

REDUCTION OF VIBRATIONAL ENERGY IN COMPLEX STRUCTURES

by

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ABSTRACT

The general equations of motion of a structure are developed into those of a set of lumped parameter oscillators, each representing a mode of vibration. A system of connected structures is thus transformed into a system of coupled oscillators. By dividing the parameters describing the coupling by the parameters of the corresponding oscillators, the equations of motion are expressed in the form of a matrix of coupling factors.

For a system comprising excited, intermediate and receiving structures, the dependence of the overall energy ratio on modal oscillator mass, resonant frequency, loss factor and coupling factor is investigated.

It is shown how the performance of complex multimodal connected structures may be analysed using the concept of transmission paths, where a transmission path comprises one oscillator from each. The response of a receiving oscillator is found to be approximately equal to the sum of the responses from each path considered separately. This enables large systems to be analysed without the need to solve high order matrices, which can be prohibitively time consuming.

An experimental method of estimating modal oscillator parameters and coupling factors from point and transfer dynamic response tests is also described.

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CHAPTER ONE

LITERATURE SURVEY

1.1 INTRODUCTION AND REVIEW OF PAST WORK

To achieve a reduction in vibration transmission between connected structures, it is necessary to : (i) obtain an understanding of the nature of vibration transmission and of the basic structural parameters which govern it, (ii) to decide the design problem of reflecting or dissipating vibrational energy from structures either before or in the course of transmission to other structures..

In recent years a large body of theory has been evolved in the study of the transmission of random vibrational energy. These theories involve structures having many modes of vibration excited by wide band random forces. Skudrzyk [1]* and [2], showed that a structure with continuously distributed mass and stiffness could be represented by a 'series resonant canonical circuit', in which each mode of vibration was a lumped parameter oscillator. This method was much simpler than the classical theory where a mechanical structure was described by a system of differential equations and boundary conditions.

* References are given on page 224

Statistical theories developed in the field of room acoustics have been used amongst others by Lyon, Maidanik, and Eichler. Lyon and Maidanik [3] found that the power flow between two conservatively coupled, independently and randomly excited oscillators was proportional to the difference in their average modal energy, and applied this model to systems containing many modes. The problem of a multimodal structure excited by a reverberant sound field was also treated; each structural mode oscillator was considered in conjunction with those field oscillators lying in a narrow band centered on the natural frequency of the structural oscillator. A simple formula was developed which expressed the ratio of the spectral densities of the sound field and the acceleration of the structure in terms of the physical parameters of the two systems. The coupling between these two systems was determined by the 'radiation resistance', defined as the mean square velocity of the structure averaged in time and space. This was measured and calculated for the case of a baffled beam, good agreement was found for modes having natural frequencies above the coincidence frequency. Lyon and Eichler [4] extended this approach to two connected multimodal structures. A method was used in which the primary variable was energy, and the structure was described by its basic parameters of length, thickness, density etc. . The kinetic energy quantities were expressed as a function of mass, modal density, internal loss and coupling loss. The structures were assumed to respond linearly and their modes to be loosely coupled. Results were in the form of average levels of many modes. Two systems were treated in this way,

a beam cantilevered to a large plate, in which limited agreement between theory and experiment was observed, and that of one plate cantilevered to another, where good agreement between theory and experiment was observed at the higher frequencies where the reverberant wavefield was well established. It was considered by the authors, however, that the calculation of coupling loss factor was unreliable. Heckl's work on the absorption coefficients of plates [5] showed that where there is an interface between two plates all the bending wave energy incident at the interface was transmitted at high frequencies only. At low frequencies the proportion was approximately one third. Later work by Eichler [6] on the vibration transmission between multimodal systems showed that the products of modal density and coupling loss-factor between two connected systems were equal.

A different approach to the calculation of coupling loss-factor was presented by Lyon and Scharton [7], in which the energy transmission between a three element structure, consisting of two plates connected by a beam, was investigated. The coupling loss factor, which described the power flow between a beam and plate edge, was derived from an analysis of the impingement of flexural and torsional waves at the beam plate interface. This was calculated using admittance matrices of Wilmsers and Lambert [8] and Eichler [9], together with an assumption, that beam to plate loss factor was equal to the power transferred from the beam to plate, divided by the energy of the beam, multiplied by frequency. Good agreement was found between theoretical and experimental beam to

plate coupling factors at higher frequencies. The experimental results were obtained from a measurement of the decay rates of the beam, with plates attached. At lower frequencies the coupling of the beam to the plate appeared to be so strong that the plate internal loss factor was unavoidably measured; by using theoretical coupling loss factors to calculate the energy ratio of the plates only general agreement with experimental values was found. Newland [10] suggested that certain other assumptions were open to doubt. The symmetrical relationship that coupling loss factor between the beam and one plate is equal to the coupling loss factor between the other plate and the beam; whilst this may be true if both plates have the same modal density, it remained to be proved. An alternative definition of coupling loss factor was proposed, which complied with the symmetrical relationship, this was a more complicated expression involving the total number of coupled modes. This increased complication however was shown by Lyon and Scharton [11] to limit its application to systems with low modal densities.

A statistical energy analysis was developed by Ungar [12] to treat two coupled sets of modes, where each mode was represented in lumped parameter form. Both sets were excited by independent sources of white noise. It was shown that the power flow between mode sets was equal to the average mode to mode flow multiplied by the number of mode pairs between which power flow occurred. The total number of modes in a structure was designated an ensemble, and therefore an ensemble in general contained several sets. It was found that the ensemble average of the steady state time average power flow from

one set of an ensemble to a set of another ensemble was proportional to the difference between the ensemble average of the set average modal kinetic energies provided; the mode to mode coupling enabled the power flow between modes of the two sets to be proportional to the modal kinetic energy differences; the mode to mode coupling coefficients and modal kinetic energies are statistically independent; and that the modal kinetic energies in the ensemble average are uniformly distributed within a set. A formula was developed which related the energy levels of two coupled structures in terms of mass, number of modes and internal loss factor of each structure, coupling was taken into account by a power flow coefficient, although details of how this was to be calculated or measured were not given.

The isolation provided by a mounting system when supported on any non-rigid foundation was considered by Snowdon [13]. A knowledge of the mechanical impedance of the foundation was necessary, and the effectiveness of such a system was considerably lower than that predicted by its transmissibility curve. This was more marked with small ratios of foundation mass to mounted mass. The effectiveness of such a system could be improved by adding mass to the foundation. Ungar and Dietrich [14] showed that when a foundation, or receiving structure, had a finite impedance it was essential to incorporate some mass in the isolation system since both springs and dampers transmit all the force applied to them. The inclusion of a mass between two resilient elements was more efficient as an isolation system than by adding mass directly to the receiving structure;

both resilient elements should have lower impedances than both source and receiver. Optimum isolation was achieved by the use of resilient elements of equal impedance.

Mounting systems of very high efficiency could be produced by a two stage system, Gorman [15], without incurring the penalty of very low stiffness necessary with a single stage mounting. Skudrzyk [16] also showed that the effectiveness of an isolation system could be seriously reduced by resonances of the vibrating structure itself. Improved isolation was obtainable if the vibrating structure was mounted on an intermediate structure which had few modes by which energy could be transmitted. The coupling between modes of connected structures was found to be dependent on their relative resonant frequencies, Newland [17], and it was shown that mismatching of resonant frequencies substantially reduced the flow of energy between them.

In order to dissipate energy from a structure some form of damping must be incorporated. However, Ungar and Kerwin [28] showed that if highly damped elements were used in a vibration isolation system, they could not contribute significantly to the total loss factor if they did not also participate considerably in the total stored energy of the system. If a structure is composed of metal, some damping material must be added in order that usefully large amounts of energy can be dissipated. Long chain polymer rubbers and plastics are commonly used for this purpose by laminating with the metal. This type of damping can be applied either as an unconstrained or as a constrained layer. The former was first developed

by Oberst [18], but the latter has since proved to be more effective. Kerwin [19] developed a theory for analysing the performance of three layered damped systems when the constraining layer was thin. It was shown that the optimum overall damping obtainable was limited by the lack of extensional stiffness of the constraining layer.

Ruzicka [20] reviewed the basic concepts of structural damping finding that to obtain a high overall loss factor several criteria had to be met. These were that the loss factor of the visco-elastic material must be high, the shear parameter should be near its optimum value. The shear parameter was defined as a function of the storage modulus and amount of visco-elastic material used, the weight loading on the structural member, the flexural rigidity, extensional stiffness, geometry of cross section and frequency of vibration. Also a geometrical parameter should have a high value. This was a function of the geometry of cross section. A high value corresponding with a low value of static stiffness of the structure, indicating that in general a compromise between stiffness and damping was likely.

It was shown by Ungar [21] that all visco-elastically damped structures, whether they are plates or beams of any configuration obey very similar relationships, in particular a general structure with a thin visco-elastic layer behaves indentially to a laminated plate.

Di Taranto and Blasingame [22] studied the performance of three and five-layered beams. In both cases the overall loss factor

reached a peak value at one particular frequency. An approximate analysis based on the limiting case of a very thin visco-elastic layer was used to obtain the peak damping frequency, which also showed that below this frequency the gradient of the logarithm of loss factor to the logarithm of frequency was unity, and above this frequency the gradient was minus unity. The optimum overall loss factor was shown to be one third of the material loss factor. For equal overall beam thickness and equal visco-elastic material thickness, a three layered beam provided higher damping at lower frequencies, whereas a five layered beam produced higher damping at higher frequencies. Blasingame et al. [23] showed experimentally that a mounting structure consisting of members laminated with visco-elastic material was effective in reducing vibration transmission.

The analysis of beams using the limiting case of a thin visco-elastic layer could be in considerable error if applied to those with thick layers. Agbasiere and Grootenhuis [24] developed an accurate analysis of multi-layered beams with any thickness of visco-elastic material. This showed that beam end conditions had a considerable influence on the overall loss factor. A beam clamped at both ends had the widest range of shear parameter over which high damping was obtainable, whilst a cantilever had the narrowest.

High damping can also occur by transverse deformations of an attached visco-elastic layer. Ungar and Kerwin [25] found this to occur at the standing wave resonant frequencies of the damping layer and that the greater the mass of the damping layer the greater the damping of the system.

Nelson [26] showed that even with heavy damping a simply supported beam at resonance yielded a forced mode shape almost identical to the undamped natural mode shape. This was true as long as the modes were not closely spaced; if the modes had close resonant frequencies intermode coupling would cause mode shape impurity.

Gray [27] found that a resonating damped beam behaved similarly to a simple oscillator in that the kinetic and potential energy levels were the same, and had the same average value when the excitation was broadband random.

1.2 SCOPE OF PRESENT WORK

It became clear after a study of the literature on the subject, that the theory of vibration isolation with random excitation was not yet sufficiently developed for immediate calculations on practical problems. The relatively simple formulae developed by Lyon et al, involving only the basic parameters of the coupled structures, whilst being highly attractive had two limitations, (i) that they were applicable only to structures of high modal density where the average response levels of several modes did not span too wide a frequency interval, (ii) that the important parameter called the coupling loss factor, defied calculation in all but the simplest cases, and was also extremely difficult to measure accurately.

The work by Skudrzyk showed that the concept of reducing the

modes of vibration of a continuous system to a series of lumped parameter oscillators was feasible and straightforward. However, the problem of coupling the modal oscillators of connected structures remained. The degree of coupling of a modal oscillator would clearly depend on whether the forcing point was at a node, an anti-node, or somewhere in between. It could therefore only be determined by experiment, unless the structure was very simple with accurately known boundary conditions. Hence it was realised that there was a need for a technique for finding the lumped parameter modal oscillators of a structure, together with their respective coupling parameters. Part of this project was devoted therefore to a method of obtaining these parameters from impedance tests on real structures, and another major part was concerned with the development of a computer programme which could evaluate the response levels of all oscillators in any coupled network, with broadband random excitation.

In chapter two, part one, the equations of motion of a general continuous system are stated and reduced to those of a set of lumped parameter modal oscillators. In part two the equations of a system of three coupled structures are analysed. Many engineering systems fall into this class which consists of machinery mounted by an intermediate structure on a non-rigid foundation. Methods of deducing the strongest transmission paths and of simplifying energy transmission computations by considering only the most important ones are developed. The excitation-response relationships of a randomly excited lumped parameter system are derived in part three. In part

four two numerical methods are presented, which were used in computer programs, for obtaining the frequency response functions of a coupled oscillator system. These were the Gauss's successive elimination method, and a method using pole-zero analysis.

Chapter three contains a series of parameter surveys of coupled oscillator systems, which illustrate basic criteria for improving and optimising the vibration isolation characteristics of an intermediate structure.

An analysis is developed in chapter four for obtaining values of coupling and modal parameters of a structure from its point and transfer responses.

In chapter five, part one, the results of experiments with an electrical circuit analogue of a system of lumped parameter oscillators are presented. These were carried out to check that the computer programs were functioning correctly. Experiments are described in part two in which a model of a typical system, a machine mounted on a non-rigid foundation, was set up. This system was represented by a vibrator mounted on a large damped plate by means of rubber springs and damped beams. The point and transfer responses of the plate were measured and were used in the computation of the response ratio between the vibrator and plate. Computed and experimental response ratios are compared. In part three experimental and theoretical response ratios between two edge connected plates are compared. The object of this comparison was to show that, using a modal oscillator approach, the response ratio between two structures when connected by an intermediate structure could be calculated

from a knowledge of the response ratio between the two structures when connected directly together.

General conclusions are presented in chapter six.

Appendices one and two respectively contain details of computer programs for evaluating the response level of any coupled oscillator network, and the analysis of structural point and transfer response data to yield oscillator parameters.

CHAPTER TWO

DERIVATION OF THEORY

INTRODUCTION

The first step in the analysis of any system is the establishment of its equations of motion. In part one of this chapter the differential equation of motion of a general continuous system is set out and developed into those of a set of lumped parameter, mass-spring-damper oscillators. Each mode of vibration of the system is represented by one such oscillator. It is then shown how these oscillators may be considered to be coupled to any point at which the structure is excited. (An analysis is developed which enables these lumped parameters, including those of coupling, to be found from measurements of the point and transfer impedances of the structure, in chapter four.)

A general analysis of a system comprising three coupled sets of oscillators is presented in part two, which by means of matrix arithmetic derives expressions from which the degree of coupling between oscillators may be calculated as well as the relative strengths of transmission paths. A transmission path is defined as a system of series coupled oscillators to which each structure contributes one. It is shown how a transmission path may be represented in terms of coupling coefficients, and that the frequency response of an oscillator is a function of the coupling coefficients

of the transmission paths to which it is connected. The extent to which the response of a receiving oscillator can be considered to be made up of the sum of the individual transmission paths, and the error introduced by ignoring weakly coupled paths is also investigated.

The coupling coefficients can only be evaluated at discrete frequencies, and it is unlikely in practice that any machine vibrates at a single or even a number of frequencies with pure simple harmonic motion. Nearly all machines will produce a noise spectrum which consists of a range of continuously varying frequencies; such a source of vibration can be said to be random. The excitation and response relationships of a randomly excited lumped parameter system are developed in part three.

In part four two numerical methods are shown for evaluating the response levels of a coupled oscillator network. These involve the simultaneous solution of the coupled equations of motion to evaluate the complex frequency response functions of each oscillator. The former, the successive elimination method of Gauss in which the equations of the second order system are solved for each frequency at which the response function is required; the latter, a method using pole-zero analysis in which the equations of the equivalent first order system are solved once only to yield eigenvalues and eigenvectors of the system, from which the oscillator responses are calculated.

2.1 TRANSFORMATION OF CONTINUOUS SYSTEM TO MODAL OSCILLATORS

For harmonic motion the differential equation of motion of a continuous system such as a beam or plate, may be expressed as follows, (see [1]) :

$$m.\ddot{\phi} + K'.L\left(\frac{\partial}{\partial t_1}, \frac{\partial}{\partial t_2}, \frac{\partial}{\partial t_3}\right)\phi = f.e^{j.w.t}$$

where m is the mass per unit area

K' is the complex stiffness

$$K' = K_0(1+j\eta)$$

t_1, t_2, t_3 are the spacial co-ordinates

f is the external force per unit area

$$\phi = \phi_0 e^{j.w.t}$$

$\phi(t_1, t_2, t_3)$ is the displacement

L is a linear self adjoint differential operator, defined such that when applied to the displacement it gives the effective strain of the material.

If for example the structure is a shaft excited torsionally, m is the inertia per unit length, f the torque per unit length, ϕ the displacement as a function of axial distance and angular displacement, L is of correspondingly reduced order.

$$\dot{\phi} = \frac{d\phi}{dt}, \quad \ddot{\phi} = \frac{d^2\phi}{dt^2}$$

$$\text{therefore } \ddot{\phi} = -w^2.\phi.e^{j.w.t}$$

hence

$$-m.w^2.\phi + K'.L\left(\frac{\partial}{\partial t_1}, \frac{\partial}{\partial t_2}, \frac{\partial}{\partial t_3}\right)\phi = f \dots\dots\dots (1)$$

The response of the system can be defined as the sum of the natural functions, or normal modes. A mode of vibration of a continuous system occurs when all points move in unison sinusoidally, thus

$$\phi = \sum_i \phi_i(t_1, t_2, t_3) \dots\dots\dots (2)$$

Each term ϕ_i is a natural function which satisfies the homogeneous equation of the undamped system, defined by $K'=K_0$

hence,

$$K_0.L\left(\frac{\partial}{\partial t_1}, \frac{\partial}{\partial t_2}, \frac{\partial}{\partial t_3}\right)\phi_i(t_1, t_2, t_3) = m.w_i^2.\phi_i(t_1, t_2, t_3) \dots\dots\dots (3)$$

where w_i is the natural frequency of the i th mode.

The natural functions ϕ_i satisfy the orthogonality relationship when integrated over the system.

Let ϕ_i and ϕ_k , both functions of (t_1, t_2, t_3) Be two natural functions of the system. Therefore from equation (3) we have :

$$K_0.L\left(\frac{\partial}{\partial t_1}, \frac{\partial}{\partial t_2}, \frac{\partial}{\partial t_3}\right)\phi_i = m.w_i^2.\phi_i \dots\dots\dots (4)$$

$$\text{Ko.L} \left(\frac{\partial}{\partial \xi_1}, \frac{\partial}{\partial \xi_2}, \frac{\partial}{\partial \xi_3} \right) \phi_k = m \cdot w_k^2 \cdot \phi_k \dots\dots\dots (5)$$

Multiplying equation (4) by ϕ_k and equation (5) by ϕ_1 ,
subtracting and integrating over the system :

$$\begin{aligned} \int_{\sigma} \text{Ko.L} \left(\frac{\partial}{\partial \xi_1}, \frac{\partial}{\partial \xi_2}, \frac{\partial}{\partial \xi_3} \right) \phi_1 \cdot \phi_k - \text{Ko.L} \left(\frac{\partial}{\partial \xi_1}, \frac{\partial}{\partial \xi_2}, \frac{\partial}{\partial \xi_3} \right) \phi_1 \phi_k &= \\ = \int_{\sigma} m(\phi_1 \cdot \phi_k)(w_1^2 - w_k^2) &\dots\dots\dots (6) \end{aligned}$$

The left hand side of equation (6) equals zero; unless $w_1^2 = w_k^2$ the integral $\int_{\sigma} m(\phi_1 \cdot \phi_k)$ must equal zero.

Hence

$$\begin{aligned} \frac{1}{\sigma} \int_{\sigma} \phi_1 \cdot \phi_k \cdot d\sigma &= \langle \phi_1^2 \rangle \quad \text{for } i=k \\ &= 0 \quad \text{for } i \neq k \end{aligned} \dots\dots\dots (7)$$

$\langle \phi_1^2 \rangle$ is the average square of ϕ_1 over the system, and σ is the area of integration. Combining equations (1), (2) and (3)

$$m \left[-w^2 \sum_i \phi_i + \sum_i w_i^2 \cdot \phi_i (1+j\eta) \right] = f \dots\dots\dots (8)$$

Multiplying by ϕ_1 and integrating over the area of the system :

$$M \left[-w^2 \langle \phi_1^2 \rangle + w_1^2 \langle \phi_1^2 \rangle (1 + j\eta) \right] = \int_{\sigma} f \cdot \phi_1 \cdot d\sigma \dots\dots\dots (9)$$

Equation (9) gives the mean square amplitude as a function of the force distribution.

Let $\phi_{i(p)}$ be the displacement of the i th mode at a point P. Dividing equation (9) by $\phi_{i(p)}$ gives :

$$M \frac{\langle \phi_1^2 \rangle}{\phi_{i(p)}^2} \left[-w^2 + w_1^2 (1 + j\eta) \right] \phi_{i(p)} = \frac{\int_{\sigma} f \cdot \phi_1 \cdot d\sigma}{\phi_{i(p)}} \dots\dots\dots (10)$$

but $\int_{\sigma} f \cdot d\sigma = f_0$, the total force.

$$\text{Let } R_{i(p)} = \frac{\int_{\sigma} f \cdot \phi_1}{f_0 \cdot \phi_{i(p)}} \cdot d\sigma$$

$R_{i(p)}$ is the fraction of the total force that excites the i th mode.

$$\text{therefore } M \frac{\langle \phi_1^2 \rangle}{\phi_{i(p)}^2} \left[-w^2 + w_1^2 (1 + j\eta) \right] \phi_{i(p)} = f_0 \cdot R_{i(p)} \dots\dots (11)$$

Every term of equation (11) is analogous to that of a single mass-spring-damper oscillator. The overall response of the system is clearly the sum of the contributions of every normal mode which is excited by the external force. It is also seen that the equations of motion of each oscillator are independent.

Equation (11) may be re-written in terms of the maximum modal displacement and the effective, or modal, mass.

Therefore $(-M_i \cdot \omega^2 + j \cdot \omega \cdot C_i + K_i) X_i = f_o \cdot R_i(p) \dots\dots\dots (12)$

where M_i is the modal mass
 C_i is the modal damping coefficient
 K_i is the modal stiffness
 X_i is the modal displacement

It is now assumed that the right hand side of equation (12) is brought about by the existence of a coupling element between the excitation point and the modal mass. The coupling element may be conservative and/or dissipative. In the case of conservative coupling, equation (12) becomes:

$$(-M_i \omega^2 + j \cdot \omega \cdot C_i + K_i) X_i = Kc_i \cdot X_p \dots\dots\dots (13)$$

where Kc_i is the coupling stiffness to the i th mode
 X_p is the displacement of the excitation point

The entire set of modal oscillators, coupled to the excitation point may be represented diagrammatically as in fig.2.01. The equations of motion of this system are;

$$\left. \begin{aligned} M_1 \ddot{x}_1 + C_1 \dot{x}_1 + K_1 x_1 &= Kc_1 x_p \\ M_2 \ddot{x}_2 + C_2 \dot{x}_2 + K_2 x_2 &= Kc_2 x_p \\ M_i \ddot{x}_i + C_i \dot{x}_i + K_i x_i &= Kc_i x_p \\ M_n \ddot{x}_n + C_n \dot{x}_n + K_n x_n &= Kc_n x_p \end{aligned} \right\} \dots\dots\dots (14)$$

The modal stiffness K_i is composed of the sum of the grounded

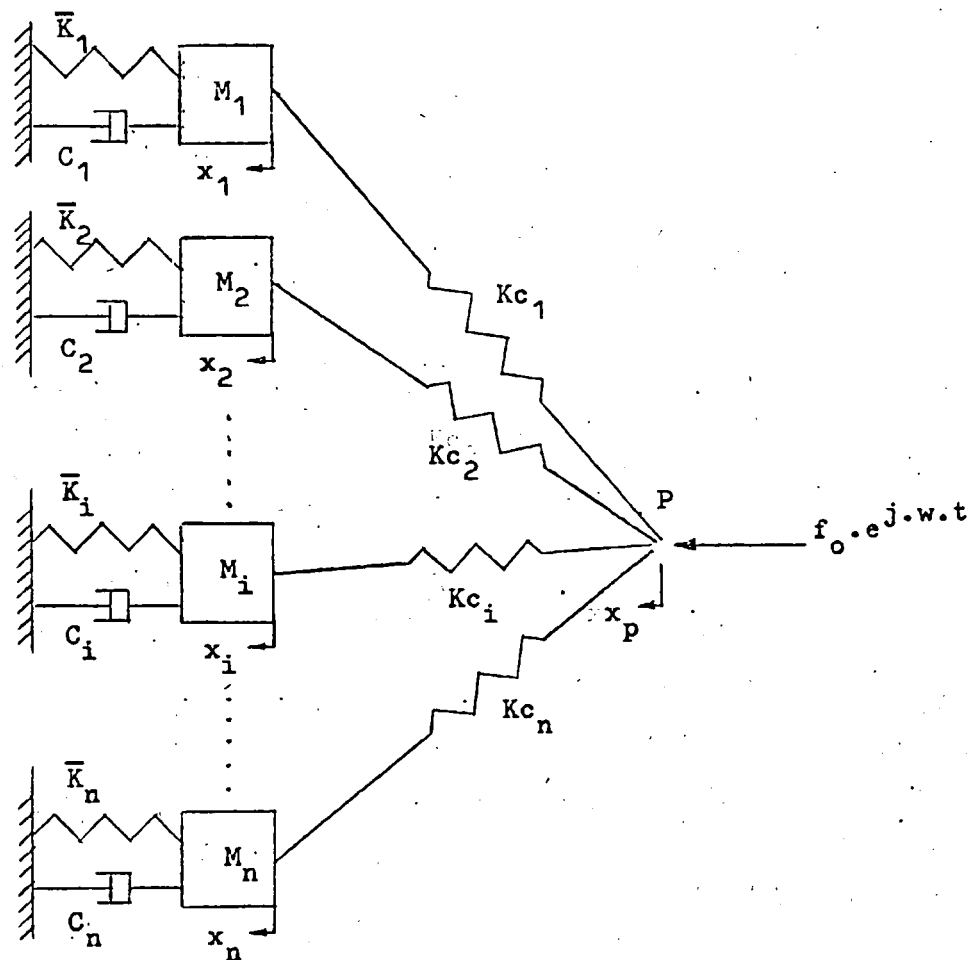


FIG 2.01 DIAGRAM OF STRUCTURAL OSCILLATORS CONSERVATIVELY COUPLED TO EXCITATION POINT

stiffness $\bar{K}_i$ and the coupling stiffness Kc_i , i.e. $K_i = (\bar{K}_i + Kc_i)$

Let $x_i = X_i \cdot e^{j \cdot \omega \cdot t}$;

$$\text{From (14)} \quad X_i = \frac{Kc_i \cdot X_p}{M_i \cdot \omega^2 + j \cdot \omega \cdot C_i + K_i} \quad \dots\dots\dots (15)$$

Equating forces at point P:

$$f_o = X_p \sum_{i=1}^n Kc_i - \sum_{i=1}^n Kc_i \cdot X_i \quad \dots\dots\dots (16)$$

Substituting (15) into (16) and defining $\frac{f_o}{X_p}$ as the
'point stiffness' K_p we have:

$$K_p = \sum_{i=1}^n Kc_i - \sum_{i=1}^n \left[\frac{Kc_i^2}{-M_i \cdot \omega^2 + j \cdot \omega \cdot C_i + K_i} \right] \quad \dots\dots (17)$$

Substituting w_i^2 for $\frac{K_i}{M_i}$ and η_i for $\frac{C_i}{w_i \cdot M_i}$ gives;

$$K_p = \sum_{i=1}^n Kc_i - \sum_{i=1}^n \left[\frac{Kc_i^2}{w_i^2 \cdot M_i \cdot \left(1 - \frac{w^2}{w_i^2} + j \cdot \eta_i \cdot \frac{w}{w_i} \right)} \right] \quad \dots\dots (18)$$

An expression for the transfer stiffness can also be obtained.
The transfer stiffness may be defined as the force exciting the
structure, divided by the sum of the modal response displacements
at some remote point.

$$\text{i.e. } K_t = \frac{f_o}{\sum_{i=1}^n X_i} \quad \dots\dots\dots (19)$$

From equation (15):

$$\sum_{i=1}^n X_i = X_p \cdot \sum_{i=1}^n \left[\frac{Kc_i}{W_i^2 \cdot M_i \left(1 - \frac{W^2}{W_i^2} + j \cdot \eta_i \cdot \frac{W}{W_i} \right)} \right] \dots\dots\dots (20)$$

Combining equations (18) and (20);

$$K_t = \frac{K_p}{\sum_{i=1}^n \left[\frac{Kc_i}{W_i^2 \cdot M_i \left(1 - \frac{W^2}{W_i^2} + j \cdot \eta_i \cdot \frac{W}{W_i} \right)} \right]} \dots\dots\dots (21)$$

OR

$$K_t = \frac{\sum_{i=1}^n Kc_i \cdot \sum_{i=1}^n \left[\frac{Kc_i^2}{W_i^2 \cdot M_i \left(1 - \frac{W^2}{W_i^2} + j \cdot \eta_i \cdot \frac{W}{W_i} \right)} \right]}{\sum_{i=1}^n \left[\frac{Kc_i}{W_i^2 \cdot M_i \left(1 - \frac{W^2}{W_i^2} + j \cdot \eta_i \cdot \frac{W}{W_i} \right)} \right]} \dots\dots\dots (22)$$

A system with dissipative coupling is shown in fig.2.02 .

The general equation of motion is:

$$M_i \ddot{x}_i + C_i \dot{x}_i + K_i x_i = Cc_i \dot{x}_p \dots\dots\dots (23)$$

The modal damping C_i is composed of the sum of the grounded and coupling damping coefficients, thus $C_i = (\bar{C}_i + Cc_i)$.

Let $\dot{x}_i = j\dot{X}_i \cdot e^{j \cdot \omega \cdot t}$

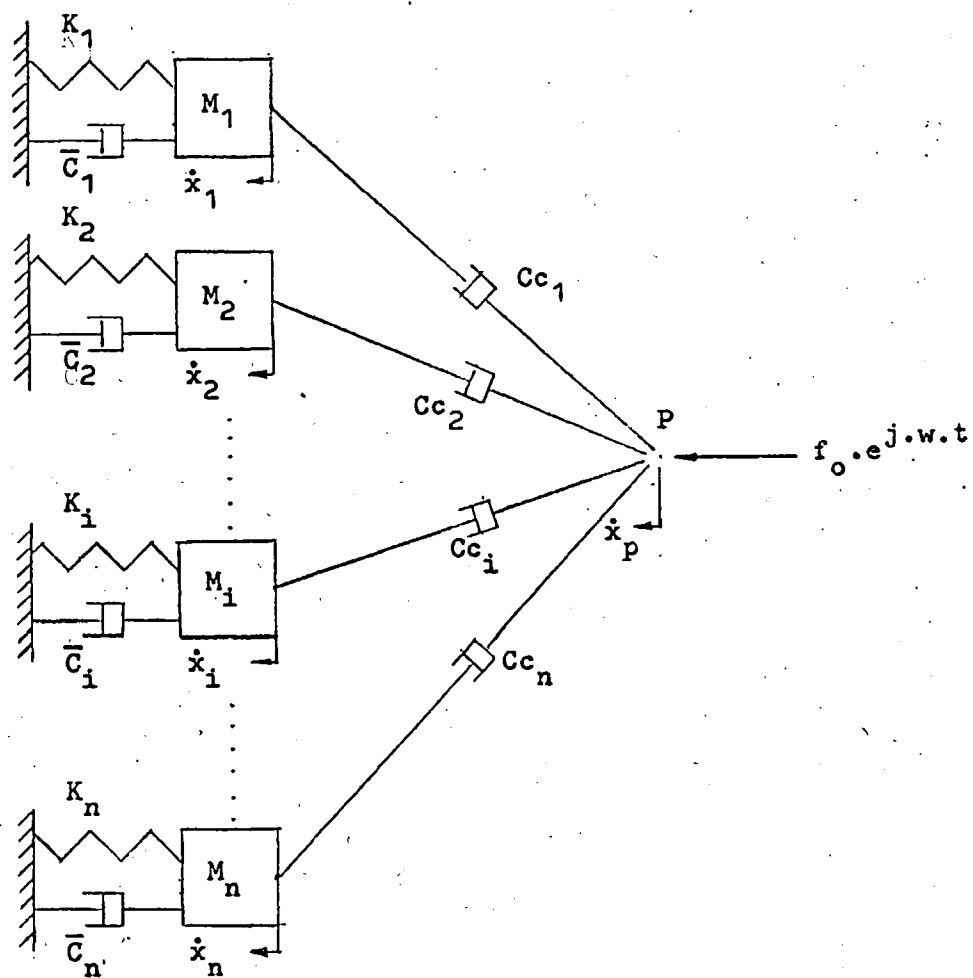


FIG 2.02 DIAGRAM OF STRUCTURAL OSCILLATORS DISSIPATIVELY COUPLED TO EXCITATION POINT

Therefore

$$\dot{X}_1 = \frac{Cc_1 \cdot \dot{X}_p}{j(M_1 \cdot \omega - \frac{1}{\omega}) + C_1} \dots\dots\dots (24)$$

Equating forces at the excitation point:

$$f_o = \dot{X}_p \sum_{i=1}^n Cc_i - \sum_{i=1}^n Cc_i \cdot \dot{X}_1 \dots\dots\dots (25)$$

Substituting (24) in (25) and defining the point damping,
or impedance, as $\frac{f_o}{\dot{X}_p}$.

$$\text{Let } I_p = \frac{f_o}{\dot{X}_p}$$

$$\text{Thus } I_p = \sum_{i=1}^n Cc_i - \sum_{i=1}^n \left[\frac{Cc_i^2}{W_i \cdot M_i \left[j \left(\frac{\omega}{W_i} - \frac{W_i}{\omega} \right) + \eta_i \right]} \right] \dots\dots (26)$$

Similarly the transfer impedance may be defined as: follows:

$$I_t = \frac{f_o}{\sum_{i=1}^n \dot{X}_i}$$

$$\text{Therefore } I_t = \frac{I_p}{\sum_{i=1}^n \left[\frac{Cc_i}{W_i \cdot M_i \left[j \left(\frac{\omega}{W_i} - \frac{W_i}{\omega} \right) + \eta_i \right]} \right]} \dots\dots\dots (27)$$

Which may be expanded as:

$$It = \frac{\sum_{i=1}^n Cc_i - \sum_{i=1}^n \left[\frac{Cc_i^2}{W_i \cdot M_i \left[j \left(\frac{W}{W_i} - \frac{W_i}{W} \right) + \gamma_i \right]} \right]}{\sum_{i=1}^n \left[\frac{Cc_i}{W_i \cdot M_i \left[j \left(\frac{W}{W_i} - \frac{W_i}{W} \right) + \gamma_i \right]} \right]} \dots\dots (28)$$

2.2 GENERAL ANALYSIS OF COUPLED OSCILLATOR NETWORKS

A large number of vibratory mechanical systems consist of three coupled structures. Typically these are a machine mounted on a non-rigid foundation by some form of supporting structure. This section will therefore concern itself with systems of this type, although any configuration could similarly be treated. The foregoing analysis has shown that the modes of vibration of a structure are independent, and hence vibration transmission is achieved by modal coupling between structures. There is no general rule about the numbers of modal oscillators within each structure. Several cases will be examined in which each structure has a different number of oscillators.

The equations of motion of a coupled network of oscillators form a series of simultaneous differential equations.

$$\begin{aligned}
\text{i.e. } M_1 \ddot{x}_1 + C_1 \dot{x}_1 + K_1 x_1 - K_{1,2} x_2 - K_{1,3} x_3 - \dots K_{1,n} x_n - \\
- C_{1,2} \dot{x}_2 - C_{1,3} \dot{x}_3 - \dots C_{1,n} \dot{x}_n = f_1 e^{j.w.t} \\
\vdots \\
M_i \ddot{x}_i + C_i \dot{x}_i + K_i x_i - K_{i,1} x_1 - K_{i,2} x_2 - \dots K_{i,n} x_n - \\
- C_{i,1} \dot{x}_1 - C_{i,2} \dot{x}_2 - \dots C_{i,n} \dot{x}_n = f_i e^{j.w.t}
\end{aligned} \tag{29}$$

Where $K_{u,v}$ and $C_{u,v}$ represent coupling stiffness and damping coefficients respectively between oscillators 'u' and 'v'.

Dividing each equation of (29) by f_1 , and letting

$$\begin{aligned}
H_i = \frac{x_i}{f_1} \text{ and } \bar{F}_i = \frac{f_i}{f_1}; \text{ they may be expressed in matrix form} \\
\text{as } [A][H] = [\bar{F}] \dots \dots \dots \tag{30}
\end{aligned}$$

$$\text{i.e. } \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \dots & a_{n,n} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{bmatrix} = \begin{bmatrix} \bar{F}_1 \\ \bar{F}_2 \\ \vdots \\ \bar{F}_n \end{bmatrix} \dots \dots \dots \tag{31}$$

$$\begin{aligned}
\text{Where } a_{u,u} &= (K_u - M_u \cdot w^2) + j.w.C_u \\
a_{u,v} &= K_{u,v} + j.w.C_{u,v} \dots \dots \dots \tag{32}
\end{aligned}$$

The frequency response functions ' H_v ' may be found using Cramer's rule:

$$H_v = \frac{\sum_{u=1}^n \bar{G}_{u,v} \cdot \bar{F}_u}{D} \dots\dots\dots (33)$$

$\bar{G}_{u,v}$ is the co-factor of the element $a_{u,v}$. The co-factor is defined as $(-1)^{u+v} \cdot \bar{M}_{u,v}$: $\bar{M}_{u,v}$ is the determinant of the minor matrix on element $a_{u,v}$.

D is the determinant of the matrix [A] and may be expressed in terms of the sum of the products of each element of a row or column and its co-factor. Thus (33) may be expressed as :

$$H_v = \frac{\sum_{u=1}^n \bar{G}_{u,v} \cdot \bar{F}_u}{\sum_{u=1}^n a_{u,v} \cdot \bar{G}_{u,v}} \dots\dots\dots (34)$$

If the elements in each row of A are divided by the leading diagonal element of that row to form another matrix B we have:

$$[B][H] = [F]$$

i.e.
$$\begin{bmatrix} 1 & b_{1,2} & \dots & b_{1,n} \\ b_{2,1} & 1 & \dots & b_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n,1} & b_{n,2} & \dots & 1 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{bmatrix} \dots\dots\dots (35)$$

$$\text{Where } b_{u,v} = \frac{a_{u,v}}{a_{u,u}}, \text{ and } F_u = \frac{\bar{F}_u}{a_{u,u}}$$

$$\text{Therefore } H_v = \frac{\sum_{u=1}^n G_{u,v} \cdot F_u}{\sum_{u=1}^n b_{u,v} \cdot G_{u,v}} \dots\dots\dots (36)$$

F_u is zero if oscillator 'u' is not externally excited. At

any particular value of frequency the ratio of the oscillator responses is independent of the absolute value of F_u .

The element $b_{u,v}$ may be called a 'coupling factor' as it is the ratio of the coupling parameters connecting oscillator 'u' to oscillator 'v', divided by the modal parameters of oscillator 'u'.

For a system of connected structures the simplest arrangement of oscillators is that comprising one from each structure. It is helpful to term such an arrangement a 'transmission path'. Consider the system shown in fig.2.03 comprising three series coupled oscillators. The corresponding matrix of coupling factors is:

$$\text{is: } \begin{bmatrix} 1 & b_{1,2} & 0 \\ b_{2,1} & 1 & b_{2,3} \\ 0 & b_{3,2} & 1 \end{bmatrix}$$

Let oscillators 1,2,3 be termed excited, intermediate, and receiving respectively. The excited to receiving oscillator response ratio is given by $\frac{H_1}{H_3}$

$$\text{From (36) } H_1 = \frac{G_{1,1} \cdot F_1}{G_{1,1} \cdot b_{1,1} - G_{2,1} \cdot b_{2,1}} \quad \dots\dots\dots (37)$$

$$\text{and } H_3 = \frac{G_{1,3} \cdot F_1}{G_{1,1} \cdot b_{1,1} - G_{2,1} \cdot b_{2,1}} \quad \dots\dots\dots (38)$$

$$\text{therefore } \frac{H_1}{H_3} = \frac{G_{1,1}}{G_{1,3}} = \frac{1 - b_{3,2} \cdot b_{2,3}}{b_{2,1} \cdot b_{3,2}}$$

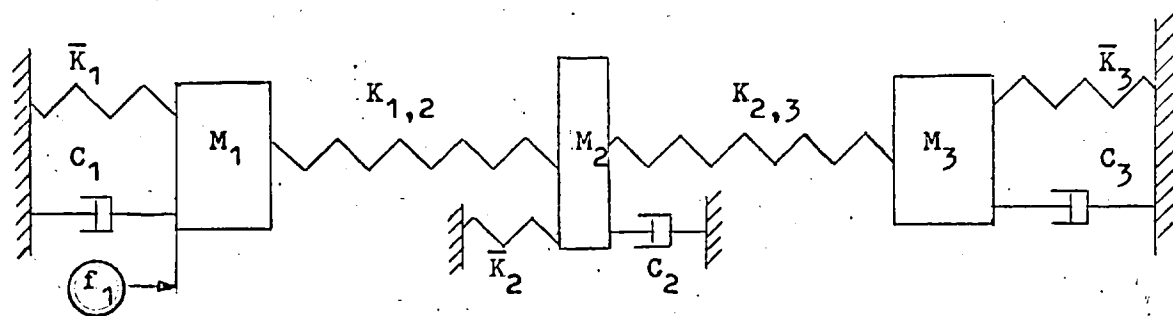


FIG 2.03 CONSERVATIVELY COUPLED THREE OSCILLATOR SYSTEM

It will be noted that the terms $b_{u,v}$ and $b_{v,u}$ are closely related, being the coupling parameters connecting oscillators 'u' and 'v' divided by the modal parameter of oscillators 'u' and 'v' respectively. Hence a further term will be defined as the product of these two; 'coupling coefficient':

$$\text{coupling coefficient} = \gamma_{u,v} = b_{u,v} \cdot b_{v,u}$$

$$\text{clearly } \gamma_{u,v} = \gamma_{v,u}$$

$$\text{and therefore } \frac{H_1}{H_3} = \frac{1 - \gamma_{2,3}}{b_{2,1} \cdot b_{3,2}} \dots\dots\dots (39)$$

The response ratio between intermediate and receiving oscillators is given by:

$$\frac{H_2}{H_3} = \frac{G_{2,1}}{G_{3,1}} = \frac{b_{1,2}}{b_{2,1} \cdot b_{3,2}} \dots\dots\dots (40)$$

The response of each oscillator may be found quantitatively from the response of the receiving oscillator.

$$H_3 = \frac{b_{2,1} \cdot b_{3,2} \cdot F_1}{(1 - b_{3,2} \cdot b_{2,3}) - b_{1,2} \cdot b_{2,1}} \dots\dots (41)$$

$$\text{therefore } H_3 = \frac{b_{2,1} \cdot b_{3,2} \cdot F_1}{1 - \gamma_{1,2} - \gamma_{2,3}} \dots\dots\dots (42)$$

2.2.1 First type of coupled oscillator network

Consider the system shown in fig.2.04, consisting of one excited oscillator coupled by a number of intermediate oscillators to a single receiver. The equations of motion may be expressed by the following matrices of coupling factors and response functions.

$$\begin{bmatrix} 1 & b_{1,2} & 0 & b_{1,4} & \dots & b_{1,n} \\ b_{2,1} & 1 & b_{2,3} & 0 & \dots & 0 \\ 0 & b_{3,2} & 1 & b_{3,4} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n,1} & 0 & b_{n,3} & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ \vdots \\ H_n \end{bmatrix} = \begin{bmatrix} F_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \dots \quad (43)$$

There are (n-2) transmission paths from oscillator 1 to oscillator 3 which will be designated as follows:

Let path 'A' comprise oscillators 1,2 and 3

Let path 'B' comprise oscillators 1,4 and 3

Let path 'C' comprise oscillators 1,5 and 3

etc.

Consider each transmission path in isolation from the rest, then from (42) we have:

$$H_{3(A)} = \frac{b_{2,1} \cdot b_{3,2} \cdot F_1}{1 - \gamma_{1,2} - \gamma_{2,3}} \quad \dots \quad (44)$$

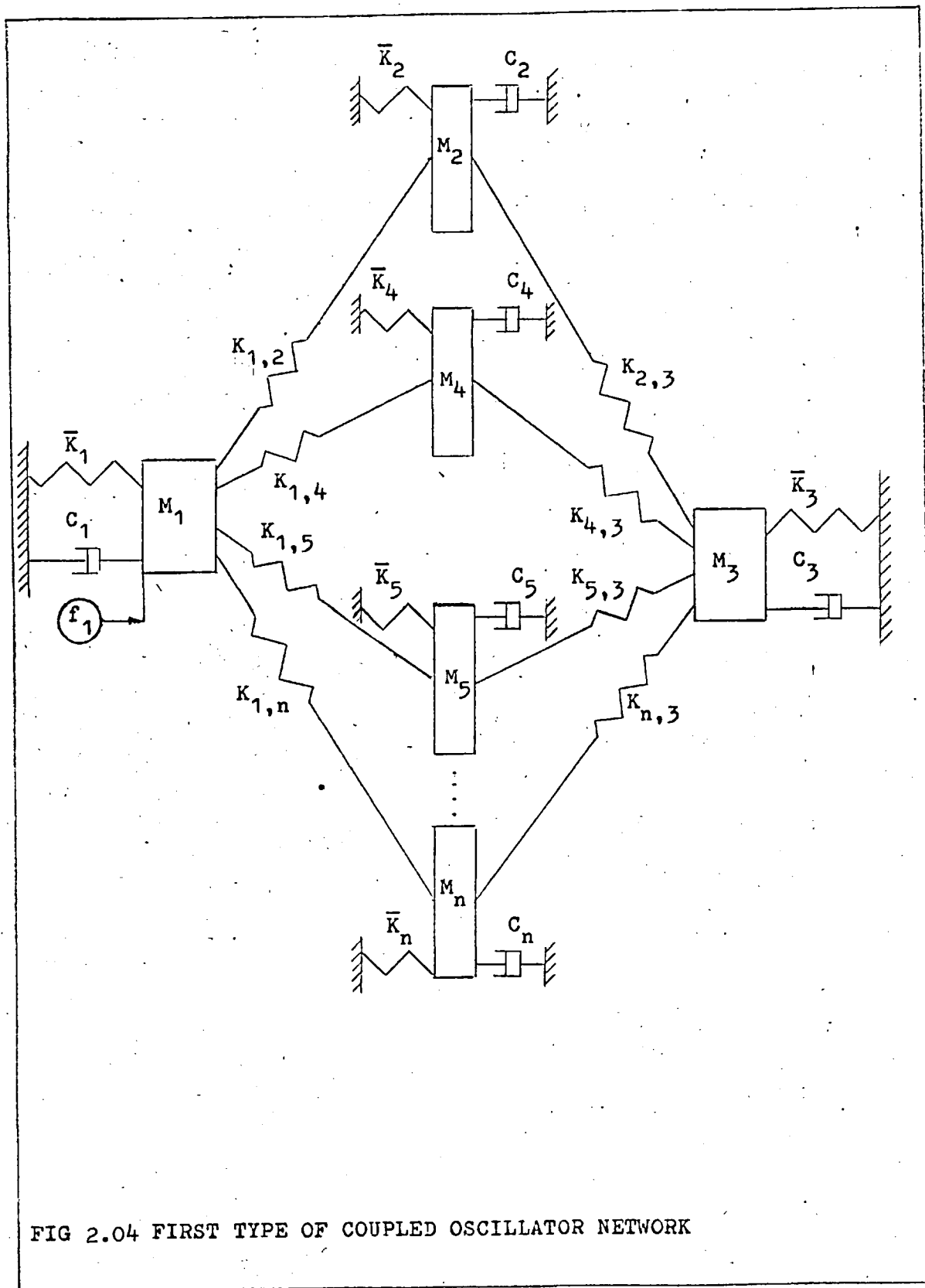


FIG 2.04 FIRST TYPE OF COUPLED OSCILLATOR NETWORK

where $H_3(A)$ is the response of oscillator 3 due to transmission through path 'A'.

$$\text{Similarly } H_3(B) = \frac{b_{4,1} \cdot b_{3,4} \cdot F_1}{1 - \gamma_{1,4} - \gamma_{4,3}} \dots\dots\dots (45)$$

The response of oscillator 3 due to all paths transmitting simultaneously is:

$$H_3 = \frac{G_{1,3} \cdot F_1}{D} \dots\dots\dots (46)$$

From the matrix B of (43):

$$G_{1,3} = b_{2,1} b_{3,2} + b_{4,1} b_{3,4} + b_{5,1} b_{3,5} + \dots b_{n,1} b_{3,n} \quad (47)$$

The determinant D may be expressed as a series involving the determinants of lesser order matrices.

$$D_n = D_{(n-1)} + b_{3,n} \cdot G_{3,n} + b_{1,n} \cdot G_{1,n} \dots\dots\dots (48)$$

Where the subscripts of D denote the order of the matrix. Expanding equation (48) in terms of coupling factors and coefficients:

$$\begin{aligned} D_n = & D_{(n-1)} - \gamma_{1,n} - \gamma_{n,3} + \gamma_{n,3} (\gamma_{1,2} + \gamma_{1,4} + \dots \gamma_{1,(n-1)}) + \\ & + \gamma_{1,n} (\gamma_{2,3} + \gamma_{4,3} + \dots \gamma_{(n-1),3}) - 2 [(b_{n,1} \cdot b_{1,2} \cdot \\ & \cdot b_{2,3} \cdot b_{3,n}) + (b_{n,1} \cdot b_{1,4} \cdot b_{4,3} \cdot b_{3,n}) + \dots \\ & \dots + (b_{n,1} \cdot b_{1,(n-1)} \cdot b_{(n-1),3} \cdot b_{3,n})] \dots\dots\dots (49) \end{aligned}$$

It will be noted that in equation (49) the second order terms made up of coupling factors and coefficients are a measure of the relative coupling of each path. If all paths have the same coupling factor, the second order terms disappear. If all intermediate oscillators have parameters of the same order of magnitude, the sum of these terms is still negligibly small. Hence equation (49) may be expressed more simply by the approximate formula:

$$D_n = D_{(n-1)} - \gamma_{1,n} - \gamma_{n,3} \dots \dots \dots (50)$$

$$\text{therefore } D_n = 1 - \gamma_{1,2} - \gamma_{2,3} - \gamma_{1,4} - \gamma_{4,3} - \dots - \gamma_{1,n} - \gamma_{n,3} \quad (51)$$

equation (46) becomes:

$$H_3 = \frac{(b_{2,1} \cdot b_{3,2} + b_{4,1} \cdot b_{3,4} + \dots b_{n,1} \cdot b_{3,n}) F_1}{1 - \gamma_{1,2} - \gamma_{2,3} - \gamma_{1,4} - \gamma_{4,3} - \dots - \gamma_{1,n} - \gamma_{n,3}} \dots (52)$$

If the relationships expressed in equations (44) and (45) are substituted in (52):

$$H_3 = \frac{H_{3(A)}(1 - \gamma_{1,2} - \gamma_{2,3}) + H_{3(B)}(1 - \gamma_{1,4} - \gamma_{4,3}) + \dots}{1 - \gamma_{1,2} - \gamma_{2,3} - \gamma_{1,4} - \gamma_{4,3} - \dots - \gamma_{1,n} - \gamma_{n,3}} \quad (53)$$

Therefore the response of a receiving oscillator driven from one excited oscillator by way of several intermediate oscillators is a function of the sum of the individual transmission path responses.

2.2.ii Second type of coupled oscillator network

Consider a system as shown in fig.2.05, comprising a number of excited oscillators driving a number of receivers through a single intermediate oscillator. The equations of motion using the matrix of coupling factors are:

$$\begin{bmatrix} 1 & b_{1,2} & 0 & 0 & \dots & 0 \\ b_{2,1} & 1 & b_{2,3} & b_{2,4} & \dots & b_{2,n} \\ 0 & b_{3,2} & 1 & 0 & \dots & 0 \\ 0 & b_{4,2} & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & b_{n,2} & 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ \vdots \\ H_n \end{bmatrix} = \begin{bmatrix} F_1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ F_n \end{bmatrix} \quad \dots \quad (54)$$

The response of a receiver, for example oscillator 3 is given by:

$$H_3 = \frac{\sum_e G_{e,3} \cdot F_e}{D} \quad \dots \quad (55)$$

where 'e' denotes the externally excited oscillators.

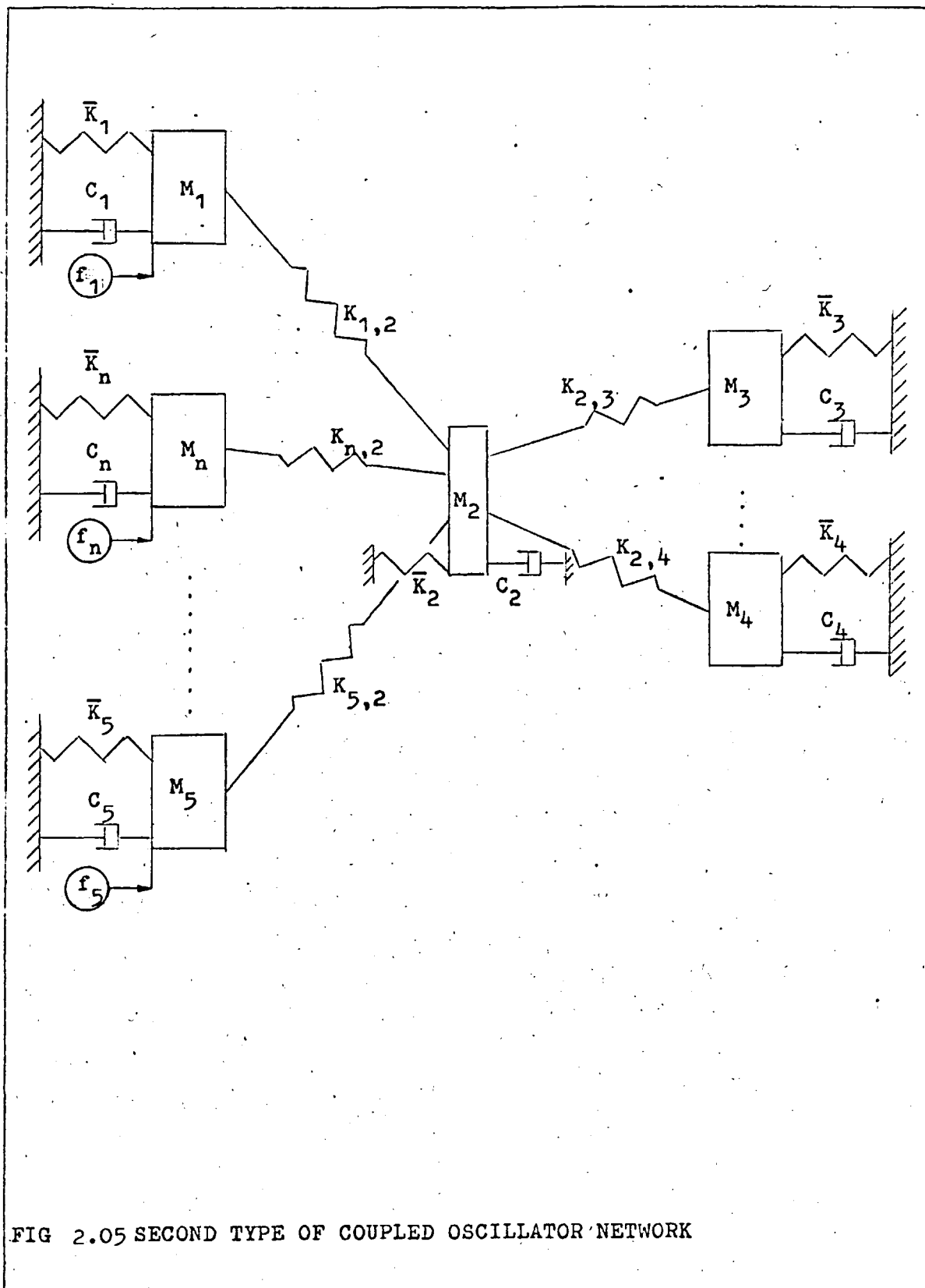
From (54) it is seen that:

$$G_{e,3} = b_{2,e} \cdot b_{3,2} \quad \dots \quad (56)$$

The determinant D may be expressed as:

$$D = G_{1,1} + b_{2,1} \cdot G_{2,1} \quad \dots \quad (57)$$

$G_{1,1}$ may be expanded as:



$$G_{1,1(n)} = G_{1,1(n-1)} - b_{2,n} \cdot b_{n,2} \quad \dots\dots\dots (58)$$

where 'n' and '(n-1)' denote the matrix order; 'n' may have any value from 4 upwards, in which case:

$$G_{1,1(4)} = G_{1,1(3)} - \gamma_{2,4} ; \quad G_{1,1(3)} = (1 - \gamma_{2,3}) \dots (59)$$

When $G_{2,1}$ is expanded it is found that:

$$G_{2,1} = b_{1,2}$$

$$\text{Therefore } D = 1 - \gamma_{2,3} - \sum_e \gamma_{e,2} \quad \dots\dots\dots (60)$$

Substituting equations (56) and (60) in (55):

$$H_3 = \frac{\sum_e b_{2,e} \cdot b_{3,2} \cdot F_e}{1 - \gamma_{2,3} - \sum_e \gamma_{e,2}} \quad \dots\dots\dots (61)$$

$$\text{but } b_{2,1} \cdot b_{3,2} = \frac{H_{3(A)} (1 - \gamma_{1,2} - \gamma_{2,3})}{F_1} \text{ etc.}$$

$$\text{Therefore } H_3 = \frac{\sum_e H_{3(e)} (1 - \gamma_{e,2} - \gamma_{2,3})}{1 - \gamma_{2,3} - \sum_e \gamma_{e,2}} \quad \dots\dots\dots (62)$$

Where $H_{3(e)}$ is the response due to the transmission path from oscillator 'e' to oscillator 3.

The response of any receiving oscillator in this network is again a function of the sum of the responses from each transmission path.

2.2.iii Third type of coupled oscillator network

This type of network is shown in fig.2.06 and consists of one receiver driven by several intermediate oscillators, each of which is in turn driven by one excited oscillator. The system equations expressed by the coupling factor matrix are:

$$\begin{bmatrix} 1 & b_{1,2} & 0 & 0 & \dots\dots\dots 0 \\ b_{2,1} & 1 & b_{2,3} & 0 & \dots\dots\dots 0 \\ 0 & b_{3,2} & 1 & 0 & b_{3,5} & \dots & b_{3,n} \\ 0 & 0 & 0 & 1 & b_{4,5} & \dots & 0 \\ \vdots & & & & & & \\ 0 & 0 & b_{n,3} & 0 & \dots & b_{n,(n-1)} & 1 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ \vdots \\ H_n \end{bmatrix} = \begin{bmatrix} F_1 \\ 0 \\ 0 \\ F_4 \\ \vdots \\ 0 \end{bmatrix} \quad \dots (63)$$

The response of the receiving oscillator (3) is:

$$H_3 = \frac{\sum_e G_{e,3} \cdot F_e}{D} \quad \dots\dots\dots (64)$$

Where 'e' denotes the externally excited oscillators.

Expanding $G_{1,3}$ which is typical of any of the $G_{e,3}$ terms:

$$G_{1,3} = b_{2,1} \cdot b_{3,2} (1 - \gamma_{4,5}) \dots (1 - \gamma_{(n-1),n}) \quad \dots\dots (65)$$

$$\text{and } D = G_{1,1} + b_{2,1} \cdot G_{2,1} \quad \dots\dots\dots (66)$$

$G_{1,1}$ may be expressed in terms of determinants of lesser order matrices thus:

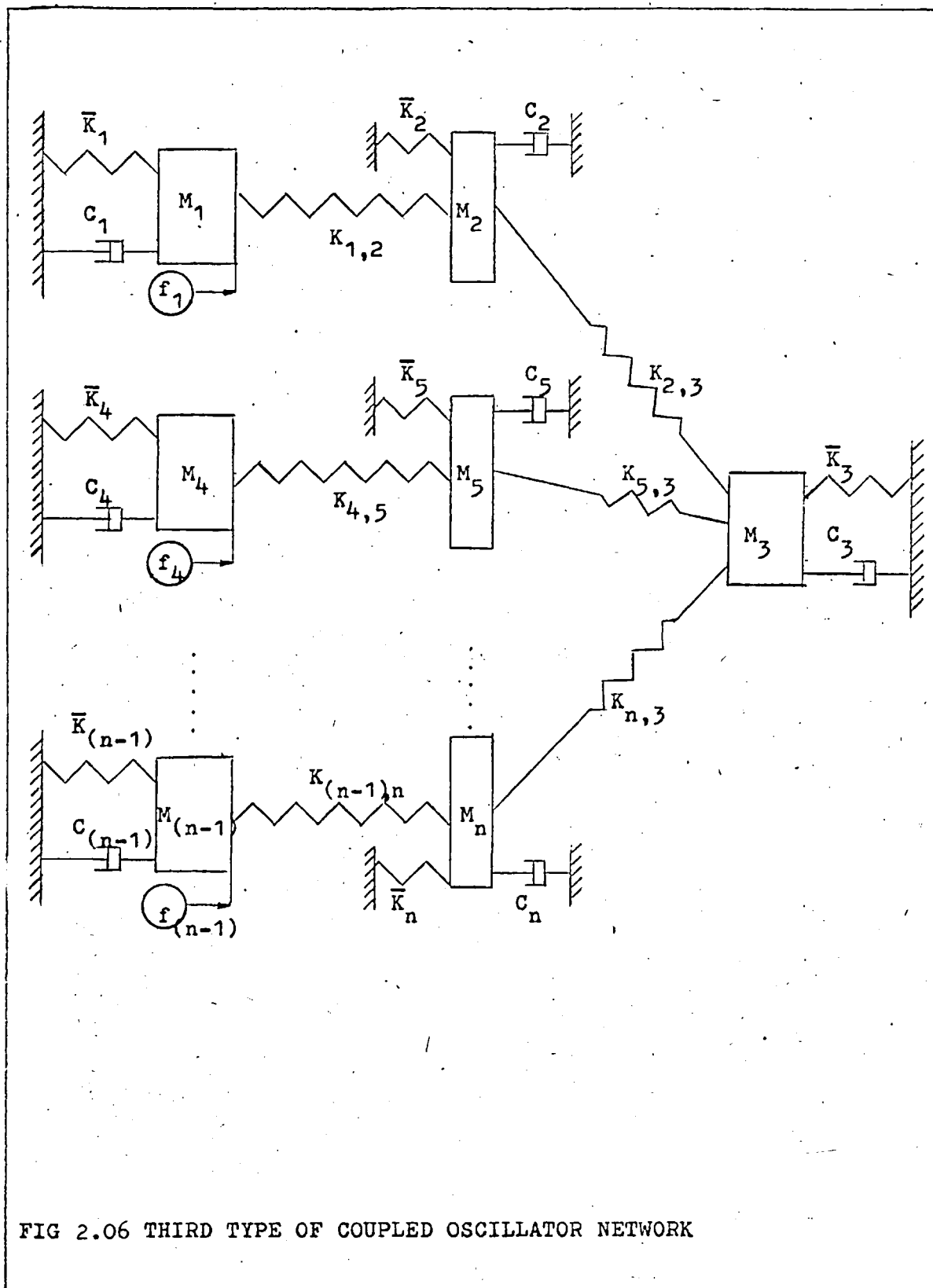


FIG 2.06 THIRD TYPE OF COUPLED OSCILLATOR NETWORK

$$G_{1,1(n)} = G_{1,1(n-2)} (1 - \gamma_{(n-1),n}) - \gamma_{n,3} (1 - \gamma_{4,5}) \dots \dots (1 - \gamma_{(n-3),(n-2)}) \quad (67)$$

The second term in equation (67) is not applicable if $n=3$, and reduces to $\gamma_{5,3}$ if $n=5$. Expanding $G_{2,1}$ in a similar manner:

$$G_{2,1(n)} = G_{2,1(n-2)} (1 - \gamma_{(n-1),n}) + b_{1,2} \gamma_{3,n} (1 - \gamma_{4,5}) (1 - \gamma_{(n-3),(n-2)}) \dots \dots \dots (68)$$

Substituting equations (67) and (68) in (66):

$$D = (1 - \gamma_{4,5}) (1 - \gamma_{6,7}) \dots (1 - \gamma_{(n-1),n}) \left\{ (1 - \gamma_{2,3}) - (1 - \gamma_{1,2}) \sum_{i=5}^{i=n} \left[\frac{\gamma_{3,i}}{(1 - \gamma_{(i-1),i})} \right] - \gamma_{1,2} \right\} \dots \dots (69)$$

Where 'i' denotes each intermediate oscillator within the range specified.

Substituting equations (65), (66), (67), (68) in (64):

$$H_3 = \frac{b_{2,1} \cdot b_{3,2} (1 - \gamma_{4,5}) \dots (1 - \gamma_{(n-1),n}) F_1 + b_{5,4} \cdot b_{3,5} (1 - \gamma_{1,2}) \dots (1 - \gamma_{(n-1),n}) F_4}{D} \quad (70)$$

Let path 'A' comprise oscillators 1,2 and 3, path 'B' oscillators 4,5 and 3 etc., then (70) becomes:

$$\begin{aligned}
& H_3(A)(1 - \gamma_{1,2} - \gamma_{2,3})(1 - \gamma_{4,5}) \dots (1 - \gamma_{(n-1),n}) + \\
& + H_3(B)(1 - \gamma_{4,5} - \gamma_{5,3})(1 - \gamma_{1,2}) \dots (1 - \gamma_{(n-1),n}) + \dots \\
H_3 = & \frac{\quad}{(1 - \gamma_{1,2})(1 - \gamma_{4,5}) \dots (1 - \gamma_{(n-1),n}) \left[1 - \frac{\sum_{i=2}^{i=n} \gamma_{i,3}}{(1 - \gamma_{(i-1),i})} \right]} \quad (71)
\end{aligned}$$

'i' takes the values 2,5,7...n.

Hence as with the previous networks the receiver response is a function of the individual transmission path responses.

2.2.iv Fourth type of coupled oscillator network

Consider the network shown in fig.2.07 consisting of one excited oscillator driving a number of intermediate oscillators, each of which is coupled to one receiver. This system is clearly identical in layout to the third type but is excited from the opposite end. The numbering of oscillators 1 and 3 is interchanged so as to keep oscillator 1 an excited oscillator, and oscillator 3 a receiver. The elements of the coupling factor matrix are identical to those of the third type of network, but with rows and columns 1 and 3 reversed. The determinant of the system will therefore be unchanged. The coupling factor matrix equation is:

$$\begin{bmatrix}
1 & b_{1,2} & 0 & 0 & b_{1,5} & 0 & b_{1,n} \\
b_{2,1} & 1 & b_{2,3} & 0 & 0 & 0 & \dots & 0 \\
0 & b_{3,2} & 1 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 1 & b_{4,5} & 0 & \dots & 0 \\
\vdots & & & & & & & \\
b_{n,1} & & & & b_{(n-1),n} & 1 & &
\end{bmatrix}
\begin{bmatrix}
H_1 \\
H_2 \\
H_3 \\
H_4 \\
\vdots \\
H_n
\end{bmatrix}
=
\begin{bmatrix}
F_1 \\
0 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix} \quad \dots (72)$$

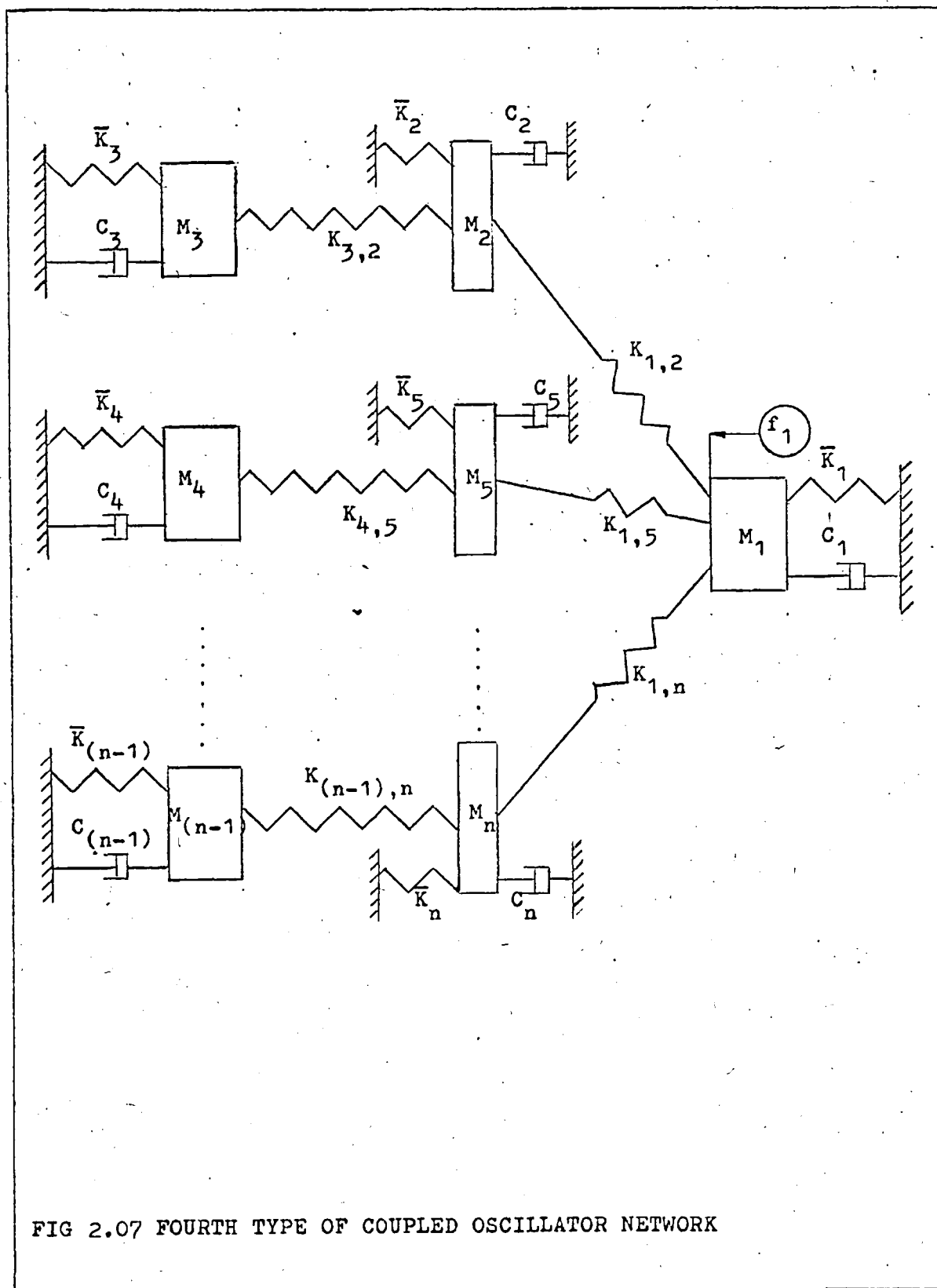


FIG 2.07 FOURTH TYPE OF COUPLED OSCILLATOR NETWORK

$$H_3 = \frac{G_{1,3} \cdot F_1}{D} \dots\dots\dots (73)$$

$$\text{Where } G_{1,3} = b_{2,1} \cdot b_{3,2} (1 - \gamma_{4,5}) \dots (1 - \gamma_{(n-1),n}) \dots (74)$$

Substituting (74) and the expression for D in (71) into (73) gives:

$$H_3 = \frac{b_{2,1} \cdot b_{3,2} (1 - \gamma_{4,5}) \dots (1 - \gamma_{(n-1),n}) F_1}{(1 - \gamma_{3,2}) (1 - \gamma_{4,5}) \dots (1 - \gamma_{(n-1),n})} \left[1 - \frac{\sum_{i=2}^{i=n} \gamma_{i,1}}{(1 - \gamma_{(i-1),i})} \right] \dots (75)$$

$$i = 2, 5, 7, \dots n$$

$$\text{Let } b_{2,1} \cdot b_{3,2} = \frac{H_{3(A)}}{F_1} (1 - \gamma_{1,2} - \gamma_{2,3})$$

Therefore:

$$H_3 = \frac{H_{3(A)} (1 - \gamma_{1,2} - \gamma_{2,3})}{(1 - \gamma_{1,2} - \gamma_{2,3}) - \frac{\sum_{i=5}^{i=n} \gamma_{i,1} (1 - \gamma_{2,3})}{(1 - \gamma_{(i-1),i})}} \dots (76)$$

The response of oscillator 1 is given by:

$$H_1 = \frac{G_{1,1} \cdot F_1}{D}$$

$$\frac{H_1}{H_3} = \frac{G_{1,1}}{G_{1,3}}$$

From (72) $G_{1,1} = (1 - \gamma_{2,3})(1 - \gamma_{4,5}) \dots (1 - \gamma_{(n-1),n})$

therefore
$$\frac{H_1}{H_3} = \frac{(1 - \gamma_{2,3})(1 - \gamma_{4,5}) \dots (1 - \gamma_{(n-1),n})}{b_{2,1} \cdot b_{3,2} (1 - \gamma_{4,5}) \dots (1 - \gamma_{(n-1),n})} \dots (77)$$

and hence
$$\frac{H_1}{H_3} = \frac{1 - \gamma_{2,3}}{b_{2,1} \cdot b_{3,2}} \dots (78)$$

Equation (78) is identical to the ratio $\frac{H_1}{H_3}$ of a three series oscillator system. It may therefore be concluded when one excited oscillator drives any number of independent transmission paths, the response ratio between the excited and receiving oscillators is exactly the same as that ratio calculated from the transmission path in isolation.

2.2.v Coupling factors

The analysis of the different types of oscillator networks, has shown that in all cases receiver response is a function of the sum of individual transmission path responses. However the smaller the coupling coefficients of the system, the nearer the simple arithmetic sum approaches the exact solution. Therefore the magnitudes of the coupling coefficients will now be examined.

The coupling coefficient between two oscillators was defined as the product of their coupling factors.

i.e. $\gamma_{u,v} = b_{u,v} \cdot b_{v,u}$

$$\text{where } b_{u,v} = \frac{K_{u,v} + j \cdot \omega \cdot C_{u,v}}{(K_u - \omega^2 \cdot M_u) + j \cdot \omega \cdot C_u}$$

Let the ratio of coupling to modal stiffness be designated

$$R_{u,v}, \quad \text{i.e. } R_{u,v} = \frac{K_{u,v}}{K_u} \quad \dots\dots\dots (79a)$$

$$\text{and similarly let } R'_{u,v} = \frac{C_{u,v}}{C_u} \quad \dots\dots\dots; \dots\dots\dots (79b)$$

$$\text{therefore } b_{u,v} = \frac{R_{u,v} \cdot K_u + j \cdot \omega \cdot R'_{u,v} \cdot C_u}{(K_u - \omega^2 \cdot M_u) + j \cdot \omega \cdot C_u} \quad \dots\dots\dots (80)$$

$$\text{when } \omega \text{ is small, } b_{u,v} \doteq R_{u,v} \quad \dots\dots\dots (81)$$

when ω is large i.e. $\omega \gg \omega_u$

$$b_{u,v} = \frac{j \cdot \omega \cdot R'_{u,v} \cdot C_u}{-\omega^2 \cdot M_u + j \cdot \omega \cdot C_u} = \frac{j \cdot R'_{u,v} \cdot \omega_u \cdot \eta_u}{-\omega + j \cdot \omega_u \cdot \eta_u}$$

$$\text{where } \eta_u = \frac{C_u}{\omega_u \cdot M_u}$$

$$\text{therefore } b_{u,v} \doteq \frac{j \cdot R'_{u,v} \cdot \omega_u \cdot \eta_u}{-\omega} \quad \dots\dots\dots (82)$$

$b_{u,v}$ has a maximum value when $\omega = \omega_u$

$$\begin{aligned} b_{u,v} &= \frac{R_{u,v} \cdot K_u + j \cdot R'_{u,v} \cdot \omega_u \cdot C_u}{j \cdot \omega_u \cdot C_u} \quad \dots\dots\dots (83) \\ (\text{max}) & \end{aligned}$$

$$\text{and hence } b_{u,v} = \frac{R_{u,v} + j \cdot R'_{u,v} \cdot \eta_u}{j \cdot \eta_u} \dots\dots\dots (84)$$

$$(max)$$

$$\text{from which } b_{u,v} = R'_{u,v} - \frac{j \cdot R_{u,v}}{\eta_u} \dots\dots\dots (85)$$

The maximum possible value of $\gamma_{u,v}$ occurs when both oscillators 'u' and 'v' are resonant at the same frequency.

If the coupling is conservative;

$$b_{u,v} = \frac{-j \cdot R_{u,v}}{\eta_u}, \text{ and } \gamma_{u,v} = \frac{-R_{u,v} \cdot R_{v,u}}{\eta_u \cdot \eta_v} \dots\dots\dots (86)$$

$$(max) \quad (max)$$

For dissipative coupling;

$$b_{u,v} = R'_{u,v}, \text{ and } \gamma_{u,v} = R'_{u,v} \cdot R'_{v,u} \dots\dots\dots (87)$$

$$(max) \quad (max)$$

It will be noted that for dissipatively coupled oscillators $\gamma_{u,v}$ must always be less than unity, but for conservatively coupled oscillators $\gamma_{u,v}$ may be greater or less than unity.

Many practical situations are likely to arise where two modal oscillators of slightly differing resonant frequencies are coupled. The dependence of coupling factor on frequencies near resonance will now be examined.

$$\text{From (80) } b_{u,v} = \frac{R_{u,v} + j \cdot R'_{u,v} \cdot \frac{\omega}{\omega_u} \cdot \eta_u}{1 - \frac{\omega^2}{\omega_u^2} + j \cdot \frac{\omega}{\omega_u} \cdot \eta_u} \dots\dots\dots (88)$$

If w is expressed in terms of the number of 'bandwidths' above or below w_u . i.e. $w = w_u(1 \pm J_u \cdot \eta_u)$, equation (88) becomes:

$$b_{u,v} = \frac{R_{u,v} + j \cdot R'_{u,v} (1 \pm J_u \cdot \eta_u) \eta_u}{(1 - (1 \pm J_u \cdot \eta_u)^2) + j(1 \pm J_u \cdot \eta_u) \eta_u} \dots\dots\dots (89)$$

Expanding (89) and ignoring terms containing products of loss factor, we have for frequencies near w_u .

$$b_{u,v} \doteq \frac{R_{u,v} + j \cdot R'_{u,v} \cdot \eta_u}{\pm 2 \cdot J_u \cdot \eta_u - j \cdot \eta_u} \dots\dots\dots (90)$$

A further approximation may be made if w is at least several bandwidths from w_u .

$$b_{u,v} \doteq \frac{R_{u,v} + j \cdot R'_{u,v} \cdot \eta_u}{\pm 2 \cdot J_u \cdot \eta_u}$$

Therefore for purely conservative coupling:

$$b_{u,v} \doteq \frac{R_{u,v}}{\pm 2 \cdot J_u \cdot \eta_u} \dots\dots\dots (91)$$

For purely dissipative coupling:

$$b_{u,v} \doteq \frac{j \cdot R'_{u,v}}{\pm 2 \cdot J_u} \dots\dots\dots (92)$$

From equations (91) and (92) it is seen that the coupling factor diminishes approximately linearly with frequency above and

below the oscillator resonant frequency. This decline continues until the low and high frequency asymptotic solutions of equation (80) apply.

2.2.vi Response nullification

From the foregoing analysis it was found that coupling factors are dependent both on the magnitude of the coupling parameters themselves (i.e. spring constants, damping coefficients) and also frequency. This frequency dependence entails a sign change for frequencies above and below the oscillator resonant frequency respectively. The sign becomes purely imaginary at resonance. The following analysis will show that if a receiving oscillator is driven by more than one transmission path, the response due to paths consisting of oscillators having higher resonant frequencies is counteracted by those paths having lower resonant frequencies.

Consider the first type of network. To reduce the algebra so as not to obscure the underlying simplicity of the analysis, only two transmission paths will be examined. As before, path 'A' consists of oscillators 1,2 and 3 and path 'B' 1,4 and 3. The response of the receiver, oscillator 3, is given by (47) and (49), thus:-

$$\frac{H_3}{F_1} = \frac{b_{2,1} \cdot b_{3,2} + b_{4,1} \cdot b_{3,4}}{1 - \gamma_{1,2} - \gamma_{2,3} - \gamma_{1,4} - \gamma_{4,3} + \gamma_{1,2} \cdot \gamma_{4,3} + \gamma_{1,4} \cdot \gamma_{2,3} - 2(b_{4,1} \cdot b_{1,2} \cdot b_{2,3} \cdot b_{3,4})} \dots\dots (93)$$

Clearly if $b_{2,1} \cdot b_{3,2} = -b_{4,1} \cdot b_{3,4}$ (94)

then $H_3 = 0$

If coupling is conservative:

$$b_{2,1} \propto \pm \frac{R_{2,1}}{\eta_1} \quad b_{3,2} \propto \pm \frac{R_{3,2}}{\eta_2}$$

$$b_{4,1} \propto \pm \frac{R_{4,1}}{\eta_1} \quad b_{3,4} \propto \pm \frac{R_{3,4}}{\eta_4}$$

therefore $b_{2,1} \cdot b_{3,2} \propto \left(\pm \frac{R_{2,1}}{\eta_1} \right) \left(\pm \frac{R_{3,2}}{\eta_2} \right)$

$$b_{4,1} \cdot b_{3,4} \propto \left(\pm \frac{R_{4,1}}{\eta_1} \right) \left(\pm \frac{R_{3,4}}{\eta_4} \right)$$

The signs of $b_{2,1}$ and $b_{4,1}$ are the same as both depend on the difference between W_1 and w . The signs of $b_{3,2}$ and $b_{3,4}$ will be opposite for w between W_2 and W_4 . Therefore a solution to equation (94) is possible at a frequency between the resonant frequencies of the intermediate oscillators.

If a third transmission path is introduced between oscillators 1 and 3, designated as path 'C', with an intermediate oscillator number 5, it will be shown that when the resonant frequency of oscillator 5 lies between those of the other intermediate oscillators, the receiver response is due solely to path 'C' under certain conditions of coupling.

For this system the response equation from (47) and (49) is:-

$$\begin{aligned}
H_3 = & \frac{H_3(A)(1-\gamma_{1,2}-\gamma_{2,3})+H_3(B)(1-\gamma_{1,4}-\gamma_{4,3})+H_3(C)(1-\gamma_{1,5}-\gamma_{5,3})}{1-\gamma_{1,2}-\gamma_{2,3}-\gamma_{1,4}-\gamma_{4,3}-\gamma_{1,5}-\gamma_{5,3}+\gamma_{1,2}\cdot\gamma_{4,3}+} \\
& +\gamma_{1,4}\cdot\gamma_{2,3}+\gamma_{5,3}(\gamma_{1,2}+\gamma_{1,4})+\gamma_{1,5}(\gamma_{2,3}+\gamma_{4,3})- \\
& -2(b_{4,1}\cdot b_{1,2}\cdot b_{2,3}\cdot b_{3,4})-2(b_{5,1}\cdot b_{1,2}\cdot b_{2,3}\cdot b_{3,5})- \\
& -2(b_{5,1}\cdot b_{1,4}\cdot b_{4,3}\cdot b_{3,5}) \quad \dots\dots\dots (95)
\end{aligned}$$

In order that $H_3 = H_3(C)$ it is necessary that:

$$b_{2,1}\cdot b_{3,2} + b_{4,1}\cdot b_{3,4} = 0$$

and that the denominator reduce to: $(1 - \gamma_{1,5} - \gamma_{5,3})$.

Therefore it is also necessary that

$$\gamma_{1,2} + \gamma_{2,3} + \gamma_{1,4} + \gamma_{4,3} = 0 \quad \dots\dots\dots (96)$$

If both the above conditions are true:

$$b_{1,2}\cdot b_{2,3} + b_{1,4}\cdot b_{4,3} = 0 \quad \dots\dots\dots (97)$$

Which implies:

$$\gamma_{1,2}\cdot\gamma_{2,3} - \gamma_{1,4}\cdot\gamma_{4,3} = 0 \quad \dots\dots\dots (98)$$

Solving equations (96) and (98) simultaneously:

$$\begin{aligned} \gamma_{1,2} &= -\gamma_{1,4} \text{ or } -\gamma_{4,3} \\ \text{and } \gamma_{2,3} &= -\gamma_{4,3} \text{ or } -\gamma_{1,4} \end{aligned} \quad \dots\dots\dots (99)$$

If the former solution of (99) is substituted in the denominator of (95) together with (94), (97), (98) then it reduces to:

$(1 - \gamma_{1,5} - \gamma_{5,3})$, therefore $H_3 = H_3(c)$ under the conditions that; $\gamma_{1,2} = -\gamma_{1,4}$ and $\gamma_{2,3} = -\gamma_{4,3}$. Substituting the latter solution of (99) and evaluating the denominator of (95) gives:

$$\begin{aligned} 1 - \gamma_{1,5} - \gamma_{5,3} - \gamma_{1,2}^2 - \gamma_{2,3}^2 + \gamma_{5,3}(\gamma_{1,2} - \gamma_{2,3}) + \\ + \gamma_{1,5}(\gamma_{2,3} - \gamma_{1,2}) + 2 \cdot \gamma_{1,2} \cdot \gamma_{2,3} \end{aligned}$$

This expression only reduces to $(1 - \gamma_{1,5} - \gamma_{5,3})$ if $\gamma_{1,2} = \gamma_{2,3}$, which gives the less general condition that $H_3 = H_3(c)$ if:

$$\gamma_{1,2} = \gamma_{2,3} = -\gamma_{1,4} = -\gamma_{4,3}.$$

Hence under certain conditions of coupling the resultant vibration transmission from paths 'A' and 'B' is zero.

Response nullification may also occur in the second type of network. To reduce the algebra analysis will be limited to a system with six oscillators; as in fig.2.05 with $n = 6$.

Let path 'A' consist of oscillators 1,2 and 3, path 'B' oscillators 5,2 and 3, path 'C' oscillators 6,2 and 3.

The response of the receiving oscillator 3 is given by equation (62):

$$H_3 = \frac{H_{3(A)}(1-\gamma_{1,2}-\gamma_{2,3}) + H_{3(B)}(1-\gamma_{5,2}-\gamma_{2,3}) + H_{3(C)}(1-\gamma_{6,2}-\gamma_{2,3})}{1 - \gamma_{1,2} - \gamma_{2,3} - \gamma_{2,4} - \gamma_{2,5} - \gamma_{2,6}} \quad \dots (100)$$

In order that the numerator reduce to; $H_{3(C)}(1-\gamma_{6,2}-\gamma_{2,3})$ it is necessary that $b_{2,1} = -b_{2,5} \quad \dots (101)$

and for the denominator to reduce to $(1 - \gamma_{6,2} - \gamma_{2,3})$ then:

$$\gamma_{1,2} + \gamma_{2,4} + \gamma_{2,5} = 0 \quad \dots (102)$$

Therefore in general the response of a receiver may be calculated by taking only one transmission path if: (i) the sum of all other coupling coefficients is zero, (ii) the sum of all other coupling factors from intermediate to excited oscillators is also zero.

If we now consider the third type of network, again limited to three transmission paths for simplicity as in fig. 2.06 where $n = 7$. Let path 'A' consist of oscillators 1,2 and 3, path 'B' oscillators 4,5 and 3, and path 'C' oscillators 6,7 and 3.

The response of the receiving oscillator 3 is given by (71):

$$\begin{aligned}
& H_{3(A)}(1 - \gamma_{1,2} - \gamma_{2,3})(1 - \gamma_{4,5})(1 - \gamma_{6,7}) + \\
& + H_{3(B)}(1 - \gamma_{4,5} - \gamma_{5,3})(1 - \gamma_{1,2})(1 - \gamma_{6,7}) + \\
& + H_{3(C)}(1 - \gamma_{6,7} - \gamma_{7,3})(1 - \gamma_{1,2})(1 - \gamma_{4,5}) \\
H_3 = & \frac{(1 - \gamma_{1,2})(1 - \gamma_{4,5})(1 - \gamma_{6,7}) \left[1 - \frac{\gamma_{2,3}}{(1 - \gamma_{1,2})} - \frac{\gamma_{5,3}}{(1 - \gamma_{4,5})} - \frac{\gamma_{7,3}}{(1 - \gamma_{6,7})} \right]}{(1 - \gamma_{1,2})(1 - \gamma_{4,5})(1 - \gamma_{6,7})} \dots (103)
\end{aligned}$$

When $b_{2,1} \cdot b_{3,2} = -b_{5,4} \cdot b_{3,5}$ the numerator is equal to;

$$\begin{aligned}
& H_{3(C)}(1 - \gamma_{6,7} - \gamma_{7,3})(1 - \gamma_{1,2})(1 - \gamma_{4,5}) + \\
& + H_{3(A)}(1 - \gamma_{1,2} - \gamma_{2,3})(1 - \gamma_{4,5})(1 - \gamma_{6,7})
\end{aligned}$$

and it is necessary that $\gamma_{1,2} - \gamma_{4,5} = 0$, to eliminate the

$H_{3(A)}$ term.

The denominator reduces to;

$$\begin{aligned}
& (1 - \gamma_{1,2})(1 - \gamma_{4,5})(1 - \gamma_{6,7} - \gamma_{7,3}) \\
\text{if } & \frac{\gamma_{2,3}}{(1 - \gamma_{1,2})} + \frac{\gamma_{5,3}}{(1 - \gamma_{4,5})} = 0
\end{aligned}$$

which occurs if $\gamma_{2,3} + \gamma_{5,3} = 0$.

Therefore equation (103) reduces to $H_3 = H_{3(C)}$

$$\text{when } b_{2,1} \cdot b_{3,2} + b_{5,4} \cdot b_{3,5} = 0$$

$$\gamma_{1,2} - \gamma_{4,5} = 0$$

$$\gamma_{2,3} + \gamma_{5,3} = 0$$

In general the response of a receiver is equal to that from one transmission path if; (i) the sum of the products of the intermediate to excited, and receiver to intermediate coupling factors is zero, (ii) the coupling coefficients between the excited and intermediate oscillators are equal, (iii) the sum of the intermediate to receiver coupling coefficients is zero for all other transmission paths. However, condition (ii) can only be true if excited oscillators have a common resonant frequency, and intermediate oscillators have a common resonant frequency.

For the fourth type of network the analysis is similar to that of the third, but with oscillator numbers 1 and 3 interchanged. Retaining the same transmission path identification we have:

$$H_3 = \frac{H_3(A)(1 - \gamma_{1,2} - \gamma_{2,3})}{(1 - \gamma_{2,3}) \left[1 - \frac{\gamma_{1,2}}{(1 - \gamma_{2,3})} - \frac{\gamma_{1,5}}{(1 - \gamma_{4,5})} - \frac{\gamma_{1,7}}{(1 - \gamma_{6,7})} \right]} \quad \dots (104)$$

clearly if $\frac{\gamma_{1,5}}{(1 - \gamma_{4,5})} + \frac{\gamma_{1,7}}{(1 - \gamma_{6,7})} = 0$, then $H_3 = H_3(A)$.

The receiver response is therefore equal to that from one transmission path only, if the sum of the ratios of excited to intermediate divided by one minus intermediate to receiver coupling coefficients is zero for all other paths.

2.2.vii Application to large multimodal structures

A large number of systems in practise will comprise structures

having possibly many thousands of modes of vibration. Assuming the modal and coupling parameters were known, the solution of the coupling factor matrix would present an all but impossible task. A digital computer having several million storage registers would be necessary, and at the present time such machines are not available. It is desirable therefore to know if any transmission paths can be ignored due to response nullification or if they do not cancel completely, what error can be expected by assuming they do.

If the excited and receiving structures have modes of vibration which are closely spaced in frequency, each mode of an intermediate structure will thus form a transmission path with approximately coincident resonant frequencies. It has been shown that such a path will have much stronger coupling than any other transmission path between these particular excited and receiving oscillators; assuming that modal and coupling parameters do not vary widely from mode to mode. If the resonant frequency of the intermediate oscillator is not the highest or the lowest of its structure, the vibration transmission from all other paths will tend to cancel itself out at this receiver, and would do so completely if for example all excited to intermediate coupling coefficients were equal, and all intermediate to receiver coupling coefficients were equal. However it is improbable that any coupling coefficient will exactly equal another, but all other paths are likely to have coefficients which are smaller than those of the coincident path. The effect of these non-coincident paths will be to add significantly

smaller responses to the coincident path response. This system forms a network of the first type. Keeping the same transmission path notation as before, the receiver response is given by:

$$H_3 = \frac{H_3(A)(1-\gamma_{1,2}-\gamma_{2,3}) + H_3(B)(1-\gamma_{1,4}-\gamma_{4,3}) + H_3(C)(1-\gamma_{1,5}-\gamma_{5,3}) + \dots}{1 - \gamma_{1,2} - \gamma_{2,3} - \gamma_{1,4} - \gamma_{4,3} - \gamma_{1,5} - \gamma_{5,3} - \dots} \quad (105)$$

ignoring product terms of coupling coefficients which will be small. Let path 'C' be a coincident path. For incomplete nullification of paths 'A', 'B', etc. Let;

$$\frac{b_{2,1} \cdot b_{3,2} + b_{4,1} \cdot b_{3,4} + \dots}{b_{5,1} \cdot b_{3,5}} = j \cdot \delta$$

hence the numerator of (105) becomes:

$$H_3(C)(1-\gamma_{1,5}-\gamma_{5,3})(1+j \cdot \delta)$$

$$\text{if } \frac{b_{1,2}}{b_{3,2}} = \frac{b_{1,4}}{b_{3,4}} = \dots = \frac{b_{1,n}}{b_{3,n}},$$

$$\text{then } \gamma_{1,2} + \gamma_{1,4} + \dots = j \cdot \delta \cdot \gamma_{1,5}$$

$$\text{and } \gamma_{2,3} + \gamma_{4,3} + \dots = j \cdot \delta \cdot \gamma_{5,3}$$

$$\text{Let } (\gamma_{1,5} + \gamma_{5,3}) = \nabla$$

then equation (105) may be re-written as:

$$\frac{H_3}{H_3(C)} = \frac{(1 - \nabla)(1 + j \cdot \delta)}{1 - \nabla - j \cdot \delta \cdot \nabla} \quad \dots \dots \dots (106)$$

Equation (106) was evaluated for several values of the ratio $\frac{6}{\nabla}$. The results are presented in fig.2.08, from which it is seen that the ratio $\frac{H_3}{H_3(C)}$ is within 33% of unity when ∇ is less than -3.

If the modal oscillators of the intermediate structure are widely spaced in frequency, the excited and receiving oscillators whose resonant frequencies are close to an intermediate oscillator will be more strongly coupled by it than any other. The analysis of the second type of network can be applied. A receiver in this case is driven by two 'semi-coincident' paths, see fig 2.09, and several non-coincident paths. The semi-coincident paths, which will in this case have the strongest coupling, consist of (i) an excited oscillator of the same resonant frequency as the receiver, plus the intermediate, i.e. oscillators 1,2 and 3, designated path 'A', (ii) an excited oscillator of the same resonant frequency as the intermediate, plus the receiver, i.e. oscillators 8,2 and 3, designated path 'B'.

The response of the receiver 3, ignoring non-coincident paths is:

$$H_3 = \frac{H_{3(A)}(1 - \gamma_{1,2} - \gamma_{2,3}) + H_{3(B)}(1 - \gamma_{8,2} - \gamma_{2,3})}{1 - \gamma_{1,2} - \gamma_{2,3} - \gamma_{8,2}} \quad \dots\dots\dots (107)$$

$$H_{3(A)}(1 - \gamma_{1,2} - \gamma_{2,3}) = b_{2,1} \cdot b_{3,2} \cdot F_1 = \frac{b_{2,1} \cdot b_{3,2}}{a_{1,1}}$$

$$H_{3(B)}(1 - \gamma_{8,2} - \gamma_{2,3}) = b_{2,8} \cdot b_{3,2} \cdot F_8 = \frac{b_{2,8} \cdot b_{3,2}}{a_{8,8}}$$

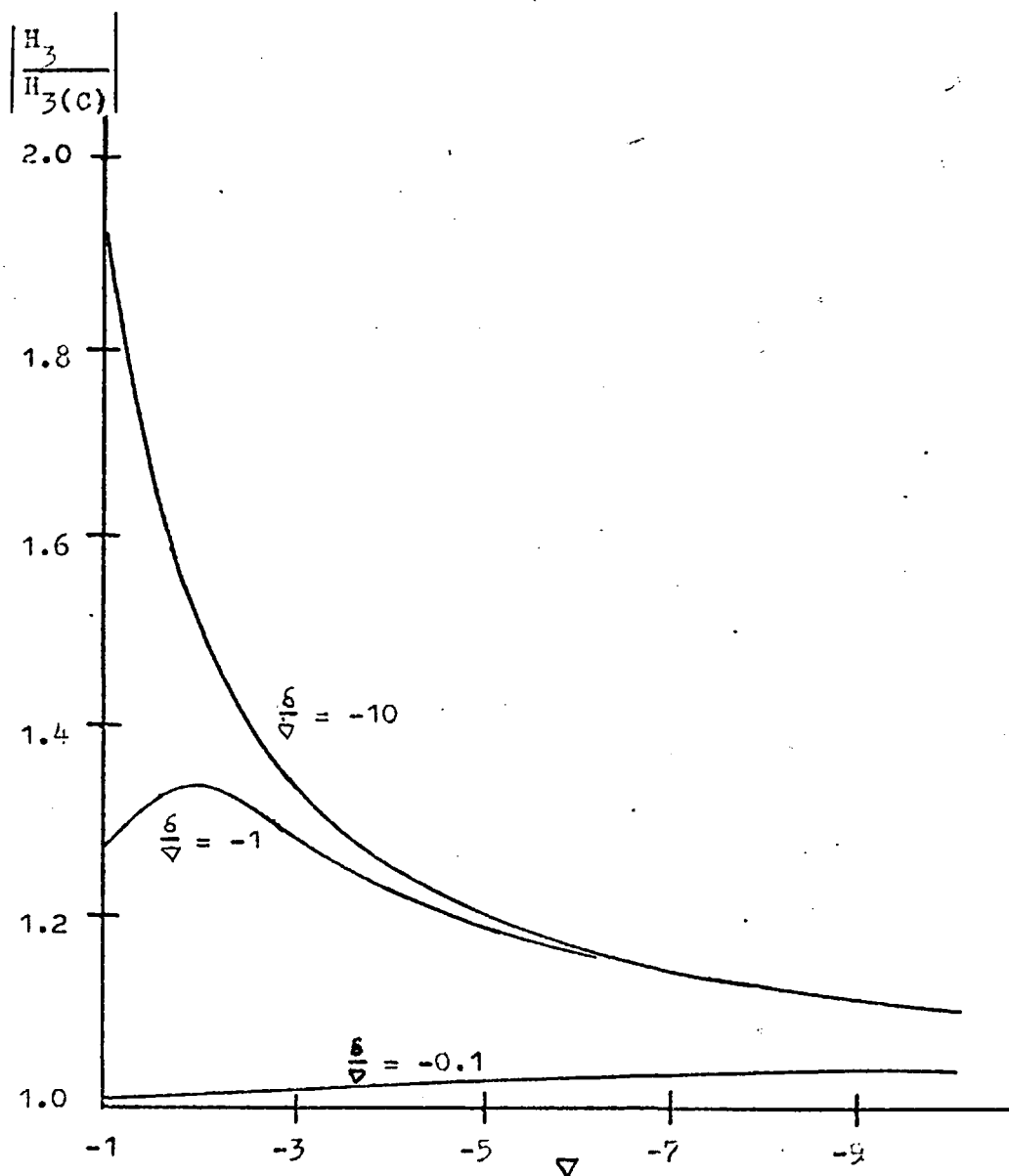


FIG 2.C8 SOLUTION OF $\frac{H_3}{H_{3(C)}} = \frac{(1 - \Delta)(1 + j.6)}{1 - \Delta - j.6.\Delta}$ PLOTTED AGAINST Δ

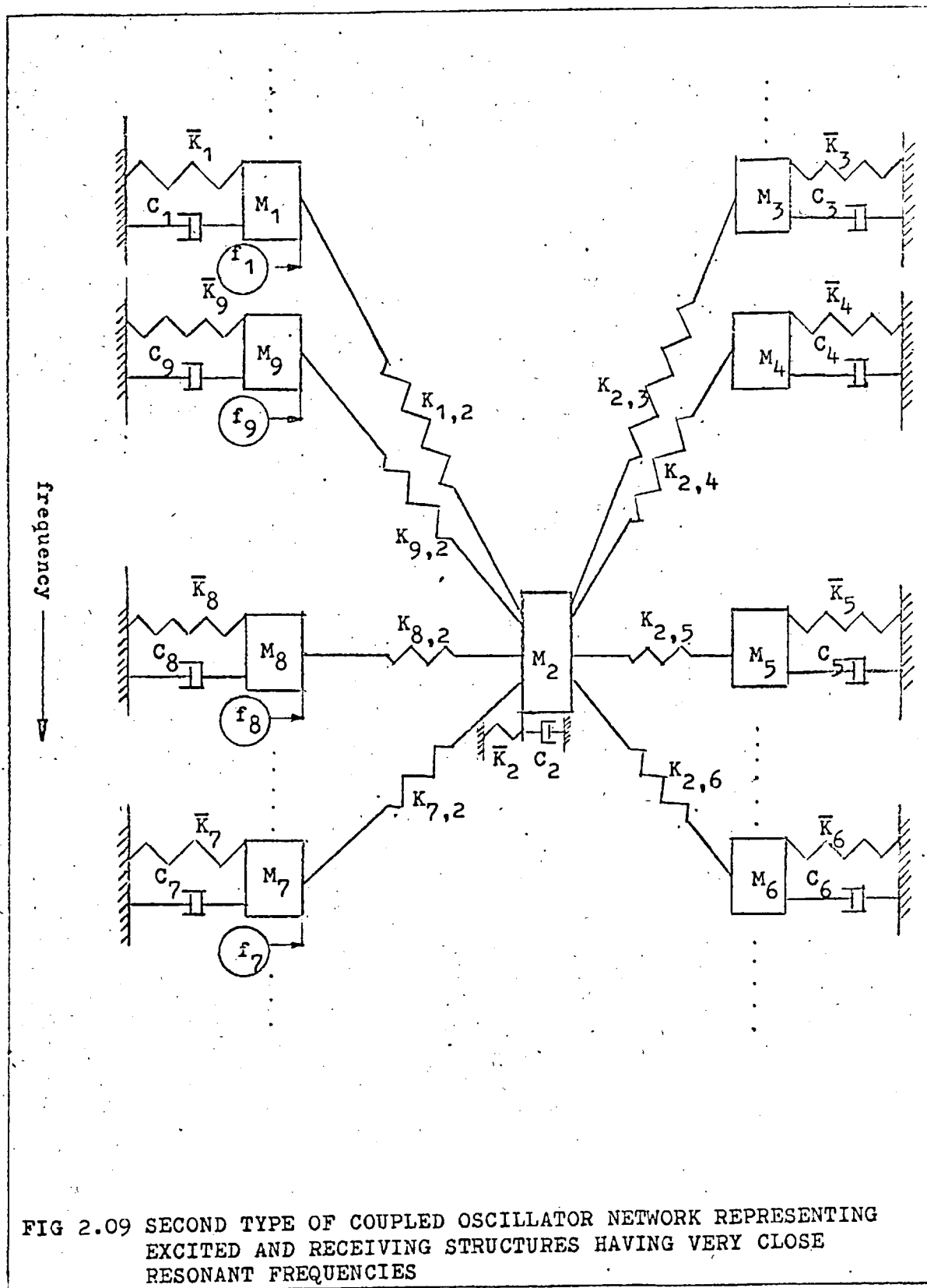


FIG 2.09 SECOND TYPE OF COUPLED OSCILLATOR NETWORK REPRESENTING EXCITED AND RECEIVING STRUCTURES HAVING VERY CLOSE RESONANT FREQUENCIES

At the frequency $\omega = \omega_3$, let
$$\frac{H_3(B)(1-\gamma_{8,2}-\gamma_{2,3})}{H_3(A)(1-\gamma_{1,2}-\gamma_{2,3})} = j.\delta$$

$$\text{therefore } \frac{b_{8,2}}{b_{1,2}} = j.\delta \quad \dots\dots\dots (108)$$

$$\text{and } \frac{\gamma_{8,2}}{\gamma_{1,2}} = j.\delta \cdot \frac{a_{8,2}}{a_{1,2}} \quad \dots\dots\dots (109)$$

Equation (107) may therefore be re-written as:

$$H_3 = \frac{H_3(A)(1-\gamma_{1,2}-\gamma_{2,3})(1+j.\delta)}{1-\gamma_{2,3}-\gamma_{1,2}(1+j.\delta \cdot \frac{a_{8,2}}{a_{1,2}})} \quad \dots\dots\dots (110)$$

from equation (108) it is seen that δ is a frequency dependent term.

$$\text{i.e. } j.\delta = \frac{a_{8,2}}{a_{8,8}} \cdot \frac{a_{1,1}}{a_{1,2}} \quad \dots\dots\dots (111)$$

$$\text{where } a_{1,1} = \omega_1^2 \cdot M_1 \left(\left(1 - \frac{\omega^2}{\omega_1^2} \right) + j \cdot \frac{\omega}{\omega_1} \cdot \eta_1 \right) \quad \dots\dots\dots (112)$$

$$\text{and } a_{8,8} = \omega_8^2 \cdot M_8 \left(\left(1 - \frac{\omega^2}{\omega_8^2} \right) + j \cdot \frac{\omega}{\omega_8} \cdot \eta_8 \right) \quad \dots\dots\dots (113)$$

The value of δ passes through a minimum and is considerably less than unity if excited and receiving oscillators have the same resonant frequency. The value of δ is a maximum at the resonant frequency of oscillator 8. The relative responses levels of the receiver at these two frequencies will be investigated.

If the coupling parameters $a_{8,2}$ and $a_{1,2}$ are approximately

equal, then at the frequency $\omega = \omega_1$ δ is likely to be of the same order of magnitude as η_1 , as long as ω_1 and ω_3 are well separated.

$$\text{Therefore} \quad H_3 \quad = \quad H_3(A) \quad \dots\dots\dots (114)$$

$$(\text{at } \omega=\omega_1) \quad (\text{at } \omega=\omega_1)$$

The coupling coefficients $\gamma_{2,3}$ and $\gamma_{1,2}$ are small as there is no resonant frequency coincidence. Therefore from (110):

$$H_3 \quad \doteq \quad H_3(A) \quad \cdot j \cdot \frac{1}{\delta} \quad \dots\dots\dots (115)$$

$$(\text{at } \omega=\omega_2) \quad (\text{at } \omega=\omega_2)$$

$$H_3(A) = \frac{b_{2,1} \cdot b_{3,2} \cdot F_1}{(1 - \gamma_{1,2} - \gamma_{2,3})} \quad , \text{ and assuming conservative coupling:}$$

$$b_{u,v} \doteq \frac{R_{u,v}}{2 \cdot J_u \cdot \eta_u} ;$$

$$(1 - \gamma_{1,2} - \gamma_{2,3})_{(\text{at } \omega=\omega_1)} = 1 - j \cdot \frac{R_{1,2} \cdot R_{2,1}}{\eta_1 \cdot 2 \cdot J_2 \cdot \eta_2} - j \cdot \frac{R_{2,3} \cdot R_{3,2}}{2 \cdot J_2 \cdot \eta_2 \cdot \eta_3} \quad \dots\dots(116)$$

$$(1 - \gamma_{1,2} - \gamma_{2,3})_{(\text{at } \omega=\omega_2)} = 1 - j \cdot \frac{R_{1,2} \cdot R_{2,1}}{2 \cdot J_1 \cdot \eta_1 \cdot \eta_2} - j \cdot \frac{R_{2,3} \cdot R_{3,2}}{2 \cdot J_3 \cdot \eta_3 \cdot \eta_2} \quad \dots\dots(117)$$

Due to the previous assumptions the ratios $\frac{J_2}{J_1}$ and $\frac{J_2}{J_3}$

may be taken as unity. Therefore:

$$(1 - \gamma_{1,2} - \gamma_{2,3})_{(\text{at } \omega=\omega_1)} \doteq (1 - \gamma_{1,2} - \gamma_{2,3})_{(\text{at } \omega=\omega_2)}$$

$$\frac{H_3(A)}{(\text{at } w=w_2)} = \frac{j \cdot R_{2,1} \cdot R_{3,2} \cdot 2 \cdot J_2 \cdot \eta_2 \cdot \eta_3 \cdot W_1^2 \cdot M_1(j \cdot \eta_1)}{j \cdot R_{2,1} \cdot R_{3,2} \cdot 2 \cdot J_3 \cdot \eta_2 \cdot \eta_3 \cdot W_1^2 \cdot M_1((1 - \frac{W_2^2}{W_1^2}) + j \cdot \frac{W_2}{W_1} \cdot \eta_1)}$$

Therefore $\frac{H_3(A)}{(\text{at } w=w_2)} = j \cdot \eta_1$ (118)

$\frac{H_3(A)}{(\text{at } w=w_1)}$

From equations (114), (115) and (118):

$$\frac{H_3}{(\text{at } w=w_2)} = j \cdot \eta_1 \cdot \frac{j}{8} \cdot \frac{H_3}{(\text{at } w=w_1)} = -\frac{H_3}{(\text{at } w=w_1)} \quad \text{..... (119)}$$

From equation (119) it is concluded that for this system, receiver responses of the same order of magnitude may be expected at both the receiver and intermediate oscillator resonant frequencies.

2.3 EXCITATION AND RESPONSE RELATIONSHIPS FOR A RANDOMLY EXCITED SYSTEM

The following assumptions about the random excitation will be made:

- (i) The random vibration is a stationary process. This implies that its probability distributions are invariant with time.
- (ii) The random process is normal.

Let $p(x)$ be the probability density of x , where x is a randomly varying quantity.

Let $E[x]$ be the expected mean value of x .

$$\text{therefore } E[x] = \int_{-\infty}^{\infty} x \cdot p(x) \cdot dx \quad \dots\dots\dots (120)$$

Let x measured at time t_1 be called x_1 and at time t_2 be called x_2 .

$$E[x_1 \cdot x_2] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1 \cdot x_2 \cdot p(x_1, x_2) \cdot dx_1 \cdot dx_2 \quad \dots\dots\dots (121)$$

$E[x_1 \cdot x_2]$ may be defined as the autocorrelation function.

The second order probability density $p(x_1, x_2)$ is independent of time if the process is stationary. It is solely a function of the time difference between t_1 and t_2 .

Let $t_1 - t_2 = T$

$$\text{then } E[x_1 \cdot x_2] = E[x(t) \cdot x(t+T)] = R(T) \quad \dots\dots\dots (122)$$

Let the Fourier transform of $R(T)$ be called $S(w)$.

$$S(w) = \frac{1}{2 \cdot \pi} \int_{-\infty}^{\infty} R(T) \cdot e^{-j \cdot w \cdot T} \cdot dT \quad \dots\dots\dots (123)$$

hence by the inverse transform:

$$R(T) = \int_{-\infty}^{\infty} S(w) \cdot e^{j \cdot w \cdot T} \cdot dw \quad \dots\dots\dots (124)$$

when $T = 0$, $E[x_1 \cdot x_2] = E[x^2]$

$$\text{therefore } E[x^2] = R(0) = \int_{-\infty}^{\infty} S(w) \cdot dw \quad \dots\dots\dots (125)$$

Therefore the expected mean square of the process equals the sum over all frequencies of $S(w).dw$. $S(w)$ may be termed the mean square spectral density.

If a system has an excitation $f(t)$ and a response $x(t)$,

$$x(t) = H(w).f(t)$$

where $H(w)$ is the complex frequency response function.

For linear systems the principle of superposition using Fourier's method can be applied. When $f(t)$ is a periodic function it can be decomposed into sinusoids to form a Fourier series. The response to each individual sinusoid is given by $H(w)$.

Let the Fourier transform of $f(t)$ be $f(w)$.

$$\text{i.e. } f(w) = \int_{-\infty}^{\infty} f(t).e^{-j.w.t}.dt \quad \dots\dots\dots (126)$$

$$\text{therefore } x(w) = H(w).f(w) \quad \dots\dots\dots (127)$$

$$\text{also } x(t) = \frac{1}{2.\pi} \int_{-\infty}^{\infty} x(w).e^{j.w.t}.dw \quad \dots\dots\dots (128)$$

$$\text{therefore } x(t) = \frac{1}{2.\pi} \int_{-\infty}^{\infty} H(w).e^{j.w.t}.dw. \int_{-\infty}^{\infty} f(T).e^{-j.w.T}.dT \quad \dots\dots(129)$$

Let $h(t - T)$ be the impulse response function, where T is the time at which the impulse was applied. The response at time t is:

$$(f(T).dT)h(t - T) \quad \dots\dots\dots (130)$$

The response at time t to all such impulses is:

$$x(t) = \int_{-\infty}^t f(T).h(t - T).dT \quad \dots\dots\dots (131)$$

The upper intergration limit in equation (131) may changed to ' ∞ ' without changing the value, as the function does not exist for t less than T .

$$\text{therefore } x(t) = \int_{-\infty}^{\infty} f(T).h(t - T).dT \quad \dots\dots\dots (132)$$

From (129) and (131) it is seen that if $f(T)$ is a unit impulse (T) , the response is given by:

$$h(t) = \frac{1}{2.\pi} \int_{-\infty}^{\infty} H(w).e^{j.w.t}.dw \quad \dots\dots\dots (133)$$

$$\text{and conversely, } H(w) = \int_{-\infty}^{\infty} h(t).\bar{e}^{j.w.t}.dt \quad \dots\dots\dots (134)$$

Let the mean value of the response be $E[x]$. Substituting $\theta = (t - T)$ in (132) gives:

$$x(t) = \int_{-\infty}^{\infty} f(t - \theta).h(\theta).d\theta \quad \dots\dots\dots (135)$$

$$\text{therefore } E[x] = E\left[\int_{-\infty}^{\infty} f(t - \theta).h(\theta).d\theta\right] \quad \dots\dots\dots (136)$$

The mathematical operations in the above equation are linear and may be interchanged.

$$E[x] = \int_{-\infty}^{\infty} E[f(t - \theta)].h(\theta).d\theta \quad \dots\dots\dots (137)$$

As $f(t)$ is a stationary process its mean value is not a function of time, hence

$$E[x] = E[f] \int_{-\infty}^{\infty} h(\theta) \cdot d\theta \quad \dots\dots\dots (138)$$

The integral in equation (138) is a constant of the system and is equal to the solution of equation (134) for $w = 0$.

therefore $E[x] = E[f] \cdot H(0)$

Similarly for the autocorrelation function:

$$x(t) \cdot x(t+T) = \int_{-\infty}^{\infty} f(t-\theta_1) \cdot h(\theta_1) \cdot d\theta_1 \cdot \int_{-\infty}^{\infty} f(t+T-\theta_2) \cdot h(\theta_2) \cdot d\theta_2 \quad \dots (139)$$

therefore

$$E[x(t) \cdot x(t+T)] = E \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(t-\theta_1) \cdot x(t+T-\theta_2) \cdot h(\theta_1) \cdot h(\theta_2) \cdot d\theta_1 \cdot d\theta_2 \right] \quad \dots (140)$$

applying the relationship of equation (122)

$$R_x(T) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_f(T+\theta_1-\theta_2) \cdot h(\theta_1) \cdot h(\theta_2) \cdot d\theta_1 \cdot d\theta_2 \quad \dots\dots\dots (141)$$

hence from equation (123)

$$S_x(w) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-j \cdot w \cdot T} \cdot R_f(T+\theta_1-\theta_2) \cdot h(\theta_1) \cdot h(\theta_2) \cdot d\theta_1 \cdot d\theta_2 \cdot dT \quad \dots (142)$$

If equation (142) is re-written with the inclusion of the factors

$e^{j \cdot w \cdot \theta_1}$, $e^{-j \cdot w \cdot \theta_2}$, $e^{-j \cdot w(\theta_1-\theta_2)}$ whose product is unity we have:

$$S_x(w) = \int_{-\infty}^{\infty} h(\theta_1) \cdot e^{j \cdot w \cdot \theta_1} \cdot d\theta_1 \cdot \int_{-\infty}^{\infty} h(\theta_2) \cdot e^{-j \cdot w \cdot \theta_2} \cdot d\theta_2 \cdot \frac{1}{2 \cdot \pi} \int_{-\infty}^{\infty} R_f(T + \theta_1 - \theta_2) \cdot e^{-j \cdot w(T + \theta_1 - \theta_2)} \cdot dT \dots (143)$$

$$\text{Thus } S_x(w) = H(-w) \cdot H(w) \cdot S_f(w) \dots \dots \dots (144)$$

$$\text{and therefore } S_x(w) = |H(w)|^2 \cdot S_f(w)$$

From equation (125) the mean square response may be found in terms of the spectral density of the input.

$$E[x^2] = \int_{-\infty}^{\infty} S_x(w) \cdot dw = \int_{-\infty}^{\infty} |H(w)|^2 \cdot S_f(w) \cdot dw \dots \dots \dots (145)$$

For the special case when the input is a source of 'white noise'

$$E[x^2] = S_o \int_{-\infty}^{\infty} |H(w)|^2 \cdot dw \dots \dots \dots (146)$$

The kinetic energy of an oscillator is proportional to its velocity squared. The mean square velocity is given by:

$$E[\dot{x}^2] = S_o \int_{-\infty}^{\infty} |j \cdot w \cdot H(w)|^2 \cdot dw \dots \dots \dots (147)$$

2.4 EVALUATION OF FREQUENCY RESPONSE FUNCTIONS

2.4.i Gauss elimination method

In section 2.2 , the equations of motion of a coupled network of oscillators were expressed in the following general

matrix form i.e. equation (30)

$$[A][H] = [\bar{F}]$$

The leading diagonal elements contain the 'modal parameters' and the off-diagonal elements the coupling parameters. The matrix $[A]$ is symmetric, which enables the number of computing operations to be reduced by a factor of two, compared with the solution of the coupling factor matrix equation (35).

The method involves the calculation of two auxiliary matrices from the system matrices $[A]$ and $[\bar{F}]$. The first auxiliary matrix is made up of two sub-matrices $[g]$ and $[h]$; $[g]$ is a lower triangular matrix containing the leading diagonal; $[h]$ is an upper triangular matrix. These are found using the following relationships.

$$g_{u,u} = a_{u,u} - \sum_{m=1}^{m=u-1} g_{u,m} \cdot h_{m,u}$$

$$g_{v,u} = a_{v,u} - \sum_{m=1}^{m=u-1} g_{v,m} \cdot h_{m,u}$$

$$h_{u,v} = a_{u,v} - \sum_{m=1}^{m=u-1} g_{u,m} \cdot h_{m,v}$$

The matrix $[\bar{F}]$ is formed into an auxiliary matrix $[F']$ by the following relationship.

$$F'_u = \frac{F_u - \sum_{m=1}^{m=u-1} g_{u,m} \cdot F'_m}{g_{u,u}}$$

After computing these auxiliary matrices, the response functions H_u are found by a process of back substitution.

$$H_u = F'_u - \sum_{m=u+1}^{m=n} h_{u,m} \cdot F'_m$$

2.4.ii Pole zero method

The equation of motion of a general linear system may be expressed as:

$$\alpha_m \cdot \frac{d^m x}{dt^m} + \alpha_{(m-1)} \frac{d^{(m-1)} x}{dt^{(m-1)}} + \dots \alpha_0 \cdot x = f \cdot e^{p \cdot t}$$

If a complex frequency form is assumed for x we have; $x = X \cdot e^{p \cdot t}$

Let the transfer function of the system be $T(p)$.

$$\text{then } T(p) = \frac{X(p)}{f(p)} = \frac{1}{\alpha_m \cdot p^m + \alpha_{(m-1)} \cdot p^{(m-1)} + \dots \alpha_0} \quad \dots (148)$$

The coefficients of the above polynomial can be factorised into the form:

$$T(p) = \frac{1}{\alpha_m (p-p_1)(p-p_2) \dots} \quad \dots (149)$$

The specific values of $p_1, p_2, \dots$ are the roots or eigenvalues of the system.

If the general parameter p moves only along the frequency

axis i.e. $p = j.w$:

$$T(p) = T(j.w) = \frac{1}{\alpha_m(j.w - p_1)(j.w - p_2)....} \quad \dots\dots (150)$$

$$T(j.w) = H(w)$$

where $H(w)$ is the previously defined frequency response function.

Equation (150) shows that the frequency response function may be readily calculated if the eigenvalues are known. The eigenvalues of a coupled oscillator network can be found as follows.

Let the equations of motion be:

$$\underline{M}.\ddot{x} + \underline{C}.\dot{x} + \underline{K}.x = \underline{f}.e^{j.w.t} \quad \dots\dots\dots (151)$$

where $\underline{M}$ is an n by n mass matrix.

$\underline{C}$ is an n by n damping coefficient matrix.

$\underline{K}$ is an n by n stiffness matrix.

x is an n by 1 displacement vector.

$\underline{f}$ is an n by 1 force vector.

w is the angular frequency.

t is time.

n is the number of oscillators.

Let a state vector y be defined as:

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \quad \dots\dots\dots (152)$$

The following partitioned set of matrix equations may thus

be formed.

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\underline{M}^{-1} \cdot \underline{K} & -\underline{M}^{-1} \cdot \underline{C} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \underline{f} \end{bmatrix} e^{j \cdot \omega \cdot t} \quad \dots (153)$$

which may be expressed as:

$$\begin{bmatrix} \dot{Y} \end{bmatrix} = \begin{bmatrix} \underline{A} \end{bmatrix} \begin{bmatrix} Y \end{bmatrix} + \begin{bmatrix} \underline{E} \end{bmatrix} e^{j \cdot \omega \cdot t} \quad \dots (154)$$

Let $m = 2 \cdot n$

$\begin{bmatrix} \underline{A} \end{bmatrix}$ is an m by m matrix of the first order equivalent system.

$\begin{bmatrix} \underline{E} \end{bmatrix}$ is an m by 1 force vector.

$\begin{bmatrix} Y \end{bmatrix}$ is an m by 1 state vector.

Let the eigenvalues of matrix $\begin{bmatrix} \underline{A} \end{bmatrix}$ be λ_i , which are found from the solution of the equation:

$$\det.(\underline{A} - \lambda_i \cdot \underline{I}) = 0 \quad \dots (155)$$

where $\underline{I}$ is a unit matrix.

The corresponding eigenvectors are found by solving the equation:

$$(\underline{A} - \lambda_i \cdot \underline{I}) Y_i = 0 \quad \dots (156)$$

for each value of λ_i .

Let $\lambda_i = r_i + j \cdot \omega_i$ and $Y_i = s_i + j \cdot q_i$

Substituting these expressions in (156) gives:

$$\underline{A} (s_i + j \cdot q_i) = (r_i + j \cdot \omega_i) \underline{I} (s_i + j \cdot q_i) \quad \dots (157)$$

Equating real and imaginary parts of (157):

$$\left[\begin{array}{c|c} (\underline{A} - (r_i \cdot I)) & w_i \cdot I \\ \hline -w_i \cdot I & (\underline{A} - (r_i \cdot I)) \end{array} \right] \begin{bmatrix} s_i \\ q_i \end{bmatrix} = 0 \quad \dots (158)$$

The elements of an eigenvector bear only a fixed ratio to one another, and have no absolute values. Therefore the last element may be arbitrarily set to $1 + j.0$. There is now one less of the original equations to be solved, and the submatrices of (158) may be re-partitioned by inserting zeros in the m th row of $(\underline{A} - (r_i \cdot I))$.

Let the resulting matrices of order $(m-1)$ be denoted by $(\underline{A} - (r_i \cdot I))_{(m-1)}$, and the m th column of $(\underline{A} - (r_i \cdot I))$ by the vector $\underline{A}_m$. Similarly the m th row of $(w_i \cdot I)$ is replaced by zeros. The resulting partitioned matrix equation is:

$$\left[\begin{array}{c|c|c|c} (\underline{A} - (r_i \cdot I))_{(m-1)} & \underline{A}_m & w_i \cdot I_{(m-1)} & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline -w_i \cdot I_{(m-1)} & 0 & (\underline{A} - (r_i \cdot I))_{(m-1)} & \underline{A}_m \\ \hline 0 & 0 & 0 & 0 \end{array} \right] \begin{bmatrix} s_{i(m-1)} \\ 1 \\ q_{i(m-1)} \\ 0 \end{bmatrix} = 0 \quad \dots (159)$$

Equation (159) may be written more simply by letting:

$$\underline{B}_i = (\underline{A} - (r_i \cdot \underline{I}))_{(m-1)}$$

$$\underline{C}_i = \underline{A}_m$$

$$\underline{D}_i = w_i \cdot \underline{I}_{(m-1)}$$

$$\underline{E}_i = s_i_{(m-1)}$$

$$\underline{F}_i = q_i_{(m-1)}$$

thus

$$\begin{bmatrix} \underline{B}_i & \underline{C}_i & \underline{D}_i & 0 \\ 0 & 0 & 0 & 0 \\ -\underline{D}_i & 0 & \underline{B}_i & \underline{C}_i \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{E}_i \\ 1 \\ \underline{F}_i \\ 0 \end{bmatrix} = 0 \quad \dots\dots\dots(160)$$

The following two simultaneous matrix equations are obtained.

$$\underline{B}_i \cdot \underline{E}_i + \underline{C}_i + \underline{D}_i \cdot \underline{F}_i = 0 \quad \dots\dots\dots(161)$$

$$-\underline{D}_i \cdot \underline{E}_i + \underline{B}_i \cdot \underline{F}_i = 0 \quad \dots\dots\dots(162)$$

$$\text{therefore } \underline{E}_i = \underline{D}_i^{-1} \cdot \underline{B}_i \cdot \underline{F}_i \quad \dots\dots\dots(163)$$

$$\underline{F}_i = -(\underline{B}_i \cdot \underline{D}_i^{-1} \cdot \underline{B}_i + \underline{D}_i)^{-1} \cdot \underline{C}_i \quad \dots\dots\dots(164)$$

but $\underline{D}_i = w_i \cdot I$ a scalar matrix,

$$\text{therefore } \underline{E}_i = \frac{1}{w_i} \cdot \underline{B}_i \cdot \underline{F}_i \quad \dots\dots\dots (165)$$

$$\text{and } \underline{F}_i = (-\frac{1}{w_i} \cdot \underline{B}_i^2 + w_i \cdot I)^{-1} \cdot \underline{C}_i \quad \dots\dots\dots (166)$$

Hence the elements of the eigenvectors are found.

Let $[U]$ be a square matrix of the eigenvectors of the system.

$$\text{i.e. } [U] = [U_1 \ U_2 \ U_3 \ \dots \ U_m]$$

where $U_1, U_2, \dots$ etc. are the eigenvectors.

$$\text{Let } [V] = [U]^{-1} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_m \end{bmatrix}$$

$$\text{Let } y = U \cdot z$$

Substituting this expression into (154) we have:

$$U \cdot \dot{z} = \underline{A} \cdot U \cdot z + \underline{E} \cdot e^{j \cdot w \cdot t} \quad \dots\dots\dots (167)$$

$$\dot{z} = U^{-1} \cdot \underline{A} \cdot U \cdot z + U^{-1} \cdot \underline{E} \cdot e^{j \cdot w \cdot t}$$

$$\dot{z} = V \cdot \underline{A} \cdot U \cdot z + V \cdot \underline{E} \cdot e^{j \cdot w \cdot t} \quad \dots\dots\dots (168)$$

$$\text{Let } P = V \cdot \underline{A} \cdot U$$

P is an m by m diagonal matrix the elements of which are the

system eigenvalues;

$$P = \begin{bmatrix} P_1 & 0 & \dots & 0 \\ 0 & P_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & & & P_m \end{bmatrix}$$

Substituting P in (168) and letting $z = Z.e^{j.w.t}$ gives.

$$j.w.Z = P.Z + V.E \quad \dots\dots\dots (169)$$

$$\text{hence } (j.w.I - P)Z = V.E$$

$$\text{therefore } Z = (j.w.I - P)^{-1}.V.E \quad \dots\dots\dots (170)$$

$$\text{but } Z = U^{-1}.Y = V.Y$$

$$\text{therefore } V.Y = (j.w.I - P)^{-1}.V.E \quad \dots\dots\dots (171)$$

$$\text{hence } Y = U(j.w.I - P)^{-1}.V.E \quad \dots\dots\dots (172)$$

Equation (172) yields the steady state response vector in terms of the modal properties of the system.

Since $(j.w.I - P)$ is a diagonal matrix, its inverse is found by inverting each element.

$$\text{i.e. } (j.w.I - P)^{-1} = \begin{bmatrix} \frac{1}{(j.w - P_1)} & 0 & \dots \\ 0 & \frac{1}{(j.w - P_2)} & \\ \vdots & & \ddots \end{bmatrix}$$

$$\text{therefore } Y = U \begin{bmatrix} \frac{1}{(j\omega - P_1)} \cdot V_1 \cdot \underline{E} \\ \frac{1}{(j\omega - P_2)} \cdot V_2 \cdot \underline{E} \\ \vdots \\ \frac{1}{(j\omega - P_m)} \cdot V_m \cdot \underline{E} \end{bmatrix} \quad \dots\dots\dots (173)$$

In equation (173) the scalars $V_i \cdot \underline{E}$ are the components of the force vector which excites the i th mode of the system. The scalar $\frac{1}{(j\omega - P_i)} \cdot V_i \cdot \underline{E}$ represents the response of the i th mode at the frequency ω . The vector Z represents the modal frequency response of the system at the frequency ω .

2.5 CONCLUDING REMARKS

The modes of vibration of a continuous structure can be represented by lumped parameter modal oscillators. When structures are connected, coupled sets of oscillators are formed. Coupling is frequency dependent and is strongest between oscillators of the same resonant frequency. If a system consists of excited, intermediate and receiving structures the response of a receiving oscillator at any particular exciting frequency, can be calculated approximately by summing its response from each transmission path. The approximation becomes more accurate for weaker coupling, i.e. smaller coupling coefficients. However, when transmission paths

have common excited or intermediate oscillators, the receiver response from paths having oscillators with higher resonant frequencies is in anti-phase with the response from paths with lower resonant frequencies.

If the excitation is a source of random vibration, the mean square response is found by integrating the square of the modulus of the frequency response function, over the frequency range of the source.

The matrix of the system may be set up with its frequency dependent terms evaluated i.e. $M \cdot \omega^2$, $j \cdot \omega \cdot C$, and solved directly to yield values of the frequency response functions. The system may alternatively be described by separate mass, stiffness and damping coefficient sub-matrices and solved to yield eigenvalues and eigenvectors. The response of each oscillator is then found by summing each eigenvector at each frequency. The former method has the advantage that it requires less computer storage space and less compiling due to the simplicity of the numerical formulae involved. The latter uses fewer operations to compute oscillator responses once the eigenvalues and eigenvectors have been found.

CHAPTER THREE

PARAMETER SURVEY

INTRODUCTION

The analysis of chapter two shows how the performance of complex multimodal coupled structures may be investigated, using the concept of transmission paths. A transmission path comprises one oscillator from each structure, and the total response of an oscillator is the sum of its responses due to each path which contains it.

It is common in practise for vibration isolation systems to comprise three structures (excited, intermediate and receiving), and the dependence of transmission path energy flow on the parameters of its three constituent oscillators is investigated in this chapter.

It was also shown in the previous chapter that the response of an oscillator is a function of four parameters (i) mass, (M_u), (ii) resonant frequency (ω_u), (iii) loss factor (η_u) defined as $C_u/\omega_u \cdot M_u$, where C_u is damping coefficient, (iv) coupling factor ($b_{u,v}$), which for conservative coupling is defined as the ratio of coupling to total oscillator stiffness divided by loss factor, and for dissipative coupling is the ratio of coupling to total oscillator damping coefficients; subscript 'v' refers to the oscillator which is coupled to oscillator 'u'.

3.1 SURVEYS

The following surveys were obtained using program 'MAG'.

In all figures the ordinate is excited to receiving oscillator energy ratio, the parameters investigated are presented as follows:

- Fig. 3.01 Coupling factor at resonance; for both conservative and dissipative coupling.
- Fig. 3.02 Intermediate to excited oscillator resonant frequency ratio; for three mass ratios.
- Fig. 3.03 Intermediate to excited resonant frequency ratio; for two values of intermediate loss factor.
- Fig. 3.04 Intermediate loss factor; for two values of intermediate to excited mass ratio.
- Fig. 3.05 Intermediate loss factor; showing comparison of intermediate mass changes with intermediate-receiving coupling factor changes.
- Fig. 3.06 The ratio of excited-intermediate to intermediate-receiving coupling factor; for two values of intermediate mass.
- Fig. 3.07 Receiving to excited resonant frequency ratio.
- Fig. 3.08 Intermediate to excited resonant frequency ratio for a four oscillator system comprising two intermediate oscillators; for two values of intermediate loss factor.
- Fig. 3.09 Coupling factor at resonance; for two identical oscillators.

FIG 3.01 ENERGY RATIO v COUPLING FACTOR AT RESONANCE: (THREE OSCILLATOR SYSTEM), EXCITED AND RECEIVING OSCILLATOR COUPLING FACTORS EQUAL.

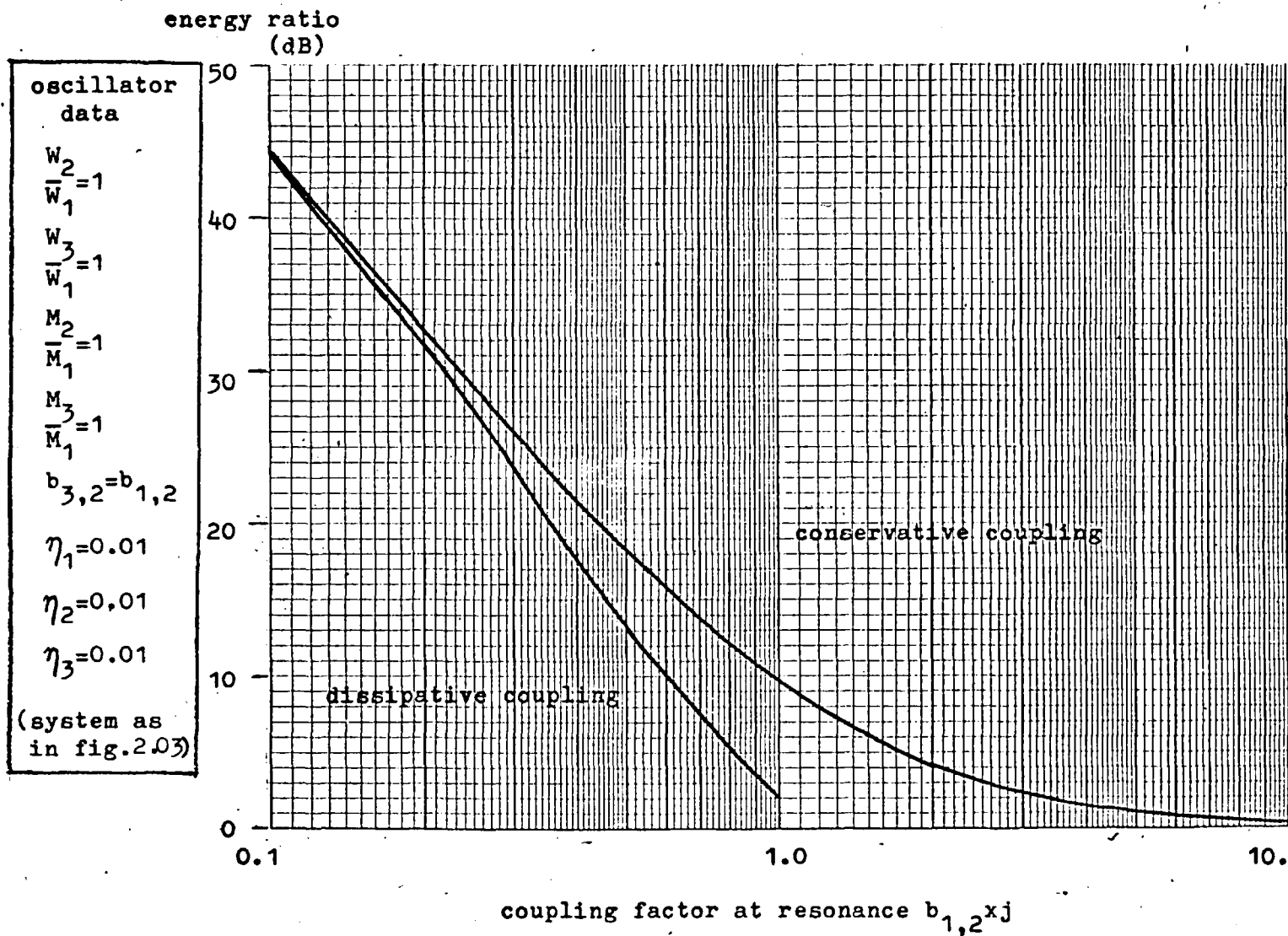


FIG 3.02 ENERGY RATIO v INTERMEDIATE TO EXCITED OSCILLATOR
 RESONANT FREQUENCY RATIO: (THREE OSCILLATOR SYSTEM)

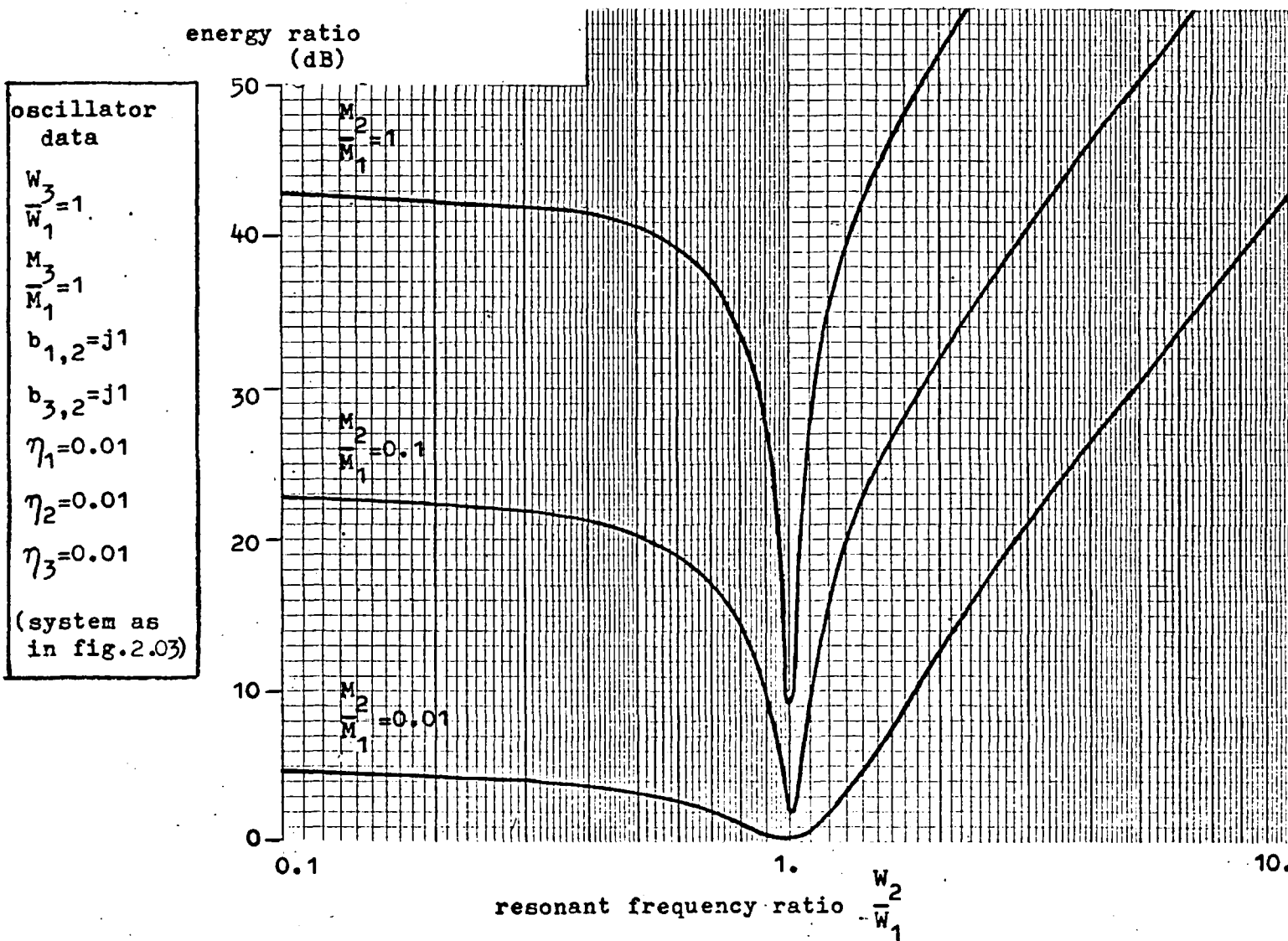


FIG 3.03 ENERGY RATIO v INTERMEDIATE TO EXCITED OSCILLATOR
 RESONANT FREQUENCY RATIO: (THREE OSCILLATOR SYSTEM)

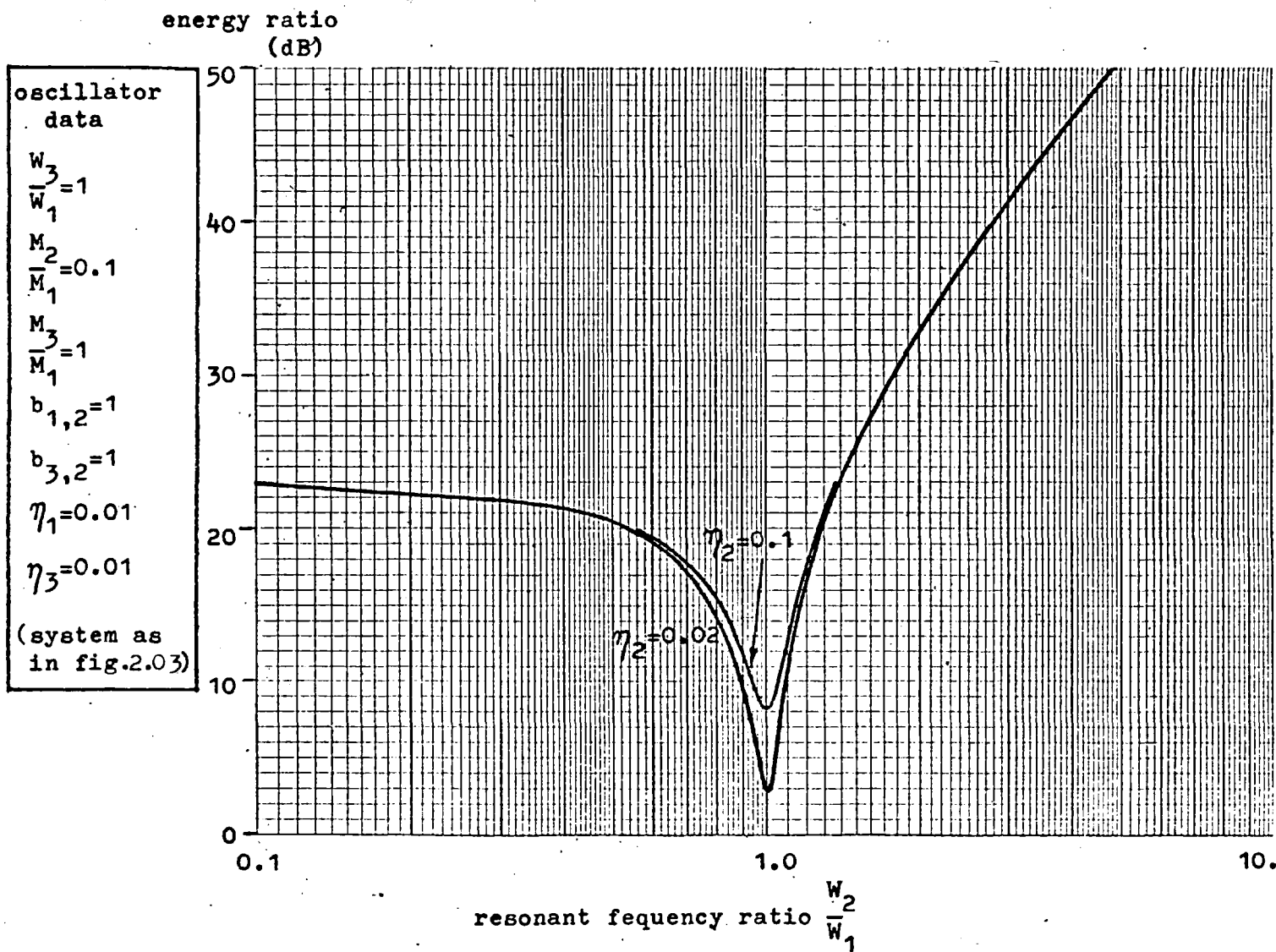


FIG 3.04 ENERGY RATIO & INTERMEDIATE OSCILLATOR LOSS FACTOR:
(THREE OSCILLATOR SYSTEM).

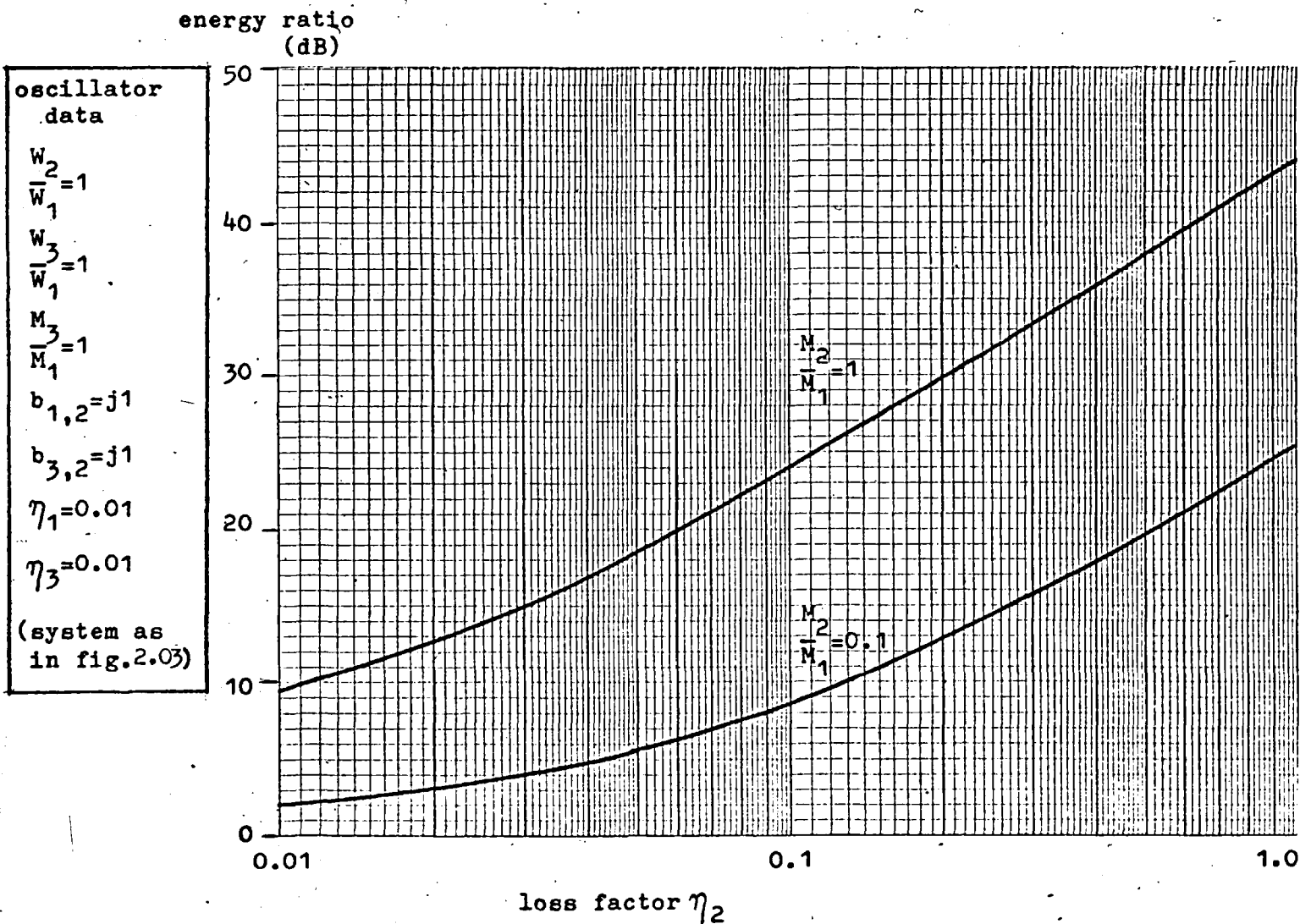


FIG 3.05 ENERGY RATIO v INTERMEDIATE OSCILLATOR LOSS FACTOR:
(THREE OSCILLATOR SYSTEM).

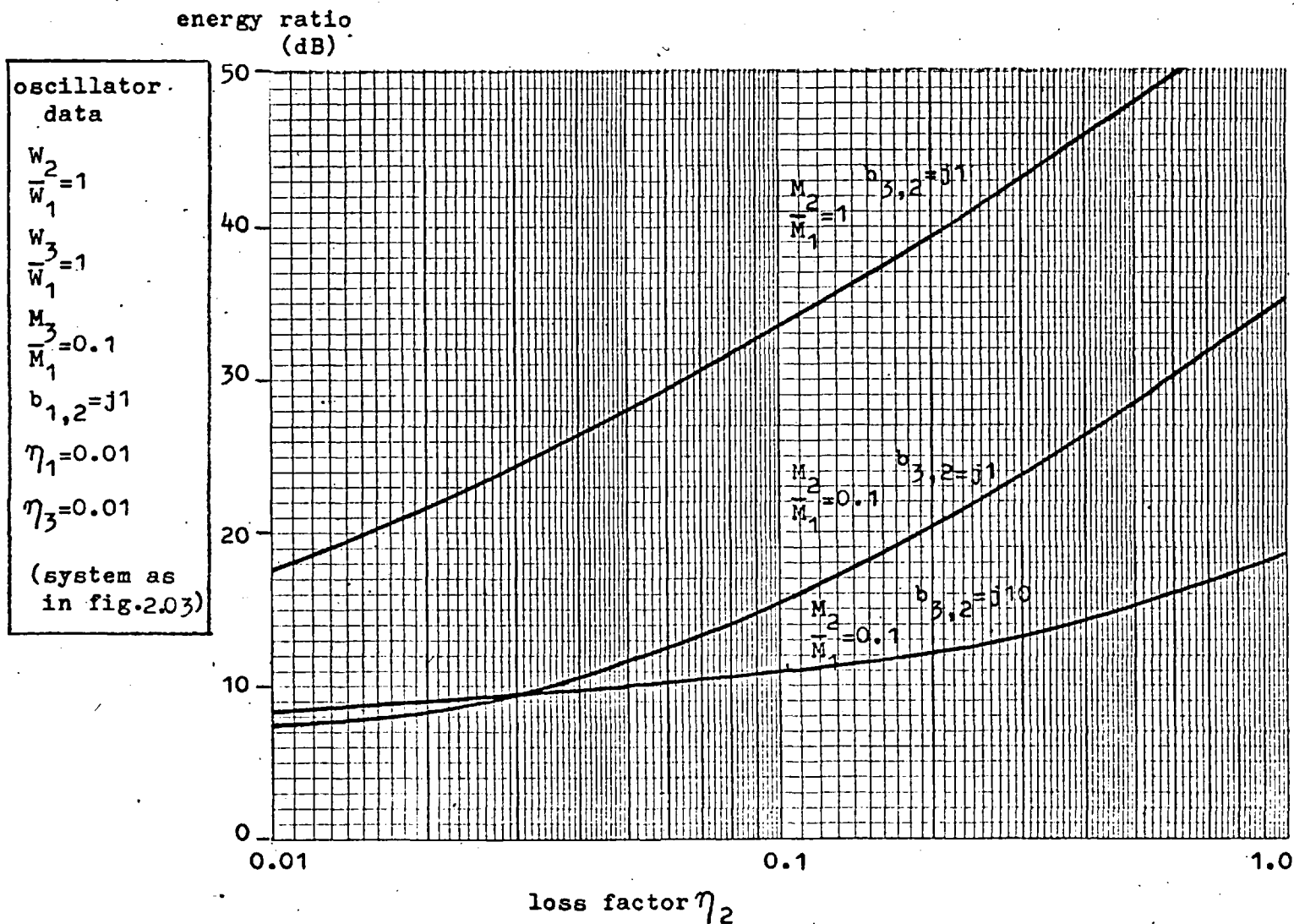


FIG 3.06 ENERGY RATIO ν EXCITED TO RECEIVING OSCILLATOR COUPLING FACTOR RATIO:(THREE OSCILLATOR SYSTEM).

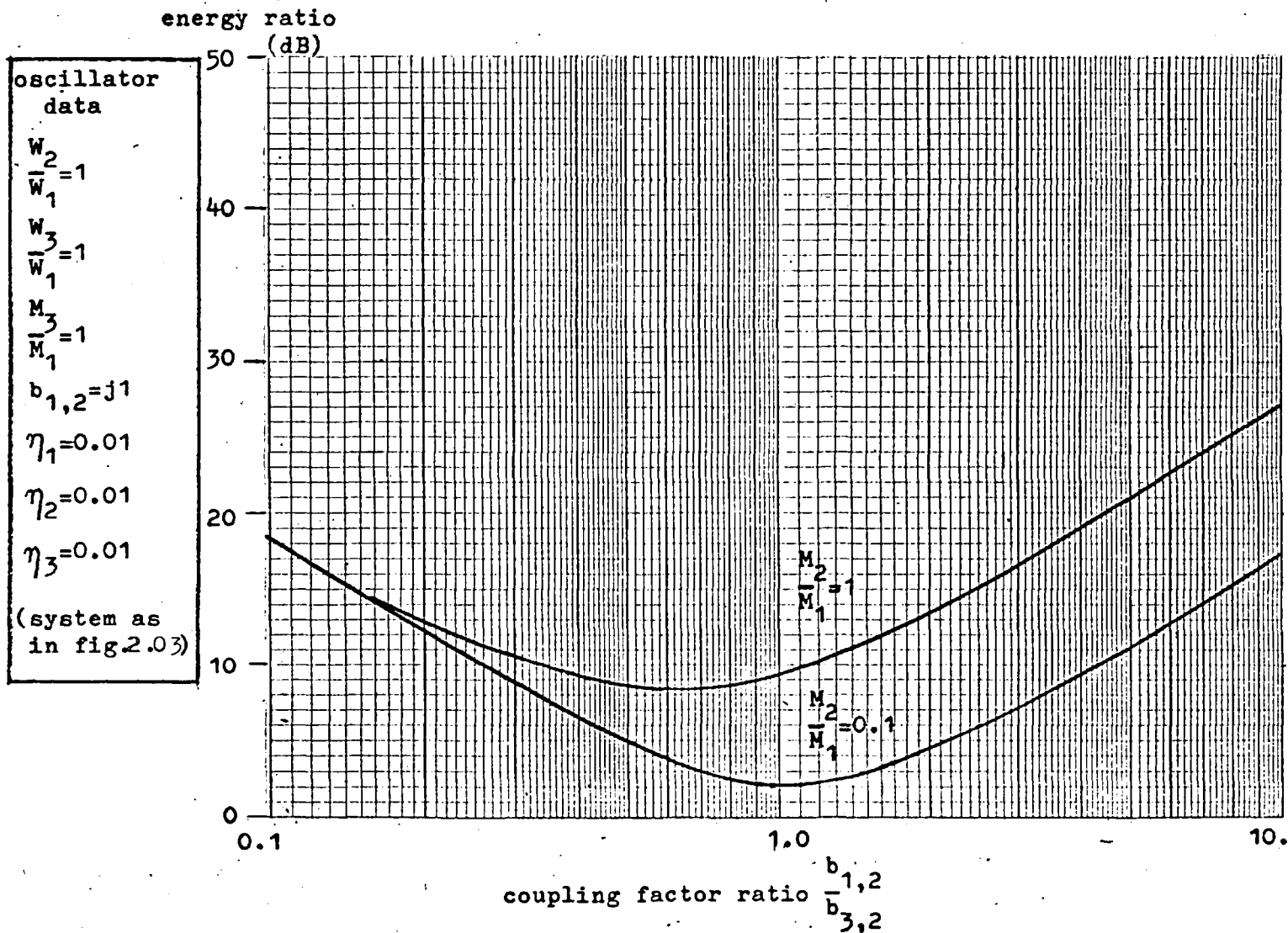


FIG 3.07 ENERGY RATIO v RECEIVING TO EXCITED OSCILLATOR RESONANT
FREQUENCY RATIO: (THREE OSCILLATOR SYSTEM)

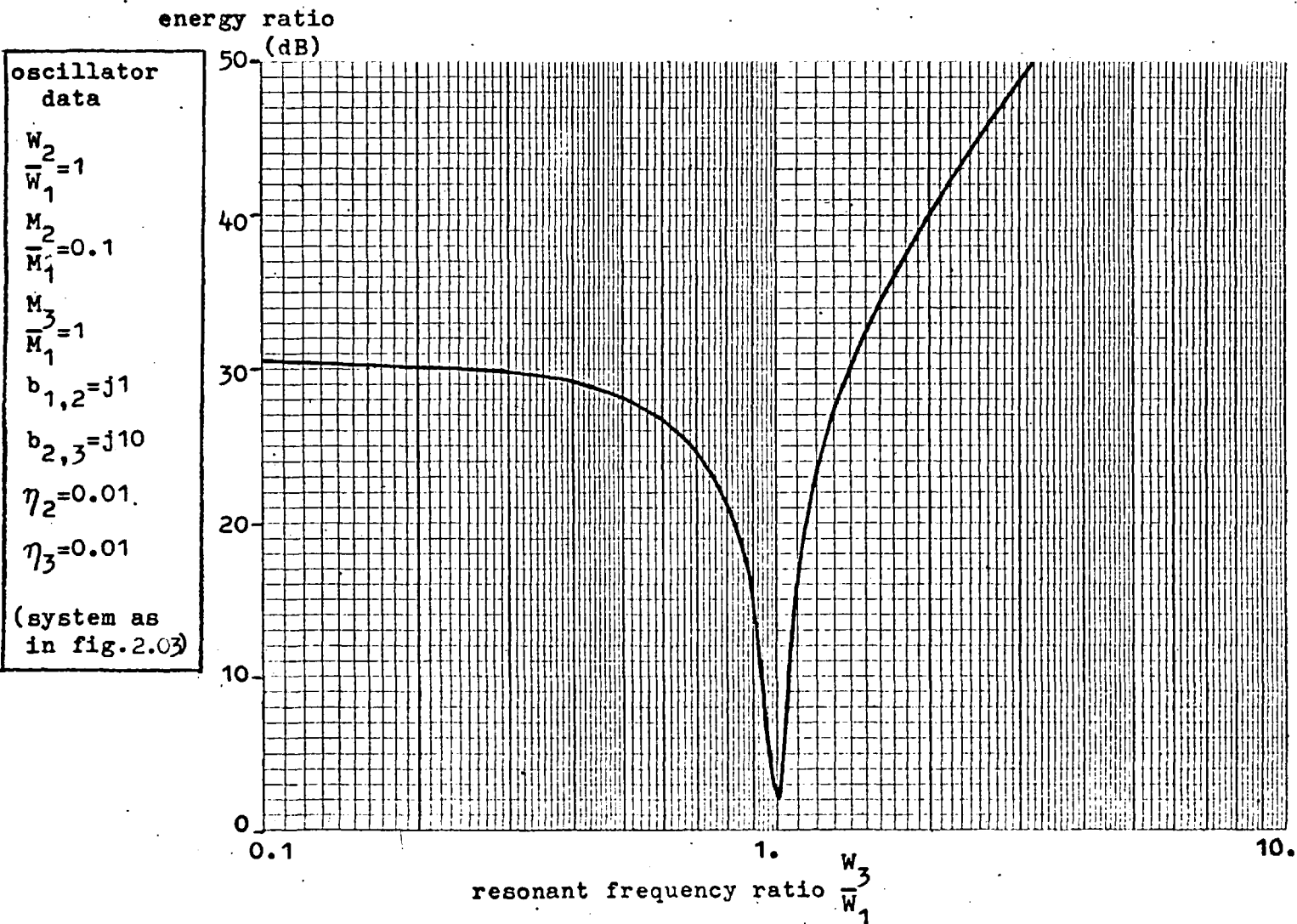
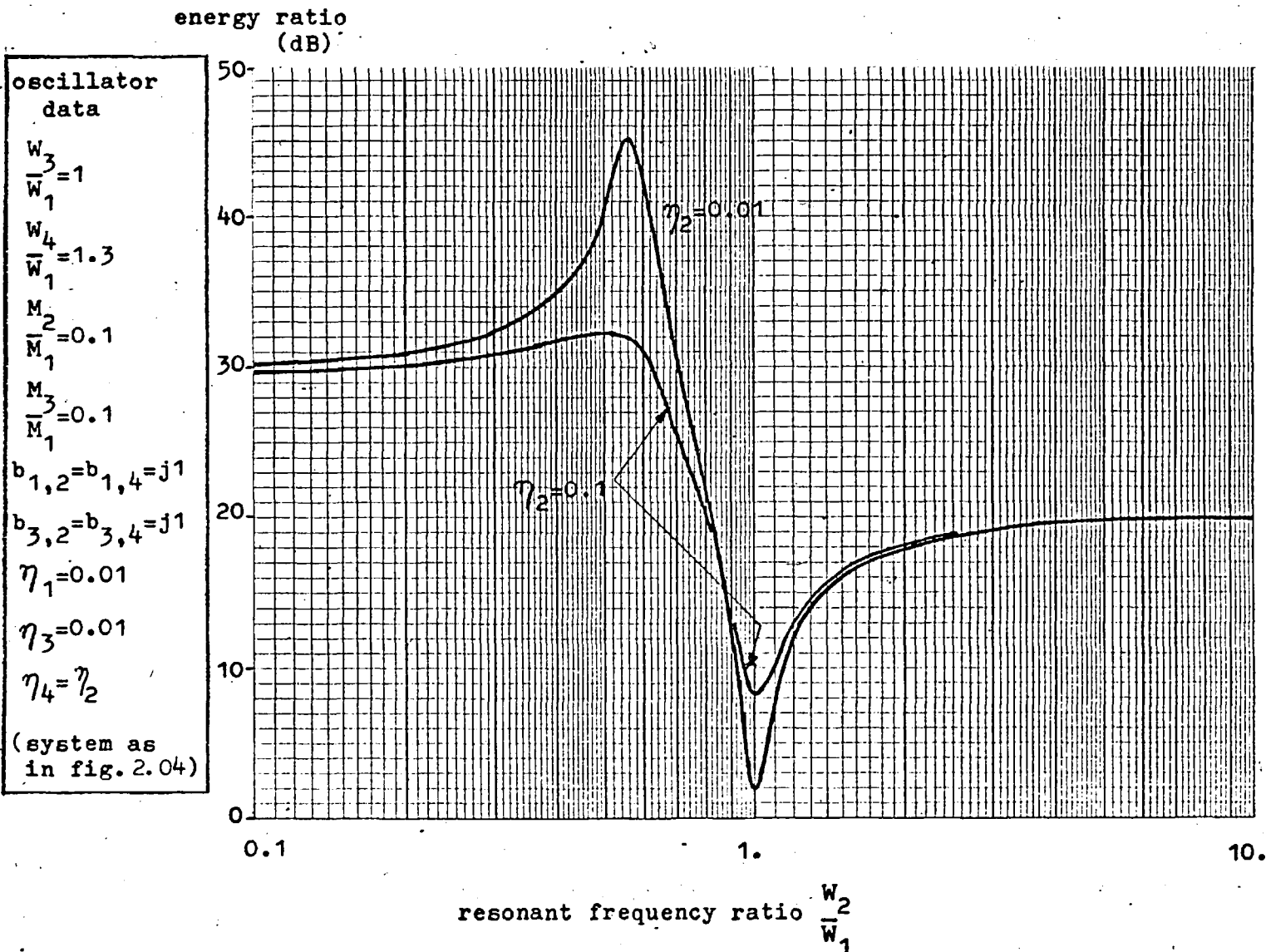


FIG 3.08 ENERGY RATIO v INTERMEDIATE TO EXCITED OSCILLATOR
 RESONANT FREQUENCY RATIO: (FOUR OSCILLATOR SYSTEM)



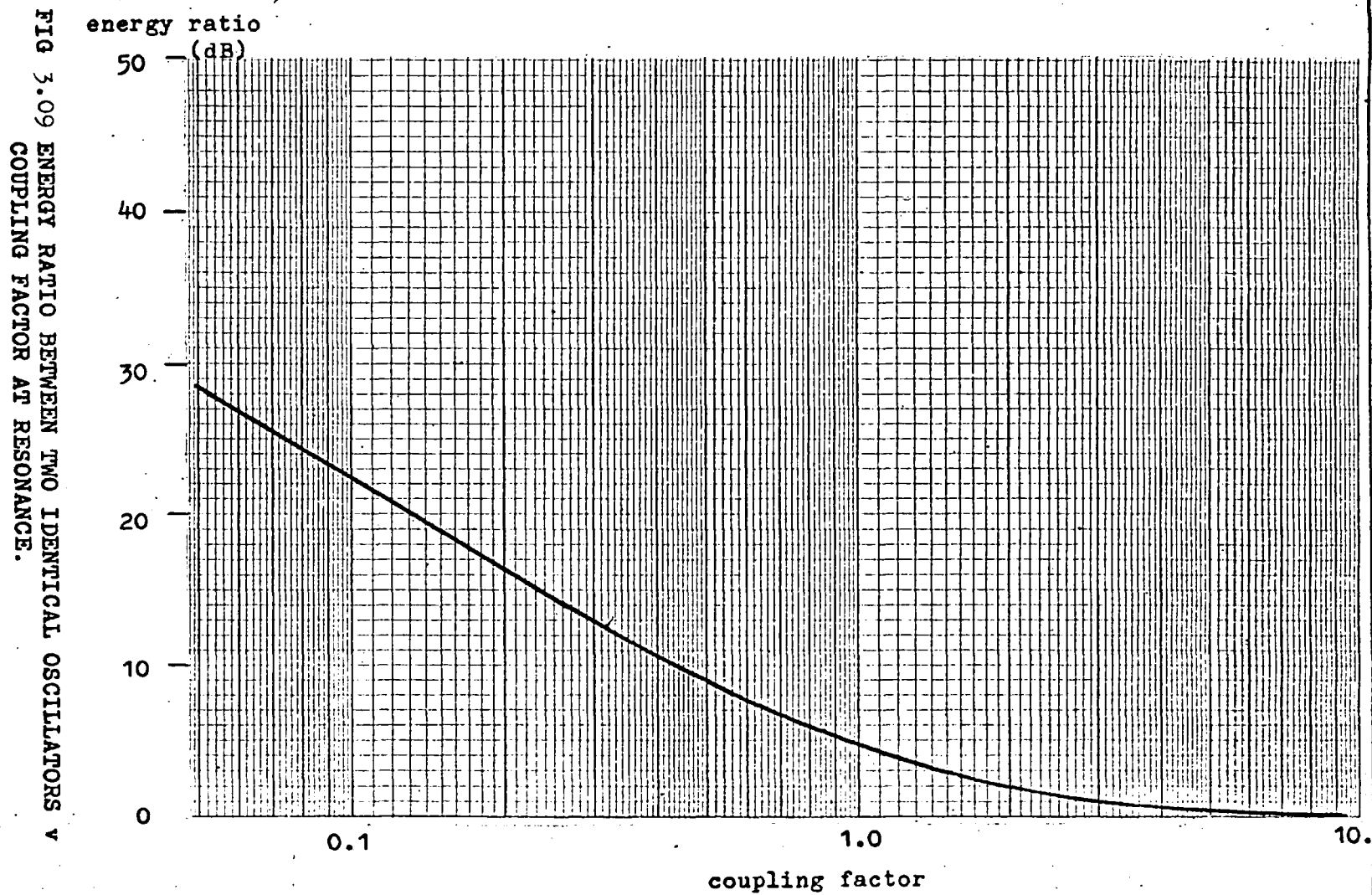


FIG 3.09 ENERGY RATIO BETWEEN TWO IDENTICAL OSCILLATORS v
COUPLING FACTOR AT RESONANCE.

3. 2 DISCUSSION

In the majority of vibration isolation problems the response or energy ratio between excited and receiving oscillators will be of primary interest, whilst improvements in this ratio will usually be most conveniently obtained in practice by modifying the intermediate oscillators. A three oscillator system, representing a typical transmission path contains eleven parameters; three pertaining to each oscillator and two coupling parameters. It is useful to take as a basis for discussion the case of identical oscillators having conservative coupling, with both coupling factors (at resonance) equal to unity. No matter what actual values are ascribed to these parameters, provided their ratios remain unchanged, the energy ratio is 9.4 dB ($\pm 1/2$ dB), see fig. 3.01. In the following discussion, unless otherwise stated, energy ratio implies that between excited and receiving oscillators, 'coupling factor' implies coupling factor at resonance. The energy ratio is greatly increased by detuning the intermediate oscillator (see fig. 3.02). With otherwise identical oscillators an increase of intermediate to excited oscillator resonance frequency ratio to 1.3 or reduction to 0.5 brings about an increase in energy ratio of 32 dB. Above and below these limits respectively the energy ratio is proportional to the square of the intermediate oscillator mass. This proportionality however ceases to apply between the above frequency limits or when the intermediate mass is small and permits low energy ratios.

Changes in intermediate oscillator loss factor have only a negligible effect on energy ratio when its resonant frequency is outside the limits of 1.3 and 0.5 (see fig. 3.03). The effects of mass and loss factor of the intermediate oscillator are directly interchangeable when all oscillators have the same resonant frequency. Compare figs. 3.02 and 3.04. Energy ratio is proportional to the square of the intermediate loss factor, but only when the energy ratio exceeds 20 dB, which occurs when the loss factor exceeds 0.06 (see fig. 3.04). For smaller loss factors greater changes are required for a given change in energy ratio. A six fold increase from 0.01 to 0.06 is necessary to increase energy ratio from 10 dB to 20 dB, but an increase by a factor of only three and one third from 0.06 to 0.2 increases energy ratio from 20 dB to 30 dB. The surveys also show that the response of a receiving oscillator is proportional to its coupling factor, whilst its energy level for a given coupling factor is proportional to its mass (see fig. 3.05).

Energy ratio is proportional to the fourth power of the excited or receiver coupling factors when both are equal and, with three identical oscillators, less than 0.4 (see fig. 3.01); this produces an energy ratio of 21.5 dB. For higher coupling factors the energy ratio approaches the asymptote of 0 dB, and correspondingly larger changes are required for a specified change in energy ratio.

A divergence between the effects of conservative and dissipative coupling becomes apparent for coupling factors exceeding 0.1. This is because a large proportion of the intermediate oscillator

damping is contained in the coupling parameter at the expense of the 'grounded' damping. With coupling factors above 0.5 the loss factor of the intermediate oscillator is therefore caused to increase because the combined coupling damping exceeds the original total oscillator damping. However, as the maximum dissipative coupling factor can not exceed unity the minimum energy ratio is shown to be limited to 2 dB.

The effects of changes in the receiver coupling factor are shown in fig. 3.06, when the excited oscillator coupling factor was held constant at unity. An increase or decrease in receiver coupling factor from this value caused an increase in energy ratio. This is explained by an analysis of the equation developed in chapter two expressing the response ratio as a function of coupling factors.

From equation (39) chapter two, $\frac{H_1}{H_3} = \frac{1 - \gamma_{2,3}}{b_{2,1} \cdot b_{3,2}} = \frac{1}{b_{2,1}} \left(\frac{1}{b_{3,2}} - b_{2,3} \right)$

$$b_{2,1} = b_{1,2} \cdot \frac{W_1^2 \cdot M_1 \cdot \eta_1}{W_2^2 \cdot M_2 \cdot \eta_2}, \quad b_{2,3} = b_{3,2} \cdot \frac{W_3^2 \cdot M_3 \cdot \eta_3}{W_2^2 \cdot M_2 \cdot \eta_2}$$

therefore for equal resonant frequencies,

$$\frac{H_1}{H_3} = \frac{M_2 \cdot \eta_2}{M_1 \cdot \eta_1} \cdot \frac{1}{b_{1,2}} \left(\frac{1}{b_{3,2}} - b_{3,2} \cdot \frac{M_2 \cdot \eta_2}{M_3 \cdot \eta_3} \right)$$

Clearly the response ratio is inversely proportional to the excited oscillator coupling factor (b_{12}), but the effect of receiver coupling factors either greater or less than unity produces the same increase in energy ratio.

The effect of receiving oscillator resonant frequency changes is shown in fig. 3.07. With two of the three system resonant frequencies identical, a greater energy ratio is obtained by changing the receiver than the intermediate resonant frequency. This is because in the former case the excited oscillator has a smaller response in the region of its own resonant frequency, due to the higher coupling between it and the intermediate oscillator.

It is seen from fig. 3.08 that when an excited oscillator is connected to a receiver by an intermediate oscillator of dissimilar resonant frequency, it is possible by suitable tuning to connect a second intermediate oscillator, between excited and receiver, which nullifies the response of the latter. It was shown in chapter two that the responses of individual transmission paths may be summed, and this nulling effect occurs when the receiver responses from each path are essentially equal in magnitude and opposite in phase. Given the data of one intermediate oscillator, the required resonant frequency of the other may be found from a plot of energy ratio against intermediate oscillator resonant frequency. A line of constant energy ratio will intersect the plot at the two values of resonant frequency (see fig. 3.02).

From figs. 3.01 and 3.09 it is seen that the insertion of an intermediate oscillator between two previously coupled oscillators may cause either an increase or decrease in their energy ratios. If the inserted oscillator is identical to the others and the magnitude of the original coupling factor was 0.5, the net effect on the

energy ratio is zero. It will be noted that the resulting three oscillator system has coupling factors of magnitude unity.

An approximate formula, expressing in terms of coupling factors the effect of inserting an intermediate oscillator, is derived as follows for the case of all oscillators having the same resonant frequency. It is assumed that the integrals in the frequency domain of similar response functions, are proportional to their respective ordinates.

The response ratio of a two oscillator system is:

$$\frac{H_1}{H_2} = \frac{1}{\bar{b}_{2,1}}$$

and of a three oscillator system:

$$\frac{H_1}{H_3} = \frac{1 - b_{2,3} \cdot b_{3,2}}{b_{2,1} \cdot b_{3,2}}$$

an increase in response ratio occurs if

$$\frac{1 - b_{2,3} \cdot b_{3,2}}{b_{2,1} \cdot b_{3,2}} \cdot \bar{b}_{2,1} > 1$$

or

$$\frac{1 - b_{3,2}^2 \cdot \frac{M_3}{M_2} \cdot \frac{\eta_3}{\eta_2}}{b_{1,2} \cdot b_{3,2} \cdot \frac{M_1}{M_2} \cdot \frac{\eta_1}{\eta_2}} \cdot \bar{b}_{2,1} > 1$$

The effect of reduced response ratio caused by the insertion of an oscillator may be palliated by making the excited and receiving oscillator coupling factors unequal, the former smaller than the latter.

let $\frac{b_{12}}{b_{32}} = Cr$ be less than unity,

$$\frac{1 - b_{3,2}^2 \cdot \frac{M_3}{M_2} \cdot \frac{\eta_3}{\eta_2}}{Cr \cdot b_{3,2}^2 \cdot \frac{M_1}{M_2} \cdot \frac{\eta_1}{\eta_2}} \cdot \bar{b}_{2,1} > 1$$

3.3 CONCLUDING REMARKS

From the parameter surveys as presented the performance of a wide range of similar vibration transmission systems may be predicted.

For optimum vibration isolation the intermediate oscillator mass should be as large as possible, and its resonant frequency not close to excited or receiving oscillator. Its loss factor should be as high as possible if resonant frequencies are close. Systems with coincident resonant frequencies should be avoided as it is then impossible for an extra intermediate oscillator to be inserted between excited and receiving oscillators to nullify the response of the latter.

Least energy transmission occurs when coupling factors are

as low as possible. A critical value of unity exists for the receiving oscillator coupling factor, about which both smaller and larger values cause a reduction in energy transmission.

CHAPTER FOUR

ANALYSIS OF STRUCTURAL POINT AND TRANSFER RESPONSE MEASUREMENTS

INTRODUCTION

In chapter two it was shown that the modes of vibration of a structure could be represented by a set of lumped parameter modal oscillators, coupled to the point at which the structure was excited.

However, whilst modal masses of structure such as plates and beams may be calculated if their dimensions are known, there is no reliable method of calculating coupling parameters. Modal coupling depends on whether there is a node, antinode or some intermediate displacement at the excitation point. Nodal patterns may only be predicted if the plan form is described by a geometrical figure such as a rectangle, circle, triangle, etc., and the boundary conditions are known explicitly.

An experimental technique is therefore necessary for determining modal and coupling parameters when the structure is non-uniform, irregular, with boundaries which are not ideal.

4.1 ANALYSIS FOR CONSERVATIVE AND DISSIPATIVE COUPLING

From equation (18) of chapter two the point stiffness of a structure with conservatively coupled modes is:

$$K_p = \sum_{i=1}^n Kc_i - \sum_{i=1}^n \left[\frac{Kc_i^2}{W_i^2 \cdot M_i \left[\left(1 - \frac{w^2}{W_i^2}\right) + j \cdot \eta_i \cdot \frac{w}{W_i} \right]} \right] \quad \dots\dots (1)$$

and from equation (26) of chapter two, if the coupling is dissipative.

$$I_p = \sum_{i=1}^n Cc_i - \sum_{i=1}^n \left[\frac{Cc_i^2}{W_i \cdot M_i \left[j \left(\frac{w}{W_i} - \frac{W_i}{w} \right) + \eta_i \right]} \right] \quad \dots\dots (2)$$

If a structure is excited by a point sinusoidal force whose frequency is swept slowly, it is seen by the above equations that the point stiffness or impedance will undergo perturbations about the constant value $\sum_{i=1}^n Kc_i$, or $\sum_{i=1}^n Cc_i$. The perturbations will occur as the exciting frequency passes through each oscillator resonance.

i.e. from equation (1), at a frequency $w = W_i$, the term

$$\left[\frac{Kc_i^2}{W_i^2 \cdot M_i \left[\left(1 - \frac{w^2}{W_i^2}\right) + j \cdot \eta_i \cdot \frac{w}{W_i} \right]} \right]$$

becomes a maximum value of

$$W_i^2 \cdot M_i \cdot j \cdot \eta_i$$

or $\frac{Kc_1 \cdot R_1}{j \cdot \eta_1}$ where R_1 , as in chapter two, is the ratio of coupling to modal stiffness. However, this imaginary maximum value has only a small effect on the modulus of K_p because $\sum_{i=1}^n Kc_i$ is real, and considerably larger unless the loss factor η_1 is extremely small. Above and below resonance the term will contain a real but smaller component. The sign of this component will be positive below resonance and negative above, and therefore the perturbation of the point stiffness will be a minimum value just below resonance and a maximum just above. These maxima and minima occur when the real and imaginary parts of the denominator of

$$\frac{Kc_1 \cdot R_1}{\left(1 - \frac{w^2}{w_i^2}\right) + j \cdot \eta_1 \cdot \frac{w}{w_i}}$$

are numerically equal, i.e.

$$\begin{aligned} 1 - \frac{w^2}{w_i^2} &= \pm \eta_1 \frac{w}{w_i} \\ \text{therefore} \quad \frac{w}{w_i} &= \frac{\pm \eta_1 \pm \sqrt{\eta_1^2 + 4}}{2} \dots\dots\dots (3) \end{aligned}$$

if η_1^2 is assumed to be negligibly small

$$\text{therefore} \quad \frac{w}{w_i} = \frac{\pm \eta_1}{2} \pm 1 \dots\dots\dots (4)$$

w_i (max or min)

if the responses of all other oscillators are assumed to be negligibly small at these frequencies the maximum and minimum perturbations of K_p near the resonant frequency of the i th oscillator are:

$$K_p \begin{matrix} \text{(max or min)} \end{matrix} = \sum_{i=1}^n Kc_i - \frac{Kc_i \cdot R'_i}{\bar{\eta}_i + j \cdot \eta_i} \dots\dots\dots (5)$$

A plot of the point stiffness against frequency will yield the sum of all coupling stiffnesses as the mean value of all perturbations. From equation (4) the difference between $w_{(\max)}$ and $w_{(\min)}$ can be used to calculate the loss factor of the i th oscillator.

$$\eta_i = \frac{2(w_{(\max)} - w_{(\min)})}{(w_{(\max)} + w_{(\min)})} \dots\dots\dots (6)$$

However if the coupling is dissipative, the point impedance passes through a minimum value at each oscillator resonance frequency, and there are no maximum and minimum values either side. At the frequency ratios of, $\frac{w}{w_i} = 1 \pm \frac{\eta_i}{2}$, and making the assumption that other oscillator responses are small, the point impedance value is the arithmetic mean of the minimum value at resonance;

$$\begin{aligned} \text{i.e. } I_{p_{\left(\frac{w}{w_i}=1\right)}} &\doteq \sum_{i=1}^n Cc_i - Cc_i \cdot R'_i, \text{ and the constant value } \sum_{i=1}^n Cc_i \\ I_{p_{\left(\frac{w}{w_i}=1 \pm \frac{\eta_i}{2}\right)}} &\doteq \sum_{i=1}^n Cc_i - \frac{Cc_i \cdot R'_i}{\bar{\eta}_i + 1} = \sum_{i=1}^n Cc_i - \frac{Cc_i \cdot R'_i (\bar{\eta}_i + 1)}{2} \\ &\dots\dots\dots (7) \end{aligned}$$

It is seen that the real part of equation (7) is substantially greater than the imaginary part;

$$\text{therefore } I_{p_{\left(\frac{w}{w_i}=1 \pm \frac{\eta_i}{2}\right)}} \doteq \sum_{i=1}^n Cc_i - \frac{Cc_i \cdot R'_i}{2}$$

From equation (22) of chapter two the transfer stiffness is given by:

$$K_t = \frac{\sum_{i=1}^n Kc_i - \sum_{i=1}^n \frac{Kc_i \cdot R_i}{(1 - \frac{w^2}{w_i^2}) + j \cdot \eta_i \cdot \frac{w}{w_i}}}{\sum_{i=1}^n \frac{R_i}{(1 - \frac{w^2}{w_i^2}) + j \cdot \eta_i \cdot \frac{w}{w_i}}} \dots\dots (8)$$

and similarly the transfer impedance:

$$I_t = \frac{\sum_{i=1}^n Cc_i - \sum_{i=1}^n \frac{Cc_i \cdot R'_i \cdot \eta_i}{j(\frac{w}{w_i} - \frac{w_i}{w}) + \eta_i}}{\sum_{i=1}^n \frac{R'_i \cdot \eta_i}{j(\frac{w}{w_i} - \frac{w_i}{w}) + \eta_i}} \dots\dots (9)$$

Ignoring the responses of non-resonant oscillators, equation (8) can be expressed as:

$$K_{t_i} = \frac{\sum_{i=1}^n Kc_i - \frac{Kc_i \cdot R_i}{j \cdot \eta_i}}{\frac{R_i}{j \cdot \eta_i}} \dots\dots\dots (10)$$

where K_{t_i} is the transfer stiffness at the resonant frequency of the i th oscillator.

$$K_{t_i} = j \cdot \frac{\eta_i}{R_i} \cdot \sum_{i=1}^n Kc_i - Kc_i \dots\dots\dots (11)$$

It will be noted that $\frac{R_i}{\eta_i}$ is equal to the value of coupling factor at resonance as defined in chapter two equation (86).

If the coupling is dissipative, equation (9) becomes:

$$It_i = \frac{\sum_{i=1}^n Cc_i - Cc_i \cdot R'_i}{R'_i} \dots\dots\dots (12)$$

$$\text{therefore } It_i = \frac{\sum_{i=1}^n Cc_i}{R'_i} \dots\dots(13), \quad R'_i = \frac{\sum_{i=1}^n Cc_i}{It_i} \dots\dots (14)$$

R_i also equals the value of coupling factor at resonance.

Relative response levels of oscillators can be determined solely from their coupling factors. Absolute values of coupling and modal parameters are not required. However, some values have to be assigned to these quantities in order to perform computations by the methods described in chapter two.

To obtain the absolute values of the coupling parameters, the modal mass and resonant frequency of each mode must be known. At the resonant frequency of each mode of vibration the transfer stiffness or impedance plotted against frequency passes through a minimum value. Therefore the transfer response and resonant frequency of any oscillator are readily obtainable from such a plot.

If the modal masses and loss factors are known by calculation, Kc_i may be approximately evaluated from (11).

$$Kc_i = \frac{j \left[\sum_{i=1}^n Kc_i \right] \eta_i \cdot W_i^2 \cdot M_i}{Kt_i} \dots\dots\dots (15)$$

and from (13) the coupling damper coefficient is given by:

$$C_{c_i} = \frac{\left[\sum_{i=1}^n C_{c_i} \right] \eta_i \cdot W_i \cdot M_i}{I t_i} \dots \dots \dots (16)$$

However, it is probable that in many circumstances the modal masses and loss factors are not known.

4.2 ITERATION TECHNIQUE TO COMPUTE MODAL AND COUPLING PARAMETERS WHEN MODAL MASS AND LOSS FACTOR ARE UNKNOWN

From equations (15) and (16) it is seen that to compute values of coupling parameters only the product of modal mass and loss factor is required, apart from the quantities measured from the point and transfer plots. It may be assumed that all modal masses of a structure are equal, as the absolute values are not directly relevant to vibration transmission. If the loss factor of each mode is initially given some arbitrary value and equation (15) is summed for all modes then.

$$\sum_{i=1}^n K c_i = \sum_{i=1}^n \left\{ \frac{j \left[\sum_{i=1}^n K c_i \right] \eta_i \cdot \omega_i^2}{K t_i} \right\} M_i \dots \dots \dots (17)$$

$$M_i = \frac{1}{\sum_{i=1}^n \frac{\eta_i \cdot \omega_i^2}{K t_i}} \dots \dots \dots (18)$$

similarly by summing equation (16) for all modes:

$$M_i = \frac{1}{\sum_{i=1}^n \frac{\eta_i \cdot W_i}{I t_i}} \dots \dots \dots (19)$$

The response of an oscillator at frequencies close to resonance is greatly affected by its bandwidth, or loss factor. By substituting the modal and coupling parameters obtained by (15) and (18) back into equation (8), the transfer stiffnesses so calculated may be compared with the measured values. Any discrepancies are due to incorrect assumptions of loss factors. The effect of specifying a loss factor which is too high is to proportionally increase an oscillator's response at the resonant frequency of another, therefore the following iteration formula may be used to recalculate loss factors from an initially assumed value:

$$\eta'_i = \frac{\eta_i \cdot Kt_i(\text{calculated})}{Kt_i(\text{measured})}$$

where η'_i is the 'new' value of loss factor of the i th mode.

η_i is the original value of loss factor. Similarly with dissipative coupling.

$$\eta'_i = \frac{\eta_i \cdot It_i(\text{calculated})}{It_i(\text{measured})}$$

4. 3 EVALUATION OF MODE TO MODE COUPLING BETWEEN TWO CONNECTED STRUCTURES

The mode to mode coupling between two structures, connected at a point, may be found from a knowledge of the coupling parameters of each mode and the structure point responses.

Let the structures be denoted 'a' and 'b'.

Also let CP_a be the coupling parameter from the point of contact to a mode of structure 'a'.

PR_a be the point response of structure 'a'.

CP_b be the coupling parameter from the point of contact to a mode of structure 'b'.

PR_b be the point response of structure 'b'.

By equating forces and displacements at the point of contact, the equivalent coupling parameter from a mode of structure 'a' to a mode of structure 'b' is:

$$CP_{a,b} = \frac{CP_a \cdot CP_b}{PR_a + PR_b}$$

4.4 CONCLUDING REMARKS

In order to compute the relative responses of coupled oscillators only their coupling factors need be known. These may be obtained from measurements of point and transfer responses of the particular structures. Indeed, in chapter two it was shown that to calculate the quantitative response of oscillators in a coupled system of structure, only the absolute parameters of oscillators in the externally excited structure, or structures need be known. However, to make use of the considerable saving in computing operations by solving a symmetric matrix of system parameters, values of coupling and modal parameters of all oscillators are required. These may be found using a straightforward iteration procedure, although in conjunction with this an assumption is made that all modal masses of a structure are equal. This assumption is valid for both plates and beams, and in any case the computed

response levels of non-externally excited structures are independent of modal mass values.

A computer program designed to carry out the analysis of structural point and transfer responses is described in appendix two.

CHAPTER FIVE

EXPERIMENTAL VERIFICATION OF ANALYSIS

INTRODUCTION

To verify the theoretical analysis developed in chapters two and four, three series of experiments were done whose objectives were as follows: (i) to check the proper functioning of the computer program, (ii) to verify the theoretical procedure for determining coupling and modal parameters of a structure, (iii) to compare experimental and computed predictions of the effect of reducing the mass and area of contact of an intermediate structure.

The first series of experiments was done using an electric circuit analogue of a coupled oscillator system. An analogue was used because of the ease and speed with which parameter values could be changed, and also because oscillator response levels could be measured by taking direct voltage readings across capacitors, and passing the signal straight into analysing equipment, thus avoiding the need for transducers.

In the second series, the point and transfer responses of a large steel plate of dimensions 48in x 48in x 3/16in were measured. From these tests data was obtained to compute modal and coupling parameters using program 'SIGMA'. The plate was then used as a receiving structure to which vibration was transmitted by an excited structure consisting simply of an electro-magnetic vibrator. The

effect on the response of the plate of various intermediate structures of both reflective and dissipative characteristics was measured. A comparison was then made with computed predictions of the responses of vibrator and plate. The reflective intermediate structures consisted of standard rubber mounts, two sets were used which gave the vibrator a mounted resonant frequency of 4Hz and 21Hz respectively. A dissipative structure consisting of highly damped visco-elastic laminated beams was used in conjunction with each set of rubber mountings and separately.

For the third series of tests an experimental system of two coupled plates was set up. A large aluminium plate of dimensions 72in x 36in x 1/8in was cut across its centre parallel to the shorter side, thus forming two plates joined by a bridge. Initially a narrow cut was made, and the response ratios between plates were measured. A comparison was made between the response ratio of the plates and that of a simple two oscillator system. Previous work [31] had shown that reducing bridge width had little effect on the transmitted vibration. The present work investigated the effect of reducing both the size of an intermediate structure as well as its contact width. The intermediate resonant structure was introduced by widening the cut to form a beam-like bridge. Bridge widths were again reduced in stages and the response ratios between plates measured. The flexural resonant frequencies of the beam were independent of its width, and therefore the response ratio between plates as a function of contact width and mass was obtained.

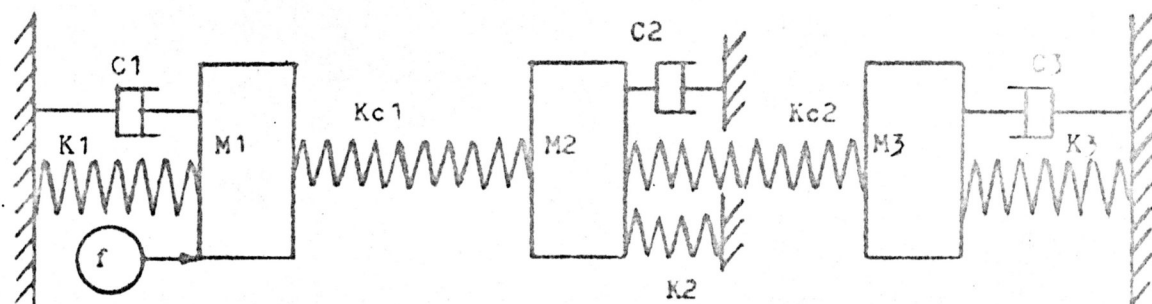
5.1 TESTS WITH ELECTRIC CIRCUIT ANALOGUE

5.1.i Details of apparatus

The analogue circuits were set up on an insulating board, made of perspex. The force-voltage analogy was used, where mass is proportional to inductance, stiffness proportional to the inverse of capacity, and damping proportional to resistance (see fig 5.01). Inductors being relatively large and heavy components and tag strips were screwed to the board, the capacitors and resistors were then soldered to the tag strips. A system of three series conservatively coupled oscillators was set up. The parameters were arbitrarily chosen, excited and receiving oscillator masses (1 and 3) were made identical, and the mass of the intermediate nominally one tenth of theirs. A block diagram of the apparatus is shown in fig 5.02.

5.1.ii Determination of oscillator parameters

In order to calibrate an electrical oscillator three parameters must be known, which are: inductance or capacitance, resonant frequency, Q factor. Q factor is defined as the magnification factor at resonance and is equal to the inverse of loss factor. The inductor values were measured by connecting each in turn in series with a standard capacitor, to form a series resonant circuit. By measuring the resonant frequency of this circuit, (see fig 5.03), and the voltage across the capacitor, the magnification factor at resonance was found. The inductance was then found from the following equation:



mechanical parameter

M

C

X

equivalent electrical
parameter

L

R

$1/C$

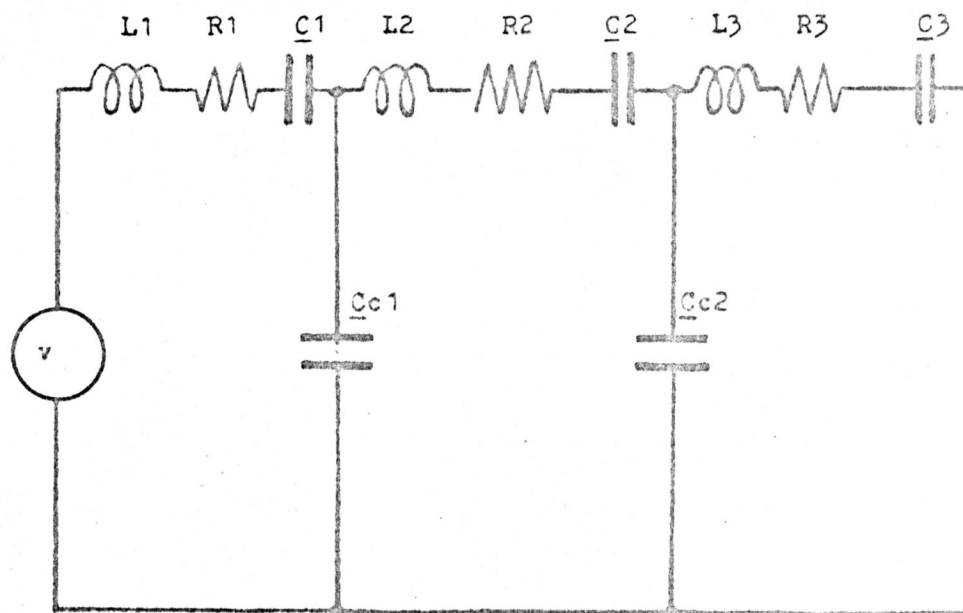


FIG 5.01 THREE OSCILLATOR MECHANICAL SYSTEM AND ELECTRICAL ANALOGUE

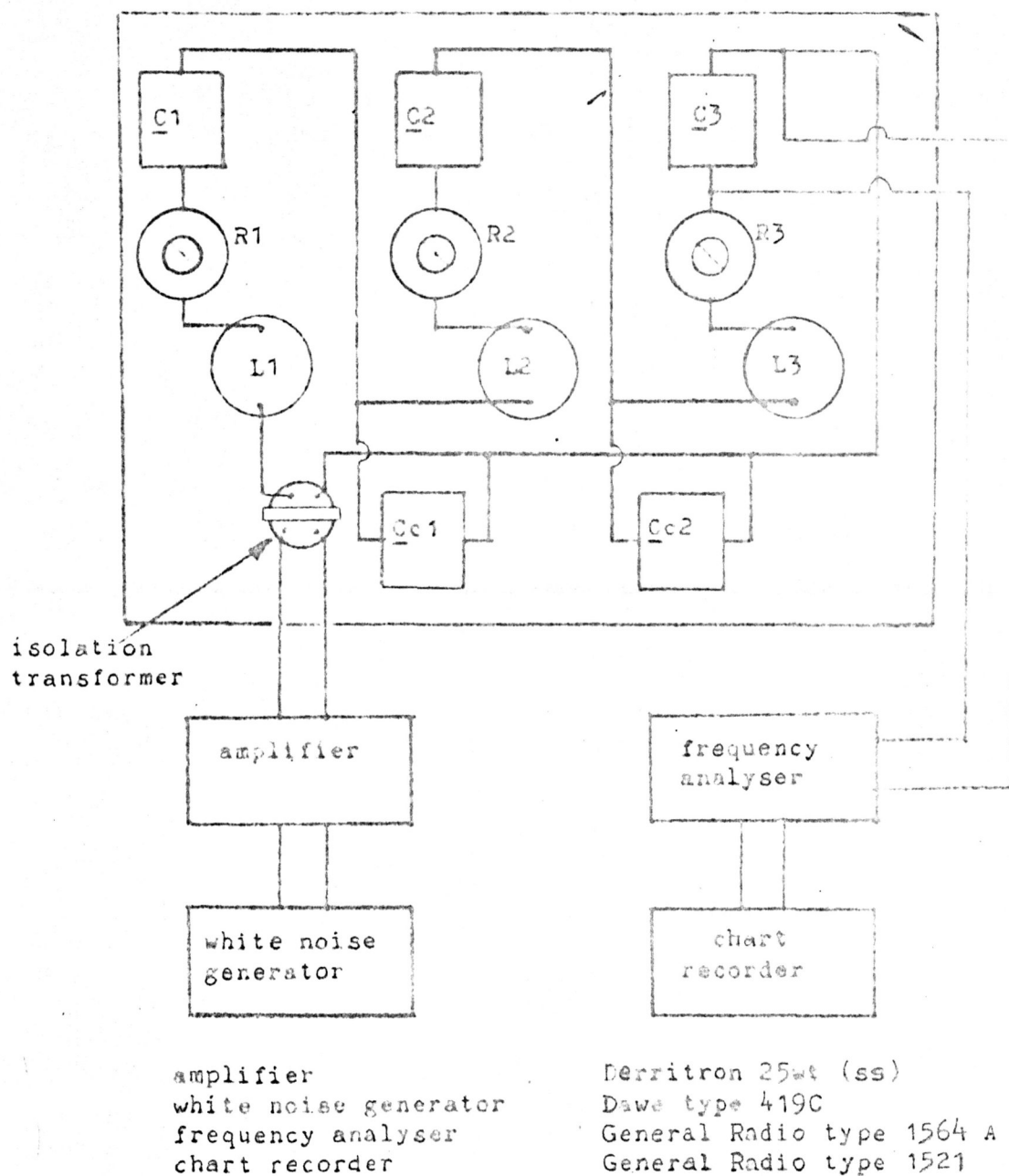
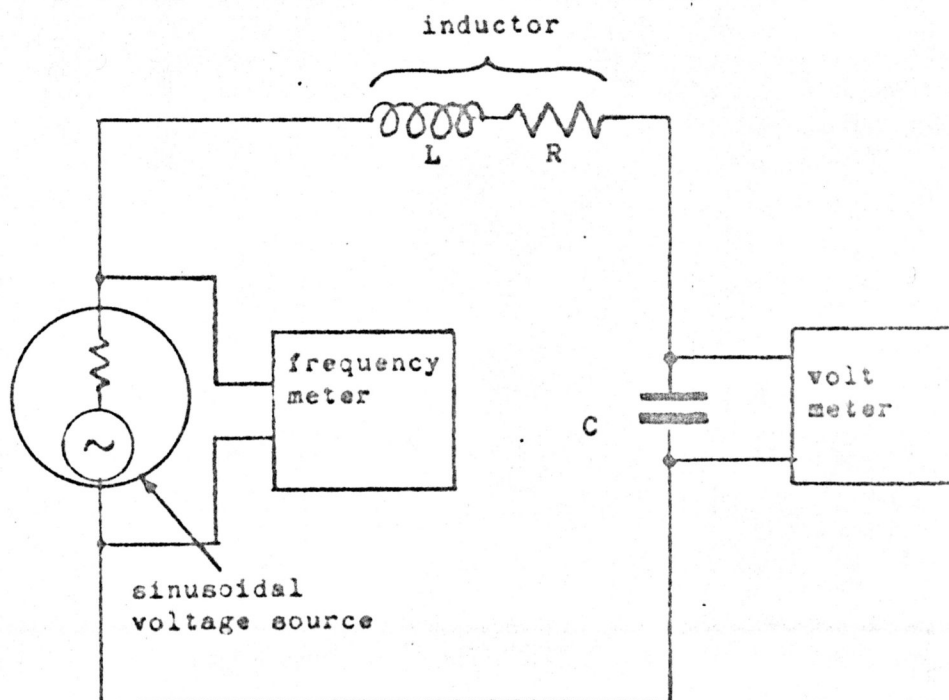


FIG 5.02 BLOCK DIAGRAM OF THREE OSCILLATOR ANALOGUE AND EXPERIMENTAL APPARATUS



frequency meter

volt meter

sinusoidal voltage source

Racal type SA520

General Radio type 1564 A

Derritron 25wt (ss)

FIG 5.03 INDUCTANCE CALIBRATION CIRCUIT

$$L = \frac{1}{2.f.C_s} \quad \text{Henries} \quad \dots\dots\dots (1)$$

$$\text{where } f = \frac{f_m}{\sqrt{1 - 1/(4.Q^2)}} \quad \dots\dots\dots (2)$$

f_m was the measured resonant frequency (Hz).

C_s was the standard capacitance (Farads).

Any inherent inductance at the output terminals of the sinusoidal voltage source was checked for by connecting the standard capacitor directly across it to form a series resonant circuit. No significant inductance existed, as no voltage increase across the capacitor was noticed when the frequency was swept up to the maximum limit of the instrument.

Coupling capacitor values were measured by connecting each in a series resonant circuit with a previously calibrated inductor. The above equations 1 and 2 were then evaluated for C_s . This technique had the advantage that values of all components were based on the same standard, thus making calibration of the standard itself unnecessary. It was possible to measure resonant frequencies very precisely by using the digital frequency meter. The capacitor voltage peaked sharply as the Q factors of the resonant circuits were high, i.e. in excess of 100. The standard capacitance was selected to cause resonance of the circuits at frequencies of the order of several kilo-Hertz, thus the frequency meter accuracy limit of $\pm 1\text{Hz}$ implied less than 0.1% error.

The maximum Q factor of an oscillator was limited by the amount of inherent resistance in the inductor, and also in the case

of the externally excited oscillator, the resistance in the secondary windings of the isolation transformer. The following technique was used to determine the Q factor of an oscillator due to the inductor resistance only. Fig 5.04 shows the equivalent circuit of the secondary windings of the isolation transformer, connected to an oscillator.

Let: i be the current flowing round the circuit,

v be the source voltage,

r be the internal resistance of secondary winding,

R be the internal resistance of the inductor,

V_c be the voltage measured across the capacitor,

V_t be the voltage measured across transformer output terminals.

As the current flowing through any component equals the voltage across divided by its impedance,

$$i = \frac{V_c}{Z_c} \quad \text{where } Z_c = \frac{1}{j \cdot \omega \cdot C},$$

$$\text{at resonance} \quad i = \frac{V_t}{R}$$

$$\text{therefore} \quad R = \frac{Z_c \cdot V_t}{V_c} \quad \dots\dots\dots 3$$

V_t and V_c are ninety degrees out of phase at resonance,

$$\text{therefore} \quad R = \frac{V_t}{V_c \cdot \omega \cdot C}$$

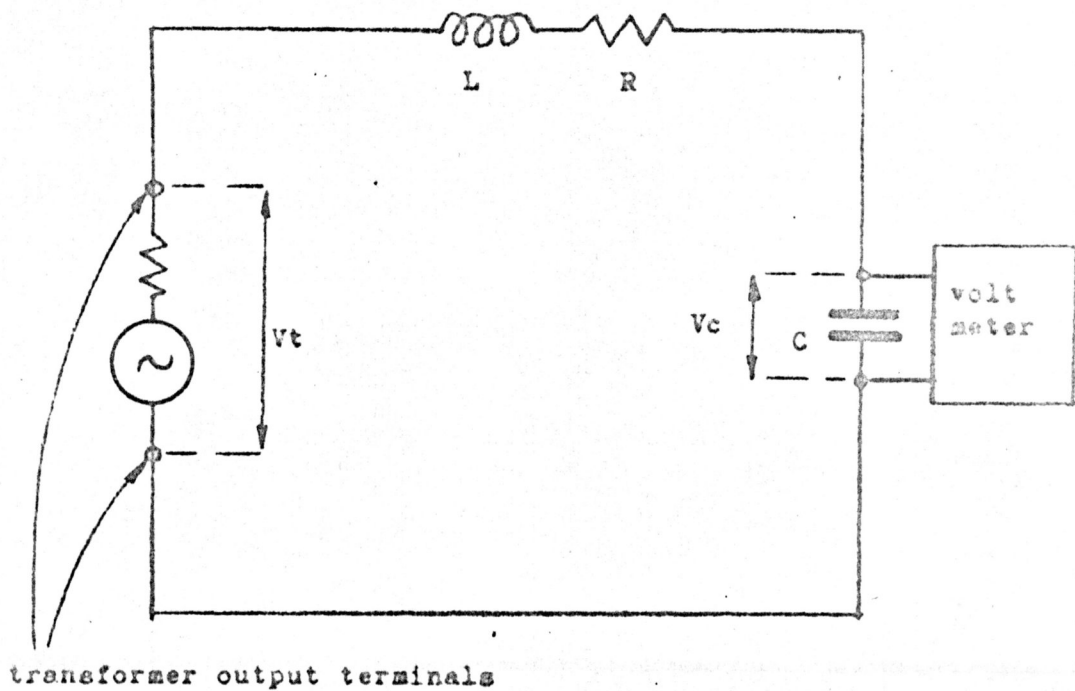


FIG 5.04 EQUIVALENT CIRCUIT OF OSCILLATOR CONNECTED ACROSS TRANSFORMER SECONDARY

where W is the resonant frequency in rad/sec.

$$\text{but } Q = \frac{W \cdot L}{R}, \quad W = W_n \sqrt{1 - \frac{1}{4Q^2}}, \quad \text{and } W_n = \sqrt{\frac{1}{L \cdot C}},$$

$$\text{therefore } \frac{Q^2 \cdot R^2 \cdot C}{L} + \frac{1}{4 \cdot Q^2} = 1$$

$$\text{therefore } Q^2 = \frac{1 \pm \sqrt{1 - \left(\frac{R^2 \cdot C}{L}\right)}}{\frac{R^2 \cdot C}{2 \left(\frac{1}{L}\right)}} \dots\dots\dots (4)$$

5.1.iii Measuring techniques

Recordings of oscillator responses were made by measuring the voltage across the capacitor. Unlike an inductor a capacitor contains negligible resistance and therefore the voltage across it was directly proportional to the charge, or in equivalent mechanical terms, the displacement. The voltage across any extra series resistor in the circuit would also have been a measure of the response, but in terms of current, or velocity. However, at resonance the capacitor voltage is a factor of Q times the voltage across the total resistance, therefore an improvement in signal to instrument back-ground noise level is achieved by the former method. The capacitor voltage was directly proportional to the oscillator potential energy level, and not kinetic energy, except at resonance. The following method was used to obtain the kinetic energy at any frequency. The mean kinetic energy level of a randomly excited electrical oscil-

lator may be expressed as:

$$\frac{1}{2} \cdot L \cdot E[i^2]$$

where $E[i^2]$ is the expected mean value of the squared current.

Therefore, if the above expression is written in terms of the voltage measured across the capacitor, at resonance kinetic and potential energies may be equated:

$$\text{i.e.:} \quad \frac{1}{2} \cdot L \cdot E[i^2] = \frac{1}{2} \cdot \frac{1}{C} \cdot E[q^2]$$

where $E[q^2]$ is the expected mean squared value of charge.

$$\text{but} \quad q = V_c \cdot C$$

$$\text{therefore} \quad \frac{1}{2} \cdot L \cdot E[i^2] = \frac{1}{2} \cdot C \cdot E[V_c^2] = \frac{1}{2} \cdot \frac{1}{L \cdot \omega_n^2} \cdot E[V_c^2]$$

If an oscillator is excited by a source of white noise it responds primarily at its own resonant frequency and the mean kinetic energy is proportional to the mean voltage across the capacitor. If the excitation has some other spectrum it is necessary also to obtain the spectrum of the capacitor voltage response.

Let ω_a be the frequency analyser centre frequency,

Let E_n be the kinetic energy of the oscillator,

$$\text{therefore} \quad \text{En}_{(w_a)} = \frac{1}{2} \cdot L \cdot E[i_a^2]$$

where i_a is the component of current at the frequency w_a

$$i_a = Vc_a \cdot w_a \cdot \underline{C}$$

where Vc_a is the capacitor voltage at w_a

$$\begin{aligned} \text{therefore} \quad \text{En}_{(w_a)} &= \frac{1}{2} \cdot L \cdot E[Vc_a^2 \cdot w_a^2 \cdot \underline{C}^2] \\ &= \frac{1}{2} \cdot L \cdot \underline{C}^2 \cdot w_a^2 \cdot E[Vc_a^2] \end{aligned}$$

$$\text{but} \quad \underline{C} = \frac{1}{L \cdot w_n^2}$$

$$\text{therefore} \quad \text{En}_{(w_a)} = \frac{1}{2} \cdot \frac{w_a^2}{L \cdot w_n^4} \cdot E[Vc_a^2] \quad \dots\dots\dots (5)$$

Hence the kinetic energy level may be found in terms of the analyser centre frequency and the mean squared capacitor voltage.

5.1.iv Results

A specimen chart recording of the capacitor voltages of the excited and receiving oscillator, numbers 1 and 3 respectively is shown in fig 5.05. Comparisons with energy ratios computed by program 'MAG' (see Appendix 1), for several intermediate oscillator Q factors and resonant frequencies are shown in figs 5.06 to 5.09.

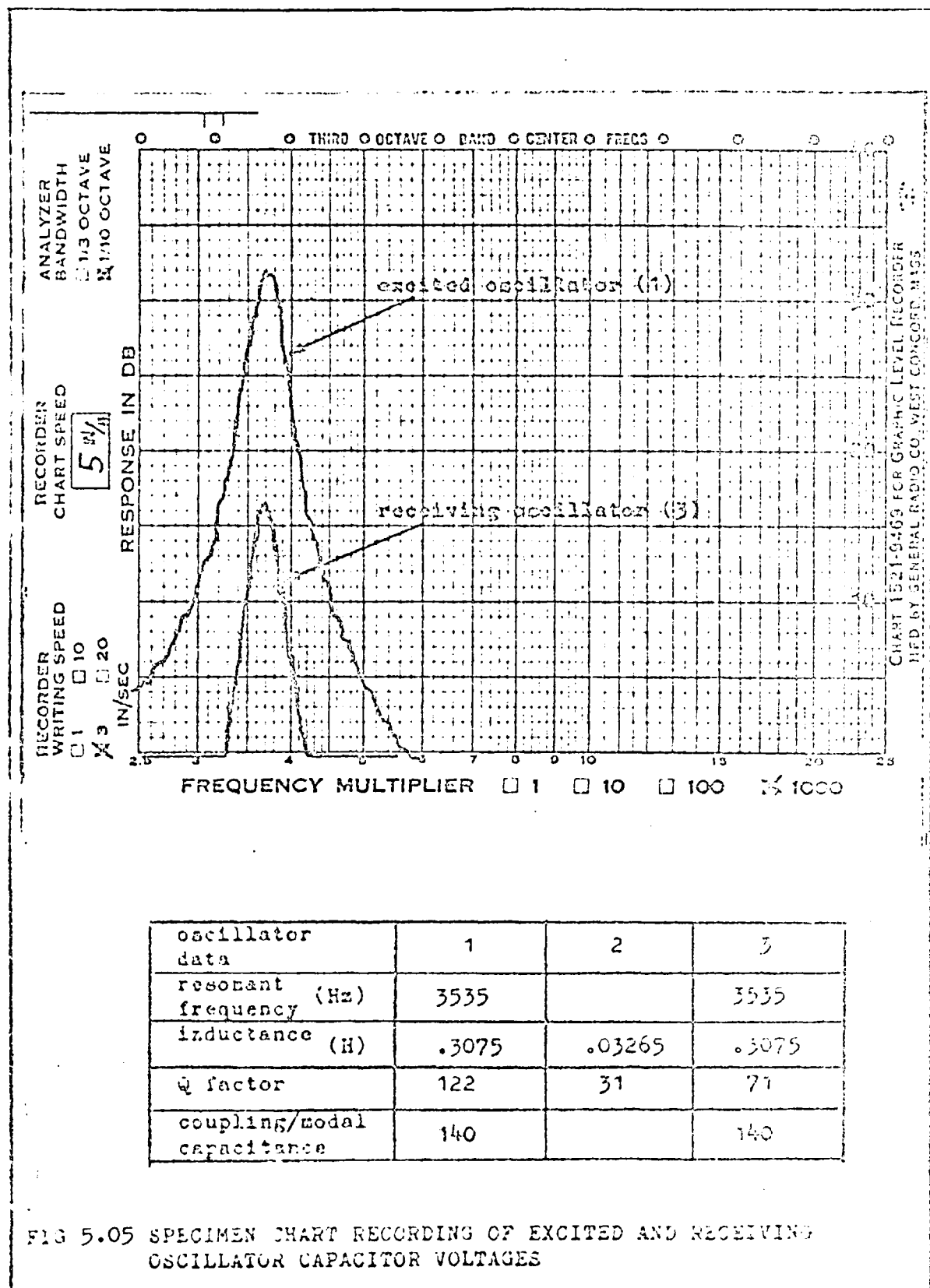
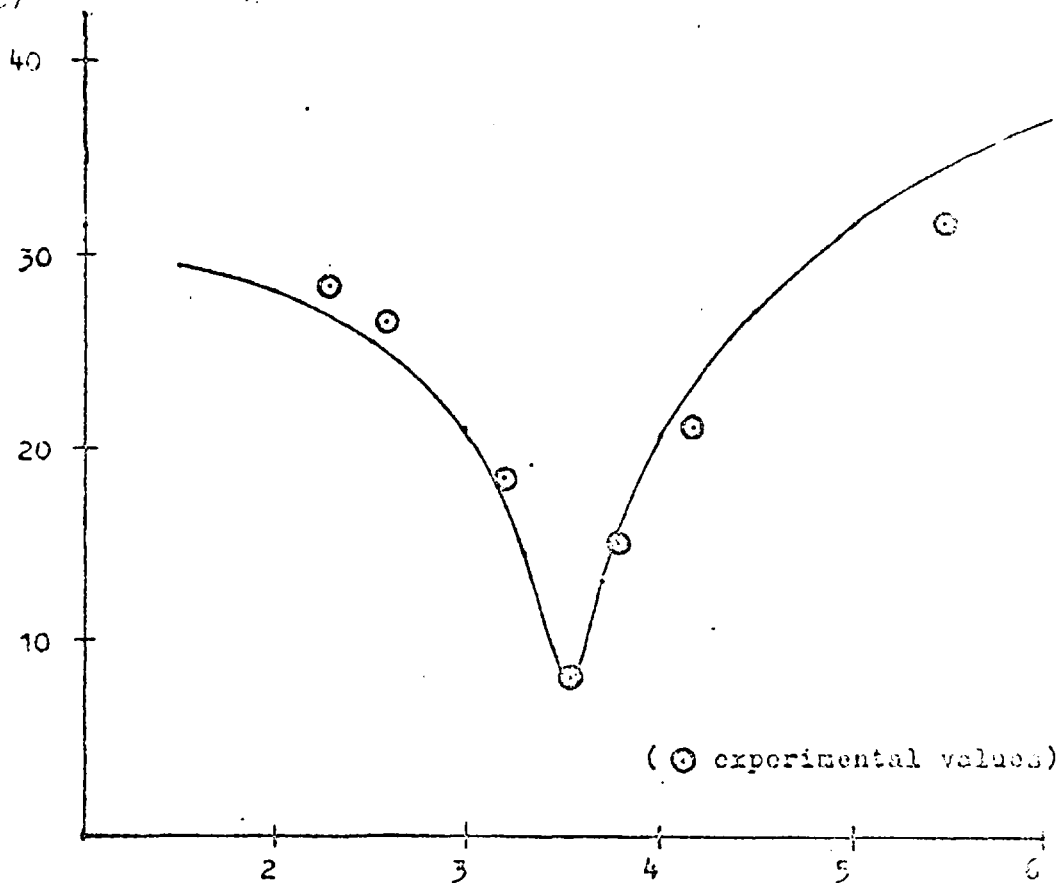


FIG 5.05 SPECIMEN CHART RECORDING OF EXCITED AND RECEIVING OSCILLATOR CAPACITOR VOLTAGES

excited to receiving oscillator
energy ratio
(dB)

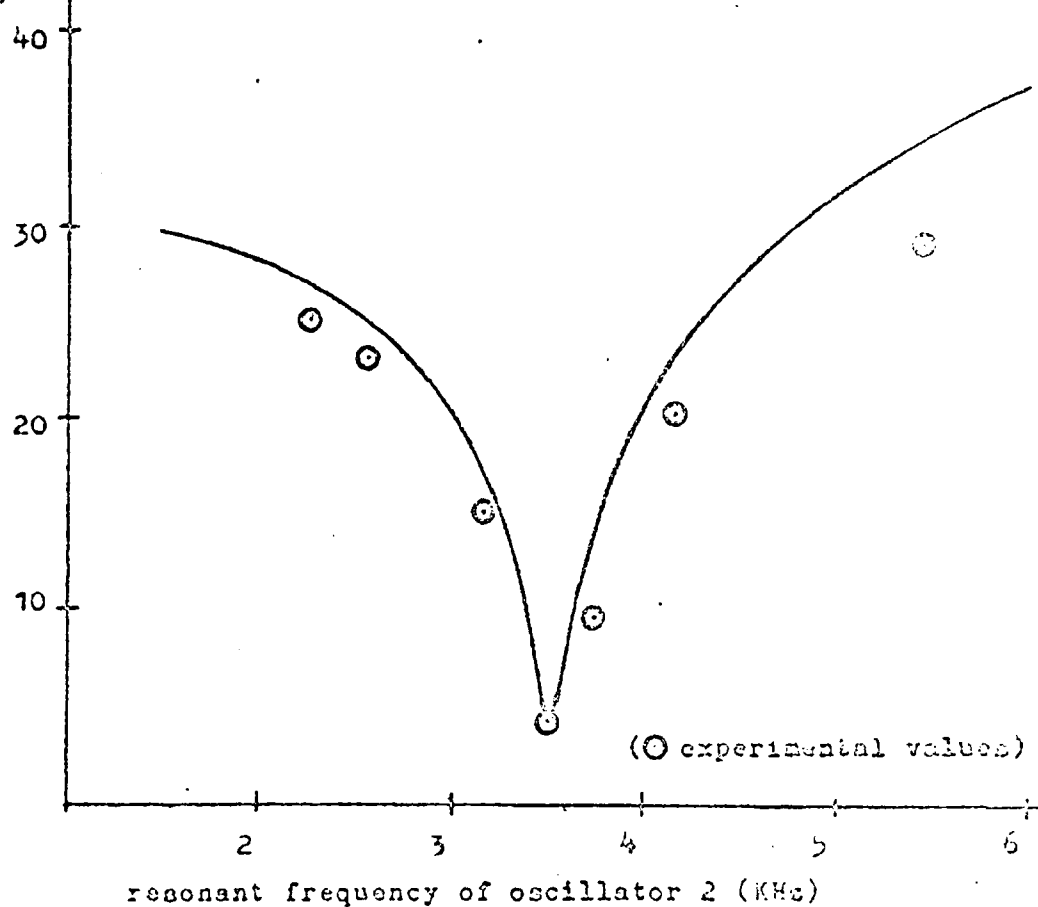


resonant frequency of oscillator 2 (KHz)

oscillator data	1	2	3
resonant frequency (Hz)	3535		3535
inductance (H)	.3075	.03265	.3075
Q factor	122	31	71
coupling/modal capacitance	140		140

FIG 5.06 COMPARISION OF COMPUTED AND EXPERIMENTAL ENERGY RATIOS

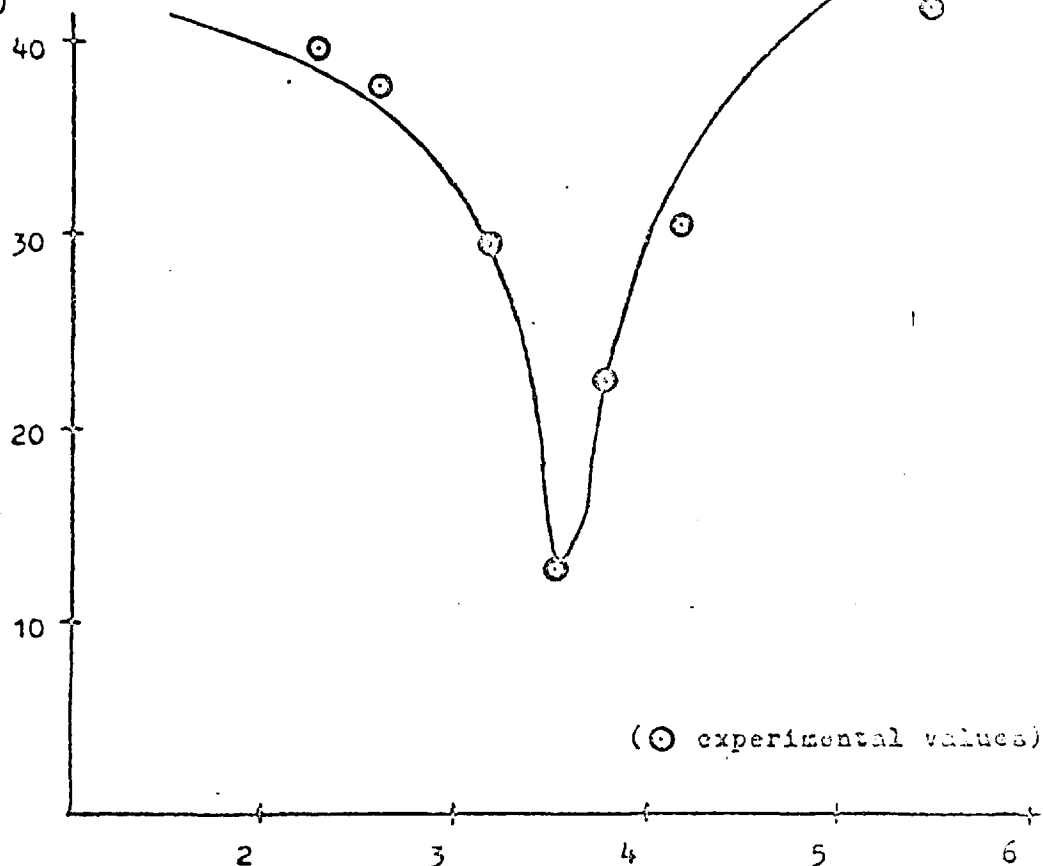
excited to receiving oscillator
energy ratio
(dB)



oscillator data	1	2	3
resonant frequency (Hz)	3535		3535
inductance (H)	.3075	.03265	.3075
Q factor	122	88	71
coupling/modal capacitance	140		140

FIG 5.07 COMPARISON OF COMPUTED AND EXPERIMENTAL ENERGY RATIOS

excited to receiving oscillator
energy ratio
(dB)



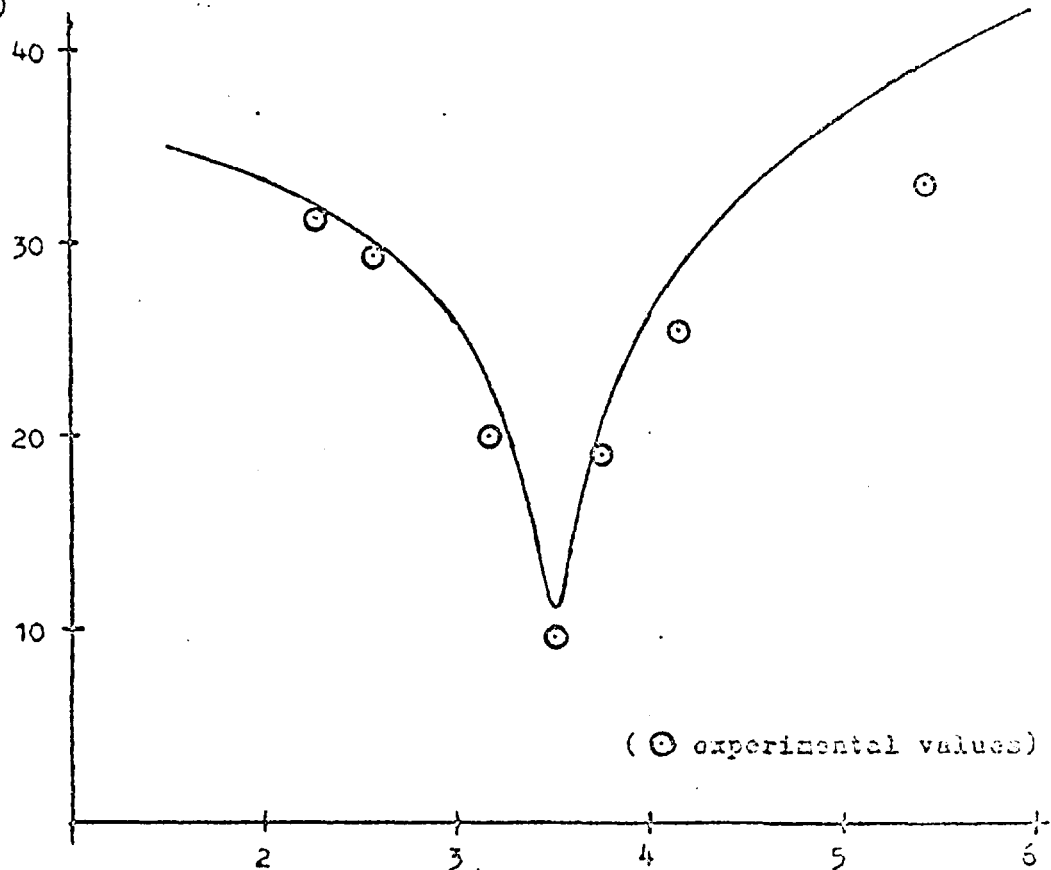
(⊗ experimental values)

resonant frequency of oscillator 2 (KHz)

oscillator data	1	2	3
resonant frequency (Hz)	3535		3690
inductance (H)	.3075	.03265	.3075
Q factor	122	88	122
coupling/modal capacitance	140		140

FIG 5.08 COMPARISON OF COMPUTED AND EXPERIMENTAL ENERGY RATIOS

excited to receiving oscillator
energy ratio
(dB)



(⊙ experimental values)

resonant frequency of oscillator 2 (KHz)

oscillator data	1	2	3
resonant frequency (Hz)	3535		3535
inductance (H)	.3075	.03265	.3075
Q factor	122	31	35
coupling/modal capacitance	140		140

FIG 5.09 COMPARISON OF COMPUTED AND EXPERIMENTAL ENERGY RATIOS

intermediate oscillator resonant frequency (Hz)	computed energy ratio (dB)	
	program MAG	program HEIG
1535	29.689	29.677
2035	28.022	28.010
2535	25.285	25.214
3035	20.148	19.997
3135	18.479	18.315
3235	16.387	16.212
3335	13.685	13.546
3435	10.293	10.224
3535	7.944	7.959
3635	10.414	10.487
3735	14.067	14.151
3835	17.039	17.128
3935	19.392	19.465
4035	21.321	21.338
4535	27.729	27.676
5035	31.743	31.712
5535	34.724	34.697
6035	37.124	37.100

oscillator data	1	2	3
resonant frequency (Hz)	3535		3535
inductance (H)	.3075	.03265	.3075
Q factor	122	31	71
coupling/modal capacitance	140		140

FIG 5.10 COMPARISON OF EXCITED TO RECEIVING OSCILLATOR ENERGY RATIOS COMPUTED BY PROGRAMS 'MAG' AND 'HEIG'

Energy ratios computed by programs 'MAG' and 'MEIG' (see Appendix 1) are compared with each other in fig 5.10.

5.1.v Discussion of results

There was good agreement in the comparisons of experimental analogue and computed energy ratio. The agreement was usually within one decibel but was sometimes as great as three decibels when the energy ratio was large. The accuracy of the frequency analyser and recording equipment was plus or minus one decibel. The experimental energy ratios were the mean ratio of two constantly fluctuating quantities, due to the random excitation. These fluctuations were of the order of about one to two decibels, therefore the overall accuracy of the experimental results was plus or minus two decibels. Another source of discrepancy between experimental and computed results lay in the numerical integration process employed in the latter, but it was found that even with the integration step length reduced by a factor of four, energy ratio did not change by more than half a decibel.

The most likely cause of these discrepancies was imperfections and non linearities of the electrical analogue components themselves. However, the general agreement was sufficiently close to confirm the proper functioning of the computer program.

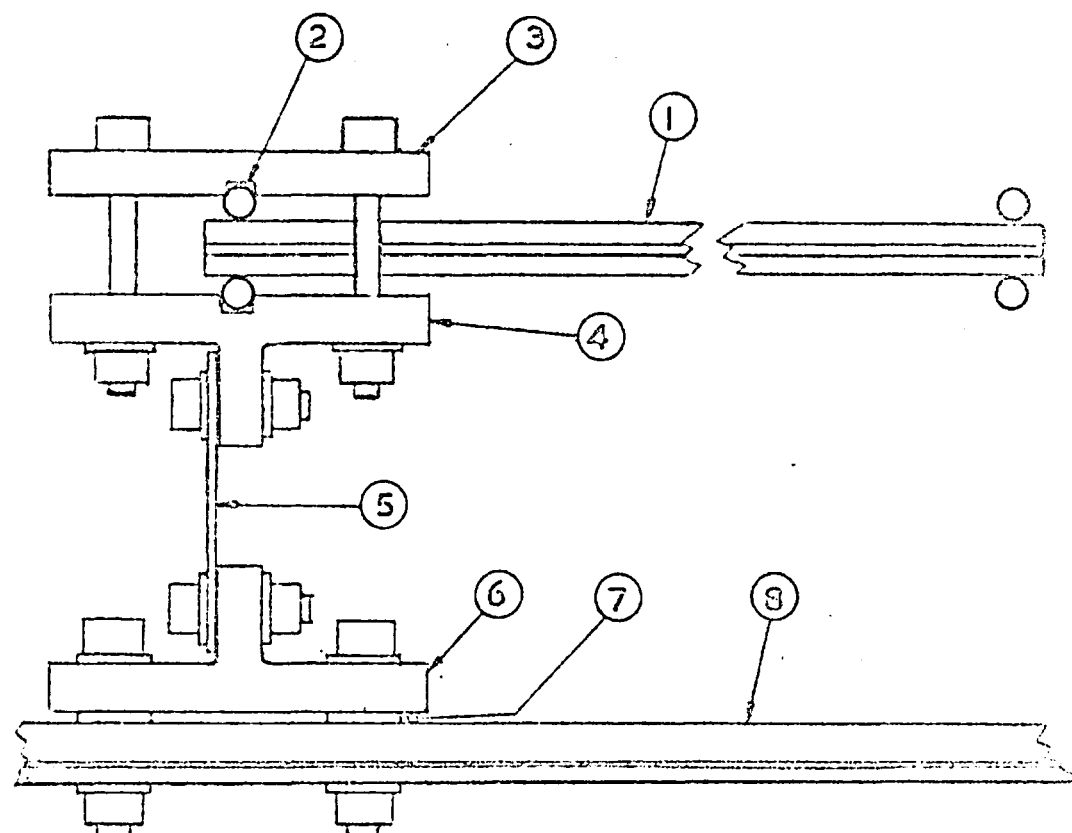
5.2 TESTS WITH VIBRATOR MOUNTED ON 'RECEIVING' PLATE

5.2.1 Details of apparatus

The large plate 48in x 48in x 3/16in used as the receiving

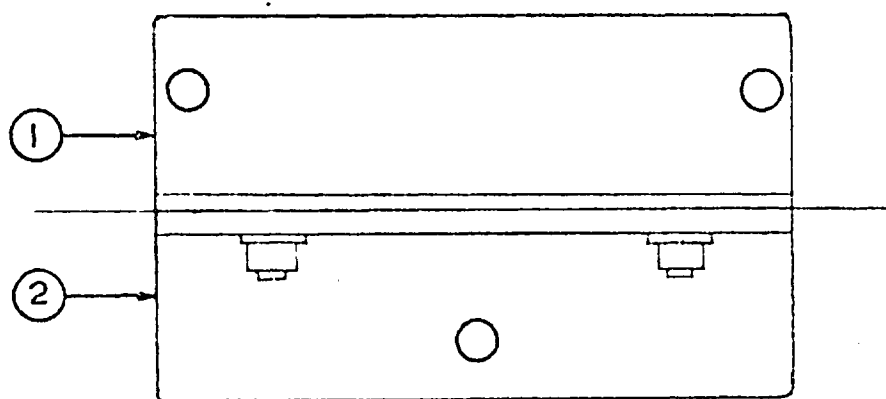
structure was made by laminating two steel plates, one $1/8$ in thick, the other $1/16$ in thick either side of a thin layer of visco-elastic damping material. This damped plate was used because preliminary tests with an undamped plate of the same dimensions, with the vibrator freely suspended above, had indicated significant acoustic coupling at frequencies within the range of the experiments. The plate was mounted in a horizontal plane with supports under each corner. The supports consisted of a block of wood approximately a cube of 3 in side, mounted on a piece of sponge rubber. The purpose of the sponge rubber was to allow the block to take up the curvature of the plate, due to the load of the vibrator on it. The rubber mountings were of the 'double U' shear type.

The damped laminated beams were made from two layers of steel each $1/16$ in thick enclosing one layer of a plasticised P.V.C. material $1/32$ in thick, length and width were 10 in and 2 in respectively. Bonding between layers was by an epoxy resin adhesive (Araldite AV100, HV100). In order that the mode shapes and resonant frequencies of the beams should conform to theoretical predictions they were mounted on flexible supports designed to give 'simply-supported' end conditions, see fig 5.11, 5.12. The vibrator used was a standard moving coil-permanent magnet type, capable of a maximum force of 25 lb (Derriton VP4). Several arrangements of the intermediate structures were used. These arrangements consisted of (i) laminated beams, (ii) 4 Hz mountings plus laminated beams, (iii) 21 Hz mountings plus laminated beams. The centre of the laminated beams was used



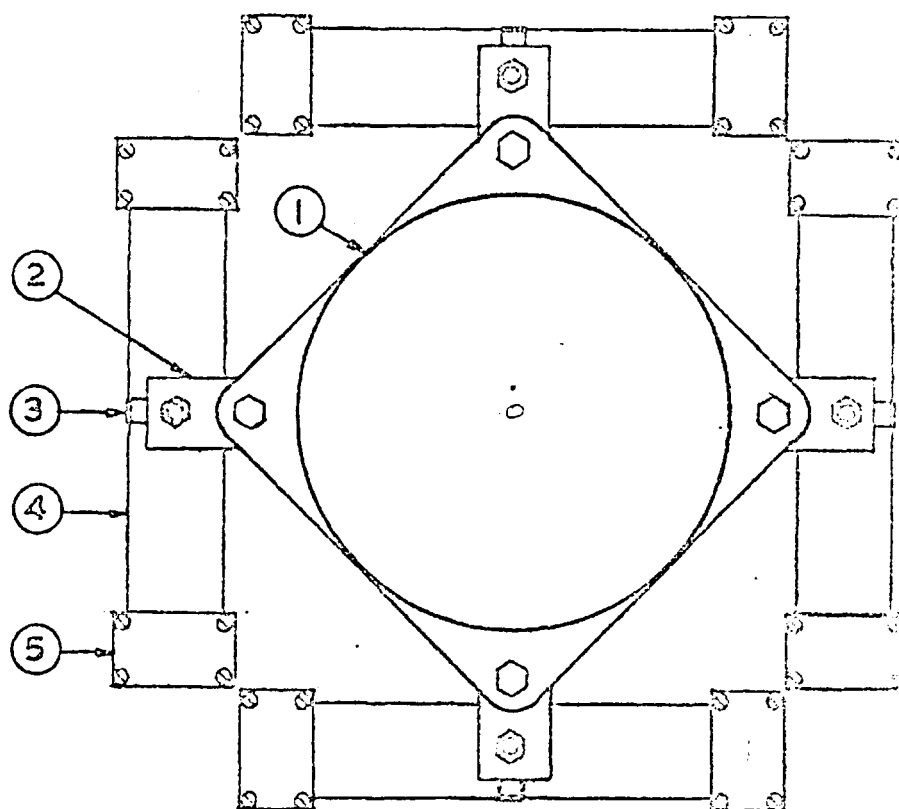
- ① laminated beam (steel - PVC)
- ② roller (silver steel)
- ③ end support top clamp (aluminium)
- ④ end support lower clamp (aluminium)
- ⑤ plate (steel 0.010 in. thick)
- ⑥ end support base (aluminium)
- ⑦ washer (steel)
- ⑧ receiving plate (steel - viscoelastic laminate)

FIG 5.11 DETAILS OF END SUPPORT OF LAMINATED BEAM



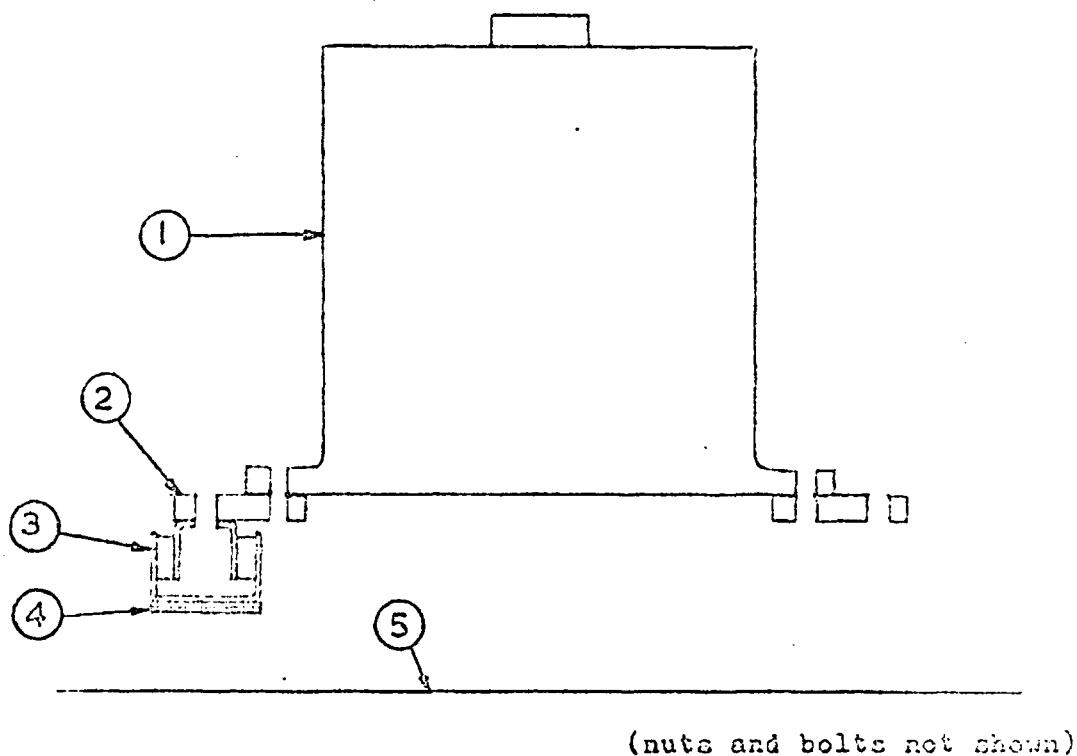
- ① support lower clamp
- ② support base

FIG 5.12 PLAN VIEW OF LOWER CLAMP AND BASE OF LAMINATED BEAM
END SUPPORT



- ① vibrator
- ② spacer
- ③ rubber mounting (double U shear type)
- ④ laminated beam
- ⑤ laminated beam end support

FIG 5.13 PLAN VIEW OF VIBRATOR ON RUBBER MOUNTINGS AND LAMINATED BEAMS



- ① vibrator
- ② spacer
- ③ rubber mounting
- ④ laminated beam
- ⑤ receiving plate

FIG 5.14 SIDE SECTION VIEW OF VIBRATOR ON RUBBER MOUNTINGS
AND LAMINATED BEAMS

as the mounting point for the rubber mountings in arrangements (ii) and (iii) and the vibrator in arrangement (i). Tests were also done with the vibrator mounted directly on the plate. The complete system with intermediate structure arrangement (ii) is shown in figs 5.13, 5.14.

5.2.ii Determination of modal oscillator parameters

a) Plate

The point responses of the large steel plate were measured at four positions corresponding to the corners of the square formed by the four laminated beams, see fig 5.13. The point responses plotted as apparent mass (i.e. force divided by acceleration) against frequency are shown in figs 5.20, 5.21. Transfer response measurements were taken with force excitation at these locations, and acceleration responses at the positions indicated in fig 5.19. These were also plotted as apparent mass, see figs 5.22 to 5.26. Calibrations and specifications of the measuring equipment used for these tests is given in [30]. From these point and transfer response curves, data was obtained for program 'SIGMA' (see Appendix 2), which computed the modal and coupling parameters of the plate. These are presented in fig 5.27.

b) Laminated beams

The modal mass of a simply supported beam is equal to 0.493268 of the mass. This figure derived as follows: from [29] the first natural frequency of a simply supported beam is,

$$f_1 = C_1 \sqrt{\frac{E \cdot I}{\rho \cdot A \cdot L^4}} \text{ Hz}$$

where: E is the Young's modulus of the beam material,

I is the second moment of area of the beam section,

ρ is the density of the beam,

A is the cross sectional area of the beam,

L is the length of the beam,

C_1 is a constant and equals 1.57.

Let the beam be represented by an equivalent mass and stiffness at its antinode (in this case the centre).

$$f_1 = \frac{1}{2 \cdot \pi} \sqrt{\frac{K}{M}}$$

where K is the modal stiffness, M is the modal mass.

$$\text{therefore } C_1 \sqrt{\frac{E \cdot I}{\rho \cdot A \cdot L^4}} = \frac{1}{2 \cdot \pi} \sqrt{\frac{K}{M}}$$

From the bending theory of beams [29]

$$K = \frac{48 \cdot E \cdot I}{L^3}$$

$$\text{therefore } M = \frac{48 \cdot \rho \cdot A \cdot L}{C_1^2 \cdot (2 \cdot \pi)^2}, \quad \rho \cdot A \cdot L = \text{total mass } M_t$$

$$\text{hence } M = 0.493268 M_t$$

The modal mass of higher modes is also 0.493268.Mt. At higher modes the beam may be considered to be made up of a series of simply supported beams running from node to node, because the same end conditions apply at a node as at a simple support, namely zero lateral deflection and restraining moment. The coupling parameters connecting a support to the modal mass are, for the first mode of vibration, equal to half the modal stiffness and half the modal damping. For higher modes of vibration, the coupling parameters at each support are equal to half the modal stiffness and damping of a simply supported beam, whose span runs from the support to the adjacent node.

$$\text{thus} \quad Kc_n = \frac{Wn^2.M}{2.n} \quad \dots\dots\dots (6)$$

$$\text{and} \quad Cc_n = \frac{Wn.M.\eta_n}{2.n} \quad \dots\dots\dots (7)$$

where Kc_n is the coupling stiffness, Wn is the natural frequency of the n th node, i.e.:

$$Wn = \frac{W}{\sqrt{1 - \frac{\eta_n^2}{4}}}$$

W is the resonant frequency of the n th mode,

n is the mode number,

η_n is the loss factor of the n th mode.

The loss factors of the laminated beams were found from their steady state response to a sinusoidal excitation. Each beam with its

flexible end supports was clamped to the top flange of a large aluminium 'I' section beam (see fig 5.15). The 'I' section beam was suspended from soft elastic cords and bolted at its base flange to the table of a vibrator. The tension in the cords was adjusted to give zero static load on the vibrator. An accelerometer was mounted on the 'I' beam adjacent to one of the flexible supports of the laminated beam. Another accelerometer was positioned on the centre of the laminated beam. The ratio of these accelerations was measured as the excitation frequency was slowly swept, and the loss factor of each mode evaluated as follows:

Let the supported beam be represented by a lumped parameter system as shown in fig 5.16. The equation of motion of such a system is:

$$M.\ddot{x} + \bar{K}.x + C.\dot{x} + K_c.x + C_c\dot{y} = K_c.y + C_c.\dot{y}$$

$$K_c y + C_c \dot{y} = F$$

where x is the displacement of the mass,

where y is the displacement of the support,

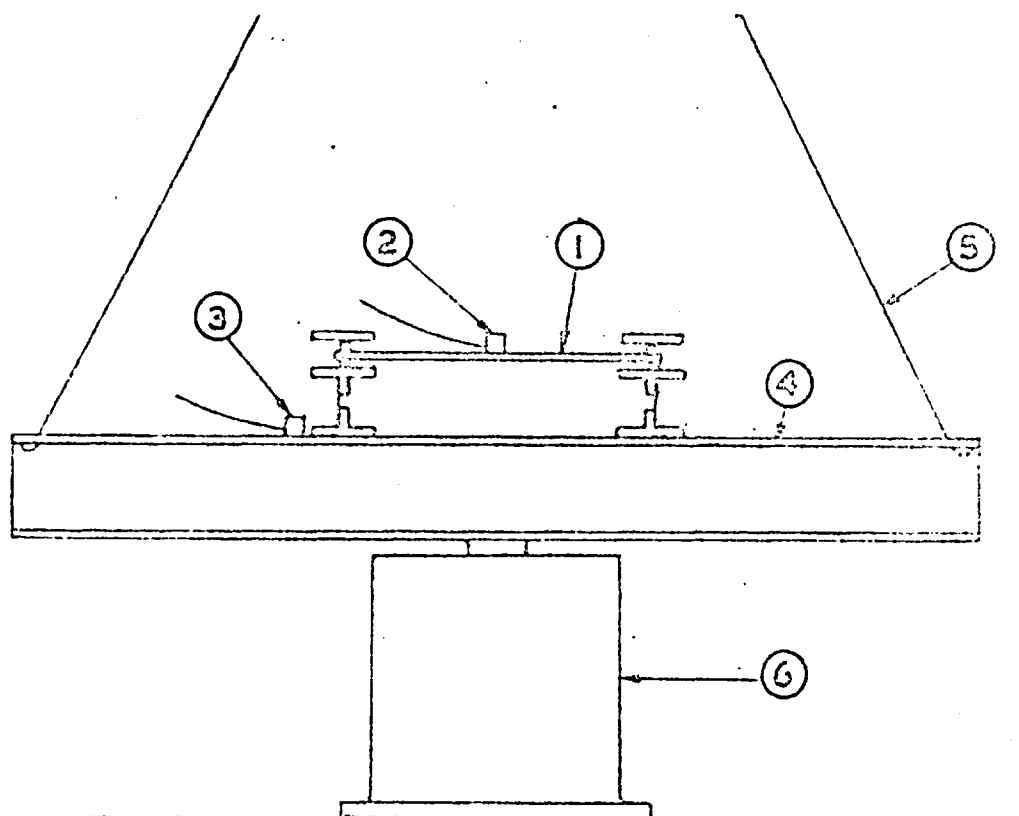
K_c and C_c represent the coupling parameters connecting the beam support to the modal mass.

$$\text{Let } x = X.e^{j.w.t}, y = Y.e^{j.w.t} \text{ and } Z = \frac{X}{Y}$$

$$\text{therefore } -M.w^2.Z + j(\bar{C} + C_c)w.Z + (\bar{K} + K_c)Z = K_c + j.w.C_c$$

$$\text{therefore } Z = \frac{K_c + j.w.C_c}{(\bar{K} + K_c) - M.w^2 + j.w(\bar{C} + C_c)}$$

$$\text{hence } Z = \frac{K_c + j.w.C_c}{(K + K_c) \left[\left(1 - \frac{w^2}{\omega_n^2}\right) + j.\eta.\frac{w}{\omega_n} \right]} \quad \dots\dots (8)$$



- ① laminated beam
- ② accelerometer (Environmental Equipments type AQ5)
- ③ accelerometer (General Radio type 1560-P52)
- ④ 'I' section beam (aluminium)
- ⑤ suspension cords
- ⑥ vibrator (Derritron type VP4)

FIG 5.15 RIG FOR MEASURING FREQUENCY RESPONSE OF LAMINATED BEAMS

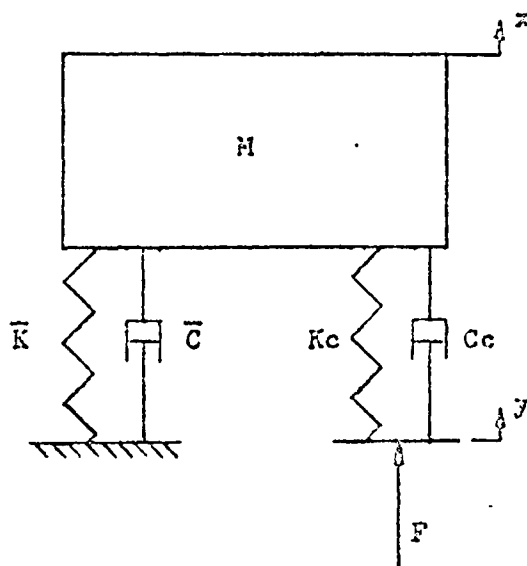


FIG 5.16 DIAGRAM OF MODAL AND COUPLING PARAMETERS OF SUPPORTED BEAM

where W is the natural frequency $= \sqrt{\frac{\bar{K} + Kc}{M}}$

is the loss factor $= \left(\frac{\bar{C} + Cc}{W.M} \right)$

therefore $Cc = \frac{C + Cc}{n}$, and $Kc = \frac{\bar{K} + Kc}{n}$

If Z is measured at each resonant frequency, the X and Y vectors are ninety degrees out of phase, therefore at these frequencies,

Let $Z = j.Z_n$

therefore $j.Z_n = \frac{\left(\frac{\bar{K} + Kc}{n} \right) + j \cdot \frac{\eta_n}{n} \cdot W_n^2 \cdot M}{j \cdot \eta_n (\bar{K} + Kc)}$

hence $-Z_n = \frac{\frac{j}{n} - \frac{\eta_n}{n}}{\eta_n}$

and $\eta_n = \frac{j}{1 + Z_n \cdot n}$ (9)

A plot of Z against frequency of beam 'A' is shown in fig 5.28 . The values of Z at the first and third modes of the four laminated beams are given in fig 5.29. Note, only modes having an even number of nodes could respond to this form of support excitation. Similarly, only these modes were produced by the central excitation of beams when used to mount the vibrator on the plate.

c) Rubber mountings

The double 'U' shear mountings were used as shown in fig 5.14. The characteristics of these mountings were similar to those of a

spring and damper in parallel. The resonant frequency of the mounted vibrator was found by setting it up on the laboratory floor, and supplying a sinusoidal excitation. The response of the body of the vibrator was measured from an accelerometer attached to its base, (see fig 5.30). The softer mountings produced a resonant frequency of 4Hz and the harder ones 21Hz. The stiffness of each of the four mountings was then found from the equation:

$$K_m = \frac{Wn^2 \cdot M_v}{4}, \quad Wn = \frac{W}{\sqrt{1 - \frac{\eta^2}{4}}}$$

Where M_v was the vibrator mass,

Wn the undamped natural frequency,

W the resonant frequency; K_m the mounting stiffness

In order to find the undamped natural frequency from the resonant frequency the loss factor must also be known. It was not possible to take reliable measurements of the magnification factor at the low resonance of 4Hz due to the non-linearity of the power amplifier and accelerometer which were only linear down to 5Hz and 6Hz respectively. The dynamics of the vibrator armature suspension produce a resonance at 30Hz, and this precluded accurate measurements at 21Hz. However, manufacturers data indicated that a loss factor of the order of 0.05 was to be expected. The resulting stiffnesses obtained by this method agreed with manufacturers data.

d) Vibrator

The vibrator was represented by a single mass equal to its

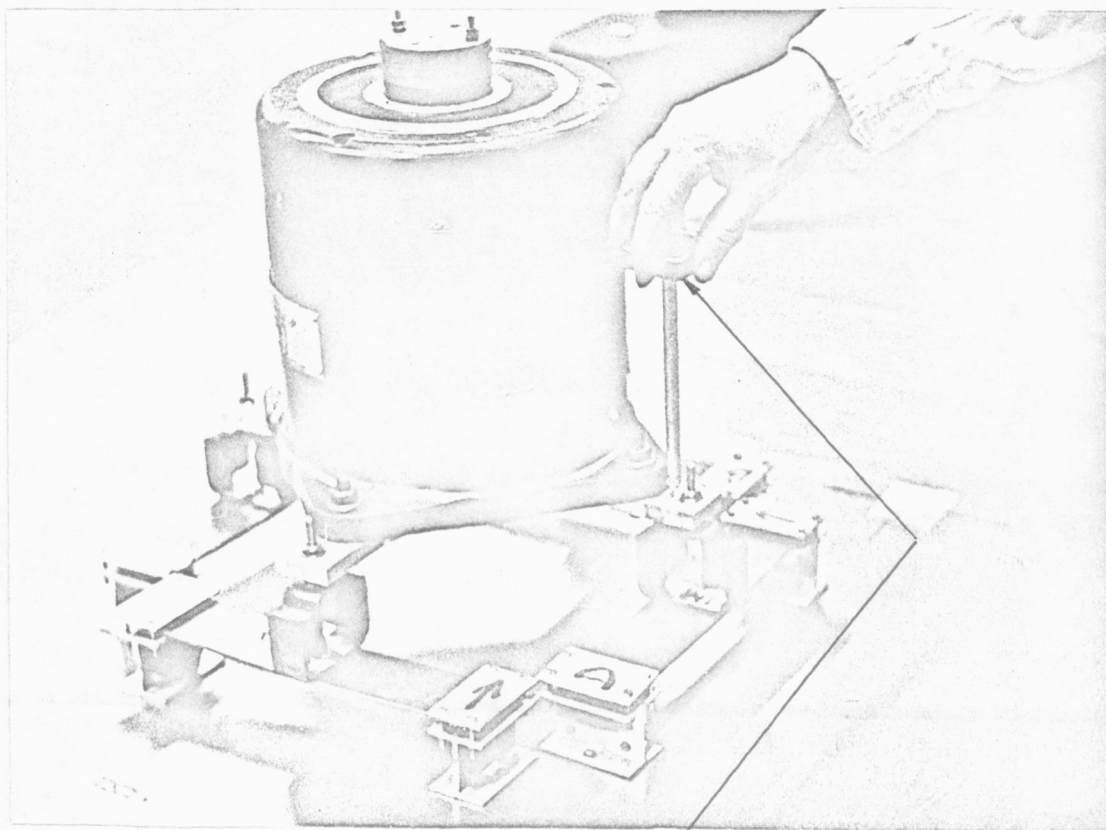
own weight. A swept sine wave test with the vibrator suspended from soft elastic cords, and an accelerometer fixed to its base, produced generally flat response (see fig 5.31). This showed that the vibrator was essentially mass-like at these frequencies, i.e. up to 500Hz.

5.2.iii Measuring technique

System responses were taken as the acceleration of the vibrator and the acceleration of the plate. It was only necessary to use one measurement position on the plate (A), as the point and transfer responses did not vary significantly from one position to another. The accelerometer used for these measurements, a General Radio 1560-P52 was tape mounted on the plate. To measure the vibrator acceleration the accelerometer was fitted with a probe and hand held on one foot of the vibrator (see figs 5.17, 5.18). Each foot was found to have the same response, which was to be expected from the symmetry of construction. The upper frequency limit of experimental measurements was 500Hz due to resonance of the General Radio 1560-P52 accelerometer at 3kHz. Accurate calibration of the accelerometer was unnecessary as the same one was used for both measurements, and only the relative values of these measurements were required.

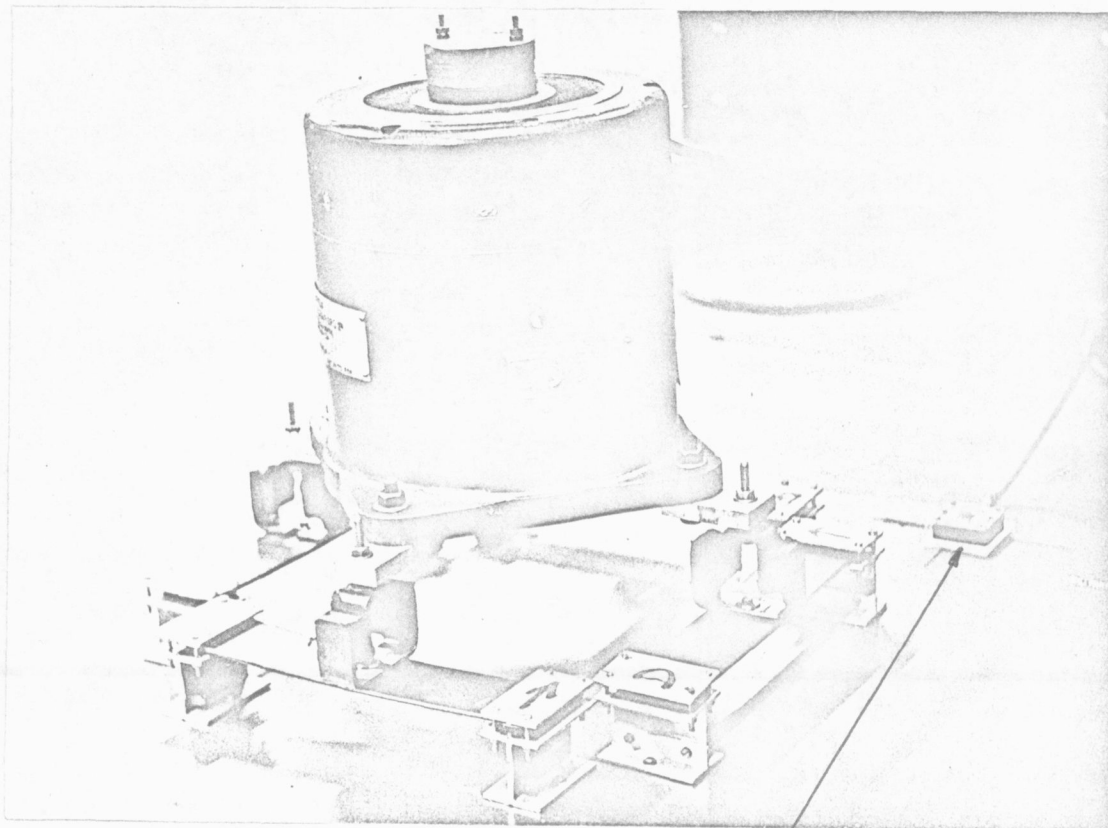
5.2.iv Results

The lumped parameter system representing the vibrator on rubber mountings and laminated beams comprised four oscillators (see fig 5.32). The first oscillator was formed by the vibrator mass on the rubber mountings. Because each laminated beam was identical their



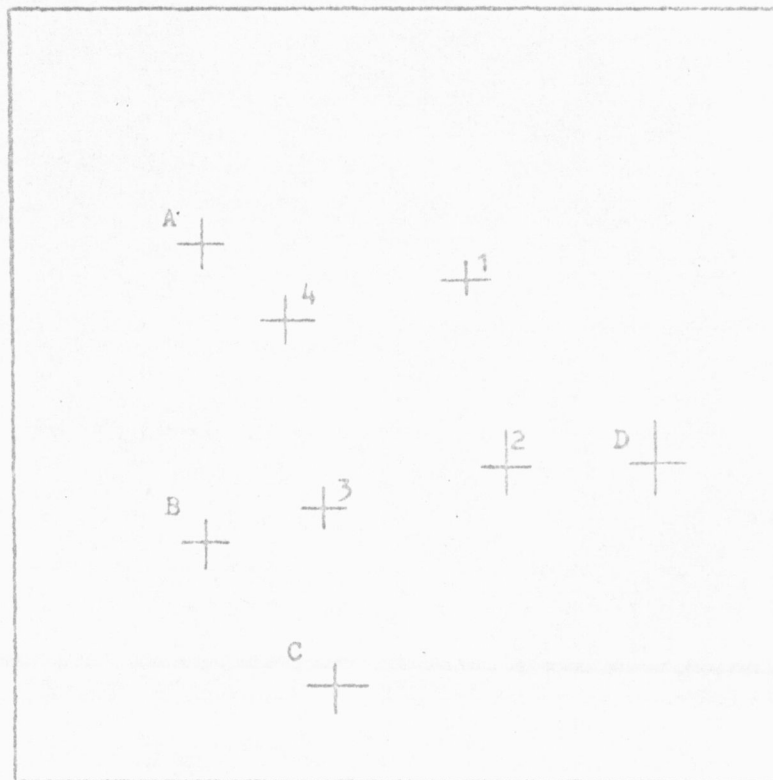
accelerometer in position for measuring vibrator response

FIG 5.17 VIBRATOR ON RUBBER MOUNTINGS AND LAMINATED BEAMS



accelerometer in position A for measuring plate response

FIG 5.18 VIBRATOR ON RUBBER MOUNTINGS AND LAMINATED BEAMS



force excitation at positions 1,2,3,4
acceleration response at positions 1,2,3,4,A,B,C,D

FIG 5.19 PLAN VIEW OF PLATE SHOWING POSITIONS OF POINT AND
TRANSFER RESPONSE MEASUREMENTS

apparent mass
(dB re 1lb)

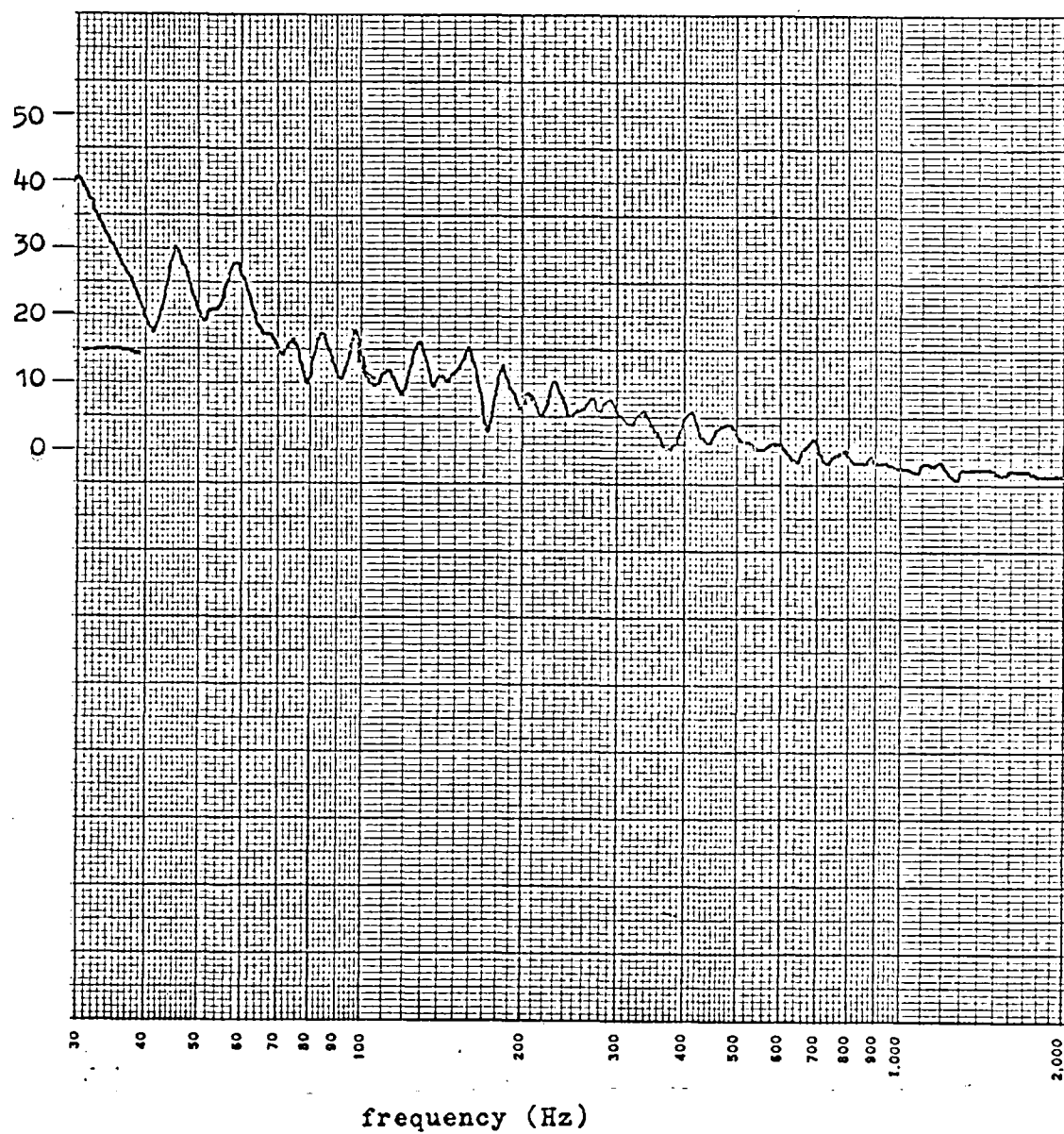


FIG 5.20 POINT RESPONSE OF RECEIVING PLATE AT POSITION 1

apparent mass
(dB re 1lb)

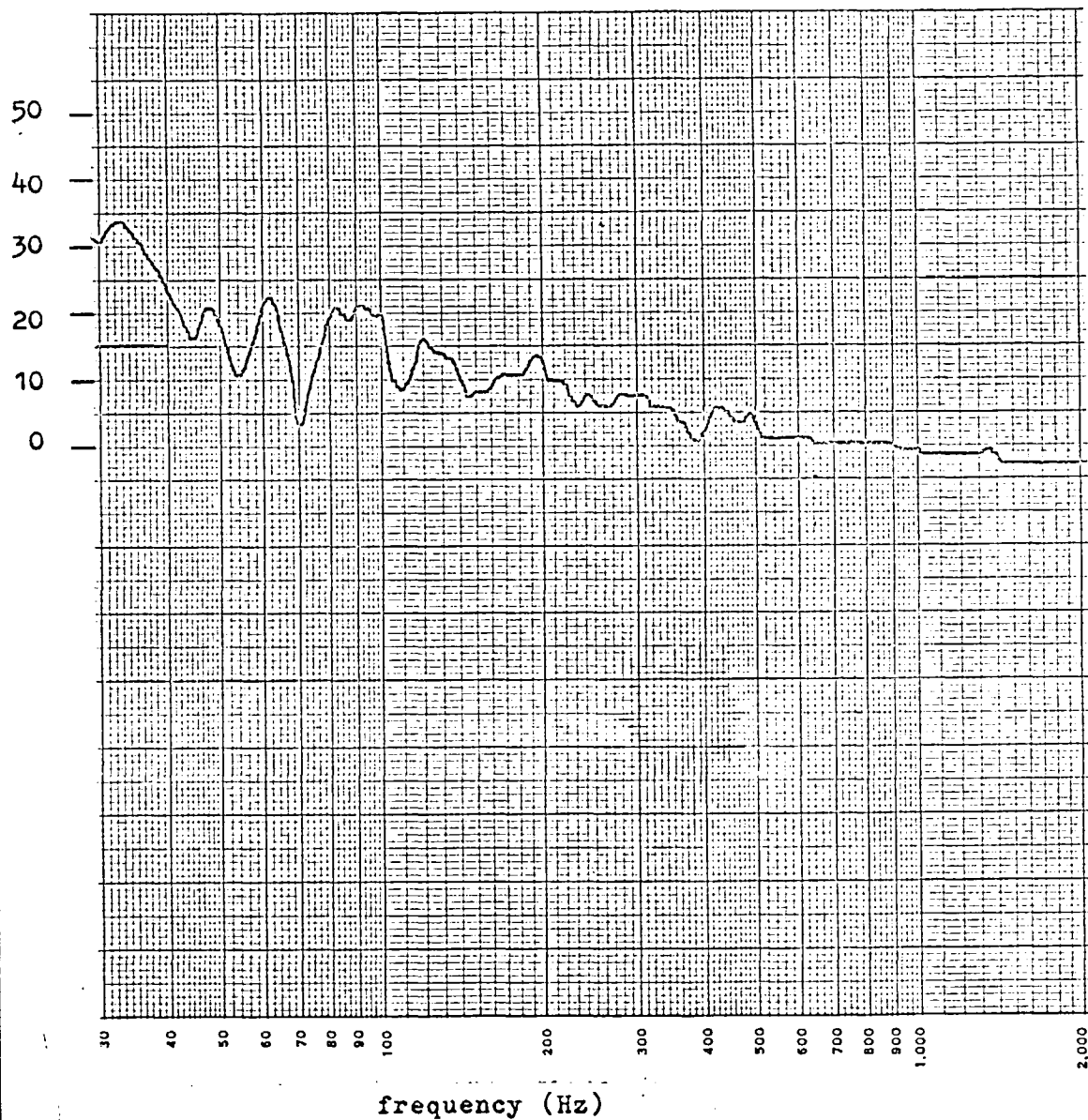


FIG 5.21 POINT RESPONSE OF RECEIVING PLATE AT POSITION 2

apparent mass
(dB re 1lb)

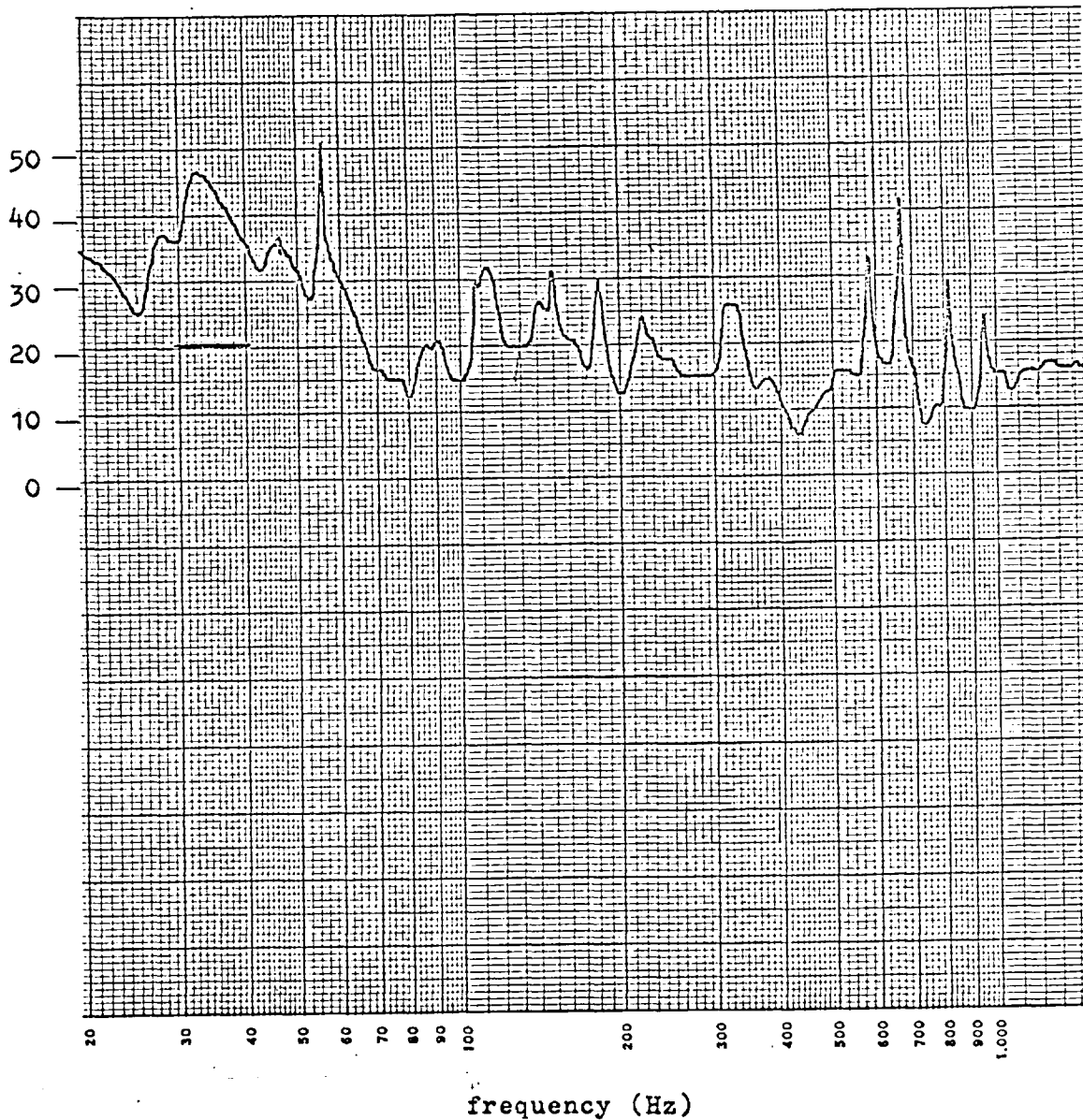


FIG 5.22 TRANSFER RESPONSE OF RECEIVING PLATE AT POSITION A;
EXCITATION AT POSITION 1

apparent mass
(dB re 11b)

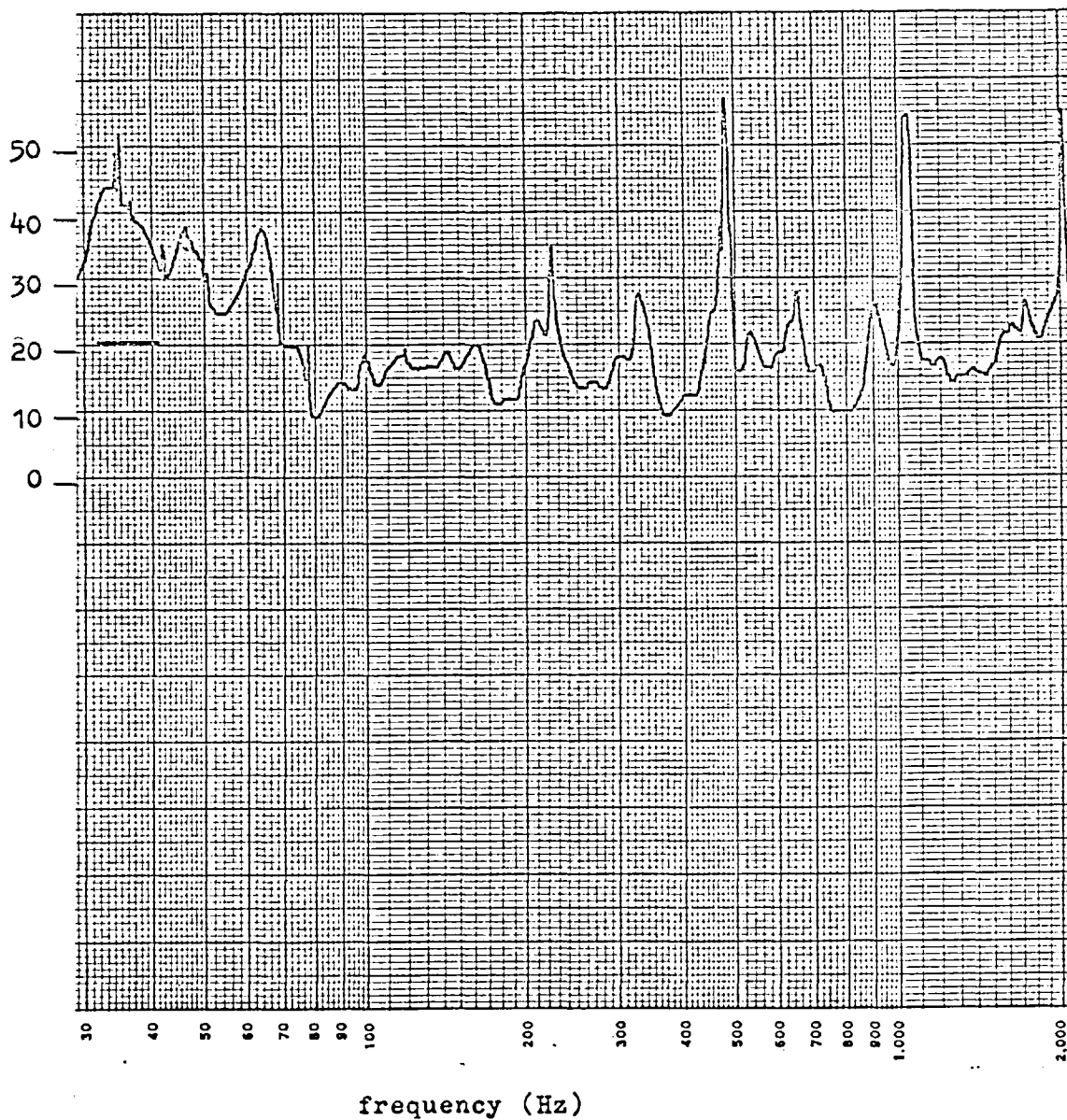


FIG 5.23 TRANSFER RESPONSE OF RECEIVING PLATE AT POSITION B;
EXCITATION AT POSITION 1

apparent mass
(dB re 1lb)

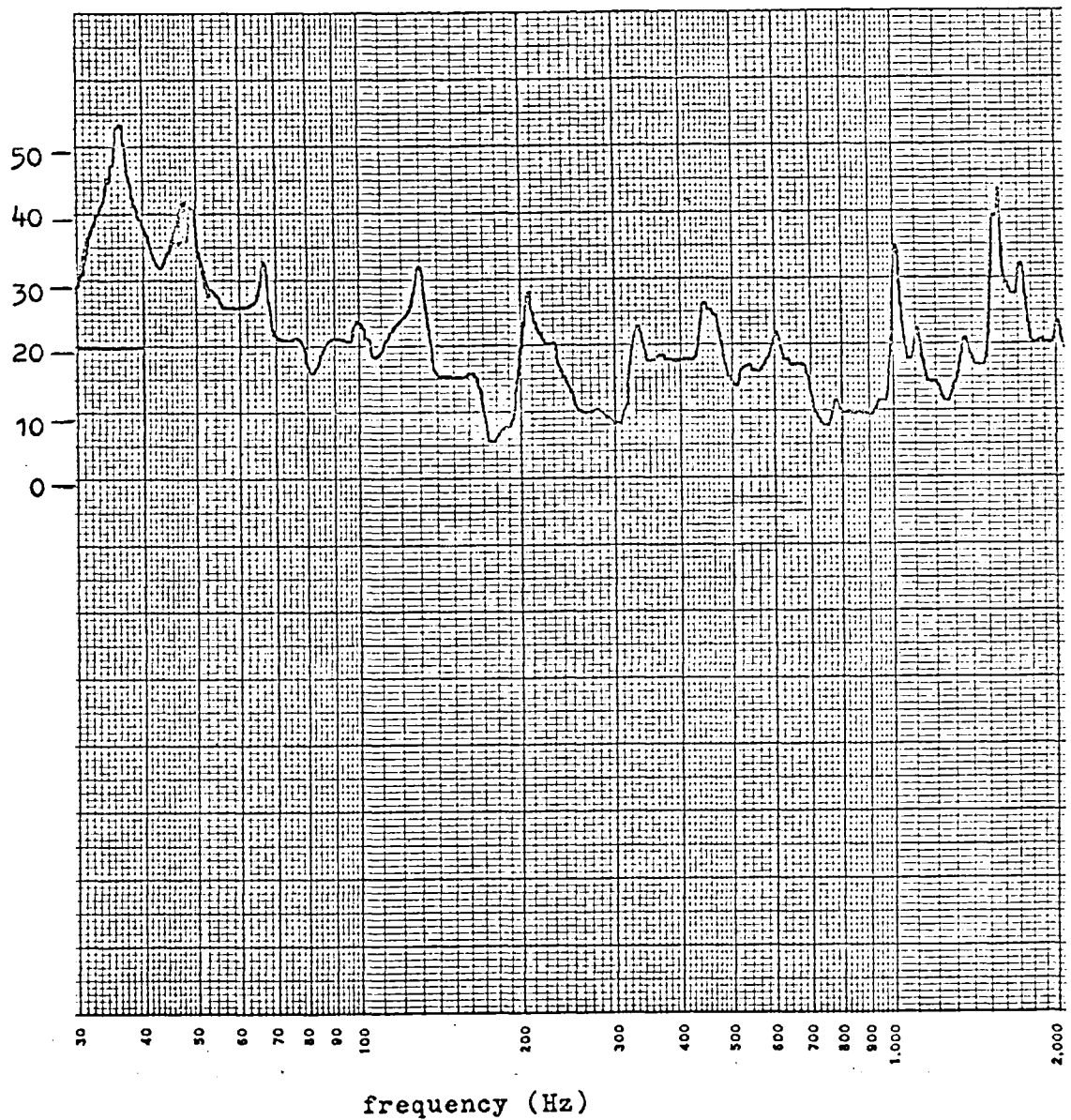


FIG 5.24 TRANSFER RESPONSE OF RECEIVING PLATE AT POSITION C;
EXCITATION AT POSITION 1

apparent mass
(dB re 11b)

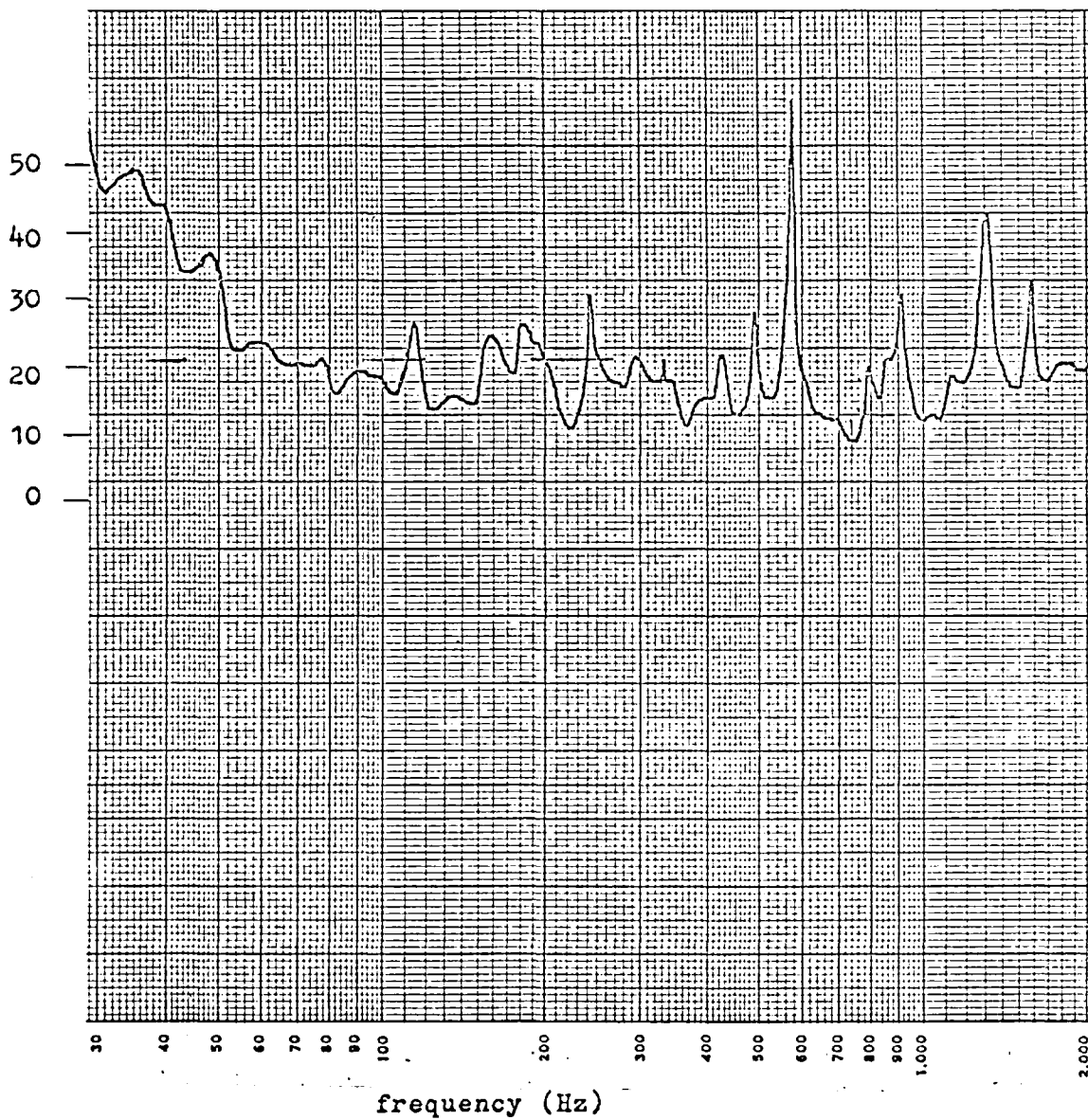


FIG 5.25 TRANSFER RESPONSE OF RECEIVING PLATE AT POSITION D;
EXCITATION AT POSITION 1

apparent mass
(dB re 1lb)

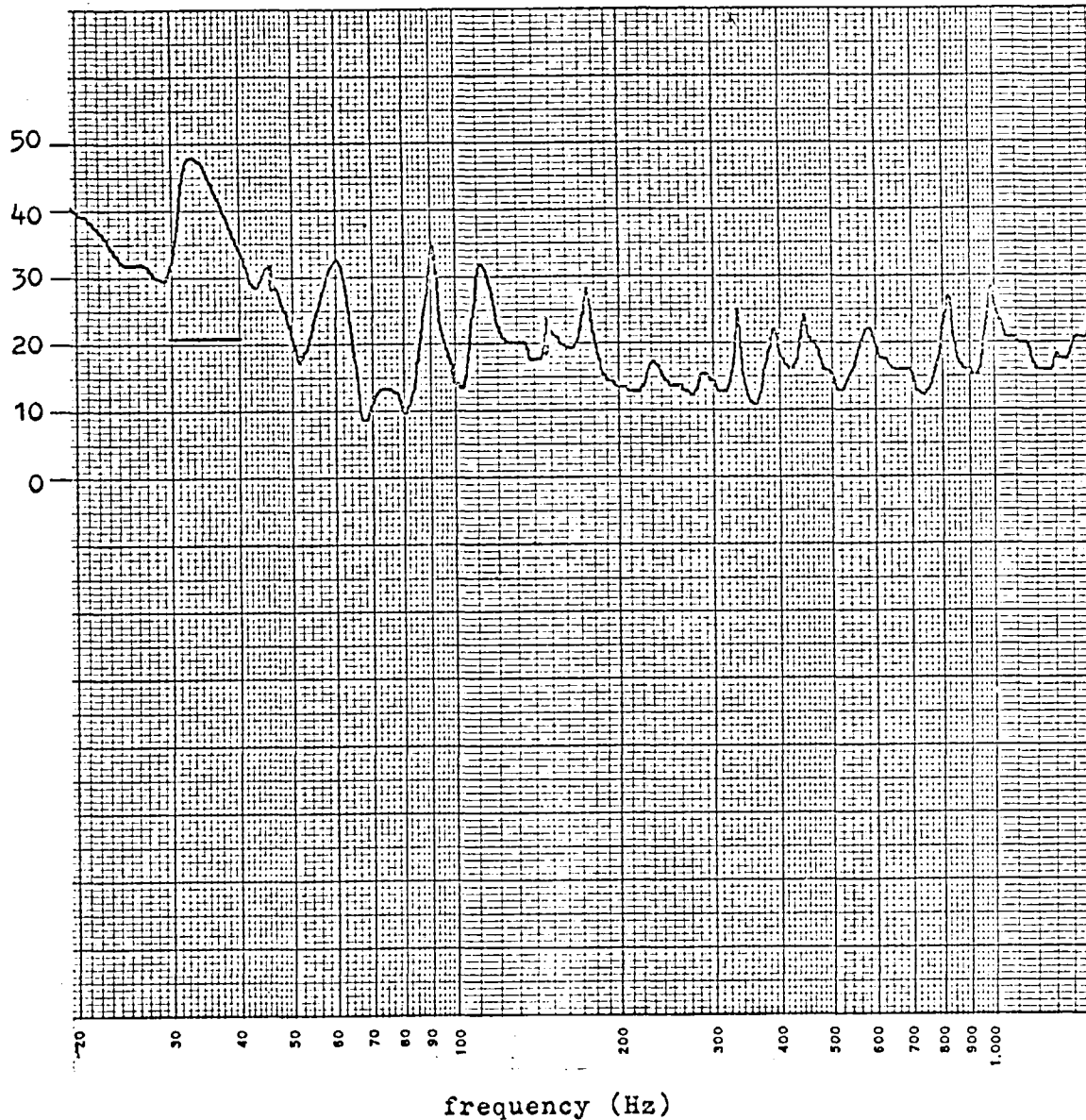


FIG 5.26 TRANSFER RESPONSE OF RECEIVING PLATE AT POSITION B;
EXCITATION AT POSITION 2

mode resonant frequency (Hz)	coupling parameter (lbf.sec/in)	coupling/modal parameter	computed/measured transfer impedance
25	0.10050	0.058490	0.8799
30	0.01709	0.008289	0.5558
42	0.02393	0.008289	0.4483
52	0.04695	0.013140	0.5790
68	0.21670	0.046360	0.7803
80	0.47980	0.087260	0.8708
100	0.40330	0.058670	0.9107
125	0.16750	0.019500	0.8863
145	0.05847	0.005867	0.7162
175	0.20920	0.017400	0.7945
200	0.45610	0.033180	0.9127
225	0.10530	0.006810	0.7778
275	0.36150	0.019130	0.9099
330	0.05939	0.002618	0.4083
360	0.27130	0.010970	0.7987
420	0.92680	0.032110	0.8719
450	0.35380	0.011440	0.7240
540	0.34000	0.009160	0.9043

computed modal mass 4.223 lbm

FIG 5.27 PLATE COUPLING AND MODAL PARAMETERS
COMPUTED BY PROGRAM 'SIGMA'

centre/support
response

10.0

1.0

0.1

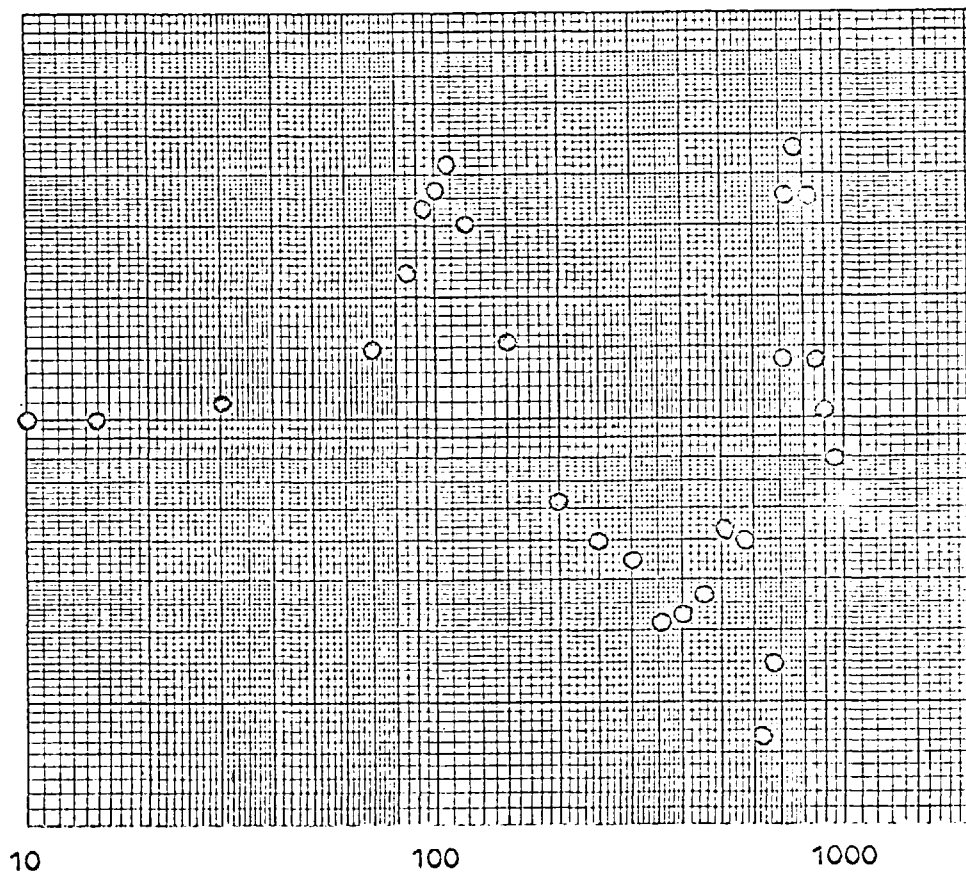


FIG 5.28 CENTRE/SUPPORT RESPONSE OF LAMINATED BEAM
PLOTTED AGAINST FREQUENCY

	centre/support response		loss factor	
	1st mode (110 Hz)	3rd mode (780 Hz)	1st mode	3rd mode
BEAM A	4.20	4.60	0.1923	0.06757
BEAM B	3.85	3.30	0.2062	0.09174
BEAM C	4.40	4.90	0.1852	0.06369
BEAM D	3.85	4.70	0.2062	0.06623

FIG 5.29 MODAL LOSS FACTORS OF LAMINATED BEAMS

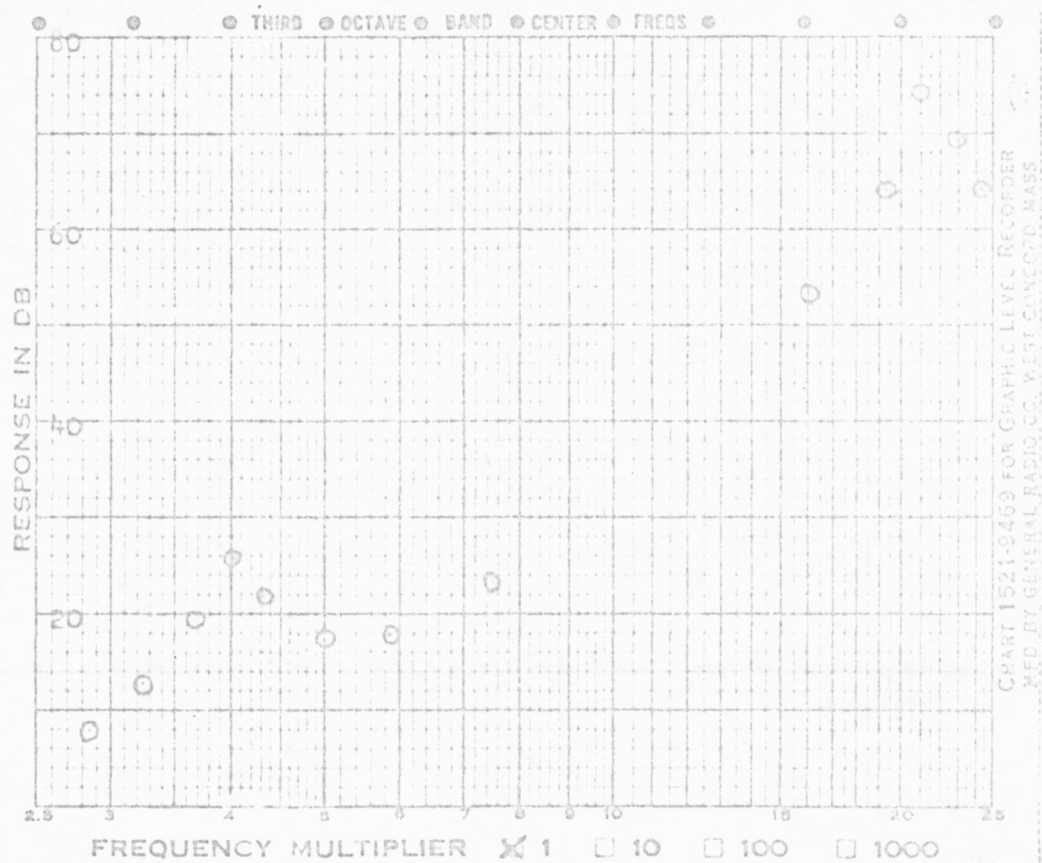


FIG 5.30 RESPONSE OF VIBRATOR ON EACH SET OF RUBBER MOUNTINGS
TO SINUSOIDAL EXCITATION

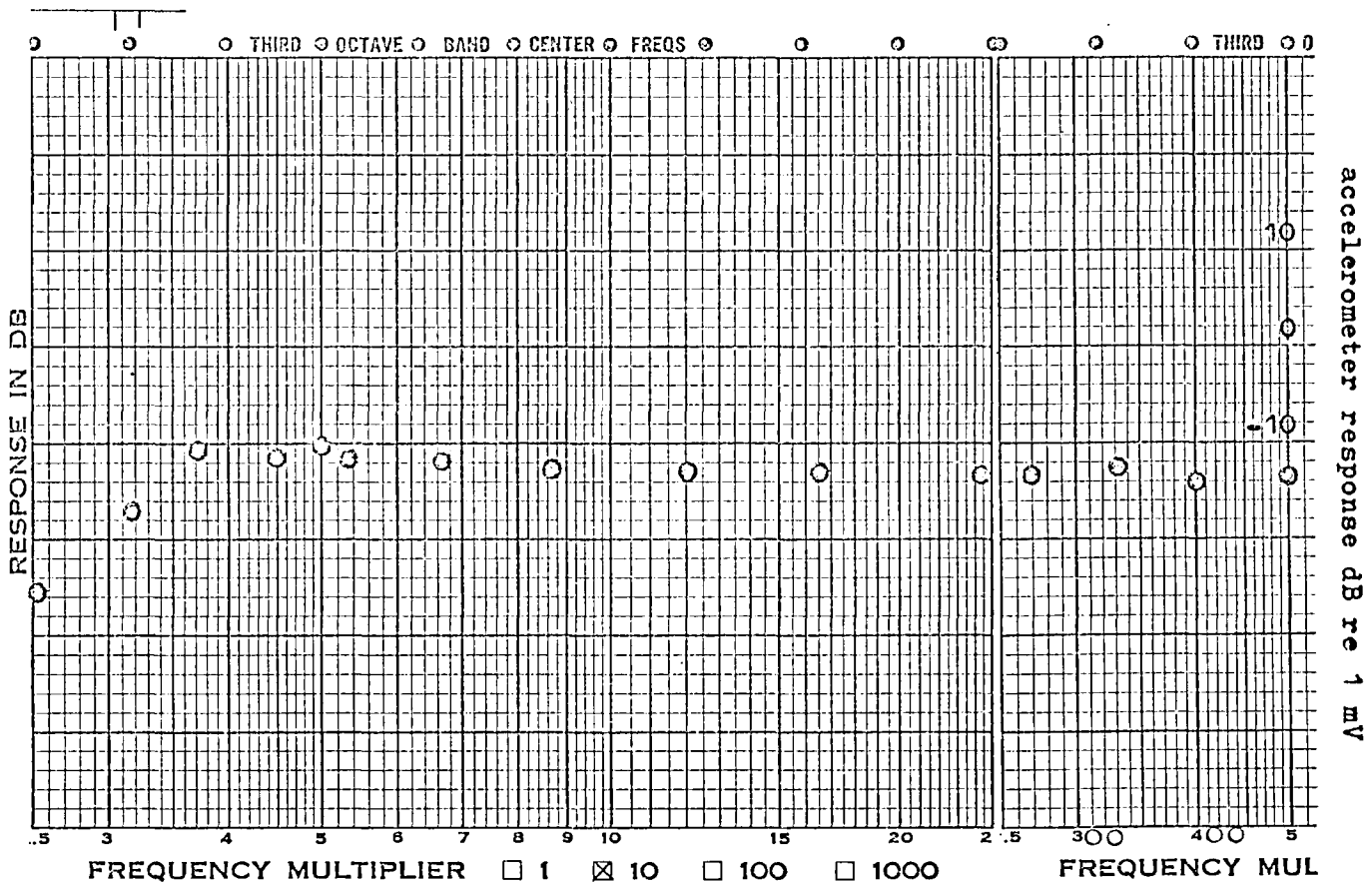


FIG 5.31 RESPONSE OF VIBRATOR ON SOFT ELASTIC SUSPENSION
TO SINUSOIDAL EXCITATION

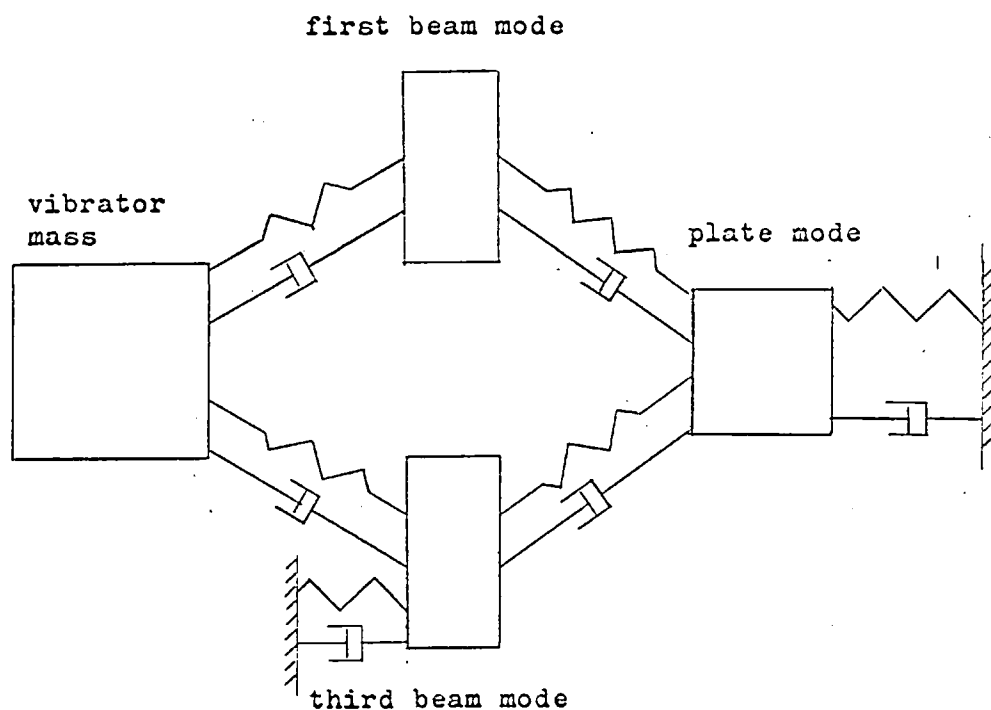


FIG 5.32 DIAGRAM OF VIBRATION TRANSMISSION PATHS FROM VIBRATOR TO PLATE THROUGH FIRST AND THIRD BEAM MODES

Accelerometer response dB re 1 mV

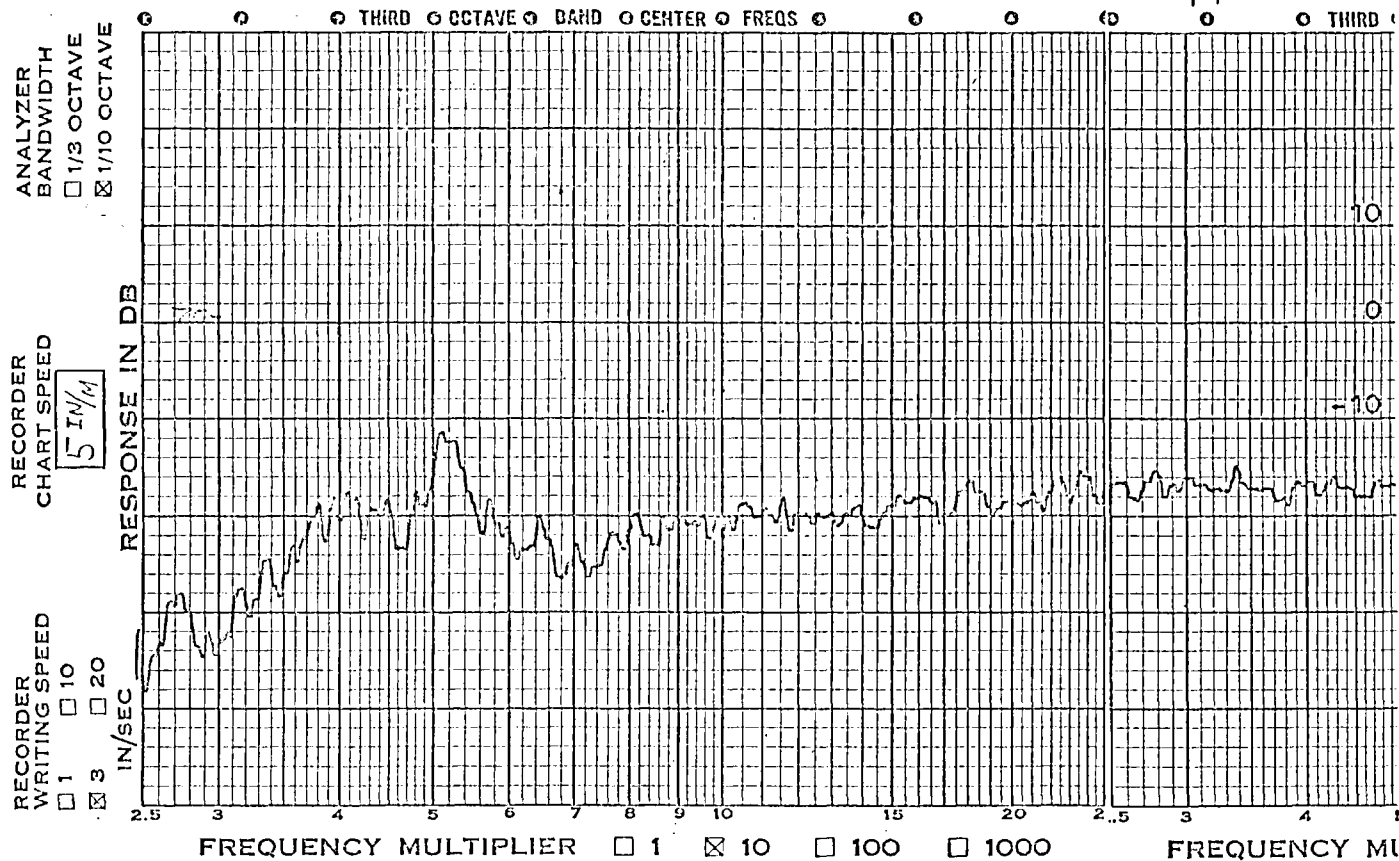


FIG 5.33 CHART RECORDING OF VIBRATOR FOOT RESPONSE

mounting system - 4Hz rubber mountings on laminated beams

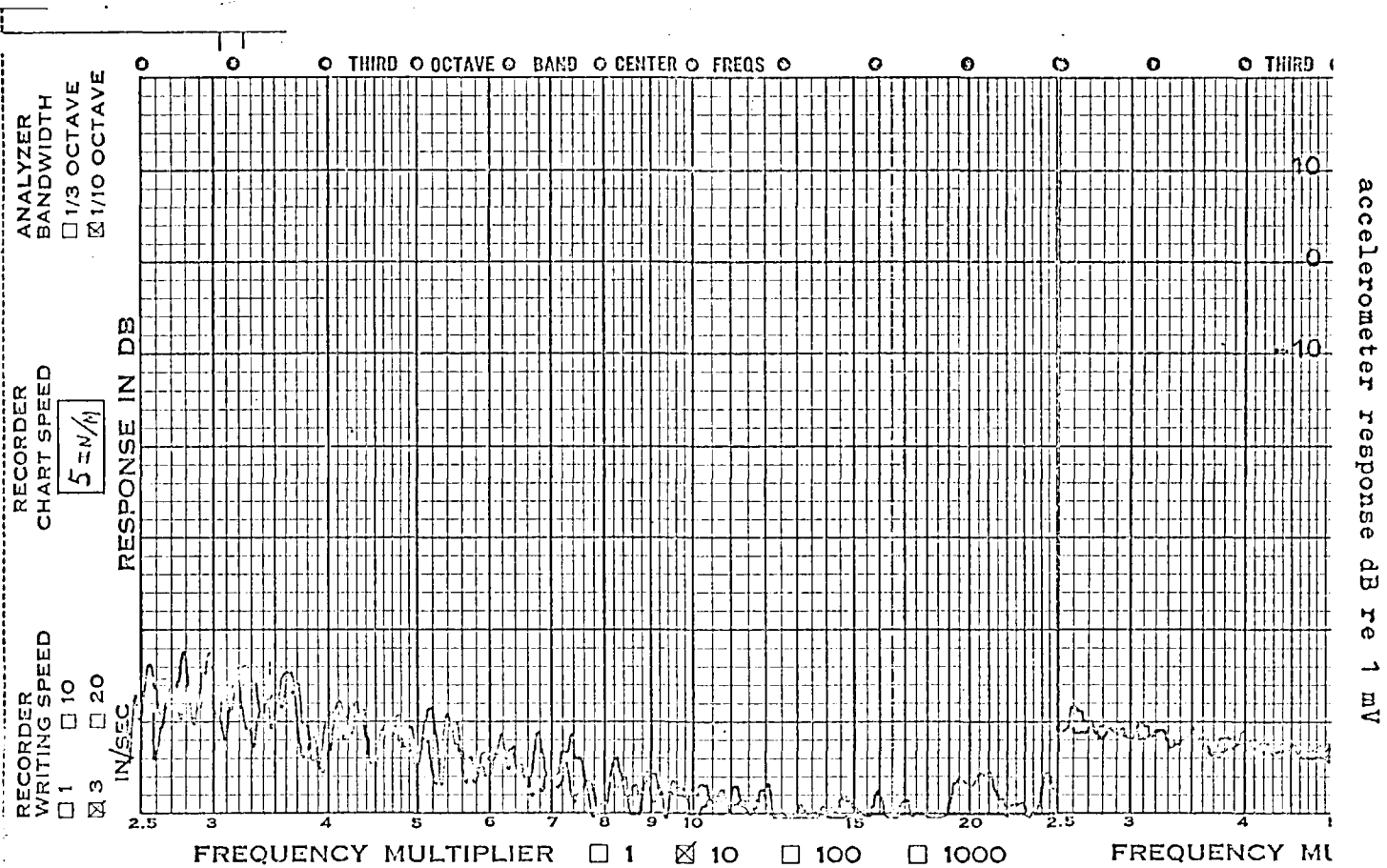
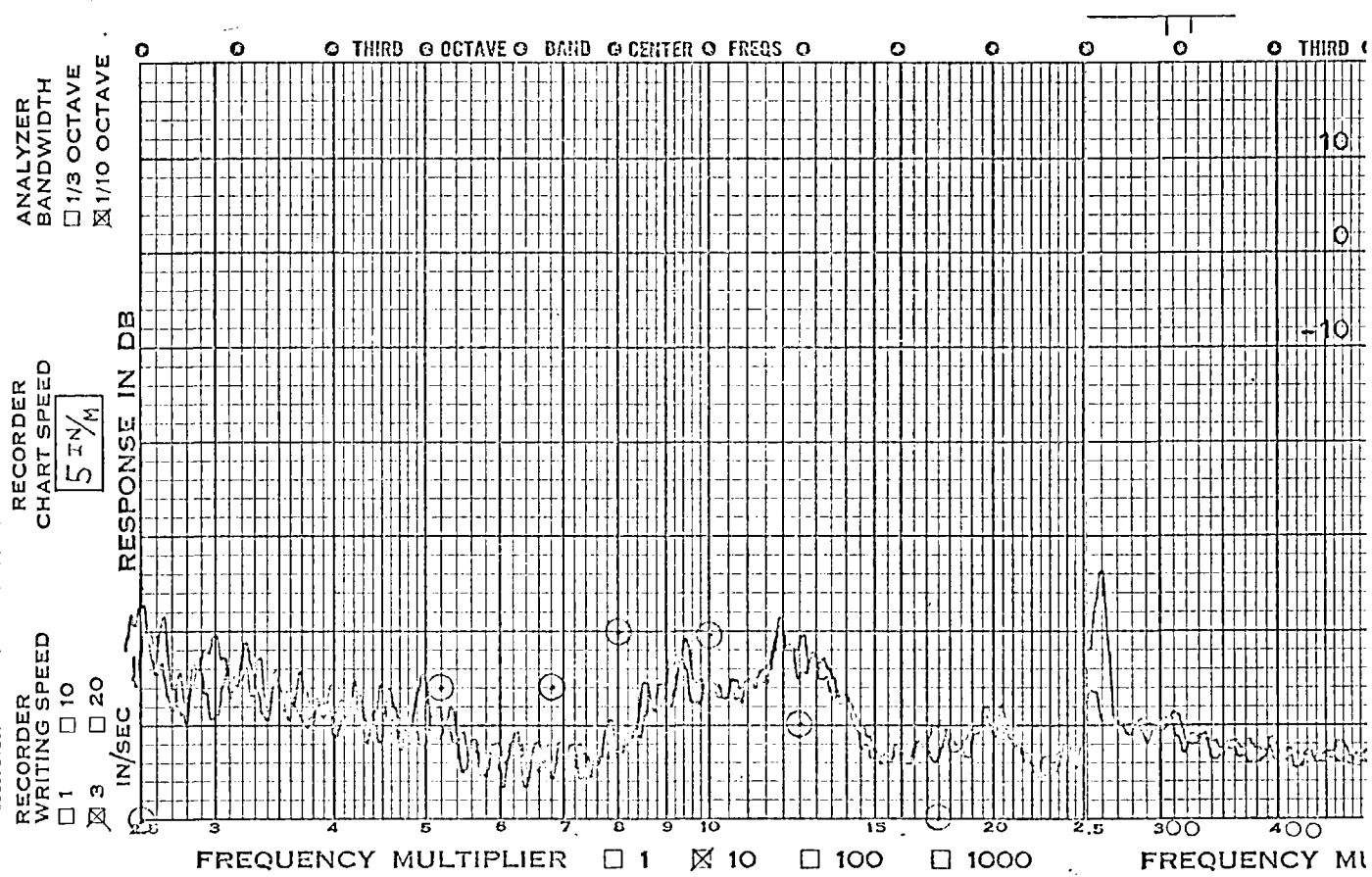


FIG 5.34 CHART RECORDING OF PLATE ACCELERATION RESPONSE
AT POSITION A

Accelerometer response dB re 1 mV

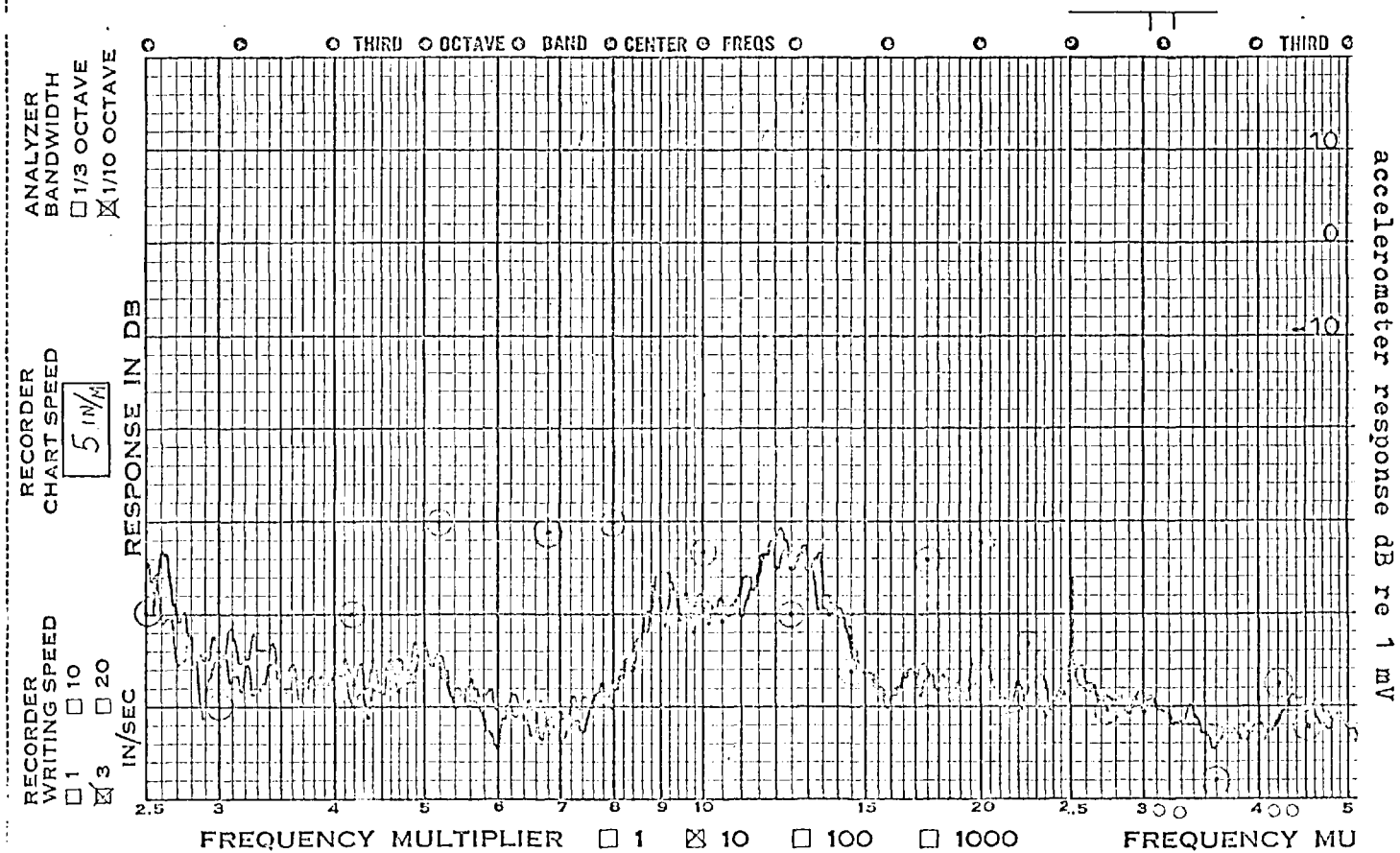


⊙ computed values

mounting system - 4Hz rubber mountings on laminated beams

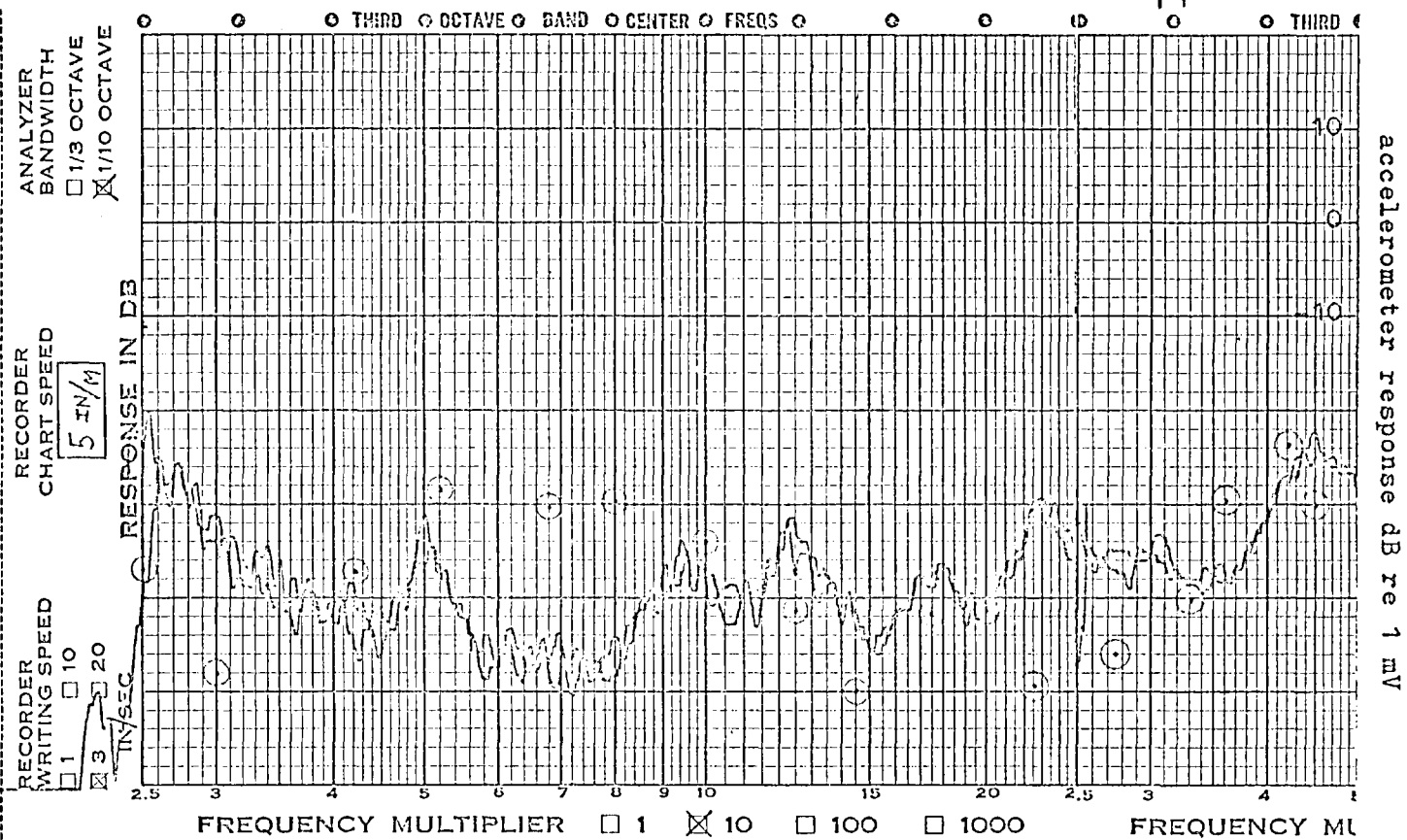
FIG 5.35 CHART RECORDING OF PLATE ACCELERATION RESPONSE AT POSITION A

FIG 5.36 CHART RECORDING OF PLATE ACCELERATION RESPONSE
AT POSITION A



⊙ computed values

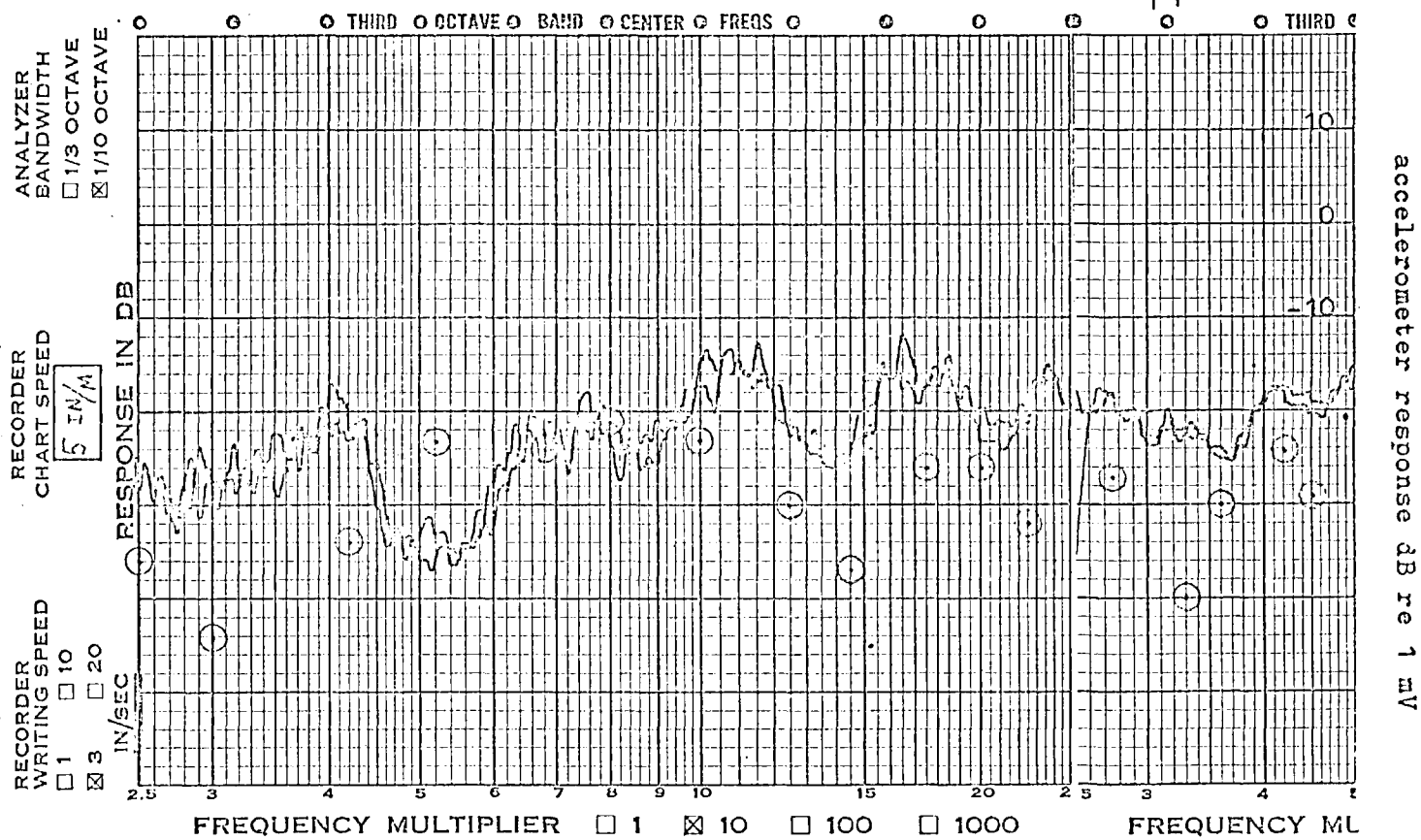
mounting system - 21Hz rubber mountings on laminated beams



⊙ computed values

mounting system - laminated beams

FIG 5.37 CHART RECORDING OF PLATE ACCELERATION RESPONSE AT POSITION A



⊙ computed values

mounting system - vibrator mounted directly on plate

FIG 5.38 CHART RECORDING OF PLATE ACCELERATION RESPONSE
AT POSITION A

modal parameters were combined to form one oscillator for each mode, the first and third modes of the beams therefore formed two intermediate oscillators. Each receiving plate oscillator was considered separately and the frequency range of excitation of this system was taken as the bandwidth of the receiving oscillator. Thus, the average vibrator to plate response ratio at each plate oscillator resonant frequency was obtained. The computed and experimental plate responses were compared by taking the computed plate response as the computed ratio of the experimental vibrator response (see figs 5.33 to 5.36).

Two oscillator systems were used to represent the vibrator attached directly to the laminated beams. The mass of the first oscillator was the sum of the vibrator mass and beam modal masses, the resonant frequency and coupling to the plate were determined by the beam mode stiffness. Thus two separate computations were necessary to accommodate the 1st and 3rd modes of the beams. The plate responses from each computation were then summed, to compare with experimental values (see fig 5.37).

The response of the plate to the vibrator directly mounted on it was also computed using a two oscillator system. The first oscillator was formed by the mass of the vibrator with the stiffness of the plate, and the second by each plate oscillator in turn, Coupling from vibrator mass to plate modal mass was dissipative, the parameter values were those obtained directly from the analysis of the point and transfer impedance. The computed plate response

levels were obtained from the computed ratio of the experimental vibrator response (see fig 5.38).

5.2.v Discussion of results

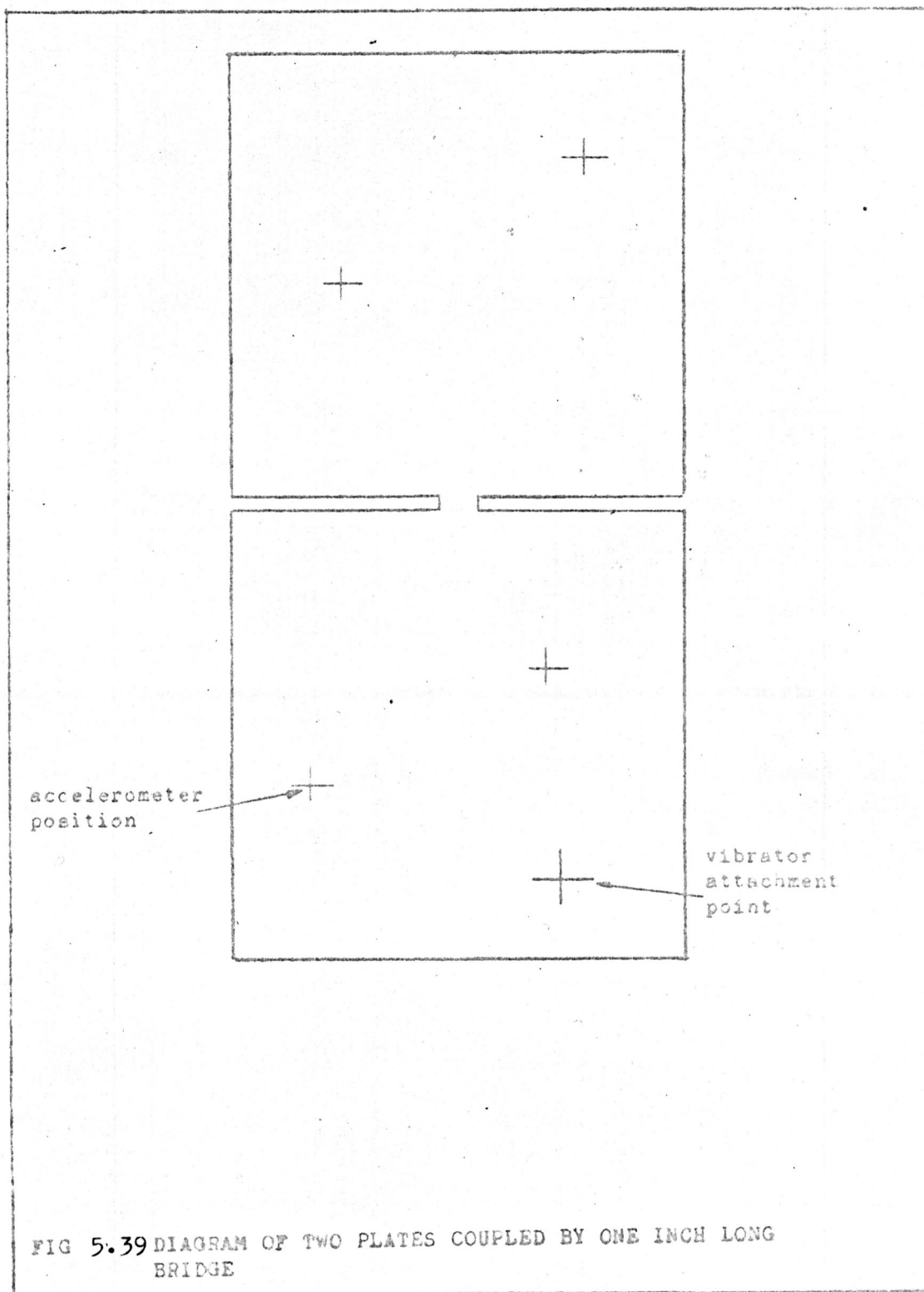
The comparison between computed and experimental plate response levels was generally good, although there were some large discrepancies. The computed results of the vibrator directly mounted on the plate should have shown good agreement as they only involved directly measured parameters. The reason for this discrepancy was thought to be due to changes in nodal patterns and resonant frequencies caused both by deformation of the plate under static load and multiple point excitation by the beam supports. Each of the beam supports was subject to the same excitation, but the point responses of the plate at these positions were measured independently. The impedance due to the mass of the beam supports was small with respect to the plate point impedance, (1.38 lbf.sec/in at 500Hz in comparison to 8.141 lbf.sec/in). Therefore this was not a cause of significant error.

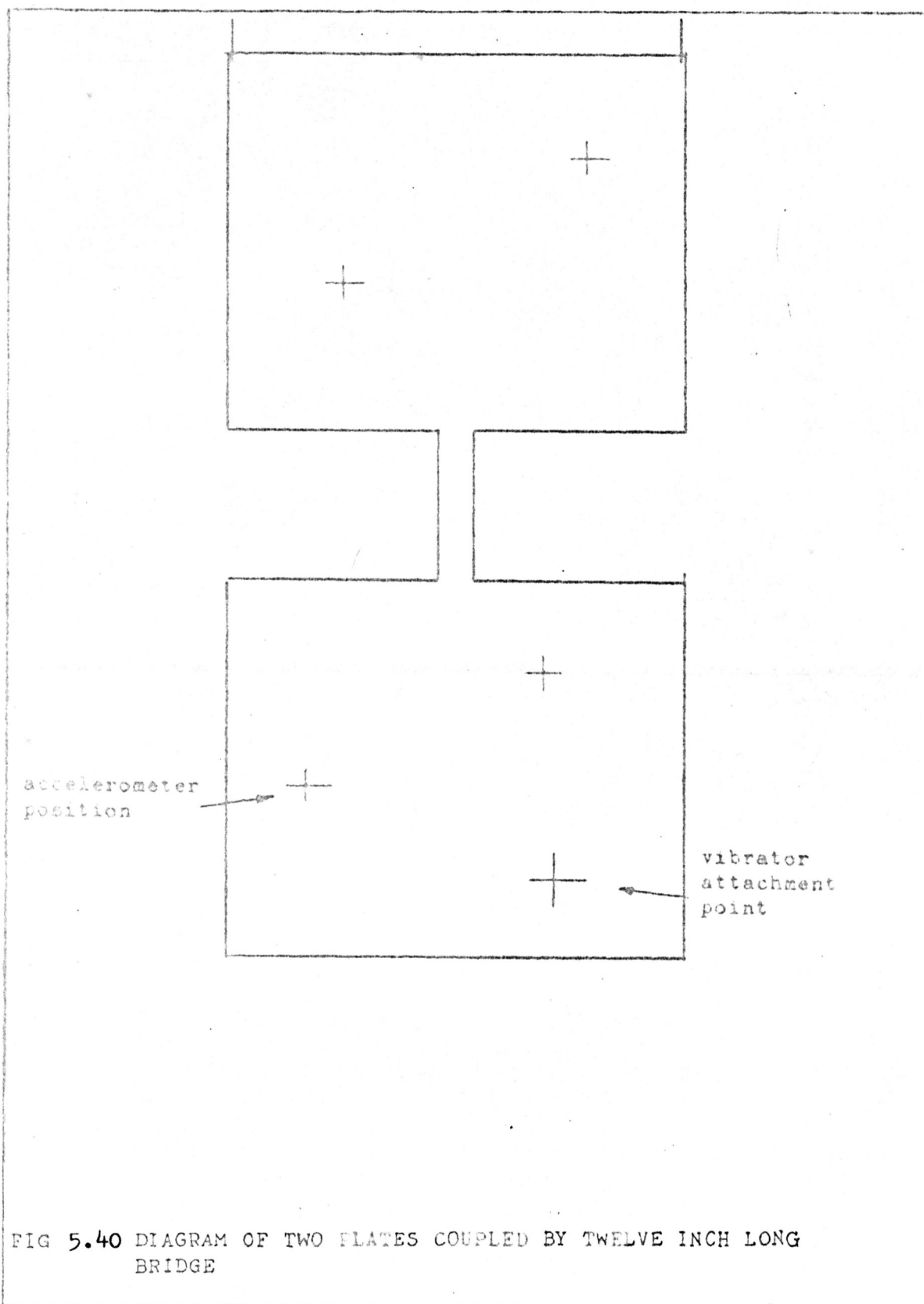
The vibrator response levels were the same for all mounting systems, because the plate impedance was much lower than that of the vibrator. The vibrator impedance at 25Hz was 36.6 lbf.sec/in, and at 500Hz was 732 lbf.sec/in.

5.3 TESTS WITH TWO EDGE COUPLED PLATES

5.3.1 Details of apparatus

A system of two coupled plates was formed by cutting a large





aluminium plate 72in x 36in x 1/8in across its centre parallel to the shorter side. Initially a short connecting bridge was produced by taking a cut one inch wide. The width of the bridge was then reduced to 3in and the vibration response levels of the plates were measured.

A bridge exhibiting resonant frequencies over a wide frequency range was produced by widening the cut to twelve inches. This lengthened bridge thus became a beam-like intermediate structure. The width of the bridge was again reduced in stages to one quarter of an inch. The plates were suspended vertically, one above the other by soft elastic cords attached to the top corners of the upper plate (see figs 5.39, 5.40). The lower plate was excited with white noise laterally by an electro-magnetic vibrator which rested on the laboratory floor. The vibrator attachment point was chosen to avoid exciting the plate at a position where a large number of nodal lines existed, and therefore did not lie close to an edge, diagonal or centre line. The accelerometer positions were also chosen according to this criteria.

5.3.ii Determination of modal oscillator parameters

a) Plate

It was not possible to measure the point and transfer responses of either plate edge due to the requirement of a 3/8in diameter hole for the impedance head stud. However, as the vibration transmission characteristics between structural oscillators are dependent only on relative parameters, the use of two identical plates implied

identical parameters and made their measurement unnecessary. Each plate mode was considered to be independent of all others because of wide frequency separation. From [7] the modal frequency separation was given by the relationship:

$$df = \frac{h \cdot C_1}{\sqrt{3 \cdot A}}$$

where df was the modal frequency separation,

h was the plate thickness,

C_1 was the flexural wave speed,

A was the plate area.

Thus the modal spacing was 13.85Hz, and due to the low damping the bandwidths of modes up to 20 Hz were less than 1Hz. Each mode of one plate was considered to have negligible coupling to all except the same mode of the other plate, hence the system of two coupled plates was represented by an array of two oscillator systems. The only parameter which governed the response ratio of this system was the ratio of coupling to modal parameter divided by modal loss factor, which was also the value of the coupling factor at resonance. Two identical dissipatively coupled oscillators must have a response ratio of 5dB or more, because the ratio of coupling to modal damping can not exceed unity, (see fig 3.07). Fig 5.43 shows the response ratio of the plates when connected by one inch long bridge which was three inches wide, it was seen that the minimum response ratio was close to 0dB and the average ratio from 30Hz to 2KHz was 3.5dB.

Although it was possible for this coupling to consist of both conservative and dissipative elements, in the absence of a method for calculating the relative contribution of each, it was assumed that the plates were conservatively coupled.

When the bridge length was increased from one inch to twelve the dimensions of the plates were also altered. Therefore mode shapes and coupling between them and the plate edge were also affected. The mean plate response ratio was therefore used in conjunction with fig (3.09), for finding the mode coupling factor. It was permissible to use the mean of these ratios as they were evenly distributed and did not differ from the mean by more than 4dB. Response ratios of a one inch long by 3/16 inch wide bridge were taken from results in [31], in order to estimate the average response ratio of the 1/4 inch wide bridge used in the present work. This value was 7dB.

It was necessary to know the loss factor of the plate so that the resultant plate to bridge coupling parameter could be calculated. Plate loss factors were obtained by measuring the rate of decay of free vibrations. The plate was struck a sharp blow, and the accelerometer response passed through a 1/3rd octave filter to the chart recorder (see fig 5.42). The logarithmic movement of the pen produced a straight line the gradient of which was proportional to loss factor. The loss factor was obtained from the following relationship:

$$\eta = \frac{13.8}{w \cdot T}$$

where w was the centre frequency of the filter (rad. per sec.),
 T was the time (secs.) for the acceleration level to fall
 60 dB.

The modal mass of a lightly damped plate was shown in [1] to be one quarter of the total mass, this value was used in the present work.

b) Bridge

The modal and coupling parameters of the twelve inch long bridge were found as follows. The bridge and plates were of aluminium alloy which had very low damping; therefore it was assumed the bridge exhibited conservative coupling. The value of the coupling parameter at either end of the bridge was found from equation(6).

Bridge loss factors were assumed to be the same as those for the plate. As they were low anyway a negligible amount of energy was dissipated and only a small error was involved.

Bridge resonant frequencies were assumed to be those produced with simply supported ends. It was expected that these mode frequencies would be too low, but it was not possible to assess accurately the constraining effect at the bridge ends. Precise values of bridge resonant frequencies were not essential for evaluating magnitudes of response levels, only the ratios of plate to bridge mode frequencies were relevant.

The modal mass of the bridge was taken as 0.49 of the total mass, which gave, with respect to the plate, a modal mass ratio of

15:1 and 180:1 for the three inch and one quarter inch widths.

5.3.iii Measuring technique

The technique for measuring the plate responses was completely straightforward. A block diagram of the experimental apparatus and measuring equipment is shown in fig (5.41). The plate was excited with a steady white noise excitation and the responses of the plates measured at two positions with an accelerometer. The plate response was taken as the mean of these two readings. It was found that there was very little advantage in using more positions as the mean value obtained from more than two positions was seldom more than 1/2 dB different. One third octave filtering was used.

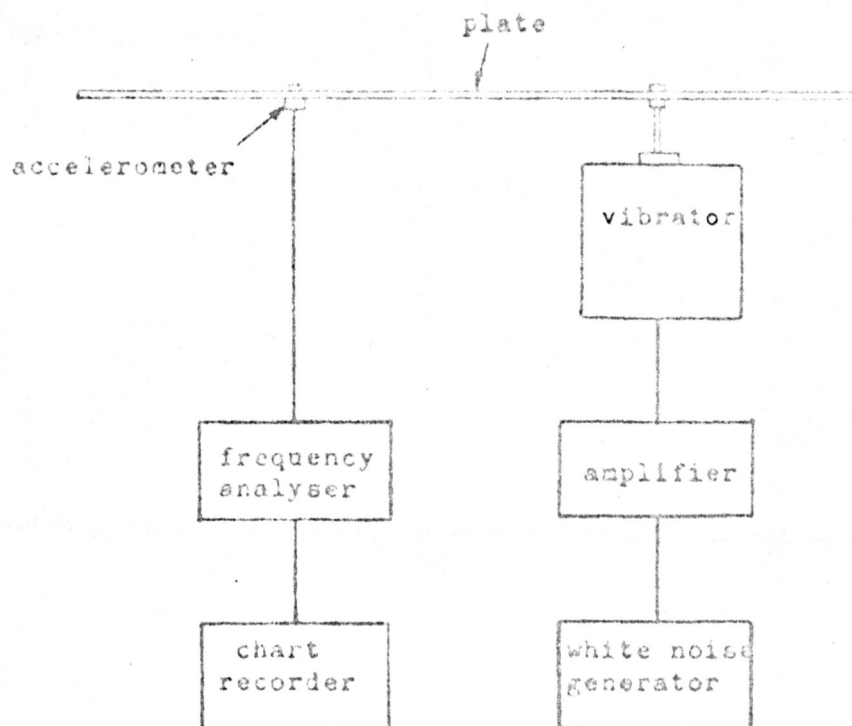
5.3.iv Results

The plate response ratio when connected by a bridge one inch long and three inches wide is shown in fig (5.43). The coupling factors at resonance which were derived from these ratios and those obtained from [31] are shown in fig (5.44).

A three oscillator system representing modes from both plates with an intermediate bridge mode was used to compute theoretical plate response ratios. The energy flowing between plate modes was evaluated by summing the flow through the transmission paths formed by the two adjacent bridge modes. Computed and experimental plate response ratios are compared for the three and one quarter inch bridge widths in figs (5.45, 5.46).

5.3.v Discussion of results

From fig (5.45), it is seen that with the plates connected by



white noise generator
 amplifier
 vibrator
 accelerometer
 frequency analyser
 chart recorder

Dawe type 419C
 Derritron type 250 WLF
 Derritron type VP4
 Endevco type
 General Radio type 1564 A
 General Radio type 1521

FIG 5.41 DIAGRAM OF PLATE EXCITATION AND RESPONSE MEASUREMENT APPARATUS

loss factor

0.1

0.01

0.001

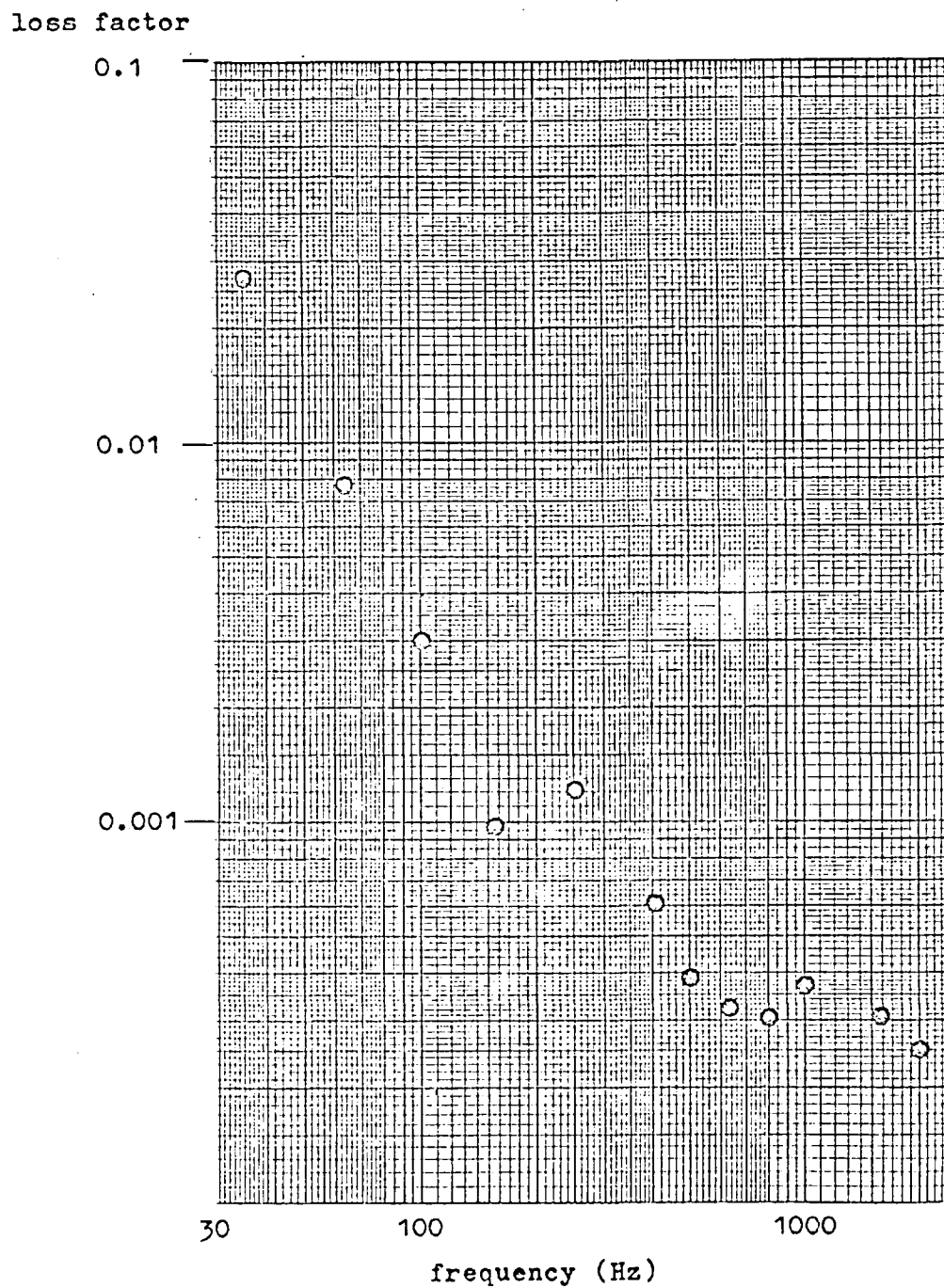
30

100

1000

frequency (Hz)

FIG 5.42 PLATE LOSS FACTOR PLOTTED AGAINST FREQUENCY



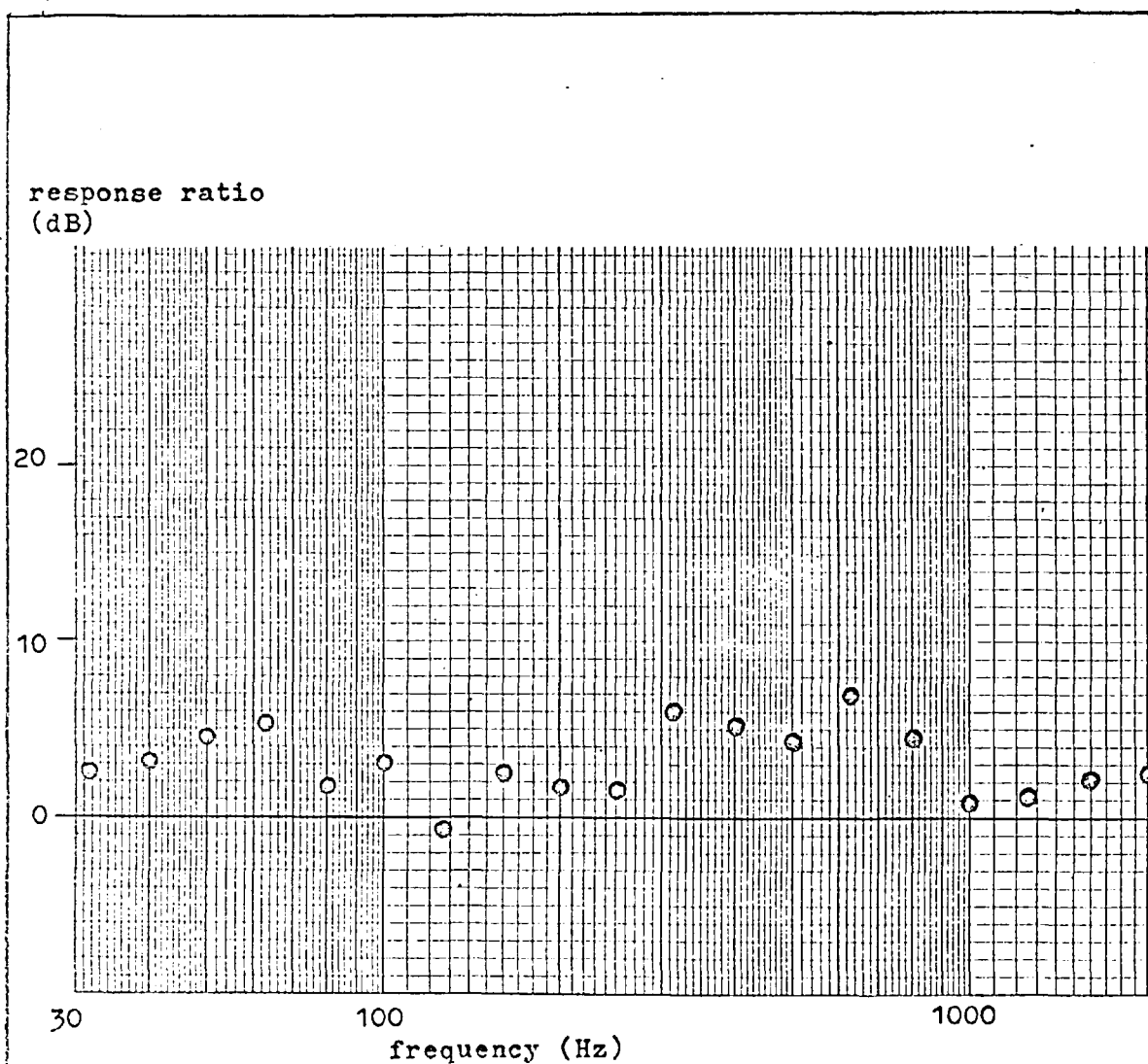


FIG 5.43 PLATE RESPONSE RATIO WHEN CONNECTED BY A THREE INCH WIDE BY ONE INCH LONG BRIDGE, PLOTTED AT THIRD OCTAVE CENTRE FREQUENCIES

bridge resonant frequency (Hz)	mode number	mean plate coupling factor	
		3in. bridge width	1in. bridge width
78	1	2.420	1.220
312	2	2.360	1.060
701	3	2.290	0.919
1248	4	2.230	0.815
1950	5	2.160	0.730

FIG 5.44 MEAN PLATE COUPLING FACTORS AT RESONANT FREQUENCIES
OF TWELVE INCH BRIDGE

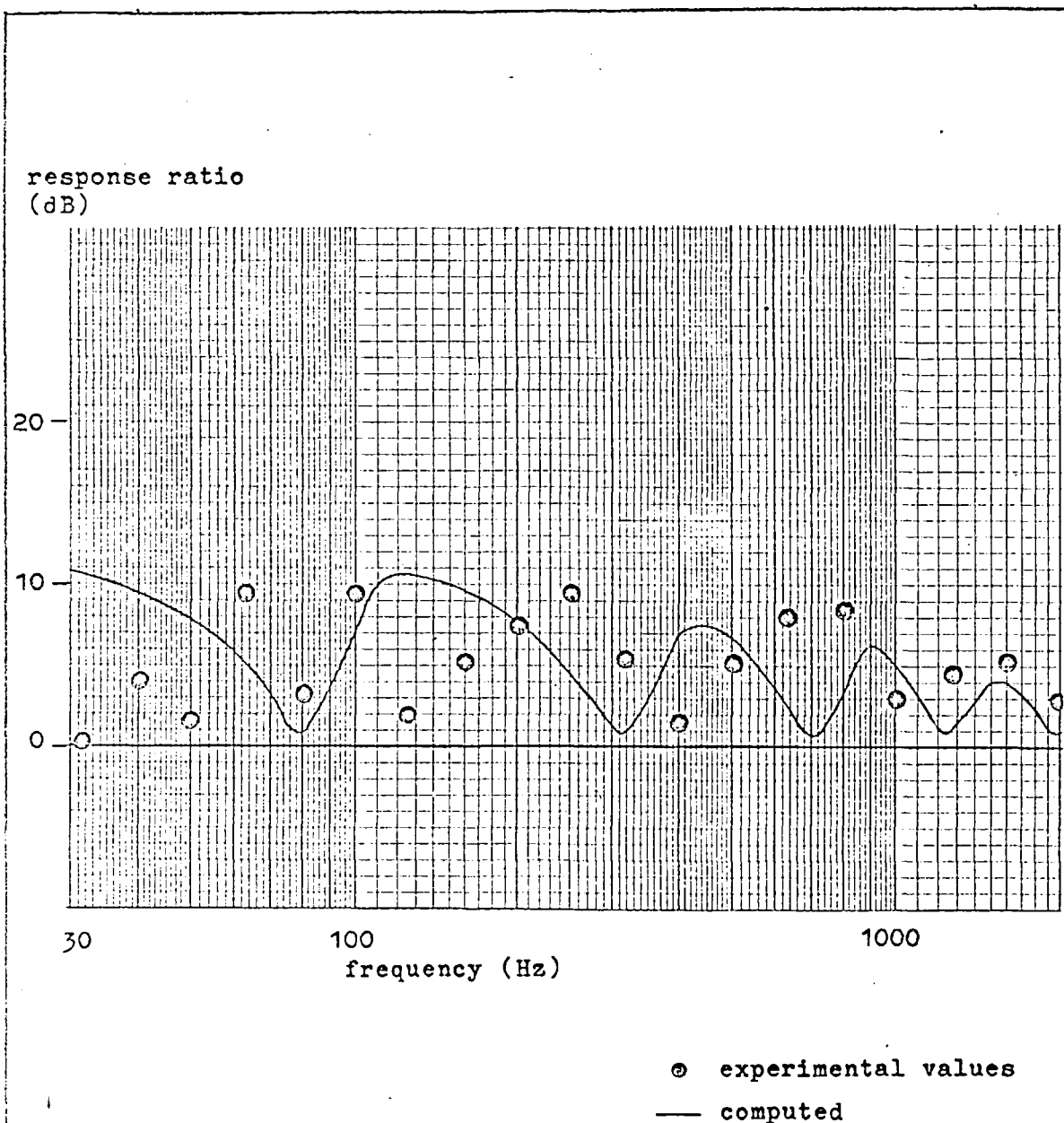


FIG 5.45 PLATE RESPONSE RATIO WHEN CONNECTED BY A THREE INCH WIDE BY TWELVE INCH LONG BRIDGE, PLOTTED AT THIRD OCTAVE CENTRE FREQUENCIES

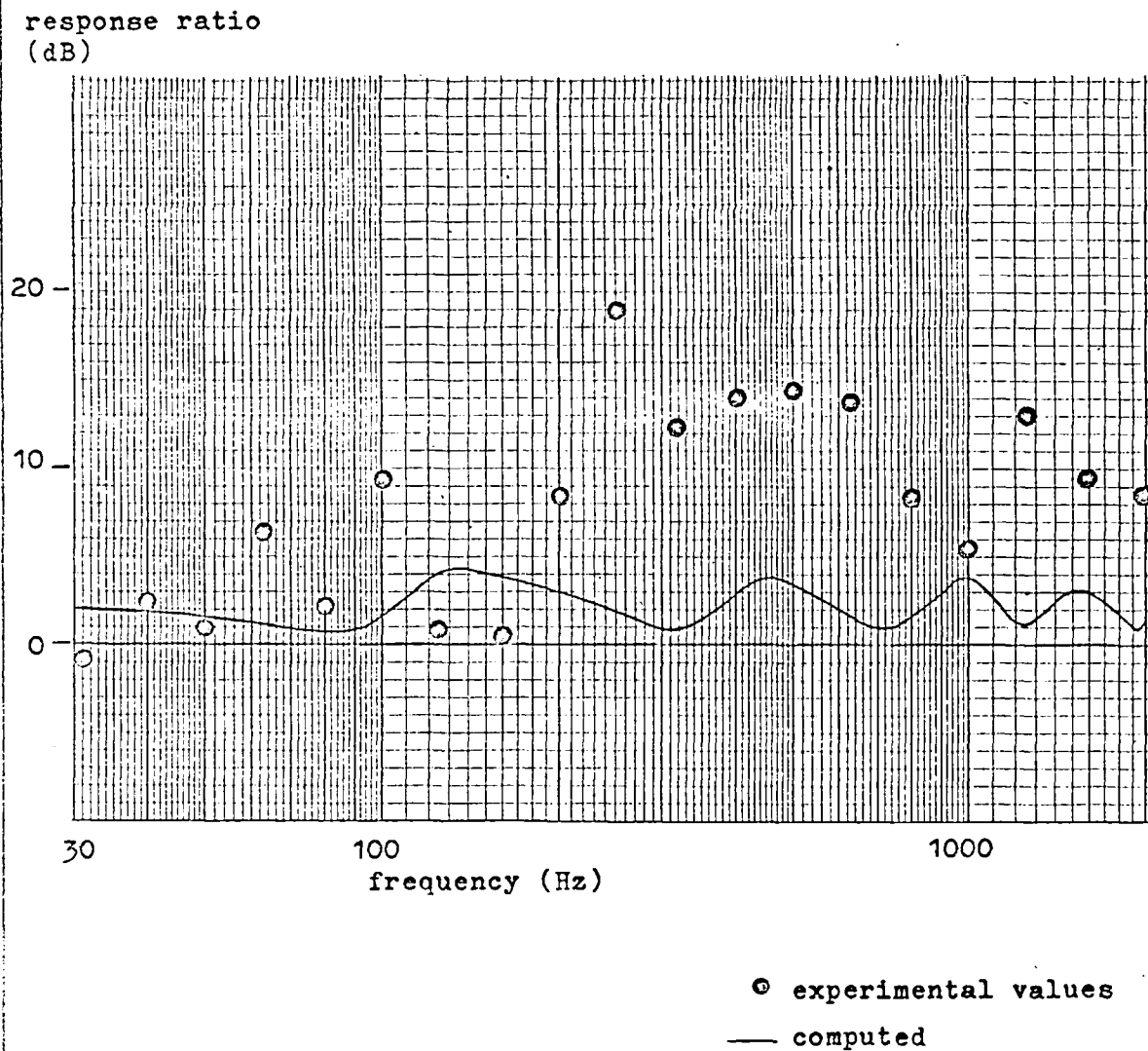


FIG 5.46 PLATE RESPONSE RATIO WHEN CONNECTED BY A QUARTER INCH WIDE BY TWELVE INCH LONG BRIDGE, PLOTTED AT THIRD OCTAVE CENTRE FREQUENCIES

a three inch wide by twelve inch long bridge, the computed and experimental plate response ratios compared well. The frequencies at which the experimental ratio reached a minimum value did not however coincide with computed predictions. This was due to the assumption that bridge mode frequencies corresponded to those produced with simple supports. The constraint imposed by the plate edge clearly caused an increase in the bridge resonant frequencies, but the frequency interval between successive minimums which would not have been affected did correspond closely. Allowing for this frequency discrepancy the magnitudes of the ratios of experimental and computed results agreed well. However, a major cause of the discrepancy between experimental and computed results may have been due to coupling by torsional modes of the bridge. It was impossible to deduce the relative effects of flexural and torsional bridge bending from the response ratios of the plates connected by the short one inch bridge. The coupling factors derived from these ratios therefore contained the combined effect of flexing and torsion. The theoretical first torsional mode of the three inch wide bridge (free or encastre ends) occurred at 434Hz, but there was no evidence of a significant decrease in plate response ratio at this or even the higher torsional mode frequencies. With the plates connected by the one quarter of an inch bridge, agreement between computed and experimental results was not as good. However, the trend of smaller response ratios at lower frequencies was correctly predicted. At these frequencies the vibration transmission was proportional to

the mass of the intermediate oscillator as the coupling factor of the one quarter of an inch bridge widths were only slightly different.

The flexural coupling stiffness of the one quarter of an inch bridge, unlike the three inch bridge was low enough to cause significant reduction in the plate coupling factor at the higher modes. This caused the computed response ratios to increase in the middle range of plate mode frequencies. The ratios however became smaller at the higher frequencies due to the convergence of bridge flexural mode frequencies.

The experimental results show a marked increase in response ratio in the middle range, but a much larger increase than predicted, this may have been due to the large reduction in torsional stiffness.

There were no torsional modes of the one quarter of an inch bridge within the range of experiment, (the first mode occurred at 4.67 kHz), therefore the torsional stiffness of the bridge was just reduced by a factor of 144 due to its length increase from one to twelve inches.

5.4 CONCLUDING REMARKS

The foregoing experiments have shown that the analysis developed in the previous chapters is valid.

The modal and coupling properties of a structure may be obtained by a technique utilising its point and transfer responses.

These data then enable the response of a coupled structure to be predicted, provided they are not altered by the process of coupling. For example, large bolts may introduce a significant mass at the structural interface, or the structures may be fastened together over a much larger area than was used for the preliminary tests.

The tests with two plates connected by a bridge showed that a simple system, which comprised one modal oscillator from each structure, could be used to analyse their response levels. The modal frequency separation must be wide with respect to each mode bandwidth otherwise more oscillators from each structure would be required. It was shown to be possible to estimate the effect of introducing an intermediate structure, provided its parameters were known, from the response ratio of two structures when first connected directly to each other.

However, it was shown in the experiments that difficulties may arise if there is more than one process of coupling, for example flexing and torsion, whose individual effects can not be deduced.

CHAPTER SIX

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

6.1 CONCLUSIONS

It was shown in chapter two that the general equation of motion of a continuous structure could be developed into a series of equations, each being the equation of motion of a lumped parameter modal oscillator. This set of equations was such that the modal excitation could be expressed in terms of the force applied at a point of the structure, and a coupling element transmitting a part of this force to each mode. A system of connected structures is thus transformed into a system of coupled oscillators.

A transmission path was defined as a series of oscillators comprising one from each structure, each oscillator was therefore in general a constituent of many transmission paths. The response of an oscillator could be calculated approximately by superposition of the responses from each of these transmission paths. The superpositioning became more accurate with weaker coupling.

To calculate the oscillator and coupling parameters of a structure, a method was presented, in chapter four, which utilized data from point and transfer response tests. The method was not capable of evaluating from this data alone the absolute values of these parameters but only their ratio. However, it was shown in chapter

two that this ratio was sufficient for the calculation of response levels of intermediate and receiving oscillators as a ratio of the excited oscillator response.

When two structures are connected together their respective oscillator coupling parameters are joined in series and it is necessary that the relative magnitudes of their coupling parameters be known, in order to calculate the resultant coupling parameter. This then requires further data, either oscillator loss factors or masses for all modes of vibration.

However, the number of modes is likely in most cases to be extremely large, and many will have resonant frequencies outside the range of any response measuring equipment. Therefore a knowledge of the parameters of a limited number of modes enables only their response ratios to be calculated.

It is desirable that if any improvement in vibration isolation efficiency of a system is to be undertaken, the modal coupling factors be known. In some circumstances, as was shown in chapter three, the apparently anomalous effect of greater vibration transmission with weaker coupling, may take place when receiving oscillator coupling factors (at resonance) have magnitudes greater than unity.

The equations of motion of a coupled oscillator system may be solved for random excitation on a computer, however the large numbers of modes of vibration present in many typical structures pose problems as regards the size of computer. In chapter two it was shown that the response of an oscillator in a receiving struc-

ture could be calculated by summing the energy passing to it from all transmission paths. This technique enables the response of large systems with many oscillators, requiring more computer storage space than available, for example, to be calculated. Also, the computing time required to solve the equations of motion is proportional to at least the square of the number of oscillators, (depending on the method used), and it is more economical in computing time to solve all but the smallest systems by the transmission path approach.

6.2 SUGGESTIONS FOR FURTHER WORK

The analysis presented in this work enables the response levels of structural vibration to be calculated, with the aid of an experimental method to obtain data of coupling and modal parameters. Clearly a major weakness at present is the necessity of this experimental stage. This can be a time consuming procedure lasting days, where as the associated computations are carried out in minutes. A detailed study of the dynamic characteristics of structures to permit modal and coupling parameters themselves to be calculated precisely would be a major step forward. Also it has been shown that very high vibration attenuation between structures can be achieved, over a wide frequency band, by intermediate modes of vibration moving out of phase with respect to a receiving mode. Therefore ways of incorporating this in practical structures would be highly desirable.

APPENDIX ONE

COMPUTER PROGRAMS FOR EVALUATING THE RESPONSE LEVELS OF COUPLED
OSCILLATORS

The development of a general method of solution of the equations of motion of a coupled oscillator system, which is accurate and economical in both computing time and storage locations has been a major theme of this project.

The response of an oscillator when excited by a broadband random force was shown in chapter two part 2.3 to require the squared modulus of the frequency response function to be integrated over the bandwidth of the excitation. Clearly the amount of computation will be a function of the size of the system matrix, or number of oscillators, and the width of the excitation spectrum. To find the exact response levels in a system it is necessary to solve the complete matrix, and perform the numerical integration at as many frequencies as is possible without incurring significant round-off error. Alternatively if the system is to be evaluated in terms of individual transmission path response, a method of solving small matrices quickly is required.

A program solving the system matrices by the Gauss elimination method was written and given the name 'MAG'. This program was more suitable both for small and very large systems, because it required little compiling. A small system could therefore be

executed quickly, whilst with a large system more core locations were left available for data.

A program using the pole-zero analysis method was written and named 'MEIG'. Use was made of several routines developed at Imperial College, for finding the eigenvalues and eigenvectors of mechanical systems. Full details of these are given in [32]. The performance of this program was such that it was best suited to medium sized systems. It required a greater amount of compiling, and most of the execution time was spent evaluating the eigenvalues and eigenvectors of the system, but very little extra time was required to compute oscillator response values and to integrate them over the excitation frequency bandwidth (see fig. A1.1).

Description of Program 'MAG'

Main Program

The main program is divided into seven sections.

1) In this section the basic constants of the computation are specified.

These constants denote the number of oscillators in the system, the number of times the program is to be executed, the frequency range of the random excitation and the frequency scaling factor. Frequency scaling is employed so that large numerical values are avoided in the computation as far as possible. The squared modulus of the acceleration response in particular may reach large values in the case of a lightly damped, strongly excited oscillator of high resonant frequency.

2) In this section the data defining the oscillators of the system is read in.

This data is taken in in the form of resonant frequency, mass and Q factor. The value of K , the excitation constant, will be non-zero for each excited oscillator, the actual value being immaterial.

3) In this section the oscillator data is processed to obtain damping coefficient, which is then used to calculate the undamped natural frequency. From this and the mass, the oscillator stiffness is also defined.

4) The coupling parameters are read-in to the program in the form of coupling stiffness and damping coefficients. Two two-dimensional arrays are thus formed which may then be printed out for inspection.

5) The frequencies at which the complex system matrix will be solved, and the associated step lengths are established. The frequency dependent terms, damping and inertia, are then evaluated.

6) This section contains the Gauss elimination method routine, which solves the symmetric system matrix to obtain a one dimensional array of the frequency response function of each oscillator at a specified frequency.

7) The frequency responses from the previous section, which are in units of displacement, are multiplied by frequency (expressed in imaginary form) to obtain velocity and acceleration. These quantities are stored after computation until further values at two higher equi-spaced frequencies have been computed. The integra-

tion process using Simpson's method is now carried out. The value of this integral is then added to all previously computed ones, and the program then re-cycles until the upper frequency limit of integration is reached. The output data is expressed in the form of mean value of oscillator kinetic energy, displacement velocity squared, acceleration, and power dissipated.

Description of sub-routines

Sub-routine SBTFL

This sub-routine sets the frequency limits for integration. The square of the frequency response modulus is a function consisting of sharp peaks separated along the frequency axis by distances large in comparison to the width of the peaks. A small step length is necessary when integrating the function in the vicinity of its peak values, but would involve an excessive number of evaluations of the function whilst integrating the relatively flat portion between peaks. The location of the peaks of the response is known. A peak value occurs at the resonant frequency of each oscillator in the coupled system. The width of each peak will depend on the Q factor of that particular oscillator.

Section 1 of this sub-routine evaluates the bandwidth of each oscillator, defined as resonant frequency, divided by Q factor. Upper and lower limits of frequency between which small integration step lengths are used are specified as ten bandwidths above and below the oscillator resonant frequency. The value ten is purely

arbitrary and chosen to include a large proportion of the 'lower slopes' of the function peak.

In section 2 the bandwidths, found in section 1, are re-arranged using sub-routine 'SORT' in descending order of magnitude, and their corresponding upper and lower frequencies re-numbered accordingly. This data is used at a later stage in the computation by sub-routine ADJSL.

Sub-routine ADJSL

The purpose of this sub-routine is to adjust the step length of the integration process, which depends on whether the function is being evaluated in the vicinity of a peak value or not. The sub-routine initially sets the step length to the course value of one fifteenth of the frequency. This value is overridden and replaced by a finer value, if the succeeding logic statements show that the frequency lies within the upper and lower frequency limits computed in subroutine SETFL.

If this is the case, the step length will thus be set to the bandwidth of the corresponding oscillators. However, the frequency may lie inside the limits of more than one oscillator. By testing to see if the frequency lies within the widest limits first down to the narrowest limits, the step length finally chosen will be the smallest of the possible alternatives, hence ensuring that no peak is inadvertantly passed over. Safeguards are also taken to prevent the possibility of the course step length completely spanning a region of peaks.

Sub-routine SORT

This is a short general purpose sub-routine written to arrange an array of variables in ascending order of magnitude

List of symbols

The following is a list of those variables which are specified as data, or contain oscillator response quantities. (see P.203)

<u>Fortran symbol</u>	<u>Meaning</u>
N	Number of oscillators
NI	Do loop parameter which specifies number of computation cycles
NJ	Do loop parameter which specifies number of computation cycles.
KPRINT	Specifies number of cycles the integrated variables PD, PV, PA, are printed out
NU	Specifies the maximum number of integration step lengths required.
FSCALE	Internal frequency scaling factor
ALHZ	Value of frequency at which integration starts (Hz)
WTHZ	Final value of frequency at which integration stops (Hz)
K	One dimensional complex array of order N; forcing vector
WNRZ	One dimensional array of order N; modal resonant frequency (Hz)

MS	One dimensional array of order N; modal mass
Q	One dimensional array of order N; modal Q factor
VA, VB, VC	Values of WNHZ, MS, Q for oscillator N (changed from cycle to cycle to obtain parameter survey)
XK	Two dimensional array of order N x N; modal and coupling stiffness
XC	Two dimensional array of order N x N; modal and coupling damping coefficient
PD	Two dimensional array order 3 x N; squared modulus of displacement
PV	Two dimensional array order 3 x N; squared modulus of velocity
PA	Two dimensional array order 3 x N; squared modulus of acceleration
DW	Integration step length
INTD, INTV, INTA.	The integrals over one frequency step length of PD, PV and PA respectively
AAD, AAU, AAA ...	One dimensional arrays of order N; sum of all previous values of INTD, INTV, INTA respectively
EX	Modal displacement
EVS	Modal velocity squared
EA	Modal acceleration
EN	Modal kinetic energy level

ENL EN in logarithmic form (dB)
 EAL EA in logarithmic form (dB)
 PWR Power dissipated from each modal oscillator

Description of Program 'NEIG'

Main Program

As in the case of program 'MAG', the main program may be divided into seven sections.

Sections 1), 2), 3), 4) are the same as the corresponding sections in 'MAG'.

5) In this section the control passes from the main program to the sub-routines. The system matrix is operated on to yield its eigenvalues and eigenvectors. The modal forcing array, VF, and the small step frequency integration limits are then set using this data.

6) In this section the responses of the system oscillators are found, by subtracting the eigenvalues from the imaginary frequency. These responses then appear in the form of a state vector of displacements and velocities.

7) The integration of the evaluated response functions is performed using Simpson's method after each set of three responses has been computed by the previous sections. These are progressively summed to give the total integral. The latter half of the response state vector contains the velocity terms, and the program as listed is set up to integrate these terms only, yielding the kinetic energy levels of the system oscillators.

Description of sub-routines (see fig.A1.2)

(For complete listings of the following see [32])

Sub-routine MODAL

This sub-routine sets up the oscillator and coupling data into the first order equivalent system matrix. A square matrix of zero coefficients, and a unit matrix are fed into the upper half of the first order matrix. The mass matrix of the system is then inverted and multiplied by the stiffness and damping matrices respectively, which are then fed into the lower half of the first order matrix.

Sub-routine INVERT

General routine for matrix inversion.

Sub-routine MULTI

General routine for matrix multiplication.

Sub-routine CMULTI

General routine for complex matrix multiplication.

Sub-routine EIGSYS

Calling routine for sub-routine EIG6, EIGVEC.

Sub-routine EIG6

This sub-routine computes trace of given matrix and trace of Hessenberg form of matrix. As a computational check these are compared with the sum of the eigenvalues. All three values should be equal. The determinant of the system matrix is found for comparison with the product of the eigenvalues. Both these values should be equal. The eigenvalues are computed by calling sub-routine QREIG.

Sub-routine DETMT

This sub-routine computes the determinant of a matrix.

Sub-routine QREIG

Calling routine for sub-routine QRT.

Sub-routine QRT

Sub-routine for finding the eigenvalues of a matrix using the QR transform method of J.G. Francis.

Sub-routine EIGVEC

This sub-routine computes eigenvectors of matrix using the previously computed eigenvalues. Sub-routine 'MATINV' is used in this operation. Real and imaginary parts are treated separately, and double precision arithmetic is used. The last element of the eigenvector is arbitrarily set to 1 + j.0, if this element should be 0 + j.0, overflow will occur, causing the computation to stop. The accuracy of the eigenvector calculation is checked, by comparing the products of the matrix multiplied by the eigenvectors with the eigenvalues multiplied by the eigenvectors, both sets should be equal.

Sub-routine MATINV

Inverts matrix and solves the resulting sets of simultaneous linear equations. Double precision arithmetic is used.

Sub-routine EIGOUT

Sub-routine for printing out results of the eigenvector computation.

Sub-routine ROWEIG

Inverts the complex eigencolumn matrix to give eigenrow matrix.

List of symbols

The following is a list of those variables which are specified as data, or contain oscillator response quantities. (see P.212)

<u>Fortran symbol</u>	<u>Meaning</u>
N	Number of oscillators
NJ	Do loop parameter which specifies number of computation cycles.
KPRINT	Specifies number of cycles the integrated variable P is printed out
NU	Specifies the maximum number of integration step lengths required
PSCALE	Internal frequency scaling factor
WLHZ	Value of frequency at which integration starts (Hz)
WTHZ	Final value of frequency at which integration stops (Hz)
F	One dimensional complex array of order $2 \times N$; forcing vector
WNHZ	One dimensional array of order N; modal resonant frequency (Hz)
MS	One dimensional array of order N; modal mass
Q	One dimensional array of order N; modal Q factor

VA, VB, VC	Values of WNEZ, MS, Q for oscillator N (changed from cycle to cycle to obtain parameter survey)
XM	Two dimensional array of order $N \times N$; modal mass
XK	Two dimensional array of order $N \times N$; modal and coupling stiffness
XC	Two dimensional array of order $N \times N$; modal and coupling damping coefficient
IVEC	Constant controlling computation of eigenvectors, non-zero value causes eigenvectors to be computed
IPRINT	Constant controlling printing, non-zero value causes eigenvalues and eigenvectors to be printed out
ICHECK	Constant controlling internal check routine, non-zero value causes check procedure to be carried out
ROOTR	One dimensional array of order $2 \times N$; real parts of eigenvalues
ROOTI	One dimensional array of order $2 \times N$; imaginary parts of eigenvalues
ROOT	One dimensional complex array of order $2 \times N$; eigenvalues

XUZ	One dimensional complex array of order $2 \times N$; response vector of system
P	Two dimensional array of order $3 \times N$; squared modulus of velocity
INT	The integral over one frequency step length of P
AA	One dimensional array of order N; some of all previous values of INT
EVS	Modal velocity squared
EN	Modal kinetic energy level
ENL	EN in logarithmic form (dB)

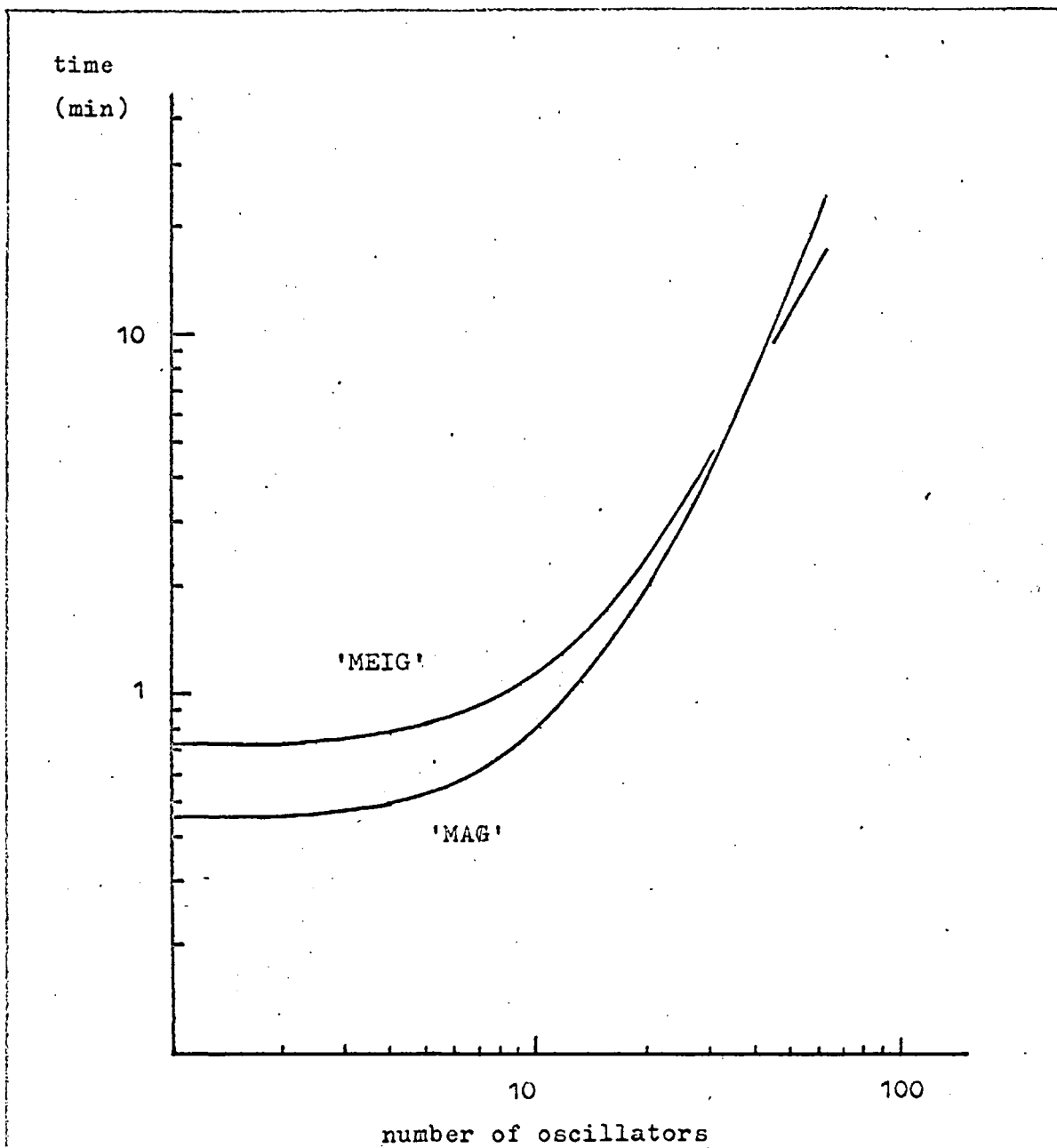
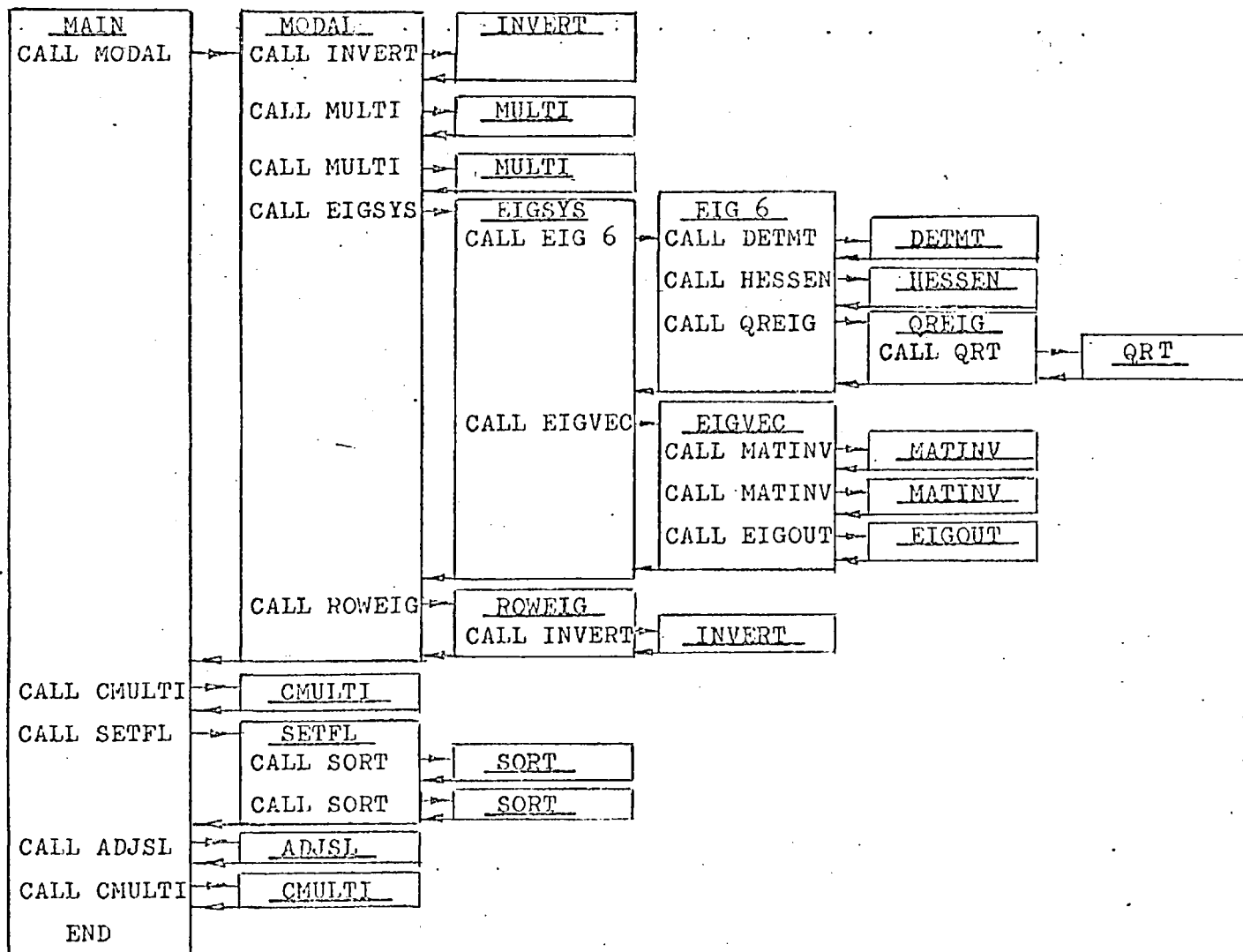


FIG A1.1 COMPARISON BETWEEN PROGRAMS 'MAG' AND 'MEIG' OF TIME TO COMPUTE OSCILLATOR RESPONSE LEVELS, (IBM 7094)

FIG A1.2 SUBROUTINE CALLING SEQUENCE IN PROGRAM 'WEIG'



```

PROGRAM MAG(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
$1BFTC DECK 1
COMMON /BLK1/ X(100),XX(100),J(100),N
COMMON /BLK2/ DW,W,LFF(21),BA(21),LA(21),UA(21),WN(21),Q(21),WT,WL
DIMENSION A(21,21),K(21),MS(21),G(21,21),H(21,21),GH(21),KK(21),
1HW(21),WNHZ(21),XK(21,21),XC(21,21),WXC(21,21),PD(3,21),
2PV(3,21),PA(3,21),AAD(21),AAV(21),AAA(21),PMA(21),WNAT(21)
COMPLEX AHW,WHW,WWHW,AD,A,G,GGH,H,GH,KKG,KG,KK,HHW,HW,HWW,AW,K,
1AKK
REAL MS,IHW,IWHW,IWWHW,MODSD,MODSV,MODSA,INTD,INTV,INTA

$
$ SECTION 1
$
N=3
NJ=7
NI=3
KPRINT=2
FSCALE=1.
NU=500
WLHZ=0.030
WTHZ=30.0
WL=WLHZ*6.28318*FSCALE
WT=WTHZ*6.28318*FSCALE
NM1=N-1

$
$ SECTION 2
$
DO 35 III=1,NI
READ(5,100)(WNHZ(I),I=1,NM1)
READ(5,100)(MS(I),I=1,NM1)
READ(5,100)(Q(I),I=1,NM1)
READ(5,100)(K(I),I=1,N)
DO 17 JJJ=1,NJ
READ(5,100) VA,VB,VC
WNHZ(N)=VA
MS(N)=VB
Q(N)=VC

$
$ SECTION 3
$
DO 20 I=1,N
WN(I)=WNHZ(I)*FSCALE*6.283185
BT=1./(2.*Q(I))
BTSQ=BT*BT
WNAT(I)=WN(I)/SQRT(1.-BTSQ)
WRITE(6,109) I,WNHZ(I),WN(I),MS(I),Q(I),K(I)
S=WNAT(I)*WNAT(I)*MS(I)
WRITE(6,101) I,S
PMA(1)=0.
AAD(1)=0.
AAV(1)=0.

```

```

AAA(I)=0.
20 CONTINUE
$
$ SECTION 4
$
DO 29 I=1,NM1
  IP1=I+1
  READ(5,100)(XK(I,JD),JD=IP1,N)
  READ(5,100)(XC(I,JD),JD=IP1,N)
  DO 28 JE=IP1,N
    XK(I,JE)=XK(I,JE)*FSCALE*FSCALE
    XC(I,JE)=XC(I,JE)*FSCALE
    XC(JE,I)=XC(I,JE)
28 XK(JE,I)=XK(I,JE)
29 CONTINUE
27 CONTINUE
DO 30 I=1,N,4
DO 30 JD=1,N
  I1=I+1
  I2=I+2
  I3=I+3
  IF(I1.GT.N) GO TO 31
  IF(I2.GT.N) GO TO 32
  IF(I3.GT.N) GO TO 33
  WRITE(6,102) JD,I,XK(JD,I),JD,I1,XK(JD,I1),JD,I2,XK(JD,I2),JD,I3,X
1K(JD,I3)
  GO TO 30
31 WRITE(6,131) JD,I,XK(JD,I)
  GO TO 30
32 WRITE(6,132) JD,I,XK(JD,I),JD,I1,XK(JD,I1)
  GO TO 30
33 WRITE(6,133) JD,I,XK(JD,I),JD,I1,XK(JD,I1),JD,I2,XK(JD,I2)
30 CONTINUE
DO 40 I=1,N,4
DO 40 JD=1,N
  I1=I+1
  I2=I+2
  I3=I+3
  IF(I1.GT.N) GO TO 41
  IF(I2.GT.N) GO TO 42
  IF(I3.GT.N) GO TO 43
  WRITE(6,142) JD,I,XC(JD,I),JD,I1,XC(JD,I1),JD,I2,XC(JD,I2),JD,I3,X
1C(JD,I3)
  GO TO 40
41 WRITE(6,141) JD,I,XC(JD,I)
  GO TO 40
42 WRITE(6,143) JD,I,XC(JD,I),JD,I1,XC(JD,I1)
  GO TO 40
43 WRITE(6,144) JD,I,XC(JD,I),JD,I1,XC(JD,I1),JD,I2,XC(JD,I2)
40 CONTINUE
44 CONTINUE
$
$ SECTION 5
$

```



```

CALL SETFL
W=WL
KTR=0
DO 91 IWW=1,NU
  IF(IWW.EQ.NU) WRITE(6,106) NU
  CALL ADJSL
  DO 9 IW=1,3
    KTR=KTR+1
    IF(IWW.GT.1.AND.IW.EQ.1) GO TO 16
    DO 21 I=1,N
      DO 21 JD=1,N
        WXC(I,JD)=W*XC(I,JD)
21  A(I,JD)=CMPLX(XK(I,JD),WXC(I,JD))
      DO 15 ID=1,N
        ADR=MS(ID)*(WNAT(ID)*WNAT(ID)-W*W)
        ADI=MS(ID)*WNAT(ID)*W/Q(ID)
        A(ID,ID)=CMPLX(ADR,ADI)
15  CONTINUE
      IF(IWW.GT.1) GO TO 152
      DO 300 I=1,N,3
        DO 300 JD=1,N
          I1=I+1
          I2=I+2
          IF(I1.GT.N) GO TO 310
          IF(I2.GT.N) GO TO 320
          WRITE(6,153) JD,I,A(JD,I),JD,I1,A(JD,I1),JD,I2,A(JD,I2)
          GO TO 300
310  WRITE(6,154) JD,I,A(JD,I)
          GO TO 300
320  WRITE(6,155) JD,I,A(JD,I),JD,I1,A(JD,I1)
300  CONTINUE
152  CONTINUE
      IF(IWW.EQ.1.AND.IW.EQ.1) WRITE(6,128)

```

\$
\$ SECTION 6
\$

```

DO 3 L=1,N
DO 2 MM=L,N
  GH=(0.,0.)
  IF(L.EQ.1) GO TO 61
  LL=L-1
  DO 1 JB=1,LL
    GGH=G(MM,JB)*H(JB,L)
    GH=GH+GGH
1  CONTINUE
61  CONTINUE
  G(MM,L)=A(MM,L)-GH
  H(L,MM)=G(MM,L)/G(L,L)
2  CONTINUE
  KG=(0.,0.)
  IF(L.EQ.1) GO TO 410

```

```

DO 4 JC=1,LL
KKG=G(L,JC)*KK(JC)
KG=KG+KKG
4 CONTINUE
410 CONTINUE
AKK=K(L)-KG
KK(L)=AKK/G(L,L)
3 CONTINUE
DO 5 II=1,N
IB=N+1-II
HWW=(0.,0.)
IF(II.EQ.1) GO TO 60
JJ=II-1
DO 6 IJ=1,JJ
NN=N+1-IJ
HHW=H(IB,NN)*HW(NN)
HWW=HWW+HHW
6 CONTINUE
60 CONTINUE
HW(IB)=KK(IB)-HWW
5 CONTINUE

```

\$
\$ SECTION 7
\$

```

Z=0.
DO 7 IC=1,N
AW=CMPLX(Z,W)
AHW=HW(IC)
WHW=AW*HW(IC)
WWHW=AW*AW*HW(IC)
RHW=REAL(AHW)
IHW=AIMAG(AHW)
RWHW=REAL(WHW)
IWHW=AIMAG(WHW)
RWWHW=REAL(WWHW)
IWWHW=AIMAG(WWHW)
MODSD=RHW*RHW+IHW*IHW
MODSV=RWHW*RWHW+IWHW*IWHW
MODSA=RWWHW*RWWHW+IWWHW*IWWHW
PD(IW,IC)=MODSD
PV(IW,IC)=MODSV
PA(IW,IC)=MODSA
IF(PV(IW,IC).GT.PMAX(IC)) PMAX(IC)=PV(IW,IC)
7 CONTINUE
GO TO 18
16 CONTINUE
DO 10 IE=1,N
PD(1,IE)=PD(3,IE)
PV(1,IE)=PV(3,IE)
PA(1,IE)=PA(3,IE)
10 CONTINUE

```

```

18  CONTINUE
    IF(IWW.GT.KPRINT) GO TO 51
    WRITE(6,129)(PV(IW,I),I=1,N)
    WRITE(6,134) IW,W
51  CONTINUE
19  CONTINUE
    IF(IW.EQ.3) GO TO 9
    W=W+DW/2.000
9   CONTINUE
    DO 11 I=1,N
        INTD=DW*(PD(1,I)+4.*PD(2,I)+PD(3,I))/6.
        INTV=DW*(PV(1,I)+4.*PV(2,I)+PV(3,I))/6.
        INTA=DW*(PA(1,I)+4.*PA(2,I)+PA(3,I))/6.
        AAD(I)=AAD(I)+INTD
        AAV(I)=AAV(I)+INTV
        AAA(I)=AAA(I)+INTA
        INTD=0.
        INTV=0.
        INTA=0.
11  CONTINUE
    IF(W.GT.WT) GO TO 171
91  CONTINUE
171 CONTINUE
    WRITE(6,120) IWW,IW,W
    WRITE(6,127)
    WRITE(6,129)(PV(IW,I),I=1,N)
    WRITE(6,135)
    WRITE(6,129)(PMA(X(I),I=1,N)
    WRITE(6,115)

    DO 12 I=1,N
        EX=SQRT(AAD(I)*2.)
        EVS=AAV(I)*2.
        EA=SQRT(AAA(I)*2.)
        EN=0.5*MS(I)*EVS
        IF(EN.LT.1..OR.EA.LT.1.) WRITE(6,103) EN,EA,I
        ENL=10.*ALOG10(EN)
        EAL=20.*ALOG10(EA)
        PWR=EN*2.*WN(I)/(Q(I)*8.84)
        WRITE(6,105) I,EN,ENL,EX,EVS,EA,EAL,PWR
        AAD(I)=0.
        AAV(I)=0.
        AAA(I)=0.
12  CONTINUE
    WRITE(6,108)
17  CONTINUE
35  CONTINUE
    STOP
100 FORMAT(6E11.4)
103 FORMAT(1H E11.4,1X,E11.4,1X,I3)
106 FORMAT(15H WARNING IWW = 14)
107 FORMAT(5H A = 2E11.4,1X,5HJA = 13,1X,5HIA = 13)

```

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```

109 FORMAT(11H OSC NO. = I3.1X.7HWNHZ = E11.4.1X.5HWN = E11.4.1X.5HMS 01
1= E11.4.1X.4HQ = E11.4.1X.4HK = 2E11.4) 01
105 FORMAT(3H I3.2X.E11.4.3X.F8.3.E12.4.E13.4.E13.4.F8.3.E13.4)
108 FORMAT(1H /50H *****)
115 FORMAT(89H OSC NO ENERGY LEVEL DB DISP VEL SQRD
1ACCN LEVEL DB POWER DISS W /)
128 FORMAT(31H THE INITIAL VALUES OF PV WERE /)
129 FORMAT(10E13.4)
134 FORMAT(6H IW = I3.1X.4HW = E15.8)
127 FORMAT(29H THE FINAL VALUES OF PV WERE /)
120 FORMAT(7H IWW = I3.1X.5HIW = I3.1X.4HW = E15.8)
131 FORMAT(4H XK(I2.1H.I2.2H)=E11.4)
132 FORMAT(4H XK(I2.1H.I2.2H)=E11.4.5X.3HXK(I2.1H.I2.2H)=E11.4)
133 FORMAT(4H XK(I2.1H.I2.2H)=E11.4.5X.3HXK(I2.1H.I2.2H)=E11.4.5X.3HXK
1(I2.1H.I2.2H)=E11.4)
135 FORMAT(27H THE MAX VALUES OF PV WERE /)
102 FORMAT(4H XK(I2.1H.I2.2H)=E11.4.5X.3HXK(I2.1H.I2.2H)=E11.4.5X.3HXK
1(I2.1H.I2.2H)=E11.4.5X.3HXK(I2.1H.I2.2H)=E11.4)
141 FORMAT(4H XC(I2.1H.I2.2H)=E11.4)
143 FORMAT(4H XC(I2.1H.I2.2H)=E11.4.5X.3HXC(I2.1H.I2.2H)=E11.4)
144 FORMAT(4H XC(I2.1H.I2.2H)=E11.4.5X.3HXC(I2.1H.I2.2H)=E11.4.5X.3HXC
1(I2.1H.I2.2H)=E11.4)
142 FORMAT(4H XC(I2.1H.I2.2H)=E11.4.5X.3HXC(I2.1H.I2.2H)=E11.4.5X.3HXC
1(I2.1H.I2.2H)=E11.4.5X.3HXC(I2.1H.I2.2H)=E11.4)
153 FORMAT(3H A(I2.1H.I2.2H)=2E11.4.5X.2HA(I2.1H.I2.2H)=2E11.4.5X.2HA(
1I2.1H.I2.2H)=2E11.4)
154 FORMAT(3H A(I2.1H.I2.2H)=2E11.4)
155 FORMAT(3H A(I2.1H.I2.2H)=2E11.4.5X.2HA(I2.1H.I2.2H)=2E11.4)
156 FORMAT(7H IWW = I4)
101 FORMAT(5H J = I3.1X.4HS = E11.4)
END

```

\$IBFTC DECK 2

SUBROUTINE SETFL

COMMON /BLK2/ DW,W,LFF(21),BA(21),LA(21),UA(21),WN(21),Q(21),WT,WL

COMMON /BLK1/ X(100),XX(100),J(100),N

DIMENSION BW(23),LF(23),UF(23)

REAL LF,LFF,LA

\$

\$ SECTION 1

\$

DO 1 I=1,N

BW(I)=WN(I)/Q(I)

LF(I)=WN(I)-10.*BW(I)

UF(I)=WN(I)+10.*BW(I)

X(I)=LF(I)

WRITE(6,104) BW(I),LF(I),UF(I),I

1 CONTINUE

\$

\$ SECTION 2

\$

CALL SORT

DO 2 I=1,N

NV=N+1-I

LFF(I)=XX(NV)

WRITE(6,101) LFF(I),I

2 CONTINUE

DO 3 I=1,N

X(I)=BW(I)

3 CONTINUE

CALL SORT

DO 4 I=1,N

NW=N+1-I

BA(I)=XX(NW)

JIX=J(NW)

LA(I)=LF(JIX)

UA(I)=UF(JIX)

WRITE(6,102) BA(I),JIX,LA(I),UA(I),I

4 CONTINUE

RETURN

104 FORMAT(6H BW = E11.4,1X,5HLF = E11.4,1X,5HUF = E11.4,1X,4HI = 13)

101 FORMAT(7H LFF = E11.4,1X,4HI = 13)

102 FORMAT(6H BA = E11.4,1X,6HJIX = 13,1X,5HLA = E11.4,1X,5HUA = E11.4
1,1X,4HI = 13)

END

\$IBFTC DECK 3

SUBROUTINE SORT

COMMON /BLK1/ X(100),XX(100),J(100),N

DO 211 JX=1,N

XLL=0.

DO 210 IX=1,N

IF(XLL.LT.X(IX)) GO TO 209

GO TO 210

209 XLL=X(IX)

KX=IX

210 CONTINUE

NJX=1+N-JX

XX(NJX)=XLL

X(KX)=0.

J(NJX)=KX

211 CONTINUE

RETURN

END

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SIBFTC DECK 4

SUBROUTINE ADJSL

COMMON /BLK2/ DW,W,LFF(21),BA(21),LA(21),UA(21),WN(21),Q(21),WT,WL

COMMON /BLK1/ X(100),XX(100),J(100),N

REAL LFF,LA

DW=W/15.

DO 5 I=1,N

IF(W+DW.GE.LFF(I).AND.W.LT.LFF(I)) DW=LFF(I)-W

5 CONTINUE

DO 6 IF=1,N

IF(W+BA(IF).GE.LA(IF).AND.W+BA(IF).LE.UA(IF)) DW=BA(IF)

IFP1=IF+1

DO 7 IG=IFP1,N

IF(W+BA(IG).GT.LA(IG).AND.W.LT.LA(IG)) DW=LA(IG)-W

7 CONTINUE

6 CONTINUE

IF(DW.LT.BA(N)) DW=BA(N)

RETURN

END

```

PROGRAM MEIG(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
$IBFTC DECK 1 RMAP
  DIMENSION XM(4,4),XC(4,4),XK(4,4),A(8,8),B(8,8),C(8,8)
  1 D(8,8),CC(16,16),ROOTR(8),ROOTI(8),U(8,8),V(8,8),ROOT(8)
  2 E(8,8),F(8,8),VF(8,8),Z(8,8),GZ(8,8),XUZ(8,8),P(3,8),AA(8,8),WNAT(4
  3 ),MS(4),KSC(4),Q(4),WNHZ(4),LFF(4),LA(4),UA(4),BA(4),WN(4
  4 ),TEMP(4,4),PMA(8),TEMP2(4,4),EX(4)
  COMMON IRANK
C
  COMPLEX U,V,ROOT,F,VF,GZ,Z,XUZ,AW
C
  DOUBLE PRECISION CC
  REAL MS,KSC,LFF,LA,IXD,MODSQ,INT
C PROGRAMME CALCULATES THE ENERGY LEVEL OF 'N' COUPLED OSCILLATORS FOR
C WHITE NOISE INPUT
$
$ SECTION 1
$
  N=4
  NJ=20
  KPRINT=2
  NU=2000
  FSCALE=0.0010
  WLHZ=10.
  WTHZ=2500.
  WL=WLHZ*6.283185*FSCALE
  WT=WTHZ*6.283185*FSCALE
  M=2*N
  M2=2*M
  NN=N
  IONE=1
  ZERO=0.0
  NM1=N-1
$
$ SECTION 2
$
  READ(5,100)(WNHZ(I),I=1,NM1)
  READ(5,100)(MS(I),I=1,NM1)
  READ(5,100)(Q(I),I=1,NM1)
  READ(5,100)(F(I),I=1,M)
  WNHZ(1)=4000.
  DO 60 I=1,NM1
60 WNHZ(I)=WNHZ(I)*FSCALE
  DO 17 JJJ=1,NJ
  READ(5,100) VA,VB,VC
  VA=VA*FSCALE
  WNHZ(N)=VA
  MS(N)=VB
  Q(N)=VC
  DO 70 I=1,N
  AMQ=MS(I)/Q(I)

```



```
PMAX(I)=0.
```

```
WRITE(6,109) I,WNHZ(I),MS(I),Q(I),AMQ
```

```
AA(I)=0.
```

```
70 CONTINUE
```

```
DO 71 I=1,M
```

```
WRITE(6,119) F(I)
```

```
71 CONTINUE
```

```
$
```

```
$ SECTION 3
```

```
$
```

```
DO 1 I=1,N
```

```
BT=1./(2.*Q(I))
```

```
BTSQ=BT*BT
```

```
WN(I)=WNHZ(I)*6.28318
```

```
WNAT(I)=WN(I)/SQRT(1.-BTSQ)
```

```
KSC(I)=MS(I)*WNAT(I)*WNAT(I)
```

```
XM(I,1)=MS(I)
```

```
XC(I,1)=WNAT(I)*MS(I)/Q(I)
```

```
XK(I,1)=KSC(I)
```

```
WRITE(6,101) I,WN(I),WNAT(I),XM(I,1),XC(I,1),XK(I,1)
```

```
1 CONTINUE
```

```
C SECTION 2 OFF-DIAGONAL COUPLING STIFFNESS TERMS
```

```
$
```

```
$ SECTION 4
```

```
$
```

```
DO 29 I=1,NM1
```

```
IP1=I+1
```

```
READ(5,100)(XK(I,J),J=IP1,N)
```

```
READ(5,100)(XC(I,J),J=IP1,N)
```

```
DO 28 JJ=IP1,N
```

```
XK(I,JJ)=XK(I,JJ)*FSCALE*FSCALE
```

```
XC(I,JJ)=XC(I,JJ)*FSCALE
```

```
IF(I.GE.2) XK(I,JJ)=XK(I,JJ)/6.
```

```
IF(I.GE.2) XC(I,JJ)=XC(I,JJ)/6.
```

```
XC(JJ,I)=XC(I,JJ)
```

```
28 XK(JJ,I)=XK(I,JJ)
```

```
29 CONTINUE
```

```
DO 30 I=1,N*4
```

```
DO 30 J=1,N
```

```
I1=I+1
```

```
I2=I+2
```

```
I3=I+3
```

```
IF(I1.GT.N) GO TO 31
```

```
IF(I2.GT.N) GO TO 32
```

```
IF(I3.GT.N) GO TO 33
```

```
WRITE(6,102) J,I,XK(J,I),J,I1,XK(J,I1),J,I2,XK(J,I2),J,I3,XK(J,I3)
```

```
GO TO 30
```

```
31 WRITE(6,131) J,I,XK(J,I)
```

```
GO TO 30
```

```
32 WRITE(6,132) J,I,XK(J,I),J,I1,XK(J,I1)
```

```
GO TO 30
```

```

33 WRITE(6,133) J,I,XK(J,I),J,I1,XK(J,I1),J,I2,XK(J,I2)
30 CONTINUE
34 CONTINUE
DO 40 I=1,N,4
DO 40 J=1,N
I1=I+1
I2=I+2
I3=I+3
IF(I1.GT.N) GO TO 41
IF(I2.GT.N) GO TO 42
IF(I3.GT.N) GO TO 43
WRITE(6,142) J,I,XC(J,I),J,I1,XC(J,I1),J,I2,XC(J,I2),J,I3,XC(J,I3)
GO TO 40
41 WRITE(6,141) J,I,XC(J,I)
GO TO 40
42 WRITE(6,143) J,I,XC(J,I),J,I1,XC(J,I1)
GO TO 40
43 WRITE(6,144) J,I,XC(J,I),J,I1,XC(J,I1),J,I2,XC(J,I2)
40 CONTINUE

```

\$
\$ SECTION 5
\$

```

IVEC=1
IPRINT=1
IF(JJJ.GT.2) IPRINT=0
ICHECK=1
IF(JJJ.GT.2) ICHECK=0
CALL MODAL(XM,XC,XK,A,B,C,D,CC,ROOTR,ROOTI,U,V,ROOT,N,M,M2,IPRINT,
1 ICHECK,E,TEMP,TEMP2)
CALL CMULTI(V,F,VF,M,M,IONE)
CALL SETFL(ROOTR,ROOTI,LFF,LA,UA,BA,M,NN)
W=WL
DO 99 IWW=1,NU
IF(IWW.EQ.1) WRITE(6,128)
CALL ADJSL(W,LFF,LA,UA,BA,DW,N)

```

\$
\$ SECTION 6
\$

```

DO 98 IW=1,3
IF(IWW.GT.1.AND.IW.EQ.1) GO TO 16
DO 11 I=1,M
AW=CMPLX(ZERO,W)
GZ(I)=1./(AW-ROOT(I))
Z(I)=GZ(I)*VF(I)
11 CONTINUE
CALL CMULTI(U,Z,XUZ,M,M,IONE)
DO 12 I=1,M
II=I
RXD=REAL(XUZ(II))
IXD=AIMAG(XUZ(II))
MODSQ=RXD*RXD+IXD*IXD

```

```

P(IW,I)=MODSQ
IF(P(IW,I).GT.PMAX(I)) PMAX(I)=P(IW,I)
12 CONTINUE
GO TO 18
16 CONTINUE
DO 13 IE=1,M
P(I,IE)=P(3,IE)
13 CONTINUE
18 CONTINUE
IF(IWW.GT.KPRINT) GO TO 51
WRITE(6,129)(P(IW,I),I=1,M)
WRITE(6,134) IW,W
51 CONTINUE
IF(IW.EQ.3) GO TO 98
W=W+DW/2.000
98 CONTINUE
$
$ SECTION 7
$
DO 20 I=1,M
INT=DW*(P(1,I)+4.*P(2,I)+P(3,I))/6.
AA(I)=AA(I)+INT
INT=0.0
20 CONTINUE
IF(W.GT.WT) GO TO 171
99 CONTINUE
171 CONTINUE
WRITE(6,127)
WRITE(6,129)(P(IW,I),I=1,M)
WRITE(6,120) IWW,IW,W
WRITE(6,135)
WRITE(6,129)(PMAX(I),I=1,M)
DO 21 I=1,N
EX(I)=SQRT(AA(I)*2.)
21 AA(I)=0.
NP1=N+1
WRITE(6,115)
DO 22 I=NP1,M
IMN=I-N
EXD=AA(I)*2.
EN=0.5*MS(IMN)*EXD
ENL=10.*ALOG10(EN)
PWR=EN*2.*WN(IMN)/(Q(IMN)*8.84)
EXDD=EXD/EX(IMN)
EXDDL=20.*ALOG10(EXDD)
WRITE(6,105) IMN,EN,ENL,EX(IMN),EXD,EXDD,EXDDL,PWR
AA(I)=0.
22 CONTINUE
WHZ=WN(3)/6.28318
WRITE(6,110)
17 CONTINUE

```

STOP

```

100 FORMAT(6E11.4)
102 FORMAT(4H XK(I2.1H,I2.2H)=E11.4,5X,3H XK(I2.1H,I2.2H)=E11.4,5X,3H XK
1(I2.1H,I2.2H)=E11.4,5X,3H XK(I2.1H,I2.2H)=E11.4)
101 FORMAT(5H I = I3.1X,5HWN = E11.4,1X,7HWNAT = E11.4,1X,5HXM = E11.4
1,1X,5HXC = E11.4,1X,5H XK = E11.4)
109 FORMAT(11H, OSC NO. = I3.1X,7H,WNHZ = E11.4,1X,7H,MASS = E11.4,1X,
14H,Q =E11.4,1X,6H,M/Q =E11.4)
119 FORMAT(5H F = 2E11.4)
120 FORMAT(7H IWW = I3.1X,5HIW = I3.1X,4HW = E15.8)
131 FORMAT(4H XK(I2.1H,I2.2H)=E11.4)
132 FORMAT(4H XK(I2.1H,I2.2H)=E11.4,5X,3H XK(I2.1H,I2.2H)=E11.4)
133 FORMAT(4H XK(I2.1H,I2.2H)=E11.4,5X,3H XK(I2.1H,I2.2H)=E11.4,5X,3H XK
1(I2.1H,I2.2H)=E11.4)
129 FORMAT(10E13.4)
127 FORMAT(28H THE FINAL VALUES OF P WERE /)
128 FORMAT(30H THE INITIAL VALUES OF P WERE /)
135 FORMAT(26H THE MAX VALUES OF P WERE /)
134 FORMAT(6H IW = I3.1X,4HW = E15.8)
110 FORMAT(1H //)
142 FORMAT(4H XC(I2.1H,I2.2H)=E11.4,5X,3H XC(I2.1H,I2.2H)=E11.4,5X,3H XC
1(I2.1H,I2.2H)=E11.4,5X,3H XC(I2.1H,I2.2H)=E11.4)
141 FORMAT(4H XC(I2.1H,I2.2H)=E11.4)
143 FORMAT(4H XC(I2.1H,I2.2H)=E11.4,5X,3H XC(I2.1H,I2.2H)=E11.4)
144 FORMAT(4H XC(I2.1H,I2.2H)=E11.4,5X,3H XC(I2.1H,I2.2H)=E11.4,5X,3H XC
1(I2.1H,I2.2H)=E11.4)
105 FORMAT(3H I3.2X,E11.4,3X,F8.3,E12.4,E13.4,E13.4,F8.3,E13.4)
115 FORMAT(89H OSC NO ENERGY LEVEL DB DISP VEL SQRD
1ACCN LEVEL DB POWER DISS W /)
END

```

APPENDIX TWO

COMPUTER PROGRAM FOR EVALUATING MODAL AND COUPLING PARAMETERS
FROM STRUCTURAL POINT AND TRANSFER RESPONSES

This computer program, named 'SIGMA', follows the analysis of chapter four very closely. It is divided into four sections.

1) The input data required by the program is read in. This consists of the point stiffness or impedance; in the case of the former the resonant frequency and associated loss factor if an attached mass at the structure excitation point causes resonance. The transfer stiffness or impedance at the resonant frequency of each mode, and an initial value of loss factor for all modal oscillators, is also required.

2) The modal mass (constant for each mode) is computed.

3) The transfer response of the derived system is computed and composed with the measured values.

4) The modal loss factors are multiplied by the ratio of computed to measured transfer response. The program is then recycled until the above ratios are within one per cent of unity.

List of symbols

The following is a list of those variables which are read in as data or contain mode parameters or responses. (see P.220)

<u>Fortran symbol</u>	<u>Meaning</u>
N	Number of modal oscillators

ID	Constant; a value of 1 specifies dissipative coupling
NJ	Do loop parameter which specifies number of computation cycles
ETAL	Lower limit of modal loss factor
ETAU	Upper limit of modal loss factor
KBASE	Value from which transfer responses are measured in dB
SIGKC	Mean point response
WNH2O	Resonant frequency (Hz) in point response
ETAO	Loss factor at resonant frequency in point response
KT	One dimensional array of order N; transfer response
WNHZ	One dimensional array of order N; modal resonant frequency
ETA	One dimensional array of order N; modal loss factor
MS	One dimensional array of order N; modal mass
KK	One dimensional array of order N; modal stiffness
KC	One dimensional array of order N; coupling stiffness
CC	One dimensional array of order N; coupling damper coefficient

R	Ratio of coupling to total parameter
RQ	Ratio of R divided by loss factor if coupling is conservative : equals R if coupling is dissipative
RA	One dimensional array of order N; ratio of computed to measured transfer response

```
PROGRAM SIGMA(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
```

```
$TRJOB
```

```
$IRFEC DECK 1
```

```
  DIMENSION KT(100),WNHZ(100),C(100),A(100),KK(100),KC(100),WN(100),  
  1ETA(100),MS(100),RA(100),CC(100)  
  REAL KT,KK,KC,MASS,KR,KBASE,MS,MSA  
  COMPLEX SIGKCC,SIG1,SIG2,D,EC,ED,ECC,FED,KTC,ABC
```

```
C PROGRAM TO CALCULATE MODE PARAMETERS FROM POINT + TRANSFER STIFFNESS
```

```
C OR POINT + TRANSFER DAMPING
```

```
C ID IS SET TO 1 IF DATA IS IN THE FORM OF POINT + TRANSFER DAMPING
```

```
$
```

```
$ SECTION 1
```

```
$
```

```
  N=33
```

```
  ID=0
```

```
  ID=1
```

```
  NJ=1
```

```
  ETAL=0.033
```

```
  ETAU=0.125
```

```
  IR=N
```

```
  DO 8 JUJ=1,NJ
```

```
    READ(5,102) KBASE,SIGKC,WNHZ0,ETA0
```

```
    WRITE(6,200)KBASE,SIGKC,WNHZ0,ETA0
```

```
    WNHZOS=WNHZ0*WNHZ0
```

```
    READ(5,101) (KT(I),I=1,N)
```

```
    READ(5,101) (WNHZ(I),I=1,N)
```

```
    WRITE(6,109) (KT(I),I=1,N)
```

```
    DO 12 I=1,N
```

```
      ETA(I)=0.1
```

```
      PR=KT(I)/20.
```

```
      KR=10.**PR
```

```
      KT(I)=KR*KBASE
```

```
  12 CONTINUE
```

```
  11 CONTINUE
```

```
  KX=0
```

```
  70 CONTINUE
```

```
$
```

```
$ SECTION 2
```

```
$
```

```
  SIGA=0.
```

```
  DO 1 I=1,N
```

```
    Q=1./ETA(I)
```

```
    WRITE(6,121) ETA(I),I,Q
```

```
    WZ=WNHZ(I)*WNHZ(I)/WNHZOS
```

```
    WZZ=ETA0*WNHZ(I)/WNHZ0
```

```
    AA=SQRT((1.-WZ)*(1.-WZ)+WZZ*WZZ)
```

```
    C(I)=KT(I)/(ETA(I)*SIGKC*AA)
```

```
    WN(I)=WNHZ(I)*6.28318
```

```
    A(I)=ETA(I)*WN(I)*WN(I)*AA/(KT(I)*396.)
```

```
    IF(ID.EQ.1) A(I)=A(I)/WN(I)
```

```
    SIGA=SIGA+A(I)
```



```

MSA=1./SIGA
WRITE(6,113) MSA,I
1 CONTINUE
MASS=1./SIGA
DO 20 I=1,N
20 MS(I)=MASS
10 CONTINUE
TSIG=0.

# # SECTION 3
#
DO 2 I=1,N
KK(I)=WN(I)*WN(I)*MS(I)/386.
KC(I)=KK(I)/C(I)
IF(ID.EQ.1) KC(I)=0.
CC(I)=WN(I)*MS(I)/(386.*C(I))
IF(ID.NE.1) CC(I)=0.
TSIG=TSIG+KC(I)+CC(I)
R=1./C(I)
WRITE(6,110) I,WNHZ(I),R,KK(I),KC(I),CC(I)
2 CONTINUE
WRITE(6,120) TSIG,MASS
IF(TSIG.GT.1.01*SIGKC.OR.TSIG.LT.0.99*SIGKC) GO TO 40
GO TO 5
40 CONTINUE
CONST=TSIG/SIGKC
DO 30 I=1,N
C(I)=C(I)*CONST
30 CONTINUE
GO TO 10
5 CONTINUE
KX=KX+1
WRITE(6,125) KX
SIGKCI=0.
SIGKCC=CMPLX(SIGKC,SIGKCI)
IF(ID.NE.1) WRITE(6,124)
IF(ID.EQ.1) WRITE(6,126)
JR=0
DO 4 J=1,N
W=WN(J)
SIG1=(0.,0.)
SIG2=(0.,0.)
IX=J-IR
IF(J.LE.IR) IX=1
IY=J+IR
IF(J+IR.GT.N) IY=N
DO 3 I=IX,IY
DR=1.-W*W/(WN(I)*WN(I))
DI=ETA(I)*W/WN(I)
D=CMPLX(DR,DI)
IF(ID.EQ.1) DR=-DR

```

```

IF (ID.EQ.1) D=CMPLX(DI,DI)
IF (ID.NE.1) ER=KC(I)*KC(I)*386./(MS(I)*WN(I)*WN(I))
IF (ID.EQ.1) ER=CC(I)*CC(I)*386./(MS(I)*WN(I))
EI=0.
EC=CMPLX(ER,EI)
ED=EC/D
IF (ID.NE.1) EER=ER/KC(I)
IF (ID.EQ.1) EER=ER/CC(I)
EEI=0.
EEC=CMPLX(EER,EI)
EED=EEC/D
SIG1=SIG1+ED
SIG2=SIG2+EED
3 CONTINUE
AR=ARS(1.-WNHZ(J)*WNHZ(J)/WNHZOS)
ART=WNHZ(J)*ETA0/WNHZO
ARC=CMPLX(AR,ART)
KTC=(SIGKCC*ARC-SIG1)/SIG2
AKTR=REAL(KTC)
AKTI=AIMAG(KTC)
AMOD=SQRT(AKTR*AKTR+AKTI*AKTI)
RA(J)=AMOD/KT(J)
WRITE(6,111) J,WNHZ(J),KTC,AMOD,KT(J),RA(J)
IF (RA(J).LT.1.01.AND.RA(J).GT.0.99) JR=JR+1
IF (JR.EQ.N) GO TO 7
4 CONTINUE
IF (KY.GT.5) GO TO 7
$
$ SECTION 4
$
DO 5 I=1,N
ETA(I)=ETA(I)*PA(I)
IF (ETA(I).LT.ETA1) ETA(I)=ETA1
IF (ETA(I).GT.ETAU) ETA(I)=ETAU
6 CONTINUE
DO 64 I=1,N
RQ=1./C(I)
IF (ID.EQ.0) RQ=RQ*Q
Q=1./ETA(I)
64 WRITE(6,122) ETA(I),I,Q,MS(I),RQ
GO TO 70
7 CONTINUE
8 CONTINUE
101 FORMAT(12F6.1)
102 FORMAT(6E11.4)
109 FORMAT(1H, 25F5.1)
110 FORMAT(10H, OSC NO = I3,1X,7H,WNHZ = E11.4,1X,4H,R = E11.4,1X,5H,K
1K = E11.4,1X,5H,KC = E11.4,1X,5H,CC = E11.4)
113 FORMAT(7H,MSA = E11.4,1X,4H,I = I4)
120 FORMAT(24H, CHECK VALUE OF SIGKCC = E11.4,1X,13H,MODAL MASS = E11.4
1)

```

```

121 FORMAT(7H ETA = E11.4,1X,4HT = I3,1X,4HQ = E11.4)
122 FORMAT(7H ETA = E11.4,1X,4HI = I3,1X,4HQ = E11.4,1X,5HMS = E11.4,1
1X,5HRO = E11.4)
124 FORMAT(77H, OSC NO.      WNHZ      KT COMPLEX      KT MOD
1TRUE KT      RATIO M/T  /)
125 FORMAT(6H KX = I4)
126 FORMAT(77H  OSC NO.      WNHZ      DT COMPLEX      DT MOD
1TRUE DT      RATIO M/T  /)
111 FORMAT(1H I3,3X,E11.4,1X,2E11.4,1X,E11.4,1X,E11.4,1X,E11.4)
200 FORMAT(10E12.4)

```

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