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Magnetised Transport in Laser Produced Plasmas

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Abstract

Imperfections in the individual laser beams used in inertial confinement fusion have been overcome, to a certain extent, using laser beam smoothing techniques. However, we are still unable to achieve the necessary degree of implosion symmetry required for direct drive. This is thought to be due to nonuniformities in the drive during the start-up phase and is known as imprint.

Up until now there have been few studies of how magnetic fields influence the transport process when density gradients are large. Magnetic fields lead to reduced lateral thermal transport and can sustain regions of very high temperatures known as hot spots.

We develop a hydrodynamic code to investigate the involvement of magnetised transport in the imprint phenomenon. An algorithm for generating random multivariate correlated numbers is also devised and this is applied to model ISI light. From these algorithms we find that filamentary structures are created in densities above critical at laser intensities $I = 10^{14} \text{ Wcm}^{-2}$ but not at $I = 10^{13} \text{ Wcm}^{-2}$.

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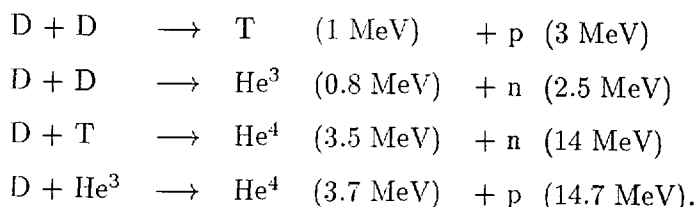
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CHAPTER 1

INTRODUCTION

One of the primary objectives of studies of laser produced plasmas is inertial confinement fusion (ICF). This is a programme for commercial generation of energy by thermonuclear reactions of heavy isotopes of hydrogen and helium: deuterium (D or H^2), tritium (T or H^3) and the helium isotopes He^3 , He^4 :



As can be seen, the energy released from such reactions is very high. However, the electrostatic Coulomb repulsion force between the positively charged nuclei has to be overcome before fusion can take place. Clearly the harder they collide the more chance there is of fusion; high thermal temperatures are necessary. The third reaction, $D + T \rightarrow He^4 + n + 17.5 \text{ MeV}$, has a higher cross-section than the other three reactions at 20 keV and below this temperature the other cross-sections are at least two orders of magnitude smaller. Significant burn occurs in the third reaction at ion temperatures of 10 keV.

Conditions for achieving breakeven for any fusion device were set out by Lawson [1]. For an ion number density n_i and energy confinement time τ , the product must satisfy $n_i \tau > 10^{14} \text{ cm}^3 \text{ s}$ and the temperature must exceed 10 keV. The specific energy required to heat DT to 10 keV is $1.2 \times 10^9 \text{ J g}^{-1}$ and the maximum possible thermonuclear specific energy release is $3.5 \times 10^{11} \text{ J g}^{-1}$, giving roughly 300-fold gain.

We can improve the gain if we *compress* the fuel. Inertial confinement fusion is one way of attaining compression. A small pellet of fuel is contained within a thin shell. The outer surface of the shell is irradiated with intense laser light and matter continually ablates outwards. Consequently the remainder of the shell is accelerated inwards compressing the fuel. While the pellet implodes the fuel changes into a hot plasma and thermonuclear reactions occur.

The minimum energy required to reach this state of affairs and satisfy the Lawson criterion is

$$E = 4\pi (n_i \tau)^3 \frac{k_B T v_i^3}{n_i^2}$$

where v_i is the ion thermal speed. We see that for fixed $n_i \tau$ the minimum energy actually decreases with density.

Lasers are inefficient and we can only expect about a third of the fuel to undergo burn before the pellet disintegrates. Also, the hydrodynamic efficiency to compress the fuel is around 15 %. Thus, it is imperative we increase the efficiency somehow. By pulse shaping the implosion we can actually contrive to improve the gain even

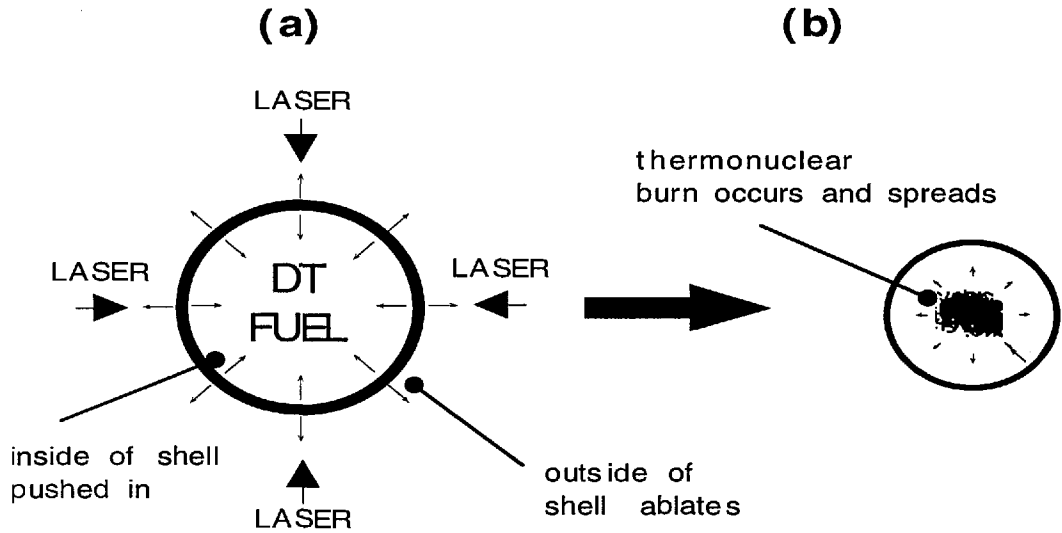


Figure 1.1: ICF by bootstrap heating: (a) DT fuel is compressed by an imploding and ablating shell (b) Part of the DT fuel starts to burn and alpha particles heat the rest of the fuel.

further. We heat only the the centre of the fuel to ignition temperatures. The rest of the compressed fuel is kept at a low temperature (but high density) and is heated by the alpha particles from the thermonuclear reactions in the centre. Burn emanates from there. This is known as bootstrap heating and the scheme can increase the gain 25-fold. Indications are that a plasma density of around 300 g cm^{-3} ($\sim 1000 \times$ solid density) is required. Temperatures must not be much higher than 10 keV otherwise the alpha particle mean free path becomes larger than the pellet radius.

The group at Osaka in Japan have been able to compress *hollow* shell targets to

500-1000 times initial solid density [2] and high densities have been attained using the NOVA laser at LLNL in the USA. However, temperatures were well below 10 keV. It is expected that future higher powered laser systems will be able to simultaneously meet both the density and the temperature requirements.

A thorough discussion of the conditions for fusion outlined above is given by Bodner [3].

1.1 Ablation Pressure and Drive Symmetry Requirements for ICF

A high pressure of approximately a few tens of megabars is generated by the lasers making the outer shell ablate and consequently causing the rest of the shell to be accelerated inwards. We shall call this driving term the ablation pressure P_a . It can be amplified $10^3 - 10^4$ -fold to give a core pressure of 800 Gbar. This core pressure easily satisfies the requirement of a density of the order of 100 g cm^{-3} at 10 keV. In short, a high ablation pressure is essential at the pellet surface so that a high density fuel can be created at the target core.

An insight into the relationship between the ablation pressure, laser wavelength λ_{las} and laser intensity I_{inc} has been obtained [4]

$$P_a = \left(\frac{\lambda_{las}}{\mu\text{m}} \right)^{-2/3} \left(\frac{I_{inc}}{10^{14} \text{ W cm}^{-2}} \right)^{2/3} 14 \text{ Mbar} \quad (1.1)$$

This relationship demonstrates that intensities of $10^{14} - 10^{15} \text{ W cm}^{-2}$ are needed to attain the necessary level of ablative pressure for fusion conditions. It is also seen that it is desirable to work with shorter wavelength light to increase the ablative pressure.

Amplification of P_a is through convergence effects inherent from the spherical geometry of the target. Also, the shell is accelerated over a long time ($\sim 10 \text{ ns}$) and the momentum is released during the relatively short time of the implosion (0.1 ns). In Figure 1.2 we can see the trajectories of the inner and outer surface of an ICF capsule passing through four phases. During the start-up phase the laser intensity is very low. This is to ensure strong shocks do not develop and preheat the target core. After a period of a few hundred picoseconds the laser intensity is increased dramatically. This drives the shell during an acceleration phase. The shell attains a maximum velocity

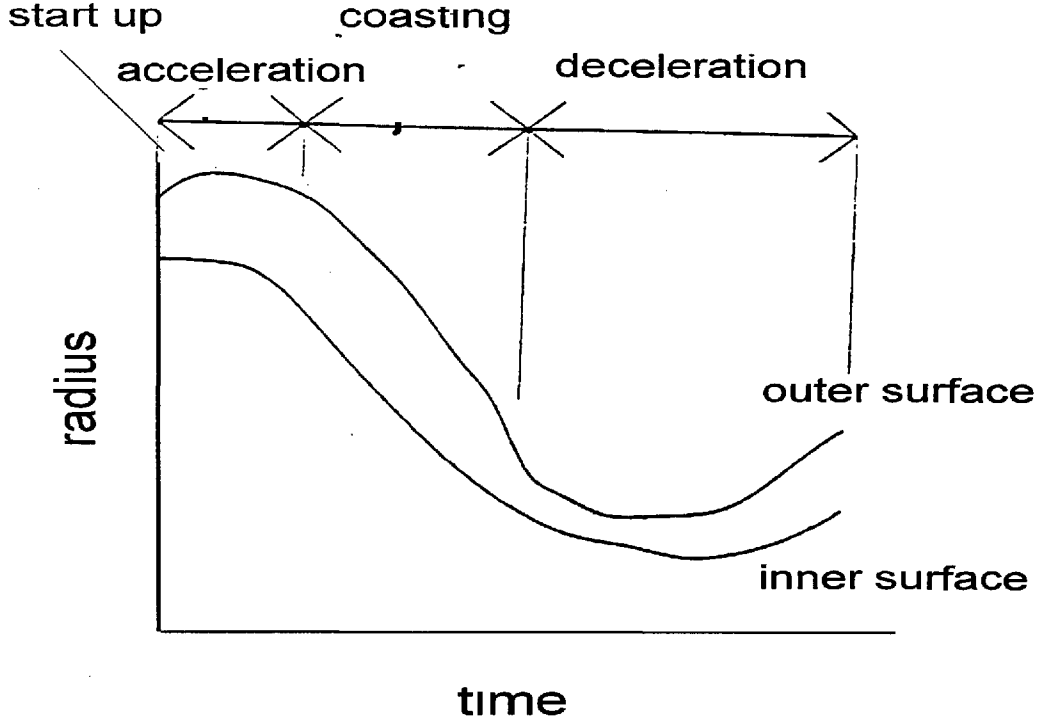


Figure 1.2: Trajectories of the inner and outer surfaces of the shell.

and then starts to coast, compressing the fuel within. In the end the pressure of the contained fuel builds up to such an extent that it decelerates the shell.

High gain laser fusion requires a very uniform ablation pressure at the pellet surface [3] because ICF targets have to be imploded very uniformly. Let us estimate how much ablation uniformity we require. First we consider a shell of radius R and implode it to a final radius R/C_R , where C_R is the radial convergence. The implosion time is $t_{imp} = (2R/g)^{1/2}$ where g is the acceleration. Using the momentum equation in Lagrangian form $\rho dv/dt = -\nabla P$ we arrive at the relationship

$$R \sim \frac{1}{2} \frac{P_a t_{imp}^2}{\rho \Delta P} \quad (1.2)$$

where ΔP is the pressure scale length in the direction of the implosion. If we apply a perturbation to the ablation pressure ΔP_a of a particular shell element, that element will be at a different radius ΔR at bang time. A differential relationship exists: $\Delta R/R = \Delta P_a/P_a$. Too much distortion of the shell can be avoided, if

$$\frac{\Delta R}{R} C_R < \frac{1}{2} \quad \text{i.e.} \quad \frac{\Delta P_a}{P_a} < \frac{1}{2C_R}.$$

If we assume $P_a \propto I^x$ with $0.6 \leq x \leq 0.97$ such as in Equation 1.1, the laser intensity uniformity constraint is

$$\frac{\Delta I}{I} \leq \frac{1}{2xC_R}.$$

Therefore, for a convergence ratio of roughly 30 the intensity uniformity has to be better than 1/60 - quite an austere requirement.

1.2 The Rayleigh-Taylor Instability

Technically it is desirable to have a thin shell so we can (a) obtain the required linear compression of approximately 30 and (b) reach large enough shell velocity at implosion time. The problem is thin shells are particularly vulnerable to the Rayleigh-Taylor (RT) instability - as well as growth through convergence effects mentioned above.

The low density ablating plasma accelerates the denser shell inwards. Inward acceleration is equivalent to outward gravity g . In the frame accelerating with the shell, the heavy, cold shell fluid is effectively supported by the low density, hot ablating fluid in a pseudo-gravitational field. The RT instability is the mechanism by which the two fluids exchange position in this frame. Any perturbation at the interface of the two fluids grows unstably in a bubble-spike manner. ‘Bubbles’ of low density fluid rise and ‘spikes’ of high density fluid fall. This instability, which operates at the ablation surface, can lead to shell disintegration in ICF [5,6].

It is well known that flow through a density gradient can lead to ablative stabilisation of the RT instability [7–9]. The growth rate of the ablative RT instability is given by the modified Takabe equation

$$\text{growth rate} = \left[\frac{kg}{1 + kL} \right]^{1/2} - \frac{3k\dot{M}}{\rho_{max}}$$

where $\dot{M} = \rho v_a$ is the mass ablation rate per unit area and is generally constant in the unstable region. v_a is the flow velocity at the ablation front.

Even with the help of ablative stabilisation the growth rate is still greater than half classical for the dangerous modes [10–12], which have wavelengths of the order of the shell thickness [8,13].

The general consensus concerning RT growth is optimistic. If one can guarantee

an initial shell surface finish better than 1%, then the RT instability can just about be endured. We are now left with the need for laser beam uniformity and this will be studied in detail later in this thesis.

1.3 The Richtmyer-Meshkov Instability

Whereas the RT instability results when an interface is subjected to a constant acceleration, the Richtmyer-Meshkov (RM) instability [14, 15] occurs when an interface is subjected to an impulsive acceleration. For both instabilities the interface separates fluids of different density. The limiting case of an impulsive acceleration is the acceleration due to a shock. The growth rate of the RM instability is governed by the equation

$$\frac{\partial \eta}{\partial t} = \frac{2\pi}{\lambda} \eta_0 A U_i \left(1 - \frac{U_i}{U_s} \right).$$

A perturbation of wavelength λ and initial amplitude η_0 at an interface, grows immediately and at a constant rate when an impulsive shock of velocity U_s gives that interface a velocity U_i . The Atwood number is $A = (\rho_+ - \rho_-)/(\rho_+ + \rho_-)$, where ρ_+, ρ_- are the densities at the interface just after the passage of the shock. For ICF $A \approx 1$ so

$$\frac{\Delta \eta}{\eta_0} = \frac{2\pi}{\lambda} U_i \left(1 - \frac{U_i}{U_s} \right) \Delta t$$

A shock will traverse a target of thickness Δx in time $\Delta t = \Delta x/U_s$. From the continuity equation we obtain $U_i/U_s = (1 - \rho_d/\rho_u)$, where $\rho_d(\rho_u)$ is the density downstream (upstream) of the shock. In the case of a strong shock, $\rho_d/\rho_u = (\gamma - 1)/(\gamma + 1) = 1/4$, and so for D-T

$$\frac{\Delta \eta}{\eta_0} = \frac{3\pi \Delta x}{8\lambda}$$

For example, Richtmyer-Meshkov shock driven growth can amplify perturbations of wavelengths 40 – 50 μm by up to 2e-foldings of growth for a target of thickness 300 μm . This growth occurs before the acceleration phase.

Figure 1.3 shows the results of a 1D simulation of an implosion using the **MEDUSA** code (taken from Reference [16]). The radius of each of the Lagrangian zones is plotted

as a function of time with the laser incident from the top of the figure. Horizontal lines

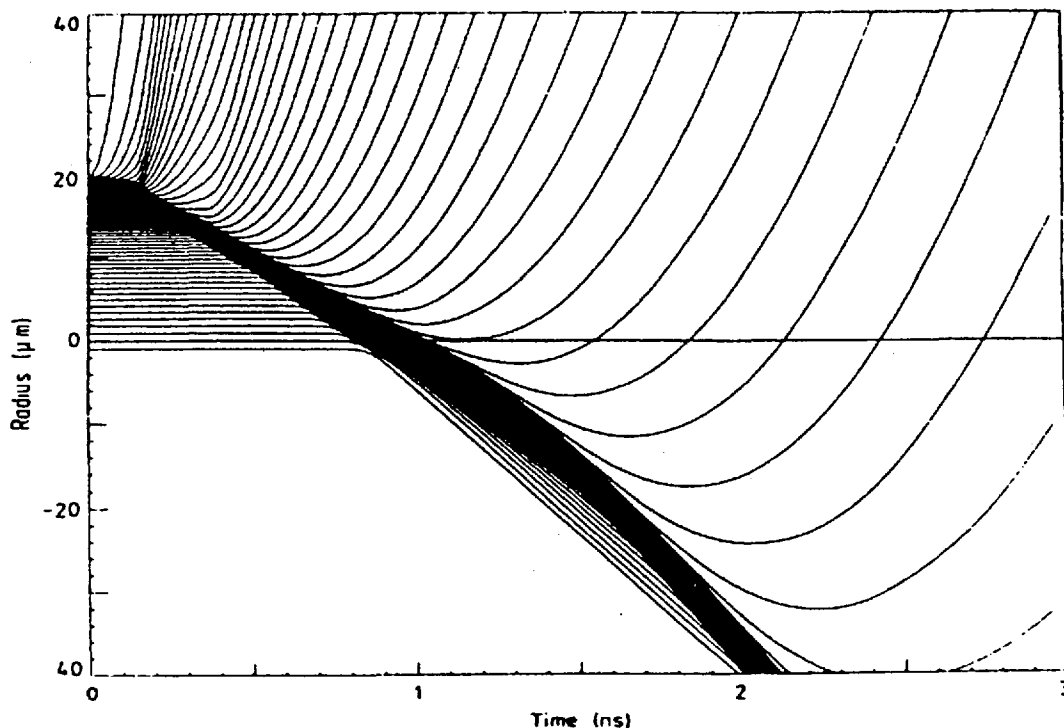


Figure 1.3: Motion of the Lagrangian mesh points in a typical MEDUSA simulation in plane geometry

represent stationary material. The Lagrangian zones are closer in the darker regions where the density is higher. The ablating material appears as lines moving towards the upper right-hand corner of the figure. After about 1.5 ns the ablation velocity is seen to be roughly constant.

Each shell element has a breakpoint when the element suddenly begins to move due to the passage of the shock wave driven by the laser. The locus of the breakpoints gives the velocity of the shock. The slope of the dark band represents the velocity behind the shock front. This is constant in the early phase. Essentially the shell material is being accelerated by the shock front.

It is only after the shock has traversed the shell that true ablative acceleration begins (after about 0.95 ns in Figure 1.3.) - primarily due to increasing laser intensity. This is when the RT instability begins to operate. By this time residual mass variations have undergone 1-2 e -foldings of growth in a RM manner. Any mass variations at this time will grow during the acceleration phase proper. For this reason mass variations at the peak of the laser pulse must be $\ll 1\%$.

1.4 The Imprint Phenomenon and the Start-Up Phase

Jet-like structures have been observed in the subcritical region [17]. We require uniformity in the blow off plasma to avoid filamentation and self focusing of the laser beam. However, it is more important that the denser region at the ablation surface is uniform so as to mitigate the RT instability. There are other hydrodynamic, thermomagnetic [18–21] and radiation rate dependent instabilities [22] which feed on the nonuniformity at the ablation surface.

The ablation surface is particularly vulnerable to laser nonuniformities at early times. In most reactor-like laser pulses the fuel has to be kept on a low adiabat. This requires a low laser intensity of around $\leq 10^{13} \text{ W cm}^{-2}$ for the first few nanoseconds, a relatively long period of time. During this time the absorption-ablation surfaces are typically only a few microns apart. This is quite a serious problem as it implies transverse thermal smoothing cannot be effected. At early times, the laser energy is deposited directly on the target surface and it is natural for the target to follow any perturbations in the laser. It is thought the earliest perturbations in the laser imprint themselves onto the target and linger on the longer nanosecond time scale.

The imprint takes the form of a shock [23]. As this shock propagates through the target it leaves residual mass variations in its wake. After about 1 ns these mass perturbations begin to grow linearly with time in a Richtmyer-Meshkov manner. They can later seed RT growth during the acceleration phase. In this light we see the first shock is a very important factor during the implosion. This phenomenon has been modelled by the group at Imperial College and others [24].

The mass variations which originate from the first shock do not succumb to subsequent laser drive. They persist and are not smoothed out thereafter. **Why this is so is the question this thesis will attempt to answer.**

The density gradient is very steep at the beginning since there is hardly any corona and the light introduces temperature gradients in the lateral direction. These conditions are favourable for the generation of magnetic fields. These fields offer an explanation of why the perturbations persist. It is well known from simulations and experiment ¹ that an isolated laser hot spot can produce jets in the ablating plasma through the

¹Although let it be said that laser intensities were much higher, scale lengths were longer and the time scale was shorter.

influence of self generated magnetic fields [25, 26]. Fields may also inhibit the flow of heat out of the hot spot [27–29]. It could be that magnetic fields are an important aspect of the imprint phenomenon.

Various laser beam smoothing methods are employed to reduce the nonuniformity at the critical surface [30–33]. For example, light produced by ISI (Induced Spatial Incoherence) [34–36] has time-integrated smoothness over nanosecond periods. Any residual structures present over this time scale are curtailed by lateral thermal smoothing between the critical and ablation surfaces, providing there is sufficient distance between these two surfaces. If the sound transit time in this region is larger than the ISI averaging time then the target effectively sees a smooth laser and the rapidly varying ISI light cannot cause perturbations. Of course, this requires a long scale length, uniform plasma.

However, compared to normal laser light ISI light is very nonuniform on shorter time scales of the order of picoseconds. This poses grave difficulties during the start-up phase as the perturbations in the first shock are attenuated. The group at Imperial College London has been able to obtain high resolution, time-resolved images close to the ablation front [37]. They used ISI-like laser light.

Figure 1.4(a) (taken from Reference [38]) reveals the blow off is not uniform. A 20 μm gold wire is irradiated from the left by defocused ISI light and from the right by tight-focused ISI light. $f/10$ optics, a laser wavelength $\lambda_{las} = 0.53 \mu\text{m}$ and intensity $I = 5 \times 10^{13} \text{ W cm}^{-2}$ were used throughout. Regularly spaced nonuniformities (corresponding to individual beamlets) are clearly visible on the defocused ISI left side. On the tight-focused right side nonuniformities are also visible but are of random nature because all the beamlets are overlapped. Figure 1.4(b) shows the result of ISI+RPP (Random Phase Plate) hybrid radiation. Gross nonuniformities in self emission and the blowoff plasma are still clearly visible in this X-ray image. The arrows indicate the initial left and right hand edges of the target.

The perturbations present in ISI-like light imprint themselves on the cold target surface during the first few picoseconds of the laser pulse. This seems reasonable since at early times the critical and ablation surface are close together and there is not much thermal smoothing.

It is well known that one can produce a uniform intense source of soft X-rays by irradiation of a burnthrough foil target. Such a foil was placed 250 μm away from a

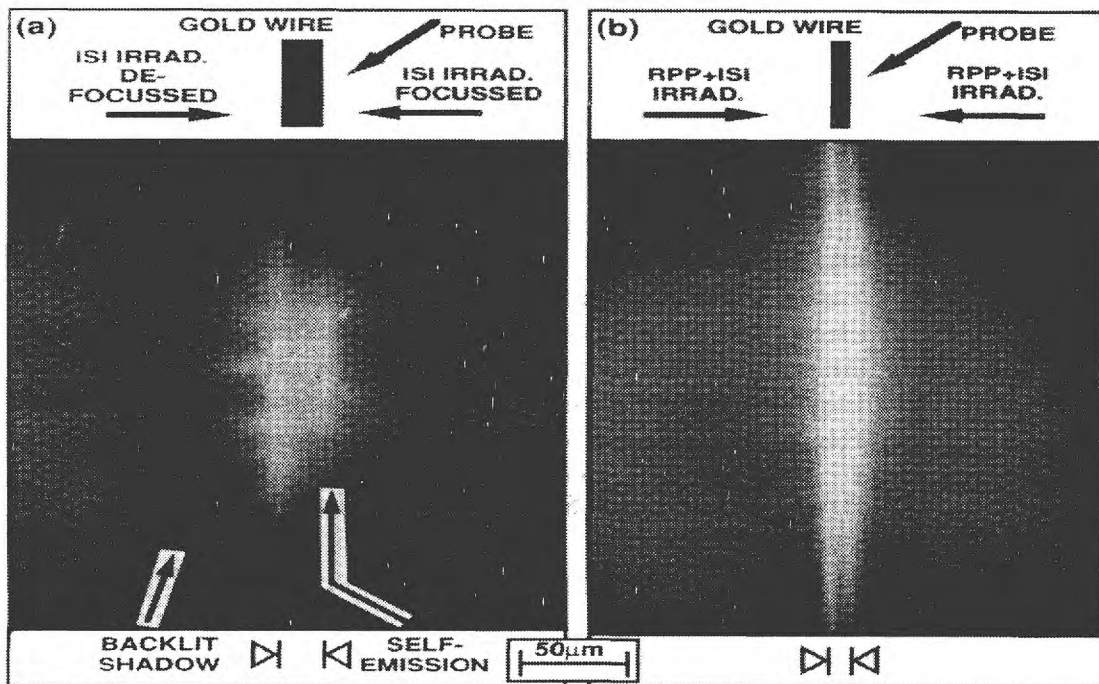


Fig 1.4 (a) Defocused and tight focused ISI irradiation of a copper wire. (b) Hybrid ISI+RPP irradiation of a copper wire.

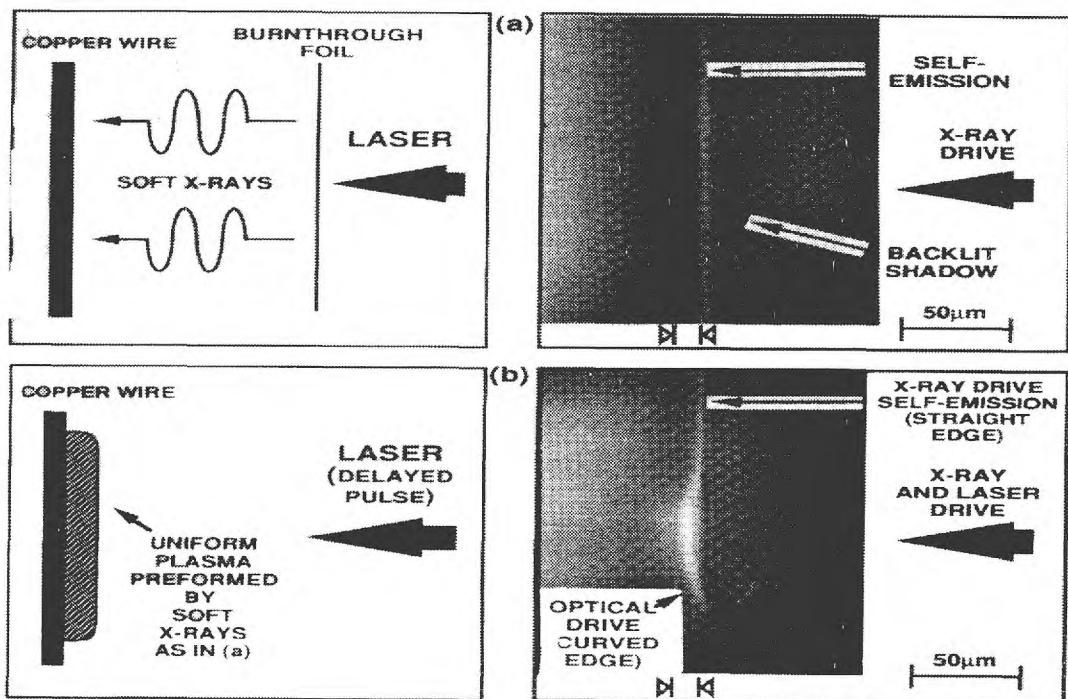


Fig 1.5 (a) Copper wire irradiated with soft x-rays. (b) ...and later directly by laser light.

10 μm copper wire target and is shown in Figure 1.5(a) (taken from Reference [38]). The X-ray irradiation was uniform and estimated to be around $10^{13} \text{ W cm}^{-2}$. The self emission and backlit shadow show that the plasma blow off is uniform in contrast to the jet-like structures observed by direct illumination.

When we irradiate this preformed uniform plasma with direct ISI-like light (delayed by 1.2 ns and $\lambda_{\text{IIS}} = 0.53\mu\text{m}$) there are no visible jet-like structures produced. See Figure 1.5(b). Clearly this is due to the critical and ablation surfaces being far apart and there is considerable thermal smoothing of the nonuniformity in the interacting ISI-like beam. Long scale length plasmas are not exposed to imprint. These results were reproduced in the simulations carried out by Emery et al [23].

1.5 Motivation behind the Thesis

It is apparent that thermal transport has an important role to play in the imprint process. Our goal is to model the transport and the hydrodynamics of the start-up phase, including the effect of magnetic fields which can influence both the heat flow and velocity flow profiles. We intend to determine whether these fields are responsible for perpetuating the first shock profile.

It is desirable to use ISI light. In comparison to normal laser light it is very nonuniform on picosecond time scales. This will serve to emphasise the coupling of early laser drive to the first shock because the collective effect of later drive will be screened out by temporal smoothing.

1.6 Outline of the Thesis

The first three chapters are a review of the relevant background physics needed if we are to comprehend what mechanisms are responsible for imprint. These chapters do not represent the author's own work.

Chapter 1 gives an outline of how fusion could be attained by inertial confinement of a pellet of fuel. The nature of this imprint is investigated.

Chapter 2 gives the theory of transport in unmagnetised plasmas with particular reference to lateral thermal smoothing. This effect requires an ablation-critical distance comparable to the perturbation scale length.

In Chapter 3 we extend our review to magnetised plasmas. The possibility of bifurcation of solutions and hydrodynamic coupling is considered. In particular the Nernst effect leads to convection of fields with the heat flow.

Beam smoothing techniques are discussed in Chapter 4. Our emphasis is on ISI light as a seed for instability with respect to the first shock. We present the algorithm we have developed to produce random bivariate correlated numbers.

Our basic assumptions are given in Chapter 5. We decide which effects to include and which geometry to use. We present several computational methods of solution of the MHD equations. The final version of our code **STAMP** (**S**tudy of a **M**agnetised **P**lasma) is selected and detailed.

The code (in one form or another) is tested against standard problems. All results of testing are contained in Chapter 6. The code's limitations are determined.

In Chapters 7 and 8 we perform calculations using our code and our ISI algorithm in various parameter regimes.

A discussion of the results of this thesis is laid out in Chapter 9 and conclusions are drawn.

Finally in Chapter 10 we suggest refinements that could be made to the code and try to anticipate what bearing they will have on the results and conclusions.

CHAPTER 2

TRANSPORT IN UNMAGNETISED PLASMAS

The laser cannot penetrate beyond the critical surface which occurs at an electron density of $n_{crit} = 10^{21}(\lambda_{las}/\mu\text{m})^{-2}\text{cm}^{-3}$, where λ_{las} is the laser wavelength. Typically this is around 0.01-0.1 times solid density. Absorption occurs in the subcritical region of the corona and any remaining energy in the beam is either used to support plasma waves at the critical surface or is reflected. Energy has to be transported to the ablation surface so the shell can implode. This is done by electron thermal conduction². *For this reason transport plays a crucial role in the efficient coupling to the solid of energy deposited by the laser.* The temperature, density and heat flow profiles in steady state are shown in Figure 2.6.

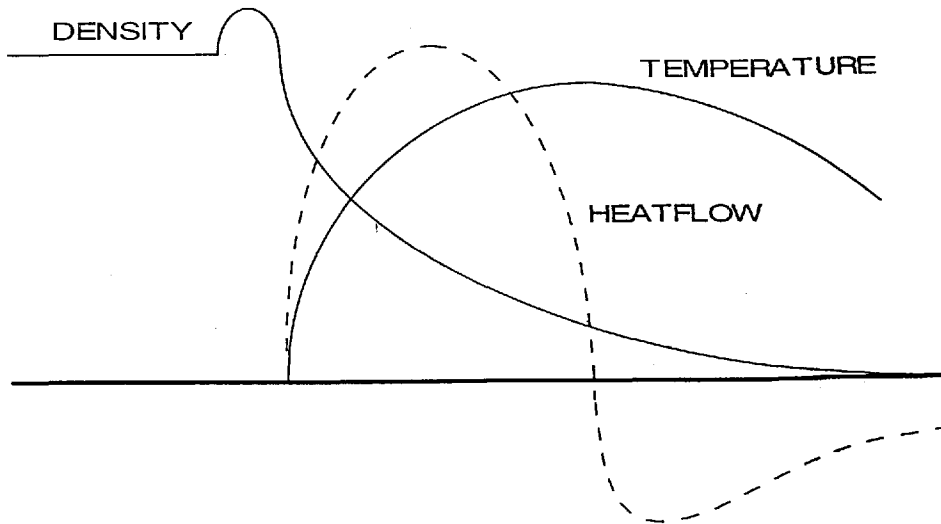


Figure 2.6: Density, temperature and heat flow profiles in a laser driven ablation process. The laser is incident from the right.

In steady state, the temperature maximum occurs at the critical surface and the plasma flow is sonic there. There is a natural bound on the heat flow called the free streaming limit:

²Radiation transport is important at high density or when dealing with high Z materials though this will not be discussed in this thesis.

$$Q_f = n_e kT \left(\frac{kT}{m_e} \right)^{\frac{1}{2}} \quad (2.3)$$

representing the heat flow carried by electrons flowing freely with the mean thermal energy down a temperature gradient at the mean thermal speed. The heat flow is maximum at the critical surface. In steady state an energy balance analysis given by Bell [39] determines the heat flow to be $0.05 Q_f$ there. This analysis makes no reference to thermal conductivities.

2.1 Spitzer-Harm Theory

Classical heat flow is essentially a diffusive process. In laser produced plasmas the electrons at the critical surface are very hot and have relatively long mean free paths. They diffuse down the temperature gradient into the denser solid. A return current of colder electrons flows towards lower densities. The plasma conductivity κ_e is strongly temperature dependent, $\kappa_e \propto T_e^{5/2}$. Spitzer-Harm Theory (SH) [40] has an electron heat flow given by $Q_e = -\kappa_e \nabla T_e$. This leads to very steep gradients in temperature and heat flow at the front of a heat wave.

In SH theory the plasma electron distribution is expressed as $f = f_0 + f_1$, where f_0 is Maxwellian and f_1 is the anisotropic part which carries the heat flow and satisfies $f_1 \ll f_0$. Using the Chapman-Enskog expansion³ one obtains $f_1 \sim (\lambda_{mfp}/L) f_0$, where λ_{mfp} is the mean free path of an electron with thermal energy $k_B T$, and L is the scale length of the temperature profile ($L = |T/\nabla T|$). From this the SH heat flow takes the form

$$|Q_{SH}| = \gamma_0 \left(\frac{\lambda_{mfp}}{L} \right) Q_f \quad (2.4)$$

with $3 \leq \gamma_0 \leq 13$. The behaviour of the plasma when the temperature scale length and the electron mean free path are comparable is of particular interest. The form of Equation 2.4 suggests the SH heat flow approaches the free streaming limit in this case. This is not borne out in experiments.

³In fact the distribution function and the heat flow can be written as an asymptotic expansion in λ_{mfp}/L .

Early experiments showed an anomalously high ablation velocity. This implied that the sound speed (and consequently the temperature) in the low density corona was unusually high. In other words energy was being partially trapped in the corona and was not reaching the ablation surface. Computer codes could not simulate this inhibited heat flow without artificially limiting it to some predetermined value. It became customary to set the maximum heat flow that could be carried by the plasma to some fraction f of the local free streaming limit Q_f [41,42]. By solving the Fokker-Planck equation (see for example References [39] and [43]) a value of $f \approx 0.1$ was obtained [44]. This estimate was later confirmed by experiment.

2.2 The Breakdown of SH Conductivity and Nonlocality

In a laser produced ablating plasma the heat flow can be a substantial fraction of the free streaming limit. This means a significant portion of the electrons is moving in the same direction and this is not compatible with the SH theory which is based upon diffusion. The heat flow is better modelled by the Fokker-Planck equation for electrons.

The heat flow is strongly influenced by the electron velocity v . We may reformulate the SH model so $f = f_0 + f_1 v_x/v$ where it is assumed $f_1 \ll f_0$ and f_0 is Maxwellian. At a threshold velocity $v = 2\sqrt{2}v_t$, the anisotropic part of the distribution is comparable to the Maxwellian there so $f_1 = f_0$. Clearly this model is not well defined at very high velocities since we cannot have $f < 0$ there. Since f_1 is negative below this threshold only the highest velocity electrons are carrying heat flow. We refer to this species as the hot electrons.

The electron mean free path is proportional to v^4 so these hot electrons are very mobile. Consequently the Maxwellian tail of f_0 in this high velocity region is depleted: *nonlinearity is due to f_0 departing strongly from a Maxwellian.* The hot electrons pour into the colder regions enhancing the tail of the distribution there. Thus, gradients in f_0 , which drive the diffusion process, are effectively less than in the SH theory. The SH theory overestimates the heat flow. The number density of hot electrons is relatively small so the maximum heat flow they can carry is in reality much smaller than the free streaming limit. In fact, these two effects combine to cause anomalous behaviour at relatively low levels of heat flow.

The hot electrons at the threshold energy have mean free paths comparable to

the temperature scale length when $L/\lambda_{mfp} \sim 150$ (λ_{mfp} is the mean free path of the thermal electrons travelling at speed v_t) and this is verified experimentally in Reference [45]. One must understand that the heat flow can be very much less than the free streaming limit in this argument. The point is that it is the temperature scale length and not the heat flow which is important. The effective temperature scale length of these hot electrons is much larger than the local scale length and this introduces a nonlocal effect. Thus we expect inhibited heat flow whenever the mean free path of the energetic electrons is of the order of the temperature scale length [46] not because of nonlinearity through saturated heat flow but because of nonlocality.

Assuming sonic flow from the ablation surface the outward enthalpy flux ranges from $0.07 Q_f$ for high Z to $0.28 Q_f$ for low Z plasmas. For steady state ablation the inward heat flux must be considerably larger than this. We already know that SH theory can breakdown as low as $0.03 Q_f$ from Reference [45]. It is becoming apparent that nonlocal transport cannot be ignored in ICF simulations.

Transport effects are known to affect the thermal filamentation process too. As a laser beam traverses the hot, low density corona it sometimes breaks up into filaments. These filaments lead to nonuniformity at the critical surface and this causes poor implosion. The appropriate temperature scale length is the width of the filament and this is much less than the mean free path of the hot electrons. The heat flowing out of the filament is less than SH theory so the growth rate is anomalously larger [47].

To summarise the SH theory works for gentle temperature gradients which are much larger than the mean free path of the hot electrons and these conditions are not satisfied in laser produced plasmas.

2.3 Lateral Thermal Smoothing

Electron transport is of prime importance in direct drive ICF for conveying energy from the critical surface to the ablation surface. In the process transport also spreads out the energy before it reaches the ablation front because heat flow is diffusive in character. Nonuniformities in the subcritical region can be smoothed out laterally as long as sufficient overdense plasma exists between the two surfaces.

A crude understanding of this mechanism is provided by the cloudy day model [48]. We start with a uniform, stationary plasma in steady state so the solution satisfies

$\nabla^2 T = 0$. Imagine we impose a small sinusoidal temperature perturbation at the critical surface. If T_1 is the temperature perturbation amplitude at critical and T_0 is the background temperature, this gives

$$T = T_0 + T_1 \sin(kx) \exp(-kz). \quad (2.5)$$

If the critical and ablation surfaces are a fixed distance D_{ac} apart, then the amplitude of the perturbation decays by a factor $\exp(-kD_{ac})$ between these two surfaces. In other words, lateral thermal smoothing gives an e -folding reduction in nonuniformity when the scale length of perturbations at critical, k^{-1} , is comparable to the distance D_{ac} . Smaller scale length perturbations do not reach the ablation surface though larger scale length modes do.

A more sophisticated analysis shows that diffusion lessens the effect of intensity nonuniformities $\Delta I/I$ on ablation pressure by a factor α

$$\left. \frac{\Delta P}{P} \right|_{abl} = \alpha \left. \frac{\Delta I}{I} \right|_{crit} \quad (2.6)$$

with

$$\alpha = \exp(-n\pi D_{ac}/\lambda_{pert}) \quad (2.7)$$

where λ_{pert} is the perturbation wavelength. Gardner and Bodner [49] give $n = 2$. Smoothing is useful though at early times (when the corona is just forming) we do not have a steady state and D_{ac} is small. Obviously lateral thermal smoothing cannot take place as there is no overdense medium present.

Since this effect may be important for real laser systems we have plotted the trajectories of the critical surface and the ablation surfaces (defined as $v = 0$) for various laser wavelengths and intensities in Figure 2.7 (taken from Reference [16]). The separation of these surfaces increases with wavelength and with increasing intensity. An approximate expression for $D_{ac}(t)$ is given by Key [50]

$$D_{ac}(t) = \left(\frac{I}{10^{14} \text{W cm}^{-2}} \right)^{1/3} \left(\frac{\lambda}{\mu\text{m}} \right)^{2/3} \left(\frac{t}{\text{ps}} \right) 0.08 \mu\text{m} \quad (2.8)$$

which is a numerical fit to spherical expansion at long times.

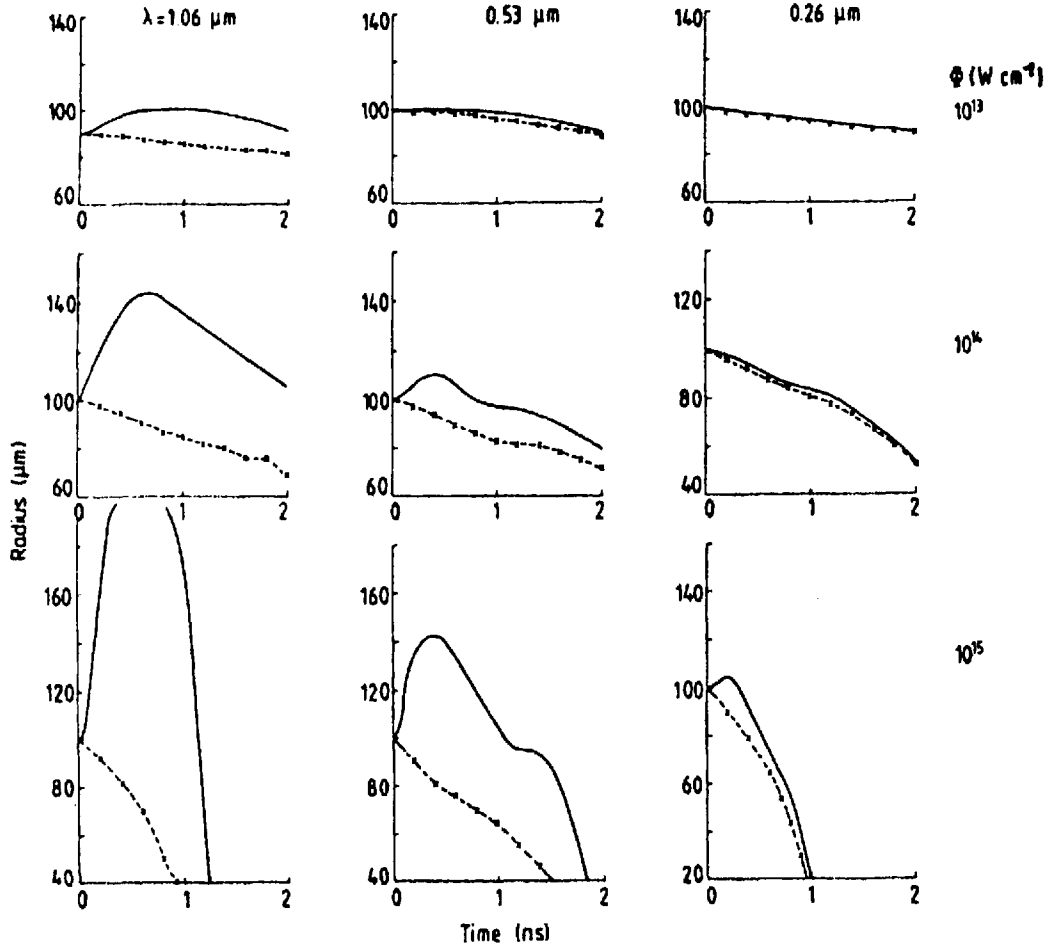


Figure 2.7: Relative motion of critical density and ablation surfaces as a function of laser wavelength and intensity.

A typical laser beam has irradiation nonuniformities with scale lengths of tens of microns and it takes tens or hundreds of picoseconds for D_{ac} to grow to this size. Even if a plasma corona has formed, reduced lateral heat flow is effected because of large mean free paths and the consequent nonlocality issue. In two dimensions there is better lateral thermal smoothing of small scale temperature perturbations with a SH model than with a FP model [51]. However, FP models give increasingly smoother profiles relative to SH models as lateral scale lengths increase, especially at early times. The FP heat flow responds more slowly than the SH heat flow and this results in smaller ablation pressure variations with the FP model than with the SH model for the same applied laser intensities. The nonlocal model has *thermal inertia* because the slower response enables more time for intensity time-averaging to take place [52].

CHAPTER 3

MAGNETISED TRANSPORT

In general when a short, powerful laser pulse is focused tightly onto a solid target large megagauss magnetic fields are produced. For ICF the fields are produced in the lower density absorption region by inhomogeneities in the laser. For higher irradiance experiments significant fields are produced at the critical surface by resonance absorption. These fields can affect how the laser interacts with the plasma and the plasma itself by reducing thermal transport. For ICF this is precarious as it can alter the pellet implosion symmetry. We provide a discussion of the primary mechanism for field generation, how these fields affect the transport, how they can lead to instabilities and when they influence the hydrodynamics.

3.1 Generation of Magnetic Field

Magnetic fields of the order of a megagauss have been observed in laser produced plasmas [53]. Generation of these fields is given by Faraday's Law

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \wedge \mathbf{E}. \quad (3.9)$$

The electric field can be determined from the momentum equation of the electrons

$$m_e n_e \frac{D\mathbf{v}_e}{Dt} = -en_e (\mathbf{E} + \mathbf{v}_e \wedge \mathbf{B}) - \nabla P_e - \nabla \mathcal{P} + \mathbf{R}_T + \mathbf{R}_u.$$

Electron viscosity, finite Larmor radius effects, magnetic field stress tensor and the ponderomotive force are all included in the pressure tensor $\mathcal{P}$. The thermal force $\mathbf{R}_T$ and the friction force $\mathbf{R}_u$ are given by Braginskii [54]. One now modifies the $\mathbf{v}_e \wedge \mathbf{B}$ term

$$\begin{aligned} [\mathbf{v}_i - (\mathbf{v}_i - \mathbf{v}_e)] \wedge \mathbf{B} &= \mathbf{v}_i \wedge \mathbf{B} - (\mathbf{J} \wedge \mathbf{B} / n_e e) \\ &= \mathbf{v}_i \wedge \mathbf{B} - (1/\mu_0 n_e e) (\nabla \wedge \mathbf{B}) \wedge \mathbf{B} \end{aligned}$$

by using $\mathbf{J} = e(Zn_i\mathbf{v}_i - n_e\mathbf{v}_e) = en_e(\mathbf{v}_i - \mathbf{v}_e)$ and $\mathbf{J} = \mu_0^{-1}\nabla \wedge \mathbf{B}$ (the displacement current is neglected since we are dealing with low frequency phenomena). Substitution into Faraday's law gives the induction equation for magnetic fields

$$\begin{aligned} \frac{\partial \mathbf{B}}{\partial t} = \nabla \wedge (\mathbf{v}_i \wedge \mathbf{B}) &+ \frac{1}{e} \left[m_e \nabla \wedge \frac{D\mathbf{v}_e}{Dt} - \nabla \wedge \left(\frac{(\nabla \wedge \mathbf{B}) \wedge \mathbf{B}}{\mu_0 n_e} \right) + \nabla \wedge \left(\frac{\nabla \mathcal{P}}{n_e} \right) \right. \\ &\left. - \nabla \wedge \left(\frac{\mathbf{R}_T + \mathbf{R}_u}{n_e} \right) + \nabla \wedge \left(\frac{\nabla P_e}{n_e} \right) \right]. \end{aligned} \quad (3.10)$$

The first term represents convection of magnetic fields by fluid motion, or conversion of flow energy into magnetic energy. The second term is an inertial term and can sometimes suppress growth of magnetic fields. It can be added to the left-hand side to give conservation of generalised vorticity $\partial/\partial t(\mathbf{B} - m_e \nabla \wedge \mathbf{v}_e/e)$. The third term is caused by the $\mathbf{J} \wedge \mathbf{B}$ effect. It is called the Hall term and produces fields normal to those already present. Diffusion of magnetic field is expressed in $\mathbf{R}_u$ and results in field energy being converted to thermal energy. The last term $\nabla P_e/n_e e$ is usually dominant. Including only this term represents the case where the electron pressure gradient is balanced by an axial electric field. One can then simplify the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} \approx \nabla \wedge \left(\frac{\nabla P_e}{n_e e} \right) = \frac{k \nabla T_e \wedge \nabla n_e}{n_e e} \quad (3.11)$$

which says magnetic fields are generated when temperature and density gradients are not parallel. This form of the induction equation becomes increasingly relevant as the focal spot is made tighter. In fact, any nonuniformity in the laser or in the target surface will make fields through this process.

It has been shown that the thermoelectric term $\mathbf{R}_T$ contributes significantly [55, 56] and this will be discussed later along with the Hall term. For now we assume a simplified electric field of the form

$$\mathbf{E} = -\frac{1}{ne} \nabla \cdot \underline{\mathbf{P}}_e - \underline{\boldsymbol{\beta}} \cdot \left(\frac{k \nabla T_e}{e} \right) + \underline{\boldsymbol{\eta}} \cdot \mathbf{J}$$

where $\underline{\mathbf{P}}_e$ contains all electron pressure components, $\underline{\boldsymbol{\beta}}$ is the thermoelectric tensor with $\mathbf{R}_T = -n_e \underline{\boldsymbol{\beta}} \cdot \nabla (kT)$ and $\underline{\boldsymbol{\eta}}$ is the electric resistivity with $\mathbf{R}_u = n_e e \underline{\boldsymbol{\eta}} \cdot \mathbf{J}$. In fact it is possible to incorporate thermal and pressure terms into a generalised tensor $\underline{\boldsymbol{\beta}}'$ and

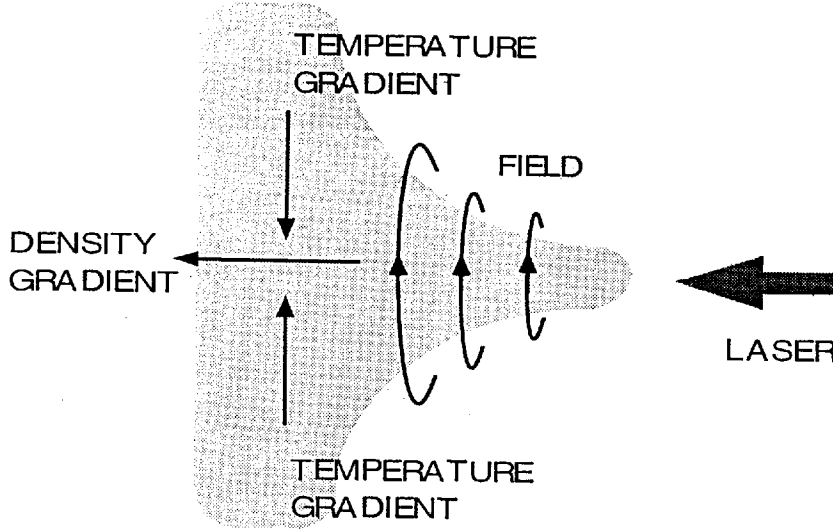


Figure 3.8: Generation of azimuthal magnetic field by the pressure gradient term

the induction equation can then be expressed as

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \wedge (\mathbf{v} \wedge \mathbf{B}) + \nabla \wedge [\underline{\eta} \cdot (\nabla \wedge \mathbf{B})] = \nabla \wedge \mathbf{S} \quad (3.12)$$

$$\mathbf{S} = \frac{1}{e} \underline{\beta}' \cdot \nabla (kT_e) \quad (3.13)$$

This form is useful for computational purposes. The nonconservative form of the electron thermal energy equation now contains contributions from current convection and magnetic source

$$\begin{aligned} \text{current convection} &= \mathbf{J} \cdot \nabla \left[\frac{3}{2} \frac{kT_e}{e} \right] \\ \text{magnetic source} &= \frac{kT_e}{e} \nabla \cdot (\mathbf{J} \cdot \underline{\beta}'). \end{aligned}$$

There are other generation mechanisms [27, 53, 56–59]. In the dense, imploding material composition discontinuities and shocks can create fields [60]. At the ablation surface the RT instability can produce a thermoelectric current which generates fields [61–63]. The fields inhibit thermal flux and so the ablation pressure is lower than usual (see for example [57]). The thermal instability [64] also operates in that region. Around the critical surface hot electron filamentation [65], thermomagnetic instabil-

ity [18–21], collisionless [66–68] and collisional Weibel instabilities [69, 70] also produce fields. Work performed by Haines has shown that a kinetic model provides additional source terms [71]. For instance turbulence can generate fields though, it has a greater effect on the transport. At high irradiances one must consider the direct effect of laser radiation. The electrons involved in resonance absorption can produce a substantial magnetic field providing the laser has the correct polarisation (see Reference [57] for an overview).

3.2 Magnetised Transport

For a random process with mean free path λ and average speed v one can define a diffusion coefficient D through

$$D \doteq \frac{\lambda v}{3}. \quad (3.14)$$

An electron in a magnetic field will gyrate around the field line with Larmor radius $r_g = (v_t)_\perp m_e / eB$, where v_t is the thermal velocity of the electron. After a collision this electron orbits a new field line, its gyrocentre having moved by one Larmor orbit. This is effectively a random walk with stepping length r_g and stepping time equal to the electron collision time τ . With these parameters one can calculate a diffusion coefficient $D_\perp$

$$D_\perp \approx \frac{r_g^2}{3\tau}.$$

Along magnetic fields or in their absence the diffusion coefficient is given by

$$D_\parallel \approx \frac{v_t^2 \tau}{3}.$$

The ratio of these two diffusion coefficients is indicative of the reduction of energy transport across magnetic fields

$$\frac{D_\perp}{D_\parallel} \approx \frac{1}{(\omega_g \tau)^2}$$

where ω_g is the thermal Larmor frequency. Defining the Hall parameter as $\chi \doteq \omega_g \tau$ we say transport is inhibited when this parameter is of order one or larger.

The reduction factor $(\omega_g \tau)^2$ applies only in a uniform magnetic field. If the field is highly disordered, then the electrons will be scattered on the time scale of one thermal Larmor orbit. In this case the step length is r_g and the stepping time is ω_g^{-1} . This is called Bohm-like transport and the diffusion coefficient is

$$D_{Bohm} = \frac{r_g \omega_g}{16} = \frac{kT}{16eB} \sim \frac{3}{16} \frac{D_{\parallel}}{\omega_g \tau}.$$

Transport across magnetic fields is enhanced in this regime in comparison to the case of a uniform field. To see this, take a magnetic field $\mathbf{B} = (B(y), 0, 0)$ in Cartesian geometry. The electrons drift in the $\hat{z}$ -direction with velocity $(r_g/L)v_t$ where L is the scale length of field variations. This drift can be very large, if r_g is not much smaller than L . The cross-field drift in the $\hat{z}$ -direction would not occur in a uniform field $\mathbf{B} = (B, 0, 0)$. Thus, if the irregularities in the magnetic field act on a scale length of a Larmor radius, then the cross-field diffusion is reduced by a factor $(\omega_g \tau)^{-1}$. If the scale length of variations in the field is much larger than r_g , then the field appears uniform to the electrons and the reduction is stronger, going like $(\omega_g \tau)^{-2}$. We call this regime Braginskii-like.

Magnetic field generation predominantly follows lateral variations in laser absorption; the magnetic field scale length is characteristic of the laser nonuniformity scale length. As discussed earlier, it is the high energy electrons which carry the heat flow and one must be aware of the Larmor radius of these electrons compared to the variations in the laser.

The transport theory developed by Braginskii [54] was improved upon by Epperlein & Haines [43]. In particular they find large errors in thermal conductivity for Hall parameters in the range $0.3 \ll \chi \ll 30$.

3.3 Bifurcation of Solution in a Magnetised Plasma

When the Hall parameter χ approaches unity energy transport across magnetic fields is reduced. The temperature within the focal spot becomes larger and so the temperature gradients become steeper. From the induction equation (Equation 3.11) this leads to generation of more magnetic field and so the Hall parameter grows. This is an instability and positive feedback can occur to produce what is known as a hot spot. These structures were first reported by Craxton & Haines [27]. Bell [28] has done a

one-dimensional perturbation analysis and has estimated the growth time to be

$$\text{growth time} = (1 + \chi^2)^{1/2} \left(\frac{T}{\text{keV}} \right)^{-3/2} \left(\frac{n_e}{4 \times 10^{21}} \right)^{1/2} 0.9 \text{ ps.}$$

Let L_z represent the axial scale length and L_x represent the lateral scale length. It is often the case that hot spots occur when $L_z \gg L_x$, i.e. for tight focus irradiation. Yabe [29] carried out calculations starting with this assumption. He discovered that there exists a bifurcation in the solutions. A low magnetic field state exists where heat flow is uninhibited and energy flows out of the focal spot. This reduces temperature gradients and these gradients are never large enough to produce strong magnetic fields. The second solution corresponds to the unstable hot spot mentioned above. Strong magnetic fields inhibit electron diffusion and the state perpetuates. The parameter which characterised the plasma behaviour was the laser intensity. At high intensities hot spots were the norm. The weak field solution tended to become the strong field solution if the initial conditions deviated much from the steady state.

In Craxton & Haines' paper [27] the hot spot overlapped with the laser beam. Yabe reports that the hot spot can be larger than the beam spot because of Nernst convection (see Section 3.6). This indicates that bifurcation can still occur even when $L_z \sim L_x$. It should be pointed out that incorporating the thermal force term leads to smoother magnetic structures [56].

3.4 Laser Produced Jets

3.4.1 High Irradiances

Experiments have been conducted [25] where solid targets were irradiated with short (2.1-7.5 ps), high powered ($1.0\text{-}12.0 \times 10^{17} \text{ W cm}^{-2}$) laser pulses in tight focus ($\approx 50 \mu\text{m}$ diameter). Here the duration of the laser is small compared to the hydrodynamic time scale of the plasma, heating is at almost solid density and the system is far from steady state. All of these conditions make these experiments very different to those of ICF-type.

A single jet was often seen to emerge from the laser spot and ion emission was confined to a half-angle 10° cone. In addition, the ions had a characteristic temperature of 100 keV. These results suggest that the flow was being collimated by some mechanism

and that heat transport out of the laser spot was being severely restricted. Collimated flow was also observed by Burgess [72].

Calculations by Bell [25, 26] suggest magnetic fields are responsible for the plasma behaviour. These fields are rapidly generated by the $\nabla T \wedge \nabla n$ term.

Neglecting resistive and convective losses we can estimate the field to be

$$B \sim \left(\frac{t_L}{2 \text{ ps}} \right) \left(\frac{kT}{100 \text{ keV}} \right) \left(\frac{L_T}{25 \text{ } \mu\text{m}} \right)^{-1} \left(\frac{L_n}{\mu\text{m}} \right)^{-1} 80 \text{ MG.} \quad (3.15)$$

Here t_L is the pulse length. The plasma beta is then

$$\beta = \frac{P}{B^2/2\mu_0} \sim \left(\frac{t_L}{2 \text{ ps}} \right)^{-2} \left(\frac{kT}{100 \text{ keV}} \right)^{-1} \left(\frac{L_T}{25 \text{ } \mu\text{m}} \right)^2 \left(\frac{L_n}{\mu\text{m}} \right)^2 \left(\frac{n_e}{n_{crit}} \right) 0.6$$

indicating that both magnetic and thermal pressure are of comparable magnitude. This result suggests the field is capable of pinching the ablating plasma onto the axis to produce collimated flow. It follows that the electron gyroradius is

$$r_g = \left(\frac{kT}{100 \text{ keV}} \right)^{1/2} \left(\frac{B}{80 \text{ MG}} \right)^{-1} 0.1 \text{ } \mu\text{m}.$$

The plasma ablates around $5 \text{ } \mu\text{m}$ during the laser pulse which is larger than r_g so magnetic fields can inhibit the lateral flow of energy, from the discussion of Section 3.2. The Hall parameter is

$$\omega\tau_{ei} \sim \left(\frac{t_L}{2\text{ps}} \right) \left(\frac{kT}{100 \text{ keV}} \right)^{5/2} \left(\frac{Z}{4} \right)^{-1} \left(\frac{L_T}{25 \text{ } \mu\text{m}} \right)^{-1} \left(\frac{L_n}{\mu\text{m}} \right)^{-1} \left(\frac{n_e}{n_{crit}} \right)^{-1} 8 \times 10^5.$$

Energy is going to be severely bottled up in the laser spot if this estimate is correct. Bearing in mind that the full Braginskii reduction in conductivity $(\omega\tau_e)^2$ may not be applicable and the field may be disordered this estimate for the Hall parameter still seems large enough to indicate low lateral heat flow. It is thought that only a thin surface layer can be heated so that the particles ejected from that region have characteristic energies reflecting the unusually high temperature of that region [25]. This could be the reason for the 100 keV ions and 1 MeV electrons that were detected in the experiments.

The **MH2D** code [25] was employed to calculate the role magnetic fields have in these experiments. Figure 3.9 (taken from Reference [25]) shows state variables 30

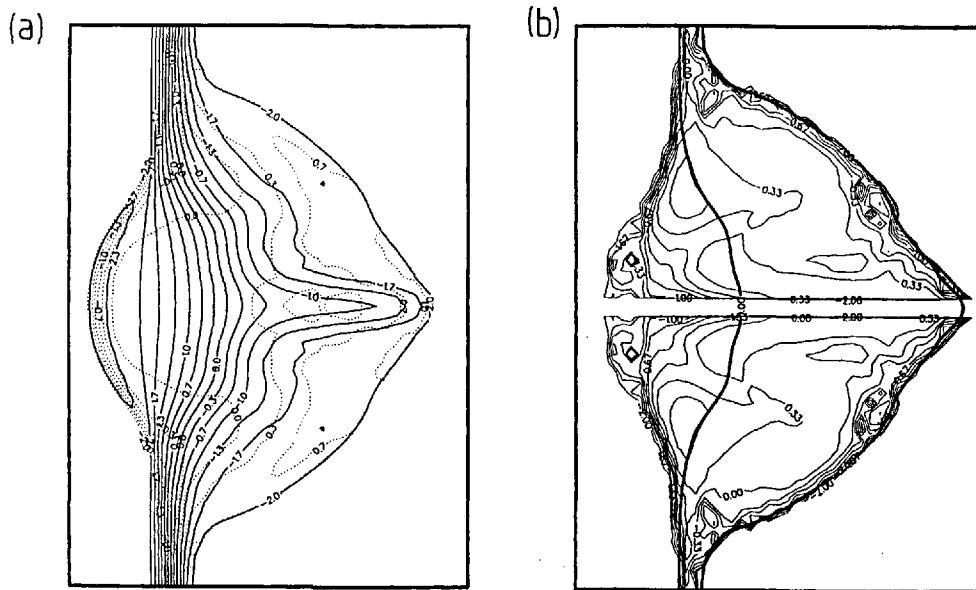


Figure 3.9: Logarithmic contour plots of (a) density (in units of 10^{21} cm^{-3}) and temperature (keV) and (b) magnetic field (in MG) 28 ps after the peak of the laser pulse. The thick contour lines in (b) give the positions of the critical surface and the plasma-vacuum interface.

ps after the peak of the laser pulse. A jet of approximately $100 \mu\text{m}$ in length was seen with a temperature of 10 keV and 30 MG fields. No collimation was observed if magnetic fields were omitted. The results were sensitive to the type of magnetic field inhibition. $(\omega\tau_e)^{-3/2}$ was used and the Hall parameter was limited to a maximum value of 10^4 . Full Braginskii $(\omega\tau_e)^{-2}$ reduction led to unrealistically high temperatures and weaker collimation, whereas Bohm diffusion $(\omega\tau_e)^{-1}$ caused no collimation at all and had temperatures two orders of magnitude less than that observed experimentally.

MH2D being a fluid code describes only the bulk properties. The electron collision times are very long in this regime so a particle code should be used. Wilks [73] using a PIC code observes 250 MG fields although his irradiance is an order of magnitude larger. Forslund & Brackbill [74] have also carried out particle simulations and find magnetic fields are an essential component in these high $I\lambda^2$ experiments.

3.4.2 Low Irradiances

It has been established that laser produced plasmas tend to break up into jets [38, 75] under ICF related conditions. Apart from the radiation cooling instability [22], the proposed instabilities responsible for this behaviour involve magnetic fields: electrother-

mal [64], thermomagnetic [18–21] and Weibel [70] instabilities. However, the growth rates of all of these mechanisms are not large enough or sufficiently free from nonlinear saturation to produce jets.

Although the laser intensity is lower and the laser pulse is longer in the parameter regime we are dealing with here it may be the case that the same mechanism which produced jets in the previous subsection could produce jets under these less severe conditions. It could be the case that the nonuniformities in the beam during the start-up phase of ICF drive many smaller hot spots. Each of these is similar to the tightly focused, high intensity beam and so jets could result. Admittedly, the experimental conditions are obviously dissimilar though parameterisation of both regimes might reveal they are of the same kind.

Bell [26] has done some calculations in this respect. **MH2D** was employed using a laser wavelength of $0.5\ \mu\text{m}$, with peak intensity in hot spot $3 \times 10^{15}\ \text{W cm}^{-2}$, peak intensity outside hot spot $10^{14}\ \text{W cm}^{-2}$, diameter of hot spot $5\ \mu\text{m}$ and an axial density scale length $1.25\ \mu\text{m}$. After 60 ps a jet of around $50\ \mu\text{m}$ length had formed with peak density within the jet 0.4 of critical. The field around the jet was around 3 MG falling to 1 MG close to the laser spot. The plasma beta was $\beta = 0.4$ indicating the hydrodynamic importance of magnetic field in producing the jet. The jet was not formed if the field was omitted from the calculations. In addition, jets were only formed if the spatial contrast in the beam was greater than 3:1 and one should also note that at $t = 60\ \text{ps}$, χ was larger than unity throughout the hot spot. Jets did not form if χ was less than this. This suggests that even in this regime the Yabe-type instability mentioned in Section 3.3 is increasing temperature and magnetic field to produce jets. If it could be possible, one should determine if the jets that occur in the ICF experiments correlate spatially to the hot spots in the laser beam at early times. Naturally one would then enquire about the stability properties of these jets, if that were the case.

3.5 Cross Field Effects and Field Generating Instabilities

Braginskii's transport coefficients $\underline{\kappa}$, $\underline{\beta}$ and $\underline{\eta}$ are tensors which contain skew symmetric components relative to some background magnetic field. Through the action of the Lorentz force, these off diagonal components can produce magnetic fields. Considering

just the collisional electric field $\mathbf{E}^*$ we write

$$\begin{aligned} en\mathbf{E}_\wedge^* &= \mathbf{J} \wedge \mathbf{B} - n\beta_\wedge \mathbf{b} \wedge \nabla(kT). \\ \text{Hall} & \qquad \qquad \text{Nernst} \end{aligned}$$

where we have ignored the resistivity. $\mathbf{b}$ is the unit vector in the direction of the field. Both contributions to the electric field have been accounted for in Equation 3.10. The Nernst effect comes about from the action of the thermal force $\mathbf{R}_T = -n\underline{\beta} \cdot \nabla(kT)$ in Equation 3.10. It can lead to instability because it produces fields in the same direction as the original field. In this context it is viewed as a source of magnetic field but later we will see that it has a convective form.

The heat flow in a magnetised plasma is that given by thermal conduction and a contribution from the thermoelectric effect

$$\mathbf{Q} = -\underline{\kappa} \cdot \nabla T - \frac{kT}{e} \underline{\beta} \cdot \mathbf{J}$$

The cross field flow is given by

$$\begin{aligned} \mathbf{Q}_\wedge &= -\beta_\wedge kT \mathbf{b} \wedge \mathbf{J}/e - \kappa_\wedge \mathbf{b} \wedge \nabla(kT). \\ \text{Ettingshausen} & \qquad \qquad \text{Righi-Leduc} \end{aligned}$$

The Ettingshausen effect produces flow of heat perpendicular to the field and along the temperature gradient [57]. It redirects convected heat flux. The Righi-Leduc effect produces heat flux perpendicular to both the magnetic field and the temperature gradient. As a consequence this effect redirects conducted heat flow [57]. It plays an important role in the thermomagnetic instability [18–21] where a small perturbation $\nabla_\perp$ normal to ∇n is applied to parallel density and temperature gradients. This creates a thermally generated electric field which in turn leads to a cross field heat flux along $(\nabla T \cdot \nabla n) \nabla_\perp T$. Thus the perturbation increases when $\nabla T \cdot \nabla n > 0$.

Epperlein [70] carried out a detailed perturbation analysis of the distribution function f where he assumed $f = f_0 + \mathbf{v} \cdot \mathbf{f}_1/v + \mathbf{v}\mathbf{v} \cdot \mathbf{f}_2/v^2 + \mathbf{v}\mathbf{v}\mathbf{v} \cdot \mathbf{f}_3/v^3 + \dots$. Calculations up to f_1 gave growth only if $\nabla T \cdot \nabla n < 0$. Agreement with fluid theory was restored when the expansion included f_2 .

The Ohmically driven thermal instability operates in the region around the ablation surface [64]. As mentioned earlier heat flow is thought to be carried by hot electrons with a cold electron return current. When the temperature increases the cold electron current increases resulting in Joule heating in that region. The local temperature increases further. Defining $\phi \doteq \lambda_{csd}/\lambda_{mfp}$ where λ_{csd} is the collisionless skin depth a necessary condition for thermal instability is that $\phi \gg 1$. This can only occur at the ablation surface.

Of particular importance are the small scale jet-like structures of Subsection 3.4.2 first observed by Willi, Rumsby & Duncan [17]. These jets were more pronounced with short wavelength lasers and high Z targets. They are thought to originate from densities above critical (for $1\ \mu\text{m}$ laser irradiation). They are not caused by a self focusing effect. The thermal instability could be a candidate for these jets because it has fast growth rate for short wavelength modes.

Perhaps a better candidate is the collisional Weibel instability (CWI) which was investigated by Epperlein [70]. He finds the CWI has fastest growth for high Z targets. Also, all of the electromagnetic instabilities are limited because convection of fields (by both advection and heat flux) transports fields away faster than they can be generated. In particular nonlocal effects cause complete damping of the CWI: there exists a critical value of ϕ below which the CWI is always damped. The thermomagnetic, CWI and Nernst instabilities require $\phi \ll 1$ and this in general is true in the low density, hot coronal matter. However, Epperlein finds the CWI instability can stem from the overdense regions (with $\phi \ll 1$ satisfied), explaining the small scale jets.

In general the Hall parameter χ can be very large in the critical region and it can reduce the lateral heat flow significantly. In the ablation region it is at most of order unity though cross field effects can enhance its value.

3.6 The Nernst Effect

If one neglects the thermal force (the fifth term in Equation 3.10), then the magnetic field is convected towards the vacuum and is of little significance. However, the thermal force term has a simple convective form [76] given by $-\nabla \cdot (\mathbf{v}_N B)$ where

$$\mathbf{v}_N = -(\tau_e/m_e) \left[(\beta_1'' \chi^2 + \beta_0'')/\Delta \right] \nabla (kT_e).$$

$\mathbf{v}_N$ is called the Nernst velocity. The coefficients β and Δ are given by Braginskii [54]. A term which effectively pinches the magnetic fields has been omitted [65].

3.6.1 Heat Flux Effects

Let $\mathbf{Q}_e$ be the electron heat flux measured in the centre of mass frame of the electrons. One can define a propagation velocity of thermal energy $\mathbf{v}_W = (\gamma - 1)\mathbf{Q}_e/n_e k T_e$. It can be shown that the Nernst convection velocity is similar to this thermal velocity [76]: $|\mathbf{v}_N|/|\mathbf{v}_W| \sim 1$. It follows, a good approximation for the Nernst velocity is

$$\mathbf{v}_N \approx \frac{\mathbf{Q}_e^\perp}{\frac{5}{2}n_e k T_e}. \quad (3.16)$$

To see this, let us say that the collision frequency depends upon the velocity through the relationship $\tau_{ei} \propto v^n$. Then for $n = 0$ there is no Nernst term, yet for $n = 2$ we have [77]

$$\frac{\mathbf{J}}{\sigma} = \mathbf{E} + \mathbf{v} \wedge \mathbf{B} + \frac{\nabla Q_e}{n_e e} + \frac{\nabla(kT_e)}{e} + \frac{2}{5} \frac{\mathbf{Q}_e \wedge \mathbf{B}}{n_e k T_e}. \quad (3.17)$$

Consider the right-hand side. The $2\mathbf{Q}_e/5n_e k T_e$ factor which is crossed with the magnetic field in the last term can be thought of as an additional velocity to be added to the fluid velocity. Simple arguments have magnetic field being convected with the electron centre of mass velocity $\mathbf{v}_e$, but when we include Nernst convection we revise this to $\mathbf{v}_e + \frac{2}{5}\mathbf{Q}_e/n_e k T_e$. It is the hotter electrons which carry the heat flow and the magnetic field is frozen to them. The colder electrons are more collisional and the field simply diffuses through them. A more realistic collision frequency has $n = 3$ which will lead to even faster convection of magnetic field. Presenting the generalised Ohm's Law

$$\eta \mathbf{J} + \frac{m_e}{n_e e^2} \frac{\partial \mathbf{J}}{\partial t} = \mathbf{E} + \mathbf{v} \wedge \mathbf{B} - \frac{\mathbf{J} \wedge \mathbf{B}}{n e} + \frac{\nabla P_e}{n_e e} + \frac{\mathbf{Q}_e \wedge \mathbf{B}}{\frac{5}{2} P_e} + \text{ponderomotive and radiation pressure terms.}$$

The second term on the left hand-side describes the effect of electron inertia and is important only at very high frequencies, so it can be ignored.

The Nernst effect is of particular relevance in laser fusion experiments. The outward enthalpy flow $\frac{5}{2}n_e k T_e \mathbf{v}$ (where $\mathbf{v}$ is the centre of mass ablation velocity) must be smaller than the heat flow from critical to ablation surfaces: $\mathbf{v}_W$ and hence $\mathbf{v}_N$ will be much

larger than the fluid velocity. Thus, transverse magnetic field is convected from the hot regions to the colder regions.

3.6.2 Amplification of Fields in Overdense Regions

Just below the critical density the Nernst velocity is directed toward the underdense region and slightly above critical density it points towards the ablation surface [76]. This means any magnetic field in the overdense region is convected towards the ablation surface. Amplification occurs in this region because of the compression associated with accumulation. By this mechanism the fields at the ablation surface can be an order or two larger than that at the critical surface. Even if the magnetic fields are generated in a subcritical region they can leak into the overdense region through diffusion. This was observed by Forslund & Brackbill in their collisionless PIC code [74]. They had self generated fields being carried by hot electrons along the surface of the target, in a transverse direction, in a layer of thickness a few electron collisionless skin depths.

A diffusion term such as $\nabla(D\nabla B)$ can be written in the form

$$-v_\alpha \nabla B + D \nabla^2 B \quad (3.18)$$

where $v_\alpha = -\nabla D$. This suggests an additional convection associated with diffusion. The total convection velocity in our present model is now $\mathbf{v} + \mathbf{v}_N + \mathbf{v}_\alpha$. Concentrating on the overdense region between ablation and critical, the fluid velocity $\mathbf{v}$ is directed towards the underdense side and the Nernst velocity towards the ablation surface. Thus magnetic field is convected from both surfaces towards the point where $\mathbf{v} + \mathbf{v}_N + \mathbf{v}_\alpha = 0$ [78]. The magnitude of the field here is determined by the balance between diffusion and convection. When the Hall parameter is small this point coincides with the point where the mean free path λ_{mfp} approaches the collisionless skin depth λ_{csd} [78]. Since the fluid velocity v is smaller than the others, in this region, $|\mathbf{v}_N|/|\mathbf{v}_\alpha| \approx (\lambda_{mfp}/\lambda_{csd})^2$. Thus, convective amplification is saturated by diffusion when $\lambda_{mfp} = \lambda_{csd}$ [78].

CHAPTER 4

BEAM SMOOTHING

In the introductory chapter we defined the need for smooth laser drive to control the hydrodynamic instabilities of ICF. It is well known that it is possible to achieve uniform illumination of a spherical surface by overlapping around twenty perfectly balanced, smooth beams [79–81]. Thus, it becomes a matter of whether we can obtain smooth and reproducible single beams. The very best of high power laser systems has peak-to-valley nonuniformities in the beam of the order of 10%. Each stage of a high power laser system introduces nonlinearities such as small phase aberrations and these add up. Also, there are imperfections in the amplifying and delivery optics [82].

After overviewing a couple of beam smoothing techniques that others have devised we present the algorithm that we have developed for generating multivariate random numbers with fixed correlation in each dimension. The algorithm is also tested to see if it can reproduce the intensity pattern of induced spatial incoherence schemes (ISI).

4.1 Random Phase Plates

A simple way to smooth a laser beam involves breaking the beam up into higher k mode structures which would then be ironed out by thermal conduction - a technique known as spatial smoothing. The earliest method used random phase plates (RPP) [32, 33].

A RPP is a two-dimensional array of transmitting elements, each of which applies a phase shift, usually 0 or π , to incoming laser light. The distribution of elements in the array is random. Each element maps the light onto the target. The superimposed beam at target contains a speckle pattern due to the interference between light from each element. Also, the far field intensity envelope is characteristic of a single element.

Although the far field pattern contains very small scale nonuniformities there is a very long scale length structure and this cannot be controlled by thermal smoothing. It is also clear that apart from a small amount of aberration the interference pattern is time independent. However, at later times (after a corona has formed) oblique laser incidence at target may allow refraction of the beamlets as the target atmosphere expands and this would lead to a time-varying pattern in the absorption region. Another

problem with RPP patterns is that only approximately 82 % of the incident energy on the plate is contained in the central lobe of the far field pattern. The rest of the energy is scattered and can damage nearby equipment as well as adding to energy costs.

One may also use lenslet arrays to spatially smooth a beam [83, 84].

4.2 Induced Spatial Incoherence

The induced spatial incoherence scheme [34–36] uses a broadband laser. This means the coherence time satisfies $\tau_c \ll t_L$, where t_L is the laser pulse width. Informally, one can define the coherence time τ_c of a signal ψ through the condition:

$$\psi(t + \tau) \begin{cases} \text{is correlated to } \psi(t) & \text{if } |\tau| < \tau_c \\ \text{is not correlated to } \psi(t) & \text{if } |\tau| > \tau_c. \end{cases}$$

At the heart of an ISI system is an echelon of elements each introducing a delay in the beam greater than the coherence time. Thus, the beam is disassembled into a large number N of independent beamlets. The beamlets are mutually incoherent so the laser beam has transverse spatial incoherence.

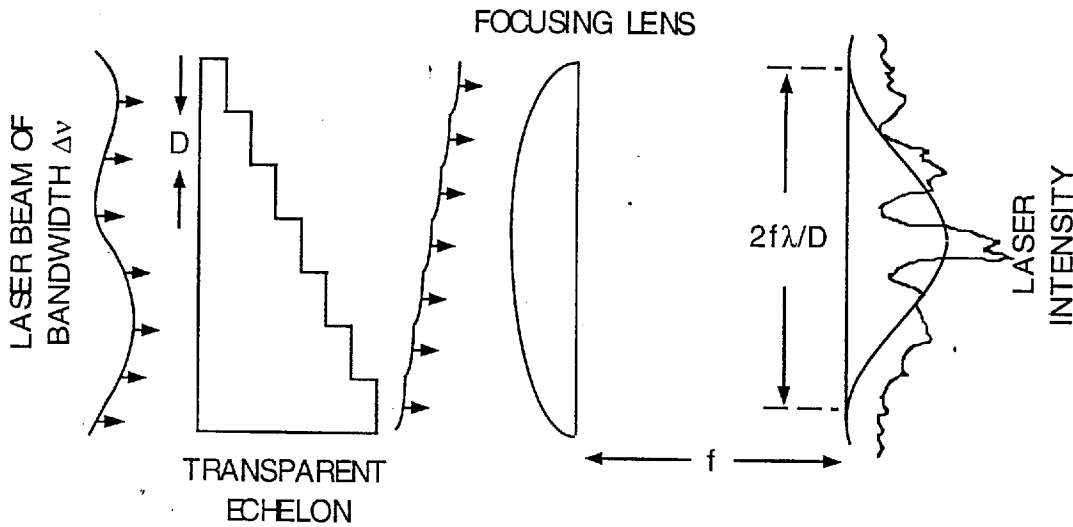


Figure 4.10: Use of spatial incoherence induced by a transparent echelon to smooth the focal spot irradiance of a broadband laser.

The light from each element is mapped on to the target surface. During a single coherence time the relative phase between each beamlet is roughly constant so at focus

there is a complicated interference pattern. The envelope of the pattern is characterised by the diffraction of an individual beamlet.

Figure 4.11 shows four instantaneous intensity profiles typical of an ISI pattern near focus. There are large amplitude variations. For a focal length f the diffraction envelope has width $d_{00} = 2f\lambda_{las}/D$, where D is the echelon step size and λ_{las} is the laser wavelength. $\Lambda_n = f\lambda_{las}/(nD)$ is the length of the n th transverse mode. The longest mode is $d_{00}/2$ and the shortest mode is $d_{00}/(2N)$. *ISI light is very 'messy' on time scales of the order of a laser coherence time.*

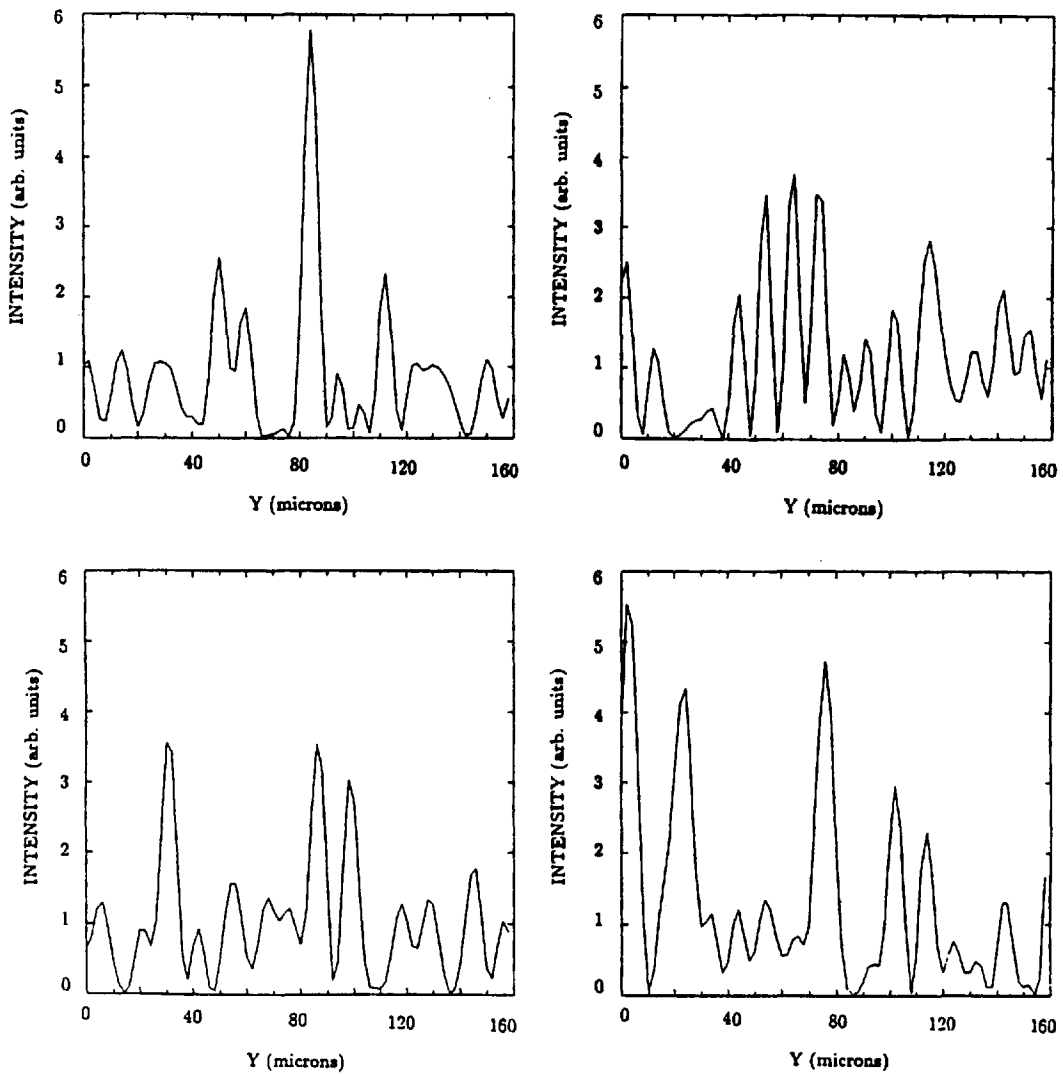


Figure 4.11: Plot of the instantaneous incident ISI laser intensity (arb units) vs the transverse dimension (in microns) at four different times.

Let M be an off axis mode. The relative magnitude of the rms energy fluctuations in all off axis modes up to M is

$$R_M \propto \left(\frac{\tau_c}{t}\right)^{1/2} \frac{d}{N\Lambda_M} \quad (4.19)$$

where d is the focal spot diameter (or pellet diameter). After each coherence time the relative phases change and the pattern at focus regenerates. In other words the pattern at focus fluctuates randomly on the time scale of τ_c . The $\frac{\tau_c}{t}$ term in Relation 4.19 indicates how many different profiles there are within the interval t . For $\tau_c \approx 1$ ps and an averaging time of $t = 400$ ps, R_M is reduced twenty-fold by this time-averaging factor.

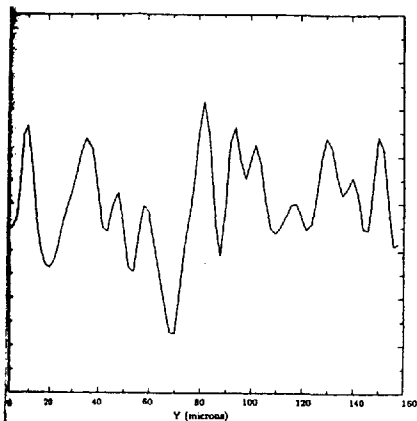


Figure 4.12: Plot of the laser intensity which has been averaged over 100 coherence times. The rms variation is 10 %.

On time scales long in comparison with τ_c the time-averaged profile converges towards that of the smooth diffraction envelope. After 1 ns $\sigma_{rms} = 0.037$ with a maximum amplitude variation of 8.6%. See Figure 4.13.

The crux of the scheme is this: If the hydrodynamic response time t_h satisfies $t_h \gg \tau_c$ the plasma will ignore the rapidly changing interference pattern and will respond only to the smooth time-averaged diffraction profile.

The $\frac{d}{N\Lambda_M}$ term in Relation 4.19 channels energy into more favourable shorter transverse scale length modes which can be smoothed out by thermal conduction. However, there are long scale length modes in the time averaged profile and these may not be removed by thermal smoothing. If we expand the profile in terms of Legendre polynomials P_l , then dominant modes have $l \leq 4$. The important modes for the RT instability have $l \approx 10 - 100$. These modes (at least in the range $l \approx 10 - 30$) have coefficients at least an order of magnitude smaller than those in the dominant modes.

The diameter and spread of the beam profile is affected by the size and tilt of the echelon steps. By manipulating the echelon Schmitt & Gardner [85] have calculated it is possible to achieve less than 2% of illumination nonuniformity at target. This corresponds to ablation pressure nonuniformity $\Delta P / \langle P \rangle < 2\%$. Their analysis of the

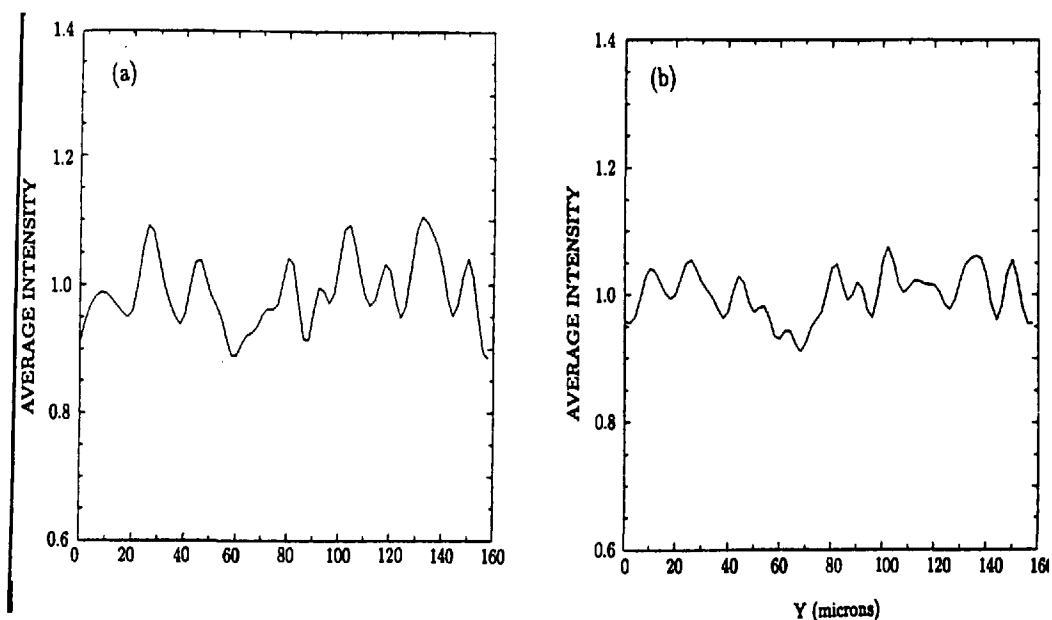


Figure 4.13: (a) Plot of the laser intensity profile averaged over 500 coherence times, $\sigma_{rms} = 0.045$. (b) Laser intensity profile averaged over 1000 coherence times $\sigma_{rms} = 0.037$

pattern at target found it to contain mainly long scale length modes with maximum magnitudes less than 0.5%. A high level of laser energy absorption (75%) was also predicted. These figures are very promising though it should be mentioned that it is very difficult to frequency-triple ISI light because of the broad spectral bandwidth inherent in the laser.

4.3 Generating Correlated Random Numbers

The impetus behind this section is the need to generate ISI-like light. A signal from such light typically has a correlation length on the target surface of around $10 \mu\text{m}$ and correlation time 1 ps. That being the case we require an algorithm to generate a bivariate random variable with fixed correlation in each dimension. In addition, it may be necessary to resolve a hot spot properly so we may need to vary the number of grid points within a correlation length. Likewise, we would like to have control over the number of temporal grid points in one correlation time. The latter two requirements can be realised through spatial-temporal normalisation so the essence of this problem is having correlation across a predetermined number of points. The algorithm is well known in the field of Signal Processing (though the author was unable to find any

source of material).

4.3.1 The Univariate Algorithm

We start by describing the algorithm for generating correlated random numbers in one dimension. A column filled with random numbers $x_1, x_2, x_3, \dots$ from the uniform distribution on the interval $[0,1]$. The column is the first in a matrix $\mathcal{A}$, and generates other elements in that array. The last column of the array $\mathcal{A}$ will contain our random correlated numbers.

Apart from the first column each element of the array $\mathcal{A}_i^j$ is the mean of the previous element in that row $\mathcal{A}_i^{j-1}$ and the number above that $\mathcal{A}_{i-1}^{j-1}$, i.e. $\mathcal{A}_i^j = (\mathcal{A}_i^{j-1} + \mathcal{A}_{i-1}^{j-1})/2.0$.

A general formula for a matrix element is

$$\mathcal{A}_i^j = \frac{1}{2^{j-1}} \sum_{n=0}^{j-1} \binom{j-1}{n} x_{i-j+n+1}. \quad (4.20)$$

x_1	$\dots$	$\dots$	$\dots$	$\dots$
x_2	$(x_1 + x_2)/2$	$\dots$	$\dots$	$\dots$
x_3	$(x_2 + x_3)/2$	$(x_1 + 2x_2 + x_3)/4$	$\dots$	$\dots$
x_4	$(x_3 + x_4)/2$	$(x_2 + 2x_3 + x_4)/4$	$(x_1 + 3x_2 + 3x_3 + x_4)/8$	$\dots$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$

Table 4.1: The algorithm describing the structure of the array $\mathcal{A}$ used to generate correlated random numbers.

Say we choose to have M columns in our matrix. Focusing on the M th column of $\mathcal{A}$, a particular element of this column $\mathcal{A}_i^M$ shall be called X_i from hereon. X_i is a linear combination of only the M numbers $x_i, x_{i-1}, \dots, x_{i-M}$.

As an example we apply this algorithm using 400 columns. To get the algorithm going we need to provide random numbers for the first column *and numbers for the first row*. We choose to fill the first row with random numbers from $\mathcal{U}[0, 1]$. Note these numbers will generate elements which are auxiliary to the algorithm. The auxiliary array elements are filtered out of the algorithm after some rows. They occupy the top right-hand corner of the array structure given in Table 4.1. We call the initial period noise. After some period of noise the numbers X_i should have the desired statistical property.

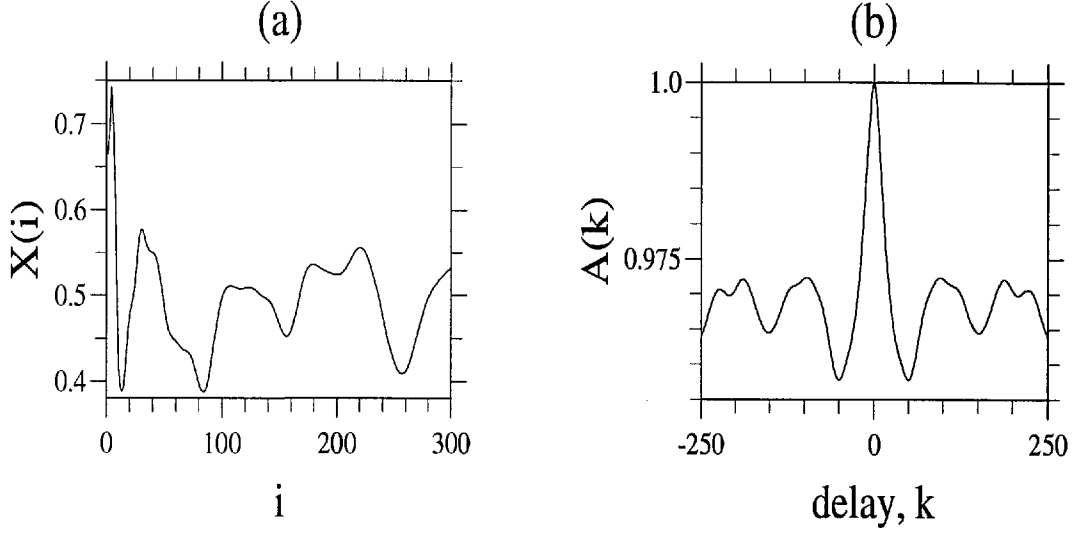


Figure 4.14: (a) Approximately 250 random numbers with correlation length of 20 units (b) Autocorrelation function of 1024 X_i elements

The results of a particular run are presented in Figure 4.14(a). Around 300 random numbers X_i have been generated. The noise can be seen in the first few values. After this noise the X_i values appear to be random with correlation of approximately 20 units. The run was extended to produce 1024 X_i values. These values were auto-correlated using the FFT routines in Numerical Recipes [86].

$$\text{Autocorrelation function} = A(k) = \frac{\sum_{i=0}^{1024} X_i (X_{i-k})}{\sum_{i=0}^{1024} X_i^2}.$$

The HWHM of the autocorrelation function is indeed approximately 20 units as seen in Figure 4.14(b).

From Figure 4.14(a) it appears the numbers X_i are no longer distributed on $\mathcal{U}[0, 1]$, but on a narrower band. This behaviour also appears for other choices of M . Due to the low contrast ratio the autocorrelation function has unusually high values outside of the HWHM.

4.3.2 Determining the Correlation Length

X_i is said to contain information only from the numbers $x_i, x_{i-1}, \dots, x_{i-M}$. The binomial coefficients act as a weighting function on these numbers. A schematic of the relationship between X_i and its x numbers is given in Figure 4.15. As M becomes

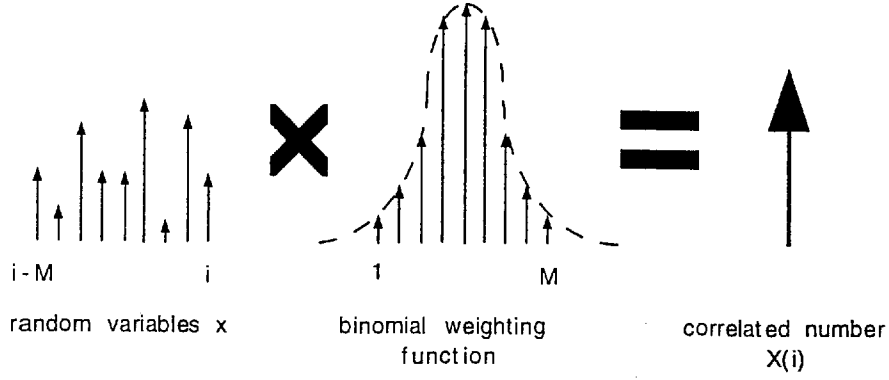


Figure 4.15: Each correlated random number is a linear combination of M specific x numbers. The binomial weighting function is symmetric.

large, the middle binomial coefficients around ${}^M C_r$ (where $r \sim M/2$) dwarf the others. This means that X_i is primarily a linear combination of the random numbers $\{x_r\}$ where $r \sim M/2$. Setting Δr to be the FWHM of the Pascal's weighting function we can say that X_i is roughly composed of the set $\mathcal{B}$ of $\{x\}$ occurring in the central band of the series of numbers $x_i, x_{i-1}, \dots, x_{i-M}$. Now let us pick another number X_k which is 'close' to X_i . By close we mean $|i - k| < \Delta r$. X_i and X_k both share one or more numbers x from the set $\mathcal{B}$. In this light we say Δr is the *correlation length of the numbers X* .

We require the following three formulae

$$\begin{aligned} \sum_{r=0}^M \binom{M}{r} &= 2^M \\ \sum_{r=0}^M r \binom{M}{r} &= M 2^{M-1} \\ \sum_{r=0}^M r^2 \binom{M}{r} &= M(M-1) 2^{M-2} + M 2^{M-1}. \end{aligned}$$

Let $f(i)$ represent our discrete, binomial weighting function on the interval $[0, M]$

$$f(i) = \frac{1}{2^M} \binom{M}{i}.$$

The variance σ^2 of this distribution is given by

$$\sigma^2 = \sum_{i=0}^M (i - \mu)^2 f(i).$$

Here μ is the mean and has the value $\mu = M/2$. We continue

$$\begin{aligned} \sigma^2 &= \frac{1}{2^M} \sum_{i=0}^M \left[i^2 - iM + \frac{M^2}{4} \right] \binom{M}{i} \\ &= \frac{M}{4} \end{aligned}$$

using the above formulae. The FWHM is roughly twice σ so we deduce that

$$\Delta r = \sqrt{M-1}$$

remembering from Equation 4.20 that the final column element is made up of a sum from 0 to $M-1$.

Conclusion *The square root of the number of columns in the array gives the correlation length between the elements in the final column.*

4.3.3 The Bivariate Algorithm

STAMP requires the illumination cross-section to be symmetric at the boundaries. The algorithm can be adapted to produce a set of correlated random numbers with zero gradient at the boundaries of that set. We simply make the sequence of random numbers x in the first column symmetric and repeat this sequence. This technique can be adapted to produce periodic boundary conditions.

We now have a means of producing correlated random numbers in one dimension. Let us assume these numbers represent, to within normalisation, the spatial signal of the laser. If a hot spot has dimension $\sim 10 \mu\text{m}$ and we wish to resolve it by at least 10 grid points we simply choose to have 100 columns in our algorithm and normalise the spatial dimension accordingly.

ISI light also has temporal correlation so we need to generate a bivariate random variable with coherence in the second dimension. Say our grid has N points in the lateral direction and we need data for T time steps. ‘X-generators’ produce correlated numbers in the lateral direction and ‘T-generators’ produce correlated random numbers

in the temporal domain. It is assumed each generator has been pre-run to cut out noise. Our goal is to produce an array $\mathcal{C}$ whose rows represent space and columns time.

Step One: Set up an X-generator to produce N random numbers with a fixed correlation length. This produces the first row of an intermediate matrix $\mathcal{C}^*$. Now discard this generator.

Step Two: Do the same for the other rows of $\mathcal{C}^*$ to produce an array of numbers correlated row-wise.

Step Three: Set up a T-generator and correlate the numbers in the first column of $\mathcal{C}^*$. Discard this generator.

Step Four: Do the same for the other columns of $\mathcal{C}^*$ to produce an array $\mathcal{C}$ of numbers correlated column-wise.

Step Five Normalise the final matrix so the temporal and spatial average is unity.

By autocorrelating row-wise or column-wise it is found that the matrix $\mathcal{C}$ has the required correlation in both columns and rows. Care must be taken not to re-use any of the generators otherwise unwanted cross-correlation can occur. The number of random numbers needed to generate a 100×100 array is immense. The linear congruential methods for producing random numbers proved inadequate as their cycles were invariably too short. Instead a subtractive method [87] was used. The subtractive method is given in Numerical Recipes [86].

4.4 Statistical Properties of ISI-like Light

Our ISI-like light is provided by an algorithm detailed in Section 4.3. It is necessary to check that the signal ⁴ produced from this algorithm has ISI-like properties. Most of the computations in this thesis are based upon the signal stored in one datafile. It contains the data for 100 ps but we present the signal for only the first 10 ps, in Figure 4.16.

Figure 4.16 contains 280 grid points in the lateral direction and 100 grid points in the time domain. This means the data is time resolved to 0.1 ps. For the purposes of the code it is necessary to interpolate the intensity profile when using a variable time step (which is usually much smaller than 0.1 ps). As expected our ISI-like light has structure over around $10 \mu\text{m}$ and a correlation time of about 1 ps.

⁴To within an arbitrary unit the signal represents the laser intensity.

ISI-like light

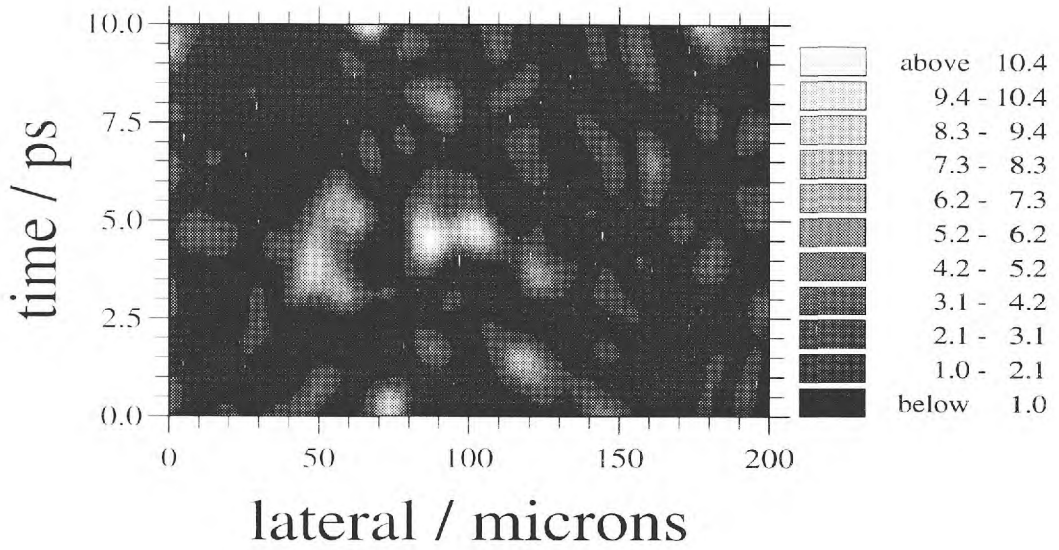


Figure 4.16: ISI-like light generated using our algorithm. Profile given for the first 10 ps.

frequency distribution

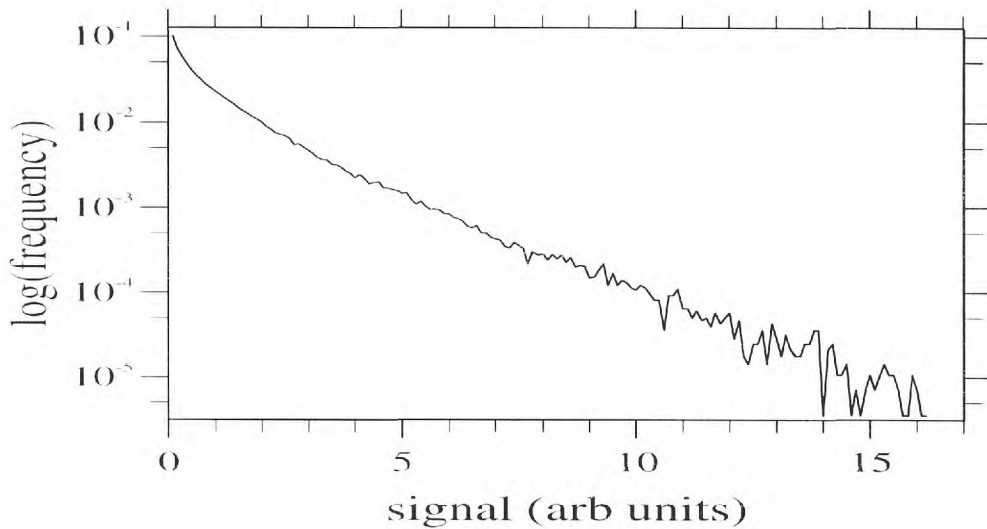


Figure 4.17: Logarithmic plot of normalised frequency $f(I)$ against intensity I for the data.

For the rest of this statistical analysis we will work on the entire signal in the datafile, i.e. a laser with cross section $200\text{ }\mu\text{m}$ and duration 100 ps . The signal has been normalised so the spatial-temporal average over the entire data is unity.

In Figure 4.17 we give a logarithmic plot of relative frequency $f(I)$ against intensity I for the data. It is evident that the distribution from the algorithm goes like $(0.1)\exp(-I/4)$.

In Figure 4.18 we plot the time-integrated average intensity at each point in the lateral direction $\bar{I}_t(x)$. The data are taken at $t = 10, 40, 70$ and 100 ps . The average intensity at each point indeed settles to unity with increasing time. Note the algorithm produces unusually high average intensity at the endpoints so avoid boundary effects we make the grid large. Alternatively, the intensity could be reduced at the boundaries by multiplying the signal by an envelope which is small there. This has not been carried out.

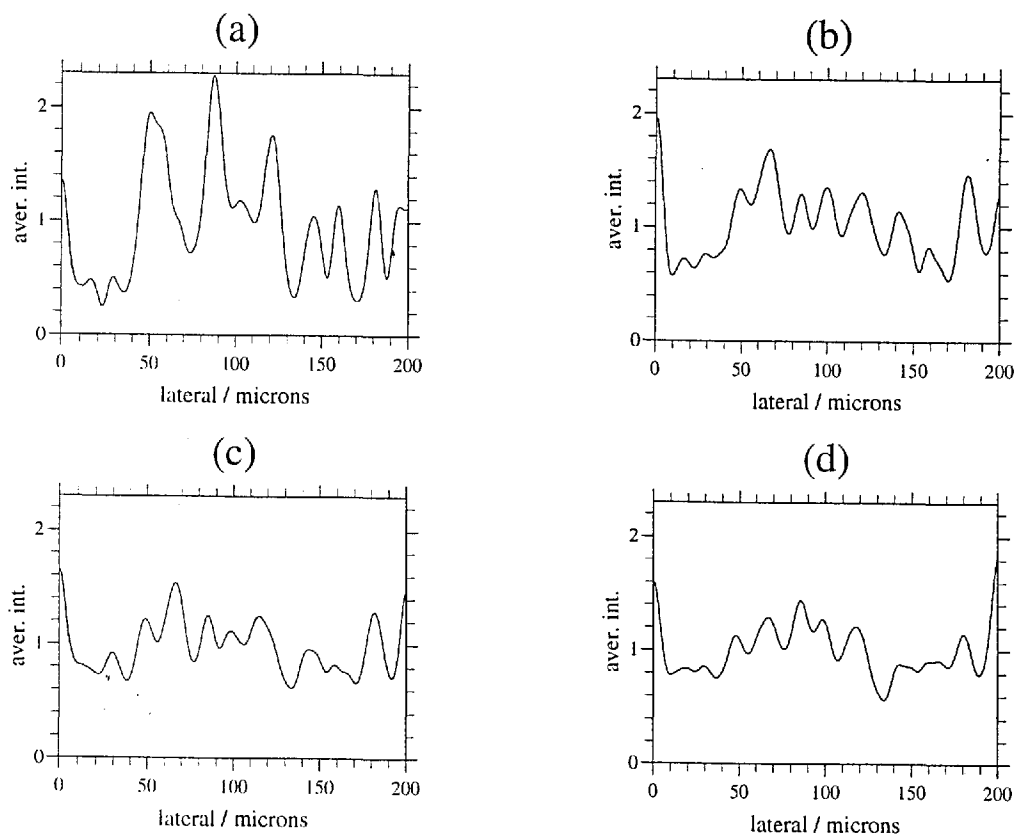


Figure 4.18: Time-integrated average of intensity profile after (a) 10 ps ($\sigma = 0.5077$), (b) 40 ps ($\sigma = 0.2841$), (c) 70 ps ($\sigma = 0.2281$) and (d) 100 ps ($\sigma = 0.2190$).

We now focus on the degree of temporal smoothing. Take the time-integrated average intensity $\bar{I}_t$ at a particular time across the target surface. We define the rms deviation σ through

$$\langle \bar{I} \rangle = \int \bar{I}(x) dx / \int dx$$

$$\sigma = \frac{1}{\langle \bar{I} \rangle} \left\{ \frac{\int dx [\bar{I}(x) - \langle \bar{I} \rangle]^2}{\int dx} \right\}^{1/2}.$$

If we assume σ follows a κt^α dependence, then the index appears as a gradient in the form $\log \sigma = \log \kappa + \alpha \log(t/\text{ps})$. For ISI light the rms falls according to Equation 4.19, i.e. $\alpha = -0.5$. The larger α is the less time-integrated smoothing.

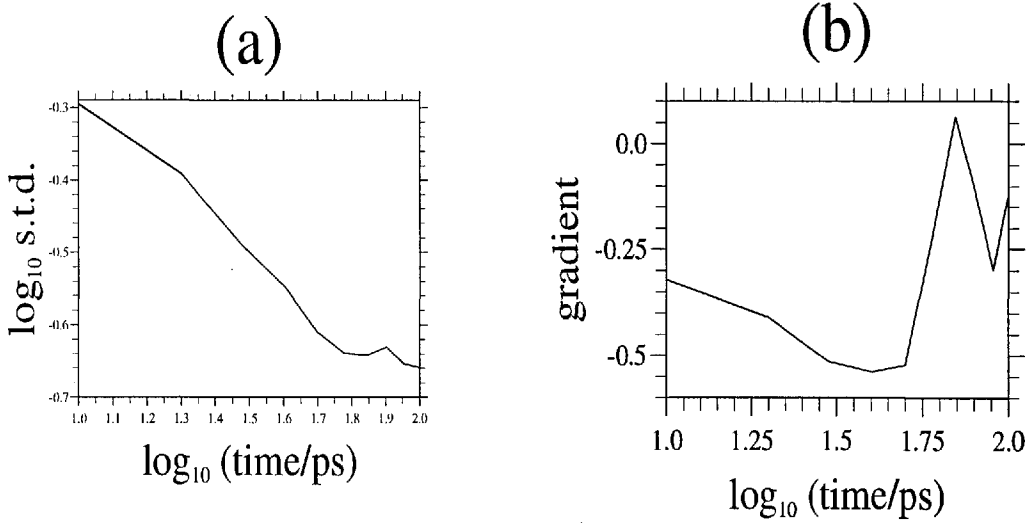


Figure 4.19: (a) $\log \sigma$ against $\log (\text{time/ps})$ and (b) Derivative of (a).

In Figure 4.19(a) we plot $\log \sigma$ against $\log(t/\text{ps})$. The rms deviation is indeed going down with time. However at later times it does not fall fast enough. Figure 4.19(b) details the gradient of $\log \sigma$ with respect to $\log(t/\text{ps})$. For times between 20-70 ps α has the desired value of about -0.5 . At especially early times α is slightly larger than this number. For large intervals (> 70 ps) time-integrated smoothing is less than that for true ISI light. Whilst the data does not smooth as fast as ISI light over very short periods (< 10 ps) or very long periods (> 70 ps) it has the desired rms decay rate for intermediate durations. Our code will be run for only 10-70 ps so for all intents and purposes the data can be thought of as ISI-like.

CHAPTER 5

CONSTRUCTING A CODE

Before deciding on the type of code we need we should consider the plasma we are trying to model. Only in the region of the corona can it be thought of as being collisionless. It is only here that a Vlasov model would apply. We are interested in the nonuniformities between the ablation surface and the critical surface. Here the plasma is rather more collisional and it seems appropriate to employ a fluid model. A better model would be a Fokker-Planck code since this would encompass both the collisionless and collisional aspects of the plasma. Unfortunately, a two-dimensional Fokker-Planck code for magnetised plasmas is beyond the scope of this thesis.

Thus, we choose a fluid model. Although very simple in form, the fluid equations do describe a diversity of complex physical phenomena. By their very nature they are highly nonlinear and sometimes there are no analytical solutions and one must resort to computational methods. No one method of solution can model the entire family of phenomena expounded by these equations; each problem requires its own carefully selected approach.

5.1 The Grid and Interpolation

Each cell is a rectangle with dimensions $\Delta z, \Delta x$. The centre of a typical cell is denoted by the indices ij and is located at $(i + 1/2)\Delta z, (j + 1/2)\Delta x$. At the beginning of every time step values of ρ density, v fluid velocity, Ψ total energy (or Θ thermal energy) and B magnetic induction are assigned to all cell centres. Other thermodynamic quantities such as temperature and sound speed are determined through the equation of state (see Section 5.2).

The laser points towards the target in the axial $\hat{z}$ direction. The plasma ablates only in the positive $\hat{z}$ direction. This means the velocity field has only a $\hat{z}$ component. All state variables are invariant in the $\hat{y}$ direction. This implies the magnetic field has only a component in the $\hat{y}$ direction as the gradients of the pressure term in the induction equation operate only in the $x - z$ plane. The early time hydrodynamic behaviour of

the plasma is principally in the $\hat{z}$ direction and so this would justify having flow in only that path. We are interested in transport across magnetic fields and the geometry described certainly encompasses this homology.

Apart from advection we employ central space differencing. A term such as $X_{i+1/2}$ is worked out as $(X_i + X_{i+1})/2$ instead of $X(z = i + 1/2)$. Products at face centres are given by

$$\begin{aligned} [XY]_{i+1/2}^j &= X_{i+1/2}^j Y_{i+1/2}^j \\ &= \frac{1}{4} (X_i^j + X_{i+1}^j) (Y_i^j + Y_{i+1}^j) \end{aligned}$$

and corner values are estimated as

$$X_{i+1/2}^{j+1/2} = \frac{1}{4} (X_{i+1}^{j+1} + X_i^{j+1} + X_i^j + X_{i+1}^j).$$

This differencing is necessary for conservation.

5.2 The Equations of Hydrodynamics

Before detailing the equations of motion we decide upon an equation of state. In our model we assume a single ideal fluid: the electron and ion temperatures are the same. The pressure P and internal energy Θ are given by

$$\begin{aligned} P &= (n_i + n_e) k_B T \\ \Theta &= \frac{3}{2} P. \end{aligned}$$

One naturally checks that these quantities never turn negative. Both of these equations can be thought of as the equation of state since the plasma is assumed to behave ideally for thermodynamic purposes.

We express the equations of ideal hydrodynamics in Eulerian form, namely in differential form with respect to some fixed frame of reference. The equations are hyperbolic; adding diffusion terms such as viscosity and thermal conduction would make them parabolic.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (5.21)$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla \cdot P \quad (5.22)$$

$$\frac{\partial \Theta}{\partial t} + \nabla \cdot (\Theta \mathbf{v}) = -P \nabla \cdot \mathbf{v}. \quad (5.23)$$

In the Eulerian scheme the independent space variables refer to a coordinate system fixed in space through which the fluid moves. One may construct an equivalent set of equations in Lagrangian form: the independent space variables refer to coordinates fixed in the fluid and so undergo all the motion of the fluid. The particles of a grid cell remain in that cell permanently and so one solves for the position in space of this cell on the grid.

Lagrangian schemes give fine resolution when fluid distortions are small. The motion of the mesh effectively concentrates cells in areas such as shocks so that the spatial accuracy is increased there. Around contact discontinuities there is no numerical diffusion on account of there being no explicit dependence on density in the equations. In short, Lagrangian methods are preferred to Eulerian schemes since interfaces between different materials are more easily followed, at least in one dimension. However, in higher dimensions the Lagrangian mesh becomes very irregular when there are large fluid distortions and large computational errors can occur.

We shall adopt the more generic Eulerian context. Our next decision will be whether to choose a conservative or nonconservative framework. Equation 5.23 is a nonconservative equation for the internal energy. Defining the momentum density $\Pi \doteq \rho v$ and the total fluid energy density $\Psi \doteq \rho (\Theta + v^2/2)$ we have

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial z} = 0$$

where

$$\mathbf{U} = \begin{bmatrix} \rho \\ \Pi \\ \Psi \end{bmatrix} \quad \mathbf{F}(\mathbf{U}) = \begin{bmatrix} \Pi \\ \Pi^2/\rho + P \\ (\Psi + P)\Pi/\rho \end{bmatrix}.$$

Each quantity in $\mathbf{U}$ is a quantity per unit volume. $\mathbf{F}(\mathbf{U})$ is the flux associated with the quantity $\mathbf{U}$. These are the Euler equations in conservative form.

In the context of linear analysis (and thus stability criteria) both conservative and nonconservative formulations are equivalent. However, spurious nonlinear effects are best treated within a conservative scheme. Conservative schemes identically satisfy the conservation laws of physics though they do not guarantee positivity of fundamental variables such as density and thermal energy. Especially in regions of supersonic flow, conservative schemes often give errors in thermal energy because of large kinetic energy terms. Our decision is best made within a MHD framework.

5.3 Computational MHD

We attempt to formulate a conservative set of MHD equations suitable for laser-plasma codes. We refer to the algorithm given by Pert [59]. The continuity equation is trivially in conservative form. The Lorentz force $\mathbf{J} \wedge \mathbf{B}$ appears in the momentum equation and can be expressed as a flux so that this equation is conservative. The induction equation has the conservative form given by Equation 3.13. The energy equation has the form

$$\begin{aligned} \frac{\partial}{\partial t} (\Theta + \rho v^2/2 + B^2/2\mu_0) + \nabla \cdot [(\Theta + P + \rho v^2/2 + B^2/\mu_0) \mathbf{v}] \\ - \nabla \cdot [(\mathbf{v} \cdot \mathbf{B})\mathbf{B}/\mu_0] + \nabla \cdot \mathbf{Q} + \nabla \cdot (\Theta_e \mathbf{v}_j) = W + \frac{1}{4} \nabla \cdot \{[\mathbf{S} - \boldsymbol{\eta} \cdot (\nabla \wedge \mathbf{B})] \wedge \mathbf{B}\}. \end{aligned}$$

W is the total external heating rate and

$$\begin{aligned} \mathbf{S} &= \frac{1}{e} [kT_e \nabla (\ln n_e) + \boldsymbol{\beta} \cdot \nabla (kT_e)] \\ \mathbf{v}_j &= \frac{-\mathbf{J}}{en_e}. \end{aligned}$$

We can discern fluxes due to the source

$$\mathbf{Q}_s = -\frac{1}{\mu_0} \mathbf{S} \wedge \mathbf{B}$$

and magnetic diffusion, or Ohmic heating

$$\mathbf{Q}_\eta = \frac{1}{\mu_0} [\boldsymbol{\eta} \cdot (\nabla \wedge \mathbf{B}) \wedge \mathbf{B}].$$

Conservation of magnetic flux is obtained by integrating the induction equation

$$\frac{\partial \Phi}{\partial t} = \int_l \mathbf{B} \cdot \mathbf{v} \wedge d\mathbf{l} + \int_l (\boldsymbol{\eta} \wedge \mathbf{B}) \cdot d\mathbf{l} = \int_l \mathbf{S} \cdot d\mathbf{l}$$

where

$$\Phi = \int \mathbf{B} \cdot d\mathbf{s}.$$

MHD problems are even more complex than hydrodynamic problems due to strong coupling between fields and fluid. It is quite possible for a state quantity to vary by many orders of magnitude within the context of one system. Sometimes it is necessary to separate regions with quite different characteristics into subsystems. For example, if the plasma density is low in the ‘vacuum’ region (or the field is very high there) then the Alfvén speed becomes very large and the time step diminishes correspondingly (see Section 5.8). This is unacceptable. To counteract this, regions where the density drops below some arbitrarily chosen minimum density ρ_{min} are treated as insulators. The resistivity is made infinite and the plasma cannot support a current: the fields are constant in a ‘vacuum’ region.

Numerical roundoff errors and the use of upwind fluxes also introduce spurious magnetic monopoles. To preserve $\nabla \cdot \mathbf{B} = 0$ one often works with an electromagnetic vector potential $\mathbf{A}$ (where $\mathbf{B} = \nabla \wedge \mathbf{A}$) rather than with the induction $\mathbf{B}$. Another scheme would be to add a corrector equation at the end of every time step. Our model implicitly satisfies $\nabla \cdot \mathbf{B} = 0$ since the field points only along the invariant $\hat{\mathbf{y}}$ -direction.

The major source of errors occurs when calculating the thermal energy from the quantity $\Theta + \rho v^2/2 + B^2/2\mu_0$. When the magnetic fields are large or when the flow is supersonic, errors in these latter quantities can lead to negative internal energy. For this reason we prefer to work with the nonconservative form of the energy equation and the induction equation

$$\frac{\partial \Theta}{\partial t} = -p \nabla \cdot \mathbf{v} - \nabla \cdot (\Theta \mathbf{v} + \mathbf{Q}) + \eta J^2.$$

We have missed out current convection and magnetic source in the energy equation. The thermal heat flux has also lost its contribution from the thermo-electric effect.

5.4 Artificial Viscosity for Shocks

A linear analysis of the hyperbolic Euler equations shows that such a system is able to support independent small amplitude waves which propagate with the sound speed and the advection velocity. However, when large amplitude variations occur nonlinearity sets in. Modes tend to couple or interact. Large amplitude waves usually steepen to produce discontinuities and shocks. On a difference mesh their energy is channelled into shorter wavelength modes. This poses a problem since no wavelength smaller than the mesh step can be described on the difference mesh. As a result large oscillations can occur between adjacent mesh points. This unphysical behaviour can distort the numerical solution over larger wavelengths. In addition, to resolve such discontinuities would require a very fine mesh and this is computationally expensive.

Both of these problems are dealt with by introducing an artificial viscous pressure [88] in regions close to shocks. Real fluids, as opposed to ideal fluids, do not have discontinuities. In reality discontinuities are very thin regions of very steep gradients: viscosity and thermal conduction act as dissipative mechanisms and they smear out shocks.

Artificial viscosity transforms the energy of large-mesh oscillation into the thermal energy of the fluid. Computationally an appropriate choice of artificial viscous term would smear out a shock by diffusion over a region of a few grid cells. In essence artificial viscosity should be wavelength dependent: being large for short wavelengths and small for longer wavelengths.

The artificial viscous pressure $q_{i+1/2}$ can take many forms. Our choice is the one given by Gentry et al [89]:

$$q_{i+1/2} = \begin{cases} \Upsilon \cdot (\rho c_s)_{i+1/2} (v_i - v_{i+1}) & \text{if Condition A} \\ 0 & \text{otherwise} \end{cases}$$

where

$$\text{Condition A} \doteq K \cdot (v^2)_{i+1/2} < (c_s^2)_{i+1/2} \text{ and } v_i > v_{i+1}.$$

Artificial viscosity is not applied at a cell interface if the Mach number there exceeds some predetermined value given by K . The magnitude of the viscous pressure term is given by Υ . If it is too small, numerical instabilities may develop; if it is too large,

small scale structure may be smeared out. It is suggested [89] that $\Upsilon < 0.5$ though in the author's opinion this is too austere. We choose $K = 1$ and find that optimal values of Υ depend on the conditions concerned. Generally the range $0.75 < \Upsilon < 3.0$ is used.

First order advection schemes such as the upwind scheme (see Subsection 5.6.1) usually do not require artificial viscosity. This is because truncation errors are usually proportional to a second order derivative. These errors are able to dampen short wavelength modes behind shocks and broaden strong gradients. On the other hand, artificial viscosity is needed for second order schemes such as van Leer's method (see Subsection 5.6.2).

There are alternative forms for artificial viscosity. For example

$$q = \Upsilon \rho (\Delta z)^2 \left(\frac{\partial v}{\partial z} \right)^2. \quad (5.24)$$

Viscosity is essentially a diffusion process so one naturally searches for a mean free path (or scale length) and a characteristic speed. The diffusion coefficient associated with Equation 5.24 is

$$D_\nu = \Upsilon (\Delta z)^2 \frac{\partial v}{\partial z}$$

Referring to equation 3.14, the general formula for a diffusion coefficient for a random process, we identify a scale length λ given by $\lambda/3 = \Upsilon \Delta z$ and a characteristic speed Δv . In this light we discern the parameter Υ determines how many grid cells the shock is spread over through the relation $\Upsilon = (\lambda/\Delta z)/3$.

5.5 Conservation, Positivity and the Split Time Step

In theory to solve the MHD equations one needs to evaluate the hydrodynamics, diffusion and source terms simultaneously. In practice one splits up the time step into individual phases, each phase dealing with a particular term in the equations.

In general the order in which the pairs appear do not commute. Our preferred choice is given. First we calculate the energy gained from the laser. Then we apply pressure to both the PdV term in the energy equation and pressure source term in the induction equation. Next we advect. Then we diffuse magnetic energy and apply ohmic heating. Finally we diffuse thermal energy.

We present our difference scheme.

Step One: Intermediate values

$$\begin{aligned}
\tilde{B}_{ij} &= B_{ij} + \frac{\Delta t}{\rho_{ij}^n} (\nabla \rho \wedge \nabla T)_{ij} \\
\tilde{v}_{ij} &= v_{ij} + \frac{\Delta t}{\rho_{ij}^n} (\tilde{J} \wedge \tilde{B})_{ij} - \frac{\Delta t}{\Delta z \rho_{ij}^n} (\tilde{P}_{i+1/2j} - \tilde{P}_{i-1/2j}) \\
\tilde{\Theta}_{ij} &= \Theta_{ij} - \frac{\partial \tilde{v}}{\partial z} \tilde{P} \Big|_{ij} \Delta t + S_{ij}^{laser} \Delta t \\
\tilde{\Psi}_{ij} &= \tilde{\Theta}_{ij} + \rho_{ij}^n \tilde{v}_{ij}^2 / 2 \\
\tilde{P} &\doteq P + q \quad , \quad \tilde{v} \doteq (v + \tilde{v}) / 2
\end{aligned}$$

where S^{laser} is the rate of laser energy absorption per unit volume.

Step Two: Advection

The intermediate velocity is to be used in the advection routines of Section 5.6, not the old velocity. So from hereon we drop the tilde sign and v is really $\tilde{v}$.

Step Three: Diffusion

$$\begin{aligned}
B^{n+1} &= B_{advection} + \overline{\mu_0^{-1} \nabla (\eta \nabla B^{n+1})} \Delta t \\
\Theta^{n+1} &= \Theta_{advection} + \overline{\eta J^2} \Delta t + \nabla (\kappa \nabla T^{n+1}) \Delta t
\end{aligned}$$

The overline refers to coupled quantities: the energy created by ohmic heating in the energy equation is identical to the loss of magnetic energy in the induction equation.

As pointed out by Pert [59] it is not possible to maintain strict positivity of all the individual energy terms along with exact energy conservation. It is possible to observe strict positivity but only if one employs a weaker energy conservation condition with some conservation error. On the other hand exact energy conservation is an option though it may be necessary to apply an explicit positivity constraint on the energy and density terms.

In his paper terms are sorted into complementary pairs, a pair consisting of terms which are coupled in two respective equations (like ohmic heating and magnetic diffusion above). The terms of a pair are applied simultaneously. With this construction the application of a pair mutually satisfies conservation in the two relevant equations. We have not adopted his scheme (which is more rigorous than ours) since many of his

terms are absent from our model.

5.6 Advection Routines

Let us start with the one-dimensional hydrodynamic problem

$$\frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial z} = 0$$

with $v > 0$ ⁵. The conservative difference equation is then

$$\frac{\rho_i^{n+1} - \rho_i^n}{\Delta t} + v \frac{\rho_{i+1/2}^n - \rho_{i-1/2}^n}{\Delta z} = 0.$$

Intuitively one expects the mass flow across the boundary of cell i during the time step to be proportional to the product of the density and the velocity at that boundary.

$$\Delta M_{i+1/2} \doteq \rho_{i+1/2} v_{i+1/2} \Delta t \Delta A_{\perp} \quad (5.25)$$

$$v_{i+1/2} \doteq \frac{1}{2} (v_i + v_{i+1}) \quad (5.26)$$

where $\Delta M_{i+1/2}$ is the mass flowing across the i^{th} cell to the $i+1^{\text{th}}$ cell in time Δt across cross-section $\Delta A_{\perp}$ ⁶. Cell i is called the upwind cell because mass flows across from cell i to cell $i+1$. This treatment requires an estimate for the density ρ at the boundary. It can be shown that if we take $\rho_{i+1/2} \doteq (\rho_i + \rho_{i+1})/2$ we arrive at a high order scheme though it is numerically unstable [89]. Instead we start with a Taylor-type expansion

$$\rho_{i+1/2} = \rho_i^n + \frac{1}{2}(1 - \theta)\Delta z D_{i+1/2} \quad (5.27)$$

with $\theta \doteq v_{i+1/2} \Delta t / \Delta z$. $D_{i+1/2} \approx \partial \rho / \partial z$ is a gradient term. Setting this to zero gives us the first order upwind method, whereas using $D_{i+1/2} = (\rho_{i+1}^n - \rho_i^n) / \Delta z$ makes the scheme second order.

⁵Note that the velocity used from hereon is the intermediate velocity calculated from the previous section.

⁶In 1D the $\Delta A_{\perp}$ term drops out of this analysis.

5.6.1 A First Order Scheme

We have a state variable ϕ_i (by volume) ⁷, fluid flow v_i and mass density ρ_i (by volume), all three at cell centres.

As mentioned before, setting the density gradient to zero gives us a first order approximation known as the upwind method (aka donor cell)

$$\rho_{i+1/2} \doteq \begin{cases} \rho_i & \text{if } v_{i+1/2} > 0 \\ \rho_{i+1} & \text{otherwise} \end{cases}$$

Using the boundary values of velocity (Equation 5.26) and the boundary densities defined above, we are able to construct mass fluxes ΔM through Equation 5.25. By assuming that the mass transported takes the donor cell value of the state variable ϕ we can apply conservation of momentum, energy and magnetic induction to obtain the new values of the state variables

$$\begin{aligned} \phi_i^{n+1} &\doteq \phi_i^n + [\Delta M_{i-1/2} \epsilon_{i-1/2} - \Delta M_{i+1/2} \epsilon_{i+1/2}] / (\Delta z \Delta A_\perp) \\ \epsilon &\doteq \phi / \rho \\ \epsilon_{m+1/2} &\doteq \phi_m / \rho_m \quad m \text{ labels the donor cell} \end{aligned}$$

This is a fluid in cell scheme (FLIC). A similar scheme is given by Gentry et al [89]. A FLIC code has the advantage that it can be adapted to different geometries. All that is needed is the volumes and areas of the grid cells. Cells can never empty themselves or contain negative density and the method is particularly stable in the far subsonic regions of the mesh. However, the scheme is very diffusive and it does not exploit all the information on the grid. For the purposes of advection the distributions are effectively approximated by flat topped cells.

It should be said one may solve the four advection equations independently using only volume fluxes. Here we have presented an alternative scheme where mass fluxes are kept consistent.

⁷This may be one of the intermediate values of: ρ density, Φ momentum, Ψ total energy, B magnetic induction.

5.6.2 A Second Order Scheme

One would think that things improve if we take into account the gradients in the donating cell while advecting. This means the density distribution is approximated by a linear curve or parabola. However, this tends to cause unphysical oscillations and positivity in density cannot be guaranteed. The problem can be overcome by introducing nonlinear cut offs. This was done by van Leer [90]. His monotonic advection scheme has less diffusion than FLIC. The scheme is outlined in Appendix B and implemented in the code through subroutine **vanleer**. Essentially the van Leer routine estimates cell boundary values of quantities that are advected: when provided with cell centre values χ_i (by mass or by volume) and velocities at boundary $v_{i+1/2}$ it returns cell boundary values $\chi_{i+1/2}$

$$\mathbf{vanleer}(\chi_i, v_{i+1/2}) \rightarrow \chi_{i+1/2}^{van}$$

We may now outline our advection scheme:

$$\begin{aligned} \rho_i^{n+1} &= \rho_i^n + \left(\Delta M_{i-1/2}^{van} - \Delta M_{i+1/2}^{van} \right) / (\Delta z \Delta A_{\perp}) \\ \Pi_i^{n+1} &= \tilde{\Pi}_i + \left(\Delta M_{i-1/2}^{van} u_{i-1/2}^{van} - \Delta M_{i+1/2}^{van} u_{i+1/2}^{van} \right) / (\Delta z \Delta A_{\perp}) \\ \Psi_i^{n+1} &= \tilde{\Psi}_i + \left(\Delta M_{i-1/2}^{van} \psi_{i-1/2}^{van} - \Delta M_{i+1/2}^{van} \psi_{i+1/2}^{van} \right) / (\Delta z \Delta A_{\perp}) \\ B_i^{n+1} &= \tilde{B}_i + \left(\Delta M_{i-1/2}^{van} b_{i-1/2}^{van} - \Delta M_{i+1/2}^{van} b_{i+1/2}^{van} \right) / (\Delta z \Delta A_{\perp}) \\ u &\doteq \tilde{\Pi} / \rho_i^n, \quad \psi \doteq \tilde{\Psi} / \rho_i^n, \quad b \doteq \tilde{B} / \rho_i^n \end{aligned}$$

The mass flux at the cell boundaries is given by

$$\Delta M_{i+1/2}^{van} \doteq \rho_{i+1/2}^{van} v_{i+1/2} \Delta t \Delta A_{\perp}$$

where the velocity at the cell boundaries is $v_{i+1/2} = (v_i + v_{i+1})/2$. **vanleer** ($\rho_i, v_{i+1/2}$) returns the estimates $\rho_{i+1/2}^{van}$ needed for $\Delta M_{i+1/2}^{van}$. We need to obtain cell boundary values for specific momentum, energy and induction for the respective equations

$$\begin{aligned} \text{momentum equation} \quad & \mathbf{vanleer}(u_i, v_{i+1/2}) \rightarrow u_{i+1/2}^{van} \\ \text{energy equation} \quad & \mathbf{vanleer}(\psi_i, v_{i+1/2}) \rightarrow \psi_{i+1/2}^{van} \\ \text{induction equation} \quad & \mathbf{vanleer}(b_i, v_{i+1/2}) \rightarrow b_{i+1/2}^{van} \end{aligned}$$

Note the peculiarity of the momentum equation. Being an advection equation we still need to apply van Leer's method. The quantity we are concerned with is *specific momentum*. This is velocity and is represented by u to distinguish from the velocity used at the boundaries for the advection, $v_{i+1/2}$. The vanleer estimate of the specific momentum at the cell boundaries $u_{i+1/2}^{van}$ is returned by `vanleer`($u_i, v_{i+1/2}$).

Alternative van Leer advection schemes are given in Appendix B.

5.7 Diffusion of Magnetic and Thermal Energy

One could end the scheme by setting

$$\begin{aligned} v_i^{n+1} &= \Pi_i^{n+1} / \rho_i^{n+1} \\ \Theta_i^{n+1} &= \Psi_i^{n+1} - \rho_i^{n+1} (v_i^{n+1})^2 / 2 \end{aligned}$$

It has been known for the thermal energy to become negative. One would naturally check for this. An equation of state may then be used to find the new temperature and pressure (see Section 5.2)

However, if one wants to incorporate diffusion in magnetic induction B and thermal energy Θ , one treats these quantities as being intermediate. An equation of state is used to find the intermediate temperature and then one uses a diffusion routine. This involves solving a large sparse matrix by the ICCG method [91].

It is well known that the diffusion equation is only unconditionally stable if a time-centred or fully implicit form is used (in fact only the time-centred form is fully conservative [59]). The choice of time step determines the accuracy.

We present an outline of the diffusion subroutine. State variables such as Θ^n , T^n and B^n refer to intermediate quantities after advection has been carried out. Coefficients refer to quantities prior to advection. The energy equation is now

$$\begin{aligned} \frac{\partial \Theta}{\partial t} &= \nabla (D \nabla T^{n+1}) = \nabla_k (D \nabla_k T) \\ \frac{\frac{3}{2} \rho^{n+1} T^{n+1} - \frac{3}{2} \rho^n T^n}{\Delta t} &= \frac{\partial}{\partial z} \left(D \frac{\partial T}{\partial z} \right) + \frac{\partial}{\partial x} \left(D \frac{\partial T}{\partial x} \right) \end{aligned}$$

$$= \frac{D}{\Delta z} \frac{\partial T}{\partial z} \Big|_{i+1/2,j} - \frac{D}{\Delta z} \frac{\partial T}{\partial z} \Big|_{i-1/2,j} + \frac{D}{\Delta x} \frac{\partial T}{\partial x} \Big|_{i,j+1/2} - \frac{D}{\Delta x} \frac{\partial T}{\partial x} \Big|_{i,j-1/2}$$

Placing terms in T^{n+1} on the left

$$\mathbf{dxm}T_{ij-1}^{n+1} + \mathbf{dxp}T_{ij+1}^{n+1} + \mathbf{dzm}T_{i-1j}^{n+1} + \mathbf{dzp}T_{i+1j}^{n+1} + \mathbf{d00}T_{ij}^{n+1} = \frac{3}{2}\rho^n T^n$$

where

$$\begin{aligned} \mathbf{dxm} &= -\frac{D_i^{j+1/2}\Delta t}{(\Delta x)^2} & \mathbf{dxp} &= -\frac{D_i^{j-1/2}\Delta t}{(\Delta x)^2} \\ \mathbf{dzm} &= -\frac{D_{i-1/2}^j\Delta t}{(\Delta z)^2} & \mathbf{dzp} &= -\frac{D_{i+1/2}^j\Delta t}{(\Delta z)^2} \\ \mathbf{d00} &= \frac{3}{2}\rho^{n+1} + \frac{D_i^{j+1/2}\Delta t}{(\Delta x)^2} + \frac{D_i^{j-1/2}\Delta t}{(\Delta x)^2} + \frac{D_{i-1/2}^j\Delta t}{(\Delta z)^2} + \frac{D_{i+1/2}^j\Delta t}{(\Delta z)^2}. \end{aligned}$$

This is almost a matrix equation of the form $\mathbf{A}T^{n+1} = \mathbf{b}$. To resolve into matrix form we need to redefine $\mathbf{dxm}$, $\mathbf{dxp}$, $\mathbf{dzm}$ and $\mathbf{dzp}$ at the boundaries. For example, if we have reflective boundary conditions, then at the right-hand boundary $T_{n_z+1}^j = T_{n_z}^j$. For this case we apply

$$\begin{aligned} \mathbf{d00}(n_z, j) &\longrightarrow \mathbf{d00}(n_z, j) + \mathbf{dzp}(n_z, j) \\ \mathbf{dzp}(n_z, j) &\longrightarrow 0. \end{aligned}$$

The scheme for magnetic diffusion follows a similar line.

$$\frac{\partial B}{\partial t} = \mu_0^{-1} \nabla (\eta \nabla B^{n+1}).$$

We simply replace D by $\mu_0^{-1}\eta$ and $\frac{3}{2}\rho T$ by B and apply

$$\begin{aligned} \mathbf{d00}(i, n_x) &\longrightarrow \mathbf{d00}(i, n_x) - \mathbf{dxp}(i, n_x) \\ \mathbf{dxp}(i, n_x) &\longrightarrow 0. \end{aligned}$$

If the density and temperature are reflective in the $\hat{\mathbf{x}}$ direction, then the fields are

antisymmetric, $B_i^{n_x+1} = -B_i^{n_x}$.

If one wanted to include current convection and residual thermo-electric flux, then this would be done in this phase in order to conserve energy. If magnetic fields are absent in the initial conditions magnetic diffusion should not be applied for the first time step, as the ICCG routine cannot cope with trivial equations.

5.8 Time Step Control

Imagine we have information flowing with velocity u . The maximum velocity the grid can adequately treat is $\Delta z/\Delta t$. Thus, for stability we insist $\Delta z/\Delta t > u$. This is known as the Courant-Friedrichs-Lewy (CFL) condition. In Eulerian codes the grid size is fixed so one constantly varies Δt such that it can encompass flow of all information. Naturally we do not want an unduly small time step because this entails more calculations. Formally, a time step constraint is obtained by executing a von Neumann stability analysis of a linear mode (see for example Reference [92]). The largest eigenvalue of the amplification matrix must be less than unity giving a condition on the time step.

For the hyperbolic system of equations (i.e. omitting diffusive terms such as viscosity, heat conduction and resistivity) there are three characteristic speeds. These are the centre of mass speed $|\mathbf{v}|$, the thermal sound speed c_s and the Alfven speed c_A .

$$c_s^2 = \frac{\gamma P}{\rho}, \quad c_A^2 = \frac{B^2}{\mu_0 \rho}$$

The magnetosonic speed v_s is given by $v_s^2 = c_s^2 + c_A^2$. The Doppler shifted magnetosonic speed is the largest speed within the plasma so a time step must obey the constraint

$$\Delta t \leq \frac{\Delta z}{|\mathbf{v}| + v_s}.$$

At the end of every time step, we find the minimum value of the right-hand side of the above inequality throughout the grid. Δt is set to a fifth of this quantity.

Note the Alfven speed is inversely proportional to the square root of the density - in low density regions the Alfven speed can be uncomfortably large leading to very small time steps. In most MHD codes special methods are required to treat these vacuum regions.

One must also bear in mind that there are additional speeds associated with the parabolic form of the equations. For example, with an artificial viscous pressure of the form given by Equation 5.24 the diffusion coefficient is given by Equation 5.25 so that it is only large in regions of rapidly varying velocity. When the parameter Υ is of order unity, the CFL constraint can be expressed as

$$(|v| + v_s) \frac{\Delta t}{\Delta z} + \frac{2\Delta t D_\nu}{(\Delta z)^2} \leq 1 \quad \implies \quad \Delta t \leq \frac{\Delta z}{2\Upsilon|\Delta v| + (|v| + v_s)}.$$

In a similar fashion there are velocities associated with thermal diffusion and electrical resistivity and these must also be considered. In addition to these there exist convection velocities due to gradients in the diffusion coefficients. See for example Equation 3.18 where we consider the velocity $v_\alpha = -\mu_0^{-1} \nabla \eta$.

5.9 Incorporating Nernst Convection

Nernst convection of fields is included by adding an additional velocity $\mathbf{v}_N$ to the fluid velocity in the magnetic field advection equation. The Nernst speed acts only in the axial direction in our model and is given by Equation 3.16. Being an additional characteristic speed it must be accounted for in the CFL condition. *The Nernst speeds can be quite large so it is artificially limited to the magnetosonic sound speed.*

An obscure point arises due to the artificial inclusion of a term like $\mathbf{v}_N$. Whilst specific density, momentum and energy are advected on the basis of common mass fluxes, the advection of fields must be done by volume. Using a mass flux that depends only on the fluid velocity $\mathbf{v}_i$ in an advection equation that depends on a velocity field $\mathbf{v}_i + \mathbf{v}_N$ is clearly incorrect. So as to compare results, the advection of fields is generally done by volume, whether there is a Nernst velocity or not.

5.10 The Complete Code

We have developed the code **STAMP** to investigate the interaction of a laser with a planar magnetised plasma. Of particular interest is the study of the heat flow and pressure perturbations produced by ISI-like light.

The plasma is a one temperature fluid governed by an ideal gas equation of state. Our heat source is a laser beam dump at the critical density (absorption fraction

being a third) spread uniformly over an axial distance of $3\text{ }\mu\text{m}$. Braginskii's transport coefficients are used though there is no flux limit on the heat flow. Diffusion of thermal and magnetic energy is carried out fully implicitly using an ICCG routine. Flow acts in the axial direction. The hydrodynamics are solved by second order van Leer differencing with artificial viscosity of variable magnitude. There is an option for Nernst convection. The third dimension is invariant and magnetic fields are only allowed in this direction.

We are able to use reflective or free boundary conditions. Energy conservation is monitored and is found to be well behaved. With the nonconservative form of the energy equation we have not needed to artificially reset quantities to maintain positivity. The underlying equations have been renormalised so it is possible to investigate different parameter regimes with great ease or switch between different sets of units.

CHAPTER 6

CODE TESTING

The tests presented here are just a sample of the many tests that do exist. The hydrodynamics is probably the most important aspect of the code. In our first test we investigate the advection of a square pulse; it turns out that the donor cell scheme is too diffusive. The second test concerns the motion of a sound wave and demonstrates the codes ability to handle coupled nonlinear equations. Our final hydrodynamic tests deal with the standard Sod's problem, a collapsing fluid problem and a MHD Riemann problem. These three tests will serve to help us understand the limitations of our code. It is necessary to check the execution of the ICCG routine using a diffusion test.

6.1 Advection of a Square Pulse

This test demonstrates the improvement one obtains when employing a higher order advection scheme. There is only one equation of motion

$$\frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial z} = 0$$

and v is a constant. Dealing with the domain $0 \leq z \leq 1$ we use 100 equal grid cells, with periodic boundary conditions. Initially the density is unity in the region $0 \leq z \leq 0.2$ and it is 0.1 elsewhere. The velocity is set to unity throughout the grid. We compare the donor cell advection scheme to that of van Leer. The results at time $t = 1.5$ are presented in Figure 6.20.

The van Leer scheme seemed to cope well with the steep density gradient. On the other hand the numerical diffusion associated with the donor cell scheme is quite apparent from curve (a) in Figure 6.20 - a spread is present even though artificial viscosity has been omitted. It only remains to check that similar results are obtained when the flow velocity is reversed. This was indeed the case. In the light of these results STAMP will be based upon a van Leer advection scheme.

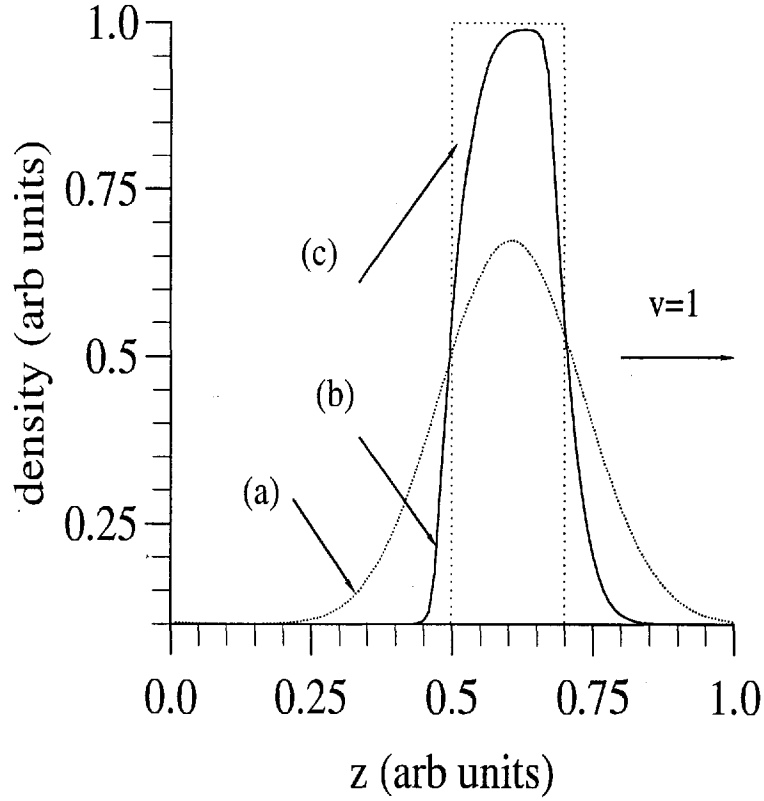


Figure 6.20: Advection of a square pulse using (a) donor cell and (b) van Leer. The exact result is given by the square pulse (c).

6.2 The Standing Sound Wave Test

We begin with an ideal, homogeneous fluid. We apply small perturbations in the three state variables ρ density, v velocity and T temperature.

$$\begin{aligned}\rho &= \rho_0 + \rho_1(z, t), & \rho_0 &= 1 \\ v &= v_0 + v_1(z, t), & v_0 &= 0 \\ T &= T_0 + T_1(z, t), & T_0 &= 1\end{aligned}$$

The hydrodynamic equations are then

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \frac{\partial (v\rho)}{\partial z} &= 0 \\ \frac{\partial \rho v}{\partial t} + \frac{\partial (v\rho v)}{\partial z} &= -\frac{\partial P}{\partial z} \\ \frac{\partial \Theta}{\partial t} + \frac{\partial (v\Theta)}{\partial z} &= -P\frac{\partial v}{\partial z}\end{aligned}$$

along with an equation of state of the form $\Theta = 3\rho T/2$. To first order the equations are

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial v_1}{\partial z} = 0 \quad (6.28)$$

$$\frac{\partial v_1}{\partial t} = - \left(\frac{\partial \rho_1}{\partial z} + \frac{\partial T_1}{\partial z} \right) \quad (6.29)$$

$$\frac{\partial(\rho_1 + T_1)}{\partial t} = -\frac{5}{3} \frac{\partial v_1}{\partial z} \quad (6.30)$$

The three coupled equations, Equations 6.28, 6.29 and 6.30, may be solved by using Fourier transforms.

$$\begin{aligned} \hat{\Phi}(k, t) &= \int \Phi(z, t) e^{ikz} dz \\ \Phi(z, t) &= \frac{1}{2\pi} \int \hat{\Phi}(k, t) e^{-ikz} dk \end{aligned}$$

In Fourier space the three coupled equations become (dropping subscripts):

$$\begin{aligned} \frac{\partial \hat{\rho}}{\partial t} &= ik\hat{v} \\ \frac{\partial \hat{v}}{\partial t} &= ik(\hat{\rho} + \hat{T}) \\ \frac{\partial(\hat{\rho} + \hat{T})}{\partial t} &= \frac{5}{3} ik\hat{v} \end{aligned}$$

The last two equations are used to obtain a simple second order differential equation in $\hat{v}$. The solutions are

$$\begin{aligned} \hat{v} &= \hat{F}(k) \sin(\alpha kt) + \hat{G}(k) \cos(\alpha kt) \\ \hat{\rho} &= -\frac{i}{\alpha} \left[\hat{F}(k) \cos(\alpha kt) - \hat{G}(k) \sin(\alpha kt) \right] \end{aligned}$$

where $\alpha \doteq \sqrt{5/3}$. The velocity perturbation must be antisymmetric at time $t = 0$ so we set $\rho_1 = 0$, $T_1 = 0$ and $v_1 = \phi \sin(2\pi z)$ at that time. This implies $\hat{\rho} = 0$ at $t = 0$,

in other words $\hat{F}(k) \equiv 0$. This gives

$$\begin{aligned}\hat{\rho} &= \frac{i\hat{G}(k)}{\alpha} \sin(\alpha kt) \\ \hat{v} &= \hat{G}(k) \cos(\alpha kt)\end{aligned}$$

We solve for $\hat{G}(k)$ by substituting in the inverse Fourier transform of the velocity at time $t = 0$.

$$\begin{aligned}\phi \sin(2\pi z) &= \frac{1}{2\pi} \int \hat{G}(k) e^{-ikz} dk \\ \Rightarrow G(z) &= \phi \sin(2\pi z) \\ \Rightarrow \hat{G}(k) &= \int \phi \sin(2\pi z) e^{ikz} dz \\ &= \frac{\pi\phi}{i} [\delta(2\pi + k) - \delta(2\pi - k)]\end{aligned}$$

using $2\pi\delta(x-a) = \int ds \exp(is(x-a))$. After some more manipulation the final solution is

$$\begin{aligned}\rho_1 &= -\frac{\phi}{\alpha} \sin(2\pi\alpha t) \cos(2\pi z) \\ v_1 &= \phi \cos(2\pi\alpha t) \sin(2\pi z).\end{aligned}$$

A one-dimensional version of the code was run with 100 equal grid cells and reflective boundary conditions on the domain $0 \leq z \leq 1$. The initial conditions were as detailed above with $\phi = 0.01$. Van Leer advection and artificial viscosity ($\Upsilon = 1.5$) were applied. The time step was approximately 0.002 throughout the run. The results are presented in Figure 6.21.

We should see a standing wave with two nodal points at $z = 0.25$ and $z = 0.75$. These are seen. The fluid code seems to lose some resolution after three periods. This is attributed to numerical diffusion. The period of the oscillation is indeed close to $\sqrt{5/3}$.

Result *The fluid code follows the analytic solution to first order.*

If we assume the exact solution to the nonlinear problem is represented well by the analytic solution to the linearised equations, at least at early times, then we can get a perspective on how errors behave. In Figure 6.22(a) we plot the density profile at

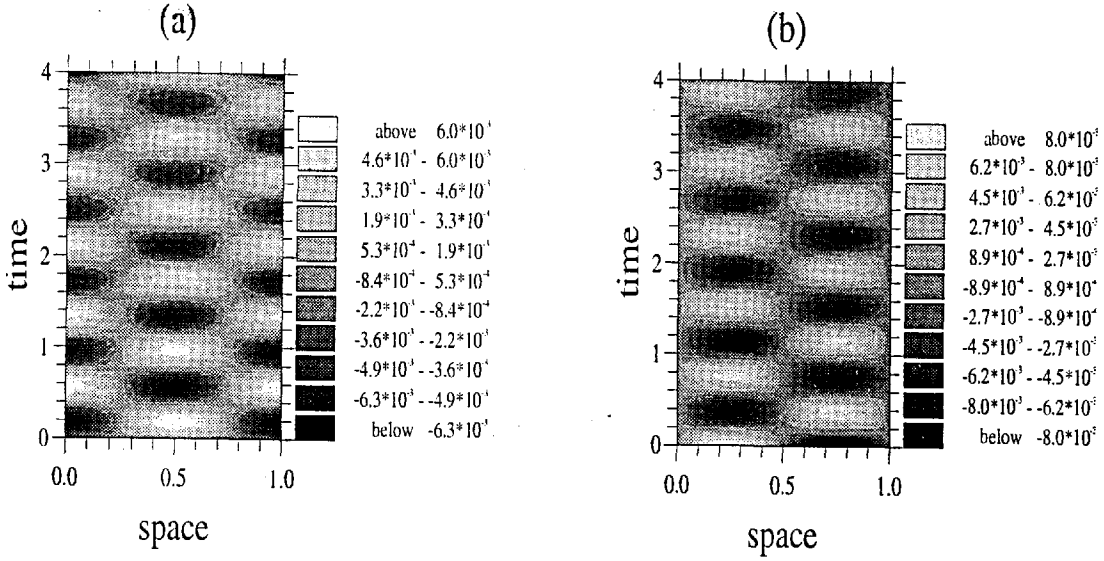


Figure 6.21: Development of (a) density and (b) velocity perturbation amplitudes in the standing sound wave test.

time $t = 1.0$ for various grid cell sizes, $dz = 0.005, 0.01, 0.02, 0.05$ and 0.1 . The lowest amplitude profile corresponds to the largest grid size $dz = 0.1$; the profiles approach the analytic solution (the largest amplitude wave) as dz becomes smaller. We may define an error as $\text{error} = \text{analytic value} - \text{code value}$. In Figure 6.22(b) we plot this error at the point $(z, t) = (0.5, 1.0)$ against choice of grid size. The error appears to grow linearly.

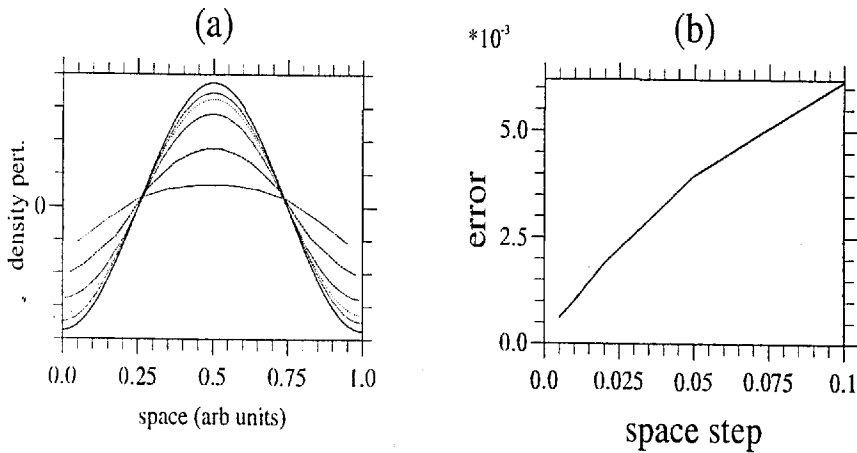


Figure 6.22: Behaviour of the standing wave solution at time $t = 1$ with respect to the grid size. (a) Density profile and (b) variations in error at the point $z = 0.5$.

6.3 The Riemann Problem

It is necessary to test the ability of one's code to manage contact discontinuities, shocks and rarefaction waves. In this respect the Riemann problem is very relevant because it is endowed with all of these phenomena.

A Riemann problem consists of a fluid with a discontinuity. At time $t = 0$ there is a left state L and a right state R separated by a contact discontinuity C . The solution develops into a wave travelling to the left $W1$ and another travelling to the right $W2$. A new state is created between the two waves and this middle state is characterised by constant pressure and velocity though there is a discontinuity in energy and density. This jump still occurs at the contact surface C and divides two substates L^* and R^* within the middle region. The pair of waves $W1$ and $W2$ are either two shocks, two rarefaction waves or a shock and a rarefaction wave.

The Riemann problem is a very simple, nonlinear, initial value problem and an exact analytic solution is available. It is the basis of the standard hydrodynamic test known as Sod's problem [93].

$$\begin{aligned} \rho &= 1, & P &= 1, & v &= 0 & 0 < z \leq 0.5 \\ \rho &= 0.125, & P &= 0.1, & v &= 0 & 0.5 < z \leq 1. \end{aligned}$$

In this particular problem the initial discontinuity becomes a weak shock: a contact discontinuity propagating to the right and a rarefaction wave moving to the left.

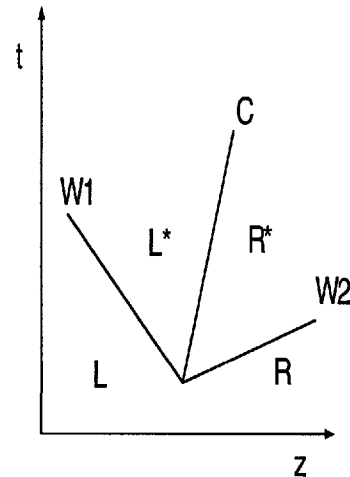


Figure 6.23: Riemann's Problem.

We run the code with 100 uniformly spaced zones with reflective boundary conditions. The time step is constrained to a fifth of the minimum Courant time step over the grid. Artificial viscosity is included ($\Upsilon = 1.5$) which, for the conditions of this problem, turns on only in the few cells nearest the shock front. We examine the results at $t = 0.143$ (when the shock front should be located at $z = 0.75$). The state variables at this time are plotted in Figures 6.24 and 6.25.

Apart from a minor amount of oscillation behind the shock front and at the head of the rarefaction wave the density profile in Figure 6.24(b) agrees very well with the

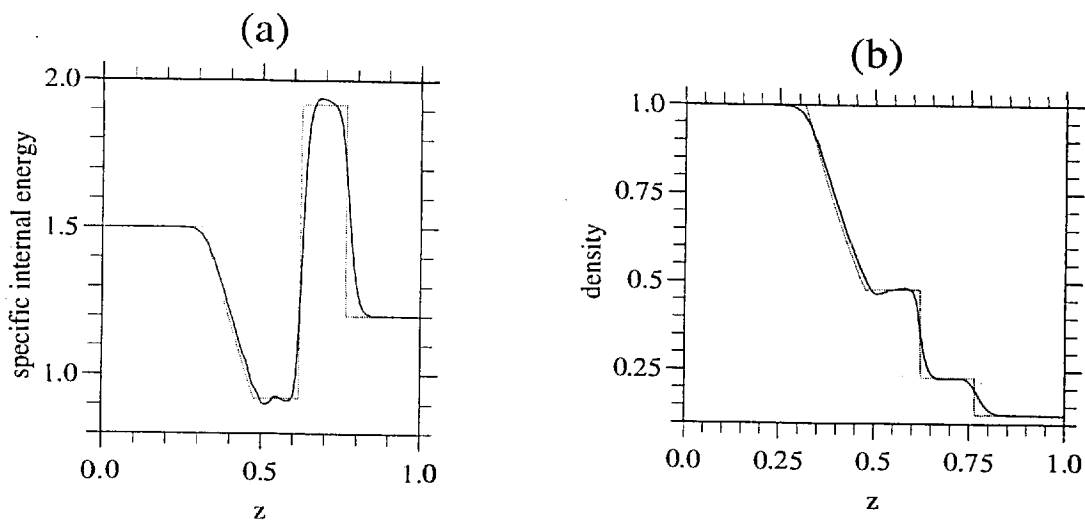


Figure 6.24: (a) Specific internal energy and (b) density profiles for Sod's problem at time $t = 0.143$, using van Leer advection (solid line) and the exact solution (dashed line).

analytic solution. The contact discontinuity (also visible in the specific internal energy plot) is smeared out over five to ten numerical grid cells. The propagation velocities, see Figure 6.25(b), also follow the analytic solution very well.

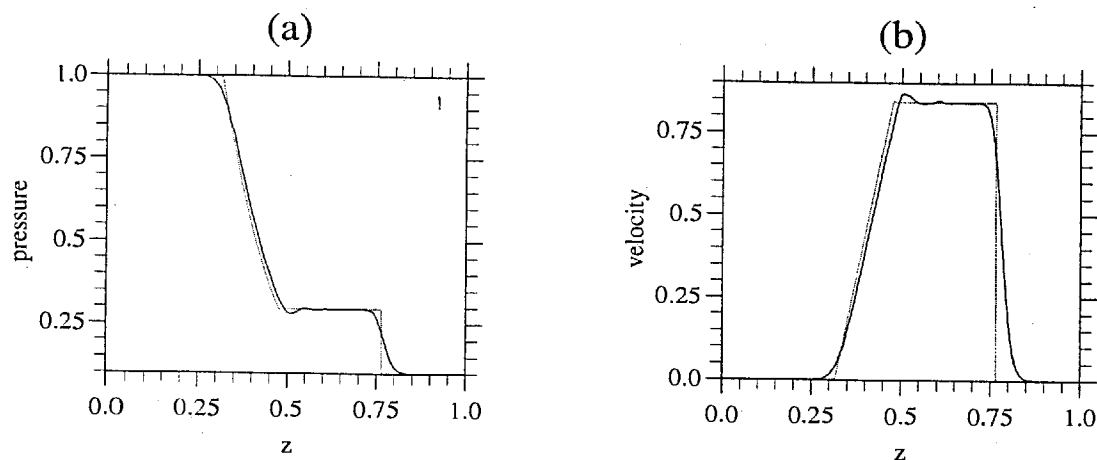


Figure 6.25: (a) Pressure and (b) velocity profiles for Sod's problem at time $t = 0.143$, using van Leer advection (solid line) and exact solution (dashed line).

The results are to be compared with the donor cell scheme results of Figure 6.26. It is evident from this figure that several structures are not being resolved well. In particular the contact discontinuity is very broad so the middle region of the Riemann problem has all but disappeared. In comparison the contact discontinuity is distin-

guishable in the van Leer scheme. The minor post shock oscillations present in the van Leer method are not as prominent in the donor cell scheme because the latter has more numerical damping associated with it.

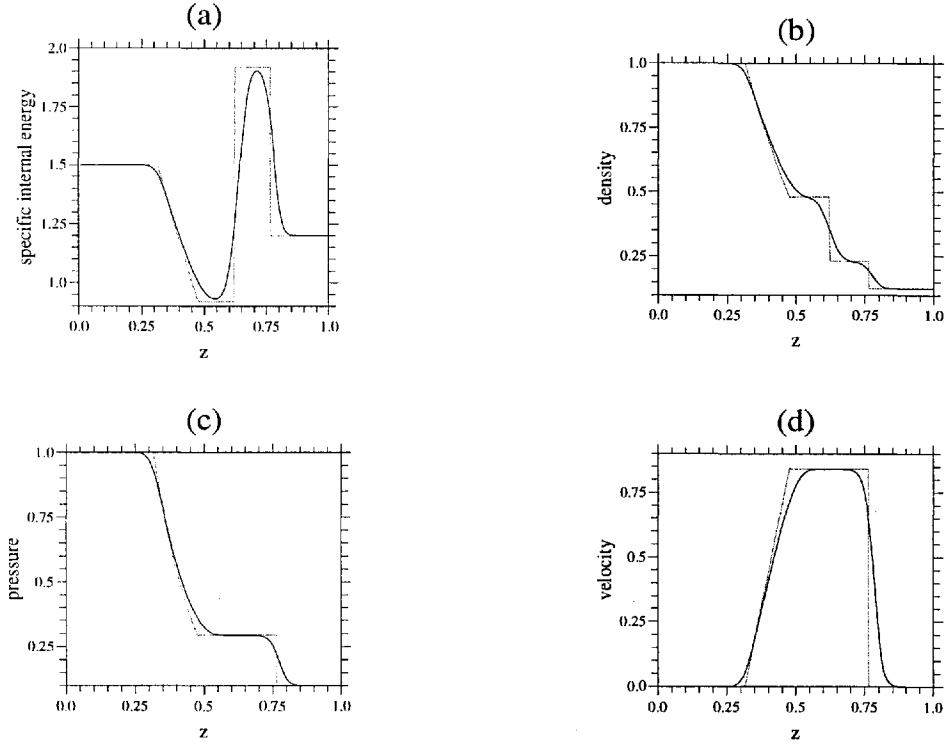


Figure 6.26: States variables at time $t = 0.143$ for Sod's problem. Solid line (dashed line) corresponds to donor cell advection (the exact solution).

Whilst shocks are captured adequately in Eulerian finite difference schemes, contact discontinuities are harder to resolve. For a shock the natural compression mechanism that is present helps to resolve the shock distinctly. However, the pressure is continuous across a contact discontinuity and the in-built dissipation of the Eulerian scheme smears out the structure there.

In Figure 6.27 we plot state variables at time $t = 0.375$ when the shock wave has reflected off the right-hand boundary and moved back into the modified gas. The pressure at the right-hand boundary is 2.4 times larger than the pressure in the oncoming fluid. Being a colliding shock problem it is very similar to Noh's problem (see Section 6.4). In Noh's problem the density drops at the rigid wall though in this case it should not do so. Examining the post reflection states at the right-hand boundary we note there is a slight drop in density right at the wall. Of more importance the density trough at $z \approx 0.8$ is not well described by the code. The structure is better resolved

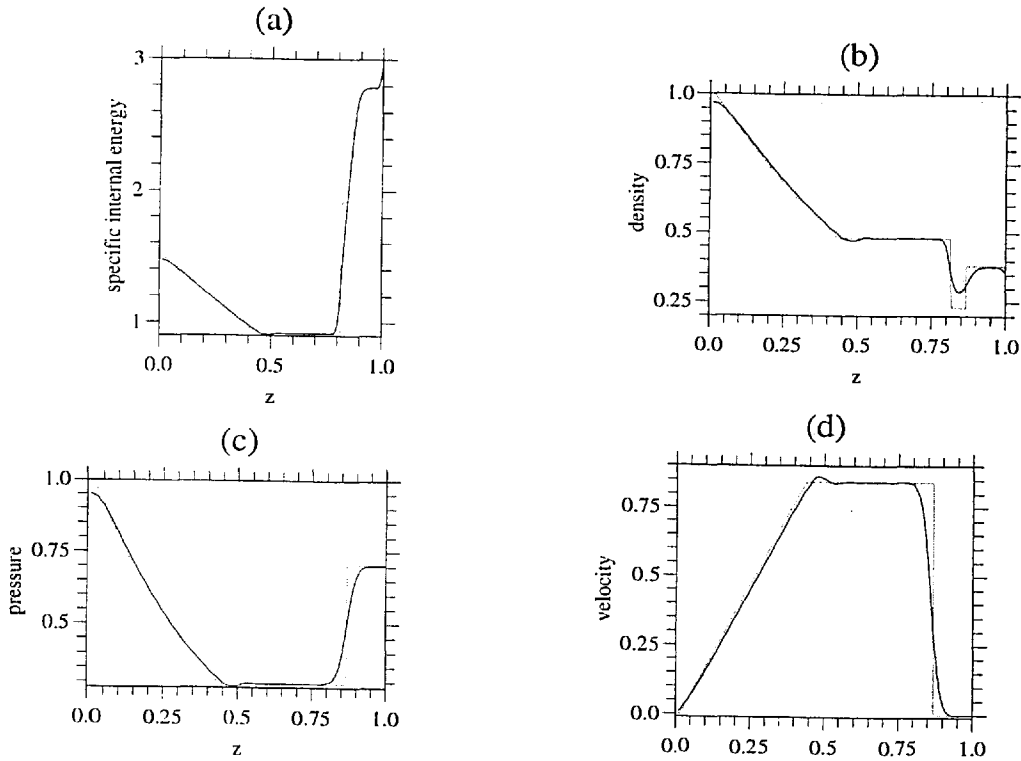


Figure 6.27: State variables at time $t = 0.375$ for Sod's problem when the shock has reflected off the right-hand boundary. Solid line (dashed line) corresponds to van Leer advection (the exact solution).

by increasing the number of grid cells.

6.4 Noh's Wall-Heating Problem

Noh's wall-heating problem is used to examine numerically generated excess heating due to intense shock formation. A constant state, constant velocity shock impinges on a fluid that has zero preshock pressure. Thus the shock has infinite strength. The test is realised by bringing a cold, perfect gas ($\gamma = 5/3, P = 0$) with constant velocity to rest at a fixed wall. This generates a new constant state headed by a reflected shock, propagating back into the oncoming, original preshock fluid. Another way of looking at it is as a constant velocity piston compressing a fluid. In our Galilean frame the piston is at rest (say a rigid wall at $z = 1$).

The initial conditions and exact solution at $t = 0.6$ are given.

$$\begin{aligned}
 \rho &= 1, \quad \Theta/\rho = 0, \quad P = 0, \quad v = 1 \quad \text{at } t = 0 && \text{throughout the fluid} \\
 \rho &= 4, \quad \Theta/\rho = 1/2, \quad P = 4/3, \quad v = 0 \quad \text{at } t = 0.6 && \text{in the post shock region.}
 \end{aligned}$$

At $t = 0.6$ the shock wave is located at $z = 0.8$ and is moving leftward with shock speed $1/3$.

The code was run with 1000 grid cells and the time step was constrained to a fifth of the time varying minimum zonal Courant time step. Before giving the results we should state that this problem focuses on the form of artificial viscosity being employed. If it is solely based upon the sound speed then there can be no artificial viscosity since there is zero pressure in the moving fluid. Instead we choose an artificial viscosity of the form given by Equation 5.24, i.e. of the form

$$\text{artificial viscosity} \propto (v_i - v_{i+1})^2 \quad (6.31)$$

for zero temperature fluid. This is applied only to the right-hand boundary where the flow is antisymmetric. The artificial viscosity generates heat at the right-hand boundary. When the temperature rises we use the familiar form $(v_i - v_{i+1})c_s$ for the artificial viscosity instead of that given by Relation 6.31. Note the condition that the fluid flow has to be less than some Mach number has been dropped as the sound speed is very low in this region.

The results are presented in Figure 6.28(a) where $\Upsilon = 0.75$. The density is roughly 4.5 when it should be 4.0 and the specific internal energy is underpredicted. There are considerable oscillations in state variables at the head of the post shock material. Also the shock wave has not reflected sufficiently far back, i.e the shock wave is not centred at $z = 0.8$.

The post shock state is inherently coupled with artificial viscosity and so the smaller Υ becomes the less post shock material and pressure is created. In fact making Υ larger makes the post shock state variables approach the correct result. The oscillation at the shock front lessens though the oscillations at the right-hand boundary become worse. See Figure 6.28(b) where $\Upsilon = 3.0$.

The density dip at the right-hand boundary is due to excessive internal energy there. Noh's problem is thus very relevant to ICF simulations where numerical heating at a centre of a pellet could lead to problems. It is also interesting to note that when we reduce the number of grid cells to 100, $\Upsilon = 1.5$ gives better results than $\Upsilon = 3.0$. This suggests the magnitude of artificial viscosity depends on the refinement of the grid.

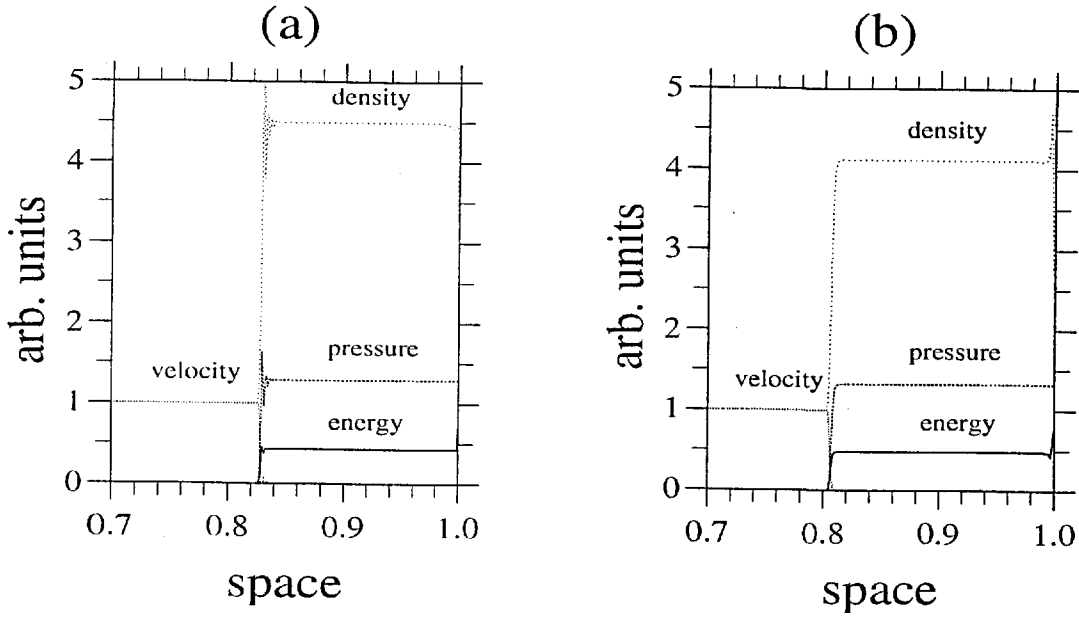


Figure 6.28: State variables of Noh's problem at time $t = 0.6$ with (a) $\Upsilon = 0.75$ (b) $\Upsilon = 3.0$

The author notes that the hybrid van Leer-donor cell advection scheme outlined in Appendix B gives the best results. The shock front is centred exactly at $z = 0.8$ and the post shock states have the correct value. The reason for the good results is because of the inherent numerical diffusion within the scheme in conjunction with the second order nature of mass advection.

This test reminds us that the best advection and artificial viscosity algorithms for a given problem really depend on the nature of that problem. The intense shocks described here are not expected to occur during imprint so we have no reason to doubt the validity of our code.

6.5 MHD Riemann Tests

Let us start with an initial value problem where we have a state specified by $U(z) = U_L$ $z < 0$ and $U(z) = U_R$ $z > 0$. U_L and U_R are any given left and right states and they are completely specified by seven variables $\rho, p, v_x, v_y, v_z, B_y, B_x$ and three constants, the ratio of specific heats γ , gas constant R and the longitudinal component of the magnetic field B_z . All variables depend on only z and t .

This is the Ideal MHD extension of the hydrodynamic Riemann problem and the solutions are more complex. Sometimes there is no unique solution and one takes the appropriate weak wave limit [94]. As with the hydrodynamic case, the initial discon-

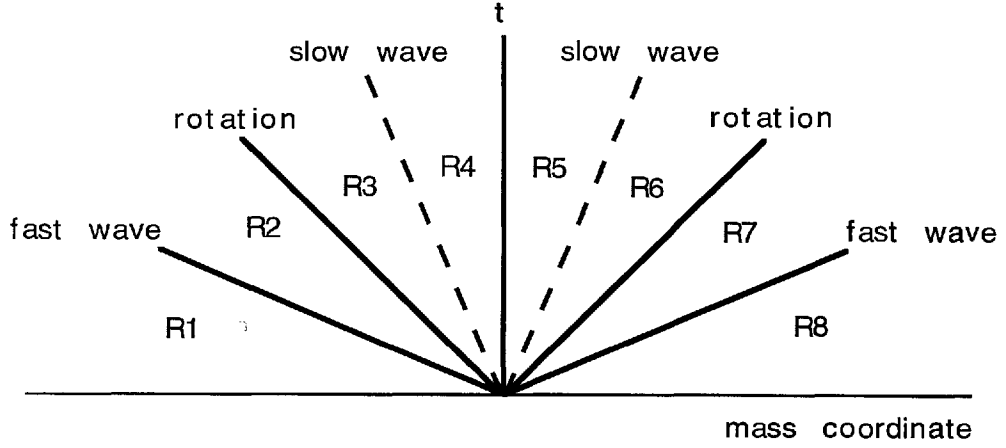


Figure 6.29: All possible waves generated from a MHD Riemann problem. There are eight possible different states divided by wave fronts.

tinuity breaks into leftward and rightward moving waves which are each separated by constant surfaces. The waves take the form of shocks, rarefactions or rotational discontinuities and there may be discontinuities at each contact surface. The number of constant states, their nature and the behaviour of state variables at the contact surfaces is dependent on the initial conditions.

In Reference [94] the authors present an approximate Riemann solver which has a general solution composed of two fast waves (either shock or rarefaction), two rotational discontinuities, two slow waves (either shock or rarefaction) and one contact discontinuity. Rotational discontinuities can only be seen in a three-dimensional model.

Our test is based upon solutions presented in Reference [94]. In that paper the hyperbolic MHD equations were solved using a Lagrangian formulation and conservative differencing. We will tackle the hyperbolic MHD equations in nonconservative form using an Eulerian scheme and compare results.

Let us start with a Riemann-type problem with only axial flow and transverse magnetic fields. Furthermore, state variables depend only on the axial component z : the partial derivatives in both transverse directions are zero.

It can be shown that no axial fields are formed. Under these conditions the $\mathbf{J} \wedge \mathbf{B}$ force (or magnetic pressure gradient) acts only in the axial direction since the only gradients are in the $\hat{z}$ direction. Looking at the momentum equation we infer that

transverse velocity cannot form and that the whole dynamical system is closed.

We now present our test which is about magnetosonic shocks. The left and right states for our initial value problem are

$$(\rho, P, v_z, B_y, B_x)_L = (0.1, 0.4, 50, -2, -1)$$

$$(\rho, P, v_z, B_y, B_x)_R = (0.1, 0.2, 0, 2, 1).$$

This work is carried out in Gaussian units. The single discontinuity will generate two strong magnetosonic shocks one moving to the right with its Mach number 12.4 and the other moving to the left with its Mach number 10.3. The states in R2 and R3 are the same as R4 and the states in region R6 and R7 are the same as in R5 according to Figure 6.29. Both transverse components of the magnetic field increase across both shocks from the preshock state to the post shock state.

STAMP was modified to simulate this one-dimensional Riemann problem. A 500 cm tube was split up into 1000 equally spaced grid cells. The time step was limited to half of the magnetosonic sound speed. Artificial viscosity was not used. The boundary conditions were fixed, i.e. throughout the simulation state variables were held constant in the ghost cells. **STAMP** was easily renormalised so code variables were explicitly in Gaussian units. Adding another dimension to the magnetic fields, B_x , in the code was not difficult.

Fluid enters the grid from the left-hand boundary and collides with a stationary fluid at lower pressure at the point $z = 5$ cm. A contact discontinuity is formed and is pushed to the right. Two constant states are formed on either side of the contact. In the frame moving with the contact, a leftward and a rightward moving wave are observed. In Figure 6.30 we plot the density profile at three different times. The constant states are clearly visible and distinguishable.

In Figure 6.31 we plot the four other state variables at time $t = 14$ s in the region around the contact. There is oscillation in the rightward moving shock but none in the leftward moving shock. The contact discontinuity is spread over around 50 grid cells whereas the shocks are spread over around ten grid cells.

The data is summarised in Table 6.2 which is to be compared with Table 6.3, the exact solution presented in Reference [94]. The density is overestimated and the pressure is underestimated in the constant states. The transverse fields are also too

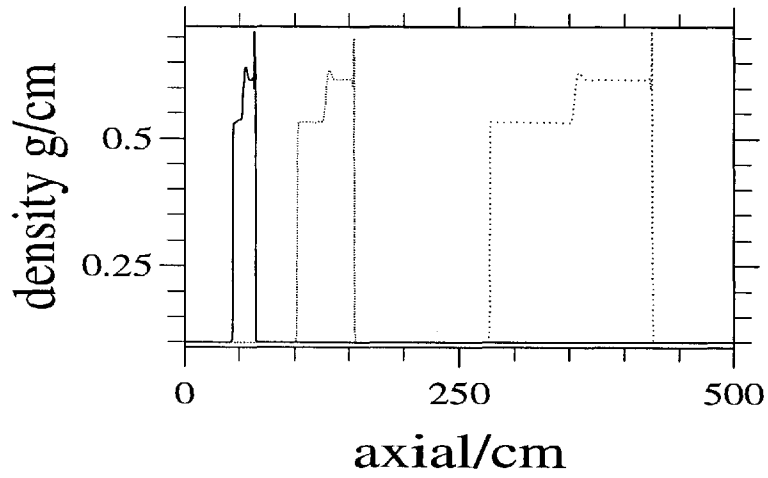


Figure 6.30: Advection of a contact discontinuity from left to right for times $t = 2$ s (solid), 5 s (dashed), 14 s (half-dashed).

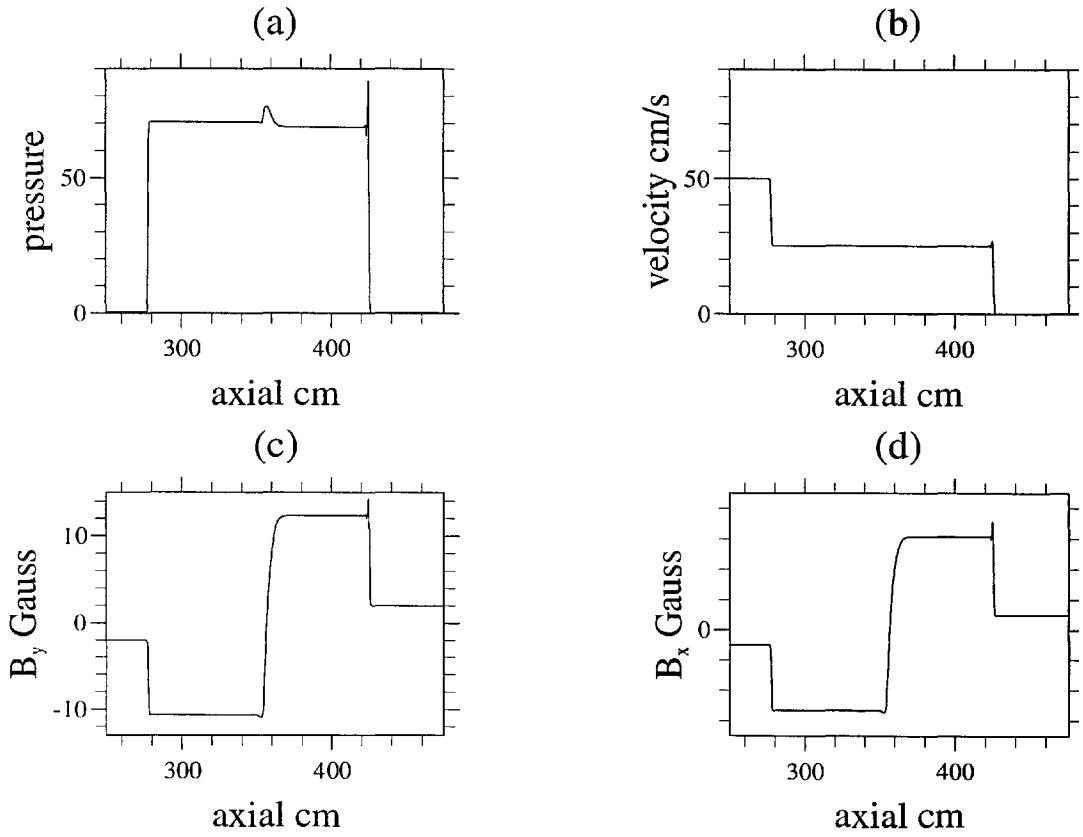


Figure 6.31: Results of MHD Riemann test at time $t = 14$ s. The constant states lie inbetween the preshock regions of the boundary. (a) Pressure, (b) axial velocity, (c) transverse magnetic field B_y and (d) transverse magnetic field B_x .

large. Only the velocity profile bears any resemblance to the exact solution. These results are similar to our test of Noh’s problem.

Region	ρ	p	v_z	B_y	B_x
R1	0.1000	0.4000	50.00	-2.000	-1.000
R4	0.5322	70.60	25.21	-10.64	-5.322
R5	0.6168	68.66	25.21	12.34	6.168
R8	0.1000	0.2000	0.0000	2.000	1.000

Table 6.2: Computed solutions for the MHD Riemann problem.

If the states were not constant but deviated away from the exact solution with time then one could compare errors to MHD growth times. However, these are constant states and the errors are there from the start and they do not change.

With errors of around 16-40 % these results do not bode well for our code. Any serious study using the results from our code would have to be based upon a regime where the magnetic fields do not affect the hydrodynamics. In this context we may still allow the fields to affect the transport.

Region	ρ	p	v_z	B_y	B_x
R1	0.1000	0.4000	50.00	-2.000	-1.000
R4	0.3871	81.66	25.03	-7.742	-3.871
R5	0.3904	81.61	25.03	7.809	3.904
R8	0.1000	0.2000	0.0000	2.000	1.000

Table 6.3: Exact solution for the MHD Riemann problem.

A thorough exposition on MHD Riemann problems is given by Tanake [95]. It is also possible to test one’s MHD differencing by modelling thermal magnetic waves [59].

6.6 Testing the Diffusion

We focus our attention on the energy equation. First we switch off the hydrodynamics, the laser, and the magnetic induction: $v(t) \equiv 0$, $S_{las} \equiv 0$, $B(t) \equiv 0$. We introduce a small temperature perturbation $T(t) = 1 + T_1(t)$ with $T_1 \ll 1$. The density is unity $\rho(t) \equiv 1$ in our region of interest $x, z \in [0, 1]$. Choosing a heat flow of the form

$$Q = -T^{5/2} \nabla T$$

the MHD equations are reduced to the single energy equation

$$\begin{aligned}\frac{\partial T}{\partial t} &= \frac{2}{3} \nabla \left(T^{5/2} \nabla T \right) \\ \Rightarrow \frac{\partial T_1}{\partial t} &= \frac{2}{3} \nabla \left((1 + T_1)^{5/2} \nabla T_1 \right) \approx \frac{2}{3} \nabla^2 T_1.\end{aligned}$$

An approximate analytic solution to the two-dimensional diffusion equation is avail-

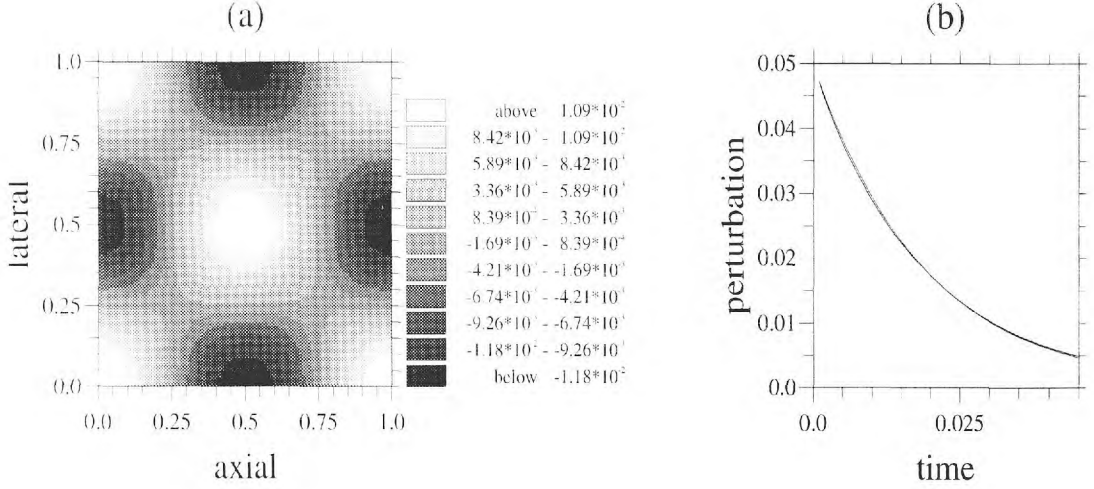


Figure 6.32: The temperature perturbation throughout the grid at time $t = 0.025$ and (b) the time development of the computed (analytic) perturbation at the point $(x, z) = (0.5, 0.5)$ is given by the solid line (dashed line).

able, $T_1(x, z, t) \approx T_1(x, z, 0) \exp(-16\pi^2 t/3)$. The code was run with initial conditions $T_1(x, z, 0) \approx 0.01 \cos(2\pi x) \cos(2\pi z)$. The grid size was $dx = dz = 0.01$, the time step was $dt = 0.001$ and the perturbation amplitude was $\phi = 0.05$. In Figure 6.32(a) we plot the temperature perturbation profile at time $t = 0.025$ and it is seen that this follows the analytic solution well. The extent of this agreement is portrayed in Figure 6.32(b) where we plot the time development of T_1 for $t < 0.04$ at the point $(x, z) = (0.5, 0.5)$.

Conclusion *The ICCG routine appears to be working adequately for the purposes of the code.*

Taking just advection and diffusion in conjunction, extrema generally decay. It is necessary to check that the ICCG routine does not spontaneously produce new extrema.

CHAPTER 7

RESULTS I - STATE VARIABLES

We present numerical results using the code **STAMP**. Our general approach will be to consider the behaviour of the plasma when: (a) there are no magnetic fields, (b) fields are generated, affecting the thermal transport and hydrodynamics and (c) a Nernst velocity for the convection of magnetic fields is included. We follow the plasma at laser intensities of LOW: $I = 10^{13}$ and HIGH: $I = 10^{14}$ W cm⁻². The absorbed intensity is a third of these values. Our laser is ISI-like as given in Section 4.3 with a coherence time of 1 ps and speckle size $\lambda_{pert} = 10$ μ m. In the penultimate section we investigate the single hot spot problem driven by a Gaussian laser profile. We close this chapter with a summary.

7.1 Model and Geometry

The laser wavelength was 1.06 μ m. The author acknowledges that the current vogue is to use a third micron or half micron laser light and hopes investigations using either of these wavelengths will be performed later. Shorter wavelength lasers entail less thermal smoothing because heating is at higher densities. Since this work focuses on the effect of magnetic fields on lateral thermal smoothing we dispense with other candidates which might influence our results, short wavelength lasers being one of them. We can proceed with other parameter regimes once we ascertain what influence magnetic fields have in the best case scenario which has long wavelength light producing more room for lateral smoothing than shorter wavelength light.

A fairly large grid is required if one wants to avoid boundary effects swamping the results. For 10^{14} W cm⁻² our grid extends over 200 μ m in the lateral direction with 280 grid points. In the axial direction the range is 26 μ m over 260 grid points. Calculations were performed on a Digital DEC 3000 and usually ran for around 4-5 days performing around 900 time steps. For 10^{13} W cm⁻² the cell sizes were the same as above but the grid extended over 24 μ m in the axial direction. Due to the smaller grid and smaller characteristic speeds the calculations lasted for 1.5 days executing 795 time steps.

The target has ionic charge equal to four and a mass number of eight. It was initialised with zero magnetic field and no velocity. The initial density and temperature variations were as given by Figure 7.33. Jumps in the state variables followed a supergaussian ramp extending over a few microns. The density in the ‘vacuum’ region was a tenth of critical. At the start all state variables were invariant in the lateral direction along the target. The right boundary was kept free and the left boundary was reflective.

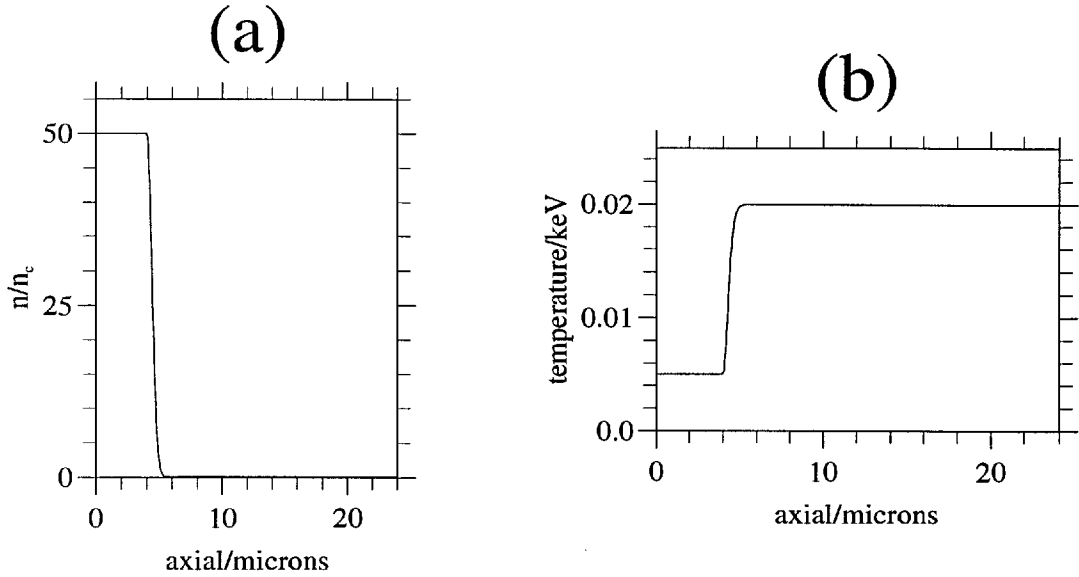


Figure 7.33: (a) Initial density variation along the axial direction, normalised to critical density. ‘Vacuum’ density is 0.1 critical. (b) Initial temperature variation along axial direction.

7.2 Structural Interchanges in the Plasma with Time

Before presenting the results we describe the time-dependent disposition of our plasma for the case $I = 10^{14} \text{ W cm}^{-2}$. This is done by identifying the ablation surface (*see Section 8.3 for prescription*), critical surface and the plasma edge. The last of these is defined in this work as the surface where $\nabla(\ln \rho)$ is maximum and negative in the subcritical region.

The plasma is initialised with these surfaces at $z \approx 6 \mu\text{m}$ and by time $t = 40 \text{ ps}$ they have diverged as shown in Figure 7.34(a). It is useful to define the regions as

- Solid : to the left of the ablation surface.
- Overdense : just to the right of the ablation surface.
- Supercritical : just to the left of the critical surface.
- Subcritical : just to the right of the critical surface.
- Coronal : to the left of the plasma edge.

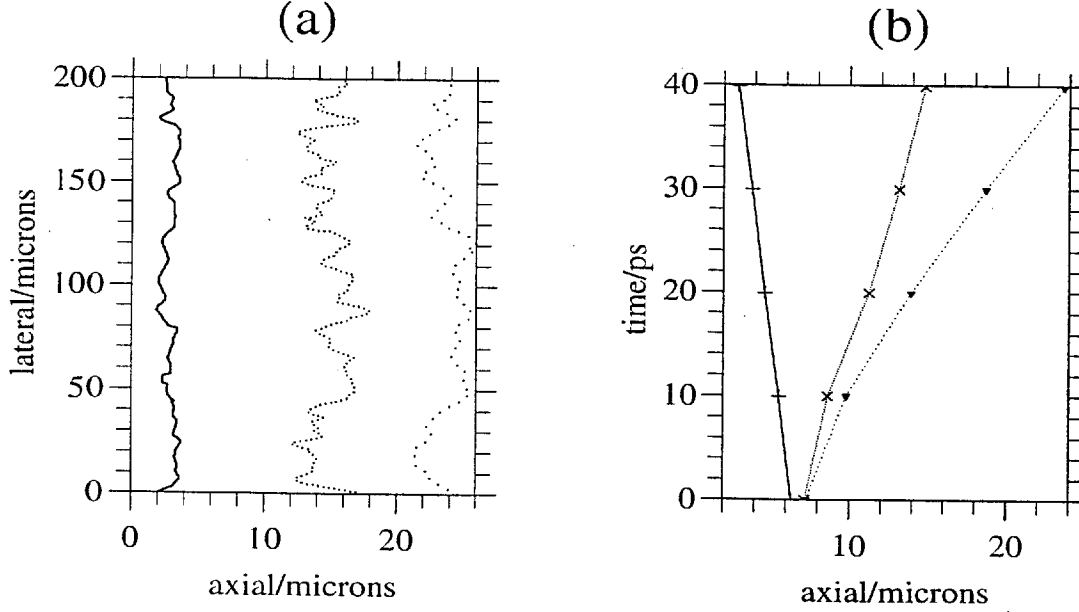


Figure 7.34: High intensity. (a) At time $t = 40$ ps: positions of ablation surface (solid line), critical surface (dashed line) and plasma edge (semi-dashed line). (b) Mean axial position as a function of time (ps) of ablation surface (+), critical surface (x) and plasma edge (▼).

Turning to the mean axial position of these surfaces we see from Figure 7.34(b) that the ablation surface moves inwards with constant speed. The critical-ablation distance $D_{ac}(t)$ grows almost linearly with time in agreement with Equation 2.8. However, when we put in the parameters $I = 10^{14} \text{ W cm}^{-2}$, $\lambda_{las} = 1.06 \mu\text{m}$ and $t = 40$ ps into that equation we arrive at $D_{ac} = 3.2 \mu\text{m}$. Figure 7.34(b) puts this quantity at $\approx 12 \mu\text{m}$, which is four times larger.

It is inappropriate to rely to heavily on Equation 2.8 since it uses different conditions and makes different assumptions to our model. The initial conditions such as the density and temperature profile have to be accounted for to arrive at the correct result for the true growth rate of D_{ac} . Assuming that D_{ac} grows at the sound speed the author finds that $D_{ac} \approx 10 \mu\text{m}$ at $t = 40$ ps. The temperature plot of Figure 7.40(a) was used to arrive at this estimate.

7.3 State Variables at $I = 10^{13} \text{ W cm}^{-2}$

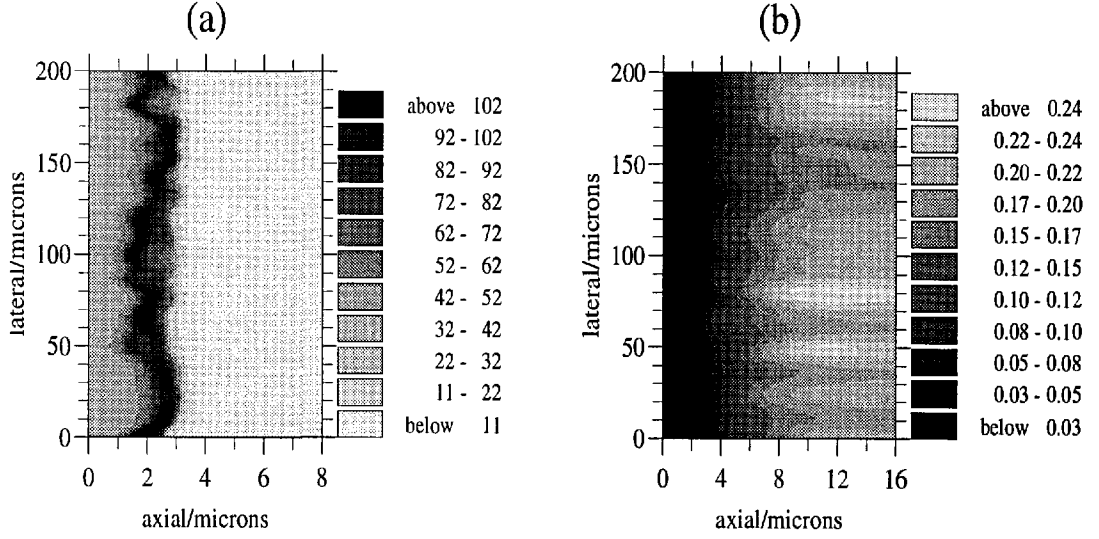


Figure 7.35: Low intensity at time $t = 60 \text{ ps}$. State variables for the unmagnetised plasma. (a) Density variation in overdense region, normalised to critical density. (b) Temperature variation across grid in keV.

STAMP was used to model 60 ps drive at $10^{13} \text{ W cm}^{-2}$ for an unmagnetised plasma. The density around the ablation surface is given in Figure 7.35(a). The laser is incident from the right. One can see the emergence of a shock wave moving inwards (towards the left). The temperature at this time is shown in Figure 7.35(b). Further into the target, in particular near the ablation surface, we see the heat wave has been smoothed out by lateral thermal conduction and in the main only long scale length modes are left.

We also present results where magnetised transport was used and magnetic fields were convected along with a Nernst velocity. To all intents and purposes the results for magnetised transport omitting Nernst convection are the same as for magnetised transport with Nernst convection, at least at this intensity. Therefore, we do not give the results for the former conditions. Naturally the fields are closer to the ablation surface when Nernst convection is employed but this does not appear to have any appreciable effect on other state variables.

The Hall parameter was limited to $\sqrt{1000}$ and the Nernst speed was artificially limited to the magnetosonic sound speed. It turns out that at this intensity, $I = 10^{13} \text{ W cm}^{-2}$, the results were identical to other runs where the Hall parameter was limited

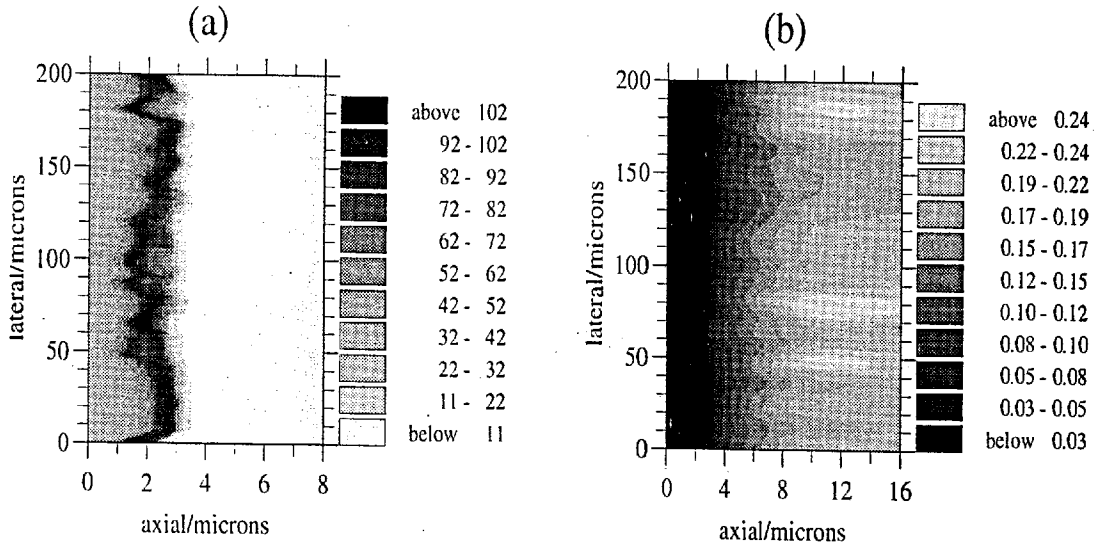


Figure 7.36: Low intensity at time $t = 60$ ps. State variables for the magnetised plasma with Nernst convection. (a) Density variation in overdense region, normalised to critical density. (b) Temperature variation across grid in keV.

to merely $\sqrt{10}$. *The Hall limit does not appear to be important.*

The density distribution is shown in Figure 7.36(a) and temperature distribution is given in Figure 7.36(b) for Nernst convected fields.

Comparing the results for magnetised transport and unmagnetised transport we conclude there is little difference in the density and temperature distributions for the two cases.

7.4 State Variables at $I = 10^{14} \text{ W cm}^{-2}$

We present state variables at time $t = 40$ ps for a plasma irradiated with $I = 10^{14} \text{ W cm}^{-2}$ light.

At high intensity the density of the inward moving shock ($\rho/\rho_c \approx 250$ in some places) is almost five times that of the preshock material (for which $\rho/\rho_c = 50$). This is shown in Figures 7.37(b). The Rankine-Hugoniot (RH) conditions limit the ratio to four. Of course the RH relations apply only in steady state (which we don't have here) and only to the hyperbolic fluid equations. For the parabolic equations the RH jump conditions are altered to include the effects of viscosity and heat conduction, see for example Reference [96].

Setting aside viscosity, the heat flow causes the temperature within the shock to decrease going from hyperbolic to parabolic forms. The shock must still satisfy the jump condition

$$\frac{d}{dz} (P + \rho v^2) = 0$$

The state variables of the preshock material are invariant. Assuming the kinetic energy of the shock is unaffected we see the pressure of the shock must remain the same. This being the case, the density needs to be higher in the shock (for the parabolic equations compared to the hyperbolic equations) to compensate for the reduced temperature of the shock.

7.4.1 Without Magnetic Fields

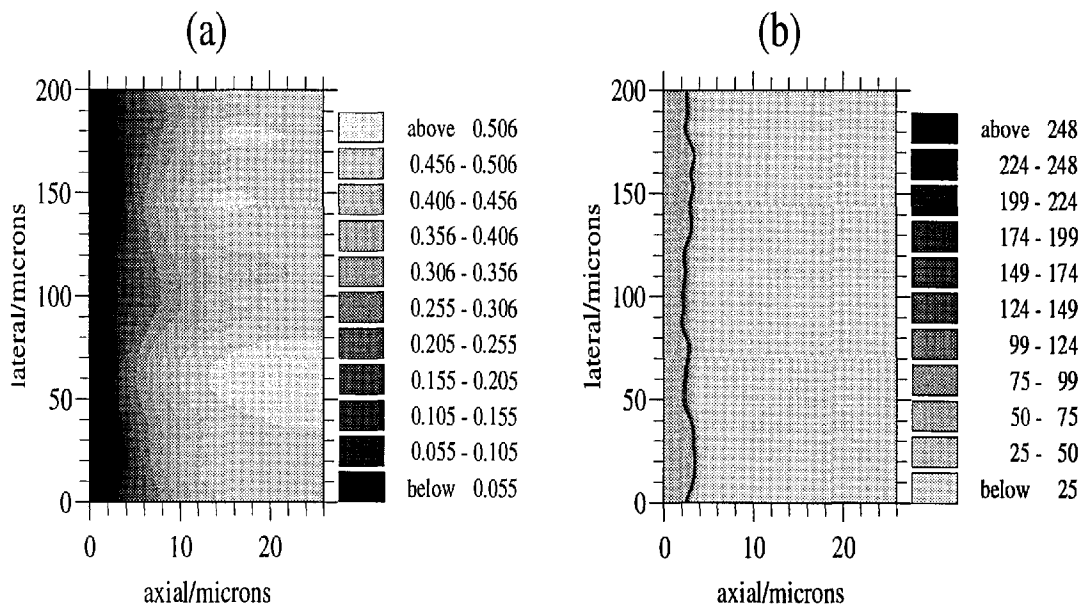


Figure 7.37: High intensity at time $t = 40$ ps without magnetic field. (a) Temperature across the grid in keV. (b) Density across the grid normalised to critical density.

The temperature and density distributions are given in Figure 7.37. By the time it has reached the ablation region the heat wave has been smoothed out by lateral thermal conduction and in the main only long scale length modes are left.

7.4.2 With Magnetic Fields, no Nernst Term

The temperature and magnetic field distributions are presented in Figures 7.38.

It was necessary to limit the Hall parameter to $\sqrt{10}$, otherwise the code became unstable. For the case $I = 10^{13} \text{ W cm}^{-2}$ the magnitude of the Hall limit did not have a physical bearing on the results. The author reasons this is also true at this new level of intensity. The important consideration is to have some form of heat flow reduction: a limit of $\sqrt{10}$ still allows for a $\approx 90\%$ decrease in flow.

An artificial limit only really affects the heat flow in the coronal and subcritical regions. The Hall parameter is more or less unchanged in the overdense regions when this limit is applied. This is because it is usually ≤ 3 anyway. Consequently we do not expect lateral thermal smoothing to be dependent on the limit χ_{max} , i.e. imprint calculations should be the same for $\chi_{max} = \sqrt{10}$ and $\chi_{max} = \sqrt{1000}$.

These arguments are in contrast to simulations by Bell [25, 26] of high irradiance laser produced plasmas (see Section 3.4.1). His Hall limit was $\chi_{max} = 10^5$ which was much larger than ours. This was because the low density matter forms the substance of the jets he observed and it was important to model the transport therewithin correctly; a Hall parameter around 10^5 is likely in the corona.

In Figure 7.39 we compare the density profile in front of the ablation surface for cases (a) unmagnetised transport and (b) magnetised transport without Nernst convection. Magnetised transport produces greater perturbations in the density profile, particularly in the centre (around $x = 100 \mu\text{m}$), away from the boundaries.

7.4.3 With Nernst Convection

Figure 7.40(a), the temperature distribution, demonstrates hot spots in the plasma penetrating right up to the ablation surface. These filamentary structures are very dangerous for ICF as they mean pressure nonuniformities at the ablation surface.

From the plot of the density, Figure 7.41(a), we see short wavelength modes have survived lateral thermal diffusion and are present at the ablation surface. The shock wave looks as if it may break up at $x \approx 180 \mu\text{m}$.

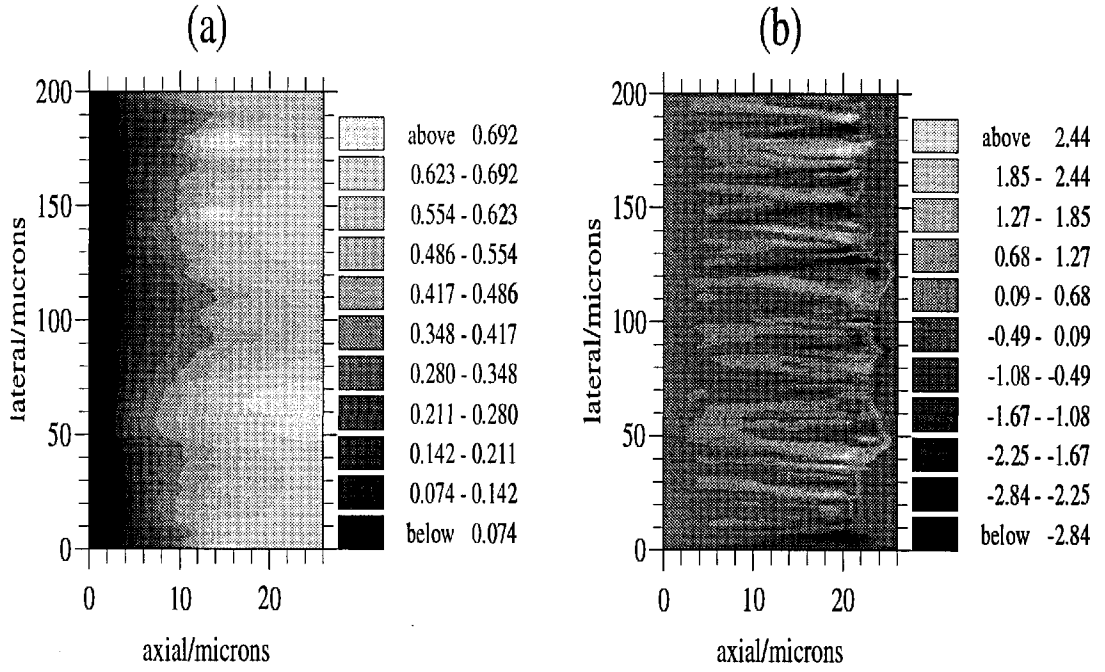


Figure 7.38: High intensity, magnetised transport at time $t = 40$ ps. (a) Temperature across the grid in keV. (b) Magnetic fields across the grid in MG.

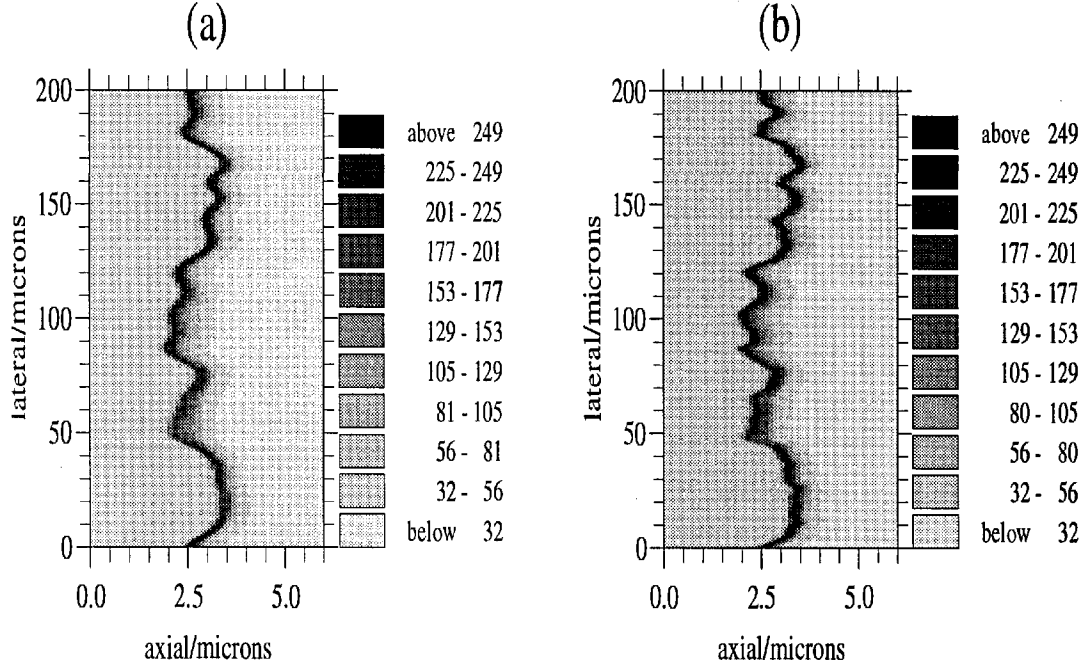


Figure 7.39: High intensity at time $t = 40$ ps. Density around the ablation surface (a) with no magnetic fields, (b) with magnetic fields but no Nernst term.

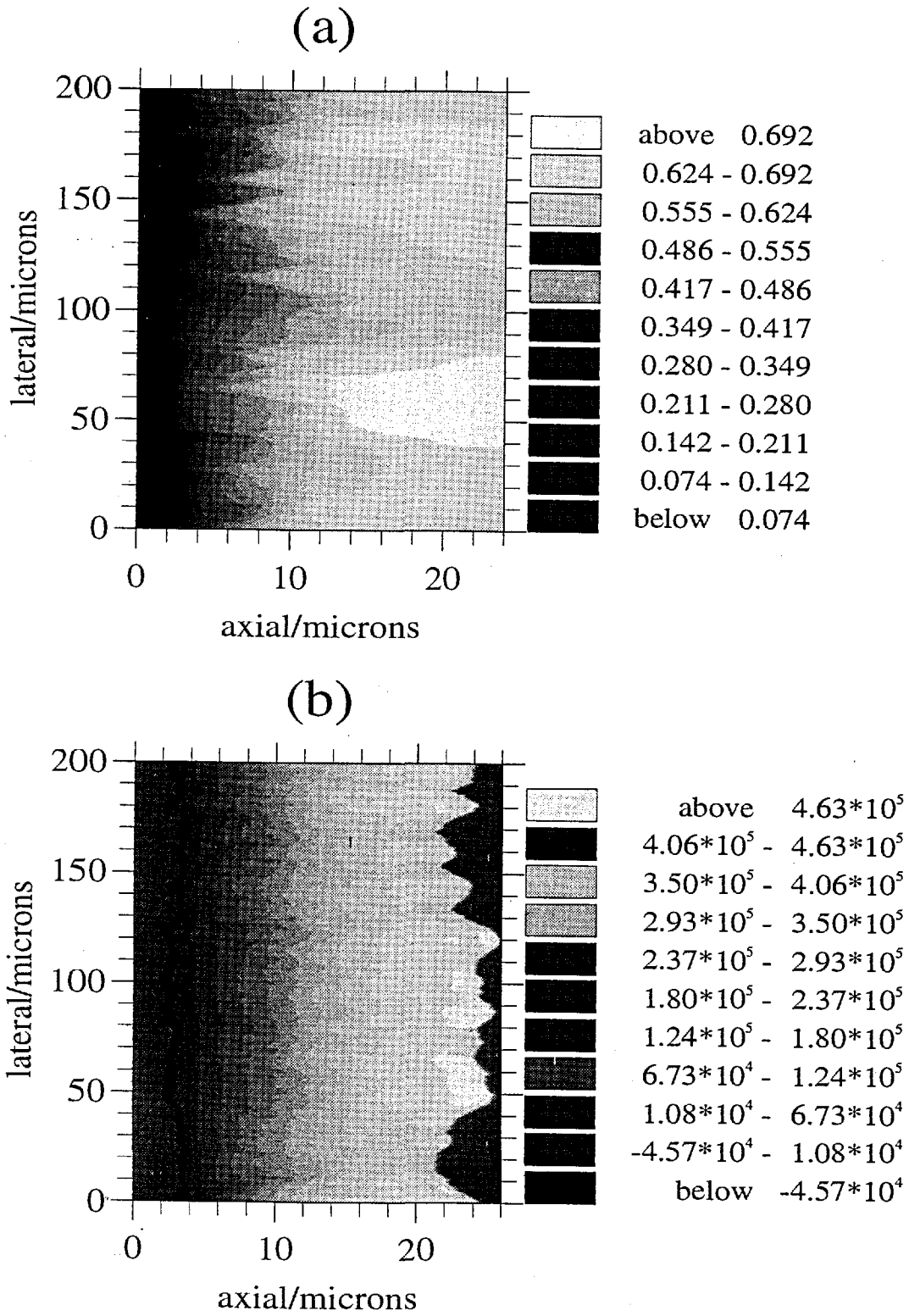


Figure 7.40: High intensity, Nernst convected transport at time $t = 40$ ps. (a) Temperature across the grid in keV. (b) Velocity in ms^{-1} .

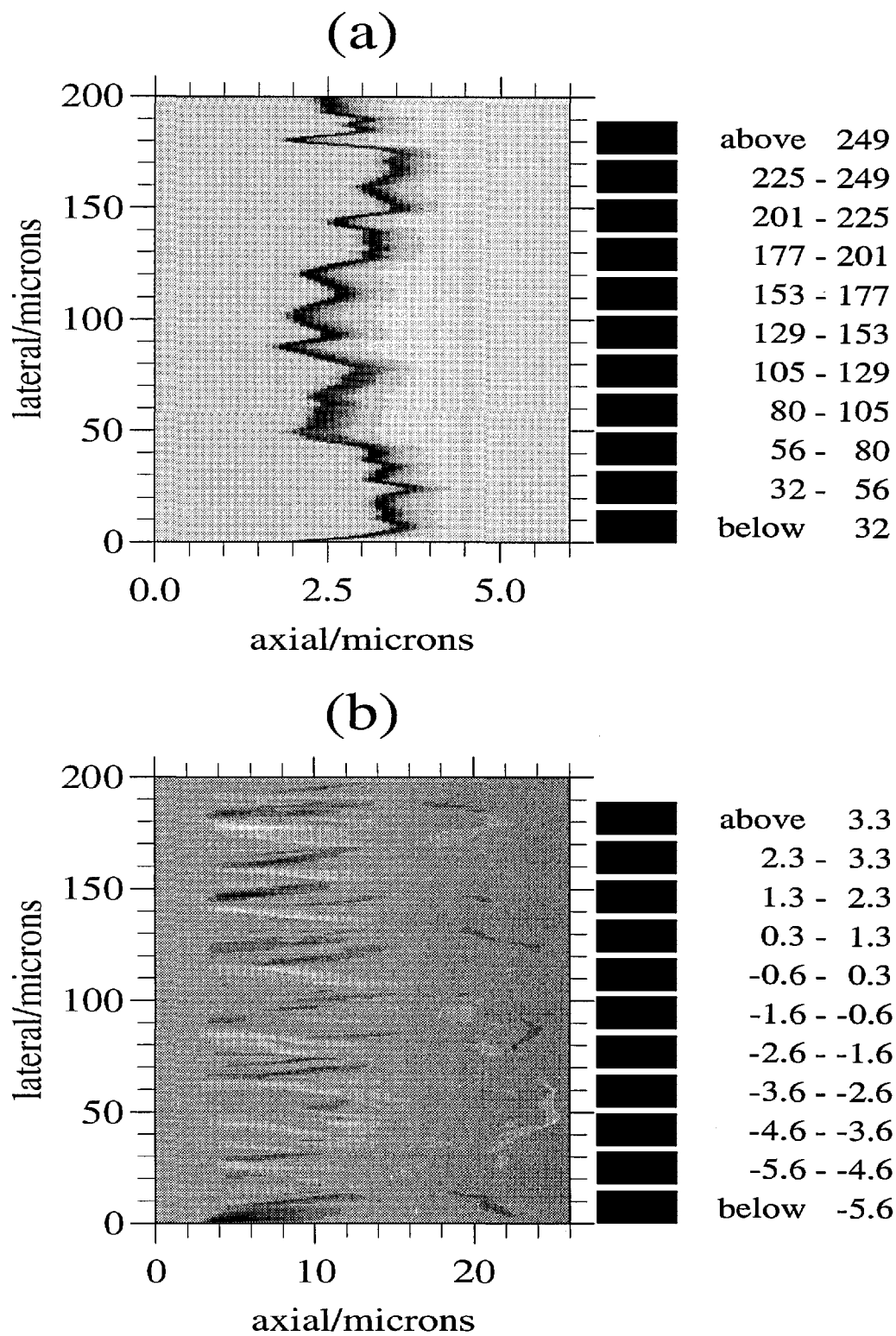


Figure 7.41: High intensity, Nernst convected transport at time $t = 40$ ps. (a) Density normalised to critical. (b) Magnetic field in MG.

7.5 Containment of Heat and Anomaly in the Density Profile

Figure 7.42 shows the logarithm of the density in the region $7 < z < 10 \mu\text{m}$, $100 < x < 150 \mu\text{m}$. Each vertical bar represents one grid cell. We observe density troughs emerging from higher densities. These troughs widen at lower densities and are bounded by density ridges which are one cell thick. Examining the magnetic field in this region we see that the strongest fields are located near these density ridges.

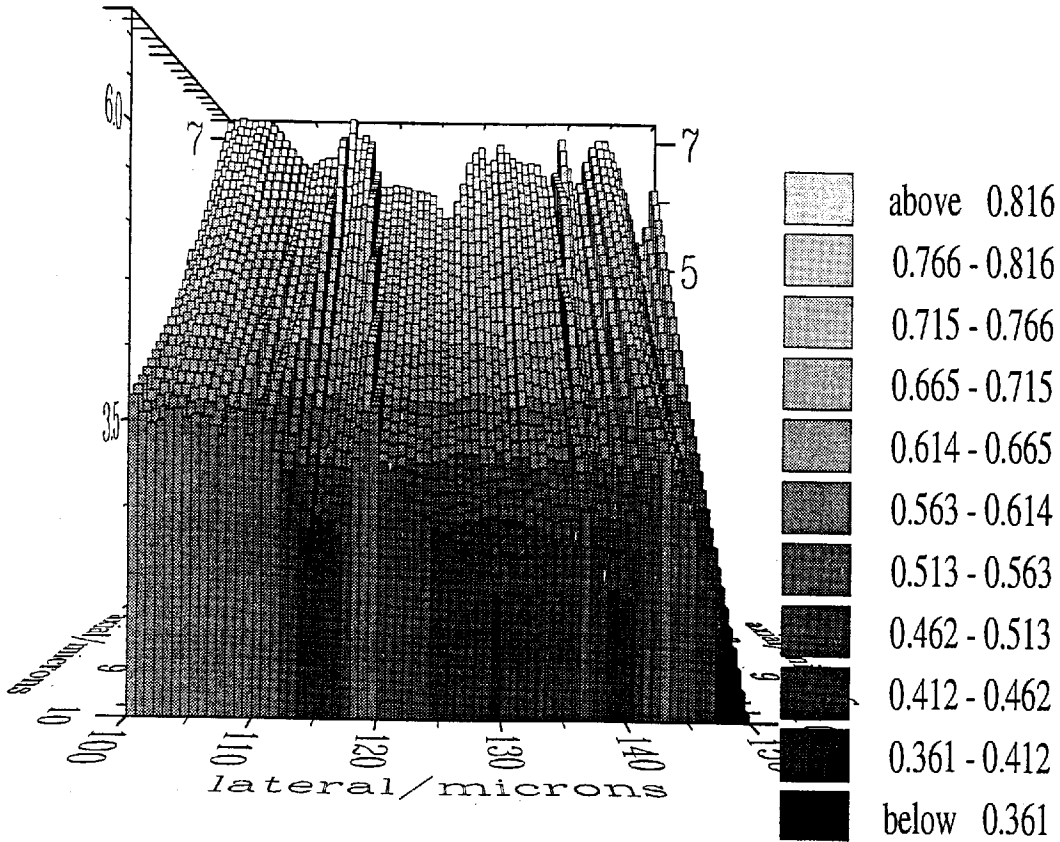


Figure 7.42: High intensity at $t = 40 \text{ ps}$ with Nernst convected fields. The key gives the logarithm of density in the region $7 < z < 10 \mu\text{m}$, $100 < x < 150 \mu\text{m}$. Actual density given by the vertical axis labels.

To scrutinise this hypothesis further we examine quantities in the plane $z = 6 \mu\text{m}$ for the top half of the grid, $100 < x < 200 \mu\text{m}$. In Figure 7.43 we plot the normalised density against the normalised $|B|$. There exists a mild correlation in the position of the density peaks and the peaks in the field. A very strong correlation is visible in Figure 7.44 where we do the same for temperature and fields. The hot spots in temperature are clearly contained within peaks in $|B|$.

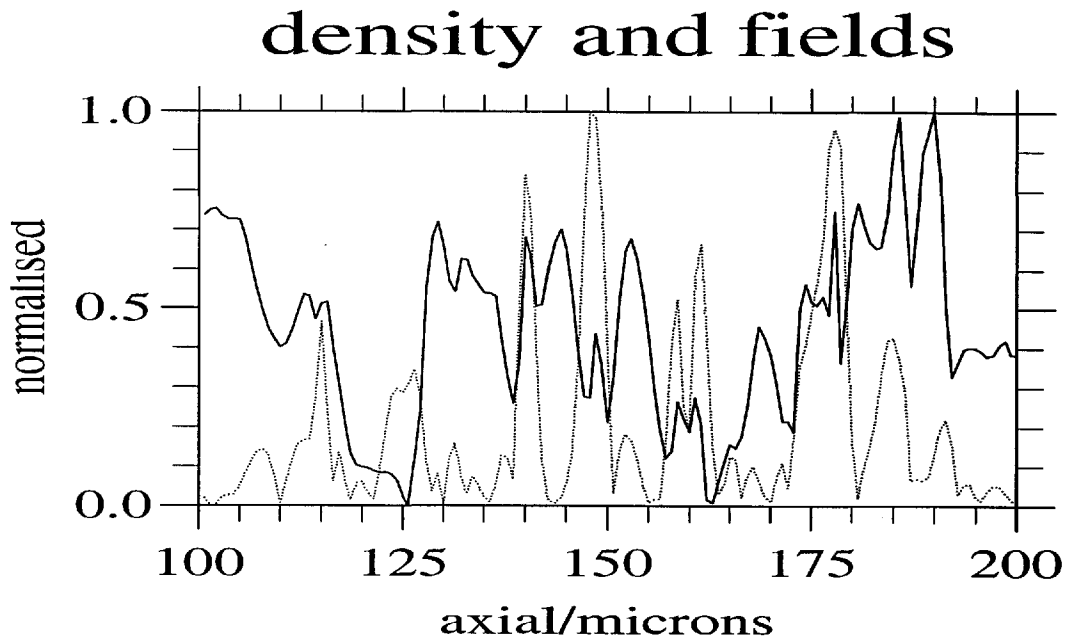


Figure 7.43: High intensity at time $t = 40$ ps with Nernst convected fields. Normalised density (solid line) and normalised magnitude of magnetic fields (dashed line) along the plane $z = 6 \mu\text{m}$.

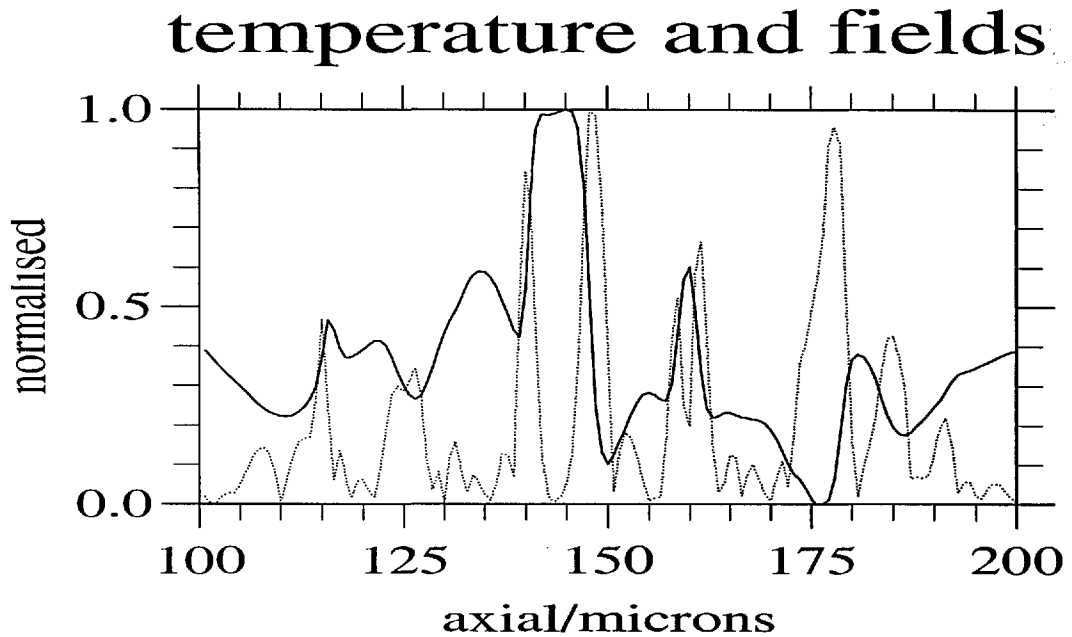


Figure 7.44: High intensity at time $t = 40$ ps with Nernst convected fields. Normalised temperature (solid line) and normalised magnitude of magnetic fields (dashed line) along the plane $z = 6 \mu\text{m}$.

7.6 The Prime Result: Heat Flow Conduits

Heat flow Q can be described as having the character of a fluid by imagining streamlines of energy flux traversing the plasma. The flow tends to circumvent ‘obstacles’ such as transverse magnetic fields.

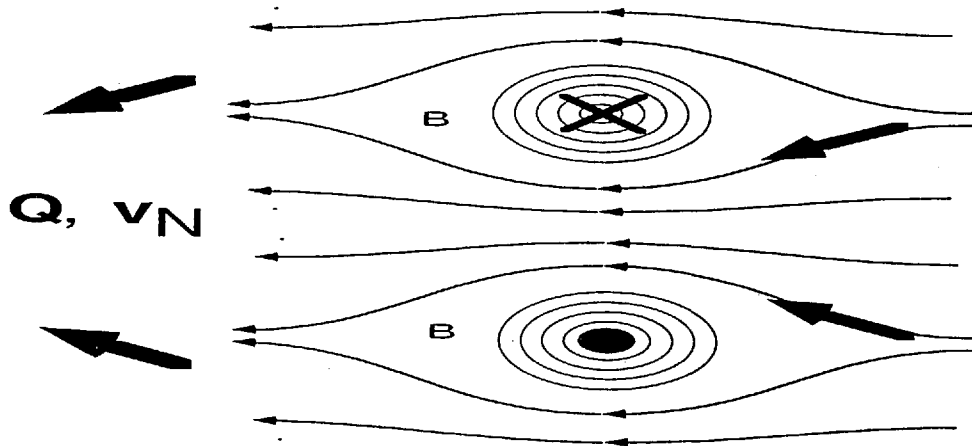


Figure 7.45: Heat flow Q and Nernst velocity v_N around a hot spot.

Figure 7.45 shows magnetic fields around a hot spot (fields pointing in and out of the paper) and how an axial flow of heat responds to the fields. The Nernst velocity, being in the same direction as the heat flow, also has the same pattern of streamlines.

The prime result of this work is the discovery of heat flow conduits which are contained by large megagauss fields. These conduits stretch from the subcritical/coronal regions right into the overdense region and sometimes up to the ablation surface itself. The cross-section of these conduits decreases further into the target so the temperature distribution has wedge-like structures within it. Although not present at low intensity they do exist at high intensity, creating perturbations at the ablation surface (see Section 8.3). Figures 7.36(b) & 7.40(a) are plots of plasma temperature at low and high intensities, respectively. The wedge-like structures are only observable at high intensity.

Perturbations at critical can now give rise to ‘hot spots’ at the ablation surface through these conduits. On the whole it is now clear that we are dealing with a phenomenon which has a filamentary nature in the supercritical/overdense region.

7.7 The Single Hot Spot Problem

In order to gain a better understanding of the physics involved in our calculations we now concern ourselves with the problem of a single hot spot caused by a time-independent perturbation in laser drive I - we monitor the development of a single speckle in a RPP set-up. The intensity is given by

$$I = \left[1 + A \exp \left(\frac{-x^2}{4\sigma^2} \right) \right] I_0 I_{amp}$$

and drives the plasma for 100 ps. The lateral scale length for perturbations is $L = 2\sigma$. In practice this will be linearly proportional to the laser wavelength. The laser intensity is normalised to $I = 10^{13} \text{ W cm}^{-2}$ by choosing I_0 such that $\langle I \rangle_x = 1$ with $I_{amp} = 1$. The parameter A can be adjusted to give the desired contrast.

For a given σ and A we may vary the parameter I_{amp} to compare results at different intensities. There is the possibility that the problem is overspecified: instead of the three parameters σ , A and I_0 , the underlying physics of bifurcation is given by two or perhaps even one sole parameter. It is already clear from the above prescription for I_0 that this parameter is actually determined by A and σ .

STAMP was used to model the behaviour of a single hot spot with the parameters $2\sigma = 10 \text{ } \mu\text{m}$, $A = 3$ and a laser intensity of $I_{amp} = 10$, i.e. $I = 10^{14} \text{ W cm}^{-2}$. Due to symmetry it was only necessary to model half a hot spot. The grid had 80 cells laterally ($40 \text{ } \mu\text{m}$) and 260 grid points axially ($52 \text{ } \mu\text{m}$). The initial density profile was as before but with the ramp located at $z = 12 \text{ } \mu\text{m}$ and the temperature of the solid much colder at 5 meV. Other parameters such as laser wavelength were as before. **The hot spot axis is located at the bottom of each of the following figures, (at $x = 0 \text{ } \mu\text{m}$), the laser having a HWHM of $5 \text{ } \mu\text{m}$ from there.**

It is important to have laser drive to provide for continued ablation pressure. This effectively means that the $3 \text{ } \mu\text{m}$ layer of laser absorption around the critical surface is well within the grid at all times. The region of fastest ablation is from the centre of the hot spot. After $t = 70 \text{ ps}$ the outward moving shock following the axis $x = 0 \text{ } \mu\text{m}$ has reached the free right-hand boundary, $z = 52 \text{ } \mu\text{m}$, though at $t = 100 \text{ ps}$ the critical surface is still located well within the grid at $z = 40 \text{ } \mu\text{m}$.

We present only the time-development of the magnetic field (in MG) in Figure 7.7. The strongest fields (which we shall call the bulk fields) are located in the overdense

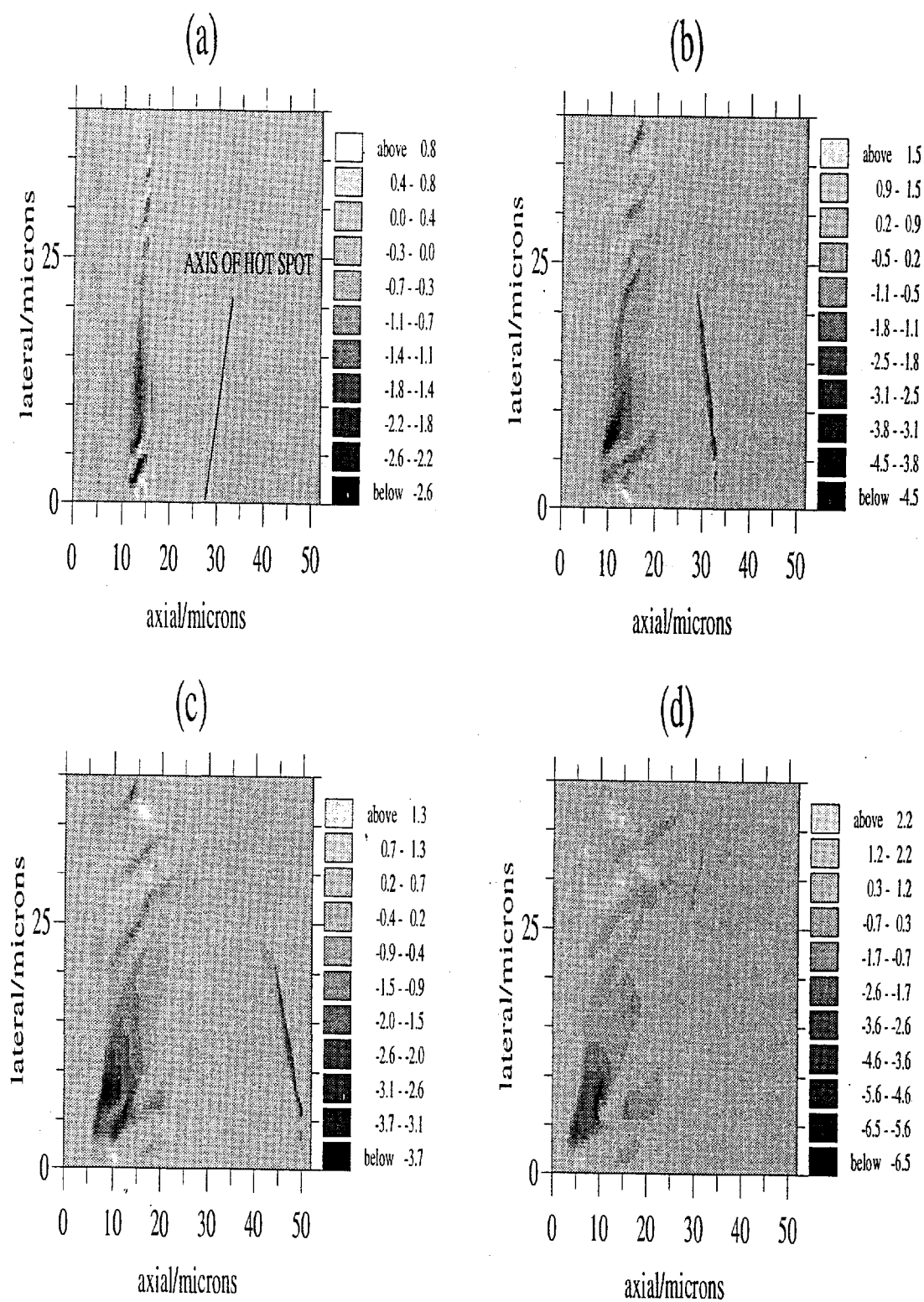


Figure 7.46: Magnetic fields in MG in the top half of a hot spot after (a) 10, (b) 40, (c) 70 and (d) 100 ps. Laser is incident from the right. The axis of the hot spot is given by $x = 0$ and below this the fields are antisymmetric.

region just outside of the ablation surface. *They do not appear to move from this vicinity*, i.e. they tend to follow the ablation surface and they point into the paper. We observe a wave of magnetic fields moving outward, following the plasma edge. It too points into the paper.

In between, in the supercritical region, there do not appear to be any fields. This is unlike our previous results where fields are present in the supercritical region.

There are also fields pointing out of the paper at the ablation surface: (a) the outer transverse fields and (b) the fields near the axis $x = 0 \mu\text{m}$. The latter are present right up until $t = 90 \text{ ps}$. These oppositely pointing fields are unexpected as traditionally theory mentions only one ring of magnetic flux around a hot spot.

The ablation fields in the centre of the hot spot have been convected in further than the outer transverse fields. This is due to two effects: (a) the heat flow (and Nernst velocity) is larger in the centre and (b) the ablation surface (source of ablation fields) moves inwards faster in the centre than in the outer transverse regions.

If we take the axial plane $z = 16 \mu\text{m}$ and move laterally inwards towards the centre of the hot spot, *the temperature actually decreases*. The reversal of ∇T and B near axis has been observed by Craxton & Haines [58]. It is thought to be due to the fields and consequently the Hall parameter being zero on axis leading to larger heat flow and lower temperatures. The reversal also adds a different dimension to the discussions of Section 8.8 (which follows). This temperature drop on axis is observable in our previous calculations by examining the axis of the conduits of Figure 7.40(a).

Given that the intensity profile is rather smooth, one may want to question why *the fields have so much structure around the ablation surface*. A numerical or physical instability may be the cause. Simply adding a Nernst velocity to the fluid velocity when convecting the fields is rather crude and the method has not been tested. In fact the criterion of positivity when advecting mass must have a counterpart for advection of field : unphysical oscillations may occur in our differencing.

Attempting to address these concerns, the author would like to point out that rich magnetic structure evolving from smooth sources is typical in many calculations (see for example the simulations by Pert [59].) With 280 grid cells in the axial direction and 80 laterally these structures are larger than Δz or Δx : they are being resolved adequately. The author believes numerical problems are not the cause. ⁸

⁸STAMP is a Cartesian code and this problem is best handled using cylindrical geometry.

Summarising this section we note that a single hot spot may have concentric rings of magnetic flux, where the fields point in opposite direction for different flux tubes. The temperature actually drops on the axis of a hot spot. Strong magnetic fields are able to remain at the ablation surface. The fields around a hot spot are rich in structure.

Although there is a bifurcation in the solution (of our ISI calculations) with laser intensity it is not clear whether this is the sole parameter which determines the behaviour of the results. Parameterisation of the single hot spot problem may lead to a clearer understanding of this bifurcation. The scenario detailed in this section would help in this respect - immediate, useful results of studies in this area can be obtained by using **STAMP**⁹. The author recommends that this be a starting point of future work.

7.8 Summary

We have attempted to numerically model ISI-like light impinging on a plasma with a very steep density gradient. Our efforts have been directed towards the first tens of picoseconds of laser drive. The laser intensity was categorised as either LOW: $I = 10^{13}$ or HIGH: $I = 10^{14}$ W cm⁻².

At low intensity magnetic fields do not have much bearing on the results.

At high intensity filamentary structures are created in densities above critical. These structures are conduits of high temperature contained within magnetic fields and are observable only if Nernst convection is included. They stretch sometimes from coronal regions right up to the ablation surface and can create hot spots there.

⁹Ideally we would want to convert to cylindrical geometry.

CHAPTER 8

RESULTS II - FURTHER ANALYSIS

We examine the pressure profile and pressure gradients that are present. Next we try to determine whether magnetised transport actually has an appreciable effect coupling laser drive to the ablation surface during imprint. A Fourier analysis of the wavelengths at the ablation surface is executed. Having scrutinised this surface we turn to the overdense filaments, examining the magnetic field generation and advection mechanisms and the heat flow. A mechanism for generating overdense filaments is presented. After commenting on the results we suggest subjects for further study.

8.1 Pressure Variations across the Grid

Figure 8.47 displays the thermal pressure at time $t = 40$ ps at high intensity for unmagnetised transport and Nernst convected fields. The pressure distribution is smooth in the absence of magnetic fields. With Nernst convected fields there are jet-like structures emanating from the ablation surface which are 4-10 μm long.

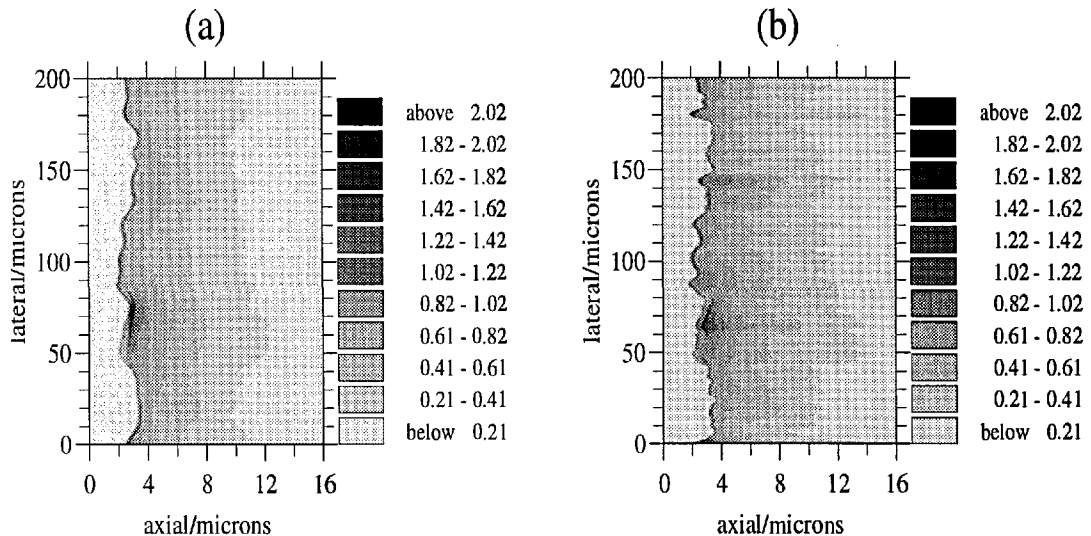


Figure 8.47: Thermal pressure in TPa at high intensity at time $t = 40$ ps for (a) no magnetic fields, (b) Nernst convected fields.

At low intensity there is very little difference between unmagnetised and Nernst convected transport.

8.1.1 Pressure Gradients

Magnetic pressure gradients are present throughout the grid. They are especially prominent in the overdense region. See Figure 8.48.

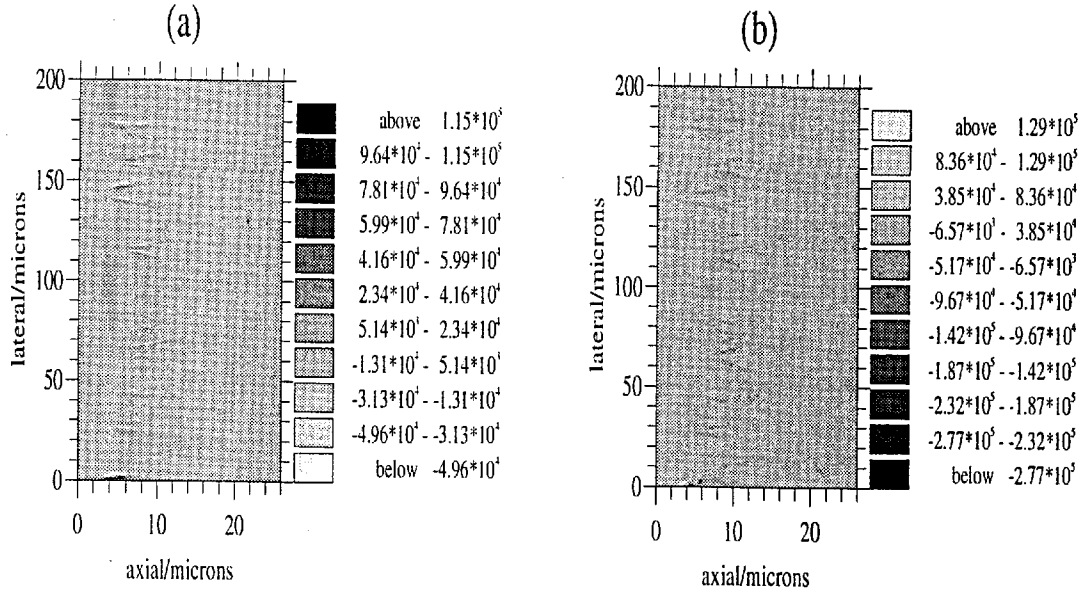


Figure 8.48: High intensity at time $t = 40$ ps with Nernst convected fields. Magnetic pressure gradients in TPa m^{-1} (a) laterally and (b) axially.

The thermal pressure gradients, plotted in Figure 8.49, are significant only around the ablation surface. The lateral pressure gradients are on the whole 10-30 % of the axial pressure gradient. To some extent this justifies having flow only in the axial direction. The pressure gradients appear to be collimated to the right of the ablation surface, another indication of jet-like behaviour. The thermal pressure gradients are generally an order of magnitude larger than the Lorentz forces.

8.1.2 The Hydrodynamic Significance of Magnetic Fields

Figure 8.50(a) shows the ratio of magnetic pressure to thermal pressure in the overdense region. Since this ratio can be as large as 60 % in some regions we say the magnetic fields may be hydrodynamically important there. At the plasma edge (not shown in the figure) this ratio is as large as 1.5. The fields act through the Lorentz force, shown in Figure 7.35(b). The data is given for the region $0 < z < 26 \mu\text{m}$ and $130 < x < 156 \mu\text{m}$.

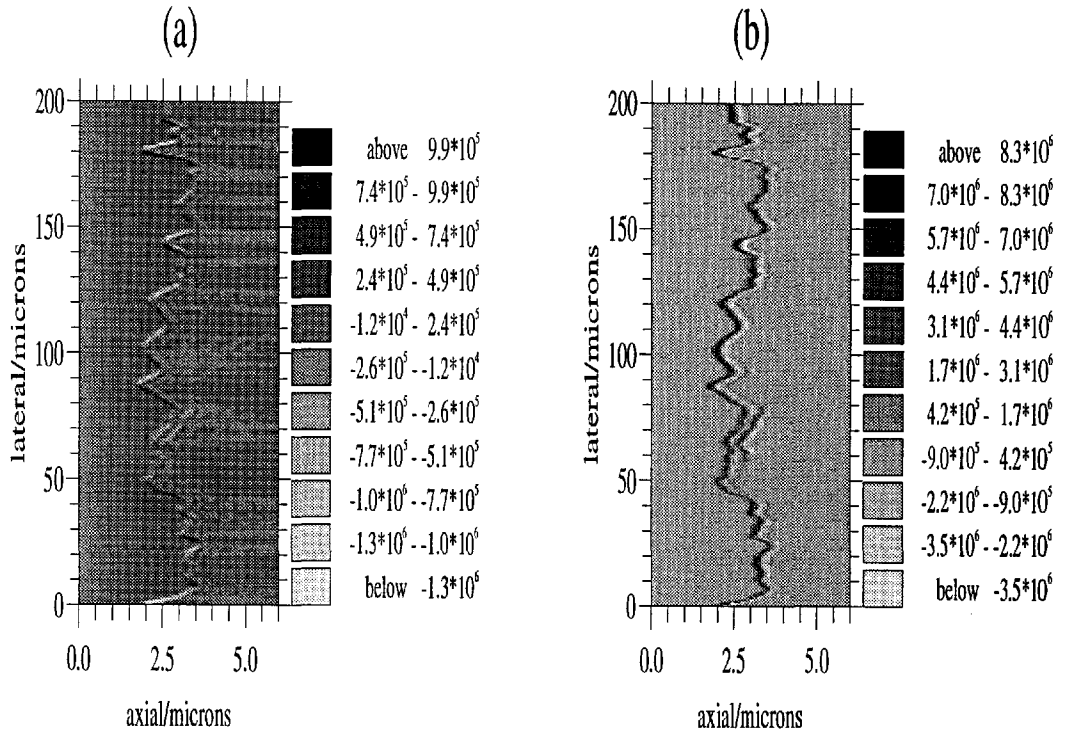


Figure 8.49: High intensity with Nernst convected fields at time $t = 40$ ps. Thermal pressure gradients in TPa m^{-1} (a) laterally and (b) axially.

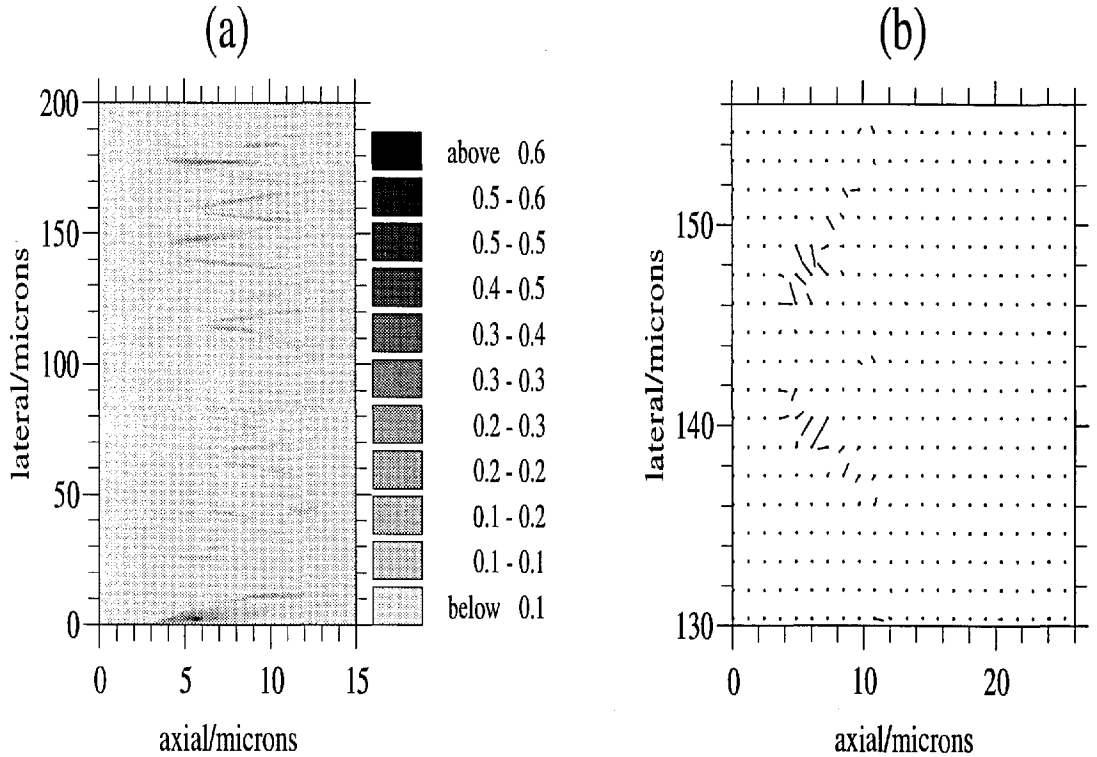


Figure 8.50: High intensity with Nernst convected fields at time $t = 40$ ps. (a) Ratio of magnetic pressure to thermal pressure in the overdense region. (b) The Lorentz force (arbitrary units) in the region $0 < z < 26 \mu\text{m}$, $130 < x < 156 \mu\text{m}$.

8.2 Imprint from Laser Drive

We hypothesise that the spatial profile of the inward moving shock is similar to laser drive, to within a reflection and compression transformation. Examining the velocity distribution in Figure 7.40(b) we define the axial position of the shock Z_i as the location of maximum inward velocity, $i = 1, \dots, n_x$ labelling laterally along the surface. In Figure 8.51(a) we plot $Z_i - \langle Z \rangle_x$ at time $t = 40$ ps, high intensity, for both Nernst convected and unmagnetised transport. The variations from the Nernst model shadow those from unmagnetised transport but they have slightly greater amplitudes.

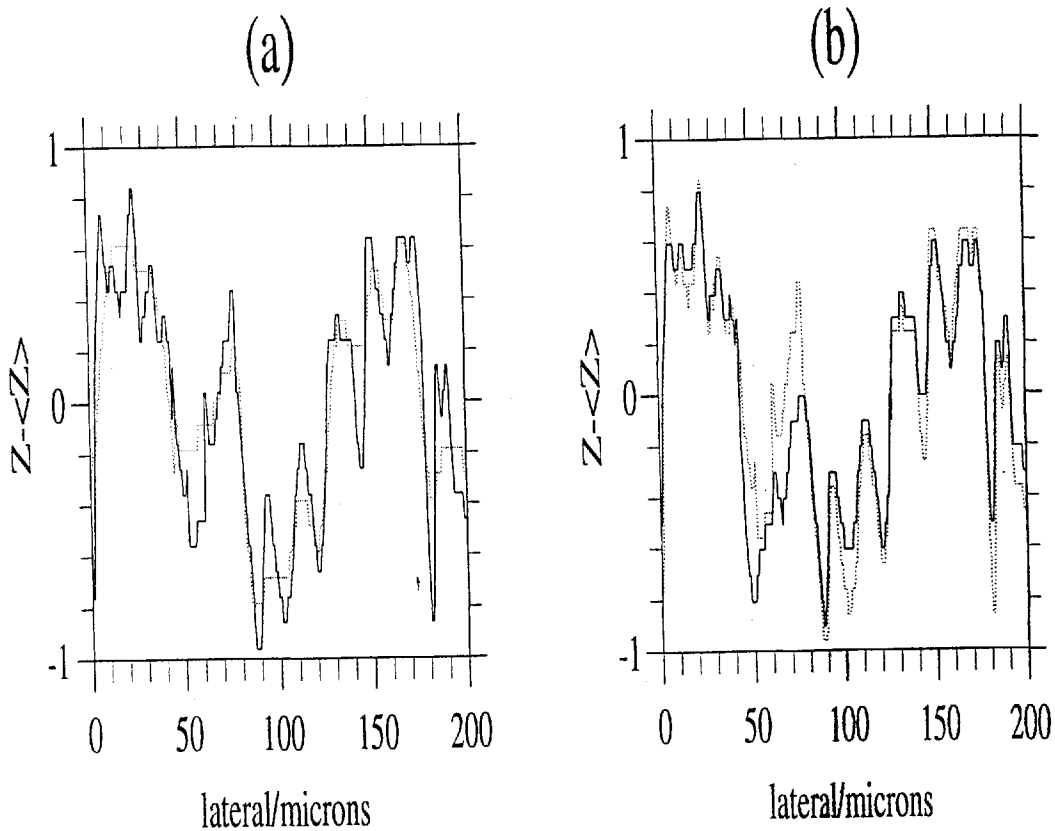


Figure 8.51: High intensity. Perturbations about mean axial position Z of the ablation surface. (a) Inward moving shock for Nernst convected (solid) and unmagnetised transport (dashed) both at time $t = 40$ ps. (b) Nernst convected fields at $t = 40$ ps (solid line) against $t = 30$ ps (dashed line).

In Figure 8.51(b) we focus on our Nernst model and plot the shape of the inward moving shock at both $t = 30$ and $t = 40$ ps. The shapes are similar and the envelopes for both profiles are almost the same. This implies that the shape of the ablation surface has not changed in time - a form of imprint. Evidently this imprint is caused by the laser.

8.2.1 Preliminary Results From a Simple Model

We assume information about the laser passes only axially towards the ablation surface. A simple model has the spatial profile of the inward moving shock indicative of time-integrated laser drive (and other effects). Normalising a profile allows us to compare both shock and laser drive. We define the normalised form $\xi(x)$ of a profile $Y(x)$ as

$$\xi \doteq \frac{Y_i - \min Y_i}{\max Y_i - \min Y_i}$$

In Figure 8.52(a) we plot the normalised position of the inward moving shock ξ_{in} (having reflected it in the axial direction) at time $t = 40$ ps (for Nernst convected fields) against normalised time-averaged intensity $\langle I(x) \rangle_t^{norm}$ across the surface after 20 ps. There is a clear similarity between the two profiles. We make the same comparison for unmagnetised transport in Figure 8.52(b). Again there is a congruence.

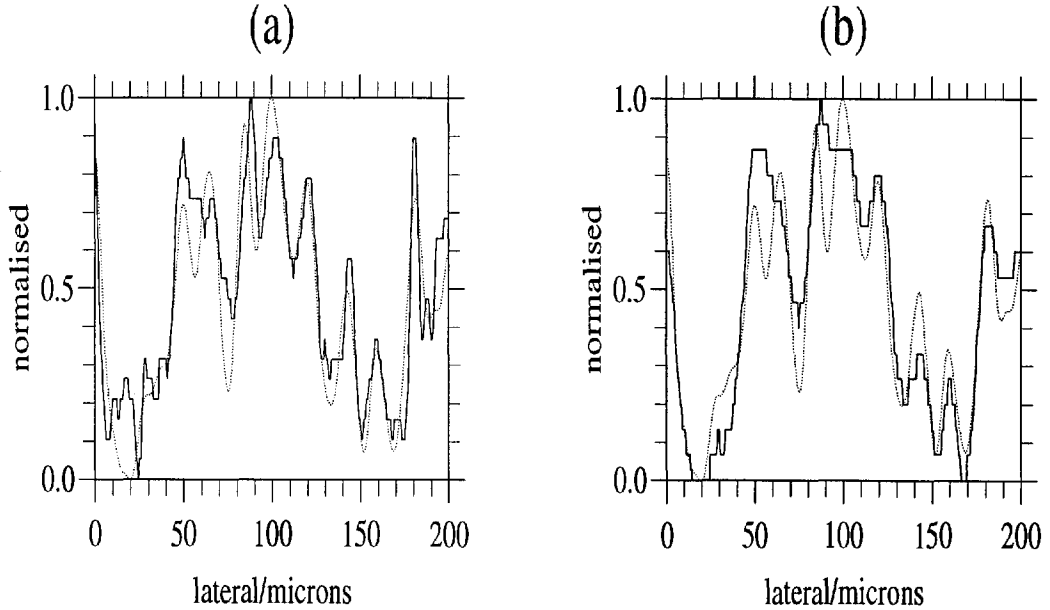


Figure 8.52: High intensity. ξ_{in} (solid line) at $t = 40$ ps along with normalised time-integrated average of laser drive after 20 ps (dashed line) for (a) Nernst convected and (b) unmagnetised transport.

The likeness is less for time-integrated drive after 10 ps, as seen in Figure 8.53(a), and after 30 ps, as seen in Figure 8.53(b). This implies, the shape of the inward moving shock at 40 ps is most similar to aggregate laser drive for the first 20 ps. Later laser drive does not affect this shape. Again comparable results are obtained for unmagnetised transport. To sum up *the effect of the laser is complete after around 20 ps* for both

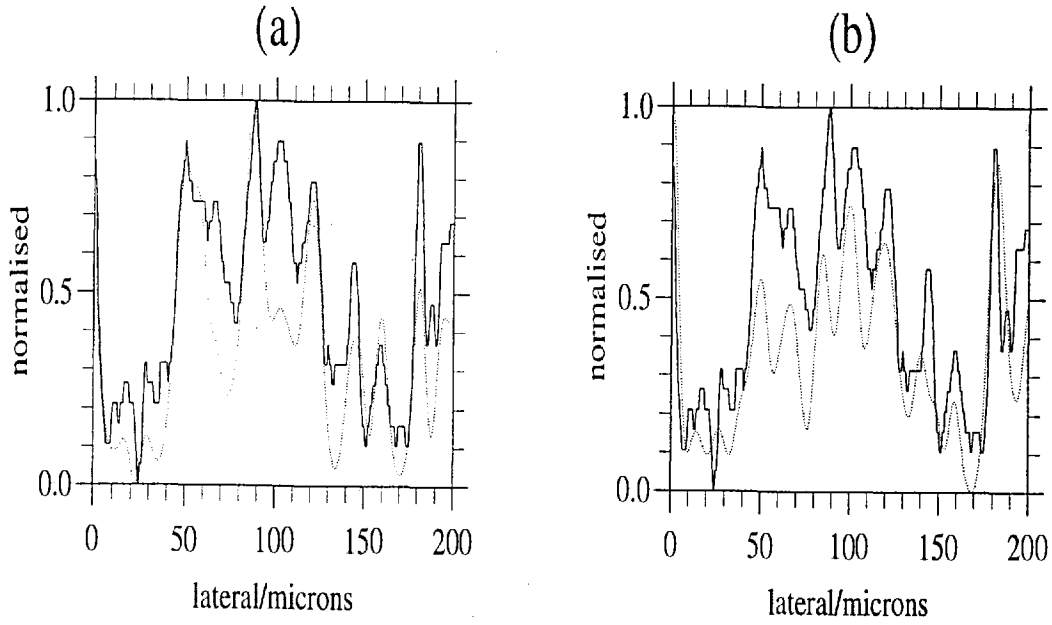


Figure 8.53: High intensity with Nernst convected fields. ξ_{in} (solid line) at $t = 40$ ps along with normalised time-integrated average of laser drive (dashed line) after (a) 10 ps and (b) 30 ps.

unmagnetised and Nernst convected transport when considering the ablation surface at $t = 40$ ps¹⁰.

Since unmagnetised transport gives almost identical results to Nernst convected transport one may initially surmise there is little difference between the two transport models. However, we have lost some information when we normalised the inward shock profiles of both models, namely perturbation amplitudes. *The Nernst model is slightly more coupled to laser drive* because its perturbation amplitudes are larger than for unmagnetised transport as demonstrated in Figure 8.51(a). However, the difference is rather small so we proceed with a more rigorous method.

8.2.2 A More Thorough Approach

Our latest result could be a temporal-connection effect and not be due to imprint. After all we cannot expect the ablation surface shape at $t = 40$ ps to be affected by the intensity at that time at critical since it takes a finite duration for information to traverse both surfaces. Secondly, our simple model does not weight the intensities according to time. Clearly laser intensities at some times will have a greater effect

¹⁰Of course it could be that the velocity profile after say 100 ps is most similar to cumulative drive after say 40 ps. Unfortunately, we do not have the data to substantiate such a claim.

than intensities at other times and simply taking a time average does not account for this. Thirdly, the amplitude of variations in aggregate laser drive lessen with time so normalising this average profile loses this information. Having made these remarks the similarity between the perturbations and integrated laser drive is still remarkable.

A more analytical measure of the correlation between the perturbations in ablation surface shape at time t to earlier laser drive $I(t - \tau)$ is

$$S(\tau) = \sum_{x_i} \left[\frac{I(x_i, t - \tau) - \langle I(t - \tau) \rangle_x}{\langle I(t - \tau) \rangle_x} \cdot \frac{\langle Z(t) \rangle_x - Z(x_i, t)}{\langle Z(t) \rangle_x} \right]$$

where for a function $Y(x, t)$

$$\langle Y(t) \rangle_x \doteq \frac{1}{n_x} \sum_{x_i} Y(x_i, t)$$

Due to temporal connection we expect $S(\tau)$ to be close to zero for small delays. Assuming there is imprint for early drive, $S(\tau)$ should be maximum for largest delays.

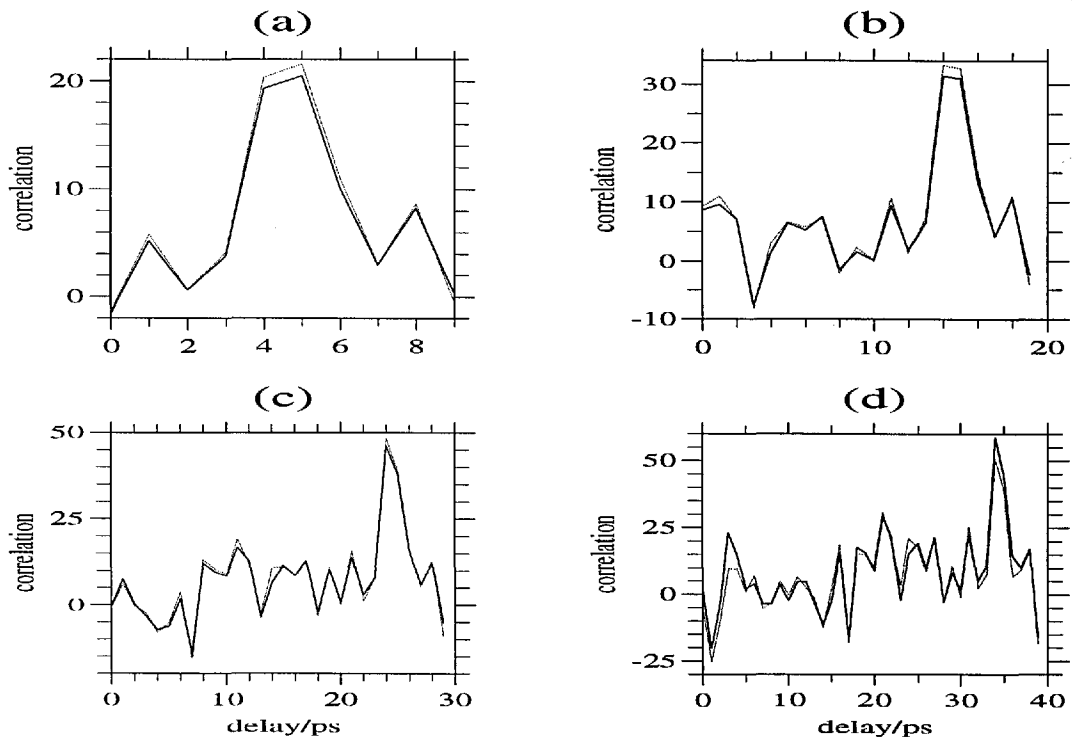


Figure 8.54: High intensity with Nernst convected (solid line) and unmagnetised transport (dashed line). Correlation between inverted ablation surface shape at time t and laser intensity at delayed time $t - \tau$.

Figure 8.54(a) correlates the shape of the ablation surface at time $t = 10$ ps against

instantaneous intensity $I(t - \tau)$ for various delays τ . There appears to be a correlation to the laser drive at $t = 5$ ps. The correlation is almost the same for unmagnetised transport (dashed line). The other figures, (b)-(d), refer to the ablation surface shape at $t =$ (b) 20, (c) 30 and (d) 40 ps. The influence of laser drive at $t = 5$ ps persists.

To conclude the results of this subsection indicate that there is little difference between Nerust convected and unmagnetised transport in the study of the coupling of laser drive to the ablation surface in the imprint process.

8.3 Ablation Pressure Nonuniformity for Different Intensities.

The ablation pressure distribution is fundamental in the physics of ICF. Shocks are driven by ablation pressure which in turn is limited by the laser intensity. These relationships are set out in Equation 1.1. It is evident from the density jump that high intensity drives a strong shock through the material, whilst low intensity does not do so.

8.3.1 Theory and Definitions

There are several means of determining the position and strength of the ablation surface.

We could choose the ablation surface to be the position in the ablative region where the fluid velocity is zero. Computationally this would require interpolation as cell velocities cannot be expected to be zero. Perhaps the cell with the minimum speed would do. An algorithm could be devised though the solution would not be straightforward, and maybe not even well defined, because of the development of secondary shocks (see for example the region $x \approx 70 \mu\text{m}$ in Figure 8.47).

A simple alternative is to define the ablation pressure for a lateral point x as the maximum pressure along the axial path $\hat{z}$. This is our chosen method. Again it is possible to drive a strong shock and then have a secondary shock with higher pressure following it. Following the ablation front laterally its axial position may change suddenly and its magnitude may not be smooth. We accept this as being part of the imperfections of the laser drive itself. Note this prescription is different to the first as turning points in velocity do not necessarily coincide with pressure maxima.

We choose the second scheme due to its clarity. Afterall we are not really concerned with the magnitude or the position of the front, rather with its degree of nonuniformity. Given a lateral ablation profile $P_a(x)$ we may obtain the mean ablative pressure $\langle P_a \rangle$ and a measure of nonuniformity σ_{rms}

$$\langle P_a \rangle = \int P_a(x) dx / \int dx \quad , \quad \sigma_{rms} = \frac{\sqrt{\langle P_a^2 \rangle - \langle P_a \rangle^2}}{\langle P_a \rangle}.$$

8.3.2 At Low Intensity

The transverse variation in ablation pressure is given in Figure 8.55. There is very little difference between the three transport models.

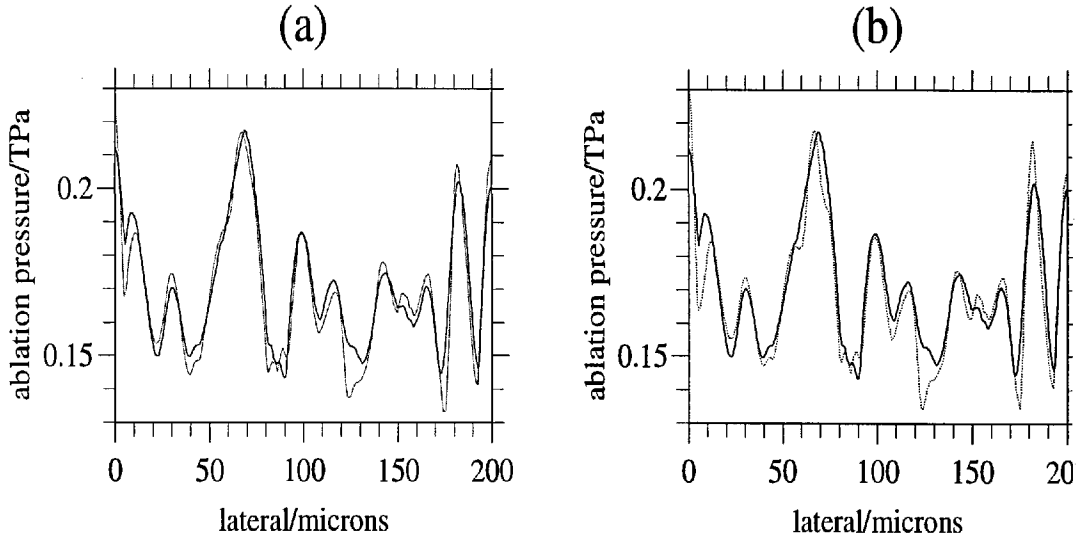


Figure 8.55: Low intensity at time $t = 60$ ps. Transverse variation of ablation pressure (in TPa) without magnetic fields (solid line) and with (a) magnetised, (b) Nernst convected transport (dashed lines).

For the case with no magnetic fields the variation in ablation pressure was $\sigma_{rms} = 0.106$. With magnetised transport $\sigma_{rms} = 0.116$, but when the fields were Nernst convected $\sigma_{rms} = 0.124$. The mean ablation pressure in all cases was unchanged at around $\langle P_a \rangle = 0.170$ TPa. See Figure 8.56(a).

Therefore, there is a slight increase in ablation pressure nonuniformity going from unmagnetised to Nernst convected transport. Using the value $I = 10^{13} \text{ W cm}^{-2}$ and $\lambda_{las} = 1.06 \text{ } \mu\text{m}$ in Equation 1.1 we obtain a theoretical value of $P_a = 0.290$ TPa. If we include the absorption fraction of a third this value is altered, $P_a = 0.139$ TPa. The

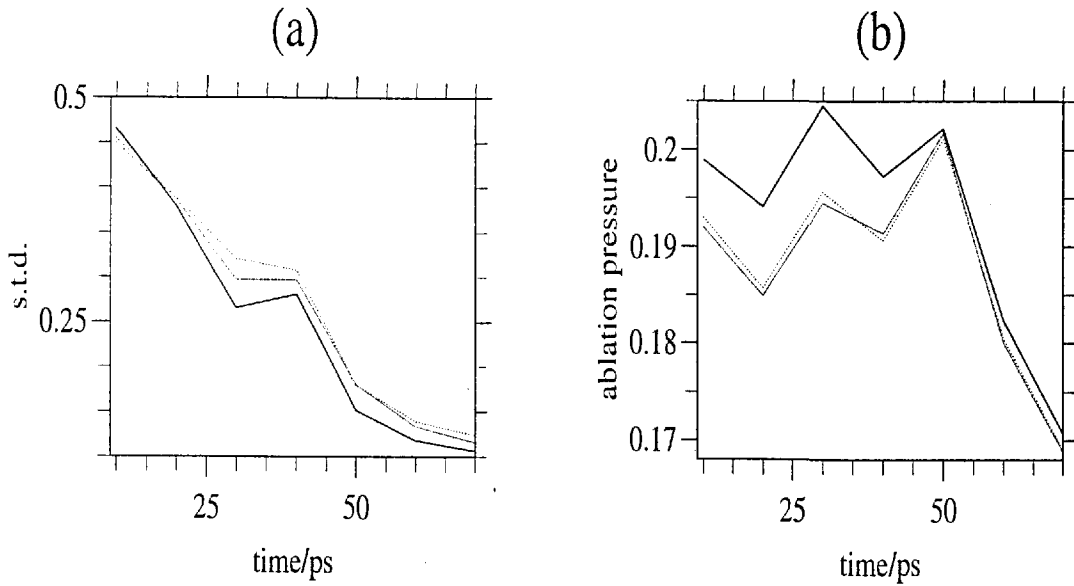


Figure 8.56: Low intensity, with no magnetic fields (solid line), with fields (dashed line) and with Nernst convected fields (semi-dashed line) (a) σ_{rms} of ablation pressure profile in lateral direction (b) mean ablation pressure in TPa, both as a function of time. Each line has seven points at $t = 10, 20, \dots, 70$ ps.

ablation pressure for all three cases is indeed of the same order as that given by theory. It should be remembered that the value given by theory refers to the ablation pressure in steady state which the state at $t = 60$ ps is not. See Figure 8.56(b).

Magnetised transport does not affect the ablation pressure at low intensity though it can lead to a slight enhancement of the perturbations that are present in unmagnetised transport.

8.3.3 At High Intensity

In Figure 8.57 we plot the ablation pressure at high intensity at time $t = 40$ ps and compare the different cases: no field, magnetised transport and Nernst convected fields. In general unmagnetised transport has long scale length modes with very short wavelength modes ($\approx 1 - 2 \mu\text{m}$) superimposed. Nernst convected transport tends to have perturbations in medium scale length modes.

Referring to Figure 8.58(a) we see that after $t = 40$ ps for no magnetic fields the variation in ablation pressure was $\sigma_{rms} = 0.163$. With magnetised transport $\sigma_{rms} = 0.171$, but when the fields were Nernst convected $\sigma_{rms} = 0.197$.

There is a slight reduction in mean ablation pressure when we compare results for

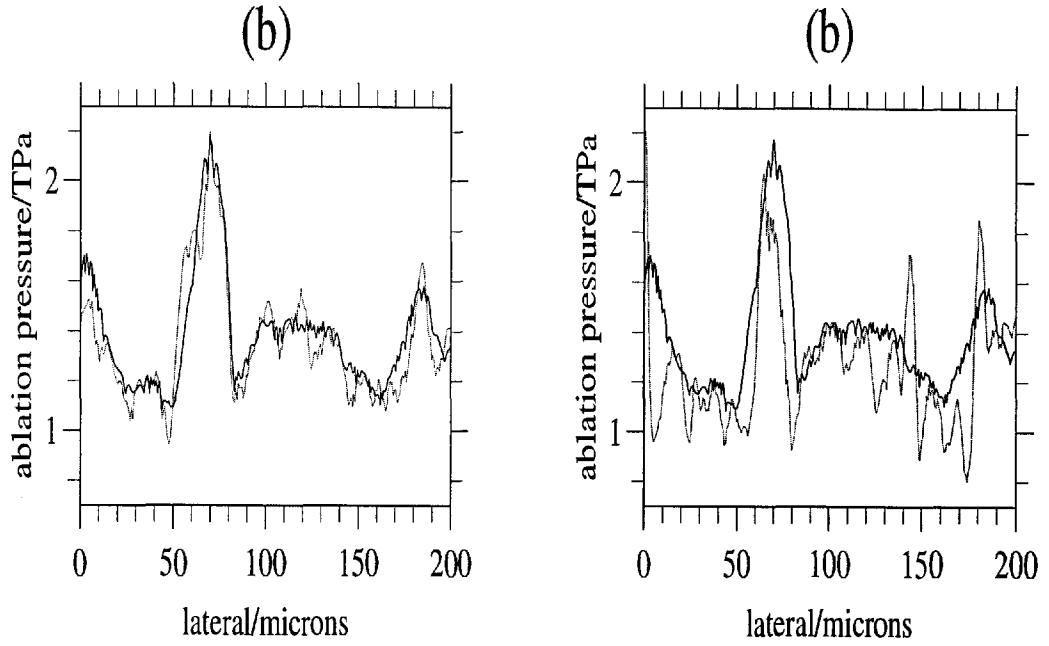


Figure 8.57: High intensity at time $t = 40$ ps. Ablation pressure in TPa for no magnetic fields (solid line) compared with (a) magnetised transport and (b) Nernst convected fields (dashed lines).

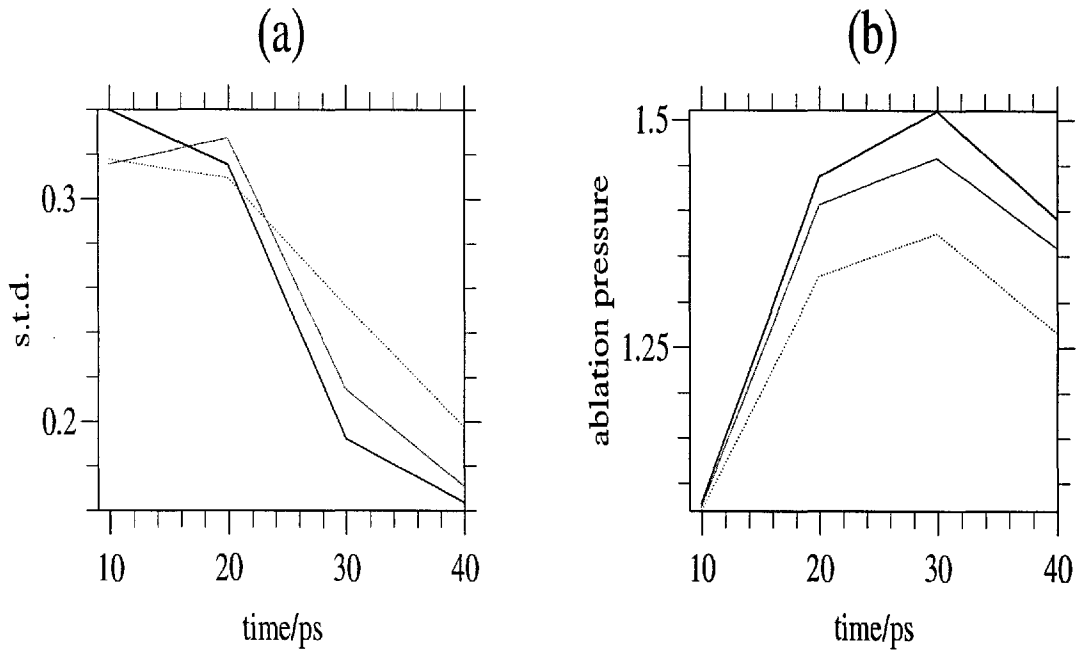


Figure 8.58: High intensity, with no magnetic fields (solid line), with fields (dashed line) and with Nernst convected fields (semi-dashed line) (a) σ_{rms} of ablation pressure profile in lateral direction $P_a(x)$. (b) Mean ablation pressure in TPa, both as a function of time. Each line has four points at $t = 10, 20, 30$ and 40 ps.

magnetised transport with unmagnetised transport, detailed in Figure 8.58(b). For unmagnetised transport the mean ablation pressure was 1.39 TPa. This agrees well with the theoretical value of 1.34 TPa obtained from Equation 1.1 when we let $I = 10^{14}$ W cm⁻² and $\lambda_{las} = 1.06 \mu\text{m}$. Including the absorption fraction of a third gives a theoretical value of $\langle P_a \rangle = 0.647$ TPa.

8.3.4 Discrete Fourier Transform of Ablation Pressure Profile

To quantify the perturbation wavelengths which manifest themselves in the ablation pressure we take the discrete Fourier transform of the central $N = 256$ grid points. This represents a range of $L = \frac{256}{280} \times 200 \mu\text{m} \approx 183 \mu\text{m}$. We do this because the fast Fourier routines we use require 2^m grid points ($m \in \mathbb{Z}^+$) and we are more concerned with what is happening in the centre than at the boundaries. The Fourier representation is given by

$$P_a(x) \approx a_0 + \sum_{m=1}^{N-1} \left(a_m \cos \frac{2\pi mx}{L} + b_m \sin \frac{2\pi mx}{L} \right)$$

We define the amplitude as $c_m = \sqrt{a_m^2 + b_m^2}$ and plot this amplitude for the various cases in Figure 8.59, in the range $1 \leq m \leq 30$. For example, in the case of unmagnetised transport the peak occurs at $m = 3$ and has an amplitude of 0.21 TPa. This means the dominant sinusoidal mode has three wavelengths in the range $L = 183 \mu\text{m}$, i.e. a wavelength of $61 \mu\text{m}$.

Nernst convected transport reduces the amplitude of the three lowest modes and increases the higher modes. In particular there is increased activity around $m = 16$ which corresponds to a wavelength of $183 \mu\text{m}/16 = 11.4 \mu\text{m}$. The author hopes that the growth rates of the individual modes will be examined in future work.

8.3.5 Discrete Fourier Transform of Ablation Front Position

It is possible to have P_a constant along the ablation front but in different $\hat{z}$ positions. This is a pressure nonuniformity as ideally we want the ablation front equidistant from the target core. In Figure 8.60 we plot the axial position $Z(x)$ of the ablation front relative to the left-hand boundary.

We perform a discrete Fourier expansion of the position $Z(x)$ in a similar manner to Equation 8.32. In Figure 8.61 we plot the amplitude (in microns) of the sinusoidal

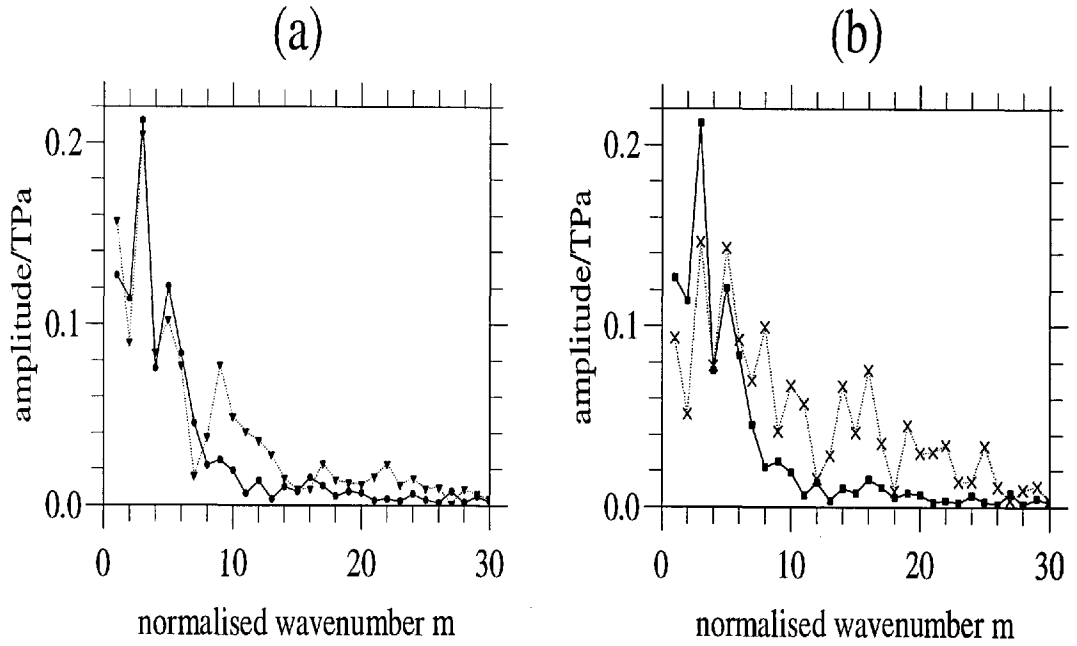


Figure 8.59: High intensity at time $t = 40$ ps. Amplitude (in TPa) against normalised wave number after taking discrete Fourier transform of the ablation pressure profile for the case of no magnetic fields (■) compared with (a) magnetised transport (▼) and (b) Nernst convected fields (×).

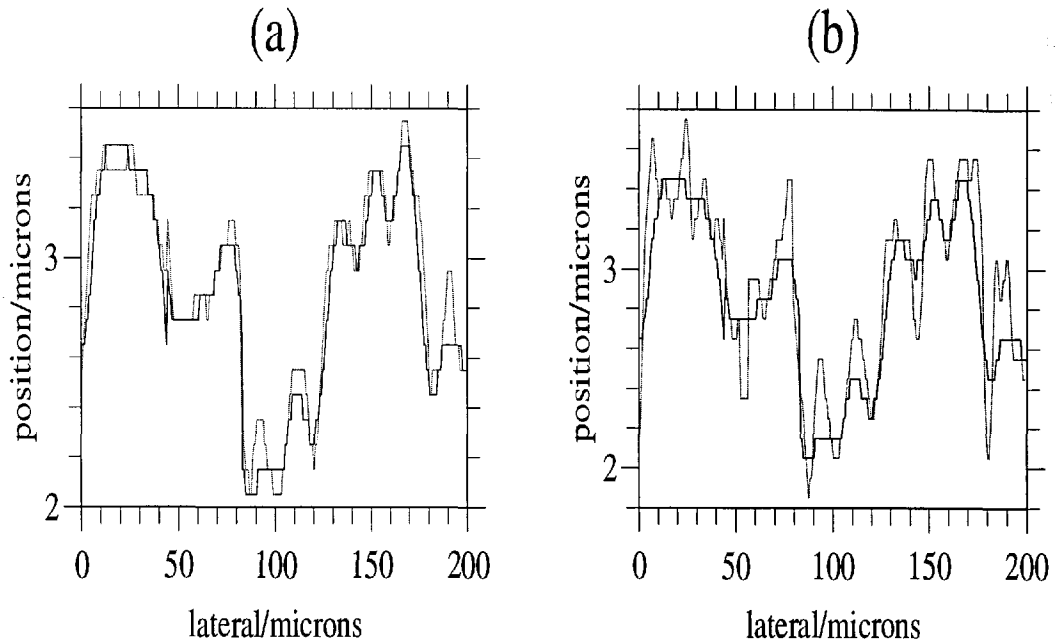


Figure 8.60: High intensity at time $t = 40$ ps. Position of ablation surface from left-hand boundary in μm for the case of no magnetic fields (solid line) compared with (a) magnetised transport (dashed line) and (b) Nernst convected fields (dashed line).

modes m . Comparing unmagnetised transport with Nernst convected transport we see the dominant $m = 1$ mode is unchanged (along with other lower modes) whereas the higher modes increase in amplitude. In particular there is enhancement of $m = 10$ and 16 corresponding to wavelengths $\lambda = 18.3$ and 11.4, respectively.

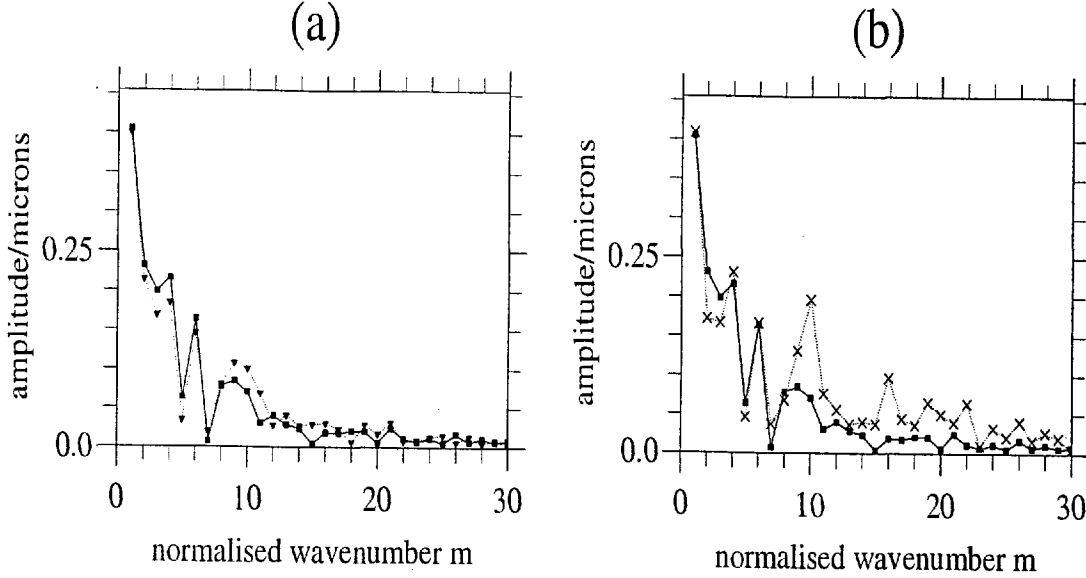


Figure 8.61: High intensity at time $t = 40$ ps. Amplitude (in microns) against normalised wave number after taking discrete Fourier transform of the ablation surface axial position for the case of no magnetic fields (■) compared with (a) magnetised transport (▼) and (b) Nernst convected fields (×).

The results are on the pessimistic side as two-dimensional flow would tend to flatten the ablation surface.

8.4 The Generation of Magnetic Fields and Their Convection

In theory fields are produced globally but it turns out that the dominant contribution in low density material is from the plasma edge, not from the critical surface. The density term of the induction equation goes like $\nabla(\ln \rho)$ and this quantity has the largest magnitude at the plasma edge. Any temperature variations at critical quickly appear at the plasma edge as the density in the subcritical region is very low. This explains why the principal source of fields is in this subcritical region.

In fact, it was the generation of a large localised field at the plasma edge which led the author to introduce resistivity into the calculations. Although computationally

expensive, resistivity is necessary. Naturally, the choice of initial density profile has a strong influence on the results.

8.4.1 At Low Intensity

Magnetised transport

Initially the plasma edge is located at $z \approx 6 \mu\text{m}$ and starts to move rightwards, continually generating fields in the regions it traverses. The edge is situated at $z \approx 21 \mu\text{m}$ after $t = 60 \text{ ps}$.

There are two magnetic field generation mechanisms to consider. Firstly, fields are produced at the plasma edge which is close to the ablation surface at early times. As this edge propagates outwards, it leaves residual fields behind due to resistive decay (diffusion). Secondly, fields are also generated in the overdense region close to the ablation surface and these sources correlate laterally with the creation of fields at the plasma edge. ‘Ablation’ fields have a higher resistive decay rate due to the low temperature in the overdense region.

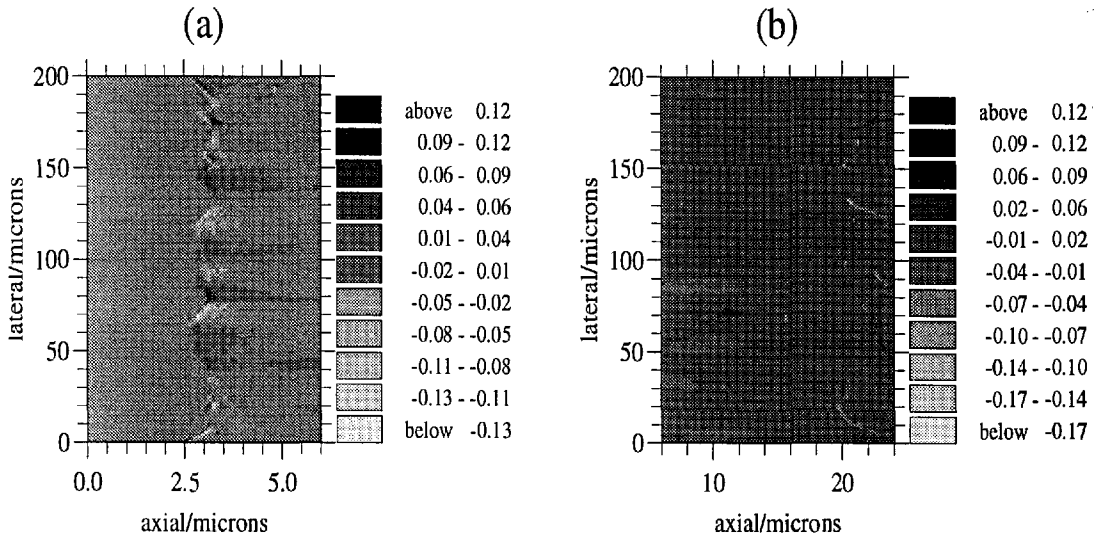


Figure 8.62: Low intensity at time $t = 60 \text{ ps}$, magnetised transport. Generation of magnetic fields in MG ps^{-1} (a) around the ablation surface (b) in the super/sub critical regions.

From Figure 8.62 we see that at time $t = 60 \text{ ps}$ the ‘ablation’ source is as large as the plasma edge source. Figure 8.63 (a) illustrates that the fields have a long axial scale length structure stretching right up to the ablation surface, in some parts.

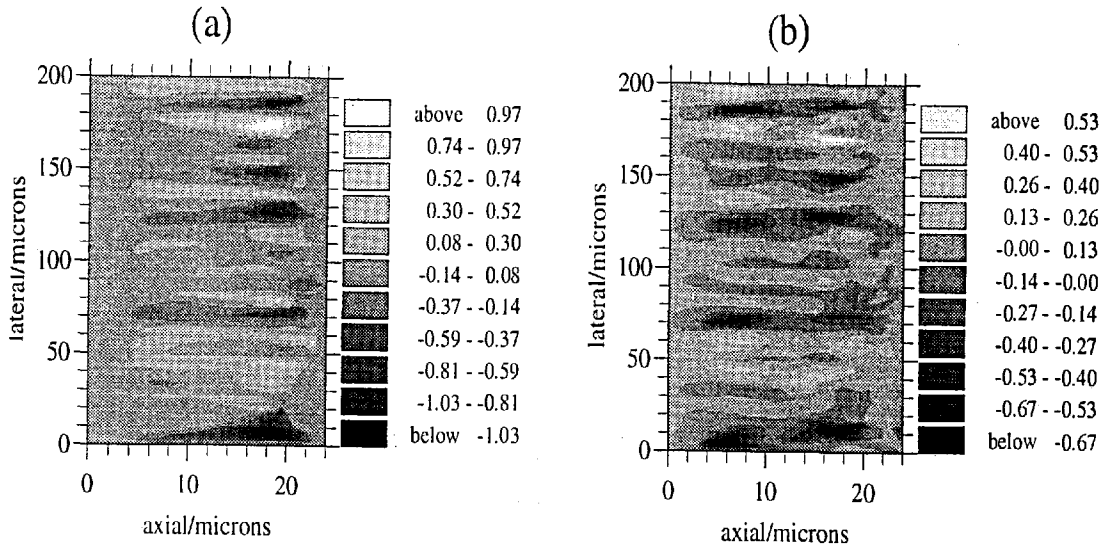


Figure 8.63: Low intensity at time $t = 60$ ps. Magnetic fields in MG (a) magnetised transport (b) Nernst convected transport.

In general the time scale for magnetic fields at the plasma edge is much larger than that for other state variables because fields come about from the cumulative effect of temperature variations. These perturbations are in turn contingent upon time-averaged laser drive. In addition the time scale of fields at the ablation surface is even larger still because of the effect of lateral thermal smoothing. As time proceeds the critical-ablation distance increases which implies that the temperature perturbations at ablation become smoother. Therefore, the magnetic source at ablation is more dependent on early laser drive.

The arguments of the last paragraph allude to a form of imprint though one must recall that magnetised transport at low intensity does not lead to results that differ much from unmagnetised transport.

Nernst Transport

We now account for Nernst convection of the fields. From Figure 8.63(b) we see the magnetic fields appear to be located near the ablation surface and near the plasma edge, with little field in between. This is in contrast to the magnetised transport model illustrated in Figure 8.63(a) in which the fields are principally situated just behind the plasma edge. We note for the Nernst model the fields at the plasma edge are smaller than those for the magnetised transport model. The converse is true in the

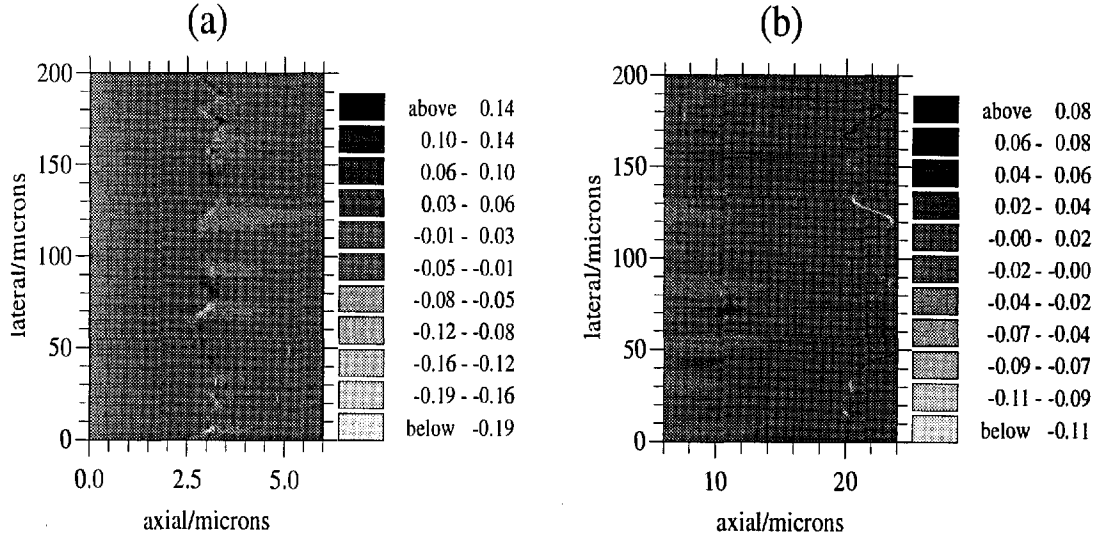


Figure 8.64: Low intensity at time $t = 60$ ps, Nernst convected fields. Generation of magnetic fields in MG ps^{-1} (a) around the ablation surface (b) in the super/sub critical regions.

overdense region.

There is $\approx 33\%$ reduction in field generation at the plasma edge and a slight increase in the source at the ablation surface. Since the fields in both models are spread over the same region there is no compression effect. Nernst convection just relocates fields placing them closer to the ablation surface. We can say this because there is little difference in the generation mechanisms between the two models.

Analysis carried out by Haines and others [76,78] (see section 3.6) suggests that the magnetic fields are located around the region where the total convective velocity $v_i + v_N + v_\alpha$ is zero. This should also be the point where the electron mean free path λ_{mfp} equals the collisionless skin depth λ_{csd} .

In the coronal region λ_{mfp} is not very much larger than λ_{csd} and both are roughly of the same order in the sub/supercritical regions. The electromagnetic instabilities of Section 3.5 (which require $\lambda_{mfp} \gg \lambda_{csd}$ to operate) are not expected to have a role in our model. In particular if the plasma Z were higher and the laser wavelength were shorter the necessary and sufficient conditions for the collisional Weibel instability may be met. Nevertheless, it cannot be the cause for the filamentary structure we observe in our results because it can only be seen in a kinetic model [70].

From Figure 8.63(b) we see the field maxima and minima in the overdense region

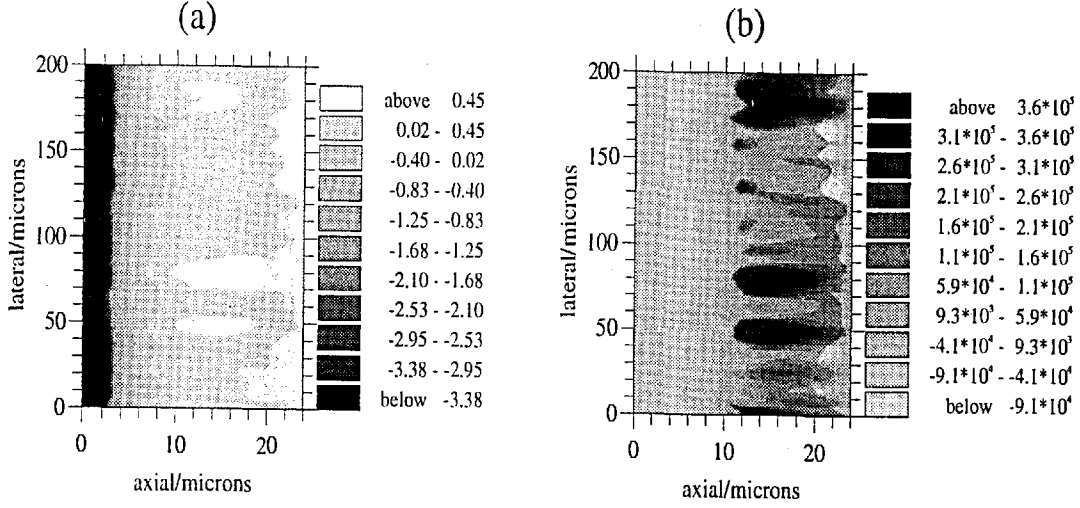


Figure 8.65: Low intensity at time $t = 60$ ps with Nernst convected fields. (a) Logarithm of the ratio $\lambda_{mfp}/\lambda_{csd}$. (b) Sum of fluid, Nernst and convective diffusion velocities $v_i + v_N + v_\alpha$ in ms^{-1} .

are located almost exactly in the plane $z = 6 \mu\text{m}$ (apart from the field at the boundary $x \approx 0 \mu\text{m}$). The logarithm of the ratio $\lambda_{mfp}/\lambda_{csd}$ is illustrated in Figure 8.65(a). We see $\lambda_{mfp} = \lambda_{csd}$ at $z = 6 \mu\text{m}$. Also this is the surface where the total convective velocity of the fields is zero, from Figure 8.65(b). The plane $z = 6 \mu\text{m}$ happens to be where the plasma edge was located originally.

We note the total convective velocity is negative in parts of the ‘vacuum’ region near the plasma edge. Of more importance, fields in the overdense region near the ablation surface are actually convected inwards. Any fields which are created here cannot be convected away though they can diffuse away because of the high resistivity.

8.4.2 At High Intensity

The fields at time $t = 40$ ps for magnetised transport and Nernst convected transport are shown in Figure 7.38(b) and Figure 7.41(b), respectively. For the Nernst model the fields are practically contained within the overdense region and are also 50-80 % larger. This latter point could be due to compression; for the case with no Nernst term the fields extend over most of the grid (around $18 \mu\text{m}$) and with the Nernst term these fields are spread over approximately $10 \mu\text{m}$. However, one must question whether the magnetic field generation is the same in both scenarios.

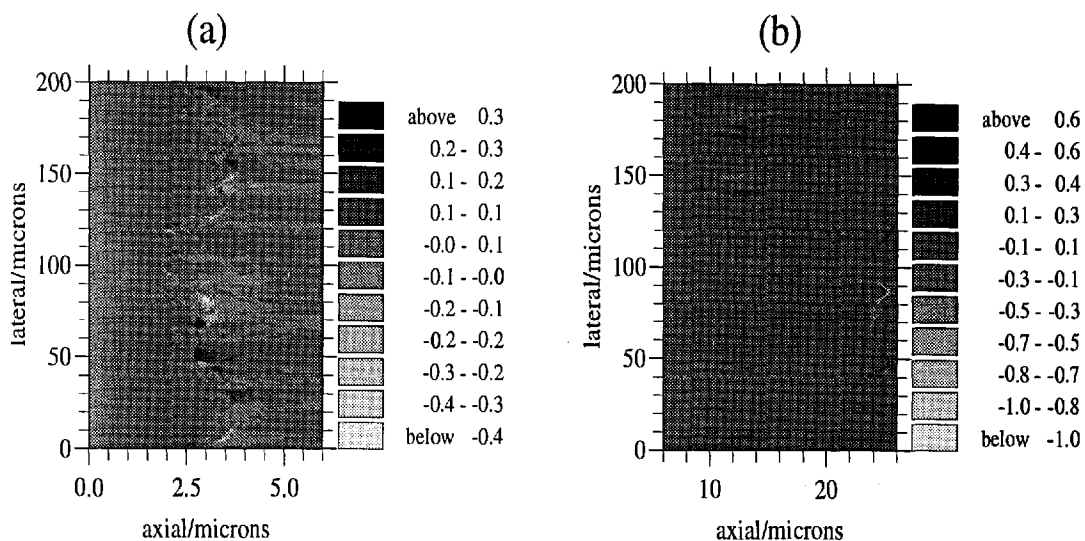


Figure 8.66: High intensity at time $t = 40$ ps with magnetised transport. Generation of magnetic fields in MG ps^{-1} (a) around the ablation surface (b) in the super/sub critical regions.

The magnetic sources for the two models are shown in in Figure 8.66 and Figure 8.67, respectively. Comparing the Nernst model with the magnetised transport model the source at the plasma edge is almost unaffected whilst the source at the ablation surface is over an order of magnitude larger. Therefore, we state that the larger fields are due to a combination of spatial compression and a partial relocation of field provenance. The source term has a definite localised sign at the plasma edge but it has a wedge-like structure at the ablation surface. Laterally its sign varies between adjacent cell groups here.

There is the question of why we see compression at high intensity and not a low intensity. Compression occurs at higher intensities because the heat flow is naturally larger. This causes larger Nernst velocities. (Of course it could be the case that at low intensity the fields will be compressed if we carry out the calculations past $t = 60$ ps.)

In Figure 8.68 we decompose the source term into its components, that due to gradients in (a) temperature laterally and density axially and (b) temperature axially and density laterally. The source term at the plasma edge is seen to be due to the lateral temperature and the axial density gradients as expected. However, both components exhibit a kinked, ‘solenoidal’ form near the ablation surface with the product of the axial temperature and lateral density gradients being slightly larger in magnitude. It

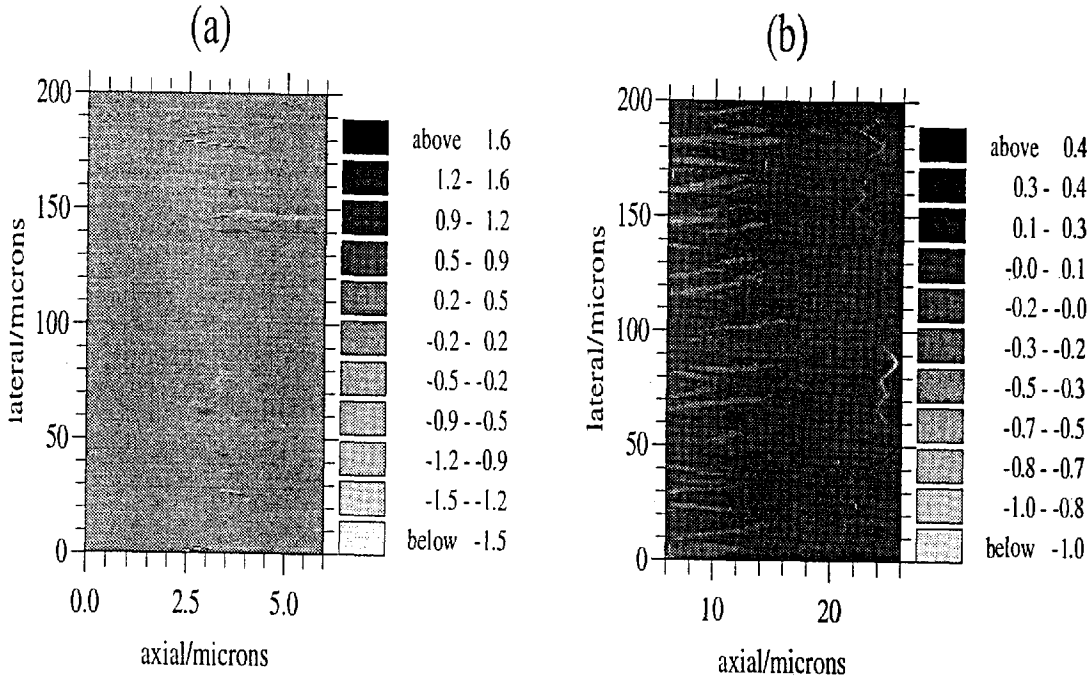


Figure 8.67: High intensity at time $t = 40$ ps with Nernst convected fields. Generation of magnetic fields in MG ps^{-1} (a) around the ablation surface (b) in the super/sub critical regions.

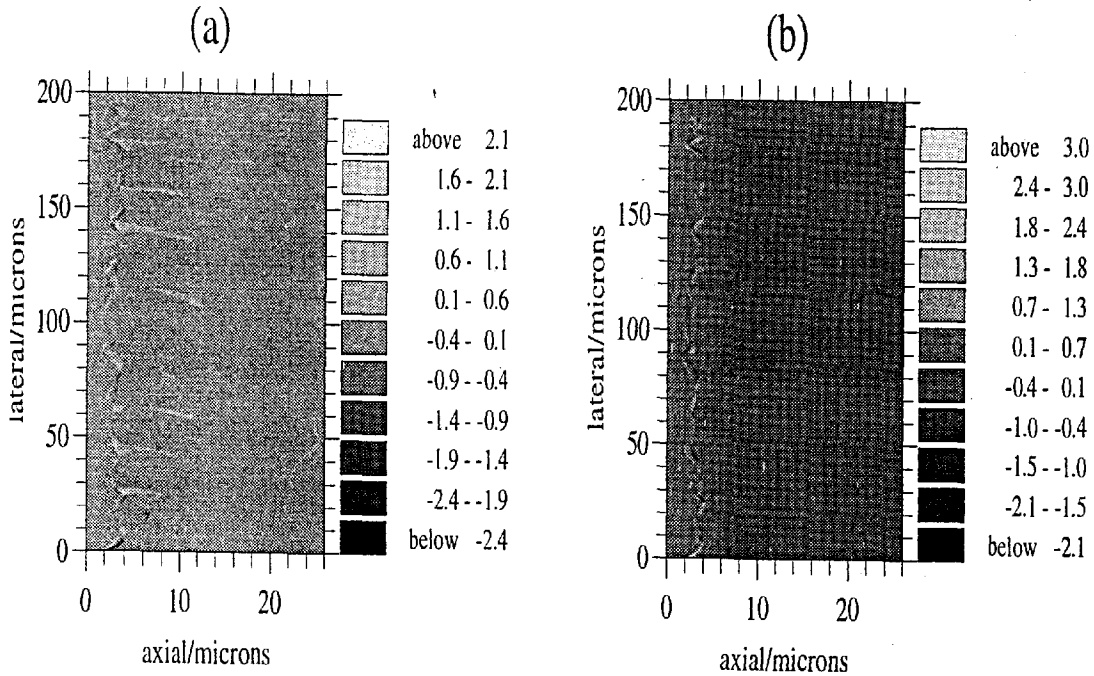


Figure 8.68: High intensity at time $t = 40$ ps, Nernst convected fields. Generation of magnetic fields in MG ps^{-1} by (a) $\nabla_x T \cdot \nabla_x \rho$ (b) $\nabla_z T \cdot \nabla_x \rho$ terms.

might be possible to associate currents with these structures.

The theory of Haines and others [76, 78] (see section 3.6) describes the Nernst velocity carrying fields outwards away from the critical surface into the subcritical region. This is not so in our model. For reasons that will be detailed in our discussion on heat flow, Section 8.6, the coronal matter is actually at a higher temperature. The Nernst velocity is directed away from the plasma edge towards the critical surface, enhancing the fields within the plasma, instead of convecting them away.

Our final comment refers to the general trail of magnetic fields in the inwards direction. The trend is laterally downwards for negative magnetic fields and laterally upwards for positive fields. This is unexpected. An attempt to explain this behaviour will be given in Section 8.8.

8.5 Can the Currents Sustain the Fields?

The current is physically limited in magnitude by $n_e e v_t$. We plot $|\mathbf{J}|/n_e e v_t$ in Figure 8.69(a) to check that the electrons are able to sustain such currents. In general the drift speed is 10^{-4} of the thermal speed: electrons can support the magnetic fields.

It is also necessary to check the drift speed in comparison to the ion sound speed. See Figure 8.69(b). This ratio is at most of order 10^{-2} . This corroborates current consensus that anomalous processes caused by electrostatic or electromagnetic turbulence are absent in ICF: ion acoustic effects and anomalous resistivity are of little consequence.

8.6 Heat Flow through a Conduit

After some time the temperature maxima are not located at the critical surface, rather in the coronal region. The heat flow is certainly at maximum at the critical surface when dealing with a planar plasma in steady state but our plasma is far from being so. In our code laser energy is dumped over a region of axial width $3 \mu\text{m}$. Some of the low density coronal matter near the plasma edge is heated to a very high temperature because the ramp spans only $\approx 2 \mu\text{m}$. This matter is convected outwards and always remains at a higher temperature. Also, ohmic heating occurs at the plasma edge where the fields are large, the plasma edge being further out than critical.

Higher coronal temperatures could be eliminated if we employed a temporal ramp for the laser intensity. Now heat in the corona would be due to earlier drive, i.e. lower

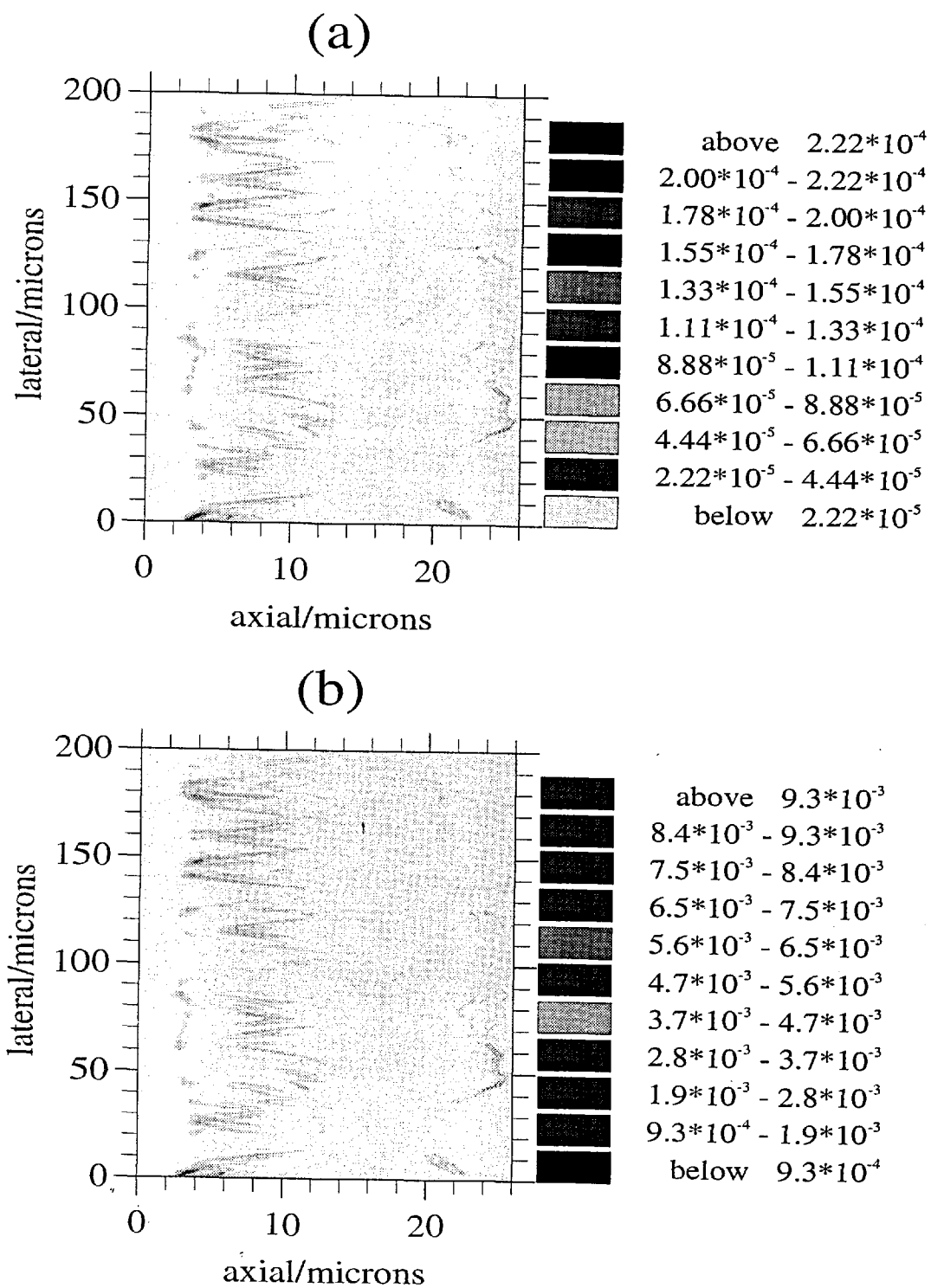


Figure 8.69: High intensity at time $t = 40$ ps with Nernst convected fields. Ratio of the electron drift speed to the (a) electron thermal speed, (b) to the ion acoustic speed.

intensities. Consequently the coronal temperature would be lower than ours. Having reported our effect we point out that the heat flow from corona to critical is dwarfed by that from the critical into overdense regions which is the domain we are more concerned with in our investigations.

In Figure 8.70 we plot the Hall parameter χ across the grid for (a) high and (b) low intensity. χ was artificially limited to $\sqrt{10}$. Without this limit the Hall parameter would be of order 10^4 or higher at the plasma edge. The critical surface is at $z \approx 14 \mu\text{m}$ in both cases.

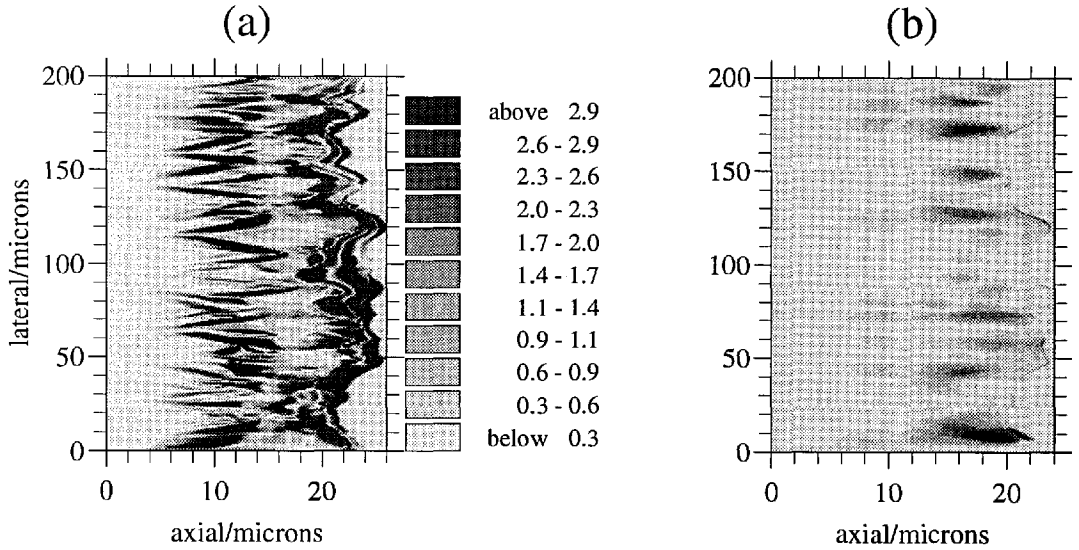


Figure 8.70: Hall parameter across the grid for (a) high intensity at $t = 40$ ps and (b) low intensity at time $t = 60$ ps.

Lateral thermal smoothing depends upon the Hall parameter. For this reason we are concerned with the value of χ in the supercritical/overdense region. For low intensity χ is of order one in the subcritical and coronal regions but at higher densities it is generally less than one. However, at high intensity χ is of order one in the supercritical/overdense region¹¹; lateral thermal transport is influenced strongly by magnetic fields at high intensity whilst it is not at low intensity.

At higher laser intensity the temperatures go up and naturally so does the Hall parameter. Whereas the perpendicular conductivity is strongly dependent on χ , the heat flow Q is not. This is because the temperature gradients increase so Q is not

¹¹Incidentally this includes regions where $\lambda_{mfp} > \lambda_{csd}$.

as affected as $\kappa_{\perp}$. In fact any decrease in Q must reduce magnetic field amplification because the Nernst velocity is dependent on the heat flow.

Figure 8.71 concentrates upon the funnel-like structure in the Hall parameter distribution centred upon $x = 115 \mu\text{m}$ at high intensity. The heat flow (arbitrary units) is superimposed in (a). It points inwards for most of the grid, being rather small around the plasma edge because of the smaller temperature gradients there. The vectors are convergent towards the inlet of the funnel and divergent thereafter.

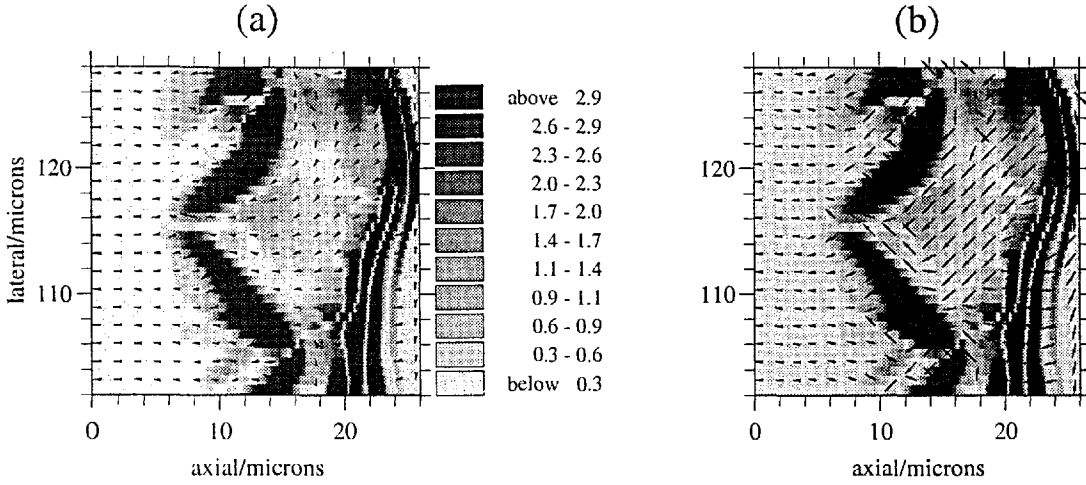


Figure 8.71: High intensity at $t' = 40$ ps. Funnel-like structure in region given. (a) Heat flow Q (in arb units) (b) Nernst velocity v_N (in arb units). The class scaling which both figures share refers to the underlying Hall parameter.

The superimposed Nernst velocity is also plotted in (b). The velocity is artificially bounded by the magnetosonic sound speed in our code otherwise prohibitively small time steps are needed. In general the Nernst velocity points in the same direction as the heat flow though its magnitude varies because of the pressure dependence $v_N \propto Q/P_e$. Our model uses only the axial component of this velocity.

We remain with the funnel and concentrate on a plane of fixed axial z . The inward, axial Nernst speed is on the whole larger in the regions close to the axis of the funnel, which is at $x = 115 \mu\text{m}$, than at other outer transverse regions. The contrast is even more apparent near the inlet of the funnel. This is illustrated in Figure 8.72(b).

The heat flow is well behaved, being smaller than 30 % of the free streaming limit. This does not necessarily preclude a heat flow limit, it is merely a good sign. One needs

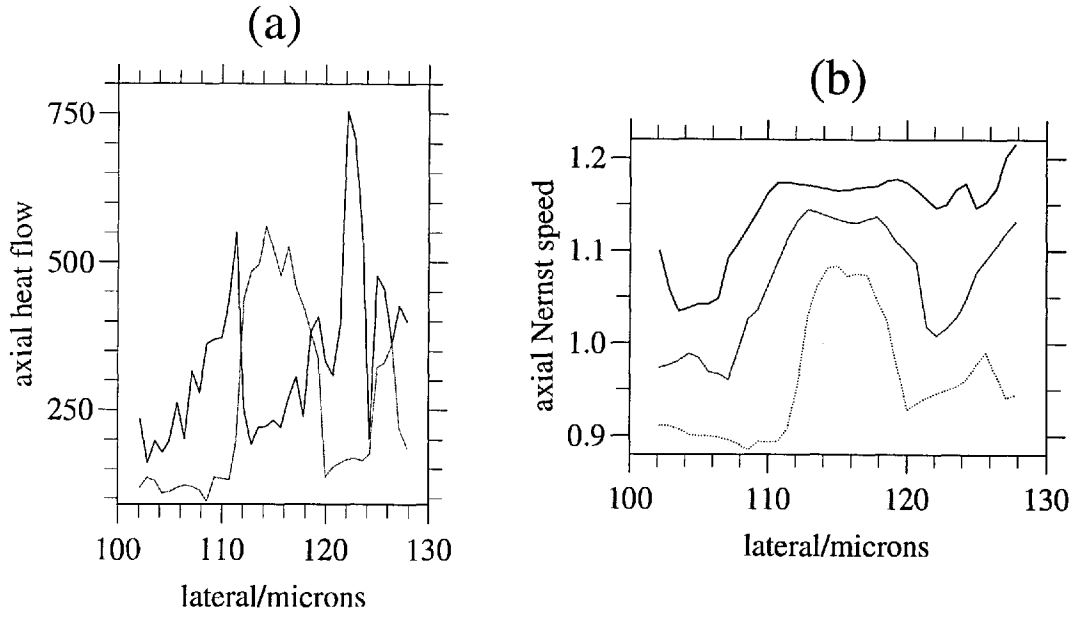


Figure 8.72: High intensity at $t = 40$ ps. Lateral profile of inward axial (a) heat flow Q (in arb units) in plane $z = 12 \mu\text{m}$ (solid curve) and plane $z = 9 \mu\text{m}$ (dashed curve) and (b) Nernst velocity v_N (in arb units) in plane $z = 13 \mu\text{m}$ (top), $z = 11 \mu\text{m}$ (middle), $z = 9 \mu\text{m}$ (bottom).

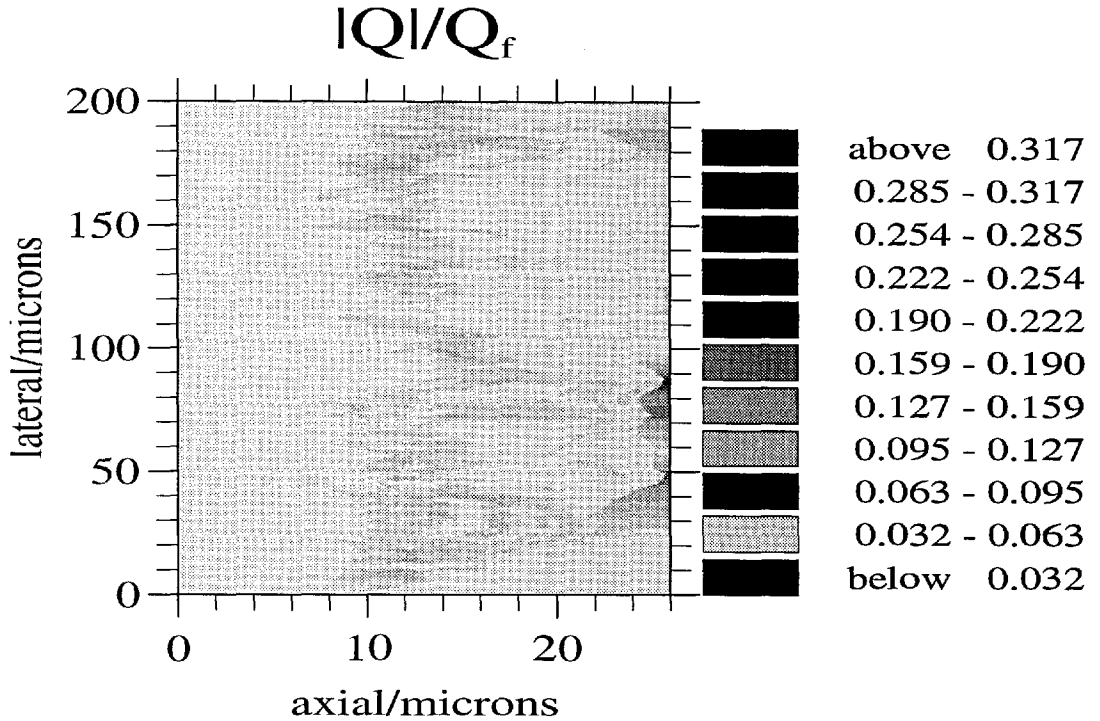


Figure 8.73: High intensity at time $t = 40$ ps with Nernst convected fields. Heat flow normalised to local free streaming limit.

to check that the ratio of the electron mean free path to the temperature gradients is not too large otherwise the heat flow becomes nonlocal.

In the coronal and subcritical region $\lambda_{mfp} < 40 \mu\text{m}$. For the supercritical region $\lambda_{mfp} < 4 \mu\text{m}$. For the overdense region $\lambda_{mfp} < 0.1 \mu\text{m}$. In Figure 8.74(a) we plot the logarithm to base ten of the ratio $L_x(T)/(150\lambda_{mfp})$. When this ratio is less than unity we expect nonlocal heat flow. The ratio is less than unity in most places. For instance in the plane $z \approx 11 \mu\text{m}$ the electron mean free path is about an order of magnitude smaller than the temperature scale length. The heat flow is just about nonlocal here.

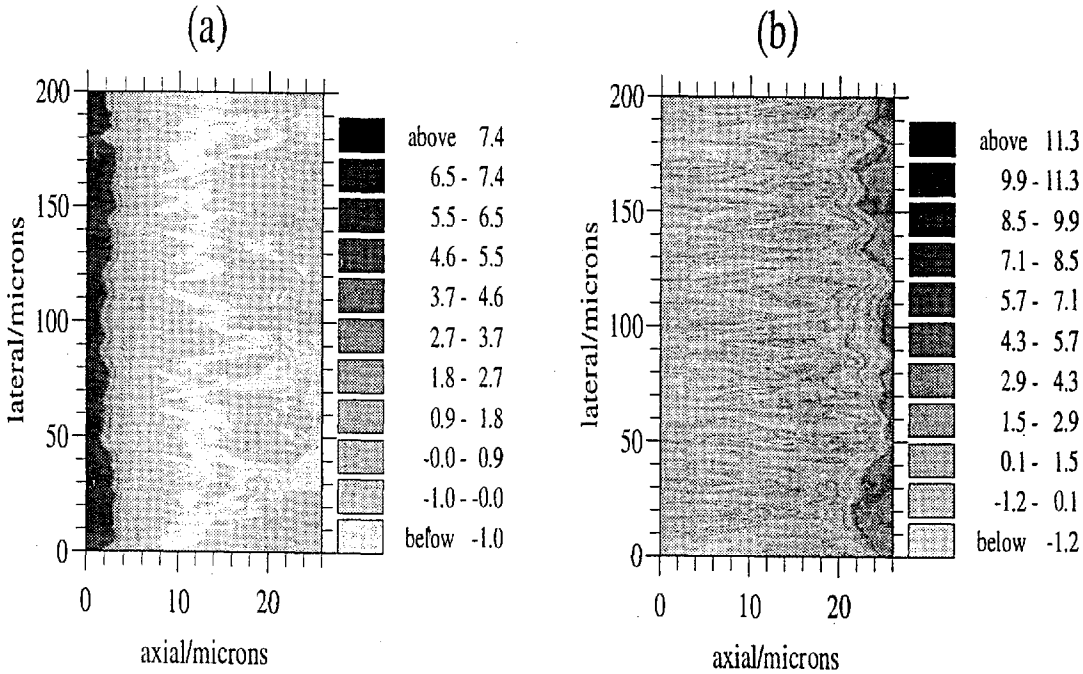


Figure 8.74: High intensity at time $t = 40 \text{ ps}$ with Nernst convected fields. (a) Logarithm (base 10) of the ratio of temperature scale length to $150 \times$ the electron mean free path, (b) logarithm (base 10) of the ratio of the electron Larmor radius to the magnetic field scale length.

The electron Larmor radius typically varies between $10^{-1} - 10^7 \mu\text{m}$. In Figure 8.74(b) we plot the logarithm to base ten of the ratio of the Larmor radius to the magnetic field scale length $L(B)$. For the regions where the strongest fields exist, this ratio is less than 0.1. The electrons effectively see a homogeneous field during one Larmor orbit. The general random nature of the fields and a large ratio suggests using Bohm-like diffusion instead of Braginskii-like diffusion.

8.7 Energy Conservation of the Code

Data was collected for 900 time steps which represented a laser duration of approximately $t_L = 44$ ps. We define a quantity X which is the sum of thermal, kinetic and magnetic energies. We calculate the energy lost δX to the right-hand boundary over the time duration t . The total energy at time $t = 0$ is X_0 . Next we calculate the energy deposited by the laser Y over the period t . The percentage error is given by

$$\% \text{ error}(t) = \frac{X + \delta X - X_0}{Y} \times 100 \%$$

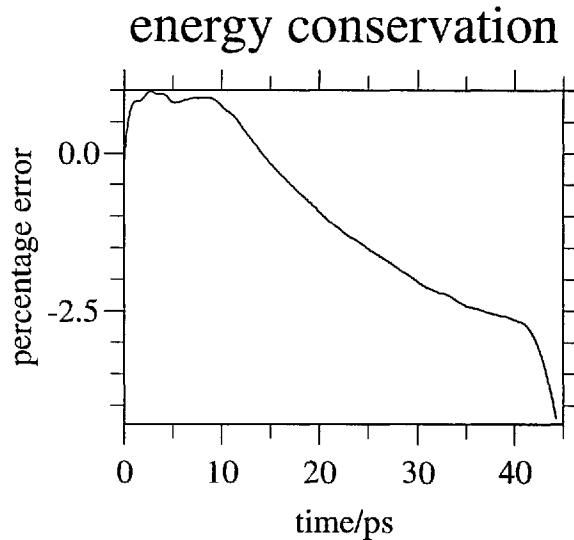


Figure 8.75: High intensity, Nernst convected fields. Percentage error in energy conservation.

The error, shown in Figure 8.75, is less than 5 % for the duration of the run t_L .

8.8 A Mechanism for Forming a Heat Flow Conduit

Any proposed mechanism for overdense filaments has to involve Nernst convection of magnetic fields as these filaments do not exist without this term. It would also have to explain the wedge-like formation of the filaments. The author gives his mechanism in outline.

It turns out that the inward, axial Nernst speed is largest at the confluence of a

heat flow conduit. This is demonstrated in Figure 8.72(b). Although the pressure is higher at the confluence, the axial heat flow is even larger.

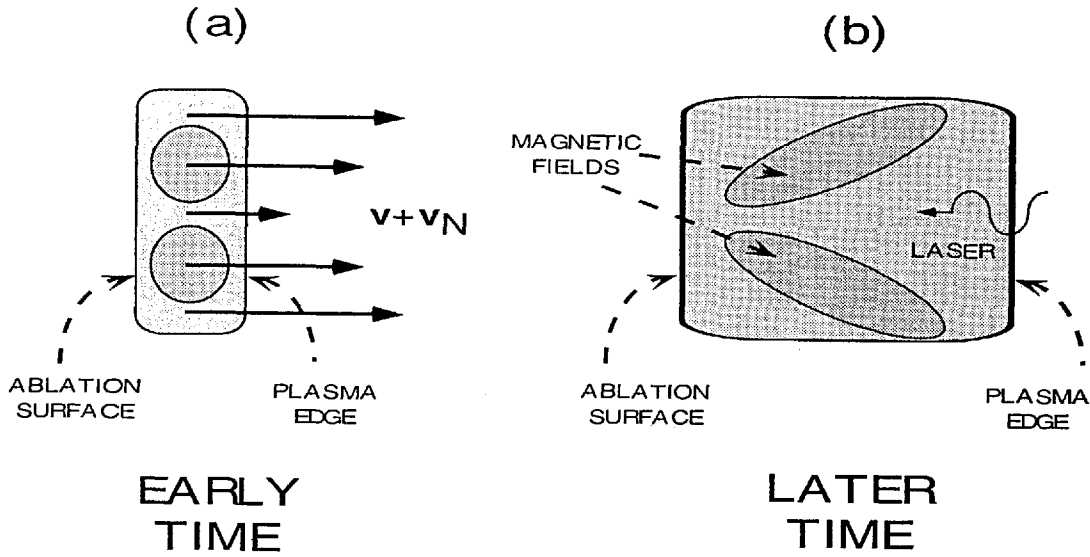


Figure 8.76: Effect of the total convective velocity on the magnetic fields around a hot spot. The fields closer to the axis of the hot spot lag behind.

Thinking of the Nernst term as a form of impedance on ablative convection of fields, the fields closer to the centre of a hot spot are dragged behind those further away from the hot spot. This explains the wedge-like structure of the conduits. A schematic of this mechanism is shown in Figure 8.76. The plasma edge and the ablation surface are shown very close together at early times. To a first approximation we assume that the fluid speed v is invariant laterally. The two surfaces are advected with the fluid velocity and so remain uniform. At later times the wedge-like formation of magnetic fields is seen.

One must understand that those fields which are convected further into the target decay at a faster rate because of lower temperatures. This would be the end of it if it were not for the 'isotropic' contribution, $\eta \nabla^2 B$, of the resistivity term $\nabla (\eta \nabla B)$. The temperature increases monotonically moving laterally into a hot spot. This implies that the resistivity decreases in that direction: central fields decay at a slower rate than outer fields. This mechanism collaborates with lateral gradients in axial Nernst speed to realise wedge-like structures.

Why are conduits seen at high intensity and not at low intensity? It is clear that higher intensities entail higher temperatures. At low intensity some fields are convected

inwards but because of the lower temperature further into the plasma, they resistively decay. At high intensity they are convected inwards but the temperatures are generally higher. Not only do the higher temperatures increase the growth rate of magnetic fields (compare Figure 8.64 with Figure 8.67) but they also increase the collision time. The latter leads to a larger Hall parameter (compare Figures 8.70 (a) and (b)) and more reduced heat flow. All these factors suggest an instability at higher temperatures.

Having given our mechanism we discuss an important component of the physics which enhances our effect. This is the characteristic velocity associated with diffusion.

8.8.1 Diffusive Convection

The term we are concerned with here is the ‘nonisotropic’ part, $\nabla\eta \cdot \nabla B$, of the diffusion term $\nabla(\eta \nabla B)$. It has the character of convection, with a velocity $\mathbf{v}_\alpha = \nabla\eta$, and may be viewed locally as a source term (the $\eta \nabla^2 B$ converts magnetic energy into thermal energy, locally and globally, and is a sink). In the following we make no reference to the magnitude of gradients, only to their direction. This is done to add clarity to the arguments though the author recognises a more rigorous discussion would have to take variations in gradient into account.

Diffusive convection generally creates larger maxima and minima at new locations in the plasma. Diffusion cannot do this so we must remember to add on the effect of the isotropic part of the resistivity in order to be physically consistent. In the diagrams that follow the new fields will have larger maxima and minima than the old fields. They will actually be smaller after including isotropic diffusion. If we do this we see that diffusive convection leads to preferential decay rates of certain magnetic fields depending upon their location relative to the centre of a hot spot.

Axially

The temperature decreases as we move axially into a hot spot. This implies that the resistivity gradient points in the opposite direction. Taking a single region of positive fields the preferential action of $\mathbf{v}_\alpha$ causes outer fields to be enhanced at the expense of inner fields. This is shown in Figure 8.77.

The same result is true if we reverse the polarity of the fields: the outer fields become more negative, whilst the inner fields decay.

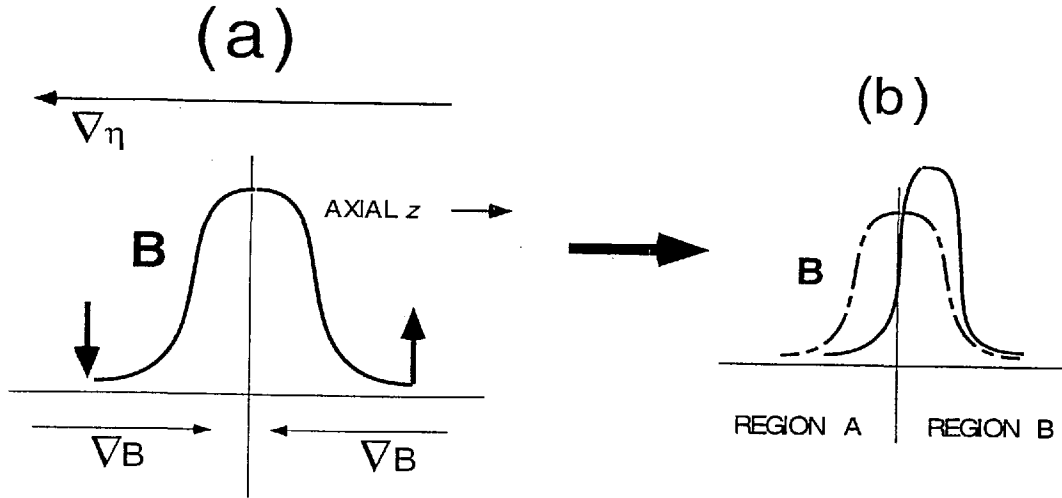


Figure 8.77: (a) Axial profile of a region of field with positive polarity. The resistivity increases from right to left. (b) Fields lying nearer to the right are preferentially enhanced at the expense of inner fields.

Laterally

Studies carried out by Nishiguchi, Yabe and Haines (see Section 3.6) have been concerned with the axial component of $\mathbf{v}_\alpha$ in describing where the fields are located (where $\mathbf{v}_i + \mathbf{v}_N + \mathbf{v}_o = 0$). However, its lateral component turns out to be quite an important ingredient in the description of the wedge-like structure of conduits.

We start with a hot spot surrounded by fields as shown in Figure 8.78(a). The temperature increases as one moves laterally towards the centre of the hot spot. This implies the resistivity gradients point outwards away from the hot spot. With respect to magnitudes, when $\mathbf{v}_\alpha$ is combined with gradients in magnetic field we see that the inner fields are enhanced and the outer fields decrease. This is demonstrated in Figure 8.78(b). Again, reversing the polarity of the fields does not change the result: fields move closer together.

We conclude that the effect of lateral and axial diffusive convection is to promote fields that are closer to the hot spot centre. In addition the isotropic decay rate is less within the hot spot and the inward Nernst velocity is larger. The total effect is that fields move closer together as they are convected inwards.

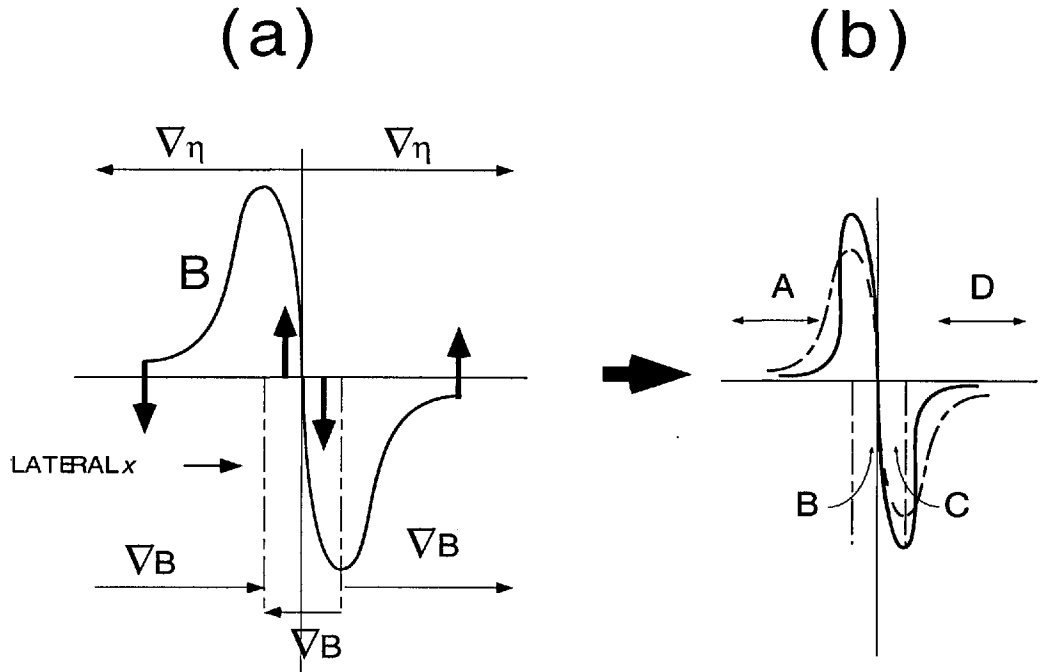


Figure 8.78: (a) Lateral profile of fields surrounding a hot spot. The resistivity increases away from the centre. (b) The inner fields are enhanced at the expense of outer fields.

8.9 Comments on Results and Direction of Future Work

Having proposed a self consistent mechanism for our overdense filaments we briefly discuss the effect future refinements to our code might have on our wedge-like structures.

We first turn our attention to lateral Nernst convection of magnetic fields. The direction of Nernst flow v_N follows that of heat flow Q and is shown in Figure 7.45. There are regions where the flow is convergent and other regions where it is divergent. Of course, the Nernst effect is only one component in the convection of magnetic fields. In conjunction with a fluid velocity v_i , it may be that the total convective velocity is divergent in the direction of the laser. Since our model omits lateral flow of fluid and magnetic fields it did not appear in our proposed mechanism described in Section 8.8.

Another mechanism which could not be a candidate for our results is isotropic expansion, again because of lack of lateral fluid flow. It is well known heat can be contained within a ring of magnetic fields (see Section 3.3). Isotropic hydrodynamic expansion of plasma from regions of high pressure (the hot spot) to regions of low pressure (the vacuum) occurs. The fields are naturally convected with fluid flow, i.e.

divergently. In this argument the fluid flow dictates the location of the fields. This is in contrast with the discussion of Subsection 3.4.1 concerning laser produced jets where the magnetic fields *influenced the location of the plasma* by collimating the ablation.

Let us not forget that the heat flow is nonlocal and that the width of our filaments is less than the electron mean free path. This means that the heat flow out of our conduits will be reduced. The whole configuration is analogous in many respects to thermal filamentation (see Section 2.2). The author expects a Fokker-Planck code to give greater growth rates for his overdense filaments.

Since the resistivity appears to have a significant role in our proposed mechanism it would be useful to carry out calculations with Nernst convected transport omitting resistivity. This would highlight the relative importance of diffusive convection in comparison to the axial Nernst speed in our mechanism. On the subject of Nernst convection, it has been pointed out by Haines that we should include its counterpart, the Ettingshausen effect.

Turning to the laser, it would be trivial to employ a temporal ramp for the laser intensity. Perhaps we could manipulate this ramp somehow to avoid forming filaments and strong shocks. Technologically, this might be difficult as our phenomenon appears to operate on picosecond time scales. Since we observe filaments at $I = 10^{14} \text{ Wcm}^{-2}$ but not at $I = 10^{13} \text{ Wcm}^{-2}$ we state there is a bifurcation in the solution for some intermediate intensity. We are also reminded that early laser drive for ICF conditions is somewhere between $I = 10^{12}$ and 10^{13} Wcm^{-2} and that imprint does not occur for long scale length, hot plasmas (see section 1.4). Ideally we require a graph where laser intensity is plotted against axial scale length, noting when bifurcation occurs.

Although using shorter wavelength light would lead to absorption at higher densities, this does not affect the *location* of the absorption region. This is because large changes in density occur only over very small distances in our model. Clearly the important parameter is the width of the laser absorption region which will depend upon the electron mean free path.

Since the purpose of this thesis is to investigate the imprint phenomenon the effect of surface imperfections should be studied. This can be done easily by imposing a fixed sinusoidal density variation.

8.10 Summary

Various postprocessed diagnostics were carried out. We find that the code conserves energy well: ≈ 2.5 % energy loss after 40 ps (900 steps).

Our correlation calculations show that imprint occurs with both (Nernst convected) magnetised transport and unmagnetised transport with very little difference between the two models. The ablation pressure nonuniformity generally increases as we go from unmagnetised to magnetised to Nernst convected transport. The ablation pressure profile has an increased presence of short wavelength modes (in particular around the the speckle scale length) when we include Nernst convection.

At low intensity magnetic fields do not have much bearing on the results. The heat flow is only partially restricted because the Hall parameter is small in the overdense region.

At high intensity there is considerable generation of fields near the ablation surface and these do not decay so quickly because of the higher temperatures. The fields tend to be compressed closer to this surface and are only a few grid cells thick in the lateral direction. There are overdense regions where the magnetic pressure is comparable to the thermal pressure and may be hydrodynamically important. Lateral heat flow is certainly affected as the Hall parameter is between one and three in most of the overdense region right up to the ablation surface.

In all our calculations fields are convected to the point where the total convective velocity is zero and this is also the surface where the collisionless skin depth is equal to the electron mean free path.

The author's mechanism for generating overdense filaments is presented. It is dependent on lateral gradients in axial Nernst speed. Diffusive convection is likely to be an auxiliary effect.

In short our calculations show that hot spots can prevail at the ablation surface at intensities of $I = 10^{14}$ Wcm $^{-2}$ providing we allow for Nernst convection of fields by heat flow.

CHAPTER 9

CONCLUSIONS

9.1 Summary of Thesis

We give a summary of the work that has been carried out in this thesis. The first three chapters are an overview of the relevant background physics that we need if we are to comprehend what mechanisms are responsible for imprint. These three chapters do not represent the author's own work.

Chapter 1 gives an outline of how fusion could be attained by inertial confinement of a pellet of fuel. A necessary condition is to have smooth ablation pressure. Lack of uniformity at the ablation surface instigates hydrodynamic instabilities there. Acceptable well contained growth rates for these effects are achievable providing the laser drive nonuniformity is kept to within 1-2 %. Even so the outcome is likely to depend upon the degree of imprint during the start-up phase. The nature of this imprint is investigated. It could be due to the first shock leaving mass perturbations in its wake or it could be that magnetised thermal transport is responsible. In this respect our efforts are directed towards ISI light.

Chapter 2 reviews transport theory in unmagnetised plasmas and determines characteristic scale lengths, with particular reference to lateral thermal smoothing. This effect requires an ablation-critical distance comparable to the perturbation scale length. This is not so at early times with a steep density gradient. The Spitzer-Harm theory proves inadequate in describing electron thermal transport. Transport is nonlocal.

Chapter 3 gives the principal generation, loss and convection mechanisms for magnetic fields. They have a considerable bearing on the transport process. This is dangerous for ICF as it can alter the pellet implosion symmetry. The predominant source term is that due to nonparallel density and temperature gradients. The possibility of bifurcation of solutions and hydrodynamic coupling is considered with reference to creation of jets at various laser intensities. A handful of electromagnetic instabilities are considered. In particular the Nernst effect leads to convection of fields with the heat

flow. Amplification of fields can occur in overdense regions where lateral smoothing is required.

In Chapter 4 we return to the subject of beam uniformity and review the contemporary schemes that are available. The Induced Spatial Incoherence [34–36] scheme can achieve less than 2% of illumination nonuniformity at target [85] though it is very ‘messy’ on time scales of the order of a laser coherence time. Early imprint could be a seed for instability with respect to the first shock. We present the algorithm we have developed to produce random, bivariate, correlated numbers. Its statistical properties are investigated. It can produce a signal with essentially the same characteristics as real ISI - fixed coherence time and fixed speckle size. Whilst the time-integrated signal profile does not relax to a more smooth state as fast as ISI light over very short periods (< 10 ps) or very long periods (> 70 ps) it has the desired rms decay rate for intermediate durations.

Our basic assumptions are given in Chapter 5. We start with a planar plasma with fluid flow only in the axial direction. We have developed an Eulerian fluid code to solve the MHD equations in nonconservative form with Cartesian geometry. Advection is carried out using second order van Leer along with artificial viscosity. The induction equation is advected by volume whilst the hydrodynamic equations are advected by mass. We use ICCG to execute two-dimensional diffusion of magnetic and thermal energies.

It is necessary to check the code is functioning properly and this is done in Chapter 6. The code gives reasonable results for the advection of a square pulse test, sound wave test and Sod’s problem. However when we turn to more extreme conditions our code shows up some of its failings. Our artificial viscosity algorithm is scrutinised in the collapsing fluid problem and proves to be inadequate for those conditions. In addition, the code returns large errors, 16-40 %, in state variables for the MHD Riemann problem. The severe conditions of the last two tests are not expected during the start-up phase.

In Chapter 7 we perform calculations using our code and our ISI algorithm. At low intensity magnetic fields do not have a bearing on the results. At high intensity filamentary structures are created in densities above critical. These structures are conduits of high temperature contained within magnetic fields and are observable only if Nernst convection is included.

Chapter 8 contains the author’s mechanism for creating these conduits. Lateral

variations in axial Nernst speed are likely to be a cause. The ablation pressure nonuniformity generally increases as we go from unmagnetised to magnetised to Nernst convected transport and as we increase the laser intensity. The ablation pressure profile has an increased presence of speckle scale length modes when we include Nernst convection. For hard drive imprint is thought to occur after approximately 20 ps whether magnetised transport is used or not.

9.2 Conclusion

We have attempted to numerically model ISI light impinging on a plasma with a very steep density gradient during the start-up phase.

Our code is capable of causing imprint in that the signature of the inward moving shock after 40 ps is determined by cumulative laser drive after only 20 ps. The actual shape of the ablation surface is thought to be particularly correlated to the laser drive at 5 ps, independent of whether a magnetised transport model is included.

Nernst convected fields are an essential component of the start-up phase. The ablation pressure profile has an increased presence of speckle width modes and the ablation pressure nonuniformity generally increases when we compare results using Nernst convected fields with results using unmagnetised plasmas. At high intensity lateral heat flow is certainly affected as the Hall parameter is as large as three in most of the overdense region right up to the ablation surface.

Hot spots can prevail at the ablation surface. They lie at the confluence of filamentary structures that exist in densities above critical. The filaments are conduits of high temperature contained within magnetic fields. They are created at intensities of $I = 10^{14} \text{ Wcm}^{-2}$ but not at $I = 10^{13} \text{ Wcm}^{-2}$ and only if the Nernst model is employed.

In conclusion, magnetic fields cannot be neglected when modelling plasmas with steep density gradients, if we are to view overdense filaments with concern.

CHAPTER 10

FUTURE REFINEMENTS TO THE CODE

The results and conclusions from this work represent preliminary findings. This is because some relevant physics have been omitted; including these would transform our calculations into a simulation. However, the code does have one advantage in its present form. MHD problems generally evolve into fine scale structures (see the single hot spot problem of Section 7.7). The code is able to resolve these features by simply increasing the number of grid cells. The improvements mentioned in this chapter will be computationally expensive.

We detail how the improvements will be incorporated into the code and concentrate on the bearing they will have on the imprint process. Our principal concern is whether the outcome will be contingent on the presence of magnetised transport in the simulation. In this respect our arguments will be based upon the effect these improvements will have on lateral thermal smoothing.

10.1 Two-Dimensional Advection

In this work the transverse pressure gradients have been at least an order of magnitude lower than the axial pressure gradients (see Section 8.1). This concurs with the assumption that at early times most of the fluid flow is driven in the axial direction. Therefore, it is reasonable to try to model the plasma behaviour in the start-up phase using only one velocity component. However, if a hot spot were to develop on the target surface, then lateral fluid flow may be useful in reducing the pressure gradients there.

This thesis is also concerned with the nature of the shock wave which is driven into the target. When a perturbed shock front enters a denser region it tends to straighten out due to dissipation. The crest of the shock is divergent and the trough is convergent. The crest tends to slow down relative to the average velocity of the shock and the trough speeds up. To capture such behaviour requires two-dimensional flow. We give a means of introducing two-dimensional flow taken from Youngs [90].

Suppose we want to model flow in both $\hat{\mathbf{x}}$ and $\hat{\mathbf{z}}$ directions. The order of advection is z first, x second and x first z second for alternate time steps. Let us assume v_z and v_x are both positive and that we are in a time step when $\hat{\mathbf{z}}$ advection comes first. The van Leer advection of cell boundary quantities ρ results in new values ρ^* satisfying the following monotonicity condition

$$\min(\rho_{j-1/2,k+1/2}^n, \rho_{j+1/2,k+1/2}^n) \leq \rho_{j+1/2,k+1/2}^* \leq \max(\rho_{j-1/2,k+1/2}^n, \rho_{j+1/2,k+1/2}^n).$$

Next we van Leer advect the new ρ^* values in the $\hat{\mathbf{x}}$ direction to obtain ρ^{n+1} . Again monotonicity is maintained

$$\min(\rho_{j+1/2,k-1/2}^*, \rho_{j+1/2,k+1/2}^*) \leq \rho_{j+1/2,k+1/2}^{n+1} \leq \max(\rho_{j+1/2,k+1/2}^*, \rho_{j+1/2,k-1/2}^*).$$

Then for the advection phase as a whole $\rho_{j+1/2,k+1/2}^{n+1}$ lies in between the maximum and minimum of $\rho_{j-1/2,k-1/2}^n$, $\rho_{j-1/2,k+1/2}^n$, $\rho_{j+1/2,k-1/2}^n$ and $\rho_{j+1/2,k+1/2}^n$.

At the moment **STAMP** has a finer grid in the axial direction than in the lateral direction. This is acceptable with one-dimensional flow, though when going to higher dimensions it is best to have square grid cells, i.e. $\Delta x \equiv \Delta z$. The artificial viscosity should be treated as a tensor.

Whether this scheme can accurately treat shocks is open to debate. In all respects results are good when the shock wave is aligned to the computational mesh. However, when the shock traverses the grid obliquely problems arise as the fluid approaches steady state [97].

The scheme outlined above has split up the advection operator into one-dimensional operators. Neither of these operators is able to identify an oblique shock in equilibrium. The van Leer cut-offs to the gradient in the one-dimensional scheme have no counterpart in higher dimensions in the context of split directional operator schemes. These schemes usually have problems dealing with incompressible flow models [98].

An unsplit Godunov scheme [99] is available for higher dimensions [98]. The current trend is to use Godunov-van Leer hybrid schemes. In view of the above arguments we still recommend implementing the above scheme. For the start-up phase the shock front is aligned mainly in the lateral direction and the fluid is far from steady state. The scheme detailed above should prove to be adequate.

10.2 A Realistic Equation of State

Laser produced plasmas are comparatively dense with respect to other plasmas. In higher density regions the particles are closer together and are structured unlike real fluids. At these densities the interparticle electrostatic energy approaches the thermal energy. The Debye shielding length is on a par with the interparticle distance and the plasma begins to adopt solid characteristics. This naturally affects the transport properties.

A realistic equation of state should have zero pressure for low temperature, high density materials such as cold solids. These are the conditions before a solid target is irradiated with laser light. Such a target cannot expand. Unfortunately, the target in **STAMP** has the undesirable property that it expands through its own pressure. This is because an ideal gas equation of state is used. Energy is advected away instead of being bottled up and this reduces hot spot growth as well as providing more room for lateral thermal smoothing. The latter point causes the author to anticipate greater presence of overdense filaments with an improved equation of state.

As a start we can consider the ions as a perfect, nondegenerate gas and the electrons as a perfect gas with no, partial or full degeneracy. This is the treatment employed in the **MEDUSA** code [100].

10.3 Flux-Limited Heat Flow

At present **STAMP** does not limit the heat flow in anyway. Since there is no restriction on flow, **STAMP** is dealing with a ‘best case’ scenario. Reduced heat flow will lead to higher temperatures and this can only increase temperature gradients and magnetic fields. Flux-limited heat flow could exacerbate the effect we have found.

A method of incorporating reduced heat flow is outlined in Reference [100]. In that paper diffusion is dealt with explicitly. It is rather difficult to imagine how such a scheme would fit in with an implicit approach. The very nature of the matrix equation

$$\underline{A} \cdot T^{n+1} = \underline{b}$$

implies there is a unique solution and imposing a condition on the gradients of T^{n+1} (i.e. the heat flow) is not well defined. If we were prepared to work with explicit

coefficients Γ^n in $Q = -\Gamma^n \nabla T^{n+1}$, then a possible scheme would be to use the harmonic mean where

$$\Gamma = \left[\left(\frac{1}{\kappa |\nabla T|} \right)^s + \left(\frac{1}{f Q_f} \right)^s \right]^{-1/s} \frac{1}{|\nabla T|}.$$

s is a positive number, usually one. Alternatively we could employ non-linear cut-offs $\Gamma = \min(\kappa, f Q_f / |\nabla T|)$. With the advent of contemporary Fokker-Planck algorithms it is now possible to solve the coupled transport and hydrodynamic equations fast and efficiently in both one and two dimensions. The concept of a flux limit for more sophisticated codes is now becoming obsolete.

10.4 On an Iterative Scheme

Iteration is useful when solving nonlinear differential equations. For example, take the simple diffusion equation

$$\frac{\partial T}{\partial t} = D(T) \nabla^2 T.$$

Solving the right-hand side implicitly we have previously taken $\nabla^2 T = \nabla^2 T^{n+1}$ and have used the present values for the nonlinear diffusion coefficient, $D(T) = D(T^n)$. The fully implicit scheme would be to use $D(T) = D(T^{n+1})$. In this respect an estimate for T^{n+1} is required for the nonlinear coefficient in order to solve for T^{n+1} . This can only be done by employing iteration.

Our first estimate for the forward value is $T^p = T^n$. This is used to obtain a value for the coefficient D . The diffusion equation can now be solved for T^{p+1} . The first iterative step has been completed. We now use the new forward value for T^{p+1} to obtain a better estimate for the coefficient and solve the diffusion equation again, thus completing the second iterative step. We continue this iterative process in the same fashion. The solutions T^{p+1} should converge to the actual answer. Usually only two or three iterations are required. Convergence is not guaranteed, however, so one must check that the iterative process is working. This iterative approach can be incorporated fully into the other difference equations. For a MHD problem there will be four convergence conditions; one for each of density, temperature, velocity and magnetic induction.

Suppose we choose to time-centre our equations. For the simple equation above this implies the diffusion coefficient has the form $D = D(T^{n+1/2})$. The proper way to treat this term is as $D = (D(T^n) + D(T^{n+1}))/2.0$, not as $D = D((T^n + T^{n+1})/2.0)$. The right-hand side of the diffusion equation will contain both present and forward temperatures and the method is similar to the Crank-Nicholson scheme so it is unconditionally stable. Pert [59] reports that his time-centred difference scheme gives better results than either a fully explicit or fully implicit scheme particularly with respect to conservation and stability properties.

10.5 Validity of Current Model and Summary

In its present form the code can only be used for low intensities below around 10^{14} W cm⁻². Above this the artificial Hall limit has to be lowered to maintain numerical stability. It is fortunate that for this work fast electron transport can be neglected as the laser irradiance is comparatively low. Up until now we have also ignored radiation transport which is important for high Z targets.

An improved equation of state and flux-limited heat flow would serve to attenuate the effect we are looking for - the author expects that overdense filaments will be more prominent in models with either of these two ingredients. Iteration would improve the overall accuracy of the results though the author does not envisage the conclusion being affected. Two-dimensional flow would help to smooth out lateral pressure variations and so would help to mitigate our effect.

If we are to search for mechanisms which would cause us to reconsider our conclusions, we must respect the need to keep computational cost low while simultaneously preserving grid resolution. In this light the author recommends that of the four improvements mentioned in this chapter, two-dimensional fluid flow should have immediate priority. However, the lack of any significant lateral pressure gradients in the present calculations tends to suggest this particular implementation will not have a bearing on our conclusions.

APPENDIX A

THE NORMALISATION SCHEME

We present the single fluid MHD equations used in the code **STAMP**. The results of Chapter 7 are based upon these.

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \frac{\partial (v\rho)}{\partial z} &= 0 \\ \frac{\partial \Pi}{\partial t} + \frac{\partial (v\Pi)}{\partial z} &= -\frac{\partial \tilde{P}}{\partial z} + J \wedge B \\ \frac{\partial \Theta}{\partial t} + \frac{\partial (v\Theta)}{\partial z} &= -\tilde{P} \frac{\partial v}{\partial z} + \nabla Q + \eta J^2 + S_{las} \\ \frac{\partial B}{\partial t} + \frac{\partial (vB)}{\partial z} &= \frac{k_B}{e} \frac{\nabla T \wedge \nabla \rho}{\rho} + \mu_0^{-1} \nabla (\eta \nabla B)\end{aligned}$$

The equations are in non-conservative form. It transpires that a suitable normalisation scheme can be conceived on the basis of a simplified Hall parameter and equation of state.

The mass density is primarily due to the ions

$$\rho = n_i m_i, \quad n_i = n_e / Z.$$

The thermal pressure is the sum of the partial pressures of each species both considered to be ideal gases

$$P = n_i k_B T_i + n_e k_B T_e$$

and the thermal energy per unit volume is given by

$$\Theta = \frac{3}{2} P.$$

The pressure $\tilde{P}$ in the energy and momentum equations is the sum of the thermal term P and an artificial viscous term q

$$\tilde{P} = P + q.$$

The current density is defined by Ampere's Law (neglecting the displacement current),
 $\mathbf{J} = \mu_0^{-1} \nabla \wedge \mathbf{B}$.

The equations are in MKSA units. We have to normalise so variables fall within a suitable range for the purposes of the code. In the following bold variables represent MKSA quantities and normal type represents variables in normalised form. For example

$$\boldsymbol{\rho} = \text{mass density in MKSA units}$$

$$\boldsymbol{\rho} = \bar{\rho} \rho$$

where ρ is the dimensionless representation of $\boldsymbol{\rho}$ in the code. We choose to normalise relative to the mass density at critical

$$\bar{\rho} = n_i m_i |_{critical} = \frac{m_i n_e}{Z} |_{critical}$$

$$\bar{\rho} \approx \frac{A m_p}{Z} \frac{1.1 \times 10^{27} \text{ m}^{-3}}{(\lambda_{las}/\mu\text{m})^2} \approx 1.84 \frac{A}{Z (\lambda_{las}/\mu\text{m})^2} \text{ kg m}^{-3}.$$

For a given A and Z the critical mass density depends only on the laser wavelength.

Length scales are normalised to ten microns

$$\bar{z} = 10 \times 10^{-6} \text{ m}$$

and the time scale to fifty picoseconds

$$\bar{t} = 50 \times 10^{-12} \text{ s}.$$

We need to normalise the current to complete the scheme. Note we may implicitly do this by defining the scaling for the induction.

$$\mathbf{B} = \bar{B} B.$$

The heat flux is said to be inhibited when the Hall parameter χ is of order one or

greater. If we choose

$$\bar{B}^{-1} = \frac{e}{m_e} \frac{6\pi\sqrt{2\pi}}{0.327} \frac{\epsilon_0^2 m_e^{1/2}}{e^4 l n \Lambda} \frac{A m_p k_B^{3/2}}{Z^2} \frac{\bar{T}^{3/2}}{\bar{\rho}} \quad (11.32)$$

then the Hall parameter simply becomes

$$\chi = \chi = \frac{BT^{3/2}}{\rho}$$

in the code. Being a dimensionless parameter it is unaffected by scaling.

Space, time and charge have all been rescaled though the induction scaling is still undetermined through $\bar{T}$. We require the scaling for mass and this is done by normalising the continuity equation

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\mathbf{v}\rho)}{\partial \mathbf{z}} \implies \frac{\bar{\rho}}{\bar{t}} \frac{\partial \rho}{\partial t} = -\frac{\bar{\rho}\bar{v}}{\bar{z}} \frac{\partial(v\rho)}{\partial z}.$$

If we impose the normalisation condition $\bar{v} = \bar{z}/\bar{t}$, we obtain the new continuity equation

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(v\rho)}{\partial z}.$$

Dealing with the momentum equation (and ignoring the $J \wedge B$ force)

$$\frac{\bar{\rho}\bar{v}}{\bar{t}} \frac{\partial \Pi}{\partial t} + \frac{\bar{\rho}\bar{v}\bar{v}}{\bar{z}} \frac{\partial(v\Pi)}{\partial z} = \frac{\bar{P}}{\bar{z}} \frac{\partial P}{\partial z}.$$

Thus, if we define

$$\bar{P} \doteq \bar{\rho}\bar{v}\bar{v} \quad (11.33)$$

then it follows

$$\frac{\partial \Pi}{\partial t} + \frac{\partial(v\Pi)}{\partial z} = -\frac{\partial P}{\partial z}.$$

In our model the thermal pressure is the sum of the ion and electron pressures

$$\mathbf{P}_i = \frac{k_B}{m_p} \frac{1}{A} \rho \mathbf{T}_i \quad \text{and} \quad \mathbf{P}_e = \frac{k_B}{m_p} \frac{Z}{A} \rho \mathbf{T}_e.$$

Setting $T_e = T_i = T$ we observe

$$\mathbf{P} = \frac{k_B}{m_p} \frac{1+Z}{A} \bar{\rho} \bar{T} \rho T.$$

The pressure connection is given by $\mathbf{P} = \bar{P} P$ so we are free to choose a normalised equation of state of the form $P = \rho T$, if we have

$$\bar{P} = \frac{k_B}{m_p} \frac{1+Z}{A} \bar{\rho} \bar{T}.$$

Using Equation 11.33 we reach the scaling for the temperature

$$\bar{T} = \frac{A m_p}{(1+Z) k_B} \bar{v} \bar{v}. \quad (11.34)$$

This gives us the scaling for the induction through Equation 11.32. **We have now replaced MKSA units in favour of the chosen $\bar{z}$, $\bar{\rho}$, $\bar{t}$ and $\bar{B}$.** In the process we have reached a very simple equation of state. It should be noted that it is now trivial to switch between different sets of units. For example, by setting $\bar{z} = 0.01$, $\bar{\rho} = 1000$, $\bar{t} = 1$, $\bar{B} = .0001$ the variables in the code will automatically be in Gaussian units.

We now detail how the individual terms are rescaled. This work is done so that evaluation of coefficients is performed prior to running the code and so the equations have a simple form. We demonstrate by example - adding the pressure source term in the induction equation.

Ignoring advection and diffusion for now we have

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{k_B}{e} \frac{\nabla \mathbf{T} \wedge \nabla \rho}{\rho} \Rightarrow \frac{\bar{B}}{\bar{t}} \frac{\partial B}{\partial t} = \frac{k_B}{e} \frac{\bar{T}}{\bar{z}^2} \frac{1}{\rho} \left(\frac{\partial T}{\partial z} \frac{\partial \rho}{\partial x} - \frac{\partial T}{\partial x} \frac{\partial \rho}{\partial z} \right).$$

Defining the following coefficient

$$baro_{coef} = \frac{k_B}{e} \frac{\bar{T}}{\bar{z}^2} \frac{\bar{t}}{\bar{B}}$$

we have our new induction equation

$$\frac{\partial B}{\partial t} = baro_{coef} \times \frac{1}{\rho} \left(\frac{\partial T}{\partial z} \frac{\partial \rho}{\partial x} - \frac{\partial T}{\partial x} \frac{\partial \rho}{\partial z} \right).$$

Other terms in the four MHD equations are added in a similar manner.

APPENDIX B

VAN LEER ADVECTION

This work is based upon the van Leer advection method [101, 102] of Youngs [90]. Given a velocity profile $u_{i+1/2}$ (at cell boundaries) and a state variable Φ_i at (cell centres) the **vanleer** subroutine returns state values $\Phi_{i+1/2}$ at the cell boundaries suitable for advection. We start with a dimensionless velocity parameter θ

$$\theta \doteq \frac{u_{i+1/2}\Delta t}{\Delta z}.$$

Say $\theta > 0$. Then

$$\begin{aligned}\Phi_{i+1/2} &\doteq \Phi_i + \frac{1}{2}(1 - \theta)(\Phi_{i+1} - \Phi_i) \\ &= \frac{1}{2}(1 + \theta)\Phi_i + \frac{1}{2}(1 - \theta)\Phi_{i+1}\end{aligned}$$

so $\Phi_{i+1/2}$ is more influenced by Φ_i if $\theta > 0$.

Now take $\theta < 0$

$$\begin{aligned}\Phi_{i+1/2} &\doteq \Phi_{i+1} + \frac{1}{2}(1 - |\theta|)(\Phi_i - \Phi_{i+1}) \\ &= \frac{1}{2}(1 - |\theta|)\Phi_i + \frac{1}{2}(1 + |\theta|)\Phi_{i+1}.\end{aligned}$$

Remark *In outline this is how we take into account the gradient of Φ at the boundary. The scheme is second order.*

The above is to be compared with the simple donor cell scheme which is only first order. In that scheme the amount of mass flowing across a boundary is proportional to the mass in the donor cell and the donor cell can never empty itself. This new scheme does not come with that guarantee; positivity is no longer assured. Another problem with this higher order scheme is that it introduces unphysical oscillations: a monotonic initial distribution must remain monotonic. One requires a monotonic advection method and such a method was provided by van Leer. Monotonicity was achieved by introducing nonlinear cut offs D in the gradients of Φ at the boundaries.

Case 1. $\theta > 0$ If we define a ϖ through

$$\varpi \doteq \begin{cases} 0 & \text{if } \text{sign}(\Phi_{i+1} - \Phi_i) \neq \text{sign}(\Phi_i - \Phi_{i-1}) \\ \text{sign}(\Phi_{i+1} - \Phi_i) & \text{otherwise} \end{cases}$$

then the Van Leer estimate is given by

$$\begin{aligned} D_{i+1/2} &= \varpi \times \min\{|\Phi_{i+1} - \Phi_i|, 2|\Phi_i - \Phi_{i-1}|\} \\ \Phi_{i+1/2} &= \Phi_i + \frac{1}{2}(1 - |\theta|) D_{i+1/2}. \end{aligned}$$

Case 2. $\theta < 0$ If we define a ϖ through

$$\varpi \doteq \begin{cases} 0 & \text{if } \text{sign}(\Phi_i - \Phi_{i+1}) \neq \text{sign}(\Phi_{i+1} - \Phi_{i+2}) \\ \text{sign}(\Phi_i - \Phi_{i+1}) & \text{otherwise} \end{cases}$$

then the Van Leer estimate is given by

$$\begin{aligned} D_{i+1/2} &= \varpi \times \min\{|\Phi_i - \Phi_{i+1}|, 2|\Phi_{i+1} - \Phi_{i+2}|\} \\ \Phi_{i+1/2} &= \Phi_{i+1} + \frac{1}{2}(1 - |\theta|) D_{i+1/2}. \end{aligned}$$

Alternative methods for calculating D are given by van Albada [103].

The upwind method is usually adequate when gradients in density and internal energy are small. When applied to the momentum equation the scheme introduces numerical diffusion and dissipates kinetic energy. A higher order scheme such as van Leer copes with steep gradients adequately. Contact discontinuities and the shock fronts are better resolved. Also, rarefaction waves are closer to the actual solution.

In Section 5.6 we outlined the scheme for van Leer advection by mass, which we use in the code for the hydrodynamics. Here is the alternative scheme where we apply van Leer's method with no reference to specific quantities:

$$\begin{aligned} \rho_i^{n+1} &= \rho_i^n + \Delta t \left(\rho_{i-1/2}^{van} v_{i-1/2} - \rho_{i+1/2}^{van} v_{i+1/2} \right) / \Delta z \\ \Pi_i^{n+1} &= \tilde{\Pi}_i^n + \Delta t \left(\Pi_{i-1/2}^{van} v_{i-1/2} - \Pi_{i+1/2}^{van} v_{i+1/2} \right) / \Delta z \\ \Psi_i^{n+1} &= \tilde{\Psi}_i^n + \Delta t \left(\Psi_{i-1/2}^{van} v_{i-1/2} - \Psi_{i+1/2}^{van} v_{i+1/2} \right) / \Delta z \\ B_i^{n+1} &= \tilde{B}_i^n + \Delta t \left(B_{i-1/2}^{van} v_{i-1/2} - B_{i+1/2}^{van} v_{i+1/2} \right) / \Delta z \end{aligned}$$

$$\begin{aligned}
\text{continuity equation} \quad & \text{vanleer}(\rho_i, v_{i+1/2}) \longrightarrow \rho_{i+1/2}^{van} \\
\text{momentum equation} \quad & \text{vanleer}(\tilde{\Pi}_i, v_{i+1/2}) \longrightarrow \Pi_{i+1/2}^{van} \\
\text{energy equation} \quad & \text{vanleer}(\tilde{\Psi}_i, v_{i+1/2}) \longrightarrow \Psi_{i+1/2}^{van} \\
\text{induction equation} \quad & \text{vanleer}(\tilde{B}_i, v_{i+1/2}) \longrightarrow B_{i+1/2}^{van}.
\end{aligned}$$

This scheme is referred to as van Leer advection by volume. The two schemes are not equivalent. Van Leer advecting by mass keeps mass fluxes consistent *and* takes into account the gradients in the four equations. Advecting by volume treats the equations as independent. For instance take the continuity and energy equation. Van Leer advecting the first equation by volume gives us mass fluxes implicitly. Van Leer advecting the energy equation by volume gives us the energy fluxes at the boundaries *with no reference to the amount of matter that has crossed that boundary*. In the authors opinion this appears to be inconsistent and so advection by mass is preferred.

A third advection scheme involves applying van Leer to only the continuity equation to work out the mass fluxes. The remaining equations are differenced in a donor cell manner. Denoting a specific quantity by ϕ , the fluxes and advection are given by

$$\begin{aligned}
\Delta\phi_{i+1/2} &= \begin{cases} \Delta M_{i+1/2}^{van} \phi_i & \text{if } \Delta M_{i+1/2}^{van} > 0 \\ \Delta M_{i+1/2}^{van} \phi_{i+1} & \text{otherwise} \end{cases} \\
\phi_i^{n+1} &\doteq \phi_i^n + [\Delta\phi_{i-1/2} - \Delta\phi_{i+1/2}] / (\Delta z \Delta A_{\perp}).
\end{aligned}$$

In many respects this third scheme gives results on a par with the first order donor cell scheme, i.e. it is diffusive. However, the author notes that this method gives the best results for Noh's problem of Section 6.4.

APPENDIX C

THE POWER SPECTRUM OF ISI

In this appendix we show that an echelon alters the power spectrum of the light that traverses it. We start with an electric field E entering a one-dimensional echelon with N steps. Each step introduces a delay $\bar{\tau}$ in the light and each beamlet is focused to a point Q. The Fourier decomposition of the signal at the echelon is

$$E(t) = \int f(\omega) e^{i\omega t} d\omega.$$

An expression for the electric field at Q (to within a normalisation constant) is

$$E_Q \approx E(t) + E(t - \bar{\tau}) + E(t - 2\bar{\tau}) + \dots + E(t - N\bar{\tau}).$$

The autocorrelation function at Q is by definition

$$A_Q(\tau) \doteq \int E_Q(t) E_Q^*(t - \tau) dt.$$

Substituting for the individual electric signals

$$\begin{aligned} A_Q(\tau) &= \int dt \left\{ \int f(\omega_1) e^{i\omega_1 t} d\omega_1 + \dots + \int f(\omega_N) e^{i\omega_N(t - N\bar{\tau})} d\omega_N \right\} \\ &\times \left\{ \int f^*(\nu_1) e^{-i\nu_1(t - \tau)} d\nu_1 + \dots + \int f^*(\nu_N) e^{-i\nu_N(t - N\bar{\tau} - \tau)} d\nu_N \right\} \\ &= \int dt \{\}_1 \{\}_2. \end{aligned}$$

Taking the product of the first integral of $\{\}_1$ with all the integrals in $\{\}_2$ and integrating over time yields:

$$\begin{aligned} &\int dt \int \int d\omega_1 d\nu_1 f(\omega_1) f^*(\nu_1) e^{i\nu_1 \tau} e^{i(\omega_1 - \nu_1)t} + \\ &\int dt \int \int d\omega_1 d\nu_2 f(\omega_1) f^*(\nu_2) e^{i\nu_2 \tau} e^{i(\omega_1 - \nu_2)t} e^{i\nu_2 \bar{\tau}} + \\ &\int dt \int \int d\omega_1 d\nu_3 f(\omega_1) f^*(\nu_3) e^{i\nu_3 \tau} e^{i(\omega_1 - \nu_3)t} e^{2i\nu_3 \bar{\tau}} + \dots \end{aligned}$$

Each double integral in (ω_1, ν_j) may be reduced to a single integral in ω due to the delta function action of $\int dt e^{i(\omega_1 - \nu_j)t}$. We may sum over the last term in each integral

$$1 + e^{i\omega\bar{\tau}} + \dots + e^{Ni\omega\bar{\tau}} = \frac{1 - e^{(N+1)i\omega\bar{\tau}}}{1 - e^{i\omega\bar{\tau}}}$$

so the product of the first integral of $\{\}_1$ with all the integrals in $\{\}_2$ yields

$$\int \frac{1 - e^{(N+1)i\omega\bar{\tau}}}{1 - e^{i\omega\bar{\tau}}} |f(\omega)|^2 e^{i\omega\tau} d\omega.$$

The product of the second integral of $\{\}_1$ with $\{\}_2$ leads to a similar expression. The summation here is

$$e^{-i\omega\bar{\tau}} + 1 + e^{i\omega\bar{\tau}} + \dots + e^{(N-1)i\omega\bar{\tau}} = \frac{1 - e^{(N+1)i\omega\bar{\tau}}}{1 - e^{i\omega\bar{\tau}}} e^{-i\omega\bar{\tau}}.$$

Combining all the integrals we arrive at

$$\begin{aligned} A_Q(\tau) &= \int \frac{1 - e^{(N+1)i\omega\bar{\tau}}}{1 - e^{i\omega\bar{\tau}}} (1 + e^{i\omega\bar{\tau}} + \dots + e^{Ni\omega\bar{\tau}}) |f(\omega)|^2 e^{i\omega\tau} d\omega \\ &= \int \frac{1 - \cos[(N+1)\omega\bar{\tau}]}{1 - \cos[\omega\bar{\tau}]} |f(\omega)|^2 e^{i\omega\tau} d\omega. \end{aligned}$$

The autocorrelation function is the Fourier transform of the power spectrum for any signal. The power spectrum of the undisturbed beam is given by $G(\nu) = |f(\nu)|^2$ so the power spectrum at the focus Q is given by

$$G_Q(\nu) = \frac{1 - \cos[(N+1)\nu\bar{\tau}]}{1 - \cos[\nu\bar{\tau}]} |f(\nu)|^2. \quad (13.35)$$

In Figure 13.79 we see how the power spectrum of a beam can be fundamentally altered by an echelon. Two effects become relevant at Q. First is the possibility that beating between two modes will produce a low frequency electromagnetic wave. The echelon has enhanced certain spectral components which are equally spaced apart. Typical parameters are $\bar{\tau} = 5$ ps, $N = 100$. If $\delta\nu$ is the distance between peaks in $G_Q(\nu)$ then $\delta\nu(N+1)\bar{\tau} = 2\pi$. This gives $\delta\nu \approx 10^{10}$ Hz. As far as we know there are no electromagnetic instabilities which operate in this regime. Second, it is also clear that the bandwidth of the original beam has changed significantly at Q: the coherence time

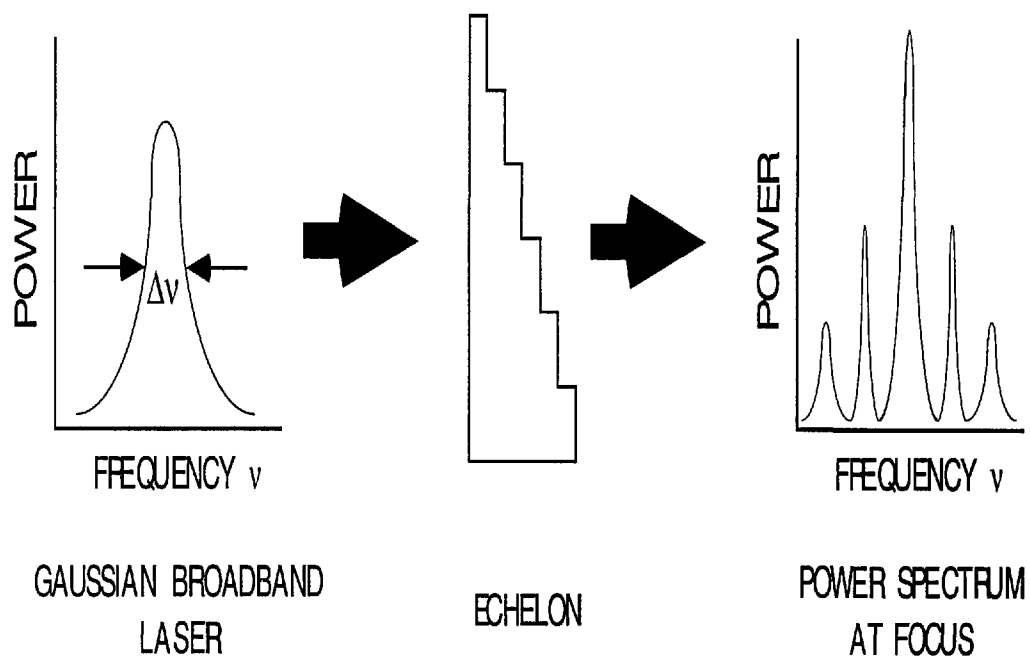


Figure 13.79: Gaussian power spectrum of a laser beam being altered by an echelon: cross-spectral purity.

has altered there.

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