

**FATIGUE CRACK PROPAGATION IN
LOW DENSITY POLYETHYLENE**

by

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ABSTRACT

Fatigue crack propagation (FCP) has been studied in two grades of Low Density and one grade of Linear Low Density Polyethylene (LDPE & LLDPE respectively). All three grades have the same nominal density (920 kg/m^3), but the single grade of LLDPE differs substantially from the LDPE grades in terms of molecular structure. Centre Cracked Tension (CCT) specimens were used and all specimens were 12 mm thick. However, shorter specimens were used in negative R-ratio tests and consequently a height correction had to be made in the analysis. Tests were conducted under sinusoidal load control at $+23^\circ\text{C}$ and -20°C . The study concentrated mainly on R-ratio and frequency effects ($R\text{-ratio} = \sigma_{\min}/\sigma_{\max}$). Test frequencies were between 0.2 and 15.0 Hz and R-ratios ranged from -2.0 to 0.6.

The testing procedure was automated with the employment of two novel techniques. Firstly, crack growth was monitored by Time-lapse photography, i.e the crack was photographed at a pre-set time interval. Secondly, low temperature tests were conducted in a domestic freezer. The domestic freezer was modified by drilling holes both on the top and bottom for the pull-rods to pass through and by fitting a double-glazed window to the front door. This allowed for long-term testing in a controlled environment.

The FCP rate per cycle, da/dN , was correlated with the range of the stress intensity factor, ΔK , using the Paris representation. The Low Density Polyethylene grades (at room temperature) showed a transitional behaviour in their FCP results. In fact there were three distinct regions of crack propagation and in the second region deceleration of growth rates was observed. For all three grades FCP rates were observed to decrease with an increase in both the R-ratio and test frequency.

It was possible to divide the total da/dN into two components: cycle-dependent and time-dependent crack propagation. In all cases it was found that the cycle-dependent component was always greater than the time-dependent component. Furthermore greater time-dependent effects were observed with an increase in R-ratio. However, in all three grades, a crack could not be grown under an applied static load (equal to $P_{\max}$ in the cyclic test) at either test temperature.

Crack tip temperature variations were also studied with the aid of a computerised thermovision system. Temperature rises were in general found to be small (under stable crack growth conditions) in tests conducted at low R-ratios (0.1 & 0.2) and at a low test frequency (3.0 Hz). However, a temperature rise of 20.9°C was observed in a test conducted at both a high R-ratio and a high test frequency ($R=0.6$ - $F=15.0\text{Hz}$).

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LIST OF SYMBOLS

a	Semi - Crack length
A, B, C	Constants
B	Specimen thickness
α	Damage factor in Two Stage line Zone Model
β	Constant
β_r	Secondary molecular-relaxation transition
da/dN	Crack growth per cycle
da/dt	Crack growth per second
δ	Crack opening displacement / Deflection
δ_c	Critical crack opening displacement (at fracture)
E	Young's modulus
$F(R)$	Function which describes the different ways in which the minimum load is supported by the zone
f	Frequency
G	Strain energy release rate
G_{IC}	Critical strain energy release rate
γ_s	Surface energy
γ_p	Plastic surface work
H	Specimen height
I	Intercept
K	Stress intensity factor
K_c	Fracture toughness
K_{max}	Maximum stress intensity factor
K_{min}	Minimum stress intensity factor
K_m	Mean stress intensity factor
K_o	A factor in two stage line zone model
K_{op}	Stress intensity factor when crack opens
K_{th}	Threshold stress intensity factor

ΔK	Range of stress intensity factor
ΔK_{ef}	Effective range of stress intensity factor
λ	$K_{max}^2 - K_{min}^2$, in Arad's equation
M	Molecular weight
m	Exponent in Paris law
n	Constant
N	Number of cycles
ν	Poisson's ratio
P	Load
R	R-ratio
r_y	Plastic zone length
r_o	Size of the zone tip in the two stage line zone
r_l	Total zone length for a two stage line zone
r_c	Critical plastic zone length
S	Slope
σ	Stress
σ_c	Craze stress
σ_m	Mean stress
σ_y	Yield stress
σ_{op}	Stress at crack closure
$\sigma_x, \sigma_y, \sigma_z$	Normal stresses in the x,y and z directions
T	Temperature
$\tau_{xy}, \tau_{yz}, \tau_{zx}$	Shear stresses
W	Specimen width
Y	Geometry factor

LIST OF ABBREVIATIONS

CCT	Centre cracked tension specimen
COD	Crack opening displacement
FCP	Fatigue crack propagation
FSF	Frequency sensitivity factor
HDPE	High density polyethylene
LEFM	Linear elastic fracture mechanics
LDPE	Low density polyethylene
LLDPE	Linear low density polyethylene
MFI	Melt flow index
PC	Polycarbonate
PMMA	Polymethylmethacrylate
PS	Polystyrene
PSF	Polysulfone
PVC	Poly(vinyl chloride)
SIF	Stress intensity factor

CHAPTER 1

INTRODUCTION

In recent years there has been a great need to understand fatigue behaviour in polymers because of their wider use in load bearing applications. Hence it is important, since fatigue is known to cause failure, that data are acquired to assist the engineer in characterising a material under different loading and environmental conditions. However, in spite of this need, the subject has received little fundamental attention and not as much engineering attention as is warranted. The problem is compounded by the facts that the existence of fatigue loading in service is not always recognised and all materials contain flaws, cracks or inhomogeneities which will ultimately propagate and cause failure.

Such pre-existent defects include gas or shrinkage pores and as a result of mechanical processing, scratches, gouges and other markings that introduce stress concentrations on the surface. However, even if the material has satisfactory properties and contains no adventitious defects, cracks can be introduced into the material during service as a result of repeated loading at stresses below the nominal yield strength. Consequently, under fluctuating loads, polymers fail at stress levels much lower than they can withstand under monotonic loading conditions. Also for a given alternating stress amplitude, they will fail in a time much less than the creep rupture time for a constant applied stress of the same magnitude.

In the light of this situation it is necessary to gain a better understanding of the mechanisms involved in the fatigue failure of polymers. Such an understanding can be achieved by investigating the influence of important external or experimental variables such as alternating stress amplitude, stress intensity factor range, frequency, mean stress, temperature and environment. However, in the case of polymeric materials an understanding of polymer structure and viscoelastic characteristics is also needed.

The main objective of the research programme was to obtain fracture toughness values for low and linear low density polyethylene. Both materials are tough and inherently difficult to test because of that. Fatigue crack growth tests have been

chosen due to their ability in characterising the toughness of viscoelastic materials, i.e cyclic loading has the effect of resharpening the crack tip and consequently brittle cracks may be grown. The Fatigue Crack Propagation (FCP) rate per cycle, da/dN , has been correlated with the range of the stress intensity factor, ΔK , using the Paris representation.

CHAPTER 2

LITERATURE SURVEY

2.1 INTRODUCTION

For the past one hundred years engineers have sought to identify the major variables responsible for fatigue failure, and although many studies have been carried out, the problem still remains unsolved. Most of these studies have been based on the fatigue fracture process in metals. However, in recent years polymers are being utilized in the manufacture of an increasing number of load bearing structural components and hence increasing attention is now being devoted to the fatigue fracture of engineering plastics. It was pointed out by Griffiths [1] that all materials contain flaws, cracks or inhomogeneities which propagate to cause failure. Hence a major design dilemma is whether one should assume that a certain structural component does or does not contain defects when first placed in service ? Consequently, if the likelihood of a starting defect is ignored, it would be reasonable to make useful service life estimates from experimental data derived from unnotched laboratory samples. Here most of the fatigue life would be consumed during crack initiation. By contrast, if we assume a starting defect exists in the component, service life estimations should be dominated by crack propagation considerations. In this case service life estimates should be based on experimental data derived from precracked samples.

In the early stages a Wohler [2] type of analysis was used where plate bending, rotational bending and reversed shear (torsion) unnotched specimens were tested at different stress levels and the fatigue life measured. However, the test method has some important drawbacks. First of all the use of unnotched specimens makes it difficult to apply the results to real structures in which stress concentrations exist because of design requirements. Secondly, as pointed out before material inhomogeneities and inherent flaws must be expected, particularly in large structures. In this case fatigue failure cannot be isolated using a S-N type of test, as fatigue failure is now dependent on the number of cycles required to nucleate and propagate a defect to a critical size. That is, with S-N plots

one records the number of loading cycles needed to break the sample at any given stress level for a wide variety of notch geometries. However, one would still not know with any degree of certainty how much cyclic life would remain after the initial defect had grown to some larger dimension. Recognising this limitation, engineers and researchers have sought to characterise the kinetics of the crack growth process, and in the last two decades a considerable effort has been devoted to the subject. Much of this information has dealt with an analysis of fatigue cracking rates in terms of fracture mechanics parameters.

For example, Paris [3] and others showed that the crack growth rate could be related to such a fracture mechanics parameter, assuming that only one parameter is required to define the stress field in the zone ahead of the crack tip and that this parameter controls the fracture process. This concept has proved to be quite successful in the analysis of fracture in statically loaded structures. However, a number of problems have been encountered with its application to fatigue. For, example a given ΔK level does not identify the stress system applied to the specimen. To do this two parameters are required (i.e ΔK or K_{max} and $R=K_{min}/K_{max}$) and the way in which the second parameter is going to affect the fracture process is dependent on the material under consideration. Also, though many studies have verified the log-log linear relationship between da/dN and ΔK , others have shown FCP plots to assume a sigmoidal shape (Fig 2.1). For the upper part of the curve, the deviation is not important in an engineering sense since it occurs at very high crack growth rates and so is near to the final failure. However, the deviation for low K values suggests that a threshold for fatigue fracture exists and this is important in materials selection. Finally, the Paris law exponent has been found to change significantly for different materials and there is no physical model which can predict such variations.

2.2 LINEAR ELASTIC FRACTURE MECHANICS

Fatigue crack propagation (FCP) studies have mainly been concerned with the tensile (mode I) type of fracture. However there are three modes of fracture: tensile (mode I), shear (mode II) and antiplane strain (mode III) as shown in Fig 2.2. Griffith (1920) [1] laid down the foundations for linear elastic fracture mechanics (LEFM) and his criterion is presented in many standard texts on fracture mechanics, Williams [4], Knott [5] and Broek [6]. He proposed that every body contains a distribution of imperfections or flaws and that failure occurs at the largest of these. Larger bodies have a greater likelihood of containing bigger flaws and will thus fail at lower stresses. Furthermore, he noted that when a flaw grew in a stressed body, there was a decrease in the potential energy of the body, and so energy was released to be available to form the new surfaces of the growing flaw. Therefore, strain energy released is equal to the surface energy, γ_s , required to create the new fracture surfaces, and furthermore, Griffith showed that the crack would grow in an unstable manner when

$$\sigma = (2E \gamma_s / \pi a)^{1/2} \quad \text{Eqn 2.1}$$

where σ is the failure stress, a the flaw size and E the Young's modulus. However, all engineering materials exhibit some plasticity, and Orowan [7] suggested that equation 2.1 could be extended to include the energy sink term associated with plastic deformation, γ_p

$$\sigma = (2E (\gamma_s + \gamma_p) / \pi a)^{1/2} \quad \text{Eqn 2.2}$$

Irwin [8] also considered the fracture of solids process from a thermodynamic point of view and arrived at a relationship similar to that by Griffith (given below)

$$\sigma = (EG / \pi a)^{1/2} \quad \text{Eqn 2.3}$$

where G is the strain energy release rate. By comparing equations 2.2 and 2.3, when fracture occurs the term $2(\gamma_s + \gamma_p)$ is normally called the critical strain energy release rate G_{IC} .

Then attention was directed towards crack tip characterizing parameters as measures of the susceptibility of materials to fracture. By using Westergaard's [9] solution for stresses near a crack, Irwin [10] was able to describe the linear elastic stress field at the tip of a sharp crack as :

$$\sigma_y = (K_I / \sqrt{2\pi r}) \cos \theta/2 [1 + \sin \theta/2 \sin 3\theta/2] + \dots$$

$$\sigma_x = (K_I / \sqrt{2\pi r}) \cos \theta/2 [1 - \sin \theta/2 \sin 3\theta/2] + \dots$$

$$\tau_{xy} = (K_I / \sqrt{2\pi r}) \cos \theta/2 \cos 3\theta/2 \sin \theta/2 + \dots$$

$$\tau_{xz} = \tau_{yz} = 0$$

$$\sigma_z = 0 \quad (\text{plane stress})$$

$$\sigma_z = \nu (\sigma_y + \sigma_x) \quad (\text{plane strain}) \quad \text{Eqns 2.4}$$

where K_I is the stress intensity factor and ν the Poisson's ratio. The stress distribution around a crack tip is shown in Fig 2.3.

The factor K_I is a function of σ and a and the relation is as follows :

$$K_I = Y \sigma (\pi a)^{1/2} \quad \text{Eqn 2.5}$$

where Y is a geometry factor.

Under plane strain test conditions attained with adherence to specific test procedures noted in ASTM E399-78, when fracture occurs, K_I is known as the critical fracture toughness

of the material, K_{IC} .

It is clear from equations 2.4 that as r approaches zero the local stresses will rise to extremely high levels. However, as this cannot occur in practice, the material yields at the crack tip. Linear elastic fracture mechanics (LEFM) will only apply if the extent of yielding at the crack tip is small compared to the highly stressed region near the crack tip. The region of yielding that occurs at the crack tip is called the plastic zone.

Irwin [10], by setting the local stress equal to the yield strength in Eqns 2.4, for σ_y and taking the plane $\theta=0$, calculated the plane stress plastic zone size as:

$$r_y = (1 / 2\pi) [K / \sigma_y]^2 \quad (\text{plane stress}) \quad \text{Eqn 2.6}$$

where σ_y is the yield strength.

As a result of the effective yield stress in plane strain being larger than the uniaxial yield stress, the plane strain plastic zone is significantly smaller than the plane stress plastic zone. Therefore a plastic constraint factor (p.c.f) is introduced and defined as a ratio of the maximum stress in the plane strain plastic zone to the uniaxial yield stress i.e

$$\text{p.c.f} = \sigma_{\max} / \sigma_y \quad \text{Eqn 2.7}$$

and can be estimated as follows :

Firstly, the Von Mises yield criterion, in terms of the principal stresses is given by :

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2.\sigma_y^2 \quad \text{Eqn 2.8}$$

Then by taking $\sigma_2 = n\sigma_1$ and $\sigma_3 = m\sigma_1$, Eqn 2.8 can be rewritten as :

$$[(1-n)^2 + (n-m)^2 + (1-m)^2].\sigma_1^2 = 2.\sigma_y^2$$

and rearranged to :

$$\text{p.c.f} = \sigma_1 / \sigma_y = (1 - n - m + n^2 + m^2 - mn)^{-1/2} \quad \text{Eqn 2.9}$$

Therefore, from Eqn 2.9 the p.c.f can be calculated at any location of the crack tip region, since from Eqns 2.4

$$n = (1 - \sin\theta/2 \cdot \sin 3\theta/2) / (1 + \sin\theta/2 \cdot \sin 3\theta/2)$$

and

$$m = v \cdot (1 + n)$$

Hence, knowledge of the plastic constraint factor enables derivation of the plastic zone size for plane strain. Irwin and Paris [11] assumed that p.c.f takes the value $\sqrt{3}$, i.e. $\sigma_1 = \sqrt{3} \cdot \sigma_y$ and therefore the length of the plane strain plastic zone is given by :

$$\begin{aligned} r_y &= (1/2\pi) [K / (\sqrt{3} \cdot \sigma_y)]^2 \\ &= (1/6\pi) [K / \sigma_y]^2 \quad (\text{plane strain}) \end{aligned} \quad \text{Eqn 2.10}$$

Whether plane strain or plane stress conditions will apply in a given situation depends upon the size of the plastic zone relative to the component thickness and actual crack length. In adherence to ASTM E399-70 it is recommended that (for plane strain conditions) both thickness, B, and crack length, a, should exceed the following size :

$$B \text{ and } a > 2.5 (K_{IC} / \sigma_y)^2 \quad \text{Eqn 2.11}$$

Also plane strain fracture conditions are considered to be present when the plastic zone size, calculated from Eqn 2.10, is equal to or less than 2% of both the component thickness and crack length (ASTM E399 - 78).

A different approach to finding the extent of the plastic zone was followed by Dugdale [12], where he considers an effective crack which is longer than the physical crack shown in Fig 2.5. That is, it is assumed that yielding of the material at the

crack tip makes the crack longer by the length of the plastic zone, r_y . The zone of yielded material in front of the physical crack is assumed to be at a uniform stress σ_y which tends to close the crack. The length r_y is thus calculated by assuming that a crack of length $2a+2r_y$ exists in the sheet and that the forces acting over the length r_y reduce the elastic stresses at the tip of the crack to σ_y . The exact solution is given by :

$$r_y = (a / \cos [\pi \sigma / 2 \sigma_y]) - a \quad \text{Eqn 2.12}$$

assuming that σ is less than $0.3\sigma_y$, this equation can be reduced to :

$$r_y = (\pi/8).(K / \sigma_y)^2 \quad \text{Eqn 2.13}$$

This may be compared to Eqn 2.6 and it is clear that the Dugdale line zone model predicts a considerably greater extent of plasticity ahead of the crack tip than the zone model in Eqn 2.6, i.e

$$r_y (\text{Dugdale}) = 2.5 r_y (\text{plane stress}) \quad \text{Eqn 2.14}$$

Burdekin and Stone [13] derived an expression for the crack opening displacement δ at the tip of the crack. Their analysis made use of the Westergaard stress functions appropriate to the yield loads across the plastic zones and to the applied stress acting on a crack of length $2a+2r_y$.

Then δ is given by :

$$\delta = (8 \sigma_y a / (\pi E)) \ln [\sec (\pi \sigma / (2 \sigma_y))] \quad \text{Eqn 2.15}$$

at very low stresses, this may be simplified to give :

$$\delta = \sigma^2 \pi a / (\sigma_y E) = K^2 / (E \sigma_y) \quad \text{Eqn 2.16}$$

The value of critical crack opening displacement at which fracture will occur is denoted by δ_c . There also exists a relationship between the two fracture criteria G_{IC} and K_{IC} where for mode I :

$$\begin{aligned} G_{IC} &= K_{IC}^2 / E & (\text{plane stress}) \\ G_{IC} &= (K_{IC}^2 / E) \cdot (1 - \nu^2) & (\text{plane strain}) \end{aligned} \quad \text{Eqns 2.17}$$

For non - LEFM the concept of G is still valid and of practical benefit, but the behaviour of the crack will no longer be controlled by the stress intensity factor and hence Eqns 2.17 are inapplicable.

2.3 FATIGUE CRACK PROPAGATION

Fatigue failure usually occurs as a result of stresses that are small in comparison to those required for failure due to a monotonic stress. This was proven over a century ago by Wohler 1870 [2] when he published results from rotating bend tests on steels. Goodman [14] modified the test procedure to push-pull loading and examined the effects of the cyclic mean stress upon the cycles to failure. A typical S-N curve is shown in Fig 2.6 illustrating that a material subjected to a cyclic stress will fail at progressively shorter times, the higher the applied stress. In most cases there is also a limiting low stress ("fatigue endurance limit") below which failure does not occur on any realistic time scale.

The Wohler test has been applied successfully to polymers by Constable et al [15] to examine the effects of notch root radii in fatigue. However, such tests and resulting data do not generally provide much information concerning the mechanisms of the fatigue process. For example, they do not establish whether crack initiation or propagation is the dominant phase in the fatigue life nor do they provide a quantitative assessment of the fatigue life of a component or structure. Therefore, over the last decade or so fracture mechanics principles have been increasingly applied to the dynamic fatigue

of polymers so as to gain an insight into both the kinetics and the mechanisms of fatigue crack growth. A major objective of any fracture theory is to identify a quantitative relationship between the stresses acting on a structure, the dimensions and configuration of a crack existing in that structure and the relevant mechanical properties of the material in question. Irwin's [10] stress intensity factor, K , and the tearing energy, T , proposed by Rivlin and Thomas [16] for non-linearly elastic materials, have been correlated with fatigue crack propagation rates. Andrews and Walker [17] used the tearing energy theory in studies on FCP in low density polyethylene but this approach has been almost exclusively confined to FCP in rubbers.

Numerous relationships have been proposed to describe the FCP behaviour of metallic and polymeric solids based on fracture mechanics principles. Paris [18] postulated that the stress intensity factor, itself a function of stress and crack length, was the major controlling factor in the FCP process. The following relationship between the crack growth per cycle da/dN and the stress intensity factor K was suggested :

$$da/dN = A \Delta K^m \quad \text{Eqn 2.18}$$

$$\text{where } \Delta K = K_{\max} - K_{\min} \quad \text{Eqn 2.19}$$

and A and m are material constants determined experimentally.

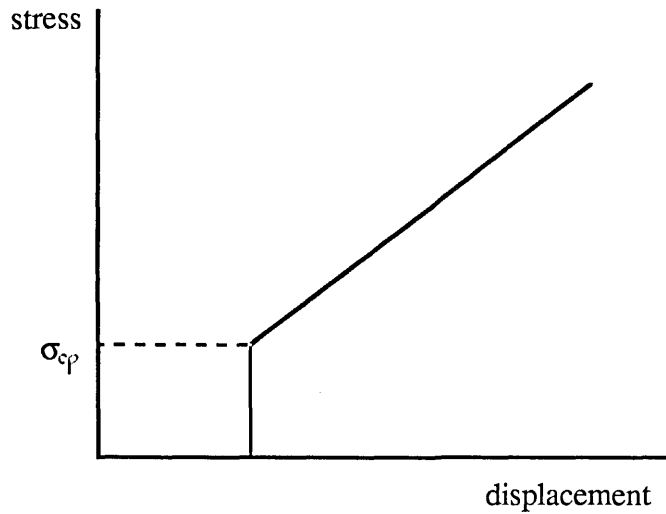
Although many studies have verified the log-log linear relationship between da/dN and ΔK , for several polymeric solids, others have shown FCP plots to assume a sigmoidal shape such as that shown in Fig 2.1. It is obvious that the above relationship describes only the central portion of the sigmoidal curve. Crack growth rates are sometimes found to decrease to vanishingly low values at the lowest value of ΔK . This value of ΔK is known as the threshold stress intensity range ΔK_{th} and for polymers values of less than $0.5 \text{ MPa } \sqrt{\text{m}}$ are typical.

However, as mentioned earlier the application of the above concept to fatigue is difficult. Firstly, a given ΔK level does not identify the stress system applied to the specimen. Hence a second parameter is needed, i.e $K_{\max}$ or $R=K_{\min}/K_{\max}$. One

way of overcoming such difficulties was proposed by Elber [19]. The physical argument in the analysis is that the amount of plastic deformation left in the wake of the crack can be high enough to close the crack during part of the fatigue cycle and in these circumstances there is a difference between the applied stress intensity factor range and its effective value. This is defined as the factor K corresponding to an open crack ($\Delta K_{ef} = K_{max} - K_{Op}$). Hence the Paris law is modified to :

$$da/dN = A \Delta K_{ef}^m \quad \text{Eqn 2.20}$$

Hence, by monitoring the applied stress against local displacement curve and detecting the moment of crack closure, the stress σ_{Op} can be obtained (shown below). Thus K_{Op} can be calculated.



2.4 EFFECT OF TEST VARIABLES

2.4.1 FREQUENCY

The influence of the test frequency depends greatly upon the viscoelastic nature of the polymer in the temperature range being investigated. For example it was reported by Hertzberg and Manson [20] that fatigue crack growth rates for PMMA, polystyrene and PVC decreased with increasing frequency over the range 0.1 to 100 Hz while those for polycarbonate, polysulphone, nylon 66 and poly (vinylidene fluoride) showed no change.

It was further suggested by Hertzberg and Manson that the attenuation of fatigue crack propagation rates occurs when the test frequency is similar to the frequency of the secondary (β_r) molecular - relaxation transition of the polymer. Their interpretation was that under these conditions maximum hysteresis and associated heating occur but that these effects are confined to the immediate vicinity of the crack tip. Therefore the local yield stress falls causing a greater extent of plastic deformation and crack tip blunting. Thus, while hysteretic heating on a large scale may contribute to thermal failure, localised heating at the crack tip may result in lower fatigue crack growth rates. On the other hand Williams [21] has explained the above frequency effect using his two-stage Dugdale model by assuming that the time dependence of the craze surface stress, σ_{cs} may be described by a power - law relation. This results in :

$$da/dN \propto f^{-m.n} \quad \text{Eqn 2.21}$$

where f is frequency, m is the Paris law exponent and

$$n = d \ln E / d \ln t \quad \text{Eqn 2.22}$$

where E is the Young's Modulus and t the time scale ; n is assumed to be approximately constant for a given material. Thus the fatigue crack growth rate is predicted to decrease

as the frequency increases. This is complimented by the fact that lower frequencies mean more time for each load application and consequently the presence of a 'creep' component, a function of chain mobility, cannot be excluded.

2.4.2 WAVEFORM

Changes in the load waveform at a fixed frequency will affect both the strain rate at which the maximum load is attained and the integrated area under one complete loading cycle. For example, loading rates could be increased by switching from a positive sawtooth waveform to triangular, sinusoidal, negative sawtooth and square waveforms in that order. However, the integrated area under these curves is not always the same, thereby making it difficult to isolate the effects of strain rate from the effects of possible creep on FCP behaviour. For example, triangular and positive and negative sawtooth waveforms have the same integrated area and can be used to estimate the effect of strain rate, while square and negative sawtooth have the same loading rate and different integrated areas and can be used to estimate the creep contribution.

Harris and Ward [22] conducted tests on a vinyl urethane, and with a square waveform, no frequency effects were detected for frequencies from 5 Hz down to one cycle every two minutes. However, when a triangular waveform was applied, the crack growth rate increased by a factor of six. They argued that the improved fatigue response of this material with the square waveform was due to stiffening and strengthening in the rubbery material associated with the very high loading rate of the square wave. Conflicting results have been found for PMMA [23], where FCP associated with a square wave profile was 2.4 times greater than that associated with a triangular waveform. Manson, Hertzberg and Skibo went on to confirm the existence of a creep effect. Square and negative sawtooth waveforms (same loading rate) were compared and crack growth rates were found to be consistently higher with the square wave which possesses twice the load - time integrated area as compared with the sawtooth waveform.

2.4.3 TEMPERATURE

The effect of temperature on fatigue crack propagation rates is rather confusing. Furthermore, the limited amount of available data are not self-consistent because of different test methods used and different materials possessing different viscoelastic and deformation characteristics. For example, in some polymers, polystyrene and PMMA the rates may decrease with decreasing temperature [24,25] while in others, such as polycarbonate and polysulphone, the rates may pass through maximum values at particular temperatures [26,27]. It should be noted that other experimental variables, such as frequency, will influence the type of behaviour observed. That is a change in frequency can change both the temperature of the specimen, particularly at the crack tip, and consequently the Young's modulus of the material. In fact tests carried out on PMMA at 1Hz, 5Hz and 100Hz under fixed load conditions show that the fatigue crack propagation rates at a given ΔK decrease monotonically with decreasing temperature. Conflicting results were found by Kurobe and Wakashima [28,29] when they reported that FCP rates in PMMA increased with decreasing temperature over a testing range of 50 to -10°C. Fatigue crack growth rates in polystyrene under fixed loading test conditions have also been shown to decrease with decreasing test temperature. Mai and Williams [24] reported a 20 - fold decrease in FCP rates at 0.15Hz when the test temperature was reduced from 60 to -60°C.

For PC [30,31] and PSF [30,32] at 1Hz the behaviour is much different since the fatigue crack propagation rate first increases as the temperature is lowered from 20 to -50°C, but as the temperature is decreased still further, a monotonic decrease in the fatigue crack propagation response is reported.

Therefore, the temperature dependence on FCP can be complex and further experimentation is required. However, the decrease in chain mobility as we move down to lower temperatures must play an important role near threshold conditions. On the other hand, as the temperature is increased, the yield stress decreases and a blunting process or an increase in shear lip size may affect the degree of stress concentration.

2.4.4 ENVIRONMENT

When predictions of crack propagation have to be made, data should be available relevant to the conditions prevailing in service. Fatigue crack propagation is affected by an endless number of properties, and the circumstances during the test will seldom be the same as in service. The influence of the environment on the rate of crack growth is the most conspicuous and has been the subject of many investigations on a variety of materials. It has been shown, for Aluminum alloys that the rate of fatigue crack propagation in normal wet air can be an order of magnitude higher than in vacuum [33,34]. Hartman [35], carried out tests on 2024-T3 alclad sheet material and observed equal rates of growth in wet argon and wet oxygen, and again equal, but much lower rates of growth in dry argon and dry oxygen. On the other hand, El-Hakeem and Culver [36] noted that fatigue crack growth rates in LDPE were greatly enhanced when the crack tip was immersed in a methanol bath as opposed to laboratory air.

At this point it should be noted that environmental effects are also affected by changes in frequency and molecular weight. For example, Marshall et al [37] reported that the rate of crack extension at a given stress intensity factor level decreased with decreasing melt flow index (increasing molecular weight). Martin and Johnson [38] conducted tests in dry nitrogen and nitrogen saturated with both water and ethanol vapours, so as to examine the effect of molecular weight and vapour environment on the fatigue life of PVC films. The results show that there is an approximate eight to tenfold decrease in fatigue life when ethanol vapours are added to a nitrogen environment. Also, fatigue life increased by four orders of magnitude when the molecular weight was increased by sixfold. Hence, by using a higher molecular weight polymer, the deleterious effect of an aggressive environment on fatigue life can be partially offset.

Frequency and environmental effects were examined by Mai [39], when he investigated the fatigue crack propagation behaviour of PMMA in ethanol and carbon tetrachloride at different cyclic frequencies. As expected, he found that crack growth rates per cycle increased with decreasing test frequency with a frequency sensitivity factor (FSF) of 20 (FSF is defined as the multiple by which the FCP rate changes per decade

change in test frequency). Note that this frequency sensitivity is much higher than that obtained from tests conducted in laboratory air.

Finally, it can be concluded that there is not just one explanation on the influence of the environment on the rate of propagation of fatigue cracks. It is likely that different explanations will apply to different materials. The effect is probably a result of degradation of the polymer and as such it is time - dependent, hence the frequency effect.

2.4.5 MEAN STRESS

In most cases fatigue testing involves stress cycling about a non - zero stationary stress and the magnitude of this stress would be expected to alter the fatigue crack propagation rate. This has been proven in metals [40] where an increase in the mean stress, or stress ratio, was reported in general to lead to an increase in growth rates. However, in polymers the effect of mean stress is much more complex and crack propagation response will depend not only on the particular material under consideration but also on test variables, such as temperature, environment and frequency. With this in mind a number of studies have been conducted to evaluate the effect of mean stress on the rate of fatigue crack propagation at different ΔK levels. Furthermore, a considerable number of semi-empirical crack propagation laws have been proposed to account for such effects [41,42,43,44,45,46].

For example, Arad et al [42] proposed that fatigue crack propagation rates could be related to the crack tip stress intensity factor by the following relationship:

$$\begin{aligned} da/dN &= \beta \lambda^n = \beta (K_{\max}^2 - K_{\min}^2)^n \\ da/dN &= \beta (2 \Delta K \cdot K_{\text{mean}})^n \end{aligned} \quad \text{Eqn 2.23}$$

where β is a function of loading frequency, material properties and environment. n is a constant .

The above relationship identifies the mean stress effect in direct contrast with the Paris type of dependence. Furthermore, it predicts that the fatigue crack growth

rate at a constant ΔK level will increase with increasing mean stress (or increasing stress ratio R). This was confirmed by Murkherjee and Burns [43] who reported that a log-linear relationship between da/dN and λ does exist at a given frequency for PMMA. In contrast, there are reports of fatigue crack propagation data at varying mean stress levels that cannot be normalized with the λ term. For example, for most ΔK levels examined [41], the crack growth rate in polycarbonate decreased with increasing mean stress level. Hence the fatigue data are better correlated by using the ΔK parameter rather than λ . Mai and Williams [24] reached the same conclusion when they examined mean stress effects on FCP behaviour in polystyrene. They reported that by increasing the load ratio R from 0.0 to 0.6, da/dN values increased by about two - to threefold at a given ΔK level (crack growth data plotted against ΔK).

In general brittle materials which have a high yield stress tend to show a strong mean stress effect and the fatigue crack propagation response is therefore strongly dependent on the maximum stress intensity factor in the cycle. On the other hand, the presence of shear yielding or the ability of the material to craze at the crack tip is better described in terms of ΔK .

Finally, Hertzberg and Manson [20] suggest that mean stress effects are a result of two conflicting processes. On one hand, there may be a tendency for higher crack growth rates with higher mean stress levels as a result of a creep component, and also because the maximum stress intensity factor in the cycle approaches the limit condition. Alternatively, there may be a strain-induced modification of the polymer structure, making the fatigue process more difficult and resulting in lower fatigue crack growth rates.

2.5 DEFORMATION PROPERTIES OF POLYMERS

A vital consideration for the use of polymers in engineering applications is the need for knowledge of the way they respond to mechanical deformation. In fact, polymers are organic long chain materials which show a certain degree of viscoelasticity depending on the particular composition of the material, the molecular structure, the time scale and the temperature. Consequently, if the stress concentration at the tip of a crack is sufficiently high, the yield stress of the material may be reached. And in contrast to metals, yielding is controlled by chain mobility rather than slip along the planes of maximum shear stress.

Three types of yielding may occur in polymeric materials, homogeneous shear yielding, shear bands and crazing. Shear bands and crazes are similar in that a high degree of molecular orientation is developed in each. In shear bands, which usually develop at an angle between about 38 and 45° with respect to the stress axis, molecules are highly orientated at an angle between that of the band itself and the stress axis. In crazes, on the other hand, the molecules become orientated in fibrils at approximately 0° with respect to the stress axis. The deformation of glassy polymers has been interpreted on a molecular level through the theoretical approaches of Bowden and Raha [47] and Argon [48].

2.5.1 HOMOGENEOUS SHEAR YIELDING

A polymer must have a high yield stress in order to be strong and if bulk homogeneous yielding does occur the polymer is likely to be tough. Even brittle crack propagation in polymers usually involves localised viscoelastic and plastic energy dissipative processes occurring in the vicinity of the crack tip. The main difference between shear yielding and crazing is that shear yielding occurs essentially at constant volume whereas crazing occurs with an increase in volume. Also, unlike shear yielding, crazing is a cavitation process in which the initiation step requires the presence of a dilatational component to the stress tensor. However, both involve localised, or

inhomogeneous, plastic deformation of the material which arises from strain softening and geometric considerations.

Sternstein [49] has argued that homogeneous shear yielding is associated with high chain mobility and is therefore related to low strain rates, high temperatures and the presence of plasticisers.

2.5.2 SHEAR BANDS

Shear bands are a particular type of shear yielding and are associated with an inhomogeneous type of deformation. In contrast to homogeneous shear yielding, high values of stress concentrations, high strain rates and factors hindering segmental mobility will favour shear band formation. And as mentioned earlier shear bands are usually orientated at an angle between 38 and 45° with respect to the stress axis. Amorphous glassy polymers, particularly when loaded in compression, have a tendency to form shear bands and since compressive stresses obviously inhibit craze formation, the likelihood of shear band formation is further increased. It has been shown [50] that different materials form shear bands which are intermediate in character. Bowden [51] suggested that the difference in character of the bands arises from the differing rates at which they propagate, which is largely controlled by the rate of strain softening and strain rate sensitivity of the yield stress. More detailed work on the structure of shear bands has been conducted by Whitney [52]. His work on compression tests on polystyrene demonstrated that shear bands observed at room temperature may consist of microshear bands about 0.5 to 1µm thick. He further noted that packets of such microshear bands propagate across the sample at an angle of 38° to the compression direction.

A second form of shear band has been observed in polystyrene by Kramer [53,54]. He studied the diffuse shear zones generated in notched bars of polystyrene subjected to uniaxial compression, and reported that the diffuse shear zones were viscoelastic, rather than plastic in character, and the strain in them was relatively low. Other interesting findings have been published by Wu and Li [55], Bowden and Raha [56] and Argon [57], and can be referred to by the reader.

2.5.3 CRAZING

A craze is initiated when an applied tensile stress causes microvoids to nucleate at points of high stress concentrations in the polymer. The microvoids develop in a plane perpendicular to the stress axis, but do not coalesce to form a true crack since they become stabilised by plastically deformed polymeric material. Like the formation of shear bands, crazing involves localised or inhomogeneous plastic deformation of the material which arises from strain softening. However, in contrast to shear yielding which occurs essentially at constant volume, crazing is a cavitation process and hence occurs with an increase in volume. Furthermore, brittle crazing mechanisms are essentially favoured by low temperatures, high strain - rates, aggressive liquid environments and a plane strain stress state. Triaxial tensile stresses, which exist ahead of flaws in relatively thick specimens in plane strain can also enhance craze initiation. Thus it can be concluded that a bulk shear yielding mechanism will lead to a ductile failure, whilst the presence of sharp cracks or defects in thick specimens (associated with a plane strain state), will favour craze initiation leading to a brittle failure.

Several informative reviews concerning crazing have been published by Kambour and Robertson [58], Rabinowitz and Beardmore [59] and Bucknall [60]. The review on deformation by Sternstein [50] also contains useful information. However, the details of the mechanisms of craze advance are not well established, but various explanations have been offered by Argon and Salama [61]. They rejected the hypothesis that crazes grow by the repeated nucleation of microvoids ahead of the main craze, and claimed that this was inconsistent with both the relative low stress concentration at the craze tip and the craze microstructure observed. They suggested that craze growth occurs by the mechanism of "meniscus instability" in which the yielded polymer at the concave air / polymer interface at the craze tip breaks up and produces new craze matter by repeated convolutions. Donald and Kramer [62] confirmed this theory when they obtained stereo-pairs of transmission electron micrographs of craze growth in polystyrene. Furthermore, Argon and Salama [61] showed that craze growth rate via the meniscus instability mechanism was theoretically and experimentally governed only by the value of

the maximum principal tensile stress.

Thus far we have presented the deformation mechanisms available and the most important variables which, in the case of fatigue cycling, can affect the segmental motion in the zone ahead of the crack tip. The effects of these variables will determine the contribution of each deformation mechanism and ultimately will define the degree of stress concentration and the engineering strength of the deformed region. Thus, in these conditions, the stress intensity factor cannot completely identify the deformation and fracture processes which take place in the plastic zone.

2.5.4 DISCONTINUOUS FATIGUE CRACK PROPAGATION

At first it was assumed due to the existence of considerable evidence of the one-load-cycle-one-fracture-band relationship in metals that the distance between the striations on fatigue fracture surfaces represented the fatigue crack propagation per cycle. However, in some polymers, under certain test conditions, fatigue cracks had progressed across the specimen in discontinuous increments associated with a large number of loading cycles. This was proven by Elinck [63] when he conducted tests on FCP rates in PVC and found that many hundreds of loading cycles could occur over the distance between the striation spacings. Doll et al [64] and Rimnac et al [65] have since shown that the striation spacing is the result of a crack jump during a single loading cycle, but many hundreds of loading cycles may be required to condition the crack tip before this jump may occur. Hence the crack tip remains stationary for many loading cycles and then at some particular cycle it advances by an increment, thus forming the discontinuous band spacing. Also Skibo [66] showed that between successive crack jumps a craze grows continuously up to its end size and then the crack jumps up to the craze tip, thus end craze length and band spacing are reported to be equal in size. However, except at very low ratios of lower to upper load, the crack does not jump to the full length of the existing craze. Hertzberg and Manson [41] have also observed the above effect of continuous craze growth and discontinuous crack growth, shown in Fig 2.7. Initially the crack is stationary, and the craze grows continuously to a critical length during many loading

cycles, then during a single loading cycle the crack rips through the majority of the craze and the process begins again.

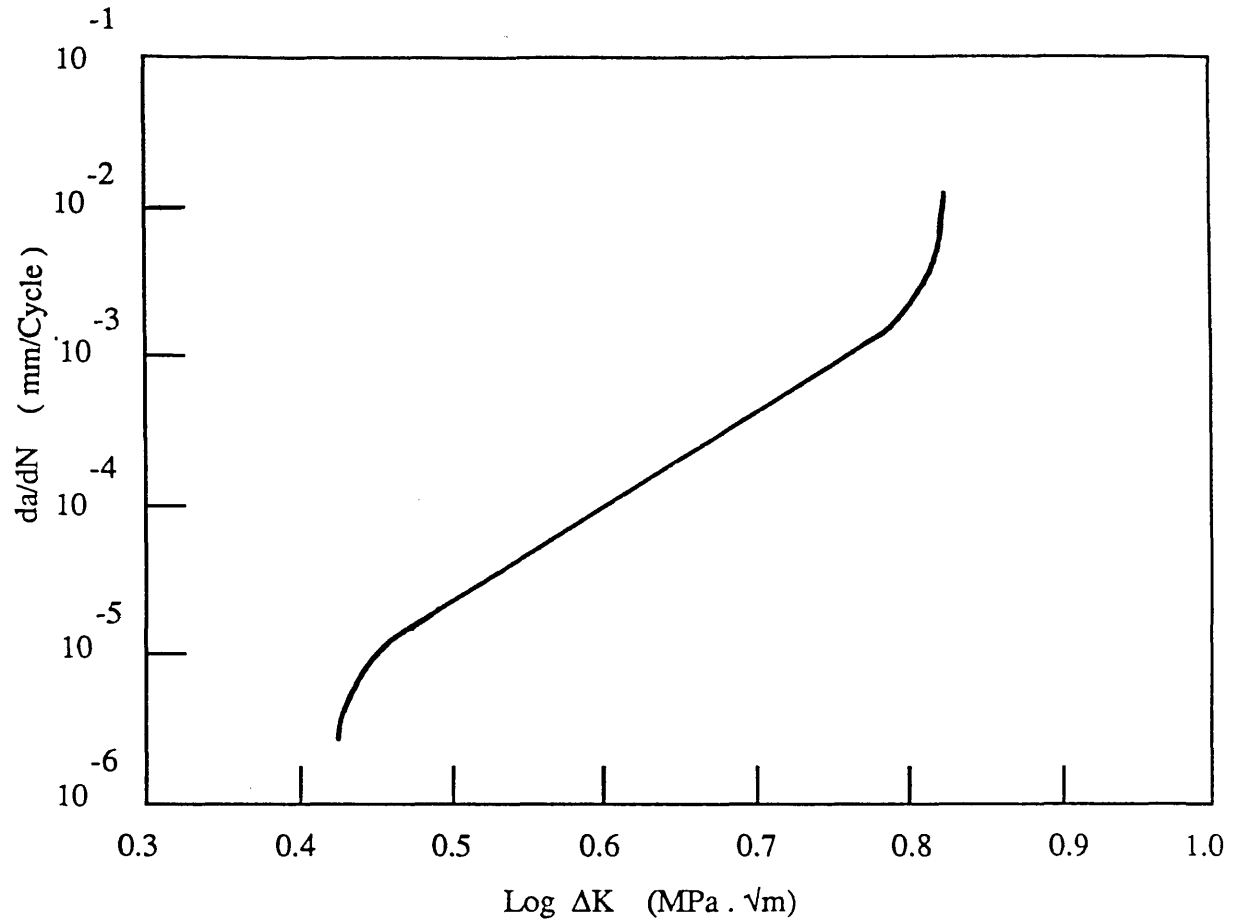


Fig 2.1 A typical Paris plot of $\text{Log } da/dN$ against $\text{Log } \Delta K$

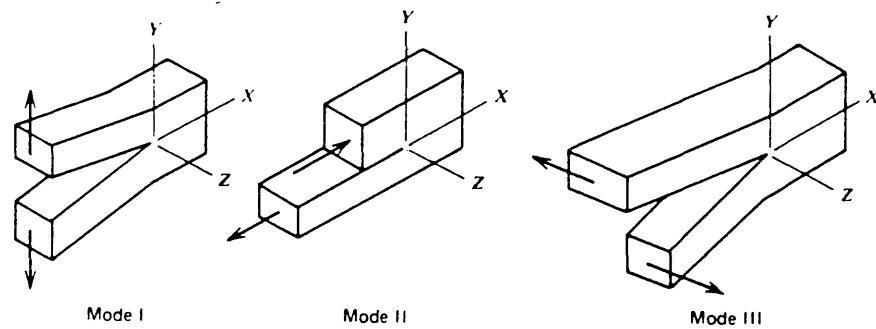


Fig 2.2 The three modes of failure

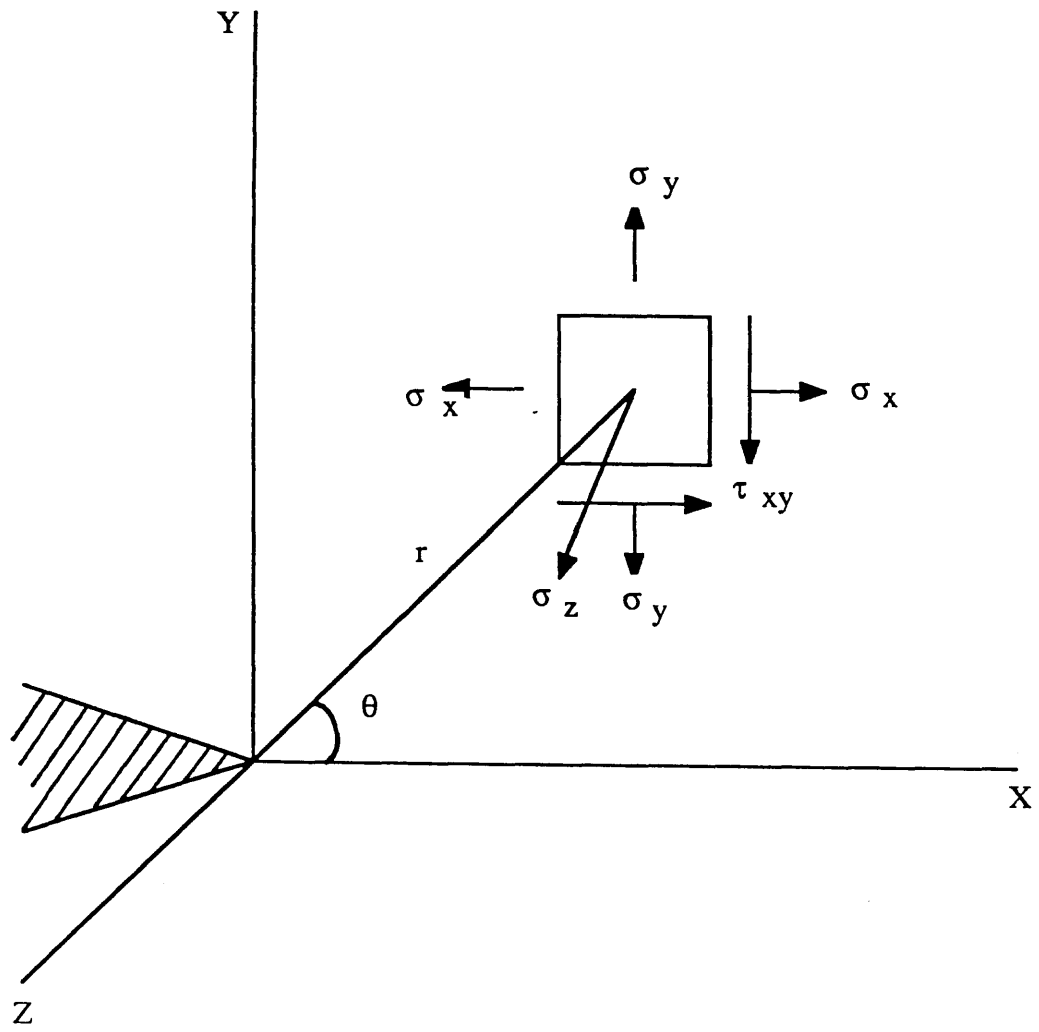


Fig 2.3 The Stress Distribution around the crack tip

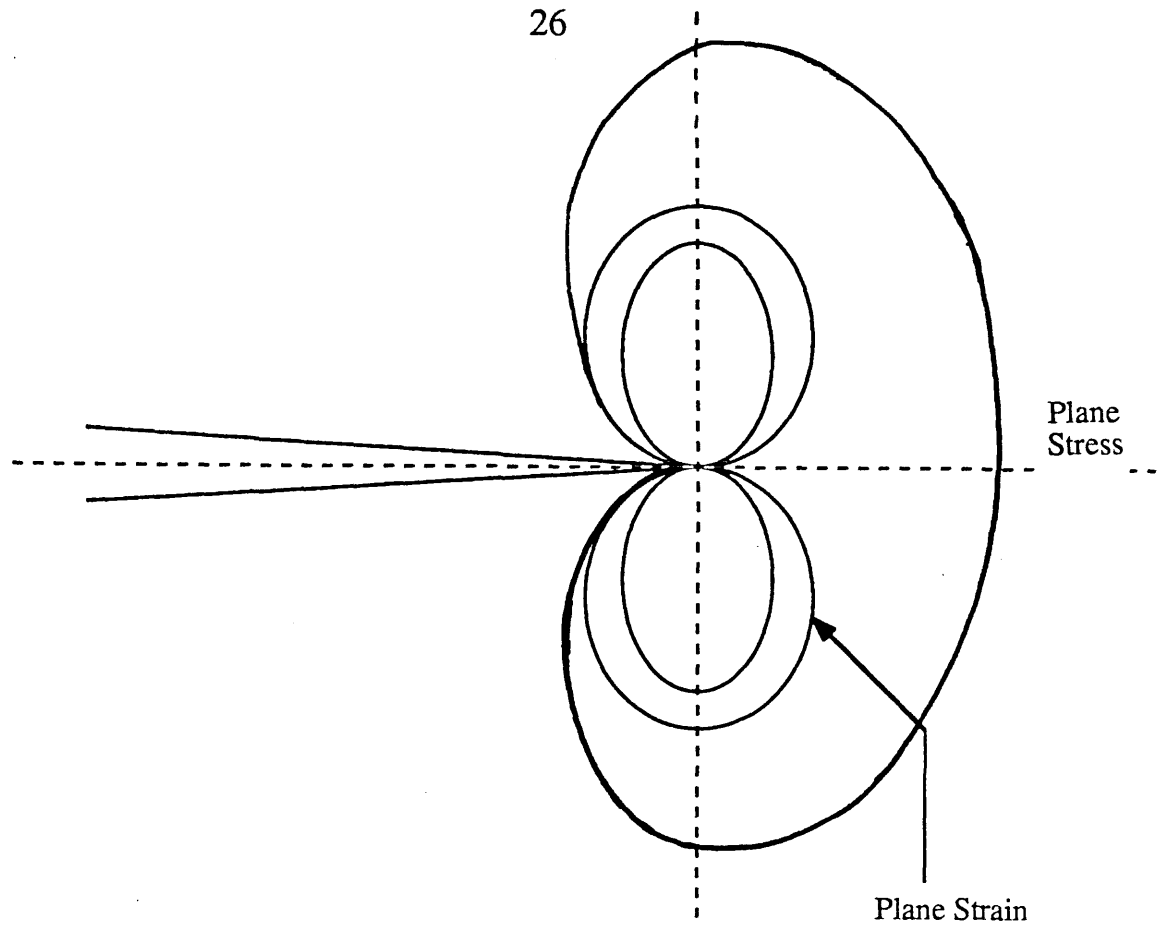


Fig 2.4 The shapes of the two plastic zones
(plane strain and plane stress)

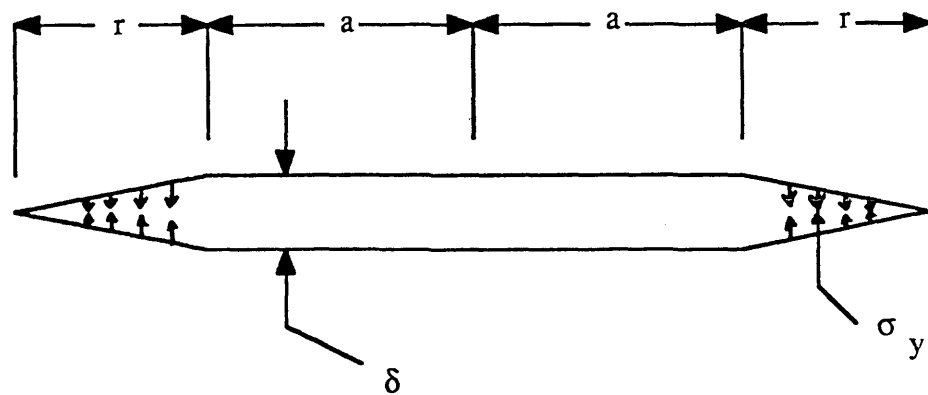


Fig 2.5 The Dugdale Zone

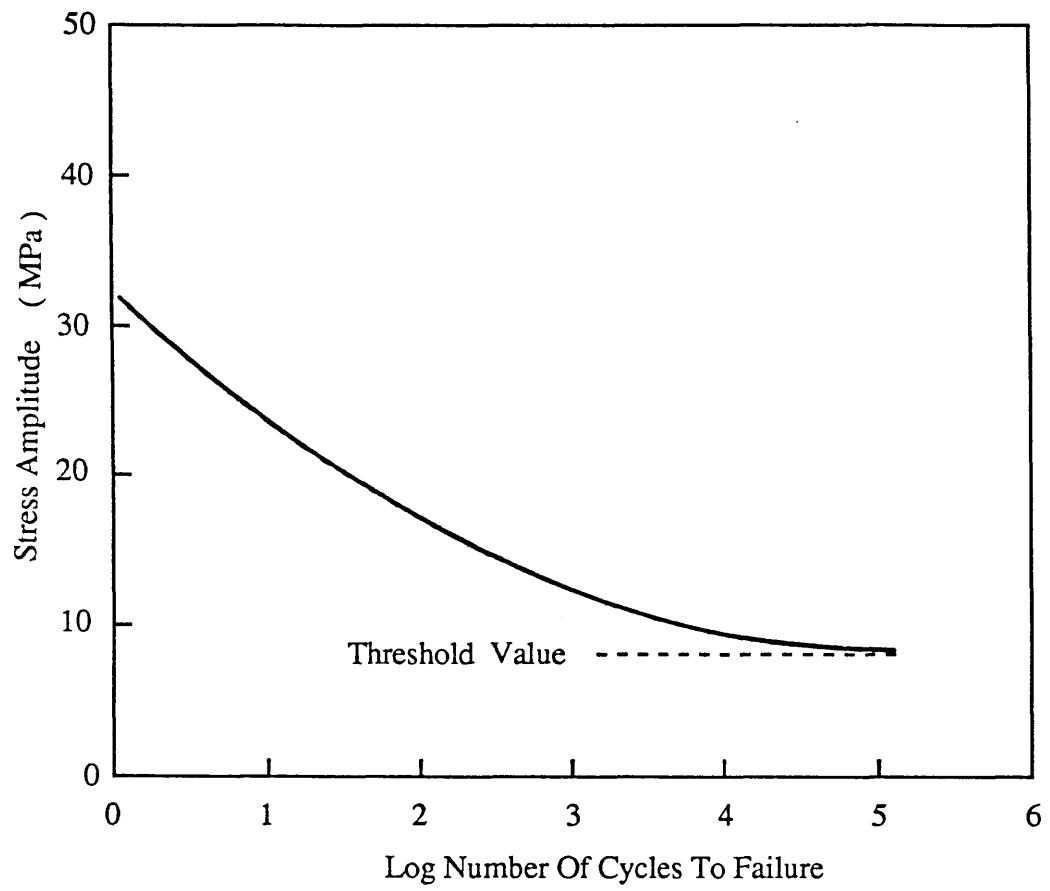


Fig 2.6 A Typical SN Curve

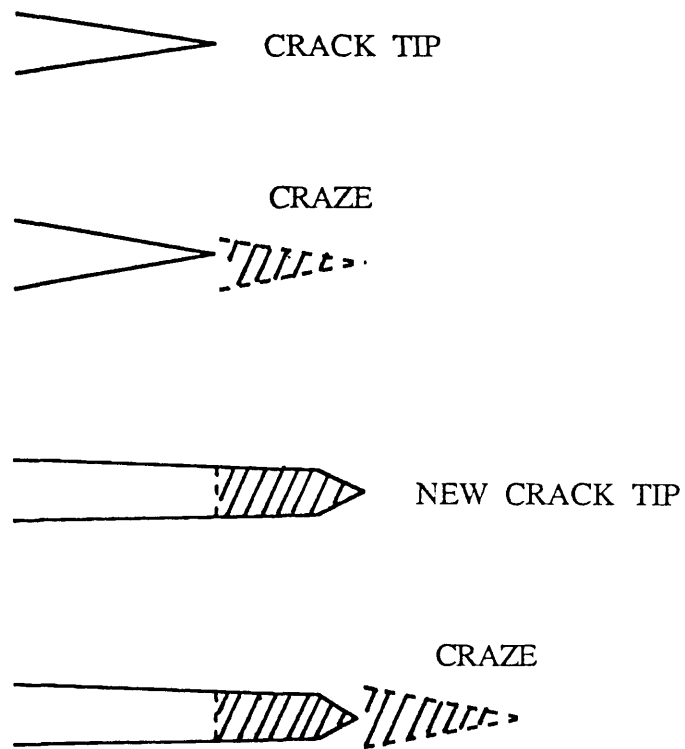


Fig 2.7 The Discontinuous Crack Growth Process

CHAPTER 3

FATIGUE CRACK PROPAGATION MODELS

3.1 INTRODUCTION

The most widely accepted relationship used to represent and predict FCP behaviour has been the Paris representation given by Eqn 3.1 :

$$da/dN = A \Delta K^m \quad \text{Eqn 3.1}$$

This representation is only applicable to LEFM conditions where the stress intensity factor is a sufficient parameter to describe the whole stress field at the tip of a crack. However, when the size of the plastic zone at the crack tip is small compared to the crack length and specimen thickness, the stress intensity factor may still give a good indication of the stress environment of the crack tip. Furthermore, the representation is based on the theory that if two different cracks have the same stress environment, i.e the same stress intensity factor, they behave in the same manner and show equal rates of growth. Therefore data obtained from specimens tested at various mean stress levels should all lie on a single straight line when plotted on a double logarithmic plot of da/dN against ΔK .

However, it has been shown throughout this research programme that Eqn 3.1 does not fully represent reality. In fact FCP rates have been seen to decrease with increasing ΔK and FCP resistance has been enhanced with increasing R-ratio. This is due to the fact that many more parameters are involved than are accounted for by Eqn 3.1 (i.e $K_{\max}$, K_{mean} , temperature, test frequency and molecular structure). The effects of these parameters are further enhanced due to the viscoelastic nature of polymeric materials. Several investigators have tried to establish empirical relations which attempt to incorporate the effect of such parameters. Some of these empirical relations will be discussed in this chapter and will be used to describe FCP behaviour for low and linear low density polyethylene.

3.2 FATIGUE CRACK PROPAGATION MODELS

3.2.1 Empirical model for FCP in polymers by H.M.El-Hakeem and L.E.Culver.

The empirical modelling approach to FCP reported by El-Hakeem and Culver (1981) [67] was proposed for a HDPE. However, their main objective was to produce a physically rational model explaining the behaviour of dynamic fatigue cracks in polymers. The authors assumed that FCP rates are controlled by the condition of the plastic zone at the crack tip. They represented the condition of the plastic zone by the COD and/or the plastic zone size, both of which are proportional to K^2 .

The relationship proposed is given below :

$$da/dN = A\Psi^L \quad \text{Eqn 3.2}$$

where

$$\begin{aligned} \Psi &= (K_{\max}^2 - K_{\min}^2)^{\alpha/L} (K_m^2)^{\beta/L} \quad \text{where } L=\alpha+\beta \\ &= (\Delta K)^2 [(0.25)^\beta ((1+R)/(1-R))^{\alpha+2\beta}]^{1/(\alpha+\beta)} \end{aligned} \quad \text{Eqn 3.3}$$

where A , α and β are functions of test frequency, environment and material properties, and K_m is the mean stress intensity factor. The authors went on to convert Eqn 3.3 into a form based on COD, the cyclic crack tip plastic zone size and the energy available for crack propagation. Therefore, the model can be used in any of the above forms depending on which term is viewed as the governing parameter of crack growth.

3.2.2 FCP model by R.G.Forman , V.E.Kearney and R.M.Engle .

Forman [68] suggested that, essentially, two effects occur which are not taken into account by the Paris representation. One is the variation in FCP rates owing to the load ratio, R , and the other is the instability of the crack growth when the stress intensity factor approaches the K_{IC} -value for the material. Based on data presented for 7075-T6 and 2024-T3 aluminum-alloy plates the author set out to modify the Paris exponential crack propagation equation to cover both the above effects.

It was proposed that a valid crack-growth law should have the criterion

$$\lim_{K_{max} \rightarrow K_{IC}} da/dN = \infty$$

and by making the substitution of

$$K_{max} = \Delta K / (1-R)$$

$$\lim_{\Delta K \rightarrow (1-R)K_{IC}} da/dN = \infty$$

Then by assuming that a correct FCP equation has an exponential form, and also a singularity at $(1-R)K_{IC} - \Delta K$, the author proposed the following improved equation :

$$da/dN = A \Delta K^m / [(1-R) K_{IC} - \Delta K] \quad \text{Eqn 3.4}$$

However, from the above equation and for a given ΔK range, the model predicts faster FCP rates with increasing R -ratios. Therefore, this model does not take into account possible crack tip blunting due to higher mean stresses causing a more severe condition at the crack tip.

3.2.3 FCP Model by K.Walker .

This model is again based on data presented for 7075-T6 and 2024-T3 aluminum [69]. The author's objective was to present a simple expression accounting for stress - ratio effects. It was shown that an effective stress, given by :

$$\bar{\sigma} = \sigma_{\max}^{1-m} \Delta \sigma^m \quad \text{Eqn 3.5}$$

where m is a material constant, can predict the effect of stress ratio on crack propagation and fatigue life for the two aluminum alloys. Since the product of the two stress terms in Eqn 3.5 has the dimensions of stress, the author defined an effective stress as :

$$\bar{S} = S_{\max}^{1-m} \Delta S^m = S_{\max} (1-R)^m$$

Therefore, an effective stress intensity factor range was defined as :

$$\begin{aligned} \overline{\Delta K} &= S_{\max} (1-R)^m . Y . \sqrt{a} \\ &= K_{\max} (1-R)^m \end{aligned} \quad \text{Eqn 3.6}$$

and an equation for crack growth was written in the form :

$$da/dN = A \overline{\Delta K}^n \quad \text{Eqn 3.7}$$

Finally, it should be pointed out that the above analysis was based on and is a modification of the more general equation

$$da/dN = A K_{\max}^m \Delta K^n \quad \text{Eqn 3.8}$$

Therefore, the exponent m is found by plotting $\log da/dN$ against $\log K_{\max}$ for a constant

value of ΔK . For the model to work effectively, FCP rates must increase with increasing R-ratio and therefore with increasing $K_{\max}$ values (i.e positive m value). This model, therefore, again assumes higher FCP rates with increasing R-ratio and consequently ignores the possibility of crack tip blunting.

3.3 FATIGUE CRACK GROWTH MODEL BY J.G.WILLIAMS (1977)

This model is based on a line plastic zone analysis. However, the tip of the fatigue crack is assumed to comprise of two zones and hence the model is referred to as the two stage line zone model [70]. The zone furthest from the crack tip has a craze stress σ_c which is subsequently reduced to a lower level $\alpha\sigma_c$ by cyclic loading. This is due to the ligament material between the voids in the craze being damaged by cyclic loading thus reducing its load carrying capacity. For simplicity it is assumed that this damage to a limiting value occurs in a single loading cycle, although in reality, several cycles would be required. Furthermore, the craze formed at the tip of a sharp notch, during the first loading cycle, will be much smaller than a craze formed during subsequent cycles (which comprises material with a low load carrying capacity). For the two stage line zone to remain compatible with the applied load, additional crazing at the high craze stress must occur during the second cycle of loading.

The process of craze extension continues with a corresponding increase in crack opening displacement (COD). When the crack opening displacement reaches a critical value, crack growth occurs. When crack growth has initiated, the rate of growth is approximately equal to the length of the high load carrying capacity craze, r_0 (i.e $r_0=da/dN$). This is related to the applied stress intensity factor and the critical static value. Therefore, for a simple plastic zone (Fig 3.1), the length r is given by :

$$r = \pi/8 [K/\sigma_c]^2 \quad \text{Eqn 3.9}$$

where σ_c is the craze stress.

Eqn 3.9 may be rewritten as :

$$\sigma_c \sqrt{r} = K. \sqrt{(\pi/8)} \quad \text{Eqn 3.10}$$

For the two stage line zone model, illustrated in Fig 3.2, the stress is σ_c over a region of length r_0 at the zone tip, and $\alpha\sigma_c$ over the remaining length. Therefore the equivalent relationship to Eqn 3.10 is :

$$(1 - \alpha). \sigma_c \sqrt{r_0} + \alpha. \sigma_c \sqrt{r_1} = K. \sqrt{(\pi/8)} = \sigma_c \sqrt{r}$$

$$\text{i.e.} \quad \sqrt{r} = \alpha \sqrt{r_1} + (1 - \alpha) \sqrt{r_0} \quad \text{Eqn 3.11}$$

From the crack opening displacement, δ , given in Eqn 2.16 by :

$$\delta = K^2 / (E \sigma_c)$$

and from Eqn 2.13

$$K^2 = 8.r.\sigma_c^2 / \pi$$

δ can be rewritten as :

$$\delta = 8.\sigma_c.r / (\pi.E) \quad \text{Eqn 3.12}$$

However, for the two stage line zone model, the equivalent relationship to Eqn 3.12 is :

$$\delta = 8\sigma_c r_1 [\alpha + (1 - \alpha) \{ y + (1/2).(1 - y^2). \ln((1 + y)/(1 - y)) \}] / (\pi E)$$

where $y^2 = r_0 / r_1$

For $y \ll 1$, the above equation can be reduced to :

$$\delta = 8\sigma_c [\alpha r_1 + 2(1 - \alpha) \sqrt{r_1}.\sqrt{r_0}] / (\pi E) \quad \text{Eqn 3.13}$$

Taking $\delta = \delta_c$ at fracture, then

$$\delta_c = 8\sigma_c r_c / (\pi E)$$

where r_c is the critical single stage zone length at fracture and from Eqn 3.13 is given by :

$$r_c = \alpha \cdot r_1 + 2 \cdot (1 - \alpha) \cdot \sqrt{r_1} \cdot \sqrt{r_0} \quad \text{Eqn 3.14}$$

By obtaining an expression for $\sqrt{r_0}$ from Eqn 3.11 and substituting it into Eqn 3.14, we have

$$r_c = 2 \cdot \sqrt{r_1} \cdot \sqrt{r} - \alpha r_1 \quad \text{Eqn 3.15}$$

Then by obtaining an expression for $\sqrt{r_1}$ from Eqn 3.11 and substituting it into Eqn 3.15, the tip zone size at fracture may also be obtained and is given by :

$$(r_0 / r) = [1 - \alpha r_c / r] / (1 - \alpha)^2$$

and therefore

$$r_0 = [r - \alpha r_c] / (1 - \alpha)^2$$

As mentioned earlier, crack growth rate is equal to r_0 , and since $r = \pi K_{\max}^2 / (8 \cdot \sigma_c^2)$ and $r_c = \pi K_c^2 / (8 \cdot \sigma_c^2)$ the relationship for the FCP rate obtained from the above equation is given by :

$$da/dN = \pi (K_{\max}^2 - \alpha K_c^2) / [8 (1 - \alpha)^2 \sigma_c^2] \quad \text{Eqn 3.16}$$

3.3.1 THE TWO STAGE LINE ZONE MODEL (1981)

This is a modification of the 1977 model taking into account both the premature crack closure concept and a more complex two stage line zone. The shortcomings which came to light with the original version were that σ_c , determined from the slopes of the various da/dN versus K^2 lines, increased rapidly with R .

Therefore, in the modified version [71] R -ratio effects were taken into consideration. It was assumed that partial unloading results in the complete unloading of part of the zone and that α and σ_c are the same for this fully unloaded section and the loaded section. Furthermore, fracture occurs at the same K_c condition as in the completely unloaded case. Eqn 3.16 does not give

$$da/dN = r_c = \pi/8 K_c^2 / \sigma_c^2$$

for the limiting case of $K=K_c$. The appropriate approximation proposed, which takes into consideration the above limiting case is :

$$da/dN = \pi (K - \alpha K_c)^2 / [8 \cdot (1 - \alpha)^2 \cdot \sigma_c^2] \quad \text{Eqn 3.17}$$

Further assumptions made were that K^2 takes the form of an effective value dependent on R and that there is a threshold value of K , K_0 , below which there is no growth.

The final form of the equation is :

$$da/dn = \pi/8 (1/(1 - \alpha)^2 \sigma_c^2) [F(R) (K^2 - K_0^2) - \alpha K_c^2] \quad \text{Eqn 3.18}$$

where $F(0) = 1$

Basically a premature crack closure concept has been introduced in the form of a constant factor K_0 which acts to decrease the applied K_{max} . Also the K_{max} term in Eqn 3.16 has

been replaced by $K_{\max}^2 - K_0^2$.

Figs 3.3(a) and 3.3(b) illustrate the possible types of behaviour which determine $F(R)$. In the first condition (Fig 3.3(a)), the tip of the two stage line zone remains loaded at σ_c when the stress intensity falls to $K_{\min}$. In the second condition illustrated by Fig 3.3(b), a zone near the crack tip sustains a stress σ_c at $K_{\min}$.

For cases (a) and (b) $F(R)$ is given by the following relationships :

$$\begin{aligned} F(a) &= 1 - (1-\alpha)^2 R^2 \\ F(b) &= (1-R)^2 + \alpha R^2 \end{aligned} \quad \text{Eqns 3.19}$$

The parameters in Eqn 3.15 were then conveniently expressed in terms of the slopes of the da/dN versus K^2 lines, S and their intercepts K_{th}^2 as :

$$S = \pi F(R) / [8 (1-\alpha)^2 \cdot \sigma_c^2] \quad \text{Eqn 3.20}$$

$$K_{th}^2 = [\alpha K_c^2 / F(R)] + K_0^2 \quad \text{Eqn 3.21}$$

and $F(R)$ may be eliminated to give :

$$S \cdot K_{th}^2 = [\pi \cdot \alpha \cdot K_c^2 / (8 \cdot (1-\alpha)^2 \cdot \sigma_c^2)] + S \cdot K_0^2 \quad \text{Eqn 3.22}$$

The factor K_0^2 can be determined from the slope of a plot of $S \cdot K_{th}^2$ against S .

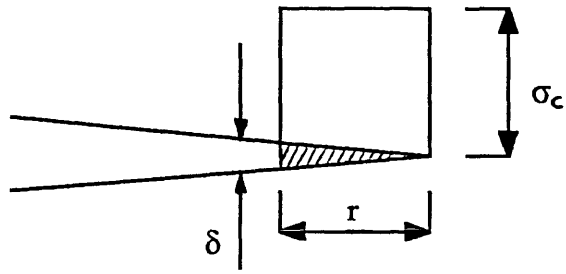


Fig 3.1 Simple Plastic Zone

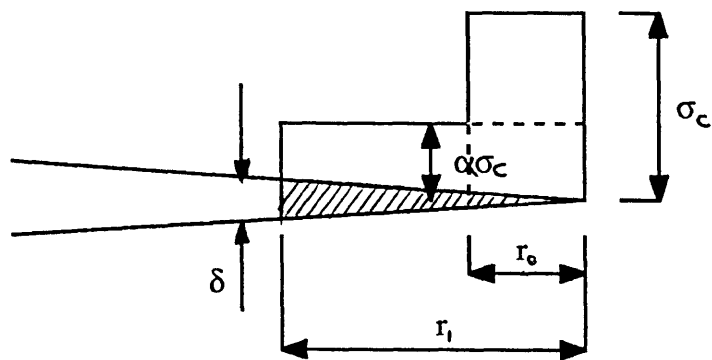


Fig 3.2 The two stage line zone .

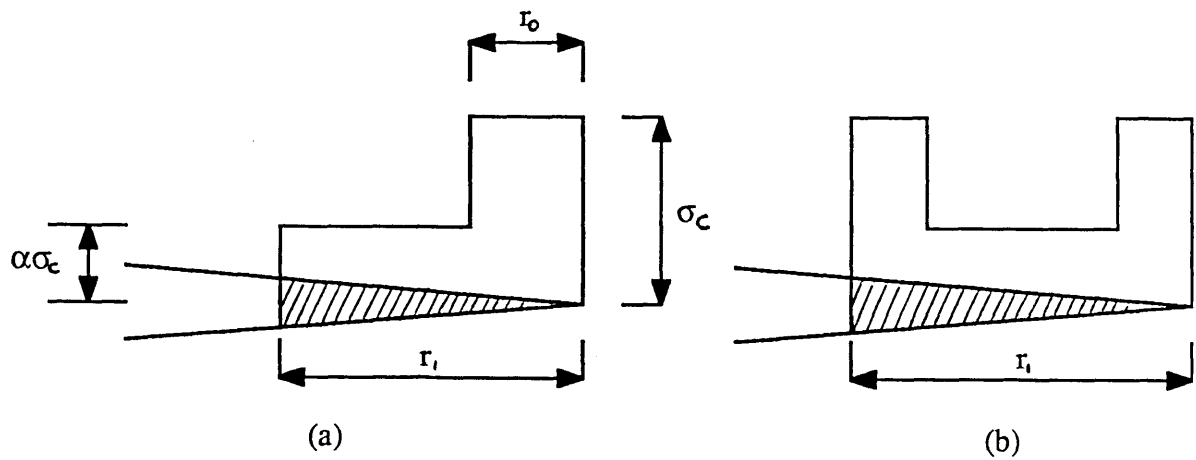


Fig 3.3 Possible types of the two stage line zone.
 (a) Zone tip sustained
 (b) Crack tip sustained

CHAPTER 4

MATERIALS AND EXPERIMENTS

4.1 INTRODUCTION

This chapter describes material characteristics, material properties and experimental details used throughout the course of this research programme.

4.2 MATERIALS

This project set out to determine the toughness and fatigue crack propagation data for linear low density polyethylene (LLDPE) and low density polyethylene (LDPE). Two grades of LDPE and one of LLDPE were supplied by BP Chemicals Ltd. Each material will be referred to by a coded number, the two grades of LDPE are designated by DFDG 4262 and BP33 and the LLDPE by LL101AA. All three polyethylenes have the same nominal density of 920 kg/m^3 but different values of Melt Flow Index (MFI). These values are 1.0 g/10min, 0.3 g/10min and 0.9 g/10min for DFDG 4262, BP33 and LL101AA respectively. The mechanical properties and physical characteristics of each grade are given in Table 4.1.

Polyethylene has a unique structural configuration and it is useful to consider in more detail the nature of the chains and how they are built up. During the polymerisation of the monomer ethylene, the double bond is opened out enabling the carbon single bonds to link up with neighbouring units to form a long chain of CH_2 groups, shown in Fig 4.1.

For a molecular weight of 300,000 there are about 21,000 carbon atoms along the backbone of the chain. Apart from antioxidants, which the suppliers were not willing to identify, the polyethylenes contain no additives. All three grades tested are semi-crystalline and their percentage of crystallinity which can affect their properties is listed in Table 4.1. Furthermore, all experiments were conducted on a single batch of each grade of polyethylene.

Fatigue tests carried out on these materials showed that the linear low

density polyethylene (LL101AA) has a superior toughness to the low density polyethylenes (BP33 and DFDG 4262) - see Chapter 5. An attempt to explain this phenomenon is complicated due to all three materials having the same nominal density, almost equal values of % crystallinity and by the DFDG 4262 and LL101AA materials having almost equal Melt Flow Indexes. Furthermore, the LL101AA material which, as already mentioned is the toughest of the three has the lowest value of Molecular Weight (125,000) and is almost half that for BP33 (240,000). This is surprising and contradicts the model proposed by J.Michel [72] who suggests that an increase in molecular weight leads to an increase in fracture toughness. However, this model was applied to amorphous polymers and cannot necessarily be applied to semi-crystalline polymers where many other factors may influence the material's toughness, such as chain branching, degree of entanglement and size of molecules.

Therefore, it must be realised that each chain must co-exist with other chains in the bulk material and the arrangement and interaction of the chains has a considerable effect on the properties of the material. Also the linear low and low density polyethylenes tested in this research programme can be termed as different classes of material since they have a completely different molecular structure.

The LDPE (BP33) is produced by high pressure, free radical initiated stirred autoclave polymerisation of ethylene. The exact free radical catalyst used for this polymer is not known but the types of catalyst typically used are organic peroxides with suitable decomposition temperatures. The free radical polymerisation process is carried out at pressures of up to 3000 bars and temperatures of up to 250°C. Under these conditions a branched polyethylene is produced. That is, a free radical is generated in the main chain which results in an active site allowing reaction with other neighbouring chains and ultimately branching can occur at these sites. The polymers produced by this process contain both short branches, typically C_2H_5 and C_4H_9 , and also much larger branches ; the overall branching concentration is of the order of 20 branches per 1000 carbon atoms.

In contrast the LLDPE (LL101AA) is an ethylene / butene-1 copolymer

produced over a Ziegler catalyst in a fluidized bed reactor. This process is carried out at much lower pressures of typically 15 to 30 bars and much lower temperatures of between 60 and 100°C. Under these conditions the only type of branching present is C_2H_5 ; there are no long branches as in the LDPE (BP33). The chain backbones are linear and this is why the material is called linear low density polyethylene.

The basic difference in molecular structure may be illustrated diagrammatically as shown in Fig 4.2.

4.3 COMPRESSION MOULDING

The 12 mm sheets of LDPE and LLDPE were compression moulded by BP Chemicals Ltd. The moulding procedure is as follows :

- (i) Preheat mould and material at $170 \pm 5^\circ\text{C}$ for 30 minutes with a very low applied pressure (under weight of ram).
- (ii) Cool mould at $5^\circ\text{C}/\text{min}$ and slowly increase the pressure in stages i.e

120°C	10 Tonnes
110°C	20 Tonnes
100°C	30 Tonnes
90°C	40 Tonnes
80°C	50 Tonnes

- (iii) At 80°C increase cooling rate to maximum ($20^\circ\text{C}/\text{min}$ attained by opening water valves to maximum).

4.4 SPECIMENS

Only one type of specimen geometry was used, centrally cracked tension (CCT), cut from the 12 mm thick compression moulded sheets. However, for tests carried out at negative R-ratios, shorter specimens were used so as to avoid buckling due to compressive forces. The dimensions for both these specimens and for dumbbell

specimens used to determine yield stresses are shown in Fig 4.3. The notches were introduced into the specimens by first drilling a 6 mm diameter hole through the centre and then, using a junior hack saw, a 15 mm long notch was cut on both sides of the hole. Finally, the notches were sharpened with a razor blade. A CCT specimen mounted onto the Mayes servohydraulic machine is shown in Fig 4.4.

4.5 STRESS INTENSITY FACTORS.

Although only one type of specimen geometry was used, specimens of different height, H , were used depending on whether positive or negative R -ratios were employed (see Fig 4.3). Therefore a height correction had to be made to the geometry factor Y for the shorter specimens. For the long CCT specimens ($H=200$ mm between the grips), the geometry factor Y is given by :

$$Y = [\pi (2W/\pi a) \tan (\pi a/2W)]^{1/2} \quad \text{Eqn 4.1}$$

where a is the half crack length and W is the half width of the specimen.

The Y calibration used for the short CCT specimens ($H=70$ mm between the grips) was obtained by fitting a fifth order polynomial to the results given by Rooke and Cartwright [73] and is given by the following equation :

$$Y = 1.0398 + 0.1167x - 1.9133x^2 + 6.075x^3 - 7.0575x^4 + 4.8269x^5 \quad \text{Eqn 4.2}$$

where x is given by a/W , and a and W are the half crack length and half specimen width respectively. The equation is valid for a/W less than 0.7.

The stress intensity factor K is given by :

$$K = Y \cdot \sigma \sqrt{(\pi a)} \quad \text{Eqn 4.3}$$

where σ is the nominal stress applied to the specimen.

4.6 FATIGUE TESTING

4.6.1 THE FATIGUE TESTING MACHINE.

All fatigue tests were carried out on a Mayes servohydraulic testing machine equipped with a closed-loop control system. Three load cells are available (2.5, 25 and 250 kN) and four ranges (10%, 20%, 40% and 100%) can be selected on the control panel. A digital voltmeter displays the feedback signal (from the load, strain or crosshead modules) as a percentage of the selected range and this arrangement provides a high precision for virtually any value of load in the range 0 to 250 kN. The control unit incorporates a function generator and square, triangular and sine waveforms can be selected with frequencies ranging from 0.01 to 100 Hz. Furthermore the machine can cycle specimens under either load or displacement control. Throughout the programme, tests were conducted under sinusoidal load control at temperatures of $23\pm 1^\circ\text{C}$ and $-20\pm 1^\circ\text{C}$ and frequencies between 0.2 and 10 Hz.

In the early stages of the low temperature testing programme, such tests were conducted using a temperature box. This was equipped with a circulating fan, a heating element and a solenoid valve, coupled with a pump unit with pressure control and a Eurotherm unit to monitor the supply of gas and heating. The cooling gas was liquid nitrogen supplied in dewars of 50 or 75 litres. The system controls the temperature to within $\pm 1^\circ\text{C}$ but it has its limitations in that it only allows continuous use of up to 12 or 18 hours. Hence a different system had to be developed which would allow continuous use of up to 30 or 40 days, which is the time needed for a low frequency test (0.2 Hz). This was done by purchasing a domestic freezer which had its cooling pipes incorporated within the inner shelves. This meant that holes could be drilled on the top and bottom of the freezer (for the pull-rods to pass through) without causing any damage to its cooling system. In order to minimize heat loss, bellows were fitted to both the top and bottom holes filled with insulating wool and strapped tightly around the pull-rods (Fig 4.5).

Finally, a double glazed window was fitted to the front door and a Eurotherm control unit connected so as to switch the system on and off at the set temperature. Thus a low temperature test could be run continuously for as long as it took

to break. Fig 4.6 shows the freezer in its adapted form mounted into the Mayes servohydraulic machine and clearly illustrating the double glazed window and how the pipes are incorporated within the three inner shelves. Finally Fig 4.7 shows the complete set-up with the camera used to measure crack lengths (see section 4.6.2) mounted into a security box.

Yield stress tests were conducted on a screw-driven Instron testing machine. Ten different crosshead speeds from 0.05 to 50 cm/min can be selected and the machine incorporates a chart recorder which records load against deflection. In the present programme all yield stress tests were conducted using a crosshead speed of 5 cm/min.

4.6.2 FATIGUE CRACK LENGTH MEASUREMENTS.

Many different techniques have been used to measure crack length with the most popular being the travelling microscope. Although this technique can give accurate results it means that vital measurements can be lost when tests are run overnight. Hence automatic crack length monitoring techniques were considered. One of these was the electrical compliance method, where a graphite coating is used on the polymer in the region of crack propagation. This is better known as the electrical potential method and an electric field is formed by passing a constant current through the specimen. The electrical field changes as the boundary conditions such as crack length change. The change in the field is measured by determining the change in potential at a predetermined point on the specimen. In practice this is done by measuring the potential difference between similar points on opposite sides of the axis of symmetry of the specimen. Four leads are thus attached to each specimen ; one pair of current leads and one pair of sensing leads. The change in potential difference across the sensing leads is continuously monitored on a chart recorder. Finally, the potential difference measured across the sensing leads is related to the crack length a , by a calibration curve and hence crack length values can be obtained.

However, this technique was considered undesirable as the graphite coating has to be sprayed on to the specimen in the order of 5 μm thick. Hence, when

used in conjunction with ductile materials where yielding occurs at the crack tip, the thin layer of coating can crack ahead of the crack tip. Thus false readings will be obtained. Furthermore, false readings will also be obtained if tunnelling occurs because crack lengths are only measured at the surface of a specimen.

Another method available for obtaining crack length values is the compliance method. Compliance, C is given by :

$$C = \delta / P \quad \text{Eqn 4.4}$$

where δ is the deflection and P the maximum applied load. The change in deflection can be continuously monitored by the use of a transducer and a chart recorder. Therefore, by obtaining a calibration curve of deflection, δ , against crack length, a , crack length values can be obtained at any particular value of deflection, δ , (Fig 4.8). Again this technique was considered undesirable since a given calibration curve at a particular R -ratio cannot describe crack propagation at a different R -ratio. That is an increase in R -ratio (and hence in $K_{\max}$) would lead to a greater deflection at corresponding values of crack length. This is because high R -ratios have been found to create greater COD's which blunt the crack tip and consequently give rise to greater deflections at even lower values of crack length. Therefore, this method would require a different calibration curve for each R -ratio which would have to be obtained by the travelling microscope method. Clearly, this is not a very convenient method and also vital measurements will again be lost due to tests being run overnight.

Crack lengths were thus measured using a novel automated process which employs time lapse photography. This consists of a camera (Canon T70) equipped with a Canon 70 command back which can be programmed to take pictures at any set time. That is it can be set to take a picture after a set time of zero seconds to a maximum of 23 hours and 59 seconds, and at an interval of one second to a maximum of 23 hours and 59 seconds. The number of frames to be taken can also be set. Fig 4.9 shows the Canon 70 command back attached on the camera. Another important feature of the Canon 70

command back is that it prints on each frame the date and time at which it was taken. Therefore the calculating procedure for the number of cycles counted is made easy. An extension tube is used on the camera (FD15) which gives a field of view of about 120mm. Lighting is provided by means of a flash gun which is switched on and off by a time switch. This allows the test to be run overnight without losing vital data. Finally, the camera is housed in a security box and the complete set-up is shown in Fig 4.10. The film is then developed, the negatives mounted into slides, projected onto a screen and the crack lengths measured.

Having obtained crack length measurements, da/dN , the crack propagation rate is obtained from slopes of the a - N curve using a computer programme. This employs a curve fitting procedure which involves fitting a second-order polynomial (parabola) to sets of $(2n+1)$ successive data points, where n is usually 1,2,3 or 4. The form of the equation for the local fit is as follows :

$$a_i = b_0 + b_1((N_i - C_1)/C_2) + b_2((N_i - C_1)/C_2)^2 \quad \text{Eqn 4.5}$$

where

$$-1 < ((N_i - C_1)/C_2) < +1$$

b_0, b_1, b_2 are the regression parameters that are determined by the least squares method over the range $a_{i-n} < a < a_{i+n}$ and a_i is the fitted value of crack length at N_i . The parameters $C_1 = 1/2 (N_{i-n} + N_{i+n})$ and $C_2 = (N_{i+n} - N_{i-n})$ are used to scale the input data. The rate of crack growth at N_i is obtained from the derivative of the above parabola which is given by the following expression :

$$(da/dN)_a = b_1/C_2 + 2b_2 (N_i - C_1) / C_2^2 \quad \text{Eqn 4.6}$$

Hence seven successive data points are needed by the programme to fit a value for crack length and in turn from the derivative calculate a value for da/dN .

The computer programme is listed in Appendix 1 and the procedure for

determining da/dN values is as follows. Firstly a data file is opened (also shown in Appendix 1) and successive values for crack length a , number of cycles N , $P_{\max}$, $P_{\min}$ and specimen geometry are typed in. This provides all the information needed by the programme to calculate corresponding values of da/dN and furthermore values for $K_{\max}$ and ΔK . An example of the output from the programme is given in Table 4.2. Information on the specimen and loading variables are listed in the output along with tabulated values of the raw data and processed data. A (MEAS) and A (REG) are values of total crack length obtained from measurement and from the regression equation (Eqn 4.5), respectively. Values of ΔK (and of $K_{\max}$ and $K_{\min}$) and da/dN are given in the same units as the input variables and are $\text{MPa}\sqrt{\text{m}}$ and mm/Cycle respectively.

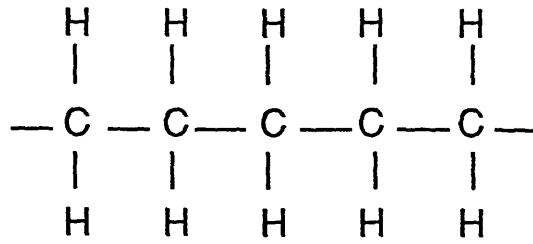
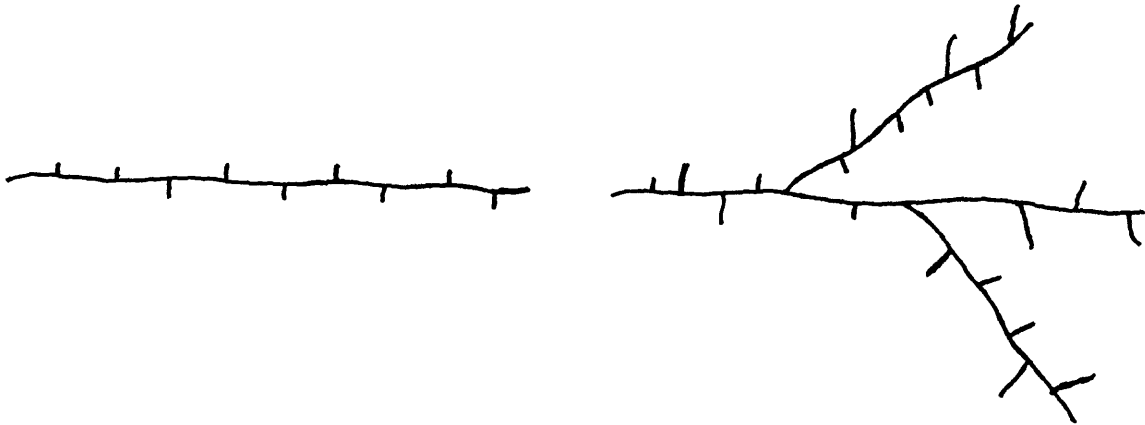


Fig 4.1 Polyethylene molecule.



LLDPE (LL101AA). Linear backbone.
All branches C_2H_5 .

LDPE (BP33). Long chain branched backbone.
Short chain branches of various lengths i.e
 C_2H_5 , C_4H_9 , C_6H_{13} etc.

Fig 4.2

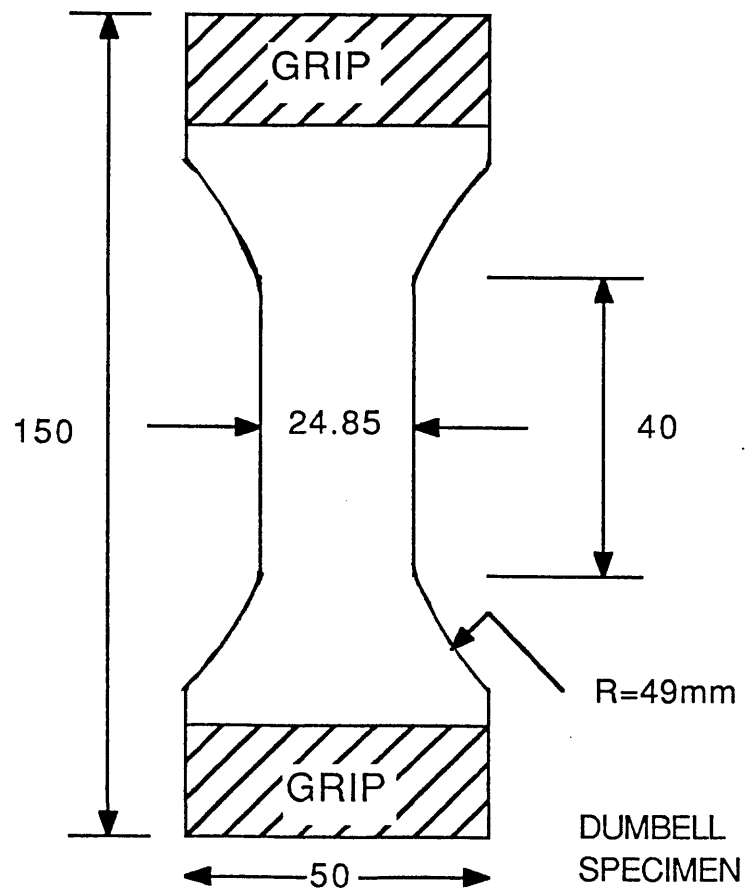
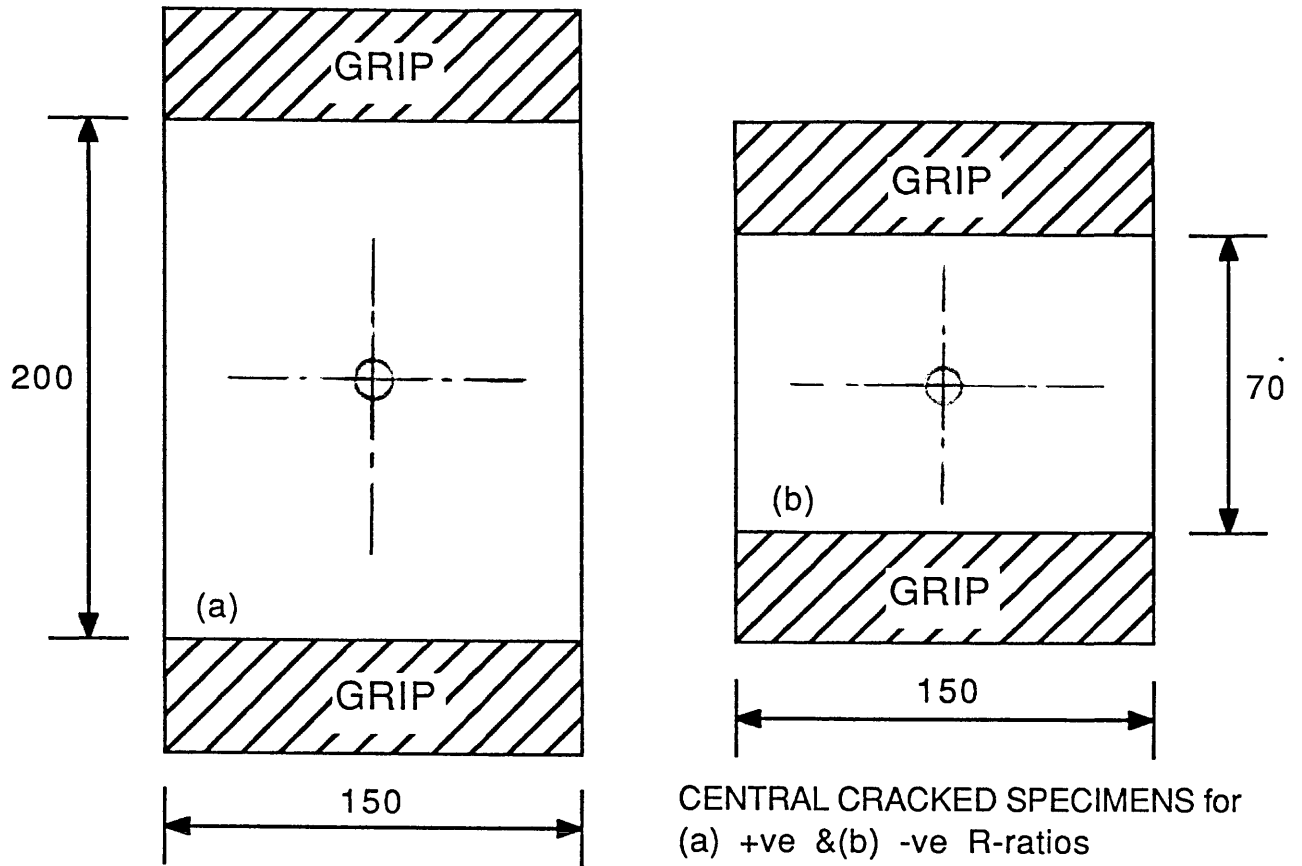


Fig 4.3 Central Cracked and Dumbell specimens.

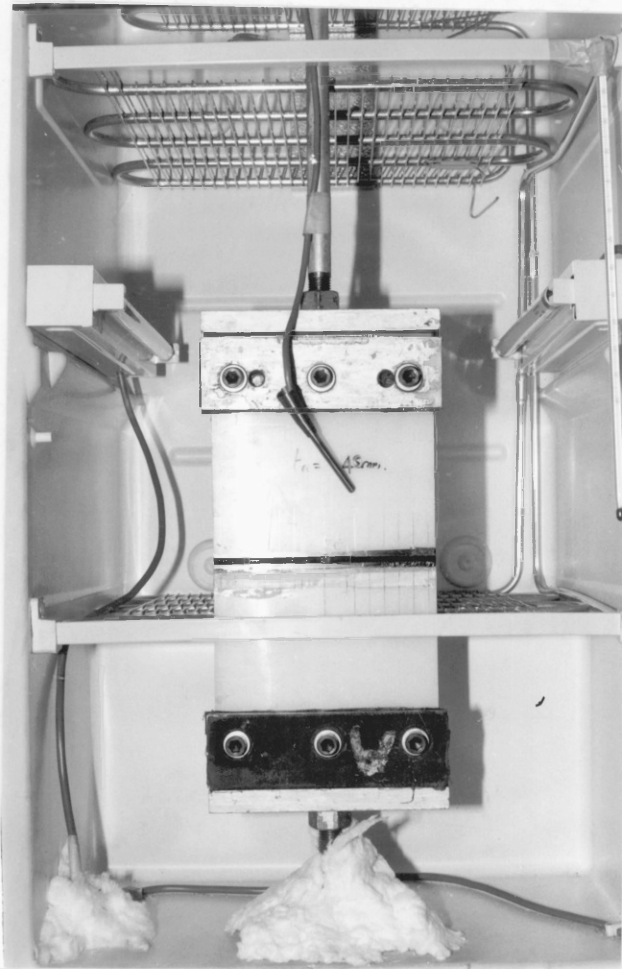


Fig 4.4 A centrally cracked specimen in position on the fatigue testing machine.



Fig 4.5 Bellows fitted so as to minimise heat loss.



Fig 4.6 Freezer in its adapted form mounted into the Mayes servo-hydraulic machine.



Fig 4.7 Complete testing set-up with freezer and camera used to measure crack lengths.

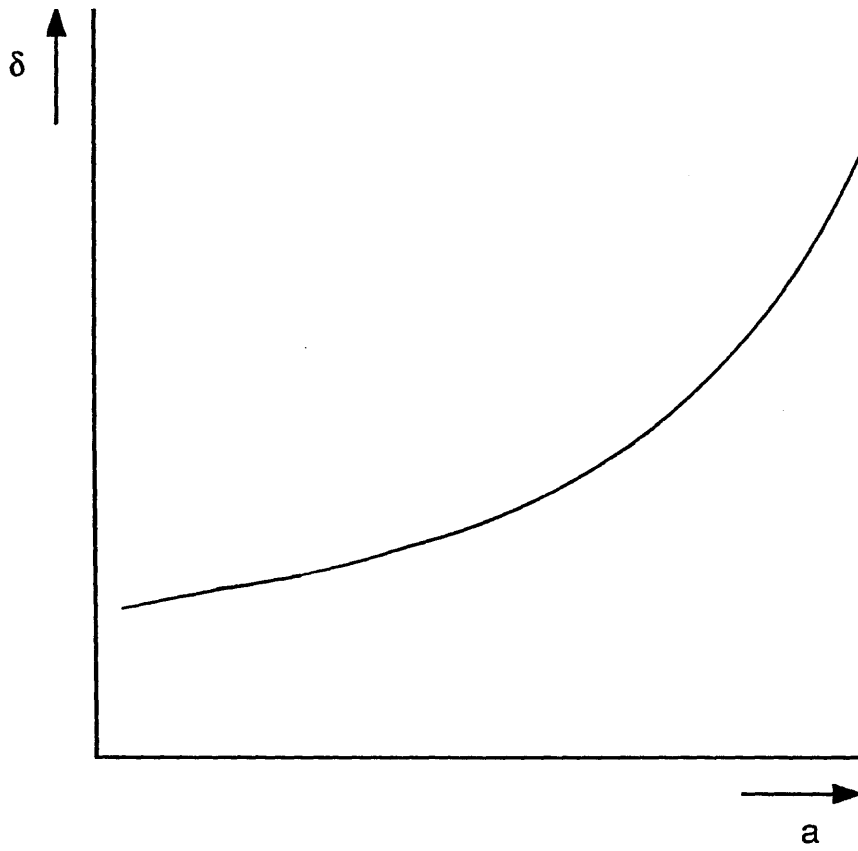


Fig 4.8 Compliance Calibration curve.



Fig 4.9 Canon 70 command back.

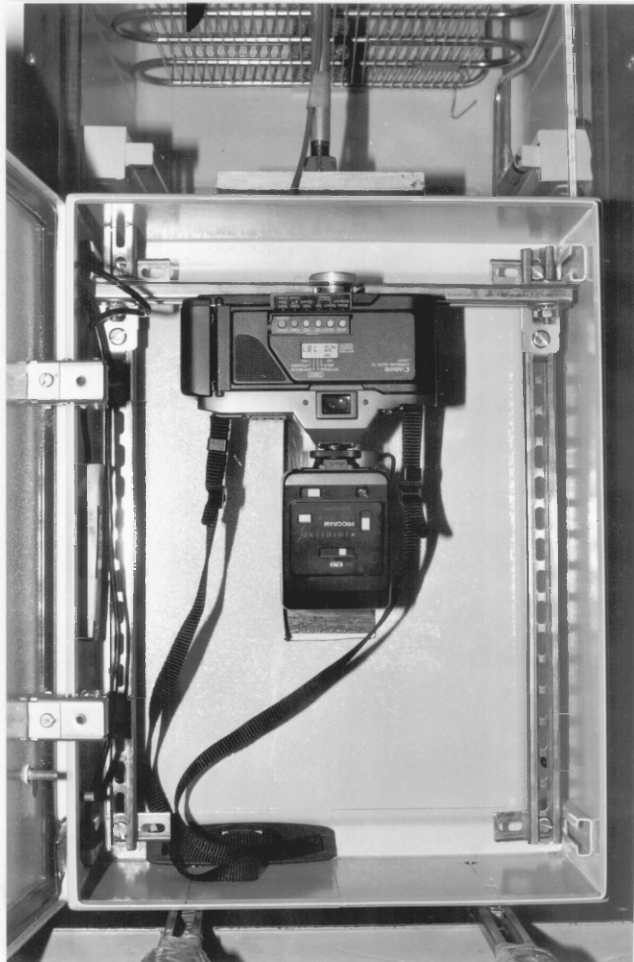


Fig 4.10 Canon T70 camera mounted into a security box.

Grade	Density kg/m ³	Molecular Wt (*10 ³)		Yield Stress (MPa)	Apparent 'K _{IC} ' (MPa.√m)
		Wt.Av	No.Av		
DFDG 4262	920	148	14	9.50	1.05
BP33	920	240	23	9.69	1.12
LL101AA	920	125	28	10.42	1.82

Cont'd ↓

Grade	Melting Temp. °C	% Crystallinity	M.F.I (g/10min)
DFDG 4262	113.3	47.0	1.0
BP33	112.7	45.7	0.3
LL101AA	127.3	46.4	0.9

Table 4.1 LDPE & LLDPE properties

1SEVEN POINT INCREMENTAL POLYNOMIAL METHOD FOR

DETERMINING DA/DN

SPEC. NO. LDPE(BP33). NO. OF POINTS= 32
 CN SPECIMEN B= 12.500 MM. W= 150.000 MM. T= 24.000 DEG C.
 PMIN= .250KN PMAX= 2.500KN R= .100 TEST FREQ= 3.000HZ SMAX=.1333E+01MN/M2 SMIN=.1333E+00MN/M2.

OBS.NO	CYCLES	A(MEAS.)	A(REG.)	KMAX	SQKMAX	LOGDK	NETS	DELTAK	DA/DN	LOGKMAX
1	0.	16.400								
2	6000.	16.580								
3	10000.	16.650								
4	14000.	16.800	16.801	.3129E+00	.9791E-01	2.450	.1718E+01	.2816E+00	.322E-04	2.495
5	20000.	17.000	16.987	.3148E+00	.9909E-01	2.452	.1724E+01	.2833E+00	.348E-04	2.498
6	30000.	17.400	17.357	.3185E+00	.1014E+00	2.457	.1735E+01	.2867E+00	.424E-04	2.503
7	40000.	17.800	17.864	.3236E+00	.1047E+00	2.464	.1750E+01	.2912E+00	.511E-04	2.510
8	60000.	19.000	19.076	.3356E+00	.1126E+00	2.480	.1788E+01	.3020E+00	.652E-04	2.526
9	80000.	20.600	20.559	.3500E+00	.1225E+00	2.498	.1837E+01	.3150E+00	.758E-04	2.544
10	100000.	22.200	22.210	.3658E+00	.1338E+00	2.518	.1894E+01	.3293E+00	.834E-04	2.563
11	120000.	24.000	23.946	.3824E+00	.1462E+00	2.537	.1959E+01	.3441E+00	.902E-04	2.582
12	140000.	25.800	25.819	.4001E+00	.1601E+00	2.556	.2033E+01	.3601E+00	.951E-04	2.602
13	160000.	27.700	27.818	.4192E+00	.1757E+00	2.577	.2119E+01	.3773E+00	.100E-03	2.622
14	200000.	32.100	32.050	.4602E+00	.2118E+00	2.617	.2328E+01	.4142E+00	.108E-03	2.663
15	240000.	36.600	36.522	.5059E+00	.2560E+00	2.658	.2599E+01	.4553E+00	.114E-03	2.704
16	260000.	38.800	38.845	.5312E+00	.2821E+00	2.679	.2766E+01	.4780E+00	.119E-03	2.725
17	280000.	41.100	41.214	.5583E+00	.3118E+00	2.701	.2960E+01	.5025E+00	.123E-03	2.747
18	300000.	43.700	43.719	.5891E+00	.3471E+00	2.724	.3197E+01	.5302E+00	.126E-03	2.770
19	320000.	46.400	46.381	.6247E+00	.3903E+00	2.750	.3494E+01	.5623E+00	.126E-03	2.796
20	340000.	49.000	49.014	.6638E+00	.4406E+00	2.776	.3848E+01	.5974E+00	.121E-03	2.822
21	360000.	51.500	51.405	.7035E+00	.4950E+00	2.802	.4238E+01	.6332E+00	.113E-03	2.847
22	380000.	53.600	53.519	.7431E+00	.5522E+00	2.825	.4655E+01	.6688E+00	.106E-03	2.871
23	400000.	55.400	55.424	.7833E+00	.6135E+00	2.848	.5108E+01	.7050E+00	.104E-03	2.894
24	420000.	57.300	57.321	.8288E+00	.6870E+00	2.873	.5656E+01	.7460E+00	.112E-03	2.918
25	440000.	59.400	59.434	.8881E+00	.7888E+00	2.903	.6424E+01	.7993E+00	.141E-03	2.948
26	460000.	62.000	62.450	.9955E+00	.9911E+00	2.952	.7968E+01	.8960E+00	.197E-03	2.998
27	470000.	64.000	64.429	.1088E+01	.1185E+01	2.991	.9460E+01	.9794E+00	.260E-03	3.037
28	480000.	67.000	67.457	.1294E+01	.1674E+01	3.066	.1326E+02	.1164E+01	.397E-03	3.112
29	484000.	68.500	69.240	.1483E+01	.2200E+01	3.125	.1736E+02	.1335E+01	.658E-03	3.171
30	486000.	69.700								
31	488000.	72.000								
32	490200.	75.000								

0.128 CP SECONDS EXECUTION TIME.

TABLE 4.2 Example output from incremental polynomial computer program.

CHAPTER 5

FATIGUE CRACK PROPAGATION IN LOW AND LINEAR LOW DENSITY POLYETHYLENE

5.1 INTRODUCTION.

The fatigue crack propagation (FCP) results on all three grades of polyethylene are presented in this chapter. In each grade, mean stress, frequency and temperature effects have been studied. All experiments were conducted on CCT specimens 150 mm wide by 12 mm thick, with a grip separation of 200 mm for positive R-ratios and 70 mm for negative R-ratios.

As stated earlier, tests were conducted at two different temperatures, $23 \pm 1^\circ\text{C}$ and $-20 \pm 1^\circ\text{C}$ and the specimens were cycled under sinusoidal load control at R-ratios ranging from -2.0 to 0.6. When the test is first started, for a short period no visible changes occur at the crack tip, then a craze-like deformation zone can be seen to extend at the crack tip with the crack itself remaining stationary. Finally, both craze and crack grow at a steady rate. For this reason crack length measurements were not taken until the fatigue crack had grown one millimetre.

Furthermore, one of the great difficulties encountered with centrally cracked specimens is the difficulty of making identical notches on both sides of the hole. Thus crack growth can sometimes start a little earlier on one side. However, the difference in crack lengths was found to be very small (1 to 2 mm) and crack propagation 'rates' were found to differ by only 0.05×10^{-5} mm/Cycle which can be neglected.

Measurements of crack length were made with the aid of time lapse photography with an accuracy of ± 0.2 mm. The FCP rates were calculated from the experimentally determined curves of crack length a , against loading cycles, N using a computer programme (see chapter 4) and ΔK is given by :

$$\Delta K = Y.(\sigma_{\max} - \sigma_{\min}).\sqrt{a} \quad \text{Eqn 5.1}$$

Finally the data analysis is based on the conventional Paris representation given by :

$$da/dN = A.\Delta K^m \quad \text{Eqn 5.2}$$

where a is crack length, N is the number of cycles and A and m are constants.

The FCP results presented in this chapter show that the LLDPE has a greater resistance to FCP than the LDPE grades and in general FCP resistance increases with increasing R-ratio.

Finally, the FCP results are discussed in relation to the FCP models presented in chapter 3 .

5.2 RESULTS AND DISCUSSION OF THE FATIGUE CRACK PROPAGATION EXPERIMENTS

5.2.1 LOW DENSITY POLYETHYLENES

5.2.1.1 Fatigue Crack Propagation in DFDG 4262.

Only mean stress effects have been studied for this grade as it was later replaced by the grade BP33. The results from tests conducted on DFDG 4262 at a frequency of 10 Hz and R-ratios ranging from 0.1 to 0.6 are shown in Fig 5.1. As can clearly be seen the results do not correspond to the conventional linear Paris representation which should be a single straight line on a double logarithmic plot. Instead there are three distinct regions of crack propagation where in the second region a deceleration of growth rate is observed [74].

In the first region (I) the crack propagation rate, da/dN increases with increasing ΔK in the form of a straight line. However, the slope of this line is steeper than expected (compared to ref [74]) due to the crack advancing through a very sharp notch under plane strain conditions. Furthermore, the crack propagates after applying a ΔK value of about $0.22 \text{ MPa } \sqrt{\text{m}}$, and since the crack will not propagate if a lower value of ΔK was applied it can be assumed that $0.22 \text{ MPa } \sqrt{\text{m}}$ is the threshold value for this material. In region II the crack propagation rate decelerates to some minimum value which occurs at a higher value of da/dN as the R-ratio decreases. When the crack has grown to about 60 mm, the growth becomes unstable and ductile tearing occurs (stage III).

These three regions mark a transition between three types of crack propagation which can be explained by referring to the fracture surface of the specimen shown in DIAG 5.1. In all cases stage I of fatigue crack propagation is very similar having a steep slope, and thus a high m value (m being the coefficient in the Paris law ; m values for both stage I and stage III for each of the six conditions ($R=0.1$ to $R=0.6$) are given in Table 5.1). This is because in the early stages the crack tip is razor sharp and no significant plastic zone has yet formed. Hence, after a small initiation period, the crack grows rapidly by 1 to 3 mm under plane strain conditions. These three millimetres of

crack growth are represented by the steep slope on the da/dN vs ΔK graph. From DIAG 5.1 this stage corresponds to the clear fracture surface before the shear lip effect occurs. As the crack propagates the slope of the curves decreases which corresponds to the rough fibrous tree type fracture surface appearing on the lips of the specimen. It is clear that this type of fracture surface is associated with a higher energy absorption and results in slower crack growth rates. However, it could be argued that the crack grows faster at the centre of the specimen (tunnelling) as is suggested by the clear fracture surface.

In stage II the crack growth rate decreases as ΔK increases. This corresponds to the rough fibrous tree type surfaces at the lips joining up at the centre of the specimen. The crack then propagates at a slower rate (due to the greater resistance offered by the tree type surface) until a minimum value of growth rate is reached. This minimum value is reached at a lower value of da/dN as the R-ratio increases. At this stage the crack length has propagated to about 55 mm and the material's resistance to crack growth has been diminished. Hence crack growth rates are then seen to increase with increasing ΔK corresponding to Paris's representation (stage III). When the crack length has propagated to about 60 mm the growth becomes unstable and ductile tearing occurs.

The most important factor which arises from this set of data is the mean stress effect. It can be seen that an increase in mean stress leads to a decrease in crack growth rates. It has already been shown that a loss in constraint, that is the transition from plane strain to plane stress, produces a deceleration in growth rates. In the same way an increase in mean stress leads to larger plastic zone sizes and thus slower crack growth rates. This effect can be seen from Fig 5.2 where the LEFM plastic zone size criterion $r(\text{LEFM})$ which is given by :

$$r(\text{LEFM}) = 2.5.(K_{\max} / \sigma_y)^2 \quad \text{Eqn 5.3}$$

has been plotted against da/dN . The vertical line drawn through the graph at the 12 mm mark represents the transition from plane strain to plane stress. It is clear that for the higher mean stress values crack propagation initiates much closer to the transition line.

Hence plane stress conditions will prevail much earlier in the life of a specimen tested at higher R-ratios. Also shear lips and the rough fibrous tree type fracture surface will form much earlier which ultimately leads to slower crack growth rates. This effect is illustrated in DIAG 5.2 where the fracture surfaces for the six conditions are shown. Clearly, the tree type fracture surface does occur earlier in the life of a specimen with increasing R-ratio and at an R-ratio of 0.6 it occurs almost immediately after initiation. Fig 5.3 illustrates the same effect by plotting crack length, a , against da/dN . This shows clearly how at high R-ratios the curves reach their minimum values of da/dN at lower values of crack length, a . As mentioned earlier this is due to the fact that high mean stresses lead to a bigger plastic zone size which occurs much earlier in the life of the specimen. Hence the early onset of plane stress conditions lead to much slower FCP rates observed at high R-ratios.

Greater hysteretic heating effects also prevail at high mean stresses (see chapter 6). An increase in temperature at the crack tip brings about a decrease in both the yield stress and the Young's Modulus of the material. This leads to a loss in constraint i.e deformation occurs in the z-direction and therefore a plane stress condition is set up.

5.2.1.2 Fatigue Crack Propagation in BP33.

The results from tests conducted on BP33 at a frequency of 3.0 Hz and R-ratios ranging from -2.0 to 0.5 are shown in Fig 5.4. The positive R-ratio conditions show the same pattern as was observed with the DFDG 4262 grade. The crack again advances through a very sharp notch under plane strain conditions represented by the steep slope in region I of Fig 5.4. However, the threshold value for this grade is slightly higher at $0.28 \text{ MPa } \sqrt{\text{m}}$. In region II the propagation rate decelerates to some minimum value which as before occurs at a lower value of da/dN as the R-ratio increases. When the crack has grown to about 60 mm, the growth becomes unstable and ductile tearing occurs (stage III). This type of behaviour can again be characterised by the fracture surface model shown in DIAG 5.1. Consequently, slower crack growth rates are due to the rough fibrous tree type fracture surface appearing on the lips of the specimen. The size of these lips increases with crack propagation and finally they join up at the centre of the specimen. The crack then propagates at a slower rate until a minimum value of growth rate is reached. It is clear, therefore, that the tree type fracture surface is associated with a higher energy absorption which obviously offers a greater resistance to crack propagation. Furthermore, by closely examining Fig 5.4 it can be seen that as the R-ratio increases from 0.1 to 0.5 the curves not only reach their minimum value at a lower value of da/dN but also at a smaller value of ΔK . To explain this effect it is necessary to point out that all tests were carried out at approximately an equal value of initial ΔK (i.e. $\Delta K_{\text{initial}} = \text{Constant}$ for all tests). Thus by increasing the R-ratio a bigger value of K_{max} and hence P_{max} had to be used, since :

$$\begin{aligned}
 & R = K_{\text{min}} / K_{\text{max}} \quad \text{and} \quad \Delta K = K_{\text{max}} - K_{\text{min}} \\
 \text{therefore} \quad & R = 1 - \Delta K / K_{\text{max}} \\
 \text{and} \quad & \Delta K = K_{\text{max}} (1 - R)
 \end{aligned}
 \tag{Eqn 5.4}$$

The increase in initial load thus led to a plastic zone forming at an earlier stage in the life of the specimen, which in turn offered a greater resistance to crack propagation. This is

further confirmed by figures 5.5 and 5.6 where the crack growth rate is plotted against the LEFM plastic zone size criterion given by :

$$r \text{ (LEFM)} = 2.5 (K_{\max} / \sigma_y)^2 \quad \text{Eqn 5.3}$$

and crack length, a , respectively. The line drawn through the middle of Fig 5.5 represents the transition from plane strain to plane stress. Hence it is clear that for high R-ratios plane stress conditions will prevail much earlier in the life of a specimen. Consequently shear lips and the rough fibrous tree type fracture surface will also form much earlier as illustrated in DIAG 5.2. Finally Fig 5.6 also illustrates the above effect where an increase in R-ratio produces decelerating crack growth rates much earlier in the life of a specimen.

As mentioned earlier an increase in R-ratio led to a test being started with a bigger value of $K_{\max}$. This is illustrated in Fig 5.7 where $K_{\max}$ is plotted against da/dN . By closer examination it can be seen that all the positive R-ratio curves reach their minimum values at the same value of $K_{\max}$ (0.81 MPa $\sqrt{m}$). This suggests that the rate of fatigue crack propagation per cycle, da/dN , is not governed by the stress intensity factor range ΔK . Furthermore the Paris representation cannot be used to define this material's behaviour. This is due to the fact that many more parameters are involved than are accounted for by Paris. Further work to derive an equation with a sound physical basis is certainly needed : it must be anticipated that this final equation will be a complicated one if it is to have a general validity.

Net section stress effects have also been considered in an attempt to characterise the transition from stage I to stage II and the point at which it occurs. Figs 5.8 and 5.9 represent tests carried out at constant R-ratios, 0.1 and 0.2 respectively at a frequency of 3.0 Hz and at different initial values of crack length. da/dN and net section stress are plotted against ΔK . Although all tests were carried out at a constant initial value of ΔK , specimens having a larger initial crack length have to endure a higher net section stress (Figs 5.10 and 5.11). From Figs 5.8 and 5.9 it is clear that in the same way as $K_{\max}$, higher net section stresses also reduce crack propagation rates and the curves

reach their minimum values earlier in the life of a specimen. Furthermore a clearer picture is established by representing the data in the form of da/dN against net section stress. The results obtained are very interesting in that for all tests (Figs 5.12 , 5.13 and 5.14), the curve's minimum value occurs at a net section stress of between 4.25 and 5.25 MPa. Therefore, it has been established that the transition from stage II to stage III occurs at a constant value of K_{max} and net section stress which can be said to be very important material characteristics.

To examine R-ratio effects further, tests were conducted on this grade with negative R-ratios between -0.2 and -2.0 (including $R=0.0$). The results at a test frequency of 3.0 Hz are shown in Fig 5.4 (together with the previous positive R-ratio tests) on a plot of $\log da/dN$ against ΔK . It is obvious that the effects of negative R-ratio are greater than those for positive R-ratios. The FCP rate increased by a factor of about three when the R-ratio decreased from 0.1 to -0.5. This increase in FCP rates is due to the fibres firstly being subjected to high tensile stresses during the tensile portion of the loading cycle which subsequently elongates them. They are then crushed during the compression part of the cycle, within the reverse plastic zone. As a result, the fibrils flatten progressively in the crack propagation direction and eventually become so thin that they offer little resistance to fatigue crack propagation. P.E.Bretz, R.W.Hertzberg and J.A.Manson [77] in a study on the modification of the fibrillar structure during cyclic loading have provided evidence of the above mechanism.

Further increases in the FCP rate were observed when the R-ratio fell from -0.5 to -1.0 , but reductions in the R-ratio below -1.0 have very little effect. Presumably, FCP rates would have increased still further but a greater degree of hysteretic heating would be expected to occur at an R-ratio of -2.0 (see chapter 6). The temperature rises that would be expected to occur are such that they act to offset any further increase in FCP rates. The effect of R-ratio on FCP rate is shown in Fig 5.15 where $\log da/dN$ is plotted against R-ratio at a constant ΔK value of $0.363 \text{ MPa } \sqrt{\text{m}}$. There are two distinct regimes : the first for R-ratios between 0.1 and 0.5 where there exists a linear relationship between decreasing FCP rates and increasing R-ratios. The

second is for R-ratios less than -1.0 where in this regime the FCP rate remains constant at about $3.7 \cdot 10^{-4}$.

To examine plane strain to plane stress transitional effects further, tests were conducted on BP33 at low temperature (-20°C) between -1.0 and 0.5. The results at a test frequency of 3.0 Hz are shown in Fig 5.16 on a plot of $\log da/dN$ against $\log \Delta K$. The coefficients A and m in the Paris law for each R-ratio are given in Table 5.2. The results for R-ratios between 0.0 and 0.5 conform to Paris's representation with ΔK being the governing factor. However, the effects of negative R-ratio are much larger with the FCP rate increasing by a factor of about ten when the R-ratio decreased from zero to -0.75. Further reductions in the R-ratio below -0.75 have very little effect because, as mentioned earlier, an increase in hysteretic heating occurs which offsets any further increase in FCP rates.

The most important factor from this set of results is that the transition from plane strain to plane stress has been eliminated. This is due to the material having both a higher yield stress and a higher modulus at low temperatures which effectively reduces the plastic zone size at the crack tip. Therefore, plane strain conditions only exist, which is confirmed by Fig 5.17 where da/dN is plotted against the LEFM plastic zone size criterion given by Eqn 5.3. The vertical line drawn through the middle of Fig 5.17 represents the transition from plane strain to plane stress and clearly all the relevant data points (i.e before the material's resistance to FCP diminishes and crack growth becomes unstable) lie in the plane strain region. The effect of R-ratio, although small for positive R-ratio tests, is shown in Fig 5.18 where $\log da/dN$ is plotted against R-ratio at a constant ΔK value of $0.912 \text{ MPa } \sqrt{\text{m}}$. In this case there are three regimes: the first for R-ratios between -1.0 and -0.75, where the FCP rate remains almost constant; the second for R-ratios between -0.75 and 0.0, where there exists a linear relationship between decreasing FCP rates and increasing R-ratios; and finally, for R-ratios between 0.0 and 0.5 the FCP rates increase slightly with increasing R-ratio but are effectively ΔK controlled.

However, having said this, there still exists a small mean stress effect

which is illustrated by Fig 5.19 in a plot of $\log da/dN$ against $\log K_{\max}$. The results show that higher values of mean stress produce slower FCP rates which again can be attributed to the effects of plasticity. That is high mean stresses produce larger plastic zone sizes which blunt the crack and offer a greater resistance to crack propagation. Again $\log da/dN$ is plotted against R-ratio, Fig 5.20 at a constant $K_{\max}$ value of $1.40 \text{ MPa } \sqrt{\text{m}}$, which clearly illustrates the mean stress effect. Furthermore, at a value of $K_{\max}$ of $1.41 \text{ MPa } \sqrt{\text{m}}$ in Fig 5.16, the slopes of the lines increase which is the point at which the crack growth becomes unstable and ductile tearing occurs. Therefore, this value of $K_{\max}$ can be taken to be the 'apparent' fracture toughness for this material at -20°C denoted by ' K_{IC} '.

The slopes of the Paris plots in Fig 5.16 are plotted against the R-ratio in Fig 5.21. There are two distinct regimes : the first for R-ratios between -1.0 and -0.75, where the slope m remains constant as the R-ratio increases ; the second is for R-ratios between -0.5 and 0.5, where the slope starts off at a higher value but decreases and finally remains constant for R-ratios between 0.0 and 0.5. This suggests that there is little difference in crack tip condition between the crack tips of the positive R-ratio dynamic fatigue cracks. But for R-ratios below 0.0 the slopes provide evidence of considerable differences in crack tip condition with decreasing R-ratio. On the other hand evidence of considerable difference in crack tip condition was found between dynamic fatigue cracks and static fatigue cracks ($R=1.0$). A crack was grown in a specimen to 35 mm from an initial value of 15 mm. A static load was then applied equivalent to the maximum load in the cyclic test ($R=1.0$) and although this load was sustained for two weeks the crack did not propagate. This confirms the fact that cyclic loading has a tendency to keep resharpener the crack tip, thus eliminating crack arrest due to blunting.

The FCP results for an R-ratio of 0.1 are shown in Fig 5.22 on a plot of $\log da/dN$ against $\log \Delta K$ for frequencies of 0.2, 0.3, 0.5, 3.0, 5.0 and 10.0 Hz. It can be seen that higher frequencies have the effect of reducing fatigue crack propagation rates. This is because when the duration of the loading cycle decreases the hysteretic heating in the specimen increases. When the specimen temperature increases the material's

properties change, that is both Young's Modulus and yield stress decrease. This leads to considerable differences in crack tip condition, with higher frequencies producing larger plastic zones at the crack tip (see crack tip temperature analysis in chapter 6). Clearly larger plastic zone sizes increase the material's resistance to crack propagation and therefore slower FCP rates are produced.

In Fig 5.23 (a-f) the crack growth per cycle at various values of ΔK is plotted against the reciprocal frequency (Dumpleton's representation, [75]). The experimental points do not give a precise linear curve, but a straight line drawn through the points (the solid curve) does give a reasonable fit. This curve shows the FCP per cycle plotted against the duration of the loading cycle. Curves at different values of ΔK are taken because the lines in Fig 5.22 are not parallel and hence slightly different results can be obtained at different values of ΔK . From Fig 5.23 (d) it can be seen that the data are in two distinct regimes : low frequency producing higher FCP rates and high frequency producing lower FCP rates.

Furthermore, these two regimes can be represented by two different lines as shown in Fig 5.23 (d) (solid curve and dotted curve). It will be shown later that during cyclic loading, a large proportion of the work done on a specimen is converted to hysteretic heating. Hence, at a test frequency of 5.0 or 10.0 Hz, the region of the specimen in the vicinity of the crack tip could be appreciably hotter than ambient. Therefore the results at 0.2, 0.3 and 0.5 Hz relate to a test temperature of about 23°C (room temperature with no change due to cyclic loading), whereas the results at 3.0, 5.0 and 10.0 Hz relate to a crack tip temperature somewhat higher than 23 °C (see chapter 6). Hence the dotted line on Fig 5.23 (d) could be the FCP curve at ambient temperature (23°C).

The total da/dN can be treated as comprising a time-dependent component and a cycle-dependent component [75].

(a) Cycle-dependent FCP (FCP (n))

This component is assumed to be independent of test frequency and

hysteretic heating has been ignored. It is equal to da/dN at a value of the reciprocal frequency equal to zero. Therefore from Fig 5.23 (d) the cycle-dependent FCP per cycle is equal to 9.8×10^{-5} mm/Cycle or 23×10^{-5} mm/Cycle at ambient temperature.

(b) Time-dependent dynamic FCP (FCP (t))

This component is analogous to the static FCP that occurs under the action of an applied static load . Although, as mentioned earlier, it was not possible to propagate a fatigue crack under an applied static load, time-dependent FCP can occur in the duration of the loading cycle. This is because cyclic loading has the effect of maintaining a sharp crack tip thus eliminating crack arrest due to blunting. The time-dependent FCP is obtained from the slope of the curve of da/dN against reciprocal frequency and is given by $(1/f) \cdot (\partial a / \partial t)_N$ since :

$$da = (\partial a / \partial N)_t \cdot dN + (\partial a / \partial t)_N \cdot dt$$

$$da/dN = (\partial a / \partial N)_t + (\partial a / \partial t)_N \cdot (dt/dN)$$

and since $dt/dN = 1/f$

$$da/dN = (\partial a / \partial N)_t + (1/f) \cdot (\partial a / \partial t)_N$$

$$da/dN = \text{Cycle-dependent FCP} + \text{Time-dependent FCP}$$

$$da/dN = B \cdot \Delta K^m + (1/f) C \cdot K_{\max}^m \quad \text{Eqn 5.5}$$

where f is the test frequency, and B and C are constants .

The slope on the curve of observed FCP (t) in Fig 5.23 (d) is equal to 4.7×10^{-5} mm/sec. Hence the total da/dN at a test frequency of 1.0 Hz is equal to $(8.4+4.7) \times 10^{-5}$ mm/Cycle for the test conditions relevant to Fig 5.23 (d). Therefore, having found numerical values for cycle-dependent and time-dependent FCP rates, the constants B and C can be evaluated. However, it was not possible to evaluate the constants B and C for this grade since it does not conform to the Paris representation (i.e the exponent m could not be found).

Further tests with varying frequency were also carried out at R-ratios of 0.2 and 0.5. The results are shown in Figs 5.24 and 5.26 respectively at frequencies between 0.2 and 10.0 Hz. The results at an R-ratio of 0.2 are very similar to those at an R-ratio of 0.1 with higher frequencies producing slower FCP rates. Again the crack growth per cycle, at various values of ΔK , is plotted against the reciprocal frequency (Figs 5.25(a-f)). Although, some scatter is observed, a straight line drawn through the points does give a reasonable fit. However, Fig 5.25 (f) confirms that the results can be represented by two different lines with the dotted line representing fatigue crack propagation at ambient temperature (23°C). This effect is not taken into account by the Dumbleton equation since hysteretic heating has been ignored. Furthermore, it could be argued that if the crack tip temperature remained constant even at high frequencies then the results would be represented by a graph of the form shown in Fig 5.28. In this case only a fatigue mechanism will be in operation.

The results at an R-ratio of 0.5, however, are slightly different than the other two cases in that there are greater differences between the low and high frequency FCP rates throughout the ΔK range. This is illustrated by Fig 5.29 where $\log da/dN$ is plotted against $\log$ frequency, f , for the three different cases of $R=0.1$, 0.2 and 0.5. It is clear that for an R-ratio of 0.5 there is a six fold decrease in FCP rates with increasing frequency while there is only a three fold and 2.3 fold decrease in FCP rates at R-ratios of 0.1 and 0.2 respectively. This is due to the combination of a high mean stress and high frequencies having a greater effect on crack tip conditions. That is both high mean stresses and high frequencies have the effect of increasing the temperature at the crack tip which decreases the yield stress and Young's modulus of the material and leads to larger plastic zone sizes. Hence the material's resistance to FCP is increased and slower growth rates are observed.

Fig 5.29 is based on the representation proposed by Webster [76] where he presented data on graphs of $\log da/dN$ vs $\log f$. By conducting tests at very low frequencies (0.001 Hz) he found that only time-dependent crack propagation took place. This was represented by a straight line on the graph of da/dN against f and in each case

the slope of this line was found to be -1. At high frequencies the line was found to reach a minimum value where da/dN values remain constant with further increases in frequency. Therefore, it was concluded that very low frequencies produced straight lines having a slope of -1 representing time-dependent crack propagation and very high frequency tests produced constant da/dN zero slope lines representing cycle-dependent crack propagation. However, in an intermediate range of test frequency it was found that both a time-dependent and cycle-dependent component existed, which is also the case for the data presented here for the range of test frequency used. This has already been confirmed by the data represented on graphs of da/dN against the reciprocal frequency.

Therefore, by referring again to Fig 5.29, it can be seen that the line representing lower frequencies at an R-ratio of 0.5 has a bigger slope than the other two conditions. Therefore, it can be concluded that time-dependent effects are greater at an R-ratio of 0.5 than at either 0.1 or 0.2, hence the greater differences in FCP rates observed at an R-ratio of 0.5. This is not surprising since at low frequencies, where the duration of the cycle is longer, high R-ratios and therefore high mean stresses are obviously going to have a greater effect on time-dependent crack growth.

As mentioned earlier numerical values for cycle-dependent and time-dependent FCP rates can be found from the graphs of da/dN against reciprocal frequency (Figs 5.23 (a-f), Figs 5.25 (a-f) and Figs 5.27 (a-f) for R-ratios of 0.1, 0.2 and 0.5 respectively). These values at different values of ΔK are given in Table 5.4 and are represented on a graph of $\log da/dN$ against $\log \Delta K$ in Fig 5.30. For each condition tested it can be seen that cycle-dependent crack growth is always greater than time-dependent crack growth.

The cycle-dependent and time-dependent components of FCP rates at a K_{max} value of $0.6 \text{ MPa } \sqrt{\text{m}}$ and a frequency of 1.0 Hz are plotted against the R-ratio in Fig 5.31(b). The most significant factor seen here is that the cycle-dependent component decreases drastically with increasing R-ratio and at an R-ratio of 0.5 it almost equals the time-dependent component. On the other hand, and as expected, the time-dependent component also decreases slightly with increasing R-ratio due to the decrease in overall

FCP rates at high R-ratios. However, and of greater importance, the time-dependent component holds up well at an R-ratio of 0.5 and is almost equal to the cycle-dependent component. Therefore as with Webster's representation in Fig 5.29 the above also confirms that time-dependent effects, although having a lower numerical value due to a lower overall value of FCP rate, are greater at an R-ratio of 0.5 than at either 0.1 or 0.2. Therefore, time-dependent effects are enhanced with an increase in R-ratio due to higher mean stresses acting more effectively in the duration of the loading cycle.

Finally, it should be pointed out that the increase in FCP rates at low frequencies is due to two important factors. Firstly, significant increases in crack tip temperature do not occur at such low frequencies and therefore smaller plastic zone sizes are formed which only offer a small resistance to crack propagation. The second factor is that at low frequencies where the duration of the cycle is longer time-dependent crack propagation is greater and by combining with fatigue-dependent crack propagation gives higher overall FCP rates for a particular R-ratio.

Frequency effects have also been studied at low temperature (-20°C) and the FCP results for an R-ratio of 0.1 are shown in Fig 5.32 on a plot of $\log da/dN$ against $\log \Delta K$ for frequencies of 0.2, 0.3, 3.0 and 10.0 Hz. Again higher frequencies have the effect of reducing fatigue crack propagation rates. Therefore, even at -20°C , crack tip temperature can still increase with increasing frequency. This leads to larger plastic zones forming at the crack tip due to a decrease in the material's yield stress and modulus which effectively increases the material's resistance to crack propagation and therefore slower FCP rates are produced.

In Fig 5.33 (a-d) the crack growth per cycle, at various values of ΔK , is plotted against the reciprocal frequency. The experimental points do not give a precise linear curve, but a straight line drawn through the points (the solid curve) does give a reasonable fit. This curve shows the FCP per cycle plotted against the duration of the loading cycle. From Fig 5.33(a) it can be seen that the data are in two distinct regimes which can be represented by two different lines; a solid curve representing high frequency data and a dotted curve representing low frequency data. As mentioned earlier,

the results at 3.0 and 10.0 Hz relate to a crack tip temperature somewhat higher than -20°C whereas the results at 0.2 and 0.3 Hz (dotted line) represent FCP at -20°C . Therefore, if the crack tip temperature remained constant even at high frequencies the results would be represented by the dotted line in Fig 5.33(a) and only a fatigue mechanism will be in operation .

Further tests with varying frequency were also carried out at R-ratios of 0.2 and 0.5. The results are shown in Figs 5.34 and 5.36 respectively at frequencies between 0.2 and 10.0 Hz. Their corresponding graphs of crack growth per cycle, at various values of ΔK , plotted against the reciprocal frequency are given by Figs 5.35 (a-d) and 5.37 (a-d). Furthermore, the dotted lines in Figs 5.35(a) and 5.37(a) represent fatigue crack propagation at -20°C . The dotted line in Fig 5.35(a) ($R=0.2$), as with that at an R-ratio of 0.1, is nearly horizontal and therefore represents only fatigue-dependent crack propagation. However, at an R-ratio of 0.5 (Fig 5.37(a)) the dotted line represents both fatigue and time-dependent crack growth. This can be attributed to the fact that even at low frequencies high mean stresses can still increase the temperature at the crack tip. Therefore, a small increase in frequency (0.2 to 0.3 Hz) will have a greater effect at an R-ratio of 0.5 than at either R-ratios of 0.1 and 0.2.

R-ratio effects are further highlighted by closer examination of Fig 5.36. It can be seen that there are greater differences in FCP rates with increasing frequency throughout the ΔK range. This is better illustrated by Fig 5.38 where $\log da/dN$ is plotted against $\log$ frequency, f , for the three different cases of $R=0.1$, 0.2 and 0.5. As mentioned earlier this is Webster's representation where at low frequencies the lines have a slope of -1 representing time-dependent crack propagation and high frequency tests produce constant da/dN zero slope lines representing fatigue-dependent crack propagation. It can be seen that for the range of test frequency used the lines lie in the intermediate range where both a time-dependent and fatigue-dependent component exist. This has already been confirmed by the data represented on graphs of da/dN against the reciprocal frequency. Also from Fig 5.38 it can be seen that at low frequencies the results for R-ratios of 0.1 and 0.2 are very similar. That is, they both have the same slope and

therefore equal time-dependent effects. However, at an R-ratio of 0.5 higher overall FCP rates are observed and the slope of the line is greater. Therefore time-dependent effects are greater at an R-ratio of 0.5. This is because at low frequencies where the duration of the cycle is longer higher mean stresses have a greater effect on time-dependent crack propagation.

The results at high frequencies, however, are of even greater interest. It can be seen that overall FCP rates decrease with increasing R-ratio. And of even greater importance is the drastic fall in FCP rates at an R-ratio of 0.5. As mentioned earlier, when the duration of the cycle decreases, the hysteretic heating in the specimen increases. High mean stresses also have the effect of increasing the hysteretic heating in the specimen. Therefore, the combination of having both high frequencies and high mean stresses has had a drastic effect on the FCP rates at an R-ratio of 0.5.

Numerical values for cycle-dependent and time-dependent FCP rates have been found from the graphs of da/dN against reciprocal frequency (Figs 5.33(a-d), 5.35(a-d) and 5.37(a-d) for R-ratios of 0.1, 0.2 and 0.5 respectively). These values at different values of ΔK are given in Table 5.5 and are represented on a graph of $\log da/dN$ against $\log \Delta K$ in Fig 5.39. For each condition tested it can be seen that cycle-dependent crack growth is always greater than time-dependent crack growth. Furthermore, fatigue-dependent crack growth decreases with increasing R-ratio while time-dependent crack growth increases and at an R-ratio of 0.5 is nearly equal to fatigue-dependent crack growth .

The cycle-dependent and time-dependent components of FCP rates at a $K_{\max}$ value of $1.585 \text{ MPa } \sqrt{\text{m}}$ and a frequency of 1.0 Hz are plotted against the R-ratio in Fig 5.41. The cycle-dependent component decreases drastically with increasing R-ratio and at an R-ratio of 0.5 it almost equals the time-dependent component. On the other hand, and as expected, the time-dependent component also decreases slightly with increasing R-ratio due to the decrease in overall FCP rates which occur with increasing R-ratio. However, and of greater importance, the time-dependent component holds up well at an R-ratio of 0.5 and is almost equal to the cycle-dependent component.

Therefore, as with Webster's representation in Fig 5.38, the above also confirms that time-dependent effects are greater at an R-ratio of 0.5 than at either 0.1 or 0.2.

5.2.1.3 SUMMARY OF FREQUENCY EFFECTS.

By varying the frequency at a constant R-ratio it was found that FCP rates increased with decreasing frequency. This was attributed to the fact that at low frequency crack tip temperature rises are not significant. This leads to smaller plastic zone sizes which consequently offer a smaller resistance to crack propagation. Also at low frequencies where the duration of the cycle is longer the effect of time-dependent crack growth is greater which combines with fatigue-dependent crack propagation to give higher overall FCP rates. Also time-dependent crack propagation was further enhanced by an increase in R-ratio due to higher mean stresses acting more effectively in the duration of the loading cycle.

Slower FCP rates were observed at high R-ratios and high frequencies due to both these factors combining to drastically increase the temperature at the crack tip (see chapter 6). Therefore, plasticity at the crack tip was enhanced which ultimately increased the material's resistance to crack propagation.

5.2.2 LINEAR LOW DENSITY POLYETHYLENE

5.2.2.1 Fatigue Crack Propagation in LL101AA.

The results from tests conducted on LL101AA at a frequency of 3.0 Hz and R-ratios ranging from -2.0 to 0.3 are shown in Fig 5.42 on a plot of $\log da/dN$ against $\log \Delta K$. The coefficients A and m in the Paris law for each R-ratio are given in Table 5.6. The results for R-ratios between 0.0 and 0.5 conform to Paris's representation with ΔK being the governing factor. However, the effects of negative R-ratio are much greater with the FCP rate increasing by a factor of about 168 when the R-ratio decreased from zero to -1.0. Also a further reduction in the R-ratio from -1.0 to -2.0 increased FCP rates even further by an additional factor of 17. Therefore, there is a massive increase in FCP rates at negative R-ratios. This suggests that there is a considerable difference in crack tip condition between the crack tips at positive R-ratio and those at negative R-ratios.

This is because during cyclic loading at a negative R-ratio the fibres are firstly subjected to high tensile stresses during the tensile portion of the loading cycle which subsequently elongates them. They are then crushed, during the compression part of the cycle, within the reverse plastic zone. As a result, the fibrils flatten progressively in the crack propagation direction and eventually become so thin that they offer little resistance to fatigue crack propagation [77].

The effect of R-ratio, although small for positive R-ratio tests is shown in Fig 5.43 where $\log da/dN$ is plotted against R-ratio at a constant ΔK value of $1.15 \text{ MPa}\sqrt{\text{m}}$. It can be seen that FCP rates decrease drastically at negative R-ratios but between 0.0 and 0.3 there is only a small decrease in FCP rates (factor of 1.8) and therefore FCP rates are effectively ΔK controlled.

However, by representing the data in the form of $\log da/dN$ against $\log K_{\max}$ in Fig 5.44 it can be seen that there exists a small mean stress effect even at positive R-ratios. This is due to tests carried out at higher R-ratios being initiated at higher values of $K_{\max}$. This introduced larger plastic zone sizes which effectively create a more

severe blunting condition at the crack tip and thus offered a greater resistance to crack propagation. This effect is better illustrated by Fig 5.45 where $\log da/dN$ is plotted against R-ratio at a constant K_{max} value of $1.5 \text{ MPa } \sqrt{\text{m}}$. The drastic reduction in FCP rates is illustrated and there exists a linear relationship between $\log da/dN$ and R-ratio for positive R-ratios. Furthermore, at a value of K_{max} of $1.82 \text{ MPa } \sqrt{\text{m}}$ in Fig 5.44 the slope of the lines representing positive R-ratios increases. This is the point at which the material becomes unstable and ductile tearing occurs. Therefore, this value of K_{max} ($1.82 \text{ MPa } \sqrt{\text{m}}$) can be taken to be the 'apparent' fracture toughness for this material denoted by ' K_{IC} '.

The slopes of the Paris plots in Fig 5.42 are plotted against the R-ratio in Fig 5.46. Again there are two distinct regimes : the first for R-ratios between -2.0 and -1.0 where the slopes remain almost constant with increasing R-ratio but have much greater values ; the second is for R-ratios between -0.75 and 0.3 where effectively the slopes remain constant at about 5.7. This suggests that there is little difference in crack tip condition between the crack tips for R-ratios between -0.75 and 0.3. However, for R-ratios below -0.75 the slopes provide evidence of considerable differences in crack tip condition. As with the BP33 material it was not possible to grow a fatigue crack under an applied static load ($R=1.0$). This further suggests that there are considerable differences in crack tip condition between dynamic fatigue cracks and static fatigue cracks ($R=1.0$). Therefore, this confirms the fact that cyclic loading has a tendency to keep resharping the crack tip, thus eliminating crack arrest due to blunting.

To examine crack tip conditions and mean stress effects further , tests were conducted on LL101AA at low temperature (-20°C) with R-ratios between -1.0 and 0.4. The results at a test frequency of 3.0 Hz are shown in Fig 5.47 on a plot of $\log da/dN$ against $\log \Delta K$. The results for R-ratios between 0.0 and 0.4 are basically governed by ΔK , having very small differences in FCP rates (factor of 2.6) due to a change in R-ratio. However, the effects of negative R-ratio are much greater with the FCP rate increasing by a factor of about seven when the R-ratio decreased from zero to -0.75. Also a further reduction in the R-ratio from -0.75 to -1.0 increased FCP rates even further by an

additional factor of four. This, as mentioned earlier, is due to the fibrils being crushed and thus becoming thinner within the reverse plastic zone during the compression part of the cycle. Therefore, there exists a considerable difference in crack tip condition between the crack tips at positive R-ratio and those at negative R-ratios.

R-ratio effects are better illustrated by Fig 5.48 where $\log da/dN$ is plotted against R-ratio at a constant ΔK value of $2.20 \text{ MPa } \sqrt{\text{m}}$. There are two distinct regimes : the first represents the drastic effect of negative R-ratios between -1.0 and -0.2 and is in the form of a straight line relationship ; in the second, for R-ratios between -0.2 and 0.4, FCP rates again decrease but in a more controlled manner and are effectively ΔK controlled.

Mean stress effects are highlighted by representing the data in the form of $\log da/dN$ against $\log K_{\text{max}}$ in Fig 5.49. It can be seen that mean stress effects exist even at positive R-ratios due to high R-ratio tests being initiated at higher values of K_{max} . This led to a more severe crack tip condition being created at the crack tip (i.e larger plastic zones) which effectively blunts the crack tip and offers a greater resistance to crack propagation. Furthermore, the above effect is illustrated by Fig 5.50 on a plot of $\log da/dN$ against R-ratio at a constant K_{max} value of $3.10 \text{ MPa } \sqrt{\text{m}}$. The points at both negative and positive R-ratios are represented by straight line relationships with increasing R-ratio producing decreasing FCP rates. Finally, from Fig 5.49 at a value of K_{max} of $3.84 \text{ MPa } \sqrt{\text{m}}$, the slope of the lines representing positive R-ratios increases. This is the point at which the crack growth becomes unstable and ductile tearing occurs. Therefore, this value of K_{max} ($3.84 \text{ MPa } \sqrt{\text{m}}$) can be taken to be the 'apparent' fracture toughness for this material at -20°C denoted by ' K_{IC} '.

The slopes of the Paris plots in Fig 5.47 are plotted against the R-ratio in Fig 5.51. It is seen that the slopes at negative R-ratios have greater values and decrease drastically from -1.0 to -0.5 and then more steadily to an R-ratio of 0.2. Also the slopes have greater values at R-ratios of 0.3 and 0.4. This suggests that there is a considerable difference in crack tip condition between the crack tips for R-ratios in the range -1.0 to -0.5 and 0.2 to 0.4. However, for R-ratios between -0.5 and 0.2, the slopes suggest that

there is little difference in crack tip condition. Furthermore, as with previous conditions, it was not possible to grow a fatigue crack under an applied static load ($R=1.0$). Therefore, this suggests that there are considerable differences in crack tip condition between dynamic fatigue cracks and static fatigue cracks. This is because cyclic loading has a tendency to keep resharpening the crack tip, thus eliminating crack arrest due to blunting.

The FCP results for an R -ratio of 0.1 and a temperature of -20°C are shown in Fig 5.52 on a plot of $\log da/dN$ against $\log \Delta K$ for frequencies of 0.2, 0.5, 3.0 and 6.0 Hz. Again higher frequencies have the effect of reducing fatigue crack propagation rates due to hysteretic heating. That is an increase in frequency leads to an increase in crack tip temperature which effectively reduces both the yield stress and the modulus of the material. This leads to considerable differences in crack tip condition, with larger plastic zones being formed at the crack tip. Clearly larger plastic zone sizes increase the materials resistance to crack propagation and therefore slower FCP rates are produced.

Again Dumbleton's representation has been used to separate the total da/dN into a time-dependent and a cycle-dependent component. This is shown in Fig 5.53(a-d) where the crack growth per cycle at various values of ΔK is plotted against the reciprocal frequency. These curves show the FCP per cycle plotted against the duration of the loading cycle. From Fig 5.53(a) the values for cycle-dependent and time-dependent crack propagation at 1 Hz are 9.6×10^{-6} mm/Cycle and 2.4×10^{-6} mm/sec respectively. Furthermore, the results for high and low frequencies can be represented by two different lines as shown in Fig 5.53(a) (solid and dotted curves). This is because at a test frequency of 3.0 or 6.0 Hz the region of the specimen in the vicinity of the crack tip could be appreciably hotter than the test temperature (-20°C). Therefore, the results at 0.2 and 0.5 Hz relate to a test temperature of about -20°C (i.e no significant temperature rise) whereas the results at 3.0 and 6.0 Hz relate to a crack tip temperature somewhat higher than -20°C (see chapter 6). Hence the dotted line on Fig 5.53(a) represents FCP at -20°C and suggests that if the crack tip temperature remained constant FCP will almost only be due to fatigue-dependent crack propagation.

Previous work with the BP33 material has established that frequency

effects are enhanced with an increase in mean stress. Therefore, further tests with varying frequency were carried out at an R-ratio of 0.3. The results are shown in Fig 5.54 at frequencies of 0.2, 0.5, 3.0 and 6.0 Hz. The effect of increased mean stresses is apparent with greater differences in FCP rates being observed due to increasing frequency. Corresponding graphs of crack growth per cycle, at various values of ΔK , plotted against the reciprocal frequency are given in Fig 5.55 (a-d). However, the dotted line in Fig 5.55(a) is not horizontal and, therefore, represents both fatigue and time-dependent crack growth. Furthermore, it cannot be assumed that this dotted line represents fatigue crack propagation at -20°C . This is because even at low frequencies (represented by the dotted line), higher mean stresses can lead to an increase in crack tip temperature. Therefore, a small increase in frequency (0.2 to 0.5 Hz) will have a greater effect on crack tip temperature at an R-ratio of 0.3 than at an R-ratio of 0.1. This obviously leads to slower FCP rates due to changes in crack tip condition.

R-ratio effects are further highlighted by Fig 5.56 where $\log da/dN$ is plotted against $\log$ frequency, f , for the two different cases of $R=0.1$ and 0.3 . As mentioned earlier, on such a plot low frequencies will produce lines having a slope of -1 representing time-dependent crack propagation and high frequencies will produce constant da/dN zero slope lines representing fatigue-dependent crack propagation. For the range of test frequency used in this programme the lines lie in the intermediate range where both a time-dependent and fatigue-dependent component exists. This has already been confirmed by the data represented on graphs of da/dN against the reciprocal frequency.

Furthermore, and of greater significance, Fig 5.56 shows that at low frequencies, higher overall FCP rates are observed at an R-ratio of 0.3, and the slope of the line is steeper. Therefore, time-dependent effects are greater at an R-ratio of 0.3. This is due to the higher mean stresses acting more effectively at the lower frequencies where the duration of the loading cycle is longer.

However, the results at high frequencies are of even greater interest. Although at low frequencies, higher FCP rates were produced with an increase in R-ratio, high frequencies have had a drastic effect on FCP rates. In fact at a frequency of 6.0 Hz,

FCP rates are lower by a factor of 1.6 at an R-ratio of 0.3. This effect has been explained earlier and has been attributed to an increase in hysteretic heating in the specimen with a decrease in the duration of the cycle. High mean stresses also have the effect of increasing the hysteretic heating in the specimen. Therefore, the combination of having both a high mean stress and high frequencies has had the effect of drastically increasing the temperature at the crack tip (see chapter 6). Consequently, this enhanced the plasticity at the crack tip (i.e larger plastic zones) which ultimately led to the increase in the material's resistance to crack propagation.

Numerical values for cycle-dependent and time-dependent FCP rates have been found from the graphs of da/dN against reciprocal frequency (Fig 5.53 (a-d) and 5.55 (a-d) for R-ratios of 0.1 and 0.3 respectively). These values at various constant ΔK values are given in Table 5.7 and are represented on a graph of $\log da/dN$ against $\log \Delta K$ in Fig 5.57. As with previous cases (Figs 5.30 and 5.39), at 1 Hz, cycle-dependent crack growth is always greater than time-dependent crack growth. Furthermore, fatigue-dependent crack growth decreases with increasing R-ratio while time-dependent crack growth increases and at an R-ratio of 0.3 is nearly equal to the fatigue-dependent component.

Finally, the cycle and time-dependent components of FCP rates at a K_{max} value of $3.467 \text{ MPa } \sqrt{\text{m}}$ and a frequency of 1.0 Hz are plotted against the R-ratio in Fig 5.59. As expected due to the overall decrease in FCP rates, which occurs with increasing R-ratio, both the cycle-dependent and time-dependent components decrease with increasing R-ratio. However, the decrease in the cycle-dependent component is much more severe than that of the time-dependent component. Furthermore, at an R-ratio of 0.3, the time-dependent component almost equals the cycle-dependent component. This further confirms that time-dependent effects are greater at higher R-ratios which is attributed to higher mean stresses acting more effectively in the duration of the loading cycle.

5.3 GENERAL DISCUSSION

The data presented in this chapter have depicted the strong effects of both R-ratio and frequency.

Throughout the testing programme on the three materials (DFDG 4262, BP33 and LL101AA), a clear picture was established where an increase in R-ratio led to slower FCP rates. This has been attributed to the higher initial values of K_{max} used in tests at high R-ratios. Consequently the increase in initial load led to a plastic zone forming at an earlier stage in the life of the specimen which in turn offered a greater resistance to crack propagation. Furthermore, shear lips were observed to form earlier in the life of a specimen with increasing R-ratio. This confirmed that a transition from plane strain to plane stress conditions occurred and more significantly that this transition occurred earlier in the life of a specimen with increasing R-ratio. This is further confirmed by Fig 5.5 where the crack growth rate is plotted against the LEFM plastic zone size criterion. The line drawn through the middle of this graph represents the transition from plane strain to plane stress. It is clear that, for high R-ratios, plane stress conditions will prevail much earlier in the life of a specimen.

Although the LEFM transition criterion is not precise, it should be pointed out that, the yield stress used to calculate the LEFM criterion is taken as the U.T.S value (ultimate tensile strength). However, from the stress-strain curve in Fig 5.5(b) it can be seen that both materials yield at lower stress values and therefore a more valid yield stress would be 4.0 MPa and 5.0 MPa for BP33 and LL101AA respectively (instead of 9.69 MPa and 10.42 MPa respectively).

This suggests that a much more severe condition exists at the crack tip and obviously plane stress conditions will prevail even earlier. Such a condition would be of the form of much larger plastic zone sizes which ultimately offer a greater resistance to crack propagation. Furthermore, graphs plotted against net section stress (Figs 5.12, 5.13 and 5.14) have shown that the curve's minimum values occur at a net section stress of about 4.5 MPa. Therefore, a yield stress value of 4 MPa (quoted above) would be in agreement with the above phenomenon. That is the ligament material would yield at about

4 MPa, thus offering a greater resistance to crack propagation and consequently the FCP rates decrease and reach a minimum value at a net section stress of 4.5 MPa. Also higher initial net section stress values are applied in tests with higher R-ratios. In fact the test at an R-ratio of 0.5 was started at a net section stress of 3.25 MPa (Fig 5.14). Thereafter the ligament material yields with the crack growing by only 13 mm compared to 37 mm (crack growth to yield) at an R-ratio of 0.1 (initial net section stress = 1.8 MPa).

Therefore, in view of the above considerations it is not surprising that slower FCP rates are observed with increasing R-ratio.

Finally, the effect of increased hysteretic heating with increasing R-ratio should not be excluded, especially in view of the viscoelastic nature of polymeric materials, that is, the time and temperature dependence of mechanical properties. Although this effect was not extensively investigated with the thermovision equipment, a single case was examined where the load was increased from $P_{\max}=6.0\text{kN}$ to $P_{\max}=7.0\text{kN}$ at an R-ratio of 0.6 and $f=10\text{ Hz}$ (Figs 6.3 (c-d)). This is analagous to the higher loads used in tests at higher R-ratios. It was observed that an increase in load of only 1.0 kN led to the crack tip temperature increasing by 3.7°C in only 3 hours of cycling. Considering the time scale of a fatigue test (2 - 5 days depending on R-ratio) it can be argued that greater variations in crack tip temperature would occur. Furthermore, temperature rises of up to 12.2°C were observed (Fig 6.3(d)), which can decrease both the yield stress and the Young's modulus of the material and ultimately lead to a change in crack tip condition. Consequently, a more severe plane stress condition is set up due to larger plastic zone sizes and will prevail even earlier in the life of a specimen. Ultimately higher R-ratios would lead to the ligament material yielding at lower values of net section stress, thus offering a greater resistance to FCP.

Crack growth rates were also found to decrease with increasing frequency for these materials. This is because when the duration of the loading cycle decreases the hysteretic heating in the specimen increases. As mentioned above, polymeric materials are viscoelastic and, therefore, an increase in crack tip temperature would lead to a change in the effective specimen modulus. This will in turn affect the

cyclic stresses and strains locally at the crack tip and therefore also the plastic zone size. Specifically, a rise in crack tip temperature leads to a decrease in the effective specimen modulus which ultimately results in larger crack tip strains. Furthermore, as a result of a lower specimen stiffness, a loss in constraint occurs which combining with the deleterious effect of decreasing yield stress (i.e. larger plastic zone sizes) leads to slower FCP rates. Finally, it can also be said that the beneficial effect of increasing frequency on FCP resistance is a result of a localised temperature rise at the crack tip which blunts the crack and results in a lower effective ΔK .

Using Dumbleton's representation it was possible to divide the total da/dN in BP33 and LL101AA into two components. One component was cycle-dependent and the other time-dependent. In all tests carried out it was found that cycle-dependent crack growth is always greater than time-dependent crack growth. Furthermore, it was not possible to grow a crack under an applied static load. This suggests that time-dependent crack growth occurs within the duration of the cycle due to cyclic loading having the tendency to keep resharping the crack tip, thus eliminating crack arrest due to blunting.

Frequency variations have also been studied at different R-ratios and by using Dumbleton's and Webster's representations, the following conclusions were reached. Firstly, higher R-ratios ($R=0.5$) were found to produce a greater sensitivity to FCP rates (i.e. greater differences) with increasing frequency. Webster's representation (Fig 5.29) showed that the line representing the low frequencies at an R-ratio of 0.5 had a bigger slope than the other two conditions. This suggests that time-dependent effects are greater at an R-ratio of 0.5 than at either 0.1 or 0.2.

This was also confirmed by Dumbleton's representation (Fig 5.31(b)) where the cycle-dependent component decreases drastically with increasing R-ratio. The time-dependent component also decreases slightly with increasing R-ratio which is understandable due to the decrease in overall FCP rates at high R-ratios. However, and of greater importance, at an R-ratio of 0.5 the time-dependent component is almost equal to the cycle-dependent component. Hence, it can be concluded that there are greater

time-dependent effects at an R-ratio of 0.5. Therefore, time-dependent effects are enhanced with an increase in R-ratio and is attributed to higher mean stresses acting more effectively in the duration of the loading cycle.

Fig 5.38 illustrates the drastic effects of having both a high mean stress and high frequencies. It can be seen that at an R-ratio of 0.5, and at low frequencies, FCP rates are greater than at an R-ratio of 0.1 and 0.2. Again this line has a bigger slope and therefore greater time-dependent effects. However, high frequencies (in combination with a high mean stress) have had a dramatic effect on FCP rates and they are seen to decrease drastically. As mentioned earlier high frequencies have the effect of increasing the hysteretic heating in the specimen. On the other hand high mean stresses produce a more severe condition at the crack tip (i.e high net section stresses which yield the ligament material and consequently offer a greater resistance to crack propagation). Furthermore, the earlier suggestion of high mean stresses, also increasing the hysteretic heating in the specimen, cannot be excluded.

Therefore, the deleterious effects of all the above considerations are in agreement and hence offer an explanation to the drastic decrease in FCP rates observed at high R-ratios and high frequencies.

5.4 THE APPLICATION OF MODELS FOR FATIGUE CRACK PROPAGATION

5.4.1 The H.M.El-Hakeem and L.E.Culver Model .

An empirical modelling approach to FCP was reported by El-Hakeem and Culver (1981) [67] in an attempt to produce a physically rational model explaining the behaviour of dynamic fatigue cracks in polymers. The relationship proposed between da/dN and the factors of K is given below :

$$da/dN = A. \Psi^L$$

$$\begin{aligned} \text{where } \Psi &= (K_{\max}^2 - K_{\min}^2)^{\alpha/L} (K_m^2)^{\beta/L} \quad \text{and} \quad L = \alpha + \beta \\ &= (\Delta K)^2 [0.25^{\beta} ((1+R)/(1-R))^{\alpha+2\beta}]^{1/(\alpha+\beta)} \end{aligned} \quad \text{Eqn 5.6}$$

where A , α and β are functions of test frequency, environment and material properties, and K_m is the mean stress intensity factor. The condition of the plastic zone at the crack tip was assumed to control FCP rates and this condition was represented by the COD and/or the plastic zone size. However, the authors did not state why they chose to represent the level of the COD and the plastic zone size with K_m and not $K_{\max}$.

The FCP results obtained in the present research programme can be compared with the model by first determining the parameter Ψ in Eqn 5.6. To do this both α and β need to be known . The value of $2(\alpha + \beta)$ is taken to be equal to the mean value of the slopes of the lines representing the experimental results at different R-ratios on the $\log da/dN / \Delta K$ graphs (e.g Fig 5.16). Also the value of $(\alpha + 2\beta)$ is taken as the mean value of the slopes of the lines representing the relation of $\log da/dN$ to $\log((1+R)/(1-R))$ at different constant ΔK values. Plots of $\log da/dN$ against $\log((1+R)/(1-R))$ are shown in Figs 5.60, 5.61 and 5.62 for the three cases considered, e.g BP33-T=-20°C, LL101AA-T=+23°C and LL101AA-T=-20°C respectively.

From Figs 5.61 and 5.62 it can be seen that the data show a high degree

of scatter and, even more significantly, negative slopes would be obtained if a straight line could be drawn through the points. Since the model assumes that the FCP results are controlled by the level of K it consequently predicts higher FCP rates with increasing R -ratio for a given ΔK . Therefore, even if a straight line could be drawn through the points, thus determining α and β , the model cannot be used effectively to predict FCP rates in LL101AA at either test temperature.

However, for BP33 at $T=-20^{\circ}\text{C}$ (Fig 5.60) it was possible, although again some degree of scatter was observed, to draw a straight line through the points and a positive slope was obtained. Therefore α and β were determined and consequently a graph of $\log da/dN / \Psi$ plotted (Fig 5.63). The objective of the model is to produce a linear relationship between $\log da/dN$ and $\log \Psi$ irrespective of the particular values of R . However, from Fig 5.63 it can be seen that variations in FCP rates exist with a change in R -ratio. These variations are similar and even more severe than those observed with the Paris representation (Fig 5.64) and therefore it can be concluded that Eqn 5.6 does not correlate da/dN in BP33 with the values of K .

5.4.2 The R.G.Forman , V.E.Kearney and R.M.Engle Model .

As mentioned earlier in chapter 3 this model is a modification of the Paris exponential crack propagation equation . In its modified form it takes account of both the variation in FCP rates owing to the load ratio, R , and the instability of the crack growth when the stress intensity factor approaches the K_{IC} -value for the material.

The proposed equation is given by :

$$da/dN = A.\Delta K^m / [(1-R)K_{IC} - \Delta K] \quad \text{Eqn 5.7}$$

In order to calculate da/dN and hence obtain the theoretical curves, the material constants, K_{IC} , A and m need to be known. The values for A and m are taken to be equal to the mean values of the intercepts and slopes respectively of the lines representing the experimental results at different R -ratios on the $\log da/dN / \Delta K$ graphs (e.g Fig 5.16).

Also 'apparent' K_{IC} values have been obtained from the experimental results.

Therefore, the material constants used for calculating the theoretical curves in Figs 5.65-5.70 for the three cases considered are :

(1) BP33 at $T=-20^{\circ}\text{C}$

$$K_{IC} = 1.41 \text{ MPa } \sqrt{\text{m}}$$

$$A = 7.55 * 10^{-4}$$

$$m = 4.40$$

(2) LL101AA at $T=+23^{\circ}\text{C}$

$$K_{IC} = 1.82 \text{ MPa } \sqrt{\text{m}}$$

$$A = 7.67 * 10^{-6}$$

$$m = 5.71$$

(3) LL101AA at $T=-20^{\circ}\text{C}$

$$K_{IC} = 3.84 \text{ MPa } \sqrt{\text{m}}$$

$$A = 1.08 * 10^{-6}$$

$$m = 4.73$$

The theoretical curves (Eqn 5.7) at different R-ratios are plotted in Figs 5.65, 5.66 and 5.67 for case (1), case (2) and case (3) respectively. From these graphs it can be seen that in each case an increase in R-ratio produces an increase in FCP rates. Furthermore, the theoretical curves at each R-ratio are compared with test results in Figs 5.68, 5.69 and 5.70, again for case (1), case (2) and case (3) respectively. These figures show clearly how the Forman model predicts faster FCP rates with increasing R-ratio. Therefore, the model cannot be used to predict the behaviour of dynamic fatigue cracks in either low or linear low density polyethylene.

This is because, although the model takes account of the variation in FCP rates owing to the load ratio, R, it assumes that an increase in the load ratio will ultimately lead to an increase in FCP rates. However, and due to the viscoelastic nature of polymeric materials the opposite is seen to occur (i.e a decrease in FCP rates with increasing R-ratio). Consequently, the model does not take into account possible crack tip

blunting which occurs and in a more severe condition with increasing R-ratio.

5.4.3 The K.Walker Model .

The model proposed by Walker [69] is an empirical correlation for FCP rates in 7075-T6 and 2024-T3 aluminium. The relation which attempts to incorporate stress-ratio effects is given below :

$$da/dN = A.K_{\max}^m \Delta K^n \quad \text{Eqn 5.8}$$

The author modified this general equation by introducing an effective ΔK , yielding :

$$da/dN = A.\Delta K^n$$

$$\text{where} \quad \Delta K = K_{\max} (1-R)^m \quad \text{Eqn 5.9}$$

The exponent m can be found by plotting $\log da/dN$ against $\log K_{\max}$ for a constant value of ΔK . These plots are shown in Figs 5.71(a), (b) and (c) for the three cases considered (i.e BP33-T=-20°C, LL101AA-T=+23°C and LL101AA-T=-20°C) respectively, at varying R-ratio and a test frequency of 3.0 Hz. From Figs 5.71(b) and (c) it can be seen that, although some degree of scatter occurs, straight lines could be drawn through the points. However, negative slopes are obtained which contradict the assumption made by the model, i.e increasing FCP rates with increasing R-ratio. For BP33 at T=-20°C, however, a straight line drawn through the experimental points does produce a positive slope (Fig 5.71(a)). Therefore, a positive m value is obtained and is equal to 0.436.

The FCP rates obtained by Eqn 5.9, with values of m equal to 0.44, -1.45 and -1.78 for cases (a), (b) and (c) are plotted in the form $\log da/dN$ against $\log \Delta K$ in Figs 5.72, 5.73 and 5.74 respectively. From Figs 5.73 and 5.74, it can be seen that the Walker model has had a drastic effect in attempting to predict FCP behaviour in LL101AA. This is due to the negative slopes obtained in Figs 5.71(b) and (c) which were

in turn due to decreasing FCP rates with increasing R-ratio. The model's basic assumption, however, is that an increase in R-ratio will lead to faster FCP rates. Consequently, the model does not take into account possible crack tip blunting which can occur due to higher loads (used with higher R-ratios) causing a yielded condition in the ligament material. Therefore, it can be concluded that the Walker model cannot be used to predict the FCP behaviour of LL101AA at either room temperature or at -20°C .

However, for BP33 at $T=-20^{\circ}\text{C}$, a positive slope was obtained from Fig 5.71(a) (exponent $m=0.44$) and therefore conforms with Walker's assumption. Fig 5.72 represents the FCP rates predicted by Eqn 5.9 in the form of $\log da/dN$ against $\log \Delta K$. Furthermore, the FCP rates predicted by Eqn 5.9, at each R-ratio, are compared with the observed Paris relationships in Figs 5.75 (a-f). The plots of $\log da/dN$ against $\log \Delta K$ and ΔK show reasonable agreement between the correlation and the experimental Paris relationship for R-ratios of 0.0 and 0.1. However, the agreement is poor for R-ratios between 0.2 and 0.5. Therefore, it can be concluded that the Walker model does not give an entirely satisfactory correlation of the FCP results. The value of the exponent m , however, is quite informative in that it suggests that the sensitivity of FCP to $K_{\max}$ is small (i.e $m=0.44$) compared to the sensitivity of FCP to ΔK ($n=4.22$, in Eqn 5.8).

5.4.4 The Two Stage Line Zone Model.

This model is based on the fracture criterion that the craze at the tip of a fatigue crack will fail at a critical COD. The craze at the tip of the crack is assumed to comprise material which at the tip of the craze has a high load carrying capacity and a low load carrying capacity near the crack. The reduction in load carrying capacity is due to the ligament material between the voids (within the craze) being damaged by cyclic loading. For simplicity, cyclic damage incurred by the fresh craze material is assumed to occur in a single loading cycle.

For comparison with experimental data, the relationship proposed by Williams [71] between da/dN and the loading parameters is given in Eqn 3.15. The FCP results obtained in the present research programme for BP33 at a test frequency of 3.0Hz, a test temperature of -20°C and for R-ratios between -1.0 and 0.5 are plotted in the form da/dN against K_{max}^2 in Fig 5.76. This figure confirms Eqn 3.15 which predicts a linear relationship between da/dN and K_{max}^2 . However, the FCP results exhibit a transition from one linear relationship to another steeper linear relationship. In 1977 Williams [70] found the same transitional behaviour in a grade of Nylon 66. He proposed that this transition represented a change from FCP dominated by plane strain to FCP dominated by plane stress and he provided experimental evidence to support this view.

The model assumes that the applied K is reduced by a constant value of K_O where K_O is the crack opening stress intensity factor. Eqn 3.19 gives the relationship between the value of K_O and the slope, S of the curves of da/dN against K_{max}^2 .

$$S.K_{th}^2 = \pi.\alpha.K_c^2 / [8.(1-\alpha)^2.\sigma_c^2] + S.K_O^2 \quad \text{Eqn 3.19}$$

where K_{th} is the threshold stress intensity factor.

The model can be compared to the data by plotting the factor SK_{th}^2 for each R-ratio against S . Figs 5.77(a) and (b) represent such plots for crack growth before (Stage 1) and after (Stage 2) the transition respectively. From both these plots it can be seen that the data show some degree of scatter, but reasonable straight lines could be drawn through the

points. From Eqn 3.19 the slope of the curves (on a plot of SK_{th}^2 against S) gives the value of K_O^2 and the intercept gives :

$$\alpha.r_c / (1-\alpha)^2$$

$$\text{where } r_c = (\pi/8).[K_c/\sigma_c]^2 \quad \text{Eqn 3.20}$$

Therefore, from Fig 5.77(a) and 5.77(b) which represent stage 1 and stage 2 respectively, the corresponding values are $K_O^2(1) = 0.10 \text{ MPa}^2.\text{m}$, $\alpha r_c / (1-\alpha)^2(1) = 292 \text{ nm}$, $K_O^2(2) = 0.26 \text{ MPa}^2.\text{m}$ and $\alpha r_c / (1-\alpha)^2(2) = 598 \text{ nm}$.

The factor α can be calculated from the following relationship :

$$K_{th}^2 = [\alpha.K_c / F(R)] + K_O^2 \quad \text{Eqn 3.21}$$

However, as mentioned earlier in chapter 3, there are two possible types of behaviour which determine $F(R)$ (Figs 3.3(a) and 3.3(b)) i.e

$$\begin{aligned} F(a) &= 1-(1-\alpha)^2.R^2 \\ F(b) &= (1-R^2) + \alpha R^2 \end{aligned} \quad \text{Eqns 3.16}$$

Therefore, before continuing it has to be decided which type of behaviour exists ahead of the crack tip for each condition investigated.

Now, $F(R)$ may be expressed in terms of the slope at $R=0.0$ since $F(0)=1$, so that :

$$F(R) = S / S_O \quad \text{Eqn 3.22}$$

Fig 5.78 shows slope data for the three conditions investigated plotted as S / S_O versus R in accordance with Eqn 3.22. Also shown are lines corresponding to equations 3.16 for various values of α . From this figure it is clear that all three conditions investigated give a

close fit to type (b) with a low α value.

Therefore Eqn 3.21 becomes :

$$K_{th}^2 = [\alpha.K_c^2 / ((1-R)^2 + \alpha R^2)] + K_o^2 \quad \text{Eqn 3.21}$$

Consequently the factor α can be calculated and for BP33 at $T=-20^\circ\text{C}$ is found to be equal to 0.13. Thereafter from :

$$\alpha r_c / (1-\alpha)^2 = 292 \text{ nm}$$

$$r_c = 1.70 \mu\text{m}$$

$$\text{and from Eqn 3.20} \quad \sigma_c = 679 \text{ MPa}$$

This value of craze stress is remarkably high and is a direct consequence of the very small r_c value determined, $1.70 \mu\text{m}$. Williams [70,71] also found very high values for σ_c in Nylon 66, PMMA and PC but it is still not known what these values really mean. However, it can be said that this is due to the formation of very small high load carrying capacity crazes.

Therefore, having obtained values for K_o^2 , α and σ_c , predicted da/dN values can be calculated and compared with experimental data from the following equation:

$$da/dN = \pi / (8.(1-\alpha)^2 \sigma_c^2) [F(R).(K^2 - K_o^2) - \alpha.K_c^2] \quad \text{Eqn 3.15}$$

$$\text{where } F(0)=1$$

Figs 5.79 (a-e) represent data predicted by Eqn 3.15 and compared with experimental data for BP33 at $T=-20^\circ\text{C}$ and $F=3.0 \text{ Hz}$, for R-ratios between -1.0 and 0.5. The correlation is seen to be very good for small R-ratios, i.e 0.0 , 0.1 and 0.2 but the curves deviate with increasing R-ratio. Specifically at high R-ratios (0.3, 0.4 and 0.5) the model predicts faster FCP rates. At this point it is necessary to point out that the last two or three points

in the experimental data (i.e high K values) should be ignored. This is because these points are due to unstable crack growth, i.e ductile tearing, and consequently a change in slope is observed in the curves.

Even at negative R-ratios the correlation is very good for $R=-0.2$ and -0.5 but for $R=-0.75$ and -1.0 the model predicts slower FCP rates. Therefore, the model is seen to fail at the two extremes of high positive and high negative R-ratios. In the case of high positive R-ratios the model predicts slightly faster FCP rates. This can be attributed to the fact that the model does not take account of hysteretic heating at the crack tip which can occur at high R-ratios due to higher applied K values. Consequently, although the model predicts some degree of crack tip blunting (i.e the crack tip sustains a stress σ_c at K_{min}) slower FCP rates are produced due to a higher degree of crack tip blunting. For the case of high negative R-ratios (-0.75 and -1.0) the model predicts slower FCP rates. This suggests that the model does not foresee the enormous extent of the damage that high compressive stresses can have on fibres within the polymer structure. That is the fibres are first elongated in the tensile portion of the cycle and then they are crushed and become thinner due to compressive stresses in the compressive portion of the cycle. Consequently, the fibres will fracture at a faster rate the higher the compressive stresses. The model has also been used to predict crack growth rates in LL101AA at both temperatures, i.e $T=+23^\circ\text{C}$ and $T=-20^\circ\text{C}$. For the former case (LL101AA , $T=+23^\circ\text{C}$) the FCP results obtained for R-ratios between 0.0 and 0.3 are plotted in the form da/dN against K_{max}^2 in Fig 5.80. The corresponding plots of SK_{th}^2 against S for crack growth before (Stage 1) and after (Stage 2) the transition are shown in Fig 5.81(a) and 5.81(b) respectively. From these figures it can be seen that in each case a negative intercept is obtained which would correspond to a negative value of r_c ,

i.e $\alpha r_c / (1-\alpha)^2 < 0$ which is impossible .

Therefore, it can be concluded that the model is inadequate in predicting FCP rates in LL101AA at $T=+23^\circ\text{C}$.

For the latter case (LL101AA-T=-20°C) the FCP results obtained for R-ratios between -1.0 and 0.4 are plotted in the form da/dN against K_{max}^2 in Fig 5.82. Furthermore, the slopes and intercepts obtained from these curves are plotted in the form of SK_{th}^2 against S in Fig 5.83. It is seen that the data exhibit some degree of scatter but a reasonable straight line could be drawn through the points. From this line K_O^2 and $\alpha r_c / (1 - \alpha)^2$ can be obtained and were found to be equal to 1.0 MPa².m and 25.0 nm respectively. The factor α can thus be calculated from :

$$K_{th}^2 = [\alpha . K_c^2 / F(R)] + K_O^2$$

and was found to be equal to 0.11.

Therefore, predicted da/dN values could be calculated from Eqn 3.15 and compared with experimental data. The compared data are represented in Fig 5.84(a-e) for LL101AA at T=-20°C and F=3.0 Hz, for R-ratios between -1.0 and 0.4. Again the correlation is seen to be very good for small positive and small negative R-ratios. As with predictions made for BP33 at T=-20°C the model fails at the two extremes of high positive and high negative R-ratios. Again, for both high positive and high negative R-ratios, the model predicts faster and slower FCP rates respectively. These effects were explained earlier and were attributed to hysteretic heating at the crack tip and to the extent of damage incurred by the fibres, due to compressive stresses respectively.

It is known that the model does not take into account time-dependent dynamic FCP. However, it was possible in earlier analysis to divide the total da/dN into two components (Dumpleton's representation), cycle-dependent and time dependent crack propagation. Therefore, the cycle-dependent crack propagation data obtained earlier can be used in conjunction with the model which could hopefully produce a better fit than in the previous cases.

Cycle-dependent crack propagation results obtained earlier for BP33 at a test temperature of -20°C and for R-ratios of 0.1, 0.2 and 0.5 are plotted in the form da/dN against K_{max}^2 in Fig 5.85. Furthermore, Fig 5.86 represents the slopes and

intercepts obtained from these curves (Fig 5.85) plotted in the form SK_{th}^2 against S . The data are seen to give a much better agreement than previously, although fewer points are available. Therefore, it can be said that the model looks better when time-dependent effects have been eliminated. Consequently, a good straight line can be drawn through the data points and the values of K_O^2 and $\alpha r_c / (1 - \alpha)^2$ can be found. These were found to be equal to $0.25 \text{ MPa}^2 \cdot \text{m}$ and 157 nm respectively. Furthermore, the factor α calculated from Eqn 3.21 was found to be equal to 0.075.

Predicted da/dN values calculated from Eqn 3.15, compared with experimental data, are shown in Fig 5.87(a-c). It can clearly be seen that for R-ratios of 0.1 and 0.2 (Figs 5.87(a) and (b)) the model is very good in predicting FCP rates. However, at an R-ratio of 0.5 the model fails and predicts faster FCP rates. Again this can be attributed to hysteretic heating causing a more severe, blunting condition at the crack tip. Also high mean stresses lead to larger plastic zones at the crack tip which ultimately lead to slower FCP rates. Although the model is based on the accumulation of damage at the crack tip, the materials tested here have a high compliance. Consequently, high mean stresses lead to more severe crack tip conditions than are foreseen by the model which was found to work better for brittle materials, such as PMMA (Williams [70,71]).

Finally, it was also possible to divide the total da/dN into a cycle-dependent and a time-dependent component for LL101AA at a test temperature of -20°C . Limited data are available since variable frequency tests were only carried out at an R-ratio of 0.1 and 0.3. However, the cycle-dependent crack growth data are plotted in the form of da/dN against K_{max}^2 in Fig 5.88. The corresponding plot of SK_{th}^2 against S is shown in Fig 5.89 and the straight line drawn through the limited data gives values for K_O^2 and $\alpha r_c / (1 - \alpha)^2$ of $0.05 \text{ MPa}^2 \cdot \text{m}$ and 34 nm respectively.

Therefore, predicted da/dN values could be calculated from Eqn 3.15 (the factor α calculated from Eqn 3.21 = 0.172) and compared with experimental data. The compared data for LL101AA at $T = -20^\circ\text{C}$ and for R-ratios of 0.1 and 0.3 are shown in Fig 5.90(a) and (b). As with previous cases, the correlation is very good for small R-ratios i.e $R=0.1$, but for an R-ratio of 0.3 the model again predicts slightly faster FCP

rates. These effects have been observed throughout this investigation and have been explained earlier. Finally, Table 5.8 lists the required variables required by the model to predict FCP rates for the five test conditions investigated.

5.4.5 Conclusions on Fatigue Crack Propagation Models.

It has conclusively been shown that FCP in low and linear low density polyethylene cannot be correlated satisfactorily with the Hakeem and Culver , Forman and Walker models . This is because many more parameters are involved than are accounted for by the above authors and also due to these models assuming that an increase in the load ratio will ultimately lead to an increase in FCP rates. However, and due to the viscoelastic nature of polymeric materials the opposite is seen to occur. Consequently the models do not take into account possible crack tip blunting which occurs and in a more severe condition with increasing R-ratio.

However, the Williams model which is based on the accumulation of damage at the crack tip was able to predict FCP rates at small positive and small negative R-ratios. This is because the data that the model is found to predict are effectively ΔK controlled and thus crack tip blunting is not severe. Therefore, it was not surprising that the model failed at higher R-ratios. Firstly, although the model takes into account possible crack tip conditions, the materials tested in the present research programme have a high compliance. Therefore, at high mean stresses a greater degree of damage can exist at the crack tip than anticipated by the model. Also the model does not take into account hysteretic heating at the crack tip which can occur at high mean stresses thus leading to a higher degree of crack tip blunting.

Finally, time-dependent effects also increase with increasing R-ratio and since it was possible to separate time-dependent effects the data have been analysed. However, the model predicted FCP rates just as well for the total da/dN data, but less scatter was observed in plots of SK_{th}^2 against S . However, only limited data were available and therefore further variable frequency work is required to explain time-dependent effects in conjunction with the model.

5.5 COMPARISON OF MATERIALS

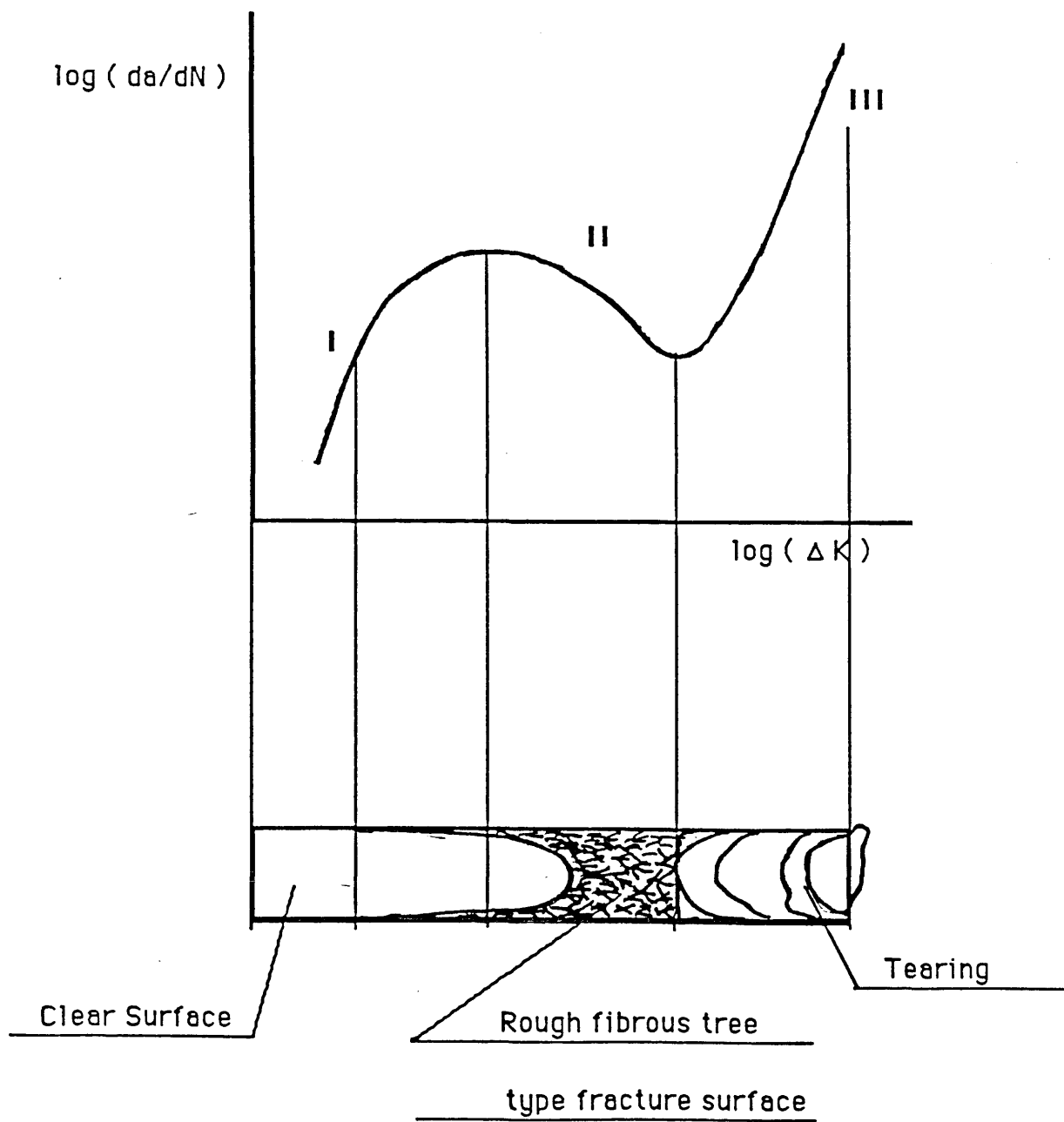
It has already been mentioned (chapter 4) that the linear low density polyethylene (LL101AA) has a superior toughness to the low density polyethylene. This section sets out to quantify the difference in fracture toughness between the two materials at the two test temperatures investigated.

The results from tests conducted on BP33 and LL101AA at a frequency of 3.0 Hz, an R-ratio of 0.1 and a test temperature of +23°C are shown in Fig 5.91. It can clearly be seen that the LLDPE has a much higher threshold value than the LDPE. Specifically, the values are 1.10 MPa. $\sqrt{m}$ and 0.28 MPa. $\sqrt{m}$ respectively which is a difference of 0.82 MPa. $\sqrt{m}$. Furthermore, at a constant ΔK value of 1.20 MPa. $\sqrt{m}$ the FCP rate for the LDPE is faster by a factor of 18 than that for the LLDPE. This difference is quite substantial and conclusively shows that the LLDPE has a superior resistance to fatigue crack propagation. Finally, the superior toughness of the LLDPE has been quantified and has a K_{IC} value of 1.82 MPa. $\sqrt{m}$ compared to 1.12 MPa. $\sqrt{m}$ for the LDPE.

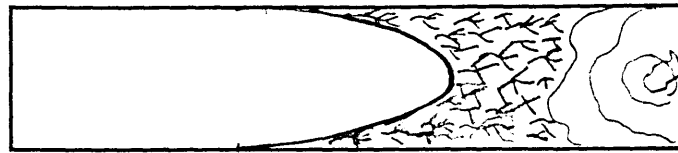
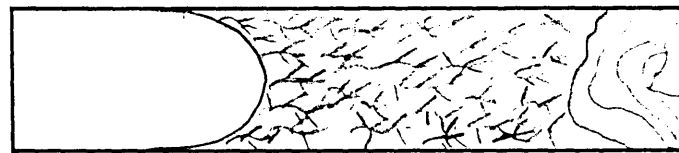
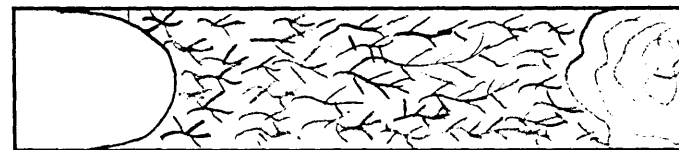
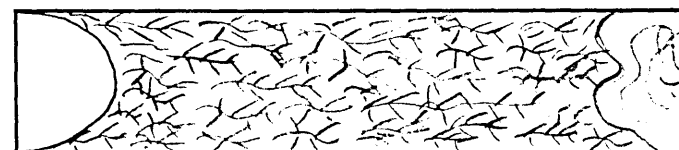
Low temperature tests have also been conducted (-20°C) and the results at a frequency of 3.0 Hz and an R-ratio of 0.1 are shown in Fig 5.92. In this case the threshold values for LLDPE and LDPE are 1.73 MPa. $\sqrt{m}$ and 0.73 MPa. $\sqrt{m}$ respectively. This gives a difference of 1.0 MPa. $\sqrt{m}$ which is slightly higher than the room temperature case. At a constant ΔK value of 2.19 MPa. $\sqrt{m}$ the FCP rate for the LDPE is again faster but in this case (-20°C) by a factor of 1765 than that for the LLDPE. Also there exists a greater difference in K_{IC} values for the two materials at low temperature. Specifically, the K_{IC} values for LLDPE and LDPE are 3.84 MPa. $\sqrt{m}$ and 1.41 MPa. $\sqrt{m}$ respectively. This gives a difference of 2.43 MPa. $\sqrt{m}$ compared to a difference of only 0.7 MPa. $\sqrt{m}$ for the room temperature case.

Therefore, it has conclusively been shown that not only is the LLDPE superior in toughness and FCP resistance but that its superiority is enhanced at low temperature. At low temperatures, a decrease in chain mobility occurs which clearly leads to an increase in fracture toughness. However, in the case of the LLDPE a more

complicated mechanism may be in operation and further work is needed to explain the above observations.



DIAG 5.1

 $R = 0.1$  $R = 0.2$  $R = 0.3$  $R = 0.4$  $R = 0.5$

DIAG 5.2

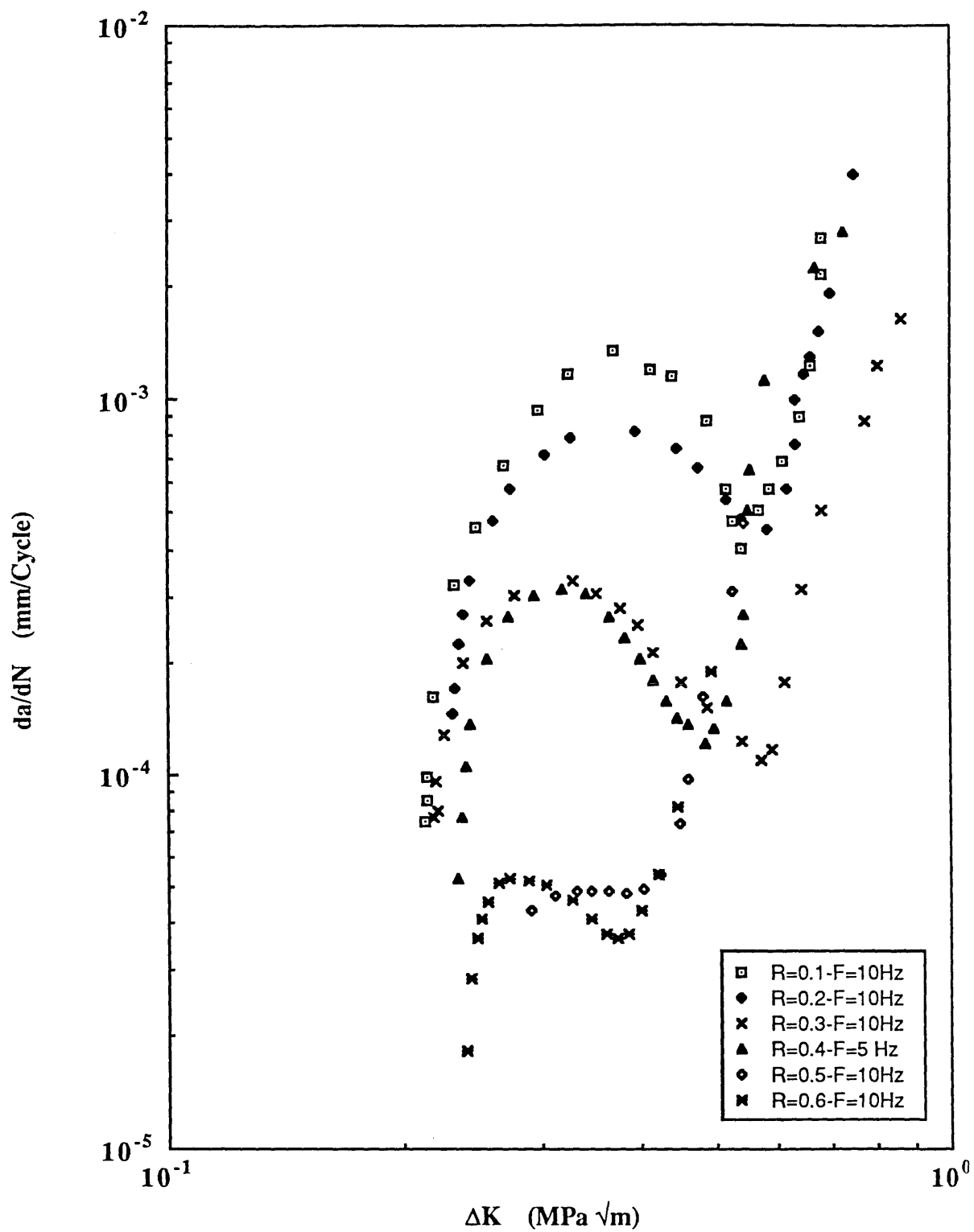


Fig 5.1 Fatigue crack propagation rates in DFDG 4262 at various R-ratios.
T=+23°C

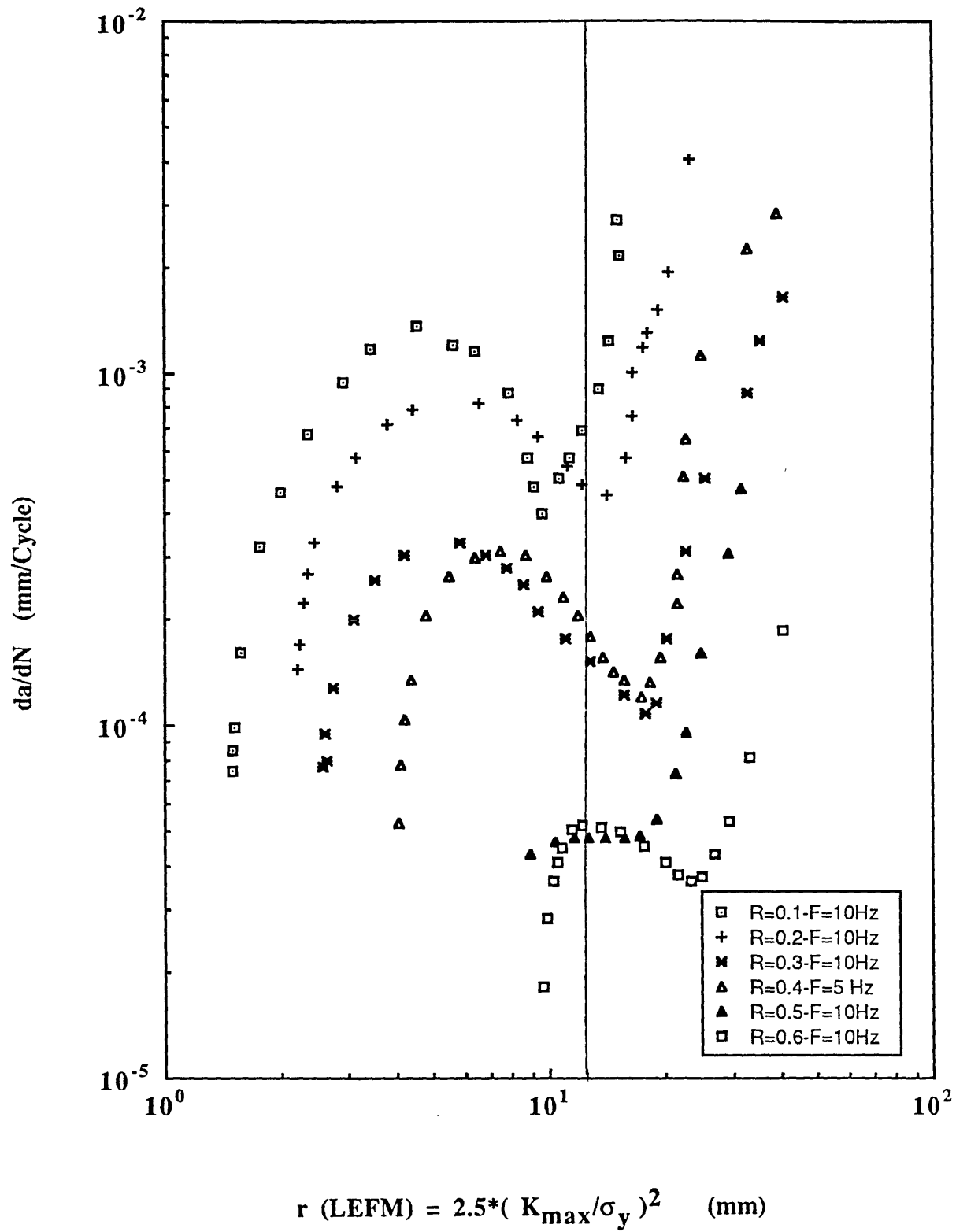


Fig 5.2 FCP rates in DFDG 4262 against LEFM plastic zone size criterion.
 $T=+23^\circ\text{C}$

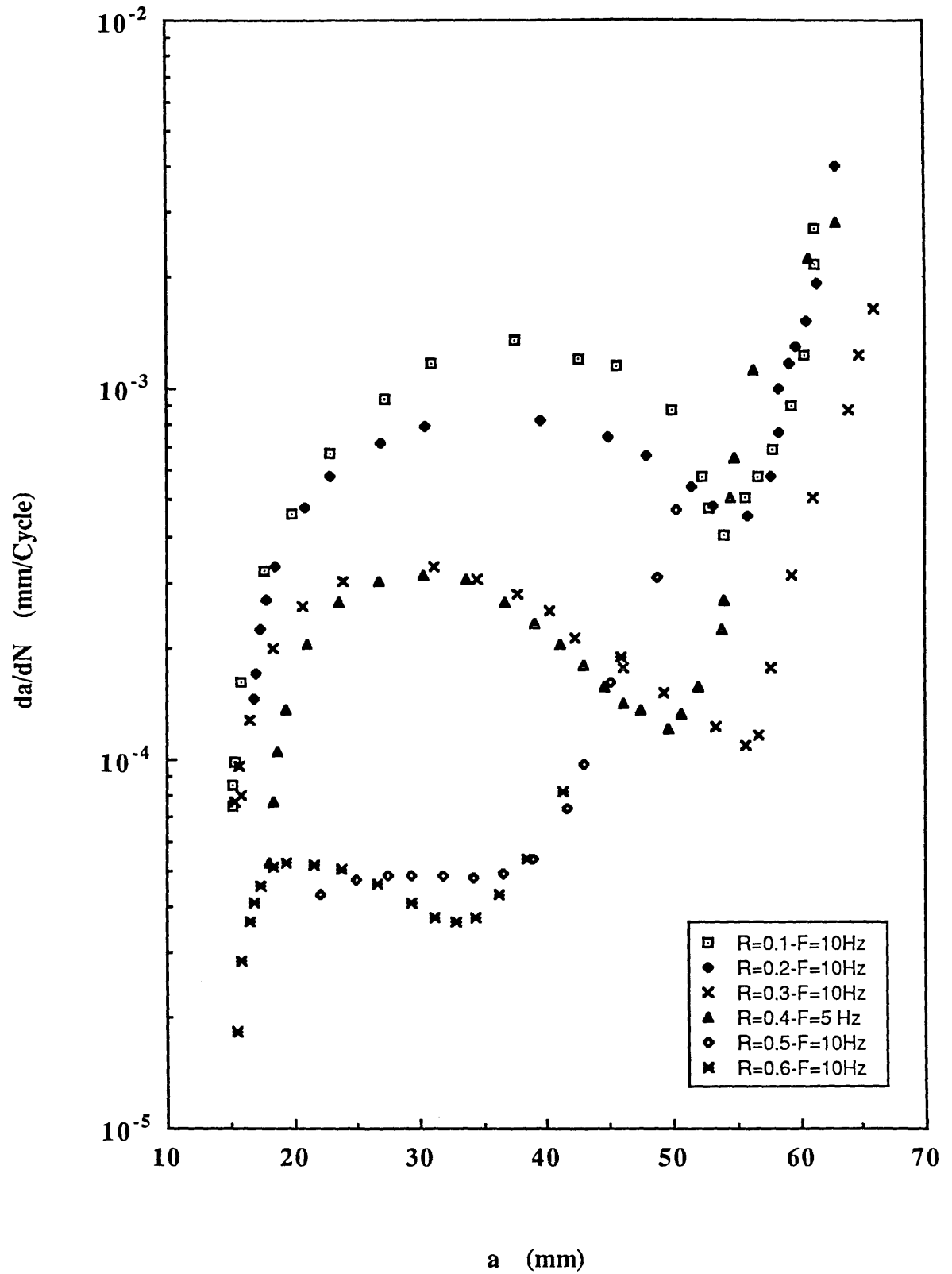


Fig 5.3 FCP rates in DFDG 4262 against crack length.
 $T=+23^{\circ}\text{C}$

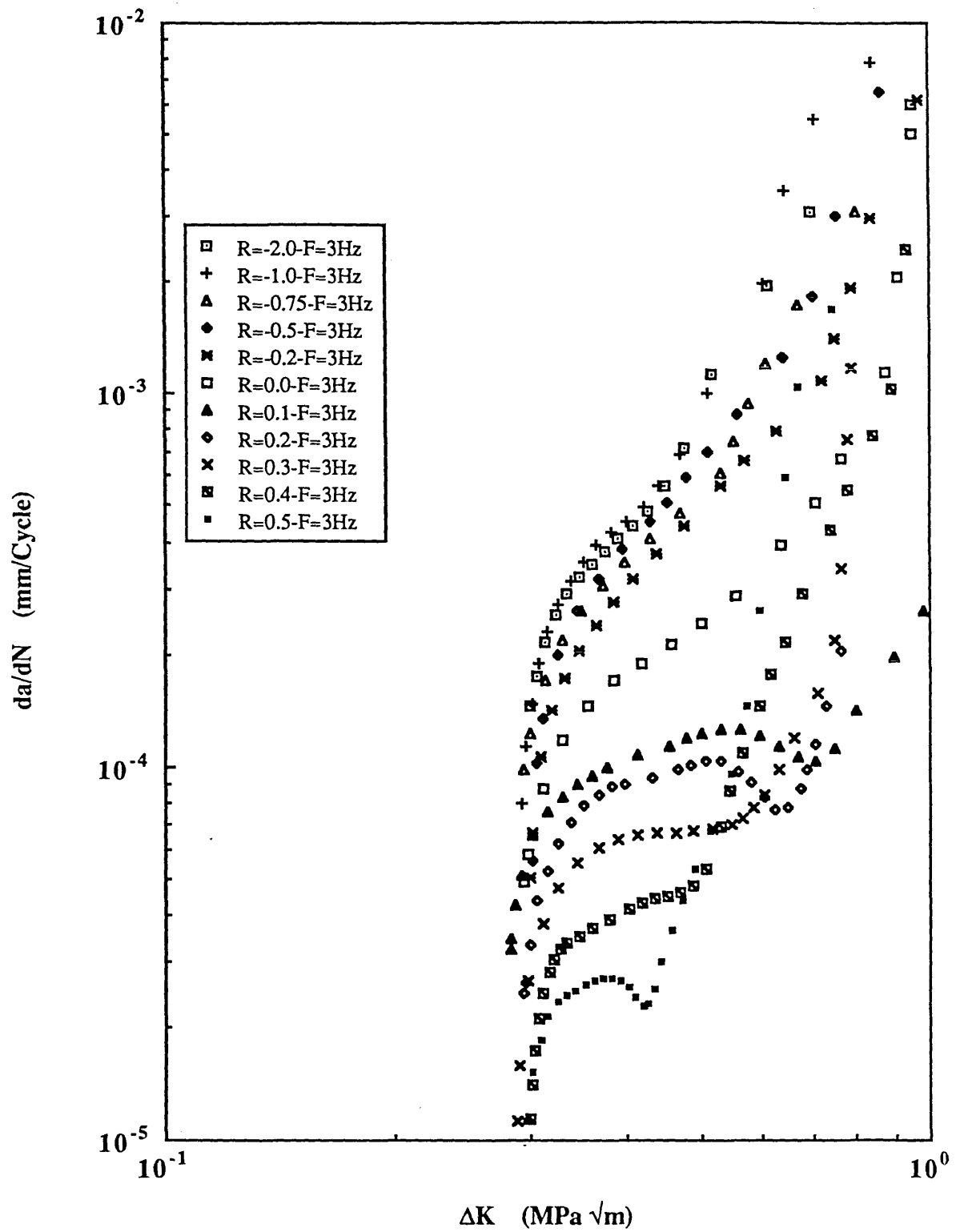


Fig 5.4 Fatigue crack propagation rates in BP33 at various R-ratios.
 $T=+23^{\circ}\text{C}$

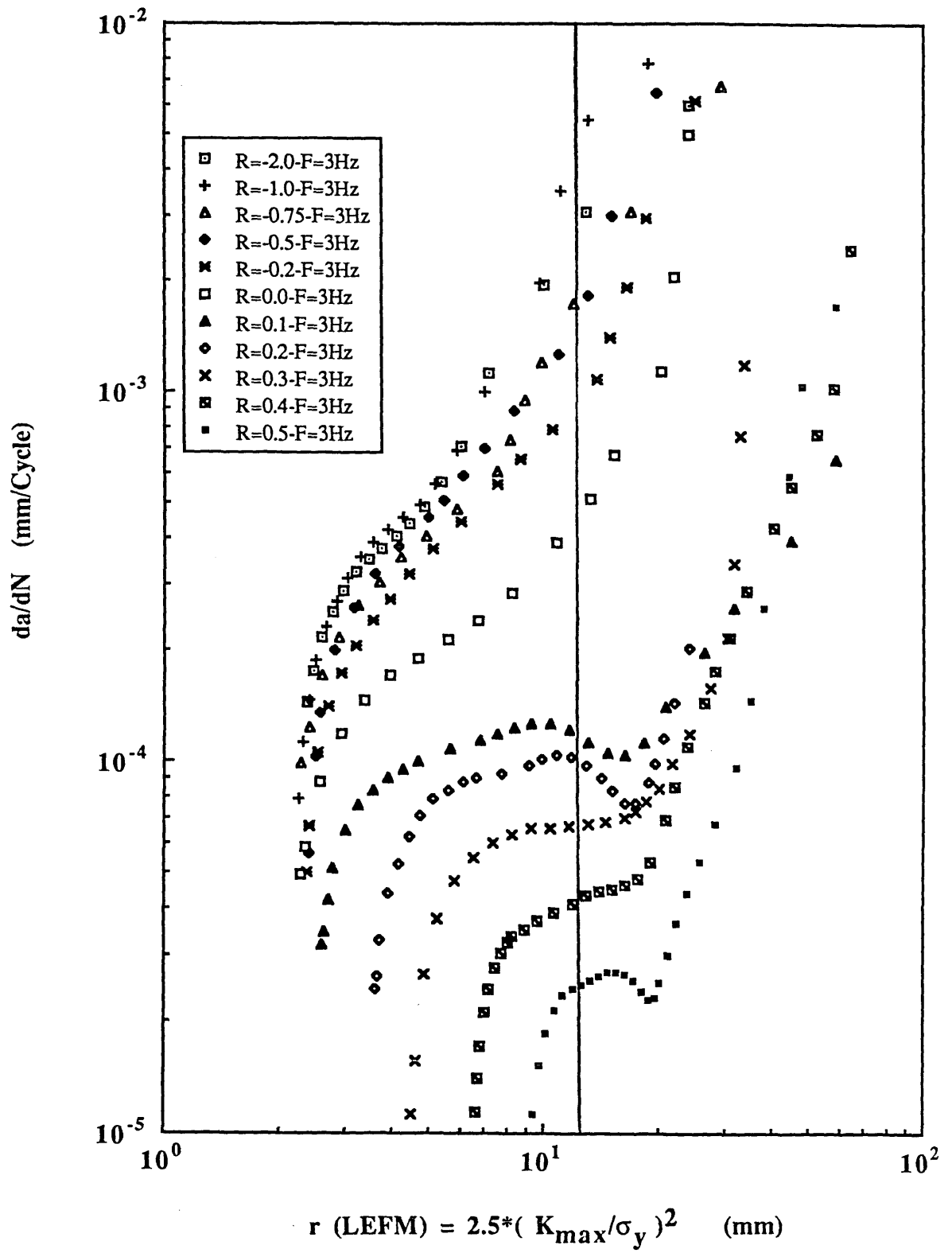


Fig 5.5 FCP rates in BP33 against LEFM plastic zone size criterion.
 $T=+23^{\circ}C$

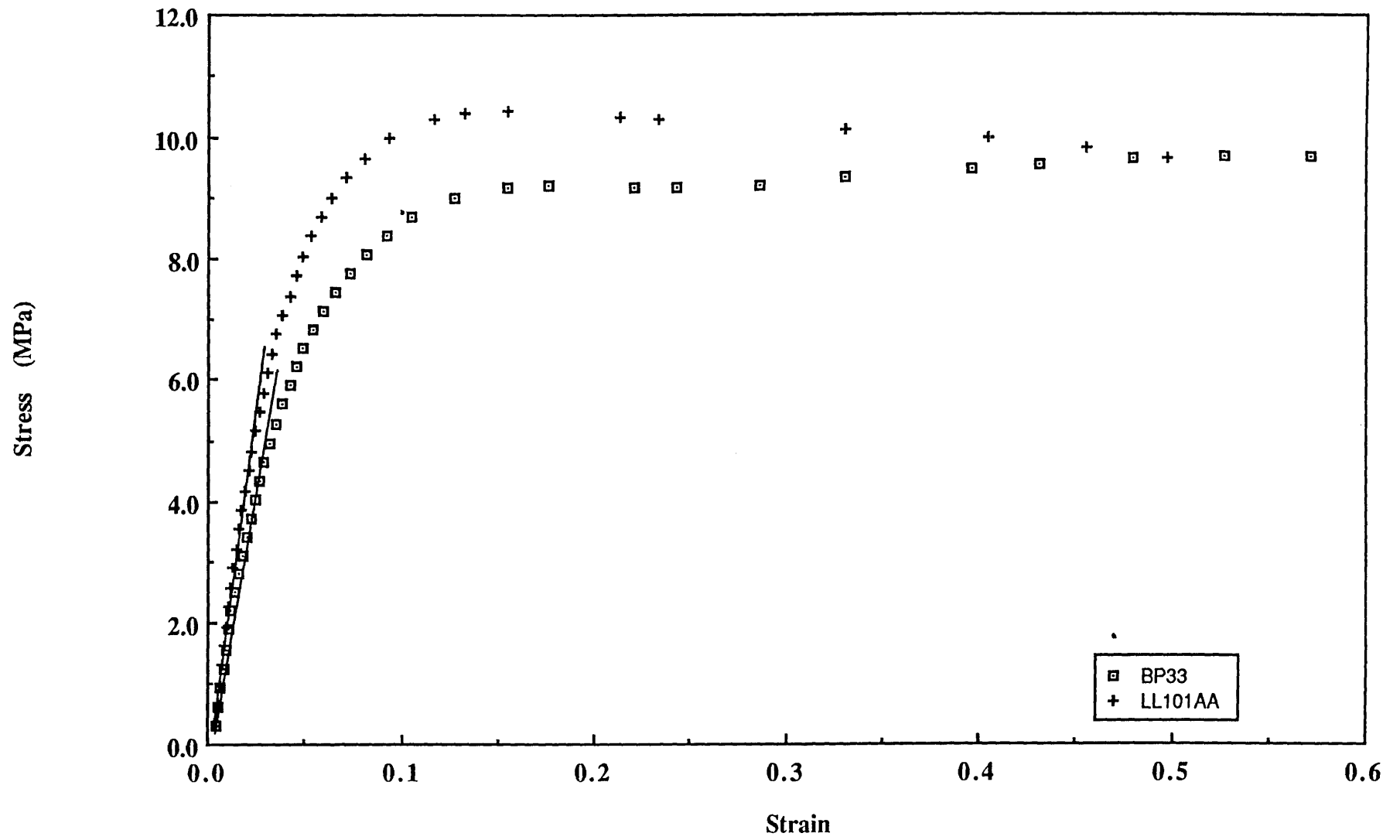


Fig 5.5 (b) Stress against Strain

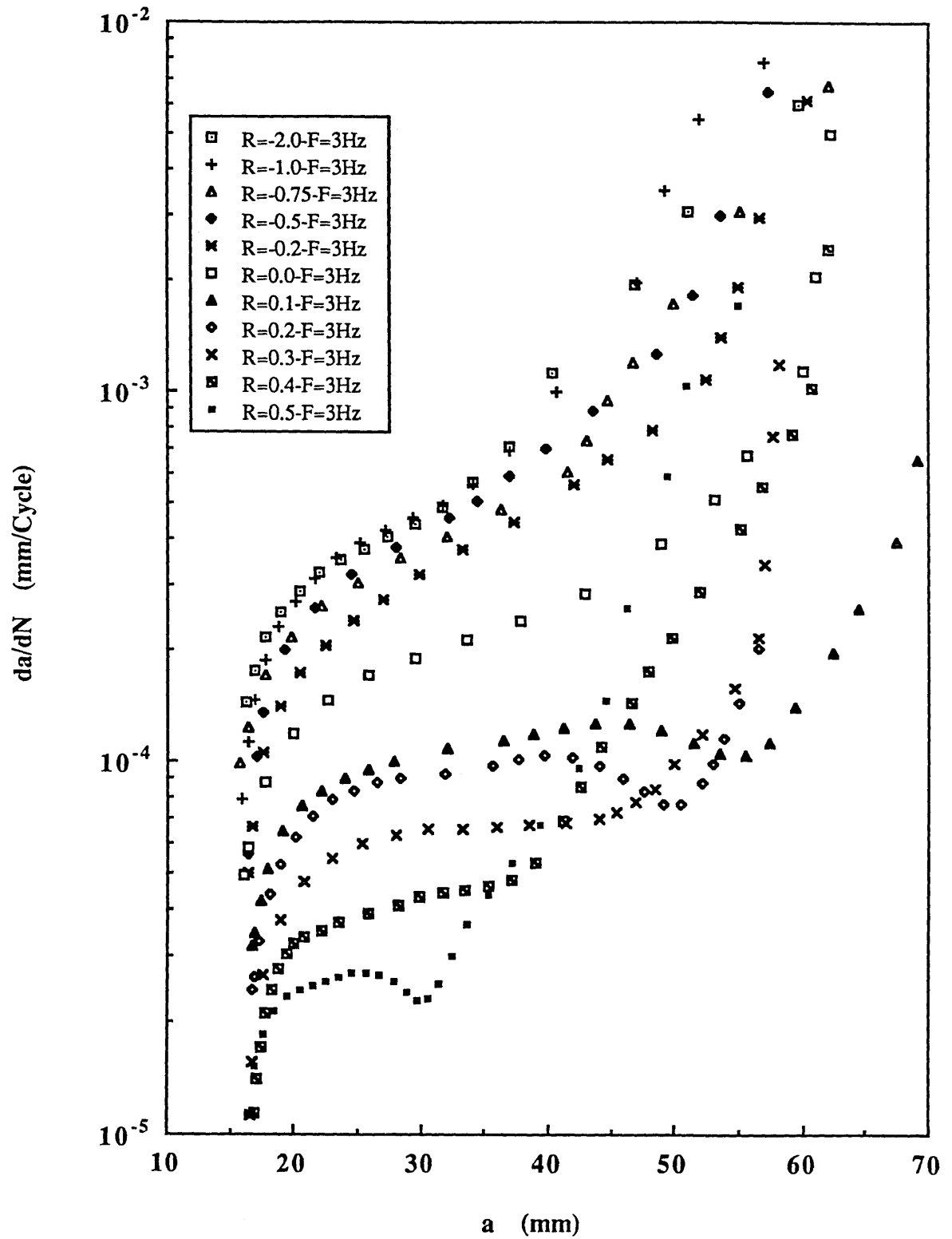


Fig 5.6 FCP rates in BP33 against crack length.
T=+23°C

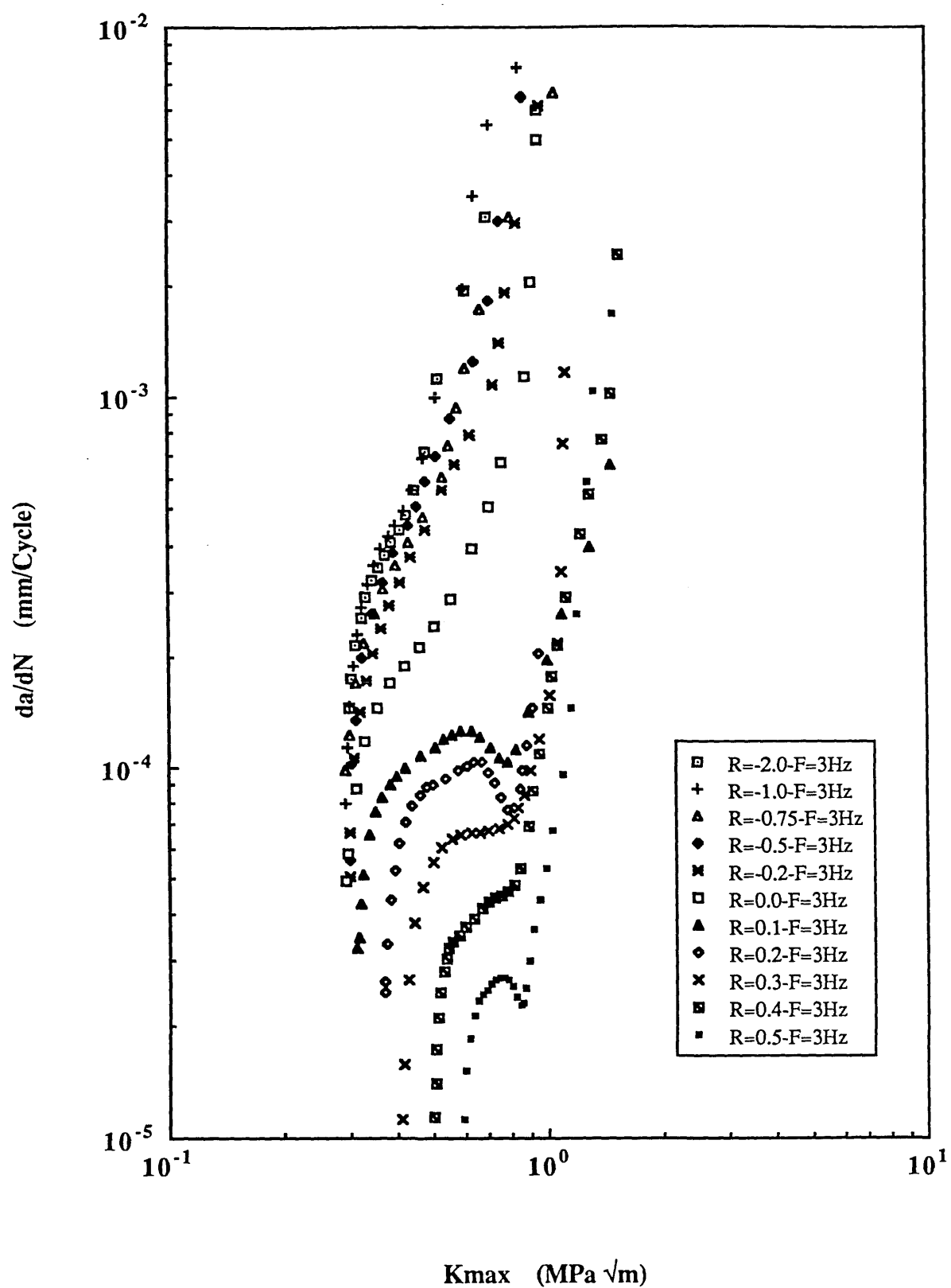


Fig 5.7 FCP rates in BP33 against Kmax.
T=+23°C

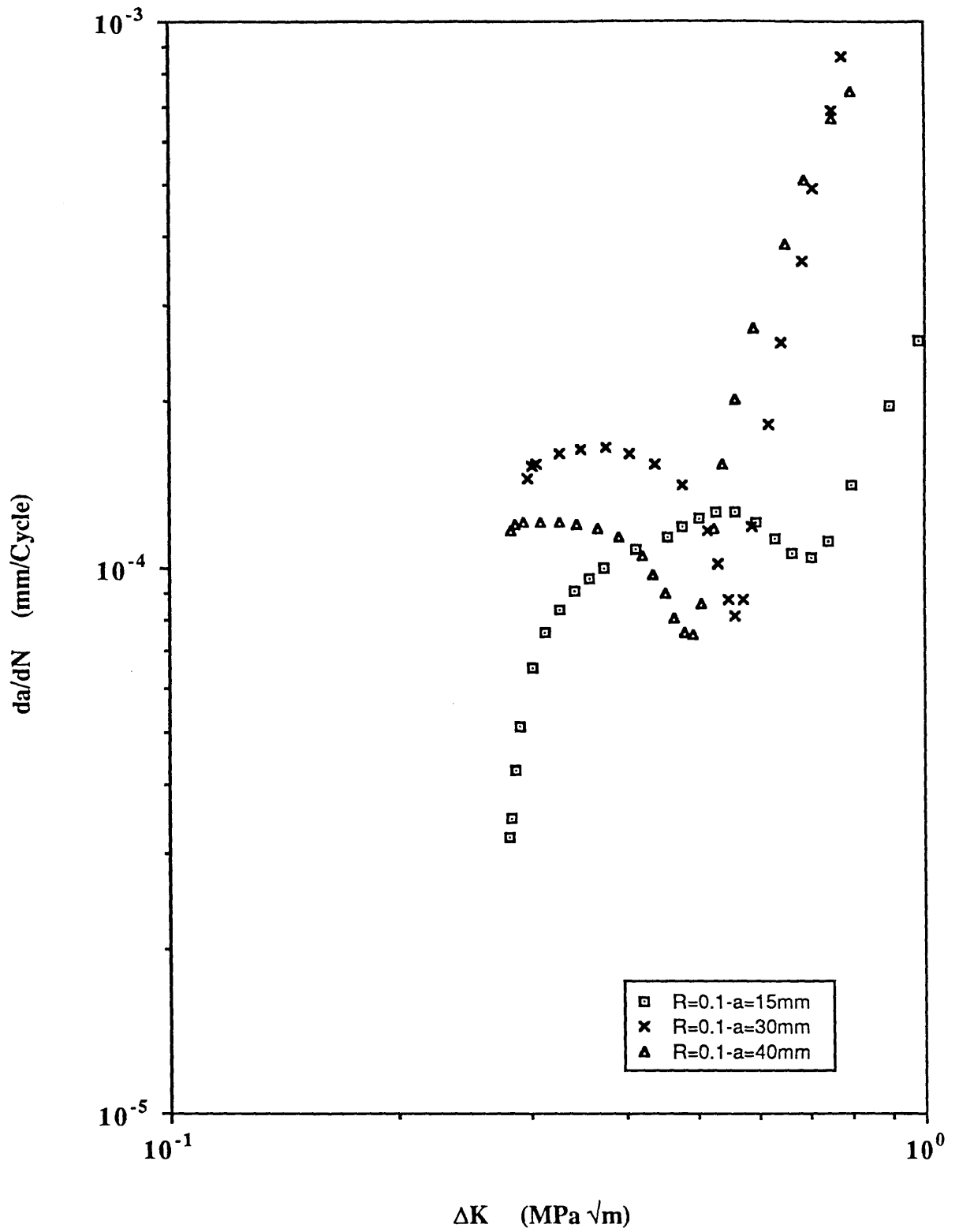


Fig 5.8 Tests carried out in BP33 with different initial crack lengths so as to study Net section stress effects.
 $R=0.1$ - $F=3.0\text{Hz}$ - $T=+23^\circ\text{C}$

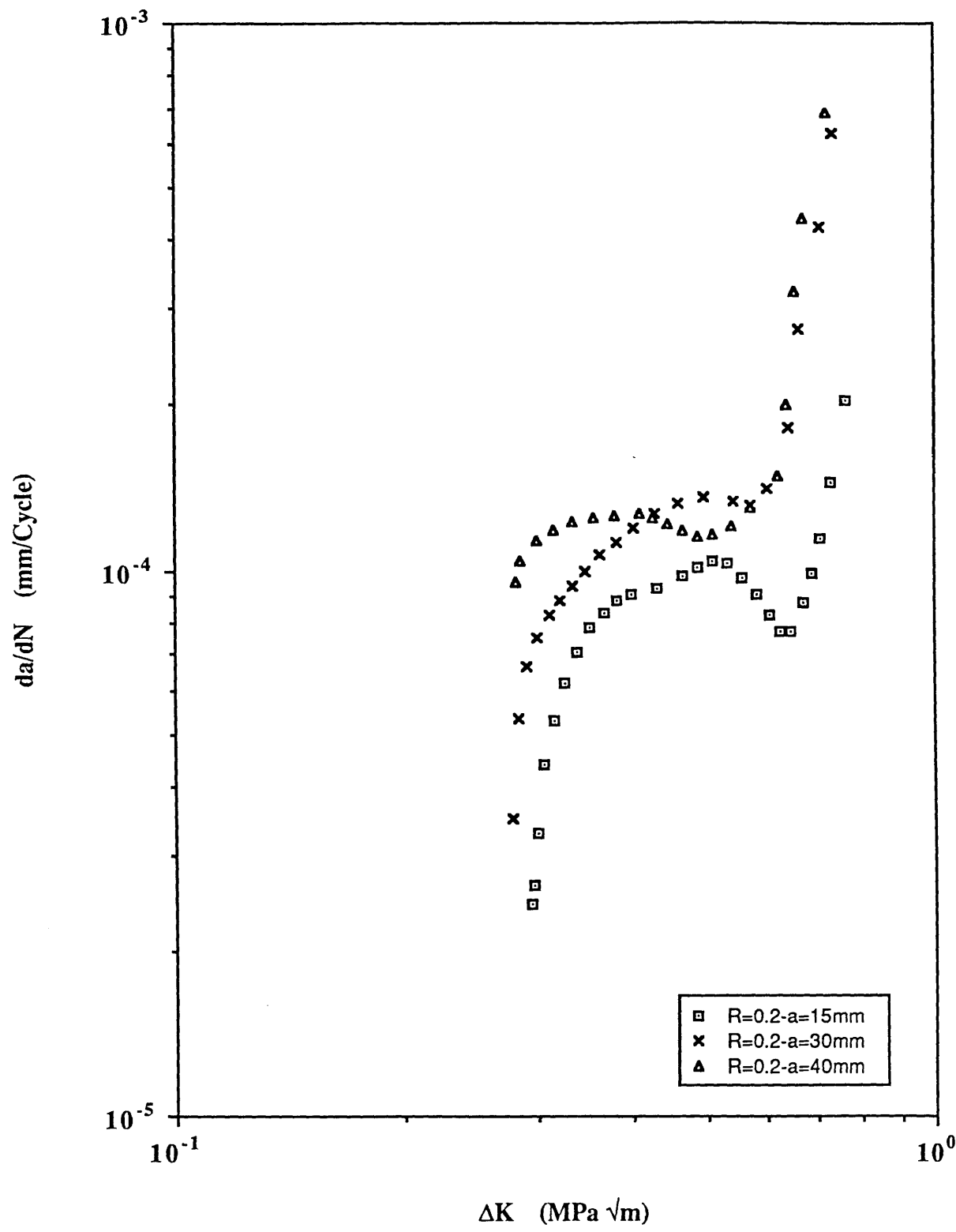


Fig 5.9 Net section stress effects in BP33.
R=0.2-F=3.0Hz-T=+23°C

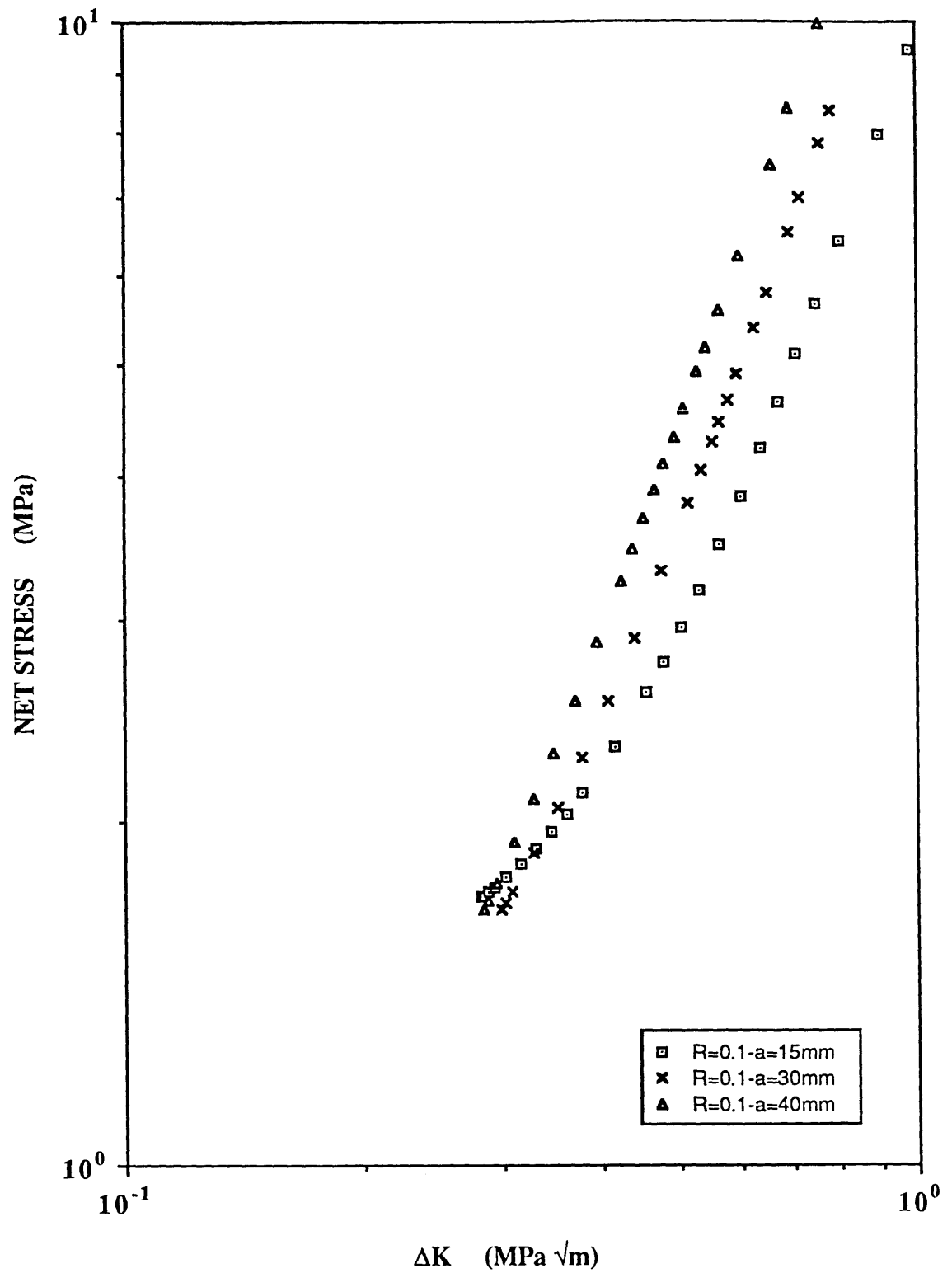


Fig 5.10 Data in Fig 5.8 plotted in the form of Net section stress against ΔK .
 $R=0.1-F=3.0\text{Hz}-T=+23^\circ\text{C}$

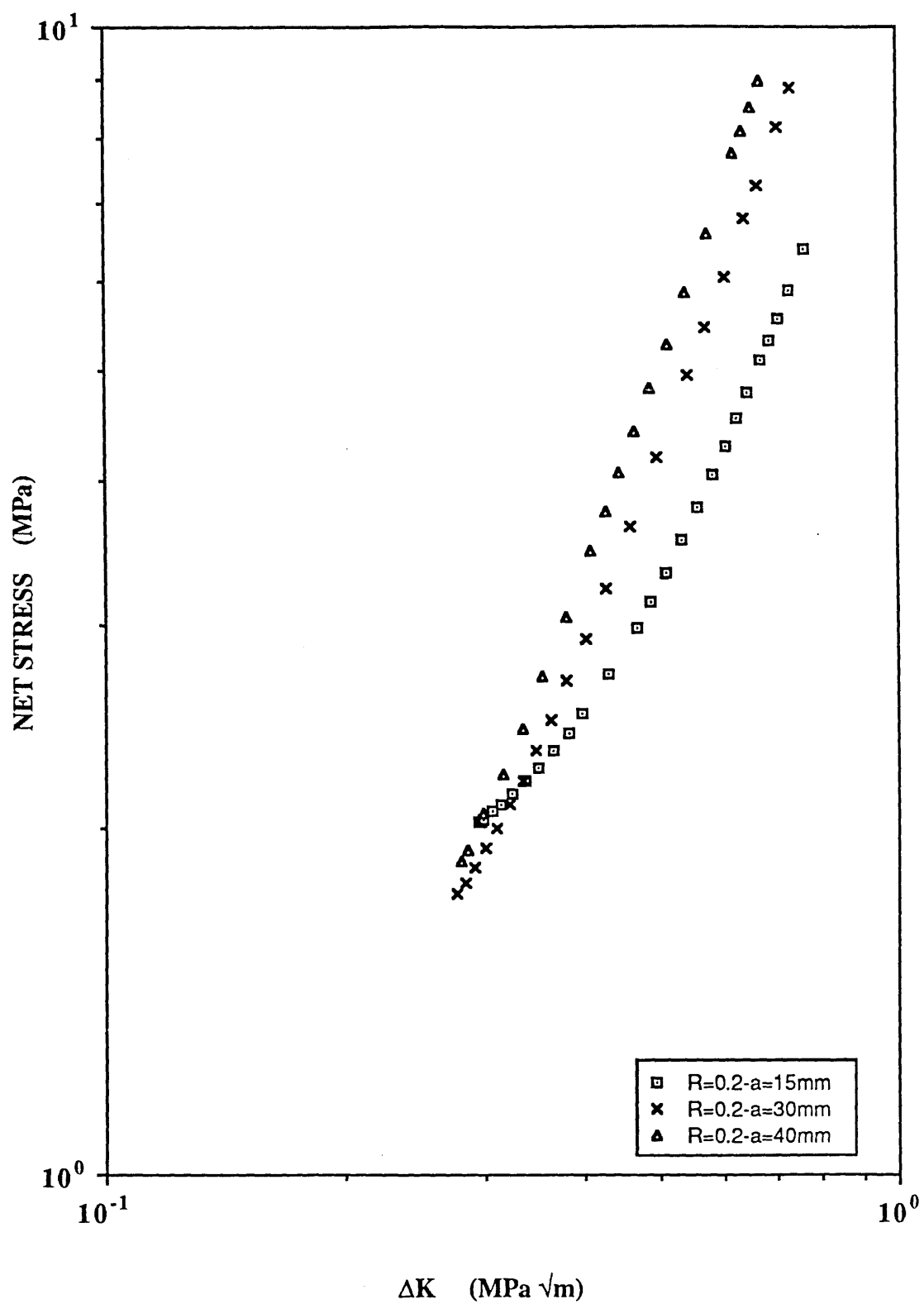


Fig 5.11 Data in Fig 5.9 plotted in the form of Net section stress against ΔK .
 $R=0.2-F=3.0\text{Hz}-T=+23^\circ\text{C}$

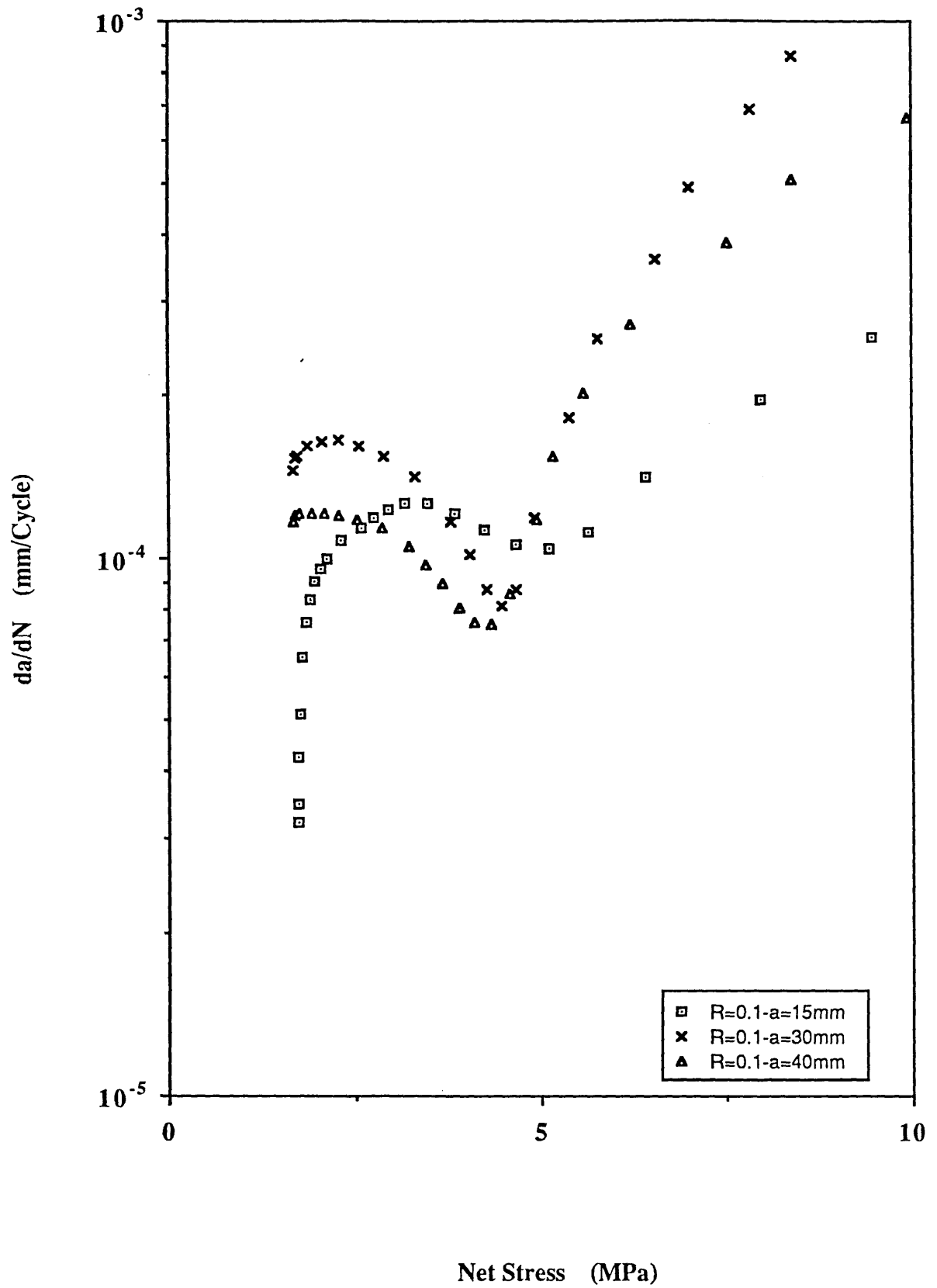


Fig 5.12 FCP rates in BP33 plotted against Net section stress from Data in Fig 5.8.
 $R=0.1-F=3.0\text{Hz}-T=+23^\circ\text{C}$

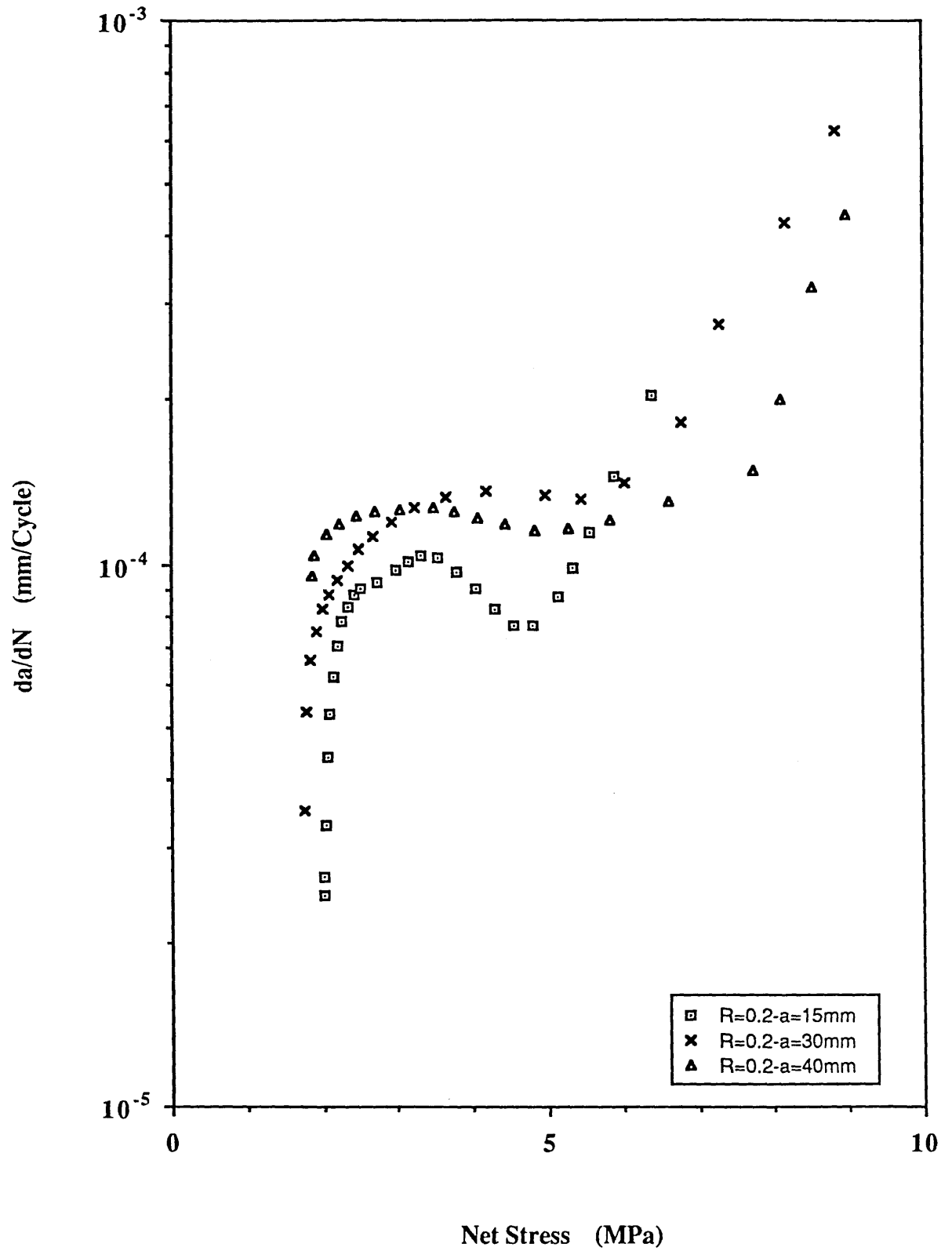


Fig 5.13 FCP rates in BP33 plotted against Net section stress from Data in Fig 5.9.
R=0.2-F=3.0Hz-T=+23°C

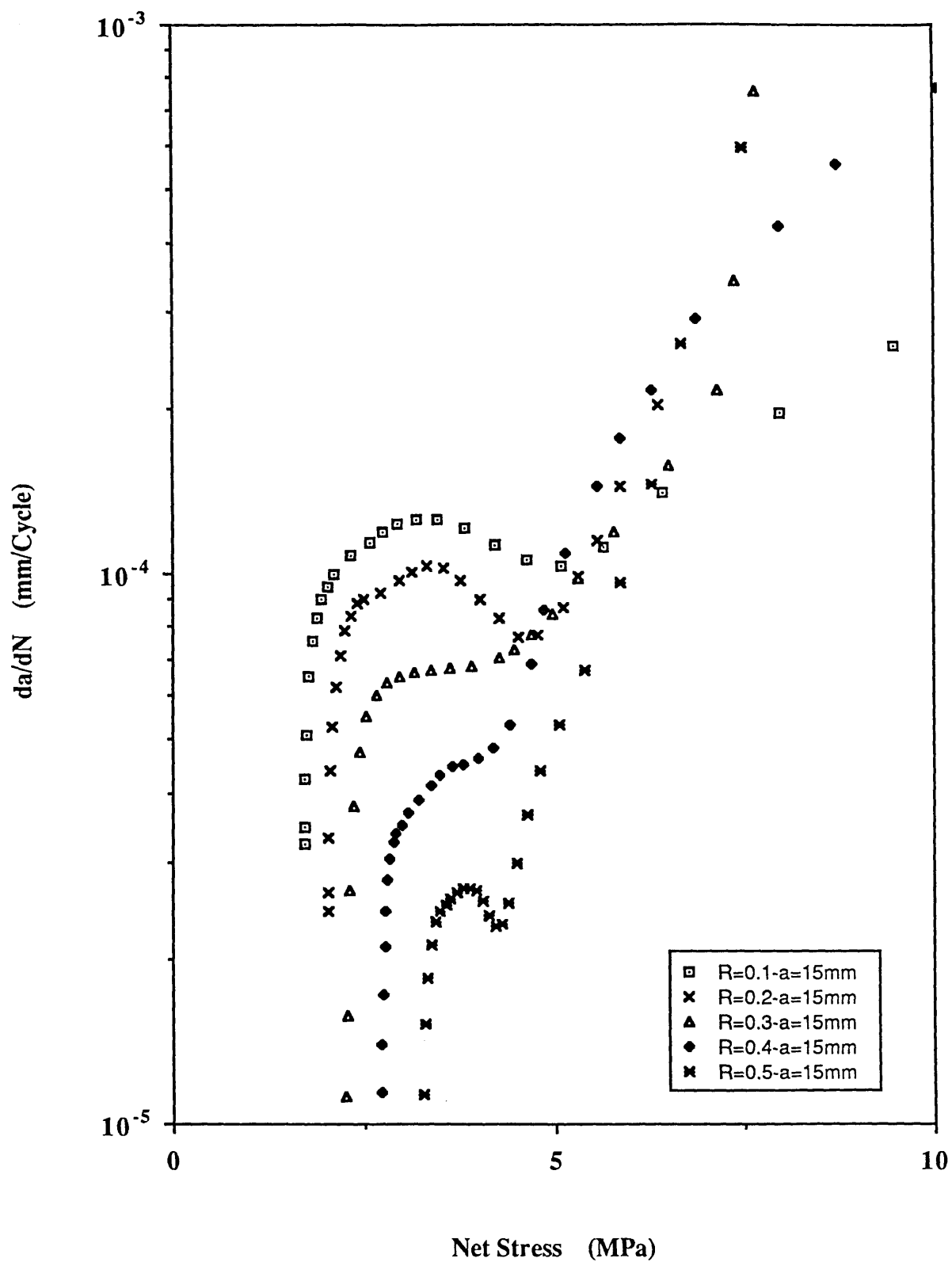


Fig 5.14 Data in Fig 5.4 plotted in the form of FCP rates against Net section stress. $F=3.0\text{Hz}$ - $T=+23^\circ\text{C}$ at various R-ratios

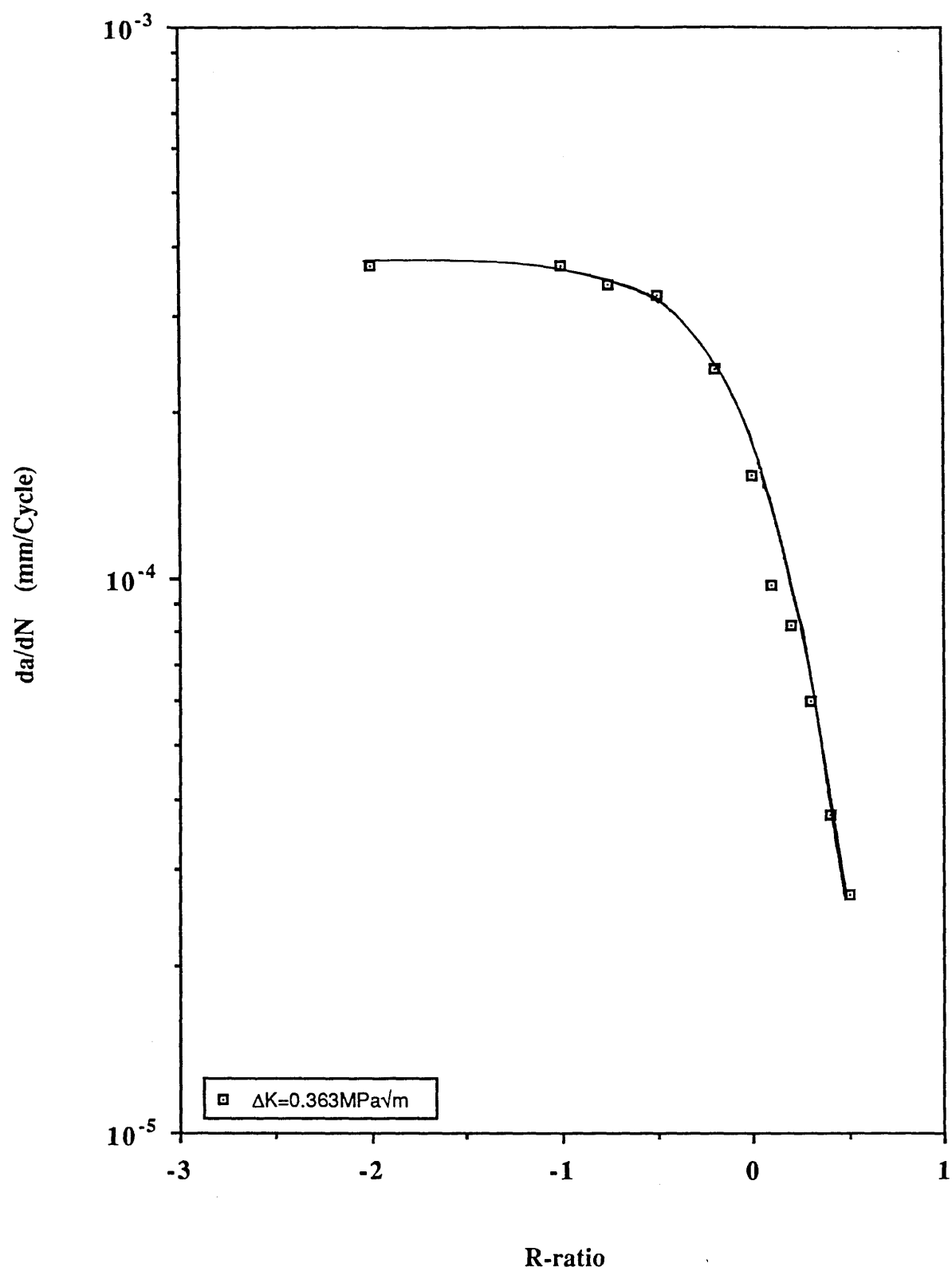


Fig 5.15 FCP rates plotted against R-ratio at a constant ΔK value of $0.363 \text{ MPa}\sqrt{\text{m}}$.
 $F=3.0\text{Hz}$ - $T=+23^\circ\text{C}$

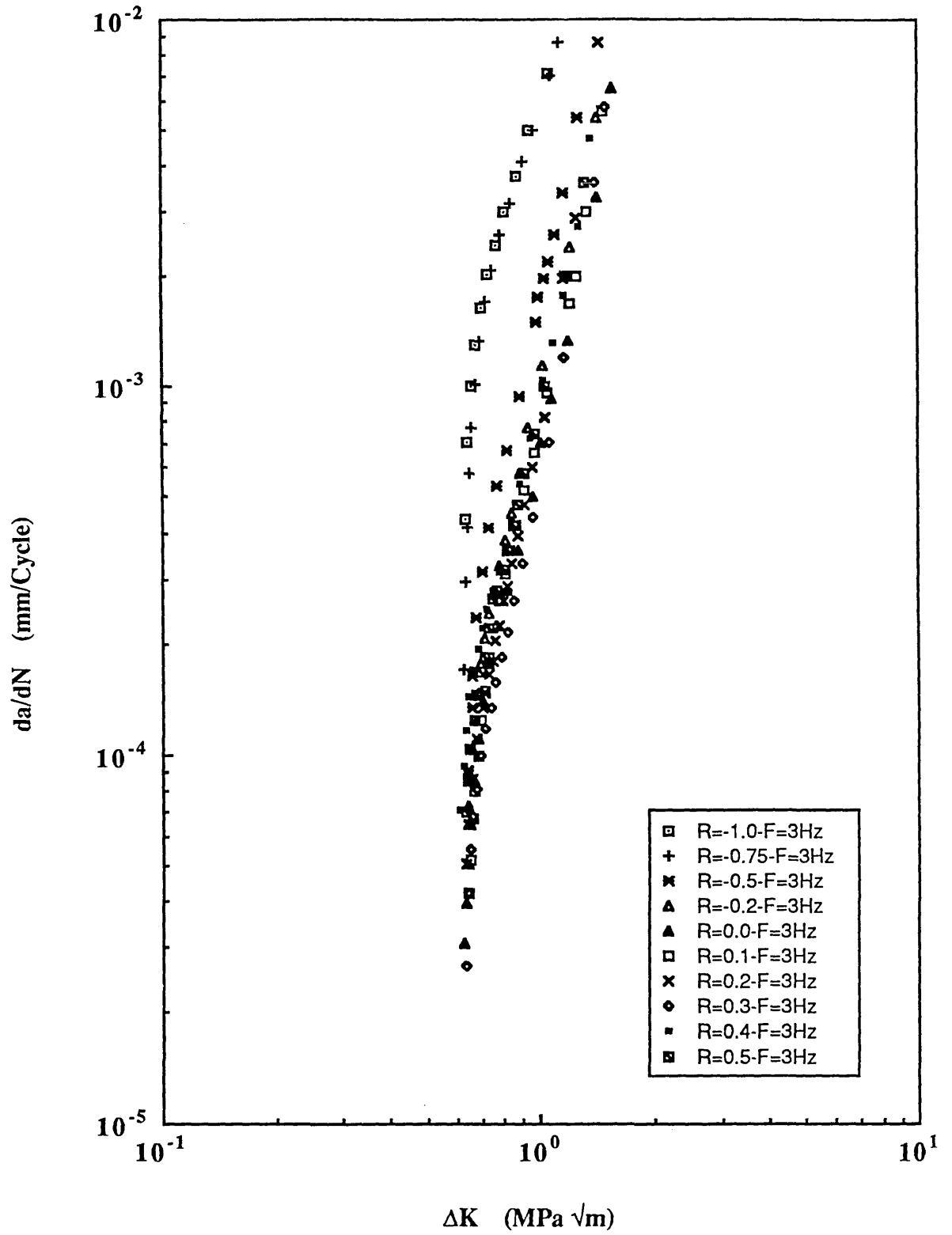
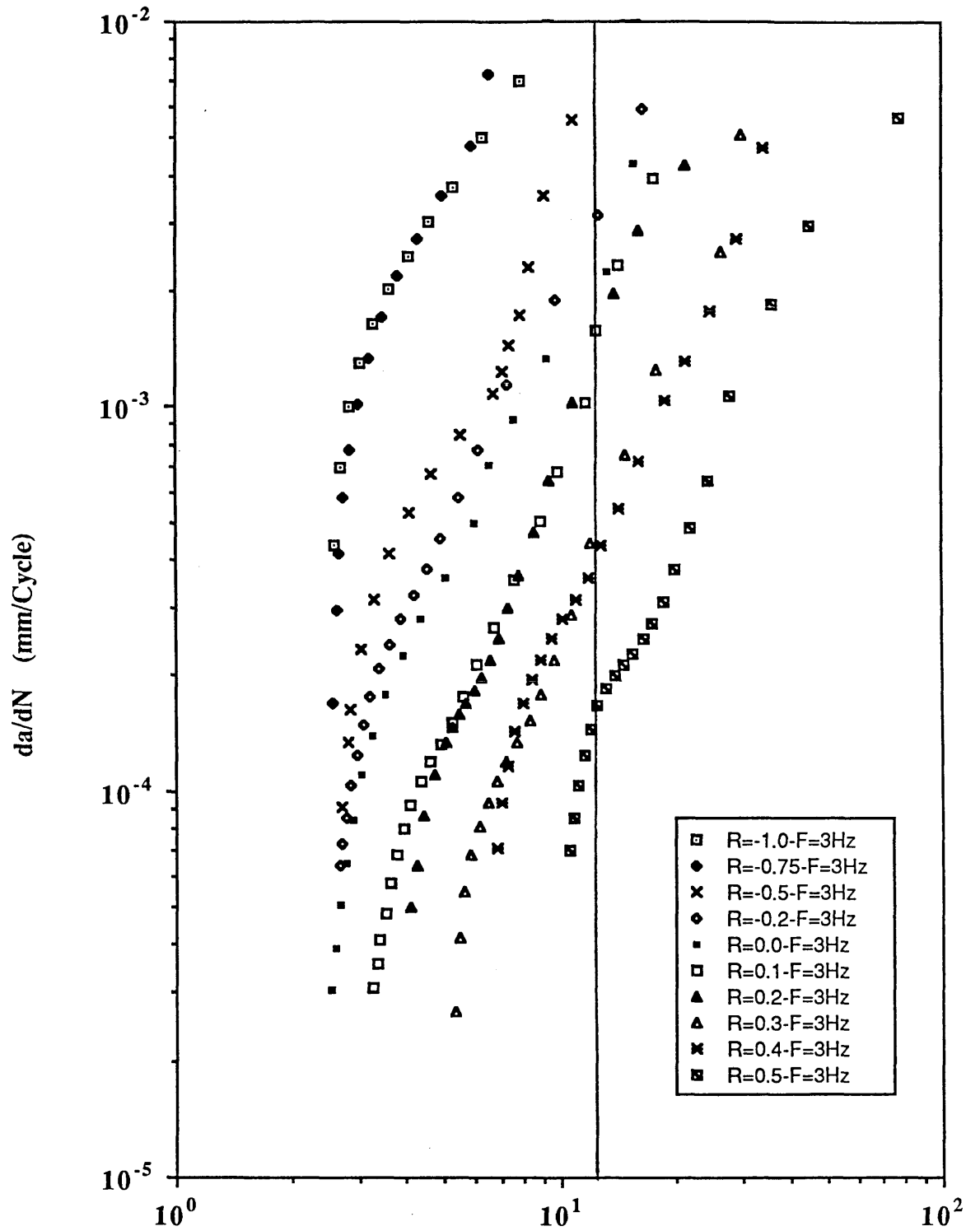


Fig 5.16 Fatigue crack propagation rates in BP33 at various R-ratios.
T=-20°C



$$r \text{ (LEFM)} = 2.5 * (K_{\max} / \sigma_y)^2 \quad (\text{mm})$$

Fig 5.17 FCP rates in BP33 against LEFM plastic zone size criterion.
T=-20°C

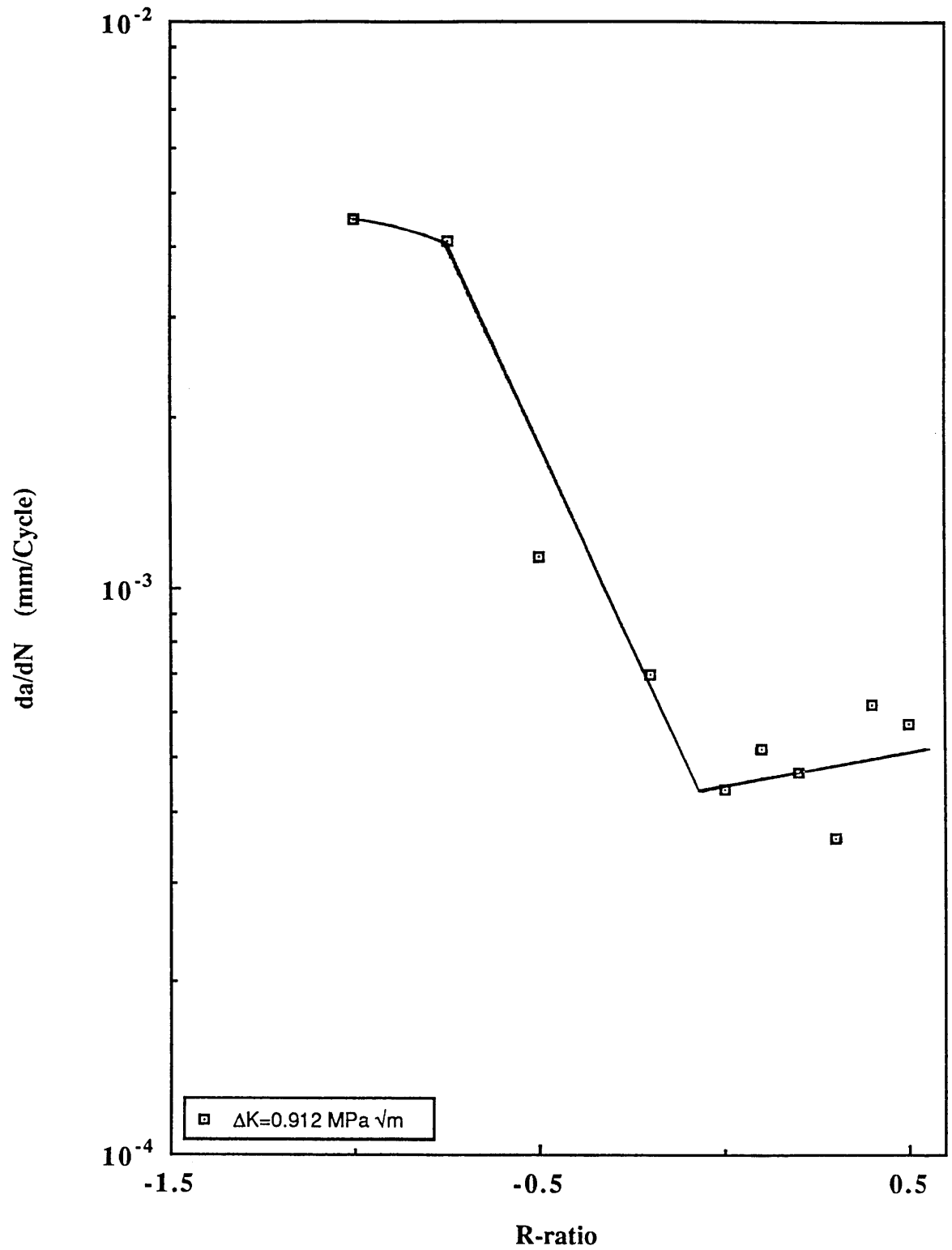


Fig 5.18 FCP rates plotted against R-ratio at a constant ΔK value of $0.912 \text{ MPa}\sqrt{\text{m}}$.
 $F=3.0\text{Hz}$ - $T=-20^\circ\text{C}$

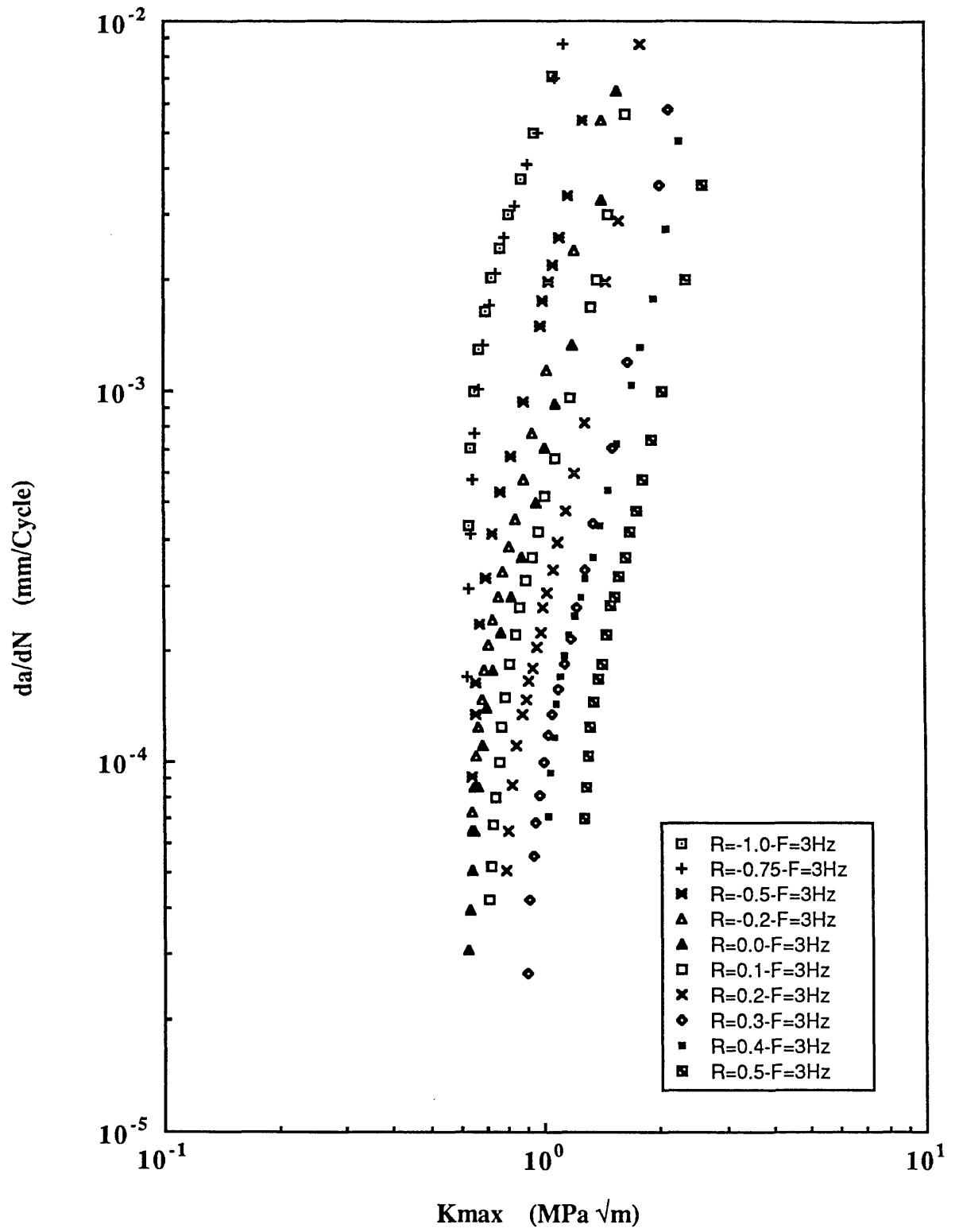


Fig 5.19 FCP rates in BP33 against Kmax.
T=-20°C

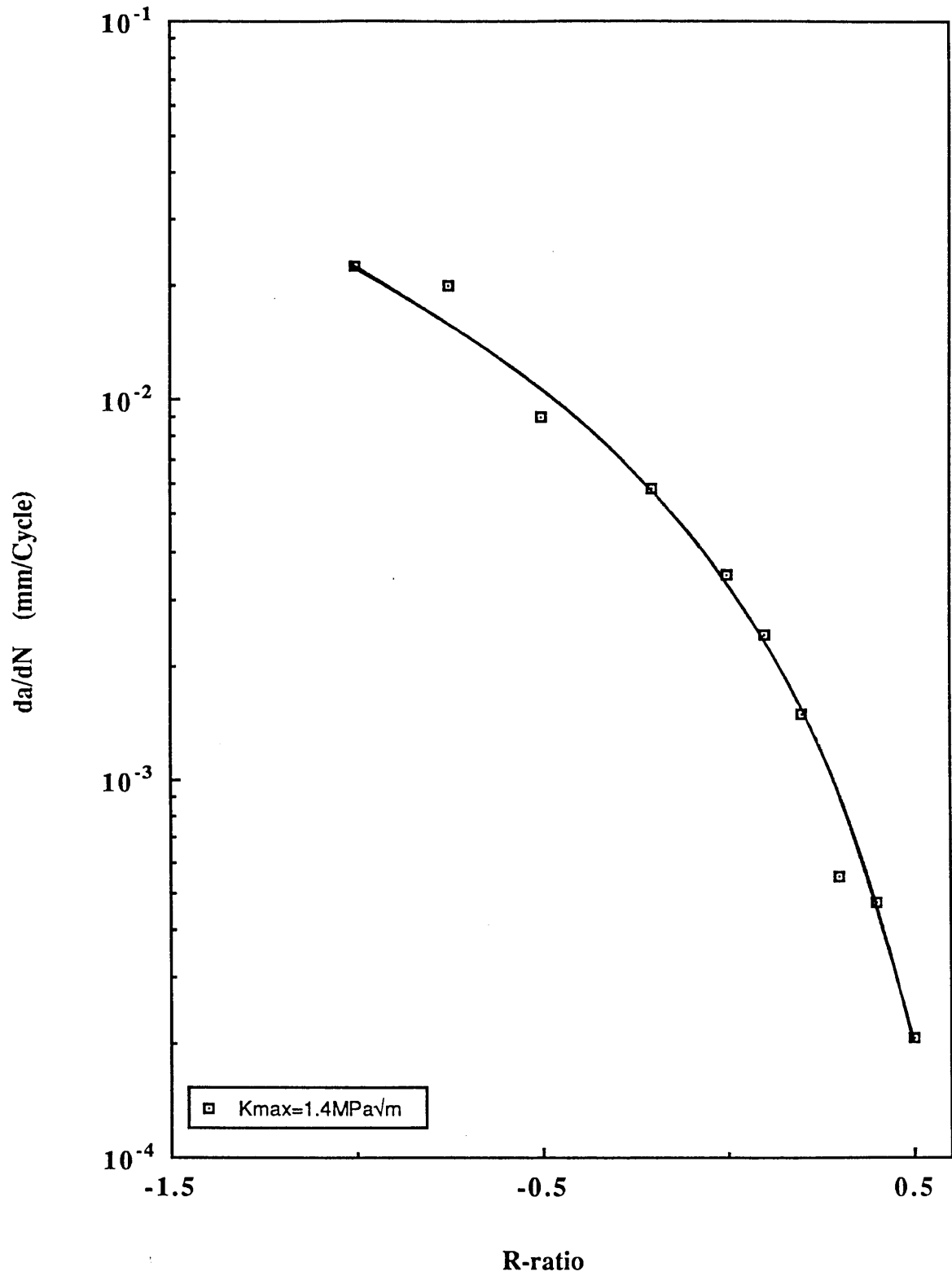


Fig 5.20 FCP rates plotted against R-ratio at a constant K_{max} value of $1.40 MPa \sqrt{m}$.
 $F=3.0Hz$ - $T=-20^{\circ}C$

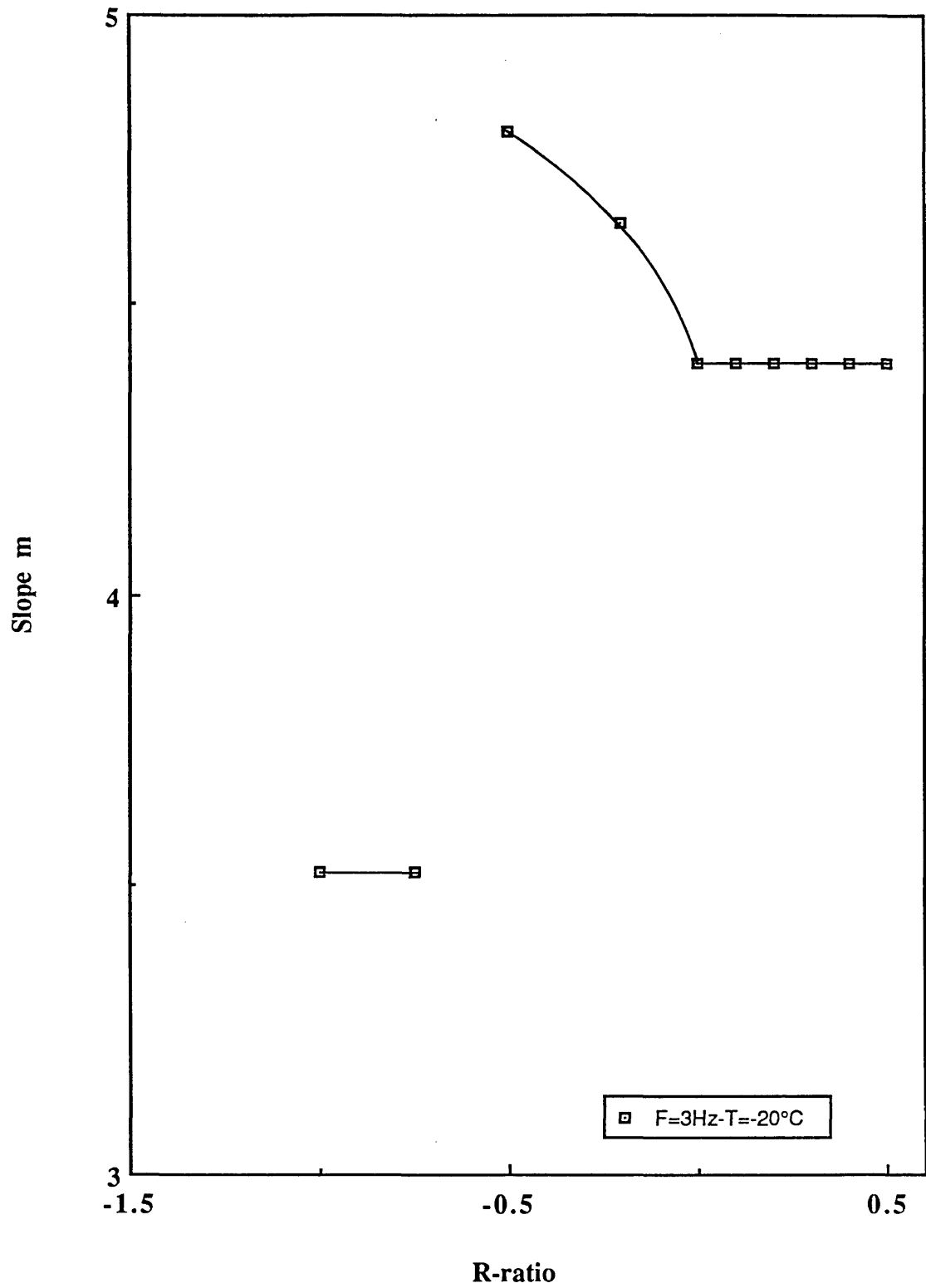


Fig 5.21 The slopes of Paris plots in BP33 against R-ratio.
 $F=3.0\text{Hz}$ - $T=-20^{\circ}\text{C}$

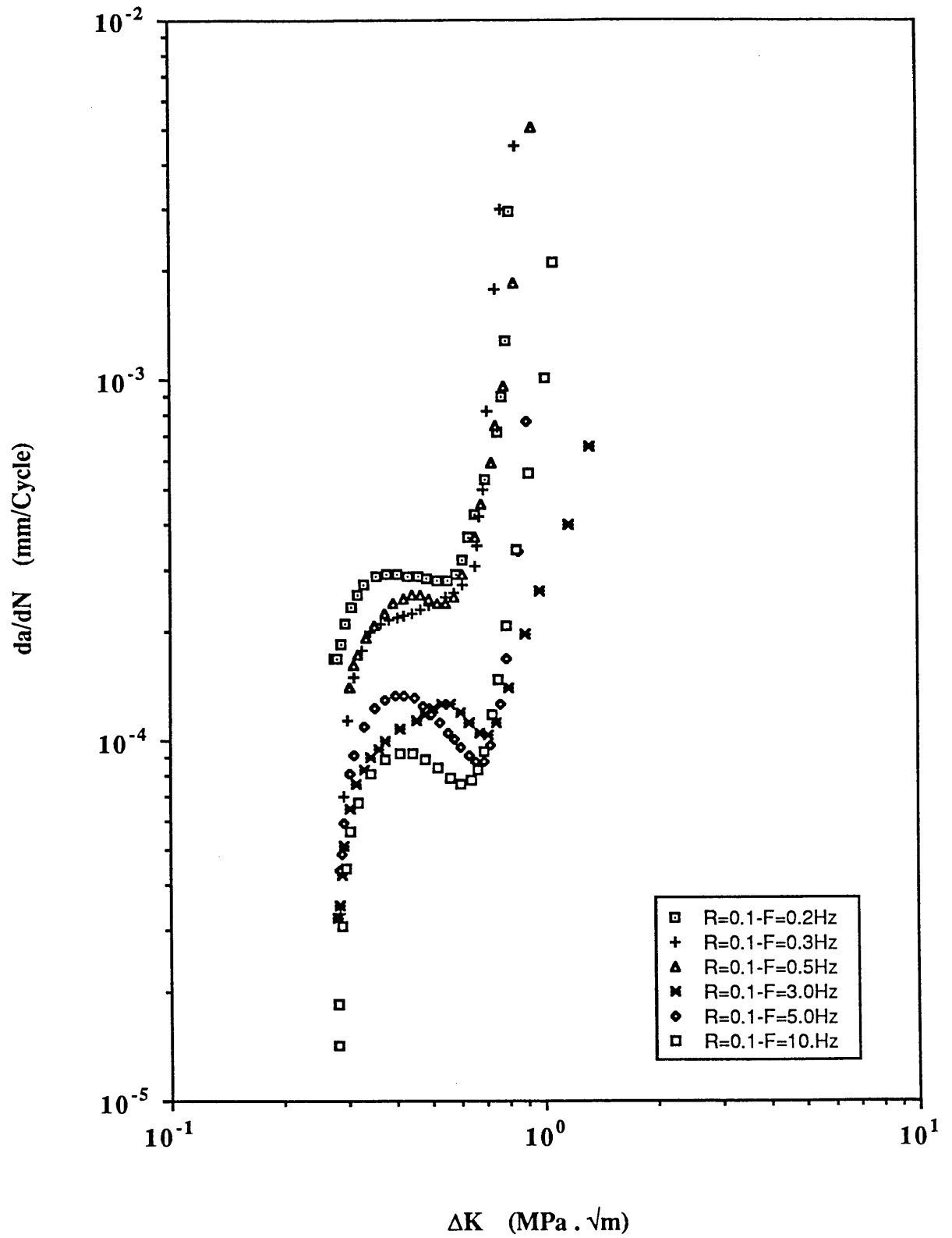


Fig 5.22 Fatigue crack propagation rates in BP33 at 0.2, 0.3, 0.5, 3.0, 5.0 and 10.0Hz.
R-ratio=0.1 - T=+23°C

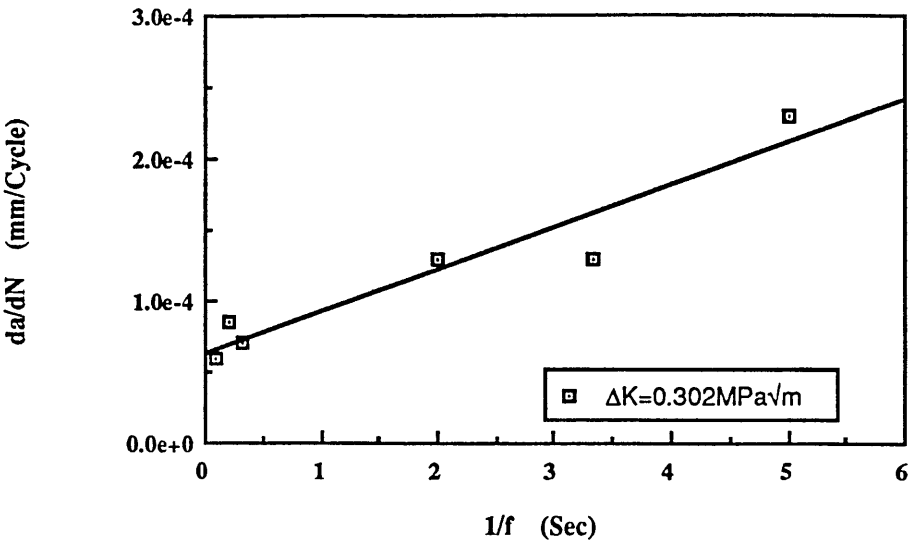


Fig 5.23 (a)

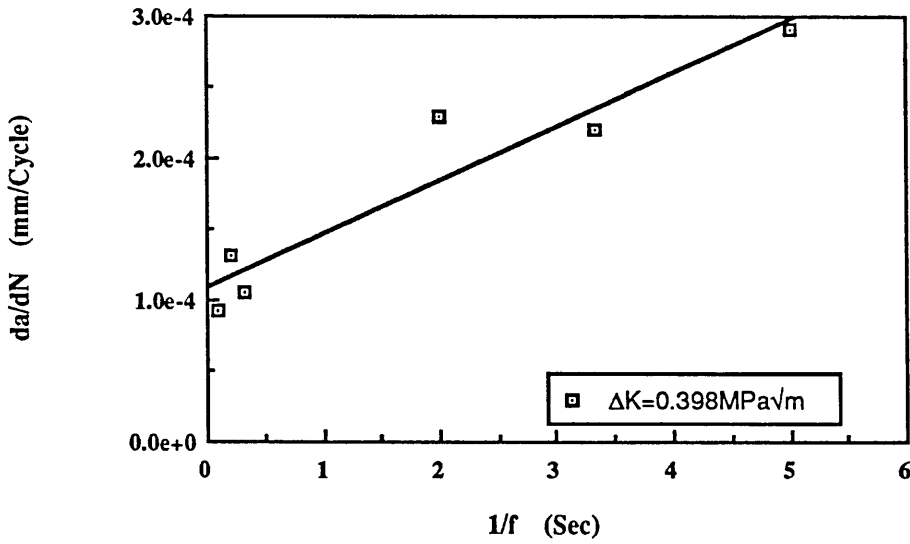


Fig 5.23 (b)

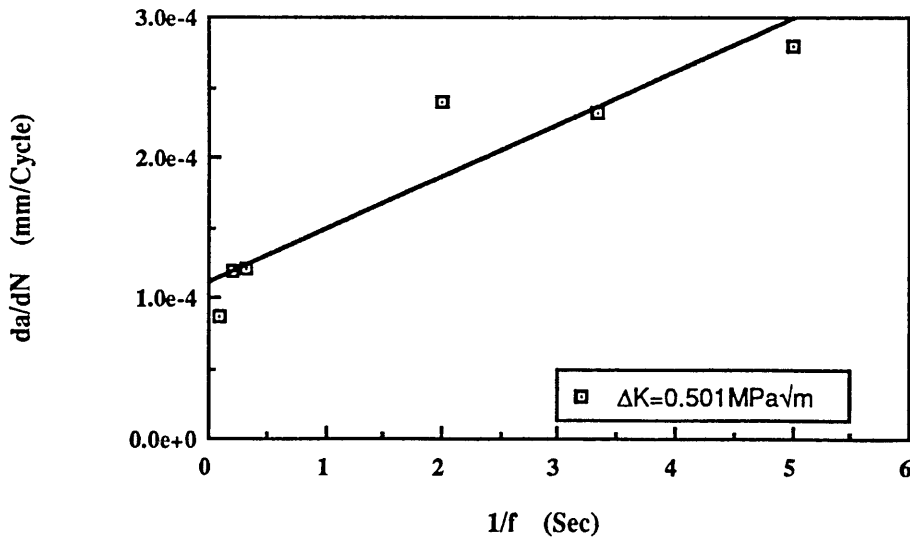


Fig 5.23 (c)

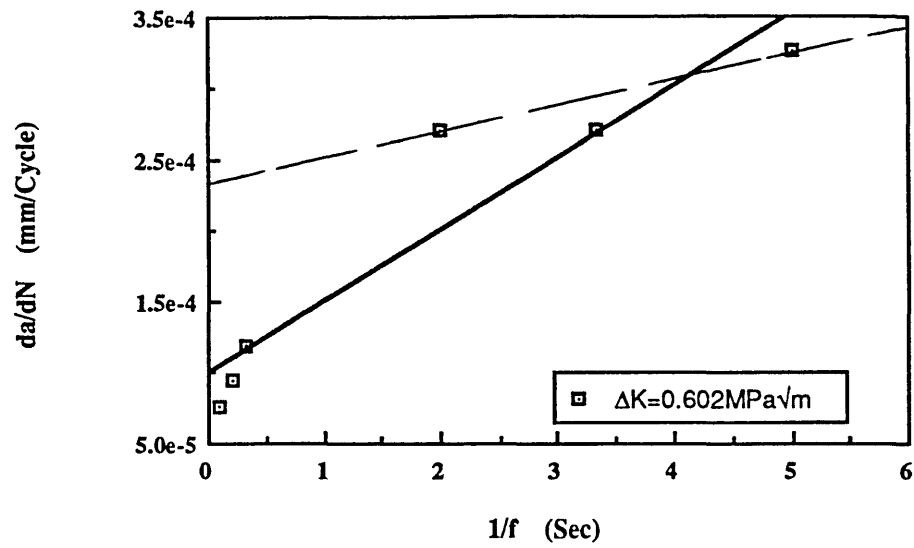


Fig 5.23 (d)

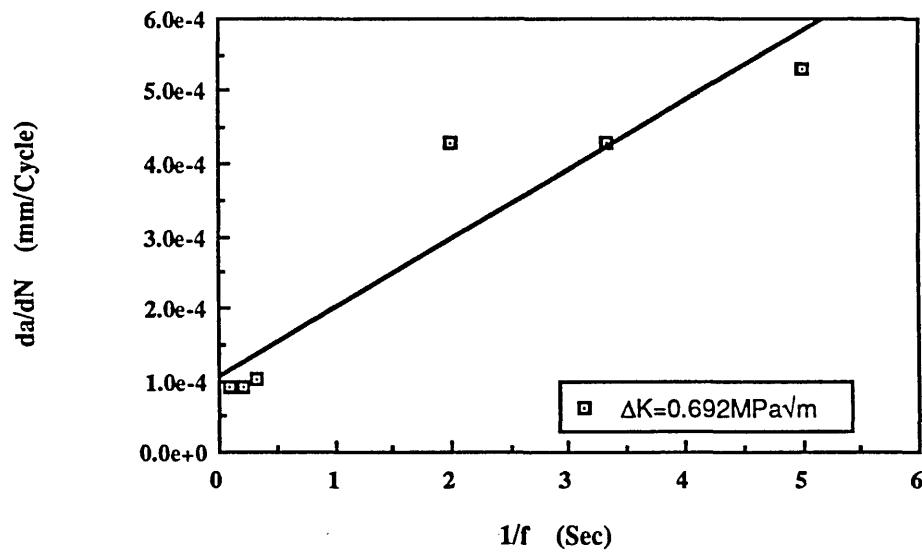


Fig 5.23 (e)

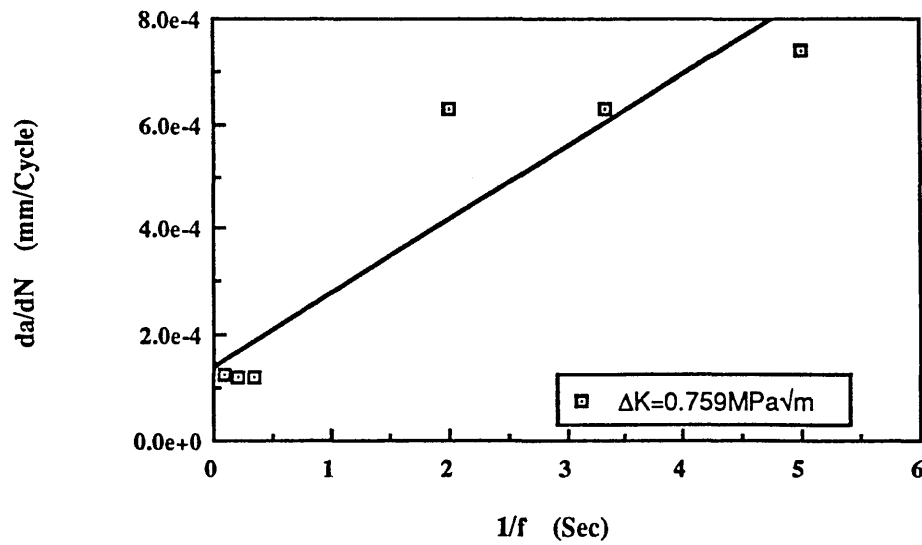


Fig 5.23 (f)

Fig 5.23(a-f) FCP rates in BP33 against reciprocal frequency at various values of ΔK .
R-ratio=0.1-T=+23°C

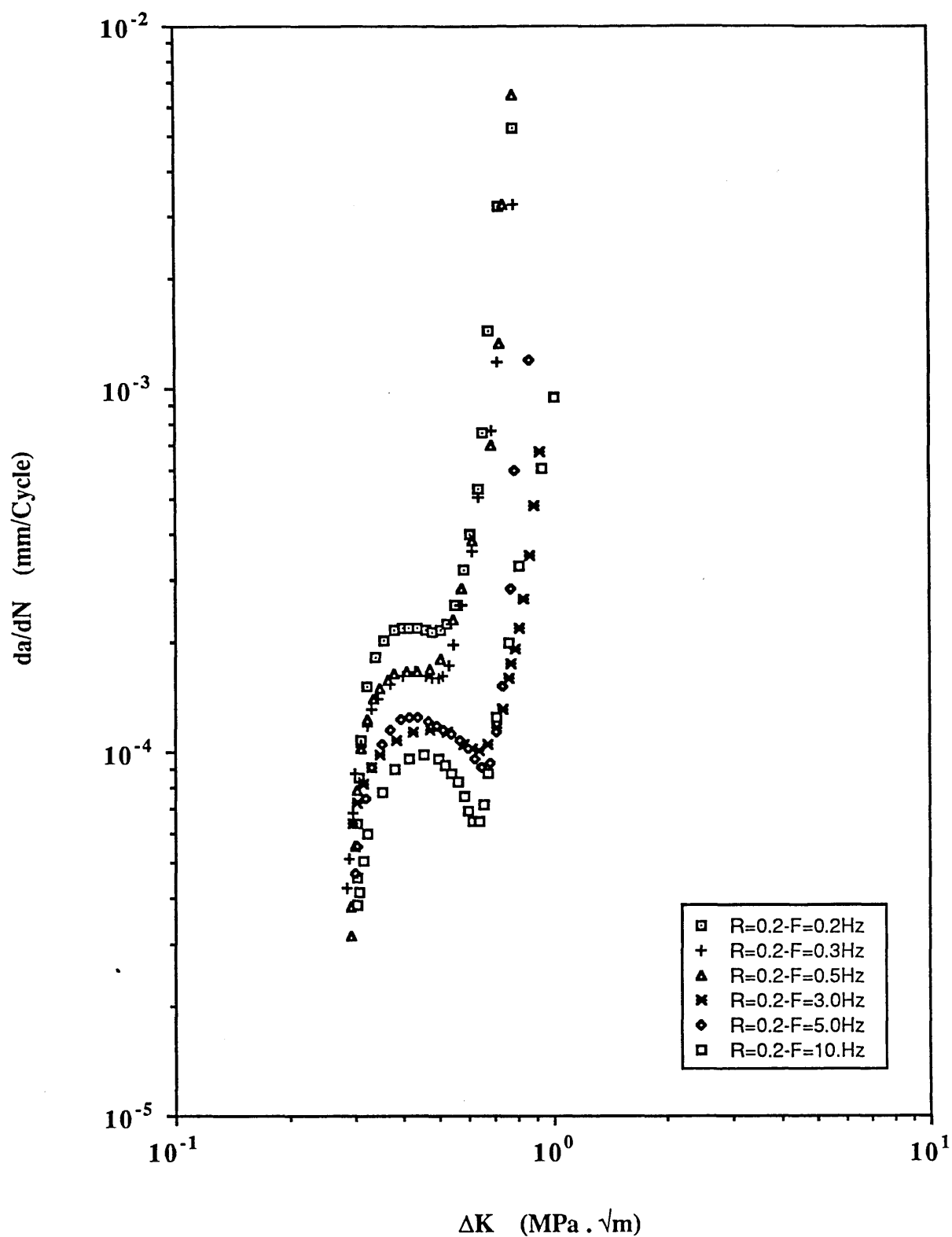


Fig 5.24 Fatigue crack propagation rates in BP33 at 0.2, 0.3, 0.5, 3.0, 5.0 and 10.0Hz.
R-ratio=0.2 - T=+23°C

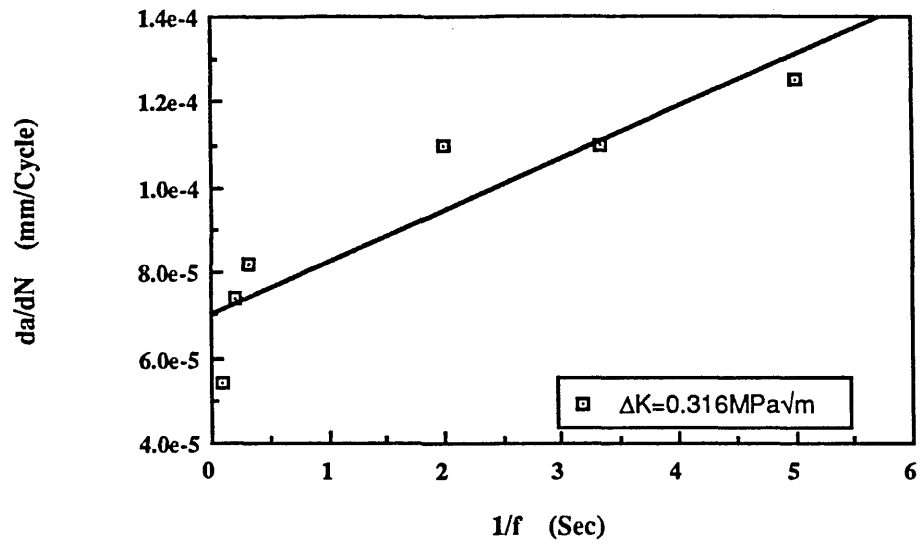


Fig 5.25 (a)

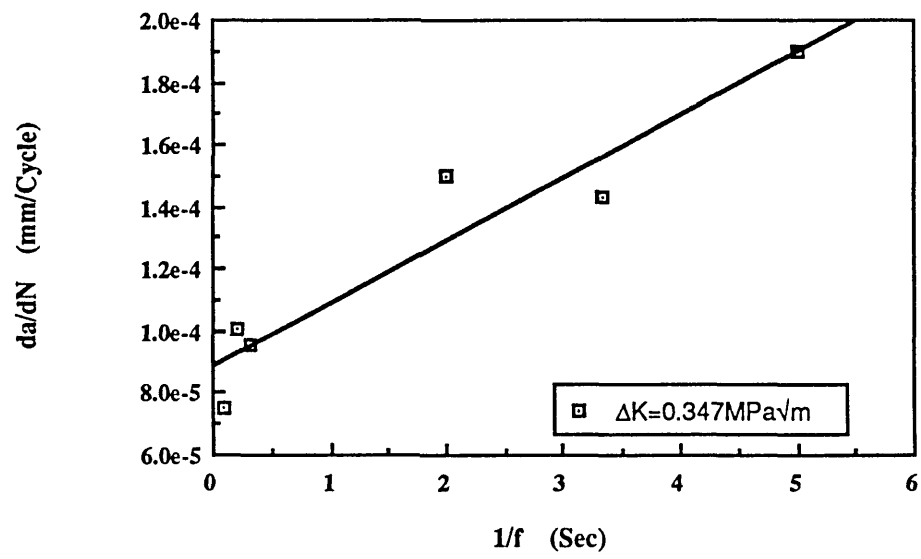


Fig 5.25 (b)

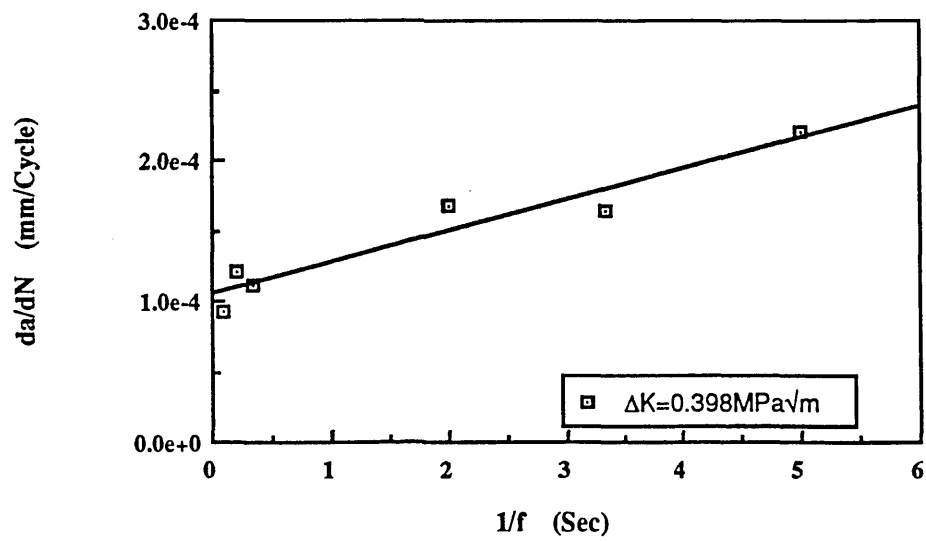


Fig 5.25 (c)

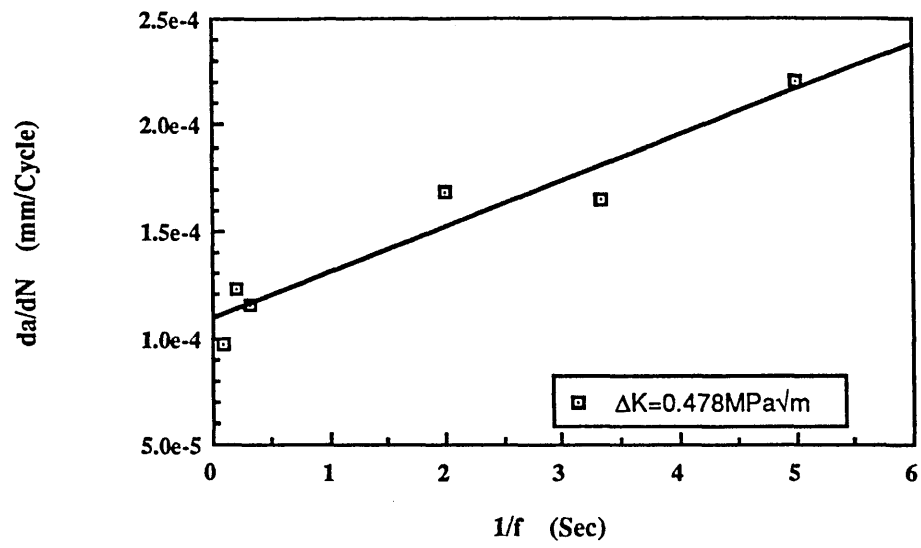


Fig 5.25 (d)

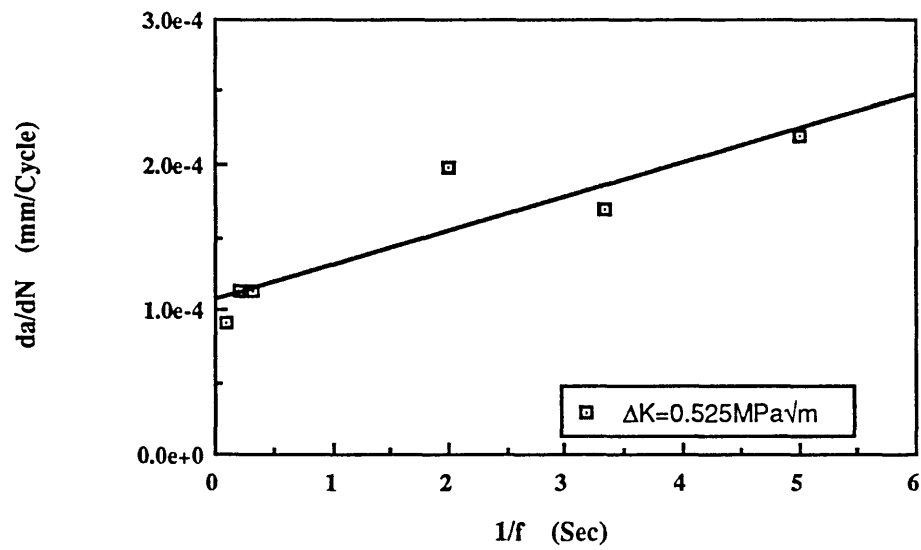


Fig 5.25 (e)

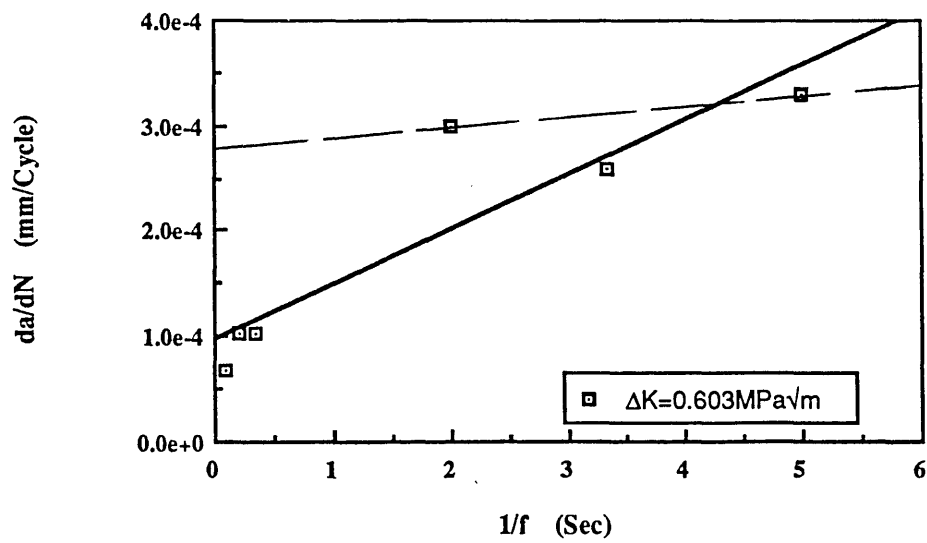


Fig 5.25 (f)

Fig 5.25(a-f) FCP rates in BP33 against reciprocal frequency at various values of ΔK .
 R-ratio=0.2-T=+23°C

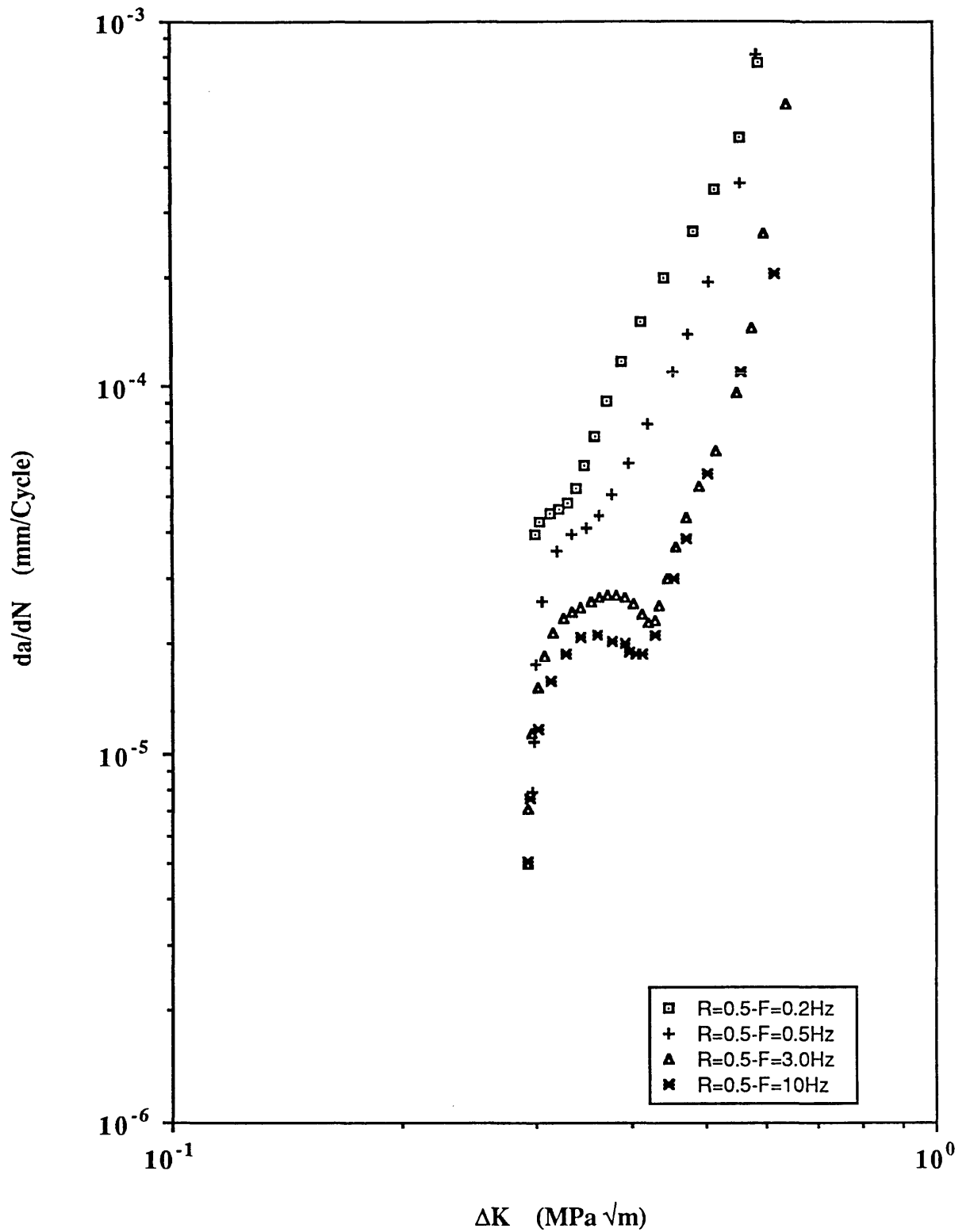


Fig 5.26 Fatigue crack propagation rates in BP33 at 0.2, 0.5, 3.0 and 10.0Hz.
R-ratio=0.5 - T=+23°C

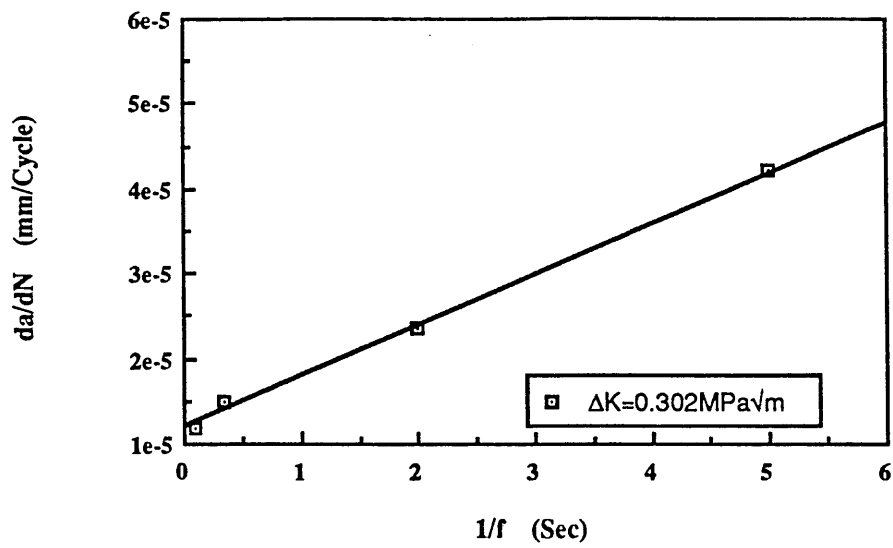


Fig 5.27 (a)

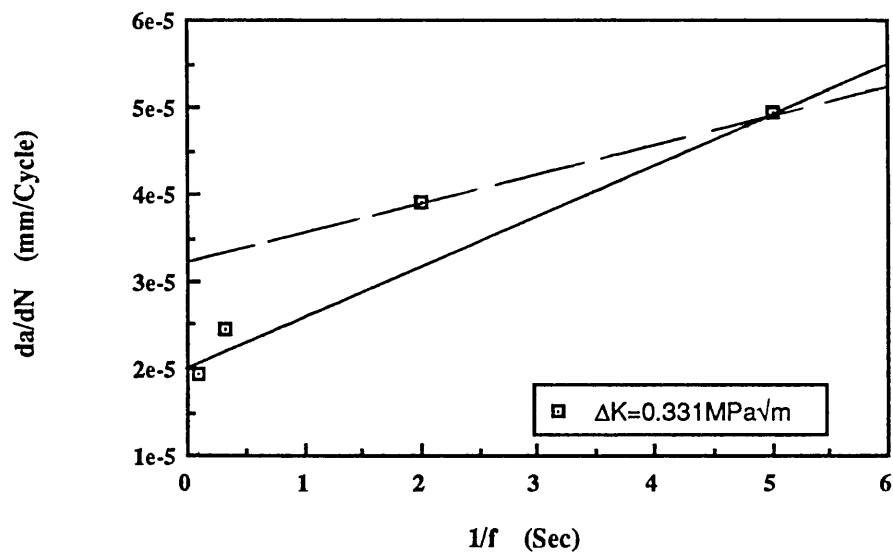


Fig 5.27 (b)

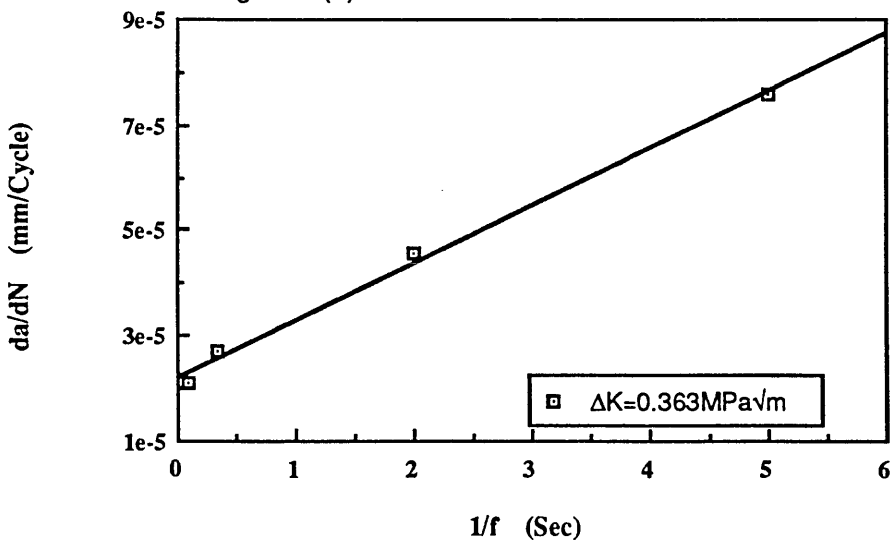


Fig 5.27 (c)

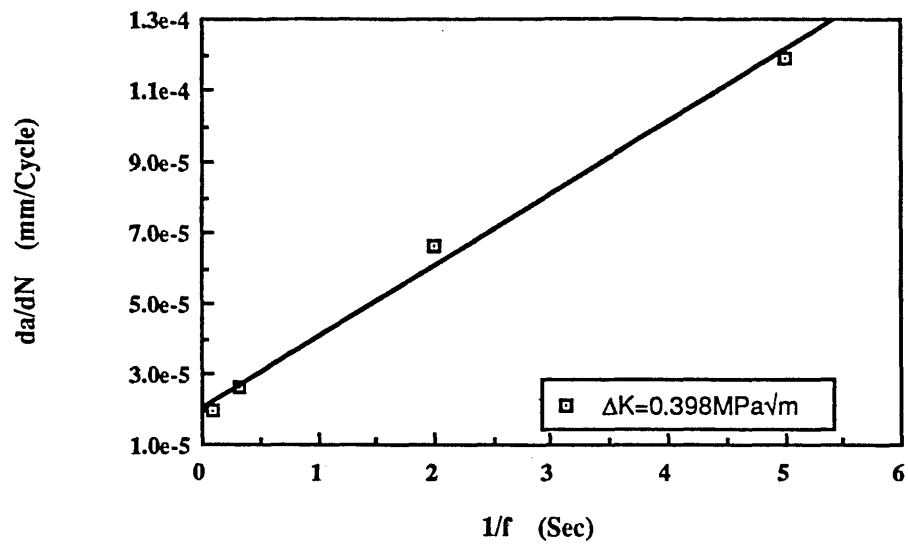


Fig 5.27 (d)

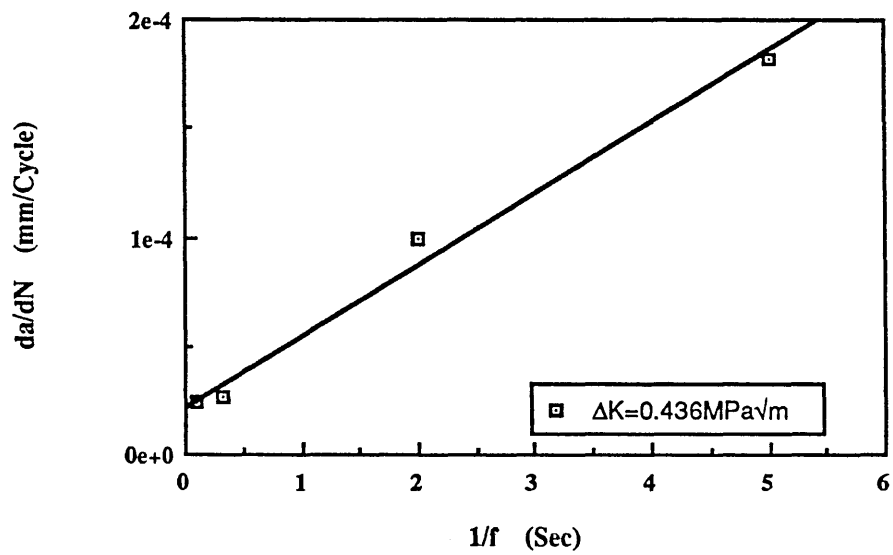


Fig 5.27 (e)

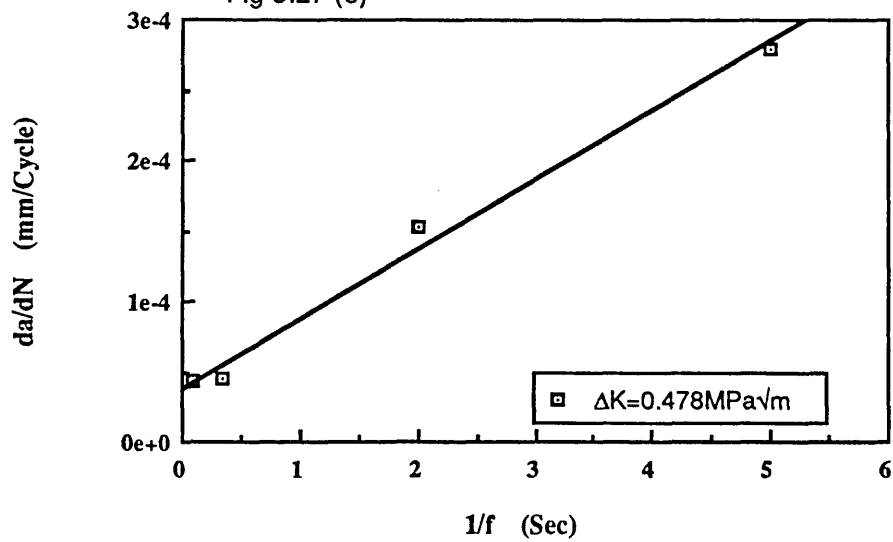


Fig 5.27 (f)

Fig 5.27(a-f) FCP rates in BP33 against reciprocal frequency at various values of ΔK .
R-ratio=0.5-T=+23°C

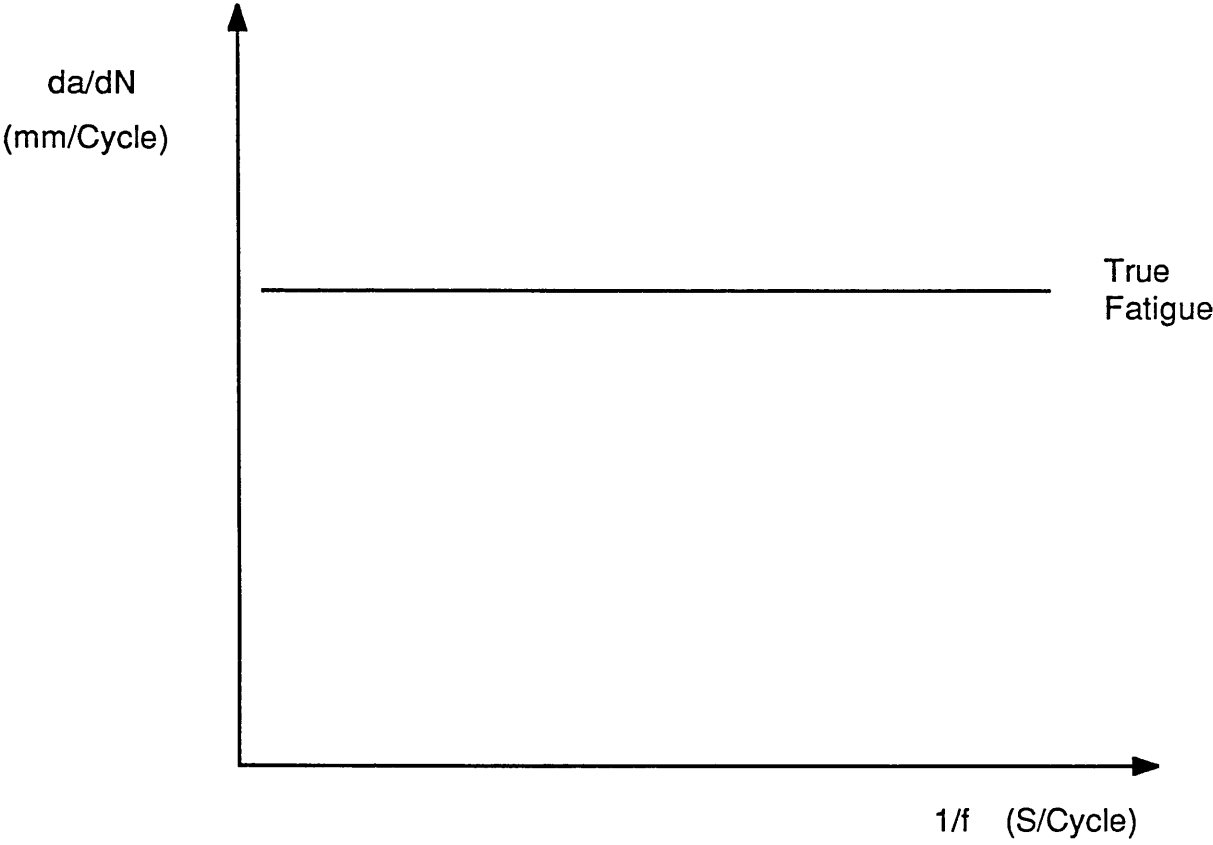


Fig 5.28

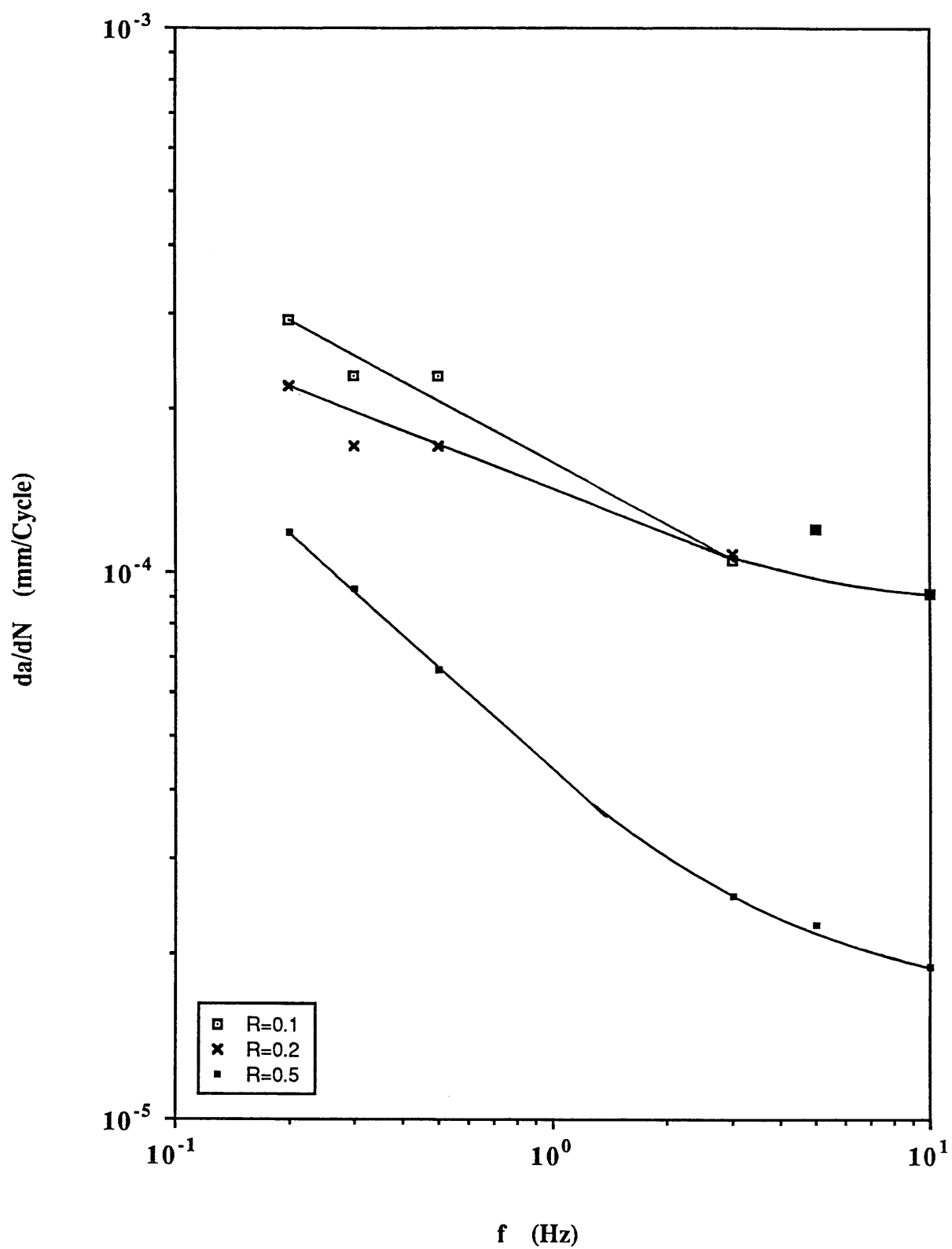


Fig 5.29 FCP rates in BP33 against frequency at a constant ΔK value of $0.398 \text{ MPa}\sqrt{\text{m}}$.
 $T=+23^\circ\text{C}$

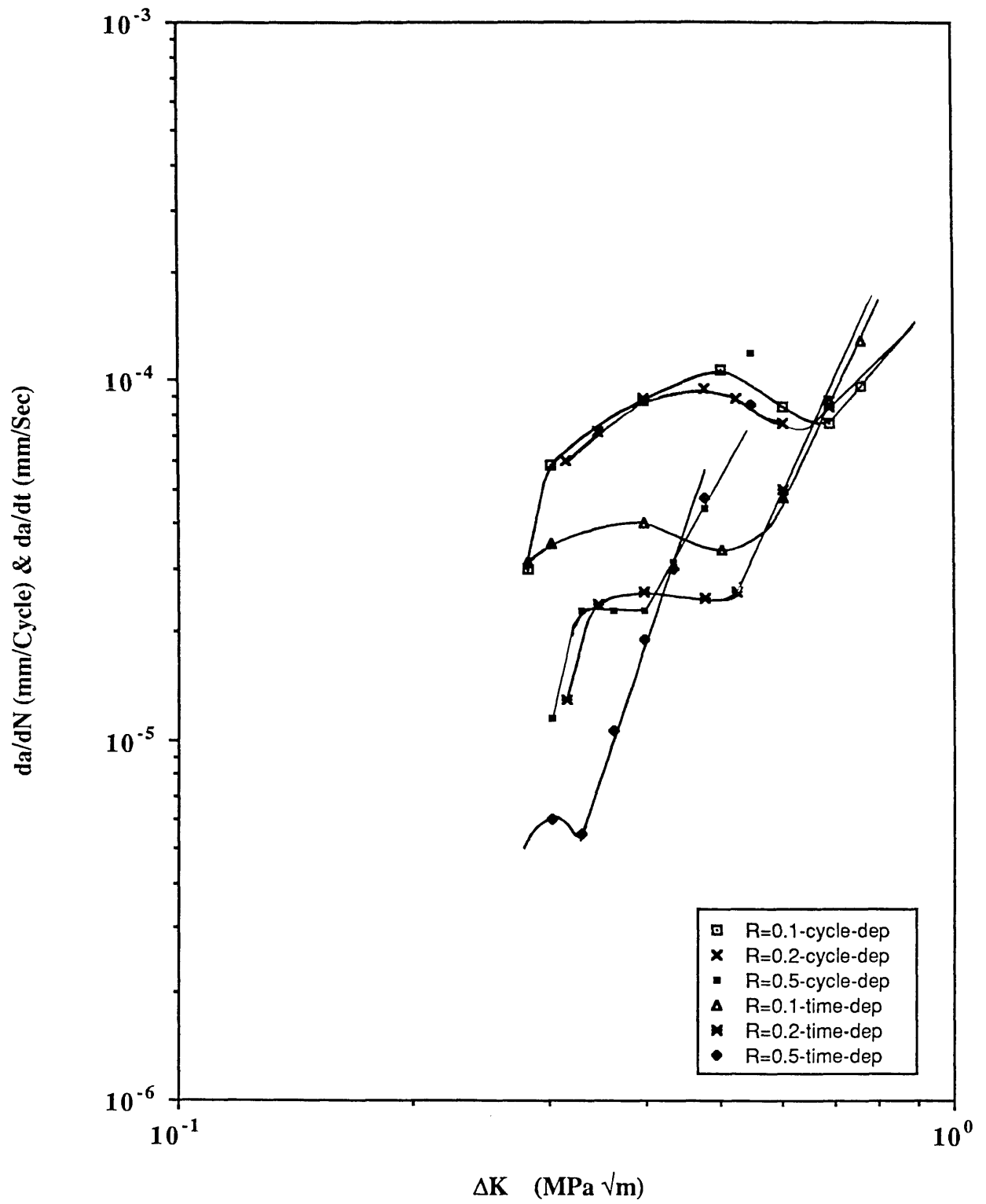


Fig 5.30 FCP rates in BP33 against ΔK representing Cycle and Time-dependent FCP at various R-ratios.
 $T=+23^{\circ}\text{C}$

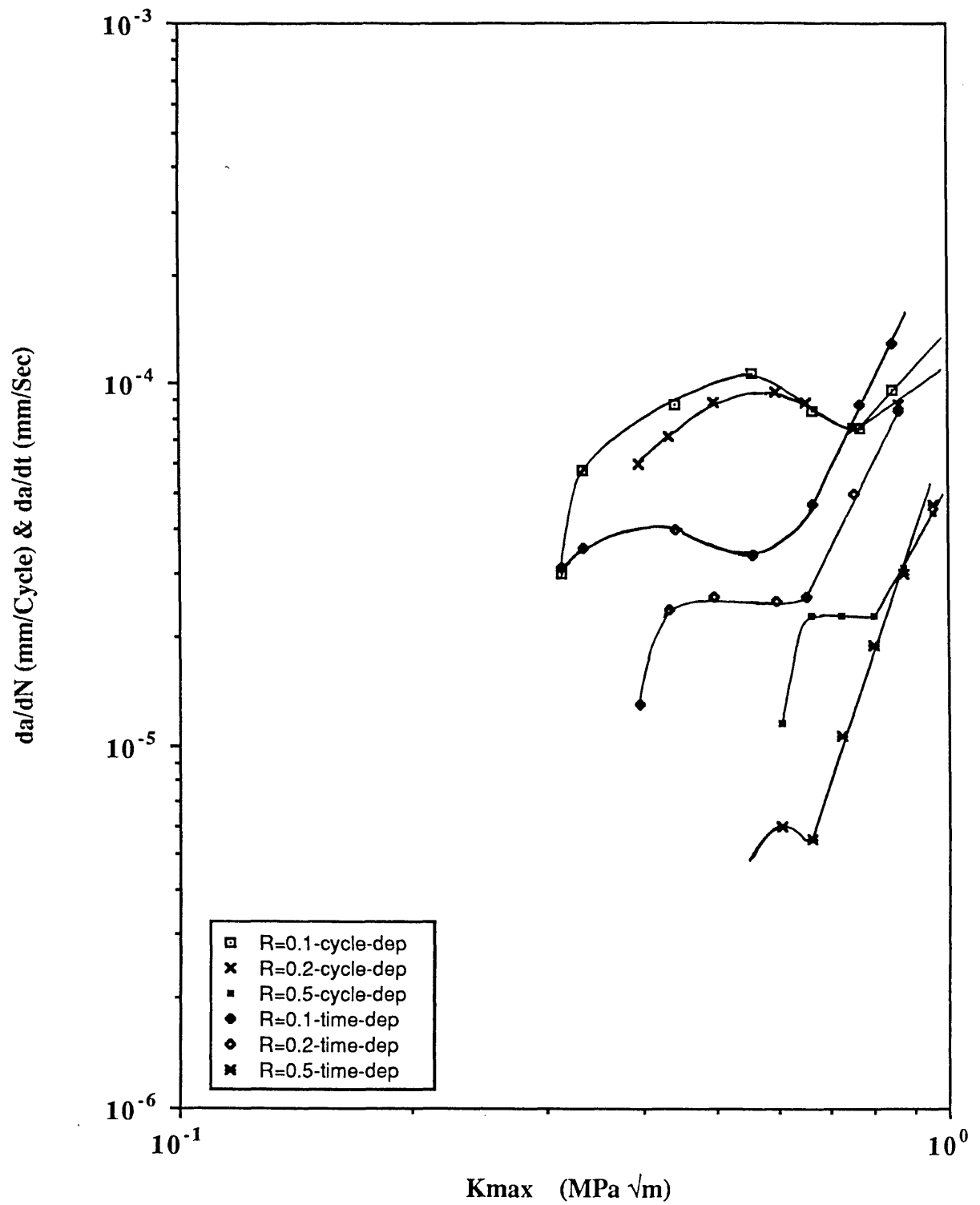


Fig 5.31(a) FCP rates in BP33 against K_{max} representing Cycle and Time-dependent FCP at various R-ratios.
 $T=+23^{\circ}C$

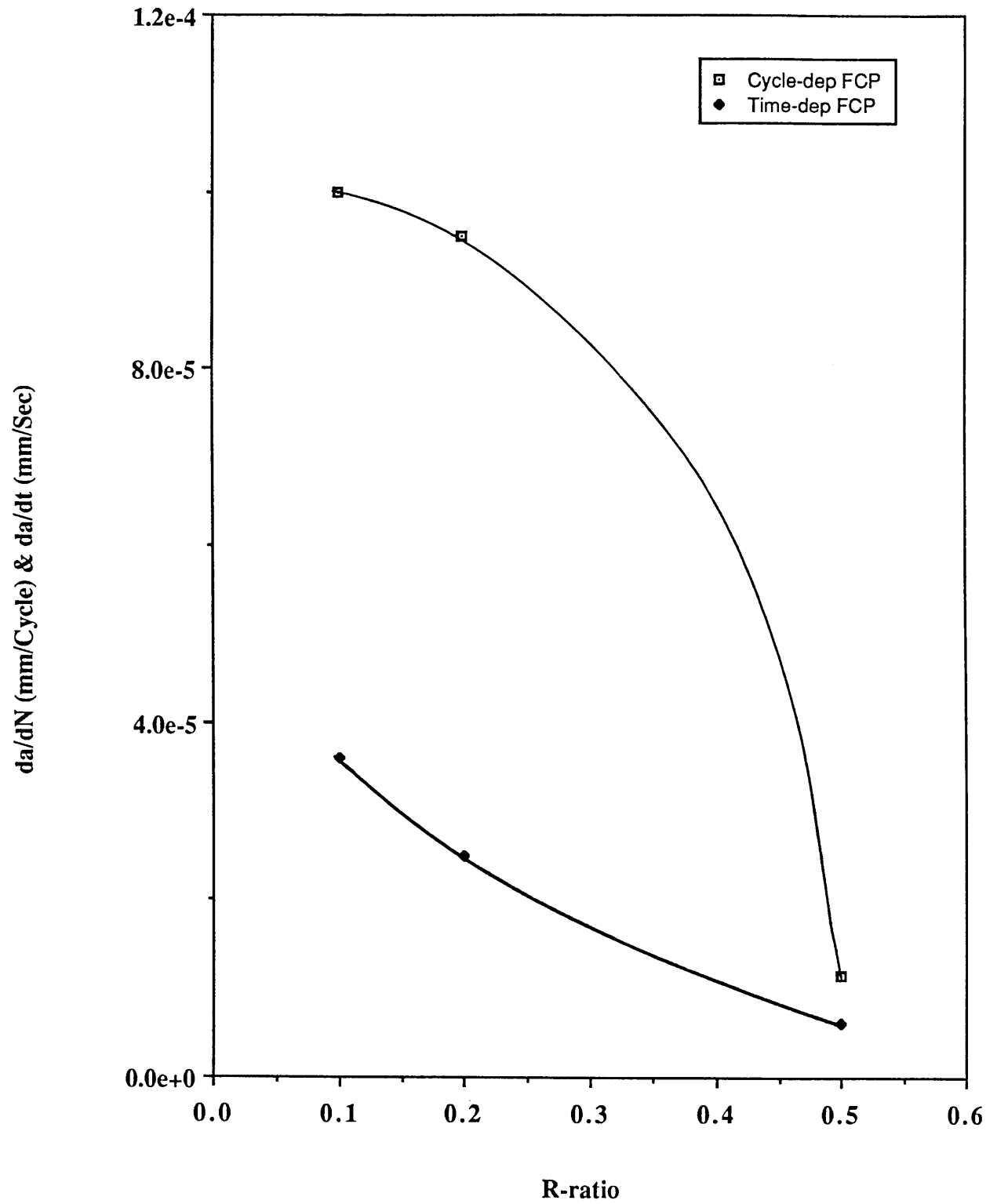


Fig 5.31(b) The Cycle and Time-dependent components in BP33 plotted against R-ratio.
Frequency=1.0Hz - $K_{max}=0.602 \text{ MPa } \sqrt{\text{m}}$

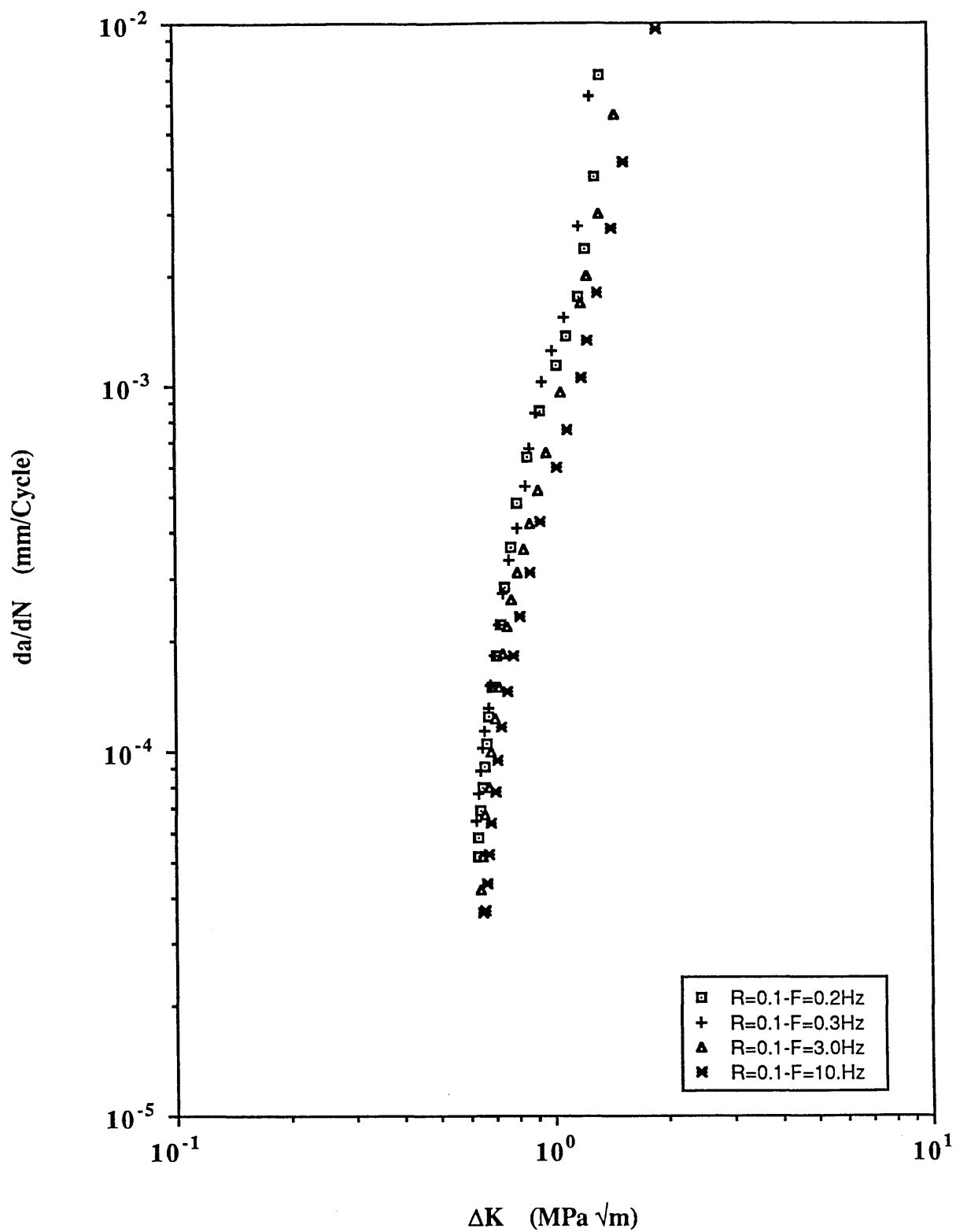


Fig 5.32 Fatigue crack propagation rates in BP33 at 0.2, 0.3, 3.0 and 10.0Hz.
R-ratio=0.1 - T=-20°C

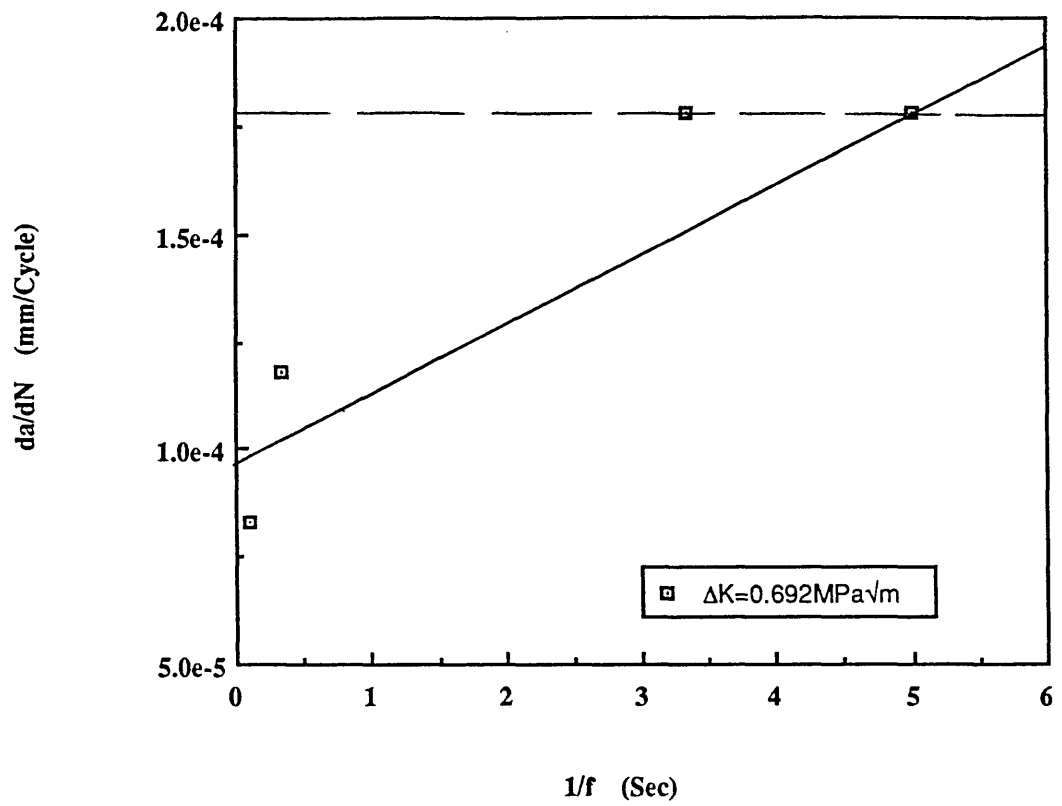


Fig 5.33 (a)

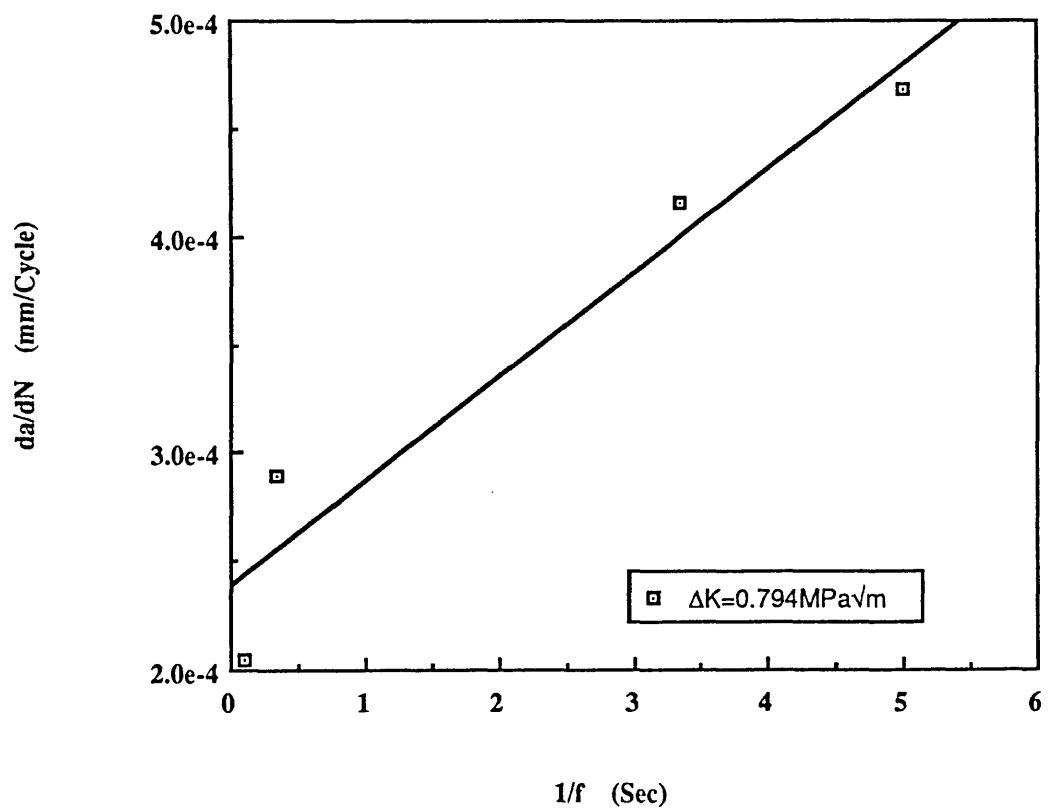


Fig 5.33 (b)

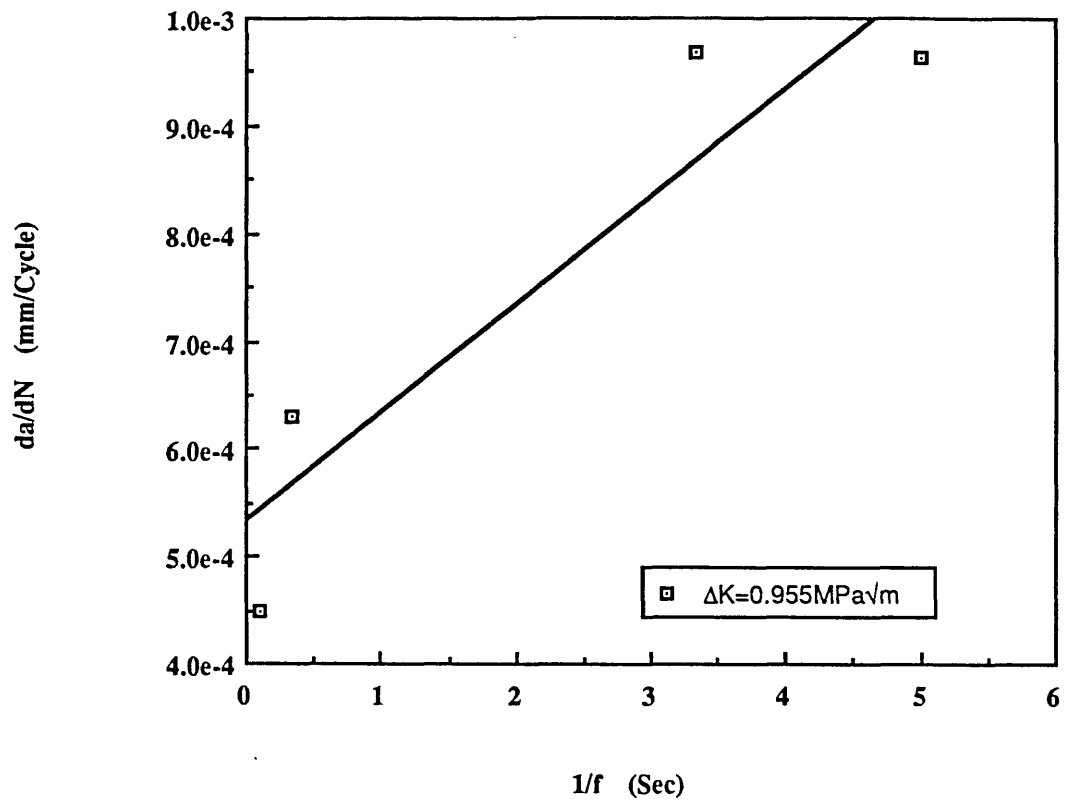


Fig 5.33 (c)

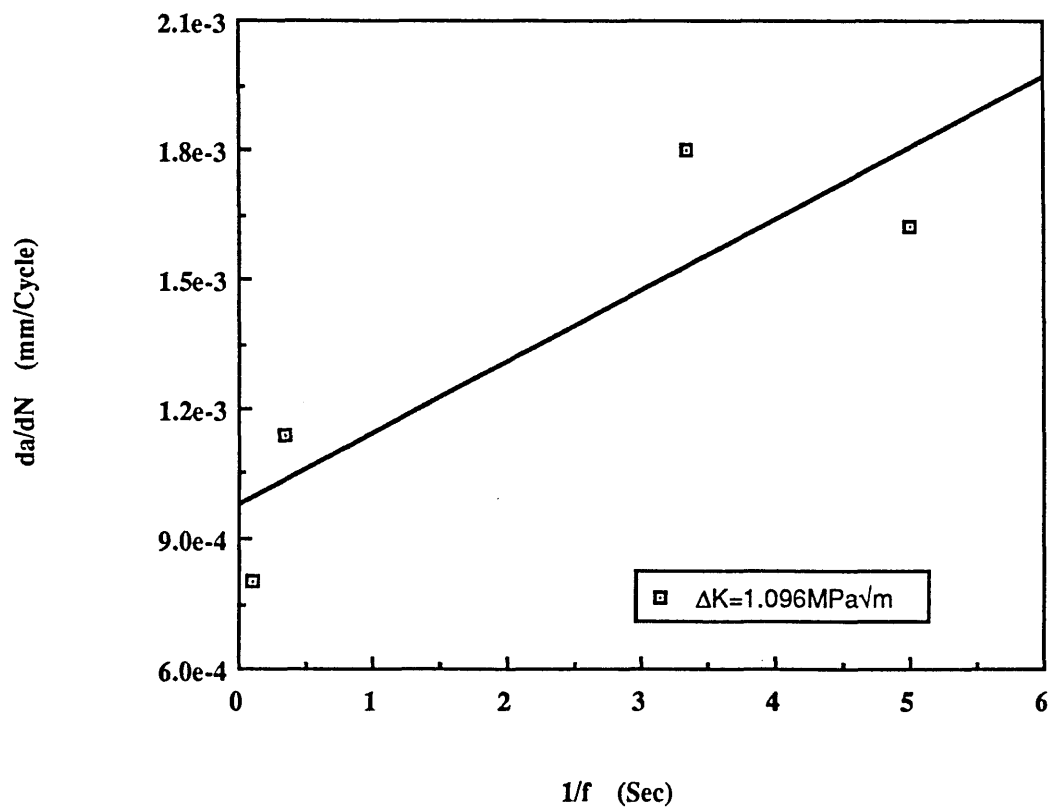


Fig 5.33 (d)

Fig 5.33(a-d) FCP rates in BP33 against reciprocal frequency at various values of ΔK .
 R-ratio=0.1-T=-20°C

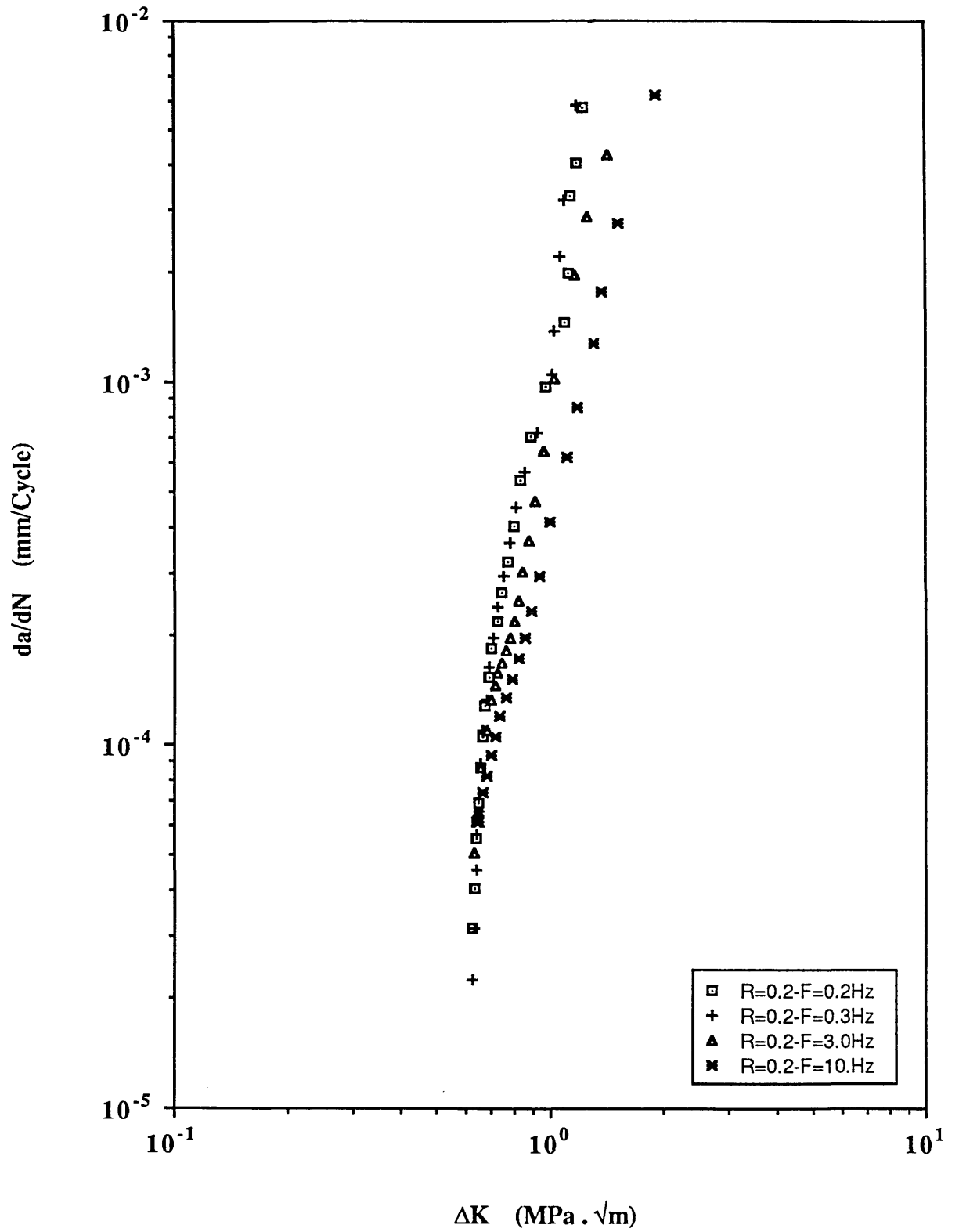


Fig 5.34 Fatigue crack propagation rates in BP33 at 0.2, 0.3, 3.0 and 10.0Hz.
R-ratio=0.2 - T=-20°C

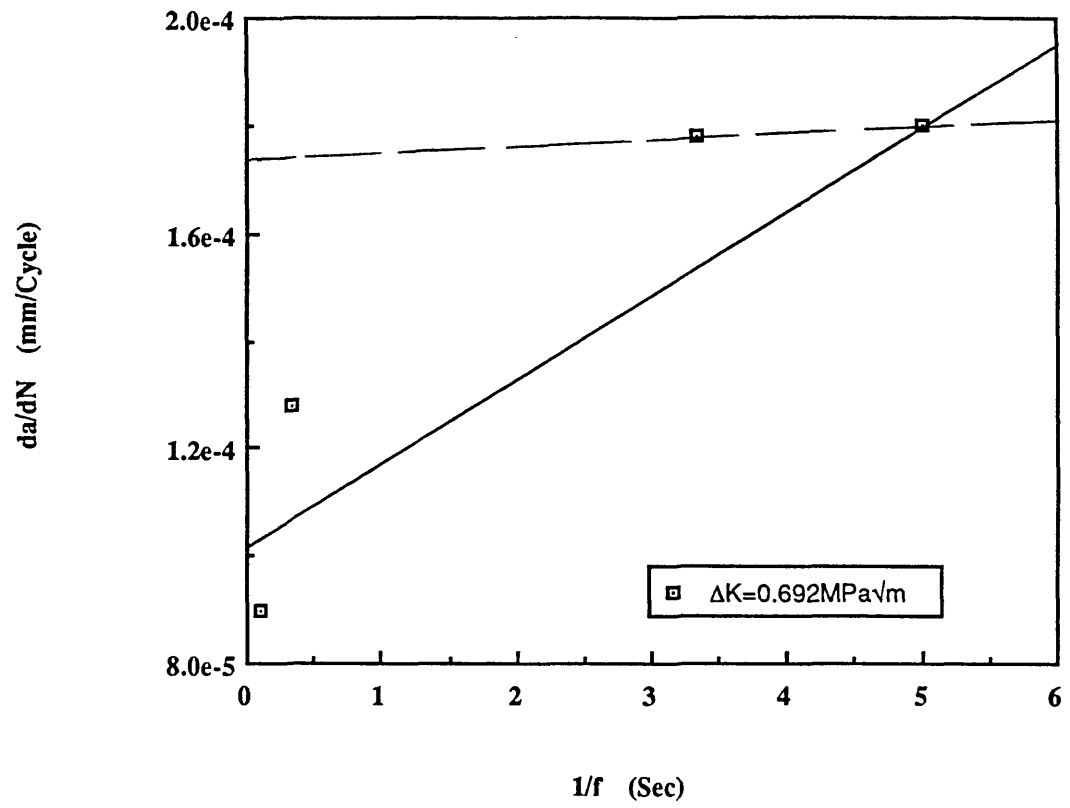


Fig 5.35 (a)

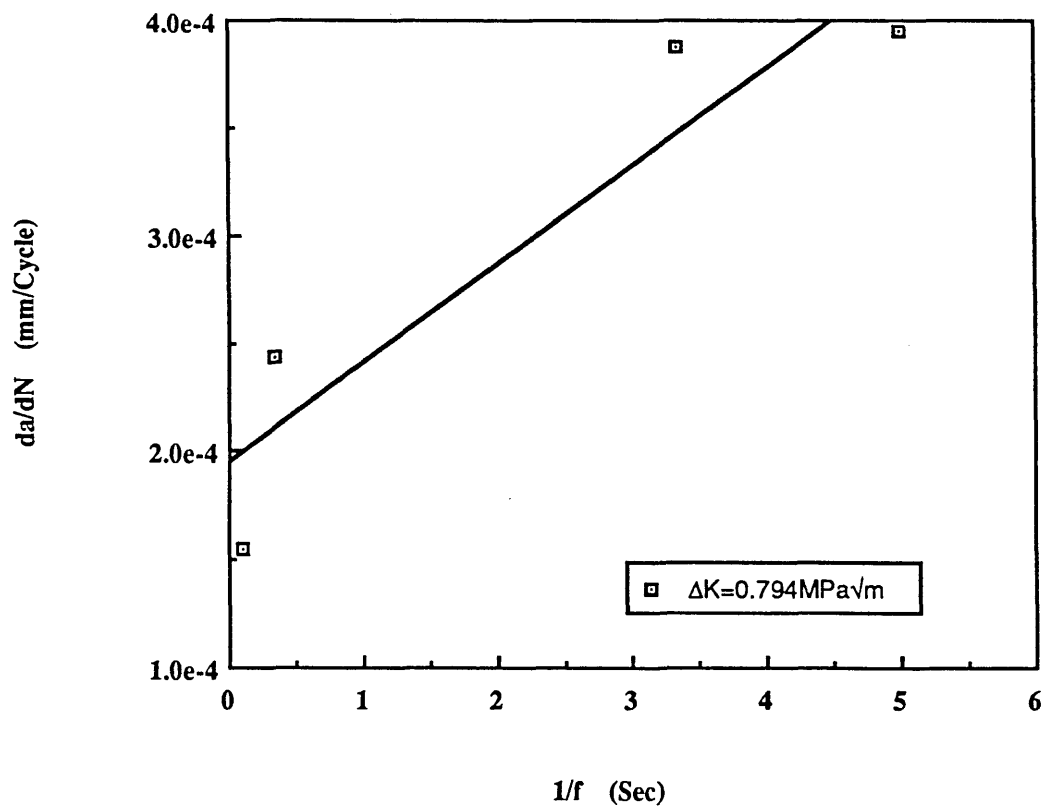


Fig 5.35 (b)

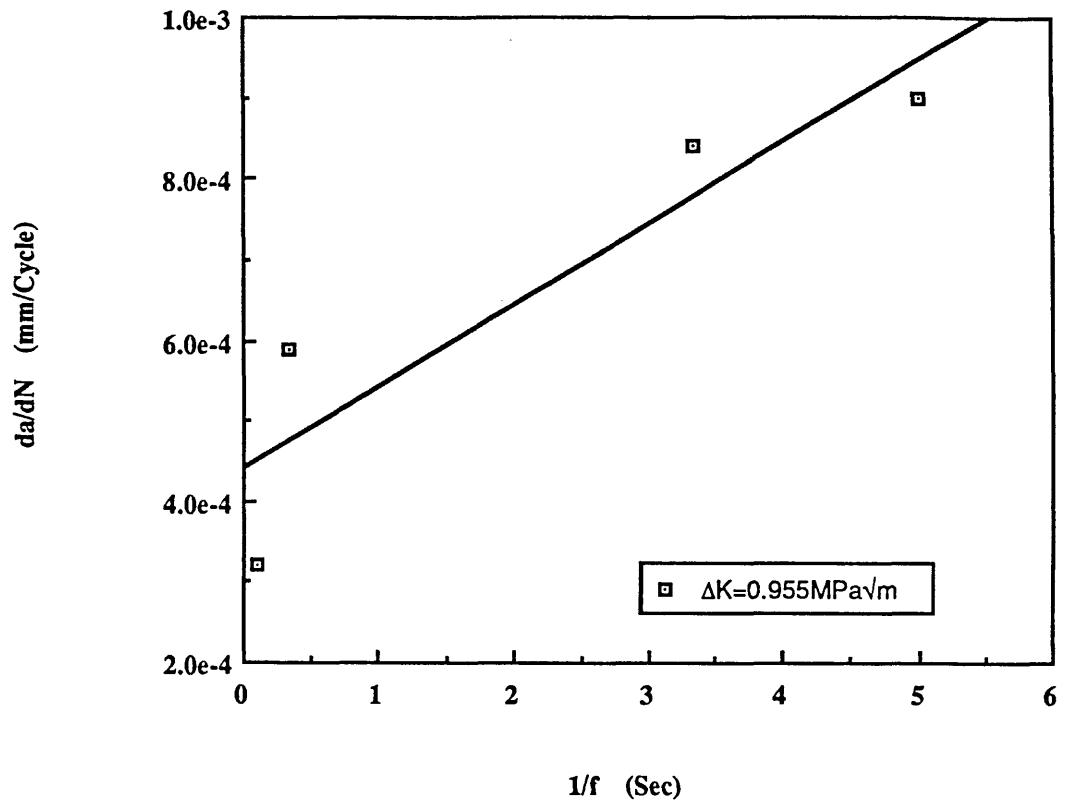


Fig 5.35 (c)

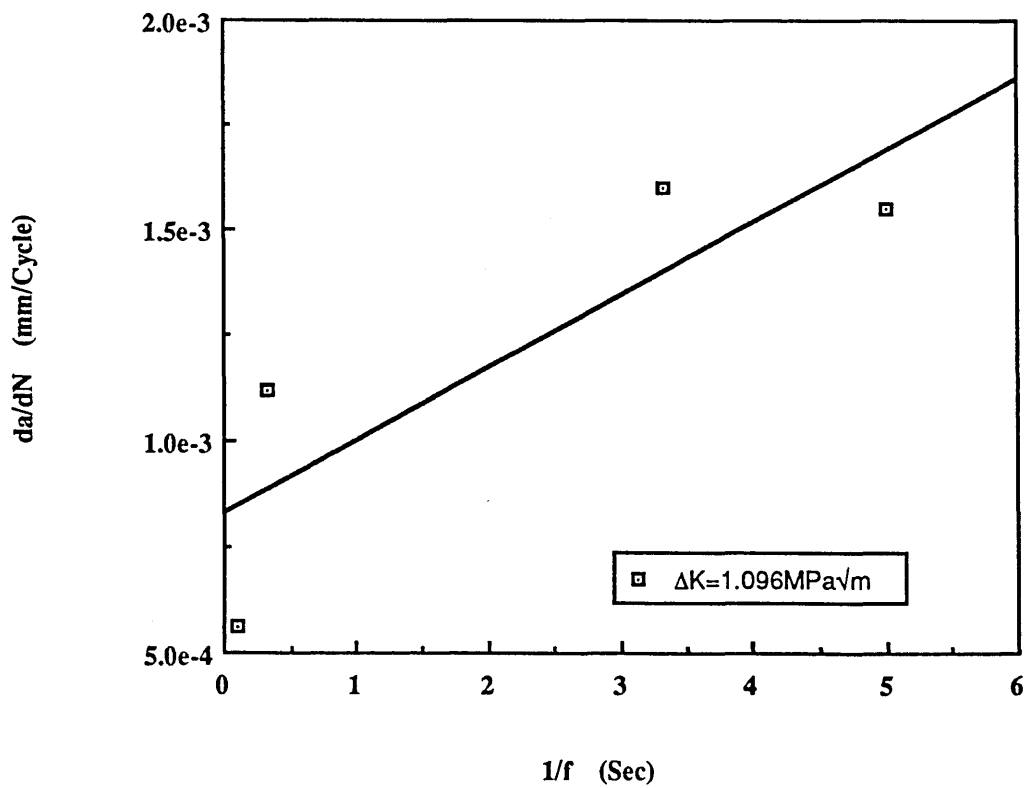


Fig 5.35 (d)

Fig 5.35(a-d) FCP rates in BP33 against reciprocal frequency at various values of ΔK .
 R-ratio=0.2-T=-20°C

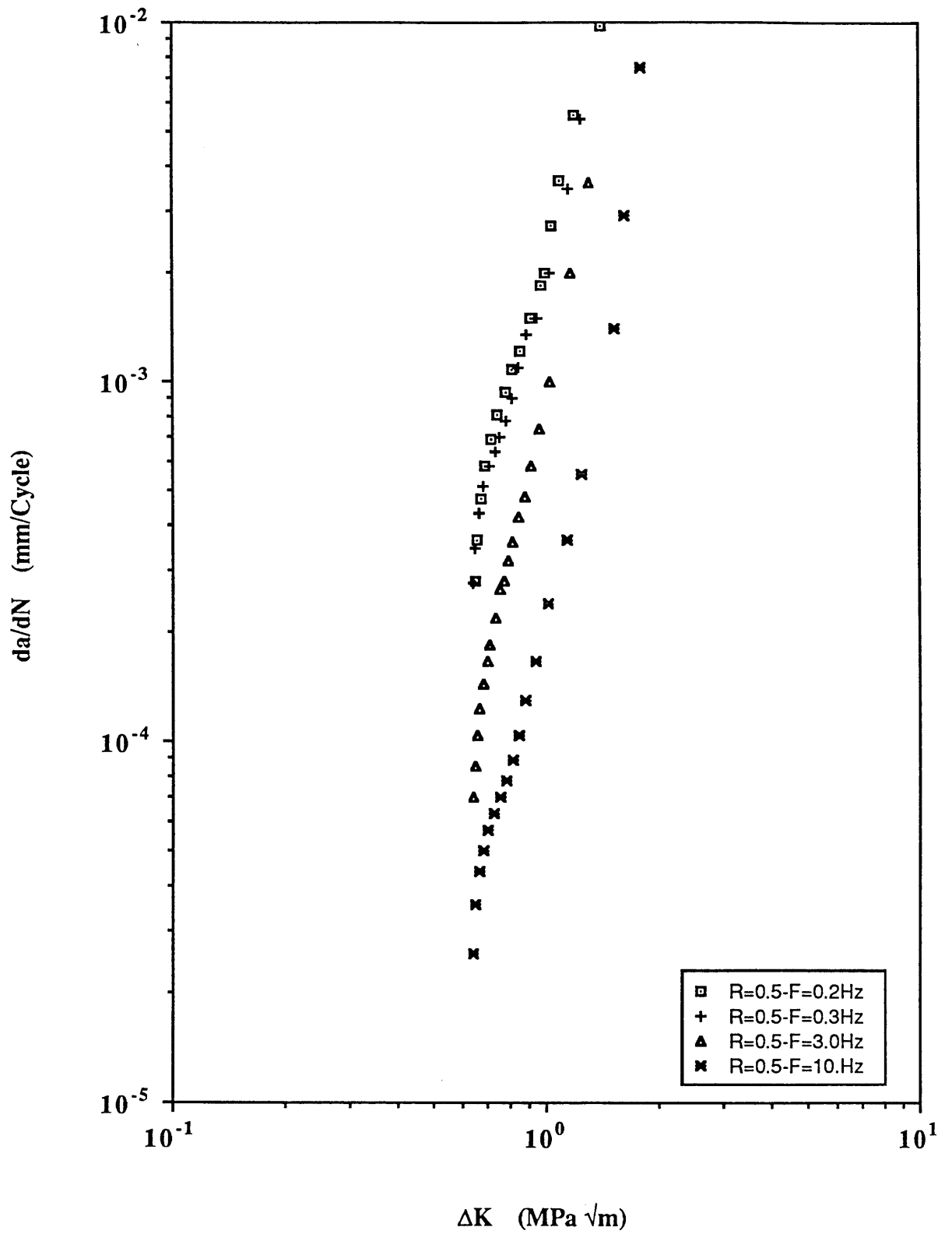


Fig 5.36 Fatigue crack propagation rates in BP33 at 0.2, 0.3, 3.0 and 10.0Hz.
R-ratio=0.5 - T=-20°C

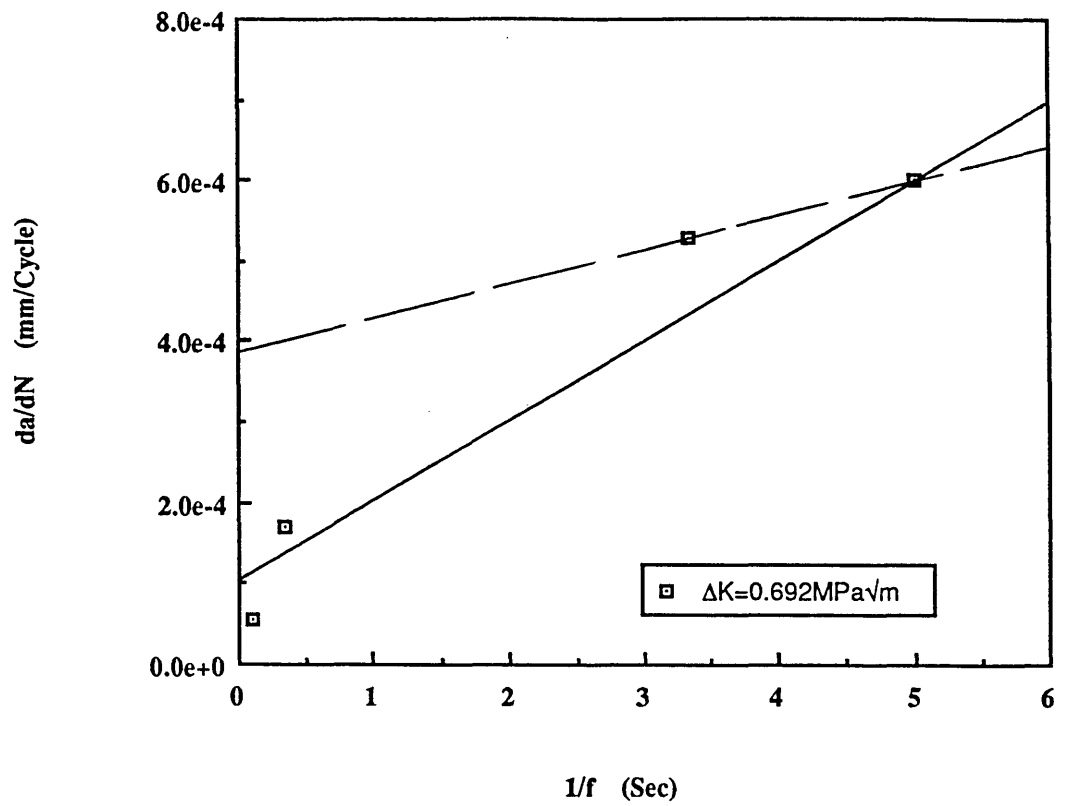


Fig 5.37 (a)

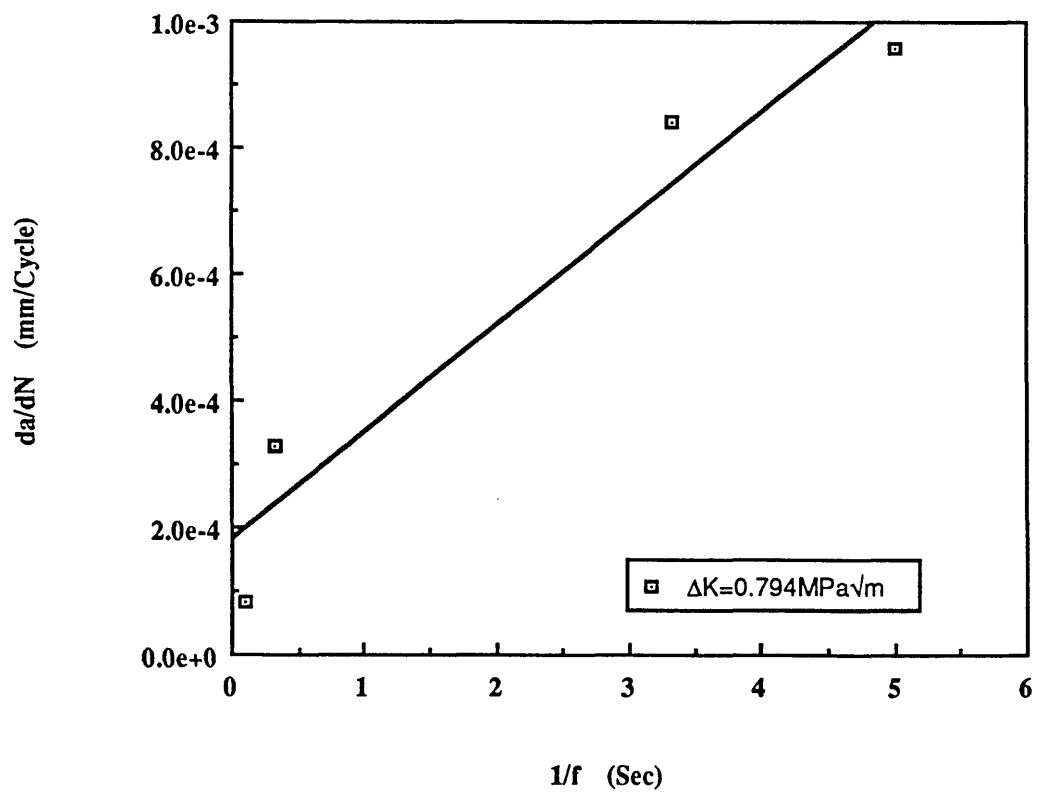


Fig 5.37 (b)

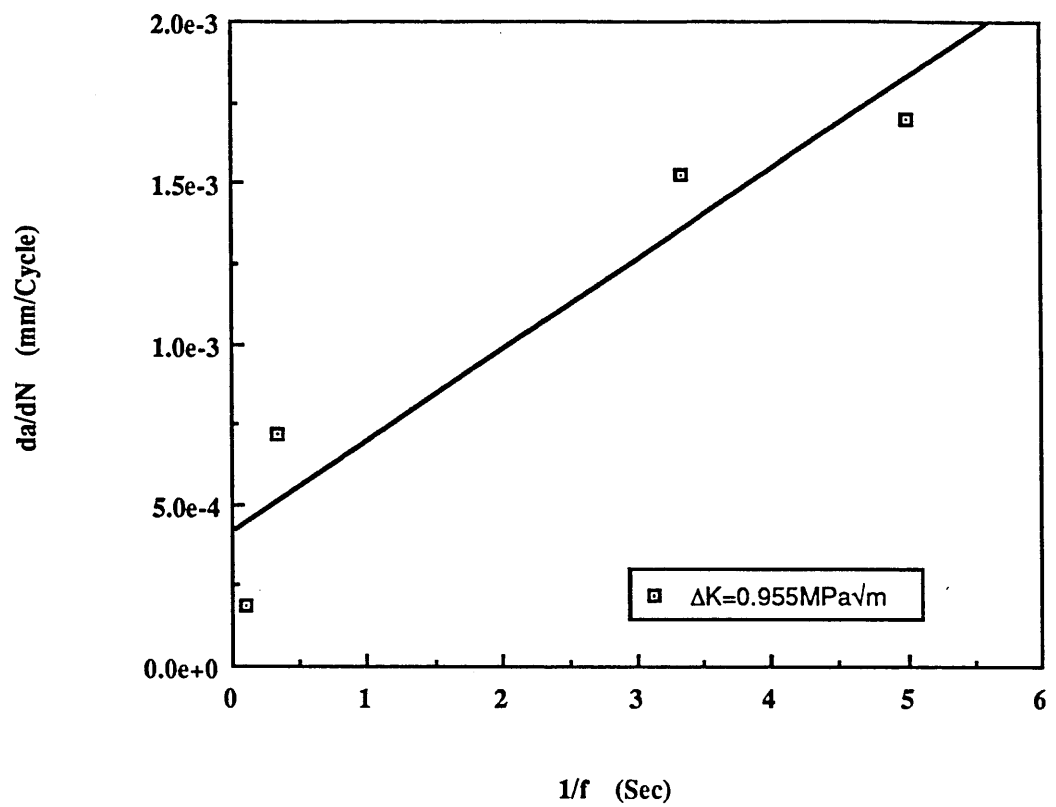


Fig 5.37 (c)

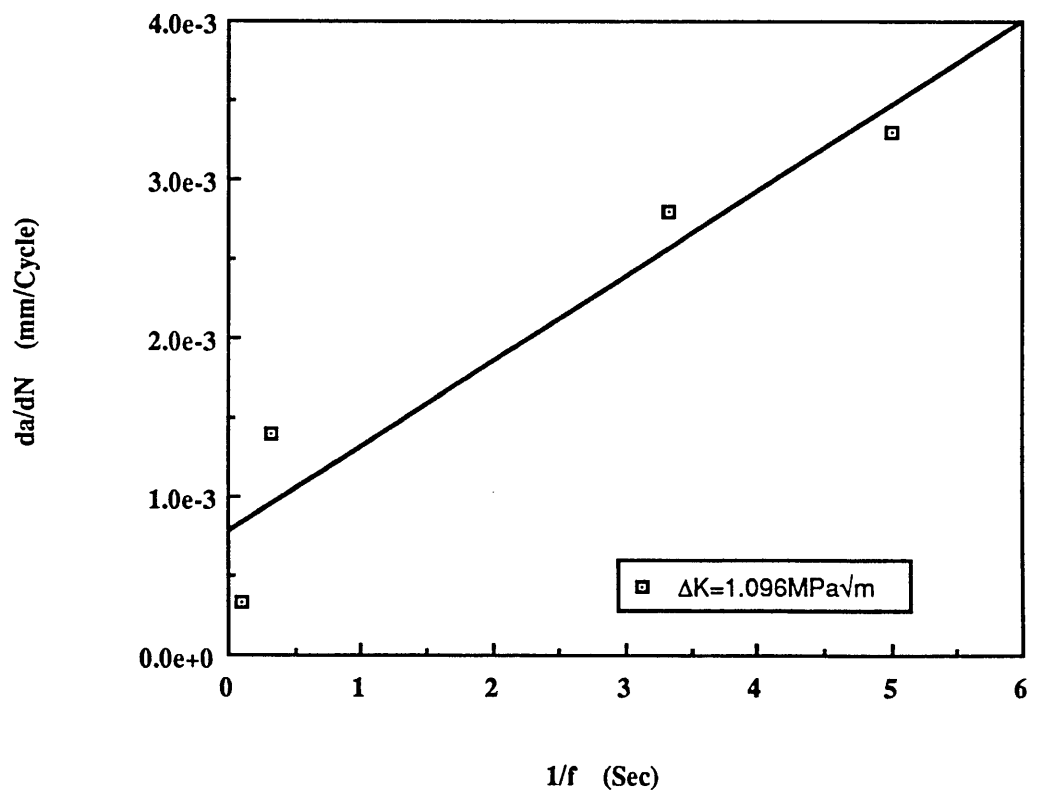


Fig 5.37 (d)

Fig 5.37(a-d) FCP rates in BP33 against reciprocal frequency at various values of ΔK .
R-ratio=0.5-T=-20°C

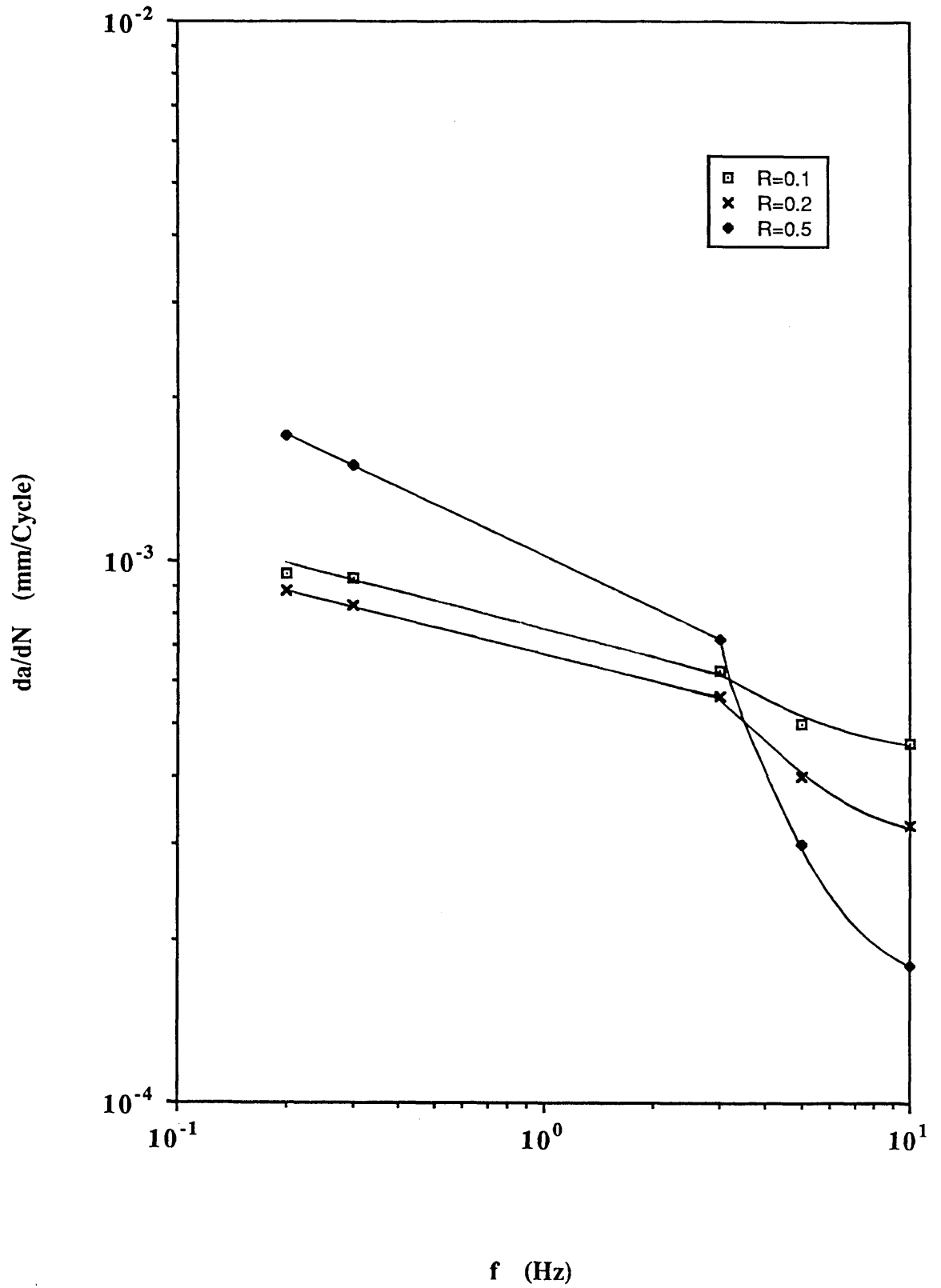


Fig 5.38 FCP rates in BP33 against frequency at a constant ΔK value of $0.955 \text{ MPa}\sqrt{\text{m}}$.
 $T=-20^\circ\text{C}$

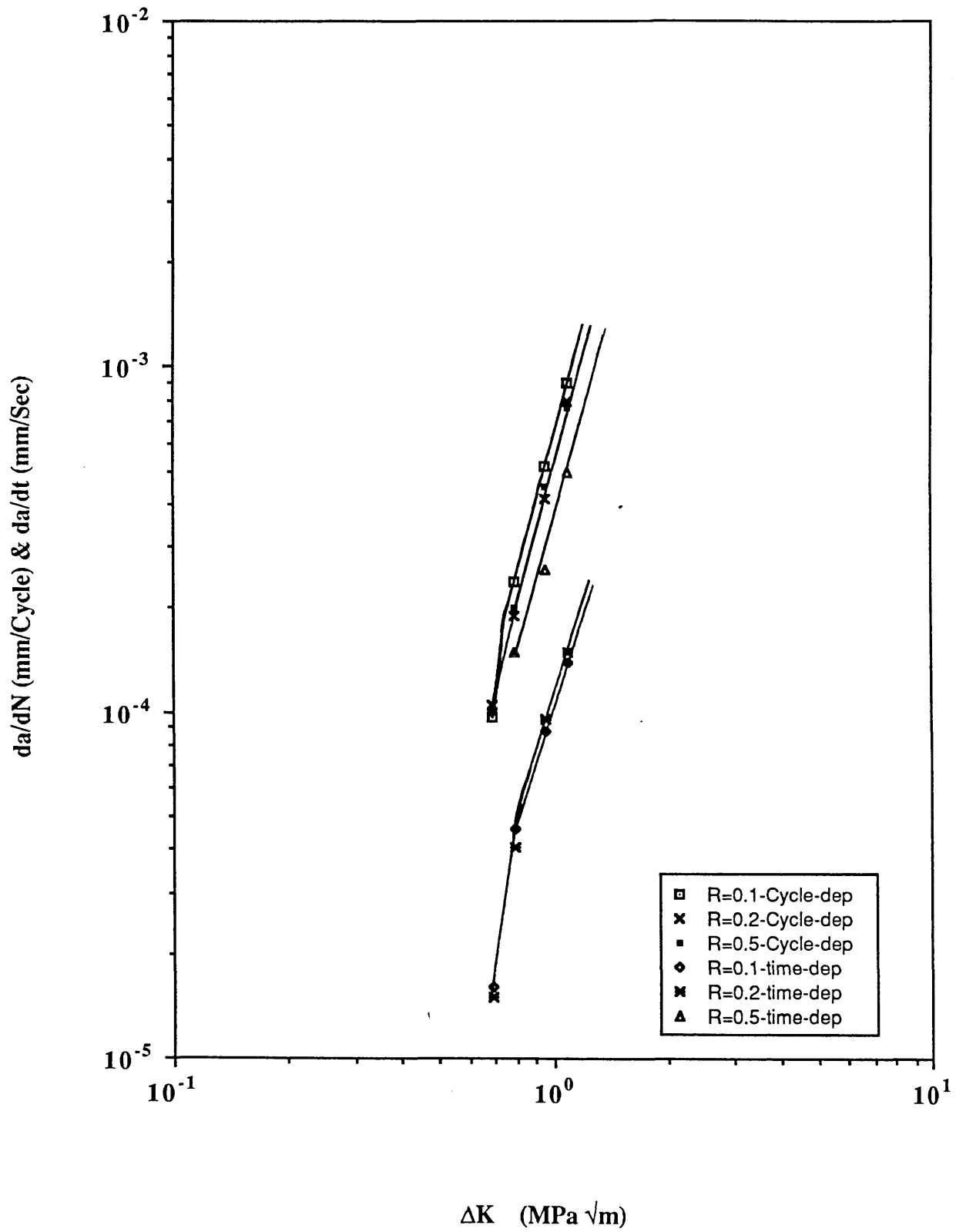


Fig 5.39 FCP rates in BP33 against ΔK representing Cycle and Time-dependent FCP at various R-ratios.
 $T = -20^\circ\text{C}$

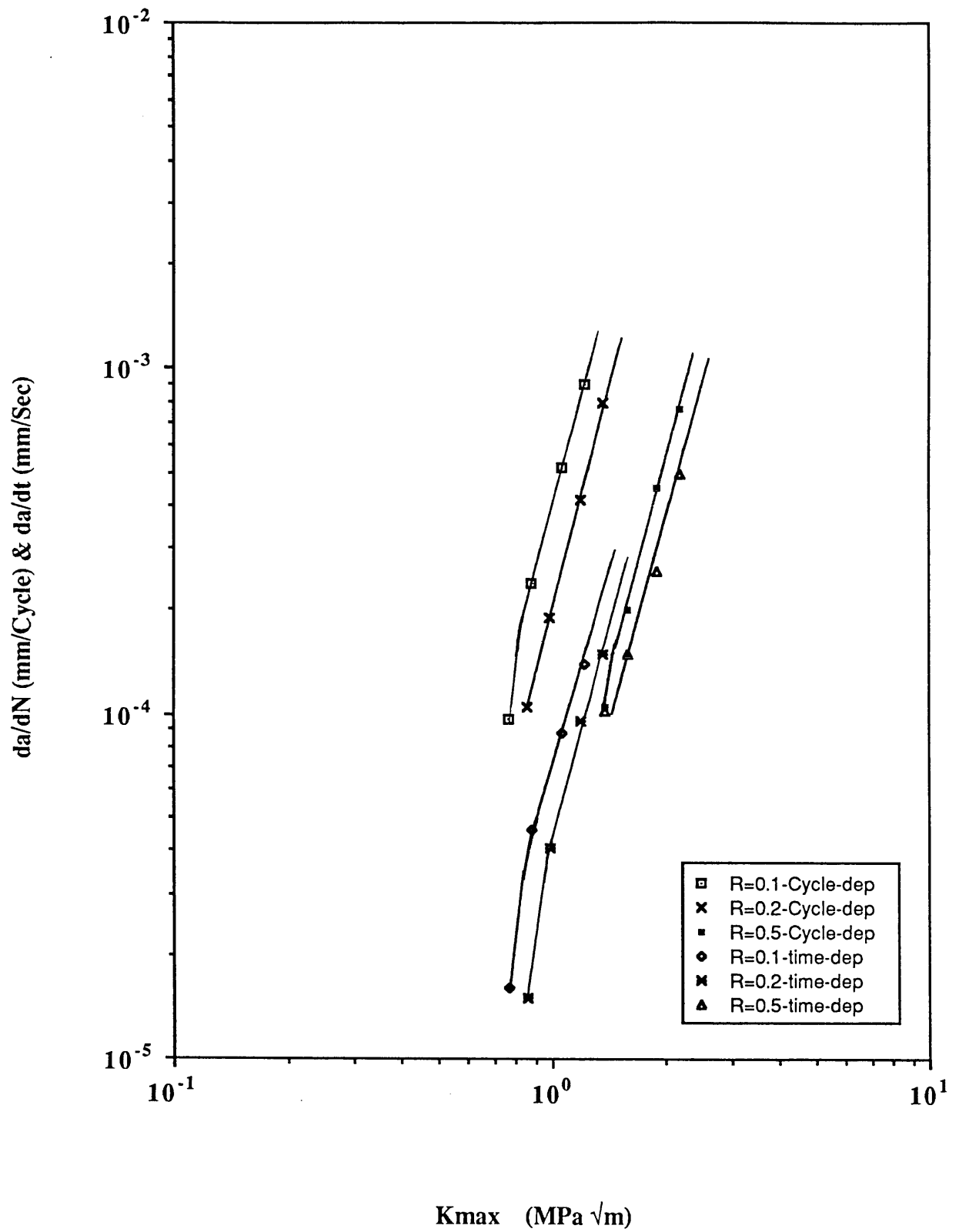


Fig 5.40 FCP rates in BP33 against K_{max} representing Cycle and Time-dependent FCP at various R-ratios.
 $T = -20^\circ\text{C}$

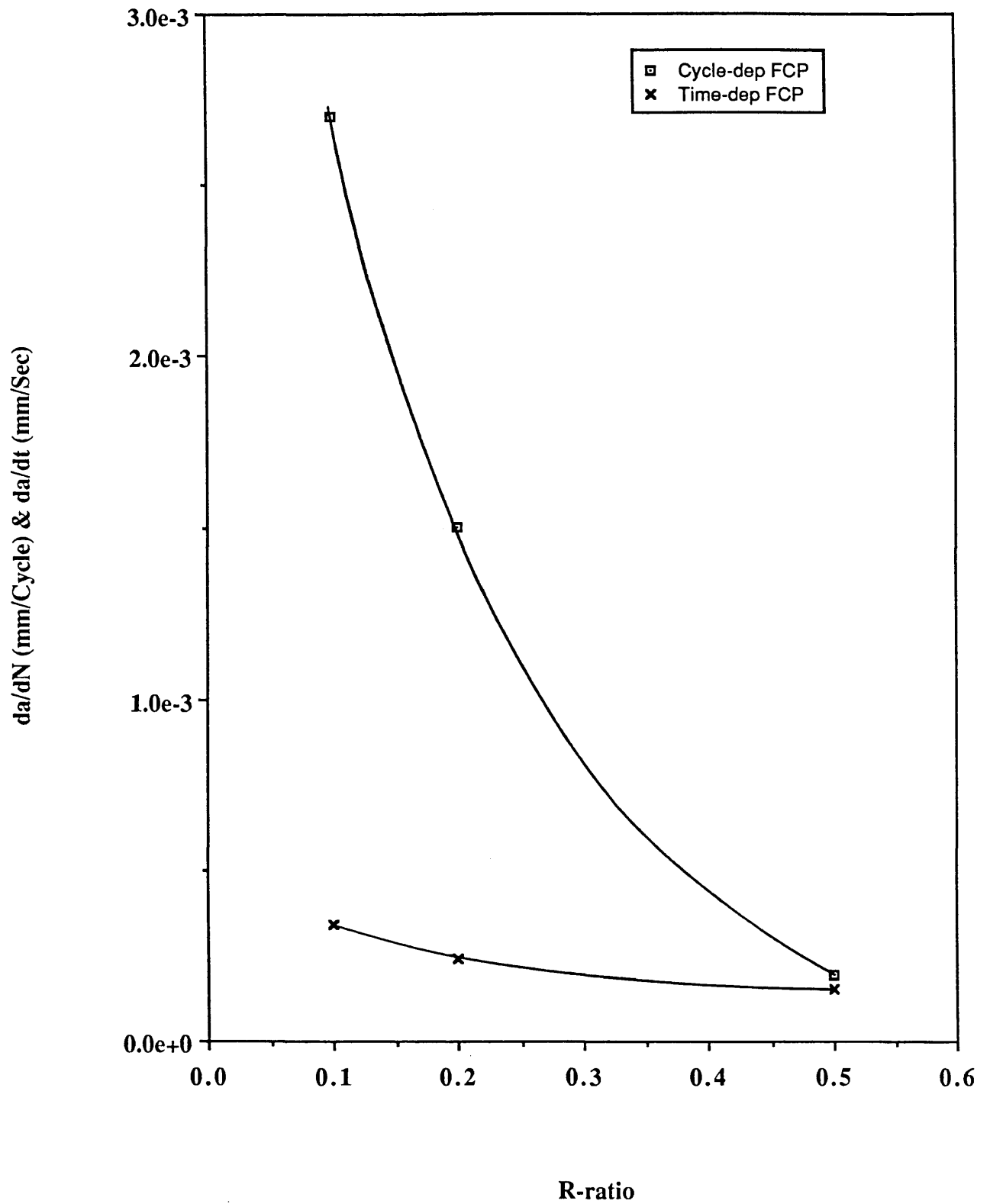


Fig 5.41 The Cycle and Time-dependent components in BP33 plotted against R-ratio. Frequency=1.0Hz - $K_{max}=1.585 \text{ MPa } \sqrt{\text{m}}$

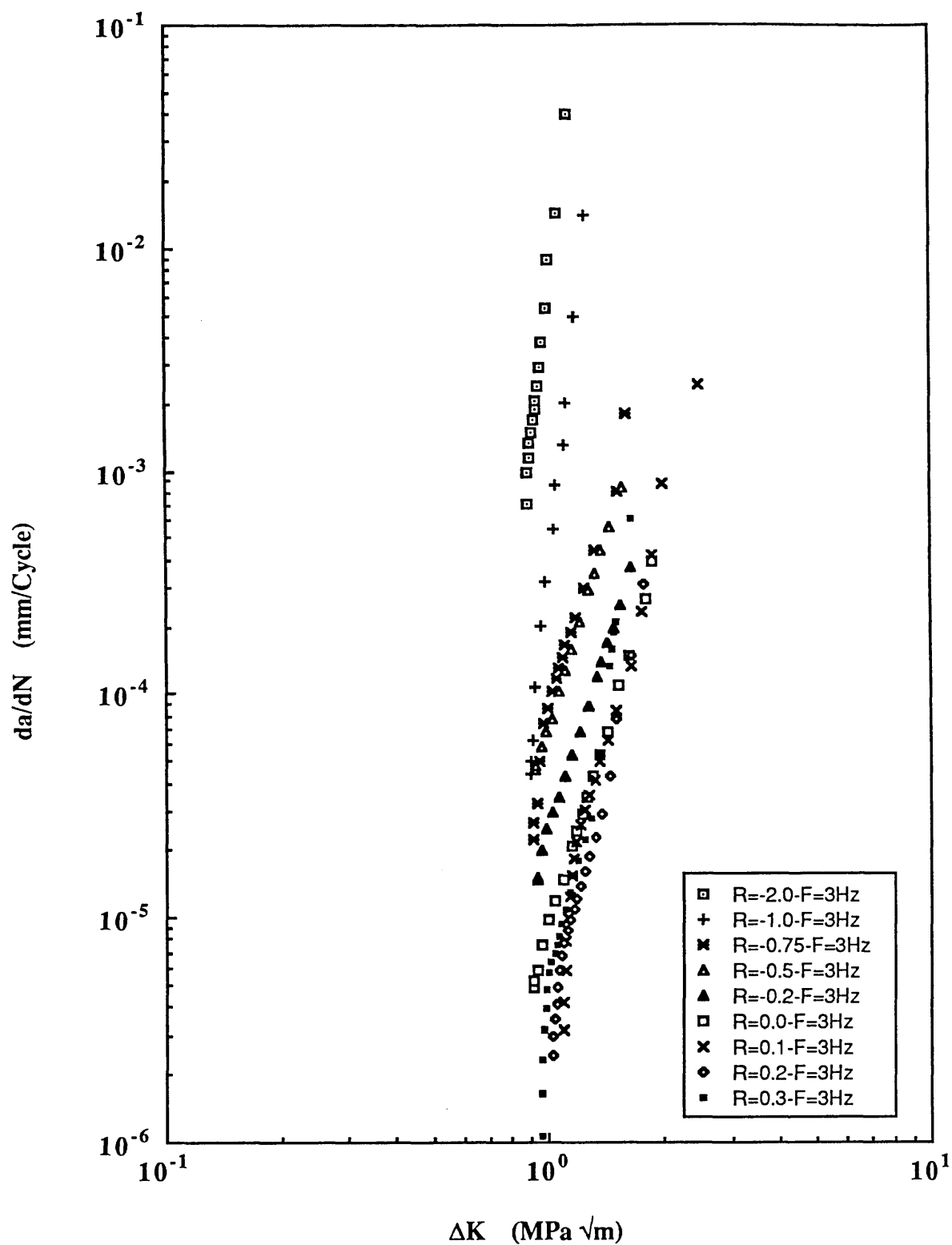


Fig 5.42 Fatigue crack propagation rates in LL101AA at various R-ratios.
T=+23°C

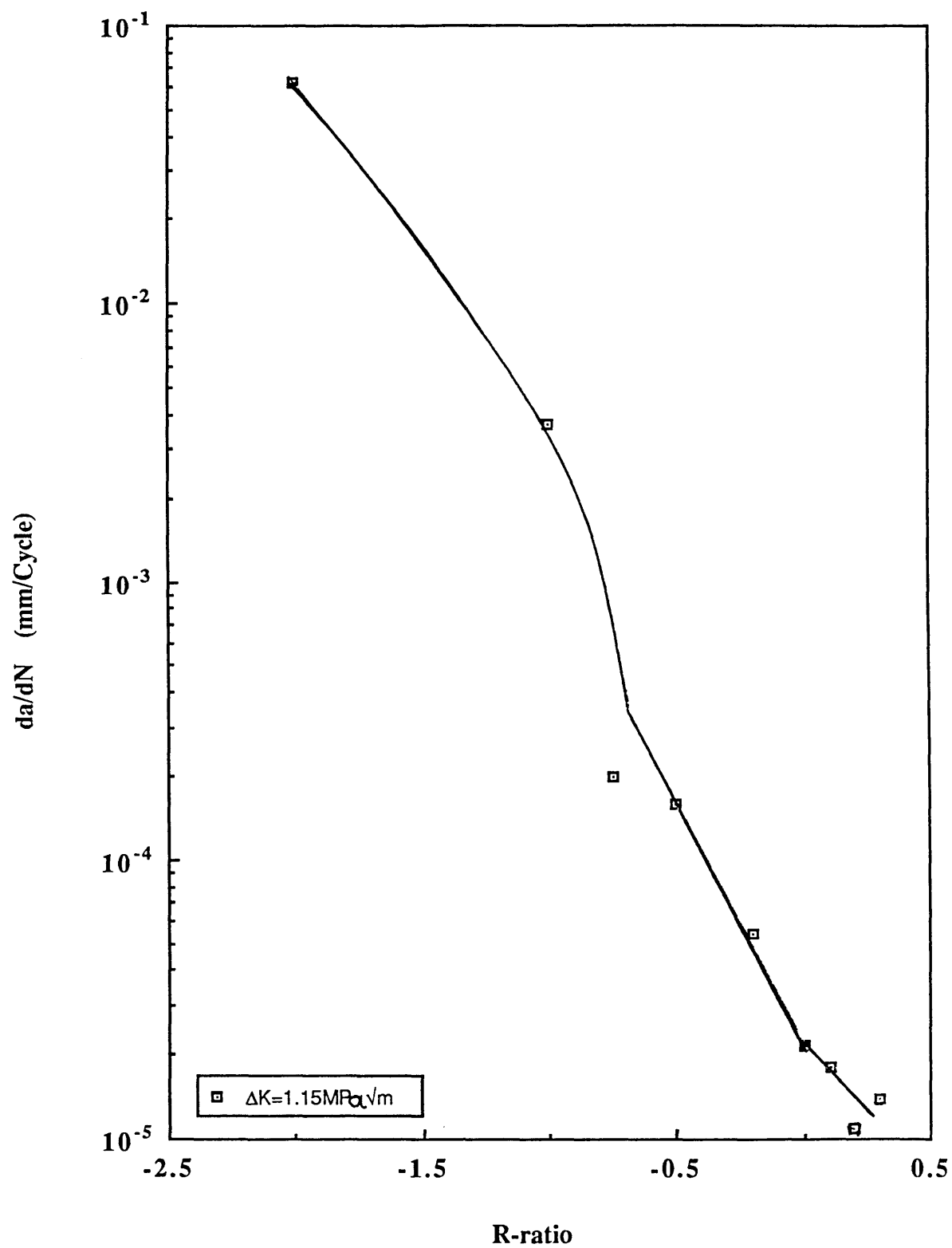


Fig 5.43 FCP rates in LL101AA plotted against R-ratio at a constant ΔK value of $1.15 \text{ MPa}\sqrt{\text{m}}$.
 $F=3.0\text{Hz}$ - $T=+23^\circ\text{C}$

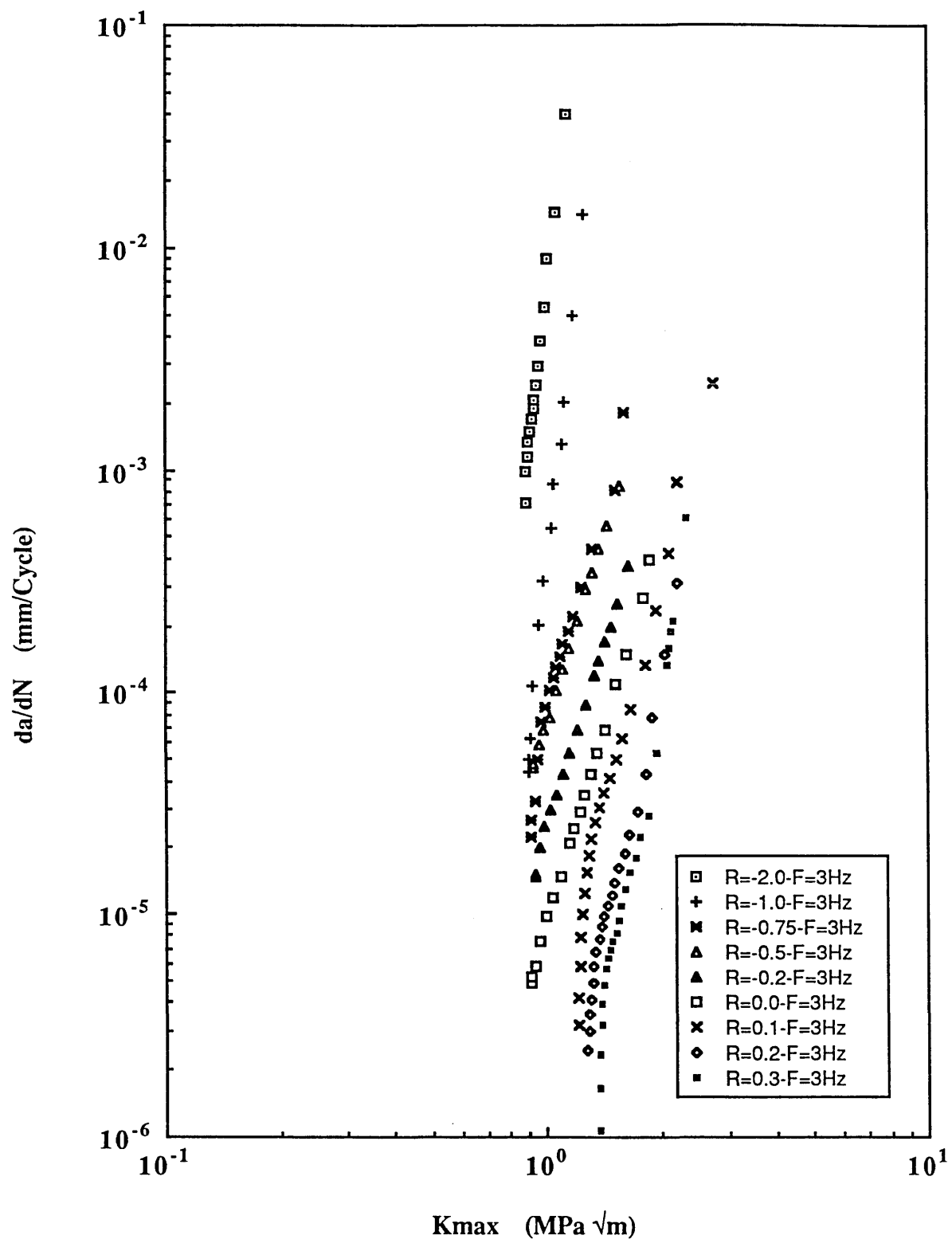


Fig 5.44 FCP rates in LL101AA against K_{max} .
 $T=+23^{\circ}\text{C}$

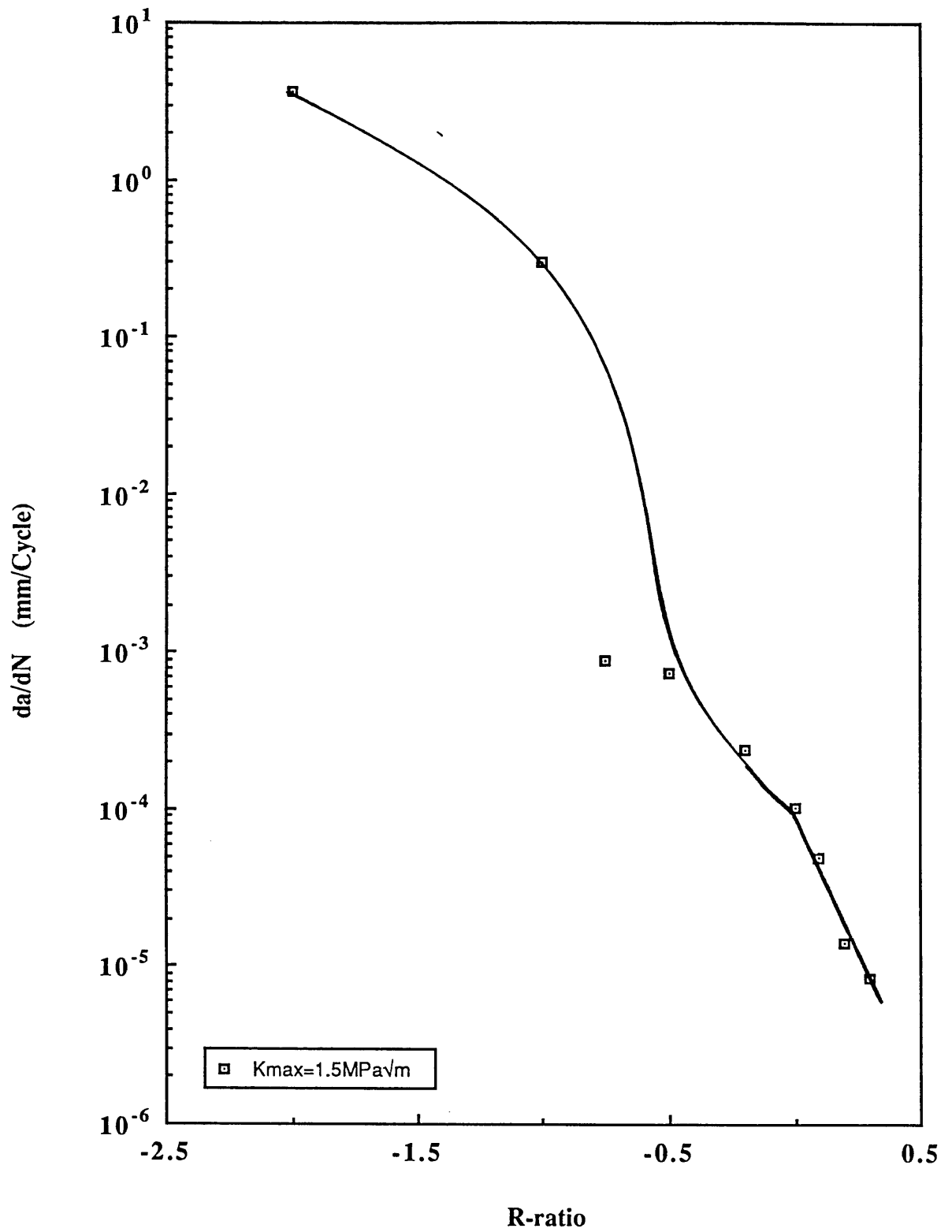


Fig 5.45 FCP rates in LL101AA plotted against R-ratio at a constant K_{max} value of $1.50\text{ MPa}\sqrt{\text{m}}$.
 $F=3.0\text{Hz}$ - $T=+23^\circ\text{C}$

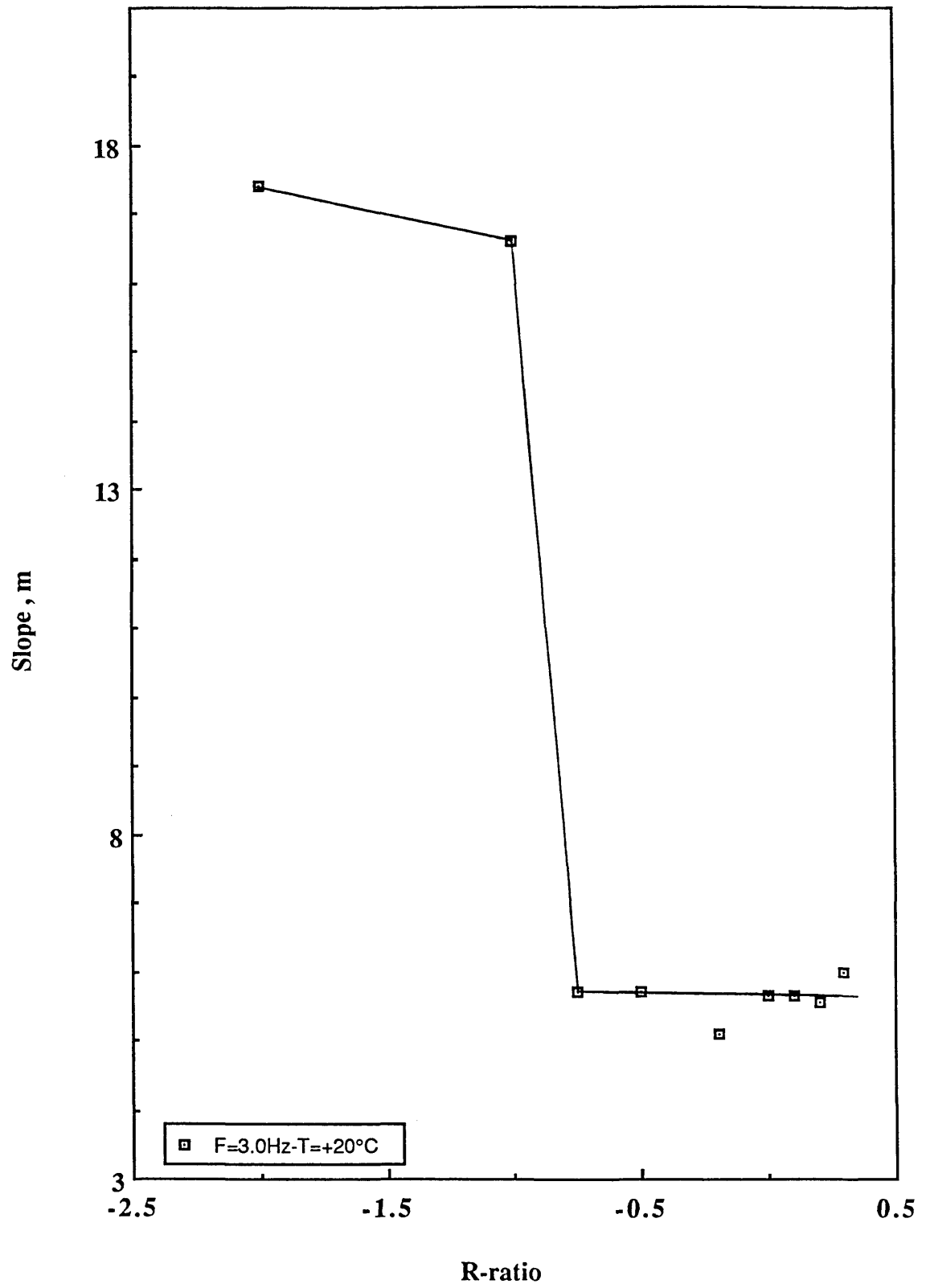


Fig 5.46 The slopes of Paris plots in LL101AA against R-ratio.
 $F=3.0\text{Hz}$ - $T=+23^{\circ}\text{C}$

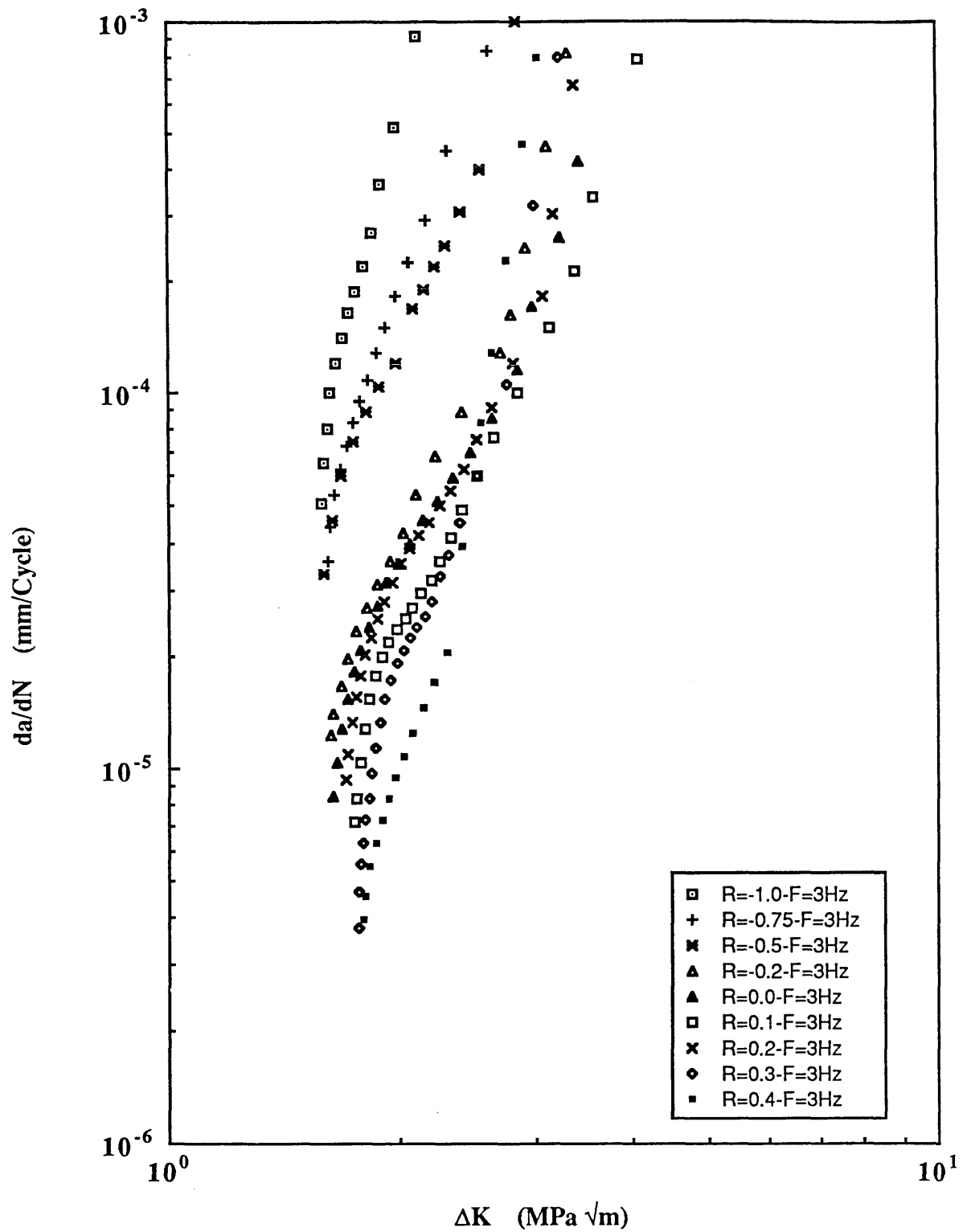


Fig 5.47 Fatigue crack propagation rates in LL101AA at various R-ratios.
T=20°C

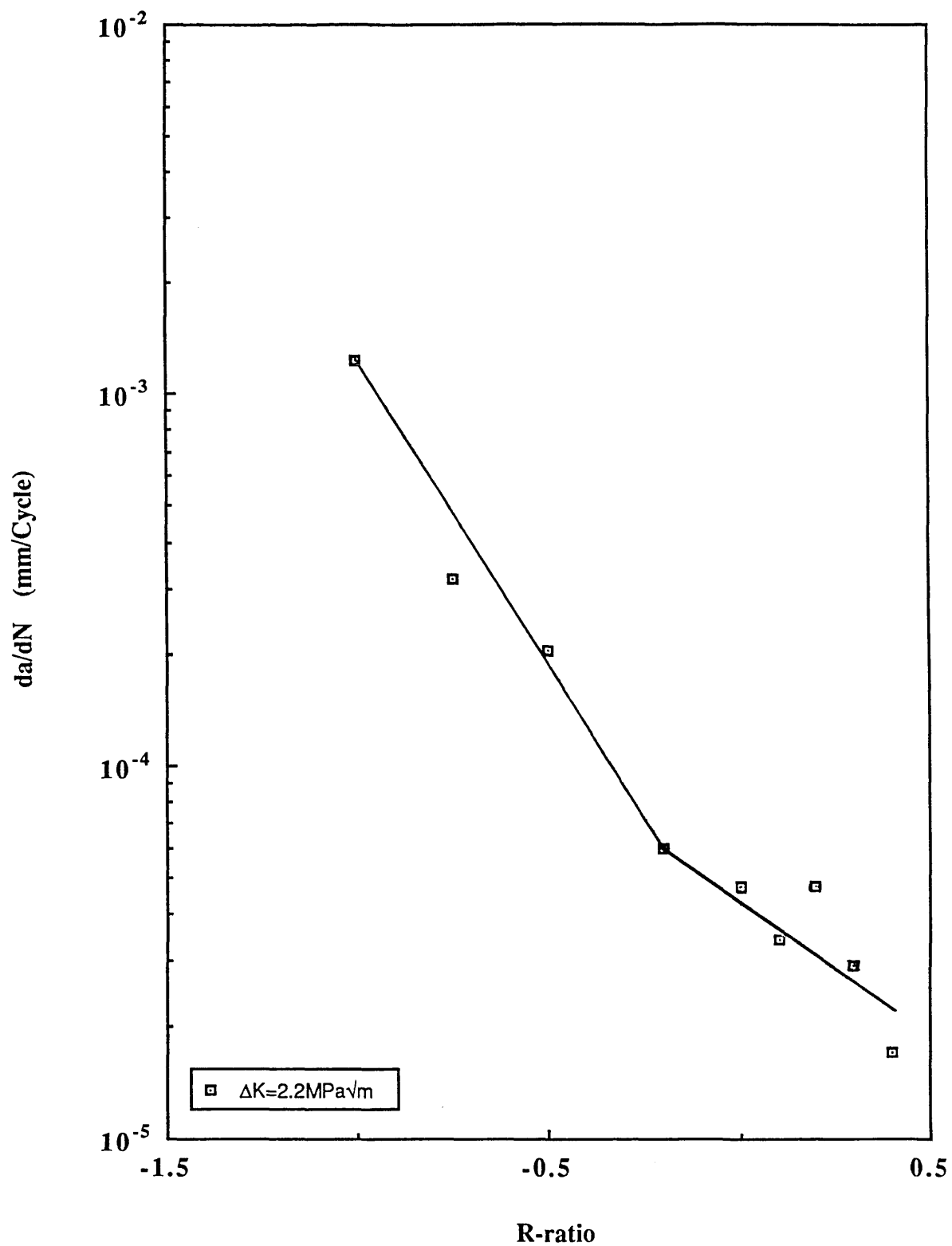


Fig 5.48 FCP rates in LL101AA plotted against R-ratio at a constant ΔK value of 2.20 MPa $\sqrt{m}$.
 $F=3.0\text{Hz}$ - $T=-20^{\circ}\text{C}$

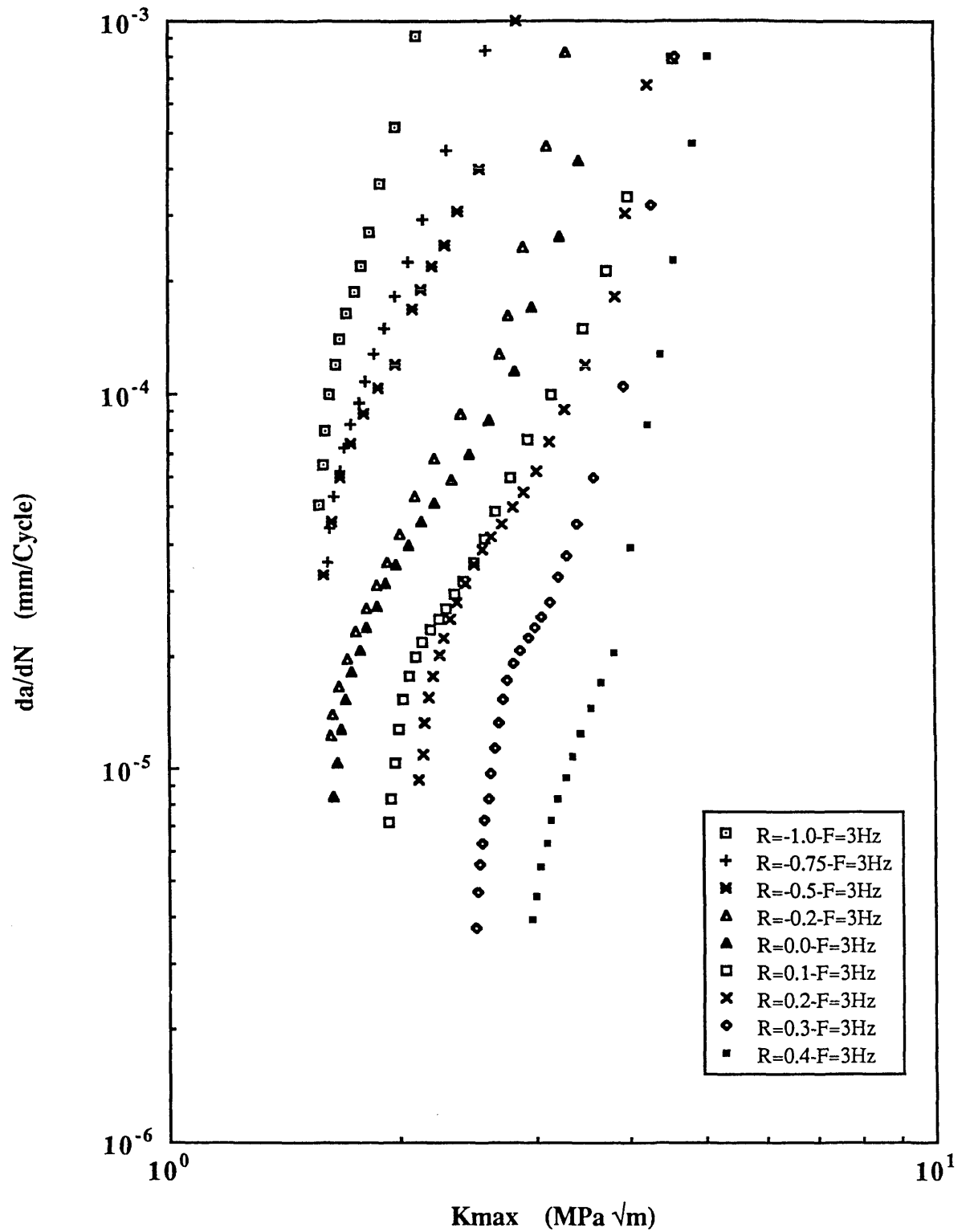


Fig 5.49 FCP rates in LL101AA against Kmax.
T=-20°C

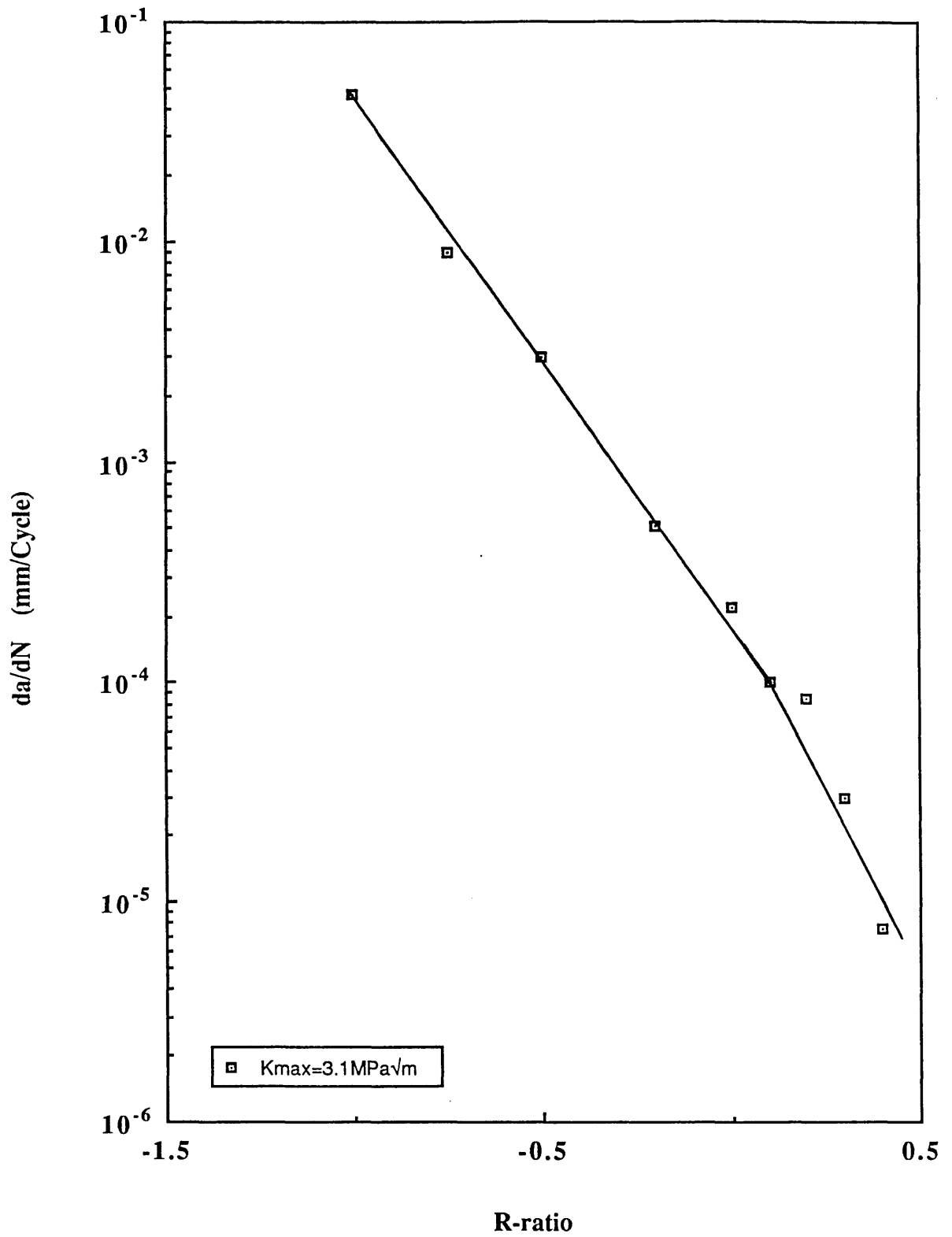


Fig 5.50 FCP rates in LL101AA plotted against R-ratio at a constant K_{max} value of $3.10 \text{ MPa}\sqrt{\text{m}}$.
 $F=3.0\text{Hz}$ - $T=-20^{\circ}\text{C}$

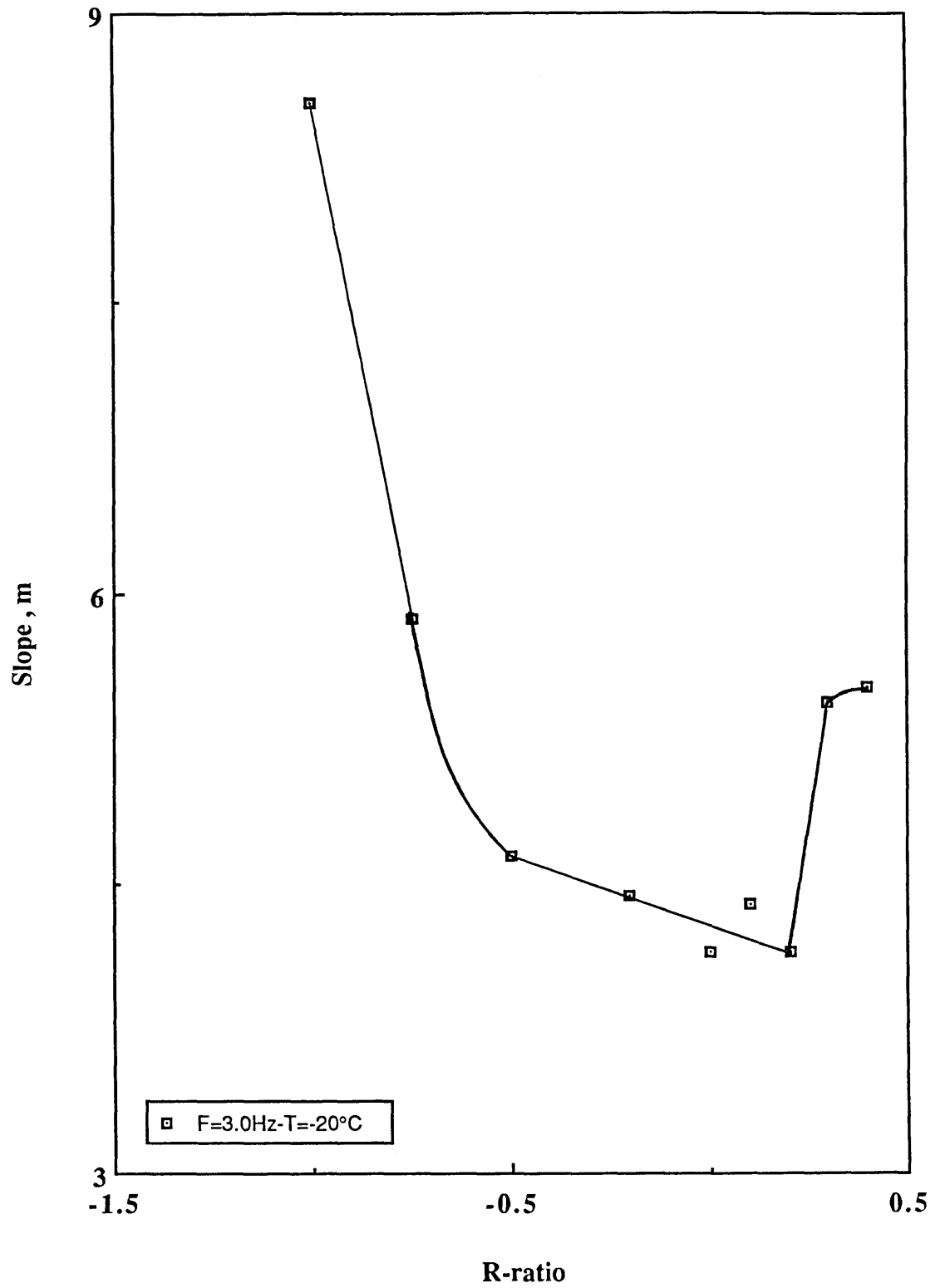


Fig 5.51 The slopes of Paris plots in LL101AA against R-ratio.
F=3.0Hz-T=-20°C

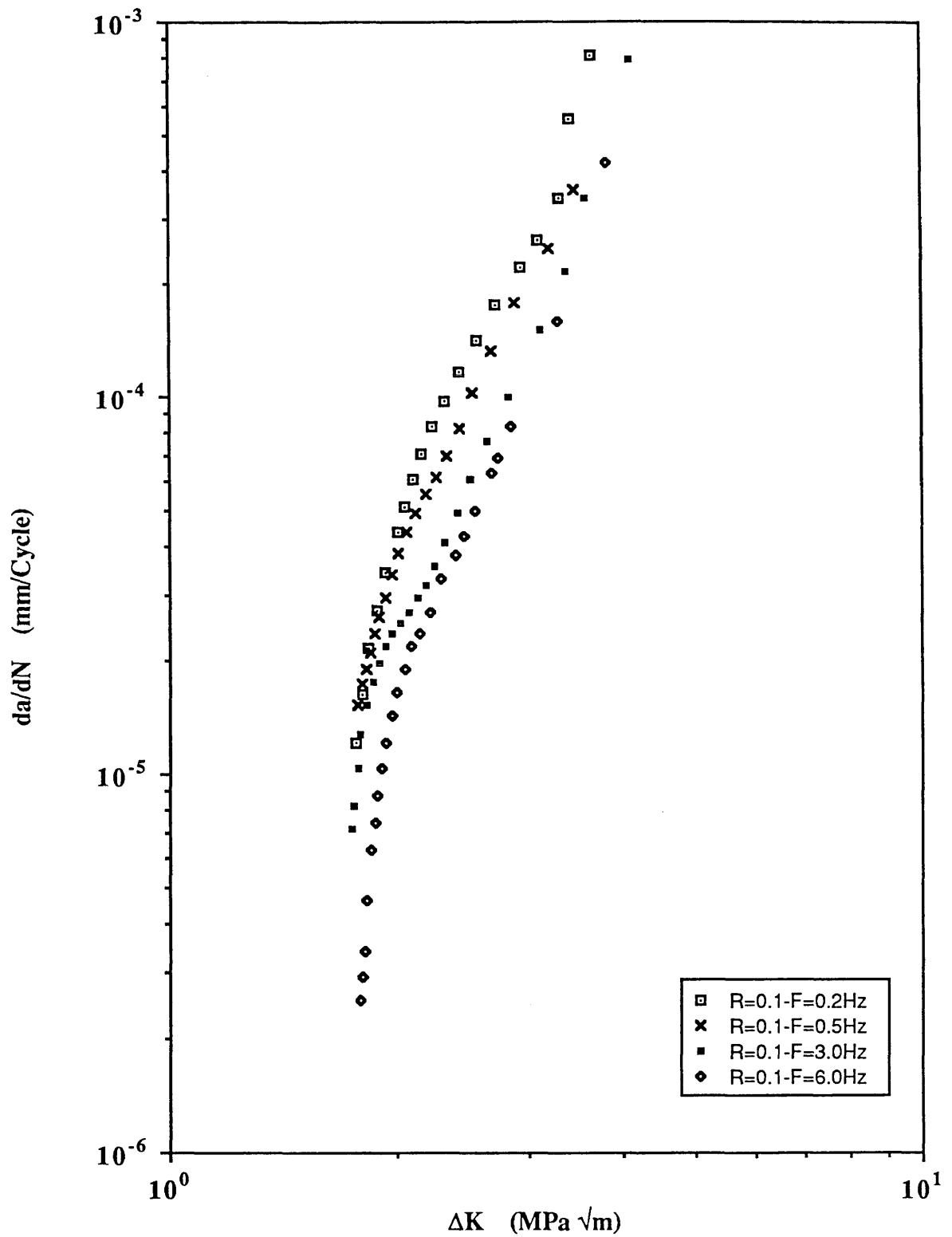


Fig 5.52 Fatigue crack propagation rates in LL101AA at 0.2, 0.5, 3.0 and 6.0Hz.
R-ratio=0.1 - $T=-20^{\circ}\text{C}$

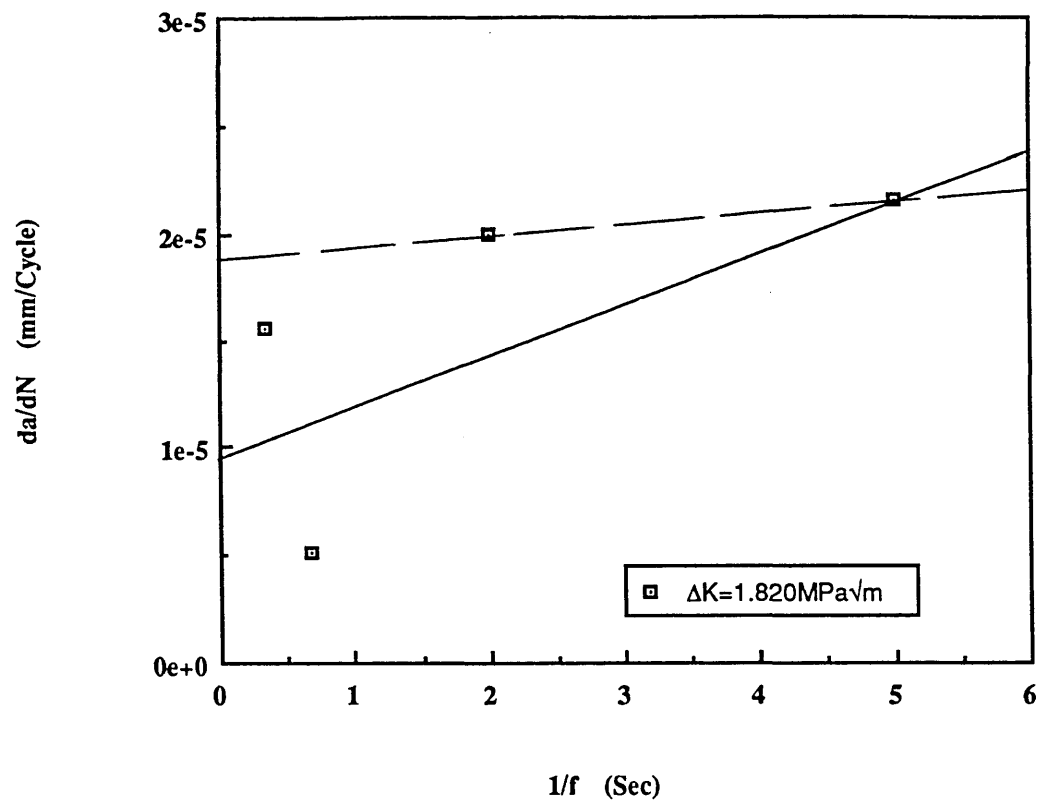


Fig 5.53 (a)

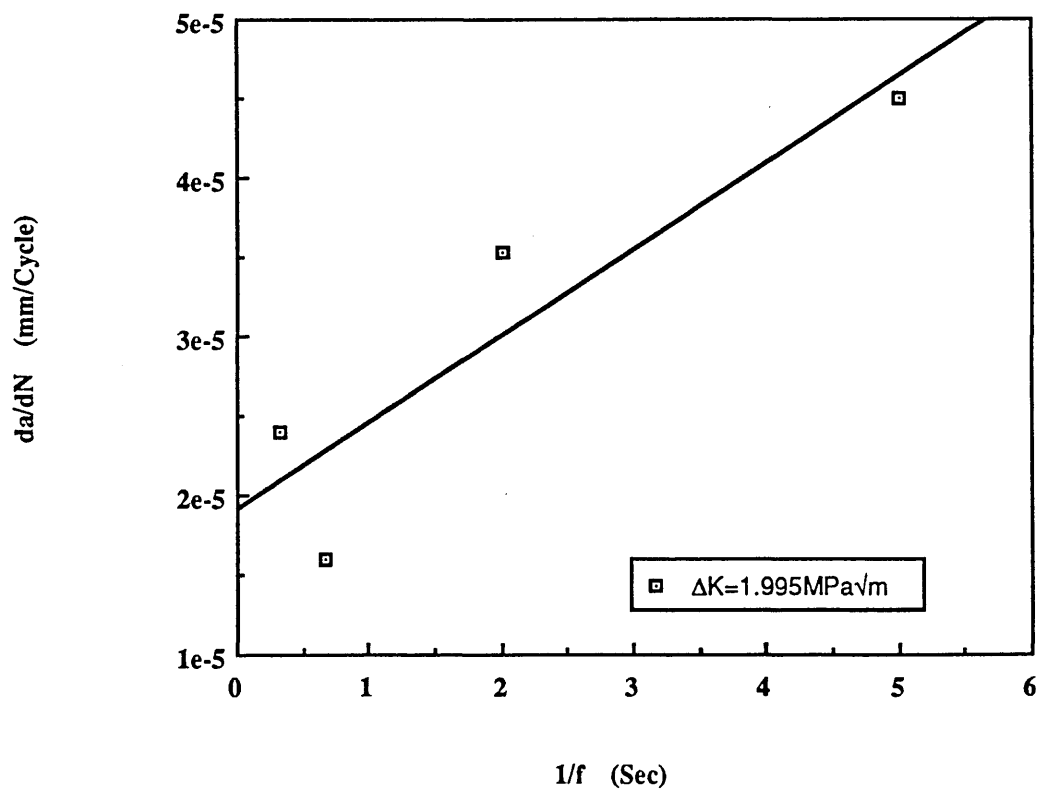


Fig 5.53 (b)

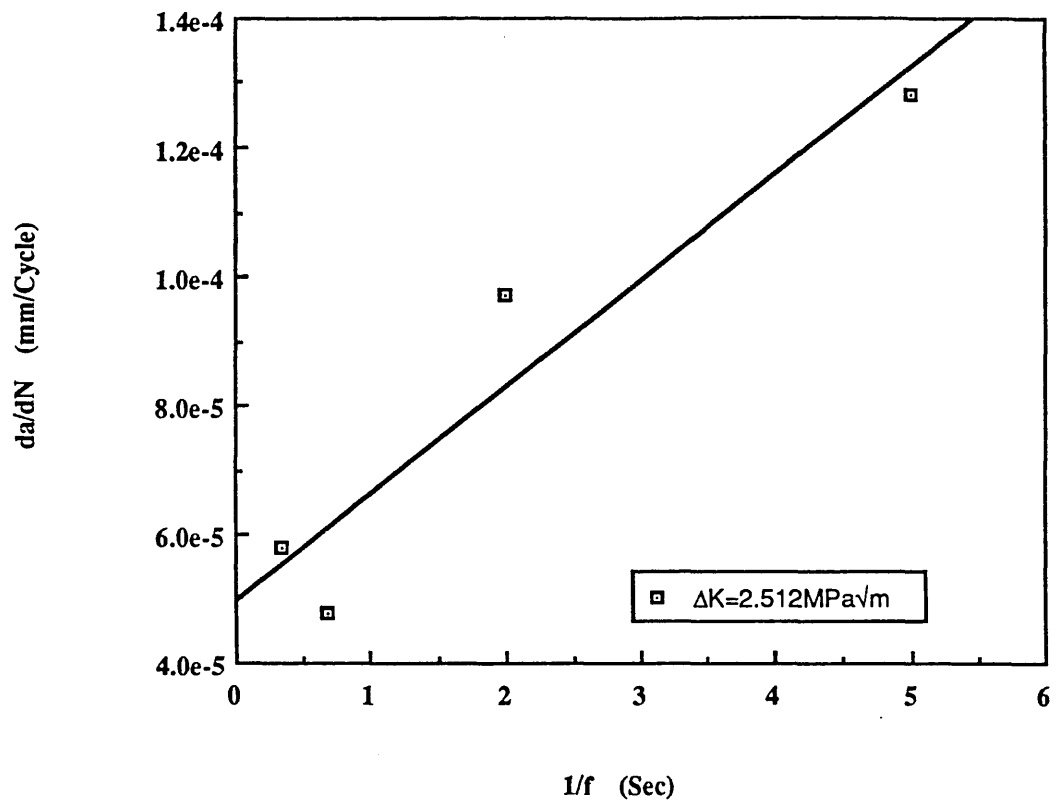


Fig 5.53 (c)

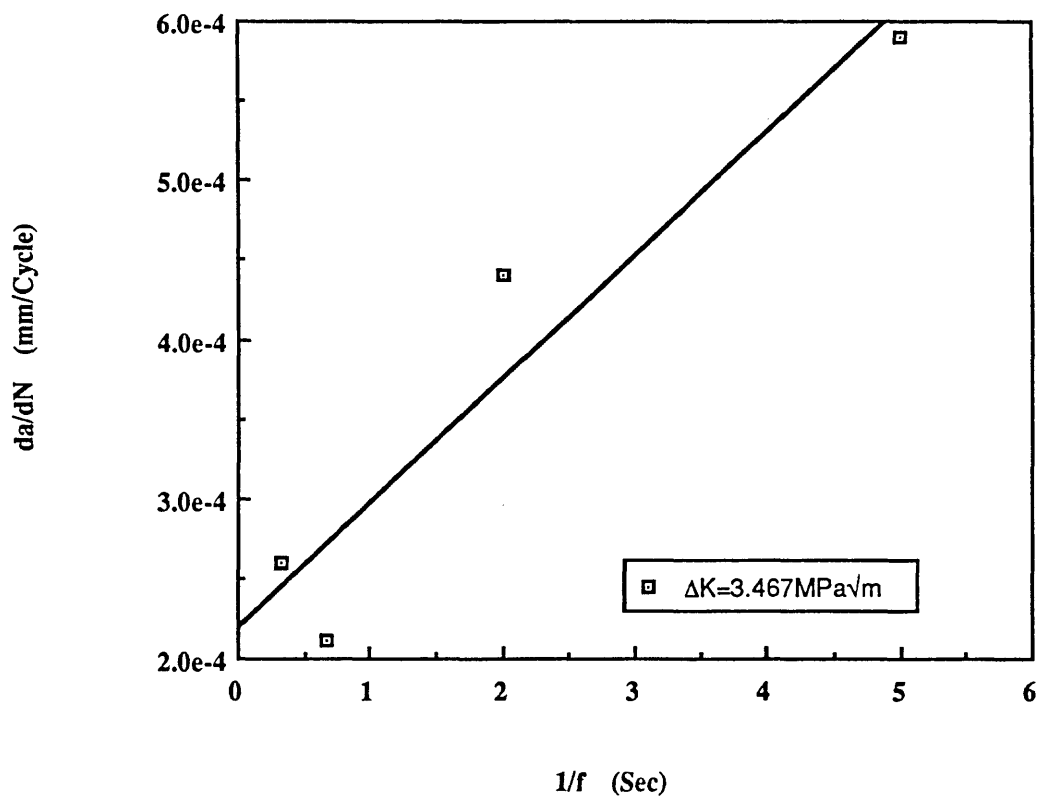


Fig 5.53 (d)

Fig 5.53(a-d) FCP rates in LL101AA against reciprocal frequency at various values of ΔK .
 R-ratio=0.1-T=-20°C

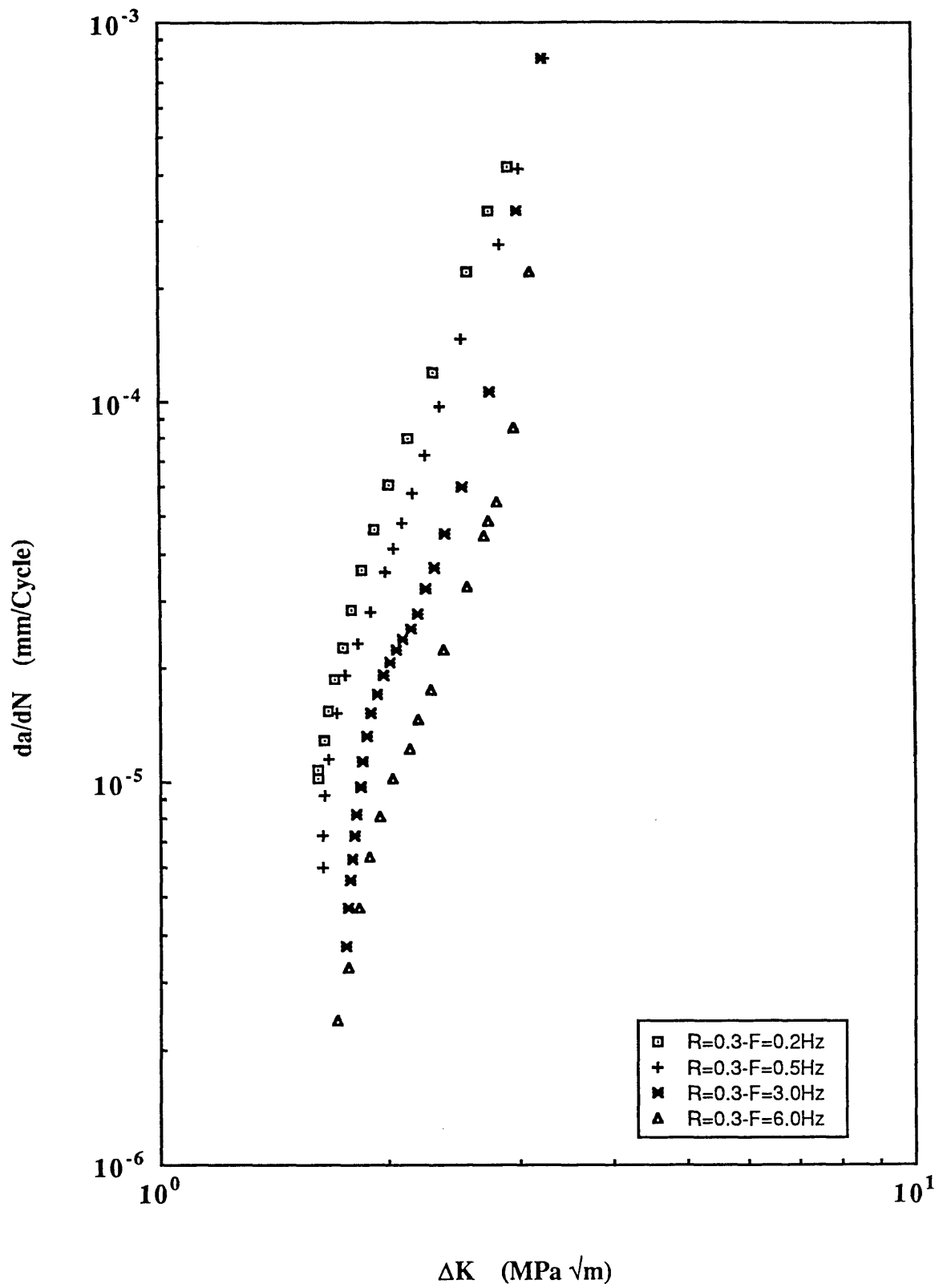


Fig 5.54 Fatigue crack propagation rates in LL101AA at 0.2, 0.5, 3.0 and 6.0Hz.
R-ratio=0.3 - T=-20°C

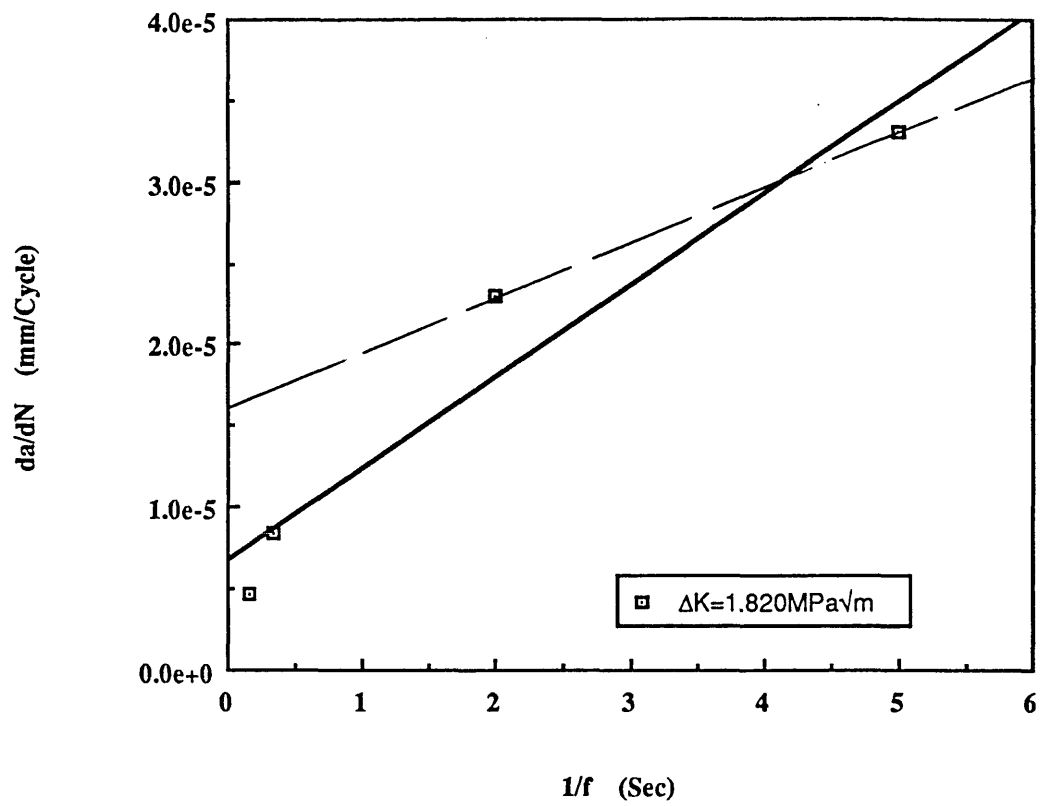


Fig 5.55 (a)

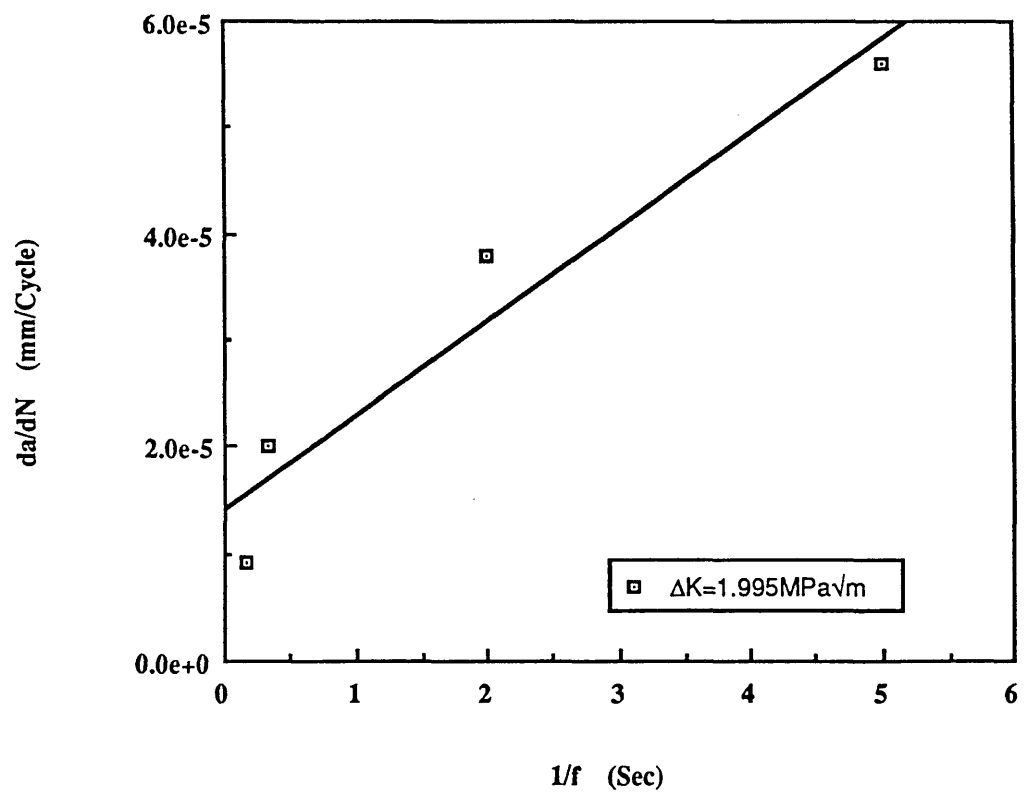


Fig 5.55 (b)

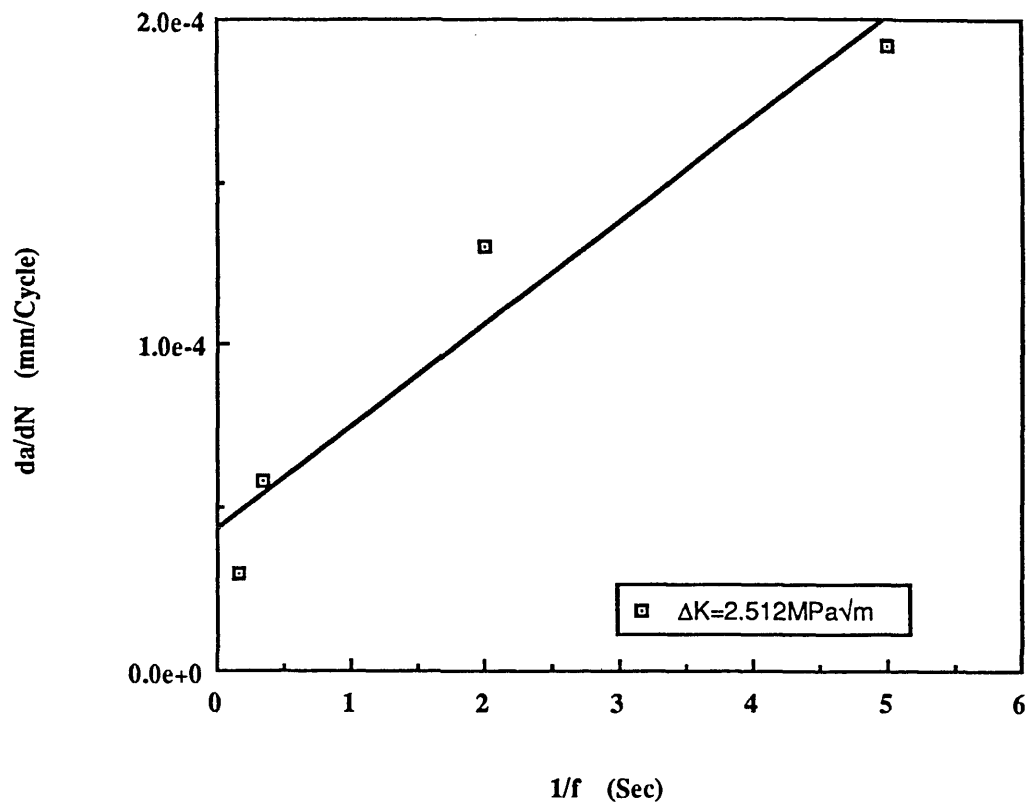


Fig 5.55 (c)

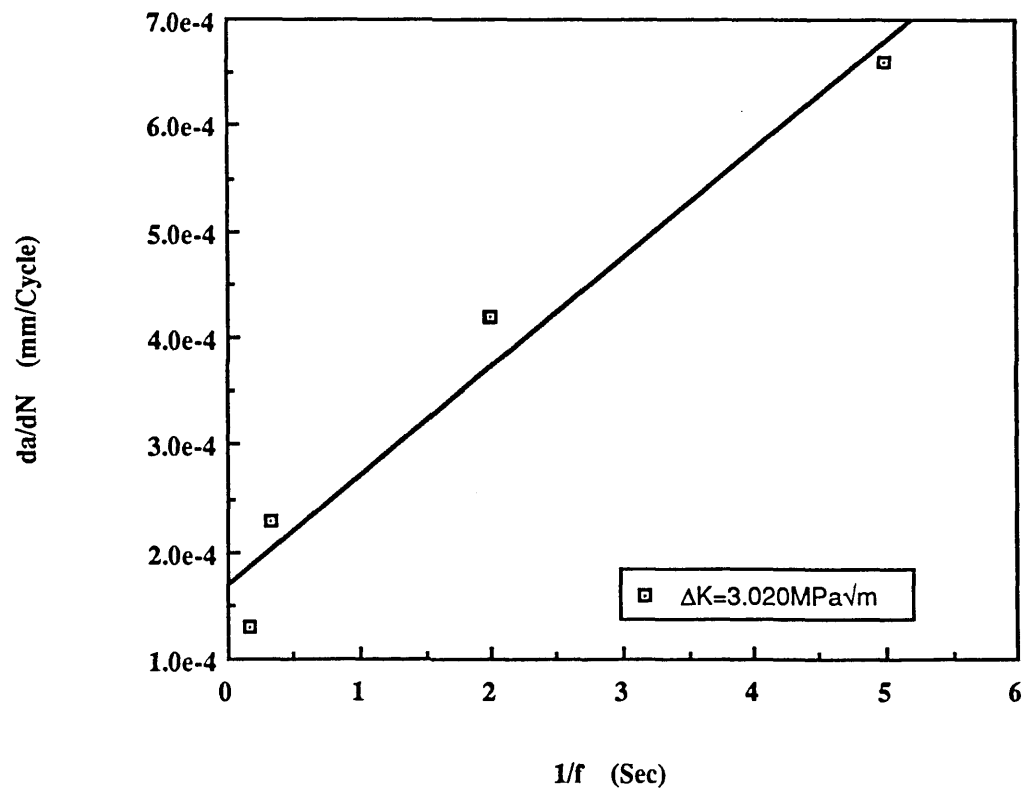


Fig 5.55 (d)

Fig 5.55(a-d) FCP rates in LL101AA against reciprocal frequency at various values of ΔK .
 R-ratio=0.3-T=-20°C

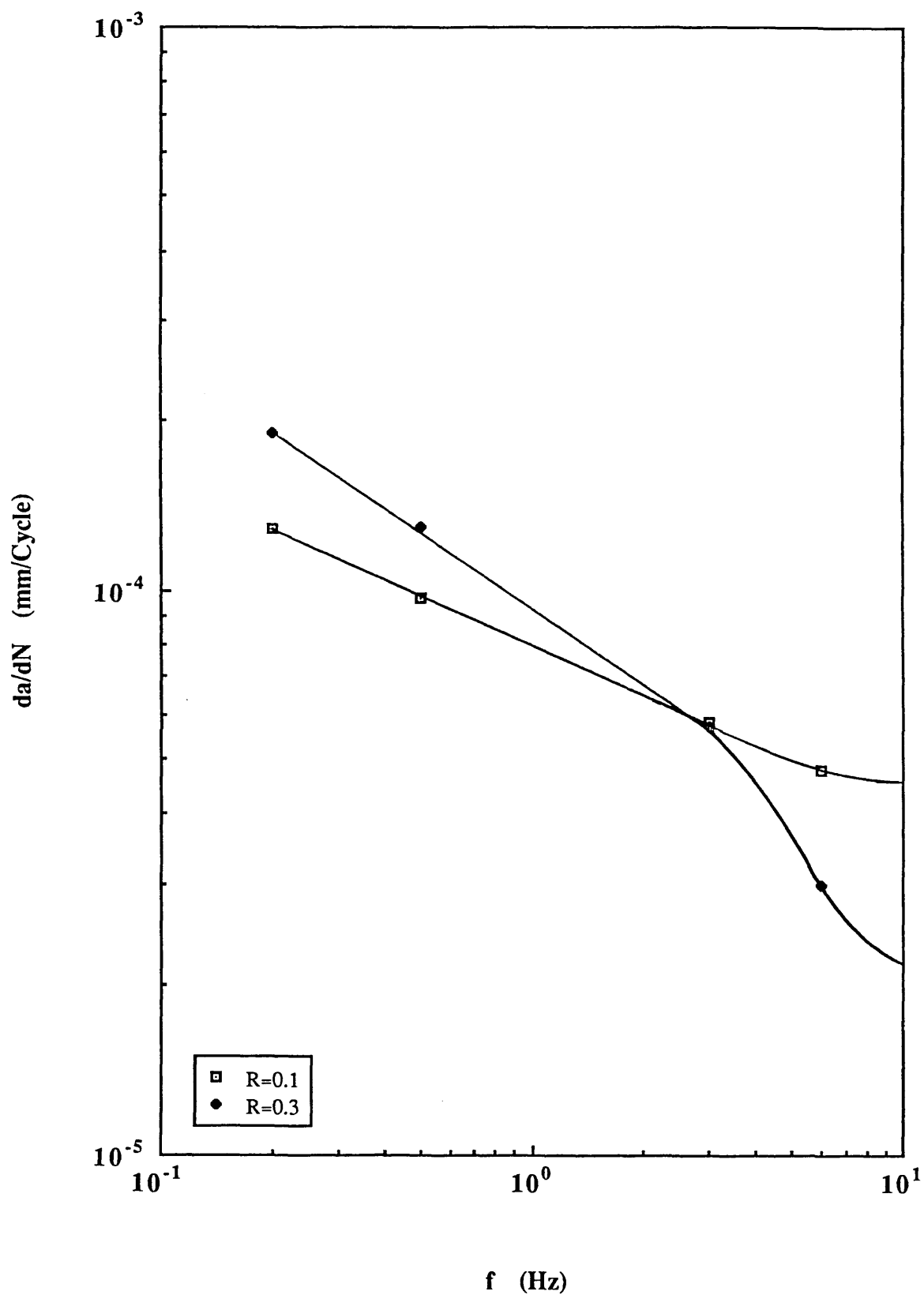


Fig 5.56 FCP rates in LL101AA against frequency at a constant ΔK value of $2.512 \text{ MPa}\sqrt{\text{m}}$.
 $T = -20^\circ\text{C}$

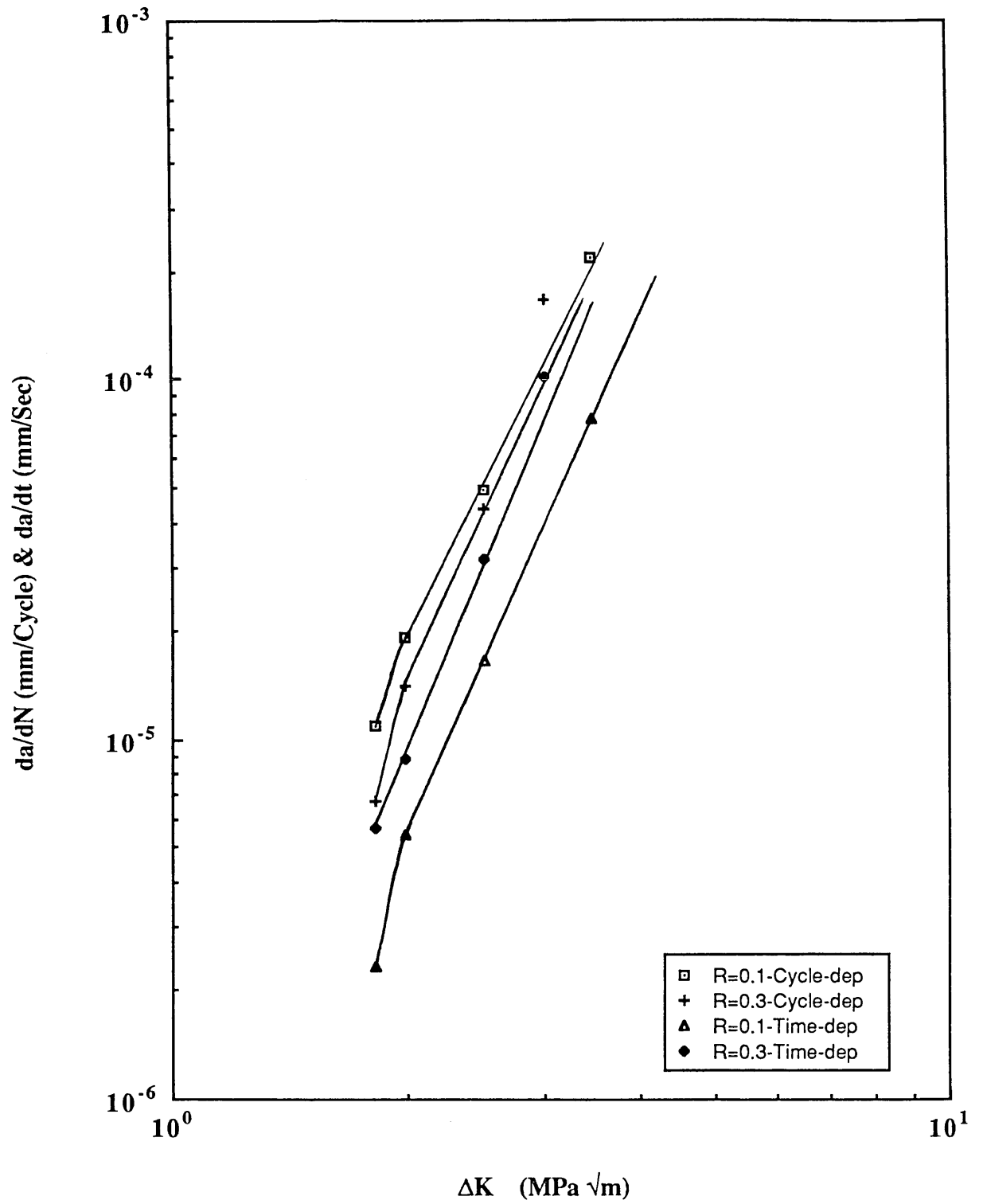


Fig 5.57 FCP rates in LL101AA against ΔK representing Cycle and Time dependent FCP at various R-ratios.
T=-20°C

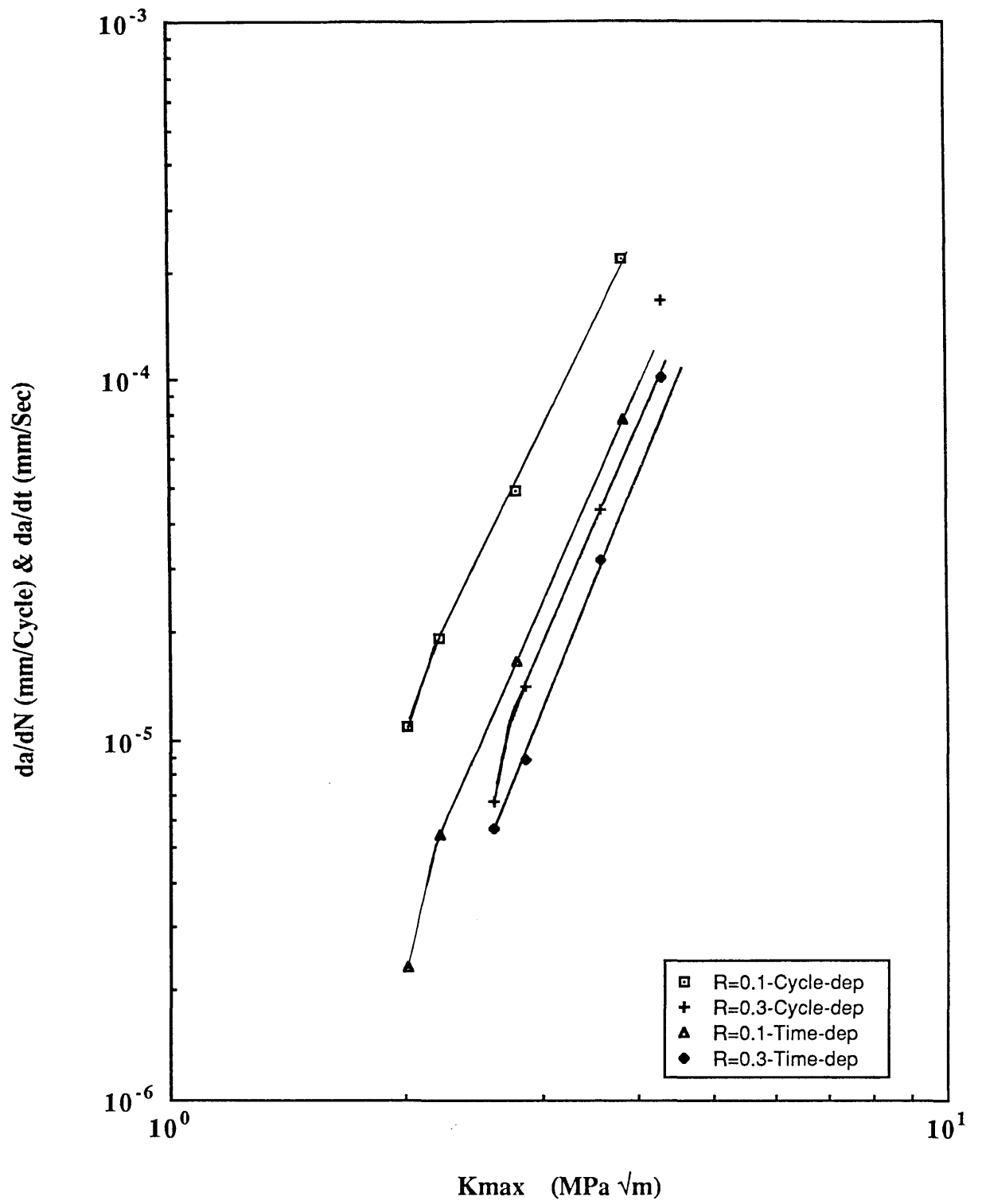


Fig 5.58 FCP rates in LL101AA against K_{max} representing Cycle and Time dependent FCP at various R-ratios.
 $T=-20^{\circ}\text{C}$

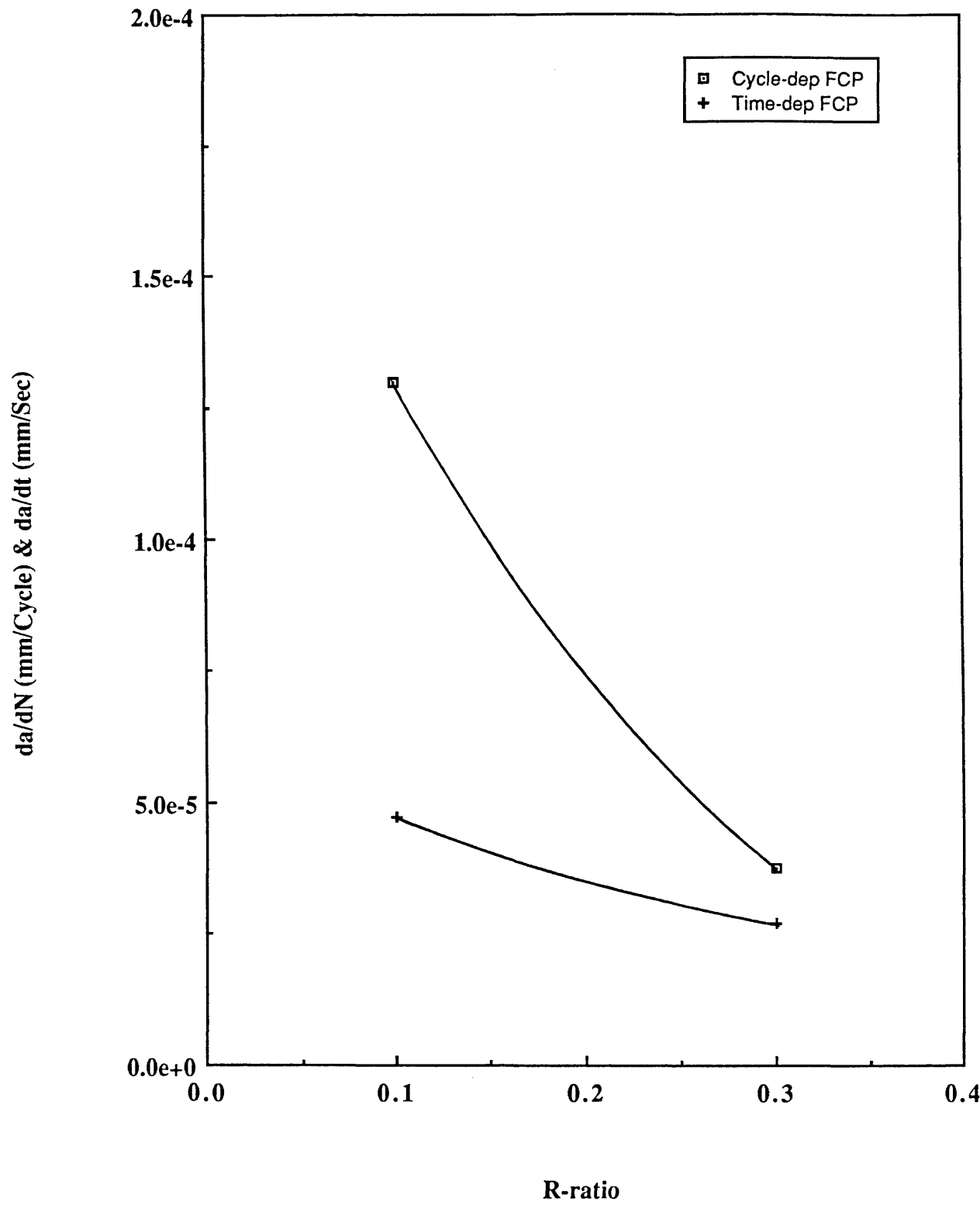


Fig 5.59 The Cycle and Time-dependent components in LL101AA plotted against R-ratio. Frequency=1.0Hz - Kmax=3.467 MPa $\sqrt{m}$

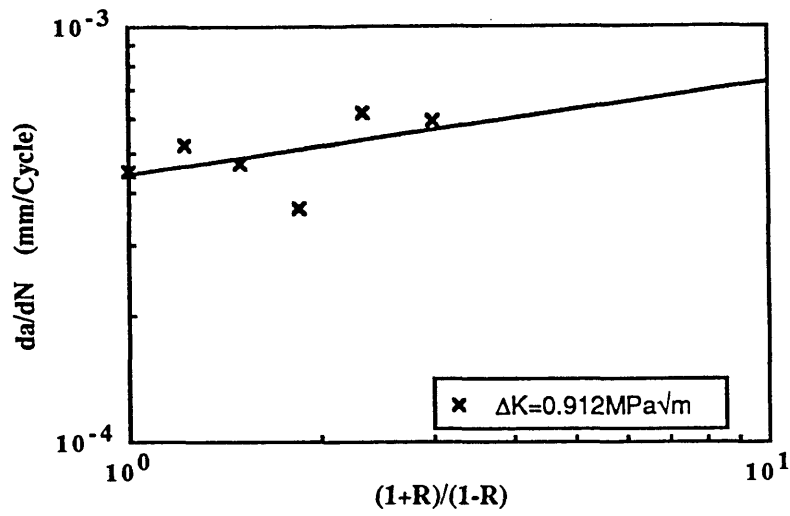


Fig 5.60 da/dN plotted against $(1+R)/(1-R)$ in accordance with the El Hakeem-Culver Model for FCP in BP33. $F=3.0\text{Hz}$ - $T=-20^\circ\text{C}$

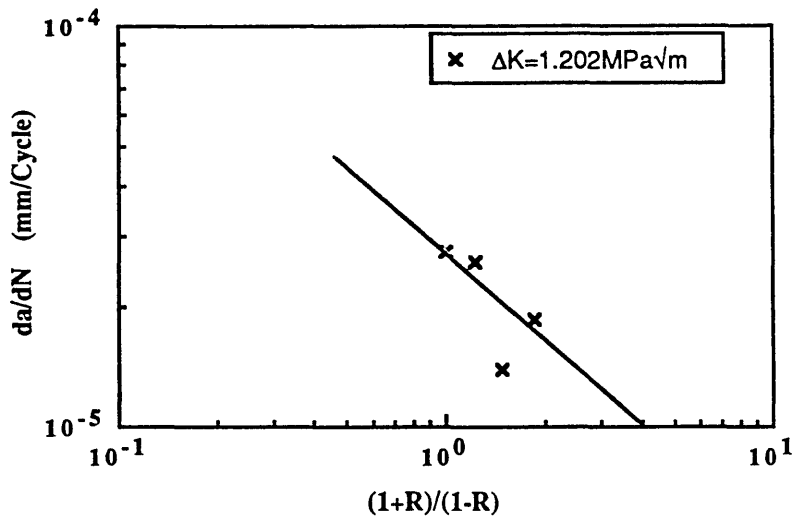


Fig 5.61 da/dN plotted against $(1+R)/(1-R)$ in accordance with the El Hakeem-Culver Model for FCP in LL101AA. $F=3.0\text{Hz}$ - $T=+23^\circ\text{C}$

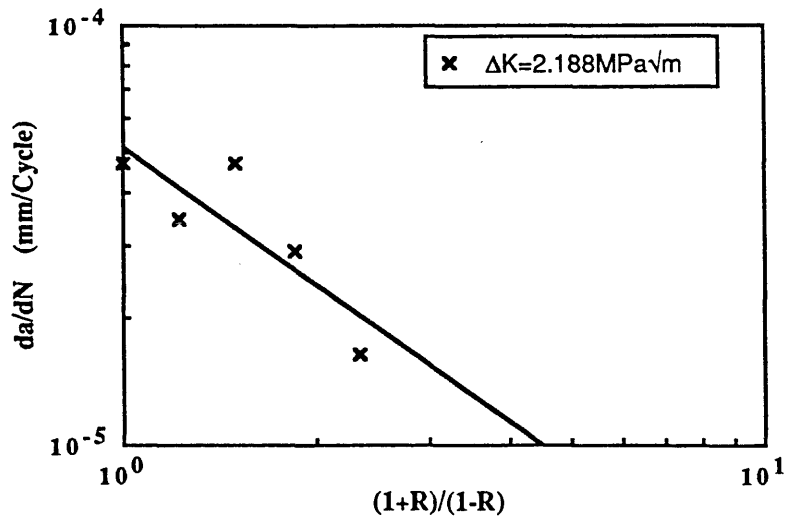


Fig 5.62 da/dN plotted against $(1+R)/(1-R)$ in accordance with the El Hakeem-Culver Model for FCP in LL101AA. $F=3.0\text{Hz}$ - $T=-20^\circ\text{C}$

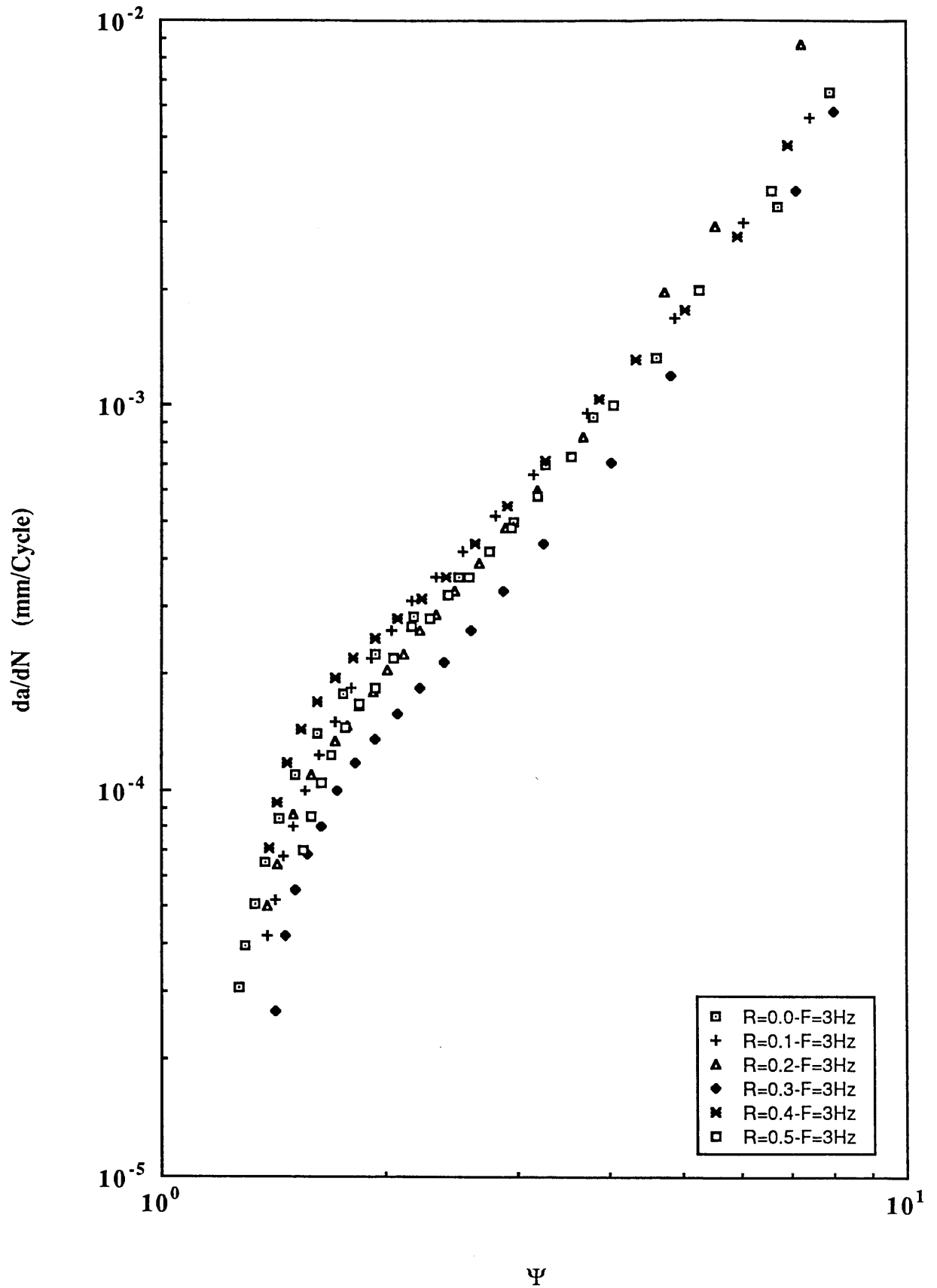


Fig 5.63 da/dN plotted against the factor Ψ in accordance with the El Hakeem-Culver Model for FCP in BP33.
 $F=3.0\text{Hz}$ - $T=-20^\circ\text{C}$

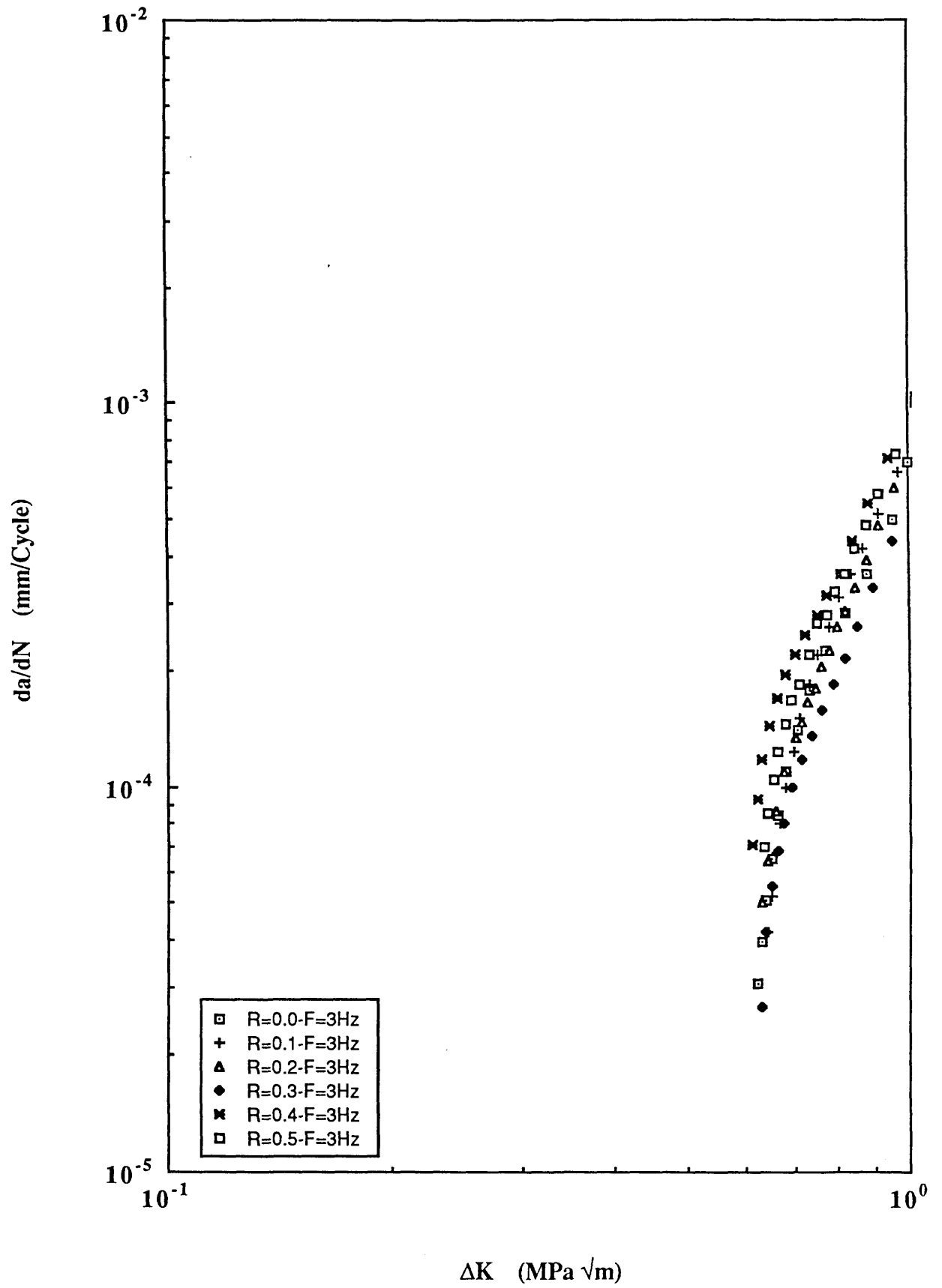


Fig 5.64 Paris representation of FCP rates in BP33.
F=3.0Hz-T=-20°C

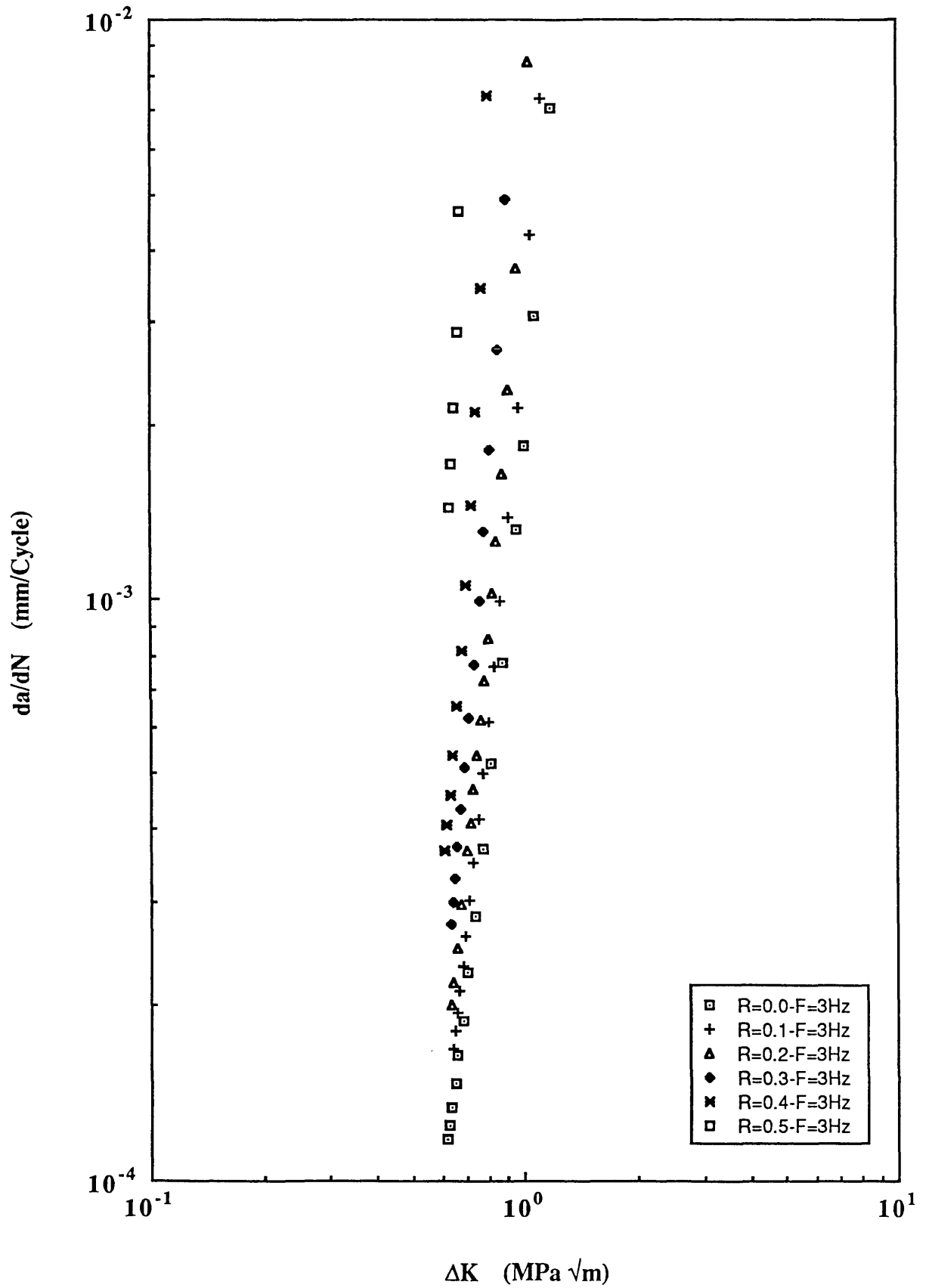


Fig 5.65 FCP rates in BP33 plotted in accordance with the Forman Model.
 $F=3.0Hz$ - $T=-20^{\circ}C$

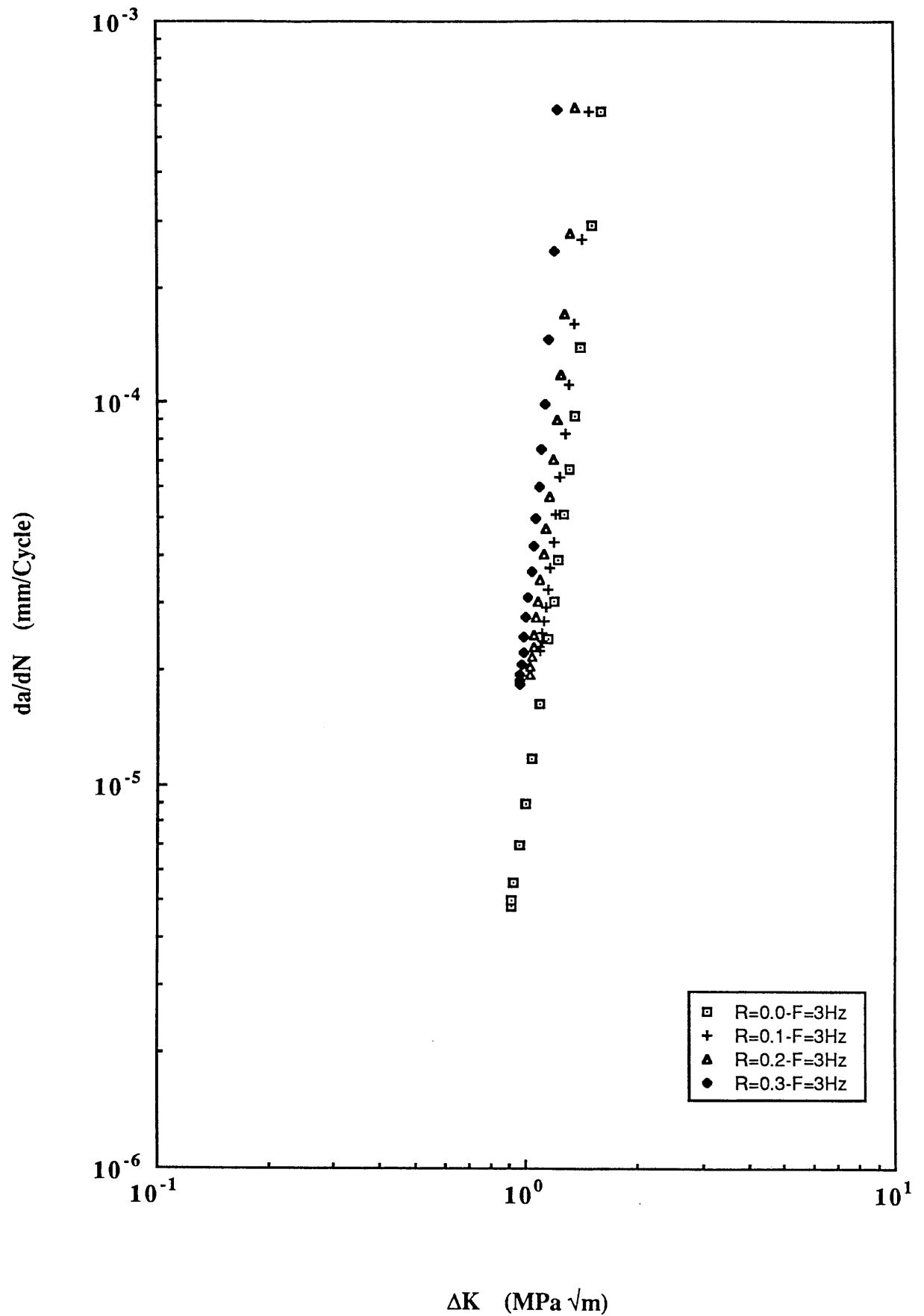


Fig 5.66 FCP rates in LL101AA plotted in accordance with the Forman Model.
F=3.0Hz-T=+23°C

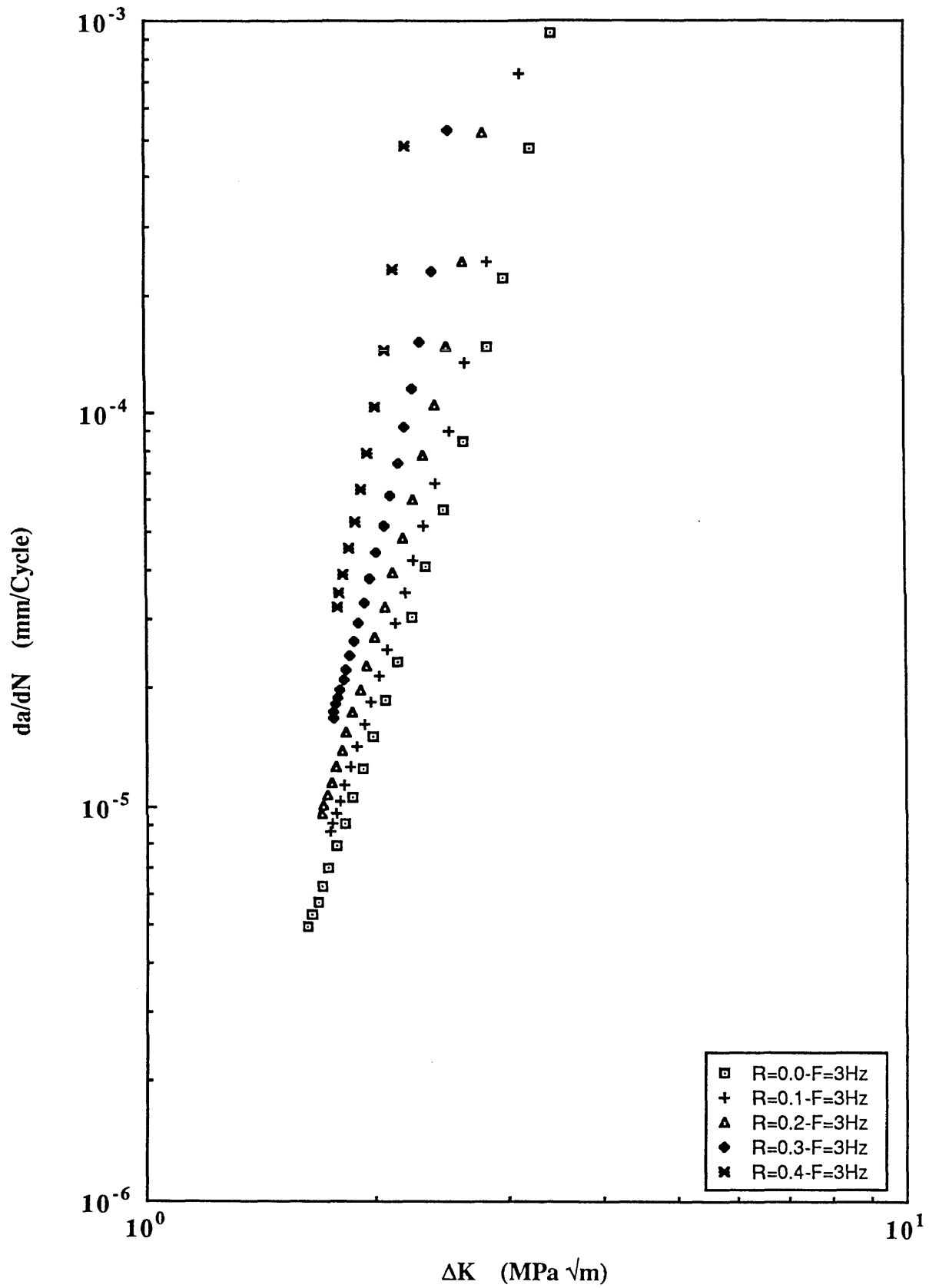


Fig 5.67 FCP rates in LL101AA plotted in accordance with the Forman Model.
F=3.0Hz-T=-20°C

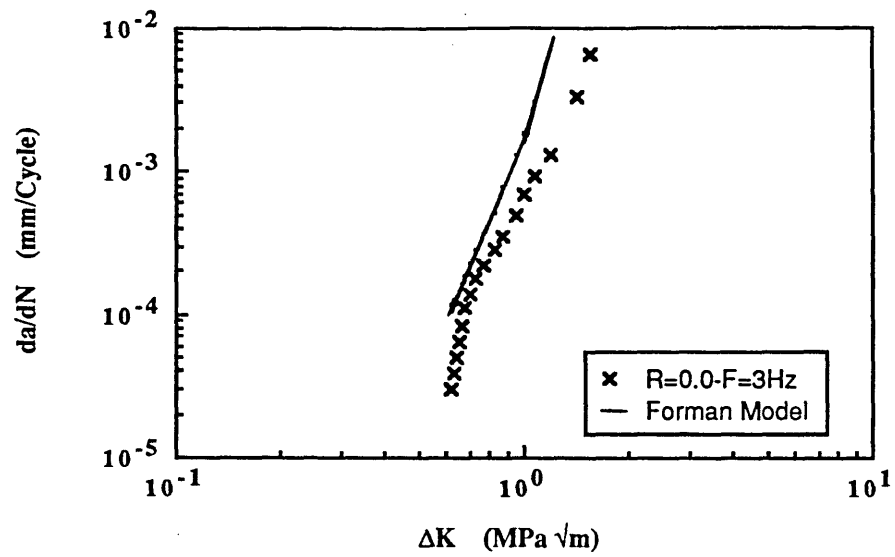


Fig 5.68 (a)

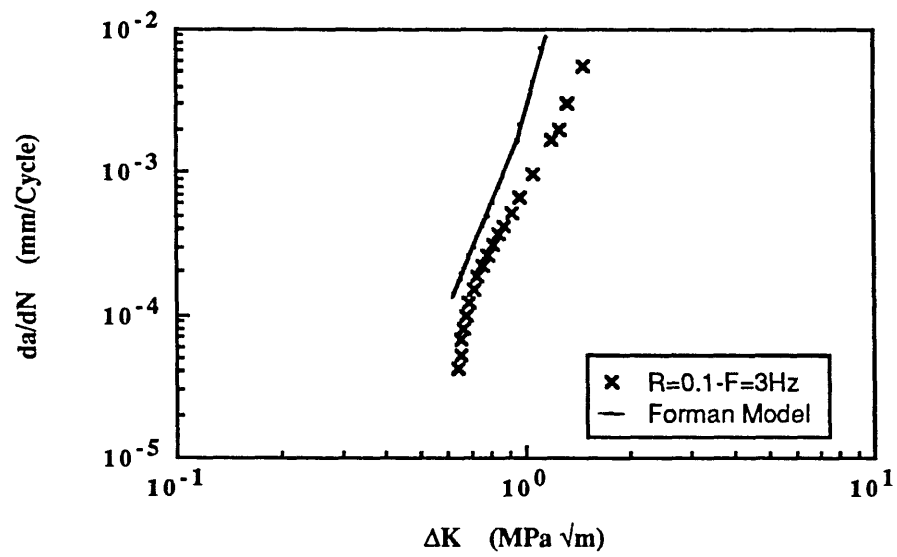


Fig 5.68 (b)

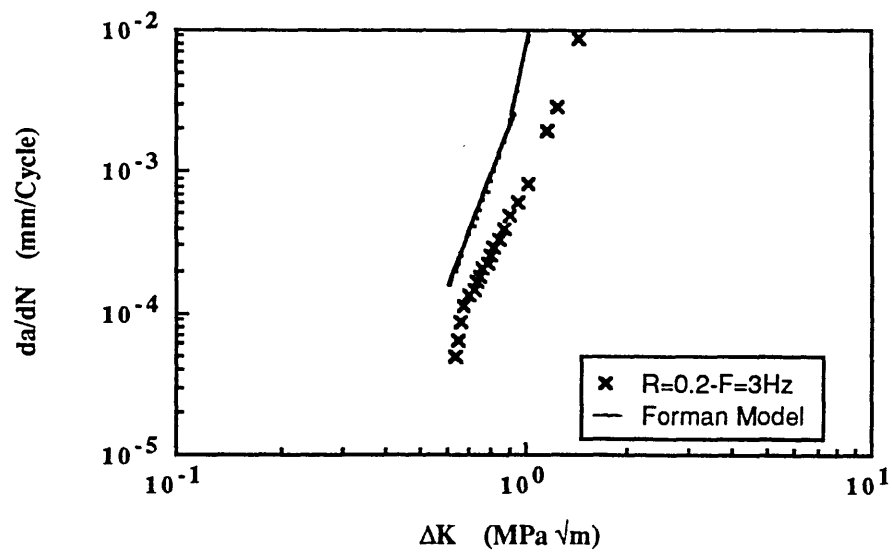


Fig 5.68 (c)

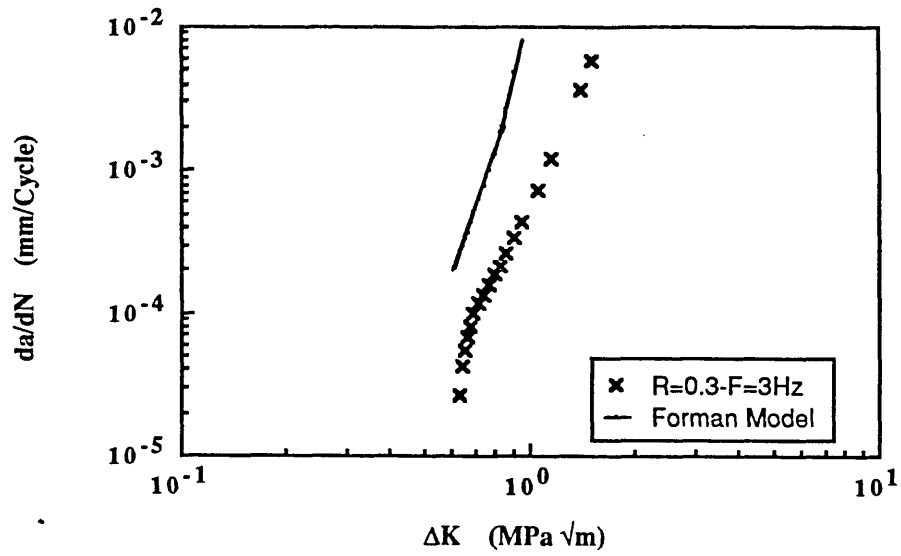


Fig 5.68 (d)

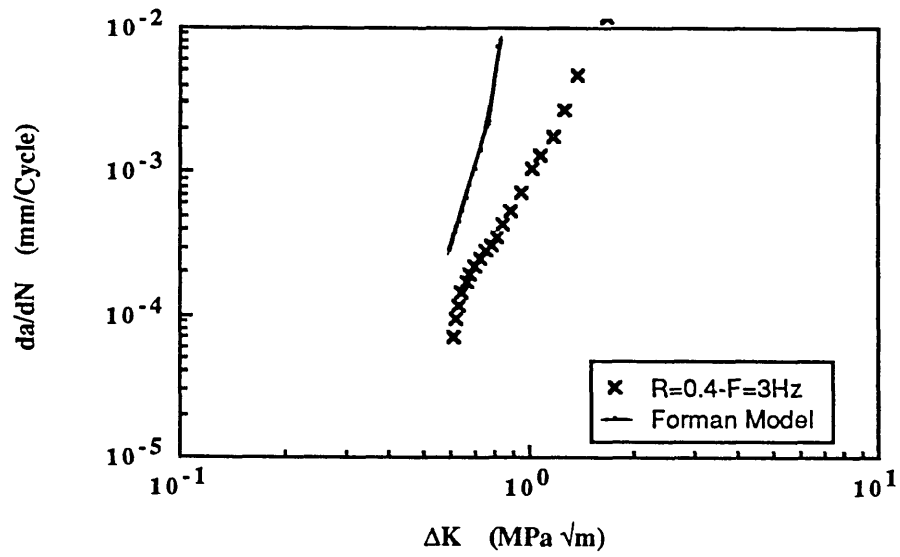


Fig 5.68 (e)

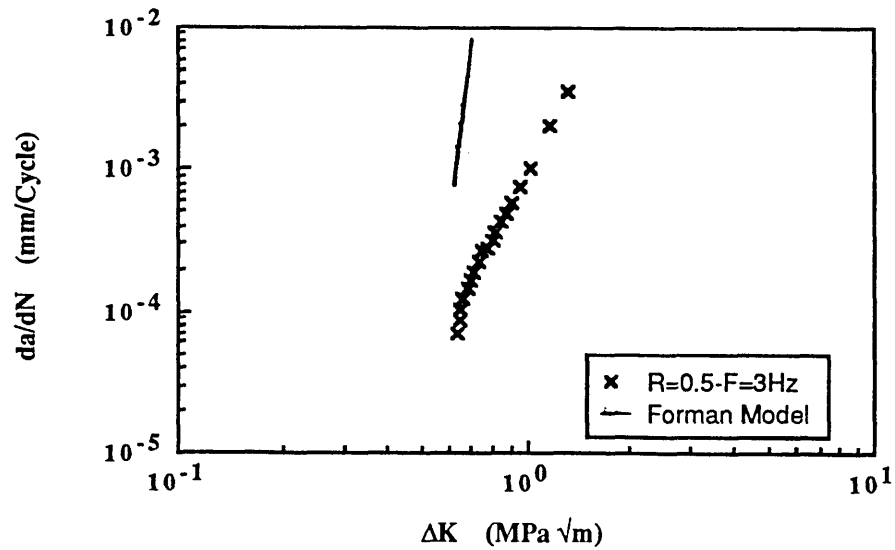


Fig 5.68 (f)

Fig 5.68 (a-f) FCP rates in BP33 predicted by the Forman Model compared with experimental data for R-ratios between 0.0 and 0.5.
 $F=3.0\text{Hz}$ - $T=-20^\circ\text{C}$

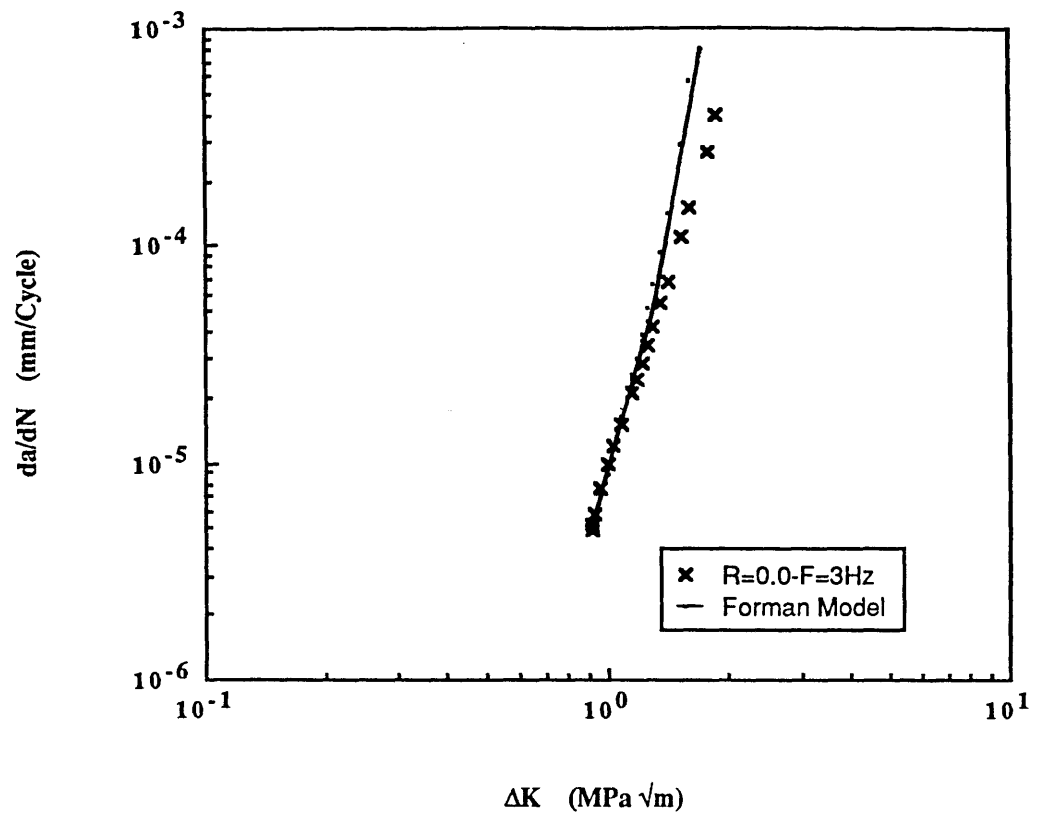


Fig 5.69 (a)

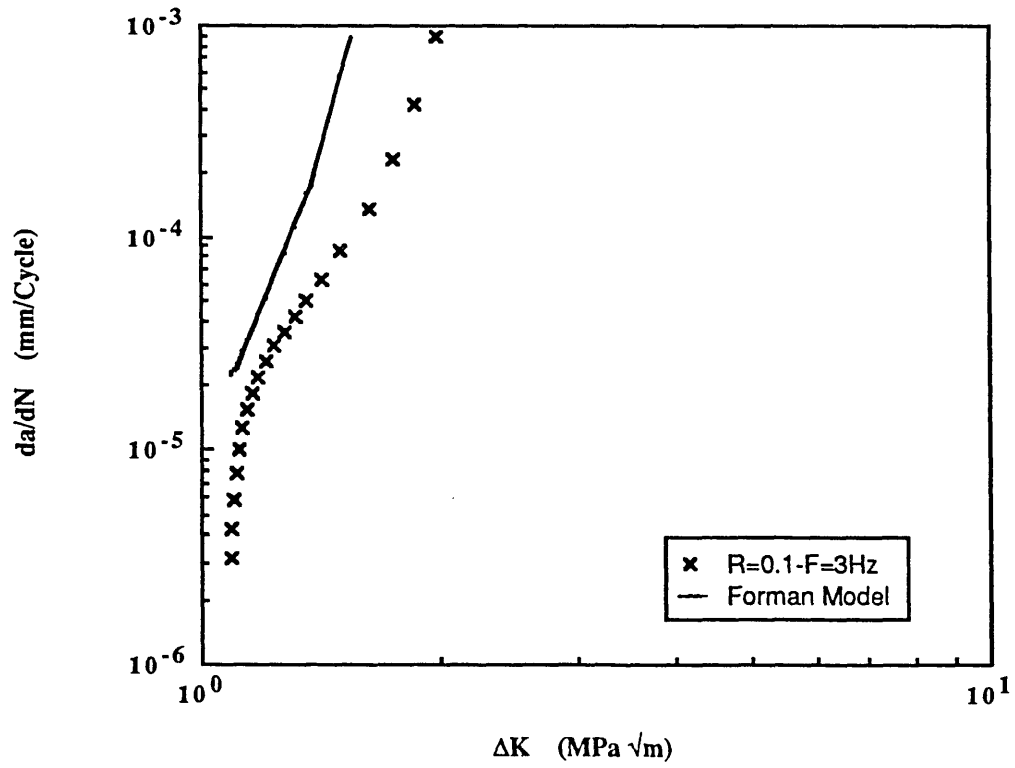


Fig 5.69 (b)

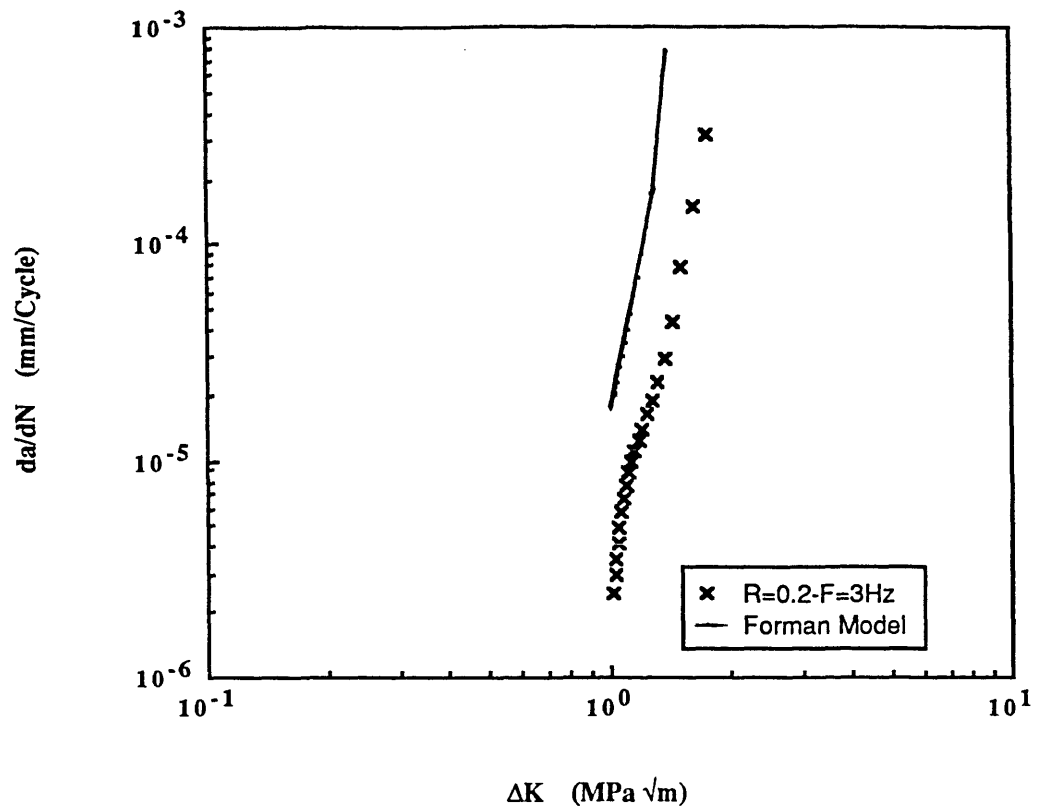


Fig 5.69 (c)

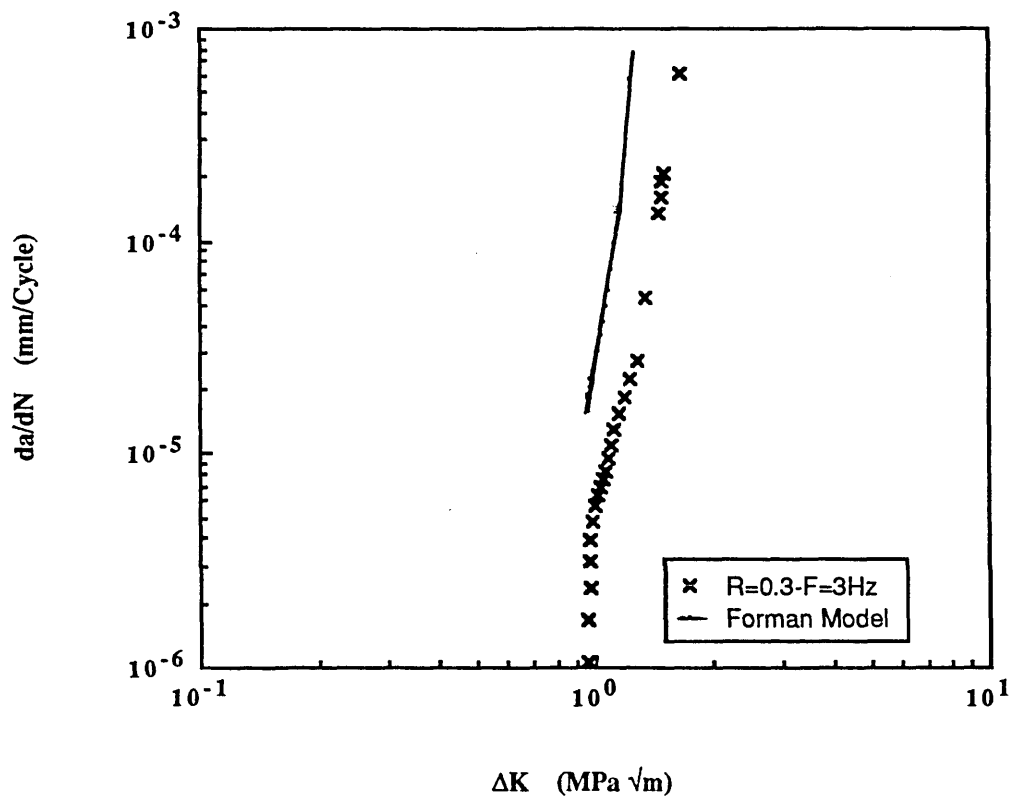


Fig 5.69 (d)

Fig 5.69 (a-d) FCP rates in LL101AA predicted by the Forman Model compared with experimental data for R-ratios between 0.0 and 0.3. F=3.0Hz-T=+23°C

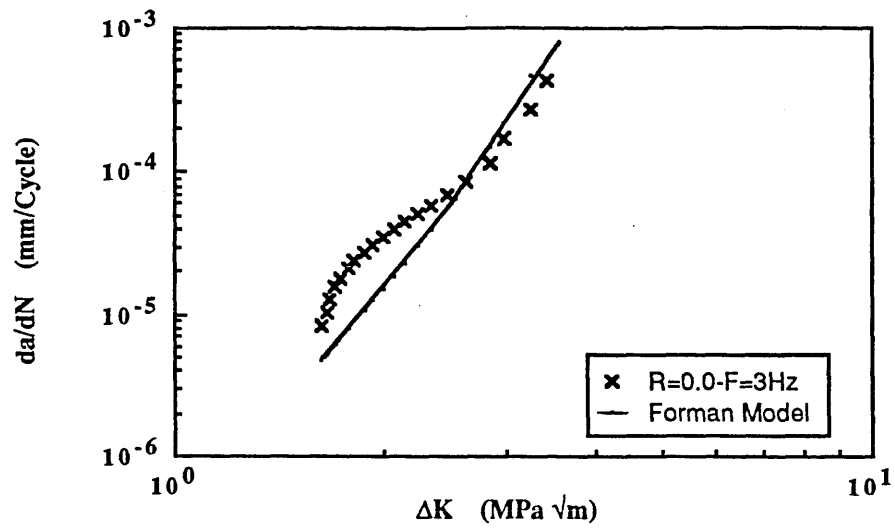


Fig 5.70 (a)

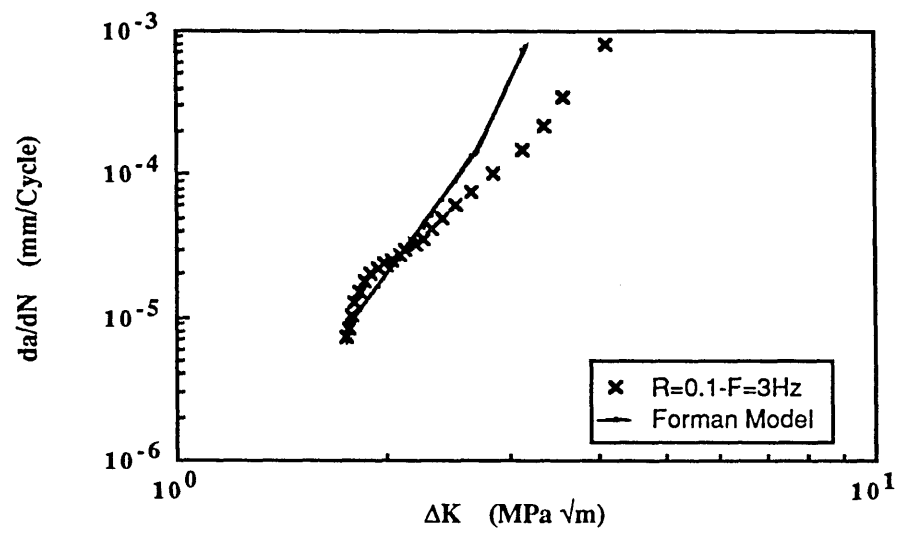


Fig 5.70 (b)

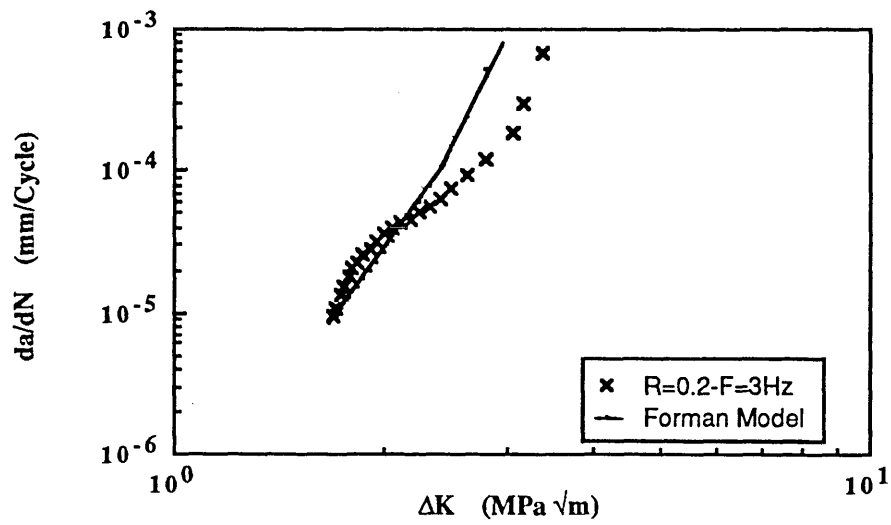


Fig 5.70 (c)

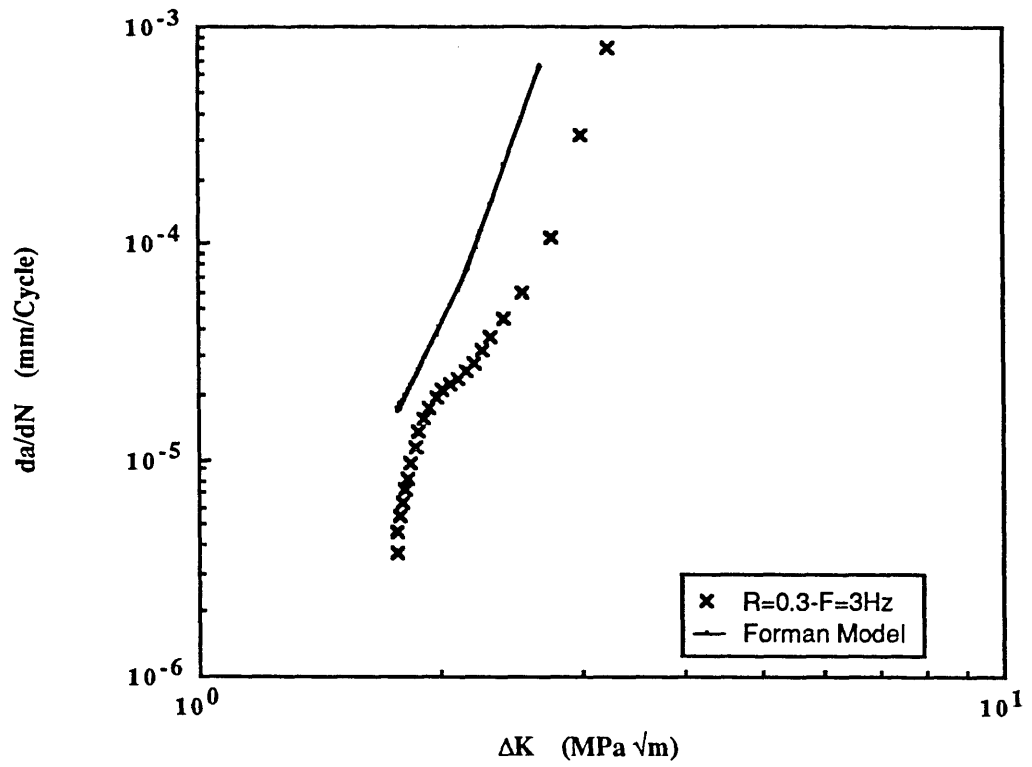


Fig 5.70 (d)

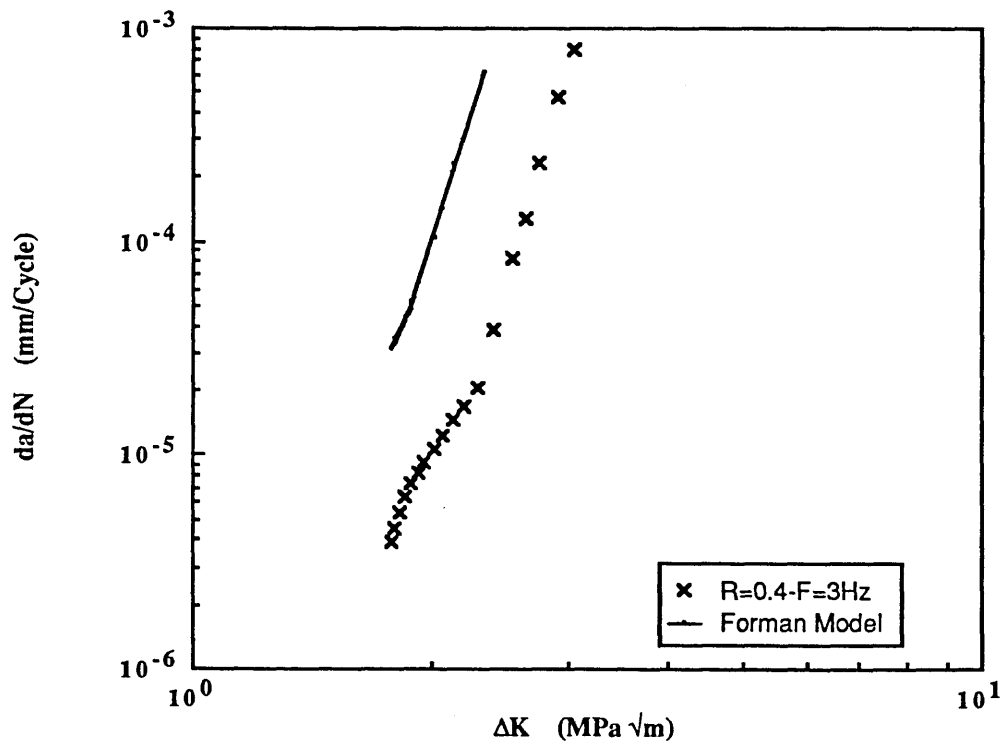


Fig 5.70 (e)

Fig 5.70 (a-e) FCP rates in LL101AA predicted by the Forman Model compared with experimental data for R-ratios between 0.0 and 0.4. F=3.0Hz-T=-20°C

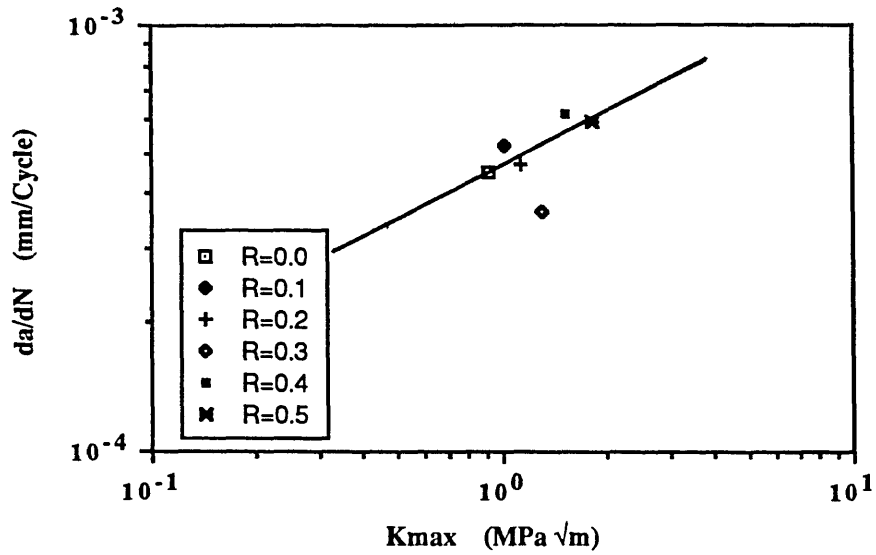


Fig 5.71 (a) BP33- $T = -20^\circ\text{C}$. ΔK constant = $0.912 \text{ MPa } \sqrt{m}$.

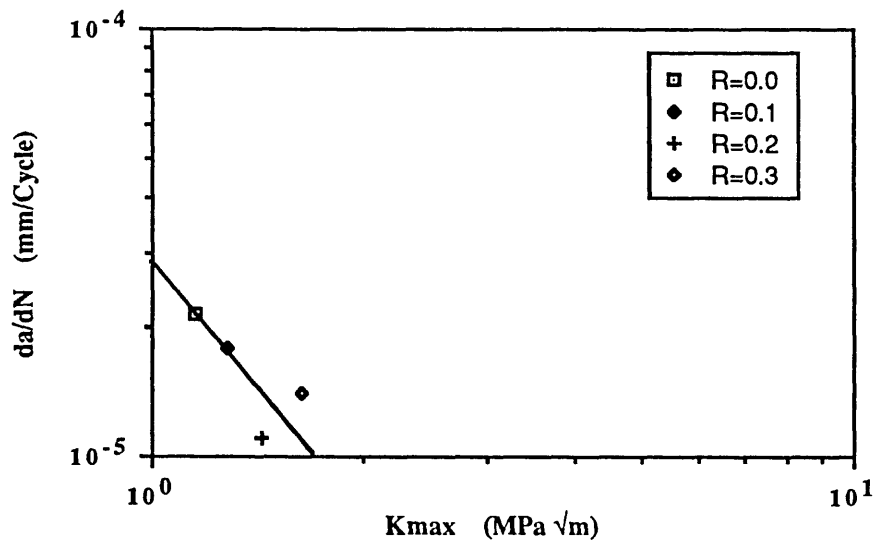


Fig 5.71 (b) LL101AA- $T = +23^\circ\text{C}$. ΔK constant = $1.148 \text{ MPa } \sqrt{m}$.

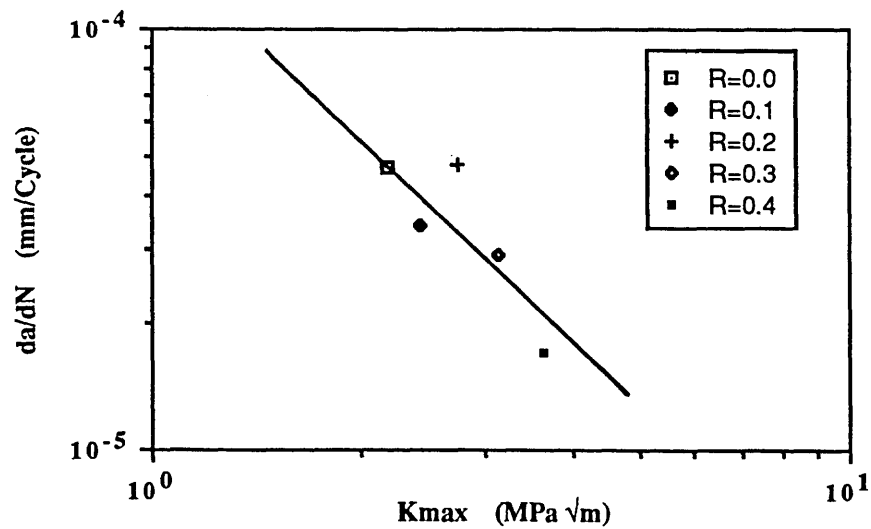


Fig 5.71 (c) LL101AA- $T = -20^\circ\text{C}$. ΔK constant = $2.188 \text{ MPa } \sqrt{m}$.

Fig 5.71 (a-c) FCP rates plotted against K_{max} in accordance with the Walker Model. $F = 3.0 \text{ Hz}$

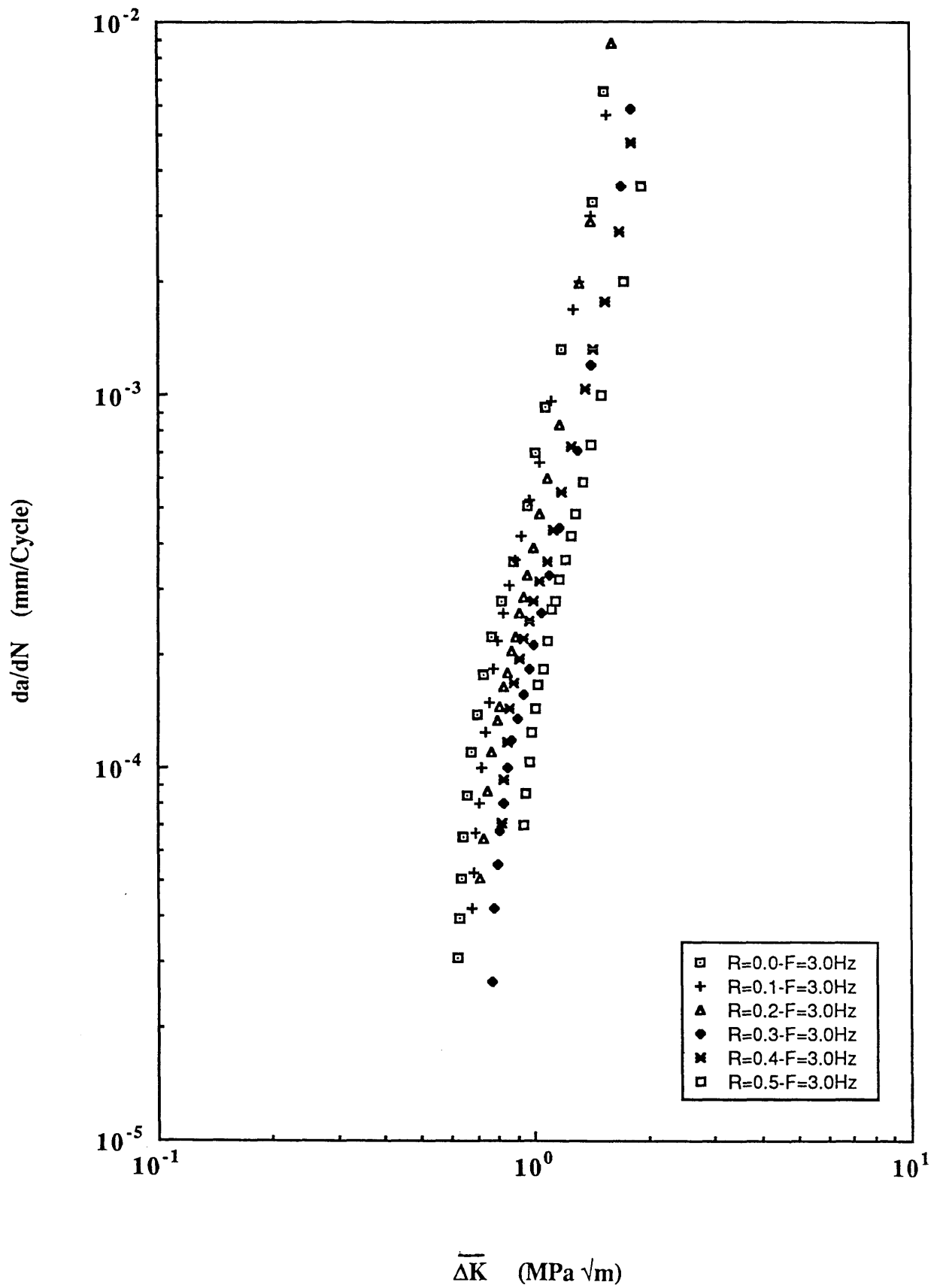


Fig 5.72 FCP rates in BP33 plotted against ΔK in accordance with the Walker Model.
 $F=3.0\text{Hz}$ - $T=-20^\circ\text{C}$

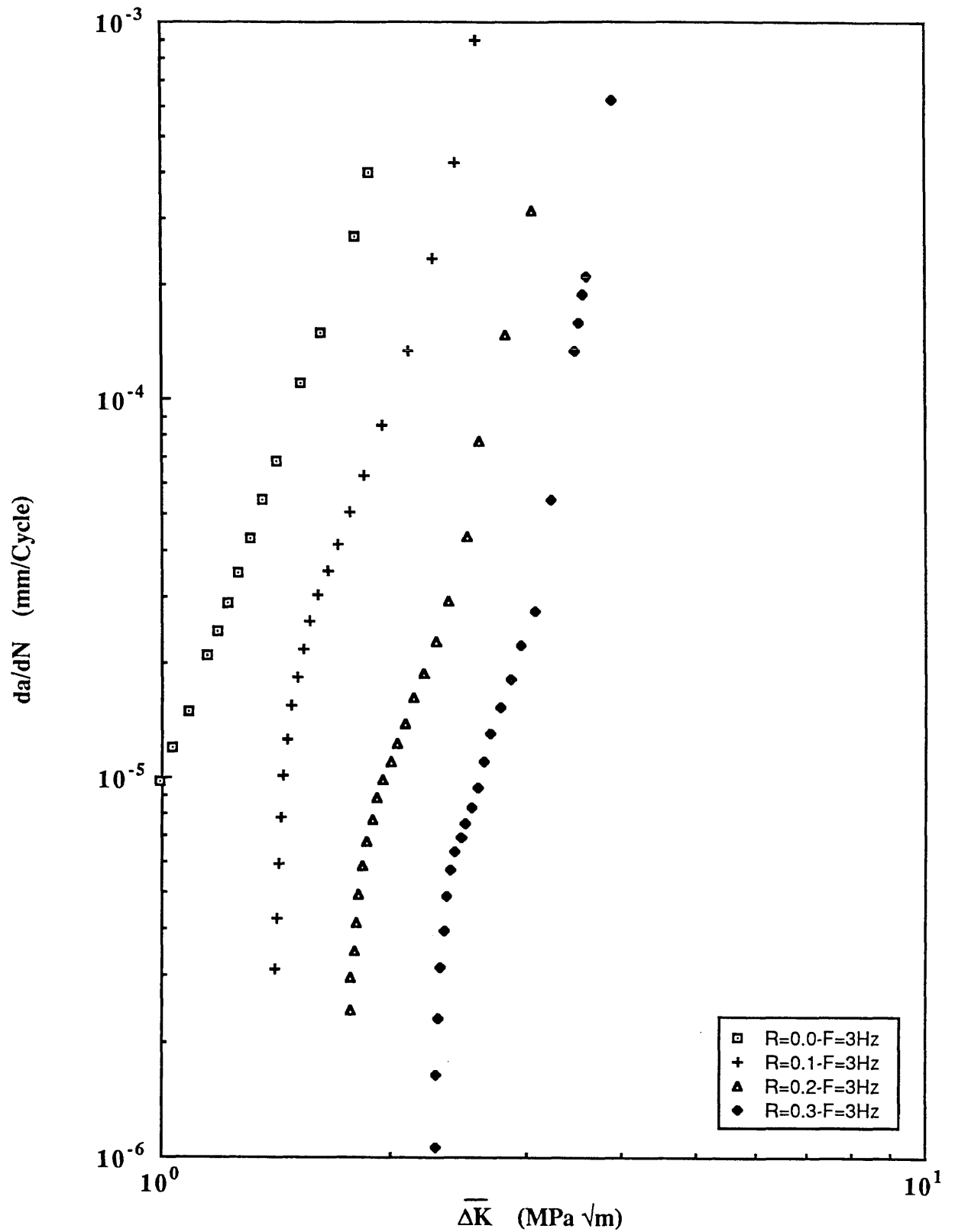


Fig 5.73 FCP rates in LL101AA plotted against ΔK in accordance with the Walker Model.
 $F=3.0\text{Hz}$ - $T=+23^\circ\text{C}$

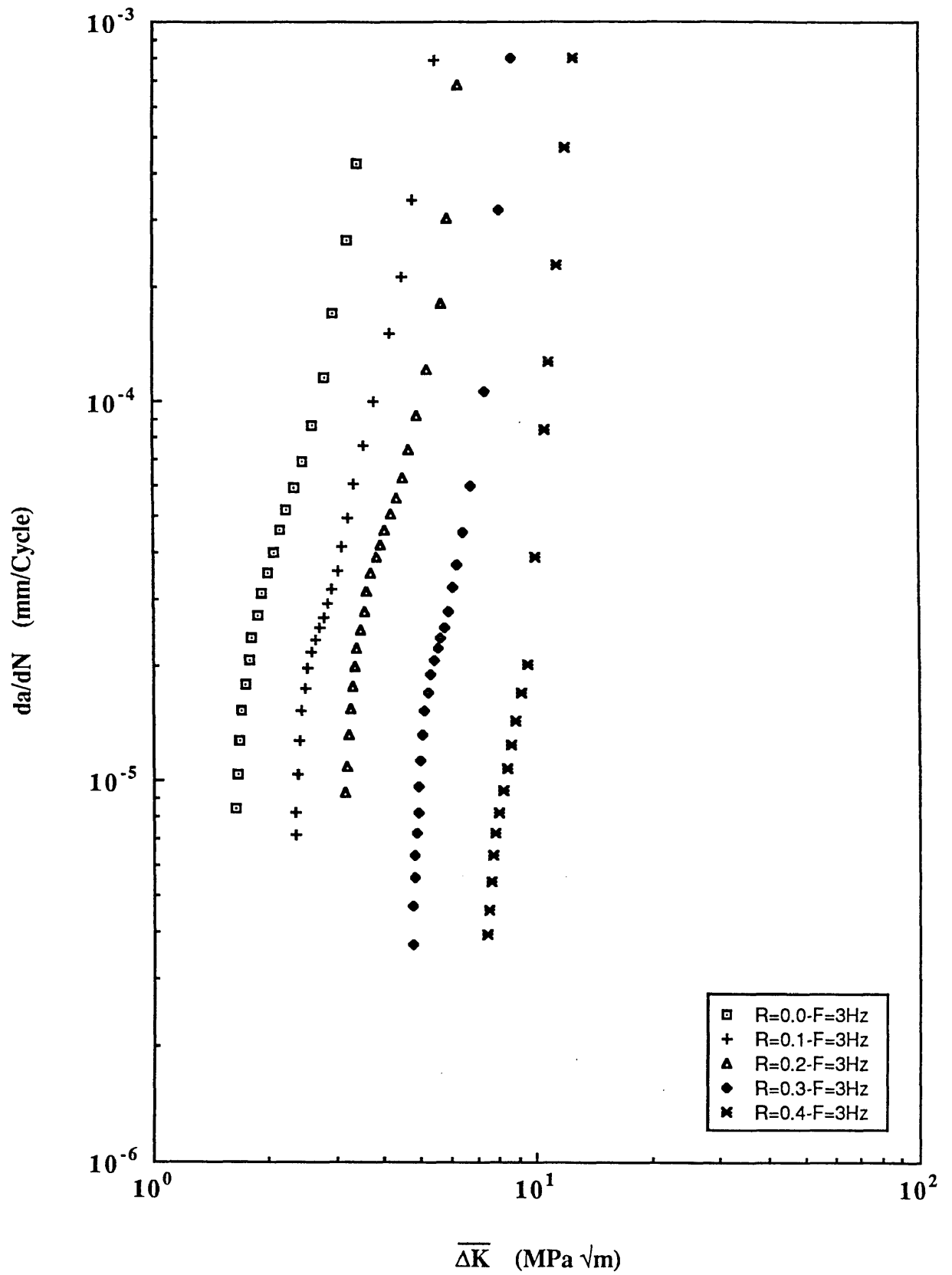


Fig 5.74 FCP rates in LL101AA plotted against ΔK in accordance with the Walker Model.
F=3.0Hz-T=-20°C

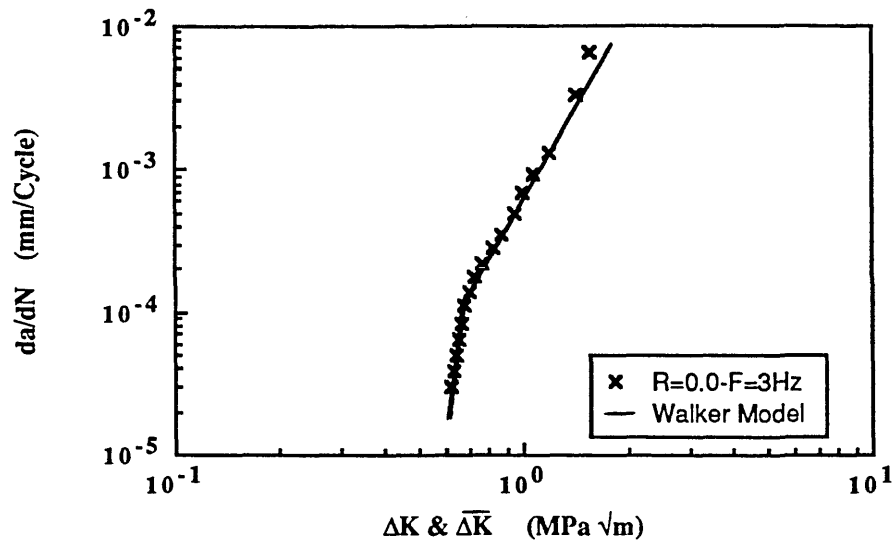


Fig 5.75 (a)

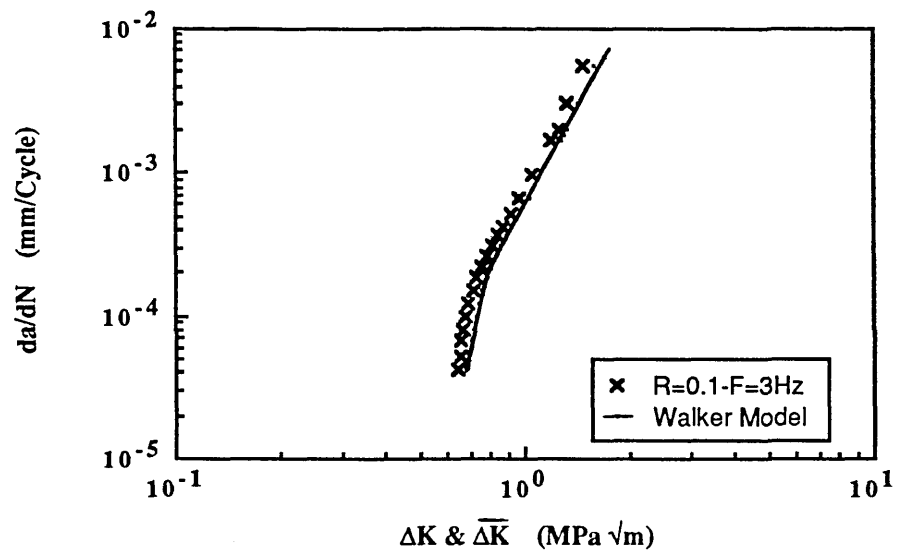


Fig 5.75 (b)

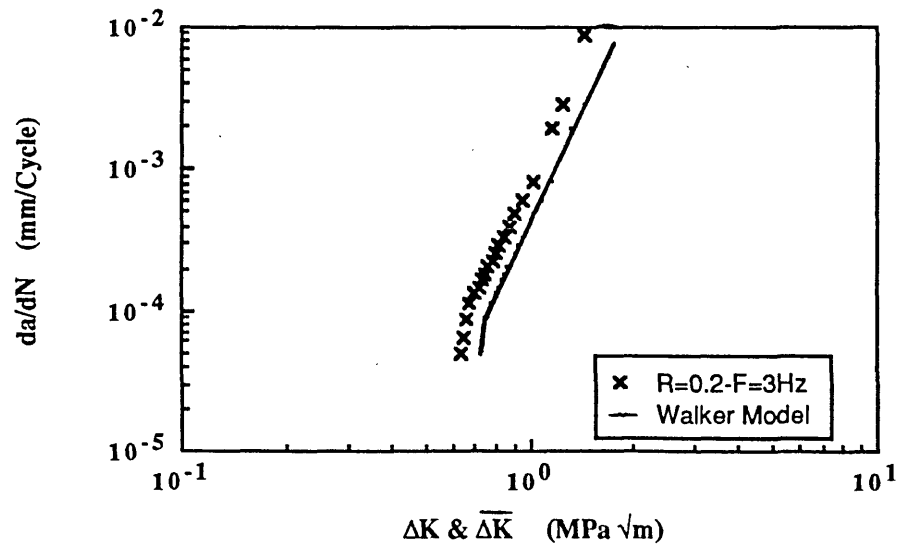


Fig 5.75 (c)

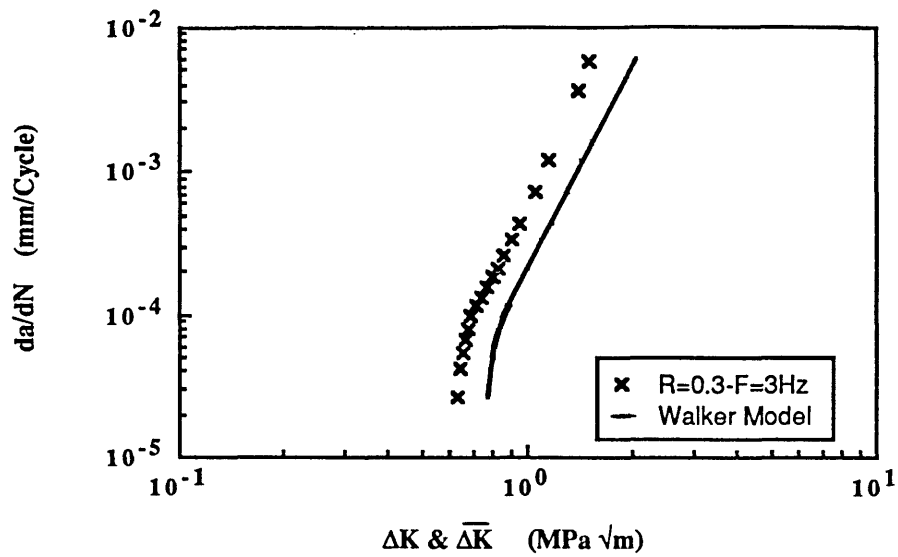


Fig 5.75 (d)

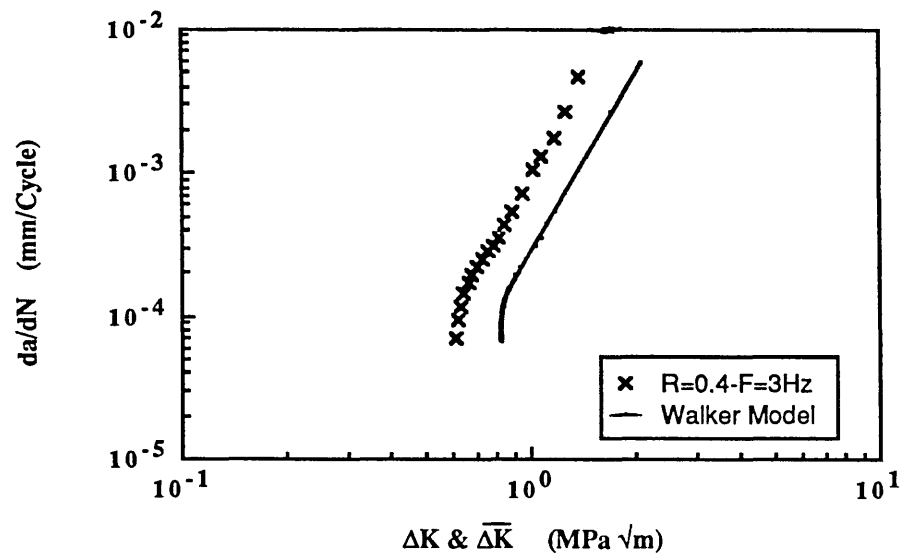


Fig 5.75 (e)

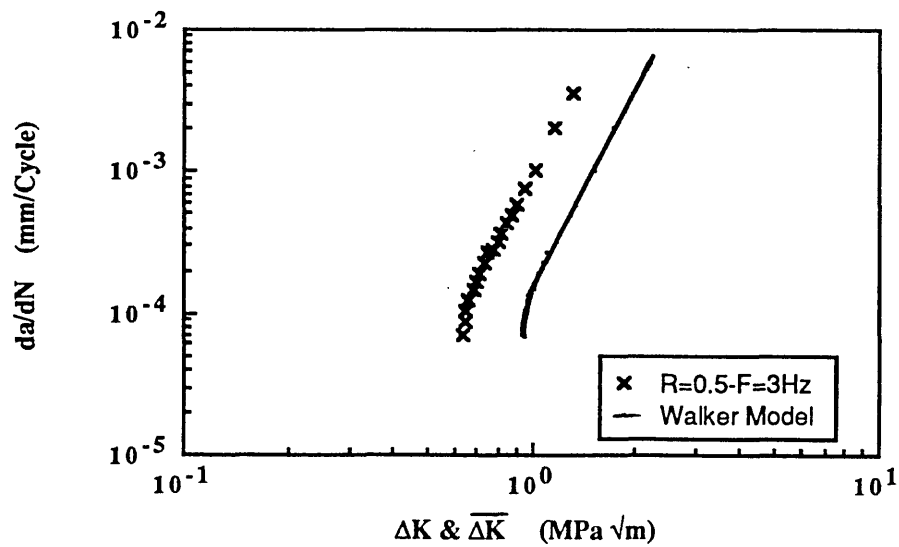


Fig 5.75 (f)

Fig 5.75 (a-f) FCP rates in BP33 predicted by the Walker Model compared with experimental data for R-ratios between 0.0 and 0.5.
 $F=3.0\text{Hz}$ - $T=-20^{\circ}\text{C}$

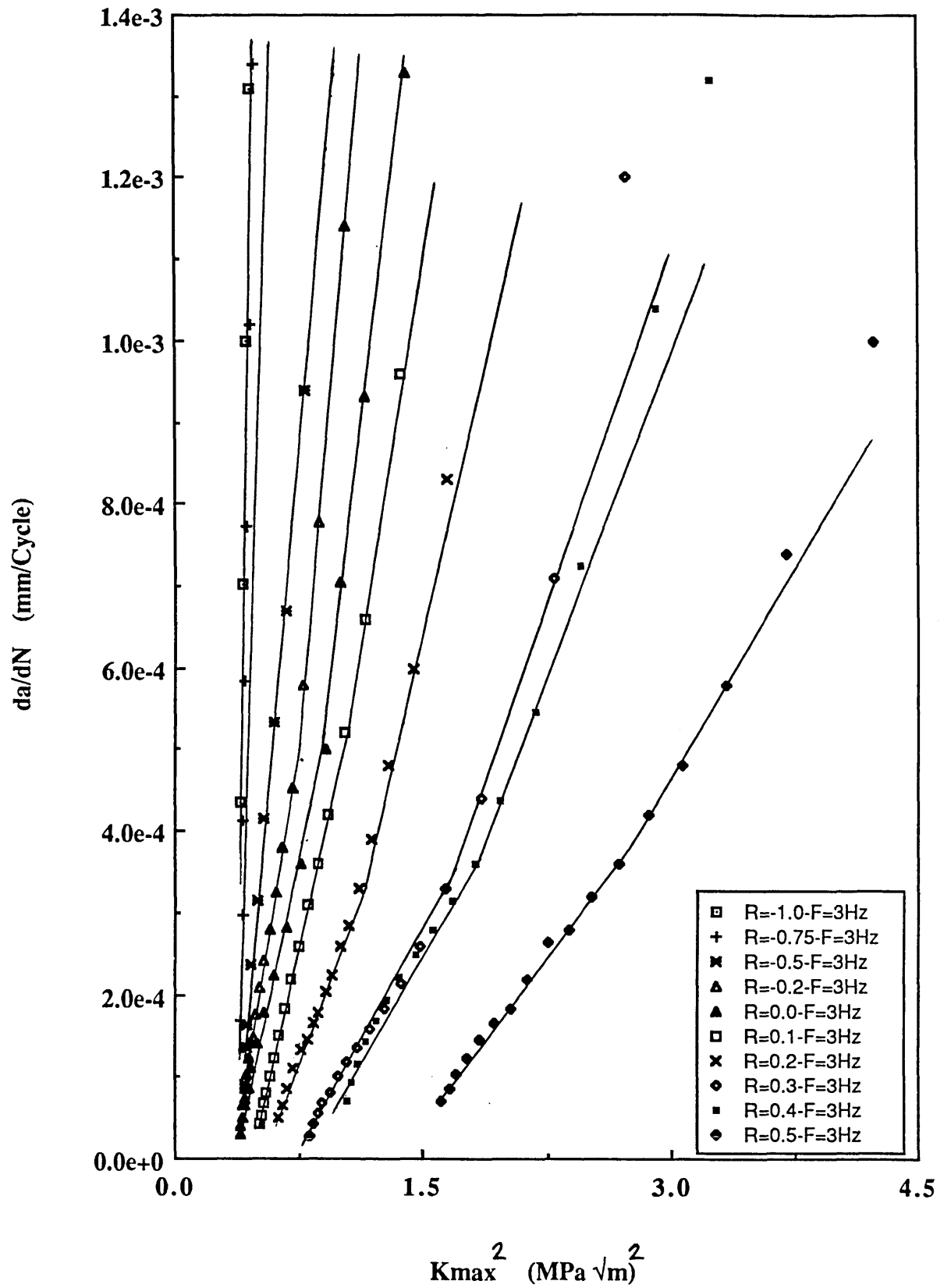


Fig 5.76 FCP rates in BP33 plotted against K_{max}^2 .
 $F=3.0\text{Hz}$ - $T=-20^\circ\text{C}$

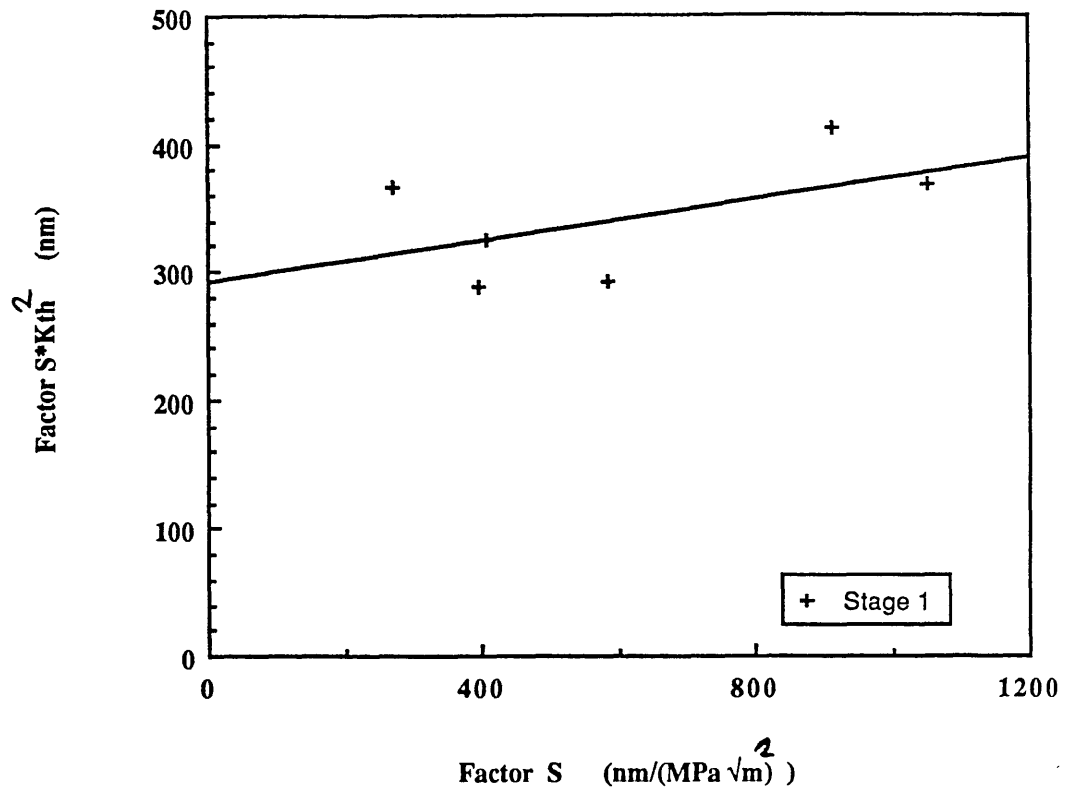


Fig 5.77 (a)

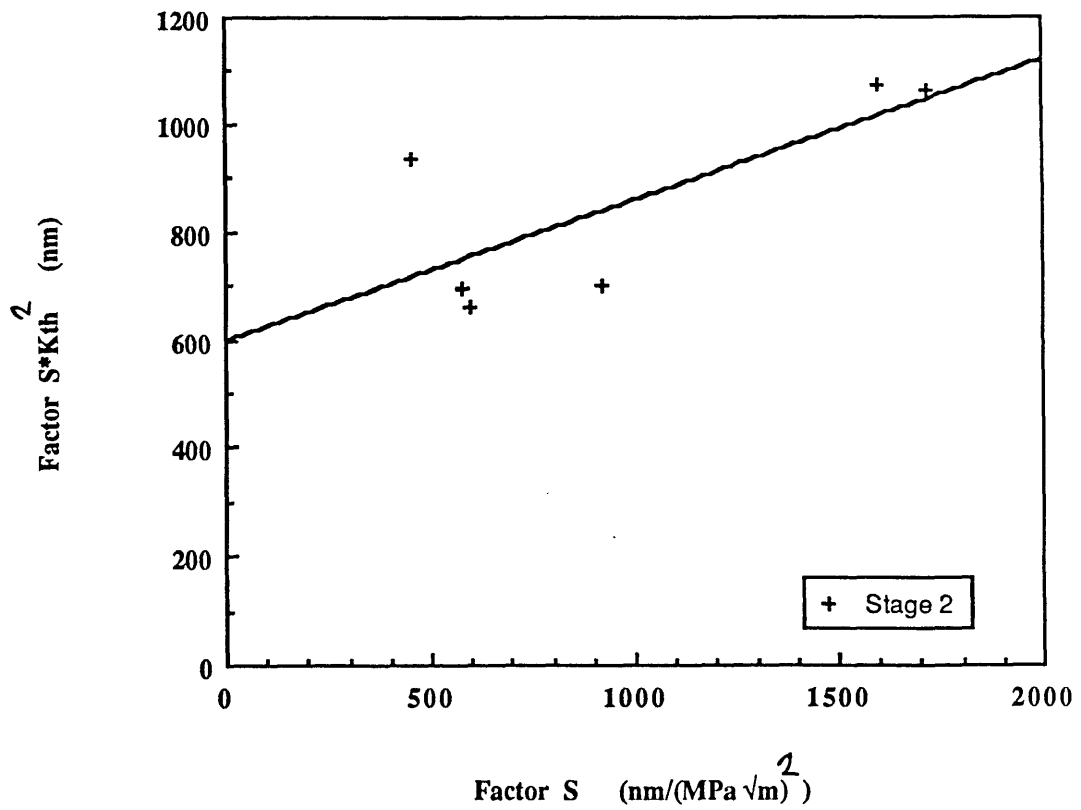


Fig 5.77 (b)

Fig 5.77(a) & (b) The factor SK_{th}^2 plotted against S in accordance with the Two Stage Line Zone Model for FCP in BP33 before and after the transition on Fig 5.76 respectively.
 $F=3.0\text{Hz}$ - $T=-20^\circ\text{C}$

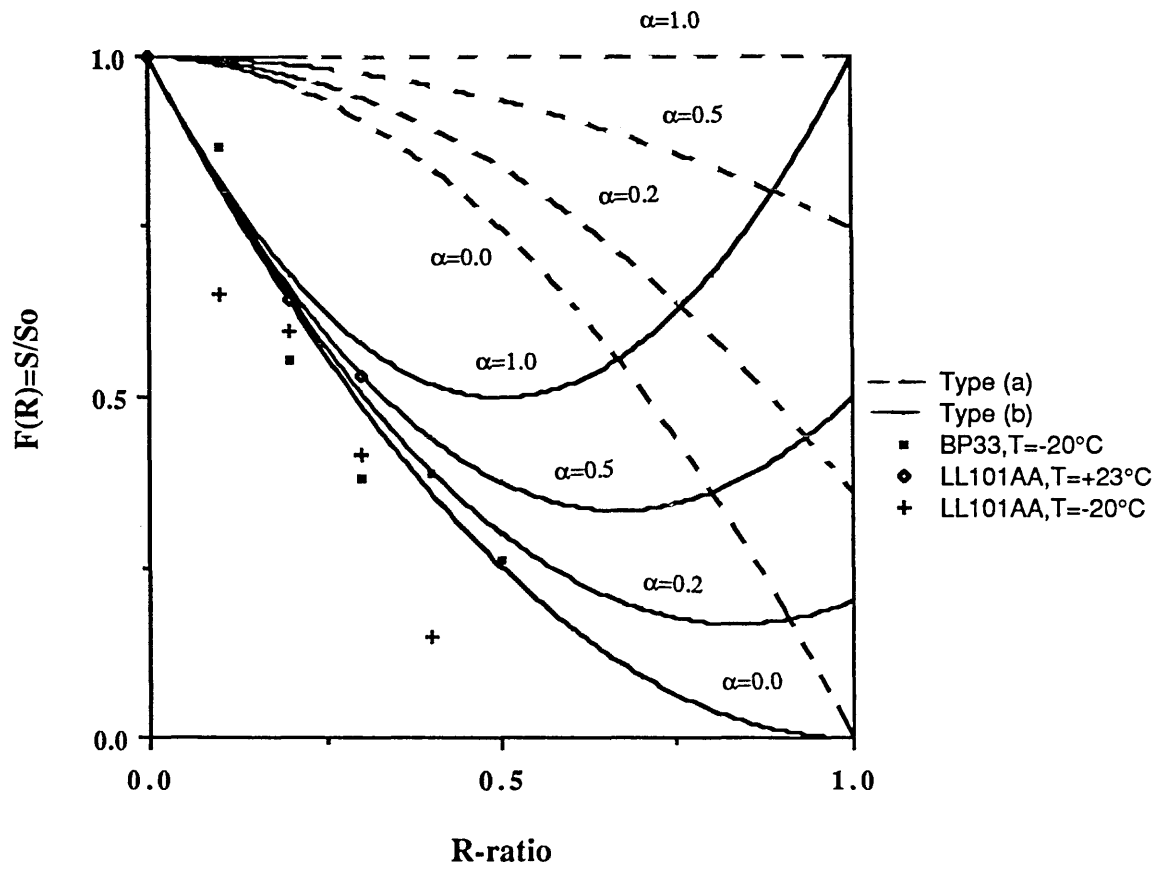


Fig 5.78 The factors $F(R)$ given by Eqn 3.16 and S/S_0 plotted against R-ratio for the three test conditions investigated.

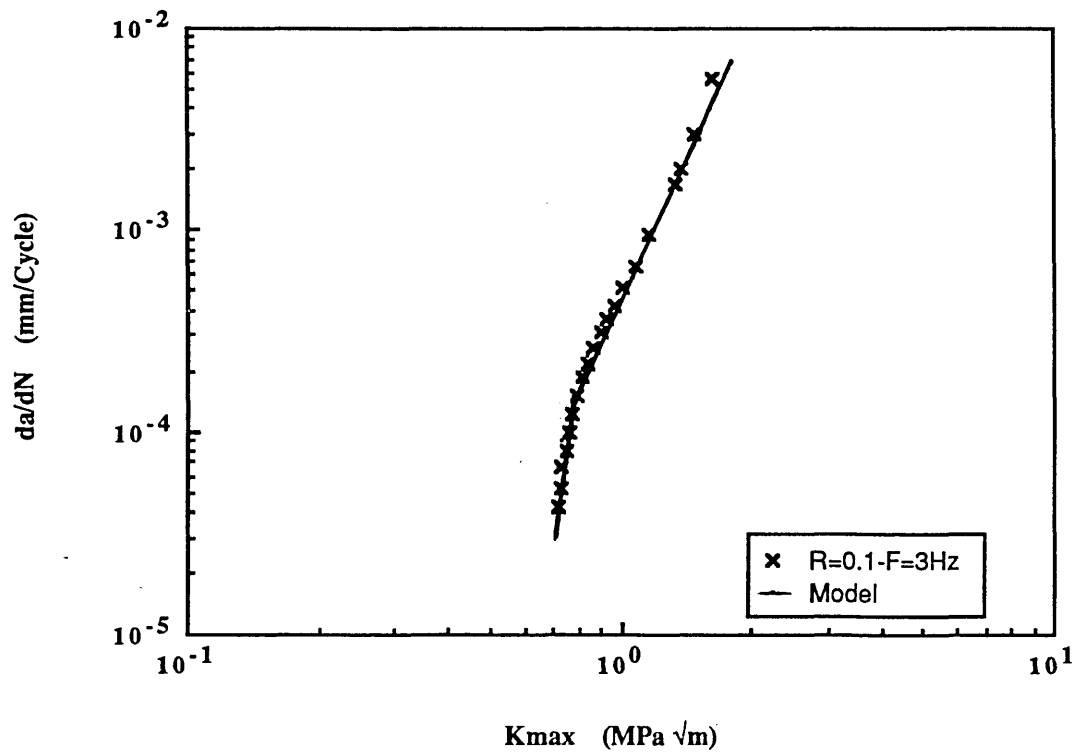
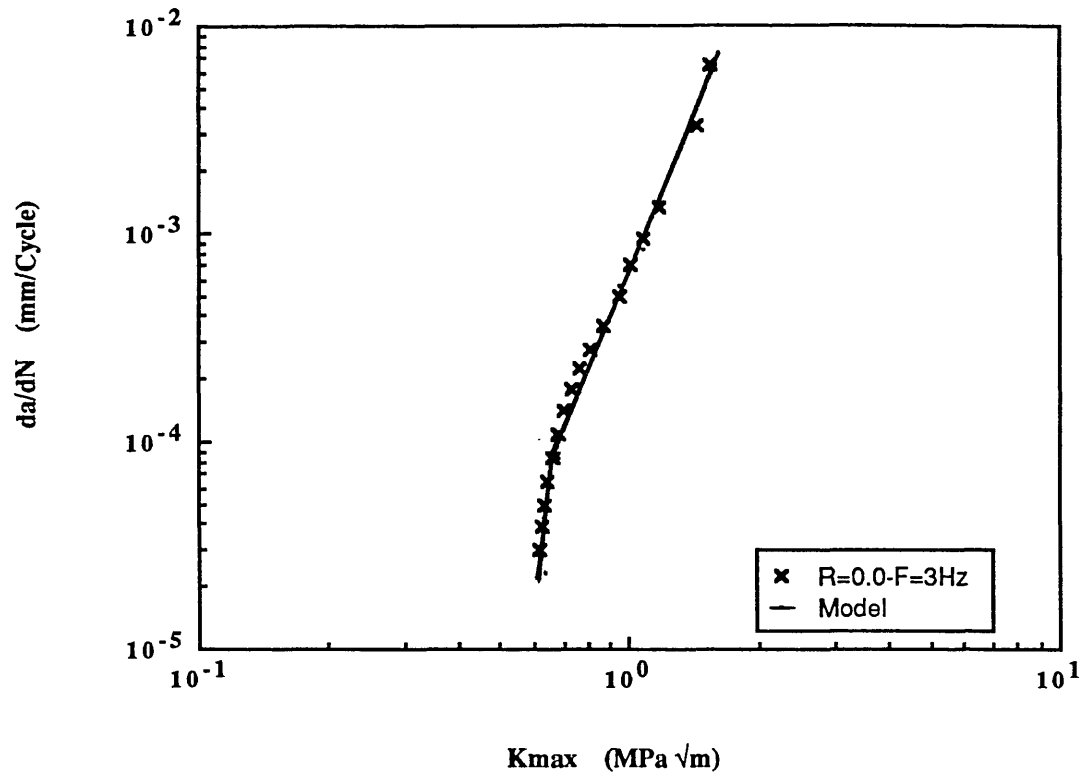


Fig 5.79 (a)

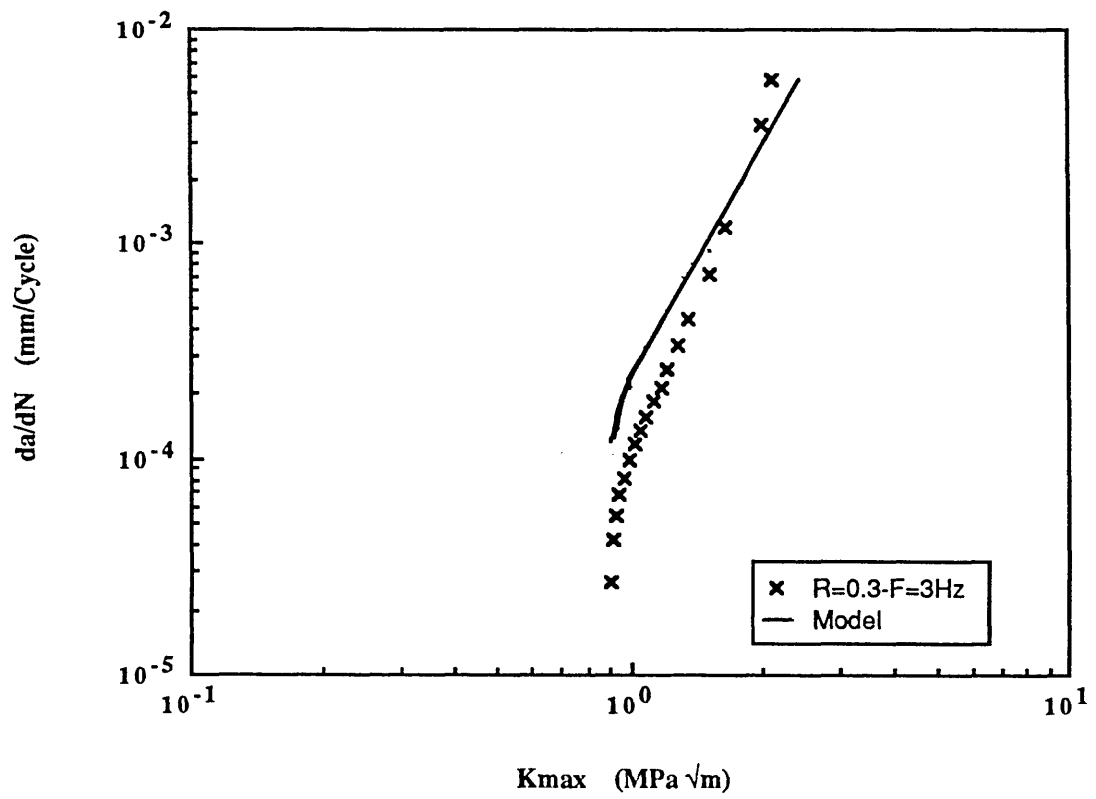
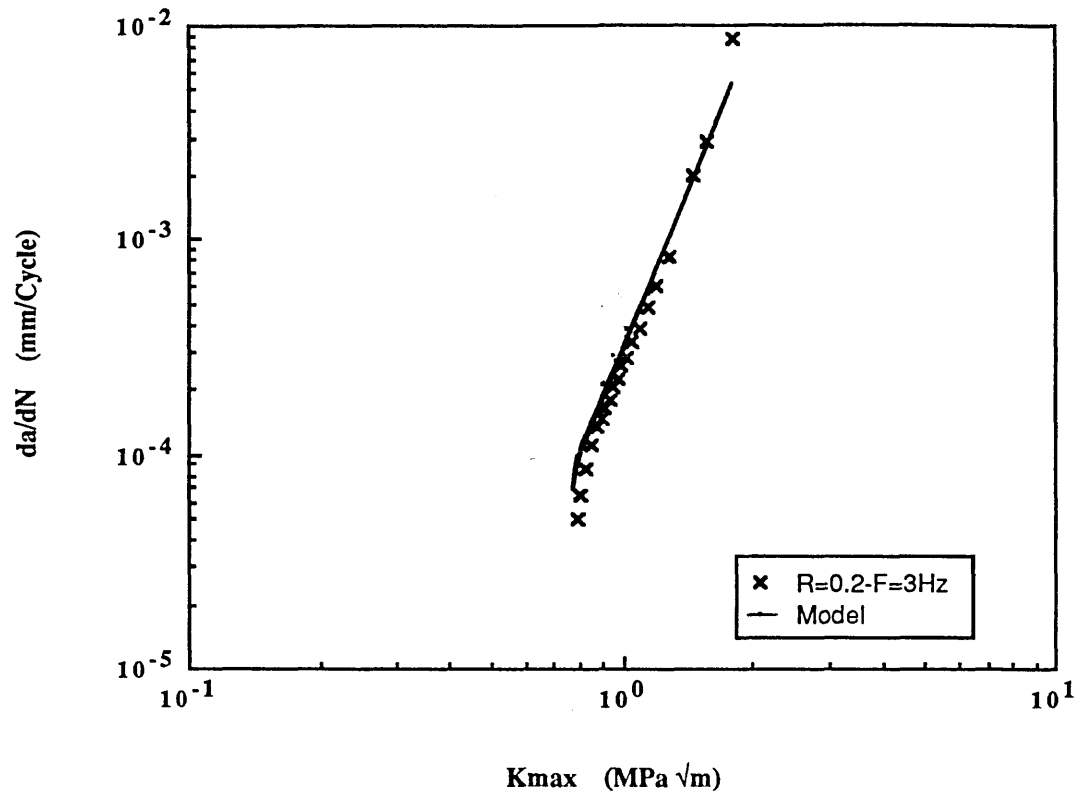


Fig 5.79 (b)

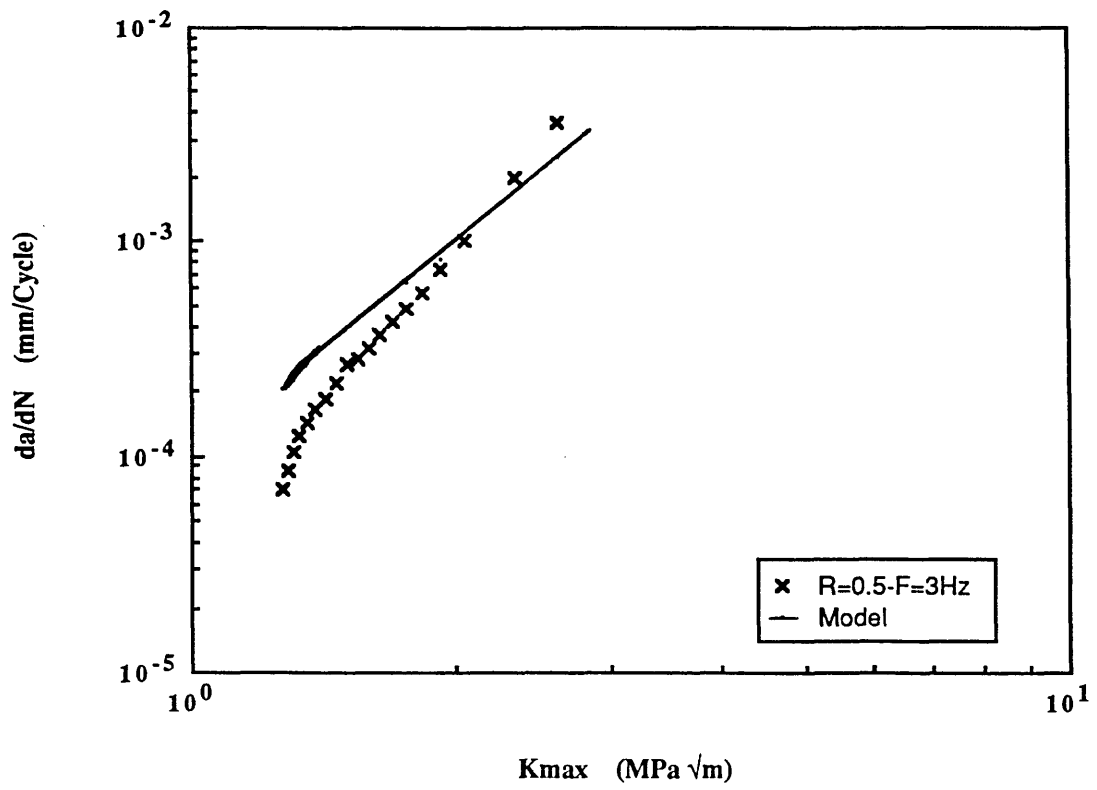
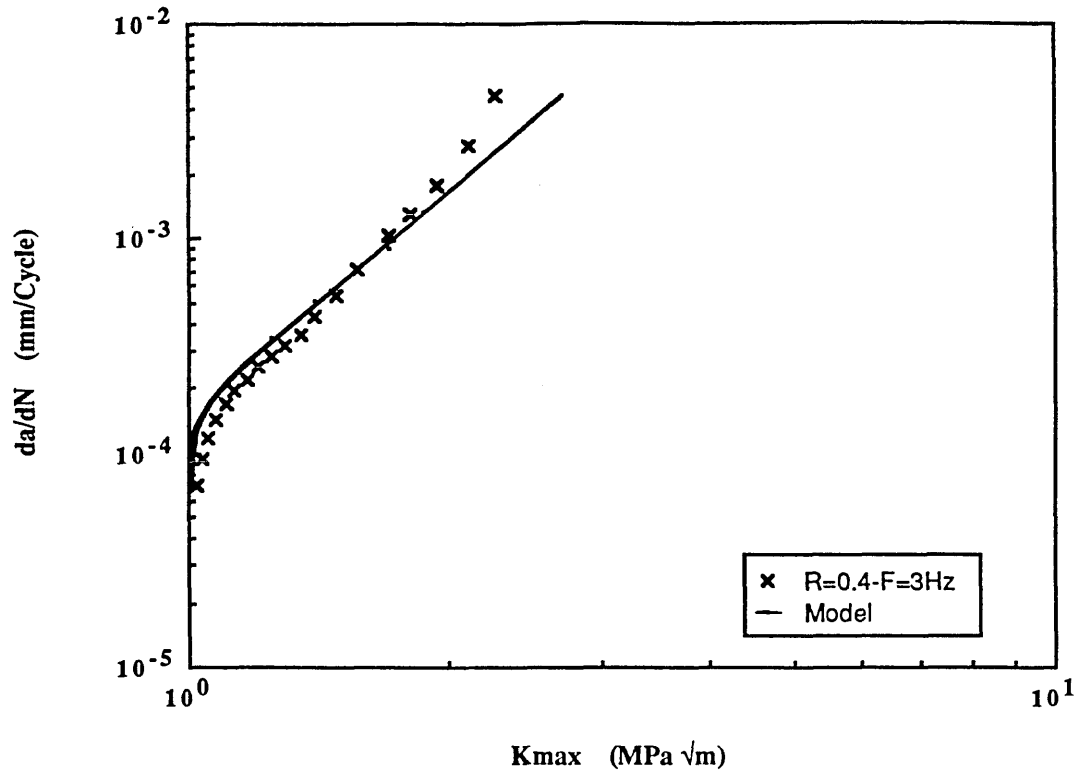


Fig 5.79 (c)

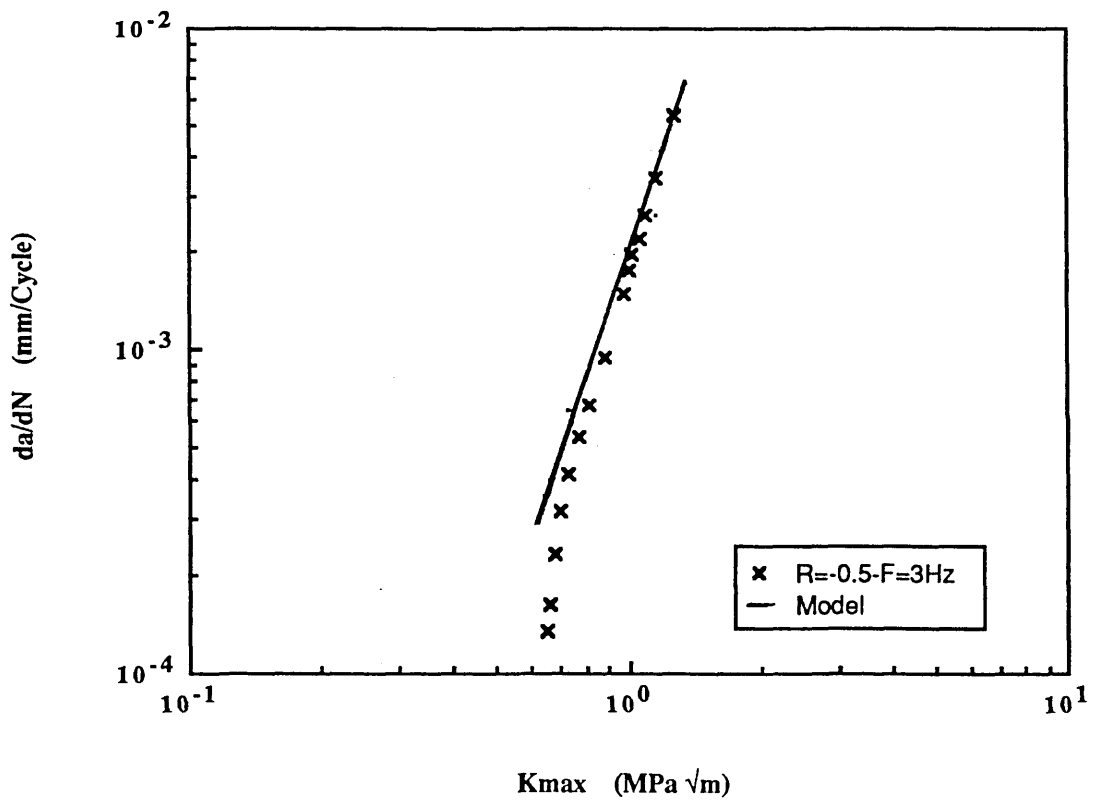
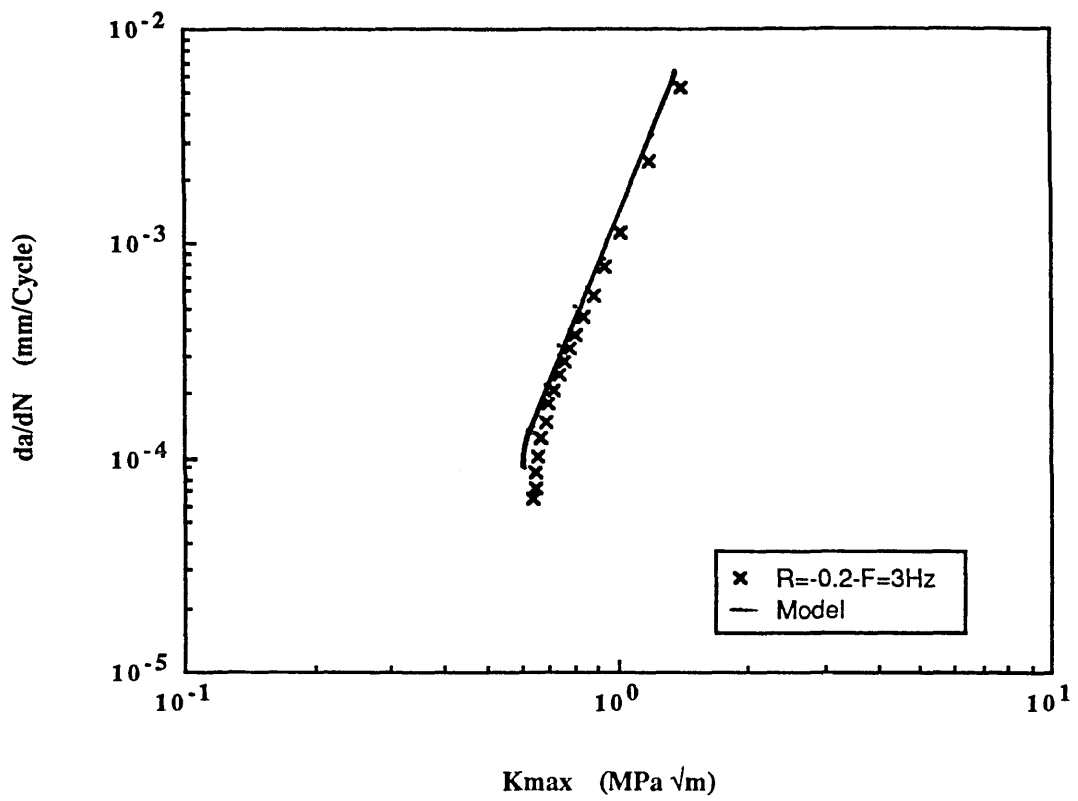


Fig 5.79 (d)

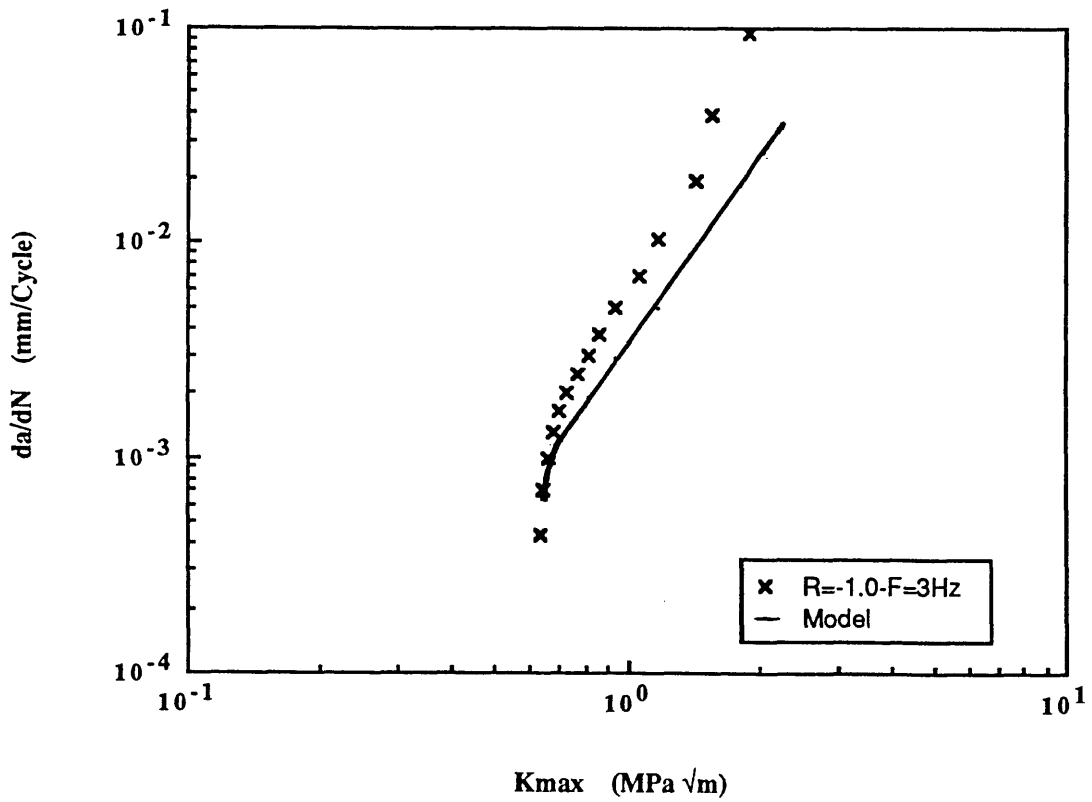
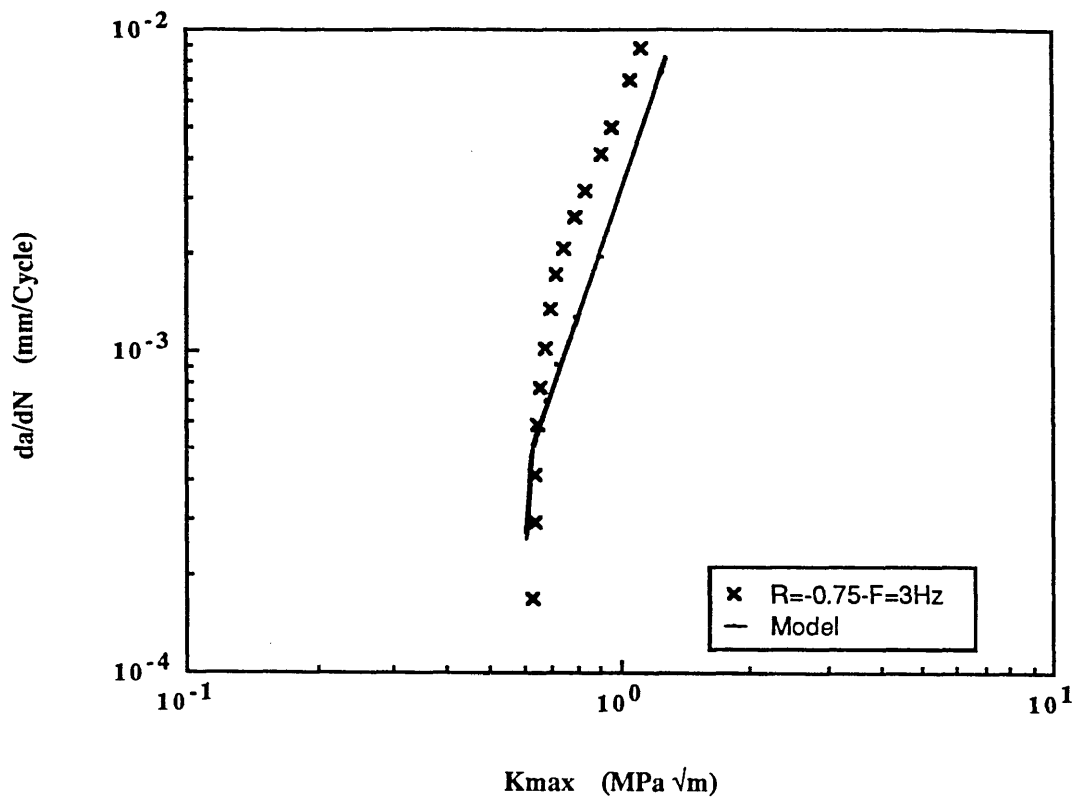


Fig 5.79 (e)

Fig 5.79 (a-e) FCP rates in BP33 predicted by the Two Stage Line Zone Model compared with experimental data for R-ratios between -1.0 and 0.5. $F=3.0Hz-T=-20^{\circ}C$

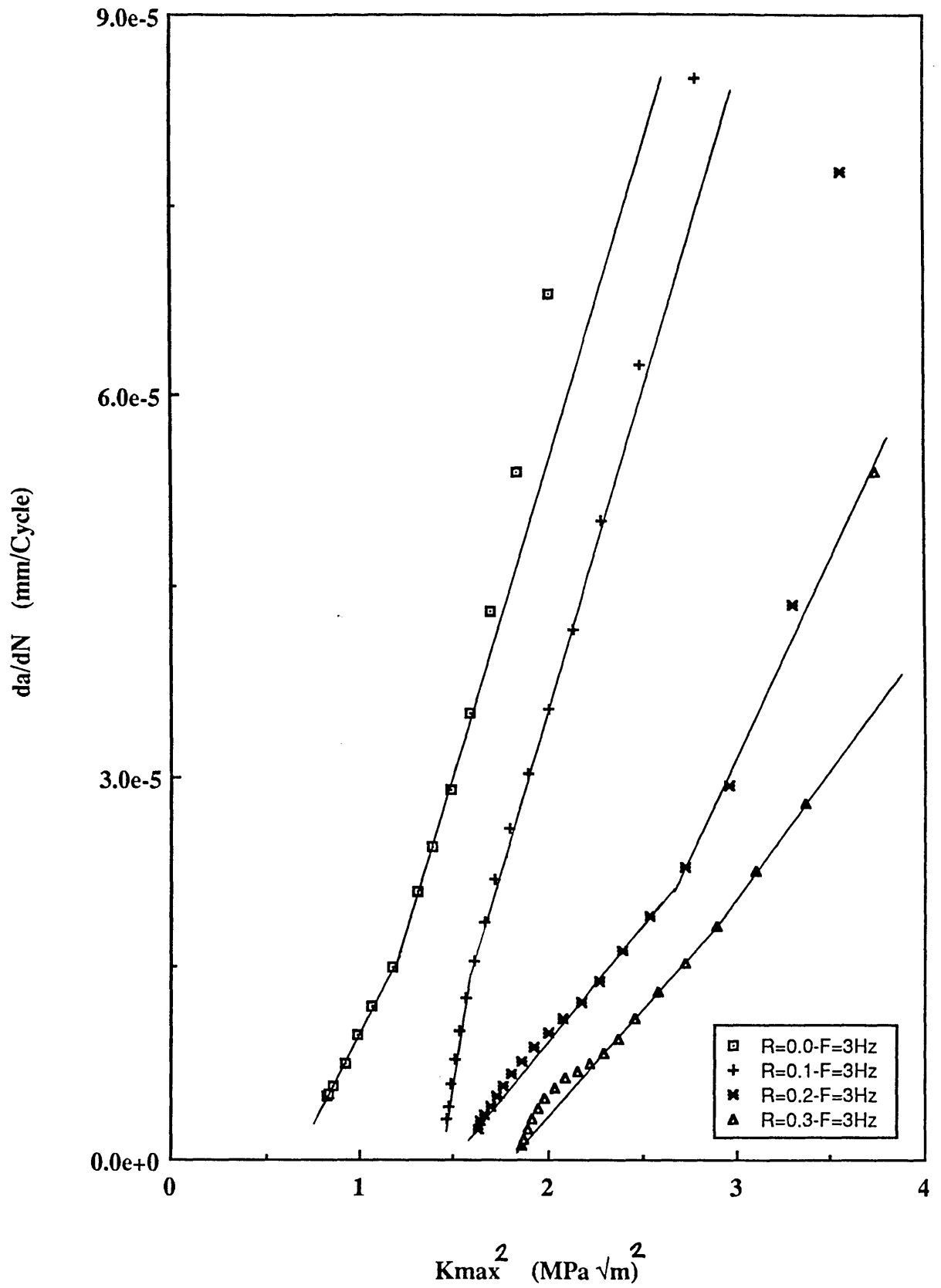


Fig 5.80 FCP rates in LL101AA plotted against K_{max}^2 .
 $F=3.0\text{Hz}$ - $T=+23^\circ\text{C}$

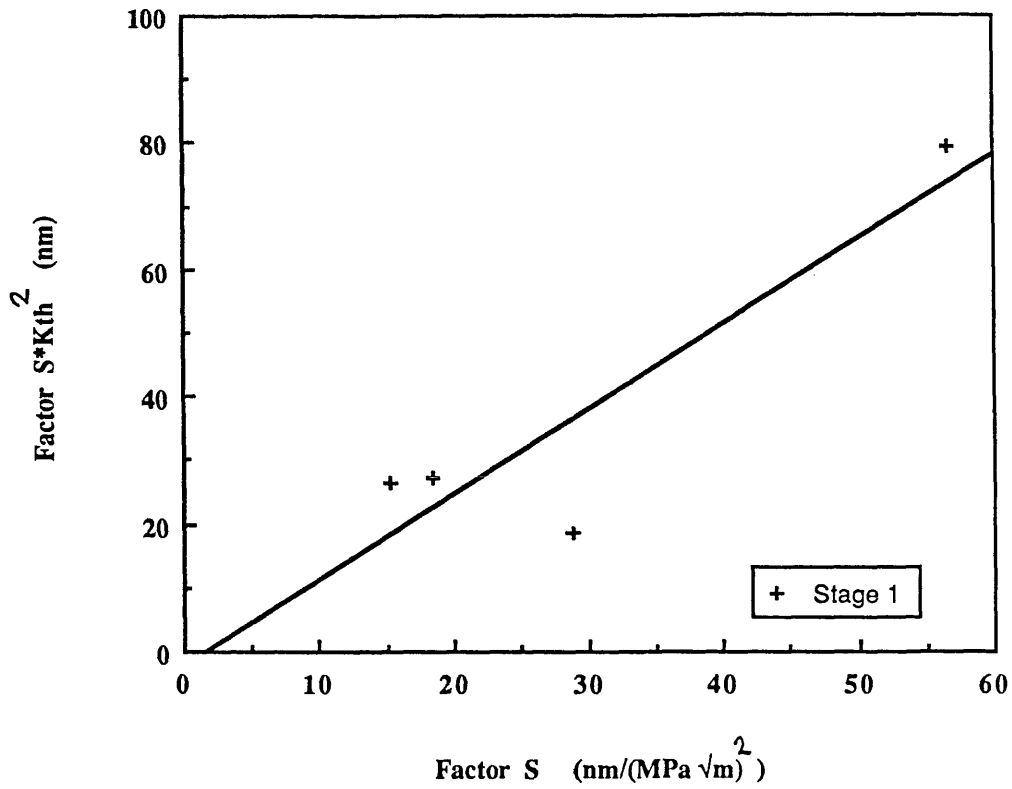


Fig 5.81 (a)

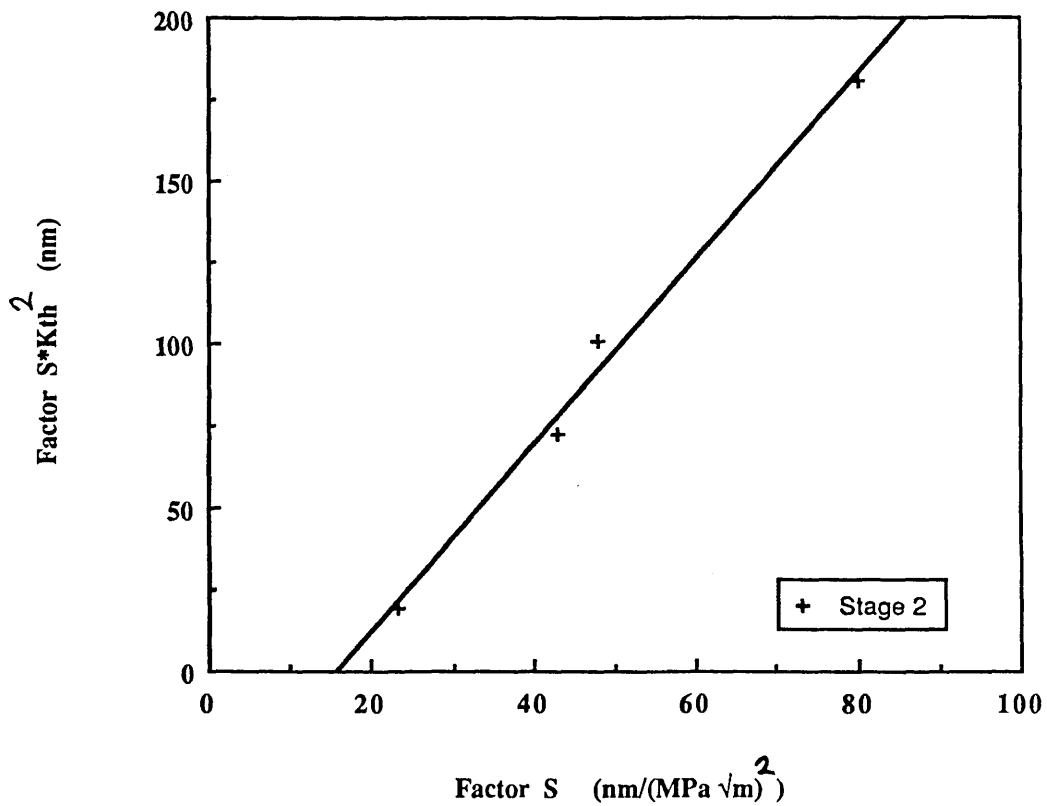


Fig 5.81 (b)

Fig 5.81(a) & (b) The factor SK_{th}^2 plotted against S in accordance with the Two Stage Line Zone Model for FCP in LL101AA before and after the transition on Fig 5.80 respectively.
 $F=3.0\text{Hz}$ - $T=+23^\circ\text{C}$

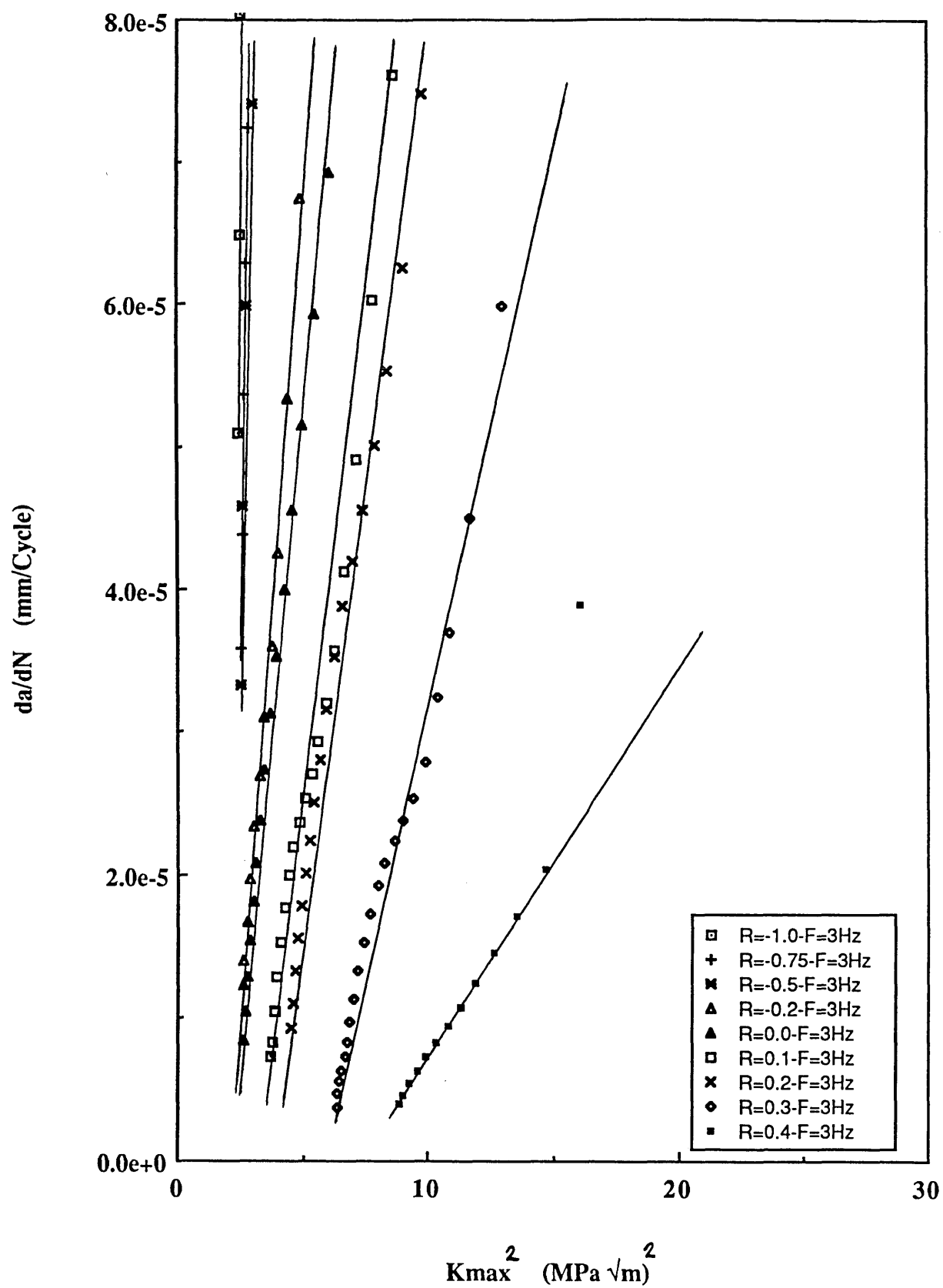


Fig 5.82 FCP rates in LL101AA plotted against K_{max}^2 .
 $F=3.0Hz$ - $T=-20^{\circ}C$

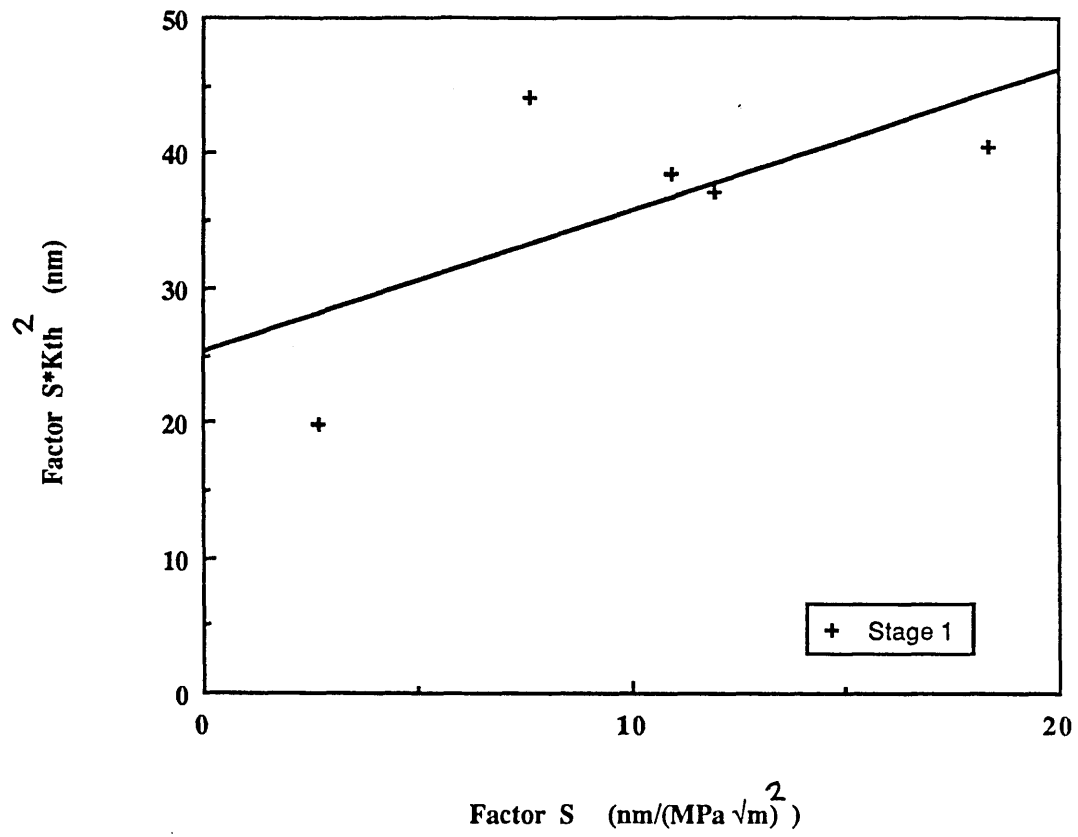


Fig 5.83 The factor SK_{th}^2 plotted against S in accordance with the Two Stage Line Zone Model for FCP in LL101AA .
 $F=3.0\text{Hz}$ - $T=+23^\circ\text{C}$

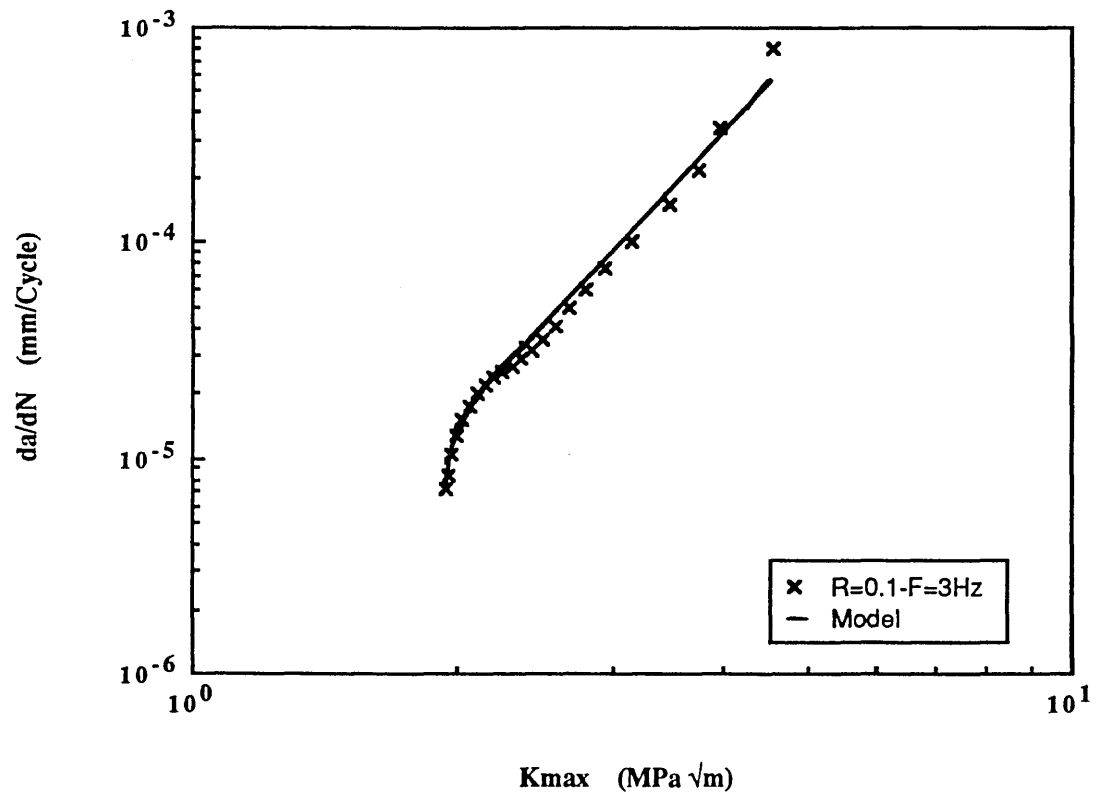
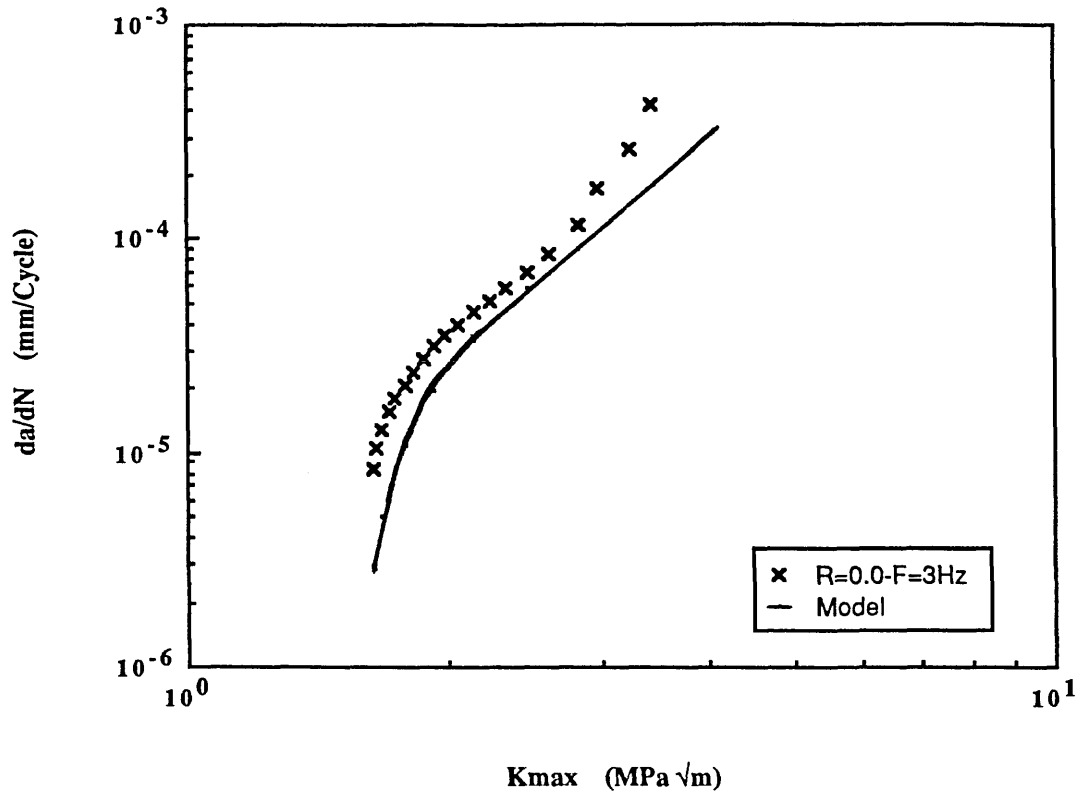


Fig 5.84 (a)

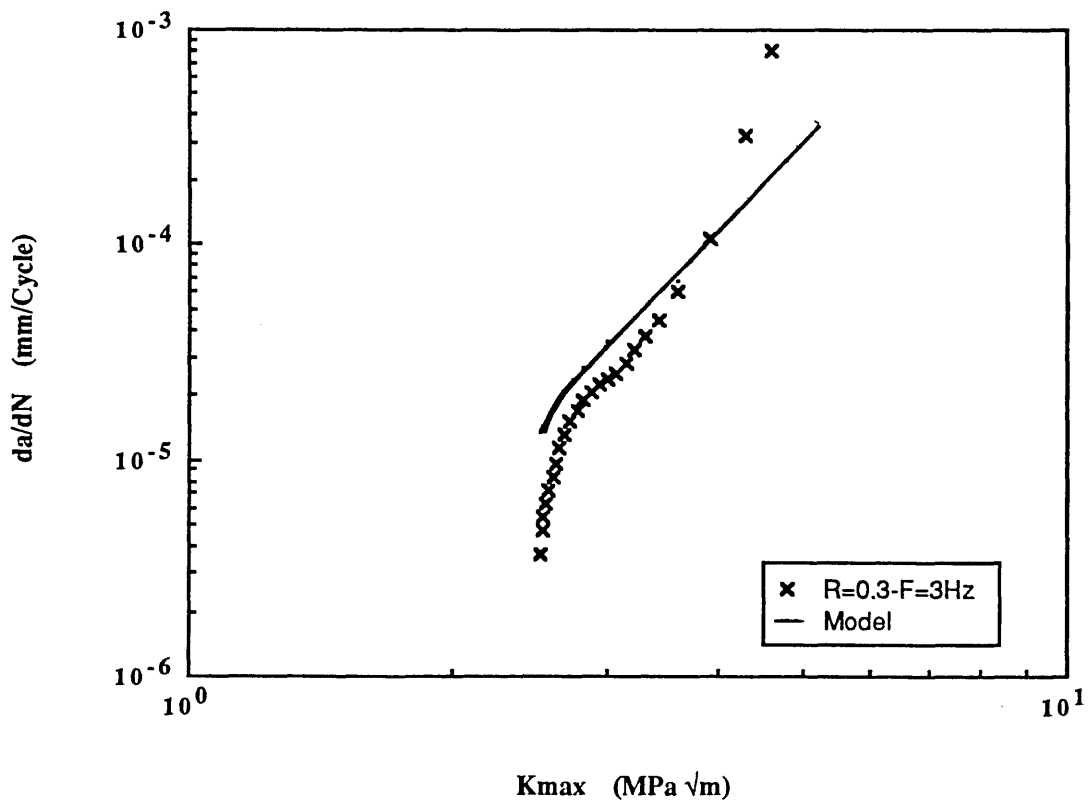
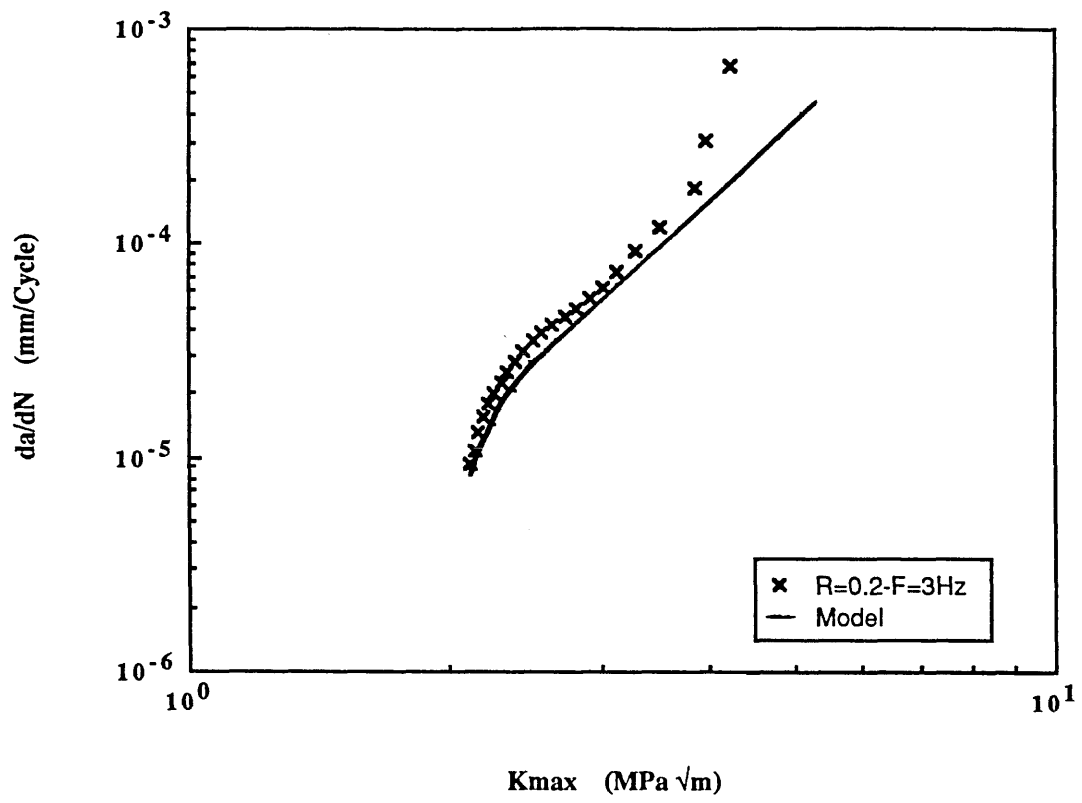


Fig 5.84 (b)

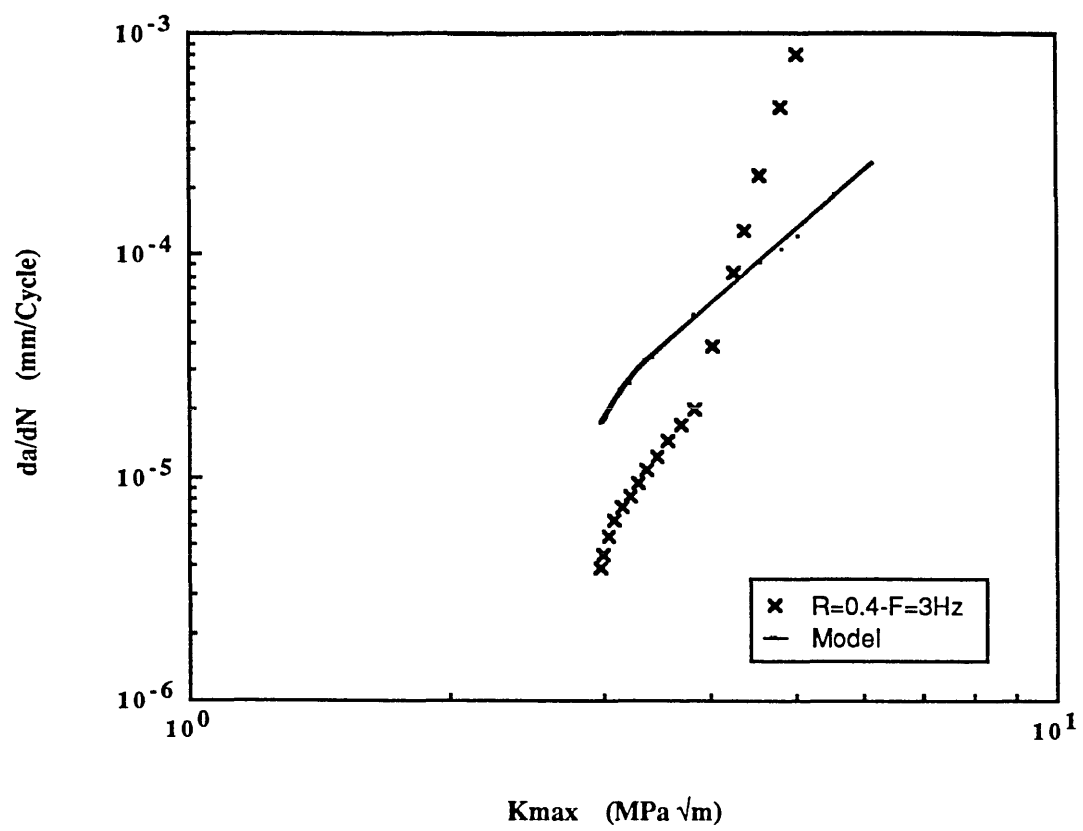


Fig 5.84 (c)

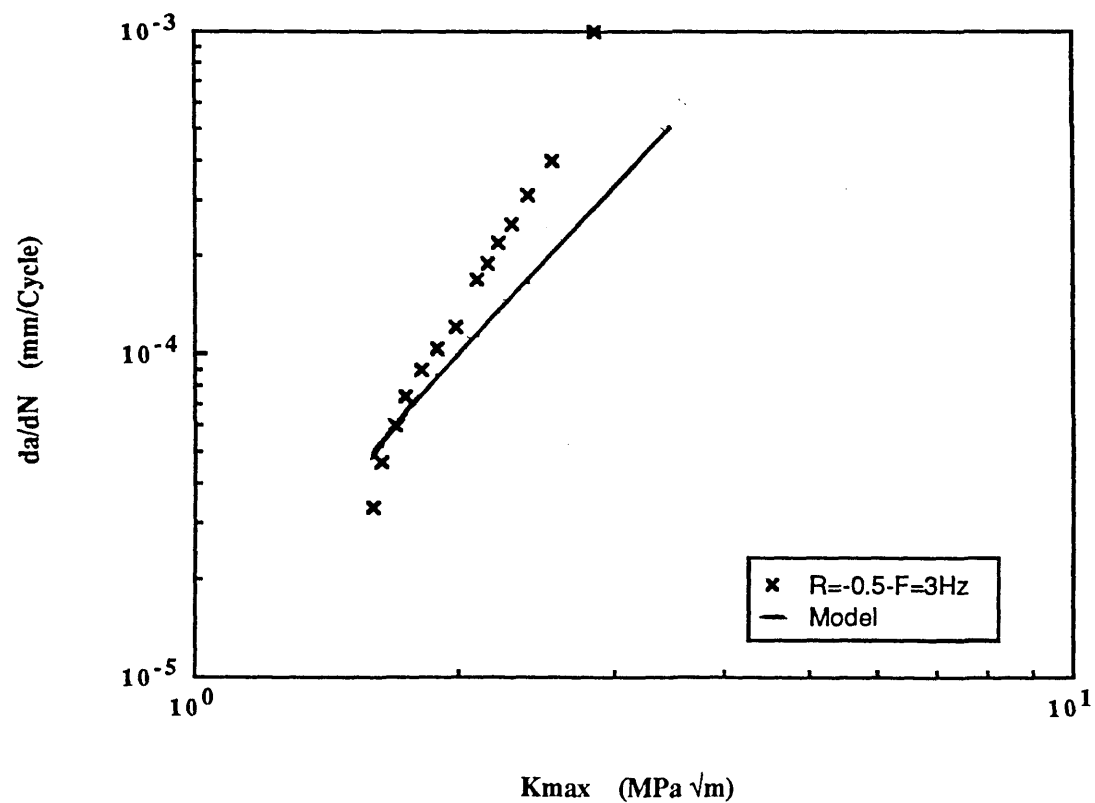
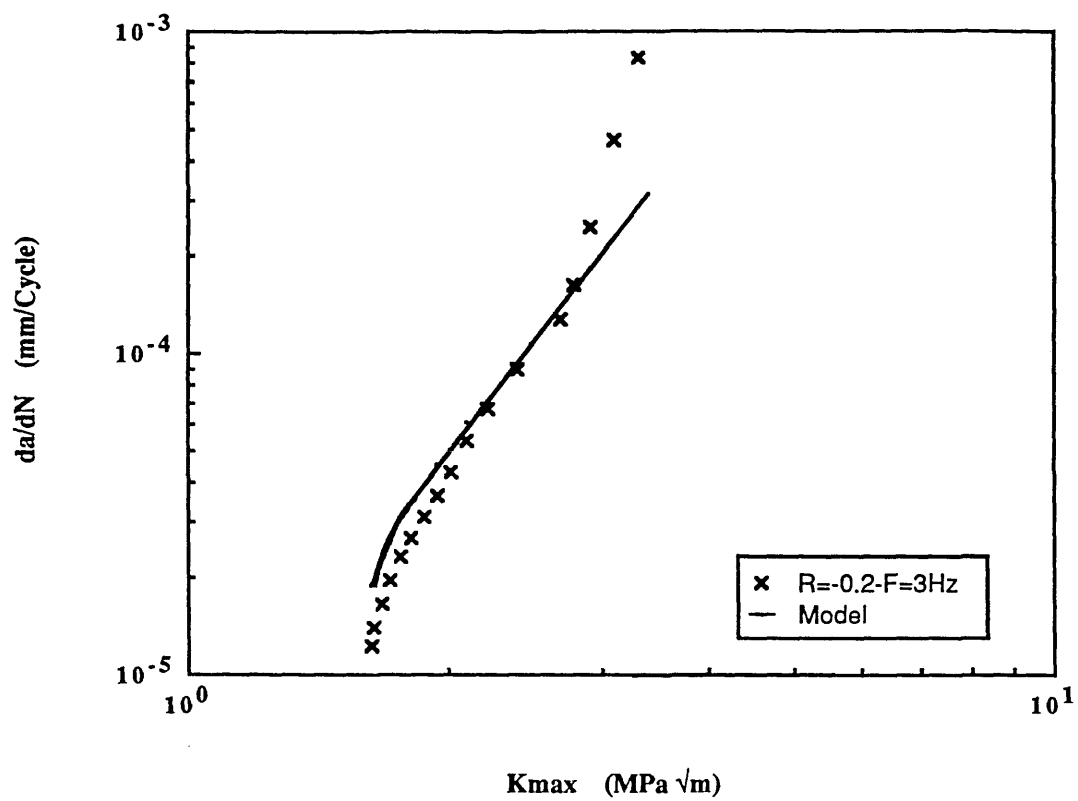


Fig 5.84 (d)

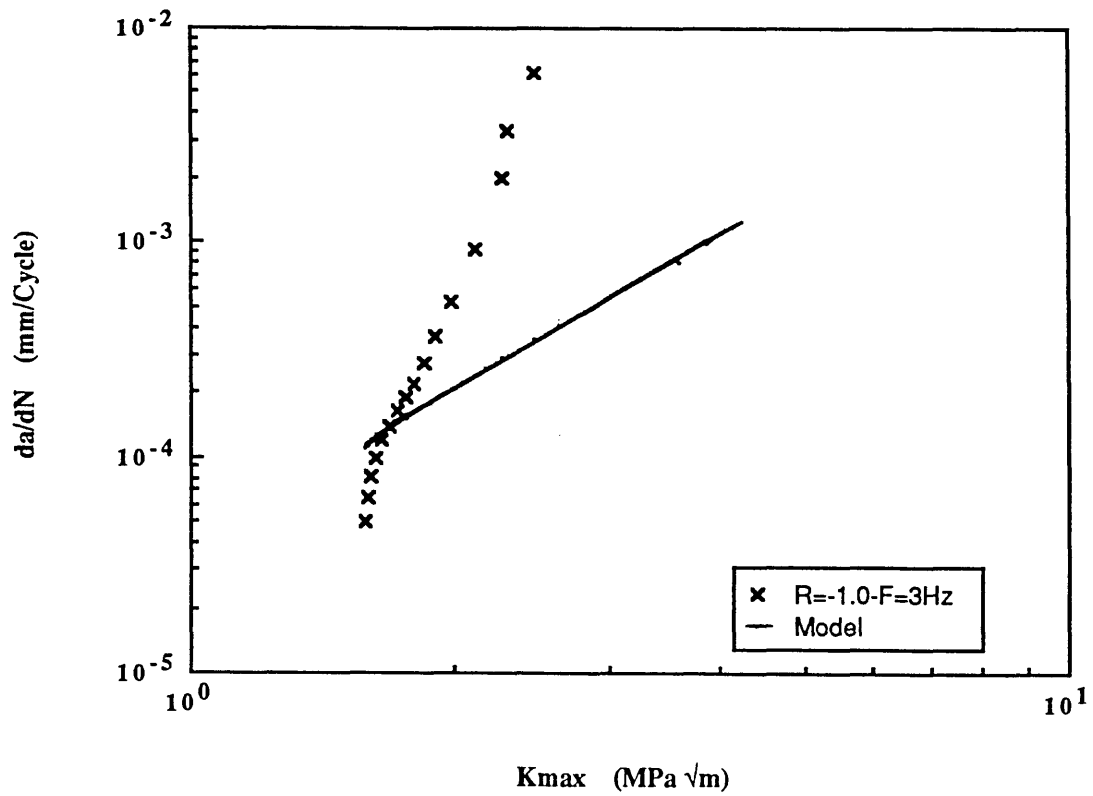
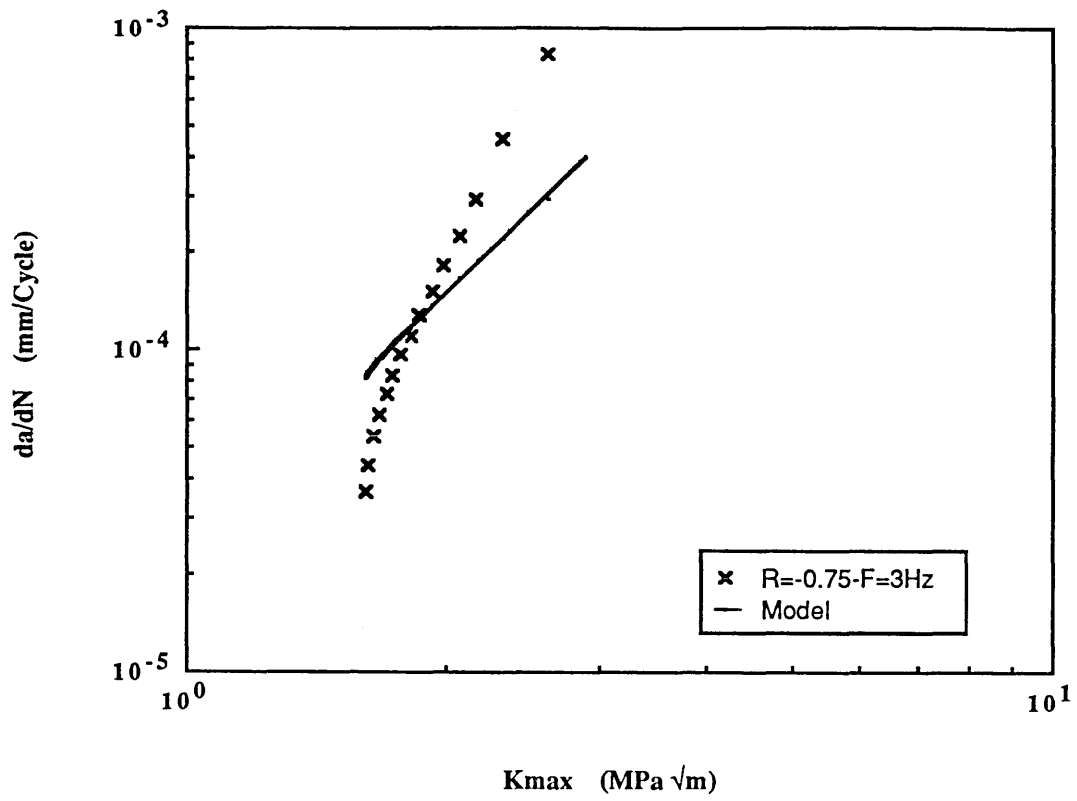


Fig 5.84 (e)

Fig 5.84 (a-e) FCP rates in LL101AA predicted by the Two Stage Line Zone Model compared with experimental data for R-ratios between -1.0 and 0.4. $F=3.0Hz-T=-20^{\circ}C$

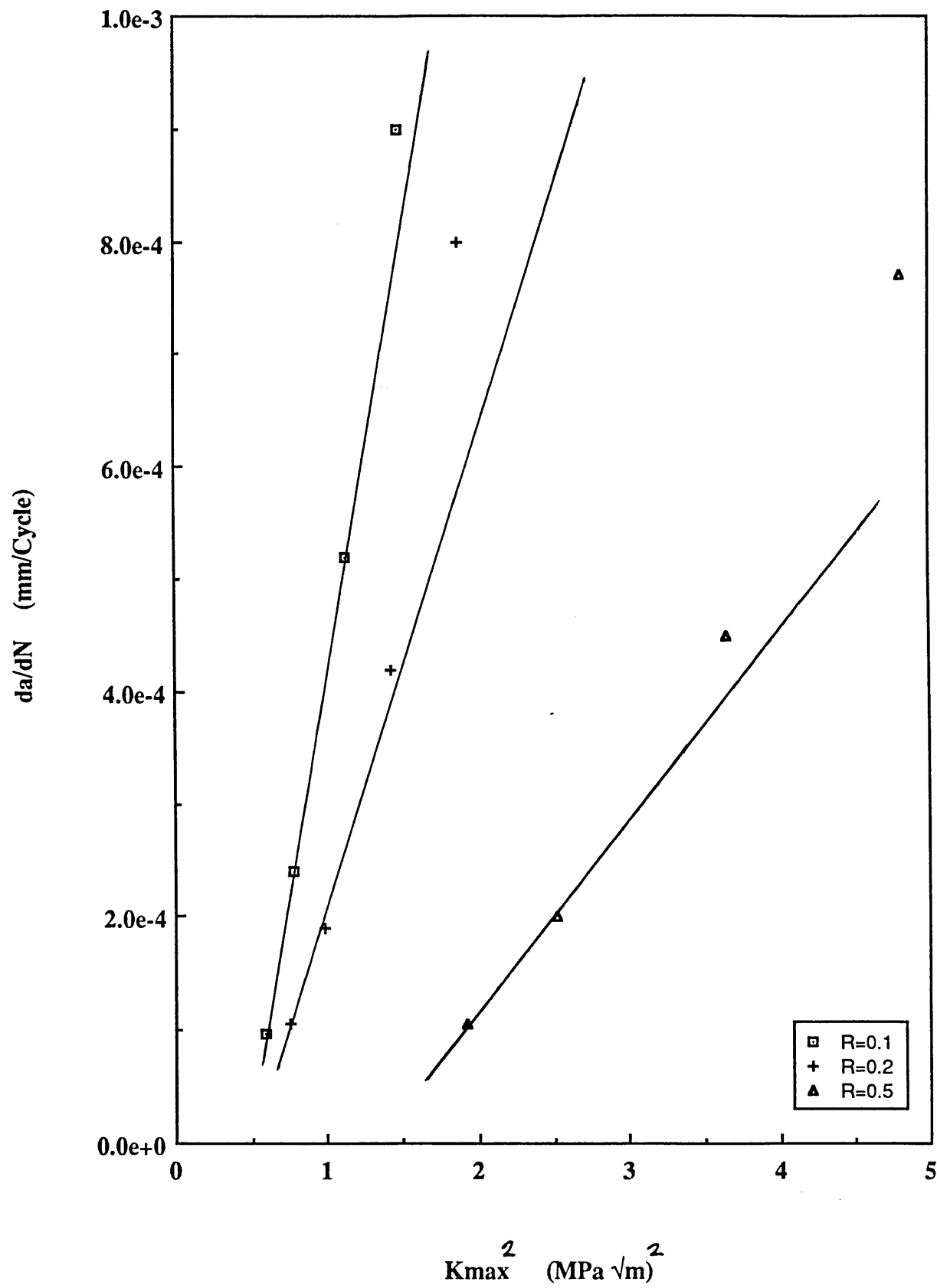


Fig 5.85 Cycle-dependent crack growth in BP33 plotted against K_{max}^2 .
 $T=-20^{\circ}C$

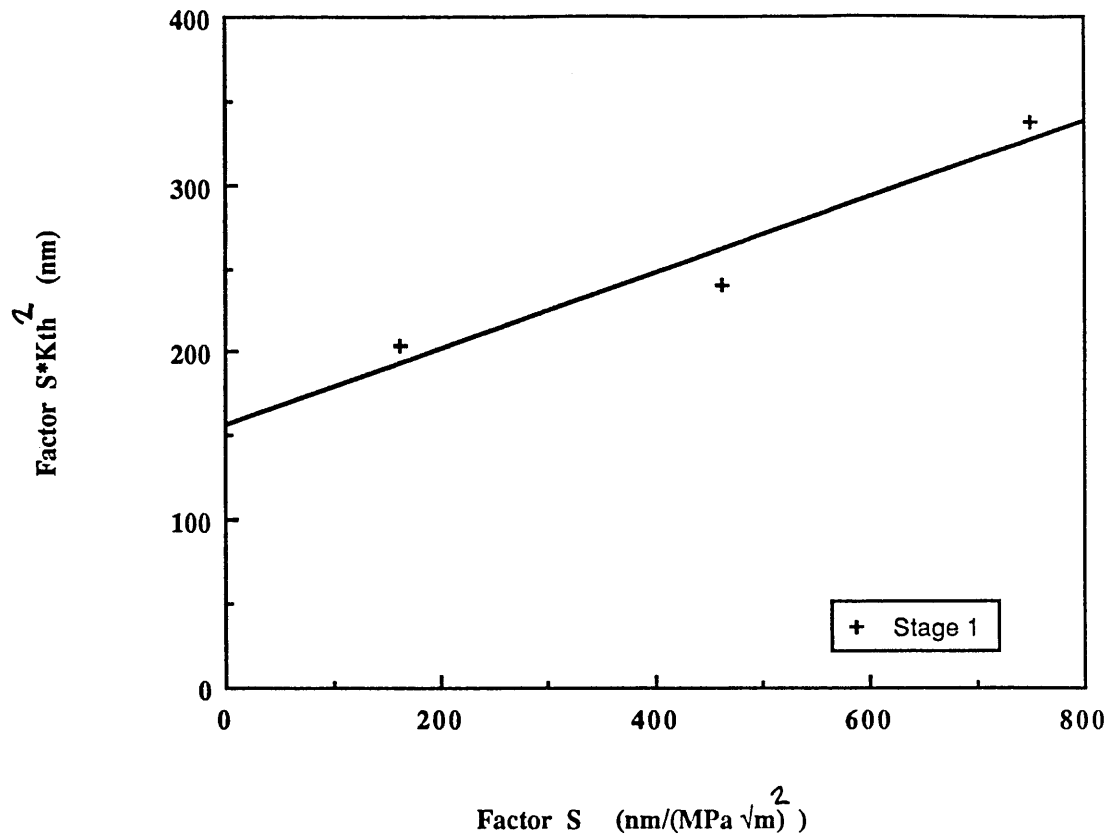


Fig 5.86 The factor SK_{th}^2 plotted against S in accordance with the Two Stage Line Zone Model for Cycle-dependent crack growth in BP33 .
 $T = -20^\circ\text{C}$

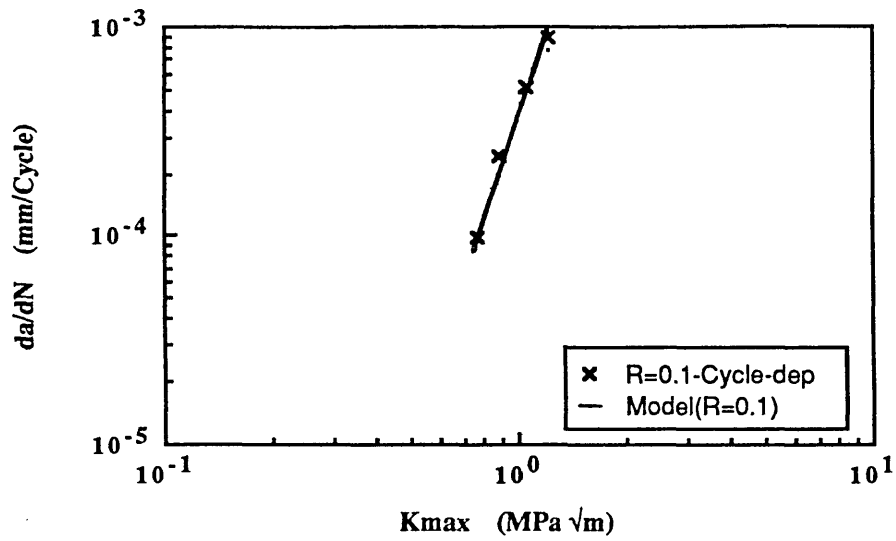


Fig 5.87 (a)

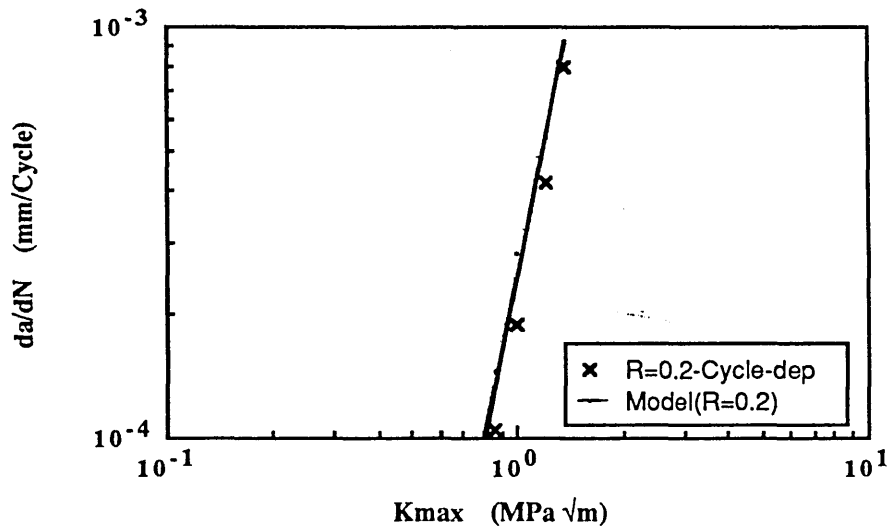


Fig 5.87 (b)

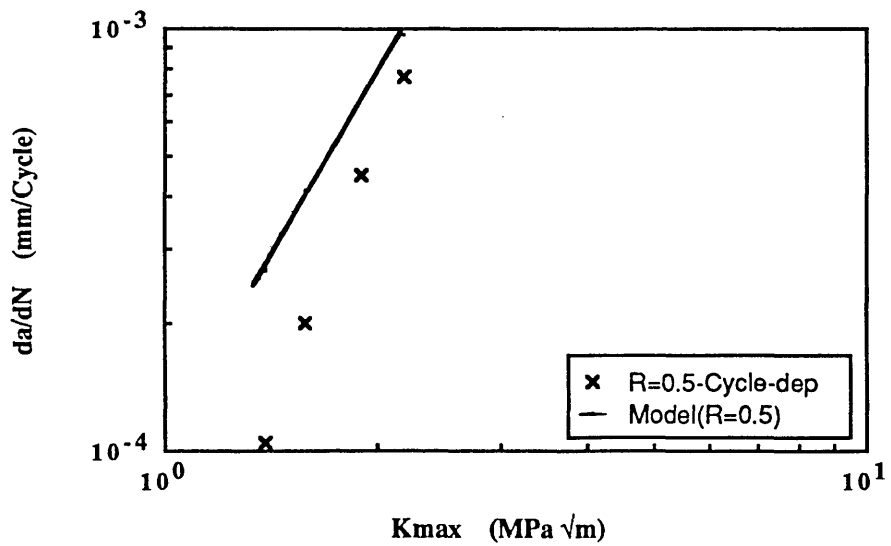


Fig 5.87 (c)

Fig 5.87 (a-c) Cycle-dependent crack growth rates in BP33 predicted by the Two Stage Line Zone Model compared with experimental data for R-ratios of 0.1, 0.2 & 0.5.
 $T = -20^{\circ}\text{C}$

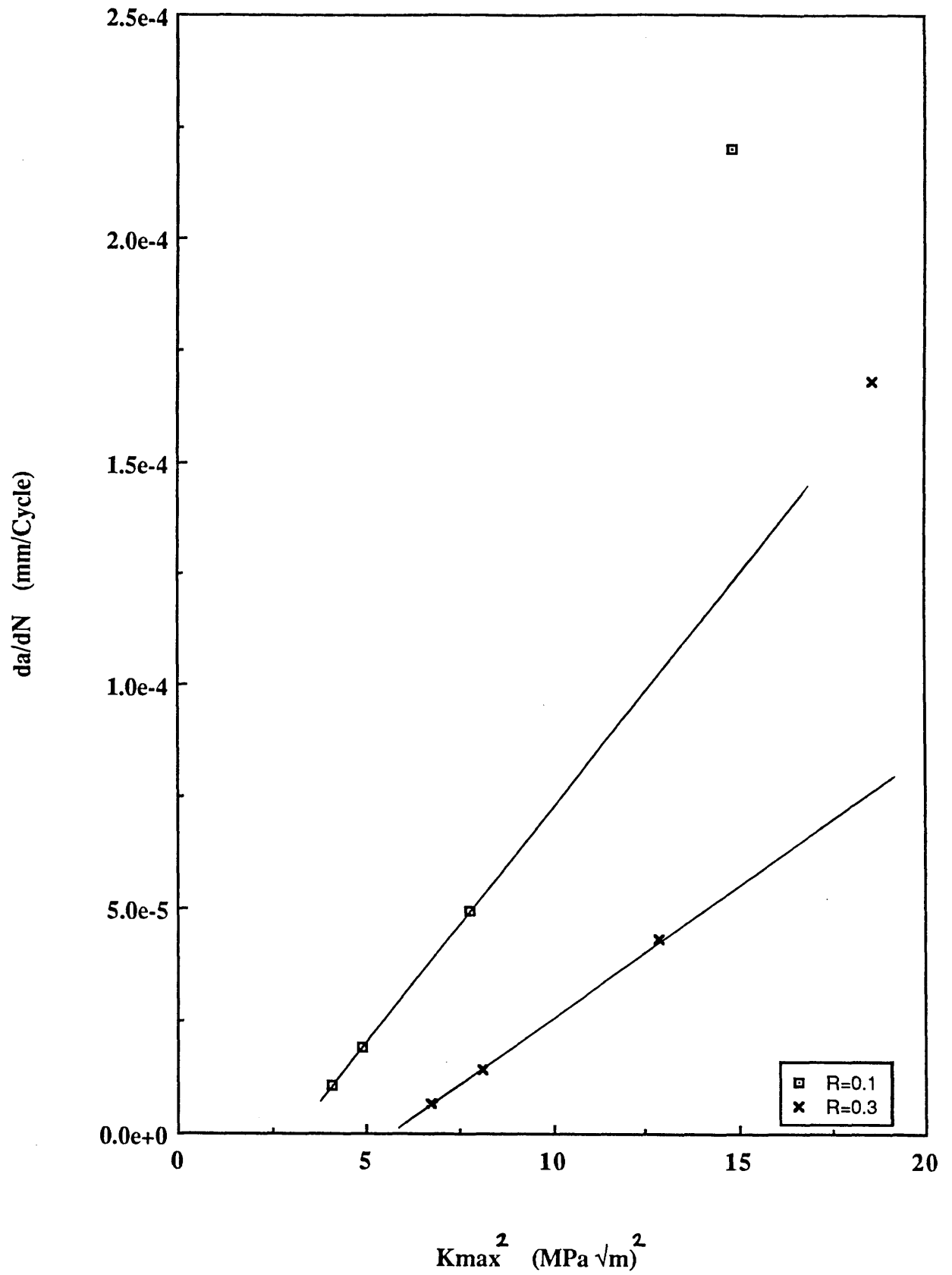


Fig 5.88 Cycle-dependent crack growth in LL101AA plotted against K_{max}^2 .
 $T=-20^{\circ}\text{C}$

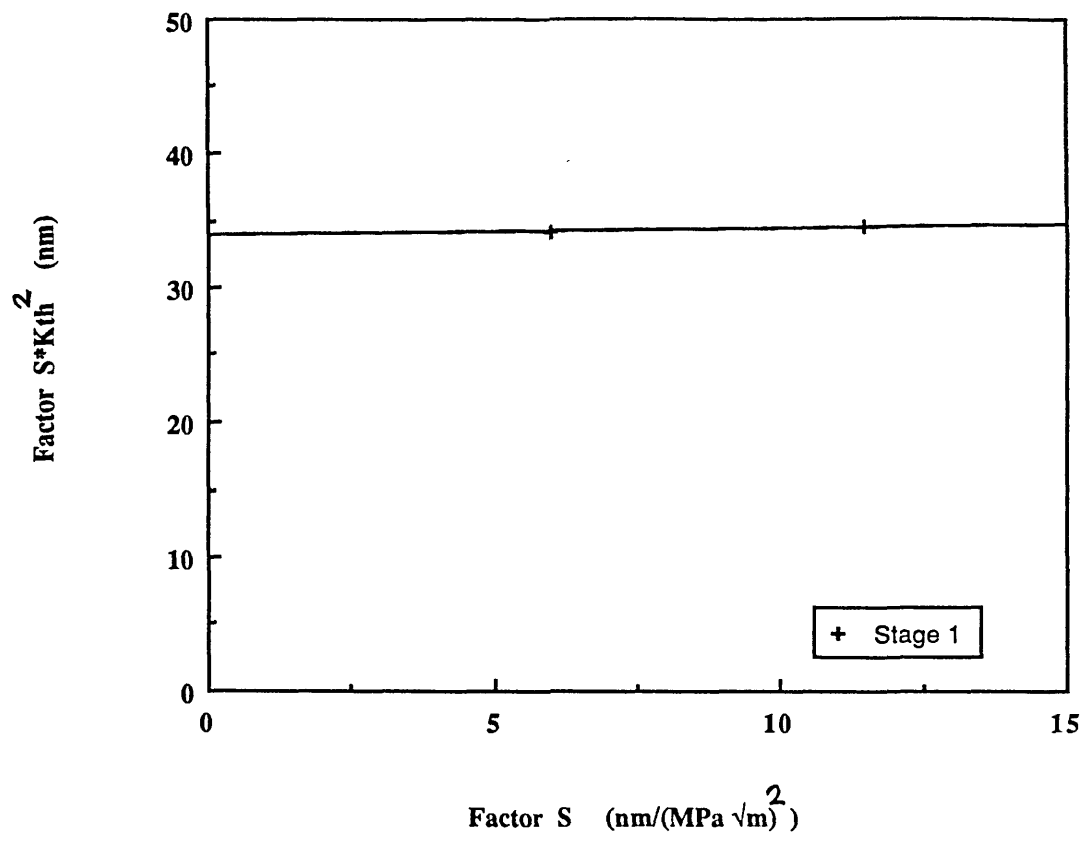


Fig 5.89 The factor SK_{th}^2 plotted against S in accordance with the Two Stage Line Zone Model for Cycle-dependent crack growth in LL101AA
 $T = -20^\circ\text{C}$

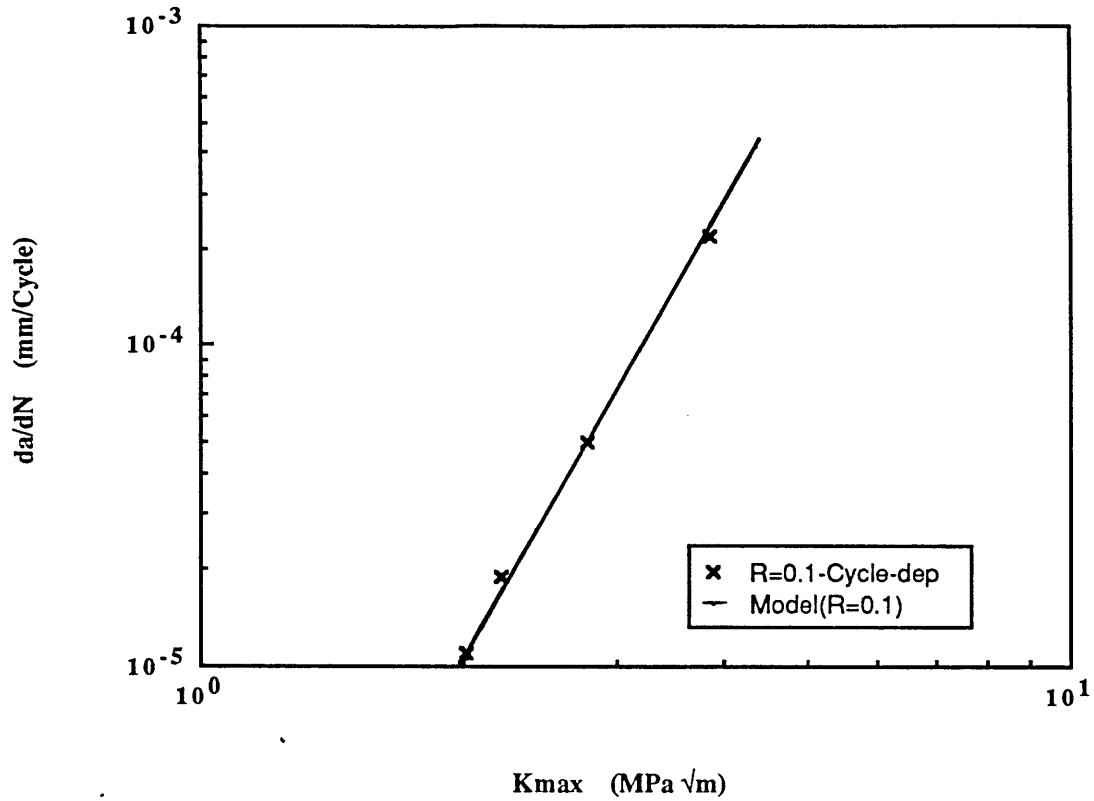


Fig 5.90 (a)

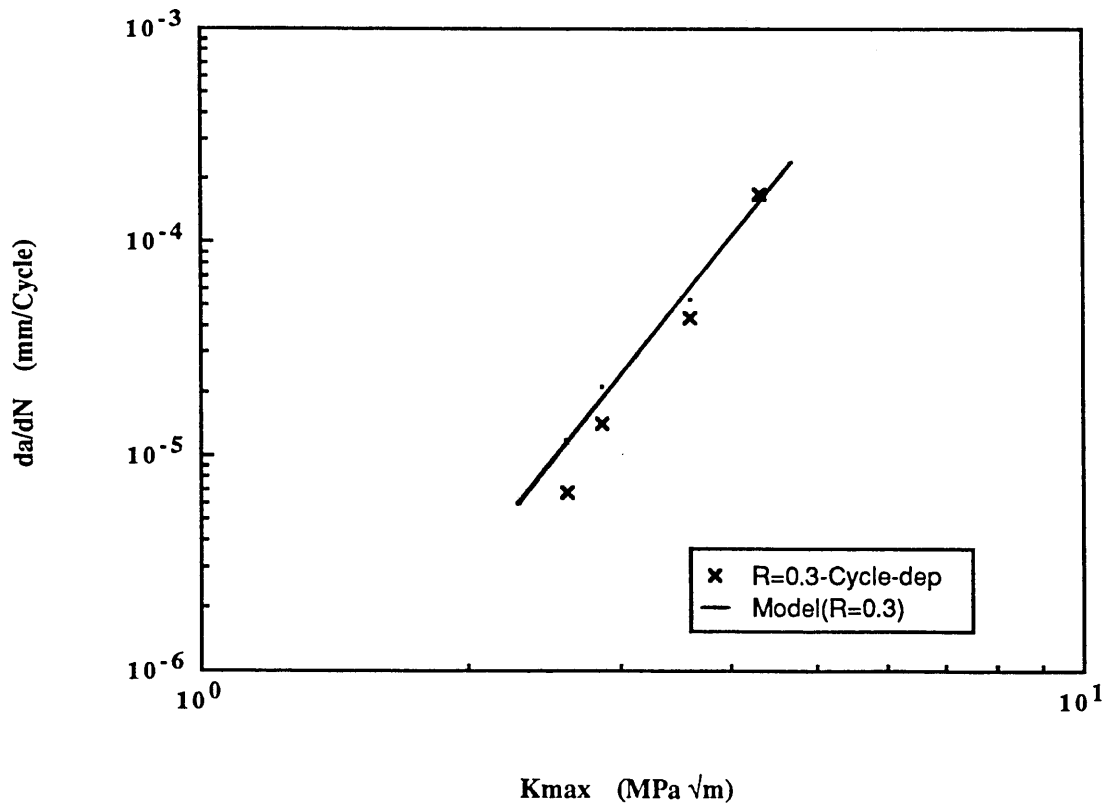


Fig 5.90 (b)

Fig 5.90 (a) & (b) Cycle-dependent crack growth rates in LL101AA predicted by the Two Stage Line Zone Model compared with experimental data for R-ratios of 0.1 & 0.3.
 $T = -20^{\circ}\text{C}$

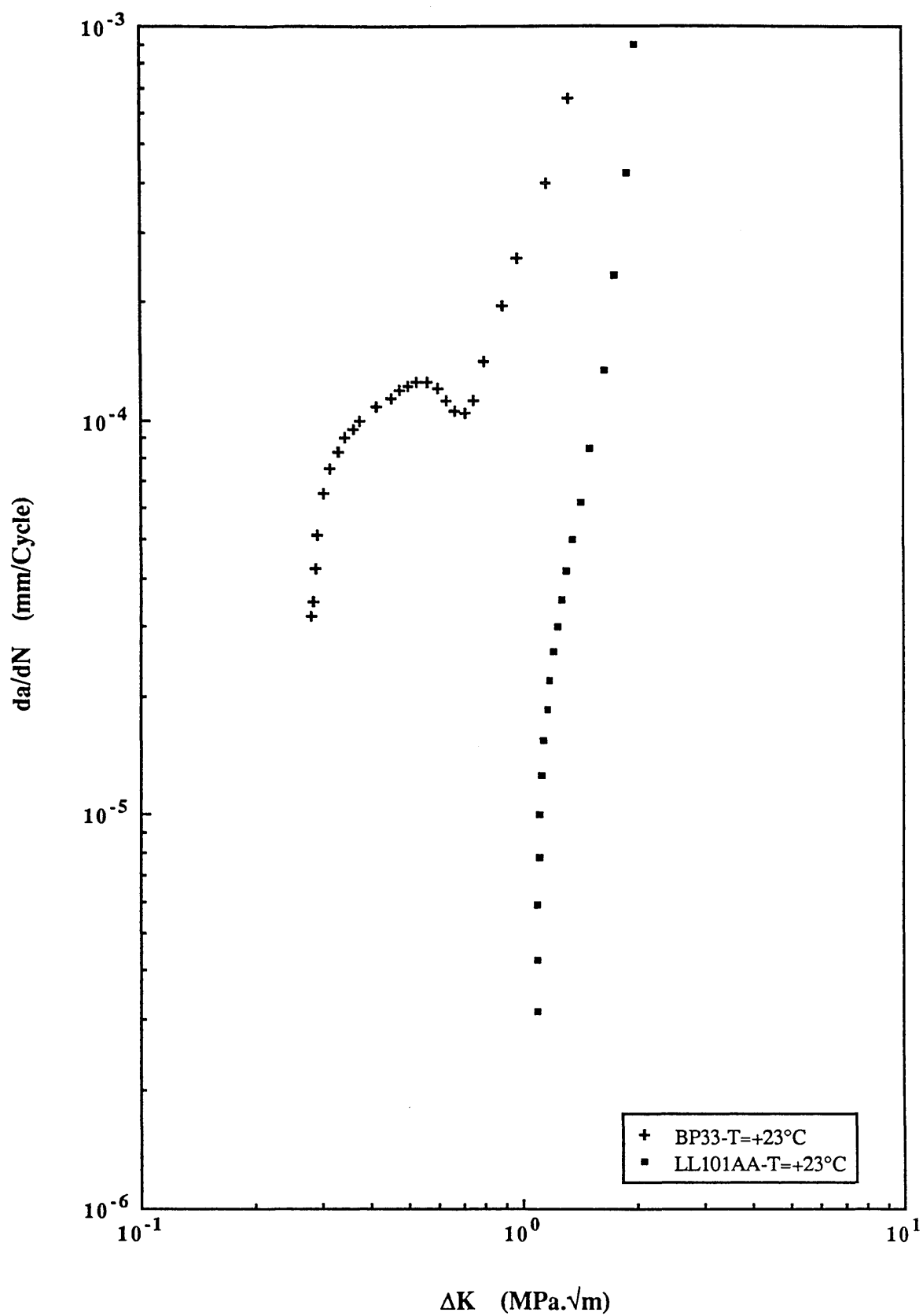


Fig 5.91 Comparison of FCP rates in BP33 and LL101AA at $T=+23^\circ\text{C}$, $F=3.0$ Hz and $R=0.1$.

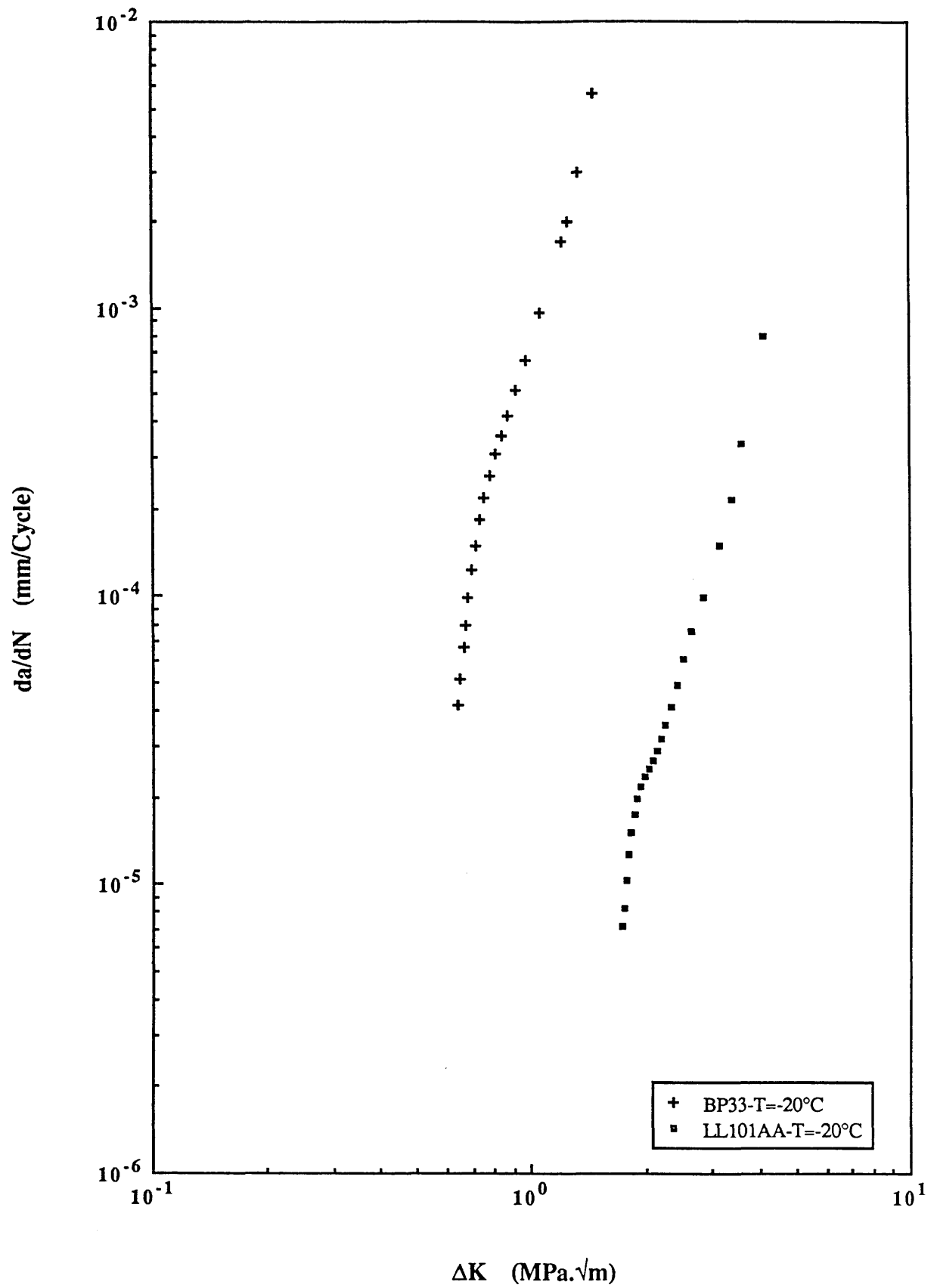


Fig 5.92 Comparison of FCP rates in BP33 and LL101AA at $T=-20^{\circ}\text{C}$, $F=3.0$ Hz and $R=0.1$.

R-ratio	Frequency (Hz)	Stage I m	Stage III m
0.1	10.0	27.60	4.86
0.2	10.0	19.00	4.25
0.3	10.0	18.50	6.00
0.4	5.0	22.50	4.03
0.5	10.0	—	8.50
0.6	10.0	31.30	8.50

Table 5.1 Paris law coefficients for DFDG 4262
 $\frac{da}{dN} = A \Delta K^m$

R-ratio	Frequency (Hz)	m	A ($\times 10^{-4}$)
-1.00	3.0	3.52	60.90
-0.75	3.0	3.52	57.00
-0.50	3.0	4.80	17.80
-0.20	3.0	4.64	10.70
0.00	3.0	4.40	6.70
0.10	3.0	4.40	7.80
0.20	3.0	4.40	7.00
0.30	3.0	4.40	5.60
0.40	3.0	4.40	9.40
0.50	3.0	4.40	8.80

Table 5.2 Paris law coefficients for BP33 at T=-20°C

ΔK / (MPa $\sqrt{m}$)	da/dn / (mm/Cycle)	da/dt / (mm/Sec)
0.283	3.00E-05	3.10E-05
0.302	5.80E-05	3.50E-05
0.398	8.80E-05	4.00E-05
0.501	1.08E-04	3.40E-05
0.602	8.40E-05	4.70E-05
0.692	7.60E-05	8.80E-05
0.759	9.60E-05	1.30E-04

R-ratio=0.1

ΔK / (MPa $\sqrt{m}$)	da/dN / (mm/Cycle)	da/dt / (mm/Sec)
0.316	6.00E-05	1.30E-05
0.347	7.20E-05	2.40E-05
0.398	8.90E-05	2.60E-05
0.478	9.50E-05	2.50E-05
0.525	8.90E-05	2.60E-05
0.603	7.60E-05	5.00E-05
0.692	8.80E-05	8.40E-05

R-ratio=0.2

ΔK / (MPa $\sqrt{m}$)	da/dN / (mm/Cycle)	da/dt / (mm/Sec)
0.302	1.15E-05	6.00E-06
0.331	2.30E-05	5.50E-06
0.363	2.30E-05	1.07E-05
0.398	2.30E-05	1.90E-05
0.436	3.10E-05	3.00E-05
0.479	4.40E-05	4.70E-05
0.549	1.20E-04	8.60E-05

R-ratio=0.5

Table 5.3 Cycle & Time-dependent components of FCP rates at 1 Hz. BP33-T=+23°C

R-ratio=0.1

ΔK / (MPa $\sqrt{m}$)	da/dN / (mm/Cycle)	da/dt / (mm/Sec)
0.692	9.75E-05	1.60E-05
0.794	2.40E-04	4.60E-05
0.955	5.20E-04	8.80E-05
1.096	9.00E-04	1.40E-04

R-ratio=0.2

ΔK / (MPa $\sqrt{m}$)	da/dN / (mm/Cycle)	da/dt / (mm/Sec)
0.692	1.05E-04	1.50E-05
0.794	1.90E-04	4.10E-05
0.955	4.20E-04	9.60E-05
1.096	8.00E-04	1.50E-04

R-ratio=0.5

ΔK / (MPa $\sqrt{m}$)	da/dN / (mm/Cycle)	da/dt / (mm/Sec)
0.692	1.05E-04	1.02E-04
0.794	2.00E-04	1.50E-04
0.955	4.50E-04	2.60E-04
1.096	7.70E-04	5.00E-04

Table 5.4 Cycle & Time-dependent components of FCP rates at 1 Hz. BP33-T=-20°C

R-ratio	Frequency (Hz)	m	A
-2.00	3.0	17.40	6.70E-03
-1.00	3.0	16.60	3.70E-04
-0.75	3.0	5.70	9.60E-05
-0.50	3.0	5.70	7.70E-05
-0.20	3.0	5.10	2.67E-05
0.00	3.0	5.65	1.00E-05
0.10	3.0	5.65	9.50E-06
0.20	3.0	5.55	5.20E-06
0.30	3.0	6.00	6.00E-06

Table 5.5 Paris law coefficients for LL101AA at T=+23°C

R-ratio	Frequency (Hz)	m	A
-1.00	3.0	8.53	1.53E-06
-0.75	3.0	5.87	3.23E-06
-0.50	3.0	4.64	5.42E-06
-0.20	3.0	4.44	1.85E-06
0.00	3.0	4.14	1.84E-06
0.10	3.0	4.40	1.09E-06
0.20	3.0	4.14	1.86E-06
0.30	3.0	5.44	4.10E-07
0.40	3.0	5.52	2.19E-07

Table 5.6 Paris law coefficients for LL101AA at T=-20°C

R-ratio=0.1

ΔK / (MPa $\sqrt{m}$)	da/dN / (mm/Cycle)	da/dt / (mm/Sec)
1.820	1.09E-05	2.34E-06
1.995	1.92E-05	5.43E-06
2.512	4.96E-05	1.65E-05
3.467	2.20E-04	7.78E-05

R-ratio=0.3

ΔK / (MPa $\sqrt{m}$)	da/dN / (mm/Cycle)	da/dt / (mm/Sec)
1.820	6.75E-06	5.63E-06
1.995	1.42E-05	8.85E-06
2.512	4.34E-05	3.15E-05
3.020	1.68E-04	1.02E-04

Table 5.7 Cycle & Time-dependent components of FCP rates at 1 Hz. LL101AA-T=-20°C

Material and Test Environment	K_c MPa $\sqrt{m}$	K_o^2 MPa $^2.m$	$\alpha r_c / (1-\alpha)^2$ nm	S_o nm/MPa $^2.m$	αK_c^2 MPa $^2.m$	α	r_c μm	σ_c MPa
BP33 - T=-20°C F=3.0Hz	1.41	0.10	292	1050	0.26	0.13	1.70	679
LL101AA T=+23°C F=3.0Hz	1.82	-----	-----	-----	-----	-----	-----	-----
LL101AA T=-20°C F=3.0Hz	3.84	1.0	25.0	18.40	1.62	0.11	0.18	5667
↓ Cycle - dependent Crack Growth Only ↓								
BP33 T=-20°C F=1.0Hz	1.41	0.250	157	-----	0.15	0.075	1.79	661
LL101AA T=-20°C F=1.0Hz	3.84	0.05	34	-----	2.53	0.172	0.136	6531

Table 5.8 Derived values from crack propagation data (Williams Model)

CHAPTER 6

CRACK TIP TEMPERATURE ANALYSIS

6.1 INTRODUCTION

During cyclic loading a proportion of the work done on a specimen is converted into hysteretic heating. At high loads used with higher R-ratio tests the work done on a specimen is greater and therefore the proportion converted into hysteretic heating will be greater. Furthermore, hysteretic heating in the specimen increases when the duration of the loading cycle decreases (i.e at high frequencies). This chapter shows how both an increase in mean stress and frequency can increase the hysteretic heating at the crack tip of a specimen.

However, the first objective of this work was to establish whether temperature variations around the crack tip could be used to distinguish between the three distinct regions of crack propagation in Fig 5.4. It was expected that deceleration of FCP rates would be marked by a significant increase in temperature around the crack tip due to the applied energy being absorbed and transformed into heat energy, instead of being used to propagate the crack.

6.2 EXPERIMENTAL

The equipment consisted of an infra-red camera coupled to a computer controlled thermovision system. Computer software was provided which enabled the operator to analyse the temperature variations throughout the specimen to a very high resolution (0.1 of a degree). A colour printer, video recorder and a disk drive provided means for data storage.

The thermovision system was used to study the temperature variations around the crack tip, and from the Figures obtained each temperature value is represented by a specific colour. A temperature scale is provided on the left hand side of the picture. The graph on the right hand side is of a temperature analysis carried out within the white perimeter marked on the picture, and indicates the relative frequency at which each

temperature value occurs . Maximum , minimum and mean temperature values are also given , and each picture is labelled with the time and date at which it was taken . If the image was recorded on video then the time at which it was recorded is shown below the image .

6.3 RESULTS AND DISCUSSION

Tests were carried out on BP33 in order to establish whether temperature variations around the crack tip could be used to distinguish between the three distinct regions of crack propagation in Fig 5.4. That is, it was expected that the transition from stage I to stage II (where deceleration of FCP rate occurs) would be marked by a significant increase in temperature around the crack tip, due to the applied energy being absorbed and transformed into heat energy, instead of being used to propagate the crack.

The results at an R-ratio of 0.1 and a frequency of 3.0Hz are shown in Figs 6.1 (a-e) (Room Temperature = 17.7°C). Figs 6.1(a) and 6.1(b) represent the crack in its early stages and although crack deceleration had already started by Fig 6.1(b) there is no significant temperature rise at the crack tip. In fact, there is only a 1.9°C temperature rise which can be neglected. Crack tip temperature continued to increase slightly but even when the crack length had reached 55 mm (Fig 6.1(c)) there was only a 3.0°C temperature difference at the crack tip.

Fig 6.1 (d) represents stage III on the da/dN vs ΔK graph where FCP rates increase with increasing ΔK (due to the material's resistance being diminished) but before unstable crack growth occurred. At this stage a temperature difference of 6.5°C was observed at the crack tip. In general, once the crack length had grown into stage III, higher temperature differences were observed due to the much higher net section stresses being applied. Fig 6.1(e) represents the point at which the crack growth became unstable and as can be seen the temperature at the crack tip is 7.5°C higher than room temperature.

Hence it was not possible to distinguish between the three distinct regions of crack propagation by temperature analysis, since the temperature difference around the crack tip remained almost constant even when growth rate deceleration had

occurred. Significant temperature differences occurred only in the later stages of crack propagation (stage III) and more so when crack growth became unstable.

Figs 6.2 (a-e) represent crack tip temperature variations at an R-ratio of 0.2 and a frequency of 3.0 Hz. In the early stages of crack propagation represented by Figs 6.2 (a) and 6.2 (b) the crack tip temperature increased by 3.3°C. Again a 3.3°C temperature rise, although higher (at the corresponding stage) than at an R-ratio of 0.1 is not high enough to warrant deceleration in FCP rates. Also, higher temperature differences of up to 8.4°C were again observed once the crack had grown into stage III (Fig 6.2(c)). However, the temperature differences of up to 13°C which occurred once the crack growth became unstable and fractured, represented by Figs 6.2(d) and 6.2(e) respectively, are much higher than the corresponding values at an R-ratio of 0.1.

In general, it was found that throughout the test, at an R-ratio of 0.2, temperature differences at corresponding stages in the life of the specimen were higher than at an R-ratio of 0.1. This is because the work done at an R-ratio of 0.2 is greater due to higher mean stresses (i.e greater energy input) and therefore a greater proportion of this is converted into hysteretic heating.

Furthermore, temperature differences were studied at an R-ratio of 0.6 at a frequency of 10.0 Hz, and the thermovision pictures are shown in Figs 6.3 (a-d). The combined effects of both a high R-ratio and a high frequency have had a drastic effect on crack tip temperature. In fact a 5.3 °C temperature rise had occurred after only five minutes of cycling (Fig 6.3(a)). The crack temperature continued to increase with further cycling and after 4 hours and 20 minutes a crack tip temperature of 6.3°C above room temperature had been established (Fig 6.3(b)). It should be pointed out that the crack remained stationary throughout this test. Therefore, at 11.00 o'clock the next day, Fri 18th / Oct / 1985 (shown on the top left of the picture) the load was increased to $P_{\max}=7.0$ kN and $P_{\min}=4.2$ kN from $P_{\max}=6.0$ kN and $P_{\min}=3.6$ kN. After only 32 minutes of cycling the crack tip temperature had increased by a further 2.2°C, Fig 6.3(c). Further cycling produced much greater variations in crack tip temperature and in fact after a further 3 hours the crack tip temperature had risen to 29.9°C (Fig 6.3(d)), which is

12.2°C above room temperature. Therefore, this confirms that an increase in mean stress (even at the same R-ratio) can have a drastic effect in crack tip temperature. An increase of 12.2°C at the crack tip can decrease both the yield stress and the Young's modulus of the material by a substantial amount which completely changes the condition at the crack tip. Furthermore, it should be noted that throughout this test the crack did not propagate even at such high loads due to the severe conditions prevailing at the crack tip (set up by the high crack tip temperatures). Therefore, a general yielding condition is set up within the ligament material (i.e plastic flow is no longer contained) and consequently the material's resistance to crack propagation is enhanced.

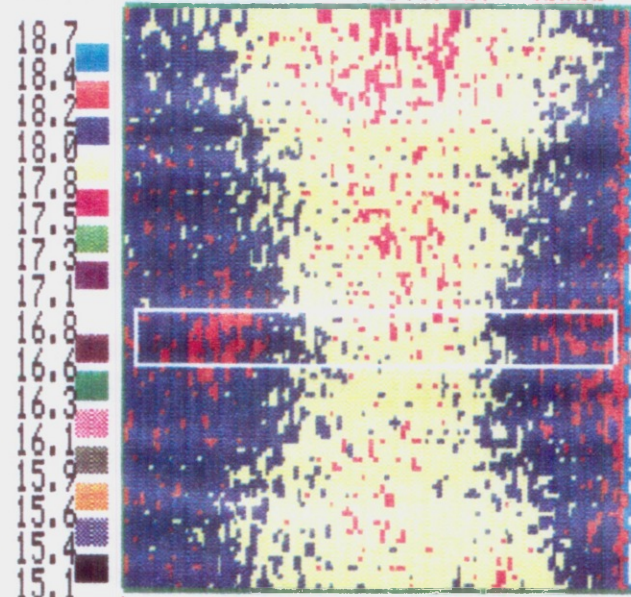
Finally at 14.45 on Fri 18th / Oct / 1985 the frequency was increased to 15 Hz at an R-ratio of 0.6 with the same loading conditions as above ($P_{\max}=7.0$ kN and $P_{\min}=4.2$ kN), and the results are shown in Figs 6.4 (a-c). After only five minutes the temperature increased rapidly to 31.4°C (Fig 6.4(a)), a further increase of 1.5°C, and continued to increase at a dramatic rate. At 15.46, after only one hour of cycling at 15 Hz the crack tip temperature had risen to 37.8°C which is 20.1°C above room temperature (Fig 6.4(b)). The consequences of this can be seen from Fig 6.4(b) where the crack tip opening displacement is enormous due to the material becoming softer in the region of the crack. Furthermore, it should be noted that by this time the crack had propagated by about 5 mm in a tearing mode. Further cycling had increased both the crack length (tearing) and the crack tip opening displacement due to a further increase in crack tip temperature of 0.7°C (a total of 20.8°C above room temperature), (Fig 6.4(c)).

Therefore, this proves and shows how an increase in frequency can increase the temperature at the crack tip of a specimen. Furthermore, it is not surprising that temperature variations of the magnitude observed here can significantly affect the conditions at the crack tip. Firstly, the crack tip opening displacement increases due to the general yielding of the ligament material which means that the plastic flow is no longer contained. Consequently, the plastic zone spreads through the entire ligament material and the fracture condition can only be reached upon the occurrence of a sufficiently large strain (ductile tearing). This effect enhances the material's resistance to fatigue crack

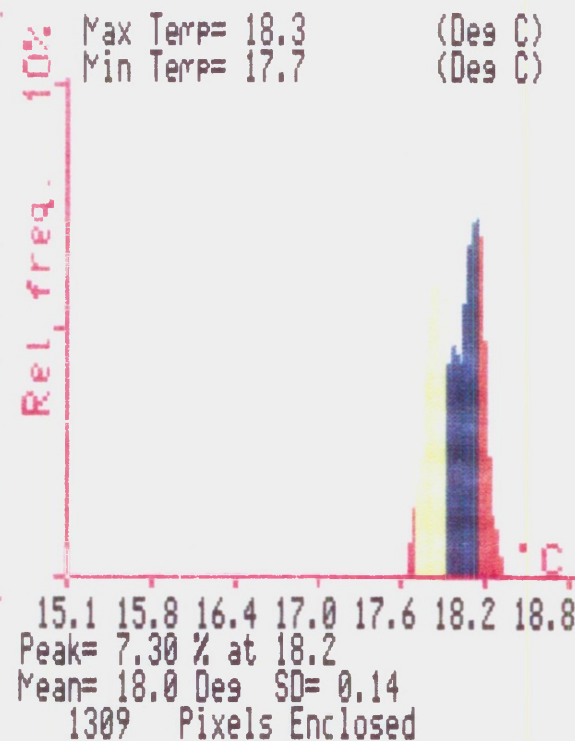
propagation.

Finally, this chapter has established that an increase in both the R-ratio and frequency leads to considerable increases in crack tip temperature. Therefore, in conclusion it can be said that this confirms and goes some way in explaining the slower FCP rates observed by an increase in both the above variables.

16:56 Tue 29Oct85 Thermoteknix V1.18



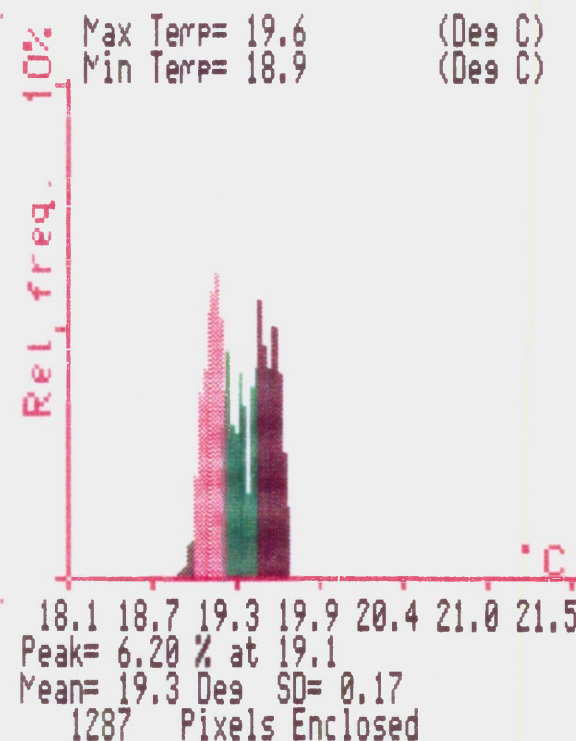
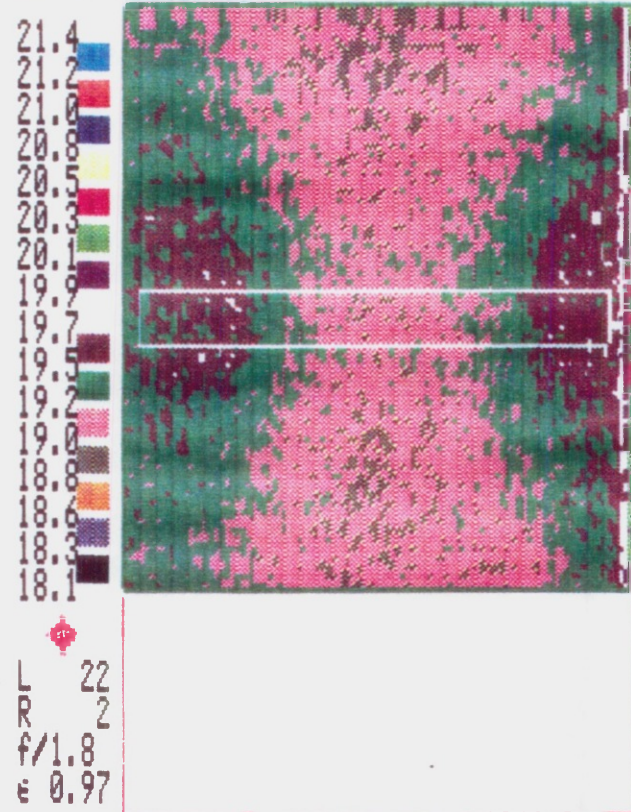
L 20
R 2
f/1.8
e 0.97



TEST 23 LDPE-BP33-12.45 Mon 28 Oct 85-T23B-

Fig 6.1 (a) R=0.1 - F=3.0 Hz

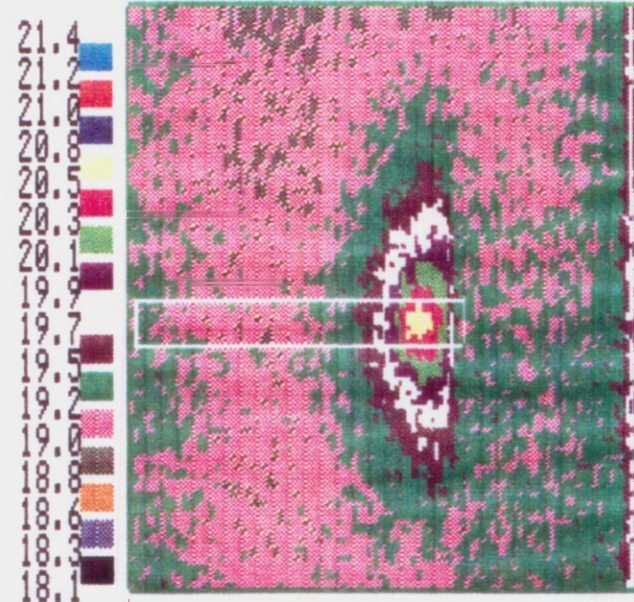
17:47 Tue 29Oct85 Thermotax 01.10



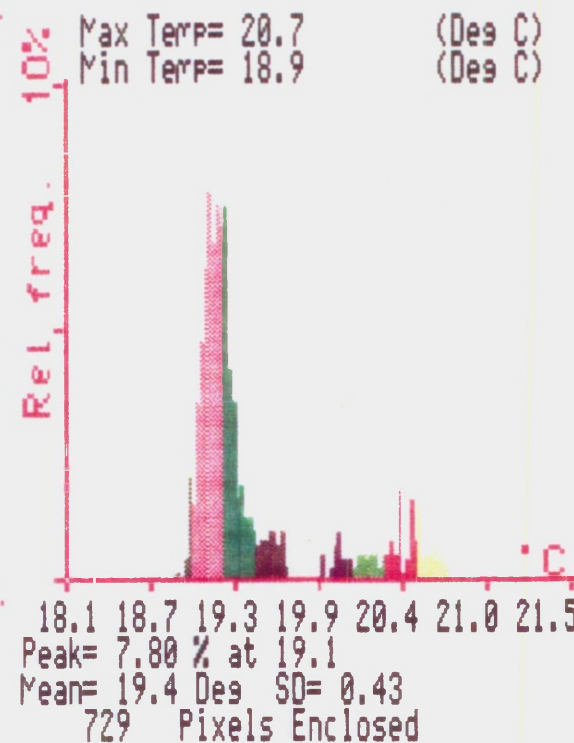
TEST 23 LDPE-BP33 17.55 Mon 28 Oct 85-T23G-

Fig 6.1 (b)

18:01 Tue 29Oct85 Thermoteknix 01.18



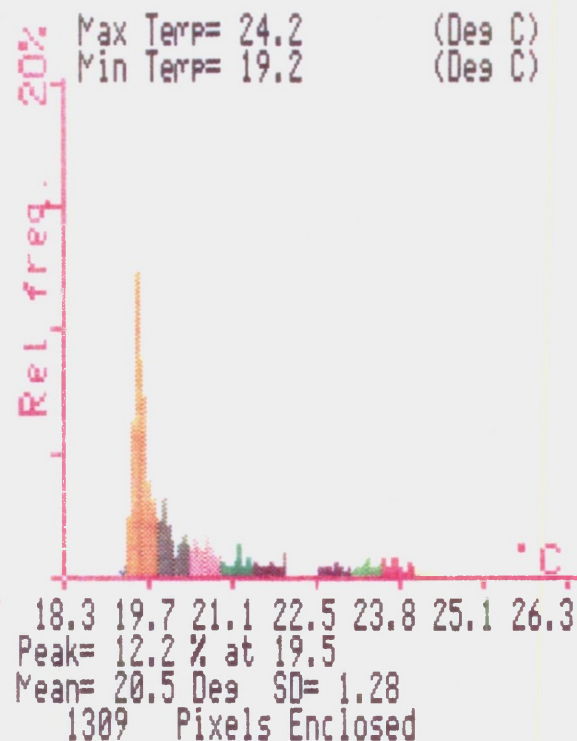
L 22
R 2
f/1.8
e 0.97



TEST 23 LDPE-BP33-11.34 Tue 29 oct 85-T23H-

Fig 6.1 (c)

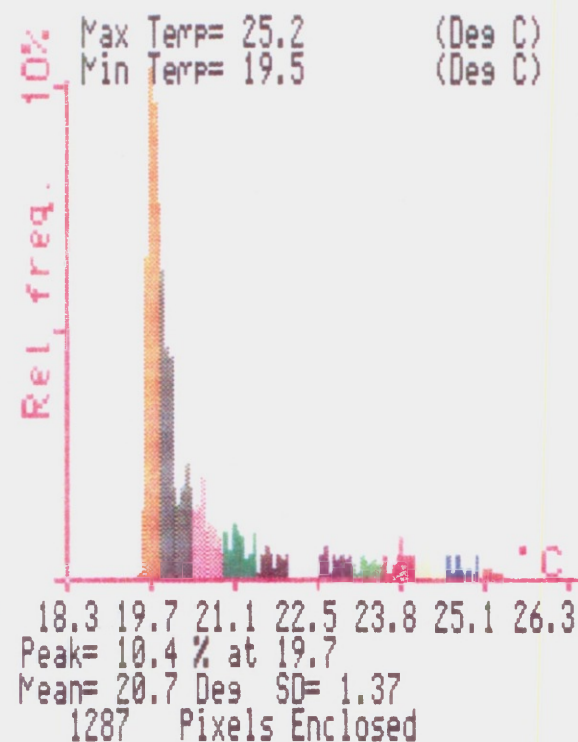
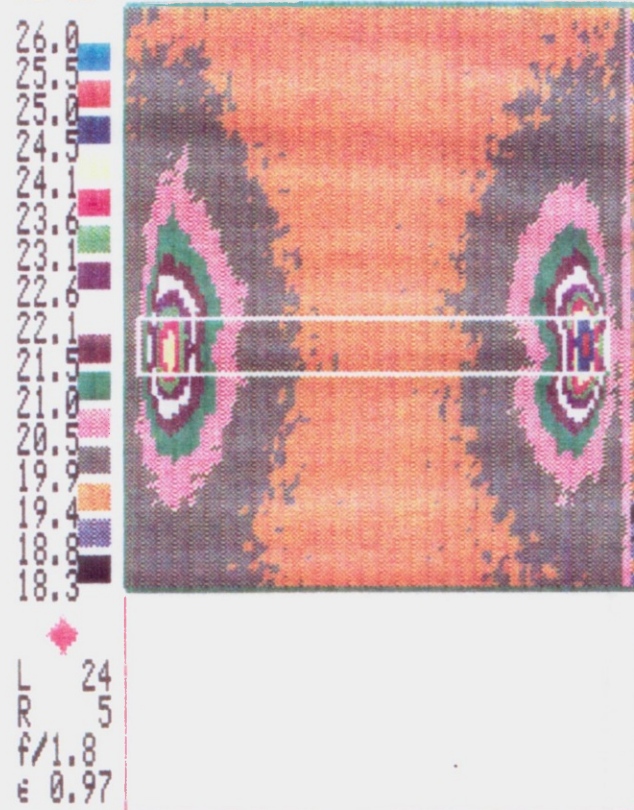
18:40 Wed 30Oct85 Thermoteknix U1.18



TEST 23 LDPE-BP33 13.40 Tue 29 Oct 85 -T23T-

Fig 6.1 (d)

12:06 Thu 31Oct85 Thermoteknix V1.10



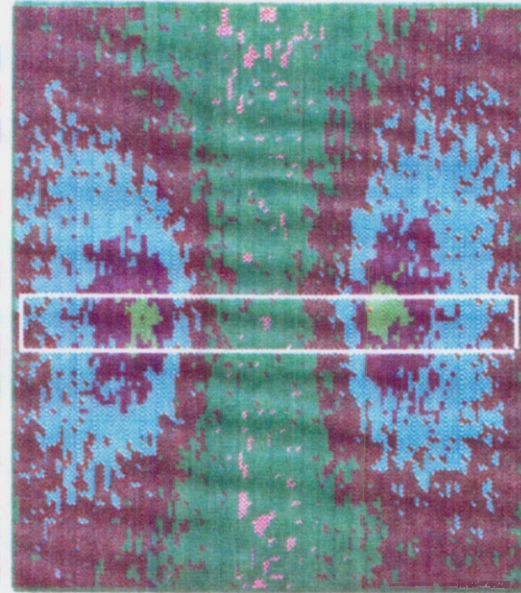
TEST 23 LDPE-BP33 13.48 Tue 29 Oct 85 -T23U-

Fig 6.1 (e)

11:06 Wed 23Oct85 Thermoteknix V1.10

20.1
19.8
19.6
19.4
19.2
19.0
18.7
18.5
18.3
18.0
17.8
17.6
17.3
17.1
16.9
16.6

L 21
R 2
f/1.8
e 0.97



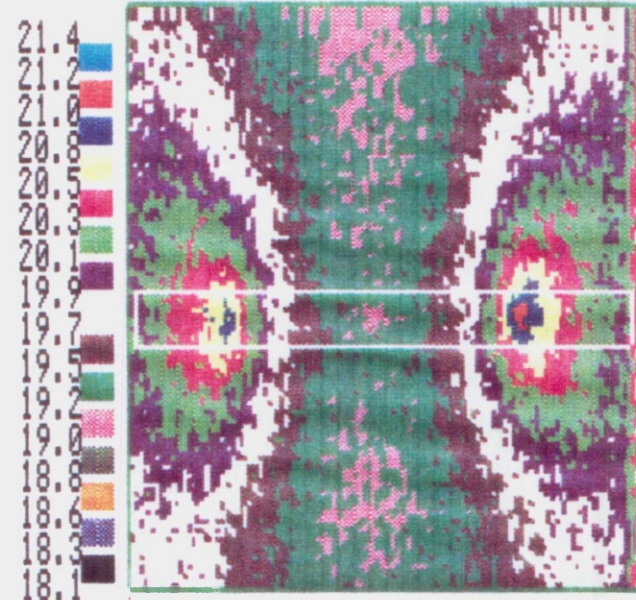
5%
Rel freq.
Max Temp= 18.9 (Deg C)
Min Temp= 17.7 (Deg C)

16.6 17.3 17.9 18.5 19.1 19.6 20.2
Peak= 4.50 % at 18.5
Mean= 18.3 Deg SD= 0.24
1353 Pixels Enclosed

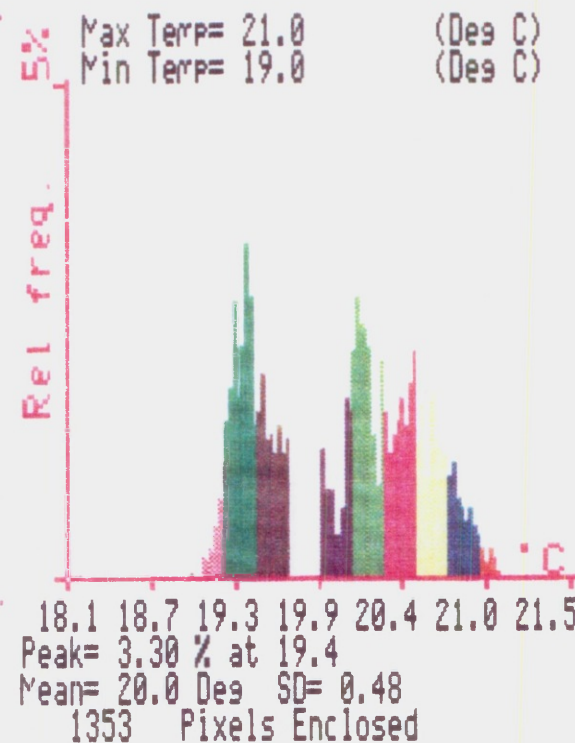
Clear | Box | Pts | Param | Temp | Grad | File | Inase | Exit | Whole

Fig 6.2 (a) R=0.2 - F=3.0 Hz

15:03 Thu 24Oct85 Thermoteknix 01.10



L 22
R 2
f/1.8
ε 0.97

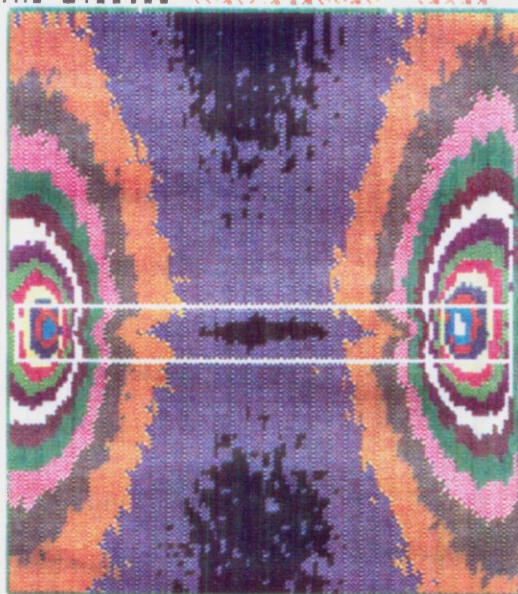


TEST 22 T22E-LDPE-BP33-18.05 WED 23 OCT 85

Fig 6.2 (b)

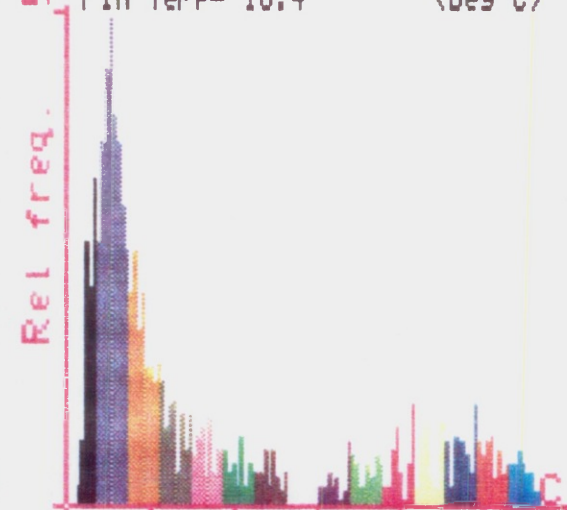
15:46 Thu 24Oct85 Thermatix V1.18

26.0
25.5
25.0
24.5
24.1
23.6
23.1
22.6
22.1
21.5
21.0
20.5
19.9
19.4
18.8
18.3



L 24
R 5
f/1.8
e 0.97

5% Max Temp= 26.1 (Deg C)
Min Temp= 18.4 (Deg C)

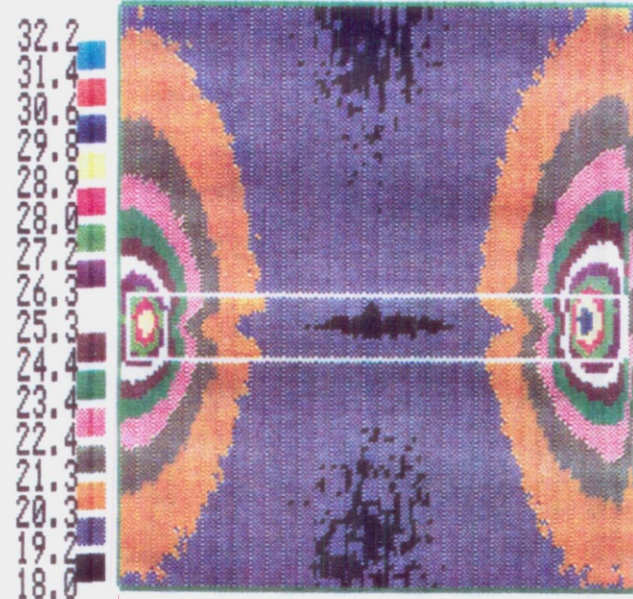


18.3 19.7 21.1 22.5 23.8 25.1 26.3
Peak= 4.90 % at 19.0
Mean= 20.9 Deg SD= 2.24
1353 Pixels Enclosed

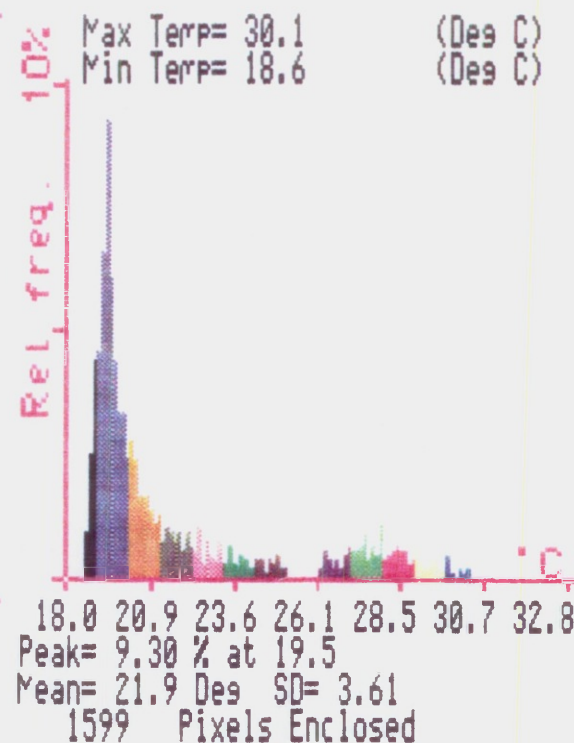
TEST 22-T22J-LDPE-BP33-11.29 THU 24 OCT 85

Fig 6.2 (c)

16:24 Thu 24Oct85 Thermoteknix U1.18



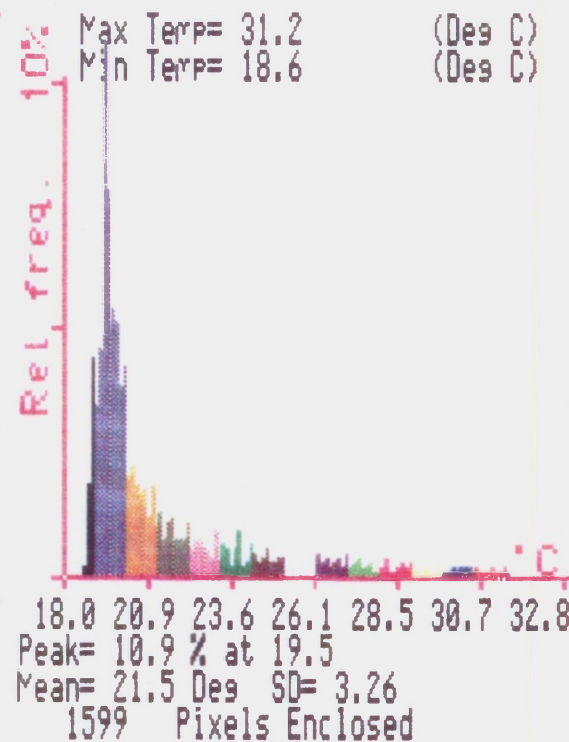
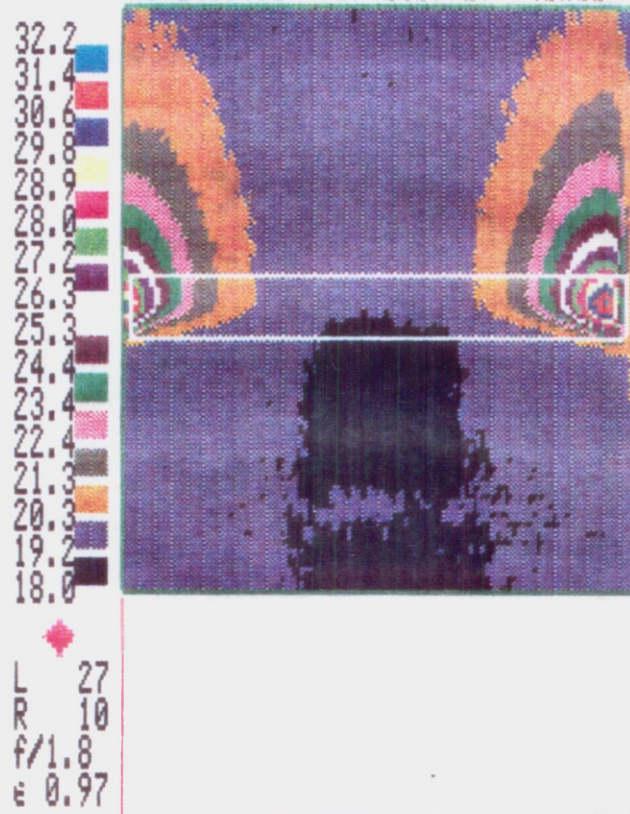
L 27
R 10
f/1.8
e 0.97



TEST 22-T22M-LDPE-BP33-12.00 Thu 24 Oct 85

Fig 6.2 (d)

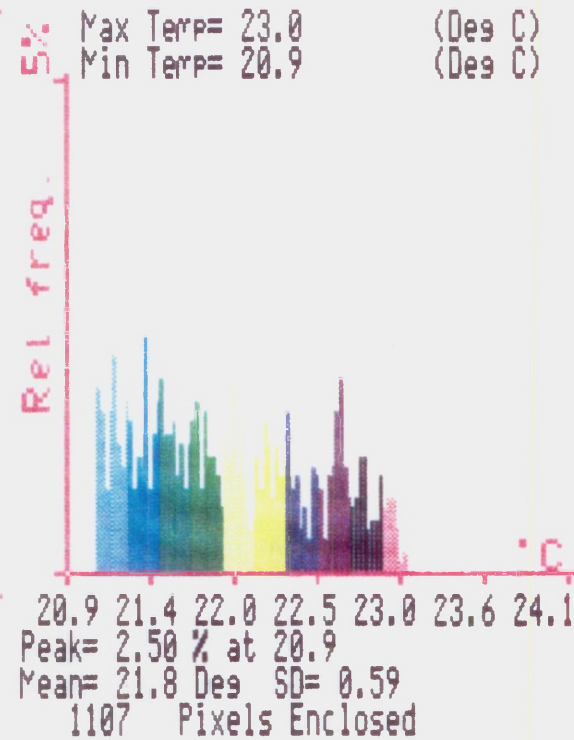
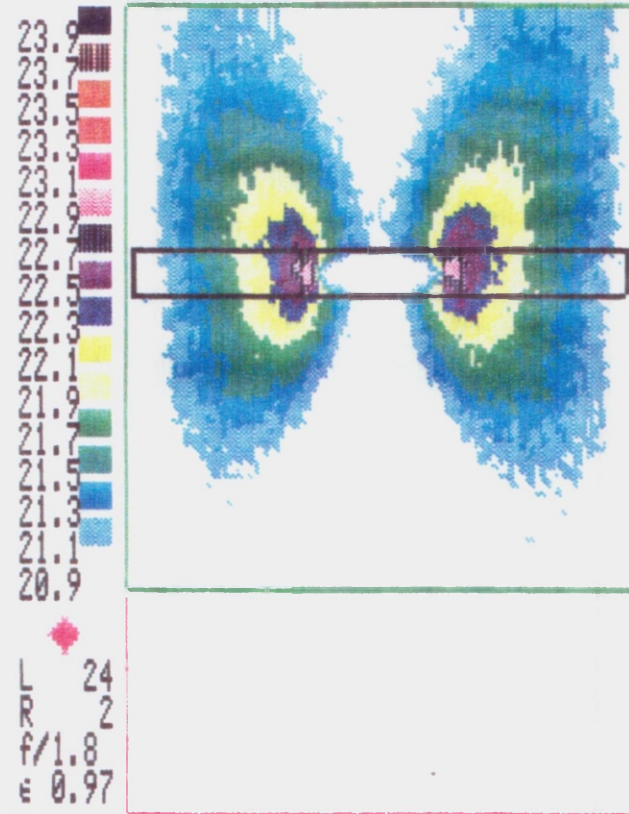
16:35 Thu 24Oct85 Thermoteknix V1.18



TEST 22-T22N-LDPE-BP33-12.11 Thu 24 Oct 85

Fig 6.2 (e)

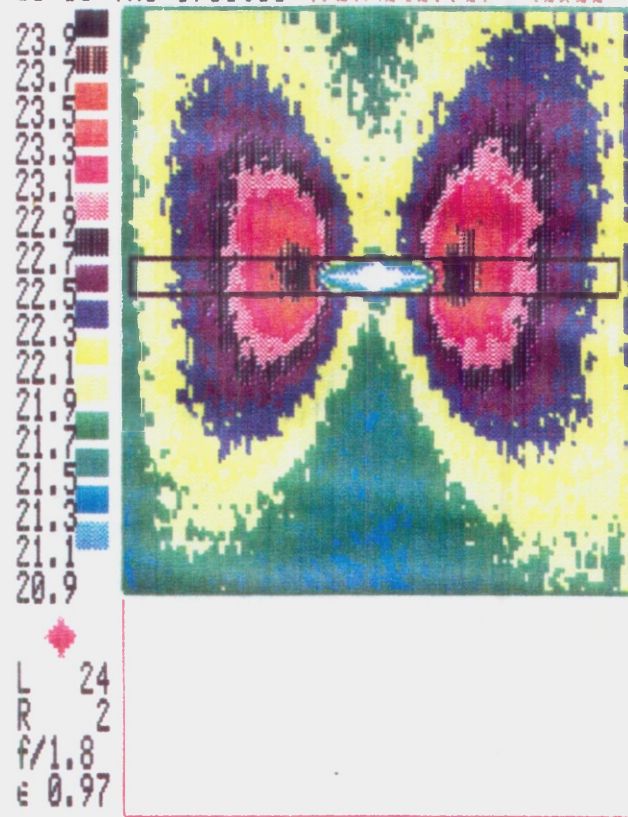
10:45 Thu 17Oct85 Thermoteknix V1.10



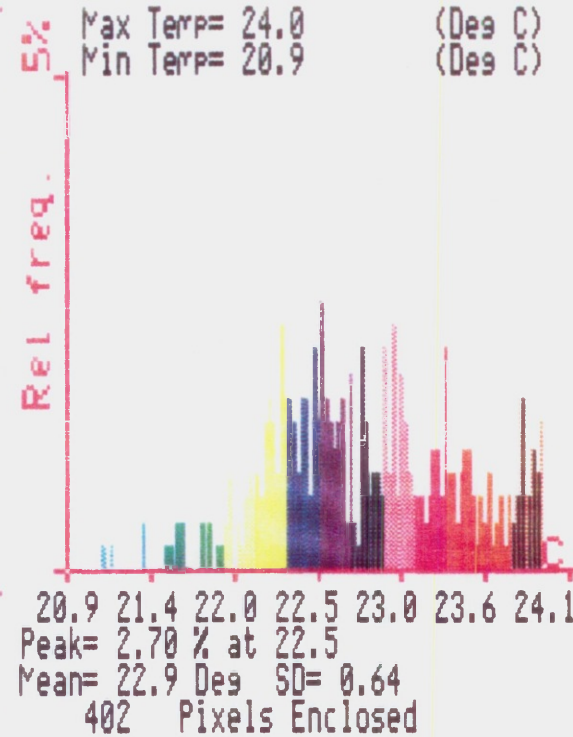
Clear | Box | Pts | Params | Temp | Grad | File | Inase | Exit | Whole

Fig 6.3 (a) $R=0.6$ - $F=10.0$ Hz ($P_{\max}=6.0$ kN , $P_{\min}=3.6$ kN)

15:00 Thu 17Oct85 Thermoteknix V1.18



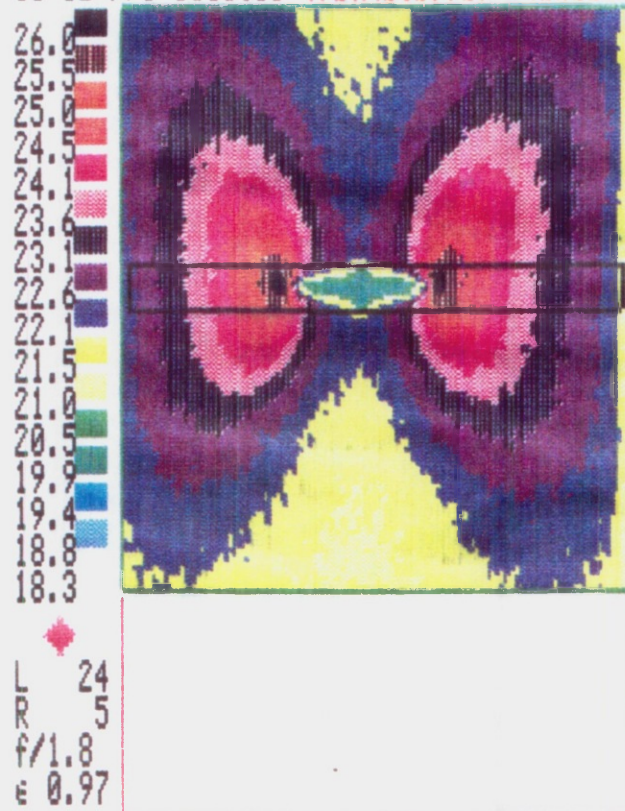
5% Max Temp= 24.0 (Deg C)
Min Temp= 20.9 (Deg C)



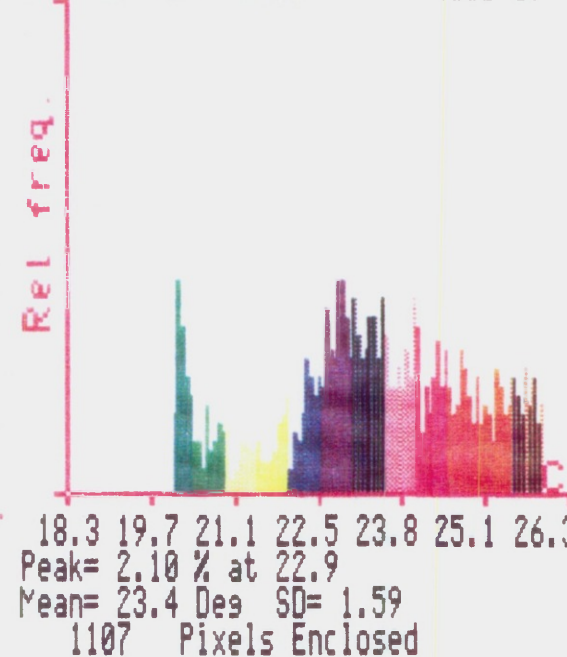
Clear | Box | Pts | Focus | Temp | Grad | File | Inase | Exit | Whole

Fig 6.3 (b)

11:32 Fri 18Oct85 Thermoteknix V1.16



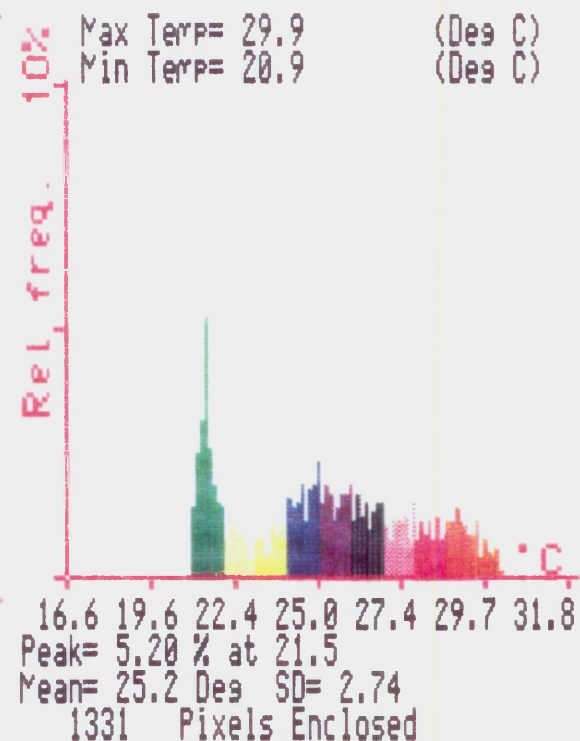
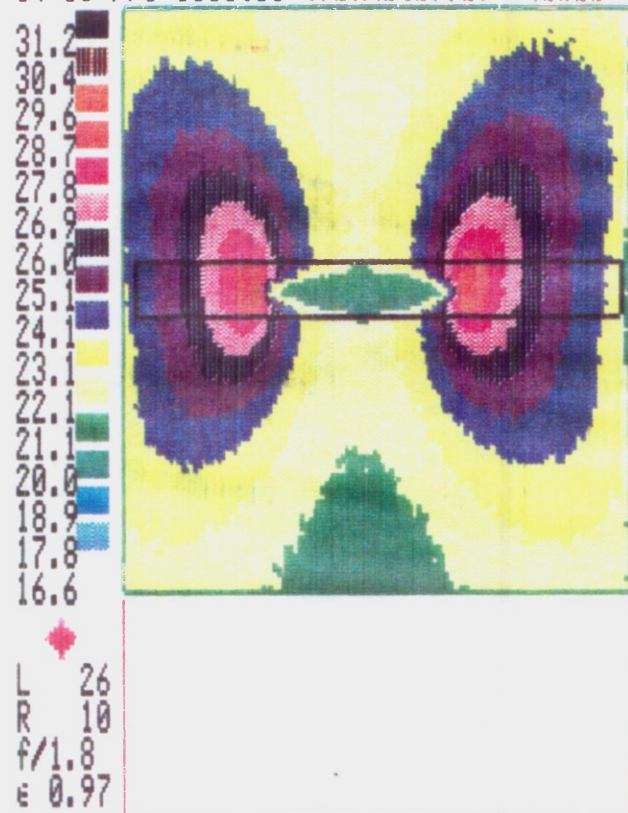
5% Max Temp= 26.2 (Deg C)
Min Temp= 20.1 (Deg C)



Clear | Box | Pts | Param | Temp | Grad | File | Inase | Exit | Whole

Fig 6.3 (c) $R=0.6$ - $F=10.0$ Hz ($P_{\max}=7.0$ kN , $P_{\min}=4.2$ kN)

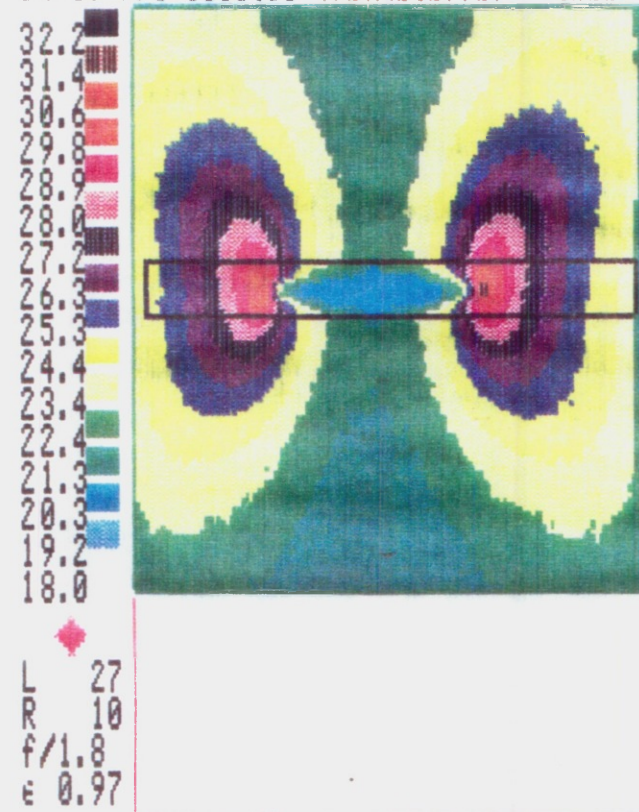
14:35 Fri 18Oct85 Thermoteknix 01.18



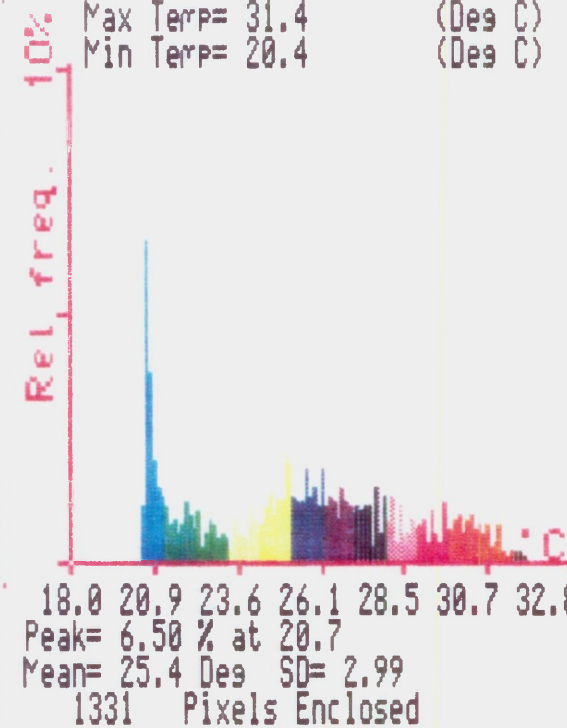
Clear | Box | Pts | Param | Temp | Grad | File | Inase | Exit | Whole

Fig 6.3 (d)

14:50 Fri 18Oct85 Thermoteknia V1.10



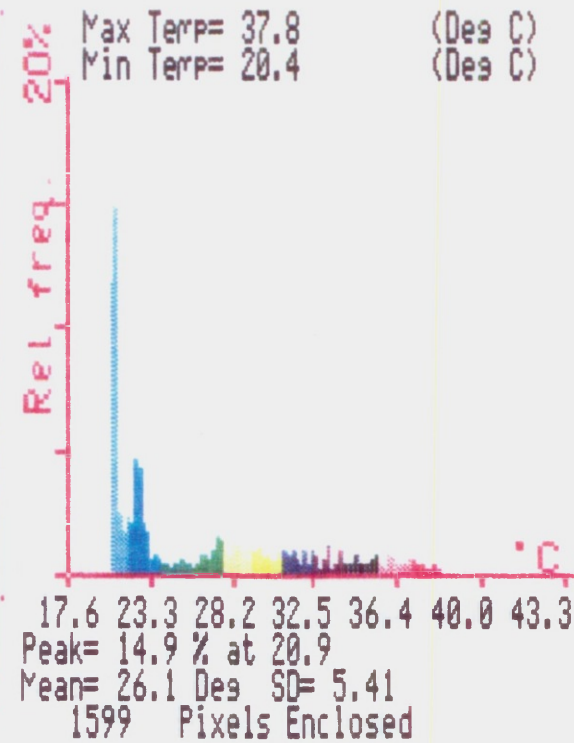
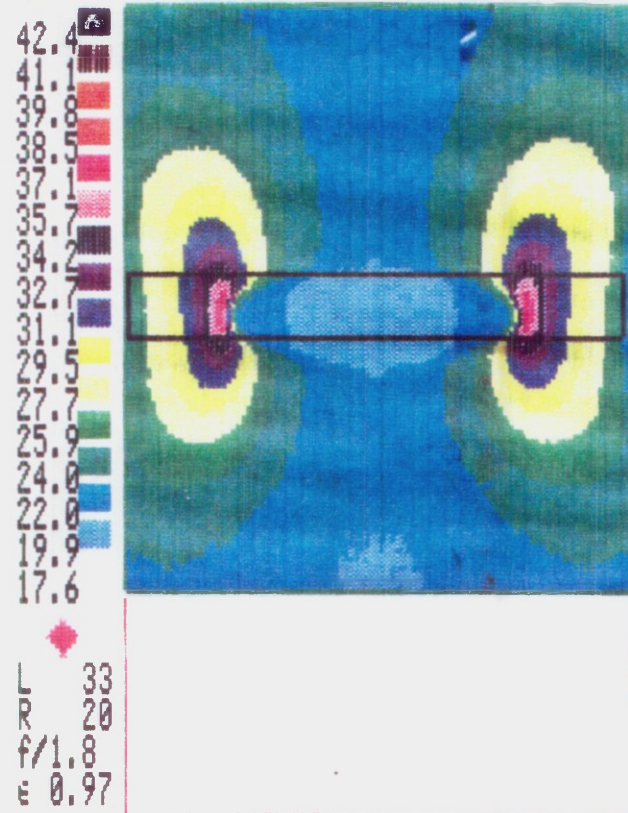
Max Temp= 31.4 (Deg C)
Min Temp= 20.4 (Deg C)



Clear | Box | Pts | Faras | Temp | Grad | File | Inase | Exit | Whole

Fig 6.4 (a) $R=0.6$ - $F=15.0$ Hz ($P_{\max}=7.0$ kN , $P_{\min}=4.2$ kN)

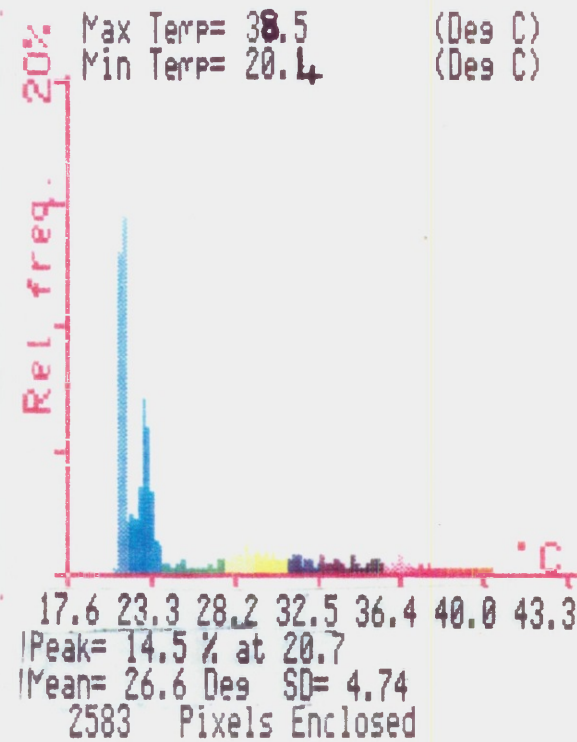
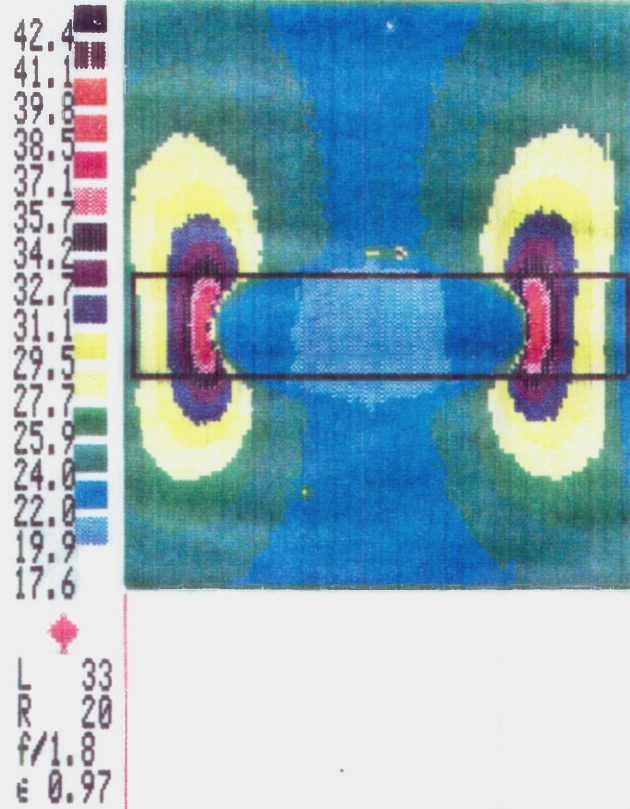
15:46 Fri 18Oct85 Thermoteknix 01.18



Clear | Box | Pts | Faras | Temp | Grad | File | Inase | Exit | Whole

Fig 6.4 (b)

16:09 Fri 18Oct85 Thermoteknix 01.18



Clear | Box | Pts | Faras | Temp | Grad | File | Inase | Exit | Whole

Fig 6.4 (c)

CHAPTER 7

CONCLUSIONS

7.1 INTRODUCTION

The main objective of the research programme was to obtain fracture toughness values and to gain some understanding of the fatigue behaviour of low and linear low density polyethylene. Fracture toughness values were obtained by fatigue crack growth tests due to their ability in characterising the toughness of viscoelastic materials, i.e cyclic loading has the effect of resharpener the crack tip and consequently brittle cracks may be grown. Throughout the research programme, tests were conducted under sinusoidal load control and for each grade, mean stress, frequency and temperature effects have been studied.

Furthermore, the first requirement of the testing programme was to automate the crack length monitoring technique. This was done by using a novel automated process which employs time lapse photography. This consists of a camera (Canon T70) equipped with a Canon 70 command back which can be programmed to take pictures at any set time. Lighting was provided by means of a flash gun which was switched on and off by a time switch. The film was then developed, the negatives mounted into slides, projected onto a screen and the crack lengths measured.

7.2 R-RATIO EFFECTS ON FCP

Fatigue crack propagation rates in low and linear low density polyethylene were correlated with the range of the stress intensity factor, ΔK , using the Paris model (Eqn 5.2). This was only found to be successful in predicting FCP rates in BP33, at R-ratios above zero, at a test temperature of -20°C and a constant test frequency of 3.0 Hz (Fig 5.10). In all other cases there is a different Paris relationship at different R-ratios. Specifically it was found that an increase in R-ratio led to slower FCP rates. In contrast, an increase in negative R-ratio (i.e higher compressive stresses) led to much higher growth rates. Also, decelerating growth rates were observed in BP33 and DFDG

4262 at room temperature. Several other FCP theories and some empirical models were examined. Only the Williams two stage line zone model was found to give a more satisfactory correlation of the FCP results than the simple Paris model. However, this model failed at the two extremes of high positive and high negative R-ratios.

7.3 FREQUENCY EFFECTS ON FCP

In all cases it was found that variable frequency led to a different Paris relationship. Specifically slower crack growth rates were observed with increasing frequency.

Using Dumbleton's representation it was possible to divide the total fatigue crack propagation in BP33 and LL101AA into two components; cycle-dependent FCP (FCP(n)) and time-dependent dynamic FCP (FCP(t)). In all cases, cycle-dependent crack growth was greater than time-dependent crack growth. Furthermore, it was not possible to grow a crack under an applied static load. Therefore, time-dependent crack growth occurs within the duration of the cycle due to cyclic loading having the tendency to keep resharping the crack tip, thus eliminating crack arrest due to blunting.

Time-dependent effects were enhanced with an increase in R-ratio, i.e at an R-ratio of 0.5 the time-dependent component was almost equal to the cycle-dependent component. This was due to higher mean stresses acting more effectively in the duration of the loading cycle.

7.4 CRACK TIP TEMPERATURE ANALYSIS

Temperature variations around the crack tip were monitored but were found inadequate in distinguishing between the three distinct regions of crack propagation. That is, deceleration of FCP rates occurred without a significant increase in temperature around the crack tip. However, higher R-ratios were found to produce higher temperature differences at the crack tip. This is because the work done at higher R-ratios is greater due to higher mean stresses and therefore a greater proportion of this is converted into hysteretic heating.

Much higher temperature differences were observed in tests carried out at both a high R-ratio and a high frequency. Specifically a temperature rise of 20.9°C was observed in a test carried out at an R-ratio of 0.6 and a frequency of 15 Hz. It was also observed that the crack tip opening displacement was very large and the crack had propagated by about 5mm in a tearing mode.

7.5 RECOMMENDATIONS FOR FURTHER WORK

7.5.1 Variable Temperature Tests

The results from the tests conducted on BP33 and LL101AA at room temperature have shown a plane strain to plane stress transition. However, at a test temperature of -20°C this transition was eliminated. Therefore it would be worthwhile carrying out tests at intermediate test temperatures (i.e -5°C , -10°C and -15°C) so as to establish the exact point at which only plane strain conditions prevail.

Furthermore, it was established that an increase in R-ratio led to slower growth rates. This has been attributed to higher mean stresses causing a more severe blunting condition at the crack tip. However, it has also been established that higher R-ratios lead to greater crack tip temperature variations which can also cause a more severe blunting condition at the crack tip. Therefore, if experiments could be conducted with a constant crack tip temperature it would be interesting to see the difference in FCP behaviour. That is, it is anticipated that smaller differences in growth rates would be

observed with increasing R-ratio due to a less severe blunting condition, caused solely by higher mean stresses (with increasing R-ratio).

7.5.2 Time- dependent Fatigue Crack Propagation

Time-dependent effects were enhanced with an increase in R-ratio. However, the time-dependent component was always lower than the cycle-dependent component, but at an R-ratio of 0.5 they were almost equal. Therefore, it would be worthwhile conducting tests at higher R-ratios, i.e 0.6 and 0.7, to see whether the time-dependent component would be greater than the cycle-dependent component.

Also from the graphs of da/dN against the reciprocal frequency it can be seen that the results can be represented by two different lines (solid curve and dotted curve). This is due to a thermal effect where the results at higher frequencies (i.e $>3.0\text{Hz}$) relate to a crack tip temperature somewhat higher than the test temperature. Therefore, again tests should be carried out at a constant crack tip temperature to see whether time-dependent effects will be eliminated. That is, it is anticipated that under such conditions the material's FCP behaviour will be independent of test frequency.

7.5.3 Crack Tip Temperature Analysis

A lot more work needs to be done with the thermovision equipment so as to establish exact temperature variations occurring at the crack tip for each test condition considered in the research programme. Therefore temperature rises occurring due to any increases in R-ratio or frequency will be known. Then tests can be carried out to find out how the material's properties (i.e yield stress and Young's Modulus) change with such temperature rises.

With this information the investigator will know a lot more about crack tip condition and what transition the ligament material is undergoing.

7.5.4 Initiation

Initiation has not been considered in this research programme but two initiation tests were carried out. The experiments were on identical specimens tested at identical loading conditions. However, the number of cycles required to initiate the crack were found to vary (between the two tests) by a substantial amount (52000 cycles). This is due to the difficulty in making identical notches in the specimens. Therefore, a more severe yielding condition can be introduced by the wedging action of the razor blade at the crack tip of a specimen, thus leading to a greater initiation period.

Therefore, if initiation is to be considered, the notching process has to be reviewed. One way of achieving better results is to razor notch at very low temperatures thus reducing the amount of yielding at the crack tip. Also a mechanical device is needed (Jig) so that all notches made are identical.

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APPENDIX 1

Definition of Input Variables for Fortran Program.

Fortran Code	Variable Definition
ID(I)	Specimen identification , for example material Code Number.
NPTS	Number of paired (a, N) data points.
PMIN	Minimum load, Pmin, in kN.
PMAX	Maximum load, Pmax, in kN.
F	Test frequency.
B	Specimen thickness, B.
W	Specimen width, W.
T	Test temperature, °C.
SMIN	Minimum applied stress, Smin, in kPa.
SMAX	Maximum applied stress, Smax, in kPa.
KIND	Specimen type, CCT.
NETS	Net Section Stress, σ_N , in kPa.
A(I)	Crack length, a, measured from cyclic tests.
N(I)	Elapsed cycles, N.

FORTRAN COMPUTER PROGRAM.

```

PROGRAM FIT(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7)
DIMENSION A(200),N(200),BB(3),DADN(200),ID(7)
DIMENSION AA(10),NN(10)
REAL N
REAL NN,LDELTAK,KMAX,NETS,SMAX,SMIN,LKMAX,ROD
INTEGER QQ
500 READ(5,30,END=1000) (ID(I),I=1,7),NPTS
30  FORMAT(6A6,A4,I6)
    READ(5,99)PMIN,PMAX,F,B,W
99  FORMAT(F8.1,F8.1,F8.3,F8.3,F8.3)
    READ(5,25)KIND,T
25  FORMAT(A6,F8.3)
    READ(5,*) (A(I),N(I),I=1,NPTS)
    WRITE(6,15)
15  FORMAT(1H1,"SEVEN POINT INCREMENTAL POLYNOMIAL METHOD FOR
+DETERMINING DA/DN")
    WRITE(6,35) (ID(II),II=1,7),NPTS
35  FORMAT(///11H SPEC. NO. ,6A6,A4,10X,15H NO. OF POINTS=,I3)
    WRITE(6,10)KIND,B,W,T
10  FORMAT(/A6,9H SPECIMEN,5X,2HB=,F8.3,5H MM. ,5X,
+2HW=,F8.3,5H MM. ,5X,2HT=,F8.3,8H DEG C . )
    SMAX=PMAX*1000./(B*W)
    SMIN=PMIN*1000./(B*W)
C
    R=PMIN/PMAX
    WRITE(6,20)PMIN,PMAX,R,F,SMAX,SMIN
20  FORMAT(/6H PMIN=,F8.3,3HKN ,5X,5HFMAX=,F8.3,3HKN ,5X,2HR=,
+F6.3,5X,10HTEST FREQ=,F8.3,3HHZ ,5X,6H SMAX=,E9.4,5HMM/M2,
+5X,6H SMIN=,E9.4,6HMM/M2.)
    WRITE(6,55)
55  FORMAT(///6H OBS.NO,3X,6HCYCLES,4X,8HA(MEAS.),4X,7HA(REG.),
+5X,4HKMAX,7X,6HSQKMAX,6X,5HLOGDK,7X,4HNETS,7X,6HDELTAK,5X,
+5HDA/DN,5X,7HLOGKMAX)
    WRITE(6,200)
200 FORMAT(//10H
    K=0
    DO 110 I=1,3
    WRITE(6,95) I,N(I),A(I)
110 CONTINUE
95  FORMAT(I4,3X,F8.0,3X,F8.3)
    NPTS=NPTS-6
    DO 100 I=1,NPTS
    L=0
    K=K+1
    K1=K+6
    DO 60 J=K,K1
    L=L+1
    AA(L)=A(J)
    NN(L)=N(J)
60  CONTINUE
    C1=0.5*(NN(1)+NN(7))
    C2=0.5*(NN(7)-NN(1))
    SX=0.
    SX2=0.
    SX3=0.
    SX4=0.
    SY=0.
    SYX=0.
    SYX2=0.
    DO 70 J=1,7
    X=(NN(J)-C1)/C2

```

Cont'd overleaf

```

YY=AA(J)
SX=SX+X
SX2=SX2+X**2
SX3=SX3+X**3
SX4=SX4+X**4
SY=SY+YY
SYX=SYX+X*YY
SYX2=SYX2+YY*X**2
70  CONTINUE
DEN=7.0*(SX2*SX4-SX3**2)-SX*(SX*SX4-SX2*SX3)+SX2*(SX*SX3-SX2**2)
T2=SY*(SX2*SX4-SX3**2)-SYX*(SX*SX4-SX2*SX3)+SYX2*(SX*SX3-SX2**2)
BB(1)=T2/DEN
T3=7.0*(SYX*SX4-SYX2*SX3)-SX*(SY*SX4-SYX2*SX2)+SX2*(SY*SX3-SYX*SX2
+)
BB(2)=T3/DEN
T4=7.0*(SX2*SYX2-SX3*SYX)-SX*(SX*SYX2-SX3*SY)+SX2*(SX*SYX-SX2*SY)
BB(3)=T4/DEN
YB=SY/7.0
RSS=0.
TSS=0.
DO 75 J=1,7
X=(NN(J)-C1)/C2
YHAT=BB(1)+BB(2)*X+BB(3)*X**2
RSS=RSS+(AA(J)-YHAT)**2
TSS=TSS+(AA(J)-YB)**2
75  CONTINUE
R2=1.0-RSS/TSS
DADN(I)=BB(2)/C2+2.0*BB(3)*(NN(4)-C1)/C2**2
X=(NN(4)-C1)/C2
AR=BB(1)+BB(2)*X+BB(3)*X**2
SIGMAX=FMAX/(W*B)*1000.
SIGMIN=FMIN/(W*B)*1000.
P=4.0*ATAN(1.0)
Y=(P*W/(P*AR)*TAN(P*AR/W))**(0.5)
CRACK=AR/1000.
DELTAK=(SIGMAX-SIGMIN)*Y*CRACK**(0.5)
DK=DELTAK*1000.
LDELTAK=ALOG10(DK)
KMAX=SIGMAX*Y*CRACK**(0.5)
ROD=KMAX*1000.
LKMAX=ALOG10(ROD)
NETS=SIGMAX/(1-(2.0*AR/W))
SQKMAX=KMAX**2
QQ=I+3
WRITE(6,92)QQ,N(QQ),A(QQ),AR,KMAX,SQKMAX,LDELTAK,NETS,DELTAK,
+DADN(I),LKMAX
92  FORMAT(I4,3X,F8.0,3X,F8.3,3X,F8.3,3X,E9.4,3X,E9.4,2X,F8.3,
+4X,E9.4,3X,E9.4,3X,E8.3,3X,F8.3)
WRITE(7,90)AR,DELTAK,LDELTAK,NETS,KMAX,LKMAX,SQKMAX,DADN(I)
90  FORMAT(5X,F8.3,5X,E9.4,5X,F8.3,5X,E9.4,5X,E9.4,5X,F8.3,5X,E9.4,
+5X,E8.3)
100  CONTINUE
J=NPTS+4
K=NPTS+6
DO 120 I=J,K
WRITE(6,95)I,N(I),A(I)
120  CONTINUE
WRITE(6,17)
17  FORMAT(/)
GO TO 500
1000 STOP
END

```

FORTRAN DATA FILE

32

LDPE(BF33).

	0.25	2.5	03.000	12.500	150.000
CN	+24.0		9.69		
16.40	000000.				
16.58	6000.00				
16.65	10000.0				
16.80	14000.0				
17.00	20000.0				
17.40	30000.0				
17.80	40000.0				
19.00	60000.0				
20.60	80000.0				
22.20	100000.				
24.00	120000.				
25.80	140000.				
27.70	160000.				
32.10	200000.				
36.60	240000.				
38.80	260000.				
41.10	280000.				
43.70	300000.				
46.40	320000.				
49.00	340000.0				
51.50	360000.0				
53.60	380000.0				
55.40	400000.0				
57.30	420000.0				
59.40	440000.0				
62.00	460000.0				
64.00	470000.0				
67.00	480000.0				
68.50	484000.0				
69.70	486000.0				
72.00	488000.0				
75.00	490200.0				