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Doctor of Philosophy

Quasi-steady quasi-homogenous
(QSQH) theory of the
relationship between large-scale
and small-scale motions in
near-wall turbulence

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Declaration of Originality

I hereby declare that the material presented in this thesis is the original work carried out by me in the Department of Aeronautics at Imperial College London. To the best of my knowledge, I have clearly referenced the contributions of others in both the text and the bibliography. Any contribution and advice related to the research from my supervisors, colleagues and other academic staffs with whom I have worked at Imperial College London or elsewhere, during the period of PhD studying, is kindly acknowledged.

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Abstract

The Quasi-Steady Quasi-Homogeneous (QSQH) theory describes the nature of the relationship between large-scale and small-scale structures in near-wall turbulent flows. In the present study, by introducing a notion of an ideal large-scale filter, the QSQH theory is stated in a rigorous form. A method is proposed for selecting a large-scale Fourier filter by multi-objective optimisation, with the first objective being a large correlation between large-scale fluctuations near the wall and in the layer at a certain finite distance from the wall, and the second objective being a small correlation between the small-scales in the same layers. The filter is demonstrated to give good results. Within the QSQH theory expansions in the amplitude of the large-scale fluctuations (assumed to be small) are found for a number of flow characteristics. Taking into account more terms of the expansion was demonstrated to improve the agreement between the theory and the results of direct numerical simulations available in the literature. It is proposed to use the QSQH theory for extrapolating the results of numerical and physical experiments from moderate to higher values of the Reynolds number. Two specific extrapolation methods based on the QSQH theory were developed, and their advantages and disadvantages explored. For near-wall region within about 100 wall units the accuracy of about 10% was achieved when extrapolating from the friction Reynolds number $Re_\tau = 950$ to $Re_\tau = 4200$. For using the technique in experiments, a special probe capable of measuring the large-scale motion, as it is defined in the QSQH theory, is required. Such a probe was designed and tested, so far in numerical simulations only. The main overall result of the present work is further validation of the QSQH theory and the development of the extrapolation method on its basis.

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Basic Nomenclature

There are many symbols and variables in this thesis. To help comprehending the meanings of those, a brief list of the commonly used superscript and subscript is provided here.

Superscript

*	Dimensional quantity
+	Non-dimensional quantity in classic wall units
'	Fluctuation of the quantity
no superscript	Non-dimensional quantity in QSQH wall units (see Chapter 3)

Subscript

L	Large-scale filtered quantity
S	Small-scale filtered quantity
c	Cut-off length
x	Streamwise component of a vector or another quantity related to the streamwise direction
z	Spanwise component of a vector or another quantity related to the spanwise direction
rms	Root-mean-square of the quantity

Chapter 1

Introduction

1.1 Motivation

First clear observations and specific studies of the relationships between the large-scale and small-scale motions in near-wall turbulence appeared in 1970s [14, 21, 95]. Recent reviews [61, 78, 107] provide a good introduction to the current state of the art. This topic grew in importance with continuing [70] accumulation of the evidence of the second peak (or plateau) in turbulence energy distribution at high Reynolds numbers Re . Due to the known limitations, both direct numerical simulations and wind-tunnel experiments on near-wall turbulence have to be done at the values of Re well below the values of Re characteristic of major aerospace and marine applications. This necessitates extrapolating the results to large Re . Prior to the discovery of the second peak and outer-inner coupling the extrapolation was thought to be justified by the universality of near-wall turbulence. Universality means that, if expressed in wall units that are units based on the mean friction at the wall, the statistical characteristics of the near-wall turbulence are virtually independent of Re . However, the coupling of large and small-scales means that the near-wall turbulence is not universal in the above sense, and the existence of the

second, outer, peak in turbulence energy, emerging as Re increases, means that even if the outer-inner interaction is not quantitatively large in direct numerical simulations and wind-tunnel experiments, it might be large or even dominant in the flows encountered in practice.

This situation is exemplified by the current state of research on turbulent skin friction drag reduction. The now-known methods of achieving significant energy savings require active open-loop control by in-plane wall motion [94], which is difficult to implement in practice. Developing alternative, equally effective but practically feasible methods of drag reduction requires theory yielding to intuitive understanding, and such theories recently appeared, making it possible to approximately predict certain organised structures (streaks) present in the flow in the regime with drag reduction [19] and the drag reduction itself [43, 86], and proposals of practically-feasible methods of drag reduction started to emerge [27]. However, all these results are verified only by comparisons with direct numerical simulations conducted at moderate values of Re . From the hypothesis of the universality of near-wall turbulence it follows that as Re increases the drag reduction should decrease slowly [75], but direct numerical simulations [56] appear to indicate a noticeably faster decrease. An explanation of this faster decrease based on the assumption of the quasi-steady nature of scale interaction was proposed [26], but no quantitative analysis was done. Overall, the state of the art in drug reduction studies required the issue of extrapolating to large Re to be clarified. This was the initial motivation for the research described in the present work and for focusing the attention on the near-wall region.

1.2 The origin and first confirmations of the QSQH theory

In a series of studies Marusic with colleagues [51, 58, 77, 81, 82] obtained results implying a particular way of extrapolation to large Re . The basic setup of their

work involves two probes measuring simultaneously the velocity at two points in the flow, the wall probe located more close to the wall than the outer probe. The signal obtained from each probe is then filtered, so that the velocity can be represented as a sum of a large-scale component and a small-scale component. Then the relationship

$$u'^+(y^+) = \alpha(y^+)u'_L^+(y_o^+) + \left(1 + \beta(y^+)u'_L^+(y_o^+)\right)u'_u^+(y^+), \quad (1.1)$$

is introduced, where u'^+ is the velocity fluctuation, u'_L^+ is its large-scale component, y^+ is the distance from the wall to the wall probe, and y_o^+ is the distance from the wall to the outer probe. The quantities expressed in wall units are marked with $+$ superscript, and all fluctuations, that is the deviations from the average, are marked with primes. The functions u'^+ , u'_L^+ , and u'_u^+ depend also on time and the other two coordinates omitted for brevity. The basic empirical observation is that it is possible to find such universal, that is Re-independent, functions $\alpha(y^+)$ and $\beta(y^+)$, and such dependence of y_o^+ on Re, that the statistical characteristics of the "universal" velocity field u'_u^+ defined by (1.1) are independent of Re.

If this observation is correct, it resolves the issue of extrapolating to large Re. For example, one can perform a direct numerical simulation of the flow in the drag-reducing regime at a moderate Re and determine the statistical properties of the corresponding u'_u^+ from (1.1). Then in a physical experiment at high Re the statistical properties of $u'_L^+(y_o^+)$ can be measured. This is easier than measuring the statistical properties of $u'^+(y^+)$, since the outer probe is located much farther away from the wall, and since the large-scale component has a lower frequency range. Then the statistical properties of $u'^+(y)$ at this high Re can be recovered from (1.1).

Formula (1.1) was found to be reasonably accurate, but more detailed studies [6, 51] revealed that the response of the near-wall turbulence to positive and negative fluctuations of the large-scale velocity u'_L^+ is not symmetric, contrary to (1.1).

Also, the empirical relation (1.1) was verified only against cases in which the full signal from both probes was available. Hence, extrapolating to Re so high that the signal from the near-wall probe cannot be obtained still required a leap of faith. A theoretical justification for (1.1) would increase the level of confidence in its validity and allow refining it.

As an answer to this demand, Chernyshenko *et al.* [28] proposed a theory that later became known as the Quasi-Steady-Quasi-Homogeneous (QSQH) theory of the effect of large-scale motions on near-wall turbulence. It was then further developed in the course of the present study, that resulted in this thesis, two papers [31, 127], and several conference presentations [29, 30, 122, 123, 124, 125, 126]. One more paper will be submitted soon.

The essence of the QSQH theory consists in the hypothesis that the effect of the large-scale motion on the small-scale motion is of a quasi-steady nature. The quasi-steadiness hypothesis is natural, since it can be expected that the characteristic time scale of the large-scale component of the velocity field is large as compared to the characteristic time scale of the small-scale component of the velocity field. Without stating it explicitly, various aspects of it were discussed on the basis of numerical [61] and experimental [51] results. The analysis of the phase shift between various properties of the large-scale and small-scale fluctuations, done in these works, is particularly illuminating. This analysis supports, at least approximately, the quasi-steady hypothesis only in the area close enough to the wall. For example, the frequency modulation of small-scales by large-scales, predicted by the quasi-steady hypothesis, is observed only for $y^+ < 100$ [51]. Fortunately, the drag reduction mechanism is concentrated in the area below 100 wall units [6, 56, 94]. Measuring the large-scale fluctuations at $y^+ = 100$ for the values of Re typical for aeronautical applications is a realistic task. Given the particular motivation of the present study, it is concentrated on an analysis of the near-wall region.

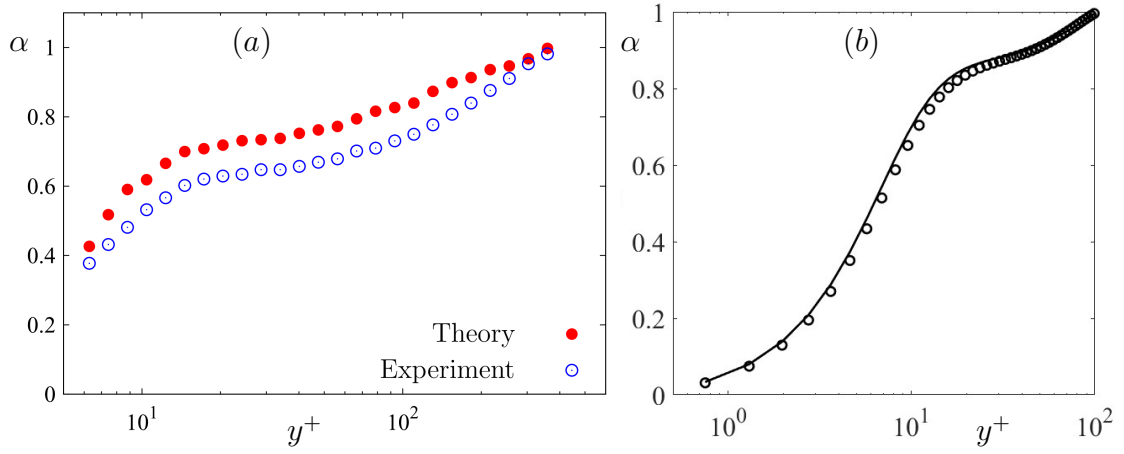


Figure 1.1: Coefficients α : (a) is the direct reproduction of the figure from [28]. ‘Theory’ is the QSQH result (1.3) and ‘Experiment’ is the data from [81]; (b) is the present work, the curve is (1.3), the symbols are derived from the data in a direct numerical simulation database at $\text{Re}_\tau = 4200$. The better agreement in (b) is due to a better large-scale filter developed in the present work.

The first comparisons supporting the QSQH theory were presented in the same paper [28] where the theory was first proposed. In this paper, the formula (1.1) was derived from the QSQH theory with an additional assumption that the amplitude of the large-scale fluctuations is small, so that linearization is possible. This allowed to obtain the first confirmations of the QSQH theory by comparing its predictions with the aggregate of experimental and numerical results represented by the empirical formula (1.1). For example, the superposition coefficient $\alpha(y, y_o)$ in (1.1) was introduced by [81] as

$$\alpha = \frac{\langle u'_L(y)u'_L(y_o) \rangle}{\langle u'^2_L(y_o) \rangle} \quad (1.2)$$

where u'_L is the large-scale-filtered velocity fluctuation, and $\langle \cdot \rangle$ denotes averaging. In [28] (see also Section 3.2.3 and (3.19)) it was shown that if the QSQH theory is valid then $\alpha(y, y_o)$ and the mean velocity profile $U(y)$ are related, namely

$$\alpha \approx \frac{\tilde{U}(y) + y d\tilde{U}/dy}{\tilde{U}(y_o) + y_o d\tilde{U}/dy_o} \quad (1.3)$$

The first comparison is presented in Figure 1.1(a). Several other comparisons were also made in [28], with about the same level of agreement. Since then, this level

was improved by improving the large-scale filter defining the large-scale motion, as it is indicated by Figure 1.1(b). More detail will be given in Chapter 5, but for completeness we indicate here that Figure 1.1(b) is a channel flow case, that $y_o^+ = 100$, and that for Figure 1.1(b) the large-scale motion is defined using the Fourier cut-off filter in streamwise (cut-off length $X_c^+ \approx 4000$) and spanwise (cut-off length $Z_c^+ \approx 450$) directions, with no cut-off in time. Improving the large-scale filter of the QSQH theory was the starting point of author's study, and is described in detail in Chapter 4.

Since the QSQH theory forms the foundation of the work described in this thesis, we will also continue to refer to more and more of its detail throughout the thesis.

1.3 The specific objectives and the structure of the thesis

In the context of the QSQH theory it is convenient to perform linearization with respect to small amplitude of the large-scale fluctuations, but this introduces an error. Quantifying this error is one of the goals of the present study. It is achieved by calculating the higher-order terms of the expansion in the amplitude of large-scale motions. In [28] the derivations were based on the intuitive idea that the large-scales are much larger than the small-scales, and were, in this respect, not fully rigorous. This in itself is acceptable when dealing with the quasi-steady hypothesis, which is only approximate. However, when derivations become cumbersome, as it is the case with the higher-order terms, relying on intuitive notions is difficult. Introducing a rigorous framework making derivations easier. This was achieved by formally postulating the necessary and sufficient properties of the filter. The second goal of the present study is to propose a method of selecting a filter most suitable for application with the theory. To this end we will consider a certain multi-objective optimisation problem. After a proper verification of the QSQH

theory using available numerical and experimental data, we will show how to use it to extrapolate statistical properties of fluid flows from moderate to high-Re, which is the final goal of this work.

The thesis is organised in the following way. In Chapter 2, we first review the history of wall-turbulence research, and then the previous studies on the large-scale motions and their interaction with the small-scale motions in near-wall turbulent flows. Chapter 3 begins with a formal statement of the two hypotheses of QSQH theory and its underlying assumptions described in Section 3.1 and continues with derivation of a number of relationships for various statistical properties of velocity field. Chapter 4 introduces the method and constraints for selecting the large-scale filter to be used with the QSQH theory. In Chapter 5, the QSQH theory is validated using the data from direct numerical simulations. In Section 5.2, for the first hypothesis in particular, the predictions of the so-called superposition and modulation coefficients derived in Section 3.2 are compared with the DNS data. The comparisons in this section are mostly concentrated on the results of the QSQH theory that can be verified with the data for a single value of Re. In Section 5.3, the second hypothesis is examined without specifying the large-scale filter. Further validation of this hypothesis by extrapolating flow statistics from moderate to high-Re is described in Chapter 6. Two methods of extrapolating are developed and compared in Sections 6.2 and 6.3. Examples of extrapolating by these methods are presented in Section 6.4. A rack of probes for measuring large-scale motions for QSQH theory is analysed in Section 6.6. The inclination of the wall-turbulence structures is not taken into account in the current version of QSQH theory. However, it does influence the accuracy of the predictions of the theory. Therefore, we performed investigations on this issue and report the results in Chapter 7. In Chapter 8, we discuss three critical points of QSQH theory and finally draw the conclusions. Some complicated derivations omitted in the main

text are detailed in the appendix.

1.4 Databases used in the study

Database	Actual Re_τ	Nominal Re_τ	Domain length $\times$ height $\times$ width	No of snapshots	Source
1	1015	1000	$4\pi h \times 2h \times 2\pi h$	2000	[7]
2	934	950	$8\pi h \times 2h \times 3\pi h$	20	[74]
3	2009	2000	$2\pi h \times 2h \times \pi h$	75	[74]
4	4179	4200	$2\pi h \times 2h \times \pi h$	40	[74]

Table 1.1: Table of the DNS channel flow databases used in the work

This study relied heavily on the publicly available data and the data provided to us. The parameters of the DNS channel flow databases used in our study are listed in Table 1.1. Also, we will use the flow statistics documented in some other databases in the later text, and their sources will be introduced by then.

1.5 Contributions of the author

During my PhD study, Prof. Chernyshenko and I had numerous, often lengthy, discussions, and the ideas born during those discussions cannot be attributed solely to one of us. My second supervisor Prof. Michael Leschziner often joined our discussions and gave his genuine and critical options. Some limited contributions were also made by MSc and undergraduate students who worked in our group. Therefore, all the achievements here should be regarded as an outcome of a group collaboration. To keep the thesis comprehensive and consistent, and as self-contained as is practical, the description of the entire study is included. Given the nature

of this thesis, it however appears to be appropriate to clarify in more detail which contributions belong largely to me. This is done below¹.

Note that I am the first author of [127], the second author of [31], and will be the first author of the paper on the specific extrapolation techniques, which is being prepared now.

In essence, my contribution can be described in the following way. The QSQH theory itself and, with rare exceptions, all the theoretical derivations on the basis of this theory were done by Prof. Chernyshenko. On the other hand, also with rare exceptions, I did all the data processing and algorithms development, and thus may claim a major contribution to the results of such processing and a significant contribution to the analysis of these results. In particular,

- My results demonstrated that the QSQH theory can give a very accurate prediction for the superposition coefficient in the well-known empirical formula describing scale interactions in near-wall turbulence.
- My results demonstrated that there are important situations when the QSQH theory gives quite different, and more accurate, results than the above-mentioned empirical formula or its equivalent linearised version of the QSQH theory.
- I performed the large-scale filter selection that made obtaining the above results possible.
- I developed the two practically feasible procedures of extrapolation from medium to high Re described in the thesis.
- I further confirmed the validity of the QSQH theory by performing such an extrapolation for two-point correlations.

¹The idea of including this section is inspired by the PhD thesis of Margit Vallikivi [115].

- The small-small correlation study was done entirely by me on my own initiative.
- I took a significant part in the analysis of the results and the supervision of the MSc and undergraduate students.

In more detail, with the references to the sections of the thesis and to the publications, my contributions are detailed below. The ‘calculations’ referred to below mostly consisted of a large volume of data processing.

Chapter 1: This chapter (and Chapter 2) mostly contain introductory material.

The text was written by me, with the use of elements of our joint papers with Profs. Chernyshenko and Leschziner.

Chapter 3: This chapter is largely based on a part of the paper [127], the text of which was written by me and edited by Profs. Chernyshenko and Leschziner. The original idea of the QSQH theory was proposed by Prof. Chernyshenko in [28], and its formalised statement in Section 3.1 was also done mostly by Prof. Chernyshenko. My contribution to Section 3.1 was limited to the discussion of the properties of the ideal large-scale filter. The majority of the functional relationships in Section 3.2 were derived by Prof. Chernyshenko and independently verified by me, and for a few of them this was reversed, so that a part of these relationships are a result of our joint work. Note that the idea of expansion in the amplitude of the large-scale fluctuations (subsection 3.2.2) was introduced by Prof. Chernyshenko in [28], but the actual results in [28] were limited to the main terms of the expansion, so that the higher-order terms derivation has my contribution, even though for some of them it was limited only to verification of the derivations.

Chapter 4: Section 4.1 was based on a part of the paper [31]. The introduction of the multi-objective optimisation approach was from the paper [127]. All the calculations and analysis in this chapter were conducted by me.

Chapter 5: Section 5.1 and a part of Section 5.2 were based on the paper [127], with all the calculations and analysis of the calculation results done by me. The idea of subsection 5.3.1 belongs to Prof. Chernyshenko. For this subsection I only produced the numerical data. Subsection 5.3.2 was the result of a long discussion lasted years between Prof. Chernyshenko and me. All other content of this chapter is the result of my own work.

Chapter 6: The idea of extrapolation to higher Re was implicit in the pioneering works of the Melbourne group [51, 58, 77, 81, 82], and was explicitly proposed by Prof. Chernyshenko in the context of the QSQH theory. Apart from this generic idea, all the calculations and the analysis in this chapter was conducted by me. The specific extrapolation methods were developed by me mostly, with only a limited advice from Prof. Chernyshenko. The idea of the probe for measuring large-scales was proposed by Prof. Chernyshenko. The probe optimisation studies were done by an MSc student Mr. Hamza Butt [23] and an UG student Mr. Mohammad Beit-Sadi [16], to whom I provided only advice and data during their studies. Some of their results are included in [29, 30] and [31], which I co-authored, but largely with different contributions. None of the results of [16] and [23] are claimed by me as contributing towards fulfilling the requirements for awarding a PhD degree. Subsection providing an explanation to some observations about the results of [16, 23] is my work.

This chapter, together with contents of Chapter 4 and Section 5.3, will be the basis of a paper to be submitted in the near future.

Chapter 7: The discussion of correlation reversal in Section 7.1 was a joint work of Prof. Chernyshenko and me. The other results presented in this chapter are my work.

Chapter 8: The conclusions were written by me.

Chapter 2

Literature review

In this chapter, a systematical review of the past studies in the area of near-wall turbulence will be presented. It will start with the introduction of the important findings that expanded the horizon of knowledge of the community of researchers working on near-wall turbulence. Then, two topics closely related to this work, namely, the large-scale motions and the interaction between the scales in the turbulent boundary layer, will be reviewed in detail. The literature is introduced mainly chronologically, and we will focus on the incompressible flow with zero-pressure gradient and smooth wall conditions.

2.1 A brief history of near-wall turbulence study

Based on the official figures [109], approximately 25% of the industry and commerce energy is consumed in fluid-flow related transportation, such as water pipes, aircraft, oil tanker, etc. Moreover, 25% of that energy is dissipated by the turbulent motions in the very near-wall region [62]. Therefore, wall-bounded turbulence, with its great importance in scientific research and engineering application, catches particular attention and interests from the researchers all over the world. How-

ever, compared to the other classical physics problems, the progress of near-wall turbulence research has been relatively slow. It is due to the difficulty in acquiring high-quality databases that contain all the possible information on turbulent motions, and also the lack of generally accepted model/theory of the highly complex phenomenon [62, 79]. Our study will be devoted mainly to theoretical research. Before any details of our work are introduced, let us go back to the beginning of the 20th century and embark on an over-a-century journey of research on near-wall turbulence .

In 1904, the great physicist Ludwig Prandtl published his groundbreaking study on the equations of fluid motions over solid surfaces [93]. In the paper he described the idea of the boundary layer and its importance to the drag and streamlining. After that, many studies followed this topic, especially for the turbulent boundary layer, which results from the transition to turbulence of the laminar flow at a sufficiently large Reynolds number, and which appears in the majority of the fluid applications. However, the next significant progress was made only two decades later. At the first international Congress for Applied Mechanics, Burgers first showed the measurements of the mean velocity profile in a turbulent boundary with hot-wire anemometry technique [22]. Several years later, Fage and Townend developed a novel visualised experimental technique to observe the turbulent trajectories in the square pipe flow [46]. They successfully obtained the distributions of the mean velocity and the maximum velocity fluctuations, that was calculated by the classic Reynolds decomposition [96], of all three directions. They also reported that the flow was both smooth and twisted in the region very close to the wall. This region is now called a viscous sublayer.

The next breakthrough was made in 1938, when Millikan proposed an expression for the mean velocity in the overlap region of the wall and outer flow [85]. The expression showed a logarithmic relation between the mean velocity and the

wall-normal distance, with both being normalised by friction velocity and kinematic viscosity. This relationship is now known as the famous ‘law of the wall’. Much later, Coles validated this expression by plotting several sets of experimental data at that time [33]. Furthermore, in the same paper he proposed his own ‘law of the wake’ functions to characterise the mean velocity of the outer flow region. In the same year of Millikan’s work, Taylor proposed a hypothesis to describe the exchangeable nature between the temporal and spatial characteristics of the turbulent motions with small intensity [110]. Since then, this hypothesis has been widely applied to the turbulence-related calculations and derivatives, even though many situations were found that contradict Taylor’s hypothesis, and various corrections were suggested [39, 71, 73, 120].

After the world war, 1950’s saw the blooming of the innovative studies and observations in the area of near-wall turbulence. Townsend in the University of Cambridge, and Klebanoff in National Bureau of Standards of United States, both measured the intensities of the turbulent velocity fluctuations and the turbulent kinetic energy by the hot-wire technique [66, 113]. The former one first sketched the eddy structure in the turbulent boundary layer. Theodorsen was perhaps the first person to consider the concept of near-wall vortex loops [111]. In his hypothesis, the momentum of near-wall turbulent motions is carried by those ‘horseshoes’ vortices of different scales to the higher regions. This ‘mind experiment’ was far ahead of the later experimental observations of the harping vortex motions. Corrsin and Kistler studied the boundary between the turbulent flow and potential flow [36]. They also discussed how the rotating turbulent motions were diffused into the irrotational flow. In this period, people also started to investigate the correlation between the turbulent motions. Favre *et al.* performed a two-point correlation experiment to obtain the space-time correlation maps of two-dimensional streamwise-cross (x - y) and cross-spanwise (y - z) planes [47]. Grants did similar

measurement to determine the correlation tensor for the velocity fluctuations of all three directions, then tried to draw the eddies that shaped those correlation curves [52].

In the last six decades, with the continuing increase of the public funding (see the statistics in the web pages <https://www.aaas.org/page/historical-trends-federal-rd> and <https://data.oecd.org/rd/gross-domestic-spending-on-r-d.htm>), more and more people have joined the research community. This resulted in the prosperity of studies of near-wall turbulence and led to an explosion of academic papers. Hence, starting from here we will only introduce those groundbreaking studies that created new research areas or reshaped other's understanding of the near-wall turbulence.

With the advance of the boundary layer visualisation techniques from the 1950s to 60s, many people observed the near-wall streaky motions with the high- and low-momentum regions appeared alternately. These streaks of low-momentum would suddenly raise up and go away from the wall. This process produced a significant amount of turbulent kinetic energy. This phenomenon was found in different types of wall-bounded flows [20, 34, 35, 48, 68, 97]. Particularly in [34], the authors developed a novel experiment to identify the motions of marked fluid particles. As a result, they further observed the dropping motion of those particles of higher momentum. They named the outward motion 'ejection event', and the inward motion 'sweep event'. The ejections were found to contribute most of the Reynolds shear stress in the turbulent boundary layer.

In 1970s, following the previous studies on the ejections and sweeps, Wallace *et al.* divided the streamwise and wall-normal velocity fluctuations regarding to their signs into four quadrants [117], namely Q1 ($+u, +v$), Q2 ($-u, +v$), Q3 ($-u, -v$) and Q4 ($+u, -v$). The Q2 and Q4 were roughly equivalent to the ejections and sweeps. They found that the Q4 sweeps contribute the most to the Reynolds shear

stress at the region of $y^+ < 12$. Blackwelder and Kaplan created so-called VITA criteria to detect the extreme energetic motion, which was named ‘bursts’ at the time [18]. They put a rake of hot-wire at the different wall-normal distances and these bursts were found existing vertically up to $y^+ = 100$. Also, Q2 and Q4 events were identified by this method at $y^+ = 15$. The downstream incline at a higher wall-normal distance of the burst was found by the following studies [8, 25]. At the same time, Brown and Thomas found the high correlation between the large-scale filtered velocity fluctuations at 0.25δ and the wall shear stress fluctuations [21], where δ is the thickness of the boundary layer. A phase relationship was reported between the small-scale signals that were removed through filtering and the large-scale signals. Recently, the phenomenon was further investigated at higher Re, and the conclusions with a more profound understanding of the coupling in the turbulent boundary layer were achieved. These state-of-the-art findings will be introduced in detail in Section 2.3.

Meanwhile, Townsend, in his book [114], proposed the ‘attached eddy hypothesis’. He described the shape of the attached eddy as a double cone, rolling in the cross-spanwise plane and propagating downstream. However, this model failed in predicting the correct value of the streamwise velocity component. In the beginning of next decades, inspired by Townsend’s idea and previous experimental observation of hairpin-like eddy [55], Perry and Chong developed their version of attached eddy model [90]. They described a collection of hierarchical Λ -shape self-similar eddies of different lengths inclined downstream in the turbulent boundary layer. The smallest length scale of the eddies is 100 wall units, and the largest length scale is the outer scale of the flow. The ratio of the length between two scales is fixed, so there is a limited number of hierarchy members. This model successfully predicted the log law of mean velocity profile and the streamwise and spanwise turbulence intensities. However, the discrete lengths of eddies raise the

issue of complex mathematical derivatives in calculations at the time. Therefore, a few years later, the authors developed a continuous method for the model, which significantly reduce the calculations complexity [91].

Until the early years of the 1980s, the majority of the studies of near-wall turbulence focused on statistics due to the single-point measurements in the experiments. A multi-dimensional flow field was always desired to study the structures of the turbulent boundary layer at the time. Large-eddy simulation method, which provided such data has been proposed [72, 106]. However, by their very nature these calculations neither resolved the small-scale motions and near-wall structures nor were very reliable due to the use of empirical sub-grid models. With the computing power increasing exponentially with time in agreement with the famous Moore’s law [102], simulating the fully-resolved wall-bounded turbulent motions in a computer became possible. From later 1980s to the end of the century, a series of DNS algorithms were developed and validated by the experimental results [88, 92]. Since then, the ‘virtual wind tunnel’ gradually became one of the most powerful and reliable research tools. Together with particle image velocimetry experiments (PIV), DNS provided abundance of information about the structure of the turbulent boundary layer and greatly accelerated the research progress.

The first direct numerical simulation (DNS) of the turbulent boundary layer was performed by Kim and Moin [64]. A turbulent channel flow of $Re_\tau = 180$ was simulated. They calculated various flow statistical properties and compared them with the experimental results. A reasonably good agreement was achieved. By utilising the advantage of DNS in flow visualisation, the authors further presented the three-dimensional shape of the hairpin-like vortex at different viewing angles by conditionally extracting the vorticity line in the Q2 ejections, and confirmed the assumption of the aforementioned attached eddy model [91]. The low momentum near-wall streaky motions were also visualised, and hence convinced the research

community that DNS is capable to simulate the near-wall turbulence. One year later, Spalart conducted another DNS calculation [108] of turbulent boundary layer. The periodic streamwise boundary conditions were first used. Similar to the previous DNS calculation, he compared the results with experiments [55] by visualizing the isolines of vorticity in the streamwise-spanwise plane. Important progress in DNS emerged very recently, when Wu and Moin conducted the DNS that contained all the spatial states from the Blasius profile, through transition, to fully developed turbulent boundary layer [118] at $Re_\theta \approx 900$. They reported that hairpin-like loops existed everywhere in both transition and fully developed states. A follow-up DNS at $Re_\theta \approx 1950$ was performed by the same authors [119], but the flow field was too complicated to confirm their previous finding. Another group did a similar DNS at higher Re , and such numerous vortices were not found [104].

While the numerical study of wall turbulence was becoming popular, experimental techniques were also developing. Adrian made a significant contribution to the development of planar particle image velocimetry (PIV) [2], in which a section of wind tunnel is illuminated by the laser from the top, and the scattered light of the flow particles is captured by the camera located at a distance in front of the section. This technique has been proved to be one of the most powerful tools to study the structure of the turbulent boundary layer [3]. Adrian and his colleagues conducted a series of validations by comparing their PIV results to the benchmark hot-wire measurements and DNS databases [4]. In this study, they observed that the hairpin vortices were raised by inclined high shear, aligned in ‘packets’ and propagated downstream. The combination of these phenomena was later called ‘Large-scale motions’ (LSM) [53]. LSM will be further discussed in Section 2.2. Recently, PIV has evolved into more advanced types, such as holographic particle tracking velocimetry (PTV) [105] and tomographic PIV [45], and revealed more details of the three-dimensional flow field. However, the Reynolds numbers

achievable in wind tunnels are limited. Fortunately, under certain conditions the surface layer of the atmospheric boundary layer will have the similar characteristics to the laboratory-based experimental turbulent boundary layer. Klewicki *et al.* conducted measurements of the atmospheric boundary layer in a wide flat terrain located southwest of Salt Lake City, Utah [67]. They successfully observed the low-momentum streaky motions. In a later paper [84], Metzger and Klewicki plotted the maximum value of the root-mean-square of the streamwise velocity fluctuations in wall units for various measurements including the atmospheric one. It was no doubt that the value increases with higher Re. They also drew the space-time correlation coefficients contours. The angle of inclination of maximum correlation was found to be Re-dependent above $y^+ = 10$. Although the structure information is limited, the statistics of very-high-Re datasets can still serve as a reference in the future studies of Reynolds number influence on the turbulent motions.

In the 1990s, there were also a few important theoretical studies of the near-wall turbulence. Barenblatt *et al.* proposed a Re-dependent power law instead of the classical logarithmic law of the wall [59]. This was a first attempt to add a Reynolds number effect to the widely accepted Re-independent mean velocity profile of a turbulent boundary layer. In their later paper [15], the authors showed good agreement between the mean profile curves of different types of the turbulent boundary layer and the curve of their power law function. To examine this idea, Zagarola and Smits conducted experiments within a wide range of Reynolds number in the well-known ‘superpipe’ facility [121]. The results indicated two overlap regions. The one for $50 < y^+ < 500$ followed the power law and the one for $500 < y^+ < 0.1r^+$ (pipe radius) followed the logarithmic law. They hypothesized that the power law was due to the partial similarity in the near-wall region. Saddoughi and Veeravalli experimented to validate the ‘isotropic small-scale’ that

can be inferred from the well-known Kolmogorov’s theory [69], at high Re in a large wind tunnel [100]. The compensated frequency spectra agreed with the Kolmogorov’s theory very well over the inertial subrange. Jeong *et al.* extracted so-called ‘quasi-streamwise vortices’, other than the hairpin-like vortices, by a criterion of the dominant rotation over strain motions [60]. They found that these vortices induce a spatial phase difference between the streamwise and wall-normal velocity fluctuations and are responsible for the Reynolds shear stress of ejections and sweeps.

The innovation of the scientific instruments brought new research tools, such as above introduced DNS and PIV, and thus pushed the boundaries of the wall turbulence research in two aspects. The first is the shift in focus from the statistics to the structures and motions, and the second is the transition from low to high Reynolds number. By following both routes, many new turbulent flow phenomena were observed. Among them, the most critical finding is perhaps the existence of the very large-scale motions, which was first observed by Kim and Adrian [65]. The studies related to this discovery will be discussed in more detail in Section 2.2. In general, in recent years the advances of the turbulent flow study relied more and more on the development of research tools, either numerical or experimental. However, observations do not automatically lead to the understanding of the phenomenon. The correlation between the structures and/or motions might be detected by merely processing the data, but the causation can only be revealed under the guide of particular models or theories. Therefore, more theoretical work is required to clarify the possible reasons behind colossal flow data sets, which is one of the primary goals of the present study.

2.2 Large-scale motions and Very-large-scale motions in a turbulent boundary layer

One of the most essential characteristics of turbulence is the multi-scale structures and motions. In a turbulent flow, the kinetic energy is transferred from the largest eddies that are comparable to the flow scale to the smallest eddies of Kolmogorov scale and then dissipated at the molecular level. This process is the well-known ‘energy cascade’, which was first introduced by Richardson [98] and then poetically described again by him [99]. In the turbulent boundary layer, the small-scale structures have been well studied and understood, as introduced in the last section. On the other extreme, since an early time researchers have observed large-scale structures, originally dubbed ‘bulges’, of the length between one and three times the boundary layer thickness δ . However, ‘bulges’ were not paid much attention at the time. Only recently, with the advance of new experimental, numerical and visualisation techniques, the importance of so-called Large-scale motions (LSM), together with Very-large-scale motions (VLSM), was recognised and emphasised. The commonly adopted criterion of distinguishing the LSM and VLSM is quite simple. Those eddies of the order of $2-3\delta$ in streamwise direction are the LSM. Any eddy longer than 3δ is the VLSM [13]. However, the nature of the two motions is entirely different. The LSM consists of a series of hairpin-like vortices coherently organised along the streamwise direction and inclined to the wall [13, 53]. Recent studies showed that detached eddies were contained in the LSM in the outer region of the turbulent boundary layer [11, 12]. Such detached eddies have to be regarded as another type of an LSM, which does not have relationships to the near-wall motions.

From the end of the last century, more and more studies have focused on the VLSM in all types of wall-bounded flows. Kim and Adrian were perhaps

the first to measure VLSM structures in the internal flow [65]. They reported that the length scale of those structures could reach 12-14 times of the pipe radius, which spread in streamwise direction over a distance 10 times longer than the length of the largest integral scale of the flow. Subsequently, various experimental and numerical studies also discovered VLSM structures of high- and low-speed regions that developed in the logarithmic layer in various wall-bounded flow types [3, 4, 12, 37, 53, 57, 76, 89, 112] and even in the natural atmospheric surface layer [76]. The nature of the VLSM is still not clearly understood. Kim and Adrian suggested that the VLSM was merely a visual effect of collections of LSM [65]. Alamo and Jiménez proposed a possible VLSM formation mechanism from a solution of the Orr–Sommerfeld–Squire equations [38]. Flores *et al.* conducted numerical studies by artificially removing the wall, and hence diminishing the near-wall viscous-induced structures [49, 50]. Interestingly, the VLSM still existed in the outer region of the turbulent boundary layer. Combining with the above mentioned LSM of detached eddies, it implies that the VLSM might consist of the outer layer LSM, and behave as a macro-scale event. From spectral analysis, Balakumar and Adrian reported that the LSM and VLSM contributed more than half of the turbulent kinetic energy and Reynolds shear stress in the pipe flow for $4000 < Re_\tau < 8000$ [13]. Hutchins and Marusic found that VLSM become more ‘energetic and prominent’ when the Reynolds number increases and form a secondary peak in the logarithmic layer, namely ‘outer-peak’, in the energy spectra density map [57]. In particular, some studies found that these VLSM could influence the near-wall flow, which effect will be reviewed in the next section.

Most recently, different views on the length of very-large-scale motions were expressed. As described in the aforementioned studies, the length of VLSM structures was believed to be over 10δ when they were first observed, and the following studies also confirmed this conclusion. Hutchins and Marusic [57] and Monty *et*

al. [89] even claimed that the length of VLSM structures could exceed 20δ in the turbulent flows of various Re_τ number. Duan and Jiménez conducted DNS channel flow calculations with different domain size. The largest domain had the stream-wise and spanwise sizes of $60\pi\delta \times 6\pi\delta$ [74]. From the two-dimensional premultiplied streamwise velocity spectra, they concluded that 20% of the kinetic energy was contained in the motions with the streamwise wavelength higher than 100δ . This is the latest lower bound of the possible length of VLSM. A different opinion was expressed by Dennis and Nickles [41, 42], who conducted a high-speed stereoscopic PIV experiment to obtain the velocity information of the whole three-dimensional flow field of a turbulent boundary layer. They visualised VLSM structures by constructing the iso-surfaces of stream-wise velocity fluctuation. Then, the length of each VLSM structures was defined as the length of each iso-surface in the stream-wise direction. Histograms of the length of VLSM structures showed that around 50% of the structures are below the length of 3δ and 97% of the structures are below the length of 7δ . There are indeed small gaps between two of these less-long structures in the streamwise direction. The authors further inferred that those very long structures observed before are made up of several shorter structures aligned with each other. Due to the limitations of two-dimensional PIV images and poor spatial resolution, previous studies did not identify them. Nevertheless, further studies are needed to confirm which of these findings are valid.

2.3 Interaction between near-wall and outer regions of turbulent boundary layer

In wall-bounded flow, such as channel flow and pipe flow, it is now well known that the outer region, which covers the logarithmic layer and the above region, influences the near-wall region, which covers the viscous and buffer layer. The very

first study in this area dates back to 1971. Rao, Narasimha and Narayanan [95] performed an experiment to examine the characteristics of a turbulent signal which has been filtered by a low-pass filter. They found that even in the viscous layer or at the wall, the mean period of this filtered signal could be better scaled with outer-flow rather than inner-flow properties. This result indicated that there is an interaction between the inner viscous layer and outer layer in a turbulent boundary layer. Several years later, Brown and Thomas [21] measured both velocity and wall shear stress by an array of hot wires and wall-shear-stress probes. Slow-varying components were observed in the wall shear stress. The signals of these components were very similar to those of large-scale motions of organised structures in the outer layer. In particular, they reported that a fluctuation with large amplitude and high frequency was generated at the maximum value of these components. It was the first time when the indication of amplitude and frequency modulation was recognised. Bandyopadhyay and Hussain [14] investigated the short-time correlation between small and large-scale motions in several types of shear flow such as boundary layers, mixing layers, plane wakes and the downstream-wise regions of plane and jets. They obtained the correlation between low-frequency and the envelope of the high-frequency component of stream-wise velocity and found that these two components are highly correlated in all types of flow. These early studies, although limited by experimental devices, established the foundation in this research area and inspired further studies.

Recently, with the advance of computing capability, direct numerical simulation (DNS), which provides more details in the near-wall viscous layer and three-dimensional information than the experiment, has been widely used to study turbulent boundary layer. Abe, Kawamura and Choi [1] performed a DNS investigation of a turbulent channel flow at three Reynolds numbers, Re_τ equal to 180, 395 and 640. By plotting the contours of filtered wall shear stress and high-fluctuation

regions at different wall normal distances, they confirmed the observations of previous studies that similar large-scale structures existed in both outer and inner layers, and even at the wall. Hutchins and Marusic [58] conducted an experiment to examine the influence of the outer-layer VLSM structures on the near-wall small-scale structures. Preliminary evidence of amplitude modulation was found when the velocity signals of two structures in the middle of the log-law region and near-wall were compared. Soon after, based on the earlier research, Mathis, Hutchins and Marusic [80] proposed a quantitative criterion to measure the degree of amplitude modulation of the near-wall small-scale motion by the outer-layer large-scale motion. This was defined by the correlation coefficient between the large-scale signal and the envelope of the small-scale signal at any chosen wall-normal distance. The former was obtained by a low-pass Fourier filter, and the latter was obtained by the Hilbert transform, which is a commonly-used tool to extract the envelope of a signal. The correlation was found quite strong in the inner layer, but weaker when the wall-normal distance increased. It became negative around the middle point of the logarithmic region where the outer peak resides. Mathis et al. also determined that the degree of amplitude modulation changes with the Reynolds number. The same authors [77, 81] further proposed an empirical mathematical model for predicting the statistical properties of the near-wall flow field when only the large-scale velocity signal at one point in the outer layer is known. Predictions of this model agree well with experiments and DNS data at both high and low Reynolds number flow. This model was extended to predict the statistical properties of the wall shear stress in a zero-pressure-gradient turbulent boundary layer [82].

Chernyshenko *et al.* [28], as we already mentioned, put forward a hypothesis that the influence of large scales on near-wall turbulence has a quasi-steady nature. Mathematically, this amounted to replacing the wall units based on time-averaged

skin friction with similar units based on large-scale-filtered skin friction. Physically, this hypothesis is noticeably different from the empirical model of [77, 81] in that it does not contain terms that can be interpreted as a superposition of large and small scales, which is present in the empirical model of [77, 81], but instead it includes a wall-normal scale modulation effect, which is not present explicitly in the empirical model of [77, 81]. Remarkably, however, it was demonstrated that in the case when the amplitude of large-scale fluctuation is small, the theory of [28] can be simplified by neglecting all but the main terms of the Taylor expansion and brought to the form of the empirical model of [77, 81], with explicit formulae expressing the empirical superposition and modulation coefficients in [77, 81] via the mean velocity profile and the root mean square velocity fluctuation profile. Notably, the superposition coefficient in [77, 81] turned out to be a linearised approximation of both amplitude and scale modulation effects in the quasi-steady theory of [28]. Later work, including the present study, and the work of other authors however demonstrated that there are situations when linearisation introduces a large error, and the full nonlinear quasi-steady theory has to be used instead of the empirical relation of [77, 81]. In particular, Agostini *et al.* [5, 6] separated the extreme positive and negative regions of large-scale velocity fluctuations, thus considering the case when large-scale fluctuations are not small. In the present study, part of which was published in [127], the deviation from the empirical model of [77, 81] was due to the higher-order terms of the small-amplitude expansion of the quasi-steady theory [28] (which were neglected when the empirical model of [77, 81] was derived from the quasi-steady theory) being multiplied by large coefficients for certain (but not every) statistical characteristic of the near-wall flow. In [127], Zhang and Chernyshenko formalised the idea of [28], and proposed to call it a quasi-steady quasi-homogeneous (QSQH) theory. They also found a way to improve the selection of the large-scale filter effectively defining the large-scales within the

QSQH theory, obtaining a much better agreement with DNS than the agreement obtained in [28]. Since [127] (and another our joint paper [31]) form a part of this thesis, without giving further detail we now move on to the relevant work of other authors.

Chung and McKeon [32] examined the VLSM structures in a channel flow by large-eddy simulation (LES) at $Re_\tau=2000$ and 20000 . They observed the same interaction between small and large-scale as Mathis *et al.* [80]. Guala, Metzger and McKeon [54] reached the same conclusion by measuring the stream-wise velocity fluctuation in the atmospheric surface layer. They reported that VLSM structures influence the inner region over the entire range of scales from large eddies to dissipative scale. Ganapathisubramani *et al.* [51] also confirmed the presence of amplitude modulation effect in high Reynolds number turbulent boundary layers. By counting the event of increasing and decreasing of small-scale velocity signal, the frequency modulation was identified. However, this effect was not observed in the region above $y^+ = 100$ in their study. Schlatter and Örlü [103] demonstrated that the skewness of the original velocity signal is related to the correlation coefficient in amplitude modulation. This was also confirmed by the authors of the model in the following study [83]. An extensive review of the most recent work on scale interaction is contained in [9].

Chapter 3

Quasi-steady quasi-homogeneous theory

This chapter describes the purely theoretical part of the study. In this PhD thesis, it is of intermediate nature between an introduction and a description of the research done (see Section 1.5 describing author's contribution to the study). The reader who do not wish to go through all mathematical derivations can read only subsection 3.1.1 for the fundamental concepts of the theory, and proceed directly to Chapter 4. Then, if necessary, the reader can refer back to this chapter for clarification of specific expressions.

3.1 Main hypotheses

3.1.1 Quasi-steadiness quasi-homogeneity and Reynolds-number universality hypotheses

The present analysis is restricted to flows statistically stationary in time and statistically homogeneous in the wall-parallel directions, such as a channel flow. Then the mean quantities depend only on the wall-normal coordinate. Let t^* denote time, y^* denote the wall-normal coordinate, and x^* and z^* denote the two remaining coordinates, x^* being in the direction of the mean skin friction. Let

$u^*(t^*, x^*, y^*, z^*)$ denote the x -component of the velocity. The superscript $*$ marks dimensional quantities. Within this work the attention is limited to the properties of $u^*(t^*, x^*, y^*, z^*)$.

According to the classical view on the universality of near-wall turbulence, near the wall and at sufficiently large Re all statistical characteristics of the velocity field are independent of Re provided that all the variables are in wall units. This is not true if the influence of the outer structures does not tend to zero as $Re \rightarrow \infty$. Hence, an alternative is required. We will now present in a refined, more rigorous, form the alternative assumption proposed in [28].

We assume that a large-scale filter is defined, that is an operator with certain properties to be described in subsection 3.1.2. Applied to any function depending on t^* , x^* , y^* , and z^* this operator gives another function of the same arguments, which we will call a large-scale component. The large-scale components will be denoted by a subscript L . In particular, τ_L^* denotes the large-scale component of the skin friction τ^* . Let

$$u_{\tau_L}^*(t^*, x^*, z^*) = \sqrt{\tau_L^*(t^*, x^*, z^*)/\rho^*}, \quad (3.1)$$

where ρ^* is the fluid density. Let

$$\tilde{u} \left(\frac{t^* u_{\tau_L}^{*2}}{\nu^*}, \frac{x^* u_{\tau_L}^*}{\nu^*}, \frac{y^* u_{\tau_L}^*}{\nu^*}, \frac{z^* u_{\tau_L}^*}{\nu^*} \right) = \frac{u^*(t^*, x^*, y^*, z^*)}{u_{\tau_L}^*(t^*, x^*, z^*)}. \quad (3.2)$$

Here all the quantities are total, not fluctuations, that is their mean values are not necessarily zero, and ν^* is the kinematic viscosity.

We replace the classical universality hypothesis with the hypothesis that near the wall at sufficiently high Re the statistical characteristics of $\tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$ for constant $\tilde{t}$, $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ are independent of Re , so that Re affects only the statistical properties of $u_{\tau_L}^*(t^*, x^*, z^*)$.

Note that if $u_{\tau_L}^*(t^*, x^*, z^*)$ is replaced with the mean friction velocity then (3.2) becomes the classical universality statement. Hence, the physical meaning of the

proposed hypothesis is that rescaling should be done with large-scale-filtered skin friction rather than the mean skin friction. Then $u_{\tau_L}^*(t^*, x^*, z^*)$ and $\tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$ are expected to be statistically independent because the variation of the large-scale components with t^* , x^* , and z^* is slow. Accordingly, this hypothesis will be called the quasi-steady quasi-homogeneous (QSQH) universality hypothesis, and the corresponding body of results that can be derived from it will be called the QSQH universality theory.

Non-dimensional variables based on ν^* and $\langle u_{\tau_L}^* \rangle$ will be used. This is similar to wall units, but with $\langle u_{\tau_L}^* \rangle$ instead of the dynamic velocity. In particular,

$$u = \frac{u^*}{\langle u_{\tau_L}^* \rangle}, \quad u_{\tau_L} = \frac{u_{\tau_L}^*}{\langle u_{\tau_L}^* \rangle}, \quad t = \frac{t^* \langle u_{\tau_L}^* \rangle^2}{\nu^*}, \quad (x, y, z) = (x^*, y^*, z^*) \frac{\langle u_{\tau_L}^* \rangle}{\nu^*}. \quad (3.3)$$

In these variables the QSQH hypothesis (3.2) can be written as

$$u(t, x, y, z) = u_{\tau_L}(t, x, z) \tilde{u}(tu_{\tau_L}^2, xu_{\tau_L}, yu_{\tau_L}, zu_{\tau_L}). \quad (3.4)$$

Since the statistical properties of $u_{\tau_L}(t, x, z)$ depend on Re and the statistical properties of $\tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$ do not depend on Re , these quantities have to be statistically independent. Note our use of mute variables, $\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}$, in this statement. In contrast, $u_{\tau_L}(t, x, z)$ and $\tilde{u}(tu_{\tau_L}^2, xu_{\tau_L}, yu_{\tau_L}, zu_{\tau_L})$ with constant t , x , y , and z are statistically dependent, the dependence entering via the arguments of $\tilde{u}$. This feature is the main source of interesting relationships to be derived. Statistical independence is understood here in the standard sense. In particular, the joint probability density function of the statistically-independent variables is equal to the product of their marginal density probability functions.

The QSQH universality hypothesis can be subdivided in two. The first hypothesis is the statistical independence of $u_{\tau_L}(t, x, z)$ and $\tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$ at each particular Re . This can be called the QSQH hypothesis per se. The second hypothesis is that the statistical properties of $\tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$ are independent of Re . This can be called the hypothesis of QSQH Re -universality.

For very large values of t , x , and z the scaled variables might respond strongly to the fluctuations of u_{τ_L} . Where this is important, the arguments of $\tilde{u}$ in (3.4) should, therefore, be replaced with a set of deformed arguments in the spirit of the classic asymptotic theories, that is the definition of t , x , and z given by (3.3) should be replaced with $\partial t/\partial t^* = \langle u_{\tau_L}^* \rangle^2/\nu^*$, $\partial x/\partial x^* = \partial z/\partial z^* = \langle u_{\tau_L}^* \rangle/\nu^*$. Also, both u and u_{τ_L} should actually be considered as vector quantities, and the hypothesis should be refined accordingly, but we will not go further into these issues.

3.1.2 The ideal large-scale filter

To complete the formal introduction of the QSQH hypothesis, the properties of the large-scale filter L should be defined. Applied to any function this filter generates another function of the same variables: $f_L(t, x, y, z) = L f(t, x, y, z)$. The following properties are postulated:

1. Linearity: $L(a f + b g) = a L f + b L g$ for any a , b , $f(t, x, y, z)$, and $g(t, x, y, z)$.
2. Invariance of averages; the averaged variables are large-scale: $L \langle f \rangle = \langle f \rangle$.
3. Projection property; the large-scale filter does not change an already large-scale-filtered function: $L L f = L f$.
4. Commuting with averaging: $\langle L f \rangle = L \langle f \rangle = \langle f \rangle$. The last of these two qualities is in fact Property 2, but we add it here because of the frequent use of these two properties together in our derivations, in particular in the form $\langle f \rangle = \langle L f \rangle \equiv \langle f_L \rangle$.
5. Scale-separation property: applying the large-scale filter to any function of t , x , y , z , and other arguments that are large-scale-filtered variables is equivalent to averaging over the homogeneous directions and/or time, t , x , and z with the other arguments held constant:

$$L f(t, x, y, z, L g_1, \dots, L g_n) = \langle f(t, x, y, z, \xi_1, \dots, \xi) \rangle_{\xi_1=L g_1, \dots, \xi_n=L g_n}$$

Finding a filter satisfying Properties 1-4 is possible. A cut-off in the Fourier space is an example. Property 5 is intimately related to the question of representing the flow field as a sum of large-scale and small-scale motions, that is to the question of defining rigorously what large-scale motions are, which is a big challenge. For functions having a two-scale structure, such as $f(t, \epsilon t)$ with $\epsilon \ll 1$, it is easy to verify that averaging over an intermediate interval, that is taking

$$L f(t, \epsilon t) = \frac{1}{2T} \int_{t-T}^{t+T} f(\tilde{t}, \epsilon \tilde{t}) d\tilde{t}, \quad 1 \ll T \ll 1/\epsilon$$

satisfies asymptotically all the required properties. This shows that the postulated properties of the filter are not self-contradictory. This also indicates that Property 5 implies that in a certain sense the filter is equivalent to averaging over interval or volume large as compared to small-scales but small as compared to large-scales. Turbulent flow fields, however, have multi-scale structure, and the question of finding a filter having all the required properties at least asymptotically as Re tends to infinity for turbulent flow fields remains open. One can, however, hope that there are filters that meet these requirements approximately.

The stated filter properties will be used for rigorous derivations, then a filter will be chosen and comparisons made.

3.2 Functional relationships derived from the hypotheses

Suppose now that a function $u(t, x, y, z)$ does have a form of (3.4), the operator L does have Properties 1-5, and $u_{\tau_L}(t, x, z)$ is a large-scale-filtered quantity with a unit average. Then, regardless of the physical nature of these variables and the filter, a number of statements can be rigorously proved. This is the main goal of the present section. It is beneficial to clearly distinguish between the physical

hypotheses of Section 3.1 and the rigorous mathematical theory of the present section, applicable to any functions and filters of the form discussed, since the majority of the rigorous results of this section apply not only to the longitudinal velocity but also to other variables.

We will also derive and discuss here a few results specific for the case when u is the longitudinal velocity, and u_{τ_L} is defined via large-scale-filtered skin friction as in (3.1), and we will use the terminology specific for this case, so as to keep it consistent throughout the thesis.

3.2.1 One-point statistics

The mean velocity is defined as $U(y) = \langle u(t, x, y, z) \rangle$. From (3.4) one then gets $U(y) = \langle u_{\tau_L} \tilde{u}(tu_{\tau_L}^2, xu_{\tau_L}, yu_{\tau_L}, zu_{\tau_L}) \rangle$. From now on the arguments of $u_{\tau_L}(t, x, z)$ (and of its fluctuation $u'_{\tau_L}(t, x, z)$) will be omitted for brevity.

Property 4 allows to insert the large-scale filter before averaging:

$$U(y) = \langle \mathcal{L} u_{\tau_L} \tilde{u}(tu_{\tau_L}^2, xu_{\tau_L}, yu_{\tau_L}, zu_{\tau_L}) \rangle.$$

Property 5 then gives

$$U(y) = \left\langle u_{\tau_L} \tilde{U}(yu_{\tau_L}) \right\rangle, \quad (3.5)$$

where $\tilde{U}(\tilde{y})$ is the average of $\tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$ over $\tilde{t}$, $\tilde{x}$, and $\tilde{z}$ with $\tilde{y}$ held constant: $\tilde{U}(\tilde{y}) = \langle \tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}) \rangle$. Note again the use of mute variables in the definition of $\tilde{U}$ but not in (3.5), where $\tilde{U}$ has a specific argument.

Note the particular use of Properties 4 and 5 in calculating the average: it is done in two steps, first performing large-scale filtering and then replacing the large-scale filtering with averaging with u_{τ_L} kept constant. This technique is widely used in the derivations in the present work.

Differentiating (3.5) with respect to y at $y = 0$ gives

$$\frac{dU}{dy}(0) = \langle u_{\tau_L}^2 \rangle \frac{d\tilde{U}}{dy}(0). \quad (3.6)$$

Since $\tau^*/\rho^* = \nu^* dU^*/dy^*$, after simple substitutions (3.6) gives that $d\tilde{U}/dy(0) = 1$. This is similar to the well-known identity $dU^+/dy^+(0) = 1$. Here we do use the physical meaning of $u_{\tau_L}^*$ and u^* . If, for example, u^* stood for the spanwise velocity component then $\tilde{U}$ would be zero and so would be its derivative.

Since the statistical properties of $\tilde{u}$ are independent of Re , $\tilde{U}$ is also independent of Re , thus representing the mean universal velocity profile. Since the statistical properties of u_{τ_L} depend on Re , (3.5) describes the dependence of the mean velocity on Re . If one assumes that in a certain range of y the universal velocity profile has a logarithmic behaviour $\tilde{U}(y) = \frac{1}{\varkappa} \log y + B$ then the mean velocity profile $U(y)$ will also have a logarithmic behaviour given by the formula

$$U(y) = \frac{1}{\varkappa} \log y + B + \frac{1}{\varkappa} \langle u_{\tau_L} \log u_{\tau_L} \rangle \quad (3.7)$$

(since $\langle u_{\tau_L} \rangle = 1$ by (3.3)). The logarithmic law is usually written in wall units, that is non-dimensional units based on the dynamic velocity $u_\tau^* = \sqrt{\langle \tau^* \rangle / \rho^*}$. The relationship between the wall units and the units (3.3) used in the present work is

$$u = u^+ / \langle u_{\tau_L}^+ \rangle, \quad t = t^+ \langle u_{\tau_L}^+ \rangle^2, \quad (x, y, z) = (x^+, y^+, z^+) \langle u_{\tau_L}^+ \rangle. \quad (3.8)$$

Note that

$$\langle u_{\tau_L}^+ \rangle = 1 / \sqrt{\langle u_{\tau_L}^2 \rangle} = 1 / \sqrt{1 + \langle u_{\tau_L}'^2 \rangle}, \quad (3.9)$$

where u_{τ_L}' is the fluctuation with zero mean so that $u_{\tau_L} = 1 + u_{\tau_L}'$. This follows from another useful relation, $\langle u_{\tau_L}^+{}^2 \rangle = 1$, which can be obtained from the definitions of $u_{\tau_L}^*$ and wall units. In wall units (3.7) becomes

$$U^+(y^+) = \frac{\langle u_{\tau_L}^+ \rangle}{\varkappa} \log y^+ + B + \frac{1}{\varkappa} \langle u_{\tau_L}^+ \log u_{\tau_L}^+ \rangle.$$

Assuming that $u_{\tau_L}' \ll 1$ allows further simplification. We will, however, only conclude that the above analysis gives a rational justification for the dependence of the logarithmic-law constants on Re and allows to relate them to the parameters of the large-scale fluctuations.

By Property 5 applying the large-scale filter to (3.4) gives

$$u_L(t, x, y, z) = u_{\tau_L} \tilde{U}(yu_{\tau_L}), \quad (3.10)$$

where $u_L(t, x, y, z)$ is the large-scale-filtered velocity. This includes the mean velocity, so that the large-scale fluctuation velocity is obtained by subtracting (3.5) from (3.10):

$$u'_L(t, x, y, z) = u_L(t, x, y, z) - U(y) = u_{\tau_L} \tilde{U}(yu_{\tau_L}) - \langle u_{\tau_L} \tilde{U}(yu_{\tau_L}) \rangle. \quad (3.11)$$

The squared root mean square of the large-scale-filtered velocity is therefore

$$u_{L,\text{rms}}^2(y) = \langle u_L'^2 \rangle = \langle u_{\tau_L}^2 \tilde{U}^2(yu_{\tau_L}) \rangle - \langle u_{\tau_L} \tilde{U}(yu_{\tau_L}) \rangle^2.$$

Subtracting (3.5) from (3.4) gives the total fluctuation velocity

$$u' = u - U = u_{\tau_L} \tilde{u}(tu_{\tau_L}^2, xu_{\tau_L}, yu_{\tau_L}, zu_{\tau_L}) - \langle u_{\tau_L} \tilde{U}(yu_{\tau_L}) \rangle,$$

and by definition the small-scale fluctuation velocity is $u'_S = u' - u'_L$. Obviously, $Lu'_S = 0$, from which and Property 5 it follows that

$$\langle u'_S u'_L \rangle = 0, \quad (3.12)$$

that is small-scale and large-scale fluctuation velocities do not correlate. We define the universal fluctuation velocity as $\tilde{u}'(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}) = \tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}) - \tilde{U}(\tilde{y})$, and define the square of the root mean square universal velocity fluctuation as $\tilde{u}_{\text{rms}}^2(\tilde{y}) = \langle \tilde{u}'^2(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}) \rangle$, where the averaging is done over the statistical ensemble. The averaged value depends on y but it is independent of $\tilde{t}$, $\tilde{x}$, and $\tilde{z}$ as these directions (including time) are considered to be statistically-homogeneous, so that the ensemble averaging is equivalent to averaging over any one or any combination of these variables.

The small-scale velocity fluctuation can now be expressed as

$$u'_S = u_{\tau_L} \tilde{u}'(tu_{\tau_L}^2, xu_{\tau_L}, yu_{\tau_L}, zu_{\tau_L}). \quad (3.13)$$

The squared root mean square of the small-scale fluctuating velocity is therefore $u_{S,\text{rms}}^2(y) = \langle u_{\tau_L}^2 \tilde{u}_{\text{rms}}^2(yu_{\tau_L}) \rangle$. Taking into account (3.12) gives that $u_{\text{rms}}^2(y) = u_{S,\text{rms}}^2(y) + u_{L,\text{rms}}^2(y)$.

Equations (3.11) and (3.13) also allow to express two-point correlations via the properties of the universal functions and u_{τ_L} . For example,

$$\langle u'_L(t, x, y, z) u'_L(t, x, y_o, z) \rangle = \left\langle u_{\tau_L}^2 \tilde{U}(yu_{\tau_L}) \tilde{U}(y_o u_{\tau_L}) \right\rangle - \left\langle u_{\tau_L} \tilde{U}(yu_{\tau_L}) \right\rangle \left\langle u_{\tau_L} \tilde{U}(y_o u_{\tau_L}) \right\rangle$$

and

$$\left\langle u'_L(t, x, y_o, z) u'^2_S(t, x, y_o, z) \right\rangle = \left\langle \left(u_{\tau_L} \tilde{U}(y_o) - \left\langle u_{\tau_L} \tilde{U}(y_o) \right\rangle \right) u_{\tau_L}^2 \tilde{u}_{\text{rms}}^2(yu_{\tau_L}) \right\rangle.$$

In these cases the averaged characteristics of the actual velocity field can be expressed via the properties of u_{τ_L} and the averaged characteristics of the universal field $\tilde{u}$, while the full knowledge of $\tilde{u}$ is not required. However, knowledge of $\tilde{U}$ and $\tilde{u}_{\text{rms}}$ is not always enough: for example, the formula for two-point correlation of the small-scale velocity $\langle u'_S(t, x, y, z) u'_S(t, x, y_o, z) \rangle$ involves the two-point correlation of $\tilde{u}$. The above derivations illustrate the general approach, while the particular formulae will be used in the following sections.

3.2.2 Expansion in the amplitude of large-scale fluctuations

Equation (3.4), expressing the QSQH hypothesis, and formulae that can be derived from it for arbitrary u_{τ_L} , as for example, those in subsection 3.2.1, are neat and universal. However, they are difficult to interpret and use, since determining the statistical characteristics of $\tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$ is not easy. Fortunately, in many flows the amplitude of the fluctuation of the large-scale component of the skin friction is not large. This allows significant simplifications.

Since $\langle u_{\tau_L} \rangle = 1$ by the non-dimensionalization, the fluctuation $u'_{\tau_L} = u_{\tau_L} - 1$.

Assuming $u'_{\tau_L} \ll 1$, one can use asymptotic expansions. Thus, (3.5) reduces to

$$U(y) = \tilde{U}(y) + \langle u'^2_{\tau_L} \rangle \frac{d}{dy} \left(\frac{y^2}{2} \frac{d\tilde{U}}{dy} \right) + \dots, \quad (3.14)$$

and (3.10) reduces to

$$u'_L = u'_{\tau_L} \left(\tilde{U}(y) + y \frac{d\tilde{U}}{dy} \right) + \left(u'^2_{\tau_L} - \langle u'^2_{\tau_L} \rangle \right) \frac{d}{dy} \left(\frac{y^2}{2} \frac{d\tilde{U}}{dy} \right) + \dots \quad (3.15)$$

The square of the root mean square velocity fluctuation can be expressed as

$$u^2_{\text{rms}}(y) = \tilde{u}^2_{\text{rms}}(y) + \langle u'^2_{\tau_L} \rangle \left[\left(\tilde{U}(y) + y \frac{d\tilde{U}}{dy} \right)^2 + \frac{1}{2} \frac{d^2 (y^2 \tilde{u}^2_{\text{rms}})}{dy^2} \right] + \dots \quad (3.16)$$

Detailed derivation process of the formulae for $U(y)$ and $u^2_{\text{rms}}(y)$ can be found in the appendix. Chernyshenko *et al.* [28] neglected all but the main terms. This leads to $\tilde{U}(y) \approx U(y)$ and $\tilde{u}^2_{\text{rms}}(y) \approx u^2_{\text{rms}}(y)$, thus allowing to find $\tilde{U}(y)$ and $\tilde{u}^2_{\text{rms}}(y)$. In fact, we will see that at least for the set of data and the filter we will use only the first of these approximate equalities is satisfied with good accuracy, while the second is not: the first term in the square bracket of (3.16) cannot be neglected. We will also see that neglecting the second-order terms in (3.15) is justified. This leads to

$$u'_L \approx u'_{\tau_L} \left(\tilde{U}(y) + y \frac{d\tilde{U}}{dy} \right). \quad (3.17)$$

These observations will affect our choice of the filter.

A similar expansion for the small-scale fluctuation (3.13) leads to

$$u'_S = (1 + u'_{\tau_L}) \left[\tilde{u}'(t, x, y, z) + u'_{\tau_L} \left(2t \frac{\partial \tilde{u}'}{\partial t} + x \frac{\partial \tilde{u}'}{\partial x} + y \frac{\partial \tilde{u}'}{\partial y} + z \frac{\partial \tilde{u}'}{\partial z} \right) + \dots \right],$$

and we face the difficulty of non-convergence at large values of t or spatial coordinates. For homogeneous directions and time this could be circumvented by selecting the origin of t , x , and z to be at the point of interest, at least as far as the multi-point statistics of interest have only y different at these points, as it is

the case in the present work. The problem with the wall-normal direction is more serious. Our comparisons will often be done up to and somewhat beyond $y = 100$. This is not a problem for large-scale or averaged quantities, because, for example, the term $y d\tilde{U}/dy$ is not large for large y , since $d\tilde{U}/dy$ is small where y is large. However, $\partial\tilde{u}'/\partial y$ might not be small at high y , and this might be even more true for the higher derivatives. For this reason for statistics involving the small-scale component it is safer first to obtain the general formula for it and only then perform the expansion, as this, for example, were done in deriving (3.16). Hence, in this work we will use the full expression (3.13) for the small-scale fluctuations even when an expansion in small u'_{τ_L} is considered.

3.2.3 Comparison with empirical relationships

Neglecting the higher-order terms, the QSQH hypothesis can be expressed [28] in the form very similar to the empirical formula (1.1). For this, (3.17) should be used at the location $y = y_o$ of the outer probe to express u'_{τ_L} via $u'^+_L(y_o^+)$: $u'_{\tau_L} = u'^+_L(y_o^+)/(\tilde{U}(y_o) + y_o d\tilde{U}/dy_o)$. Then a variable $u'_{u, QS}$ is introduced satisfying the equation

$$\tilde{u}(tu_{\tau_L}^2, xu_{\tau_L}, yu_{\tau_L}, zu_{\tau_L}) = \frac{\tilde{u}_{rms}(yu_{\tau_L})}{\tilde{u}_{rms}(y)} u'_{u, QS}. \quad (3.18)$$

The idea is that the amplitude of the fluctuations of $u'_{u, QS}$ will be unaffected by u'_{τ_L} , which is the expected property of u'_u in (1.1). These equations are then substituted into the expressions (3.13) and (3.17) for small- and large-scale fluctuations, which then are added together, $\tilde{u}_{rms}(yu_{\tau_L})$ is expanded in u'_{τ_L} , and only the linear terms are kept. From (3.8) and (3.9) it follows that up to the higher-order terms the units (3.3) used in the present work and the wall units coincide. This leads to the expression for the velocity fluctuation coinciding with (1.1), provided that

$$u'_{\text{u, QS}} = u'_{\text{u}},$$

$$\alpha(y) = \frac{\tilde{U}(y) + y \frac{d\tilde{U}}{dy}}{\tilde{U}(y_o) + y_o \frac{d\tilde{U}}{dy_o}}, \quad \text{and} \quad \beta(y) = \frac{1 + \frac{y}{\tilde{u}_{\text{rms}}(y)} \frac{d\tilde{u}_{\text{rms}}(y)}{dy}}{\tilde{U}(y_o) + y_o \frac{d\tilde{U}}{dy_o}}. \quad (3.19)$$

Assuming that $u'_{\text{u, QS}} = u'_{\text{u}}$ is not rigorous. Chernyshenko *et al.* [28] commented that u'_{u} in (1.1) is Re-independent, while $u'_{\text{u, QS}}$ is frequency- and scale-modulated. Frequency modulation was also considered by Ganapathisubramani *et al.* [51]. Another subtle difference lies in the definition of the amplitude of the fluctuations, which is usually defined via the Hilbert transform in the context of (1.1) and as $u_{\tau_L} \tilde{u}_{\text{rms}}(y u_{\tau_L})$ in (3.18), which definitions, while similar, are not exactly the same. The relationships between various indicators of the amplitude of fluctuation in the present context were discussed in a number of works [17, 44, 83, 103]. In spite of these subtleties, it is clear that the linearized QSQH hypothesis does provide a theoretical justification for the empirical model (1.1). A more complicated empirical model proposed in [6] is nonlinear, and if it can also be explained by QSQH hypothesis, this will be more complicated.

Chapter 4

Large-scale filter

The motivation of investigating the possibility to use the QSQH hypothesis for developing means of extrapolating the results of direct numerical simulations of flows with drag reduction to higher Re led us to choose first the set of data containing the drag-reduction case, described in detail in [7] and provided to us by the authors of that paper. We used only the part of database describing the reference flow with no drag reduction, which is the canonical channel flow at $Re_\tau = 1000$ in a computational domain of the length $\times$ height $\times$ width $= 4\pi h \times 2h \times 2\pi h$, with the duration of about $T^+ = 2600$, and containing 2000 time snapshots. This duration is somewhat short for certain tasks: therefore, we used two approximately equal halves of it separately, and compared the results, to make sure that the averaging error is sufficiently small as not to affect the conclusions of this work. Figure 4.1 gives a comparison of the mean velocity profile and the root-mean-square fluctuation obtained for the two parts of this database with the data available in the literature. The discrepancy gives an idea about the magnitude of the error of this database. Due to the insufficient wall-normal resolution of this database, we will also use the DNS channel flow databases of $Re_\tau = 934$, 2009 and 4179 from [74] to perform the calculations that require the wall-normal derivatives. However, the

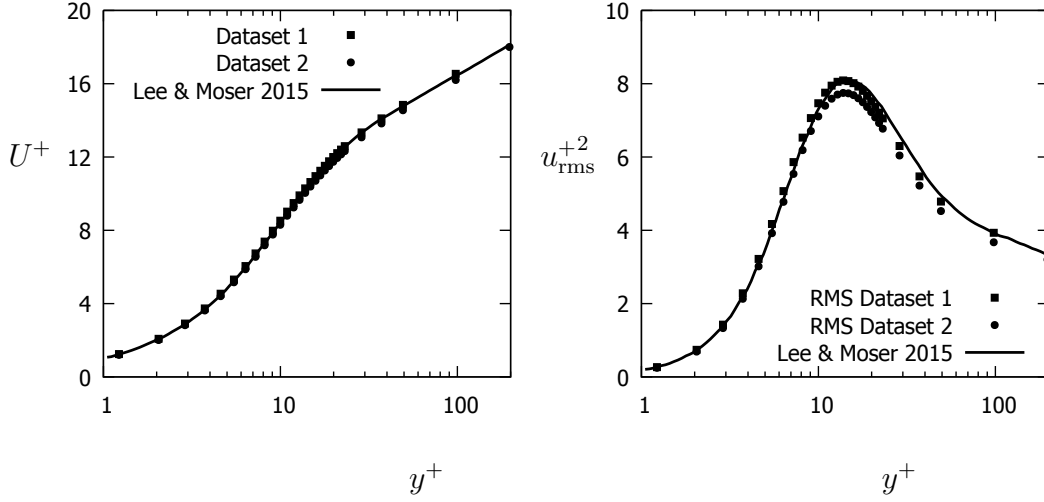


Figure 4.1: Mean velocity U^+ and root-mean-square fluctuations u_{rms}^{+2} as a function of wall distance: the points are obtained from the $Re_\tau = 1000$ database, and the curves are from [70].

numbers of time snapshots for these databases are minimal, which prevent most of the temporal calculations, such as time filtering, being available. The parameters of four DNS databases were listed in Table 1.1.

4.1 Why the ideal properties cannot be fully satisfied

The QSQH theory postulates that the large-scale filter has specific five properties. If such a filter is impossible, the entire theory would be self-contradictory, but an example of such a filter has already been given in subsection 3.1.2, confirming the self-consistency of the theory. Four of these properties, namely, linearity, invariance of averages (the large-scale filter applied to an averaged quantity does not change it), the projection property (the large-scale filter does not change an already large-scale-filtered variable), and commuting with averaging (the average of large-scale-filtered variable equals the average of the unfiltered variable) are

easier to understand, and easier to satisfy. However, the fifth property, namely scale-separation property, can be satisfied only by rather special filters. The fifth property is formulated in subsection 3.1.2 in the following way: “Applying the large-scale filter to any function of t , x , y , z , and other arguments that are large-scale-filtered variables is equivalent to averaging over the homogeneous directions and/or time t , x and z with the other arguments held constant. For the mathematical derivations in Chapter 3, this property is used formally: where convenient, the large-scale-filtered function is replaced with its average exactly as stated. It is not known whether a filter exactly satisfying all the five properties and at the same time useful for the study of the effect of large-scale motion on the near-wall turbulence actually exists. In practice, the end user of the theory needs to select a filter satisfying this property at least approximately, and for this an intuitive understanding of this property is desirable. This can be helped by two examples.

The first example is an infinite time averaging used as a large-scale filter. In this case verifying the five properties is trivial. However, in this case the large-scale quantities do not fluctuate at all, $u'_{\tau_L} = 0$; and the QSQH hypothesis reduces to the classical universality hypothesis, which is known to be inaccurate and which the QSQH theory is supposed to improve. A finite-time averaging over a sliding window, the size of which is much smaller than the characteristic time scale of the large-scale motions, can be better from the viewpoint of satisfying the QSQH hypothesis, but it does not exactly satisfy, for example, the projection property.

The second example is a Fourier cut-off filter. The cut-offs could be selected in such a way that the fifth property would be satisfied if the actual velocity distribution had an infinitely large (say, tending to infinity as $Re \rightarrow \infty$) spectral gap between the large and small-scales. Note also that the main QSQH hypothesis (3.4) also implies such a gap, in the sense that for it to be precisely accurate, the large-scales should be much larger than the small-scales, and there should be no

intermediate scales at all. It is equivalent also to averaging over a sliding window the size of which is much greater than the largest timescale of the small-scale motions and at the same time much smaller than the smallest time scale of the large-scale motions. Instead of time scales, spatial scales would also be acceptable here. These requirements on the relation of scales might be considered as one of the most intuitively obvious ways of satisfying the fifth property of the ideal filter. To satisfy these requirements, the large-scales should be much larger than the small-scales. The ratio of largest scales to smallest scales in turbulent flows increases as Re increases, which means that the QSQH theory can be expected to give better results for higher Re . However, for actual turbulence, a Fourier cut-off filter also cannot exactly satisfy all the requirements, because the turbulence spectrum is continuous, that is the largest of the small-scales coincide with the smallest of the large-scales. Therefore, it seems highly likely that a filter exactly satisfying all these requirements and also ensuring that the QSQH hypothesis is exactly accurate does not exist. Hence, in specific applications selecting the filter has to involve a compromise, which implies a multi-objective optimisation. It is also a constrained optimisation, since not every filter imaginable can be applied to every case in question. For example, if the only data that is available is a single-point time series then a spanwise-direction Fourier cut-off filter cannot be applied, while the time Fourier cut-off filter can.

4.2 An optimised filter

The spectral filter suggested by Hutchins and Marusic [58] cuts off the Fourier components with the streamwise wave-lengths λ_x^+ that are than the channel half-width. The cut-off length is thus between the wave-length of inner and outer spectral peaks, which correspond to the dominant small and large-scales of near-wall turbulence. Also, a Fourier filter has the required Properties 1-4 of an ideal

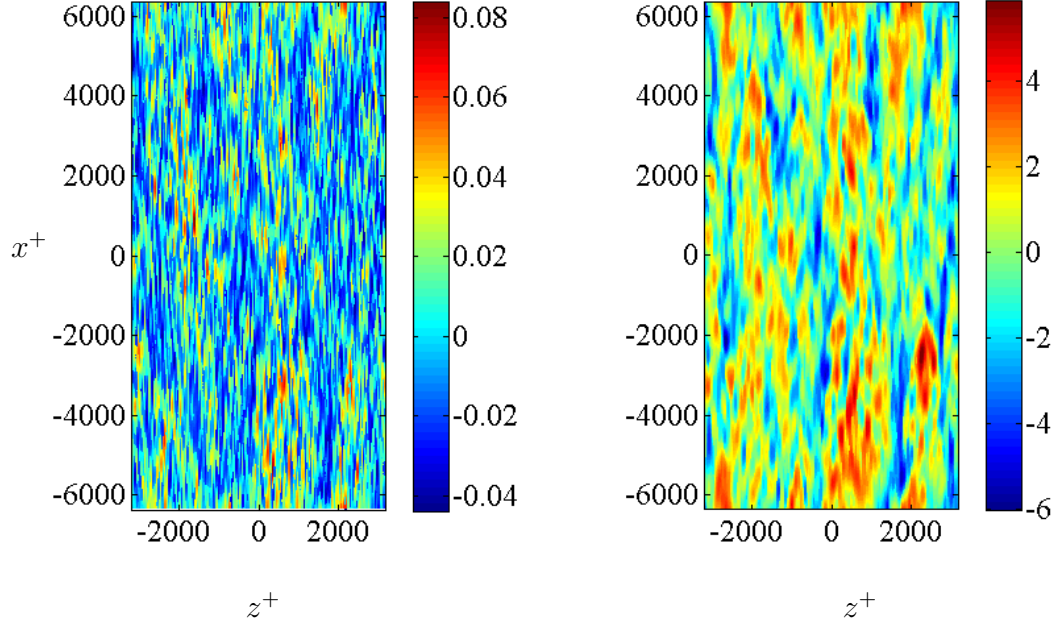


Figure 4.2: Large-scale velocity fluctuations obtained by applying the Hutchins and Marusic [58] filter at the layers of $y^+ = 0$ (left) and $y^+ = 100$ (right) for the $Re_\tau = 1000$ database.

filter. Hutchins and Marusic suggested using this filter at $Re_\tau > 1700$ because the large and small-scale components are not fully separated in low- Re_τ flows. Comparing the predictions of the QSQH theory with empirical data for $\alpha(y^+)$ and $\beta(y^+)$ in the empirical formula (1.1) that were obtained by using this filter, gave a reasonable, but not perfect, agreement [28]. We performed the same comparisons for our first database ($Re_\tau = 1000$), but the agreement with the QSQH theory was less good. To further examine the properties of this filter we selected two wall-parallel layers. The first layer was very close to the wall (at the nearest grid layer, represented below as $y^+ = 0$), and the second layer was at $y^+ = 100$. Figure 4.2 shows the instantaneous large-scale fluctuation of u given by this filter at $y^+ = 0$ and $y^+ = 100$. According to (3.17) the large-scale-filtered velocity at different y

should differ by a factor independent of t , x , and z . This is clearly not the case in Figure 4.2. The details of these pictures reveal the reasons for this: 1) There is no filtering in the spanwise direction and, hence, it does not remove the small-scale structures in this direction. 2) The cutoff length is not large enough compared to the length of near-wall streaks, and hence it does not remove them. For high-Re databases, such a filter might work, but to apply the QSQH theory for the case in question (that is at $Re_\tau = 1000$), a different filter is needed. Fortunately, since we have access to a database of a direct numerical simulation containing the entire velocity field, this turned out to be possible.

4.2.1 The multi-objective optimisation approach

The available numerical data allowed to add cut-off Fourier filtering in wall-parallel directions to cut-off Fourier filtering in time. This requires defining three thresholds. We define the large scale filter with several cut-offs as such a filter that passes through only those Fourier modes that has all the wavelengths greater than the corresponding cut-offs. For any cut-offs, a Fourier filter satisfies the first four requirements stated in subsection 3.1.2. Property 5 can be satisfied only approximately. As a quantitative measure of how well it is satisfied, we selected the correlation coefficient $r_{LL} = \langle u'_L|_{y^+=0^+} \cdot u'_L|_{y^+=y_o^+} \rangle / \sqrt{\langle u'^2_L \rangle \langle u'^2_L \rangle}$ between the large-scale field very close to the wall and the large-scale field at the "outer" layer at $y^+ = y_o^+ = 100$, since we know that it is a sensitive measure of the filter quality. From (3.17) it follows that this correlation coefficient should be close to 1, provided that Property 5 is satisfied and that $\langle u'^2_{\tau_L} \rangle$ is small enough. We observed that by increasing the threshold levels it is possible to make the correlation coefficient to be very close to 1, however, increasing the thresholds decreases the energy of the large-scale fluctuations, thus seemingly making the results less physically significant. As we will see later when considering extrapolation, the actually important

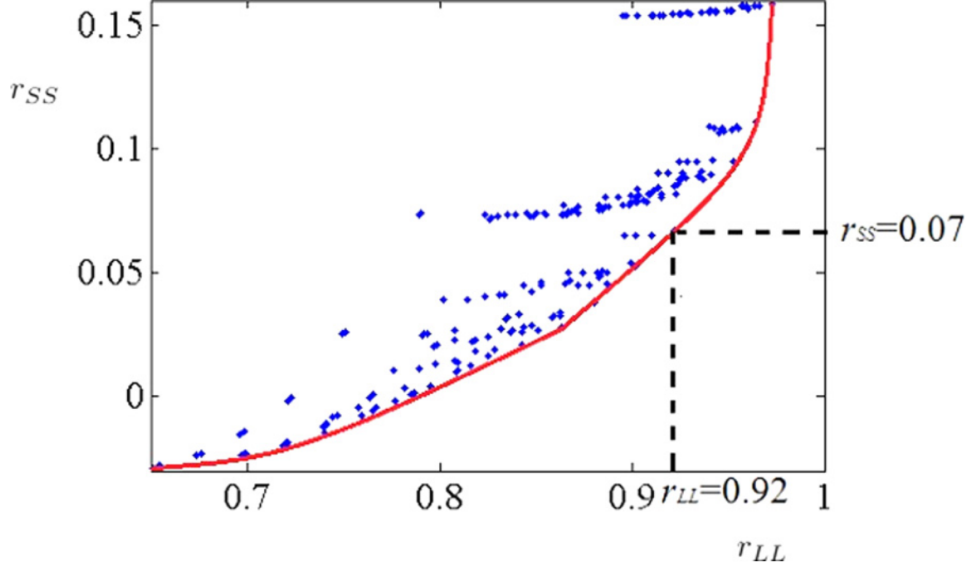


Figure 4.3: Pareto front of large-large and small-small correlation coefficients, where each point represents a combination of cut-off frequency and wave-numbers. Data from the $Re_\tau = 1000$ database.

requirement should be that the Reynolds-number effects are in a certain sense constrained to large scales, which imposes a limit to how large the threshold can be. We will not, however, discuss this in more detail now. Another obvious effect of increasing the thresholds is an increase of the correlation between the small-scale component at the wall and the small-scale component at the outer layer, $r_{SS} = \langle u'_S|_{y^+=0^+} \cdot u'_S|_{y^+=y_o^+} \rangle / \sqrt{\langle u'^2_S \rangle \langle u'^2_S \rangle}$. The absence of correlation between the small-scale motions at the points separated by large distance is naturally expected. Hence, we sought a compromise between r_{LL} being as close to 1 as possible and r_{SS} being small. In other words the ‘good’ filters correspond to the points more close to the bottom-right corner of the plot.

Selecting a particular combination of the cut-offs in time and two wall-parallel directions fully specifies the filter and allows to calculate r_{LL} and r_{SS} , which is

then plotted as a point in the r_{LL} - r_{SS} plane. This is repeated for a considerable number of the cut-off combinations, thus obtaining a cluster of points, and the Pareto front based on these points is shown in Figure 4.3. Any combination of r_{LL} and r_{SS} above and to the left of this Pareto front could be obtained with at least one Fourier cut-off filter, while no filter could give the combination of r_{LL} and r_{SS} to the right and below the Pareto front. Thus, the Pareto front is a set of optimal points. The choice of the particular point on the Pareto front, and hence of the filter, has to be based on additional considerations. Along the Pareto front, the values of large-scale to large-scale correlation vary between 0.65 and 0.97 and small-scale to small-scale correlation vary between -0.03 and 0.15 . We selected the point shown in Figure 4.3 as a reasonable compromise. At this point the frequency and wave-number cut-offs in Fourier-transform space correspond to the time scale $T_c^+ = 260$, the longitudinal length $X_c^+ = 1000\pi$, and the spanwise length $Z_c^+ = 125\pi$.

Applying the new filter leads to a noticeable improvement, as shown in Figure 4.4. The thin streaks that appeared in Figure 4.2 are not observed any more. The size, shape and position of the large-scale structures at the near-wall layer and the outer layer are close.

For our filter, $\langle u'_{\tau_L}{}^2 \rangle = 0.004364$, which means that (3.17) is a good approximation of (3.15). From (3.17) it follows that $r_{u_L u_{\tau_L}}(y) = \langle u'_L u'_{\tau_L} \rangle / \sqrt{\langle u'_L{}^2 \rangle \langle u'_{\tau_L}{}^2 \rangle} \approx 1$. Figure 4.5 shows that the discrepancy is within 10%. This level of accuracy is acceptable for studying physical mechanisms of the flow. However, it should be admitted that the choice of the current large-scale filter is still somewhat arbitrary. Therefore we will also examine the influence of the filter on the accuracy of QSQH predictions in the next chapter, where we will show results for another filter corresponding to the point on the Pareto front at $r_{LL} = 0.78$. Additional possible constraints on selecting the cut-off lengths will be discussed in Section 4.3.

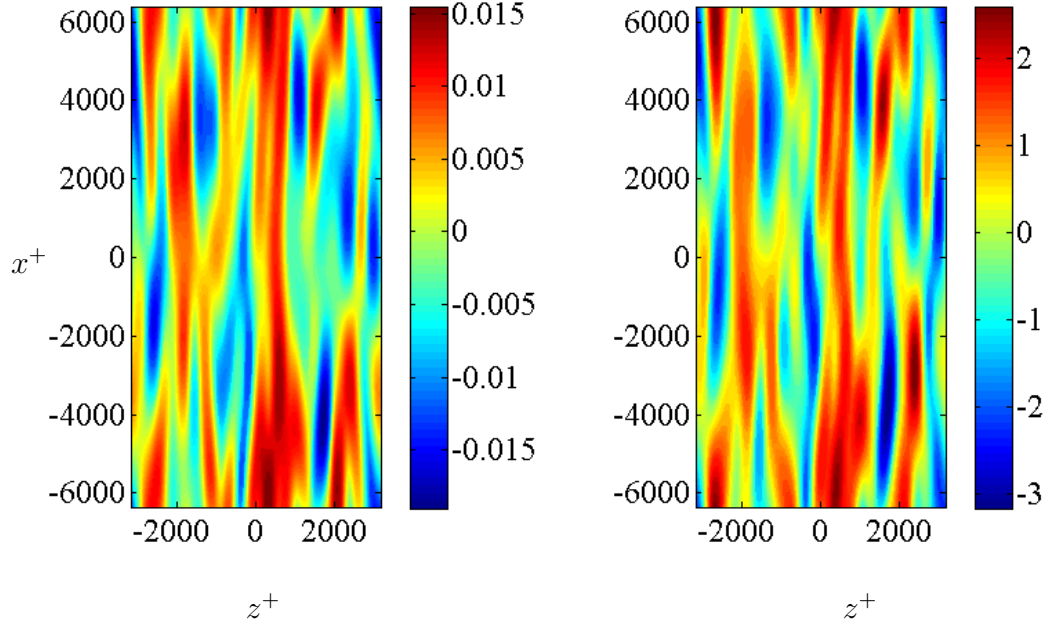


Figure 4.4: Large-scale velocity fluctuations obtained by the new filter at the layers of $y^+ = 0$ (left) and $y^+ = 100$ (right).

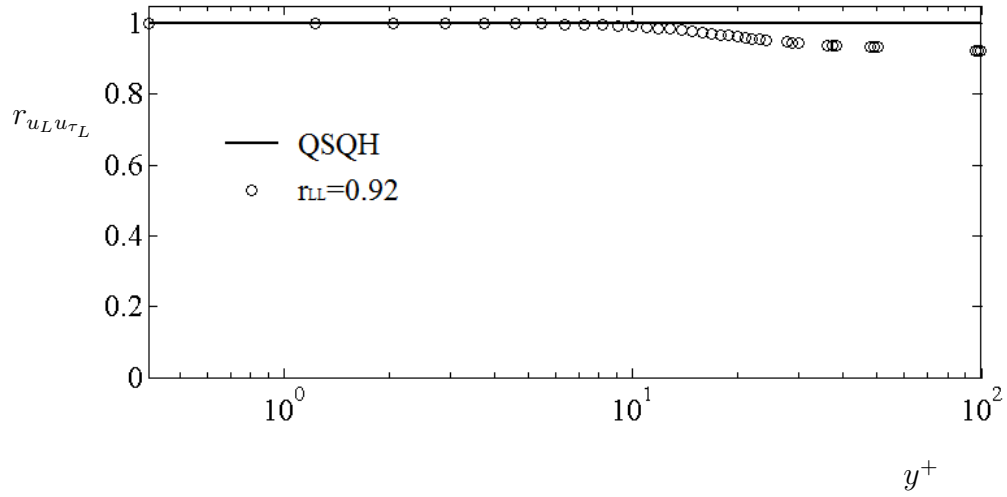


Figure 4.5: Comparison of $r_{u_L u_{\tau_L}}$ as given by the theory (line) and the numerical data of $r_{LL} = 0.92$ (circle).

It is also appropriate to mention that for $y^+ > 100$ the quality of the agreement deteriorates, but further details have to be postponed until later, see, in particular, Figure 5.4.

4.2.2 Reynolds number effect on the Pareto front

In this section we will concentrate on the effect of the Reynolds number on the filter selection and properties. To this end we will use the databases of the DNS channel flow of $Re_\tau = 950$, $Re_\tau = 2000$ and $Re_\tau = 4200$ [74]. The $Re_\tau = 950$ database contains 20 time snapshots of the computational domain of the size $\text{length} \times \text{height} \times \text{width} = 8\pi h \times 2h \times 3\pi h$, $Re_\tau = 2000$ database has 75 snapshots in $2\pi h \times 2h \times \pi h$ domain, and $Re_\tau = 4200$ database has 40 snapshots in $2\pi h \times 2h \times \pi h$ domain. Because in these databases the number of time snapshots is limited, time filtering can not be performed properly here. For this reason we regenerated the Pareto front by filtering in streamwise and spanwise directions. Assuming that the Taylor's hypothesis applies, time filtering should be redundant when streamwise filtering is performed. However, it is certainly was not the case when we tried to check this. We calculated the Pareto fronts with time filtering and streamwise filtering only and found that the difference between the two was quite large. More details of the difference between filtering in different directions will be given in the next subsection. Here we focus on the Re effect on the Pareto front, which provides information on the degree of separation between the large and small-scale motions.

Figure 4.6 shows the Pareto fronts for the current databases of $Re_\tau = 950$, $Re_\tau = 2000$ and $Re_\tau = 4200$. The outer probe is located at $y^+ = 100$ for all three databases. It is apparent that the Pareto front shifts towards the bottom-right corner of the plot when Re_τ increases. Therefore, for higher Re_τ better filters become available. The movement implies that at higher Re_τ large-scale and

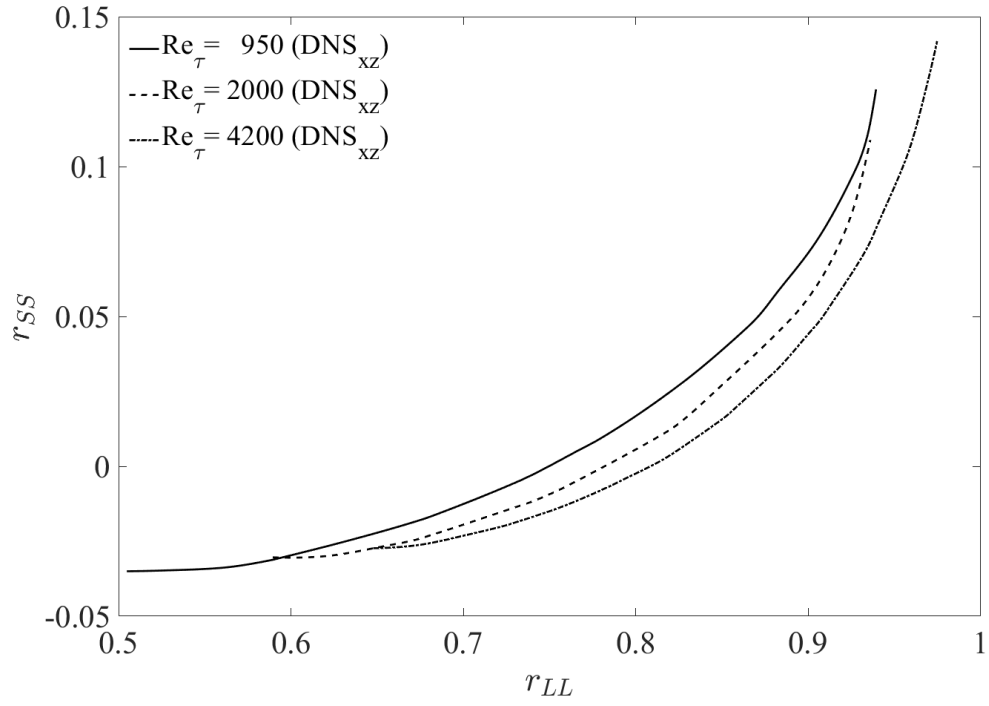


Figure 4.6: Pareto fronts of large-large and small-small correlation coefficients for the database of $Re_\tau = 950$, $Re_\tau = 2000$ and $Re_\tau = 4200$ [74], filtering in spanwise and streamwise directions.

small-scale components of the near-wall turbulent flow can be better separated by the Fourier cut-off filter, thus allowing more accurate predictions by QSQH theory. The result also conform with the observations that the outer peak will move farther away from the inner peak in the spectral diagram as Re increases.

4.2.3 Filtering in different directions

The better results obtained in the last two subsections for the case when spatial cut-offs were used instead of a time cut-off suggest that the relative significance of different directions for large-scale filtering should be investigated. Naturally, optimising the Fourier cut-off filter over all available directions, that is time, streamwise, and spanwise coordinates, would give the best results. However, it is not always possible to filter in all temporal and spatial directions. Most of the experiments will obtain the time series, which can only be filtered in time, while some of the DNS databases are not well time-resolved, and can only be filtered in streamwise and spanwise directions. Thus, it naturally raises the question of applicability to the QSQH theory with various databases. Here, by using the Pareto front of large-large and small-small correlation coefficients as a tool, a systematic investigation of the influence of the filtering directions on the performance of the filter is conducted.

Figure 4.7 shows the comparisons between the Pareto fronts of single-direction filtering for various Re_τ and the triple-direction filtering for $Re_\tau = 1000$. First, by comparing those Pareto fronts of the same cut-off direction, the conclusion of the last subsection that higher Re leads to better filter is further confirmed. Then, by comparing the four Pareto fronts of $Re_\tau = 1000$, the importance of the filtering directions in detecting the large-scale motions is revealed. It turns out that streamwise-only-cut-off filter is better than the time-cut-off filter, which in turn is better than the spanwise-only-cut-off filter. Experimental data usually

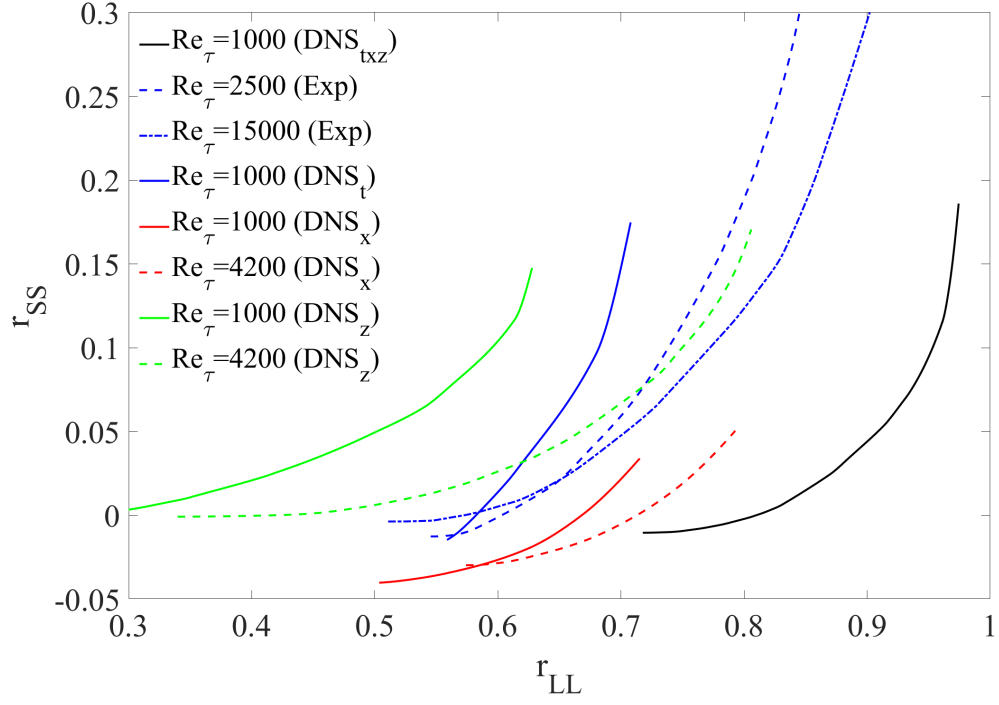


Figure 4.7: Comparisons of the Pareto fronts of large-large and small-small correlation coefficients with single-direction cut-off for various Re_τ . The black curve represents the Pareto front for triple-direction cut-off at $Re_\tau = 1000$. The blue curves represent the Pareto fronts of time cut-off. The red curves represent the Pareto fronts of streamwise cut-off. The green curves represent the Pareto fronts of spanwise cut-off.

contain only time series, but, as we can see from the figure, this is not enough to properly filter the large-scale motion, even at $Re_\tau = 15000$. Assuming that the Taylor hypothesis is approximately valid for large-scale motions, cut-offs in time and the streamwise directions should be equivalent. However, the comparison between the Pareto fronts of streamwise filtering and time filtering do not support the above statement. In our calculations, we assumed that the convection velocities of the large-scale motions are the same across the regions, so the same time cut-off lengths were applied to the two layers. Obviously, with the cut-off moving to a smaller length, this assumption will gradually become invalid. It might explain the observed difference. Also, the recent studies [87] and [120] suggested that the convection velocity should be measured locally to accurately transform between the temporal and spatial fluctuations. Further investigation is required to find how to properly compare temporal and streamwise filtering in the present context. In the present study we will continue using the current time filtering method.

Figure 4.8 shows the comparison between the Pareto fronts of double-direction filtering for various Re_τ and also the triple-direction filtering for $Re_\tau = 1000$. It can be concluded that none of the double-direction filter can be as good as the triple-direction filter. Time-streamwise filtering has the worst performance among the three, while the time-spanwise and streamwise-spanwise filtering being approximately the same and quite close to the triple-direction filtering. Hence, it is the loss of the cut-off in the spanwise direction that leads to a rather substantial deterioration of the quality of time-only filter, although for uni-cut-off filters spanwise is the least important filtering direction. The importance of the spanwise direction for adequately detecting large-scale motion was also noticed in [63], where a complicated experimental and data-processing technique was proposed. Another possible solution to detect in an experiment the large-scale motions most compatible with the QSQH theory will be described in Section 6.6.

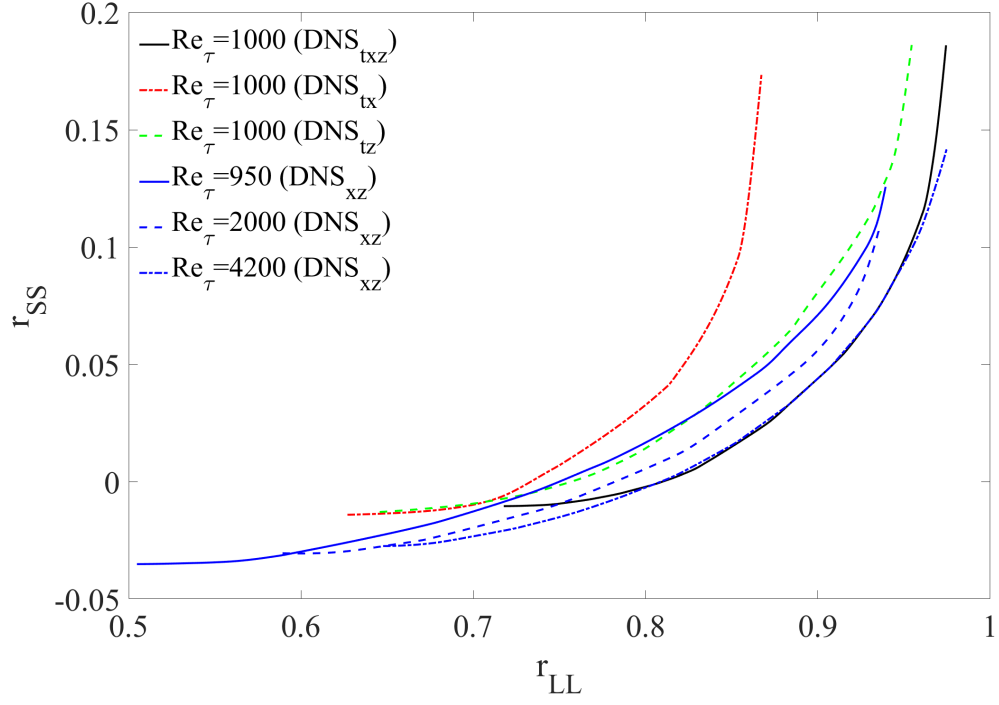


Figure 4.8: Comparisons of the Pareto fronts of large-large and small-small correlation coefficients with double-direction cut-offs for various Re_τ . The black curve represents the Pareto front of all triple-direction cut-offs at $Re_\tau = 1000$. The blue curves represent the Pareto fronts of streamwise and spanwise cut-offs. The red curve represents the Pareto front of time and streamwise cut-offs. The green curve represents the Pareto front of time and spanwise cut-offs.

Based on above analysis, we can provide the following guidelines for using Fourier filtering to extract large-scale turbulent motions: a) to obtain the representation of large-scale motions in the near-wall turbulence conforming best to the QSQH theory, filtering in the temporal and both wall-parallel spatial directions should be performed; b) any single-direction filtering is not recommended; c) time-spanwise and streamwise-spanwise filtering can be a substitute when triple-direction filtering is not available.

It is interesting to note that for a certain range of the cut-offs, the value of r_{SS} is negative. By examining the Pareto fronts of single-direction filtering in Figure 4.7, only the value of r_{SS} of spanwise filtering gradually approaches to zero without becoming negative anywhere. This suggests that the cause of the negative correlation is related to the motions in streamwise direction only. We conducted further studies to determine the possible reasons behind it, and attributed this behaviour to the time lag between the large and small-scales of the two layers. Since the negative r_{SS} correlation does not affect the selection of the filter, we will not further discuss this issue here, and return to it later in Section 7.2.

4.3 Kinetic energy of the large-scale fluctuations

In the last section the selection of the cut-off lengths of the large-scale filter was reduced by multi-objective optimisation to the points along a Pareto front. However, the selection of the point on the Pareto front still remains somewhat arbitrary. In this section, we will discuss how to further reduce the range of selection via additional constraints. Based on our understanding of the filtering, it should be clarified first that, in our opinion, there are no unique optimal cut-offs, and that there are no universal objectives which would provide a method of selecting such unique optimal cut-offs. Instead, the appropriate cut-offs should be determined on case-by-case basis, depending on the particular research objectives and flow

properties. Prior to discussing this further, we will however investigate first an alternative choice of the parameters of a two-parametric Pareto front.

Selecting r_{LL} and r_{SS} as the parameters of the Pareto front was justified by the desire to apply in the future the QSQH theory to studying the drug reduction problem. For validating the QSQH theory and using it as a tool to extrapolate statistical properties of the flow to higher Re , r_{SS} might be of less interest. On the other hand, the fluctuation amplitude of the large-scale motions plays a vital role both in validating the QSQH theory and in the mechanism of extrapolating flow statistics. Indeed, if this amplitude is zero, the QSQH theory reduces to the classical universality hypothesis. The fluctuation amplitude of the large-scale motions depends on the filter and, therefore, the filter has to be selected in such a way that this amplitude is large enough. The relationships obtained by expansion of the QSQH theory assuming that the amplitude of the large-scale fluctuations is small imply that the kinetic energy of large scales at the distance from the wall that represents interest can serve as the appropriate measure of the amplitude of the large-scale fluctuations. For example, the first term in the square brackets in (3.16) corresponds exactly to the kinetic energy of large scales, as it can be seen from (3.17). Equation (3.16) describes the behaviour of the root mean square velocity and, therefore, if this is the quantity of interest at, for example, $y^+ = 100$ the reasonable objectives of the optimisation can be r_{LL} and u_{Lrms}^2/u_{rms}^2 , with both being aimed to be large. For the current databases of $Re_\tau = 950$ and $Re_\tau = 4200$, the Pareto fronts of such objectives are shown in Figure 4.9.

These u_{Lrms}^2/u_{rms}^2 vs r_{LL} Pareto fronts can be compared with the proportion of the kinetic energy of the large-scale motions calculated along the r_{SS} vs r_{LL} Pareto fronts we calculated earlier, shown in Figure 4.10. Naturally, the u_{Lrms}^2/u_{rms}^2 vs r_{LL} Pareto fronts, by its very nature, lie higher than the corresponding u_{Lrms}^2/u_{rms}^2 vs r_{LL} curves in Figure 4.10, but, interestingly, not so much higher. This means

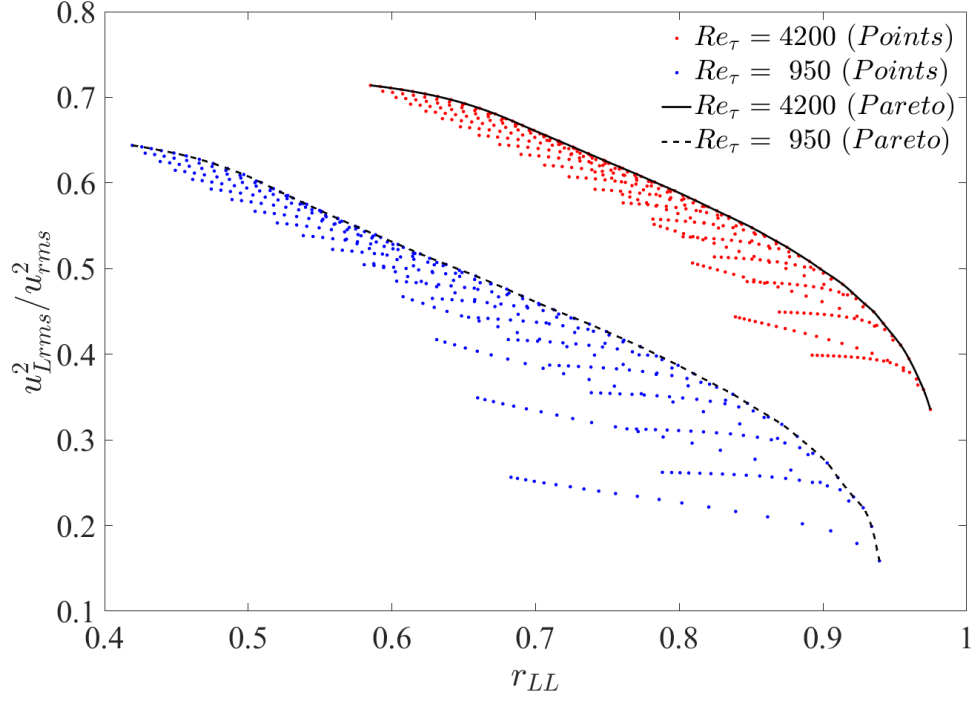


Figure 4.9: Pareto fronts of large-large correlation coefficients and the ratio of the mean large-scale velocity fluctuation energy to the total turbulent kinetic energy at $y^+ = 100$ for $Re_\tau = 950$ (dash line) and $Re_\tau = 4200$ (solid line). Each point represents a combination of the objectives.

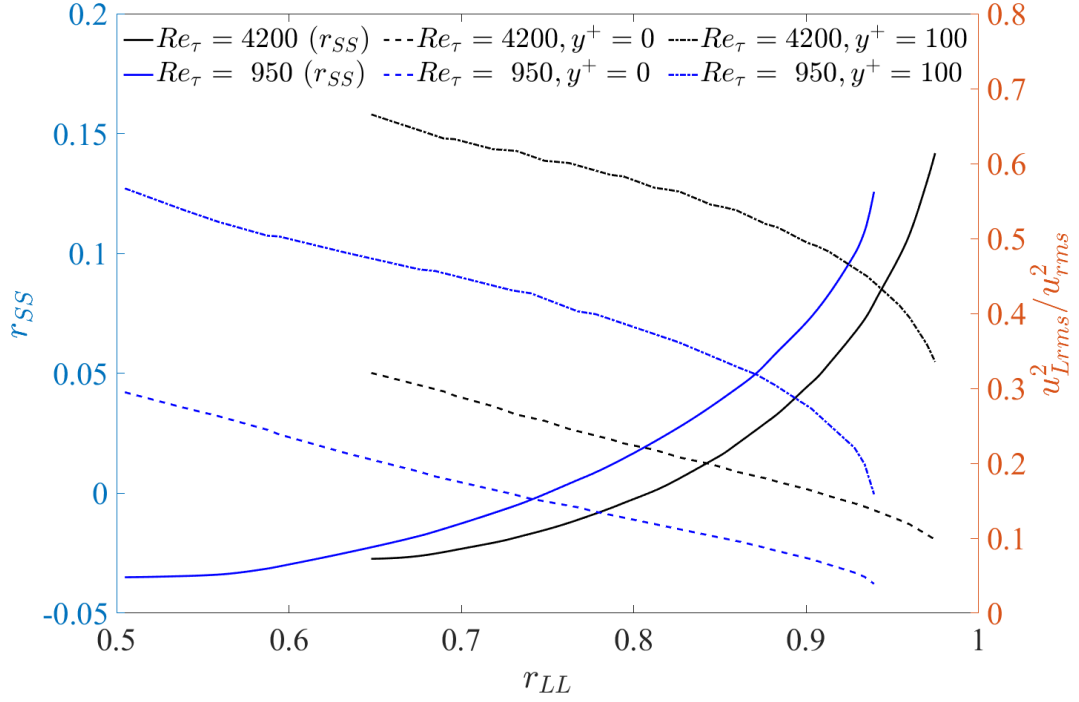


Figure 4.10: Pareto fronts of large-large and small-small correlation coefficients for $Re_\tau = 950$ (solid blue) and $Re_\tau = 4200$ (solid black) and the ratio of the mean large-scale velocity fluctuation energy to the total turbulent kinetic energy along the corresponding Pareto fronts, at $y^+ = 0$ (dash) and $y^+ = 100$ (dash-dot).

that using the r_{SS} vs r_{LL} Pareto fronts for selecting a filter does not lead to a noticeable reduction in u_{Lrms}^2/u_{rms}^2 at the same r_{LL} . This justifies the use of the r_{SS} vs r_{LL} Pareto fronts in the following chapters.

Still, there remains the question of the actual values of u_{Lrms}^2/u_{rms}^2 along the Pareto fronts. Here we focus on the high r_{LL} segment, that is the right end of the curves to meet the requirement of the QSQH theory. In addition to the values of u_{Lrms}^2/u_{rms}^2 at $y^+ = 100$, Figure 4.10 also shows the values immediately at the wall. From the figure, the large-scale motions only contribute a small portion of the energy in the very near wall regions at $Re_\tau = 950$. On the other hand, the proportion of the energy of the large-scale motions is greater at $y^+ = 100$, reaching around 30% of the total energy for $Re_\tau = 950$ and 50% for $Re_\tau = 4200$. This is a significant increase, and it can be expected that the large-scale motions might even dominate the near-wall regions in the very high-Re to which we wish to extrapolate.

4.4 Spectral constraints on the point on the Pareto front

Since Pareto fronts based on different objectives can produce similar optimal sets of filters, as it might have happened with the two sets of objectives we considered so far in Sections 4.2 and 4.3, selecting a specific point on the Pareto front requires additional consideration. In the turbulent boundary layer different coherent structures have different length scales. Hence, one can attempt to identify large-scale motions directly from the energy spectra of the turbulent motion. With the goal of selecting the spanwise and streamwise cut-offs in mind, we calculated the pre-multiplied two-dimensional wall-parallel energy spectra density maps at $y^+ = 0.3$ and $y^+ = 100$ for the database of $Re_\tau = 950$ and $Re_\tau = 4200$, shown in Figure 4.11 and Figure 4.12 respectively. The idea was that if we could observe two distinctive spectra peaks, we could select the point on the Pareto front in such a way that

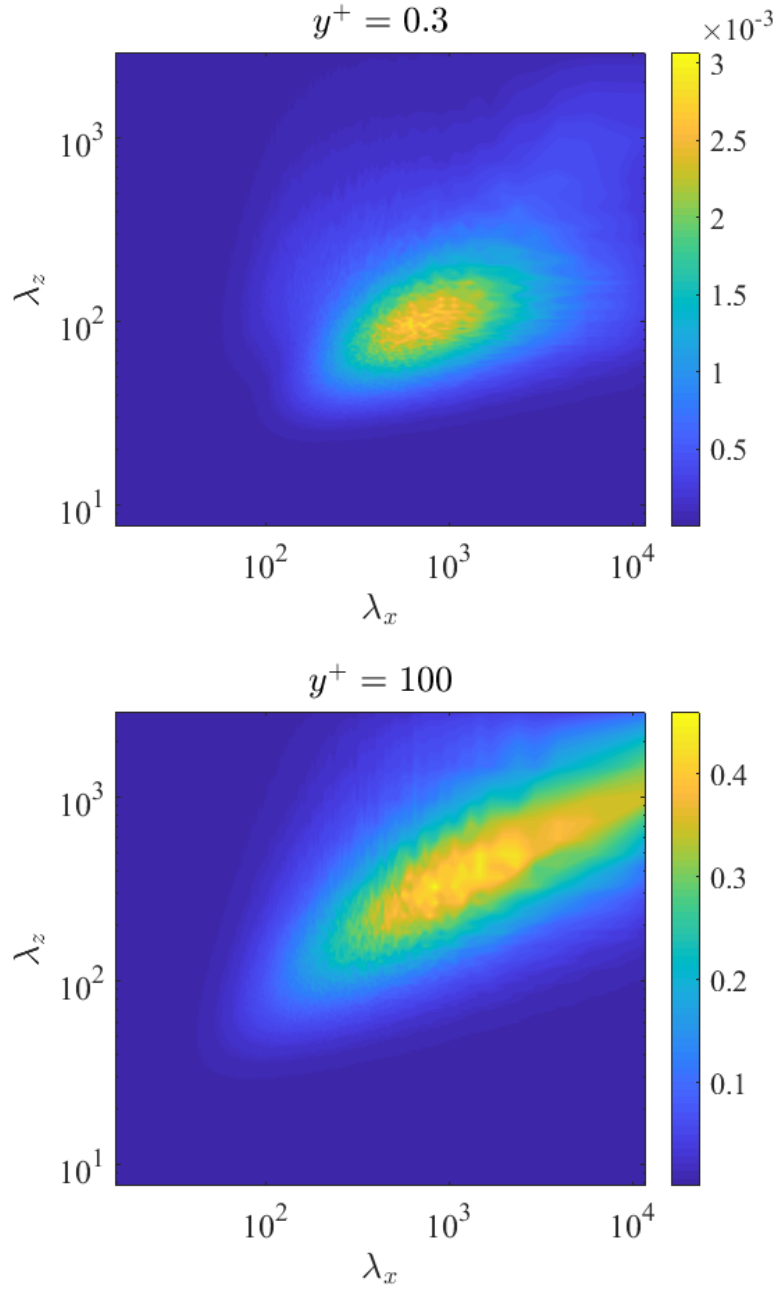


Figure 4.11: Two-dimensional pre-multiplied energy spectra density map $k_x k_y \langle \hat{u} \hat{u}^* \rangle$ of $Re_\tau = 950$ at $y^+ = 0.3$ and $y^+ = 100$.

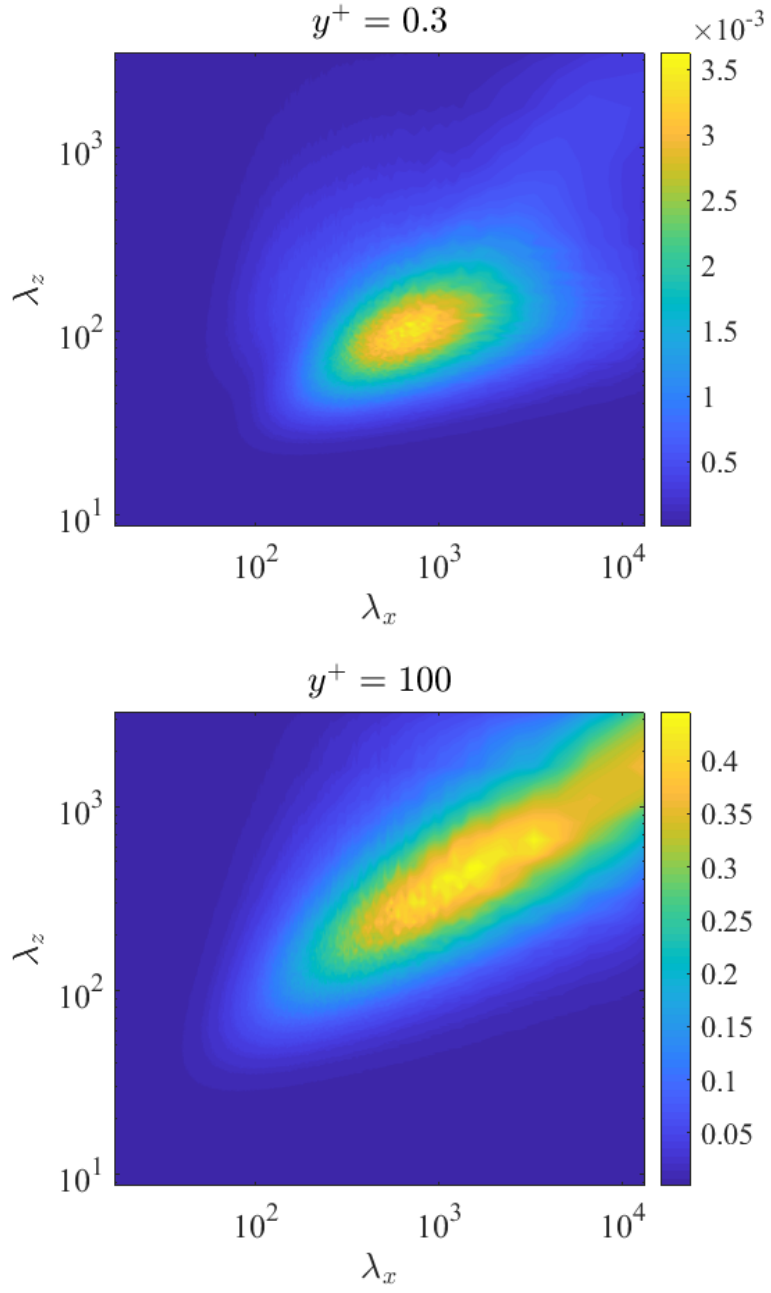


Figure 4.12: Two-dimensional pre-multiplied energy spectra density map $k_x k_y \langle \hat{u} \hat{u}^* \rangle$ of $Re_\tau = 4200$ at $y^+ = 0.3$ and $y^+ = 100$.

the corresponding filter cut-offs in x and z , that is in the streamwise and spanwise directions, will separate the peaks, and this will be the correct choice of the point on the Pareto front. For $y^+ = 0.3$, the energy peak located on $X_c^+ \approx 1000$ and $Z_c^+ \approx 100$ at both values of Re_τ considered corresponds to the near-wall streaky motions. These motions scaled with inner units should be Re-independent after the large-scale effect is removed. However, the large-scale motions that should be located in the top-right area of the plot are too weak to recognise. Consequently, the proper cut-offs are hard to determine from this spectrum.

As the wall distance increases, the energy peak gradually shifts to higher spanwise wavelengths and extends into higher streamwise wavelengths, but even for the higher $Re_\tau = 4200$ case the second peak cannot be separated from the lower peak. This is somewhat different from the more usual plots of 1D spectra, with the second coordinate being the distance to the wall, where the separation of the peaks is easier to observe. For the spectrum at $y^+ = 100$ certain previous studies [40] found that the left side of the peak corresponds to various hairpin vortexes and can be expected to scale with the wall-normal distance and have Re-independent intensity, while the right side corresponds to the large-scale motions, should be scaled with the outer length units, and has higher energy intensities with Re being larger. There is an overlapping region in the middle. We should further mention that the left-side motions shall also contain a certain degree of the large-scale effect from the right-side motions due to the amplitude modulation. Overall, Figures 4.11 and 4.12 suggest that, due to the merging of large and small scales, the appropriate cut-off lengths can not be definitely determined from these figures.

The attempt to determine the point of the Pareto front we just described was based on the intuitive idea that the large scales are large and the small scales are small, and this turned out to be not enough. We now will explain how to use the spectra to determine the point of the Pareto front based on the idea that

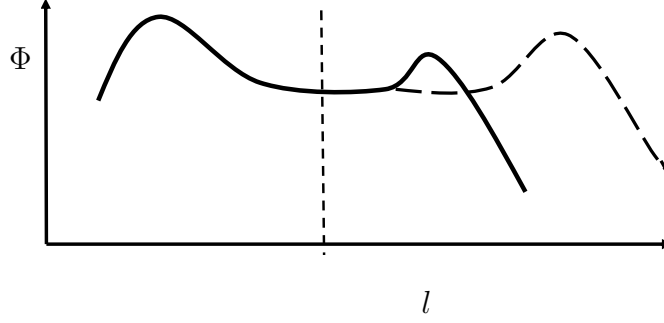


Figure 4.13: Schematic representation of the energy spectral density Φ as a function of the length scale l for two values of Re .

the small scale should be Re -independent while large scales are Re -dependent (in wall units). That this is indeed so is implied by a number of recent studies. The properties of the large-scale motions are usually measured further away from the wall, and often, when the effect of the Reynolds number is studied, the distance to the wall is taken to depend on the Reynolds number so that this distance corresponds to the middle of the logarithmic layer. The QSQH theory, on the other hand, requires to know the properties of large-scale component of the skin friction. Fortunately, Chandran et al. [24] also analyse the behaviour of the 2D (wall parallel) energy spectra at $y^+ = 150$ which is close enough to the wall for our purpose of elucidating the likely features of the large scales. They found that at large Reynolds numbers the boundaries of the region of high energy is located along the lines of the form $\lambda_z \sim \lambda_x$, which implies that the eddies within this region have the same energy regardless of their size. At the same time, Vallikivi et al. [116] proposed the scaling law of the outer spectral peak. Namely, they concluded that as Re_τ increases, the magnitude of the inner-scaled outer spectral peak increases slowly and reaches its upper bound when $Re_\tau \approx 20000$, whereas the streamwise

wavelength of the outer spectral peak is proportional to $Re_\tau^{0.5}$. It can therefore be inferred that with an increase of Re_τ , the outer spectral peak will move up and to the right comparatively fast in the inner-scaled two-dimensional spectra map, while its intensity raises slowly. This leads us to the idea of an approach to identify the boundary of the large-scale motions, which we illustrate in Figure 4.13¹ It shows the spectra schematically, and in 1D form, for two different Re . The vertical line represents the cut-off length of the filter. The area beneath the curves and to the right of the cut-off represents the energy of large-scale. Where extrapolation from the smaller Re to the larger Re is to be involved, the natural choice of the cut-off is to place it as far to the right as possible without putting it into the position when the small scales would be affected by Re . This position can be detected by considering the behavior of the difference of the two spectra, $\phi_{Re_{high}} - \phi_{Re_{low}}$. To the right side of the such naturally-selected cut-off one can expect a small dip followed by a big bump. Apparently, the upper bound of the filtering length should be the starting point of the dip, which is a reasonable constraint for the extrapolating and also the low Re cases.

In order to implement the above idea, we interpolated the two-dimensional energy spectra of $Re_\tau = 950$ and $Re_\tau = 4200$ in the arguments k_x , k_y and y^+ in order to represent them on the same grid. Then the difference between the premultiplied spectra was calculated. The resulting colour-coded maps for $y^+ = 1$ and $y^+ = 100$ are shown in Figure 4.14. The colour levels are centered at zero to better identify the positive and negative values. As we expected, there is a small area of negative amplitude and a large area of positive amplitude in the top-right corner for both contours. At $y^+ = 1$ despite the patchy error due to insufficient averaging, one can also see a positive region and a negative region located in the

¹This idea turned out to be fruitful, and we will reproduce this figure later in one of the follow-on chapters, to illustrate yet another point.

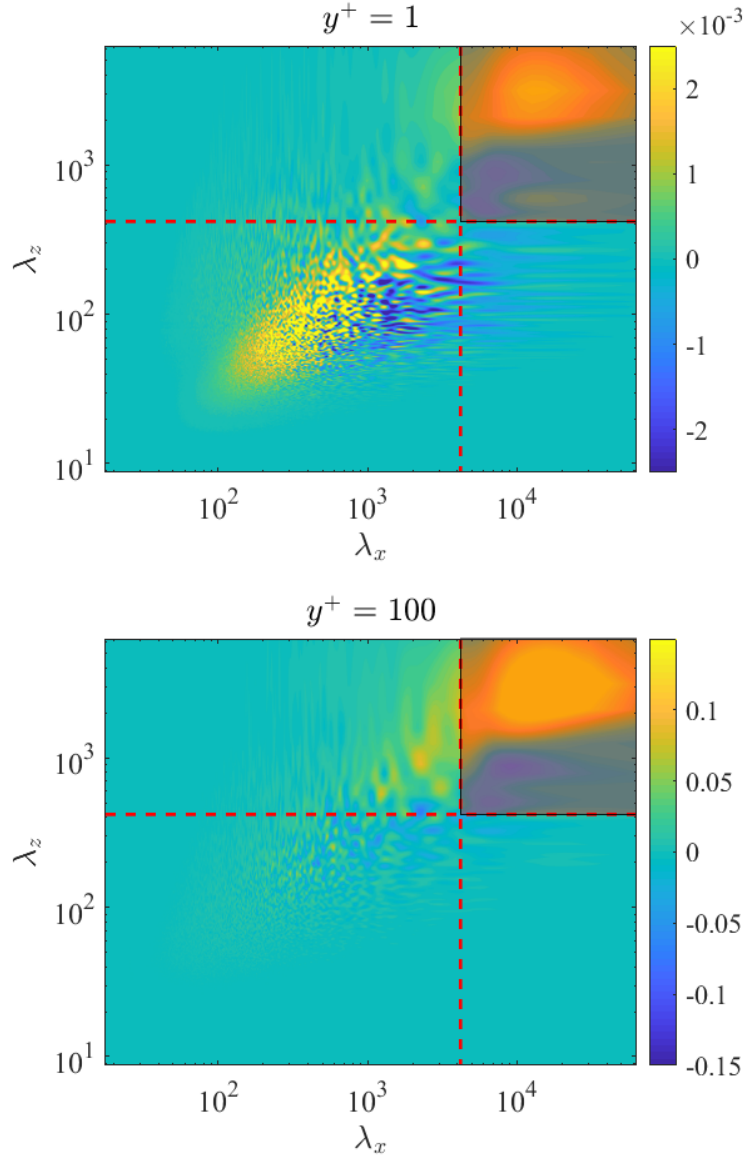


Figure 4.14: The difference between the interpolated two-dimensional premultiplied energy spectra density map of $Re_\tau = 950$ and $Re_\tau = 4200$ at $y^+ = 1$ and $y^+ = 100$. The red dash lines correspond to the cut-off lengths of $X_c^+ \approx 4000$ and $Z_c^+ \approx 450$. The shaded right-top area corresponds to the filtered large-scale motions.

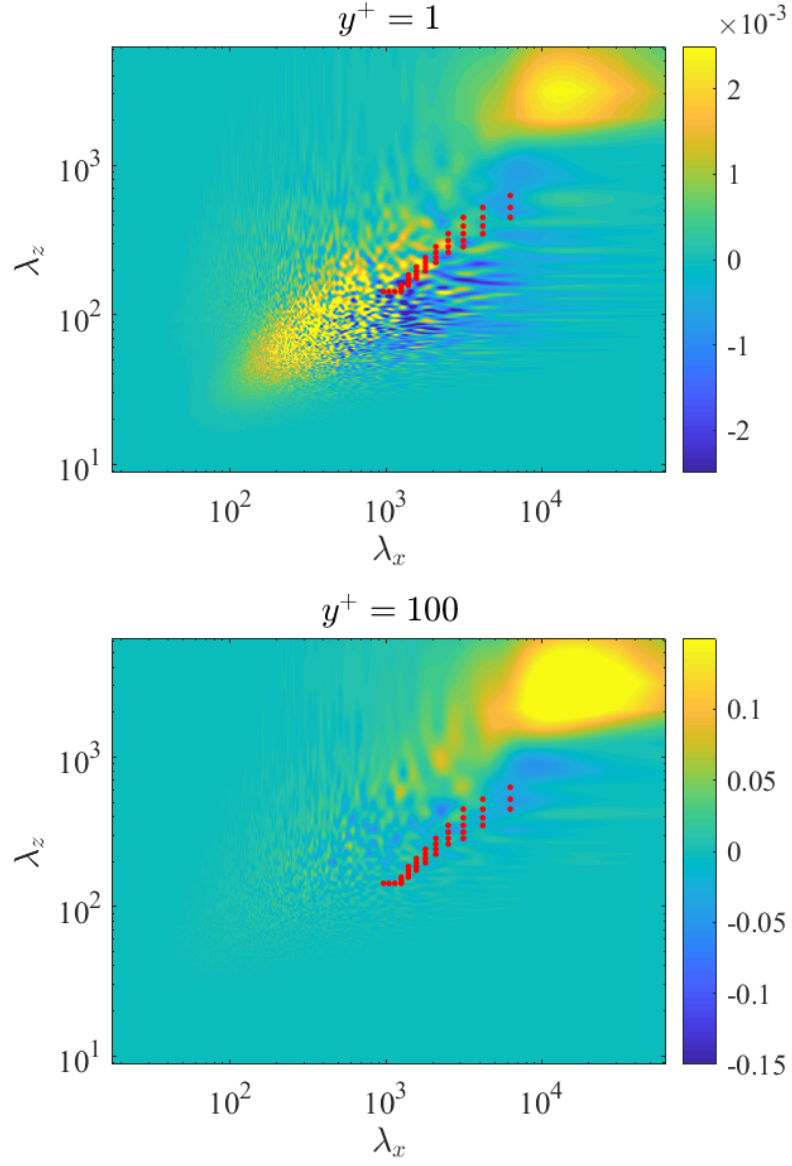


Figure 4.15: The map of the increase in the interpolated two-dimensional premultiplied energy spectra density at $y^+ = 1$ and $y^+ = 100$ as Re_τ increases from 950 to 4200. The red points represent all available combinations of cut-off lengths on the Pareto front.

area of streaky motions. We believe that this indicates the effect of the large-scale motions on the small-scale (mostly streaks) motions.

For the contour of $y^+ = 100$, the area that can be attributed to small scale motion is located more close to the area of the large-scale motion but in both $y^+ = 1$ and $y^+ = 100$ cases the plots of the spectra variation allow to clearly separate the scales using the two cut-off lengths, as it is shown in Figure 4.14. A more detailed study requires a better-converged two-dimensional spectra for two databases of different Re. This is not available yet. Based on these observations, we now suggest two new constraints for the cut-off lengths for extrapolating flow statistics, which are

1. The upper bounds of the cut-off lengths of low-Re should be limited to the edge of the area of the small negative and the large positive value of the difference between the energy spectrum of low and high-Re.
2. The lower bounds of the cut-off lengths should be above the area corresponding to the near-wall streaks.

These requirements should be combined with the requirement that the cut-off lengths are on the Pareto front. For the Pareto front with objectives we use in the present work, in Figure 4.15 we plot all the points corresponding to the Pareto fronts for $Re_\tau = 950$ and for $Re_\tau = 4200$ on the spectral difference map. These points form an narrow elongated area. For $y^+ = 1$ this area connects the region of streaky motions and the region of large-scale motions. As we discussed earlier, all the points in these regions should be excluded from the set of cut-off lengths suitable for the QSQH filter. Therefore, only limited choices of the cut-off lengths are available. For $y^+ = 100$, it is interesting to note that the Pareto-front area is just underneath the region of high energy. The implications of this observation will be considered in the future studies.

Together with the Pareto fronts, the new constraints significantly reduce the range of selection of the cut-off lengths. Following these rules, for databases of $Re_\tau = 950$ and $Re_\tau = 4200$ the optimal filter cut-off lengths were selected to be $X_c^+ \approx 4000$ and $Z_c^+ \approx 450$. Using the same cut-off values for both databases implies that the definition of the small scale is the same for both databases, which is necessary for validating the second (that is Re-independence) hypothesis of the QSQH theory.

Determining at least one appropriate combination of the cut-off values is sufficient for our next step, namely, validating the QSQH theory by comparisons. An interesting and important question of whether the same cut-offs should be used in all possible situations will be considered in subsection 5.3.2 (see in particular page 106).

4.4.1 Dependence of the Pareto front on the outer probe location

In [77, 81] the authors suggested that the large-scale motions should be extracted from the middle of the logarithm layer, which they defined as $y_o^+ = 3.9Re_\tau^{1/2}$. The large-scale velocity entering their empirical formula (1.1) is taken at this distance from the wall. In contrast, the QSQH theory uses the large-scale wall friction to describe the large-scale effects on the near-wall turbulence and, hence, on its own does not depend on the choice of the distance from the wall where large scale motion are to be detected. However, the multi-objective optimisation method of selecting the large-scale filter, described in subsection 4.2.1, does involve an “outer” layer at a certain distance from the wall. We performed a study on how the height of the outer layer affects the Pareto front. Six Pareto fronts for outer layers at different distances from the wall are plotted in Figure 4.16 for the DNS database of $Re_\tau = 4200$. For this Re the middle of the log layer is at $y^+ = 253$. The middle four Pareto fronts for the outer layers at $y^+ = 150, 200, 250$ and 300

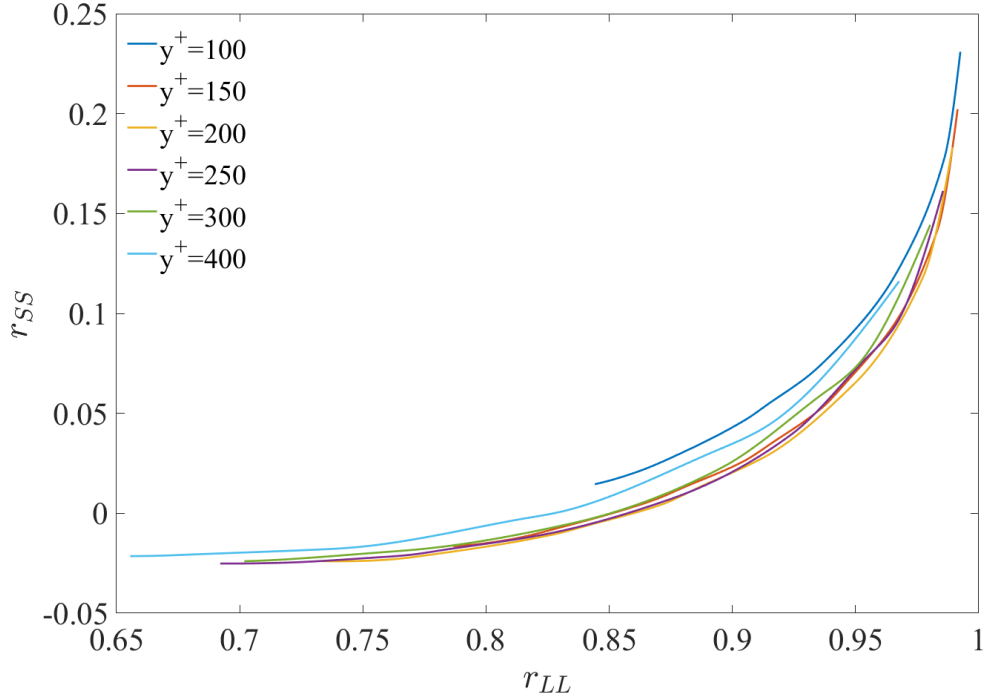


Figure 4.16: Pareto fronts of large-large and small-small correlation coefficients for the database of $Re_\tau = 4200$ for the outer layers at different wall-normal distances.

are very close to each other. The Pareto fronts for $y^+ = 100$ and $y^+ = 400$ are above and to the left. This means that if the Pareto front shape and location were the only criteria, the outer layer could be placed within a range of distances from the wall. This range, however, depends on Re , and the outer layer has to be closer to the wall for smaller Re . In this work, our outer layer locates at $y^+ = 100$, and we will consider the region with $y^+ \leq 100$ when validating the theory and extrapolating flow statistics in the near-wall region.

Chapter 5

Verification of the quasi-steady quasi-homogeneous theory

5.1 Preliminaries

5.1.1 Units

In this chapter, the QSQH theory will be validated by the comparisons of its predictions with the data obtained from the databases of direct numerical simulations of channel flows. In particular, comparisons will be made for the formulae obtained in Chapter 3 by expansion in the magnitude of the fluctuations of the large-scale component of skin friction, characterised by $\langle u'_{\tau_L}{}^2 \rangle$. For the databases and filters we will use in the comparisons, this quantity is quite small, but, as we will see, there will be cases when it is multiplied by a large other quantity and cannot be neglected. However, the non-dimensional units (3.3), convenient for QSQH theory, differ from the wall units (see (3.8) and (3.9)) by a factor $\sqrt{1 + \langle u'_{\tau_L}{}^2 \rangle}$, which is in all cases very close to 1 (≈ 1.002 for the $Re_\tau = 1000$ database and the corresponding filter, 1.0015 for $Re_\tau = 950$, 1.0022 for $Re_\tau = 2000$, and 1.0031 for $Re_\tau = 4200$). Such a small difference is not visible on the plots and will be

ignored: the plot axes will refer to the familiar wall units.

5.1.2 The mean universal velocity profile and the mean universal root-mean-square fluctuation

Many of the formulae obtained by expansion in the magnitude of the fluctuations contain the universal mean velocity profile $\tilde{U}(y)$ and the root means square fluctuation amplitude $\tilde{u}_{\text{rms}}(y)$ of the universal velocity function. For the purposes of the comparisons in this chapter, the universal mean velocity profile $\tilde{U}(y)$ can be found from (3.14), in which the term quadratic in u'_{τ_L} can be neglected. To estimate this quadratic term, we can assume that $\tilde{U}(y) \approx \frac{1}{\varkappa} \log y + B$ away from the wall. Then, for $\varkappa = 0.4$, this term equals $\langle u'_{\tau_L}{}^2 \rangle / (2\varkappa) \approx 0.0055$ for the $Re_\tau = 1000$ database case. This is much smaller than $U(y)$, and this is also true for other cases we will consider. One can use (3.6) to demonstrate that the quadratic term is also negligible near the wall. Hence, during the comparisons with good accuracy we will take $\tilde{U}(y) = U(y)$. This also justifies the use of the linearized expression (3.17) for the large-scale fluctuation.

The situation changes for $\tilde{u}_{\text{rms}}(y)$. In this case, see (3.16), the quadratic term involving $\tilde{u}_{\text{rms}}(y)$ can indeed be neglected, but $\langle u'_{\tau_L}{}^2 \rangle \left(\tilde{U}(y) + y d\tilde{U}/dy \right)^2$ is relatively large. At $y = 100$, taking $B = 5$, the logarithmic law gives a value of about 1.6 for this term, while the maximum value of $u_{\text{rms}}^2(y)$ is below 10 for $Re_\tau = 1000$. Hence, we take

$$\tilde{u}_{\text{rms}}^2(y) \approx u_{\text{rms}}^2(y) - \langle u'_{\tau_L}{}^2 \rangle \left(\tilde{U}(y) + y \frac{d\tilde{U}}{dy} \right)^2. \quad (5.1)$$

The large factor $\left(\tilde{U}(y) + y d\tilde{U}/dy \right)^2$ will also enter other formulas, where, too, it cannot be neglected. As we will see, this makes a significant difference.

Figure 5.1 shows $\tilde{u}_{\text{rms}}^{+2}$ given by (5.1) for the two parts of the $Re_\tau = 1000$ dataset: the difference between the two sets of points indicates the magnitude of

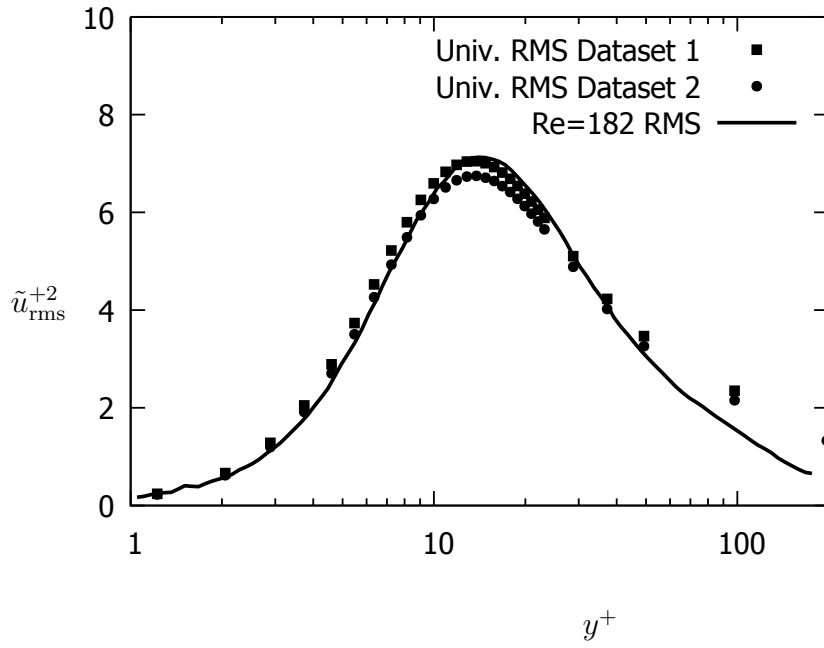


Figure 5.1: Universal rms fluctuations $\tilde{u}_{rms}^{+2}$ by (5.1) for two data subsets (points) and actual rms fluctuations $\tilde{u}_{rms}^{+2}$ for $\text{Re}_\tau = 182$ from [70] (curve).

the averaging error. Comparing Figure 5.1 to Figure 4.1 one can see that at the peak the contribution of the term $\langle u'_{\tau_L}{}^2 \rangle \left(\tilde{U}(y) + y \frac{d\tilde{U}}{dy} \right)^2$ is about 1. Naturally it becomes greater further away from the wall. Note that $\tilde{u}_{\text{rms}}^{+2}(y)$ turns out to be quite close to the actual root-mean-square fluctuations at $\text{Re}_\tau = 182$, plotted using the data of [70], also shown in the figure. One can argue that this could be expected because at such a low Re there are no large-scale fluctuations. However, more in-depth analysis, given in subsection 5.3.2, reveals a more complicated picture, further discussion of which has now to be postponed.

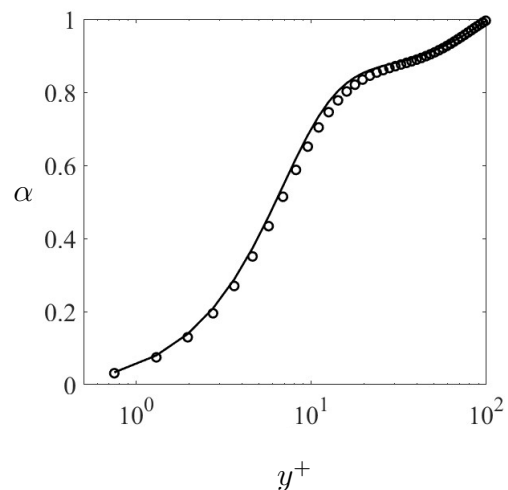
5.2 The quasi-steadiness quasi-homogeneity hypothesis

5.2.1 Two-point correlation of the large-scale motions

We have already made one comparison related to two-point correlation of large-scale motion, namely, in Figure 4.5 while developing an optimised filter we checked that $r_{u_L u_{\tau_L}}(y) = \langle u'_L u'_{\tau_L} \rangle / \sqrt{\langle u'^2_L \rangle \langle u'^2_{\tau_L} \rangle} \approx 1$, as predicted by the linearised version of the QSQH theory. The next and the most obvious target for comparisons is the coefficient $\alpha(y)$ in (1.1) introduced by Hutchins and Marusic [58] ((1.2) in the introduction, repeated here for convenience):

$$\alpha(y) = \frac{\langle u'_L(y) u'_L(y_0) \rangle}{\langle u'^2_L(y_0) \rangle}$$

(we ignore here the small difference between y and y^+). This coefficient is the two-point correlation of the large-scale velocity normalised in a certain way,



Reproduction of Figure 1.1(b)

with y and y_o usually interpreted as the position of the wall probe and the position of the outer probe respectively. In Figure 1.1(b), which we reproduce here for the convenience of the reader, we have already given the comparison for this formula.

The agreement with the linearized QSQH theory prediction (1.3):

$$\alpha(y) \approx \left(\tilde{U}(y) + y d\tilde{U}/dy \right) / \left(\tilde{U}(y_o) + y_o d\tilde{U}/dy_o \right),$$

as shown in Figure 1.1(b), is rather good. However, such good agreement in Figure 1.1(b) is in part coincidental. The coefficient $\alpha(y)$ defined by (1.2) is such that $\alpha(0) \equiv 0$ and $\alpha(y_o) \equiv 1$. Accordingly, any reasonable theory will have zero errors at these two points, and this reduces also the error in between. To obtain a measure of the error independent of this coincidence, several other related non-dimensional quantities can be used. We introduce $\alpha_0, \alpha_1, \alpha_2$, and α_3 as

$$\begin{aligned} \alpha_0(y) &= \left\langle u'_L(y) u'_L(y_o) \right\rangle / \left\langle u'^2_L(y_o) \right\rangle \left(U(y_o) + y_o \frac{dU}{dy_o} \right), \\ \alpha_1(y) &= \left\langle u'_{\tau_L} u'_L(y) \right\rangle / \left\langle u'^2_{\tau_L} \right\rangle, \\ \alpha_2(y) &= \sqrt{\left\langle u'^2_L(y) \right\rangle / \left\langle u'^2_{\tau_L} \right\rangle}, \\ \alpha_3(y) &= \sqrt{\left\langle u'^2_L(y) \right\rangle / \left\langle u'^2_L(y_o) \right\rangle} \left(U(y_o) + y_o \frac{dU}{dy_o} \right). \end{aligned} \tag{5.2}$$

For these functions the higher-order terms of the expansion in u'_{τ_L} are negligible, and the linearized QSQH theory gives the same expression:

$$\alpha_0(y) \approx \alpha_1(y) \approx \alpha_2(y) \approx \alpha_3(y) \approx \tilde{U}(y) + y \frac{d\tilde{U}}{dy}. \tag{5.3}$$

Figure 5.2 shows the comparison. Figure 5.2 and Figure 4.5 give a more fair illustration of the accuracy of the QSQH hypothesis for the filter we use than Figure 1.1(b). The error is about 10% for $y^+ \leq 100$, and in fact, for Figure 4.5 it is the value we ourselves selected by choosing a point on the Pareto front in

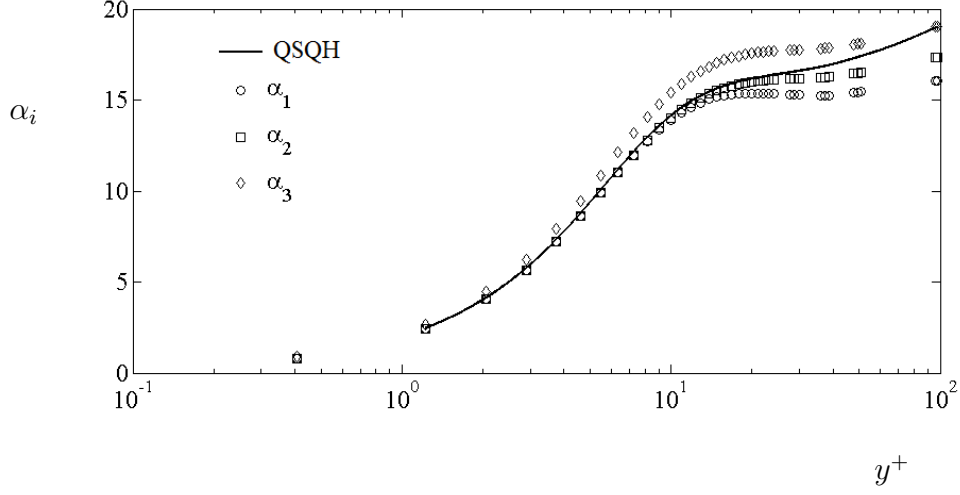


Figure 5.2: Comparison, $Re_\tau = 1000$: solid curve is the linearized QSQH theory prediction (5.3), points are α_1 (circle), α_2 (square), and α_3 (diamond). The (high) quality of the agreement for α_0 can be inferred from Figure 1.1(b).

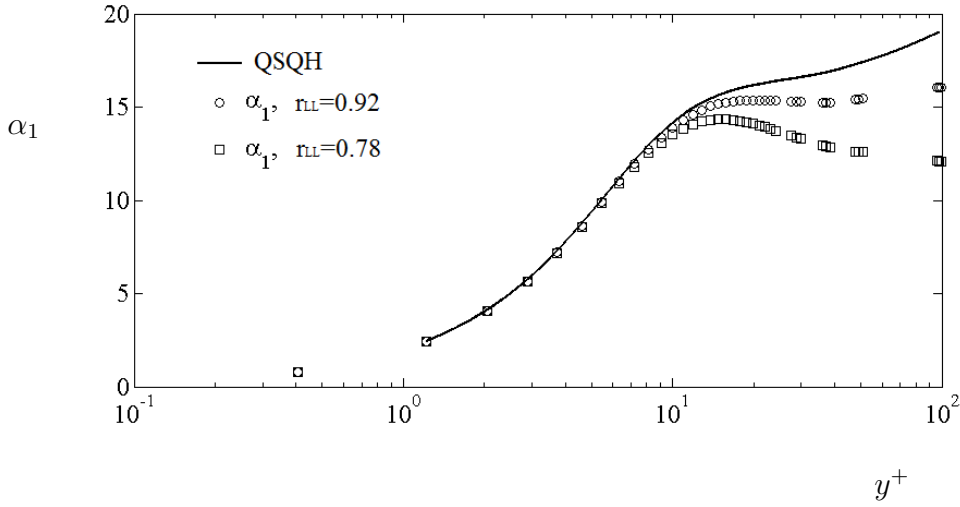


Figure 5.3: Comparison, $Re_\tau = 1000$: solid curve is the linearized QSQH theory prediction (5.3). Circles and squares are the α_1 calculated with the large-scale filters of $r_{LL}=0.92$ and $r_{LL}=0.78$.

Figure 4.3 such that r_{LL} was 0.92. What should be mentioned is that adding the second-order term into calculations has no noticeable effect on Figure 5.2. However, if we choose a point with lower r_{LL} on the left side of the Pareto front, the agreement between α_1 function and the QSQH prediction will deteriorate, as shown in Figure 5.3. Here the r_{LL} and r_{SS} of the second point are 0.78 and 0.

It should be expected that the performance of the QSQH theory improves with Re . To check this, the comparisons at higher Re should be made. However, we do not have a DNS database of high- Re with the sufficient number of snapshots sufficient time resolution to use the time cut-off. The databases of $Re_\tau = 950$ and $Re_\tau = 4200$ with limited number of time snapshots will be used. For these databases, the large-scale filter with streamwise and spanwise filtering, discussed in Section 4.3, with the cut-off lengths $X_c^+ \approx 4000$ and $Z_c^+ \approx 450$ will be used for both databases. For this filter, the corresponding point on the Pareto front are at $r_{LL} = 0.89$ and $r_{SS} = 0.068$ for $Re_\tau = 950$ and $r_{LL} = 0.94$ and $r_{SS} = 0.087$ for $Re_\tau = 4200$. Using the same cut-off at different Re is the requirement of extrapolating from one Re to another Re , see page 106 in subsection 5.3.2, but such a filter can and will be used here also for validating purposes.

Figure 5.4 shows the comparisons between the α coefficients defined by (5.2) and calculated by the data from the databases of $Re_\tau = 950$ and $Re_\tau = 4200$, and the linearised QSQH prediction (5.3). The error plot shows the relative error in α_0 , α_1 , α_2 and α_3 . The discrepancy is within 15% for $Re_\tau = 950$ and 10% for $Re_\tau = 4200$. The agreement improves as Re_τ increases. Since the α coefficients represent the two-point correlations of the large-scale motions, their error levels can be expected to behave as the parameter r_{LL} of the Pareto front. The observed behaviour is consistent with this argument.

We can conclude that as far as two-point large scale correlations are concerned, as Re increases, the QSQH theory gives at least 10% accuracy within 100

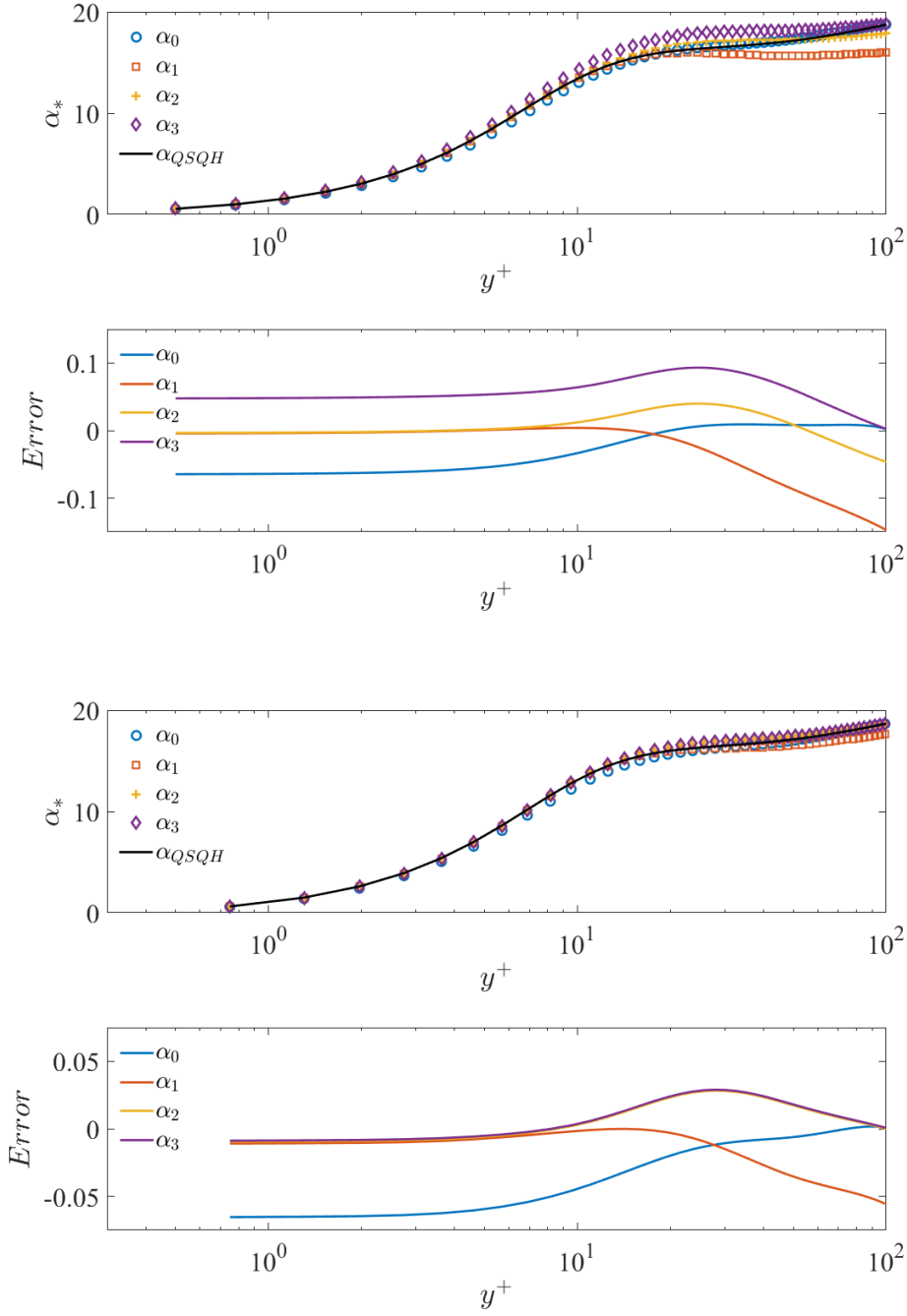


Figure 5.4: Coefficients α for $Re_\tau = 950$ (upper) and $Re_\tau = 4200$ (lower). For each case, the upper sub-plot shows the comparison, solid curve is the linearized QSQH theory prediction (5.2), points are α_0 (circle), α_1 (square), α_2 (plus), and α_3 (diamond). The lower sub-plot shows the relative error of the predictions.

wall units from the wall.

5.2.2 Modulation of fluctuation amplitude by large-scale motions

Various measures of the modulation of small-scale fluctuations by large-scales can be used and were discussed [17, 44, 80, 83, 103]. We calculated the correlations between u'_{τ_L} and the squares of small-scale fluctuation and the total fluctuations, normalised with $\langle u'^2_{\tau_L} \rangle$:

$$\gamma_1(y) = \frac{\langle u'_{\tau_L} u'^2_S \rangle}{\langle u'^2_{\tau_L} \rangle} \quad \text{and} \quad \gamma_2(y) = \frac{\langle u'_{\tau_L} (u'^2 - \overline{u'^2})_L \rangle}{\langle u'^2_{\tau_L} \rangle}, \quad (5.4)$$

where, as everywhere, the subscript L denotes a large-scale-filtered component. Recollect that the velocity is non-dimensionalized with $\langle u_{\tau_L} \rangle$, so that γ_1 can also be written in the form $\gamma_1 = \langle u'_{\tau_L} u'^2_S \rangle / (\langle u'^2_{\tau_L} \rangle \langle u_{\tau_L} \rangle)$ independent of the units of measure. Using the derivation techniques of Chapter 3 gives

$$\gamma_1(y) = 2\tilde{u}_{\text{rms}}^2 + y \frac{d\tilde{u}_{\text{rms}}^2}{dy} + \frac{1}{2} \frac{\langle u'^3_{\tau_L} \rangle}{\langle u'^2_{\tau_L} \rangle} \frac{d^2(y^2 \tilde{u}_{\text{rms}}^2)}{dy^2} + \dots \quad (5.5)$$

$$\gamma_2(y) = \gamma_1(y) + \frac{\langle u'^3_{\tau_L} \rangle}{\langle u'^2_{\tau_L} \rangle} \left(\tilde{U} + y \frac{d\tilde{U}}{dy} \right)^2 + \dots \quad (5.6)$$

Note that the difference between γ_1 and γ_2 is proportional to the normalised skewness of u'_{τ_L} , which can be regarded as a first-order non-linear term. The relation of the skewness and the measures of the amplitude modulation of the small-scales is well known [17, 83, 103]. The last term in (5.5) is the same as the last term in the square bracket in (3.16). For the datasets and the filters we are using this term turns out to be negligible. We can now use (5.1) to eliminate $\tilde{u}_{\text{rms}}$ from (5.5) (with the second term in the square bracket neglected), arriving at

$$\gamma_1(y) \approx \frac{1}{y} \frac{d}{dy} \left\{ y^2 \left[u_{\text{rms}}^2 - \langle u'^2_{\tau_L} \rangle \left(\tilde{U} + y \frac{d\tilde{U}}{dy} \right)^2 \right] \right\}. \quad (5.7)$$

Here the last term inside the bracket is a second-order non-linear term. If only the main term of the expansion is kept, (5.6) and (5.7) reduce to

$$\gamma_1(y) \approx \gamma_2(y) \approx \gamma(y) = \frac{1}{y} \frac{d}{dy} (y^2 u_{\text{rms}}^2).$$

Comparison with (3.19) shows that

$$\gamma(y) = 2\beta(y)u_{\text{rms}} \left(\tilde{U}(y_o) + y_o \frac{d\tilde{U}}{dy_o} \right). \quad (5.8)$$

Hence, a separate comparison for $\beta(y)$ is not necessary.

Figure 5.5 shows the comparisons for $Re_\tau = 1000$ database. For two large-scale filters with different values of r_{LL} , the behaviour of γ differs from the behaviour of α in two respects. The first respect is that taking into account the higher-order terms leads to a substantial change of the results. Moreover, we found that the accuracy of QSQH predictions of γ_2 is better than that of γ_1 . The difference between the linear and non-linear curves of γ_1 (contain first-order correction only) and γ_2 (contain both first and second-order corrections) show that the contribution of the second-order non-linear term, while smaller, remains comparable to the contribution of the first-order non-linear term. This indicates that taking into account more higher-order terms might improve the agreement. The second respect is that choosing a different large-scale filter will affect not only the actual value of γ obtained from DNS using (5.4), but also the QSQH predictions given by (5.5) and (5.6). It is interesting to note that in (5.7) the only term depending on the filter is $\langle u'_{\tau_L}{}^2 \rangle$, and, as it is obvious from the shape of the curves, adjusting the filter might result in a close agreement for γ_1 . This, however, will also affect γ_2 . Just as the figure shows, when the large-scale filter with different r_{LL} was applied, the prediction error for γ_1 decreased and that for γ_2 increased.

Figure 5.6 shows the comparisons of γ coefficients for the databases with $Re_\tau = 950$ and $Re_\tau = 4200$. Similar to the α coefficients, increasing Re_τ have obvious

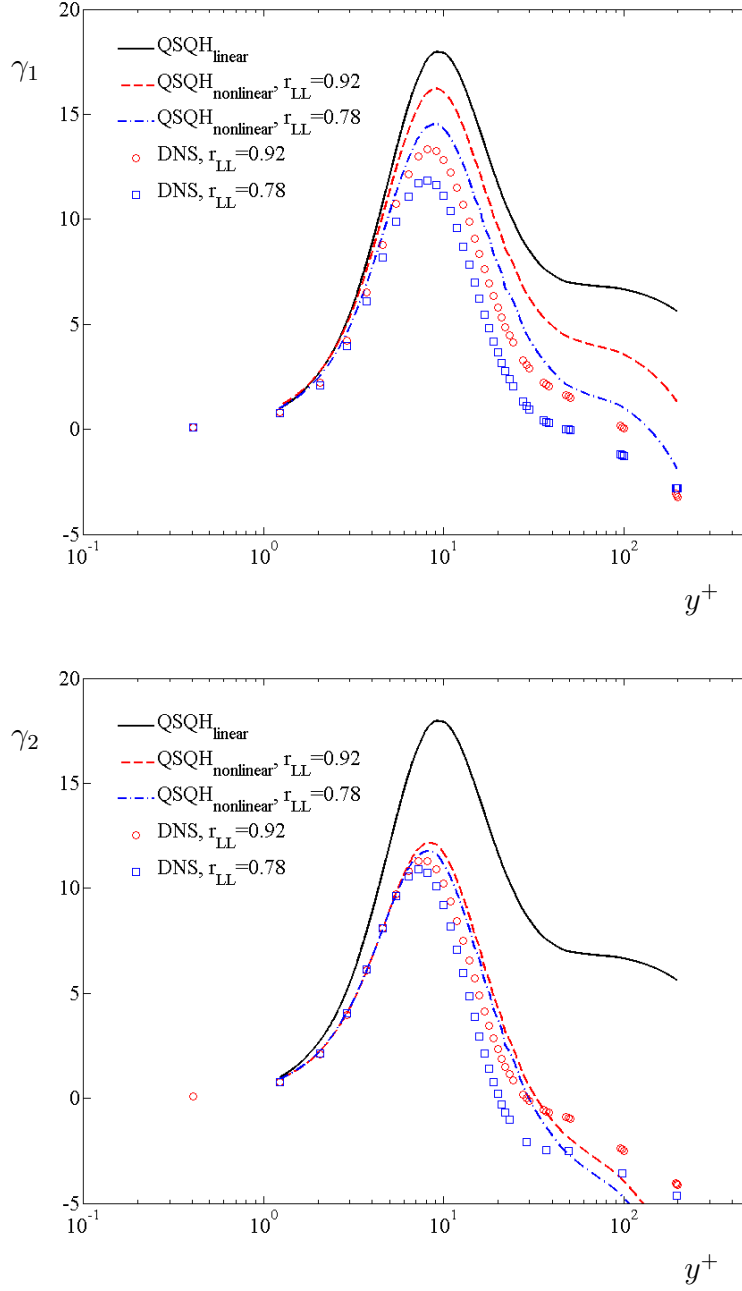


Figure 5.5: The black curve is γ , that is the main term for γ_1 and γ_2 . The red dashed curves are for expansions including the quadratic terms for γ_1 and γ_2 calculated with the large-scale filters of $r_{LL}=0.92$. The blue dash-dot curves are for these of $r_{LL}=0.78$. Red circles and blue square are Eq. (5.4) calculated from DNS with the large-scale filters of $r_{LL}=0.92$ and $r_{LL}=0.78$.

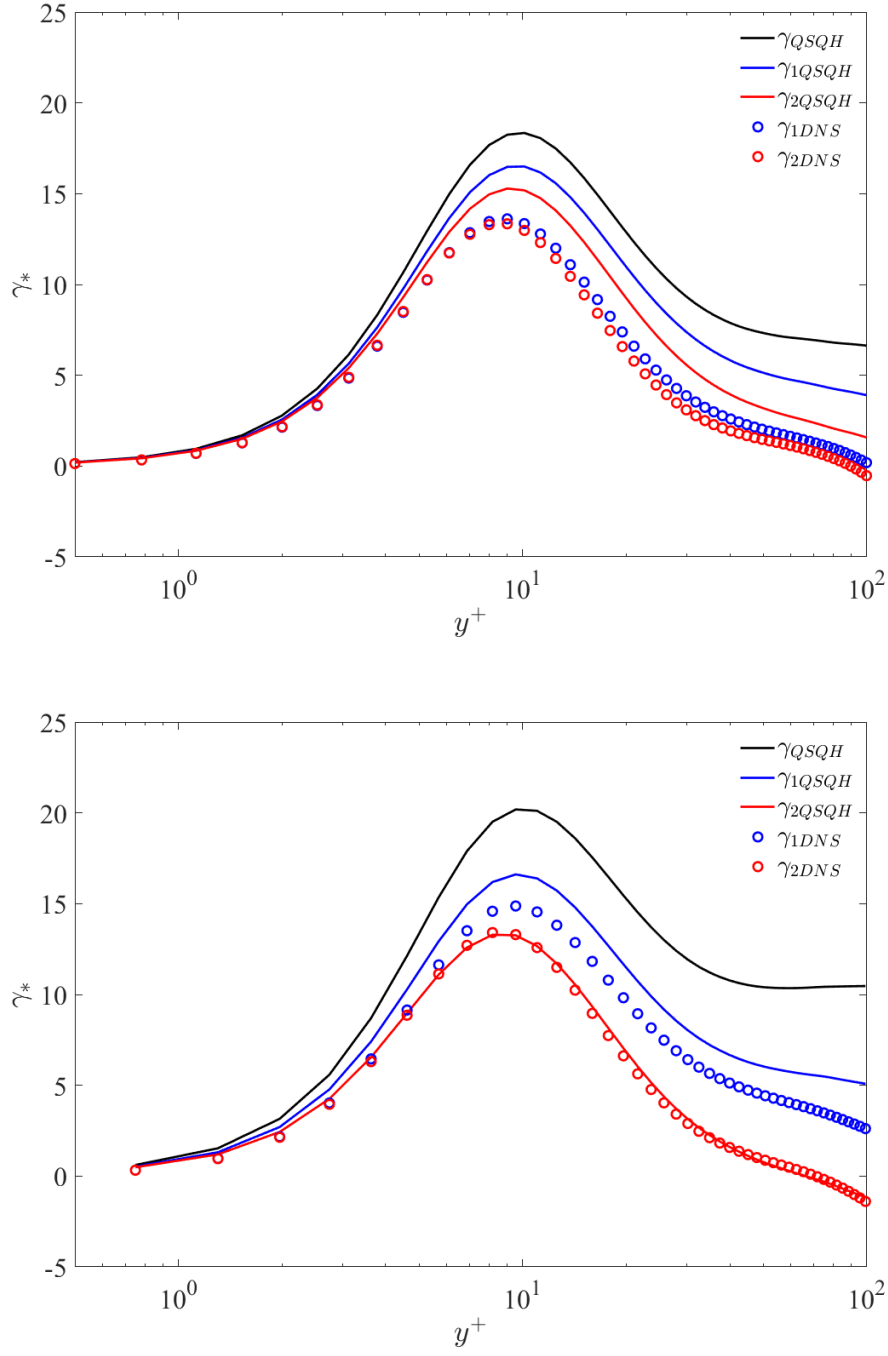


Figure 5.6: γ coefficients of $Re_\tau = 950$ (upper) and $Re_\tau = 4200$ (lower). The black curve is γ , that is the main term for γ_1 and γ_2 . The blue and red curves are the predictions including the quadratic terms for γ_1 and γ_2 . The blue and red circles are the statistical properties of γ_1 and γ_2 .

influence on the predictions of modulation coefficients. For $Re_\tau = 950$, for both $\gamma_1(y)$ and $\gamma_2(y)$ the agreement is not so particularly good. For $Re_\tau = 4200$, the agreement of $\gamma_1(y)$ is reasonably good and the agreement of $\gamma_2(y)$ is almost perfect. By comparing with the previous results, one can see that the agreement of γ predictions for $Re_\tau = 1000$ is better than that of $Re_\tau = 950$, and has approximately the same accuracy as $Re_\tau = 4200$. It can be explained by the Pareto fronts of these databases. In Figure 4.8, the Pareto front of $Re_\tau = 1000$ with triple-direction filtering is quite close to the Pareto front of $Re_\tau = 4200$ with streamwise-spanwise filtering. Both Pareto fronts lie on the bottom-right side of the Pareto fronts of $Re_\tau = 950$. This indicates a higher degree of separation between the large- and small-scale motions, which results in the observed more accurate predictions for $\gamma_1(y)$ and $\gamma_2(y)$ at higher Re .

An important observation is the significant improvement of the agreement when more terms of the expansion in the magnitude of large-scale fluctuations are taken into account. As it was shown in Chapter 3, the empirical model of [81] can be derived from the QSQH theory as the main term of this expansion. Noticeably different, and better, results obtained with more terms of the expansion shows that the QSQH theory is not equivalent, and can be more accurate than the empirical model of [81]. It should, of course, be remembered that with the empirical model one can use the freely-adjustable modulation coefficient $\beta(y)$ (closely related to $\gamma_1(y)$ and $\gamma_2(y)$) to improve the agreement, while the expansions of the QSQH theory express $\gamma_1(y)$ and $\gamma_2(y)$ uniquely via other parameters of the flow.

Overall, the error of the QSQH predictions for $\gamma_1(y)$ and $\gamma_2(y)$ is somewhat greater than the error for $\alpha_1(y)$ functions, but unlike the α functions, it is improving when more terms of the expansion are included, and it can be expected to be improved further by taking a full account of the nonlinearity of the theory.

As a whole, the comparisons in this chapter support the hypothesis that the

effect of large-scale motions on the near-wall turbulence is of quasi-steady quasi-homogeneous nature.

5.3 The Reynolds-number universality hypothesis

5.3.1 The first test of the Reynolds-number universality

The studies performed in Section 5.2 correspond to a single value of Re . As a result, the comparisons in that section can validate only the quasi-steadiness quasi-homogeneity hypothesis, but not the hypothesis that the statistical properties of $\tilde{u}(\tilde{\tau}, \tilde{\xi}, \tilde{\eta}, \tilde{\zeta})$ are independent of Re . It turns out that it is possible to test this hypothesis even without specifying explicitly the large-scale filter. To this end one can subtract the values of u_{rms}^2 for two different Re as given by (3.16), neglecting also the higher terms of the expansion denoted with dots in (3.16). Then if, and only if, $\tilde{u}_{\text{rms}}$ is indeed independent of Re , as it follows from the Reynolds-number universality hypothesis, this gives

$$u_{\text{rms}}^2|_{\text{Re}_1} - u_{\text{rms}}^2|_{\text{Re}_2} = \left(\left\langle u'^2_{\tau_L} \right\rangle_{\text{Re}_1} - \left\langle u'^2_{\tau_L} \right\rangle_{\text{Re}_2} \right) \left[\left(\tilde{U} + y \frac{d\tilde{U}}{dy} \right)^2 + \frac{1}{2} \frac{d^2 y^2 \tilde{u}_{\text{rms}}^2}{dy^2} \right]. \quad (5.9)$$

Note that $\langle u'^2_{\tau_L} \rangle$ is independent of y . Hence, taking a logarithm, taking a derivative, multiplying by y , and introducing a function

$$D(y, \text{Re}_1, \text{Re}_2) = y \frac{d}{dy} \ln \left(u_{\text{rms}}^2|_{\text{Re}_1} - u_{\text{rms}}^2|_{\text{Re}_2} \right) \quad (5.10)$$

gives

$$D(y, \text{Re}_1, \text{Re}_2) = y \frac{d}{dy} \ln \left[\left(\tilde{U} + y \frac{d\tilde{U}}{dy} \right)^2 + \frac{1}{2} \frac{d^2 y^2 \tilde{u}_{\text{rms}}^2}{dy^2} \right]. \quad (5.11)$$

Multiplying by y is desirable because otherwise the resulting function would tend to infinity as $y \rightarrow 0$, thus making plotting more difficult.

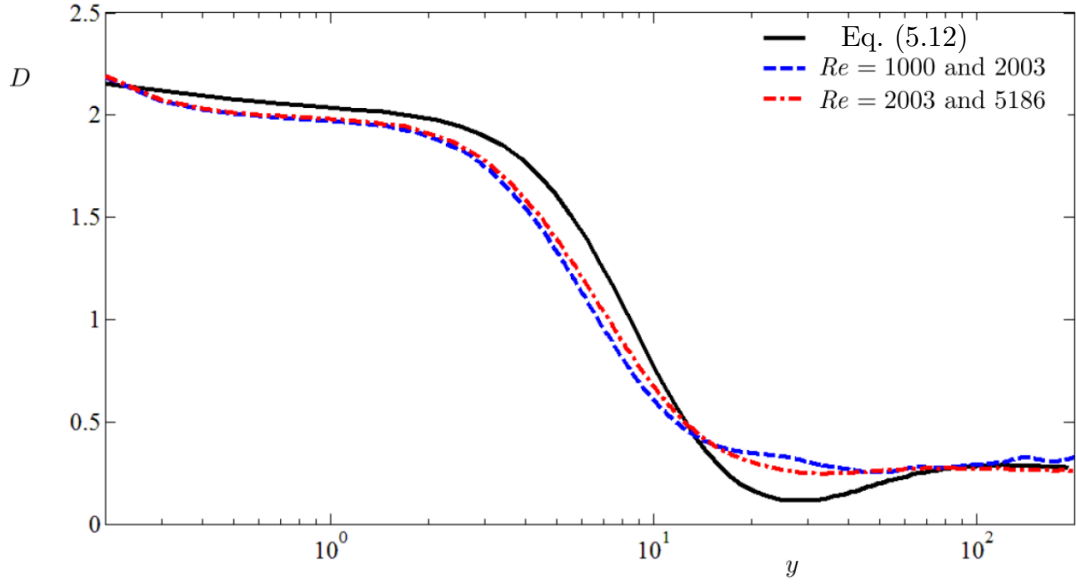


Figure 5.7: The y -multiplied logarithmic derivative $D(y, Re_1, Re_2)$ (5.10) of the increment with Re of the rms fluctuation velocity: the approximate QSQH prediction (5.12) (solid) and the numerical data [70] for Re increasing from 1000 to 2003 (dashed) and from 2003 to 5186 (dot-dashed).

Neglecting the last term in square bracket gives also an approximate formula

$$D(y, \text{Re}_1, \text{Re}_2) \approx y \frac{d}{dy} \ln \left[\left(\tilde{U} + y \frac{d\tilde{U}}{dy} \right)^2 \right], \quad (5.12)$$

which can be checked without knowing $\tilde{u}_{\text{rms}}(y)$.

Figure 5.7 shows the comparisons for the direct numerical simulation data of [70]. The prediction that $D(y, \text{Re}_1, \text{Re}_2)$ as defined by (5.10) is independent of Re_1 and Re_2 is confirmed very convincingly, and the less precise prediction (5.12) is also confirmed. As far as it would be difficult to imagine these results without the Reynolds universality hypothesis, the hypothesis itself is also confirmed.

Note that the definition (5.10) of $D(y, \text{Re}_1, \text{Re}_2)$ does not involve even a notion of a large-scale filter. Hence, the statement of $D(y, \text{Re}_1, \text{Re}_2)$ being independent of Re_1 and Re_2 is a statement about the objective properties of the turbulent flow itself, regardless of any theories or filters. This important point deserves further investigation, the first step of which is done in the following section.

5.3.2 Subjective nature of the universal velocity field in the QSQH theory

The main relationship (3.4) of the QSQH theory allows to calculate the statistical properties of the universal velocity $\tilde{u}$ provided that the large-scale filter is defined. An important question is whether there is only one suitable filter. If yes, then the universal velocity is an objective element of the physical world. If there are many equally suitable filters then the definition of $\tilde{u}$ involves a subjective selection of one of the filters by the researcher. If, as it is the case, the theory is approximate, and in practice the filters satisfy the required properties only approximately, the question of the uniqueness of the best filter becomes particularly important. Moreover, if there are many equally suitable filters then different suitable filters should lead to

the same predictions about the physical reality, and an explanation of how this can be possible is required.

While these questions are yet far from being fully answered, certain observations suggest a negative answer. In other words, it is likely that there is no unique best, or “correct”, filter. In this section these observations are described and a conceptual explanation of the possibility to obtain results independent of the subjective choice of a particular filter among the suitable filters is given. This explanation will also cover the issue of how the QSQH theory can give reasonable results in spite of the continuity of the turbulence spectrum and the energy contained in the intermediate scales not being small, that is the actual situation being rather different from the situation with a spectral gap.

We will continue to consider the example of using the QSQH theory for the analysis of the dependence of the root mean square fluctuation velocity on Re started in subsection 5.3.1. For simplicity, we will continue to assume that $u'_{\tau_L} \ll u_{\tau_L}$, which allows to use equation (3.16), which we will now rewrite as

$$u_{\text{rms}}^2(y) = \tilde{u}_{\text{rms}}^2(y) + \langle u'_{\tau_L}{}^2 \rangle \left[\left(U(y) + y \frac{dU}{dy} \right)^2 + \frac{1}{2} \frac{d^2(y^2 \tilde{u}_{\text{rms}}^2)}{dy^2} \right] + \dots, \quad (5.13)$$

in which we replaced the universal mean velocity $\tilde{U}$ with the mean velocity profile U , since they are very close when $u'_{\tau_L} \ll u_{\tau_L}$. In (5.13) $\tilde{u}_{\text{rms}}^2(y)$ is the value at $\eta = y$ of the root mean square of the fluctuations of $\tilde{u}(\tilde{t}, \tilde{x}, \eta, \tilde{z})$ (with the mean taken over $\tilde{t}$, $\tilde{x}$ and $\tilde{z}$ while η is held constant). Therefore, if the filter is known one can calculate both $\langle u'_{\tau_L}{}^2 \rangle$ and $\tilde{u}_{\text{rms}}^2(y)$ and verify (5.13). Related comparisons, with satisfactory results, will be presented in the following sections, see, for example, Figures 5.7 and 6.2. These comparisons will, however, require to define a particular large-scale filter. At this point, however, we will start from scratch, even before the filter is selected, and, hence, $\langle u'_{\tau_L}{}^2 \rangle$ and $\tilde{u}_{\text{rms}}$ are yet undefined. This, conveniently, will allow us to use the data on $u_{\text{rms}}(y)$ available in the literature. In particular,

we will use the DNS data of Lee and Moser [70] for $u_{\text{rms}}(y)$ at a variety of the Reynolds number values. Given $u_{\text{rms}}^2(y)$ for a particular Reynolds number, one can select arbitrarily the value of $\langle u'_{\tau_L}{}^2 \rangle$ and find $\tilde{u}_{\text{rms}}(y)$ from (5.13) by solving the differential equation with appropriate boundary conditions, or, easier, by just fitting a reasonable representation of $\tilde{u}_{\text{rms}}(y)$ to the data. We used the formula

$$\tilde{u}_{\text{rms}}^2 = \frac{a_2 y^2 + a_1 y + a_0}{y^3 + b_2 y^2 + b_1 y + b_0}.$$

We assumed that $\langle u'_{\tau_L}{}^2 \rangle = 0.01$ at $Re_\tau = 2000$, and used the least squares method to determine the values of a_0 , a_1 , a_2 , b_0 , b_1 , and b_2 ensuring the best fit of $u_{\text{rms}}(y)$ as given by (5.13) to the $u_{\text{rms}}(y)$ DNS data for $Re_\tau = 2000$ in [70]. The success of such fitting depends only on the number of the adjustable parameters and in itself does not tell anything. The next step is to try to fit $u_{\text{rms}}(y)$ as given by (5.13) to the $u_{\text{rms}}(y)$ DNS data at different values of Re_τ by adjusting $\langle u'_{\tau_L}{}^2 \rangle$ only while using the same $\tilde{u}_{\text{rms}}^2(y)$. Since here we are trying to fit a function of y by adjusting a single parameter, which is not possible in general, the success of this next step will confirm (5.13) and, hence, the QSQH theory. The result of such a procedure is shown in Figure 5.8.

While the agreement for $Re_\tau = 2000$ follows from the fitting procedure, the agreement for $Re_\tau = 1000$ and 5200 confirms the QSQH theory and may seem to suggest that the choice of $\langle u'_{\tau_L}{}^2 \rangle = 0.0078$ for $Re_\tau = 1000$, $\langle u'_{\tau_L}{}^2 \rangle = 0.01$ for $Re_\tau = 2000$, and $\langle u'_{\tau_L}{}^2 \rangle = 0.0125$ for $Re_\tau = 5200$ is appropriate. Judging by Figure 5.8 alone, one can decide that it is necessary to select the filter to ensure these values of $\langle u'_{\tau_L}{}^2 \rangle$ and thus define uniquely the universal function $\tilde{u}(\cdot, \cdot, \cdot, \cdot)$. However, investigating the other choices of $\langle u'_{\tau_L}{}^2 \rangle$ at $Re_\tau = 2000$ reveals a drastically different picture.

Figure 5.9 shows the result of the same procedure but for $\langle u'_{\tau_L}{}^2 \rangle = -0.01$ at $Re_\tau = 2000$ and Figure 5.10 shows the same for $\langle u'_{\tau_L}{}^2 \rangle = 0.03$. Both these choices are unfeasible, since both imply negative values for strictly positive quantities,

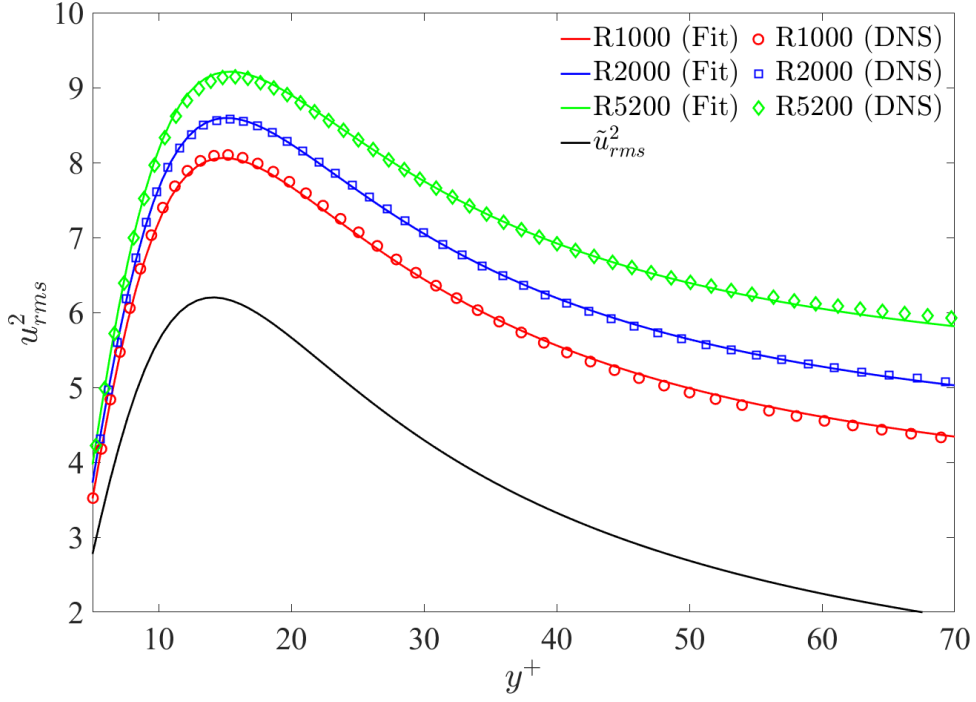


Figure 5.8: Squared rms of u . Symbols are the DNS data [70] for $Re_\tau = 1000$, 2000 and 5200. The curves are: $\tilde{u}_{rms}^2$ obtained by taking $\langle u'_{\tau_L}{}^2 \rangle = 0.01$ and fitting the DNS data for $Re_\tau = 2000$ via $\tilde{u}_{rms}^2$, and u_{rms}^2 as given by (5.13) with $\langle u'_{\tau_L}{}^2 \rangle = 0.0078$ for $Re_\tau = 1000$, $\langle u'_{\tau_L}{}^2 \rangle = 0.01$ for $Re_\tau = 2000$, and $\langle u'_{\tau_L}{}^2 \rangle = 0.0125$ for $Re_\tau = 5200$.

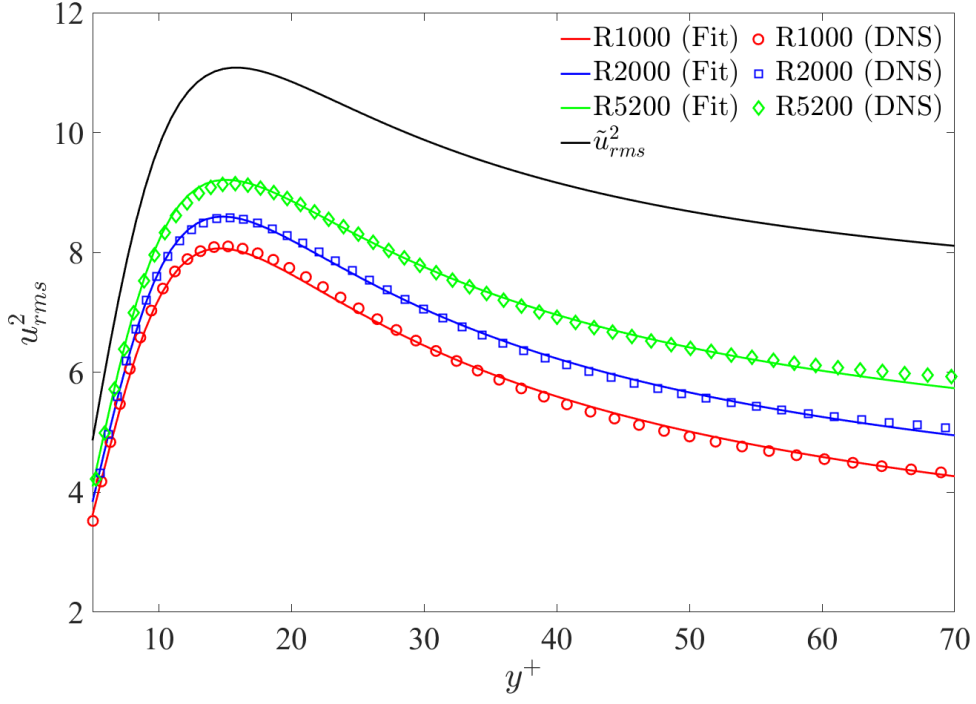


Figure 5.9: Squared rms of u . Symbols are the DNS data [70] for $Re_\tau = 1000$, 2000 and 5200. The curves are: $\tilde{u}_{rms}^2$ obtained by taking $\langle u'_{\tau_L}{}^2 \rangle = -0.01$ and fitting the DNS data for $Re_\tau = 2000$ via $\tilde{u}_{rms}^2$, and u_{rms}^2 as given by (5.13) with $\langle u'_{\tau_L}{}^2 \rangle = -0.0122$ for $Re_\tau = 1000$, $\langle u'_{\tau_L}{}^2 \rangle = -0.01$ for $Re_\tau = 2000$, and $\langle u'_{\tau_L}{}^2 \rangle = -0.0075$ for $Re_\tau = 5200$.

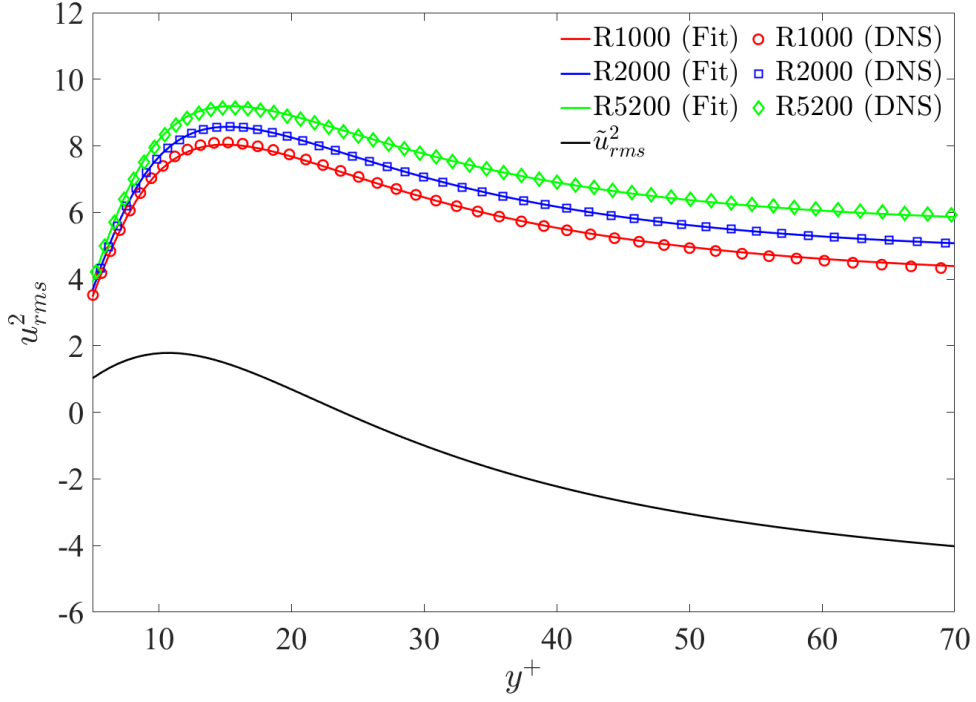


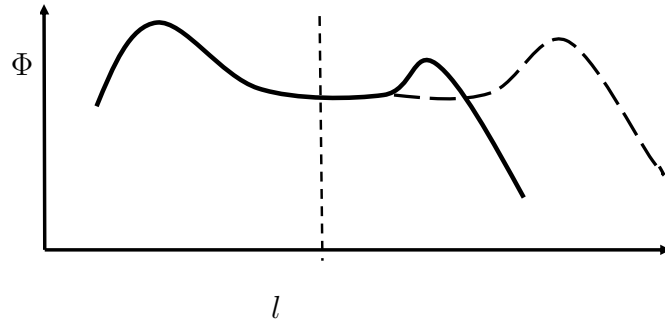
Figure 5.10: Squared rms of u . Symbols are the DNS data [70] for $Re_\tau = 1000$, 2000 and 5200. The curves are: $\tilde{u}_{rms}^2$ obtained by taking $\langle u'_{\tau_L}{}^2 \rangle = 0.03$ and fitting the DNS data for $Re_\tau = 2000$ via $\tilde{u}_{rms}^2$, and u_{rms}^2 as given by (5.13) with $\langle u'_{\tau_L}{}^2 \rangle = 0.0277$ for $Re_\tau = 1000$, $\langle u'_{\tau_L}{}^2 \rangle = 0.03$ for $Re_\tau = 2000$, and $\langle u'_{\tau_L}{}^2 \rangle = 0.0326$ for $Re_\tau = 5200$.

namely $\langle u'_{\tau_L}{}^2 \rangle$ in the first case and $\tilde{u}_{\text{rms}}^2$ in the second case. In spite of this, the quality of the agreement with the DNS data remains quite similar to the quality of the agreement in Figure 5.8.

This observation shows that the subjective choice of the filter affects $\tilde{u}_{\text{rms}}^2(y)$ much stronger than it affects the predictions of the QSQH theory related to the real physical world. It might well, therefore, be possible that not only $\tilde{u}_{\text{rms}}^2(y)$, but also the universal velocity $\tilde{u}$ itself is only a subjective intermediate construct of the theory, which, via this construct, reflects, however approximately, the objective properties of the real velocity fields.

A further, and most conclusive, confirmation of the Reynolds universality hypothesis is the success of the extrapolation procedure described in Chapter 6. The content of this section, however, helps to understand how the extrapolation can be successful even though there is no spectral gap between what is assumed to be small scales and large scales in the QSQH theory. For example, (5.9) shows that if u_{rms}^2 at $\text{Re} = \text{Re}_1$ is given then for predicting u_{rms}^2 at $\text{Re} = \text{Re}_2$ one needs to know only the difference between $\langle u'_{\tau_L}{}^2 \rangle_{\text{Re}_1}$ and $\langle u'_{\tau_L}{}^2 \rangle_{\text{Re}_2}$ but not each of these values separately. We can illustrate this by reproducing here Figure 4.13 showing the sketch of the energy spectrum. In this figure $\langle u'_{\tau_L}{}^2 \rangle$ schematically corresponds to the area below the curve and to the right from the dashed line indicating the cut-off filter. The absence of the spectral gap means that the curve is not near the zero axis in the vicinity of the cut-off. If it were then changing the cut-off within the spectral gap would not change $\langle u'_{\tau_L}{}^2 \rangle$ and thus would not change the predictions. However, as far as the cut-off is to the left from the point where the two spectra for different Re , shown with the solid and dashed curves, first diverge, the difference in the area below the curve and to the right of the cut-off for the solid and dashed curve is independent of the cut-off position even though there is no spectral gap.

In other words, as far as the changes due to changing Re are restricted, by the choice of the filter, to large scales, the extrapolation based on the use of the same cut-off filter for both Re might be successful even when there is no spectral gap. In extrapolation examples we will always use the same cut-offs for both Re . The extrapolation is the subject of the next chapter.



Reproduction of figure 4.13

Chapter 6

Extrapolation

6.1 Motivation

Developing the method of extrapolating the statistics of turbulent flows from medium to high Reynolds numbers and demonstrating its efficiency is an important component of validating the second hypothesis of the QSQH theory.

There is, however, one more reason for such a development. Direct numerical simulations and wind-tunnel experiments are typically done at the values of the Reynolds number below those characteristic for the practical applications. Therefore, in order to be able to use the results of such numerical calculations and experiments for predicting the behaviour of flows in practical applications, extrapolation is required. Over a very long time such extrapolation was done on the basis of the classical universality hypothesis. That is, the extrapolation amounted to expressing the results in wall units, and assuming that in this scaling the results will be independent of the Reynolds number. However, as discussed in Section 2.2, the large-scale structures are not Re -independent, they grow as Re increases, and they affect the near-wall part of the flow. Hence, the classical universality hypothesis does not hold, and there is a need for an alternative way of extrapolating

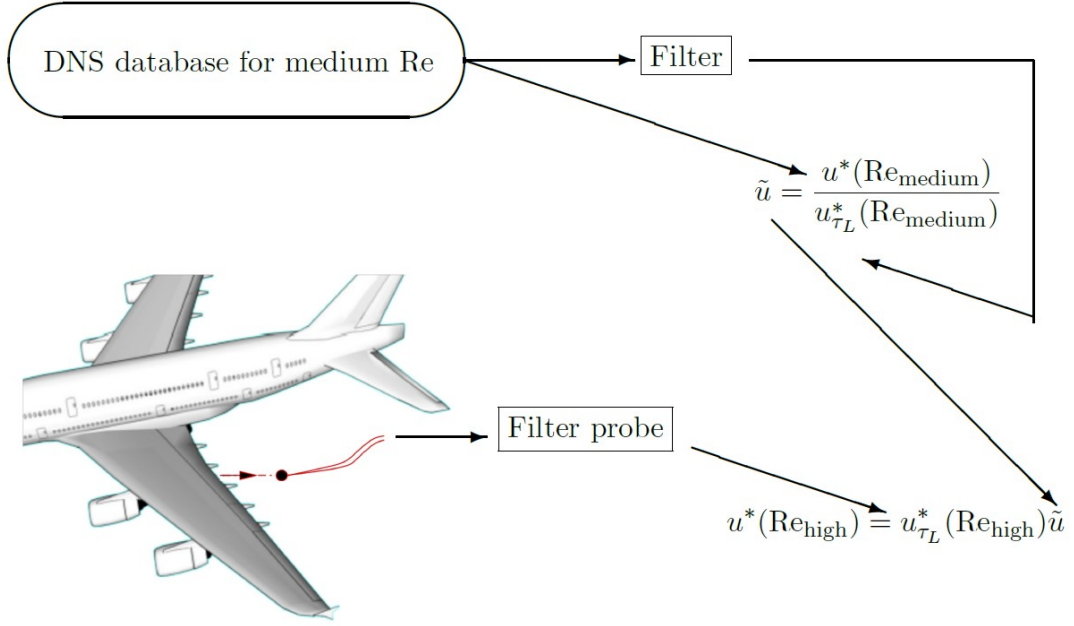


Figure 6.1: Extrapolation idea. Both formulas are versions of the same formula (3.2) expressing the main QSQH hypothesis.

turbulence statistics to higher Re . The QSQH theory provides such an alternative. The main idea is to use the data at a medium Re to determine the statistical properties of the universal, that is Re -independent, velocity $\tilde{u}$, and then, at high Re , to measure only the large-scale component. This is easier because the large-scale component can be determined from measurements further away from the wall using the QSQH theory and because, being large-scale, it requires less spatial and temporal resolution. Schematically, this procedure is illustrated in Figure 6.1.

The procedure itself can be formalised as a sequence of the following steps.

Step 1. An experiment or DNS is performed at a moderate $\text{Re} = \text{Re}_m$ to obtain

$$u(t, x, y, z, \text{Re}_m) \text{ and } u_{\tau_L}(t, x, z, \text{Re}_m).$$

Step 2. The statistical properties of $\tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$ are determined from the main

relationship of the QSQH theory (3.2).

Step 3. Measurements of $u_{\tau_L}(t, x, z, \text{Re})$ are made at the high Re .

Step 4. The statistical properties of $u(t, x, y, z, \text{Re})$ at the higher Re are obtained from (3.2).

At the first glance implementing this process appears to be straightforward. Namely, the most efficient way of representing the full set of statistics of a turbulent flow in a computer is simply to store a long enough sequence of the instantaneous snapshots of the velocity field. This is the method used in DNS databases of turbulent flows. Accordingly, one can attempt to use (3.4) to record a long enough sequence of the instantaneous snapshots of the velocity field $\tilde{u}$. In practice, however, this turns out to be difficult. The reason for this is the difference between the arguments of the velocity distribution calculated in DNS, which is often just a set of grid points equally spaced in x and z directions and organised into a wall-parallel flat grid layers with constant values of the wall-normal coordinate. The corresponding points in the arguments of $\tilde{u}$ are at xu_{τ_L} , yu_{τ_L} , and zu_{τ_L} . Since u_{τ_L} itself is a function of time and wall-parallel coordinates, under transformation the original structured mesh becomes unstructured, and in some cases even the order of neighboring grid points will reverse. It is almost impossible to reconstruct the universal flow field with such low-quality mesh. Therefore, two alternative approaches were developed. Both approaches are based on the idea that the necessary statistics at the higher Re should be expressed in terms of the statistics of the universal velocity and the statistics of the large-scale component, and then the resulting formula should be evaluated numerically on the basis of the medium- Re database and (3.4). The first approach, which will be called Taylor's expansion method, is based on the Taylor expansion with respect to the amplitude of the large-scale fluctuations, while the second approach, which will be

called coordinate transformation method, does not use the assumption of large-scale fluctuation magnitude being small. The coordinate transformation method does imply the coordinate transformation in the wall-normal direction, and deals with the wall-parallel directions differently, as explained below in Section 6.3.

6.2 Taylor's expansion method

The Taylor expansion in the powers of the amplitude of the large-scale fluctuations has already been considered in Section 3.2. The main idea is to use the truncated Taylor expansion of the statistic of interest for extrapolation. The extrapolating process of Taylor's expansion method can be divided into the following steps:

- Step 1. Express the target statistics via the statistics of the universal velocity field by applying the Taylor's expansion in powers of u'_{τ_L} to the first hypothesis of the QSQH theory. The series will normally contain the universal form of the target statistics as its main term and other statistics of the universal velocity field in the high-order terms.
- Step 2. In a compromise between accuracy and computational complexity, neglect those high-order terms that are relatively small as compared to the main terms.
- Step 3. Express the universal statistics entering the Taylor expansion via the moderate-Re data.
- Step 4. Substitute the expression for the universal statistics via the moderate-Re data into the truncated Taylor expansion.
- Step 5. Using the high-Re data on the large scales, calculate the large-scale-dependent quantities entering the Taylor expansion.

Let us consider $u_{\text{rms}}^2(y)$ as an example. The related expressions for the statistics required at Step 1 are (3.14) and (3.16). At Step 2, the higher-order term in (3.14) was found to be at least three orders of magnitude smaller than the main term. Thus the expression can be reduced to

$$U(y) \approx \tilde{U}(y) \quad (6.1)$$

In (3.16), the first term in the square bracket is too large to neglect, while the second term is relatively small. Removing this second term will only slightly affect the accuracy but greatly reduce the amount of calculations. Neglecting this term and then substituting (6.1) into (3.16) gives

$$u_{\text{rms}}^2(y) \approx \tilde{u}_{\text{rms}}^2(y) + \left\langle u'_{\tau_L}{}^2 \right\rangle \left(U(y) + y \frac{dU}{dy} \right)^2. \quad (6.2)$$

As it is well known, near the wall $U(y)$ is virtually independent of Re , and, therefore, $U(y) + y \frac{dU}{dy}$ can be calculated from the the medium- Re data. Subtracting the expression (6.2) applied at the moderate Re from the same expression for higher Re eliminates $\tilde{u}_{\text{rms}}^2(y)$, giving

$$u_{\text{rms-high}}^2(y) = u_{\text{rms-mod}}^2(y) + \left(\left\langle u'_{\tau_{L\text{high}}}{}^2 \right\rangle - \left\langle u'_{\tau_{L\text{mod}}}{}^2 \right\rangle \right) \left(U(y) + y \frac{dU}{dy} \right)^2 \quad (6.3)$$

This is an efficient implementation of Steps 3 and 4. The remaining Step 5 requires calculating only $\left\langle u'_{\tau_{L\text{high}}}{}^2 \right\rangle$ and $\left\langle u'_{\tau_{L\text{mod}}}{}^2 \right\rangle$.

Using (6.3), $u_{\text{rms}}^2(y)$ can be easily extrapolated from moderate to high Re_τ with only very simple calculations. As one can see, in this method the majority of the work consists in developing the extrapolating equation, while the computing is relatively easy and cheap. This kind of simplicity in calculating is the biggest advantage of this method. On the other hand, Taylor's expansion method has three serious disadvantages. First, expressions of complicated statistical properties are hard to derive. Applying Taylor's expansion to the high-order statistics creates

dozens of terms after expanding, and many of them cannot be eliminated or combined in the derivation process. A more thorny problem is that some high-order terms might not be very small compared to the main terms. If such terms are neglected to simplify the equation, that is what we already did in extrapolating $u_{\text{rms}}^2(y)$, the overall accuracy of the extrapolating will be sacrificed. This is the second disadvantage. Taylor's expansion method also requires the u'_{τ_L} being very small. From the databases we used and with a typical large-scale filter, we calculated the ratio between $\sqrt{\langle u'^2_{\tau_L} \rangle}$ and $\langle u_{\tau_L} \rangle$ to be 0.056 for $Re_\tau = 950$ and 0.080 for $Re_\tau = 4200$. It can be inferred that the accuracy of this method will deteriorate with an increase in Re , which is the third disadvantage. These disadvantages can prevent Taylor's expansion method to be used widely.

6.3 Coordinate transformation method

To circumvent the disadvantages of Taylor's expansion method, a method based on QSQH theory but not using Taylor's expansion is required. The main idea of this method consists in the following. Rather than trying to represent in full the statistical properties of the universal velocity $\tilde{u}$, only the statistics of $\tilde{u}$ that are required for calculating the target velocity statistics are determined from the database for the medium Re on the basis of the main QSQH hypothesis (3.4). By statistics we understand here any specific average property, such as, for example, the mean velocity profile, the root mean square fluctuation, multi-point correlations and other similar quantities. Note that the statistics can be functions of coordinates.

The discussion in subsection 3.2.1 provides a number of examples of how the statistics of the velocity field can be expressed via the statistical properties of u_{τ_L} and a limited (in each particular case) set of the statistics of the universal velocity field. Nevertheless, deriving the representation of the target statistics via

the specific statistics of u_{τ_L} and $\tilde{u}$ is the main difficulty of this method. (Note that these derivations are still much easier than the derivations required in Taylor's expansion method.)

The extrapolating process of the coordinate transformation method can be broken down into the following steps:

Step 1. Derive algebraically from (3.4) the general expression for the target statistics via the specific statistics of $\tilde{u}$ and u_{τ_L} .

Step 2. Find numerically these specific statistics of $\tilde{u}$, as functions of their arguments, from the database of moderate Re using (3.4). They should be universal by the second hypothesis of QSQH theory.

Step 3. Calculate from the expression obtained in Step 1 the target statistics using u_{τ_L} for the target Re and the specific statistics of $\tilde{u}$ obtained in Step 2.

Step 1 can be illustrated by considering again $u_{\text{rms}}^2(y)$. Let $\tilde{U}_{\text{sq}}(\tilde{y})$ be the average of $\tilde{u}^2(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$ over $\tilde{t}$, $\tilde{x}$, and $\tilde{z}$ with $\tilde{y}$ held constant: $\tilde{U}_{\text{sq}}(\tilde{y}) = \langle \tilde{u}^2(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}) \rangle$. This definition is similar to the definition of in subsection 3.2.1. Then from (3.4) one can derive that

$$\begin{aligned} u_{\text{rms}}^2(y) &= \langle (u(y) - U(y))^2 \rangle = \langle (u_{\tau_L} \tilde{u}(yu_{\tau_L}) - U(y))^2 \rangle \\ &= U^2(y) - 2U(y) \langle u_{\tau_L} \tilde{U}(yu_{\tau_L}) \rangle + \langle u_{\tau_L}^2 \tilde{U}_{\text{sq}}(yu_{\tau_L}) \rangle, \end{aligned} \quad (6.4)$$

(Note that the large-scale filter Property 4, namely, that $\langle Lf \rangle = L \langle f \rangle = \langle f \rangle$, was used in the final step of the derivation.) Therefore, $\tilde{U}(\tilde{y})$ and $\tilde{U}_{\text{sq}}(\tilde{y})$ are the two statistics, which need to be calculated in Step 2.

In practice, Step 2 can be implemented in the following way. The range of interest of $\tilde{y}$ is divided into a set of intervals B_k , which we will call bins, with centers at points $\tilde{y}_k$, $k = 1, \dots, N_{\text{bins}}$, where N_{bins} is the number of the bins. The moderate-Re database used as the basis of extrapolation consists of the set of values of

$u(t_i, x_i, y_i, z_i)$, $i = 1, \dots, N_{\text{data}}$, where N_{data} is the total number of the data points in the entire database, that is at all grids point and all time frames. This is complemented with the set of values of $u_{\tau_L i} = u_{\tau_L}(t_i, x_i, z_i)$, obtained using the large-scale filter. For each of these values we also introduce the corresponding universal coordinates as $\tilde{t}_i = t_i/u_{\tau_L i}^2$, $\tilde{x}_i = x_i/u_{\tau_L i}$, $\tilde{y}_i = y_i/u_{\tau_L i}$, $\tilde{z}_i = z_i/u_{\tau_L i}$. This allows to obtain the set of values of $\tilde{u}_i = \tilde{u}(\tilde{t}_i, \tilde{x}_i, \tilde{y}_i, \tilde{z}_i) = u(t_i, x_i, y_i, z_i)/u_{\tau_L}(t_i, x_i, z_i)$, by the main QSQH hypothesis (3.4). Then, for each bin B_k we select those database points for which $\tilde{y}_i u_{\tau_L i}$ is inside this bin, and assign the average of $\tilde{u}_i$ over these points to be the value of $\tilde{U}$ at the center of this bin, and the corresponding average of $\tilde{u}_i^2$ to be the value of $\tilde{U}_{\text{sq}}$ at the center of this bin:

Then, for each bin B_k we select those database points for which $\tilde{y}_i u_{\tau_L i}$ is inside this bin, and assign the average of $\tilde{u}_i$ over these points to be the value of $\tilde{U}$ at the center of this bin, and the corresponding average of $\tilde{u}_i^2$ to be the value of $\tilde{U}_{\text{sq}}$ at the center of this bin:

$$\begin{aligned}\tilde{U}(\tilde{y}_k) &= \frac{1}{N_k} \sum_{i: \tilde{y}_i \in B_k} \tilde{u}_i, \\ \tilde{U}_{\text{sq}}(\tilde{y}_k) &= \frac{1}{N_k} \sum_{i: \tilde{y}_i \in B_k} \tilde{u}_i^2,\end{aligned}\tag{6.5}$$

where N_k is the number of points for which $\tilde{y}_i$ is inside B_k . This procedure neglects deviations due to the distortion of the uniformity of the database grid when transformation to the universal variables is made. This issue is discussed in subsection 6.5.2. For the numerical examples in our work this effect turned out to be negligible indeed. With these discrete points obtained, the universal functions $\tilde{U}(\tilde{y})$ and $\tilde{U}_{\text{sq}}(\tilde{y})$ can be obtained by interpolation. In the present study cubic spline interpolation was used.

The range of $\tilde{y}$ covered by the bins can be determined from the following considerations. First, one should decide on the distance to the wall y_0 below which the QSQH theory is expected to provide sufficient accuracy. This depends on the

desired accuracy and the Reynolds number of the database that is being used as the basis for extrapolations. Then the bins should cover the range of $\tilde{y}$ from zero at the wall to $u_{\tau_L max}$, where $u_{\tau_L max}$ is the maximum value of $u_{\tau_L}(x, y, z)$ at the value of Re for which the prediction is to be made. Selection of the number and the size of the bins involves a compromise between the several types of errors involved and the computational effort required for obtaining the universal statistics. Fewer but larger bins imply larger separation between $\tilde{y}_k$, and larger distances between $\tilde{y}_k$ and $\tilde{y}_i$ inside, which will inevitably reduce the accuracy of interpolation. On the other hand, with few large bins the calculations will be cheaper, and a larger number of points inside each bin reduces the error of averaging over a finite number of points in the bin. With many smaller bins these trends reverse.

For the statistics depending on two or more variables, such as two-point correlations, the coordinate transformation method can also be used, with multi-dimensional bins. Each dimension will correspond to a particular variable. This dramatically increases the volume of the numerical calculations required, but it remains doable, as we will demonstrate in Section 6.4.

Note that this particular method has so far been implemented only for the statistics that depend only on the wall-normal coordinate(s), such as $u_{rms}^2(y)$ or two-point velocity correlations when the two points has the same x and z coordinates. It can be extended to the case of statistics depending on points with different x and z , provided that these point are not too far apart. In general, however, calculating multi-point statistics with large separation of points in space and/or time requires developing a more general version of the QSQH theory, see the last paragraph of subsection 3.1.1.

Step 3 hardly requires further explanations. The full set of values of u_{τ_L} is taken for the target- Re case, substituted into (6.4) or its equivalent if other statistics are being extrapolated, and averaged over all available values of u_{τ_L} . It is useful to

note here that the target-Re values of u_{τ_L} should be obtained with the same filter (same cut-offs for a Fourier-cut-off filter) that was used to determine u_{τ_L} for the database on which the extrapolation is based. The reason for this is discussed in Section 4.3. In the next section, we will show examples of extrapolating between two DNS databases.

6.4 Examples

In this section we will apply to specific examples the two methods of extrapolating statistical properties of near-wall turbulent flow from moderate Re_τ to high Re_τ introduced in the last section. The extrapolation will be performed on the basis of the aforementioned DNS channel flow database of $Re_\tau = 950$, and the predictions will be made for the statistics at $Re_\tau = 4200$. The $Re_\tau = 4200$ database, which we also already described, will be used in two ways. First, it will provide the large set of values of u_{τ_L} at $Re = 4200$, which is the only information required about the $Re = 4200$ for the extrapolation. Second, the $Re = 4200$ data will be used for comparisons with the extrapolation results.

Two statistics will be extrapolated by applying the two methods. We begin with extrapolating u_{rms}^2 . For u_{rms}^2 the extrapolation equation (6.3) of Taylor's expansion method is:

$$u_{rms4200}^2(y) = u_{rms950}^2(y) + \left(\langle u_{\tau_L4200}'^2 \rangle - \langle u_{\tau_L950}'^2 \rangle \right) \left(\tilde{U}(y) + y \frac{d\tilde{U}}{dy} \right)^2, \quad (6.6)$$

where $\tilde{U}(y)$ approximately equals $U(y)$, that is the universal mean velocity profile. The values of $\langle u_{\tau_L4200}'^2 \rangle$ and $\langle u_{\tau_L950}'^2 \rangle$ are 0.0064 and 0.0031 respectively based on our large-scale filter. The cut-off parameters are $X_c^+ \approx 4000$ and $Z_c^+ \approx 450$ for both databases. In terms of the coordinate transformation method, the extrapolating

equation is

$$u_{\text{rms}4200}^2(y) = U^2(y) - 2U(y) \left\langle u_{\tau_L 4200} \tilde{U}(yu_{\tau_L}) \right\rangle + \left\langle u_{\tau_L 4200}^2 \tilde{U}_{\text{sq}}(yu_{\tau_L}) \right\rangle \quad (6.7)$$

where the functions $\tilde{U}$ and $\tilde{U}_{\text{sq}}$ are the universal statistical properties obtained from the database of $Re_\tau = 950$. The number of bins was selected to be 35 and the length of each bin corresponds to about twice the spacing between the corresponding wall-parallel grid layers of the DNS database on which the extrapolation is based. The bins cover the region from the wall to $y^+ = 150$. The functions of $u_{\text{rms}}^2(y)$ at $Re_\tau = 4200$ was extrapolated by (6.6) and (6.7) in the range $0 \leq y^+ \leq 100$. The results are shown in Figure 6.2. The blue dash line and red solid line represent the $u_{\text{rms}}^2(y)$ calculated directly from the databases. The circle and square points are the predictions by Taylor's expansion method and the coordinate transformation method respectively. For the former method, there is a small gap between the predictions and the actual value. For the latter method, the two curves agree well. As we mentioned, the difference between the predictions of the two methods is mainly caused by neglecting the high-order terms in Taylor's expansion method.

Recollect that we have already hypothesized that the discrepancy in the prediction of the modulation coefficient γ_1 (see Figure 5.5) was also due to neglecting the higher-order terms of the Taylor expansion. Since the results suggest that the coordinate transformation method is more accurate than Taylor's expansion method, it can be considered as further evidence in favour of that conjecture.

The second statistics, which we will extrapolate, is the two-point correlation $\langle u'(y_1)u'(y_2) \rangle$. Here we will use only the coordinate transformation method, because in this case the derivations required by the Taylor's expansion method are too complicated. The extrapolating equation is shown below:

$$\begin{aligned} \langle u'(y_1)u'(y_2) \rangle_{4200} = & U(y_1)U(y_2) - U(y_2) \left\langle u_{\tau_L 4200} \tilde{U}(y_1 u_{\tau_L 4200}) \right\rangle - \\ & U(y_1) \left\langle u_{\tau_L 4200} \tilde{U}(y_2 u_{\tau_L 4200}) \right\rangle + \left\langle u_{\tau_L 4200}^2 \tilde{R}(y_1 u_{\tau_L 4200}, y_2 u_{\tau_L 4200}) \right\rangle \end{aligned} \quad (6.8)$$

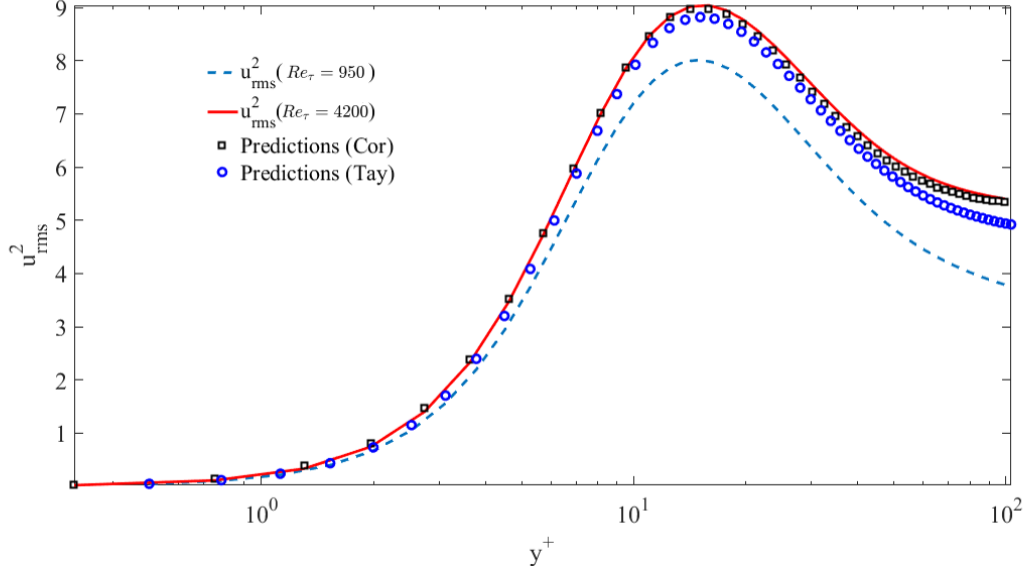


Figure 6.2: Extrapolating u_{rms}^2 from $Re_\tau = 950$ to $Re_\tau = 4200$ by Taylor's expansion method and the coordinate transformation method.

where $u_{\tau_L 4200}$ is u_{τ_L} at $Re_\tau = 4200$, $\tilde{R}(\tilde{y}_1, \tilde{y}_2)$ is the two-point correlation of $\tilde{u}$, defined as $\tilde{R}(\tilde{y}_1, \tilde{y}_2) = \langle \tilde{u}(\tilde{y}_1) \tilde{u}(\tilde{y}_2) \rangle$, where the averaging is done with $\tilde{y}_1$ and $\tilde{y}_2$ held constant. The detailed derivation of (6.8) is given in the appendix.

The universal function $\tilde{R}(\tilde{y}_1, \tilde{y}_2)$ can be determined from the database of $Re_\tau = 950$. We generated 35 bins for $\tilde{u}(y_1 u_{\tau_L})$ and 35 bins $\tilde{u}(y_2 u_{\tau_L})$, which compose total 630 two-dimensional bins to contain the transformed points. The sizes, positions, and number of the one-dimensional bins were the same as that used in prediction u_{rms} . Figure 6.3 shows the comparison between the predictions and actual value of $\langle u'(y_1) u'(y_2) \rangle$ at $Re_\tau = 4200$ in the contour plots with the same colour range. These contour plots appear to be identical. However, since the difference in the two-point correlations at these two Reynolds numbers is not too large, the possible error of the prediction would better be revealed by comparing the predicted difference between the two-point correlations at $Re_\tau = 4200$ and $Re_\tau = 2000$, each predicted

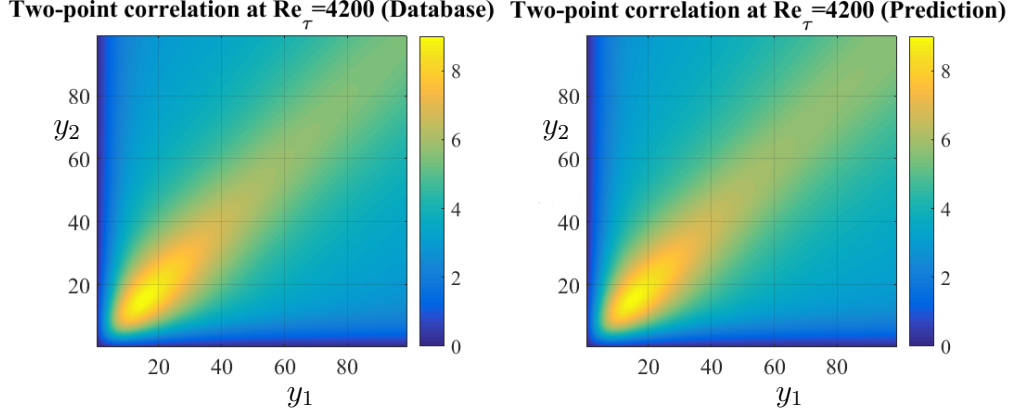


Figure 6.3: Comparison between the two-point correlation $\langle u'(y_1)u'(y_2) \rangle$ at $Re_\tau = 4200$ and its values extrapolated from $Re_\tau = 950$ using the coordinate transformation method.

on the basis of the $Re_\tau = 950$ dataset, with the actual difference between them as obtained from direct numerical simulations.

Such a comparison is provided in Figure 6.4. This sensitive test does confirm that the extrapolation was indeed successful.

To examine the accuracy of extrapolating quantitatively, we calculated the absolute value of the relative error. The results is shown in Figure 6.5. Except a small vicinity of the wall, the relative error is under 10%, and even drops below 5% further away from the wall. The relatively large discrepancy in the very near-wall region is likely to be caused by a non-uniform distribution of points $\tilde{y}_i$ in the near-wall bins. For this reason more of this points turn out to be on one side of the bin center than the other and, hence, the average over these points would better be attributed not the the centre of the bin but to a point appropriately shifted. We did not pursue this possible improvement, however.

We also extracted the prediction for $\langle u(y)u(y) \rangle$, which is also the prediction

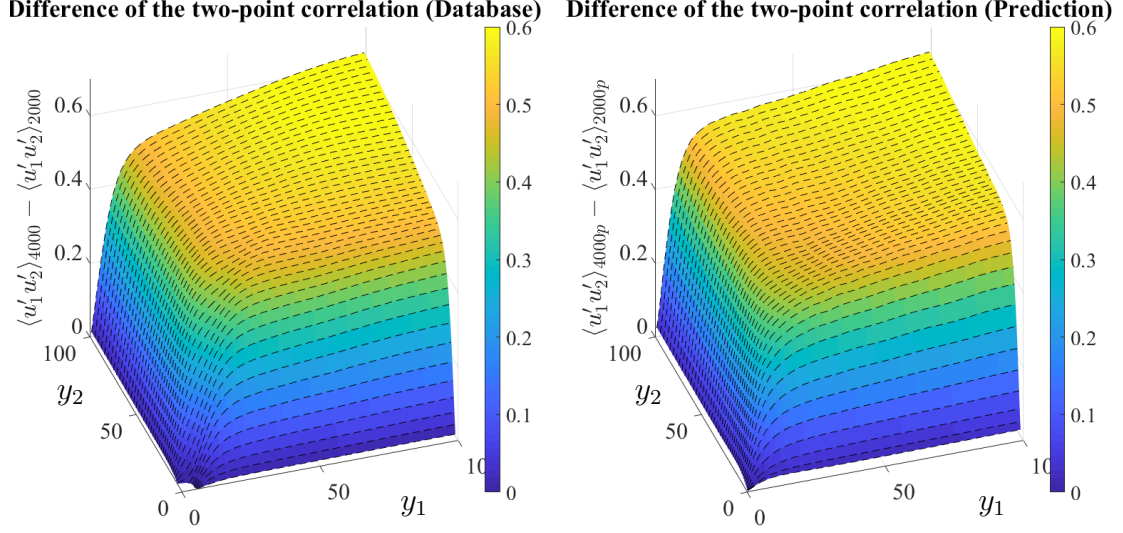


Figure 6.4: Actual (left) and extrapolated (right) distribution of $\langle u'(y_1)u'(y_2) \rangle|_{\text{Re}_\tau=4200} - \langle u'(y_1)u'(y_2) \rangle|_{\text{Re}_\tau=2000}$.

of $u_{\text{rms}}^2(y)$, but obtained with two-dimensional bins. The prediction was plotted together with the previous predictions of one-dimensional bins and the actual value of $u_{\text{rms}}^2(y)$, and it is shown in Figure 6.6. In the region below $y^+ = 50$, just like one-dimensional predictions, the two-dimensional predictions agree well with the actual value. Farther away from the wall the error of the 2D prediction increases, but it remains still quite small even at $y^+ = 100$. The curves obtained with 1D bins and 2D bin do not coincide exactly. This is due to the smaller number of samples in two-dimensional bins leading to higher averaging error. Therefore, a larger database, which provides more data points and thus allows more bins, would improve the accuracy of the two-dimensional predictions by the coordinate transformation method.

The coordinate transformation method was developed to resolve the three disadvantages of Taylor's expansion method. The successful prediction of $u_{\text{rms}}^2(y)$

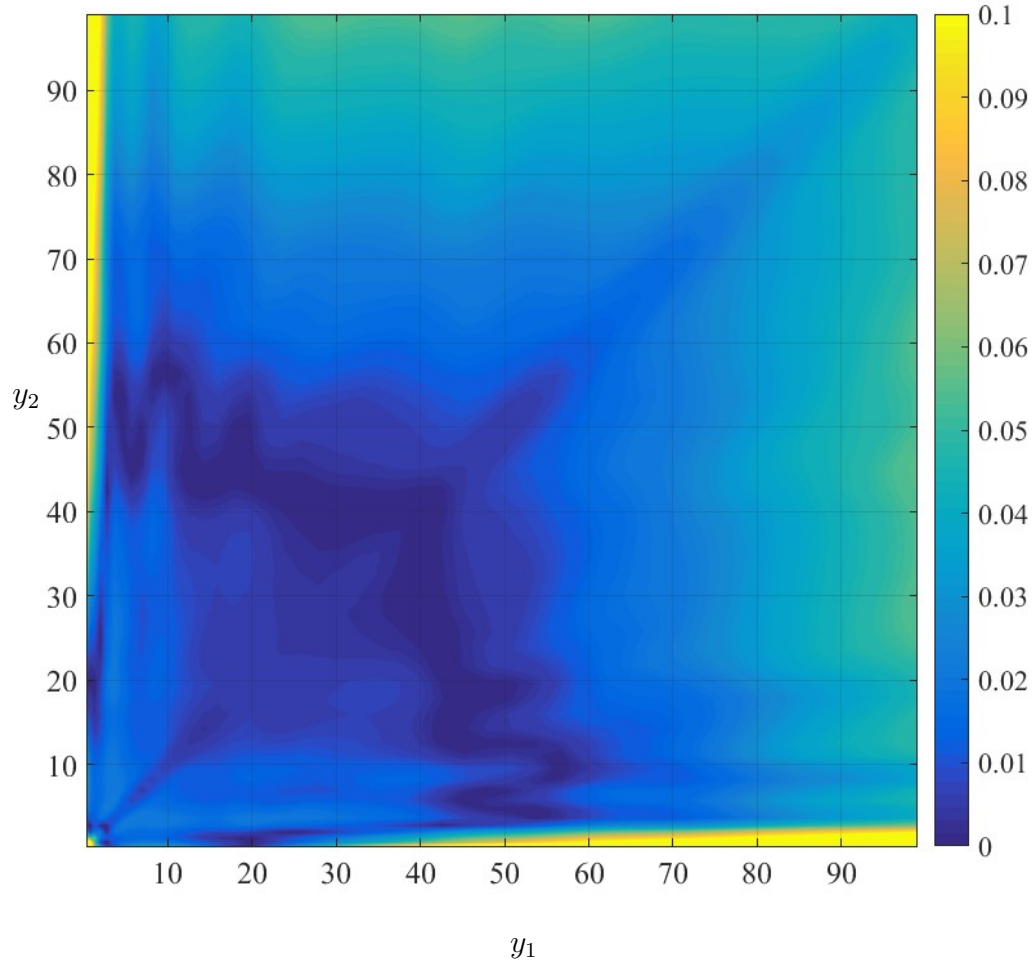


Figure 6.5: The absolute value of the relative error between the two-point correlation $\langle u'(y_1)u'(y_2) \rangle$ at $Re_\tau = 4200$ and its prediction by extrapolation from $Re_\tau = 950$ by the coordinate transformation method.

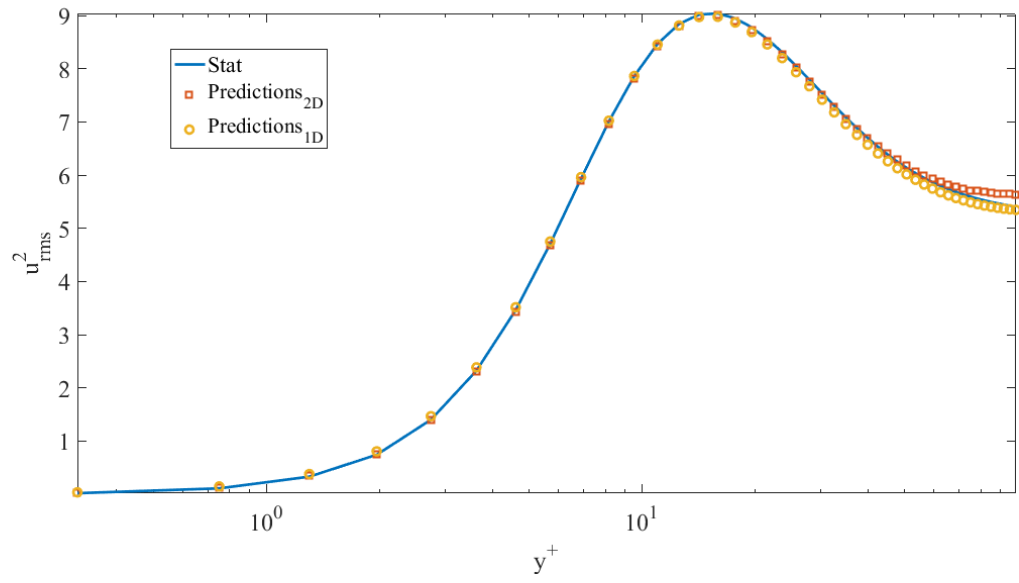


Figure 6.6: Comparison between the 1D and 2D predictions, and the actual value of $u_{\text{rms}}^2(y)$ at $Re_\tau = 4200$ by the coordinate transformation method.

confirms that the coordinate transformation method is indeed more accurate. The good agreement in extrapolating $\langle u'(y_1)u'(y_2) \rangle$ confirms that the method can also be used for predicting more complicated statistical properties. The assumption that u'_{τ_L} is small, which is not the case in the flow of high Re_τ , is no longer required in this method. However, despite these advantages, the coordinate transformation method still have its drawback. It requires the set of $u_{\tau_L}(t, x, z)$ at the target Reynolds number to be sufficiently large, while only the values of a few statistics of u_{τ_L} , such as $\langle u'_{\tau_L}{}^2 \rangle$ and $\langle u'_{\tau_L}{}^3 \rangle$, are required in Taylor's expansion method. It also takes longer to perform the calculations by the coordinate transformation method. Therefore, the selection of extrapolating method actually depends on the practical requirements of the user. If one has only a limited information of the large-scale flow field at high Re or has limited computing resources, the Taylor's expansion method might be a good choice, and vice versa.

The overall results of this section also can be regarded as a validation of the second hypothesis of the QSQH theory.

6.5 Further comments concerning extrapolation

6.5.1 Domain length requirement for the extrapolation database

We conducted the extrapolation based on the DNS databases of $Re_\tau = 950$, in order to predict the flow statistics at $Re_\tau = 4200$. In Table 1.1 a DNS database for $Re_\tau = 2000$ is also listed. However, we decided not to use this database in the validation and the extrapolation due to its relatively short domain length. For the range of flows we consider we found that the optimal streamwise cut-off length is $X_c^+ \approx 4000$, which is one-third of the domain length of the $Re_\tau = 2000$ database. If we would applied our large-scale filter with this database, only the first and second modes of Fourier series in streamwise would remain after cutting-off. It is quite

unlikely that the shape of large-scale structures can be precisely captured with only two sinusoidal functions. Therefore, to investigate the near-wall large-scale structures in a turbulent boundary layer, a relatively longer domain is required. We suggest that the length of the domain should be at least 12 times the channel half-width.

6.5.2 One source of error of coordinate transformation method

Although the extrapolation was proved to be very accurate, in general using the coordinate transformation method requires a certain caution. At one of the steps, the coordinate transformation method involves averaging over the homogeneous directions and/or time. For example, if the extrapolation data set consists of only one time snapshot, then the universal mean profile is defined as

$$\tilde{U}(\tilde{y}) = \frac{1}{T X Z} \int_{\tilde{t}=0}^{\tilde{t}=T} \int_{\tilde{x}=0}^{\tilde{x}=X} \int_{\tilde{z}=0}^{\tilde{z}=Z} \tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}) d\tilde{t} d\tilde{x} d\tilde{z},$$

where T , X , and Z are the dimensions of the box over which the averaging is performed. If the $\tilde{t}, \tilde{x}, \tilde{z}$ grid were uniform then the average could be approximated as the average of $\tilde{u}_i = \tilde{u}(\tilde{t}_i, \tilde{x}_i, \tilde{y}, \tilde{z})$ over all the grid points.

However, since $\tilde{t} = tu_{\tau_L}^2$, $\tilde{x} = xu_{\tau_L}$ and $\tilde{z} = zu_{\tau_L}$, the $\tilde{t}, \tilde{x}, \tilde{z}$ grid is not uniform. To rectify this problem, in calculating the integral as the average of all $\tilde{u}_i$ one can replace $\Delta\tilde{t}_i \Delta\tilde{x}_i \Delta\tilde{z}_i$ with $u_{\tau_L i}^4 \Delta t_i \Delta x_i \Delta z_i$ and arriving at the weighted average

$$\tilde{U}(\tilde{y}_k) = \frac{\sum_{i:\tilde{y}_i \in B_k} u_{\tau_L i}^4 \tilde{u}_i}{\sum_{i:\tilde{y}_i \in B_k} u_{\tau_L i}^4}$$

instead of (6.5). In the case when the extrapolation dataset consists of only one time snapshot, $u_{\tau_L i}^4$ should be replaced with $u_{\tau_L i}^2$. The same applies also to calculating all other similar averages in the coordinate transformation method.

When the Reynolds number of the extrapolation dataset is not high, u_{τ_L} rarely deviates far from one, and neglecting this deviation as it was done in (6.5) might

not lead to a large error. We checked this on several examples of our calculations and did not find much difference between the numerical values of weighted and unweighted averages: the universal functions were almost the same. In order to save the computing power, we decided not to include the weights in the extrapolating for the results presented in this work. Nevertheless, the possible discrepancy caused by this issue should always be examined, especially when the extrapolation dataset has higher Re .

6.6 A probe for measuring the large scale motions

This section is devoted to a method for measuring the large-scale motions in the turbulent boundary layer using a rake of specially designed probe. The method and the probe design are described in [16, 23, 29, 30] and [31]. Subsection 6.6.2 gives an overview of the results of these works largely following the more detailed exhibition given in [16] and [23]¹

6.6.1 The need for a special probe for measuring large-scale motions

The quality of extrapolation in Section 6.4 benefited from an accurate knowledge of the large-scale motions in the high- Re case. This was available because the high- Re case was represented by the DNS database consisting of the snapshots of the velocity in wall-parallel planes. This made it possible to apply a Fourier cut-off filter in two wall-parallel directions. However, obtaining such snapshots in experiments at high- Re would be very challenging. It might require PIV mea-

¹The results of the UG and MSc theses [16] and [23] presented here are not claimed as a part of the work presented in this thesis for fulfilling the requirements for a PhD degree. This material is provided here as the necessary background to the new results in subsection 6.6.3. Then in Subsection 6.6.3 we present the new results obtained in the present work. These new results are purely analytical and explain certain features of the results of [16] and [23].

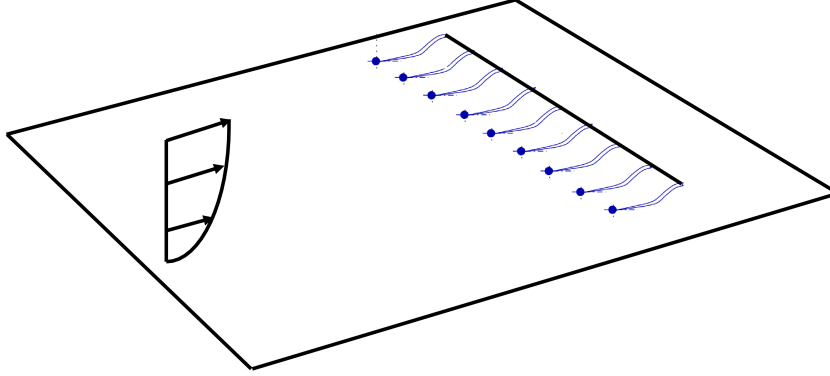


Figure 6.7: The probe consisting of a rake of velocity sensors

surement in wall-parallel planes in windows larger than or at least comparable to the characteristic size of large-scale motion. Hence, an alternative is required. If experimental measurements are to be made with sensors, obtaining long time series is feasible. In subsection 4.2.3, we concluded that any single-direction filtering is not enough for applications of the QSQH theory, and triple-direction filtering was recommended. We also concluded that time-spanwise filtering is a reasonable substitute of the triple-direction filtering. This following subsection describes a probe that is realistically feasible and that allows to approximate the result of exact time-spanwise filtering with reasonable accuracy [16, 23].

6.6.2 Review of the previous results

In order to apply the QSQH theory to the experimental data, it was proposed [16, 23] to use a probe array consisting of a spanwise rake of sensors as shown in Figure 6.7. The probe measures approximately the large-scale-filtered velocity, and the large-scale friction is then obtained with the help of the QSQH theory. The probe should be as close to the wall as possible. (Ideally, it should measure the wall friction.) In practice, the sensors can be located at the distance y_o^+ up to about 100 wall units from the wall, since the large-scale motions are reasonably well

correlated in wall normal direction up to this distance and since the QSQH theory provides a reasonably accurate relation between the large-scale velocity at this distance to the wall and the large-scale skin friction (see for example Figures 4.5 and 5.4 of the present thesis).

The probe gives the time series of the velocity measured by each sensor. The coordinates of the center probe are denoted (x^+, y_o^+, z^+) , and the coordinates of the k -th sensor are $(x^+, y_o^+, z^+ + \Delta z_k^+)$. The time-dependent velocity measured by each sensor is filtered using a Fourier cut-off filter in time, with a prescribed cut-off time T . Determining the cut-offs was one of the subjects of Chapter 4 of the present thesis, and the appropriate cut-offs were assumed to be known in [16, 23]. The result of this filtering can be denoted $u_T(t^+, x^+, y_o^+, z^+ + \Delta z_k^+)$. Then the approximated values of the large-scale velocity at the center of the probe are calculated as a weighted average of the time-filtered signals from all n probes:

$$u_{Lp}(t^+, x^+, y_o^+, z^+) = \sum_{k=1}^n w_k u_T(t^+, x^+, y_o^+, z^+ + \Delta z_k^+). \quad (6.9)$$

The weights w_k and the sensor locations Δz_k^+ were optimised in [16, 23] in such a way that the approximation (6.9) is as accurate as possible.

In [16] the optimal weights and the optimal sensor locations with respect to the probe were determined assuming that the data from the plane channel flow database for $\text{Re}_\tau = 4200$ would be used as the basic set of data for extrapolating to higher Re . It is the same database as one of the databases used in the present thesis. In general, the optimal probe parameters depend on the basis set of data and the particular filter used to determine the statistics of the universal velocity field $\tilde{u}$. If a different database is to be used as the basis for extrapolations then the optimization procedure will need to be repeated for this database and the corresponding filter.

The database for $\text{Re}_\tau = 4200$ does not contain enough time frames for per-

forming time filtering. However, using the Taylor hypothesis for determining the weights w_k and the sensor locations Δz_k^+ time was replaced with the streamwise coordinate x . One particular time frame was selected and a spatial Fourier cut-off filter with the cut-off lengths $L_x^+ \approx 4000$ and $L_z^+ \approx 450$, which are the same² as we used for this database in the present thesis, to obtain the actual large-scale filtered velocity $u_L(x^+, y_o^+, z^+)$. Note that the frames in this database are large enough to be representative. Similarly to the case of a probe in a physical experiment, it was assumed that the probe gives the data for the entire set of x values in the database, but only at the spanwise locations of the sensors. Hence, in this case the streamwise Fourier-cut-of filtering of the sensor signals could be performed by using the same cut-off length $L_x^+ \approx 4000$. This gives a partially filtered field, which will be denoted $u_{Lx}(t^+, x^+, y_o^+, z^+)$. Then the approximation for the large-scale filtered field was obtained from the formula

$$u_{Lp}(x^+, y_o^+, z^+) = \sum_{k=1}^n w_k u_{Lx}(x^+, y_o^+, z^+ + \Delta z_k^+). \quad (6.10)$$

There is no time arguments in (6.10) since only one time frame is being used. Then, w_k and Δz_k^+ were optimised so as to minimize the difference between the actual large-scale field $u_L(x^+, y_o^+, z^+)$ and its approximation $u_{Lp}(x^+, y_o^+, z^+)$ obtained with the probe.

Specifically, the grid layer at $y_o^+ = 100$ provided a slice of the velocity in the form $u = u(x_i^+, z_j^+)$, where x_i^+ and z_j^+ are the grid line coordinates. Applying to this slice the Fourier cut-off filter described above gave the corresponding large-scale component $u_L(x_i^+, z_j^+)$. The number of sensors $n = 9$ was assumed after consultations³ concerning the possible future experimental implementation. The optimal weights w_k and sensor displacements Δz_k^+ were found by minimizing over

²Studies [16, 23] were done with the help from the author of the present thesis, and their authors had access to and made extensive use of the results described in the present thesis.

³with Dr. O.Buxton.

w_k and Δz_k^+ the L_2 norm

$$\sum_{i,j} (u_L(x_i^+, z_j^+) - u_{Lp}(x_i^+, z_j^+))^2$$

of the difference between u_L and u_{Lp} , with u_{Lp} defined by (6.10). Optimisation over w_k is a simple quadratic optimisation. In the first instance, the sensors were assumed to be evenly spaced, $\Delta z_k^+ = (k - 5)\Delta z^+$, and the optimisation over Δz^+ was done by a line search. In the second approach, the sensor displacements were optimised using the genetic optimisation algorithm included in MATLAB. The sensor displacements were assumed to be symmetric with respect to the center sensor, but the weights were optimised for all nine sensors, and the symmetry of the obtained weights served as an additional test for the consistency of the optimisation procedure. The actual large-scale field u'_L and its approximation u'_{Lp} were then plotted side-by-side and found to be very similar, see Figure 6.8, which is a reproduction of Figures 11 and 12 from [16]. The relative error $\|u'_L - u'_{Lp}\|/\|u'_L\|$ was about 0.07 for the genetically optimised probe and about 0.08 for a constant spacing probe. Then out-of-sample test was performed: the optimised probe was used to obtain an approximation for the large-scale field from a different time snapshot in the database, and this gave the relative errors of about 0.08 and 0.09 respectively. For uniformly-spaced probes, the sensor spacing was found to be $\Delta z^+ = 64.13$. The weights and the relative positions of the sensors of the uniformly-spaced optimised probe and the genetically optimised probe are given in Table 6.1 and illustrated in Figure 6.9. The results were given with 4 digits, which are valid for a particular time frame from the database. However, by observing the variation of the results from one time frame to another, the authors believed that only two digits here are significant.

The same procedure was repeated for the database for $\text{Re}_\tau = 2000$, but for uniformly-spaced probes only. Importantly, it turns out that the Reynolds number

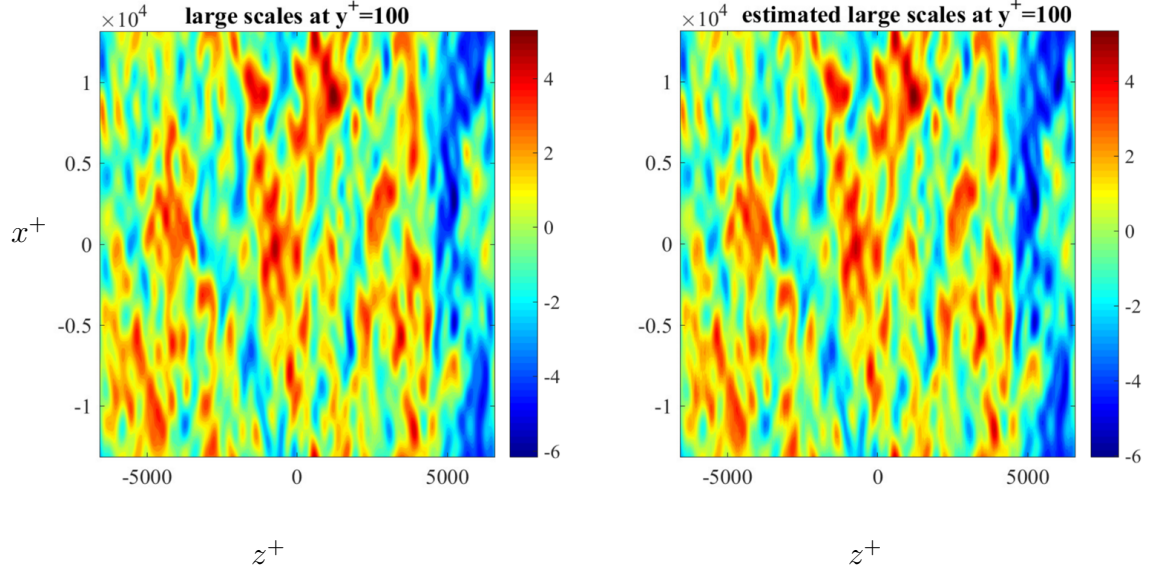


Figure 6.8: Comparison of the actual large-scale velocity distribution with the velocity distribution obtained with the probe, Figures 11 and 12 from [16].

	Uniform spacing		Non-uniform spacing	
k	Δz_k^+	w_k	Δz_k^+	w_k
5	0	0.3306	0	0.2564
4 and 6	± 64.13	0.2783	± 51.3	0.2482
3 and 7	± 128.26	0.1369	± 111.6	0.1979
2 and 8	± 192.39	0.0050	± 260.8	-0.0999
1 and 9	± 256.52	-0.0932	± 478.8	0.0322

Table 6.1: Positions and weights of sensors of the optimised probes, approximating the Fourier cut-off filter with the cut-off lengths $L_x^+ \approx 4000$ and $L_z^+ \approx 450$. Only two digits are expected to be significant, and the relative approximation error is believed to be about 0.08 for the non-uniformly spaced sensors and 0.09 for the uniformly spaced sensors [16].

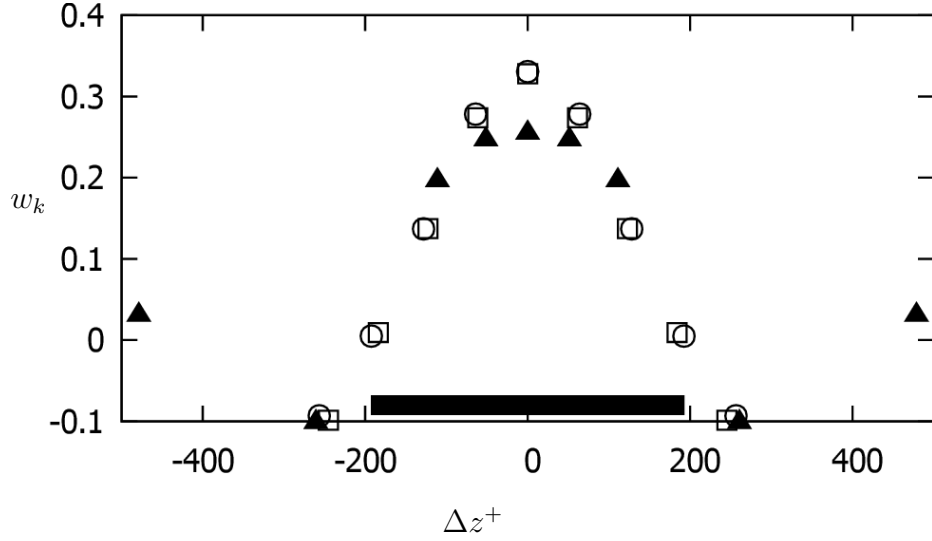


Figure 6.9: Weights: $\circ$ uniform $\text{Re}_\tau = 4200$, $\square$ uniform $\text{Re}_\tau = 2000$, $\blacktriangle$ genetically optimised $\text{Re}_\tau = 4200$. The horizontal bar shows the cut-off length [16].

has little effect on the optimal probe. The main effect of the genetic optimisation amounts to moving the sensor that has almost zero weight in the uniformly-spaced probe to a new position. Calculations with different cut-off lengths within a reasonable range were also done, and did not lead to large variations of the optimal probe. It can be concluded that the proposed procedure of selecting the parameters of the optimal probe can give an accuracy better than 10%, which is better than the accuracy of the majority of predictions of the QSQH theory achieved so far. Hence, with such a probe the QSQH theory can be used for extrapolation to the conditions in which the large-scale component is obtained from an experiment.

Finally, Figure 6.10 ([30], see also Figure 15 in [16]) shows the gain of the probe filter as compared with the gain of the cut-off filter, and a suitably-scaled premultiplied velocity spectrum. It can be seen that the optimization procedure trades off the agreement of the gains at the wavenumbers with low energy for the better agreement of the gain at the higher energy, as it can indeed be expected.

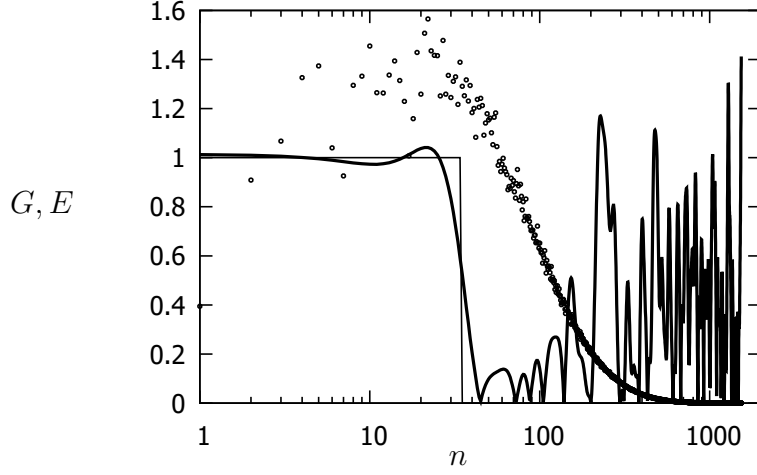


Figure 6.10: Filter gains G and premultiplied spectrum E versus the wavenumber n [16, 30]. Straight lines show the gain of the Fourier cut-off filter, the curve is the gain of the genetically-optimized 9-probe filter, and symbols are a sample of a premultiplied velocity spectrum.

6.6.3 Determining the weights analytically

The results reviewed in the previous subsection demonstrate the possibility to apply the QSQH theory in the experimental data. There remains, however, two issues. For the first one, as we can see in Figure 6.9, the weights for uniformly-spaced sensors are different from the weights for a genetically optimised probe. In a real experiment it will not be always possible to adjust the sensor spacing for different flow conditions. For the second one, a previous study [24] suggested that the minimum spanwise measuring gap between two convectional hot-wire sensors is about 0.01δ . Even the state-of-the-art nano-scale thermal anemometry probe can only reduce this gap by one order of magnitude [101]. Therefore, the optimised probe spacing, that is about 50 to 200 wall units, might not be practicable in the real applications where the Re_τ reaches the order of $10^5 - 10^7$. It would, therefore, be beneficial, if it is at all possible, to be able to determine the weights for a given sensor spacing without conducting additional studies. This subsection makes a step in this direction.

Any linear filter can be implemented by convolving the input signal with the filter impulse response, expressed as:

$$u_L(z) = w(z) \otimes u(z) = \int_{-\infty}^{\infty} w(\zeta) \cdot u(z - \zeta) d\zeta, \quad (6.11)$$

where $w(z)$ is the system's response to an impulse signal $u(\zeta)$, and $u_L(z)$ is the filtered signal. For a specific filter, the corresponding impulse response is unique and independent of the input signals. Therefore, we can try to interpret (6.10) as the discrete form of the convolution (6.11)

$$\sum_{k=1}^n w_k u_{Lx}(z_k^+) \approx \int_{-\infty}^{\infty} w(z^+) u_{Lx}(\zeta^+ - z^+) dz^+ \approx \sum_{k=1}^n w(z_k^+) u_{Lx}(z_k^+) \Delta z_k^+,$$

where Δz_k^+ are dependent on the quadrature used to approximate the integral. If this interpretation is valid, the weights should be related to the sensor locations and the impulse response function by the formula

$$w_k = w(z_k^+) \Delta z_k^+.$$

This can be verified by plotting $w_k / \Delta z_k^+$ for various cases, and checking that the results overlap. Moreover, the impulse response w_F of the Fourier cut-off filter, also known as the ideal low-pass filter, can be expressed via the sinc function:

$$w_F(z^+) = 2f_c \operatorname{sinc}(2f_c z^+) = \frac{\sin(2\pi f_c z^+)}{\pi z^+},$$

where f_c is the cut-off frequency/wave number. For the uniformly spaced sensors the Δz_k^+ for each weight naturally equals the fixed probe spacing. For the genetically optimised results, we let $\Delta z_k^+ = (z_{k+1}^+ - z_{k-1}^+)/2$ for the interior weights, and $\Delta z_1^+ = z_2^+ - z_1^+, \Delta z_{end}^+ = z_{end}^+ - z_{end-1}^+$ for the boundary weights.

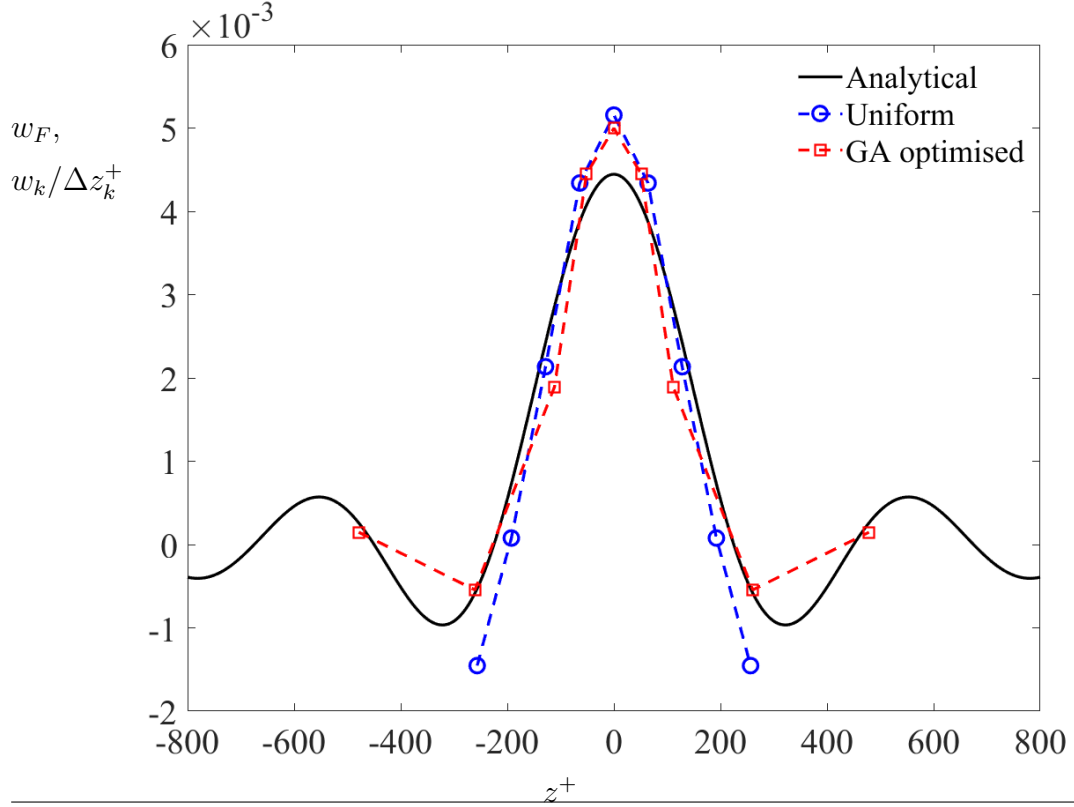


Figure 6.11: Comparison of the analytical expression for the impulse response w_F of the Fourier cut-off filter and the normalised optimal weights $w_k/\Delta z_k^+$ for the uniformly spaced and genetically optimised nonuniformly spaced sensors. Blue circles and red squares represent respectively the normalised optimal weights for uniformly spaced and nonuniformly spaces sensors.

Figure 6.11 shows the comparison of w_F and $w_k/\Delta z_k^+$. In general, the agreement confirms our interpretation of (6.10) as a discrete approximation of (6.11). The deviation near the edges of the probe might be due to the quality of the quadrature formula we use. It is apparent that the values of $w_k/\Delta z_k^+$ collapse better than the values of w_k , but that they are somewhat different from w_F . The explanation for this follows from Figure 6.10: $w_k/\Delta z_k^+$ are more likely to approximate the impulse response of the optimised 9-mode filter than the impulse response

w_F of the ideal Fourier cut-off filter.

It can be concluded that, if the probe has been optimised for the specific experiment conditions, designed, and manufactured, but has then to be used at somewhat different conditions and hence different sensor spacing as measured in wall units, the sensor weights can and should be readjusted to correspond to the same impulse response w .

Chapter 7

On the borderline of the QSQH theory: inclination of structures

7.1 Correlation reversal

Intuitively, when the large-scale velocity increases the fluctuation intensity should increase, too. However, the experiments and numerical calculations reveal a different picture. For example, the modulation coefficient $\beta(y)$ in (1.1), found by fitting experimental data [81] is positive near the wall but becomes negative farther away from it. This counter-intuitive reversal of the correlation between large-scale fluctuations and the intensity of small-scale fluctuations is now well ascertained by experiments and direct numerical simulations.

One well-known explanation of this phenomenon is illustrated in Figure 7.1, which is a modified fragment of Figure 13(a) from [51]. (In addition to what is included [51], in Figure 7.1 the inner and outer probes and the layer in which the QSQH theory applies are also shown.) This explanation relies on the fact that the structures are inclined. As a result, when the distance between the probes located one above the other as shown in Figure 7.1 equals half the characteristic streamwise

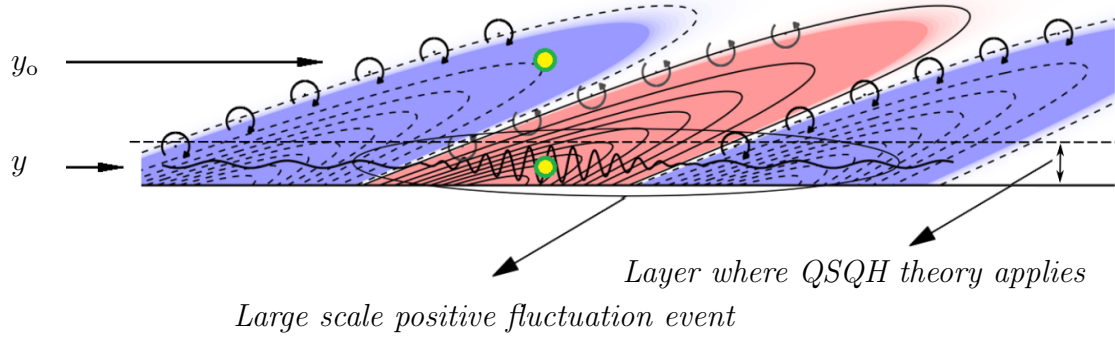


Figure 7.1: Schematic of inclined large-scale structures, with outer (at y_o) and inner (at y) probe positions marked with green circles. The figure is built on Figure 13(a) in [51].

period of the large scale structures multiplied by the tangent of the inclination angle, the correlation between the velocities measured by probes will be negative. The near-wall small-scale fluctuations can be expected to correlate positively with the large-scale velocity near the wall and, therefore, it should correlate negatively with the velocity measured by the outer probe.

Within the QSQH theory the effect of large-scale structures on the near-wall turbulence occurs via the proxy of the wall shear only and in a way that neglects the inclination of the large-scale structures. In Figure 7.1 this is reflected by the thickness of the layer where the QSQH theory applies being noticeably smaller than the characteristic streamwise dimension of the large-scale structures. As a result, the QSQH theory does not include the phase shift mechanism described above. However, the QSQH theory suggests [127] an alternative physical mechanism of the correlation reversal. This mechanism is illustrated in Figure 7.2. According to (3.13), the effect of the change in the large-scale fluctuation is twofold: an amplitude modulation and a frequency/scale modulation. When u_{τ_L} increases, the maximum of the fluctuation intensity curve also increases due to amplitude

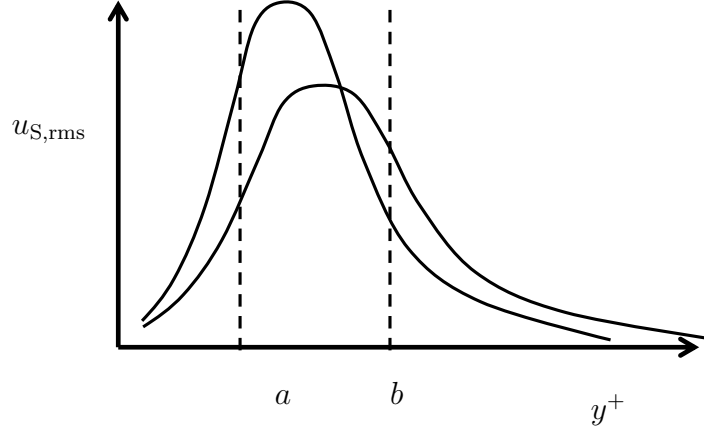


Figure 7.2: The physical mechanism of the correlation reversal: the curve with the higher maximum corresponds to positive large-scale fluctuation.

modulation, but it also shifts towards the wall due to the modulation of the wall-normal length scale. Near the wall ($y = a$ in Figure 7.2) both effects increase the fluctuation intensity at a fixed distance to the wall, but farther away where the intensity decreases with wall distance ($y = b$ in Figure 7.2), the scale-modulation effect works towards decreasing the fluctuation intensity. A somewhat similar mechanism was proposed in [61] for the reversal of the correlation of the intensity of the wall-normal fluctuations, where the effect of scale modulation was suggested to be leading to correlation reversal via the proxy of the wall shear, with which the fluctuation intensity was assumed to be in equilibrium.

In subsection 5.2.2 we introduced coefficients $\gamma_1(y)$ and $\gamma_2(y)$ characterizing the correlation between the wall shear and the amplitude of small-scale fluctuations. The quantitative comparisons of the QSQH theory predictions for these coefficients with their values obtained in DNS, see Figures 5.5 and 5.6, support the QSQH mechanism of the correlation reversal.

On the other hand, there is a discrepancy between the implications of the QSQH mechanism of Figure 7.2 and the observations. Namely, the observations show that the distance from the wall to the correlation reversal point, expressed in wall units, increases quite significantly with Re increasing, see, for example, Figure 4 in [32]. Applicability of a QSQH hypothesis at large distance from the wall can be questioned. Within the quadratic QSQH theory the Re effect on γ_1 is reduced to the change in $\langle u'_{\tau_L}{}^3 \rangle / \langle u'_{\tau_L}{}^2 \rangle$, which depends on the filter. Hence, at this point the question of whether the observed correlation reversal can be explained within the QSQH theory remains open.

An interesting question is whether another correlation reversal point will appear much closer to the wall for other Re and a different filter. Figure 1 in [83] shows the behaviour consistent with such an idea, namely, there is a minimum of the correlation coefficient as a function of the wall distance near $y^+ = 30$, that is where the slope of $\tilde{u}_{rms}(y)$ is negative. The quasi-steady mechanism of Figure 7.2 might provide an explanation of this minimum.

The considerations in this section assume a rather simplified picture of the inclination of the structures, and these considerations indicate that the issue need to be looked at in more detail, as it is done in the rest of this chapter.

7.1.1 Inclination of large-scale structures

Inclination of the large-scale motions might be one of the mechanisms reducing the accuracy of the QSQH theory. In the empirical model (1.1) of [77] the inner and outer probe large-scale signals were taken with a certain time shift that allowed the maximum correlation coefficient between them. In this section we will attempt to lay the first foundation of the possibility of a similar improvement of the QSQH theory by simply analyzing the inclination as a function of the length scale. Because the DNS databases we use have the periodic boundary

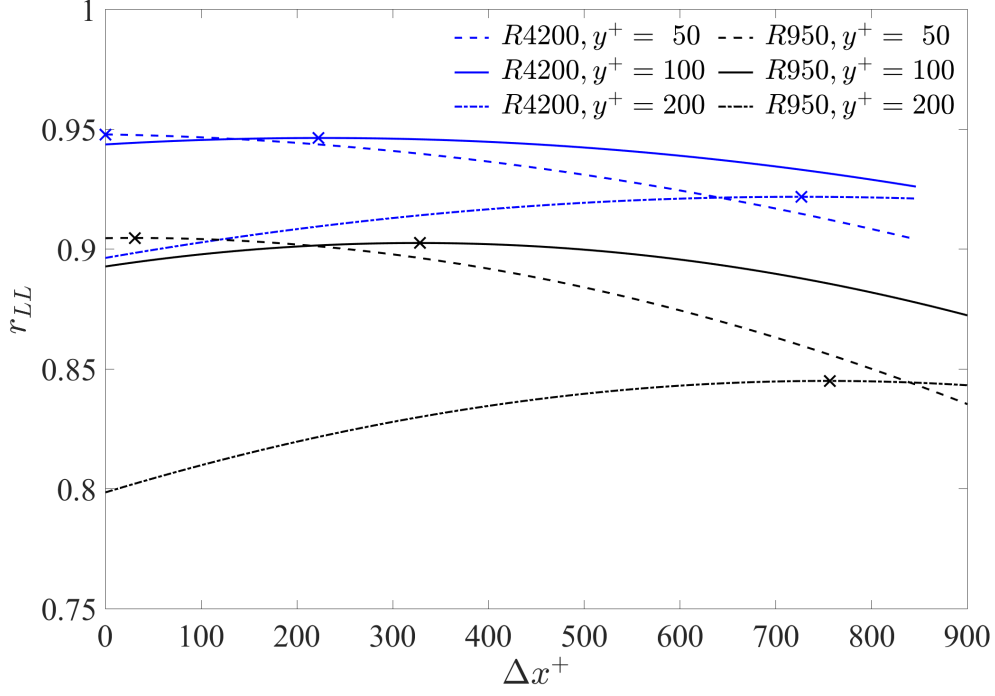


Figure 7.3: Correlation coefficients between the large-scale motions $u_L(x^+, +0)$ and $u_L(x^+ + \Delta x^+, y^+)$ for $y^+ = 50, 100, 200$ for the databases of $Re_\tau = 950$ and $Re_\tau = 4200$. The cross symbols indicate the maximum of each curve.

condition in the streamwise direction, it is much easier to calculate the correlation coefficients between the large-scale motions of the certain streamwise shift, as shown in Figure 7.3. The large-scale filter defining the large scales for this figure is the same as used throughout this thesis for the databases of $Re_\tau = 950$ and $Re_\tau = 4200$. The results show that the curves for different Re but the same wall-normal distance have a similar behaviour and reach the highest values at approximately the same streamwise shift distances. However, the shift maximising the correlation depends on the distance to the wall. For $Re_\tau = 4200$, the shift distances of highest correlation coefficient are about 200 wall units for $y^+ = 100$ and 700 wall units for

$y^+ = 200$, which indicate an inclination angle of approximately 12 degrees. There is no increase of the correlation coefficients to the case of $y^+ = 50$ with the shift. These observations are consistent with the results from the previous experimental studies [9, 51]. For $Re_\tau = 950$, the shift distances at $y^+ = 100$ and $y^+ = 200$ are slightly larger than those of $Re_\tau = 4200$, and optimizing the shift led to a small rise of the correlation coefficients even at $y^+ = 50$. The reasons for this behaviour might be insufficient separation between the small and large-scales, or a lower outer-peak for the flow at moderate-Re. However, the maximal increase in the correlation is only about 1% at $y^+ = 100$ and it is below 5% at $y^+ = 200$ for both cases. It can be concluded that considering the shift of large-scale motions has little benefit for the predicting accuracy of QSQH theory below $y^+ = 100$. A considerable improvement might be achieved at the higher region in the logarithmic layer, but this is not considered in the present study. The results also imply that the shift distances are Re-dependent at moderate-Re for the large-scale motions obtained by the multi-optimisation method.

7.2 Negative small-small correlation

In subsection 4.2.3 we mentioned that the small-small correlation coefficients r_{SS} have negative values over a part of many Pareto fronts. Here we will further investigate this behaviour. We start with comparing a uni-directional streamwise Fourier cut-off filter with a uni-directional spanwise Fourier cut-off filter by plotting r_{SS} vs r_{LL} as parametrised by the single cut-offs, see Figure 7.4. Since it is a single-direction filtering, strictly speaking these curves are not Pareto fronts, because all possible combinations of r_{SS} and r_{LL} are on the curve, not to one side of it. However, for uniformity of terminology we will refer to them as Pareto fronts. For the spanwise filtering curve, r_{SS} gradually approaches zero as r_{LL} decreases, remaining always positive. For the streamwise filtering curve, as r_{LL} decreases r_{SS}

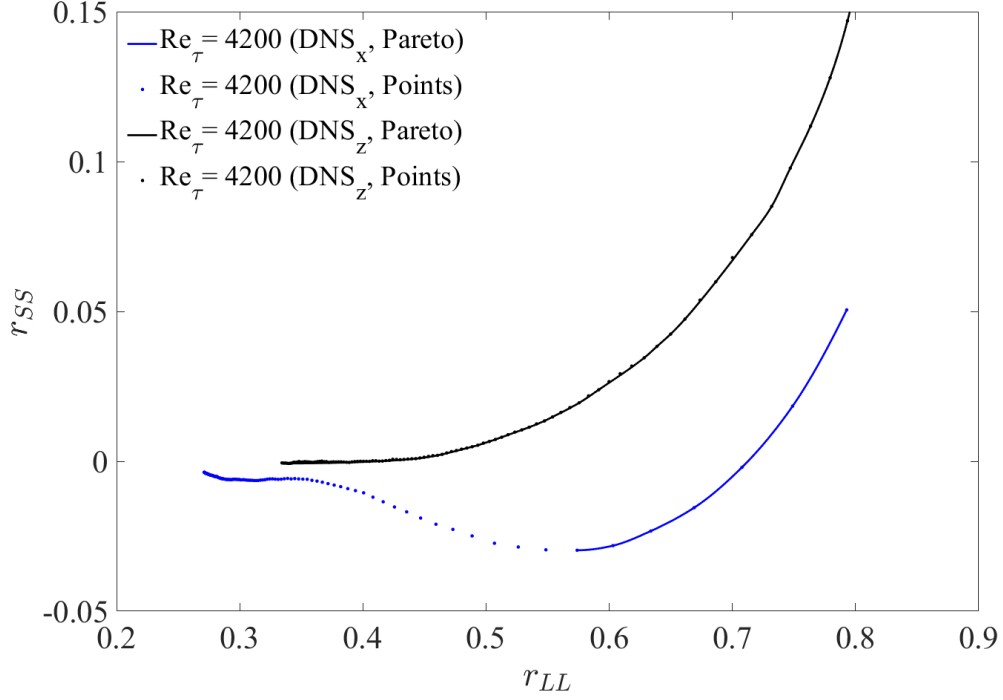


Figure 7.4: Pareto fronts (solid line) of large-large and small-small correlation coefficients of streamwise only and spanwise only filtering for the database of $Re_\tau = 4200$, with $y_o^+ = 100$. Each point represents a specific length of the cut-offs.

drops below zero at $r_{LL} \approx 0.7$, attains a minimum at $r_{LL} \approx 0.58$, and recovers back to zero as r_{LL} decreases further. Also, there is a tiny drop of r_{SS} at $r_{LL} \approx 0.3$. Moving from right to left along single-direction filtering curves corresponds to a reduction in the filtering length. Therefore, these observations suggest that the contribution of streamwise motions with certain wavelength oscillate as a function of the wavelength. However, from Figure 7.4 it is hard to determine the wavelength of particular motions that contribute to the negative correlation. To investigate the possible reason for negative correlation in detail, we designed a special correlation

function

$$r_{k_x k_z}(y) = \langle u'_m(k_x, k_z, y_{\approx 0}) u'_m(k_x, k_z, y) \rangle / \sqrt{\langle u'^2_m(k_x, k_z, y_{\approx 0}) \rangle \langle u'^2_m(k_x, k_z, y) \rangle}, \quad (7.1)$$

where $y_{\approx 0}$ is a point very close to the wall, and u'_m is the velocity component consisting only of the Fourier modes with the wavenumbers k_x and k_z . It is obtained by calculating the 2D (x - z) Fourier transform $\hat{u}'(\hat{k}_x, \hat{k}_z)$ of the original velocity fluctuation $u'(x, y, z)$, setting to zero the amplitude of all the Fourier modes except those with $\hat{k}_x = \pm k_x$ and $\hat{k}_z = \pm k_z$, and calculating the inverse Fourier transform. Hence, $r_{k_x k_z}$ is the correlation coefficient of the sum of the $\pm k_x, \pm k_z$ Fourier modes of the skin friction and the sum of the $\pm k_x, \pm k_z$ Fourier modes of the velocity at the distance y from the wall. As a result, r_{SS} and r_{LL} can be obtained as sums of $r_{k_x k_z}$ over k_x, k_z corresponding to small and large scales respectively.

Even for the $Re_\tau = 1000$ database containing a large number of time frames the contours of $r_{k_x k_z}$ are too irregular due to large averaging errors. To smooth the result to some extent, in Figures 7.5, 7.6 and 7.7 we plot the average over three neighbouring wavenumbers, namely, $(r_{k_x k_z-1} + r_{k_x k_z} + r_{k_x k_z+1})/3$, for different wall-normal distances. Negative correlation regions can be identified in all three cases. The minimal values are located roughly between $\lambda_x = 800$ to $\lambda_x = 1800$ and extend up to $\lambda_z = 60$ for $y^+ = 50$ case, centering at about $\lambda_x = 1600$ and extend up to $\lambda_z = 120$ for $y^+ = 100$ case, and at about $\lambda_x = 2800$ and extend up to $\lambda_z = 250$ for $y^+ = 200$ case. It seems that the size of the motions that are responsible for the negative correlation increases proportionally to wall-normal distance, and is close to the size of LSM. It is likely that these objects are the same as the 'wall-attached self-similar structures' proposed by Baars *et al.* [10]. They found that the streamwise/wall-normal aspect ratio of the structures is about 14, which agrees with the dimensions of the motions discussed here. An intuitive explanation of the observed behaviour is the phase difference between the motions at the wall and

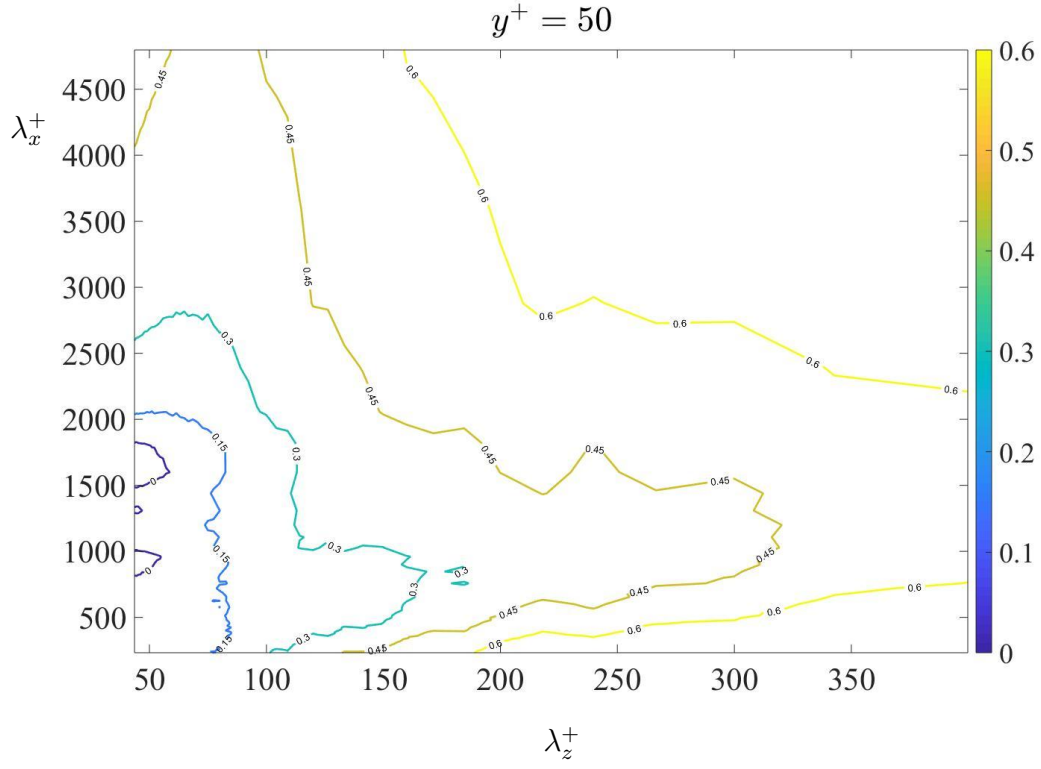


Figure 7.5: Averaged coefficient $(r_{k_x k_z - 1}(y^+) + r_{k_x k_z}(y^+) + r_{k_x k_z + 1}(y^+))/3$ at $y^+ = 50$ for the database of $Re_\tau = 1000$, plotted in $(\lambda_x^+, \lambda_z^+)$ space, where $\lambda_x^+ = 2\pi/k_x^+$ and $\lambda_z^+ = 2\pi/k_z^+$.

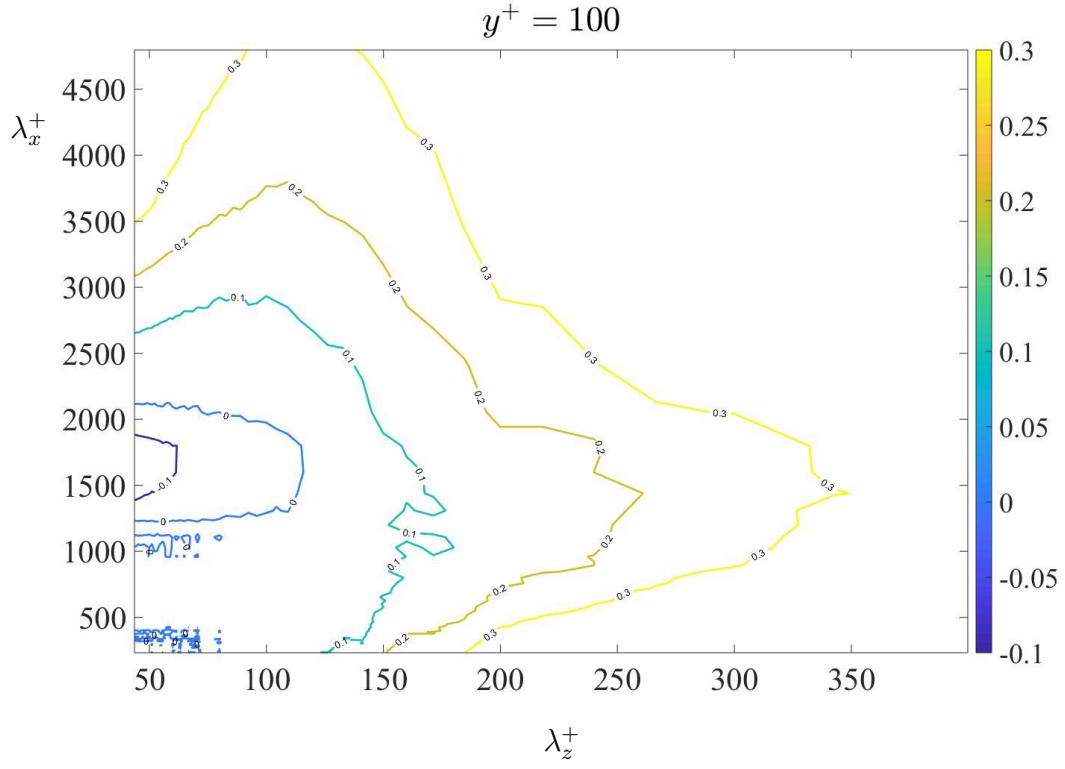


Figure 7.6: Averaged coefficient $(r_{k_x k_z - 1}(y^+) + r_{k_x k_z}(y^+) + r_{k_x k_z + 1}(y^+))/3$ at $y^+ = 100$ for the database of $Re_\tau = 1000$, plotted in $(\lambda_x^+, \lambda_z^+)$ space, where $\lambda_x^+ = 2\pi/k_x^+$ and $\lambda_z^+ = 2\pi/k_z^+$.

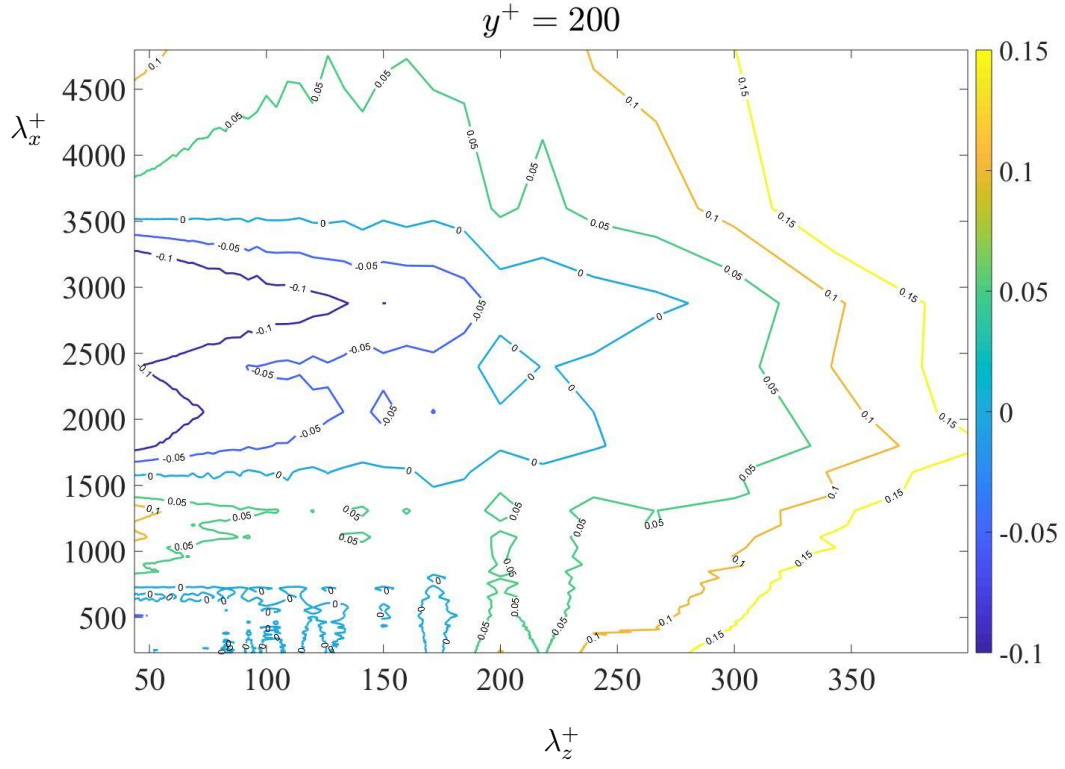


Figure 7.7: Averaged coefficient $(r_{k_x k_z - 1}(y^+) + r_{k_x k_z}(y^+) + r_{k_x k_z + 1}(y^+))/3$ at $y^+ = 200$ for the database of $Re_\tau = 1000$, plotted in $(\lambda_x^+, \lambda_z^+)$ space, where $\lambda_x^+ = 2\pi/k_x^+$ and $\lambda_z^+ = 2\pi/k_z^+$.

outer layers of certain wavelengths. If so, the phase shift distances should be half of the wavelengths, that is about $\Delta_x = 800$ for $y^+ = 100$ case and about $\Delta_x = 1400$ for $y^+ = 200$ case. Since the negative regions of $y^+ = 50$ case are ambiguous, it will not be included in the following analysis. However, the results imply that the near-wall small-scale structures may also be influenced by the medium-scale motions below $y^+ = 100$.

Due to insufficient time snapshots in the other databases, we cannot obtain a well-converged 2D correlation contours at higher Re. Instead of that, the mode-by-mode correlation coefficients $r_{k_x}(y)$ of the streamwise Fourier transform of u' at different wall-normal distances for both Re were calculated, defined as

$$r_{k_x}(y) = \langle u''_m(k_x, y_{\approx 0}) u''_m(k_x, y) \rangle / \sqrt{\langle u''_m{}^2(k_x, y_{\approx 0}) \rangle \langle u''_m{}^2(k_x, y) \rangle},$$

where $u''_m(k_x, y)$ is obtained in a way similar to the way $u'_m(k_x, k_z, y)$ in (7.1) but using a 1D Fourier transform in the streamwise direction. The plots of $r_{k_x}(y)$ are shown in Figure 7.8.

Since for each streamwise mode all the spanwise modes were accounted for in $r_{k_x}(y)$, the contributions of the negative correlations between the motions of certain streamwise and spanwise wavelengths is less than the positive contributions, and $r_{k_x}(y)$ is positive for all k_x . However, for each curve, there is still a local minimal value at a certain wavelength. The results still imply the existence of the streamwise phase shift, which also can be assumed to be approximately equal to the half of the wavelength minimising $r_{k_x}(y)$. All the results on the streamwise phase shift are summarized in Table 7.1 and Figure 7.9.

Figure 7.9 provides a convenient way of comparing all the results. First of all, the comparison shows that the Reynolds number dependence of the inclination angle of the QSQH large scale structures mentioned in subsection 7.1.1 is comparatively small, and that their angle is close to the widely cited 12° angle of the outer structures, which confirms, as could be anticipated, that the QSQH

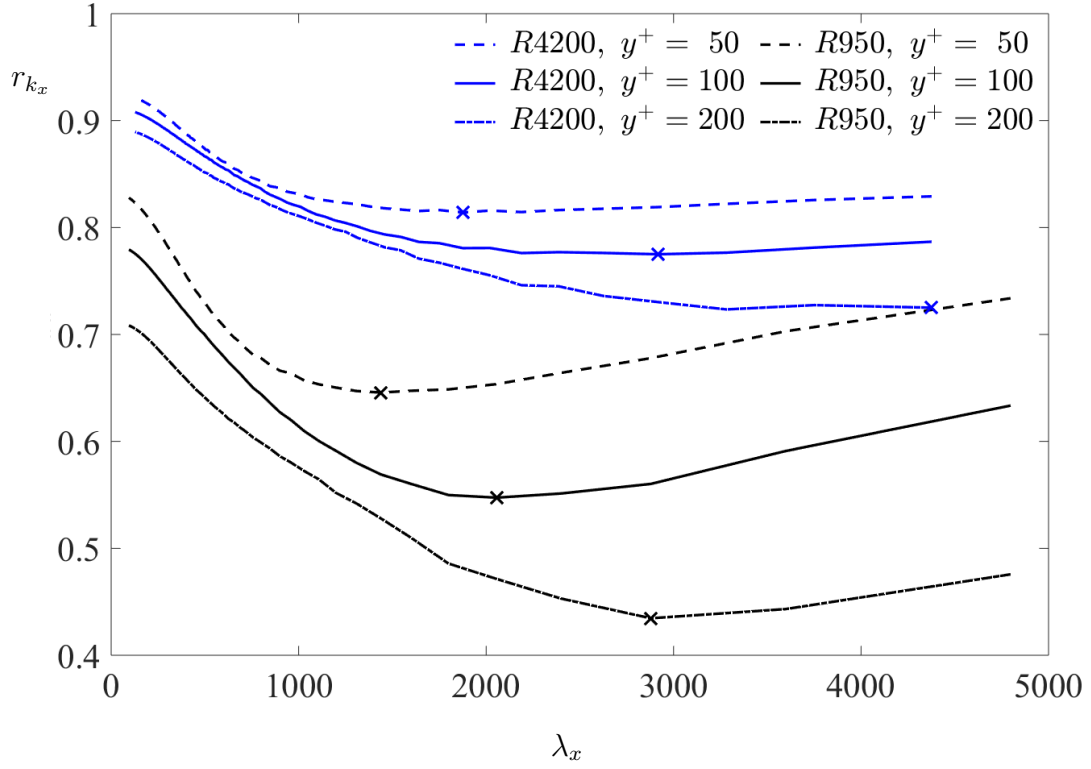


Figure 7.8: Coefficient $r_{k_x}(y^+)$ at $y^+ = 50, 100, 200$ for the databases of $Re_\tau = 950$ and $Re_\tau = 4200$, plotted against $\lambda_x = 2\pi/k_x$. The cross symbols mark the minimums.

Fig.	Type	$\lambda_x^+ = 2\pi/k_x^+$	$\lambda_z^+ = 2\pi/k_z^+$	Re_τ	y^+	$\approx \Delta_x^+$
7.3	LL	all > 4000	all > 450	950	50	50
7.3	LL	all > 4000	all > 450	950	100	350
7.3	LL	all > 4000	all > 450	950	200	750
7.3	LL	all > 4000	all > 450	4200	50	0
7.3	LL	all > 4000	all > 450	4200	100	200
7.3	LL	all > 4000	all > 450	4200	200	700
7.6	2Dmode	1600	all < 120	1000	100	800
7.7	2Dmode	2800	all < 250	1000	200	1400
7.8	1Dmode	1400	all	950	50	700
7.8	1Dmode	2100	all	950	100	1050
7.8	1Dmode	2900	all	950	200	1450
7.8	1Dmode	1900	all	4200	50	950
7.8	1Dmode	2900	all	4200	100	1450
7.8	1Dmode	4400	all	4200	200	2200

Table 7.1: Table of the shifts

large scale structures as defined by the specific cut-off filter we used effectively correspond to the outer structures.

Second, from the two-dimensional mode results (see the diamond symbols in Figure 7.9) we can see that the structures contributing most to the negative correlation are rather elongated, with the length-to-width ratio (that is λ_x/λ_z) being larger than 10. Third, from the one-dimensional mode results (see the square symbols in Figure 7.9) we can see that length of the structures contributing most to the minimal correlation is somewhat greater than 1000 wall units and increases no-

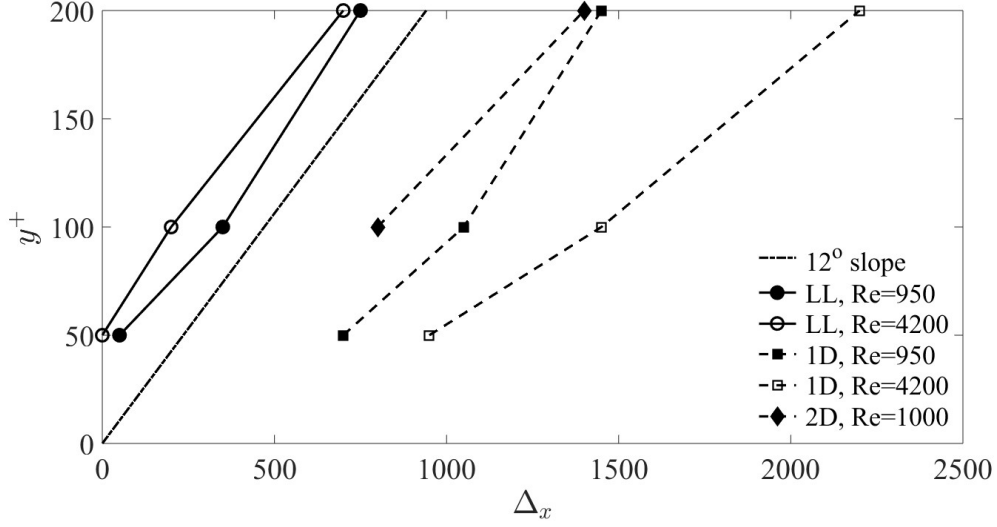


Figure 7.9: Streamwise phase shifts from Table 7.1. Note that the symbols connected by dashed lines do not correspond to the same wavelengths. Circle symbols correspond to the large scales of the QSQH theory. Other symbols are for modes responsible for negative or minimal correlations. Squares and diamonds are for the particular wavelengths shown, diamonds correspond to two-dimensional wavenumbers while squares are ‘spanwise-averaged’ and correspond to single streamwise wavelengths. Open symbols correspond to higher Re.

ticeably as Re increases. Therefore, these correlation-reducing structures are not the classical near-wall streaks. Physically, they are neither purely large-scale nor purely small-scale, since their characteristic streamwise dimension is more close to the large-scale dimension but their spanwise dimension is more close to the small-scale dimensions.

This observation brings attention to an issue that was not discussed so far. Namely, even if the QSQH large-scale filter is selected to be a Fourier cut-off filter and the particular values of the cut-offs are determined, there remains a choice to be made. The convention adopted in the present work is that the large scales

consist of the modes with all the wavelengths being greater than the corresponding cut-off wavelength. As a result, the correlation-reducing structures just considered turn out to be a part of the small scales. It is, however, possible to define the large scale motion to consist of all the modes with at least one wavelength greater than the corresponding cut-off. Then these structures will be a part of large scales. In any case, being physically neither purely large nor purely small, these structures should contribute to the deviation of the QSQH theory from the actual flow behaviour. Admittedly, this contribution is not very large, as demonstrated by the success of the comparisons we made. Still, if, very crudely, we assume that this contribution is measured by the greatest in absolute value negative correlation $r_{SS} \approx -0.03$, see Figure 7.4, that is 3% of the maximum possible value of $r_{SS} = 1$, and compare it with the characteristic error of the QSQH theory at the same Re , which again very crudely, is about 10% on average over the comparisons made, then this contribution turns out to be about one third of the total error. Hence, it can be concluded that as well as taking into account the inclination of large-scale structures, optimizing the definition of the large-scale filter with a view to these intermediate structures can be one of the routes of further improving the accuracy of the predictions of the QSQH theory.

Chapter 8

Conclusions

Overall, the three main overarching outcomes of the present work is the development of the technique of selecting the large-scale filter for applications of the QSQH theory, the confirmation of the QSQH theory, and the development of the techniques of extrapolating the statistics of near-wall turbulent flows from medium to higher Reynolds numbers on the basis of the QSQH theory. They will be concluded respectively in following text.

8.1 Developing the large-scale filter

In general, the multi-objective optimisation approach developed in the present study for determining the large-scale filter to be used in applications of the QSQH theory consists in the following. The specific optimisation objectives and the type of the filter should be selected on the case-by-case basis depending on the goal of the particular application and the availability of the filter. Then the Pareto front in the space of the filter parameters is to be found on the basis of the available data. Finally, the point on the Pareto front, and thus the filter, is selected on the case-by-case by application-specific considerations. The specific multi-objective

optimisation approach used in the present study had two objectives: a large correlation between large-scale fluctuations near the wall and in the layer at a certain finite distance from the wall and a small correlation between the small-scales in the same layers. The large-scale filters used in the present study were Fourier cut-off filters in time, streamwise, and spanwise directions. The cut-offs were the parameters that were optimized using the Pareto front. The specific considerations determining the point on the Pareto front were the performance of the QSQH theory in predicting relations between various statistics of the near-wall turbulence and the quality of extrapolation of near-wall turbulence statistics to higher Reynolds number. For the latter, additional constraints of the filtering lengths were found based on the energy spectra analysis. In essence, for the success of extrapolation the point on the Pareto front should be selected to correspond to the largest scales such that the change of the Reynolds number required in the extrapolation would not affect the small scales as described by the resulting universal velocity distribution of the QSQH theory. Several other findings useful in selecting the large-scale filter for a particular application were also made. The Pareto fronts with the objective of correlation between the small scales being replaced with the large-scale fluctuation energy were found to be close to the plots of the large-scale fluctuation energy versus the correlation between large scales. This suggests that using such a Pareto front instead of the Pareto fronts used in the present study can be expected to make little difference.

It was found that while filtering in time and wall-parallel directions gives the best results, using only wall-parallel filtering also gives satisfactory results provided that the data samples available are sufficiently large. In all cases the spanwise filtering is found to be essential. Contrary to what can be expected on the basis of the Taylor hypothesis, the Pareto front obtained using time filtering noticeably differs from the Pareto front obtained using the streamwise filtering. It was shown

that as Re increases, the Pareto fronts move in the direction which indicates that the predictions of QSQH theory might become more accurate. Finally, by calculating the Pareto fronts with the outer layer placed at different wall-normal distances, it was demonstrated that in a wide range near the middle of logarithm layer, the optimal solutions do not depend strongly on the location of the outer layer used for defining the optimisation objectives. However, the researcher can be limited to the outer layer located more close to the wall by the Reynolds number of the database used for extrapolation: moving the outer layer further away from the wall requires this Reynolds number to be higher. With the database $Re_\tau = 950$ the region of applicability of the QSQH theory with the error of the order of 10% was found to be the near-wall region with $y^+ < 100$. In summary, selection of the cut-off lengths for the Fourier cut-off filter should satisfy the following constraints:

- The cut-off lengths should correspond to the points on the Pareto front the objectives of which are selected for the specific research purpose for which the filter will be used.
- The upper bound of the cut-off length is at the edge of the area of the small negative and the large positive value of the difference between the two-dimensional energy spectrum of high and low Re .
- The lower bound of the cut-off length is outside the region corresponding to the near-wall streaky motions in the two-dimensional energy spectrum.
- The cut-off lengths should not be close to the wavelengths of the motions exhibiting negative correlations between the wall friction and the velocity away from the wall.
- The large-scale motions obtained by the filter should not contain any visible small-scale motion in any transverse plane from the wall to the logarithmic layer.

8.2 Validating the QSQH theory

The results concerning the validity of the QSQH theory constitute a second significant part of the present study. In general, these results confirmed both major hypotheses constituting the theory, demonstrated that there are cases when the full nonlinear version of the theory produces results significantly more accurate than the results of its linearised version (which gives a theoretical derivation of the well-known empirical predictive model proposed by the group led by Prof. Marusic), and revealed more detail of the effects causing the (not so large) deviation of the predictions of the theory from reality.

In particular, expansions for various quantities were found with respect to the amplitude of the large-scale fluctuations. It was shown that taking into account more terms of the expansion improves the agreement between the theory and the numerical data. The comparisons for investigating the accuracy of truncated expansions were done on the basis of the data from the DNS channel flow databases of $Re_\tau = 950$ and $Re_\tau = 4200$. In particular, the well-known so-called ‘superposition’ coefficient entering the aforementioned predictive model was shown to be predicted by the theory quite accurately, but other similar measures of the correlation between the large-scale fluctuations at the wall and at 100 wall units from the wall were predicted only within 15% accuracy for the database of $Re_\tau = 950$, and within 10% accuracy for the database of $Re_\tau = 4200$. Quantitative results on the prediction of the known effect of the reversal of the correlation between the large-scale velocity and the fluctuation intensity were obtained. It was shown that the predictions obtained under the assumption that the fluctuations of the large-scale signal are small improved when more terms of the expansion were taken into account. The discrepancy was too large to make definite conclusions about the correlation reversal mechanism, thus leaving open the question of whether the

explanation on the basis of the quasi-steady quasi-homogeneous theory is correct or another physical mechanism is involved. It was observed that the discrepancy was less related to the variation in the Reynolds number than to the variation of the shape of the Pareto front. As a by-product of the calculations of the Pareto front, it was observed that for certain combinations of the filter cut-offs on the Pareto front the correlation between small scale motions near the wall and small scale motions at the outer layer used to construct the Pareto front are negative. A detailed investigation of this effect and its relation to the inclination of the near-wall structures revealed that this negative correlations were due to the structures (or modes) with the streamwise length of the order close to the dimension of the large scales but of width of the order close to the dimension of the small scales. This suggested, that including these structures into the small scales, as it is done by the cut-off filter used, can be a source of the error in the predictions of the QSQH theory in addition to the inclination of the large-scale structures and the finite value of the Reynolds number.

8.3 Extrapolating the statistics

The third significant part of the present study concerns extrapolation of the statistics of near-wall turbulence from medium to high Reynolds number. In general, the results of this part confirmed further the QSQH theory especially with respect to Reynolds-number independence of the universal statistics, revealed the subjective nature of the universal velocity of the QSQH theory while explaining at the same time why the QSQH-theory-based extrapolation can be successful in spite of the continuity of the spectrum of turbulent fluctuations, and provided the specific methodology of such an extrapolation, including the theoretical basis for the design of the large-scale probe that can be used in a real experiment.

In particular, it was discovered that the variation of the large-scale filter cut-

offs affects the root mean square of the universal velocity much stronger than it affects the prediction of the QSQH theory for the variation of the actual (full) root mean square velocity with the Reynolds number. Further analysis led to the conclusion that as far as the changes caused by a change in the Reynolds number are restricted, by the choice of the filter, to large scales, the extrapolation based on the use of the same cut-off filter for both the database from which extrapolation is done and the target Reynolds number, the extrapolation can be successful even when there is no spectral gap. The hypothesis that the statistical properties of the universal small-scale signal defined in the quasi-steady quasi-homogeneous theory are independent of Re was validated by extrapolating certain statistical properties from moderate to high- Re (using the databases of $Re_\tau = 950$ and $Re_\tau = 4200$). Two methods based on the QSQH theory were developed. The general idea of both methods is that the full data at a moderate Re are used to obtain the statistical properties of the universal, that is Re -independent, quantities and the predictions are made based on the knowledge of the large-scale signal only at larger Re . The first method utilizes Taylor's expansion in the magnitude of the fluctuations of the large-scale motion. This allows to derive the expressions for the statistical quantity of interest, which are then used in extrapolations. As an example of this method, extrapolating u_{rms}^2 from $Re_\tau = 950$ to $Re_\tau = 4200$ within the error of 10% was presented. This method is convenient, but its accuracy is lowered by the errors introduced by truncating the Taylor expansion. In the second method, the particular statistics of interest is expressed using the QSQH theory via the corresponding statistics of the universal field and properties of large-scale motion. Then this corresponding statistics is calculated using the available data at moderate Reynolds number. Finally, substituting the large-scale skin-friction velocity of the high- Re flow into the obtained expression, the desired statistics of the high- Re flow are obtained. In our examples, the u_{rms}^2 and the two-point

correlation $\langle u'(y_1)u'(y_2) \rangle$ were extrapolated from $Re_\tau = 950$ to $Re_\tau = 4200$ with the overall error under 5%. The predictions of the second method are obviously more accurate. However, it involves time-consuming computations. Therefore, the selection of the extrapolating method will depend on the compromise of target statistical properties, accuracy, and computing resources requirements in practical applications. A numerical study was conducted to examine the possibility of reconstructing large-scale motions from the signal collected by a spanwise rake of 9 velocity probes. DNS channel flow databases of $Re_\tau = 2000$ and $Re_\tau = 4200$ were used to simulate the experimental condition. As a result, a relative error of 8% between the large-scale sample motions and reconstructed motions was achieved. The accuracy of reconstructing was found to be practically independent of the Reynolds number.

The QSQH theory presents a vast area of research, only a small part of which could be covered in the course of the present study. However, the results reported herein constitute a significant contribution to this field.

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Appendix

This appendix gives a detailed derivation of certain statistical properties.

For the function $\tilde{U}(y)$ in Taylor's expansion method, we start with

$$\begin{aligned} u(y) &= u_{\tau_L} \tilde{u}(\tilde{y}) = (1 + u'_{\tau_L}) \left(\tilde{u}(y) + y u'_{\tau_L} \frac{d\tilde{u}(y)}{dy} + \frac{1}{2} (y u'_{\tau_L})^2 \frac{d^2 \tilde{u}(y)}{dy^2} + \dots \right) \\ &= \tilde{u}(y) + u'_{\tau_L} \left(\tilde{u}(y) + y \frac{d\tilde{u}(y)}{dy} \right) + u'^2_{\tau_L} \left(\frac{y^2}{2} \frac{d^2 \tilde{u}(y)}{dy^2} + y \frac{d\tilde{u}(y)}{dy} \right) + \dots \end{aligned}$$

Then, averaging both sides and applying the property 4 of the large-scale filter gives

$$\begin{aligned} U(y) &= \tilde{U}(y) + \left\langle u'_{\tau_L} \left(U(y) + \left\langle y \frac{d\tilde{u}(y)}{dy} \right\rangle \right) \right\rangle + \left\langle u'^2_{\tau_L} \left(\left\langle \frac{y^2}{2} \frac{d^2 \tilde{u}(y)}{dy^2} \right\rangle + \left\langle y \frac{d\tilde{u}(y)}{dy} \right\rangle \right) \right\rangle + \dots \\ &= \tilde{U}(y) + \left\langle u'^2_{\tau_L} \right\rangle \frac{d}{dy} \left(\frac{y^2}{2} \frac{d\tilde{U}}{dy} \right) + \dots \end{aligned}$$

Neglecting the higher order terms gives $U(y) \approx \tilde{U}(y)$.

For the function $u_{rms}^2(y)$ in Taylor's expansion method,.

$$\begin{aligned} (u(y) - U(y))^2 &= \left(\tilde{u}(y) + u'_{\tau_L} \left(\tilde{u}(y) + y \frac{d\tilde{u}(y)}{dy} \right) + u'^2_{\tau_L} \left(\frac{y^2}{2} \frac{d^2 \tilde{u}(y)}{dy^2} + y \frac{d\tilde{u}(y)}{dy} \right) - \tilde{U}(y) + \dots \right)^2 \\ &= \tilde{u}'^2(y) + A u'_{\tau_L} + B u'^2_{\tau_L} + \dots, \end{aligned}$$

where $\tilde{u}'(y) = \tilde{u}(y) - \tilde{U}(y)$. Since $\langle u'_{\tau_L} \rangle = 0$, the second term will become zero after

averaging. Hence, we calculate only B and also neglect the higher-order terms.

$$\begin{aligned}
B &= \left(\tilde{u}'(y) + \tilde{U}(y) + y \frac{d\tilde{u}(y)}{dy} \right)^2 + 2\tilde{u}(y) \left(\frac{y^2}{2} \frac{d^2\tilde{u}(y)}{dy^2} + y \frac{d\tilde{u}(y)}{dy} \right) \\
&= \left(\tilde{U}(y) + y \frac{d(\tilde{U}(y) + \tilde{u}'(y))}{dy} \right)^2 + \left(\tilde{u}'^2(y) + 4\tilde{u}'(y)y \frac{d(\tilde{U}(y) + \tilde{u}'(y))}{dy} + \tilde{u}'(y)y^2 \frac{d^2(\tilde{U}(y) + \tilde{u}'(y))}{dy^2} \right) + \\
&\quad 2\tilde{u}'(y)\tilde{U}(y) = \left(\tilde{U}(y) + y \frac{d\tilde{U}(y)}{dy} \right)^2 + \frac{1}{2} \frac{d^2(y^2\tilde{u}'^2(y))}{dy^2} + C\tilde{u}'(y) + D \frac{d\tilde{u}'(y)}{dy}.
\end{aligned}$$

Here C and D are constants. Since $\langle \tilde{u}'(y) \rangle = 0$ and $\left\langle \frac{d\tilde{u}'(y)}{dy} \right\rangle = 0$, the last two terms will become zero after averaging. By taking the average of both sides of the previous equation, applying the property 4 of the large-scale filter and neglecting the high-order terms, we arrive at the expression

$$u_{\text{rms}}^2(y) \approx \tilde{u}_{\text{rms}}^2(y) + \left\langle u_{\tau_L}'^2 \right\rangle \left[\left(\tilde{U}(y) + y \frac{d\tilde{U}}{dy} \right)^2 + \frac{1}{2} \frac{d^2(y^2\tilde{u}_{\text{rms}}^2)}{dy^2} \right].$$

For the function $\langle u'(y_1)u'(y_2) \rangle$ in the coordinate transformation method,

$$\begin{aligned}
\langle u'(y_1)u'(y_2) \rangle &= \langle (u_{\tau_L}\tilde{u}(\tilde{y}_1) - U(y_1))(u_{\tau_L}\tilde{u}(\tilde{y}_2) - U(y_2)) \rangle \\
&= \langle U(y_1)U(y_2) - u_{\tau_L}U(y_1)u(\tilde{y}_2) - u_{\tau_L}U(y_2)u(\tilde{y}_1) + u_{\tau_L}^2\tilde{u}(\tilde{y}_1)\tilde{u}(\tilde{y}_2) \rangle \\
&= U(y_1)U(y_2) - U(y_1)\langle \mathbf{L} u_{\tau_L} u(\tilde{y}_2) \rangle - U(y_2)\langle \mathbf{L} u_{\tau_L} u(\tilde{y}_1) \rangle + \langle \mathbf{L} u_{\tau_L}^2 \tilde{u}(\tilde{y}_1)\tilde{u}(\tilde{y}_2) \rangle \\
&= U(y_1)U(y_2) - U(y_2)\langle u_{\tau_L}\tilde{U}(y_1u_{\tau_L}) \rangle - U(y_1)\langle u_{\tau_L}\tilde{U}(y_2u_{\tau_L}) \rangle + \langle u_{\tau_L}^2 \langle \tilde{u}(y_1u_{\tau_L})\tilde{u}(y_2u_{\tau_L}) \rangle \rangle.
\end{aligned}$$