

Fracture effects in seismic attenuation images reconstructed by waveform tomography

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ABSTRACT

We have investigated seismic waveform tomography to characterize fractures in petroleum reservoirs. Seismic reflection data are used in a frequency-domain inversion to reconstruct subsurface attenuation images. The images show fracture distributions, from which fracture density is estimated. Fractures smaller than or equal to a half-wavelength of seismic act as single scatterers, producing images of strong attenuation ellipses and from which fracture density can be estimated. When fracture size approaches one wavelength, fracture orientation affects the attenuation image. Horizontal fractures act as individual reflectors and produce strong tomographic attenuation images from which fracture density can be estimated. The strength of the attenuation image decreases when the fracture angle relative to horizontal increases; vertical fractures produce the weakest attenuation image. Consequently, the accuracy of fracture density measurements decreases with increased fracture angle unless waveform tomography includes different seismic modes acquired from several directions.

INTRODUCTION

Fractures are important in reservoir characterization and development, especially because they can act as fluid pathways (Liu et al., 2003). We investigate attenuation effects resulting from fractures in subsurface media. We generate the seismic response with reflection geometry from fractured media and then invert them using waveform tomography to produce the attenuation images. Because waveform tomography can produce attenuation images with adequate high resolution, we can compare attenuation characteristics and fracture properties directly, including distribution and density.

Seismic waveform inversion can be implemented in the time domain or the frequency domain, but one advantage of frequency-do-

main waveform tomography is its efficiency. Marfurt (1984) suggests that the frequency domain can be the method of choice for finite-difference/finite-element modeling if a significant number of source locations are involved. Pratt and Worthington (1990) point out that large-aperture seismic surveys can be inverted effectively using only a limited number of frequency components. Sirgue and Pratt (2004) and Wang and Rao (2006) further show that a frequency-domain waveform inversion using only a few frequencies can yield a result comparable to full waveform inversion in the time domain. Another advantage of frequency-domain implementation is that waveform tomography can account for the earth attenuation effect easily by replacing the real-valued velocity variable with a complex-valued velocity. In this paper, we use frequency-domain waveform tomography to invert the attenuation effect Q^{-1} within fractured media.

It is possible to study directly the fracture effect on the attenuation factor without using waveform tomography. Vlastos et al. (2003) show the effects of fractures on seismic wavefronts through forward modeling. Vlastos et al. (2007) further investigate scattering attenuation at different stages of fracture evolution. Both papers base their analysis on forward-modeling seismograms. However, in this paper, we try to investigate the ability of waveform inversion to generate the attenuation image from fractures.

We design a group of models with fractured media and generate wavefields by finite-difference forward modeling. In the forward wavefield simulation, we implement fractures on finite-difference grids using an effective medium theory (Coates and Schoenberg, 1995; Liu et al., 2000; Vlastos et al., 2006). This method can deal with multiple scattering cases without limiting the number of fractures in the medium. We consider reflection seismic geometry and use waveform tomography to invert for Q^{-1} caused by fractures in the media. Reconstructed attenuation models represent the effect of fractures on seismic energy, so we attempt to assess the potential of these attenuation images in the quantitative density estimation of fractures with different scales and orientations in 2D media.

Manuscript received by the Editor 21 August 2008; revised manuscript received 18 March 2009; published online 5 June 2009.

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WAVEFORM TOMOGRAPHY ON FRACTURED MEDIA

Waveform tomography for the attenuation

For seismic tomography, we use Tarantola's (1984) inversion method, which implements waveform residual back propagation through forward modeling. The objective is to minimize the data misfit between the synthetic data generated by a given model and the real data presented in the frequency domain (Pratt and Worthington, 1990; Pratt et al., 1998). The inversion procedure can be divided into four steps.

- 1) For a given model estimate, calculate the synthetic wavefield.
- 2) Use the data residual between the synthetic and real data as virtual sources to generate a back-propagation wavefield.
- 3) Crosscorrelate the synthetic wavefield and the back-propagation wavefield to get the gradient direction.
- 4) Estimate an optimal model update along the gradient direction to minimize the misfit between synthetic and real data.

This four-step procedure is performed iteratively. For implementing waveform tomography in the reflection geometry, refer to Wang and Rao (2009).

In the acoustic case, the mass density is assumed to be constant and the model is defined by the velocity field only. In frequency-domain waveform tomography (Song et al., 1995), the attenuation effect is considered by using complex-valued velocity instead of conventional real-valued velocity. Consider a complex-valued velocity function

$$c(\mathbf{x}) = c_{\text{Re}}(\mathbf{x}) + ic_{\text{Im}}(\mathbf{x}), \quad (1)$$

where $c_{\text{Re}}(\mathbf{x})$ and $c_{\text{Im}}(\mathbf{x})$ are the real and imaginary parts, respectively, of the complex velocity $c(\mathbf{x})$ and $\mathbf{x} = (x, z)$ is the spatial position. We have a complex-valued wavenumber corresponding as

$$k = \frac{\omega c_{\text{Re}}}{c_{\text{Re}}^2 + c_{\text{Im}}^2} - i \frac{\omega c_{\text{Im}}}{c_{\text{Re}}^2 + c_{\text{Im}}^2}. \quad (2)$$

Then, we may express the frequency component of a plane wave at distance r as

$$A(r) = A_0 \exp(ikr) = A_0 \exp\left(i \frac{\omega c_{\text{Re}} r}{c_{\text{Re}}^2 + c_{\text{Im}}^2}\right) \exp\left(-\frac{\omega c_{\text{Im}} r}{c_{\text{Re}}^2 + c_{\text{Im}}^2}\right), \quad (3)$$

where A_0 is the initial amplitude of the plane wave and $A(r)$ is the amplitude at distance r . The first and second exponential factors in the right-hand side of equation 3 represent the phase change and the amplitude attenuation effect along r .

According to Aki and Richards (1980), we can define the quality factor Q using

$$A(r) = A_0 \exp\left(-\frac{\omega r}{2c_{\text{Re}}Q}\right). \quad (4)$$

Following the amplitude attenuation in equations 3 and 4, and considering that in practice $c_{\text{Re}}^2 \gg c_{\text{Im}}^2$, we can describe the relationship between Q and the complex velocity field as

$$Q^{-1} \approx -\frac{2c_{\text{Im}}}{c_{\text{Re}}}, \quad (5)$$

where c_{Im} is negative valued.

In waveform tomography, we need forward synthetics to match seismic data. For forward modeling with a given real-velocity model and an attenuation model, we use equation 5 to calculate c_{Im} , the imaginary part of the complex velocity. We then solve the wave equation with the complex-valued velocity model using a finite-difference scheme.

In the inversion, we use a sequential inversion scheme for the velocity and attenuation models (Wang, 2008). We first reconstruct c_{Re} , the real part of the velocity, and assume $c_{\text{Im}} = 0$ (i.e., $Q^{-1} = 0$). We then invert for c_{Im} and calculate the effective attenuation field Q^{-1} based on the complex-valued velocity field. This sequential inversion scheme for waveform tomography is justified because the real-part velocity and the attenuation have five orders of magnitude difference. We also conducted the resolution analysis on the trade-off between real-part velocity and attenuation model and found that a small velocity perturbation can change the attenuation value but has minimal effect on the structure of the attenuation distribution (Rao and Wang, 2008).

The Q^{-1} model is frequency independent, and we do not address the attenuation mechanism in this paper. However, based on equation 3, the amplitude and the phase factors are frequency dependent. Therefore, frequency-domain tomography attempts to match a frequency component of synthetics with a component of real data in amplitudes and phases.

Fractures in the finite-difference implementation

We use the equivalent medium theory to represent the fractured medium (Schoenberg, 1980; Coates and Schoenberg, 1995; Liu et al., 2000). In this representation, the fractured medium is expressed as an anisotropic medium, and all parameters have physical meaning.

The stiffness tensor for an anisotropic elastic medium is expressed as

$$c_{k\ell} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{21} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}. \quad (6)$$

When taking fractures into account, the elastic constants in the stiffness tensor are expressed as

$$\begin{aligned}
 c_{11} &= (\lambda + 2\mu) \left(1 + \frac{\lambda + 2\mu}{\mu} \varepsilon U_{33} \right)^{-1}, \\
 c_{22} &= c_{33} = [(\lambda + 2\mu) + 4(\lambda + \mu) \varepsilon U_{33}] \\
 &\quad \times \left(1 + \frac{\lambda + 2\mu}{\mu} \varepsilon U_{33} \right)^{-1}, \\
 c_{12} &= c_{13} = c_{21} = c_{31} = \lambda \left(1 + \frac{\lambda + 2\mu}{\mu} \varepsilon U_{33} \right)^{-1}, \\
 c_{23} &= c_{32} = \lambda (1 + 2\varepsilon U_{33}) \left(1 + \frac{\lambda + 2\mu}{\mu} \varepsilon U_{33} \right)^{-1}, \\
 c_{44} &= \frac{c_{22} - c_{23}}{2} = \mu, \\
 c_{55} &= c_{66} = \mu (1 + \varepsilon U_{11})^{-1}, \tag{7}
 \end{aligned}$$

where λ and μ are the Lamé coefficients, ε is the crack density, and U_{11} and U_{33} are two elastic constants representing a cluster of cracks in a compact area, given by Hudson (1981) for dry cracks as

$$U_{11} = \frac{16}{3} \frac{\lambda + 2\mu}{3\lambda + 4\mu} \quad \text{and} \quad U_{33} = \frac{4}{3} \frac{\lambda + 2\mu}{\lambda + \mu}. \tag{8}$$

The parameter ε in expression 7 represents the effect of fractures. For instance, if $\varepsilon = 0$, the tensor represents the elastic medium outside of the two fracture tip points. At the two tips, we apply a hyperbolic taper to the ε , following Vlastos et al. (2003). With this tapering implementation, the value of the stiffness constant in the center of the fracture is a maximum of one, and the value toward the fracture tips gradually goes to zero.

We represent the fracture as a boundary with characteristics of discontinuous displacement and continuous stress (Schoenberg, 1980). This boundary condition can be expressed as

$$[u] = \mathbf{Z}\boldsymbol{\tau}, \tag{9}$$

where $[u]$ is the average displacement discontinuity, $\boldsymbol{\tau}$ is the traction acting across the fracture, and $\mathbf{Z}$ is the fracture compliance tensor, which is the inverse of the stiffness tensor $c_{k\ell}$. In the area surrounding a fracture, the compliance tensor has two parts: the compliance tensor in the absence of any fractures and the extra compliance tensor resulting from the fractures (Liu et al., 2000).

We use a finite-difference method to simulate the seismic response and study the effects of different attributes of fractures on wavefield characteristics. The finite-difference scheme is a staggered-grid, second-order in time, and fourth-order in space scheme (Levander, 1988), and it is stable for all values of Poisson's ratio. We set the elastic parameters as the equivalent anisotropic medium within the cells surrounding a fracture of any size, provided the grid size is smaller than the fracture length (Vlastos et al., 2003).

Elastic forward modeling and acoustic inversion

Fractures in the media cause amplitude attenuation in seismic waves. For waveform tomography, if it does not consider the attenuation effect, we should perform inverse Q filtering to compensate the amplitudes (Wang, 2008) before we input seismic data to the inversion to reconstruct a velocity image. In this paper, however, we assume the background velocity is accurately known and use waveform tomography to invert for the attenuation model caused by fractures.

For forward modeling, we use the elastic wave equations in the time domain. For waveform tomography, because we invert for media attenuation resulting from fractures, we use the acoustic wave equation in the frequency domain. Thus, we need to convert the two-component elastic wavefield generated from forward modeling to the pressure wavefield suitable for the acoustic wave equation inversion. We also need to convert the stress source for elastic wave equation modeling to a source waveform for the acoustic wave equation inversion.

In the acoustic assumption, $c_{11} = c_{33} = c_{13} = \lambda$ and $c_{55} = c_{15} = c_{35} = 0$. The elastic wave equation can be written as

$$\begin{aligned}
 -\rho \frac{\partial v_x}{\partial t} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} &= 0, \\
 -\rho \frac{\partial v_z}{\partial t} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} &= 0, \\
 -\frac{\partial \tau_{xx}}{\partial t} + \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right) &= S(t), \\
 -\frac{\partial \tau_{zz}}{\partial t} + \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right) &= S(t), \\
 \frac{\partial \tau_{xz}}{\partial t} &= 0, \tag{10}
 \end{aligned}$$

where (v_x, v_z) is the particle velocity vector and $S(t)$ is the stress source used for forward modeling. According to Newton's second law, the relationship between particle velocity and pressure is

$$\begin{aligned}
 \rho \frac{\partial v_x}{\partial t} &= -\frac{\partial P}{\partial x}, \\
 \rho \frac{\partial v_z}{\partial t} &= -\frac{\partial P}{\partial z}. \tag{11}
 \end{aligned}$$

Combining equations 10 and 11, we obtain

$$\frac{\partial P}{\partial t} + \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right) = S(t). \tag{12}$$

Then we combine equations 10 and 12 to get the acoustic wave equation:

$$-\frac{\partial^2 P}{\partial t^2} + \frac{\lambda}{\rho} \left(\frac{\partial}{\partial x} \frac{\partial P}{\partial x} + \frac{\partial}{\partial z} \frac{\partial P}{\partial z} \right) = \frac{\partial S(t)}{\partial t}. \tag{13}$$

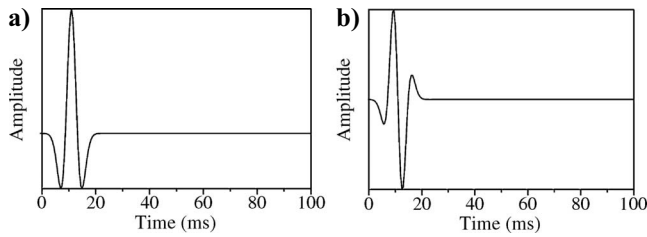


Figure 1. (a) Ricker wavelet for forward modeling. (b) The equivalent source wavelet for waveform tomography.

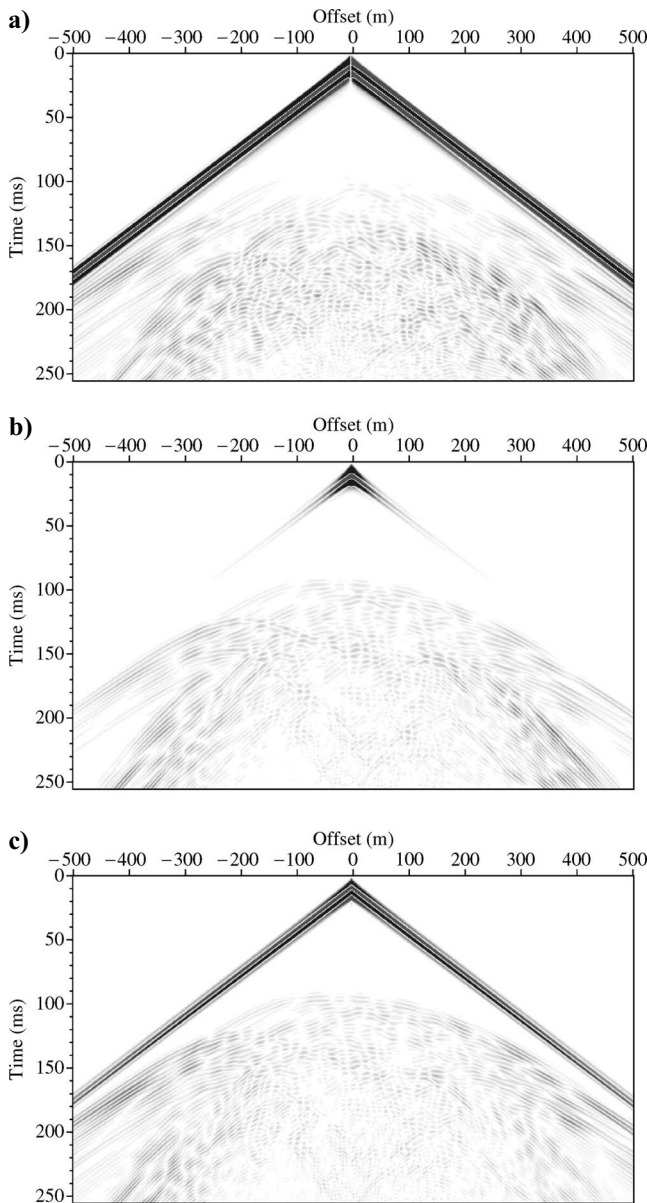


Figure 2. (a) Horizontal component of a shot gather. (b) The vertical component of a shot gather. (c) The shot gather of the pressure wavefield, converted from the two particle-velocity components.

In forward modeling, the source type is a stress source $S(t)$ on τ_{xx} and τ_{zz} , and the waveform is a Ricker wavelet with a dominant frequency of 100 Hz (Figure 1a). For the acoustic wave equation inversion, according to equation 13, we calculate the derivative of $S(t)$ and use it as a source signal (Figure 1b).

We also convert the particle-velocity wavefield to the pressure wavefield. Figure 2a and b shows shot gathers of two particle-velocity components, horizontal component v_x and vertical component v_z , respectively. To convert v_x and v_z in the elastic wave equation to pressure P in the acoustic wave equation, we calculate the partial derivatives of v_x and v_z and obtain pressure projection P_x and P_z in the x - and z -directions, respectively. We can find the pressure from $P = P_x \cos \theta + P_z \sin \theta$, where θ is the incident reflection angle in the receiver position. Figure 2c shows a converted shot gather from the two particle-velocity components in Figure 2a and b.

THE ATTENUATION IMAGES OF FRACTURED MEDIA

To investigate the relationship between fracture properties and an attenuation model reconstructed from seismic tomography, we design a group of models with different fracture scales and orientations. The research area is 540 m laterally and 300 m in depth. A line of 380 sources and a line of 380 receivers are placed evenly at the surface, spreading from -300 to 840 m in the x -direction, with 300 extra source/receiver points outside of two ends of the research area $[0, 540 \text{ m}]$ to ensure adequate ray coverage on the target area. The model is discretized into $3 \times 3\text{-m}^2$ cells to satisfy the criteria of forward modeling and waveform tomography. To simulate wave propagation, the background P- and S-wave velocities are 3000 and 1700 m/s, respectively, and density ρ is 2300 kg/m³. Each model has 52 fractures distributed randomly between 120 and 210 m in depth.

Figure 3a and b shows horizontal and vertical fracture models, respectively, in which fractures are distributed randomly and uncorrelated; the fracture size is equal to half the wavelength ($\lambda/2 = 15 \text{ m}$). Figure 3c and d shows the corresponding reconstructed attenuation (Q^{-1}) models. Areas of strong attenuation in the reconstructed models correspond to fracture distributions. The fracture size is equal to $\lambda/2$, so fractures act as diffraction points in the seismic response, and it is impossible to distinguish the fracture orientation from the attenuation models. Therefore, in practice, with a limited frequency band, a significant number of small fractures may be missed. A joint inversion using surface and crosshole seismic data is needed to improve fracture prediction because borehole seismic data usually have much higher frequencies than surface seismic data.

Figure 4 shows models containing randomly distributed fractures with size equal to λ (30 m) and the corresponding reconstructed attenuation models. For horizontal fractures (Figure 4a), the attenuation (Figure 4c) is stronger than for a fracture size of $\lambda/2$ (Figure 3c). But for vertical fractures (Figure 4b), the attenuation (Figure 4d) becomes much weaker than horizontal fractures of the same size (Figure 4c) and of half the size (Figure 3d). Strong attenuation areas in the reconstructed model (Figure 4d) only correspond to the fracture tips.

We can derive further clues from the seismic response of forward modeling. Figure 5 displays vertical-component seismic responses

of the four fractured models shown in Figures 3 and 4. Comparing Figure 5a and c, the seismic responses of the two horizontal fracture models of Figures 3a and 4a, we can observe that the fracture size has a strong effect on the seismic response. Figure 5a shows that more scattering occurs if fracture size is $\lambda/2$. When the size of horizontal fractures is λ , as shown in Figure 5c, the fractures change the apparent properties of the whole material. The long fractures act as individual reflection boundaries. The interference between the various reflections has a strong effect on the amplitudes of reflected waves, as the strong coherence energies from horizontal reflections show in Figure 5c. Hence, the different scales of horizontal fractures can be shown sensitively in the reconstructed attenuation factor models.

Shot records displayed in Figure 5 also suggest that the reconstructed physical properties of the fractures depend on the orientations. The shot gathers corresponding to the $\lambda/2$ horizontal and vertical fracture models in Figure 5a and b, respectively, are quite similar from 0 to 240 ms. This is because the $\lambda/2$ fractures act mainly as scattering points in the seismic response; hence, the corresponding attenuation models show little difference. When the fracture size is equal to λ , the tips of vertical fractures become strong diffraction points.

However, strong side reflections from vertical fractures can be seen only from snapshots of wave motion, as shown in Vlastos et al.

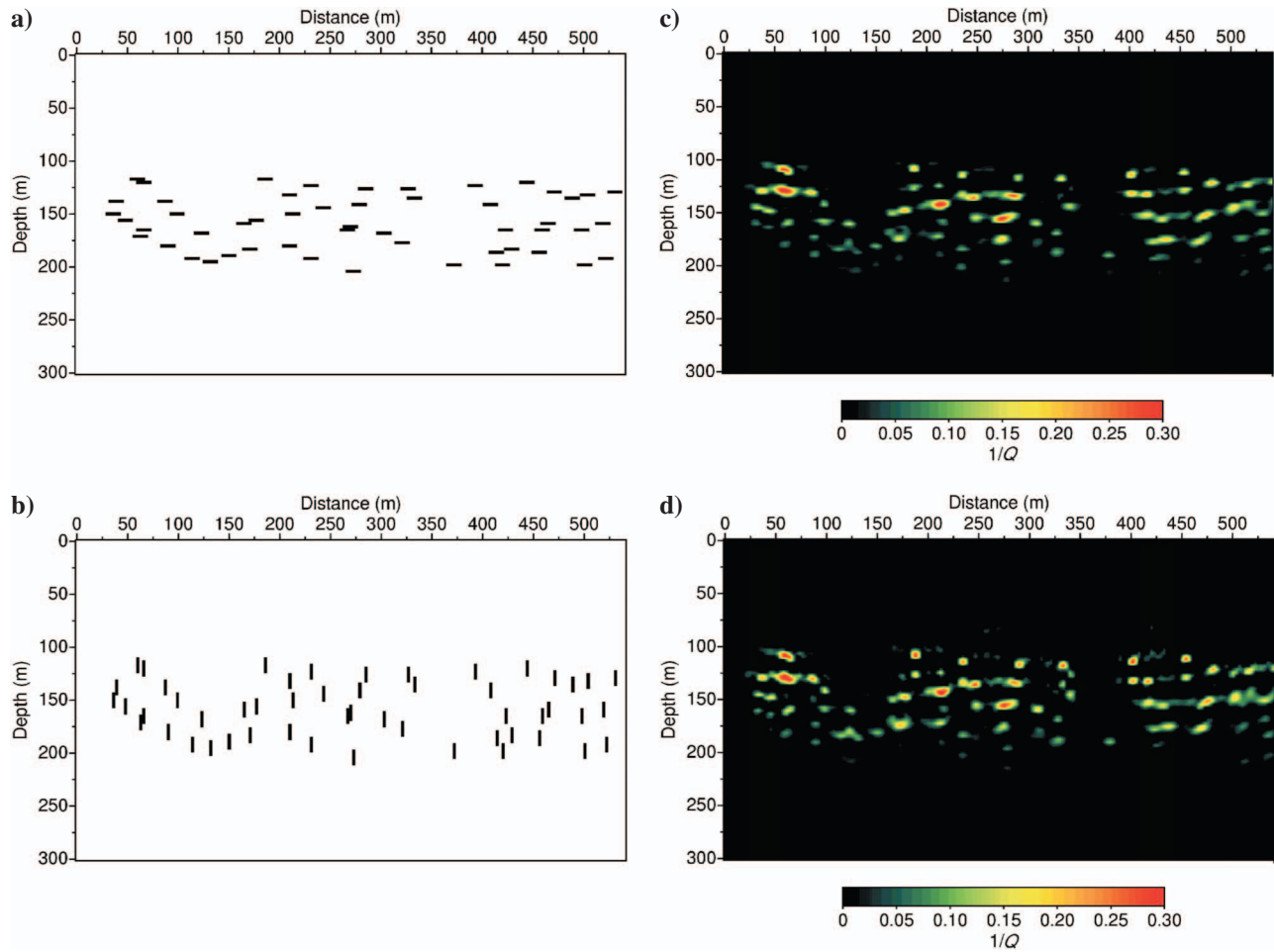


Figure 3. (a, b) Fracture models with horizontal and vertical fractures; fracture size of 0.5λ . (c, d) The attenuation models reconstructed by waveform tomography, corresponding to the horizontal and vertical fracture models, respectively.

(2003), and cannot be seen from seismograms recorded by receivers spread along the surface. Waveform tomography, which uses reflections received at the surface, cannot image these vertical fractures in the model. Thus, a joint inversion with borehole seismic data is needed to include side reflections in the data, apart from the frequency-band expansion.

We also test two models with different fracture orientations: a 45° angle fracture model (Figure 6a) and a 135° fracture model (Figure 6b). The reconstructed attenuation models are shown in Figure 6c and d. The attenuation corresponding to these fractures is weaker than in the horizontally fractured model but stronger than in the vertically fractured example. These orientation tests indicate that we

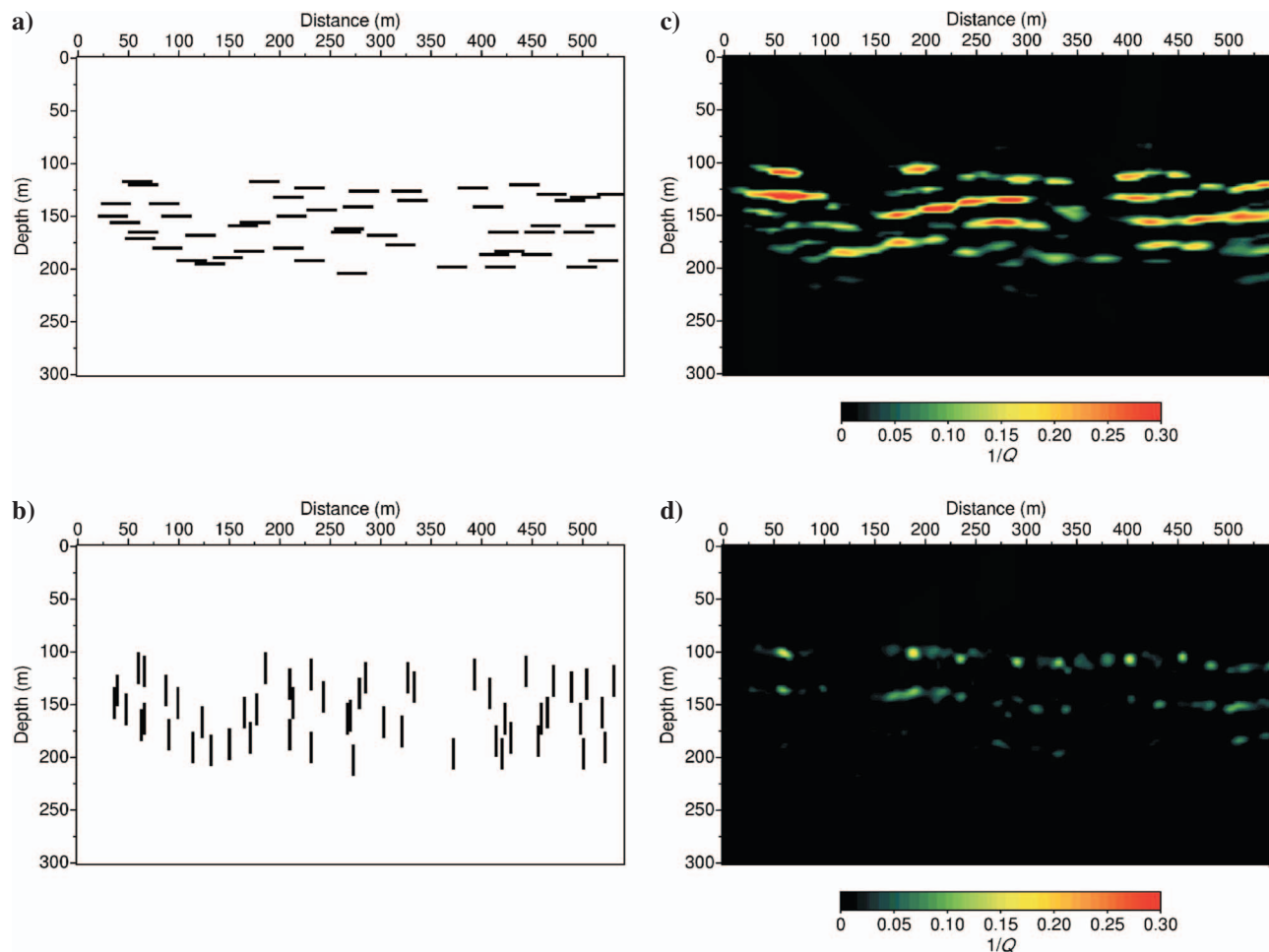


Figure 4. (a, b) Fracture model with horizontal and vertical fractures; fracture size of λ . (c, d) The attenuation models reconstructed by waveform tomography, corresponding to the horizontal and vertical fracture models, respectively.

can recover horizontal fracture in an attenuation model. We also can recover fractures with steeper dips, such as 45° and 135° . However, from an inverted attenuation model alone, it is difficult to judge if the weak attenuation areas correspond to fractures with weak attenuation or to vertical fractures even with longer fracture length.

The reflection seismic data include P-wave as well as PS-wave energy. Figure 7a shows a shot gather simulated from model Figure 4a, with horizontal fractures and a fracture size of λ . The dipping coherent events marked by circles are the converted S-waves. Figure 7b is

the same shot gather after PS-wave energy attenuation, on which the PS-wave reflections have been attenuated. Figure 7c is the reconstructed attenuation model, obtained by using the seismic data after PS-wave attenuation. However, there is no clear difference between Figures 7c and 4c. Reflection waveform inversion mainly matches the strong P-wave reflections in the data, and the weak PS-wave energy has little effect on the image result.

Therefore, in the previous tests, even if the PS-wave energy remains in the input to tomography, it has little effect on the final re

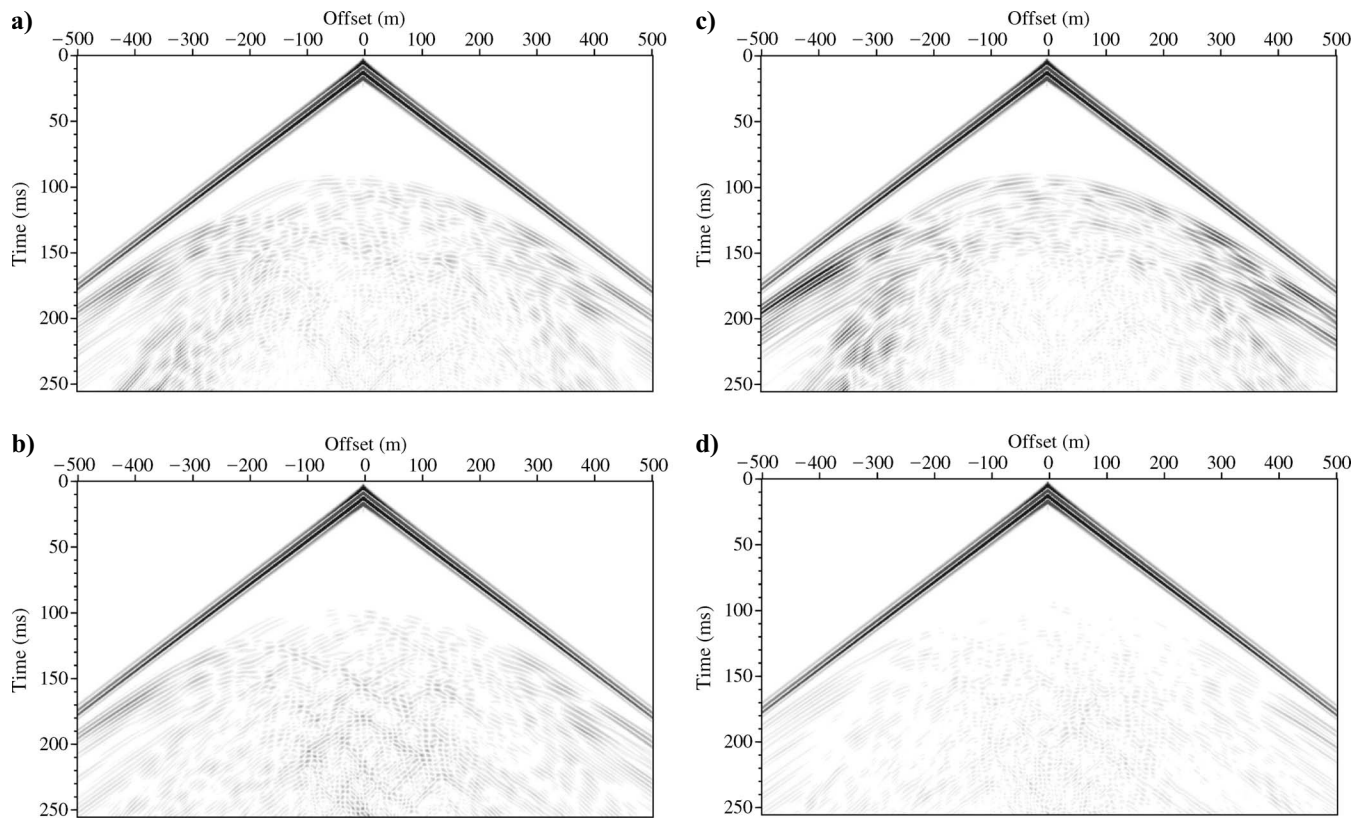


Figure 5. (a) A shot gather of the seismic response of the horizontal fracture model (fracture size equal to $\lambda/2$) in Figure 3a. (b) A shot gather of the seismic response of the vertical fracture model (fracture size equal to $\lambda/2$) in Figure 3b. (c) A shot gather of the seismic response of the horizontal fracture model (fracture size equal to λ) in Figure 4a. (d) A shot gather of the seismic response of the vertical fracture model (fracture size equal to λ) in Figure 4b.

constructed model. However, fractures may have different effects on the P-, S-, and PS-waves. Including other wave modes in the inversion may improve the attenuation image obtained by using P-wave reflections only in the waveform tomography.

In application, fracture distribution density is an important parameter for reservoir characterization. Table 1 compares fracture distribution densities of true models and recovered densities of reconstructed images. All of the estimates are conducted within a rectangle between 120- and 210-m depth and 0 and 540 m in space. The

density is calculated with weighting as

$$d = \frac{1}{S} \sum_{i,j} w_{ij} s_{ij}, \quad (14)$$

where S is the total area of the rectangle, $s_{i,j}$ is the area of the (i,j) th cell, and $w_{i,j}$ is the weight associated with the attenuation within the cell. A least-squares inversion of waveform tomography produces a smooth image with Gaussian distribution. Thus, in density estima-

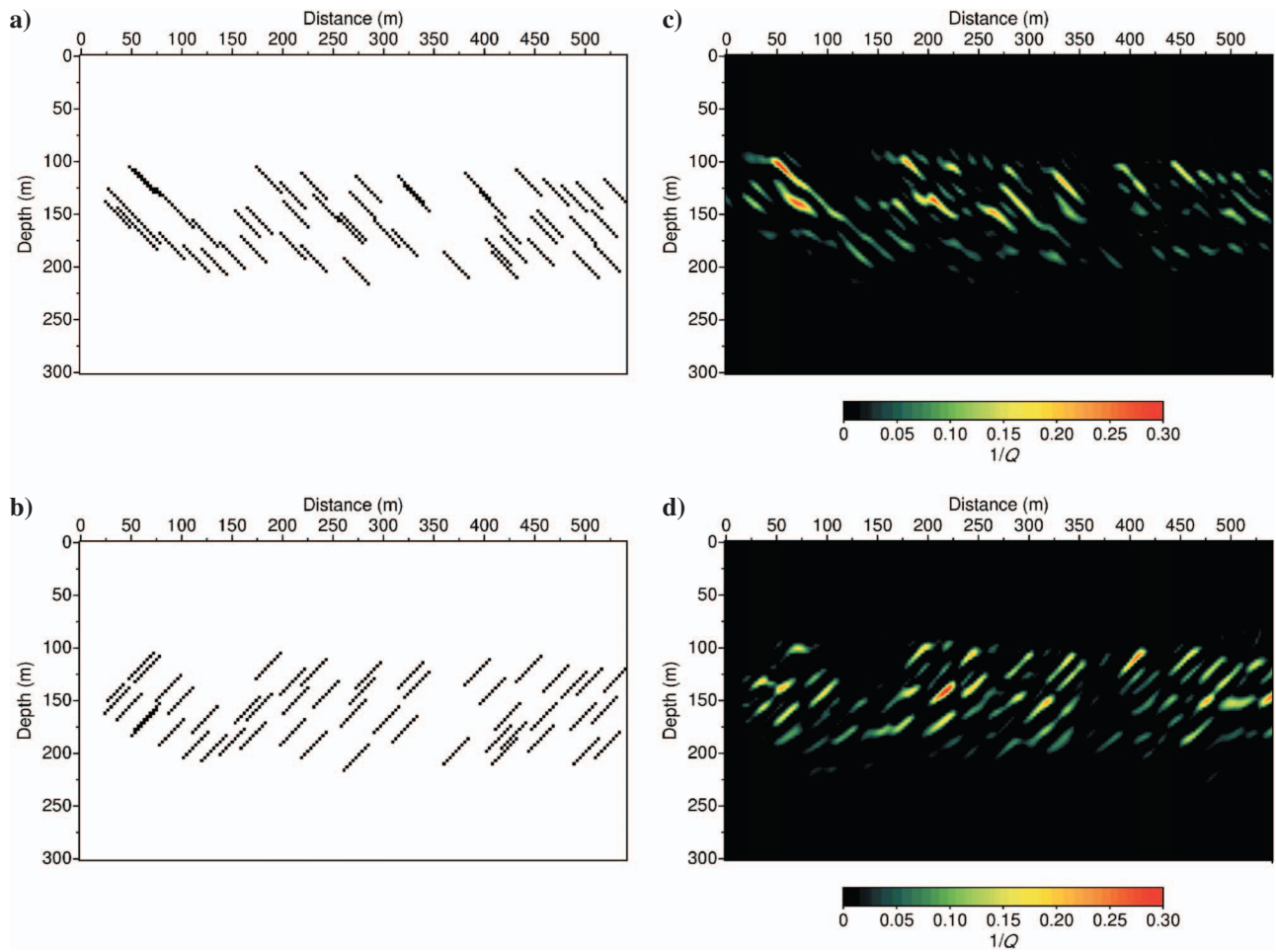


Figure 6. (a) Fracture model with 45° fractures and fracture size of λ . (b) Fracture model with 135° fractures and fracture size of λ . (c) Reconstructed attenuation model from waveform tomography, corresponding to fracture model (a). (d) The reconstructed attenuation model corresponding to fracture model (b).

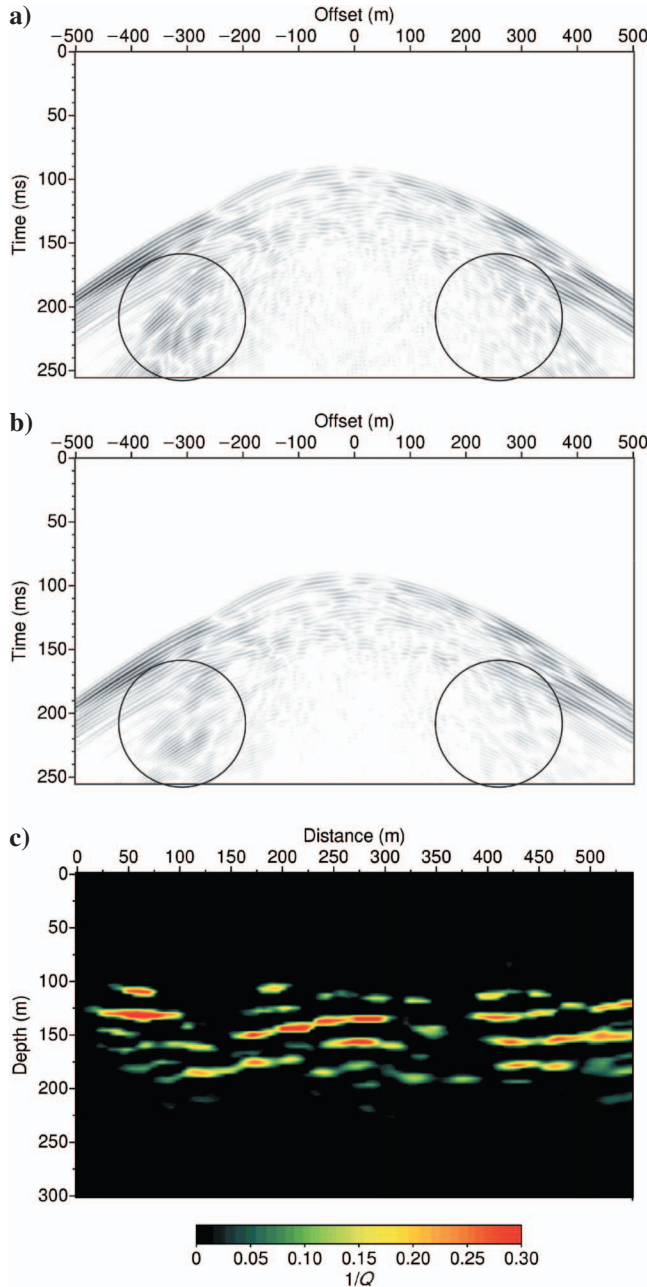


Figure 7. (a, b) Shot gather of the pressure wavefield before and after attenuation of the coherent PS-wave energy (marked by circles). (c) The attenuation model (corresponding to the fracture model in Figure 4a) reconstructed from waveform tomography, obtained by using seismic data after PS-wave energy attenuation.

tion, we set the weighting to be proportional to the attenuation value:

$$w_{i,j} = \begin{cases} (Q_{ij}^{-1}/0.3)^2, & \text{for } 0 \leq Q_{ij}^{-1} < 0.3, \\ 1, & Q_{ij}^{-1} \geq 0.3. \end{cases} \quad (15)$$

Table 1 indicates quantitatively that the vertical fractures (with size of λ) can be poorly recovered, compared to the horizontal fractures. For horizontal fractures of length λ and for vertical/horizontal fractures of length $\lambda/2$, waveform tomography generates attenua-

Table 1. Fracture distribution densities of true synthetic models and reconstructed attenuation images.

Fracture models	True fracture density	Recovered fracture density
Model 1 (horizontal fractures, size = $\lambda/2$)	4.98% (Figure 3a)	4.49% (Figure 3c)
Model 2 (vertical fractures, size = $\lambda/2$)	4.98% (Figure 3b)	5.08% (Figure 3d)
Model 3 (horizontal fractures, size = λ)	9.96% (Figure 4a)	10.18% (Figure 4c)
Model 4 (vertical fractures, size = λ)	9.96% (Figure 4b)	2.37% (Figure 4d)
Model 5 (45°, fractures, size = λ)	9.96% (Figure 6a)	5.31% (Figure 6c)
Model 6 (135°, fractures, size = λ)	9.96% (Figure 6b)	6.05% (Figure 6d)

tion images that have a fracture density value close to the true fracture distribution.

CONCLUSIONS

Whether reflection seismic waveform tomography can be used to reconstruct fracture distributions depends on the size and orientation of the fractures in 2D media. When the fracture size is less than or equal to half of the wavelength, fractures act as scattering points. Strong attenuation images can indicate the positions of fractures clearly and can be used to estimate fracture density accurately. When the horizontal fracture size is equal to one wavelength, fractures act as thin layers with strong attenuation in the reconstructed attenuation model, so the fracture density can be estimated. However, for vertical fractures of such length, the attenuation images are weak and cannot be used to estimate density. In fact, it is difficult to separate the attenuation images of vertical fractures from the images of small sized fractures.

This conclusion complements the forward-modeling observations of side reflections generated by vertical fractures. These side reflections will not contribute to inversion imaging because they are not included in seismograms constructed from reflections received at the surface. Therefore, a joint inversion of surface and borehole seismic data is required to properly image the attenuation associated with fractures. Such an inversion might then include reflections from different directions, different propagation modes, and a broader frequency range.

ACKNOWLEDGMENTS

We are grateful to the sponsors of the Centre for Reservoir Geophysics, Imperial College London, for supporting this research.

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