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Introduction to the Special Issue on Global and Robust Optimization of Dynamic Systems

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Providing certificates of global optimality or robustness in dynamic optimization problems remains a formidable challenge. The development of reliable methods to tackle these problems, such as complete-search algorithms, as well as the analysis of these methods constitutes an exiting research topic. A particular focus over the past few decades has been on computing both tight approximations and rigorous enclosures for the reachable set of parametric / uncertain dynamic systems, with developments scattered across the control, optimization and reliable computing literature. Despite significant advances, only simple dynamic systems comprising either a small number of states or subject to a small level of uncertainty can be tackled with these methods to this day. Attempts have also been made to embed these methods within global and robust dynamic optimization solvers, but these approaches are themselves restricted to simple, small-scale problems only. In this special issue of *Optimal Control Applications and Methods*, we are providing a selection of articles that feature novel algorithmic developments to address some of these outstanding challenges.

The paper by Harwood and Barton [1] presents a novel approach to computing affine relaxations on the solutions of constrained parametric ordinary differential equations (ODEs). They develop an extension of differential inequality-based comparison theorems, whereby a polyhedral outer-approximation of the reachable set of the constrained ODE system is constructed under uncertain time-varying inputs and initial conditions. In particular, the new relaxation method is capable of incorporating information from extra state constraints. They show by means of several numerical examples, that incorporating state bound and constraint information leads to tighter enclosures, and that this extra information may also speed up the computation of the relaxations.

Diedam and Sager [2] propose to solve global optimal control problems with a new algorithm that combines ideas from Bock's direct multiple shooting method for locally optimal control with the αBB method. The introduction of additional variables to lift the dynamic constraints leads to different algorithmic behavior, both in local and in global optimization. The authors show that the multiple shooting approach may have computational advantages compared to the single shooting approach. In addition to the well-known advantages of multiple shooting for each local optimal control problem that needs to be solved in a branch-and-bound tree, the overall number of nodes may be significantly reduced due to stronger bounds. The authors prove that the right branching strategy guarantees that there is no increase in the number of nodes and formulate assumptions for an improvement of the bounds resulting from convex under- and concave overestimations. This surprising result is mainly due to the effect that the additional variables reset the propagated bounds on the values of states and sensitivities, and hence avoid an exponential growth in time.

The paper by Bregner and Kirches [3] is concerned with the optimization dynamic systems with parametric uncertainty subject to chance constraints. They develop a method to approximate the chance constraints by propagating a polynomial chaos expansion of the reachable set. Theoretical guarantees on the level of constraint satisfaction are established from an a priori error estimate on the reachable set approximation, which is shown to converge exponentially with the size of the expansion for sufficiently smooth solution trajectories. A particular appeal of their approach lies in the fact that it is non-intrusive, and can thus be implemented seamlessly on top of state-of-the-art (local or global) dynamic optimization solvers. The methodology is demonstrated on two real-world nonlinear uncertain optimal control problems, a semibatch fermentation process and a container crane control problem, using Monte Carlo sampling to verify the robustness level in terms of constraint satisfaction.

Houska, Li and Chachuat [4] present a rigorous approach to robust optimal control in the presence of bounded time-varying uncertainty and nonlinear ODEs with affine uncertainty dependence. Their algorithm combines a high-order moment expansion of the time-varying uncertainty, with uncertainty set-propagation based on ellipsoidal calculus for bounding the effect of the (infinite-dimensional) remainder term in this expansion. The paper proves that the error introduced by the expansion converges to zero on adding more moments, and exploits this property for the construction of a conservative, yet computationally tractable, robust optimization problem whose solution values are converging to those of the original robust optimal control problem. The approach and its convergence properties are illustrated with the case study of a motorized robot arm.

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