic and R.N. Mohapatra, Phys. Rev. D12, 502 (1975).

9. Provisionally, one may still permit left $\leftrightarrow$ right asymmetric scalar mass terms in the lagrangian, if one wishes to, without spoiling "natural" left $\leftrightarrow$ right -symmetry and without introducing parity non conservation of any measurable consequence except to the extent it arises through the gauge mass-inequality $m_{W_R} \neq m_{W_L}$ (see R.N. Mohapatra and J.C. Pati, Phys. Rev. D11, 2557 (1975)).
10. Parity violation also arises if the many Cabibbo-like angles, generated by the spontaneously induced Fermi-mass matrix, turn out to be unequal for the left and right handed fermions. This can be a major cause of parity violation, if quarks carry more than four flavours.
11. R.N. Mohapatra and J.C. Pati, Phys. Rev. D11, 566 (1975).
12. "Light" in the present context denotes a mass-scale of order 50 to 100 Gev.
13. See later.
14. H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
For E_7 , see Ref. 5.
15. It should be understood (though we do not exhibit it in the chart above), that there is the distinct possibility that the superunifying group G could descend directly into either $\bar{G}$ or G_0 (through essentially

one superheavy mass-scale m_1 rather than two). Also there is a range of intermediate possibilities corresponding to G descending to G_0 passing through $\bar{G}$, for example, via mass scales as shown below:

$$\begin{array}{cccc} G > 10^4 \text{ GeV} & \bar{G} \sim 10^3 \text{ GeV} & G_0 \sim 10^2 \text{ GeV} & U(1)_F \times SU(3)_{L+R}' \\ < D > & < B > & < C >, < A > \end{array}$$

where $D = (1, 1, 15)$, $B = (1, 2+2, \bar{4})$, $C = (2+2, 1, \bar{4})$ and $A = (2+2, 2+2, 1)$. The pattern of spontaneous symmetry breaking induced through the scalar multiplets A, B, C, D (Ref. 6) and the intermediate possibilities, such as mentioned above, are discussed in Sections II, III and IV.

16. R.N. Mohapatra and D.P. Sidhu, Phys. Rev. Lett. 38, 667 (1977).
17. A. De Rujula, H. Georgi and S.L. Glashow, Harvard Preprint, (1977).
18. This overlap and correspondence is not unexpected, since the origin of the basic considerations of the MS-scheme may be traced to our Ref. 6 (vide Ref. 13 of the MS paper).
19. The appearance of two coupling constants g and f associated with the gauge group G reflect the passage of a higher unifying symmetry G down to G through spontaneous symmetry breaking with the effective coupling constants associated with $SU(2)_{L,R}$ $SU(4)_{L+R}'$ differing from each other through finite renormalization effects.
20. Since $\langle E_L \rangle$ contributes to the masses of the charged $W_L^\pm$ only, but not to the neutral W_L^3 , allowing for $\langle E_L \rangle \neq 0$ would mean that all our expressions for the neutral-current amplitudes (to be presented in sections III and IV) would simply be multiplied by a uniform factor deviating from unity depending upon $\langle E_L \rangle / m_{W_L}^+$. See remarks later for the experimental implications of $\langle E_L \rangle \neq 0$.
21. Most notably $(2, 2, 15)$ or $(2+2, 2+2, 15)$ (see Ref. 6).
22. We, of course, do not neglect c_1 (for the case of integercharge quarks), in those considerations, where its contribution is

crucial, such as in the mixing of gluons with the W's as well as in W-X-mixing.

23. J.C. Pati and Abdus Salam, Phys. Rev. D8, 1240 (1973),
J.C. Pati in Gauge Theory and Modern Field Theory, edited by
R. Arnowitt and P. Nath (M.I.T., Cambridge, 1975), p27.
24. V. Elias, Univ. of Maryland Tech. Rep. No. 77-049 and references therein; Q. Shafi, Univ. of Freiburg Preprint THEP 76/5.

Following normalisation group methods Elias obtains a

$$\frac{4}{3} e^2 < f_{15}^2 < 4 e^2, \text{ subject to certain assumptions of hierarchical}$$

breaking of the unifying symmetry G containing the minimal

symmetry $G = SU(2)_L \times SU(2)_R \times SU(4)_{L+R}'$ of the model of Ref. 6.

25. Within extended models, with more than four flavors, for example the mirror-model, in which quarks carry eight flavours altogether, (J.C. Pati and A. Salam, Physics Letters 58B, 333 (1975)), it may be possible to arrange the Fermi-mass matrix such that the $W_R^\pm$ are coupled to (V+A)-currents such as $\bar{p} \gamma_\mu (1 - \gamma_5) n' (\theta_R) + \bar{\nu}_e \gamma_\mu (1 - \gamma_5) E + \dots$, where n' and E denote a heavy quark and a heavy lepton respectively. In this case, suppression of (V+A)-amplitudes in β -decay and μ -decay may simply be a consequence of heaviness of the new fermions and the skewness of the Fermi-mass matrix, rather than of $m_{W_R} \gg m_{W_L}$.
26. Assuming SU(3)-symmetric matrix elements of vector and axial vector-currents, the present experimental values of g_V and g_A for Λ - β -decay seems to suggest that (within the four flavour-model) $(g^2/m_{W_R}^2 +) \sin^2 \theta_R < \frac{1}{3} (g^2/m_{W_L}^2 +) \sin^2 \theta_L$, which together with EQ.(11) leads to the inequality $\tan^2 \theta_R < 3 \tan^2 \theta_L$; in turn this implies that right-handed neutrinos may be emitted in strangeness-changing processes ($\Lambda \rightarrow p e^- \bar{\nu}_e$ and $K \rightarrow \mu + \nu_\mu$ etc.) perhaps as frequently as 10% of the times, even though in strangeness conserving processes their relative rate of emission is known to be $< 1\%$. This may be of experimental consequence in eventually detecting the right-handed

neutrinos and left-handed antineutrinos, which are expected to exist within the model. In this regard it is worth noting that the model in general may permit the emergence of either (i) light four-component ν_e and ν_μ , or (ii) massless two component ($\nu_{eL}, \nu_{\mu L}$) as well as their two-component right-handed partners ($\nu_{eR}, \nu_{\mu R}$), such masslessness would be preserved provided the eventual mechanism of spontaneous symmetry breaking is constrained to respect γ_5 -symmetry for the neutrinos. At any rate it should be stressed that the doubling of helicities with left-handed fermions being always balanced by the existence of right-handed counterparts, even for neutrinos, is one crucial prediction of the left $\leftrightarrow$ right symmetric theory (Ref. 6).

27. Using the mass-matrix (9.2)-(9.3), it is easy to verify with $b \neq 0$, but $a = c = 0$, that in addition to the triplet $(W^\pm, W^3)_L$ and the octet of gluons $(v(8))$, the particular combination $B^0 (f_{15} W_R^3 / -2 / 3g v_{15}) / (f_{15}^2 + \frac{2}{3} g^2)^{\frac{1}{2}}$ remains massless, the corresponding residual local symmetry thus being $SU(2)_L \times SU(3)_{L+R}' \times U(1)$.

28. We may now point out the one-one correspondence referred to before in Sec. I. between the scheme of spontaneous symmetry breaking proposed in Ref. 6 (see Eq. (17) and Footnote 21 of this Paper) and the one being considered by Mohapatra and Sidhu (ref. 16). ($b^2 + c^2$) of Ref. 16 corresponds to a^2 of Ref. 6; likewise a^2 of the former to $(b_4^2 + c_4^2)/2$ of the latter and zero of the former $(b_4 - c_4)$ of the latter. The gauge coupling constants g and g' of Ref. 16 correspond to g and $(\frac{\sqrt{3}}{2})f_{15}$ respectively of Ref. 6.

29. If $\epsilon_L \neq 0$, as stated earlier, all neutral-current amplitudes given in Secs. III and IV would simply be multiplied by an uniform enhancement factor $(\frac{c^2 + a^2 + \epsilon_L^2}{c^2 + a^2}) = (4 m_{WL}^2 + \epsilon_L^2) / (4 m_{WL}^2 + g^2 \epsilon_L^2)$.

Such a case offers the possibility (for a range of values of ϵ , and r), see Eq. (28)) that one of the neutral gauge particles can be even lighter than the values presented in sec.V (for $\epsilon_L = 0$). This

possibility will be considered in more detail elsewhere.

30. For many purposes it is convenient to use an alternative parametrisation $b^2 \equiv c^2 (1+x)$; $a^2 \equiv b^2 y$ with $-1 < x < \infty$, $0 < y < \infty$. Thus $x = (\eta - \zeta)/\zeta$ and $y = (1 - \zeta)/\eta$. See discussions in Section V for the usefulness of x and y .
31. See for example, J. Bernabeu and C. Jarkskog, CERN Preprint (Th 2206 - 1976).
32. Note, of all the scalar multiplets introduced (A, B, C, D, E_L, E_R), only the multiplet A can have Yukawa coupling with the fermions.
33. This dichotomy between neutrino versus electron-induced processes has once again its root simply in the fact that available neutrinos are essentially two component objects (ν_L 's or ν 's), while electrons or atoms are four-component objects.
34. V. Elias and A.R. Swift, Phys. Rev. D13, 2083 (1976).
35. It should be noted that Case B is, of course, not necessarily exclusive of Case A. One may satisfy the requirements of both cases by setting $a^2 = 0$ (i.e. $\zeta = 1$) and simultaneously setting $\eta = b^2/(c^2 + a^2) \gg 1$.
36. Note that the point $r = f_{15}^2/g^2 = \frac{1}{3}$ corresponds (via Eq. (13.a.) to $(f_{15}^2/e^2) = (\frac{4}{3})$, which is the lower limit of the range $(\frac{4}{3} e^2 < f_{15}^2 < 4e^2)$ suggested by the renormalization group considerations (Ref. 24).
37. See for example, S. Sehgal, Phys. Lett. B48, 133 (1974); V. Barger and D.V. Nanopoulos, Wisconsin Preprint C00-562 (1976). The definitions of leptonic scattering parameters $C_{\nu_\mu e}$, $C_{\bar{\nu}_\mu e}$ and $C_{\bar{\nu}_e e}$ follow those of B.W. Lee, Rapporteur Talk at the Aachen International Conference (1976), (To appear in the Proceedings). For example $\sigma_{\nu_\mu e} \equiv C_{\nu_\mu e} (10^{-41} \times E) \text{ cm}^2/\text{Gev}$, similarly for $C_{\bar{\nu}_\mu e}$ and $C_{\bar{\nu}_e e}$.
38. This can introduce an error (5 to 10%).
39. Our inferences are, however, based on a much wider range of variation of the three relevant parameters $r = (f_{15}^2/g^2)$, $\zeta = c^2/(c^2 + a^2)$

and $\eta = b^2/(c^2+a^2)$ than those exhibited in the tables and the figures.

40. J. Blietschau, et al., CERN Preprint, 1976, submitted to Nucl. Phys.; P. Musset, report at the Tbilisi Conference (1976).

41. L. Stutte, in Proceedings of the Conference on Particles with New Quantum Numbers, Madison (1976).

42. A. Venvenuti, et al., Proceedings of the Conference on Particles with New Quantum Numbers, Madison (1976).

43. H. Faissner et al., reports at Aachen and Tbilisi Conferences.

44. F. Reines, H. Curr and H. Sobel, Phys. Rev. Lett. 37, 315 (1976).

45. W. Lee et al., Phys. Rev. Lett. 37, 186 (1976).

46. D. Cline et al., Phys. Rev. Lett. 37, 648 (1976).

47. The comparison of theory with experiments for leptonic processes is still premature due to large errors in the experimental values.

As regards elastic p and p^- scattering parameters, the comparison is somewhat premature, since the theoretical predictions depend upon the knowledge of ν , $\bar{\nu}$ -spectra and the mass characterising the axial-form-factor. We have followed

D.P. Sidhu, Phys. Rev. D14, 2235 (1976); C. Albright, C. Quigg,

J. Smith and R. Schrock, Phys. Rev. D14, 1780 (1976), R.M. Barnett,

Harvard Preprint (1976); and V. Barger and D.V. Nanopoulos (Ref. 37)

in taking these features into account. We thank Professor Vernon

Barger for most helpful communications in this regard.

48. In Figs. 1,2 and 3, we have exhibited the results for a fixed

$r = (f_{15}^2/g^2) = 0.44$. For a given $y = (1 - \xi)/\eta$, the neutral current cross section ratio $R = (\sigma_{Nc}^{\bar{\nu}N} / \sigma_{Nc}^{\nu N})$ increases with increasing r

(see Table I). However as long as the coupling constant ratio r stays within the range 0.33 to 0.66 (the reasons for this restriction is discussed later), the cross-section ratio R decreases below the GGM-range (0.59 ± 0.14) once y exceeds ~ 0.28 . (see Fig. 3). We should also remark that it is possible to obtain a fit for some of the parameter $C_{\mu e}^{22}$ (Ref. 37) becomes too small compared to the present experimental range, or R becomes too small, or else R becomes too large. (Note that in the $SU(2) \times U(1)$ -limit ($\eta \gg 1$), $y = (1 - \zeta)/\eta$ vanishes; on the other hand $x = (\eta - \zeta)/5 = (b^2 - c^2)/c^2$ can be large (by $\zeta \rightarrow 0$) without y vanishing. Thus $x \rightarrow \infty$ is not necessarily the $SU(2) \times U(1)$ -limit only provided $\zeta = c^2/(c^2 + a^2)$ is non zero).

49. The answer does not depend sensitively on ζ near $\zeta \sim 1$.

50. While $m_{W_L^+}$ depends on r , the ratio $(m_{N_1}/m_{W_L^+})$ is not so sensitively dependent on r (see Table II).

51. Allowing for $(g^2/4) (a_1^2 + a_1 a_4) \ll \overset{\text{and}}{\frac{1}{4} m_{W_R^+}^2} \gg 10 m_{W_L^+}^2$, the lowering of W_L^+ mass due to $W_L^- - W_R^-$ mixing (see (9.2)) is given by

$$\frac{g^2}{4} \frac{(a_1^2 + a_1 a_4)^2}{a^2 + b^2 + e_R^2} = \left(\leq \frac{1}{4} m_{W_L^+}^2 \right) \frac{\leq \frac{1}{4} m_{W_L^+}^2}{\geq 10 m_{W_L^+}^2} \leq m_{W_L^+}^2 / 160$$

which is a small effect. Likewise the $W_L^+ - W_R^+$ mixing angle is given by

$$2(a_1^2 + a_1 a_4) / (a^2 + b^2 + e_R^2 - a^2 - c^2 - e_L^2) = 2 \left(\frac{\leq \frac{1}{4} m_{W_L^+}^2}{\geq 10 m_{W_L^+}^2} \right) \leq \frac{1}{20}$$

which also is a small effect for most purposes.

52. This is similar to the case studied by De Rujula et al..(Ref.17).

TABLE I Prediction of Neutrino-Parameters in the Chiral Limit $a =$, or -1 , or a

$$\eta = \frac{b^2}{c^2 + a^2} = 0 \text{ to } \infty.$$

PARAMETERS	PREDICTIONS $(r = f_{15}^2/g^2)$						EXPERIMENT		
	0.11	0.22	0.33	0.44	0.77	1	G G M ⁴⁰	C I T F ⁴¹	H P W F ⁴²
R_V	0.39	0.33	0.30	0.27	0.24	0.23	0.25 ± 0.04	0.24 ± 0.02	0.29 ± 0.04
$R_{\bar{V}}$	0.41	0.39	0.39	0.40	0.42	0.44	0.39 ± 0.06	0.34 ± 0.09	0.39 ± 0.10
R	0.35	0.39	0.44	0.48	0.53	0.64	0.59 ± 0.14	-	$\leq 0.61 \pm 0.25$
$C_{\bar{V}_\mu e}$	0.11	0.12	0.14	0.17	0.22	0.25	$\left. \begin{array}{l} + 0.21 \\ - 0.09 \end{array} \right\} \text{GGM}^{40} \quad 0.54 \pm 0.17^{43}$ (Aachen Padova)		
$C_{V_\mu e}$	0.25	0.18	0.14	0.13	0.11	0.11	0.24 ± 0.12 Aachen Padova ⁴³		
$C_{\bar{V}_e e}/0.57$	0.58	0.81	1.00	1.15	1.45	1.58	0.87 ± 0.25 (1.5 ~ 3 Mev) 1.70 ± 0.44 (3 ~ 4 Mev) Reines et al ⁴⁴		
R_V^{el}	0.19	0.15	0.13	0.12	0.10	0.10	"	CIR ⁴⁵ 0.23 ± 0.09	HPWF ⁴⁶ 0.17 ± 0.05
$R_{\bar{V}}^{\text{el}}$	0.19	0.17	0.18	0.19	0.22	0.24	"	-	0.20 ± 0.10
R^{el}	0.34	0.39	0.46	0.53	0.72	0.81	"	-	0.4 ± 0.2

FOOTNOTE TO TABLE I

The above predictions in respect of neutrino-parameters hold for all values of η ($\equiv b_7^2/(c^2 + a^2)$) = 0 to ∞ in the chiral limit (i.e. $\xi = 1$ or $a^2 = 0$). Thus the same predictions can also be obtained for the $SU(2) \times U(1)$ theory (corresponding to the limit $\eta \gg 1$), provided we substitute in this case $\sin^2 \theta_W = \frac{3r}{6r+2}$ ($= \frac{1}{8}$ to $\frac{3}{8}$ for r varying from $\frac{1}{9}$ to 1). However the predictions of the left-right symmetric theory agreeing with them only in the limit $\eta = \frac{b^2}{c^2 + a^2} \gg 1$ (see Table II). (The η -dependence of the above predictions with regard to neutrino-parameters is fairly stable even if we depart from the chiral limit by allowing $a^2 \neq 0$, i.e. $\xi < 1$ (see Table III)). We define the experimental parameters by $R_{\nu} = (\sigma_{Nc}^{\nu N} / \sigma_{cc}^{\nu N})$; $R_{\bar{\nu}} = (\sigma_{Nc}^{\bar{\nu} N} / \sigma_{cc}^{\bar{\nu} N})$, $R = (\sigma_{Nc}^{\bar{\nu} N} / \sigma_{Nc}^{\nu N})$ and similarly R^{el} , R_{-}^{el} , and R^{el} , where Nc and cc refer to neutral and charged current-cross sections respectively. The parameters $C_{\nu e}$, $C_{\bar{\nu} e}$ and $C_{\bar{e} e}$ have been defined in Ref. 37. The predictions of elastic scattering parameters are for a choice of $M_A^2 = 1.15$ Gev, where M_A^2 characterises the axial vector form-factor. The predictions would be somewhat lower (by about 15-20%) if we choose $M_A^2 = 1$ Gev.

TABLE II Predictions of the masses of N_1 and N_2 and the atomic parity violation - parameter in the chiral limit ($a^2 = 0$ or $\zeta = 1$) as a function of $(\eta - \zeta)/\zeta$.

$r = \frac{f_{15}^2}{g^2}$	$\frac{\eta - \zeta}{\zeta} = \frac{b^2 - c^2}{c^2}$	$K_1 = \frac{M_{W_L}^{2+}}{M_{N_1}^2}$	$K_2 = \frac{M_{W_L}^{2+}}{M_{N_2}^2}$	Q_W	$M_{W_L}^+ \approx 53 (1 + \frac{1}{3r})^{\frac{1}{2}}$
0.33	- 0.5	1.64	0.61	126	75 GeV
	-0.375	1.36	0.59	75	
	0.0	1.00	0.50	0	
	1.0	0.82	0.34	- 63	
SU(2) $\times$ U(1) limit	99	0.75	0.007	- 125	$\sin^2 \theta_W = 0.25$
0.44	-0.5	1.61	0.53	133	70 GeV
	-0.375	1.34	0.51	83	
	0.0	1.00	0.43	0	
	1.0	0.81	0.27	- 69	
SU(2) $\times$ U(1) limit	99	0.72	0.006	- 136.5	$\sin^2 \theta_W = 0.29$

TABLE III Predictions of All Neutral Current Parameters for the Non-Chiral case

 $(a^2 \neq 0, \text{ i.e. } \zeta \neq 1), r = .44$

PARAMETERS	$\zeta = 0.9$					$\zeta = 0.8$				
$(1-\zeta)/\eta \equiv y$	0.5	0.16	.10	.05	.03	0.5	0.16	.10	.05	.03
R_v	0.22	0.25	0.26	0.26	0.27	0.22	0.24	0.25	0.26	0.27
$R_{\bar{v}}$	0.24	0.31	0.34	0.36	0.37	0.26	0.30	0.31	0.34	0.35
R	0.37	0.42	0.44	0.46	0.46	0.40	0.38	0.41	0.43	0.45
C_{ν_e}	0.09	0.13	0.14	0.15	0.16	0.07	0.12	0.13	0.14	0.15
$C_{\bar{\nu}_e}$	0.05	0.09	0.10	0.11	0.12	0.03	0.07	0.09	0.10	0.11
$C_{\nu_e}/0.57$	1.22	1.17	1.16	1.16	1.15	1.29	1.20	1.18	1.17	1.16
R_v^{el}	0.08	0.10	0.11	0.11	0.12	0.07	0.09	0.10	0.11	0.11
$R_{\bar{v}}^{el}$	0.08	0.14	0.15	0.17	0.18	0.07	0.11	0.13	0.15	0.17
R^{el}	0.35	0.45	0.48	0.50	0.51	0.30	0.39	0.43	0.47	0.50
$(m_{W_L} + m_{N_1})^2$	2.39	1.12	0.45	0.77	0.74	1.80	0.94	0.79	0.74	0.73
$(m_{W_L} + m_{N_2})^2$	0.62	0.55	4.07	0.29	0.19	0.66	0.60	0.47	0.30	0.19
Q_W	360	.60	-13.0	-65	-94.0	230	36	-24	-70	-96

G. FIGURE-CAPTIONS

Figure 1 Variation of R_{ν} as a function of $x = (\eta - \zeta)/\zeta = (b^2 - c^2)/c^2$ for $y = (1 - \zeta)/\zeta = a^2/b^2 = 0.0, 0.1$ and 0.18 with the coupling constant-ratio r fixed at 0.44 .

Figure 2 Variation of $R_{\bar{\nu}}$ as a function of x for $y = 0.0, 0.1$ and 0.18 with r fixed at 0.44 .

Figure 3 Variation of $R = (\sigma_{\bar{\nu}N}^{\nu}/\sigma_{Nc}^N)$ as a function of y with $r = 0.44$ and 0.66 . Note that R is independent of $x = (\eta - \zeta)/\zeta = (b^2 - c^2)/c^2$.

Figure 4 R_{ν} versus $R_{\bar{\nu}}$ for varying coupling constant-ratio r at the chiral symmetric point $\zeta = c^2/(c^2 + a^2) = 1$, where all neutrino-amplitudes are functions of the single parameter r . The allowed range of the coupling constant-ratio r (following from comparison of the predicted values with experiments) is not altered appreciably by departing from the chiral-limit ($\zeta = 1$) and varying $\eta = b^2/(c^2 + a^2)$ as well.

Figure 5 Variation of the atomic parity violation-parameter Q_W with $x = (\eta - \zeta)/\zeta = (b^2 - c^2)/c^2$ for the cases of exact or approximate chiral symmetry (i.e. $1 - \zeta = 0$ and 0.1) with fixed coupling constant ratio r .

Figure 6 Variation of the predicted masses of the two neutral gauge particles N_1 and N_2 with $x = (\eta - \zeta)/\zeta$ for the chiral symmetric point ($\zeta = 1$) with fixed coupling constant ratio r . These predictions are insensitive to small departures from the chiral symmetric point (i.e. ζ varying say between 0.8 to 1) even at relatively small x .

Figure 7 Variation of the predicted masses of the two neutral gauge bosons with $Q_W/(Q_W)_0$. Here $(Q_W)_0$ is the $SU(2) \times U(1)$ limit of Q_W . The coupling constant ratio r is fixed.

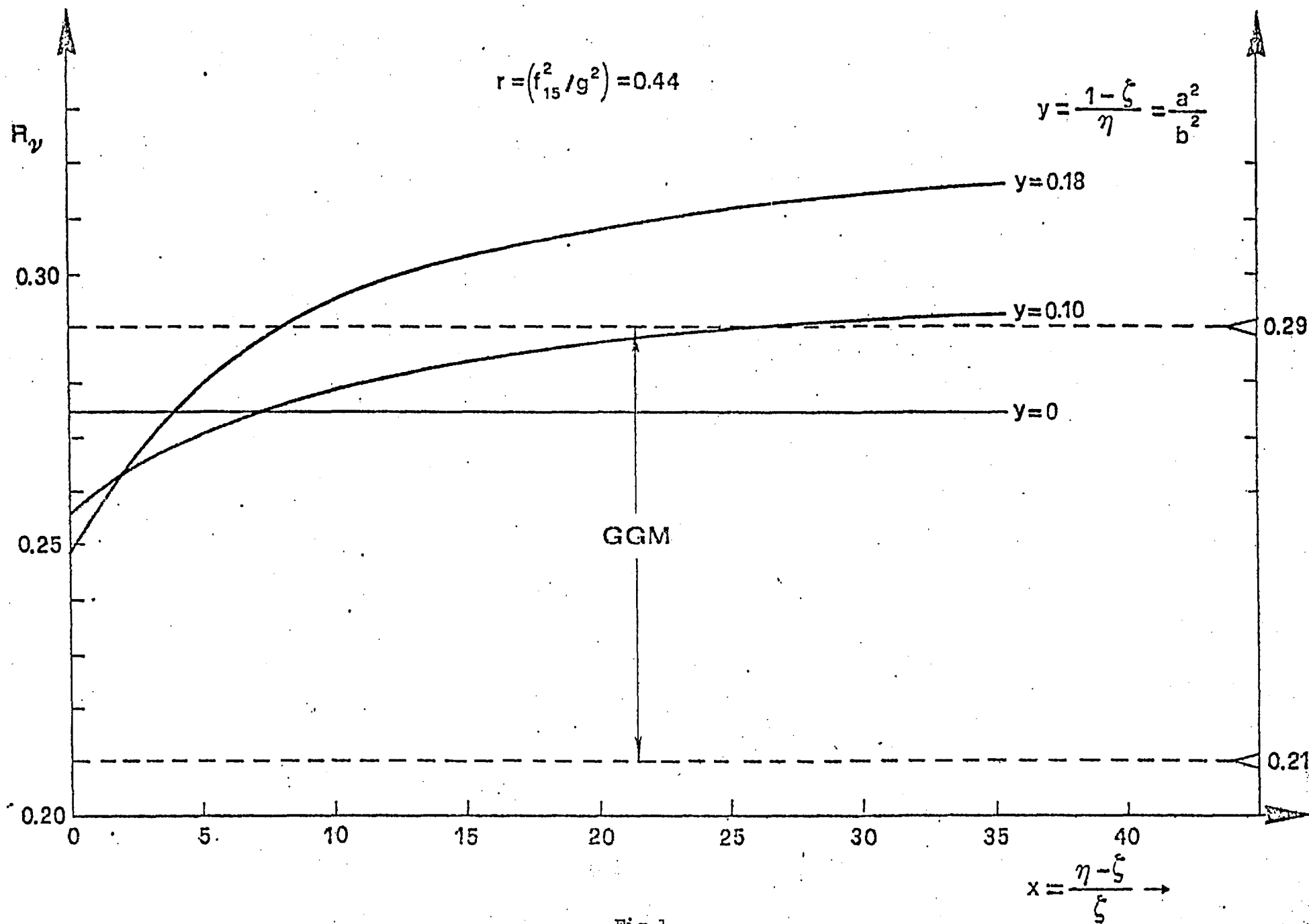


Fig.1

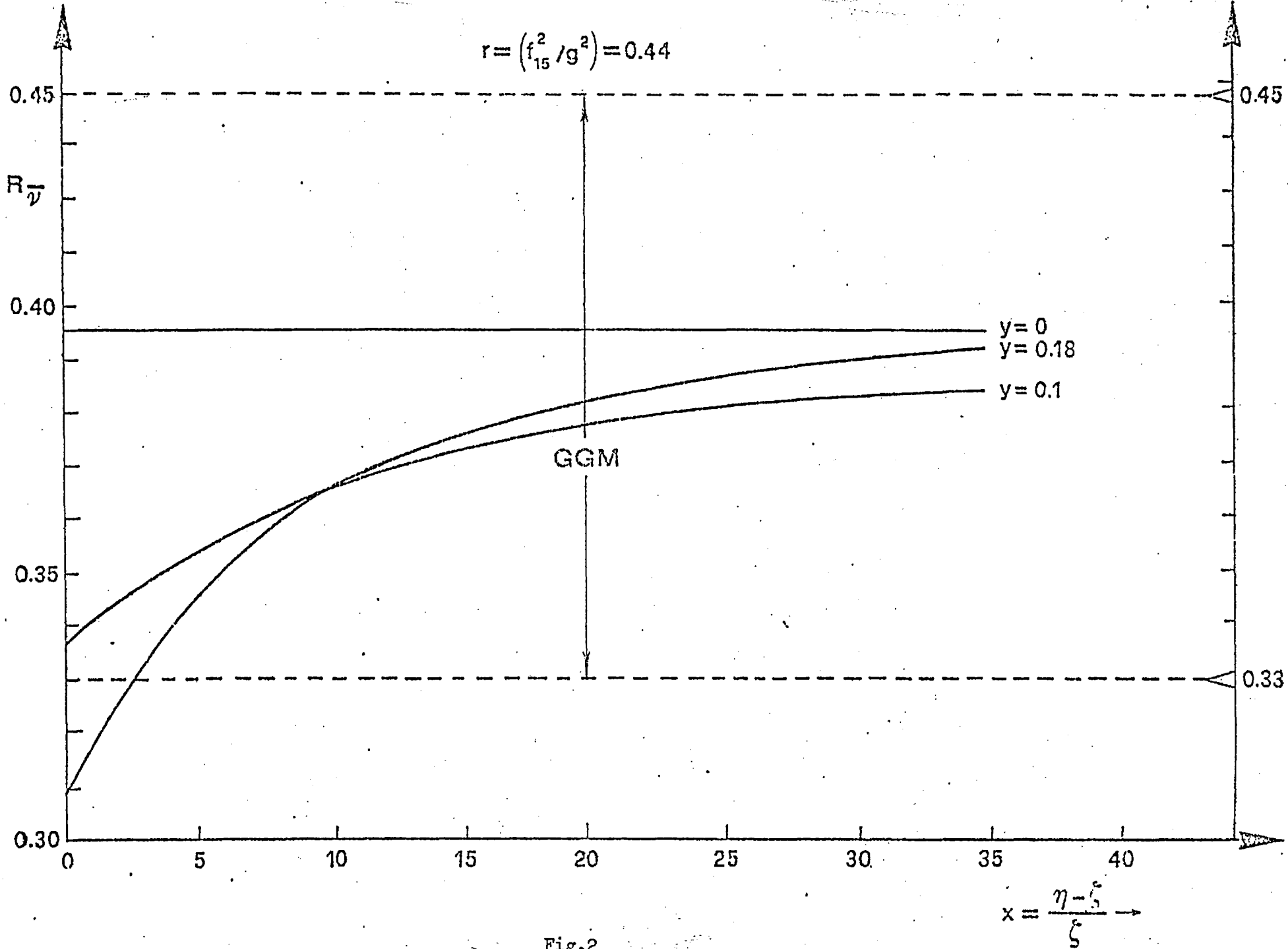
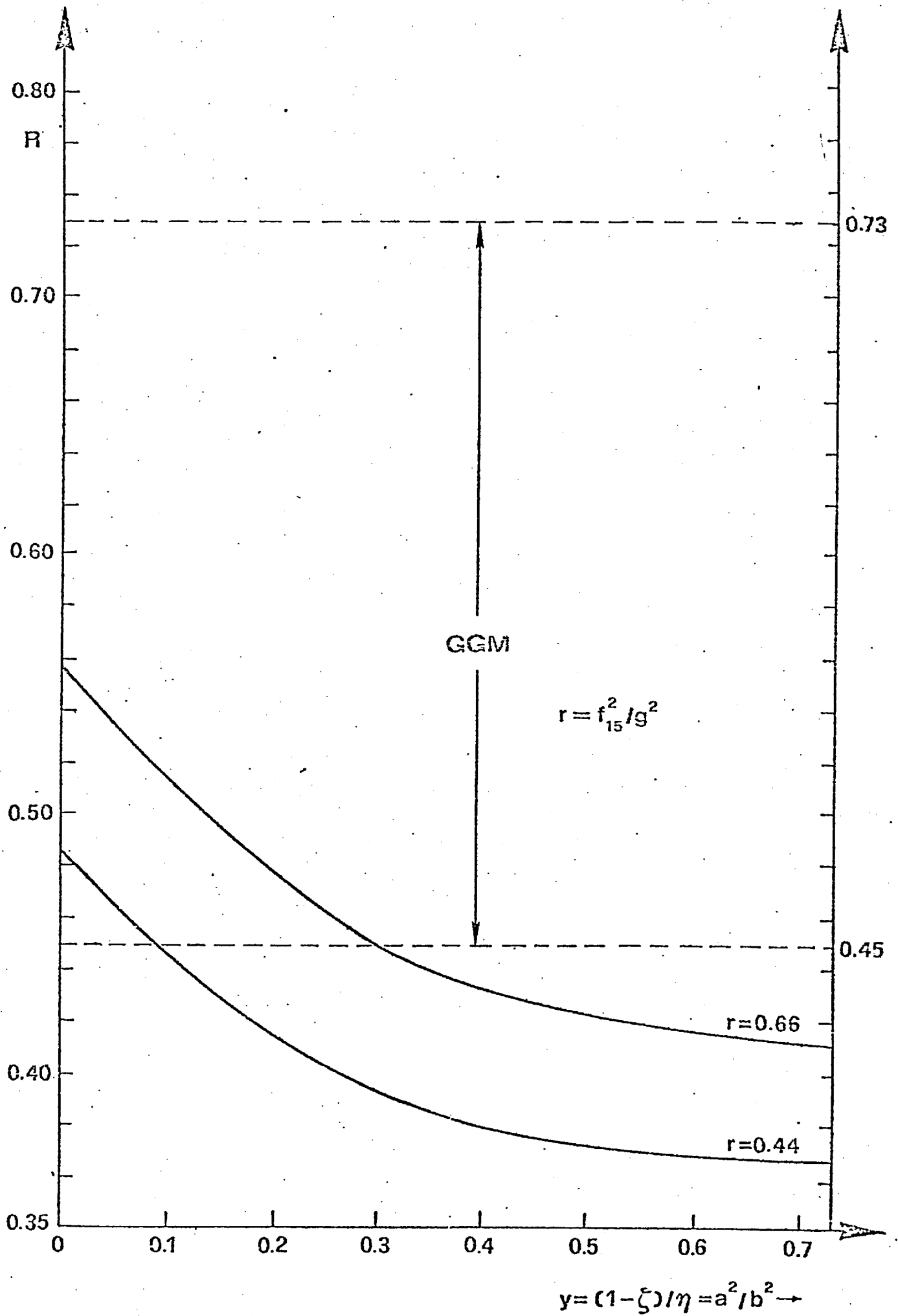


Fig.2



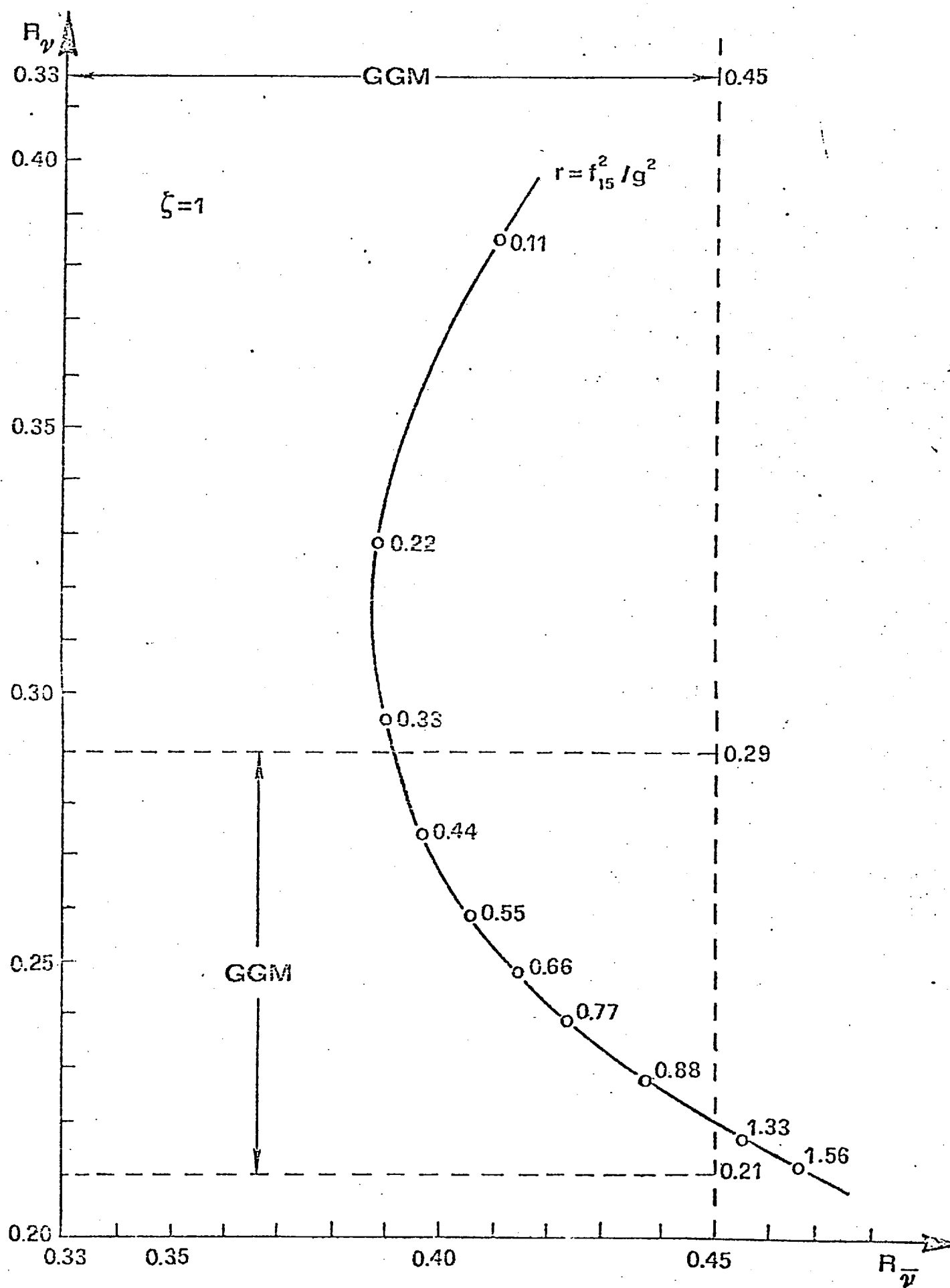


Fig. 4

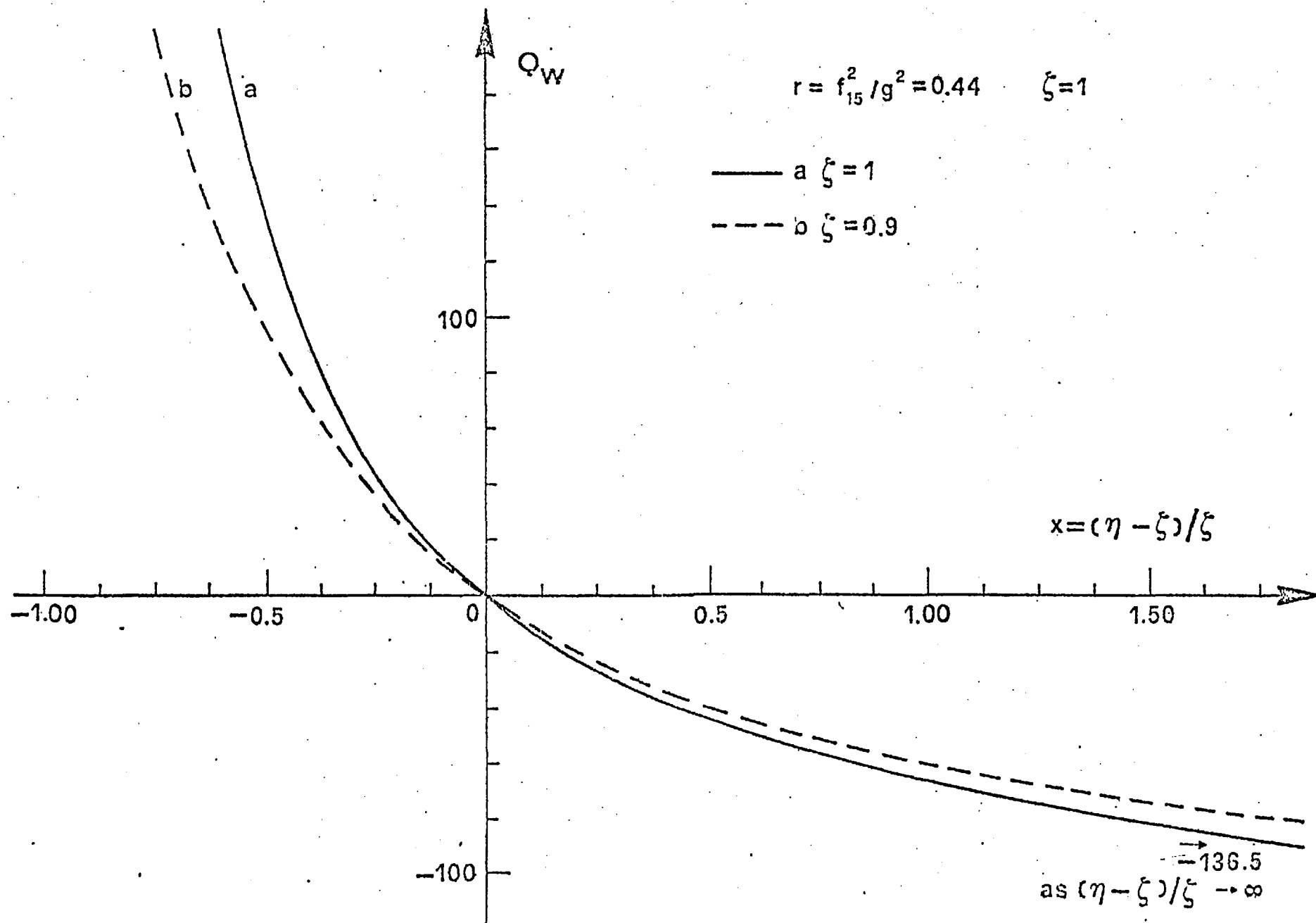


Fig.5

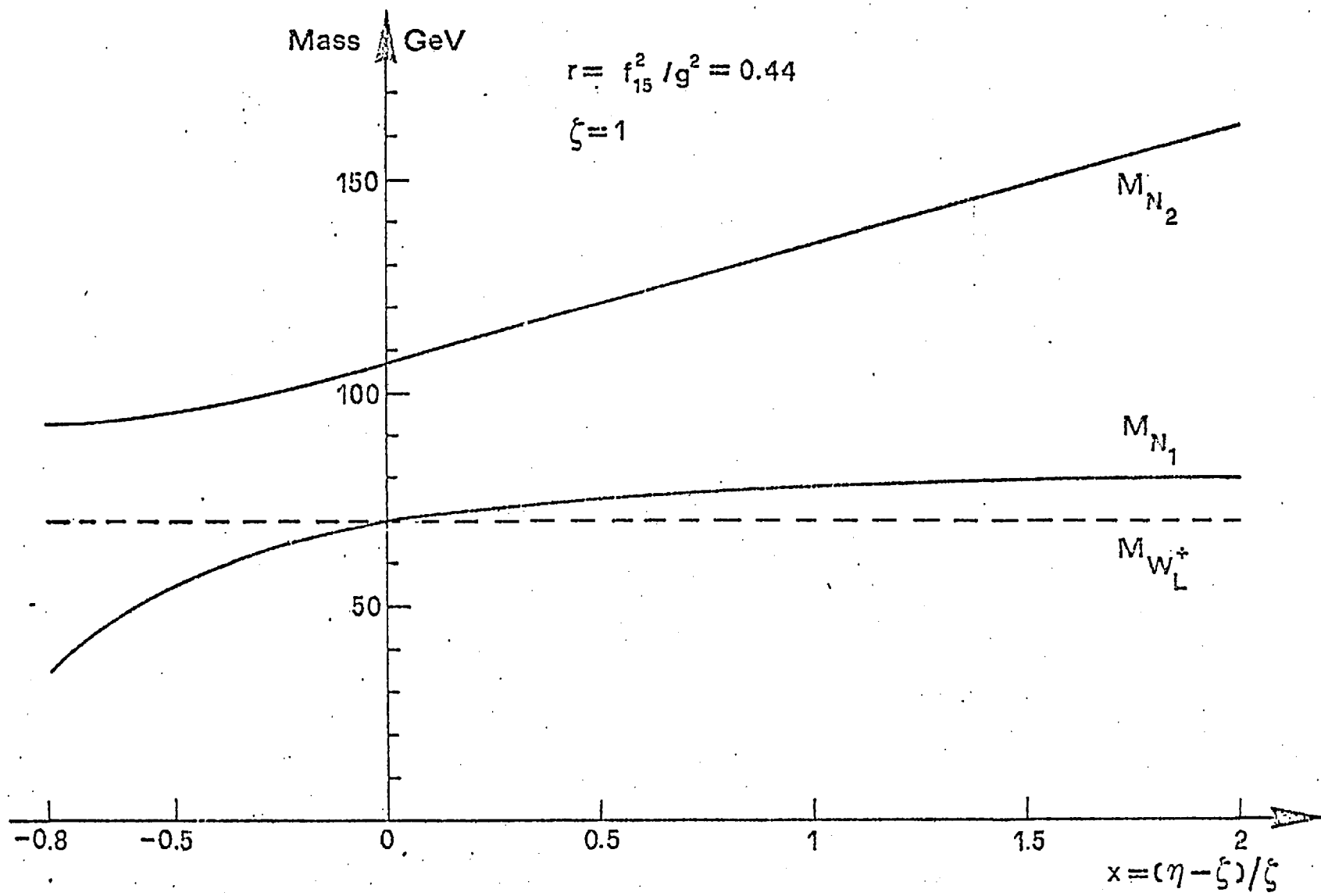


Fig.6

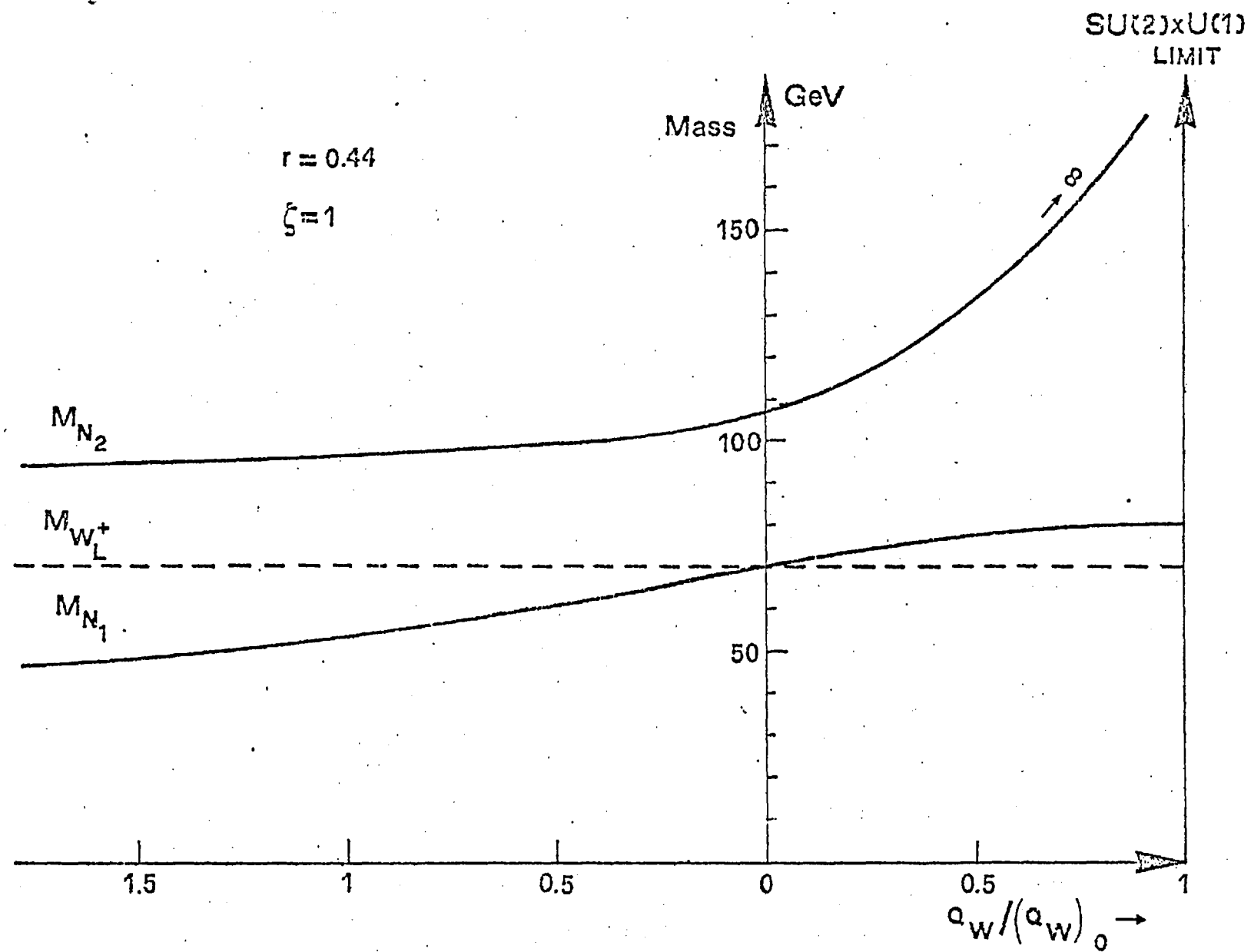


Fig.7

P A R T III

EFFECTS OF NEUTRAL WEAK CURRENTS IN ELECTRON-POSITRON
ANNIHILATION

A. INTRODUCTION

The last section was devoted to constructing a model of weak and electromagnetic interactions based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$. The motivation for the introduction of this "left-right" symmetric structure was provided by the findings of atomic parity non violation in Bismuth ⁽¹⁾. These experiments could be indicating the correctness of an idea proposed some time ago ⁽²⁾ that right handed and left handed gauge currents enter into the basic Lagrangian in a parallel fashion. The proposal had three ingredients:

1. Two sets of charged $W_L^\pm, W_R^\pm$ and two neutral gauge bosons N_1 and N_2 .
2. A one-one correspondence between left and right matter, entering the respective currents and in particular between a set of right handed neutrinos $\nu_{eR}, \nu_{\mu R}$ with their familiar left-handed counterparts $\nu_{eL}, \nu_{\mu L}$.
3. The prediction that parity-violation might only be a transient low-energy phenomenon, which disappears at asymptotic energies ^{2,4}.

Such a left-right symmetric theory ⁵ permits one of the two neutral gauge bosons (N_1) to be particularly light (perhaps as light as 50 GeV) compatible with all neutrino data as well as with the value of the atomic parity violating parameter Q_W , given that Q_W may be small and even opposite in sign compared to its $SU(2)_L \times U(1)$ value. In view of such a light mass being in the vicinity of PEP and PETRA energy ranges, it is of special interest to examine the consequences of the left-right symmetric theory with such a light mass for e^+e^- annihilation. We show that e^+e^- asymmetry measurements even at PEP and PETRA

energies will provide a sharp distinction between the left-right symmetric theory with the two relatively light neutral gauge bosons on the one hand and the left-handed $SU(2)_L \times U(1)$ theory with just one relevant neutral gauge particle Z^0 on the other. We stress that atomic parity experiments, although indicative of a left-right theory, may not be regarded as definitive largely due to theoretical uncertainties associated with the relevant atomic calculations. The $e^+e^- \rightarrow \mu^+\mu^-$ asymmetry calculations are totally devoid of any such dynamical uncertainty; thus such measurements of PEP and PETRA would be crucial to a clear distinction between the left-right symmetric theory with the solution of two light neutral particles on the one hand and the left-handed $SU(2)_L \times U(1)$ theory on the other.

The simplest local gauge structures incorporating the basic hypothesis of a discrete symmetry between left and right matter and consistent with the phenomenology of known interactions are

$$\overline{G} = SU(2)_L \times SU(2)_R \times U(1)_{L+R} \times SU(3)_{L+R}^c$$

or its quark-lepton unifying extension

$$G = SU(2)_L \times SU(2)_R \times SU(4)_{L+R}^c$$

based on four quark flavours (P, N, λ , C)_{L,R} with three colours and four leptons⁶

($\nu_e, e^-, \mu^-, \nu_\mu$)_{L,R}. Both these local symmetries with manifest and (as it turns

out⁷ "natural") left-right symmetry were proposed in Ref. 2. It was shown

that it is possible to choose the pattern of spontaneous symmetry breaking (SSB)

and the parameters in a Higgs potential in such a manner that one pair of charged

W's ($W_R^\pm$ with predominant right handed coupling) and one of the two neutral gauge

particles, N_2 , are much heavier ($\sim 10^3 \sim 10^5$ GeV) than the " $W_L^\pm$ " and N_1 . With this

choice, the effective gauge group for weak and electromagnetic interactions

relevant for relatively low energy experiments ($E \lesssim 10^3$ GeV) would be familiar

$SU(2)_L \times U(1)$ with N_1 identified as Z^0 . It was also noted⁸ in Ref. 2, that

one could equally well make an alternative choice of Higgs-Kibble parameters

such that the masses of all W's as well as the two neutral gauge bosons N_1 and N_2

were comparable to each other. It is this version of the theory, which has recently been examined by several authors ^{5,9,10} stimulated by the near null result of atomic parity experiments.

Now there are two interesting consequences of the larger model $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ which the treatment of Reference 5 has particularly highlighted.

(a) For all processes initiated by the left-handed neutrinos (or right handed anti neutrinos), the predictions of the larger model amazingly coincide with those of the restricted $SU(2)_L \times U(1)$ in the chiral limit which neglects masses of quarks, leptons as well as W_L - W_R mass mixing compared with M_{W_L} .

(b) The atomic parity violating parameter Q_W , on the one hand (for the larger $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ symmetry) can in general be much smaller than its value for the $SU(2)_L \times U(1)$ limit and even opposite in sign. If Q_W is opposite in sign then for the $SU(2)_L \times U(1)$ prediction, there is the exciting possibility that $M_{N_1} < M_{W_L}$. In the (perhaps unlikely) event that Q_W turns out to be ≈ 440 for Bismuth, we would predict $M_{N_1} \approx 39$ GeV so that PEP and PETRA might just produce N_1 as a physical object. We now turn to the calculation of the various asymmetries in e^+e^- annihilation.

B. CHARGE ASYMMETRY IN $e^+e^- \rightarrow \mu^+\mu^-$.

In general, the photon-neutral weak bosons interference influences the e^+e^- annihilation in such a way that characteristic asymmetries should grow with the centre-of mass energy squared, therefore the high energy region is most promising for finding effects caused by the neutral gauge bosons N_1 and N_2 . Weak effects due to photon-weak interference, cause forward backward and charge asymmetries, as well as outgoing particles that appear with non vanishing average helicities.

The interaction Lagrangian due to the photon, N_1 and N_2 gauge bosons is

$$\mathcal{J}_\mu^{em} = \frac{2}{3} \bar{p} \gamma_\mu p + \frac{2}{3} \bar{c} \gamma_\mu c - \frac{1}{3} \bar{n} \gamma_\mu n - \frac{1}{3} \bar{\lambda} \gamma_\mu \lambda \\ - \bar{e} \gamma_\mu e - \bar{\mu} \gamma_\mu \mu$$

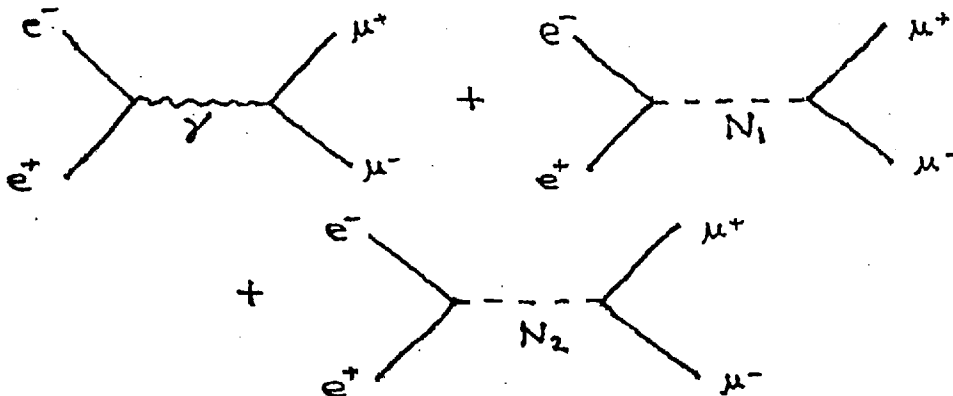
where

$$\mathcal{J}_\mu^{N_1} = \frac{g}{2\sqrt{2}} \left[(\bar{p} \gamma_\mu \gamma_5 p - \bar{n} \gamma_\mu \gamma_5 n - \bar{\lambda} \gamma_\mu \gamma_5 \lambda + \bar{c} \gamma_\mu \gamma_5 c) \cos \beta \right. \\ \left. - \frac{(1-r) \sin \beta}{(3r+1)^{1/2}} (\bar{p} \gamma_\mu p + \bar{c} \gamma_\mu c) - \frac{(1+r) \sin \beta}{(3r+1)^{1/2}} (\bar{n} \gamma_\mu n + \bar{\lambda} \gamma_\mu \lambda) \right. \\ \left. + [(3r+1)^{1/2} \sin \beta + \cos \beta] (\bar{\nu}_{eL} \gamma_\mu \nu_{eL} + \bar{\nu}_{\mu L} \gamma_\mu \nu_{\mu L}) - (\bar{e} \gamma_\mu \gamma_5 e + \bar{\mu} \gamma_\mu \gamma_5 \mu) \cos \beta \right. \\ \left. + [(3r+1)^{1/2} \sin \beta - \cos \beta] (\bar{\nu}_{eR} \gamma_\mu \nu_{eR} + \bar{\nu}_{\mu R} \gamma_\mu \nu_{\mu R}) \right. \\ \left. + \frac{(3r-1) \sin \beta}{(3r+1)^{1/2}} (\bar{e} \gamma_\mu e + \bar{\mu} \gamma_\mu \mu) \right]$$

and

$$\mathcal{J}_\mu^{N_2} = \frac{g}{2\sqrt{2}} \left[(-\bar{p} \gamma_\mu \gamma_5 p - \bar{c} \gamma_\mu \gamma_5 c + \bar{n} \gamma_\mu \gamma_5 n + \bar{\lambda} \gamma_\mu \gamma_5 \lambda) \sin \beta \right. \\ \left. + \frac{(1-r) \cos \beta}{(3r+1)^{1/2}} (\bar{p} \gamma_\mu p + \bar{c} \gamma_\mu c) - \frac{(1+r) \cos \beta}{(3r+1)^{1/2}} (\bar{n} \gamma_\mu n + \bar{\lambda} \gamma_\mu \lambda) \right. \\ \left. + [(3r+1)^{1/2} \cos \beta - \sin \beta] (\bar{\nu}_{eL} \gamma_\mu \nu_{eL} + \bar{\nu}_{\mu L} \gamma_\mu \nu_{\mu L}) \right. \\ \left. + [(3r+1)^{1/2} \cos \beta + \sin \beta] (\bar{\nu}_{eR} \gamma_\mu \nu_{eR} + \bar{\nu}_{\mu R} \gamma_\mu \nu_{\mu R}) \right. \\ \left. + \frac{(3r-1) \cos \beta}{(3r+1)^{1/2}} (\bar{e} \gamma_\mu e + \bar{\mu} \gamma_\mu \mu) + (\bar{e} \gamma_\mu \gamma_5 e + \bar{\mu} \gamma_\mu \gamma_5 \mu) \sin \beta \right]$$

The relevant graphs contributing to the process $e^+ e^- \rightarrow \mu^+ \mu^-$ are



The reduced amplitude or equivalently the effective current Lagrangian is

$$\frac{M_{fi}}{-i} = \frac{e^2}{S} \bar{e} \gamma_\alpha e \cdot \bar{\mu} \gamma_\alpha \mu + \sum_{i=1,2} (g_{\alpha\beta} - \frac{q_\alpha q_\beta}{M_{N_i}^2}) \frac{\bar{e} \gamma_\alpha (g_V^{\alpha\omega} + g_A^{\alpha\omega} \gamma_5) e \cdot \bar{\mu} \gamma_\beta (g_V^{\mu\omega} + g_A^{\mu\omega} \gamma_5) \mu}{(q^2 - M_{N_i}^2)}$$

The term $q_\alpha q_\beta$ in the boson propagator just gives contributions proportional to $\underline{Me}$ and will be neglected. This simplifies the

$$M_{Ni}^2$$

matrix element to

$$\frac{M_{fi}}{-i} = G_\gamma(S) \bar{e} \gamma_\alpha e \cdot \bar{\mu} \gamma_\alpha \mu + G_{VV}^{(\omega)} \bar{e} \gamma_\alpha e \cdot \bar{\mu} \gamma_\alpha \mu + G_{VA}^{(\omega)} \bar{e} \gamma_\alpha e \cdot \bar{\mu} \gamma_\alpha \gamma_5 \mu + G_{AV}^{(\omega)} \bar{e} \gamma_\alpha \gamma_5 e \cdot \bar{\mu} \gamma_\alpha \mu + G_{AA}^{(\omega)} \bar{e} \gamma_\alpha \gamma_5 e \cdot \bar{\mu} \gamma_\alpha \gamma_5 \mu.$$

where

$$G_\gamma(S) = \frac{e^2}{S} = \frac{G_F}{\sqrt{2}} \cdot \frac{M_W^2}{S} \cdot \frac{12r}{3r+1}$$

$$G_{VV}^{(\omega)}(S) = \frac{G_F}{\sqrt{2}} \cdot M_W^2 \cdot \frac{(3r-1)^4}{(3r+1)} \cdot [G_1(S) \sin^2 \beta + G_2(S) \cos^2 \beta]$$

$$G_{VA}^{(\omega)} = \frac{G_F}{\sqrt{2}} \cdot M_W^2 \cdot \frac{(3r-1)}{(3r+1)^{1/2}} [G_2(S) - G_1(S)] \sin \beta \cos \beta$$

$$G_{AA}^{(\omega)} = \frac{G_F}{\sqrt{2}} \cdot M_W^2 \cdot [G_1(S) \cos^2 \beta + G_2(S) \sin^2 \beta]$$

$$G_{AV}^{(\omega)} = G_{VA}^{(\omega)}.$$

and $G_2(S) =$

$$\frac{1}{[M_{N_1}^2 - S - i\Gamma_1 M_{N_1}]}$$

$$G_2(S) = \frac{1}{[M_{N_2}^2 - S - i\Gamma_2 M_{N_2}]}$$

The couplings parameters g and γ are related to e^2 , G_F and M_{W1} by the relations

$$\frac{g^2}{8M_{W1}^2} = \frac{G_F}{\sqrt{2}} \quad ; \quad \frac{1}{e^2} = \frac{1}{g^2} \left(\frac{6r+2}{3r} \right)$$

for fractionally charged quarks while the masses M_{N_1} and M_{N_2} and the angle β are specified by two independent Higgs-Kibble parameters n, ζ (with $0 < n < \infty$, $0 < \zeta < 1$) as follows

$$\tan 2\beta = \frac{2(3r+1)^{\frac{1}{2}}(\eta-5)}{3r(\eta+5) - 4(1-5)}$$

$$4(M_{N_1}^2, M_{N_2}^2) = (3r+2)(\eta+5) + 4(1-5) \mp [3r(\eta+5) - 4(1-5)] \sec 2\beta$$

In terms of ζ and η , the chiral limit of the full left-right symmetric theory is given by $\zeta=1$ while the $SU(2)_L \times U(1)$ limit of the theory is given by $\eta \rightarrow \infty$.

If the e^- beam is incident in the positive direction, $P_1 = (E, 0, 0, +E)$ and in the natural centre of mass system of the storage ring $P_2 = (E, 0, 0, -E)$, $P_4 = P_3$. The production angles θ, ϕ of μ^- are defined by $P_3 = (E, E \sin \theta \cos \phi, E \sin \theta \sin \phi, E \cos \theta)$, $S = 4E^2$. The differential cross-section is ¹²

$$\begin{aligned} \frac{d\sigma}{d\Omega} = & \frac{\alpha^2}{4s} \cdot \frac{1}{G_Y^2} \left[F_1(s) (1 + \cos^2 \theta) (1 + P_+ P_- \cos \theta + \cos \theta_-) \right. \\ & + F_2(s) \sin^2 \theta P_+ P_- \sin \theta_+ \sin \theta_- \cos (2\phi - \phi_+ - \phi_-) \\ & + 2 F_3(s) \cos \theta (1 + P_+ P_- \cos \theta + \cos \theta_-) \\ & + \left\{ F_4(s) (1 + \cos^2 \theta) + 2 F_5 \cos \theta \right\} (P_+ \cos \theta_+ + P_- \cos \theta_-) \\ & \left. - F_6(s) \sin^2 \theta P_+ P_- \sin \theta_+ \sin \theta_- \sin (2\phi - \phi_+ - \phi_-) \right]. \end{aligned}$$

θ^+, θ^- give the orientation of the Polarisation (P^+, P^-) in the c.m. frame of the colliding beams.

TABLE 1

 e^+e^- ASYMMETRIES AS A FUNCTION OF Q_W^+

	SU(2) _L x SU(2) _R x U(1) THEORY					SU(2)xU(1) LIMIT	
Q _w INPUT	245	147	88	0	-49	-146	
M _{N₁} (Gev)	47	53	58	67	73	81	
M _{N₂} (Gev)	95	98	100	110	124	∞	
L _p ^{μμ}	√s=28	4%	2.2%	1.3%	0	-0.5%	-2.0%
	√s=38	4%	41%	2.6%	0	-1.3%	-4%
A ^{μμ}	√s=28	-38%	-28%	-23%	-17%	-14%	-8%
	√s=38	-42%	-68%	-55%	-37%	-30%	-16.5%

+ The predictions are for a representative set of values of $r = .55$ and $\zeta=1$ suggested by neutrino-data (see text). $\sqrt{s}$ is given in Gev.

The F_i 's ($i = 1, 6$) are defined as

$$F_1(s) = (G_\gamma^{(u)} + G_{VV}^{(u)})^2 + 2 G_{VA}^{(u)2} + G_{AA}^{(u)2}$$

$$F_2(s) = (G_\gamma^{(u)} + G_{VV}^{(u)})^2 - G_{AA}^{(u)2}$$

$$F_3(s) = 2 G_\gamma^{(u)} G_{AA}^{(u)} + 2 G_{VV}^{(u)} G_{AA}^{(u)} + 2 G_{AV}^{(u)} G_{VA}^{(u)}$$

$$F_4(s) = 2 G_{VA}^{(u)} (G_\gamma^{(u)} + G_{VV}^{(u)} + G_{AA}^{(u)})$$

$$F_5(s) = F_4(s)$$

and

$$F_6 = 0 \quad \text{since time reversal invariance is assumed.}$$

The charge asymmetry is defined to be

$$A_W^c = \left(\frac{d\sigma(\theta)}{d\Omega} - \frac{d\sigma(\pi-\theta)}{d\Omega} \right) / \left(\frac{d\sigma(\theta)}{d\Omega} + \frac{d\sigma(\pi-\theta)}{d\Omega} \right)$$

Measuring differential cross-section in one direction θ and then in the opposite direction $(\pi - \theta)$ projects out the parity violating weak interference terms. Summing over $\mu\pm$ helicities

$$A_W^c = 2 F_3 \cos \theta / [F_1(1 + \cos^2 \theta) - F_5 \sin^2 \theta \cos 2\phi]$$

The maximum value of A_W^c occurs at

$$\cos^2 \theta = \frac{1 - (F_2/F_1) |P_+ P_-| \cos 2\phi}{1 + (F_2/F_1) |P_+ P_-| \cos 2\phi}$$

and is

$$\hat{A}_W^c = \frac{F_3}{F_1} \cdot \left(1 - \left(\frac{F_2}{F_1} |P_+ P_-| \cos 2\phi \right)^2 \right)^{-\frac{1}{2}}$$

A measure of $\frac{F_3}{F_1}$ gives the charge asymmetry.

The significance of the weak charge asymmetry becomes apparent once one calculates the forward-backward asymmetry. This is defined as

$$\frac{N_F - N_B}{N_F + N_B} = A^{\mu\mu} = \frac{3}{4} \frac{F_3}{F_1}.$$

where

$$N_F = \int_0^{2\pi} d\phi \int_0^1 d(\cos\theta) \frac{d\sigma}{d\Omega} \quad \begin{array}{l} \text{no of } \mu^+ \text{ in the} \\ \text{forward hemisphere} \end{array}$$

$$N_B = \int_0^{2\pi} d\phi \int_{-1}^0 d(\cos\theta) \frac{d\sigma}{d\Omega} \quad \begin{array}{l} \text{no of } \mu^+ \text{ in the} \\ \text{backward hemisphere} \end{array}$$

The numbers N_F and N_B of μ^+ being measured relative to the incident beam. Thus the forward-backward asymmetry is directly related to the charges. In this analysis we have neglected radiative corrections which arise from the interference of one photon and two photon exchange diagrams as well as from the bremsstrahlung contributions. These are expected to be small at the high energies at which the beams collide.

C. AVERAGE HELICITY OF μ^-

The effect of the interfering weak neutral current term with the photon can be studied by measuring the average helicity of the μ^- .

This is defined as

$$\langle h_3 \rangle = \frac{\sum \frac{d\sigma}{d\Omega}(1, h_4) - \sum \frac{d\sigma}{d\Omega}(-1, h_4)}{\sum \frac{d\sigma}{d\Omega}(1, h_4) + \sum \frac{d\sigma}{d\Omega}(-1, h_4)}$$

The helicity (h_4) of the μ^+ is summed over. From the expression of the differential cross-section

$$\langle h_3 \rangle = \frac{F_4(1 + \cos^2\theta) + F_5 P_+ P_- \sin^2\theta \cos 2\phi + F_6 \cos\theta}{F_1(1 + \cos^2\theta) + F_2 P_+ P_- \sin^2\theta \cos 2\phi + 2F_3 \cos\theta} \quad \begin{matrix} F_5 = 2F_4 \\ F_1 \gg F_2 \end{matrix}$$

The beams are transversely polarised in the direction of the x-axis.

The maximum effect for unpolarised beams is

$$\langle h_3 \rangle_{\max} = 2 \cdot \frac{F_4}{F_1}$$

There is an electromagnetic background to $\langle h_3 \rangle$ originating from $\gamma\text{-}2\gamma$ exchange (order α^3). In certain angular regions the background vanishes i.e. $\theta = \pi/2$, $\phi = 0$. In this case for unpolarised initial beams, we get

$$\langle h_3 \rangle = \frac{F_4}{F_1}$$

D. LONGITUDINALLY POLARISED BEAMS

This method is particularly suitable for identifying parity violating terms in the cross-section. Here the longitudinal asymmetry

is defined as

$$L^{\mu\mu} = \frac{\frac{d\sigma}{d\Omega}(\rightarrow\rightarrow) - \frac{d\sigma}{d\Omega}(\leftarrow\leftarrow)}{\frac{d\sigma}{d\Omega}(\rightarrow\rightarrow) + \frac{d\sigma}{d\Omega}(\leftarrow\leftarrow)}$$

$$L^{\mu\mu} = \left(\frac{P_+ + P_-}{1 + P_+ P_-} \right) \frac{2 F_3 \cos \theta + (1 + \cos^2 \theta) F_4}{(1 + \cos^2 \theta) F_1 + 2 \cos \theta F_3}$$

The advantage this asymmetry has over the other asymmetries is that it is a purely parity violating asymmetry, therefore the electromagnetic background which is parity conserving will not disturb it.

In the table (B₃-101), the expected results for two centre of mass energies $\sqrt{s}=28$ and 38 for a variety of values of Q_w and the corresponding predicted values of M_{N_1} and M_{N_2} with $\gamma = 0.55$ and $\zeta = 1$ are presented. Since the average helicity and longitudinal asymmetries are related and the same goes for the charge and forward backward asymmetries, we only present the longitudinal and forward-backward asymmetries for $\mu^+ \mu^-$. Similar predictions for the $SU(2)_L \times U(1)$ model are compared.

E. EXCLUSIVE HADRONIC FINAL STATES

The measurability of two-hadron final state at 20 to 40 GeV depends on the energy dependence of the electromagnetic form factors which is not quite known. Therefore it is not known a priori that weak effects due to N_1 and N_2 boson exchange will not manifest themselves at such high energies. The $\frac{1}{E^2}$ decrease for the electromagnetic form factors of the pion, high rate of events at low energies would transform into a few events per day.

In the process¹³ $e^+e^- \rightarrow \pi^+ \pi^-$, three form factors enter into the relevant reduced matrix elements

$$\langle \pi^+ \pi^- | J_\mu^{\text{em}} | 0 \rangle = (P_{\pi^+} - P_{\pi^-}) F_\pi(q^2)$$

$$\langle \pi^+ \pi^- | J_\mu^{\text{NC}} | 0 \rangle = (P_{\pi^+} - P_{\pi^-}) F_1^{\text{NC}}(q^2) + (P_{\pi^+} + P_{\pi^-}) F_2^{\text{NC}}(q^2).$$

In order to be able to use the above relations meaningfully, the current J^{em} and J^{NC} are the quark currents. Only the isospin one contributions are relevant for the process we are considering.

The effective interaction Lagrangian is

$$\begin{aligned} \mathcal{L} = & \frac{e^2}{s} \bar{e} \gamma_\mu e \left[-\frac{2}{3} \bar{p} \gamma_\mu p + \frac{1}{3} \bar{n} \gamma_\mu n \right] \\ & + \bar{e} \gamma_\mu e \left[G_{VV}^p \bar{p} \gamma_\mu p + G_W^n \bar{n} \gamma_\mu n \right] \\ & + \bar{e} \gamma_\mu \gamma_5 e \left[G_{AV}^p \bar{p} \gamma_\mu p + G_{AV}^n \bar{n} \gamma_\mu n \right] \\ & + \bar{e} \gamma_\mu e \left[G_{VA}^p \bar{p} \gamma_\mu \gamma_5 p + G_{VA}^n \bar{n} \gamma_\mu \gamma_5 n \right] \\ & + \bar{e} \gamma_\mu \gamma_5 e \left[G_{AA}^p \bar{p} \gamma_\mu \gamma_5 p + G_{AA}^n \bar{n} \gamma_\mu \gamma_5 n \right] \end{aligned}$$

Only the first three terms are relevant for $e^+e^- \rightarrow \pi^+ \pi^-$. The isospin 1

contribution are

$$\begin{aligned}
 \mathcal{L} &= -\frac{e^2}{s} \bar{e} \gamma_\mu e J_\pi^{I=1} + G_{VV}^\pi \bar{e} \gamma_\mu e J_\pi^{I=1} + G_{AV}^\pi \bar{e} \gamma_\mu \gamma_5 e J_\pi^{I=1} \\
 &= \left(-\frac{e^2}{s} + G_{VV}^\pi \right) \bar{e} \gamma_\mu e J_\pi^{I=1} + G_{AV}^\pi \bar{e} \gamma_\mu \gamma_5 e J_\pi^{I=1}
 \end{aligned}$$

The effective couplings G_{VV}^P , G_{AV}^P etc. are tabulated below. $G_{VV}^\pi =$

$$G_{VV}^P - G_{VV}^N \text{ and } G_{AV}^\pi = G_{AV}^P - G_{AV}^N.$$

$$G_\gamma^{(P)} = -\frac{G_F}{\sqrt{2}} \cdot \frac{m_W^2}{s} \cdot \frac{8r}{3r+1}$$

$$G_{VV}^{(P)} = -\frac{(3r-1)(r-1)}{(3r+1)} \cdot \frac{G_F}{\sqrt{2}} \cdot m_W^2 [D_1 \sin^2 \beta + D_2 \cos^2 \beta]$$

$$G_{VA}^P = \frac{(3r-1)}{(3r+1)^{1/2}} \cdot \frac{G_F}{\sqrt{2}} \cdot m_W^2 \cdot \sin \beta \cos \beta [D_1 - D_2]$$

$$G_{AV}^P = \frac{(r-1)}{(3r+1)^{1/2}} \cdot \frac{G_F}{\sqrt{2}} \cdot m_W^2 \cdot \sin \beta \cos \beta [D_1 - D_2]$$

$$G_{AA}^P = -\frac{G_F}{\sqrt{2}} \cdot m_W^2 \cdot [D_1 \cos^2 \beta + D_2 \sin^2 \beta]$$

$$G_\gamma^N = -\frac{1}{2} G_\gamma^P$$

$$G_{VV}^N = \left(\frac{r+1}{r-1} \right) \cdot G_{VV}^P$$

$$G_{AV}^N = \left(\frac{r+1}{r-1} \right) G_{AV}^P$$

$$G_{VA}^N = -G_{VA}^P$$

$$G_{AA}^N = -G_{AA}^P$$

$$D_1(s) = (s - m_{N_1}^2 + i\Gamma_1 m_{N_1})^{-1}; \quad D_2(s) = (s - m_{N_2}^2 + i\Gamma_2 m_{N_2})^{-1}.$$

The reduced matrix element then is

$$\frac{M}{-i} = \left[\left(-\frac{e^2}{s} + G_{VV}^\pi \right) \bar{e} \gamma_\mu e + G_{AV}^\pi \bar{e} \gamma_\mu \gamma_5 e \right] F(q^2) (P_{\pi^+} - P_{\pi^-})_\mu$$

and the differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4e^2 q^2} \left(1 - \frac{4m_\pi^2}{s} \right)^{\frac{1}{2}} T$$

$$\begin{aligned}
 T &= \left(\frac{s}{2} - 2m_\pi^2 \right) q^2 \sin^2 \theta \left\{ 1 + n_z n_z' + (n_y n_y' - n_x n_x') \cos 2\phi \right. \\
 &\quad \left. - (n_x n_y' + n_y n_x') \sin 2\phi \right\} A + 2(1 + n_z n_z') \\
 &\quad + (n_z + n_z') \text{ Real } B.
 \end{aligned}$$

Here terms proportioned to $\frac{M_e}{\sqrt{s}}$ have been neglected. This eliminates $F_S(q^2)$. Also any imaginary term falls out on the assumption of T reversal invariance

$$A = \left\{ \left| -\frac{e^2}{s} + G_{VV}^\pi \right|^2 - |G_{AV}^\pi|^2 \right\} \frac{F_\pi(q^2)^2}{4}$$

$$B = -2 G_{AV}^\pi \left[-\frac{e^2}{s} + G_{VV}^\pi \right]$$

There are two cases:

(a) Transverse polarisations. This case will be ignored completely as the electromagnetic background is substantial and the signal therefore is too contaminated to be of practical value.

(b) Longitudinal polarisations. Here the longitudinal asymmetry is again defined as

$$\rho^{\pi\pi} = [T(\rightarrow\rightarrow) - T(\leftarrow\leftarrow)] / [T(\rightarrow\rightarrow) + T(\leftarrow\leftarrow)]$$

$$= \frac{P_+ + P_-}{1 + P_+ P_-} \cdot \frac{2 G_{AV}^\pi \left[\frac{e^2}{s} - G_{VV}^\pi \right]}{\left| \frac{e^2}{s} - G_{VV}^\pi \right|^2 + |G_{AV}^\pi|^2}$$

The graph on the next page gives a plot of $\rho^{\pi\pi}$ against $\sqrt{s}$. for various values of Q_W .

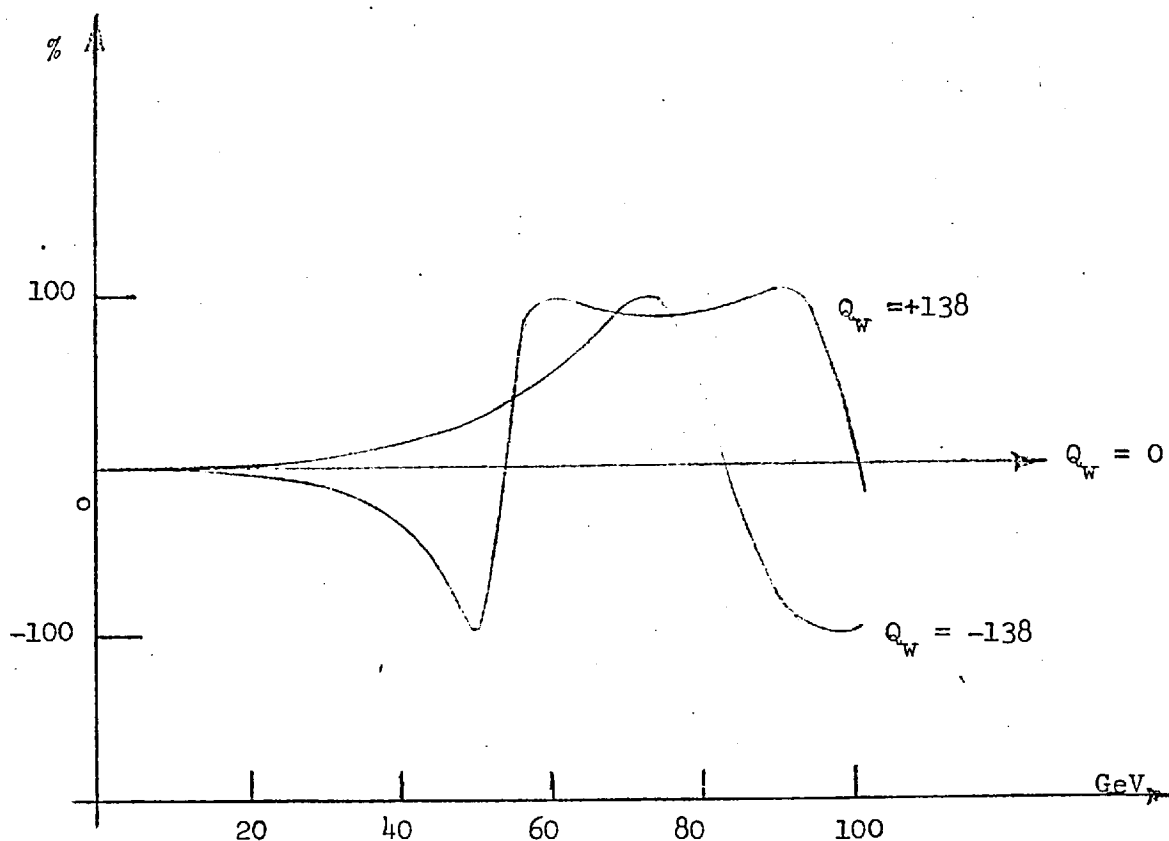
F. DISCUSSION

(a) Note that as long as Q_W is either small in magnitude (compared to its $SU(2) \times U(1)$ value) or opposite in sign, the left-right symmetric theory would inevitably predict that the parity conserving forward-backward asymmetry parameter (A_{FB}) must be strongly enhanced by a factor (2 to 5) compared to its $SU(2) \times U(1)$ value even at

PEP and PETRA energies.

(b) In general, there is a correlation between the sign of the parity violating asymmetry parameters for $e^+e^- \rightarrow \mu^+\mu^-$ and $\pi^+\pi^-$ on one hand and the sign of Q_W on the other.

Thus e^+e^- asymmetry parameters (and also atomic parity violation parameter Q_W) unlike neutrino data, provide clear distinctions between the left-right symmetric theory with a light mass on the one hand and the $SU(2) \times U(1)$ theory on the other. Should experiments show relatively large forward backward asymmetry or parity violating asymmetry parameters with signs opposite to those of $SU(2) \times U(1)$, they would provide clear signatures in favour of the left-right symmetric theory together with the characteristic predictions of a light N_1 .



$\rho^{\pi\pi}$ against $\sqrt{s}$

G. FOOTNOTES

1. The quoted results of the Washington and Oxford experiments (E.N. Fortson et al., Nature, 264, 528 (1976) for the optical rotation parameter R in atomic Bismuth are $R_{876\text{nm}} = (-8 \pm 3) \times 10^{-8}$ (Washington) $R_{648\text{nm}} = (+10 \pm 8) \times 10^{-8}$ (Oxford) with a systematic uncertainty of $(\pm 10 \times 10^{-8})$. If one uses recent estimates of the purely atomic factor as given by R. Cowan, S. Meshkov and S.P. Rosen, Preprint (1977) and also take the systematic experimental uncertainties into account, the two experiments (Washington and Oxford) appear not to be incompatible with $-150 < Q_w < 300$, (with $Q_w \approx 0$ the most probable). For the present theory this implies a range of masses for (N_1, N_2) lying between the $SU(2) \times U(1)$ prediction of (81 GeV, ∞) down to (45 GeV, 95 GeV) at the two extremes.
2. J.C. Pati and Abdus Salam, Phys. Rev. D10, 275 (1974).
3. Thus either the observed neutrinos in such a theory are truly light four component objects, acquiring mass through spontaneous symmetry breaking (SSB) or else the right and left components remain disjoint (and therefore massless) despite SSB.
4. In a different context, see M.A.B. Beg, A.A. Zee, Phys. Rev. Lett. Vol. 30, 675 (1973).
5. J.C. Pati, S. Rajpoot and A. Salam, Imperial College Preprint ICTP/76/11.
6. Straightforward extensions are possible for more than four flavours and four colours, (for example the mirror model, J. Pati and A. Salam, Phys. Lett. Vol. 58B, 333, (1975)).
7. The model as proposed in Ref. 2 satisfies the criterion of natural left-right symmetry stated by R.N. Mohapatra and J.C. Pati, Phys. Rev. D11, 2558 (1976).

8. Sec Sec. 6 and Footnote 21 of Ref. 2.
9. R.N. Mohapatra and D.P. Sidhu, Phys. Rev. Letters 38 667 (1977) and BNL-preprint (22561), who have considered a special case ($Q_W \approx 0$) arising within the model of Ref. 2.
10. A. De Rujula, H. Georgi and S.L. Glashow, Harvard Preprint (1977). The light mass gauge neutral boson ($< m_{W_L}^+$) is not available in their model, nor in the special case considered in Re. 9.
11. The familiar $SU(2) \times U(1)$ is the limit of the larger theory when $\eta \rightarrow \infty$, for which $m_{N_1}^2 / m_{W_L}^2 \rightarrow (6r+2)/(3r+2) (1 + O(\eta^{-1}))$, $m_{N_2}^2 / m_{W_L}^2 = (3r+2)/2\eta (1+O(\eta^{-1}))$, $\sin^2 \epsilon_W = 3r/(6r+2)$.
12. See, for example, R. Budny, Physics Letters 55B, 227 (1975) and A. McDonald, Nuclear Physics, B75, 343 (1974).
13. E. Lendari and G. Pocsik, Physics Letters 56B, 462 (1975).

P A R T I V

DILEPTON PRODUCTION IN PP AND $\bar{P}P$ COLLISIONS AS A PROBE
TO THE NATURE OF THE NEUTRAL CURRENT INTERACTION

A. INTRODUCTION

As was shown in Section 2, the near null results of atomic parity violation experiments⁽¹⁾, taken at their face value, appear to suggest that the underlying low-energy limit of the unified theory of weak and electromagnetic interactions may be the left-right symmetric⁽²⁾ $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ theory rather than its left-handed limit $SU(2)_L \times U(1)$ ⁽³⁾. The left-right symmetric theory displays the interesting feature that one of its two neutral gauge bosons may be reasonably light - of the order of 50-70 GeV in mass (corresponding to Q_W for Bismuth of order +130 and Zero, respectively) - to be compared to neutral mass for $SU(2)_L \times U(1)$ of order 80 GeV (corresponding to $Q_S \approx 130$ and $\sin^2 \theta_W \approx 0.28$). The colliding beams at PEP and PETRA fall short of producing the lowest allowed mass (assuming it exists). Its possible existence could be studied indirectly by interpreting the various asymmetries that could be measured at PEP and PETRA in the e^+e^- annihilation channel. This was elaborately discussed in the last section.

However, with the proton-proton and proton-antiproton colliding beams of centre of mass energies up to 800 GeV, not only will the light N_1 be produced but also the N_2 and the charged gauge bosons will be produced (perhaps more new physics also). This will not only distinguish between the left-right symmetric theory and the $SU(2) \times U(1)$ theory⁽⁴⁾, but also shed light on the correctness of the idea of vector bosons as

mediators of the weak force.

In what follows it is of interest to examine the cross-sections for massive μ -pair production and asymmetries in the reactions like

- (i) Proton + Proton $\rightarrow \mu^+ \mu^- + \text{anything}$
- (ii) Proton + antiproton $\rightarrow \mu^+ \mu^- \rightarrow \text{anything}$

In this section we calculate

(a) $\frac{d\sigma}{dm_{\mu\mu}}$, where $M_{\mu\mu}$ is the invariant mass of the $(\mu^+ \mu^-)$ pair

(b) The energy asymmetry $\langle \Delta E \rangle = \langle E^+ \rangle - \langle E^- \rangle$ and the related ratio

$$\frac{\langle \Delta E \rangle}{N} = \frac{n(E^+ > E^-) - n(E^- > E^+)}{n(E^+ > E^-) + n(E^- > E^+)}$$

where E^+ and E^- refer to the energies of μ^+ and μ^- from the same event in $\bar{P}P$ collisions

(c) Forward-backward asymmetry

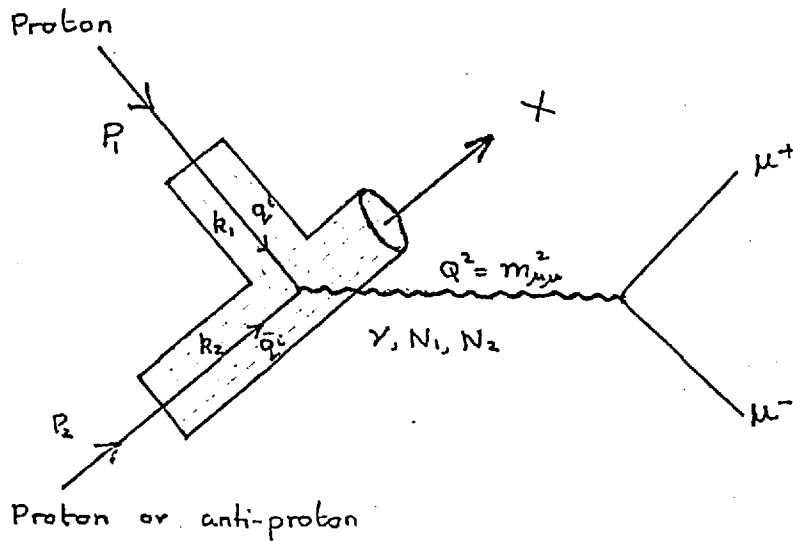
$$A_{\mu\mu} = (N_f - N_b) / (N_f + N_b)$$

in $\bar{P}P$ collision, where N_f and N_b refer to the numbers of μ 's travelling in the forward or the backward hemispheres relative to $\bar{P}$.

B. MASSIVE μ -PAIR PRODUCTION

In order to study the massive $\mu^+ \mu^-$ pairs in high energy proton-proton

and proton-antiproton collisions Drell and Yan⁵ use the parton model coupled with the hypothesis of scaling.



DRELL-YAN PICTURE FOR MASSIVE MUON PAIR PRODUCTION

Here P_1, P_2 = four momentum of colliding hadrons
 $S = (P_1 + P_2)^2$
 Q = four momentum of the virtual γ, N_1 , or N_2
 k_1, k_2 = four moment of the annihilating partons
 $k_1 = x_1 P_1$; $k_2 = x_2 P_2$

If quarks are identified with partons, then in the $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ theory, the effective interaction Lagrangian, $\mathcal{L}_{eff}$, arising due to the virtual exchange of the photon, N_1 and N_2 neutral gauge bosons is

$$\begin{aligned}
 -\mathcal{L}_{eff} = & (G_\gamma^V + G_{VV}^P) \bar{\mu} \gamma_\mu \mu \cdot \bar{p} \gamma_\mu p \\
 & + G_{VA}^P \bar{\mu} \gamma_\mu \mu \cdot \bar{p} \gamma_\mu \gamma_5 p + G_{AV}^P \bar{\mu} \gamma_\mu \gamma_5 \mu \cdot \bar{p} \gamma_\mu p \\
 & + G_{AA}^P \bar{\mu} \gamma_\mu \gamma_5 \mu \cdot \bar{p} \gamma_\mu \gamma_5 p + (p \rightarrow n) \\
 & + (n \rightarrow \lambda) + (\lambda \rightarrow c) .
 \end{aligned}$$

r and β are two parameters of the theory ⁽⁶⁾; here the widths of N_1 and N_2 are assumed to be 1 Gev.

The Drell-Yan massive μ -pair production cross-section is computed to be

$$\frac{d\sigma}{dm_{\mu\mu}} = \frac{4\pi\alpha^2}{3m_{\mu\mu}^2} \sum_{i=\substack{p \\ n,\lambda}} \int_0^1 dx_1 \int_0^1 dx_2 f_A^i(x_1) f_B^i(x_2) A^i(m_{\mu\mu}) \tau$$

where

$$A^i(m_{\mu\mu}) = (q^i + \frac{m_{\mu\mu}^2}{e^2} G_W^i)^2 + \frac{m_{\mu\mu}^2}{e^2} (G_{AV}^{i^2} + G_{VA}^{i^2} + G_{AA}^{i^2})$$

q^i = quark charges, $f_a^i(x)$ is the quark structure functions for the i^{th} quark in particle A and A stands for either a proton or anti-proton. $\tau = \frac{M_{\mu\mu}^2}{s}$ and $\sqrt{s}$ is the centre of mass energy. The sum i extends over all quarks and anti-quarks.

C. ENERGY ASYMMETRY

Here we project out the average value of the four momentum of the $\mu^+\mu^-$ particles in the direction of the unit vector n_μ which we eventually choose to be $\equiv (1,0,0,0)$. The energy asymmetry is defined to be

$$\langle E_+ \rangle - \langle E_- \rangle = \frac{\sum_i E_+^i \frac{d\sigma^i}{dm_{\mu\mu}} - \sum_i E_-^i \frac{d\sigma^i}{dm_{\mu\mu}}}{\sum_i \frac{d\sigma^i}{dm_{\mu\mu}}}$$

We know the expression for $\frac{d\sigma^i}{dm_{\mu\mu}}$ from Drell-Yan. Therefore

$$\begin{aligned} \langle E_+ \rangle - \langle E_- \rangle &= \frac{1}{2} \frac{d\sigma}{d\sigma} \cdot \frac{\alpha}{6\sqrt{s}} \sum_i \int_0^1 dx_1 \int_0^1 dx_2 \delta(x_1 x_2 - \tau) \\ &\times \text{Real} [(G_V^i + G_W^i) G_{AA}^i + G_{AV}^i G_{VA}^i] \frac{m_{\mu\mu}^2}{e^2} \\ &\times (x_1 - x_2) [f_1^{q^i}(x_1) f_2^{\bar{q}^i}(x_2) + f_1^{\bar{q}^i}(x_1) f_2^{q^i}(x_2)] \end{aligned}$$

Here p, n, λ, c refer to the usual quarks. The eight functions $G_W^p, \dots$ etc. are given in terms of the masses M_{N_1} and M_{N_2} of the neutral gauge bosons and the square of the momentum transfer $q^2 = m_{\mu\mu}^2$.

These are tabulated below.

$$G_P^Y = -\frac{2}{3} \frac{e^2}{q^2} = -\frac{G_F}{\sqrt{2}} \frac{m_W^2}{s} \cdot \frac{8r}{3r+1}$$

$$G_{VV}^P = -\frac{(3r-1)(r-1)}{(3r+1)} \frac{G_F m_W^2}{\sqrt{2}} \left\{ D_1(q^2) \sin^2 \beta + D_2(q^2) \cos^2 \beta \right\}$$

$$G_{VA}^P = \frac{(3r-1)}{(3r+1)^{\frac{1}{2}}} \frac{G_F m_W^2}{\sqrt{2}} \sin \beta \cos \beta \left\{ D_1(q^2) - D_2(q^2) \right\}$$

$$G_{AV}^P = \frac{(r-1)}{(3r+1)^{\frac{1}{2}}} \frac{G_F m_W^2}{\sqrt{2}} \sin \beta \cos \beta \left\{ D_1(q^2) - D_2(q^2) \right\}$$

$$G_{AA}^P = -\frac{G_F}{\sqrt{2}} m_W^2 \left\{ D_1(q^2) \cos^2 \beta + D_2(q^2) \sin^2 \beta \right\}$$

couplings are same for C quarks.

$$G_n^Y = \frac{1}{3} \frac{e^2}{q^2} = \frac{G_F}{\sqrt{2}} m_W^2 \frac{4r}{3r+1}$$

$$G_{VV}^n = -\frac{G_F}{\sqrt{2}} m_W^2 \frac{(3r-1)(r+1)}{3r+1} \left\{ D_1(q^2) \sin^2 \beta + D_2(q^2) \cos^2 \beta \right\}$$

$$G_{VA}^n = -\frac{G_F}{\sqrt{2}} m_W^2 \frac{3r-1}{(3r+1)^{\frac{1}{2}}} \sin \beta \cos \beta \left\{ D_1(q^2) - D_2(q^2) \right\}$$

$$G_{AV}^n = \frac{G_F}{\sqrt{2}} m_W^2 \frac{r+1}{(3r+1)^{\frac{1}{2}}} \sin \beta \cos \beta \left\{ D_1(q^2) - D_2(q^2) \right\}$$

$$G_{AA}^n = -G_{AA}^P$$

Same for the λ quarks.

The function $F_{1,2}(X_1, X_2, M_{\mu\mu})$ is different for the proton-proton and proton-antiproton case.

$$F_{12} = f_1^{q_i}(x_1) f_2^{\bar{q}_i}(x_2) + f_1^{\bar{q}_i}(x_1) f_2^{q_i}(x_2)$$

If 1 = Proton

2 = Proton

$$F_{pp} = f_p^{q_i}(x_1) f_p^{\bar{q}_i}(x_2) + f_p^{q_i}(x_2) f_p^{\bar{q}_i}(x_1)$$

If 1 = Proton, 2 = antiproton

$$\begin{aligned} F_{p\bar{p}} &= f_p^{q_i}(x_1) f_{\bar{p}}^{\bar{q}_i}(x_2) + f_{\bar{p}}^{q_i}(x_1) f_p^{\bar{q}_i}(x_2) \\ &= f_p^{q_i}(x_1) f_{\bar{p}}^{\bar{q}_i}(x_2) - f_p^{q_i}(x_2) f_{\bar{p}}^{\bar{q}_i}(x_1) \\ &\equiv 0 \end{aligned}$$

The proton-antiproton case is symmetric in x_1 and x_2 but anti-symmetric in exchange of fermions.

D. FORWARD-BACKWARD ASYMMETRY

This asymmetry is defined in terms of the net flux of particles of one type minus the flux of particles of another type. The type is usually specified by the charge they carry. In this case it is the number of μ^+ in the forward hemisphere minus the number of μ^- in the backward hemisphere with respect to the proton beam in the centre of mass frame.

To do this we choose a space-like unit vector $\hat{\mu}$ which we eventually choose to be (0,0,0,1) in the centre of mass frame of the experiment. In choosing this particular direction we have calculated the following

$$\begin{aligned} &\frac{1}{2} (n_f(\mu^+) - n_b(\mu^+)) - \frac{1}{2} (n_f(\mu^-) - n_b(\mu^-)) \\ &\equiv (N_f - N_b) \end{aligned}$$

Therefore

$$\begin{aligned}
 A_{\mu\mu} &= (N_f - N_b) / (N_f + N_b) \\
 &= \frac{2\alpha e}{S} \int_e^1 \frac{dx}{x} \cdot \frac{(x + \frac{e}{x})}{[4e + (x - \frac{e}{x})^2]^{\frac{3}{2}}} \cdot F(x, e, m_{\mu\mu}) \\
 F(x, e, m_{\mu\mu}) &= \sum_i (f_A^{q_i}(x) f_B^{\bar{q}_i}(\frac{e}{x}) - f_A^{\bar{q}_i}(\frac{e}{x}) f_B^{q_i}(x)) A^i(m_{\mu\mu}) \\
 A^i(m_{\mu\mu}) &= \text{Re} \left[G_{AA}^i (q_L^i + G_{VV}^i \frac{m_{\mu\mu}^2}{e^2}) + \frac{m_{\mu\mu}^2}{e^2} G_{AV}^i G_{VA}^i \right]
 \end{aligned}$$

Finally it turns out that

$$\begin{aligned}
 &\frac{n(E_+ \rightarrow E_-) - n(E_- \rightarrow E_+)}{n(E_+ \rightarrow E_-) + n(E_- \rightarrow E_+)} \\
 &= \frac{3}{2m_{\mu\mu}} (\langle E_+ \rangle - \langle E_- \rangle) \\
 &= \frac{3}{2m_{\mu\mu}} \cdot \text{Energy asymmetry.}
 \end{aligned}$$

E. REMARKS

1. For the structure functions, summed over three colours, we have taken

$$\begin{aligned}
 S(x) &= 0.15 (1-x)^7 / x = \bar{S}(x) = \bar{d}(x) = c(x) \\
 u(x) &= 1.79 (1-x)^3 (1+2.3x) / \sqrt{x} + S(x) \\
 d(x) &= 1.07 (1-x)^{3.1} / \sqrt{x} + S(x).
 \end{aligned}$$

bar denotes the sea.

2. The energy asymmetry $\langle E^+ \rangle - \langle E^- \rangle$ for the PP has its origin in the presence of VV as well as AA interactions, such that the $\mu^+ \mu^-$ pair is produced both in $C = -1$ as well as $C = +1$ eigenstates. The interference (7) between these two gives rise to the effect of energy asymmetry. For the proton-antiproton system, $f_p^i(x) = f_{\bar{p}}^i(x)$ so that we obtain vanishing energy asymmetry between $\mu^+ \mu^-$ consistent with CP conservation.

3. The forward-backward asymmetry vanishes for the proton-proton system, since we are dealing here with identical particles.

F. DISCUSSION

In the accompanying figures 1, 2 and 3, the results of our calculations at $\sqrt{s} = 200$ GeV and 400 GeV are plotted, for a range of $m_{\mu\mu}$ lying between 30 and 100 GeV. The results are presented for three values of Q_W for Bismuth ($Q_W = +138$, $Q_W = 0$ and $Q_W = -138$). The last value of Q_W corresponds to the $SU_L(2) \times U(1)$ limit. The corresponding expected masses (m_{N_1}, m_{N_2}) in the theory are (55, 96), (70, 107) and (83, ∞) GeV, respectively, when $\sin^2 \theta_W = 0.28$ (i.e. $r = \frac{4}{9}$). For the range of $m_{\mu\mu}$ considered, we find numerically that $|G_{AA}(m_{\mu\mu}^2)|$ predominates over the other functions $|G_{VV}|$, $|G_{AV}|$ and $|G_{VA}|$ by one to two orders of magnitude, so that the function $A_i(m_{\mu\mu}^2)$ occurring in the asymmetry expressions has the characteristic form:

$$\frac{\cos^2 \beta (m_{\mu\mu}^2 - m_{N_1}^2)}{[(m_{\mu\mu}^2 - m_{N_1}^2) + m_{N_1}^2 \Gamma_1^2]} + \frac{\sin^2 \beta (m_{\mu\mu}^2 - m_{N_2}^2)}{[(m_{\mu\mu}^2 - m_{N_2}^2) + m_{N_2}^2 \Gamma_2^2]}$$

For small β , $A_i(m_{\mu\mu}^2) \approx 0$ for $m_{\mu\mu} = m_{N_1}$. It changes sign around this point and with it the asymmetries. The noteworthy features of the results

are:

i) There are the characteristic π rather high peaks (three orders of magnitude) which are rather broad at the base (≈ 25 GeV wide) - in $\frac{d\sigma}{dm_{\mu\mu}}$. As expected, these are centered at m_{N_1} . Note the manner in which the purely electromagnetic values of the cross-section provide an asymptotic cushion on the lower side of the peaks. We have also computed $\frac{d\sigma}{dm_{\mu\mu}}$ for ISR energies, but find its magnitude rather small ($\approx 4 \times 10^{-40} \text{ cm}^2/\text{GeV}$ for $m_{\mu\mu} \approx 30$ GeV).

ii) The energy asymmetry in PP increases with energy from a few percent at $\sqrt{s} \approx 60$ GeV to $\approx -15\%$ for $\sqrt{s} = 400$ GeV. It does not seem to rise above this value in the energy range considered.

iii) The angular asymmetry in $P\bar{P}$ is large, varying between 20% to -30% as $m_{\mu\mu}$ varies between 30 to 100 GeV at $\sqrt{s} = 200$ GeV. This asymmetry is relatively large even at $\sqrt{s} = 60$ GeV. For example, at $m_{\mu\mu} \approx 40$ GeV, $A^{\mu\mu} = 0.45, 0.28$ and 0.15 for $Q_W = 138, 0$ and -132 , respectively. However, the cross-section $\frac{d\sigma}{dm_{\mu\mu}}$ at $\sqrt{s} = 60$ GeV is around $7 \times 10^{-40} \text{ cm}^2/\text{GeV}$ for $m_{\mu\mu} = 40$ GeV. When $\sqrt{s}$ is increased to 400 GeV, $A_{\mu\mu}$ decreases and is down to levels between +10% and -10% for $30 < m_{\mu\mu} < 100$ GeV. It would thus appear that $\sqrt{s} \approx 200$ GeV represents an optimum energy for this effect.

G. REFERENCES AND FOOTNOTES

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- 5) J.C. Pati, S. Rajpoot and Abdus Salam, Imperial College, London, preprint ICTP/76/15.
- 6) S.D. Drell and T.M. Yan, Phys. Rev. Letters 25, 316 (1970) and Ann. Phys. (NY) 66, 578 (1971).

- 7) These effects are produced even if P and C are conserved. They are, for example, produced also by 2γ exchanges producing the $\mu^+\mu^-$ pair. These effects have been studied by R. Gatto and G. Preparata, *Lettere al Nuovo Cimento* 7, 89 (1973). We expect their magnitude for our energies to be small though they have not yet been fully calculated.

H. FIGURE CAPTIONS

Figure 1a A plot of $\frac{d\sigma}{dm_{\mu\mu}}$ vs. $m_{\mu\mu}$ for $PP \rightarrow \mu^+ \mu^- X$ ($\sqrt{s} = 200$ GeV).

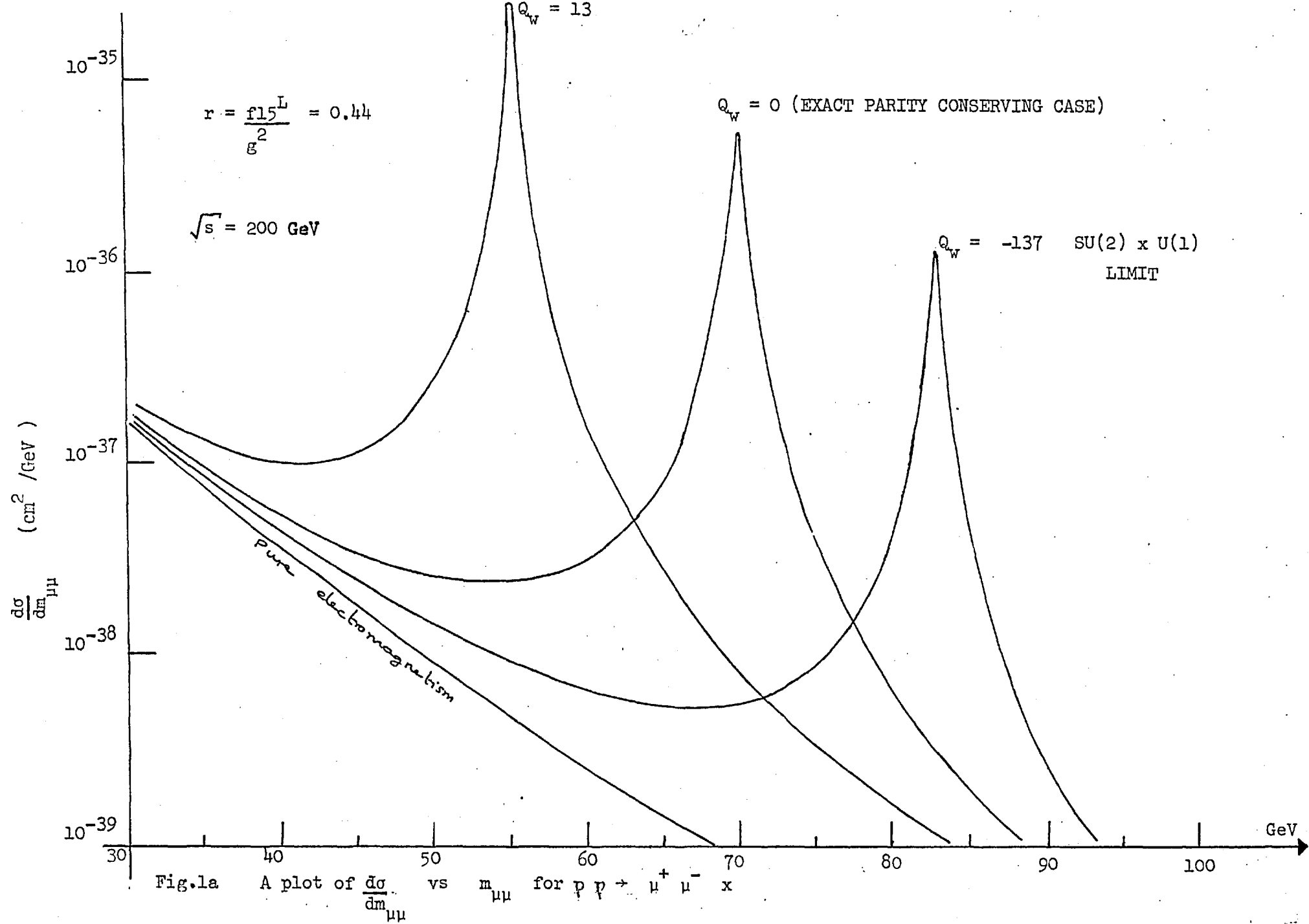
Figure 1b A plot of $\frac{d\sigma}{dm_{\mu\mu}}$ vs. $m_{\mu\mu}$ for $PP \rightarrow \mu^+ \mu^- X$ ($\sqrt{s} = 400$ GeV).

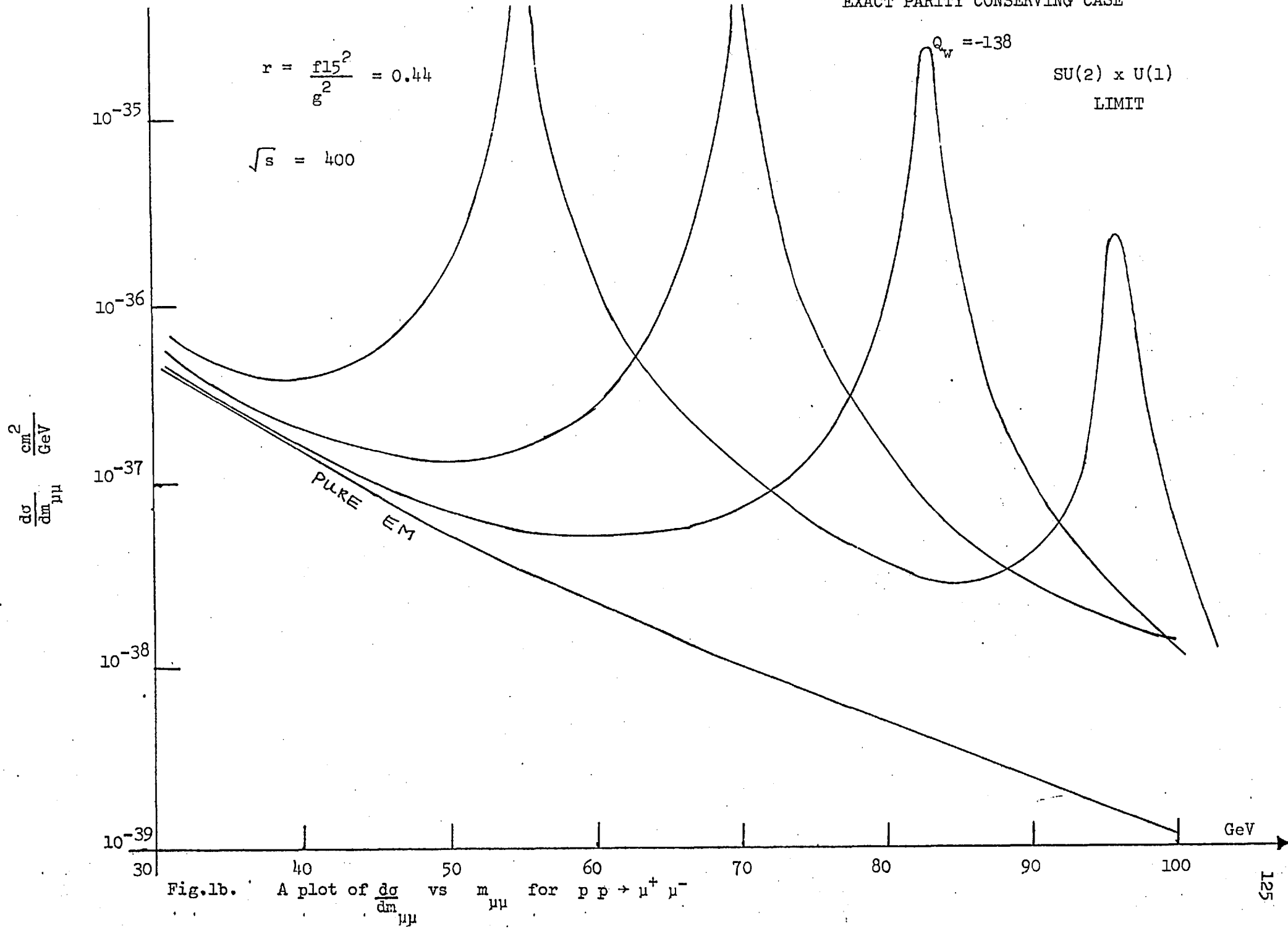
Figure 2a A plot of $\frac{d\sigma}{dm_{\mu\mu}}$ vs. $m_{\mu\mu}$ for $P\bar{P} \rightarrow \mu^+ \mu^- X$ ($\sqrt{s} = 200$ GeV).

Figure 2b A plot of $\frac{d\sigma}{dm_{\mu\mu}}$ vs. $m_{\mu\mu}$ for $P\bar{P} \rightarrow \mu^+ \mu^- X$ ($\sqrt{s} = 400$ GeV).

Figure 3a A plot of $A^{\mu\mu}$ vs. $m_{\mu\mu}$ for $P\bar{P} \rightarrow \mu^+ \mu^- X$ ($\sqrt{s} = 200$ GeV).

Figure 3b A plot of $A^{\mu\mu}$ vs. $m_{\mu\mu}$ for $PP \rightarrow \mu^+ \mu^- X$ ($\sqrt{s} = 400$ GeV).





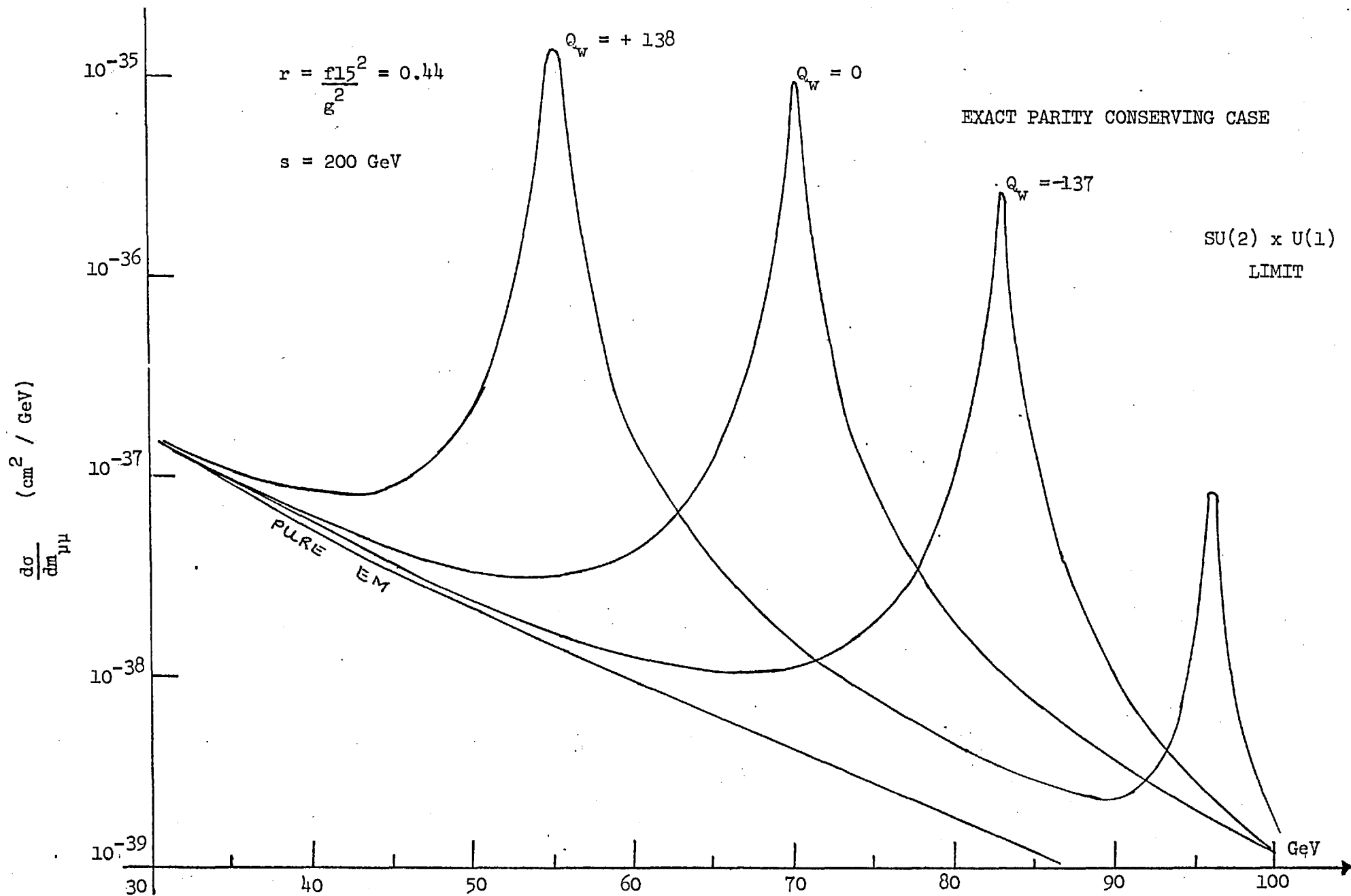
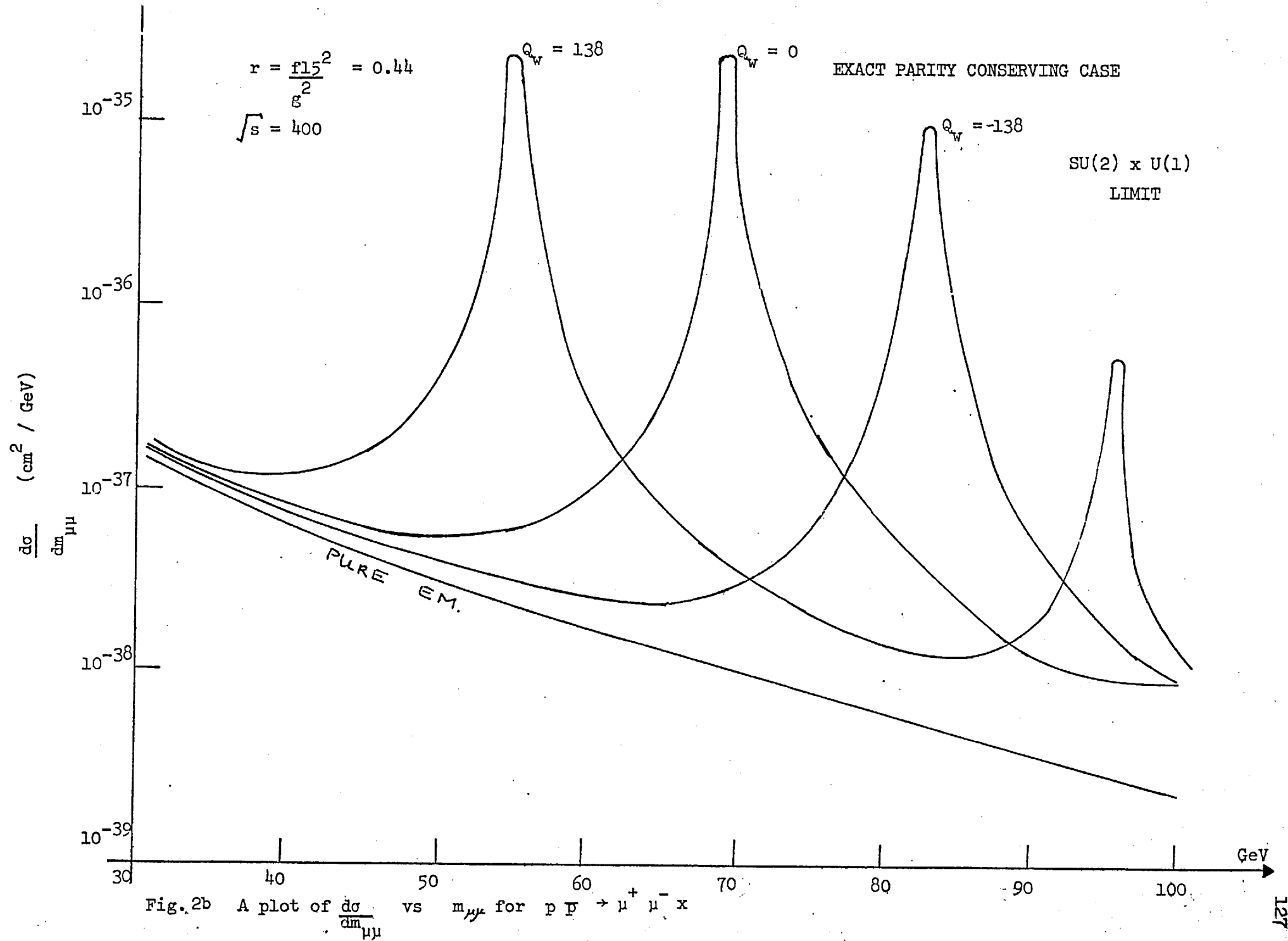


Fig. 2a A plot of $\frac{d\sigma}{dm_{\mu\mu}}$ vs $m_{\mu\mu}$ for $p p \rightarrow \mu^+ \mu^-$



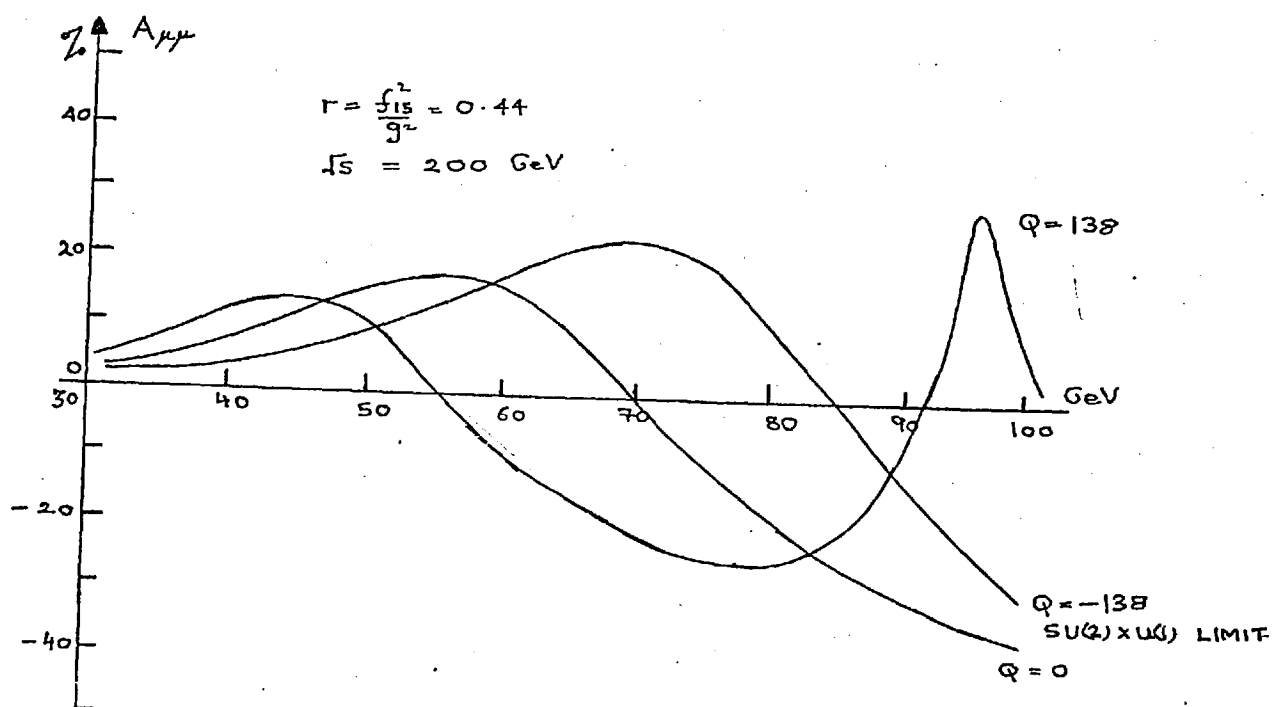


Fig. 3a. A plot of $A_{\mu\mu}$ vs $m_{\mu\mu}$ for $\bar{P}P \rightarrow \mu^+\mu^-X$

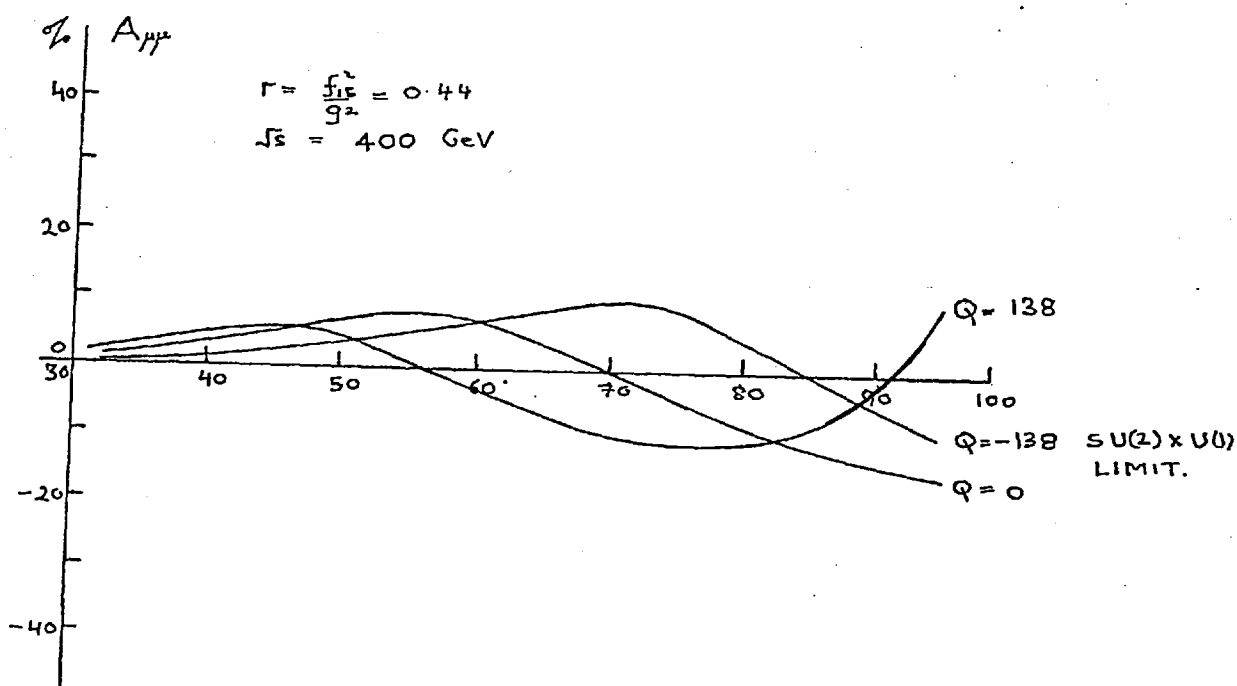


Fig. 3b. A plot of $A_{\mu\mu}$ vs. $m_{\mu\mu}$ for $P\bar{P} \rightarrow \mu^+\mu^-X$