

THE DETERMINATION OF ACOUSTIC OUTPUT POWER
BY MEANS OF THE RADIATION PRESSURE BALANCE

by

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Submitted to the University of London for
the degree of Doctor of Philosophy, and
representing work carried out at the Royal
Postgraduate Medical School and the
National Physical Laboratory during the
years 1978-84

1984

ABSTRACT

The total power in an ultrasonic field is frequently assessed by the measurement of the radiation pressure, although the nature of this phenomenon continues to be the subject of some controversy. This thesis presents a theoretical treatment of radiation pressure together with a discussion of the design and evaluation of a radiation pressure balance.

The different techniques used for the determination of ultrasonic fields are discussed in Part I. In particular, the various radiation pressure balances described in the current literature are reviewed.

Several apparently contradictory definitions for radiation pressure are in current use, and a number of authors have commented on the confusion that exists within the literature. In Part II of this thesis, a unified and simple treatment of radiation pressure is developed, and, for the one-dimensional case, the time-averaged force per unit area on the surface of an obstacle placed in the field is shown to be exactly related (i.e. to all orders of magnitude) to the time-averaged momentum flux of the wave, or equivalently the time-averaged pressure in the field expressed in Lagrangian coordinates. Unlike all previous treatments, which are based on expansions to second order, the result derived here is valid to all orders of approximation. As it is usual to relate the radiation pressure to other field quantities, particularly the energy density, a relation between these two parameters is also derived.

In Part III of the thesis, the design and evaluation of a radiation pressure balance is described, with particular emphasis on a realistic assessment of the sources of uncertainty. The various factors considered in the design of the microbalance system and target are discussed, and this is followed by an evaluation of the effects of phenomena such as surface tension, streaming, convection currents and target stability.

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PART I

Determination of Ultrasonic Fields: Concepts and Techniques

1. THE NEED FOR FIELD MEASUREMENT

Reliable and accurate measurements of ultrasonic fields are required in many areas. Ultrasound is now used routinely in hospitals both as a diagnostic and a therapeutic tool, and many health authorities feel the need to be able to monitor patient exposure levels. The American Institute of Ultrasound in Medicine recommends (HHS Publication) that ultrasonic diagnostic equipment should be labelled to indicate the power and intensity emitted, and manufacturers clearly need unified methods of providing these figures. Field measurements also figure prominently in the design and evaluation of ultrasonic transducers. Research groups investigating the interaction of ultrasound with tissue may need to combine any comment on the biological effect of the ultrasound with accurate measurement of the applied field, particularly in the evaluation of possible threshold levels. Similarly, in the non-destructive testing of materials, reliable information about, for example, defects in a metal can be obtained only with a good knowledge of the interrogating ultrasonic beam.

Most of the impetus for field measurements has so far been directed towards the determination of the ultrasonic fields used in medicine, and therefore this thesis will tend to concentrate on this subject. The techniques described could, however, in general be equally well applied to the determination of the fields used in non-destructive testing.

Medical ultrasound is generally restricted to the frequency range approximately 0.5 MHz to 10 MHz. The lower limit is dictated by the velocity of ultrasound in tissue (approximately 1500 m/s) which makes the wavelength and hence the resolution, or localisation, at lower frequencies unacceptable. The upper limit is determined by the attenuation of the ultrasound which increases with frequency.

The use of ultrasound in medicine can be divided into three categories: imaging, monitoring and therapy, and each of these utilises, in general, a different form of transducer excitation and tends to have its own specific field measurement problems. Imaging is based on the pulse-echo technique, where a pulse of one or two cycles duration is sent into the body, and the scattered and reflected acoustic signals are detected by the same transducer. The total time-averaged ultrasonic power used in imaging is low (typically less than 50 mW), but, as the pulses are very short, very high peak pressures (tens of atmospheres) are produced and nonlinear propagational effects may become significant. The second category, which utilises the Doppler effect, in general uses continuous waves with the ultrasound transmitted from one transducer and received at another (mounted close to the transmitter). Any ultrasound reflected from a moving surface is frequency shifted and analysis of the frequency shift is used to give information, for example, on the velocity of the movement. The total ultrasonic power in Doppler equipment is similar to that from imaging, but, as the ultrasound is continuous, the acoustic pressures are much lower (generally less than 0.1 atmospheres). Therapeutic equipment uses either continuous wave ultrasound or relatively long pulses (well over 100 cycles) and high levels of ultrasonic power (typically greater

than 1 W).

Measurements on medical ultrasonic fields are generally made with the transducer radiating into water. There are several reasons for this. Water is an appropriate medium to use. It is plentiful and cheap and can, in many circumstances, be treated as an ideal fluid. Water has a low coefficient of attenuation (0.00025 neper/cm at 1 MHz and 0.025 neper/cm at 10 MHz, both determined at 20°C, Pinkerton 1947), and it is generally considered that water may be regarded as non-viscous in many applications. The ultrasonic wave in water is therefore often treated as longitudinal with no shearing component. In general, the measurement of ultrasonic fields in water is seen as the first step, the starting point towards the understanding of the more complex fields present in non-homogeneous tissue.

This thesis is based on a discussion of the nature of radiation pressure in water together with a description of the development of a sensitive radiation pressure balance for the measurement of low ultrasonic power levels. In the first part of the thesis, the different techniques used to provide measurements on the fields from medical ultrasonic transducers will be described. Some of the techniques are comparable to the radiation pressure balance and these will be discussed in some detail, whereas others will be mentioned only briefly. The techniques are, in general, intended for the determination of low power fields, but can often be extended to therapeutic levels, although, as will be indicated, this often introduces special problems.

2. QUALITATIVE TECHNIQUES

In general, it is not possible to perceive the effects of a water-borne ultrasonic field directly, although at high intensities (a few W/cm^2) the effects of streaming and cavitation can often be seen. A field of the intensity normally used in medical diagnostic applications (typically $40 \text{ mW}/\text{cm}^2$ on time average) may, however, be visualised, and its qualitative character studied, using relatively simple methods.

The Schlieren technique, highly developed for the dynamic study of air flow patterns (Barnes and Bellinger 1945), can be used for viewing ultrasonic fields in water. With continuous wave ultrasound and steady illumination, only coarse detail of the near-field structure can be seen and a projection-type display of the general beam shape established. Pulsed ultrasound may be visualised more effectively by using optical pulses with a duration shorter than the period of the highest frequency component in the ultrasonic field. If the repetition frequency of the optical pulses is synchronised to that of the ultrasound, the positions of the ultrasonic pulses can be "frozen" in the Schlieren image, and their propagation studied by varying the time delay between the optical and ultrasonic pulses (Hunter et.al. 1964).

Another common method of obtaining information on the distribution of energy within a pulsed ultrasonic field is to use the transducer in the 'pulse-echo' mode, normally associated with the B-scan imaging technique. An effective cross-section of the beam can be built up by recording the echo signals received from a fine wire or small spherical target as it is moved over an appropriate plane within the ultrasonic beam (Gordon 1964).

Clearly the 'picture' of the beam obtained by this technique will be dependent on the size and type of target used, and also on the criteria used to analyse the echo signal. A plot of the peak-to-peak received signal may be very different from that for the pulse average of the received signal. This technique measures some function of the performance of the transducer as both a transmitter and a receiver (the pulse-echo loop), rather than the transmitted field characteristics.

3. SPECIFYING AN ULTRASONIC FIELD: BASIC FIELD VARIABLES

Qualitative techniques have a limited application and only provide details of the extent and shape of an ultrasonic field or basic system performance. In general, to specify or characterise the ultrasonic field, more quantitative information is required.

There are basically two ways to approach the problem of field measurement. The field can be characterised by measuring the time dependence of quantities that vary point by point in space. Alternatively, the time and space variation can be removed and a conserved quantity that has the same value at all time anywhere in the ultrasonic field can be determined. Clearly, however, the first requirement is a description of the form of the ultrasonic field in water, and we will start by obtaining the equations for a more general fluid and then discuss the approximations that can be made when dealing with water-borne ultrasound. Some properties of water are given in Appendix A.

The treatment broadly follows that of Landau and Lifshitz (1959) and Temkin (1981).

3.1 Conservation of Mass

The first general equation arises from conservation of mass. If the fluid is continuous and there are no 'sources' or 'sinks' of matter within a volume V fixed in space, the decrease per unit time of mass in this volume must equal the rate at which mass flows out of the volume, and so

$$\int_V [\partial \rho / \partial t + \nabla \cdot (\rho \underline{u})] dV = 0,$$

where ρ is the fluid density and $\underline{u}$ is the fluid velocity.

This is known as the equation of continuity, and, since the volume V may be any volume within the fluid, can also be written in differential form as

$$\partial \rho / \partial t + \nabla \cdot (\rho \underline{u}) = 0. \quad \dots(3.1)$$

3.2 Conservation of Momentum

Consider a volume V , surrounded by a surface S , fixed not in space but on the individual moving elements of the fluid. By Newton's second law

$$(d/dt) \left(\int_V \rho \underline{u} dV \right) = \int_S \sigma_{ij} \hat{n}_j dS, \quad \dots(3.2)$$

where the first term denotes the rate of change of momentum of the fluid volume, and the second the total force on the volume. Here σ_{ij} is a tensor representing the stress on the fluid volume, and $\hat{n}_j$ is unit normal to the surface element. σ_{ij} is the i th component of the force per unit area exerted across a plane surface element normal to the j th direction.

Already two assumptions have been made in obtaining this formula. Firstly, the 'body' forces such as gravity have been neglected. Lighthill (1978) shows that any sound waves with a

wavelength less than (c^2/g) are negligibly affected by gravity. Here c is the velocity of sound and g is the acceleration due to gravity. For water the ratio (c^2/g) is approximately 200 km and this rules out any influence of gravity on sound waves in practical situations. The second assumption is that the surface forces on a volume element can be represented by a stress tensor. Batchelor (1967) describes a proof of the correctness of this statement and also proves that the stress tensor σ_{ij} is symmetric, i.e. $\sigma_{ij} = \sigma_{ji}$.

The left hand side of equation (3.2) can be written (Batchelor 1967) as

$$(d/dt)\left(\int_V \rho \underline{u} dV\right) = \int_V (d/dt)(\rho \underline{u}) dV + \int_V \rho \underline{u} \nabla \cdot \underline{u} dV.$$

Using the general expression

$$dh/dt = \partial h / \partial t + \underline{u} \cdot \nabla h,$$

which relates time-differentiation following the motion of the fluid (d/dt) to that at a fixed point in space $(\partial/\partial t)$, together with the equation of continuity, gives

$$(d/dt)\left(\int_V \rho \underline{u} dV\right) = \int_V \rho (d\underline{u}/dt) dV.$$

Applying the divergence theorem to the right hand side of equation (3.2)

$$\int_S \sigma_{ij} \hat{n}_j dS = \int_V (\partial \sigma_{ij} / \partial x_j) dV,$$

where $\partial \sigma_{ij} / \partial x_j$ is shorthand notation for $\sum_{j=1}^3 (\partial \sigma_{ij} / \partial x_j)$, and hence

$$\int_V \rho (du_i/dt) dV = \int_V (\partial \sigma_{ij} / \partial x_j) dV.$$

Since the choice of the volume V is arbitrary, this equation must be true at every point in the fluid, and therefore

$$\rho du_i/dt = \partial \sigma_{ij} / \partial x_j, \quad \dots(3.3)$$

an expression for the equation for motion of the fluid.

Clearly this is not a useful equation of motion until the stress tensor σ_{ij} has been specified. The diagonal components of σ_{ij} are normal stresses and each of them gives the normal component of the force acting across a plane surface element parallel to one of the coordinate planes. The six non-diagonal components of σ_{ij} are tangential stresses and are set up by shearing motion in which parallel layers of the fluid slide relative to one another. Clearly in a liquid these shearing stresses would be expected to be small.

In an ideal non-viscous fluid, no tangential stresses occur, the forces on a small element are all normal to the surface of the element, and, at a given point, have the same magnitude independent of direction. The stress tensor can therefore be written as

$$\sigma_{ij} = -p\delta_{ij},$$

where the negative sign is due to the convention that a positive stress tensor denotes tension. The parameter p is the pressure at a point in the fluid. It may be a function of the position in the fluid, and in an ideal fluid is identical to the thermodynamic pressure.

In a real fluid there will, in general, be tangential stresses, and in this case the stress tensor can be written as

$$\sigma_{ij} = -P\delta_{ij} + d_{ij},$$

where P is defined as $-(1/3)\sum_{i=1}^3 \sigma_{ii}$, and is a purely mechanical definition of pressure, not necessarily equal to the usual

thermodynamic definition. d_{ij} is known as the deviatoric stress tensor (Batchelor 1967), with $\sum_{i=1}^3 d_{ii} = 0$.

For Newtonian fluids, which have an isotropic structure, the relationship between the deviatoric stress tensor and the rate of strain tensors is linear. According to Temkin (1981), water follows this relationship quite well, and in this case

$$d_{ij} = 2\mu(e_{ij} - \Delta\delta_{ij}/3),$$

where μ is the shear viscosity of the fluid, e_{ij} is the rate of strain tensor defined by

$$e_{ij} = (\partial u_i / \partial x_j + \partial u_j / \partial x_i) / 2,$$

and Δ is the rate of expansion given by

$$\Delta = \sum_{i=1}^3 e_{ii}.$$

Hence the stress tensor for an isotropic Newtonian fluid is

$$\sigma_{ij} = -P\delta_{ij} + 2\mu(e_{ij} - \Delta\delta_{ij}/3). \quad \dots(3.4)$$

Substituting in the equation of motion (3.3)

$$\rho du_i / dt = -\partial P / \partial x_i + (\partial / \partial x_j) [2\mu(e_{ij} - \Delta\delta_{ij}/3)],$$

which is usually known as the Navier-Stokes equation of motion.

In many fluids, the shear viscosity μ depends significantly on temperature, and, when appreciable temperature differences exist in the fluid, μ can no longer be regarded as constant. The variation of μ with temperature for water (at atmospheric pressure) is given in table 1 (CRC Handbook of Chemistry and Physics, 58th edition), and it can be seen that for water μ decreases at the rate of approximately 2.5% per degree rise in temperature in the neighbourhood of ordinary temperatures.

Temperature (°C)	Shear Viscosity μ (g cm ⁻¹ s ⁻¹)
15	1.139 x 10 ⁻²
16	1.109
17	1.081
18	1.053
19	1.027
20	1.002
21	0.978
22	0.955
23	0.933
24	0.911
25	0.890

Table. 1: The Variation of the Shear Viscosity With Temperature.

We can obtain an approximate value for the temperature differences occurring in an ultrasonic wave by considering the simplified situation of one-dimensional wave propagation in a non-viscous lossless fluid. For a unit mass element of the fluid, the energy change per unit mass dE due to compression can be expressed as

$$dE = -pdV = p(dp/\rho^2).$$

If there are no losses of energy within the fluid, this change in energy will alter the temperature of the fluid element by an amount dT given by

$$c_v dT = dE = p(dp/\rho^2),$$

where c_v is the specific heat at constant volume of the fluid. This expression can be used to estimate the temperature change occurring in water due to the propagation of an ultrasonic wave, and gives a figure of approximately $2 \times 10^{-6} \text{ }^\circ\text{C}$ for a sinusoidal wave of intensity 10^3 W/m^2 in water: clearly a very small effect.

An alternative source of temperature differences in a fluid may arise from absorption of the ultrasonic wave as it passes through the fluid. To a first order of approximation, the intensity I of a one-dimensional ultrasonic wave can be expressed by

$$I = I_0 \exp(-2\alpha x),$$

where α is the amplitude attenuation coefficient (equal to 0.025 neper/cm in water at 10 MHz), x is the distance travelled and I_0 is the intensity at the position $x = 0$.

Energy is lost from the sound field at the rate $-dI/dx = 2\alpha I$ per unit volume. If it is assumed that all this energy is converted into heat, the rate of heat production per unit volume for a wave of frequency 10 MHz and intensity 10^3 W/m^2 travelling in water is approximately $5 \times 10^3 \text{ J m}^{-3} \text{ s}^{-1}$. This gives a rate of temperature rise (assuming no dissipation of heat) of 0.001°C/s .

In an ultrasonic wave in water the differences in temperature are therefore generally much less than a degree, and it is reasonable to assume that μ is uniform over the fluid. Hence equation (3.4) becomes

$$\rho du_i/dt = -\partial P/\partial x_i + \mu[\partial^2 u_i/\partial x_j \partial x_j + (1/3)\partial \Delta/\partial x_i]. \quad \dots(3.5)$$

As was mentioned earlier, the variable P represents the mechanical pressure, and may be different from the thermodynamic pressure p . It can be shown (Batchelor 1967) that P and p can be related using the equation

$$P - p = - \eta \Delta, \quad \dots(3.6)$$

where η is the bulk viscosity of the fluid.

This equation can be interpreted physically in terms of relaxational phenomena. As well as translational energy, molecules of a fluid also have rotational and (if the temperature is high enough) vibrational modes of motion. The difference between the thermodynamic and mechanical pressures is due to the delay in establishing equipartition of energy between the various modes of motion of a molecule. For water at 20°C, the bulk viscosity η is equal to $2.8 \times 10^{-2} \text{ g cm}^{-1} \text{ s}^{-1}$, and is of the same order of magnitude as the shear viscosity.

This is the first, and the last, time in this thesis that the molecules of the fluid will be considered. In general, the continuum model can be applied, which means that an elemental volume must be large enough for an average over the molecules in the volume to be independent of their number. For most laboratory experiments with fluids, very little variation of the physical and dynamical properties of the fluid occurs over distances less than 10^{-3} cm and hence volumes of 10^{-9} cc give a measure of a local property. This size of volume contains approximately 3×10^{13} molecules of water at normal temperatures and pressures, which is large enough, by a very wide margin, for the continuum model to be applicable.

Using the expression for the mechanical pressure P in equation (3.5), and assuming that the bulk viscosity is constant over the fluid

$$\rho du_i/dt = - \partial p / \partial x_i + \mu \partial^2 u_i / \partial x_j \partial x_j + (\mu/3 + \eta) \partial \Delta / \partial x_i,$$

expressed in vector notation as

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + (\mu/3 + \eta) \nabla \nabla \cdot \mathbf{u}. \quad \dots(3.7)$$

3.3 Conservation of Energy

Further insight into the effect of forces on a fluid may be obtained by considering the energy balance for an element of the fluid, of volume V contained within a surface S . Work is done on this volume of fluid by surface forces, and the volume may also be gaining heat by transfer across the surfaces. Some of this total gain of energy is manifested as an increase in the kinetic energy of the fluid while the remainder appears as an increase in the internal energy.

The rate at which work is done on the element is

$$\begin{aligned} \int_S \sigma_{ij} \hat{n}_j u_i dS &= \int_V (\partial/\partial x_j)(u_i \sigma_{ij}) dV, \\ &= \int_V (u_i \partial \sigma_{ij} / \partial x_j + \sigma_{ij} \partial u_i / \partial x_j) dV, \end{aligned}$$

and from equation (3.3) this is equal to

$$\int_V (\rho u_i \frac{du_i}{dt} + \sigma_{ij} \partial u_i / \partial x_j) dV.$$

The first term represents the increase in kinetic energy of the fluid, whereas the second term represents the work done in deforming the element without changing its velocity.

The energy of the fluid element may also change due to heat transfer, and the rate of gain of heat by a mass of fluid by conduction across its surface is

$$\int_S K (\partial T / \partial x_i) \hat{n}_i dS = \int_V (\partial/\partial x_i)(K \partial T / \partial x_i) dV,$$

where T is the local temperature and K is the thermal conductivity of the fluid (equal to $5.9 \times 10^{-3} \text{ W cm}^{-1} \text{ }^\circ\text{C}^{-1}$ for water at 17°C). Hence the rate of change of internal energy of the fluid per unit

mass $d\epsilon/dt$ is given by

$$d\epsilon/dt = (\sigma_{ij}/\rho)\partial u_i/\partial x_j + (1/\rho)(\partial/\partial x_i)(K\partial T/\partial x_i).$$

Substituting for the stress tensor from equation (3.4)

$$d\epsilon/dt = -P\Delta/\rho + (2\mu/\rho)(e_{ij}e_{ij} - \Delta^2/3) + (1/\rho)(\partial/\partial x_i)(K\partial T/\partial x_i).$$

The second term on the right hand side of this equation is the work done in deforming the element by shearing motion. This term can never be negative, which demonstrates that any shearing motion in the fluid is necessarily accompanied by a transfer of energy to the internal energy of the fluid. This is to be expected from the nature of the shear stress.

It is convenient to define the viscous dissipation factor

$$\Phi = (2\mu/\rho)(e_{ij}e_{ij} - \Delta^2/3),$$

which is the rate of dissipation of mechanical energy, per unit mass of the fluid, due to viscosity.

Using this together with the expression for P (equation 3.6) gives

$$d\epsilon/dt = -p\Delta/\rho + \Phi + \eta\Delta^2/\rho + (1/\rho)(\partial/\partial x_i)(K\partial T/\partial x_i). \quad \dots(3.8)$$

The first law of thermodynamics for a reversible process

$$TdS = d\epsilon + pd(1/\rho),$$

where S is the entropy per unit mass, together with the equation of continuity, gives

$$TdS/dt = \eta\Delta^2/\rho + \Phi + (1/\rho)(\partial/\partial x_i)(K\partial T/\partial x_i). \quad \dots(3.9)$$

3.4 Equation of State

For the circumstances that are of interest here, and for fluids of uniform composition, the number of independent properties needed to specify the state of the fluid is two (Batchelor 1967, Temkin 1981). All other quantities describing the state of the fluid are functions of these two parameters of state. Any two properties of the fluid can be chosen, and the choice depends on the particular requirements. We will use the density and entropy per unit mass as the independent variables, and then the pressure will have a value given by

$$p = p(\rho, S), \quad \dots(3.10)$$

which is an equation of state of the fluid.

3.5 Summary

We have obtained six equations to describe the state of the fluid:

the equation of continuity

$$\partial \rho / \partial t + \nabla \cdot (\rho \underline{u}) = 0, \quad \dots(3.1)$$

the equation of motion (three equations)

$$\rho d\underline{u}/dt = -\nabla p + \mu \nabla^2 \underline{u} + (\mu/3 + \eta) \nabla \nabla \cdot \underline{u}, \quad \dots(3.7)$$

the equation of conservation of energy

$$T dS/dt = \eta \Delta^2 / \rho + \Phi + (1/\rho) (\partial / \partial x_i) (K \partial T / \partial x_i), \quad \dots(3.9)$$

and the equation of state

$$p = p(\rho, S). \quad \dots(3.10)$$

There are six unknowns p , ρ , S and $\underline{u}$ in these six equations, and

therefore, in principle, if suitable initial and boundary conditions are stated, and if the value of one of the unknown parameters is measured at all points in the fluid at all time, this gives a complete determination of the ultrasonic field. An ultrasonic field may, therefore, be completely specified by measuring the value of a parameter such as the acoustic pressure or the fluid particle displacement amplitude, and several techniques can, in principle at least, provide the direct determination of this type of basic field variable. Information on the fluid particle displacement amplitude can be obtained from the measurement of the displacement of an acoustically transparent membrane, as mentioned below. There are several types of pressure-measuring devices such as strain gauges and capacitance probes, but, for the determination of the pressure in an ultrasonic field in water, hydrophones are convenient and are in widespread use, and will be discussed later. Thermal probes which can be used to determine the intensity at a point will be left to another section.

3.6 Optical Techniques

A measure of the displacement of the particles of a fluid can be made using a thin membrane that is acoustically transparent but optically reflecting (Mezrich et.al. 1975). This membrane is put into the ultrasonic field and moves with the fluid particles in contact with its surfaces. The movements can be measured at any point using a laser interferometer, and a knowledge of the particle displacement amplitude distribution throughout the ultrasonic field can be obtained. An instrument to perform this type of measurement will, however, be complex and expensive. The technique does not measure the actual displacement of the

particles of the fluid but determines the component of the displacement parallel to the membrane surface. Also a number of corrections must be applied to the results; the most important being an allowance for the interaction of the optical beam with the ultrasonic field. The laser beam must pass through the ultrasound in order to record the movements of the membrane, and the refractive index changes from the ultrasonic field will distort the optical beam.

3.7 Hydrophones

Miniature transducers designed for use as detectors are often referred to as hydrophones, and typically consist of a small piezoelectric element, in the form of a thin disc, mounted on the end of a needle-like probe. An approximate description of the behaviour of a hydrophone is usually obtained from the simple and standard concepts given below. These equations are included for completeness, but it is worth mentioning that they are only approximations to the true situation. A more complete treatment of the behaviour of hydrophones is outside the scope of this thesis. However, it should be noted that it is usually considered that the force on the hydrophone surface can be directly related to 'pressure' without being specific as to the nature of this pressure.

If the force on the surface of the hydrophone and the resultant change in thickness are not too great, Hooke's law can be used to give

$$F_h/A = c_s \epsilon_h / t_h,$$

where F_h is the force on the hydrophone surface of area A , ϵ_h is the decrease in the thickness t_h of the hydrophone, and c_s is a

stiffness constant (Heuter and Bolt 1955).

If the output from the hydrophone is short-circuited, the charge Q developed is approximately proportional to the strain, i.e.

$$\begin{aligned} Q &= eA\xi_h/t_h, \\ &= eF_h/c_s, \end{aligned}$$

where e is a stress constant.

The capacitance C of the hydrophone can be approximated by

$$C = A\epsilon_h/t_h,$$

where ϵ_h is the dielectric constant of the hydrophone material (assumed to be an insulator).

Hence, the voltage V_h generated across the electrodes of the hydrophone can be given by

$$\begin{aligned} V_h &= Q/C, \\ &= eF_h t_h / (c_s A \epsilon_h). \end{aligned}$$

Now the force F_h on the hydrophone surface is equal to the pressure p in the ultrasonic wave integrated over the surface, and therefore

$$V_h = \int p dA [et_h / (c_s A \epsilon_h)].$$

Hence a plane ultrasonic wave impinging on the surface of a piezoelectric disc generates a voltage across the electrodes of the disc proportional to the acoustic pressure in the ultrasonic field.

As the piezoelectric disc integrates the pressure over its surface, it is difficult to interpret the measurements unless the phase of the ultrasound is constant over the front surface of the hydrophone. The phase varies only over dimensions typically

greater than a wavelength, and hence a hydrophone with a diameter of the order of a wavelength or less should be used. Clearly this is not practicable at high frequencies where the wavelength is approximately 0.1 mm and in this case some averaging occurs over the element. Another problem occurs with the averaging over the front face of the element if the disc is mounted so that, for example, the edges are effectively clamped.

Ideally, if the ultrasonic pressure is to be reproduced in the voltage signal without distortion, the hydrophone should respond equally to pressure variations occurring at any frequency, but variations in the frequency response of a hydrophone are caused by two effects. Firstly, the piezoelectric element will resonate at its fundamental thickness mode resonance, which for a 0.2 mm thick element of PZT4 occurs at approximately 10 MHz. Clearly, a disc could be chosen so thin that this resonance point occurs above any frequency of interest. The second variation in frequency response is due to radial resonances. As the diameter of the piezoelectric disc is reduced to minimise phase changes across it, the frequency of the radial resonance increases. For example, the fundamental radial resonance for a 0.5 mm diameter disc of PZT4 occurs in the region of 4 MHz.

A recent development (Bacon 1982) is the membrane hydrophone, which consists of a thin film of piezoelectric plastic (polyvinylidene fluoride, pvdf) stretched across an annular ring. The diameter of this ring (typically 100 mm) is chosen to be greater than the width of the ultrasonic beam. Electrodes are applied to a small central region of the plastic film and these are poled, producing a piezoelectrically active central element. The membrane is thin (25 μm or less) giving a fundamental thickness mode resonance frequency of approximately 53 MHz for the

25 μm film, a much higher frequency than is normally used in medical applications. The amplitude reflection coefficient for a 25 μm thick membrane is approximately 0.07 at 1 MHz, and the variation of frequency response from radial resonances which is troublesome in ceramic hydrophones causes no difficulty with the membrane hydrophone as it occurs typically at approximately 10 kHz.

Hydrophones are particularly suitable for measuring pulsed fields where any reflections, for example from the hydrophone support, can be eliminated from the measurements using time-gating (providing the pulse is sufficiently short). For an accurate representation of the pulses from medical diagnostic equipment, however, the hydrophone must have a broad frequency response as nonlinear distortion of the pulse may result in significant high frequency components.

Hydrophone measurements of continuous wave fields are more difficult. Reflections from both the hydrophone mounting, in probe devices, and the membrane itself, in membrane hydrophones, together with reflections from the walls and surface of the water tank, may interfere with the primary signal. Great care is therefore needed in interpreting the measurements obtained using a hydrophone in a continuous wave field. An additional problem may occur when measuring therapy fields as the relatively high ultrasonic intensities may severely damage the hydrophone.

Hydrophones are not absolute devices, but the relationship between the applied acoustic pressure and the voltage produced by the hydrophone can be determined by calibration. One method of hydrophone calibration consists of placing the hydrophone in an ultrasonic field which has been measured using a reciprocity technique, and one of these techniques, self-reciprocity, is

discussed later in this thesis. An alternative method of calibration, also mentioned later, is to scan the hydrophone over a plane perpendicular to the direction of propagation of the ultrasonic wave. The acoustic pressure measured at every point on this plane can be used to obtain a value for the total power in the ultrasonic field. This total power can then be measured using an independent technique, and the hydrophone calibrated by comparing the results. It is worth noting that the relationship between the pressure measured by the hydrophone and the intensity in the ultrasonic field is required in this method of hydrophone calibration. The relationship generally used is a first order approximation for plane sinusoidal wave motion.

4. MEASUREMENT OF CONSERVED QUANTITIES

Measurement of the acoustic pressure or the particle displacement amplitude at a point in the ultrasonic field is sufficient in some applications, but a complete specification of the field can be achieved only if the time variation of these parameters is measured at all points in space - clearly a time-consuming and complex exercise. Rather than this total point-by-point specification, an easier option would be to characterise the field using the measurement of a parameter which has the same value anywhere in the ultrasonic field. This suggests measurement of a conserved quantity, and there are several parameters of an ultrasonic field that obey conservation laws. Measurement of the time-averaged flux of two of these conserved quantities, energy and momentum, is often used for field characterisation, and we now proceed to discuss the concepts and techniques used. Clearly the time-averaging is an important

requirement for these conserved quantities otherwise the particular measurement time and the time behaviour would need to be specified. Here time-averaging is interpreted as a linear average either over an infinitely long time or over a single cycle if the quantity has cyclic behaviour. For pulsed quantities, it is assumed that the pulse has a definite beginning and end, and the time average is taken over the pulse length.

5. ENERGY AND ENERGY FLUX

In this section, the fluid will be assumed to be non-viscous and lossless, as this assumption simplifies the equations considerably and gives a good illustration of the basic physical concepts.

The energy E of unit volume element fixed in space can be given by

$$E = \rho \underline{u}^2 / 2 + \rho \epsilon,$$

where the first term is the kinetic energy and the second the internal energy. As before ϵ is the internal energy per unit mass.

The change in this energy is given by

$$\partial E / \partial t = (\underline{u}^2 / 2) \partial \rho / \partial t + \rho \underline{u} \cdot \partial \underline{u} / \partial t + \partial (\rho \epsilon) / \partial t. \quad \dots (5.1)$$

Applying equation (3.8) to a non-viscous lossless fluid,

$$d\epsilon / dt = - p \Delta / \rho = - (p / \rho) \nabla \cdot \underline{u}.$$

Hence,

$$\partial \epsilon / \partial t = - (p / \rho) \nabla \cdot \underline{u} - \underline{u} \cdot \nabla \epsilon,$$

and substituting for $\nabla \cdot \underline{u}$ from the equation of continuity (3.1) gives

$$\partial \epsilon / \partial t = (p/\rho^2) \partial \rho / \partial t + (p/\rho^2) \underline{u} \cdot \nabla \rho - \underline{u} \cdot \nabla \epsilon .$$

Substituting in equation (5.1) gives

$$\begin{aligned} \partial E / \partial t = & (u^2/2) \partial \rho / \partial t + \rho \underline{u} \cdot \partial \underline{u} / \partial t + \epsilon \partial \rho / \partial t + (p/\rho) \partial \rho / \partial t \\ & + (p/\rho) \underline{u} \cdot \nabla \rho - \rho \underline{u} \cdot \nabla \epsilon . \end{aligned} \quad \dots(5.2)$$

From equation (3.7), the equation of motion for a non-viscous lossless fluid is

$$-\nabla p = \rho \partial \underline{u} / \partial t + \rho \underline{u} \cdot \nabla \underline{u} ,$$

and using this in equation (5.2) together with the equation of continuity (3.1) gives

$$\partial E / \partial t + \nabla \cdot [\underline{u}(E + p)] = 0 .$$

The meaning of this equation becomes clear if we integrate over some volume V fixed in space, and then transform into a surface integral to obtain

$$(\partial / \partial t) \int_V E dV = - \int_S [\underline{u}(E + p)] \cdot \hat{n} dS, \quad \dots(5.3)$$

where $\hat{n}$ is the unit outward normal for an element of surface dS .

This equation is physically self-evident and has the form of a conservation equation. The left hand side is the rate of change of total energy in the volume V , whereas the first term on the right hand side is the amount of energy transported into the volume, and the second term is the rate of working on the volume. The right hand side is the total amount of energy flowing out of the volume in unit time, and the term $\underline{u}(E + p)$ is therefore the energy flux, rigorously defined as the rate of transfer of energy per unit area

normal to the direction of motion. We will call the time-averaged energy flux the intensity in accord with many definitions. It is worth noting that from this definition, intensity is a vector and has the same direction as the velocity $\underline{u}$.

The equation for the change in the total energy of the fluid derived here includes terms which represent the work done by the 'undisturbed' constant hydrostatic pressure p_0 (the pressure in the fluid in the absence of the acoustic field). These additional terms can be expressed in the form of an energy continuity equation as

$$(\partial/\partial t) \int_V E' dV = - \int_S [\underline{u}(E' + p_0)] \cdot \hat{n} dS,$$

where E' is the energy associated with the work done by the undisturbed pressure p_0 .

This equation can then be subtracted from the total energy continuity equation (5.3) to obtain a useable expression for the energy and intensity of an acoustic wave

$$(\partial/\partial t) \int_V E_A dV = - \int_S [\underline{u}(E_A + p - p_0)] \cdot \hat{n} dS,$$

where $E_A = E - E'$ is the energy density of the acoustic field. The equation for the intensity $\underline{I}$ of the acoustic field is therefore

$$\underline{I} = \overline{\underline{u}(E_A + p - p_0)}. \quad \dots(5.4)$$

This is an exact expression for the intensity of an acoustic field in a non-viscous lossless fluid. No assumptions have been made about the equation of state of the fluid medium, and the result is not limited to small displacements.

6. MEASUREMENT OF INTENSITY

The rate of transfer of energy across a boundary in an acoustic field varies throughout the acoustic cycle. This quantity is seldom of practical interest, what is required is the time-averaged value over a number of cycles, and there are two different types of measurements that can be made to determine the time-averaged energy flux (intensity) in an ultrasonic field. These techniques will not be described in great detail, but are mentioned here for completeness.

6.1 Energy Conversion

Measurement of the rate at which electrical energy is converted into ultrasonic energy can be used to provide an indication of the intensity of an ultrasonic field. A method used at the US National Bureau of Standards (Zapf 1974) measures the susceptance and conductance of a transducer operating under different load conditions; unloaded (transducer in air), loaded (transducer in water) and effectively clamped (transducer driven at twice the resonance frequency). It is then possible to identify a resistive component which represents the apparent load on the electrical drive circuit imposed by the conversion of electrical energy into ultrasound. Unfortunately, this method uses a simplified equivalent circuit, which is valid only for some quartz transducers. This equivalent circuit assumes that losses due to the mounting of the transducer and within the transducer itself are negligible, which is not true for piezoelectric ceramic transducers.

6.2 Energy Absorption

The second method of determining the intensity in an ultrasonic field is the measurement of the rate at which energy is received by an absorbing target. It is useful to distinguish between calorimeters, which average over the whole field, and probes which interrogate a small element of the field.

(i) Calorimeters

These are instruments which provide a measure of the intensity averaged over the whole ultrasonic beam or over a large part of the field. The calorimeter is intended to absorb all the ultrasonic energy, and a precise determination of the associated rate of temperature rise gives an value for the intensity of the ultrasonic field. The system is calibrated by applying a known amount of energy to the absorbing target using an electrical heater.

The construction of a practical calorimeter for operation at the power levels from ultrasonic diagnostic equipment is, however, far from simple, and there are a number of problems that should be overcome. All the ultrasonic energy should enter the calorimeter and be absorbed there. Reflections from either the calorimeter entrance or the absorber should be minimal. The ultrasound should be absorbed uniformly to produce uniform temperature rises, and the heat dissipation should be minimal or calculable. For total absorption, a large absorber is often used and this gives small temperature rises which are difficult to measure. The design most favoured so far (Torr and Watmough 1977, Zapf et.al. 1976) uses a fluid absorber such as castor oil, which flows through an absorption chamber at a steady rate while the input and output temperatures of the fluid are monitored.

(ii) Thermal Probes

There have been attempts to develop small temperature sensing probes to measure the spatial distribution of intensity within an ultrasonic field. These typically consist of a small thermistor (Szilard 1974) or thermocouple (Fry and Fry 1954) embedded in an absorbing medium (usually castor oil or polythene), which is chosen to have an acoustic impedance close to that of water and also a high absorption coefficient. The temperature rise of the probe in the ultrasonic field is recorded and can be related to the intensity. As with calorimeters there are a number of factors that must be considered in the design of thermal probes. Differences between the acoustic impedance of the absorbing material and that of water will produce reflections from the probe. Unless it is smaller than a wavelength, the probe will interfere with the ultrasonic field, and measure a spatially averaged quantity. The temperature rise should be the same throughout the probe, and heat should not escape to the surrounding fluid.

7. MOMENTUM AND MOMENTUM FLUX

Travelling waves carry energy and have a momentum associated with the energy transfer. Any object which absorbs or reflects the ultrasonic waves, and hence changes the wave momentum, will experience a force equal to the rate of change of that momentum. This force, commonly given the name radiation pressure, has, as is demonstrated later, been the source of much confusion within the literature, and forms the basis for this thesis. It will be discussed in detail in Part II.

8. MEASUREMENT OF RADIATION PRESSURE

As mentioned earlier, conservation of energy means that the time-averaged energy flux (intensity) is a suitable quantity for field characterisation. Similarly, as momentum is conserved, the time-averaged momentum flux (radiation pressure) can also be used. In the measurement of intensity, calorimeters provide a space-averaged value, while for spatial information thermal probes are used. Similarly with the measurement of radiation pressure, there are two different techniques; one giving a value averaged over the whole ultrasonic beam, the other interrogating a point in space.

8.1 Radiation Pressure Balances

Most ultrasonic transducers used in medical diagnosis operate at frequencies between 1 MHz and 10 MHz and have diameters of 5 mm to 25 mm. Hence, unlike the situation that exists for acoustical sources in the audible range, a target that intercepts the entire ultrasonic beam need not be of an unreasonable size.

The first instrument used to determine total output power by the measurement of radiation pressure was described by Altberg in 1903. Since then many different measurement devices have been mentioned in the literature and these can be divided into two categories:

- uncompensating - the target is allowed to move and the amount of movement is noted,
- compensating - the force required to keep the target stationary is measured.

In this section, the various different types of measurement devices will be described and their specific problems discussed in the context of the design of the radiation pressure balance

treated in Part III.

(i) Uncompensating Devices

Systems with moving targets, in general, tend to be insensitive and can only be used at therapy power levels as the force, and hence the movement produced, is small. There are two basic designs in this category.

(a) Floating Target Devices

These measure the deflection obtained when ultrasound is applied to a floating target. Kossoff (1962) describes a thin conical target (0.005 in. bronze), which is air-backed and has an attached stem. The target is placed in water and floats at a point of neutral buoyancy with the stem immersed in a beaker of carbon tetrachloride. When exposed to incident ultrasound, the radiation pressure forces the float and stem deeper into the carbon tetrachloride; the displacement of the float being proportional to the incident ultrasonic power. This system can only be used for high power levels (0.5 W to 25 W) and Kossoff estimates the accuracy as $\pm 5\%$. There are several practical difficulties associated with this type of float device. Firstly, surface tension effects at the water/carbon tetrachloride interface may cause erratic behaviour. Also, although the target will self-centre on the ultrasonic beam, there are no restraining forces on the float when the ultrasound is switched off, and the float may tend to drift.

A floating target device of a somewhat different design has been built at the National Physical Laboratory (NPL) (Shotton 1980), and consists of a target held in place within a water bath by three light silver chains linking the body of the target to the walls of the bath. Each of the chains hangs in a loop, and the fraction of the weight of the chains supported by the target

increases as the target rises, with the result that the target assumes an equilibrium position. The ultrasonic transducer is directed downwards towards the target, and the additional force exerted by the ultrasound is compensated by a downward movement of the target to a new equilibrium position. The displacement can be measured and related to the total time-averaged power from the transducer. This device can be used over the same power range as the Kossoff float with an estimated accuracy of $\pm 5\%$. It has several advantages as it does not require any liquid other than water, there are no surface tension effects and the chains keep the float centered.

(b) Moving Target Devices

The second group of the devices which relate the movement of a target to the radiation pressure includes equipment described by Newell (1963). He uses a torsion balance, similar to that designed by Gabrielli and Iernetti (1963), completely immersed in water. A wedge-shaped target, consisting of two 0.2 mm brass sheets with an air gap, is on one arm of the balance. The ultrasound displaces the target and the deflection can be measured and related to the total ultrasonic power, with a sensitivity of 1 W/cm. Clearly this device is insensitive, is only suitable for therapy power levels, and does not have any advantages over the float devices which are simpler and easier to use. A similar design used by Lunt and Ashley (1979) consists of a perspex beam mounted on a pivot with an elliptical target at 45° on one arm of the beam. Their target is rather unusual, consisting of a 1 mm thick perspex plate with 1 mm of rubber on the back, and is designed to reflect some of the ultrasound and absorb the remainder. They have an estimated error of $\pm 15\%$ and are able to measure powers as low as 1.8 mW. For perspex, the critical angle for longitudinal waves is 33° whereas,

as the velocity of shear waves in perspex is only 1330 m/s (compared with a velocity of longitudinal waves in water of 1484 m/s at 20°C), there is no critical angle for shear wave propagation. Hence, for a perspex target at 45°, all the longitudinal waves will be reflected, but there will be some shear wave transmission. Another device is described by Wells et.al. (1963), who also use the measurement of the deflection of a reflecting vane hanging from long wires (Wells et.al. 1964). All the systems in this group have two disadvantages. Firstly, they are rather insensitive as they use the measurement of target deflection, which for diagnostic power levels is typically 0.1 mm or less. Secondly, their calibration, achieved by measuring weights in water, is rather inaccurate and will be temperature dependent.

A type of balance arrangement which is slightly different from those described above is used by Hill (1970). This has a reflecting target suspended by three nylon threads from one arm of a chemical microbalance with the transducer mounted vertically above the reflector. He estimates that this will measure powers to a precision of ± 0.3 mW (standard error on the mean for ten measurements), which is clearly just the measurement reproducibility and is far below the systematic uncertainties. The major disadvantage of this device is the movement of the suspension wires through the air/water interface where surface tension effects will be a problem.

(ii) Compensating Devices

In the second class of systems the force required to keep the target fixed is measured, and this largely overcomes any problem of surface tension variations. A detailed description of his equipment is given by Rooney (1973), who uses a servocontrolled

balance with a 2 mm thick SOAB absorbing target suspended into water from one arm. The water bath consists of two vessels. The target is inside an inner vessel, which is closed at the bottom by a thin membrane to shield the target from streaming, and the transducer is mounted against another membrane at the bottom of the outer vessel. Rooney claims to measure powers as low as $30 \mu\text{W}$ with a standard deviation of $5 \mu\text{W}$. Two problems are immediately evident in this design of balance. Firstly the target is not a perfect absorber. In Part III of this thesis, measurements on a 12 mm thick slab of SOAB are described. Rooney's target was slightly different from our slab of SOAB where one surface was plane and the other face had been cut to reduce the reflection of the ultrasound. At 1 MHz, the plane face had an amplitude reflection coefficient of approximately 0.14, whereas cutting the surface reduced this to 0.033. The amplitude transmission coefficient for the 12 mm slab was 0.019. Clearly, as Rooney's target is plane, a significant fraction of the ultrasound will be reflected and also, as his target is only 2 mm thick, some of the ultrasound will be transmitted. Hence, it can be roughly estimated that at 1 MHz Rooney's target absorbs less than 75% of the incident ultrasonic energy. The second problem with this balance design is the two membranes which are positioned perpendicular to the incident ultrasound. These will cause standing waves to be set up, as even a $6 \mu\text{m}$ thick membrane has an amplitude reflection coefficient ranging from 0.02 at 1 MHz to 0.17 at 10 MHz. Hence the fraction of total ultrasonic energy reaching the target will vary between approximately 99.9% at 1 MHz to 94% at 10 MHz. A possible disadvantage of all designs using an absorbing target is that heating of the target and surrounding water will introduce buoyancy changes and convection currents. A similar system with

the same design of absorbing target is described by Carson et.al. (1978), who achieve a reproducibility of 14%. They use a brass reflector to channel the ultrasound towards the target from their transducer; the obvious disadvantage being that any ultrasound reflected from the brass will strike the absorber at a different angle, and introduce a different component to the radiation pressure.

Measurement systems using a servocontrolled balance and a reflecting target are described by Kossoff (1965), and also by Miyasaki et.al. (1972), who use a wedge-shaped reflector to measure powers down to 0.5 mW with an accuracy of 10-15%. Wemlen (1968) uses a torsion balance submerged in an oil bath with a reflector on one plate of a capacitor. The radiation pressure tends to move this reflector, the change of capacitance is measured by a transformer bridge and provides the input signal for a servosystem which keeps the reflector fixed. Wemlen estimates the error on a reading of 5 mW as less than 5%. This is quite a nice design; the main disadvantage being the necessity for an oil bath which will have a high attenuation coefficient.

A portable device for the routine measurement of diagnostic equipment has been built by Farmery and Whittingham (1978), who use a moving-coil galvanometer with an optical sensing system to keep a reflecting target in a fixed position. The whole device is immersed in liquid paraffin and the ultrasound is applied through a polythene window. They estimate the accuracy of their instrument to be $\pm 15\%$ with a sensitivity of 0.1 mW. Again the major disadvantage of this equipment is the high attenuation coefficient of liquid paraffin. Also, as the transducer is mounted against a thin pliable membrane window, its position and hence the ultrasonic path length are ill-defined giving an uncertainty in

the correction for attenuation.

A different technique used at the National Bureau of Standards (Greenspan et.al. 1978) is the detection of modulated radiation pressure. The input to a transducer is amplitude modulated at a low frequency (35 Hz), and hence the radiation pressure on a reflecting target has a component at this frequency. The target is rigidly connected to an electromagnetic vibration detector and associated electronics, which captures the modulated signal, produces a servocontrolling signal and provides a measure of the radiation pressure. This technique discriminates against extraneous environmental noise and has the potential for measuring very low ultrasonic power levels. It is used for powers from 10 μ W to 10 W with an estimated error of $\pm 3\%$; the main source of uncertainty is in the calibration of the electromagnetic receiver. The main disadvantage with this system is the need for signal modulation which means that it cannot readily be used to measure the output from medical ultrasonic equipment.

8.2 Radiation Pressure on a Small Sphere

An alternative approach to the measurement of radiation pressure, which allows a study of the spatial variations in the ultrasonic field, is the measurement of the deflection of a small target suspended in the sound field. Intensity distributions can, in principle, be obtained using a target much smaller than the cross-section of the ultrasonic beam. Such a target necessarily has dimensions comparable with the wavelength of the ultrasound, and the measurement technique therefore requires diffractive effects to be considered. In practice, small spherical targets are used (Fry and Dunn 1962), and the force exerted on them by an incident ultrasonic wave (assumed to be plane) can be calculated.

The deflection of the suspended target from the vertical must be kept small and, in this case, the intensity I of the sound field is given by

$$I = mgxc/\pi l r^2 Y,$$

where m is the mass of the sphere, g the acceleration due to gravity, x the displacement of the sphere from the vertical, c the velocity of ultrasound in water, l the length of the suspension, r the radius of the sphere and Y (Hasegawa and Yoshioka 1969) is a function depending on the material and dimensions of the sphere.

This technique has a number of major disadvantages. The Y value depends strongly on the radius of the sphere (assumed to be perfectly spherical) and on the characteristics of the sphere material. It is also highly dependent on frequency. The theory used to calculate the relation between the deflection and the intensity assumes the incident ultrasonic wave is plane and also neglects any nonlinear propagation of the ultrasound. The technique itself is relatively insensitive, for example, a 0.5 mm diameter stainless steel sphere suspended from a thread 1 m long in a 2 MHz ultrasonic field of 10 mW/cm^2 intensity will be deflected approximately 2 mm. Practical problems occur with air currents around the supporting wire and surface tension effects as the wire move through the air/water interface. Even mounting the sphere on the end of the wire will be difficult; the usual method is to use glue but the amount of glue must be very small otherwise the sphericity of the target will be affected. In general, the suspended sphere technique has no advantages over a hydrophone for spatial measurements other than it measures, in principle at least, another parameter of the ultrasonic field.

9. CONCLUSION

The different techniques used to characterise an ultrasonic field have been described. A hydrophone can be used for the direct measurement of the acoustic pressure at a point in the field, while the particle displacement amplitude can be determined with an acoustically transparent membrane and an interferometer. For a complete specification of the ultrasonic field, the measurement of a conserved quantity is preferred as this will have the same value anywhere in the field. A calorimeter or a radiation pressure balance can be used to determine the total power, whereas for determination of the spatial distribution of the intensity, a thermal probe or a small suspended target is required.

PART II

Theoretical Treatment of Acoustic Radiation Pressure

1. INTRODUCTION

In this part of the thesis, the relationship between the force measured by a radiation pressure balance and various properties of an ultrasonic field is discussed. The measurements are normally made in distilled degassed water, a medium which may be considered to a reasonable degree of approximation as both non-viscous and lossless as mentioned in Part I. In a non-viscous medium, the local force on a small element of an obstacle within the fluid must necessarily be normal to its surface and hence indistinguishable from the effect of a hydrostatic pressure. It is for this reason that the force associated with the presence of an acoustic field has often been given the name radiation pressure, an unfortunate use of the term pressure which is normally reserved for a scalar quantity, one of the variables used to describe the state of the fluid medium. In this thesis, however, we shall use the term radiation pressure, despite its disadvantages.

1.1 Definitions of Radiation Pressure

Several apparently contradictory definitions for radiation pressure are in current use, and a number of authors have commented on the confusion which exists within the literature concerning the subject of radiation pressure (Beyer 1978, Post 1953, Woodcock 1979, Zieniuk and Chivers 1976). It is generally thought that this uncertainty in the relation between the measured force on an obstacle in the field and some other field quantity is

due to the different approximations made in calculations. It will be shown, however, that this is only partly true: in some cases the conflicting results have to be expected because different authors start from different definitions of radiation pressure. The various approaches followed are reviewed here and it is demonstrated that, while most of the apparently unrelated definitions are, in fact, equivalent, this is not always the case.

The values of the field quantities obtained from any practicable measurement of radiation pressure are not the instantaneous levels, but are the time-averaged quantities for which the time-averaging takes place over a period much larger than that of a single oscillation of the ultrasonic wave. Hence any definition of radiation pressure, which is to be of practical significance, should contain time-averaging. This is generally the case and there appears to be only one exception to this (see Table 1 in Zieniuk and Chivers 1976), a concept which will not be considered further in this discussion.

The most common definition of radiation pressure found in the literature is the time-averaged force per unit area on a large fixed obstacle which intercepts the entire ultrasonic beam (Awatani 1955, Beyer 1978, Biquard 1932, Bopp 1940, Borgnis 1953a, Brillouin 1925, Goldberg 1971, Goberman 1968, Hertz and Mende 1939, Kanevskii 1961, Lindsay 1960, Olsen et.al. 1958, Richter 1940, Rooney and Nyborg 1972, Stapper 1978, Wells et.al. 1977, Westervelt 1957) The presence of the obstacle is therefore an inherent part of this concept. Some authors, however, define radiation pressure in terms of a particular variable of the ultrasonic field. One example is the time-averaged pressure experienced by a moving fluid element (Beyer 1978, Borgnis 1953a, Hertz and Mende 1939), while another is the time-averaged momentum

flux of the ultrasonic wave (Brillouin 1925, Freeman 1963, Goldberg 1971, Kanevskii 1961, Morse and Ingard 1968, Westervelt 1957). In some cases (Awatani 1955, Lindsay 1960, Mendousse 1950, Stapper 1978), the name radiation pressure is given to the time-averaged pressure at a point fixed in space. The relationship between these different 'field definitions' and their relation to the 'force-on-an-obstacle' concept have not been clearly explored in the literature, and, occasionally, the differences have led to the impression that different physical phenomena are being considered.

1.2 Rayleigh and Langevin Radiation Pressures

A further source of confusion is the assertion sometimes made that there are two distinct types of radiation pressure, the Langevin and the Rayleigh. The time-averaged force on an obstacle is called the Langevin radiation pressure by some authors (Awatani 1955, Beyer 1978, Lindsay 1960, Stapper 1978), with the name Rayleigh reserved either for the time-averaged pressure on a moving fluid element (Beyer 1978), or for the time-averaged pressure at a fixed point in space (Awatani 1955, Lindsay 1960, Stapper 1978). By the last notion, the Rayleigh radiation pressure (but not the Langevin radiation pressure) is a field quantity, deducible without reference to the type, shape, or even presence of an obstacle. In fact, Rayleigh and Langevin dealt with the same physical effect but obtained different results by adopting different boundary conditions, as was pointed out as early as 1940 by Bopp (1940). Beyer (1978) reports that Langevin's work was never published (and we have certainly failed to uncover any such reference), although a presumably accurate account of his derivation was written by Biquard (1932).

In this thesis, a unified and simple treatment of the subject is developed, and radiation pressure is shown to be a single physical phenomenon, a characteristic of an ultrasonic field as meaningful as intensity or energy density. It is shown that this definition of radiation pressure is consistent with most of the definitions mentioned above and that much of the apparent contradiction is illusory.

In previous treatments of radiation pressure theory, expansions are made only to second order, usually with respect to pressure or density, or, occasionally, displacement amplitude. However, the theory presented here is correct to all orders of magnitude, but to simplify the analysis it is initially assumed that viscosity and attenuation are negligible. This assumption, which is adopted by most authors, does not affect the physical interpretation: modifications of the basic equations to take account of the effects of viscosity, and the consequences for radiation pressure have been studied, to second order only, by Goldberg (1971). Our present treatment is also applicable only to large targets which intercept the whole acoustic beam, as the effects of diffraction and scattering must be allowed for in any investigation of radiation pressure in relation to small obstacles.

A summary of the significant contributions from different authors to the theory of radiation pressure is given in Appendix B.

2. RADIATION PRESSURE FOR ONE-DIMENSIONAL MOTION IN A LOSSLESS FLUID: EXACT THEORY

2.1 General Derivation

We will start by considering one-dimensional motion in a lossless fluid as this is the simplest situation and provides a clear picture of the nature of radiation pressure.

The one-dimensional lossless forms of the continuity equation (I.3.1) and the equation of motion (I.3.7)

$$\partial \rho / \partial t + \partial(\rho u) / \partial x = 0, \quad \dots(2.1)$$

$$- \partial p / \partial x = \rho \partial u / \partial t + \rho u \partial u / \partial x, \quad \dots(2.2)$$

can be combined to give

$$\partial(\rho u) / \partial t + \partial(p + \rho u^2) / \partial x = 0. \quad \dots(2.3)$$

This may be time-averaged to

$$(\partial / \partial x)(\bar{p} + \overline{\rho u^2}) = 0, \quad \dots(2.4)$$

where the bar indicates the time-averaged value

$$\bar{f} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(t) dt.$$

Clearly, the time average of df/dt must vanish if the smooth function $f(t)$ remains bounded.

On integrating equation (2.4) with respect to x

$$\bar{p} + \overline{\rho u^2} = R, \quad \dots(2.5a)$$

where R is a constant.

Averaging this equation over any region fixed in space gives

$$\langle \bar{p} \rangle + \langle \overline{\rho u^2} \rangle = R, \quad \dots(2.5b)$$

where $\langle \rangle$ indicates the space-averaged value.

The equations (2.5) are perfectly general and true to all orders of magnitude, valid for all fluids which obey equations (2.1) and (2.2). The constant R may be considered a fundamental property of the acoustic field and can be regarded as a parameter characterising the field. We suggest that the quantity $(R - p_0)$ be called the "radiation pressure", where p_0 is the (constant) hydrostatic pressure of the fluid in the absence of the acoustic field. The justification for this definition lies in the fact that it will be shown to be compatible with, and also a generalisation of, the majority of the (second order) definitions advocated in the literature.

2.2 Radiation Pressure for a Rigid Target

The physical significance of the constant R is particularly clear for the case of a rigid target fixed in the acoustic field. Applying equation (2.5a) on the surface of the target gives

$$R = (\bar{p})_{\text{surf}} + (\overline{\rho u^2})_{\text{surf}}.$$

The values of p , ρ and u at the target surface are the values of the field variables obtained by solving equations (2.1) and (2.2) subject to the appropriate boundary conditions. A full solution of these equations will, in general, demand the specification of another relationship, such as the equation of state of the fluid.

For a rigid target, $(u)_{\text{surf}} = 0$, and therefore

$$R = (\overline{p})_{\text{surf}}.$$

Hence R is the time-averaged pressure exerted by the fluid on the surface of a rigid target. It is clear from this equation that R includes a constant term, $(\overline{p}_0)_{\text{surf}} \equiv p_0$, from the constant hydrostatic pressure. Hence $(R - p_0)$ is the time-averaged pressure exerted by the acoustic wave on the rigid target surface. As far as the target is concerned, this is experienced as a force, and so can be identified with one of the usual definitions of radiation pressure, namely the time-averaged force per unit area on an appropriate target which intercepts the whole beam.

2.3 Eulerian and Lagrangian Coordinates

The hydrodynamic equations we have used are written in an Eulerian formalism, which expresses the motion with respect to coordinates fixed in space. The pressure at a fixed point in space is therefore the pressure in Eulerian coordinates, written from now on as p_E . An alternative representation is to express all quantities in terms of coordinates fixed, not in space, but in the individual elements of the fluid; in this case the coordinates are known as Lagrangian, and quantities expressed in this system will be given the subscript L . The pressure experienced by a particular moving fluid element can therefore be written as p_L .

At the surface of a rigid wall, the Eulerian and Lagrangian coordinate systems coincide and, in this special situation,

$$R = (\overline{p}_E)_{\text{surf}} = (\overline{p}_L)_{\text{surf}}.$$

2.4 One-Dimensional Equations of Continuity and Motion in Lagrangian Coordinates

Before considering the radiation pressure on non-rigid targets, we need to derive expressions for the equations of continuity and motion using the Lagrangian coordinate system.

Consider an element of fluid which occupies the space $dx dy dz$ when at rest, and a plane wave travelling through the fluid so that, at a given instant, the fluid particles originally at x are displaced a distance ξ , and those originally at $x + dx$ are displaced a distance $\xi + d\xi$. The new boundaries of the fluid element could have been called a and $a + da$.

The original volume of the fluid element was $dx dy dz$ giving a mass of $\rho_0 dx dy dz$, where ρ_0 is the (constant) density of the fluid in the absence of the acoustic field. When the plane wave is present, the volume and mass of the fluid element become $da dy dz$ and $\rho_L da dy dz$ respectively. Mass is conserved, and therefore the mass must be the same with or without the acoustic wave, i.e., $\rho_L da dy dz = \rho_0 dx dy dz$. But, $a = x + \xi$, and so

$$\rho_L = \rho_0 (1 + \partial \xi / \partial x)^{-1},$$

which is the equation of continuity expressed in Lagrangian coordinates.

If the pressure on one side of the fluid element is p_L , and that on the other is $p_L + (\partial p_L / \partial a) da$, the net force on the element is $-(\partial p_L / \partial a) da dy dz$. Equating this force to the product of mass and acceleration gives

$$-(\partial p_L / \partial a) da dy dz = \rho_0 dx dy dz (\partial^2 \xi / \partial t^2),$$

or

$$-\partial p_L / \partial x = \rho_0 \partial^2 \xi / \partial t^2, \quad \dots (2.6)$$

which is the equation of motion expressed in Lagrangian coordinates.

2.5 Radiation Pressure for Non-rigid Targets

An extension of the interpretation of R given in section (2.2) to non-rigid targets requires more subtle arguments. Consider a target placed in the path of the acoustic beam. The time-averaged external force (per unit area) experienced by the front wall of the target, located at A , is $(\overline{p_L})_A$, where this is calculated in the fluid immediately adjacent to the wall. This statement merely expresses the meaning of the Lagrangian coordinate system. Clearly, $\overline{p_L}$ may be identified with the "force-on-an-target" definition of radiation pressure, and its relationship to R needs to be established. It should be pointed out that, in an experiment designed to measure the radiation pressure using a non-rigid target, the actual total force on the target would be given by

$$F = (\overline{p_L})_A - (\overline{p_L})_B,$$

where B is the location of the rear wall of the target.

F can then be related to acoustic field values only if the transmission and reflection properties of the target are known (e.g. for a perfectly absorbing surface at A , the target could be chosen thick enough that $(\overline{p_L})_B = p_o$.)

The one-dimensional equation of motion expressed in Lagrangian coordinates (equation 2.6) can be time-averaged to give

$$-\overline{\partial p_L / \partial x} = 0,$$

and integration gives that $\overline{p_L}$ is a constant of the motion. This again suggests that $\overline{p_L}$ and R may be related. They are, in fact, identical, as may be proved as follows.

From the definitions of the Lagrangian and Eulerian coordinate systems

$$p_L(x, t) = p_E(x + \xi, t), \quad \dots(2.7)$$

where x is the coordinate of a particular fluid particle at time $t=0$ and $\xi(x, t)$ is the displacement of that particle at time t .

Writing equation (2.7) as a Taylor expansion gives

$$\begin{aligned} p_L &= p_E + \xi \partial p_E / \partial x + (1/2) \xi^2 \partial^2 p_E / \partial x^2 + \dots \\ &= p_E + \sum_{k=1}^{\infty} \frac{\xi^k}{k!} D^k[p_E], \end{aligned} \quad \dots(2.8)$$

where $D^k[p_E]$ is a shorthand notation for the differential operation $\partial^k p_E / \partial x^k$.

Expressing equation (2.3) in this form gives,

$$D[p_E] = -(\rho_E u_E)_t - D[\rho_E u_E^2],$$

where $()_t$ denotes differentiation with respect to time.

Substituting this expression in equation (2.8) gives

$$\begin{aligned} p_L &= p_E - \sum_{k=1}^{\infty} \frac{\xi^k}{k!} D^{k-1}[(\rho_E u_E)_t] - \sum_{k=1}^{\infty} \frac{\xi^k}{k!} D^k[\rho_E u_E^2], \\ &= p_E + \rho_E u_E^2 - \left(\sum_{k=1}^{\infty} \frac{\xi^k}{k!} D^{k-1}[\rho_E u_E^2] \right)_t + (\xi)_t \sum_{k=1}^{\infty} \frac{\xi^{k-1}}{(k-1)!} D^{k-1}[\rho_E u_E^2] \\ &\quad - e^{\xi D}[\rho_E u_E^2], \end{aligned}$$

where $e^{\xi D}[\rho_E u_E^2]$ is notation for $\rho_E u_E^2 + \sum_{k=1}^{\infty} \frac{\xi^k}{k!} D^k[\rho_E u_E^2]$.

But

$$\begin{aligned} (\xi)_t &\equiv u_L = u_E + \sum_{k=1}^{\infty} \frac{\xi^k}{k!} D^k[u_E], \\ &= e^{\xi D}[u_E], \end{aligned}$$

and so

$$\begin{aligned}
 p_L &= p_E + \rho_E u_E^2 - \left(\sum_{k=1}^{\infty} \frac{\xi^k}{k!} D^{k-1} [\rho_E u_E] \right)_t + (\xi)_t (\rho_E u_E + \sum_{l=1}^{\infty} \frac{\xi^l}{l!} D^l [\rho_E u_E]) \\
 &\quad - e^{\xi D} [\rho_E u_E^2], \\
 &= p_E + \rho_E u_E^2 - \left(\sum_{k=1}^{\infty} \frac{\xi^k}{k!} D^{k-1} [\rho_E u_E] \right)_t + e^{\xi D} [u_E] e^{\xi D} [\rho_E u_E] \\
 &\quad - e^{\xi D} [\rho_E u_E^2]. \quad \dots(2.9)
 \end{aligned}$$

Now

$$\begin{aligned}
 e^{\xi D} [u_E] e^{\xi D} [\rho_E u_E] &= (u_E + \xi \partial u_E / \partial x + \dots) [\rho_E u_E + \xi \partial (\rho_E u_E) / \partial x + \dots], \\
 &= \rho_E u_E^2 + \xi \partial (\rho_E u_E^2) / \partial x + \dots \\
 &= e^{\xi D} [\rho_E u_E^2].
 \end{aligned}$$

Substituting into equation (2.9) gives

$$p_L = p_E + \rho_E u_E^2 - \left(\sum_{k=1}^{\infty} \frac{\xi^k}{k!} D^{k-1} [\rho_E u_E] \right)_t.$$

Taking time averages of this expression gives

$$\overline{p_L} = \overline{p_E} + \overline{\rho u_E^2} = R. \quad \dots(2.10)$$

An alternative derivation of this equation can be obtained by integrating equation (2.2) between two planes in the fluid medium, one at η , fixed in laboratory coordinates, and the other at λ moving with the medium,

$$(p_E)_{\eta} - (p_E)_{\lambda} = \int_{\eta}^{\lambda} \rho_E (\partial u_E / \partial t) dx + \int_{\eta}^{\lambda} \rho_E u_E (\partial u_E / \partial x) dx. \quad \dots(2.11)$$

Now

$$(\partial / \partial t) \left(\int_{\eta}^{\lambda} \rho_E u_E dx \right) = \int_{\eta}^{\lambda} \rho_E (\partial u_E / \partial t) dx + \int_{\eta}^{\lambda} u_E (\partial \rho_E / \partial t) dx + (\rho_E u_E^2)_{\lambda},$$

and substituting in equation (2.11)

$$\begin{aligned} (p_E)_\eta - (p_E)_\lambda &= (\partial/\partial t) \left(\int_\eta^\lambda \rho_E u_E dx \right) + \int_\eta^\lambda (\rho_E u_E \partial u_E / \partial x - u_E \partial \rho_E / \partial t) dx \\ &\quad - (\rho_E u_E^2)_\lambda. \end{aligned} \quad \dots(2.12)$$

From the equation of continuity (equation 2.1)

$$\int_\eta^\lambda u_E (\partial \rho_E / \partial t) dx = - \int_\eta^\lambda u_E [\partial (\rho_E u_E) / \partial x] dx,$$

and substituting in equation (2.12)

$$(p_E)_\eta - (p_E)_\lambda = (\partial/\partial t) \left(\int_\eta^\lambda \rho_E u_E dx \right) - (\rho_E u_E^2)_\eta.$$

Taking time averages gives

$$(\overline{p_E})_\lambda = (\overline{p_E} + \overline{\rho_E u_E^2})_\eta. \quad \dots(2.13)$$

The right-hand side of this equation must be independent of η , since it is equal to R . Also, the pressure $\overline{p_E}$ time-averaged on a plane moving with the medium must, by the definition of the Eulerian and Lagrangian coordinate systems, be identically equal with $\overline{p_L}$. Since the choice of the moving plane λ is quite arbitrary, equation (2.13) reduces to equation (2.10).

This relationship was demonstrated, to a second order approximation only, by Borgnis (1953b) using a rather different method of proof. He also demonstrated that a less useful relation

$$(\overline{p_L})_{x_1} - (\overline{p_L})_{x_2} = (\overline{p_E} + \overline{\rho_E u_E^2})_{x_1} - (\overline{p_E} + \overline{\rho_E u_E^2})_{x_2},$$

holds exactly, for any two planes at x_1 and x_2 in the medium. He then erroneously concluded that this result implied the validity of equation (2.10).

2.6 Other Definitions of Radiation Pressure

We have shown that once the field equations have been solved subject to the appropriate boundary conditions, then $\overline{p_L}$ calculated at any point in the field will give the time-averaged force on the surface of the target; similarly so will the quantity $(\overline{p_E} + \overline{\rho_E u_E^2})$, calculated anywhere in the field. Both these quantities include a constant term, $\overline{p_0} \equiv p_0$, which will be present in the absence of the acoustic field. Hence the time-averaged force on the surface of the target from the acoustic field is $(\overline{p_L} - p_0)$, or equivalently $(\overline{p_E} + \overline{\rho_E u_E^2} - p_0)$. This justifies the naming of $(R - p_0)$ as the radiation pressure. Clearly, in general, the time-averaged pressure at a fixed point, $\overline{p_E}$, is not equal to R , and so the results obtained by those authors (Awatani 1955, Mendousse 1950, Stapper 1978), who proceed from this definition cannot be expected to agree with other expressions for the radiation pressure.

Another of the definitions of radiation pressure is in terms of the time-averaged momentum flux of the wave (Brillouin 1925). We can investigate this by returning to equation (2.3), which has the form of a continuity equation (Landau and Lifshitz 1959). The first term is the time derivative of momentum density, and hence, since total momentum is conserved, the second term $(\overline{p_E} + \overline{\rho_E u_E^2})$ must represent the momentum flux, where the term "flux" denotes the rate of transfer of a quantity per unit area normal to the direction of flow. Thus in equation (2.5a), R is the time-averaged momentum flux. That the radiation pressure in a wave arises from the momentum carried was pointed out by Rayleigh (1905), who refers to an earlier statement by Poynting containing the idea.

2.7 Summary

To summarise, we have defined radiation pressure in a unified way, correct to all orders of magnitude, and demonstrated the equivalence of the following:

- (1) the time-averaged force per unit area on the surface of a target,
- (2) the time-averaged pressure in the field expressed in Lagrangian coordinates,
- (3) the time-averaged momentum flux.

The alternative definition of radiation pressure as the time-averaged pressure in Eulerian coordinates produces a result compatible with the above only for the case of a perfect (rigid) reflector, and only provided that $\overline{p_E}$ is calculated on the reflector surface itself. In general, the association of the term radiation pressure with $\overline{p_E}$ does not seem justified, since its value would vary from point to point in the field. A value equal to R would be obtained only for the rather special case above, in circumstances which must unavoidably perturb the original field.

3. CALCULATION OF R

Differences between authors often occur in the methods by which they calculate R , and this may well have given rise to the impression that each is dealing with a different concept of radiation pressure. The calculation can be done in one of three ways.

3.1 From $R = \overline{p_E} + \overline{\rho_E u_E^2} = \overline{p_L}$ (Local method)

To apply this equation it is necessary to know either the values of p_E , ρ_E and u_E at any one point in space, or the value of the pressure p_L on any particular moving fluid element. These

values are obtained by solving equations (2.1) and (2.2) subject to the appropriate boundary conditions for the particular physical system under consideration, or by solving the equivalent problem expressed in a Lagrangian coordinate system. The two equations, however, are functions of three field variables and so a third equation is required for their solution, the equation of state of the medium.

It is mentioned in Part I (section 3.4) that only two independent properties are needed to specify the state of a fluid, and density and entropy are chosen as the independent variables giving an equation of state of the form $p = p(\rho, S)$. If the fluid can be assumed to be adiabatic, this simply becomes $p = p(\rho)$. It is equally valid to choose the density and the temperature as the independent variables, with the corresponding equation of state $p = p(\rho, T)$. Hence the relation $p = p(\rho)$ is valid as an equation of state either for adiabatic or isothermal motion. We will assume that suitable conditions exist in the lossless non-viscous medium so that $p = p(\rho)$ is an appropriate equation of state.

3.2 From $R = \langle \bar{p}_E \rangle + \langle \bar{\rho}_E u_E^2 \rangle$. (Non-local method)

This method presents the difficulty that the field needs to be known everywhere throughout the fixed region over which the spatial average is taken. In practice, however, this may be no more difficult than obtaining the field at a particular point (as for the local method), but the space-averaging involves an additional computational step. On the other hand, special boundary conditions may drastically simplify the method, as will be indicated later.

3.3 From Momentum Flux Arguments

The values of p , ρ and u used in the equations above refer to the total motion, which in the presence of a reflector is the incident plus the scattered field. It is sometimes difficult to solve the equations for the total field and an alternative approach is to investigate the momentum transferred to the obstacle. In this case, it is simpler to depart from a one-dimensional treatment and consider an imaginary surface around the reflector. The incident wave with its associated momentum m_{in} , passes inwards through the surface and an amount of momentum m_{out} emerges outwards with the reflected wave. A quantity of momentum $(m_{in} - m_{out})$ is transferred to the interior of the surface. In the absence of absorption, all this momentum must be transferred to the reflector, which therefore experiences a force. If the reflecting properties of the obstacle are known, and there is no interaction between incident and reflected waves, m_{out} can be estimated directly from m_{in} , and the radiation pressure associated with the field before insertion of the obstacle may be calculated.

A powerful extension of the method was developed by Westervelt (1957), who considered a surface so far from the obstacle that the problem becomes one of scattering theory. This approach is suitable for calculating forces on small obstacles. It should be clear that, although this method is computationally quite unlike the others reviewed above, it does not attach a different meaning to the concept of radiation pressure.

4. RELATIONSHIP OF RADIATION PRESSURE TO OTHER QUANTITIES

We have shown that radiation pressure as we define it is a single physical phenomenon, a parameter of the acoustic field, and a direct consequence of the hydrodynamic equations. The time-averaged force per unit area on the surface of an obstacle placed in the field is exactly related to the time-averaged momentum flux density of the wave, or the time-averaged pressure in the field expressed in Lagrangian coordinates. This result is valid for a non-viscous lossless fluid to all orders of approximation, and is independent of the equation of state of the medium (subject to $p = p(\rho)$ being applicable). Since this field quantity, radiation pressure, takes on the same value anywhere within the field, it may justifiably be called a constant of the motion, and its measurement adds an important element to our knowledge of the field. In practice, however, this immediate and important result, obtainable in a conceptually simple measurement, seems to have been disregarded in favour of attempts to relate the measured force to various other quantities, particularly the energy density. It is at this stage that various additional simplifying approximations need to be made. Clearly, the relationship of the measured force to the desired field quantity depends on the circumstances of the experiment, but it is worth emphasising again that the measured force always relates to the time-averaged momentum flux: what may change from one measurement system to another, is the relationship of other field quantities to that flux.

As mentioned, the majority of authors do not use the direct relationship between the radiation pressure and the time-averaged momentum flux or the time-averaged pressure in Lagrangian

coordinates. Instead they try to relate the measured force to either the energy density or the intensity. These cannot be related directly and various additional assumptions have to be used. There are two particular "boundary conditions" that have been used in the literature, and these are first discussed. These are not boundary conditions in the usual sense of an equation valid on a particular surface in the fluid, but are much more general, being constraints on the behaviour of a particular variable of the acoustic field.

4.1 Open Vessel Boundary Condition

Equation (2.10) can be space averaged to give

$$R = \langle \overline{p_E} \rangle + \langle \overline{\rho_E u_E^2} \rangle = \langle \overline{p_L} \rangle.$$

The term $\langle \overline{\rho_E u_E^2} \rangle$ is, by definition, equal to $2\langle \overline{T_E} \rangle$, where T_E is the kinetic energy density, and so

$$R = \langle \overline{p_E} \rangle + 2\langle \overline{T_E} \rangle. \quad \dots(4.1)$$

Several authors (Bopp 1940, Borgnis 1953, Rooney and Nyborg 1972), have suggested the following constraint for measurements conducted in an open vessel:

$$\langle \overline{p_E} \rangle = p_o, \quad \dots(4.2)$$

where p_o is the static pressure in the absence of ultrasound and is the value assumed to hold in the unperturbed medium outside the beam.

Use of this "boundary condition" in equation (4.1) gives

$$R - p_o = 2\langle \overline{T_E} \rangle,$$

an exact result, true to all orders of magnitude.

Hence, measurements of radiation pressure may, under certain circumstances, produce values for the space-time averaged kinetic energy density of the acoustic field. This is a very powerful result; it is valid to all orders of magnitude and is independent of both the equation of state of the fluid and of the type of obstacle intercepting the field, the only restraint being the validity of equation (4.2). It is well to emphasise under what conditions this result holds: the time average must be taken over a time T either infinitely long or such that $(1/2T) \int_{-T}^T \partial(\rho_E u_E)/\partial t = 0$, and the space average must be taken over a volume such that $\langle \overline{p_E} \rangle = p_0$. Any volume will not do. In addition, although condition (4.2) is often referred to as the "open-vessel" condition, it has little to do with whether the vessel is open or closed, and depends only on whether the region containing the sound field is able to interact with an outside unperturbed region (Borgnis 1953a). The boundary condition (4.2) has been regarded as the condition necessary for characterising the so-called Langevin radiation pressure (see, for example, Rooney and Nyborg 1972), but as already emphasised such distinctions are artificial.

4.2 Closed Vessel Boundary Condition

The second boundary condition in the literature is

$$\langle \rho_E \rangle = \rho_0, \quad \dots(4.3)$$

which was used by Rayleigh (1905) and is frequently seen as that characterising the so-called Rayleigh radiation pressure, valid for measurements conducted within a closed vessel. However, a careful examination of Rayleigh's original work reveals that equation (4.3) is merely the introduction of the equation of

continuity into his treatment, in a form appropriate to the problem he considers: a one-dimensional (standing wave) field contained within a box with rigid end walls. Equation (4.3) is not in general equivalent to the equation of continuity and may even be in conflict with it. To see this, consider the equation of continuity space-averaged over a fixed length L , which gives

$$\delta \langle \rho_E \rangle / \delta t = (1/L) [(\rho_E u_E)_L - (\rho_E u_E)_0].$$

Hence, for any fixed volume, $\langle \rho_E \rangle$ is a constant only if $\rho_E u_E$ is always instantaneously equal at both the planes 0 and L (i.e. matter is entering the region as fast as it is leaving at every moment). In general, therefore $\langle \rho_E \rangle \neq \rho_0$ when the averaging is performed over some arbitrary, fixed volume in space, even for the closed vessel situation, unless (as in Rayleigh's treatment), the container has rigid static walls ($(u_E)_L = (u_E)_0 = 0$) and the volume is chosen to be that enclosed by the walls.

Rooney and Nyborg (1972) incorrectly used equation (4.3) for a volume bounded on one side by an absorber. This must, by its very nature, be non-rigid and hence their container did not have the rigid static end walls required for equation (4.3) to be valid. Under these circumstances it is not surprising that their result differs from treatments of the same problem by Beyer (1978) and Westervelt (1950). Condition (4.3) (when it is applicable) contains no additional restrictions if the continuity equation has been incorporated right from the start, as in this treatment. For those calculations where it is used as a substitute for the equation of continuity (i.e. conservation of mass), the volume over which the averaging is implemented must be chosen with care.

4.3 Potential Energy Density

It is clear that the constant density boundary condition can only be used in a very special situation, and the open vessel constraint, while effecting significant simplification, may not be generally valid. We will therefore, attempt to obtain a relation between radiation pressure and intensity or energy density for one-dimensional motion in a non-viscous lossless fluid without the use of any simplifying boundary conditions. We have already derived an expression for the intensity of an acoustic field (equation I.5.4), but need a general equation for the energy density. The first law of thermodynamics gives

$$T\partial S/\partial x = \partial \epsilon/\partial x + p\partial(1/\rho)/\partial x,$$

where ϵ is the internal energy per unit mass, S is the entropy and T is the temperature.

We have already mentioned that $p = p(\rho)$ is valid as an equation of state for both adiabatic and isothermal motion, but it is now necessary to restrict this treatment slightly further and assume that the motion is adiabatic. In this case $\partial S/\partial x = 0$, and hence

$$\begin{aligned}\partial \epsilon/\partial x &= - p\partial(1/\rho)/\partial x, \\ &= - \partial(p/\rho)/\partial x + (1/\rho)\partial p/\partial x. \quad \dots(4.4)\end{aligned}$$

For a non-viscous fluid, Kelvin's circulation theorem can be used to show that a body in irrotational motion continues to move irrotationally (Batchelor 1967), and Batchelor also points out that when a motion is generated from rest in a non-viscous fluid it will necessarily be irrotational. Hence it is reasonable to assume in our present treatment that the motion is irrotational, and therefore we can express the velocity of a particle in the

fluid in terms of a velocity potential ϕ using the expression

$$u = - \partial \phi / \partial x.$$

This equation for u , together with the one-dimensional equation of motion (2.2), can then be substituted into equation (4.4) to give

$$\partial \epsilon / \partial x = - \partial (p/\rho) / \partial x + \partial^2 \phi / \partial x \partial t - \partial (u^2/2) / \partial x,$$

and on integration

$$\rho \epsilon = - p + \rho \partial \phi / \partial t - \rho u^2/2 + \text{const.} \quad \dots(4.5)$$

We can obtain a value for the constant by considering the state of equation (4.5) at some time t when no acoustic field is present. All the terms in equation (4.5) must be zero except for the pressure p which will equal p_0 , the constant hydrostatic pressure of the fluid in the absence of the acoustic field.

Hence

$$\rho \epsilon = - p + p_0 + \rho \partial \phi / \partial t - \rho u^2/2.$$

Substituting into the equation for the energy E_A of the acoustic field

$$E_A = \rho \epsilon + \rho u^2/2,$$

gives

$$E_A = \rho \partial \phi / \partial t - (p - p_0), \quad \dots(4.6)$$

which is an exact expression for the energy density of a one-dimensional acoustic wave in a non-viscous lossless medium if it can be assumed that the motion is adiabatic.

An alternative expression for the intensity of an acoustic wave can be obtained by substituting for E_A from equation (4.6) into

the one-dimensional form of equation (I.5.4) to obtain

$$I = \overline{\rho u \partial \phi / \partial t}.$$

4.4 Expansion in Orders of Magnitude

We have derived expressions for the radiation pressure ($R - p_0$), the intensity (I) and the energy density (E). It may be possible to obtain a general relation between these three quantities, but we have been unable to do so, and have found no such treatment in the literature. Instead, an indication of the form of the relationship is obtained by expanding the equations in orders of magnitude with the aid of a perturbation parameter N (Eckart 1948). The variables of an acoustic field p , ρ and u and the velocity potential ϕ are expanded as

$$p = p_0 + Np_1 + N^2p_2 + \dots$$

$$\rho = \rho_0 + N\rho_1 + N^2\rho_2 + \dots$$

$$u = Nu_1 + N^2u_2 + \dots$$

$$\phi = N\phi_1 + N^2\phi_2 + \dots$$

where p_1, ρ_1, u_1, ϕ_1 are the first order contributions to p, ρ, u and ϕ , p_2, ρ_2, u_2, ϕ_2 the second order, etc.

These expressions are then substituted into Euler's equation, the equation of continuity, and the equations for ($R - p_0$), I and E . The different orders of magnitude forms of each equation are obtained by equating coefficients of powers of N .

However, a further relationship is required; the equation of state of the fluid medium. To keep this treatment general, we will use the equation $p = p(\rho)$, which is valid for adiabatic motion in all lossless non-viscous fluids.

The specific details of the expansions are given in Appendix C.

In this part of the thesis, the results obtained for different orders of magnitudes are discussed. It is worthwhile, at this stage, to indicate the notation that will be used for the expansions. These will be of the form

$$\{Q\}_m = Q_0 + NQ_1 + N^2Q_2 + \dots + N^mQ_m,$$

where Q_m is the mth order contribution to the variable Q , and $\{Q\}_m$ indicates the value of the variable Q expanded upto and including the mth order contribution.

4.5 First Order (coefficients of N)

The first order equations can be solved to obtain

$$p_1 = \rho_0 \partial \phi_1 / \partial t,$$

$$\rho_1 = p_1 / c_0^2,$$

$$\{R - p_0\}_1 = 0,$$

$$E_1 = 0,$$

$$I_1 = 0,$$

which illustrates that there is no first order contribution to the radiation pressure, energy density or intensity. (Here $c_0^2 = (dp/d\rho)_0$.)

We can substitute for u_1 and ρ_1 in equation (C.2) to obtain

$$\partial^2 \phi_1 / \partial t^2 = c_0^2 \partial^2 \phi_1 / \partial x^2,$$

which is the first order wave equation with general solution

$$\phi_1 = Df_1(x - c_0 t) + Ef_2(x + c_0 t),$$

where f_1 and f_2 are arbitrary functions, and D and E are

constants.

For the special case of a wave travelling in the +x direction only, this becomes

$$\phi_1 = Df(x - c_o t),$$

which obeys the equation

$$\partial\phi_1/\partial x = - (1/c_o)\partial\phi_1/\partial t, \quad \dots(4.7)$$

or equivalently

$$u_1 = p_1/(\rho_o c_o).$$

4.6 Second Order (coefficients of N^2)

The second order equations can be solved to obtain

$$R_2 = \overline{p_1^2}/(2\rho_o c_o^2) + \overline{\rho_o u_1^2}/2,$$

$$E_2 = p_1^2/(2\rho_o c_o^2) + \rho_o u_1^2/2,$$

$$I_2 = \overline{p_1 u_1}.$$

Illustrating that to second order

$$\{R - p_o\}_2 = \{\overline{E}\}_2.$$

This is a powerful result and demonstrates that, to second order, for any type of wave motion in a lossless non-viscous medium under adiabatic conditions, the radiation pressure is equal to the time average of the energy density. In general, however, the radiation pressure is not equal to the intensity, even to second order.

For the special case of a wave travelling in the +x direction, however, we can use equation (4.7) to give

$$\{R - p_o\}_2 = \{\overline{E}\}_2 = \overline{\rho_o u_1^2} = \{I\}_2/c_o.$$

This is the expression normally used to relate radiation pressure, energy density and intensity, and was obtained by several authors (Beyer 1950, Borgnis 1953c, Cady and Gittings 1953, Freeman 1963). It is clearly a useful result, but it must be emphasised that it is only valid to second order and only for a travelling wave in a lossless non-viscous fluid under adiabatic conditions.

In a radiation pressure balance, the total force F on a target is equal to the radiation pressure $(R - p_0)$ multiplied by the area A of the target, and therefore to second order for a travelling wave

$$F \equiv \{R - p_0\}_2 A = \{I\}_2 A / c_0 = W / c_0, \quad \dots(4.8)$$

where $W = \{I\}_2 A$ is the total power in the ultrasonic beam to second order.

This is the widely-used relation between the force measured by a radiation pressure balance and the total power in the ultrasonic beam.

The equality (to second order) between the radiation pressure and the time-averaged energy density has not always been appreciated. In some cases, authors consider the force on a target and distinguish between a perfectly absorbing and a perfectly reflecting target. They produce the result $R - p_0 = \bar{E}$ for the absorber, and $R - p_0 = 2\bar{E}$ for the reflector. This is confusing and stems from a misunderstanding of the meaning of the term E . If E is taken to be equal to the energy density of the wave, then $\{R - p_0\}_2 = \{\bar{E}\}_2$ regardless of the type of target. This can be demonstrated quite simply by considering sinusoidal excitation of the ultrasonic transducer.

For a perfectly absorbing target, a travelling wave will exist,

and the first order component of the velocity potential will be of the form

$$\phi_1 = D \sin(x - c_0 t). \quad \dots(4.9)$$

In this case

$$\{E\}_2 = \rho_0 D^2 \cos^2(x - c_0 t),$$

and

$$\{R - p_0\}_2 = \{\bar{E}\}_2 = \rho_0 D^2 / 2.$$

For a perfectly reflecting target perpendicular to the direction of wave motion, however, as the wave is reflected back, standing waves will be set up of the form

$$\phi_1 = D \sin(x - c_0 t) + D \sin(x + c_0 t + \theta),$$

where θ indicates the phase difference between the incident and reflected waves.

In this case

$$\begin{aligned} \{E\}_2 &= \rho_0 D^2 \cos^2(x - c_0 t) + \rho_0 D^2 \cos^2(x + c_0 t + \theta) \\ &\quad + 2\rho_0 D^2 \cos(x - c_0 t) \cos(x + c_0 t + \theta), \end{aligned}$$

and

$$\{R - p_0\}_2 = \{\bar{E}\}_2 = \rho_0 D^2.$$

It is clear, therefore, that the force on a perfectly reflecting target (perpendicular to the direction of the wave motion) will be twice the force on a perfect absorber. This is not due to a different relation between radiation pressure and time-averaged energy density in the two cases, but simply because the time-averaged energy density of the field produced by the

reflecting target is twice that for the absorber.

We can obtain a general second order wave equation by substituting into equation (C.3) to give

$$\begin{aligned} \partial^2 \phi_2 / \partial t^2 - 2(\partial \phi_1 / \partial x)(\partial^2 \phi_1 / \partial x \partial t) + (1/c_o^2)(\partial \phi_1 / \partial t)(\partial^2 \phi_1 / \partial t^2) \\ - (\partial \phi_1 / \partial t)(\partial^2 \phi_1 / \partial x^2) - (\rho_o/c_o^4)(\partial \phi_1 / \partial t)(\partial^2 \phi_1 / \partial t^2)(d^2 p / d\rho^2)_o \\ = c_o^2 \partial^2 \phi_2 / \partial x^2. \end{aligned}$$

For a travelling wave obeying equation (4.7) this becomes

$$\begin{aligned} \partial^2 \phi_2 / \partial t^2 - [2/c_o^2 + (\rho_o/c_o^4)(d^2 p / d\rho^2)_o](\partial \phi_1 / \partial t)(\partial^2 \phi_1 / \partial t^2) \\ = c_o^2 \partial^2 \phi_2 / \partial x^2, \end{aligned}$$

which has a solution

$$\phi_2 = (D^2/2c_o)[1 + (\rho_o/2c_o^2)(d^2 p / d\rho^2)_o]x[f'(x - c_o t)]^2. \quad \dots(4.10)$$

This is a general expression for the second order component of the velocity potential of a travelling wave.

4.7 Third Order (coefficients of N^3)

The third order equations can be solved to obtain

$$\begin{aligned} R_3 = \overline{p_1 p_2} / (\rho_o c_o^2) - \overline{p_1^3} / (3\rho_o^2 c_o^4) [1 + (\rho_o/2c_o^2)(d^2 p / d\rho^2)_o] + \overline{\rho_o u_1 u_2} \\ + \overline{p_1 u_1^2} / c_o^2, \end{aligned}$$

$$= \overline{E}_3 - \overline{p_1^3} / (6\rho_o^2 c_o^4) [1 - (\rho_o/c_o^2)(d^2 p / d\rho^2)_o] + \overline{p_1 u_1^2} / (2c_o^2),$$

$$I_3 = \overline{p_2 u_1} + \overline{p_1 u_2} + \overline{u_1 p_1^2} / (2\rho_o c_o^2) + \rho_o \overline{u_1^3} / 2.$$

Hence, to third order, for wave motion in a non-viscous lossless fluid under adiabatic conditions, the radiation pressure is not equal to the time average of the energy density.

Again, for the special case of a travelling wave, we can use equation (4.7) to obtain

$$\begin{aligned}\{R - p_o\}_3 &= \{\bar{E}\}_3 + \bar{p}_1^3 / (3\rho_o^2 c_o^4) [1 + (\rho_o / 2c_o^2) (d^2 p / dp^2)_o], \\ &= \{I\}_3 / c_o - \bar{p}_1^3 / (3\rho_o^2 c_o^4) [1 + (\rho_o / 2c_o^2) (d^2 p / dp^2)_o],\end{aligned}$$

which demonstrates that, even for a travelling wave, the radiation pressure is not equal to the time average of the energy density or the intensity unless $\bar{p}_1^3 = 0$.

Now, for a travelling wave

$$p_1 = \rho_o c_o u_1 = - \rho_o c_o \partial \phi_1 / \partial x,$$

and hence

$$\bar{p}_1^3 = - \rho_o^3 c_o^3 (\overline{\partial \phi_1 / \partial x})^3.$$

Clearly, if the first order component of the velocity potential is a sinusoidal wave obeying equation (4.9), then

$$(\overline{\partial \phi_1 / \partial x})^3 = D^3 \overline{\cos^3(x - c_o t)} = 0,$$

and in this special case

$$\{R - p_o\}_3 = \{\bar{E}\}_3 = \{I\}_3 / c_o,$$

upto and including third order terms.

4.8 Fourth Order (coefficients of N^4)

The fourth order equations are complex and long, and are written in full in Appendix C.5. The radiation pressure is not equal to the time average of the energy density or the intensity for any general type of wave motion.

For a wave travelling in the +x direction, the fourth order terms give

$$\begin{aligned}\{R - p_o\}_4 &= \{\bar{E}\}_4 + \bar{p}_1^3/(3\rho_o^2 c_o^4)[1 + (\rho_o/2c_o^2)(d^2 p/d\rho^2)_o] \\ &\quad - \bar{p}_1^4/(3\rho_o^3 c_o^6)[1 + (5\rho_o/4c_o^2)(d^2 p/d\rho^2)_o - (\rho_o^2/4c_o^2)(d^3 p/d\rho^3)_o \\ &\quad + (3\rho_o^2/4c_o^4)(d^2 p/d\rho^2)_o^2] + \overline{p_2 p_1^2}/(\rho_o^2 c_o^4)[1 + (\rho_o/2c_o^2)(d^2 p/d\rho^2)_o], \\ &= \{I\}_4/c_o - \bar{p}_1^3/(3\rho_o^2 c_o^4)[1 + (\rho_o/2c_o^2)(d^2 p/d\rho^2)_o] \\ &\quad + \bar{p}_1^4/(24\rho_o^3 c_o^6)[1 + (5\rho_o/c_o^2)(d^2 p/d\rho^2)_o - (\rho_o^2/c_o^2)(d^3 p/d\rho^3)_o \\ &\quad + (15\rho_o^2/4c_o^4)(d^2 p/d\rho^2)_o^2] - \overline{p_2 p_1^2}/(\rho_o^2 c_o^4)[1 + (\rho_o/2c_o^2)(d^2 p/d\rho^2)_o].\end{aligned}$$

Clearly, these equations are still very complicated, but significantly further simplification occurs if we assume, as for the third order case, that the first order component of the velocity potential is a sinusoidal wave obeying equation (4.9). In this case, using equation (4.10), $\partial\phi_2/\partial t$ is proportional to $\cos(x - c_o t)\sin(x - c_o t)$, and, as $p_2 = \rho_o \partial\phi_2/\partial t$ for a travelling wave,

$$\overline{p_2 p_1^2} = 0,$$

and

$$\bar{p}_1^4 = \rho_o^4 c_o^4 D^4 \overline{\cos^4(x - c_o t)} = 3\rho_o^4 c_o^4 D^4/8.$$

These relations give

$$\begin{aligned}\{R - p_o\}_4 &= \{\bar{E}\}_4 - [\rho_o D^4/(8c_o^2)][1 + (5\rho_o/4c_o^2)](d^2 p/d\rho^2)_o \\ &\quad - (\rho_o^2/4c_o^2)(d^3 p/d\rho^3)_o + (3\rho_o^2/4c_o^4)(d^2 p/d\rho^2)_o^2, \\ \{R - p_o\}_4 &= \{I\}_4/c_o + [\rho_o D^4/(64c_o^2)][1 + (5\rho_o/c_o^2)(d^2 p/d\rho^2)_o \\ &\quad - (\rho_o^2/c_o^2)(d^3 p/d\rho^3)_o + (15\rho_o^2/4c_o^4)(d^2 p/d\rho^2)_o^2].\end{aligned}$$

4.9 Magnitude Of Terms

We have produced expansions and relations between the radiation pressure and the intensity or the energy density upto and including fourth order terms. The relation

$$F = W/c_o,$$

between the total force (F) on a target in a radiation pressure balance and the total power (W) in the ultrasonic beam is generally used in experimental situations, and as shown in equation (4.8), this is equivalent to assuming that

$$R - p_o = I/c_o. \quad \dots(4.11)$$

We have shown, however, that for the case of a plane sinusoidal travelling wave in a non-viscous lossless medium, this is only true upto third order. It is therefore useful to use the previous expansions to obtain an estimate of the error introduced in a typical experimental situation by the use of relation (4.11).

Firstly, however, we need to obtain values for $(d^2p/d\rho^2)_o$ and $(d^3p/d\rho^3)_o$ applicable for water. In many treatments $p = f(\rho)$ is expanded in the form

$$p = p_o + A[(\rho - \rho_o)/\rho_o] + (B/2!)[(\rho - \rho_o)/\rho_o]^2 \\ + (C/3!)[(\rho - \rho_o)/\rho_o]^3 + \dots$$

and Beyer (1974) states that, for water at 20°C, $B/A = 5.0$ and C/A is negligible.

Comparing this with our treatment gives

$$A = \rho_o c_o^2, \quad B = \rho_o^2 (d^2p/d\rho^2)_o, \quad C = \rho_o^3 (d^3p/d\rho^3)_o.$$

Hence, for water at 20°C, $(\rho_o/c_o^2)(d^2p/d\rho^2)_o = 5.0$ and $(\rho_o^2/c_o^2)(d^3p/d\rho^3)_o$ is negligible.

Substituting these values into the fourth order relations between $(R - p_o)$, I and E gives

$$\{\bar{E}\}_4 - \{R - p_o\}_4 = 13\rho_o D^4/(4c_o^2),$$

$$\{R - p_o\}_4 - \{I\}_4/c_o = 479\rho_o D^4/(256c_o^2).$$

For a wave obeying equation (4.9), with a sinusoidal first order component of the velocity potential, the second order expression for the intensity $\{I\}_2$ gives

$$\{I\}_2 = \overline{p_1 u_1} = \rho_o c_o D^2/2.$$

Hence to fourth order,

$$\{\bar{E}\}_4 - \{R - p_o\}_4 = 13\{I\}_2^2/(\rho_o c_o^4),$$

$$\{R - p_o\}_4 - \{I\}_4/c_o = 479\{I\}_2^2/(64\rho_o c_o^4).$$

Therefore, for a wave of intensity $\{I\}_2 = 10^3 \text{ W/m}^2$ travelling in water at 20°C ($\rho_o = 998.2 \text{ kg/m}^3$ and $c_o = 1482 \text{ m/s}$), the error in assuming $R - p = \bar{I}/c_o$ is approximately $0.2 \times 10^{-6}\%$. For an intensity of 10^6 W/m^2 this increases to 0.2% . It is therefore reasonable to assume that for a transducer driven sinusoidally and emitting plane waves into a water bath, at the power levels normally encountered, the radiation pressure is equal to the time-averaged energy density or the intensity divided by c_o . We have clearly made the assumption that water can be treated as a non-viscous lossless fluid and that suitable conditions exist for the motion to be adiabatic. The validity of this assumption will be discussed later in section 6.

4.10 Kinetic and Potential Energy Density

It is often assumed (Borignis 1953a, Rooney and Nyborg 1972, Wach 1981) that the time-averaged values of the kinetic and potential energy densities are equal. This assumption can be investigated using the order of magnitude expansions.

The kinetic energy density T and the potential energy density W are respectively

$$T = \rho u^2/2,$$

and

$$W = E - T = \rho \partial \phi / \partial t - (p - p_0) - \rho u^2/2.$$

To first order, as expected, there is no contribution to either of these terms. The second order expansions give

$$\{T\}_2 = \rho_0 u_1^2/2,$$

and

$$\{W\}_2 = p_1^2/(2\rho_0 c_0^2),$$

which demonstrates that for wave motion in a non-viscous fluid under adiabatic conditions these two quantities are not equal. However, for the case of a wave travelling in the $+x$ direction we can use equation (4.7) to give $\{T\}_2 = \{W\}_2$, and hence the kinetic and potential energy densities are equal in magnitude to second order for a travelling wave.

5. THREE-DIMENSIONAL MOTION: FORCE ON A TARGET IN A LOSSLESS FLUID

The three-dimensional forms of the equation of continuity (I.3.1) and the lossless form of the equation of motion (I.3.7) can be combined to give

$$\partial(\rho \underline{u})/\partial t + \underline{u} \cdot \nabla \rho \underline{u} + \rho \underline{u} \cdot \nabla \underline{u} + \nabla p = 0. \quad \dots(5.1)$$

If the velocity $\underline{u}$ is expressed in the form

$$\underline{u} = \underline{i}u_x + \underline{j}u_y + \underline{k}u_z,$$

where $\underline{i}$, $\underline{j}$ and $\underline{k}$ are unit vectors in the x,y and z directions, then equation (5.1) consists of three equations of the form

$$\partial(\rho u_x)/\partial t + (\partial/\partial x)(p + \rho u_x^2) + (\partial/\partial y)(\rho u_x u_y) + (\partial/\partial z)(\rho u_x u_z) = 0,$$

with similar expressions for u_y and u_z .

This is a momentum continuity equation. The first term is the time derivative of the momentum density in the x direction, and hence, since total momentum is conserved, the remaining terms must give the amount of momentum flowing in unit time across a plane perpendicular to the x direction. This concept is most conveniently expressed in terms of a momentum flux density tensor Π , and the equation of conservation of momentum then becomes

$$\partial(\rho u_i)/\partial t + \sum_k \partial \Pi_{ik} / \partial x_k = 0.$$

The components of the tensor Π can be expressed as

$p + \rho u_x^2$	$\rho u_x u_y$	$\rho u_y u_z$
$\rho u_x u_y$	$p + \rho u_y^2$	$\rho u_y u_z$
$\rho u_x u_z$	$\rho u_y u_z$	$p + \rho u_z^2$

Although it may be possible to obtain equations for a general type

of wave motion, these will be complex and will therefore not be attempted here. Instead the case of a plane wave travelling in the x direction with $u_x = u$, $u_y = u_z = 0$ will be considered, and the momentum flux density tensor Π then becomes

$$\begin{array}{ccc} p + \rho u^2 & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{array} \quad \dots(5.2)$$

As was mentioned earlier in section (3.3), one possible method of treating the radiation pressure on a target is to use momentum flux arguments. If an incident wave with associated momentum m_{in} passes through a surface around the target, and a reflected wave with momentum m_{out} emerges from the surface, then a quantity of momentum ($m_{in} - m_{out}$) will be transferred to the interior of the surface.

We are treating the case of a lossless fluid surrounding the target and, as none of the momentum will be absorbed by the fluid, all the momentum ($m_{in} - m_{out}$) will be transferred to the target. The momentum transfer concept is clearly equally applicable to lossy fluids, where some of the momentum will be transferred to the fluid and the remainder to the target.

The situation in the lossless case is therefore one of an incident wave with a momentum flux density tensor given by equation (5.2) travelling through a surface, and a reflected wave with a momentum flux density tensor also given by equation (5.2) emerging through the surface. Here it has been assumed that the reflected wave is travelling in one direction only, which implies that the reflecting part of the target must be a plane surface. Clearly there is room for confusion in this concept as both the incident and reflected waves have the same momentum flux density

tensor. This is because, for each wave, the coordinate system is chosen to coincide with the direction of motion of the wave, with the incident and reflected waves therefore having different coordinate systems.

For a perfectly absorbing target inside the surface, the situation is straightforward as, in this case, all the momentum carried by the incident wave is directly transferred to the target, which will therefore experience a force per unit area $(\bar{p} + \bar{p}u^2)$ in the direction of the incident ultrasonic beam. As demonstrated in section (4.6), to second order this is equal to the time-averaged energy density in the ultrasonic wave.

For the more complex case of a perfectly reflecting target, the momentum flux density tensors for the incident and reflected waves must be transformed to the same coordinate system before the momentum transferred to the target can be calculated. Let us consider a plane reflecting target with the normal to the reflecting surface at an angle ϕ to the incident ultrasonic wave. Let (x_1, x_2, x_3) represent a rectangular coordinate system based on the surface of the reflector with the axes x_1 normal to the reflector surface, and chosen so that the plane x_1 - x_2 contains the direction of the incident wave. If (x'_1, x'_2, x'_3) is the coordinate system for the incident wave, then the unprimed set of coordinate axes (x_1, x_2, x_3) is obtained by rotating the primed set (x'_1, x'_2, x'_3) through an angle ϕ about the x_3 axis. This is a coordinate transform using a transformation tensor A

$$A = \begin{matrix} & \begin{matrix} a_{11} & a_{12} & a_{13} \end{matrix} \\ \begin{matrix} a_{21} \\ a_{31} \end{matrix} & \begin{matrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{matrix} \end{matrix}$$

where $a_{ij} = \cos(x_i, x'_j)$ denotes the cosine of the angle between the x_i and x'_j coordinate axes.

For the transformation between the coordinate systems based on the incident ultrasonic wave and the target, A becomes

$$\begin{array}{ccc} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{array}$$

and the components of the primed tensor T'_{ij} are transformed to the unprimed coordinates T_{ij} using

$$T_{ij} = \sum_{kl} a_{ik} a_{jl} T'_{kl}.$$

The momentum flux density tensor Π_i of the incident wave therefore becomes (expressed in the coordinate system based of the target)

$$\Pi_i = \begin{array}{ccc} p + \rho u^2 \cos^2\theta & \rho u^2 \cos\theta \sin\theta & 0 \\ \rho u^2 \cos\theta \sin\theta & p + \rho u^2 \sin^2\theta & 0 \\ 0 & 0 & p \end{array}$$

The components of the first row of this tensor give the rate of transfer of momentum to the target from the incident wave. $(p + \rho u^2 \cos^2\theta)$ is the rate of transfer of momentum in the x_1 direction (normal to the target) and $(\rho u^2 \cos\theta \sin\theta)$ is the corresponding value for the x_2 direction (tangential to the target). Hence the time-averaged force per unit area produced on the target by the incident wave in a direction normal to the target is $(\bar{p} + \overline{\rho u^2 \cos^2\theta})$.

The same treatment can be used for the reflected wave with coordinate axes (x'_1, x'_2, x'_3) , which will be reflected at an angle θ from the plane reflector. In this case the transformation

tensor A will be

$$A = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and the momentum flux density tensor Π_r of the reflected wave becomes

$$\Pi_r = \begin{pmatrix} p + \rho u^2 \cos^2\theta & -\rho u^2 \cos\theta \sin\theta & 0 \\ \rho u^2 \cos\theta \sin\theta & p + \rho u^2 \sin^2\theta & 0 \\ 0 & 0 & p \end{pmatrix}$$

For the reflected wave, the components of the first row of this tensor give the rate of transfer of momentum to the target. $(p + \rho u^2 \cos^2\theta)$ is the rate of transfer of momentum in the x_1 direction, and $(-\rho u^2 \cos\theta \sin\theta)$ the corresponding value in the x_2 direction.

To calculate the total force on the target produced by the incident and reflected waves, the contributions from the individual waves must be summed together. This gives a time-averaged force per unit area of $2(\bar{p} + \overline{\rho u^2} \cos^2\theta)$ in the x_1 direction, and no resultant force in the x_2 and x_3 directions. Hence, the time-averaged force per unit area produced on a plane perfectly reflecting surface at an angle θ to a plane ultrasonic wave is exactly equal to $2(\bar{p} + \overline{\rho u^2} \cos^2\theta)$. As can be shown using Appendix C.3, $\{\bar{p}\}_2 = p_0$ and $\{\overline{\rho u^2}\}_2 = \{\bar{E}\}_2$ and hence, to second order, the time-averaged force per unit area on the target is $2\bar{E} \cos^2\theta$.

In this section, two cases have been discussed: a perfectly absorbing target and a perfect reflector. A perfect absorber will experience a force per unit area equal to $(\bar{p} + \overline{\rho u^2})$ in the

direction of the incident ultrasonic beam. While for a perfectly reflecting target at an angle ϕ to the incident beam, the force will be $2(\bar{p} + \bar{\rho}u^2 \cos^2 \phi)$ normal to the reflecting surface.

If the reflector is normal to the incident ultrasound ($\phi = 0$), then the force on the reflector will be twice that produced on an absorbing target, as expected from momentum transfer concepts.

If the reflector is angled at 45° to the ultrasonic beam, the force normal to the reflector surface will be equal to $(2\bar{p} + \bar{\rho}u^2)$. For the case of an ultrasonic beam directed vertically onto a 45° target, all the reflected ultrasound will travel horizontally, and hence the force produced on the reflector surface in the vertical direction will be equal to the time-averaged momentum flux density of the incident ultrasonic beam $(\bar{p} + \bar{\rho}u^2)$.

6. RADIATION PRESSURE IN A VISCOUS FLUID

6.1 Velocity in a Viscous Fluid

Any vector function of position, such as the velocity $\underline{u}$, can be uniquely separated into a longitudinal part $\underline{u}_l$ (for which $\text{curl} \underline{u}_l = 0$) and a rotational part $\underline{u}_r$ (for which $\text{div} \underline{u}_r = 0$) (Morse and Ingard 1968).

Substituting for $\underline{u}$ into the equation of motion for a viscous fluid (I.3.7) gives

$$\rho d(\underline{u}_l + \underline{u}_r)/dt = - \nabla p + \mu \nabla^2 (\underline{u}_l + \underline{u}_r) + (\mu/3 + \eta) \nabla \nabla \cdot (\underline{u}_l + \underline{u}_r).$$

The vector identity,

$$\nabla \nabla \cdot \underline{v} = \nabla \times \nabla \times \underline{v} + \nabla^2 \underline{v},$$

applicable to any vector $\underline{v}$, can be used to give

$$\rho d(\underline{u}_1 + \underline{u}_r)/dt = - \nabla p + (4\mu/3 + \eta) \nabla^2 \underline{u}_1 - \mu \nabla \times \nabla \times \underline{u}_r.$$

As $\nabla \times \nabla p = 0$, this can be split into two separate equations

$$\rho d\underline{u}_1/dt = - \nabla p + (4\mu/3 + \eta) \nabla^2 \underline{u}_1, \quad \dots(6.1)$$

and

$$\rho d\underline{u}_r/dt = - \mu \nabla \times \nabla \times \underline{u}_r.$$

The first of these equations relates the longitudinal part of the velocity to the pressure, while the other gives the behaviour of the rotational part of $\underline{u}$. For the longitudinal part of $\underline{u}$, we can introduce the velocity potential ϕ for which

$$\underline{u}_1 = - \nabla \phi.$$

Morse and Ingard (1968) show that a possible solution for the rotational part $\underline{u}_r$ of the velocity is in the form of a shear wave, and it therefore seems reasonable to neglect this contribution to the velocity when considering one-dimensional motion. Hence for motion in one dimension we will use the one-dimensional form of equation (6.1) together with

$$\underline{u} = - \partial \phi / \partial x. \quad \dots(6.2)$$

6.2 One-Dimensional Motion in a Viscous Fluid

The one-dimensional form of the equation of motion for a viscous fluid (6.1) is

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = - \frac{\partial p}{\partial x} + (4\mu/3 + \eta) \frac{\partial^2 u}{\partial x^2}.$$

This can be combined with the equation of continuity to give

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial}{\partial x} [p + \rho u^2 - (4\mu/3 + \eta) \frac{\partial u}{\partial x}] = 0.$$

This is a momentum continuity equation. The first term is the time derivative of the momentum density and hence, as total momentum is conserved, the second term must be the rate of transfer of momentum per unit area - the momentum flux. It seems reasonable to use the same definition in the viscous case as in the non-viscous, and we will therefore define the radiation pressure in a viscous fluid as the time-averaged momentum flux.

Hence for a viscous fluid

$$R - p_0 = \overline{p} + \overline{\rho u^2} - (4\mu/3 + \eta) \overline{\frac{\partial u}{\partial x}}. \quad \dots(6.3)$$

This is a general expression for the radiation pressure in a viscous fluid and is true to all orders of magnitude, applicable to all fluids for which equation (6.1) is a valid equation of motion. It is clearly an important expression which can be derived directly from the basic equations of fluid motion. We have, however, been unable to find any derivation of this kind in the literature.

As in the non-viscous case, we will now proceed to relate the radiation pressure in a fluid to the intensity, but first need to derive an expression for the intensity in a viscous fluid. We can follow the same reasoning as in the non-viscous case (section

I.5).

Using equation (I.5.1), the change in the energy E of unit volume element fixed in space can be given by

$$\partial E / \partial t = (u^2/2) \partial \rho / \partial t + \rho u \partial u / \partial t + \rho \partial \epsilon / \partial t + \epsilon \partial \rho / \partial t, \quad \dots (6.4)$$

where ϵ is the internal energy per unit mass.

The first law of thermodynamics for a reversible process can be used to give

$$T dS/dt = d\epsilon/dt + p d(1/\rho)/dt,$$

and the one-dimensional form of the equation of conservation of energy in a viscous fluid (I.3.9) is

$$T dS/dt = (4\mu/3 + \eta)(1/\rho)(\partial u/\partial x)^2 + (1/\rho)(\partial/\partial x)(K \partial T/\partial x).$$

Using these two equations gives

$$\begin{aligned} \partial \epsilon / \partial t = & (p/\rho^2) \partial \rho / \partial t + (up/\rho^2) \partial \rho / \partial x - u \partial \epsilon / \partial x \\ & + (4\mu/3 + \eta)(1/\rho)(\partial u/\partial x)^2 + (1/\rho)(\partial/\partial x)(K \partial T/\partial x), \end{aligned}$$

and substituting in equation (6.4)

$$\begin{aligned} \partial E / \partial t = & (u^2/2) \partial \rho / \partial t + \rho u \partial u / \partial t + \epsilon \partial \rho / \partial t + (p/\rho) \partial \rho / \partial t + (pu/\rho) \partial \rho / \partial x \\ & - \rho u \partial \epsilon / \partial x + (4\mu/3 + \eta)(\partial u/\partial x)^2 + (\partial/\partial x)(K \partial T/\partial x). \end{aligned}$$

We can use the equations of motion and continuity to give expressions for $\partial u/\partial t$ and $\partial \rho/\partial t$, and hence

$$\partial E / \partial t = - (\partial/\partial x)[pu + \rho u^3/2 + \rho u \epsilon - (4\mu/3 + \eta)u \partial u/\partial x - K \partial T/\partial x].$$

This is in the form of an energy continuity equation. As in the non-viscous case, we now subtract a continuity equation for the energy associated with the work done by the undisturbed pressure p_0 to give

$$\partial E_A / \partial t + (\partial / \partial x)[u(p - p_0 + E_A) - (4\mu/3 + \eta)u \partial u / \partial x - K \partial T / \partial x] = 0,$$

where E_A is the energy density of the acoustic field.

The equation for the intensity I of the acoustic field in a viscous fluid is therefore

$$I = \overline{u(p - p_0 + E_A) - (4\mu/3 + \eta)u \partial u / \partial x - K \partial T / \partial x}.$$

This is an exact expression for the intensity of an acoustic field in a viscous fluid, true to all orders of magnitude.

As in the non-viscous case, we have been unable to derive an exact relationship between the radiation pressure and the intensity, and need to resort to an expansion in orders of magnitude. The equations for a viscous heat-conducting fluid given in section (I.3.5) can be expanded with the aid of a perturbation parameter as for the non-viscous case.

The first order one-dimensional forms of these equations are:

the equation of continuity

$$\partial p_1 / \partial t + \rho_0 \partial u_1 / \partial x = 0, \quad \dots(6.5)$$

the equation of motion

$$\rho_0 \partial u_1 / \partial t = - \partial p_1 / \partial x + (4\mu/3 + \eta) \partial^2 u_1 / \partial x^2, \quad \dots(6.6)$$

the equation of conservation of energy

$$T_0 \partial S_1 / \partial t = (K_0 / \rho_0) \partial^2 T_1 / \partial x^2, \quad \dots(6.7)$$

and the equation of state $p = p(\rho, S)$ becomes

$$p_1 = c_o^2 \rho_1 + (\partial p / \partial S)_o S_1,$$

where p_1 , ρ_1 , u_1 , T_1 and S_1 are the first order contributions to p , ρ , u , T and S .

These four equations are functions of five variables, and clearly to solve these equations we need to reduce them to functions of only four variables and we will choose p_1 , ρ_1 , u_1 and T_1 as the four variables.

As mentioned in section (I.3.4), any two parameters of state can be chosen for use in the equation of state of the fluid, and, instead of ρ and S , we can equally choose ρ and T . The equation of state can then be written as $p = p(\rho, T)$ with a first order form

$$p_1 = (\partial p / \partial \rho)_T \rho_1 + (\partial p / \partial T)_o T_1. \quad \dots(6.8)$$

Using Maxwell's relations

$$\begin{aligned} (\partial p / \partial \rho)_T &= (\partial p / \partial \rho)_S (\partial S / \partial T)_V (\partial T / \partial S)_p, \\ &= c^2 / \gamma, \end{aligned}$$

where $c^2 = (\partial p / \partial \rho)_S$, γ is the ratio of the specific heats c_p and c_v , $c_p = T(\partial S / \partial T)_p$ is the specific heat at constant pressure, and $c_v = T(\partial S / \partial T)_v$ is the specific heat at constant volume.

Hence $(\partial p / \partial \rho)_T = c_o^2 / \gamma$, where $c_o^2 = (\partial p / \partial \rho)_S$.

Similarly

$$\begin{aligned} (\partial p / \partial T)_\rho &= - (c^2 / \gamma) (\partial \rho / \partial T)_p, \\ &= \beta \rho c^2 / \gamma, \end{aligned}$$

where $\beta = - (1/\rho) (\partial \rho / \partial T)_p$ is the coefficient of thermal expansion.

Hence $(\partial p / \partial T)_o = \beta_o \rho_o c_o^2 / \gamma$, and substituting in equation (6.8)

gives

$$p_1 = (c_o^2/\gamma)\rho_1 + \rho_o\beta_o(c_o^2/\gamma)T_1. \quad \dots(6.9)$$

In addition, if the entropy S is considered to be a function of temperature and pressure, $S = S(p,T)$, then

$$\begin{aligned} dS/dt &= (\partial S/\partial T)_p dT/dt + (\partial S/\partial p)_T dp/dt, \\ &= (c_p/T)dT/dt - (\beta/\rho)dp/dt. \end{aligned}$$

Hence,

$$dS_1/dt = (c_{po}/T_o)dT_1/dt - (\beta_o/\rho_o)dp_1/dt,$$

and substituting in equation (6.7)

$$\rho_o c_{po} dT_1/dt - T_o \beta_o dp_1/dt = K_o \partial^2 T_1 / \partial x^2. \quad \dots(6.10)$$

We now have four equations, (6.5), (6.6), (6.9) and (6.10) which are functions of four variables p_1 , ρ_1 , u_1 and T_1 .

We can simplify these equations by using the nomenclature of Temkin (1981) and substitute for

$$T_v = (4\mu/3 + \eta)/(\rho_o c_o^2),$$

and

$$T_T = K_o/(\rho_o c_o^2 c_{po}),$$

where T_v and T_T are known as the viscous and thermal relaxation times.

The equations can then be combined to give a single equation which is a function of one variable only. p_1 is chosen for simplicity. We will restrict the treatment to sinusoidal wave motion, and would expect that the effects of heat conduction and viscosity

will both attenuate the waves and also change their propagation velocity. These effects are taken into account by assuming that p_1 depends on distance and time as

$$p_1 = \text{Re}\{P' \exp[i(\omega t - kx)]\}. \quad \dots(6.11)$$

where $\text{Re}\{ \}$ denotes the real part of the expression and P' is independent of x and t .

Using this form of solution gives

$$(k^2 c_o^2 / \omega^2)(1 - i T_T c_o^2 k^2 / \omega) = (1 - i T_V c_o^2 k^2 / \omega)(1 - i \gamma T_T c_o^2 k^2 / \omega).$$

As is mentioned by Temkin (1981), this is a dispersion relation for k and possesses two roots, one representing highly attenuated waves, the other slightly attenuated. In an ideal fluid, $k = \omega / c_o$, and following the treatment of Temkin (1981), as we expect both viscosity and heat conduction to have a small effect on the waves, we obtain a solution for k of the form

$$k^2 c_o^2 / \omega^2 = (1 - i \omega T_V)(1 - i \gamma \omega T_T) / (1 - i \omega T_T). \quad \dots(6.12)$$

It is worth considering the magnitude of the terms in this equation to check the validity of the assumption that viscosity and heat conduction have a small effect. For water at 20°C, $T_V = 1.9 \times 10^{-12}$ s and $T_T = 6.4 \times 10^{-14}$ s, and hence, even at 15 MHz, all the terms ωT_V , ωT_T and $\gamma \omega T_T$ are much less than 1. Hence we can approximate equation (6.12) to

$$k = (\omega / c_o)[1 - i \omega T_V / 2 - i \omega (\gamma - 1) T_T / 2]. \quad \dots(6.13)$$

Hence the sinusoidal expression for the wave motion (equation 6.11) becomes

$$\exp[-\omega^2 T_V x / (2 c_o) - \omega^2 (\gamma - 1) T_T x / (2 c_o)] \exp[i(\omega t - \omega x / c_o)].$$

This represents wave propagation at a wave velocity c_0 with an attenuation coefficient α given by

$$\alpha = w^2 T_v / (2c_0) + w^2 (\gamma - 1) T_T / (2c_0). \quad \dots(6.14)$$

Hence the speed of propagation of the wave is the same as in a non-viscous fluid. In addition, it can be seen from equation (6.14) that the effects of viscocity and heat conduction are separable. There is a viscous attenuation coefficient α_v given by

$$\alpha_v = w^2 T_v / (2c_0),$$

and a thermal attenuation coefficient α_T

$$\alpha_T = w^2 (\gamma - 1) T_T / (2c_0).$$

For water at 20°C, $\gamma - 1 = 0.0058$ (Batchelor 1967), and hence at 1 MHz, $\alpha = 0.025 \text{ m}^{-1}$ with $\alpha_v = 0.025 \text{ m}^{-1}$ and $\alpha_T = 4.9 \times 10^{-6} \text{ m}^{-1}$. This value for the attenuation coefficient α is in excellent agreement with the value of 0.0253 m^{-1} obtained by Pinkerton (1947). Also the effects of heat conduction can be seen to be significantly less than those from viscosity, and it therefore seems reasonable to neglect any heat conduction in the remainder of this section.

If we neglect heat conduction, the expression for the radiation pressure in a viscous fluid is the same as equation (6.3). The intensity, however, becomes

$$I = \overline{u(p - p_0 + E_A)} - (4\mu/3 + \eta) \overline{u \partial u / \partial x}.$$

The first order expressions for the radiation pressure and the intensity are

$$R_1 = \overline{p_1} - (4\mu/3 + \eta) \overline{\delta u_1 / \delta x},$$

and

$$I_1 = 0.$$

The first order form of the one-dimensional equation of motion is

$$\rho_0 \partial u_1 / \partial t = - \partial p_1 / \partial x + (4\mu/3 + \eta) \partial^2 u_1 / \partial x^2. \quad \dots(6.6)$$

Now, we can express the velocity in terms of a velocity potential as in equation (6.2) and put

$$u_1 = - \partial \phi_1 / \partial x.$$

Substituting for u_1 in equation (6.6) and integrating with respect to x gives

$$p_1 = \rho_0 \partial \phi_1 / \partial t + (4\mu/3 + \eta) \partial u_1 / \partial x,$$

and hence

$$R_1 = 0.$$

We will now assume that the first order expression for the energy density E_1 is also equal to zero. This was the case for a non-viscous fluid and should be valid for a viscous fluid as it is unlikely that viscosity would introduce a first order component to the energy density.

The second order expressions for the radiation pressure and intensity are therefore

$$R_2 = \overline{p_2} + \rho_0 \overline{u_1^2} - (4\mu/3 + \eta) \overline{\partial u_1 / \partial x},$$

$$I_2 = \overline{p_1 u_1} - (4\mu/3 + \eta) \overline{u_1 \partial u_1 / \partial x}.$$

The second order form of the equation of motion is

$$- \partial p_2 / \partial x = \rho_0 \partial u_2 / \partial t + \rho_1 \partial u_1 / \partial t + \rho_0 u_1 \partial u_1 / \partial x - (4\mu/3 + \eta) \partial^2 u_2 / \partial x^2. \quad \dots(6.15)$$

As in the first order case, we can use the substitution $u_2 = - \partial \phi_2 / \partial x$, and hence we can integrate equation (6.15) to give

$$p_2 = \rho_0 \partial \phi_2 / \partial t - \left(\int \rho_1 \partial u_1 / \partial t \, dx \right) - \rho_0 u_1^2 / 2 + (4\mu/3 + \eta) \partial u_2 / \partial x.$$

Hence

$$R_2 = \rho_0 \overline{u_1^2} / 2 - \overline{\left(\int \rho_1 \partial u_1 / \partial t \, dx \right)}. \quad \dots(6.16)$$

In addition, if we restrict our treatment to sinusoidal wave motion, as p_1 and ρ_1 are in phase we can use the expressions

$$p_1 = P' \exp(-\alpha x) \cos[w(t - x/c_0)],$$

and

$$\rho_1 = D' \exp(-\alpha x) \cos[w(t - x/c_0)],$$

where P' and D' are both independent of x and t .

From the equation of continuity (equation 6.5), an expression for u_1 can be obtained, and this is in the form

$$u_1 = U' \exp(-\alpha x) \{ \alpha \sin[w(t - x/c_0)] - (w/c_0) \cos[w(t - x/c_0)] \},$$

where U' is independent of x and t .

Hence we can substitute for ρ_1 and u_1 in equation (6.16) to give

$$R_2 = \rho_0 w^2 U'^2 \exp(-2\alpha x) (1 + \alpha^2 c_0^2 / w^2) / (2c_0^2),$$

Similarly, using the first order equations gives

$$I_2 = \rho_0 w^2 U'^2 \exp(-2\alpha x) (1 + \alpha^2 c_0^2 / w^2) (1 - \alpha^2 c_0^2 / w^2) / (2c_0),$$

and hence

$$\{R - p_o\}_2 = \{I\}_2 / [c_o(1 - \alpha^2 c_o^2 / w^2)].$$

Hence for a viscous fluid, the radiation pressure is not equal to the intensity divided by c_o even to second order for sinusoidal wave motion. The magnitude of the difference between these two quantities is however approximately equal to $\alpha^2 c_o^2 / w^2$, which at 15 MHz is equal to 8×10^{-9} . Hence for sinusoidal motion in water at the frequencies used in medical diagnosis, it is reasonable to neglect any effect of viscosity on the relation between radiation pressure and intensity. The equation

$$R - p_o = \{I\} / c_o,$$

can be used.

As in the non-viscous case, we can relate the force F on a target to the total power W in the ultrasonic beam using the equation $F = W / c_o$. It is worth pointing out, however, that this relation is between the force on a target at a particular point in the ultrasonic field and the total ultrasonic power at the same point in the field. If a relation between the force F and the output power from the transducer is required then it is necessary to correct for the attenuation of the ultrasonic beam. Hence if W_o is the power at the transducer, which we will put at a distance $x=0$, then the power W at a distance x from the transducer is equal to $W_o \exp(-2\alpha x)$. Hence, the relation between the force F on the target and the output power W_o from a transducer is given by

$$F = (W_o / c_o) \exp(-2\alpha x). \quad \dots(6.17)$$

7. CONCLUSION

We have shown that radiation pressure as we define it is a single physical phenomenon, a parameter of the acoustic field, and a direct consequence of the hydrodynamic equations. In a non-viscous lossless fluid, the time-averaged force per unit area on the surface of an obstacle placed in the field is exactly related to either the time-averaged momentum flux of the wave, or the time-averaged pressure in the field expressed in Lagrangian coordinates. This result is valid to all orders of approximation, and is independent of the equation of state of the medium (subject to $p = p(\rho)$ being applicable). These two field quantities are constants of the motion, and take on a single value for any field configuration.

The majority of authors prefer to relate radiation pressure to other field quantities, particularly the energy density. We have shown that, in this case, second-order approximations must be made and that it is necessary to consider the experimental situation very carefully. Moreover, various additional assumptions have to be used. Two particular boundary conditions have been discussed in some detail. One, $\langle \overline{p_E} \rangle = p_0$, was shown to effect significant simplification leading to a value for the radiation pressure expressed in terms of the kinetic energy density of the beam (correct to all orders). It is indicated that another boundary condition, $\langle \rho_E \rangle = \rho_0$, is just the continuity equation applied to a special situation. We have also demonstrated how these different boundary conditions are on occasions chosen inappropriately, leading to incorrect expressions for the radiation pressure.

The general one-dimensional expressions for the radiation pressure, intensity and energy density in a lossless non-viscous

medium have been derived, and an indication of the relationship between these parameters has been obtained by expanding the equations in orders of magnitude. To second order for any type of wave motion, the radiation pressure is shown to be equal to the time-averaged energy density, but it is also shown that only for a travelling wave is the radiation pressure directly related to the intensity to second order.

Three-dimensional motion in a non-viscous fluid has also been discussed, and we have demonstrated that for a plane wave incident on a perfectly reflecting target at an angle ϕ to the ultrasonic beam, the time-averaged force per unit area normal to the target surface is exactly equal to $2(\bar{p} + \bar{\rho}u^2\cos^2\phi)$. In addition, a one-dimensional expression true to all orders of magnitude for the radiation pressure in a viscous heat-conducting fluid has been obtained, and it has been shown that, in a viscous fluid, the radiation pressure is not equal to I/c_0 even to second order for sinusoidal wave motion. However, the magnitude of the difference between these two quantities has been shown to be negligible in water at the frequencies used in medical diagnosis.

In conclusion, we have demonstrated that for plane sinusoidal travelling waves at the powers and frequencies used in medical diagnosis, it is reasonable to assume that the force F on a target is related to the total ultrasonic power W_0 from a transducer using the equation $F = (W_0/c_0)\exp(-2\alpha x)$, where α is the amplitude attenuation coefficient and x is the separation between the transducer and the target.

PART III

Radiation Pressure Balance

1. INTRODUCTION

In the first two parts of this thesis the different quantities that can be measured to characterise an ultrasonic field have been discussed, and the relationship between one of these quantities, radiation pressure, and other variables of the ultrasonic field has been derived. Measurement of the radiation pressure is often used as a determination of the total power in an ultrasonic beam, and we have determined the validity of the assumption that these two quantities are equal. This part of the thesis describes the design and evaluation of a sensitive radiation pressure balance for the determination of the power from medical diagnostic equipment. A full investigation of all the different phenomena occurring in the radiation pressure balance is outside the scope of this thesis, but it is intended to provide a realistic assessment of the sources of uncertainty in the measurements.

There are two different quantities that can be characterised using a radiation pressure balance. It is possible to determine the ultrasonic field incident on a target at a particular point or plane in the field. Alternatively, as in this thesis, the radiation pressure balance can be used to provide a measure of the output power from an ultrasonic transducer.

Radiation pressure balances can be designed to intercept either the whole or a known fraction of the ultrasonic beam. In the latter case, it is necessary to use either a target smaller than

the beam or a restricting window, and with either of these options the fraction of the ultrasonic beam intercepted will depend on both the ultrasonic beam shape and the separation between the transducer and the target. It was therefore decided to use a target large enough to intercept the whole ultrasonic beam.

2. THE MICROBALANCE

2.1 Sensitivity and Accuracy

The radiation pressure balance is intended to be used for the determination of the total output power from medical diagnostic equipment, for comparison with other techniques and, together with beam plotting, for hydrophone calibration. The total power from diagnostic equipment may be as low as 1 mW and the balance must be able to measure down to this level. An ultrasonic beam of 1 mW total power incident normally on a totally absorbing target produces a change in the apparent weight of the target of approximately 69 μg . Hence, a microbalance for the accurate determination of total diagnostic ultrasonic powers must have a sensitivity of 1 μg or better.

In Part I of this thesis, the radiation pressure balances described in the literature are shown to have very different estimated accuracies. We will aim towards a 5% limit on the overall uncertainty in a determination of the total output power from an ultrasonic transducer: an uncertainty figure corresponding to a determination of the apparent change in weight of the target to within approximately 3 μg .

2.2 Choice of Microbalance

The radiation pressure balance was designed and built in 1977, and at that time only four commercial manufacturers of suitable

<u>Manufacturer/Model</u>	<u>Sensitivity</u>	<u>Accuracy</u>	<u>Capacity</u>	<u>Price(1977)</u>
Metler ME22	0.1 μ g	0.1 μ g	3 g	£5297
Sartorius 4433	0.1 μ g	0.3 μ g	3 g	£3980
CI Mark 2	0.1 μ g	0.5 μ g	1 g	£1369
Cahn 1000	0.5 μ g	1.5 μ g	100 g	£7876

Table 2: A Comparison of Different Microbalances.

microbalances were found (see table 2). Two microbalances, the Sartorius 4433 and the CI Mark 2, were investigated and the source of their price difference was immediately apparent: the different methods of suspending the balance pans. In the Sartorius microbalance, there was a sophisticated mounting system with a knife-edge arrangement to ensure that the point of suspension of the balance pans stayed fixed. The CI balance, however, had a crude suspension system consisting of interlocked wire loops and, if the balance pan was disturbed, the position of the suspension point on the balance arm and hence the absolute balance calibration would change. This would be a major source of error if, for example, the suspension point changed when a weight was added to the balance pan, but it was felt that this would not necessarily be a major disadvantage in the proposed radiation pressure measurement system. The balance would have a target on one arm, ultrasound would be applied to the target and the apparent change in the target weight measured. This apparent weight change would be small compared with the weight of the target itself, and hence would be unlikely to disturb the suspension point. For this reason, it was decided to purchase the cheaper CI balance and hope that any problems could be overcome.

2.3 CI Microbalance

The microbalance consists of a galvanometer coil and balance arm (figure 1). A shutter on the balance arm is interposed between a lamp and a pair of silicon photocells, and when the balance arm is central, the photocells are equally illuminated and have equal resistance. Any small displacement of the arm, however, changes the relative illumination on the photocells, unbalancing their resistances and causing a current to flow. This current is amplified and passed through the galvanometer coil, restoring the balance arm to its original central position. In addition to generating a current exactly proportional to the applied weight (displacement), the servocontrol should ensure a rapid response and make the balance insensitive to external vibrations.

The CI microbalance was supplied with the balance head inside a glass flask. The head was attached by a multiway cable to a control cabinet containing the servocontrol electronics and a current-indicating meter. At the rear of the cabinet was a jack socket which provided output to a chart recorder where the current necessary to keep the balance arm central could be monitored.

2.4 Transducer Position

There are four conceivable orientations of the transducer in a radiation pressure balance: directed vertically upwards or downwards, horizontal or inclined at an angle. The microbalance is to be used above a water bath, and it is therefore virtually necessary for the ultrasonic beam to be vertical, as microbalances are only sensitive to forces in a vertical direction. Any horizontal ultrasonic beam would produce a net horizontal deflection of the target and this would not be subject to a restoring force from the microbalance servocontrol. Inclining the

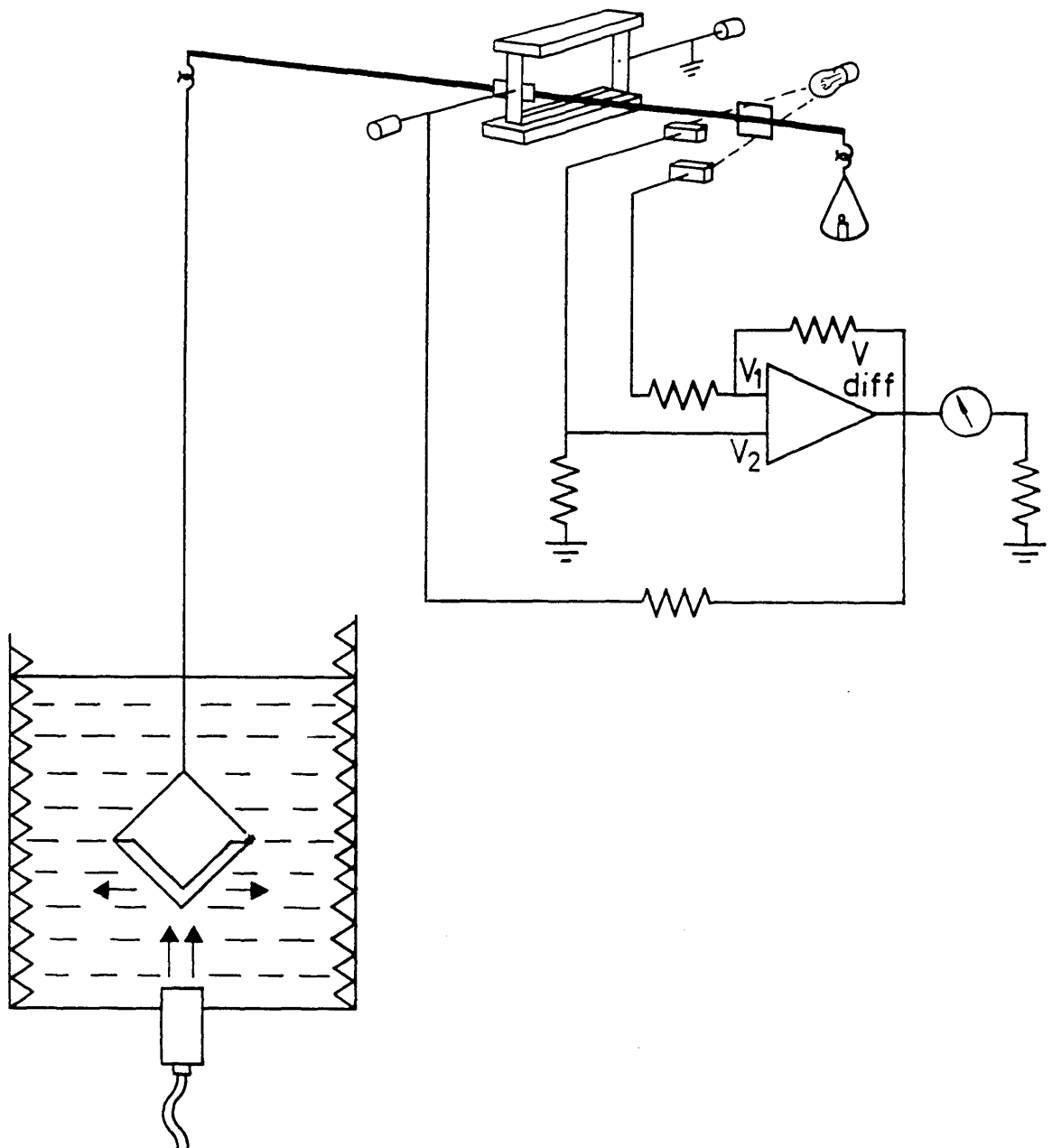


Fig. 1: A Schematic Diagram of the Radiation Pressure Balance.

transducer at an angle would require an additional plate to reflect the ultrasound towards the target which would add to the complexity of the system. It was therefore decided to mount the transducer vertically. It would be most convenient to position the transducer at the top of the tank where it could easily be changed and no special water tank would be required. In this position, however, the transducer would be in a direct line from the balance to the target, and a complex target suspension would be required, increasing any surface tension forces. The transducer was therefore placed at the bottom of a special tank on a removeable mounting plate. The alternative of placing the transducer against a thin membrane in the bottom of the tank was rejected because of the possibility of reflections from the membrane, and also because such a membrane would not locate the transducer firmly.

3. THE TARGET

3.1 Reflection and Transmission Coefficients

In this section, the reflection and transmission coefficients that are needed in any discussion of possible target design will be derived. The treatment is along similar lines to that of Brekhovskikh (1960), who considers multi-layered media.

As explained in section (II.6.1), the particle velocity $\underline{u}$ at any point can be expressed through a scalar and a vector potential by

$$\underline{u} = \text{grad } \phi + \text{curl } \psi,$$

where ϕ is the velocity potential of the longitudinal wave and ψ the velocity potential of the shear wave.

In this section plane waves will be considered, and therefore the

coordinate system can be chosen so all the quantities depend only on x and z , and all the particle trajectories lie in the $(x-z)$ plane. Additionally, the velocity potential ϕ can be chosen so that $\phi = \phi_y$ with all the other components equal to zero.

Then

$$\text{grad}\phi = \underline{i}(\partial\phi/\partial x) + \underline{k}(\partial\phi/\partial z),$$

and

$$\text{curl} = -\underline{i}(\partial\psi/\partial z) + \underline{k}(\partial\psi/\partial x),$$

where $\underline{i}$ and $\underline{k}$ are unit vectors in the x and z directions respectively.

Hence

$$\underline{u} = \underline{i}[(\partial\phi/\partial x) - (\partial\psi/\partial z)] + \underline{k}[(\partial\phi/\partial z) + (\partial\psi/\partial x)]. \quad \dots(3.1)$$

(i) Boundary Conditions

Let us initially consider a boundary between a non-viscous liquid (medium 1) and a solid (medium 2). A longitudinal wave ϕ_1 exists in the liquid, and in the solid there is both a longitudinal wave ϕ_2 and a shear wave ψ_2 . It can be implicitly assumed that the liquid and solid media do not separate, but, since both the media may move, their motion must be such that the normal component of the particle velocity is continuous. If it is assumed that the z axis is normal to the liquid-solid boundary, then the continuation of the normal component of the particle velocity gives

$$(\partial\phi_1/\partial z) = (\partial\phi_2/\partial z) + (\partial\psi_2/\partial x). \quad \dots(3.2)$$

In addition, there are two boundary conditions on the stress tensor as both the normal and tangential components of this tensor

must be continuous (Brekhovskikh 1960). Goberman (1968) shows that for a plane wave in the z direction the normal component σ_{zz} of the stress tensor can be given by

$$\sigma_{zz} = \lambda [(\partial \epsilon_x / \partial x) + (\partial \epsilon_z / \partial z)] + 2\mu' (\partial \epsilon_z / \partial z),$$

where λ and μ' are Lamé's constants, and ϵ_x and ϵ_z are the displacements in the x and z directions respectively.

Similarly, the tangential component σ_{zx} of the stress tensor is

$$\sigma_{zx} = \mu' [(\partial \epsilon_x / \partial z) + (\partial \epsilon_z / \partial x)].$$

We will introduce an additional assumption that the motion is harmonic with a time dependence given by $e^{-i\omega t}$, where ω is the angular frequency of the wave. Hence

$$u_x = \partial \epsilon_x / \partial t = -i\omega \epsilon_x,$$

and

$$u_z = -i\omega \epsilon_z,$$

where u_x and u_z are the components of the velocity $\underline{u}$ in the x and z directions respectively.

Substituting for u_x and u_z from equation (3.1) gives

$$\sigma_{zz} = (i/\omega) \{ \lambda [(\partial^2 \phi / \partial x^2) + (\partial^2 \phi / \partial z^2)] + 2\mu' [(\partial^2 \phi / \partial z^2) + (\partial^2 \psi / \partial x \partial z)] \},$$

$$\sigma_{zx} = (i/\omega) \mu' [(2\partial^2 \phi / \partial x \partial z) + (\partial^2 \psi / \partial x^2) - (\partial^2 \psi / \partial z^2)].$$

As $\mu' = 0$ in the liquid, the tangential stress tensor σ_{zx} must be zero at the solid boundary, that is

$$2(\partial^2 \phi_2 / \partial x \partial z) + (\partial^2 \psi_2 / \partial x^2) - (\partial^2 \psi_2 / \partial z^2) = 0. \quad \dots(3.3)$$

Continuity of the normal component σ_{zz} of the stress tensor at the liquid-solid boundary gives

$$\lambda_1[(\partial^2 \phi_1 / \partial x^2) + (\partial^2 \phi_1 / \partial z^2)] = \lambda_2[(\partial^2 \phi_2 / \partial x^2) + (\partial^2 \phi_2 / \partial z^2)] \\ + 2\mu'[(\partial^2 \phi_2 / \partial z^2) + (\partial^2 \psi_2 / \partial x \partial z)]. \quad \dots(3.4)$$

Gooberman (1968) shows that the longitudinal wave in a solid travels with a wave number k given by

$$(w/k) = [(\lambda + 2\mu')/\rho]^{1/2},$$

where ρ is the density of the solid.

The shear wave has a corresponding wave number k given by

$$(w/k) = (\mu'/\rho)^{1/2}.$$

We will assume only the first order components of the disturbance need to be considered, and, in this case, the velocity potentials obey the wave equation

$$\nabla^2 \phi = (k/w)^2 \partial^2 \phi / \partial t^2, \\ = -k^2 \phi. \quad (\text{for harmonic wave motion})$$

Equation (3.4) can therefore be written as

$$\rho_1 \phi_1 = \rho_2 \phi_2 - 2(\rho_2/k^2)[(\partial^2 \psi_2 / \partial x \partial z) - (\partial^2 \phi_2 / \partial x^2)] \quad \dots(3.5)$$

In this section, three boundary conditions (equations 3.2, 3.3 and 3.5) have been derived, and these equations must be obeyed at a boundary between a liquid and a solid.

(ii) Solid plate in a fluid

Consider a solid plate (medium 2) of thickness d , separating two non-viscous fluids (media 1 and 3). The density of the media are ρ_1 , ρ_2 and ρ_3 respectively. The longitudinal wave numbers in

the media are k_1 , k_2 and k_3 , and the shear wave number in the solid is k .

Assuming the motion is sinusoidal, and omitting the $e^{-i\omega t}$ time dependence for simplicity, the incident wave ϕ_{1i} in medium 1 can be written as

$$\phi_{1i} = Ae^{[ik_1(x\sin\theta_1 - z\cos\theta_1)]},$$

where A is a constant and θ_1 is the angle of incidence.

Similarly the reflected wave ϕ_{1r} in medium 1 can be written as

$$\phi_{1r} = Be^{[ik_1(x\sin\theta_1 + z\cos\theta_1)]},$$

where B is a constant.

Hence the total disturbance ϕ_1 in medium 1 is

$$\begin{aligned}\phi_1 &= \phi_{1i} + \phi_{1r}, \\ &= e^{(ik_1x\sin\theta_1)}[Ae^{(-ik_1z\cos\theta_1)} + Be^{(ik_1z\cos\theta_1)}].\end{aligned}$$

Similarly, the total longitudinal disturbance ϕ_2 in the solid (medium 2) is

$$\phi_2 = e^{(ik_2x\sin\theta_2)}[Ce^{(-ik_2z\cos\theta_2)} + De^{(ik_2z\cos\theta_2)}],$$

where C and D are constants, and θ_2 is the angle between the z axis and the normal to the longitudinal wavefront in the solid (the angle of refraction).

The total shear disturbance ψ_2 in the solid is

$$\psi_2 = e^{(ikx\sin\gamma_2)}[Ee^{(-ikz\cos\gamma_2)} + Fe^{(ikz\cos\gamma_2)}],$$

where E and F are constants, and γ_2 is the angle between the z axis and the normal to the shear wavefront.

Finally in the other fluid (medium 3) at a distance d away from

medium 1, the total disturbance ϕ_3 is

$$\phi_3 = e^{(ik_3 x \sin \theta_3)} G e^{-ik_3(z-d) \cos \theta_3},$$

where G is a constant and θ_3 is the angle of transmission.

These longitudinal and shear waves must obey the three boundary conditions given above at $z = 0$ (for ϕ_1 , ϕ_2 and ψ_2) and at $z = d$ (for ϕ_2 , ψ_2 and ϕ_3).

The first boundary condition (equation 3.2) is

$$(\partial \phi_1 / \partial z) = (\partial \phi_2 / \partial z) + (\partial \psi_2 / \partial x),$$

at $z = 0$, and using the expressions for ϕ_1 , ϕ_2 and ψ_2 , this can be written as

$$e^{(ik_1 x \sin \theta_1)} (ik_1 \cos \theta_2) (B - A) = e^{(ik_2 x \sin \theta_2)} (ik_2 \cos \theta_2) (D - C) + e^{(ik x \sin \gamma_2)} (ik \sin \gamma_2) (E + F).$$

This equation must be valid anywhere on the plane $z = 0$, that is it must be true for all x , and therefore

$$k_1 \sin \theta_1 = k_2 \sin \theta_2 = k \sin \gamma_2,$$

the law of refraction.

Using this gives

$$k_1 \cos \theta_1 (B - A) = k_2 \cos \theta_2 (D - C) + k \sin \gamma_2 (E + F).$$

Similarly, using the first boundary condition at $z = d$ gives

$$k_2 \sin \theta_2 = k \sin \gamma_2 = k_3 \sin \theta_3,$$

and

$$-k_3 \cos \theta_3 G = k_2 \cos \theta_2 (-C e^{-iP} + D e^{iP}) + k \sin \gamma_2 (E e^{-iQ} + F e^{iQ}),$$

where, to simplify the equation, we have used the notation

$$P = k_2 d \cos \theta_2,$$

and

$$Q = k d \cos \gamma_2.$$

The second boundary condition (equation 3.3) gives

$$2(\partial^2 \phi_2 / \partial x \partial z) + (\partial^2 \psi_2 / \partial x^2) - (\partial^2 \psi_2 / \partial z^2) = 0,$$

at $z = 0$ and $z = d$.

At $z = 0$ this gives

$$k_2^2 \sin \theta_2 \cos \theta_2 (D - C) + k^2 (\cos^2 \gamma_2 - \sin^2 \gamma_2) (E + F) = 0,$$

and at $z = d$

$$k_2^2 \sin \theta_2 \cos \theta_2 (D e^{iP} - C e^{-iP}) + k^2 (\cos^2 \gamma_2 - \sin^2 \gamma_2) (E e^{-iQ} + F e^{iQ}) = 0.$$

The final boundary condition (equation 3.5) is

$$\rho_1 \phi_1 = \rho_2 \phi_2 - 2(\rho_2 / k^2) [(\partial^2 \psi_2 / \partial x \partial z) - (\partial^2 \phi_2 / \partial x^2)],$$

at $z = 0$, and this gives

$$\rho_1 (A+B) = \rho_2 (C+D) - 2(\rho_2 / k^2) [-k^2 \sin \gamma_2 \cos \gamma_2 (F-E) + k_2^2 \sin^2 \theta_2 (C+D)].$$

Using the final boundary condition at $z = d$

$$\begin{aligned} \rho_3 G = \rho_2 (C e^{-iP} + D e^{iP}) \\ - 2(\rho_2 / k^2) [-k^2 \sin \gamma_2 \cos \gamma_2 (F e^{iQ} - E e^{-iQ}) + k_2^2 \sin^2 \theta_2 (C e^{-iP} + D e^{iP})]. \end{aligned}$$

The transmission coefficient T can be defined as the ratio of the acoustic pressures in the transmitted and incident waves,

$$T = (\rho_3 G) / (\rho_1 A),$$

and similarly the reflection coefficient R is the ratio of the acoustic pressures in the reflected and incident waves,

$$R = B/A.$$

The six simultaneous equations produced above can be solved to give

$$T = 2N/\{M(1+Z_1/Z_3) + i[1+(N^2-M^2)Z_1/Z_3]\}, \quad \dots(3.6)$$

$$R = [1-(N^2-M^2)Z_1/Z_3+iM(1-Z_1/Z_3)]/[iM(1+Z_1/Z_3)-1-(N^2-M^2)Z_1/Z_3], \quad \dots(3.7)$$

where, for simplicity, we have used the following notation:

$$\begin{aligned} Z_1 &= \rho_1/(k_1 \cos \theta_1), & Z_2 &= \rho_2/(k_2 \cos \theta_2), \\ Z_{2s} &= \rho_2/(k \cos \gamma_2), & Z_3 &= \rho_3/(k_3 \cos \theta_3), \\ N &= (Z_2/Z_1)(\cos^2 \gamma_2 / \sin P) + (Z_{2s}/Z_1)(\sin^2 \gamma_2 / \sin Q), \\ M &= (Z_2/Z_1)(\cos^2 \gamma_2 / \tan P) + (Z_{2s}/Z_1)(\sin^2 \gamma_2 / \tan Q). \end{aligned}$$

3.2 Type of Target

(i) Partial Reflector

A solid plate immersed in the water tank is one possible design of target. Some of the ultrasound will be reflected from the plate and the remainder will travel through. The possible problems of this type of target can be investigated using the reflection and transmission coefficients derived above.

As the plate is immersed in water, $Z_1 = Z_3$, and in this case

$$T = 2N/[2M + i(1 + N^2 - M^2)],$$

$$R = [1 - (N^2 - M^2)]/[2iM - 1 - (N^2 - M^2)].$$

For simplicity, we will consider a plate normal to the incident ultrasonic beam ($\theta_1 = 0$), and then

$$T = 2Z_1Z_2/[2Z_1Z_2\cos k_2d + i(Z_1^2 + Z_2^2)\sin k_2d], \quad \dots(3.8)$$

$$R = (Z_1^2 - Z_2^2)\sin k_2d/[2iZ_1Z_2\cos k_2d - (Z_1^2 + Z_2^2)\sin k_2d]. \quad \dots(3.9)$$

It is clear from these two equations that the single plate will have reflection and transmission coefficients dependent on k_2d . This means that these coefficients will be frequency dependent. Ideally, the amount of reflection from a target in a radiation pressure balance should be independent of frequency otherwise some form of frequency dependent correction will be required.

In the general equations for the transmission and reflection coefficients (equations 3.6 and 3.7), both N and M are frequency dependent, and hence, for any general situation, the transmission and reflection coefficients will be frequency dependent. If, however, the reflection is almost total ($R \approx 1$), or is almost negligible ($R \ll 1$), then the frequency dependence will be significantly reduced. These two options are idealised through the concepts of a totally reflecting or a perfectly absorbing target, although neither is possible in practice.

(ii) Absorbing Target

For an absorbing target, a block of material from which the reflection coefficient R is very low is required. Equations (3.8) and (3.9) give values for the transmission and reflection coefficients for a slab of a material at normal incidence.

In the radiation pressure balance, medium 1 will be water, which will be assumed to be non-absorbing with $k_1 = \omega/c_1$, where c_1 is the velocity of sound in water (equal to 1482 m/s at 20°C). Medium 2 is the target material, which may be absorbing and, in

this case, as Brekhovskikh (1960) mentions, the presence of absorption may be taken into account by letting the wave numbers k_2 and k be complex. Hence, we can write $k_2 = w/c_2 + i\alpha$, where α is the amplitude attenuation coefficient for longitudinal waves in medium 2. For ultrasound at normal incidence, no shear waves are produced.

Initially, we neglect absorption by putting $\alpha = 0$, and then $k_2 = w/c_2$ and

$$R = (\rho_1^2 c_1^2 - \rho_2^2 c_2^2) \sin(wd/c_2) / [2i\rho_1 c_1 \rho_2 c_2 \cos(wd/c_2) - (\rho_1^2 c_1^2 + \rho_2^2 c_2^2) \sin(wd/c_2)],$$

In this case if $\rho_1 c_1 = \rho_2 c_2$, then $R = 0$, and hence, if the attenuation in the solid can be neglected, a perfectly absorbing target is one for which the acoustic impedance equals that of water.

If $\rho_1 c_1 = \rho_2 c_2$, and in addition $\alpha = 1.2 \text{ cm}^{-1}$ (the amplitude attenuation coefficient for natural rubber at 1 MHz), then for a 1 cm thick slab at 1 MHz, the reflection coefficient R is approximately equal to 0.024. Hence, a finite amplitude attenuation coefficient has a significant effect on the reflection coefficient. It seems, however, reasonable to attempt to choose an absorber which has a close impedance match to water, and also a high ultrasonic absorption coefficient. Both the amount of absorption and the reflection coefficient will depend on the thickness of the material.

A rubbery substance called SOAB is claimed (Rooney 1973) to be a good approximation to a perfect absorber, although no figures for the absorption or reflection coefficients of this material are given. It was decided to obtain some measure of the reflection and transmission coefficients for a sample of SOAB.

Measurements were performed on a 12 mm thick slab of SOAB which had one plane surface and the other surface had been cut to try to reduce reflections. To determine the reflection coefficient, a membrane hydrophone was used to measure the reflected signal from the front surface of a thick plate of stainless steel. This plate was then removed and replaced by the absorber, and the signals reflected from the absorber were determined.

At 1 MHz the reflected signals were:-

stainless steel	- 88.8 mV,
SOAB (plane surface)	- 13.6 mV,
SOAB (cut surface)	- 3.17 mV.

The reflection coefficient from the front surface of the stainless steel plate was calculated to be 0.937 (using a density of $7.9 \times 10^3 \text{ kg/m}^3$ and a velocity of sound of 5790 m/s). Hence the reflection coefficients for the sample of SOAB are 0.144 for the plane surface and 0.033 for the cut surface. At 5 MHz, the corresponding values were 0.150 and 0.06.

The transmission coefficient of the sample was determined by measuring the signal at the hydrophone, firstly with water between the transducer and the hydrophone, and secondly with the SOAB absorber placed in between.

At 1 MHz the transmitted signals were:-

water	- 85.1 mV,
SOAB (plane face towards transducer)	- 1.63 mV,
SOAB (cut face towards transducer)	- 1.59 mV.

This gives an amplitude transmission coefficient of 0.019 for the SOAB sample with either the plane face towards the transducer or the cut face. This result is rather unexpected in the light of the

difference in the measured reflection coefficients for the cut and plane surfaces of the SOAB, but can be explained if the cut surface is scattering the incident ultrasound rather than reducing the total reflection.

Despite the significant reflection coefficient, the slab of SOAB was made into an absorbing target. The cut surface was at the bottom and a glass dome was mounted against the plane surface (figure 2). The dome contained air and was used to reduce the apparent weight of the absorbing target in water to below 1 g.

The absorbing target was used to investigate some of the sources of uncertainty in the measurements made with the reflecting target described later. There are many problems with

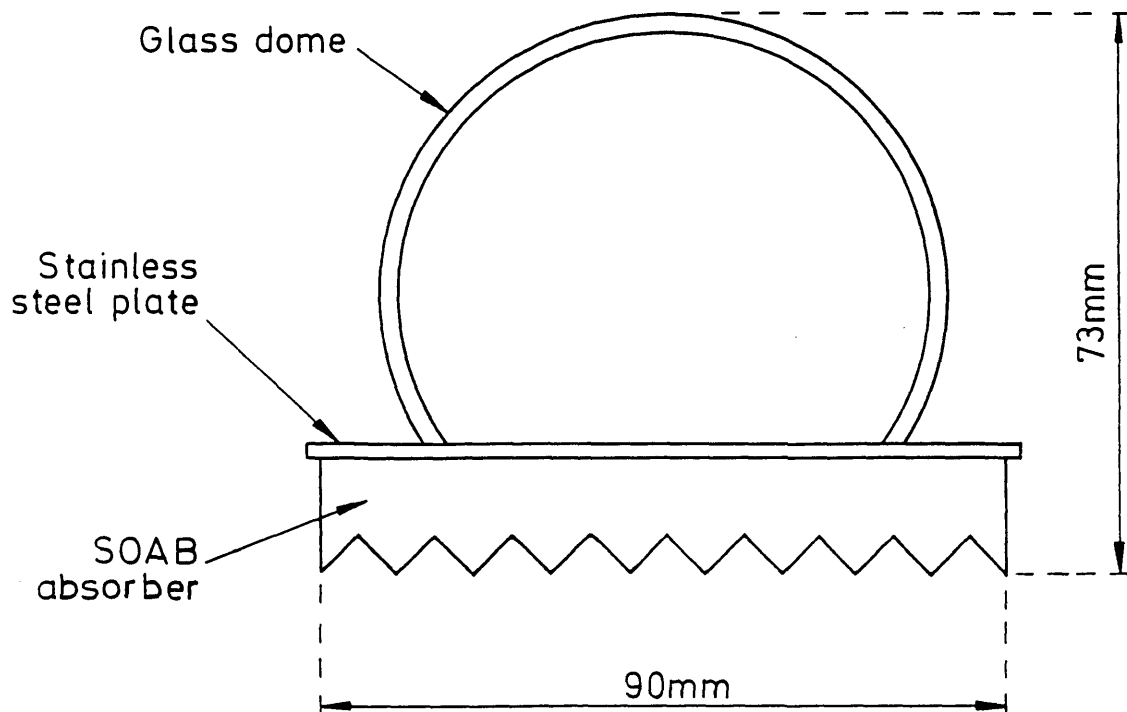


Fig. 2: The Absorbing Target.

absorbing targets. Firstly, for a slab of material both the transmission and reflection coefficients will in general be dependent on frequency. Secondly, the absorption coefficient of the material will be strongly frequency dependent. In addition, heat will be produced by the absorbed ultrasound, and the consequent temperature rise will cause buoyancy changes and may also set up convection currents.

(iii) Reflecting Target

For a single interface between two media with $Z_1 = \rho_1/k_1 \cos \theta_1$ and $Z_2 = \rho_2/k_2 \cos \theta_2$, the transmission and reflection coefficients T and R are given by

$$T = 2Z_2/(Z_2 + Z_1) \qquad R = (Z_2 - Z_1)/(Z_2 + Z_1).$$

Let us consider a water/air interface at normal incidence ($\theta_1 = 0$), and neglect any attenuation in the two media. Using the values for the density and the velocity of sound in water of 10^3 kg/m^3 and 1482 m/s at 20°C , and the corresponding values of 1.2 kg/m^3 and 330 m/s for air, gives $T = 0.00053$ and $R = 0.99947$.

Hence an air/water interface is a possible approximation to a perfectly reflecting target as this is an excellent reflector of ultrasound. The lack of rigidity of this type of target would, however, cause several problems. Firstly, the target volume must be kept constant, as changes in the volume will alter the weight of the target in water. In addition, as corrections must be made for the attenuation of the water, the position of the reflecting interface must be known. The best approximation to a rigid perfect reflector is therefore a water/rigid material/air interface. The choice of the rigid material is not critical, but it must remain stable and rust free in water, and have a low coefficient of thermal expansion. Several materials, aluminium, brass, copper,

glass, gold, perspex, polysterene, rubber, silver and stainless steel could be considered possible options for the target material. The appropriate values of the density, the velocity of longitudinal and shear waves, the coefficient of thermal expansion and the bulk and shear moduli are given in table 3. Also given (in table 4) are the transmission and reflection coefficients calculated for a wave of frequency 1 MHz incident at 45° on a 1 mm thick slab of the material. It can be seen that all the materials, except perspex, polysterene and rubber, have a transmission coefficient of less than 0.01%. The transmission and reflection coefficients in table 4 were determined without including any attenuation of either the longitudinal or the shear waves. The attenuation was neglected firstly for ease of calculation, secondly because it does not appear to have a very significant effect, and thirdly because very few values for the attenuation coefficients, particularly for shear waves, could be found in the literature.

Sharpe (1970) gives values of 0.14 cm^{-1} and 0.56 cm^{-1} respectively for the longitudinal and the shear wave amplitude attenuation coefficients in stainless steel at 8 MHz. If these coefficients are assumed to be directly proportional to frequency, the values at 1 MHz become 0.017 cm^{-1} and 0.071 cm^{-1} . Using these values for the attenuation coefficients in the calculations in table 4 gives $T = 2.8 \times 10^{-5}$ and $R = 1.0037$ compared with 2.7×10^{-5} and 0.9999961 if attenuation is neglected. Hence the attenuation in the stainless steel has a small, but significant, effect on the transmission and reflection coefficients from this material.

Other important considerations in the choice of target material are temperature dependence and rigidity. Let us first consider

	<u>Longitudinal</u> <u>Wave Velocity</u> m/s	<u>Shear Wave</u> <u>Velocity</u> m/s	<u>Density</u> kg/m ³ (x10 ³)
Aluminium	6374	3111	2.7
Brass	4372	2100	8.5
Copper	4759	2325	8.9
Glass (crown)	5660	3420	2.5
Gold	3240	1200	19.3
Perspex	2700	1330	1.2
Polysterene	2350	1120	1.1
Rubber (natural)	1600	-	1.3
Silver	3704	1698	10.5
Stainless Steel	5980	3297	7.7

	<u>Coefficient</u> <u>of Linear Expansion</u> °C ⁻¹ (x10 ⁻⁶)	<u>Bulk Modulus</u> N/m ² (x10 ¹⁰)	<u>Shear Modulus</u> N/m ² (x10 ¹⁰)
Aluminium	23	7.5	2.6
Brass	19	11.2	3.7
Copper	17	13.8	4.8
Glass (crown)	8	4.1	2.9
Gold	14	21.7	2.7
Perspex	85	0.60	0.23
Polysterene	80	0.40	0.14
Rubber (natural)	200		
Silver	19	10.4	3.0
Stainless Steel	16	16.6	8.4

(All values from Kaye and Laby 1966).

Table 3: Longitudinal and Shear Wave Velocities, Densities, Linear Expansion Coefficients and Bulk and Shear Moduli of Various Materials.

	<u>Transmission Coefficient</u>	<u>Reflection Coefficient</u>
Aluminium	9.2×10^{-5}	0.999959
Brass	4.0×10^{-5}	0.9999989
Copper	4.9×10^{-5}	0.9999983
Glass (crown)	8.6×10^{-7}	0.99999913
Gold	2.9×10^{-5}	0.99999994
Perspex	3.4×10^{-4}	0.99976
Polysterene	4.8×10^{-4}	0.99974
Rubber (natural)	4.7×10^{-4}	0.99962
Silver	4.2×10^{-5}	0.9999980
Stainless Steel	2.7×10^{-5}	0.9999961

Table 4: The Transmission and Reflection Coefficients Calculated for a Wave of Frequency 1 MHz Incident at 45° on a 1 mm Thick Slab of the Material with Water on One Side and Air on the Other.

temperature dependence. The apparent weight of a target in water is equal to $(\rho - \rho_w)V$, where ρ and ρ_w are the densities of the target and the water respectively, and V is the target volume. For temperatures around 20°C , a 1°C change in temperature alters the density of water by 0.2 kg/m^3 . Hence the change in the apparent weight of a target caused by a 1°C change in temperature can be calculated. For a target consisting of a plate 10 mm thick and 100 mm in diameter (both arbitrary values chosen to demonstrate the order of magnitude of the effects), the change in the apparent weight of the different target materials for a 1°C change in temperature is given in table 5.

It can be seen that, due to their relatively high densities, the apparent weight change is lowest for gold and silver targets, but, except for glass and aluminium which have a high apparent weight change, there is little variation between the materials.

<u>Material</u>	<u>Percentage Change in</u> <u>Apparent Weight</u>	<u>Weight Change</u> <u>μg</u>
Aluminium	0.012	9.2
Brass	0.003	2.1
Copper	0.003	2.0
Glass (crown)	0.013	10.5
Gold	0.001	0.9
Silver	0.002	1.7
Stainless Steel	0.003	2.3

Table 5: The Change in the Apparent Weight of Different Target Materials for a 1°C Change in Temperature.

The second consideration in the choice of target material is rigidity, and this is related to the bulk and shear moduli. The bulk modulus is the ratio of the applied pressure to the fractional change in volume, while the shear modulus is the ratio of the shear stress to the shear strain. For all the materials discussed here, the bulk modulus is larger than the shear, and we will therefore consider this form of deformation. In section (2.1), it was shown that the apparent weight of the target needs to be known to within approximately 3 μg. For a plate 10 mm thick and 100 mm in diameter, the change in target volume producing a change in the apparent weight of 3 μg can be calculated, and, using the bulk modulus for the material, the applied pressure required to produce this volume change can then be determined. The results for different materials are given in table 6.

The higher the pressure required to produce the apparent weight change of 3 μg, the more suitable the material is for use as a target in terms of its rigidity. Table 6 therefore shows that aluminium, followed by glass and stainless steel, is the best

<u>Material</u>	<u>Volume Change</u>	<u>Fractional Change</u>	<u>Applied Pressure</u>
	$\text{m}^3 (\times 10^{-12})$	$\frac{\text{in Volume}}{(\times 10^{-9})}$	$\frac{\text{Required}}{\text{N/m}^2}$
Aluminium	1.8	22	1650
Brass	0.4	5.1	570
Copper	0.4	4.8	660
Glass (crown)	2.0	25	1030
Gold	0.2	2.1	460
Silver	0.3	4.0	420
Stainless Steel	0.4	5.7	950

Table 6: The Applied Pressure Required to Produce an Apparent Weight Change of 3 μg .

choice for a target material in this respect.

Taking the two considerations of temperature dependence and rigidity into account, stainless steel appears to be a good choice of target material. Ease of machining should also be considered as the target should be a good specular reflector rather than a scatterer of the ultrasound. This implies a surface flatness of better than a tenth of a wavelength. The longitudinal wavelength ranges from 6.4 mm in aluminium at 1 MHz to 0.3 mm in gold at 10 MHz, and hence the surface of the target should be smooth to within approximately 0.01 mm. Of the materials discussed here, glass is by far the most difficult to machine, and hence for any target design except a flat plate, glass should be discounted. In addition, the target material should be very stable in water, and for this reason neither aluminium nor copper is a particularly suitable choice. Hence for the many reasons given here, together with one of cost, stainless steel was chosen for the reflecting target.

The thickness of the target was the next consideration, and a

number of factors were taken into account. Firstly, the volume of the target must remain constant despite both the force from the ultrasound and the change in the hydrostatic pressure as the vertical position of the target is altered. The amount of deflection y produced by a force at the centre of a circular plate simply supported at the edge can be calculated using a standard formula (Roark and Young 1975)

$$y = [Wa^2/(16\pi D)](3 + \nu)/(1 + \nu),$$

where $W = q\pi r_o^2$ is the force at the centre of the plate, q is the force per unit area, r_o is the radius of action of the force, a is the radius of the plate, ν is Poisson's ratio (0.27 for stainless steel), and D is a plate constant given by

$$D = Et^3/[12(1 - \nu^2)],$$

where E is Young's modulus (28×10^6 lb/in² for stainless steel), and t is the thickness of the plate.

A force at the centre of the plate produces a deflection y and we can therefore calculate an approximate value for the change in the volume and the consequent change in the apparent weight of the target caused by this deflection. The formula for the volume of a segment is

$$(\pi y^2/3)[3(r^2 + y^2)/(2y) - y],$$

where r is the radius of the segment and y the height.

Consider a circular plate of stainless steel, 30 mm in radius and t mm thick, and an ultrasonic beam of 500 mW total power and 10 mm radius incident on the centre of the stainless steel plate. The peak pressure in the ultrasonic beam is approximately 7×10^4 Pa, and the force at the centre of the plate produced by

this pressure is approximately 20 N. The deflection of the plate produced by a force of this magnitude and the consequent change in the apparent weight of the target can be calculated, as given in table 7.

An ultrasonic beam with a total power of 500 mW incident on a plane totally reflecting target produces a change in the apparent weight of the target of approximately $3 \times 10^4 \mu\text{g}$. Hence, from table 7, it can be seen that, for a 1 mm thick plate, the change in the apparent weight produced by an applied pressure equal to the peak pressure in the ultrasonic wave is approximately twice the apparent weight change produced by the ultrasonic radiation pressure. Clearly, however, the pressure in an ultrasonic wave is oscillatory and changes from positive to negative over a very short time scale depending on the period of the ultrasonic wave (typically between 0.1 and 1 μs). Therefore, the deflection of the plate produced by the peak pressures in an ultrasonic wave is likely to be much smaller. It does seem advisable, however, to choose a plate sufficiently thick to reduce the apparent weight

<u>Plate thickness</u>	<u>Deflection</u>	<u>Apparent Weight Change</u>
t mm	y μm	μg
1	55	7.8×10^4
2	6.5	9.2×10^3
3	2.0	2.8×10^3
4	0.85	1.2×10^3
5	0.43	610
6	0.25	350
7	0.16	230
8	0.11	160
9	0.08	106
10	0.06	78

Table 7: The Apparent Weight Change Produced by an Ultrasonic Beam of 500 mW Total Power and 10 mm Radius.

change caused by a pressure equal to the peak ultrasonic pressure to below 1% of the total apparent weight change. For a ultrasonic beam of 10 mm radius and 500 mW total power, this occurs for a plate greater than 6 mm thick.

The change in the volume of the target due to changes in hydrostatic pressure as the vertical position of the target is altered is, as mentioned earlier, related to the bulk modulus of the target material. As the radiation pressure balance will be a null device with the servocontrol bringing the arm back to a central rest position, the variation in vertical position of the target during the measurements will be very small, much less than 1 mm. A 1 mm change in vertical position produces a change in hydrostatic pressure of approximately 10 Pa, which, using the bulk modulus of stainless steel, produces a fractional change in target volume of 6×10^{-11} . Hence any variation in hydrostatic pressure can be neglected.

Another consideration is the capacity of the microbalance. The CI balance has a maximum capacity of 1 g, and the target must have sufficient volume to reduce its apparent weight in water to below this value. This is done by backing the target with a low density material, such as glass, enclosing an air volume. If the reflecting material has a large mass, a large volume target is required, and changes in the density of the water will make the weight of the target very sensitive to temperature. It is therefore necessary to reach a compromise and use a target which has a low mass and a consequent low volume, but is rigid.

With a reflecting target, the reflected wave must be considered, and this must not return and interfere with either the target or the transducer. Clearly, a plane reflector parallel to the transducer face will produce standing waves. If the plane

reflector is angled at, for example, 45° to the transducer front surface, all the ultrasound will be reflected away from the transducer, but there will be a horizontal force on the target moving it out of the ultrasonic beam and also altering the surface tension forces. Another possible design of reflecting target is an upturned large-angle cone hanging with its apex at the top (figure 3). This target will self-centre over the ultrasonic beam. The only problem is that the reflected wave, which is sent to be absorbed below the target, may produce convection currents. For a right-angled cone with its tip pointing downwards or an upturned rooftop, however, all the reflected ultrasound goes to the sides of the water tank where it can be absorbed with minimal disturbance to the target. The main disadvantage of these two designs is that the target will not be self-centering, and any small disturbance will tend to knock the target off the centre of the ultrasonic beam. This will produce a net sideways radiation force away from the beam centre opposed only by the weight of the target, making it oscillate. This instability sets an upper limit to the measurable ultrasonic power. Additional disadvantages with the upturned rooftop are the larger volume and the long line along the apex which could reflect some of the ultrasound back towards the target.

All these factors were considered when the type of target was chosen, and it was finally decided to build a right-angled conical reflector. This was constructed from a stainless steel cone with a thin glass dome on top sealing air inside the cone (figure 4). The target was suspended from the balance arm using a thin nylon thread; the load on the thread being between 300 mg and 700 mg according to temperature.

As a right-angled cone is not self-centering, it is necessary

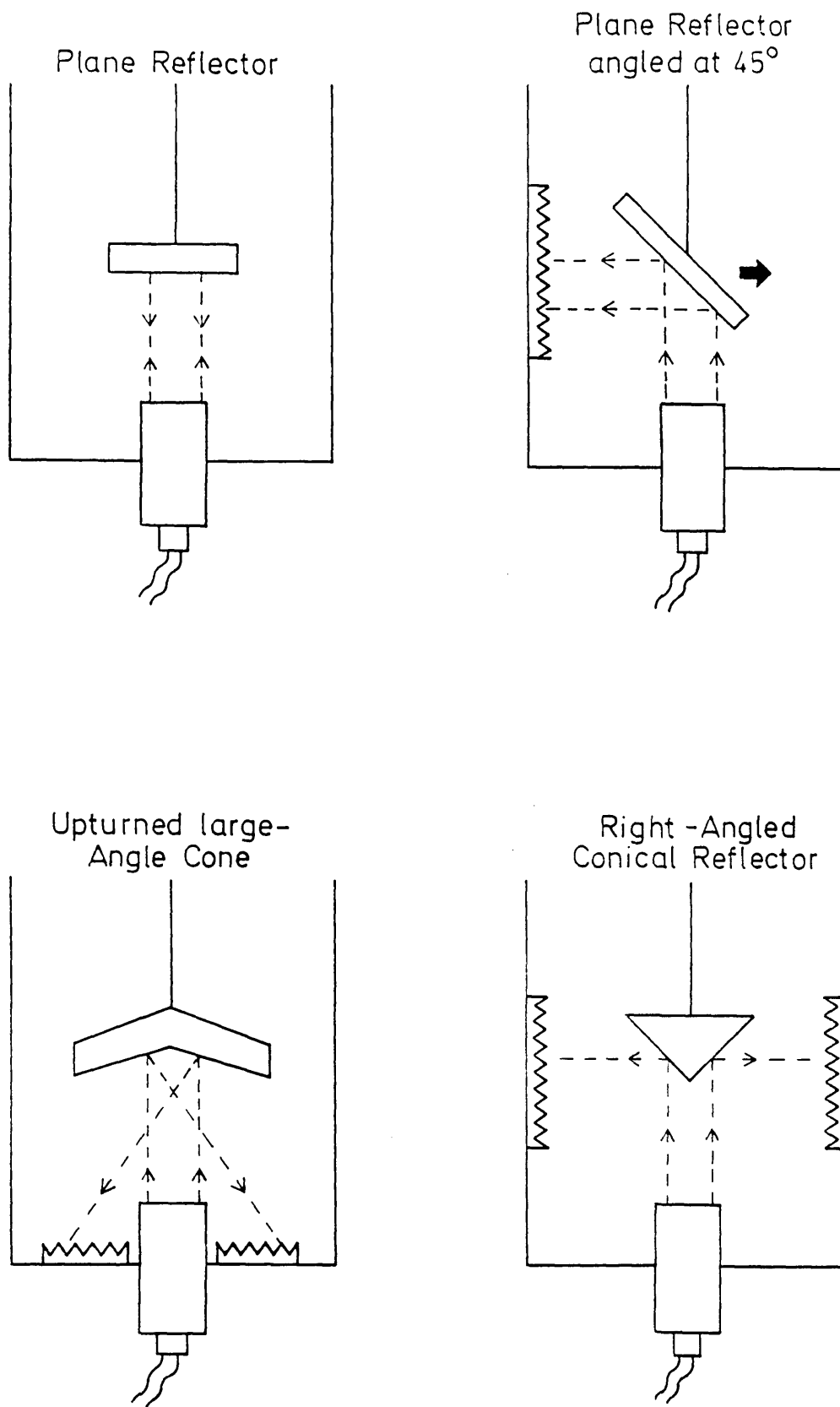


Fig. 3: Possible Designs of Reflecting Target.

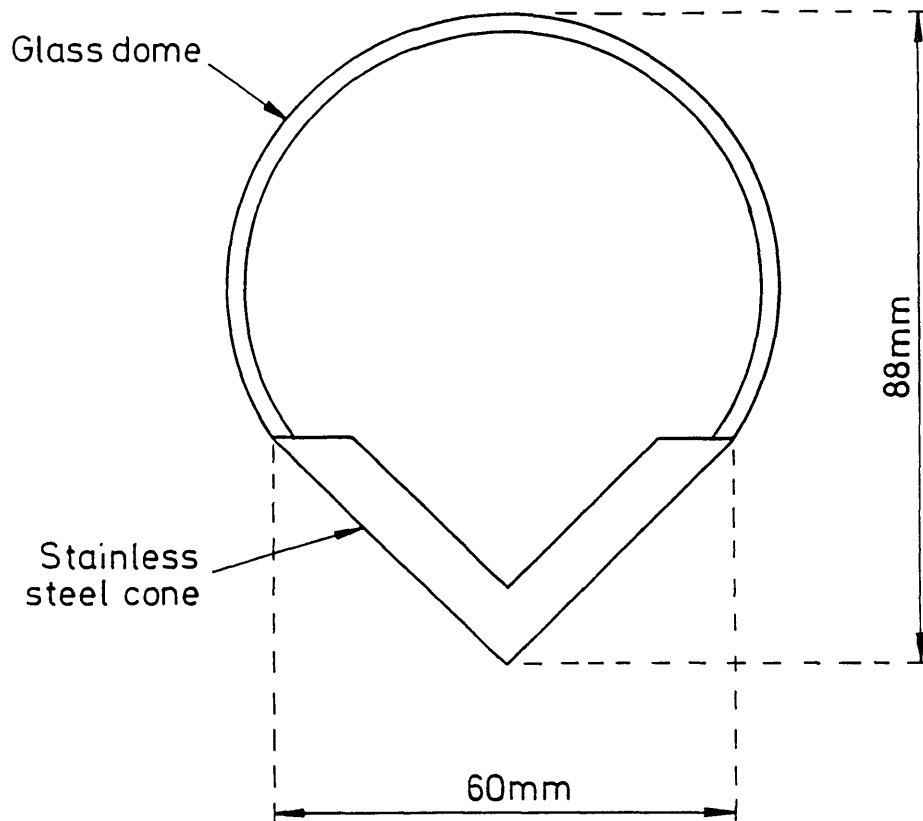


Fig. 4: The Reflecting Target.

to position the transducer accurately beneath the target. This was done by mounting the tank and transducer on x and y micropositioning devices. The microbalance with its suspended target was fixed in a support arrangement with the tank and transducer able to move underneath. In addition the geometric and acoustic axes of a transducer may not be in the same line, so the tank was mounted on a tilting table to allow the target to be aligned parallel to the acoustic beam axis. It was also considered desirable to have a variable transducer/target separation, and the transducer, tank and xy positioners were mounted on a jacking

device which could move them up and down.

3.3 Change in Target Weight

The microbalance records the apparent weight of the target in the water bath, and all the measurements are based on a determination of the change in this apparent weight produced by the ultrasound. For convenience, however, throughout the remainder of this thesis, the term "weight" will, in many cases, be used in preference to the more cumbersome term "apparent weight", with the meaning and significance of the term made clear through its context.

3.4 Relation Between Change in Target Weight and Total Power

The relation

$$F = (W_0/c_0)\exp(-2\alpha x), \quad \dots(\text{II.6.17})$$

given in section (II.6.2) between the total ultrasonic output power W_0 from a transducer and the force F on a target will be assumed to apply for the measurements described in this part of the thesis. This relation is valid for plane sinusoidal one-dimensional wave motion in a viscous fluid, and is applicable in water at the frequencies and output powers used in medical diagnosis.

It is worth mentioning that equation (II.6.17) is only applicable to travelling waves and may not apply if standing waves are set up, for example, when the target is a plane reflector parallel to the ultrasonic beam. It is, however, quite valid for a plane perfectly absorbing target parallel to the ultrasonic beam. For a conical perfectly reflecting target hanging vertically with its reflecting surface angled at 45° to the vertical ultrasonic beam, as is mentioned in section (II.5), all the reflected

ultrasound will travel horizontally and the vertical forces on the target will equal the time-averaged momentum flux density of the incident ultrasonic beam. Hence equation (II.6.17) can be used to relate the vertical force on the conical reflecting target to the total ultrasonic output power. This equation can also be written as

$$W_o = c_o g dw \exp(-2\alpha x),$$

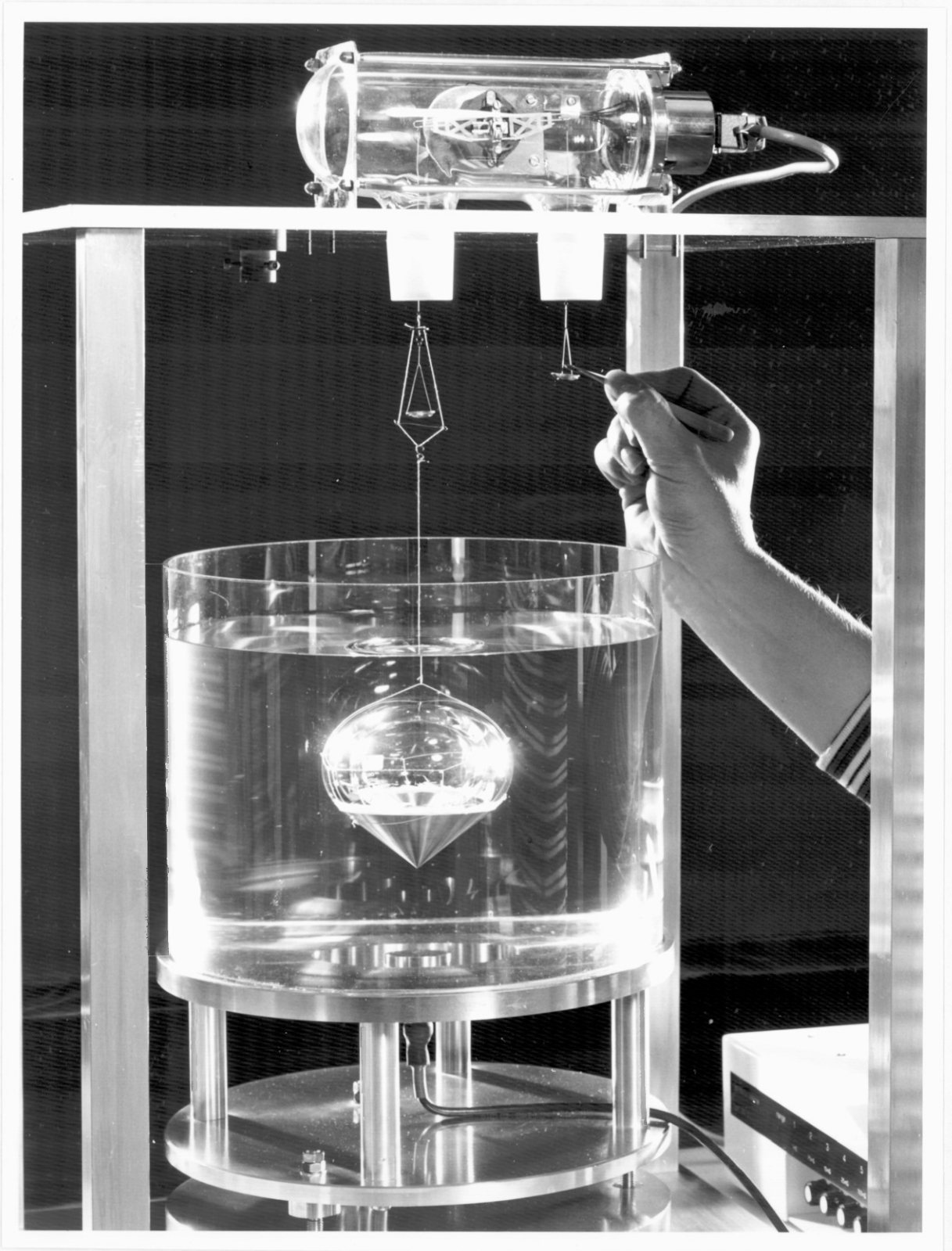
where dw is the change in the apparent weight of the target and g is the acceleration due to gravity.

4. OPERATION OF THE BALANCE

4.1 Balance Servocontrol

The microbalance was initially set up with the balance head on a mounting plate above a water tank, and the reflecting target was suspended from one arm of the balance beam (photograph 1). When the servocontrol was switched on, the balance arm went into severe oscillations. If the target was removed, however, and normal weights applied, the servocontrol performed perfectly adequately and the weights could be weighed. This implied that the servocontrol could not be used for the high mass and large damping of the target in water.

A servocontrol system provides a signal 180 degrees out of phase with the disturbing signal, and the amount of compensation provided depends on the frequency of the disturbance. The frequency response of the balance servocontrol system was determined by applying a perturbing signal of variable frequency. The amount of movement of the balance arm in response to this perturbing signal was monitored by measuring the current at a



Photograph 1: The Initial Mounting Arrangement for the
Microbalance and the Target above a Water tank.

particular point in the servocontrol circuit, and the frequency response curves were obtained for several different cases (figure 5). It can be seen that, when compared with the other configurations, the target on the balance has the lowest resonant frequency. This resonant frequency is important in servocontrol systems as it is where the mechanical system is most likely to oscillate. The original microbalance servocontrol (figure 6) had been designed to provide a mechanical resonant frequency greater than 0.6 Hz. The target, however, has a resonant frequency of approximately 0.2 Hz, and with the original servocontrol circuitry the system was only very loosely locked. Any small disturbance caused the target and balance arm to oscillate. It was decided, therefore, to build another servocontrol circuit able to control either the target or the two balance pans (figure 7). When the servocontrol had been rebuilt, the current required to keep the balance arm stationary was monitored by measuring the voltage across a resistance with both a digital voltmeter and a chart recorder. The complete balance system with the new servocontrol circuitry, digital voltmeter and chart recorder can be seen in photograph (2).

4.2 Measurement of Temperature

A platinum resistance thermometer (P.R.T.), consisting of a 100 ohm resistance sensor connected to a control box with digital display, was purchased. The display meter had 0.1 °C resolution over the range -160 to +199 °C, and the accuracy of the thermometer was determined using conventional techniques available in the Thermometry Section at NPL. Two standard calibrated thermometers were used to record temperatures between 17 and 30 °C; achieved using a thermostatically-controlled water bath.

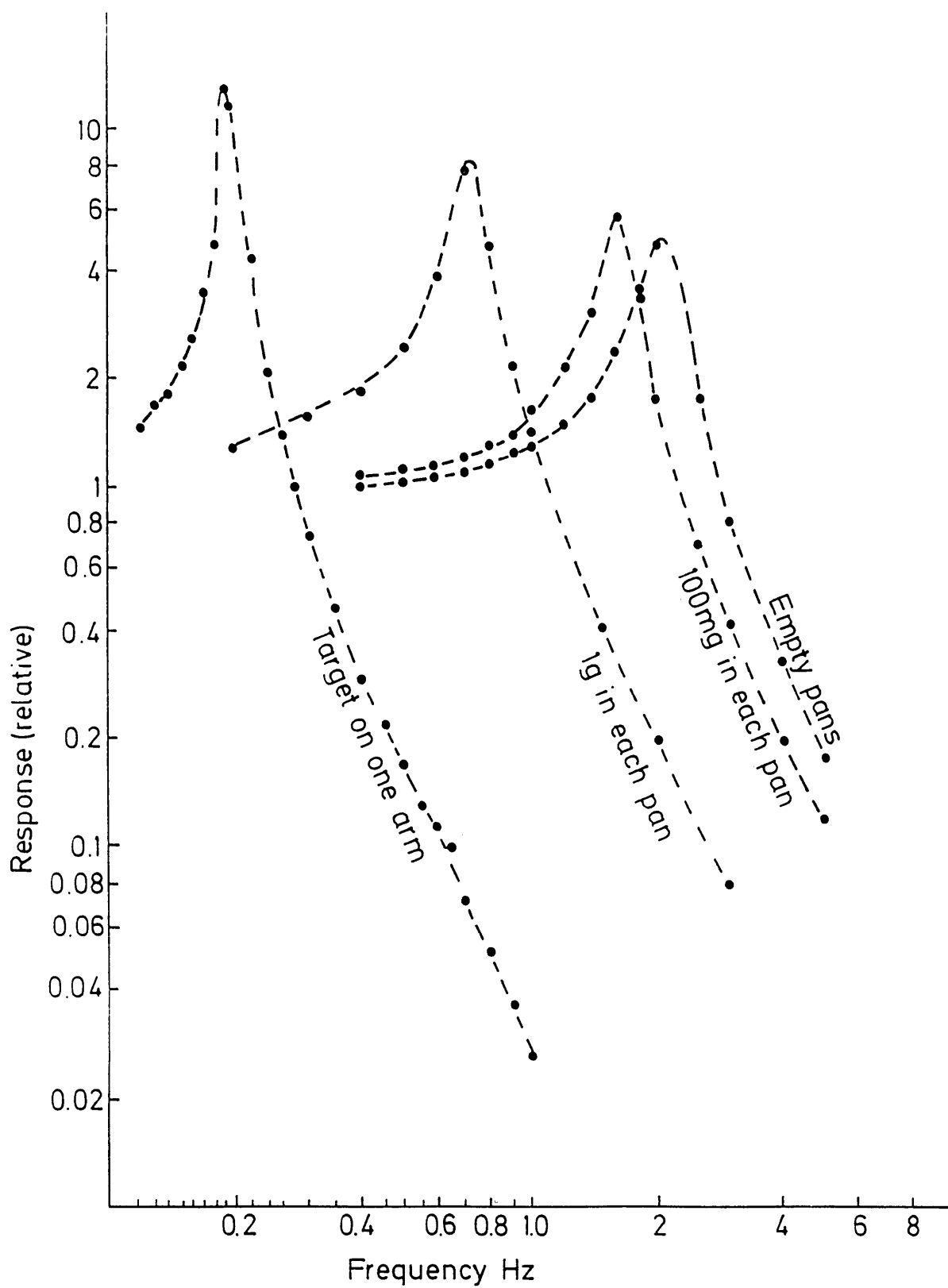


Fig. 5: The Balance Servocontrol - Frequency Response Curves.

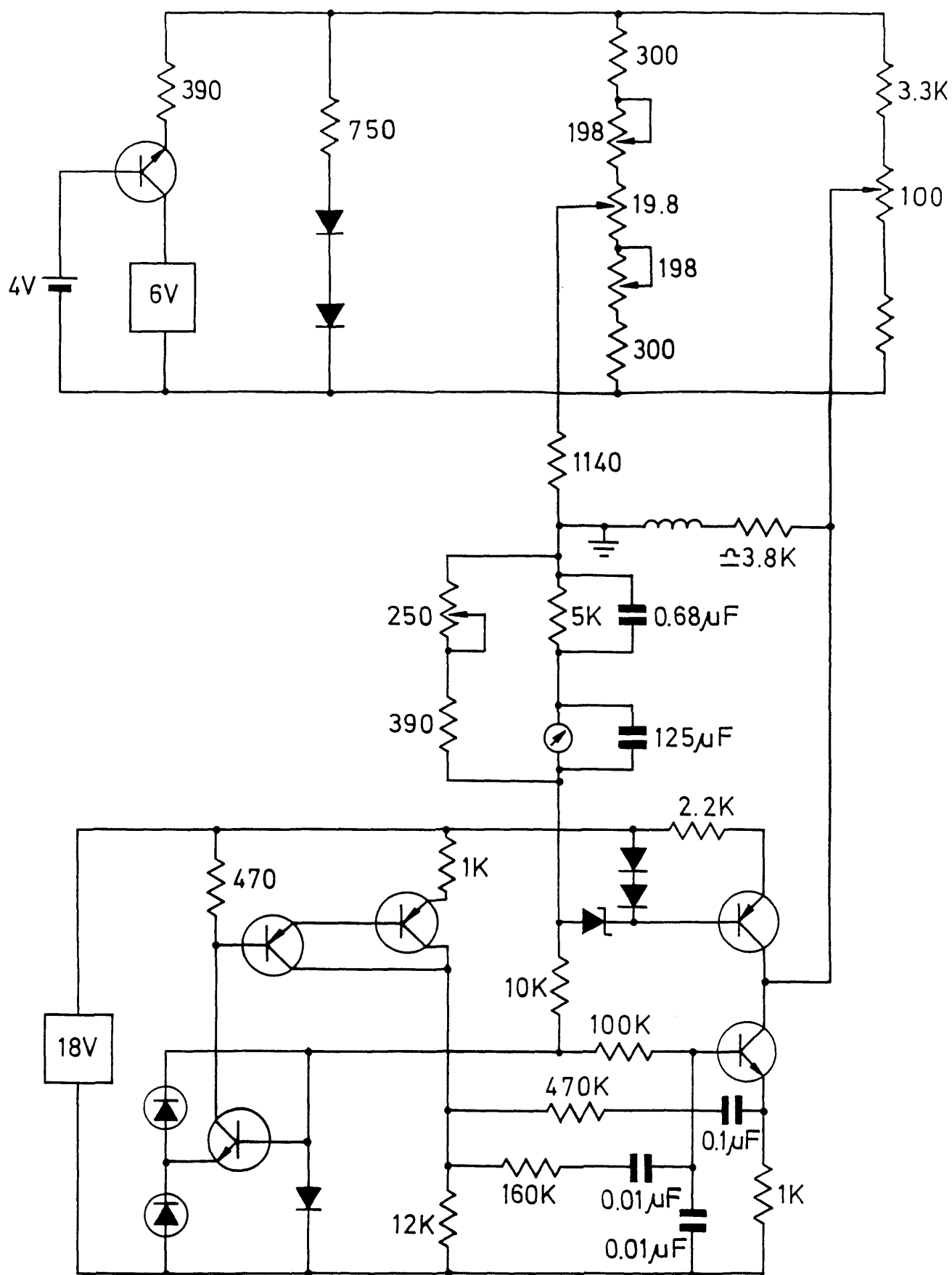
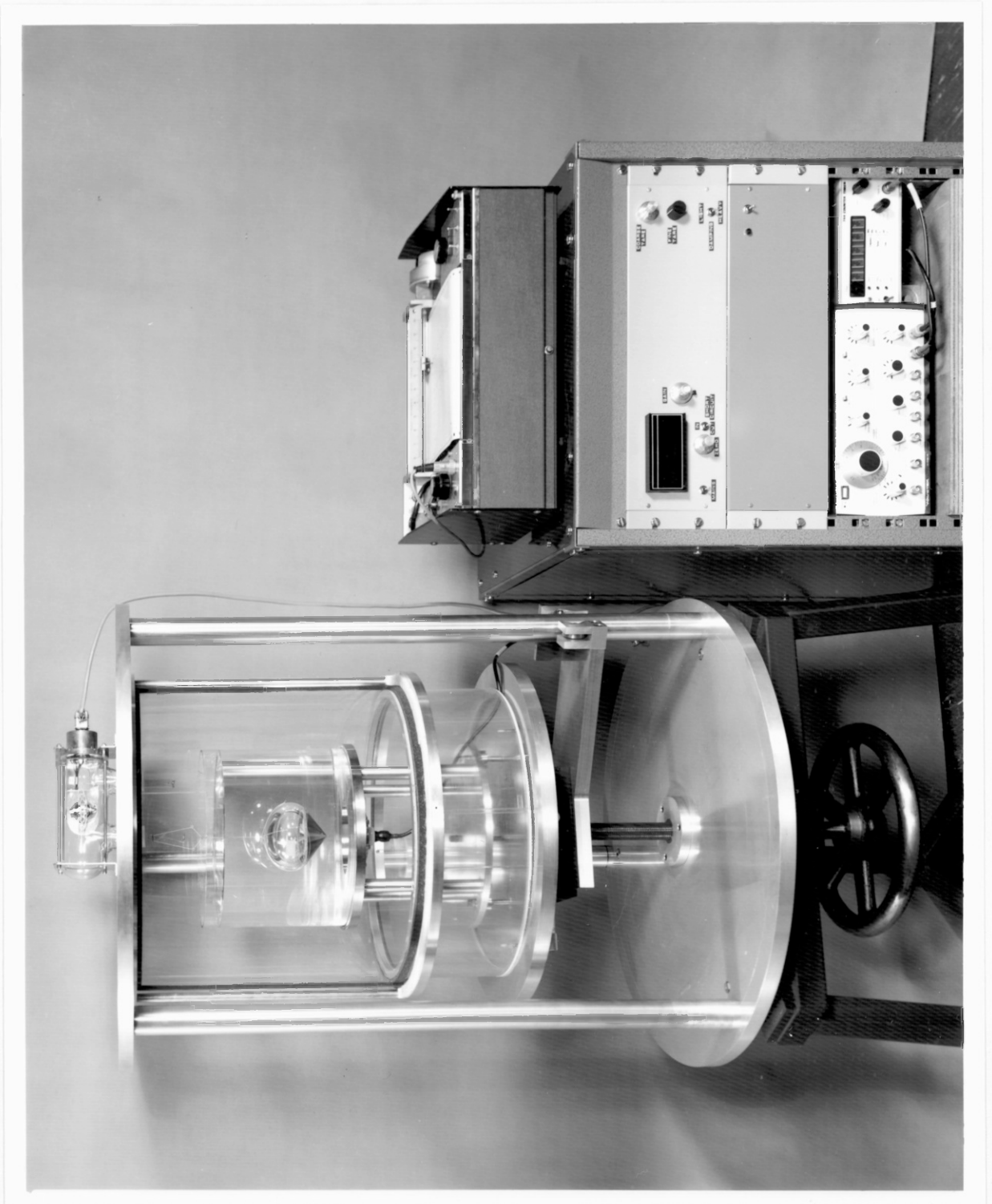


Fig. 6: The Balance Servocontrol - Original Microbalance Circuitry.



Photograph 2: The Complete Balance System with the New Servocontrol Circuitry, Digital Voltmeter and Chart Recorder.

<u>Thermometer A</u>	<u>Thermometer B</u>	<u>P.R.T.</u>
$^{\circ}\text{C}$	$^{\circ}\text{C}$	$^{\circ}\text{C}$
17.08	17.10	17.0
19.79	19.77	19.7
22.60	22.59	22.5
26.47	26.49	26.4
29.86	29.87	29.8

Table 8: Calibration of the Platinum Resistance Thermometer.

The results are given in table 8. It is clear that the platinum resistance thermometer is, in general, reading approximately $+ 0.1^{\circ}\text{C}$ low. All the temperatures recorded by the device were therefore increased by 0.1°C .

As a check on stability, the control box for the platinum resistance thermometer was heated to 35°C and the measurements repeated. No significant difference was found between the results at this temperature and the previous data obtained with the control box at room temperature (18°C).

4.3 Calibration of the Balance

The balance was calibrated by noting the reading on the chart recorder when a calibrated 20 mg weight was applied to one balance pan. The initial method of calibration consisted of suspending a pan from the balance arm, adding the calibrated weight and relating the change in weight recorded by the chart recorder to the known calibrated weight. The pan was then removed and the target suspended from the balance arm. This technique, however, produced a variation of upto $\pm 2\%$ in the calibration figure obtained. It was thought that this could be due to a disturbance of the suspension point on the balance beam (as mentioned earlier)

when either the pan was added and removed, or as the target was applied. To investigate this possibility, some measurements were made on the balance, as follows:

- (i) pans were put on both arms of the balance, and a reading taken,
- (ii) a 100 mg weight was added to one pan and a reading taken,
- (ii) the weight was removed and a reading taken,
- (iv) the pan was removed and then replaced, and a reading was taken,
- (v) steps (ii) to (iv) were repeated.

This gave a number of calibration figures for the balance. The complete procedure was then repeated with an extra 500 mg weight in the pan during the whole exercise. It was thought that the first set of measurements would correspond to the pan being added or removed in the initial calibration procedure, whereas the second set with the 500 mg weight corresponded to adding or removing the target.

The different calibration figures obtained for both sets were plotted (figure 8). For the measurements with the pan alone, the calibration figures were reasonably consistent with a standard error on the mean for 40 measurements of 0.02%. The results for the pan plus 500 mg were more scattered with a standard error on the mean for the same number of readings of 0.16%. More than one calibration figure obtained with 500 mg in the pan was 3% different from the mean value, and, if this sort of effect should occur when the target was applied, sizeable errors would be introduced into the total power measurements. It was felt that the target and pan should both be kept on the balance arm during the measurements (figure 9), and the balance calibrated by adding and

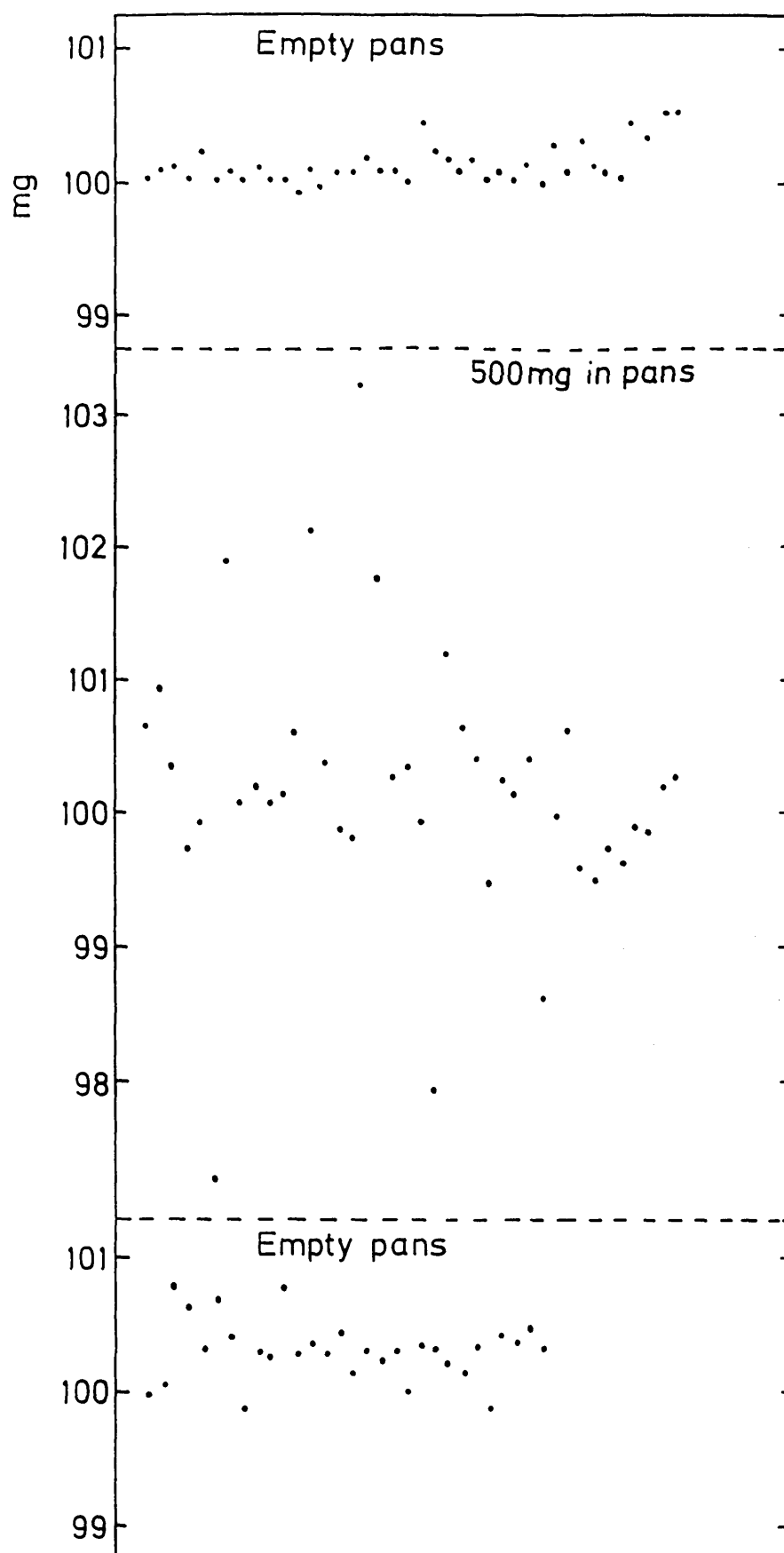


Fig. 8: Calibration of the Balance - Calibration Figures Obtained for Different Configurations.

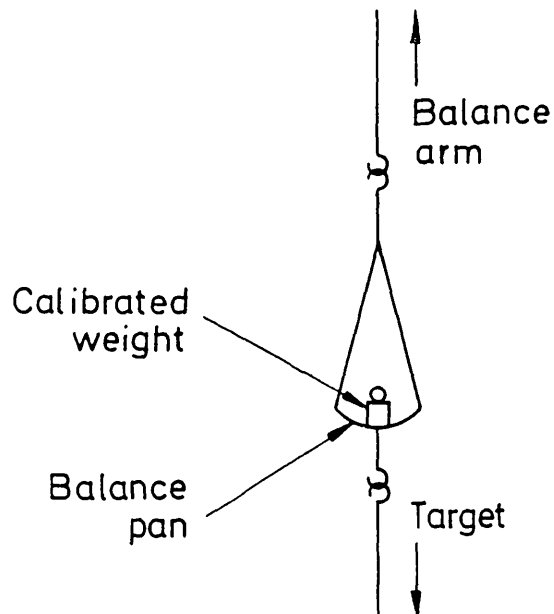


Fig. 9: Calibration of the Balance - Target Suspension Arrangement.

removing a small 20 mg weight. This should not disturb the suspension point on the balance beam.

4.4 Absorption of the Reflected Wave

With a conical reflecting target, the ultrasound is reflected to the sides of the water tank, where it must be absorbed to prevent reflections from the tank walls returning the ultrasound towards the target and the transducer. A variety of absorbers have been used, including rubber (Kossoff 1965, Greenspan et.al. 1978, Wells et.al. 1964), brushes (Newell 1963) and carpet (Wemlen 1968), and rubber car matting and woollen long pile carpet were selected for study. The acoustic impedance of these materials is different from water, but the rubber spikes and carpet pile may

present on average a gradual impedance change to the ultrasonic wave, and hence reduce the reflections.

The amount of ultrasound that is specularly reflected from an absorber can be measured using a hydrophone, but it is likely that a significant quantity of ultrasound will be scattered either from the surface or the interior of the absorber, and this presents a more complex measurement problem. It was decided to look at the absorbers using a Schlieren system which, although not providing quantitative results, can give a good indication of the absorption properties of a material.

The absorber to be investigated was placed in the Schlieren tank with an ultrasonic transducer mounted above, and the voltage applied to the transducer was monitored. Initially, this voltage was increased until a Schlieren image of the ultrasound became visible. The voltage on the transducer was noted, and the reflection and transmission properties of the absorber were then studied by increasing the transducer voltage. The reflected, scattered and transmitted waves from the rubber car matting were clearly visible. With the carpet in the Schlieren tank, however, no waves could be seen even when the transducer voltage was increased to greater than ten times the minimum voltage required to produce a Schlieren image.

Many authors, including Gooberman (1968) and Heuter and Bolt (1955), demonstrate that the output power from an ultrasonic transducer is proportional to the square of the applied electrical voltage. To obtain this relationship, they make several assumptions: that only the thickness of the piezoelectric disc changes under the influence of the electric field, that the applied electric field is sinusoidal, and that the piezoelectric material is an insulator. These assumptions appear to be valid for

most piezoelectric ultrasonic transducers, particularly if the transducers are driven at their thickness mode resonant frequency. Hence it seems reasonable to assume that increasing the voltage on a transducer by a factor of ten is approximately equivalent to a one hundredfold increase in output power from the transducer. The Schlieren measurements, therefore indicate that any reflected, scattered or transmitted waves from the carpet are well below the 1% level. Long pile carpet was therefore chosen for the absorber around the inside of the radiation pressure balance tank.

Photograph (3) shows the radiation pressure balance tank and support arrangement with the microbalance at the top. The carpet absorber around the edge of the cylindrical water tank can be seen. In addition, there are water-filled coils around the inside of the water tank between the absorbing carpet and the glass wall of the tank. These coils are connected to a temperature regulation system and, if water at a constant temperature is pumped through the coils, the temperature of the water in the balance tank can be controlled.

4.5 Variation in the Apparent Weight of the Target

It was found that the apparent weight of the target measured by the microbalance varied with time and, although this effect was thought due to temperature variations, some investigation was required to ensure that there was no contribution from other sources. An approximate calculation was used to give an indication of the relation between temperature and target weight, and this was compared with some experimental measurements.

At temperatures around 20°C, a one degree temperature change alters the density of water by 0.0002 g/cm³. The volume of the target was calculated to be 210 cm³, and hence the weight of this



Photograph 3: The Radiation Pressure Balance.

The carpet absorber and water-filled coils around the inside of the water tank are clearly visible.

target should change by approximately 42 mg for a one degree change in temperature. The coefficients of expansion of stainless steel and glass are such that any effect due to the volume expansion of the target is negligible compared with variations due to the changes in the density of water. To experimentally determine the change in the apparent weight of the target with temperature, a thermistor bridge and chart recorder were used to record the temperature, while the target weight was measured on the radiation balance chart recorder. A linear relation between the temperature change and the target weight change was obtained (figure 10), and this showed that the apparent weight of the target changed by 43.8 mg for a one degree change in temperature. There is, therefore, good agreement between the measured value and that calculated from the target volume and the change in the density of water, indicating that no other effects are contributing to the change in the target weight with time.

4.6 Operation of the System

Once the balance and the target had been constructed and various preliminary investigations made, it was necessary to decide upon the most effective method of use of the system. A transducer was mounted in the bottom of the balance tank and, after filling the tank with water, the target was suspended from the balance arm. There was significant bulk movement of the water directly after the tank had been filled and the system was left to stabilise. The current required to keep the balance arm fixed was monitored on a chart recorder, and the complete system was assumed to be stable once the chart recorder trace, and hence the target, was steady. This took approximately half an hour. The transducer was then excited using a sinusoidal electrical signal, and the

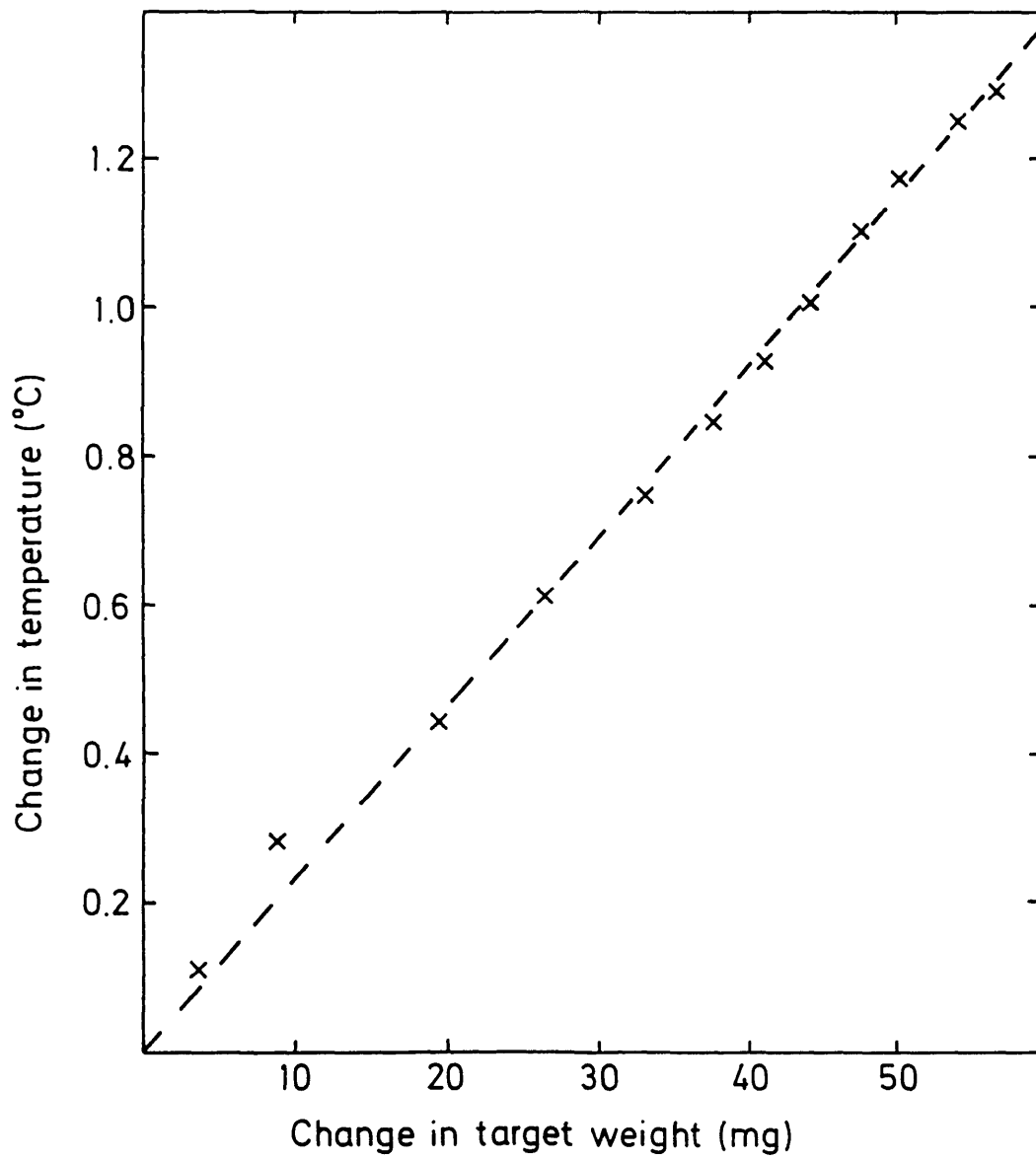


Fig. 10: Relation Between a Change in Temperature and a Change in the Apparent Weight of the Target.

(dashed line is a least squares fit through the experimental points)

ultrasound from the transducer caused a decrease in the apparent weight of the target and a consequent sharp deflection in the chart recorder trace. The ultrasound was left on for approximately one minute until the chart recorder trace had become stable and was then switched off. This produced an increase in the apparent target weight and the chart recorder trace returned to its original position. Repetition of this procedure (figure 11), yielded a series of steps superimposed upon a general drift in the apparent weight of the target due to temperature change. The size of the chart recorder step can easily be measured even from a single excitation of the transducer (figure 12). Switching the transducer on and off was used to give two values for the change in the apparent weight of the target.

Two effects were studied by switching the transducer on and off a number of times. Firstly, any difference between the on and the off values might indicate the presence or build-up of some effect due to the ultrasound. Secondly, any drift in the balance performance would produce a variation in the mean of the on and off values with time. Measurements were made on three transducers of frequencies 1, 5 and 10 MHz, and the results are given in table 9. It can be seen from these results that, at all frequencies, the on and the off deflections are equal within the measurement uncertainties, and also there is no significant change in the mean deflection with time.

As the weight of the target is temperature dependent, to minimise any temperature variations, the measurement time should be as short as possible. A single excitation of the transducer requires the shortest measurement time, but it was decided to use two or three transducer excitations, so an incorrect value for the change in the apparent weight of the target caused, for example,

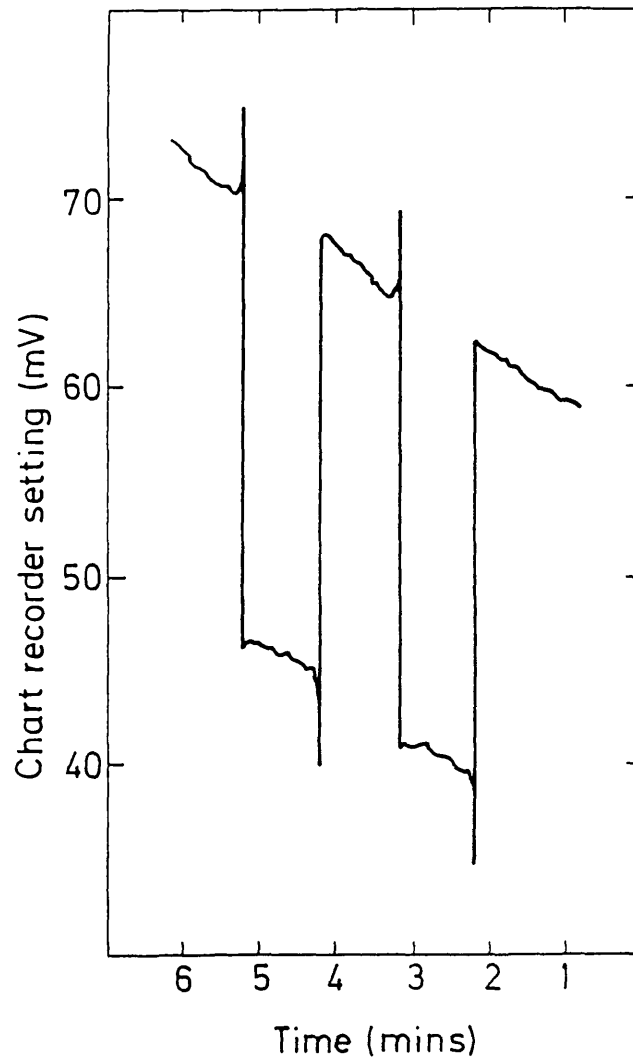


Fig. 11: Operation of the System - Typical Chart Recorder Trace.

(transducer switched on and off twice)

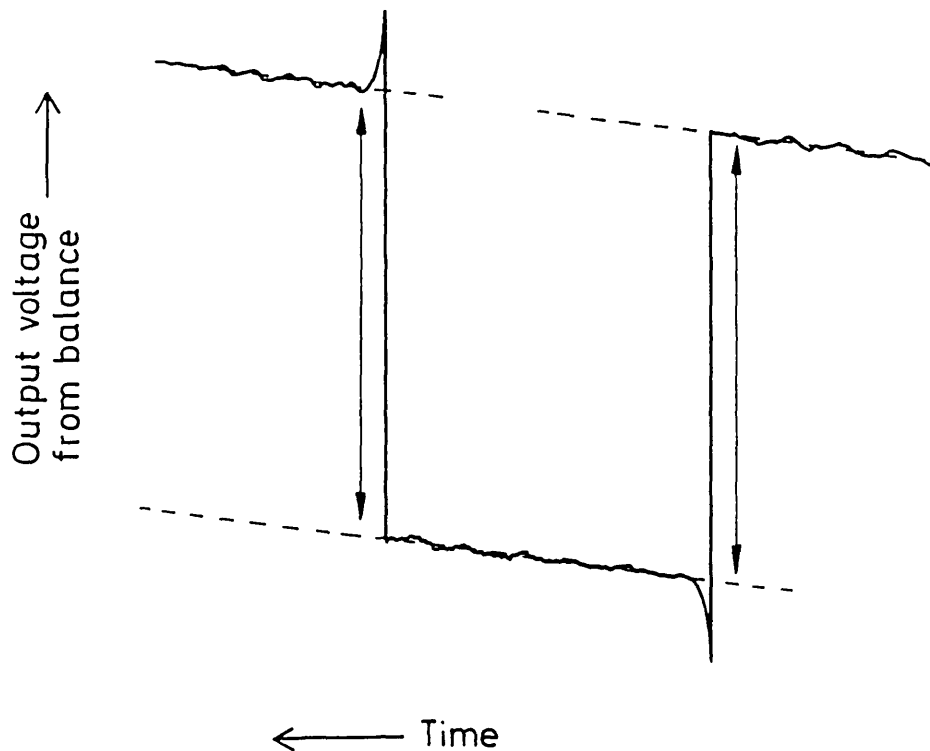


Fig. 12: Operation of the System - Determination of the Change in Target Weight from the Chart Recorder Trace.

by sudden changes in the surface tension force could be identified, and, if necessary, not be included in the final result. The method of operation of the system for the measurements described in this thesis therefore consists of switching the transducer on and off two or three times to obtain a value for the change in target weight corresponding to a particular ultrasonic power.

Frequency = 1 MHz.

<u>Chart Recorder Deflection (mV)</u>		<u>Running Mean</u>
<u>on</u>	<u>off</u>	
3.265	3.291	3.278
3.262	3.287	3.276
3.314	3.293	3.285
3.288	3.273	3.284
3.232	3.237	3.274
3.298	3.298	3.278
3.290	3.276	3.279
3.229	3.238	3.273

Mean on deflection = 3.272 ± 0.011 mV.
Mean off deflection = 3.274 ± 0.009 mV.

Frequency = 5 MHz.

<u>Chart Recorder Deflection (mV)</u>		<u>Running Mean</u>
<u>on</u>	<u>off</u>	
3.131	3.051	3.091
2.967	2.966	3.029
3.058	2.967	3.023
2.837	3.021	3.000
2.885	2.975	2.986
3.004	2.905	2.981
2.857	3.012	2.974
2.977	3.046	2.979

Mean on deflection = 2.965 ± 0.036 mV.
Mean off deflection = 2.993 ± 0.017 mV.

Frequency = 10 MHz.

<u>Chart Recorder Deflection (mV)</u>		<u>Running Mean</u>
<u>on</u>	<u>off</u>	
1.777	1.800	1.789
1.801	1.713	1.773
1.714	1.732	1.756
1.673	1.776	1.748
1.695	1.684	1.737
1.646	1.748	1.730
1.679	1.694	1.724
1.831	1.763	1.733

Mean on deflection = 1.727 ± 0.024 mV.
Mean off deflection = 1.739 ± 0.014 mV.

Table 9: Transducer On and Off Values.

4.7 Automatic System

In the last section, the method used to determine the change in the apparent weight of the target for most of the measurements in this thesis was described. This technique involves the extrapolation of the chart recorder trace by eye, and is consequently both time-consuming and operator dependent. Particularly at low power levels, where there is significant noise, or at high powers, where the target oscillates, this method is not accurate or easy to perform. To overcome these problems, the measurement system was automated using a microcomputer and a programmable function generator.

In this new system, the voltage (current) required to keep the balance arm fixed is directly recorded using an Amplicon Model 54 digital panel meter, while the temperature of the water bath is monitored with a platinum resistance thermometer connected to an Analogic Model AN2576 digital panel meter. Both these panel meters have a BCD output which is used to feed information into a PET microcomputer via an instrument coupler (figure 13). Also used are a cassette recorder to store the microcomputer programs and a printer to provide an output of the results.

A program in Basic was written for the PET to accept and analyse the data received from the voltage and the temperature monitors. The PET receives this data in the form of a string variable containing nine characters: the first five being the voltage reading, the next three the temperature, and the last the polarity of the voltage. One problem occurred with the timing of the signals as the read-ready signal from the Amplicon meter is activated approximately four times per second and the data transferred to the instrument coupler at this rate. The PET, however, has a built-in 65 ms time-out period and, if data is not

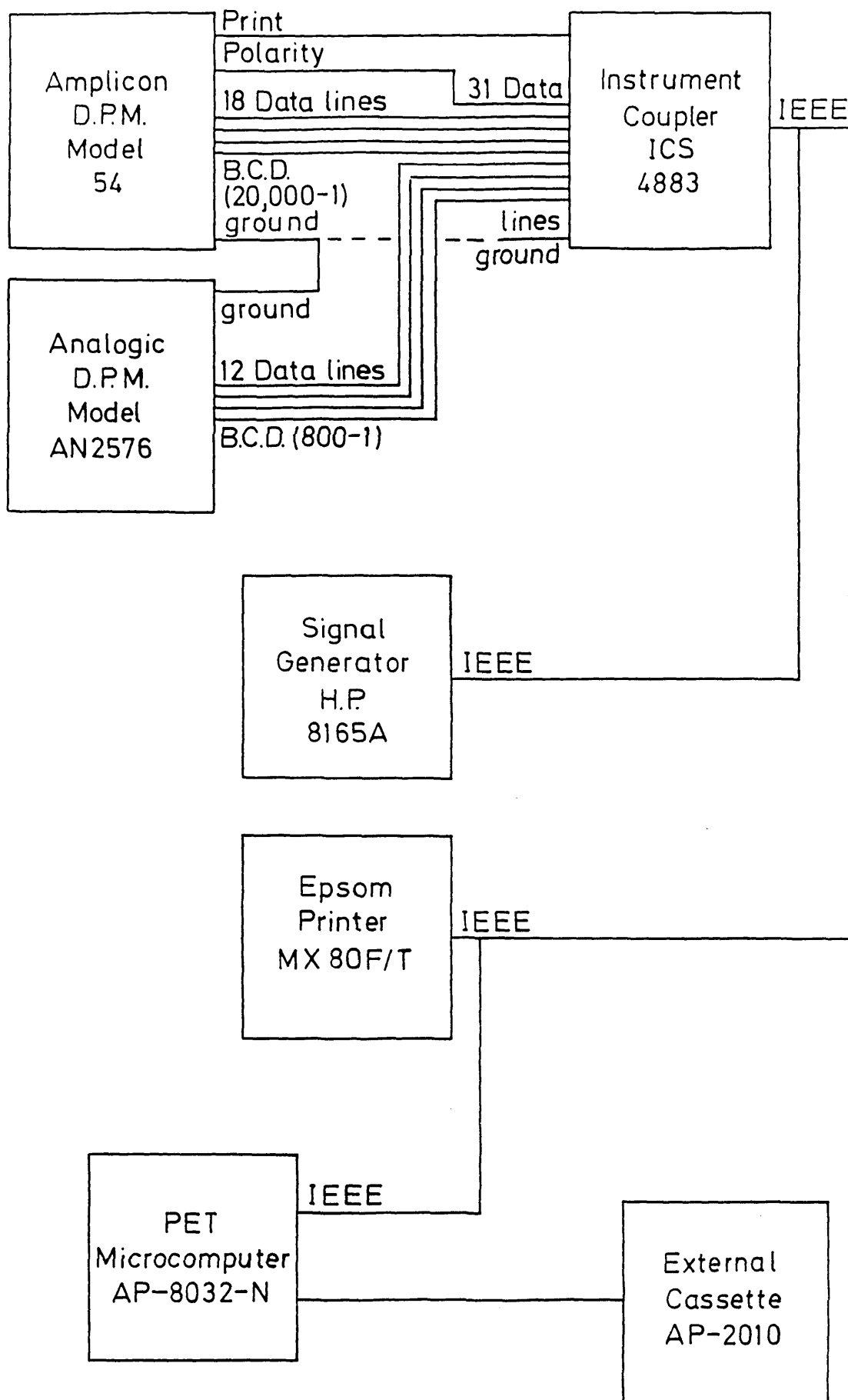


Fig. 13: Schematic Diagram of the Automatic System.

received within this time, the PET registers a time-out status. The program in the PET therefore checks the status of the PET during the input of data and, if a time-out occurs, returns the PET to the input routine.

The analysis of the data consists of treating the mean of twenty voltage readings (v) as one element (1,1) of a 10x2 element array. The mean time (t) of this particular voltage measurement is treated as the other element (1,2) of the row in the array. The procedure is then repeated by taking the mean of each subsequent set of twenty voltage readings until the array is complete. A straight line $v = at + b$ is then fitted through each set of voltage and time points (v,t) in the array. This fit is tested by examining the uncertainties on the values of the gradient a and the intercept b , and, if these errors are not acceptable, a further five sets of elements (v,t) are added to the array, and the fitting and the stability check repeated. This program can be used either during the calibration of the balance, where a calibrated weight is added to the balance pan, or during the output power measurements, where an electrical signal is applied to the transducer.

To calculate the change in the apparent weight of the target produced by the calibration or the power measurement, the straight line $v = at + b$ is extrapolated back to the instant in time at which the weight or the electrical signal was applied. The difference between this extrapolated value and the value at the instant before the change occurred is used as a measure of the weight change.

The temperature data is analysed by computing the mean of all the values of the temperature recorded during the measurements. Values for the velocity of sound in water and the amplitude

attenuation coefficient of water corresponding to this particular temperature are then selected.

The Basic program for the PET allows various types of measurements to be made automatically. The first of these is the calibration of the balance where the program determines the deflection produced by the applied calibrated weight and gives the option of applying the weight a number of times to obtain a mean for the calibration figure. Once the balance has been calibrated, the next section of the program provides output power measurements and offers three options:

- (i) For the investigation of the stability of a transducer - measurements at a fixed voltage and frequency at different times,
- (ii) For the investigation of the frequency response of a transducer - measurements at a fixed voltage but variable frequency,
- (iii) For the investigation of the linearity of a transducer - measurements at a fixed frequency but variable voltage.

These options use a Hewlett Packard 8165A programmable function generator which is directly connected to the PET and allows electrical signals to be automatically applied to a transducer. Depending on which option has been selected, the PET program sends the relevant information to the function generator and sets up the necessary drive configuration. Under program control, the drive to the transducer is switched on and off, and the change in the apparent weight of the target determined. The program uses equation (II.6.17) together with the appropriate values of the velocity of sound and the attenuation coefficient of water, to calculate the corresponding output power from the transducer. The complete radiation pressure measurement system including the PET

microcomputer and programmable function generator are shown in photograph (4).

5. PERFORMANCE OF THE BALANCE: SYSTEMATIC ERRORS

5.1 Linearity

Clearly, an important consideration is the linearity of the radiation pressure measurement system. In most cases, it can be assumed that the output power from a transducer is proportional to the square of the applied electrical voltage (Gooberman 1960, Heuter and Bolt 1955), and it was decided to investigate both this assumption and the linearity of the balance with a series of measurements.

Three transducers of different frequencies were mounted in the balance and their output power measured as a function of the drive voltage. This voltage applied to the transducer was measured using a high impedance probe connected to an oscilloscope. Figure (14) shows the measured total power plotted as a function of the drive voltage for the three different frequencies. It can be seen that, within the measurement uncertainties (approximately $\pm 1\%$), there is no evidence of any deviation from a linear dependence between the power and the square of the voltage of the electrical signal applied to the transducer. This suggests that the power measurement system is linear for the range of powers used: between 1 mW and 1 W. It is perhaps conceivable that a particular transducer and the radiation pressure balance could have nonlinearities which are equal and opposite and cancel each other out producing a linear result. But the three plots obtained were for three transducers from different manufacturers, one (the 2 MHz) consisting of a quartz crystal, the others ceramic. It is



Photograph 4: The Complete Radiation Pressure Measurement System
Including the PET Microcomputer and the Programmable
Function Generator.

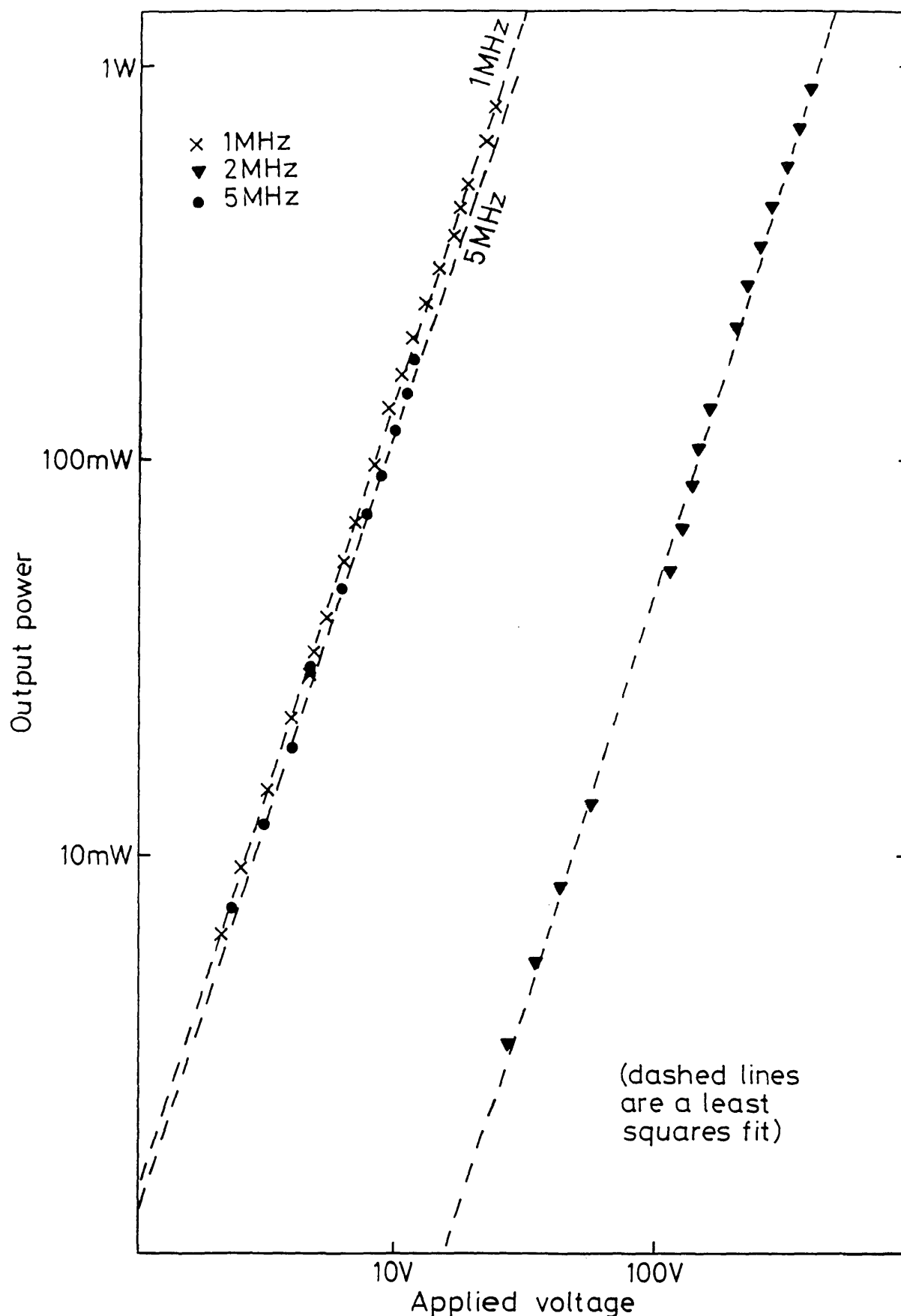


Fig. 14: The Measured Total Output Power Plotted Against the Applied Drive Voltage for Three Different Frequencies.

extremely unlikely that these transducers would have the same degree of nonlinearity between the output power and the input drive voltage.

5.2 Force on the Target

It was mentioned in the section on target design (section 3.2) that stability is a problem with a conical target. This type of target is not self-centering and any small disturbance will tend to knock the target off the centre of the ultrasonic beam. The value of the ultrasonic power at which the system will become unstable can be estimated as is shown below.

If the conical target is displaced a distance d from the centre of the ultrasonic beam, the restoring force W due to the weight of the target can be calculated as shown in figure (15). Equating forces in the vertical and horizontal directions

$$mg = T \cos \theta,$$

$$W = T \sin \theta = mg \tan \theta,$$

where T is the tension in the suspension wire, θ is the angular displacement of the target from the vertical and mg is the weight of the target.

Now

$$\tan \theta = d/l,$$

where l is the distance between the target and the suspension point on the balance arm.

Hence

$$W = mgd/l.$$

The target has a weight in water of approximately 500 mg and l is

approximately 300 mm, giving

$$W = 5d/3 \text{ mg/mm.}$$

With the target displaced a distance d from the centre of the ultrasonic beam, the restoring force W due to the weight of the target is opposed by a net sideways radiation pressure F_A acting away from the centre of the ultrasonic beam. If F is the total force from the ultrasound and r the radius of the ultrasonic beam, then every element of area dA on the surface of the target will experience a force $FdA/(\pi r^2)$ acting horizontally towards the centre of the target. To calculate the value of the total outwards force F_A for the target displaced a distance d from the ultrasonic beam centre, it is necessary to determine the components of the force in the direction of d (figure 16). Each element of area contributes a force $FdA\cos\phi/(\pi r^2)$ along the direction of the displacement d . To perform the summation of these forces it is simplest to divide the ultrasonic beam into two segments (figure 17). The elemental area dA is equal to the segmental area $1/2 f^2 d\phi$, and the total horizontal force F_H due to the ultrasound over each segment is therefore

$$\begin{aligned} F_H &= \int_A F\cos\phi/(\pi r^2) dA, \\ &= \int_{-\pi/2}^{\pi/2} 1/2 f^2 F\cos\phi/(\pi r^2) d\phi, \\ &= F/(\pi r^2) \int_0^{\pi/2} f^2 \cos\phi d\phi. \end{aligned}$$

For the smaller segment

$$\begin{aligned} r^2 &= f^2 \sin^2\phi + (d + f\cos\phi)^2, \\ &= f^2 + 2df\cos\phi + d^2, \end{aligned}$$

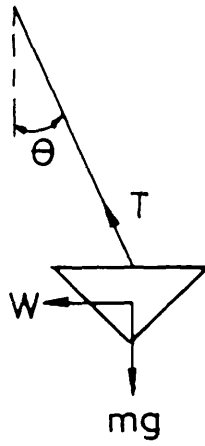


Fig. 15: Forces on the Target.

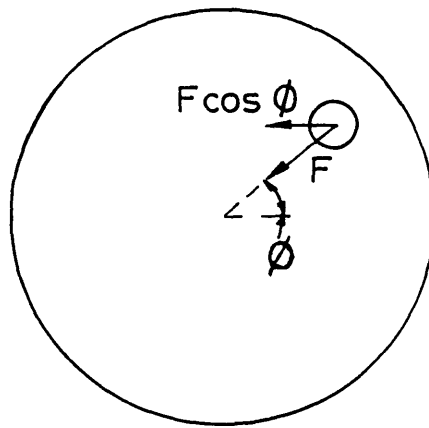


Fig. 16: The Components of the Force on the Target in the Direction of the Displacement d .

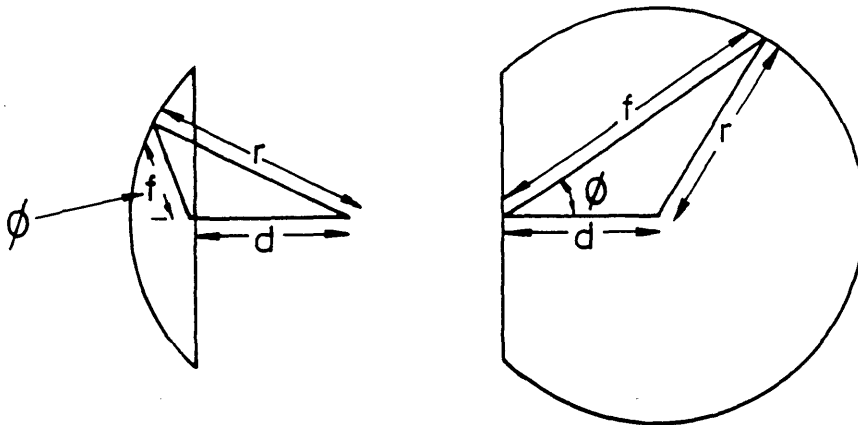


Fig. 17: The Summation Over the Two Segmental Areas.

which gives

$$f^2 = d^2 \cos^2 \phi - 2d \cos \phi (r^2 - d^2 \sin^2 \phi)^{1/2} + r^2 - d^2 \sin^2 \phi,$$

and therefore the total horizontal force F_1 on the smaller segment is

$$F_1 = F/(\pi r^2) \int_0^{\pi/2} [d^2 \cos^3 \phi - 2d \cos^2 \phi (r^2 - d^2 \sin^2 \phi)^{1/2} + r^2 \cos \phi - d^2 \sin^2 \phi \cos \phi] d\phi.$$

Using the notation $k = d/r$ gives

$$\begin{aligned} F_1 &= F/(\pi) \int_0^{\pi/2} (k^2 \cos^3 \phi - 2k \cos^2 \phi (1 - k^2 \sin^2 \phi)^{1/2} + \cos \phi - k^2 \cos \phi \sin^2 \phi) d\phi, \\ &= F/(\pi) [(k^2/3) \sin \phi (2 + \cos^2 \phi) + \sin \phi - k^2 \sin^3 \phi / 3] \\ &\quad - F/(\pi) \int_0^{\pi/2} 2k \cos^2 \phi (1 - k^2 \sin^2 \phi)^{1/2} d\phi, \\ &= F/(\pi) \{1 + k^2/3 - 2/(3k) [(1+k^2)E(k) - (1-k^2)K(k)]\}, \end{aligned}$$

where

$$E(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \phi)^{1/2} d\phi,$$

and

$$K(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \phi)^{-1/2} d\phi,$$

are elliptical integrals.

For the larger segment

$$r^2 = f^2 \sin^2 \phi + (d - f \cos \phi)^2,$$

and a similar procedure gives the total horizontal force F_2 on the larger segment as

$$F_2 = F/(\pi) \{1 + k^2/3 + 2/(3k) [(1+k^2)E(k) - (1-k^2)K(k)]\}.$$

The net horizontal force F_A on the target due to the ultrasound is therefore

$$F_A = F_2 - F_1 = 4F/(3\pi k)[(1+k^2)E(k) - (1-k^2)K(k)].$$

This is the net sideways radiation pressure tending to push the conical target away from the centre of the ultrasonic beam when the target is displaced a distance d from the centre. In figure (18), the relation between the net sideways radiation pressure F_A and the displacement d of the target is plotted for different values of both the total force F from the ultrasound and the beam radius r . Also indicated is a line marked W which corresponds to the restoring force due to the weight of the target. Above the line W , the target is unstable. If any small disturbance knocks the target off the centre of the ultrasonic beam, the net force on the target will tend to push the target further away from the ultrasonic beam centre. Below the line W , the target is stable with the net force on the target acting towards the centre of the ultrasonic beam.

It may be expected that at higher powers the target will never become stable, but will be pushed further and further away from the centre of the ultrasonic beam. However, the target is only 60 mm in diameter, and eventually some of the ultrasound will miss the target completely. A situation will occur where the opposition force due to the fraction of the ultrasound striking the target will be balanced by the mass of the target, and the change in the target weight recorded by the balance will no longer be directly related to the total ultrasonic power.

Throughout all the analysis in this section, it has been assumed that the target does not tilt when the ultrasound is

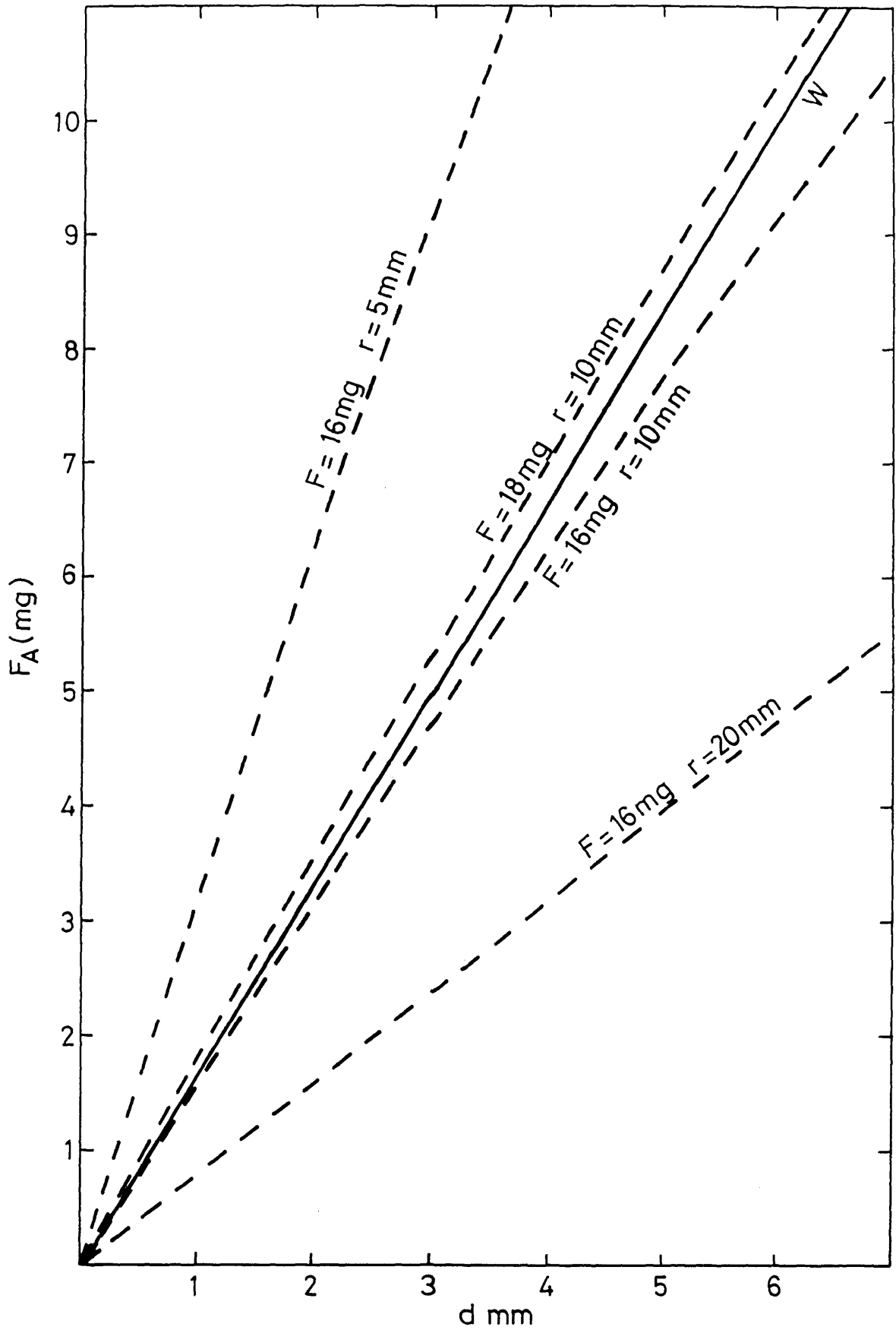


Fig. 18: Force on the Target - The Relation Between the Net Sideways Radiation Pressure F_A and the Displacement d of the Target Plotted for Different Values of the Total Force F from the Ultrasound and the Beam Radius r .

(The line marked W corresponds to the restoring force due to the weight of the target.)

incident on the side of the cone. Clearly this assumption needs to be investigated, but first the centres of buoyancy and mass of the target must be determined.

The centre of buoyancy of any object in water is the centre of mass of the displaced volume of water, and, for the target, this can be calculated by considering the individual centres of mass of the conical and the hemispherical dome volumes. The right-angled cone has an external volume of $\pi r^3/3$, where r is the cone radius, and the centre of mass of the solid conical volume is $r/4$ from the flat top of the cone. For the glass dome, which was made by slicing a segment from a glass sphere, the external volume is equal to

$$2\pi r_1^3 - 4\pi r_1 r_2^2/3 + (\pi/3)(4r_1^2 - r_2^2)(r_1^2 - r_2^2)^{1/2},$$

where r_1 is the radius of the glass sphere and r_2 is the radius of the base of the dome.

Using measured values of r_1 and r_2 , the centre of mass of the dome volume can be calculated to be at 24.8 mm from the base of the dome. A value for the centre of mass of a volume of water equivalent to the volume of the complete target is obtained by taking moments for the conical and dome volumes about a point above the join between the cone and the dome. This gives that the centre of buoyancy of the target is 20.2 mm above the join between the cone and the dome, or equivalently 50.2 mm from the tip of the cone.

For the centre of mass of the target itself, the calculations are more complicated, but the centre of gravity of the hollow cone is found to be at 7.5 mm from the flat top of the cone, and similarly for the hollow dome the centre of gravity is 29 mm from its base. Taking moments as before about a point above the base of

the dome gives that the centre of mass of the target is 0.5 mm above the base of the dome, or equivalently at 30.5 mm from the tip of the cone.

These results demonstrate that the centre of mass of the target is 19.7 mm below its centre of buoyancy, and hence the target sits in stable equilibrium in the water. It is now possible to determine the angle of tilt produced by a force on the side of the target. Consider a force F acting at a distance d from the centre of the target, and assume this causes the target to tilt to an angle θ with the vertical. Taking moments about the centre of buoyancy gives

$$F(d + 50.2\sin\theta) = W19.7\sin\theta,$$

where W is the weight of the target in air, and d is the separation in mm between the point of action of the force F and the centre of the target.

Consider an extreme case and let F be equal to the force from a 500 mW beam of ultrasound (3.4×10^{-4} N), and consider this force to be acting at 30 mm from the tip of the cone ($d = 30$). The weight of the target in air is approximately 210 g, and hence under the action of the force F the target tilts through an angle θ of 0.014° . There is therefore negligible tilting of the target even when the ultrasound is acting on the edge of the cone, and it can be assumed that the target remains vertical throughout the measurements. This is a great advantage and is due to the large separation between the centres of mass and buoyancy which makes the target very stable. Hence, for a vertical ultrasonic beam, it can always be assumed that the ultrasound is incident at 45° on the surface of the target, whether or not the target is at the centre of the ultrasonic beam. This means that the target gives a

reliable measure of the radiation pressure from the ultrasound for larger values of the total ultrasonic power than previously determined from considerations of the forces on the target. The radiation pressure balance can be used for all power measurements unless a fraction of the ultrasound misses the edge of the target.

The previous results can be used to calculate the values of the total power and the transducer radius at which the target may be pushed out to such a distance that some of the ultrasound misses the target. This gives the relation

$$F_{\max} = 5\pi(30 - r)/8,$$

where r is the radius of the ultrasonic beam in mm, and $F_{\max}$ is the force (in mg) for which the target may be pushed out to such a distance that the ultrasonic beam will hit the edge of the target. Hence for a beam of 5 mm radius, $F_{\max}$ is equal to 49 mg, equivalent to a total ultrasonic power of approximately 710 mW. The relationship between $F_{\max}$ and r is plotted in figure (19).

5.3 Surface Tension

If the target moves, the suspension wire will move through the water surface and the surface tension force may change. A crude indication of the magnitude of this force can be obtained by multiplying the circumference of the suspension wire (approximately 0.032 cm) by the surface tension coefficient of water (73×10^{-5} N/cm), and this gives a force of 2.3×10^{-5} N. This is equivalent to 2.4 mg weight, the force exerted on the reflecting target by an ultrasonic beam of approximately 35 mW total power. Hence, the magnitude of the surface tension force is comparable with the radiation pressure from the ultrasound. This demonstrates that the effects of surface tension should be made as

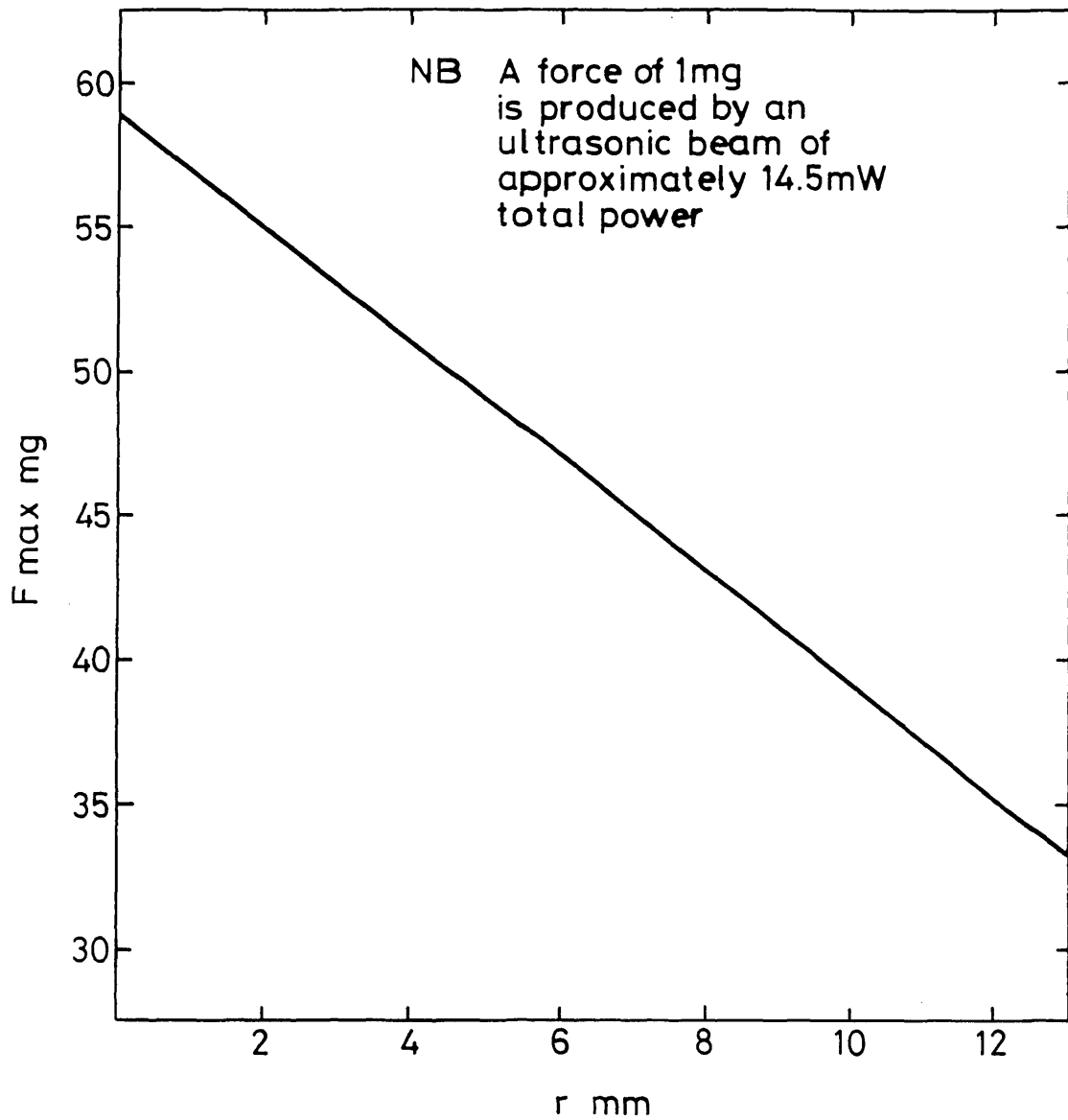


Fig. 19: Force on the Target - The Relation Between the Beam Radius r and the Force $F_{\max}$ at which the Target will be pushed out to such a distance that the ultrasonic beam will hit the edge of the target.

small as possible, or alternatively the surface tension force should remain constant throughout the measurements.

A servocontrolled microbalance does not keep its balance arm, and hence the suspension wire, rigidly fixed, typical movements being of the order of μm . An estimation of the magnitude of any error caused by changes in the surface tension can be obtained by keeping the measurement system and procedure constant, but altering the level of the water surface. Changing this level should transfer the water meniscus to a different point on the suspension wire, where either the radius or the angle of contact may be different, leading to a change in the surface tension force. Figure (20) shows the results of measurements of the output power from a transducer with different levels of water in the tank. It can be seen that, within the measurement uncertainties, an alteration in the water level has no effect on the measured total power. It is therefore reasonable to assume, although this test is not conclusive, that surface tension problems do not affect the results of a power measurement.

Other possible changes in the surface tension force can be caused by changes in environmental factors, such as dust on the water surface etc, but these effects are only relevant in an absolute measurement system. The radiation pressure balance is used to determine changes in apparent weight, and no problems will occur unless the surface tension force changes during a measurement.

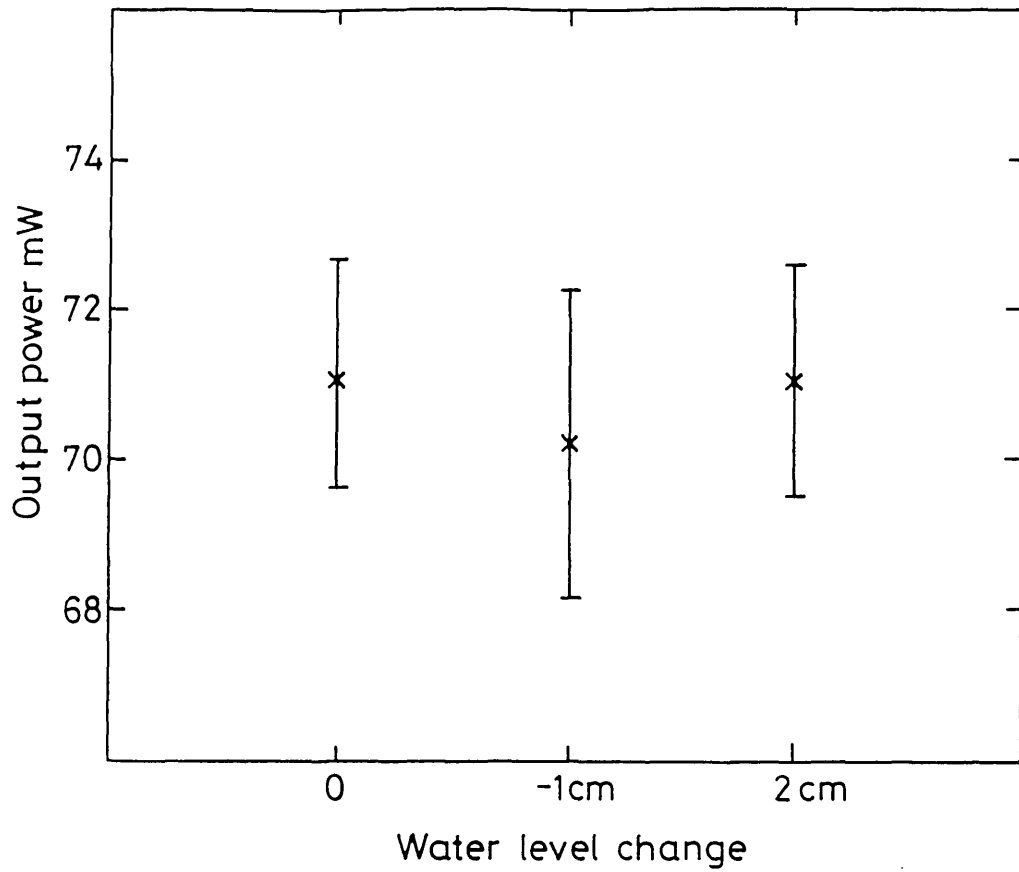


Fig. 20: Variation of the Measured Output Power from a Transducer with the Level of the Water in the Balance Tank.

(The error bars denote the standard error on the mean of the measured values.)

5.4 Variation of the Measured Power with Transducer/Target Separation

The radiation balance is intended to be used to determine the output power from an ultrasonic transducer. Clearly a major requirement of the system is that the value of output power obtained for a particular transducer should be independent of the separation between the transducer and the target.

As mentioned earlier in section (II.6.2), the power W in an ultrasonic beam varies with the distance x from the transducer as

$$W = W_0 \exp(-2\alpha x), \quad \dots(5.1)$$

where α is the amplitude attenuation coefficient and W_0 is the output power from the transducer.

Hence, the values of the power determined with the radiation pressure balance should decrease exponentially with increased separation between the transducer and the target. Alternatively, taking logarithms of equation (5.1) gives

$$\ln W = \ln W_0 - 2\alpha x,$$

and hence a plot of $\ln W$ against x should produce a straight line of intercept $\ln W_0$ and gradient 2α .

To determine if the power measured with the radiation pressure balance decreases exponentially with distance, measurements of the power were made over a range of transducer/target separations. In all cases, the separation between the transducer and the target was taken to be the distance between the front surface of the transducer and the tip of the cone of the target. Transducers of four different frequencies 1, 5, 10 and 15 MHz were used, and the values of the power determined with the radiation pressure balance are plotted on a logarithmic scale against the distance from the transducer in figures (21) to (24). The results obtained from a least squares fit through the points at each frequency are tabulated overleaf. The uncertainty figures are the standard error on the mean.

<u>Frequency</u> MHz	$\frac{W_o}{mW}$	$\frac{2\alpha}{mm^{-1}}$
1	47.92 $\pm$ 0.28	- 0.00022 $\pm$ 0.00011
5	45.00 $\pm$ 0.40	0.00135 $\pm$ 0.00015
10	29.90 $\pm$ 0.30	0.00342 $\pm$ 0.00020
15	42.99 $\pm$ 0.53	0.00312 $\pm$ 0.00024

Pinkerton (1947) gives a value for the attenuation coefficient in water at 20°C over this range of frequencies of $2.53 \times 10^{-17} \times f^2 \text{ mm}^{-1}$, where f is the frequency. The values of the attenuation coefficient determined by Pinkerton (which we will call α_p) can therefore be compared with the experimental values α_E required to correct the values of power measured with the radiation pressure balance.

<u>Frequency</u>	α_p	α_E
1	0.000025	- 0.00011 $\pm$ 0.00006
5	0.00063	0.00067 $\pm$ 0.00008
10	0.0025	0.0017 $\pm$ 0.0001
15	0.0057	0.0016 $\pm$ 0.0001

It is clear that at 5 MHz and below, within the experimental errors, it is reasonable to use the amplitude attenuation coefficient α_p determined by Pinkerton to correct the values for the total power measured with the radiation pressure balance. Above 5 MHz, however, the attenuation coefficient α_E required is significantly lower than α_p (32% $\pm$ 4% at 10 MHz and 72% $\pm$ 2% at 15 MHz). There are two possible explanations for this difference: either the attenuation coefficient determined by Pinkerton is not applicable to this particular experimental situation, or an additional frequency dependent effect is occurring which produces an increase with distance in the measured value of the power.

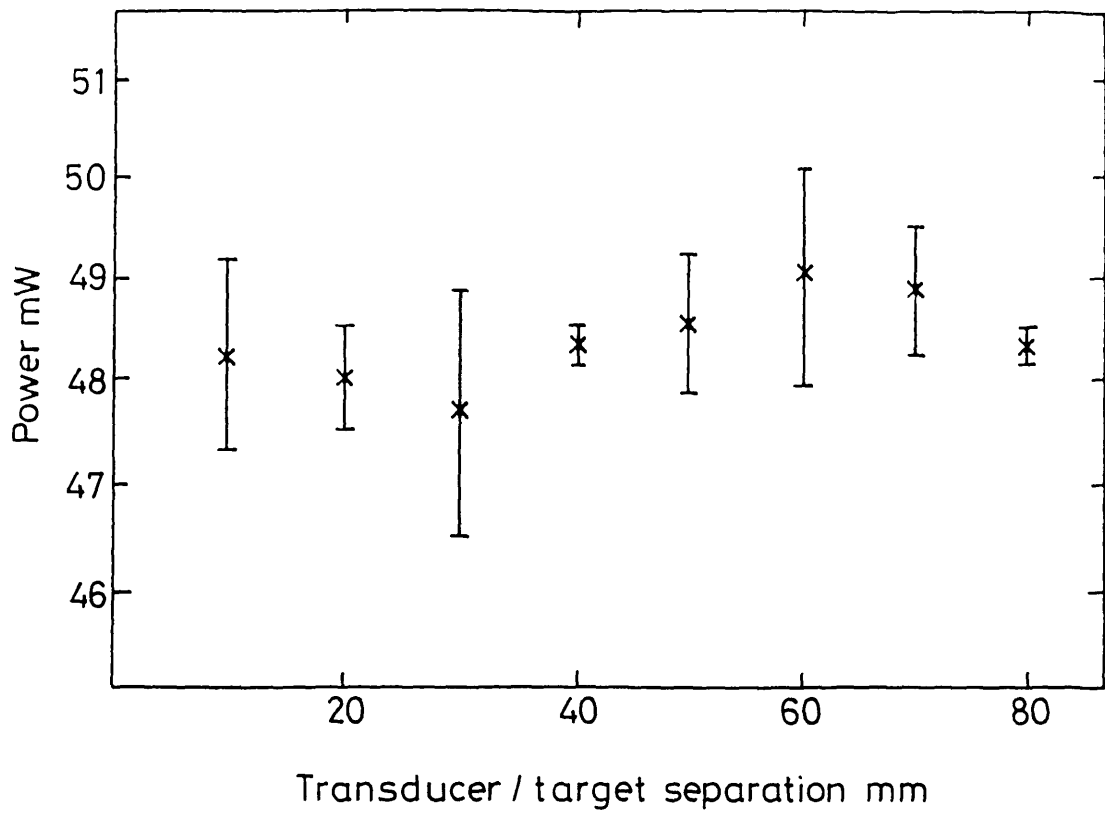


Fig. 21: The Power Determined with the Radiation Pressure Balance
Plotted Against the Distance from a 1 MHz Transducer.

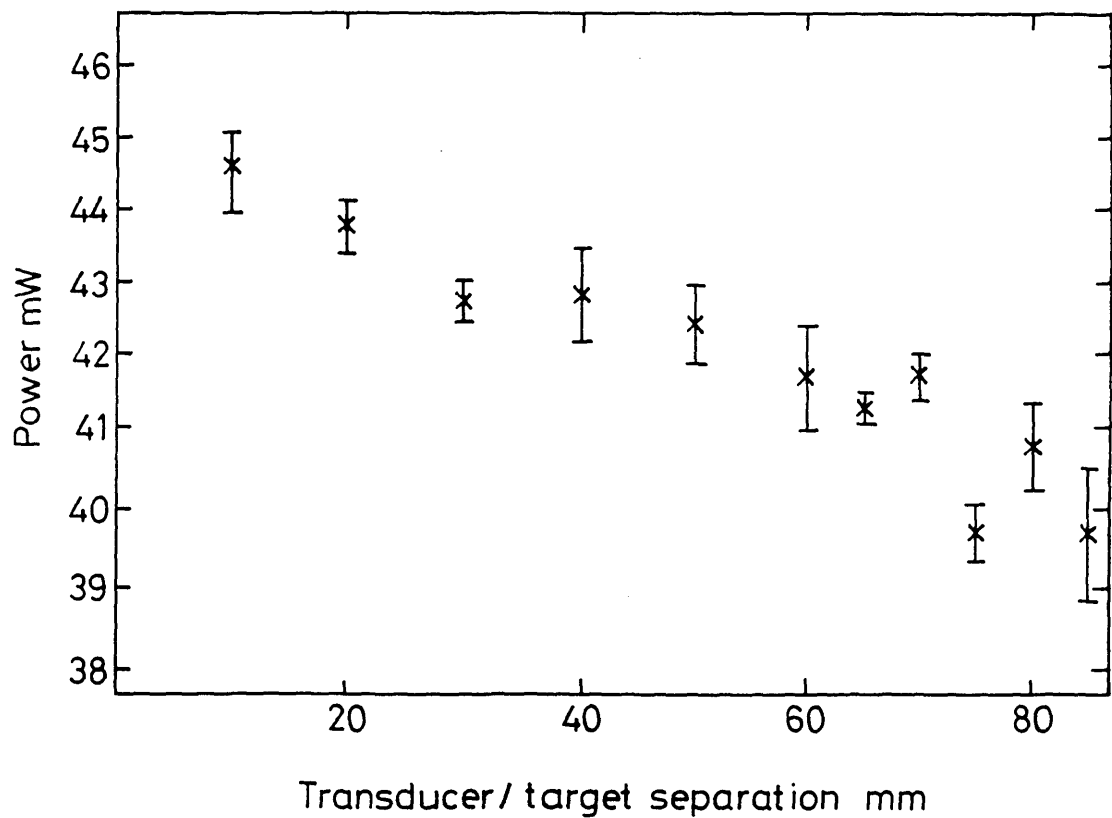


Fig. 22: The Power Determined with the Radiation Pressure Balance
Plotted Against the Distance from a 5 MHz Transducer.

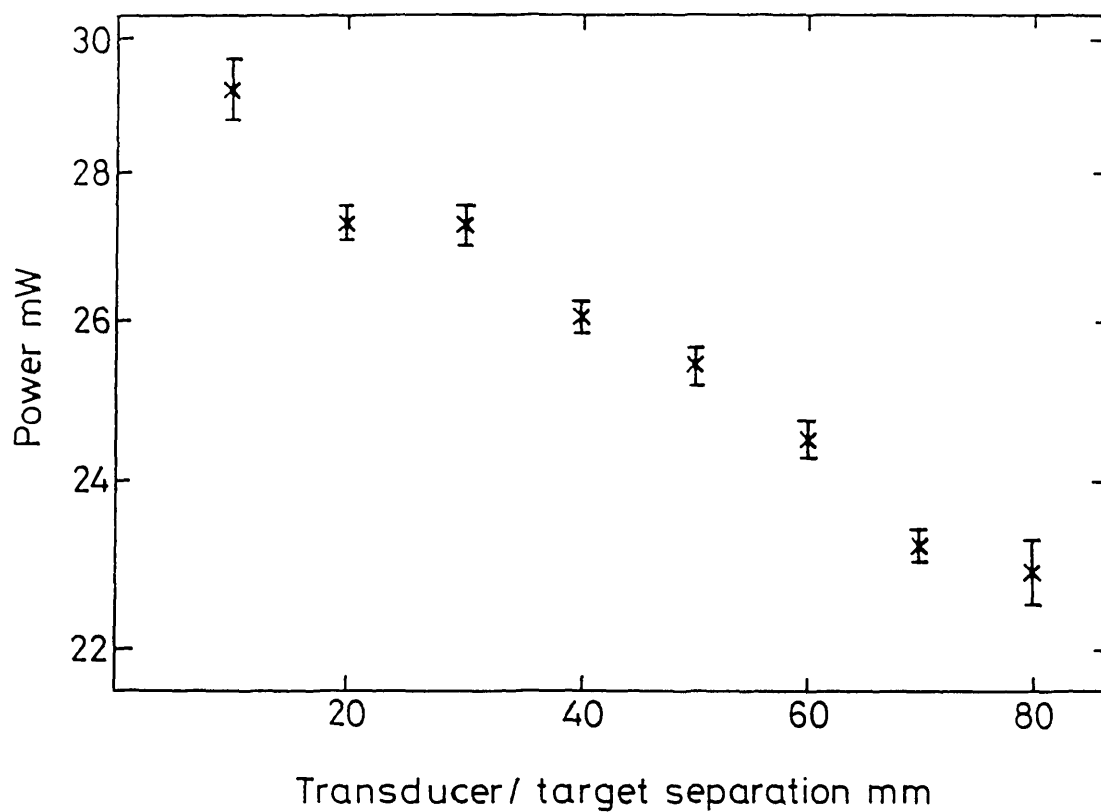


Fig. 23: The Power Determined with the Radiation Pressure Balance
Plotted Against the Distance from a 10 MHz Transducer.

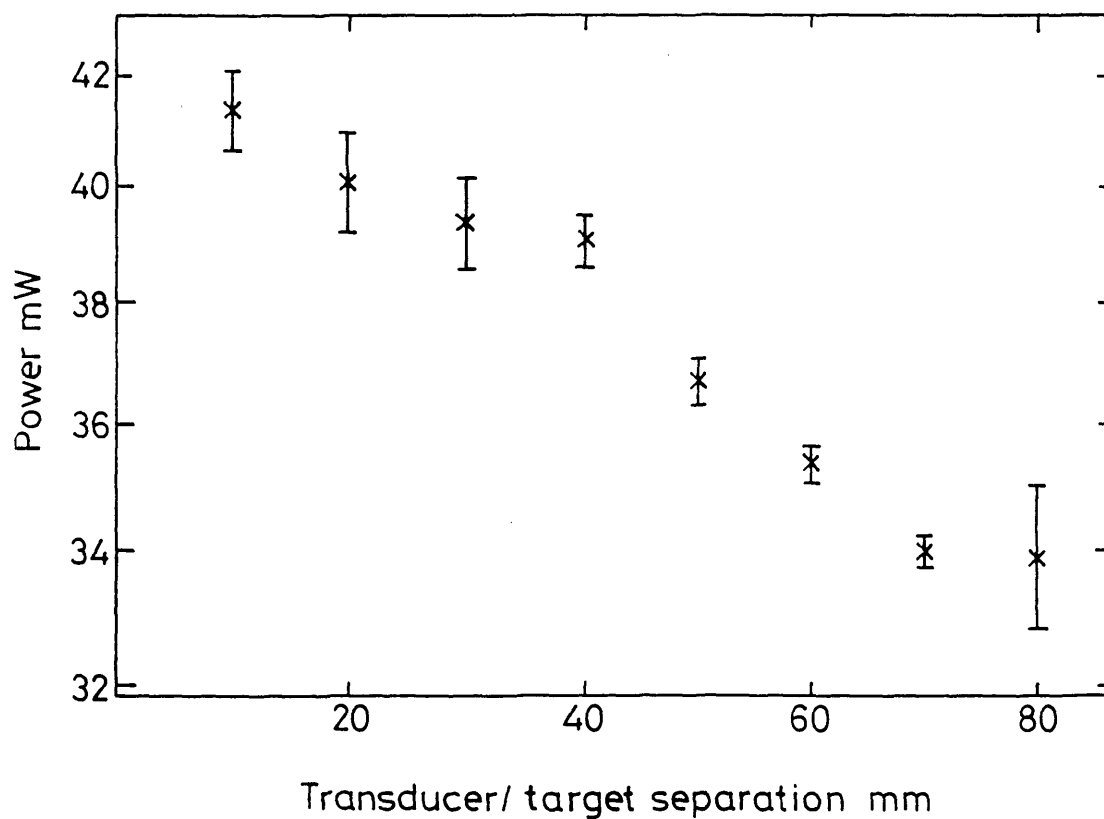


Fig. 24: The Power Determined with the Radiation Pressure Balance
Plotted Against the Distance from a 15 MHz Transducer.

The attenuation coefficient α_E required in the radiation pressure balance is not directly related to frequency, and appears to decrease with frequency above 10 MHz. It is difficult to conceive that the attenuation of the ultrasonic beam could reduce as the frequency increases, and no liquids have been found to show this effect (Herzfeld and Litovitz 1959). In fact, the standard model for attenuation in water (Herzfeld and Litovitz 1959) predicts an attenuation coefficient which increases as the square of the frequency over the ultrasonic frequency range. Hence an additional frequency dependent effect appears to be taking place in the radiation pressure balance, and this produces the observed increase in the measured power with distance. There are two mechanisms that could produce a frequency dependent effect and these will now be discussed.

5.5 Streaming and Convection Currents

Streaming in the water bath of the radiation pressure balance may be expected to occur if there is absorption of the ultrasonic wave; momentum is necessarily conveyed to the medium as it is lost from the travelling ultrasonic wave. In several radiation pressure balances (Carson et.al. 1978, Rooney 1973), a thin membrane is placed close to the target. This is intended to stop the bulk movement of the fluid and therefore reduce any effect of streaming on the target.

Convection currents may also occur in the water above the ultrasonic transducer. These arise from two distinct sources. Firstly, most transducers are inefficient and a large proportion of the electrical power applied to the transducer is transformed into heat, and this may be transferred into the water bath and set up convection currents. Secondly, convection currents may result

from the transformation of any absorbed ultrasonic energy into heat and the consequent temperature rises and changes in density of the water. If the currents from either of these sources reach the target, the associated temperature and density changes would cause the apparent weight of the target to change. This effect from convection currents will tend to occur only when the transducer is excited, giving a larger change in target weight than that caused by the ultrasonic power alone. The effects of both streaming and convection will increase as the frequency of the ultrasonic wave increases.

A membrane set low in the water bath, close to the ultrasonic transducer should reduce any convection currents from the transducer itself. This is unlikely to affect any convection from absorption in the bulk of the fluid or any streaming that may be present. Any effects on the target from both streaming and convection currents should be reduced, however, by a high membrane close to the target.

A series of experiments were performed using the transducers of 10 and 15 MHz as in the previous measurements. A membrane was placed in the water bath between the transducer and the target, and the separation between the membrane and the target varied. The membrane consisted of a thin polythene sheet approximately 50 μm thick stretched across a ring (approximately 100 mm in diameter), and angled with the normal to the membrane surface at 30° to the ultrasonic beam. An angled membrane was chosen so standing waves were not set up between the transducer and the front surface of the membrane.

Before starting the set of experiments, the intensity transmission coefficient of the membrane was determined at the two measurement frequencies using a hydrophone. At each frequency,

measurements were made with the transducer driven in tone-burst mode. The hydrophone was placed in the ultrasonic field at a certain distance away from the transducer and the peak-to-peak signal received at the hydrophone was determined. The membrane was then inserted between the transducer and the hydrophone, and the decrease in the signal received by the hydrophone noted. The ratio of the received signal with the membrane to that without was calculated, and used to give a value for the intensity transmission coefficient of the membrane as tabulated below.

<u>Frequency (MHz)</u>	<u>Intensity Transmission Coefficient</u>
10	0.639 ± 0.007
15	0.430 ± 0.006

(In all cases the error denotes the standard error of the mean of the measured values).

All the values of the ultrasonic power determined with the membrane above the transducer were divided by the intensity transmission coefficient to take account of reflections from the membrane.

Measurements were first performed using the 10 MHz transducer. The membrane was positioned at approximately 26 mm above the front surface of the transducer, and the power in the ultrasonic beam was determined for various values of the transducer/target separation. The membrane was then moved to 46 mm and the ultrasonic power was again determined with the target at several distances from the transducer. Finally the membrane was moved to 66 mm above the transducer and the power determined with the target at both 70 and 80 mm. The ultrasonic power determined from this set of measurements is plotted on a logarithmic scale against the transducer/target separation in figure (25). Also included are

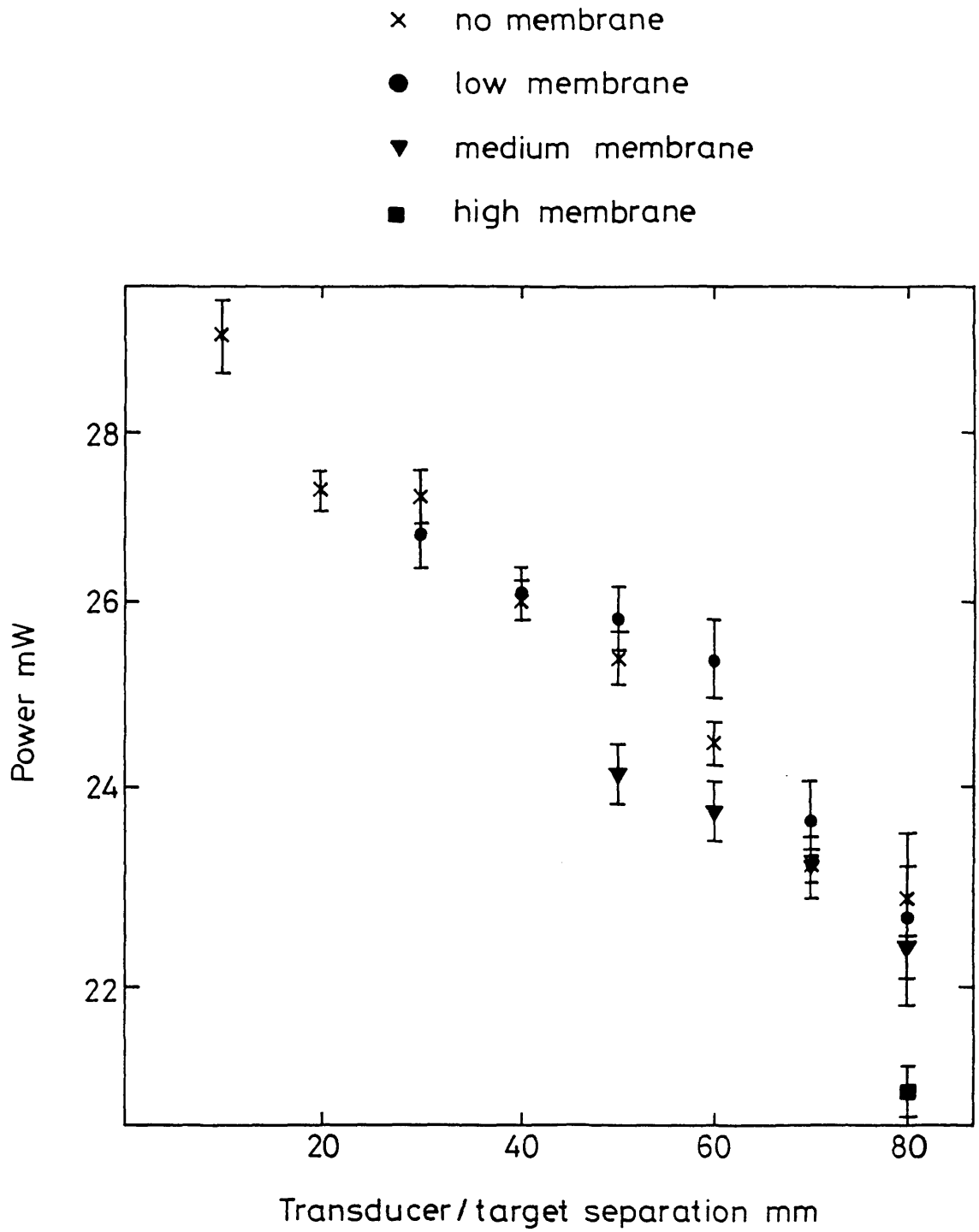


Fig. 25: The Power Determined with the Radiation Pressure Balance Plotted Against the Distance from a 10 MHz transducer with, in some cases, a Membrane Between the Transducer and the Target.

the results obtained from the previous measurements without the membrane. These measurements were repeated using the transducer of 15 MHz frequency, and the results obtained are plotted in figure (26). At both frequencies the results show some rather unexpected features and need to be examined in some detail.

We will first consider the results obtained at 15 MHz as these show the largest effects. Four sets of results have been obtained: without a membrane, and with a membrane at 36, 56 and 76 mm above the transducer. As mentioned in section (5.4), it appears that in the radiation pressure balance the ultrasonic power is being attenuated with an attenuation coefficient given by $2\alpha_p$ (equal to 0.0114 mm^{-1} at 15 MHz), but an additional effect is increasing the value of the power determined by the balance. For measurements without a membrane, the power $W_b(x)$ determined by the balance at a distance x from the transducer is given by

$$W_b(x) = W_o \exp(-2\alpha_E x),$$

and a least squares fit to the measurements at 15 MHz gives $W_o = 43.0 \pm 0.5 \text{ mW}$, $2\alpha_E = 0.0031 \pm 0.0002 \text{ mm}^{-1}$.

If a membrane is present between the transducer and the target, this will stop any streaming or convection currents. Hence if streaming and convection are affecting the power measurements, the power W_m determined by the radiation pressure balance when the target is at the membrane should equal the true ultrasonic power at that point, $W_o \exp(-2\alpha_p x_m)$, where x_m is the separation between the transducer and the membrane. Above the membrane, however, the streaming and convection will restart, and the power determined by the balance will be greater than the true ultrasonic power. We can

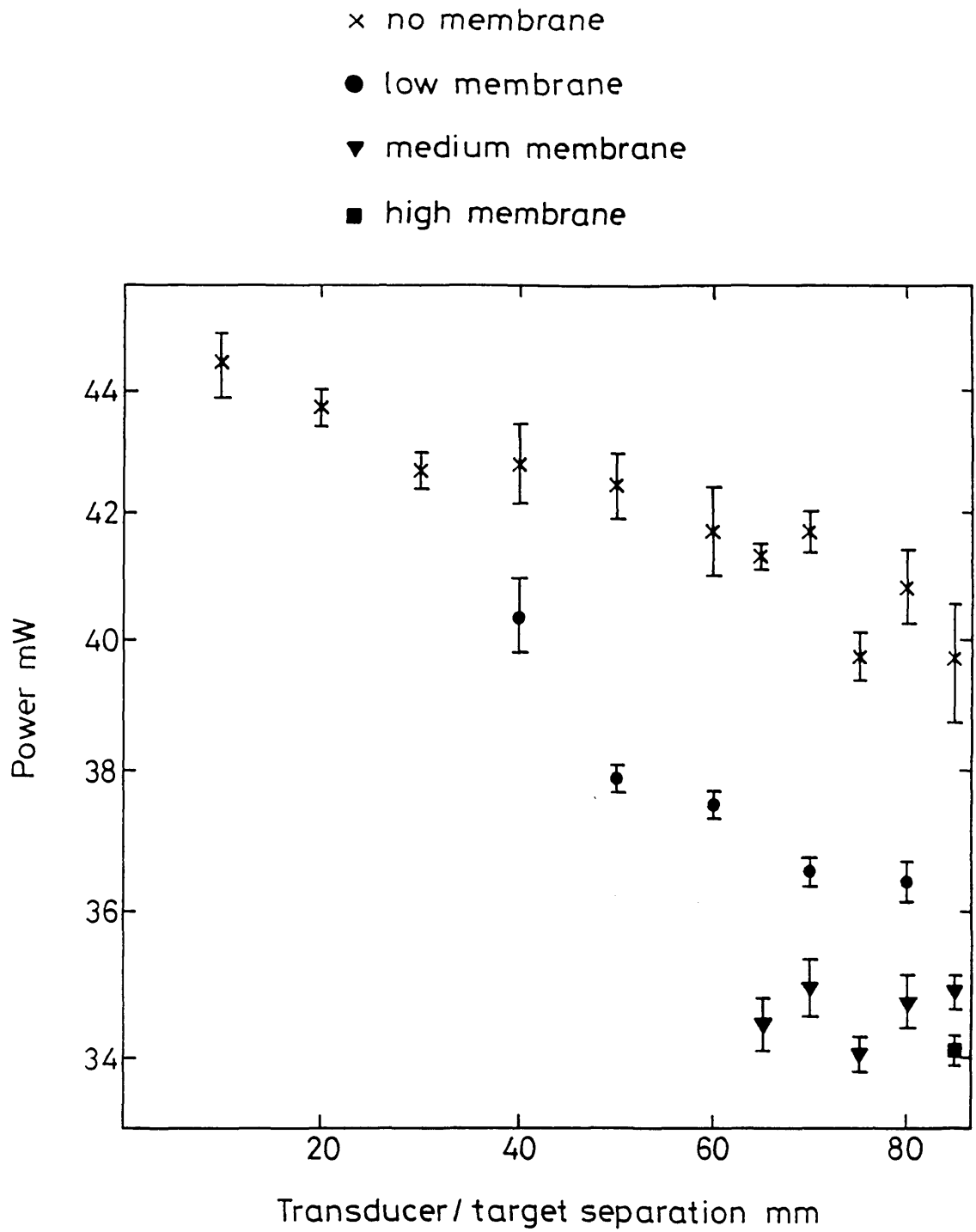


Fig. 26: The Power Determined with the Radiation Pressure Balance Plotted Against the Distance from a 15 MHz transducer with, in some cases, a Membrane Between the Transducer and the Target.

write

$$W_{bm}(x-x_m) = W_m \exp[-2\alpha_E(x-x_m)],$$

where $W_{bm}(x-x_m)$ is the power determined by the balance at a distance $(x-x_m)$ above the membrane, and W_m is the ultrasonic power at the membrane.

Taking logarithms gives

$$\ln W_{bm}(x-x_m) = \ln W_m - 2\alpha_E(x-x_m),$$

and therefore, for measurements with the membrane, a plot of $\ln W_{bm}(x-x_m)$ against $(x-x_m)$ should give a straight line of gradient $-2\alpha_E$ and intercept $\ln W_m$.

For the measurements with the membrane at 36 mm from the transducer, a least squares fit gives $\ln W_m = 3.40 \pm 0.03$, $2\alpha_E = 0.0028 \pm 0.0012$. Hence within the measurement uncertainties, the same value of α_E is obtained for measurements both without the membrane and with the membrane at 36 mm. This demonstrates that the streaming or convection continues or restarts directly after the membrane.

If the power W_m at the membrane is the true ultrasonic power then $W_m = W_o \exp(-2\alpha_P x_m)$, and we can use the value of α_P given by Pinkerton together with W_m calculated above to give a value for W_o . This gives $W_o = 45.1 \pm 1.3$ mW, which is in reasonable agreement with the value obtained from the measurements without the membrane. With the membrane at 56 mm from the transducer, similar calculations give $2\alpha_E = 0.0004 \pm 0.0013$ and $W_o = 43.2 \pm 0.9$ mW. Hence the same value for the output power W_o from the transducer is obtained from the three sets of measurements: without the membrane and with the membrane at 36 and 56 mm from the transducer. This demonstrates more or less

conclusively that the ultrasonic power is being attenuated with an attenuation coefficient given by $2\alpha_p$ and an additional effect is increasing the value of power determined by the radiation pressure balance. This additional effect is stopped by the membrane but restarts directly afterwards.

As the measured value of the power is increased, not only between the transducer and the target, but also between the membrane and the target, convection currents from the transducer itself cannot be affecting the power measurements. These convection currents would be stopped by the membrane and would not restart after it. Both streaming and convection currents from absorption in the bulk of the fluid could be expected to be stopped by the membrane, but would restart directly afterwards.

It is strange that the value of α_E obtained from measurements with the membrane at 56 mm from the transducer is significantly less than that obtained for the other two sets of results. There is, however, a large uncertainty on the value of α_E , which was obtained by a straight line fit through only three points.

We can analyse the results obtained from the measurements at 10 MHz in a similar fashion, and the results at both 10 and 15 MHz can be summarised as follows:

Frequency = 10 MHz

<u>Situation</u>	$\frac{W}{O}$ mW	α_E mm ⁻¹
no membrane	29.9 $\pm$ 0.3	0.0017 $\pm$ 0.0002
membrane at 26 mm	31.4 $\pm$ 0.5	0.0016 $\pm$ 0.0002
membrane at 46 mm	30.9 $\pm$ 0.2	0.0012 $\pm$ 0.0002
membrane at 66 mm	29.1 $\pm$ 2.2	0.0008 $\pm$ 0.019

Frequency = 15 MHz

<u>Situation</u>	$\frac{W_0}{\text{mW}}$	$\frac{\alpha_E}{\text{mm}^{-1}}$
no membrane	43.0 ± 0.5	0.0016 ± 0.0001
membrane at 36 mm	45.1 ± 1.3	0.0014 ± 0.0006
membrane at 56 mm	43.2 ± 0.9	0.0002 ± 0.0007

Hence within the measurement uncertainties, measurements both with and without the membrane give the same value for the output power W_0 from the transducer. To obtain a value for the output power from a transducer using the radiation pressure balance at these high frequencies, it is therefore acceptable to determine the power $W(x)$ at various distances x from the transducer and then fit these results to

$$\ln W(x) = \ln W_0 - Kx,$$

where K is a constant.

The value of the output power from the transducer can be determined using this technique to within approximately 2%, but clearly the uncertainty in the final value will depend on the number of different distances used.

We have shown that the power determined by the radiation pressure balance is equal to the true ultrasonic power plus an additional term due to other effects such as streaming and convection in the water. It is difficult to conceive of any experiment that could be done with the radiation pressure balance to distinguish between the effects of streaming and convection currents. The best solution would be to direct the ultrasonic beam horizontally and measure the horizontal force on a target. The streaming would still occur in the horizontal direction, but any

convection currents would travel vertically and not affect the power measurements. It would be necessary, however, to build a new complete radiation pressure measurement system.

Instead some theoretical models were used to estimate the magnitude of the effects from streaming and convection. A brief summary of the calculations is given in Appendix D. It is shown that both streaming and convection currents could change the apparent weight of the target by a sufficient amount to produce the observed effects. The models are only approximate, however, and it is not possible to conclude whether streaming or convection is the more dominant.

As far as this thesis is concerned, it is necessary to decide if the radiation pressure balance can be used to determine the output power from an ultrasonic transducer. Clearly at 5 MHz and below as shown in section (5.4), there is little attenuation of the ultrasonic beam, and the attenuation coefficient of water as determined by Pinkerton can be used to correct for the variation of output power with distance. At higher frequencies, however, a large fraction of the ultrasonic energy is dissipated in the water bath, and the consequent streaming or convection produces an additional change in the apparent weight of the target. The power determined by the balance is no longer equal to the true power in the ultrasonic beam. It has, however, been shown that the power $W(x)$ determined by the radiation pressure balance is given by $W(x) = W_0 \exp(-Kx)$, where W_0 is the total ultrasonic power from the transducer, x is the separation between the transducer and the target and K is a constant.

5.6 Comparison of the Absorbing and Reflecting Targets

Measurements were made on four different transducers using both the plane absorbing and the conical reflecting targets, and a value of output power was determined for each transducer using each target. The results obtained with the absorbing target were corrected for the reflection from the target using the measured values for the reflection coefficient of SOAB obtained in section (3.2). The ratio of the output power measured with the absorbing and the reflecting targets was then calculated and the results are plotted in figure (27). There are a number of conclusions that can be drawn from these results. Firstly, the measurements were at two frequencies, 2 MHz and 5 MHz, and taking the mean of the ratios at each frequency gives values of 1.019 ± 0.012 at 2 MHz and 1.021 ± 0.021 at 5 MHz (the uncertainty figures are the standard error on the mean). This demonstrates that there is no significant difference between the results obtained at the two different frequencies.

Secondly, the mean of all the results give a value of 1.020 ± 0.010 for the ratio of the output power measured by the plane absorbing target to that for the conical reflector. On average, the values of the total output power obtained with the absorbing target are, therefore, approximately 2% higher than those for the reflector.

In figure (28) the same ratios are plotted for three different power levels. The measurements were made at powers of approximately 10 mW, 100 mW and 1 W, and it can be seen that there is no significant difference between these sets of measurements. It appears that for all values of the output power the absorbing target measures approximately 2% higher than the reflector.

There are several possible reasons for this difference between

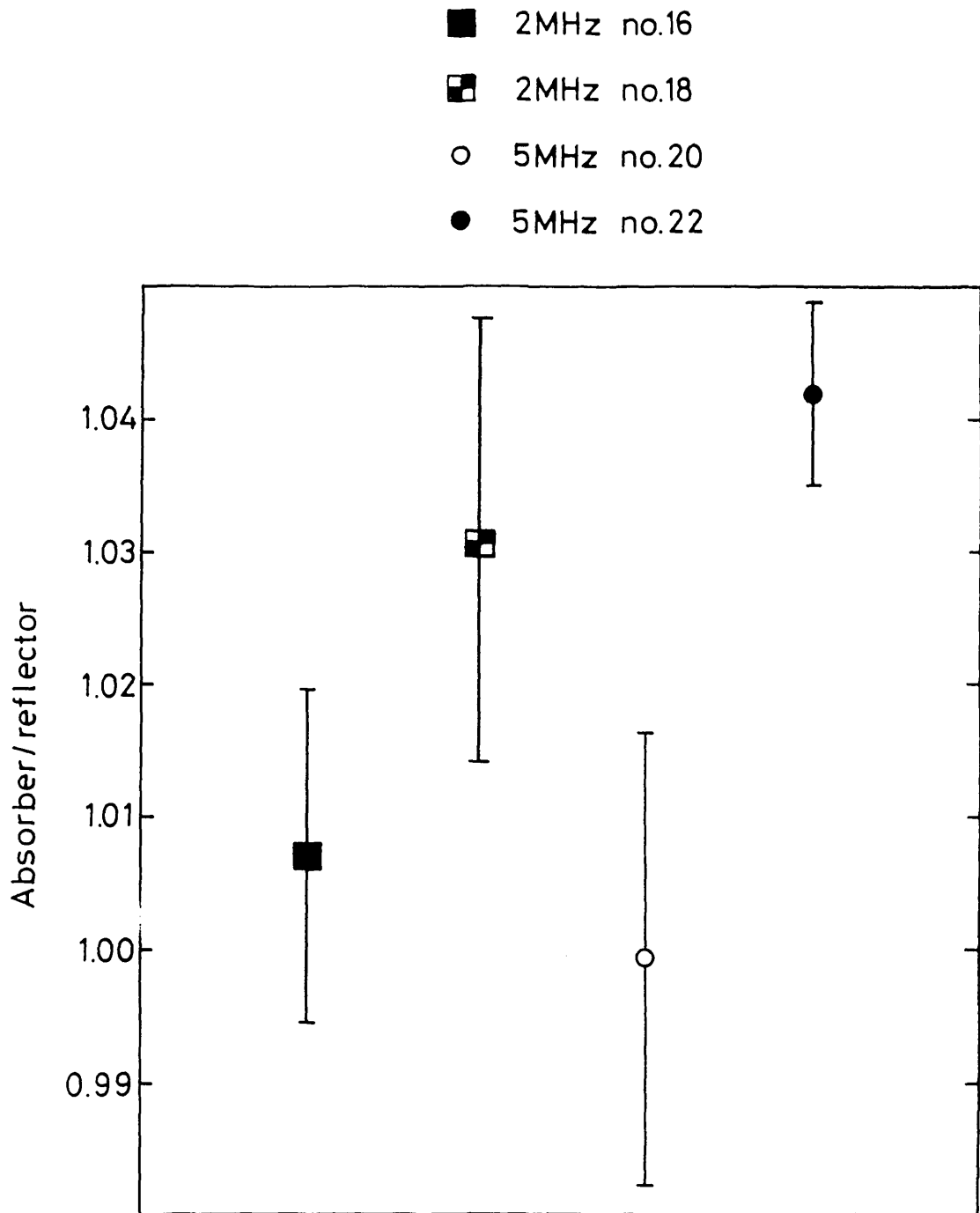


Fig. 27: The Ratio of the Output Power Measured with the Absorbing and the Reflecting Targets.

(The error bars denote the standard error on the mean of the measured values.)

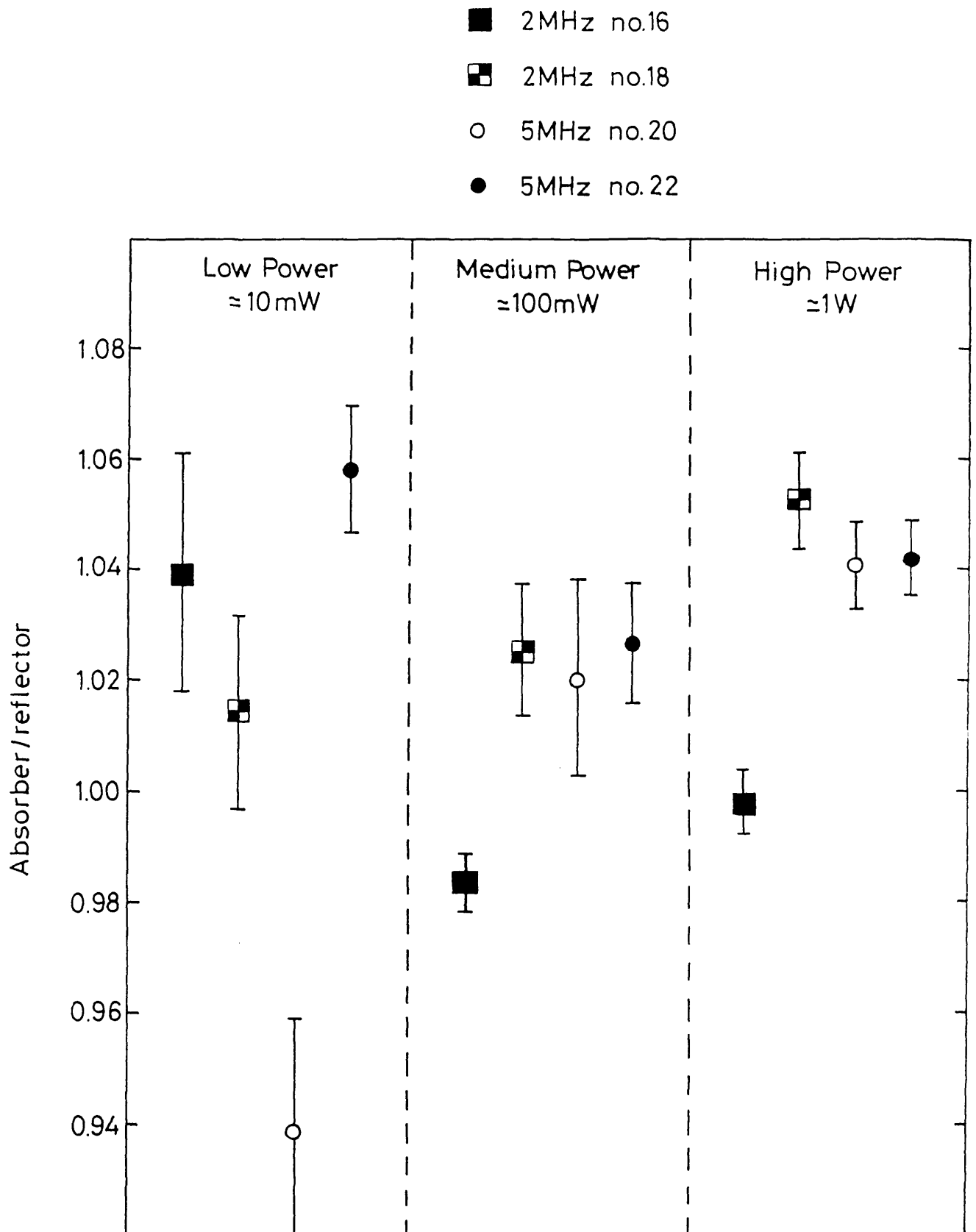


Fig. 28: The Ratio of the Output Power Measured with the Absorbing and the Reflecting Targets at Three Different Power Levels.

(The error bars denote the standard error on the mean of the measured values.)

the absorbing and the reflecting targets. Firstly, the reflection coefficient of the SOAB absorber obtained from the measurements with a hydrophone is only the specular coefficient of reflection. Any ultrasound which is scattered from the target in directions other than backwards will not be included in this reflection coefficient. Secondly, the comparison of the two types of target may confirm to some extent the earlier discussion on streaming. It is reasonable to expect that the absorbing target, being a plane interface, will effectively stop all the streaming, and much of the momentum carried by the streaming water particles will be transferred to the absorbing target. The conical reflector is, however, much more streamlined, and it is difficult to imagine that this type of target would necessarily stop all the streaming. Some of the streaming fluid particles would be expected to continue and flow along the sides of the cone. Both an increased reflection coefficient for the SOAB and the different effect of streaming on the absorbing target would produce a higher value for the output power determined with the absorbing target than that obtained with the conical reflector.

5.7 Systematic Uncertainties in the Determination of Output Power with the Radiation Pressure Balance

(i) Reflection Coefficient of the Target

The 45° angle for the conical target is beyond the critical angle for transmission of both the longitudinal and the shear waves through stainless steel. (The critical angle at a water/stainless steel interface is approximately 14° for longitudinal waves and 26° for shear.) It is therefore reasonable to assume that any ultrasound incident on the surface of the target at an angle greater than 26° will be reflected. For angles

less than 26° , the amplitude transmission coefficient of the target can be calculated as demonstrated in section (3.2). For normal incidence, the amplitude transmission coefficient is equal to 0.002% at 1 MHz and 0.005% at 15 MHz. Hence it can be assumed that any uncertainty from transmission through the target is less than 0.005% and is therefore negligible compared with the other systematic uncertainties given below.

(ii) Relation Between the Change in Target Weight and the Total Output Power

It is shown in section (II.6.2) that at the frequencies and output powers used in medical diagnostic equipment,

$$F = (W_o/c_o) \exp(-2\alpha x),$$

where F is the force on the target, W_o is the total ultrasonic output power from the transducer, c_o is the velocity of sound, α is the amplitude attenuation coefficient and x is the separation between the transducer and the target.

In the radiation pressure balance, the separation between the transducer and the target is determined by lowering the target down in the water bath until the tip of the target touches the front surface of the transducer. The reading on a vernier scale mounted on the balance is then noted. The target is then moved upwards until the vernier reading has changed by an amount equal to the required separation between the transducer and the target. This procedure enables the separation between the tip of the target and the transducer to be determined to within approximately ± 1 mm. This gives an uncertainty in the attenuation correction $\exp(-2\alpha x)$ which, for a transducer/target separation of 30 mm, ranges from $\pm 0.005\%$ at 1 MHz to $\pm 1.1\%$ at 15 MHz.

In addition, there is an uncertainty in the attenuation

correction from the conical shape of the target. The ultrasound at the extreme edges of the ultrasonic beam will travel further before it hits the target than that at the centre of the beam, which will hit the tip of the cone. For a 5 mm radius beam, this means that the ultrasound at the extreme edges of the beam will travel an extra 5 mm before it hits the target. The uncertainty in the attenuation correction from this effect clearly depends on the beam radius, but for a 5 mm radius beam at 30 mm from the transducer increases from 0.025% at 1 MHz to 0.63% at 5MHz, 2.5% at 10 MHz and 5.8% at 15 MHz.

(iii) Calibration of the Balance

It is shown in section (4.3) that, if the target is kept on the balance arm during a calibration, the calibration figure obtained has an error of approximately $\pm 0.1\%$. If the target is removed from the arm, this error increases to approximately $\pm 1\%$. Therefore, provided the target is not removed from the balance arm, the uncertainty in the calibration figure is negligible compared with other sources of uncertainty.

(iv) Absorption of the Reflected Wave

A Schlieren visualisation system was used, as described in section (4.4), to investigate the reflection properties of the carpet around the inside of the balance tank. Any reflected or scattered waves from the carpet were found to be well below the 1% level.

(v) Linearity

The linearity of the radiation pressure balance with applied ultrasonic power was confirmed using the square law relationship between the output power from a transducer and the applied electrical drive voltage, as indicated in section (5.1). Within the measurement uncertainties of approximately $\pm 1\%$, there was no

evidence of any nonlinearities.

(vi) Tilting of the Target

In section (5.2), it is calculated that, even when the ultrasound is acting on the extreme edge of the cone, there is negligible tilting of the target, and it can be assumed that the target remains vertical throughout the measurements. It is, however, necessary to ensure that the ultrasonic beam radius is less than 30 mm otherwise some of the ultrasound will miss the target completely. In addition, there is a limit to the power that can be determined using the radiation pressure balance, as above a certain power level the target may be pushed out to such a distance that some of the ultrasound does not hit the target. This power level depends on the ultrasonic beam radius and ranges from 710 mW for a 5 mm radius beam to 280 mW for a beam of 20 mm radius.

(vii) Surface Tension

The variation of the measured value of output power with different levels of water in the balance tank was checked in section (5.3). Within the measurement uncertainties of $\pm 1\%$, an alteration in the water level had no effect on the measured value of total power.

(viii) Streaming and Convection Currents

Streaming and convection currents have little effect at frequencies of 5 MHz and below, and it is shown in section (5.5), that the attenuation coefficient of water determined by Pinkerton (1947) can be used to correct for the variation of power with distance. This gives a standard error on the mean of the measured values of less than $\pm 1\%$.

Above 5 MHz, the power determined by the balance is no longer equal to the true power in the ultrasonic beam. It has been

	1 MHz	5 MHz	10 MHz	15 MHz
Reflection Coefficient of Target	-	-	-	-
Determination of Transducer/Target Separation	0.005%	0.13%	0.5%	1.1%
Effect of Target Shape on Attenuation Correction	0.025%	0.63%	2.5%	5.8%
Calibration of the Balance	0.1%	0.1%	0.1%	0.1%
Absorption of the Reflected Wave	1%	1%	1%	1%
Linearity	1%	1%	1%	1%
Tilting of the Target	-	-	-	-
Surface Tension	1%	1%	1%	1%
Streaming and Convection Currents	1%	1%	2%	5%
TOTAL	4.1%	4.9%	8.1%	15%

Table 10: Summary of the Systematic Uncertainties in the
Determination of Output Power with the Radiation Pressure
Balance.

demonstrated in section (5.5), however, that the power $W(x)$ determined by the radiation pressure balance can be given by

$$W(x) = W_0 \exp(-Kx), \quad \dots(5.3)$$

where W_0 is the total ultrasonic power from the transducer, x is the separation between the transducer and the target, and K is a constant.

At 10 MHz, if the output power is determined at two distances and the results fitted to equation (5.3), this gives an uncertainty in the value of the output power W_0 of approximately $\pm 2\%$. At 15 MHz, this uncertainty increases to approximately $\pm 5\%$.

6. INTERNATIONAL INTERCOMPARISON

In 1979, the National Physical Laboratory (NPL) took part in an international intercomparison of output power measurements organised by the National Bureau of Standards (NBS). Measurements were also made at NBS in Colorado and Washington, the Bureau of Radiological Health, the National Research Council and the Radiation Protection Bureau in Canada, the Ultrasonics Institute in Australia and the Physikalisch-Technische Bundesanstalt in Germany. Four ultrasonic transducers, two of 2 MHz resonant frequency and the others of 5 MHz, were shipped to each of the participating laboratories in turn, and each laboratory used its own method to obtain a measure of the total ultrasonic output power from the transducers. Each transducer was measured at three different power levels: low (2.5 - 25 mW), medium (25 - 250 mW) and high (250 - 2500 mW).

At NPL, measurements were made on each transducer using the radiation pressure balance over a period of three months between

December 1979 and February 1980. Typically for each transducer fifteen sets of measurements of the total output power were made. The values of output power divided by the square of the applied electrical voltage (output power/volt²) obtained for one of the 5 MHz transducers are shown in figure (29). For each transducer, all the measured values of the output power were combined to obtain a mean value of the total output power/volt² at each of the three different power levels. The total output power from one of the 2 MHz transducers (3-18) was, however, found to change during the period of the measurements, and the output powers measured during December were significantly different from those obtained in January and February. Therefore for this particular transducer, two values of the mean output power/volt² were reported for each power level. A summary of the results obtained for all the four transducers is given in figure (30).

The National Bureau of Standards collated all the results from the participating laboratories, and a draft report was received at NPL in 1982. The final results obtained from all the laboratories for two of the transducers are summarised in figures (31) and (32), with the values obtained by NPL clearly marked. In all cases, the error bars denote the estimated total systematic uncertainty. For both transducers, the results obtained using the radiation pressure balance at NPL are in excellent agreement with those from the vast majority of the other participants. This agreement is within the estimated uncertainties of the NPL measurements. The results from this intercomparison, therefore, give some confidence to the measurements made with the NPL radiation pressure balance, with the systematic uncertainties being shown to be good estimates of the total measurement uncertainty. In addition, as the intercomparison included other

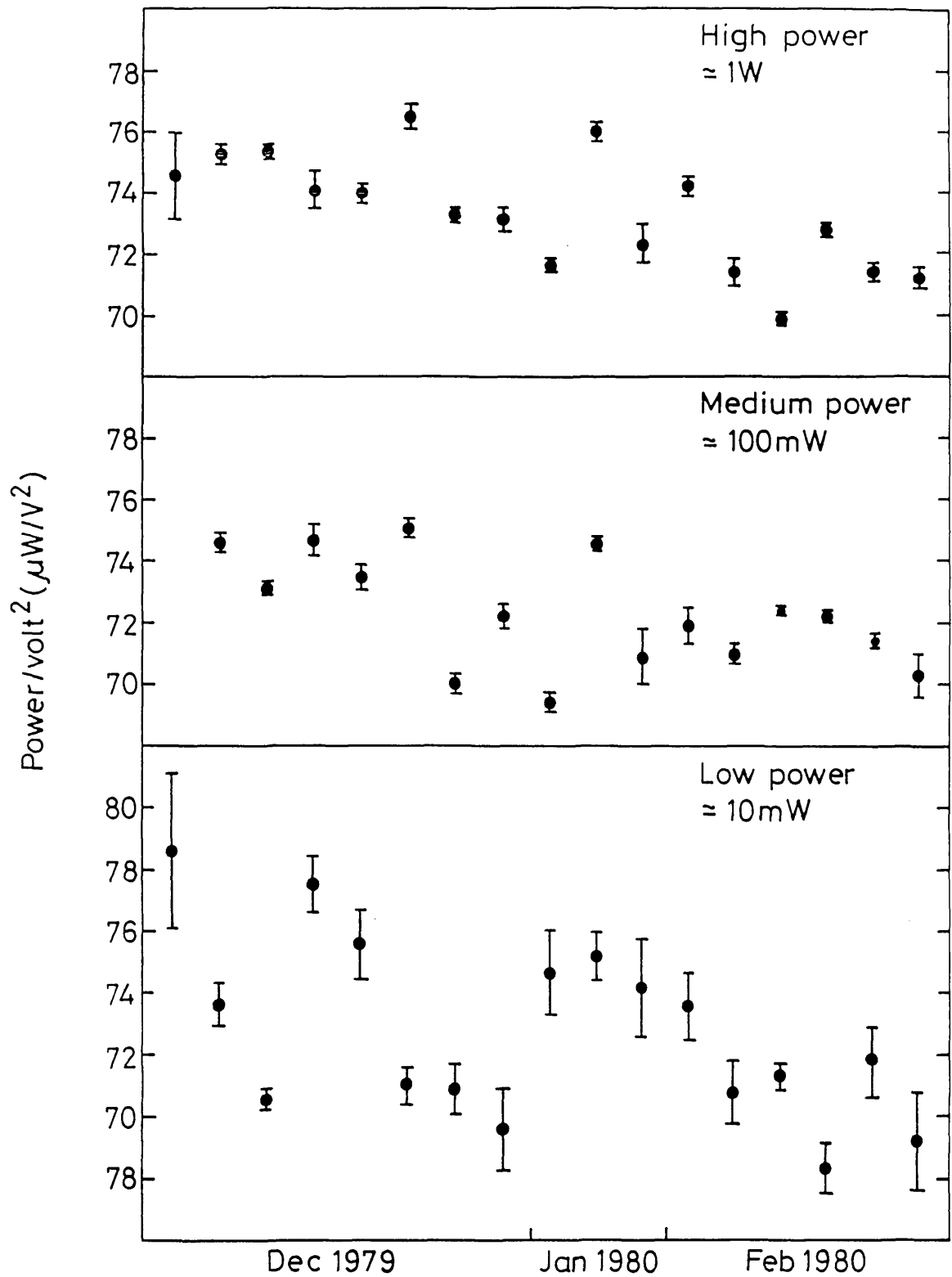


Fig. 29: International Intercomparison - Results for the 5 MHz Transducer no.3-20.

(The error bars denote the standard error on the mean of the measured values.)

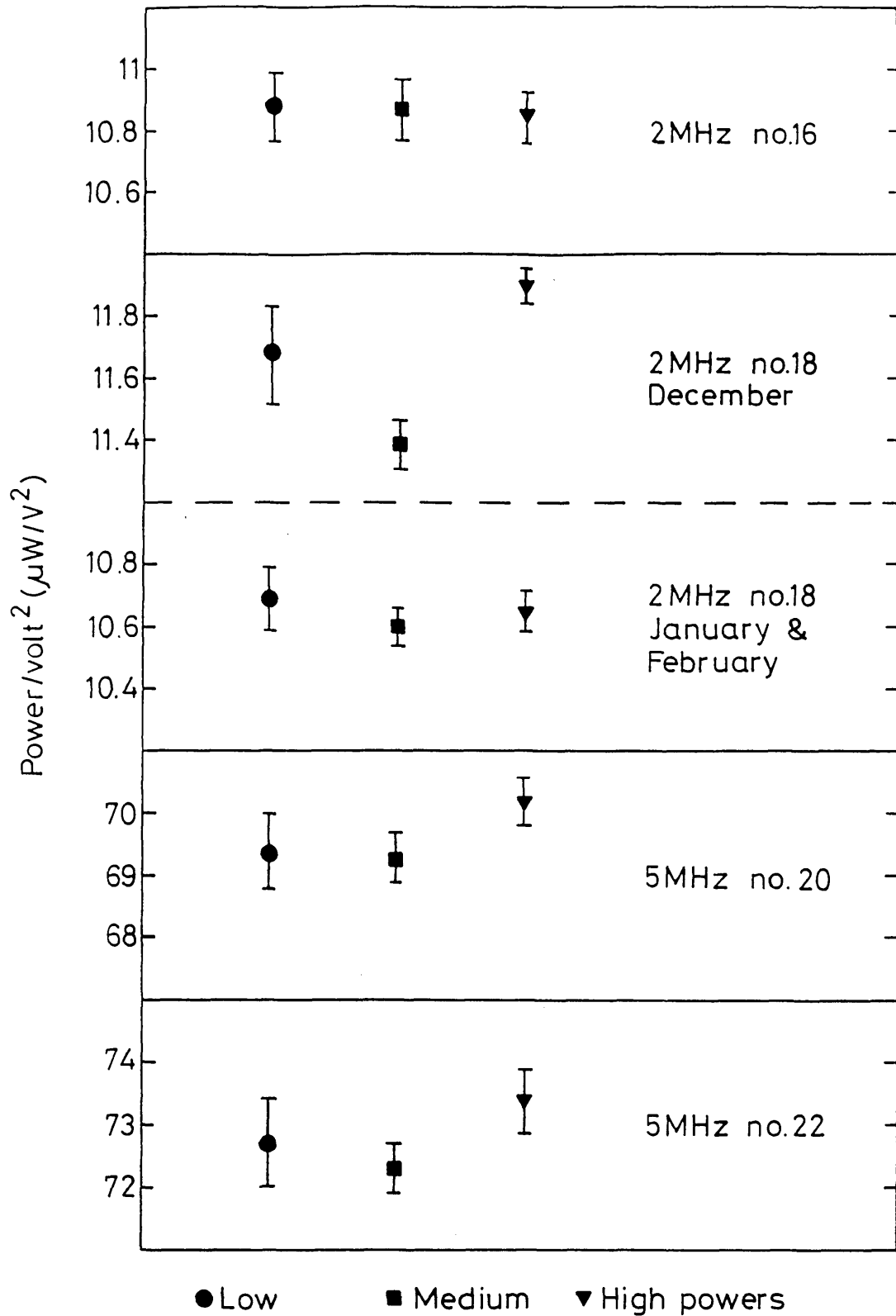


Fig. 30: International Intercomparison - Mean Values of Power/Volt² for all Four NBS Transducers.

(The error bars denote the standard error on the mean of the measured values.)

□ Balance-direct

■ Balance-direct

▣ Balance-direct

▤ Balance-direct

● Balance-feedback

◐ Balance-feedback

○ Balance-feedback

⊕ Balance-modulated

▽ Calorimeter

▼ Optical

× Reciprocity

- 194 -

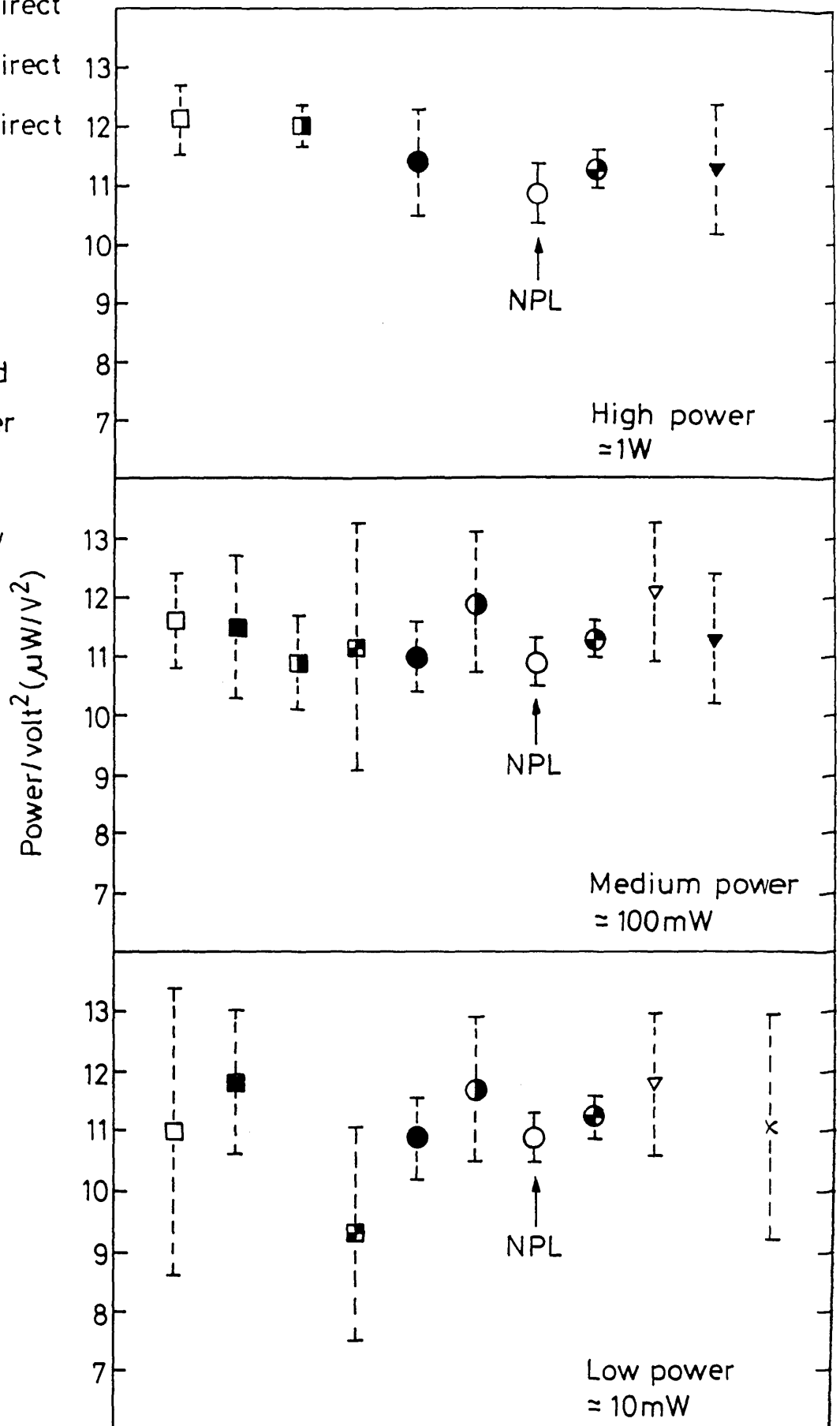


Fig. 31: International Intercomparison - Summary of the Values of Power/Volt² Obtained by all the Participating Laboratories for the 2 MHz Transducer no.3-16. (The error bars denote the estimated total systematic uncertainty.)

- Balance-direct
- Balance direct
- ▣ Balance direct
- ▤ Balance direct
- Balance-feedback
- ◐ Balance-feedback
- Balance-feedback
- ◑ Balance-modulated
- ▽ Calorimeter
- × Reciprocity

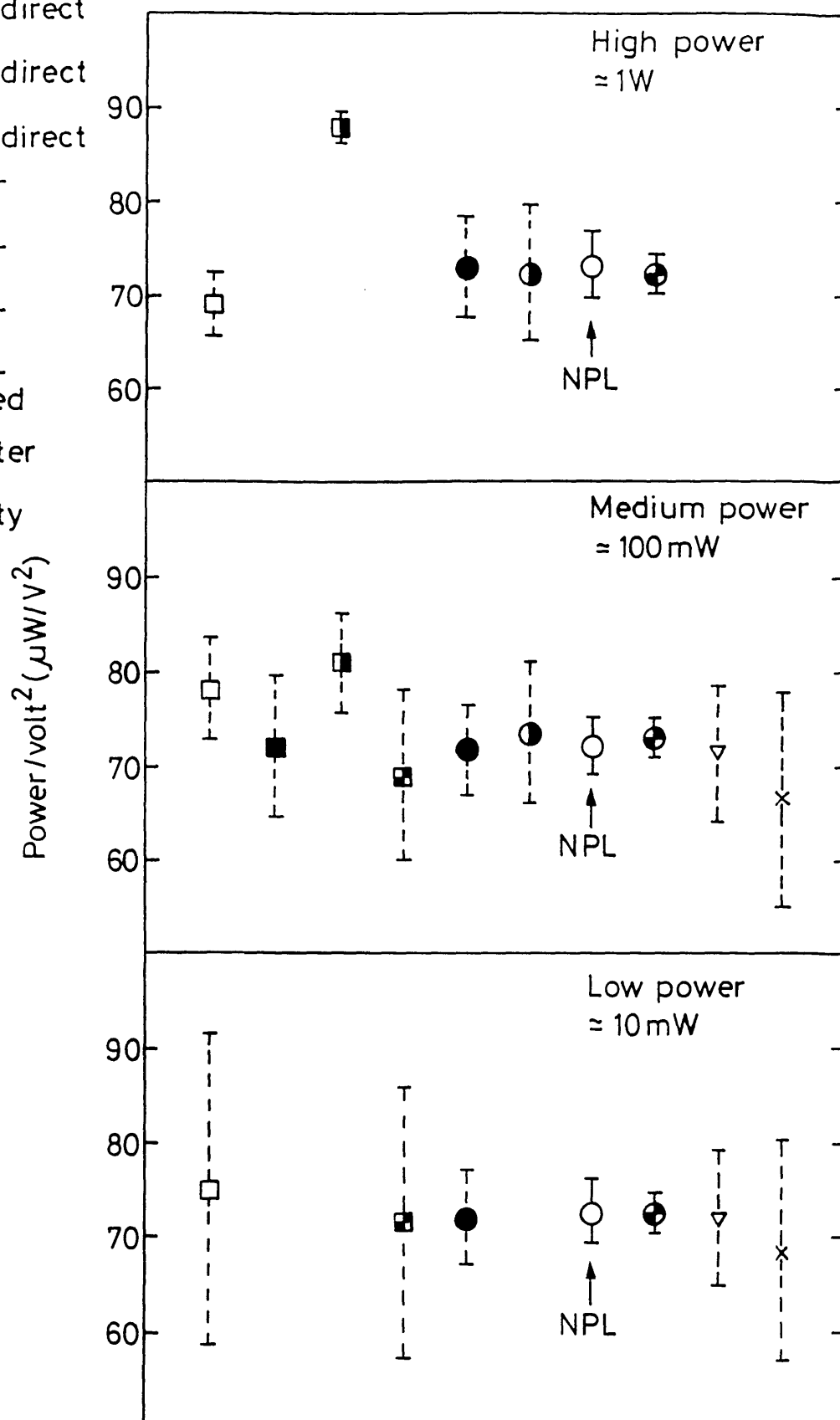


Fig. 32: International Intercomparison - Summary of the Values of Power/Volt² Obtained by all the Participating Laboratories for the 5 MHz Transducer no.3-22. (The error bars denote the estimated total systematic uncertainty.)

power-determination techniques such as calorimetry, optical interferometry and self-reciprocity, the results give added weight to the belief that a radiation pressure balance can be used to give a true measure of the total output power from an ultrasonic transducer.

7. ADDITIONAL INVESTIGATIONS

7.1 Transducer Soaking

A unusual effect, which has not been mentioned in the literature, was found to occur with all the ultrasonic transducers used at NPL. The value of the output power from the transducers determined with the radiation pressure balance was found to vary in a consistent manner. Initially, the measured value of the power increased with time in the balance until, after a period of typically thirty hours, it became stable at a value upto 10% higher than that initially determined.

At first, it was thought that this effect was due to a change in some property of the water in the balance tank, but changing the water had no effect on the measured value of the power from a transducer. Another possibility was that some alteration was occurring in the state of the transducer. Each transducer had been left in air before being placed in the balance tank, and the water in the tank could possibly take some time to saturate the transducer surface. To investigate this possibility, a transducer was placed in the balance tank and its output power was measured until a maximum value was reached. The tank was then emptied and the transducer was dried and left uncovered in the tank for sixteen hours. The tank was then refilled with water, and the output power from the transducer was again determined as a

function of time. A graph of output power against time was plotted (figure 33) and, although the measurement uncertainties are quite large, it can be seen that the power from the transducer decreased by approximately 4% after the transducer had been left in air, and then took approximately twenty hours to return to the previous maximum value. As a check, another transducer was covered with water for one week and then placed in the radiation pressure balance tank, where the measured value of output power from this transducer did not change with time.

These two measurements indicate that the change in the measured value of the output power from a transducer is due to water affecting the surface of the transducer. The transducers used at NPL have a front surface coating of epoxy and this may contain small holes. These holes will normally contain air, and when the transducer is placed in water it takes a finite length of time for the air to escape and be replaced by the water.

Soaking a transducer in water could conceivably affect the electrical impedance of the transducer. If any penetration of the water through the epoxy surface of the transducer changes the electrical impedance of the piezoelectric crystal, this will affect the total output power from the transducer. The impedance of a transducer was, therefore, monitored as the output power from this transducer was determined. A graph of electrical impedance against transducer output power was obtained (figure 34), and within the measurement uncertainties, which were rather high due to the difficulty of obtaining reliable electrical impedance measurements, there was no correlation between the output power from the transducer and its electrical impedance.

It was decided that the variation of the output power from a transducer could be avoided by keeping all the transducers out of

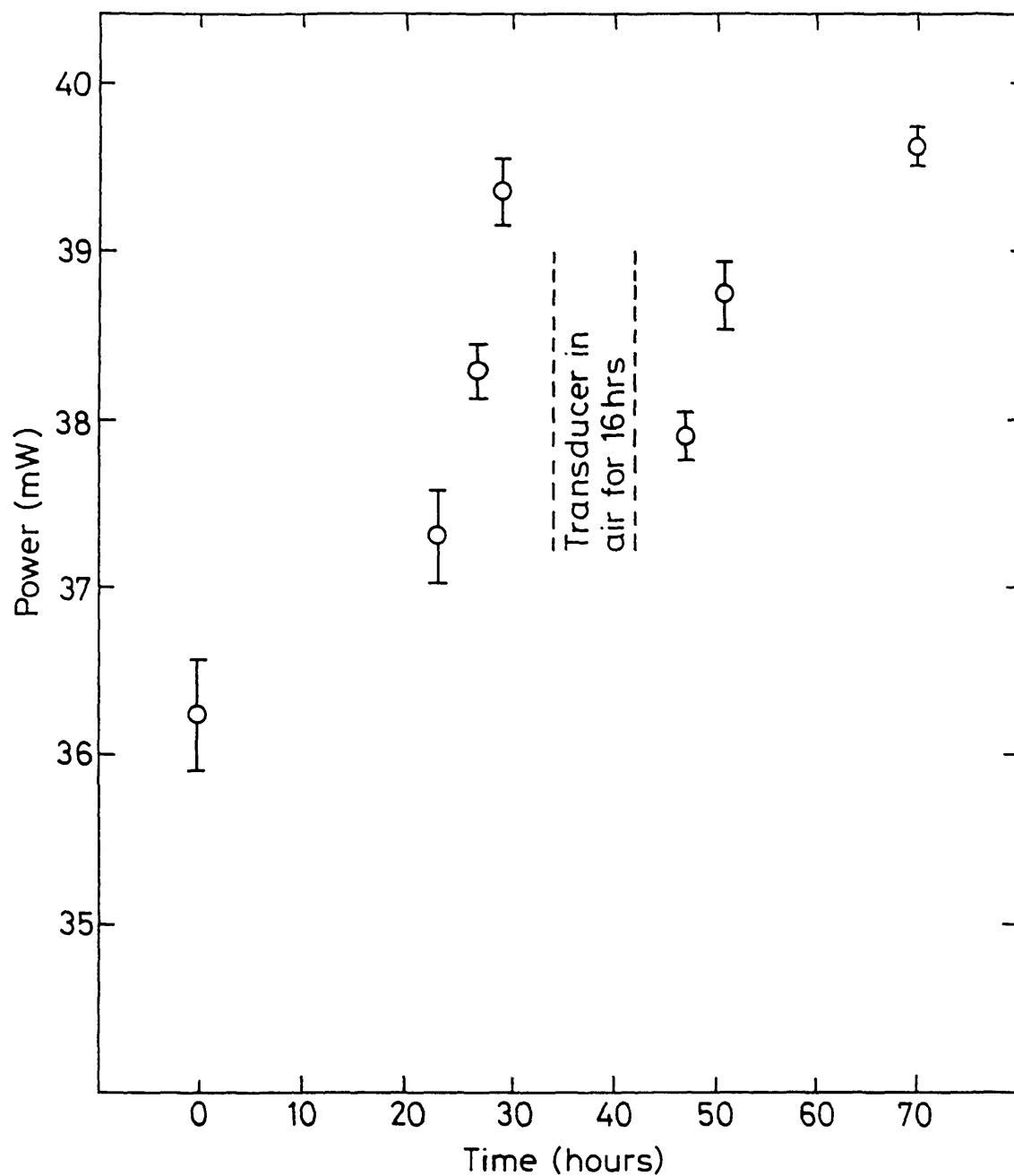


Fig. 33: Transducer Soaking - Variation of the Output Power
From a Transducer with Time.

(The error bars denote the standard error on the mean
of the measured values.)

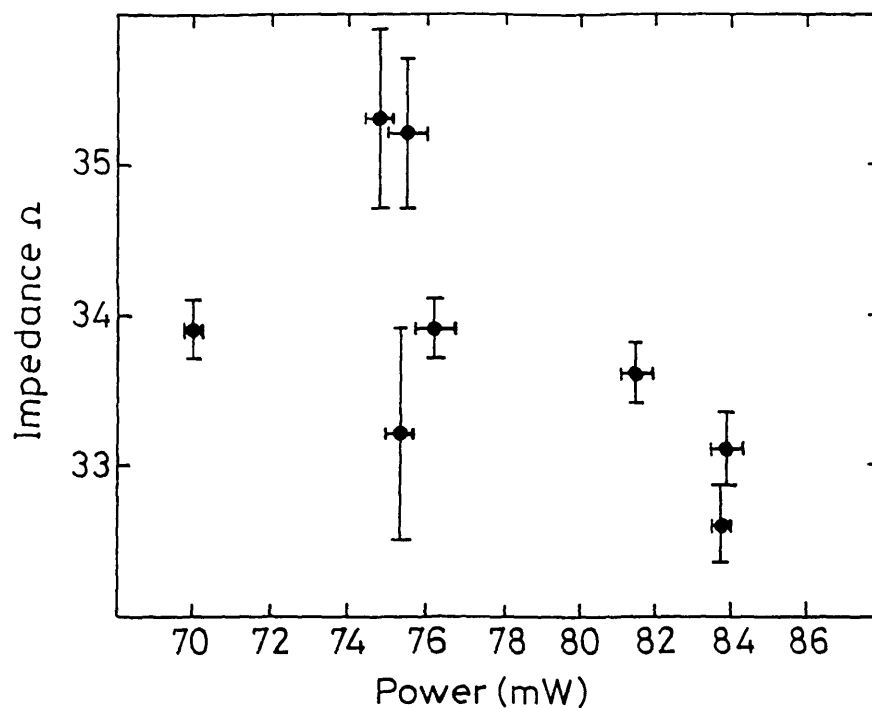


Fig. 34: Transducer Soaking - Variation of Transducer Output Power with the Measured Electrical Impedance.

(The error bars denote the standard error on the mean of the measured values.)

the air. As water deteriorates with time, this was achieved by soaking the front surface of each transducer in glycerine and then washing the surface before use. This procedure gave values for the total output power from a transducer which did not vary with the time of immersion in the balance tank.

Further evidence of the problem with the epoxy coating on the surface of the transducers was obtained when different transducers were measured in the radiation pressure balance as part of an international intercomparison. These transducers consisted of quartz crystals with a gold plating on the front surface, and showed no variation of output power with immersion in water.

7.2 Pulsed Fields

There is little experimental or theoretical evidence in the literature to indicate that a radiation pressure balance will correctly determine the power in a pulsed ultrasonic field. It was therefore decided to perform a detailed set of measurements using different lengths of ultrasonic tone bursts and various duty cycles.

A 1MHz transducer was used, and initially this transducer was driven using continuous wave excitation, and a value for the output power from the transducer was determined. The electrical drive voltage applied to the transducer was then gated to produce tone bursts, and the value of the total ultrasonic power obtained using the radiation pressure balance was determined. Values for the pulse length and the duty cycle were obtained by monitoring the voltage signal applied to the transducer with a high impedance probe connected to an oscilloscope.

Measurements were made using eight different pulse lengths of between 1 μ s and 208 μ s, and with duty cycles of 0.5, 0.25, 0.125 and 0.0625. Here the duty cycle is defined as the fraction of the total time for which the ultrasonic transducer is excited. The power determined by the radiation pressure balance was then divided by the duty cycle. This gave a value of power/duty cycle which was found to be constant within the measurement uncertainties for the four different duty cycles used.

If the power from the transducer was directly proportional to the applied pulse length, then the power/duty cycle should be constant for all pulse lengths. There was, however, found to be a significant difference between the measured value of power/duty cycle for the different pulse lengths (see table 11). This result

<u>Applied Pulse Length (μs)</u>	<u>Power/duty cycle (mW)</u>	<u>Power/duty cycle corrected to allow for ringing (mW)</u>
cont.	91.87 ± 1.48	91.87 ± 1.48
208	92.8 ± 2.3	93.1 ± 2.3
98	91.9 ± 2.0	92.5 ± 2.0
48	89.4 ± 1.6	90.5 ± 1.6
24	89.5 ± 0.7	91.7 ± 0.7
12	87.6 ± 1.3	92.0 ± 1.4
6	81.5 ± 1.3	90.3 ± 1.4
3	73.4 ± 0.3	90.7 ± 0.4
2	63.4 ± 1.1	88.8 ± 1.5
1	42.4 ± 2.0	90.6 ± 4.3

Table 11: The Variation Of Power/Duty Cycle with Applied Pulse Length

(The errors denote the standard error on the mean of the measured values.)

was not unexpected, as the ultrasound from a transducer is not an exact reproduction of the applied voltage signal, and some ringing occurs in the transducer at the start and at the end of the applied electrical signal.

To obtain an estimate of the ringing, the transducer was placed in a scanning tank and the ultrasonic signal produced using different electrical pulse lengths was recorded using a hydrophone. A hydrophone gives a measure of the pressure in the ultrasonic beam, and as the power in an ultrasonic field is related to the square of the pressure integrated over the area of the beam, it was therefore possible to obtain an approximate estimate of the effect of ringing in the transducer. If no ringing was occurring at the start or at the end of the applied

electrical signal such as would occur from an ideal transducer, the signal at the hydrophone would consist of a certain number of cycles of equal magnitude. From a real transducer, however, the ultrasonic signal builds up to a maximum value and then reduces again. An estimate of the difference between the actual power in the ultrasonic beam and the power that would have been present in the field from an ideal transducer can be obtained by summing the square of the contribution to the hydrophone signal from each individual half cycle. For example, when a tone burst of 6 μ s pulse length was applied to the transducer, the signal measured by the hydrophone was found to consist of 9 half cycles of 16.5 volts, 2 of 13.5 volts, 2 of 8 volts and 2 of 2 volts. Summing the squares of these contributions gives 2952 volt², and hence the power in the ultrasonic beam can be assumed to be approximately proportional to 2952 volt². From an ideal transducer with no ringing at the start or at the end of the pulse, the signal at the hydrophone would have consisted of 12 half cycles of 16.5 volts giving a total power approximately proportional to 3267 volt². Hence the values of power/duty cycle measured with the 6 μ s nominal pulse length need to be multiplied by $2952/3267 = 0.903$ to allow for the ringing in the pulse. The values of the output power measured for the different pulse lengths were all corrected in this way. It can then be seen (table 11) that the measured values of power/duty cycle all agree within the measurement uncertainties, and this indicates that the radiation pressure balance gives a correct determination of the total power in a pulsed ultrasonic field.

8. ABSOLUTE HYDROPHONE CALIBRATION

8.1 Introduction

The most widely used and convenient method of determining the spatial and temporal distribution of acoustic pressure within an ultrasonic field is by means of miniature hydrophones. These hydrophones are not absolute devices, and the relation between the applied pressure in the ultrasonic field and the voltage produced by the hydrophone must be determined by calibration. Hydrophone calibration is therefore interpreted as a determination of the response of the hydrophone, as a function of frequency, to an applied acoustic pressure. Two techniques for hydrophone calibration, total power measurement combined with beam plotting, and reciprocity, are commonly used, and this section of the thesis describes the use of these two techniques to calibrate one particular hydrophone, with emphasis on a determination of the associated measurement uncertainties. At the end of the section, the results from the two techniques are compared.

The hydrophone used in this study was a polyvinylidene fluoride (pvdf) membrane hydrophone manufactured by GEC-Marconi Electronics Ltd, and consisted of a 9 μm thick film of pvdf stretched across an annular ring (100 mm internal diameter). A small central region (1 mm diameter) of the film was piezoelectrically active.

8.2 Total Output Power and Beam Plotting

The sensitivity of a hydrophone can be determined by scanning the hydrophone over the ultrasonic beam, and dividing the integral of the hydrophone signal by the total power in the beam (Jones et.al. 1981).

(i) Measurement Method

Measurements were made using seven transducers of frequencies ranging from 0.5 MHz to 15 MHz, and the total power from each transducer was first determined using the radiation pressure balance. The total output power from the transducer was determined at different applied voltages, under continuous electrical excitation, giving a determination of output power/volt².

The transducer was then mounted horizontally in a beam plotting tank, and, after an initial setting-up procedure, the hydrophone was moved horizontally across a line perpendicular to the direction of the ultrasonic beam. With tone-burst excitation of the transducer, the received signal at the hydrophone was measured as a function of the position of the hydrophone in the ultrasonic beam. This produced a beam scan in one direction. The scanning was performed using an automatic two-axis scanning system with the data being collected using a Tektronix transient digitiser and a 4052 controller. Beam scans were made with the hydrophone positioned at distances from the transducer equal to two and three times the near-field distance. (Here the near-field distance is defined as the distance between the transducer and the last maximum in the axial pressure distribution). The transducer was then rotated about its acoustic axis through an angle of 90° and another horizontal beam scan obtained. Finally, the output power from the transducer was re-determined using the radiation balance.

(ii) Determination of Hydrophone Sensitivity

The sensitivity $S(f)$ of the hydrophone at a frequency f was determined using the equation

$$S(f)^2 = \int V^2 dA \exp(2\alpha d) / (8QZ),$$

where $\int V^2 dA$ is the square of the received hydrophone signal V

integrated over the area of the ultrasonic beam, $\exp(2\alpha d)$ is a correction for the attenuation of the water path of length d between the transducer and the hydrophone, Q is the total power in the ultrasonic beam, ρ is the acoustic impedance of water and α is its amplitude attenuation coefficient.

(iii) Corrections and Sources of Uncertainty

A value for the total output power Q from the transducer during the beam scan was obtained by multiplying the mean of the two values of output power/volt² measured in the radiation pressure balance before and after the beam scan by the square of the voltage of the tone-burst signal applied to the transducer during the beam scan. Two systematic uncertainties occurred in this determination of the total output power. The first was the overall systematic uncertainty on the determination of the total output power from a transducer using the radiation pressure balance, estimated as in section (5.7). The other was the variation in the total output power from the transducer during the scanning procedure, and was estimated by taking half of the difference between the output powers measured before and after the beam plot. This was typically $\pm 2\%$ for transducers of frequencies between 0.5 MHz and 5 MHz, and $\pm 9\%$ between 7.5 MHz and 15 MHz.

The correction for the attenuation of the water was calculated using an attenuation coefficient given in the literature (Pinkerton 1947), with the transducer/hydrophone separation d obtained from measurements of the time delay between the transmitted and the received signals. An estimate of the overall systematic uncertainty on this attenuation correction was obtained by considering the effects of any temperature variations (typically $\pm 0.5^\circ\text{C}$) on the determination of d , and typically increased from $\pm 0.01\%$ at 0.5 MHz to $\pm 2\%$ at 10 MHz.

At low frequencies, the measurements were corrected for the directional response of the hydrophone. The alternative of rotating the hydrophone for maximum signal at each point in the scan was used for the transducers of 10 MHz and 15 MHz, but, as this was very time-consuming, it was not thought necessary at the lower frequencies where the effect of the hydrophone directional response was less. The systematic uncertainty from this correction was negligible compared with the contributions from other sources.

Each half of each beam scan was analysed separately. The square of the hydrophone voltage in each case was integrated and treated as a quadrant of the ultrasonic beam. Variation in the values between the four quadrants was used to estimate the uncertainty in the assumption of cylindrical symmetry. This was typically $\pm 1\%$ at 0.5 MHz, $\pm 4\%$ at 1 and 2.25 MHz, $\pm 12\%$ at 5 and 7.5 MHz, and $\pm 15\%$ at 10 MHz.

Although the beam scans were taken out to a distance where no signal was observed above the noise, it is quite possible that signals below the noise level may have been present at these distances, potentially representing a significant contribution to the total integral. It was decided that the best estimate of the magnitude of these contributions would be obtained using a theoretical model of the ultrasonic beam. The model used was for a circular plane piston source for which the pressure at a point P is proportional to $2J_1(ka\sin\theta)/(ka\sin\theta)$, where k is the wave number, a is the transducer radius, θ is the angle between the line connecting the point P to the centre of the transducer and the beam axis, and J_1 is a Bessel function of the first kind. Any contribution to the integral from outside the scanning region was then calculated from this equation and was found to be less than 0.5% for all frequencies.

Another possible source of uncertainty arose from the finite size of the hydrophone, as the hydrophone will measure the pressure integrated over its surface and not the pressure at a point. An estimate of the magnitude of this effect was obtained from a convolution of the theoretical beam profile given above with a square function representing the hydrophone, and taking the difference between the integral obtained for the theoretical and the convoluted profiles. This difference was less than 0.5% at all frequencies.

Two sources of uncertainty came from the measurements of the received hydrophone signal. The first of these was the fundamental accuracy of the voltage measurement system, typically $\pm 2\%$. The second was caused by noise on the received signal, as the signal including the noise was digitised and the peak-to-peak value determined. Clearly, this resulted in an overestimate of the true peak-to-peak value by an amount dependent on the magnitude of the noise, with the overall effect on the integral depending to a large extent on the signal-to-noise ratio. For frequencies between 0.5 MHz and 2.25 MHz the contribution to the uncertainty on the integral was typically 2.5%. This increased to 5% at 5 MHz, 35% at 7.5 MHz, 5% at 10 MHz and 40% at 15 MHz.

A small random uncertainty in the hydrophone calibration occurred in the initial setting-up procedure, which consisted of positioning the hydrophone at a point corresponding to the peak pressure. This was typically $\pm 1\%$ from 0.5 MHz to 5 MHz and $\pm 2.5\%$ from 7.5 MHz to 15 MHz.

The values for the systematic uncertainties given in this section are contributions to $S(f)^2$, however, and to obtain a value for the uncertainty in the hydrophone sensitivity $S(f)$, these were therefore halved. All the systematic uncertainties were combined

linearly to give absolute upper and lower bounds on the total systematic uncertainty, and this ranged from 7% at 0.5 MHz to 25% at 10 MHz and 50% at 15 MHz, as shown in figure (35). In some cases, more than one calibration figure is given at each frequency. For 2.25 MHz, this represents an exact repeat of the hydrophone calibration procedure using the same transducer, whereas at 1 MHz, 5 MHz and 7.5 MHz, the calibration was repeated using another transducer of the same frequency.

8.3 Reciprocity

The technique used for hydrophone calibration was essentially the two-transducer reciprocity method described by Brendel and Ludwig (1976) and proposed in the draft IEC document 29D(CO)19. This technique can be divided into two parts. Firstly a transducer is calibrated by the self-reciprocity method, which gives a value for the acoustic pressure at a point in the field transmitted by the transducer as a function of the drive current. The hydrophone to be calibrated is then placed in the field and its output voltage determined.

(i) Measurement Method

The measurements were made using nine transducers of frequencies ranging from 0.5 MHz to 15 MHz. The transducer was mounted with its acoustic axis horizontal in a water tank, and a repetitive electrical signal of between 10 and 20 cycles was applied. The ultrasonic signal from the transducer was reflected from a 20 mm thick stainless steel plate, with the position and the orientation of the transducer and the reflecting plate adjusted for maximum received echo signal at the transducer. The drive current applied to the transducer and the voltage of the received signal were then determined. The time delay between the

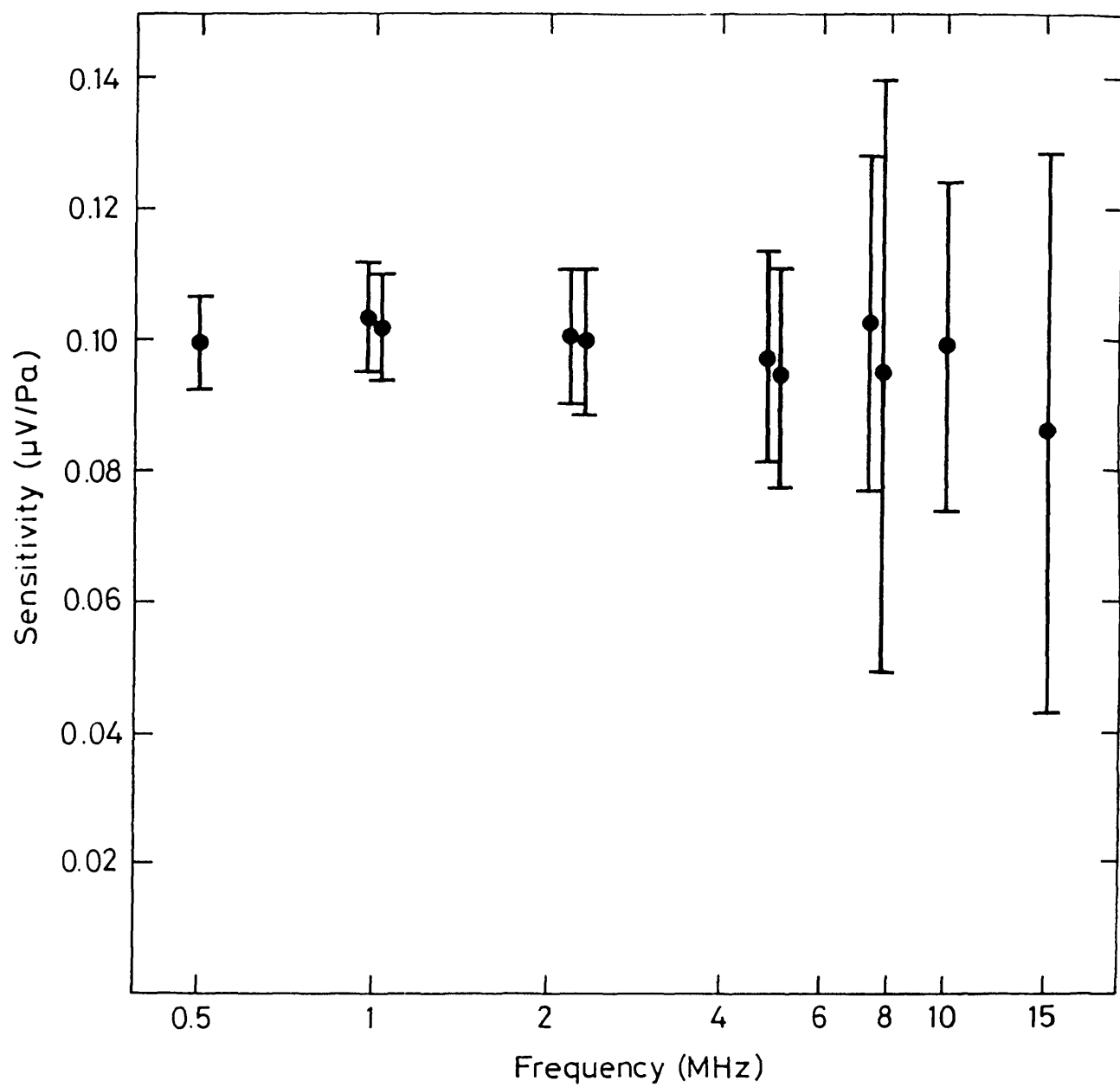


Fig. 35: Sensitivity of the Hydrophone Determined Using the Total Output Power and Beam Plotting Technique.

(The error bars denote the total systematic uncertainty in the measurements.)

transmitted and the received signals was measured and used to calculate the total length d of the sound path. Measurements of the received signal were made for several values of d between one and five near-field distances.

The reflecting plate was then removed, and the hydrophone to be calibrated was placed in the ultrasonic field at each of the different distances d from the transducer. At each distance, the orientation of the hydrophone was adjusted for maximum signal and the received voltage measured. As a check on stability and reproducibility, the stainless steel reflector was then placed in the tank and the signal received at the transducer remeasured.

(ii) Determination of Hydrophone Sensitivity

This technique assumes that the transmitting response of the transducer to drive current is equal to its receiving sensitivity to acoustic pressure, i.e. the transducer acts as a reciprocal device. This is expressed as

$$v/I = V_o/F, \quad \dots(8.1)$$

where (in transmission) v is the velocity of the front surface of the transducer and I is the drive current, and (in reception) V_o is the open-circuit received voltage and F is the force on the transducer surface. Some discussion of this assumption is given by Beissner (1980). For a plane wave (to first order),

$$p_t = vZ,$$

where p_t is the pressure in the acoustic wave directly in front of the transducer, and Z is the acoustic impedance of the medium.

In reception, the pressure p_r at the transducer is

$$p_r = p_t r G_1 \exp(-\alpha d),$$

where r is the amplitude reflection coefficient for the water/reflector interface, G_1 is a correction for any diffraction effects during self-reciprocity, $\exp(-\alpha d)$ is a correction for the attenuation of the water path of length d , and α the amplitude attenuation coefficient of water.

The force F on the surface of the transducer is therefore

$$F = 2Ap_r = 2Ap_t r G_1 \exp(-\alpha d),$$

where A is the effective receiving area of the transducer.

Substituting for v and F in equation (8.1)

$$p_t = \{V_o IZ / [2Ar G_1 \exp(-\alpha d)]\}^{1/2},$$

which is the pressure in the ultrasonic field directly in front of the transducer obtained from the self-reciprocity measurement.

For the hydrophone, the pressure at the hydrophone p_h is

$$p_h = p_t G_2 \exp(-\alpha d),$$

where G_2 is a correction for the diffraction effects in the hydrophone calibration.

The sensitivity $S(f)$ of the hydrophone is therefore

$$S(f) = V_h / p_h = (V_h / G_2) \{2Ar G_1 \exp(\alpha d) / [V_o IZ]\}^{1/2},$$

where V_h is the open-circuit voltage received by the hydrophone.

(iii) Corrections and Sources of Uncertainty

Several corrections were applied to the experimental results due to both diffraction and attenuation losses and the electrical load on the transducer.

For the transducer, a major correction arose from the electrical loading of the transmitting and the receiving

circuitry. The electrical drive signal to the transducer was obtained from a function generator and passed through a FET gate. This gate presented negligible impedance to the applied tone burst, but was switched to provide a high impedance (approximately 10 Kohm) load for the received signal. The current applied to the transducer was monitored using a current probe (Tektronix CT-2), while the received voltage was determined using an oscilloscope, and the measured values of current and voltage were corrected using equivalent circuits. The loading due to the various components such as the current probe, oscilloscope, gate and interconnecting cables was calculated, and many were found to be significant at the higher frequencies. For example, the FET gate used had a capacitance of 27 pF; equivalent to approximately 400 ohm at 15 MHz.

Two corrections were calculated from the equivalent circuits. One was for the difference between the current measured by the current probe and the drive current applied to the transducer, and was estimated to range from + 0.05% ($\pm 0.02\%$) at 0.5 MHz, to - 3.6% ($\pm 0.2\%$) at 15 MHz. The other correction was due to the difference between the measured received voltage and the voltage that would have been received into an open-circuit load. This was calculated to be between + 5.3% ($\pm 0.5\%$) at 0.5 MHz and - 48% ($\pm 2\%$) at 15 MHz. The estimated systematic uncertainties, given in brackets, arose from the uncertainty in the value of the impedance of the different circuit components, and from the uncertainty in the validity of the equivalent circuits.

The values of the diffraction corrections, G_1 and G_2 , were equal to the ratio of the received acoustic pressure averaged over the surface of the transducer (for G_1) or the hydrophone (for G_2) to the pressure in a plane wave equivalent to the transmitted wave

immediately in front of the transducer. A plot of this ratio against the normalised distance was derived theoretically for an ideal piston-like source with various transmitter/receiver diameter ratios by Fay (see Brendel and Ludwig, 1976). The normalised distance used was the transmitter/receiver separation d divided by the near-field distance. Typically, at two near-field distances for a 5 MHz transducer of 11.5 mm diameter, G_1 was 0.75 and, for the 1 mm diameter hydrophone, G_2 was 1.47.

There were several possible sources of uncertainty in the values of G_1 and G_2 . The most difficult to quantify was the applicability of the theoretical model, but this was tested using the beam plotting data from section (8.2) to obtain values for G_2 , and gave a difference between the theoretical and the measured values of G_2 ranging from 3% at 0.5 MHz to 7% at 10 MHz. The other major source of uncertainty in the values of G_1 and G_2 arose in the determination of the normalised distance, as this was related to the near-field distance, and required a knowledge of the effective radius of the transducer acting as a transmitter. A figure for this effective radius was obtained by plotting the acoustic pressure amplitude measured by a hydrophone as a function of the position along the transducer beam axis. This experimentally-determined pressure distribution was then compared with that predicted for a perfect piston-like source, and the radius of the theoretical source was adjusted for optimum agreement between the experimental data and the theoretical model. The difference between the effective radius calculated using this technique and the physical size of the transducer varied between transducers: for the 1MHz transducer the difference was + 8% and for the 15 MHz transducer it was - 12%. These measured values of the effective radius were used to obtain the values for G_1 and G_2

with the uncertainties on G_1 and G_2 calculated from the uncertainty in the determination of the effective radius. The contribution to the systematic uncertainty in the hydrophone sensitivity from the estimated uncertainties in G_1 and G_2 was typically between $\pm 1\%$ and $\pm 5\%$.

A further problem occurred in the determination of a value for the effective radius of the transducer acting as a receiver, which may not have been the same as the effective transmitting radius determined above. For the calculation of hydrophone sensitivity, however, it was assumed that these two areas were equal, but the difference between the physical area of the transducer and the effective transmitting area was used as a measure of the uncertainty in the receiving area.

The correction for the attenuation of the water was calculated using the same values as for the beam plotting technique (section 8.2), and the overall systematic uncertainty on the correction was estimated to increase from $\pm 0.1\%$ at 0.5 MHz to $\pm 2\%$ at 15 MHz.

Two sources contributed to the random uncertainty in the measurements. One of these was determined from the variation in the results for different separations between the transducer and the hydrophone, and was typically $\pm 2\%$. The other source of random uncertainty was the reproducibility of the measurements: typically $\pm 1\%$. As with the beam plotting technique, the total random uncertainty was, therefore, significantly less than the overall systematic uncertainty. All the systematic uncertainties were combined to give absolute upper and lower bounds for the total systematic uncertainty. Typically this ranged from 8% at 0.5 MHz to 12% at 10 MHz and 20% at 15 MHz, and the results of the calibration are shown in figure (36). At some frequencies, the calibration procedure was repeated using the same transducer,

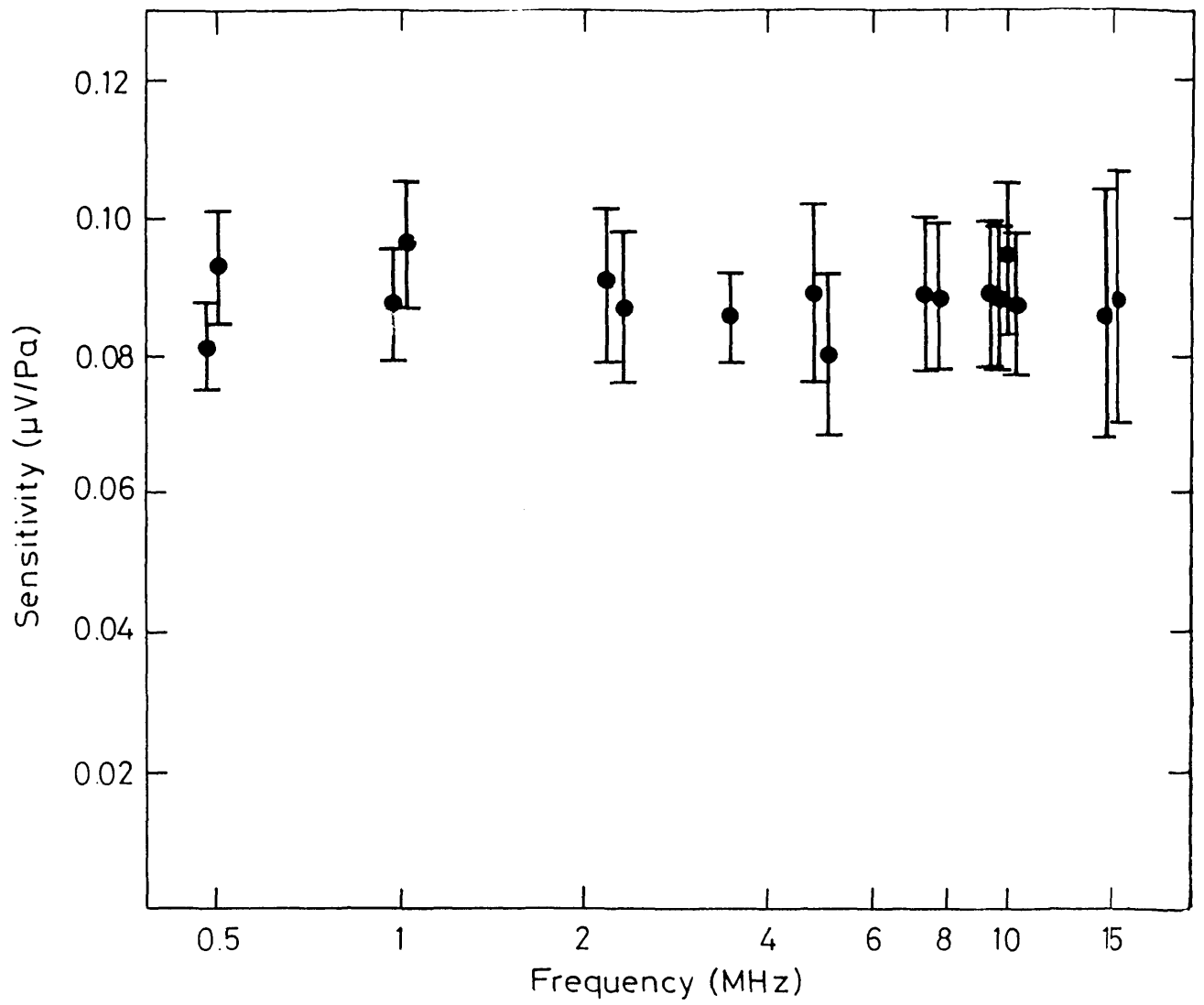


Fig. 36: Sensitivity of the Hydrophone Determined Using the Reciprocity Technique.

(The error bars denote the total systematic uncertainty in the measurements.)

whereas, at 10 MHz, a repeat was made using another transducer of the same frequency.

8.4 Conclusion

Two techniques have been used to provide an absolute calibration of a hydrophone, and the results are shown in figure (37). These calibrations agree within the overall uncertainties, thereby demonstrating satisfactory estimates of the individual uncertainties in the two methods. Although the two techniques are very different, there remains a possibility that common systematic errors may be occurring. The same system of hydrophone and associated amplifier were used and the measurements were performed over the same range of positions in the transducer field.

The total uncertainties associated with both techniques, especially at high frequencies, are large. This is not particularly satisfactory for hydrophone calibration as, for example, hydrophones are often used to determine the intensity in an ultrasonic field. A 20% uncertainty in a hydrophone calibration gives a 40% uncertainty in the determination of intensity (assumed to be proportional to pressure squared). It is therefore relevant to discuss the possible means of reducing the measurement uncertainties, although this will in general add to the complexity of the technique.

For the total power and beam plotting results, the main systematic uncertainties arose from three sources: variations in the total output power from the transducer during the scanning procedure, lack of cylindrical symmetry, and noise on the received signal. The first of these was dependent on the type of transducer used as, mainly at high frequencies, the transducers were not particularly stable and the power measured after the beam scanning

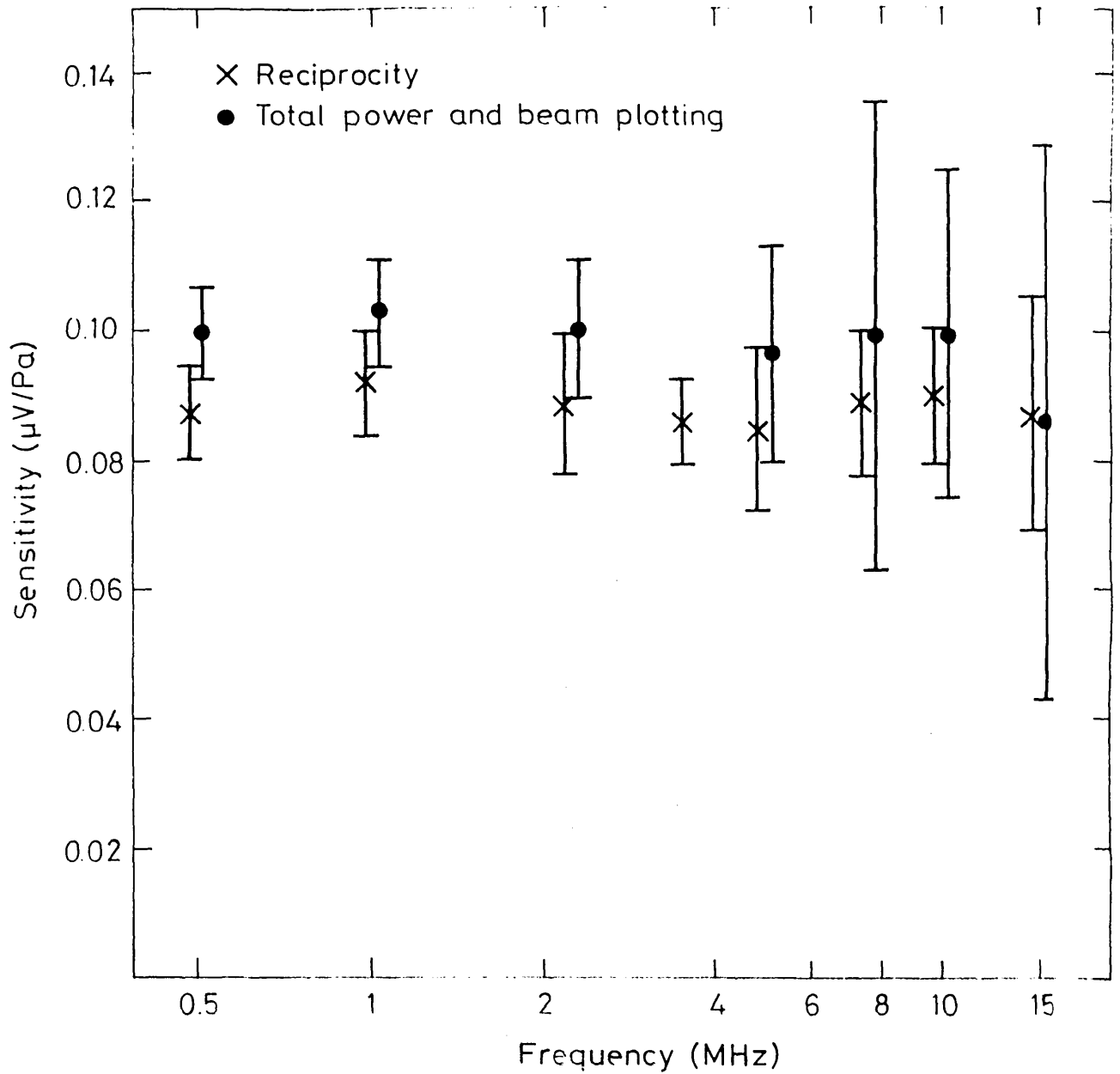


Fig. 37: Comparison of the Absolute Calibration of a Hydrophone Using Two Different Techniques.

(The error bars denote the total systematic uncertainty in the measurements.)

was as much as 10% different from that previously determined. Clearly it may be possible to reduce this contribution to the total uncertainty if transducers with a more stable output can be found. Contributions from cylindrical symmetry depend to some extent on the transducer, but are also a reflection of the measurement technique. With more complex measurements, for example, a raster scan over a plane in the ultrasonic beam, the uncertainty due to lack of cylindrical symmetry could be considerably reduced. The uncertainty due to noise on the received signal was due to the measurement equipment and the signal-to-noise ratio. This could possibly be reduced by using a system with greater resolution together with various techniques to reduce the total noise on the received signal. There is therefore potential for reducing the systematic uncertainties associated with the total power measurement and beam plotting technique.

The main sources of uncertainty in the reciprocity technique arose from the electrical and the diffraction corrections, and the uncertainty in the receiving area of the transducer, and there are a number of ways in which these could possibly be reduced. The electrical corrections were due to loading on the circuit, and it may be possible to build a system in which the measured values of the current and the voltage are less affected by the load of the other components. Alternatively, the uncertainty could be reduced using more complex equivalent circuits which are closer to the true situation together with a more accurate determination of the impedance of the individual circuit components. For the diffraction corrections, improvements in the theoretical model might reduce this uncertainty, whereas for the receiving area of the transducer it may be possible to devise a technique to provide a direct determination of this parameter.

Clearly it should be possible, by developing specialised measurement and analysis techniques, to reduce the measurement uncertainties associated with the two techniques described. However, the merit of the two methods is that they use readily-available laboratory equipment and are therefore generally applicable.

9. CONCLUSION

The design and evaluation of a radiation pressure balance for the determination of the output power from ultrasonic diagnostic equipment has been described. The various factors considered in the design of a microbalance system have been discussed together with a realistic assessment of the sources of uncertainty in the measurements.

The target is a crucial part of any radiation pressure balance, and particular emphasis has been placed on the design of the target. Clearly, if the radiation pressure of a field before the insertion of an obstacle is required, a perfectly absorbing target should ideally be used. All other types of target will unavoidably perturb the original acoustic field. By considering the reflected and transmitted waves from a solid plate in a fluid, it has been shown, however, that a finite attenuation coefficient has a significant effect on the reflection coefficient of a material. This makes the choice of a suitable absorbing target more difficult. The alternative target of the perfectly reflecting type will necessarily perturb the acoustic field, but a suitable design of target can be chosen so the force measured on the target is exactly equal to the radiation pressure of the ultrasonic wave. A

right-angled conical reflector is an appropriate choice, and the design used at NPL is, in addition, extremely stable in water. There is negligible tilting of the target even when the ultrasound is incident on the edge of the cone.

Various studies have been made to assess the sources of uncertainty in the determination of output power. The linearity of the balance with applied ultrasonic power was confirmed using the square law relationship between the output power from a transducer and the applied electrical drive voltage. Possible errors caused by changes in surface tension forces were also investigated and found to be small.

An absorbing target was constructed to investigate some of the sources of uncertainty and, on average, values of total power determined with the absorbing target were 2% higher than those for the reflector. This difference is relatively small and could be due either to reflections from the absorber or streaming in the water bath.

Both streaming and convection currents in the water bath were studied using a membrane at different distances from the target. It was found that, at frequencies of 5 MHz and below, the attenuation coefficient of water as determined by Pinkerton (1947) can be used to correct for the variation of power with distance. At higher frequencies, a large fraction of the ultrasonic energy is dissipated in the water bath, and the consequent effects produce an additional change in the apparent weight of the target. It has been shown, however, that the power determined with the radiation pressure balance can be directly related to the total ultrasonic power from the transducer.

From the detailed investigations of the performance of the radiation pressure balance, the overall systematic uncertainty on

the measurement of total power at frequencies of 5 MHz and below is estimated to be $\pm 5\%$. This uncertainty figure has been confirmed in an international intercomparison of techniques for the measurement of ultrasonic power organised by the National Bureau of Standards in the United States. This intercomparison included other techniques such as calorimetry, and the NPL radiation pressure balance was found to give agreement to better than $\pm 5\%$ with the mean total output power obtained from all the different techniques.

APPENDIX A

Some Properties of Water

DENSITY

(CRC Handbook of Chemistry and Physics, 58th edition)

at atmospheric	T (°C)	15	20	25
pressure	ρ ($\times 10^3$ kg/m ³)	0.9991	0.9982	0.9970

VELOCITY OF SOUND

(CRC Handbook of Chemistry and Physics, 58th edition)

at atmospheric	T (°C)	15	20	25
pressure	c (m/s)	1466	1482	1496

(American Institute of Physics Handbook, 2nd edition)

at 30°C	p (atm)	1	140	270
	c (m/s)	1510	1533	1566

ATTENUATION OF SOUND

(CRC Handbook of Chemistry and Physics, 58th edition)

at atmospheric	T (°C)	15	20	25
pressure	α ($\times 10^{-15}$ s ² /cm)	29.6	25.3	22.2

(American Institute of Physics Handbook, 2nd edition)

at 30°C	p (atm)	1	500
	α ($\times 10^{-15}$ s ² /cm)	18.5	15.4

THERMAL CONDUCTIVITY

(Handbook of Chemistry and Physics, 58th edition)

T (°C)	7	17	27
K ($\times 10^{-3}$ W cm $^{-1}$ °C $^{-1}$)	5.74	5.92	6.09

SHEAR VISCOSITY

(Kaye and Laby, 13th edition)

at atmospheric pressure	T (°C)	15	20	25
	μ ($\times 10^{-2}$ g cm $^{-1}$ s $^{-1}$)	1.139	1.002	0.890

BULK VISCOSITY

(Nyborg 1978)

at 20°C	η	2.8×10^{-2} g cm $^{-1}$ s $^{-1}$
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BULK MODULUS

(Kaye and Laby, 13th edition)

at 15°C	2.05×10^9 N/m 2
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APPENDIX B

Different Treatments of Radiation Pressure

Many different treatments of radiation pressure can be found in the literature, and this appendix is a summary of the main methods used. The results given here are chosen either because they represent the first use of a particular treatment or assumption, or because they are often quoted. No attempt is made to indicate inaccuracies or mistakes in a particular treatment; where appropriate, mention is made of this in the main body of the thesis.

AUTHOR	<u>RAYLEIGH</u> (1905)
DEFINITION	p_{RAY} is the force on a perfect reflector in a closed box.
EXPERIMENTAL SITUATION	Standing waves between a fixed piston and a rigid perfect reflector.
EQUATIONS USED	Euler's equation.
METHOD	1. Time averages Euler's equation and integrates to give $\bar{w} = \int dp/\rho = \bar{u}^2/2 + \text{const.}$ 2. Expands w in terms of $f(\rho)$. At the piston $u = 0$ and $\rho = \rho_1$ etc. Obtains value for $\text{const.} = \bar{w}_1$ in terms of ρ_1's and $f(\rho)$'s. 3. Space averages to obtain $\langle \bar{w} \rangle = \text{const.} + \langle \bar{u}^2 \rangle / 2 = \bar{w}_1 + \langle \bar{u}^2 \rangle / 2.$ 4. Uses the boundary condition and $\bar{p}_1 - p_o = \overline{f(\rho_1)} - f(\rho_o).$ 5. Applies the standing wave solution and obtains $\bar{p}_1 - p_o = \{ \rho_o + [\rho_o^2 f''(\rho_o)] / [2f'(\rho_o)] \} \langle \bar{u}^2 \rangle.$ 6. Uses the adiabatic law to obtain $\bar{p}_1 - p_o = (\gamma + 1) \rho_o \langle \bar{u}^2 \rangle / 2.$
APPROXIMATIONS OR ASSUMPTIONS	1. The equation of state is $p = f(\rho)$. 2. Expands $w = \int dp/\rho = \int [f'(\rho)/\rho] d\rho$ to second order. 3. Uses a standing wave solution of the form $\phi = \cos(\pi x/l) \cos(\pi \omega t/l).$ 4. The adiabatic law, $p = p_o(\rho/\rho_o)^\gamma$. 5. $\langle \bar{E} \rangle = \rho_o \langle \bar{u}^2 \rangle$.
BOUNDARY CONDITIONS	$\langle \bar{\rho} - \rho_o \rangle = 0.$
FINAL RESULT	$p_{\text{RAY}} = \bar{p}_1 - p_o = (\gamma + 1) \langle \bar{E} \rangle / 2.$

AUTHOR	<u>BIQUARD</u> (1932)
DEFINITION	The radiation pressure is the time-averaged pressure on an obstacle.
EXPERIMENTAL SITUATION	A totally reflecting plane surface.
EQUATIONS USED	The theorem of conservation of circulation and Euler's equation.
METHOD	1. Uses the theorem of conservation of circulation to obtain a similar result to Rayleigh for two surfaces 1 and 2, $(\overline{u^2/2 - w})_2 - (\overline{u^2/2 - w})_1 = 0.$ 2. Expands p in terms of ρ to second order. 3. Substitutes in the above equation to give $\overline{\Delta p}_1 - \overline{\Delta p}_2 = \overline{E}_1 - \overline{E}_2.$ 4. Puts the surface 2 outside the disturbance (total reflector) and obtains $\overline{\Delta p}_1 = \overline{E}_1$.
APPROXIMATIONS OR ASSUMPTIONS	Uses second order expressions.
BOUNDARY CONDITIONS	
FINAL RESULT	$\overline{\Delta p}_1 = \overline{E}_1.$

AUTHOR	<u>HERTZ and MENDE</u> (1939)
DEFINITION	P_{LAN} is the difference between the pressure at a wall and the pressure in the medium at rest behind the wall.
EXPERIMENTAL SITUATION	A plane wave.
EQUATIONS USED	The equation of motion in Lagrangian coordinates.
METHOD	1. Uses the equation of motion and Hooke's Law to obtain an equation for ξ. 2. Assumes a solution of the form $\xi = \xi_0 \sin(\omega t - kx)$ and obtains a value for p^*. 3. Transforms p^* to p and space averages to give $\langle p \rangle = p_0 - \langle E \rangle$. 4. Considers a fixed piston A in the end of a closed tube and a movable piston B in the wall of the tube. If B was fixed, then $\langle p_B \rangle = p_0 - \langle E \rangle$, but B can move and will move until $\langle p_B \rangle = p_0$. This compresses the fluid. p_A is $\langle E \rangle$ units larger than p_B, and therefore $p_A = p_0 + \langle E \rangle$.
APPROXIMATIONS OR ASSUMPTIONS	1. An elastic fluid, $p^* - p_0 = (-1/\beta)\delta\xi/\delta a$. 2. The solution for ξ has the form $\xi = \xi_0 \sin(\omega t - kx).$ 3. The transformation from Lagrangian to Eulerian coordinates is to first order.
BOUNDARY CONDITIONS	$\langle p \rangle = p_0$.
FINAL RESULT	$P_{LAN} = p_A - p_0 = \langle E \rangle$.

AUTHOR	<u>BOPP</u> (1940)
DEFINITION	p_{LAN} is the force on a reflector in an open box.
EXPERIMENTAL SITUATION	Standing waves between a fixed piston and a rigid reflector.
EQUATIONS USED	Euler's equation and the equation of continuity.
METHOD	1. Combines Euler's equation and the equation of continuity, time averages and integrates to give $\bar{p} - p_o = C - 2\bar{T}$, where T is the kinetic energy. 2. On the reflector, $u = 0$, and so $(\overline{p - p_o})_s = C.$ 3. Space averages to obtain $\langle \overline{p - p_o} \rangle = (\overline{p - p_o})_s - 2\langle \bar{T} \rangle.$ 4. The boundary condition gives $(\overline{p - p_o})_s = 2\langle \bar{T} \rangle.$
APPROXIMATIONS OR ASSUMPTIONS	The oscillations are small and therefore $2\langle \bar{T} \rangle = \langle \bar{E} \rangle$.
BOUNDARY CONDITIONS	$\langle \overline{p - p_o} \rangle = 0.$
FINAL RESULT	$p_{LAN} = (\overline{p - p_o})_s = \langle \bar{E} \rangle.$

AUTHOR	<u>BOPP</u> (1940)
DEFINITION	p_{RAY} is the force on a reflector in a closed box.
EXPERIMENTAL SITUATION	Standing waves between a fixed piston and a rigid reflector.
EQUATIONS USED	As for p_{LAN} . Also the internal energy is $dU = -pd(1/\rho)$.
METHOD	1. Repeats treatment for p_{LAN} to obtain $\langle \overline{p - p_o} \rangle = (\overline{p - p_o})_s - 2\langle \overline{T} \rangle.$ 2. Uses the equation of state to produce a value for $\bar{U}$. 3. Puts $\langle \overline{\rho U} \rangle = \langle \overline{V} \rangle$ $= [1/(\gamma-1)]\langle \overline{p-p_o} \rangle - [p_o/\rho_o(\gamma-1)]\langle \overline{p-p_o} \rangle,$ where V is the potential energy. 4. The boundary condition gives $\langle \overline{p - p_o} \rangle = (\gamma-1)\langle \overline{V} \rangle.$ 5. Obtains $(\overline{p - p_o})_s = (\gamma-1)\langle \overline{V} \rangle + 2\langle \overline{T} \rangle.$
APPROXIMATIONS OR ASSUMPTIONS	1. The equation of state is $p = p_o(\rho/\rho_o)^\gamma$. 2. The oscillations are small, so $\langle \overline{V} \rangle = \langle \overline{T} \rangle = \langle \overline{E} \rangle/2.$
BOUNDARY CONDITIONS	$\langle \overline{p - p_o} \rangle = 0.$
FINAL RESULT	$p_{\text{RAY}} = (\overline{p - p_o})_s = (\gamma+1)\langle \overline{E} \rangle/2.$

AUTHOR	<u>RICHTER</u> (1940)
DEFINITION	p_{LAN} is the pressure on a perfect reflector in an open vessel.
EXPERIMENTAL SITUATION	Plane waves in an open vessel.
EQUATIONS USED	Euler's equation $\partial v / \partial t = - \nabla (v^2/2 + P)$ where $P = \int dp/\rho$.
METHOD	1. Time averages Euler's equation to obtain$C = \rho_0 \overline{v^2}/2 + \rho_0 \overline{P}.$ 2. Expands $p = f(\rho)$ in terms of ρ to second order. 3. Substitutes for the potential energy E_p. 4. Obtains $C = \overline{E}_k - \overline{E}_p + \overline{p} - p_0$. 5. Spaces averages, uses the boundary condition and the equation $\langle \overline{E}_k \rangle = \langle \overline{E}_p \rangle$, to obtain $C = 0$. On the reflector in a standing wave field, $\overline{E}_k = 0$ and $\overline{E}_p = \overline{E}$, and therefore$\overline{p_s} - p_0 = \overline{E}.$
APPROXIMATIONS OR ASSUMPTIONS	1. The equation of state is $p = f(\rho)$ expanded to second order 2. Uses a second order expression for the potential energy, $E_p = [f'(\rho_0)/2\rho_0](\rho - \rho_0)^2$. 3. $\langle \overline{E}_k \rangle = \langle \overline{E}_p \rangle$.
BOUNDARY CONDITIONS	$\langle \overline{p} - p_0 \rangle = 0.$
FINAL RESULT	$p_{LAN} = \overline{p_s} - p_0 = \overline{E}.$

AUTHOR	<u>RICHTER</u> (1940)
DEFINITION	p_{RAY} is the pressure on a perfect reflector in a closed vessel.
EXPERIMENTAL SITUATION	Plane waves in a closed vessel.
EQUATIONS USED	As for p_{LAN} .
METHOD	1. As in the treatment of p_{LAN}, but obtains an expression for the density, and, $C = \overline{E}_k - (1 - 2\alpha)\overline{E}_p + f'(\rho_0)(\overline{\rho} - \rho_0).$ 2. Space averages and uses the boundary condition to give $C = \alpha\overline{E}$. 3. Substitutes for C in the equation for p to give $\overline{p_s} - p_0 = (1 + \alpha)\overline{E}$, and then substitutes for α in terms of γ.
APPROXIMATIONS OR ASSUMPTIONS	As for p_{LAN} .
BOUNDARY CONDITIONS	$\langle \overline{\rho} - \rho_0 \rangle = 0.$
FINAL RESULT	$p_{\text{RAY}} = \overline{p_s} - p_0 = (1 + \gamma)\overline{E}/2.$

AUTHOR	<u>MENDOUSSE</u> (1950)
DEFINITION	The radiation pressure is the time-averaged excess pressure at a fixed point in space.
EXPERIMENTAL SITUATION	
EQUATIONS USED	The law of compressibility and Euler's equation.
METHOD	1. Expands p to second order in p using the compressibility law. 2. Time averages and substitutes for the potential energy density $\bar{e}_p$. Obtains $\bar{p} = c_o^2 \overline{\Delta \rho} + 2(D - 1) \bar{e}_p.$ 3. Expands Euler's equation to second order. Time averages and integrates to give $\overline{\Delta p} = \bar{e}_p - \bar{e}_k + \text{const.}$ 4. Combines the equations and eliminates $\overline{\Delta p}$. Integrates over a volume V fixed in space with a term to allow for the movement of the edges of the volume. Obtains a value for const. and substitutes.
APPROXIMATIONS OR ASSUMPTIONS	1. Expands to second order. 2. Uses a second order equation for the potential energy density.
BOUNDARY CONDITIONS	
FINAL RESULT	$\overline{\Delta p} = (1/V)[c_o^2 \overline{\Delta M} - (3-2D)\bar{E}_p + \bar{E}_k] + \bar{e}_p - \bar{e}_k.$

AUTHOR	<u>BORGNIS</u> (1953)
DEFINITION	P_{LAN} is the force on an obstacle from plane waves in an open box.
EXPERIMENTAL SITUATION	A plane harmonic wave striking a plane obstacle in an open vessel.
EQUATIONS USED	Euler's equation and the equation of continuity.
METHOD	1. Combines Euler's equation and the equation of continuity, time averages and integrates to give $\bar{C} = \bar{p} + \overline{\rho u^2} = \bar{P}$. 2. Space averages to give $\bar{p} + \overline{\rho u^2} = \langle \bar{p} \rangle + \langle \overline{\rho u^2} \rangle.$ 3. The boundary condition gives $\bar{P} = \langle \overline{\rho u^2} \rangle = 2 \langle \bar{E}_{kin} \rangle.$ 4. A first order solution for u gives $\langle \overline{\rho u^2} \rangle = \rho_0 \langle \bar{u}^2 \rangle = (1 + r^2) \bar{E}_i.$
APPROXIMATIONS OR ASSUMPTIONS	1. The solution for u to first order is of the form $u_1 = w \xi_0 \{ \cos(wt - ka) + r \cos(wt + ka + \theta) \}$. 2. $\bar{E}_i = \rho_0 w^2 \xi_0^2 / 2$.
BOUNDARY CONDITIONS	$\langle \bar{p} \rangle = 0$.
FINAL RESULT	$P_{LAN} = \bar{P} = (1 + r^2) \bar{E}_i$, where r is the reflection coefficient, and E_i is the incident energy density.

AUTHOR	<u>AWATANI</u> (1955)
DEFINITION	The radiation pressure is the time-averaged excess pressure on a surface.
EXPERIMENTAL SITUATION	A wave striking a surface.
EQUATIONS USED	Euler's equation and the equation of continuity.
METHOD	1. Expands Euler's equation to second order and combines it with the equation of continuity. 2. Assumes irrotational motion and introduces the velocity potential ϕ. Obtains a value for the excess pressure in terms of ϕ's. 3. Integrates the excess pressure over the surface and time averages to obtain the radiation pressure.
APPROXIMATIONS OR ASSUMPTIONS	1. Expands to second order. 2. Irrotational motion.
BOUNDARY CONDITIONS	
FINAL RESULT	$W = \langle \int \rho_0 \phi_1 \cos(n\nu) ds \rangle + \int \langle (\rho_0/2c_0^2)(\dot{\phi}_1)^2 - (\rho_0/2)(\nabla\phi_1)^2 \rangle \cos(n_0\nu) ds_0.$

AUTHOR	<u>MAWARDI</u> (1956)
DEFINITION	The radiation pressure is the time-averaged momentum flux.
EXPERIMENTAL SITUATION	
EQUATIONS USED	The equation of continuity in Lagrangian coordinates $\rho_L = \rho_0(1 + \xi_x)^{-1}$.
METHOD	1. Uses the adiabatic equation of state, $p = p_0(\rho/\rho_0)^\gamma.$ 2. Substitutes for the density from the equation of continuity. 3. Expands to second order. 4. Uses the Fubini expression for ξ. 5. Time averages.
APPROXIMATIONS OR ASSUMPTIONS	1. The equation of state is $p = p_0(\rho/\rho_0)^\gamma$. 2. Expands to second order. 3. The Fubini equation for ξ.
BOUNDARY CONDITIONS	
FINAL RESULT	$\bar{\theta}_{11} = - \{ \gamma P_0 [(\gamma+1) v_0^2 / (8c_0^2)] + P_0 \}.$

AUTHOR	<u>WESTERVELT</u> (1957)
DEFINITION	The radiation force is $\underline{F} = \int \overline{p}_{b2} d\underline{A}$, where $\overline{p}_{b2}$ is the time-averaged pressure (to second order) on an obstacle.
EXPERIMENTAL SITUATION	An obstacle in a vessel.
EQUATIONS USED	Euler's equation and the equation of continuity.
METHOD	1. Combines Euler's equation and the equation of continuity, uses $p = p(\rho)$, expands to second order and time averages to give $\overline{p}_2 = -\overline{L} + \text{const}$, where L is the Lagrangian. 2. Changes p_2 from Eulerian to Lagrangian coordinates, $\underline{F} = \int \overline{L} d\underline{A} + \int \overline{\underline{a} \cdot \nabla p_1} d\underline{A}$. 3. Uses the boundary condition and transfers the integration to an arbitrary surface R. 4. Expands $\overline{L}$ etc. in terms of the incident and scattered waves. 5. Assumes the incident wave is plane, which gives $p_i = \rho_o c_o u_i$. 6. Calculates the force in the direction of the incident wave.
APPROXIMATIONS OR ASSUMPTIONS	1. $p = p(\rho)$. 2. Expands to second order, $p = p_o + p_1 + \dots$ etc. 3. $p_L = p_E + \underline{a} \cdot \nabla p_E$. 4. The incident wave is plane.
BOUNDARY CONDITIONS	$\underline{u}$, $\underline{a}$ and $d\underline{A}$ are all parallel on the surface of the obstacle.
FINAL RESULT	$P_{LAN} = F = - (1/\rho_o c_o^2) \int \overline{p_i p_s} (1 + \cos \theta) dA$ $- (1/\rho_o c_o^2) \int \overline{(p_s)^2} \cos \theta dA,$ where θ is the angle between the incident and scattered waves.

AUTHOR KANEVSKII (1961)

DEFINITION The radiation pressure is the force on an obstacle due to the change in momentum of an acoustic wave.

EXPERIMENTAL
SITUATION

EQUATIONS USED The equation of motion in Newtonian form,
 $F = - \partial p / \partial x = \rho dv / dt.$

METHOD

1. Assumes plane waves with $\rho_1 = \rho - \rho_0 = p_1 / c_0^2.$
2. Substitutes into the equation of motion to second order in p_1 and $\rho_1.$
3. Integrates and time averages.
4. Obtains a result for a plane travelling wave.

APPROXIMATIONS
OR ASSUMPTIONS

1. A plane wave with $\rho_1 = p_1 / c_0^2.$
2. Expands to second order.
3. A travelling wave.

BOUNDARY
CONDITIONS

FINAL RESULT $\bar{p} = \overline{p_1^2} / (2\rho_0 c_0^2) + \rho_0 \overline{v^2} / 2 = \bar{E}.$

AUTHOR	<u>GOLDBERG</u> (1971)
DEFINITION	p_{RAY} is the time-averaged pressure on a reflector in a closed box.
EXPERIMENTAL SITUATION	Plane standing waves between two stationary planes in a fluid.
EQUATIONS USED	The equation of motion and the equation of continuity in Lagrangian coordinates.
METHOD	1. Time averages the equation of motion to give $\overline{p_L} = \text{const} = A$. 2. Uses the equation of state, the equation of continuity and a linear approximation for ρ_1 to obtain $\overline{p_L} = [(\gamma+1)/2]\rho_o c_o^2 \langle (\partial \xi / \partial a)^2 \rangle$. 3. Uses a standing wave solution to produce $\overline{p_L} = [(\gamma+1)/8]\rho_o v_o^2$.
APPROXIMATIONS OR ASSUMPTIONS	1. The equation of state to second order is $p_1 = c_o^2 \rho_1 + [(\gamma-1)c_o^2/2\rho_o]\rho_1^2$. 2. Linear approximation $\rho_1 = -\rho_o \partial \xi / \partial a$. 3. Linear standing wave $\xi = (v_o/w) \sin k a \cos \omega t$.
BOUNDARY CONDITIONS	
FINAL RESULT	$p_{RAY} = \overline{p_L} = [(\gamma+1)/8]\rho_o v_o^2$.

AUTHOR	<u>GOLDBERG</u> (1971)
DEFINITION	p_{LAN} is the time-averaged momentum flux per unit area of the surface of an obstacle.
EXPERIMENTAL SITUATION	A plane sound wave travelling in unbounded space.
EQUATIONS USED	Euler's equation of motion and the equation of continuity (including viscosity).
METHOD	1. Combines the equations of motion and continuity. Introduces the quantity$\nabla\psi = \rho_0 \text{div} \underline{v} - (\rho/2) \nabla \underline{v}^2.$ 2. Time-averages, and integrates to second order to obtain $\bar{p} = -\rho_0 \bar{v}^2 - \bar{\psi}$ 3. The boundary condition gives $\bar{p} = -\rho_0 \bar{v}^2$. 4. $\bar{\pi} = \bar{p} + \rho_0 \bar{v}^2 = 0$.
APPROXIMATIONS OR ASSUMPTIONS	1. Irrotational motion. 2. Neglects terms above second order.
BOUNDARY CONDITIONS	Sound wave decays to zero at infinity. ($\bar{\psi} = 0$).
FINAL RESULT	$p_{LAN} = \bar{\pi} = 0$.

AUTHOR	<u>ROONEY and NYBORG (1972)</u>
DEFINITION	p_{LAN} is the time-averaged force per unit area on an obstacle in an acoustic beam.
EXPERIMENTAL SITUATION	A plane travelling wave striking a perfect absorber in an open vessel.
EQUATIONS USED	Euler's equation and the equation of continuity.
METHOD	1. Combines Euler's equation and the equation of continuity, time averages and integrates to give $R = \bar{p} + \overline{\rho u^2}$. 2. Expands to second order to obtain $R - p_o = \bar{p}_2 + \rho_o \bar{u}_1^2.$ 3. The boundary condition and the assumption $\langle \bar{p} \rangle = \bar{p}$ give $\bar{p}_2 = 0$, and therefore $R - p_o = \rho_o \bar{u}_1^2.$
APPROXIMATIONS OR ASSUMPTIONS	1. Expands to second order, $p = p_o + p_1 + p_2$. 2. $\bar{p}_1 = 0$. 3. $\langle \bar{p} \rangle = \bar{p}$. 4. $\rho_o \bar{u}_1^2 = \bar{E}$.
BOUNDARY CONDITIONS	$\langle \bar{p} \rangle = p_o$.
FINAL RESULT	$p_{LAN} = R - p_o = \bar{E}$.

AUTHOR	<u>ROONEY and NYBORG (1972)</u>
DEFINITION	p_{RAY} is the force on an absorber in a closed box.
EXPERIMENTAL SITUATION	A plane travelling wave striking a perfect absorber in a closed vessel.
EQUATIONS USED	As for p_{LAN} .
METHOD	1. Repeats treatment for p_{LAN} with space averaging to give, $R - p_o = \langle \bar{p}_2 \rangle + \rho_o \langle \bar{u}_1^2 \rangle$. 2. The equation of state, the boundary condition and the second order approximation give $\langle \bar{p}_2 \rangle = (B/2A) \langle \bar{E} \rangle$.
APPROXIMATIONS OR ASSUMPTIONS	1. The equation of state to second order is $p = p_o + A(\rho - \rho_o)/\rho_o + (B/2)[(\rho - \rho_o)/\rho_o]^2$. 2. The second order approximation $\langle [(\rho - \rho_o)/\rho_o]^2 \rangle = (w^2 \xi^2)/(2c_o^2) = \bar{E}/A$.
BOUNDARY CONDITIONS	$\langle \bar{p} - \bar{p}_o \rangle = 0$.
FINAL RESULT	$p_{RAY} = R - p_o = (1 + B/2A) \langle \bar{E} \rangle$ $= (1+\gamma) \langle \bar{E} \rangle / 2.$

AUTHOR	<u>BEYER</u> (1978)
DEFINITION	$p_{\text{RAY}} = \bar{p}_L - p_o$
EXPERIMENTAL SITUATION	A travelling wave.
EQUATIONS USED	The equation of continuity in Lagrangian coordinates $\rho_L/\rho_o = (1 + \xi_x)^{-1}$.
METHOD	1. Uses the equation of continuity and the equation of state to obtain $p_L = p_o + \rho_o c_o^2 [-\xi_x + (\gamma+1)\xi_x^2/2].$ 2. Time averages and uses a travelling wave solution for ξ. Obtains $p_{\text{RAY}} = \bar{p}_L - p_o = (\gamma+1)\bar{E}/4.$
APPROXIMATIONS OR ASSUMPTIONS	1. The equation of state is $p = p_o(\rho/\rho_o)^\gamma$. 2. The solution for ξ has the form $\xi = \xi_o [1 - \cos(\omega t - kx)] + [M^2(\gamma+1)x/8][1 - \cos 2(\omega t - kx)],$ where M is the acoustic Mach number.
BOUNDARY CONDITIONS	
FINAL RESULT	$p_{\text{RAY}} = (1+\gamma)\bar{E}/4.$

AUTHOR	<u>BEYER</u> (1978)
DEFINITION	p_{LAN} is the difference between the pressure at a wall and the pressure in the medium at rest behind the wall.
EXPERIMENTAL SITUATION	A perfect absorber in an open vessel.
EQUATIONS USED	As for p_{RAY} .
METHOD	1. Uses the relation between the Lagrangian and Eulerian coordinate systems to obtain (using p_L as for p_{RAY}) $p_E = p_L + \rho_o c_o^2 \xi_{xx}$. 2. Time averages using the solution for ξ to give the pressure on the side wall of the closed vessel $\overline{p'_E} = p_o + (\gamma-3)\overline{E}/4 = \overline{p'_L} - \overline{E}$. 3. For an open vessel $\overline{p'_E} = \overline{p'_L} - \overline{E}' = p_o$. 4. The pressure on the absorber$\overline{p'_L} = p_o + \overline{E}' = p_o + \overline{E}.$
APPROXIMATIONS OR ASSUMPTIONS	1. As for p_{RAY}. 2. $\overline{E}' = \overline{E}$.
BOUNDARY CONDITIONS	The pressure sideways in an open vessel is p_o .
FINAL RESULT	$p_{LAN} = \overline{p'_L} - p_o = \overline{E}.$

AUTHOR	<u>STAPPER</u> (1978)
DEFINITION	p_{LAN} is the force on a boundary caused by alterations in the momentum vector of a wave.
EXPERIMENTAL SITUATION	A plane progressive wave.
EQUATIONS USED	Euler's equation and the equation of continuity.
METHOD	1. Combines Euler's equation and the equation of continuity. Uses the equation of state and the linear relation to obtain an expression for ρ in terms of v. 2. Solves the equation and expands to second order. 3. Obtains an expression for $p v$ and time averages. 4. Uses the assumption $\bar{v} = 0$. 5. Obtains $\bar{p}_L = 2\bar{w}_{kin}$ to second order.
APPROXIMATIONS OR ASSUMPTIONS	1. The equation of state is $p = p(\rho)$, $v = v(\rho)$. 2. The linear relation $(\partial x / \partial t)_v = c_0 + \alpha v$. 3. $\bar{v} = 0$ (no streaming).
BOUNDARY CONDITIONS	
FINAL RESULT	$p_{LAN} = \bar{p}_L = 2\bar{w}_{kin} = \bar{E}$.

AUTHOR	<u>STAPPER</u> (1978)
DEFINITION	p_{RAY} is the scalar hydrostatic pressure due to the non-linear equation of state.
EXPERIMENTAL SITUATION	A plane progressive wave striking a perfect absorber in a closed tube.
EQUATIONS USED	As for p_{LAN} .
METHOD	1. Uses the treatment for p_{LAN} to obtain a relation for p in terms of p. 2. Space averages and uses the boundary condition. 3. Expands to second order which gives $\langle p - p_o \rangle = (\gamma - 1)(\Delta p_{rms})^2 / (2\rho_o c_o^2)$. 4. Obtains an expression for $\langle w_{kin} \rangle$ in terms of p_{rms}, which gives $\langle p - p_o \rangle = (\gamma - 1)\langle w_{kin} \rangle$. 5. Puts $p_R = \langle p \rangle - p_o + p_L = (\gamma - 1)\langle w_{kin} \rangle + 2\bar{w}_{kin}$. 6. Proves that $\langle w_{kin} \rangle = \bar{w}_{kin}$.
APPROXIMATIONS OR ASSUMPTIONS	1. As for p_{LAN}. 2. $\langle w_{kin} \rangle = \bar{w}_{kin}$.
BOUNDARY CONDITIONS	$\langle p \rangle = p_o$.
FINAL RESULT	$p_{RAY} = p_R = \langle p \rangle - p_o + p_L = (\gamma + 1)\bar{E}/2$.

AUTHOR	<u>CHU and APFEL</u> (1982)
DEFINITION	The Rayleigh radiation pressure, $\overline{p_T - p_0}$, is the time-averaged force per unit area on a plane target in a laterally confined beam.
EXPERIMENTAL SITUATION	A plane travelling wave striking a perfectly absorbing target in a closed vessel.
EQUATIONS USED	$u = \int dp/(\rho_0 c_0).$
METHOD	1. Expands the equation for u in a Taylor series about p_0. 2. Assumes that the relation is valid in Lagrangian coordinates. 3. Introduces the dimensionless parameter$K = 1 + (\rho_0/c_0)(dc/d\rho)_0.$ 4. Obtains $\overline{p_T - p_0} = (K/2)\overline{E} = (\gamma+1)\overline{E}/4.$
APPROXIMATIONS OR ASSUMPTIONS	1. Expands to second order. 2. The relation for u is valid in Lagrangian coordinates. 3. The mean kinetic and potential energies are equal.
BOUNDARY CONDITIONS	
FINAL RESULT	$\overline{p_T - p_0} = (\gamma+1)\overline{E}/4.$

AUTHOR	<u>CHU and APFEL</u> (1982)
DEFINITION	The Langevin radiation pressure, $\overline{p_L - p_O}$, is the time-averaged force per unit area on a plane target in a laterally unconfined beam.
EXPERIMENTAL SITUATION	A laterally unconfined wave.
EQUATIONS USED	The Lagrangian form of the equation of motion.
METHOD	1. Assumes the medium is homogeneous and the equation of state is isentropic. 2. Takes time averages. 3. Uses the boundary conditions and expands to second order. 4. Obtains $\overline{p_L - p_O} = \overline{E_L}$.
APPROXIMATIONS OR ASSUMPTIONS	1. The medium is homogeneous and the equation of state is isentropic. 2. Expands to second order.
BOUNDARY CONDITIONS	In an unconfined space $p_L \rightarrow p_O$ and $v_L \rightarrow 0$ as $ a \rightarrow \infty$.
FINAL RESULT	$\overline{p_L - p_O} = \overline{E_L}$.

APPENDIX C

Expansions in Orders of Magnitude

1. GENERAL EQUATIONS

The general one-dimensional equations for a non-viscous lossless fluid obtained in the main part of this thesis are expanded with the aid of a perturbation parameter. This gives Euler's equation of motion:

$$-(\partial/\partial x)(Np_1 + N^2p_2 + \dots) = (\rho_0 + Np_1 + \dots)(\partial/\partial t)(Nu_1 + N^2u_2 + \dots) + (\rho_0 + Np_1 + \dots)(Nu_1 + N^2u_2 + \dots)(\partial/\partial x)(Nu_1 + N^2u_2 + \dots),$$

Equation of continuity:

$$(\partial/\partial t)(Np_1 + N^2p_2 + \dots) + (\partial/\partial x)[(\rho_0 + Np_1 + \dots)(Nu_1 + N^2u_2 + \dots)] = 0,$$

Velocity potential:

$$(Nu_1 + N^2u_2 + \dots) = -(\partial/\partial x)(N\phi_1 + N^2\phi_2 + \dots),$$

Radiation pressure:

$$(NR_1 + N^2R_2 + \dots) = \frac{(N\bar{p}_1 + N^2\bar{p}_2 + \dots)}{+ (\rho_0 + Np_1 + \dots)(Nu_1 + N^2u_2 + \dots)^2},$$

Energy density:

$$(NE_1 + N^2E_2 + \dots) = (\rho_0 + Np_1 + \dots)(\partial/\partial t)(N\phi_1 + N^2\phi_2 + \dots) - (Np_1 + N^2p_2 + \dots),$$

Intensity:

$$(I_1 + N^2 I_2 + \dots) = \frac{(\rho_0 + N\rho_1 + \dots)(N u_1 + N^2 u_2 + \dots)(\partial/\partial t)(N\phi_1 + N^2\phi_2 + \dots)}{(\rho_0 + N\rho_1 + \dots)(N u_1 + N^2 u_2 + \dots)(\partial/\partial t)(N\phi_1 + N^2\phi_2 + \dots)}.$$

The equation of state, $p = f(\rho)$, can be expanded in a Taylor series to obtain

$$p = p_0 + c_0^2(\rho - \rho_0) + [(\rho - \rho_0)^2/2!](d^2p/d\rho^2)_0 + [(\rho - \rho_0)^3/3!](d^3p/d\rho^3)_0 + \dots$$

where $c_0^2 = (dp/d\rho)_0$. Here $(x)_0$ is a shorthand notation for the value of x corresponding to $\rho = \rho_0$.

Using the perturbation parameter gives

$$(N\rho_1 + N^2\rho_2 + \dots) = c_0^2(N\rho_1 + N^2\rho_2 + \dots) + [(N\rho_1 + N^2\rho_2 + \dots)^2/2!](d^2p/d\rho^2)_0 + \dots$$

The different orders of magnitude forms of each equation are obtained by equating coefficients of N .

2. FIRST ORDER (COEFFICIENTS OF N)

$$-\partial p_1/\partial x = \rho_0 \partial u_1/\partial t, \quad \dots(C.1)$$

$$\partial \rho_1/\partial t + \rho_0 \partial u_1/\partial x = 0, \quad \dots(C.2)$$

$$u_1 = -\partial \phi_1/\partial x,$$

$$R_1 = \bar{p}_1,$$

$$E_1 = \rho_0 \partial \phi_1/\partial t - p_1,$$

$$I_1 = 0,$$

$$p_1 = c_0^2 \rho_1.$$

Substituting for u_1 in equation (C.1) and integrating gives

$$p_1 = \rho_0 \partial \phi_1 / \partial t.$$

Hence

$$R_1 = 0,$$

and

$$E_1 = 0.$$

3. SECOND ORDER (COEFFICIENTS OF N^2)

$$-\partial p_2 / \partial x = \rho_1 \partial u_1 / \partial t + \rho_0 \partial u_2 / \partial t + \rho_0 u_1 \partial u_1 / \partial x,$$

$$\partial p_2 / \partial t + (\partial / \partial x)(\rho_0 u_2 + \rho_1 u_1) = 0, \quad \dots (C.3)$$

$$u_2 = -\partial \phi_2 / \partial x,$$

$$R_2 = \overline{p_2} + \overline{\rho_0 u_1^2},$$

$$E_2 = \rho_0 \partial \phi_2 / \partial t + \rho_1 \partial \phi_1 / \partial t - p_2,$$

$$I_2 = \rho_0 \overline{u_1 \partial \phi_1 / \partial t},$$

$$p_2 = c_0^2 \rho_2 + (\rho_1^2 / 2)(d^2 p / dp^2)_0.$$

Solving these equations and using some of the first order expressions gives

$$p_2 = \rho_0 \partial \phi_2 / \partial t + c_0^2 \rho_1^2 / (2\rho_0) - \rho_0 u_1^2 / 2,$$

$$R_2 = \overline{\rho_0 u_1^2} / 2 + \overline{p_1^2} / (2\rho_0 c_0^2),$$

$$E_2 = \rho_0 u_1^2 / 2 + p_1^2 / (2\rho_0 c_0^2),$$

$$I_2 = \overline{p_1 u_1}.$$

4. THIRD ORDER (COEFFICIENTS OF N^3)

$$- \partial p_3 / \partial x = \rho_2 \partial u_1 / \partial t + \rho_1 \partial u_2 / \partial t + \rho_0 \partial u_3 / \partial t + \rho_0 (\partial / \partial x) (u_1 u_2) + \rho_1 u_1 \partial u_1 / \partial x,$$

$$\partial \rho_3 / \partial t + (\partial / \partial x) (\rho_0 u_3 + \rho_1 u_2 + \rho_2 u_1) = 0,$$

$$u_3 = - \partial \phi_3 / \partial x,$$

$$R_3 = \bar{p}_3 + \overline{\rho_1 u_1^2} + 2 \overline{\rho_0 u_1 u_2},$$

$$E_3 = \rho_0 \partial \phi_3 / \partial t + \rho_1 \partial \phi_2 / \partial t + \rho_2 \partial \phi_1 / \partial t - p_3,$$

$$I_3 = \overline{\rho_0 u_1 \partial \phi_2 / \partial t} + \overline{\rho_0 u_2 \partial \phi_1 / \partial t} + \overline{\rho_1 u_1 \partial \phi_1 / \partial t},$$

$$p_3 = c_0^2 \rho_3 + \rho_1 \rho_2 (d^2 p / d\rho^2)_0 + (\rho_1^3 / 3!) (d^3 p / d\rho^3)_0.$$

Solving these equations and using some of the first and second order expressions gives

$$p_3 = \rho_0 \partial \phi_3 / \partial t + c_0^2 \rho_1 \rho_2 / \rho_0 + [\rho_1^3 / (3\rho_0)] (d^2 p / d\rho^2)_0 - c_0^2 \rho_1^3 / (3\rho_0^2) - \rho_0 u_1 u_2,$$

$$R_3 = \overline{p_1 p_2} / (\rho_0 c_0^2) - \overline{p_1^3} / (3\rho_0^2 c_0^4) + \overline{p_1 u_1^2} / c_0^2 + \overline{\rho_0 u_1 u_2} - [\overline{p_1^3} / (6\rho_0 c_0^6)] (d^2 p / d\rho^2)_0,$$

$$E_3 = p_1 p_2 / (\rho_0 c_0^2) - p_1^3 / (6\rho_0^2 c_0^4) + \rho_0 u_1 u_2 + p_1 u_1^2 / (2c_0^2) - [p_1^3 / (3\rho_0 c_0^6)] (d^2 p / d\rho^2)_0,$$

$$I_3 = \overline{p_2 u_1} + \overline{p_1 u_2} + \overline{u_1 p_1^2} / (2\rho_0 c_0^2) + \rho_0 \overline{u_1^3} / 2.$$

5. FOURTH ORDER (COEFFICIENTS OF N^4)

$$- \partial p_4 / \partial x = \rho_0 \partial u_4 / \partial t + \rho_1 \partial u_3 / \partial t + \rho_2 \partial u_2 / \partial t + \rho_3 \partial u_1 / \partial t + \rho_2 u_1 \partial u_1 / \partial x + \rho_0 (\partial / \partial x) (u_1 u_3) + \rho_0 u_2 \partial u_2 / \partial x + \rho_1 (\partial / \partial x) (u_1 u_2),$$

$$\partial \rho_4 / \partial t + (\partial / \partial x)(\rho_0 u_4 + \rho_1 u_3 + \rho_2 u_2 + \rho_3 u_1) = 0,$$

$$u_4 = - \partial \phi_4 / \partial x,$$

$$R_4 = \overline{p}_4 + 2\overline{\rho_0 u_1 u_3} + \overline{\rho_0 u_2^2} + 2\overline{\rho_1 u_1 u_2} + \overline{\rho_2 u_1^2},$$

$$E_4 = \rho_0 \partial \phi_4 / \partial t + \rho_1 \partial \phi_3 / \partial t + \rho_2 \partial \phi_2 / \partial t + \rho_3 \partial \phi_1 / \partial t - p_4,$$

$$I_4 = \overline{\rho_0 u_1 \partial \phi_3 / \partial t} + \overline{\rho_0 u_2 \partial \phi_2 / \partial t} + \overline{\rho_0 u_3 \partial \phi_1 / \partial t} + \overline{\rho_1 u_1 \partial \phi_2 / \partial t} \\ + \overline{\rho_1 u_2 \partial \phi_1 / \partial t} + \overline{\rho_2 u_1 \partial \phi_1 / \partial t},$$

$$p_4 = c_0^2 \rho_4 + (\rho_2^2 / 2)(d^2 p / dp^2)_0 + \rho_1 \rho_3 (d^2 p / dp^2)_0 \\ + (\rho_1^2 \rho_2 / 2)(d^3 p / dp^3)_0 + (\rho_1^4 / 24)(d^4 p / dp^4)_0.$$

Solving these equations and using some of the first, second and third order expressions gives

$$p_4 = \rho_0 \partial \phi_4 / \partial t + c_0^2 \rho_1 \rho_3 / \rho_0 + c_0^2 \rho_2^2 / (2\rho_0) + (\rho_1^2 \rho_2 / \rho_0)(d^2 p / dp^2)_0 \\ + [\rho_1^4 / (8\rho_0)](d^3 p / dp^3)_0 - c_0^2 \rho_1^2 \rho_2 / \rho_0^2 - [\rho_1^4 / (4\rho_0^2)](d^2 p / dp^2)_0 \\ + c_0^2 \rho_1^4 / (4\rho_0^3) - \rho_0 u_1 u_3 - \rho_0 u_2^2 / 2,$$

$$R_4 = \overline{p_1 p_3} / (\rho_0 c_0^2) - [\overline{p_1^4} / (24\rho_0 c_0^8)](d^3 p / dp^3)_0 + \overline{p_2^2} / (2\rho_0 c_0^2) \\ + [\overline{p_1^4} / (8\rho_0 c_0^{10})](d^2 p / dp^2)_0^2 - [\overline{p_2 p_1^2} / (2\rho_0 c_0^6)](d^2 p / dp^2)_0 \\ - \overline{p_2 p_1^2} / (\rho_0^2 c_0^4) + [\overline{p_1^4} / (4\rho_0^2 c_0^8)](d^2 p / dp^2)_0 + \overline{p_1^4} / (4\rho_0^3 c_0^6) + \overline{\rho_0 u_1 u_3} \\ + \overline{\rho_0 u_2^2} / 2 + 2\overline{\rho_1 u_1 u_2} / c_0^2 + \overline{p_2 u_1^2} / c_0^2 - [\overline{p_1^2 u_1^2} / (2c_0^6)](d^2 p / dp^2)_0,$$

$$E_4 = p_1 p_3 / (\rho_0 c_0^2) + [\overline{p_1^4} / (6\rho_0^2 c_0^8)](d^2 p / dp^2)_0 + p_2^2 / (2\rho_0 c_0^2) \\ + [3\overline{p_1^4} / (8\rho_0 c_0^{10})](d^2 p / dp^2)_0^2 - [\overline{p_1^2 p_2} / (\rho_0 c_0^6)](d^2 p / dp^2)_0 \\ - \overline{p_1^2 p_2} / (2\rho_0^2 c_0^4) - [\overline{p_1^4} / (8\rho_0 c_0^8)](d^3 p / dp^3)_0 + \overline{p_1^4} / (12\rho_0^3 c_0^6) \\ + \rho_0 u_1 u_3 + \rho_0 u_2^2 / 2 + p_1 u_1 u_2 / c_0^2 + p_2 u_1^2 / (2c_0^2) \\ - [\overline{p_1^2 u_1^2} / (4c_0^6)](d^2 p / dp^2)_0.$$

APPENDIX D

Streaming and Convection Currents

Streaming in the water bath of the radiation pressure balance may be expected to occur if there is absorption of the ultrasonic wave; momentum is necessarily conveyed to the medium as it is lost from the travelling wave. Convection currents may also occur in the water above the ultrasonic transducer, both from the conversion of the electrical power applied to the transducer into heat, and also from the transformation of the absorbed ultrasonic energy into heat in the body of the water bath.

It is shown in section (III.5.5) that the measured value of the output power from a transducer increases, not only between the transducer and the target, but also between a membrane and the target. This demonstrates that convection currents from the transducer itself, which would be stopped by the membrane, cannot be affecting the power measurements. Both streaming and convection currents from absorption in the bulk of the fluid could be expected to be stopped by the membrane but would restart directly afterwards.

It is difficult to conceive of any experiment that could be done with the radiation pressure balance to distinguish between the effects of streaming and convection currents. Instead, theoretical models were used to estimate the magnitude of the effects from streaming and convection.

1. Streaming

Eckart (1948) gives an expression for the streaming in a cylindrical tube due to a sinusoidal travelling wave with a velocity u_1 of the form,

$$u_1 = u_0 \exp(-\alpha x) \sin(\omega t - kx), \quad \dots(D.1)$$

where u_0 is the particle velocity amplitude, α is the amplitude attenuation coefficient and k is the wave number.

The streaming velocity u is given by

$$u = G[(1-r^2/r_1^2)/2 - (1-r_1^2/2r_0^2)(1-r_2^2/r_0^2) - \ln(r_1/r_0)],$$

where r_1 is the ultrasonic beam radius (assumed to be constant), r_0 is the radius of the tube, r is the perpendicular distance from the beam axis, and

$$G = (4/3 + \eta/\mu)(1/c_0)(ku_0 r_1/2)^2.$$

At the centre of the beam ($r = 0$), the streaming velocity u is therefore given by

$$u = G[0.5 - (1 - r_1^2/2r_0^2) - \ln(r_1/r_0)]. \quad \dots(D.2)$$

In the radiation pressure balance, the water tank has a radius r_0 of approximately 110 mm. In the investigation of the variation of the measured value of the power with distance, a 15 MHz transducer was used. This transducer had a crystal radius of 3.18 mm, and the total ultrasonic power at the transducer was approximately 43 mW. If the wave velocity is given by equation (D.1) and the ultrasonic beam is assumed to be plane, the total power in the ultrasonic field can be approximately given by $\rho_0 c_0 u_0^2 A/2$, where A is the ultrasonic beam area. If the ultrasonic beam is assumed to have a radius equal to the radius of the transducer crystal, then for a

total power of 43 mW, the particle velocity amplitude u_0 is approximately 4.3×10^{-4} m/s. Substituting the values of u_0 , r_1 and r_0 into equation (D.2) gives a value for the streaming velocity u at the centre of the ultrasonic beam equal to 0.16 m/s.

The force on the target produced by the fluid particles streaming with a velocity u can be approximately given by $\rho_0 u^2 A$. Hence if $u = 0.16$ m/s, the force on the target is approximately 8.1×10^{-4} N; equivalent to the force on the target due to the radiation pressure from an ultrasonic beam of 1.2 W total power. Clearly this value for the total force from the streaming is much greater than that occurring experimentally, as the powers determined with the radiation pressure balance are all significantly less than 1 W. There are a number of possible reasons for this discrepancy. Firstly, Eckart's formula is only valid for $\alpha l \ll 1$, where l is the length of the path in the ultrasonic beam. In the radiation pressure balance, the largest value of the transducer/target separation used is 80 mm, and in this case at 15 MHz, $\alpha l = 0.91$, and hence Eckart's formula is not valid in this situation. Secondly, the formula is only applicable for plane waves and, in addition, the particle velocity amplitude has been assumed to be constant over the whole of the ultrasonic beam. Clearly these two assumptions are not generally applicable. Thirdly, even if the streaming velocity of 0.16 m/s was set up in the balance tank, many of the streaming fluid particles would continue along the sides of the conical target. Only for a plane target perpendicular to the ultrasonic beam would all the momentum from the streaming be directly transferred to the target.

It is clear that the calculation used here is not valid for the experimental situation in the radiation pressure balance. No suitable treatment has been found in the literature, however, and

a complete discussion of streaming is outside the scope of this thesis. It is not possible to conclude if streaming of the water in the balance tank is affecting the power measurements. The calculation here demonstrates, however, that any streaming velocities that are set up could be large enough to produce the observed effect.

2. Convection Currents

Convection currents may be set up from absorption of the ultrasound in the bulk of the fluid. The absorption of the ultrasound will raise the temperature of the fluid and hence change its density. The fluid will then start to move with the higher temperature fluid particles moving upwards and being replaced by fluid particles of a lower temperature. The currents produced could have two effects on the target. Firstly, there could be a force on the target from the momentum carried by the currents, and secondly, the change in the temperature of the water around the target will change the apparent weight of the target.

For a plane ultrasonic wave, the intensity I in the ultrasonic field at a distance x from the transducer can be given by $I = I_0 \exp(-2\alpha x)$, where I_0 is the intensity at the transducer. Energy is lost from the sound field at the rate $-dI/dx = 2\alpha I$ per unit volume, and hence, if q is the rate of heat production per unit volume, and assuming all the lost energy is converted into heat, then $q = 2\alpha I$. If some of this heat is lost, for example by conduction, at the rate $K\Delta T$ per unit volume, where K is a constant and ΔT is the rate of increase in the temperature per unit volume, then the remaining heat will raise the temperature of the fluid at a rate so that $2\alpha I - K\Delta T = h\Delta T$, where h is the heat capacity per unit volume (equal to $4.2 \text{ J g}^{-1} \text{ }^\circ\text{C}^{-1}$ for water). Hence the

temperature of the fluid will rise at a rate ΔT of $2\alpha I/(h+K)^{\circ}\text{C}$ per unit volume per second. If it is assumed that, in the steady state situation, the fluid moves with a velocity u , then the total temperature rise T of the fluid as it moves between the transducer and the target can be approximately given by

$$\begin{aligned} T &= \int_0^{x'/u} 2\alpha I_0 \exp(-2\alpha u t)/(h+K) dt, \\ &= I_0 [1 - \exp(-2\alpha x')]/[u(h+K)]. \end{aligned} \quad \dots(D.3)$$

where x' is the separation between the transducer and the target. This temperature rise T of the fluid will cause a change in the apparent weight of the target. It was shown in section (III.4.5) that the apparent weight of the target changes by 43.8 mg for a one degree change in temperature. Hence if the temperature of the fluid changes by 1°C during a power measurement, the value of power determined by the radiation pressure balance will change by 0.61 W.

The change in the value of the power determined by the radiation pressure balance in the investigations described in section (III.5.5) can be calculated by subtracting the measured power W at a distance x' from the transducer from $W_0 \exp(-2\alpha x')$, the actual ultrasonic power at the target. Hence, the change T in the temperature of the fluid required to produce this increase in measured value of the power can be calculated. This value for T can then be substituted in equation (D.3), together with a value for the intensity I_0 determined by dividing W_0 the power at the transducer by the area of the ultrasonic beam. At 15 MHz within the measurement uncertainties, at all the measurement distances the value for $u(h+K)$ obtained from this calculation is approximately constant and equal to $(29.8 \pm 0.7) \times 10^3$. For water, the heat capacity per unit volume, $h = 4.2 \times 10^6 \text{ J m}^{-3} \text{ }^{\circ}\text{C}^{-1}$, and

hence if $K = 0$, (equivalent to assuming that no heat is lost from the fluid), this gives $u = 7 \times 10^{-3}$ m/s. Hence if no heat is lost through conduction or radiation, the fluid moves with a velocity equal to 7×10^{-3} m/s. If all the momentum carried by the moving fluid particles is transferred to the target, the force on the target is approximately 1.6×10^{-6} N; equivalent to the radiation pressure from a beam of ultrasound of 2.3 mW total power. Clearly this calculation could proceed by an iterative process, and after subtracting 2.3 mW from the measured power, a value for u could be recalculated. It does, however, demonstrate that the force on the target from the momentum carried by the convection currents would only produce a small change in the apparent weight of the target. A much larger contribution to the apparent weight change would come from the change in the temperature of the water around the target.

These calculations demonstrate that the velocity of the moving fluid when convection currents occur is significantly less than the velocity present if streaming is the cause of the variation in the measured power with distance. It is not possible, using the approximate treatments given here, to decide whether streaming or convection currents are producing the observed effects. Significantly further work, which is outside the scope of this thesis, is clearly required. It is, however, possible to conclude that either streaming or convection currents, or as is possibly more likely, both effects, are occurring in the radiation pressure balance, and that either of these effects could cause the observed variation of the measured value of the power with distance.

APPENDIX E

The following papers have been published in the scientific literature or presented at conferences, and include some of the work contained in this thesis.

(i) 'Measurement Systems for Ultrasound' K.C.Shotton and A.J.Livett, Acoustics Bulletin, 1979 Vol.4, pp18-21.

(ii) 'Radiation Pressure: Is It or Isn't It?' A.J.Livett, NPL Radiation Science Report RS47, July 1979. Presented at the Bio-Acoustics Group Meeting in London in July 1979.

(iii) 'Development of Measurement Standards and Techniques' A.J.Livett, Proceedings of the Institute of Acoustics, Standardisation in Acoustics, October 1980, NPL, Teddington, pp47-51.

(iv) 'Acoustic Radiation Pressure' A.J.Livett, E.W.Emery and S.Leeman, Journ. Sound and Vib. 1981 Vol.76, pp1-11.

(v) 'A Review of Various Problems Encountered in the Characterisation of Ultrasonic Fields' A.J.Livett, D.R.Bacon and R.C.Preston. Presented at the Conference on Physics in Medical Ultrasound organised by the H.P.A., April 1981 in Durham.

(vi) 'The Absolute Calibration of Hydrophones and Their Use in the Measurement of the Output from Medical Ultrasonic Equipment' A.J.Livett, R.C.Preston and D.R.Bacon, Proceedings of the Institute of Acoustics, Acoustics 82, Spring Conference, University of Surrey, ppB.2.1.1-4.

(vii) 'Absolute Hydrophone Calibration' A.J.Livett, D.R.Bacon and R.C.Preston, Ultrasound 82, Proceedings of the Third Meeting of the World Federation for Ultrasound in Medicine and Biology, Brighton, July 1982. Editors R.A.Lerski and P.Morley, Pergamon Press, pp191-5.

(viii) 'Radiation Pressure and Its Measurement' A.J.Livett and S.Leeman, Proceedings of the 1983 Ultrasonics Symposium, November 1983.

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