

STUDIES ON COLLISIONAL ELECTROMAGNETIC INSTABILITIES
AND LINEAR TRANSPORT
IN LASER-PLASMAS

by

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ABSTRACT

This thesis develops a method for evaluating electron transport coefficients in fully ionized plasmas and investigates the possible occurrence of electromagnetic (EM) instabilities.

The transport coefficients are calculated by finite difference techniques, for arbitrary atomic number and magnetic field strength. By using 100 velocity mesh intervals, their numerical values are more accurate than those obtained by Laguerre expansion methods with up to 20 terms. Significant errors are found in Braginskii's coefficients for certain values of $\omega\tau$ (where ω is the electron cyclotron frequency and τ the mean electron-ion collision time).

The stability analysis is carried out by considering processes occurring on a time scale larger than τ and on a spatial scale larger than an electron mean-free-path. Instability theories based upon fluid models are extended. A more general theory is developed, where the electron Fokker-Planck equation is perturbed, and the electron distribution function expanded in cartesian tensors. The collisional Weibel instability is shown to dominate over several examined instability mechanisms. Its predictions of growth rate and optimum wavelength compare well with experimental observations of filamentary structures in laser-produced plasmas.

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LIST OF FREQUENTLY USED SYMBOLS

Latin symbols

<u>a</u>	acceleration of an electron by an electric field ($=e\mathbf{E}/m$)
<u>b</u>	unit vector in direction of magnetic field ($=\mathbf{B}/ \mathbf{B} $)
<u>B</u>	magnetic field
c	speed of light
c_s	ion sound speed $[(ZT/m_i)^{1/2}]$
C	Fokker-Planck collision operator
D	Fokker-Planck differential operator
<u>dv</u>	volume element in velocity space ($=dv_x dv_y dv_z$)
e	proton charge
<u>E</u>	electric field
f	electron distribution function
f_j	j^{th} order cartesian tensor of the distribution function
f_m	Maxwellian distribution function
F	normalized distribution function
g	Gamma function
I	Fokker-Planck integral operator
$\mathbf{I}_2$	unit dyadic
<u>j</u>	electric current
k	perturbation wave number ($=2\pi/\lambda$)
$l_{T,n}$	temperature, density scale-length
l_s	collisionless skin depth ($=c/\omega_p$)
L	dimensionless scale-length
m	electron mass
m_i	proton mass

n	electron number density
n_i	ion number density
p	electron pressure
q	electron heat flow
q_f	electron heat flow ('free-streaming limit')
$\underline{Q}$	normalized electron heat flow
r	temperature exponent of the electron energy loss
$\underline{R}$	electron-ion momentum exchange rate
s	electron energy loss
S	normalized electron energy loss
t	time
T	electron temperature (measured in enegy units)
T_i	ion temperature (measured in enegy units)
$\underline{v}$	electron velocity
$\underline{v}_i$	ion velocity
v_t	1-D r.m.s. electron velocity $[(T/m)^{1/2}]$
v_T	thermal velocity of an electron of energy T $[(2T/m)^{1/2}]$
$\underline{V}$	normalized electron velocity
$\underline{x}$	cartesian coordinates of position $[(x,y,z)]$
Z	atomic number

Greek symbols

$\underline{\alpha}$	electrical conductivity tensor
$\underline{\beta}$	thermoelectric tensor
γ	growth rate
Γ	normalized growth rate
ϵ	normalized electric field $(=eE/mv_T \nabla_T)$
θ	ratio of hot to cold electron number densities
$\underline{\kappa}$	thermal conductivity tensor

λ	perturbation wavelength
Λ	normalized perturbation wavelength
Λ_{ei}	electron-ion Coulomb logarithm
Λ_{ee}	electron-electron Coulomb logarithm
μ	velocity direction cosine
ν_{ei}	electron-ion collision frequency
ν_{ee}	electron-electron collision frequency
ν_t	mean electron-ion collision frequency ($=1/\tau$)
ν_T	electron-ion collision frequency for an electron of energy T.
ν_T^*	total collision frequency for an electron of energy T.
$\pi_{\perp}$	off-diagonal pressure tensor
σ	electrical conductivity
τ	mean electron-ion collision time ($=1/\nu_t$)
χ_m	magnetic diffusivity
ω	electron cyclotron frequency
ω_p	electron plasma frequency
Ω	normalized electron cyclotron frequency ($=\omega/\nu_T$)

Special symbols and operations

$\parallel \perp$	denotes vector parallel and perpendicular to $\underline{b}$ respectively.
$\wedge$	denotes vector in the $\underline{b} \times \nabla T$ direction.
$\langle A \rangle$	velocity average of 'A(v)' with respect to f_0 ($= \int d\underline{v} A f_0$)
∂_w	derivative with respect to a coordinate w ($= \partial / \partial w$)

UNITS

Gaussian c.g.s. units are used unless otherwise stated.

To my Parents and Monique

CHAPTER 1

Introduction**1.1 Introduction**

This thesis deals with two topics that are relevant to the physics of laser-produced plasmas. The first one is an evaluation of linear transport coefficients for arbitrary magnetic fields and atomic number. The second one is a theoretical investigation of collisional electromagnetic (EM) instabilities, occurring especially at supercritical densities.

Some aspects of these are discussed, in turn, in sections 1.2 and 1.3.

1.2 Transport

In order to assess the feasibility of laser-fusion, it is important to understand the transport processes governing the implosion of a fuel target, such as the thermal flow between the critical surface (where most of the laser energy is deposited) and the ablation surface (where the compression takes place).

Several methods exist for describing the transport in a plasma. They may involve following the particle (electron or ion) trajectories,¹ solving the Fokker-Planck equation for a given particle species,² or solving a set of fluid equations³. All of these, and particularly the first two, tend to require large amounts of numerical computation when applied to realistic transport problems. The particle approach is most appropriate for relatively hot and rare plasmas, where collisions do not play a significant role in the transport, whereas the fluid approach is more appropriate when collisions are dominant.

Solutions of the Fokker-Planck equation are sometimes used to bridge the gap between the particle and fluid predictions, and provide a check on the accuracy of the latter. In particular, one-dimensional (1-D) numerical simulations of the electron thermal flow in a plasma, by means of an expansion of the Fokker-Planck equation, indicate that the fluid description breaks down when $\lambda_t > 0.01 l_T$ (where l_T is the temperature scale-length and λ_t the electron mean-free-path⁴).² Essentially the 'non-linear' heat flow is non-local in nature, and cannot be described by local values of temperature gradients.

Transport processes in laser-plasmas are usually 2-D in nature, by virtue of uneven energy deposition on target, magnetic field generation, instabilities, etc. However, simulations of the transport

by means of 2-D particle or Fokker-Planck codes are presently prohibitive, in terms of computer resources. Thus, despite being the least accurate, 2-D fluid codes are the most commonly used for modelling laser-plasma experiments.

There have been many derivations of fluid equations, whose main differences lie in the method of evaluation and in the numerical values of the transport coefficients. A concise review on this subject is found in an article by Hochstim.⁵

One of the aims of this thesis is to calculate the transport coefficients appropriate to the electron heat flow and Ohm's Law, for a fully ionized plasma of arbitrary atomic number and magnetic field.

1.3 Instabilities

In an idealized situation, the compression of a fuel pellet by means of uniform laser irradiation should produce a perfectly symmetric implosion. One of the main obstacles towards achieving such a goal is the occurrence of plasma instabilities with subsequent disruption of the implosion symmetry. As an example of such phenomena, there is the Rayleigh-Taylor instability⁶ (observed experimentally by A. Cole et al.⁷), which describes the interchange of a dense plasma with a less dense plasma in the accelerating frame of the ablation layer.

Other types of instabilities, for which there have been several experimental reports,⁸⁻¹¹ appear to be linked with electromagnetic effects. In particular, recent experiments performed by Willi et al.⁸ at the Rutherford Appleton Laboratory show the presence of plasma jets, surrounded by megagauss magnetic fields, along the ablation of laser-driven high-Z targets (see Fig. 1.1). These filamentary structures, which measure approximately $10\mu\text{m}$ in the transverse direction, are thought to originate in the overdense plasma.

One of the main motivations for studying the stability of laser-plasmas is based on these experimental observations.

The stability analysis is carried out by means of perturbation theory, under the assumption that the relevant instability (or instabilities) occurs on a time scale that is large compared with the average collision time between electrons and ions. By further assuming that the instability mechanisms can be adequately described in terms of macroscopic plasma quantities, such as the electron number density n , temperature T and pressure p , several theories have been proposed to explain the formation of plasma jets.^{9,12-15}

Most of these instabilities are conveniently classified by separate mechanisms of magnetic B-field generation in the induction equation⁴ (introducing the notation ∂_w to describe a derivative with respect to an arbitrary variable w).

$$\partial_t \underline{B} = \frac{c}{e} \nabla_x [\nabla T \cdot \underline{\beta} + \frac{1}{n} \nabla p + \frac{1}{n} \nabla \cdot \underline{\pi} - e \underline{j} \cdot \underline{\alpha}] + \dots$$

where $\underline{\pi}$ is the electron stress tensor, and $\underline{\alpha}$ and $\underline{\beta}$ are the electrical resistivity and thermoelectric tensors respectively.

The first two terms on the R.H.S. of this equation represent the Nernst¹⁵ and thermomagnetic¹² instabilities, respectively. The third term represents the Weibel instability,⁹ and the last term the Ohmically driven thermal instability.¹³

No definite picture has yet emerged to demonstrate which of the above instabilities (if any) is responsible for the experimentally observed filamentary structures in laser-plasmas.

A purpose of this thesis is to resolve this problem by investigating and expanding the present instability models, and to suggest a new one, based upon a perturbation of the Fokker-Planck equation.¹⁶

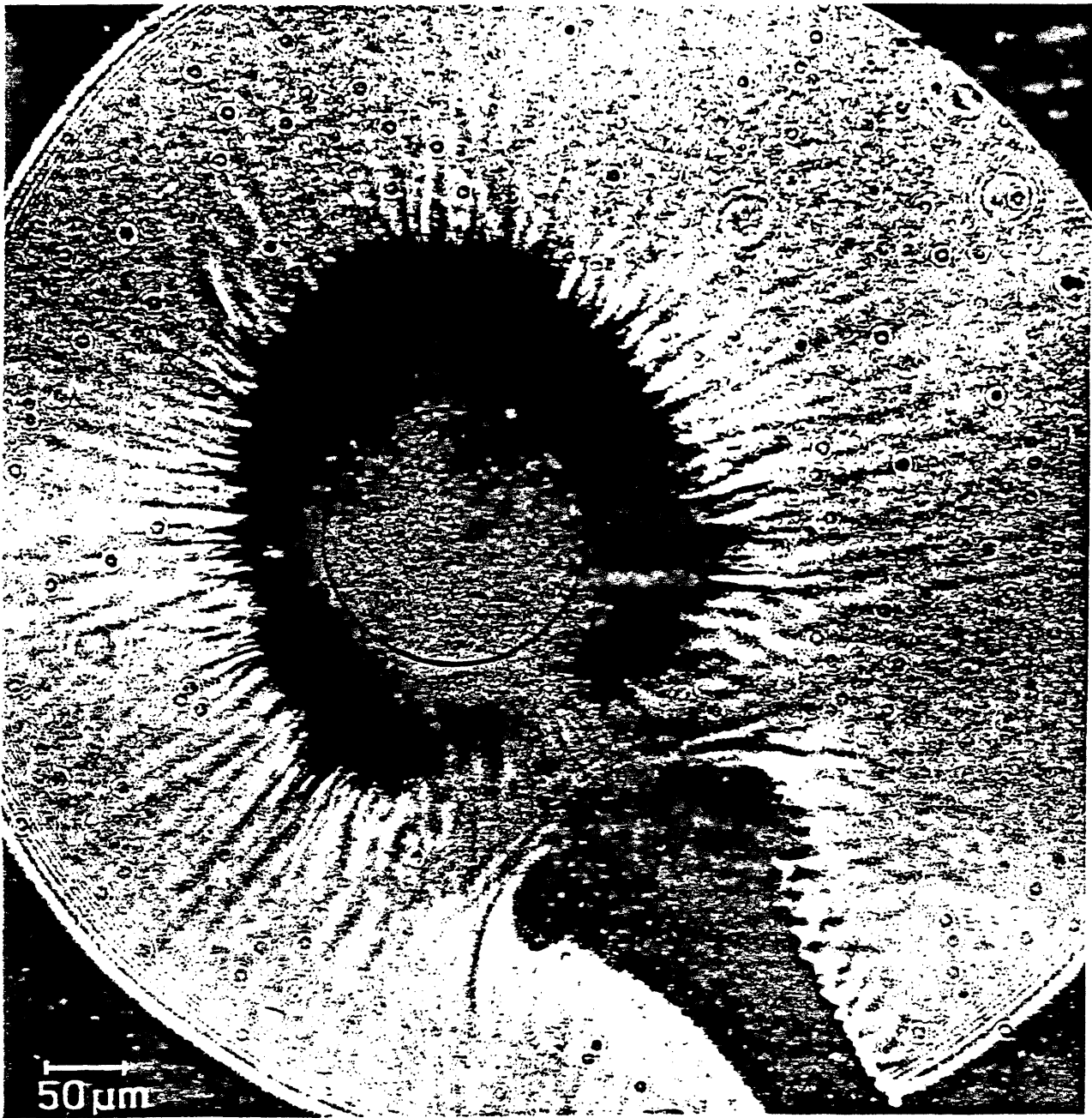


Fig. 1.1 Shadowgram of a gold-coated glass microballoon $156.4 \mu\text{m}$ diameter irradiated at an intensity of $2.5 \times 10^{14} \text{ W cm}^{-2}$ for a laser wavelength of $1.05 \mu\text{m}$, and probed 1.5 ns after the peak of the laser pulse. The circle indicates the original position of the balloon.⁴⁰

1.4 Arrangement of thesis

Chapter 2 introduces a new method for calculating electron transport coefficients for a fully ionized plasma, with arbitrary atomic number and magnetic field strength. The ensuing results are compared with those already published.

A model for studying electromagnetic (EM) instabilities in collisional plasmas is developed in chapters 3-5. Chapter 3 describes the various instability mechanisms, by means of fluid equations, and estimates the growth rates and instability criteria. A kinetic model is then constructed in chapter 4, to analyze the most unstable modes predicted from the fluid model. This theory is further developed in chapter 5, and is subsequently used in chapter 6 to compare its predictions with the filamentary structures observed in laser-plasma experiments. The conclusions drawn are presented in chapter 7.

CHAPTER 2

Linear Transport in Fully Ionized Plasmas

2.1 Introduction

One of the major problems in constructing a set of fluid equations to describe the transport in a fully ionized plasma is to calculate the transport coefficients. These have to be derived from the kinetic equation (usually with a Fokker-Planck collision term) after some suitable approximation.

The most commonly used approach consists of expanding the distribution function $f(\underline{x}, \underline{v}, t)$ in cartesian tensors¹⁷ (or the equivalent spherical harmonic expansion) and truncating the expansion after only the first two terms, i.e. $f_0(\underline{x}, \underline{v}, t) + \underline{f}_1(\underline{x}, \underline{v}, t) \cdot \underline{v}/v$. Since f_0 is isotropic in velocity space, it is $\underline{f}_1$ that describes the transport. Substituting this expansion into the Fokker-Planck equation and taking angular moments, one obtains two coupled integro-differential equations in f_0 and $\underline{f}_1$. Assuming f_0 to be a Maxwellian f_m , $\underline{f}_1$ becomes linear in E , ∇n and ∇T (the electric field, and density and temperature gradient, respectively), and hence the terminology 'linear transport'.

Spitzer and Härm¹⁸ solved the $\underline{f}_1$ equation numerically, in the absence of both magnetic fields and alternating electric fields, and derived transport coefficients for different values of Z . In the case of arbitrary magnetic fields and a.c. electric fields, many papers⁵ have expanded $\underline{f}_1$ in Laguerre (Sonine) polynomials, truncated the

expansion after N terms, and derived the transport coefficients by solving the resultant N coupled equations. Amongst the earliest derivations by this method, Cowling¹⁹ in 1945 used $N=2$, Pruginskii⁴ in 1957 used $N=4$ and Kaneko²⁰ in 1960 used $N=6$. To my knowledge, the latest and most accurate derivation is due to Hochstim (1967),⁵ who used $N=20$.

Other methods, like the one developed by Grad,²¹ involve expanding the distribution in Hermite polynomials. Herdan and Liley²² used this approach, by taking three terms in the expansion, and obtained results equivalent to those of the cartesian tensor expansion with $N=2$. Robinson and Bernstein²³ used a different approach altogether, by employing the variational method to a trial function of six polynomials.

This chapter adopts the cartesian tensor method, but solves the resultant $\underline{f}_1$ equation (with $f_0=f_m$) numerically for arbitrary magnetic fields and Z . Such an approach is equivalent to expanding $\underline{f}_1$ by an infinite number of Laguerre polynomials (i.e. $N=\infty$). For simplicity, only the transport due to electrons is dealt with, the ions being considered fixed and cold. Pressure anisotropy effects are also ignored.

Section 2.2 introduces the transport equations and main definitions. By assuming, initially, a Lorentz plasma (i.e. neglecting electron self-collisions), the $\underline{f}_1$ equation becomes purely algebraic and the transport coefficients can be evaluated by simple numerical integration. This is described in section 2.3. Section 2.4 then develops a method for solving the $\underline{f}_1$ equation with electron-electron collisions included. Discussion and conclusions are presented in section 2.5.

2.2 Transport equations and definitions

2.2.1 Electron kinetic equation

The electron kinetic equation is given by:

$$\partial_t f + \underline{v} \cdot \nabla f - (\underline{a} + \underline{v} \times \underline{\omega}) \cdot \nabla_v f = C, \quad (2.1)$$

where $\underline{a} = eF/m$ is the electron acceleration by an electric field $\underline{E}$, $\underline{\omega}$ the electron cyclotron frequency in a magnetic field $\underline{B}$, $-e$ the electron charge, m the electron mass, c the speed of light and C the rate of change of f due to collisions.

A Fokker-Planck collision term C is used, as described by Shkarofsky et al.¹⁷ Its validity is based on three main assumptions: that the collisions are elastic, that the collisional process is Markovian and that the deflections may be added linearly up to a Debye length.

2.2.2 Macroscopic quantities

Introducing the averaging operator $\langle \rangle$, defined by

$$\langle A \rangle = \frac{1}{n} \int d\underline{v} f A,$$

where $n(\underline{x}, t) = \int d\underline{v} f(\underline{x}, \underline{v}, t)$ is the electron number density, we have

$$\underline{j} = -en \langle \underline{v} \rangle,$$

$$p = nT = \frac{1}{3} mn \langle (\underline{v} - \langle \underline{v} \rangle) \cdot (\underline{v} - \langle \underline{v} \rangle) \rangle,$$

$$\underline{p}_T = mn \langle \underline{v} \underline{v} \rangle,$$

$$\underline{\pi} = \frac{1}{3} mn \langle (\underline{v} - \langle \underline{v} \rangle) (\underline{v} - \langle \underline{v} \rangle) \rangle - \underline{I} p,$$

$$\underline{q} = \frac{1}{2} mn \langle [(\underline{v} - \langle \underline{v} \rangle) \cdot (\underline{v} - \langle \underline{v} \rangle)] (\underline{v} - \langle \underline{v} \rangle) \rangle,$$

$$\underline{q}_T = \frac{1}{2}nm\langle(\underline{v}\cdot\underline{v})\underline{v}\rangle \text{ and}$$

$$U = \frac{1}{2}nm\langle\underline{v}\cdot\underline{v}\rangle ,$$

where $\underline{j}$ is the electric current, T the temperature, p the isotropic pressure, $\underline{p}_T$ the total pressure tensor, $\underline{\pi}$ the stress tensor, $\underline{q}$ the intrinsic heat flow, $\underline{q}_T$ the total heat flow and U the energy density.

Quantities $\underline{q}$, p and $\underline{\pi}$ are defined in the local reference frame of the electrons, whereas $\underline{q}_T$, $\underline{p}_T$ and U are defined in the 'lab' frame ($\underline{j}$ is identical in both cases).

A simple relationship between $\underline{q}$ and $\underline{q}_T$ may be obtained by expanding $\underline{q}$ and omitting $\langle\underline{v}\rangle^3$ terms, since they are proportional to the cube of the current.

$$\underline{q} = \underline{q}_T + \underline{j}\left(\frac{5}{6}m\langle\underline{v}\cdot\underline{v}\rangle + \underline{\pi}/n\right)/e . \quad (2.2)$$

2.2.3 Fluid equations

Multiplying equation (2.1) by 1, $m\underline{v}$ and $\frac{1}{2}m\underline{v}^2$, and integrating in velocity space, we obtain equations describing conservation of number density, momentum and energy, respectively.⁴

$$\partial_t n + \nabla \cdot (n\underline{v}) = 0 , \quad (2.3)$$

$$\partial_t (nm\langle\underline{v}\rangle) + \nabla \cdot \underline{p}_T = nm(\underline{a} + \langle\underline{v}\rangle \times \underline{\omega}) + \underline{R} , \quad (2.4)$$

$$\partial_t U + \nabla \cdot \underline{q}_T = nm\underline{a} \cdot \langle\underline{v}\rangle + H_T , \quad (2.5)$$

where $\underline{R} = \int d\underline{v} m \underline{v} C$ and $H_T = \int d\underline{v} \frac{1}{2} m \underline{v}^2 C$ are the mean rate of change of momentum and energy, respectively, due to collisions with ions (on average there is no net momentum or energy exchange between

electrons).

Equations (2.4) and (2.5) may be rearranged, with the help of the definitions presented in section 2.2.2, into the more familiar form (due to Braginskii⁴), i.e.

$$mn(\partial_t + \langle \underline{v} \rangle \cdot \nabla) \langle \underline{v} \rangle = - \nabla p - \nabla \cdot \underline{\Pi} + nm(\underline{a} + \langle \underline{v} \rangle \times \underline{\omega}) + \underline{R}, \quad (2.4)'$$

$$\frac{3}{2}n(\partial_t + \langle \underline{v} \rangle \cdot \nabla)T + p \nabla \cdot \langle \underline{v} \rangle = - \nabla \cdot \underline{q} - \underline{\Pi} : \nabla \langle \underline{v} \rangle + H, \quad (2.5)'$$

where $H = \int d\underline{v} \frac{1}{2} m [(\underline{v} - \langle \underline{v} \rangle) \cdot (\underline{v} - \langle \underline{v} \rangle)] C$.

In order to solve the above equations it is necessary to establish a relationship between $\underline{\Pi}$, $\underline{q}$ (or $\underline{q}_T$), $\underline{R}$ and H (or H_T) and the parameters n , T and $\langle \underline{v} \rangle$. This functional dependence gives rise to the transport coefficients.

2.2.4 Cartesian tensor expansion

Substituting $f = f_0 + \underline{v} \cdot \underline{f}_1 / v + \underline{v} \underline{v} : \underline{f}_2 / v^2$ into equation (2.1) and making use of the orthogonal properties of this expansion it may be shown that¹⁷

$$\partial_t f_0 + \frac{1}{3} v \nabla \cdot \underline{f}_1 - \frac{1}{3} (1/v^2) \partial_v (v^2 \underline{a} \cdot \underline{f}_1) = \underline{C}_0, \quad (2.6)$$

$$\partial_t \underline{f}_1 + v \nabla f_0 - \underline{a} \partial_v f_0 - \underline{\omega} \times \underline{f}_1 + \frac{2}{5} v \nabla \cdot \underline{f}_2 - \frac{2}{5} (1/v^3) \partial_v (v^3 \underline{a} \cdot \underline{f}_2) = \underline{C}_1, \quad (2.7)$$

where $\underline{C}_0 = (1/4\pi) \int d\underline{\Psi} C$ and $\underline{C}_1 = (1/4\pi) \int d\underline{\Psi} \underline{\Psi} C$ (with $\underline{\Psi} = \underline{v}/v$).

In the limit of $m \ll m_i$, where m_i is the ion mass, the collision term $\underline{C}_0 (= C_{ee0})$ describes the thermalization of the electrons, and $\underline{C}_1 (= C_{ee1} + C_{ei1})$ describes the momentum exchange between electrons, and electrons and ions. Both C_{ee0} and C_{ee1} (defined in appendix A.2) are

integro-differential operators, whereas $\underline{C}_{ei1}$ assumes the simple form¹⁷

$$\underline{C}_{ei1} = - \nabla_{ei} f_1, \quad (2.8)$$

where $\nabla_{ei} = 4\pi n_i (Ze^2/m)^2 \ln \Lambda_{ei}$ ($\ln \Lambda_{ei}$ being the electron-ion coulomb logarithm and n_i the ion number density).

The validity of the cartesian tensor expansion relies on the assumption that the distribution function f is nearly isotropic in velocity. This is expected to be the case when the energy acquired by the electron with collisions with ions is much less than their thermal energy.⁵

From equation (2.6) we may recover the number density and energy conservation relations (Eqs. (2.3) and (2.5)) by multiplying by $4\pi v^2 dv$ and $\frac{1}{2}mv^2 4\pi v^2 dv$, respectively, and integrating in velocity. The average of a scalar variable $A(v)$, a vector $\underline{P}(v) = P(v)\underline{v}/v$ and a dyad $\underline{CC}(v) = C^2(v)\underline{vv}/v^2$ have been taken to be

$$\langle A \rangle = (4\pi/n) \int_0^\infty A f_0 v^2 dv,$$

$$\langle \underline{P} \rangle = (4\pi/3n) \int_0^\infty B \underline{f}_1 v^2 dv \text{ and}$$

$$\langle \underline{CC} \rangle = (4\pi/3n) \int_0^\infty C^2 f_0 \underline{I} v^2 dv + (8\pi/15n) \int_0^\infty C^2 \underline{f}_2 v^2 dv.$$

Equation (2.7) describes the momentum and energy flow, as may be shown by multiplying by $(4\pi v^2/3)mv$ and integrating over velocity, to obtain equation (2.4). Here we note that the $\nabla \cdot \underline{f}_2$ term is responsible for the non-isotropic part of the pressure tensor, whereas ∇f_0 gives rise to the isotropic pressure. Since we will not be considering the influence of pressure anisotropies in the transport,

we set $\underline{f}_2=0$. By further neglecting the time evolution of $\underline{f}_1$, equation (2.7) becomes

$$[D_1 + \underline{\omega} \times] \underline{f}_1 = (v \nabla - \underline{a} \partial_v) f_0, \quad (2.7)'$$

where $D_1 \underline{f}_1 = C_1$.

Instead of solving the above equation self-consistently with equation (2.6), f_0 is assumed to be the local Maxwellian

$$f_m = (n/\pi^{3/2} v_T^3) \exp(-v^2/v_T^2)$$

(where $v_T = (2T/m)^{1/2}$), with the thermodynamic variables n and T being functions of space.

Equation (2.7)' may now be solved and the current and heat flow calculated as follows (using the definitions provided in section 2.2.2, and Eq. (2.2) without the stress tensor contribution).

$$\underline{j} = - (4\pi e/3) \int_0^\infty \underline{f}_1 v^3 dv \text{ and} \quad (2.9)$$

$$\underline{q}_T = (2\pi m/3) \int_0^\infty \underline{f}_1 v^5 dv = \underline{q} - \frac{5}{4} m v_T^2 \underline{j} / e. \quad (2.10)$$

2.2.5 Notation and coordinate system

The notations used for describing the transport fall into two main categories: one, where $\underline{j}$ and $\underline{q}_T$ are expressed in terms of ∇n , ∇T and $\underline{E}$, and the other, where $\underline{E}$ and $\underline{q}$ are expressed in terms of ∇n , ∇T and $\underline{j}$.

In typical laser-plasma simulations, by means of fluid equations, $\underline{E}$ is obtained by integrating Faraday's Law, and $\underline{j}$ by solving Ampere's Law without the displacement current. Hence, the second notation is

found to be preferable.

Braginskii's expressions for Ohm's Law and electron heat flow obey this notation, and are given by⁴

$$\underline{e}n\underline{E} = -\nabla p - en\langle\underline{v}\rangle\underline{x}E/c + \underline{\alpha}\cdot\underline{j}/ne - n\underline{\beta}\cdot\nabla T \quad \text{and} \\ \underline{q} = -\underline{\chi}\cdot\nabla T - \underline{\beta}\cdot\underline{j}T/e ,$$

where $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\chi}$ are the electrical resistivity, thermoelectric and conductivity tensors, respectively, defined in appendix A.1.

Defining dimensionless variables

$$F = (4\pi/n)v_T^3 f , \quad V = v/v_T , \quad \underline{\epsilon} = e\underline{E}/mv_T\lambda_T , \\ \underline{\Omega} = \underline{\omega}/\lambda_T , \quad \underline{\nabla} = \lambda_T \nabla , \quad \underline{J} = \underline{j}/nev_T , \quad \underline{Q} = \underline{q}/nmv_T^3 ,$$

where $v_T = (2T/m)^{1/2}$, $\underline{v}_T = \underline{v}_{ei}(v_T)$ and $\lambda_T = v_T/\underline{v}_T$, the above equations become

$$\underline{\epsilon} = -\frac{\underline{\nabla}p}{2p} + \underline{J}\underline{\Omega} + A\underline{\alpha}^c\cdot\underline{J} - \underline{\beta}^c\cdot\frac{\underline{\nabla}T}{2T} \quad \text{and} \quad (2.11)$$

$$\underline{Q} = -\frac{1}{4A}\underline{\chi}^c\cdot\underline{\nabla}T/T - \frac{1}{2}\underline{\beta}^c\cdot\underline{J} , \quad (2.12)$$

where $A = 4/3\sqrt{\pi} (=0.752)$, $\underline{\alpha}^c = \underline{\alpha}(\tau/mn)$, $\underline{\beta}^c = \underline{\beta}$ and $\underline{\chi}^c = \underline{\chi}(m/nT\tau)$ (with $\tau = 1/A \underline{v}_T$ being the mean electron-ion collision frequency, as defined by Braginskii⁴).

In the subsequent sections a cartesian system of coordinates is used, as shown in figure 2.1, where $\underline{E} = (0, C, B)$ and $\underline{\nabla}T = (\underline{\nabla}_x T, \underline{\nabla}_y T, 0)$.

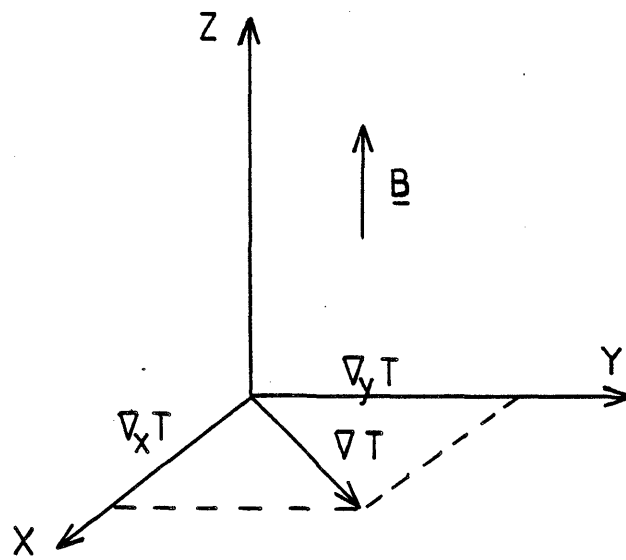


Fig. 2.1 Coordinate system.

2.3 Transport coefficients of a Lorentz plasma

In the Lorentz plasma approximation, the electrons do *only* collide with the background ions, so that equation (2.7)' becomes simply (using Eq. 2.8):

$$\begin{aligned} \nabla_{ei} f_x &= -v \nabla_x f_m - \omega f_y + a_x \partial_v f_m \quad \text{and} \\ \nabla_{ei} f_y &= -v \nabla_y f_m + \omega f_x + a_y \partial_v f_m . \end{aligned}$$

Using $\nabla f_m = \left[\frac{\nabla p}{p} - \frac{5\nabla T}{2T} + (v/v_T)^2 \frac{\nabla T}{T} \right] f_m$ and $\partial_v f_m = -2(v/v_T) f_m$, and substituting for f_x and f_y , we obtain (in dimensionless form)

$$\begin{aligned} F_x &= F_m \left\{ -v^4 \left[(\nabla_x p)/p - \frac{5}{2} (\nabla_x T)/T \right] - v^6 (\nabla_x T)/T \right. \\ &\quad \left. + \Omega v^7 \left[(\nabla_y p)/p - \frac{5}{2} (\nabla_y T)/T \right] + \Omega v^9 (\nabla_y T)/T + 2\Omega \epsilon_y v^7 - 2\epsilon_x v^4 \right\} / (1+v^6 \Omega^2) \end{aligned}$$

$$\begin{aligned} F_y &= F_m \left\{ -v^4 \left[(\nabla_y p)/p - \frac{5}{2} (\nabla_y T)/T \right] - v^6 (\nabla_y T)/T \right. \\ &\quad \left. - \Omega v^7 \left[(\nabla_x p)/p - \frac{5}{2} (\nabla_x T)/T \right] - \Omega v^9 (\nabla_x T)/T - 2\Omega \epsilon_x v^7 - 2\epsilon_y v^4 \right\} / (1+v^6 \Omega^2) \end{aligned}$$

The current $\underline{j}$ and heat flow $\underline{q}_T$, defined by equations (2.9) and (2.10), respectively, may also be written in dimensionless form as follows.

$$\underline{J} = -\frac{1}{3} \int_0^\infty \underline{F}_1 v^3 dv \quad \text{and}$$

$$\underline{Q}_T = \frac{1}{6} \int_0^\infty \underline{F}_1 v^5 dv = \underline{Q} - \frac{5}{4} \underline{J} .$$

Taking the v^3 and v^5 moments of F_x and F_y we, therefore, obtain

$$-3J_x = -\langle v_\Omega^5 \rangle \left[(\nabla_x p)/p - \frac{5}{2} (\nabla_x T)/T \right] - \langle v_\Omega^7 \rangle (\nabla_x T)/T + 2\Omega \epsilon_y \langle v_\Omega^8 \rangle$$

$$+ \Omega \langle v_{\Omega}^8 \rangle [(\nabla_{yP})/P - \frac{5}{2}(\nabla_{yT})/T] + \Omega \langle v_{\Omega}^{10} \rangle (\nabla_{yT})/T - 2\epsilon_x \langle v_{\Omega}^5 \rangle, \quad (2.13)$$

$$\begin{aligned} -3J_y &= -\langle v_{\Omega}^5 \rangle [(\nabla_{yP})/P - \frac{5}{2}(\nabla_{yT})/T] - \langle v_{\Omega}^7 \rangle (\nabla_{yT})/T - 2\Omega \epsilon_x \langle v_{\Omega}^8 \rangle \\ -\Omega \langle v_{\Omega}^8 \rangle [(\nabla_{xP})/P - \frac{5}{2}(\nabla_{xT})/T] &- \Omega \langle v_{\Omega}^{10} \rangle (\nabla_{xT})/T - 2\epsilon_y \langle v_{\Omega}^5 \rangle, \quad (2.14) \end{aligned}$$

$$\begin{aligned} Q_x &= \frac{1}{6} \{ -\langle v_{\Omega}^7 \rangle [(\nabla_{xP})/P - \frac{5}{2}(\nabla_{xT})/T] - \langle v_{\Omega}^9 \rangle (\nabla_{xT})/T \\ &+ \Omega \langle v_{\Omega}^{10} \rangle [(\nabla_{yP})/P - \frac{5}{2}(\nabla_{yT})/T] + \Omega \langle v_{\Omega}^{12} \rangle (\nabla_{yT})/T + 2\Omega \epsilon_y \langle v_{\Omega}^{10} \rangle - 2\epsilon_x \langle v_{\Omega}^7 \rangle \} \\ &+ \frac{5}{4} J_x, \quad (2.15) \end{aligned}$$

$$\begin{aligned} Q_y &= \frac{1}{6} \{ -\langle v_{\Omega}^7 \rangle [(\nabla_{yP})/P - \frac{5}{2}(\nabla_{yT})/T] - \langle v_{\Omega}^9 \rangle (\nabla_{yT})/T \\ &- \Omega \langle v_{\Omega}^{10} \rangle [(\nabla_{xP})/P - \frac{5}{2}(\nabla_{xT})/T] - \Omega \langle v_{\Omega}^{12} \rangle (\nabla_{xT})/T - 2\Omega \epsilon_x \langle v_{\Omega}^{10} \rangle - 2\epsilon_y \langle v_{\Omega}^7 \rangle \} \\ &+ \frac{5}{4} J_y, \quad (2.16) \end{aligned}$$

where the notation $\langle v_{\Omega}^j \rangle = \int_0^{\infty} dV V^{j+2} F_m / (1 + \Omega^2 V^6)$ has been introduced.

Making $\underline{\epsilon}$ the subject of equations (2.13) and (2.14), and comparing the result with equation (2.11), we obtain the following transport coefficients.

$$\alpha_1^c = \frac{3}{2} [A \langle v_{\Omega}^5 \rangle \Delta^c],$$

$$\alpha_{\Lambda}^c = \frac{\Omega}{A} \left[\frac{3}{2} \langle v_{\Omega}^8 \rangle / (\langle v_{\Omega}^5 \rangle \Delta^c) - 1 \right],$$

$$\beta_1^c = [\langle v_{\Omega}^7 \rangle + \Omega^2 \langle v_{\Omega}^8 \rangle^2 \langle v_{\Omega}^{10} \rangle] / \Delta^c - \frac{5}{2} \quad \text{and}$$

$$\beta_{\Lambda}^c = \Omega \langle v_{\Omega}^8 \rangle [\langle v_{\Omega}^{10} \rangle - \langle v_{\Omega}^7 \rangle] / \Delta^c,$$

where $\langle v_{\Omega}^m \rangle \equiv \langle v_{\Omega}^m \rangle / \langle v_{\Omega}^n \rangle$ and $\Delta^c = 1 + \Omega^2 \langle v_{\Omega}^8 \rangle^2$.

Substituting equations (2.13) and (2.14) into (2.15) and (2.16), and comparing the result with equation (2.12), we obtain

$$\chi_{\perp}^c = \frac{2}{3} A \{ \langle v_{\Omega}^9 \rangle - [\langle v_5^7 \rangle + \Omega^2 \langle v_5^8 \rangle \langle v_5^{10} \rangle] \langle v_{\Omega}^7 \rangle / \Delta^c + \Omega^2 \langle v_5^8 \rangle \langle v_5^{10} \rangle [\langle v_8^{10} \rangle - \langle v_5^7 \rangle] \langle v_{\Omega}^5 \rangle / \Delta^c \} \quad \text{and}$$

$$\chi_{\Lambda}^c = \frac{2}{3} A \Omega \{ \langle v_{\Omega}^{12} \rangle - 2 \langle v_5^{10} \rangle \langle v_5^7 \rangle \langle v_{\Omega}^5 \rangle / \Delta^c + \langle v_5^7 \rangle^2 \langle v_{\Omega}^8 \rangle / \Delta^c - \Omega^2 \langle v_5^{10} \rangle^2 \langle v_{\Omega}^8 \rangle / \Delta^c \}.$$

All these coefficients may be solved analytically in the limits $\Omega=0$ and $\Omega=\infty$, by using:

$$\langle v_{\Omega}^j \rangle = \begin{cases} \langle v^j \rangle = \frac{2}{\sqrt{\pi}} g[(j+3)/2], & \text{for } \Omega=0 \\ \langle v^j \rangle = \frac{2}{\sqrt{\pi}} g[(j-3)/2], & \text{for } \Omega=\infty \end{cases}$$

where $g[\dots]$ is the Gamma function.³⁷

Table 2.1 shows the values of the $\perp$ components, for $\Omega=0$ and $\Omega \gg 1$, and of the Λ components, for $\Omega \ll 1$ and $\Omega \gg 1$ (the notation $\omega\tau=(\Omega/A)$ has been used). Braginskii's results are displayed inside brackets.

The coefficients derived here arise from exact moments of $\underline{f}_1$, whereas Braginskii relies on a finite Laguerre expansion for $\underline{f}_1$ to evaluate the moments. His coefficients are, therefore, expected to be less accurate.

From Table 2.1, we observe that most of Braginskii's results lie close to the 'true' solutions to within 10%. The only large error arises in the χ_{Λ}^c coefficient (i.e. the Righi-Leduc heat flow coefficient), which is underestimated by a factor of about 1.6 (or a 40% error) in the limit $\omega\tau \ll 1$. The calculated values of $\beta_{\perp}^c$ and α_{Λ}^c , for $\omega\tau \gg 1$, have been denoted by a * superscript to indicate that they

do not obey the $1/(\omega\tau)^2$ dependence predicted by Priginskii. They do, however, vanish identically in the limit $\omega\tau=\infty$.

For arbitrary values of $\omega\tau$, the coefficients need to be evaluated numerically. This is done in the next section.

	$\alpha_{\perp}^c$	$\beta_{\perp}^c$	$\chi_{\perp}^c$
$\omega\tau=0$	$\frac{3\pi}{32}$ =0.2945 (0.2949)	$\frac{3}{2}$ =1.5 (1.521)	$\frac{128}{3\pi}$ =13.581 (12.471)
$\omega\tau \gg 1$	1 (1)	0^* (3.798/(\omega\tau)^2)	$3.25/(\omega\tau)^2$ (3.25/(\omega\tau)^2)

	$\alpha_{\parallel}^c$	$\beta_{\parallel}^c$	$\chi_{\parallel}^c$
$\omega\tau \ll 1$	$[\pi \frac{315}{512} - 1]\omega\tau$ =0.933 $\omega\tau$ (0.978 $\omega\tau$)	$\frac{315}{32} \omega\tau$ =9.84 $\omega\tau$ (9.13 $\omega\tau$)	$\frac{1085}{2\pi} \omega\tau$ =173 $\omega\tau$ (107 $\omega\tau$)
$\omega\tau \gg 1$	0^* (1.704/(\omega\tau)^2)	$\frac{3}{2}/\omega\tau$ ($\frac{3}{2}/\omega\tau$)	$\frac{5}{2}/\omega\tau$ ($\frac{5}{2}/\omega\tau$)

Table 2.1 Transport coefficients for a Lorentz plasma. Priginskii's coefficients are shown inside brackets. ($\omega\tau = \Omega/A$)

2.4 Transport coefficients for arbitrary Z

Writing the full equation (2.7)' in component form, we have

$$D_1 f_x = g_x + \omega f_y \quad \text{and}$$

$$D_1 f_y = g_y + \omega f_x ,$$

where $g_{x,y} = [v \nabla_{x,y} - a_{x,y} \partial_v] f_m$ and D_1 is an integro-differential operator (defined by $D_1 f_1 = C_1$, where C_1 is given in appendix A.2), which includes e-e collisional effects.

Substituting for f_x and f_y in the above equations, gives us

$$[D_1 D_1 + \omega^2] f_x = D_1 g_x + \omega g_y \quad \text{and} \quad (2.17)$$

$$[D_1 D_1 + \omega^2] f_y = D_1 g_y - \omega g_x . \quad (2.18)$$

These need to be solved numerically.

By finite differencing in velocity space, the operators D_1 and $[D_1 D_1 + \omega^2]$ become full matrices, and $g_{x,y}$ and $f_{x,y}$ become column vectors (as described in appendix A.3).

The finite difference version of equations (2.17) and (2.18) are solved directly for $f_{x,y}$ with the matrix operations (i.e. multiplication, inversion, etc.) being performed via NAG library routines, available at the Imperial College Computer Centre.

The resultant values of f_x and f_y are substituted into equations (2.9) and (2.10), and the transport coefficients derived following the procedure outlined in section 2.3. These are then plotted (solid lines) over a wide range of $\omega\tau$, in figures (2.2)-(2.7), for $Z=1$ and $Z=\infty$. The equivalent Braginskii coefficients are also plotted (dashed lines) in the same figures. When Z is finite, the relation $\nu_{ei} = Z \nu_{ee}$

is used (which involves the approximation that $\ln \Lambda_{ei} = \ln \Lambda_{ee}$). For infinite Z the results become identical to those obtained using the Lorentz plasma approximation (section 2.3).

The numerical calculations have been performed by taking $v_{\max} = 5v_T$ (=maximum velocity) and a mesh spacing $\Delta v = 0.1v_T$. The accuracy of the results was checked by increasing $v_{\max}$ and decreasing Δv .

From the figures we observe oscillations in Praginskii's results when $\omega\tau \sim 1$. This is a characteristic feature of the polynomial fit in $\omega\tau$, applied to his coefficients. Their overall accuracy depends, however, on both this oscillatory behaviour and the errors arising from the truncation of the Laguerre polynomial expansion for f_1 .

Considering initially the $Z=\infty$ results, we investigate the accuracy of each Praginskii coefficient in turn.

(i) $\alpha_{\perp}^c$. For $\omega\tau \gg 1$ and $\omega\tau \ll 1$ both results coincide to a high degree of accuracy. A maximum error of about 25% occurs for $\omega\tau \approx 10$.

(ii) $\alpha_{\parallel}^c$. The error is small for $\omega\tau \ll 1$ but there is a large discrepancy for $\omega\tau > 1$. This, however, is of no consequence, since the $\underline{J} \times \underline{\Omega}$ term in equation (2.11) dominates over the Λ component of $A\alpha_{\parallel}^c \cdot \underline{J}$ by a factor of $\omega\tau/\alpha_{\parallel}^c$.

(iii) $\beta_{\perp}^c$. As in the previous case, the error is small for $\omega\tau \ll 1$ but starts to deviate considerably when $\omega\tau > 1$. It is not clear, however, what direct consequences such a discrepancy might have on transport calculations.

(iv) $\beta_{\parallel}^c$. Within the range $0.3 < \omega\tau < 30$ the error oscillates from about -30% to 50%.

(v) $\chi_{\perp}^c$. Within the range $0.3 < \omega\tau < 30$ the error oscillates from about -35% to 65%.

(vi) $\chi_{\parallel}^c$. The error of -40% (predicted in section 2.3) persists

for $\omega\tau$ up to about 10^{-2} . As $\omega\tau$ increases beyond this value, the errors decrease to an absolute value of less than 20%.

Similar behaviour is found for $Z=1$ and other values of Z , though there is an overall improvement in the accuracy of Priginskii's coefficients as Z decreases.

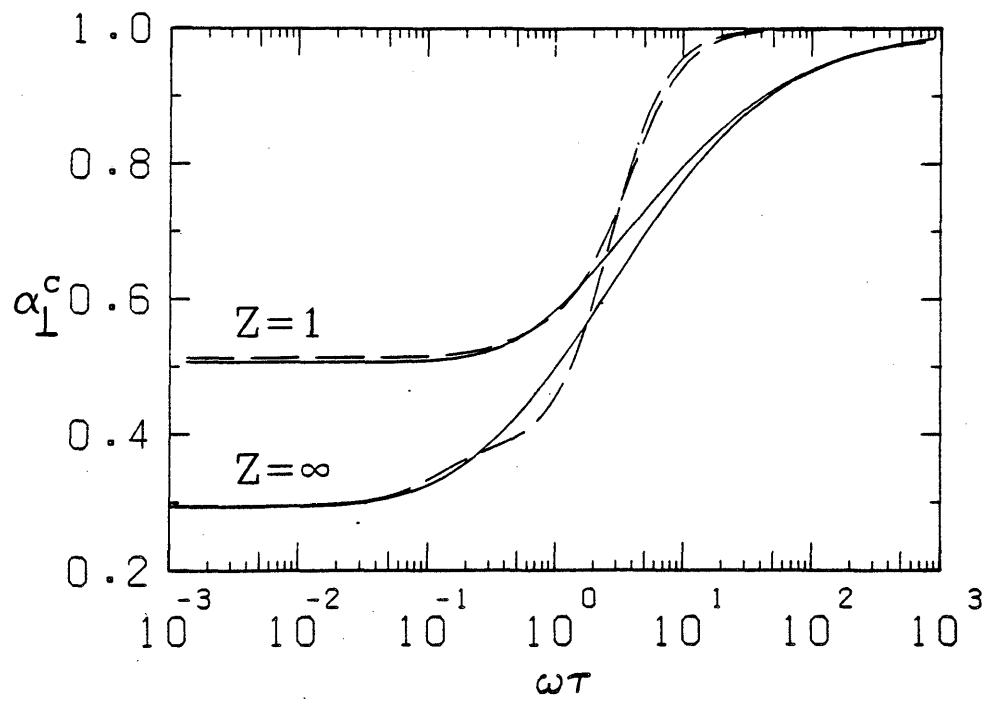


Fig. 2.2

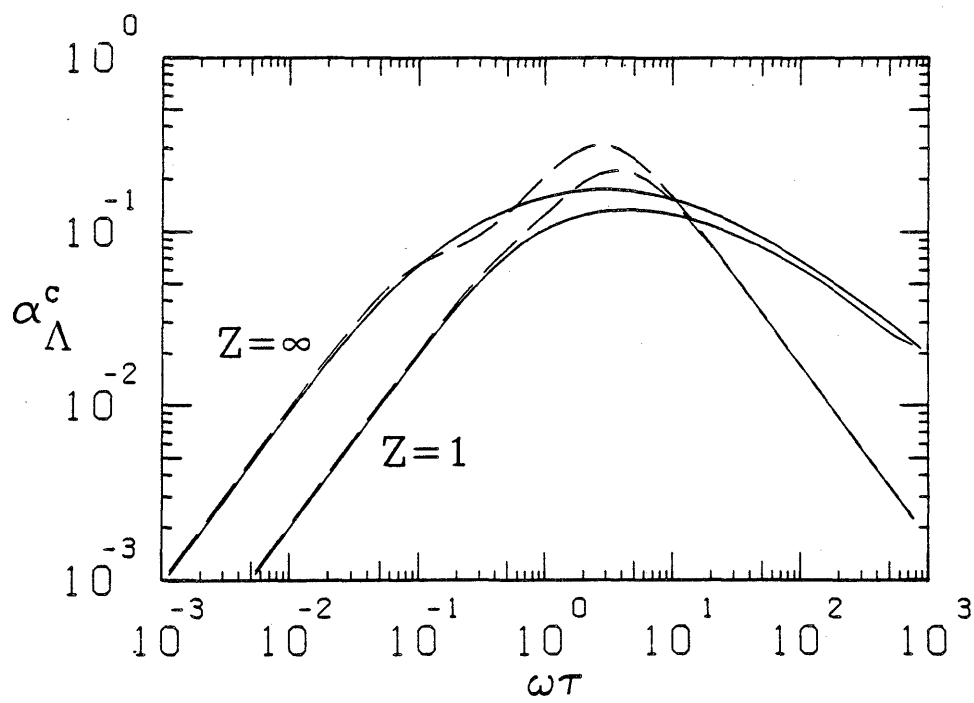


Fig. 2.3

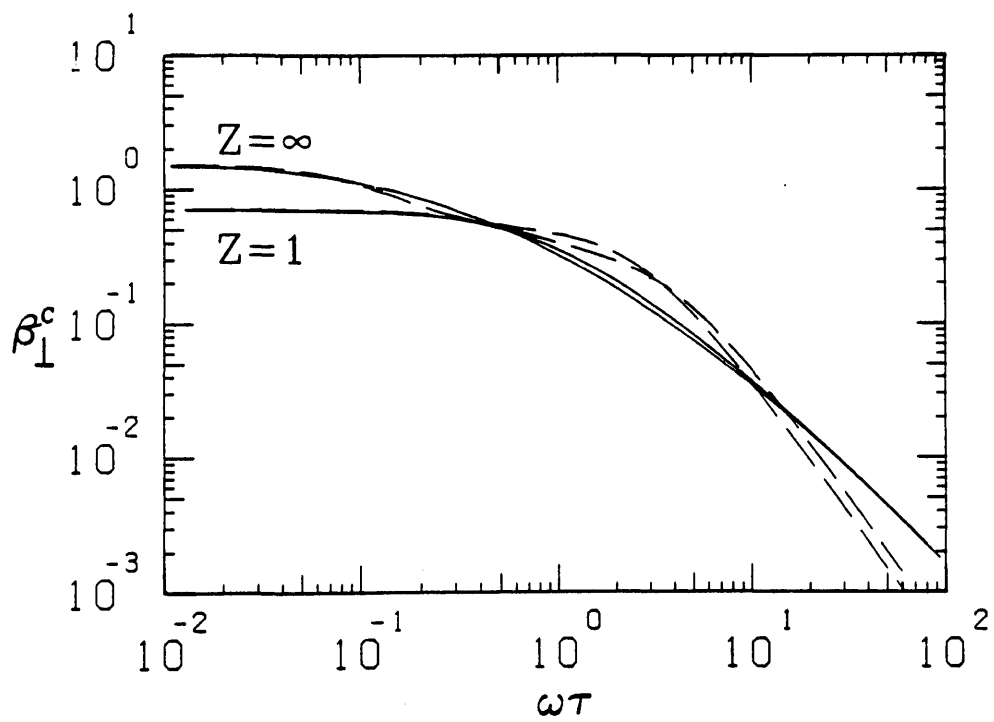


Fig. 2.4

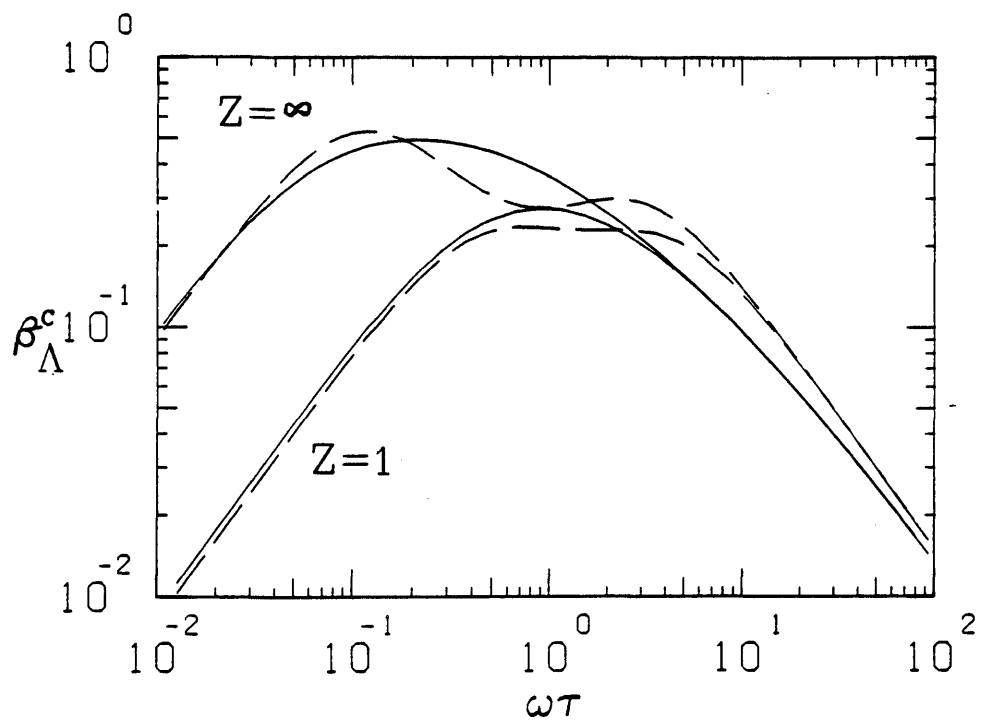


Fig. 2.5

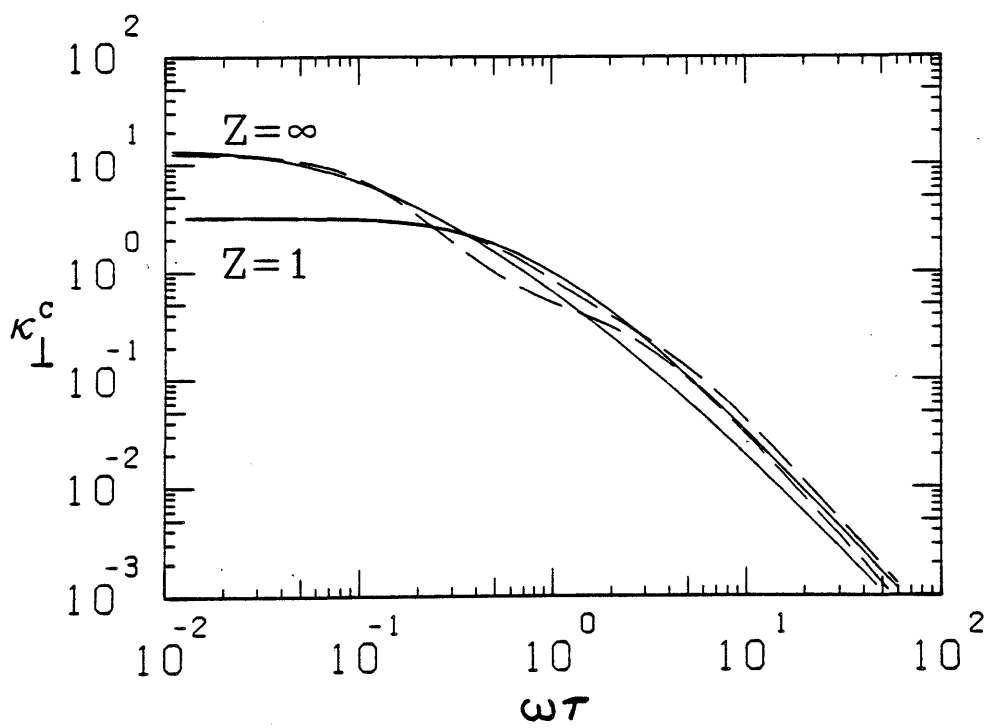


Fig. 2.6

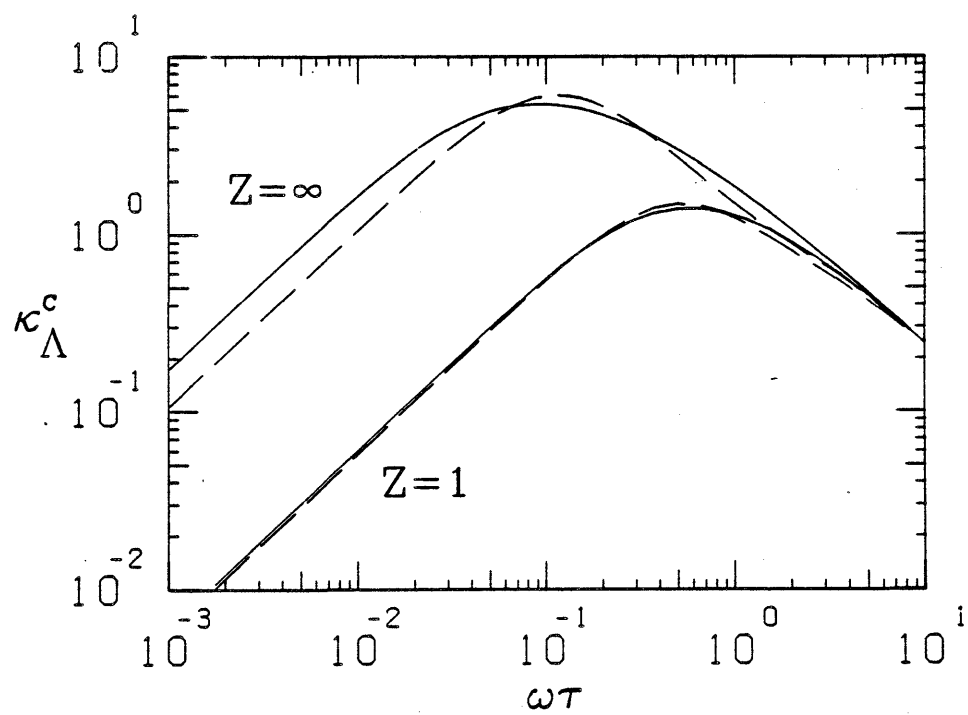


Fig. 2.7

2.5 Discussion and conclusions

In order to assess the accuracy of the coefficients evaluated in this chapter, these will be compared with Hochstim's coefficients,⁵ which were derived by means of a 20-term Laguerre expansion for f_1 .

Using Hochstim's notation, Ohm's Law and electron heat flow are expressed as follows.

$$\underline{j} = \underline{\sigma}_{dc} \cdot \underline{E} + e\underline{D} \cdot \nabla n + e\underline{\tau} \cdot \nabla T \quad \text{and}$$

$$\underline{q}_T = -\underline{\sigma}_q \cdot \underline{E} - \underline{Q} \cdot \nabla n - \underline{K} \cdot \nabla T.$$

The coefficient $\underline{\sigma}_{dc}$ represents the electric conductivity, $\underline{D}$ the current diffusion, $\underline{\tau}$ the current thermal diffusion, $\underline{\sigma}_q$ the energy electric conductivity, $\underline{Q}$ the energy diffusion and $\underline{K}$ the thermal conductivity. Their $\perp$ and $\parallel$ components are proportional to

$$\frac{g}{[g^2 + (\omega\tau)^2 h^2]} \quad \text{and}$$

$$\frac{(\omega\tau)h}{[g^2 + (\omega\tau)^2 h^2]},$$

respectively, where g and h are functions $\omega\tau$ and Z (which are usually different for the various transport coefficients).

Hochstim tabulated the quantities g and h for several values of $\omega\tau$, and for $Z=1, 2, 4, 16$ and ∞ . Table 2.2 compares our results, with Hochstim's (shown inside brackets), by considering the particular case of $Z=1$ and $\omega\tau=10^{-6}, 1$ and 10^6 .

As observed from the table, both values agree to no less than 3

significant figures. This certifies the earlier assertion that the method for evaluating transport coefficients, developed in this chapter, is equivalent to having an 'infinite' number of Laguerre polynomials. Theoretically, the finite difference technique employed in our analysis requires the mesh size Δv to be small enough to yield the desirable amount of accuracy. In practice, errors of less than 1% are expected by using only 50 mesh points of $\Delta v = 0.1v_T$.

In conclusion, a simple and efficient method has been developed for calculating transport coefficients in a magnetized fully ionized plasma of arbitrary Z .

Most Braginskii coefficients are found to be highly inaccurate over certain ranges of $\omega\tau$, especially for high- Z plasmas. In the particular case of a Lorentz plasma, the Righi-Leduc heat flow coefficient (χ_Λ^c) is underestimated by approximately 40% when $\omega\tau < 10^{-2}$. The lateral heat flow coefficient ($\chi_\perp^c$) has absolute errors of at least 35% in the range $0.3 < \omega\tau < 30$ (reaching a maximum of 65% at $\omega\tau = 5$).

$\omega\tau$	g_σ	h_σ	g_τ	h_τ
10^{-6}	0.50612 (0.50613)	1.1989 (1.1989)	0.2972 (0.2972)	0.8582 (0.8579)
1	0.5823 (0.5823)	1.1034 (1.1034)	0.2539 (0.2539)	0.8663 (0.8663)
10^6	1.0000 (1.0000)	1.000 (1.000)	-0.5000 (-0.5000)	1.0000 (1.0000)

$\omega\tau$	g_μ	h_μ	g_K	h_K
10^{-6}	0.3951 (0.3950)	1.0446 (1.0445)	0.3577 (0.3577)	1.0273 (1.0262)
1	0.4130 (0.4130)	1.0062 (1.0062)	0.3648 (0.3648)	1.0057 (1.0057)
10^6	0.4000 (0.4000)	1.0000 (1.0000)	0.38284 (0.38284)	1.0000 (1.0000)

Table 2.2 Transport coefficients using Hochstim' notation, for $Z=1$, $v_{\text{max}}=5v_T$ and $\Delta v=0.05v_T$. (Hochsim's coefficients shown inside brackets).

CHAPTER 3

A Fluid Approach to EM Instabilities

3.1 Introduction

In an attempt to explain magnetic field generation and filamentary structures observed in laser-compression experiments,⁸⁻¹¹ many electromagnetic (EM) instability theories, based on fluid models, have been developed.^{9,12,13}

The earliest theories were published independently by Tidman and Shanny and Bol'shov et al.¹² They used the model of a collisional heat carrying plasma, with fixed ions and zero magnetic field (the 1-fluid model), and carried out the stability analysis with Braginskii⁴ fluid equations, in planar geometry. The instability mechanism proposed by them relies on a local enhancement of the temperature caused by the Righi-Leduc heat flux. This heat flux, in turn, arises as a result of magnetic fields being generated from an initial temperature perturbation. A necessary condition for the instability to grow is that both density and temperature gradients be in the same direction.

Improvements on the 1-fluid EM instability theory were later achieved by including current convection, ion motion, plasma ablation, and Nernst effects.^{15,24,25}

An altogether different approach has been taken by Haines¹³ and Mochizuki et al.⁹, who both devised a model based on counter-streaming hot and cold electron fluids (the 2-fluid model). Haines' instability mechanism may be described as follows. When a temperature

perturbation is applied to a plasma, the electrical conductivity increases where the temperature is a maximum. This raises the local return current and Ohmic heating rate, and may thus lead to unstable growth of the initial perturbation. The instability theory due to Mochizuki et al.,⁹ on the other hand, is based on the fact that the counter-streaming electrons experience Lorentz forces when subjected to magnetic field perturbations. This leads to local net currents and possible unstable growth of the initial field perturbation (similar to a Weibel instability²⁶).

The aim of this chapter is to analyse and develop both 1-fluid and 2-fluid FM instability theories. In section 3.2 the basic equations are defined. Section 3.3 deals with the 1-fluid model, where the effects of using more accurate transport coefficients (derived in chapter 2), instead of Braginskii's, are investigated. Section 3.4 describes an improved theory based on the 2-fluid model. The general conclusions drawn from both models are presented in section 3.5.

For simplicity all the analyses are carried out under the assumption that the ions are fixed, cold and fully ionized. This should not greatly affect the conclusions drawn from this chapter.

3.2 Basic equations

Using Pragenskii's formalism,⁴ the electron density, momentum and energy conservation equations (neglecting viscous and radiation pressure effects) are given by:

$$\partial_t n + \nabla \cdot (n \underline{v}) = 0, \quad (3.1)$$

$$m n (\partial_t + \underline{v} \cdot \nabla) \underline{v} = -\nabla p - e n (\underline{E} + \underline{v} \times \underline{B}/c) + \underline{R}, \quad (3.2)$$

$$n (\partial_t + \underline{v} \cdot \nabla) (3T/2) = -p \nabla \cdot \underline{v} - \nabla \cdot \underline{q} + \underline{j} \cdot \underline{R}/ne - s, \quad (3.3)$$

where $n(=Zn_i)$ is the electron number density, T the electron energy, $p=nT$ the electron pressure, $\underline{v}$ the electron velocity, Ze the ionic charge, $\underline{j}=-nev$ the current, $\underline{E}$ the electric field and $\underline{B}$ the magnetic field. The electron-ion momentum exchange rate $\underline{R}$ and the intrinsic electron heat flow $\underline{q}$ are defined as follows.

$$\underline{R} = \underline{\alpha} \cdot \underline{j}/ne - n \underline{\beta} \cdot \nabla T$$

$$\underline{q} = -\underline{\chi} \cdot \nabla T - \underline{\beta} \cdot \underline{j}T/e$$

Tensors $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\chi}$, represent the electrical resistivity, thermoelectric effects and thermal conductivity, respectively. Their components, relative to $\underline{b}=\underline{E}/|\underline{E}|$, are listed in appendix A1.

Term $s \propto n_i n T^r$, in equation (3.3) represents any electron energy loss mechanism due to, for example, electron-ion collisions ($r=-1/2$) or Bremsstrahlung ($r=1/2$).

Lastly we need Faraday's and Ampere's law:

$$\partial_t \underline{B} = -c \nabla \times \underline{E} , \quad (3.4)$$

$$\nabla \times \underline{E} = -4\pi \underline{j}/c , \quad (3.5)$$

where c is the speed of light.

3.3 The 1-fluid model

3.3.1 Derivation of the dispersion relation

In the unperturbed state the plasma is assumed to be homogeneous in the x - y plane, and to have known density and temperature profiles along z . The perturbation is applied in the form $\exp(\gamma t + i k x)$, where γ and k obey the collisionality conditions:

$$\gamma \ll \nu_t \quad (3.6)$$

$$2\pi/k (= \lambda) \gg \lambda_t$$

with ν_t being the mean electron-ion collision frequency (Braginskii, 1965),⁴ $\lambda_t (= v_t / \nu_t)$ the electron mean-free-path and $v_t = (T/m)^{1/2}$ the characteristic electron thermal speed.

A further restriction on k is that:

$$2\pi/k (= \lambda) \ll l_{T,n} \quad (3.7)$$

where $l_{T,n} = [\partial_z \ln(T,n)]^{-1}$. This is called the 'local approximation', which allows us to neglect the z -dependence of the perturbed quantities.³⁶

The assumption of fixed ions restricts the theory to predicting growth times that are smaller than a sound transit time across a perturbation wavelength, i.e.

$$\gamma \gg k c_s \quad (3.8)$$

where $c_s = (ZT/m_i)^{1/2}$ is the isothermal ion sound speed.

Finally, we require the unperturbed plasma to be sufficiently collisional (i.e. $\lambda_t \ll l_{T,n}$), and to be evolving on a time scale that is large compared with the growth time of the instability (i.e. $\gamma^{-1} \ll l_{T,n}/c_s$).

Before linearizing the transport equations, it is convenient to derive the magnetic induction equation, by substituting the electron momentum equation (3.2) into Faraday's law (3.4), and neglecting the electron inertia:

$$\partial_t \underline{B} = \nabla \times (\underline{v} \times \underline{B}) - \frac{c}{ne} \nabla n \times \nabla T - c \nabla \times (\underline{j} \cdot \underline{\alpha}) + \frac{c}{e} \nabla \times (\underline{\beta} \cdot \nabla T) \quad (3.9)$$

The energy equation (3.3) may also be rearranged into a more suitable form, i.e. :

$$\begin{aligned} n \partial_t (3T/2) = & \nabla \cdot (\underline{\alpha} \cdot \nabla T) + \underline{j} \cdot \nabla (3T/2)/e + p \nabla \cdot (\underline{j}/ne) \\ & + T \nabla \cdot (\underline{\beta} \cdot \underline{j})/e + \underline{j} \cdot \underline{\alpha} \cdot \underline{j} - s \end{aligned} \quad (3.10)$$

Assuming the perturbation configuration shown in figure 3.1 (i.e. $\underline{k} \perp \underline{B}' \perp \nabla T, \nabla n$), with the dash denoting first order quantities, the above two equations and Ampere's law become:

$$\begin{aligned} \gamma \omega' = & -ikv_t (\lambda_t/l_n) \gamma_t (T'/T) + \alpha_0 ik \gamma_t (j'/ne) \\ & + \frac{\beta_0''}{\delta_0} [(\Delta + 3/2) \lambda_{t/1T} - \lambda_{t/1n}] (\lambda_{t/1T}) \gamma_t \omega', \end{aligned} \quad (3.11)$$

$$\gamma (3T'/2) = -\gamma_0 k^2 \lambda_t^2 \gamma_t T' + \frac{\gamma_0''}{\delta_0} ik \lambda_t (\lambda_{t/1T}) \omega' T, \quad (3.12)$$

$$ik \omega' = (1/l_s)^2 (j'/ne), \quad (3.13)$$

where $\omega = (eB/mc)$ and $\omega_F = (4\pi ne^2/m)^{1/2}$ are the electron cyclotron and

plasma frequency, respectively, $l_s = c/\omega_p$ is the collisionless skin depth and $\Delta = l_T^2 (\partial_z^2 T)/T$. Pragenskii coefficients α_0 , β_0'' , γ_0 , γ_0'' , and δ_0 arise from the linearization of the tensors $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\chi}$ (see appendix A1).

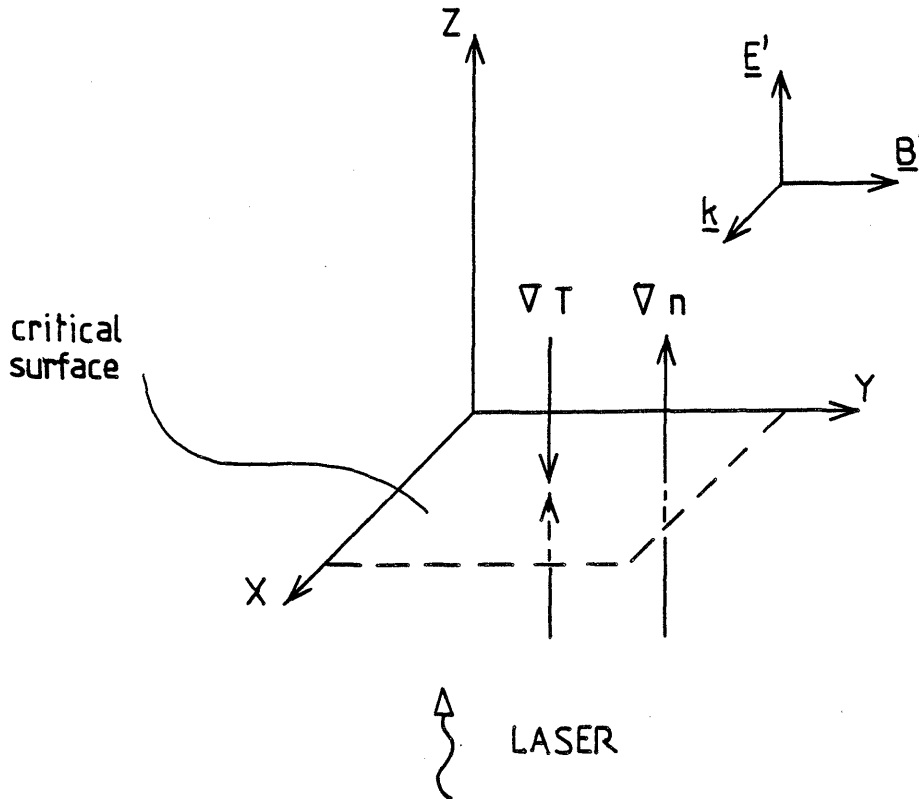


Fig. 3.1 Perturbation configuration for the 1-fluid model.

As shown above, the only term that survives from the R.H.S. of equation (3.10) is the electron diffusion $\nabla \cdot (\underline{\chi} \cdot \nabla T)$. The current convection term $\underline{j} \cdot \nabla (3T/2)/e$ and the pressure term $p \nabla \cdot (\underline{j}/ne)$ have been found by Pert²⁴ to give rise to thermal magnetic waves and possible magnetic hot spot formation. These effects are, however, only important for $\lambda \gtrsim l_{n,T}$, where our 'local approximation' breaks down. Unstable phenomena arising from the sink term s have been

investigated by Haines¹³ and Evans¹⁴, and were found to be significant for cold and dense high-Z plasmas,²⁷ where the present instability mechanism becomes weak. Its effects will be discussed in section 3.4.

Solving equations (3.11)-(3.13) we arrive at the following dispersion relation.

$$\Gamma^2 + \Gamma \{ \alpha_0 k^2 L_s^2 + (2\gamma_0/3) k^2 - S^N \} + (2\gamma_0/3) k^2 \{ \alpha_0 k^2 L_s^2 - S^N - S^T \} = 0 \quad (3.14)$$

where

$$\Gamma = \gamma / \gamma_t, \quad K = k \lambda_t, \quad L_{T,n,s} = l_{T,n,s} / \lambda_t,$$

$$S^N = (\beta_0'' / \delta_0) \{ (\Delta + 3/2) / L_T^2 - 1 / L_n L_T \}, \quad S^T = (\gamma_0'' / \gamma_0') / L_n L_T$$

are suitable dimensionless quantities.

3.3.2 Properties of the dispersion relation

In the dispersion relation (3.14), there are two possible sources of instability: S^N and S^T , where $S^N, S^T > 0$ is a necessary condition for destabilization to occur. These source terms are functions of Δ , L_T , L_n and L_s , which, in turn, determine the state of the unperturbed background plasma. If the plasma is in equilibrium and has no heat sources or sinks it must satisfy $\nabla \cdot \underline{q} = 0$ and $\nabla p = 0$, which imply that $\Delta = -5/2$ (since $q \propto T^{5/2} \nabla T$) and $L_T = -L_n$, respectively. Under these conditions, $S^N = 0$ and $S^T < 0$, and hence, no unstable modes exist. However, since we assume the evolution of the plasma to be negligible during the growth time of the instability, we are free to choose arbitrary values of Δ and $L_{n,T}$. In particular, it is expected that $\nabla \cdot \underline{q} < 0$ (i.e. $\Delta > -5/2$) and $\nabla p > 0$ (i.e. $1/L_T + 1/L_n > 0$) for a typical

laser-driven plasma.

In the range of validity of our stability analysis, i.e. $2\pi \gg K \gg 2\pi/L_{n,T}$ (conditions 3.6 and 3.7), the full instability criterion is given by: $\alpha_0 K^2 L_s^2 - S^N - S^T < 0$, or

$$(\beta_0''/\delta_0)(\Delta+3/2) + (\gamma_0''/\gamma_0' - \beta_0''/\delta_0)(L_T/L_n) > \alpha_0 (KL_T)^2 L_s^2 \quad (3.15)$$

and the maximum growth rate is approximately,

$$\begin{aligned} \Gamma &\cong S^N + S^T \\ &\cong \{ (\beta_0''/\delta_0)(\Delta+3/2) + (\gamma_0''/\gamma_0' - \beta_0''/\delta_0)(L_T/L_n) \} / L_T^2 \end{aligned} \quad (3.16)$$

where (β_0''/δ_0) and (γ_0''/γ_0') are coefficients of order one.

Assuming that $|L_T| \sim |L_n|$, Γ is found to scale roughly as $1/L_T^2$ (i.e. $\gamma \sim \nu_t (\lambda_t/l_T)^2 \propto T^{5/2}/nZ$), and reaches a maximum when $K \sim 2\pi/L_T$ (i.e. $\lambda \sim l_T$). At short wavelengths there exists a cut-off under which the unstable mode becomes damped. This cut-off wavelength $\Lambda_0 (= 2\pi/K_0)$ is defined by the inequality (3.15) and is proportional to $L_s (= l_s/\lambda_t \propto n^{1/2}Z/T^2)$.

From the above remarks it may be concluded that an instability is more likely to occur for $L_{s,T,n} \ll 1$, that is, for hot and tenuous low-Z plasmas with sharp density and temperature profiles. More precisely, however, it is necessary to satisfy the instability criterion (3.15) by inserting the appropriate transport coefficients.

On table 3.1 we tabulate α_0 , (β_0''/δ_0) and $(\gamma_0''/\gamma_0' - \beta_0''/\delta_0)$ for several values of Z , using the coefficients derived in chapter 2, as well as Braginskii's. There we note that $(\gamma_0''/\gamma_0' - \beta_0''/\delta_0)$ is always positive, which implies greater instability for $L_T/L_n > 0$ (i.e. for $\nabla n \cdot \nabla T > 0$). We also note that unless $L_t/L_n \gg 0$, the unstable mode becomes damped

for $\Delta < -3/2$. Thus, a plasma is more likely to be unstable when its temperature and density gradients are parallel, and when $\nabla \cdot \underline{q} \ll 0$.

These instability criteria have been analysed in detail in a paper by Hirao and Ogasawara.²⁵ However, since they used Braginskii coefficients (see table 3.1), they wrongly concluded that a high-Z plasma is more unstable for $L_T/L_n < 0$ (instead of $L_T/L_n > 0$).

	Z=1	Z=4	Z=∞
α_0	.506 (.513)	.375 (.375)	.295 (.295)
(β_0''/δ_0)	.883 (.810)	3.24 (3.13)	9.84 (9.13)
$(\gamma_0''/\gamma_0' - \beta_0''/\delta_0)$	1.01 (1.01)	1.76 (1.32)	2.87 (-.586)

Table 3.1 Transport coefficients derived in chapter 2 and Braginskii's (...).

3.3.3 Physical mechanisms of the instability

Two sources of instability have been identified from the dispersion relation (3.14): S^N and S^T . In order to explain their physical origin we return to the linearized form of the magnetic induction equation, i.e.

$$\partial_t \underline{B}' = (c/e) \nabla \times [\beta_\Lambda(\underline{B}') \underline{b} \times \nabla T] - (c/en) \nabla_n \times \nabla T' + \chi_m \nabla^2 \underline{B}'$$

where $\chi_m = \alpha_0 l_s^2 \gamma_t$ is the magnetic diffusivity.

We deal initially with the first term on the R.H.S. of this equation, the so called 'Nernst term'.⁴ It arises as a result of a

thermal force that is needed in order to balance the friction between electrons and ions in the $\underline{b} \times \nabla T$ direction. Using the perturbation configuration described in figure 3.1 we obtain:

$$\partial_t \underline{E}' = (\beta_0''/\delta_0) \partial_z \{ (\partial_z T)/\nabla_t \} \underline{E}'/m + \dots$$

and, hence, unstable growth of an applied B-field may occur, provided $\partial_z \{ (\partial_z T)/\nabla_t \} \propto \partial_z \{ (\partial_z T)(T^{3/2}/n) \} > 0$. This is known as the Nernst instability, and was originally investigated by Brownell, and Dolginov and Urpin.¹⁵ In the present analysis, this effect is represented by the term S^N .

Considering now the second term in the R.H.S. of the induction equation, we find that it generates B-fields as a result of a temperature perturbation perpendicular to the zeroth order density gradient. If we take into account the electron thermal transport as described by the linearized version of equation (3.10) i.e.

$$n(3/2) \partial_t \underline{T}' = \chi_{\parallel} \nabla_{\parallel}^2 \underline{T}' + \nabla \cdot [\chi_{\perp}(\underline{E}') \underline{b} \times \nabla T]$$

we note that the B-field produces a heat flux in the $\underline{b} \times \nabla T$ direction (the Righi-Leduc heat flux), the sign of which may cause the initial temperature perturbation to either grow or decay. This process is illustrated schematically in figure 3.2, where it becomes apparent that $\nabla n \cdot \nabla T > 0$ is a necessary condition for growth. The mechanism for this instability, which may be classified as 'thermomagnetic', is due to Tidman and Shanny, and Bol'shov et al.¹² Its effect is represented by the term S^T .

In practice Nernst and thermomagnetic instabilities cannot be considered separately, since they have similar strength and scaling

with n , T and Z . For short wavelengths they are both damped as a result of magnetic diffusion (last term in the induction equation). The ratio of the damping rate to the instability growth rate is approximately given by $k^2 l_T^2 (l_s / \lambda_t)^2 = (K L_T)^2 L_s^2$, where L_s^2 ($\propto n Z^2 / T^4$) may be envisaged as a ratio of magnetic to thermal diffusivities. This must be small for both instability mechanisms to be effective.

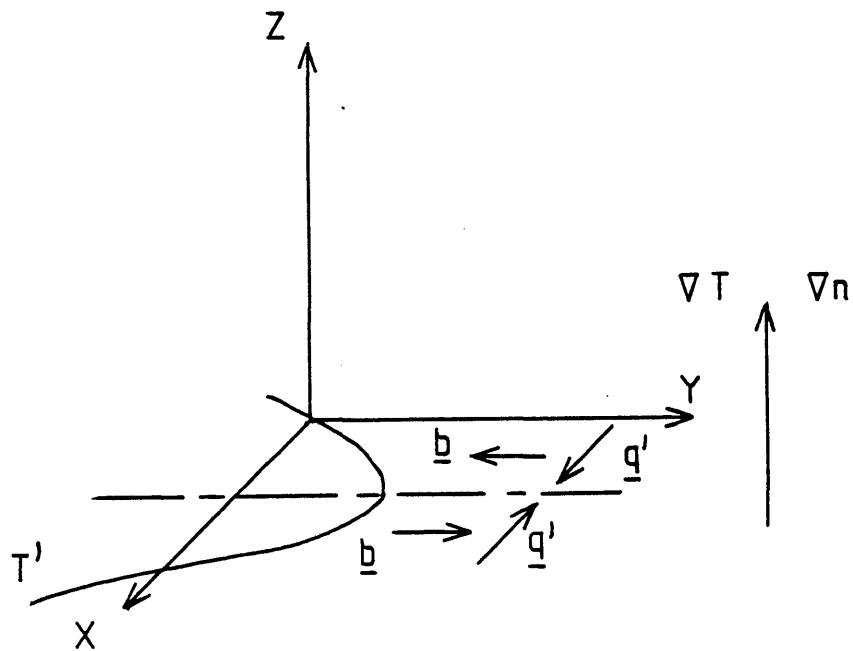


Fig. 3.2 Physical mechanism of the thermomagnetic mode.

3.3.4 Discussion

There are three main restrictions on the present stability analysis: the collisionality condition, the 'local approximation' and the neglect of hydrodynamics (conditions 3.6-3.8, respectively).

These are discussed in turn.

(i) Collisionality condition. Since the fluid description of the background plasma requires that $\lambda_t \gg l_{T,n}$ for its validity,⁴ the maximum growth rate of the instability obeys: $\gamma \sim \nu_t (\lambda_t / l_T)^2 \ll \nu_t$. If we now choose a perturbation wavelength that is larger than an electron m.f.p. (i.e. $\lambda \gg \lambda_t$), condition (3.6) is clearly satisfied.

It must be noted, however, that the collisionality condition is purely qualitative, as it may not ensure an accurate solution of the fluid equations. A more exact treatment requires the use of a kinetic model. This approach is taken in chapters 4 and 5.

(ii) 'Local approximation'. This arises because we have neglected variations in the perturbed variables along the direction of the zeroth order inhomogeneity (i.e. along z). As the growth rate tends to a maximum for $\lambda \lesssim l_{T,n}$, non-local effects may be important. Dolginov and Urpin¹⁵ investigated this, using the W.K.B. method, and found that a perturbation along z caused the unstable mode to become convective. The drift was shown to be in the $-\nabla T$ direction and originated from the Nernst term in the induction equation.

A more conclusive analysis would require the numerical solution of the full eigenvalue problem.

(iii) Neglect of hydrodynamics. This approximation may be tested by inserting the expression for the maximum growth rate (Eq. 3.16) directly into condition (3.8):

$$\{(\beta_0''/\delta_0)(\Delta + 3/2) + (\gamma_0''/\gamma_0' - \beta_0''/\delta_0)(L_T/L_n)\} > (KL_T)L_T (Zm/A m_p)^{1/2}$$

where $A (=m_i/m_p)$ is the atomic number and m_p the proton mass.

Taking the extreme case of $KL_T = 2\pi$, and assuming that $A = 2Z$, $\Delta = 0$ and $L_n = L_T$, the above inequality predicts: $L_T < 170$ for $Z = \infty$ and $L_T < 23$

for $Z=1$. Clearly, for larger values of K this condition may be easily violated.

The main effect of introducing ion perturbation is to enhance the instability,^{15,25,27} as may be demonstrated by considering the second term on the R.H.S. of the induction equation (3.9):

$$\partial_t \underline{E} = -(e/nc) \nabla n \times \nabla T + \dots$$

This provides an extra source to drive the thermomagnetic mode, namely $(e/nc) \nabla n \times \nabla T$. Referring back to figure 3.2, a density perturbation, which will now be out of phase with the temperature perturbation (to ensure pressure balance), generates a E -field that always directs the Righi-Leduc heat flux towards the temperature peaks, thus leading to an instability. It is straightforward to show that this mechanism introduces a new term in the expression for the growth rate (Eq. 3.16), of the form $(\gamma_0''/\gamma_0')/L_T^2 > 0$.

Finally, the condition for neglecting the hydrodynamic evolution of the background plasma (i.e. $\gamma > c_s/l_{T,n}$) leads to the less stringent requirement that, $L_T < 10^3$ for $Z = \infty$ and $L_T < 140$ for $Z=1$.

3.4 The 2-fluid model

3.4.1 Derivation of the dispersion relation

In the 2-fluid model the electron population is divided up into hot and cold energy groups. Each group is described by a set of the fluid equations (3.1)-(3.3), with variables identified by subscripts h(=hot) and c(=cold). By definition we have $T_h > T_c$, though it is assumed throughout the analysis that $\theta = n_h/n_c \ll 1$ and $p_h \ll p_c$.

The quasi-steady state is set up by having the hots streaming in the positive z direction with a speed v_h and the colds streaming in the opposite direction with a speed v_c , in a uniform background plasma. Preservation of charge neutrality requires that $zn_i = n_c + n_h$ and $n_h v_h = n_c v_c$.

This configuration leads to a finite heat flow along z, given by

$$q = \frac{1}{2}(mn_h v_h^3 - mn_c v_c^3) + \frac{5}{2}(mn_h v_{th}^2 v_h - mn_c v_{tc}^2 v_c) \quad \text{or}$$

$$Q = (q/q_f) = (1/2)\theta(v_h/v_{tc})[(v_h/v_{tc})^2 + 5(T_h/T_c - 1)] \quad (3.17)$$

where $v_{tc,th} = (T_{c,h}/m)^{1/2}$, and Q is the heat flow normalized to its 'free-streaming' limit, defined by $q_f = n_c m v_{tc}^3$. Total energy balance is achieved by equating the Ohmic heating of the cold return current with the electron energy loss effects, i.e. $j_c^2 / \sigma_c = s$ where $\sigma_c = n_c e^2 / m v_{tc} \alpha_0$ is the electrical conductivity for $\underline{P}=0$.

The perturbation analysis is carried out in a similar manner to that of the 1-fluid model (see Fig. 3.3), where $\gamma \ll v_{tc,th}$ and $2\pi/k(=\lambda) \gg \lambda_{tc,th}$ are the collisionality conditions, and $\gamma \gg kc_{sc,sh}$ is the criterion for neglecting the ion motion.

Applying the perturbation to equations (3.1)-(3.5), and assuming

that $T_h = \text{constant}$ (since $q_h \gg q_c$), we get:

$$\gamma n'_c = -ikv'_{cx} n_c$$

$$\gamma n'_h = -ikv'_{hx} n_h$$

$$mn_c \nabla_{tc} \alpha_{Ov'_{cx}} = -ikn'_c T_c - ikn_c T'_c - en_c E'_x - j_c B'/c$$

$$mn_h \nabla_{th} \alpha_{Ov'_{hx}} = -ikn'_h T_h - en_h E'_x + j_h B'/c$$

$$E'_z = j'_c / \sigma_c - (n'_c / n_c + 3T'_c / 2T_c) (j_c / \sigma_c)$$

$$E'_z = j'_h / \sigma_h + (n'_h / n_h) (j_h / \sigma_h)$$

$$\frac{3}{2} \gamma T'_c n_c + n_c T_c ikv'_{cx} + ikq'_{cx} = j_c E'_z + j'_c E_z - s(n'_c / n_c + rT'_c / T_c)$$

$$\gamma B' = ikE'_z$$

$$ikB'(c/4\pi) = j'_c + j'_h$$

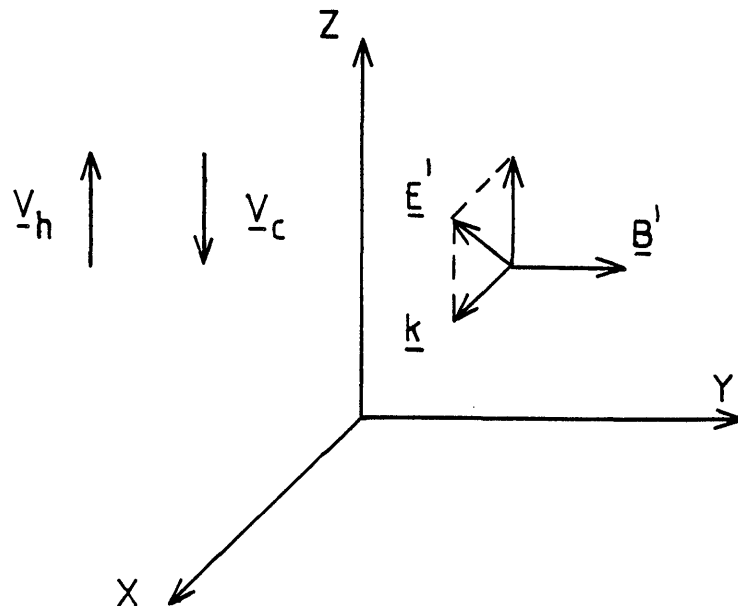


Fig. 3.3 Perturbation configuration for the 2-fluid model.

From these equations we obtain (after lengthy but straightforward algebraic manipulations) the following dispersion relation.

$$\begin{aligned}
 & \Gamma^3_{ab} + \Gamma^2 \left\{ K^2 [ab(2\gamma_0/3) + a(T_h/\theta T_c)/\alpha_0 + \alpha_0 b L_s^2] \right. \\
 & + b \alpha_0 v_c^2 [2 - a(1 - 2r/3)] \left. \right\} \\
 & + \Gamma \left\{ K^4 [(2\gamma_0/3)(a(T_h/\theta T_c)/\alpha_0 + b \alpha_0 L_s^2) + (T_h/\theta T_c) L_s^2] \right. \\
 & - K^2 [v_h^2 + \alpha_0 v_c^2 (1 - 2r/3)(a(T_h/\theta T_c)/\alpha_0 + \alpha_0 b L_s^2)] \left. \right\} \\
 & + K^6 [(2\gamma_0/3) L_s^2 (T_h/\theta T_c)] \\
 & - K^4 [(2\gamma_0/3) v_h^2 + \alpha_0 v_c^2 (1 - 2r/3)(T_h/\theta T_c) L_s^2] \\
 & + K^2 [v_h^2 \alpha_0 v_c^2 (1 - 2r/3)] = 0
 \end{aligned} \tag{3.18}$$

where

$$\Gamma = \gamma / v_{tc}, \quad K = k \lambda_{tc}, \quad L_s = l_s / \lambda_{tc},$$

$$a = 1 + \theta v_{tc} / v_{th}, \quad b = 1 + v_{th} / \theta v_{tc}, \quad S = s / (3 n_c T_c v_{tc} / 2),$$

$$v_{c,h} = v_{c,h} / v_{tc}.$$

This result was obtained in the limit of $\theta, p_h / p_c \ll 1$, and required the use of the equilibrium zero current condition $n_c v_c = n_h v_h$ and energy balance relation $j_c^2 = s \sigma_c$, which in dimensionless form become $v_c = \theta v_h$ and $v_c^2 = (3/2) S / \alpha_0$, respectively.

For simplicity, it is assumed from now on that the electrons lose their energy mainly through equipartition with 'cold' ions. This gives us $S=2m/m_i$ and $r=-1/2$, and thus $V_c^2=3m/m_i\alpha_0$.

3.4.2 Properties of the dispersion relation

The dispersion relation (3.18) may generate modes for the special case when temperature effects are negligible, by artificially setting $\gamma_0 \rightarrow \infty$ (equivalent to assuming a very large electron heat flow and thus a constant T) i.e.

$$\Gamma^2_{ab} + \Gamma \{K^2(a(T_h/\theta T_c)/\alpha_0 + \alpha_0 b L_s^2\} + K^4 L_s^2 (T_h/\theta T_c) - K^2 V_h^2 = 0 \quad (3.19)$$

Apart from the numerical factor α_0 (the electron-ion friction coefficient in Pragenskii's notation⁴), this equation is equivalent to the one derived by Mochizuki et al.,⁹ who ignored temperature perturbations altogether. It gives rise to a purely growing unstable mode, known as the 'collisional Weibel' mode, for $K < K_{01}$, where

$$K_{01}^2 (= 2\pi/\Lambda_{01})^2 = (\theta T_c/T_h) V_h^2/L_s^2 \quad (3.20)$$

with a maximum growth rate given by:

$$\Gamma \approx V_h^2/(\alpha_0 L_s^2 + a(T_h/\theta T_c)/\alpha_0) \quad (3.21)$$

at approximately $K_{01}/\sqrt{3}$.

Haines¹³ proposed a similar model, but also included electron temperature effects. In the limit of no hydrodynamic motion, and with $r=-1/2$ and $S=2m/m_i$ (loss due electron-ion energy equipartition) his dispersion relation becomes,

$$\Gamma^2 + \Gamma \{ k^2 [(2\gamma_0/3) + \alpha_0 L_s^2] + (2/3)\alpha_0 v_c^2 + \alpha_0 \theta (1 - v_h^2 T_c / T_h) \} + [\alpha_0 k^2 L_s^2 + \alpha_0 \theta (1 - v_h^2 T_c / T_h)] [(2\gamma_0/3)k^2 - (4\alpha_0/3)v_c^2] = 0 \quad (3.22)$$

which essentially describes the instability arising from the Ohmic heating of the cold return current. For $v_h^2 = T_h / T_c$ (i.e. $v_h = v_{th}$) the cut-off wavelength Λ_{02} is given by:

$$k_{02}^2 = (2\pi / \Lambda_{02})^2 = 2\alpha_0 v_c^2 / \gamma_0 \quad (3.23)$$

and the maximum growth rate by,

$$\Gamma \approx (4\alpha_0/3)v_c^2 / \{ (2\gamma_0/3\alpha_0) / L_s^2 + 1 \} \quad (3.24)$$

where $a = L_s^2 (3\alpha_0/2\gamma_0)$, in Haines' notation.

However, when we compare dispersion relation (3.22) with (3.18) we note that they are qualitatively different, even though both contain temperature effects. In order to explain this discrepancy, two basic assumptions present in Haines' model¹³ must be considered: that of collisionless hot, and that of charge neutrality between cold electrons and ions (i.e. $n_i' = n_c'$). The first of these requires for its validity that $\gamma \gg v_{th}$. This condition is violated since $\gamma \sim (2m/m_i) v_{tc} \sim (2m/m_i) (T_h/T_c)^{3/2} v_{th} \ll v_{th}$ for a typical laser-plasma. The second assumption implies that the electrical conductivity $\sigma_c (\propto T^{3/2} n_c/n_i)$ is independent of density perturbations, and also leads to $n_c' = 0$ when $n_i' = 0$, instead of $n_c' = -n_h'$ (i.e. both hot and cold electrons moving out of phase with each other so as to preserve quasi-neutrality).

The dispersion relation (3.18) also contains instability effects

arising from the sink term s .¹⁴ These manifest themselves through the factor r (the temperature exponent of s), which when less than $3/2$ leads to a further destabilization of the Ohmically driven unstable mode.¹³

We now investigate the properties of the full dispersion relation (3.18). The criterion for the onset of a non-convective unstable mode is given by:

$$K^6[(2\gamma_0/3)L_s^2(T_h/\theta T_c)] - K^4[(2\gamma_0/3)v_h^2 + (4\alpha_0/3)v_c^2(T_h/\theta T_c)L_s^2] + K^2[(4\alpha_0/3)v_h^2v_c^2] < 0$$

or $(K^2 - K_{01}^2)(K^2 - K_{02}^2) < 0$, where K_{01} and K_{02} are the cut-off wavenumbers for the collisional Weibel and Ohmic instabilities, respectively (defined by equations 3.20 and 3.23). The ratio of these two competing mechanisms is

$$(K_{01}/K_{02})^2 = (\gamma_0/2\alpha_0)(p_c/p_h)/L_s^2 \quad (3.25)$$

where $(\gamma_0/2\alpha_0) = 3.08$ and 23.1 for $Z=1$ and ∞ respectively.

Plots of the unstable modes as functions of Λ , with $T_h/T_c=2$, $\theta=0.005$, $V_c=0.015$ and $V_h=3$, are shown in figure (3.4) for: (a) $L_s=0.1$ and (b) $L_s=100$. These parameters are consistent with typical Fokker-Planck simulations of non linear heat flow in overdense laser-plasmas,²⁸ which predict a heat flux (defined by Eq. 3.17) of about 10% of the 'free-streaming' limit, and a cold electron flow at approximately the isothermal sound speed. For $Z=10$ and $A=20$ the Ohmic heating of the return current exactly balances the energy equipartition to the cold ions (with $V_c = 0.015$).

As observed from the plots, for $L_s=0.1$ (i.e. $K_{01} \gg K_{02}$) the

collisional Weibel instability prevails and is well described by Mochizuki's dispersion relation (3.19) in the region where $\Lambda_{01} < \Lambda < \Lambda_{02}$. For $\Lambda > \Lambda_{02}$, temperature effects come into play, and the Ohmic instability causes the unstable mode to become convective.

For $L_s = 100$ (i.e. $K_{01} \ll K_{02}$) the Ohmic instability is dominant in the region where $\Lambda_{02} < \Lambda < \Lambda_{01}$, though for $\Lambda > \Lambda_{01}$ the collisional Weibel instability is once again important and the unstable mode becomes complex.

Haines' dispersion relation (3.22) is also plotted for comparison in figures 3.4a and 3.4b.

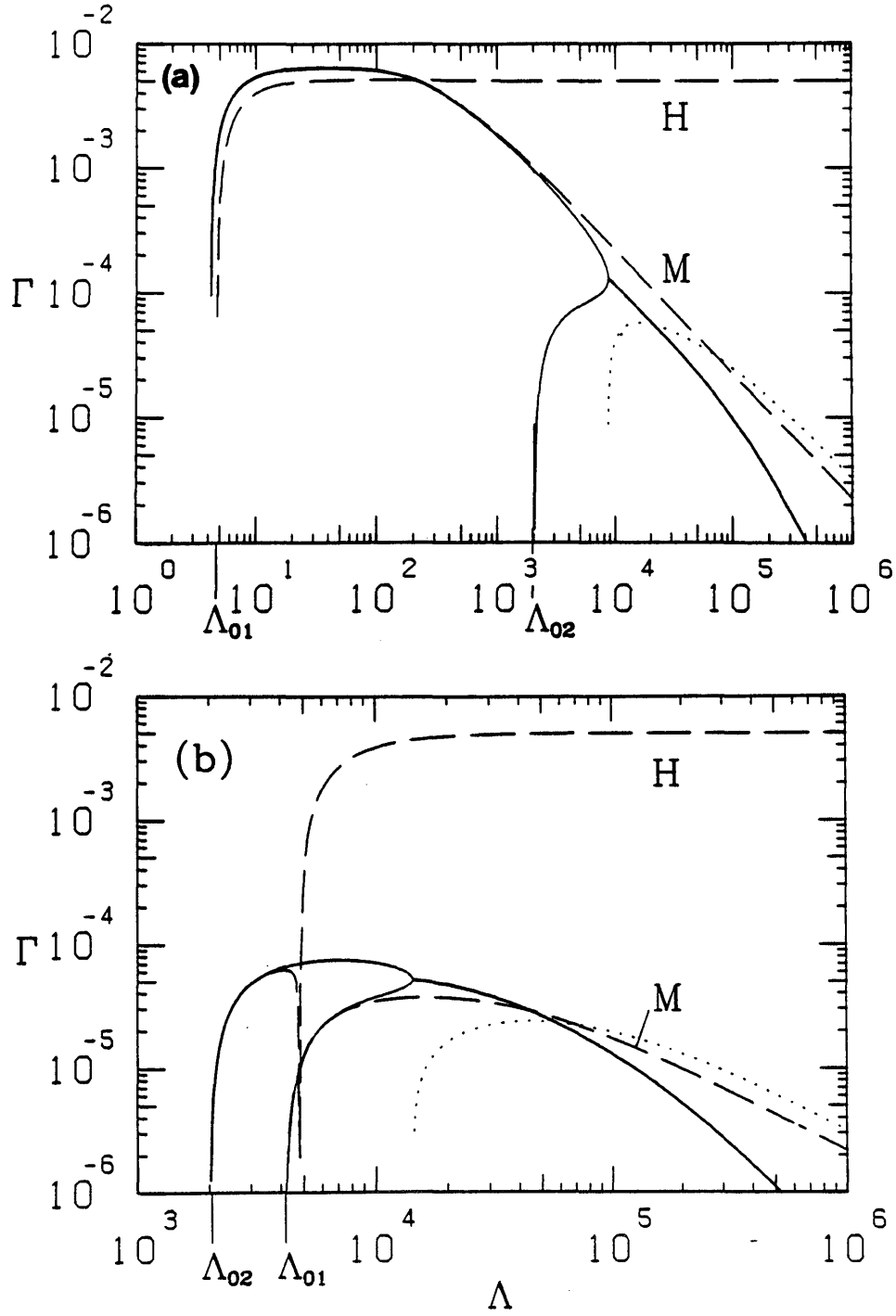


Fig. 3.4 Plot of growth rate Γ vs. wavelength Λ , for $\theta=0.005$, $T_h/T_c=2$, $V_h=3$ and $V_c=0.015$ ((a) $L_s=0.1$, (b) $L_s=100$). Solid and dotted lines represent the real and imaginary parts of Γ , respectively, and dash-dotted lines the positive roots of Mochizuki's (M) and Haines' (H) dispersion relations (Eqs. 3.19 and 3.22 respectively).

3.4.3 Physical mechanisms of the instability

Two types of unstable modes have been discussed in the previous section. They were referred to as the collisional Weibel and Ohmic mode.

The first of these derives its name from its similarity with the original collisionless EM instability devised by Weibel,²⁶ which is driven by a pressure anisotropy. In the present 2-fluid model the pressure anisotropy is attributed to the unidirectional flow of hot and cold electrons. The instability mechanism is explained as follows.⁹

Referring to figure 3.5, where the perturbation configuration is described, we see that a perturbed magnetic field pushes the electrons towards its neutral points, via the Lorentz force. There, a net current is induced (via Ampere's law), which is directed such as to amplify the initial E-field, hence causing it to grow unstable. For small perturbation wavelengths this growth is damped out by the Joule dissipation of the perturbed current, a process which is commonly known as magnetic diffusion.

The second unstable mode, as the name implies, arises as result of Ohmic dissipation of the cold return current, i.e. j_c^2 / σ_c .¹³ Its mechanism may be explained in terms of the temperature dependence of the electrical conductivity σ_c . Since $\sigma_c \propto T^{3/2}$, a perturbation in temperature will cause the cold current to increase where the temperature is highest. There the Ohmic heating rate will also increase, thus promoting a further enhancement of the temperature perturbation. Unlike the previous instability mechanism, the main source of damping at short wavelengths is thermal diffusion.

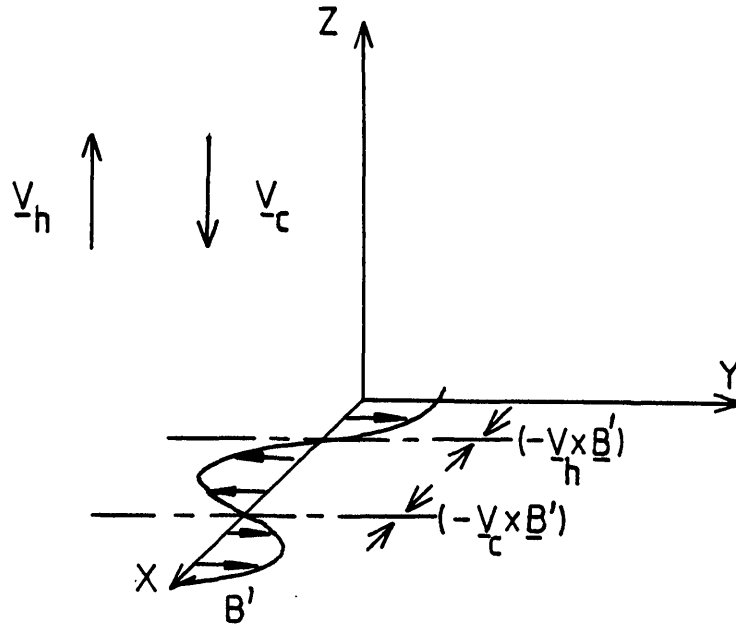


Fig. 3.5 Instability mechanism of the collisional Weibel instability.

3.4.4 Discussion

Several approximations have been made in the present 2-fluid analysis.

The first of these comes from splitting the electrons into hot and cold groups. This was done under the assumption that there were no collisions between the species, and by using single-fluid transport coefficients, which were derived in the absence of electron drifts. The validity of this approximation is investigated in chapter 5.

Secondly, we have omitted the effects of equilibrium inhomogeneities in the perturbation. This might have been 'easily' included in the analysis, as was done for the 1-fluid model. However, it is expected that the hot electrons, which are the main source of

the present instabilities, have relatively small density and temperature gradients, and may thus be considered homogeneous. The assumption of a spatially constant cold electron background may be reasonably justified provided the temperature scale-length, required to drive a classical heat flux⁴ $Q_{cl} = (128/3\pi)/L_T$ equal to Q (Eq. 3.17), is larger than the optimum wavelength of the instability Λ_{max} . For the situation depicted on figure 3.4a, this is found to be the case, since $L_T = 135 \gg \Lambda_{max}$.

The last main approximation comes from the assumption of fixed ions. The effect of neglecting their motion in perturbation may be investigated by considering the equation for total momentum conservation in the centre-of-mass frame of the plasma, i.e.

$$m_i n_i (\partial_{t-v_i+v_i} \cdot \nabla) = -\nabla \cdot (p_c + p_h + p_i)$$

where $m/m_i \ll 1$ and $Zn_i = n_c + n_h$ have been used. Linearizing this equation, as well as the ion continuity relation $\partial_t n_i + \nabla \cdot (n \underline{v}_i) = 0$, we obtain, after straightforward substitution,

$$\begin{aligned} (n'_c/n_c) = & - (n'_h/n_h) \Theta[(\gamma^2 + k^2 c_{ch}^2)/(\gamma^2 + k^2 c_{sc}^2)] \\ & - (T'_c/T_c) [k^2 c_{sc}^2/(\gamma^2 + k^2 c_{sc}^2)] \end{aligned}$$

In the limit as $\gamma^2 \gg k^2 c_{sc,sh}^2$ (i.e. negligible hydrodynamics) we recover the result $n'_c = -n'_h$, whereas in the opposite limit (i.e. strong hydrodynamics), we obtain $n'_c = -n'_h (T_h/T_c) - T'_c n_c/T_c$. If the full stability analysis was now to be repeated for the strong hydrodynamic case, one would obtain a dispersion relation that is not too different from the present one (Eq. 3.18). Indeed, the conclusions reached so far would remain practically unchanged.

3.5 Conclusions

Four main instability mechanisms have been identified in this chapter: the thermomagnetic, the Nernst, the collisional Weibel and the Ohmic. The first two of these arose from the 1-fluid model and the last two from the 2-fluid model.

Their range of applicability may be characterized by a single parameter

$$L_s = 5.83 \times 10^{-2} (n/10^{21} \text{ cm}^{-3})^{1/2} (Z/10) (\ln \Lambda_{ei}/5) (1 \text{ keV}/T)^2$$

(i.e. the ratio of the collisionless skindepth to the electron mean free path), as described by the instability criteria:

(i) thermomagnetic and Nernst (criterion 3.15)

$$L_s < \{ (\beta_0''/\delta_0)(\Delta + 3/2)/\alpha_0 + (\gamma_0''/\gamma_0' - \beta_0''/\delta_0)(L_T/L_n)/\alpha_0 \}^{1/2} (\Lambda/L_T)/2\pi$$

(ii) collisional Weibel (criterion 3.20)

$$L_s < \Lambda v_h (\theta T_c/T_h)^{1/2} / 2\pi$$

(iii) Ohmic (criteria 3.23 and 3.25)

$$L_s > \{ (\gamma_0/2\alpha_0)(p_c/p_h) \}^{1/2} \text{ and } \Lambda > 2\pi (\gamma_0/2\alpha_0)^{1/2} / v_c$$

This is demonstrated graphically on figures 3.6a,b for $Z=1$ and 10 , and

$\Lambda = L_n = L_T = 100$, and by assuming $\Delta = 0$, $\theta = 0.005$, $T_h/T_c = 2$, $v_h = 3$ and $v_c = 0.015$. There we also plot the lines for $\ln \Lambda_{ei} = 1, 5$. Since the fluid equations used in the present analysis have leading errors of $O[1/\ln \Lambda_{ei}]$, we should restrict ourselves to regions in (n_c, T_c) space well above the $\ln \Lambda_{ei} = 5$ line.

As observed, thermomagnetic, Nernst and collisional Weibel instabilities are more likely to occur for high temperature and tenuous low-Z plasmas, whereas the Ohmic instability would tend to occur in the opposite regime. In this latter regime, however, the present analysis begins to break down (as $\ln \Lambda_{ei} < 5$).

Using the same parameters as above (with $Z=10$), the maximum growth rate and optimum wavelength for each instability is approximately given by:

(i) thermomagnetic and Nernst (equations 3.15 and 3.16)

$$\gamma_{TN} \approx 18 \nu_t (\lambda_t / l_T)^2 \propto (T^{5/2} / nZ) / l_T^2$$

$$\lambda_{TN} \approx l_T$$

(ii) collisional Weibel (equations 3.20 and 3.21)

$$\gamma_{CW} \approx 0.30 \nu_{tc} (v_h / v_{th})^2 \propto (n_c Z / T_c^{5/2}) (p_c / p_h) v_c^2$$

$$\lambda_{CW} \approx 11 l_s (v_{th} / v_h) / \theta^{1/2}$$

(iii) Ohmic (equations 3.23 and 3.24)

$$\gamma_0 \approx 0.40 \nu_{tc} (v_c / v_{tc})^2 \propto (n_c Z / T_c^{5/2}) v_c^2$$

$$\lambda_0 \approx 1.3 (v_{tc} / v_c)$$

Thus, for a given l_T , γ_{TN} increases for large T and small nZ , and for a constant $v_c^2 (p_c / p_h)$, both γ_{CW} and γ_0 increase for large $n_c Z$ and small T_c .

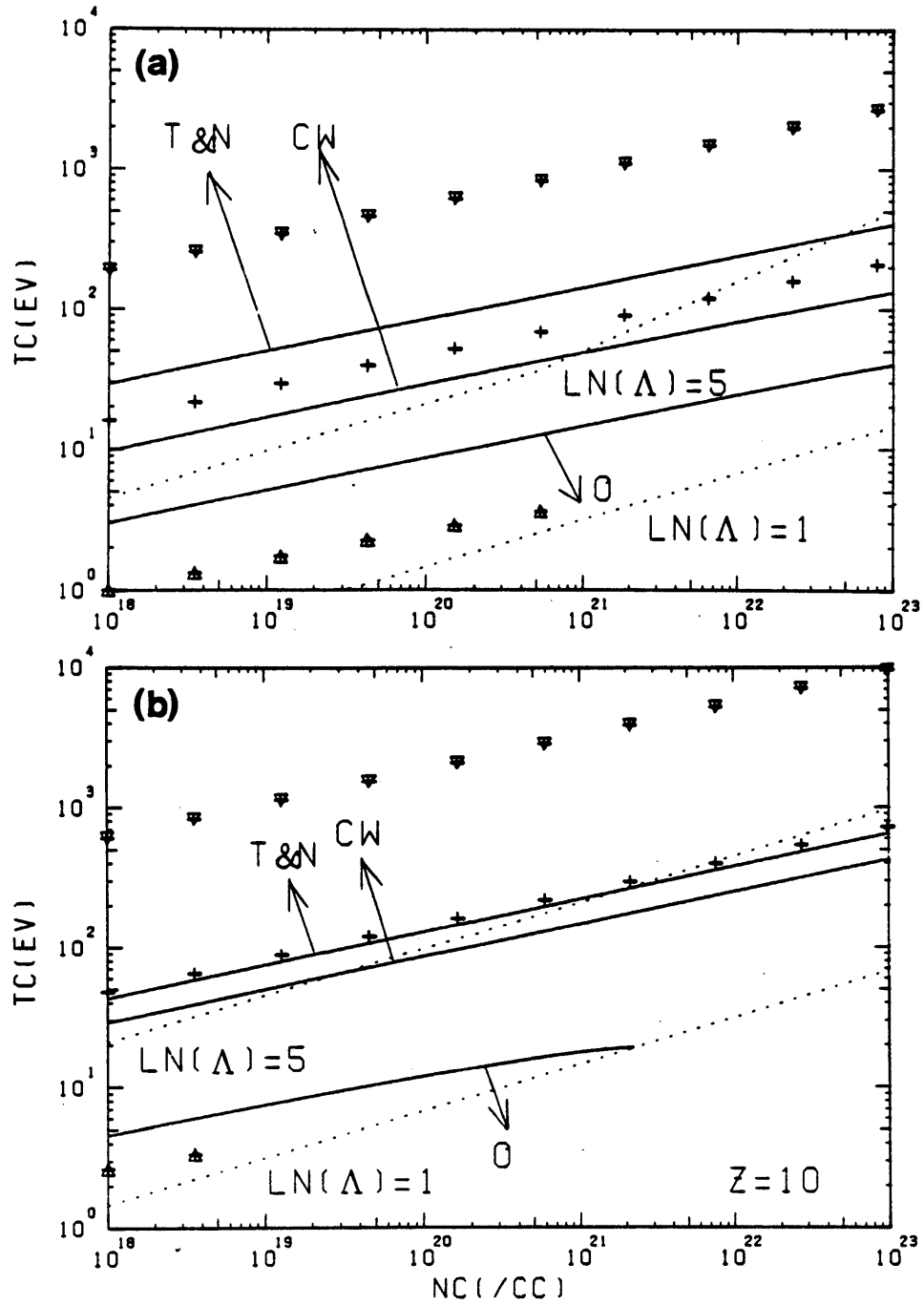


Fig. 3.6 Plot of T_c vs. n_c for $\Lambda=L_n=L_T=100$, $\Delta=0$, $\theta=0.005$, $T_h/T_c=2$, $V_h=3$ and $V_c=0.015$ ((a) $Z=1$, (b) $Z=10$), showing the regions where thermomagnetic and Nernst (TN), collisional Weibel (CW) and Ohmic (O) modes may be unstable. Dotted lines represent $\ln \Lambda_{ei}=1, 5$, and points ∇ , $+$ and Δ refer to $L_s=0.01, 1$ and 100 , respectively.

CHAPTER 4

Kinetic Treatment of EM Instabilities

(with isotropic pressure)

4.1 Introduction

In chapter 3, a 1-fluid model was used in order to investigate EM instabilities in a collisional plasma. Two types of instability mechanisms were identified: one, due to the thermal force (Nernst mode) and the other, due to the Righi-Leduc heat flow coupled with B-field generation (thermomagnetic mode). Their growth rates were shown to scale roughly as $\nu_t (\lambda_t / l_T)^2$, where ν_t is the mean electron-ion collision frequency, λ_t the electron-ion mean-free-path and l_T the temperature scale length.

Fluid equations are, however, known to be valid only when the average quantities in the plasma change slowly in space and time, i.e. when $\partial_t \ll \nu_t$ and $\nabla \ll \lambda_t^{-1}$.⁴ These conditions translate readily into the collisionality criteria for perturbation ($\lambda \gg \lambda_t$, $\gamma \ll \nu_t$) and equilibrium ($\lambda_t / l \ll 1$), presented in the stability analysis for the 1-fluid model (chapter 3, section 3.3.1). Apart from these general validity criteria, fluid theory on its own does not give us information about the accuracy of its predictions when $\partial_t \sim \nu_t$ and $\nabla \sim \lambda_t^{-1}$, though numerical simulations of 1-D plasma transport, involving the Fokker-Planck equation, suggest that the fluid description will break down when $\lambda_t / l_T > 0.01$.²

Thus, in order to investigate the validity of the 1-fluid EM

instability theory, a kinetic model will be developed in this chapter, based on a perturbation of the electron Fokker-Planck equation.

The assumptions involved are equivalent to the ones used for the 1-fluid model: fixed and cold ions, collisional electrons, local approximation, 1-D equilibrium, etc. By considering the particular case of a high-Z plasma, the electron-electron momentum transfer may be neglected in comparison with the electron-ion momentum transfer. The electron distribution function f , is solved by means of the expansion: $f = f_0 + \underline{v} \cdot \underline{f}_1 / v$, where $|f_0| \gg |f_1|$ is assumed.¹⁷ The effect of including higher order terms in this expansion is the subject of chapter 5.

A description of the model is given in section 4.2. This is followed by a derivation of the eigenvalue equation in section 4.3 and a presentation of its formal and asymptotic solutions in section 4.4. Numerical results are presented in section 4.5. The effects associated with the electron-electron energy exchange are investigated in section 4.6. Discussion and conclusions are presented in section 4.7.

In this chapter, the electron thermal velocity (v_T), mean collision frequency ($\bar{\nu}_T$) and mean-free-path (λ_T) are defined as follows.

$$v_T = (2T/m)^{1/2} = v_t \sqrt{2},$$

$$\bar{\nu}_T = n_i 4\pi (Ze^2/m)^2 \ln \Lambda_{ei} / v_T^3 = \bar{\nu}_t (3/4) \sqrt{\pi},$$

$$\lambda_T = v_T / \bar{\nu}_T = \lambda_t (32/9\pi)^{1/2},$$

where subscript 't' refer to chapter 3 definitions.

4.2 Description of the model

The basic equations needed for the stability analysis are the electron Fokker-Planck equation,

$$\partial_t f + \underline{v} \cdot \nabla f - (\underline{a} + \underline{v} \times \underline{\omega}) \cdot \nabla_v f = C ,$$

and Maxwell's equations,

$$\partial_t \underline{\omega} = -\nabla \times \underline{a} , \quad (4.1)$$

$$\nabla \times \underline{\omega} = (\omega_p/c)^2 \underline{j}/n , \quad (4.2)$$

where $\underline{a} = e\mathbf{E}/m$, $\underline{\omega} = e\mathbf{B}/mc$ is the cyclotron frequency, $\omega_p = (4\pi n e^2/m)^{1/2}$ the plasma frequency and C the collision operator (all other symbols have their usual meaning).

Substituting $f = f_0 + \underline{v} \cdot \underline{f}_1 / v$ into the Fokker-Planck equation and making use of the orthogonal properties of this expansion, we get (Shkarofski):

$$\partial_t f_0 + (v/3) \nabla \cdot \underline{f}_1 - (1/3v^2) \partial_v (v^2 \underline{a} \cdot \underline{f}_1) = C_{ee0} , \quad (4.3)$$

$$\partial_t \underline{f}_1 + v \nabla f_0 - \underline{a} \partial_v f_0 - \underline{\omega} \times \underline{f}_1 = -\gamma_{ei} \underline{f}_1 , \quad (4.4)$$

where $\gamma_{ei} = [n_i 4\pi (Ze^2/m)^2 \ln \Lambda_{ei}] / v^3$ is the electron-ion collision frequency and $\ln \Lambda_{ei}$ the respective Coulomb logarithm. The operator C_{ee0} , which represents the collisional energy exchange between electrons, is defined in appendix A.2.

In order to comply with the 1-fluid analysis of chapter 3 the same

equilibrium configuration is chosen, with inhomogeneities in n and T taken along z . As a kinetic model is used for the stability analysis, the unperturbed distribution function $f=f_0+v_z f_z/v$ needs to be specified, which ideally, should be done by solving equations (4.3) and (4.4) self-consistently. Here, however, we take f_0 to be a Maxwellian, i.e.

$$f_0 = f_m = (n/\pi^{3/2} v_T^3) e^{-v^2/v_T^2}, \quad (4.5)'$$

and substitute it into equation (5.4) to obtain (appendix A.5),

$$f_z = \left(\frac{v}{v_T}\right)^4 (4-v^2/v_T^2) (\lambda_T/l_T) f_m, \quad (4.5)''$$

$$\text{and, } a_z = -v_T^2(5/l_T + 2/l_n)/4, \quad (4.6)$$

where a_z is the electric field required to preserve quasi-neutrality and $l_{T,n} = [\partial_z \ln(T,n)]^{-1}$. This type of distribution is only valid for small temperature gradients (i.e. $\lambda_T/l_T \ll 1$), since the total f can become negative for large velocities.²⁹ Nevertheless, it has been used to derive fluid transport coefficients (for $B=0$), as described in chapter 2, and provides an equilibrium state which is equivalent to the one specified for the 1-fluid model.

No such assumptions will be made about the perturbed part of the distribution f , so that f_0 is derived from equation (4.3). This is in direct contrast with the fluid description, which always presupposes that $f_0=f_m$, even in perturbation.

4.3 Derivation of the eigenvalue equation

Using the perturbation configuration, as depicted in figure 3.1 of chapter 3, with the dash denoting the perturbed quantities, equations (4.1)-(4.4) become:

$$\gamma \omega' = i k a'_z - \partial_z a'_x, \quad (4.7)$$

$$i k \omega' = - (\omega_p/c)^2 (4\pi/3n) \int_0^\infty dv v^3 f'_z, \quad (4.8)$$

$$\begin{aligned} \gamma f'_0 + \frac{1}{3} i k v f'_x - \frac{1}{3} a'_z [(1/v^2) \partial_v (v^2 f'_z)] \\ - \frac{1}{3} a'_z [(1/v^2) \partial_v (v^2 f'_z)] = c'_{ee0}, \end{aligned} \quad (4.9)$$

$$i k v f'_0 - a'_x \partial_v f_0 - \omega' f_z = - \gamma_{ei} f'_x, \quad (4.10)$$

$$(v \partial_z - a'_z \partial_v) f'_0 - a'_z \partial_v f_0 = - \gamma_{ei} f'_z, \quad (4.11)$$

where $\gamma, kv \ll \gamma_{ei}$ has been assumed for a thermal electron, following the imposed collisionality condition, and $k l_{T,n} \gg 2\pi$, under the 'local approximation'.

Substituting equations (4.7) and (4.8) into (4.11), and defining ϕ by the relation $a'_x = \phi \omega'$ (to be later determined by quasi-neutrality), we obtain

$$\begin{aligned} f'_z - (1/\gamma_{ei}) [(\gamma + \partial_z \phi)(\partial_v f_0)] (1/k l_s)^2 (4\pi/3n) \int_0^\infty dv v^3 f'_z \\ = - (1/\gamma_{ei}) (v \partial_z - a'_z \partial_v) f'_0, \end{aligned} \quad (4.12)$$

where $\partial_z f' = 0$ (according to the local approximation) and $l_s = c/\omega_p$ is the collisionless skin depth.

The main difference between the present kinetic treatment and the 1-fluid analysis of chapter 3 lies in the term on the R.H.S. of equation (4.12). It is worth noting, however, that if we set $f'_0 = f'_m$ the two approaches yield exactly the same result, namely the dispersion relation (3.14).

In order to get an expression for f'_0 in terms of f'_z , equations (4.7), (4.8), (4.10) and (4.11) are substituted into (4.9):

$$\begin{aligned}
 & \left\{ \gamma v_{ei} + \frac{1}{3} k^2 v^2 - v_{ei} v_{ee} (D+I) \right. \\
 & + \left. \frac{1}{3} a_z (v_{ei}/v^2) \partial_v [(v^2/v_{ei}) (v \partial_z - a_z \partial_v)] \right\} f'_0 \\
 & = \left\{ k^2 v (\partial_v f'_0) \phi + v_{ei} (\partial_z \phi) (1/v^2) \partial_v (v^2 f'_z) \right. \\
 & + v_{ei} (\partial_z \phi) a_z (1/v^2) \partial_v [(v^2/v_{ei}) \partial_v f'_0] + k^2 v f'_z \\
 & + \left. \gamma v_{ei} (1/v^2) \partial_v (v^2 f'_z) + \gamma v_{ei} a_z (1/v^2) \partial_v [(v^2/v_{ei}) \partial_v f'_0] \right\} \\
 & (1/k l_s)^2 (4\pi/3n) \frac{1}{3} \int_0^\infty dv v^3 f'_z, \quad (4.13)
 \end{aligned}$$

where $v_{ee} (D+I) f'_0 = C'_{ee0}$, and D and I are differential and integral operators, respectively (defined in appendix A.2). Comparing the first, second and third terms in the R.H.S. of this equation, and using $v \sim v_T$ and $\partial_z \sim l^{-1}$ (where $|l_T| \sim |l_n| \sim |l|$), together with the fact that $f'_z \sim (\lambda_T/l) f'_0$ (Eq. (4.5)) and $a_z \sim v_T^2/l$ (Eq. 4.6), we have:

$$\frac{k^2 v (\partial_v f_0)}{\gamma_{ei} (\partial_z \phi) (1/v^2) \partial_v (v^2 f_z)} \sim k^2 l^2 \gg 1 \text{ and}$$

$$\frac{k^2 v (\partial_v f_0)}{a_z (\partial_z \phi) (\gamma_{ei}/v^2) \partial_v [(v^2/\gamma_{ei}) \partial_v f_0]} \sim k^2 l^2 \gg 1 .$$

Similarly, comparing the fourth and last two terms:

$$\frac{k^2 v f_z}{\gamma \gamma_{ei} (1/v^2) \partial_v (v^2 f_z)} \sim \frac{k^2 v f_z}{\gamma a_z (\gamma_{ei}/v^2) \partial_v [(v^2/\gamma_{ei}) \partial_v f_0]} \sim k^2 l^2 \gg 1 ,$$

where $\gamma \sim \gamma_T (\lambda_T/l)^2$ has been used (as predicted from the 1-fluid model). Lastly, comparing the second and fourth term on the L.H.S., we have:

$$\frac{k^2 v^2}{a_z (\gamma_{ei}/v^2) \partial_v [(v^2/\gamma_{ei}) (v \partial_z - a_z \partial_v)]} \sim k^2 l^2 \gg 1 ,$$

so that equation (4.13) becomes simply,

$$f'_0 = \mathcal{D}^{-1} \frac{1}{3} k^2 v^2 [(\phi/v) \partial_v f_0 + f_z/v] \int_0^\infty dv v^3 f'_z , \quad (4.13)'$$

where $\mathcal{D} = [\gamma \gamma_{ei} + \frac{1}{3} k^2 v^2 - \gamma_{ee} \gamma_{ei} (D+I)]$. This reduced equation may have been otherwise derived by neglecting the electric field contribution from equation (4.9). The equivalent approximation in the 1-fluid analysis of chapter 3 has been to neglect the E.j heating in the energy equation.

Further simplification is also achieved by setting $C'_{ee0} = 0$, which strictly would only be true if f'_0 was of a maxwellian form. Here, it

is assumed that the secular and advection terms in the f'_0 equation (4.9) dominate over the electron self-collision term. This is discussed further in section 4.6.

Substituting equation (4.13)' (with $\nabla_{ee}=0$) into (4.12), and introducing dimensionless variables, the following integro-eigenvalue equation is obtained.

$$F'_Z + (V^3/3K^2L_s^2) \left\{ - (\partial_{VF_0})(\Gamma + \partial_Z \Phi) + (V \partial_Z - \epsilon \partial_V) \left[\frac{a_1 K^2 V^5}{\Gamma + a_1 K^2 V^5} \right] [(\Phi/V) \partial_{VF_0 + F_Z/V}] \right\} \int_0^\infty dV V^3 F'_Z = 0, \quad (4.14)$$

where $a_1 = \frac{1}{3}$,

$$\Gamma = \gamma / \nu_T, \quad K = k \lambda_T, \quad V = v / \nu_T,$$

$$L_{s,T,n} = l_{s,T,n} / \lambda_T, \quad \epsilon = a_z / \nu_T \nu_T, \quad \Delta = l_T^2 (\partial_Z^2 T) / T,$$

$$\partial_Z^i \Phi = (\lambda_T^i / \nu_T) \partial_Z^i \phi, \quad \partial_Z^i F = (4\pi \nu_T^3 / n) \lambda_T^i \partial_Z^i f.$$

The distribution function f has been normalized such that $\int_0^\infty dV V^2 F_m = 1$.

Taking the v^2 moment of equation (4.13)' (with $\nabla_{ee}=0$) and using the quasi-neutrality condition $n' = 4\pi \int_0^\infty dv v^2 f'_0 = 0$, we obtain the following expression for Φ .

$$\Phi = - \frac{\int_0^\infty dV V^6 F_Z / (\Gamma + a_1 K^2 V^5)}{\int_0^\infty dV V^6 \partial_{VF_0} / (\Gamma + a_1 K^2 V^5)}. \quad (4.15)$$

4.4 Formal and asymptotic solutions

Rewriting the integro-eigenvalue equation (4.14) in the form

$$F'_Z + G \int_0^\infty dV V^3 F'_Z = 0 ,$$

where

$$G = (V^3/3K^2L_s^2) \{ - (\partial_{V^{F_0}})(\Gamma + \partial_Z \Phi) \\ + (V \partial_Z - \epsilon \partial_V) \left[\frac{a_1 K^2 V^5}{\Gamma + a_1 K^2 V^5} \right] [(\Phi/V) \partial_{V^{F_0+F_Z/V}}] \} ,$$

we recognize it as a Fredholm equation of the 1st kind.³⁰ These have non-trivial solutions given by:

$$F'_Z = \text{constant} \times G , \quad (4.16)$$

provided,

$$1 + \int_0^\infty dV V^3 G = 0 . \quad (4.17)$$

We now have an expression for the eigenvalue F'_Z , and dispersion relation. In the limit of $\Gamma \ll a_1 K^2$ the dispersion relation may be solved analytically, to give:

$$\Gamma = \left[\int_0^\infty V^6 \{ (\partial_{V^{F_0}})(\partial_Z \Phi) - (V \partial_Z - \epsilon \partial_V) [(\Phi/V) \partial_{V^{F_0+F_Z/V}}] \} dV \right. \\ \left. - 3K^2L_s^2 \right] / \left\{ - \int_0^\infty dV V^6 \partial_{V^{F_0}} \right\} ,$$

where $\Phi = -\{\int_0^\infty v F_z dv\} / \{\int_0^\infty v \partial_v F_0 dv\}$ (from Eq. 4.15).

Integrating by parts and introducing the notation

$$\partial_z^m \langle v_i^j \rangle = \int_0^\infty v^{j+2} \partial_z^m F_i dv$$

(where $m=0,1,\dots$), we get:

$$\begin{aligned} \Gamma = & \left\{ -\frac{1}{6} \partial_z \langle v_z^4 \rangle - \epsilon \langle v_z^2 \rangle + \Phi [\partial_z \langle v_0^3 \rangle + 4\epsilon \langle v_0^1 \rangle] \right. \\ & \left. - \frac{1}{2} k^2 L_s^2 \right\} / \langle v_0^3 \rangle \end{aligned} \quad (4.18)$$

and $\Phi = \langle v_z^{-1} \rangle / \langle v_0^{-2} \rangle$.

Substituting the equilibrium distribution and electric field, given by equations (4.5)', (4.5)" and (4.6), i.e.

$$\begin{aligned} F_0 &= F_m = (4/\sqrt{\pi}) e^{-v^2}, \\ F_z &= v^4 (4-v^2) F_0 / L_T \text{ and} \\ \epsilon &= -(5/L_T + 2/L_n) / 4, \end{aligned}$$

into equation (4.18) we obtain (appendix A.4):

$$\Gamma = A_1 / L_T^2 + A_2 / L_n L_T + A_3 \Delta / L_T^2 - \frac{\sqrt{\pi}}{8} k^2 L_s^2, \quad (4.19)$$

Coefficients A_1 , A_2 and A_3 are listed on table 4.1.

This dispersion relation is similar to the one derived in chapter 3 (Eq. 3.16), and for comparison, the equivalent fluid coefficients are also listed on table 4.1.

As observed, coefficient A_1 is approximately 1.5 times larger, indicating an enhanced instability. Also, the change in sign of A_2

modifies the instability criterion (Eq. 3.15) to make the plasma more unstable when $L_n L_T < 0$ (i.e. $\nabla n \cdot \nabla T < 0$), instead of $L_n L_T > 0$.

We note in the derivation of equation (4.19) (Appendix A.4) that the main contribution to A_1 comes from the $\partial_z \langle v_z^4 \rangle$ term. The relative contribution due to $\Phi[\dots]$ amounts to about 8% of the total.

Model	A_1	A_2	A_3
Kinetic	14.87	-1.454	6.543
1-Fluid	9.814	1.908	6.543
(with coefficients derived in chapter 2)			
1-Fluid	9.099	-0.3896	6.066
(with Braginskii coefficients)			

Table 4.1 Coefficients of Eq. (4.19).

4.5 Numerical results

The full dispersion relation (4.17) can only be solved numerically.

The integration is performed by means of Simpson's rule, using equally spaced velocity intervals. The unstable root, which is purely real, is evaluated by rewriting equation (4.17) as $\Gamma = \Psi(\Gamma)$ and iterating as follows (method I).

$$\Gamma_{n+1} = \Psi(\Gamma_n),$$

where $n=0,1,\dots$ and Γ_0 is the initial guess. A convergent solution Γ_c is obtained provided $|\partial \Psi / \partial \Gamma|_{\Gamma_c} < 1$.³⁷

When $|\partial \Psi / \partial \Gamma|_{\Gamma_c} > 1$, the method of 'false position' is used (method II),³⁷ which consists basically of interpolating between two initial guesses of Γ .

In figures (4.1)a and (4.1)b, the growth rate Λ is plotted against wavelength $\Lambda (=2\pi/K)$ for $\nabla_n \cdot \nabla_T < 0$ and $\nabla_n \cdot \nabla_T > 0$, respectively, with $L_s = 0.05$, $L_n = 50$, $|L_T| = 100$, and $\Delta = 0, -5/2$. Plots in figure (4.1)a, were obtained by method I and figure (4.1)b by method II.

The arrows indicate the range of validity of the theory, where the left hand boundary (i.e. $1 \ll \Lambda$) corresponds to the collisionality condition, and the right hand boundary (i.e. $\Lambda \ll |L_{n,T}|$) to the 'localization condition'. In this validity range, $\Gamma \ll a_1 k^2$, so that the growth rate is well described by equation (4.19). For $\Lambda \gg |L_{n,T}|$, the unstable mode (Fig. (4.1)a), given by $\Gamma = -\partial_z \Phi$, arises from the Nernst effect,¹⁵ and is identical in both fluid and kinetic descriptions.

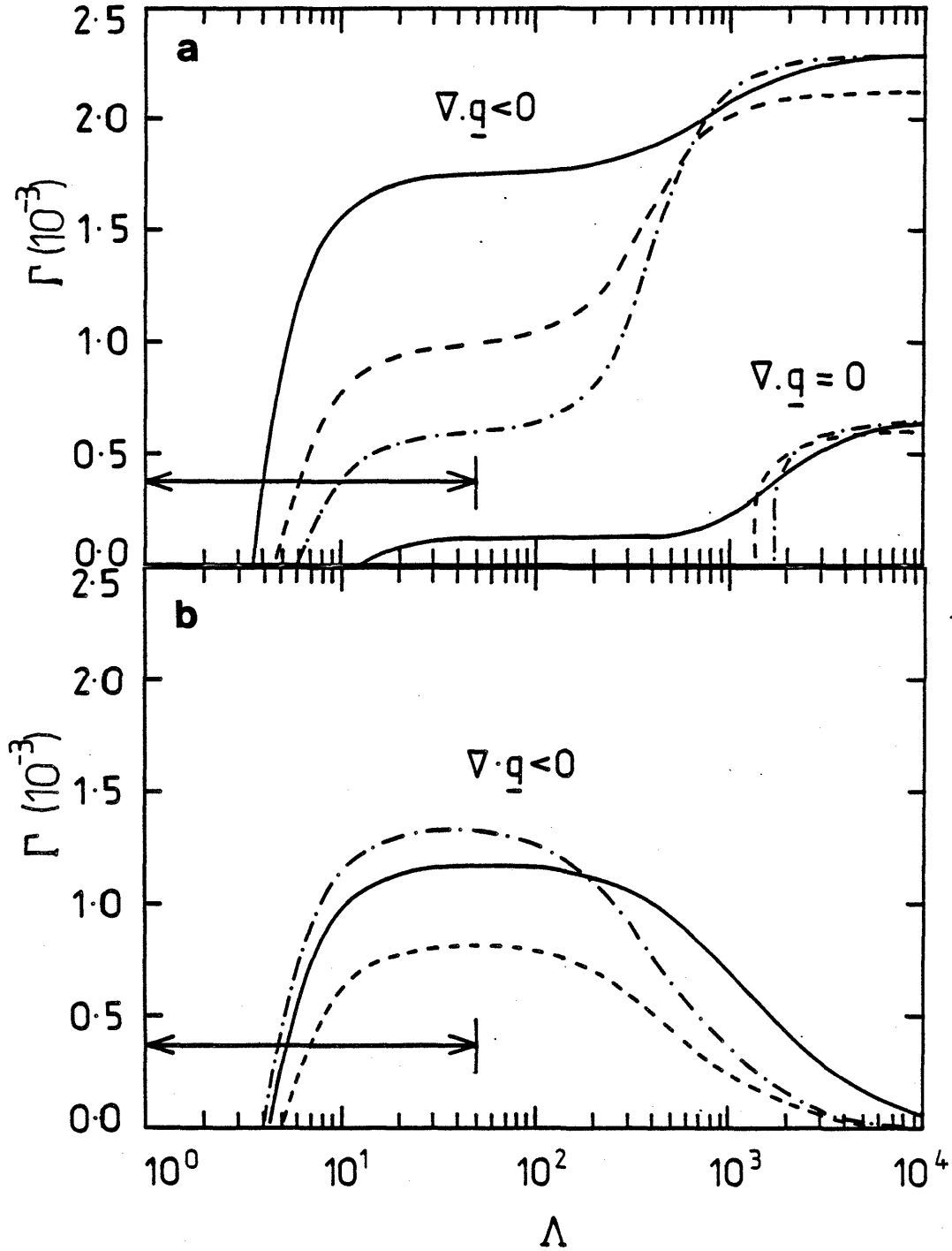


Fig. 4.1 Plot of Γ vs. Λ for $L_s=0.05$, $L_n=50$, $\Delta=-5/2, 0$ and: (a) $L_T=-100$ and (b) $L_T=100$. Solid lines represent the kinetic results, dashed lines the fluid results with Braginskii coefficients and dashed-dotted lines the fluid results with the coefficients derived in chapter 2. The arrows indicate the expected range of validity of both fluid and kinetic models.

4.6 Effect of electron self-collisions

In the present kinetic model, the electron-electron momentum and energy exchange have been neglected. The former may be approximately accounted for if we replace Z by $Z+1$ in the $\bar{V}_{ei}$ term of the $\underline{f}_1$ equation (4.4). The latter, represented by C_{ee0} in equation (4.3), cannot be so conveniently simplified.

In fluid theory it is assumed that C_{ee0} dominates over all other terms in equation (4.3), and since f_m is a solution of C_{ee0} , we then have

$$f'_{01} = f'_m = (T'/T)(v^2/v_T^2 - 3/2)f_m,$$

in perturbation.

For the theory developed in this chapter the opposite ordering has been adopted, i.e. the L.H.S of equation (4.3) is assumed to dominate over the C_{ee0} term. This gives us the following expression for f'_0 .

$$f'_{02} \propto [a_1 k^2 v^2 / (a_1 k^2 v^2 + \gamma \bar{V}_{ei})] [(\phi/v) \partial_v f_0 + f_z/v],$$

which is obtained from equation (4.13)' by setting $\bar{V}_{ee}=0$.

In figure (4.2), f'_{01} and f'_{02} (both normalized to unity) are plotted as functions of V . The dashed curve refers to f'_{01} and the dotted curve to f'_{02} . The latter corresponds to the configuration described in figure (4.1)a, with $\Delta=0$ and $\Lambda=30$.

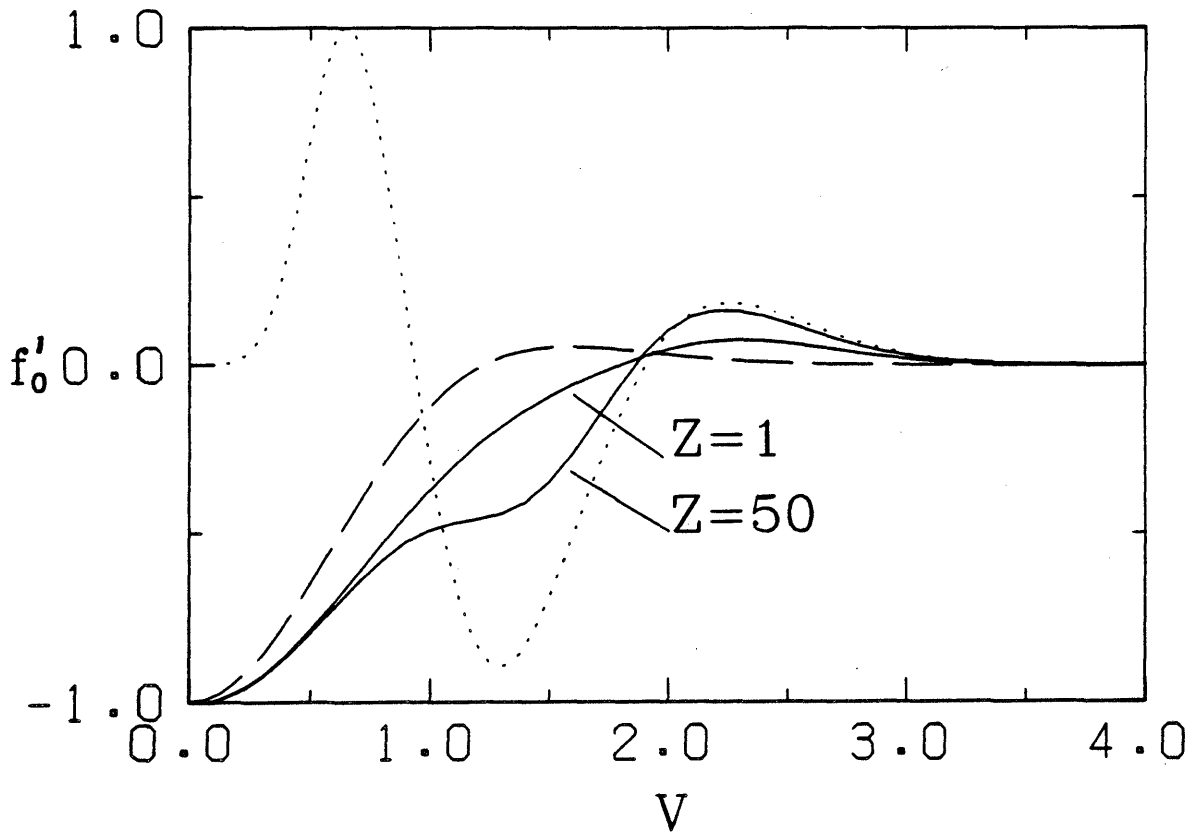


Fig. 4.2 Plot of f'_{01} (dashed line), f'_{02} (dotted line) and f'_0 (solid line) as a function of V . All curves have been normalized to unity.

As observed, there is a large discrepancy between f'_{01} and f'_{02} . In order to establish which one is closer to the 'true' solution the stability analysis should ideally be repeated with C_{ee0} included in equation (4.9). This, however, would prove to be computationally very expensive, as it involves several matrix inversions and multiplications. Here, the C_{ee0} contribution is estimated by reevaluating f'_{02} from equation (4.13)' as follows.

$$f'_0 \propto \mathcal{D}^{-1} a_1 k^2 v^2 [(\phi/v) \partial_v f_0 + f_z/v],$$

where $\mathcal{D} \approx \gamma \nabla_{ei} + a_1 k^2 v^2 - \gamma_{ee} \nabla_{ei} \cdot$. This differential operator, which neglects the perturbation of the 'field' electrons (see App. A.2), is inverted numerically by the method described in appendix A.3.

The f'_0 curve is plotted in figure (4.2) (solid line) for $Z=1$ and 50. There, we note that the $Z=1$ solution is very close to f'_{01} , indicating a strong thermalization effect arising from the electron self-collisions. The slight departure between the two curves is the result of having neglected the $\gamma_{ee} f_0$ contribution to C_{ee0} . As Z increases, the high velocity tail of f'_0 approaches the Lorentz plasma solution f'_{02} , though at low velocities the electron collisions are still sufficiently strong to keep $f'_0 \approx f'_{01}$.

A more quantitative comparison between f'_{01} , f'_{02} and f'_0 may be obtained by evaluating their velocity moments. In figure (4.3) we plot $|\langle V^j \rangle|$ against j for f'_{01} (dashed line), f'_{02} (dotted line) and f'_0 (solid line). Since the growth rate of the instability depends mainly on the $\partial_Z \langle V_z^4 \rangle$ term of equation (4.18), and hence on the $j=4$ moment of f'_0 , the omission of electron self-collisions is likely to be a good approximation, even for $Z=1$.

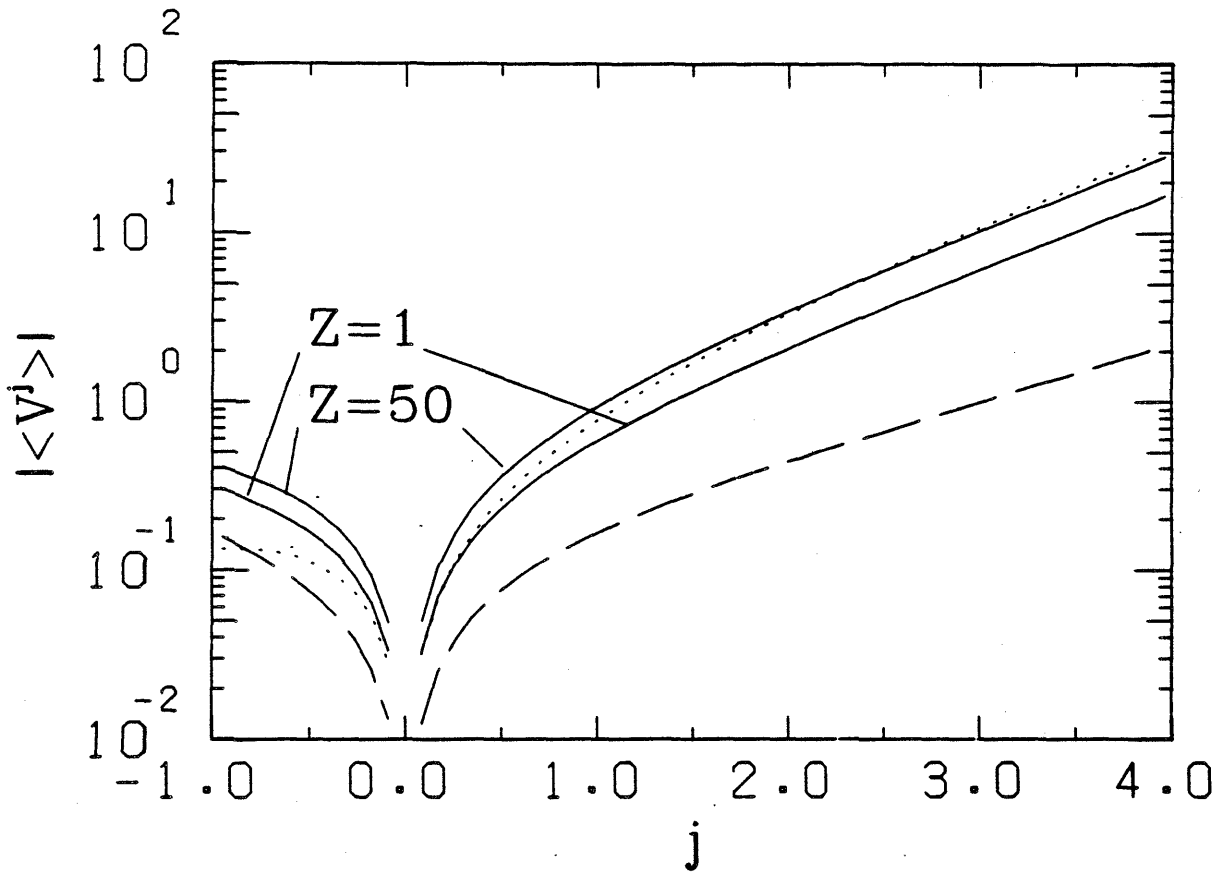


Fig. 4.3 Plot of $|\langle V^j \rangle|$ as function of j , for f'_{01} (dashed line), f'_{02} (dotted line) and f'_0 (solid line).

4.7 Discussion and conclusions

The main approximations used in the kinetic model of this chapter are very similar to the ones used in the 1-fluid model of chapter 3. They involve the omission of hydrodynamics, the 'local approximation' and the collisionality condition. The kinetic approach, however, allows us to address the question of collisionality in a more quantitative manner. This is directly related to the validity of the expansion $f = f_0 + \underline{v} \cdot \underline{f}_1 / v$ and the assumption that f_0 is a Maxwellian.

In equilibrium, a distribution of this form (Eqs. (4.5)' and (4.5)') is known to be inadequate for describing the electron heat flow q ($\propto \langle v_z^3 \rangle$), or any higher velocity moment of the distribution, when $l_T / \lambda_T < 100$.²⁹ Since the main driving term of the instability is proportional to $\langle v_z^4 \rangle$, a more realistic distribution is thus required for the theory to be applied to laser-plasmas (where $l_T > 100 \lambda_T$ is easily violated). Such a distribution may be obtained from Fokker-Planck computer codes.^{2,28,31}

In perturbation, the assumption that $f'_0 = f'_m$ (implicit in the fluid analysis) has been shown to be invalid, as the electron self-collisions are not sufficiently strong to thermalize the high velocity part of the distribution, which drives the instability. A similar effect has been found by Bell,³² in the context of electron heat flow in ion waves.

As for the higher order terms in the f expansion (i.e. $f_2, f_3, \dots$), the assumed ordering in equilibrium is that $|f_2/f_1| \sim |f_3/f_2| \sim \dots \sim \lambda_T / l_T \ll 1$. Since the instability growth rate is proportional to $(\lambda_T / l_T)^2$, the contribution due to f_2 should strictly be included in the analysis. This is done in chapter 5.

In conclusion, a kinetic model has been developed for the 1-fluid

EM instability theory, and the following points have been raised.

(i) Due to the non-thermal nature of the isotropic part of the perturbed distribution function (i.e. f'_0), the instability criterion has been modified to make the plasma more unstable when $\nabla n \cdot \nabla T < 0$ (instead of $\nabla n \cdot \nabla T > 0$, as predicted by fluid theory). Also, for $\nabla n = \nabla^2 T = 0$ the growth rate is about 1.5 times larger than the equivalent fluid prediction (using the transport coefficients derived in chapter 2).

(ii) The main driving term of the instability is found to be proportional to $\partial_z \langle v_z^4 \rangle$, thus requiring an equilibrium distribution that accurately describes the high velocity electrons. The simple distribution used in this chapter (Eqs. (4.5)' and (4.5)''') is therefore inadequate when applied to a typical laser-plasma.

(iii) Since the growth rate is proportional to $(\lambda_T / l_T)^2$, the f_2 term in the cartesian tensor expansion should not be neglected.

CHAPTER 5

Kinetic Treatment of EM Instabilities
(including pressure and heat tensor effects)

5.1 Introduction

EM instabilities driven by pressure anisotropy, like the Weibel instability,²⁶ cannot be adequately described by means of conventional fluid equations, as these breakdown in the presence of strongly anisotropic pressures. In order to avoid this difficulty a 2-fluid model, composed of hot and cold electron groups, was developed in chapter 3, which was then used to demonstrate the existence of a collisional class of the Weibel instability.⁹ This instability, as well as the thermomagnetic and Nernst instabilities discussed in the same chapter, will now be incorporated into a more general and accurate theory based on a perturbation of the Fokker-Planck equation.

The model used here is an extension of the one described in chapter 4, where an expansion of the distribution-function as $f = f_0 + \underline{v} \cdot \underline{f}_1 / v$ was adopted. In order to account for anisotropic pressure effects, however, we need the $\underline{f}_2$ term, so that the expansion of f becomes:

$$f = f_0 + \underline{v} \cdot \underline{f}_1 / v + \underline{v} \underline{v} : \underline{f}_2 / v^2 + \underline{v} \underline{v} \underline{v} : \underline{f}_3 / v^3 + \dots$$

The inclusion of the next term, $\underline{f}_3$, allows us to estimate the rate of convergence this series.

The assumptions involved in the stability analysis are similar to

the ones used in chapter 4.

Section 5.2 defines the basic equations. The eigenvalue equation is derived in section 5.3, followed by a presentation of the results for various equilibria in section 5.4. Section 5.5 discusses the general validity of the model. The conclusions are presented in section 5.6.

In this chapter, the electron thermal velocity (v_T), mean collision frequency ($\bar{\nu}_T$) and mean-free-path (λ_T) are defined as follows.

$$v_T = (2T/m)^{1/2} = v_t \sqrt{2} ,$$

$$\bar{\nu}_T = n_i 4\pi (Ze^2/m)^2 \ln \Lambda_{ei} / v_T^3 = \lambda_t (3/4) \sqrt{\pi} ,$$

$$\lambda_T = v_T / \bar{\nu}_T = \lambda_t (32/9\pi)^{1/2} ,$$

where subscript 't' refer to chapter 3 definitions.

5.2 Basic equations

The basic equations used in this chapter are the electron Fokker-Planck equation,

$$\partial_t f + \underline{v} \cdot \nabla f - (\underline{a} + \underline{v} \times \underline{\omega}) \cdot \nabla_v f = C ,$$

and Maxwell's equations,

$$\partial_t \underline{\omega} = - \nabla \times \underline{a} , \quad (5.1)$$

$$\nabla \times \underline{\omega} = (\omega_p/c)^2 \underline{j}/n , \quad (5.2)$$

where $\underline{a} = e\underline{E}/m$, $\underline{\omega} = e\underline{B}/mc$ is the cyclotron frequency, $\omega_p = (4\pi n e^2/m)^{1/2}$ the plasma frequency and C the collision operator (all other symbols have their usual meaning).

Following the cartesian tensor expansion of the distribution function f , the Fokker-Planck equation becomes a set of four coupled equations in f_0 , $\underline{f}_1$, $\underline{\underline{f}}_2$ and $\underline{\underline{\underline{f}}}_3$, given by:³³

$$\partial_t f_0 + \frac{1}{3} \underline{v} \cdot \nabla \underline{f}_1 - \frac{1}{3} (1/v^2) \partial_v (v^2 \underline{a} \cdot \underline{f}_1) = C_0 , \quad (5.3)$$

$$\partial_t \underline{f}_1 + \underline{v} \cdot \nabla f_0 - \underline{a} \partial_v f_0 - \underline{\omega} \times \underline{f}_1 + \frac{2}{5} \underline{v} \cdot \nabla \underline{\underline{f}}_2 - \frac{2}{5} (1/v^3) \partial_v (v^3 \underline{a} \cdot \underline{\underline{f}}_2) = C_1 , \quad (5.4)$$

$$\begin{aligned} & [\partial_t \underline{\underline{f}}_2 + \underline{v} \cdot (\nabla \underline{f}_1 - \frac{1}{3} \nabla \cdot \underline{f}_1 \underline{\underline{I}}_2) - \underline{v} \partial_v (\underline{a} \underline{f}_1 / v - \underline{a} \cdot \underline{f}_1 \underline{\underline{I}}_2 / 3v) - 2 \underline{\omega} \times \underline{\underline{f}}_2 - C_2 \\ & + \frac{2}{7} \{ \underline{v} \cdot \nabla \underline{\underline{\underline{f}}}_3 - (1/v^4) \partial_v (v^4 \underline{a} \cdot \underline{\underline{\underline{f}}}_3) \}]_2 = 0 \text{ and} \end{aligned} \quad (5.5)$$

$$\begin{aligned} & [\partial_t \underline{\underline{\underline{f}}}_3 + \underline{v} \cdot (\nabla \underline{\underline{f}}_2 - \frac{2}{5} \nabla \cdot \underline{\underline{f}}_2 \underline{\underline{\underline{I}}}_2) - \underline{v}^2 \partial_v \{ \underline{a} \underline{\underline{f}}_2 / v^2 - \frac{2}{5} \underline{a} \cdot \underline{\underline{f}}_2 \underline{\underline{\underline{I}}}_2 \} \\ & - 3 \underline{\omega} \times \underline{\underline{\underline{f}}}_3 - C_3]_3 = 0 . \end{aligned} \quad (5.6)$$

The operator $[]_m$ means that xyz for each element is permuted in all ways ($m!$ ways), the result added, and divided by $m!$ (to produce a completely symmetric m^{th} order tensor). $\underline{I}_m$ is the unit diagonal m^{th} order tensor.

The current is given by $\underline{j} = -(4\pi e/3) \int dv v^3 \underline{f}_1$. The term $C_m = C_{eem} + C_{eim}$ describes the collisional exchange between electrons C_{eem} and electrons and ions C_{eim} (assuming a fully ionized plasma). Under the assumption that the ions are homogeneous and cold we may write, to a very good approximation, $C_{eim} = m(m+1) \underline{V}_{ei-m} f / 2$ ($m > 0$) and $C_{ei0} = 0$, where $\underline{V}_{ei} = [n_i 4\pi (Ze^2/m)^2 \ln \Lambda_{ei}] / v^3$ ($\ln \Lambda_{ei}$ being the Coulomb logarithm).¹⁷ As for the electron-electron collisions, the approximation that $C_{eem} = m(m+1) \underline{V}_{ee-m} f / 2$ ($m > 0$), where $\underline{V}_{ee} = [n_e 4\pi (e^2/m)^2 \ln \Lambda_{ee}] / v^3$, is also used. This term will be initially neglected since $\underline{V}_{ei} / \underline{V}_{ee} \approx Z \gg 1$ for a high-Z plasma. The term C_{ee0} , which represents the energy exchange between electrons, is defined in appendix A.2.

5.2 Derivation of the eigenvalue equation

Using the same perturbation configuration as in chapters 3 and 4, equations (5.1)-(5.6) are linearized as follows (with the dash denoting perturbed quantities).

$$\gamma \omega' = ika'_z - \partial_z a'_x, \quad (5.7)$$

$$ik\omega' = -(\omega_p/c)^2 (4\pi/3n) \int_0^\infty dv v^3 f'_z, \quad (5.8)$$

$$\begin{aligned} \gamma f'_0 + \frac{1}{3} ikvf'_x - \frac{1}{3} a'_z [(1/v^2) \partial_v (v^2 f'_z)] - \\ - \frac{1}{3} a'_z [(1/v^2) \partial_v (v^2 f'_z)] = c'_{ee0}, \end{aligned} \quad (5.9)$$

$$ikvf'_0 - a'_x \partial_v f'_0 - \omega' f'_z = -\gamma_{ei} f'_x, \quad (5.10)$$

$$(v \partial_z - a'_z \partial_v) f'_0 - a'_z \partial_v f'_0 + \frac{2}{5} ikvf'_{xz} = -\gamma_{ei} f'_z, \quad (5.11)$$

$$\frac{1}{2} ikvf'_z + \frac{3}{7} ikvf'_{xxz} - \frac{1}{2} a'_x [v \partial_v (f'_z/v)] - \frac{3}{7} \omega' f'_{zz} = -3 \gamma_{ei} f'_{xz}, \quad (5.12)$$

$$\frac{8}{15} ikvf'_{xz} = -6 \gamma_{ei} f'_{xxz}. \quad (5.13)$$

In deriving the above equations it has been assumed that $\gamma, kv \ll \gamma_{ei}$ (for a thermal electron), in accordance with the collisionality condition. Furthermore, subject to the 'local approximation', $kl \gg 2\pi$, where l is a typical spatial scale-length. Lastly, the assumption that $|f_0| \gg |f_1| \gg \dots$ has been used for the equilibrium distribution.

Substituting equations (5.7), (5.8), (5.12) and (5.13) into

equation (5.11), and defining the relation $a'_x = \phi \omega'$, we obtain:

$$\begin{aligned}
 f'_z & \left[1 + \frac{a_2 k^2 v^2}{\gamma_{ei} + a_3 k^2 v^2} \right] - (1/\gamma_{ei}) \left\{ (\gamma + \partial_z \phi) (\partial_v f_0) \right. \\
 & + \gamma_{ei} \left[\frac{a_2 k^2 v^2}{\gamma_{ei} + a_3 k^2 v^2} \right] [3f_{zz}/v + \phi \partial_v (f_z/v)] \left. \right\} (1/k l_s)^2 (4\pi/3n) \int_0^\infty dv v^3 f'_z \\
 & = - (1/\gamma_{ei}) (v \partial_z - a_z \partial_v) f'_0, \quad (5.14)
 \end{aligned}$$

where $a_2 = 1/15$, $a_3 = 4/315$ and $l_s = c/\omega_p$ is the collisionless skin depth. Constants a_2 and a_3 represent the contributions due to f_2 and f_3 , respectively. If we set $a_2 = 0$, equation (5.14) reduces to equation (4.12), which was derived in chapter 4 without f_2 or higher order terms.

The equation for f'_0 may be deduced from equations (5.7)-(5.10) in the same way as was done in chapter 4, section 4.3. The result, in the absence of electron self-collisions, is given by (Eq. (4.13)'):

$$f'_0 = \left[\frac{a_1 k^2 v^2}{\gamma_{ei} + a_1 k^2 v^2} \right] [(\phi/v) \partial_v f_0 + f_z/v] \int_0^\infty dv v^3 f'_z.$$

Substituting this expression into equation (5.14), we obtain the following integro-eigenvalue equation.

$$\begin{aligned}
 F'_z & + (v^3/3k^2 L_s^2) \left[\frac{1 + a_3 k^2 v^8}{1 + (a_2 + a_3) k^2 v^8} \right] \left\{ - (\partial_v F_0) (\Gamma + \partial_z \Phi) \right. \\
 & + (v \partial_z - \epsilon \partial_v) \left[\frac{a_1 k^2 v^5}{\Gamma + a_1 k^2 v^5} \right] [(\Phi/v) \partial_v F_0 + F_z/v] \\
 & \left. - \left[\frac{a_2 k^2 v^5}{1 + a_3 k^2 v^8} \right] [\Phi \partial_v (F_z/v) + 3F_{zz}/v] \right\} \int_0^\infty dv v^3 F'_z = 0.
 \end{aligned}$$

All dimensionless variables are defined at the end of section 4.3, in

chapter 4. Apart from the terms a_2 and a_3 , this equation is equivalent to equation (4.14).

Following the procedure outlined in section 4.4 we obtain (in the limit as $\Gamma \ll a_1 K^2$):

$$\begin{aligned} \Gamma = & \left[\int_0^\infty v^6 \left[\frac{1+a_3 K^2 v^8}{1+(a_2+a_3)K^2 v^8} \right] \{ (\partial_{v F_0})(\partial_z \Phi) \right. \\ & - (v \partial_z - \epsilon \partial_v) [(\Phi/v) \partial_{v F_0 + F_z/v}] \\ & + \left. \left[\frac{a_2 K^2 v^5}{1+a_3 K^2 v^8} \right] [\Phi \partial_v (F_z/v) + 3 F_{zz}/v] \} dv - 3 K^2 L_s^2 \right] \\ & / \left\{ - \int_0^\infty dv v^6 \left[\frac{1+a_3 K^2 v^8}{1+(a_2+a_3)K^2 v^8} \right] \partial_{v F_0} \right\}, \end{aligned} \quad (5.15)$$

where $\Phi = - \{ \int_0^\infty v F_z dv \} / \{ \int_0^\infty v \partial_{v F_0} dv \}$ (from Eq. 4.15).

Two further limiting cases of the above expression for Γ may be obtained by taking $K \gg 1$ (collisionless limit) and $K \ll 1$ (collisional limit), respectively, and integrating by parts.

$$\begin{aligned} \Gamma_{K \gg 1} = & \left\{ - \frac{1}{6} \partial_z \langle v_z^4 \rangle - \epsilon \langle v_z^2 \rangle + \Phi [\partial_z \langle v_0^3 \rangle + 4 \epsilon \langle v_0^1 \rangle] \right. \\ & + (a_2/2a_3) [\langle v_{zz}^0 \rangle - \Phi \langle v_z^{-1} \rangle] - \left. \left[(a_2+a_3)/a_3 \right] \frac{1}{2} K^2 L_s^2 \right\} / \langle v_0^3 \rangle, \end{aligned} \quad (5.15)'$$

$$\begin{aligned} \Gamma_{K \ll 1} = & \left\{ - \frac{1}{6} \partial_z \langle v_z^4 \rangle - \epsilon \langle v_z^2 \rangle + \Phi [\partial_z \langle v_0^3 \rangle + 4 \epsilon \langle v_0^1 \rangle] \right. \\ & + (a_2/2) K^2 [\langle v_{zz}^8 \rangle - (11/3) \Phi \langle v_z^7 \rangle] - \left. \frac{1}{2} K^2 L_s^2 \right\} / \langle v_0^3 \rangle, \end{aligned} \quad (5.15)''$$

where $\Phi = \langle v_z^{-1} \rangle / \langle v_0^{-2} \rangle$ and $\partial_z^m \langle v_i^j \rangle = \int_0^\infty v^{j+2} \partial_{v F_i}^m dv$ ($m=0,1,\dots$).

5.4 Results for different equilibria

In a general one dimensional equilibrium, defined by a distribution function 'F', terms F_0 , F_z , F_{zz} , ..., of the cartesian tensor expansion become identical to the Legendre moments:

$$F_0 = \frac{1}{2} \int_{-1}^1 d\mu F ,$$

$$F_z = \frac{3}{2} \int_{-1}^1 d\mu \mu F ,$$

$$F_{zz} = \frac{5}{4} \int_{-1}^1 d\mu (3\mu^2 - 1) F ,$$

.

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of the expansion, $F = F_0 + \mu F_z + \frac{1}{2}(3\mu^2 - 1)F_{zz} + \dots$, where $\mu = V_z/V$.

In chapter 4, F_{zz} and higher order terms were neglected, and F_0 was assumed to be a Maxwellian. Here, this approach is extended to include F_{zz} , and to introduce also other types of equilibria by specifying the total distribution function F and evaluating the Legendre moments numerically. Once the moments are known, equations (5.15)-(5.15)" may be solved by numerical integration.

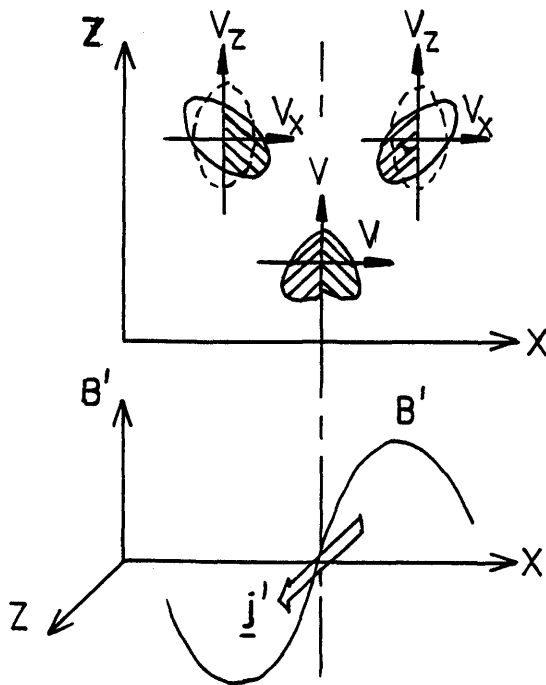
5.4.1 2-Temperature distribution

Consider a plasma that is homogeneous in space but has a pressure anisotropy characterized by the following 2-temperature Maxwellian.

$$F = \frac{4}{\sqrt{\pi}} \exp[-v_{xy}^2 - v_z^2 T/T_z] ,$$

where $v_{xy}^2 = v_x^2 + v_y^2$, and all dimensionless quantities have been normalized relative to the temperature T in the xy plane.

Such a distribution can drive a purely growing EM instability, known as the Weibel instability, provided $T_z > T$.²⁶ Its mechanism is briefly described as follows.³⁴ A perturbed magnetic field $B'(x, t)$ in the y direction, with a sinusoidal modulation along x , rotates the elliptic distribution via the Lorentz force, as seen in figure (5.1). As a result, an electric current is induced about the $B'=0$ node, which then amplifies the initial magnetic field perturbation.



5.1 Instability mechanism of the Weibel instability.

Weibel derived the growth rate for this instability as applied to a collisionless plasma.²⁶ In dimensionless form it is given by:

$$\Gamma_W = \frac{K}{\sqrt{\pi}} [T_z/T - 1 - K^2 L_s^2] T/T_z ,$$

where $T_z/T > 1 + K^2 L_s^2$ is a necessary condition for $\Gamma_W > 0$. Since collisions have been neglected, the validity of this result is restricted to $\Gamma_W \gg 1$ and $\Lambda (= 2\pi/K) \ll 1$.

The theory developed in this chapter describes the Weibel instability in the limit where collisions are strong, i.e. for $\Gamma \ll 1$ and $\Lambda \gg 1$. With the symmetric 2-temperature Maxwellian, chosen for the equilibrium, $F_z = 0$, and hence equation (5.15) becomes simply:

$$\Gamma = 3 \left[\int_0^\infty \left[\frac{a_2 K^2 v^{10}}{1 + (a_2 + a_3) K^2 v^8} \right] F_{zz} dv - K^2 L_s^2 \right] \\ / \left[- \int_0^\infty dv v^6 \left[\frac{1 + a_3 K^2 v^8}{1 + (a_2 + a_3) K^2 v^8} \right] \partial_v F_0 \right] .$$

The rotation of the electron velocity distribution, necessary to drive the Weibel mode, now arises from the xz component of the pressure tensor, represented by term f'_{xz} in equation (5.11). The time evolution of f'_{xz} is governed by equation (5.12), i.e.

$$f'_{xz} = (\omega'/7 \gamma_{ei}) f_{zz} + \dots$$

where f_{zz} describes the pressure anisotropy of the unperturbed plasma. This mechanism is different to the one required to drive the thermomagnetic mode (described in chapters 3 and 4), which relies on a perturbation of the isotropic pressure, represented by the term f'_0 in equation (5.11).

Figure (5.2)a plots Γ_W and Γ as a function of Λ , for $L_s=0$ (i.e. no short-wavelength damping), and $T_z/T=1.01$ and 2 (i.e. 1% and 100% temperature anisotropy respectively). The dotted curves refer to Γ_W and solid ones to Γ . By setting $a_3=0$ we are able to estimate the contribution that f'_3 makes towards Γ , and hence assess the convergence of the cartesian tensor expansion. As expected, the expansion deteriorates when $\Lambda < 1$, since the advection term in the Fokker-Planck equation starts to dominate over the collision term. In the opposite limit, the collisionless Weibel theory becomes increasingly inaccurate and fails to yield the correct K -dependence for the growth rate (this being proportional to K^2 when $K \sim 0$).

Figure (5.2)b plots Γ as a function of Λ , for $T_z/T=2$ and $L_s=0.05$, 0.5 and 5. As observed from the figure, the collisional results (solid lines) indicate the presence of a value of L_s above which the unstable Weibel mode becomes damped. This cut-off L_s , which does not arise in the collisionless theory, is given by

$$\begin{aligned} L_s^c &= 5.48 \times 10^{-2} (n/10^{21} \text{ cm}^{-3})^{1/2} (Z/10) (\ln \Lambda_{ei}/5) (1 \text{ keV}/T)^2 \\ &= [(1/15) \langle v_{zz}^8 \rangle]^{1/2}. \end{aligned} \quad (5.16)$$

For $T_z/T=2$ and 1.01, $L_s^c=6.8$ and 0.38, respectively.

Physically, when $L_s > L_s^c$ the joule dissipation of the induced current becomes much too strong for any B-field generation by the Weibel instability mechanism to occur.

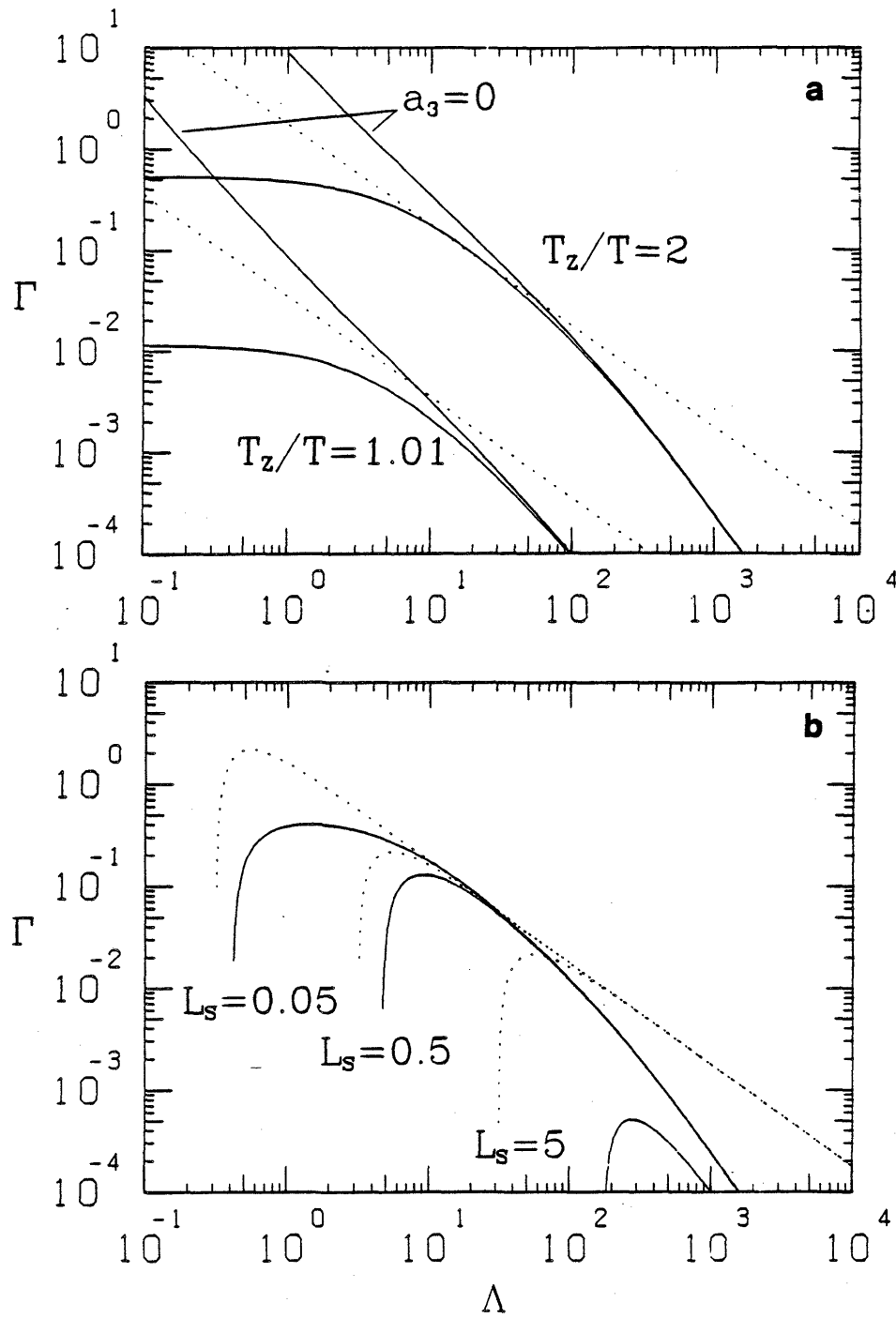


Fig. 5.2 Plot of Γ_w (dotted lines) and Γ (solid lines) as functions of Λ , for: (a) $L_s = 0$ and $T_z/T = 2$ and 1.01 , and (b) $T_z/T = 2$ and $L_s = 0.05$, 0.5 and 5 .

5.4.2 Skewed distribution

Consider the following distribution function.³⁴

$$F = \frac{4}{\sqrt{\pi}} [1 + Q(\frac{2}{5}\sqrt{2})V_z(V^2 - 5/2)] e^{-V^2}.$$

This describes a homogeneous heat carrying plasma, where the heat flow Q is measured in units of $q_f = nm(T/m)^{3/2}$ (the 'free-streaming limit').

Since $F_{zz} = 0$, the dispersion relation (5.15) reduces to

$$\Gamma = \left[\int_0^\infty \left[\frac{a_1 K^2 V^{11}}{1 + (a_2 + a_3) K^2 V^8} \right] \Phi \partial_V (F_z/V) dV - 3K^2 L_s^2 \right] \\ / \left[- \int_0^\infty dV V^6 \left[\frac{1 + a_3 K^2 V^8}{1 + (a_2 + a_3) K^2 V^8} \right] \partial_V F_0 \right],$$

which is plotted in figure (5.3) as a function of Λ , for $L_s = 0$ and $Q = 0.1$ and 0.01 .

As observed, despite the fact that the unperturbed plasma has no pressure anisotropy an unstable mode may still arise. The instability mechanism is equivalent to the original Weibel mechanism, though the perturbed xz component of the pressure tensor is now induced by the longitudinal electric field, i.e. (Eq. 5.12)

$$f'_{xz} = a'_x [v \partial_V (f_z/v)] / 6 \gamma_{ei} + \dots,$$

instead of the perturbed B -field.

It will be seen in the next section that this Weibel instability, driven by heat flow, is not as strong as the one driven by pressure anisotropy.

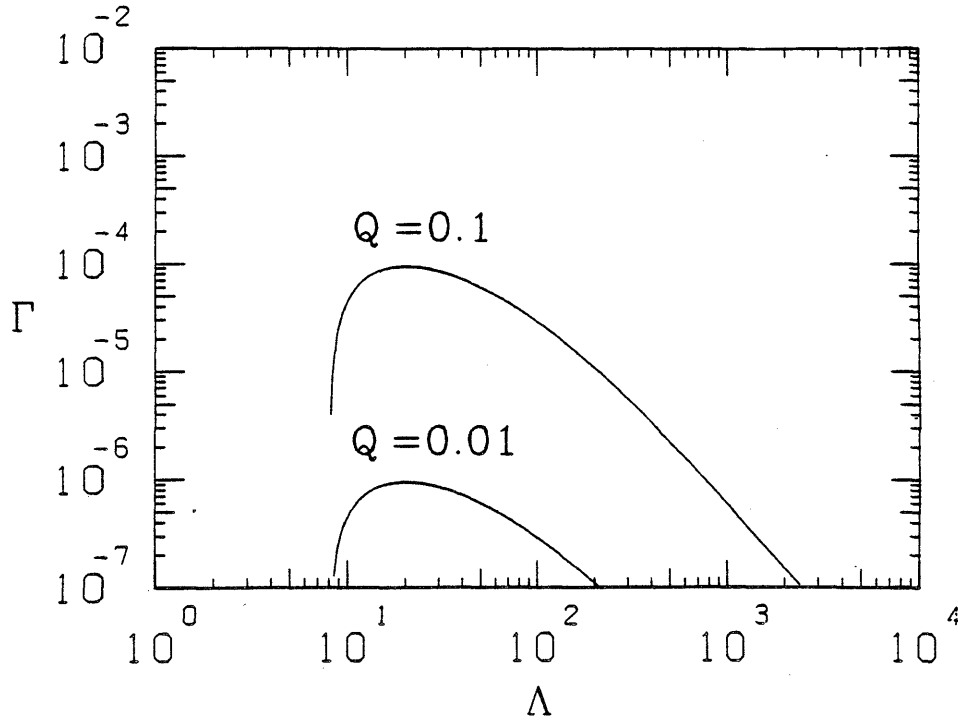


Fig. 5.3 Plot of Γ vs. Λ , for $L_s=0$ and $Q=0.01$ and 0.1 .

5.4.3 Hot and cold distribution

Here, the following equilibrium distribution is introduced.

$$F = \frac{4}{\sqrt{\pi}} \left\{ \exp[-(V_z + V_c)^2 - V_{xy}^2] + (T_c/T_h)^{3/2} \theta \exp[-((V_z - V_h)^2 + V_{xy}^2) T_c/T_h] \right\},$$

This is composed of a hot and cold electron population (identified by subscripts h and c , respectively), where $\theta = n_h/n_c \ll 1$ and $V_{xy}^2 = V_x^2 + V_y^2$.

All dimensionless quantities have been normalized relative to the cold electrons.

Such a distribution possesses the basic features of the two previously considered equilibria, in that it has a pressure anisotropy

(arising from the hot and cold electron drifts) and carries a heat flow Q , given by (Eq. 3.17),

$$Q = (q/q_f) = \Theta V_h \sqrt{2} [V_h^2 - V_c^2 + \frac{5}{2}(T_h/T_c - 1)] .$$

In order to have no net current in the equilibrium we must satisfy $V_c = \Theta V_h$.

Okada et al.³⁴ investigated the collisionless Weibel instability associated with this distribution and found the growth rate to be (in dimensionless form):

$$\Gamma_W = \frac{K}{\sqrt{\pi}} [2(V_c^2 + \Theta V_h^2) - 1 - K^2 L_s^2] / 2[V_c^2 + \Theta V_h^2 (T_c/T_h)^{3/2}] .$$

The collisional theory developed in this chapter gives rise to (from Eq. 5.15):

$$\Gamma = \left[\int_0^\infty \left[\frac{a_1 K^2 V^{11}}{1 + (a_2 + a_3) K^2 V^8} \right] [\Phi \partial_V (F_z/V) + 3F_{zz}/V] dV - 3K^2 L_s^2 \right] / \left[- \int_0^\infty dV V^6 \left[\frac{1 + a_3 K^2 V^8}{1 + (a_2 + a_3) K^2 V^8} \right] \partial_V F_0 \right] .$$

Both results are plotted in figure (5.4)a as a function of Λ , for $L_s = 0$, $\Theta = 0.005$, $T_h/T_c = 3$, $V_c = 0.01$ and $V_h = 2$ ($Q = 0.13$). The solid lines refer to Γ and the dotted lines to Γ_W . This figure shows the effect of neglecting $\underline{f}_3'$ in the analysis, by setting $a_3 = 0$, and also shows the contribution due to the skewed part of the equilibrium distribution, i.e. $\underline{f}_1$, by setting $\underline{f}_2 = 0$. We note that the contribution due to $\underline{f}_3'$ is important for $\Lambda < 10$, whereas the one due to the equilibrium $\underline{f}_1$ is small for all Λ .

Comparison between collisional (Γ) and collisionless (Γ_W)

results, shows that the latter inadequately predicts the growth rate for $\Lambda > 1$.

Figure (5.4)a also presents the result Γ_{CW} (dashed line), obtained by means of the 2-fluid model (Eq. 3.19), developed in chapter 3 (section 3.4). As observed, the fluid theory predictions are very inaccurate, even for large Λ where it is expected to yield the best results. Possible reasons for such discrepancies have been discussed in section 3.4.4.

Figure (5.4)b investigates the effect of adding short-wavelength damping by setting $L_s = 0.05, 0.5$ and 5 . As in the case of the 2-temperature equilibrium distribution (section 5.4.1), a cut-off L_s exists, above which the Joule dissipation (or magnetic diffusion) dominates over the E-field generation mechanism of the Weibel instability. Such an effect is absent in both collisional 2-fluid⁹ and collisionless Weibel³⁴ theories.

Since the F_z contribution towards Γ has been found to be negligible, L_s^C is given by equation (5.16) and is approximately equal to 6.2, for the chosen equilibrium conditions.

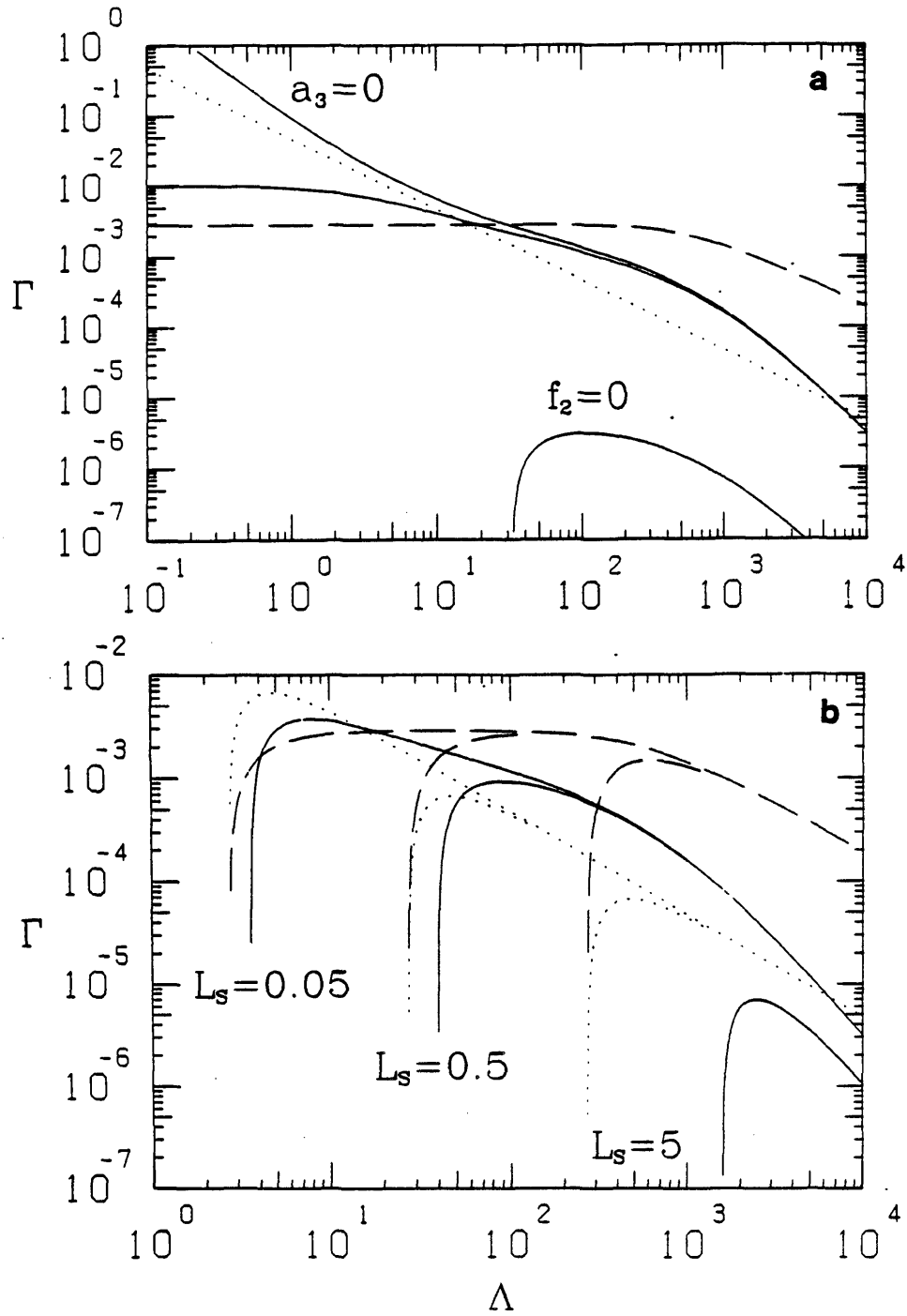


Fig. 5.4 Plot of Γ (solid lines), Γ_w (dotted lines) and Γ_{cw} (dashed lines), as functions of Λ for $\theta=0.005$, $T_h/T_c=3$, $V_c=0.01$, $V_h=2$ ($Q=0.13$) and: (a) $L_s=0$ and (b) $L_s=0.05$, 0.5 and 5 .

5.4.4 Collisional distribution of order $(\lambda_T/l_T)^2$

In chapter 4 the stability analysis was carried out by means of the following equilibrium distribution (Eqs. (4.5)' and (4.5)'').

$$F = F_0 + V_z F_z / V, \text{ where}$$

$$F_0 = F_m = (4/\sqrt{\pi}) e^{-V^2},$$

$$F_z = V^4(4-V^2)F_0/L_T \text{ and}$$

$$\epsilon = -(5/L_T + 2/L_n)/4.$$

Here we investigate the effect of introducing the next term in the expansion, namely $\frac{1}{2}(3V_z^2 - 1)F_{zz}/V^2$. The Legendre moment F_{zz} has been evaluated in appendix A.5 and is given by

$$F_{zz} = \frac{2}{9}V^6 [V^6 - V^4(10 + L_T/L_n - \Delta) + V^2(105/4 + 13L_T/2L_n - 4\Delta) - 6(5/2 + L_T/L_n)] F_0/L_T^2,$$

$$\text{where } \Delta = T(\partial_z^2 T)/(\partial_z T)^2.$$

Substituting this equilibrium distribution into equation (5.15)' we are able to obtain an analytic expression for Γ in the short-wavelength limit, i.e.

$$\Gamma_{K \gg 1} = [(A_1 + A_1')/L_T^2 + (A_2 + A_2')/L_n L_T + (A_3 + A_3')\Delta/L_T^2 - (25/4)\frac{\sqrt{\pi}}{8}K^2 L_s^2],$$

where $A_1' = 53.02$, $A_2' = -5.089$ and $A_3' = 22.90$. These coefficients represent the effect of the Weibel instability. Coefficients A_1 , A_2 and A_3 are listed in table 4.1 of chapter 4 and correspond to the thermomagnetic and Nernst instabilities (described in chapter 3). Since $A_1'/A_1 \simeq A_2'/A_2 \simeq A_3'/A_3 \simeq 3.5$, the Weibel mechanism is dominant.

The above approximate expression for the growth rate may also be compared with the following result obtained from the collisionless Weibel theory, developed by Ramani and Laval.³⁵

$$\Gamma_W = (32K/21\pi) [A_1'/L_T^2 + A_2'/L_n L_T + A_3' \Delta/L_T^2 - (21/4) \frac{\sqrt{\pi}}{8} K^2 L_s^2] .$$

Apart from the general K dependence of the growth rate, both $\Gamma_{K \gg 1}$ and Γ_W lead to similar instability criteria. This is not surprising since the Weibel instability mechanism is identical in both collisional and collisionless regimes.

A more detailed analysis of the results is obtained by solving the full dispersion relation (5.15). Figure (5.5)a plots Γ (solid lines) and Γ_W (dotted lines) against Λ , for $L_s=0$, $L_n=-L_T=100$ and $\Delta=0$ (i.e. $\nabla^2 T=0$). Dashed-dotted lines show the sole contributions of the thermomagnetic and Nernst modes (obtained by setting $F_{zz}=0$), i.e.

$$\begin{aligned} \Gamma^{TN} = & \left[\int_0^\infty v^6 \left[\frac{1+a_3 K^2 v^8}{1+(a_2+a_3)K^2 v^8} \right] \{ (\partial_v F_0)(\partial_z \Phi) \right. \\ & - (v \partial_z - \epsilon \partial_v) [(\Phi/v) \partial_v F_0 + F_z/v] - 3K^2 L_s^2 \} \\ & \left. / \left[- \int_0^\infty dv v^6 \left[\frac{1+a_3 K^2 v^8}{1+(a_2+a_3)K^2 v^8} \right] \partial_v F_0 \right] \right] \end{aligned}$$

and dashed lines those of the Weibel mode (obtained by assuming that $f_0'=0$), i.e.

$$\begin{aligned} \Gamma^W = & \left[\int_0^\infty \left[\frac{a_1 K^2 v^{11}}{1+(a_2+a_3)K^2 v^8} \right] [\Phi \partial_v (F_z/v) + 3F_{zz}/v] dv - 3K^2 L_s^2 \right] \\ & / \left[- \int_0^\infty dv v^6 \left[\frac{1+a_3 K^2 v^8}{1+(a_2+a_3)K^2 v^8} \right] \partial_v F_0 \right] . \end{aligned}$$

The former has also been evaluated for $a_3=a_2=0$ and $a_3=0$, so as to assess the contributions due f'_2 and f'_3 respectively, and the latter for $a_3=0$, so as to assess the contribution due to f'_3 . As shown in the figure, f'_3 becomes important when $\Lambda < 10$, and f'_2 when $\Lambda < 100$.

Of the three instability mechanisms, the Weibel mechanism is noted to be the dominant one for perturbation wavelengths Λ up to about $L_{n,T}$. For larger values of Λ the thermomagnetic and Nernst modes tend to dominate, though the analysis breaks down by virtue of the 'local approximation' (which requires that $\Lambda < L_{T,n}$).

Figure (5.5)b introduces short-wavelength damping by setting $L_s=0.05$ and 0.5 . From the plots we see that both collisional and collisionless results yield approximately the same cut-off wavelength, though not the same growth rates.

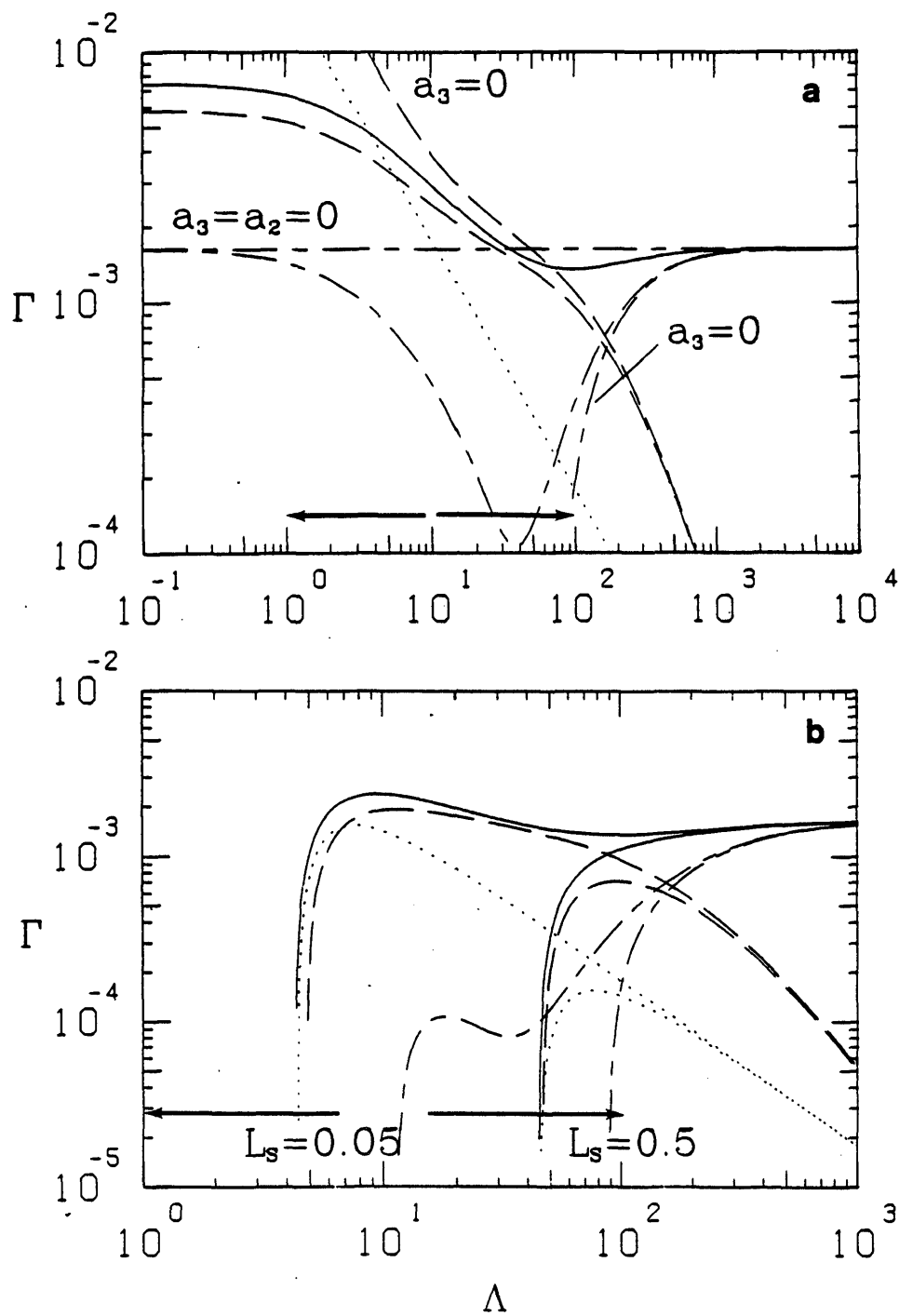


Fig. 5.5 Plot of Γ (solid lines), Γ^W (dashed-dotted lines), Γ^{TN} (dashed lines) and Γ_W (dotted lines) as functions of Λ , for $L_n = -L_T = 100$, $\Delta = 0$ and: (a) $L_s = 0$ and (b) $L_s = 0.05$ and 0.5 . The arrows indicate the expected range of validity of the collisional theories.

5.5 Discussion

Three main approximations have been made in the present stability analysis. These involve the assumption that the perturbed variables do not vary significantly along the direction of the equilibrium inhomogeneity ('local approximation'), that the distribution is fairly isotropic in velocity space (collisionality condition) and that the effect of ion motion may be neglected.

The local approximation is usually considered to be valid provided that $\lambda \ll l_{T,n}$.³⁶

The assumption that the electron distribution function f has a relatively small degree of anisotropy is closely related to the assumption that its cartesian tensor expansion converges (i.e. $|f_0| \gg |f_1| \gg \dots$). This chapter has shown that it is important to include f'_3 whenever $\lambda < 10 \lambda_T$. The f_2 term, however, must always be included (even for $\lambda > 10 \lambda_T$) in order to describe the Weibel instability.

Following the discussion presented in section 4.7, involving the thermomagnetic and Nernst instabilities, we find that the unstable Weibel mode is also driven by the high velocity electrons. This may be demonstrated by returning to equation (5.15)' and noting that $\langle v_{zz}^0 \rangle$, which is the main Weibel instability source, is proportional to $\langle v_O^j \rangle$ (where $j \geq 6$) and hence depends on those electrons moving at approximately twice their thermal speed v_T . The equilibrium distribution used in section 5.4.4 is, therefore, inadequate since it can become negative for $v \gtrsim 2.5 v_T$, when $l_T < 100 \lambda_T$.

The effect of ion motion on the thermomagnetic and Nernst

instabilities has been discussed in section 3.3.4. Its influence on the Weibel mode arises mainly through the perturbed longitudinal electric field E'_x , via a modification in the perturbed f_0 . However, as a result of the small contribution that E'_x makes towards the growth rate of the Weibel instability, a perturbation of the ions is likely to have a negligible effect on the final results. This agrees with the prediction made in section 3.4.4, based on the 2-fluid model.

For the same reasons stated above we may also neglect the effect of electron-electron energy exchange on the Weibel mechanism.

5.6 Conclusions

In this chapter it has been shown that the f_2 term in the cartesian tensor expansion of the electron distribution function cannot be omitted from the stability analysis, as it is essential for describing the Weibel instability mechanism. The contribution due to the perturbed part of the next term in the expansion, i.e. f'_3 , becomes significant when $\lambda < 10 \lambda_T$.

A comparison between the thermomagnetic, Nernst and Weibel instabilities indicates that the latter is likely to yield the highest growth rate.

The collisional Weibel instability based on the 2-fluid model, described in chapter 3, as well as the collisionless Weibel theory, yield inaccurate results for $\lambda > \lambda_T$. Both these theories also fail to predict the presence of a critical value of L_s , above which the unstable Weibel mode becomes damped (Eq. 5.16).

Finally it has been demonstrated that the Weibel instability (like the thermomagnetic and Nernst instabilities) is driven by the hot electron 'tail' of the equilibrium distribution function and hence requires a distribution that is accurate at high velocities (e.g. $v \gtrsim 2v_T$).

CHAPTER 6

FM Instabilities in Laser-Plasma Experiments

6.1 Introduction

There have been many recent experimental reports on the occurrence of filamentary structures (or plasma jets) and strong magnetic fields in the ablation of laser-driven targets.⁸⁻¹¹ These structures, which have transverse scale-lengths of about $10\mu\text{m}$, occur mainly in medium to high Z target materials (e.g. glass or gold), and have been inferred to originate at densities above critical (for $1\mu\text{m}$ laser irradiation).⁸

Figure 6.1 shows a shadowgram of a target consisting of plastic on one side and tungsten on the other, irradiated with opposing laser beams with an average intensity of $6 \times 10^{14} \text{Wcm}^{-2}$. The picture has been taken from a series of experiments performed by Willi et al.⁸ at the Rutherford Appleton Laboratory and indicates the presence of plasma jets occurring 1.1ns after the beginning of the laser pulse. It has been demonstrated experimentally that these jets are not caused by laser beam self-focusing.

Several mechanisms have been proposed to explain the appearance of these filamentary structures, such as the radiation cooling instability,¹⁴ the Ohmic instability (also with radiation cooling),¹³ the collisional Weibel instability⁹ and the thermomagnetic instability^{8,12}. The first of these gives the correct Z dependence for the growth rate (due to the large radiation loss of high- Z materials) but fails to provide a magnetic field generation mechanism.

Also, the optimum wavelength predictions based on this theory, have been made for a perturbation vector $\underline{k}$ perpendicular to the direction of plasma ablation, whereas identical results may be obtained for any direction taken for $\underline{k}$. All other instability mechanisms have been investigated in chapters 3 of this thesis, where it has been concluded that the collisional Weibel instability is likely to dominate over the Ohmic instability. The theoretical studies made in chapter 5 suggest that the collisional Weibel instability will also dominate over the thermomagnetic and Nernst instabilities.

Here, an attempt has been made to correlate the experimental observations with the theoretical predictions based on the kinetic model for the collisional Weibel instability (developed in chapter 5). Section 6.2 presents the theoretical results and section 6.3 the discussion and conclusions.

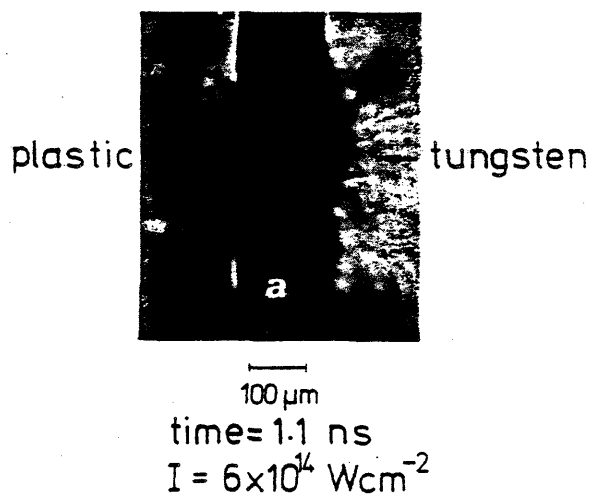


Fig. 6.1 Shadowgram of a target consisting of plastic on one side and tungsten on the other, irradiated with opposing laser beams. The average irradiance was $6 \times 10^{14} \text{ Wcm}^{-2}$ on both sides. The probe timing was 1.1 ns after the beginning of the laser pulse.

6.2 Results

The results presented here are based on solutions of the dispersion relation (5.15), for the equilibrium distribution described in section 5.4.3, where $\Theta=0.005$, $T_h/T_c=3$, $V_c=0.01$ and $V_h=2$ ($Q=0.13$).

Such an equilibrium has been chosen to model the experimental conditions, since it closely resembles the results obtained from typical Fokker-Planck simulations of transport in plasmas irradiated by $1\mu\text{m}$ laser light.²⁸ The parameters are consistent with an electron heat flow of approximately 13% of the 'free-streaming limit' (Eq. 3.17) and a cold electron drift at the isothermal sound speed $c_s=(ZT_c/m)^{1/2}$ (for $Z=10$).

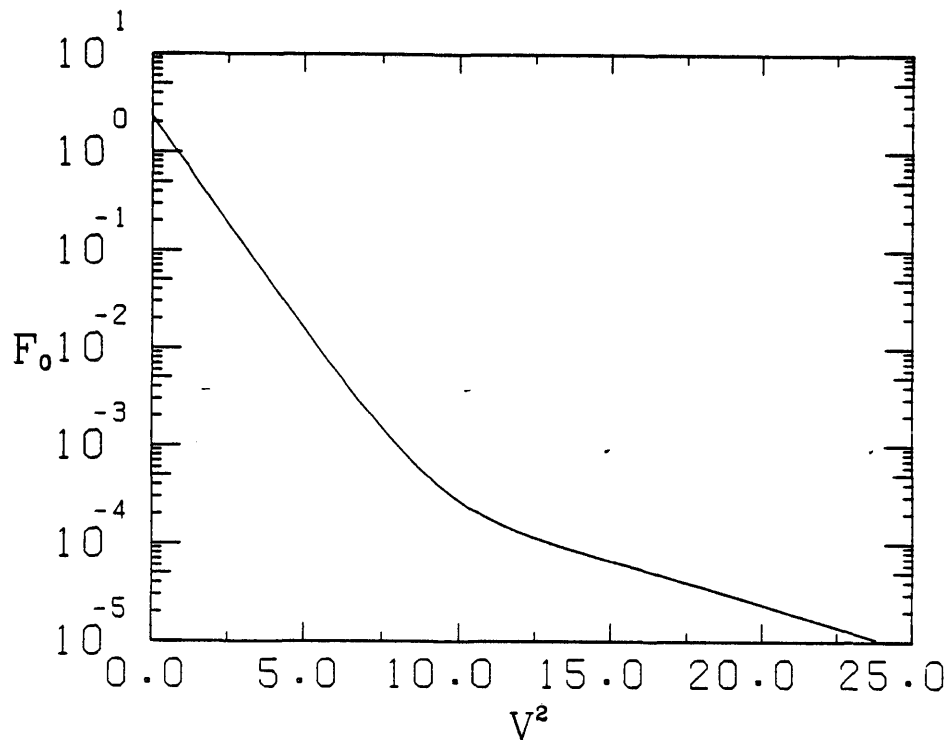


Fig. 6.2 Plot of F_0 as a function of V^2 .

Figure 6.2 plots the isotropic part of the equilibrium distribution as a function of $V^2(=v^2/v_T^2)$. Here we observe the characteristic hot electron 'tail', expected to be present in the overdense region of a laser-plasma.

Before evaluating the growth rates, the thermal collision frequency ν_T is redefined so as to approximately account for the electron-electron momentum exchange, i.e.

$$\begin{aligned}\nu_T^* &= \nu_{ei}(v_T) + \nu_{ee}(v_T) \\ &\approx n_c 4\pi (e^2/m)^2 [Z \ln \Lambda_{ei} + \ln \Lambda_{ee}] / v_T^3.\end{aligned}$$

The maximum growth rate $\gamma_{\max}$ is plotted in figure (6.3)a and corresponding perturbation wavelength $\lambda_{\max}$ in figure (6.3)b as functions of Z , for different values of n_c and T_c .

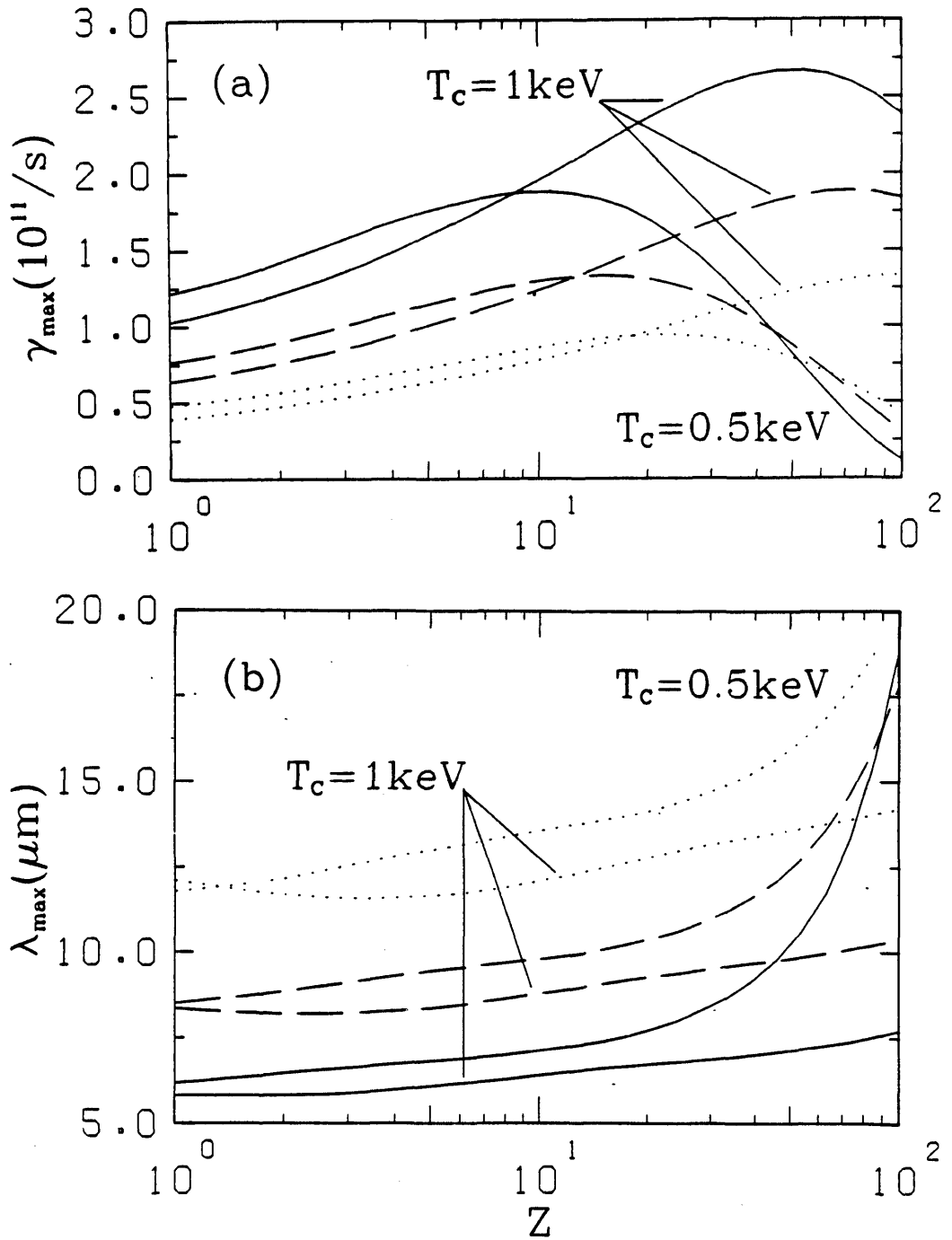


Fig. 6.3 Plot of: (a) maximum growth rate $\gamma_{\max}$ and (b) optimum wavelength $\lambda_{\max}$, as a function of Z . Dotted lines are for $n_c = 5 \times 10^{21} \text{ cm}^{-3}$, dashed lines for $n_c = 10^{22} \text{ cm}^{-3}$ and solid lines for $n_c = 2 \times 10^{22} \text{ cm}^{-3}$. The equilibrium parameters are: $\theta = 0.005$, $T_h/T_c = 3$, $V_c = 0.01$ and $V_h = 2$ ($Q = 0.13$).

6.3 Discussion and conclusions

From figure (6.3)a we note that, in general, the growth rate increases with Z . For large values of Z , L_s approaches its critical value (given by Eq. 5.16) causing severe damping of the unstable mode.

Considering a plastic ($Z \approx 4$) and a tungsten target ($Z \approx 30$) (of the type shown in Fig. 6.1), we find that after 0.1ns, an initial perturbation at $n_c = 10^{22} \text{ cm}^{-3}$ and $T_c = 1 \text{ keV}$ would experience 7.5 and 16 e-folding growths, respectively. This is consistent with the experimental observation.

The optimum wavelength, shown in figure (6.3)b, is relatively insensitive to variations in n_c , T_c and Z , and compares well with the observed spatial scale-lengths of the plasma jets.

It must be stressed, however, that these results do not allow for the n_c , T_c and Z dependence of the equilibrium plasma. Since the instability depends on a high velocity moment of the distribution, namely the hot electron tail, one should strictly perturb a self-consistent equilibrium derived from a non-linear Fokker-Planck simulation of the transport.

In conclusion, the collisional Weibel instability may explain the experimentally observed filamentary structures in laser-plasmas. This instability would appear to be most dangerous for high- Z targets.

CHAPTER 7

Conclusions

7.1 Summary

7.1.1 Linear transport

Chapter 2 dealt with the derivation of electron transport coefficients as applied to a fully ionized plasma. A short review on the various derivation methods was presented. The fluid equations were derived and the main definitions introduced. By using the Lorentz plasma approximation, transport coefficients, appropriate to Ohm's Law and electron heat flow, were calculated by direct integration of the $\underline{f}_1$ equation. A new method was subsequently developed for introducing the effect of electron self-collisions.

The coefficients were compared with the equivalent ones derived by Braginskii.⁴ Some of his results were shown to be very inaccurate over certain ranges of $\omega\tau$, with the errors being more pronounced for $Z \gg 1$. In particular, for $Z = \infty$ and $\omega\tau \ll 10^{-2}$, the Righi-Leduc heat flux coefficient was found to be about 40% below the 'exact' value. Also for $Z = \infty$ but within the range $0.3 < \omega\tau < 30$, the lateral heat flow coefficient oscillated by more than 35%. The reasons for such discrepancies were due to the Laguerre expansion for $\underline{f}_1$ and the polynomial fit in $\omega\tau$ applied to his coefficients.

Comparisons were also made with Hochstim's transport coefficients,⁵ which were based on a 20-term Laguerre expansion for $\underline{f}_1$. These were found to be in very good agreement with our results.

7.1.2 EM instabilities

All stability analysis was done by considering an electron plasma with a background of cold and fixed ions.

(i) By means of a single-fluid model, the thermomagnetic¹² and Nernst¹⁵ instabilities were investigated in chapter 3. The use of more accurate transport coefficients, derived in chapter 2, substantially modified the results based on Braginskii's coefficients. In particular, the unstable mode was favoured by having $\nabla T \cdot \nabla n > 0$, for all Z .

An improved instability theory, based on a perturbation of hot and cold electron fluids with finite cold electron heat flow, was developed to model the collisional Weibel instability⁹ and Ohmically driven thermal instability¹³.

All these instabilities were characterized by a single parameter

$$L_s = 5.83 \times 10^{-2} (n/10^{21} \text{ cm}^{-3})^{1/2} (Z/10) (\ln \Lambda_{ei}/5) (1 \text{ keV}/T)^2$$

which is the ratio of the collisionless skin depth to the electron mean-free-path. For typical plasma conditions of the overdense region of targets irradiated by $1 \mu\text{m}$ light, the thermomagnetic, Nernst and collisional Weibel instabilities were shown to be likely to occur for $L_s \ll 1$, whereas the Ohmic instability required that $L_s \gg 50$. The latter inequality could only be satisfied for very cold and dense high- Z plasmas, where the Coulomb logarithm approaches unity, and the analysis breaks down.

(ii) Chapter 4 developed a kinetic stability theory based upon a perturbation of the Fokker-Planck equation and the expansion $f = f_0 + \underline{v} \cdot \underline{f}_1$. The results were compared with the fluid theory

predictions for the thermomagnetic and Nernst instabilities. The growth rate of these was shown to be proportional to the v_y^3 moment of the equilibrium distribution function. The high velocity part of the perturbed f_0 , which drives the instability was found to be weakly thermalized by the electron self-collisions. Consequently the results differed to those based on the fluid model, as the latter always assumes the electrons to be close to thermal equilibrium.

The dependence of the instability upon the behaviour of the high velocity electrons (i.e. $v_y \gg 2v_T$) led to the conclusion that the 'local' equilibrium distribution function is inadequate when $l_T < 100\lambda_T$.

(iii) Chapter 5 extended the kinetic model of chapter 4 by including the f_2 and f_3 terms in the cartesian tensor expansion. The first term was shown to be essential for describing the collisional Weibel instability, whereas the f_3 term was found to become important for $\lambda < 10\lambda_T$. The convergence of the expansion became slow as $\lambda < \lambda_T$. Using an extension of the 'Spitzer equilibrium distribution',¹⁸ so as to include f_2 , the thermomagnetic, Nernst and collisional Weibel instabilities were compared, and the latter found to be dominant. The 2-fluid description for the collisional Weibel instability (developed in chapter 3) was shown to yield inaccurate results.

A critical value of L_s was found, above which the collisional Weibel mode is always damped.

(iv) Chapter 6 compared the predictions of maximum growth rate and optimum wavelength for the collisional Weibel instability with the experimental observations of plasma jets. They were shown to be in quantitative agreement.

The instability growth was found to generally increase with Z .

7.2 Conclusions

A new method has been developed for calculating electron transport of a magnetized, fully ionized plasma of arbitrary atomic number. The method is based upon a numerical evaluation of the f_1 equation by finite difference techniques rather than the conventional Laguerre expansion approach. The results obtained by taking 100 velocity intervals of $0.05v_T$ were found to be more accurate than the those obtained by using the first 20 terms in a Laguerre expansion.⁵ Braginskii's coefficients,⁴ which were derived by means of 2-term expansion, have significant errors over certain values of $\omega\tau$: the heat flow coefficient in the $\underline{E} \times \nabla T$ direction is underestimated by 40% for $\omega\tau < 10^2$, and the one in the $\underline{E} \times (\nabla T \times \underline{B})$ direction deviates by -35% to +65% from the 'exact' value, within $0.3 < \omega\tau < 30$.

Electromagnetic instabilities relevant to collisional laser-produced plasmas, with emphasis on those occurring at super-critical densities, have been investigated. Several instability mechanisms have been considered, such as the thermomagnetic, the Nernst, the Ohmic and the collisional Weibel mechanism, of which the latter was found to dominate.

Fluid equations were shown to be inappropriate for describing the collisional Weibel instability. Instead, a kinetic model has been used which perturbs the electron Fokker-Planck equation and expands the distribution function in cartesian tensors. From such a model it has been concluded that the instability depends on a high-velocity moment of the equilibrium distribution. Equilibria which are characterized by local thermodynamic plasma quantities are, therefore, inadequate.

The collisional Weibel instability has been shown to be the likely cause of the experimentally observed plasma jets in laser-targets.

FUTURE WORK

In view of the inaccuracies found in Braginskii's transport coefficients, and in view of their wide applicability in transport calculations, a more accurate set should be written. The method developed in this thesis is ideally suited for such a purpose. The coefficients may be fitted numerically to a rational polynomial in $\omega\tau$ (similar to Braginskii's) by the method of least squares.

The main weaknesses of the EM instability theories developed in this thesis have been the lack of an adequate equilibrium description for the plasma and the use of the 'local approximation'. A natural step towards the solution of these problems lies in the construction of a 'perturbation code', whereby the equilibrium is initially solved in 1-D via a non-linear Fokker-Planck code, and the time evolution of a perturbation is followed by means of a linearized 2-D Fokker-Planck code.

APPENDICES

A.1 Pragenskii coefficients (definitions and linearization)

Tensors $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\chi}$ represent the electric resistivity, thermoelectric effects and conductivity, respectively. Using Pragenskii's notation,⁴ their components relative to $\underline{b}(=\underline{E}/|\underline{E}|)$ are defined as follows.

$$\underline{R} = (\alpha_{\parallel} \underline{j}_{\parallel} + \alpha_{\perp} \underline{j}_{\perp} - \alpha_{\Lambda} \underline{bxj}) / ne - n(\beta_{\parallel} \nabla_{\parallel} T + \beta_{\perp} \nabla_{\perp} T + \beta_{\Lambda} \underline{bx} \nabla T),$$

$$\underline{q} = -(\chi_{\parallel} \nabla_{\parallel} T + \chi_{\perp} \nabla_{\perp} T + \chi_{\Lambda} \underline{bx} \nabla T) - (\beta_{\parallel} \underline{j}_{\parallel} + \beta_{\perp} \underline{j}_{\perp} + \beta_{\Lambda} \underline{bxj}) T / e,$$

where $\underline{u}_{\parallel} = \underline{b}(\underline{u} \cdot \underline{b})$ and $\underline{u}_{\perp} = \underline{bx}(\underline{u} \times \underline{b})$ for an arbitrary vector $\underline{u}$. The coefficients are given by

$$\alpha_{\parallel} = mn \nu_t \alpha_0, \quad \alpha_{\perp} = mn \nu_t [1 - (\alpha_1' x^2 + \alpha_0') / \Delta], \quad \alpha_{\Lambda} = mn \nu_t x (\alpha_1'' x^2 + \alpha_0'') / \Delta,$$

$$\beta_{\parallel} = \beta_0, \quad \beta_{\perp} = (\beta_1' x^2 + \beta_0') / \Delta, \quad \beta_{\Lambda} = x (\beta_1'' x^2 + \beta_0'') / \Delta,$$

$$\chi_{\parallel} = \frac{nT}{m \nu_t} \gamma_0, \quad \chi_{\perp} = \frac{nT}{m \nu_t} \gamma_0 (\gamma_1' x^2 + \gamma_0') / \Delta, \quad \chi_{\Lambda} = \frac{nT}{m \nu_t} \gamma_0 x (\gamma_1'' + \gamma_0'') / \Delta,$$

where $x = \omega / \nu_t (= \omega \tau)$, $\Delta = x^4 + \delta_1 x^2 + \delta_0$ and $\alpha_0, \beta_0, \dots$ are defined in Pragenskii's paper for various values of Z .

Linearizing by writing $T \rightarrow T + T'$, $\omega \rightarrow \omega'$, $n \rightarrow n + n'$ and $n_i \rightarrow n_i + n_i'$, we obtain

$$\alpha'_{||} = (n'/n + n'_i/n_i - \frac{3}{2}T'/T) \alpha_0,$$

$$\alpha'_{\perp} = (n'/n + n'_i/n_i - \frac{3}{2}T'/T) \alpha_0,$$

$$\alpha'_\Lambda = (\omega'/\gamma_t) \alpha_0,$$

$$\beta'_{||} = 0,$$

$$\beta'_{\perp} = 0,$$

$$\beta'_\Lambda = (\omega'/\gamma_t) \beta''_0 / \delta_0,$$

$$\chi'_{||} = (nT/m\gamma_t) (n'/n - n'_i/n_i + \frac{5}{2}T'/T) r_0,$$

$$\chi'_{\perp} = (nT/\gamma_t) (n'/n - n'_i/n_i + \frac{5}{2}T'/T) r_0 \text{ and}$$

$$\chi'_\Lambda = (\omega'/\gamma_t) (nT/m\gamma_t) r''_0 / \delta_0.$$

A.2 e-e energy and momentum exchange term: C_{ee0} and C_{ee1}

From Shkarofsky we have:¹⁷

$$C_{ee0}(f_0^p, f_0^q) = \nu_{ee} \left[\frac{v^2}{3} (I_2^0 + J_{-1}^0) \partial_v^2 + \frac{v}{3} (2J_{-1}^0 + 3I_0^0 - I_2^0) \partial_v + 4\pi v^3 f_0^p \right] f_0^q / n$$

$$\begin{aligned} C_{ee1} = & \nu_{ee} \left[\frac{v^2}{3} (I_2^0 + J_{-1}^0) \partial_v^2 f_1 + \frac{v}{3} (3I_0^0 - I_2^0 + 2J_{-2}^0) \partial_v f_1 + \frac{1}{3} (-3I_0^0 + I_2^0 - 2J_{-1}^0) f_1 \right. \\ & \left. + 8\pi v^3 f_1 f_0 + \frac{v^2}{5} (I_3^1 + J_{-2}^1) \partial_v^2 f_0 + \frac{v}{30} (-6I_3^1 + 4J_{-2}^1 + 10I_1^1) \partial_v f_0 \right] / n \end{aligned}$$

$$\text{where } \nu_{ee} = [4\pi n (e^2/m)^2 \ln \Lambda_{ee}] / v^3,$$

$$I_j^i = (4\pi / v^j) \int_0^v f_i^p v^{2+j} dv \text{ and } J_j^i = (4\pi / v^j) \int_v^\infty f_i^p v^{j+2} dv.$$

Here, f_0^q refers to the 'test' electrons and f_0^p to the 'field' electrons.

Linearizing C_{ee0} about a Maxwellian, i.e. $f_0 = f_m + f'_0$, and expressing it in dimensionless form we obtain,

$$C'_{ee0} = \nu_{ee} (D + I) f'_0, \text{ where}$$

$$D = \left\{ [\text{erf}(V)/2 - VF_m/4] \partial_V^2 + [\text{erf}(V)V - \text{erf}(V)/2V + F_m/4] \partial_V + V^3 F_m \right\},$$

$$I = \left\{ \frac{4}{3} V^2 F_m \int_0^V dV V^4 - 2V^2 F_m \int_0^V dV V^2 + V^3 \left(\frac{4V^2}{3} - 2 \right) F_m \int_V^\infty dV V \right\}$$

$$\text{and } \text{erf}(V) = \frac{2}{\sqrt{\pi}} \int_0^V e^{-x^2} dx.$$

A.3 Numerical solution of integro-differential equations

Consider the following equation.

$$(D + I_v + I^v)f(v) = g(v) , \quad (1)$$

where D and I are differential and integral operators in v , respectively, defined by,

$$D = a_2(v) \partial_v^2 + a_1(v) \partial_v + a_0(v) ,$$

$$I^v = b_n(v) \int_0^v dv v^n + b_{n-1}(v) \int_0^v dv v^{n-1} + \dots + b_0(v) \text{ and}$$

$$I_v = c_n(v) \int_v^\infty dv v^n + c_{n-1}(v) \int_v^\infty dv v^{n-1} + \dots + c_0(v) .$$

Since D is a 2^{nd} order differential operator we need to specify 2 boundary conditions for equation (1). For the problems of interest in this thesis, the appropriate ones are:

$$f(\infty) = 0 \text{ and} \quad (2)$$

$$\partial_v f|_{v=0} = 0 . \quad (3)$$

Equation (1) is solved numerically by finite differencing ' v ' within the interval $[0, v_{\max}]$, as follows.

$$v_j = jh ,$$

where $j=1,2,\dots,J$ and h is the velocity step (assumed constant).

Here, it is assumed that the contribution due to $v > v_{\max}$ is negligible.

(i) Finite differencing D

We adopt a '5-point' differencing scheme for D, such that³⁸

$$D_j f_j = a_{2j} [-f_{j+2} + 16f_{j+1} - 30f_j + 16f_{j-1} - f_{j-2}] / 12h^2 \\ + a_{1j} [-f_{j+2} + 8f_{j+1} - 8f_{j-1} + f_{j-2}] / 12h + a_{0j}, \quad (4)$$

at a point $2 < j < J-2$. This method is fourth order accurate, since the leading error is proportional to h^4 .

For $j=2$ and $J-1$ we use a '3-point' differencing scheme,

$$D_j f_j = a_{2j} [f_j - 2f_{j+1} + f_{j+2}] / h^2 + a_{1j} [f_{j+1} - f_{j-1}] / 2h + a_{0j}, \quad (5)$$

which is 2nd order accurate.

Equations (4) and (5) give rise to a quindagonal matrix for D, denoted $\underline{D}$.

(ii) Finite differencing I_v and I^v

For odd J and $J-3 > j > 3$, we use the 3rd order accurate 'trapezium rule' to describe the integrals. Hence,

$$I^v f_j = b_{nj} [v_1^n f_1 + 2(v_2^n f_2 + \dots + v_{j-1}^n f_{j-1}) + v_j^n f_j] h/2 \\ + \dots + b_{0j} \text{ and}$$

$$I_v f_j = c_{nj} [v_j^n + 2(v_{j+1}^n f_{j+1} + \dots + v_{J-1}^n f_{J-1}) + v_J^n f_J] h/2 \\ + \dots + c_{0j},$$

and similarly for $j=2$ and $J-1$.

When J is odd but j is even, we employ the 5th order accurate 'Simpson rule', where for $3 < j < J-3$ we obtain,

$$I_v^V f_j = b_{nj} \left[v_1^n f_1 + 4(v_2^n f_2 + v_4^n f_4 + \dots + v_{j-1}^n f_{j-1}) + 2(v_3^n f_3 + v_5^n f_5 + \dots + v_{j-2}^n f_{j-2}) + f_j \right] h/3 + \dots + b_{0j} \text{ and}$$

$$I_v f_j = c_{nj} \left[v_j^n f_j + 4(v_{j+1}^n f_{j+1} + v_{j+3}^n f_{j+3} + \dots + v_{J-1}^n f_{J-1}) + 2(v_{j+2}^n f_{j+2} + v_{j+4}^n f_{j+4} + \dots + v_{J-2}^n f_{J-2}) + v_J^n f_J \right] h/3 + \dots + c_{0j}.$$

The resultant matrices, $\underline{I}_v$ and $\underline{I}_v^V$, are of the upper and lower triangular type, respectively.

(iii) The boundary conditions

Equations (2) and (3) become: $f_j=0$ and $f_1=-f_2$, respectively.

(iv) Full differenced equation

Equation (1) now becomes

$$(\underline{D} + \underline{I}_v + \underline{I}_v^V) \underline{f} = \underline{g},$$

where $\underline{f}$ and $\underline{g}$ are column vectors. The operator, which is a full matrix, may be inverted by conventional means.³⁹

A.4 Derivation of the dispersion relation (4.19)

Using the relation $\langle v_0^j \rangle = \langle v_m^j \rangle = \frac{2}{\sqrt{\pi}} g[(j+3)/2]$ (where $g[\dots]$ is the Gamma function)³⁷, we have:

$$\begin{aligned} \frac{1}{6} \partial_z \langle v_z^4 \rangle &= \frac{1}{6} [\langle v_0^8 \rangle (4\Delta - 10) + \langle v_0^{10} \rangle (15/2 - \Delta) - \langle v_0^{12} \rangle] / L_T^2 \\ &= -(2835/64) [1 + \Delta/3] / L_T^2, \end{aligned}$$

$$\begin{aligned} \epsilon \langle v_z^2 \rangle &= -\frac{1}{4} (5/L_T + 2/L_n) [4\langle v_4^6 \rangle - \langle v_0^8 \rangle] / L_T \\ &= (525/64) [1 + \frac{2}{5} (L_T/L_n)] / L_T^2, \end{aligned}$$

$$\begin{aligned} \Phi [\partial_z \langle v_0^3 \rangle + 4\epsilon \langle v_0^1 \rangle] &= [(4\langle v_0^3 \rangle - \langle v_0^5 \rangle) / \langle v_0^{-2} \rangle] \\ &\times [\langle v_0^3 \rangle (L_T/L_n - 3/2) + \langle v_0^5 \rangle - \langle v_0^1 \rangle (5 + 2L_T/L_n)] / L_T^2 \\ &= -\frac{8}{\pi} / L_T^2, \end{aligned}$$

$$\langle v_0^3 \rangle = \frac{4}{\sqrt{\pi}}.$$

Substituting back into Eq. (4.18), we get,

$$\Gamma = \frac{\sqrt{\pi}}{8} \{ (105/32)(22 + 9\Delta - 2L_T/L_n) / L_T^2 - (16/\pi) / L_T^2 - k^2 L_s^2 \},$$

and hence,

$$\Gamma = \{ 14.87 / L_T^2 - 1.454 / L_T L_n + 6.543 \Delta / L_T^2 - \frac{\sqrt{\pi}}{8} k^2 L_s^2 \}.$$

A.5 Derivation of the equilibrium distribution up to f_2

The 1-D Fokker-Planck equation for a Lorentz plasma is given by ⁷

$$\partial_t f + \mu v \partial_z f - a_z [\mu \partial_v + \frac{1}{v} (1 - \mu^2) \partial_\mu] f = C,$$

where $C = \frac{1}{2} \nabla_{ei} \partial_\mu (1 - \mu^2) \partial_\mu f$ (using $m/m_i \ll 1$).

We assume that $f = f_0 + \mu f_z + \frac{1}{2} (3\mu^2 - 1) f_{zz}$, where $f_0 = f_m$. Substituting this expansion into the Fokker-Planck equation and taking angular moments of we obtain

$$\nabla_{ei} f_z = - [v \partial_z - a_z \partial_v] f_m - \frac{2}{5} [\partial_z f_{zz} - (1/v^3) \partial_v (v^3 a_z f_{zz})], \quad (1)$$

$$3 \nabla_{ei} f_{zz} = - \frac{2}{3} v [\partial_z f_z - \partial_v (a_z f_z / v)]. \quad (2)$$

By neglecting the f_{zz} contribution from equation (1), under the assumption that $|f_{zz}/f_z| \sim \lambda_{T/1T} \ll 1$, and using

$$f_m = (n/\pi^{3/2} v_T^3) \exp(-v^2/v_T^2)$$

(where $v_T = (2T/m)^{1/2}$), we derive the following expression for f_z .

$$f_z = - (v/\nabla_{ei}) [(\partial_z n)/n + (v^2/v_T^2) (\partial_z T)/T - \frac{3}{2} (\partial_z T)/T + 2a_z/v_T^2] f_m.$$

Applying the zero current condition $j_z = -(4\pi e/3) \int_0^\infty dv v^3 f_z = 0$ to the above equation, we obtain

$$f_z = (v/\nabla_{ei}) (4 - v^2/v_T^2) f_m / l_T, \quad (3)$$

where $l_{T,n} = [\partial_z \ln(T,n)]^{-1}$ and the self-consistent electric field is given by

$$a_z = -v_T^2(5/l_T + 2/l_n)/4. \quad (4)$$

Equation (3) may now be substituted into equation (2), to yield:³⁵

$$\begin{aligned} f_{zz} = & -\frac{2}{9}(v^2/\bar{v}_{ei}) \{ (v^5/v_T^2)/l_T^2 - v^3(5/l_T^2 + 2/l_n l_T)/2 \\ & + (4 - v^2/v_T^2) [\frac{3}{4}v_T^2(5/l_T^2 + 2/l_n) v + (\Delta/l_T^2 - (5/l_T^2 + 1/l_n l_T)) v^3 \\ & + (v^5/v_T^2)/l_T^2] \} f_m. \end{aligned} \quad (5)$$

where $\Delta = (\partial_z^2 T) l_T^2 / T$.

Rewriting equations (3)-(5) in dimensionless form we have,

$$F_z = v^4(4 - v^2)F_0/L_T,$$

$$\epsilon = -(5/L_T + 2/L_n)/4 \text{ and}$$

$$\begin{aligned} F_{zz} = & \frac{2}{9}v^6 [v^6 - v^4(10 + L_T/L_n - \Delta) \\ & + v^2(105/4 + 13L_T/2L_n - 4\Delta) - 6(5/2 + L_T/L_n)] F_0/L_T^2, \end{aligned}$$

where $F_0 = (4/\sqrt{\pi})e^{-v^2}$.

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REFERENCES

- 1 D.W. Forslund and J.U. Blackbill, Phys. Rev. Lett. 48, 1614 (1982).
- 2 A.R. Bell et al., Phys. Rev. Lett. 46, 243 (1981); J.P. Matte and J. Virmont, Phys. Rev. Lett. 49, 1936 (1982).
- 3 G.J. Pert, J. Comput. Phys. 43, 1 (1981).
- 4 S.I. Braginskii, in 'Reviews of Plasma Physics', edited by M.A. Leontovich (Consultants Bureau, N. Y., 1965), Vol. 1; S.I. Braginskii, Soviet Physics JETP 6, 358 (1958).
- 5 A.R. Hochstim, in 'Kinetic Processes in Gases and Plasmas', edited by A.R. Hochstim (Academic Press, N. Y., 1969), p. 141.
- 6 S. Chandrasekhar, in 'Hydrodynamics and Hydromagnetic Stability' (Clarendon, Oxford, 1961).
- 7 A.J. Cole et al., J. Phys. D: Appl. Phys. 15, 1689 (1982).
- 8 O. Willi et al., Opt. Comm. 41, 110 (1982).
- 9 T. Mochizuki et al., Jpn. J. Appl. Phys. 19, L645 (1980).
- 10 M.J. Herbst et al., Phys. Rev. Lett. 47, 91 (1981).
- 11 I.V. Aleksandrova et al., JETP Lett. 38, 68 (1983).
- 12 D.A. Tidman and R.A. Shanny, Phys. Fluids 17, 1207 (1974); L.A. Pol'shov et al., JETP Lett. 19, 168 (1974).
- 13 M.G. Haines, Phys. Rev. Lett. 47, 917 (1981).
- 14 R.G. Evans, J. Phys. D: Appl. Phys. 14, L173 (1981).
- 15 A.Z. Dolginov and V.A. Urpin, Sov. Phys. JETP 50, 912 (1979); J.H. Brownell, Comm. Plasma Phys. Cont. Fusion 4, 131 (1979).
- 16 E.M. Epperlein, 'Kinetic model for the collisional Weibel instability in laser-produced plasmas' (submitted to

- Phys. Rev. Lett., Jan. 1984).
- 17 I.P. Shkarofsky et al., in 'The Particle Kinetics of Plasmas' (Addison-Wesley, Reading, Mass., 1966).
 - 18 L. Spitzer and R. Härm, Phys. Rev. Lett. 89, 977 (1953).
 - 19 T.G. Cowling, Proc. Roy. Soc. A183, 453 (1945).
 - 20 S. Kaneko, J. Phy. Soc. Japan 15, 1685 (1960).
 - 21 H. Grad, Comm. Pure Appl. Math. 2, 325 (1949).
 - 22 R. Herdan and B. Liley, Rev. Mod. Phys. 32, 731 (1960).
 - 23 B.P. Robinson and I.B. Bernstein, Ann. Phys. 18, 110 (1962).
 - 24 G.J. Pert, J. Plasma Phys. 18, 227 (1977).
 - 25 M. Ogasawara et al., J. Phys. Soc. Japan 49, 322 (1980); A. Hirao and M. Ogasawara, J. Phys. Soc. Japan 50, 668 (1981).
 - 26 E.S. Weibel, Phys. Rev. Lett. 2, 83 (1959).
 - 27 F.M. Epperlein et al., in CECAM Workshop Report on the 'Flux Limiter and Heat Flow Instabilities', France, Oct. 1981.
 - 28 J.R. Albritton, Phys. Rev. Lett. 26, 2078 (1983); J.P. Matte et al., in 'Results from a Fokker-Planck treatment of electrons and ambipolar hydrodynamics in 1-D laser-target simulation', 1983 (unpublished).
 - 29 D.R. Gray et al., Phys. Rev. Lett. 39, 1270 (1977).
 - 30 B.L. Moiseiwitsch, in 'Integral equations' (Longman, London, 1977), p. 6.
 - 31 T.H. Kho, Ph.D. Thesis (1983).
 - 32 A.R. Bell, Phys. Fluids 26, 279 (1983).
 - 33 T.W. Johnston, Phys. Rev. 120, 1103 (1960).
 - 34 T. Okada et al., J. Phys. Soc. Japan 43, 1042 (1977); T. Okada et al., J. Plasma Phys. 20, 405 (1978).
 - 35 A. Ramani and G. Laval, Phys. Fluids 21, 980 (1978).
 - 36 N.A. Krall and A.W. Trivelpiece, in 'Principles of Plasma Physics'

(MacGraw-Hill, 1973).

- 37 B.A. Chirgwin and C. Plumpton, in 'A Course of Mathematics for Engineers and Scientists' (Pergamon Press, Oxford, 1970), Vol. 1, p. 306.
- 38 L. Fox and D.F. Mayers, in 'Computing Methods for Scientists and Engineers' (Clarendon Press, Oxford, 1968), p. 153.
- 39 D. Potter, in 'Computational Physics' (John Wiley, London, 1973).
- 40 M.G. Haines et al., presented at the 9th International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Baltimore USA, Sept 1982 (unpublished).