

To my parents.

MODELLING OF CRACK TIP BEHAVIOUR IN POLYMERS

by

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ABSTRACT

An investigation of the blunting of cracks in a wide range of polymers has been undertaken. The materials and characteristics displayed may be divided into two categories; those that blunt elastically prior to failure and those that exhibit discontinuities in what is essentially ductile deformation. Elastic materials follow a predicted blunting pattern and the failure of glassy polymers from a blunt notch is predicted by applying a fracture criterion to this blunting model. As a result the strain energy release rate at failure, calculated using the current elastic strain energy, is found to depend linearly upon crack tip radius. Of the ductile materials examined, two show evidence that if the current elastic strain energy is carefully assessed, then the ductile tearing of such materials is characterised by constancy of critical energy release rate and, furthermore, that the same linearity of this parameter with crack radius is strongly suggested.

The consequences of these results are examined in the context of the thermal blunting of tips of cracks in the impact test. Local adiabatic heating in high rate fracture tests has previously been suggested as an explanation of the elevated toughness values recorded. Simulated crack tip heating has been shown to increase toughness in low rate fracture tests, and applying a blunting analysis the results are found to agree in concept with those of high rate tests.

In conclusion, the message is that thoughtful modelling of crack tip deformation in polymers may be necessary for assessing toughness in semi-brittle failure modes and is vitally important for the further development of yielding fracture mechanics.

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CHAPTER ONE

INTRODUCTION

Increasingly thermoplastic materials are being used in load bearing applications and situations in which they are prone to failure. It has become essential, therefore, to understand their mechanical behaviour more thoroughly with a view to improved design procedures. In general, the materials are viscoelastic which often provides difficulty when attempting to predict material response from data obtained at a different deformation rate. Such a problem is usually overcome by the introduction of a rate-dependent modulus and this has proved useful e.g. in the prediction of the buckling of stacks of plastic containers (Constable and Price (1973)).

When we move to the study of fracture, however, we see immediately that such an approach will only be profitable while attention is restricted to linear elastic fracture mechanics (LEFM). Time dependence may again be introduced into the material modulus and yield stress in order to rationalise the relationship between linear elastic fracture parameters and strain rates (Williams (1978)). This technique may be regarded as an extension of LEFM to viscoelastic materials and as such has its own merits. Prospects for development along these lines are poor since solutions to fully non-linear time-dependent elasticity problems are, at best, extremely limited. Most recent developments in the field of polymer fracture are attempts to move from the sound basis of LEFM for brittle failure to regimes where these materials exhibit substantial amounts of ductility. Thus the pattern is similar to the development of studies on metal fracture.

As has been seen quite clearly in the past with metals, brittle failure is prominent in thick sections of structures where the stress field is triaxial giving high constraint on the deformation of material. It is more customary at the present time for polymers to be found in small highly stressed components such as automobile fans, gears, bearings, valve parts, pipes, etc., and as such they may be susceptible to ductile fracture. As well as the need to understand this phenomenon, there is a requirement to be able to predict the failure of an in-service component from a generally smaller test specimen.

Most of the attempts to characterise this type of fracture have involved either extension to LEFM or interpretations of elastic - plastic fracture mechanics. In this study, an approach has been developed which takes an extension to LEFM - that which describes failure from a blunt notch - and provides a framework in which to discuss and rationalise the 'local' and 'global' material softening which accompanies certain types of polymer failure. Local blunting takes place in polymer fracture when a crack is loaded rapidly; the high work rates at the tip of the crack and the low thermal conductivity of the material lead to crack tip heating and subsequent local blunting. Global softening is readily associated with unrestrained plasticity and ductile failure.

By the way of introduction, chapter two gives an overview of the theory of LEFM. Attention is focussed on the philosophy behind the development of the subject and on those aspects of LEFM fundamental to subsequent arguments put forward in this study.

Chapter three begins by examining the elastic stress field surrounding an elliptical hole in an infinite sheet. The analysis is extended to include changing geometry and may be used to model a blunting crack tip. In its basic form i.e. with small geometry change,

the model has been used to relate the fracture stress of a blunt notched test specimen to the toughness of the material. Other theories have been proposed to explain blunt notch fracture and these are compared with the theory detailed here with specific reference to polymers. Following this, a section is devoted to an investigation of the validity of the continuous blunting model. The material chosen for examination is natural rubber and good agreement is found between experiment and theory. The concluding section examines whether the model may be extended to include continuously softening materials, for example low density polyethylene. Some success is again obtained and in this case calculations of fracture parameters may be made which refer to the fracture process zone at the crack tip.

While the model proposed can rationalise certain aspects of fracture, it could not be expected to have any credibility in the regime of yielding fracture mechanics. Chapter four documents the most recent developments in this area and concentrates on the efforts being made to characterise crack tip deformation. A succinct account of the principal developments in J theory is presented which forms the basis for the discussion of elastic-plastic deformations.

Following on from the later work in chapter three, chapter five investigates crack tip behaviour in a range of ductile polymers. Prior to crack growth, the near tip deformations in yielding material show a rapid transition from the model of chapter three to behaviour which is best characterised by a constant length stretch zone at the crack tip. However, such trends are heavily obscured by the onset of ductile tearing. Behaviour of these materials following initiation of crack growth is also documented and inferences may be drawn from the work on ductile metals as to the size of the fracture process zone and the

critical requirements for fracture to proceed. The latter half of the chapter contains a detailed study of the ductile fracture of polycarbonate which relates the general observations of the preceding section to measured energy release and energy absorption rates.

Chapter six deals with local crack tip blunting. This phenomenon is restricted to high rate loading of cracked polymers and is attributable to their low thermal conductivity. Two situations may be envisaged where the effect is substantial, one of which is examined in chapter six; firstly, in the impact testing of plastics where too high a rate of loading may lead to a false impression of toughness and secondly, in fast crack propagation where an elevation in crack tip temperature with increasing crack speed may lead to complex behaviour of toughness with crack speed. The material under examination in this chapter is PMMA but this is merely for convenience and these phenomena may be detected with less glassy polymers, for example HDPE or polycarbonate.

In conclusion chapter seven draws together the results of the study and discusses their implications.

CHAPTER TWO

ELASTIC FRACTURE MECHANICS

(a) Introduction

Fracture in solids consists of the rupture of atomic bonds and the creation of new surfaces within the fractured body. The analysis of such phenomena may be traced back to early in this century, the original work being done by A. A. Griffith (circa 1920). The initial approach to the problem was an examination of the energies released and absorbed by a cracked body during extension of the crack. In view of the vast difficulties involved with any kind of atomic model, the much more tractable but less precise continuum mechanics ideas were employed, not only by Griffith but also by subsequent workers up to this present time. The limitations of such a treatment are readily apparent on realising that the most essential region relevant to crack extension is an extremely small one at the crack tip and for accurate modelling ought not to be regarded as part of a continuum.

An analysis of the stresses at the base of a crack in an elastic medium provided the next step in the development of fracture mechanics. This work, due to Irwin (circa 1955), enabled the state of a cracked body under stress to be characterised by a loading parameter and the failure to be predicted when this parameter reached a critical value. Compatibility between the approaches of Griffith and Irwin was shown to exist in cases where crack tip plasticity is limited to a very small zone.

With the realisation of the limitations of linear elastic fracture mechanics and the appreciation of the importance of the crack tip region, came several approximate models meeting with varying degrees of success.

These ideas are summarised under the heading:

Plasticity corrections.

The outline of this section, therefore, is to trace the development of elastic and "corrected" elastic fracture mechanics concluding with attempts to extend these ideas into the post yield regime.

(b). Griffith Fracture - The Energy Balance Approach

In his two papers of 1921 (Griffith (1921)) and 1924 (Griffith (1924)) Griffith presented a theory predicting the rupture of an elastic solid body which embraced the theory of minimum potential energy and a statement that if rupture is accompanied by a decrease in potential energy then the ruptured state is one of equilibrium. Contributions to the potential energy of a cracked elastic body were considered to come from the strain energy and the surface energy, this latter term being expressed as the product of surface tension or energy per unit area and the area of crack surface. Thus, the basic idea of Griffith fracture is one in which crack extension occurs when the release of stored strain energy is greater than the energy required to create new crack surface.

Using the results of Inglis (1913) for the stresses near an elliptical flaw in an infinite flat plate, Griffith evaluated the change in strain energy, W , on insertion of the flaw as

$$W = (1 + \kappa) \pi a^2 \sigma_o^2 / 8\mu \quad (2.1)$$

where ' σ_o ' is the remotely applied normal stress, 'a' is the length of the semi-major axis, ' μ ' is the shear modulus of the elastic material

and ' κ ' is given by

$$\begin{aligned}\kappa &= 3 - 4\nu && \text{for a plane strain situation} \\ &= (3 - \nu)/(1 + \nu) && \text{for a plane stress situation}\end{aligned}$$

' ν ' being the Poisson's ratio for the material. It should be noted that the strain energy is independent of the length of the semi-minor axis ' b ' of the ellipse.

The surface energy of such a flaw is given by

$$S = 4 a T \quad (2.2)$$

where ' T ' is the surface tension. Now, crack extension is proposed to occur when

$$\frac{\partial W}{\partial a} \geq \frac{\partial S}{\partial a} \quad (2.3)$$

and from the expressions quoted this leads to a critical fracture stress ' σ^* ' given as

$$\sigma^* = 4 \left\{ \mu T / \pi a (1 + \kappa) \right\}^{\frac{1}{2}} \quad (2.4)$$

Griffith provided experimental confirmation of the relation between ' σ^* ' and $(a)^{\frac{1}{2}}$ by performing work on the fracture of glass cylinders and the bursting of thin hollow glass spheres. His attempt to totally verify the above relationship by extrapolating a value for the surface tension ' T ' was not so successful but the constancy of the

product ' $\sigma^*(a)^{\frac{1}{2}}$ ', was the significant development in work on elastic fracture.

When examining the balance of energy at an extending crack in a non-elastic material, Irwin (1948) and Orowan (1949) draw attention to the fact that the energy dissipated at a plastically deforming crack tip far outweighs that needed to overcome the cohesive molecular forces at the tip. Fracture will only take place in such a situation if the externally applied forces continue to do the necessary plastic work to propagate the crack. In an attempt to model this, both Irwin and Orowan replaced the surface energy term ' S ' with a plastic energy absorption term ' γ_p ' and argued that, provided the plastic deformation at the crack tip is confined to a small zone, then the crack energy released could still be calculated using the elastic analysis to a fair degree of accuracy. Assuming the fracture process to be essentially the same for different loading conditions and specimen geometries, Irwin and Kies (1952) proposed that fracture occurs when the strain energy release rate reaches a critical value. A method for determining strain energies of cracked bodies was then presented.

For an elastic body the strain energy ' W ' can be written as

$$W = P^2 C / 2 \quad (2.5)$$

where ' P ' is the load and ' C ' the corresponding specimen compliance.

The rate of change of strain energy with crack length is given by

$$\frac{\partial W}{\partial a} = P^2 \frac{\partial C}{\partial a} / 2 \quad (2.6)$$

and thus by measuring specimen compliance for various crack lengths in a particular specimen geometry, the release rate at fracture may be determined from measurements of fracture load and final crack length. Irwin (1957), denoting the strain energy release rate by 'G', suggests that...

the fixed-grip strain energy release rate has the same role as an influence controlling time rate of crack extension as the longitudinal load has in controlling time rate of plastic extension of a tensile bar. In the latter case the force per unit area tending to cause plastic extension is the longitudinal stress. In the former case a motivating force per unit thickness can be defined quite generally in terms of the rate of conversion of strain energy to thermal energy during crack extension.

Employing the results of Westergaard (1939), Irwin examined the stress distribution near the tip of a stationary crack, expressing the stresses in terms of his crack driving force 'G'. He, then, showed that 'G' is the fixed-grip loss of energy from the strain energy field per unit crack extension.

The investigation of the stress and strain distribution at the base of a crack was the initial step in understanding the concentrating effect of flaws in materials and led to the stress intensity factor approach to fracture.

(c). Crack Tip Singularities

Although much of the credit for developing the idea of the intensity of stress at the tip of a crack has been attributed to Irwin,

two earlier papers of M. L. Williams published in 1951 and 1952 examine the stress singularities at angular corners in plates subjected to bending and tension. In both problems the character of the stress near the corner is determined using an eigenfunction expansion technique which will be discussed in more depth later. A more relevant presentation of the results, from the point of view of fracture mechanics, is that of Williams (1957) in which the characteristic stress singularity at the tip of a sharp crack in a linearly elastic, homogeneous, isotropic plate is deduced. The corresponding stress distribution is calculated and the symmetric and anti-symmetric contributions are examined.

This characteristic singularity may also be obtained by consideration of Inglis' elliptical flaw problem when the tip radius of the flaw is shrunk to zero. The stresses and displacements in the vicinity of an elastic crack loaded in tension with respect to polar co-ordinates centred at the tip are dominated by

$$\begin{aligned}
 \sigma_{xx} &= \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \cos \frac{\theta}{2} \left\{ 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right\} \\
 \sigma_{yy} &= \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \cos \frac{\theta}{2} \left\{ 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right\} \\
 \sigma_{xy} &= \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2} \quad (2.7) \\
 U_x &= \frac{K_I}{8\mu} \left(\frac{2r}{\pi} \right)^{\frac{1}{2}} \left\{ (2\kappa - 1) \cos \frac{\theta}{2} - \cos \frac{3\theta}{2} \right\} \\
 U_y &= \frac{K_I}{8\mu} \left(\frac{2r}{\pi} \right)^{\frac{1}{2}} \left\{ (2\kappa + 1) \sin \frac{\theta}{2} - \sin \frac{3\theta}{2} \right\}
 \end{aligned}$$

where ' μ ' and ' κ ' have the previously assigned meaning and ' K_I ' is the Irwin stress intensity factor appropriate to mode I, i.e. tensile, loading.

It should be pointed out at this stage that despite the predictions of infinite stresses at the elastic crack tip, this can be put in true perspective by acknowledging that the continuum rationale breaks down near the tip and thus the elastic stress field merely sets outer field behaviour in the body. Equivalence of the concept of crack driving force and stress intensity was established in Irwin (1957) using a virtual work argument, the relationship being

$$G = \frac{(1 + \kappa)}{8\mu} K^2 \quad (2.8)$$

Thus the characterisation of fracture by attainment of a critical strain energy release rate is identical to characterising fracture by the critical nature of the stress environment near the flaw.

The inverse square root nature of the elastic crack tip stress singularity is a feature emerging from all elastic crack problems. With reference to the three types of crack opening : mode I - tension; mode II - in-plane shear; mode III - anti-plane shear, the crack tip stress field is representable in the form

$$\sigma_{ij} = \frac{K_I}{r^{\frac{1}{2}}} F_{ij}^I + \frac{K_{II}}{r^{\frac{1}{2}}} F_{ij}^{II} + \frac{K_{III}}{r^{\frac{1}{2}}} F_{ij}^{III} \quad (2.9)$$

where the F's are dimensionless and dependent on the inclination angle θ only. From a dimensional point of view, if the cracked body is loaded with remote stress ' σ ' and is large enough so that the only relevant

length scale is the crack length 'a', then the stress intensity must be given by

$$K \propto \sigma a^{\frac{1}{2}} \quad (2.10)$$

Consequently, if another length scale, say 'w' the width of the body, enters the calculation it must do so via some non-dimensional corrective factor, say 'Y' and the stress intensity is given by

$$K = \sigma Y(a/w) a^{\frac{1}{2}} \quad (2.11)$$

Whilst recognising the need for geometric correction functions 'Y' to enable the stress intensity at fracture to be ascertained, it is also pertinent to ask of the effect of specimen thickness on the calculated fracture values. Irwin (1960) found the critical stress intensity at fracture in tension ' K_{Ic} ' to be strongly dependent on the material thickness. This apparent variation in what was regarded as a material property is attributed to the through-thickness change in constraint of the stress field. Thin specimens are in an approximately plane stress state allowing lateral contraction whereas the bulk of the material in thick specimens is constrained to be in a state of plane strain.

Summarising the stress intensity approach it is clear that providing the plastic zone at the crack tip is sufficiently small, the elastic field surrounding the crack will be only marginally affected by plastic relaxation. While the fracture event will be controlled by the plastic environment of the crack tip, the gross elastic field and corresponding elastic stress intensity will characterise the fracture by attainment of its critical value.

(d) Plasticity Corrections

Discussion so far has centred on purely elastic arguments.

In order to predict failure of material which in the main is linear elastic but near the crack extension zone exhibits limited plasticity, it is necessary to estimate the extent of the plasticised material and thus its effect on the body as a whole. The early work on correcting for the effect of crack tip plasticity was again done by Irwin and his contemporaries and depended on estimates of plastic zone sizes calculated using elastic stress analyses and a yield criterion.

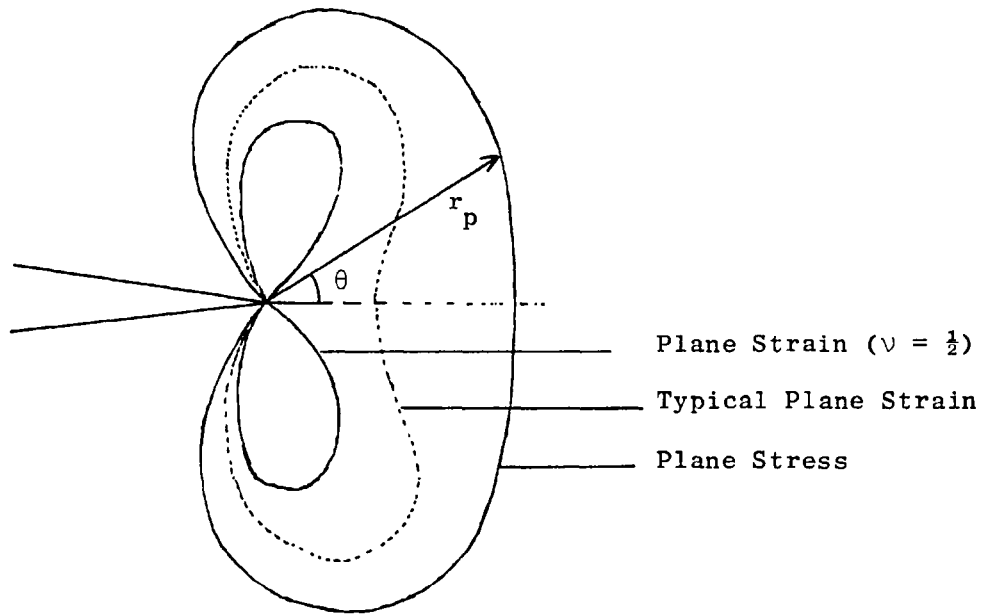
If the plastic zone at the tip of a crack is assumed to be small then it will not greatly disturb the elastic stress field, and therefore an approximation to the size and shape of the plastic zone is obtained by applying the Von Mises yield criterion to the elastic field. In the Von Mises condition, yielding is determined by the deviatoric stresses alone, and if σ_1 , σ_2 and σ_3 are the principal stresses in the body and ' σ_y ' is the yield stress in tension then the yield surface is defined by the equation

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2 \sigma_y^2 \quad (2.12)$$

With the stresses given in the previous section and a plane strain situation, in which the transverse stress σ_z is given by $\nu(\sigma_{xx} + \sigma_{yy})$, then the plastic zone is bounded by the equation

$$r_p = \frac{K_I^2}{2\pi\sigma_y^2} \cos^2 \frac{\theta}{2} \left\{ 4(1 - \nu(1 - \nu)) - 3\cos^2 \frac{\theta}{2} \right\} \quad (2.13)$$

The plane stress situation is included by substituting $\nu = 0$ and the shapes of zones are shown overleaf.



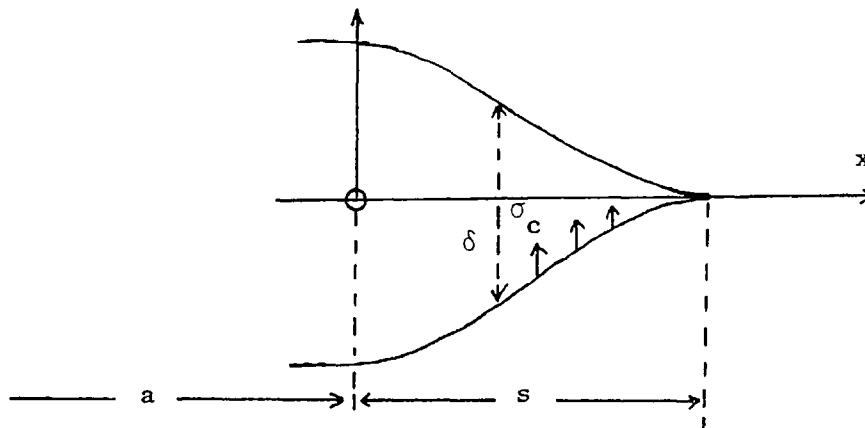
It is clearly seen that for $\nu = \frac{1}{2}$ the plastic zone consists of two lobes whereas for $0 < \nu < \frac{1}{2}$ a zone of intermediate type (dotted line) results with the limiting plane stress zone again indicated. Irwin and co-workers (1960) remark that the plastic zone at the tip of a crack "appears to the stress field as a region of somewhat relieved normal stress, an effect roughly comparable to an extra extension of the crack". They correlated their data with a 'corrected' equation which gave the stress intensity factor in terms of the effective crack length 'a*' given by

$$a^* = a + r_p^*$$

where r_p^* is known as the Irwin plastic zone correction. Knott (1973) gives a concise account of the derivation of this correction from the relaxation of the elastic stress field ahead of a sharp crack. An original estimate of the plastic zone size is calculated to be r_p^* and, following further analysis, a better approximation is found to be $2r_p^*$. This second approximation leaves the effective crack tip in the middle

of the plastic zone and effectively extends the crack length by r_p^* . Such a correction is equal to the maximum plane stress zone length as given by equation (2.13).

With specific reference to brittle and semi-brittle polymers, it is more likely that crack tip yielding will take the form of a line zone extending ahead of the crack along the direction of ensuing propagation. To this end it is proposed to discuss a crack tip model similar to that discussed by Dugdale (1960) but fully outlined by Barenblatt (1962) in which a crack of length 'a' has at its tip a zone of length 's' in which potential fracture surfaces are held together by the forces of cohesion of the material. This model is particularly applicable to the crazing of material at the tips of cracks in polymers and the cohesive forces are represented by a craze stress ' σ_c '. A diagram of the zone is shown below



In the treatment of Barenblatt, linear elasticity is maintained throughout but three assumptions are made about the cohesive field at the crack tip.

- (i) The effect of the cohesive stress is to remove the singularity normally present at the crack tip.

(ii) The length 's' of the zone is small relative to 'a'.

(iii) Under given conditions, the cohesive stresses are independent of the loading of the material.

The imposition of crack closing forces on part of the crack length creates an inverse square root singularity, and the superposition of this and the usual crack tip stress singularity must result in finite stresses near the tip. Such a description of the region neighbouring the crack tip lends itself to direct physical interpretation and further usefulness is provided when a fracture criterion is considered. Postulating a limit to the forces of cohesion, the crack tip condition when this limit is reached may be assessed and, as is particularly pertinent to the fracture of polymers, a critical crack opening displacement may be used to characterise the fracture. The displacements in the Barenblatt zone are given concisely by Rice (1968) for small-scale yielding as

$$\delta = \frac{(1 + \kappa) \sigma_c s}{\pi \mu} \left[\left(1 - \frac{x}{s}\right)^{\frac{1}{2}} - \frac{x}{2s} \ln \left\{ \frac{1 + (1 - \frac{x}{s})^{\frac{1}{2}}}{1 - (1 - \frac{x}{s})^{\frac{1}{2}}} \right\} \right] \quad (2.14)$$

where ' κ ', ' μ ' and ' σ_c ' are as previously defined, 'x' is the distance along the zone as shown in the figure and 's' is the zone length given by

$$s = \pi K_I^2 / 8 \sigma_c^2 \quad (2.15)$$

again for small scale-yielding. Thus a crack opening displacement may be defined by δ ($x = 0$) and denoted by ' δ_o '

$$\delta_o = \frac{(1 + \kappa) \sigma_c}{\pi \mu} \frac{\pi K_I^2}{8 \sigma_c^2} = \frac{(1 + \kappa) K_I^2}{8 \mu \sigma_c} \quad (2.16)$$

and for plane stress this becomes

$$\delta_o = \frac{K_I^2}{E \sigma_c} = \frac{G_I}{\sigma_c} \quad (2.17)$$

Thus for a constant craze stress ' σ_c ', a critical crack opening displacement fracture criterion becomes equivalent to the critical strain energy release rate criterion.

The postulates of the Dugdale model are similar to those of Barenblatt, the only differences being that (i) the material in the Dugdale zone is subjected to a uniform tensile yield stress and (ii) the length of the zone is not, in general, small in comparison to the crack length. Rice (1968) compares the small-scale yielding approximation to the results for the zone size and crack tip opening displacement corresponding to the perfectly plastic Dugdale-Barenblatt model which may be written in the dimensionless form

$$\frac{s}{a} = \sec \left(\frac{\pi \sigma}{2 \sigma_c} \right) - 1 \quad (2.18)$$

$$\frac{\delta_o \pi \mu}{(1 + \kappa) \sigma_c a} = \ln \left\{ \sec \left(\frac{\pi \sigma}{2 \sigma_c} \right) \right\} \quad (2.19)$$

where ' σ ' is the remote applied stress. A similar comparison is shown

by Williams (1978) but he includes a comparison of the Irwin crack length correction mentioned earlier in this section. There is good agreement between the line zone result and the r_p^* correction illustrating the usefulness of the latter.

Returning to the Dugdale line zone model, it should be noted that an alternative analysis involving a continuous array of dislocations on a plane ahead of a crack leads to the self same solutions. An investigation of these types of models is presented in the following section and includes attempted extensions of fracture mechanics into the post yield regime.

(e) Dislocation Models

Several models have been proposed in which the plastically relaxed region ahead of a crack tip is represented by a continuous distribution of dislocations in an otherwise linear elastic continuum. The plastic displacements at any point in the plastic zone are then described by the total Burgers vector at that point. As is pointed out by Bilby, Cottrell and Swinden (1963) the slip planes of these dislocations are not necessarily crystallographic but are analogous to the lines of maximum limiting shear stress defined in the mathematical theory of plasticity and thus may also relate to polymeric materials. Their paper then examines a sheared slit and the associated distribution of dislocations. It is made clear that the results for a Dugdale tensile slit are similar to those presented for the slit under shear.

Cottrell (1965) furthers these ideas in a discussion of a possible change in fracture mode from a ductile nature, in which the dislocations are confined to the slip lines near the crack tip, to brittle cleavage fracture portrayed by the model of Bilby, Cottrell and Swinden.

Just as Cottrell relates the dislocation model to the fracture mechanisms, so the work of Heald, Spink and Worthington (1972) (henceforth referred to as HSW) attempts to use the theory to extend fracture mechanics into the post yield regime. The plastic zone size and crack tip displacement are given by equations (2.18) and (2.19) respectively and applying a critical crack opening displacement fracture criterion of $\delta_o = \delta_c$ leads to a fracture stress ' σ_f ' given by

$$\sigma_f = \frac{2}{\pi} \sigma_c \cos^{-1} \left[\exp \left\{ - \frac{\pi \mu \delta_c}{4 (1 - \nu) \sigma_c a} \right\} \right] \quad (2.20)$$

for the case of plane strain. Then, expanding this expression for small values of $\frac{\pi \mu \delta_c}{4 (1 - \nu) \sigma_c a}$ gives

$$\begin{aligned} \sigma_f &= \frac{2}{\pi} \sigma_c \left\{ \frac{\pi \mu \delta_c}{2 (1 - \nu) \sigma_c a} \right\}^{\frac{1}{2}} \\ &= \left\{ \pi \frac{4 \mu}{(1 - \nu) a} \frac{1}{2} \delta_c \sigma_c \right\}^{\frac{1}{2}} \end{aligned} \quad (2.21)$$

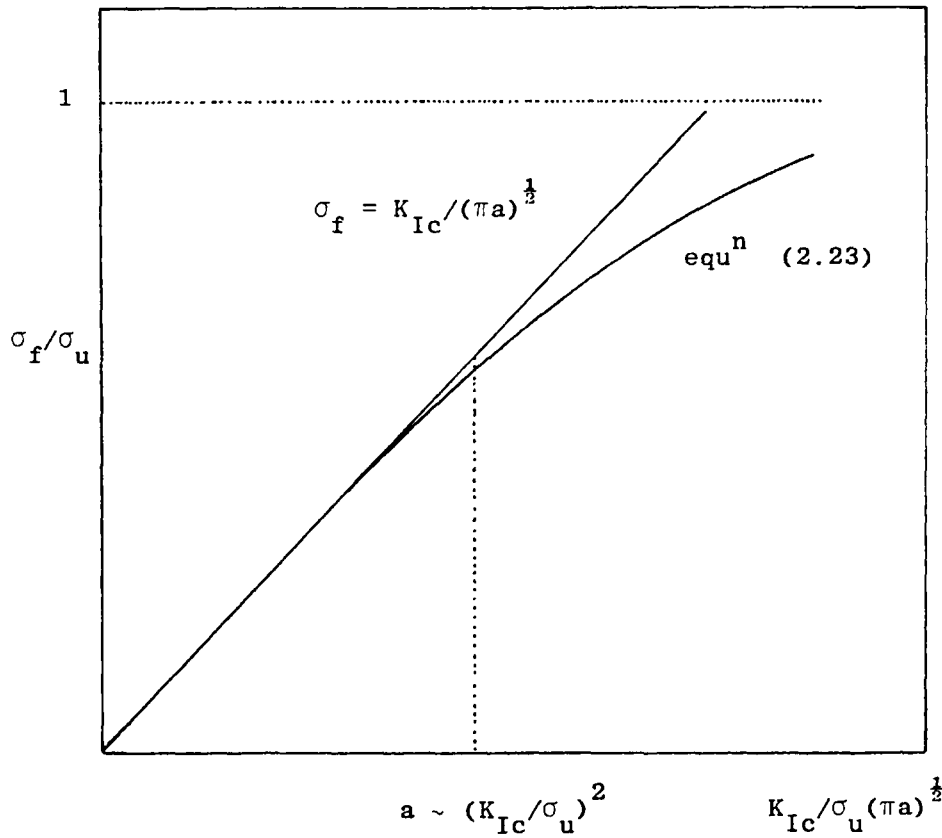
which is in agreement with the Griffith fracture condition of equation (2.4). HSW remark that the value of σ_c in equation (2.18) should be one corresponding to yielding, i.e. the yield stress or the craze stress, whereas that value pertinent to the failure condition, equation (2.20), should be an ultimate tensile stress for the material. Rewriting equation (2.20) in terms of $\sigma_c \equiv \sigma_u$, the ultimate failure stress, and the plane strain fracture toughness given by

$$K_{Ic} = \sigma_f (\pi a)^{\frac{1}{2}} = \left\{ \frac{2 \mu \delta_c \sigma_c}{1 - \nu} \right\}^{\frac{1}{2}} \quad (2.22)$$

we have

$$\sigma_f = \frac{2}{\pi} \sigma_u \cos^{-1} \left[\exp \left\{ - \frac{\pi K_{Ic}^2}{8 \sigma_u^2 a} \right\} \right] \quad (2.23)$$

As $a \rightarrow 0$ so $\sigma_f \rightarrow \sigma_u$ in contrast with the Griffith condition in which $\sigma_f \rightarrow \infty$ and thus this model provides an extension towards the post-yield regime. The following figure shows equation (2.22) and (2.23) plotted on the basis of the variables σ_f/σ_u and $K_{Ic}/\sigma_u (\pi a)^{\frac{1}{2}}$ and demonstrates the departure from linear elastic fracture mechanics for short crack lengths.



Equation (2.23) suggests a technique for assessing K_{IC} from tests in the post yield regime and an apparent toughness may be defined as

$$K_A = \sigma_f (\pi a)^{\frac{1}{2}} = 2 \sigma_u \left(\frac{a}{\pi} \right)^{\frac{1}{2}} \cos^{-1} \left[\exp \left\{ - \frac{\pi K_{IC}^2}{8 \sigma_u^2 a} \right\} \right] \quad (2.24)$$

from which it is seen that for large cracks $K_A \rightarrow K_{IC}$ and it is apparent that K_{IC} characterises the fracture behaviour even in the post yield region. HSW provide good correlation between experimental data and equation (2.24) and discuss the effects of specimen geometry and notch tip radius on equation (2.24). The latter point will be included in the next chapter.

A fair degree of success in modelling post yield fracture is gained using a dislocation model and more analytical work has been done by Vitek (1976), where the dislocation distribution is at the tip of an elliptical flaw, and by Vitek (1976a) and Riedel (1976), where slip is proposed to occur in planes at an angle to the crack direction. The usefulness of the latter work is readily apparent since many materials are observed to have plastic zones which are not co-planar extensions of the crack. Such phenomena are accountable to shear yielding as opposed to crack tip crazing. Referring back to section (d), a plane strain plastic zone calculated using an elastic stress field and a yield criterion, results in two angled lobes within which the direction of maximum increase in shear strain energy is at 70° to the line of the crack. Therefore, it is particularly useful to review the conclusions of Vitek (1976a) in such a case.

Results are presented for the dependence of the plastic zone size (in fact, its projected length on the line of the crack), the

crack opening displacement and the Rice J - Integral (see Chapter Four) on the remotely applied stress. These are plotted and compared with the results for the co-planar zone discussed previously and it is noticed that there is good agreement between the co-planar zone and an inclined zone model of between 60° and 80° . Two popular fracture criteria are examined : the critical crack opening displacement and the critical J integral value. It is found that in the region of post yield there is a substantial difference in applying these criteria and, since the crack opening displacement depends upon the inclination angle of the dislocations, the critical J criterion is held to be more appropriate. However, for angles of approximately 65° the fracture curve as plotted in the previous figure coincide with the results for the co-planar zone. The much simpler model of Bilby, Cottrell and Swinden has obvious practical advantages and its justification is supplied by these inclined zone models.

(f) Eigenfunction Expansion Technique

In concluding this chapter on elastic fracture mechanics and its extensions, it is of interest to note the possibilities presented by admitting non-homogeneous elasticity into crack problems. One technique of discussing these situations is the eigenfunction expansion method which has been thoroughly documented by Fenner (1973). The method is illustrated by Williams (1978) for a crack in a homogeneous elastic plate and requires a separable solution in terms of polar variables 'r' and ' θ ' to the biharmonic equation involving a stress function 'U' which is written

$$\nabla^4 U = 0 \quad (2.25)$$

and the particular form of solution is written

$$U(r, \theta) = r^{\lambda + 1} F(\theta, \lambda) \quad (2.26)$$

where ' λ ' is the eigenvalue and $F(\theta, \lambda)$ the corresponding eigenfunction. Such a procedure produces an ordinary differential equation for the eigenfunction F and imposition of finiteness usually restricts the possible values of λ . The dominant behaviour at the crack tip may be identified and in the homogeneous case leads to the inverse square root singularity.

When non-homogeneity is admitted, however, the equation for the stress function now includes radial and angular dependence of the material modulus and its derivatives. According to Fenner (1973), by suitable choice of the functional form of the modulus, separable solutions may still be obtained and the value of the dominant eigenvalue may be determined. For a purely radial modulus dependence the results of Fenner illustrate that, for a symmetrically loaded crack in a material in which the modulus increases with radial distance from the crack tip, the dominant singularity lies between inverse square root dependence and constancy. As shall be seen at a later stage this has similarities with elastic - plastic singularities and to a limited extent describes the effect of plasticity at a crack tip in an elastic body.

CHAPTER THREE

ELASTIC BLUNTING ANALYSIS AND APPLICATIONS

(a) Crack Tip Blunting - Introduction

Within the framework of the previous chapter and as outlined in chapter one, it is intended to show that for a number of situations arising in the fracture of polymers, limited ductility at crack tips can be successfully modelled by an elastic blunting argument. Fracture from a blunted crack tip takes place at a higher gross stress level and a consequently elevated toughness value. An examination of the stresses near a blunt notch in an elastic material provides the analytical background and this is presented in the following section.

It is important to assess the validity of this modelling argument and, to this end, three situations arise, two of which are examined in subsequent sections of the chapter. Firstly, the straightforward description of the testing of blunt notched specimens of brittle polymers is presented and results are compared with the theory and with other theories which have been proposed elsewhere. The second instance of applicability of a blunting argument is in the deformation and fracture of ductile polymers and a technique for examining such behaviour is given, with results for rubbers and other low strength materials illustrated in later sections.

Finally, an assessment of the usefulness of such an approach is presented where the limitations of the theory are discussed and to what extent it may be used in the context of elastic - plastic fracture mechanics.

Before embarking on a full account of the blunting analysis it

is considered helpful to illustrate schematically what occurs when a crack in a material blunts open under monotonically increasing load and ultimately fails. The following figures show a very general type of behaviour which will obviously be modified according to the individual nature of the material.

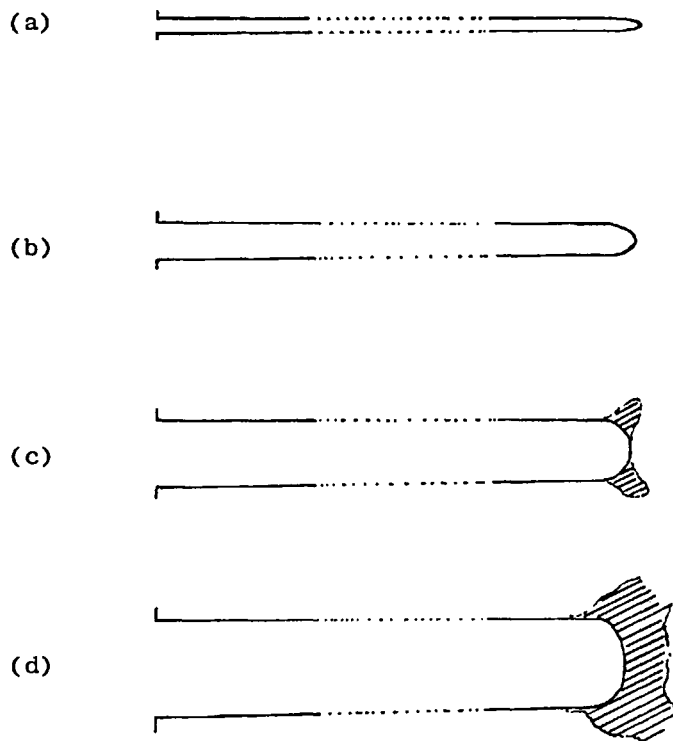


Figure (a) represents the unstressed crack, nominally sharp but actually of small initial root radius. Loading to the situation of figure (b) has produced slight blunting but no yield zone. A further increase in stress promotes contained yielding eventually resulting in the high

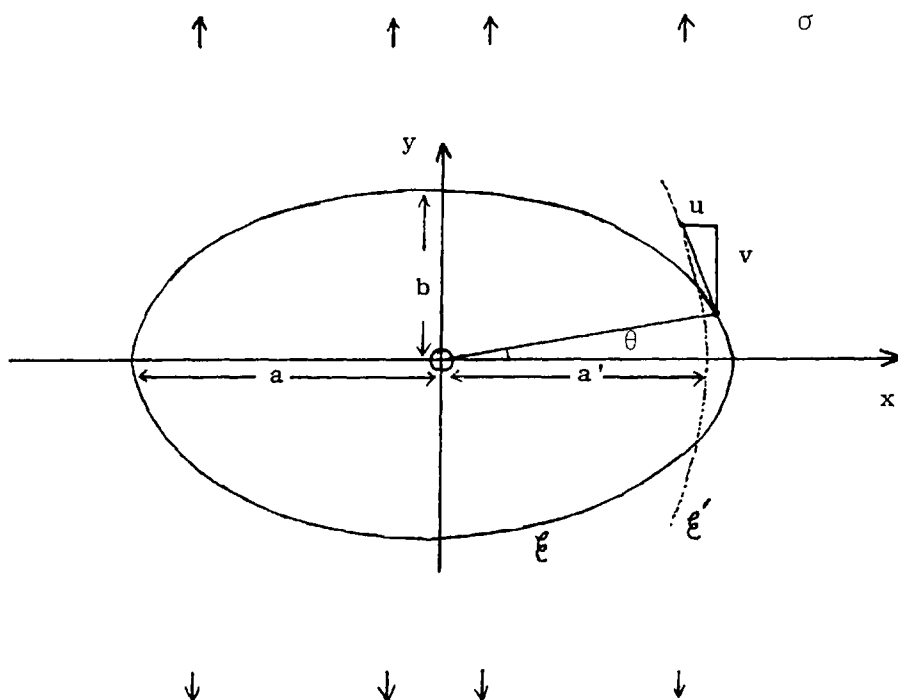
degree of yielding and crack tip blunting of figure (d). Failure may take place at any stage according to the material.

Therefore, it is apparent why an assessment of crack tip blunting is vital in interpreting fracture behaviour. The following section provides a means of doing just this.

(b) Full Elastic Analysis

The problem of the stress distribution near an elliptical flaw in a homogeneous elastic continuum was originally treated by Inglis (1913) and the results have been used in the limiting case of the very slender ellipse as previously noted. Williams (1973) summarises the results indicating their usefulness for interpreting the fracture from bluntly notched specimens in terms of two failure parameters - a critical stress attained at a critical distance ahead of the crack tip. A reassessment is warranted in respect of the fact that the results are static and do not take into account progressive blunting, and they are also limited to cases where the crack tip radius is small compared to the crack length.

The displacements of the boundary of an elliptical hole in an infinite elastic plate subjected to a uniform tension normal to the major axis (u, v) related to cartesian axes (x, y) are given below in terms of ' σ ', the remotely applied stress, ' E ' the material tensile modulus, ' θ ', the eccentric angle and ' a ' and ' b ', the semi-major and semi-minor axis of the original ellipse respectively. The figure overleaf shows this situation more clearly.



The result of applying a stress ' σ ' to the ellipse ϵ is deformation resulting in the boundary ϵ' .

Displacements (u , v) are

$$\begin{aligned} u &= -\frac{\sigma}{E} a \cos\theta \\ v &= \frac{\sigma}{E} (2a + b) \sin\theta \end{aligned} \tag{3.1}$$

for plane stress. Strictly speaking the displacements are only valid for small geometry changes since the boundaries of the ellipse move with increasing applied stress. It is readily seen, however, that any point (x , y) on the ellipse, when subjected to a small increment of stress ' $d\sigma$ ', will move to the new position (x' , y') given by

$$\begin{aligned}x' &= x - \frac{d\sigma}{E} a \cos\theta \\ \text{i.e. } x' &= x \left\{ 1 - \frac{d\sigma}{E} \right\} \\ y' &= y + \frac{d\sigma}{E} (2a + b) \sin\theta \\ \text{i.e. } y' &= y \left\{ 1 + \frac{d\sigma}{E} \left(1 + \frac{2a}{b} \right) \right\}\end{aligned}\tag{3.2}$$

We see that the ellipse defined by semi-major and semi-minor axes (a, b) has been deformed into another ellipse of characteristic axes (a', b') given by

$$\begin{aligned}a' &= a \left\{ 1 - \frac{d\sigma}{E} \right\} \\ b' &= b \left\{ 1 + \frac{d\sigma}{E} \left(1 + \frac{2a}{b} \right) \right\}\end{aligned}\tag{3.3}$$

Successive increments in load produce a series of ellipses and it is, therefore, a simple process to link the geometry change to the loading increment. Denoting a change in the semi-major axis by 'da' and a corresponding change in the semi-minor axis by 'db' we obtain

$$\begin{aligned}da &= - a \frac{d\sigma}{E} \\ db &= (b + 2a) \frac{d\sigma}{E}\end{aligned}\tag{3.4}$$

The intention is to relate the results to the opening of an

initially blunt crack. Thus, initially, 'b' is small and 'a' is large and it is found that on imposing constant crack length 'a' no significant difference is made. For a fixed crack length 'a₀' the crack tip radius 'ρ' is given by

$$\rho = b^2 / a_0 \quad (3.5)$$

$$\text{where } db = (b + 2a_0) \frac{d\sigma}{E}$$

and solving leads to the expression

$$\sigma = E \ln \left\{ 1 + \frac{1}{2} \left(\frac{\rho}{a_0} \right)^{\frac{1}{2}} \right\} \quad (3.6)$$

for the boundary condition of $b \sim 0$ for $\sigma = 0$ and for an initial notch root radius ' ρ_0 ' the corresponding expression is

$$\sigma = E \ln \left[\left\{ 1 + \frac{1}{2} \left(\frac{\rho}{a_0} \right)^{\frac{1}{2}} \right\} / \left\{ 1 + \frac{1}{2} \left(\frac{\rho_0}{a_0} \right)^{\frac{1}{2}} \right\} \right] \quad (3.7)$$

Defining an apparent stress intensity at the tip of such a crack, as was done by Heald, Spink and Worthington (1972) leads to

$$\begin{aligned} K_A &= \sigma (\pi a_0)^{\frac{1}{2}} \\ &= E (\pi a_0)^{\frac{1}{2}} \ln \left\{ 1 + \frac{1}{2} \left(\frac{\rho}{a_0} \right)^{\frac{1}{2}} \right\} \end{aligned} \quad (3.8)$$

In the limit of small tip radii this gives

$$K_A = E \frac{1}{2} \pi^{\frac{1}{2}} \rho^{\frac{1}{2}} \quad (3.9)$$

which is identical to the expression for the crack tip radius derivable from the elastic displacements (2.7). The full relation (3.8) provides the departure from (3.9) attributable to progressive blunting of the crack tip.

In order to apply these results to fracture, a criterion for failure is needed. As is stated in Williams (1973), observations of fracture behaviour in polymers indicate that the attainment of a critical stress at a critical distance ahead of the crack tip is most appropriate. This idea has several appealing features, one of which is that the stress and distance parameters may be compared to the micromechanical properties of the material. It is, therefore, desirable to calculate the stresses immediately ahead of our elliptical flaw. The stress field is given in Timoshenko and Goodier (1951) following a complex variable approach (as, in fact, are the displacements) and, denoting the normal stress at a distance 'r' ahead of the ellipse by $\sigma_{yy}(r)$ we have the following relation which is again only strictly applicable to the case of small geometry change.

$$\begin{aligned} \frac{\sigma_{yy}(r)}{\sigma} = & \left[\frac{1}{2\sqrt{2}} \left(\frac{a}{r} \right)^{\frac{1}{2}} \left(\frac{\rho}{r} - \frac{\rho}{a} \right) \left(1 + \frac{\rho}{2r} + \frac{r}{2a} \right)^{-\frac{3}{2}} \left\{ 1 - \left(\frac{\rho}{a} \right)^{\frac{1}{2}} \right\} \right. \\ & + \frac{1}{\sqrt{2}} \left(\frac{r}{a} \right)^{\frac{1}{2}} \left(1 + \frac{a}{r} \right) \left(1 + \frac{\rho}{2r} + \frac{r}{2a} \right)^{-\frac{1}{2}} \left\{ 1 - 2 \left(\frac{\rho}{a} \right)^{\frac{1}{2}} \right\} \\ & \left. + \frac{\rho}{a} \right] / \left\{ 1 - \left(\frac{\rho}{a} \right)^{\frac{1}{2}} \right\}^2 \end{aligned} \quad (3.10)$$

Simplification for the case $\rho/a \ll 1$, $r/a \ll 1$ leads to

$$\frac{\sigma_{yy}(r)}{\sigma} = \left(\frac{a}{2r} \right)^{\frac{1}{2}} \frac{1 + \rho/r}{\left(1 + \rho/2r \right)^{3/2}} \quad (3.11)$$

in agreement with the result quoted by Williams (1973). It should be noted that for the sharp crack case, $\rho = 0$, (3.11) becomes equivalently

$$\sigma (\pi a)^{\frac{1}{2}} = \sigma_{yy}(r) (2 \pi r)^{\frac{1}{2}} \quad (3.12)$$

and thus the imposition of a critical stress ' σ_c ' at a critical distance 'c' has obvious relation to the sharp crack fracture toughness K_c in the form

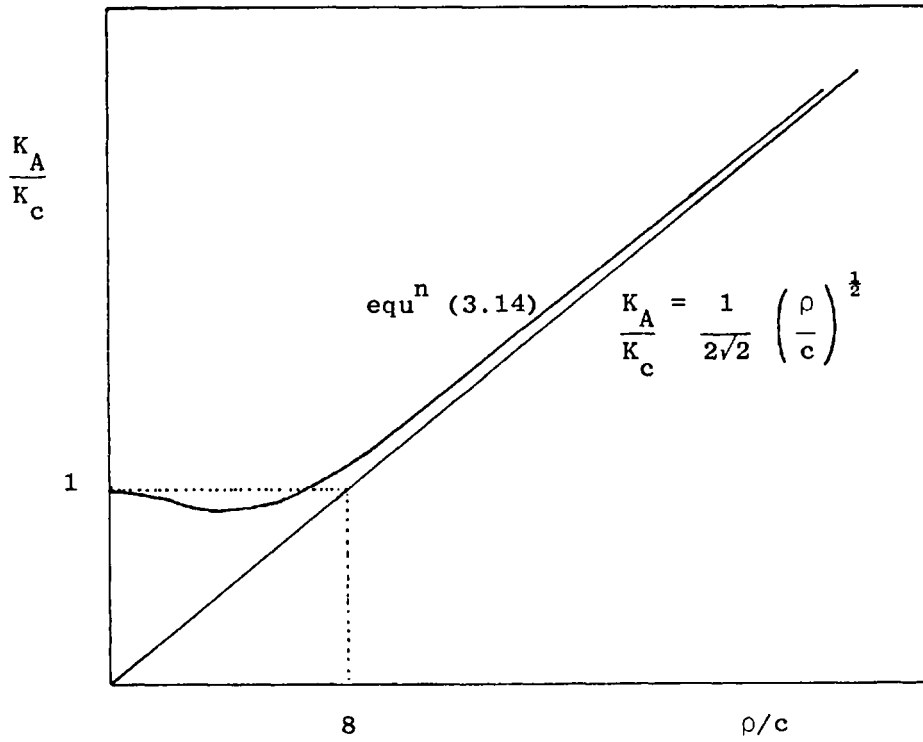
$$K_c = \sigma_c (2 \pi c)^{\frac{1}{2}} \quad (3.13)$$

which is another interesting aspect of this fracture criterion, in that it provides a sensible extension into the post yield regime.

Williams (1973) applied the fracture criterion to equation (3.11) to obtain the elevation in fracture toughness due to crack tip blunting as given by

$$\frac{K_A}{K_c} = \frac{\left(1 + \rho/2c \right)^{3/2}}{1 + \rho/c} \quad (3.14)$$

This expression is shown in the following figure as K_A/K_c plotted against ρ/c



An anomaly is present here since for a small tip radius a lower apparent toughness than that for a sharp crack is predicted. As will be shown later in this chapter, such a prediction is not supported experimentally and a more useful relationship between K_A/K_c and ρ/c is employed and is written

$$\frac{K_A}{K_c} = \begin{cases} 1 & \rho/c < 8 \\ \frac{1}{2\sqrt{2}} \left(\rho/c \right)^{\frac{1}{2}} & \rho/c \geq 8 \end{cases} \quad (3.15)$$

Strictly speaking, however, equation (3.11) only applies for small changes and by writing (3.11) as

$$\frac{d\sigma_{yy}(r)}{d\sigma} = \left(\frac{a}{2r} \right)^{\frac{1}{2}} \frac{1 + \rho/r}{(1 + \rho/2r)^{\frac{3}{2}}} \quad (3.16)$$

and recalling that for $\rho/a \ll 1$ equation (3.6) may be written

$$\sigma = \frac{E}{2} \left(\frac{\rho}{a_0} \right)^{\frac{1}{2}} \quad (3.17)$$

a fully integrated coupled blunting solution may be obtained. In view of the anomaly in the behaviour of (3.16) the linear representation of (3.15) is employed. The integrated result for the stress at a distance, $\sigma_{yy}(r)$, is used in conjunction with the fracture criterion to give a fracture at a radius determined from

$$\begin{aligned} \rho &= 8c \sigma_c^2 / E^2, & \rho < 8c \\ &8c e^{2(\sigma_c/E - 1)}, & \rho \geq 8c \end{aligned} \quad (3.18)$$

The fracture stress is determined using (3.17) and gives

$$\begin{aligned} \sigma &= \sigma_c \left(\frac{2c}{a} \right)^{\frac{1}{2}}, & \rho < 8c \\ &E \left(\frac{2c}{a} \right)^{\frac{1}{2}} e^{(\sigma_c/E - 1)}, & \rho \geq 8c \end{aligned} \quad (3.19)$$

or, in terms of toughnesses,

$$\begin{aligned} \frac{K_A}{K_c} &= 1, & \rho < 8c \\ &\frac{E}{\sigma_c} e^{(\sigma_c/E - 1)}, & \rho \geq 8c \end{aligned} \quad (3.20)$$

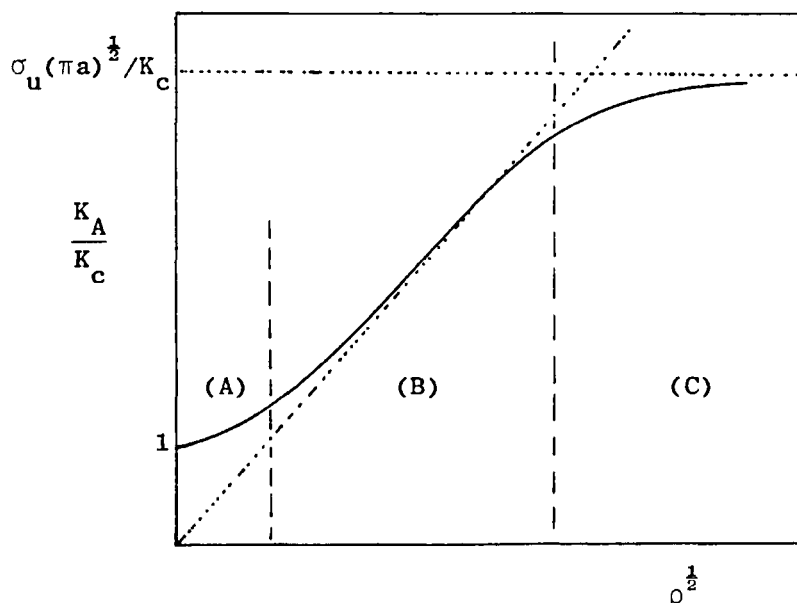
Various aspects of the theory of elastic blunting are used to model the fracture behaviour of polymers and in the subsequent sections

of this chapter these are examined. Since fracture from a blunted crack tip is the crux of the matter, it is of fundamental importance to discuss the results of fracture tests performed on specimens which have pre-machined blunt notches.

(c) Blunt-Notch Fracture of Polymers

Not only is the investigation of fracture from blunt notches a necessary precursor to discussion of the failure of materials from flaws which undergo blunting, but it is an aspect of fracture mechanics vital to the design of structures incorporating sharp indentations and other stress concentrating features.

Until recently the only available data recording the effect of notch acuity upon the fracture toughness of materials was for metals, and steels in particular. The results generally showed an elevation in toughness in proportion to the square root of the machined notch tip radius. Similar work has been done for PMMA and polyester resins which shows the same trends, the characteristics of which are outlined in the following schematic diagram.



There are quite clearly three regions of behaviour:

Region (A), where the toughness of the material is sensibly constant, the extent of which will be controlled by some microstructural parameter; Region (B), a linear dependence of toughness upon the square root of the notch tip radius and Region (C), a limiting region imposed by the ultimate strength of the material.

An early attempt to model the elastic - plastic stress distribution and ensuing fracture from a blunt notch was made by Wilshaw, Rau and Tetelman (1968). Their theoretical curve for toughness against the square root of tip radius was a straight line through the origin and gave fair agreement with their data for mild steel in region (B). The departure from linearity for large initial radii was highlighted by Spink, Worthington and Heald (1973) and, continuing the theme of a previous paper (HSW), they use an expression for the apparent toughness of a bluntly notched specimen which is derived from Smith (1967) and applies strictly to anti-plane strain deformation. The fracture criterion used is the attainment of the ultimate tensile strength within a small zone ahead of the notch and the dependence of toughness upon notch tip radius is

$$\frac{K_A}{K_C} = \left\{ 1 + \frac{\sigma_u}{K_C} (\pi \rho)^{\frac{1}{2}} \right\} / \left\{ 1 + \left(\frac{\rho}{a} \right)^{\frac{1}{2}} \right\} \quad (3.21)$$

Despite the inexactitude of this approach the description of the behaviour of material toughness for large radii is verified experimentally. In the limit $\rho \rightarrow \infty$ (3.21) reduces to

$$\sigma_f = \sigma_u \quad (3.22)$$

Figure 1 shows data obtained for a high temperature steel in three point bend tests. It should be noticed that the data is for large notch tip radii only and agreement with (3.21) for such a region is excellent. For small notch tip radii, equation (3.21) predicts an immediate toughness elevation from the sharp crack value. This, as has been stated previously, is not observed experimentally and, although the results of Oates (1967), as quoted in Spink, Worthington and Heald (1973), show agreement with equation (3.21) for small radii, the majority of data shows a near constancy in this region.

The blunting analysis of the previous section is more successful at predicting fracture from blunt notches. Equation (3.14) gives an excellent representation of the behaviour in regions (A) and (B). For large tip radii the limiting equation is

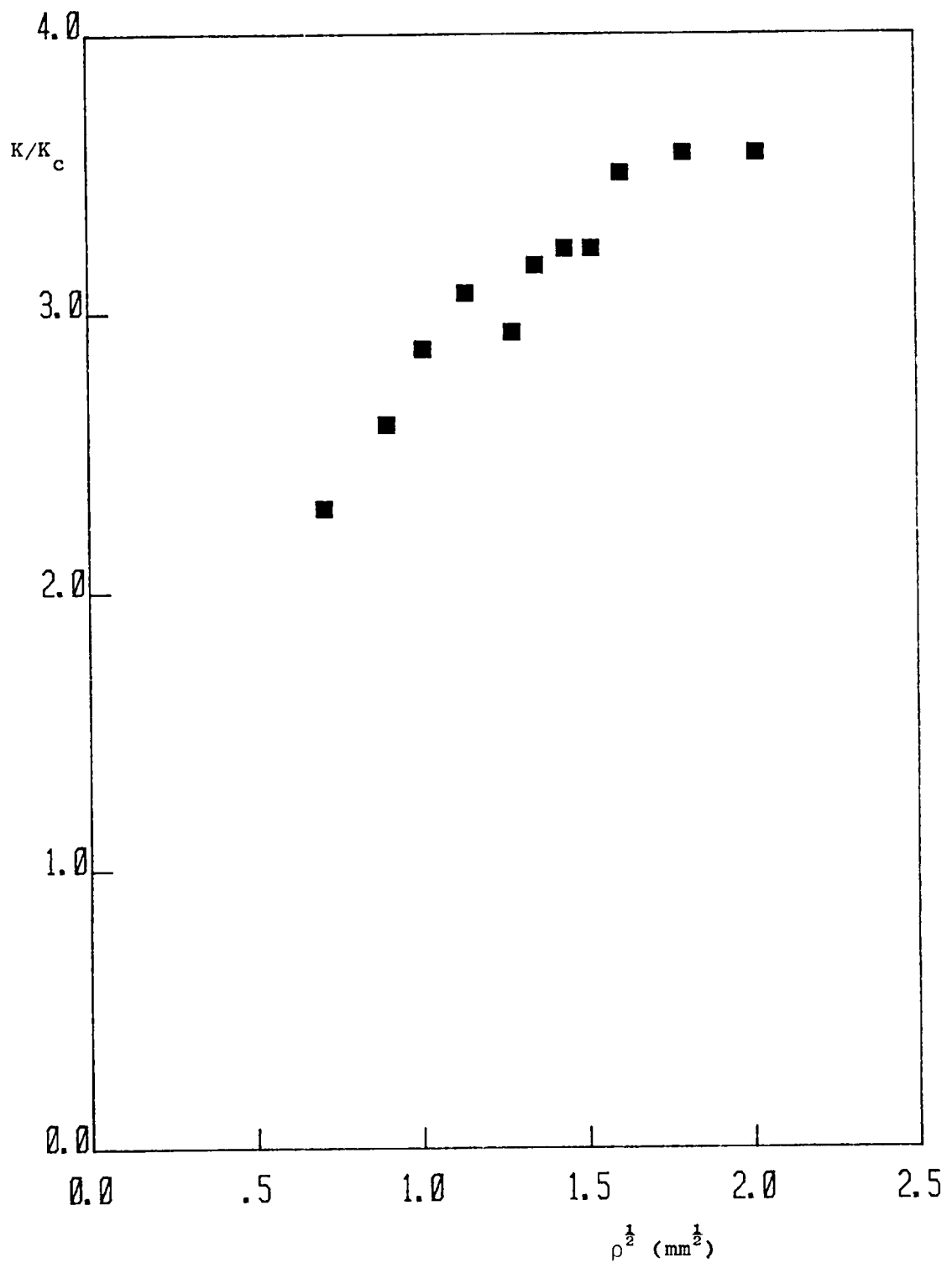
$$\frac{K_A}{K_C} \sim \frac{1}{2} \left(\frac{\rho}{2c} \right)^{\frac{1}{2}} \quad (3.23)$$

which does not model the behaviour in region (C) adequately. This is not surprising since equation (3.14) only applies for $\rho/a \ll 1$. Returning to equation (3.10), it may be shown that for $r/a \ll 1$

$$\frac{\sigma_{yy}(r)}{\sigma} \sim 2 \left(\frac{a}{\rho} \right)^{\frac{1}{2}} + 1 \quad (3.24)$$

and, as $\rho \rightarrow \infty$ so the limit $\sigma_f = \sigma_c$ is approached. Therefore, using the full analysis of the elastic situation, the departure from linearity as observed in region (C) may be predicted. Two further points worth noting with regard to equation (3.14) are that, firstly, it is strictly

FIGURE 1: VARIATION OF FRACTURE TOUGHNESS OF STEEL
WITH SQUARE ROOT OF CRACK TIP RADIUS



applicable to a fixed radius but gives good agreement because of the limited crack tip deformation in brittle materials, and secondly, the anomalous decrease in toughness for small radii is an artefact of the fracture criterion imposed but is a fair approximation to the observed constant toughness in this region.

Experimental data for various test configurations has been obtained by various workers including Ryan (1970) on PMMA for the single edge notch test-piece in tension, Plati (1975) on several polymers including PMMA and polycarbonate in impact, Kinloch (Kinloch and Williams (1980)) on an epoxy resin for compact tension specimens and Leever (1979) on several polyesters and an epoxy resin for surface notched specimens tested in tension. These materials fall neatly into two categories: those that craze (Ryan (1970), Plati (1975)) and those where crazes have not been observed (Kinloch and Williams (1980), Leever (1979)).

Dealing firstly with PMMA and polycarbonate, the critical stress at a critical distance argument gives good agreement with the data of Ryan for a critical distance of about $15\mu\text{m}$ and both the results and theoretical curve are shown in figure 2. Similar results have been obtained in the present study and these are indicated on the figure together with error bars. A critical stress of 120 MPa and critical distance of approximately $15\mu\text{m}$ are suggestive of a craze dominated failure, since they are close to the craze stress and craze length in magnitude. Similar results have been obtained by Plati (1975) as shown in figure 3 where the measured toughness is plotted against notch tip radius. The fitted line is according to equation (3.26) and indicates a critical distance of $55\mu\text{m}$. Both sets of data give a critical distance which is close to quoted craze lengths (Morgan and Ward (1977))

FIGURE 2: VARIATION OF FRACTURE TOUGHNESS OF PMMA
WITH SQUARE ROOT OF CRACK TIP RADIUS

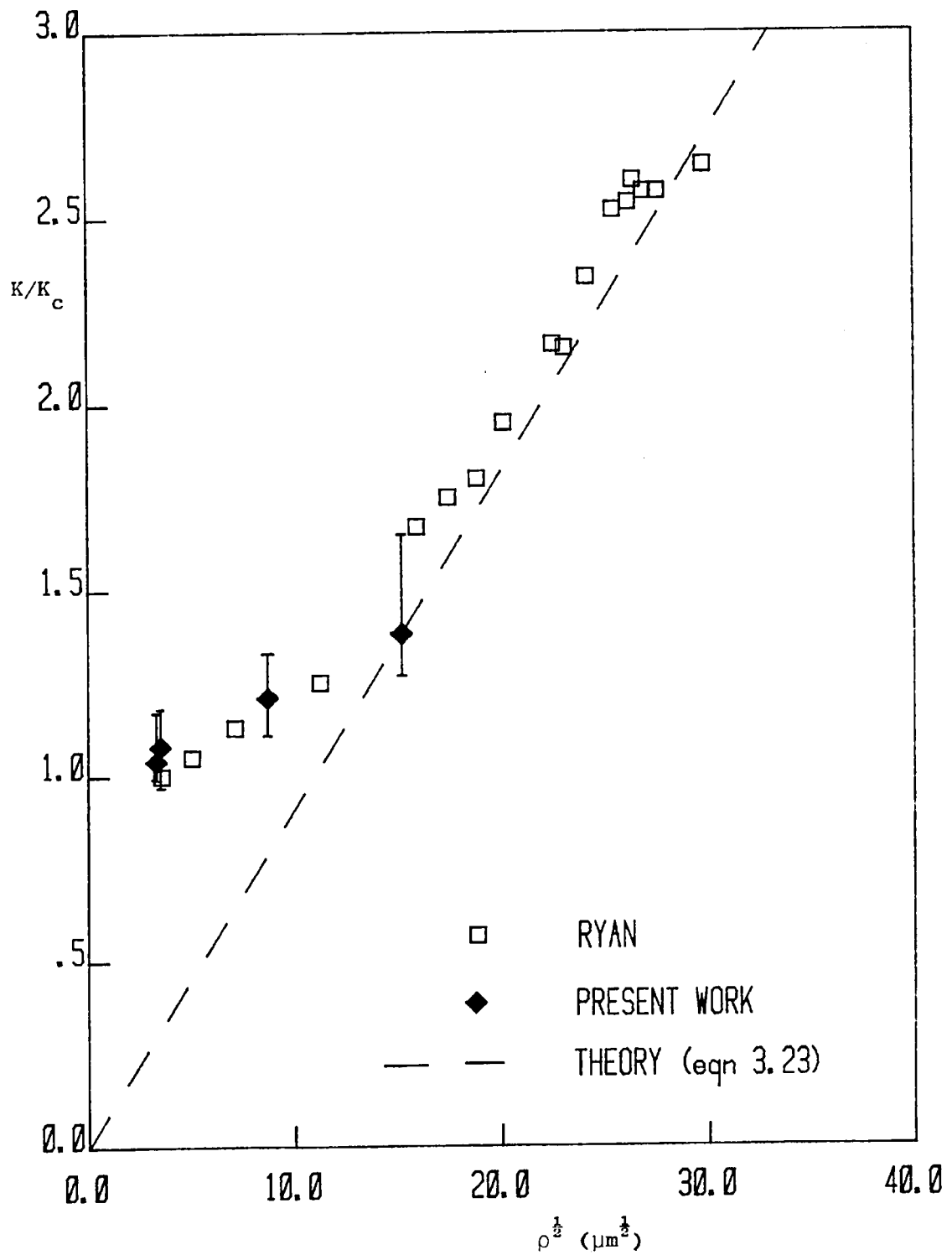
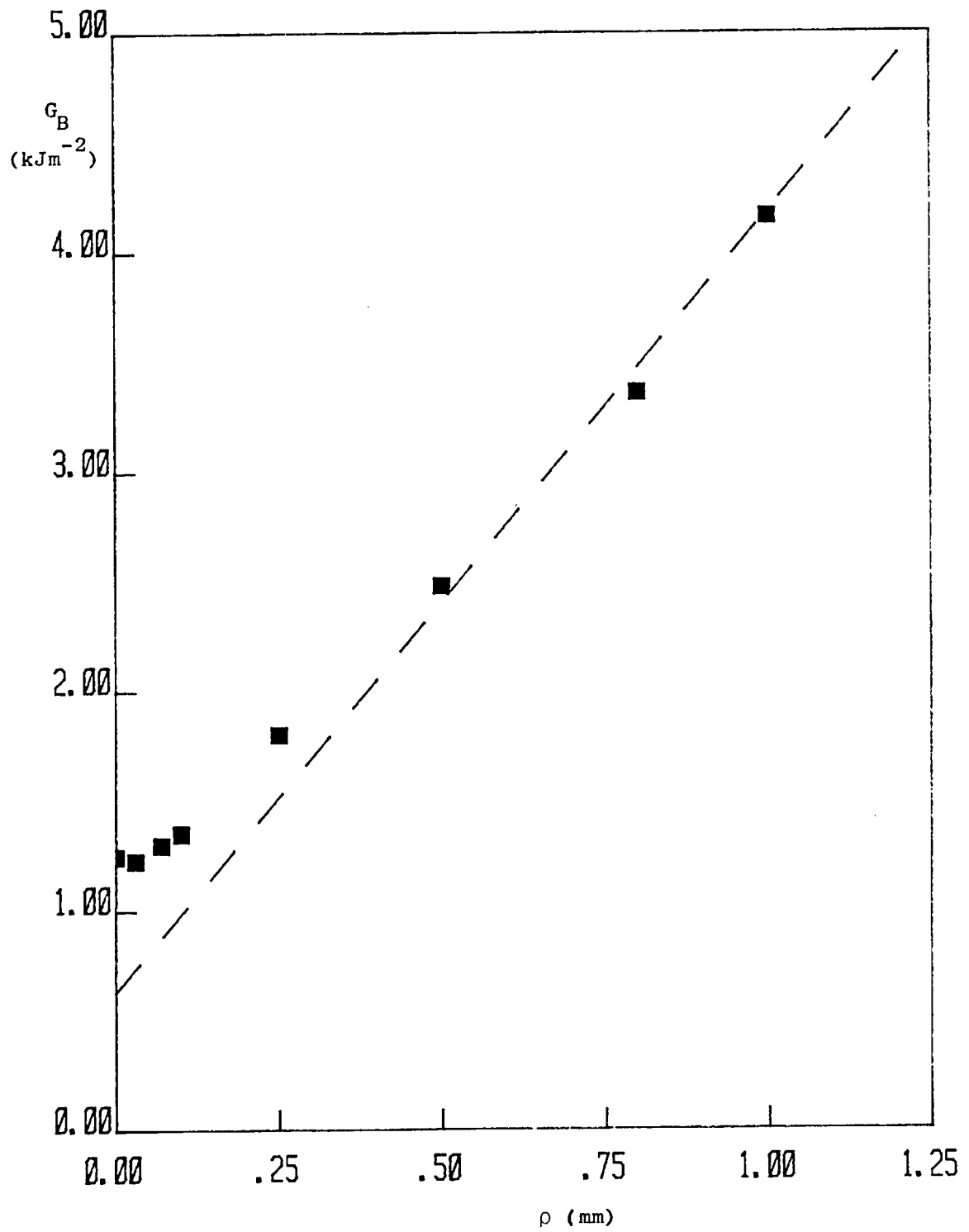


FIGURE 3: CRITICAL STRAIN ENERGY RELEASE RATE
VERSUS NOTCH TIP RADIUS FOR PMMA



and suggest crazing as the dominant fracture mechanism. The discrepancy between the results of Ryan and Plati is extremely bewildering.

Not only is it more difficult to avoid crazing ahead of the notches in the relatively deeply notched tests of Ryan, but there is also a rate effect in impact which, all things being equal, should result in higher elevations in toughness in the impact test. The opposite trend illustrated here confirms the extreme importance of careful examination of both the notching tools and the final notches in the fracture toughness testing of polymers.

In impact testing, the fracture energy is measured and equation (3.14) must be converted so that the apparent strain energy release rate ' G_A ' is predicted from the minimum or sharp crack strain energy release rate ' G_c '. Equation (2.8) demands that we square equation (3.14) to obtain

$$\frac{G_A}{G_c} = \frac{(1 + \rho/2c)^3}{(1 + \rho/c)^2} \quad (3.25)$$

and, for large radii this approximates to

$$\frac{G_A}{G_c} \sim \frac{1}{2} + \frac{\rho}{8c} \quad (3.26)$$

This explains why in figure 3 the fracture toughness is plotted against the notch tip radius and not its square root. Similar results are obtained for polycarbonate (Plati (1975)) but are not shown here.

To summarise the blunt notch testing of PMMA, therefore, it is found that regions (A) and (B) show close agreement with equations

(3.15) and (3.26) but evidence of region (C) is not strong, although Ryan's data does show tendencies in the right direction. It is presumed that region (C) would be observed for even blunter cracks than those examined by Plati and Ryan. According to any theory for blunt notch testing the maximum value of K_A is $\sigma_c (\pi a)^{\frac{1}{2}}$ and thus the behaviour of region (C) may be inherent in Plati's data since the crack length was varied in this work. However, for each tip radius an average toughness, G_A , is calculated and the effect is lost. Ryan's data for two crack lengths unfortunately only showed that deeper notches inevitably produce more crazing at the crack tip, although the trends were identical in the two cases.

Moving now to discuss the blunt notch fracture behaviour of polyester and epoxy resins. The only availability of a comparison between the data of Kinloch and Leever is for an epoxy resin. Figure 4 shows blunt notch data for compact tension specimens of DGEBA/piperidine and figure 5 consists of data for Araldite MY750/HT972 obtained from surface notched test-pieces in tension. Each figure includes a theoretical line of the form of equation (3.15) for chosen values of critical distance. The striking features are the much greater toughness elevation for the compact tension specimens showing no evidence of region (C), emphatic evidence of region (C) for the Araldite and, for region (B), agreement with the theoretical curves necessitating values of less than $1\mu\text{m}$ for the critical distance. Undoubtedly, when bending is incorporated in blunt notch testing the departure from linearity with the square root of notch tip radius is delayed, probably due to the ultimate tensile stress observed in bending being approximately twice that observed in tension. This is confirmed by the further emphatic evidence of region (C) observed in Leever's data

FIGURE 4: FRACTURE TOUGHNESS / NOTCH TIP RADIUS
DEPENDENCE FOR AN EPOXY RESIN

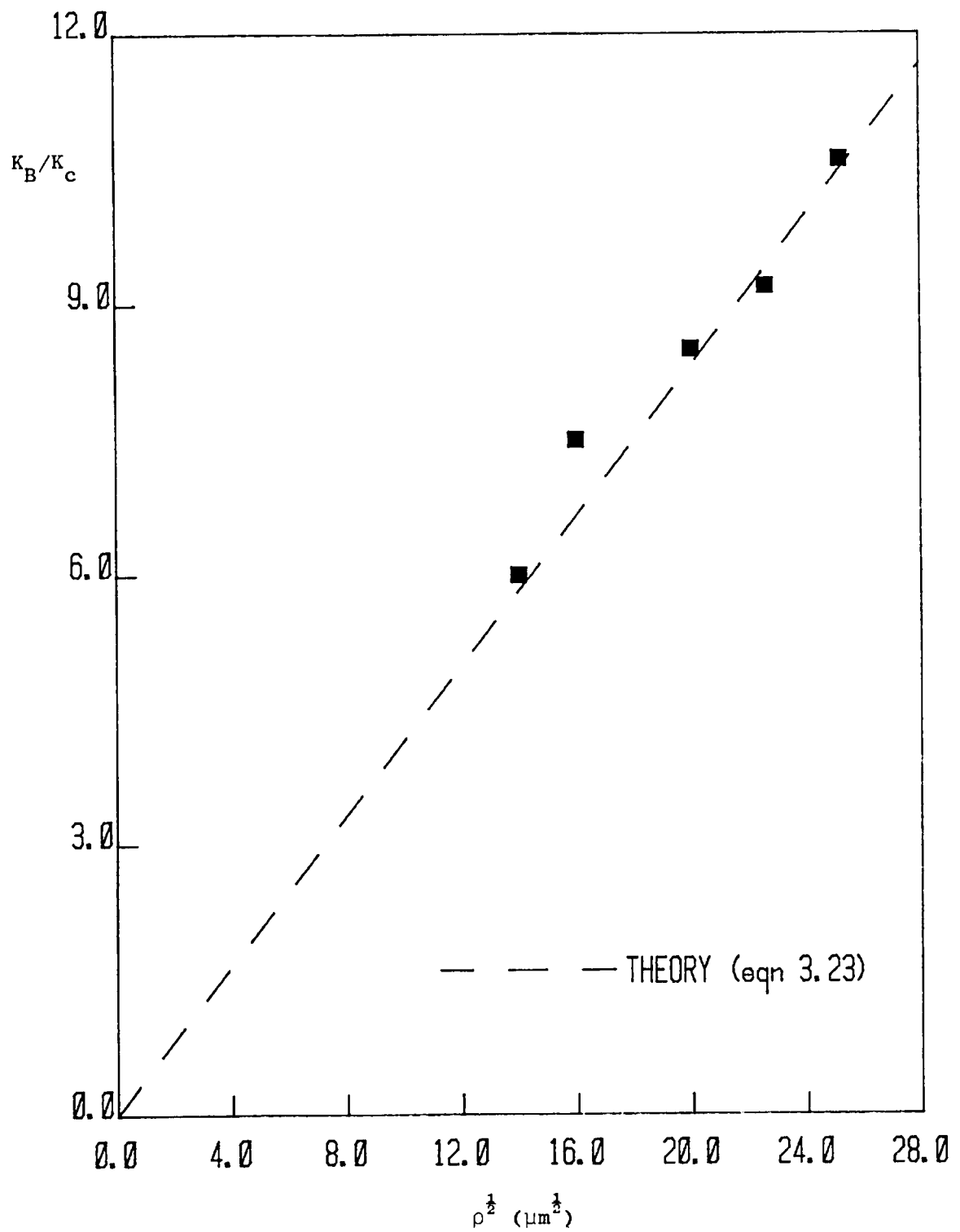
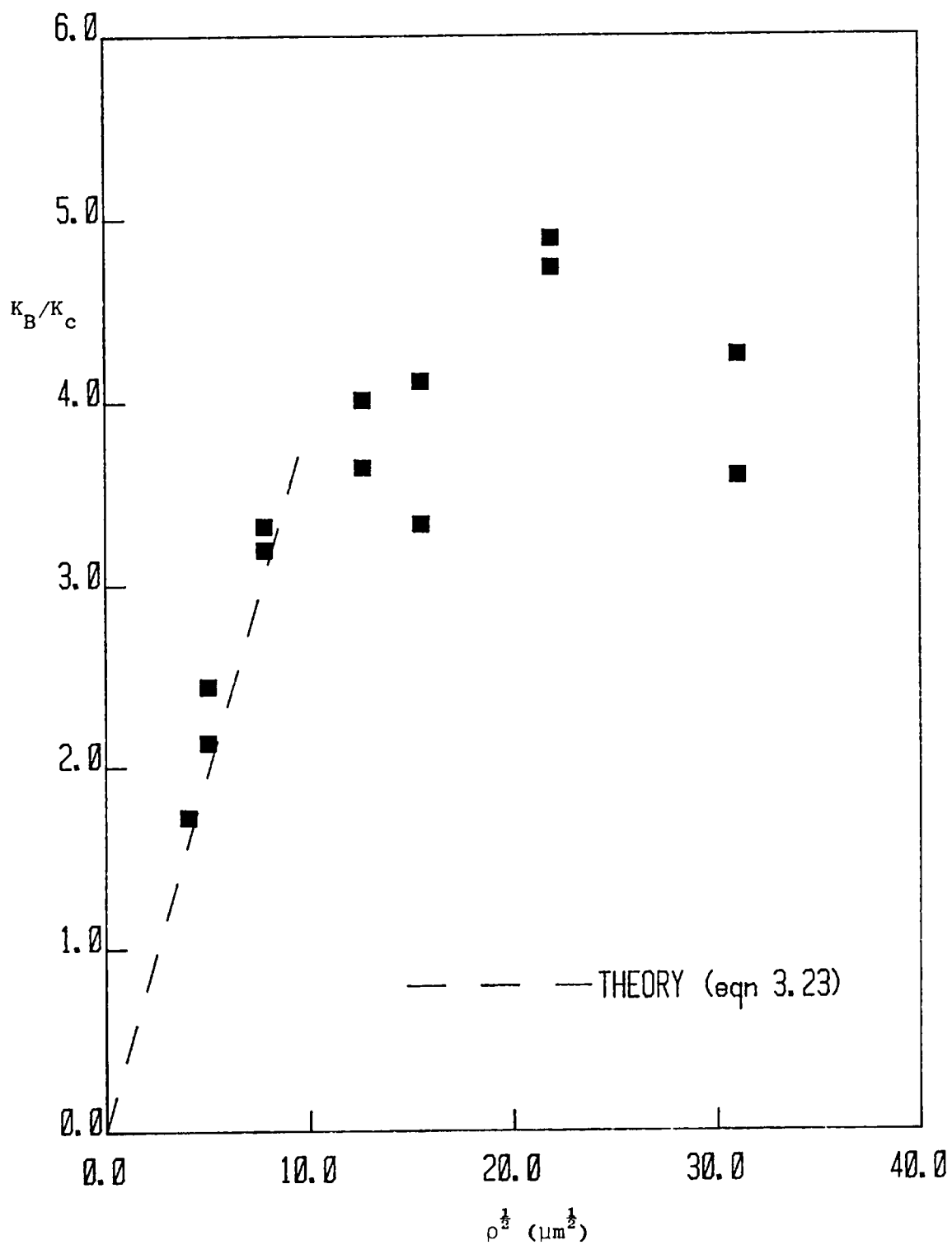


FIGURE 5: FRACTURE TOUGHNESS / NOTCH TIP RADIUS
DEPENDENCE FOR ARALDITE MY750/HT972



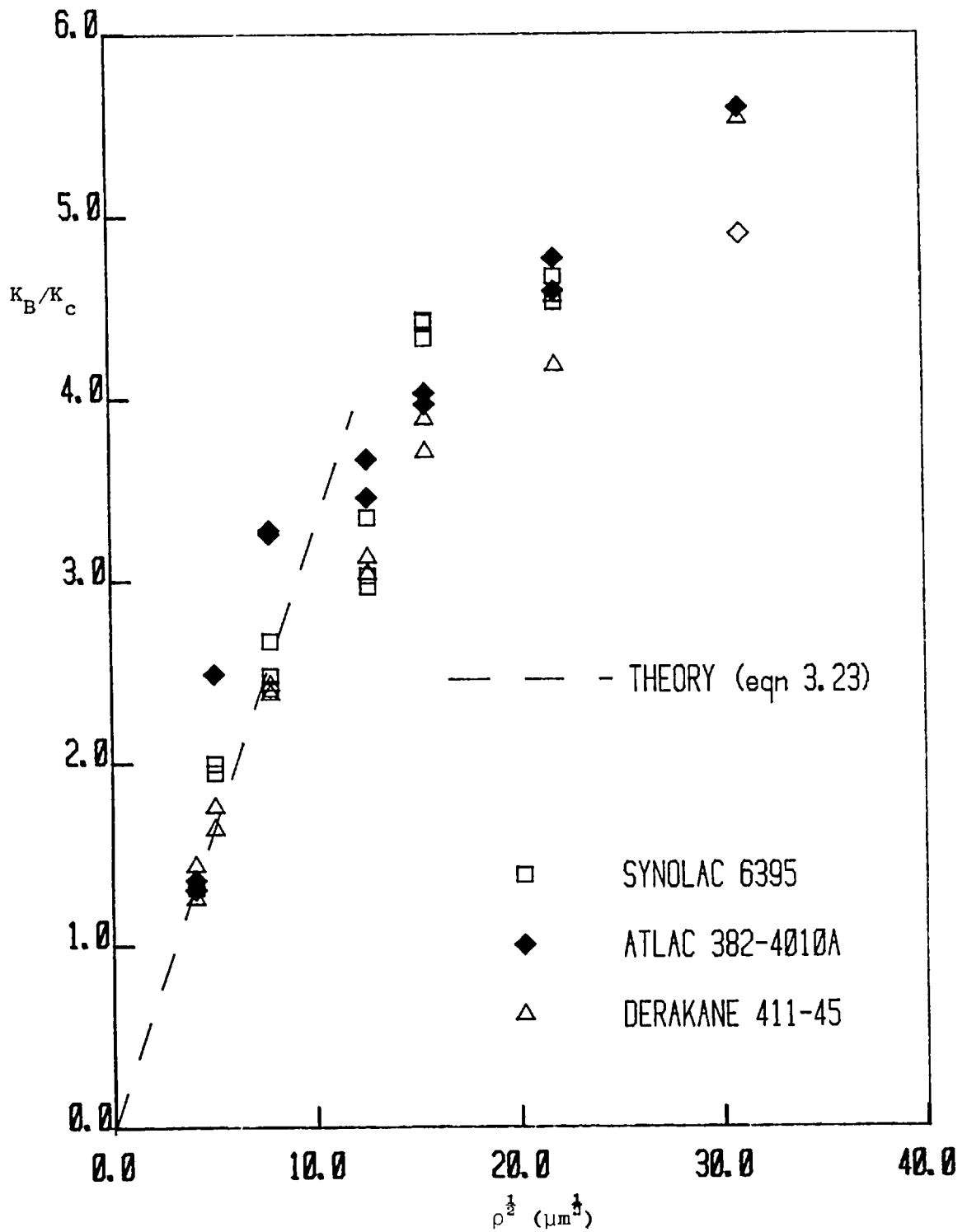
for polyesters. Three different polyester resins have been tested and the results are plotted together with a theoretical curve in figure 6. A critical distance of $1\mu\text{m}$ was chosen for the theoretical curve and agrees with the data for all three polyesters up to a crack tip radius of $100\mu\text{m}$.

One final note concerning the various models is provided by Mendez and Gasc (1977). They have re-examined the earlier results of Wilshaw et al (1968) and Ritchie and coworkers (1976) on steels, returning the verdict that the model of Heald, Spink and Worthington needs modification to account for blunt notch failure at small notch root radii. Their modification as presented is one in which the fracture toughness remains constant below a critical radius ρ_0 , and above this value the increase is given by equation (3.21) with ρ replaced by $(\rho - \rho_0)$. In order to compare this model with that due to Williams requires substitution of σ_c for σ_u in (3.21). Their model then gives linearity of K_A/K_C with $\rho^{\frac{1}{2}}$ for $\rho_0 < \rho \ll a$ and with a gradient of $1/(2c)^{\frac{1}{2}}$ whereas that of Williams gives linearity with a gradient of $1/(8c)^{\frac{1}{2}}$. Not enough quality data has been obtained on any particular material to discuss the merits of the two models except to say that the one due to Williams is the more immediately appealing since it describes behaviour in regions (A), (B) and (C).

(d) Experimental Blunting of Rubber

As has been shown conclusively in the previous section, blunt notch fracture is well characterised by the analysis of section (b). Following along these lines, the next two sections assess the validity of the deformation characteristics proposed in the analysis. In the first of these, the emphasis is on the crack tip profile of natural rubber at strain levels of up to 150%.

FIGURE 6: FRACTURE TOUGHNESS / NOTCH TIP RADIUS
DEPENDENCE FOR SEVERAL POLYESTER RESINS



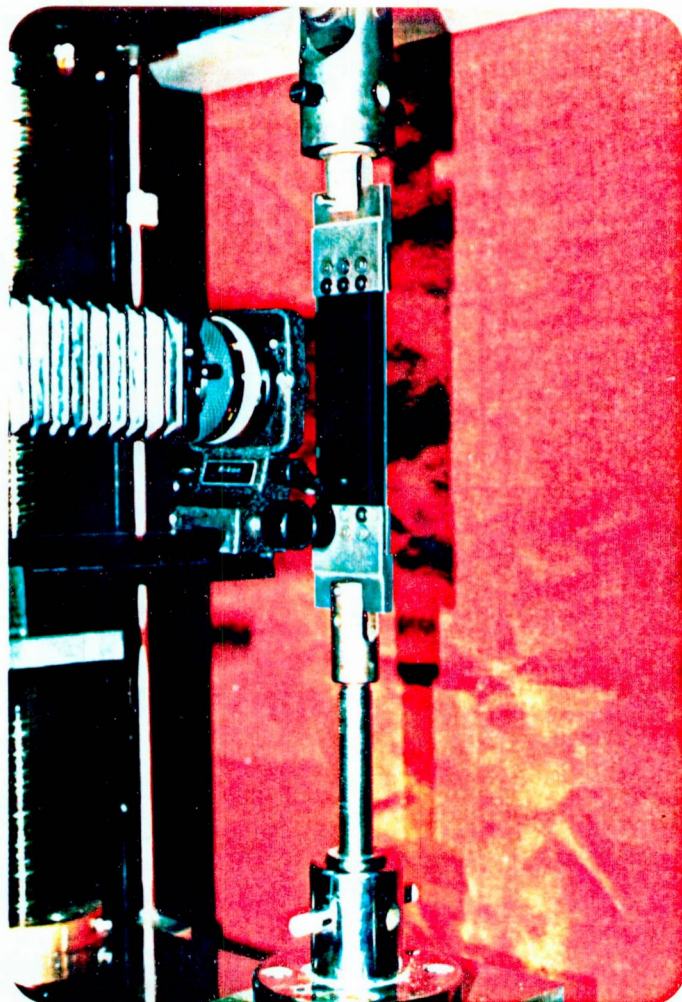
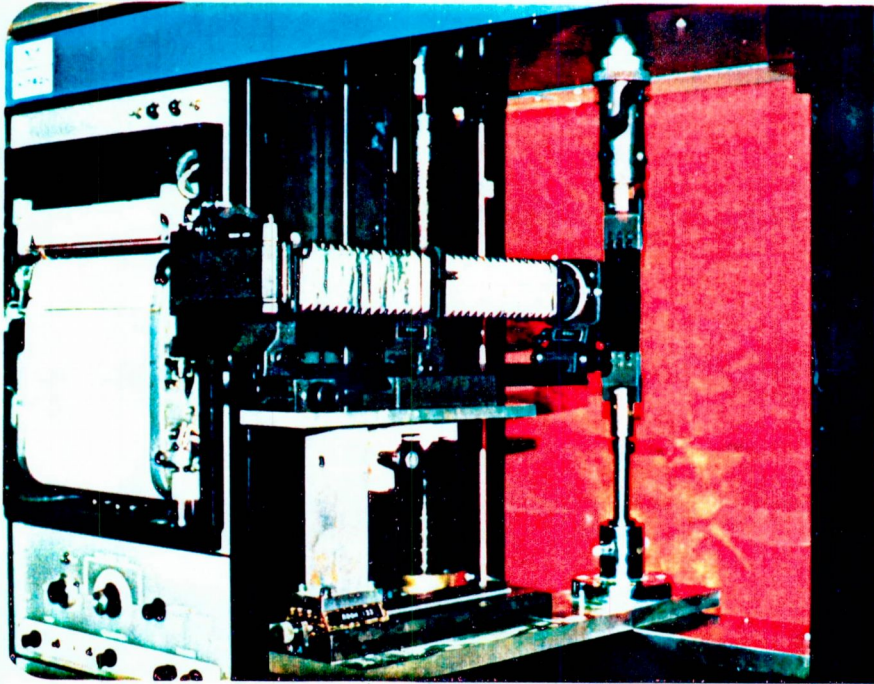
(i) Experimental Procedure

With an ultimate view to recording the near crack tip deformation of a range of materials tested in a single edge notch tensile mode, a high magnification photographic technique was employed. The experimental apparatus was as shown in the photographs of figure 7, but varied depending on the material under scrutiny. In this particular case, the material was rubber which gave large crack tip deformations not necessitating the use of either camera bellows or extension bellows for magnification purposes. The specimens, of initial width 50mm and of various thicknesses (1mm, 2mm and 4mm), were notched such that the ratio of crack depth to initial width lay in the range $0.04 < a_o/w_o < 0.40$. Slow steady cross-head displacement was applied to each specimen and the crack tip deformation was photographed. A load against strain output from the Instron chart recorder was obtained with marks indicating the state of loading corresponding to each photograph. All photographs were taken at a shutter speed of 1/30 second, an aperture setting of f3.5 and with a Kodak Ektachrome 64 colour transparency film loaded in the Nikon Nikkormat camera.

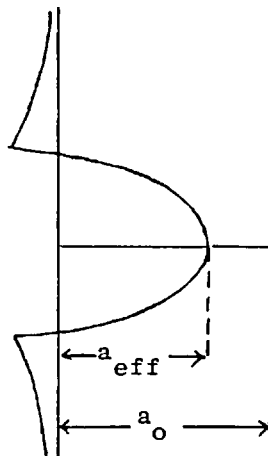
Several tests were run and it was found that strains of up to 40% could be accommodated by the apparatus before the onset of slipping in both the specimen grips and extensometer brackets. In order to obtain data for higher strains, the gauge length of the specimen was varied between 100mm and 160mm and cross-head displacement was recorded on the Instron chart. Slipping in the grips was obviated by stopping the test and tightening the grips at intervals of 20% strain.

The films monitoring crack tip deformation were sent away for processing and the resulting transparencies were projected horizontally onto a screen to enable an accurate tracing of the crack tip

FIGURE 7: APPARATUS FOR HIGH MAGNIFICATION
PHOTOGRAPHIC TECHNIQUE



profile to be made. This was undertaken at three different magnifications so that averaged values of measurements could be obtained. A radius template was used to estimate the crack tip radius; insertion of successively larger radii into the crack profile, the measured crack tip radius being the largest which formed a smooth inner tangent to the crack tip. The true crack tip radius was then calculated by reducing the measured value by the magnification factor of the projection. For the large strain tests the reduction in crack length was measured, and in these cases the effective crack length is that distance from the tip of the blunted notch to the line of extension of the fully stressed edge of the specimen as shown below.



(ii) Results

It was observed that at all stress levels the near tip crack profile was approximately elliptic and the deformation is typified by projected crack tip profiles as shown in figure 8. A typical set of photographs have been collated in figure 9 for a different test and show the same trends. Measurements of crack tip radius at strains in the range 0 - 50% were taken and the results for several specimens are tabulated in table 1. For those specimens which were strained beyond

FIGURE 8: TYPICAL TIP PROFILES IN RUBBER

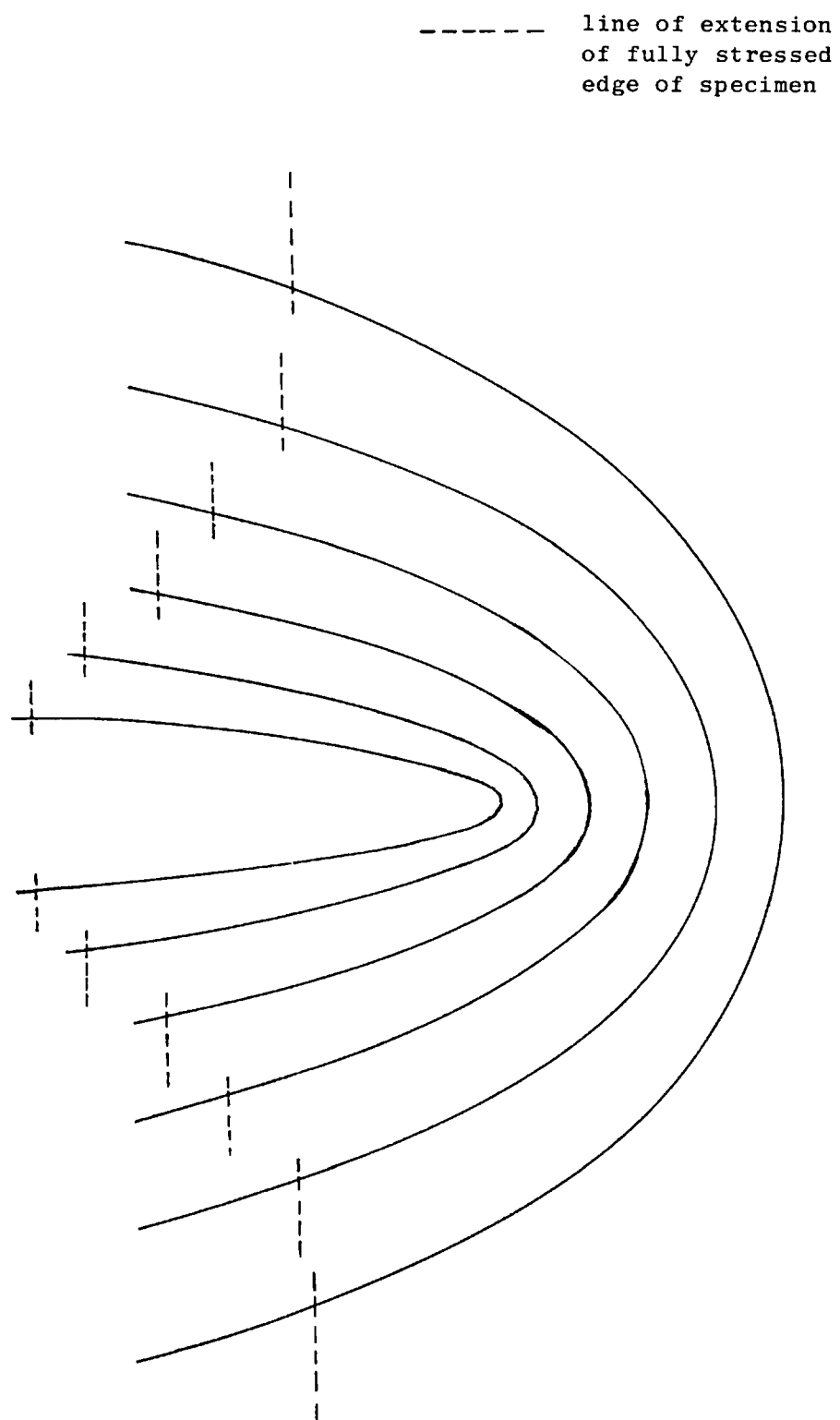


FIGURE 9: PHOTOGRAPHIC RECORD OF
BLUNTING IN RUBBER

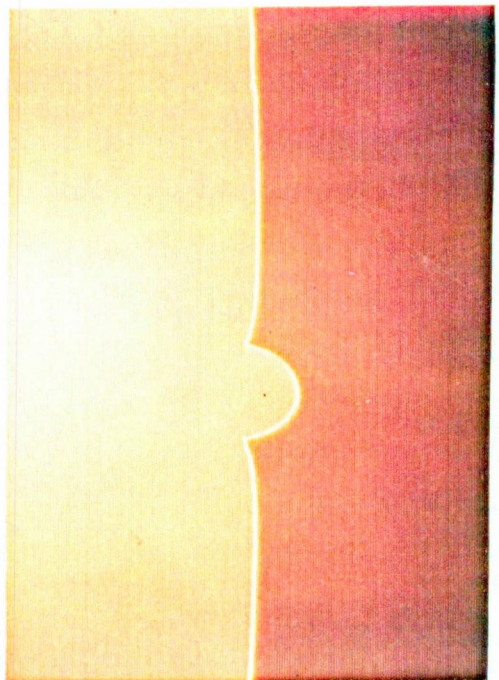
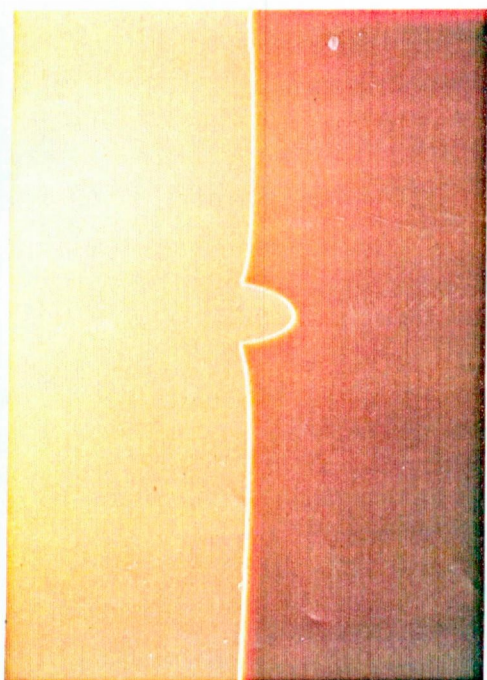
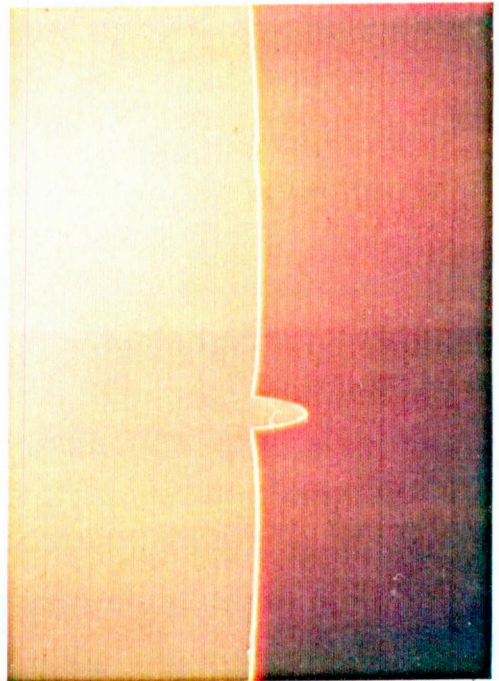
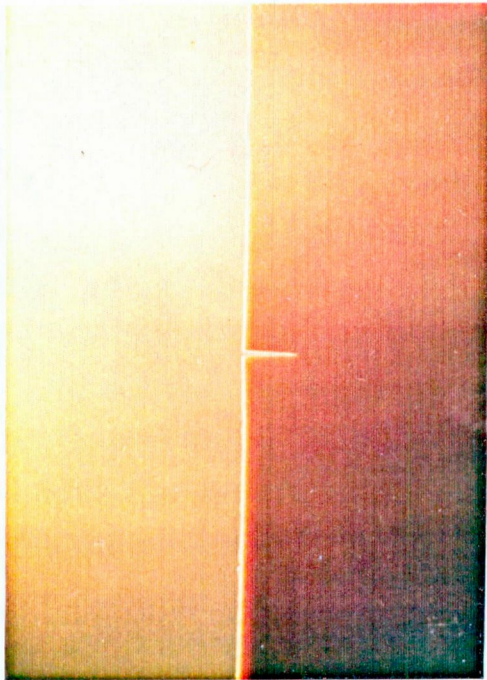


TABLE 1.

RUBBER - CRACK TIP RADII
MEASURED WITH INCREASING STRAIN

	STRAIN (%)	ρ (mm)	ρ / a_o
t = 1mm	5.9	0.290	.022
a_o = 13mm	12.5	0.720	.056
l_o = 128mm	20.7	1.440	.111
w_o = 150mm	30.9	3.250	.250
a_o/w_o = 0.09	40.6	6.500	.500
	52.0	9.020	.694
t = 4mm	7.1	0.170	.024
a_o = 7mm	13.4	0.340	.048
l_o = 121mm	23.3	0.680	.097
w_o = 150mm	32.2	1.690	.242
a_o/w_o = 0.05	43.1	3.610	.516
	51.1	5.080	.726
t = 4mm	10.9	0.100	.050
a_o = 2mm	15.8	0.170	.083
l_o = 156mm	20.5	0.330	.167
w_o = 50mm	33.3	0.670	.333
0.04	40.1	0.830	.417
	49.3	1.500	.750

TABLE 1.

(Continued)

	STRAIN (%)	ρ (mm)	ρ / a_o
t = 2mm	5.6	0.108	.013
a _o = 8mm	7.3	0.175	.021
l _o = 100mm	9.8	0.199	.024
w _o = 50mm	11.9	0.274	.033
a _o /w _o = 0.16	16.9	0.690	.083
	19.6	0.864	.104
	25.9	1.778	.214
t = 2mm	2.8	0.117	.009
a _o = 13mm	7.3	0.310	.023
l _o = 100mm	9.8	0.386	.029
w _o = 50mm	13.9	0.620	.047
a _o /w _o = 0.26	15.4	0.692	.052
	18.2	0.958	.072
	21.0	1.158	.087
	23.8	1.938	.146
	28.5	2.756	.208
	31.1	4.314	.325
	35.5	5.826	.439

TABLE 1.

(Continued)

	STRAIN (%)	ρ (mm)	ρ / a_o
t = 2mm	5.6	0.299	.016
a_o = 19mm	9.8	0.486	.026
l_o = 100mm	10.7	0.655	.035
w_o = 50mm	12.3	0.748	.040
a_o/w_o = 0.38	17.8	2.020	.108
	26.2	2.600	.139
	34.1	5.105	.273

this range, the effective crack length, a_{eff} , as indicated in the previous text figure, was measured. The results are tabulated in Table 2.

Despite the fact that the rubber specimens are cut from thin sheets the state of stress during loading is one of plane strain coupled with incompressibility, since upon fracturing a specimen the fracture surface is flat and exhibits no plastic zone. The theoretical displacements needed to analyse such a situation are derived in similar manner to those of equation (3.1) and give:

$$u = - (1 - 2\nu^2) \frac{\sigma}{E} a \cos\theta \quad (3.27)$$

$$v = \left\{ b + 2a (1 - \nu^2) \right\} \frac{\sigma}{E} \sin\theta$$

It should be noted that the plane stress results are regained upon substitution of $\nu = 0$. Rubber being an incompressible material, we choose to take $\nu = 0.5$ and proceed using the same incremental argument that was proposed in section (b). Relationships may be derived for the crack tip radius ' ρ ' and the crack depth ' a ' in terms of the gross stress ' σ ', the modulus ' E ' and the initial crack length ' a_0 '. Denoting σ/E by ε and, in keeping with large strain elasticity, writing $\lambda = 1 + \varepsilon$ the plane strain results are:

$$\frac{\rho}{a_0} = \left[\exp \left\{ \frac{3\varepsilon}{2} \right\} - 1 \right]^2 \exp \left\{ - \frac{\varepsilon}{2} \right\} \quad (3.28a)$$

$$\frac{a}{a_0} = \exp \left\{ - \frac{\varepsilon}{2} \right\} = \exp \left\{ - \frac{(\lambda - 1)}{2} \right\} \quad (3.28b)$$

TABLE 2.

RUBBER - DATA FOR STRAINS BEYOND 50%

	STRAIN (%)	$\pi a_{eff}/a_o$
$a_o = 2mm$ $a_o/w_o = 0.04$	58.0	2.34
	68.6	2.34
	80.4	2.10
	98.1	1.94
$a_o = 4mm$ $a_o/w_o = 0.08$	21.0	2.80
	40.0	2.54
	59.0	2.44
	78.0	2.25
	95.0	2.08
	115.0	1.97
	133.0	1.97
	145.0	1.89

Figure 10 illustrates the correlation obtained between the measured crack tip radius and equation (3.28a) for strains less than 35%, and indicates the departure for higher strains which is primarily due to the discrepancy in geometry between analysis and experiment. The trends are consistent for various values of the ratio of initial crack length to initial width and it is concluded that this model predicts the blunting of rubber very well for strains up to 35%.

(iii) Large Strain Behaviour

It is relevant to discuss the blunting pattern at large strains in the terms of some results due to Greensmith (1963) for the change in stored energy on the introduction of a cut into the edge of a rubber test-piece held in simple extension. These results apply for strains of the order of 100%. As described in Greensmith (1963), the change in stored energy, $W_i - W_f$, is related to the cut length, a_o , the thickness, h_o , and energy density $\mathcal{E}$ by

$$W_i - W_f = K(\lambda) a_o^2 h_o \mathcal{E} \quad (3.29)$$

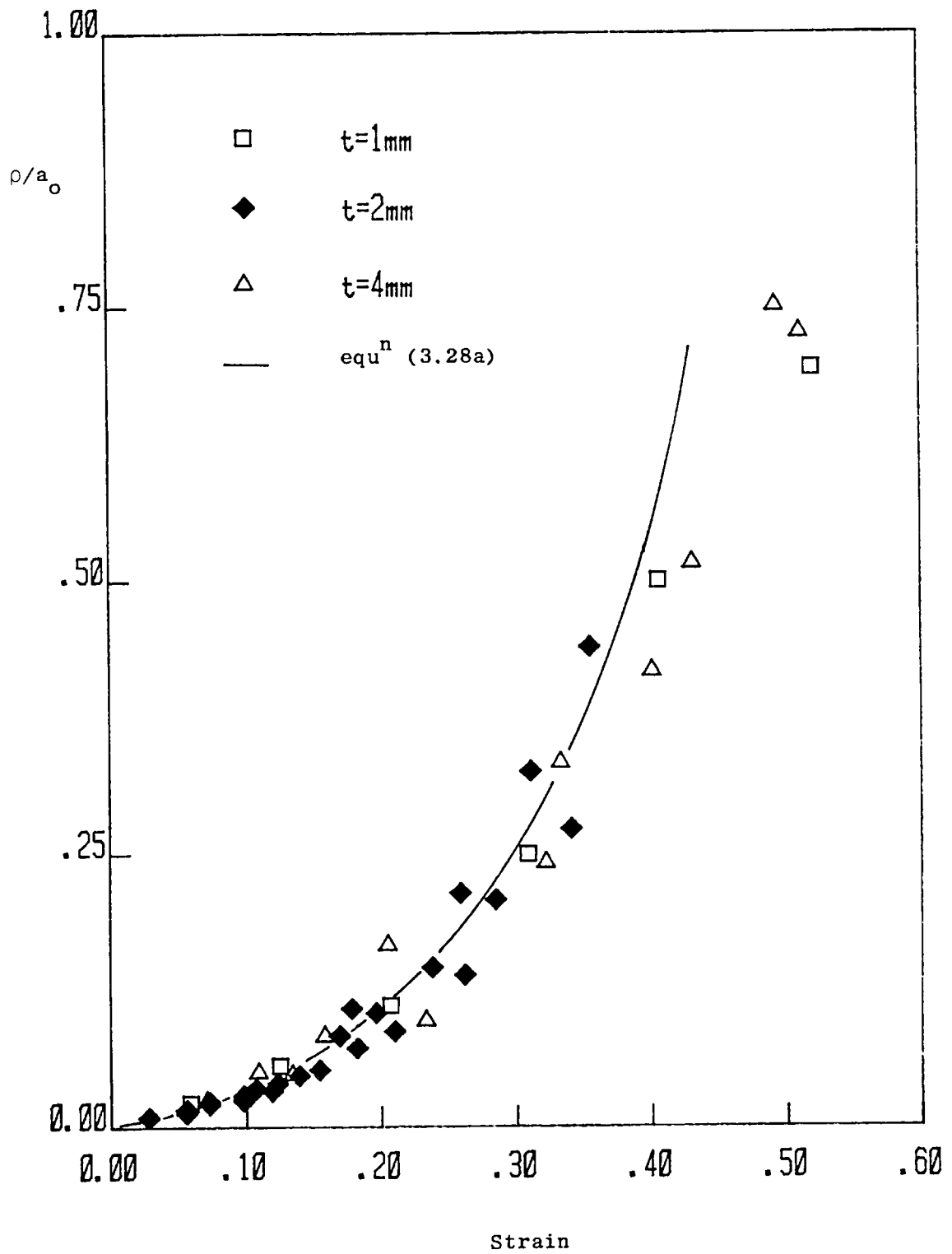
where $K(\lambda)$ is a numerical factor dependent on extension ratio λ .

Thus for a small change in crack length the usual energy release rate appropriate to crack extension, G , may be derived by differentiation, and, writing the energy density $\mathcal{E}$, as its approximate linear elastic form

$$\mathcal{E} = \frac{1}{2} \frac{\sigma^2}{E} \quad (3.30)$$

we readily obtain

FIGURE 10: BLUNTING OF CRACKS IN RUBBER



$$G = \frac{\delta W}{\delta A} = \frac{K(\lambda)}{\pi} \frac{\sigma^2 \pi}{2E} a_0 \quad (3.31)$$

The latter part of the right hand side of (3.31) is instantly recognised as being the energy release rate for small strain linear elasticity.

Postulating fracture to take place at a critical strain energy release rate, G_c , and denoting the apparent strain energy release rate by G_B we obtain

$$G_c = \frac{K(\lambda)}{\pi} G_B$$

or (3.32)

$$\frac{G_B}{G_c} = \frac{\pi}{K(\lambda)}$$

Now, as documented by Greensmith (1963), $K(\lambda)$ is a decreasing function of λ and physically denotes the decrease in stress intensity at the tip of the crack requiring higher energies for fracture. Two points should be emphasised, however: firstly, the crack length used to determine G_B is the original value and secondly, $K(\lambda)$ has only been determined empirically.

According to the analysis of elastic blunting for a totally elastic material, the initially sharp crack of length a_0 blunts into one of length a given by equation (3.28b). Introducing this into (3.31) gives

$$\frac{G_B}{G_c} = \frac{\pi}{K(\lambda)} \exp \left\{ - \frac{(\lambda - 1)}{2} \right\} \quad (3.33)$$

Clearly the criterion for fracture must be strain independent and since for brittle linear elastic materials fracture takes place when $G_B = G_c$, then for rubbery materials the following relation must hold

$$K(\lambda) = \pi \exp \left\{ - \frac{(\lambda - 1)}{2} \right\} \quad (3.34)$$

Figure 11 illustrates equation (3.34) in relation to the empirical $K(\lambda)$ derived in Greensmith (1963). Agreement is fair and reflects the discrepancy in situations between the infinite lamina of section (b) and the SEN test piece of Greensmith (1963). This is shown to be so by replacing (3.34) by the equivalent expression

$$K(\lambda) = \pi a_{\text{eff}} / a_o \quad (3.35)$$

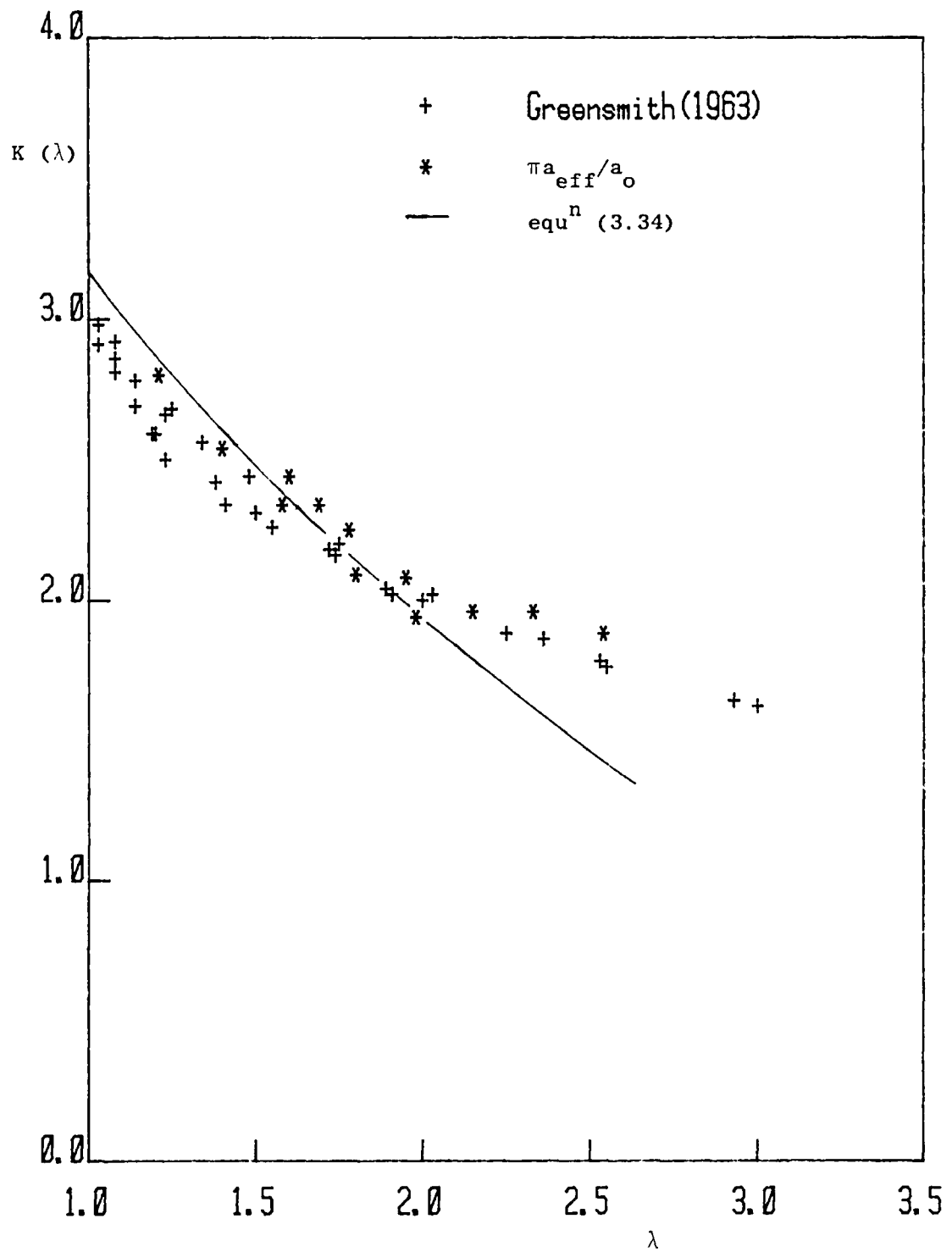
where a_{eff} is the effective crack length, and plotting the data of table 2. This more realistic comparison is also shown in figure 11 and the agreement is much improved.

These results show that the blunting model not only predicts the deformation of totally elastic materials at modest strains, but also provides a sound physical interpretation for the strain dependent parameter $K(\lambda)$ which is found in the theory of the tearing of rubber.

(e) Blunting of Low Density Polyethylene (LDPE)

To a great degree the success of the blunting analysis proposed earlier in predicting the deformation of rubber is due to the total elasticity throughout the material, that is the lack of plastic deformation at the crack tip. It is anticipated, and indeed found,

FIGURE 11: STRAIN-DEPENDENT PARAMETER $K(\lambda)$
DUE TO GREENSMITH(1963)

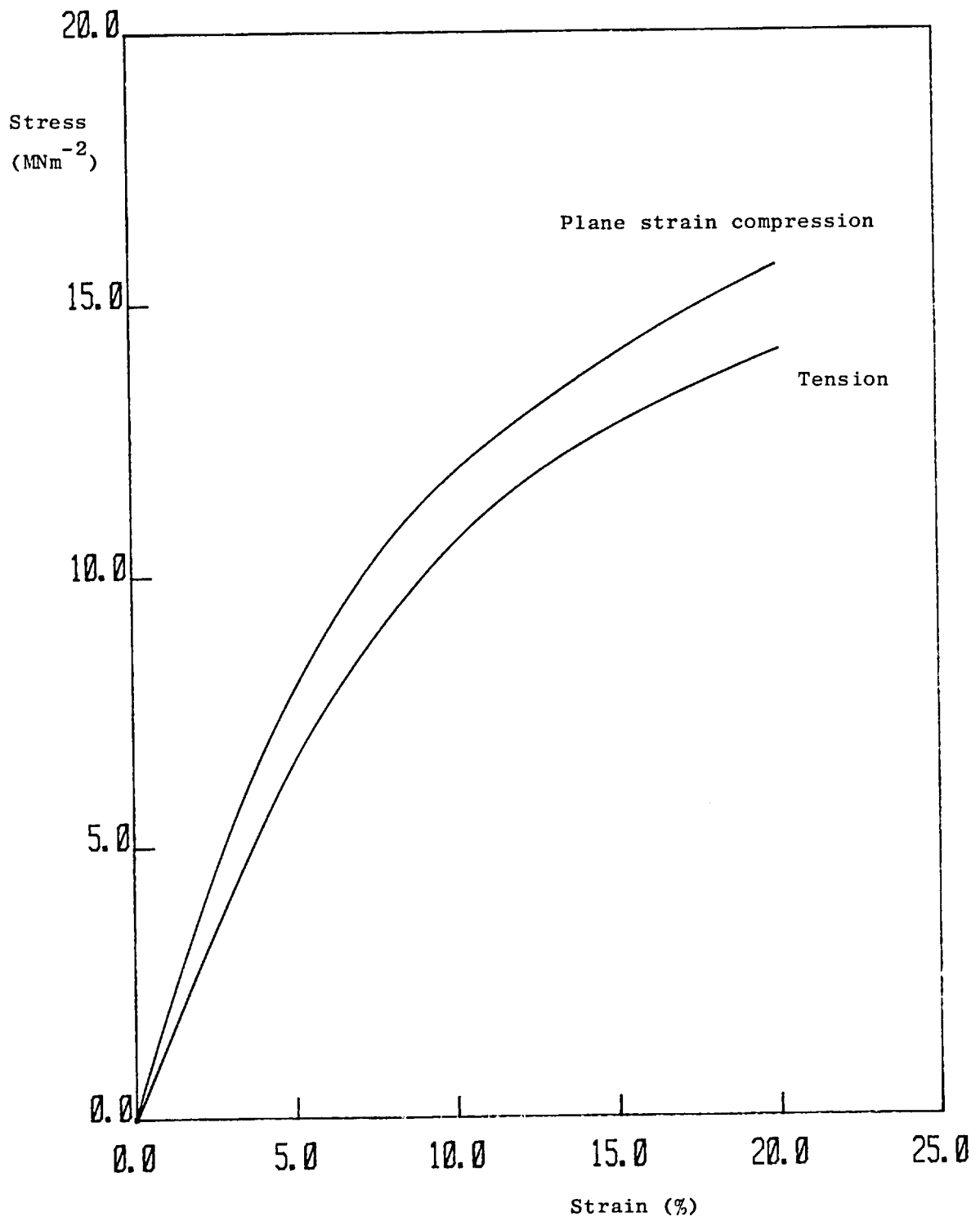


that very stiff but ductile materials which exhibit a well defined transition from elastic to plastic behaviour blunt far more intensely near the crack tip, thus illustrating departure from the elastic theory. These materials will be examined in a later chapter which also attempts to model the elastic - plastic deformation. In this section attention is focussed on LDPE which at room temperature shows no well defined yield point and can be regarded as exhibiting a large amount of continuing elasticity. Such a material was considered as ideal for future examination on the analysis of crack tip blunting.

(i) Experimental Procedure

Firstly, to realise the elasticity present in LDPE, stress-strain curves of the chosen grade were obtained, both in tension and compression and, for similar strain rates, the two curves are almost identical. Figure 12 indicates no marked yield point and a substantial degree of elasticity. As with natural rubber, SEN specimens were examined of 3mm thickness, 50mm width and 100mm gauge length. For a range of initial crack lengths the load-deflection curves were recorded and marked at intervals corresponding to initiation of crack growth and propagation through regular crack growth increments. An Instron cross-head speed of 0.5mm min^{-1} was chosen to enable the slow deformation to be photographed using a similar set up to that shown in figure 7. In this instance only the camera and bellows were used which allowed a maximum magnification of 4x. The camera settings were a shutter speed of $\frac{1}{4}$ second and an aperture of f16. Photographs were taken prior to crack initiation and corresponding gross strains were

FIGURE 12: STRESS-STRAIN CURVES FOR LDPE



calculated by dividing the deflection by the gauge length. The deformation during crack extension was recorded at 4mm intervals. In all cases, the resulting transparencies were projected onto a screen to facilitate detailed measurement of the crack tip deformation.

(ii) Results

Dealing at first with the pre-initiation behaviour, the sketches of projected slides show that the deformation follows an approximately elliptic pattern but with, as might be expected, slightly greater displacement of the crack flanks near the crack tip. Tip radii were measured for a range of five initial crack lengths and results were compared with equation (3.28) by plotting them as ρ/a_0 against gross strain ϵ , as shown in figure 13. The departure from the solid line representing totally elastic behaviour is found to be more substantial for larger strains, following a predictable pattern in view of the stress-strain relation of figure 12. Figure 14 is a collation of prints for pre-initiation behaviour where the behavioural trends are clearly illustrated.

Post initiation behaviour has been examined in an identical fashion to that proposed by Hodgkinson and Williams (1981). Photographs were taken throughout propagation of the crack, and despite an intense region of straining at the crack tip a measurable tip radius was still present. Table 3 shows crack propagation data for all five initial crack lengths and the qualitative pattern is one in which the tip blunts substantially further immediately following initiation, eventually settling down to a steady state value after roughly 10mm of crack growth.

FIGURE 13: BLUNTING OF CRACKS IN LDPE

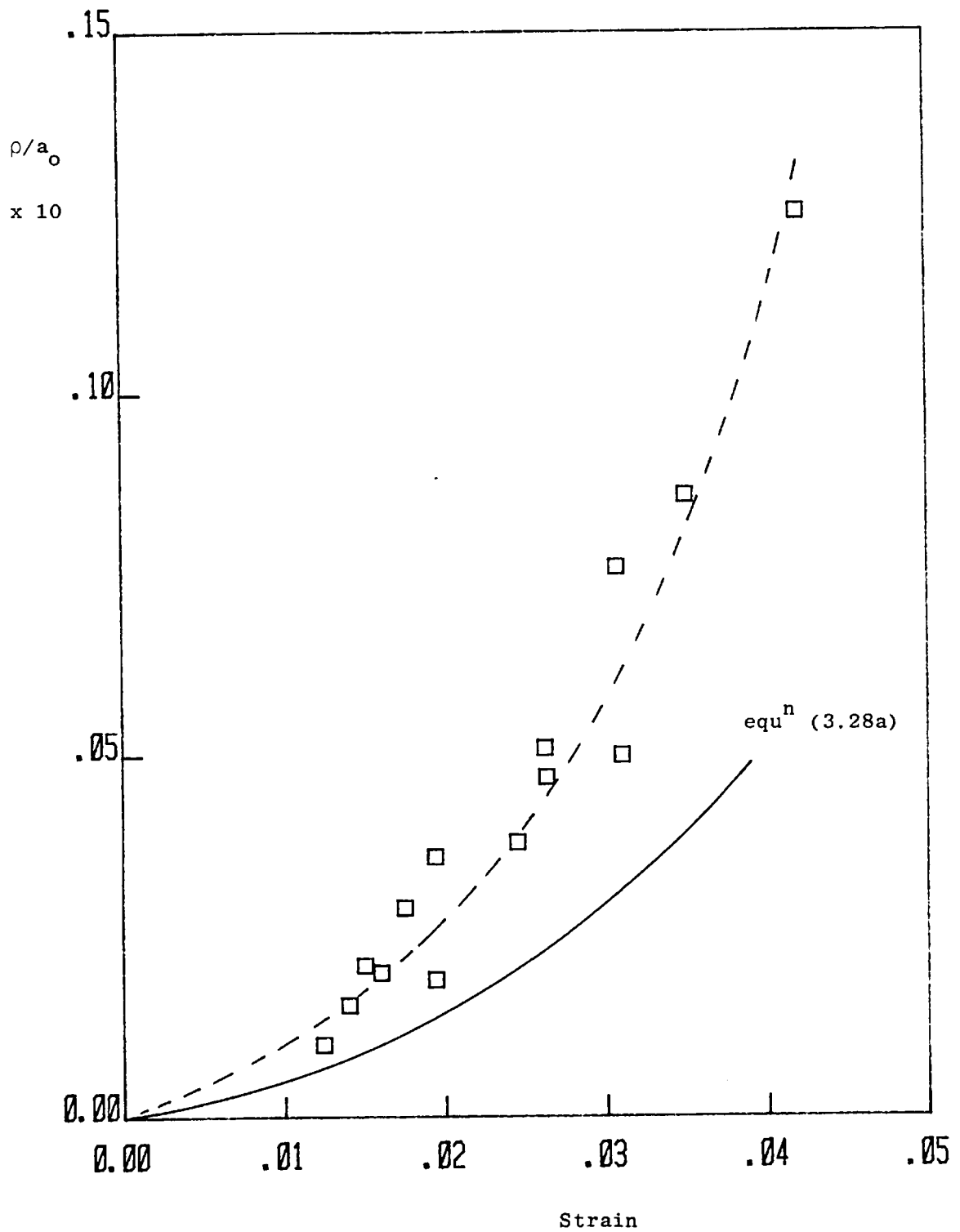


FIGURE 14: PHOTOGRAPHIC RECORD OF
BLUNTING IN LDPE

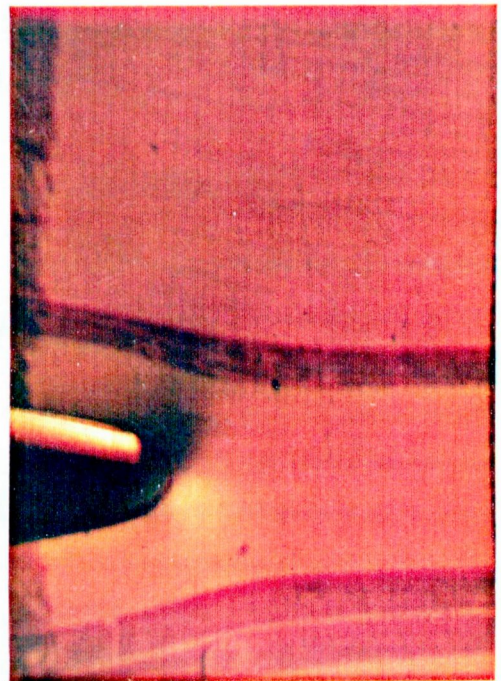
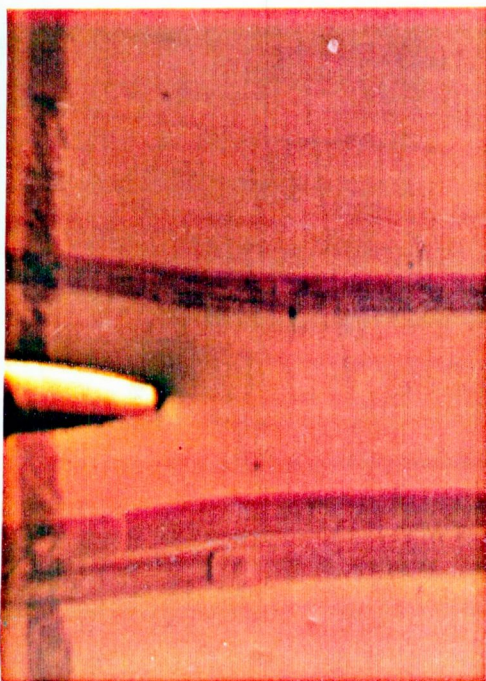
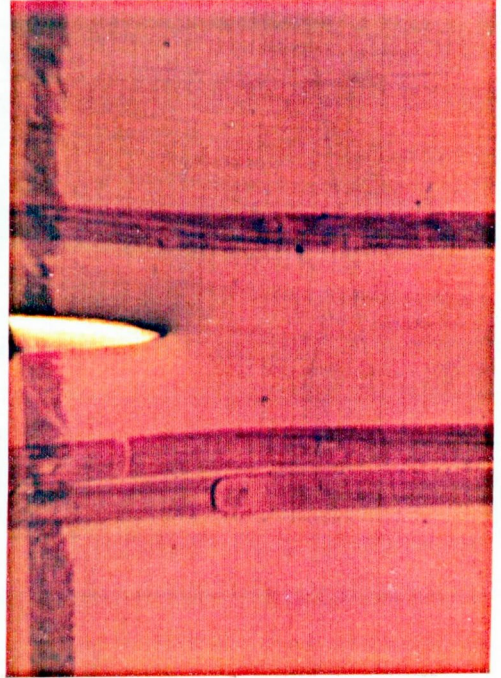
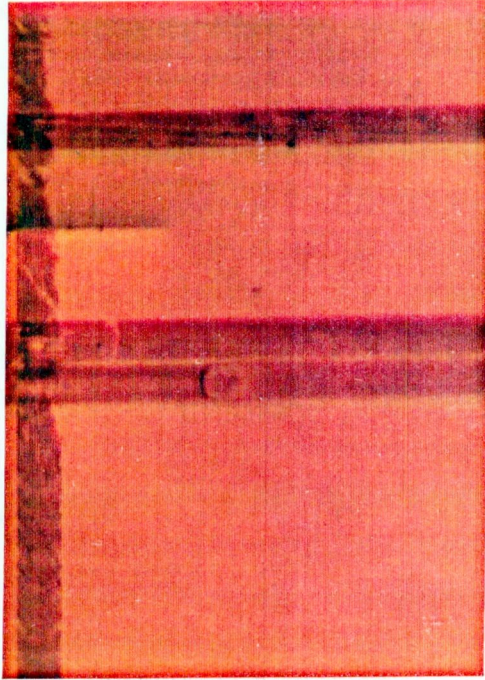


TABLE 3.

LDPE - INITIATION AND CRACK GROWTH DATA

PHOTOGRAPH NUMBER	LOAD (N)	DEFLECTION (mm)	CRACK LENGTH (mm)	CRACK TIP RADIUS (μm)
4.1	138	1.0	26	22
4.2	229	1.9	26	49
4.3	282	2.5	26	97
4.4	500	16.7	30	360
4.5	441	20.7	34	400
4.6	346	23.9	38	320
4.8	234	1.4	21	33
4.9	362	2.6	21	96
4.10	644	17.4	25	480
4.11	569	22.2	29	400
4.12	479	25.7	33	480
4.13	378	28.5	37	400
4.15	255	1.2	16	16
4.16	356	1.9	16	57
4.17	489	3.1	16	119
4.18	777	18.3	20	480
4.19	691	24.7	24	400
4.20	606	27.8	28	480
4.22	340	1.8	11	33
4.23	569	3.5	11	96
4.24	878	18.7	15	400
4.25	803	24.1	19	400
4.26	383	1.5	7	14
4.27	527	2.6	7	52
4.28	713	4.2	7	128
4.29	1000	18.5	10	720
4.30	897	25.6	14	480
4.31	851	28.9	18	480
4.32	777	31.5	22	400
4.33	665	34.8	26	400
4.34	559	37.6	30	400

(iii) Discussion

An analysis of the tearing of low density polyethylene has been proposed by Hodgkinson and Williams (1981) and in order to rationalise fully the results obtained in this study a short review is warranted. Since this analysis is also employed in chapter five, specific attention has been focussed on the calculation procedure for G as defined in Hodgkinson and Williams (1981).

Hodgkinson and Williams in analysing the tearing of LDPE use a modified form of the linear elastic fracture mechanics analysis which takes into account the irrecoverable energy dissipated in the loaded body and derive a current local elastic energy release rate at the tip of a growing crack. Their parameter, denoted by G but synonymous with I (chapter four, (d) (iii)) was calculated using the formulae

$$G = \frac{P^2}{2B} \frac{dC}{da} \left\{ \frac{2}{\eta} - 1 \right\} \quad (3.36)$$

$$\eta = \frac{(W - a)}{C} \frac{dC}{da}$$

and was found to be reasonably constant during crack propagation. P , here, is the load, B the specimen thickness, C the compliance at crack length a and η the elastic η -factor referred to in chapter four. The accuracy of the calculation is critically dependent on the determination of C and dC/da , and since this involves taking successive gradients of curves, errors can multiply. A rationale was sought by the author which would reduce these errors and provide a consistent calculation of G .

Clearly, the η -factor as derived in Hodgkinson and Williams (1981) is erroneous. At zero crack length C is non-zero and dC/da is zero, therefore η should be zero. The η -factor was recomputed for the compliance data given in Hodgkinson and Williams (1981) and compared with other elastic η -factors presented in Turner (1979). Figure 15 illustrates this comparison and a clear pattern emerges for the variation of η -factors in SEN tension with both crack length and gauge length.

The present study confirms the values of compliance and limit loads recorded in Hodgkinson and Williams (1981) and figure 16 shows typical load deflection curves for five initial crack lengths. As before, curves of constant crack length have been drawn to illustrate the limit load behaviour. Equation (3.36) is strictly valid for the limit load case and, as can be seen from figure 16, this is appropriate at most of the recorded crack increments. These increments are marked on the loading diagram with a short vertical line and the details of crack length, load, deflection and crack tip radius have been recorded in table 3. Values of G have been recalculated using the data from table 3 together with the η -factor from figure 15 and the results for propagation of the crack are tabulated in table 4 along with crack tip radius. Figure 17 is a plot of G versus crack advance for the five initial crack lengths and shows a constant value of 20kJm^{-2} during propagation. This confirms the results of Hodgkinson and Williams (1981) but has removed the scatter (and hence the uncertainty). This is entirely due to the more rational approach employed. Values of G for the initiation of tearing were calculated and gave an average value of 4.6kJm^{-2} , also in accord with this work.

From table 3 we see that at initiation the tip radius is

FIGURE 15: η -FACTOR FOR LDPE RECOMPUTED FROM
HODGKINSON AND WILLIAMS(1981)

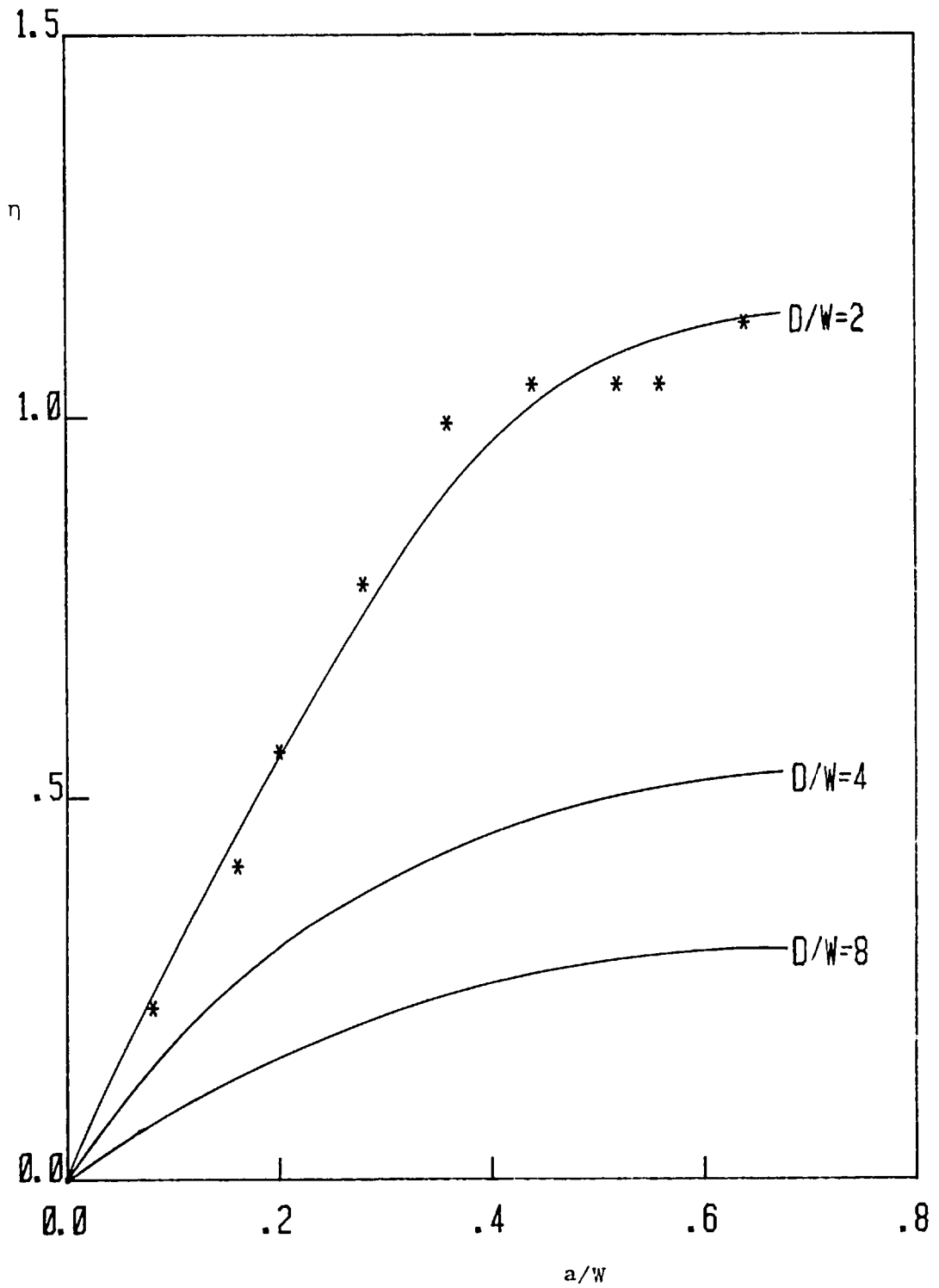


FIGURE 16: LOAD-DEFLECTION DIAGRAM FOR LDPE
INCLUDING CRACK GROWTH

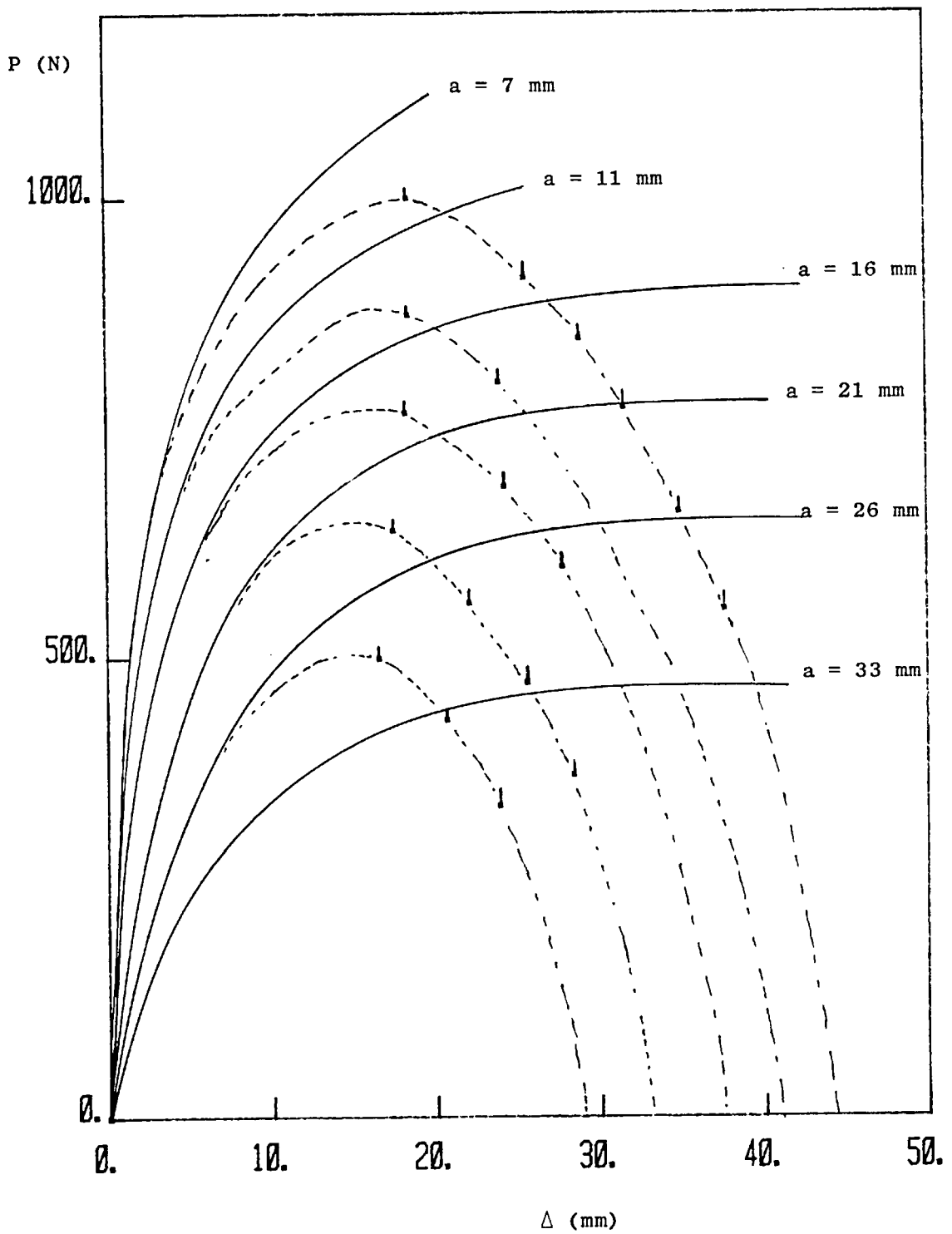
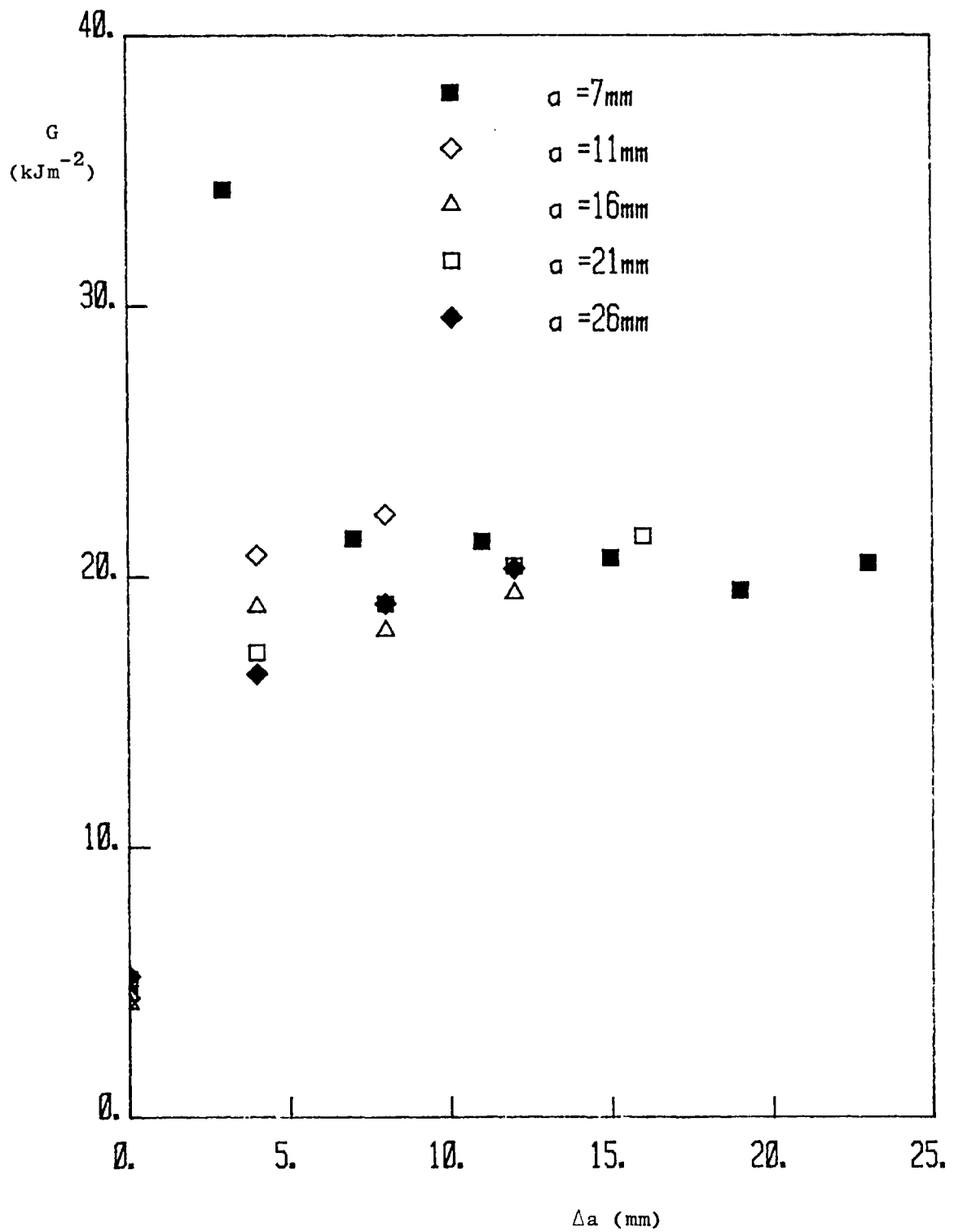


TABLE 4.

LDPE - CRACK GROWTH DATA AND
CALCULATED ENERGY RELEASE RATE

PHOTOGRAPH NUMBER	LOAD (N)	CRACK LENGTH (mm)	$\frac{dC(N^{-1} \times 10^3)}{da}$	η	$G(kJm^{-2})$	$\rho (\mu m)$
4.4	500	30	.48	1.1	16.4	360
4.5	441	34	.77	1.12	22.3	400
4.6	346	38	1.35	1.14	20.3	320
4.10	644	25	.28	1.06	17.2	480
4.11	569	29	.43	1.10	19.0	400
4.12	479	33	.68	1.12	20.4	480
4.13	378	37	1.17	1.13	21.5	400
4.18	777	20	.18	.98	18.9	480
4.19	691	24	.25	1.05	18.0	400
4.20	606	28	.38	1.09	19.4	480
4.24	878	15	.11	.81	20.8	400
4.25	803	19	.16	.95	19.0	400
4.29	1000	10	.08	.56	34.3	720
4.30	897	14	.10	.77	21.4	480
4.31	851	18	.15	.92	21.3	480
4.32	777	22	.21	1.01	20.7	400
4.33	665	26	.31	1.08	19.5	400
4.34	559	30	.48	1.10	20.5	400

FIGURE 17: VARIATION OF STRAIN ENERGY RELEASE RATE WITH CRACK GROWTH IN LDPE



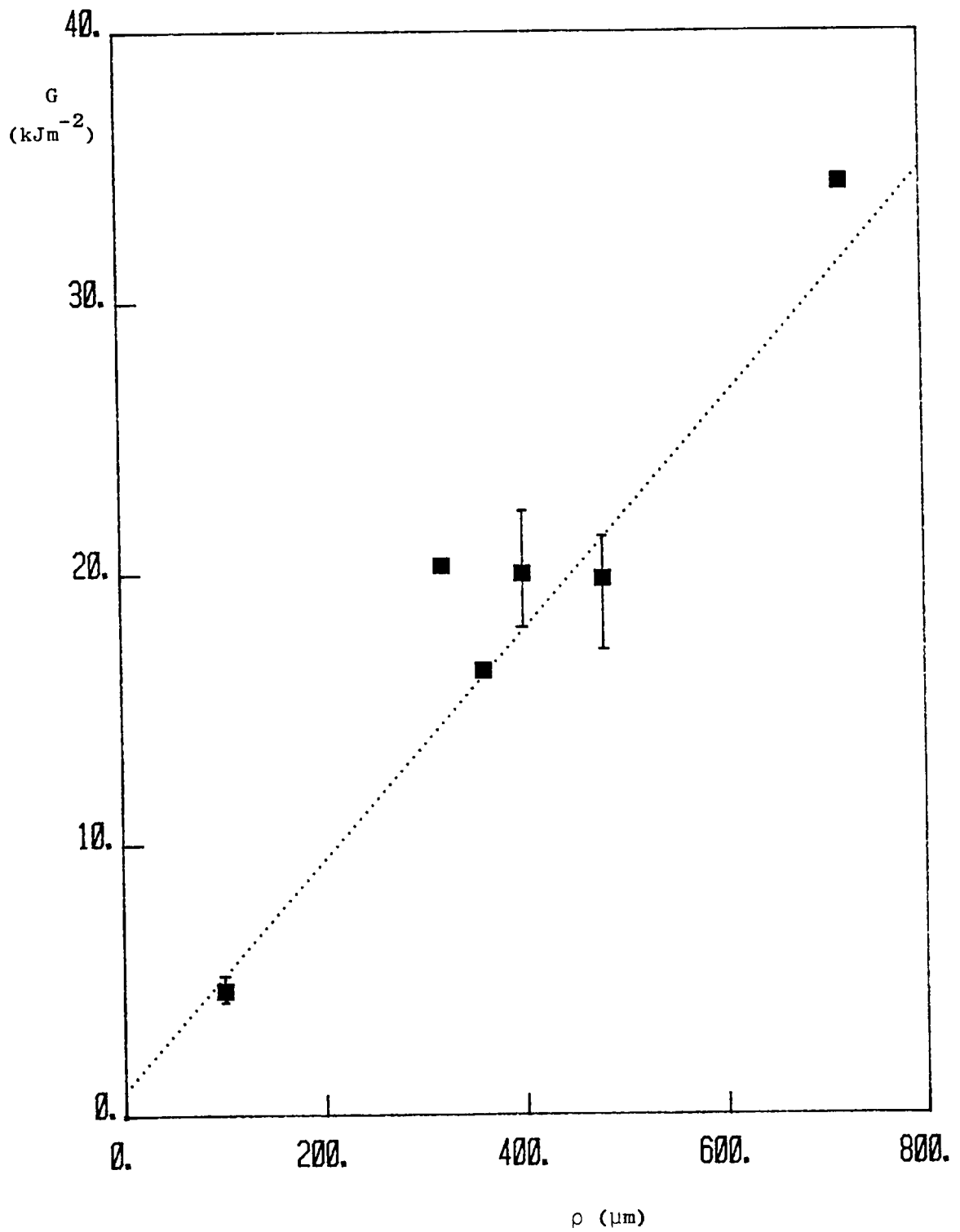
roughly $100\mu\text{m}$ and during propagation is approximately $400\mu\text{m}$.

Recourse to section (c) of this chapter provides a relationship between strain energy release rate and crack tip radius for initiation of failure in the brittle mode. It is tempting to speculate that a similar type of behaviour may be occurring here but with accompanying ductility. Figure 17 shows that for the shortest initial crack length the strain energy release rate rises rapidly for small amounts of crack growth subsequently dropping to the value for propagation. This is in agreement with the blunting of the crack tip and figure 18 is a plot of G against crack tip radius, ρ , from the data of table 4. A very reasonable linear relationship is obtained in view of the difficulties in assessing an accurate radius. It is expected that a further study concentrating on crack blunting just after initiation would confirm this relationship.

Equation (3.26) has been applied to the data and the straight line relationship involving G and ρ has been drawn in figure 18. The intercept for ρ equals zero is $\frac{1}{2} G_c$, and gives a value of G_c of approximately 1.8 kJm^{-2} and consequently, from the gradient, a critical distance of $5\mu\text{m}$. It was claimed in Hodgkinson and Williams (1981) that the initiation value of roughly 5 kJm^{-2} represented a plane strain value for the commencement of crack growth. This is clearly not the case since the crack tip was very poorly constrained in the 3mm thick SEN specimen. It is in the opinion of the author that the value quoted here is more representative of initiation in conditions of plane stress. It should be noted that it is an order of magnitude higher than that quoted by Andrews and Fukahori (1977) although further comment on this aspect will be deferred.

The conclusions to be reached are that the calculation of a

FIGURE 18: STRAIN ENERGY RELEASE RATE VERSUS
CRACK TIP RADIUS FOR LDPE



local current energy release rate at a propagating crack in a ductile polymer is useful in characterising behaviour during crack growth. The value of this parameter is sensibly constant during propagation in the steady state crack tip mode and both initiation and propagation values obey a linear relationship between energy release rate and crack tip radius. This shows that the simple model of the blunting of a crack can be used successfully in the context of plastic deformation and that for the simplest case, where the crack tip radius can be measured, a plane strain initiation value of strain energy release rate can be derived from those values obtained at the initiation of unconstrained tearing and during propagation.

CHAPTER FOUR

ELASTIC-PLASTIC FRACTURE MECHANICS

(a) General

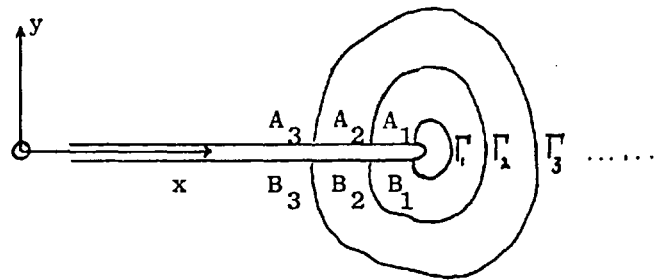
Ever since the use of structural materials, chiefly metals, in more exacting situations demanding accurate design from a safety assessment point of view, work on extending failure mechanics to include ductile phenomena has been prolific. Study directed along these lines is most easily traced through the ASTM special technical publications and the background is best summarised in McClintock and Irwin (1965). Fracture, as viewed at different scales of magnitude, is illustrated schematically and the ideology behind fracture mechanics is clearly stated; that for a complete and precise description of failure, integration of many models, each appropriate to phenomena of different size factors, is necessary. Boundary conditions in each region would be implied from the adjacent models. As a preliminary to presenting full elastic - plastic solutions to the mode III situation, the authors quote some fully plastic slip line field solutions, concluding that in contrast to elastic problems, the material will markedly influence the shape of the strain singularity at the crack tip through the strain-hardening characteristics, this effect being least evident in the plane-strain tensile mode.

Initial attempts to include crack tip plasticity in fracture analysis were merely corrections to LEFM which proved insufficient to cope with more than small scale yielding. Subsequent techniques, including the equivalent energy (EE) method and crack opening displacement (COD) method, were lacking in a formal analytical

background which consequently made their limitations unclear. The development of the J-integral technique accredited to Rice (1968), although originally derived independently by Eshelby (1956) and Cherepanov (1967), has provided an increasingly respected rationale in which to discuss material behaviour with accompanying plasticity. As has been realised, there are strict limitations to the use of this method, but at present no other technique, save possibly the generalised theory presented by Andrews discussed in section (e), offers the advantage of a treatment of non-linear materials.

(b) The J-Integral Technique

An interest in energy approaches to fracture led Rice in the mid-1960's to the identification of a line integral, which when taken along a contour such as those shown below in a non-linear elastic material containing a notch, is path-independent.



This integral is denoted J and is given by

$$J = \int_{\Gamma_n} \left\{ w dy - \sigma_{ij} n_j \frac{\partial u_i}{\partial x} ds \right\} \quad (4.1)$$

where W is the non-linear elastic strain energy density

σ_{ij} denotes the stress tensor

u_i denotes the displacement vector

n_j denotes the normal vector associated with Γ_n

ds is an incremental distance along Γ_n

and (x, y) are cartesian co-ordinates as illustrated.

The only conditions for path independence are that the crack flanks, $A_3 A_2 A_1$ and $B_3 B_2 B_1$, be stress free and parallel to the x direction. W , the strain energy density, may be written in terms of the strain tensor, ϵ_{ij} :

$$W = \int^{\epsilon} \sigma_{ij} d\epsilon_{ij} \quad (4.2)$$

and, if the contour Γ_1 in the figure is shrunk onto the crack tip keeping the points A_1 and B_1 fixed, then

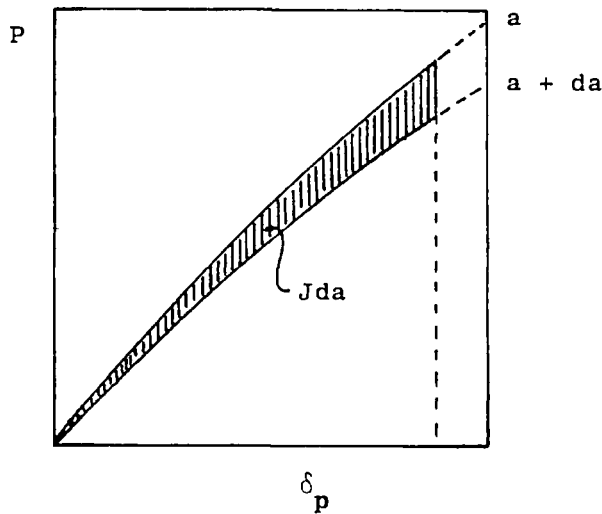
$$J = \int_{\Gamma_0} W dy \quad (4.3)$$

where Γ_0 is the contour at the tip, and thus J is an averaged measure of the strain at the crack tip.

Immediately, it is apparent that the J -integral will be an extremely useful tool in non-linear situations; not only does the path independence enable its value to be determined in the far field well away from the singular behaviour at the crack tip, but this value characterises the strain field near the tip.

More physical insight into the meaning of this parameter is given by examining the component parts of the integral when the crack is allowed to extend by a small increment from crack length

'a' to 'a + da'. The quantity $J B da$, where B is the material thickness, is the total energy transmitted through the contour during crack extension. Therefore, J is the energy available at the crack tip per unit area of crack extension and for linear elastic materials is equivalent to the Griffith energy, G . In identically the same way as G may be derived for various geometries by a compliance technique so can the appropriate value of the J -integral. Consideration of the non-linear elastic load deflection curves for crack lengths differing by 'da' leads to further definitions of J as shown schematically below



$$\begin{aligned}
 J &= - \int \frac{\partial P}{\partial a} d\delta_p \\
 &= \int \frac{\partial \delta_p}{\partial a} dP
 \end{aligned}
 \tag{4.4}$$

Limitations of the J -integral are also readily apparent.

Application should be specifically reserved for non-linear elastic, or equivalently 'deformation' plastic type materials.

True material behaviour is better modelled by incremental or flow theory plasticity in which unloading produces residual strains. Instances which require the use of such a theory, and thus cast doubts on the applicability of the J -integral technique, are fatigue, slow stable crack growth and dynamic crack propagation, all of which involve unloading of areas near the crack tip. It is generally accepted that, provided loading of the cracked body

is monotonic and no unloading occurs, representation of elastic - plastic materials by deformation plasticity and subsequent use of J theory is acceptable.

Experimental work has been performed by many workers, the most notable of whom are Landes and Begley (Landes and Begley (1972), Begley and Landes (1972)), to establish whether the attainment of a critical value of this parameter at crack initiation in metals is a valid fracture criterion. Further references to experimental determinations of critical J values are made by Turner (1979) in his comprehensive review of yielding fracture mechanics. The outcome is that no differentiation on the grounds of exactitude may be made between the critical J and the critical COD criteria.

Such evidence is for initiation of growth prior to unloading and, from this point of view, correlation of the two criteria is reconcilable. McMeeking (1977a) has revealed a path dependence of the J integral when finite deformation analysis is applied to a tensile notch in plane strain with contained yielding. Summarising the results in McMeeking (1977b), it is clear that there is no functional dependence of the J integral determined near the tip on that calculated remote from the notch; in fact, as the contour approaches the tip J tends to zero. It is thought conceivable that J calculated far from the notch is uninfluenced by the detailed deformation at the tip and thus denotes a characteristic severity of the crack tip stress and strain field. However, when the contour approaches zones of intense deformation near the tip then the J-integral becomes path dependent. Success of analysis which neglects blunting of the tip, for example Rice and Rosengren (1968), Hutchinson (1968) and Tracey (1976) in relating COD in terms of a remote J can thus be explained.

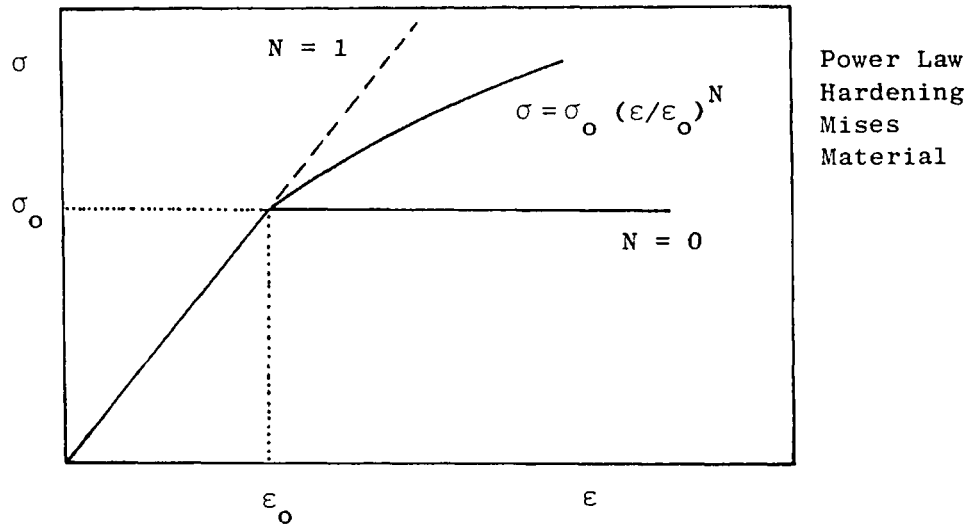
Among the efforts to characterise the stress field ahead of a

crack in an elastic plastic material, Rice and Rosengren (1968) and Hutchinson (1968) are generally regarded as cornerstones in J theory. Both investigate the dominant term in an eigenfunction expansion technique similar to that of Williams (1957) and Fenner (1973), but in Rice and Rosengren (1968) the eigenvalue (exponent of the characterising variable, r , the distance from the tip) is arrived at by intuition whereas in Hutchinson (1968), it is calculated for various power law exponents and observed to relate to the exponents via a simple formula. The stress-strain relations used are of various types; customary power law behaviour in Rice and Rosengren (1968) and Ramberg-Osgood power law or piecewise linear hardening law in Hutchinson (1968). Slightly different approaches are made in these studies and as a consequence the results are presented in differing fashions. Common to these solutions, and to many other solved crack problems, is that the product of stress and strain as the crack tip is neared becomes proportional in angular average to $1/r$. In terms of the J-integral, the results for plane strain may be written as

$$\sigma_{ij} = \sigma_o \left\{ \frac{J}{r \sigma_o \epsilon_o} \right\}^{1 + \frac{N}{1+N}} \bar{\Sigma}_{ij}(\theta, N) \quad (4.5)$$

$$\epsilon_{ij} = \epsilon_o \left\{ \frac{J}{r \sigma_o \epsilon_o} \right\}^{\frac{1}{1+N}} E_{ij}(\theta, N)$$

for a stress-strain law of the form depicted overleaf:



$\bar{\Sigma}$ and $\bar{E}$ are functions dependent only on the angular position and the degree of strain hardening. In a similar manner to the characterisation of the stress field in an elastic body by G , so J represents the degree of severity of straining at the tip of a crack in an elastic - plastic material.

Over recent years, much attention has been paid to the relationship between G , J and the work of fracture. The dominant contributor to this aspect of J theory has been Turner, whose η factor provides the link between the two and whose scenario for discussing many problems involving plasticity is by far the most enlightening and encouraging to emerge since the advent of work on the J -integral.

In the LEFM regime, the work, U_{el} , required to fracture a brittle specimen of thickness, B , and ligament length, b , is related to the energy release rate, G , by the expression:

$$G = \eta_{el} U_{el} / B b \quad (4.6)$$

where η_{el} is a factor depending on the compliance of the body and the

crack length to specimen width ratio. As stated in chapter two the release rate G may be written as:

$$G = \frac{P^2}{2B} \frac{dC}{da} = \frac{U_{e1}}{BC} \frac{dC}{da} \quad (4.7)$$

where P is the fracture load, C the compliance and $U_{e1} = P^2 C / 2$.

Consequently the η factor is expressed by:

$$\eta_{e1} = \frac{b}{C} \frac{dC}{da} = \frac{W - a}{C} \frac{dC}{da} \quad (4.8)$$

Rice et al (1973) showed that the J-integral for a deeply cracked bend specimen could be derived simply by dimensional analysis, the resulting expression being:

$$J = 2 U / Bb \quad (4.9)$$

This expression, taken with the fact that for deep notched bend specimens with a span to depth ratio of 4 the value of η_{e1} is approximately 2, led Sumpter (1973) to suggest that an overall η factor, η_o , may be defined in terms of overall work, U_o , and the constituent parts by:

$$\eta_o U_o = \eta_{e1} U_{e1} + \eta_{p1} U_{p1} \quad (4.10)$$

wherein the J-integral could be written as:

$$J = \frac{\eta_{e1} U_{e1}}{Bb} + \frac{\eta_{p1} U_{p1}}{Bb} \quad (4.11)$$

η_{pl} may be derived from limit load analysis and, if P_L is the limit load, then:

$$\eta_{pl} = - \frac{1}{B} \frac{\delta U_{pl}}{\delta a} = - \frac{\delta P_L}{B} \frac{\delta P_L}{\delta a} \quad (4.12)$$

Alternatively:

$$\eta_{pl} = \frac{bB J_{pl}}{U_{pl}} = - \frac{b}{U_{pl}} \frac{\delta U_{pl}}{\delta a} = - \frac{b}{P_L} \frac{\delta P_L}{\delta a} \quad (4.13)$$

Writing η_{pl} in this manner affords complete interpretation of the plastic η factor. Whereas in the totally elastic configuration, η_{el} depends solely on test-piece geometry, for fully plastic situations dependence upon crack tip constraint is included. Considering the cases of tension and bending, the limit load may be written as:

$$P_L = L \sigma_y B (W - a)^N / S^{N-1} \quad (4.14)$$

where L is the constraint factor, σ_y the yield stress, S the span in bending or gauge length in tension and $N = 1$ or 2 for tension or bending respectively. Combining (4.14) and (4.13) leads to:

$$\eta_{pl} = N - \frac{b}{L} \frac{\delta L}{\delta a} \quad (4.15)$$

Clearly, the constraint for deeply cracked test-pieces is less responsive to crack length changes and hence $\eta_{pl} \sim N$.

Paris, Ernst and Turner (1980) have investigated the existence of

the plastic η factor, and have reached the conclusion that η_{pl} may be determined if the dependence of applied load upon crack length and extent of deformation are separable. Deep notch bending and tension are shown to satisfy this condition, as indeed, will be the case for a power law hardening material in many practical cases.

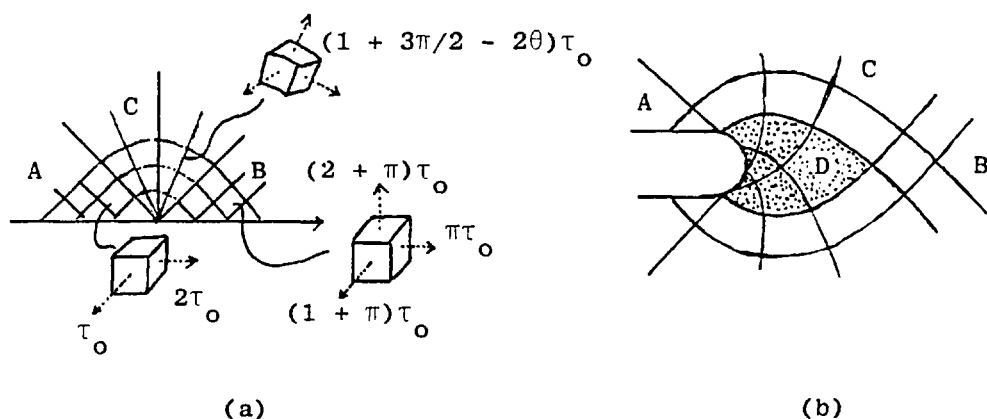
Discussion of the overall η factor, its determination for large amounts of plasticity and applications to testing and design is presented by Turner (1980) and, whilst the η factor is not essential for the determination of J , it is readily apparent that many of the diverse aspects of fracture in elastic and elastic - plastic regimes are linked by this parameter and may be understood and compared within this rationale.

J - theory has been proposed more recently for situations including crack growth (Hermann and Rice (1980), Garwood (1976)), for plane stress ductile fracture (Cotterell and Mai (1981)) and for determination of tearing instabilities (Paris et al (1979)), although doubts regarding the rigour of such applications are difficult to dispel. Several aspects of these recent works are relevant to this study and are reviewed subsequently. Of course, a full picture of deformation and fracture in elastic-plastic materials is incomplete without the treatment of large crack tip geometry changes for both elastic - plastic and fully plastic materials.

(c) Elastic - Plastic Crack Tip Blunting

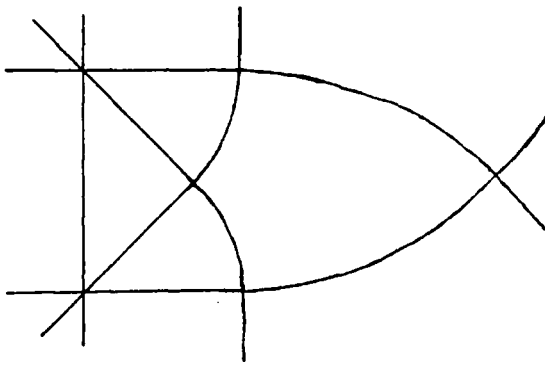
The earliest attempt to analyse the effects of large geometry changes on fracture was made by Rice and Johnson (1970). Two important differences between their results and those for small geometry changes are noted; incorporation of large geometry changes results in intense straining ahead of the tip whilst these are absent under assumption of small geometry change. The stress field changes in such a fashion that,

even in cases of strain hardening, the maximum stress ahead of the blunted tip is limited. Examination of the slip-line field near sharp and blunted cracks in perfectly plastic solids shows clearly why this is the case and these fields are illustrated below.

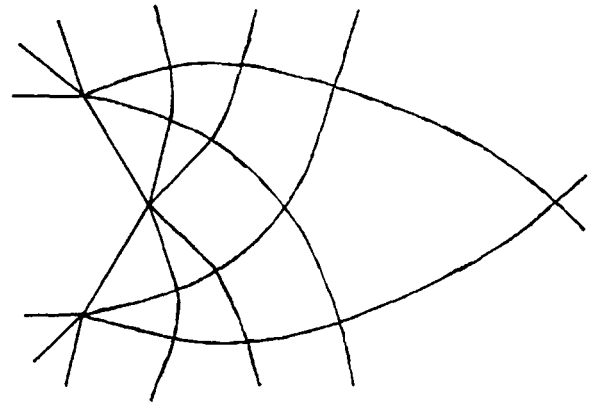


The sharp crack slip-line field (a) consists of wedge-shaped regions A and B of constant stress with a fan shaped region C where the stress has a linear dependence upon angle θ . Lack of focussing of slip lines ahead of the crack precludes the presence of intense straining in this region. The slip-line field of (b), for a semi-circular notch tip, shows how blunting of the crack tip has introduced a region of strain concentration. Whereas in (a) the fan C was centred at the crack tip, now the corresponding region has slip-lines which focus ahead of the notch in region D.

This case, (b), is for assumed smooth blunting of the crack tip. Study of the blunting of crack tips by localised shearing at corners near the tip has been performed by McClintock (1971) and slip-line fields are presented for notches with two or three corners at the notch tip as shown overleaf.

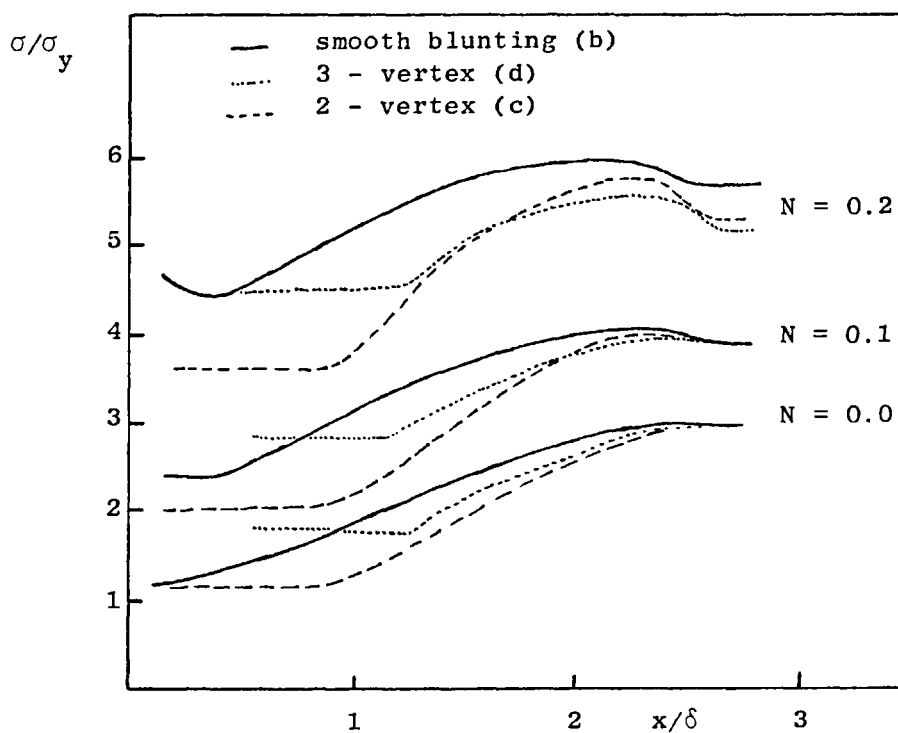


(c)



(d)

Similar slip-line fields may be developed for crack tips with multiple corners and the case of smooth blunting may be viewed as the limiting case of vertex blunting for a large number of corners. McMeeking (1977) has calculated the stress distribution ahead of such notches and the results are illustrated in the following graph of normalised tensile stress directly ahead of the tip against distance from the tip normalised with respect to crack opening displacement.



It should be noticed that, although the behaviour close to the tip varies substantially from case to case, the peak stresses and the positions at which they occur are similar for a particular yield strain and hardening exponent irrespective of the mode of blunting. Further to these results, McMeeking has applied a finite element technique which admits large geometry changes to the problem of stress and strain fields around smoothly blunting cracks in elastic plastic materials under conditions of plane strain. The tip shapes produced from such a computation are almost identical for four different materials, the basic difference being the non-parallel flanks for strain-hardening materials. Stress distributions for these cases were also calculated and the patterns of maximum stress and the distance ahead of the tip at which it occurs are similar to those produced for vertex blunting and the smoothly blunted case of Rice and Johnson.

In the author's opinion, the investigation of fully plastic and elastic - plastic blunting in plane-strain reveals that, although there may be a certain non-uniqueness in the deformation behaviour, stress ahead of the notch is dependent upon the material characteristics alone. Details of near tip stresses vary slightly but fracture, in particular failure under some critical stress criterion, should be independent of the nature of the crack tip blunting. It is clear that for moderate degrees of strain hardening the peak stresses ahead of a blunted crack tip occur at a distance largely independent of the degree of strain hardening. This suggests that in cases of contained yielding, applying a critical stress at a critical distance fracture criterion, may lead to critical distances of the same magnitude for different materials irrespective of the value of critical stress, a suggestion which is borne out by results for various polymeric materials.

(d) Slow Crack Growth and Ductile Tearing

(i) Introduction

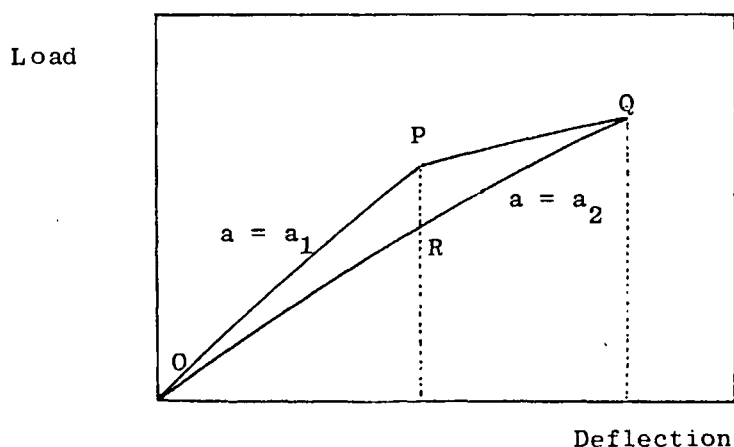
A K_{Ic} or J_{Ic} fracture criterion is specifically applicable to the initiation of failure from a highly constrained crack tip in an elastic material and, as such, has been thought to be too conservative a measure for design purposes. The need for extending the study of fracture to include moderate amounts of crack growth was of paramount importance and led to the recent work in this area. However, a further requirement was the understanding of failure in unconstrained circumstances (i.e. extensive crack tip plasticity), since catastrophic failures may also be witnessed in ductile situations. This two-fold need has been tackled in various ways, none of which are totally rigorous, but several of which give good qualitative correlation with the experimental data of the following chapter. In a few cases these models may be interrelated giving relationships between macroscopic variables (energy rates, yield stresses, etc.) and measured microscopic parameters (crack opening displacements, crack tip radii, etc.) and this provides much food for discussion. Treatment of these situations falls into four broad categories - the examination of crack tip geometry during growth, the definition of a crack resistance curve based on the J-integral, assessment of an energy absorption rate for crack growth under elastic - plastic conditions and plane stress ductile failure examined in terms of a specific work of fracture.

(ii) Crack Resistance and Crack Growth Models

The concept of crack resistance may be summarised by describing the initiation and subsequent growth of a sharp crack in a ductile material. Onset of crack extension occurs in identical circumstances, that is with a high degree of constraint at the crack tip, but the break-

down of this constraint and the development of crack tip plasticity is synonymous with the transition from flat fracture to some form of shear fracture. Breakdown of constraint and through-thickness yielding result in the loss of crack tip acuity and the need for increased energy per unit extension for continued crack propagation. Consequently, crack growth resistance has been studied extensively in plane stress situations.

Generally, the growth of resistance, R , with crack extension is a rising curve to which the curve of energy rate available must become tangential for unstable growth. For perfectly brittle materials there is no growth of resistance and the curve is flat. Several workers have attempted to derive R - curves for the slow stable tearing of ductile materials where the resistance is defined by a J - like term. Garwood et al (1975) define a parameter J_r which overcomes the most serious reservations held against the use of the J -integral in crack growth situations. A hypothetical loading curve is proposed at constant crack length as shown below



The actual loading curve is OPQ which includes crack growth from $a = a_1$ at P to $a = a_2$ at Q and ORQ is the unknown hypothetical loading curve. ORQ defines the work, U , to be used in the formula for

J (equation (4.11)). If the steps in crack growth are small and the technique is used incrementally then, as shown in Garwood et al (1975) the fictitious curve need not be known precisely for a good estimate of the modified J to be found.

Use of this parameter has been detailed by Garwood and Turner (1977) for a structural mild steel in three point bend geometry for various ligament areas and thicknesses, both with side grooves to provide flat fracture and with full thickness test pieces. Even the flat fracture results provide an increase in crack resistance of fivefold over a crack increment of 5mm, and the results for full thickness in the same test geometry exhibit a greater elevation. In order to try and clarify the behaviour the authors have used a crack infiltration technique to measure crack opening displacements both at the original tip and at the advancing crack tip. At initiation the value of c.o.d., δ_i , and the measured toughness, J_{Ic} relate via the usual expression

$$J_{Ic} = m \sigma_y \delta_i \quad (4.16)$$

with $m = 2.1$. The c.o.d. at the original tip thereafter increases linearly with crack growth, whereas at the advancing tip the c.o.d. remains constant at a value of approximately $\delta_i/4$. In terms of flank angle the behaviour is of a slight decrease in angle with increasing crack growth, although to all intents and purposes this may be regarded as constant once the growth becomes well-developed. What is particularly puzzling is that despite the c.o.d. at the advancing tip remaining constant, the toughness or crack resistance continues to increase. This can surely be explained by noting that any J - type parameter used to monitor toughness during crack growth must necessarily include energy required to deform the test-piece as the crack advances under load.

Models of crack tip deformation prior to initiation may be derived from various sources including the behaviour described in the preceding sections. Analytical models for the advancing crack tip are anything but numerous in the literature, the only relevant work at present being that due to Hermann and Rice (1980) who have analysed the quasistatic growth of a crack in an ideally plastic solid. The stress field ahead of the crack tip is only marginally different from that due to Rice and Johnson (1970) reviewed earlier and it is stated that a similar field may also apply to large scale yielding and general yielding in certain highly constrained configurations. With δ taken as the crack opening displacement between upper and lower surfaces near the tip the following relationship is true:

$$\delta = \frac{\alpha r}{\sigma_y} \frac{dJ}{da} + \frac{\beta r \sigma_y}{E} \left\{ 1 + \ln \left(\frac{R}{r} \right) \right\} \quad (4.17)$$

where r is the distance behind the crack tip, β depends upon Poisson's ratio, α depends upon the crack tip constraint and R is in some sense representative of the extent of plasticity. This may be rewritten as

$$\delta = \frac{\beta r \sigma_y}{E} \ln \left(\frac{\rho}{r} \right) \quad (4.18)$$

where ρ is given by

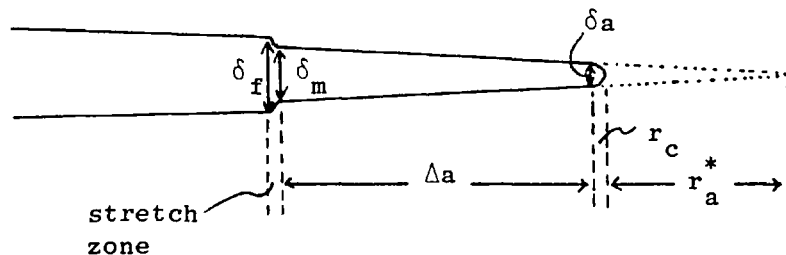
$$\rho = eR \exp \left\{ \frac{\alpha}{\beta} \frac{E}{\sigma_y^2} \frac{dJ}{da} \right\} \quad (4.19)$$

Clearly, from equation (4.18) the form of crack tip profile for small r is determined by ρ . Hence, state the authors, if a steady-state tip profile is postulated as a fracture criterion then this may be expressed as constancy of ρ . Such a statement is equivalent to:

$$\frac{dJ}{da} = \frac{\beta}{\alpha} \frac{\sigma_y^2}{E} \ln \left(\frac{\rho}{eR} \right) \quad (4.20)$$

and it is noted that if the extent of plasticity reaches a steady-state and the constraint at the crack tip is slowly varying with crack length, then J increases linearly with crack extension.

An attempt to correlate this model with measurements of both c.o.d. and J has been made by Garwood and Turner (1978). They noticed that if equation (4.17) is evaluated for r of the same order of magnitude as δ then the dominant term is that involving dJ/da . Data in the form of crack opening displacement at the original and advancing crack tips has been examined in terms of this model. A fictitious crack tip at a distance, r_a^* , beyond the point at which δ_a is measured must be introduced for the model to be sensible. Pictorially the situation is thus:



where δ_a is the c.o.d. at the advancing tip, δ_f is the c.o.d. at the original tip and $r_c \sim \sigma_y \delta_a / E$. For well established growth:

$$r_a^* = \delta_a \Delta a / (\delta_m - \delta_a) \quad (4.21)$$

$$1/\alpha = (r_a^* / \delta_a \sigma_y) \frac{dJ}{da} \quad (4.22)$$

Therefore, for constant δ_a and linear increase of δ_m with crack extension, r_a^* is roughly constant and equation (4.21) relates constraint to the gradient of the resistance curve. In Garwood and Turner (1978) the resistance curves for structural mild steel in three point bend show substantial curvature leading to variation in $1/\alpha$ from 0.16 for steady growth to 5.3 for $\Delta a = 0.2\text{mm}$ in the case of full thickness. Paris et al (1979) suggest that, schematically, the J resistance curve should consist of an initial linear portion for $J < J_{Ic}$ reflecting flow at the crack tip and creation of a stretch zone. This is followed by a region of steady tearing in which the increase in resistance is proportional to crack extension. According to equation (4.22) this would represent unvarying constraint. In Paris et al (1979) the authors define a tearing modulus, T_{mat} , based upon the constancy of dJ/da , by

$$T_{mat} = \frac{dJ}{da} \frac{E}{\sigma_y^2} \quad (4.23)$$

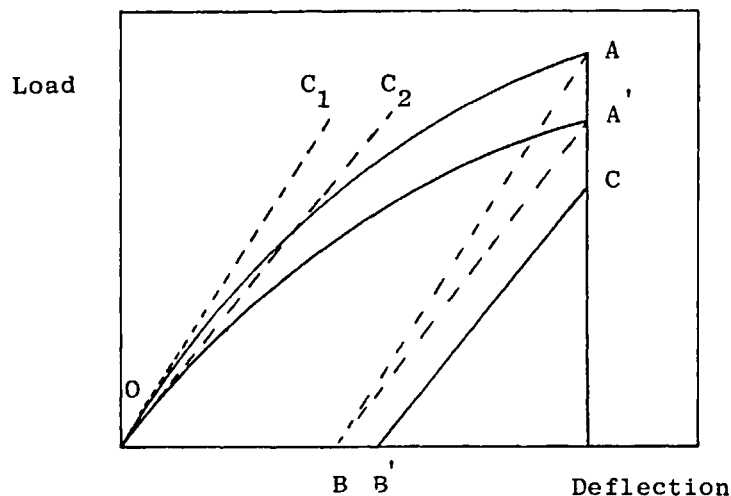
which is found to reoccur in the analysis of tearing in many geometrical configurations.

The foregoing section has shown how a J - resistance curve may be interpreted in terms of a suitable crack growth model even though the resistance curve may be for full thickness fracture with accompanying

shear lips. dJ/da is seen to be the dominant factor in crack growth assessment and, as will become apparent shortly, it also governs the material's ability to withstand unstable ductile tearing.

(iii) Energy Absorption Rate, I , during Crack Growth

Discussion of the elastic energy release rate has taken place in chapter two with extension of the concepts in terms of Turner's η factor being examined earlier in this chapter. These extensions were for non-linear elastic materials or equivalently those obeying the laws of total plasticity. True elastic - plastic materials have different loading and unloading curves, the area beneath the unloading curve being equal to the residual elastic energy in the body. Such a picture can be seen in the following figure which summarises these concepts. Suppose an elastic - plastic body of crack length a whose loading curve is OA . For ideal elastic - plastic materials, i.e. materials whose unloading curve is parallel to the initial



compliance curve (OC_1), then if no growth were to take place the unloading curve would be AB . However if the crack growth from a to $a + da$ in length, then the load will change in accordance with the

increase in net section stress and the decrease in net section area. Supposing this growth to occur at constant displacement then the reduction in load is AC and not AA'. The reasoning behind this is that A'C is the contribution due to increase in net section stress and for plasticity this is restricted to the yield stress. If the body was indeed non-linear elastic then AA' would be the decrease in load and OA' the unloading curve. In such a case the area OAA' would be JBda, where B is the thickness of the ligament. For the elastic - plastic body the curve for unloading is B'C and the area BACB' is the energy absorbed during growth denoted IBda where I is the energy absorption rate pertinent to elastic - plastic crack growth. In the totally elastic situation, the area ABA' is the elastic energy being released and is equivalent to GBda, where G has the usual meaning.

Turner (1980) has derived an expression relating I to the elastic rate, G, in terms of limit loads, elastic displacement and η factors, and in an earlier reference (Turner (1979)) notes the inequality $G < I < J$. The concept has been applied in the fracture of low density polyethylene by Hodgkinson and Williams (1981). The authors write G instead of I but the derivation is equivalent i.e.

$$I = - \frac{1}{B} \left. \frac{dU_{el}}{da} \right]_{\Delta} \quad (4.24)$$

The elastic energy, U_{el} , is equal to $P^2 C/2$ where C is the crack length dependent specimen compliance and thus I comprises two terms according to:

$$I = - \frac{P^2}{2B} \frac{dC}{da} - \frac{PC}{B} \frac{dP}{da} \left]_{\Delta} \quad (4.25)$$

Using the expression for limit loads (4.14) the second term may be simplified, provided of course that the ligament of the specimen has yielded (which was the case). An analysis was performed in Hodgkinson and Williams (1981) which suggested that I might be reasonably constant during the propagation of ductile tearing. Values of J during propagation and values of both J and G at initiation were also quoted. The conclusion was that J is a measure of total energy to propagate the crack, whereas I is the local energy dissipation to finally effect fracture. Results showed that whereas J increased monotonically with growth in typical R - curve fashion, the behaviour of I with crack extension was approximately constant. Whether this defines a useful fracture parameter is debatable.

The parameter, I , has been used by Turner (1981) to describe the point of instability in crack growth. The criterion used is for the local energy dissipation rate to exceed the total work dissipation rate or:

$$I > dw / B da \quad (4.26)$$

Expressing the total work rate in terms of the η factor and dJ/da , Turner obtains an expression involving the tearing modulus, T_{mat} , of Paris given in equation (4.23). The relation is:

$$\frac{\eta}{b} \frac{IE}{\sigma_y^2} > T_{mat} \quad (4.27)$$

Paris et al (1979) analyse various geometrical configurations in order to determine whether unstable tearing will occur. The criteria calculated for each case involve a geometrical parameter exceeding T_{mat}

for instability. Equation (4.27) is in essence the same statement but may differ slightly according to the values of η derived by estimation. By implication I is a geometry dependent parameter and depends, at least, upon the constraint reflected by the η factor.

(iv) Plane Stress Ductile Fracture

Several workers have made recent attempts to examine crack growth in ductile materials for plane stress situations. In such cases, it has been customary to relate the total work of fracture to the thickness and ligament length in the specimen. As the ligament becomes longer in such tests the shear lip effect is more pronounced and typical data follow a linear plot of specific fracture energy against ligament.

Judy and Goode (1973) have shown this relation for a range of steels. Both thickness and ligament were varied and the results compared with the relationship

$$U = R_p (W - a)^x B^y \quad (4.28)$$

where x and y were chosen for a best fit to the data. In several studies it was found that x was roughly equal to 2 whereas y varied between 0.5 and 1.0. Although a strict one to one relationship, i.e. no energy intercept, is quoted, Turner (1979) has plotted the results for one of the steels examined in Judy and Goode (1973) as $U/B(W - a)$ against $W - a$ revealing linear dependence but with a positive intercept on the $U/B(W - a)$ axis.

Recent work by Cotterell and Reddel (1977) postulates the near crack tip region to be composed of an end region very close to the tip contained within an outer plastic region which screens plastic deformation

hence facilitating large strains in the end region. In this inner region the work performed, called essential work, is regarded as being particular to the material and considered a prospective material constant. Energy expended in the outer region is termed associated plastic work and is geometry dependent. For situations of plane stress the essential work is thought to be dependent on specimen thickness.

Concentration was focussed on specimens whose ligaments yielded completely prior to failure and where, although gross yielding was observed the plastic region was contained within the specimen (cf. scheme (c), page 24 as identified in Turner (1979)). In such a test the work of fracture can be separated into two components (i) non-essential work performed within the plastic zone and (ii) essential work at the crack tip. Assuming the specific essential work remains constant, contribution from (ii) will be proportional to ligament length and contribution from (i) will be proportional to the square of the ligament length. Their model proposes:

$$U = B (W - a) w_e + B (W - a)^2 w_p \quad (4.29)$$

where w_e is the specific essential work ('specific' pertaining to unit area) and w_p is the specific non-essential work ('specific' pertaining to volume). Thus if the specific work of fracture $U/B (W - a)$ is plotted against $(W - a)$ the result should be a straight line of gradient w_p and intercept w_e . Such a model was vindicated in Cotterell and Reddel (1977) for double edge notched sheets of a low alloy steel and again by Cotterell and Mai (1981) for a strain hardening and a non-strain hardening metal.

(e) Generalised Fracture Mechanics

In spite of continued attempts to rationalise the effects of ductile fracture within the framework of existing and well established treatments, Andrews considers the present techniques for analysing ductile failures to be unsatisfactory, and has encouraged a fresh look at the subject with the proposals put forward in his theory of generalised fracture mechanics (Andrews (1974)).

From purely dimensional considerations, the stress field and local energy density at a point in a infinite cracked lamina are of the form

$$\sigma_{ij} = \sigma_o f_{ij}(x, y, \epsilon_o) \quad (4.30)$$

$$W = W_o f(x, y, \epsilon_o)$$

where (x, y) are co-ordinates normalised with respect to crack length, ϵ_o is the strain remote from the crack and σ_o, W_o are the remote stress and energy density respectively. The inclusion of ϵ_o is to enable characterisation of the overall level of constraint in the system and for elastic materials would therefore be redundant. The change in the total energy of the system, $\mathcal{E}$, with increase in crack length is then evaluated by summing the various contributions from all the volume elements of the body. For a perfectly elastic solid the expression derived is:

$$- \frac{d\mathcal{E}}{dA} = k_1(\epsilon_o) c W_o \quad (4.31)$$

where A is the interfacial crack area, c the crack length, and k a dimensionless function of strain, which is also geometry dependent for bodies of finite dimension.

When considering an elastic - plastic solid, the energy dissipated in unloading elements of the body must be taken into account. A calculation is described which expresses these losses in terms of a function β , the hysteresis ratio of the material, and the modified energy release rate is then given as:

$$- \frac{d\mathcal{E}}{dA} = k_2 (\sigma_o, T, R, \epsilon_o) c W_o \quad (4.32)$$

It is claimed that for crack propagation, the energy release rate as defined by (4.31) represents the total energy expended by the system in producing unit crack growth and thus, for an elastic system represents the true surface energy alone (i.e. that energy required to rupture the bonds across unit area of fracture plane) whilst for a ductile system it includes energy dissipated in inelastic deformation. The symbol denoting this release rate is $\mathcal{J}$. Equation (4.32), on the other hand, represents solely bond rupture energy, and the corresponding release rate $\mathcal{J}_o$ is the true surface energy regardless of the type of material. It follows that:

$$\mathcal{J} = \frac{k_1 (\epsilon_o) \mathcal{J}_o}{k_2 (\sigma_o, T, R, \epsilon_o)} = \mathcal{J}_o \phi (\sigma_o, T, R) \quad (4.33)$$

and $\phi (\sigma_o, T, R)$ is now a loss function which varies with external constraint σ_o , temperature and the rate of strain.

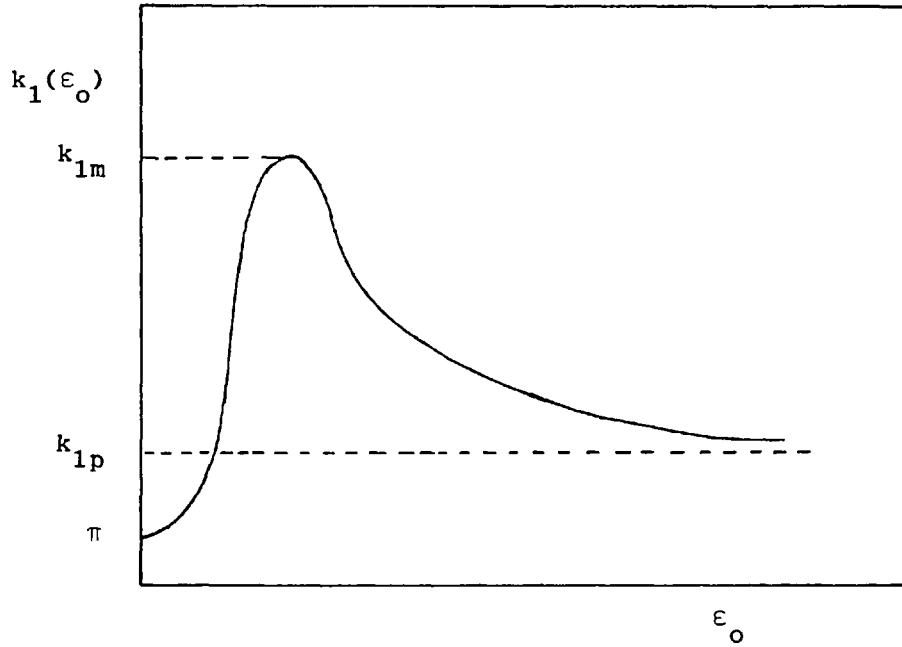
The relationship of (4.31) to established results in fracture mechanics is quite clear. For a linear material the input energy density is $\sigma_o^2 / 2E$, where E is the material modulus, and the critical form of equation (4.31) reduces immediately to Griffith's result with

$k_1 = \pi$. In the case of large strain rubber elasticity the critical form is just that quoted in the previous chapter as equation (3.31) where $k_1 (\epsilon_0)$ is to be identified as $K (\lambda)$. Compatibility with J integral theory is clear since for non-linear elastic materials $J = -2d\mathcal{E} / dA$ and hence for fracture initiation, at least, $J_{Ic} \equiv 2\mathcal{J}$. While the generalised theory reassuringly reproduces results from non-linear fracture mechanics it is suggested that the most important new features are

- (i) the explicit consideration of energy losses due to unloading
- (ii) the provision of a theoretical expression for fracture toughness in terms of 'true' surface work, and a loss function which may be derived from the material hysteresis and the crack tip strain field
- (iii) in principle, the possibility of determining the function $k_1 (\epsilon_0)$ other than by the compliance method, which would lead to simplification of the determination of J_{Ic} or $\mathcal{J}$. Andrews and Billington (1976) have shown the truth of equation (4.31) for a Copper-Beryllium alloy which shows marked elastic - plastic behaviour. Unfortunately the calculation of $k_1 (\epsilon_0)$ illuminates a high degree of geometry dependence in this function which, together with k_1 estimates for other materials, casts aspersions on the usefulness of such an approach particularly with regard to design applications.

Andrews and Fukahori (1977), in an attempt to vindicate equation (4.33), examine four materials with very individual stress-strain behaviour. Their data for k_1 follows a typical pattern, further confirmed by data for steels (Andrews and Bhatti (1980)), which may be explained in similar terms to the interpretation of $K (\lambda)$ offered for rubbers in chapter three. As shown schematically overleaf, the function $k_1 (\epsilon_0)$ for elastic - plastic materials rises rapidly from a value near π

to a peak which corresponds to the onset of gross yielding and is hence situated close to the yield strain.



The function $K(\lambda)$ examined in the previous chapter is equivalently $k_1(\epsilon_o)$ for a totally elastic material and, as was explained, drops from π to a value of about 1 due to the effective decrease in crack length with increased strain. When a plastic zone forms at the base of a crack in an elastic - plastic material, the effective length of the crack is increased just as forms the basis of the Irwin correction for plasticity. Rewriting the critical form of (4.31):

$$\mathcal{J} = k_1(\epsilon_o) c W_{oc} \quad (4.34)$$

in these terms and approximating W_{oc} by $\sigma_o^2 / 2E$ then we have

$$\frac{\sigma_o^2 \pi (c + r_p)}{2E} = k_1 (\epsilon_o) c \frac{\sigma_o^2}{2E} \quad (4.35)$$

$$\text{i.e. } 1 + \frac{r_p}{c} = \frac{k_1 (\epsilon_o)}{\pi} \quad (4.36)$$

and thus the departure from the linear elastic small strain value of π may be interpreted as a change in effective crack length. The peak value, k_{1m} , and the plateau value, k_{1p} , are in some way related to the steady state plastic zone size and the specimen geometry, but this is too complicated to assess.

The crux of the theory is the calculation of $k_1 (\epsilon_o)$ and $k_2 (\sigma_o, T, R, \epsilon_o)$ in order to determine γ_o . It is clear that $k_1 (\epsilon_o)$ may not be derived simply; in fact Andrews in Andrews and Fukahori (1977) has attempted to confirm the validity of equation (4.33) by expressing k_2 in terms of k_1 and a term representing elements of material which experience unloading. This latter term is subsequently calculated by examining specimens which have orthogonal grids drawn on them, the strain pattern giving an indication of the input local energy density. It is quite clear that such a mechanical procedure is unlikely to subtract all of the energy dissipated in plastic deformation and the derived parameter, γ_o , is therefore bound to be higher than any estimate of true surface energy. The other factor which precludes the extraction of γ_o is that energy balance for crack propagation in ductile circumstances is adequately catered for in continuum mechanics terms without the apparent need for a miniscule surface energy term. Such a term will therefore not be derivable from a macroscopic continuum approach. As illustrated in Andrews and Fukahori (1977) the values of γ_o are

generally an order of magnitude too high and the justification of such a tedious approach is not apparent.

While the theory is an attempt to generalise fracture mechanics, which is a highly commendable aim, it fails for the same reason. The introduction of an empirical function $k_1(\epsilon_0)$ which varies with geometry, material properties, crack length, etc., the parameter γ which may increase or decrease with crack growth according to local plasticity and the general complexity of deriving results all must be justified by the outcome. They are not.

CHAPTER FIVE

THE DUCTILE FRACTURE OF POLYMERS

(a) Introduction

So wide-ranging are the uses of plastics in mechanical applications, both in large structures and in small, highly stressed components, that it has become necessary to examine material behaviour under many types of service conditions. Generally speaking, these studies have involved such an environment as to provoke brittle failure e.g. low temperatures, high rates and corrosive environments.

A wide range of polymers have been tested and it is now well established that many of these materials exhibit a regime in which LEFM may be used to predict the failure of components with great accuracy; this in spite of the inherent time dependent characteristics which they possess. This regime for such materials as polypropylene, polyethylene and nylon is found at very high testing rates (impact behaviour), and at temperatures well below ambient (-100°C , say). As a consequence LEFM is more pertinent to the derivation of a material toughness for the most severe of situations; its applicability as a conservative lower bound in design procedures is clear. In circumstances allowing appreciable amounts of plastic deformation, LEFM is only valid when the plasticity is contained by high constraint at the crack tip. Polymers above their glass transition temperatures have low yield stresses and high toughnesses which would necessitate very large test specimens in order to contain the yielding. Not only that, but such a situation, although enabling the material toughness to be characterised, would not give any insight into the ductile failure mechanisms which the material would be likely to experience in service.

Very little, to date, has been written about the ductile fracture behaviour of polymers. Some extensions to LEFM have been proposed to describe behaviour in the ductile-brittle transition region notably by Parvin and Williams (1975) for polycarbonate, further substantiated by Fraser and Ward (1977), for polypropylene by Fernando and Williams (1980) and by Ferguson, Marshall and Williams (1973) for several semi-ductile polymers but with emphasis on rubber modified polystyrene. In this last work, a K_c fracture criterion is found only to be valid to the point of crack initiation and thereafter they employ a critical crack opening displacement argument, the value for this critical COD then being converted to a J-integral value for comparison between materials. Some of the ideas presented bear relation to this study and will be commented upon in a later section.

Newmann (1979) has considered the fracture of acrylonitrile butadiene styrene, a rubber modified glassy polymer in terms of linear elastic (K) and non-linear elastic (J) type models. The conclusion reached was that, in view of the lengthy calculations involved in J techniques and the higher degree of scatter compared with the K measurements, no advantage is gained over a linear elastic treatment. Not surprisingly, a fracture toughness based upon maximum load is very sensitive to specimen geometry; the higher constraint at the crack tip in bending inevitably leads to lower toughness values when growth characteristics are included. Although the plastic zone size is quoted as 1.6 mm using a Dugdale estimate, the actual size is nearer 6mm as indicated in an earlier photograph. Therefore, for this complex unconstrained system it seems entirely plausible that the initiation behaviour alone is consistent and may be modelled successfully using a Dugdale line zone and critical COD criterion. More extensive J-integral measurements

were performed on the same material by Broutman and Sridharan (1978) and their work has been reviewed in Newmann (1979).

In all of these studies, the attempt has been to correlate fracture behaviour with accepted material parameters or extensions thereof. It would seem more appropriate to examine in detail the crack tip deformation taking place before trying to implement some fracture criterion. To the author's best knowledge, no examination of the near tip deformations at cracks in polymers has been undertaken. Of course, consideration of the plastic zone and inference of properties from phenomena on the surface of fracture abounds the literature. Here, it is proposed to investigate the nature of deformations near the tips of notches in several different polymeric materials. Such work leads to a greater understanding of the physics of fracture processes. Having drawn some general conclusions concerning material behaviour at the crack tip, specific fracture criteria are discussed and the chapter is concluded with a report on the ductile tearing of polycarbonate, one of the materials whose deformation characteristics were examined.

(b) Crack Tip Deformation of Ductile Polymers

As has been shown in chapter three, a great deal of success may be obtained in describing the deformation and fracture behaviour of materials which have no well-defined yield point, but display extensive elasticity. Five more polymers are investigated in this chapter, all of which show a large amount of ductility at ambient temperature. For each material, the stress-strain curve was obtained in tension and compression and the high magnification photographic technique, as outlined in chapter three (d) (i), was employed to monitor the characteristics of crack tip deformation. The materials are dealt

with separately in the following subsections and any relevant analysis is collated in section (c).

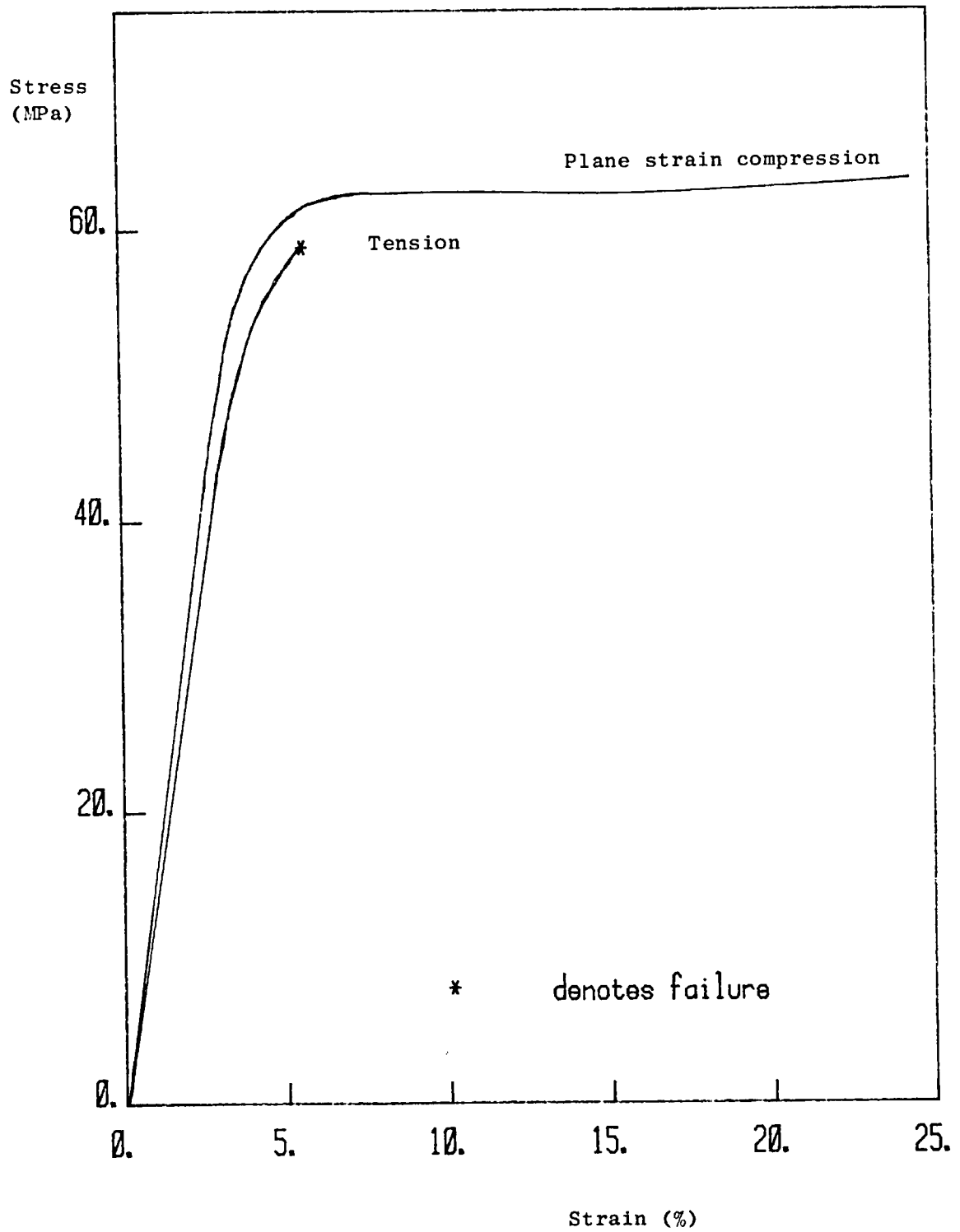
(i) Polycarbonate

Extruded sheet of 'Rhiamer' polycarbonate marketed under the trade name 'Makrolon' by Farbenfabriken Bayer AG was used in a variety of thicknesses (1mm, 3mm, 5mm). This material has good stability, a high melting point, a large modulus and toughness, and is transparent.

In tension, at a cross-head rate of 2mm min^{-1} , polycarbonate has a modulus of elasticity of 2.0 GNm^{-2} . At stresses approaching 60 MNm^{-2} the specimen necks down, usually outside the extent of the gauge length, and when the specimen fails the true stress is incalculable. For a similar strain rate in compression, the stress-strain characteristics are assessed by a technique employing the plane strain compression of 50 mm wide sheet as detailed by Williams (1967). Figure 19 shows both compressive and tensile data, the former to a strain of 25%, the latter to failure and illustrates the enhanced yield stress appropriate to necking of the tensile specimen. Post yield strain softening is followed by negligible strain hardening up to the maximum strain. The modulus recorded in both modes agrees with technical data issued for the material.

Single edge notch specimens of polycarbonate, of gauge length 100 mm, width 50 mm and various thicknesses were prepared. The notches introduced into the specimens were of three types: razor notches, impact razor notches and notches machined with a sharp single point fly cutter. A more consistent type of notch was produced by one of the razor notching techniques, although this only became apparent later (see section documenting the work on ABS), and as a result the razor notching

FIGURE 19: STRESS-STRAIN CURVES FOR POLYCARBONATE



method was predominant. Specimens with initial crack lengths ranging from 5 mm to 20 mm were examined by the photographic arrangement employing both camera bellows and extension bellows. Lighting problems were minimal because of the transparent nature of the material, but, as in all cases involving high magnification, the restricted depth of field limited the quality of the photographs. Transmitted light which was reflected internally in the specimen resulted in dark areas appearing coincident with highly strained regions. Despite these problems, crack tip deformation behaviour preceding and subsequent to initiation of crack growth was well documented.

For all thicknesses the trends were similar, and data for the intermediate thickness of 3 mm is given here. Tests were performed at an Instron cross-head rate of 0.5 mm min^{-1} and figure 20 shows gross section stress plotted against gross deflection for four crack lengths. In each case, the pattern was for a slightly non-linear loading curve until a small amount of crack growth had taken place (approximately 4 mm in most cases), whereupon the tearing reached a steady speed and progressed through the specimen at decreasing load. More detail of the mechanics of this process are presented in section (d). Photographs were taken prior to the initiation of tearing and at various stages during propagation. A full scheme of events is presented in figures 21 to 23 for an initial crack length of 10 mm and includes a loading curve marked in the appropriate positions with the relevant prints. It should be noted that the grid in close proximity to the crack is of 1 mm squares and was used to calculate the approximate magnification factors corresponding to projections of the slides.

Initial deformation as shown by photographs P2 and P3 involves separation of the crack flanks and blunting of the tip accompanied by

FIGURE 20: STRESS-DEFLECTION CURVES FOR SEN TENSION
TEST-PIECES OF POLYCARBONATE

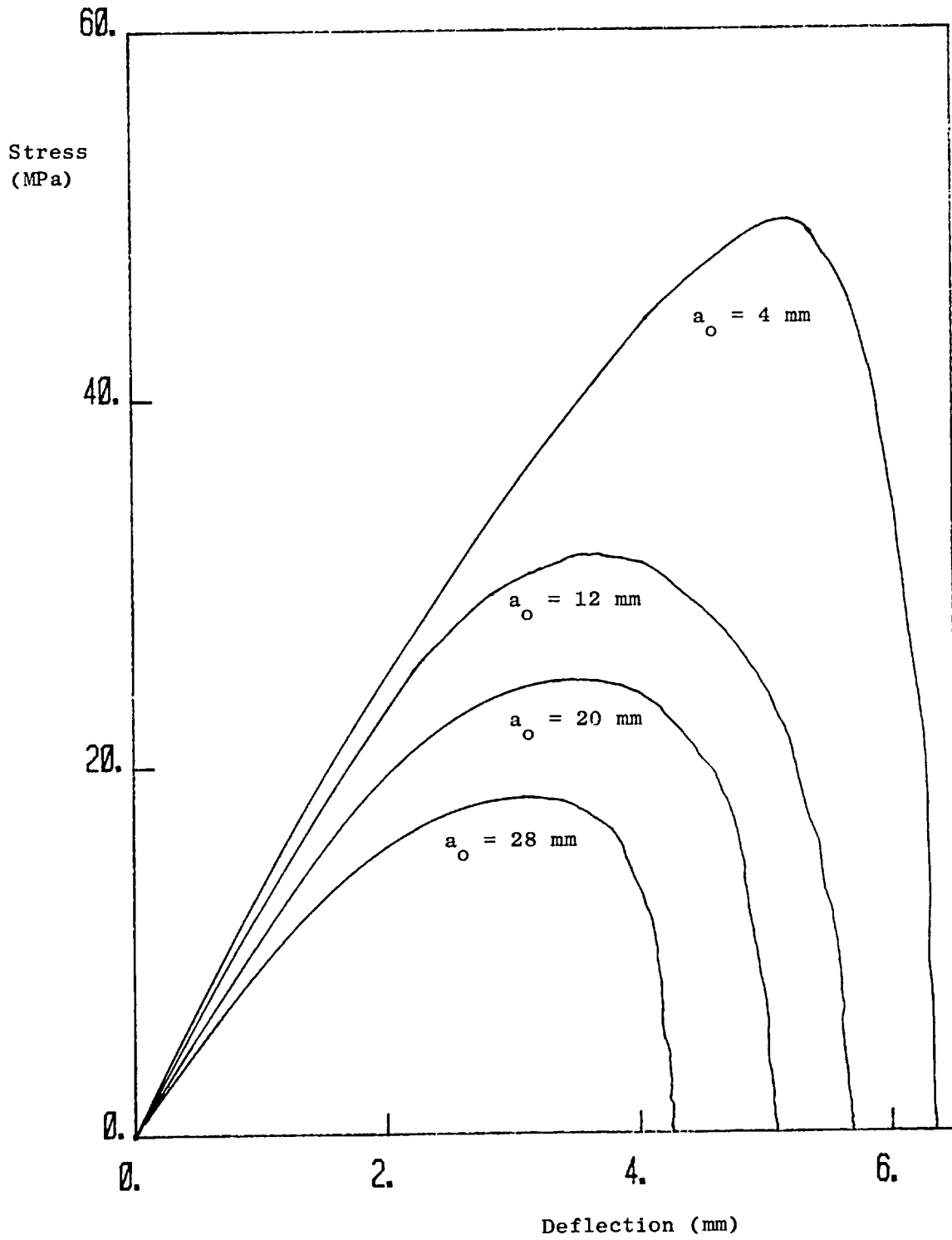


FIGURE 21: STRESS-DEFLECTION CURVE INDICATING
WHERE PHOTOGRAPHS WERE TAKEN

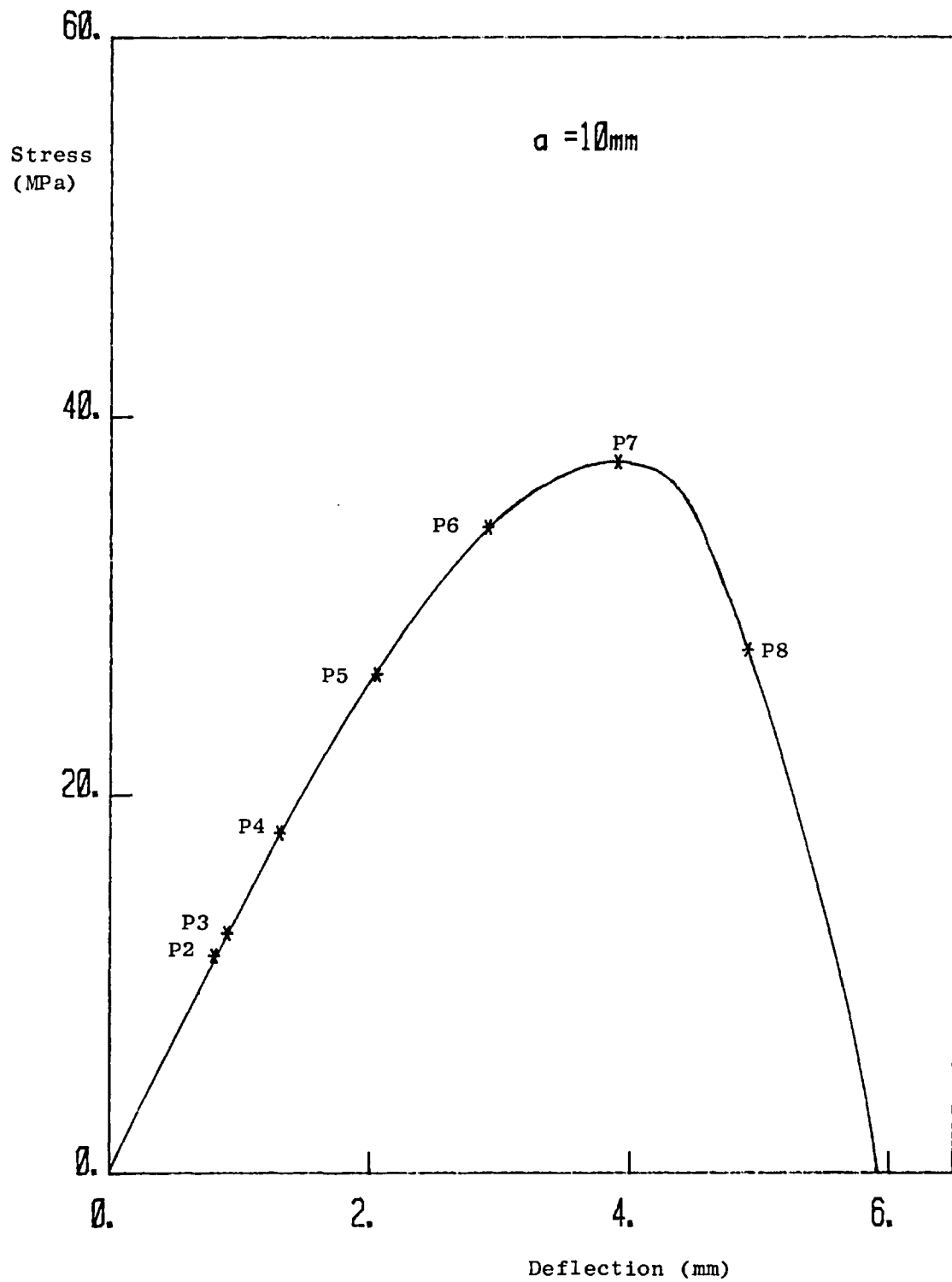
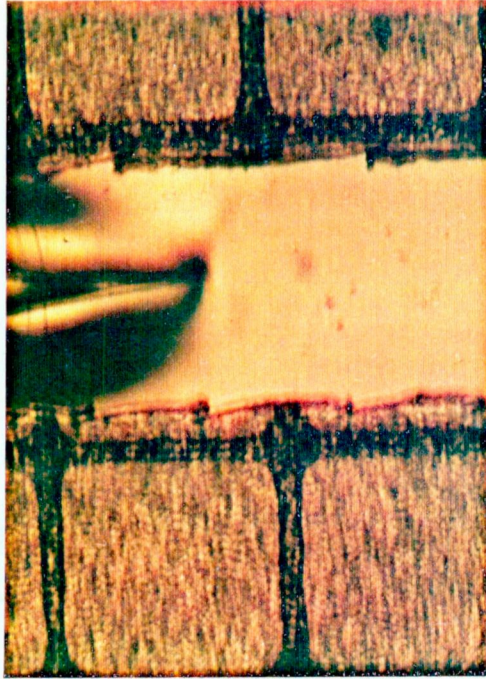
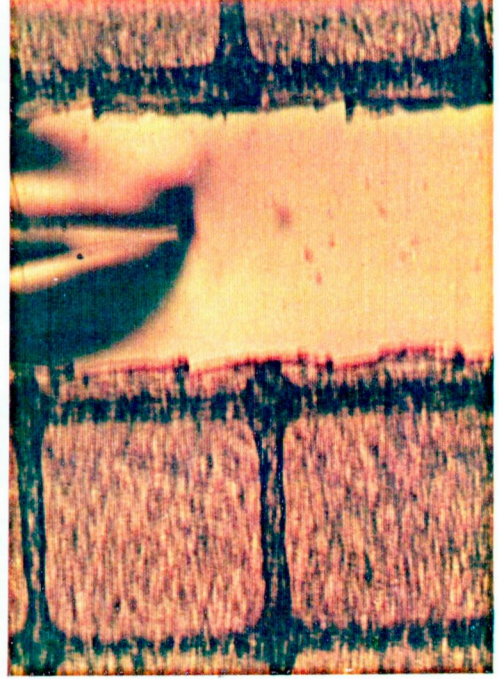


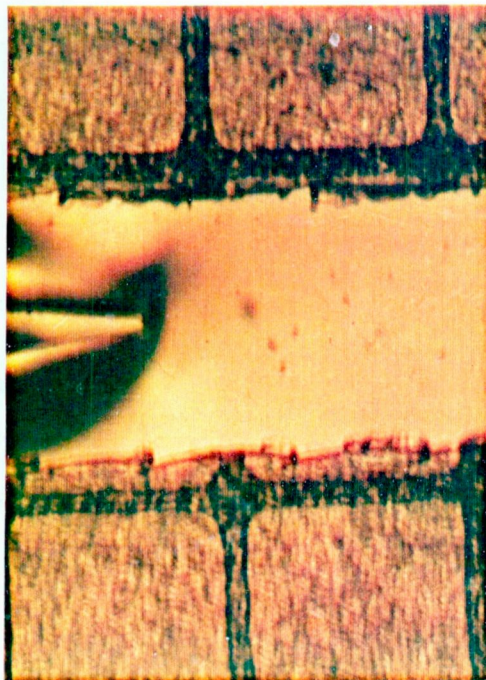
FIGURE 22: PHOTOGRAPHIC RECORD OF BLUNTING
AND CRACK GROWTH IN POLYCARBONATE



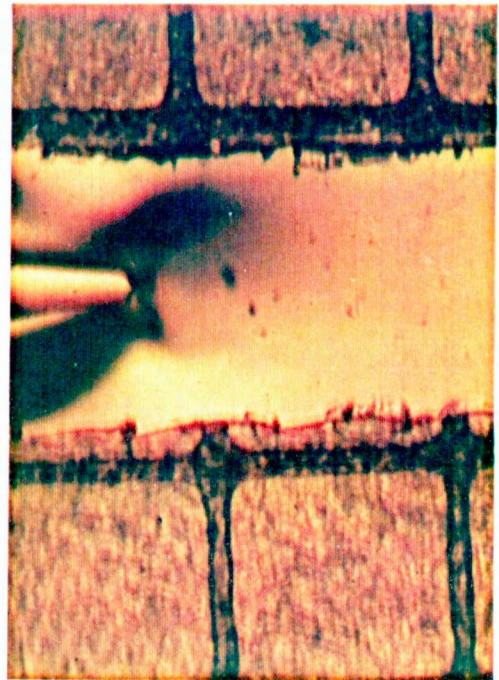
P1



P2

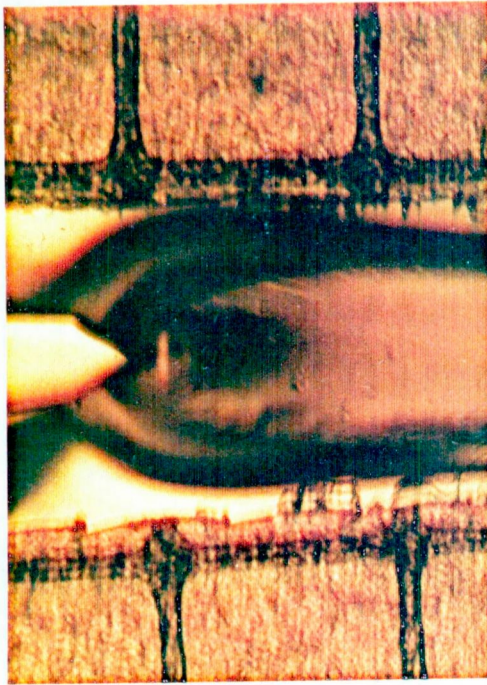


P3

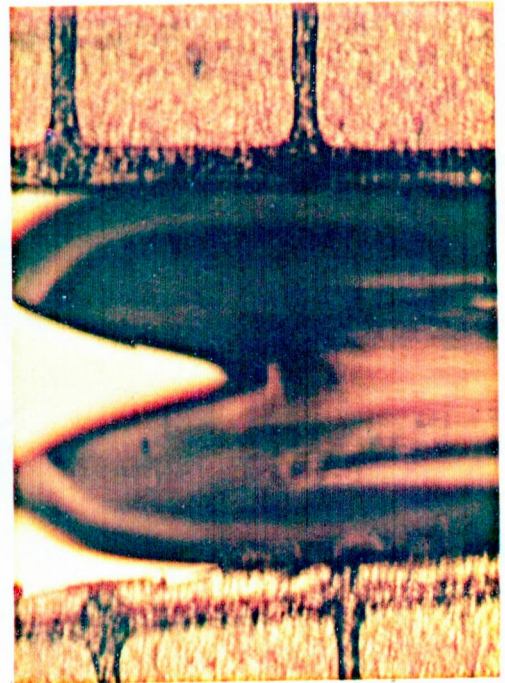


P4

FIGURE 23: PHOTOGRAPHIC RECORD OF BLUNTING
AND CRACK GROWTH IN POLYCARBONATE (cont)



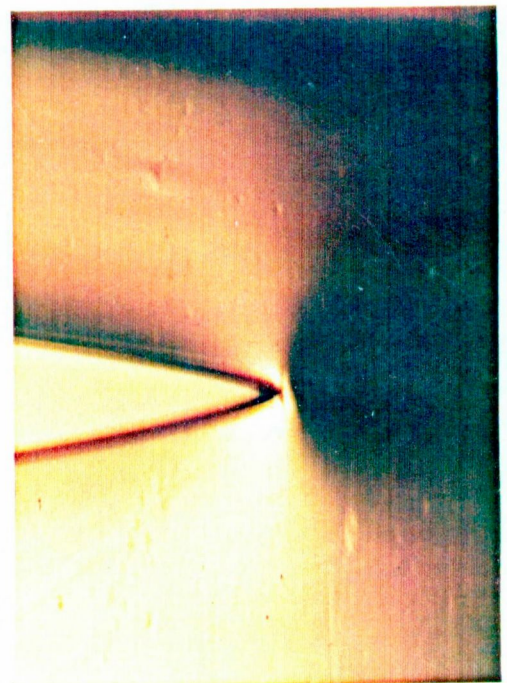
P5



P6



P7



P8

any shear deformation. It should be emphasised at this stage that crack initiation, as depicted by crack front bowing according to Parvin and Williams (1975), has presumably taken place between photographs P1 and P2 (presumably, in the sense that attention was not paid to this event). The occurrence of 'pop-in' as described in Parvin and Williams (1975) was occasionally observed, more so in thicker specimens, and such a condition would occur between photographs P2 and P3. The initiation of growth detected in this study is the onset of ductile tearing in which there is a transition from smooth tip blunting to shear band formation and crack propagation. Photographs P3 and P4 capture this transition and the initial stages of tearing are detailed in photographs P5 - P7. P8 shows the crack tip profile in the steady state situation when the plastic zone in the specimen is fully developed.

The slides, from which the prints are taken, were projected onto a screen and accurate reproductions of the crack tip profiles were drawn. For the scheme illustrated, such a technique gave the results of figure 24. From this and similar figures measurements of crack opening displacements, crack tip radii, crack flank angles and shear band angles were taken for use in conjunction with the analysis of section (c), variations with crack length being noted in each case. Similarities between all the materials tested warranted simultaneous discussion and this is combined with the analytical section.

(ii) Acrylonitrile Butadiene Styrene (ABS)

The ABS examined was a transparent grade of Cycolac produced by Borg-Warner Chemicals Ltd. with stress-strain curves in tension and compression as shown in figure 25. Failure of this material in tension is as a result of craze growth in the glassy matrix and the

FIGURE 24: TYPICAL TIP PROFILES IN POLYCARBONATE
BEFORE AND AFTER INITIATION

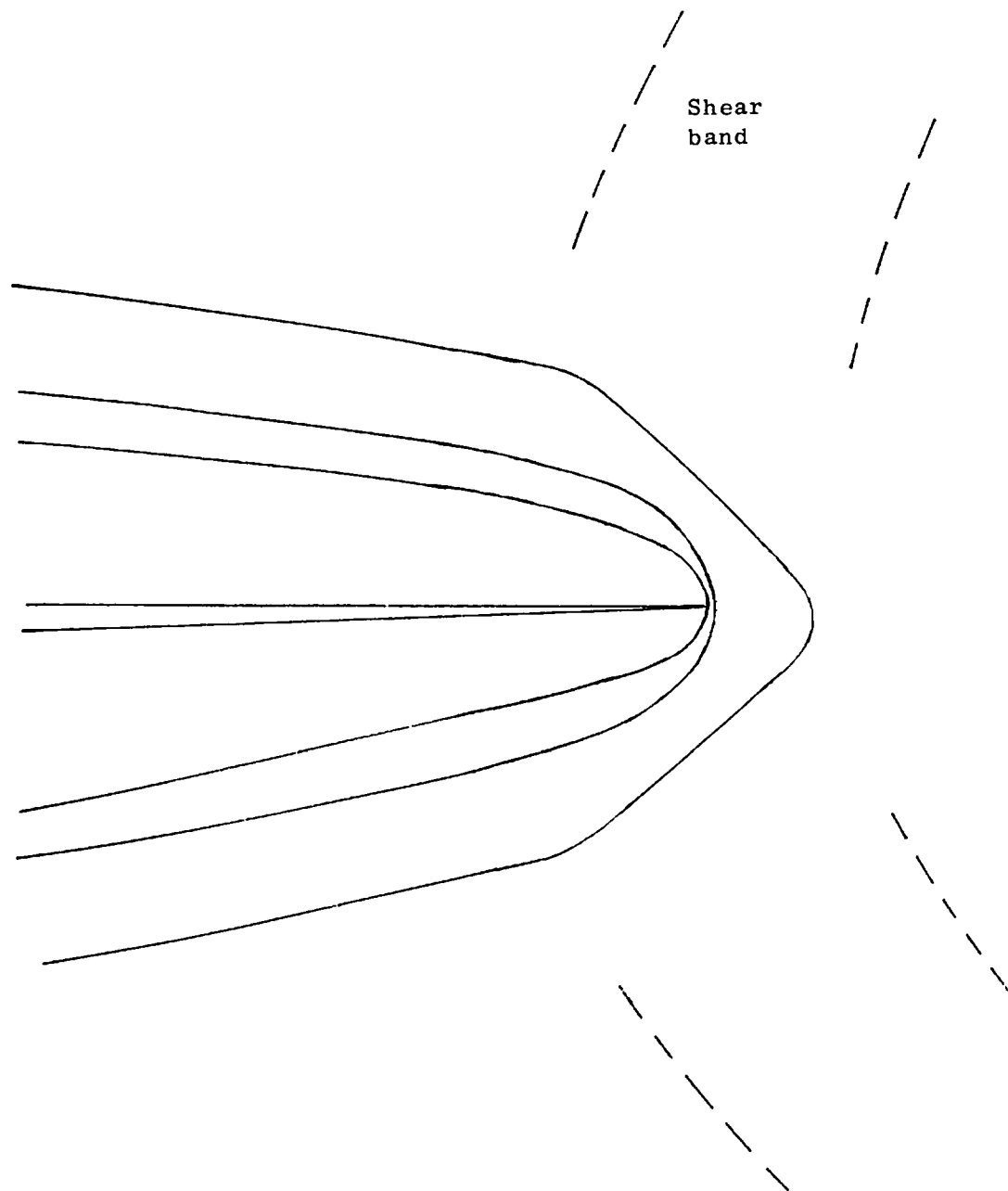
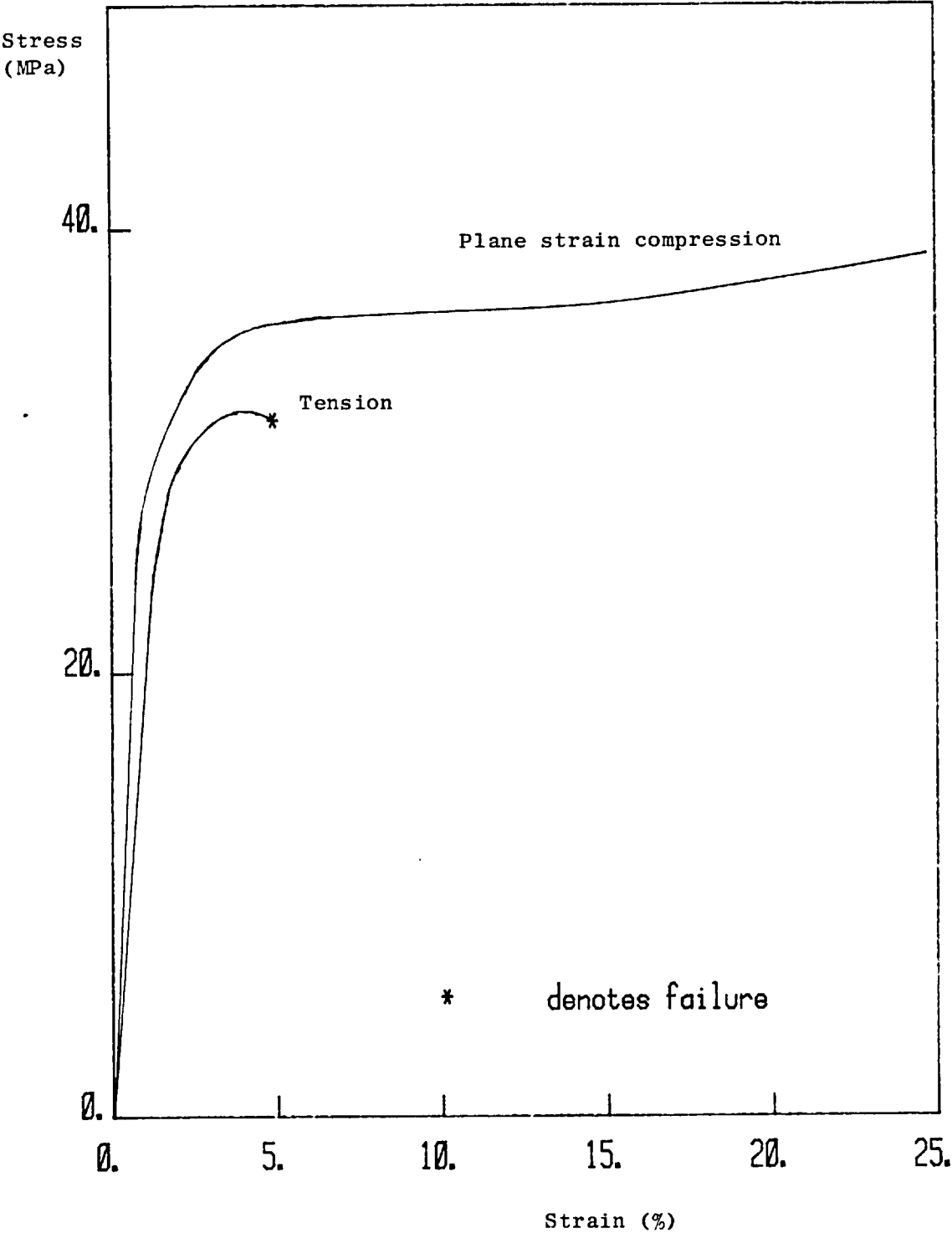


FIGURE 25: STRESS-STRAIN CURVES FOR ABS



compressive data, as was the case for polycarbonate, was used to characterise the material. A yield stress of 38.5 MNm^{-2} is quoted for this grade in Newmann (1979) and the discrepancy between this figure and that given by the tensile data shown here is most likely to be a rate effect. Compressive data shows an estimate of the yield stress if crazing were to be suppressed. Values for the modulus agree well with Newmann (1979) being in the range of 1.9 GNm^{-2} to 2.3 GNm^{-2} .

Unlike polycarbonate, ABS when razor notched produces whitening and associated irregular craze bunching at the notch tip. It was thought at this stage that notches machined with a single point cutter would have greater consistency. One thickness, 3 mm, and several crack lengths were tested in the same SEN geometry at the same rate. Again, the entire magnification apparatus was necessary and light was transmitted from behind the specimen .

Examination of the photographs of the initial notches gave much cause for concern. All the specimens had been clamped firmly in a vice on a milling bench and notched with a single point fly cutter turning at such a speed as to produce little vibration while removing material from the specimens - the standard technique for machining notches. Figures 27 and 28 show photographs for two specimens with nominally the same crack length. Variable amounts of crazing are apparent at the crack tips as indicated by darkened regions. This highlights the possible extremes of crack tip quality in machined notches in other materials and is likely to be the prime reason for experimental scatter in fracture data. It was viewed that a truer picture of crack tip deformation behaviour would be obtained from those specimens with little or no crazing near the initial tip. It is clear from comparison of figures 27 and 28 that the initial crazing leads to sharper flanks angles during

growth. Figure 26 shows gross section stress versus deflection curves for initial cracks of length 4, 6, 8 and 10 mm. As before, the grid on the specimen is of 1 mm squares enabling the slides to be projected and the measurements of deformation near the crack tip to be made.

Qualitatively the observations are similar to those for polycarbonate; the crack blunts under increasing load with, in this case, heavy crazing ahead of the tip; the initiation of tearing is synonymous with the development of shear band regions. Necking within the extent defined by these shear bands is evident in both materials, but in the case of ABS a large amount of crazing takes place in the bands as indicated by both photographs P3 and P6.

Study of the fracture of notched tensile specimens in ABS has been carried out by Truss and Chadwick (1977). They recorded crack growth through a single edge notch specimen from an initial crack length of about 0.5 mm with a movie camera. Razor notches and the accompanying craze bunches were used as starter cracks, and the Instron testing rate was 0.5 mm min^{-1} . Several features of the fracture appear to be different from those presented here, and are entirely due to crack growth through the craze bunch. Shear bands are still generated at roughly 45° to the crack direction at the initial crack tip but the crack opening displacements required to propagate the crack are much less. It is to be expected that crack growth through a crazed region which is also growing ahead of the crack will show no geometric discontinuities at the original tip. The unstable nature of propagation in such a ductile material has been observed in polycarbonate but no inconsistency is found with the behaviour during stable growth - this instability is explicable in terms of energy balance (see section (d)). Conclusions to be drawn here are that near tip conditions are crucial to the

FIGURE 26: STRESS DEFLECTION CURVES FOR SEN
TENSION TEST-PIECES OF ABS

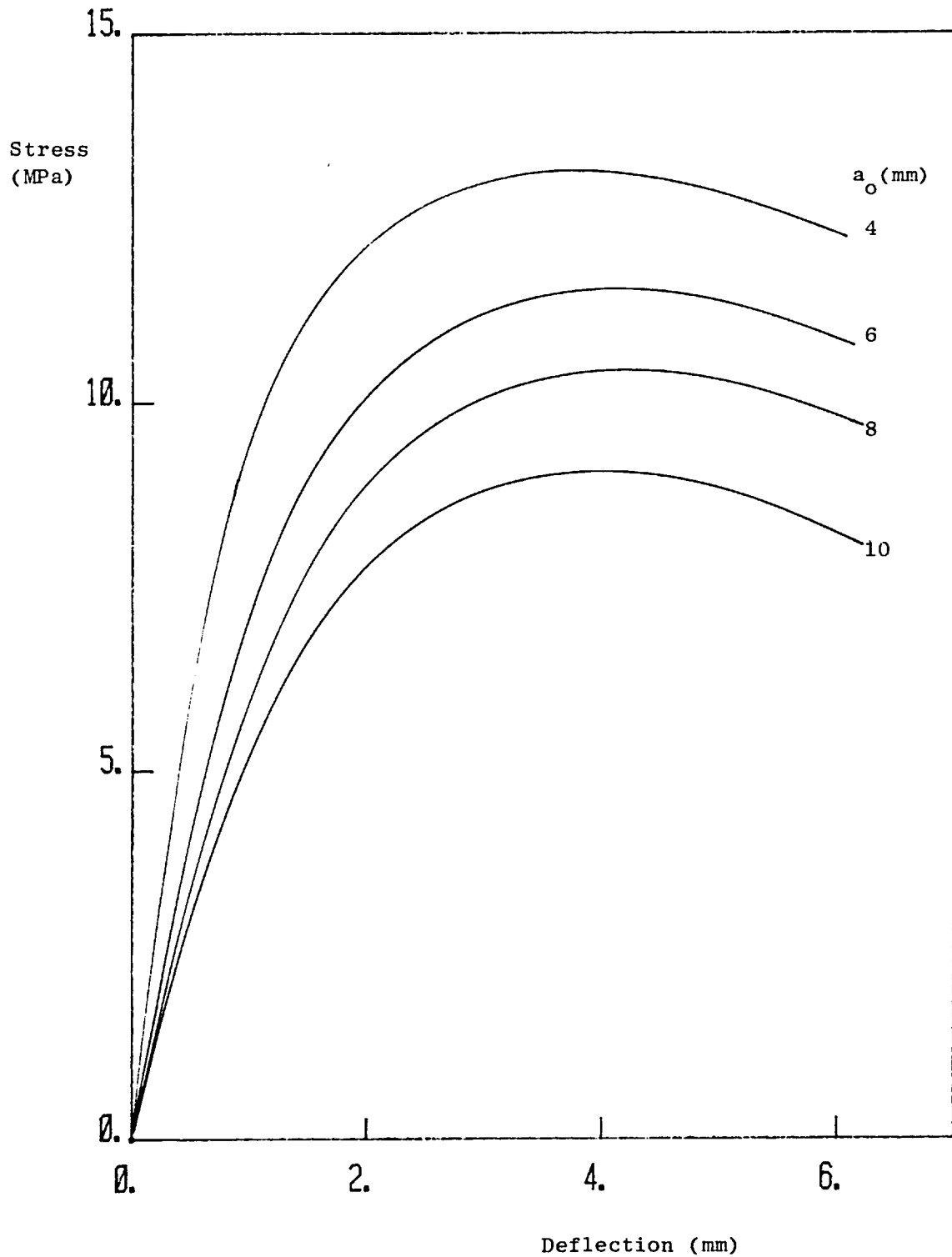


FIGURE 27: PHOTOGRAPHIC RECORD OF
CRACK GROWTH IN ABS



P1



P2



P3



P4

FIGURE 28: PHOTOGRAPHIC RECORD OF
CRACK GROWTH IN ABS(cont)



P5



P6



P7

particular mode of failure. The uncrazed notch in this study promotes failure of a plane stress type within shear bands, whereas the crazed zone in Truss and Chadwick (1977) encourages failure by craze growth with no apparent necking of the specimen.

(iii) Polyvinylchloride (PVC)

Standard grade of 'Darvic' PVC sheet as produced by ICI Ltd. was chosen for investigation since none of the transparent grade was readily available. One thickness only, 3 mm, was used. Stress-strain curves are shown for compression and tension in figure 29 and the moduli and yield stresses respectively in the two cases are 3.2 GNm^{-2} and 61 MNm^{-2} for the former mode and 3.0 GNm^{-2} and 56 MNm^{-2} for the latter. Mechanical properties quoted according to ICI Ltd., are a flexural modulus of 3.4 GNm^{-2} and a tensile strength of 55 MNm^{-2} . Substantial necking takes place in tension which accounts for the inconsistent behaviour beyond yield.

Razor notching procedures produced notches similar in quality to those made in polycarbonate. Initial notch lengths ranged from 8 mm to 16 mm, the testing conditions and test-piece dimensions remaining unaltered. Difficulties with lighting were surmounted by reflecting some light off the specimen while still admitting light from behind it. Figure 30 shows a typical fracture from a crack of length 12 mm, while figure 31, for an initial notch of length 16 mm, gives the higher magnification obtained from slide projections.

Deformation follows a similar pattern to that displayed by the previous materials; separation of the crack flanks and tip blunting precedes the initiation of tearing and the simultaneous occurrence of shear bands. Photograph P2 indicates shear band initiation at the

FIGURE 29: STRESS-STRAIN CURVES FOR μ PVC

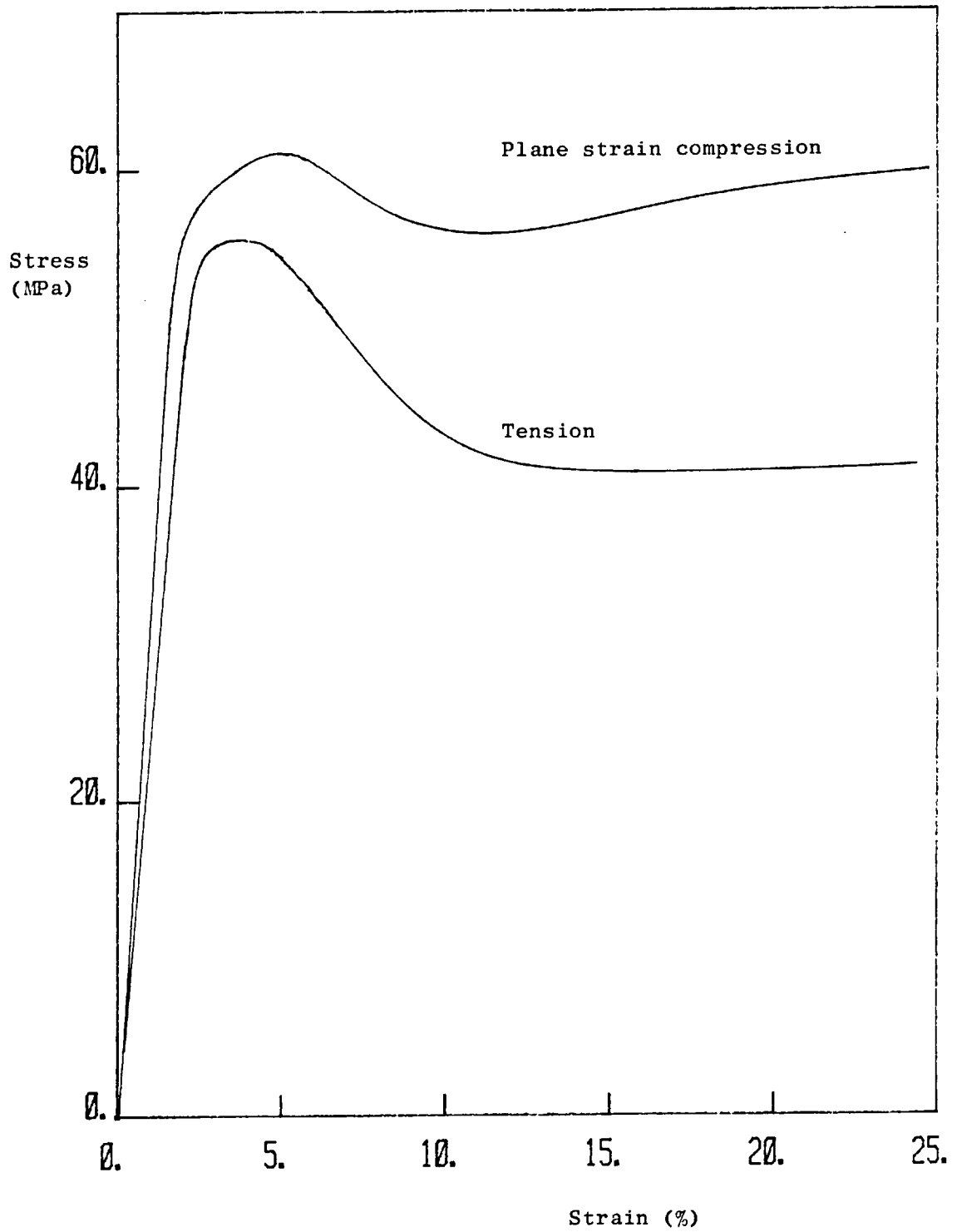
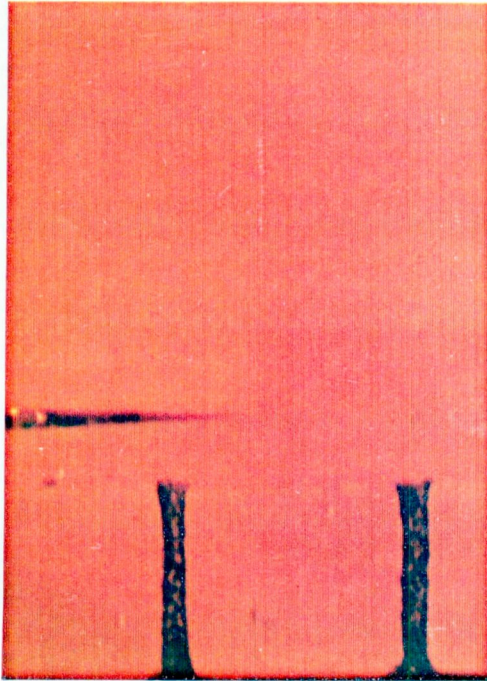
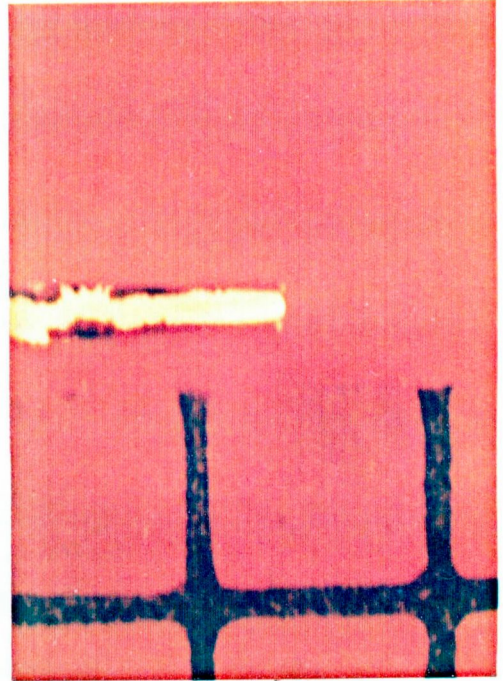


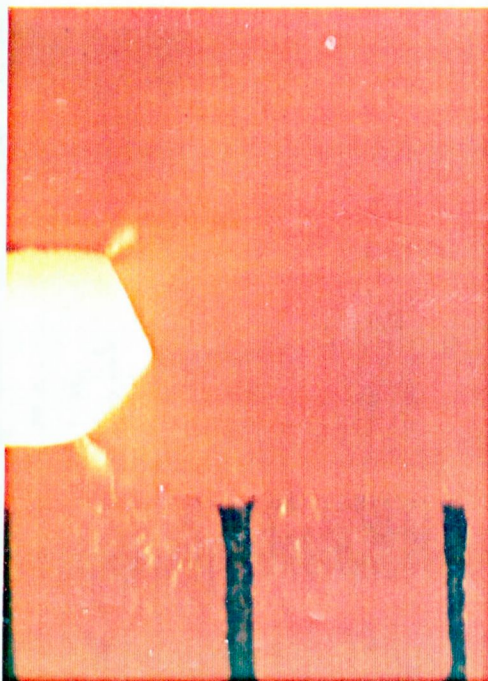
FIGURE 30: PHOTOGRAPHIC RECORD OF BLUNTING
AND CRACK GROWTH IN uPVC



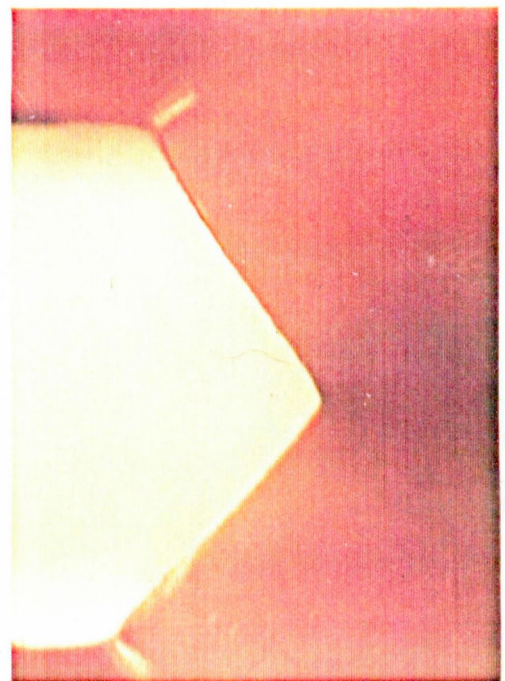
P1



P2

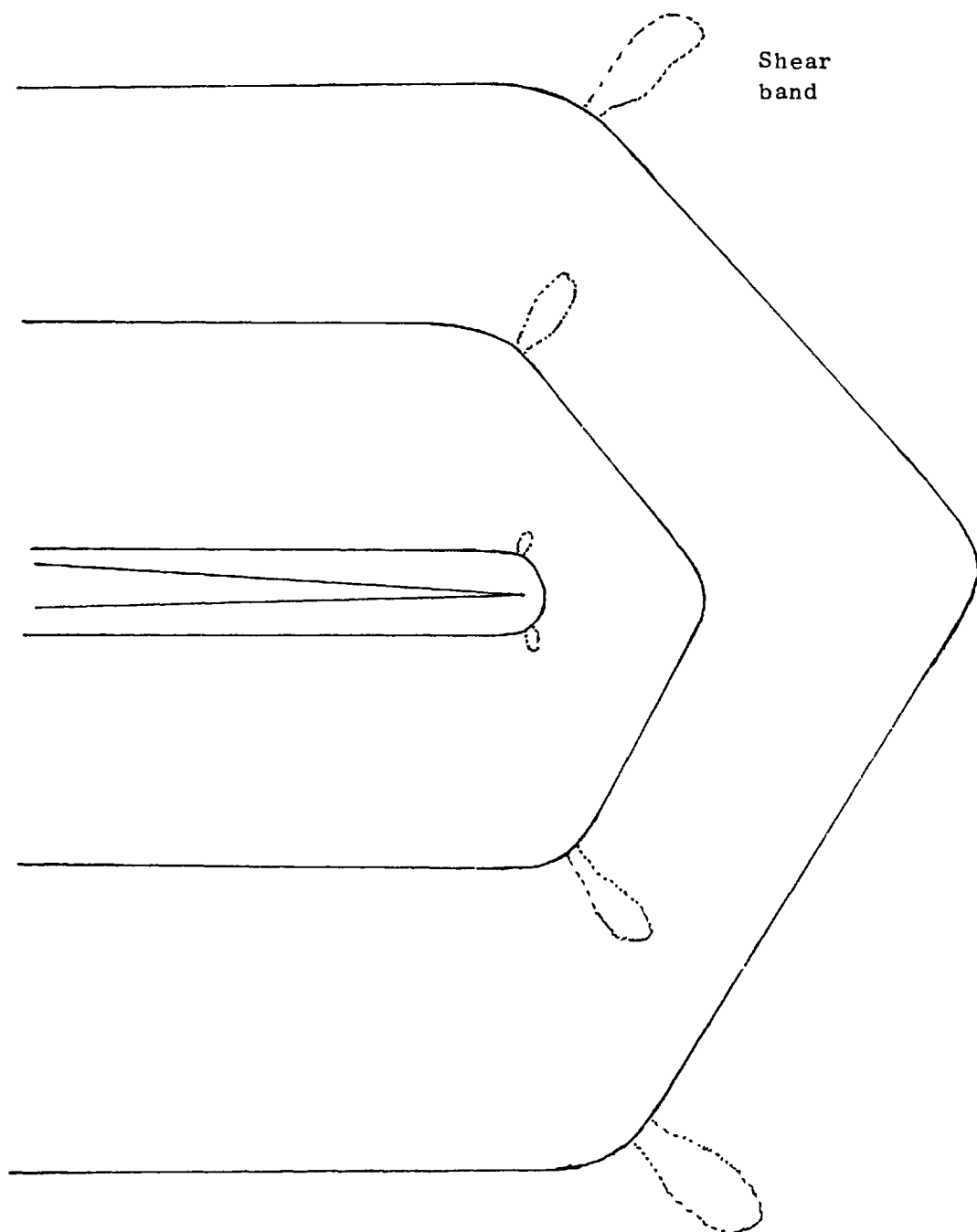


P3



P4

FIGURE 31: TYPICAL TIP PROFILES IN JPVC
BEFORE AND AFTER INITIATION



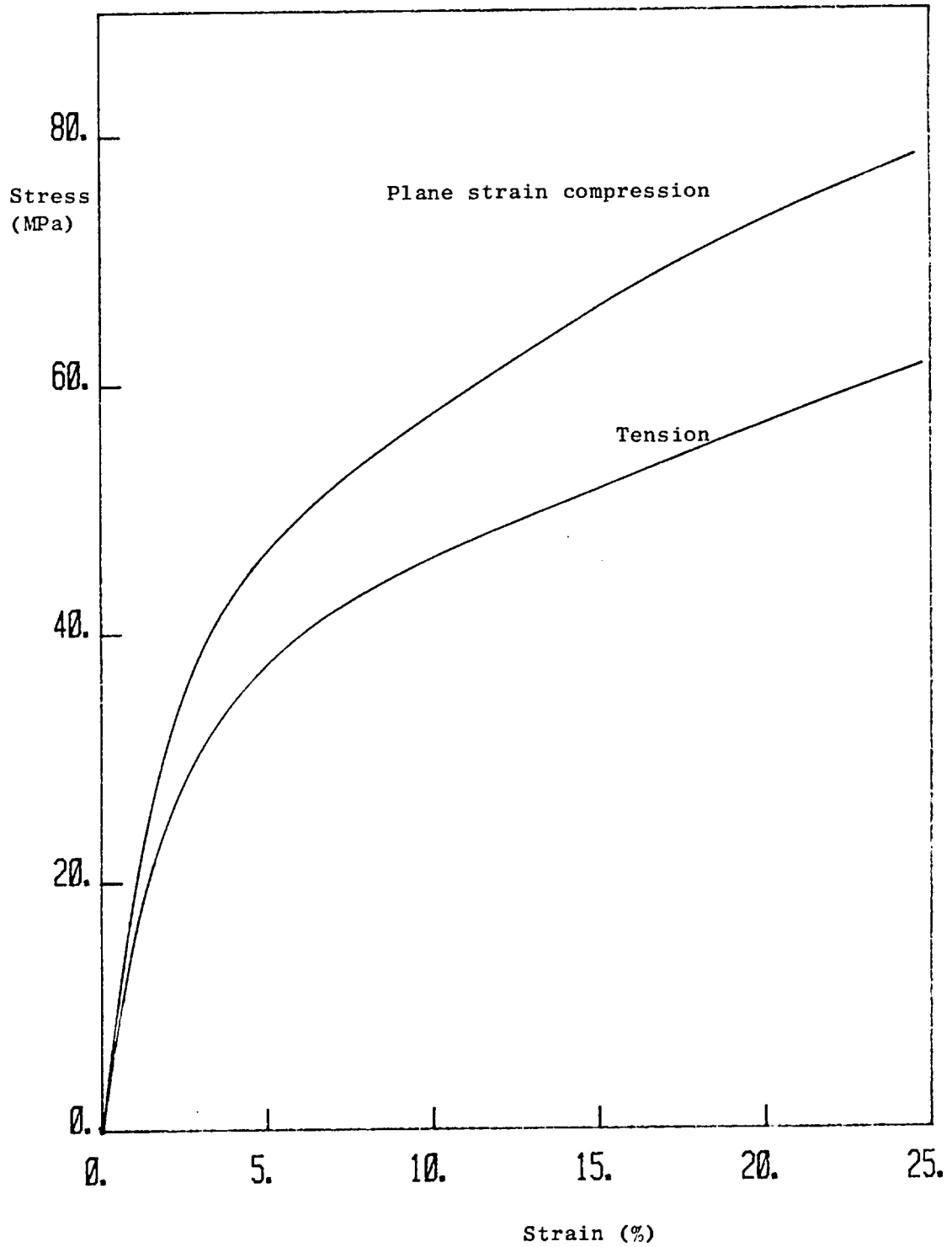
blunted crack tip. The tip remains extremely blunt after the commencement of crack growth and promotes full slip in the ligament as indicated by the slip line field of figure (d), chapter four section (c). Propagation takes place by gross tearing and the lack of constraint results in crack wandering. Another peculiarity to PVC are the points of origin of the shear bands which exhibit a sharply defined region which grows in the direction of the shear band with crack growth but is of much smaller extent than the shear yielding; whereas the yielded zone is defined approximately by 45° slip lines, these sharply defined regions are at an angle of roughly 70° to the extension of the original crack line.

(iv) Nylon 6/6

"Maranyl" nylon, code A100, as supplied in extruded sheet form by ICI Ltd. was examined in one thickness only; 3 mm. All tests on this material were performed at 20°C and in a laboratory of relative humidity of 50%. Tensile tests were made along the direction of extrusion, while compressive tests were made orthogonal to this direction.

Figure 32 shows the results of tensile and compressive stress-strain tests up to a strain level of 25%. There are discrepancies between data presented here and the type of stress-strain behaviour quoted by ICI especially with regard to the yield strain. In both tension and compression there is a modulus of $1.5 - 1.8 \text{ GNm}^{-2}$ and a distinct change in gradient of the curves at a strain of 3% and a stress level of 30 MNm^{-2} . Values of tensile strength for this grade are in the region of 70 MNm^{-2} with a yield point at 30% strain. However the data is regarded, nylon is characterised by a monotonic curve of stress against strain up to strain levels of 25%. In this

FIGURE 32: STRESS-STRAIN CURVES FOR NYLON 6/6



respect, it is a material having some qualitative similarities with low density polyethylene yet having far greater stiffness.

Notching with a razor produced a region of whitening at the crack tip, and so in this case initial cracks were machined in the specimens to depths of between 8 mm and 16 mm. Photographs were taken using both bellows and extension bellows, before and after crack initiation. Figure 33 indicates deformation trends for a crack length of 9 mm and figure 34 is a scheme from projections of slides for a 13 mm crack.

There are a few interesting observations: firstly, the curvature developed in the crack flanks prior to initiation is similar to deformation observed in low density polyethylene; there are no well defined shear bands although geometrical discontinuities at the original tip region exist throughout crack extension; 'sucking-in' ahead of the crack is greatly pronounced, much more so than in previously examined materials.

(v) Ethylene-Propylene Copolymer

A block copolymer of ethylene and propylene produced by Shell Chemicals Ltd. was examined in a thickness of 3 mm. The grade supplied had an ethylene content of 7.5% and a crystallinity of 59%. Fernando (1980) quotes the yield stress at 20°C in tension as 28 MNm^{-2} and this is borne out by the tensile and compressive data supplied here in figure 35. Both modes give a modulus in the range of $0.8 - 1.1 \text{ GNm}^{-2}$ and the post yield behaviour, i.e for strains beyond 10%, is one of no strain hardening.

As with nylon, razor notches produced too much stress whitening at the crack tip and notches were again machined. Crack

FIGURE 33: PHOTOGRAPHIC RECORD OF BLUNTING
AND CRACK GROWTH IN NYLON 66

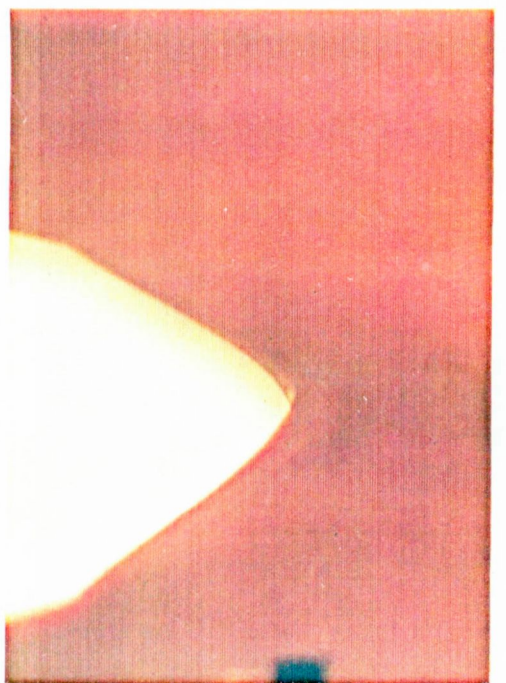
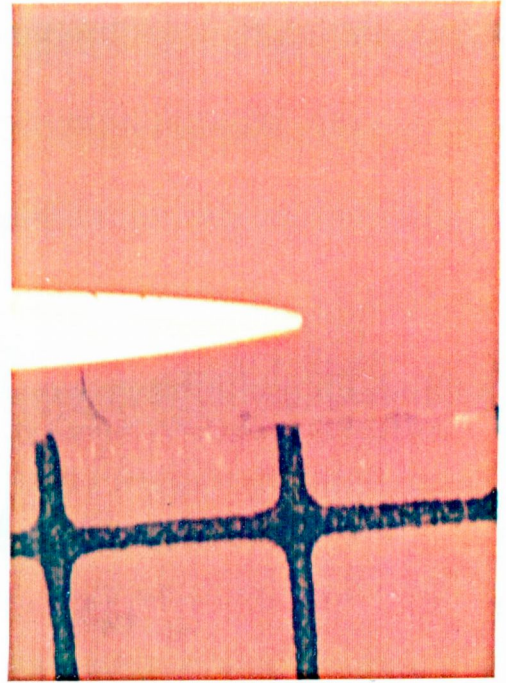
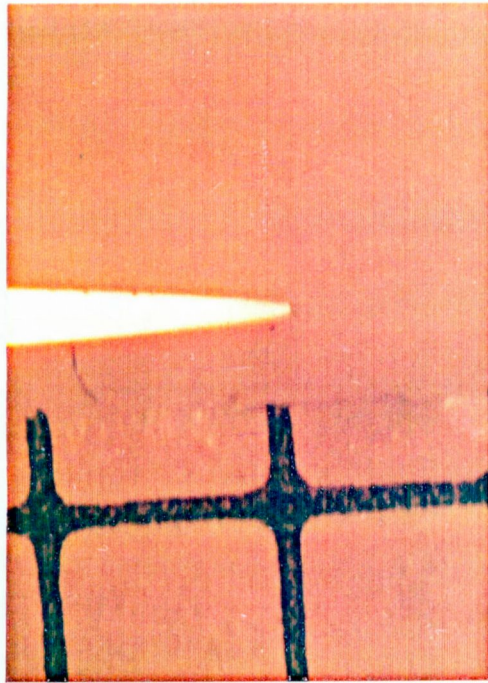


FIGURE 34: TYPICAL TIP PROFILES IN NYLON 6/6
BEFORE AND AFTER INITIATION

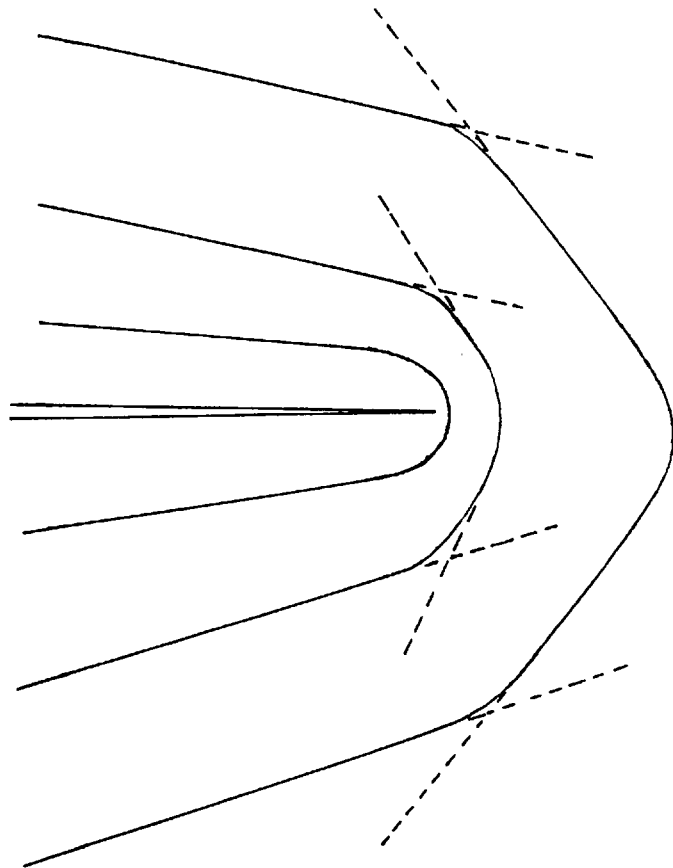
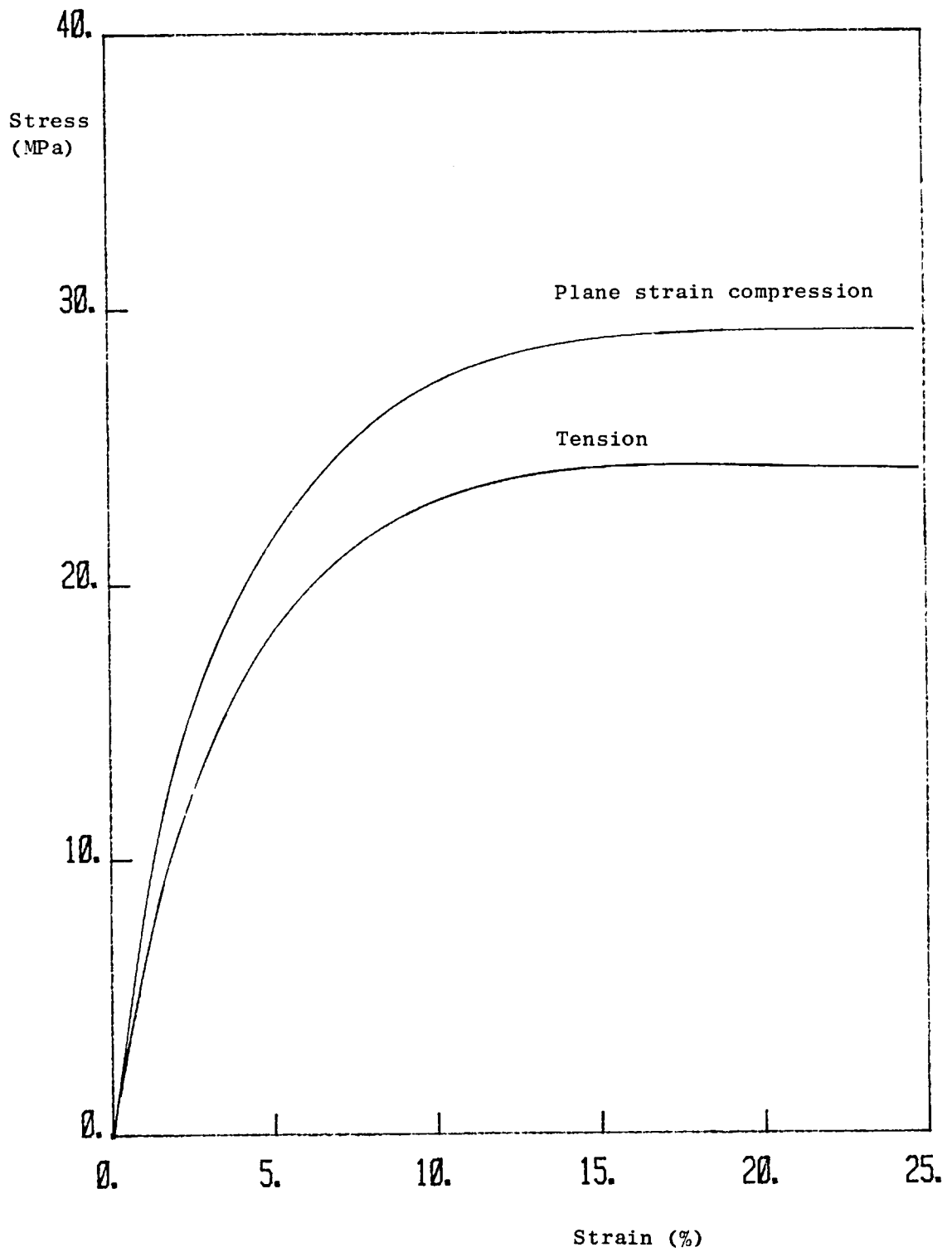


FIGURE 35: STRESS STRAIN CURVES FOR AN
ETHYLENE-PROPYLENE COPOLYMER



lengths ranged from 4 mm to 10 mm and photographs were taken during loading of the specimens with the bellows attachment only, since deformations were of an order of magnitude higher than with those previously treated. Stress-deflection curves for several initial crack lengths are shown in figure 36 and characteristic behaviour is shown in figure 37. Points marked * on the loading curves are where the propagation of tearing was observed to start.

The nature of deformation is very different to that observed with the other materials. Crack flanks separate in a parallel manner and gross blunting of the crack tip occurs. Unlike the shear banding which occurs in PVC, ABS and polycarbonate, the system of slip is more akin to that shown in chapter four for fully plastic slip ahead of a flat-nosed notch. Crazeing takes place in the zone near the tip and demarcation between the zone and the uncrazed material allows a clear definition of the crack tip opening displacement.

(c) Summary and Analysis

The materials examined constitute a broad view of plastic behaviour in polymers; of those that may be regarded as non strain-hardening, ABS is a rubber modified brittle material, ethylene-propylene copolymer is a rubber modified ductile material and polycarbonate is a tough unfilled polymer. Together with these, PVC, with its moderate strain softening region, and nylon, which strain hardens appreciably, form a well balanced selection of materials for discussion.

Distinction must be made between pre-initiation and post-initiation behaviour, the latter requiring a fracture criterion to be invoked in order to describe the resulting deformation. Pre-initiation behaviour in totally elastic materials is well characterised by the elastic blunting

FIGURE 36: STRESS-DEFLECTION CURVES FOR
ETHYLENE-PROPYLENE COPOLYMER (SEN)

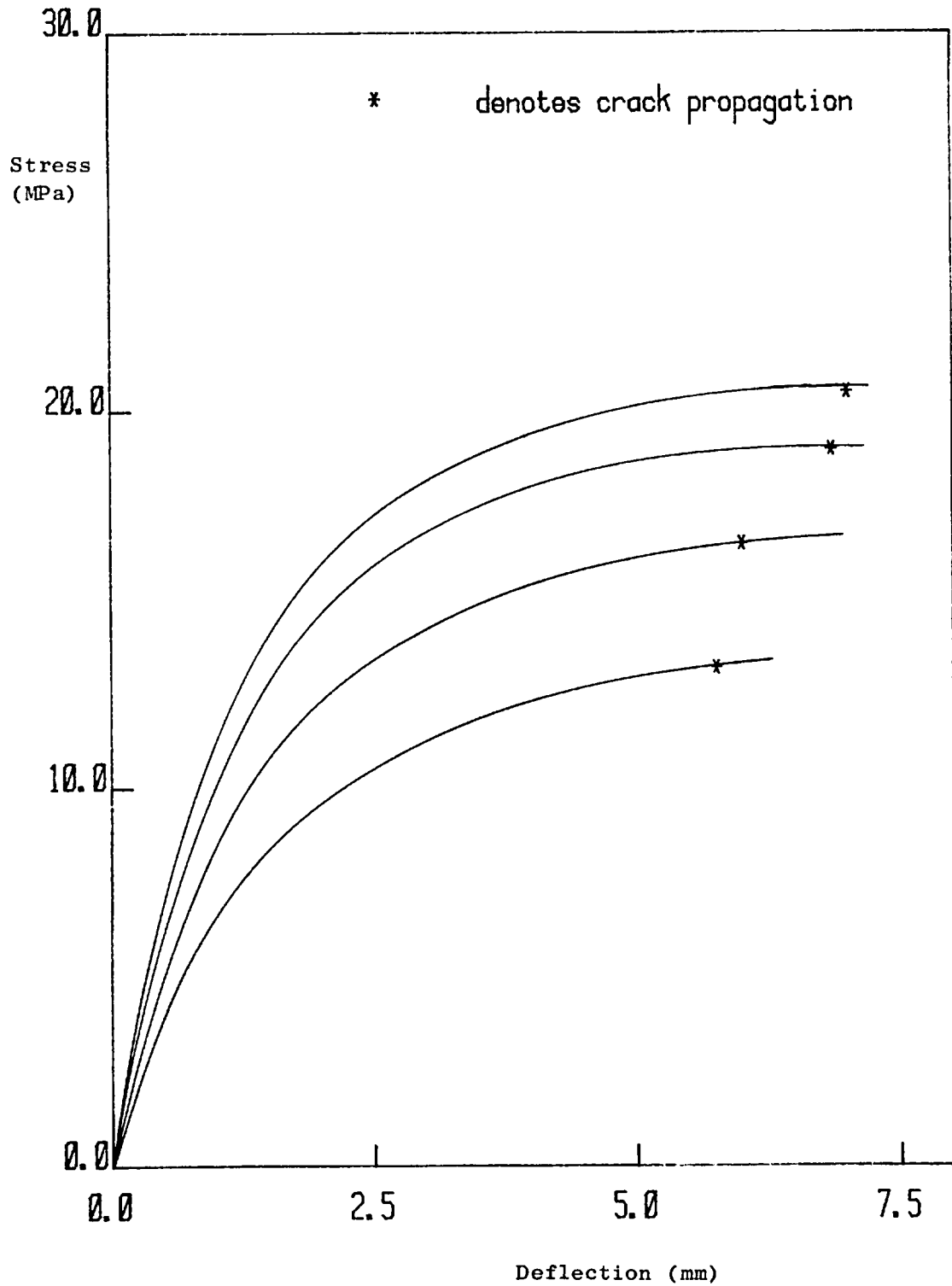
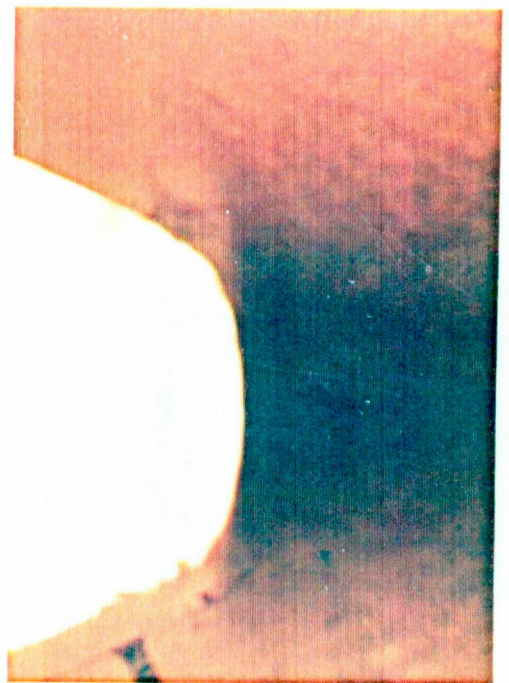
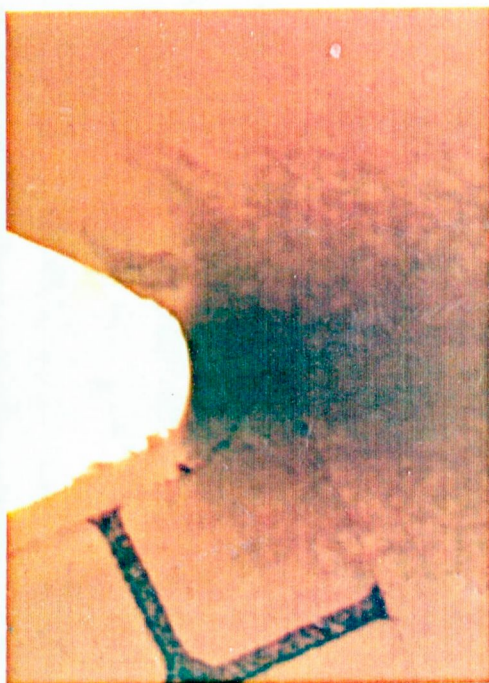
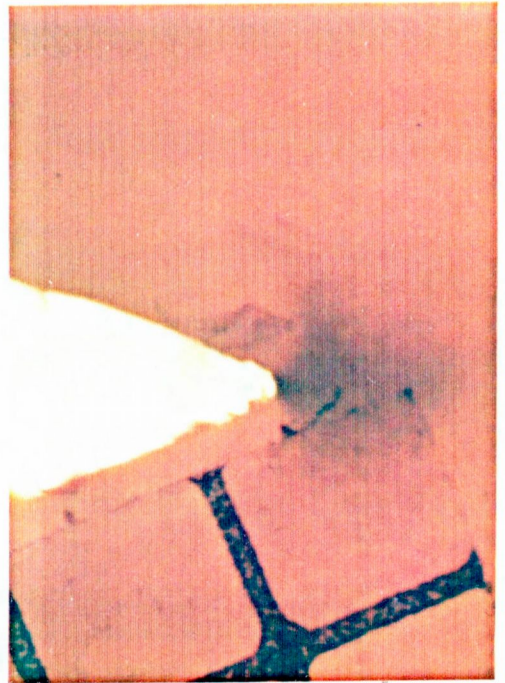
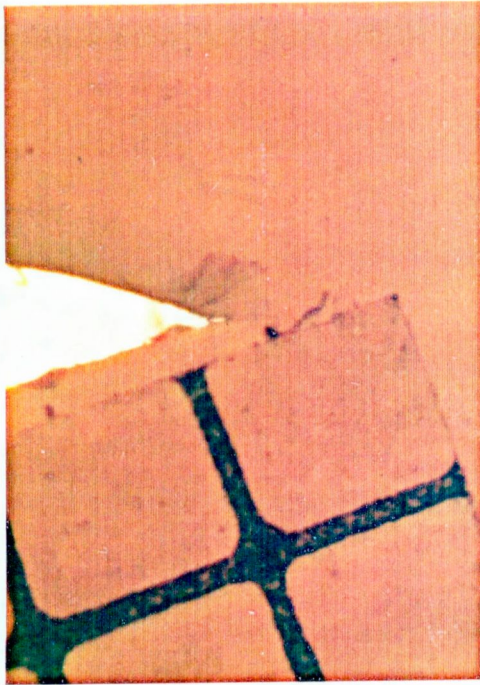


FIGURE 37: PHOTOGRAPHIC RECORD OF BLUNTING
IN AN ETHYLENE-PROPYLENE COPOLYMER



model proposed in chapter three. It is clear that this model will apply approximately to materials which have no well defined yield point and exhibit extensive elasticity. Such is the case with low density polyethylene and, to a lesser extent, with nylon. In order to indicate the departure from this regime, the values of crack tip radii prior to crack initiation were recorded for all materials excluding polypropylene and the radius to crack length ratio was plotted against strain in the same manner as for figure 10. Smooth curves were drawn through the data and are shown in figure 38, wherein it is immediately seen that polyethylene, and nylon to a certain extent, is in far better agreement with the line for total elasticity than are PVC and polycarbonate. Further confirmation of the near elastic behaviour of polyethylene and nylon is provided by the curvature present in the crack flanks both before and after crack initiation which is increasingly apparent near the tip.

Determination of the precise crack tip profiles before initiation for non-hardening materials is difficult in view of the limited amount of deformation taking place. However, the nature of the curves of crack tip radius against strain seems to be quite different. Plotting the results for all materials on a log basis, i.e. log radius versus log strain as shown by figure 39, highlights the differences in the following manner.

- (i) For polyethylene and nylon, reasonable straight lines are obtained whose gradient is close to 2. This confirms the similarities between the deformations for these two materials and totally elastic blunting.
- (ii) In the case of ethylene-propylene copolymer there is good linearity but with a gradient of approximately 4.

FIGURE 38: BLUNTING OF CRACKS IN VARIOUS MATERIALS

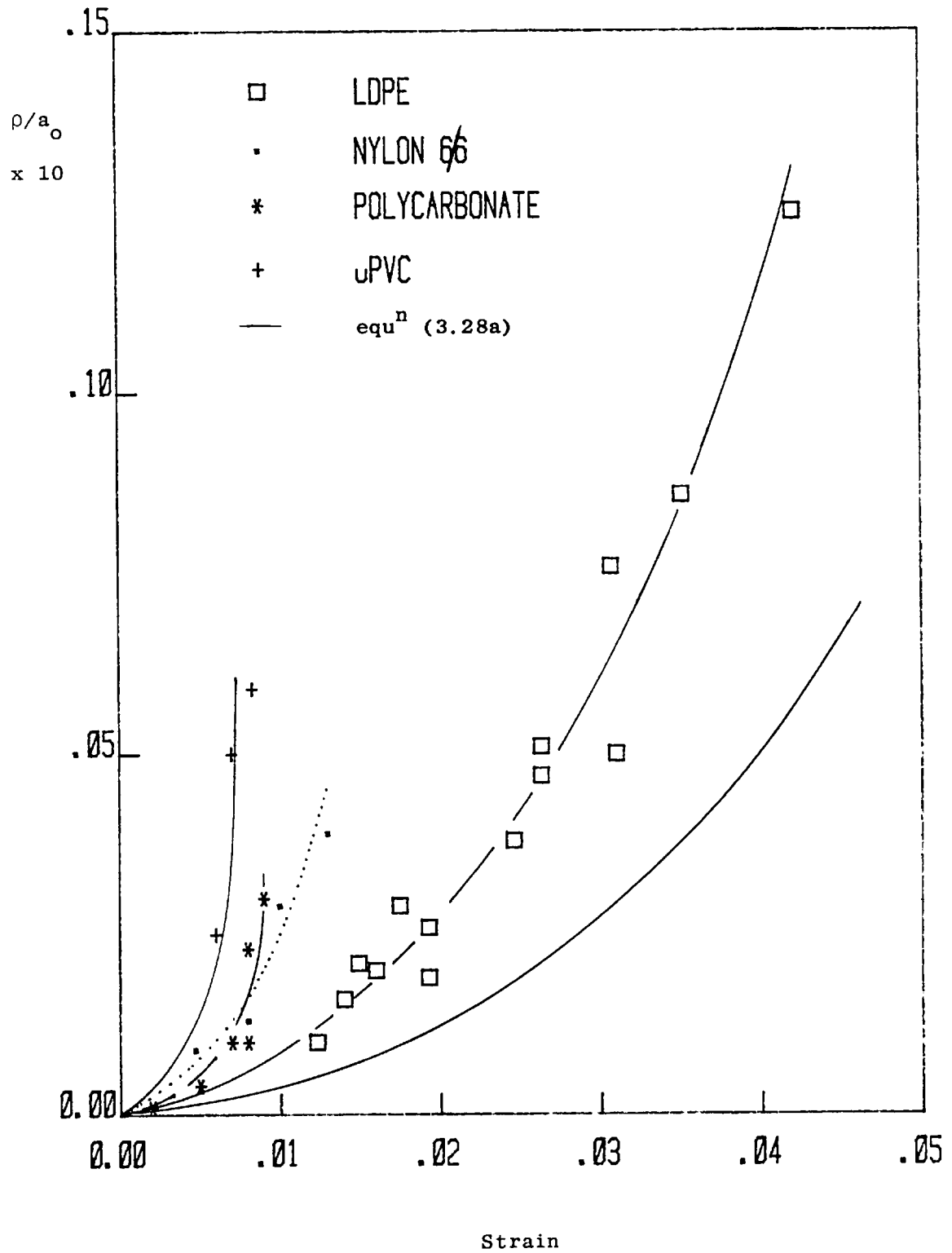
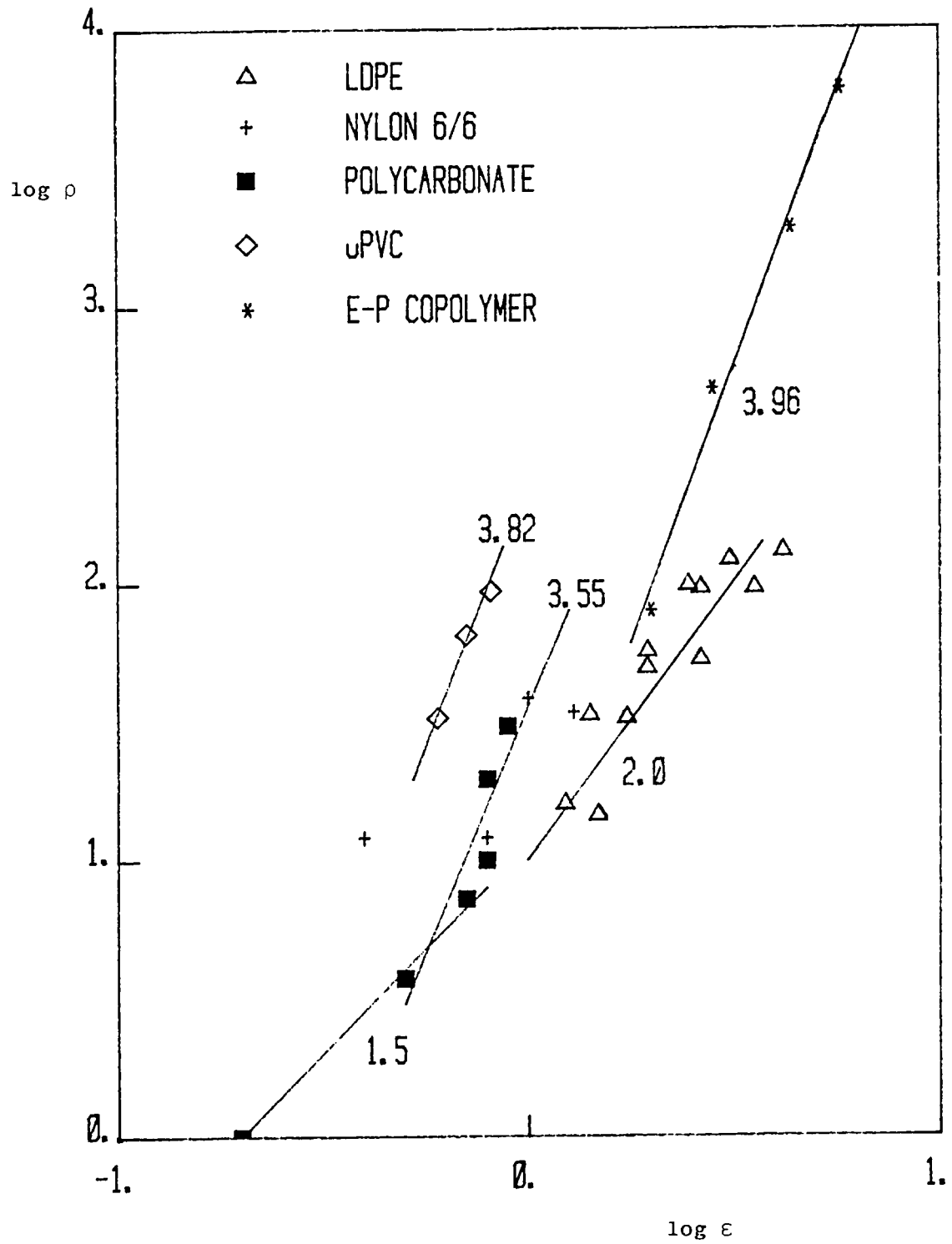


FIGURE 39: DEVELOPMENT OF CRACK BLUNTING
LOGARITHMIC PLOT



(iii) Polycarbonate and PVC show a gradual transition between the behaviour akin to case (i) and that akin to case (ii).

The behaviour of polypropylene is characterised by parallel separation of the crack flanks with severe discontinuities between the flanks and the crack front. If the stretch zone, s , as defined by figure 40 is assumed constant then the displacement, Δ , at the original crack tip is related to the radius at the tip, ρ , by the equation

$$\Delta^2 = 4 (2\rho - s) s \quad (5.1)$$

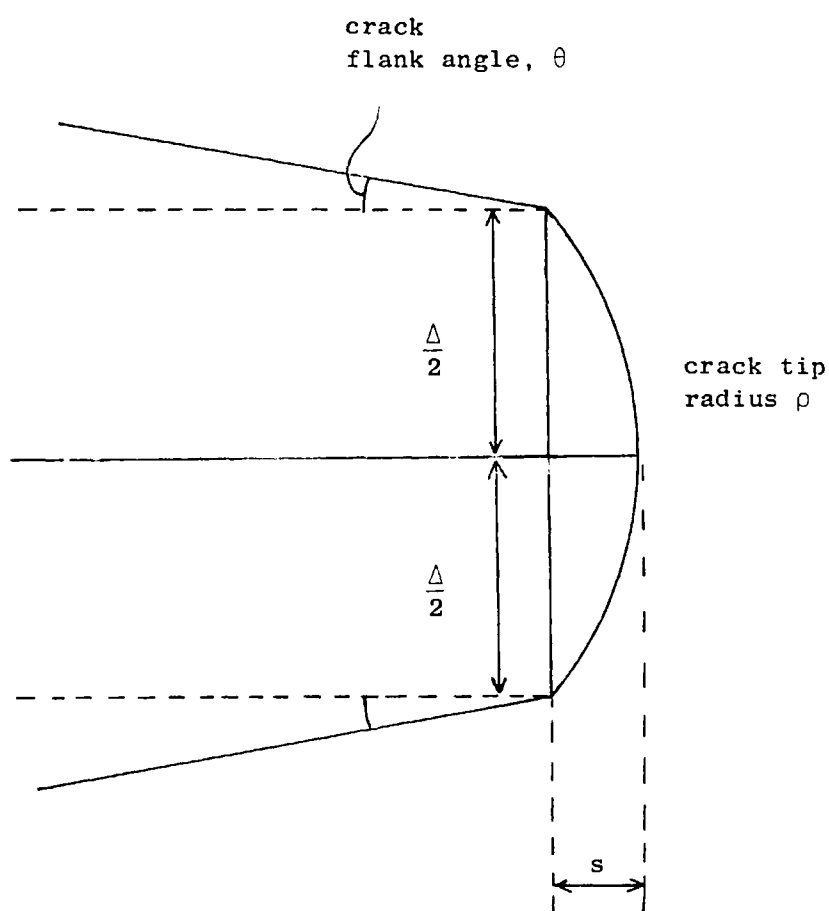
for smooth blunting at the tip. Now, the displacement Δ can be regarded as being dominated by the elastic stress field in the specimen and will therefore be proportional to the second power of the strain (as was the case for elastic blunting). Thus rearranging equation (5.1) and writing $\Delta = k\varepsilon^2$ we have

$$\rho = \frac{1}{2} s + \frac{k^2}{s} \varepsilon^4 \quad (5.2)$$

and, therefore, a log plot of ρ versus ε may be expected to yield a straight line of gradient 4.

Post initiation behaviour follows a similar pattern to that described by Garwood and Turner reviewed in chapter four section (d) (ii) in that steady state crack propagation is characterised by constant flank angle. This is true of all the materials which propagated cracks freely although particular attention was focussed on polycarbonate alone. In view of the comments to be made and their relevance to strain energy release rate, a summary of the findings is deferred until the details

FIGURE 40: CRACK TIP MODEL FOR DEFORMATION
AFTER SHEAR BAND FORMATION



$$\Delta^2 = 4 s (2\rho - s)$$

of experiments performed on this material have been presented.

(d) Crack Initiation and Tear Propagation in Polycarbonate

The brittle and ductile features inherent in the fracture of polycarbonate have been described concisely in Parvin and Williams (1975). A transition from brittle to ductile failure is observed as the thickness or strain rate is reduced and these effects are modelled using a sandwich fracture model, where the fracture surface is imagined to consist of a plane strain region between two plane stress surface regions. In the course of extending fracture mechanics analyses to describe the ductile tearing of polycarbonate further comment is made on the initiation aspects and comparisons are drawn between the results presented here and those due to Parvin and Williams (1975).

A variety of specimen dimensions were examined in this study in order that the geometrical independence of any fracture parameters might be assessed. The SEN geometry was employed throughout, however, since it enabled a clear determination of the amount of crack growth. Three types of loading system were used:-

- (i) wedge grips tightened prior to load application.
- (ii) wedge grips tightened by the initial tensile load and
- (iii) bolted plates.

It was necessary to use these different systems in order that a variety of specimen width might be examined; system (i) being used for 50 mm and 36 mm widths, system (ii) for 80 mm width and system (iii) for a width of 150 mm. Two thicknesses were used, 1mm and 3mm, but only for the 50 mm wide specimen. For each specimen geometry a range of crack lengths was employed within the limits 0.1 to 0.7 for crack length to width ratio. On individual specimens, lines perpendicular to the

anticipated line of crack growth were drawn at regular intervals in order for propagation to be monitored.

The experimental procedure was as follows.

Load versus cross-head deflection traces were obtained at an Instron cross-head speed of 0.5 mm min^{-1} for all specimen geometries until failure occurred. Several events were recorded on the load deflection trace. These were (i) 'pop-in' as described by Parvin and Williams (1975), (ii) the initiation of ductile tearing defined by the commencement of growth of shear bands at the crack tip, and (iii) successive increments of crack growth defined by the propagation of the crack past the pre-marked lines on the specimen. An event described as 'crack tip bowing' was detected by Parvin and Williams (1975) occurring before 'pop-in', but its detection in this study was omitted since it is a highly subjective matter and probably requires thicker specimens for accurate measurement.

(i) 'Pop-in Initiation'

This phenomenon was not detected for a specimen thickness of 1 mm regardless of width, but for the 3 mm thick, 50 mm wide specimens it was observed for crack lengths in excess of 12 mm. The point of 'pop-in' was noted with an event marker and figure 42 shows this for initial crack lengths ranging from 16 mm to 32 mm.

The calculation of a stress intensity factor, K , appropriate to 'pop-in' was performed as follows. Since the wedge grips employed inhibited rotation of the specimen, the most relevant finite width correction factor was thought to be that due to Harris (1967). This was further borne out by the compliance data which gave a dependence on crack length which correlated well with that derivable from the Harris

finite width factor. Values of $\sigma^2 Y^2$ were plotted against the reciprocal of crack length, as is customary, and a straight line was obtained as is shown in the graph of figure 41. The square root of the gradient gave a value for K of $3.4 \text{ MNm}^{-3/2}$.

This value conflicts with that given by Parvin and Williams (1975), in fact it is lower than the 'pop-in' value quoted for 5 mm thick specimens. Using the model presented by Parvin and Williams the predicted value for 3 mm thick specimens is $5.1 \text{ MNm}^{-3/2}$. In order to substantiate the values obtained here, values of strain energy release rate derived from load and compliance measurement were converted to values of K using a modulus of 2.0 GNm^{-2} . The results are shown in table 5 and give an average K of $3.5 \text{ MNm}^{-3/2}$.

So, there is a genuine disparity in the values of 'pop-in' toughness and the only possible explanation is one of difference in grade of material.

(ii) Tearing

Dominating the analysis of tearing in polycarbonate is a modification of the approach adopted by Hodgkinson and Williams (1981) for polyethylene. The local strain energy release rate, assuming material to be perfectly elastic-plastic, is given in chapter four by equation (4.24) to be

$$G = - \frac{P^2}{2B} \frac{dC}{da} - \frac{PC}{B} \frac{dP}{da} \quad (5.3)$$

Δ

Two cases are degenerate from this equation. Firstly, the elastic case relevant to crack initiation as described in the previous section.

FIGURE 41: DETERMINATION OF "POP-IN" TOUGHNESS
FOR POLYCARBONATE IN SEN TENSION

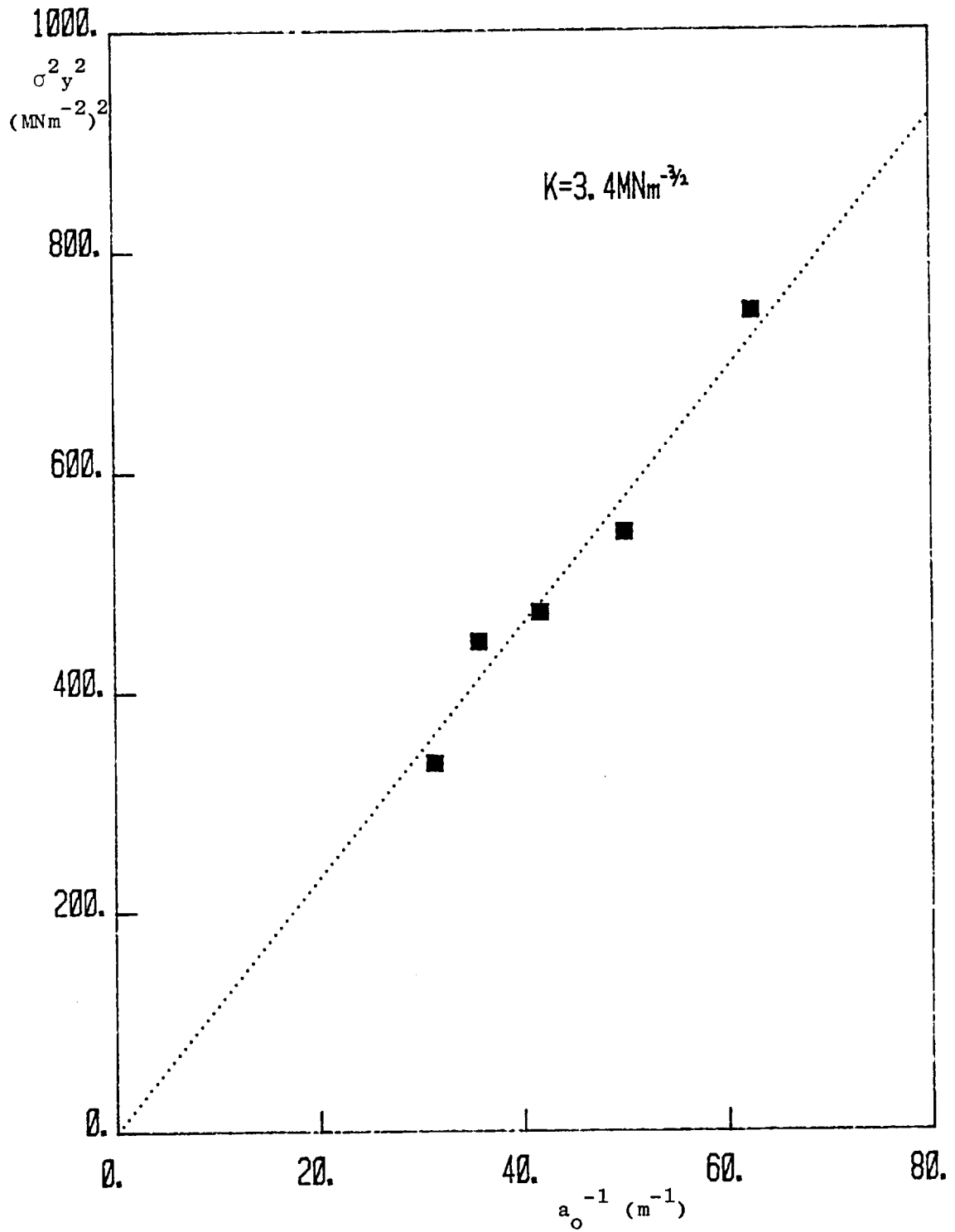


TABLE 5.

POLYCARBONATE - INITIATION DATA

AND TOUGHNESS CALCULATIONS

a_o (mm)	P (N)	$\frac{dC}{da} (N^{-1}) \times 10^3$	$G = \frac{P^2}{2B} \frac{dC}{da} (kJm^{-2})$	$K = (EG)^{\frac{1}{2}} (MNm^{-\frac{1}{2}})$
16	922	0.25	5.94	3.45
20	1158	0.34	6.92	3.72
24	1285	0.53	6.52	3.61
28	1462	0.70	5.84	3.42
32	1795	0.94	5.87	3.43

$$E = 2.0 \text{ GNm}^{-2}$$

$$B = 3\text{mm}$$

In this case the deflection Δ is given by $\Delta = PC$ and the second term in equation (5.3) may be simplified using

$$\left[\frac{dP}{da} \right]_{\Delta} = - \frac{\Delta}{C^2} \frac{dC}{da} = - \frac{P}{C} \frac{dC}{da} \quad (5.4)$$

and the usual elastic strain energy release rate at crack initiation is derived. The second case is that used in Hodgkinson and Williams (1981) for propagation in the fully yielded ligament. Using equation (4.14) the second term in (5.3) may be simplified using

$$\left[\frac{1}{P_L} \frac{dP_L}{da} \right]_{\Delta} = - \frac{1}{W - a} \quad (5.5)$$

and the final expression becomes

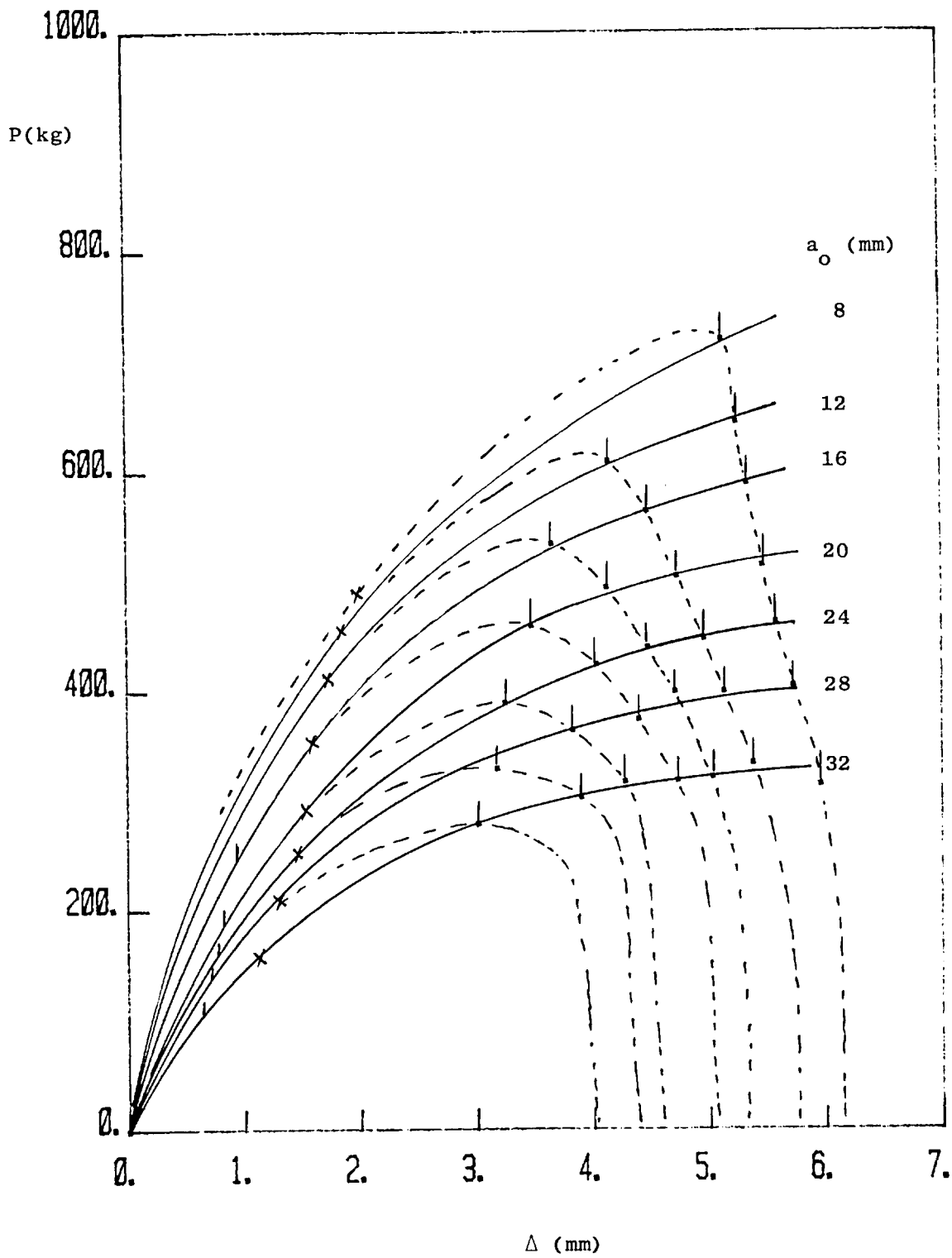
$$G = \frac{P^2}{2B} \frac{dC}{da} \left\{ \frac{2}{\eta} - 1 \right\} \quad (5.6)$$

where η is given, as before, by equation (4.7).

The use of the full equation (5.3) is necessary to define the value of G in situations intermediate to the two cases illustrated above and therefore may be used to calculate the behaviour of G during the entire crack growth regime.

On each load deflection trace, lines of constant crack length were drawn. This is illustrated for the initial case of $W = 50\text{mm}$, $B = 3\text{ mm}$ in figure 42 where the vertical event marks on each trace represent 4 mm of crack growth and the points denoting 'pop-in'

FIGURE 42: LOAD-DEFLECTION CURVES
FOR POLYCARBONATE



initiation and initiation of tearing are also marked. Using these lines of constant crack length, a value of $\left. \frac{dP}{da} \right|_{\Delta}$ may be derived by plotting load versus crack length at various values of Δ and obtaining gradients. The variation in this parameter with Δ is recorded in figure 43 and, as is to be expected from equation (4.14), a limiting value is obtained.

Two assumptions are made in implementing this analysis. Not only is the unloading curve assumed to be linear but it is also assumed to be parallel to the initial (small strain) loading curve of an initial crack of similar length. Therefore the compliance of the specimen is solely determined by crack length and is independent of the state of material in the ligament. Figure 44 shows experimental compliance data versus crack length for the case $W = 50$ mm, $B = 3$ mm plotted in the form C against a_0 where a_0 is the initial crack length. The solid line represents the compliance derived from the finite width correction factor of Harris (1967) and the good correlation is readily apparent. Values of the compliance gradient dC/da were obtained by a numerical differentiation technique.

Equation (5.3) was used to calculate values of G at the initiation of tearing and at each 4 mm increment during crack propagation. Figure 45 shows the results and the following conclusions may be drawn.

- (a) After an initial period of growth, during which time a plastic zone is developing at the crack tip, the value of G remains reasonably constant at a value of 75 kJm^{-2} .
- (b) During the initial period, which may cover 8 mm or more of crack growth, initially short cracks show a dramatic increase in G above the steady state value and a gradual fall as growth proceeds. Initially long cracks however show a gradual increase to the steady state value.

FIGURE 43: $-\left.\frac{dP}{da}\right]_{\Delta}$ VERSUS DEFLECTION, Δ ,
FOR POLYCARBONATE

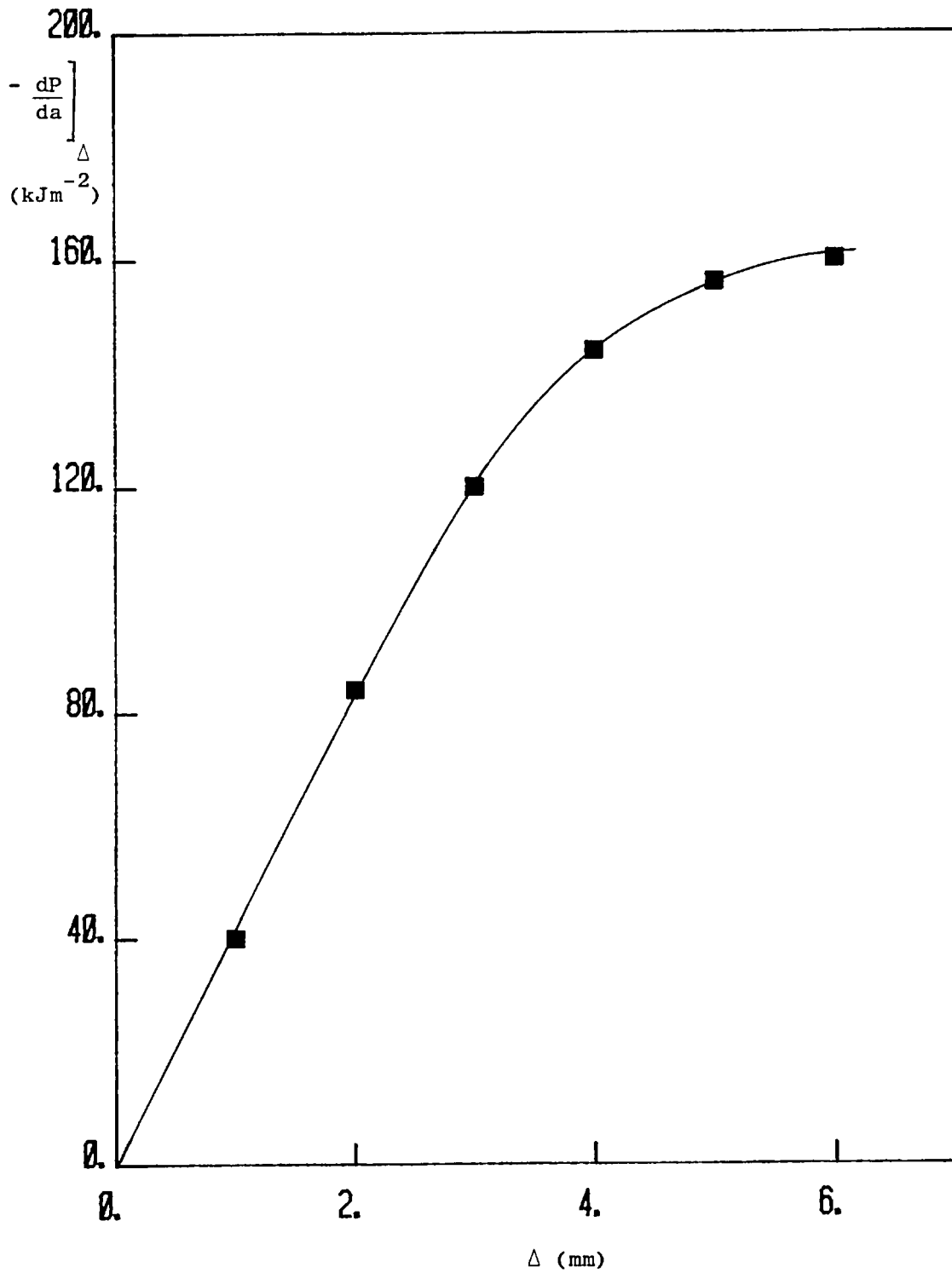


FIGURE 44: COMPLIANCE VERSUS CRACK LENGTH FOR SEN
TENSION SPECIMENS OF POLYCARBONATE

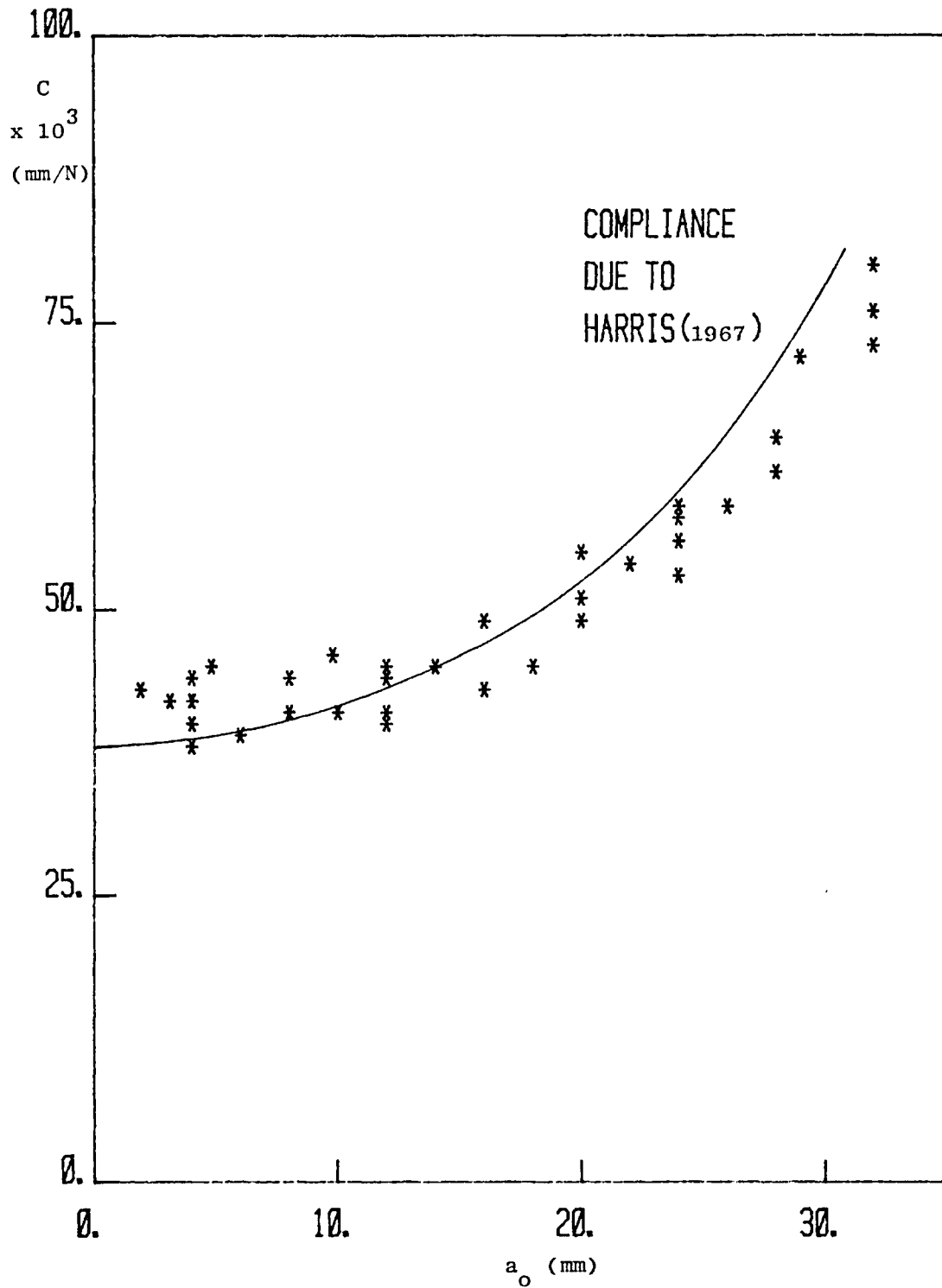
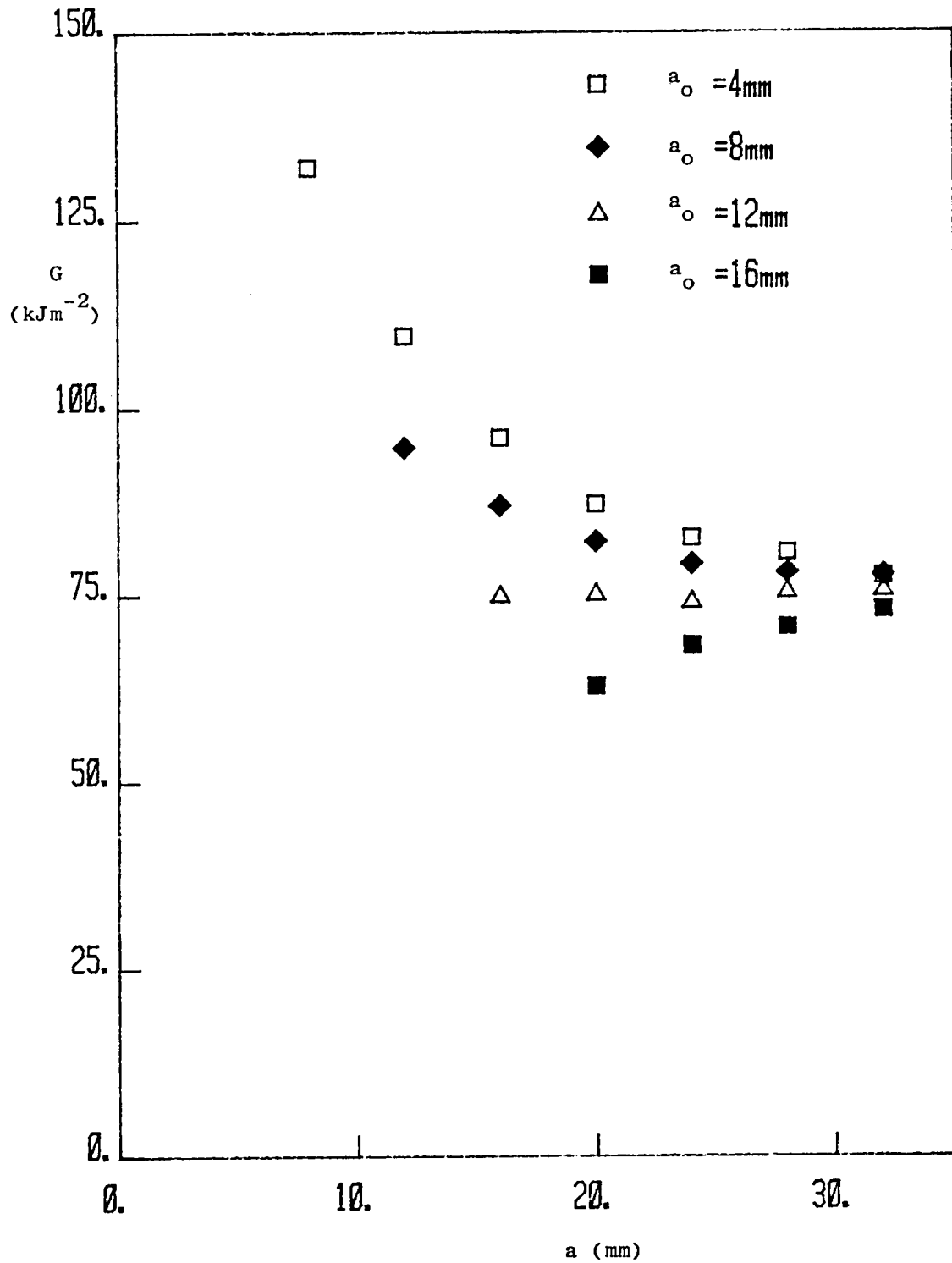


FIGURE 45: STRAIN ENERGY RELEASE RATE VERSUS
CRACK GROWTH FOR POLYCARBONATE



Crack tip radii have been measured for propagation in polycarbonate (see figure 23) and give an average value of 60µm. For initiation, the value is 25 µm which corresponds to an initiation G of roughly 25 kJm⁻². Figure 46 shows strain energy release rate plotted against crack tip radius and a reasonably linear dependence is again found.

An alternative analysis of the results is provided by J theory and was performed as follows. The energy, U, defined as the area beneath the constant crack length curves of figure 42 was measured using a planimeter for each crack length and a variety of constant deflection values. Plotting energy, U, versus crack length, a, at constant deflection gave reasonable straight lines as shown in figure 47. The gradients of these lines gave J according to the equation

$$J = - \frac{1}{B} \left. \frac{dU}{da} \right|_{\Delta} \quad (5.7)$$

Figure 48 illustrates the variation of J with deflection and the usual form is obtained; that is an initial quadratic dependence with a gradual transition to linearity. Using this figure in conjunction with figure 42, crack resistance curves were drawn for the various initial lengths and are shown in figure 49.

J resistance curves for predominantly plane stress failure have previously been produced for polyethylene in the same geometry (Hodgkinson and Williams (1981)) and, although of a different shape, gave a similar crack length dependence to that shown here (figure 49). However, they were neither explained nor utilised except for comparative purposes.

FIGURE 46: STRAIN ENERGY RELEASE RATE VERSUS
CRACK TIP RADIUS FOR POLYCARBONATE

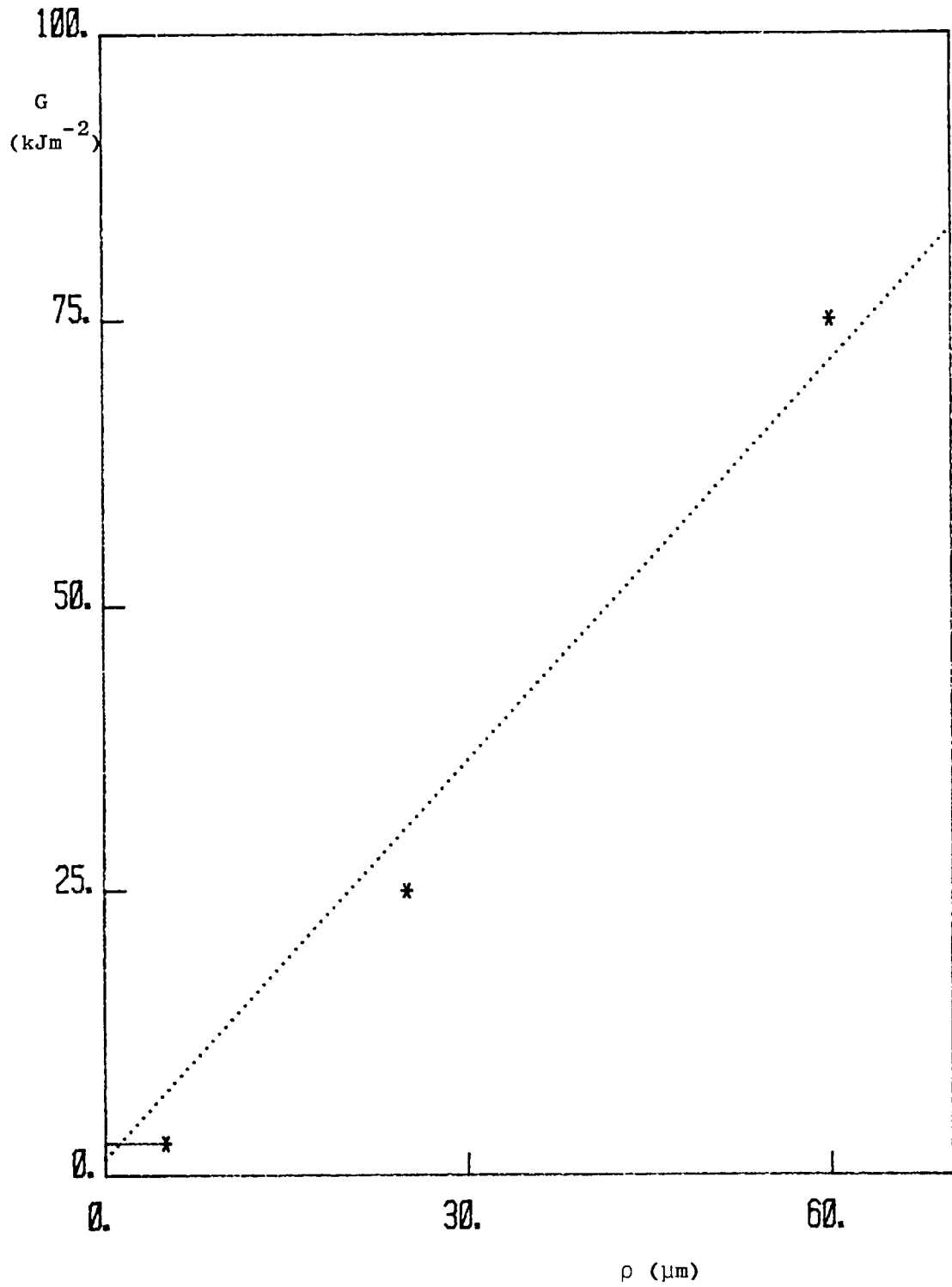


FIGURE 47: ENERGY ABSORBED AS A FUNCTION OF CRACK LENGTH AT CONSTANT DEFLECTION

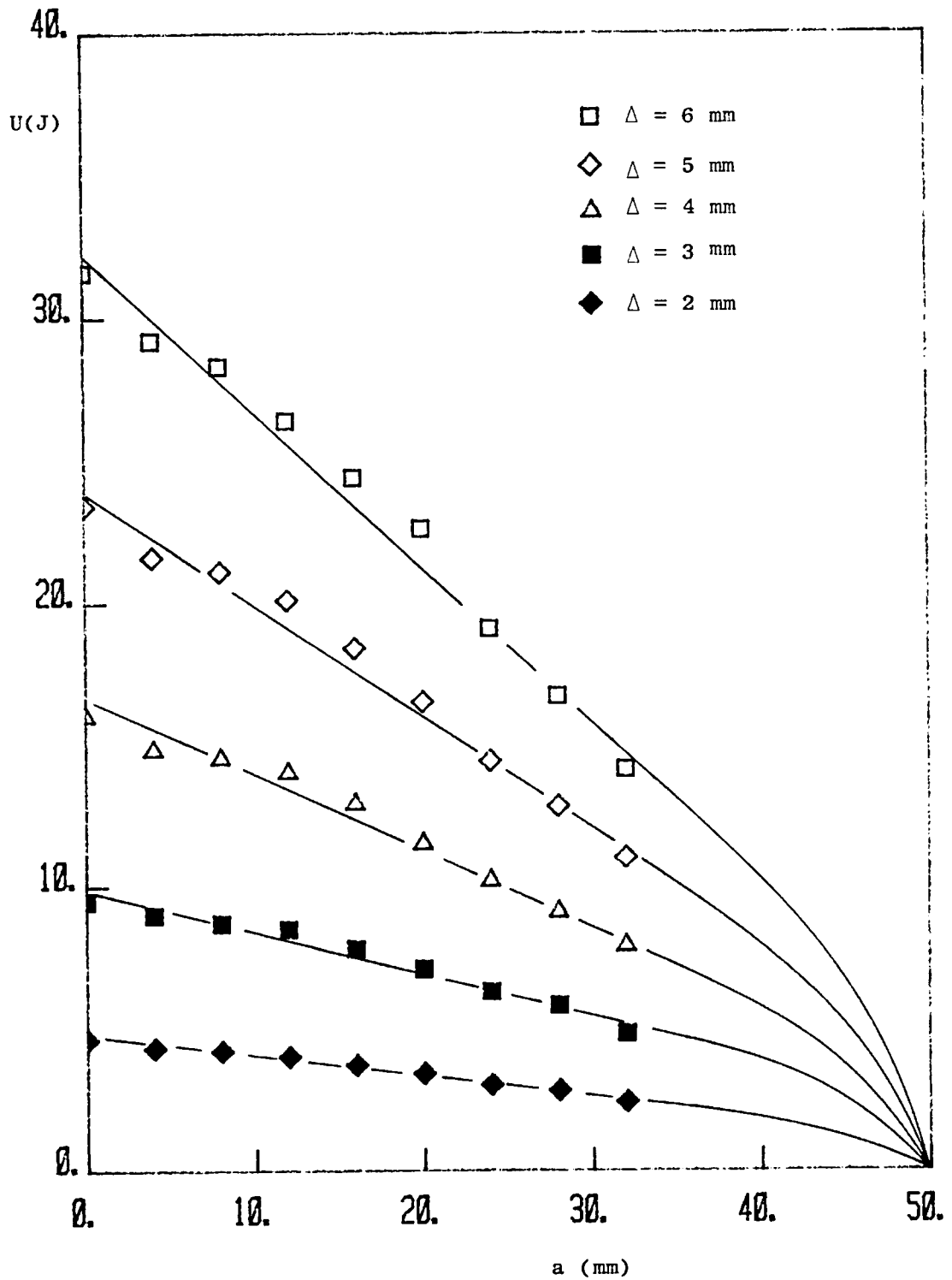


FIGURE 48: J-INTEGRAL VERSUS DEFLECTION CURVE
FOR POLYCARBONATE

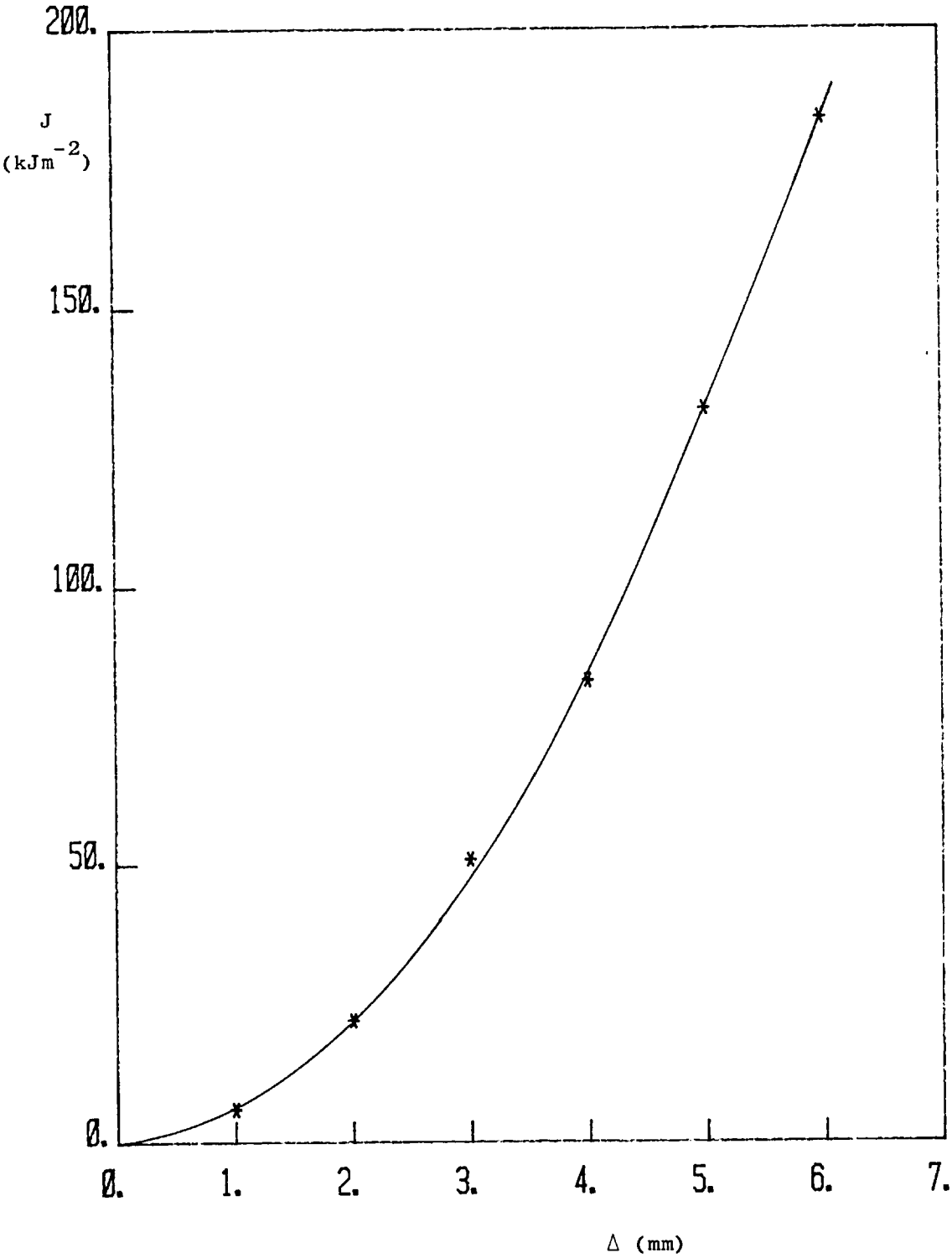
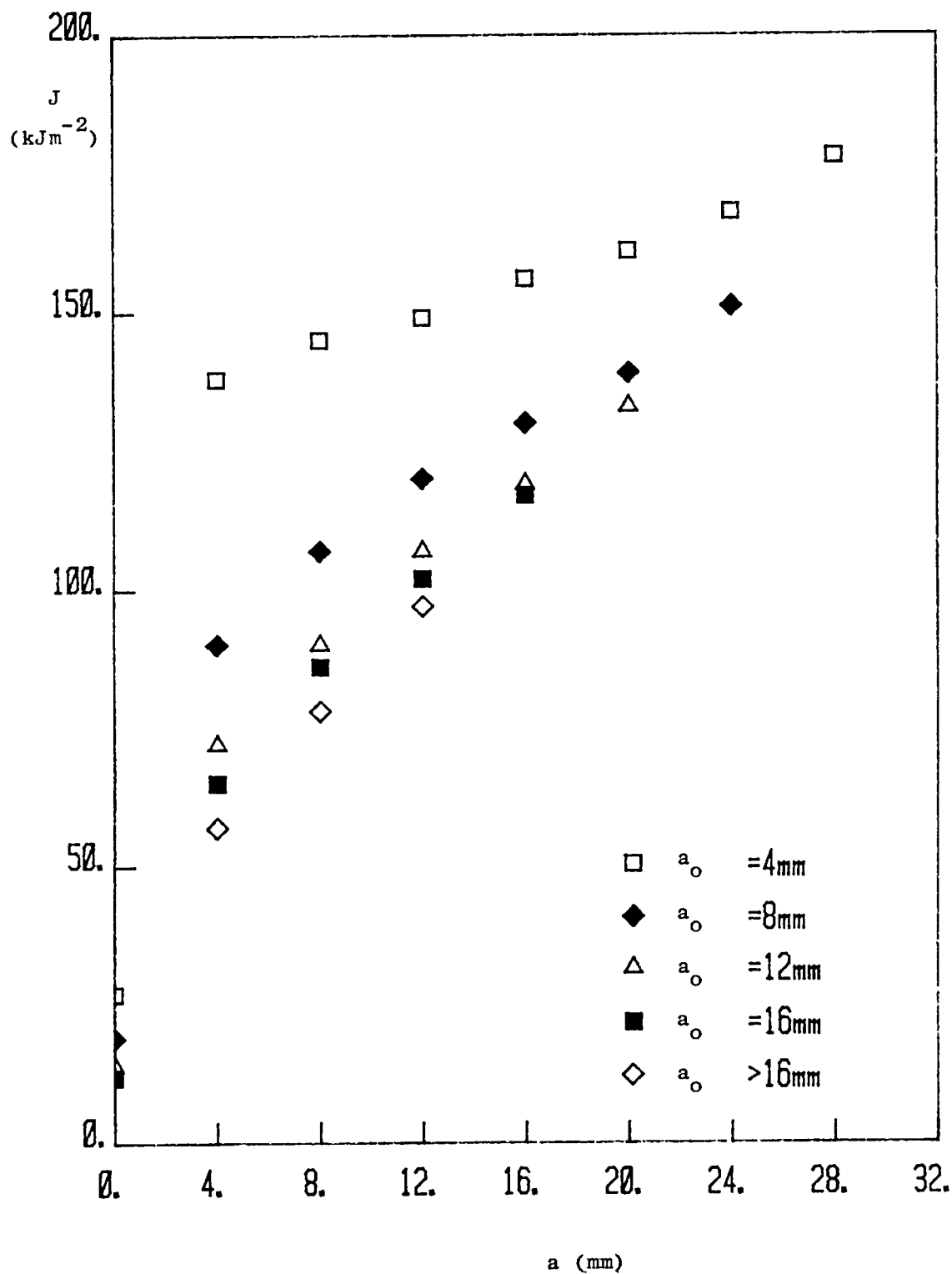


FIGURE 49: J-RESISTANCE CURVES FOR POLYCARBONATE



It has been noted by Garwood and Turner (1978) and reiterated by Turner (1979) that the crack flanks during well-established growth can be described by equation (4.22) and they have used measured values of dJ/da and crack opening displacement to derive values of the parameter α in the formula

$$\delta = \frac{\alpha r}{\sigma_y} \frac{dJ}{da} \quad (5.8)$$

which is meant to reflect the constraint at the crack tip and is, therefore, not necessarily constant. They have performed this exercise both for side-grooved flat fracture and full thickness and found the value of α to vary between 0.2 and 6.0.

When this approach is applied to the propagation of cracks in polyethylene and polycarbonate then several conclusions may be drawn. It has been shown that during the steady propagation of cracks, the crack tip shape is also steady and this coupled with a constant yield stress means that equation (5.8) demands

$$\alpha \frac{dJ}{da} = \text{constant} \quad (5.9)$$

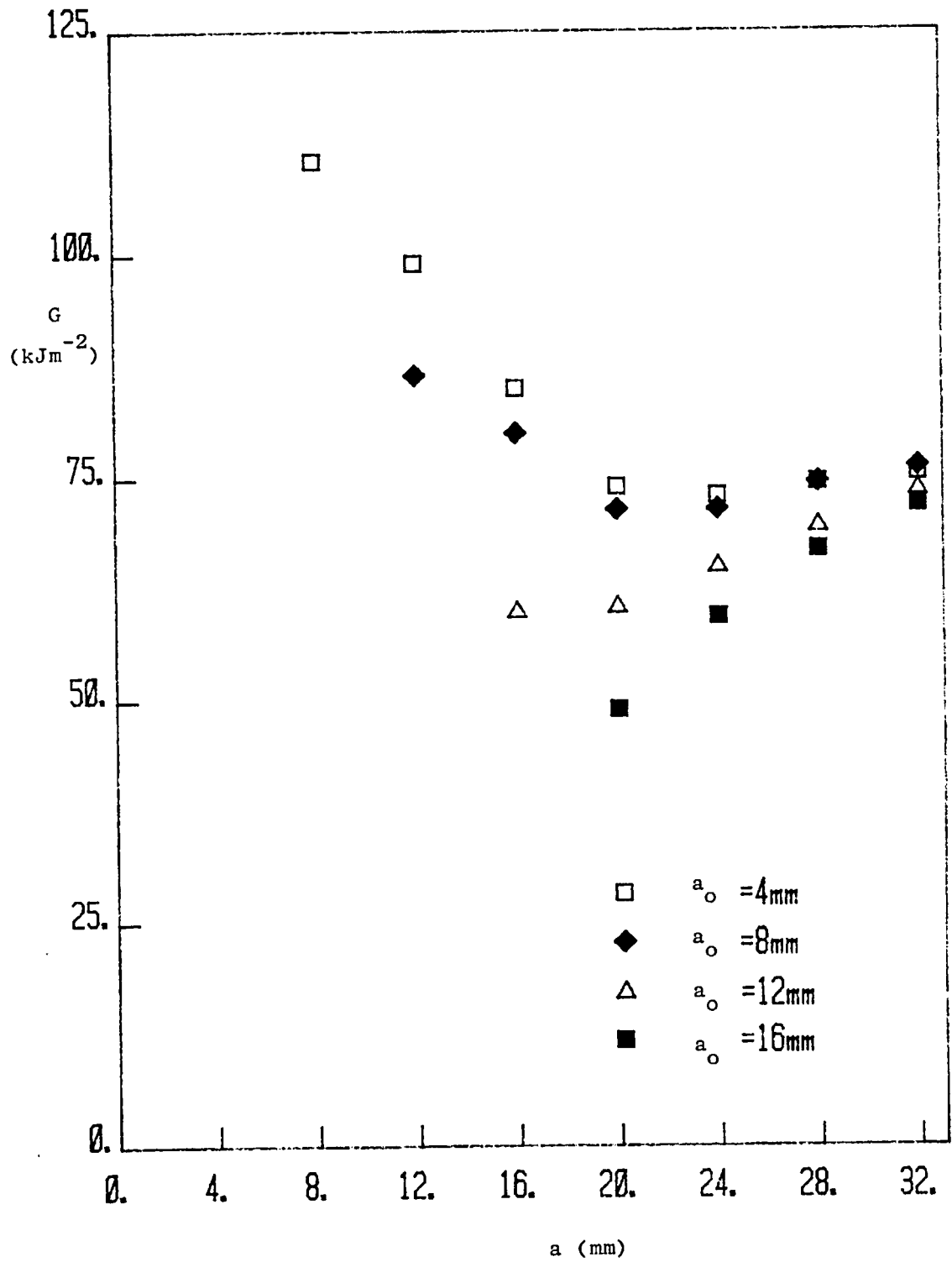
Now, α is inversely proportional to the constraint factor and thus high constraint means low α . Therefore, short initial cracks (low α) give higher values of dJ/da than do long cracks (high α). This is a qualitative explanation of the J-resistance curves. Furthermore, it was noticed that for fully developed cracks the value of dJ/da was a unique function of crack length irrespective of the initial crack length for both polyethylene and polycarbonate. This function is different for the two materials because of their differing crack tip deformation modes.

In polyethylene, dJ/da continues to decrease with increasing crack length indicating a continuing decrease in constraint with deformation, whereas in polycarbonate, dJ/da becomes constant once the plastic zone is fully developed. Comparison of the magnitudes of $\frac{1}{\sigma_y} \frac{dJ}{da}$ for the two materials is also indicative of the nature of deformation taking place. For polyethylene the values vary between 1 and 2, while for polycarbonate the value is roughly 0.05. Thus for a value of α of 6 the equivalent flank angles as derived from (5.8) are 75° for polyethylene and 10° for polycarbonate which agrees reasonably with observation. Of course, for polyethylene the flanks are curved and the model is not quite so appropriate.

(iii) Geometrical Variations

As with any proposed fracture characterising parameter, it is essential to investigate its variability when the geometry of the pre-cracked body is changed. A direct assessment of the effect of specimen thickness was obtained by comparison of the previous results with those for specimens of 1 mm thickness and 50 mm width. Quite simply, with reference to equation (5.3) it should be expected that when the thickness B is reduced by a factor of 3, the loads P should be reduced by the same factor, and the compliance and compliance gradient should increase by a factor of 3. The net effect may be expected to be zero and a constancy of G with thickness might be predicted. Figure 50 shows that this is indeed the case although the steady state values of G are roughly 7% lower. The behavioural trends of figure 45 are not so clearly portrayed by the data for the 1 mm thickness which exhibit a slight increase in G as the back face of the specimen is approached. This may be a consequence of the bending which takes place as the crack propagates through the

FIGURE 50: DEPENDENCE OF G ON CRACK GROWTH
POLYCARBONATE - 1mm THICK SHEET



specimen. Such bending would be liable to increase crack flank angle and therefore blunt the crack and, in accordance with the linearity of G and crack tip radius, would result in an increased G .

Prediction of the effects of changing specimen width is not simple. Coupled with this fact was the need to vary clamping techniques in order to accommodate the variation in width. Three additional specimen widths (36, 80 and 150 mm) were tested and each was loaded in a different set of clamps. The 36 mm width was at least gripped in the same manner as the previously discussed data i.e. wedge grips tightened prior to loading. Figures 51 and 52 show loading curves for a series of initial crack lengths and the calculated curves of G versus crack growth respectively. There are the usual patterns attributable to initial crack length but an average value of G taken during growth would be roughly 50 kJm^{-2} . This may be due to the plastic zone reaching the back face of the specimen and fully yielding the ligament; observed values of plastic zone size during propagation are of the order of 15 mm. Indeed, once the crack progresses beyond the mid-section the value of G rises to about 60 kJm^{-2} .

A set of specimens of width 80mm were tested and the results are displayed in figure 53 in the form of G versus crack length. As with the 1 mm thick data for $W = 36 \text{ mm}$ and $W = 50 \text{ mm}$ there is a 'saddle' value of G at the mid-section after which G then increases. It is quite conceivable that the thicker 3 mm specimen is less influenced by bending and responds in a more controlled manner. Nevertheless, the range of plateau values for G are $60 - 75 \text{ kJm}^{-2}$ which is in keeping with most of the data.

Data for $W = 150 \text{ mm}$ have been omitted on the grounds that the loading system was inadequate to determine an accurate compliance and consequently a precise value for G . This is shown by figure 54 which

FIGURE 51: POLYCARBONATE - LOAD DEFLECTION CURVES
1mm THICK ; 36mm WIDE SHEET

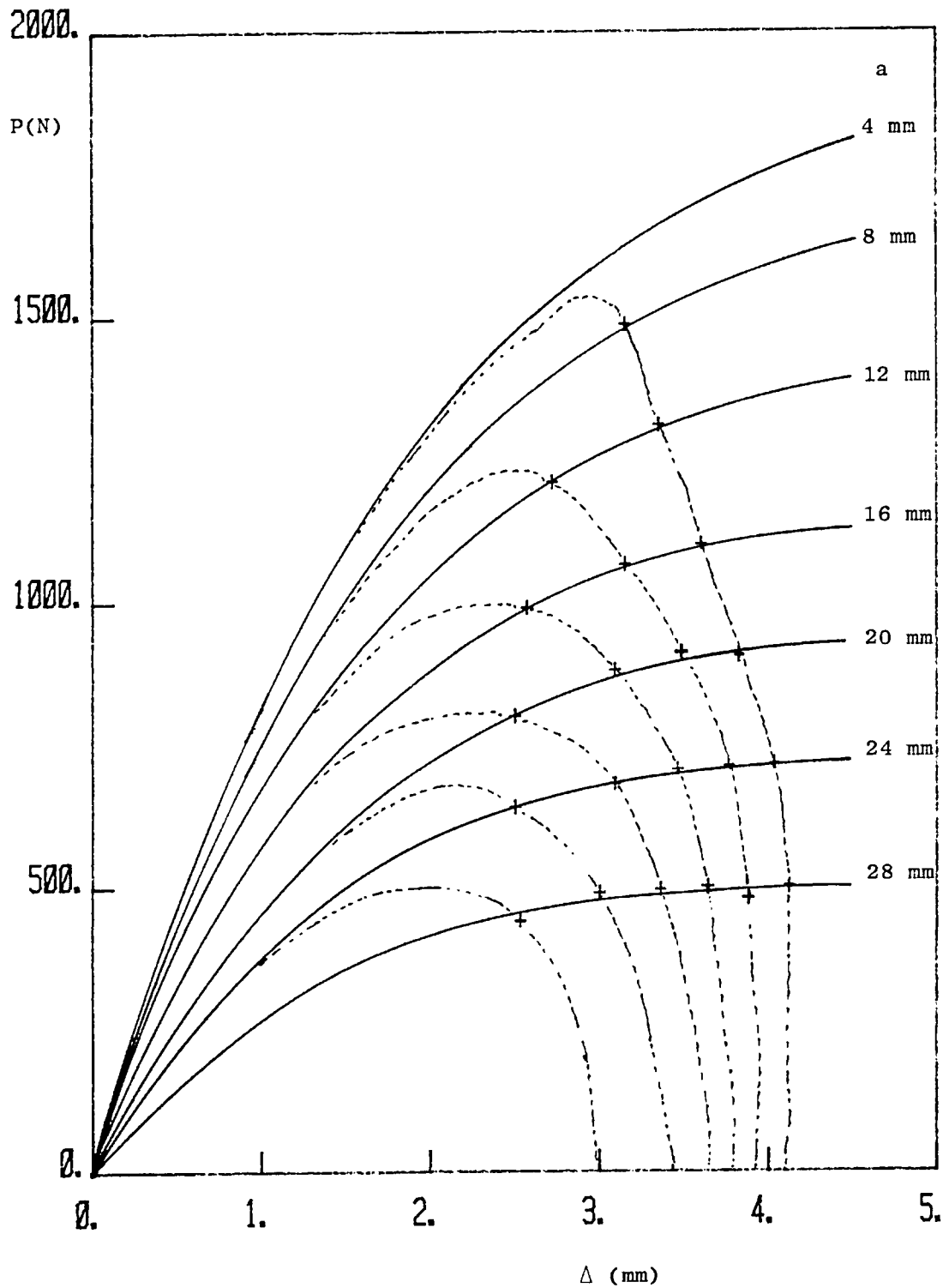


FIGURE 52: POLYCARBONATE - G VERSUS CRACK GROWTH
1mm THICK ; 36mm WIDE SHEET

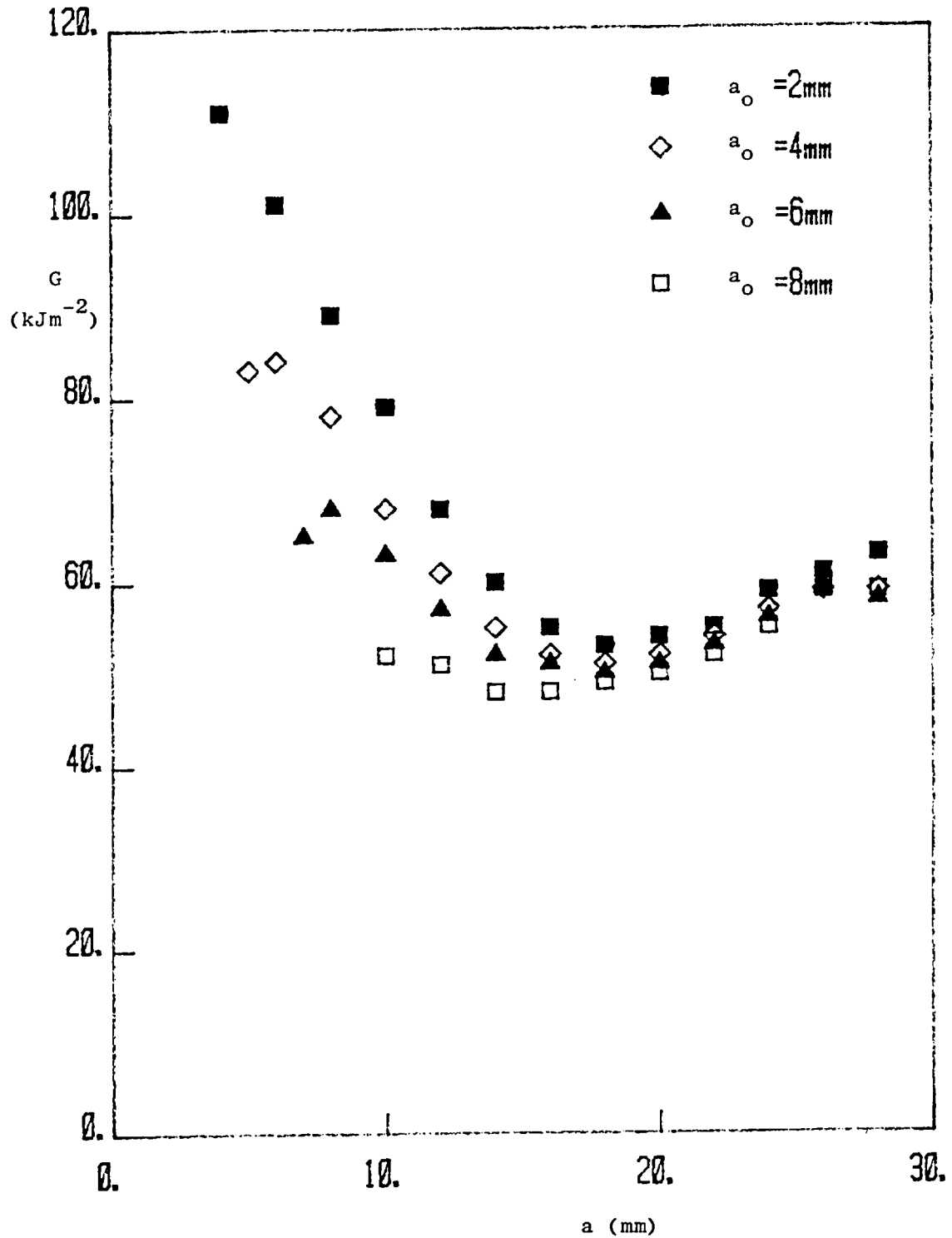


FIGURE 53: POLYCARBONATE - G VERSUS CRACK GROWTH
1mm THICK ; 80mm WIDE SHEET

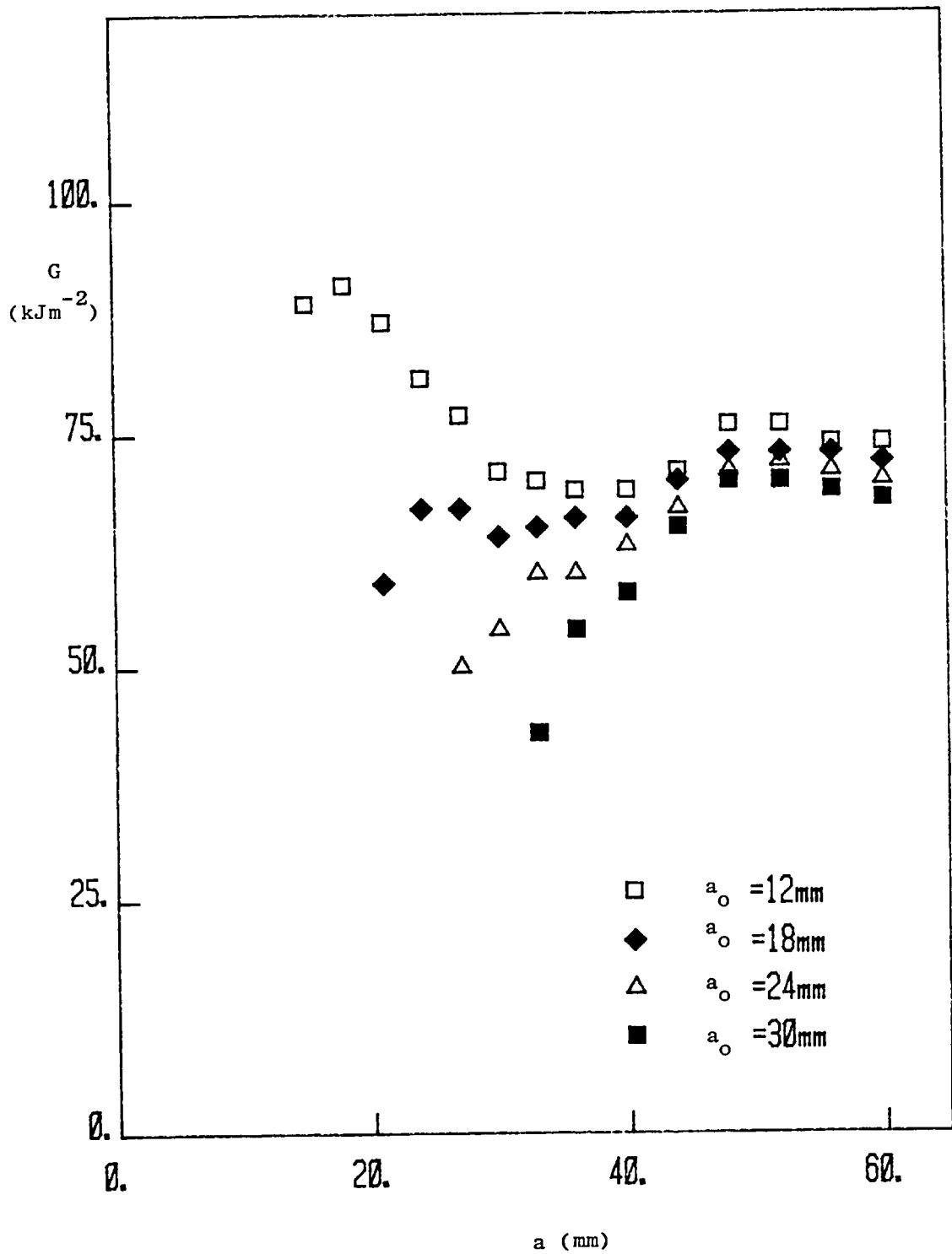
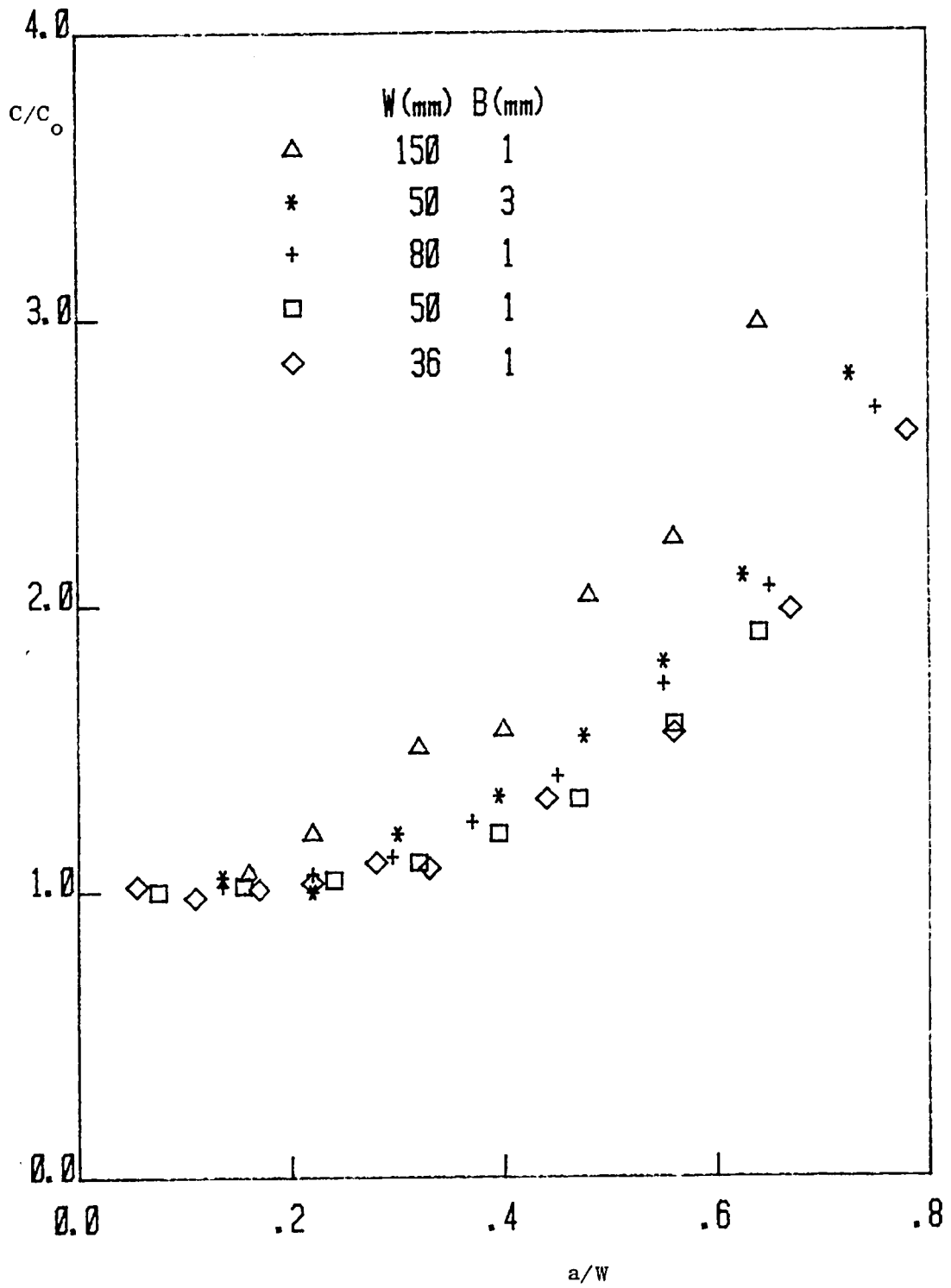


FIGURE 54: POLYCARBONATE - COMPLIANCE CURVES



summarises the compliance data for all SEN geometries examined.

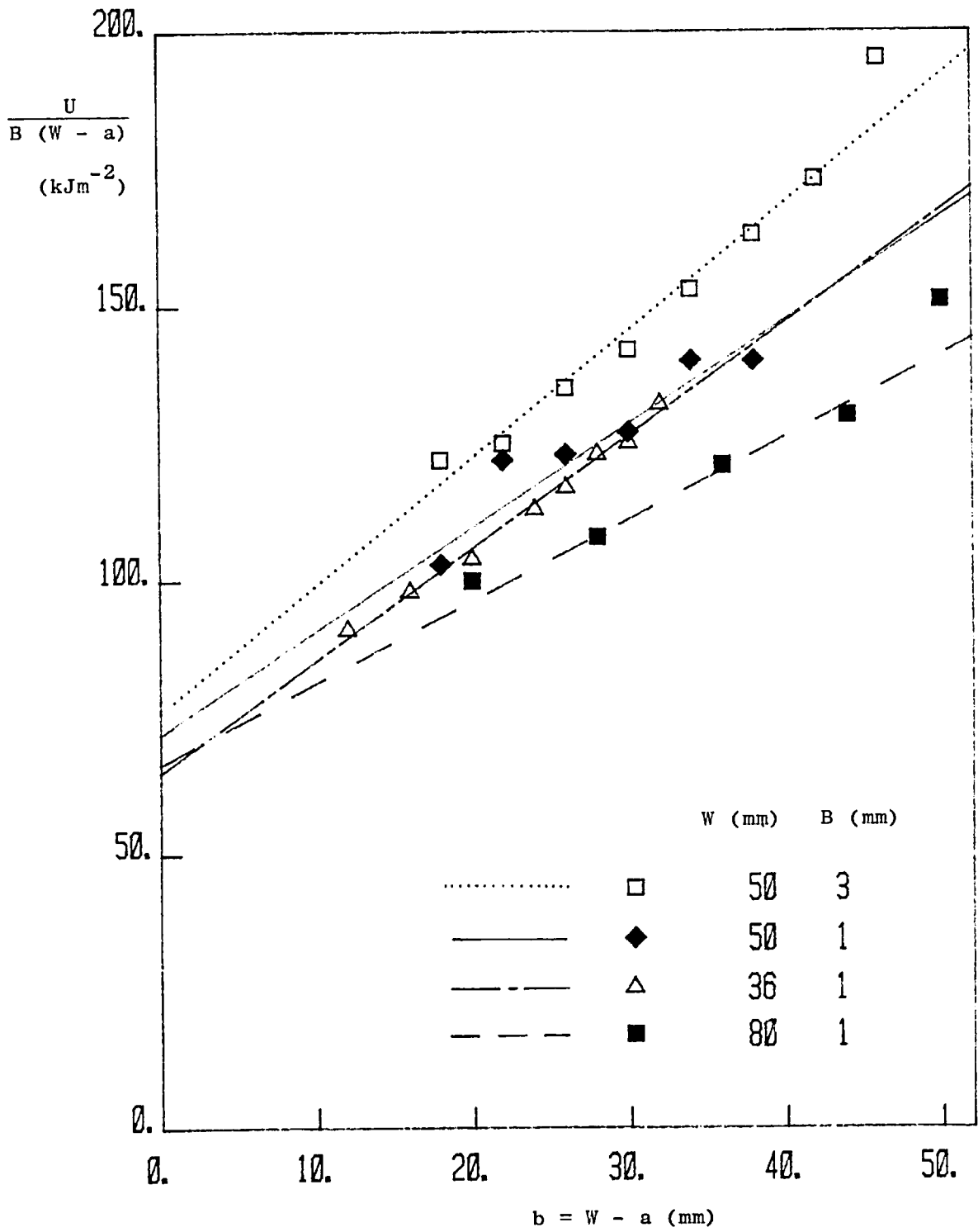
Whereas the $W = 36$ mm, $W = 50$ mm and $W = 80$ mm compliances lie in roughly the same area of the diagram, the $W = 150$ mm compliance data, when normalised, is far higher. This is not surprising since any slipping will produce lower loads and higher deflections both tending to increase measured compliance.

The geometrical variations in G , although less than 20%, are highlighted by an alternative analysis. Following the work of Cotterell and Reddel (1977) reported in the previous chapter, equation (4.28) is recalled whereby a plot of total specific energy to failure U/B ($W - a$) versus ligament length ($W - a$) gives a straight line with intercept w_e and gradient w_p . w_e is the specific essential work performed in the neighbourhood of the crack and w_p is the specific non-essential work (energy per unit volume) performed remote from the crack. Measuring the total energy U beneath each loading curve to failure using a planimeter, figure 55 illustrates the results for four different geometries. Values of w_e , which may be identified with the average propagation value of G , are not only very similar but reflect the slight geometrical variations detected previously.

(e) Conclusions

In polycarbonate, the strain energy release rate local to the crack tip defined by the currently available elastic energy is reasonably constant whatever geometry is examined. During ductile tearing, however, change in constraint of the crack tip can lead to a more blunted crack and a subsequent rise in energy release rate occurs. This is in accordance with the results obtained for polyethylene and an approximately linear relationship between energy rate and crack tip radius is again observed.

FIGURE 55: SPECIFIC ENERGY TO FRACTURE VERSUS
LIGAMENT LENGTH FOR POLYCARBONATE



Using a value of G_c of 2.8 kJm^{-2} from Parvin and Williams (1975), and the slope of the straight line in figure 46, equation (3.26) gives a value of c of $0.3 \text{ }\mu\text{m}$.

Plati (1975) has performed blunt notch impact tests on polycarbonate and the deviation from linearity occurs at a radius of between $100 \text{ }\mu\text{m}$ and $400 \text{ }\mu\text{m}$ indicating from equation (3.25) that c takes the value of between $25 \text{ }\mu\text{m}$ and $100 \text{ }\mu\text{m}$. A crazing failure is proposed in this case and these critical distances are in good agreement with craze lengths calculable from Fraser and Ward (1978). The value of $0.3 \text{ }\mu\text{m}$ presented here is indicative of failure dominated by shear yielding and tearing.

At first sight the propagation value of 70 kJm^{-2} for strain energy release rate might seem very large. However, other workers have obtained geometry dependent values of G , notably Fraser and Ward (1977), Mai (1973) and Kambour and Miller (1976), which are of a similar magnitude. Recent work has been done by Andrews and Barnes (1982) in an attempt to apply generalised fracture mechanics to the ductile failure in polycarbonate. The quoted value for τ_0 (which is equivalent to $\frac{1}{2} G_c$) is 13.3 kJm^{-2} and agreement is claimed between this and the work of Fraser and Ward (1977). Plane strain values for G_c in this latter work do not exceed 5 kJm^{-2} and are similar to those derived from the K_c quoted in Parvin and Williams (1975). It is hard to see how a value of 26.6 kJm^{-2} can be regarded as a good estimate of G_c .

The concurrent G and J analyses have proved useful in substantiating the crack growth models of figure 40 and equation (5.8). However, further photographic evidence concentrating specifically on crack propagation would be valuable in explaining the behaviour of G from initiation to propagation for various initial crack lengths. Whereas there may remain some doubt as to the geometry independence of G during

growth, it is the author's opinion that the relationship between G and crack tip radius is less disputable. Just as in LEFM sufficient crack tip constraint is necessary to ensure a plane strain fracture toughness, so the propagation value of G is likely to depend on the crack tip constraint through its intrinsic relationship with crack tip radius.

CHAPTER SIX

THERMALLY INDUCED DUCTILITY

IN HIGH RATE FRACTURE

(a) Introduction

Emphasis throughout this work has been placed on the important role of crack tip deformation in determining fracture. Accurate modelling of the situation is essential in predicting failure and some success has been achieved in isothermal cases, both brittle and ductile; that is, when the component or test-piece is at a uniform temperature. Polymers are rate dependent by nature and this factor, coupled with their low conductivity, leads to accumulation of heat wherever work rates on the body under examination are high. Such is the case at the tips of cracks in viscoelastic media undergoing fast fracture, impact loading or fatigue. Crack tip heating will undoubtedly alter fracture properties and it is this problem to which attention is now turned.

Temperature elevations have been detected at the tips of fast moving cracks in polystyrene and PMMA by Fuller, Fox and Field (1975), in steel by Weichert and Schonert (1969) and in glass by Weichert and Schonert (1978). The magnitude of such tip temperatures, which depend upon the crack velocity, the thermal constants of the material and the heat evolved, are sufficiently large in certain circumstances to plasticise the material at the crack tip and therefore increase crack resistance. Modelling this problem has been undertaken by various workers including Rice and Levy (1969), Marshall, Coutts and Williams (1974) and Weichert and Schonert (1978a).

Subjecting polymers to impact loading was originally thought to be

an ideal method of inflicting brittle failure and results gained from these tests might help determine a material's suitability in demanding practical applications. However, a test of this nature must be standardised or at least analysed accurately to remove spurious effects. Crack tip heating at high impact rates is just one of these effects and has been rationalised fully by Williams and co-workers at Imperial College. Not only has a successful model been presented but results at impact rates have been correlated with those from slow rate tests in which the crack tip region is heated independently during specimen loading.

(b) Impact Testing of Polymers

As a result of the discrepancies between the values quoted for the impact strengths of many materials, a great deal of work has been done by a succession of workers at Imperial College using a pendulum impact testing machine as shown in figure 56. Specimens of various geometries and notch depths are tested in either the Charpy or Izod mode as illustrated in figure 57. In each test, the loss in energy of the pendulum is calculated from measurement of the maximum angle of swing after fracturing the specimen. This energy comprises the fracture energy and the kinetic energy imparted to the broken specimen. Denoting these energies by W and k respectively, then the elastic strain energy release rate at fracture, G_c , is given by equation (4.6) with U_{e1} replaced by W , that is:

$$G_c = \frac{W}{BC} \frac{dC}{da} \quad (6.1)$$

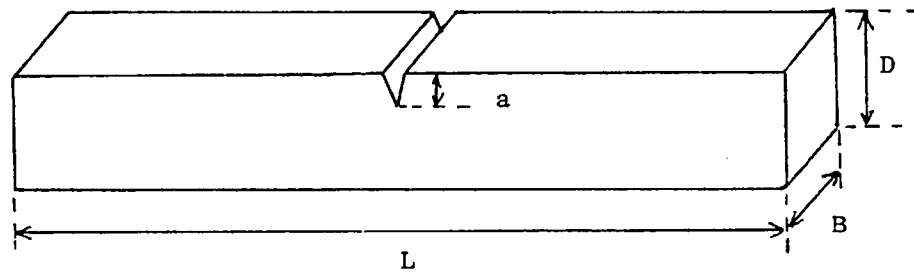
If U is the measured energy and ϕ is a calibration factor given by:

$$\phi = C \frac{dC}{d\left(\frac{a}{D}\right)} \quad (6.2)$$

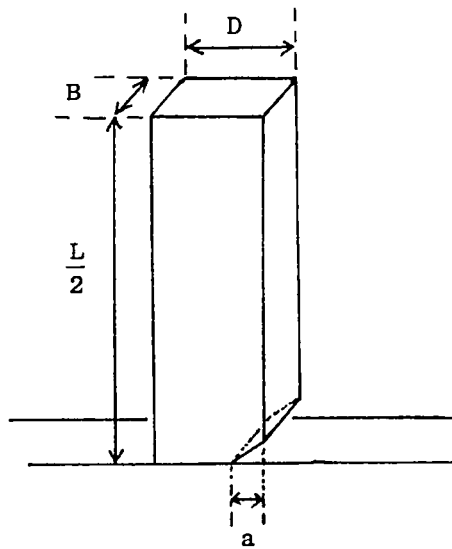
FIGURE 56: THE PENDULUM IMPACT TESTING MACHINE



FIGURE 57: IMPACT TEST - SPECIMEN GEOMETRIES



(a) Charpy



(b) Izod

then we have:

$$G_c = \frac{U - k}{BD\phi} \quad (6.3)$$

and, therefore, finally:

$$U = G_c \cdot BD\phi + k \quad (6.4)$$

In order to determine the fracture toughness, G_c , for a particular material a batch of specimens of different notch depths is tested, the energies, U , recorded and calibration factors, ϕ , determined from tables. Plati and Williams (1975) have documented the calculation of these factors for both Charpy and Izod modes enabling a graph to be drawn of U against $BD\phi$; G_c is the gradient of such a plot and k is the positive intercept on the energy axis.

There is great uncertainty as to how correction should be made for kinetic energy and three approaches have been tried.

(i) A theoretical estimate in terms of masses, velocities and the coefficient of restitution between tup and specimen has been derived (Plati and Williams (1975)).

(ii) An experimental value obtained by hitting a specimen off plain supports may be used.

(iii) The points corresponding to low $BD\phi$ values may be extrapolated linearly back to the energy axis.

In general, estimate (i) is lower than the true kinetic energy, estimate (ii) is higher and estimate (iii) by far a better approximation provided the lowest recorded value of total energy, U , is at least twice the value at the intercept.

Another technique of analysis was employed in Plati and Williams (1975) to calculate a valid fracture parameter for cases with a high degree of plasticity. This was an attempt to use J theory as had previously been done by Brown (1973) working on ABS. However, the analysis gave no dependence of fracture energy U upon crack length other than linear dependence with ligament area. This is consistent with a constant η factor of 2 and is clearly not the case since the maximum crack length to specimen depth ratio was 0.5. Although reasonable agreement is found for HIPS, there is still significant curvature in the data and application of the method to other ductile materials (Plati (1975)) has been generally unsuccessful. Possible improvement might be made by an analysis including the correct overall η factors as quoted by Turner (1979, 1980).

Furthering the early work on impact testing, Plati and Williams (1975a) have investigated the temperature dependence of low and high strength materials, Birch and Williams (1977) have discussed rate effects in terms of loss processes for PTFE and a medium density polyethylene and more recently the work of Hodgkinson and Williams (1979) has rationalised rate effects in a much more coherent fashion.

In Hodgkinson and Williams (1979) no mechanism is offered to explain the toughness elevation with rate, except to say that the ductile zone of material at the crack tip absorbs more energy. Postulating this zone to be of a certain width, then a critical loading time may be calculated below which the zone at the crack tip is regarded as being adiabatic, the material in this region reaching the softening temperature, T_s .

In the examination of this variation with rate, two parameters are of use:-

(i) A simple estimate of the strain rate; that of the maximum strain rate in an unnotched specimen is made using the velocity of the striker, V , the specimen depth, D , and the span, L , the strain rate being given by:

$$\dot{\epsilon} = 6 \left(\frac{V}{D} \right) \left(\frac{D}{L} \right)^2 \quad (6.5)$$

(ii) More precisely, the loading time of each specimen may be written:

$$t = \Delta / V \quad (6.6)$$

where Δ is the specimen deflection at fracture given by:

$$\Delta^2 = 2BDG_c \phi^2 \frac{dC}{dx} \quad (6.7)$$

where $x = a/D$, enabling the finite width correction factor, Y^2 , to be introduced via;

$$\frac{dC}{dx} = \frac{9L^2}{2BED^2} Y^2 x \quad (6.8)$$

and, therefore, finally:

$$t = \frac{1}{V} \left(\frac{G_c D}{E} \right)^{\frac{1}{2}} \cdot \frac{3L}{D} \cdot \phi \cdot (Y^2 x)^{\frac{1}{2}} \quad (6.9)$$

Williams and Hodgkinson (1981) explain the increasing toughness with rate in terms of crack tip blunting within the thermally softened zone. No solution is, at present, available to the problem of calculating stresses and strains near cracks in elastic - plastic materials when geometry changes are included. Numerical solutions have been obtained to a limited class of problem (Chapter four, section (c)) the outcome being that for low work-hardening materials the crack tip becomes flat-nosed, whereas for moderate amounts of strain-hardening a tip radius is evident. This is further confirmed by the blunting curves of chapter five.

Crack tip heating produces material of lower modulus at the tip and the ensuing fracture process may be compared with that taking place in a strain-hardening material. The zones of heating and high stressing are coincident, and thus the temperature dependence of material modulus may be regarded as providing strain-hardening behaviour of decreasing tangent modulus. Such a picture, in the light of the results of chapter five strengthens the argument for using the elastic blunting model to predict fracture, and the analysis may be outlined as follows. The elevation in toughness of a material containing a crack of radius, ρ , above the sharp crack toughness, G_c , is given by equation (3.26) as:

$$\frac{G_B}{G_c} = \frac{1}{2} + \frac{\rho}{8c} \quad \text{for } \rho > 4c \quad (6.10)$$

$$\frac{G_B}{G_c} = 1 \quad \text{for } \rho \leq 4c$$

where G_B is the elevated toughness and c is the critical distance embodied in the fracture criterion. Recalling equation (3.13) we see that:

$$K_c^2 = \sigma_c^2 2 \pi c \quad (6.11)$$

For moderate strain-hardening, the crack tip radius is of the same order of magnitude as the COD, and if we assume that the factor of proportionality is approximately one half then:

$$\rho \sim \frac{1}{2} \delta = \frac{2 K_c^2}{\pi E \sigma_y} = \frac{2 e_y}{\pi} \frac{K_c^2}{\sigma_y^2} \quad (6.12)$$

and combining equation (6.11) and (6.12) leads to

$$\frac{\rho}{c} = 4 e_y \left(\frac{\sigma_c}{\sigma_y} \right)^2 \quad (6.13)$$

It should be noted that σ_c is the fracture stress in the highly constrained zone near the crack tip and σ_y is the local value of yield stress. Under the assumptions that, firstly, the fracture stress is insensitive to temperature, and that the yield stress has a linear dependence of the form:

$$\frac{\sigma_y}{\sigma_o} = 1 - \frac{\Delta T}{\Delta T_o} \quad (6.14)$$

where σ_y has value σ_o at temperature T_o , $\Delta T = T - T_o$, $\Delta T_o = T_s - T_o$ and the value of σ_y tends to zero as T tends to T_s , then the ratio ρ/c will increase with temperature, indicating crack blunting.

Another crucial point of note is that in equation (6.12) the sharp crack initiation condition, $K_I = K_c$, is employed. This is used to estimate

the degree of blunting from which the elevated toughness results. Kinloch and Williams (1980) have discussed two types of blunting; uncoupled, in which the estimate is as above, and coupled, in which the estimate is given by substituting K_B , the elevated toughness, for K_c in equation (6.12). It is generally thought that for thermal blunting, in which the effective crack tip yield stress decreases during the loading of the specimen and the gradual dissipation of crack heat, the uncoupled behaviour is more appropriate. Changes in yield stress due to processing and curing conditions are expected to take immediate effect and coupled blunting is then applicable.

Combining equations (6.10), (6.13) and (6.14) we have:

$$\frac{G_B}{G_c} = \frac{1}{2} + \frac{1}{2} e_y \left(\frac{\sigma_c}{\sigma_o} \right)^2 \left(1 - \frac{\Delta T}{\Delta T_o} \right)^{-2} \quad (6.15)$$

which is valid provided $e_y \sigma_c^2 / \sigma_y^2 > 1$. Introducing the parameter N defined by

$$N = \left(\frac{e_y}{2} \right)^{\frac{1}{2}} \cdot \frac{\sigma_c}{\sigma_o} \quad (6.16)$$

equation (6.15) may be expressed more conveniently by:

$$\left(\frac{G_B}{G_c} - \frac{1}{2} \right)^{-\frac{1}{2}} = \frac{1}{N} \left(1 - \frac{\Delta T}{\Delta T_o} \right) \quad (6.17)$$

In order to relate the local temperature rise to the loading time of the test, Williams and Hodgkinson (1981) examine the time dependence of temperature in a strip zone with fixed rate of energy input.

For a narrow zone, the temperature rise due to the dissipation of energy, G_c , in time, t , is:

$$\Delta T = \frac{G_c}{(\pi \rho C k)^{\frac{1}{2}}} t^{-\frac{1}{2}} \quad (6.18)$$

where ρ is the density, C the specific heat and k the conductivity.

Postulating a minimum loading time, t_1 , for which $\Delta T = \Delta T_o$ then:

$$\frac{\Delta T}{\Delta T_o} = \left(\frac{t_1}{t} \right)^{\frac{1}{2}} \quad (6.19)$$

and finally :

$$\left(\frac{G_B}{G_c} - \frac{1}{2} \right)^{-\frac{1}{2}} = \frac{1}{N} \left(1 - \frac{t_1}{t} \right)^{\frac{1}{2}} \quad (6.20)$$

For $t \gg t_1$, equation (6.20) may be expanded to reveal:

$$\frac{G_B}{G_c} \sim \frac{1}{2} + N^2 + 2N^2 \left(\frac{t_1}{t} \right)^{\frac{1}{2}} \quad (6.21)$$

which illustrates the $t^{-\frac{1}{2}}$ dependence observed earlier (Williams and Birch (1977)).

Calculating the loading times from equation (6.9) Williams and Hodgkinson (1981) have plotted data for several materials in accordance with equation (6.20) and obtained linearity at several temperatures for PMMA, HDPE and PTFE. The slope of such plots of $\left(\frac{G_B}{G_c} - \frac{1}{2} \right)^{-\frac{1}{2}}$ against $t^{-\frac{1}{2}}$ is

$$\left(\frac{t_1}{N} \right)^{\frac{1}{2}} = \left(\frac{2}{e y} \right)^{\frac{1}{2}} \frac{\sigma_o}{\sigma_c} \frac{G_c}{(\pi \rho C k)^{\frac{1}{2}}} \frac{1}{T_s - T_o} \quad (6.22)$$

combining equations (6.16) and (6.18). From (6.14), however, σ_o is linearly dependent upon $T_s - T_o$ and therefore (6.22) suggests that the slope obtained should be constant for different values of T_o ; this constancy is confirmed for HDPE and PTFE.

The modelling of high rate behaviour in this manner leads to predictions of limiting loading times and calculations of softening temperatures, critical stresses and critical distances. Values of these parameters for three materials have been tabulated in Williams and Hodgkinson (1981) and several comments are appropriate. Calculation of limiting loading times are entirely dependent on the choice of T_s and agreement for PMMA and PTFE is fair. Discrepancies arise in the case of HDPE and this is claimed quite rightly to be due to an inappropriate choice of softening temperature. Since N^{-1} is proportional to σ_o , equation (6.16), linearity with T_o is predicted for this quantity with extrapolation through the point $(T_s, 0)$. Good lines were obtained for HDPE and PTFE with extrapolated values of 347°C and 464°C respectively. While the difference between these and the "true" softening temperatures may be explained as variation in σ_c , the argument for a reduced softening temperature as proposed in Williams and Hodgkinson (1981) is incorrect. As is clearly stated, for $N^{-1} < \sqrt{2}$ there is blunting without thermal effects and, postulating a transition to the type of coupled blunting observed by Kinloch and Williams (1980), then N^{-1} might be expected to behave as $(G_B/G_c)^{-\frac{1}{2}}$ and thus decreases to zero as the softening temperature is approached. The dotted line variation of N^{-1} with T_o illustrated in figure 13 of Williams and Hodgkinson (1981) may therefore be indicative of the true variation and can be rationalised in terms of a transition to coupled blunting.

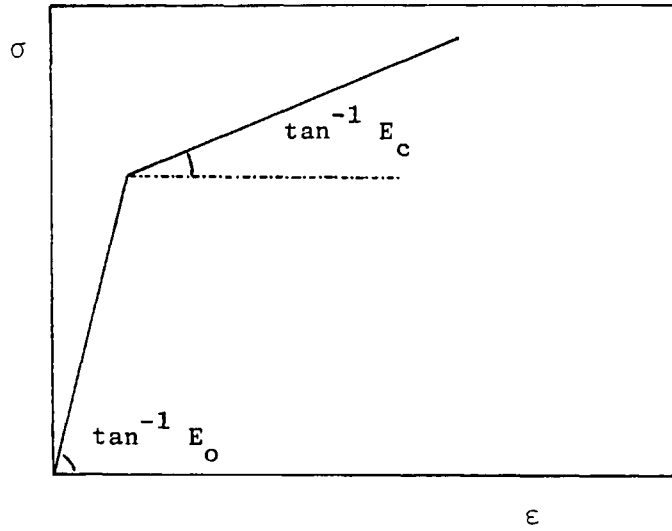
Discussion of the values of σ_c generated by such a model is

strictly only valid for $N^{-1} > \sqrt{2}$ and data for PMMA and PTFE come in this category. Critical stresses for PMMA have been quoted at values of 550 MNm^{-2} (Williams and Hodgkinson (1981)) and 1 GNm^{-2} (Clutton and Williams (1981)) and therefore constraint factors at the crack tip as derived from σ_c/σ_o vary from 4 to 10. Such figures are justified to a certain extent by the results of McMeeking as reviewed in chapter four. For PTFE using a modulus of 0.5 GNm^{-2} and yield stress of 13.0 MNm^{-2} at 20°C gives a critical stress of 59.0 MNm^{-2} . Arguments have been proposed in an attempt to rationalise physically such large stresses in terms of second order Van Der Waals forces (Williams (1980)) but it may be more realistic to say that only one of the two fracture parameters will be truly representative of behaviour and since the stress values are high then the distances revealed by the analysis are more indicative of the fracture process. As has been previously shown, fracture from a blunt notch in the absence of crazing leads to process zones which are an order of magnitude lower in size than craze zones. The blunting mechanism proposed is thus substantiated by the attainment of such low critical distance values as $3.0 \mu\text{m}$ (Williams and Hodgkinson (1981)) and $1.0 \mu\text{m}$ (Clutton and Williams (1981)) for PMMA and $45.0 \mu\text{m}$ for PTFE.

(c) Crack Tip Softening - An Alternative Approach

As stated in the previous section the softened zone present at the tip of a crack undergoing rapid deformation may be regarded as strain-hardening material of lower tangent modulus than the material in the bulk of the body. In such a way is the situation of a rapidly loaded viscoelastic body very similar to that of strain-hardening elasticity. Solution of the problem of fracture in power law hardening

materials has been broached by Rice and Rosengren (1968) and by Hutchinson (1968), the latter having also treated the case of piece-wise linear elasticity in both plane stress and plane strain. In Hutchinson (1968) for a bi-modular material, i.e. one whose stress-strain behaviour is as follows,



the dominant stress at the tip of a sharp crack in the material is enhanced by a factor of $(E_o/E_c)^{\frac{1}{2}}$. Thus, by analogy with the case of a softened crack tip zone of modulus E_c , we can derive the apparent toughness of the specimen in terms of the base toughnesses of the bulk material. The toughness of the material at the crack tip is enhanced by $(E_o/E_c)^{\frac{1}{2}}$ and, if the crack tip temperature is T_c then

$$K_B (T_o) = K_c (T_c) \cdot (E_o/E_c)^{\frac{1}{2}} \quad (6.23)$$

where $K_c (T_c)$ is the toughness of the material at the crack tip and $K_B (T_o)$ reflects the toughness of a specimen with crack tip temperature, T_c , but bulk temperature, T_o . Squaring both sides and dividing by

the crack tip modulus E_c gives

$$\frac{G_B(T_o)}{G_c(T_c)} = \frac{E_o}{E_c} \quad (6.24)$$

and using equation (5.14), or its modulus equivalent

$$G_B = G_c(T_c) \left(1 - \frac{\Delta T}{\Delta T_o} \right)^{-1} \quad (6.25)$$

where ΔT is now the temperature rise at the crack tip. It is immediately apparent that the elevation in toughness may be regarded as due to two independent effects : the toughening of the crack tip material due to local ductility and the reduction in severity of stress intensity at the tip due to decrease in local modulus. In order to investigate the validity of this approach, the dependence of base toughness G_c upon temperature must be determined.

(i) Fracture Toughness, G_c , of PMMA at Elevated Temperatures.

Impact tests were performed on PMMA specimens of span 72 mm, breadth 6 mm and depth 6 mm over a range of crack lengths. Each notch was machined to the required depth with a 60° sharp single point fly cutter of nominal tip radius 25 μ m. Batches of specimens were tested at each of the following temperatures - 20°C, 30°C, 40°C, 60°C, 80°C, 100°C, 115°C and the results, i.e. values of fracture energy and $BD\phi$, were treated in identical fashion for each temperature. That is, the points were plotted and two lines were drawn; a) one corresponding to the least squares regression line through the data and b) a second

line drawn through the lowest points in the data and denoted by the expression "minimum energy line". The justification for this second line is that for such a brittle material as PMMA, the scatter at ambient temperatures precludes both the accurate determination of G_c and the selection of a suitable energy intercept. A lower bound such as the "minimum energy line" provides possibly a true reflection of the material toughness which tends to the least squares line as the recorded energy becomes larger compared with the contribution of the kinetic term. This is readily apparent from figures 58 and 59 which show the data obtained at 40°C and 115°C respectively. There is a clear "minimum energy line" in all cases and figure 60 shows the variation of both least squares estimates and minimum energy estimates over the complete range of temperature. On the basis that the minimum energy gives the true behaviour of toughness with temperature, a smooth curve has been drawn through these points as indicated.

(ii) Discussion of Results

Several puzzling features are presented by the results, the most prominent of which is the lack of a sharp increase in toughness as the glass transition temperature is approached. The only possible explanation of this is that the rate of loading is sufficient to depress the effective test temperature by roughly 20°C. As will be shown subsequently this is entirely feasible. Testing at higher temperatures than 115°C was invalidated by warping of the specimens and so the highest test temperature reached was effectively nearer 90°C than the glass transition temperature of PMMA (110°C). Fractured specimens at 100°C and 115°C had a similar fracture surface to those at lower temperatures - a mirror smooth surface except for a small rough initiation region at the notch.

FIGURE 58: CHARPY IMPACT DATA FOR PMMA AT 40°C

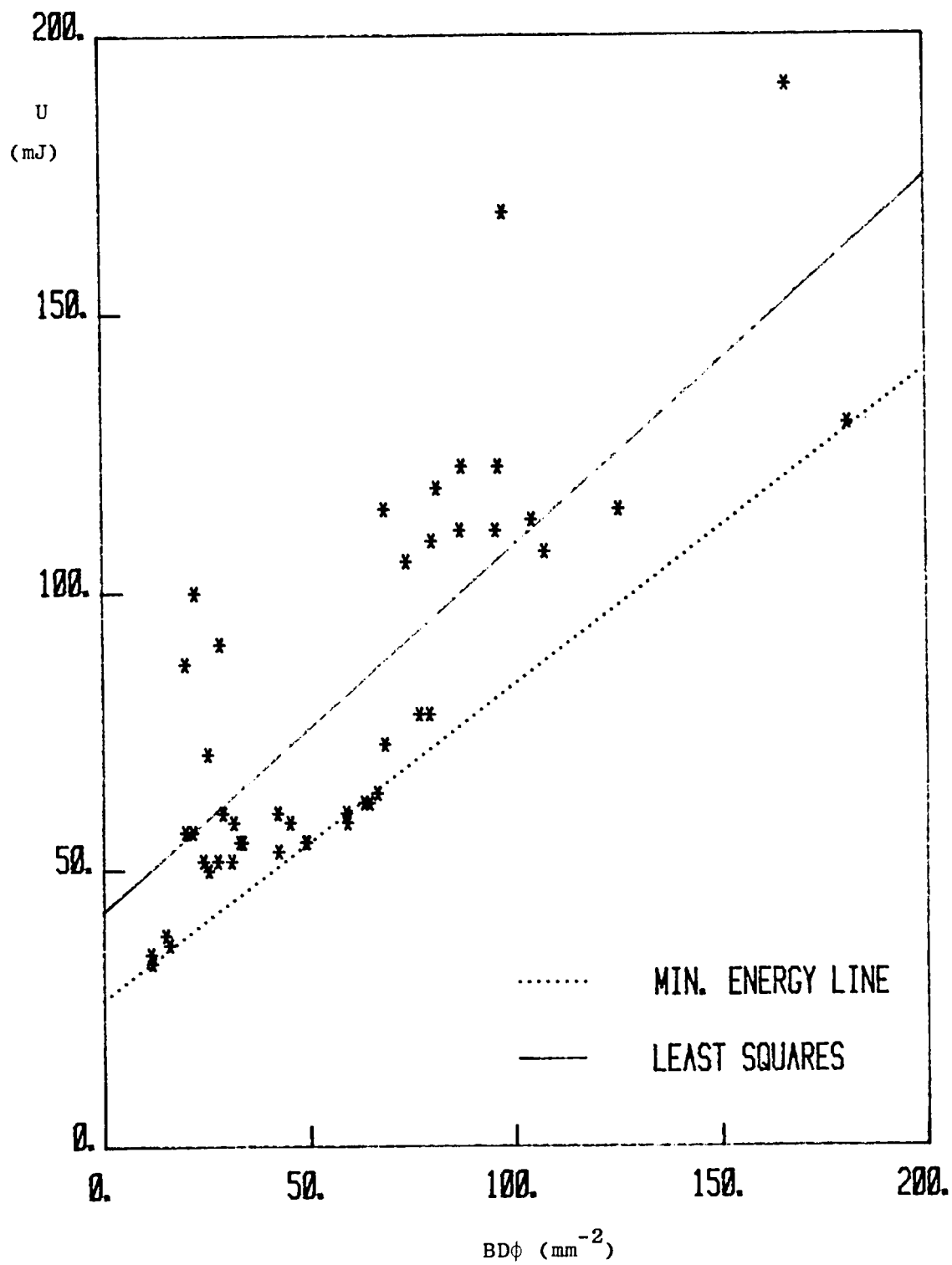


FIGURE 59: CHARPY IMPACT DATA FOR PMMA AT 115°C

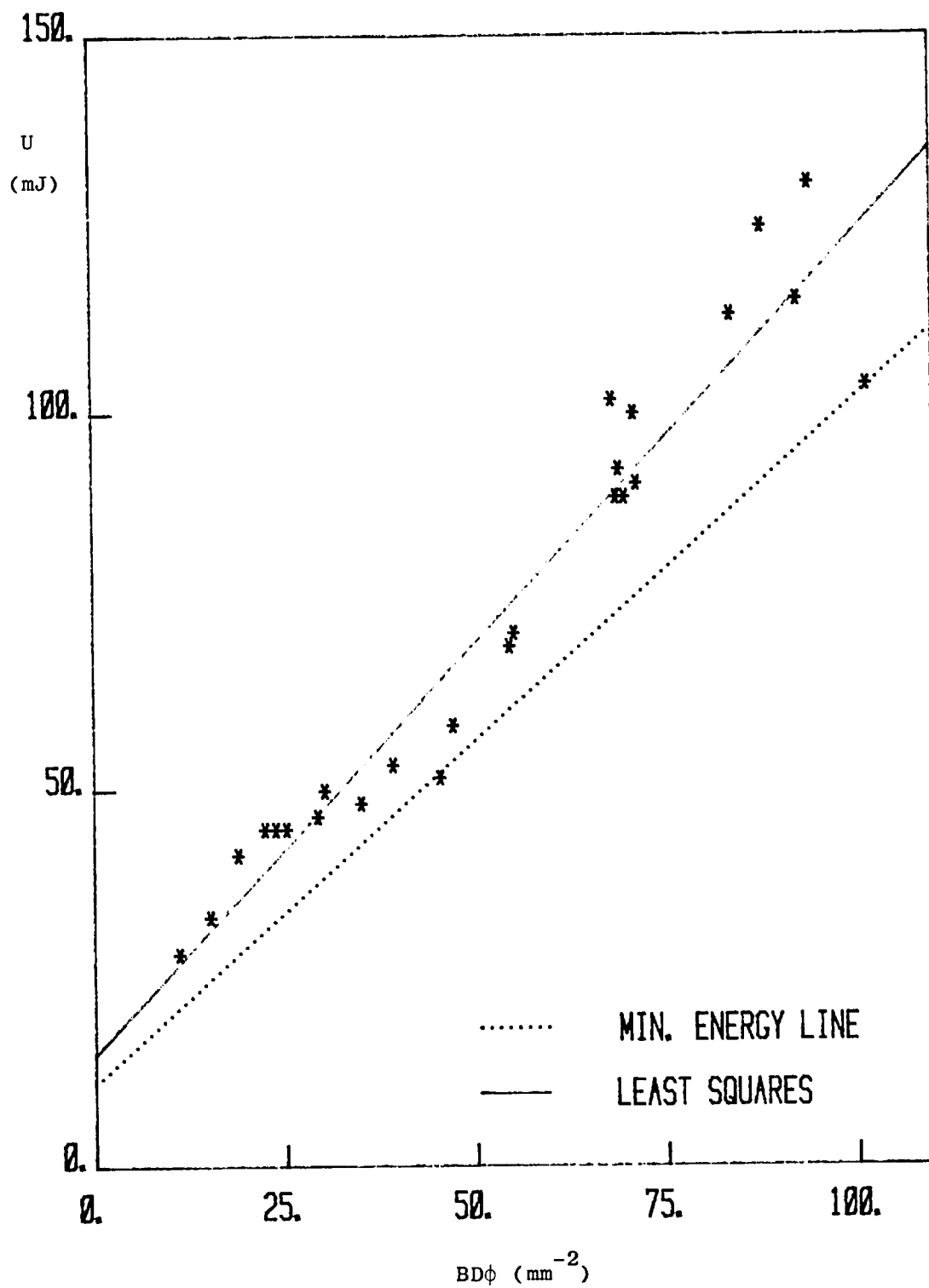
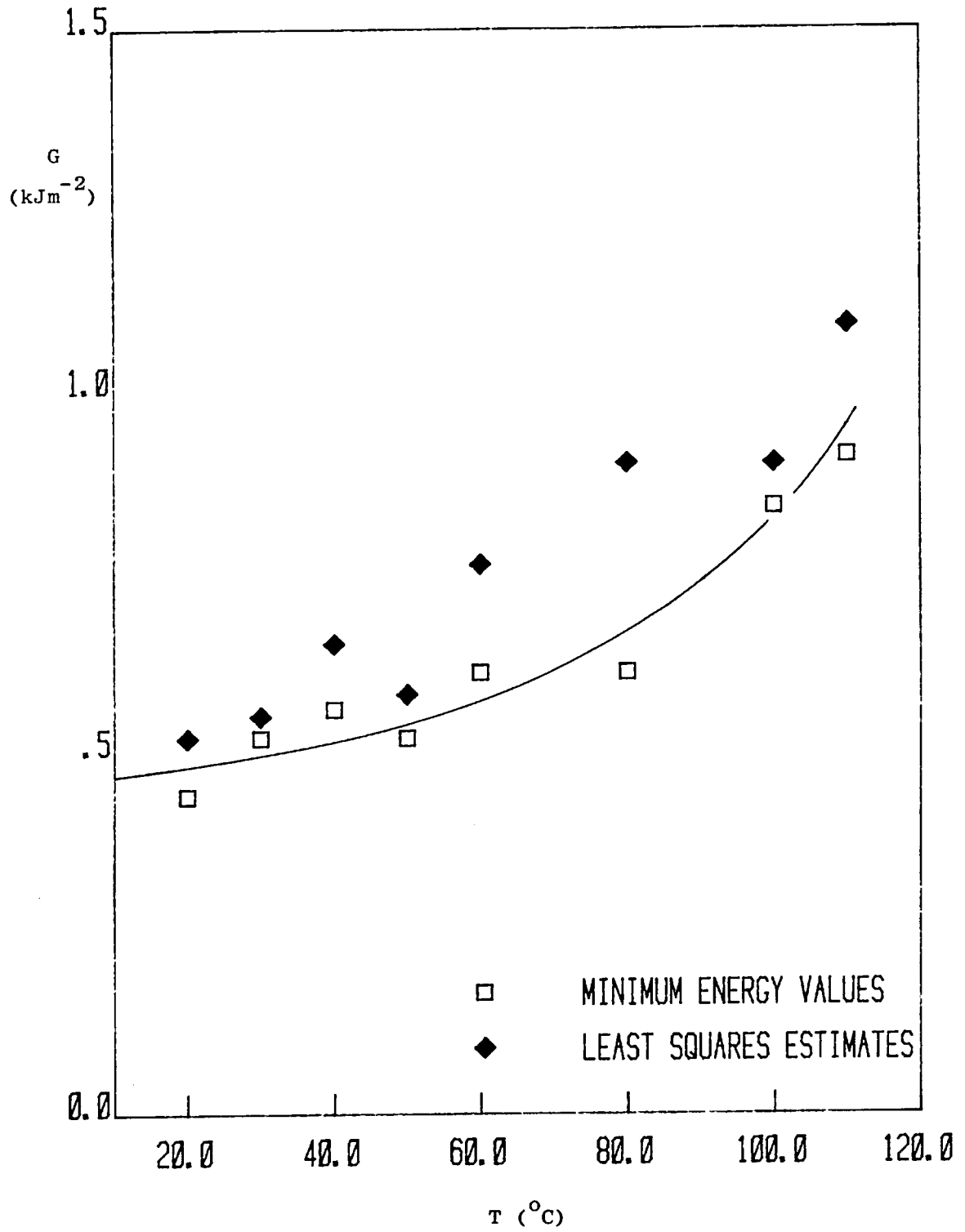


FIGURE 60: TEMPERATURE DEPENDENCE OF G FOR PMMA



Using the data of figure 60 in conjunction with equation (6.25), the dependence of toughness G_B upon crack tip temperature may be derived. Figure 61 illustrates comparison between equation (6.17) of section (b), equation (6.25) and the data from Williams and Hodgkinson (1981) which has been replotted as G_B against crack tip temperature. Clearly the alternative approach is equally valid.

(d) Crack Tip Heating - Simulated Impact

The following proposition was put forward : if crack tip heating is truly responsible for the elevation in apparent toughness during high rate impact tests then it should clearly be possible to produce the self same effect by introducing artificial local heating at the crack tip of specimens undergoing slow rate testing. An experimental programme was devised to analyse this possibility which included the assessment of slow rate fracture toughness in the isothermal situation for a range of temperatures. Again, PMMA was chosen for convenience and the results and discussion are now divided into two sections.

(i) Slow Rate Fracture Toughness, K_{IC} , of PMMA at Elevated Temperatures.

The standard single edge notched test-piece, as shown in figure 62, was used to provide the slow rate data since it provides the most expedient method for determining a stress intensity factor appropriate to the onset of unstable fracture. Two thicknesses of specimen were used 1 mm and 3 mm neither of which violates the condition for plane strain behaviour. Specimens were notched in two different manners, (a) saw-cutting followed by razor-notching to the required depth and (b) notching with an single point fly cutter as shown in figure 63 (a) and (b). At each thickness and at several temperatures, a range

FIGURE 61: COMPARISON OF MODELS WITH DATA FOR PMMA

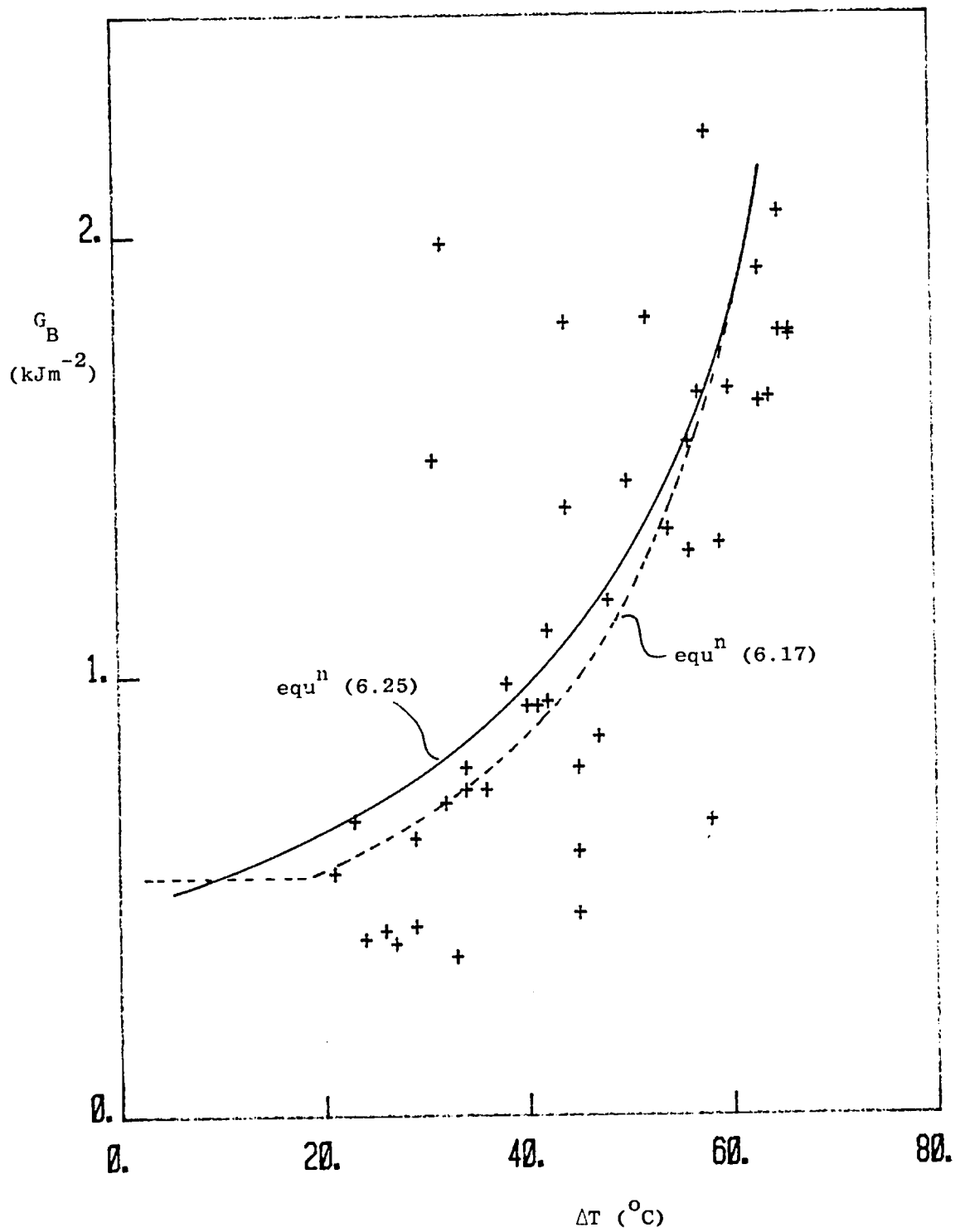


FIGURE 62: SEN TEST-PIECE DIMENSIONS

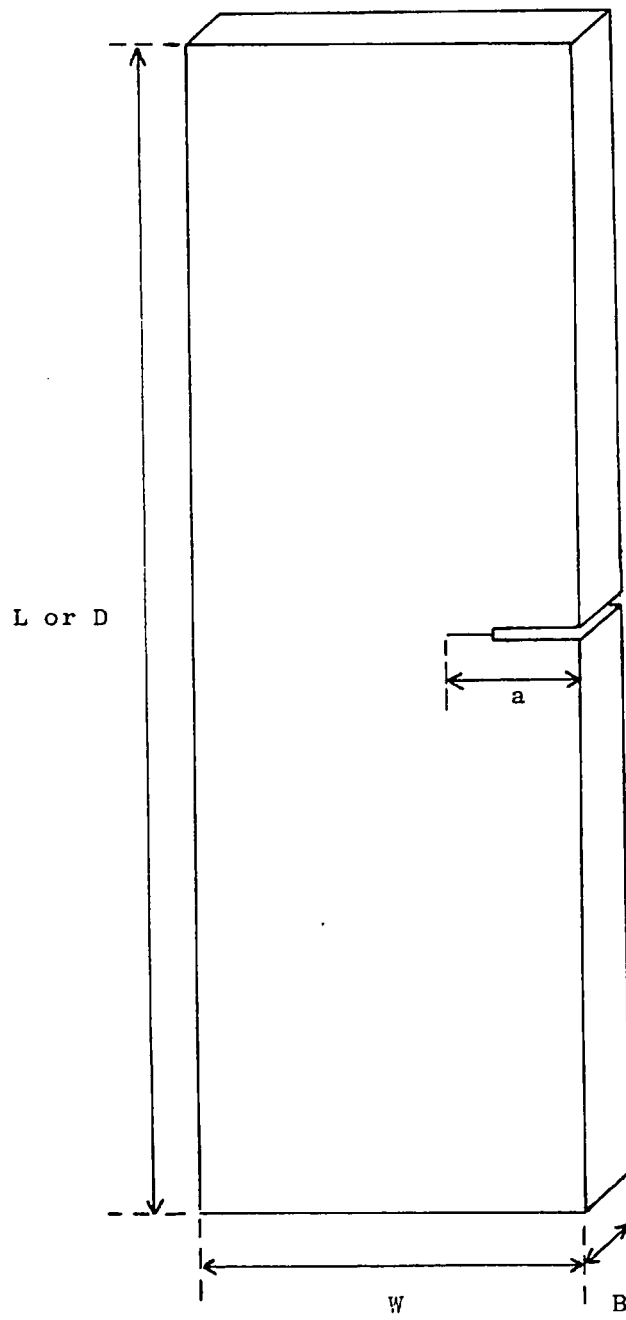
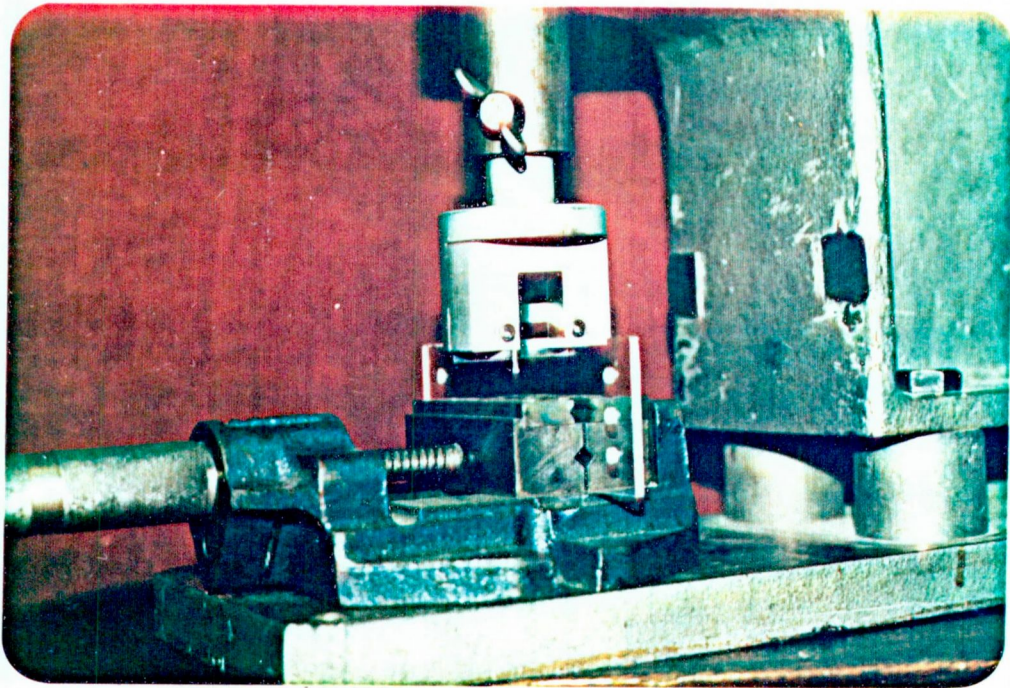
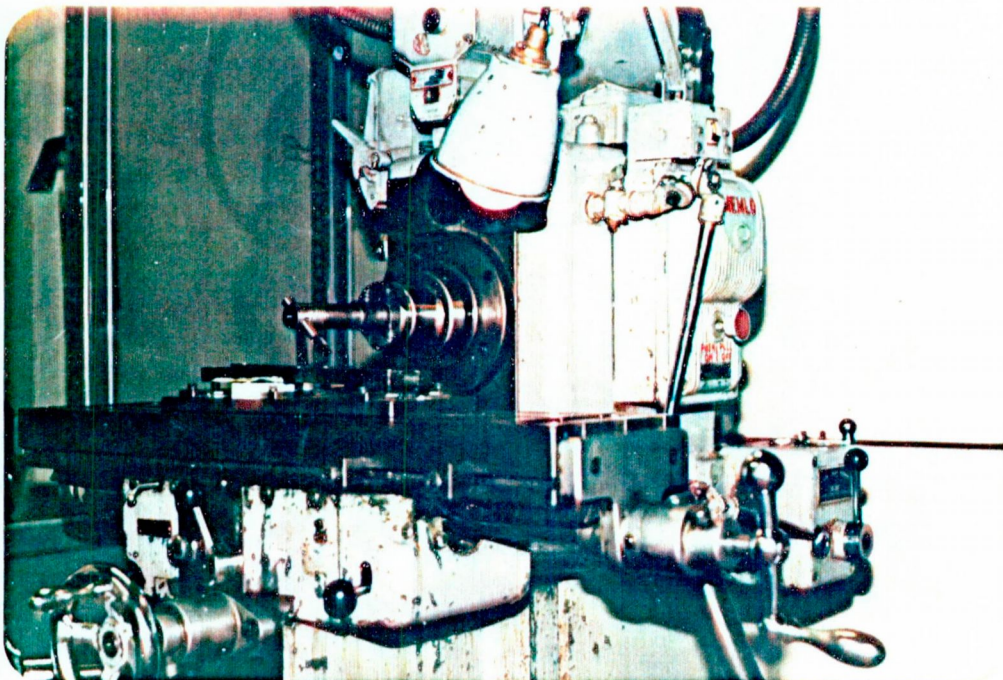


FIGURE 63: RAZOR AND MACHINED NOTCHING TECHNIQUES



(a)



(b)

of notch depths was examined, the results being duplicated for the alternative notching technique. All specimens were tested on an Instron universal testing machine at a constant cross-head rate of 5 mm min^{-1} . Temperature control was facilitated by an insulated cabinet surrounding the entire specimen, a closed system comprising liquid nitrogen coolant and heating coils, and a Eurotherm temperature control unit. This gave a deviation from the steady state of less than 1°C .

Gross section stresses at failure, σ_f , were calculated using the peak load recorded on the Instron chart divided by the cross sectional area, and crack lengths, a_f , were measured appropriate to final failure. These final crack lengths included a small portion of slow growth for the razor notched specimens but no such addition for the machine notched specimens. At elevated temperatures, i.e. 60°C and above, distinguishing the slow growth portion was difficult but under reflected light there was a colour change between the two regions (slow growth and fast fracture) enabling accurate determination of a_f . Finite plate corrections, Y , dependent upon the crack length and specimen width were calculated and with regard to the relation

$$K_c = \sigma_f Y a_f^{\frac{1}{2}} \quad (6.26)$$

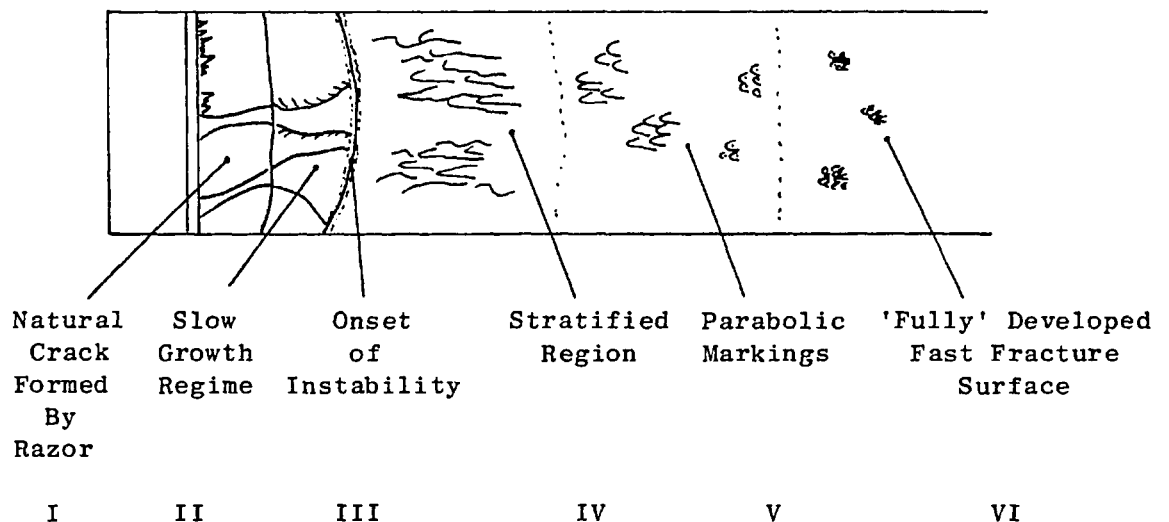
the stress intensity factor, K_c , at each condition was determined by taking the square root of the gradient of a straight line plot of $\sigma_f^2 Y^2$ against a_f^{-1} .

(a) Sharp Notch Results

At temperatures of 25, 40, 55, 70, 85, 90, 95 and 100°C the

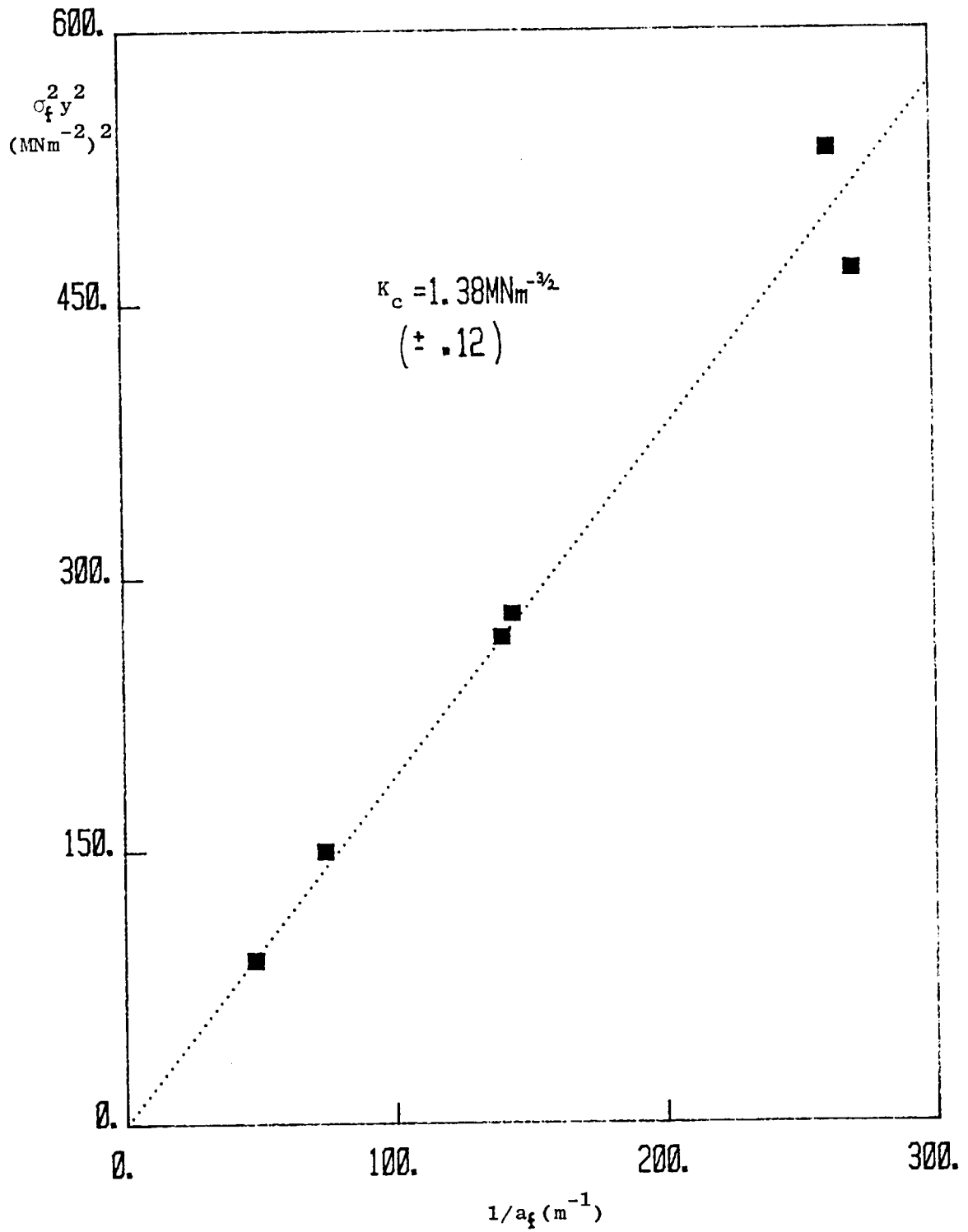
procedure overleaf was followed and figure 64 shows a typical graph of $\sigma_f^2 Y^2$ versus a_f^{-1} at 40°C . The values of K_c were determined from a least squares fit and appropriate 95% confidence limits are quoted. More data are available at the higher temperatures since it was anticipated that the majority of specimens would yield - this was found not to be the case; in fact, even at 100°C specimens with a notch length of 3.5mm fractured in a brittle manner, load/deflection traces being linear in all cases.

Fracture surfaces of PMMA and their various characteristics have been studied previously (Berry (1962)) but changes with increasing temperature have not been highlighted. With reference to the schematic diagram shown, all six regimes are present at 25°C and 40°C . Under illumination with a powerful tungsten lamp the reflected light from regions I and II show a pink tint, whereas regions V and VI show green. The intermediate stratified region IV comprises pink and green streaks.



Some unavoidable irregularities in the original crack are present as shown in regions I and II. They do not progress further than region II. At low temperatures region III is clearly discernable as a line on the fracture surface. The first region of unstable fracture consists

FIGURE 64: DETERMINATION OF TOUGHNESS K_c
FOR PMMA AT 42°C



of streaks running in the direction of the fracture and, as the region is traversed, fine parabolic marking may be detected amongst the streaks. Region V is distinguished by an increase in density of these parabolae and, as the density increases, so a small point or head appears at the focus. Region VI, which for short initial crack lengths predominates on the fracture surface, is termed the 'fully' developed fast fracture surface and is characterised by just the heads or points of the parabolae.

At temperatures of 55°C and above region VI was absent and in some cases at the higher temperatures region V could not be detected. Pin-pointing region III at elevated temperatures was difficult but reliance on the changing colour led to detection of a hazy dimpled band corresponding to this region. Slow growth regions were measured at all temperatures and the following tables shows slow growth portion/initial crack length at the various temperatures for a nominal notch length of 3.5mm.

T (°C)	25	40	55	70	85	90	95	100
$(a-a_o)/a_o$	0.51	0.70	0.78	0.80	0.83	0.87	0.47	0.35

Marshall, Coutts and Williams (1974) have studied the fracture toughness of PMMA at a range of temperatures and have measured a value for initiation of growth and another for the onset of fast fracture. They claim to observe the start of crack growth through a microscope at cross-head speeds up to and including 50 mm min⁻¹. It is the author's belief that such observations are highly dubious. In the present

study no discrimination could be made between the load at crack initiation and that at the point of instability. Such a difference is lost in the sharp peak of the load-deflection trace and would only be revealed by a machine with a faster response at these cross-head speeds. This being the case, the initiation value of fracture toughness was calculated from the peak load and length of the original razor notch.

Figure 65 shows values corresponding to initiation and to the onset of fast fracture over the whole temperature range. The solid line on the figure is due to Marshall et al. (1974) and good correlation is shown between those results and the present study. Initiation values are seen to be substantially lower than the test fracture values except at the higher temperatures where the two values converge.

(b) Machined Notch Results

Specimens were tested successfully at 25, 40, 55, 70 and 85°C, there being no indication of ductility on the fracture surface, nor deviation from a linear load-deflection curve. The fracture surfaces at all temperatures were as shown schematically below and a careful record was kept of the position with respect to specimen thickness of the initiation region.

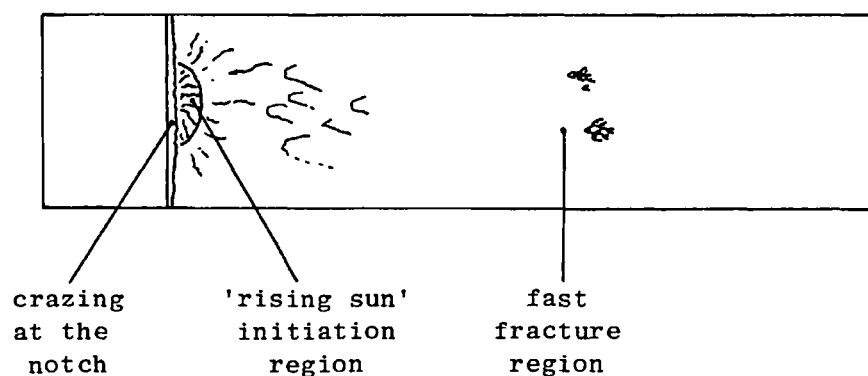
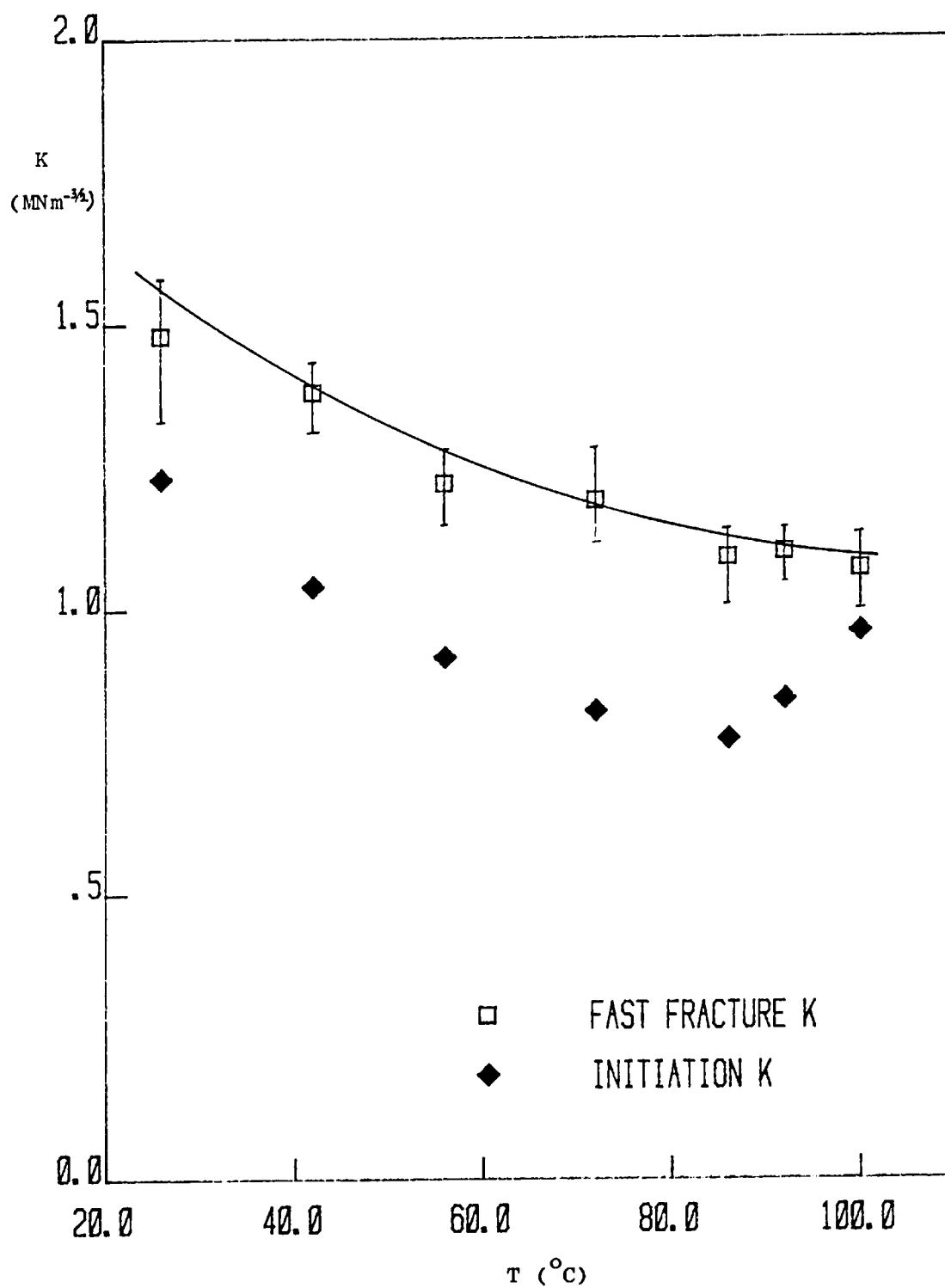


FIGURE 65: DEPENDENCE OF TOUGHNESS K
OF PMMA UPON TEMPERATURE



The value of stress intensity corresponding to the onset of catastrophic failure was totally independent of the position of the "rising sun" initiation mark and a typical graph of $\sigma_f^2 Y^2$ against a_f^{-1} at 25°C shown in figure 66.

A projection microscope was used to estimate the tip radius of the notching tool and a value of 20 μm was obtained. In view of the results due to Marshall et al (1974), in which it is found that K_c at instability is rate independent over the entire range of temperature examined here, it might be expected that some correlation should be obtained between the slow rate machined notch data and the impact data of section 6 (c) (i).

Attempts to convert critical strain energy release rates to equivalent critical stress intensities are fraught with difficulty particularly in respect of choice of modulus. Recalling equation (2.17) we may write:

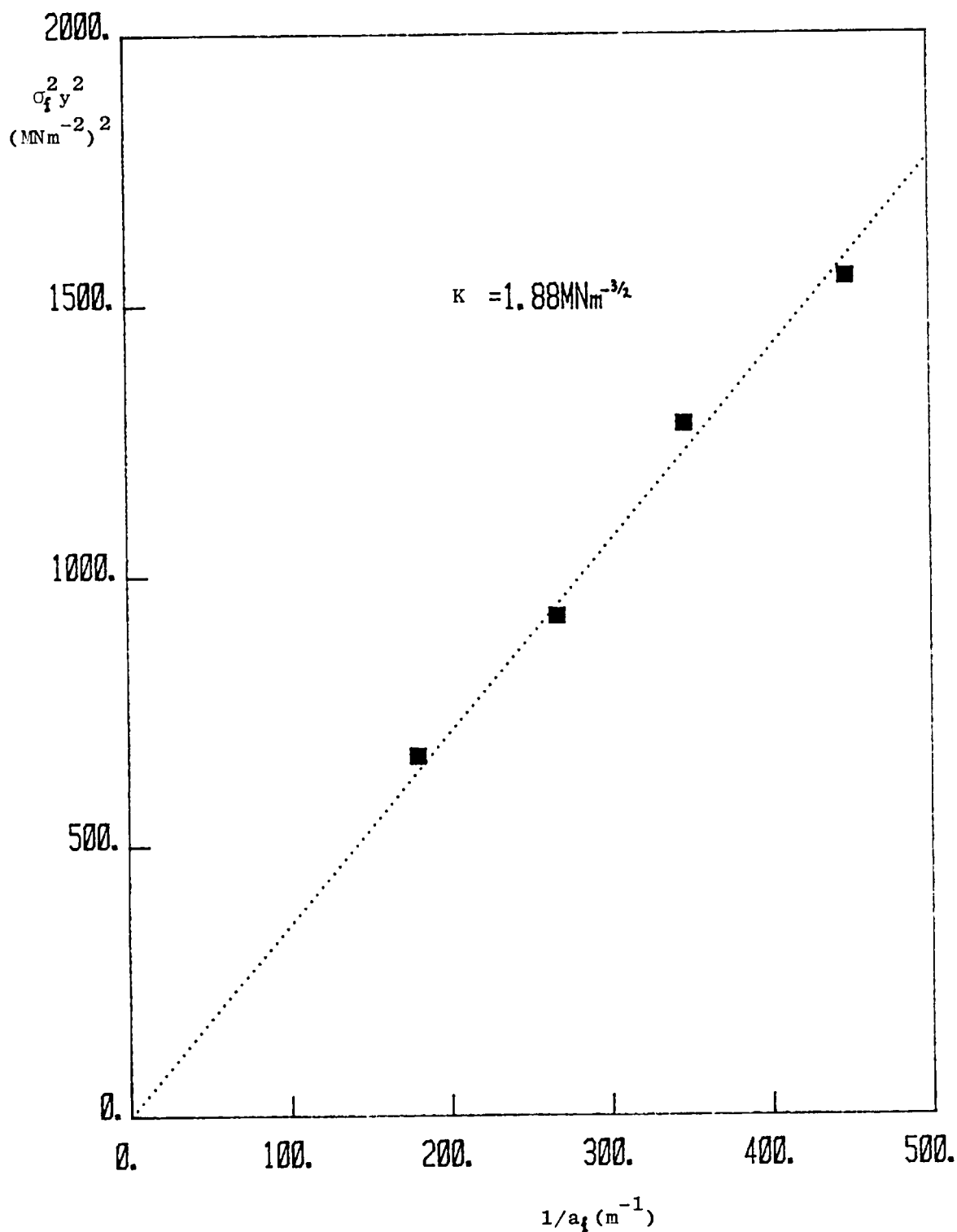
$$K = (EG)^{\frac{1}{2}} \quad (6.27)$$

or

$$G = \frac{K^2}{E} = \sigma_y \frac{K^2}{E\sigma_y} = \sigma_y \delta = E \delta e_y \quad (6.28)$$

where δ is the crack opening displacement, since the K applicable to fast fracture for PMMA is rate independent in the range of Instron cross-head speeds from 0.05 mm min.⁻¹ to 500 mm min.⁻¹ (i.e. a strain rate range of 10⁻⁵ to 10⁻¹ s⁻¹), it was thought most appropriate to convert the data of section 6 (c) (i) to equivalent critical stress

FIGURE 66: DETERMINATION OF BLUNT NOTCH TOUGHNESS
FOR PMMA AT 26°C



intensities. The problem of selecting a value for the modulus relevant to the impact of PMMA at room temperature has largely been alleviated by the recent findings of Hodgkinson, Vlachos, Whitelaw and Williams (1981) who, using a laser-doppler technique, have obtained transient velocity profiles for the impact test, and can thus calculate an appropriate elastic modulus. The average value at 20°C is 7.5 GNm⁻² and although no further tests were performed at elevated temperatures, the temperature dependence of modulus may be expected to be approximately linear, tending to zero at the glass transition temperature. As has been indicated by figure 60, the glass transition temperature appropriate to PMMA at impact rates is roughly 130°C, and finally the conversion from the data of figure 60 to K is given by

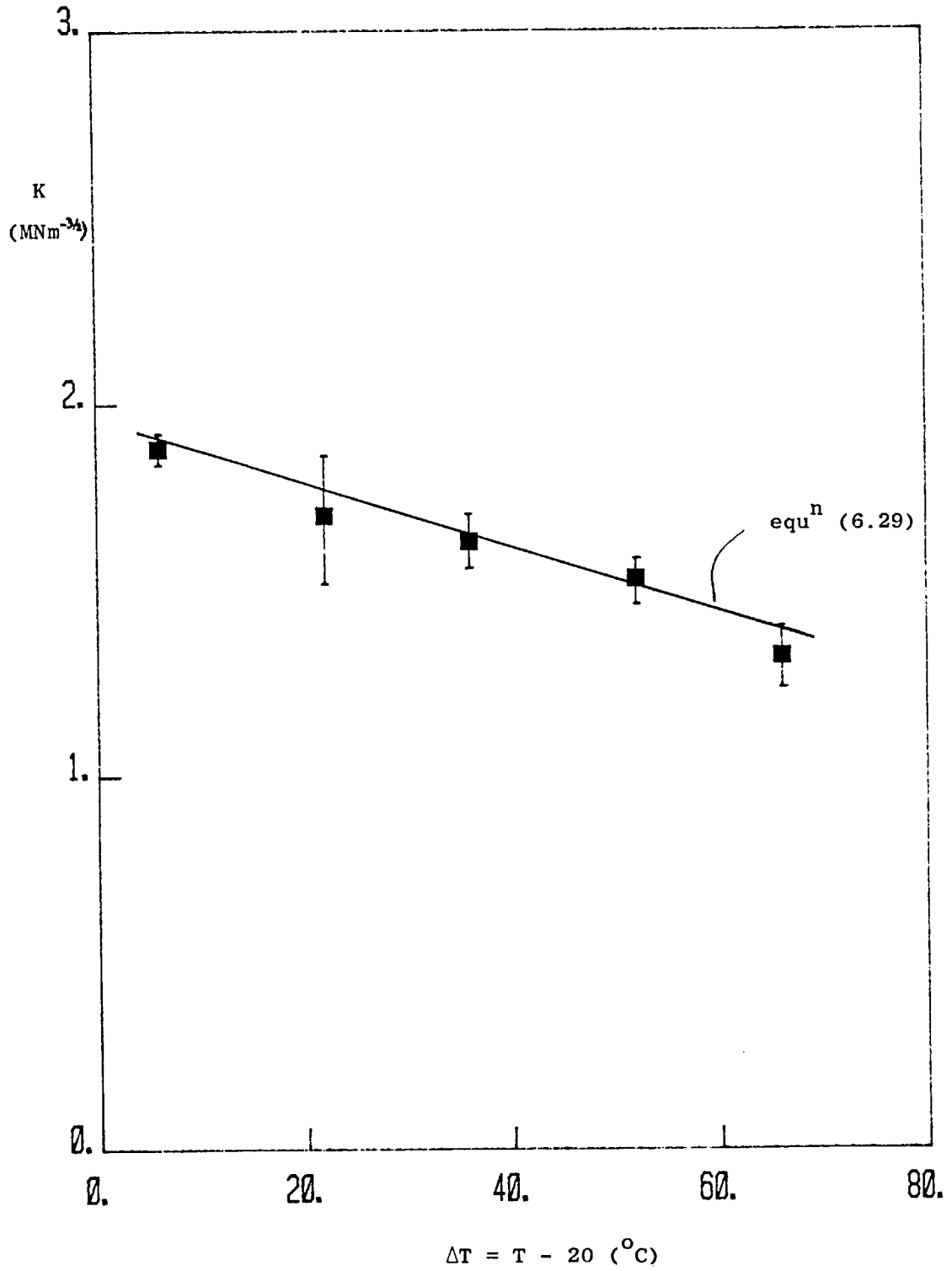
$$K_c = \left[7.5 \cdot \left\{ 1 - \frac{\Delta T}{110} \right\} \cdot \{ G_c(T) \} \right]^{\frac{1}{2}} \quad (6.29)$$

The experimental data for slow rate K_c is plotted together with 95% confidence limits in figure 67 and the solid line represents conversion of data at impact rates according to equation (6.29). Correlation is excellent with the ultimate conclusion that the critical stress intensity factor for catastrophic failure in PMMA is rate independent over the temperature range 20°C - 90°C.

(ii) Fracture Toughness of PMMA with Localised Heating at the Crack Tip

The elevation in impact fracture toughness attributable to crack tip heating and subsequent blunting should strictly be the difference in toughness between (i) a specimen whose crack tip alone has suffered adiabatic heating to a temperature T and (ii) a specimen

FIGURE 67: DEPENDENCE OF TOUGHNESS K OF PMMA
UPON TEMPERATURE - MACHINED NOTCH



whose entirety is at temperature T . This is to enable direct comparison of specimens whose fracture process zone is at a common temperature, the only difference being in the nature of the deformation at the crack tip. In Williams and Hodgkinson (1981) the authors use a base toughness, G_c , of 0.55 kJm^{-2} and thus comparisons and toughness elevations relate to this value. Since the G_c value for PMMA is roughly constant up to 70°C no discrepancy is introduced into their argument.

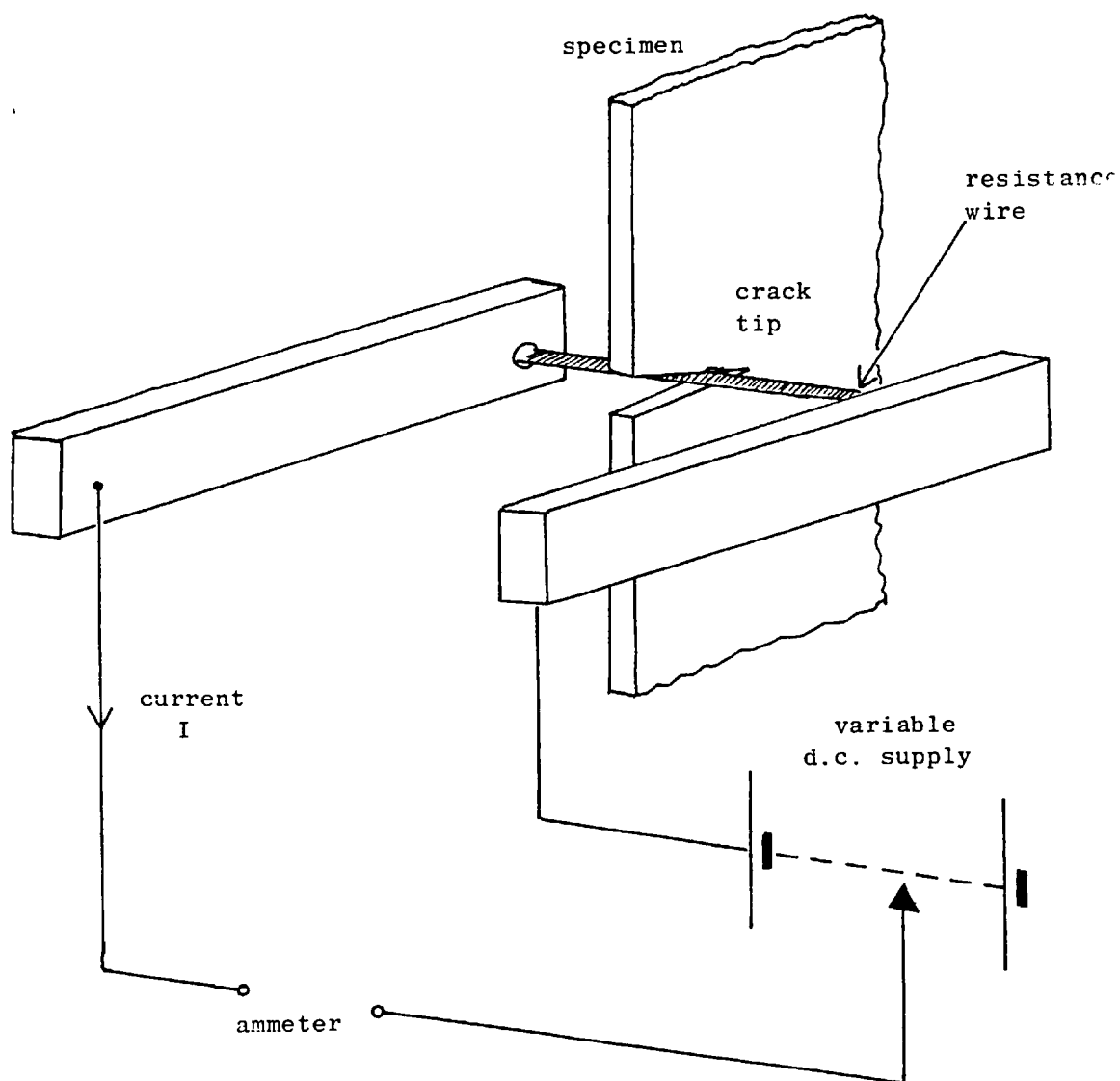
An investigation of the fracture toughness at low rates of specimens whose crack tip regions had been heated was then pursued. Localised heating was effected in the following manner. A drilled hole, of radius 0.15 mm , was used to guide a piece of resistance wire passing through an SEN specimen behind, but close to the crack tip. The thickness of the wire was chosen so that good contact was made with the material and the electrical circuit shown in figure 68 was completed. Passage of a current, I , through this circuit produced a steady-state distribution of temperature near the crack tip after a period of approximately 10 minutes had elapsed. In order to assess this distribution, thermocouples were placed in dummy specimens and the temperatures at various positions were monitored. The following analysis was offered for assessment purposes.

According to Carslaw and Jaeger (1959), the temperature distribution due to a cylindrical heat source in an infinite circular cylinder is given by:

$$T(r) - T(r_o) = - \frac{F}{2\pi k} \ln (r/r_o) \quad (6.30)$$

where $T(x)$ is the temperature at a radial distance, x , F is the

FIGURE 68: APPARATUS AND ELECTRICAL CIRCUIT FOR
ARTIFICIALLY HEATING CRACK TIPS IN PMMA



heat generation rate per unit length along $r = 0$ and k is the thermal conductivity of the material. Therefore, for a current-carrying wire of resistance per unit length, R , radius, r_w , and surface temperature, T_w

$$T(r) = T_w - \frac{I^2 R}{2\pi k} \ln (r/r_w) \quad (6.31)$$

when the current carried is I . It was found that this relationship was very closely followed in this situation and figure 69 shows temperatures recorded using thermocouples at various distances directly in front of the crack tip. A line of gradient $I^2 R/2\pi k$, where the known fixed values of R and k and the known value of I were used, was fitted to the data and the intercept $r = r_w$ gave the surface temperature of the wire. This procedure was performed for different values of I and figure 70 shows T_w plotted against I^2 . It was found that they were proportional with, perhaps not surprisingly, an intercept at room temperature for $I = 0$.

Batches of specimens with differing notch lengths were then tested for a particular value of current in the range 0.3A to 1.2A. The wire temperature in each case was known and the crack tip temperature, T_c , was calculated from the foregoing analysis together with knowledge of the distance from the wire to the crack tip. As with the isothermal tests the load-deflection trace recorded on the Instron was practically linear and a toughness, K_B , corresponding to a particular crack tip temperature was calculated.

Figure 71 illustrates the temperature dependence of K_B in comparison with the fracture toughness for the isothermal test. In the former case there is toughness elevation whereas in the latter there is a slow decrease.

FIGURE 69: RADIAL TEMPERATURE DISTRIBUTION NEAR A
HOT WIRE LOCATED IN A CRACK IN PMMA

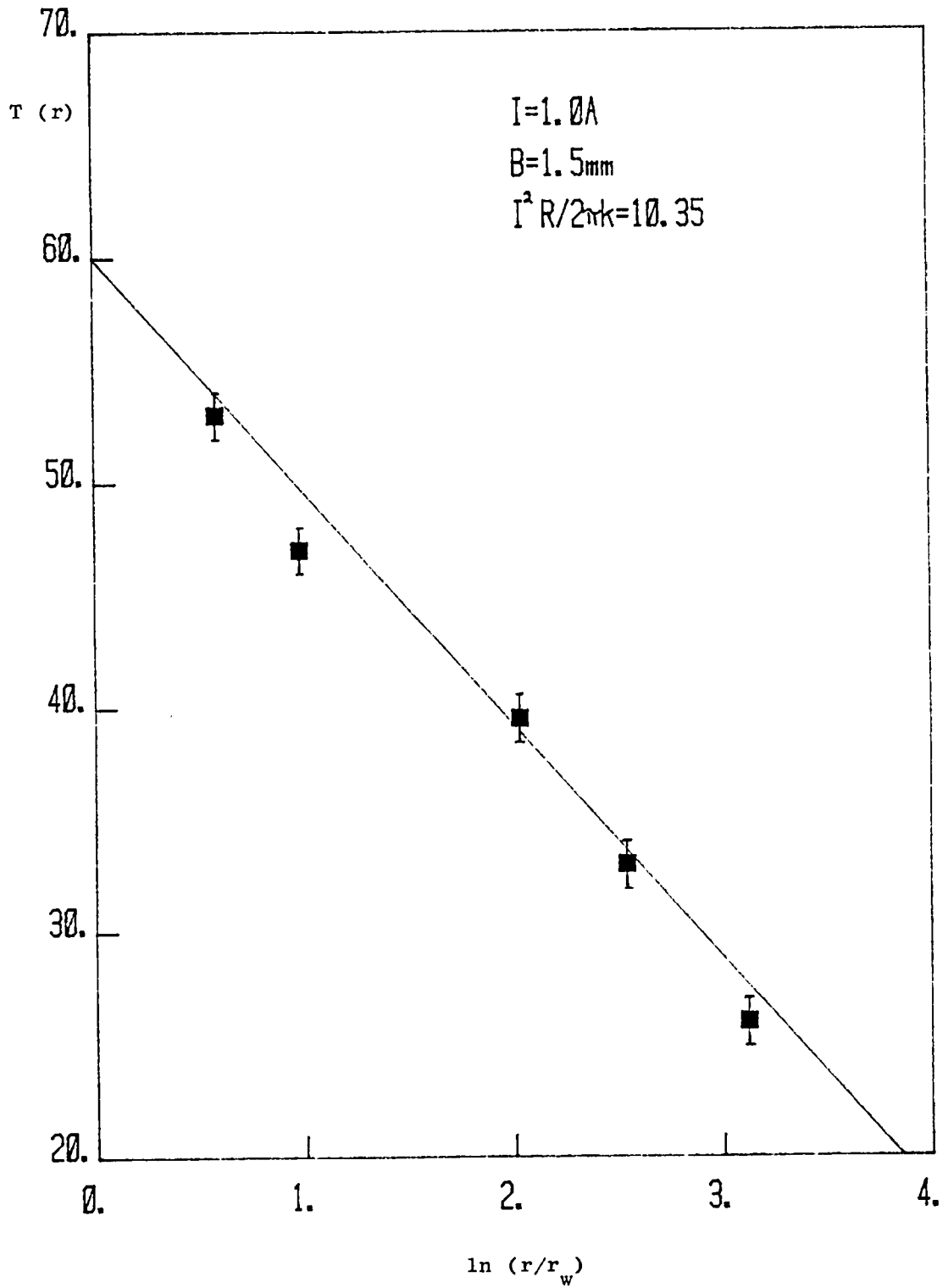


FIGURE 70: WIRE TEMPERATURE VERSUS
CURRENT SQUARED

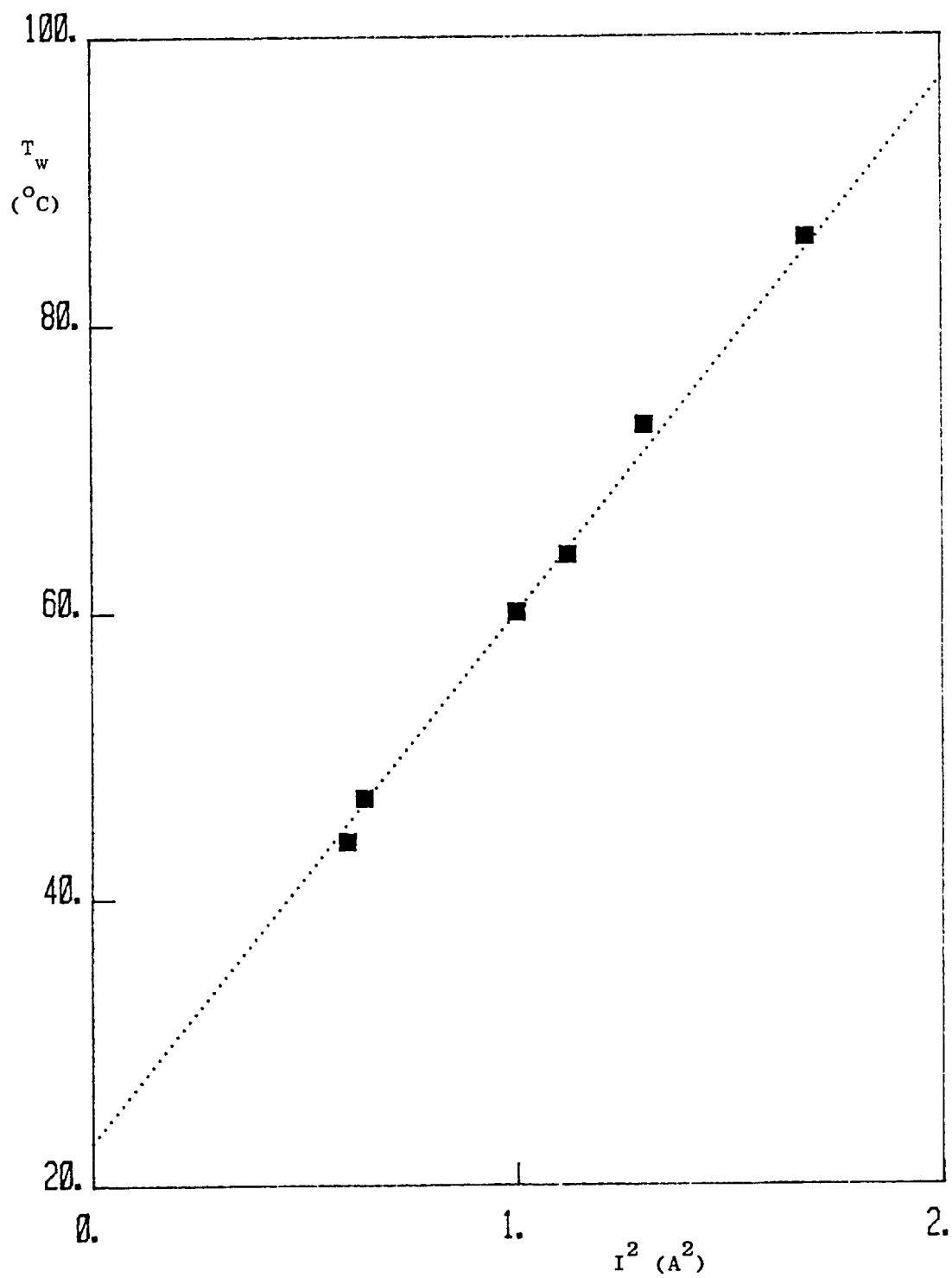
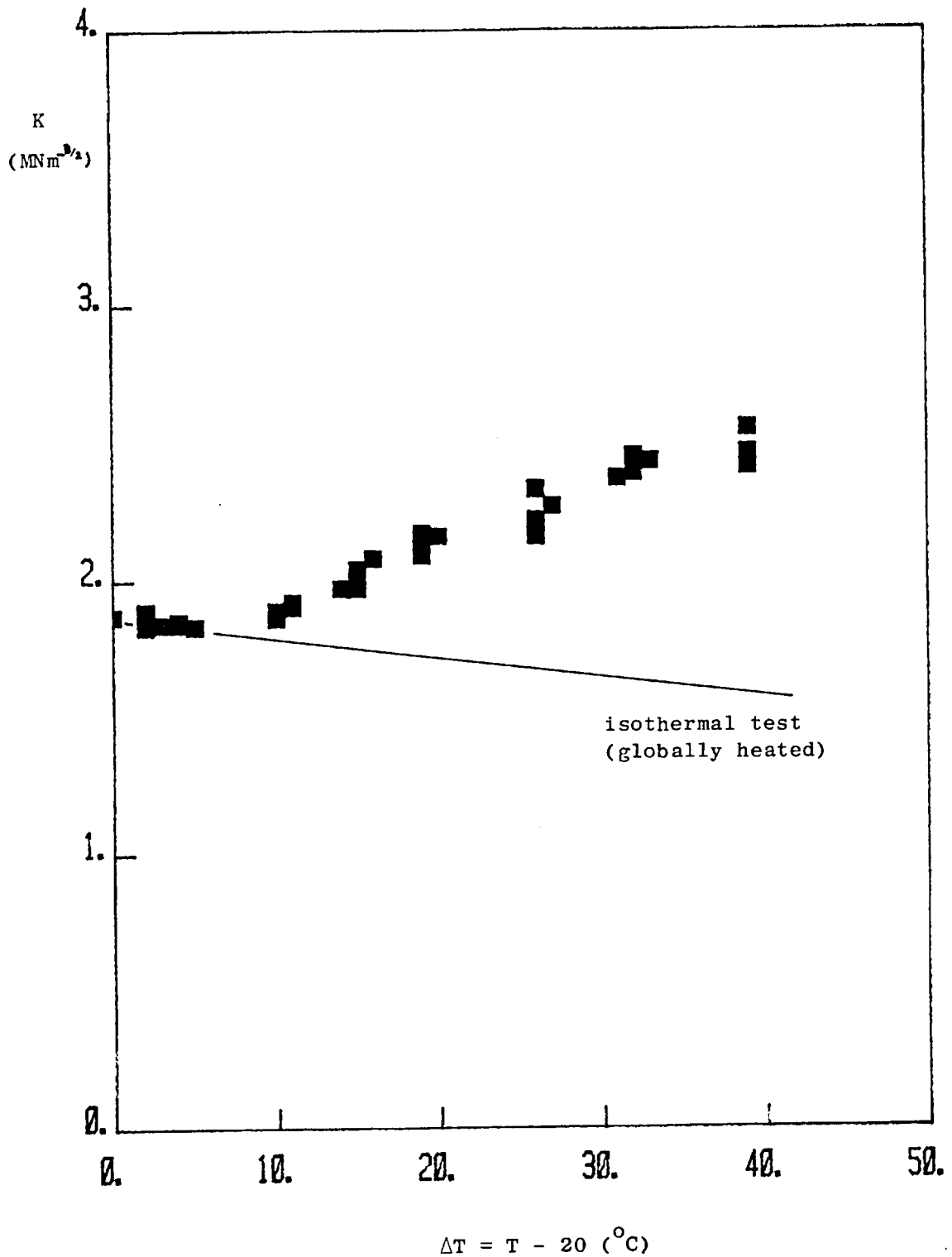


FIGURE 71: COMPARISON OF TOUGHNESS FROM LOCALLY
AND GLOBALLY HEATED SPECIMENS - PMMA



Analysis of the elevation in fracture toughness K_c to K_B follows similar lines to that for G_c to G_B but the starting point is in this case equation (3.15):

$$\frac{K_B}{K_c} = 1 \quad \text{for} \quad \rho \leq 8c \quad (6.32)$$

$$\frac{K_B}{K_c} = \left(\frac{\rho}{8c} \right)^{\frac{1}{2}} \quad \text{for} \quad \rho > 8c$$

which is readily transformed to include crack tip softening via equations (6.13), (6.14) and (6.16). The resulting equation is given as:

$$\frac{K_B}{K_c} = 1 \quad \text{for} \quad 1 - \frac{\Delta T}{\Delta T_o} \geq N \quad (6.33)$$

$$\frac{K_B}{K_c} = \frac{N}{1 - (\Delta T / \Delta T_o)} \quad \text{for} \quad 1 - \frac{\Delta T}{\Delta T_o} < N$$

where N is defined by equation (6.16) as

$$N = \left(\frac{e_y}{2} \right)^{\frac{1}{2}} \frac{\sigma_c}{\sigma_o} = \frac{1}{(2e_y)^{\frac{1}{2}}} \frac{\sigma_c}{E_o} \quad (6.34)$$

Similarly, the analysis may be couched in terms of loading time using equation (6.19) and the expressions are:

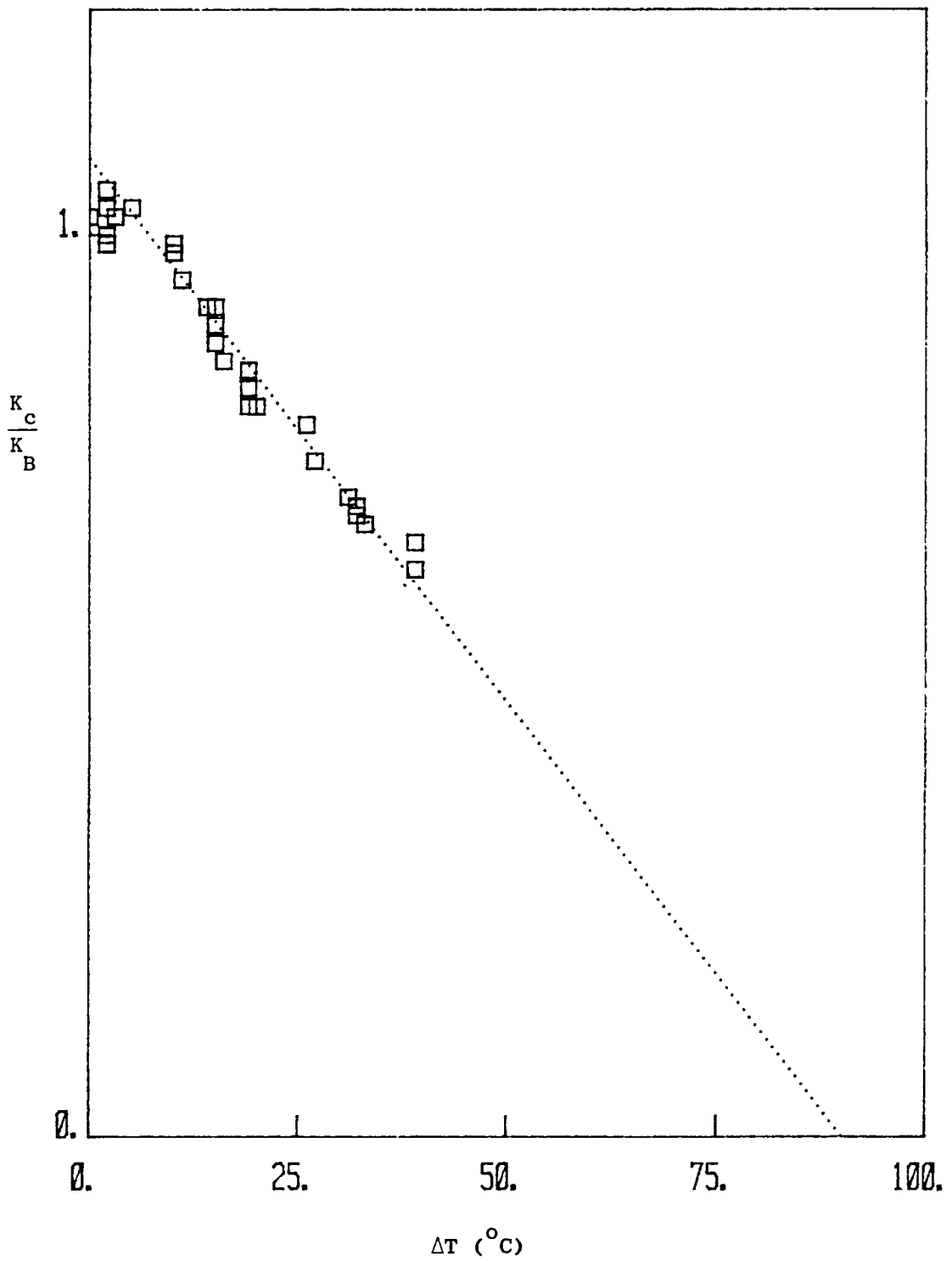
$$\frac{K_B}{K_C} = 1 \quad \text{for} \quad 1 - (t_1/t)^{\frac{1}{2}} \geq N \quad (6.35)$$

$$\frac{K_B}{K_C} = \frac{N}{1 - (t_1/t)^{\frac{1}{2}}}^{\frac{1}{2}} \quad \text{for} \quad 1 - (t_1/t)^{\frac{1}{2}} < N$$

Values of K_C/K_B , where K_C is the isothermal fracture toughness and K_B is the increased toughness due to crack tip heating, were plotted against ΔT according to equation (6.33) as illustrated in figure 72. The data shows good agreement with the relationship and the straight line shown gives intercepts corresponding to values for N of 0.92 and for ΔT_0 of 90°C .

Two sources provide information for possible correlations with this value of N . One is the original paper of Williams and Hodgkinson (1981) on crack-blunting mechanisms in impact tests; the other is by Hodgkinson, Vlachos, Whitelaw and Williams (1981) where a laser-doppler technique enables direct assessment of the forces, and consequently the stress intensities, at failure. The former yields a value of N of 0.56 whereas the latter gives 0.63. Reference to the data in these two papers shows that there is little doubt that both impact data and the slow rate data presented here agree well with the thermal blunting analysis, the only differences being in the value of N required to fit each set of data. For the three sources the yield strain, e_y , and critical stress, σ_c , are assumed identical. This is strictly because the heated crack tip zone is presumed responsible for a suppression of the crazing mechanism of fracture. The base modulus, E_0 , however is not the same for these sets of data owing to its rate dependence. In the work presented here, the value

FIGURE 72: K_c/K_b VERSUS ΔT FOR LOCALLY
HEATED SPECIMENS OF PMMA



of E_0 appropriate to the slow rates is 5 GN/m^2 , whereas the value most applicable for the average strain rate of the impact tests is approximately 7.5 GN/m^2 . The first of these values is taken from Marshall, Coutts and Williams (1974) and is a room temperature modulus suitable for instability crack speeds and the second value is an average of the calculated moduli provided by Hodgkinson, Vlachos, Whitelaw and Williams (1981). Assuming constancy of e_y and σ_c , then the product NE_0 should remain constant. This product takes the values: 4.73 for the impact data of the latter, 4.2 for the data presented in the former and 4.6 for the data presented here. Reconciliation of the discrepancy in N is thus effected through a rate dependent base modulus.

Several further aspects of these studies warrant attention. Firstly, the critical stresses, σ_c , given by equation (6.34) for the quoted values of modulus are of the order of 1 GN/m^2 - the previously reported value of 0.55 GN/m^2 Williams and Hodgkinson (1981) was obtained using an incorrect modulus value. No insight into the stress levels at an uncrazed failure is available and such high values of stress remain unsubstantiated. Using the critical stress at a critical distance criterion, however, such a stress would necessitate a distance of about $1 \mu\text{m}$. As indicated in chapter three section (c), blunt notch data on polyesters and epoxies, which do not craze, are rationalised using a critical distance of this magnitude which seems to suggest that the parameter of most physical relevance in the fracture criterion is that critical distance ahead of the tip.

Secondly, it was noted earlier in this chapter that slow rate razor-notched specimens produce substantially lower toughnesses than machine notched specimens, whereas in impact tests the two notch

types give similar results. This is entirely a rate effect due to the absence of slow crack growth in impact. Therefore, figure 71 comparing slow rate data for the isothermal and crack tip heated cases includes the complete thermal effect.

Finally, the possible effect of a region of compressive stress at the crack tip was investigated. For both an adiabatic plug of material (a zone of material at constant elevated temperature), and a zone with a temperature distribution given by equation (6.31), an upper bound to the compressive stress is estimated to be 10 MNm^{-2} . This stress will have little effect on the measured toughness, since it is local to the crack tip and negligible compared to the global stress field. However, a local compressive stress such as this may be sufficient to preclude the onset of crazing. In such a situation a transition to fracture dominated by shear yielding might be expected and the difference in behaviour between local and global heating thus explained.

CHAPTER SEVEN

CONCLUSIONS

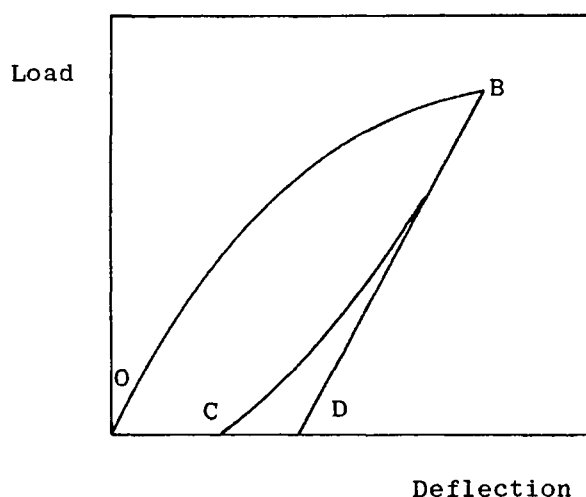
(a) Achievements

A general approach to the crack tip modelling of polymers has been presented which has been shown to be of use in deriving fracture parameters for a wide range of materials. The cornerstone of the work is the linear relationship between strain energy release rate and crack tip acuity which, although derived for elastically deforming materials, has been shown to be true for two ductile polymers. The crack tip strain energy release rate in these cases was redefined in terms of currently available elastic energy. This has far reaching consequences in predicting fracture for cases in which cracks are blunted and where the degree of blunting can be assessed. An example is given of its usefulness in predicting augmented toughness in the impact testing of polymers when crack tip softening occurs.

Further discussion of the "critical stress at a critical distance" failure criterion is warranted in the light of the values of these parameters derived in the preceding chapters. It was pointed out in chapter three that blunt notch brittle failure in glassy thermoplastics takes place according to the above criterion with values indicating craze stresses and craze lengths governing the fracture. For non-crazing materials such as epoxies and polyesters, the value of the critical distance was roughly $1\text{ }\mu\text{m}$ which in turn means a critical stress of approximately 1 GPa. This theme is further developed by the results of chapter three section (e) and chapter five. Plati (1975) has shown that impact testing of polyethylene and polycarbonate for various notch

root radii results in linear dependence of toughness upon tip radius with a critical distance indicative of craze length ($50 - 100 \mu\text{m}$). The values obtained in this study relevant to ductile failure are $5 \mu\text{m}$ and $0.3 \mu\text{m}$ respectively and again suggest stresses well in excess of yield stresses. Finally, of course, chapter six presents results for the high speed impact failure of PMMA in which crack tip softening can suppress crazing and hence reduce the critical distance to $1 \mu\text{m}$. The magnitude of predicted stresses is somewhat alarming but no indication of the tolerable stresses ahead of a crack in non-crazing material is available although these are likely to be limited by true ultimate tensile strengths.

As far as the derivation of an elastic energy release rate local to the propagating crack is concerned, there are two important assumptions which come under severe scrutiny. Firstly, the model of the material is ideal e.p.e. behaviour as defined by Turner (1979) and this is not totally accurate. Typical unloading behaviour for polyethylene or polycarbonate takes the form shown below.



BD is the assumed unloading curve according to elastic - plastic - elastic behaviour whereas BC is the actual unloading curve. The discrepancy

has not been investigated here but the model as presented is thought to be adequate in defining strain energy release rate. A second assumption is intrinsic to the model and that is, that the unloading compliance (slope of BD) is only dependent upon the current crack length and is equal to the compliance derived from initial loading curves. The validity of this is questionable and may give rise to some of the variations in strain energy release rate observed during propagation. Finally, it can be said that, in spite of these reservations, the determination of a toughness appropriate to the ductile crack propagation is a useful contribution to fracture mechanics.

Constancy of this parameter with varying geometry is suggested rather than confirmed and perhaps a stronger conclusion would be that the strain energy release rate for crack propagation is directly proportional to crack tip radius. In these circumstances, provided the crack tip deformation can either be predicted or modelled, an assessment of whether fracture will proceed can be made.

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