

Stability and ultimate behaviour of prestressed stayed beam-columns

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Abstract

The instability of beam-columns with crossarms and externally prestressed cable stays is studied analytically, where the combination of bending and compression is assumed to be derived from the system self-weight acting orthogonally to the applied axial load. Three principal zones of behaviour are identified with two of these each having two sub-zones that relate the critical buckling load to the initial prestressing force applied to the stay cables. The ultimate load-carrying capacity of the beam-columns is evaluated by conducting nonlinear finite element analysis within the commercial package ABAQUS. Results show that the analytically derived critical buckling loads generally provide safe predictions of the ultimate loads due to significant post-buckling strength. It is found that releasing the geometric double symmetry of the system can make for a significantly more efficient structure due to the effect of pre-cambering against the self-weight. The strength and efficiency of stayed beam-column systems opens up a range of potential applications, including lighter alternatives to conventional props to support wide excavations, which currently utilize very heavy steelwork.

Keywords: Prestressed stayed columns; Steel structures; Analytical modelling; Structural stability; Finite element analysis; Nonlinearity

1. Introduction

Slender struts supported with prestressed cable-stays in conjunction with periodically-spaced crossarms distributed longitudinally, offer an attractive, innovative and pragmatic solution to reducing structural self-weight while maintaining buckling strength. The use of the crossarm-stay system with pretensioning within the cable stays effectively provides intermediate supports that act to reduce the effective buckling wavelength which, in turn, results in a higher load-carrying capacity. Research on this type of structural system can be found initially from the 1960s [1, 2] with specific work on determining the buckling response

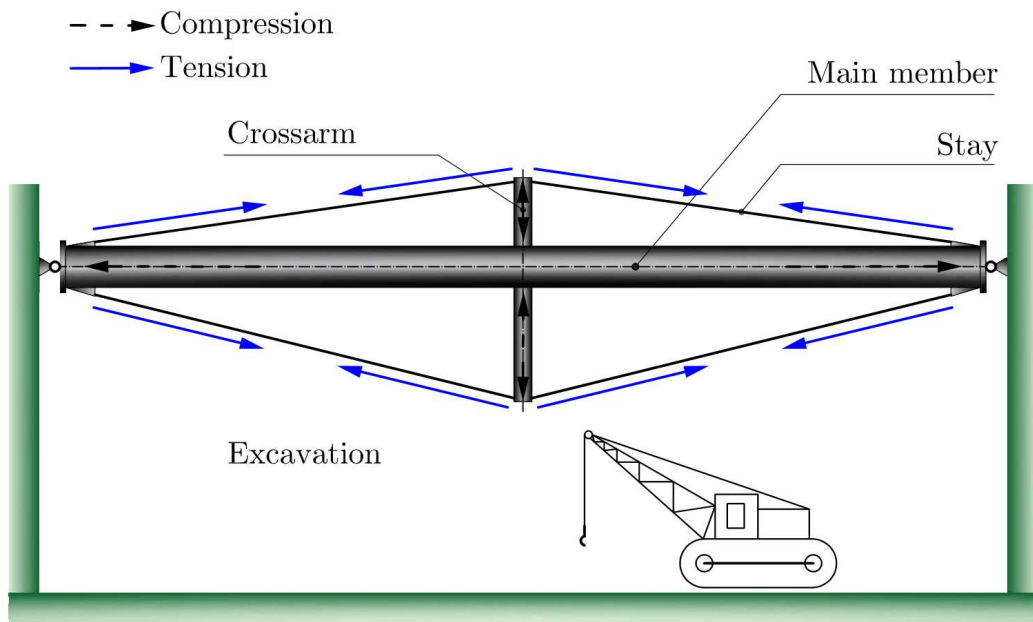
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analytically [3, 4] and the important relationship between the levels of pretensioning and the critical buckling load [5]. The work presented in [3] was verified in [6, 7] and imperfection sensitivity was investigated in [8]. Experiments and numerical studies were conducted to validate the buckling behaviour [9, 10]. Subsequently, the relationship regarding the levels of prestress and the nonlinear post-buckling behaviour of stayed struts was determined numerically [11] and then theoretically for distinct post-buckling behaviour [12, 13] and where interactive buckling occurs [14, 15, 16]. Design guidance for struts with one intermediate crossarm system was presented in [17], where the work was based primarily on experimental [18] and finite element studies [19]. In addition, a nonlinear dynamic finite element method is applied to study the static equilibrium of cable-strut system during the construction process [20].

Most of the aforementioned research has been conducted for vertical members where the direction of the self-weight and the axial load have been collinear. However, there is a potential demand for very long struts to be used in the horizontal plane, for example to support excavation works in the construction of large scale foundations, as shown in Figure 1. As can be seen in Figure 1(b), existing so-called ‘super struts’ are currently used; these are constructed from heavy tubular steel cross-sections such that the struts can withstand the earth pressures axially in conjunction with their considerable self-weight in bending. It has been demonstrated in the aforementioned research work that the most effective stayed columns can increase the load-carrying capacity of a conventional column by an order of magnitude. Hence, the current study considers the effect of placing stayed columns in the horizontal plane, thereby converting them into stayed beam-columns where the axial load and the self-weight act orthogonally to each other. This introduces new complexity to the mechanics of the system since the collinearity of the loading no longer exists. The self-weight introduces a directionality that is similar to that found in cable-stayed bridges, where stays are regularly used to reduce the self-weight deflection of the deck, thereby increasing capacity [21]. Currently, there is a similar opportunity by reducing the length of the upper portion of the crossarm, as shown in Figure 1(a), to provide a net upwards force from the intermediate restraint. This enables the main beam-column member to be pre-cambered by a controlled extent to counteract the effect of the self-weight.

In the current article, mechanical modelling is employed to determine the relationships between the prestressing forces introduced into the stayed beam-column system, the applied axial load and the residual forces within the core beam-column component and the stays. As in the work by Hafez *et al.* [5], a clear relationship between the initially applied prestressing force and the critical buckling load is established. However, unlike in [5], instead of three clearly distinct zones of behaviour, two of the three zones are each sub-divided into two further sub-zones of behaviour owing to the effect of the self-weight. The primary aim of the current work is to compare the ultimate loads of the studied members, obtained numerically within the commercial package ABAQUS [22], with their critical buckling loads, obtained from the currently presented analytical model in the different zones of behaviour that depend on the initial prestressing level. This is to establish whether there is any significant post-buckling strength. It is in fact determined that there is considerable post-buckling reserve especially at relatively higher prestressing levels and



(a) Potential application of a prestressed stayed beam-column as an excavation prop



(b) Conventional 'super struts' as excavation props

Figure 1: (a) Mechanical principles of stayed beam-columns with stays pretensioned to provide restraint against global buckling. (b) So-called 'super struts' used to support a basement excavation (source: www.theconstructionindex.co.uk/news/view/super-struts, 2012).

that reducing the ratio of the upper to lower crossarm lengths (defined as α currently in Eq. (2) below) from unity initially has a generally beneficial effect. However, an excessive reduction in α is shown to promote highly unstable post-buckling behaviour. Finally, it is demonstrated that the relationship and level of the relative prestressing forces within the upper and lower stays, determined geometrically from the current mechanical model, provides a good estimate for maximizing the load-carrying capacity.

2. General definitions and modelling

The numbering of the stays and the geometric parameters of the currently considered prestressed stayed beam-column are shown in Figure 2. The upper and lower crossarm

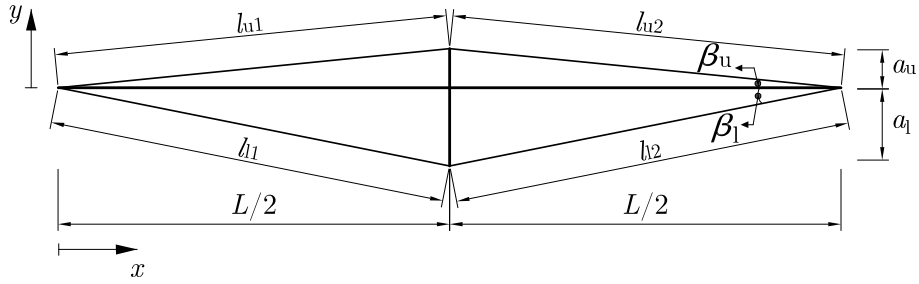


Figure 2: Stay numbering and geometric parameters of the prestressed stayed beam-column: L is the total length of the main column element; a_u and a_l are the lengths of the upper and lower crossarms respectively; β_u and β_l are the angles between the main member and the upper stay and the lower stay respectively; l_{un} are the lengths of the upper stays, and l_{ln} are the lengths of the lower stays, where $n = \{1, 2\}$. In the current study, $l_u = l_{u1} = l_{u2}$ and $l_l = l_{l1} = l_{l2}$.

ratios are defined respectively as follows:

$$\alpha_u = \frac{a_u}{L}, \quad \alpha_l = \frac{a_l}{L}, \quad (1)$$

and the crossarm-length ratio α is introduced thus:

$$\alpha = \frac{\alpha_u}{\alpha_l} = \frac{a_u}{a_l}. \quad (2)$$

The main column element has a Young's modulus E_c , cross-sectional area A_c and second moment of area I_c . Both upper and lower crossarms are assumed throughout the current work to have an identical Young's modulus and cross-sectional area E_a and A_a respectively. As in previous work [12, 19, 16], the connections between the crossarms and the main column element are assumed to be rigid, whereas the connections between the stay cables and both the main column element and the crossarms are assumed to be pinned.

It should be noted that although the current article considers essentially a two-dimensional (2D) system, in reality, it is a three-dimensional (3D) system, the beam-column must be supported by three or more crossarm elements to be stable in the third dimension. An alternative strategy is to introduce lateral braces between two adjacent props to decrease

the lateral effective length, as shown in Figure 1(b), to avoid buckling out of plane, and the problem would essentially return to 2D and the current model would be valid and circumvent the need to include imperfections out of the plane.

2.1. Loading steps and analytical model assumptions

The prestressed stayed beam-column studied herein is under bending, through the action of self-weight, in conjunction with axial compression that amplifies the bending displacement. This is naturally quite different from the previously studied prestressed stayed columns under purely axial compression [5, 12, 18, 16]. Hence, it is necessary to define the adopted loading sequence, as given below and represented in Figure 3:

Step 1 With no pre-tension applied to all the stays, owing to its self-weight, the member assumes a curved profile similar to a plain beam.

Step 2 A specific prestressing force is introduced within the lower stays to counteract the self-weight effect and the midspan vertical displacement becomes zero relative to the supports.

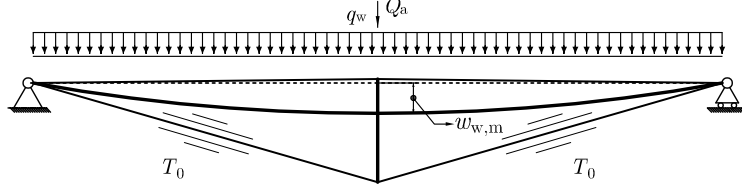
Step 3 Additional prestressing forces are applied within all the stays to improve the buckling behaviour of the system.

Step 4 Finally, an axially compressive load is applied until the system fails, usually by a combination of bending and buckling.

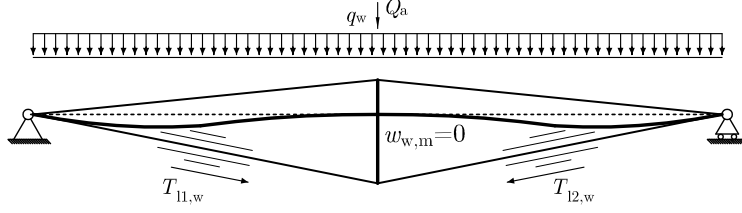
For modelling purposes, the following assumptions are made:

1. All the deflections are relatively small, although the effects of nonlinearity from combining bending and axial compression are included.
2. All materials are linearly elastic since the geometries are very slender and hence the strains are generally well below the yield strain.
3. The self-weight of all the stays is currently neglected, since the unit-length self-weight of the stays is less than 10% of the main member in most models.
4. When the self-weight is introduced in loading step 1, all the stays are assumed to be straight but without stress. Similarly, when applying the prestress within the lower stays to counteract the self-weight in loading step 2, the upper stays are straight but are assumed to have zero stress.
5. The main member is assumed to be perfectly horizontal when deducing the optimum prestressing forces and zones of behaviour. This is because after applying the self-weight and the prestressing force within the lower stays, the midspan vertical displacement of the main member returns to zero and the maximum vertical displacement of the main member remote from midspan is small.
6. When deducing the analytical expressions for the optimum prestressing forces and the boundaries demarcating the zones of behaviour, the prestressing force within the lower stays that counteracts the self-weight always exists. However, this force, alongside the self-weight, is no longer taken into account explicitly owing to the effect of them counteracting one another.

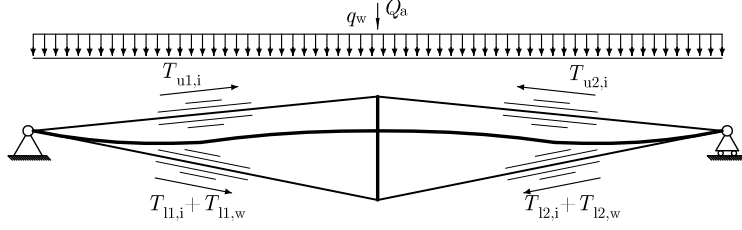
Loading step 1: Applying crossarm and main-member self-weight Q_a and q_w



Loading step 2: Prestressing to counteract self-weight; increasing tension in the lower stays to $T_{l1,w}$, $T_{l2,w}$ such that $w_{w,m}=0$



Loading step 3: Prestressing to improve buckling performance; increasing tension in all stays simultaneously to $T_{u1,i}$, $T_{u2,i}$, $T_{l1,i}$, $T_{l2,i}$ such that the axial forces within the crossarms are equal (to leading order)



Loading step 4: Applying compressive axial load P until system buckles

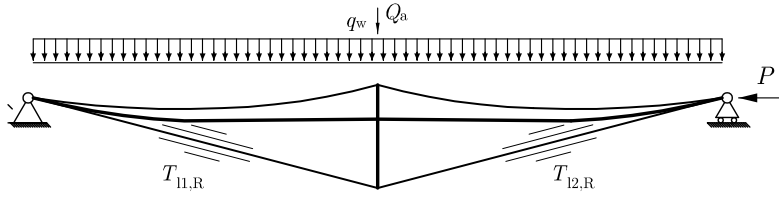


Figure 3: Loading steps for the prestressed stayed beam-column. In steps 1–3, all stays are assumed to remain straight; T_0 is the stress within the lower stays in step 1; $w_{w,m}$ is the mid-span vertical displacement after applying self-weight and $T_{l,w} = T_{l1,w} = T_{l2,w}$ are the prestressing forces within lower stays to counteract the midspan self-weight deflection; $T_{u,i} = T_{u1,i} = T_{u2,i}$ and $T_{l,i} = T_{l1,i} = T_{l2,i}$ are the prestressing forces applied within the upper and lower stays respectively to improve buckling performance; P is the compressive axial load; $T_{l,R} = T_{l1,R} = T_{l2,R}$ are the residual tensions within the lower stays when the system buckles.

2.2. Finite element model

In the current study, finite element (FE) modelling is used to verify the theoretical results obtained. Following previous numerical studies of prestressed stayed columns using the commercial software ABAQUS [14, 19, 16], the Euler–Bernoulli beam element B23 is adopted to model the main horizontal member as well as the crossarms. Since all the stays are pinned to the ends of the main member and crossarms, these are modelled with the truss element T2D2 in conjunction with the ‘no compression’ option. The main member and crossarms are meshed as beam elements each with a length of 50 mm, and each stay is meshed as an individual truss element. Buckling analysis is conducted to calculate the linear eigenvalue and eigenmode profile, and then Riks analysis is conducted to obtain the nonlinear post-buckling behaviour.

3. Analytical modelling

The principal purpose of the current analytical work is to investigate the structural response in each of the loading steps described in Figure 3 based on the aforementioned assumptions by applying mechanical principles. Subsequently, the analytical expressions of the so-called optimum prestressing forces and how these demarcate the behaviour into separate zones are deduced.

3.1. Counteracting self-weight

The applied prestressing force within the lower stays at the beginning of loading step 2 to counteract the self-weight $T_{l,w-a}$, becomes $T_{l,w}$ at the end of loading step 2 owing to the loss in prestressing from the end-shortening displacement of the main element. In order to determine the expression for $T_{l,w-a}$, the structural response in loading steps 1 and 2 is studied (see Figure 3).

In loading step 1, the stays and crossarms do not affect the structural response of the main member from the aforementioned assumptions. Hence, the main member can be considered to be a simply-supported beam under a vertical load with a distributed intensity q_w from the main member self-weight, alongside a midspan concentrated load Q_a from the crossarm self-weight. Owing to linear theory, the effect of these two types of loads can be considered simply as their linear superposition, as shown in Figure 4.

Based on Euler–Bernoulli bending theory, $w_{w,a}(x)$ and $w_{w,c}(x)$ can be expressed thus, where $x = [0, L/2]$:

$$\begin{aligned} w_{w,a}(x) &= \frac{Q_a L^2 x}{48 E_c I_c} \left(3 - \frac{4x^2}{L^2} \right), \\ w_{w,c}(x) &= \frac{q_w L^3 x}{24 E_c I_c} \left(1 - \frac{2x^2}{L^2} + \frac{x^3}{L^3} \right). \end{aligned} \quad (3)$$

The total deflection function of the main member $w_w(x)$ is the superposition of $w_{w,a}(x)$ and $w_{w,c}(x)$, thus:

$$w_w(x) = w_{w,a}(x) + w_{w,c}(x). \quad (4)$$

The deformation of the system in loading step 1 is illustrated in Figure 5 and includes

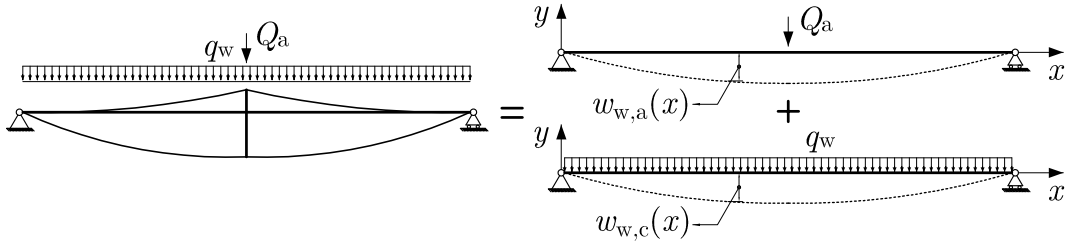


Figure 4: Simplifying the effect from the loads in loading step 1: $w_{w,a}(x)$ is the deflection function of a simply-supported beam under a midspan concentrated vertical load Q_a representing the self-weight of the crossarms; $w_{w,c}(x)$ is the deflection function of a simply-supported beam under a distributed vertical load q_w representing the self-weight from the main beam-column member.

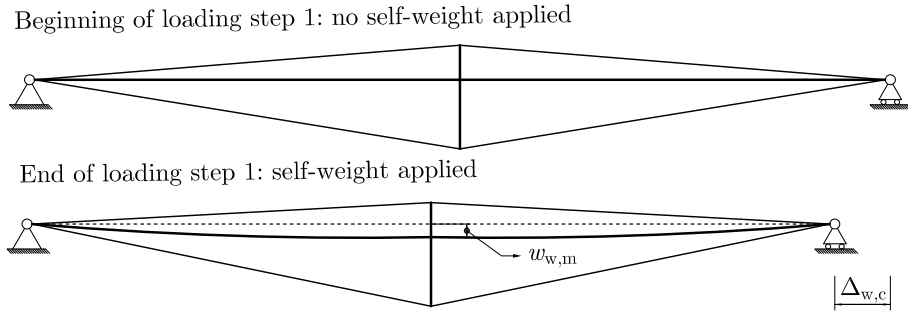


Figure 5: Deformation of the system under self-weight in loading step 1: $\Delta_{w,c}$ is the end-displacement and $w_{w,m}$ is the midspan vertical displacement of the main member.

the end-shortening displacement $\Delta_{w,c}$, arising from w_w , a leading order approximation of which is given by [23]:

$$\Delta_{w,c} = \frac{1}{2} \int_0^L w_w'^2(x) dx. \quad (5)$$

Hence, the midspan displacement $w_{w,m}$ can be expressed thus:

$$w_{w,m} = w_w(L/2) = \frac{Q_a L^3}{48 E_c I_c} + \frac{5 q_w L^4}{384 E_c I_c}. \quad (6)$$

If all the stays are straight but without stress, the length of stay n at the end of loading step 1, $l_{w,n}$, would be as follows:

$$\begin{aligned} l_{w,u} = l_{w,u1} = l_{w,u2} &= \sqrt{(a_u - w_{w,m})^2 + \left(\frac{L}{2} - \frac{\Delta_{w,c}}{2}\right)^2}, \\ l_{w,l} = l_{w,l1} = l_{w,l2} &= \sqrt{(a_l + w_{w,m})^2 + \left(\frac{L}{2} - \frac{\Delta_{w,c}}{2}\right)^2}. \end{aligned} \quad (7)$$

Equilibrium analysis at the end of loading step 2 is shown in Figure 6. The right-hand half of the main member is equivalent to a beam under a distributed load q with the left-hand support being clamped and the right-hand support being pinned with a roller

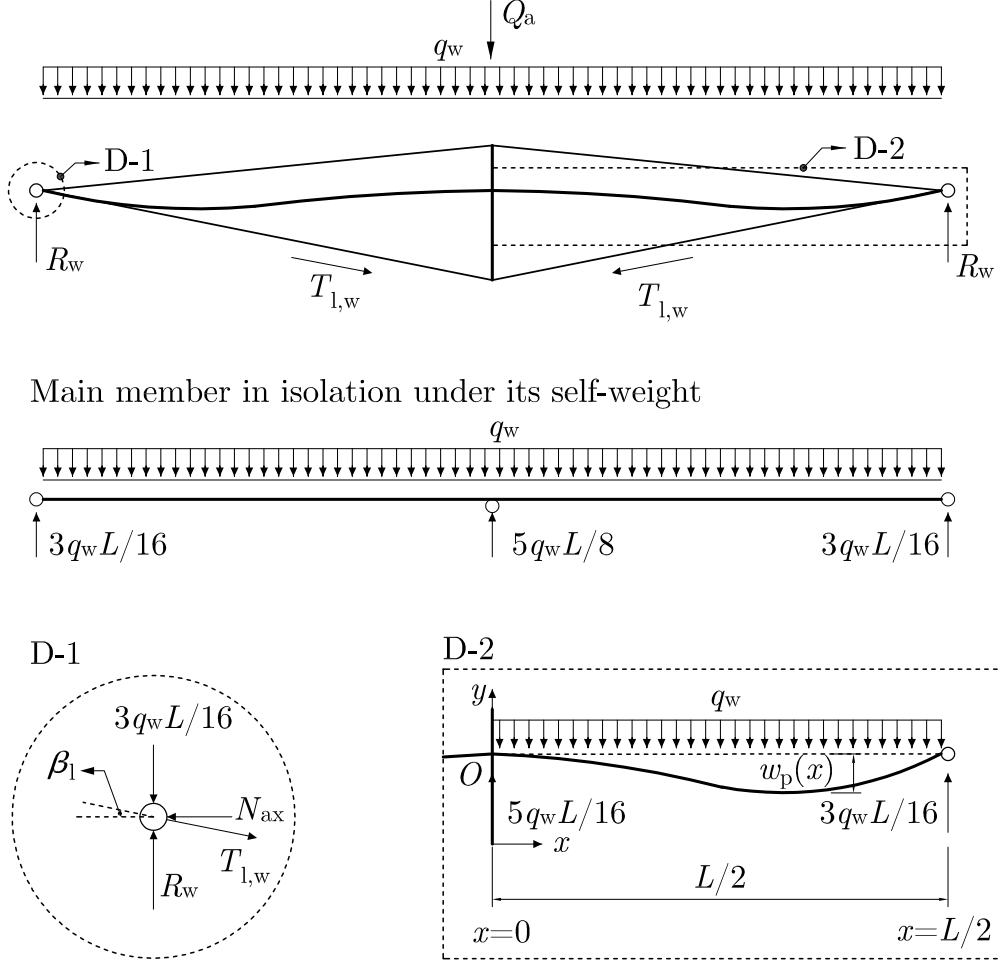


Figure 6: System at the end of loading step 2: the end reactions of main member are derived from a two-span continuous beam under a uniformly distributed load of intensity q_w ; ‘D-1’ shows the free-body diagram of the left-hand pinned support; ‘D-2’ details the simplified beam of the right-hand half of the main member; R_w is the vertical reaction force from the self-weight; N_{ax} is the axial force within the main member; $w_p(x)$ is the simplified beam deflection function for the right-hand half of the main member.

allowing motion in x . Moreover, the member has reflective symmetry about the midspan of the main member. The vertical reaction force R_w is determined thus:

$$R_w = \frac{1}{2}(q_w L + Q_a). \quad (8)$$

The static equilibrium equations for the left-hand support are as follows:

$$\begin{aligned} R_w - T_{l,w} \sin \beta_1 - \frac{3}{16} q_w L &= 0, \\ T_{l,w} \cos \beta_1 - N_{ax} &= 0. \end{aligned} \quad (9)$$

Substituting Eq. (8) into Eq. (9) and solving Eq. (9) gives:

$$N_{ax} = \frac{(5q_w L/8 + Q_a)}{2 \tan \beta_1}, \quad T_{l,w} = T_{l1,w} = T_{l2,w} = \frac{(5q_w L/8 + Q_a)}{2 \sin \beta_1}. \quad (10)$$

The deflection function of the simplified beam for the right-hand half of the main member, $w_p(x)$, can be again determined from Euler–Bernoulli bending theory thus:

$$w_p(x) = \frac{q_w L^2 x^2}{96 E_c I_c} \left(\frac{4x^2}{L^2} - \frac{5x}{L} + \frac{3}{2} \right). \quad (11)$$

The maximum vertical deflection $w_{p,\max}$ is at $x = 0.290L$, which is:

$$w_{p,\max} = \frac{q_w L^4}{2960 E_c I_c}. \quad (12)$$

In addition, the deformation after loading step 2 is illustrated in Figure 7. The dis-

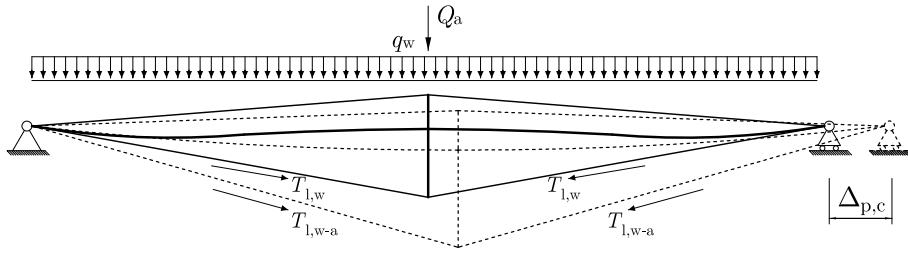


Figure 7: Deformation after loading step 2: $\Delta_{p,c}$ is the end-displacement of the main member.

placement $\Delta_{p,c}$ is given similarly to Eq. (5) but now including the axially compressive deformation, thus:

$$\Delta_{p,c} = \frac{N_{ax}}{K_c} + \frac{1}{2} \int_0^L w_p'^2(x) dx, \quad (13)$$

where the axial stiffness of the main member $K_c = E_c A_c / L$. The axially compressive deformation of the lower crossarm $\Delta_{p,al}$ in loading step 2 can be determined from Hooke's law in one-dimension thus:

$$\Delta_{p,al} = \frac{2T_{l,w} \sin \beta_1}{K_{al}}, \quad (14)$$

where the axial stiffness of the lower crossarm $K_{al} = E_a A_a / a_l$. If all the stays are straight but without stress from the aforementioned assumptions, the length of each stay would be as follows at the end of loading step 2:

$$\begin{aligned} l_{p,u} = l_{p,u1} = l_{p,u2} &= \sqrt{a_u^2 + \left(\frac{L}{2} - \frac{1}{2}\Delta_{p,c}\right)^2}, \\ l_{p,l} = l_{p,l1} = l_{p,l2} &= \sqrt{(a_l - \Delta_{p,a1})^2 + \left(\frac{L}{2} - \frac{1}{2}\Delta_{p,c}\right)^2}. \end{aligned} \quad (15)$$

Hence, the required lower stay prestressing force $T_{l,w-a}$ to achieve $T_{l,w}$ at the beginning of loading step 2, compensating for the end-shortening of the main member, can be expressed thus:

$$T_{l,w-a} = T_{l,w} + \frac{E_s A_s (l_{w,l} - l_{p,l})}{l_{w,l}}, \quad (16)$$

where $T_{l,w}$ was given in Eq. (10).

3.2. Optimum prestressing forces

In the previous sections, the prestressing forces within the lower stays that counteract the self-weight were investigated. However, to improve the buckling behaviour of the stayed beam-columns, additional prestressing forces would be required as in the case of conventional prestressed stayed columns. The processes for applying these additional prestressing forces and then the axially compressive force are in loading steps 3 and 4 respectively (see Figure 3). In the current section, the appropriate levels of the final additional prestressing forces within the upper and lower stays $T_{u,i}$ and $T_{l,i}$ in loading step 3, and the buckling behaviour in loading step 4 corresponding to specific prestressing levels are investigated, which leads to the solutions for the so-called optimum prestressing forces.

From the aforementioned assumptions, the respective prestressing forces within the upper and lower stays T_u and T_l at the end of loading step 3 can be obtained thus:

$$T_u = T_{u,i}, \quad T_l = T_{l,i} + T_{l,w}, \quad (17)$$

where $T_{u,i}$ and $T_{l,i}$ are the additional prestressing forces within the upper and lower stays respectively to improve the buckling behaviour at the end of loading step 3. It should be noted that the following relationship needs to be satisfied to ensure that the internal axial forces within the upper and lower crossarms are equal to leading order (since this equation does not take the induced end-displacement into consideration):

$$T_{u,i} \sin \beta_u = T_{l,i} \sin \beta_l. \quad (18)$$

In order to reach a specific prestressing level accurately at the end of loading step 3, the loss of prestressing owing to the end-displacement of members during loading step 3 is taken into consideration, which is different from previous studies [5, 12]. Hence, prestressing forces $T_{u,i-a}$ and $T_{l,i-a}$, are applied to the stays at the beginning of loading

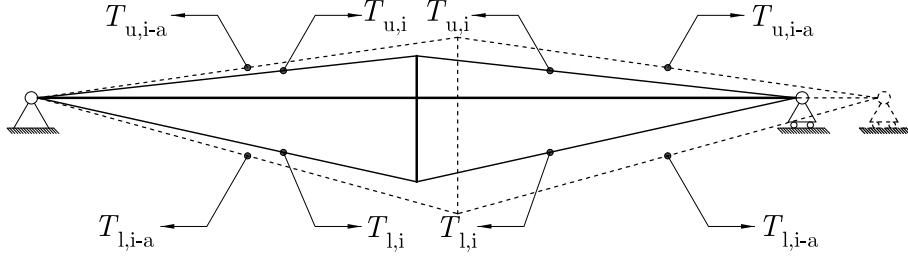


Figure 8: Initial and final prestressing forces ($T_{x,i-a}$, $T_{x,i}$) within loading step 3, where subscript ‘x’ represents either the upper (‘u’) or lower stays (‘l’).

step 3 and they become $T_{u,i}$ and $T_{l,i}$ respectively at the end of loading step 3, which is illustrated in Figure 8. The forces $T_{u,i-a}$ and $T_{l,i-a}$ can be obtained in a similar way to Eq. (16) and expressed respectively thus:

$$\begin{aligned} T_{u,i-a} &= T_{u,i} + \left[\frac{(T_{u,i} \cos \beta_u + T_{l,i} \cos \beta_l) \cos \beta_u}{2K_c} + \frac{2T_{u,i} \sin \beta_u}{K_{au}} \right] \frac{E_s A_s}{l_{p,1}}, \\ T_{l,i-a} &= T_{l,i} + \left[\frac{(T_{u,i} \cos \beta_u + T_{l,i} \cos \beta_l) \cos \beta_l}{2K_c} + \frac{2T_{l,i} \sin \beta_l}{K_{al}} \right] \frac{E_s A_s}{l_{p,3}}, \end{aligned} \quad (19)$$

where the axial stiffness of the upper crossarm is $K_{au} = E_a A_a / a_u$.

To formulate a revised general expression for the optimum prestressing forces similar to previous studies [5, 12], but that can be applied to both $\alpha = 1.0$ (i.e. equal crossarms) and $\alpha < 1.0$ (i.e. the upper crossarm is shorter than the lower crossarm) cases, the structural response in loading step 4 needs to be evaluated. The definitions of the forces in loading step 4 are illustrated in Figure 9, and the geometric details for the deformation analysis in that loading step is shown in Figure 10.

From Figures 9–10, the process of deducing the optimum prestressing forces is now presented. The extensions of the upper stays Δ_{su} and the lower stays Δ_{sl} are related to the change in the length of the main member and crossarms thus:

$$\Delta_{su} = \frac{\Delta_c \cos \beta_u}{2} - \Delta_{au} \sin \beta_u, \quad \Delta_{sl} = \frac{\Delta_c \cos \beta_l}{2} - \Delta_{al} \sin \beta_l. \quad (20)$$

The initial axial force in the main member P_i , induced by the initial prestressing forces within the stays, is:

$$P_i = T_{u,i} \cos \beta_u + T_{l,i} \cos \beta_l, \quad (21)$$

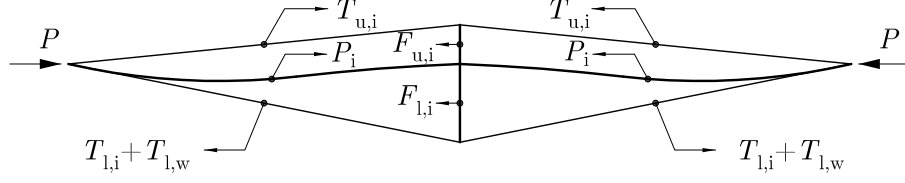
with the forces $F_{u,i}$ and $F_{l,i}$ being expressed respectively thus:

$$F_{u,i} = 2T_{u,i} \sin \beta_u, \quad F_{l,i} = 2T_{l,i} \sin \beta_l. \quad (22)$$

After applying the external axial load P , the final axial force in the main member P_f is:

$$P_f = T_{u,f} \cos \beta_u + T_{l,f} \cos \beta_l + P, \quad (23)$$

Beginning of loading step 4 (subscript i)



End of loading step 4 (subscript f)

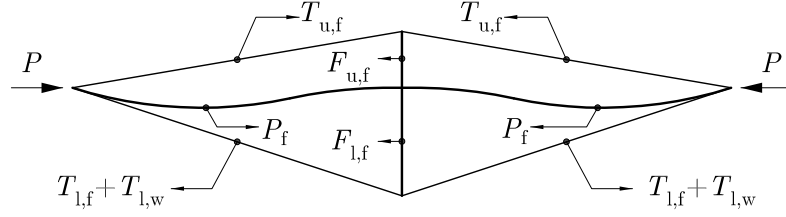


Figure 9: Axial force definitions within loading step 4: P is the externally-applied axial load; P_i and P_f are the internal axial forces within the main member; $F_{u,i}$ and $F_{u,f}$ are the internal axial forces within the upper crossarm; $F_{l,i}$ and $F_{l,f}$ are the internal axial forces within the lower crossarm; $T_{u,i}$ and $T_{u,f}$ are the applied prestressing forces within the upper stays; $T_{l,i}$ and $T_{l,f}$ are the prestressing forces within the lower stays applied in addition to $T_{l,w}$.

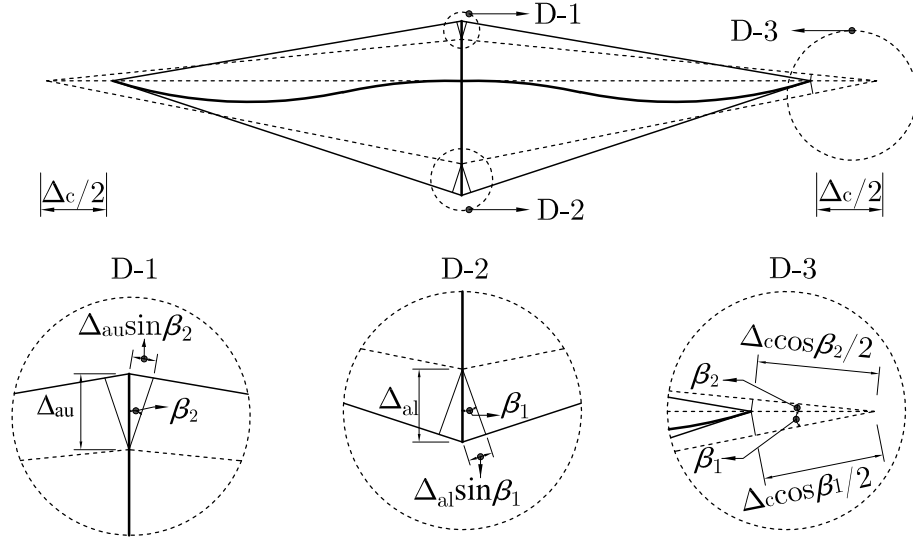


Figure 10: Geometric deflection analysis of prestressed stayed beam-columns in loading step 4: ‘D-1’ is the detailed figure for the deformation at the top of the upper crossarm; ‘D-2’ shows the bottom of the lower crossarm, and ‘D-3’ shows the right-hand pinned support of the main member; Δ_c is the total end-shortening displacement of the main member; Δ_{au} and Δ_{al} are the end-shortening displacements of the upper and lower crossarms respectively.

with the forces $F_{u,f}$ and $F_{l,f}$ being expressed respectively thus:

$$F_{u,f} = 2T_{u,f} \sin \beta_u, \quad F_{l,f} = 2T_{l,f} \sin \beta_l. \quad (24)$$

The main member axial displacement Δ_c can be expressed thus:

$$\Delta_c = \frac{P_f - P_i}{K_c}. \quad (25)$$

Substituting Eqs. (21) and (23) into Eq. (25) gives the following:

$$\Delta_c = \frac{P + (T_{u,f} - T_{u,i}) \cos \beta_u + (T_{l,f} - T_{l,i}) \cos \beta_l}{K_c}, \quad (26)$$

the upper and lower crossarm end-shortening displacements Δ_{au} and Δ_{al} can be expressed thus:

$$\Delta_{au} = \frac{F_{u,i} - F_{u,f}}{K_{au}}, \quad \Delta_{al} = \frac{F_{l,i} - F_{l,f}}{K_{al}}. \quad (27)$$

Substituting Eqs. (22) and (24) into Eq. (27) give the following:

$$\Delta_{au} = \frac{2(T_{u,i} - T_{u,f}) \sin \beta_u}{K_{au}}, \quad \Delta_{al} = \frac{2(T_{l,i} - T_{l,f}) \sin \beta_l}{K_{al}}. \quad (28)$$

The displacement of the upper stays Δ_{su} and the lower stays Δ_{sl} can be expressed thus:

$$\Delta_{su} = \frac{T_{u,i} - T_{u,f}}{K_{su}}, \quad \Delta_{sl} = \frac{T_{l,i} - T_{l,f}}{K_{sl}}, \quad (29)$$

where the stiffnesses of the upper and lower stays are $K_{su} = E_s A_s / l_u$ and $K_{sl} = E_s A_s / l_l$ respectively, with $l_u = l_{u1} = l_{u2}$ and $l_l = l_{l1} = l_{l2}$. Substituting Eqs. (26), (28) and (29) into Eq. (20) gives the following equations:

$$\begin{aligned} \frac{T_{u,i} - T_{u,f}}{K_{su}} = & \left[\frac{P + (T_{u,f} - T_{u,i}) \cos \beta_u + (T_{l,f} - T_{l,i}) \cos \beta_l}{2K_c} \right] \cos \beta_u \\ & - \frac{2(T_{u,i} - T_{u,f}) \sin^2 \beta_u}{K_{au}}. \end{aligned} \quad (30)$$

$$\begin{aligned} \frac{T_{l,i} - T_{l,f}}{K_{sl}} = & \left[\frac{P + (T_{u,f} - T_{u,i}) \cos \beta_u + (T_{l,f} - T_{l,i}) \cos \beta_l}{2K_c} \right] \cos \beta_l \\ & - \frac{2(T_{l,i} - T_{l,f}) \sin^2 \beta_l}{K_{al}}. \end{aligned} \quad (31)$$

Substituting Eqs. (26), (28) and (29) into Eq. (20) give the following equations: Solving Eqs. (30) and (31) gives the following equations:

$$\frac{T_{u,i} - T_{u,f}}{P} = \frac{C_1}{C_3}, \quad \frac{T_{l,i} - T_{l,f}}{P} = \frac{C_2}{C_3}, \quad (32)$$

where:

$$\begin{aligned}
C_1 &= K_{\text{au}} K_{\text{su}} (2K_{\text{sl}} \sin^2 \beta_1 + K_{\text{al}}) \cos \beta_u, \\
C_2 &= K_{\text{al}} K_{\text{sl}} (2K_{\text{su}} \sin^2 \beta_u + K_{\text{au}}) \cos \beta_1, \\
C_3 &= 8K_{\text{c}} K_{\text{sl}} K_{\text{su}} \sin^2 \beta_1 \sin^2 \beta_u + 2K_{\text{al}} K_{\text{sl}} K_{\text{su}} \cos^2 \beta_1 \sin^2 \beta_u \\
&\quad + 2K_{\text{au}} K_{\text{sl}} K_{\text{su}} \cos^2 \beta_u \sin^2 \beta_1 + 4K_{\text{c}} K_{\text{al}} K_{\text{su}} \sin^2 \beta_u + 4K_{\text{c}} K_{\text{au}} K_{\text{sl}} \sin^2 \beta_1 \\
&\quad + K_{\text{al}} K_{\text{au}} K_{\text{sl}} \cos^2 \beta_1 + K_{\text{al}} K_{\text{au}} K_{\text{su}} \cos^2 \beta_u + 2K_{\text{c}} K_{\text{al}} K_{\text{au}}.
\end{aligned} \tag{33}$$

Defining new quantities $C_u = C_1/C_3$ and $C_l = C_2/C_3$, allow the following simplifications:

$$\frac{T_{\text{u,i}} - T_{\text{uf}}}{P} = C_u, \quad \frac{T_{\text{l,i}} - T_{\text{lf}}}{P} = C_l. \tag{34}$$

Combining Eqs. (23) and (26) gives:

$$\Delta_{\text{c}} = \frac{P_{\text{f}} - T_{\text{u,i}} \cos \beta_u - T_{\text{l,i}} \cos \beta_1}{K_{\text{c}}}. \tag{35}$$

Substituting Eqs. (28), (29) and (35) into Eq. (20) gives:

$$\frac{T_{\text{l,i}} - T_{\text{l,f}}}{K_{\text{sl}}} = \left(\frac{P_{\text{f}} - T_{\text{u,i}} \cos \beta_u - T_{\text{l,i}} \cos \beta_1}{2K_{\text{c}}} \right) \cos \beta_1 - \frac{2(T_{\text{l,i}} - T_{\text{l,f}}) \sin^2 \beta_1}{K_{\text{al}}}, \tag{36}$$

hence:

$$T_{\text{l,f}} = -\frac{K_{\text{sl}} K_{\text{al}} [P_{\text{f}} - T_{\text{u,i}} \cos \beta_u - T_{\text{l,i}} \cos \beta_1] \cos \beta_1}{2K_{\text{c}} (2K_{\text{sl}} \sin^2 \beta_1 + K_{\text{al}})} + T_{\text{l,i}}. \tag{37}$$

Substituting Eqs. (28), (29) and (35) into Eq. (20) gives:

$$\frac{T_{\text{u,i}} - T_{\text{uf}}}{K_{\text{su}}} = \left(\frac{P_{\text{f}} - T_{\text{u,i}} \cos \beta_u - T_{\text{l,i}} \cos \beta_1}{2K_{\text{c}}} \right) \cos \beta_u - \frac{2(T_{\text{u,i}} - T_{\text{uf}}) \sin^2 \beta_u}{K_{\text{au}}}, \tag{38}$$

hence:

$$T_{\text{uf}} = T_{\text{u,i}} - \frac{K_{\text{su}} K_{\text{au}} (P_{\text{f}} - T_{\text{u,i}} \cos \beta_u - T_{\text{l,i}} \cos \beta_1) \cos \beta_u}{2K_{\text{c}} (2K_{\text{su}} \sin^2 \beta_u + K_{\text{au}})}. \tag{39}$$

Substituting Eq. (37), Eq. (39) into Eq. (23) after some manipulation gives:

$$\begin{aligned}
P = P_{\text{f}} - & \left[T_{\text{l,i}} - \frac{K_{\text{sl}} K_{\text{al}} (P_{\text{f}} - T_{\text{u,i}} \cos \beta_u - T_{\text{l,i}} \cos \beta_1) \cos \beta_1}{2K_{\text{c}} (2K_{\text{sl}} \sin^2 \beta_1 + K_{\text{al}})} \right] \cos \beta_1 \\
& - \left[T_{\text{u,i}} - \frac{K_{\text{su}} K_{\text{au}} (P_{\text{f}} - T_{\text{u,i}} \cos \beta_u - T_{\text{l,i}} \cos \beta_1) \cos \beta_u}{2K_{\text{c}} (2K_{\text{su}} \sin^2 \beta_u + K_{\text{au}})} \right] \cos \beta_u.
\end{aligned} \tag{40}$$

Defining a new quantity C_0 , thus:

$$C_0 = 1 + \frac{1}{2K_{\text{c}}} \left(\frac{K_{\text{su}} K_{\text{au}} \cos^2 \beta_u}{2K_{\text{su}} \sin^2 \beta_u + K_{\text{au}}} + \frac{K_{\text{sl}} K_{\text{al}} \cos^2 \beta_1}{2K_{\text{sl}} \sin^2 \beta_1 + K_{\text{al}}} \right), \tag{41}$$

and substituting Eq. (41) into Eq. (40), the following expression is obtained:

$$P = (P_f - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l) C_0. \quad (42)$$

From the relationships in Eq. (34), the optimum prestressing forces are determined based on the force within the lower stays being given by the condition $T_{l,f} = 0$, where the lower stay forces become $T_l = T_{l,w}$, or the upper stays becoming slack given by the condition $T_{u,f} = 0$ (since $T_u = 0$ also) when the system buckles thus:

$$T_{\text{opt1}} = P_{\text{max}}^C C_l, \quad T_{\text{opt2}} = P_{\text{max}}^C C_u. \quad (43)$$

Hence, T_{opt1} is the optimum prestressing force based on the condition that $T_{l,f} = 0$ when the beam-column buckles; conversely, T_{opt2} is the optimum prestressing force based on the condition that the upper stays become slack when the beam-column buckles. This leads to the evaluation of the maximum critical buckling load P_{max}^C , thus:

$$P_{\text{max}}^C = \frac{P_{\text{FE},T=0}^C}{C_0}, \quad (44)$$

where $P_{\text{FE},T=0}^C$ is evaluated using linear buckling analysis within FE after loading step 2; in the current case this is obtained using the commercial code ABAQUS [22] and is similar to the previously presented procedure [14].

From Eq. (18), when $T_{l,i} = T_{\text{opt1}}$, the optimum prestressing force within the upper stays is defined as T_{opt12} and can be expressed thus:

$$T_{\text{opt12}} = \frac{T_{\text{opt1}} \sin \beta_l}{\sin \beta_u}. \quad (45)$$

Similarly when $T_{u,i} = T_{\text{opt2}}$, the optimum prestressing forces within the lower stays is defined as T_{opt21} and can be expressed thus:

$$T_{\text{opt21}} = \frac{T_{\text{opt2}} \sin \beta_u}{\sin \beta_l}. \quad (46)$$

For the beam-column with double symmetry ($\alpha = 1.0$), $C_l = C_u$, and the expressions reduce to those in previous work [5, 12]. When $\alpha < 1.0$, however, $C_l < C_u$ and the currently developed expressions need to be applied.

In addition, this solution can be adjusted to include the effect of support settlement and temperature changes in practice. In terms of the axially compressive force within the main member, it can be monitored and adjusted using an active force control device to maintain the required compressive load precisely, which can counteract the axial force change owing to these two effects [24]; in terms of the stress within stays, the stress change from these two effects can be taken into account as part of an equivalent prestress, thus:

$$T_{x,i} = T_{\text{equivalent}} + T_{\text{applied}}, \quad (47)$$

where $x = \{u, l\}$; $T_{\text{equivalent}}$ is the equivalent prestressing force accounting for effects of support settlement and temperature; T_{applied} is the externally applied prestressing force, and $T_{x,i}$ can be set to the desired level as described above.

3.3. Zones of behaviour

Zones of behaviour have been identified in previous studies on prestressed stayed columns under purely axial compression [5, 12] to define regions of distinct mechanical behaviour; in the current work, a set of revised zones that are associated with distinct mechanical responses for beam-columns with double-symmetry ($\alpha = 1.0$) and with mono-symmetry ($\alpha < 1.0$) are introduced. Figure 11 provides a review of the three zones of

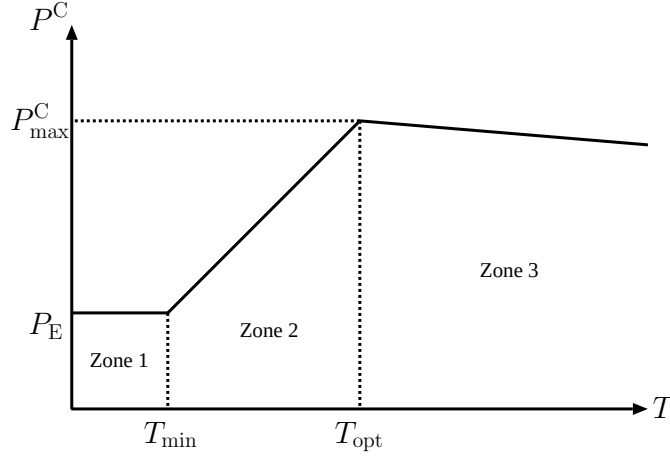


Figure 11: Zones of behaviour for prestressed stayed columns. The abscissa is the applied prestressing force within the cable stays T and the ordinate is the critical buckling load P^C .

behaviour for the prestressed stayed column that were identified previously [5, 12]:

Zone 1 (column) – defined where the initial prestressing force within the cable stays $T = [0, T_{\min}]$, where under axial compression the prestressing force within the stays is released entirely before buckling occurs and the column buckles at the Euler load of the bare column, hence: $P^C = P_E$.

Zone 2 (column) – defined where the initial prestressing force within the cable stays $T = [T_{\min}, T_{\text{opt}}]$, where the column buckles above the Euler load since the stays are still under (a reduced) tension when the axial load is P_E . Subsequently, however, the column buckles as soon as the cable tension is released completely, hence $P^C = [P_E, P_{\max}^C]$.

Zone 3 (column) – defined where the initial prestressing force within the cable stays $T > T_{\text{opt}}$, where the column buckles while there is still tension within the cable stays. However, since increasing the prestress introduces more pre-compression in the main column, the critical load reduces as T is increased, hence: $P^C \leq P_{\max}^C$.

3.3.1. Beam-columns with double-symmetry ($\alpha = 1.0$)

In terms of the specific case where $\alpha = 1.0$, the minimum prestressing force $T_{\min}$, which marks the boundary between Zones 1 and 2 (i.e. the prestressing level that ensures at least two stays are active at the point of buckling), can be determined from the relationships in

Eq. (34). This is based on the force within the lower stays being given by the condition $T_{l,f} = 0$ (where $T_l = T_{l,w}$) or the upper stays become slack $T_{u,f} = 0$ when the system buckles thus:

$$T_{\min} = P_E C_l = P_E C_u, \quad (48)$$

where P_E is the Euler buckling load for a simply-supported strut with the cross-sectional properties of the main column element:

$$P_E = \frac{\pi^2 E_c I_c}{L^2}. \quad (49)$$

The prestressing forces marking the boundary between Zones 2 and 3 (i.e. the prestressing level that ensures all stays are active at the point of buckling), $T_{\text{opt}1}$, $T_{\text{opt}2}$, $T_{\text{opt}12}$ and $T_{\text{opt}21}$ can be calculated from Eqs. (43), (45) and (46) respectively. For this case, $T_{\text{opt}} = T_{\text{opt}1} = T_{\text{opt}12} = T_{\text{opt}2} = T_{\text{opt}21}$. In Zone 1 ($T_{u,i} = T_{l,i} = [0, T_{\min}]$), the critical buckling load P_{Z1}^C is a constant, thus:

$$P_{Z1}^C = P_E. \quad (50)$$

In Zone 2, where $T_{u,i} = T_{l,i} = [T_{\min}, T_{\text{opt}}]$, $T_{u,f} = 0$ and $T_{l,f} = 0$ simultaneously when the system buckles, both the upper and lower stays provide stiffness until buckling. From Eq. (34), the critical buckling load P_{Z2}^C is given thus:

$$P_{Z2}^C = \frac{T_{u,i}}{C_u} = \frac{T_{l,i}}{C_l}. \quad (51)$$

In Zone 3, where $T_{u,i} = T_{l,i} > T_{\text{opt}}$, the critical buckling load P_{Z3}^C is given from Eq. (42), thus:

$$P_{Z3}^C = (P_{\max}^C - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l) C_0. \quad (52)$$

The critical buckling load P^C reaches the maximum $P_{\max}^C$ (from Eq. (44)) where $T_{u,i} = T_{l,i} = T_{\text{opt}}$.

3.3.2. Beam-columns with mono-symmetry ($\alpha < 1.0$)

In terms of the case where $\alpha < 1.0$, from the outlined assumptions, the reference minimum prestressing forces $T_{\min1,\text{ref}}$ and $T_{\min2,\text{ref}}$, can be determined from the relationships in Eq. (34), based on the condition where the upper stays become slack ($T_{u,f} = 0$), or the force within the lower stays is given by the condition $T_{l,f} = 0$, when the system buckles thus:

$$T_{\min1,\text{ref}} = P_E C_l, \quad T_{\min2,\text{ref}} = P_E C_u. \quad (53)$$

From Eq. (18), when $T_{l,i} = T_{\min1,\text{ref}}$, the minimum prestressing force within the upper stays, defined as $T_{\min12,\text{ref}}$, can be expressed thus:

$$T_{\min12,\text{ref}} = \frac{T_{\min1,\text{ref}} \sin \beta_l}{\sin \beta_u}. \quad (54)$$

Since $T_{\min2,\text{ref}} < T_{\min12,\text{ref}}$ when $\alpha < 1.0$, there would be a sub-zone within Zone 2 where there are residual forces within the upper stays and zero stress within the lower stays

under the externally-applied axial load $P = P_E$. In this sub-zone, the lower stays would be inactive before the system buckles while the upper stays would be active until the system buckles. For this case, the expressions for C_1 and C_u from Eq. (33), which are based on both the upper and lower stays being active, cannot be applied. Repeating the process in the previous sections gives revised expressions for parameters $C_{u,Z2}$ (without considering the lower stays and crossarm) and $C_{l,Z2}$ (without considering the upper stays and crossarm), thus:

$$\begin{aligned} C_{u,Z2} &= \frac{K_{au}K_{su} \cos \beta_u}{2K_cK_{au} + K_{au}K_{su} \cos^2 \beta_u + 4K_cK_{su} \sin^2 \beta_u}, \\ C_{l,Z2} &= \frac{K_{al}K_{sl} \cos \beta_l}{2K_cK_{al} + K_{al}K_{sl} \cos^2 \beta_l + 4K_cK_{sl} \sin^2 \beta_l}. \end{aligned} \quad (55)$$

The minimum prestressing forces $T_{\min1}$ and $T_{\min2}$, which marks the boundary between Zones 1 and 2, can be determined from the relationships in Eq. (34), based on the condition that the upper stays become slack ($T_{u,f} = 0$) or the force within the lower stays being given by the condition $T_{l,f} = 0$ when the system buckles, thus:

$$T_{\min1} = P_E C_{l,Z2}, \quad T_{\min2} = P_E C_{u,Z2}. \quad (56)$$

From Eq. (18), when $T_{l,i} = T_{\min1}$, the minimum prestressing force within the upper stays, defined as $T_{\min12}$, can be expressed thus:

$$T_{\min12} = \frac{T_{\min1} \sin \beta_l}{\sin \beta_u}. \quad (57)$$

Similarly, when $T_{u,i} = T_{\min2}$, the minimum prestressing force within the lower stays, defined as $T_{\min21}$, can be expressed thus:

$$T_{\min21} = \frac{T_{\min2} \sin \beta_u}{\sin \beta_l}. \quad (58)$$

With the prestressing levels within all the stays increasing, there is an additional sub-zone within Zone 2 where both the upper and lower stays are active until the system buckles, and the parameters obtained from the previous sections can be applied. The prestressing forces marking the boundary between Zones 2 and 3, $T_{\text{opt}1}$, $T_{\text{opt}2}$, $T_{\text{opt}12}$ and $T_{\text{opt}21}$ can be calculated from Eqs. (43), (45) and (46) respectively. In the current formulation, for most cases with $\alpha < 1.0$ (apart from $\alpha \rightarrow 0$), $T_{\min21} < T_{\min1} < T_{\text{opt}21} < T_{\text{opt}1}$ corresponding to $T_{\min2} < T_{\min12} < T_{\text{opt}2} < T_{\text{opt}12}$. The details of the responses within each zone can be described as follows:

Zone 1 – when $T_{l,i} = [0, T_{\min21}]$, corresponding to $T_{u,i} = [0, T_{\min2}]$, on applying P , the prestressing forces would disappear (excluding $T_{l,w}$ within the lower stays from the aforementioned assumptions) owing to the axial end-shortening displacement before Euler buckling occurs, and so the response replicates that of unstayed beam-columns. The expression for the buckling load P_{Z1}^C is thus:

$$P_{Z1}^C = P_E. \quad (59)$$

Zone 2 – when $T_{l,i} = [T_{\min 21}, T_{\text{opt}21}]$, corresponding to $T_{u,i} = [T_{\min 2}, T_{\text{opt}2}]$, on applying P , the prestressing forces within all the stays would disappear when the system buckles (excluding $T_{l,w}$ within the lower stays as described above), generally at a higher load compared to Zone 1. However, there are two different sub-zones within Zone 2, currently named ‘Zone 2-1’ and ‘Zone 2-2’ that are characterized from whether a residual force within the lower stays exists when the external load $P = P_E$, thus:

Zone 2-1 – when $T_{l,i} = [T_{\min 21}, T_{\min 1}]$, corresponding to $T_{u,i} = [T_{\min 2}, T_{\min 12}]$, and $P = P_E$, there would be a residual force within the upper stays (*i.e.* $T_{u,f} > 0$) while $T_{l,f} = 0$ within the lower stays. The restraining effect from the residual force within the upper stays only would increase P^C , where $P^C = [P_E, P_{\max,2-1}^C]$, with $P_{\max,2-1}^C$ being the maximum critical buckling load in this sub-zone. The expression for the buckling load P_{Z2-1}^C is:

$$P_{Z2-1}^C = \frac{T_{u,i}}{C_{u,Z2-1}}, \quad (60)$$

where $C_{u,Z2-1} = C_{u,Z2}$ in Eq. (55), and $P_{\max,2-1}^C = T_{\min 12}/C_{u,Z2-1}$.

Zone 2-2 – when $T_{l,i} = [T_{\min 1}, T_{\text{opt}21}]$, corresponding to $T_{u,i} = [T_{\min 12}, T_{\text{opt}2}]$, and $P = P_E$, there would be a residual force within both the upper and lower stays, *i.e.* $T_{u,f} > 0$ and $T_{l,f} > 0$. The restraining effect from the residual forces within both the upper and lower stays would increase P^C further, and $P^C = [P_{\max,2-1}^C, P_{\max,1}^C]$, where $P_{\max,1}^C$ is the maximum critical buckling load in this sub-zone. The expression for the buckling load P_{Z2-2}^C is determined from linear interpolation thus:

$$P_{Z2-2}^C = P_{\max,2-1}^C + \left[\frac{P_{\max,1}^C - P_{\max,2-1}^C}{T_{\text{opt}2} - T_{\min 12}} \right] (T_{u,i} - T_{\min 12}), \quad (61)$$

where:

$$P_{\max,1}^C = P_{\max}^C = \frac{P_{\text{FE},T=0}^C}{C_0}. \quad (62)$$

Zone 3 – when $T_{l,i} > T_{\text{opt}21}$, corresponding to $T_{u,i} > T_{\text{opt}2}$, on applying P , the beam-column buckles while residual forces exist within all the stays. However, in a similar fashion to Zone 2, there are two different sub-zones within Zone 3, currently named ‘Zone 3-1’ and ‘Zone 3-2’ that are characterized by whether or not a residual force within the lower stays exists at the point of buckling, thus:

Zone 3-1 – when $T_{l,i} = [T_{\text{opt}21}, T_{\text{opt}1}]$, corresponding to $T_{u,i} = [T_{\text{opt}2}, T_{\text{opt}12}]$, on applying P , the force within the lower stays would reduce to $T_{l,w}$, but there would be a residual force within the upper stays ($T_{u,f} > 0$) when the system buckles. The upper-stay residual force would decrease P^C , and $P^C = [P_{\max,2}^C, P_{\max,1}^C]$, where $P_{\max,2}^C$ marks the minimum critical buckling load in this sub-zone. The

expression for the buckling load P_{Z3-1}^C is determined from linear interpolation thus:

$$P_{Z3-1}^C = P_{\max,1}^C + \left[\frac{P_{\max,2}^C - P_{\max,1}^C}{T_{\text{opt}12} - T_{\text{opt}2}} \right] (T_{u,i} - T_{\text{opt}2}), \quad (63)$$

where the minimum critical buckling load $P_{\max,2}^C$ in this sub-zone is given from Eq. (42), thus:

$$P_{\max,2}^C = (P_{\max}^C - T_{\text{opt}12} \cos \beta_u - T_{\text{opt}1} \cos \beta_l) C_0. \quad (64)$$

Zone 3-2 – when $T_{l,i} > T_{\text{opt}1}$, corresponding to $T_{u,i} > T_{\text{opt}12}$, on applying P , the force within the lower and upper stays would both have residual forces when the system buckles, i.e. $T_{l,f} > 0$ and $T_{u,f} > 0$. Both the upper-stay and lower-stay residual forces would decrease P^C , and $P^C < P_{\max,2}^C$. The expression for the buckling load P_{Z3-2}^C can be determined from Eq. (42), thus:

$$P_{Z3-2}^C = (P_{\max}^C - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l) C_0. \quad (65)$$

The aforementioned zones of behaviour of prestressed stayed beam-columns with $\alpha < 1.0$ are illustrated in Figure 12.

4. Numerical verification

The principal purpose of the current section is to investigate the buckling and post-buckling behaviour of stayed beam-columns in the horizontal plane. It should be noted presently that initial geometric imperfections are not taken into consideration since the self-weight serves the purpose of perturbing the geometry to enable the buckling problem to be solved. Firstly, using linear buckling analysis in ABAQUS [22], the optimum prestressing forces are obtained by substituting Eq. (44) into Eq. (43), and $T_{l,i} = T_{\text{opt}1}$ is selected as the benchmark prestressing force for the lower stays corresponding to $T_{u,i} = T_{\text{opt}12}$ for the upper stays; subsequently, the nonlinear mechanical behaviour is studied using Riks analysis.

Apart from the lower-crossarm length a_l , stay diameter ϕ_s and crossarm-length ratio α , the material properties and structural dimensions of prestressed stayed beam-columns in the current study are fixed and are listed in Table 1. In terms of doubly-symmetric cases, 12 configurations, as defined in [14], listed as F1–F6 and A1–A6 are studied. In terms of the mono-symmetric cases F3.1 to F3.9 and F4.1 to F4.9 (inclusive), the crossarm-length ratio α is varied from 0.1 to 0.9 (inclusive), while the stay diameter and the lower-crossarm length remain the same as in cases F3 and F4 respectively. The detailed parameters of each studied case are listed in Tables 2, 3 and 4. Changing ϕ_s allows the stiffness of the cable to be varied; crossarm length ratio α_l , α_u and α defined in Eqs. (1) and (2) allows the angle of the stays relative to the main element to be varied, since $\beta_l = \arctan 2\alpha_l$ and $\beta_u = \arctan 2\alpha_u$.

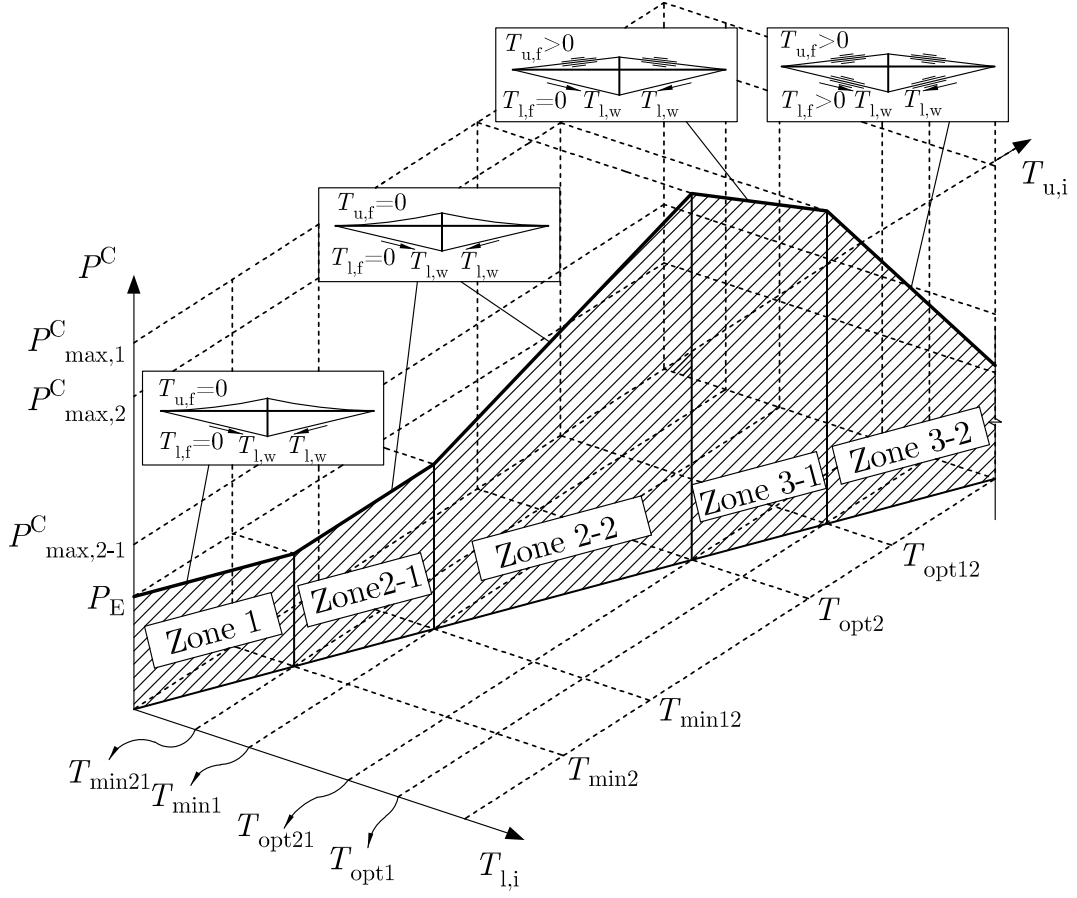
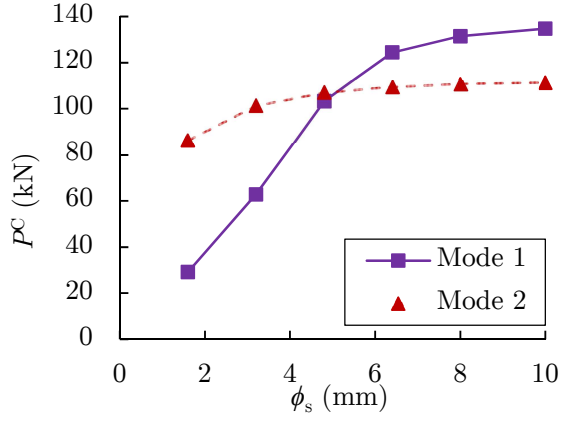


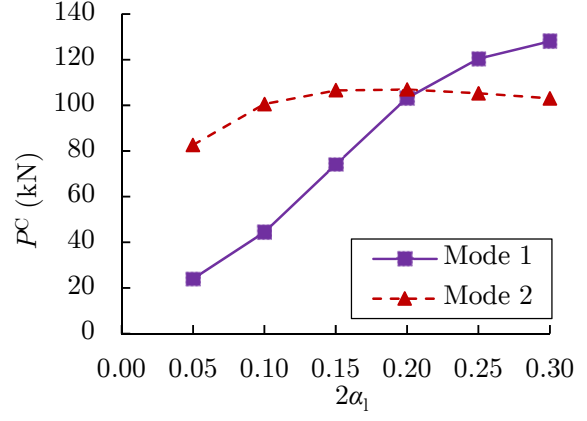
Figure 12: Zones of behaviour for prestressed stayed beam-columns with $\alpha < 1.0$: the vertical coordinate marks the critical buckling load of the beam-column for given combinations of initial prestressing forces within the upper and lower stays.

Table 1: Material properties and structural dimensions of the prestressed stayed beam-columns in the current study.

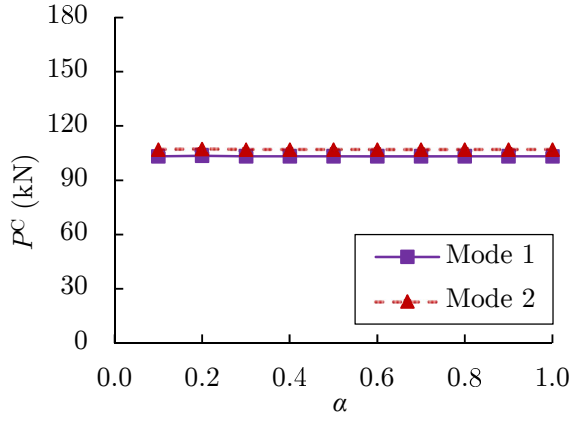
Length of main member	$L = 3050 \text{ mm}$
Main member Young's modulus	$E_c = 201 \text{ kN/mm}^2$
Crossarm Young's modulus	$E_a = 201 \text{ kN/mm}^2$
Stay Young's modulus	$E_s = 202 \text{ kN/mm}^2$
Outside diameter of main member	$\phi_{co} = 38.1 \text{ mm}$
Inside diameter of main member	$\phi_{ci} = 25.4 \text{ mm}$
Outside diameter of crossarm	$\phi_{ao} = 38.1 \text{ mm}$
Inside diameter of crossarm	$\phi_{ai} = 25.4 \text{ mm}$
Material density (steel)	$\rho = 7850 \text{ kg/m}^3$
Gravitational acceleration	$g = 9.81 \text{ m/s}^2$



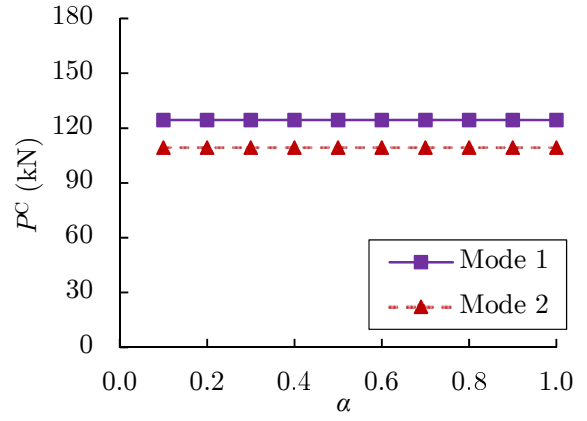
(a) F1–F6



(b) A1–A6



(c) F3.1–F3.9 & F3



(d) F4.1–F4.9 & F4

Figure 13: Linear buckling loads after applying the self-weight in loading step 2: ‘Mode 1’ represents the buckling load of the eigenmode profile that has reflective symmetry about midspan; ‘Mode 2’ represents the buckling load of the eigenmode profile that has rotational symmetry about midspan.

Table 2: Detailed parameters of studied cases where $\alpha = 1.0$. In cases F1–F6: $\alpha_1 = 0.100$ throughout; cases A1–A6: $\phi_s = 4.8$ mm throughout.

Case	F1	F2	F3	F4	F5	F6
ϕ_s (mm)	1.6	3.2	4.8	6.4	8.0	10.0
Case	A1	A2	A3	A4	A5	A6
α_1	0.025	0.05	0.075	0.100	0.125	0.150

Table 3: Detailed parameters of studied cases F3.1–F3.9 where $\phi_s = 4.8$ mm throughout.

Case	F3.1	F3.2	F3.3	F3.4	F3.5	F3.6	F3.7	F3.8	F3.9
α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

4.1. Buckling analysis

Figure 13 shows the buckling loads for ‘mode 1’ (symmetric eigenmode) and ‘mode 2’ (antisymmetric eigenmode), which are illustrated in Figure 14, after loading step 2 from linear buckling analysis within ABAQUS. From the expression of the optimum prestressing forces, the lower-stay optimum prestressing force T_{opt1} can be calculated. It should be noted that the linear buckling loads for modes 1 and 2 are used as $P_{\text{FE},T=0}^C$ in Eq. (44). Based on Figures 13(a–b), in terms of the doubly-symmetric models F1–F6 and A1–A6, it is evident that the critical buckling mode changes from mode 1 to 2 with the stay diameter increasing, and the same trend is found when the crossarm length is increased. This conclusion is similar to that drawn in previous studies of stayed columns [14, 16]. The graphs in Figures 13(c–d) show that the governing buckling mode and load for the mono-symmetric models considering the self-weight is fundamentally independent of the crossarm ratio α .

4.2. Zonal boundaries

Using the benchmark prestressing forces where $T_{l,i} = T_{\text{opt1}}$ and $T_{u,i} = T_{\text{opt12}}$, the buckling and post-buckling behaviour is investigated using Riks analysis [25] within ABAQUS. It should be noted that the four loading steps are adopted, as described in Figure 3. In addition, the following initial prestressing force levels within the lower stays are considered:

$$T_{l,i}/T_{\text{opt1}} = \{0, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75, 3.00\} \quad (66)$$

and the corresponding upper-stay prestressing forces ($T_{u,i}$) can be obtained from Eq. (18). Based on this, the ultimate load of each model is investigated using nonlinear FE modelling. To verify the profile of the response zones identified in the previous section (Figure 12),

Table 4: Detailed parameters of studied cases F4.1–F4.9 where $\phi_s = 6.4$ mm throughout.

Case	F4.1	F4.2	F4.3	F4.4	F4.5	F4.6	F4.7	F4.8	F4.9
α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

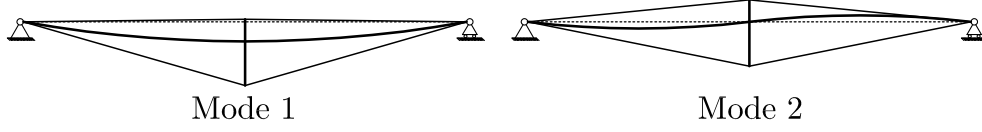


Figure 14: Illustration of buckling eigenmodes 1 (symmetric) and 2 (antisymmetric).

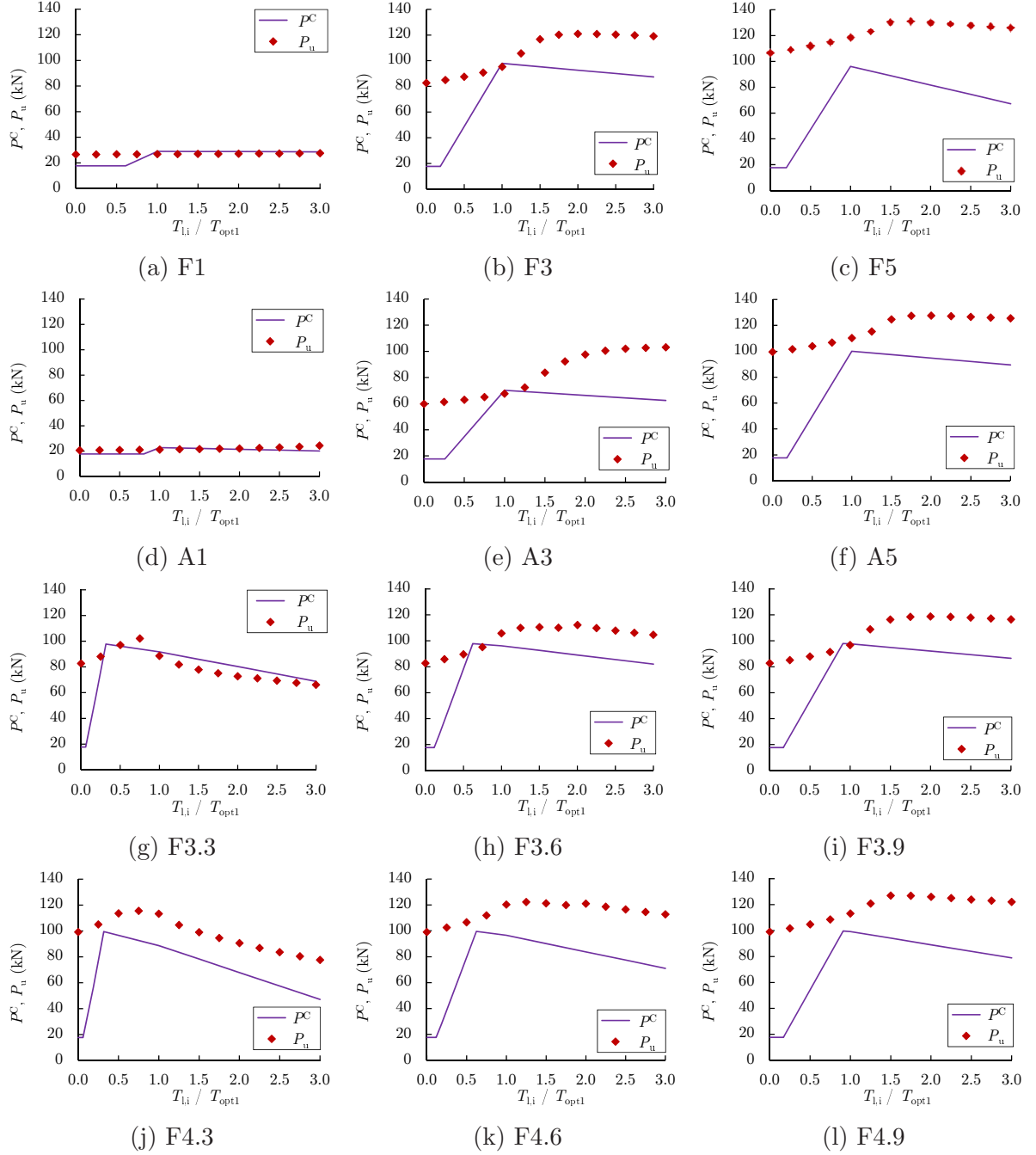


Figure 15: Plots of the analytically derived critical buckling loads P^C and numerically (FE) derived ultimate loads P_u versus the lower-stay relative prestressing level T_{li}/T_{opt1} .

both the analytical critical buckling loads P^C and the ultimate loads P_u from nonlinear FE analysis are presented in Figure 15. The methodology of using nonlinear FE analysis to verify the quality of the buckling load predictions has been used in earlier work [26]. This is because in the current system, the self-weight introduces eccentricities that initiates bending within the system as soon as the external axial load is applied. Therefore, it is not possible to detect a buckling instability in the classical (linear) sense since the behaviour within the FE model is always nonlinear. Hence, an analysis of the post-buckling response is used to infer the quality of the analytical model predictions; this also has the advantage of determining whether there is any significant post-critical stiffness and strength that can be exploited.

In the doubly-symmetric cases presented, Figures 15(a–c) show that when the stay diameter ϕ_s is relatively large, a significantly higher load-carrying capacity can be obtained if the prestressing level within the lower stays is higher than T_{opt1} . For example, in terms of F3 and F5, the ultimate loads increase significantly when $T_{\text{l,i}}/T_{\text{opt1}}$ increases, while in case F1, the ultimate loads and the critical buckling loads basically coincide for $T_{\text{l,i}}/T_{\text{opt1}} \geq 1$. As can be seen, these trends are also observed in Figures 15(d–f) respectively where the crossarm lengths are progressively increased.

In the mono-symmetric cases presented, Figures 15(g–i) show that when α is small, higher prestressing levels may lead to a significant decline of the buckling load based on both the analytical curve and nonlinear FE results, *i.e.* the ultimate load of F3.3, with $\alpha = 0.3$, decreases obviously when $T_{\text{l,i}}/T_{\text{opt1}} > 0.75$ and essentially shadows the trend of the critical buckling load. However, in terms of cases F3.6 and F3.9 with a relatively larger value of α , the ultimate load becomes significantly higher than the critical buckling load where the lower-stay initial prestressing force $T_{\text{l,i}}/T_{\text{opt1}} > 1.0$ and a clear maximum can be observed. It can also be observed that similar trends appear in Figures 15(j–l), although the stay diameter and the critical buckling mode are different to those in Figures 15(g–i). Finally, as for the case of the prestressed stayed column under pure axial compression [12, 13, 16], it is significant that T_{opt1} , obtained from purely linear analysis, is not the true optimum prestressing force for the lower stays in the sense that it does not maximize the load-carrying capacity. Hence, introducing higher prestressing levels would be recommended in practice as determined in previous studies [27, 17, 28, 29, 26].

Note that the analytical results presented in Figure 15 are critical buckling loads P^C , while the numerical results are ultimate loads P_u . The critical buckling loads serve as reference values for the basis of the establishment of future design rules, while the difference between P^C and P_u reflects the influence of geometric imperfections, residual stresses, second-order effects and post-buckling stability. An analytical model that includes second-order effects is currently under development.

4.3. Nonlinear behaviour

To examine the nonlinear equilibrium behaviour of stayed beam-columns from various perspectives, the prestressing force within the lower stays $T_{\text{l,i}}$ is now fixed to T_{opt1} and for the upper stays $T_{\text{u,i}} = T_{\text{opt12}}$. Figure 16 presents equilibrium diagrams of the externally-applied axial load P versus the corresponding end-shortening displacement $\mathcal{E}$ to show the

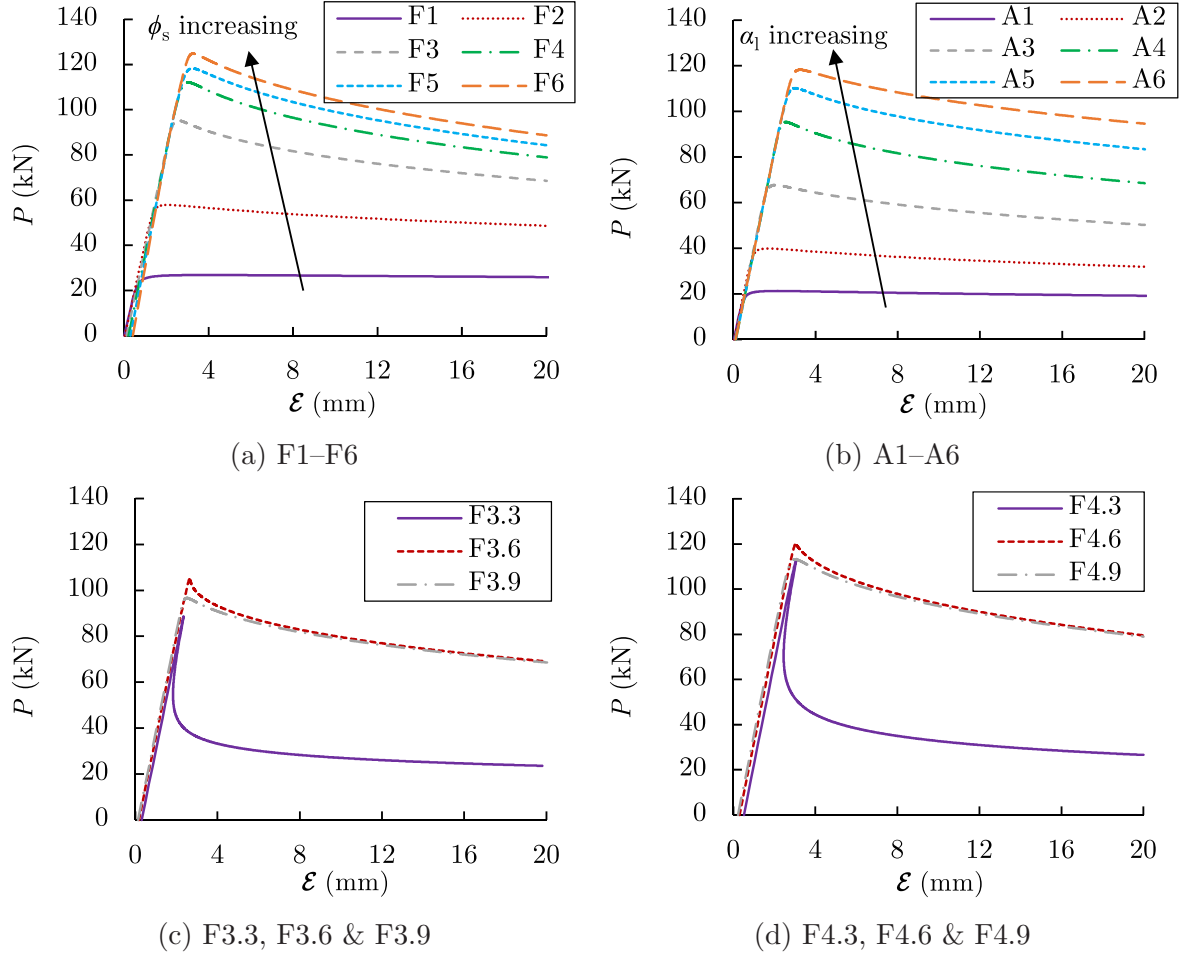


Figure 16: Externally-applied axial load P versus the end-shortening displacement ε of the main member when $T_{l,i}/T_{opt1} = 1.0$.

post-buckling behaviour of the beam-columns. For the doubly-symmetric cases, the graphs in Figures 16(a–b) indicate that increasing the stay diameter or the crossarm length both increase the ultimate load, but the post-buckling response is slightly more unstable when these two parameters are larger. For the mono-symmetric cases, it can be observed in the graphs in Figures 16(c–d) that when α is small, such as in cases F3.3 and F4.3, the post-buckling response is hugely unstable when compared to models with a larger α (cases F3.6, F3.9, F4.6 and F4.9). Tables 5 and 6 present the corresponding numerical ultimate loads.

Table 5: Ultimate loads of F1–F6 and A1–A6 when $T_{l,i}/T_{opt1} = 1.0$.

Case	F1	F2	F3	F4	F5	F6
P_u (kN)	27.5	59.2	96.7	112.7	118.6	124.8
Case	A1	A2	A3	A4	A5	A6
P_u (kN)	21.9	41.0	69.0	96.7	111.0	118.7

Table 6: Ultimate loads of cases F3.1–F3.9 and F4.1–F4.9 when $T_{l,i}/T_{opt1} = 1.0$.

Case	F3.1	F3.2	F3.3	F3.4	F3.5	F3.6	F3.7	F3.8	F3.9
P_u (kN)	34.2	60.6	85.8	106.0	107.3	107.7	105.3	101.1	98.3
Case	F4.1	F4.2	F4.3	F4.4	F4.5	F4.6	F4.7	F4.8	F4.9
P_u (kN)	42.0	84.0	112.2	117.4	120.7	121.2	117.9	115.4	113.9

It is evident that the ultimate load increases with the stay diameter or crossarm length increasing as the effectiveness of the stays in resisting structural instability is also increased. It should also be noted that in the considered mono-symmetric cases, the ultimate load reaches the maximum value when $\alpha = 0.6$, *i.e.* in cases F3.6 and F4.6.

4.3.1. Sensitivity to prestress

Since it was revealed that the buckling and post-buckling behaviour of stayed members is strongly linked to the prestressing level in previous studies [12, 14], the effect of an increase in the initial prestress is investigated currently using nonlinear FE analysis. Two different levels of prestressing forces, $T_{l,i}/T_{opt1} = \{1.0, 2.0\}$, are adopted. It should be noted that the corresponding upper-stay prestressing force satisfies the relationship in Eq. (18). The graphs in Figure 17 show comparisons of the curves of the externally-applied axial load P versus the end-shortening displacement $\mathcal{E}$ of the main member between these two cases. It is observed that in terms of the doubly-symmetric models, F1–F6 and A1–A6, in Figures 17(a–b), the higher prestressing level increases the ultimate load, which can also be observed in Figure 15. However, the post-buckling response is relatively more unstable at $T_{l,i}/T_{opt1} = 2.0$ when compared to $T_{l,i}/T_{opt1} = 1.0$ in the initial post-buckling range but the residual capacity is generally similar, if slightly higher; these conclusions are similar to those in the previous studies on the purely axially-loaded cases [12, 14].

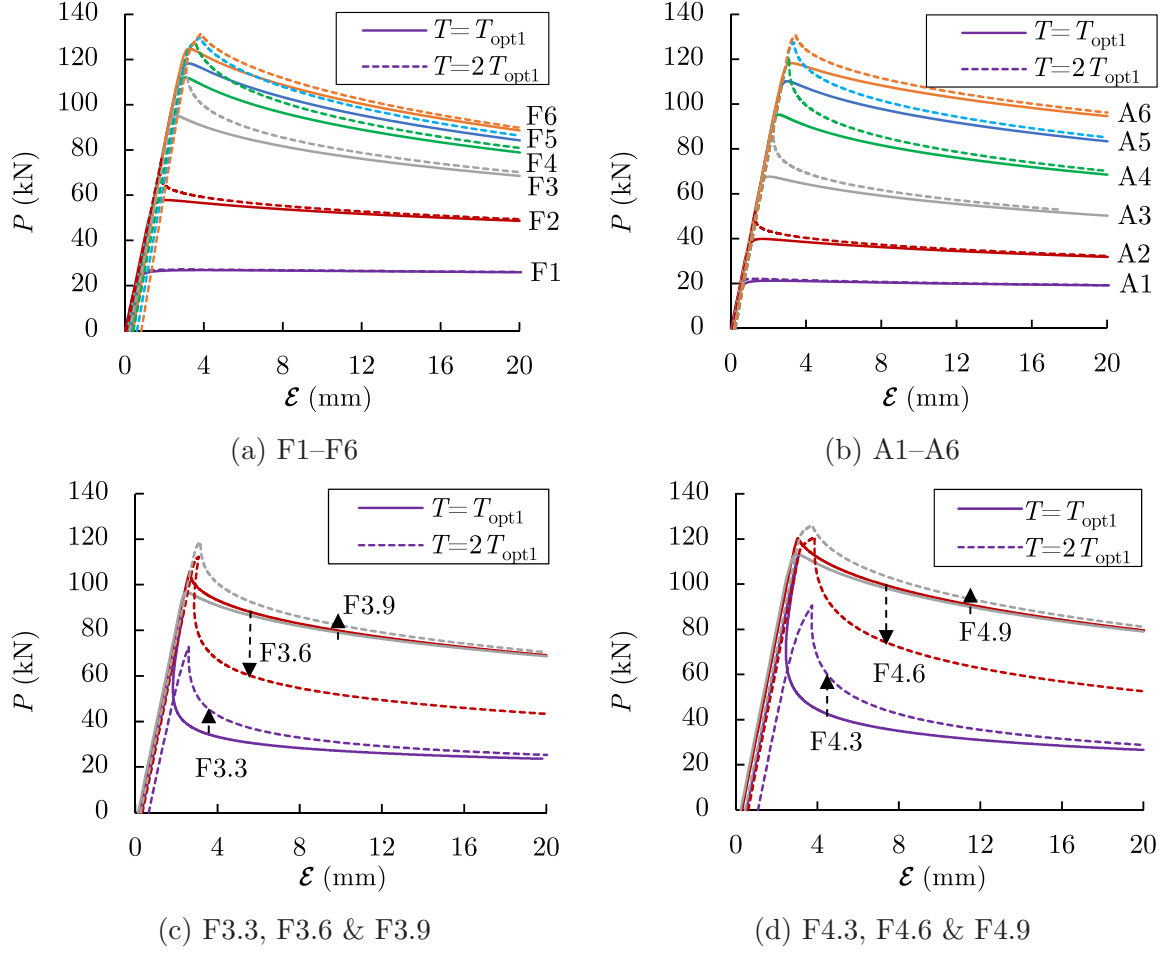


Figure 17: Comparisons of the externally-applied axial load P versus the end-shortening displacement $\mathcal{E}$ of the main member between $T_{l,i}/T_{opt1} = 1.0$ and 2.0 .

In terms of the mono-symmetric models, F3.1–F3.9 and F4.1–F4.9, when α is small, such as in cases F3.3 and F4.3, the higher prestressing level reduces the ultimate load, but increases the post-buckling stability. When α is assigned an intermediate value, such as in cases F3.6 and F4.6, where $\alpha = 0.6$, the higher prestressing level increases the ultimate load, but decreases the post-buckling stability significantly. Finally, when α is relatively large, such as in cases F3.9 and F4.9, these behave similarly to the doubly-symmetric cases. A reason for these variations in response owing to the lower values of the mono-symmetry parameter α may be attributed to the more acute pre-cambering effect at higher prestressing levels.

4.3.2. Effect of pre-cambering

To study the post-buckling response further, the vertical displacement at midspan w_m and the forces within each stay T_n , where $n = \{1, 2, u, v\}$ (see Figures 18 and 19) are

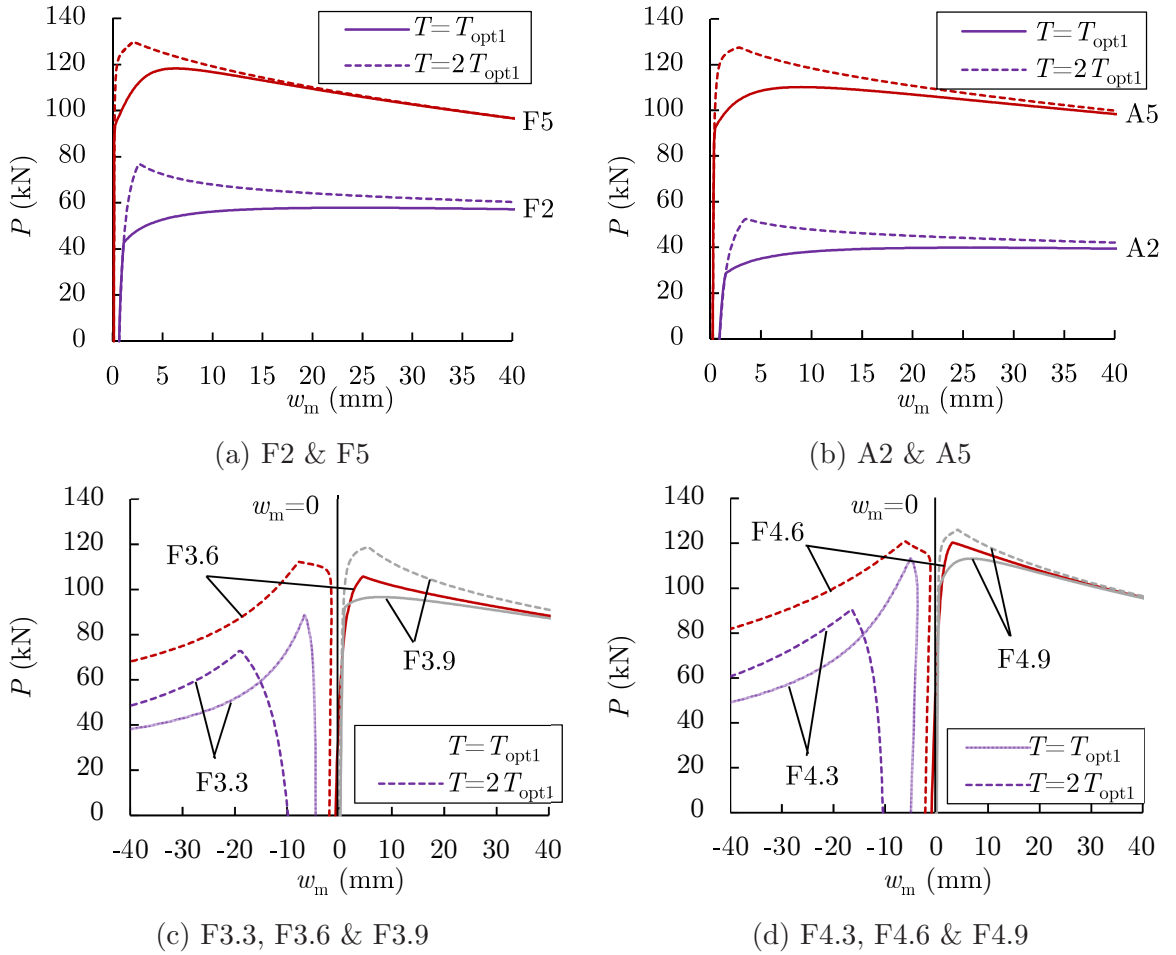


Figure 18: Comparisons of the externally-applied axial load P versus the vertical displacement at midspan w_m between $T_{1,i}/T_{opt1} = 1.0$ and 2.0 .

investigated. However, at this point it is worth recalling that Equation (18), which relates

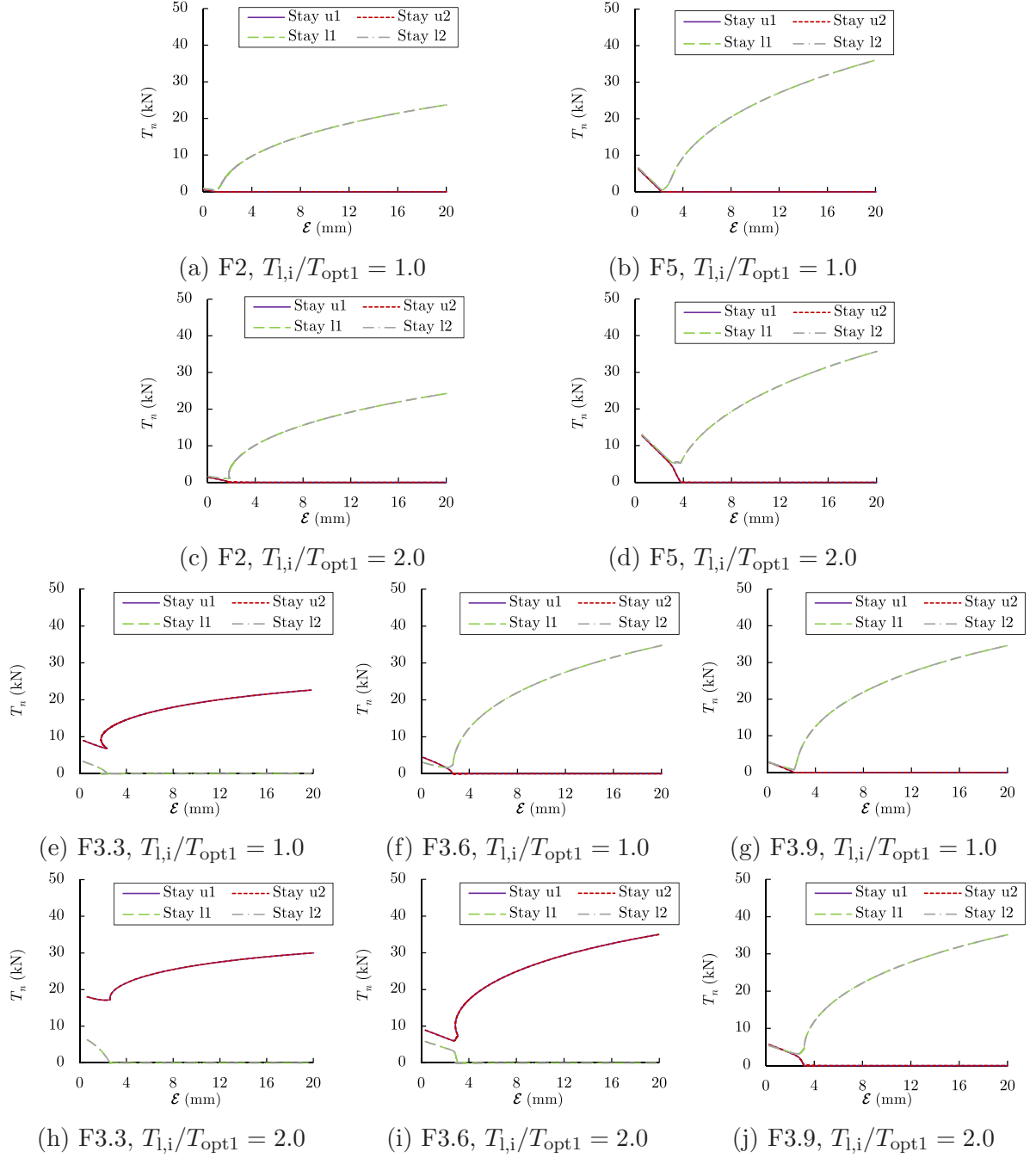


Figure 19: Forces within the individual upper and lower stays versus end-shortening displacement ε .

the initial prestressing levels within the upper and lower stays such that the restraining forces within both crossarms that inhibit the midspan displacement are equal, is only accurate to leading order. This is because that relationship deliberately neglects the induced end-displacement from the introduction of the prestressing forces through compressing and bending the main column element to maintain analytical simplicity. Hence, for cases where $\alpha < 1.0$ with higher prestressing levels, the initial lateral midspan displacement of the main column element is not necessarily zero. Therefore, there may be a significant pre-camber when the external axial load is introduced.

The graphs in Figure 18 compare the externally-applied axial load P versus the vertical displacement at midspan w_m between $T_{l,i}/T_{opt1} = 1.0$ and 2.0 . It should be noted that $w_m > 0$ represents a downward vertical displacement (*i.e.* the same direction as gravity). It is evident that in terms of the doubly-symmetric models presented in Figures 18(a–b), w_m is always positive for the downward displacement in both the $T_{l,i}/T_{opt1} = 1.0$ and 2.0 cases, which indicates that the pre-cambering effect is always smaller than the effect of the self-weight. This leads to a downward midspan displacement but higher prestressing levels increase the ultimate load.

However, the direction of w_m reverses in the case of the mono-symmetric models presented in the graphs in Figures 18(c–d). It can be seen that when α is small, such as in cases F3.3 and F4.3, $w_m < 0$ in both $T_{l,i}/T_{opt1} = 1.0$ and 2.0 cases, which shows that the pre-cambering effect is greater than the effect of the self-weight, leading to an upward midspan displacement. This is naturally associated with a potentially greater load-carrying capacity since the beam-column has to work against the gravitational force to deflect upwards. However, since $w_m < 0$ for both prestressing levels in these two cases, F3.3 and F4.3, the higher prestressing forces reduce the ultimate load because of the extra initial compression within the beam-column. For the cases where there is an intermediate level of α , such as in cases F3.6 and F4.6, $w_m > 0$ when $T_{l,i}/T_{opt1} = 1.0$ but $w_m < 0$ when $T_{l,i}/T_{opt1} = 2.0$. This demonstrates the cases where the beam-column switches from being self-weight dominant, when $T_{l,i}/T_{opt1} = 1.0$, to being pre-camber dominant, when $T_{l,i}/T_{opt1} = 2.0$. For these two cases, F3.6 and F4.6, the increase in prestressing level produces advantageous behaviour in terms of ultimate strength and initial post-buckling behaviour. Finally, when α approaches unity, *i.e.* the cases converge to the doubly-symmetric cases, such as F3.9 and F4.9, $w_m > 0$ in both prestressing levels and the behaviour is similar to the doubly-symmetric cases.

The graphs in Figure 19 confirm all the conclusions drawn from Figure 18 from the perspective of how the force changes within each stay as the end-shortening displacement is increased. It can be observed that the lower stays (l1 and l2) go slack finally in Figures 19(e), (h) and (i), which indicates that $w_m < 0$ and the pre-cambering effect dominates. The upper stays (u1 and u2) go slack in Figures 19(a), (b), (c), (d), (f), (g) and (j), which indicate that $w_m > 0$ and self-weight dominates. These results indicate that the pre-cambering effect from a small α or higher prestressing levels can affect the post-buckling response and the ultimate load significantly but there appears to be a combination of geometric and prestressing properties that maximize the effectiveness of the system.

4.4. Prestressing ratio of upper and lower stays

Hitherto, the ratio between the upper-stay and lower-stay prestressing forces is set from the geometric condition in Eq. (18). In order to verify that this ratio is approximately optimal, nonlinear FE analysis is conducted with cases F3.3, F3.5 and F3.7 being selected for this parametric study. In each model, the lower-stay prestressing force is assigned in turn from those defined in Eq. (66); once this is fixed, all of the following levels are studied for the upper-stay prestressing force in turn:

$$T_{u,i}/T_{l,i} = \{0, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75, 3.00\}. \quad (67)$$

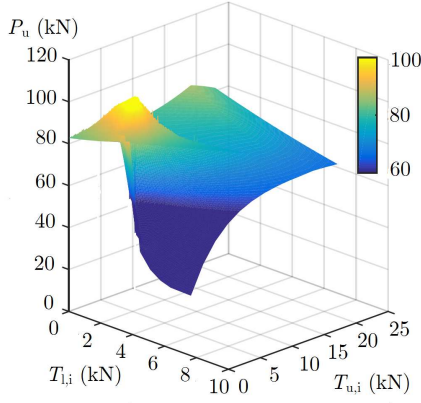
In total, there are 169 cases for each model, the results from which are compiled into the three-dimensional surfaces provided in Figures 20(a), (c) and (e) using linear interpolation. The graphs in Figures 20(b), (d) and (f) show planes of these surfaces in the plane of $T_{u,i}$ versus $T_{l,i}$. As can be seen, the larger ultimate loads P_u are numerically close to the line defined by the geometric condition $T_{u,i}/T_{l,i} = \sin \beta_l / \sin \beta_u$, which represents the prestressing ratio obtained from Eq. (18). The plots appear to suggest that this particular prestressing ratio provides a good initial estimate for maximizing the ultimate load, although further research is necessary for this to be confirmed.

5. Concluding remarks

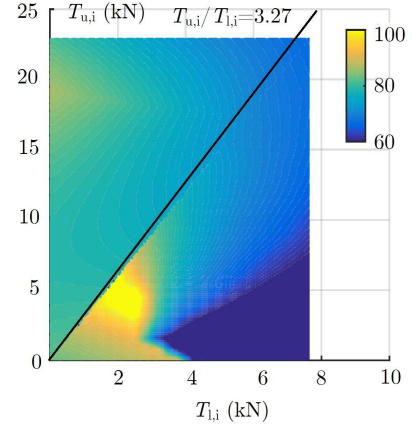
As far as the authors are aware, this is the first study to investigate the buckling and post-buckling behaviour of prestressed stayed beam-columns. Although previous studies investigated the stability of prestressed stayed columns, their focus was only on doubly-symmetric stayed columns in the vertical plane where the applied force and the principal self-weight contributions were not only in the same direction but also collinear. Putting the stayed column within the horizontal plane changes this significantly and the flexural effect of the structural self-weight cannot be neglected. This opens up a whole new set of issues, including the possible desirability of making the structure mono-symmetric by adopting different crossarm lengths at midspan such that the prestressed system counteracts the effect of self-weight. It has been shown currently that the crossarm-length ratio alongside the effect of self-weight have significant effects on the mechanical behaviour.

The study has found that the three distinct zones of behaviour that have been determined for purely axially-loaded prestressed stayed columns are significantly affected by the effect of self-weight. The principal effect is to sub-divide two of these zones into two further parts by taking the natural mono-symmetry of the geometry into account. The newly-derived zones for the beam-column have been verified by nonlinear FE results and, as has been determined for stayed columns, the true optimum prestressing level, in terms of maximizing the ultimate load-carrying capacity, is higher than the linearly-derived optimum level, which implies that the current theoretical study needs to be developed further to provide a fundamental understanding of the mechanical behaviour within the nonlinear range.

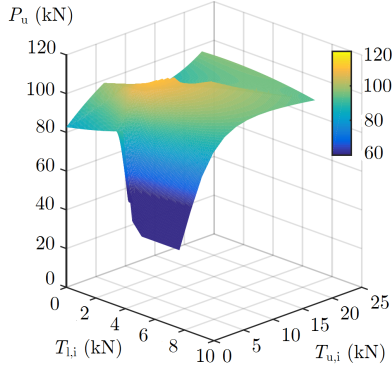
Apart from the well-known finding that increasing the stay diameter and crossarm lengths can increase the ultimate load for doubly-symmetric stayed beam-columns, as is



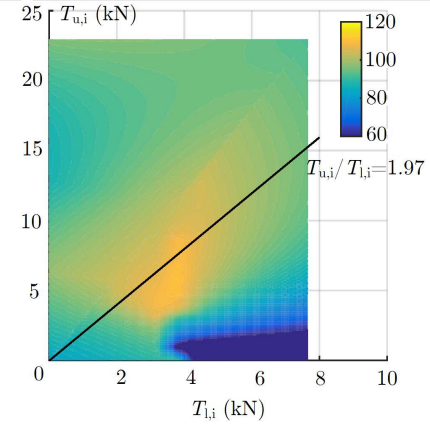
(a) F3.3–3D surface



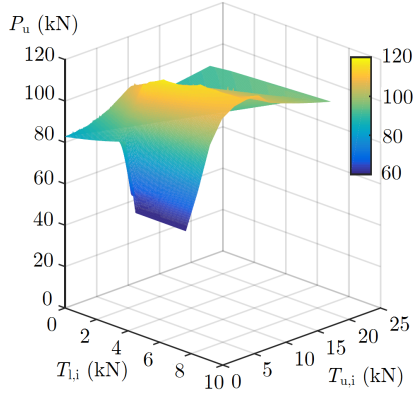
(b) F3.3–2D contour, $\sin \beta_l / \sin \beta_u = 3.27$



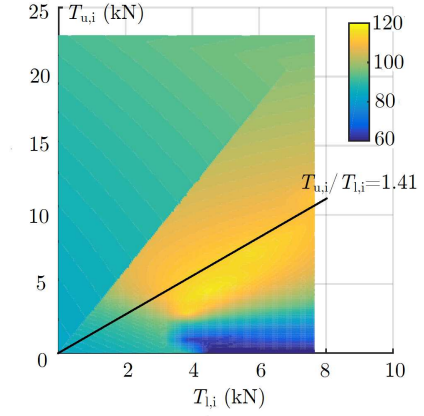
(c) F3.5–3D surface



(d) F3.5–2D contour, $\sin \beta_l / \sin \beta_u = 1.97$



(e) F3.7–3D surface



(f) F3.7–2D contour, $\sin \beta_l / \sin \beta_u = 1.41$

Figure 20: Effects from allowing different prestressing forces within the upper $T_{u,i}$ and lower stays $T_{l,i}$; the surfaces in (a), (c) and (e) are depicted as planes in (b), (d) and (f). The relationship from Eq. (18) is superimposed on the graphs in (b), (d) and (f).

case for conventional stayed columns, the crossarm length ratio is also found to be a key parameter. It transpires that small crossarm-length ratios should be avoided since this has been demonstrated to lead to extremely unstable post-buckling behaviour including snap-back responses. The current results demonstrate that intermediate crossarm-length ratios, determined herein to be approximately 0.6, gives the most favourable mechanical behaviour since it strikes a balance between the effect of self-weight and the initial pre-cambering displacement. Finally, the prestressing force distribution between the upper and lower stays, determined from a simple force balancing equation, appears to provide a sensible estimate for the combination needed to be adopted in stayed beam-columns to maximize the system effectiveness.

Notwithstanding the present findings, since the currently developed theoretical formulation is based on linear theory and it has been shown through nonlinear FE analysis that the behaviour is considerably more nuanced, further research is clearly required. Moreover, the current study is limited to perfect structures without taking imperfections into consideration. The present findings suggest that a nonlinear model to investigate imperfection sensitivity [13, 28] and the potential for interactive buckling, which are known issues in prestressed stayed columns [14, 16], would lead to predicting and understanding the practical ranges for parameters and realistic arrangements. This, in turn, would enable guidance for designers to be established in future such that the system may be safely and efficiently used in practice.

Acknowledgement

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