

Leaderless Cooperative Adaptive Cruise Control Based on Constant Time-Gap Spacing Policy

Hamed Rezaee, *Senior Member, IEEE*, Thomas Parisini, *Fellow, IEEE*,
and Marios M. Polycarpou, *Fellow, IEEE*

Abstract—Cooperative adaptive cruise control (CACC) in leaderless scenario is addressed in this paper. The common idea in CACC is that a leader vehicle determines the steady state speed of the platoon without considering feedback from other vehicles. The main contribution of this paper is to propose a control strategy such that each vehicle in a leaderless platoon keeps a desired distance from its preceding vehicle, determined based on the constant time-gap spacing policy, and simultaneously all the vehicles reach consensus upon their speeds. Moreover, whereas each vehicle needs access to the information of some neighboring vehicles (via a communication network) to achieve speed consensus, we assume that it has access to the relative distance of just one possible preceding vehicle (using a radar/lidar). Simulation results illustrate the performance of the proposed control strategy.

Index Terms—Connected vehicles, consensus, cooperative adaptive cruise control, vehicular platoons.

I. INTRODUCTION

Adaptive cruise control (ACC) is a common technology in modern road vehicles, where a cruise control system adjusts the speed of a vehicle to keep a safe distance from its preceding vehicle. The distance from the preceding vehicle is measured by a radar/lidar, and such measurement is applied as feedback for the ACC system to steer the distance to a desired value. In addition to radar/lidar measurements, more information among the vehicles can be exchanged via a communication network. Cooperative adaptive cruise control (CACC) is the extension of ACC, based on which the communicated information of other vehicles is used in cruise control. Therefore, by access to more state information of other vehicles, each vehicle can follow its preceding vehicles more precisely [1]–[3].

A. State of the Art

The popular idea in CACC is that a group of vehicles follows a leading vehicle, while keeping desired intervehicle distances. The leading vehicle is driven manually, semiautonomously, or autonomously, and depending on the traffic spacing policy, determines the trajectory of the other vehicles (a spacing policy determines the desired steady state distances among the vehicles). While each

follower vehicle can measure the distance from its preceding vehicle via its own radar/lidar, the leader sends required state information to some rear vehicles, and other vehicles also exchange state information among each other. By employing such information as feedback, each follower vehicle keeps a desired distance from its preceding vehicle such that a platoon of vehicles will be achieved. This problem has been investigated from several points of view. For instance, adaptive and robust control of vehicular platoons is considered in [4]–[6], and [7]. In [1], [8], [9], and [10], CACC in the presence of unreliable communication is investigated. Cyber-security problems in CACC are addressed in [11]–[13], and [14], and fuel efficient optimal cruise control is studied in [15]–[17], and [18].

B. Motivation

Whereas leader-following is the popular approach in CACC, a leader decreases the degree of autonomy in a network of vehicles, as it does not consider feedback from other vehicles and determines the platoon speed independently. In other words, while the leader sends its state information to other vehicles, its control law is not cooperative, and the platoon speed is just determined by the leader. In such condition, for instance, it is possible that the leader is moving with a low speed which may affect the transportation rate. Hence, the motivation of this study is to propose a framework when there is not an independent leader in the platoon. Based on such framework, the platoon speed can be determined in a cooperative way by the vehicles. Therefore, the problem is that while each vehicle keeps a desired safe distance from its preceding vehicle, how is it possible for the vehicles to simultaneously reach consensus/agreement upon their speeds? We called this problem as *leaderless CACC*.

It should be noted that the problem of velocity consensus has a long history in formation control of multiagent systems. Based on that idea, the agents reach consensus on a velocity, and simultaneously they keep a desired formation pattern [19]–[21]. Therefore, for intervehicle regulation and achieving speed consensus, the idea of formation control problem can be applicable [22]. However, there are differences between CACC and formation control of multiagent systems such that the consensus-based formation control idea may not be always practical or efficient for CACC:

- 1) In CACC, depending on the spacing policy, the desired distance of each vehicle from its preceding vehicle may be a function of its own states. For instance, to increase safety in roads traffic, the desired distance can depend on the vehicle speed [23], [24]. By contrast, in the formation control problem, the objective is keeping a fixed or time-varying geometric pattern of mobile agents, where the desired relative positions are determined as external reference signals (not based on the states).
- 2) For road vehicles, adjusting the relative distance from a rear vehicle may not be efficient. Indeed, such adjustment (by using the relative distance information of the rear vehicle in the CACC strategy) means that each vehicle tries to converge to a desired distance from its rear vehicle as well as its preceding vehicle. However, this may not be an efficient approach due to

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H. Rezaee is with the Department of Mechanical Engineering, Imperial College London, London SW7 2AZ, UK (e-mail: h.rezaee@imperial.ac.uk).

T. Parisini is with the Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2AZ, UK, with the Department of Engineering and Architecture, University of Trieste, 34127 Trieste, Italy, and also with the KIOS Research and Innovation Center of Excellence, University of Cyprus, CY-1678 Nicosia, Cyprus (e-mail: t.parisini@imperial.ac.uk).

M. M. Polycarpou is with the KIOS Research and Innovation Center of Excellence and the Department of Electrical and Computer Engineering, University of Cyprus, CY-1678 Nicosia, Cyprus (e-mail: mpolycar@ucy.ac.cy).

forward motion of the vehicles. Therefore, for CACC, it is more practical to use just the information of the distance from the preceding vehicle in the controller of each vehicle. Moreover, for agreement of all the vehicles on a common speed, each vehicle should exchange information with more vehicles via a strongly connected communication network [25]. Under this condition, despite the formation control problem, the topology of the communication network to exchange state information is not compatible with the information of intervehicle distances measured by vehicles radars/lidars.

The above-mentioned issues indicate the importance of leaderless CACC and its challenges, and motivate more study in this area.

C. Objectives and Contributions

In this paper, the problem of speed consensus problem in leaderless vehicular platoons is addressed. We consider a platoon in which all the vehicles receive communicated information from neighboring vehicles, while each vehicle measures the distance from the preceding vehicle. We introduce a set of auxiliary states updated based on the real states and exchanged among the vehicles. By employing such auxiliary states and the information of distance from the preceding vehicle, a distributed control strategy is proposed such that the vehicles speeds converge to an *a priori* unknown common value, and simultaneously the intervehicle distances converge to desired values determined by the *constant time-gap spacing policy* (as functions of the vehicles speeds) [23]. Therefore, despite the existing results in the literature, where a leader vehicle determines the platoon speed without considering feedback from other vehicles [1], [4]–[14], based on the proposed strategy in this paper, the vehicles cooperate in order to reach consensus upon their speeds.

It is worth mentioning that the notion of consensus-based CACC is considered in several studies in the literature as well (for instance, see [26]–[29]). However, in those studies, just the design of the controller is based on the standard consensus formulation, and no speed consensus control is considered.

The rest of this paper is organized as follows. Preliminaries are provided in the next section. The problem is stated in Section III. The proposed leaderless CACC strategy is presented in Section IV. Discussion on further extension of the results is given in Section V. Simulation results are provided in Section VI, and Section VII concludes the paper.

II. PRELIMINARIES

In this section, notation and some basic concepts on graph theory are introduced.

A. Notation

Let $\mathbb{R}$ and $\mathbb{R}_{>0}$ denote the sets of real and positive real numbers, respectively. $0_{n \times m}$ implies an $n \times m$ zeros matrix, 1_n implies an $n \times 1$ ones vector, and I_n is an $n \times n$ identity matrix. For two sets $\mathcal{S}_1$ and $\mathcal{S}_2$, $\mathcal{S}_1 \setminus \mathcal{S}_2$ denotes the reduction of $\mathcal{S}_1$ by $\mathcal{S}_2$. $\min\{\cdot\}$ and $\max\{\cdot\}$ stand for the minimum and maximum of a set of real numbers, respectively. $\sup\{\cdot\}$ denotes supremum. $\|\cdot\|$ stands for the 2 norm. $\|\cdot\|_{\mathcal{L}_p}$ stands for the $\mathcal{L}_p$ norm. $\text{diag}(M_1, M_2, \dots, M_n)$ is a block diagonal matrix composed of the matrices $M_1, M_2, \dots, M_n$. For a square matrix M , we say $M \leq 0$ if it is symmetric negative semidefinite. Moreover, for a real symmetric matrix $M \in \mathbb{R}^{N \times N}$, $\lambda_i(M)$ denotes the i th smallest eigenvalue of M such that

$$\lambda_1(M) \leq \lambda_2(M) \leq \dots \leq \lambda_N(M).$$

B. Graph Theory

To describe interaction among the vehicles, the theory of weighted graphs is exploited. A weighted directed graph $\mathcal{G}$ is described by the triple $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{1, 2, \dots, N\}$ is the set of N nodes (describing the vehicles) and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, where $(i, j) \in \mathcal{E}$ indicates an edge from Node i to Node j (describing the direction of information flow). Moreover, $\mathcal{A}$ is the adjacency matrix associated with $\mathcal{G}$ expressed as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$, where $a_{ij} \in \mathbb{R}_{>0}$ if $(j, i) \in \mathcal{E}$, $i \neq j$, and $a_{ij} = 0$ if $(j, i) \notin \mathcal{E}$. A weighted directed graph is *bidirected* if for any $(j, i) \in \mathcal{E}$, also $(i, j) \in \mathcal{E}$, implying that if $a_{ij} \in \mathbb{R}_{>0}$, also $a_{ji} \in \mathbb{R}_{>0}$, where a_{ij} may not be equal to a_{ji} . If for all $i, j \in \mathcal{V}$, $a_{ij} = a_{ji}$, a bidirected graph is also called *undirected*. Moreover, for Node i , we define $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$.

According to the definition of the adjacency matrix, the Laplacian matrix $\mathcal{L} = [\ell_{ij}] \in \mathbb{R}^{N \times N}$ associated with $\mathcal{G}$ is expressed as

$$\ell_{ij} = \begin{cases} \sum_{j=1, i \neq j}^N a_{ij} & i = j \\ -a_{ij} & i \neq j, \end{cases}$$

where has rows with zero entries summation, and thus it has at least one zero eigenvalue. Moreover, according to the Gerschgorin theorem, all the eigenvalues of $\mathcal{L}$ are on the closed right half-plane [25]. This implies that if the Laplacian matrix is symmetric, it is positive semidefinite.

A directed/bidirected graph is defined to be *strongly connected*, if from each node to every other node, it contains a path of edges with compatible directions. If $\mathcal{G}$ is strongly connected, $\mathcal{L}$ has one zero eigenvalue and the other eigenvalues lie on the open right half-plane. Moreover, if $\mathcal{G}$ is strongly connected, the right eigenvector associated with the zero eigenvalue of $\mathcal{L}$ is 1_N . Note that if a graph is undirected and strongly connected, it is also called *connected* [30].

III. PROBLEM STATEMENT

Consider a vehicular platoon comprising N Vehicles indexed as Vehicle i , $i \in \mathcal{V}$, in which Vehicle $i-1$, $i \in \mathcal{V} \setminus \{1\}$, is the preceding vehicle of Vehicle i . By defining $\mathcal{V}' = \mathcal{V} \setminus \{1\}$, we consider the following model for longitudinal dynamics of Vehicle i [10], [22]:

$$\begin{aligned} \dot{d}_i(t) &= v_{i-1}(t) - v_i(t), \quad i \in \mathcal{V}' \\ \dot{v}_i(t) &= u_i(t), \quad i \in \mathcal{V}, \end{aligned} \quad (1)$$

where $d_i(t)$ is the distance of Vehicle i from Vehicle $i-1$, $v_i(t)$ denotes the speed, and $u_i(t)$ is the control input. The objective is to develop a control strategy such that all the vehicles reach a common speed and simultaneously each vehicle keeps a safe distance from its preceding vehicle. There exist several policies to define safe distances in control of vehicular platoons. A common policy is the constant time-gap spacing. According to this policy, the desired distance of Vehicle i from Vehicle $i-1$, $i \in \mathcal{V}'$, is a linear function of the speed of Vehicle i as follows [23]:

$$d_{\text{des},i}(t) = r_i + h v_i(t), \quad i \in \mathcal{V}', \quad (2)$$

where r_i is the standstill desired safe distance and h is the time headway. The objective is developing a CACC scheme such that

$$\lim_{t \rightarrow \infty} (d_{\text{des},i}(t) - d_i(t)) = 0, \quad i \in \mathcal{V}' \quad (3a)$$

$$\lim_{t \rightarrow \infty} (v_i(t) - v_j(t)) = 0, \quad i, j \in \mathcal{V}. \quad (3b)$$

It should be noted that in order to regulate the intervehicle distances in desired values as (3a) and simultaneously achieving agreement upon the vehicles speeds as (3b), the vehicles longitudinal dynamics should be controllable. Here, by considering the vehicles longitudinal dynamics as (1), such controllability condition is satisfied.

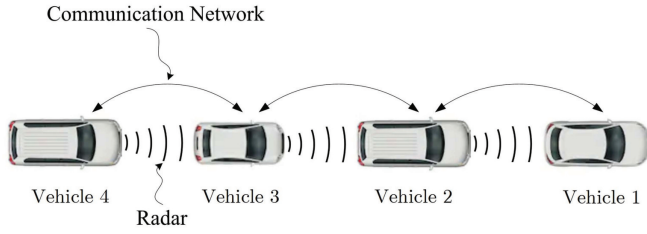


Fig. 1. A leaderless platoon of four vehicles when Vehicle 1 receives feedback from other vehicles.

For achieving consensus on a common value, it is necessary for all the vehicles to use the state information of some neighboring vehicles via a communication network. The required condition for the consensusability of a network of agents such that all the agents can affect the consensus value is that the communication graph should be strongly connected [25]. Thus, to have a strongly connected communication graph, the following assumption is considered.

Assumption 1: Under a communication network, Vehicle i , $i \in \mathcal{V} \setminus \{1, N\}$, exchanges information with Vehicle $i - 1$ and Vehicle $i + 1$, Vehicle 1 exchanges information with Vehicle 2, and Vehicle N exchanges information with Vehicle $N - 1$.

According to Assumption 1, the vehicular platoon communication graph is bidirected as demonstrated in Fig. 1. Indeed, we assume that each vehicle just communicates with its closest vehicles, which is practically justifiable.

IV. LEADERLESS COOPERATIVE ADAPTIVE CRUISE CONTROL STRATEGY

By assuming that each vehicle just has access to the relative distance of its preceding vehicle and the state information of up to two neighboring vehicles, the control law of each vehicle is proposed as

$$u_i(t) = -\sigma_i \dot{s}_i(t) - \alpha_i s_i(t) + \nu_i(t), \quad i \in \mathcal{V}, \quad (4)$$

where $\sigma_i, \alpha_i \in \mathbb{R}_{>0}$ and $s_i(t)$ is a local control surface designed as

$$\begin{aligned} s_i(t) &= \xi_{0i}(t) - \xi_{1i}(t) \\ \dot{\xi}_{0i}(t) &= v_i(t) \\ \ddot{\xi}_{1i}(t) &= \nu_i(t), \end{aligned} \quad (5)$$

in which $\xi_{0i}(t), \xi_{1i}(t) \in \mathbb{R}$ are auxiliary states with arbitrary initial values, and

$$\begin{aligned} \nu_i(t) &= \frac{1}{\gamma h} \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\xi}_{1j}(t) - \dot{\xi}_{1i}(t)), \quad i = 1 \\ \nu_i(t) &= \frac{1}{\gamma h} \left(-k(r_i + h v_i(t) - d_i(t)) \right. \\ &\quad \left. + \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\xi}_{1j}(t) - \dot{\xi}_{1i}(t)) \right), \quad i \in \mathcal{V}', \end{aligned} \quad (6)$$

in which $k, \gamma \in \mathbb{R}_{>0}$. Moreover, according to Assumption 1, $a_{ij}, i, j \in \mathcal{V}$, are designed as follows:

$$a_{ij} = \begin{cases} 2\gamma & i \in \mathcal{V}' \text{ and } j = i - 1 \\ \gamma & i \in \mathcal{V} \setminus \{N\} \text{ and } j = i + 1 \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

Note that to simplify the formulation in the rest of the paper, we have considered $1/\gamma$ as the gain of $\nu_i(t)$ in (6), whereas γ is used in a_{ij} as well.

The speed consensus problem with intervehicle distances regulation should be described by at least a second-order differential equation. The main idea of considering the auxiliary states $\xi_{1i}(t), i \in \mathcal{V}$,



Fig. 2. Weighted bidirected communication graph associated with a leaderless platoon of vehicles.

is that under the second-order differential equation $\ddot{\xi}_{1i}(t) = \nu_i(t)$, we solve the consensus problem for the states $\xi_{1i}(t), i \in \mathcal{V}$, (along with intervehicle distances regulation). On the other hand, the control law described in (4) is designed such that it guarantees the convergence of the trajectories of the real states of the vehicles to the trajectories of the auxiliary states and the objective (3) is achieved. According to (6) and (7), there is not an independent leader in the network, and all the vehicles are in bidirected interaction with neighboring vehicles. Moreover, (6) and (7) imply that the weights of interaction with the possible preceding vehicle is 2 and with the possible rear vehicle is 1. The difference between these two weights helps us in the mathematical analysis of the proposed control strategy, which will be provided later. Note that the platoon front vehicle does not have any preceding vehicle, while the vehicle at the end of the platoon does not have any rear vehicle, and this issue is considered in (7). According to the definition of $a_{ij}, i, j \in \mathcal{V}$, in (7), the weighted bidirected communication graph associated with four vehicles is depicted in Fig. 2. By considering (7) as the entries of an adjacency matrix, the associated Laplacian matrix is as follows:

$$\mathcal{L} = \mathcal{L}' + \mathcal{L}'' ,$$

where $\mathcal{L}'$ is a nonsymmetric Laplacian matrix as

$$\mathcal{L}' = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ -\gamma & \gamma & 0 & \dots & 0 & 0 \\ 0 & -\gamma & \gamma & \dots & 0 & 0 \\ 0 & 0 & -\gamma & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\gamma & \gamma \end{bmatrix} \quad (8)$$

and $\mathcal{L}''$ is a symmetric Laplacian matrix as follows:

$$\mathcal{L}'' = \begin{bmatrix} \gamma & -\gamma & 0 & \dots & 0 & 0 \\ -\gamma & 2\gamma & -\gamma & \dots & 0 & 0 \\ 0 & -\gamma & 2\gamma & \dots & 0 & 0 \\ 0 & 0 & -\gamma & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\gamma & \gamma \end{bmatrix}. \quad (9)$$

In Theorem 1, we show that under the proposed control strategy, the objective (3) is satisfied. Before presenting the theorem, the following lemma is required.

Lemma 1: Let $\mathcal{L} \in \mathbb{R}^{N \times N}$ be the Laplacian matrix associated with a connected undirected graph. For any $x \in \mathbb{R}^N$, $x^\top \mathcal{L} x = 0$ if and only if $\mathcal{L} x = 0_{N \times 1}$.

Proof: From $\mathcal{L} x = 0_{N \times 1}$ it follows that $x^\top \mathcal{L} x = 0$. Now, we show that if $x^\top \mathcal{L} x = 0$, then $\mathcal{L} x = 0_{N \times 1}$. Since the graph is connected, $\mathcal{L}$ has a zero eigenvalue and its other eigenvalues are positive real, where the right eigenvector associated with the zero eigenvalue is 1_N . Now, for a $\theta \in \mathbb{R}$, we decompose x as $x = \theta 1_N + \delta$, where $\delta \in \mathbb{R}^N$ is perpendicular to 1_N . According to this decomposition and since $\mathcal{L} 1_N = 0_{N \times 1}$, one can observe that

$$x^\top \mathcal{L} x = \delta^\top \mathcal{L} \delta. \quad (10)$$

Since δ is perpendicular to 1_N , we also have [25]

$$\lambda_2(\mathcal{L}) \delta^\top \delta \leq \delta^\top \mathcal{L} \delta. \quad (11)$$

According to (10) and (11) and since $\lambda_2(\mathcal{L}) \in \mathbb{R}_{>0}$, if $x^\top \mathcal{L}x = 0$, then $\delta = 0_{N \times 1}$. Therefore, as $\mathcal{L}1_N = 0_{N \times 1}$ and $x = \theta 1_N + \delta$, we also have $\mathcal{L}x = 0_{N \times 1}$. ■

Theorem 1: Consider the vehicular platoon (1) under the CACC strategy proposed in (4)-(7), where k satisfies the following inequality:

$$k - \frac{1}{2\varepsilon_1} - \frac{1}{2\varepsilon_2} - \frac{\varepsilon_1^2 k^2}{4} \left(\varepsilon_1 - \frac{\varepsilon_1}{2\varepsilon_3} \right)^{-1} > 0, \quad (12)$$

where $\varepsilon_1, \varepsilon_2, \varepsilon_3 \in \mathbb{R}_{>0}$, and $\varepsilon_3 > 1/2$. Then, the objective (3) is satisfied.

Proof: According to (5), it can be observed that $\ddot{s}_i(t) = \dot{v}_i(t) - \nu_i(t)$. Hence, by substituting the control law (4) into (1), one gets

$$\ddot{s}_i(t) + \sigma_i \dot{s}_i(t) + \alpha_i s_i(t) = 0, \quad i \in \mathcal{V}, \quad (13)$$

implying that $s_i(t)$ and $\dot{s}_i(t)$ remain bounded and

$$\begin{aligned} \lim_{t \rightarrow \infty} s_i(t) &= 0 \\ \lim_{t \rightarrow \infty} \dot{s}_i(t) &= 0, \quad i \in \mathcal{V}. \end{aligned} \quad (14)$$

Now, we investigate the behavior of the platoon along (14). Since the surfaces are described based on the auxiliary states, first we represent $d_i(t)$ and $d_{\text{des},i}(t)$ based on the auxiliary states as well. From (1) it follows that

$$d_i(t) = \int_0^t (v_{i-1}(\tau) - v_i(\tau)) d\tau + d_i(0). \quad (15)$$

By considering (5), there exists a constant $c_i \in \mathbb{R}$ which (15) can be restated as follows:

$$d_i(t) = s_{i-1}(t) + \xi_{1,i-1}(t) - s_i(t) - \xi_{1i}(t) + c_i. \quad (16)$$

Moreover, by considering (5), one gets

$$v_i(t) = \dot{s}_i(t) + \dot{\xi}_{1i}(t). \quad (17)$$

From (17), $d_{\text{des},i}(t), i \in \mathcal{V}'$, defined in (2) can be restated as

$$d_{\text{des},i}(t) = r_i + h\dot{s}_i(t) + h\dot{\xi}_{1i}(t). \quad (18)$$

Now, let us define the following error:

$$e_i(t) = r_i + h\dot{\xi}_{1i}(t) - \xi_{1,i-1}(t) + \xi_{1i}(t) - c_i, \quad i \in \mathcal{V}'. \quad (19)$$

From (16), (18), and (19) it follows that

$$d_{\text{des},i}(t) - d_i(t) = e_i(t) + h\dot{s}_i(t) - s_{i-1}(t) + s_i(t). \quad (20)$$

By considering (14) and (20), the objective (3a) is achieved if $e_i(t), i \in \mathcal{V}'$, converge to zero. Moreover, by considering (14) and (17), the objective (3b) is achieved if

$$\lim_{t \rightarrow \infty} (\dot{\xi}_{1i}(t) - \dot{\xi}_{1j}(t)) = 0, \quad i, j \in \mathcal{V}. \quad (21)$$

Therefore, to achieve the objective (3), we consider the following Lyapunov function:

$$V(t) = \frac{1}{2} \gamma e(t)^\top e(t) + \frac{1}{2} \varepsilon_1 \gamma h \dot{\xi}_1(t)^\top \mathcal{L}'' \dot{\xi}_1(t), \quad (22)$$

with

$$\begin{aligned} e(t) &= [e_2(t) \quad e_3(t) \quad \dots \quad e_N(t)]^\top \\ \xi_1(t) &= [\xi_{11}(t) \quad \xi_{12}(t) \quad \dots \quad \xi_{1N}(t)]^\top. \end{aligned}$$

Note that as (19) is not defined for Vehicle 1, the dimension of $e(t)$ is $(N-1) \times 1$. Since $\mathcal{L}''$ is a Laplacian matrix associated with a connected undirected graph, by invoking Lemma 1, $\dot{\xi}_1(t)^\top \mathcal{L}'' \dot{\xi}_1(t)$ converges to zero if and only if $\mathcal{L}'' \dot{\xi}_1(t)$ converges to zero. Therefore, the Lyapunov function $V(t)$ is positive definite with respect to $e(t)$ and $\mathcal{L}'' \dot{\xi}_1(t)$. Since $\mathcal{L}''$ is a Laplacian matrix associated with a

connected undirected graph, it has a zero eigenvalue and its other eigenvalues are positive real, where the right eigenvector associated with the zero eigenvalue is 1_N . Hence, if $\mathcal{L}'' \dot{\xi}_1(t)$ converges to zero, all $\dot{\xi}_{1i}(t), i \in \mathcal{V}$, converge to identical values, and then (21) is satisfied. From (5) and (6) it follows that

$$\begin{aligned} \ddot{\xi}_{1i}(t) &= \frac{1}{\gamma h} \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\xi}_{1j}(t) - \dot{\xi}_{1i}(t)), \quad i = 1 \\ \ddot{\xi}_{1i}(t) &= \frac{1}{\gamma h} \left(-k(r_i + h v_i(t) - d_i(t)) \right. \\ &\quad \left. + \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\xi}_{1j}(t) - \dot{\xi}_{1i}(t)) \right), \quad i \in \mathcal{V}'. \end{aligned} \quad (23)$$

By considering (16) and (17), we can restate (23) as follows:

$$\begin{aligned} \ddot{\xi}_{1i}(t) &= \frac{1}{\gamma h} \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\xi}_{1j}(t) - \dot{\xi}_{1i}(t)), \quad i = 1 \\ \ddot{\xi}_{1i}(t) &= \frac{1}{\gamma h} \left(-k \left(r_i + h \dot{\xi}_{1i}(t) - \xi_{1,i-1}(t) + \xi_{1i}(t) - c_i \right) \right. \\ &\quad \left. + \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\xi}_{1j}(t) - \dot{\xi}_{1i}(t)) + \rho_i(t) \right), \quad i \in \mathcal{V}', \end{aligned} \quad (24)$$

where

$$\rho_i(t) = k(s_{i-1}(t) - s_i(t) - h\dot{s}_i(t)), \quad i \in \mathcal{V}'. \quad (25)$$

According to the definition of $e_i(t)$ in (19), (24) can be rewritten as

$$\begin{aligned} \ddot{\xi}_{1i}(t) &= \frac{1}{\gamma h} \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\xi}_{1j}(t) - \dot{\xi}_{1i}(t)), \quad i = 1 \\ \ddot{\xi}_{1i}(t) &= \frac{1}{\gamma h} \left(-k e_i(t) + \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\xi}_{1j}(t) - \dot{\xi}_{1i}(t)) \right. \\ &\quad \left. + \rho_i(t) \right), \quad i \in \mathcal{V}'. \end{aligned} \quad (26)$$

Now, by considering the definition of $a_{ij}, i, j \in \mathcal{V}$, in (7) and the definition of $\mathcal{L}'$ and $\mathcal{L}''$ in (8) and (9), (26) can be written for all the platoon as follows:

$$\gamma h \ddot{\xi}_1(t) = -k S e(t) - \mathcal{L}' \dot{\xi}_1(t) - \mathcal{L}'' \dot{\xi}_1(t) + S \rho(t), \quad (27)$$

where

$$\begin{aligned} \rho(t) &= [\rho_2(t) \quad \rho_3(t) \quad \dots \quad \rho_N(t)]^\top \\ S &= \begin{bmatrix} 0_{1 \times (N-1)} \\ I_{N-1} \end{bmatrix}. \end{aligned}$$

Note that since (25) is not defined for Vehicle 1, the dimension of $\rho(t)$ is $(N-1) \times 1$. Moreover, according to the definition of $a_{ij}, i, j \in \mathcal{V}$, in (7), from (26), one obtains

$$\begin{aligned} \gamma (h \ddot{\xi}_{1i}(t) - \dot{\xi}_{1,i-1}(t) + \dot{\xi}_{1i}(t)) &= -k e_i(t) \\ &\quad + \sum_{j \in \mathcal{N}_i} \gamma (\dot{\xi}_{1j}(t) - \dot{\xi}_{1i}(t)) + \rho_i(t), \quad i \in \mathcal{V}', \end{aligned}$$

which by obtaining $\dot{e}_i(t)$ from (19), it follows that

$$\gamma \dot{e}_i(t) = -k e_i(t) + \sum_{j \in \mathcal{N}_i} \gamma (\dot{\xi}_{1j}(t) - \dot{\xi}_{1i}(t)) + \rho_i(t), \quad i \in \mathcal{V}' \quad (28)$$

Hence, for all Vehicles $i \in \mathcal{V}'$, (28) can be stated as follows:

$$\gamma \dot{e}(t) = -k e(t) - S^\top \mathcal{L}'' \dot{\xi}_1(t) + \rho(t), \quad (29)$$

where $S^\top \mathcal{L}''$ is an $(N-1) \times N$ matrix obtained by removing the first row of the matrix $\mathcal{L}''$. Indeed, $S^\top \mathcal{L}''$ contains those rows of

the Laplacian matrix $\mathcal{L}''$ associated with Vehicles $i \in \mathcal{V}$. Now, by taking the time derivative of $V(t)$ along (27) and (29), one obtains

$$\begin{aligned} \dot{V}(t) = & -ke(t)^\top e(t) - e(t)^\top S^\top \mathcal{L}'' \dot{\xi}_1(t) + e(t)^\top \rho(t) \\ & - \varepsilon_1 k \dot{\xi}_1(t)^\top \mathcal{L}'' S e(t) - \varepsilon_1 \dot{\xi}_1(t)^\top \mathcal{L}'' \mathcal{L}' \dot{\xi}_1(t) \\ & - \varepsilon_1 \dot{\xi}_1(t)^\top \mathcal{L}''^2 \dot{\xi}_1(t) + \varepsilon_1 \dot{\xi}_1(t)^\top \mathcal{L}'' S \rho(t). \end{aligned} \quad (30)$$

To continue the proof, we recall that for two vectors a and b with compatible dimensions, for each $\varepsilon \in \mathbb{R}_{>0}$ we always have

$$a^\top b \leq \frac{1}{2\varepsilon} a^\top a + \frac{\varepsilon}{2} b^\top b. \quad (31)$$

If we employ (31) to nonquadratic terms in (30), for $\varepsilon_1, \varepsilon_2, \varepsilon_3 \in \mathbb{R}_{>0}$ satisfying the inequality (12), we have

$$\begin{aligned} \dot{V}(t) \leq & -ke(t)^\top e(t) + \frac{1}{2\varepsilon_1} e(t)^\top e(t) \\ & + \frac{\varepsilon_1}{2} \dot{\xi}_1(t)^\top \mathcal{L}''^\top S S^\top \mathcal{L}'' \dot{\xi}_1(t) + \frac{1}{2\varepsilon_2} e(t)^\top e(t) \\ & + \frac{\varepsilon_2}{2} \rho(t)^\top \rho(t) - \varepsilon_1 k \dot{\xi}_1(t)^\top \mathcal{L}'' S e(t) \\ & - \varepsilon_1 \dot{\xi}_1(t)^\top \mathcal{L}'' \mathcal{L}' \dot{\xi}_1(t) - \varepsilon_1 \dot{\xi}_1(t)^\top \mathcal{L}''^2 \dot{\xi}_1(t) \\ & + \frac{\varepsilon_1}{2\varepsilon_3} \dot{\xi}_1(t)^\top \mathcal{L}'' \mathcal{L}''^\top \dot{\xi}_1(t) + \frac{\varepsilon_1 \varepsilon_3}{2} \rho(t)^\top \rho(t). \end{aligned} \quad (32)$$

According to the definition of $\mathcal{L}'$ and $\mathcal{L}''$ in (8) and (9), it can be observed that

$$\begin{aligned} & -\mathcal{L}'' \mathcal{L}' - \mathcal{L}'^\top \mathcal{L}''^\top + \mathcal{L}''^\top S S^\top \mathcal{L}'' \\ & = \text{diag}\left(\gamma^2 \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, 0_{(N-2) \times (N-2)}\right). \end{aligned} \quad (33)$$

Note that

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \leq 0,$$

and hence from (33) it follows that

$$\frac{\varepsilon_1}{2} \dot{\xi}_1(t)^\top \mathcal{L}''^\top S S^\top \mathcal{L}'' \dot{\xi}_1(t) - \varepsilon_1 \dot{\xi}_1(t)^\top \mathcal{L}'' \mathcal{L}' \dot{\xi}_1(t) \leq 0. \quad (34)$$

Therefore, from (32) and according to (34), it follows that

$$\begin{aligned} \dot{V}(t) \leq & -\begin{bmatrix} e(t) \\ \mathcal{L}'' \dot{\xi}_1(t) \end{bmatrix}^\top T \begin{bmatrix} e(t) \\ \mathcal{L}'' \dot{\xi}_1(t) \end{bmatrix} \\ & + \left(\frac{\varepsilon_2}{2} + \frac{\varepsilon_1 \varepsilon_3}{2}\right) \rho(t)^\top \rho(t), \end{aligned} \quad (35)$$

where

$$T = \begin{bmatrix} \left(k - \frac{1}{2\varepsilon_1} - \frac{1}{2\varepsilon_2}\right) I_{N-1} & \frac{1}{2} \varepsilon_1 k S^\top \\ \frac{1}{2} \varepsilon_1 k S & \left(\varepsilon_1 - \frac{\varepsilon_1}{2\varepsilon_3}\right) I_N \end{bmatrix}.$$

We recall that the right eigenvector associated with the zero eigenvalue of $\mathcal{L}''$ is 1_N . Now, for a $\theta(t) \in \mathbb{R}$, we decompose $\dot{\xi}_1(t)$ as $\dot{\xi}_1(t) = \theta(t) 1_N + \delta(t)$, where $\delta(t) \in \mathbb{R}^N$ is perpendicular to 1_N . According to this decomposition and since $\mathcal{L}'' 1_N = 0_{N \times 1}$, from (35) it follows that

$$\begin{aligned} \dot{V}(t) \leq & -\begin{bmatrix} e(t) \\ \mathcal{L}'' \delta(t) \end{bmatrix}^\top T \begin{bmatrix} e(t) \\ \mathcal{L}'' \delta(t) \end{bmatrix} \\ & + \left(\frac{\varepsilon_2}{2} + \frac{\varepsilon_1 \varepsilon_3}{2}\right) \rho(t)^\top \rho(t). \end{aligned} \quad (36)$$

According to the Schur complement [31], T is positive definite if

$$\begin{aligned} & \varepsilon_1 - \frac{\varepsilon_1}{2\varepsilon_3} > 0 \\ & \left(k - \frac{1}{2\varepsilon_1} - \frac{1}{2\varepsilon_2}\right) I_{N-1} - \frac{1}{2} \varepsilon_1 k S^\top \left(\varepsilon_1 - \frac{\varepsilon_1}{2\varepsilon_3}\right)^{-1} \frac{1}{2} \varepsilon_1 k S > 0, \end{aligned}$$

which since $S^\top S = I_{N-1}$ and $\varepsilon_3 > 1/2$, the inequality (12) guarantees these. Hence, from (36) and since $\delta(t) \in \mathbb{R}^N$ is perpendicular to the kernel of $\mathcal{L}''$, it follows that

$$\begin{aligned} \dot{V}(t) \leq & -\lambda_1(T) \left(e(t)^\top e(t) + \lambda_2(\mathcal{L}'')^2 \delta(t)^\top \delta(t) \right) \\ & + \left(\frac{\varepsilon_2}{2} + \frac{\varepsilon_1 \varepsilon_3}{2} \right) \rho(t)^\top \rho(t) \\ \leq & -\lambda_1(T) \min \left\{ 1, \lambda_2(\mathcal{L}'')^2 \right\} \left(e(t)^\top e(t) + \delta(t)^\top \delta(t) \right) \\ & + \left(\frac{\varepsilon_2}{2} + \frac{\varepsilon_1 \varepsilon_3}{2} \right) \rho(t)^\top \rho(t). \end{aligned} \quad (37)$$

By similar arguments, from (22) one gets

$$V(t) \leq \frac{1}{2} \max \left\{ \gamma, \varepsilon_1 \gamma h \lambda_N(\mathcal{L}'') \right\} \left(e(t)^\top e(t) + \delta(t)^\top \delta(t) \right). \quad (38)$$

From (37) and (38) it follows that

$$\dot{V}(t) \leq -\varrho V(t) + \left(\frac{\varepsilon_2}{2} + \frac{\varepsilon_1 \varepsilon_3}{2} \right) \rho(t)^\top \rho(t), \quad (39)$$

where

$$\varrho = \frac{2\lambda_1(T) \min \left\{ 1, \lambda_2(\mathcal{L}'')^2 \right\}}{\max \left\{ \gamma, \varepsilon_1 \gamma h \lambda_N(\mathcal{L}'') \right\}}. \quad (40)$$

Since $s_i(t)$ and $\dot{s}_i(t)$, $i \in \mathcal{V}$, remain bounded and converge to zero, $\rho(t)$ remains bounded and converges to zero. Hence, by considering (39), $V(t)$ converges to zero, and as a result,

$$\lim_{t \rightarrow \infty} e(t) = 0_{(N-1) \times 1} \quad (41a)$$

$$\lim_{t \rightarrow \infty} \mathcal{L}'' \dot{\xi}_1(t) = 0_{N \times 1}. \quad (41b)$$

By considering (20), from (14) and (41a) it follows that the objective (3a) is satisfied. Moreover, since the right eigenvector associated with the single zero eigenvalue of $\mathcal{L}''$ is 1_N , from (41b), one gets

$$\lim_{t \rightarrow \infty} (\dot{\xi}_{1i}(t) - \dot{\xi}_{1j}(t)) = 0 \quad i, j \in \mathcal{V}. \quad (42)$$

Now, from (14), (17), and (42), it follows that the objective (3b) is satisfied as well. Therefore, the proof is completed. ■

According to Theorem 1, the objective (3) is satisfied. From (3a), (23), and (42), it follows that $\dot{\xi}_{1i}(t)$, $i \in \mathcal{V}$, converge to zero. Since (23) along (1) implies a linear dynamical system, by convergence of $\dot{\xi}_{1i}(t)$, $i \in \mathcal{V}$, to zero, $\xi_{1i}(t)$, $i \in \mathcal{V}$, converge to constant values. Hence, as $\dot{s}_i(t)$, $i \in \mathcal{V}$, converge to zero, (17) implies that $v_i(t)$, $i \in \mathcal{V}$, converge to constant values as well. Therefore, according to the results of Theorem 1, all the vehicles speeds converge to a common constant value. It can be said that as the closed-loop control system described in (13), (27), and (29) is linear, the steady state variables depend on the state matrix and the initial states.

Remark 1: Based on (39), the rate of the convergence of $V(t)$ to zero depends on the magnitude of ϱ and the rate of the convergence of $\rho_i(t)$ to zero. According to (40), ϱ depends on the parameters $k, \varepsilon_1, \varepsilon_2, \varepsilon_3, \gamma$, and h , and according to (13) and (25), the rate of the convergence of $\rho_i(t)$ to zero depends on the parameters σ_i and α_i .

Remark 2: It is worth mentioning that obtaining just one solution for (12) is sufficient for all CACC scenarios, as (12) does not change as the number the vehicles in the platoon changes. For instance, the solution $k = 2$, $\varepsilon_1 = 0.5$, $\varepsilon_2 = 10$, and $\varepsilon_3 = 10$ is sufficient for all platoons. Indeed, the inequality (12) formulated sufficient solutions for Theorem 1. There can exist several ways to obtain such solutions. For instance, by selecting the parameter ε_2 large enough, the effect of the term $-1/(2\varepsilon_2)$ can be decreased or ignored in the inequality. Moreover, by selecting the parameter ε_3 large enough with respect to a ε_1 , the effect of the term $-\varepsilon_1/(2\varepsilon_3)$ can be decreased or ignored. Then, the inequality can be simplified and solved for two parameters k and ε_1 .

Remark 3: Based on Theorem 1, the obtained results are for all initial conditions. Hence, if the platoon is perturbed in finite time, the objective (3) can be satisfied eventually. Accordingly, the proposed control strategy has some flexibilities discussed below:

- It is possible for a vehicle to decrease its own speed and not to follow its preceding vehicle. In that condition, by increasing the distance of that vehicle from the preceding vehicle, the platoon will be split to two platoons. Then, under the proposed CACC strategy, the objective (3) will be satisfied for the two platoons of the vehicles. Moreover, if that vehicle again approaches the preceding platoon, it will rejoin the platoon. In general, under the proposed CACC strategy, if a vehicle approaches a vehicular platoon, it will join that platoon. Note that the proposed control strategy is independent to the number of the vehicles in the platoon.
- If a vehicle increases its speed over a finite time, it will be possible to change the steady state of the platoon and increase the agreed speed.

String stability is one of concerns in control of vehicular platoons. There are various criteria for string stability in vehicular platoons, defined based on the availability of a leader in the platoon [4]. However, some of them can also be redefined for leaderless platoons as well. In this regard, we follow the general definition of the $\mathcal{L}_p$ string stability given in [32]. The basis for the $\mathcal{L}_p$ string stability is the $\mathcal{L}_p$ -boundedness of the states of the platoon with respect to the input/disturbance of the leader. In leaderless platoons, since there is not a leader in the network, we redefine the $\mathcal{L}_p$ string stability in terms of the $\mathcal{L}_p$ -boundedness of the states of the platoon with respect to inputs/disturbances of all the vehicles as follows.

Definition 1: Let

$$y(t) = \begin{bmatrix} d_{\text{des},1}(t) - d_1(t) & \dots & d_{\text{des},N-1}(t) - d_{N-1}(t) \\ \sum_{j \in \mathcal{N}_1} a_{1j}(v_1(t) - v_j(t)) & \dots & \sum_{j \in \mathcal{N}_N} a_{Nj}(v_N(t) - v_j(t)) \\ s_1(t) & \dots & s_N(t) & \dot{s}_1(t) & \dots & \dot{s}_N(t) \end{bmatrix}^\top.$$

The leaderless vehicular platoon (1) under the control strategy (4)-(7) is $\mathcal{L}_p$ string stable if there exist class $\mathcal{K}$ functions $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$ such that for any initial state $y(0)$,

$$\|y(t)\|_{\mathcal{L}_p} \leq \alpha_1(\|u_d(t)\|_{\mathcal{L}_p}) + \alpha_2(\|y(0)\|), \quad (43)$$

where by defining $u_{di}(t) \in \mathbb{R}$ as the $\mathcal{L}_p$ -bounded input/disturbance of Vehicle i , $u_d(t)$ is as follows:

$$u_d(t) = [u_{d1}(t) \quad u_{d2}(t) \quad \dots \quad u_{dN}(t)]^\top.$$

By considering Definition 1, we show the $\mathcal{L}_\infty$ string stability of the leaderless vehicular platoon. In the presence of the $\mathcal{L}_\infty$ -bounded disturbance inputs $u_{di}(t)$ in (1), (13) becomes

$$\ddot{s}_i(t) + \sigma_i \dot{s}_i(t) + \alpha_i s_i(t) = u_{di}(t), \quad i \in \mathcal{V}. \quad (44)$$

Since (44) implies a Hurwitz linear system with $\mathcal{L}_\infty$ -bounded input, it follows that $s_i(t)$ and $\dot{s}_i(t)$ are $\mathcal{L}_\infty$ -bounded, with bounds depending on the norms of $s_i(0)$ and $\dot{s}_i(0)$ and the $\mathcal{L}_\infty$ norm of $u_{di}(t)$. Now, from (39) it follows that

$$V(t) \leq e^{-\rho t} V(0) + \int_0^t e^{-\rho(t-\tau)} \left(\frac{\varepsilon_2}{2} + \frac{\varepsilon_1 \varepsilon_3}{2} \right) \rho(\tau)^\top \rho(\tau) d\tau, \quad (45)$$

where $\rho(t)$ is $\mathcal{L}_\infty$ -bounded, whose $\mathcal{L}_\infty$ norm depends on the norms of $s_i(0)$ and $\dot{s}_i(0)$ and the $\mathcal{L}_\infty$ norm of $u_{di}(t)$, $i \in \mathcal{V}$. By considering (45) and after some manipulation one gets

$$V(t) \leq e^{-\rho t} V(0) + \frac{1}{\rho} \left(\frac{\varepsilon_2}{2} + \frac{\varepsilon_1 \varepsilon_3}{2} \right) \sup_{0 \leq \tau \leq t} (\rho(\tau)^\top \rho(\tau)). \quad (46)$$

Since the $\mathcal{L}_\infty$ norm of $\rho(t)$ depends on the norms of $s_i(0)$ and $\dot{s}_i(0)$ and the $\mathcal{L}_\infty$ norm of $u_{di}(t)$, $i \in \mathcal{V}$, by defining

$$\bar{y}(t) = \begin{bmatrix} e(t)^\top & \dot{\xi}_1(t)^\top \mathcal{L}''^\top & s(t)^\top & \dot{s}(t)^\top \end{bmatrix}^\top,$$

with

$$s(t) = [s_1(t) \quad s_2(t) \quad \dots \quad s_N(t)]^\top,$$

from (46) one gets there exist class $\mathcal{K}$ functions $\bar{\alpha}_1(\cdot)$ and $\bar{\alpha}_2(\cdot)$ such that

$$\|\bar{y}(t)\|_{\mathcal{L}_\infty} \leq \bar{\alpha}_1(\|u_d(t)\|_{\mathcal{L}_\infty}) + \bar{\alpha}_2(\|\bar{y}(0)\|). \quad (47)$$

Hence, by considering (17), (20), and (47) and the definitions of $\bar{y}(t)$ and $y(t)$, the $\mathcal{L}_\infty$ string stability of the platoon as (43) can be concluded as well.

V. EXTENSION TO VEHICLES WITH THIRD-ORDER NONLINEAR MODEL

Since the speed consensus problem with intervehicle distances regulation needs at least a second-order model, we considered a linear second-order model. Indeed, by employing such second-order models, it is assumed that the vehicles accelerations can be determined as the control inputs. However, the control strategy proposed in (4)-(7) provides a basic theory for extension to more complex scenarios. For instance, by increasing the order of the controller and modifying (4)-(6) (with more states measurement), it is possible to extend the results to platoons of vehicles with third-order and uncertain nonlinear models. In this regard, let us consider the following third-order model for the longitudinal dynamics of Vehicle i [4], [33], [34]:

$$\begin{aligned} \dot{d}_i(t) &= v_{i-1}(t) - v_i(t), \quad i \in \mathcal{V}' \\ \dot{v}_i(t) &= a_i(t), \quad i \in \mathcal{V} \\ \dot{a}_i(t) &= f_i(v_i, a_i) + \frac{1}{m_i \tau_i} u_i(t) + \Delta_i(v_i, a_i, u_i, t), \quad i \in \mathcal{V}, \end{aligned} \quad (48)$$

where $d_i(t)$ and $v_i(t)$ are the same as (1), $a_i(t)$ denotes the acceleration, m_i is the mass, τ_i is the actuators time constant, and $f_i(v_i, a_i)$ is a nonlinear function as follows:

$$f_i(v_i, a_i) = -\frac{2K_{ai}}{m_i} v_i(t) a_i(t) - \frac{1}{\tau_i} \left(a_i(t) + \frac{K_{ai}}{m_i} v_i(t)^2 + \frac{K_{mi}}{m_i} \right),$$

where K_{ai} and K_{mi} are the aerodynamic and mechanical drag coefficients, respectively. Moreover, $\Delta_i(v_i, a_i, u_i, t)$ models uncertainties in $f_i(v_i, a_i)$ and $1/(m_i \tau_i)$ as well as external disturbances (with known bounds). To extend the obtained results for (1) to (48), by increasing the order of the controller, (5) can be modified as

$$\begin{aligned} s_i(t) &= \xi_{0i}(t) - \xi_{1i}(t) \\ \dot{\xi}_{0i}(t) &= v_i(t) \\ \ddot{\xi}_{1i}(t) &= \nu_i(t), \end{aligned} \quad (49)$$

where

$$\begin{aligned} \nu_i(t) &= \frac{1}{\gamma h} \sum_{j \in \mathcal{N}_i} a_{ij} (\ddot{\xi}_{1j}(t) - \ddot{\xi}_{1i}(t)) + \\ &\quad \chi_i \left(\ddot{\xi}_{1i}(t) - \frac{1}{\gamma h} \sum_{j \in \mathcal{N}_i} a_{ij} (\ddot{\xi}_{1j}(t) - \ddot{\xi}_{1i}(t)) \right) \quad i = 1 \\ \nu_i(t) &= \frac{1}{\gamma h} \left(-k(h a_i(t) - \dot{d}_i(t)) + \sum_{j \in \mathcal{N}_i} a_{ij} (\ddot{\xi}_{1j}(t) - \ddot{\xi}_{1i}(t)) \right) \\ &\quad \chi_i \left(\ddot{\xi}_{1i}(t) - \frac{1}{\gamma h} \left(-k(r_i + h v_i(t) - d_i(t)) \right. \right. \\ &\quad \left. \left. + \sum_{j \in \mathcal{N}_i} a_{ij} (\ddot{\xi}_{1j}(t) - \ddot{\xi}_{1i}(t)) \right) \right) \quad i \in \mathcal{V}', \end{aligned} \quad (50)$$

where $\chi_i \in \mathbb{R}_{>0}$. By considering the definition of $s_i(t)$ in (49), from (48) one gets

$$\ddot{s}_i(t) = f_i(v_i, a_i) + \frac{1}{m_i \tau_i} u_i(t) + \Delta_i(v_i, a_i, u_i, t) - \nu_i(t). \quad (51)$$

The system (51) implies an input-affine nonlinear system, and control of such systems is widely studied in the literature. For instance, by considering the reference set point $(0, 0, 0)$ for the states $(s_i(t), \dot{s}_i(t), \ddot{s}_i(t))$, if we employ the robust control strategy introduced in [35], it is straightforward to design $u_i(t)$ such that $s_i(t)$, $\dot{s}_i(t)$, and $\ddot{s}_i(t)$ remain bounded and converge to zero (as such strategy is already available in the literature, it is not repeated here). Since $\xi_{1i}(t) = \nu_i(t)$, from (50) it follows that

$$\begin{aligned} \ddot{\xi}_{1i}(t) &= \frac{1}{\gamma h} \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\xi}_{1j}(t) - \dot{\xi}_{1i}(t)) + \omega_i(t), \quad i = 1 \\ \ddot{\xi}_{1i}(t) &= \frac{1}{\gamma h} \left(-k(r_i + h v_i(t) - d_i(t)) \right. \\ &\quad \left. + \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\xi}_{1j}(t) - \dot{\xi}_{1i}(t)) \right) + \omega_i(t), \quad i \in \mathcal{V}', \end{aligned}$$

where $\omega_i(t)$, $i \in \mathcal{V}$, is a bounded vanishing error. Then, following a procedure similar to the proof of Theorem 1 (after (23)), the objective (3) can be achieved.

VI. SIMULATION RESULTS

In this section, the performance of the proposed CACC strategy is investigated via numerical examples. Consider a vehicular platoon comprising six vehicle where the states $v_i(t)$ [m/s], $i \in \mathcal{V}$, and $d_i(t)$ [m], $i \in \mathcal{V}'$, are initialized as $v_1(0) = 22$, $(d_2(0), v_2(0)) = (9.5, 22.5)$, $(d_3(0), v_3(0)) = (5, 23)$, $(d_4(0), v_4(0)) = (6.7, 24)$, $(d_5(0), v_5(0)) = (7, 26)$, and $(d_6(0), v_6(0)) = (11.5, 28.5)$. We employ the proposed leaderless CACC such that the objective (3) is achieved. Accordingly, we set $h = 0.2s$ and $r_i = 3m$, $i \in \mathcal{V}'$, and the control gains are set to $k = 2$, $\gamma = 1$, $\sigma_i = 1$, and $\alpha_i = 1$, $i \in \mathcal{V}$. Now, we present the simulation results in two scenarios. In the first scenario, the vehicles reach consensus on a speed and simultaneously the intervehicle distances converge to the desired values. Then, in the second scenario, we repeat the first scenario, but after reaching the steady state, one of the vehicles tries to increase the platoon speed.

Scenario 1: Scenario 1 aims to achieve the objective (3) for the six vehicles. By employing the control strategy of Theorem 1 with the aforementioned parameters and initial condition, simulation results are depicted in Fig. 3, verifying that the objective (3) is achieved. According to Fig. 3, all the vehicles reach consensus upon their speeds, and all the spacing errors $\tilde{d}_i(t) = d_i(t) - d_{des,i}(t)$, $i \in \mathcal{V}'$, converge to zero, that is, the intervehicle distances converge to desired values defined in (2). In Fig. 3, the control inputs of the vehicles are also demonstrated.

Scenario 2: In Scenario 2, we repeat the simulation experiment of Scenario 1, while Vehicle 3 tries to affect the agreed speed and increase the speed of the platoon. To push the platoon to increase its speed, we assume that Vehicle 3 and subsequently Vehicle 2 decrease the distance from the preceding vehicle by decreasing $d_{des,i}(t)$. In this case, the simulation results of this scenario are depicted in Fig. 4. According to Fig. 4, the vehicles speeds converge to a common value until $t = 10s$. Then, decrease in the distances among the vehicles pushes the vehicles to increase their speeds (in order to compensate for the new spacing errors) such that the vehicles reach consensus on a larger speed. Moreover, all the spacing errors converge to zero until $t = 10s$. At that time, due to changes in the desired distances, there will be changes in the spacing errors. Then, again the new spacing errors asymptotically converge to zero. In Fig. 4, the control inputs of the vehicles are also demonstrated.

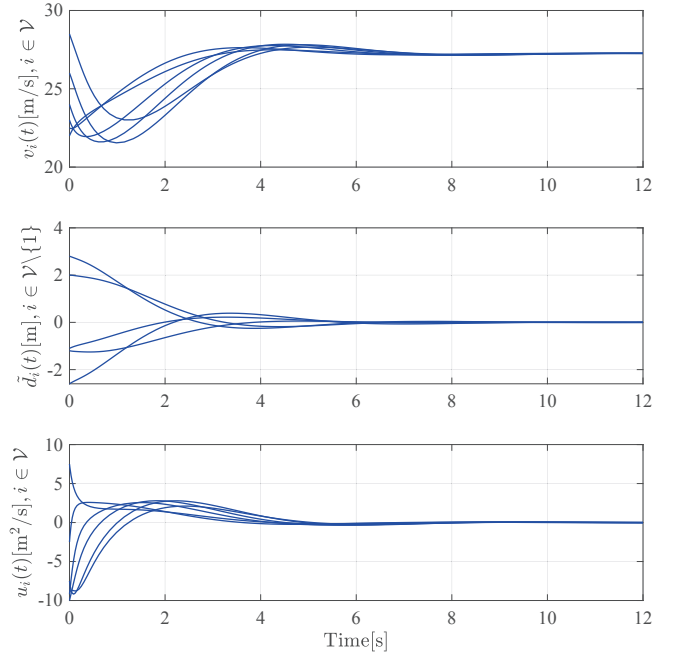


Fig. 3. Speeds, spacing errors, and control inputs of the vehicles in Scenario 1.

VII. CONCLUSION AND FUTURE WORK

Leaderless CACC in vehicular platoons was addressed in this paper. The proposed CACC strategy made it possible for vehicles to contribute toward the eventual speed of the platoon. Accordingly, we developed a control strategy such that the vehicles reached consensus upon their speeds and simultaneously they kept desired distances from their preceding vehicles determined based on the constant time-gap spacing policy. This study was a primary step in leaderless CACC and many problems remained open for future studies. For instance, extension of the results to networks with general communication topologies and networks with communication links failure, noise, and delays are few problems to be investigated as future work.

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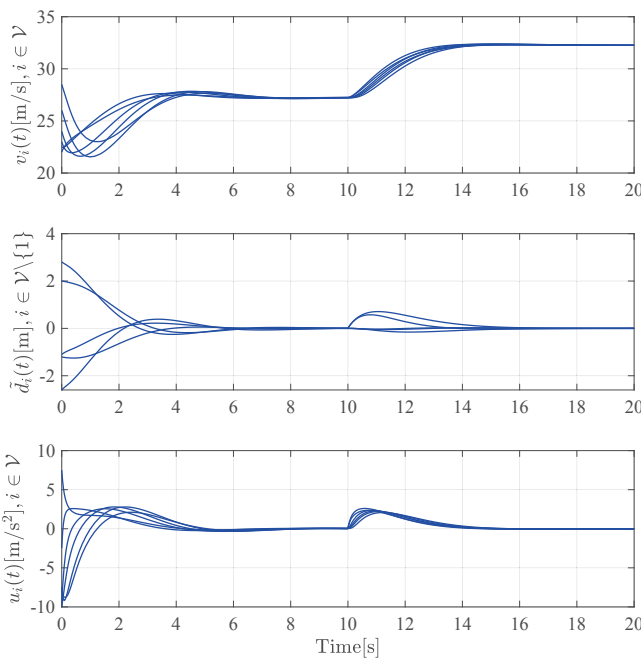


Fig. 4. Speeds, spacing errors, and control inputs of the vehicles in Scenario 2, where the platoon speed increases.

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