

HYDROSTATIC EXTRUSION OF TUBES

by

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To my Parents and my Wife

ABSTRACT

The main purpose of this thesis is to provide information for the design of fixed mandrels used in the hydrostatic extrusion-drawing of tubes.

The literature is reviewed to sum up the methods used to assess the normal and shear stresses acting at the plastically deforming metal/tool interface.

An experimental apparatus was designed for the hydrostatic extrusion-drawing of the tubes enabling the recording of the extrusion pressure, drawing force, friction force on the deforming material/mandrel interface and the hoop and axial strains in the bore of the mandrel.

Experiments were carried out for aluminium and copper extrusions under several controlled conditions.

An upper bound approach for the hydrostatic extrusion-drawing of hollow and solid specimens is presented. It is based on general velocity and strain rate fields whose formulae are obtained analytically with the assumption that the plastic deformation zone boundaries and that the streamlines in that region are straight lines. Because the derivation is analytical it has some advantages in computation time and accuracy in relation to velocity fields derived from a flow function, which usually leads to numerical differentiation.

For conical boundaries the theoretical extrusion pressure, calculated by the proposed upper bound approach is minimized with respect to two parameters.

The stresses in the plastically deformed region are found by means of the weighted residual method combined with the derived velocity and strain rate fields for conical boundaries.

The stress distributions in the mandrel are determined by a

finite element computer program and provide information for the design criteria for the mandrel.

Finally strains in the mandrel bore are computed from the stresses obtained theoretically at the deforming material/tool interface and compared with the experimental results.

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NOMENCLATURE

A	Area
ΔA	Elemental area
A_{ij}, B_{kl}, C_{mn}	Parameters in stress functions
a_i	Parameter
d, d_1, d_2	Distances
E	Young's Modulus
F	Drawing force
F_f, F_{fp}, F_{fe}, F_d	Friction forces
G	Weighting factor
l_d	Die land length
l_m	Length of elastic zone/mandrel interface
m	Friction factor
N	Total number of elements
n_n	Number of nodal points on a streamline
n_s	Number of nodal points on the boundaries
P	Pressure
R	Extrusion ratio
R_o	Mandrel ratio
R_1	Outside radius of the product
R_2	Outside radius of the billet
r	Co-ordinate in the r direction
r_1, r_2	Co-ordinate in the r direction of a point in boundaries Γ_1 and Γ_2 respectively
r_c	Element centroid radial co-ordinate
r_m	Radial co-ordinate of the middle point of the elemental segment
S	Area
ΔS	Elemental area
t	Time

v, v_i	Velocity at a point
v_1, v_2	Velocity at a point on the boundaries Γ_1 and Γ_2 respectively belonging to the deformation zone
v_{le}	Velocity of the product
v	Velocity of the billet
v_N	Velocity component normal to the boundary
Δv	Velocity discontinuity
Δv_T	Tangential velocity discontinuity
V	Volume
ΔV	Elemental volume
$\dot{V}$	Volume rate
w_k	Weighting factor
$\dot{W}$	Power
$\dot{W}_d$	Power of deformation
$\dot{W}_{r1}, \dot{W}_{r2}$	Power dissipated on surfaces Γ_1 and Γ_2 respectively
$\dot{W}_{f3}, \dot{W}_{f4}, \dot{W}_{f5}, \dot{W}_{f6}$	Power consumption on surfaces $\Gamma_3, \Gamma_4, \Gamma_5$ and Γ_6
$\dot{W}_p$	Power due to the applied pressure
$\dot{W}_{ps}$	Power due to the prescribed forces
Y	Initial yield stress
Y_{qi}	Trial functions
z	Co-ordinate in z direction
α	Semi cone angle
α_i	Undetermined parameter
Γ	Boundary surfaces
γ	Engineering strain
δ_{ij}	Kronecker delta
ϵ_{ij}	Component of the strain tensor
$\bar{\epsilon}$	Effective strain
$\dot{\epsilon}_{ij}$	Component of the strain rate tensor
$\dot{\epsilon}_r$	Radial strain rate

$\dot{\epsilon}_{\theta}$	Tangential strain rate
$\dot{\epsilon}_z$	Axial strain rate
$\dot{\epsilon}_{rz}$	Shear strain rate
$\bar{\dot{\epsilon}}$	Effective strain rate
θ	Inclination angle of a streamline
ν	Poisson's ratio
ρ, ρ_1, ρ_2	Distances
σ_{ij}	Component of the stress tensor
σ_r	Radial stress
σ_{θ}	Hoop stress
σ_z	Axial stress
σ_s	Flow stress
$\bar{\sigma}$	Equivalent stress
τ	Shear stress
ϕ_i	Trial function. Stress Function
ψ	Stream function
ψ_1, ψ_2	Angle of the normal to the boundaries Γ_1 and Γ_2 to the z-axis respectively

CHAPTER 1

INTRODUCTION

1.1 - The Hydrostatic Extrusion - Drawing Process

Although metalworking processes have been in use for several thousand years, it is only in the last two decades that considerable progress has been made in the development of new industrial processes and techniques and also in the theoretical analysis of the metal forming operations.

The continuous development of technology demands research into sophisticated variants of conventional metalworking processes as it is the case of hydrostatic extrusion.

Hydrostatic extrusion was first described by Robertson (1) in 1893, although it seems that no practical work has been carried out. Bridgman (2) appears to be the first to examine hydrostatic extrusion in a scientific manner and encountered some difficulties in his experiments, mainly related to the control of the extrusion at high pressures and to large thermal effects due to the high extrusion velocities.

Pugh (3) and his colleagues at the National Engineering Laboratory have investigated the process theoretically and experimentally in great detail, studying mainly the relationship between the extrusion pressure and extrusion ratio for a wide range of engineering materials and the resulting properties of the products, particularly those extruded into a back pressure. A comprehensive summary of this work and of other researches is presented in Ref (3,4).

Independently and simultaneously, Beresnev and his colleagues developed research work at the High Pressure Laboratory of the Academy of Sciences of Moscow. Their contribution towards the understanding of the novel process is presented in the book by Beresnev et al (5).

Many other research workers have investigated the hydrostatic extrusion process including Randall, Davis, Sievgiej and Lowestein (6) at Nuclear Metals Inc., Fiorentino, Sabroff and Boulger (7) at the Battelle Memorial Institute, Bobrowski (8) of the Pressure Technology Corporation of America, Rozner and Faupel (9) of Du Pont in U.S.A., Green (10), Slater (11), Thompson (12) and Kronberger (13) at the United Kingdom Atomic Authority, Alexander and Lengyel (14, 15) at Imperial College, Avitzur (16) at Lehigh University, Duffil and Mellor (17) at Birmingham University, Parsons, Bretherton and Cole (18) at Leeds University.

The book by Alexander and Lengyel (19) contains a detailed review of the research work in this field up to 1971.

Interest continued to grow and by 1973 a Conference on Hydrostatic Extrusion was held at the University of Stirling following the earlier Conference on High Pressure Engineering organized by the Institution of Mechanical Engineers in 1967, the C.I.R.P. Conference in Nottingham and the M.T.D.R. Conference in Birmingham in 1968.

In recent years extensive work in hydrostatic extrusion has been carried out by Japanese investigators at Kobe Steel Ltd and Hitachi Cable Ltd. Quite recently at the 20th M.T.D.R. Conference in Birmingham two papers were given dealing with hydrostatic extrusion (20, 21).

Hydrostatic extrusion is a metalworking process which essentially consists of extruding a specimen through a die using a pressurized fluid which surrounds the billet (Fig. 1.1). When the pressure reaches a specified value the billet is extruded through the die orifice into the shape determined by the die. As the billet is not in contact with the wall of the container friction is reduced. The reduction in frictional effects makes it possible to use low die angles, decreasing redundant work. Consequently larger reductions can be achieved under otherwise identical conditions than it would be possible in a conventional extrusion. The control of the rate of extrusion is difficult to achieve.

The extrusion speed is often too high and the billet and fluid are ejected violently from the container.

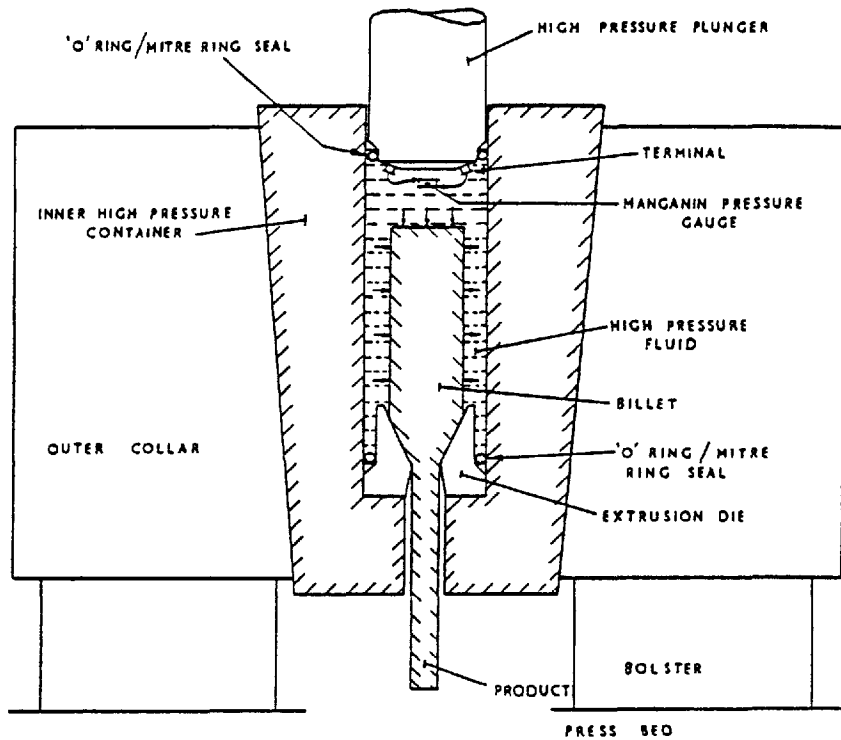


FIG. 1.1 - SIMPLE HYDROSTATIC EXTRUSION (After Ref. 4)

If the extrusion speed is not controlled stick slip instability may arise.

Control of the hydrostatic extrusion process can be achieved by either the application of a compressive force to the billet or a drawing force to the product. These techniques are often called billet and product augmentation, respectively. The latter is the procedure adopted in the experiments carried out in the present work.

Figure 1.2 illustrates the four basic systems in use.

In augmented hydrostatic extrusion the fluid pressure is raised to a value slightly below that required for extrusion and the augmentation force provides the stress necessary to make up the total stress needed for the process.

In the differential pressure extrusion or fluid to fluid extrusion (Fig. 1.2b) both billet and product are surrounded by fluid at different pressures. The effective pressure causing extrusion is the difference between the pressures of the two fluids. This system is attractive for

the extrusion of brittle materials.

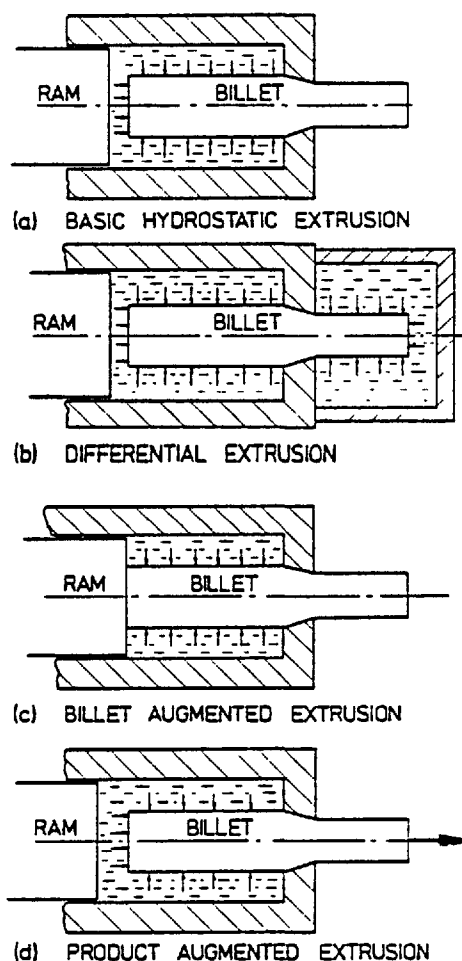
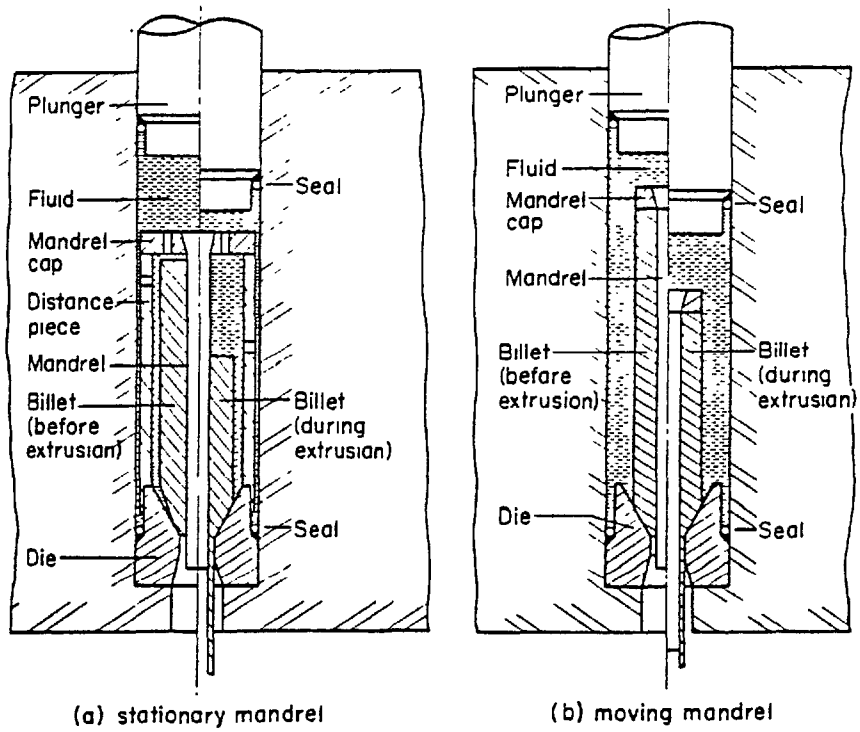


FIG. 1.2 - BASIC SYSTEMS IN HYDROSTATIC EXTRUSION (After Ref. 22)

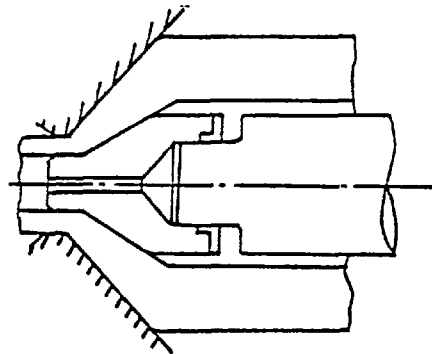
The hydrostatic extrusion process enables not only the extrusion of round bars but also billets of complex sections, particularly the extrusion of hollow sections. For the extrusion of tubes a mandrel is fitted into the hollow billet in order to size the bore of the tubular product.

Figure 1.3 shows three different arrangements of tube extrusion corresponding to different designs of the mandrel, namely the stationary, moving and floating mandrel.

In the present experimental work a hollow stationary mandrel was chosen because of the nature of the measurements required.



(After Ref. 4)



c) floating mandrel

FIG. 1.3 - ARRANGEMENTS OF TUBE EXTRUSION

1.2 - Literature Survey of Pressure Distribution and Friction Conditions at the Metal Tool/Interface

1.2.1 - Introduction

In metalworking processes where the bulk of material undergoes plastic deformation in contact with a tool which remains elastic, the frictional conditions differ from those existing when both contacting bodies remain elastic.

Friction in metal forming processes may take place under conditions ranging from boundary to hydrodynamic lubrication. In boundary lubrication, which covers the vast majority of cases in metal forming, the surfaces, are lubricated either by molecular layers adsorbed onto the surfaces, or by surface films formed by chemical reactions with the surface. Thus, the effective value of the interface shear stress is reduced. In the quasi-hydrodynamic regime hydrodynamic pockets are formed by entrapment of lubricant in troughs between the asperities, reducing friction by preventing increase in the contact area. The coefficient of friction in this type of lubrication is a function of the lubricant properties and also the conditions of the metal forming process itself. Hydrodynamic films can exist in certain metalworking processes such as rolling, wire drawing and hydrostatic extrusion. The subject has been treated theoretically by applying Reynold's equation together with the equilibrium equation, boundary conditions, yield criterion and associated flow rule.

The transition between quasi-hydrodynamic and full hydrodynamic lubrication is not very well understood. Although some theories predict the conditions for initiation of the fluid film, either constant coefficient of friction or a constant friction factor is assumed, if the fluid film thickness is not sufficient to maintain a complete separation of the surfaces.

Normal pressure and shear stresses at the metal tool interface are two interconnected parameters. Although their joint action is well recognized by the effects on tool stresses and wear, flow pattern, forming forces, surface finish and dimensional accuracy of the product, they are not easily separable.

A large number of approaches have been tried to assess the normal pressure and the coefficient of friction at the deforming material/tool interface. In what follows a review of these methods used in metal forming processes is presented.

1.2.2 - Experimental Methods

Within the experimental approaches two categories are usually considered, namely, the direct and indirect approach.

1.2.2.1 - Direct Approach

One of these experimental techniques, called the pressure sensitive pin technique, was introduced by Siebel and Lueg (23) who determined the pressure distribution along the arc of contact in rolling. In this method pressure transmission pins were embedded normally to the surface of the roll.

Later, Mc Gregor and Palme (24), with the use of strain gauges, attempted to find the normal and tangential stresses during rolling by measuring the radial and lateral deflection of a pin in the form of a cantilever. The technique was improved by van Rooyen and Backofen (25) to determine the coefficient of friction between roll and the material strip. They used a special roll containing two small pins installed at right angles to the roll axis, one in a radial direction and the other inclined to the first in an oblique direction. The coefficient of friction in this method, is a function of the ratio of the stress in

the oblique pin to that in the radial pin.

The method has been used by El-Behery, Lambie and Johnson (26) for measuring the container wall pressure and coefficient of friction in the axi-symmetric extrusion of lead. Pearsall and Backofen (27) and Nagamatsu, Murota and Jimma (28) have also used this technique for measuring the distribution of normal pressure and coefficient of friction at the metal/tool interface during the upsetting of aluminium discs.

Although the method is very effective to determine the pressure distribution and the coefficient of friction at the metal/tool interface there are, nevertheless certain sources of error in it.

The gap between the pin and the tool should be small to avoid the extrusion of the deforming material into it. On the other hand, reducing this gap the friction between the tool and pin can be very large affecting the results.

Another limitation is the possibility of binding between the pin and the tool owing to the elastic distortion of the latter. Also the protrusion or recession of the pin relative to the surface of the tool could have a marked effect on the values of the coefficient of friction measured with the oblique-radial technique. The oblique pin is rather sensitive to small amounts of protrusion or recession.

To apply the direct approach it is required to have enough space available in the tool to accomodate the pins. This fact may constitute a restriction in the use of the method as the holes to house the pins would weaken the tool.

1.2.2.2 - Indirect Approach

In this type of experimental indirect approach the values of the mean pressure and coefficient of friction are determined by measuring the values of the other process variables.

Included in this category is the method developed by Whitton and Ford (29). They evaluated the coefficient of friction in cold strip rolling without making use of any theory of the distribution of roll pressure along the arc of contact. In this technique the coefficient of friction, assumed to be constant over the arc of contact, is determined from the measured values of the roll force and torque while applying a back tension.

The split die technique, first suggested by Mac Lellan (30) to investigate the mean coefficient of friction and die pressure in drawing, is another method which does not need recourse to the theory of metal forming. Wistreich (31) utilized this technique in wire drawing determining the values of these variables by measuring the force that tends to separate the two halves and the drawing force.

While simple, this method could not be used with liquid lubricants as not only the lubricant leaks into the split die but also generates unwanted separating forces. The method was improved by Yang (32) who used a large ring to restrain the split die. He measured the die separating force with a strain gauged dynamometer housed in the retaining ring.

Majors (33) evaluated the normal force on the die by measuring the hoop strain at one point on the outer circumference of the die. The die was calibrated by applying hydraulic pressure to its inside with the help of a closing plug.

The plug drawing process, used by Blazynski and Cole (34) and Rowe and Lancaster (35), can be utilized to advantage as the geometry is simple. A mean coefficient of friction can be calculated by measuring the separating force on the die and the tension in the plug bar.

Parsons, Bretherton and Cole (18) used strain gauges attached to the periphery of the die in the hydrostatic extrusion/drawing process. To determine the effect of product augmentation on the frictional

conditions, the strain gauges were calibrated by applying a known fluid pressure over the area of the truncated cone of the die.

These methods can only assess the mean die pressure and are unsuitable for determining the distribution of the die pressure. To obtain the die pressure distribution Kamyab (36) and Alexander and Kamyab (37) have refined the strain gauged die technique. They have used long, narrow strain gauges to measure the circumferential strain at sections along the die during copper extrusion. The pressure distribution at the conical surface of the die was assumed to be a fourth order polynomial with unknown coefficients. Using a finite element program the strains produced by the assumed pressure distributions had to match the strains measured experimentally. The coefficients in the polynomial and the average coefficient of friction were evaluated by an iterative procedure. The unknown parameters in the polynomial were adjusted until the sum of the squared deviations of the calculated and experimental strains was a minimum. The coefficient of friction determined for these experiments with castor oil and MoS_2 grease as lubricant was found to be 0.03.

Said (38), studying the clamp size suitable to generate an axial frictional force sufficient to clamp the billet material during successive extrusion cycles in semi-continuous hydrostatic extrusion, adopted a similar method. His finite element analysis indicated that the axial stresses across the cross-sections of the clamp were approximately constant. This assumption together with the equilibrium equation of an element along the clamp length enabled the determination of the pressure distribution and the average coefficient of friction from the experimentally measured values of the axial and hoop strains at the outside surface of the clamp.

The strain gauge technique has been applied by Lengyel and Musuva (39) and Lengyel and Mutlu (40) to extrusion containers, experimentally to obtain the approximate pressure distribution along the container bore.

Daneshi and Hawkyard (41) used a split platen technique to find the pressure distribution in an upsetting operation. This method has been recently modified by Chitkara and Taspinar (42) to measure the pressure distribution over an unsymmetrical turbine blade section in rolling by using strain gauges embedded in the end faces of one of the split-form roll rings.

The photoelasticity method has been used for the evaluation of the coefficient of friction and the pressure distribution in plane strain drawing dies. Cook and Wistreich (43) determined the value of these variables in plane strain drawing of lead through a die made of a photoelastic material. The method was further developed by Osashi and Nishitani (44) for studying the deformation in a specimen also made of a photoelastic material as well. The application of the photoelastic technique is very limited due to the kind of materials that can be utilized. Also, only plane strain cases can be analysed by this method.

The visioplasticity technique has been used in extrusion not only to evaluate the pressure distribution and coefficient of friction but also to give a detailed information on the strain rates, total strain and state of stress throughout the deformation zone. It was first suggested by Thomsen and Lapsley (45) and Thomsen, Yang and Bierbower (46). This method requires that an experiment is performed on a split billet which was previously provided with a grid on the split plane. A flow function is defined from the steady state co-ordinates of the points in the deformed flow line. By numerical differentiation of the flow function the velocities at the points in the deformation zone are found. Numerical differentiation of the velocities enables the strain rates to be assessed. The integration of the strain rates with respect to time, along the streamlines yields the total strains. The strain rate and total strain fields are then used with the equilibrium equation, constitutive equation and a yield criterion and associated flow rule to give

the complete stress field where plastic flow has taken place. Throughout the analysis the elastic strains are neglected, as it is usual in problems of large plastic deformation.

The pressure and the coefficient of friction along the billet/die interface have been determined using the visioplasticity technique by Shabaik and Thomsen (47) in conventional extrusion and Dunn (48) and Dunn and Lengyel (49) in hydrostatic extrusion.

Although the visioplasticity method provides accurate information about the flow of deforming material it presents some difficulties in the calculation of the stresses mainly the boundaries so that the frictional conditions at the metal-tool interface obtained by this method might be far from the actual ones.

Ishikawa, Hata and Goto (50) proposed a method to describe approximately the mechanics of material flow during forwards extrusion. They assume that the flow lines can be approximated by the function of hyperbolic tangent. With this method there is no need to perform an extrusion test and the smoothing procedure, which is a feature of visioplasticity, is not necessary.

1.2.3 - Theoretical Approach

The pressure distribution and the coefficient of friction at the plastically deformed zone - tool interface can be assessed by methods which make use of the theory of the particular metalworking process.

In this category can be included the method suggested by Kunogi (51) and developed by Male and Cockroft (52) to measure the coefficient of friction at the metal/tool interface during the forging of rings. In this technique the coefficient of friction is related to the change in the internal diameter produced by a certain amount of compression in the thickness direction. Measurements of the internal diameter of compressed

rings provide a particularly sensitive means of studying the coefficient of friction, since the internal diameter increases or decreases as the coefficient of friction is small or large respectively. Male and Cockroft used the Schroder and Webster (53) analysis to compute the theoretical average pressure during the compression of rings under different conditions of friction and obtained experimental calibration curves.

The theoretical solution proposed by Alexander (54) for plane strain indentation problems has been used by Takahashi and Alexander (55) for the evaluation of friction in this process.

A large variety of upper bound solutions have been proposed to analyse metal forming processes. Coefficients of friction have been obtained by assuming these theories and finding a value which gives the closest fit to experimental results. However, this procedure gives too low coefficient of friction, as the theory itself tends to overestimate the load required in the process. A review of upper bound solutions to metal flow is given in Chapter 4.

Duffill, Hooke and Mellor (56) obtained the pressure distribution on the conical surface of a die for hydrostatic extrusion. They have used the measured extrusion pressure to calculate the die pressure at a known extrusion ratio. For this purpose they have modified the expression of die pressure utilized by Green (57) for plane strain drawing to be applied to axisymmetric cases.

1.2.4 - Numerical Methods

The application of variational principles to plasticity and the development of numerical methods allowed not only the description of the metal flow but also to obtain a complete stress solution to a large variety of metal forming problems.

In recent years extensive work has been carried out to analyse metalworking problems by the finite element method. A comprehensive review of the elastic-plastic analysis of problems of large strains is presented by Blass (58). Another numerical method that is receiving increasing attention to solve the plasticity equations is the method of weighted residuals. The present work will deal with this method.

In what follows the literature dealing with these methods, used to find the pressure distribution at the deforming material-tool interface is reviewed.

1.2.4.1 - Finite Element Methods

Iwata, Osakada and Fujino (59) presented an elastoplastic finite element analysis of hydrostatic extrusion using triangular ring elements. They found that the die pressure was high near the entry and exit of the die. The higher the friction the higher the pressure. In the analysis only small strains are reported as their method is not suitable for problems involving large deformations.

The elastoplastic finite element method for small displacements was employed by Kamyab (37) to find the pressure distribution on the die during the hydrostatic extrusion of rods. This purpose was achieved by obtaining the distribution of the yield stress over a meridional plane of the deformed billet from experimental measurements of the hardness distribution.

Lee and Kobayashi (60), to overcome some of the drawbacks of the finite element elastoplastic formulation for large deformations, devised a new technique called the matrix method. This approach uses the basic principles of the finite element method and has the advantage of assuming rigid plastic behaviour of the materials, since in most metal forming processes the elastic strains can be neglected.

The matrix method is utilized in the analysis of compression of cylindrical specimens having initial height to diameter ratios of 1.0 and 0.75 under various friction conditions by Lee and Kobayashi (61) and ratios of 2.0 and 2.5 under dry friction conditions by Shah, Lee and Kobayashi (62). For the latter cases the authors show that the maximum pressure occurs at the centre or the periphery of the contact surface as the reductions are greater or less than 0.6 respectively.

Pressure distribution along the punch-billet interface in piercing was obtained by the matrix method by Shah, Oh and Kobayashi (63). They have found that initially the pressure is higher at the centre, but as the reduction increases the pressure increases at all points, being more accentuated near the corner than the centre of the punch. The pressure distribution along the container wall in the same process is obtained by the same method. The radial pressure distribution remains constant in the undeforming zone and decreases towards the free surface. With the matrix method the analysis of conventional axisymmetric extrusion was carried out by the same authors (63). They applied the method to work-hardening and non hardening materials. The pressure distribution along the die, found by that technique, shows that the pressure is maximum at the die entrance and decreases towards the exit. Near the exit the pressure increases by a small amount. They also report that the trend of the curves for nonhardening and workhardening materials is similar differing only by a constant.

1.2.4.2 - Weighted Residual Methods

The term "Weighted Residual Methods" has been introduced by Crandall (64) to denote a general approximation procedure for solving various types of differential equations in boundary-value and initial-value problems.

In this procedure an approximate solution is proposed in a form of a set of known trial functions with undetermined parameters. By substituting an approximate solution into the governing equation a residual function is obtained which is then required to be orthogonal to a selected set of weighting functions.

The method of weighted residuals, particularly the least squares method, have been used to solve plasticity problems and to determine the stresses in the plastically deformed zones. Steck (65) has applied the method to obtain the stress distribution in the extrusion of metals through conical dies and in the upsetting of rings. The method in the field of plasticity has been used mainly by German authors although at Imperial College has also been applied.

Helman (66) employed the least squares method to the study of the co-extrusion of two metals and determined the pressure distribution along their surface of contact in the deformation zone and the temperature distribution in that region.

Ravindranath (67) found the stress distribution in the plastically deformed zone during the upsetting of aluminium and copper rings using the least squares method. The deformation was assumed to be in steps and for each step the flow stress of the material was corrected by taking into account the calculated temperature rise.

Lange, Rebholz and Roll (68) report several solutions achieved by the method of weighted residuals in forward extrusion and upsetting and compare alternative solutions found by the finite element and weighted residuals methods.

1.3 - Aims of the Present Work

The determination of the stress distribution in the plastic deformation zone and the pressure distribution at the material/tool interface

has been a matter of interest as shown in the previous section. Among the various approaches the numerical methods, with the advent of the computers, are increasingly being applied as a mathematical tool which is able to predict the stress distribution in the process. To assess their usefulness it would be necessary to know the degree of accuracy they allow.

The main objective of the present work is to compare the pressure distribution at the metal/tool interface obtained by the weighted residual method with the experimental measurements. To achieve this objective the hoop and axial strains in the bore of a hollow cylindrical mandrel during the hydrostatic extrusion of copper and aluminium tubes have been obtained by two means :

- 1 - Experimentally by bonding axial and hoop strain gauges to the bore of the mandrel and recording the corresponding strains.
- 2 - Numerically by finding the pressure distribution at the deforming material/tool interface with the help of the least squares method and introducing it as input data into a finite element program which in turn calculates the strains at the mandrel bore.

The determination of the pressure distribution is by itself an important feature due to its role in mandrel design. For the same reason the dragging force due to the friction at the material/tool interface is also the subject of this investigation.

Finally, another aim of the present work is to find the stress distributions in the mandrel to supply the designer with information which enables the development of criteria for the design of fixed mandrels in hydrostatic extrusion-drawing of tubes.

CHAPTER 2

EXPERIMENTAL WORK

2.1 Introduction

The aim of the experimental work was to measure the axial and hoop strains in the bore of a hollow mandrel, the extrusion pressure, the drawing load and the friction force on the billet/mandrel interface during the hydrostatic extrusion-drawing of tubes.

Experiments have been carried out using commercial purity aluminium and high conductivity copper specimens, for several extrusion ratios and various extrusion pressure/drawing stress conditions.

The main purpose of these experiments is to gather information concerning the design of the mandrel and check the validity of the results obtained theoretically. Thus the mandrel was made hollow and a strip of strain gauges was bonded to its bore to allow a comparison between the experimental and theoretical strain distributions.

2.2 Experimental Equipment

2.2.1 Hydrostatic Extrusion - Drawing Apparatus

Various components of the experimental apparatus, as well as the design criterion related to their manufacture, have already been described: the container by Lengyel et al (69) and Lengyel and Ashford (70), the pulling device by Dunn (48) and Helman (66).

In what follows the characteristics of those components will

be described which are relevant to the experimental work, together with the modifications which have been introduced to obtain the present set-up.

A general view of the apparatus used in the hydrostatic extrusion-drawing of tubes is shown in Figure 2.1.

2.2.1.1 Drawing Bench

Part of the drawing bench was a screw jack manufactured by Duff-Norton & Co. Ltd., driven by a 1.1kW (1 1/2 H.P.) induction motor via a Kopp variator and reduction gear box and an Autogard torque limiting device.

Although the maximum pulling force permitted by the screw jack was 50kN, the maximum drawing load, actually used in the experiments, was limited by the strength of the roller chain (Figure 2.1) and could not exceed 15kN.

The Kopp variator enabled the product velocity to be varied in the range of 0.15cm/s to 1.3cm/s.

The main purpose of the torque limiting device was to ensure that the load applied to the product was not too large so as to cause fracture.

In order to measure the drawing force a load cell was located between the screw jack and the roller chain. The latter, passing through a pulley, was connected to the prepared tag of the billet. The friction between the roller chain and the pulley was negligible as two ball bearings were used to support the pulley, enabling it to rotate freely. Therefore the pulling load measured by the load cell was taken as the drawing force acting on the billet.

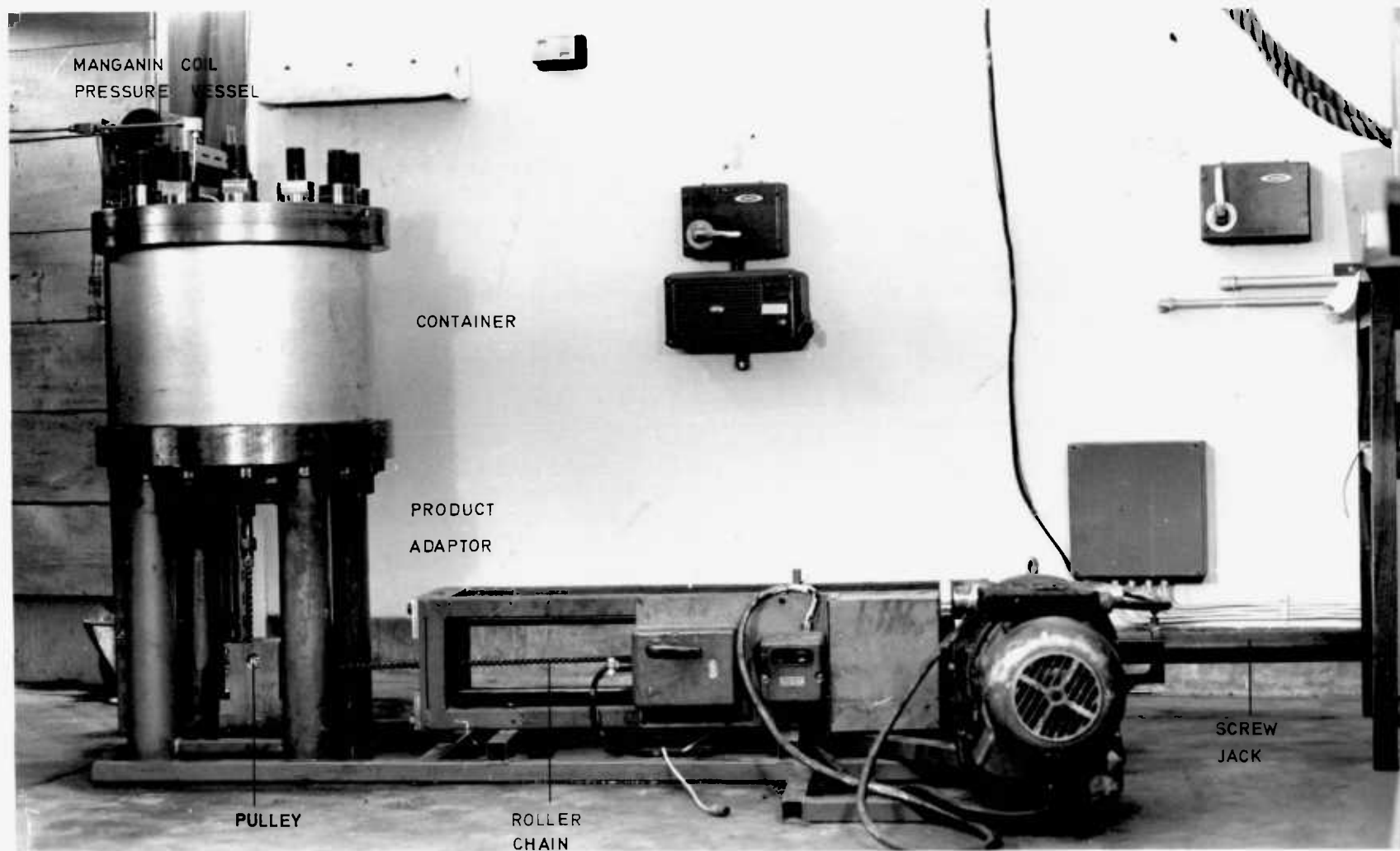


FIG. 2.1 — GENERAL VIEW OF THE APPARATUS.

2.2.1.2 Extrusion Apparatus

The experimental arrangement of the extrusion apparatus is shown in Figure 2.2.

2.2.1.2.1 Pressure Vessel

The container was a monoblock cylinder, made from a high tensile steel VH50 (0.4%C - 1.05%Si - 0.4%Mn - 5.5%Cr - 1.15%V - 1.5%Mo), which was designed to support fluid pressures up to 700MN/m².

Owing to the compressibility of liquids at high pressures, large elastic energy is stored in the container constituting a potential danger. In order to reduce this energy, the volume of fluid inside the container was reduced by using a filler cylinder, Figure 2.2. Also for safety reasons the pressure vessel was surrounded by a guard.

2.2.1.2.2 Mandrel

The thick walled cylindrical hollow mandrel (Figure 2.2) was made from KEA 145 tool steel, heat treated to 52-54 Rockwell C hardness. It was designed to remain elastic but with as thin wall as possible, to give the maximum hoop strain at the bore.

The external and internal diameters were respectively 25.4mm and 9.525mm.

To ensure that the mandrel remains elastic the fluid pressure was restricted to a maximum of 350MN/m². This pressure determines the maximum extrusion ratio for the materials in study.

Another aspect that was taken into account was the length of the mandrel. To calibrate the strain gauges bonded to its bore the

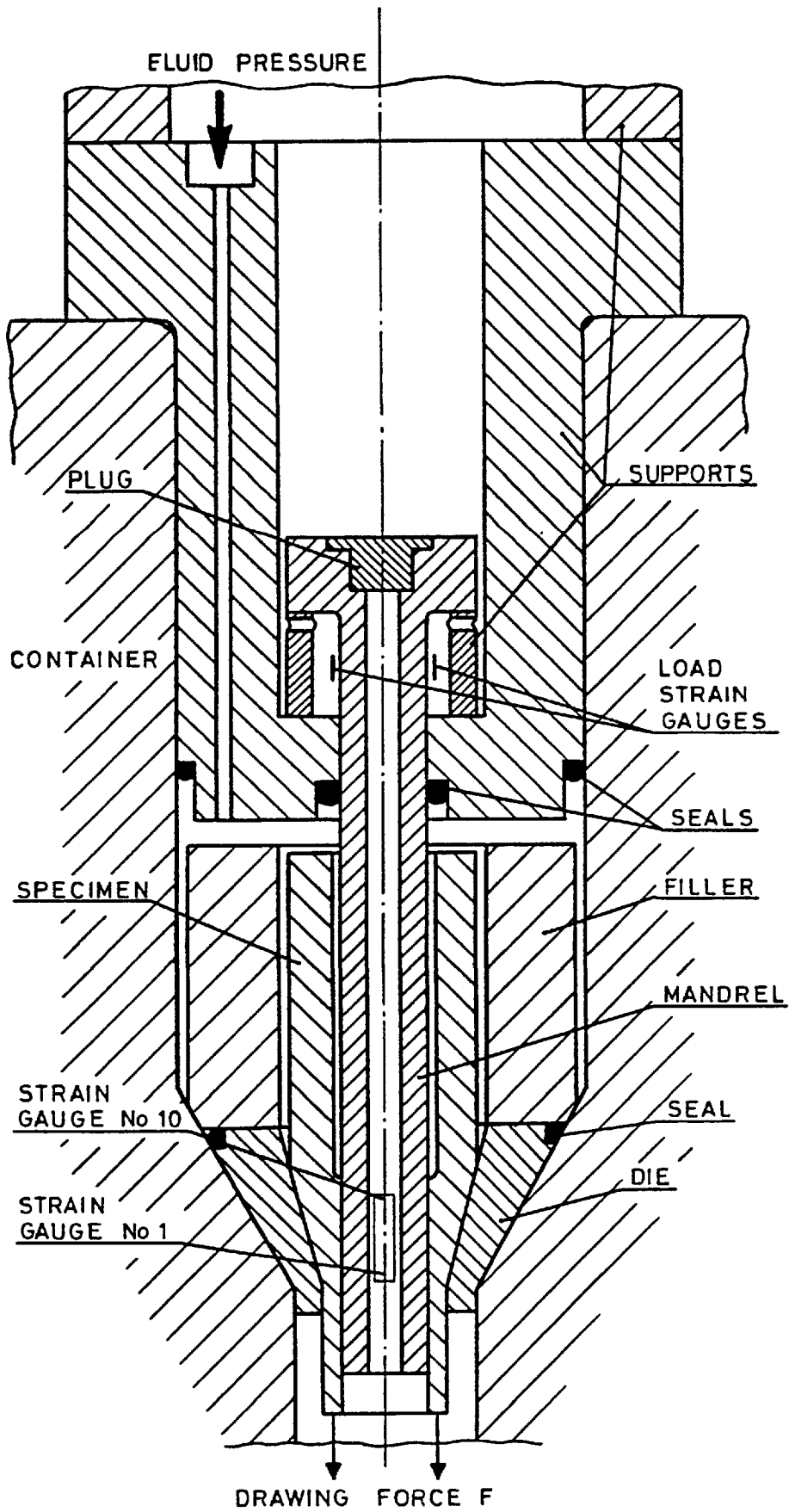


FIGURE 2.2 - EXPERIMENTAL ARRANGEMENT

mandrel was subjected to several external fluid pressures. The mandrel was designed to have a sufficient length so that the effect of seals during the strain gauge calibration could be neglected, as it will be discussed in section 2.3.5.

2.2.1.2.3 Die

The die (Figure 2.2) was made from KEA 180 tool steel heat treated to 52-54 Rockwell C hardness.

All tests were carried out using the same die, which was 45.72mm in height and 30° included angle. The die land was 2.5mm.

2.2.1.2.4 Supports

The supports shown in Figure 2.2 were made from EN30B steel heat treated to 50 Rockwell C hardness.

The mandrel support was 25.4mm high and its external and bore diameters were 45.7mm and 30.5mm respectively.

Initially it was designed to be a load cell to measure the friction force at the billet/mandrel interface. For this purpose two axial and two hoop strain gauges were bonded onto its outside surface and connected to make a full bridge with gauges of the same type in the opposite arms. The load cell was calibrated under compression in a high precision testing machine and the output signal was recorded by a Solartron data logger.

To ensure that the load cell would give the same response for the same load, irrespective of its position, four calibration tests were carried out after 90° , 180° and 270° rotation of the load cell, around its vertical axis, relative to its initial position. In each position the load cell was checked for repeatability.

It was found that, although for each position of the load cell the calibration results were repeatable, this was not the case when two different positions were compared. As it was difficult to guarantee exactly the same position of the load cell inside the support this method of measuring frictional load was abandoned. The alternative solution adopted was to bond the strain gauges to the outside surface of the mandrel as it will be discussed in section 2.3.4.

2.2.1.2.5 Seals (19)

The seal was a synthetic rubber O-ring backed up by a copper mitre ring and held by a retainer. Details of this arrangement have been omitted from Figure 2.2. To seal between the die and the container the retainer was unnecessary. This arrangement was satisfactory, there was no need to replace the seals throughout the experiments.

The main sealing problem arose in preventing the fluid leaking between the specimen and the mandrel, in the initial stages of the extrusion. To overcome this difficulty Thompson (12) developed a rear end seal for tube extrusion, Figure 2.3. In the present a narrow lip formed in the bore of the billets was found satisfactory.

2.2.1.2.6 Extrusion Fluid and Billet Lubrication

The fluid in hydrostatic extrusion is required to transmit the pressure effectively and to act as a lubricant (4,19).

Several investigators (3,49,71,72) have shown that castor oil not only transmits the pressure well (Figure 2.4) but also have lubricant properties depending on the materials to be extruded.

In the present work all the experiments were carried out using a 4:1 mixture of castor oil and methyl alcohol, with the billet coated with a graphite solution. The addition of methyl alcohol is necessary since above a certain pressure castor oil starts to freeze.

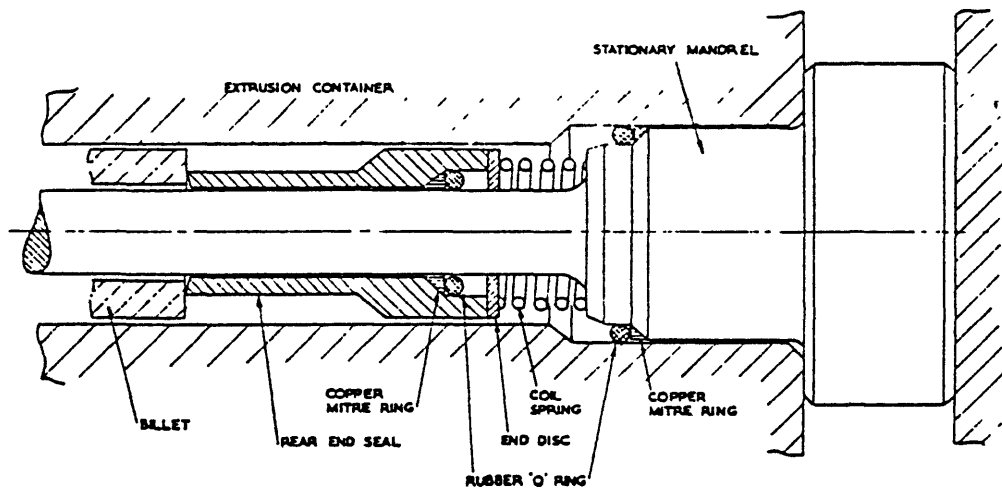


FIGURE 2.3 REAR END SEAL FOR TUBE EXTRUSION
(AFTER THOMPSON, REFERENCE 12)

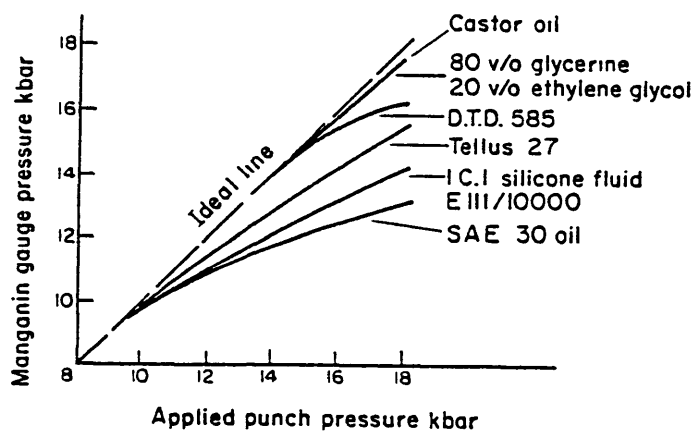


FIGURE 2.4 BEHAVIOUR OF PRESSURE TRANSMITTING FLUIDS
(AFTER LOWE ET AL, REFERENCE 71)

2.2.1.2.7 Billet Design

Figure 2.5 is a drawing of a billet used in the experiments.

The main factors that have been taken into account in the design of the billet were the following:

1. It must have a tag to allow the billet to be pulled.
2. Sealing must be ensured along the billet/mandrel and billet/die interfaces, to prevent the fluid leaking in the earlier stages of extrusion.
3. Preventing, or at least attenuating, the effects of a burst in case the fluid pressure increases to a high level in the container. The billet being shot out at an excessive speed could be dangerous and could damage the components underneath the container.
4. Allowing a small gap (0.5mm) between the mandrel and the undeformed part of the billet enables not only the extrusion fluid to reach the plastic deformation zone/mandrel interface, but avoids the friction between the mandrel and the billet before the latter reaches the die as well.

Figure 2.5 shows the tag geometry chosen in view of the length of the mandrel. The end of the billet tag is threaded to enable an adaptor to be connected. The other end of the adaptor is attached to the roller chain.

To prevent the fluid leaking along the billet/die interface, the billet was machined to the same angle as the die.

To develop sealing along the billet/mandrel interface, a few experiments had to be done to determine the appropriate size of the lip in the bore of the billet.

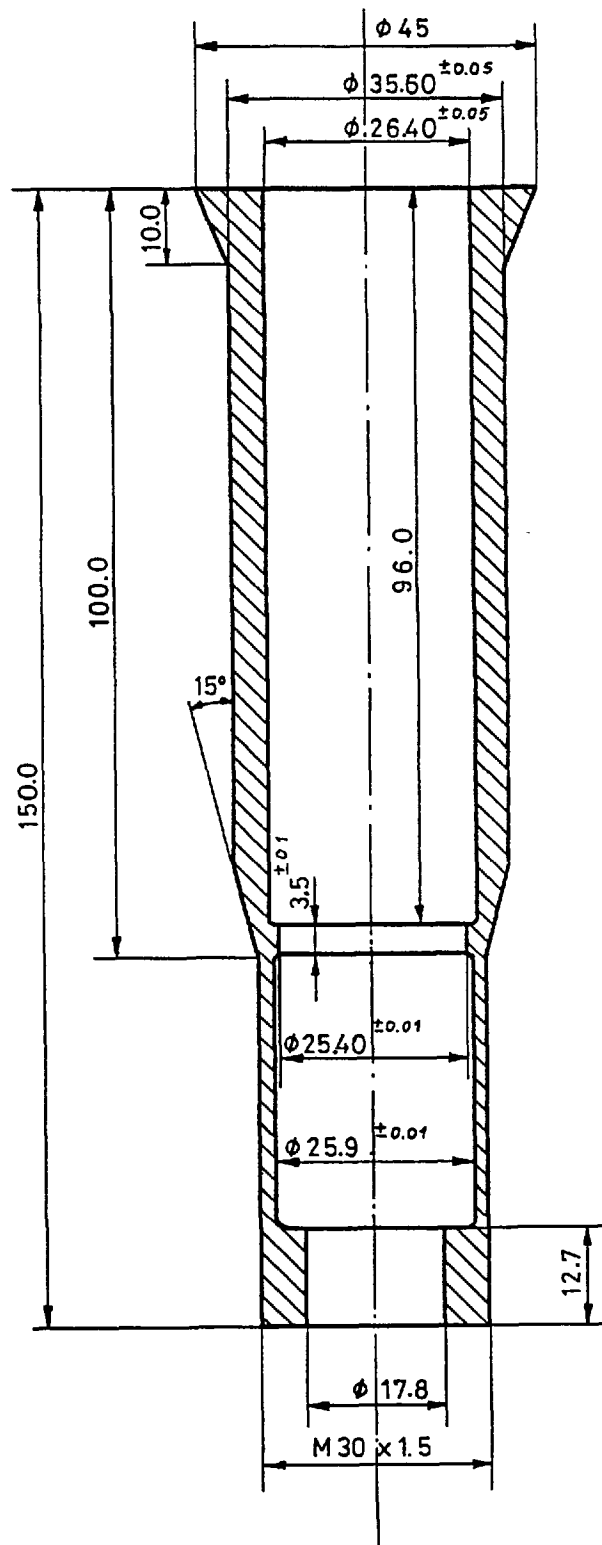


FIGURE 2.5 - SPECIMEN FOR AN EXTRUSION RATIO OF 2.0

The top end of the billet had a larger cross-section to prevent full extrusion, thus, the sudden release of the high pressure fluid.

As the same die was used in all experiments the external diameter of the billet was changed to provide various extrusion ratios.

2.2.2 Generation of Pressure

The fluid pressure was generated by a pump-intensifier unit manufactured by Stansted Filtration Limited, with two pump-intensifier sets working alternately to provide a constant pressure.

Each pump, Figure 2.6, was powered by an 0.4kW (1/2 H.P.) induction motor and could deliver oil at a rate of 410cm³/min up to a pressure of 103.4MN/m² (15000lbf/sq.inch). The pumps stopped automatically on attaining a preset pressure and held it constant, restarting when necessary to make up for losses.

The pressure was further increased by the intensifier by a factor of 37.

A control unit was used to set a pre-determined pressure.

The output signal from a manganin coil, which monitored the pressure on the high pressure side, was fed into the control unit.

In Figure 2.6 are visible a Bourdon type pressure gauge to give a quick indication of the pressure on the high pressure circuit and a release valve - which isolates the intensifier unit from the hydrostatic extrusion rig.

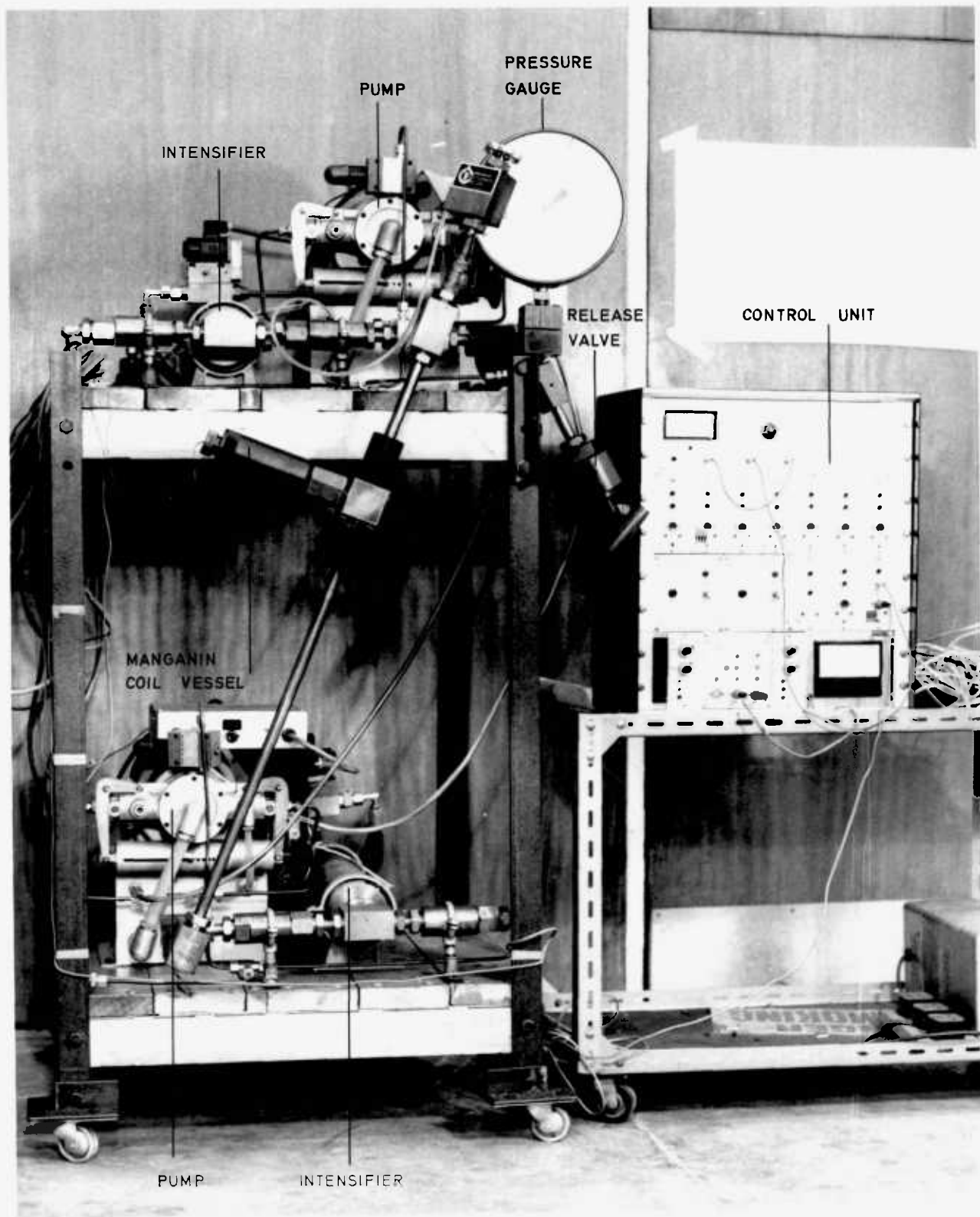


Fig. 2.6 - PUMP - INTENSIFIER AND CONTROL UNITS.

2.3 Instrumentation and Experimental Measurements

2.3.1 Data Logger

A Solartron LP 1029 digital data recorder, Figure 2.7 was utilized to record the output signals from a manganin coil and the strain gauges used in the experimental measurements.

Up to 100 analogue signals can be converted to a digital form and displayed in an inline unit. The data logger allows a scanning of all or a limited number of channels at speeds varying between one sample in every four seconds and ten samples per second.

Alternatively a single channel can be selected and continually scanned.

A high speed numeric printer and a paper tape punch connected on line with the data recorder, enable the readings and the channel identity to be either printed out on a paper strip, or punched on a paper tape, or do both simultaneously.

The smallest range of digital output was 100 millivolts and the resolution of the digital signal was 0.01 millivolts. However, in this range the data logger enables the output signals to be multiplied by four. This facility allows variations of 0.0025 millivolts to be recorded. As the maximum digital output in the experimental measurements was 1.5 millivolts, this range was selected, taking advantage of the multiplication factor feature.

2.3.2 Pressure Measurement (73)

A 100 ohms resistance coil, made from 42 S.W.G. diamel coated manganin wire, was used to monitor the pressure on the high pressure system. It was wound and loosely tied to prevent it unwinding.



FIGURE 2.7 DATA LOGGER

As recommended by the wire manufacturers, Johnson-Matthey the coil was stress relieved by heating it to 140°C and holding the temperature for 10 hours. It was then pressure seasoned by pressurising it to 800MN/m^2 a few times. After this treatment practically no zero drift was noticed at ambient pressure and temperature over the period of the experiments.

The 100 ohms manganin coil, located in a pressure vessel, Figure 2.1, inserted in the high pressure circuit near the hydrostatic extrusion container, was an arm of a d.c. full Wheatstone bridge.

This bridge was connected to one of the channels of the data recorder.

2.3.2.1 Manganin Coil Calibration

The manganin coil was calibrated by using an Harwood dead-weight tester (piston gauge), type DWT-1000. The coil was connected to the "measured pressure" side of the dead weight tester and the change of resistance, for each increment of pressure, 41.37MN/m^2 (6000lb/in^2) was printed onto a paper tape by the high speed numeric printer on line with the data logger, which was continually scanning the manganin coil channel.

Figure 2.8 shows the manganin coil calibration curve.

2.3.3 Drawing Load Measurement

A tensile load cell was used to measure the drawing load, as described in section 2.2.1.1.

Two 120 ohms strain gauges attached diametrically opposed to a 50mm long rectangular cross-section beam. The strain gauges were of

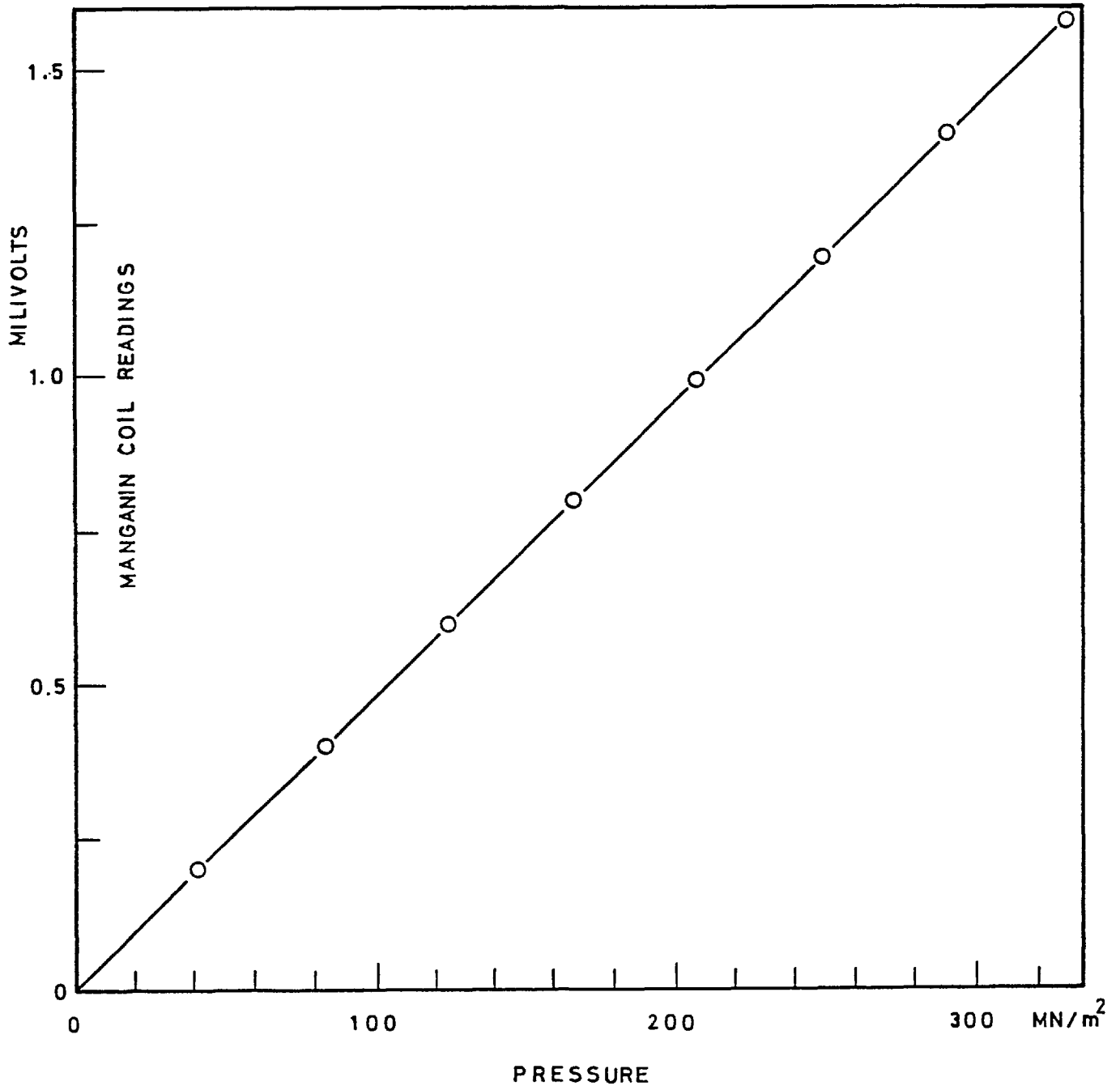


FIGURE 2.8 - MANGANIN COIL CALIBRATION CURVE

the self-compensating type for temperature, when used on steel. They were connected in series to eliminate the effects due to bending.

Continuous monitoring of the load was carried out by feeding the output from a d.c. bridge to the data logger.

2.3.3.1 Load Cell Calibration

The load cell was calibrated in a Tinius-Olsun testing machine. The load was applied in increments of 2.22KN (500lbs) up to 11.1KN (2500lbs) and the output voltage from the bridge recorded in the same manner as described for the manganin coil calibration.

Figure 2.9 illustrates the load cell calibration curve.

2.3.4 Friction Force Measurement

Since the support (Figure 2.2) was shown to be unreliable to work as a load cell, the friction force on the billet/mandrel interface was measured by four axial and four hoop strain gauges bonded to the mandrel and connected to form a d.c. Wheatstone full bridge. They were placed in such a way that the axial and hoop strain gauges alternated on the perimeter of the mandrel, at 40.0mm from the top of the mandrel.

Each arm of the d.c. full bridge was made up by two strain gauges with opposite arms having the same type of strain gauges. Thus, the bridge was compensated for temperature and bending.

The bridge was connected to a channel of the data logger.

2.3.4.1 Calibration of the Load Strain Gauges

The load strain gauges were calibrated in tension and compression.

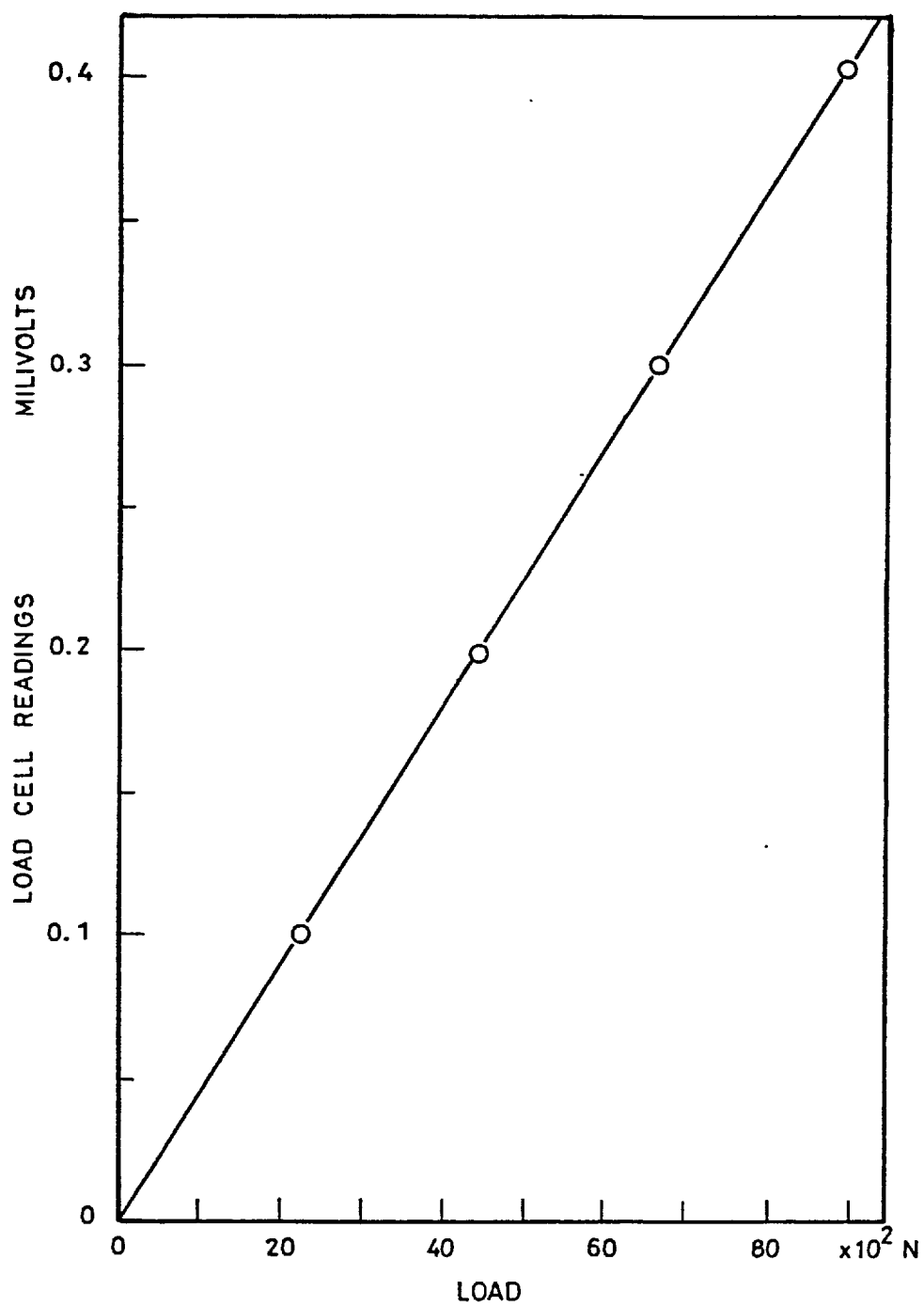


FIGURE 2.9 - LOAD CELL CALIBRATION CURVE

To avoid damage the mandrel surface by the testing machine grips, when calibrating in tension, a special tool was made, Figure 2.10.

Two halves of a 150mm long cylindrical tube, with a bore diameter slightly smaller than the mandrel external diameter, were tightened against the mandrel by means of twelve bolts as shown in Figure 2.10. The mandrel surface had previously been covered with Araldite to prevent the mandrel sliding during the tensile test.

With this arrangement it was possible to reach a calibration load up to 31kN. Above this load the mandrel slipped inside the sleeve.

The calibration curves in tension, (Figure 2.11) and in compression did not show any significant difference in the calibration constant. In both tests the load was applied in increments of 2.22kN (500lbs), and the output voltage from the bridge was recorded as described on the manganin coil calibration section.

2.3.5 Hoop and Axial Strains Measurements in the Bore of the Mandrel (74-77)

To measure the hoop and axial strains in the bore of the mandrel, caused by the normal and shear stress distribution acting on its surface, during the extrusion operation, a 20mm long strip of ten strain gauges was bonded to it. The five hoop and five axial strain gauges were placed alternately in the axial direction along the mandrel bore and bonded to it in such a way that the centre line of gauge no.1 (a hoop strain gauge) was 20mm from the product end of the mandrel, Figure 2.2.

Each of the strain gauges was 120 ohms resistance and temperature compensated when used on steel. The gauge factor and cross-sensitivity



FIGURE 2.10 TOOL TO HOLD THE MANDREL DURING THE CALIBRATION OF
THE LOAD STRAIN GAUGES

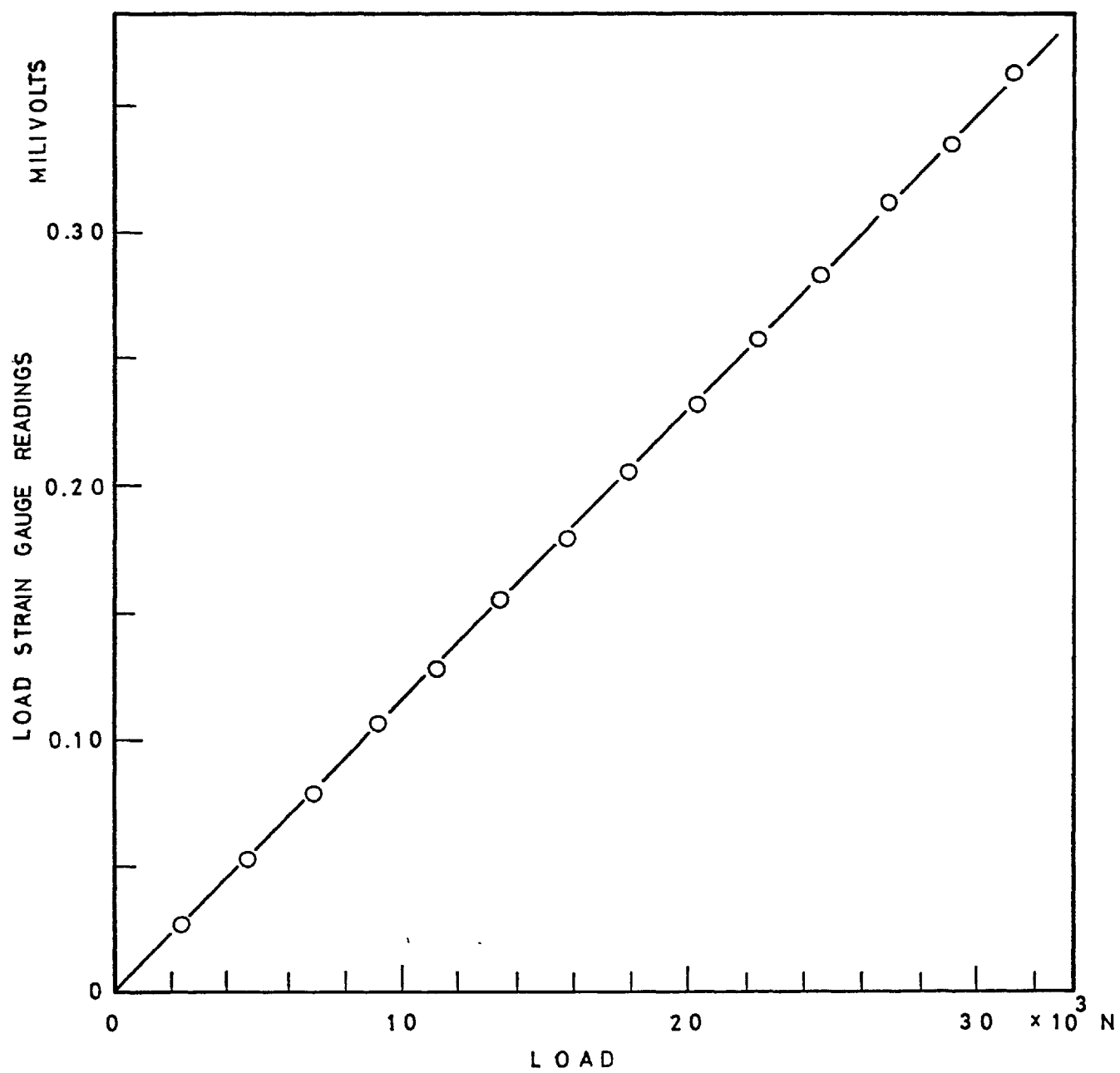


FIGURE 2.11 - LOAD STRAIN GAUGES CALIBRATION CURVE

as quoted by the manufacturers, Micro Measurements Inc., are $2.16 \pm 1.0\%$ and 1.6% for the hoop strain gauges and $2.05 \pm 1.0\%$ and 1.4% for the axial strain gauges.

The size of the strain gauges and the length of the strip were chosen to suit the length of the billet/mandrel interface and the gradient of strain in the bore of the mandrel. Compensating strain gauges were bonded to a dummy mandrel outside the pressure vessel. Compensation for the temperature increase during the extrusion operation was not possible with the present arrangement. To lessen this factor the strain gauges were chosen to be of the self-temperature compensating type and the velocity of the extrusion, controlled by the pulling device, was kept as low as the system permitted (1.5mm/s), to reduce the amount of heat generated.

The bonding of the strain gauges raised some problems, since this work was made difficult by two main factors:

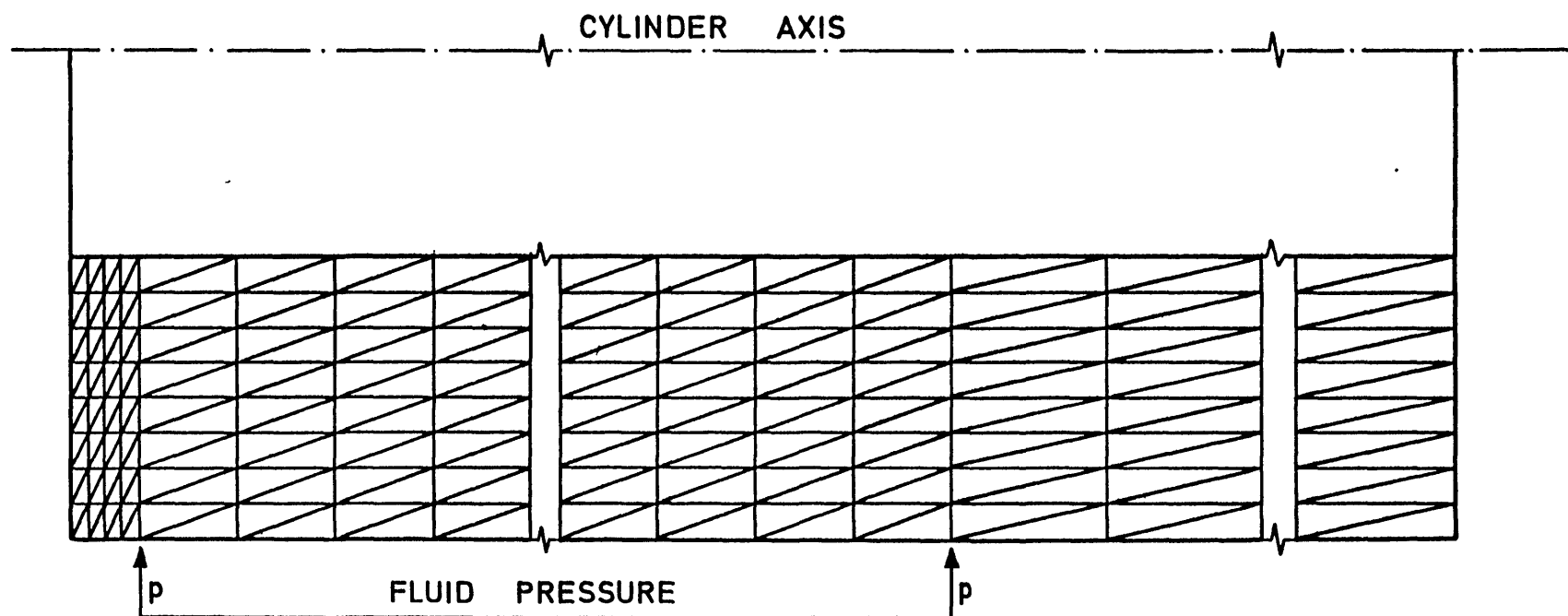
1. The small diameter of the mandrel bore, and
2. The distance from the central line of the strain gauge no. 1 to the product end of the mandrel.

The mandrel bore diameter was determined by the room available in the container and the maximum fluid pressure. Bearing this in mind the mandrel was designed to have the bore diameter as large as possible.

The second difficulty in bonding the strain gauges was that the effect of seals (78,79) needed to be considered to carry out the strain gauge calibrations.

The effect of seals on the hoop and axial strains in the bore of the mandrel, when subjected to an external pressure, was analysed by means of a finite element program (80). The mesh used in these calculations is shown in Figure 2.12.

On the seal regions the pressure drop was assumed as a step



NUMBER OF ELEMENTS — 954

NUMBER OF NODAL POINTS — 540

FIGURE 2.12 - FINITE ELEMENT MESH

function, since it corresponds to the most unfavourable condition.

Figure 2.13 illustrates the hoop and axial strain distribution on the bore of the mandrel along its axis, obtained from that computer program and the Lamé equations. For the sake of accuracy the hoop strains in the finite element analysis were calculated direct from the radial displacement of each node.

In view of these results the strip of strain gauges was bonded in the zone marked in Figure 2.13, where the effect of seals could be neglected and as near the product end of the mandrel as possible to ease the bonding operation.

The strain gauge wires were connected to the corresponding pins in a miniature plug (Figure 2.2) at the top of the mandrel.

The active and compensating gauges were joined to form ten Wheatstone half-bridges, by means of a junction box. These half-bridges were connected in parallel and energised by the data recorder supply voltage. Each one of the ten half-bridges was then connected to a channel of the data logger which made up the other half of the full bridge. Figure 2.14 illustrates the electrical circuit of one of the strain gauge bridges.

To energise the bridges the supply voltage was chosen to be 1 volt, given the small size of the strain gauge grid. In this way the amount of heat generated due to gauge resistance was negligible neither damaging the strain gauges nor affecting the readings. As the same power supply energised the manganin coil, the load cell and the load strain gauge bridges, the same voltage was used for all.

2.3.5.1 Strain Gauge Calibrations

The strain gauge calibrations were carried out using the

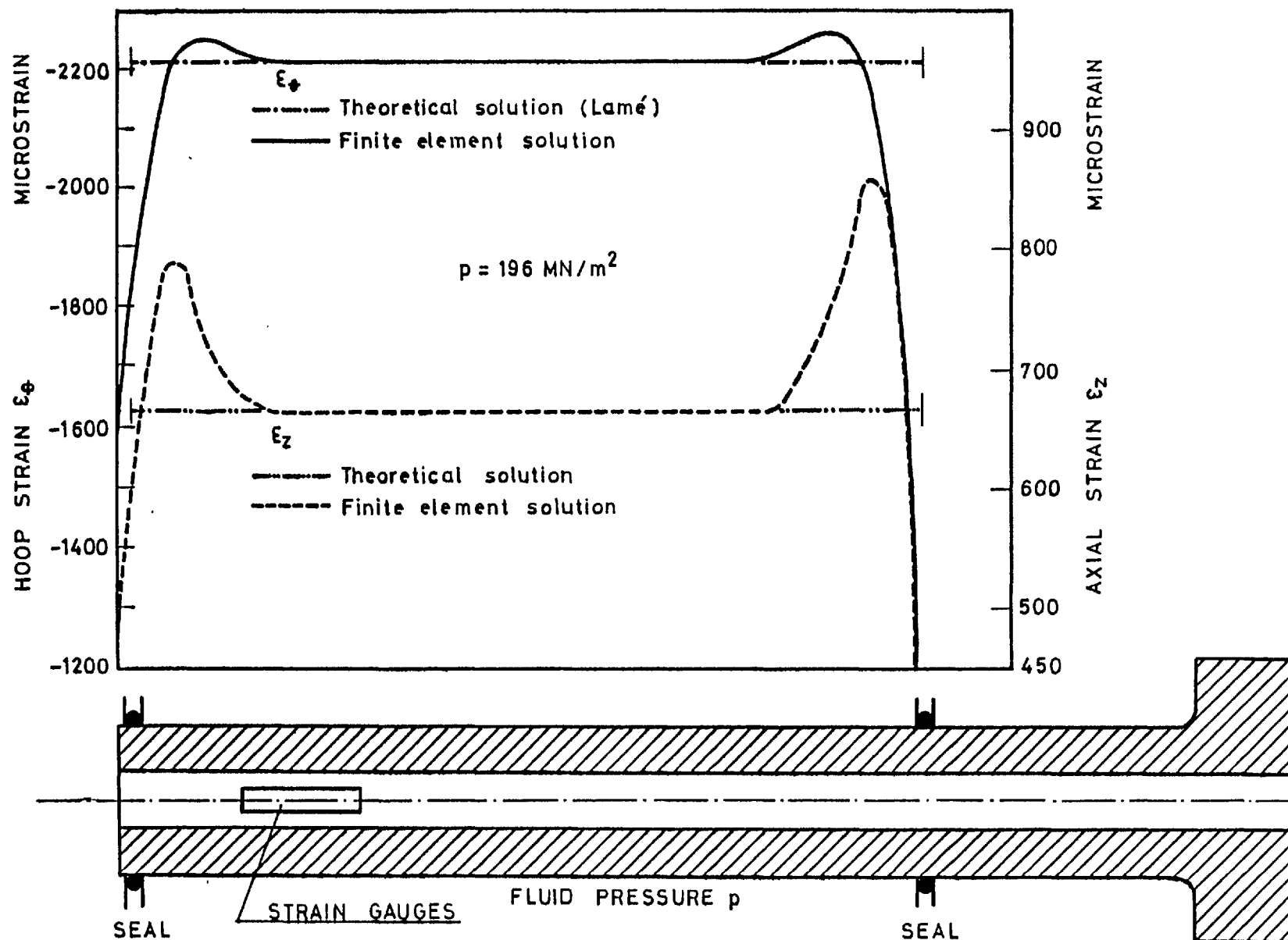


FIGURE 2.13 - AXIAL AND HOOP STRAIN DISTRIBUTIONS IN THE BORE OF THE MANDREL WHEN SUBJECTED TO AN EXTERNAL PRESSURE OF 196 MN/m^2

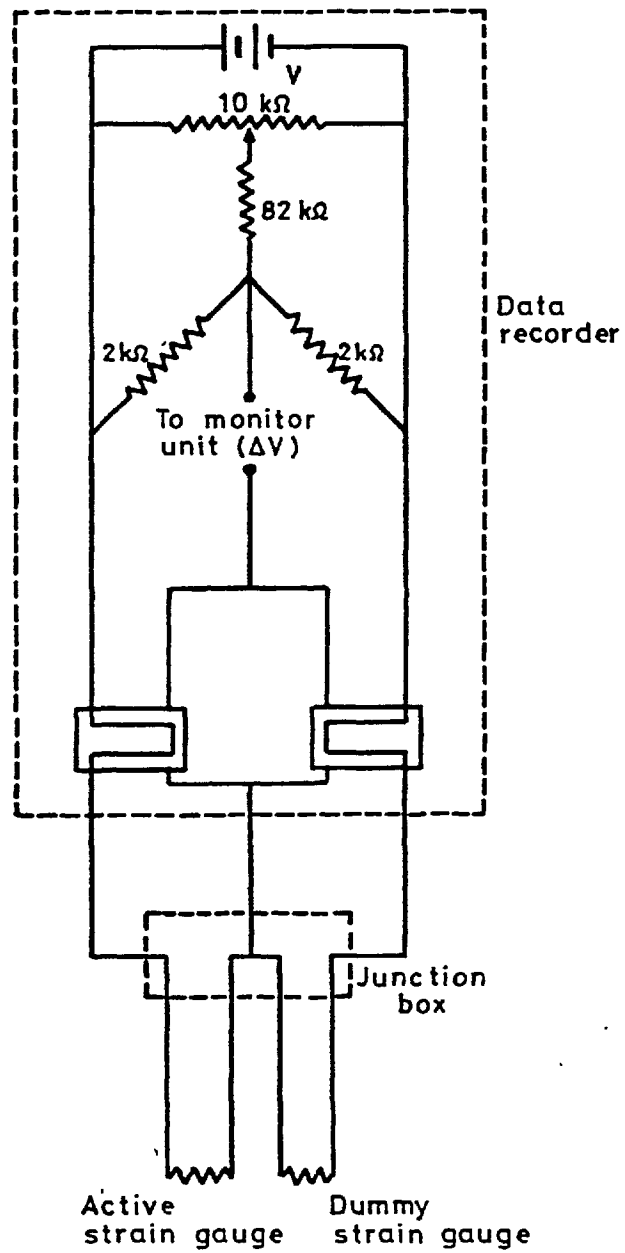


FIGURE 2.14 - ELECTRICAL CIRCUIT DIAGRAM

experimental arrangement shown in Figure 2.15. The fluid pressure was monitored by the manganin coil described in section 2.3.2.

Since the strain gauges were in a zone where the effect of seals could be neglected, the strains obtained from their output must be equal to those calculated by the Lamé equations.

The hoop (ϵ_{θ}) and axial (ϵ_z) strains obtained from the Lamé equations for a cylinder with open ends subjected to an external pressure are respectively:

$$\epsilon_{\theta} = - \frac{1}{E} \frac{p K^2}{K^2 - 1} \left\{ (1 - \nu) + \frac{r_b^2}{r^2} (1 + \nu) \right\}$$

and

$$\epsilon_z = \frac{2\nu}{E} \frac{K^2}{K^2 - 1} p$$

where

p	fluid pressure
E	Young modulus of the cylinder material
ν	Poisson's ratio of the cylinder material
K	$\frac{\text{outside radius}}{\text{inside radius}}$ of the cylinder
r_b	inside radius
r	radius

The calibrations were performed by first setting to zero the data logger channels corresponding to the manganin coil, load strain gauges and mandrel strain gauges. These readings were recorded by the data logger which was scanning these channels at a speed of four samples per second and printed them out on a paper tape by the high speed numeric printer.

The pressure was then raised by increments of 68.9MN/m² (10000lb/in²),

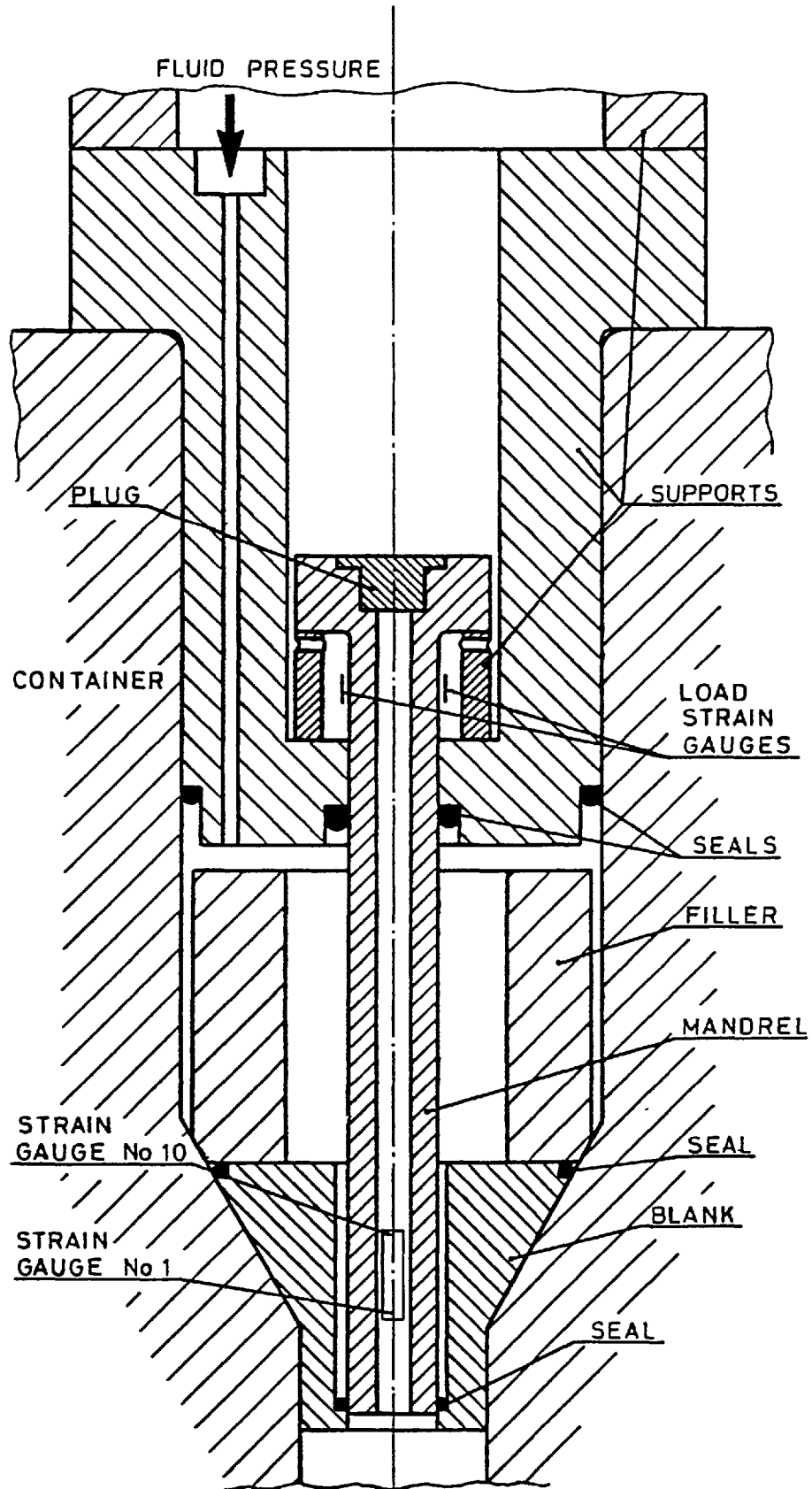


FIGURE 2.15 - STRAIN GAUGE CALIBRATIONS SET-UP

For each pressure increment the pressure was held constant while the recorded outputs from the channels were printed out as described above.

These calibrations were repeated after every five hydrostatic extrusion-drawing experiments.

The calibration curves, Figure 2.16 and 2.17, show the relationship between the output voltage from each of the ten bridges and the strains in the bore of the mandrel when it was subjected to a known external pressure.

2.4 Experimental Procedure

Hydrostatic extrusion-drawing experiments were carried out with commercial purity aluminium and high conductivity copper specimens. The extrusion ratios were 2, 2.65 and 3.5 for aluminium, 1.5 and 2 for copper. The billets were annealed, the aluminium for one hour at 300°C, the copper for two hours at 600°C.

The following procedure was used during the experimental work:

1. To obtain a good fit between billet, die and mandrel, the specimen was subjected to a compressive load, using the die and a cylinder inside the billet. The cylinder diameter was 25.30mm, a little smaller than the mandrel outside diameter.
2. After this preforming operation the billet was cleaned and degreased with carbon tetrachloride and coated with a graphite solution.
3. The torque limiter was set to the maximum admissible load.
4. The billet was assembled into the die, and the adaptor was attached to the billet tag. This assembling was then introduced into the high pressure vessel. The experimental rig was assembled by placing into the container, in succession the filler

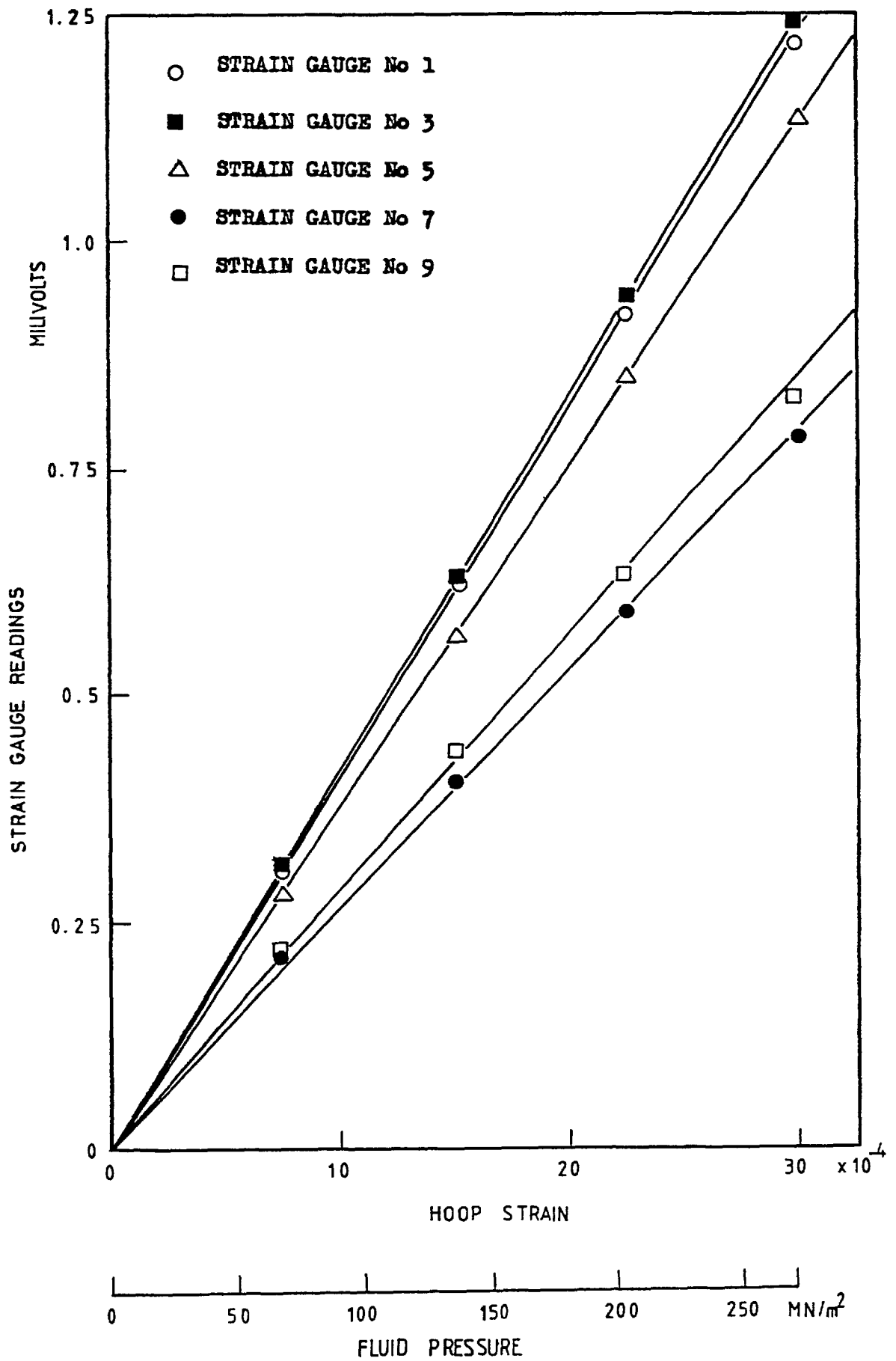


FIGURE 2.16 - HOOP STRAIN GAUGE CALIBRATION CURVE

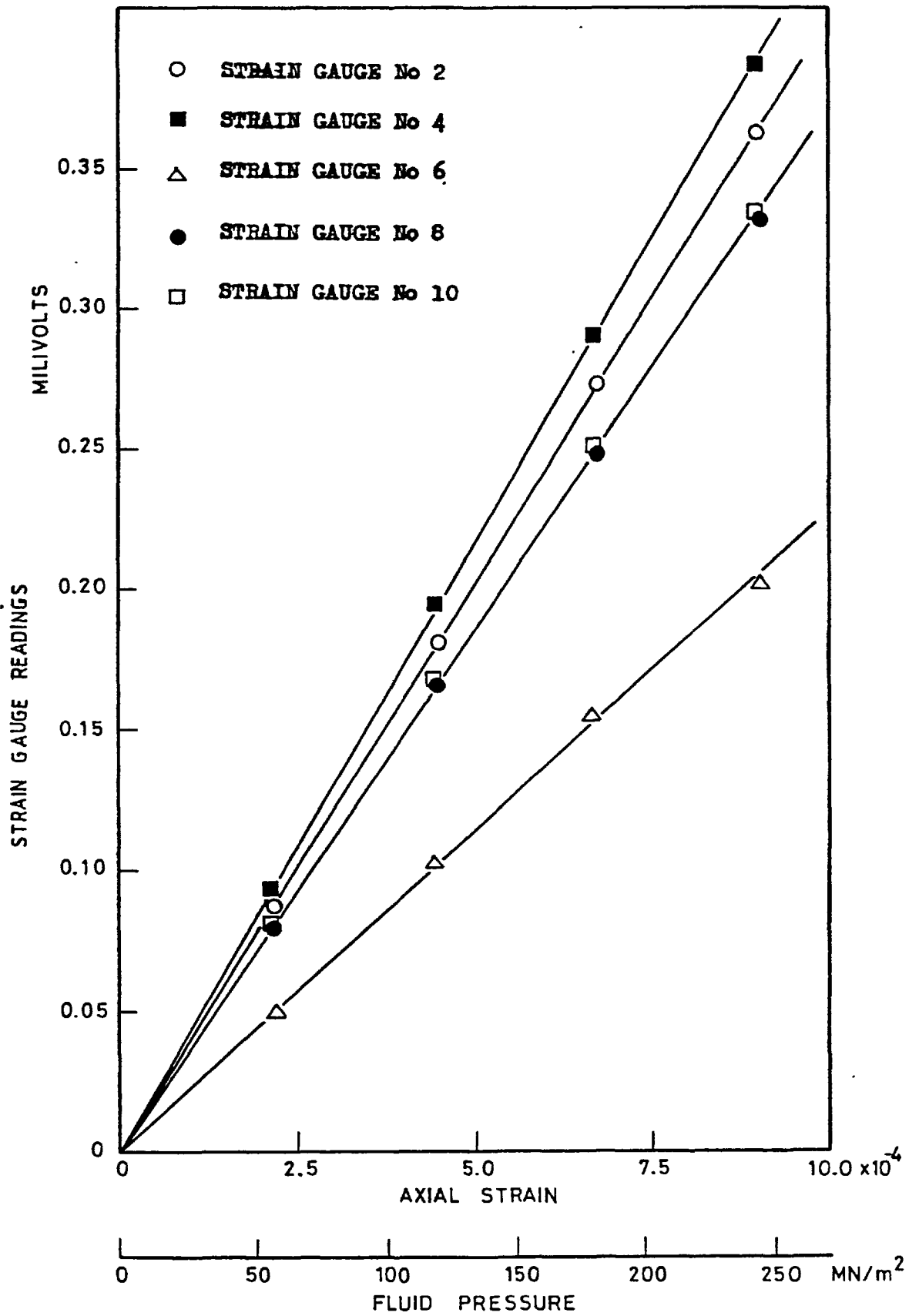


FIGURE 2.17 - AXIAL STRAIN GAUGE CALIBRATION CURVES

cylinder, the supports and the mandrel, and connecting the billet tag to the roller chain. Next the high pressure pipes were connected to the container.

5. The electrical wire connections were made at this stage.
6. The data logger channels were set to zero by means of variable resistors located on the front panel of the recorder.
A scan and print out of the channel readings at balance were carried out.
7. A small drawing load was applied to the billet to make an effective seal at the billet/mandrel and billet/die interfaces.
8. The pressure was raised to a predetermined level and kept constant. The data logger was operated to scan continually all the channels so that their outputs were printed out on a paper tape.
9. The pulling device was operated to apply the necessary drawing load to the billet tag so as to make up the total stress required for the process and pull the specimen material through the die at a constant speed.
10. The extrusion continued until a certain length of product is reached. The pulling device was then stopped, the load removed and the pressure released.
11. The channel readings continued being printed out to check for zero drifts.
12. The data logger and the high speed numeric printer were operated to stop scanning and printing out.
13. The rig was dismantled.

Figure 2.18 illustrates some of the products obtained by the hydrostatic extrusion-drawing process.

The experimental results will be given in Chapter 7 together with the theoretical predictions.



FIGURE 2.18 HYDROSTATIC EXTRUSION - DRAWING PRODUCTS

CHAPTER 3

ELEMENTS OF THE MATHEMATICAL THEORY OF PLASTICITY

3.1 Introduction

The theories of plasticity can be classified in two categories: Physical theories and mathematical theories. While the former seek an explanation for the plastic flow of metals from a microscopic view point, the latter are phenomenological in nature and attempt to put into useful form the results of macroscopic experiments.

The mathematical theories of plasticity are concerned with the formulation of mathematical relationships of stress and strain in solid bodies which deform permanently under the action of external loads.

In this chapter some of the basic concepts of the mathematical theory of plasticity are considered which are required to analyse the plastically deforming zone in the hydrostatic extrusion of tubes. Only the incremental or flow theories are referred to in the text, since the deformation or total strain theories are unsuitable to describe the plastic behaviour of a metal, except in the case of radial or proportional loading in which both theories coincide (81,82).

3.2 Stress Field (82-90)

The state of stress at a point of a body, subjected to a system of external forces, can be determined in terms of the stresses on three perpendicular planes passing through the point. In a cartesian system of co-ordinates (x_1, x_2, x_3) the complete specification of the stress at a point is given by the stress tensor σ_{ij} :

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \quad (3.1)$$

where σ_{11} , σ_{22} , σ_{33} are the normal stresses acting in those planes and σ_{12} , σ_{13} , σ_{21} , σ_{23} , σ_{31} , σ_{32} are the shear stresses, as shown in Figure (3.1).

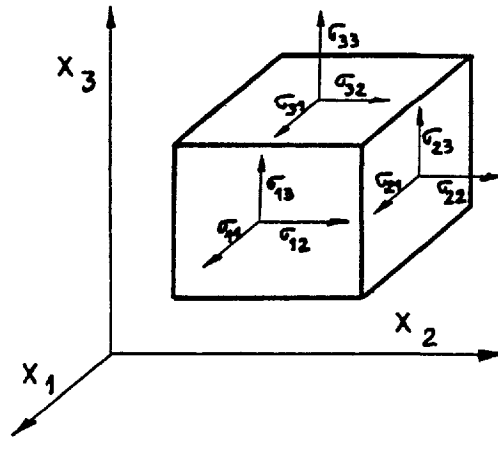


FIG. 3.1 THE COMPONENTS OF THE STRESS TENSOR

Normal stresses can be either tensile or compressive, and the convention followed in this work is that tensile stresses are positive and compressive stresses are negative.

The convention for the shear stresses is as follows: the positive directions of the components of the shear stress on any side of a cubical element are taken as being in the positive direction of the co-ordinate axis if a tensile stress on the same side would have the positive direction of the corresponding axis.

Considering the equilibrium of the forces acting on the various surfaces of an infinitesimal parallelepiped, the following relationships are obtained:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + x_i = 0 \quad (3.2)$$

where X_i are the body forces per unit of volume.

The equilibrium of an element requires also that the resultant moment should vanish. The consideration of moments leads to the conclusion that the stress tensor is symmetric,

$$\sigma_{ij} = \sigma_{ji} \quad (3.3)$$

The stress tensor has three independent invariants which are:

$$\begin{aligned} J_1 &= \sigma_{ii} \\ J_2 &= \sigma_{ij} \sigma_{ij} \\ J_3 &= \sigma_{ij} \sigma_{jk} \sigma_{ki} \end{aligned} \quad (3.4)$$

In plasticity theory it is convenient to consider the stress tensor as the sum of the spherical stress tensor and the stress deviatoric tensor.

The three normal components of a spherical stress tensor are equal and its shear components are zero.

The sum of the elements in the principal diagonal of a stress deviatoric tensor is zero. Thus,

$$\sigma_{ij} = \sigma'_{ij} + \sigma_m \delta_{ij}$$

where $\sigma_m = 1/3 \sigma_{ii}$, is known as the hydrostatic stress, and δ_{ij} is the Kronecker delta.

The deviatoric stress is

$$\sigma'_{ij} = \sigma_{ij} - \sigma_m \delta_{ij} \quad (3.5)$$

whose invariants are:

$$\begin{aligned}
 J_1' &= \sigma_{ii}' = 0 \\
 J_2' &= \frac{1}{2} \sigma_{ij}' \sigma_{ij}' \\
 J_3' &= \frac{1}{3} \sigma_{ij}' \sigma_{jk}' \sigma_{ki}'
 \end{aligned}
 \tag{3.6}$$

The equivalent stress, $\bar{\sigma}$, is defined as:

$$\bar{\sigma} = \sqrt{3/2 \sigma_{ij}' \sigma_{ij}'} = \sqrt{3J_2'}
 \tag{3.7}$$

In the hydrostatic extrusion of tubes, the deformation is axisymmetric so it is convenient to adopt a reference system of cylindrical co-ordinates (r, θ, z) Figure (3.2).

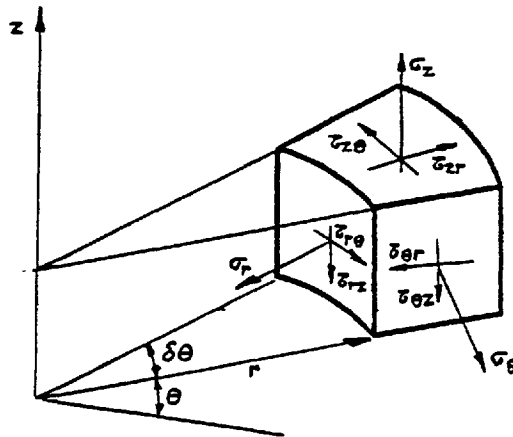


FIG. 3.2 STRESS COMPONENTS REFERRED TO CYLINDRICAL CO-ORDINATES

In this system of co-ordinates the stress tensor σ_{ij} is:

$$\begin{bmatrix}
 \sigma_r & \tau_{r\theta} & \tau_{rz} \\
 \tau_{\theta r} & \sigma_\theta & \tau_{\theta z} \\
 \tau_{zr} & \tau_{z\theta} & \sigma_z
 \end{bmatrix}
 \tag{3.8}$$

Due to axial symmetry:

$$\tau_{r\theta} = \tau_{\theta r} = \tau_{\theta z} = \tau_{z\theta} = 0$$

and the remaining components of the stress are independent of the variable θ .

The second invariant of the deviatoric stress tensor, in this system of co-ordinates, can be written as:

$$J_2' = -(\sigma_r' \sigma_\theta' + \sigma_\theta' \sigma_z' + \sigma_z' \sigma_r') + \tau_{rz}^2 \quad (3.9)$$

3.2.1 Stress Functions

The equilibrium equations (3.2) for cylindrical co-ordinates and axisymmetric conditions, neglecting body forces can be written as:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

$$\frac{\partial \sigma_\theta}{\partial \theta} = 0 \quad (3.10)$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = 0$$

By introducing the stress functions $\phi_1 = \phi_1(r, z)$ and $\phi_2 = \phi_2(r, z)$ such that:

$$\sigma_r = \frac{1}{r} \frac{\partial^2 \phi_1}{\partial z^2} + \frac{\phi_z}{r}$$

$$\sigma_\theta = \frac{\partial \phi_2}{\partial r}$$

$$\sigma_z = \frac{1}{r} \frac{\partial^2 \phi_1}{\partial r^2}$$

$$\tau_{rz} = -\frac{1}{r} \frac{\partial^2 \phi_1}{\partial r \partial z} \quad (3.11)$$

the equilibrium equations (3.10) will automatically be satisfied.

3.3 Strain Rate and Strain Field (82-90)

In a body in motion described by its velocity field the components of the strain rate tensor in a cartesian system of co-ordinates (x_1, x_2, x_3) are given by:

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (3.12)$$

where v_i are functions representing the velocity field.

In cylindrical co-ordinates and axisymmetric conditions the strain rate has the following form:

$$\dot{\epsilon}_{ij} = \begin{bmatrix} \dot{\epsilon}_r & 0 & \dot{\epsilon}_{rz} \\ 0 & \dot{\epsilon}_\theta & 0 \\ \dot{\epsilon}_{rz} & 0 & \dot{\epsilon}_z \end{bmatrix}$$

where:

$$\begin{aligned}
 \dot{\epsilon}_r &= \frac{\partial v_r}{\partial r} \\
 \dot{\epsilon}_\theta &= \frac{v_r}{r} \\
 \dot{\epsilon}_z &= \frac{\partial v_z}{\partial z} \\
 \dot{\epsilon}_{rz} &= \frac{1}{2} \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right)
 \end{aligned}
 \tag{3.13}$$

The strain rate tensor can also be split into two tensors, the deviatoric tensor $\dot{\epsilon}'_{ij}$ and the spherical tensor, $\epsilon_m \delta_{ij}$. The deviatoric tensor $\dot{\epsilon}'_{ij}$ is given by:

$$\dot{\epsilon}'_{ij} = \dot{\epsilon}_{ij} - \epsilon_m \delta_{ij}
 \tag{3.14}$$

where $\epsilon_m = \frac{1}{3} \dot{\epsilon}_{ii}$

The three invariants of $\dot{\epsilon}'_{ij}$ are:

$$\begin{aligned}
 I_1 &= \dot{\epsilon}'_{ii} = 0 \\
 I_2 &= \frac{1}{2} \dot{\epsilon}'_{ij} \dot{\epsilon}'_{ij} \\
 I_3 &= \det (\dot{\epsilon}'_{ij})
 \end{aligned}
 \tag{3.15}$$

The assumption that the material is incompressible leads to

$$\dot{\epsilon}_{ii} = 0 \quad (3.16)$$

i.e. the spherical strain rate tensor is equal to zero. Under axisymmetric conditions and in cylindrical co-ordinates the second invariant I_2 has the form:

$$I_2 = - (\dot{\epsilon}'_r \dot{\epsilon}'_\theta + \dot{\epsilon}'_\theta \dot{\epsilon}'_z + \dot{\epsilon}'_z \dot{\epsilon}'_r) + \dot{\epsilon}_{rz}^2 \quad (3.17)'$$

The equivalent or effective strain rate is defined as:

$$\dot{\bar{\epsilon}} = \sqrt{\frac{2}{3} \dot{\epsilon}_{ij} \dot{\epsilon}_{ij}} \quad (3.18)$$

and, assuming incompressibility,

$$\dot{\bar{\epsilon}} = \frac{2}{3} \sqrt{I_2} \quad (3.19)$$

When a body is described by its small displacement field the components of the strain tensor in a cartesian system of co-ordinates (x_1, x_2, x_3) are given by:

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (3.20)$$

where u are functions representing the displacement field.

For finite deformation the components of strain are calculated from the components of strain rate as follows,

$$\epsilon_{ij} = \int_{t=0}^t \dot{\epsilon}_{ij} dt \quad (3.21)$$

The total amount of plastic strain suffered by an element of material during the deformation is given by the equivalent or effective strain, $\bar{\epsilon}$, (90,91),

$$\bar{\epsilon} = \int_{t=0}^t \dot{\bar{\epsilon}} dt \quad (3.22)$$

The integrations in (3.20) and (3.21) are performed along the path of deformation.

3.4 Yield Surface (85-89)

The state of stress at any point in the material is represented by a point in a nine-dimensional Euclidean stress space whose axis are σ_{ij} ($i, j = 1, 2, 3$). Since σ_{ij} is symmetric the stress space can be reduced to a six-dimensional one.

In the stress space a variation of the stress at a material point corresponds to a change of the stress point. The region of that space where only elastic strains can occur is bounded by a surface called yield surface. The scalar function $f(\sigma_{ij}, \epsilon_{ij}^p, k)$ which describes the yield surface is denoted by yield or loading function and is assumed to be depending on the states of stress and plastic strain and the history of loading. It is also assumed that the equation

$$f(\sigma_{ij}, \epsilon_{ij}^p, k) = 0 \quad (3.23)$$

represents a closed surface in the stress space.

If $f < 0$ the stress state is an elastic one; $f = 0$ means the stress state is on the yield surface; $f > 0$ has no meaning.

To determine the changes occurring in the stress state for the case $f = 0$ let's consider the time derivative of f .

$$\dot{f} = \frac{\partial f}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\partial f}{\partial \epsilon_{ij}} \dot{\epsilon}_{ij} + \frac{\partial f}{\partial k} \dot{k} \quad (3.24)$$

Three cases can now be distinguished for a strain hardening material:

1 - $f = 0$ and $\dot{f} < 0$. This condition leads to an elastic state as in the next instant of time would be $f < 0$. This constitutes unloading, where no plastic strain occurs, $\dot{\epsilon}_{ij}^p = 0$ and $\dot{k}$ must also vanish since there is no change in the plastic strain hardening parameter. The criterion for unloading from a plastic state is thus defined by:

$$\frac{\partial f}{\partial \sigma_{ij}} \dot{\sigma}_{ij} < 0$$

which characterises unloading.

$$2 - f = 0 \quad \text{and} \quad \frac{\partial f}{\partial \sigma_{ij}} \dot{\sigma}_{ij} = 0$$

This situation is called neutral loading.

$$3 - f = 0 \quad \text{and} \quad \frac{\partial f}{\partial \sigma_{ij}} \dot{\sigma}_{ij} > 0$$

which constitutes loading.

Thus, an infinitesimal change of stress, $d\sigma$, constitutes unloading or loading or neutral loading according to whether the vector $d\sigma$ points towards the interior or the exterior of the yield surface or is tangential to it.

3.5 Flow Rule for a Work Hardening Material (85-89)

To define work hardening Drucker (92) considered an external agency which slowly applies to a given state of stress an additional set of stresses and then slowly removes them. During the application and removal of such a set of stresses the material remains in equilibrium.

The original configuration may or may not be restored after the cycle.

A work hardening material is defined to be one such that:

- (a) Positive work is done by the external agency during the application of the set of stresses; and
- (b) The network performed by the external agency over the cycle of application and removal is zero or positive.

This definition of work hardening means that useful net energy above the elastic energy cannot be extracted from the material and the system of forces acting on it, as only the elastic energy can be recovered during unloading.

When the external agency applies small surface tractions, the state of stress, σ_{ij} is altered by $d\sigma_{ij}$ and the state of strain ϵ_{ij} by $d\epsilon_{ij}$. The small strain increments $d\epsilon_{ij}$ contain an elastic part $d\epsilon_{ij}^e$ and a plastic part $d\epsilon_{ij}^p$ such that

$$d\epsilon_{ij} = d\epsilon_{ij}^e + d\epsilon_{ij}^p$$

When the external agency removes the added tractions the elastic strain increments $d\epsilon_{ij}^e$ are released.

From condition a) it follows that:

$$d\sigma_{ij} (d\epsilon_{ij}^e + d\epsilon_{ij}^p) > 0 \quad (3.25a)$$

and from condition (b)

$$d\sigma_{ij} d\epsilon_{ij}^p > 0 \quad (3.25b)$$

The latter inequality is sometimes referred as the uniqueness condition (93), and represents the scalar product of the vectors $d\sigma$ and $d\epsilon$ in a six-dimensional stress space. Therefore, the positive directions of the two vectors must make an acute angle with each other.

The consequences of this work hardening definition are (94):

(1) The yield surface and all the subsequent loading surfaces must be convex.

(2) The plastic strain rate vector $\dot{\epsilon}_{ij}^p$ must be normal to the loading surface at a smooth point,

$$\dot{\epsilon}_{ij}^p = \Lambda \frac{\partial f}{\partial \sigma_{ij}} \quad (3.26)$$

and lie between adjacent normals at a corner. In expression 3.26, Λ is a function that depends on stress, stress rate, strain and strain history. From the work hardening definition, (3.25) follows that Λ must be non-negative.

(3) The plastic strain increment is linear in the stress increment

$$d\epsilon_{ij}^p = A_{ijkl} d\sigma_{kl} \quad (3.27)$$

Proof of these corollaries were given by Drucker (92,94) and Naghdi (95).

Loading points must satisfy the equation:

$$f(\sigma_{ij}, \epsilon_{ij}^P, k) = 0$$

Prager (96) called this equation the consistency condition which means that loading from a plastic state must lead to another plastic state.

Assuming k to be a function of ϵ_{ij}^P (86), during loading it must be:

$$\dot{f} = \frac{\partial f}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\partial f}{\partial \epsilon_{ij}^P} \dot{\epsilon}_{ij}^P + \frac{\partial f}{\partial k} \frac{\partial k}{\partial \epsilon_{ij}^P} \dot{\epsilon}_{ij}^P = 0 \quad (3.28)$$

From (3.26) and (3.28) one obtains:

$$\Lambda = - \frac{\frac{\partial f}{\partial \sigma_{kl}} \dot{\sigma}_{kl}}{\left(\frac{\partial f}{\partial \epsilon_{ij}^P} + \frac{\partial f}{\partial k} \frac{\partial k}{\partial \epsilon_{ij}^P} \right) \frac{\partial f}{\partial \sigma_{ij}}} \quad (3.29)$$

Substituting (3.29) in (3.26) results

$$\dot{\epsilon}_{ij} = G \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial f}{\partial \sigma_{kl}} \dot{\sigma}_{kl} \quad (3.30)$$

where

$$G = - \frac{1}{\left(\frac{\partial f}{\partial \epsilon_{mn}^P} + \frac{\partial f}{\partial k} \frac{\partial k}{\partial \epsilon_{mn}^P} \right) \frac{\partial f}{\partial \sigma_{mn}}}$$

Equation (3.30) shows that a linear relationship exists between the strain rate and the stress rate.

Koiter (97) generalised equation (3.30) to be applied to a surface with singularity points. This theory assumes the yield surface made up of several smooth surfaces defined by several functions:

$$f_1(\sigma_{ij}), f_2(\sigma_{ij}), \dots, f_n(\sigma_{ij})$$

Koiter proposed that the loading surfaces described by $f_p = 0$ act independently and equation (3.30) is replaced by:

$$\dot{\epsilon}_{ij}^p = \sum_{p=1}^n C_p G_p \frac{\partial f_p}{\partial \sigma_{ij}} \frac{\partial f_p}{\partial \sigma_{kl}} \dot{\sigma}_{kl}$$

where:

$$C_p = 0 \quad \text{if} \quad f_p < 0 \quad \text{or} \quad \left(\frac{\partial f_p}{\partial \sigma_{ij}} \right) \dot{\sigma}_{ij} < 0$$

$$C_p = 1 \quad \text{if} \quad f_p = 0 \quad \text{or} \quad \left(\frac{\partial f_p}{\partial \sigma_{ij}} \right) \dot{\sigma}_{ij} \geq 0$$

and G_p are n positive functions of stress, strain and strain history.

3.6 Yielding Criteria (82-91)

For a material that hardens isotropically, the yield function depends only on the stress invariants (86,87).

$$f = f(J_1, J_2, J_3)$$

According to experimental results (98,99) the plastic deformation

is independent of hydrostatic pressure. In view of this conclusion the first stress invariant, J_1 , has no contribution to the plastic deformation. Therefore, the yield function can be considered as a function of the invariants of the deviatoric stress tensor:

$$f = f(J_2', J_3')$$

The two most widely used yield criteria, the von Mises criterion and Tresca's criterion, are specific cases of this expression.

The von Mises criterion is defined by:

$$J_2' - k^2 = 0 \quad (3.32)$$

where k is a scalar function depending on plastic strain history.

The Tresca yield criterion establishes that the plastic flow is initiated when the maximum shear stress reaches a certain value k^* . The yield function for Tresca's criterion has the form:

$$4J_2'^3 - 27J_3'^2 - 36k^{*2}J_2' + 96k^{*4}J_2' - 64k^{*6} = 0 \quad (3.33)$$

The results obtained by Lode (100), Taylor and Quinney (101), Ros and Eichinger (102), show that the von Mises criterion fit their experimental data considerably better than the Tresca criterion.

However, the upper yield point of annealed mild steel is said to follow Tresca's criterion (82).

3.7 Flow Rule Associated with the Von Mises Criterion

To obtain the flow rule for a von Mises material one has to consider expression (3.26) and the yield function

$$f = J_2' - k^2$$

The second invariant of the deviatoric stress tensor can be written as:

$$J_2' = \frac{1}{2} \sigma_{ij} \sigma_{ij} - \frac{1}{6} (\sigma_{kk})^2$$

Differentiating with respect to σ_{ij} leads to:

$$\frac{\partial J_2'}{\partial \sigma_{ij}} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk} = \sigma_{ij}'$$

Substituting this value into equation (3.26) one obtains:

$$\dot{\epsilon}_{ij}^p = \Lambda \sigma_{ij}' \quad (3.34)$$

which are the Reuss-Prandtl equations.

To determine the function Λ , equations (3.34) are related to the experimental uniaxial tensile stress-strain curve. Functions have to be found, so that, results obtained in different loading conditions can be correlated with that experimental curve. These functions are the effective stress and the effective strain defined by (3.7) and (3.22) respectively.

Introducing $\dot{\epsilon}_{ij}$ given by (3.34) in (3.18) it becomes:

$$\frac{\dot{\epsilon}}{\epsilon} = \Lambda \sqrt{\frac{2}{3} \sigma_{ij}' \sigma_{ij}'}$$

Using (3.7), one obtains:

$$\Lambda = \frac{3}{2} \frac{\dot{\epsilon}}{\sigma}$$

and finally:

$$\dot{\epsilon}_{ij} = \frac{3}{2} \frac{\dot{\bar{\epsilon}}}{\bar{\sigma}} \sigma'_{ij} \quad (3.35)$$

Which is the form of the Reuss-Prandtl equations to be used throughout this work.

CHAPTER 4

AN UPPER BOUND SOLUTION FOR THE HYDROSTATIC

EXTRUSION-DRAWING OF TUBES

4.1 Introduction

Mathematically exact solutions to metal forming problems are difficult to obtain, so approximate solutions have to be considered. One of the most powerful methods often used is the upper bound technique.

The upper bound theorem, which is one of the limit-load theorems, was formulated by Prager and Hodge (103) and later modified by Drucker et al (104) to include velocity discontinuities.

This theorem states:

Among all kinematically admissible strain rate fields the actual one minimizes the expression

$$J^* = \int_V \bar{\sigma} \dot{\epsilon} \, dV + \int_{S_T} \tau |\Delta v| \, dS - \int_{S_T} T_i v_i \, dS \quad (4.1)$$

where

V is the deformed volume

S_T are the surfaces of discontinuity within V

τ are the shear stresses along S_T

Δv are the velocity discontinuities tangent to S_T

S_T is the portion of the outer surface where external stresses are applied

T_i are the external stresses

v_i is the velocity of the points at S_T

Equation (4.1) shows that the upper bound on power J^* is a function of three quantities:

- 1) The internal power for deformation over the volume of the deforming body, represented by:

$$\int_V \bar{\sigma} \dot{\bar{\epsilon}} dV \quad (4.1.a)$$

- 2) The shear power over surfaces of velocity discontinuities Δv including the boundaries between tool and the material, expressed by:

$$\int_{S_\Gamma} \tau |\Delta v| dS \quad (4.1.b)$$

- 3) The power supplied by predetermined body tractions, represented by:

$$\int_{S_T} T_i v_i dS \quad (4.1.c)$$

A velocity field to be kinematically admissible is required to satisfy:

- a) The incompressibility condition throughout the volume
- b) The continuity across the velocity discontinuity surfaces
- c) The velocity boundary conditions

Hence, velocity discontinuities tangential to the boundaries are allowed and normal components of velocity across the boundaries should be continuous.

These basic requirements can be satisfied by an infinite number of velocity fields and the choice of a particular velocity field depends on additional assumptions.

For metal flow through dies several kinematically admissible velocity fields have been proposed and a brief review is given in what follows.

Johnson and Kudo (105) used extensively the upper bound theorem for a variety of solutions for different conditions in extrusion. They utilized two types of velocity fields for axisymmetric deformation, the parallel field (Figure 4.1) and the triangular velocity field (Figure 4.2).

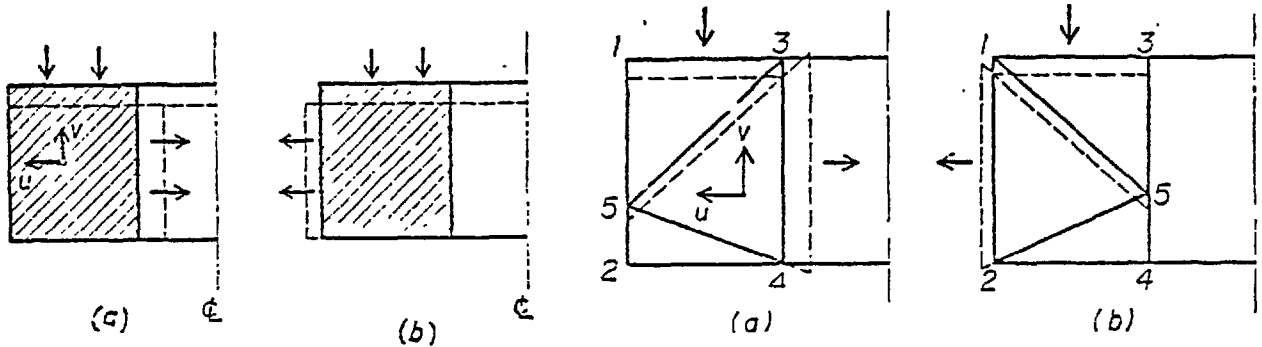
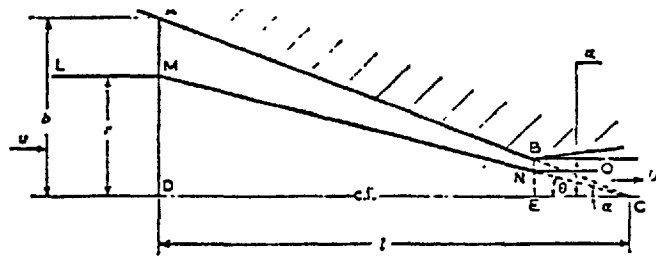


FIGURE 4.1 - PARALLEL FIELD

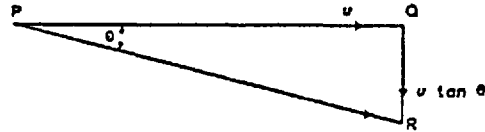
FIGURE 4.2 - TRIANGULAR VELOCITY FIELD

AFTER REFERENCE 105

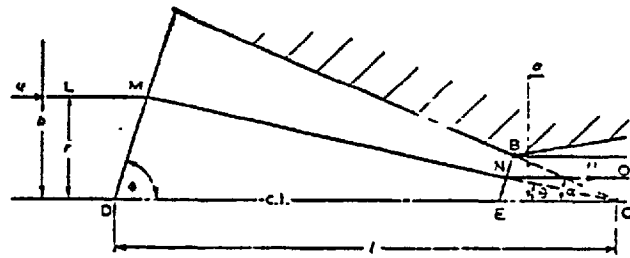
Pugh (106) in his study of the mechanics of the process of hydrostatic extrusion compared the upper bound pressure calculated by three kinematically admissible velocity fields. He assumed plane, conical and spherical boundaries and a radial flow (Figure 4.3). He concluded that the conical boundaries give a lower upper bound although he had selected in his study the spherical boundary because it seemed a more natural choice.



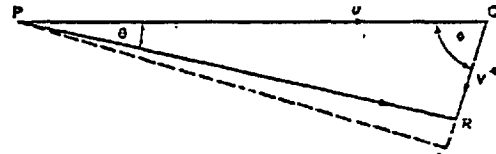
a Flow line.



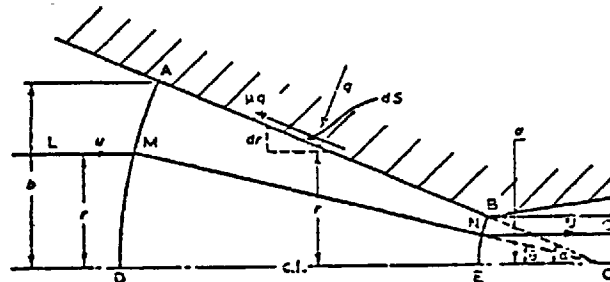
a Velocity components along flow line immediately to the right of M.



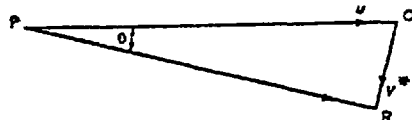
a Flow line.



b Velocity components along flow line immediately to the right of M.



a Flow line.



c Velocity components along flow line immediately to the right of M.

FIGURE 4.3 FLOW THROUGH A CONICAL DIE (AFTER REFERENCE 106)

(a) PLANE TRANSVERSE ENTRY AND EXIT BOUNDARIES

(b) CONICAL ENTRY AND EXIT BOUNDARIES

(c) SPHERICAL ENTRY AND EXIT BOUNDARIES

Avitzur (107-111) assumed radial flow through conical dies, in a spherically bounded plastic zone. The velocity at the points in this zone is directed towards the apex of the die cone and is calculated from the point co-ordinates.

He applied this velocity field to the analysis of wire drawing and extrusion (107-110), hydrostatic extrusion (16) tube sinking and expanding (111) (Figure 4.4).

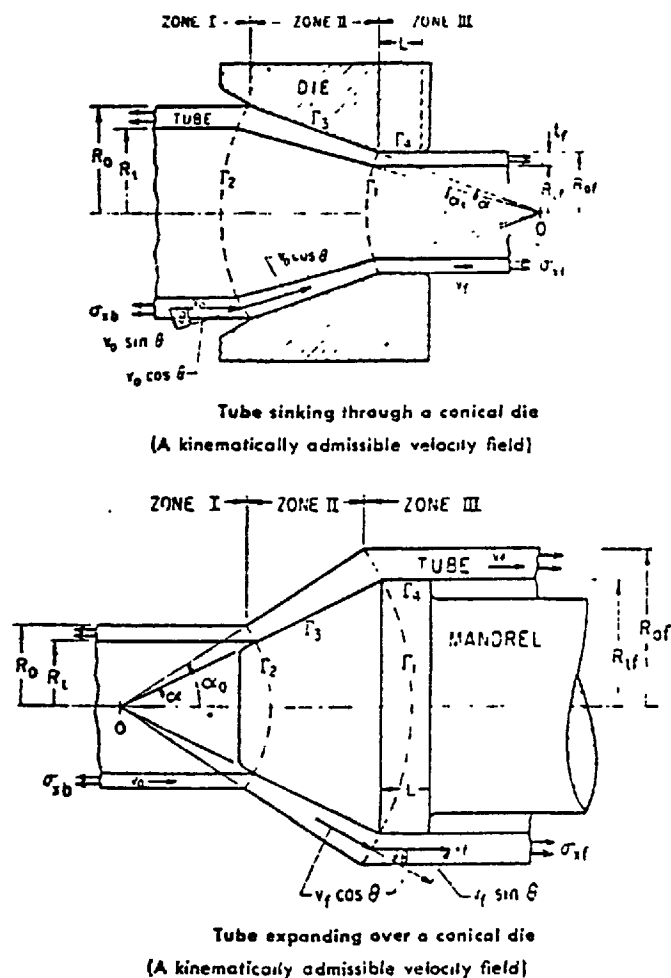


FIGURE 4.4 - KINEMATICALLY ADMISSIBLE VELOCITY FIELD FOR TUBE SINKING AND EXPANDING OVER A CONICAL DIE (AFTER REFERENCE 111)

Halling and Mitchell (112) proposed as kinematically admissible velocity field for the axisymmetric extrusion through conical dies

a flow pattern that used a double triangle arrangement of velocity discontinuities (Figure 4.5). This is an extension of an earlier triangular velocity field used by Kudo (113) for the rectangular die.

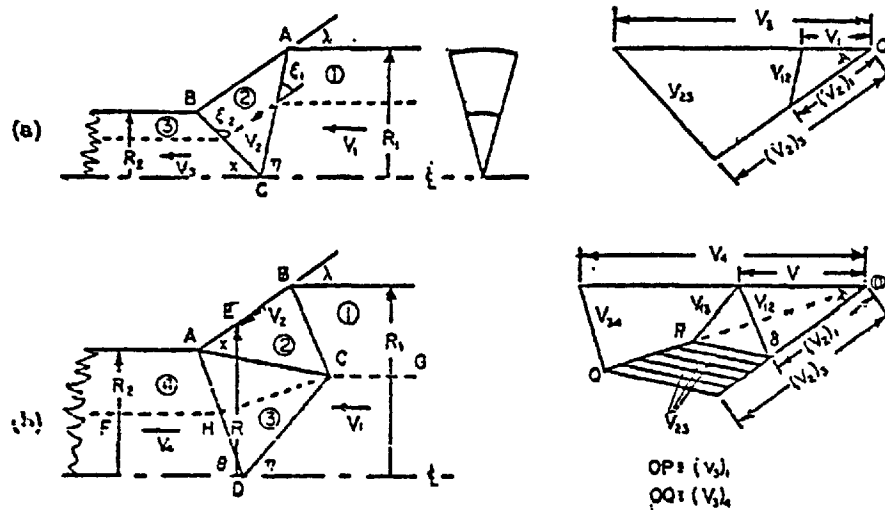


FIGURE 4.5 - VELOCITY DISCONTINUITY PATTERNS AND THEIR ASSOCIATED HODOGRAPHS (AFTER REFERENCE 112)

Lambert and Kobayashi (114) obtained an admissible velocity field without discontinuities by superposition of basic flow patterns using a flow function, for axisymmetric bar extrusion.

Mehta, Shabaik and Kobayashi (115) extended the latter velocity field to tube extrusion (Figure 4.6). Their velocity field was given as a function of three parameters which could be determined by minimizing the total energy-dissipation rate. In both tube and bar extrusion the parameters were determined in such a way that the boundaries of the deformation zone in the theoretical velocity field best approximated the observed boundaries in the experiments.

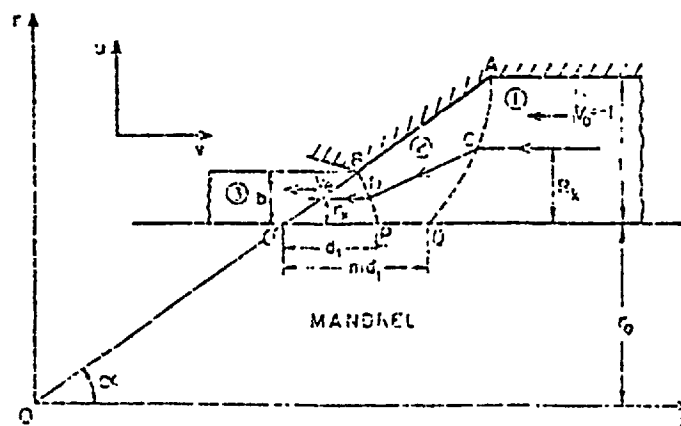


FIGURE 4.6.a
BASIC FLOW PATTERN
OF TUBE EXTRUSION
(AFTER REFERENCE 115)

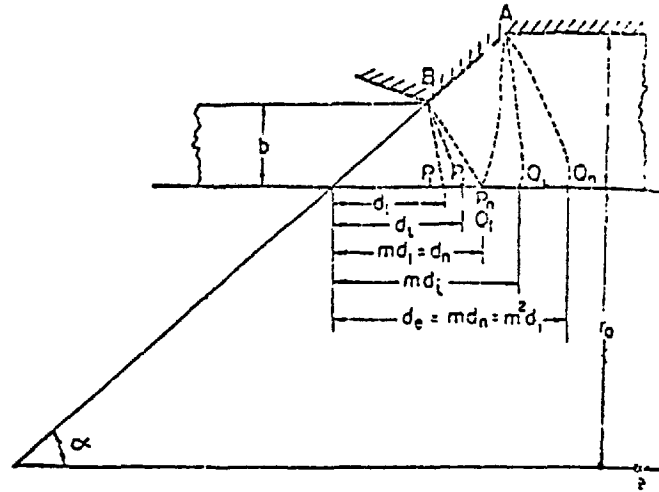


FIGURE 4.6.b SUPERPOSITION OF BASIC FLOW PATTERNS
(AFTER REFERENCE 115)

Zimmerman and Avitzur (116) using a radial flow through a conical die obtained for solid bar shapes of curved boundaries separating plastic zone from rigid zones (Figure 4.7), by minimizing the upper bound on power with respect to a parameter.

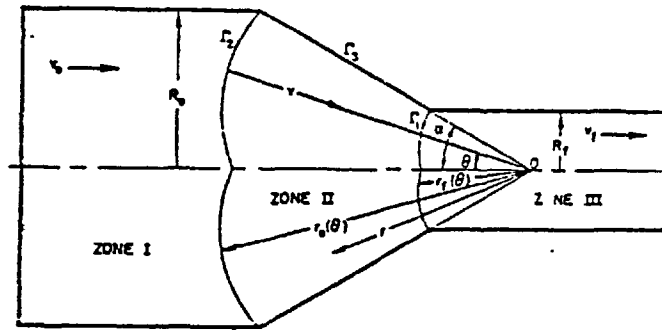


FIGURE 4.7 VELOCITY FIELD (AFTER REFERENCE 116)

Chen and Ling (117) described a method for selecting a class of admissible velocity fields by prescribing the form of the streamlines. They applied their method to the flow through cosine, elliptic, hyperbolic and square dies.

Nagpal (118) based on the latter work, derived general velocity

of the plastic boundaries can be arbitrarily selected (Figure 4.9).

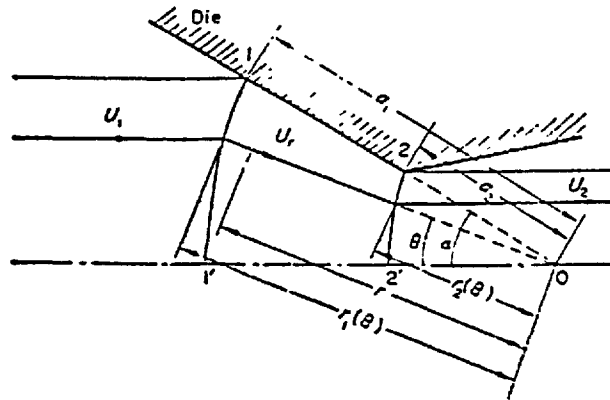


FIGURE 4.9 GENERALISED RADIAL FLOW FIELD (AFTER REFERENCE 121)

Kudo and Shinosaki (122) using the upper bound technique to analyse the extrusion of a hollow component assumed a spherical entry boundary and, in a section containing the central axis, a plastic flow converging to a point located on the mandrel surface (Figure 4.10).

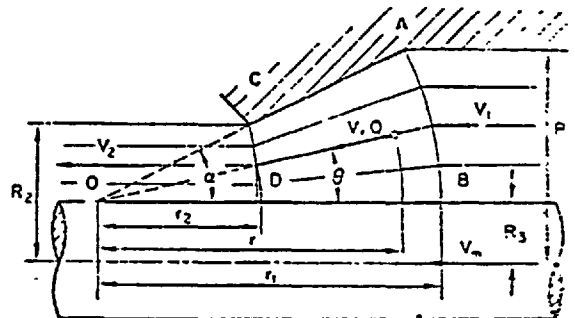


FIGURE 4.10 ADMISSIBLE VELOCITY FIELD FOR TUBE EXTRUSION (AFTER REFERENCE 122)

The last velocity field was generalised by Osakada, Limb and Mellor (123) to be applied to the analysis of hydrostatic extrusion of composite rods (Figure 4.11).

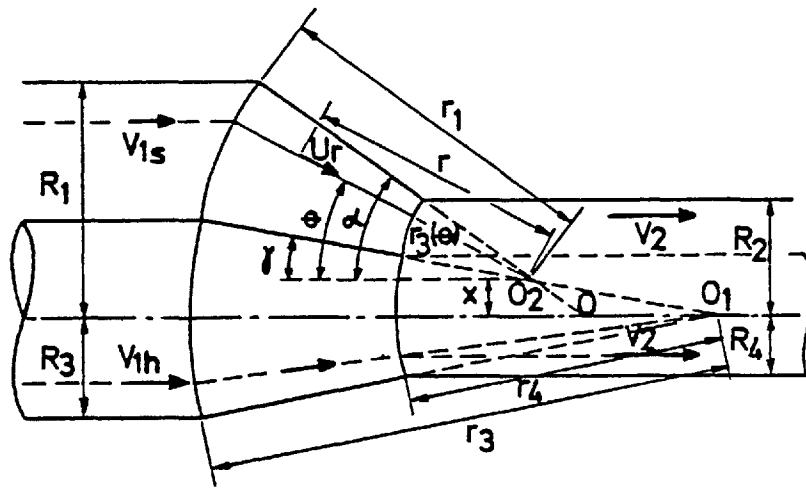
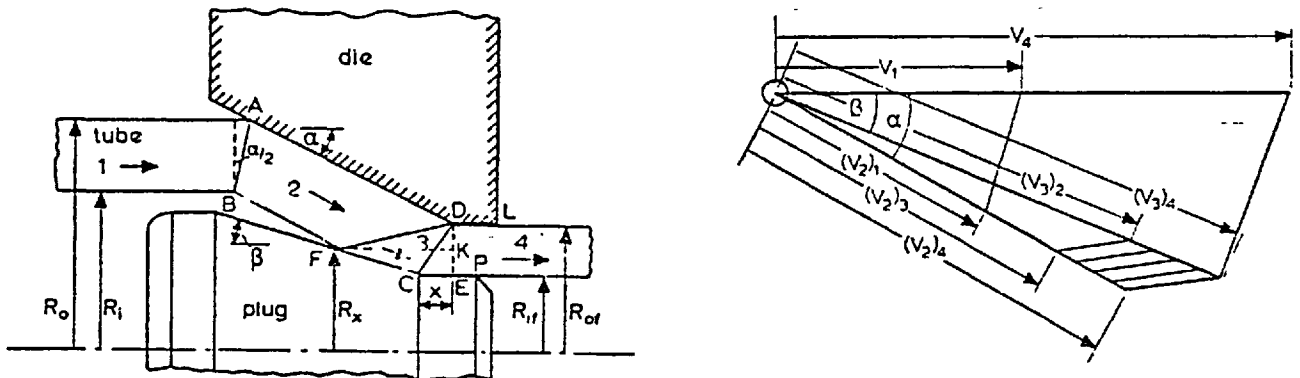


FIGURE 4.11 KINEMATICALLY ADMISSIBLE FIELD FOR EXTRUSION ON COMPOSITE RODS (AFTER REFERENCE 123)

Bramley and Smith (124) studying tube drawing with a floating plug assumed the pattern of velocity discontinuities shown in Figure 4.12. In region 1 the undrawn tube travels parallel to the drawing axis. In region 2 the material flows parallel to the die face. In region 3 the material flows parallel to the conical surface of the plug. In region 4 the drawn tube travels parallel to the drawing axis.



a) PATTERNS OF VELOCITY DISCONTINUITIES FOR TUBE DRAWING (AFTER REFERENCE 124) b) HODOGRAPH OF PATTERN DISCONTINUITIES

FIGURE 4.12

Islam and Bramley (125) extended this velocity field to the cold drawing of duplex tubes (Figure 4.13).

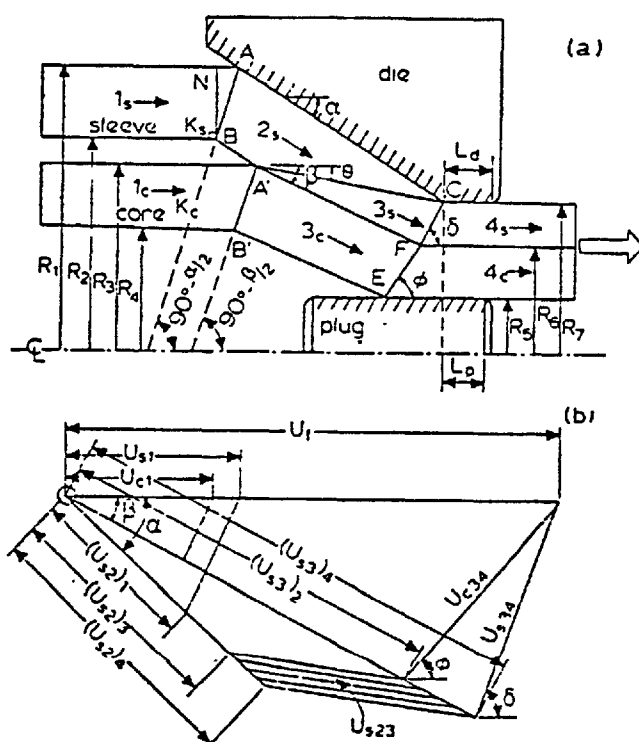


FIGURE 4.13 PATTERN OF VELOCITY DISCONTINUITIES AND HODOGRAPH FOR DUPLEX TUBE COLD DRAWING (AFTER REFERENCE 125)

From this review it is clear that most of the suggested velocity fields assumed particular surfaces of velocity discontinuity and plastic flow and few attempts have been made to minimize the total energy dissipation rate in order to obtain a lower upper bound. In the cases in which the optimization of the upper bound on power is possible, the number of free parameters with respect to that the optimum value is sought, is small. Otherwise special optimization techniques are required (121).

4.2 Assumptions and Notation

In the next section a kinematically admissible velocity field is described to be applied to the analysis of the hydrostatic extrusion drawing of tubes, using a straight mandrel and a conical die. It can also be used for the study of the extrusion of solid billets through conical dies.

In this velocity field the inlet and outlet boundaries are

arbitrarily selected and the flow lines in the plastic zone are assumed to be straight lines.

A minimization of the upper bound pressure is carried out in respect of two free parameters when conical boundaries, separating the plastic zone from the rigid zones, are prescribed.

This upper bound takes into account the strain hardening of the material during the deformation. Throughout the calculations the process is assumed to be isothermal (90,126).

In the derivation of the velocity and strain rate fields the following notation will be used:

A subscript i ($i = 1,2$) will be used to denote variables, functions or symbols related to the exit ($i = 1$) and the entry ($i = 2$) boundaries.

4.3 Velocity Field

Figure 4.14 illustrates the flow of a rigid-plastic material between a straight mandrel and a conical die. The flow is axisymmetric and the deformation takes place in the zone enclosed by the surfaces Γ_1 and Γ_2 which are represented in a longitudinal section containing the mandrel axis by the curves $z_1 = f_1(r_1)$ and $z_2 = f_2(r_2)$ respectively.

The z -axis coincides with the mandrel axis. The functions f_1 and f_2 are continuous and their second differential exist.

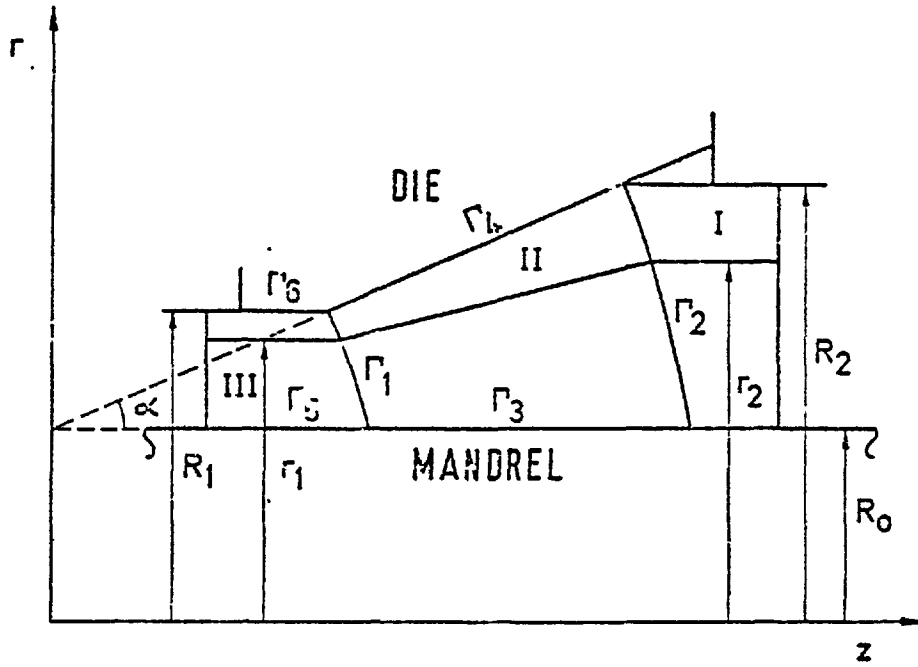


FIGURE 4.14 FLOW PATTERN

A flow line in region I is parallel to the z -axis and becomes parallel to it again in the region after the deformation (region III).

Due to incompressibility of the material, radial distances r_1 and r_2 are related to each other by the expression

$$r_1 = \sqrt{\frac{r_2^2 - R_0^2}{R}} + R_0^2 \quad (4.2)$$

where R is the extrusion ratio defined by

$$R = \frac{R_2^2 - R_0^2}{R_1^2 - R_0^2} \quad (4.3)$$

Figure 4.15 shows the metal flow between two close streamlines which have inclination angles θ and $\theta + d\theta$ with the z -axis, in the region II. The point O , where two streamlines meet, is not a fixed point in the general case. In figure 4.15 the velocity v_1 at P_1 , immediately before the flow crossing Γ_1 , and the velocity v_2 at P_2 ,

the deformation zone is:

$$\dot{V} = 2\pi r \rho d\theta v \quad (4.5)$$

Expressions (4.4) and (4.5) are identical, since the volume rate is constant along the tubular channel represented in Figure 4.15 by the two streamlines.

Due to continuity, the normal component of the velocity before and after the surfaces of discontinuity Γ_1 and Γ_2 must be equal:

$$v_{ie} \cos\psi_i = v_i \cos(\psi_i - \theta) \quad (i = 1, 2)$$

and

$$v_i = \frac{\cos\psi_i}{\cos(\psi_i - \theta)} \cdot v_{ie} \quad (i = 1, 2) \quad (4.6)$$

where ψ_1 and ψ_2 are the angles between the z-axis and the vector normal to the exit and entry boundaries respectively. They can be determined from the equations of the boundaries as follows:

$$\psi_i = \arctan\left(-\frac{dz_i}{dr_i}\right) \quad (i = 1, 2) \quad (4.7)$$

Relating v_{1e} to v_{2e} by the extrusion ratio, leads to:

$$v_{1e} = R v_{2e} \quad (4.8)$$

Combining the right hand sides of the expressions represented by (4.4) and

$$\rho_2 = \rho_1 + d \quad (4.9)$$

where

$$d = \sqrt{(r_1 - r_2)^2 + (z_1 - z_2)^2} \quad (4.10)$$

one obtains:

$$\rho_1 = \frac{d}{\frac{r_1 v_1}{r_2 v_2} - 1} \quad (4.11)$$

Calculating ρ by:

$$\rho = \rho_1 + d_1 \quad (4.12)$$

where

$$d_1 = \sqrt{(r - r_1)^2 + (z - z_1)^2} \quad (4.13)$$

and combining (4.5) and the right hand sides of the expressions (4.4) the velocity at a point in the deformation zone is obtained:

$$v = \frac{\rho_i r_i v_i}{\rho r} \quad (i = 1, 2) \quad (4.14)$$

4.4 Strain Rate and Strain Fields

To calculate the strain rates at a point in the plastically deformed zone the expressions (3.13) are applied to the components of velocity:

$$v_r = v \sin \theta$$

$$v_z = v \cos \theta$$

The strain rate field is obtained by the following expressions:

$$\dot{\epsilon}_r = \frac{\partial v}{\partial r} \sin \theta + v \cos \theta \frac{\partial \theta}{\partial r} \quad (4.15)$$

$$\dot{\epsilon}_\theta = \frac{v_r}{r}$$

$$\dot{\epsilon}_z = \frac{\partial v}{\partial z} \cos \theta - v \sin \theta \frac{\partial \theta}{\partial z}$$

$$\dot{\epsilon}_{rz} = \frac{1}{2} \left(\frac{\partial v}{\partial z} \sin \theta + v \cos \theta \frac{\partial \theta}{\partial z} + \frac{\partial v}{\partial r} \cos \theta - v \sin \theta \frac{\partial \theta}{\partial r} \right)$$

In these expressions $\frac{\partial \theta}{\partial r}$, $\frac{\partial \theta}{\partial z}$, $\frac{\partial v}{\partial r}$ and $\frac{\partial v}{\partial z}$ are unknown so far and have to be determined.

As the streamlines in region II are assumed to be straight lines, the angle θ is related to the co-ordinates of the point P and to the co-ordinates of the points P_2 and P_1 in the same streamline belonging to the entry and exit boundaries respectively.

$$\theta = \arctan \left(\frac{r - r_i}{z - z_i} \right) \quad (i = 1, 2) \quad (4.16)$$

By differentiating (4.16) with respect to r and z , $\frac{\partial \theta}{\partial r}$ and $\frac{\partial \theta}{\partial z}$ are obtained as follows:

$$\frac{\partial \theta}{\partial r} = \frac{1}{\frac{dr_i}{d\theta} (1 - \tan \theta \frac{dz_i}{dr_i}) + (z - z_i) \cdot \sec^2 \theta} \quad (i = 1, 2) \quad (4.17)$$

$$\frac{\partial \theta}{\partial z} = - \frac{\partial \theta}{\partial r} \tan \theta \quad (4.18)$$

where $\frac{dr_i}{d\theta}$ is calculated from (4.4), (4.6), (4.9), (4.11) and is given by:

$$\frac{dr_i}{d\theta} = \rho_i \frac{\cos\psi_i}{\cos(\psi_i - \theta)} v_{ie} \quad (i = 1, 2) \quad (4.19)$$

By using the chain rule one obtains

$$\frac{dz_i}{d\theta} = \frac{dz_i}{dr_i} \frac{dr_i}{d\theta} \quad (i = 1, 2) \quad (4.20)$$

In the subsequent derivation (equations (4.21) to (4.24)) to avoid repeating expressions which are formally identical the following notation will be utilized:

A superscript j ($j = 1, 2$) will be used with a variable x to represent the co-ordinates of a point in the deformation zone. Thus, the variables r and z which currently represent the co-ordinates of a point will be replaced in those equations by the variables $x^{(1)}$ and $x^{(2)}$ respectively.

Attending to (4.17), (4.18), (4.19) and (4.20) the following derivatives can be obtained:

$$\frac{\partial r_i}{\partial x^{(j)}} = \frac{dr_i}{d\theta} \frac{\partial \theta}{\partial x^{(j)}} \quad (i = 1, 2) \quad (4.21)$$

$$(j = 1, 2)$$

$$\frac{\partial z_i}{\partial x^{(j)}} = \frac{dz_i}{d\theta} \frac{\partial \theta}{\partial x^{(j)}}$$

Differentiating equation (4.7) with respect to r and z and using equation (4.21), $\frac{\partial \psi_1}{\partial r}$, $\frac{\partial \psi_1}{\partial z}$, $\frac{\partial \psi_2}{\partial r}$ and $\frac{\partial \psi_2}{\partial z}$ become

$$\frac{\partial \psi_i}{\partial x^{(j)}} = - \frac{d^2 z_i}{dr_i^2} \frac{\partial r_i}{\partial x^{(j)}} \cos^2 \psi_i \quad \begin{matrix} (i = 1, 2) \\ (j = 1, 2) \end{matrix} \quad (4.22)$$

$\frac{\partial v_1}{\partial r}, \frac{\partial v_1}{\partial z}, \frac{\partial v_2}{\partial r}$ and $\frac{\partial v_2}{\partial z}$ are obtained by differentiating (4.6) with respect to r and z and introducing equation (4.22) as follows

$$\begin{aligned} \frac{\partial v_i}{\partial x^{(j)}} = & - \left\{ \sin \theta \frac{\partial \psi_i}{\partial x^{(j)}} + \cos \psi_i \sin(\psi_i - \theta) \frac{\partial \theta}{\partial x^{(j)}} \right\} \\ & \sec^2(\psi_i - \theta) v_{ie} \end{aligned} \quad (4.23)$$

The differentiation of equation (4.12) with respect to r and z combined with equations (4.21) and (4.22) leads to

$$\begin{aligned} \frac{\partial \rho_1}{\partial x^{(j)}} = & \left\{ \frac{1}{r_2} \frac{\partial r_2}{\partial x^{(j)}} + \frac{1}{v_2} \frac{\partial v_2}{\partial x^{(j)}} + \frac{1}{d} \frac{\partial d}{\partial x^{(j)}} \right. \\ & - \left(\frac{\partial r_1}{\partial x^{(j)}} v_1 + \frac{\partial v_1}{\partial x^{(j)}} r_1 - \frac{\partial r_2}{\partial x^{(j)}} v_2 \right. \\ & \left. \left. - \frac{\partial v_2}{\partial x^{(j)}} r_2 \right) / (r_1 v_1 - r_2 v_2) \right\} \rho_1 \end{aligned} \quad (4.24)$$

where $\frac{\partial d}{\partial r}$ and $\frac{\partial d}{\partial z}$ are obtained by differentiating (4.11) with respect to r and z .

Finally differentiating (4.14) with respect to r and z one obtains

$$\begin{aligned} \frac{\partial v}{\partial r} = & \left(\frac{\partial \rho_1}{\partial r} \frac{1}{\rho_1} + \frac{\partial r_1}{\partial r} \frac{1}{r_1} + \frac{\partial v_1}{\partial r} \frac{1}{v_1} - \frac{\partial \rho}{\partial r} \frac{1}{\rho} - \frac{1}{r} \right) v \\ & (4.25) \\ \frac{\partial v}{\partial z} = & \left(\frac{\partial \rho_1}{\partial z} \frac{1}{\rho_1} + \frac{\partial r_1}{\partial z} \frac{1}{r_1} + \frac{\partial v_1}{\partial z} \frac{1}{v_1} - \frac{\partial \rho}{\partial z} \frac{1}{\rho} \right) v \end{aligned}$$

All the terms in (4.25) are known and expressions (4.15) can now be used to determine the strain rate field.

The components of strain and the effective strain at the point P are calculated by integrating the components of the strain rates and the effective strain rate along the streamlines using (3.21) and (3.22) respectively. These expressions can be written in a more suitable form to be applied to the present problem.

Since $v_z = \frac{dz}{dt}$ and for

$$t = 0 \quad z = z_2$$

$$t = t \quad z = z$$

expressions (3.21) and (3.22) can be written respectively as:

$$\epsilon_{ij} = \int_{z_2}^z \frac{\dot{\epsilon}_{ij}}{v_z} dz \quad (4.26)$$

$$\bar{\epsilon} = \int_{z_2}^z \frac{\dot{\bar{\epsilon}}}{v_z} dz \quad (4.27)$$

When the metal crosses the velocity discontinuity surfaces Γ_1 and Γ_2 it undergoes deformation which is given in terms of the engineering shear strain γ by:

$$\gamma_i = \frac{\Delta v_{Ti}}{v_{Ni}} \quad (4.28)$$

where Δv_{Ti} is the absolute value of the difference between the velocity components tangential to the boundary Γ_i before and after crossing it and is given by:

$$\Delta v_{Ti} = |v_i \sin(\psi_i - \theta) - v_{ie} \sin \psi_i| \quad (i = 1, 2) \quad (4.29)$$

v_{Ni} is the magnitude of the velocity component normal to the boundary Γ_i and is given by:

$$v_{Ni} = v_{ie} \cos \psi_i \quad (i = 1, 2) \quad (4.30)$$

The effective strains corresponding to these shear strains are:

$$\bar{\epsilon}_{\Gamma_i} = \frac{\sqrt{3}}{3} \gamma_i \quad (4.31)$$

Therefore, the effective strain in a point in region II is given by:

$$\bar{\epsilon}_t = \bar{\epsilon}_{\Gamma_2} + \bar{\epsilon} \quad (4.32)$$

and the effective strain in the product is:

$$\bar{\epsilon}_p = \bar{\epsilon}_{\Gamma_2} + \bar{\epsilon} + \bar{\epsilon}_{\Gamma_1} \quad (4.33)$$

To test the presented formulae of the velocity, strain rate and strain fields, an analytical derivation of all these fields was carried out for the case of converging flow assuming a spherical surface for the entry boundary (see Appendix A). The results obtained by these two different methods were then compared and verified that they were identical for the velocity and strain rate fields and the error, due to the numerical integration in expressions (4.26) to calculate the strain field, was negligible.

4.5 Stress-Strain Curves

To determine the effective stress in every point of the plastic deformation zone it is necessary to relate it to the effective strain. The stress-strain relationship can be obtained experimentally using various methods which are described in several works together with testing conditions and data presentation (127-135).

The experimental results can be expressed in different manners. Swedlow (129) deals with tabular data for a discrete number of values of the plastic strain and uses linear interpolation for intermediate values. Suzuki et al (133) and Lengyel and Mohitpour (134) present the results in a graphical form, while Hockett (132) considers a mathematical equation to match the experimental data.

To solve plasticity problems it is often convenient to have simple analytical expressions to represent those relationships and among them the following are frequently utilized:

Power equation	$\bar{\sigma} = A \bar{\epsilon}^m$
Ludwik's equation	$\bar{\sigma} = A + B\bar{\epsilon}^m$
Swift's equation	$\bar{\sigma} = A(B + \bar{\epsilon})^m$
Voce's equation	$\bar{\sigma} = A + (B - A) (1 - e^{-m\bar{\epsilon}})$

where A, B and m are constants and e in Voce's equation is the Napierian logarithmic basis.

In the present work the commercial purity aluminium and high conductivity copper stress-strain curves for quasi static conditions obtained by Mohitpour (135) were fitted by polynomials with coefficients and best degree found by means of a computer program using a least squares fitting library subroutine.

The equations representing those stress-strain curves (figure

4.16) and the limits of validity are as follows:

Aluminium

$$\begin{aligned} \sigma_s = & 55.771 + 268.962 \bar{\epsilon} - 664.653 \bar{\epsilon}^2 + 1243.302 \bar{\epsilon}^3 \\ & - 1332.300 \bar{\epsilon}^4 + 711.250 \bar{\epsilon}^5 - 146.540 \bar{\epsilon}^6 \text{ MN/m}^2 \quad (4.34.a) \end{aligned}$$

$$\bar{\epsilon} \leq 1.5$$

Copper

$$\begin{aligned} \sigma_s = & 53.447 + 1189.854 \bar{\epsilon} - 817.258 \bar{\epsilon}^2 - 3280.618 \bar{\epsilon}^3 \\ & + 7808.392 \bar{\epsilon}^4 - 6303.488 \bar{\epsilon}^5 + 1771.301 \bar{\epsilon}^6 \text{ MN/m}^2 \quad (4.34.b) \end{aligned}$$

$$\bar{\epsilon} \leq 1.0$$

4.6 The Upper Bound Approach

4.6.1 Discretisation

To obtain an upper bound to the extrusion pressure, the powers in the hydrostatic extrusion-drawing process are computed, as expressed in the power balance equation (4.1).

Since the analytical expressions of the variables in (4.1) are complex, a computer program was developed to calculate numerically the integrals in that equation and to obtain the upper bound pressure for given boundaries.

For this purpose n_s streamlines equally spaced in region I are considered. In region III the r_1 co-ordinate of a streamline is related to the r_2 co-ordinate in region I by the equation (4.2).

With the equations of the entry and exit boundaries assumed, the streamlines in region II are completely defined.

Each streamline in region II is divided into $n_n - 1$ segments of the same length by n_n nodal points. The volume of the plastic deformation zone is divided into triangular section toroidal elements.

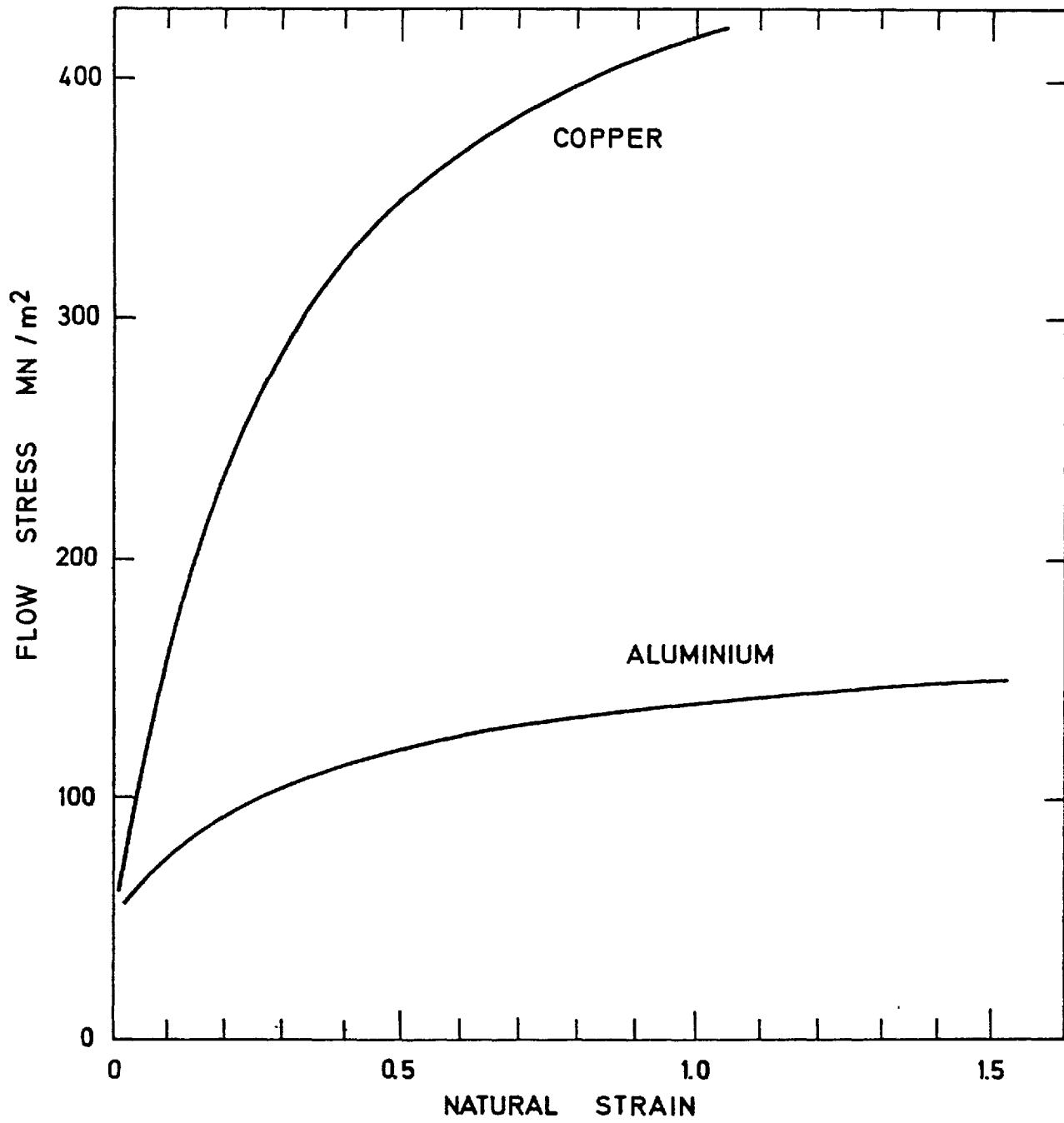


FIGURE 4.16 STRESS-STRAIN CURVES FOR COPPER AND ALUMINIUM
(AFTER REFERENCE 135)

The configuration of the mesh obtained is shown in figure 4.17.

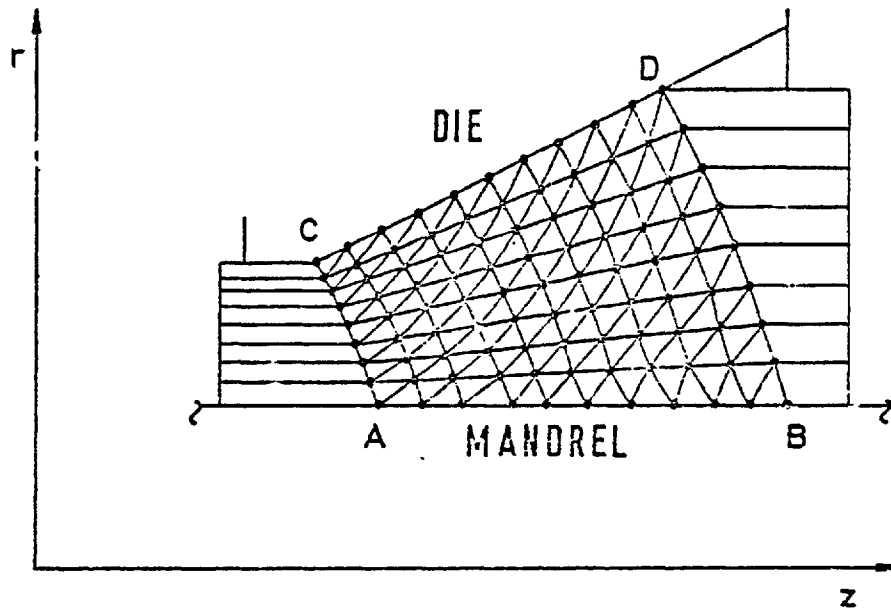


FIGURE 4.17 MESH CONFIGURATION

It is further assumed that the number of streamlines and nodal points is large enough so that the error introduced in the calculations by the discretisation is negligible.

4.6.2 Internal Power of Deformation

The internal power of deformation is given by expression (4.1.a). For all nodal points the effective strain is calculated by (4.33) or (4.32) according to whether or not they belong to the boundary Γ_1 respectively. The integration of the effective strain rate in equation (4.27) is performed along the streamline containing the nodal point, using the trapezoidal rule. By replacing the value of the effective strain in the stress-strain curve equation of the material, one obtains the effective stress at the nodal points.

The following approximation is made for an elemental torodial

volume ΔV_e :

$$\int \bar{\sigma} \dot{\bar{\epsilon}} dV_e = \frac{\Delta V_e}{3} \sum_{j=1}^3 \bar{\sigma}_j \dot{\bar{\epsilon}}_j \quad (4.35)$$

where $\bar{\sigma}_j$ and $\dot{\bar{\epsilon}}_j$ represent the effective stress and the effective strain rate, respectively, in a vertex of the triangle and

$$\Delta V_e = 2\pi r_c \Delta A_e \quad (4.36)$$

In (4.36) ΔA_e is the area of the triangular section and r_c is the radius of the triangle centroid. In Appendix B it is shown how ΔA_e and r_c are calculated from the co-ordinates of the triangle vertices.

The internal power of deformation is computed by:

$$\dot{W}_d = \int_V \bar{\sigma} \dot{\bar{\epsilon}} dV = \frac{1}{3} \sum_{e=1}^N \sum_{j=1}^3 \bar{\sigma}_j \dot{\bar{\epsilon}}_j \Delta V_e \quad (4.37)$$

where N is the total number of elements in the deformation zone and is given by:

$$N = 2(n_n - 1)(n_s - 1)$$

4.6.3 Power Dissipated over Surfaces of Velocity Discontinuity

To calculate the integral expressed by (4.1.b) the following power consumptions are analysed.

4.6.3.1 Power Dissipation on Surfaces Γ_1 and Γ_2

As the maximum value that the shear stress can have is $\frac{\bar{\sigma}}{\sqrt{3}}$ if the von Mises yield criterion is assumed the power dissipation on each of those surfaces is as follows

$$\dot{W}_{r_i} = \int_{\Gamma_i} \tau |\Delta v| dS_i = \frac{1}{\sqrt{3}} \int_{\Gamma_i} \bar{\sigma} \Delta v_{T_i} dS_i \quad (i = 1, 2) \quad (4.38)$$

where Δv_{T_i} is given by (4.29).

The surfaces Γ_i contain $(n_s - 1)$ elemental surfaces whose area is calculated by

$$\Delta S_e = 2\pi r_m l_e \quad (4.39)$$

where l_e is the length of the elemental segment and r_m is the medium radius of the segment l_e .

The integrals in (4.38) can be approximated by

$$\dot{W}_{r_i} = \frac{1}{2\sqrt{3}} \sum_{e=1}^{n_s-1} \sum_{j=1}^2 \bar{\sigma}_j v_{T_{ij}} \Delta S_e \quad (i = 1, 2) \quad (4.40)$$

where $\bar{\sigma}_1$ and $\bar{\sigma}_2$ are the effective stress in the nodal points limiting the elemental segment l_e .

4.6.3.2 Friction Losses over Surfaces $\Gamma_3, \Gamma_4, \Gamma_5, \Gamma_6$

Assuming a constant friction factor, m , at the mandrel/billet, mandrel/product, die/billet and die land/product interfaces the shear stresses acting at these surfaces can be written as:

$$\tau = m \frac{\bar{\sigma}}{\sqrt{3}}$$

The power consumption on surface Γ_3 is given by

$$\dot{W}_{f_3} = \int_{\Gamma_3} \tau |\Delta v| dS = \frac{m}{2\sqrt{3}} \sum_{e=1}^{n-1} \sum_{j=1}^2 \bar{\sigma}_j v_j \Delta S_e \quad (4.41)$$

where ΔS_e represents the elemental area on the mandrel/billet interface and v_j is the velocity at the nodal points on the same interface and is determined by (4.14).

The power consumption on the surface Γ_4 is found as follows.

$$\dot{W}_{f_4} = \int_{\Gamma_4} \tau |\Delta v| dS = \frac{m}{2\sqrt{3}} \sum_{e=1}^{n-1} \sum_{j=1}^2 \bar{\sigma}_j v_j \Delta S_e \quad (4.42)$$

where ΔS_e represents the elemental area on the die/billet interface and v_j is the velocity at the nodal points on the same interface.

The power consumption on surface Γ_5 is calculated by

$$\dot{W}_{f_5} = \int_{\Gamma_5} \tau |\Delta v| dS = 2\pi R_o l_m \cdot m \frac{\bar{\sigma}_A}{\sqrt{3}} \cdot v_{1e} \quad (4.43)$$

where l_m is the length of mandrel in contact with the product up to the die exit, and $\bar{\sigma}_A$ is the effective stress at the point A (figure 4.17).

The power consumption on surface Γ_6 is computed by

$$\dot{W}_{f_6} = \int_{\Gamma_6} \tau |\Delta v| dS = 2\pi R_1 l_d \cdot m \frac{\bar{\sigma}_C}{\sqrt{3}} \cdot v_{1e} \quad (4.44)$$

where l_d is the length of the die land and $\bar{\sigma}_C$ is the effective stress

at the point C (Figure 4.17).

4.6.4 Power Due to the Prescribed Drawing Stresses

The integral (4.1.c) is computed as follows:

$$\dot{W}_{ps} = - \int T_i v_i dS = -F v_{1e} \quad (4.45)$$

where F is the drawing force.

4.6.5 Power Due to the Applied Pressure

The power due to the applied pressure, p, becomes

$$\dot{W}_p = \pi(R_2^2 - R_o^2) \cdot p \cdot v_{2e} \quad (4.46)$$

4.6.6 Balance of Power

The equation of the balance of power can be written as

$$\dot{W}_p = \dot{W}_d + \dot{W}_{r_1} + \dot{W}_{r_2} + \dot{W}_{f_3} + \dot{W}_{f_4} + \dot{W}_{f_5} + \dot{W}_{f_6} + \dot{W}_{ps} \quad (4.47)$$

The upper value of the extrusion pressure is evaluated by solving (4.47) in respect of p.

4.7 A Lower Upper Bound Approach

So far a velocity field and an upper bound approach, based on

it, were described for a general case. So, if the equation of the entry and exit boundaries is known the upper bound value of the extrusion pressure can be determined.

A lower upper bound can be found by minimizing the upper bound pressure in respect of some parameters which are introduced in the inlet and outlet boundary equations. In this way, a more accurate information concerning the mode of deformation of the material for the family of boundaries considered, can be obtained.

A method of achieving that goal is to assume the boundary equations in a polynomial form

$$z = a_1 + a_2 r + \dots + a_n r^{n-1}$$

The minimum condition implies that:

$$\frac{\partial p}{\partial a_i} = 0 \quad (i = 1, n) \quad (4.48)$$

As it was seen in the previous sections the evaluation of the upper bound pressure utilizes complex expressions and numerical integrations which makes impossible the analytical calculation of the derivatives in (4.48) and, consequently, the solution of the system of equations.

To overcome this problem in the computer program (shown in Appendix C) is included a library subroutine (136) which uses a quasi-Newton method to minimize a function and does not need the supply of the derivatives of the function. The subroutine starts from an initial guess of the minimum position and generates a sequence of points by estimating the gradient and the curvature of the objective function.

These points are intended to converge to a local minimum of

the function.

The number of parameters in the boundary equations is reduced to two, as for a larger number the computer time would be substantially increased and the minimization would become cumbersome. Thus, the exit and entry boundaries are assumed to be conical and to contain the points C and D (Figure 4.18) respectively. In hydrostatic extrusion at high speeds it is known that the material undergoes deformation before reaching the die. For low speeds, however, Dunn and Lengyel (49) have shown that this effect is absent. In the following the assumption is made that the rigid plastic boundary passes through point D where the billet surface contacts the die, which thus neglects that deformation. As in the present work a low velocity of the product was selected (1.5mm/s), the error introduced by this assumption should be negligible.

Dunn (48) has shown, by estimating the deformation zone in the hydrostatic extrusion of bars from grid lines scribed on split billets, that the exit boundary is approximately conical and contains the material points adjacent to the die exit corner. So it is assumed in the present case, that the exit boundary passes through point C, located at the die corner as found by Dunn.

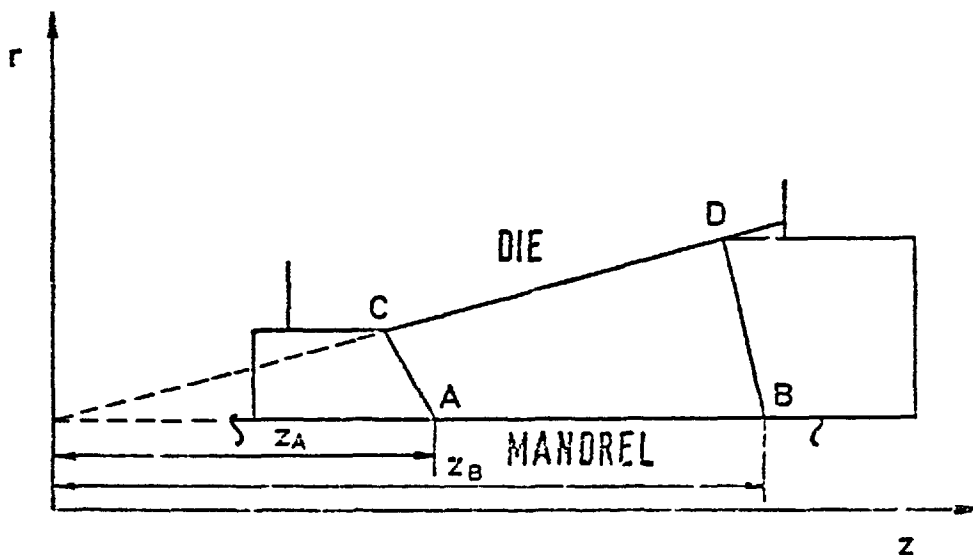


FIGURE 4.18 CONICAL SURFACES USED IN THE LOWER UPPER BOUND APPROACH

The upper bound pressure minimization is carried out with respect to the z-co-ordinates of the points A and B.

4.7. Calculation of the Friction Factor

As it was pointed out in Chapter 2 the friction force on the mandrel/billet interface was measured experimentally for several extrusion ratios and various extrusion pressure/drawing stress ratios. The friction factor can be estimated for the experimental conditions using this lower upper bound.

The measured friction force (F_f) is the result of the friction forces acting on the plastic zone/mandrel (F_{fp}) and elastic zone/mandrel (F_{fe}) interfaces, i.e.:

$$F_f = F_{fp} + F_{fe} \quad (4.49)$$

The assumption of a constant friction factor leads to:

$$1) \quad F_{fp} = \int_{S_m} \tau \, dS = \frac{m}{\sqrt{3}} \int_{S_m} \bar{\sigma} \, dS \quad (4.49.a)$$

where S_m is the area of the plastic zone/mandrel surface and

$$2) \quad F_{fe} = \frac{2\pi}{\sqrt{3}} R_o l_m \cdot m \cdot \bar{\sigma}_A \quad (4.49.b)$$

The friction factor is, therefore evaluated by:

$$m = \frac{\sqrt{3}}{2\pi R_o} \frac{F_f}{(\sigma_A l_m + \int_{z_A}^{z_B} \bar{\sigma} \, dz)} \quad (4.50)$$

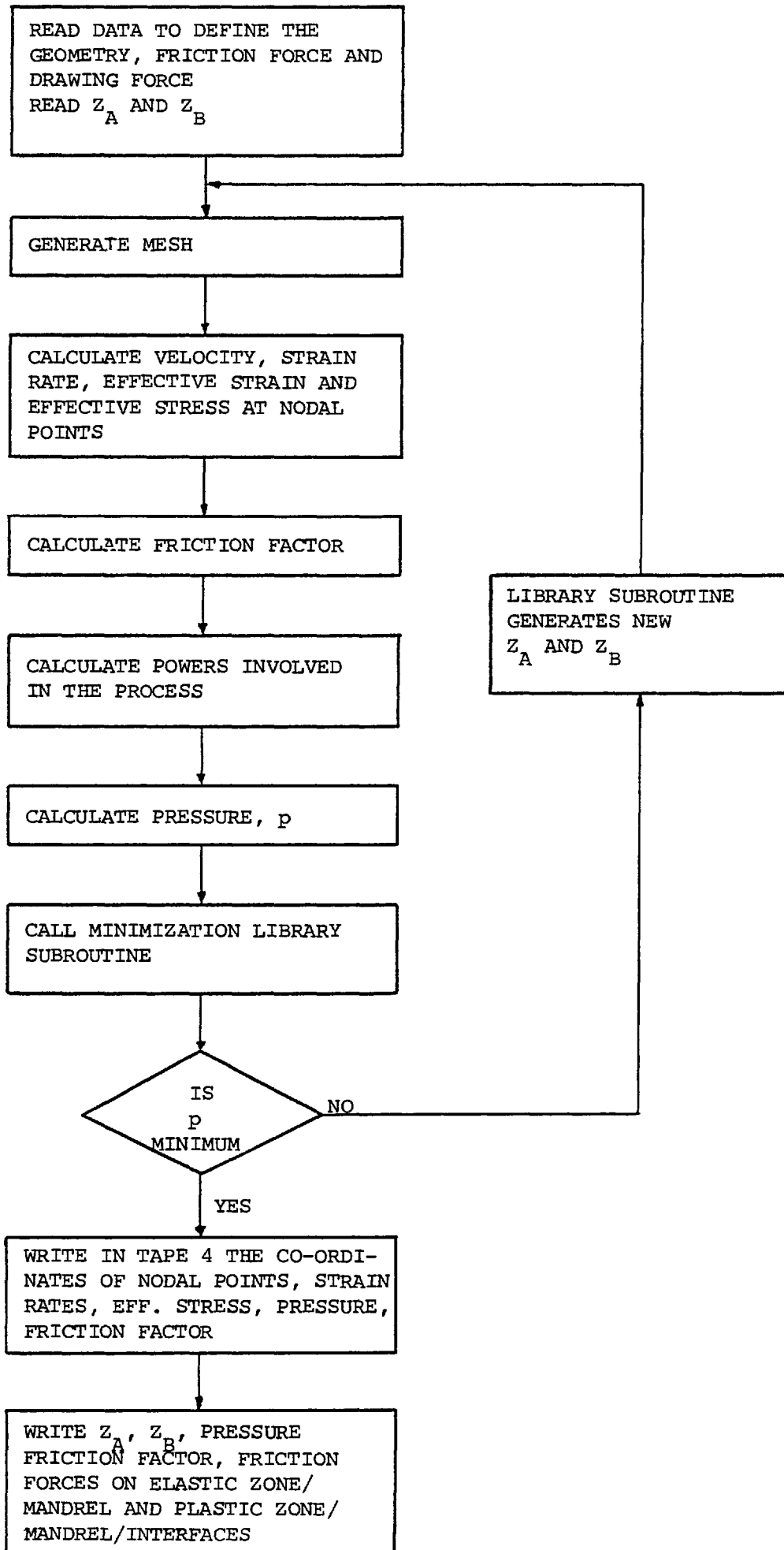


Figure 4.19 - FLOW CHART OF PROGRAM MIN

where the integral $\int_{z_A}^{z_B} \bar{\sigma} dz$ is estimated by:

$$\frac{1}{2} \sum_{e=1}^{n-1} \sum_{j=1}^2 \bar{\sigma}_j \Delta z_e \quad (4.51)$$

In (4.51) $\bar{\sigma}_1$ and $\bar{\sigma}_2$ are the effective stress at the nodal points limiting the elemental segment Δz_e .

4.7.2 Upper Bound Extrusion Pressure

The calculated value of the friction factor is introduced in equations (4.41), (4.42), (4.43) and (4.44) to enable the evaluation of the power consumptions on surfaces Γ_3 , Γ_4 , Γ_5 , and Γ_6 . The other terms in the balance of power equation (4.47) are determined as described previously.

The upper bound extrusion pressure is found as follows:

$$p = \frac{\dot{W}_p}{\pi(R_2^2 - R_0^2) v_{2e}} \quad (4.52)$$

The procedure used to obtain a lower upper bound solution is summarized in the form of a flow chart, Figure 4.19, which corresponds to the program MIN. The output values written in the data file TAPE 4 will be used as input data for the program STRESS to be discussed in the next chapter.

A listing of program MIN together with the variable name list is included in Appendix C.

4.7.3. Results

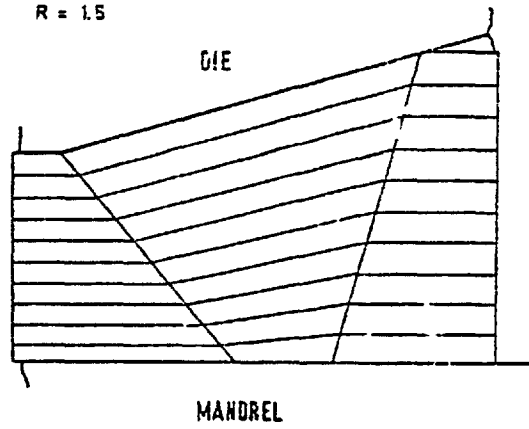
The flow patterns obtained by this lower upper bound approach are illustrated in Figure 4.20. As it will be shown later in Chapter 7, for each extrusion ratio the influence of the friction force in the theoretical flow pattern is very small for the range of extrusion pressure/drawing stress ratios used in the experiments. For this reason the flow patterns in Figure 4.20 are considered as representative of the theoretical flow for each extrusion ratio, irrespective of the extrusion pressure/drawing stress ratios. Also for the same extrusion ratio the flow pattern are assumed to be the same for both materials, aluminium and copper.

Figures 4.21 to 4.25 illustrate the velocity fields, represented by $\frac{v}{v_{1e}}$ where v_{1e} is the product velocity, the effective strain rate, the effective strain and the effective stress, represented as $\frac{\bar{\sigma}}{Y}$, obtained by means of program MIN. Y is the initial yield stress of the materials and corresponds to the first term in the right hand side of the stress-strain curve equation for aluminium and copper.

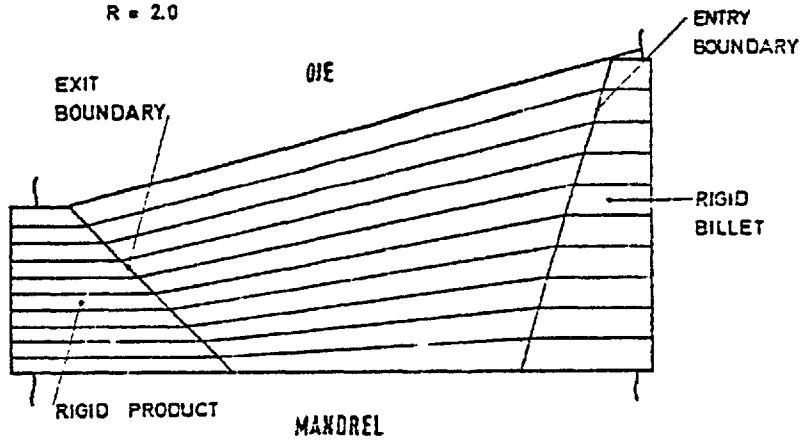
Figures 4.20 to 4.25 also show the position of the conical boundaries separating the plastic zone from the rigid zones, obtained by means of the computer program. A similar position of these surfaces of velocity discontinuity was found, for bar extrusion, by Sha, Oh and Kobayashi (63) using the matrix method (60) and Dunn (48) using the viscoplasticity technique.

The velocity distribution illustrations (Figures 4.21 to 4.25) show that the increase in velocity is greater along the mandrel surface than along the die surface. Considering cross sections, the magnitude of the total velocity increases from the mandrel to the die up to about half way along the die and decreases from the mandrel to the die towards the exit. The velocity at the points where the exit

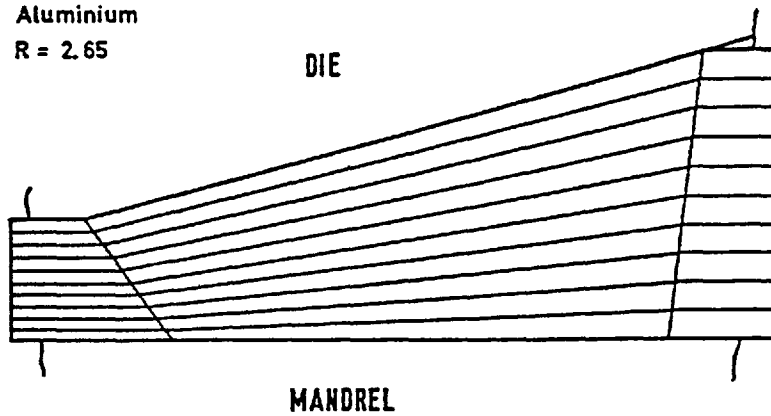
Copper
R = 1.5



Aluminium
Copper
R = 2.0



Aluminium
R = 2.65



Aluminium
R = 3.5

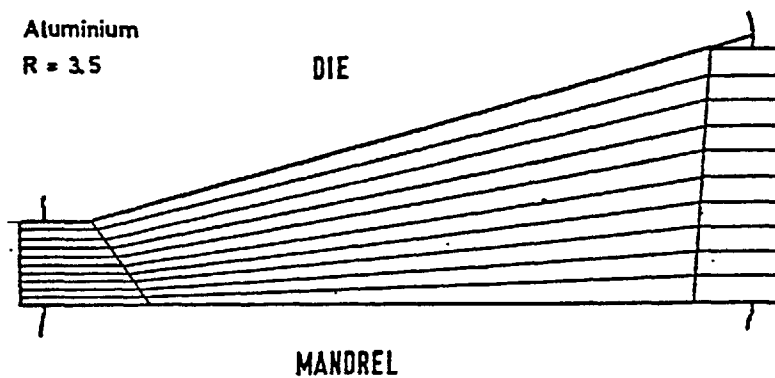


FIGURE 4.20 FLOW PATTERNS OBTAINED BY PROGRAM MIN FOR VARIOUS EXTRUSION RATIOS

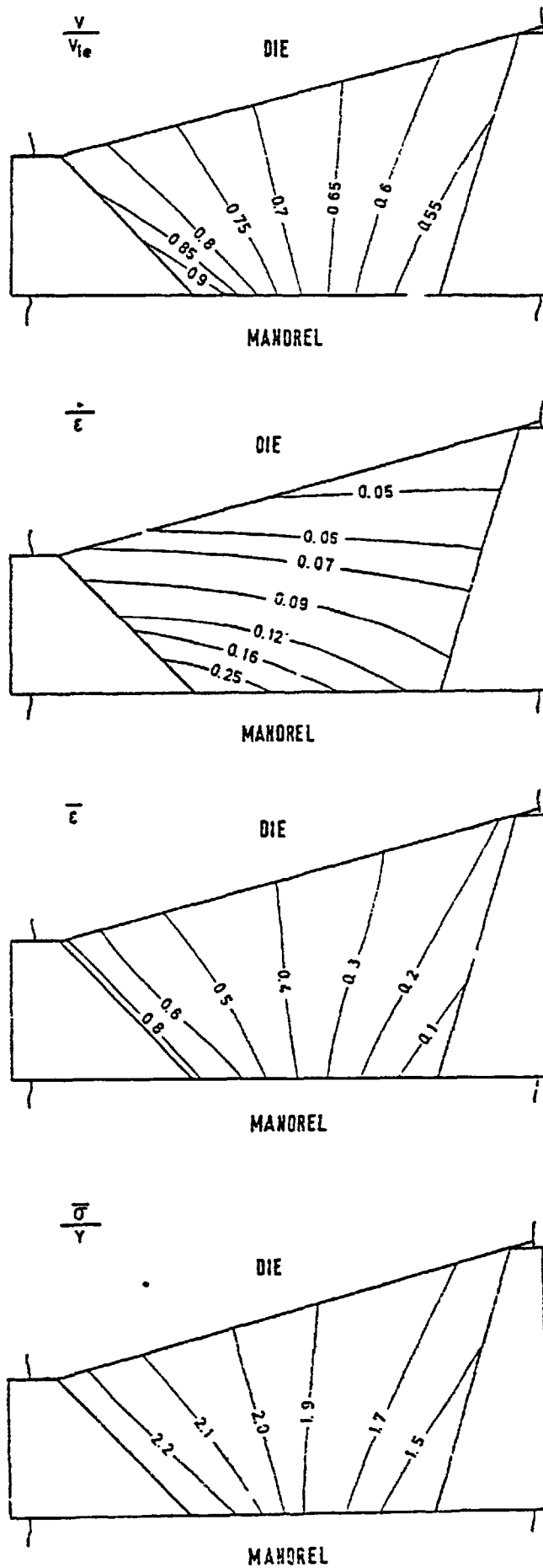


FIGURE 4.21 VELOCITY (v), EFFECTIVE STRAIN RATE ($\dot{\epsilon}$), EFFECTIVE STRAIN ($\bar{\epsilon}$) AND EQUIVALENT STRESS ($\bar{\sigma}$) DISTRIBUTIONS FOR ALUMINIUM EXTRUSION $R = 2.0$. FRICTION FACTOR $m = 0.043$

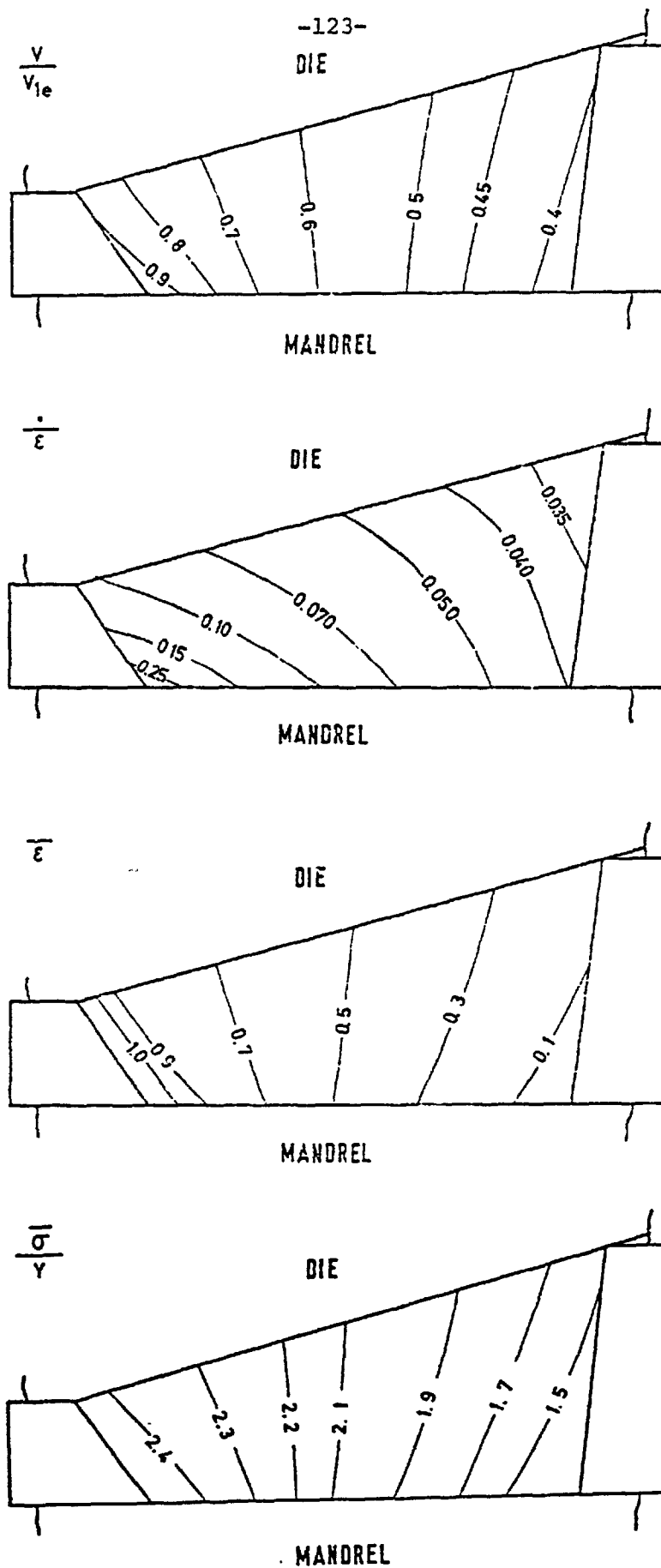


FIGURE 4.22 VELOCITY (v), EFFECTIVE STRAIN RATE ($\dot{\epsilon}$), EFFECTIVE STRAIN (ϵ) AND EQUIVALENT STRESS (σ) DISTRIBUTIONS FOR ALUMINIUM EXTRUSION $R = 2.65$. FRICTION FACTOR $m = 0.036$

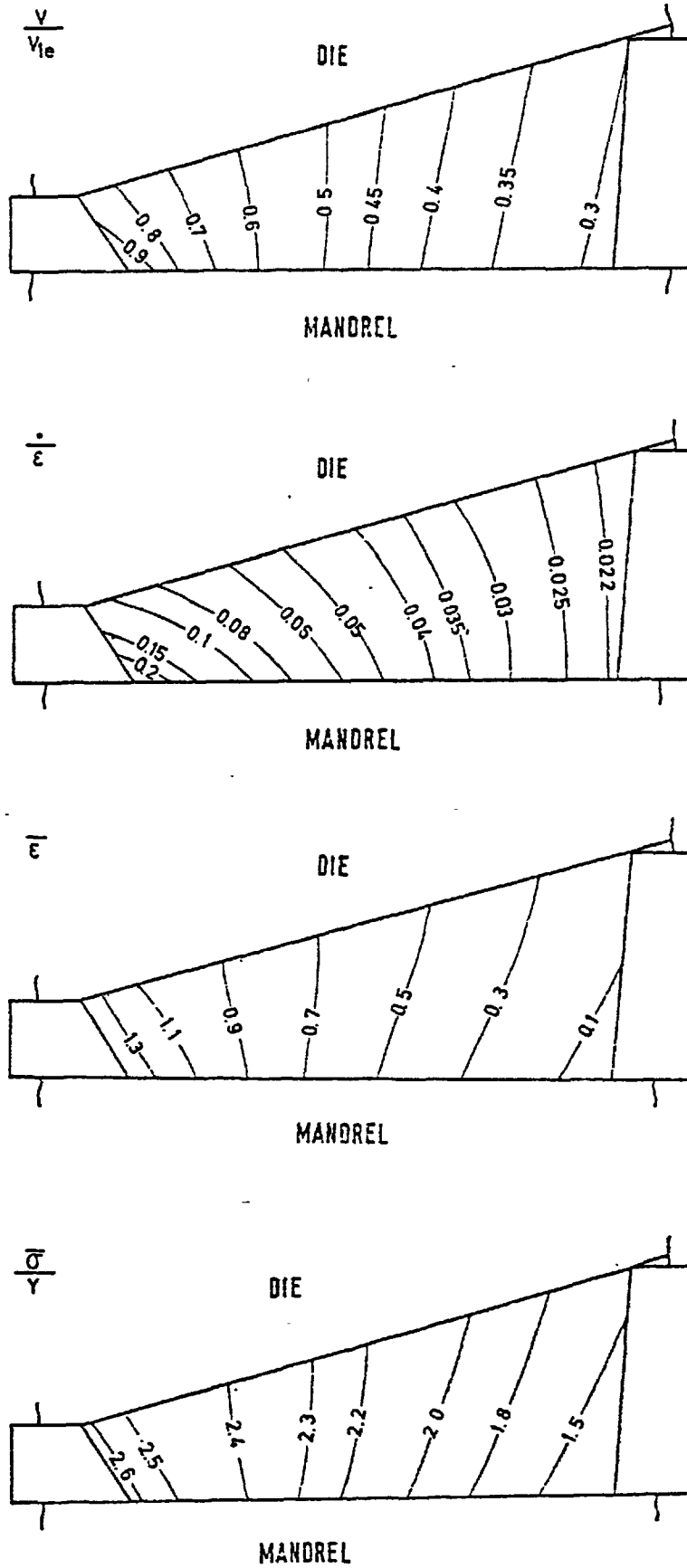


FIGURE 4.23 VELOCITY (v), EFFECTIVE STRAIN RATE ($\dot{\epsilon}$), EFFECTIVE STRAIN ($\bar{\epsilon}$) AND EQUIVALENT STRESS (σ) DISTRIBUTIONS FOR ALUMINIUM EXTRUSION $R = 3.5$. FRICTION FACTOR $m = 0.035$

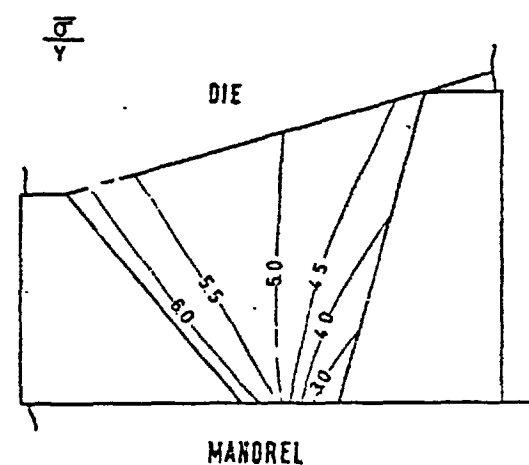
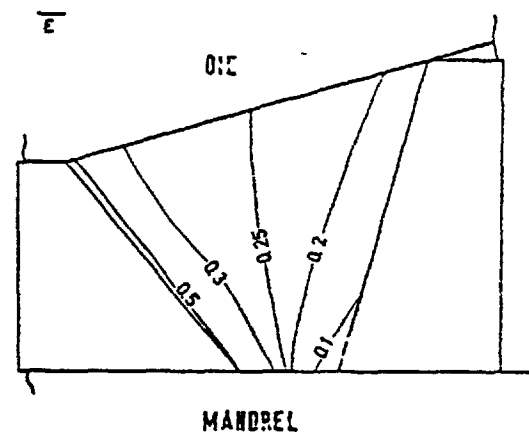
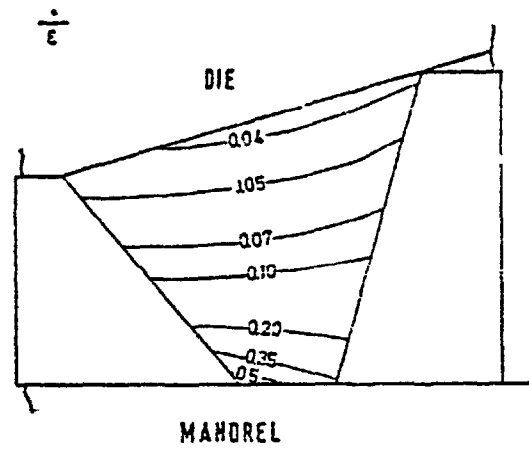
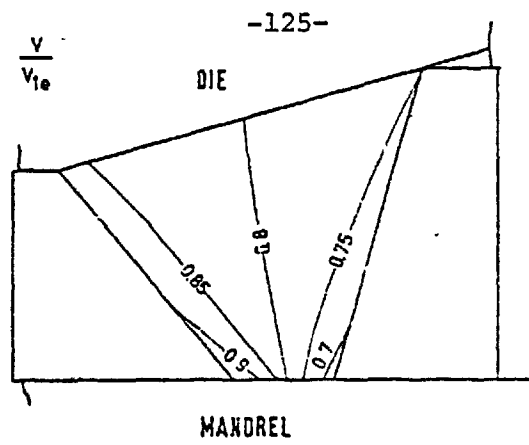


FIGURE 4.24 VELOCITY (v), EFFECTIVE STRAIN RATE ($\dot{\epsilon}$), EFFECTIVE STRAIN ($\bar{\epsilon}$) AND EQUIVALENT STRESS (σ) DISTRIBUTIONS FOR COPPER EXTRUSION $R = 1.5$. FRICTION FACTOR $m = 0.017$

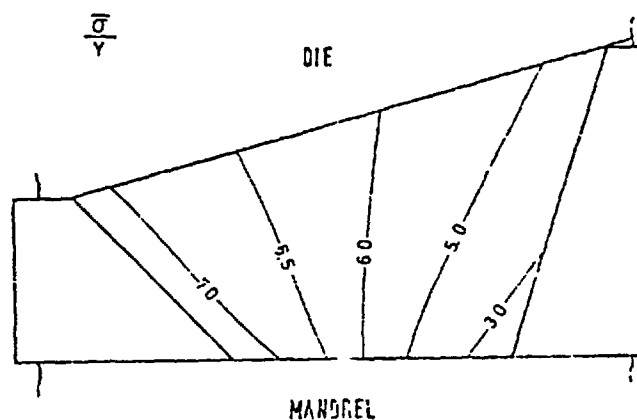
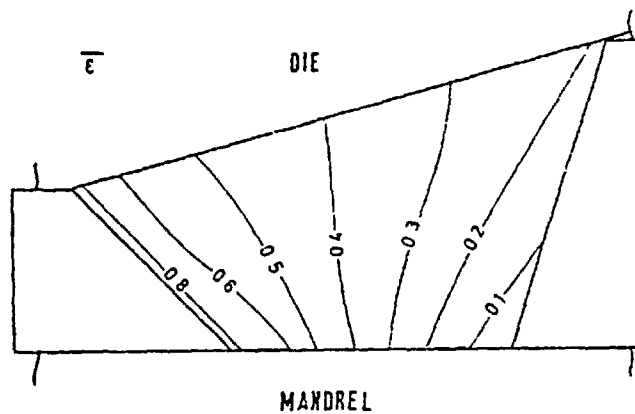
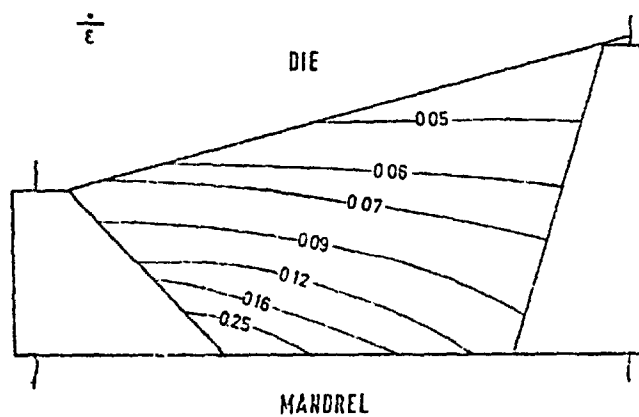
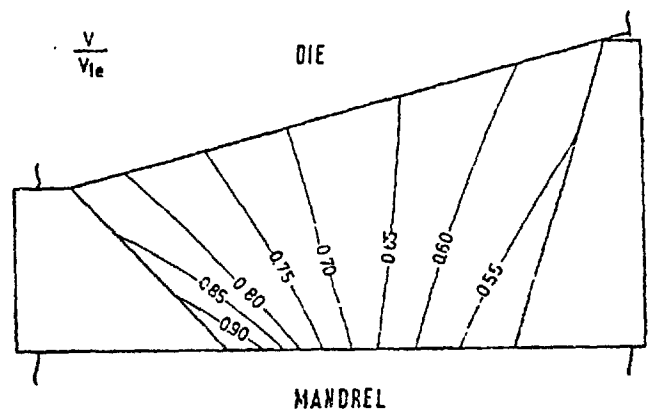


FIGURE 4.25 VELOCITY (v), EFFECTIVE STRAIN RATE ($\dot{\epsilon}$), EFFECTIVE STRAIN ($\bar{\epsilon}$) AND EQUIVALENT STRESS ($\bar{\sigma}$) DISTRIBUTIONS FOR COPPER EXTRUSION $R = 2.0$. FRICTION FACTOR $m = 0.022$

boundary meets the mandrel is v_{1e} , as no change of direction occurs along the mandrel. As expected the velocity at the entry is the highest for the lowest extrusion ratio, while at the exit it has approximately the same value for all the cases considered.

The effective strain rate increases from the entrance towards the exit. As the entry and exit boundaries are surfaces of velocity discontinuity, the effective strain rate distribution shows a sudden change from the rigid zones, where it is zero, to the plastic zones. Near the exit the effective strain rate has a sharp drop. Along the entry boundary the effective strain rate has a small variation for the higher extrusion ratios, while for the lower ones varies substantially (for $R = 1.5$ varies between 0.35 s^{-1} near the mandrel to 0.04 s^{-1} near the die). At the exit it is maximum near the mandrel and minimum at the die. The highest value occurs for extrusion ratio of 1.5 at the exit boundary. In the whole deformation zone the values of the strain rate are very small, justifying the choice of quasi-static stress strain curves.

The effective strain distributions show the same pattern for all the cases presented. At the entrance it increases from the mandrel to the die. In a cross section at half way along the plastic deformation zone/mandrel interface the effective strain becomes the highest near the mandrel, decreasing to the die. At the exit boundary the effective strain is practically constant on the exit surface. The largest values of the effective strain are found at the largest extrusion ratios as expected.

The effective stress distribution obtained from the effective strain follows the same pattern. As at the exit the equivalent stress is constant a uniform hardness in the product cross section is predicted.

As it was pointed out the effect of friction in the velocity field is negligible, although variations in the friction force on the

billet/mandrel interface were measured. However, this variation was not enough to cause significant alteration in the theoretical velocity field. On the other hand this is in agreement with the fact that in hydrostatic extrusion the friction on the plastic zone/tool interface is low.

The other relevant results, such as the upper bound extrusion pressure, are left to be analysed in Chapter 7, where they will be compared with the experimental ones.

CHAPTER 5

THE METHOD OF WEIGHTED RESIDUALS IN PLASTICITY

5.1 Introduction

Most practical problems in engineering generally are described by complex equations that cannot be solved exactly by current analytical methods. Hence it is frequently necessary to resort to methods that yield approximate solutions.

Approximate methods may be classified into three broad categories (137): iterative, asymptotic and weighted residuals.

The iterative methods are based in some form of repetitive calculations which starting from a guessed initial set of values, transform it, according to some operation, into a second set which is in turn transformed by the same operation into a third set, etc. The solution is obtained when the new set is exactly the same as its predecessor. In these methods are included the development of series, methods of successive approximations and rational approximations.

By using the asymptotic methods one seeks solutions that are approximately valid when a physical parameter of the problem is very small or very large. Typical of these methods are the perturbation procedures.

The weighted residual methods consist of proposing a solution in a form of a set of known trial functions with undetermined parameters and requiring that the difference between the approximate and exact solutions is minimized in some sense. A large variety of approximate methods can be included under the designation of weighted residual methods. Brebia (138) presents a weighted residual classification of approximate methods establishing the relationship existing between methods such as the finite element technique, the cell collocation

method, the boundary element method and some others.

As the method of weighted residuals is used in this work to find the stress field in the plastic deformation zone formed during the hydrostatic extrusion-drawing of tubes, it is discussed more fully in the following sections.

5.2 The Method of Weighted Residuals

Consider, in a domain D , the following differential equation, whose exact solution is not known

$$\mathcal{L}(u) = f(x) \quad (5.1.a)$$

and the boundary conditions

$$B_i(u) = g_i(x) \quad i = 1 \dots p \quad (5.1.b)$$

In (5.1) $\mathcal{L}$ is a differential operator, B represents the boundary conditions and f, g are functions of a set of independent variables x .

The problem to be solved by the method of weighted residuals consists of:

given a set of n trial function ϕ_i , which can describe the unknown functions u , finding the n free parameters α_i , such that the approximate solution:

$$\tilde{u}(x) = \phi_0 + \sum_{i=1}^n \alpha_i \phi_i \quad (5.2)$$

is the best approximation that can be made for the function u with the trial functions ϕ_i . The chosen trial functions are linearly independent and members of a complete set of functions. The completeness condition

ensures that the series of equation (5.2) is capable of representing the exact solution provided enough terms are used. However, this property required for mathematical purposes, is sometimes ignored in practice (139).

There are two basic types of criteria for determining the free parameter (64). In one of the methods the parameters are calculated to give a stationary value to a functional related to equation (5.1.a), which can be obtained via the calculus of variation.

In the other criterion, the weighted residual method, the parameters are chosen so as to make a weighted average of the equation residual vanish. The residual R , which is found by substituting equation (5.2) in equation (5.1.a), becomes:

$$R(\alpha, \phi) = f - \mathcal{L}(\bar{u}) = f - \mathcal{L}\{\phi_0 + \sum_{i=1}^n \alpha_i \phi_i\} \quad (5.3)$$

The equation residual can be interpreted as the error associated with the assumption that equation (5.2) represents the exact solution of equation (5.1). When $\bar{u}$ is the exact solution, R is equal to zero.

To force the residual to vanish in an average sense over the entire domain D , one demands that R be orthogonal to a set of weighting functions, W_k

$$\int_D W_k R \, dx = 0 \quad k = 1, 2, \dots, n \quad (5.4)$$

This requirement yields a set of n algebraic equations for the $n \alpha_i$ parameters.

5.2.1. Weighting Functions

Each type of weighting function introduced in (5.4) corresponds

to a different criterion of the method of weighted residuals. Some of the more important and widely used of these methods are described below.

5.2.1.1 Collocation Method

In this method n points p_i are chosen in the domain D and the weighting functions are the Dirac delta function,

$$w_k = \delta(p - p_k)$$

Equation (5.4) becomes:

$$\int_D \delta(p - p_k) R \, dx = R(p_k) = 0 \quad k = 1, 2, \dots, n \quad (5.5)$$

Therefore, as many collocation points must be chosen as there are free parameters. The point locations are arbitrary, but if no special circumstances arise equally spaced collocation points are selected.

In this method the presumption is that the residual is approximately zero in between the locations where it vanishes.

5.2.1.2 Subdomain Method

The subdomain method requires that the domain D be divided into n smaller subdomains, $D_1, D_2, \dots, D_n$, and the integral of the residual over the different regions be zero.

The weighting functions are

$$W(R) = \begin{cases} 1 & j = k \\ 0 & j \neq k \end{cases}$$

and

$$\int_{D_k} R dx = 0 \quad k = 1, 2, \dots, n \quad (5.6)$$

When the number of subdomains is increased the differential equation is satisfied on the average in smaller regions and presumably the residual approaches zero everywhere.

5.2.1.3 Galerkin's Method

In this method the weighting functions are chosen to be the trial functions ϕ_k . The Galerkin method forces the residual to be zero by making it orthogonal to each member of the complete set of trial functions. Equation (5.4) becomes:

$$\int \phi_k R dx = 0 \quad (5.7)$$

5.2.1.4 Method of Moments

The method of moments is similar to Galerkin's method, but in this case the residual is made orthogonal to members of a system of functions which are not the trial functions. The weighting functions are chosen to be $1, x, x^2, \dots, x^n$ and the equation (5.4) is forced to vanish for the first n moments of R :

$$\int_D P_k(x) R dx = 0 \quad (5.8)$$

For the first approximation the method of moments is identical to the subdomain method and is usually called the integral method.

5.2.1.5 Method of Least - Squares

The integral of the square of the residual is minimized with respect to the free parameters to yield n simultaneous algebraic equations.

Equation (5.4) becomes:

$$\frac{\partial}{\partial \alpha_k} \int_D R^2 dx = \int \frac{\partial R}{\partial \alpha_k} R dx = 0 \quad (5.9)$$

The weighting function W_k are therefore represented by $\frac{\partial R}{\partial \alpha_k}$.

The purpose of the above criteria is to select the best approximation for a given family of trial functions. Comparisons of these criteria have been carried out on the same problem. Crandall (64) states that for a particular family which satisfies all the boundary conditions of a linear equilibrium problem the weighted residual criteria will in general produce slightly different approximations and in an equivalent extremum problem the Galerkin's method provides the optimum weighted residual criterion.

Bickeley (140) considered an initial value problem of heat transfer and compared the collocation, moment and Galerkin methods. He found that the Galerkin method proved to be superior.

Ames (137) reports a similar comparison made by Collings (141) on a nonlinear problem. He concludes that the Galerkin method is, again, superior but cautions that this conclusion may not hold in general.

Becker (142) listed some conditions that a variational method must satisfy and concludes that the least - squares method is

the best general criterion of the method of weighted residuals.

Finlayson and Scriven (139) summarizing all these findings come to the conclusion that the superiority of a method can only be determined on the basis of representative applications. However, in higher approximations the choice of the method is not crucial, since all methods give similar results (143).

5.2.2 Trial Functions

In the application of the method of weighted residuals the choice of a criterion is not too crucial, while the selection of the trial functions is very important. The power of the method lies in the choice of the trial functions, as known information concerning a specific problem can be incorporated into the approximating function. However, this selection is somewhat arbitrary as it is dependent on the user's intuition and experience. This is often regarded as a major disadvantage of the method (139).

Although no rigid rules can be fixed, some general guidelines are usually followed to generate the trial functions. Some of them are listed below:

- a) The trial functions must be linearly independent and members of a complete set.
- b) Special features of the problem, as symmetry, periodicity, monotonicity, should be incorporated in the trial solution.
- c) Whenever possible the trial solution should satisfy the governing equations and the boundary conditions, independently of the undetermined parameters α_k .
- d) The form of the estimates should be kept simple, not unduly complicating the analysis. Too rigid restrictions should be

avoided even if a minor characteristic of the solution has to be lost.

When the trial solution cannot satisfy some of the boundary conditions, separate residual functions have to be constructed. In such cases the problem can be solved by considering a total weighted residual function, which is the sum of the weighted residual function in the domain D and on the boundary S multiplied by some weighting factors, G_D and G_B respectively:

$$G_D \int W_k R_D dV + G_B \int W_k R_B dS = 0 \quad (5.10)$$

where R_D is the residual corresponding to the governing equation and R_B the residual corresponding to the boundary conditions.

In this way it is ensured that the boundary error functions are taken into account in the approximate solution.

In discrete calculations, a basis for the determination of the magnitude of the weighting factors, usually adopted, is as follows:

If N_D and N_B are the number of nodal points in the domain D and on the boundary S, respectively, the weighting factors affecting the residual on the boundary is given by:

$$G_B = \frac{N_D}{N_B} \quad (5.11)$$

Another aspect that can influence the selection of the trial functions is that they should ease the computation. The choice of polynomials is often made as they are easy to handle.

5.3 Application of the Weighted Residual Method to Plasticity

The method of least-squares is applied by Steck (65) to obtain the velocity and stress fields in the deformation zone of axially symmetrical forward extrusion and upsetting. For this purpose the domain is discretised into a number of nodal points and a reference system of cylindrical co-ordinates is utilised.

A stream function ψ and two stress functions ϕ_1 and ϕ_2 are proposed as follows:

$$\psi = \psi_0(r, z) + \psi_1(r, z) \sum_i \sum_j A_{ij} r^{f_0(i)} z^{g_0(j)} \quad (5.12)$$

$$\phi_1 = \phi_0(r, z) + \sum_k \sum_l B_{kl} r^{f_1(k)} z^{g_1(l)} \quad (5.13)$$

$$\phi_2 = \sum_m \sum_n C_{mn} r^{f_2(m)} z^{g_2(n)} \quad (5.14)$$

where A_{ij} , B_{kl} and C_{mn} are unknown and ψ_0 , ψ_1 and ϕ_0 are functions chosen to satisfy some boundary conditions.

From the stream function the velocity field can be derived. The radial, v_r and axial, v_z components of the velocity are expressed by:

$$v_r = \frac{1}{r} \frac{\partial \psi}{\partial z} \quad (5.15.a)$$

$$v_z = -\frac{1}{r} \frac{\partial \psi}{\partial r} \quad (5.15.b)$$

Using expression (3.13) the strain rate field is determined as a function of the parameters A_{ij} :

$$\dot{\epsilon}_{st} = \dot{\epsilon}_{st}(r, z, A_{ij}) \quad (5.16)$$

$$\frac{\dot{\epsilon}}{\epsilon} = \frac{\dot{\epsilon}}{\epsilon}(r, z, A_{ij}) \quad (5.17)$$

Substituting the expressions of the stress functions in equations (3.11) the stress field is obtained as a function of the free parameters B_{kl} and C_{mn} :

$$\sigma_{st} = \sigma_{st}(r, z, B_{kl}, C_{mn}) \quad (5.18)$$

$$\sigma_m = \sigma_m(r, z, B_{kl}, C_{mn}) \quad (5.19)$$

The components of the deviatoric stress tensor are expressed by the following equations:

$$\sigma'_{st} = \sigma_{st}(r, z, B_{kl}, C_{mn}) - \delta_{st} \sigma_m(r, z, B_{kl}, C_{mn}) \quad (5.20)$$

Introducing equations (5.20) into the Prandtl-Reuss equations (3.35) four error functions R_{1P} , R_{2P} , R_{3P} and R_{4P} , corresponding to σ_r , σ_θ , σ_z and τ_{rz} respectively, are obtained at every nodal point P in the discretised domain:

$$\begin{aligned} R_{qp} = & \sigma_{st}(r, z, B_{kl}, C_{mn}) - \delta_{st} \sigma_m(r, z, B_{kl}, C_{mn}) \\ & - \frac{2}{3} \frac{\bar{\sigma}}{\bar{\epsilon}} \dot{\epsilon}_{st}(r, z, A_{ij}) \end{aligned} \quad (5.21)$$

with $q = 1 \dots 4$

If these errors are squared and added up for all the nodal points of the deformation zone an error function in terms of the free parameters is found:

$$R_q(A_{ij}, B_{kl}, C_{mn}) = \sum_p (R_{qp})^2 \quad (5.22)$$

As the work material is assumed to be incompressible only two of the three error functions corresponding to σ_r , σ_θ , σ_z , need to be taken into account.

A similar procedure is carried out for the boundary conditions which are not satisfied in the estimates.

The total error is obtained by adding up the individual error functions multiplied by the weighting factors G_q .

$$R_T = \sum_q R_q G_q \quad (5.23)$$

By minimizing the total error with respect to the free parameters the following system of algebraic equations is obtained:

$$\begin{aligned} \frac{\partial R_T}{\partial A_{ij}} &= 0 \\ \frac{\partial R_T}{\partial B_{kl}} &= 0 \\ \frac{\partial R_T}{\partial C_{mn}} &= 0 \end{aligned} \quad (5.24)$$

and the values of A_{ij} , B_{kl} and C_{mn} can be determined.

As the flow rule is non-linear in the strain rates an iterative procedure is employed to find the velocity and stress fluids.

For this purpose Steck writes the flow rule as follows:

$$\sigma'_{st} = \frac{k}{\sqrt{I_2}} \dot{\epsilon}_{st} \quad (5.25)$$

where k is the shear yield stress and I_2 is the second invariant of the strain rate tensor.

As a first step of iteration I_2 is assumed to be equal to unity, i.e. the system is equivalent to a viscous fluid with viscosity $\eta = \frac{k}{2}$. In the second step of iteration the strain rate determined in the first step is used to calculate a new I_2 and a new stress field.

$$\sigma'_{st}{}^{(2)} = \frac{k}{\sqrt{I_2}^{(1)}} \dot{\epsilon}_{st}^{(1)} \quad (5.26)$$

In the third step the strain field is corrected from the stress field obtained in step 2, according to:

$$\dot{\epsilon}_{st}^{(2)} = \frac{\sqrt{I_2}^{(1)}}{k} \sigma'_{st}{}^{(2)} \quad (5.27)$$

The calculations are carried out by executing the operation described in step 2. The cycle is completed when the errors in the yield criterion and in the flow rule are negligible and no changes occur in the strain rate and stress fields from one step to another.

Ravindranath (67), applying this iteration procedure to the upsetting of rings, has shown that this formulation of the problem is not satisfactory. His analysis reveals that two sets of approximating functions were proposed for the same quantity and the unknown parameters were determined by minimizing the difference between these two sets of approximating functions. He states that "mathematically there is no reason why any of these solutions should be considered unique unless a criterion with a physical meaning is established".

To overcome this difficulty the strain rate field can be derived from a predetermined velocity field.

The method of weighted residuals has been applied to obtain

stress field in the plastic deformation zone of the upsetting of rings (67) and the hydrostatic extrusion of bimetallic composites (66) utilizing the respective velocity fields proposed by Avitzur (90).

5.4 Stress Analysis of the Hydrostatic Extrusion - Drawing of Tubes by the Method of Weighted Residuals

To determine the stress field in the plastically deformed zone during the hydrostatic extrusion-drawing of tubes, the strain rate field, obtained by the lower upper bound approach described in Chapter 4, is used in conjunction with the method of least squares.

The analysis is carried out for the aluminium and copper specimens used in the experiments.

The following assumptions are made:

- a) The materials are isotropic
- b) The materials are incompressible
- c) The materials stress-strain curves are represented by equations (4.34).
- d) The elastic strains are small in comparison with the plastic strains and can be neglected.
- e) The frictional stress at the tool/billet interfaces is assumed to be proportional to the shear yield stress of the material
- f) The process is isothermal

5.4.1 Discretisation

The discretisation of the plastic deformation zone into a number of nodal points, Figure 5.1, presented in Chapter 4 is used in the following sections.

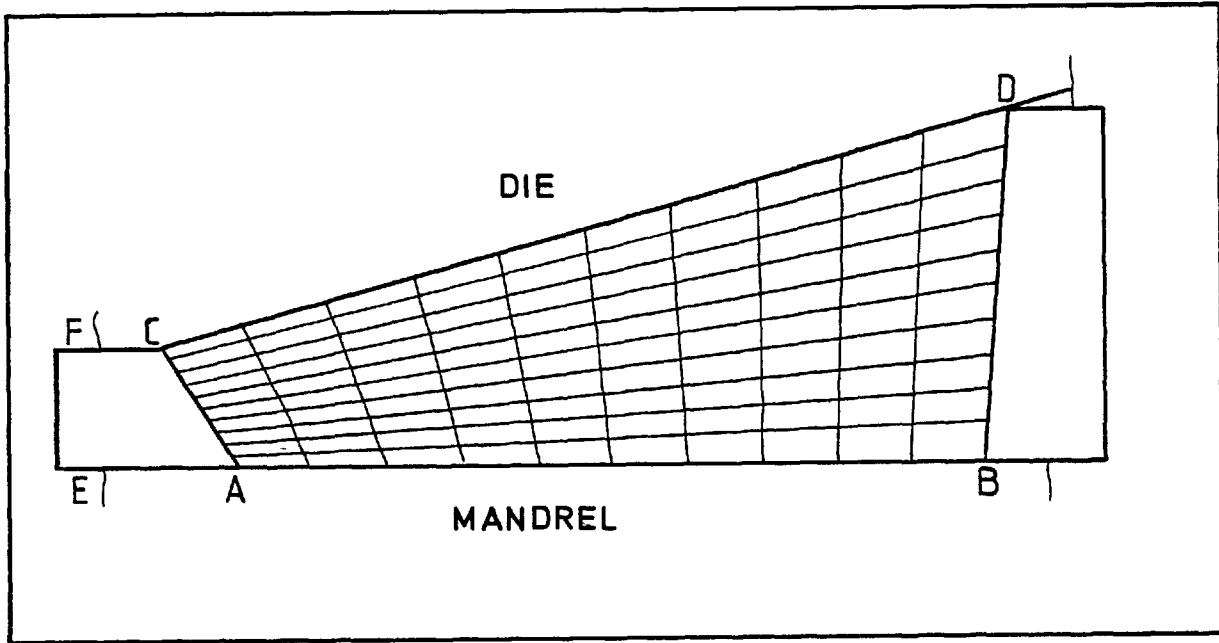


FIG. 5.1 - MESH FOR TUBE EXTRUSION. $R = 3.5$

Every nodal point is associated with the strain rates, effective strain rate and effective stress determined previously.

5.4.2 Governing Equations

The Prandtl Reuss equations (3.35) in a system of cylindrical co-ordinates can be written as:

$$\sigma_r = \sigma_m + \frac{2}{3} \frac{|\dot{\epsilon}|}{\epsilon} \dot{\epsilon}_r \quad (5.28.a)$$

$$\sigma_\theta = \sigma_m + \frac{2}{3} \frac{|\dot{\epsilon}|}{\epsilon} \dot{\epsilon}_\theta \quad (5.28.b)$$

$$\sigma_z = \sigma_m + \frac{2}{3} \frac{|\dot{\epsilon}|}{\epsilon} \dot{\epsilon}_z \quad (5.28.c)$$

$$\tau_{rz} = \frac{2}{3} \frac{|\dot{\epsilon}|}{\epsilon} \dot{\epsilon}_{rz} \quad (5.28.d)$$

where

$$\sigma_m = \frac{\sigma_r + \sigma_\theta + \sigma_z}{3}$$

Equations (5.28) represent the governing equations of the problem in study.

5.4.3 Boundary Conditions

The following boundary conditions are assumed:

- a) The frictional stress at the plastic deformation zone/mandrel interface is expressed as

$$\tau_{rz} = m \frac{\bar{\sigma}}{\sqrt{3}} \quad (5.29)$$

- b) At the deformation zone/die interface the boundary condition due to the frictional stress is written as

$$\tau = m \frac{\bar{\sigma}}{\sqrt{3}} \quad (5.30)$$

Expressing τ in terms of σ_r , σ_z , τ_{rz} and the semi cone angle of the die, α , equation (5.30) becomes

$$\frac{\sigma_r - \sigma_z}{2} \sin 2\alpha + \tau_{rz} \cos 2\alpha = m \frac{\bar{\sigma}}{\sqrt{3}} \quad (5.31)$$

- c) At the entry boundary the following approximation is made

$$\sigma_z = p \quad (5.32)$$

where p is the extrusion pressure

d) Similarly, at the exit boundary

$$\sigma_z = \sigma_d \quad (5.33)$$

where σ_d is the average axial stress applied to the exit boundary.

It can be calculated by

$$\sigma_d = \frac{F - F_d - F_{fe}}{A_p} \quad (5.34)$$

where F is the drawing force

F_d is the friction force along the die land ($\overline{CF}$ in figure 5.1)

F_{fe} is the friction force at the product/mandrel interface
($\overline{AE}$ in figure 5.1)

A_p is the cross sectional area of the product

F_d and F_{fe} were determined for each case by the lower upper bound approach.

5.4.4 Error Functions

As the strain rate field is already determined, the estimates referred in section 5.3 are reduced to the two stress functions ϕ_1 and ϕ_2 . The stresses derived from these functions through equations (3.11) satisfy the equilibrium equations (3.10). The stress functions are chosen to incorporate the following symmetry conditions:

- a) The radial stress σ_r , the axial stress σ_z and the hoop stress σ_θ are symmetrical across the z-axis.
- b) The shear stress τ_{rz} is antisymmetrical across the z-axis.

The estimates for the stress functions are as follows:

$$\phi_1 = \sum_{k=1}^{MR} \sum_{l=1}^{MZ} B_{kl} r^{2k+3} z^l \quad (5.35)$$

$$\phi_2 = \sum_{m=1}^{NR} \sum_{n=1}^{NZ} C_{mn} r^{2m+1} z^n \quad (5.36)$$

Applying equations (3.11) the trial functions for the stresses are obtained

$$\begin{aligned} \sigma_r &= \sum_k \sum_l 1 \cdot (1 - 1) B_{kl} r^{2k+2} z^{l-2} \\ &+ \sum_m \sum_n C_{mn} r^{2m} z^n \end{aligned} \quad (5.37)$$

$$\sigma_\theta = \sum_m \sum_n (2m + 1) C_{mn} r^{2m} z^n \quad (5.38)$$

$$\sigma_z = \sum_k \sum_l (2k + 3) (2k + 2) B_{kl} r^{2k} z^l \quad (5.39)$$

$$\tau_{rz} = - \sum_k \sum_l (2k + 3) \cdot 1 \cdot B_{kl} r^{2k+1} z^{l-1} \quad (5.40)$$

The trial functions are polynomials in the variables of the system of co-ordinates satisfying, therefore, the requirement of completeness. The power of the variable r is chosen to be odd in both stress functions, so that the axisymmetry conditions for the stresses are included in the trial solution.

None of the boundary conditions are incorporated in the trial functions so that separate error functions have to be constructed to take them into account. Three main reasons can be mentioned for not including the boundary conditions in the trial functions:

1. They are only approximations and their inclusion in the trial functions would represent a strong restriction.
2. The trial solution would become much more complicated.
3. The trial solution would lose its generality, since for different shapes of the entry and exit boundaries the estimates would not apply.

5.4.5 The Computer Program

To find the stress field in the plastically deformed zone of the hydrostatic extrusion of tubes a computer program, called STRESS, was produced.

The data file, generated by program MIN, which includes the co-ordinates of the mesh points, the strain rate field and the equivalent stress at all nodal points, as well as the values of the upper bound pressure, the friction factor and the axial stress at the exit boundary is transferred into program STRESS by a READ statement. The program also reads from another data file the number of terms of the stress functions and the number of error functions.

To develop the program it is convenient to express the error functions R_{qp} at a nodal point p in the following form:

$$R_{qp} = \sum_{i=1}^{NT} A_i Y_{qi}(r,z) + C_q \quad (5.41)$$

where

$$NT = MR \times MZ + NR \times NZ$$

A_i replaces the free parameter B_{kl} for $i = 1, \dots, MR \times MZ$
and the free parameters C_{mn} for $i = MR \times MZ + 1, \dots, NT$

and Y_{qi} represents the trial functions

The total error is written as

$$R_T = \sum_{p=1}^N \sum_{q=1}^{NE} G_q \left[\sum_{i=1}^{NT} A_i Y_{qi} + C_q \right]^2 \quad (5.42)$$

and minimizing expression (5.42) with respect to the free parameter A_m leads to

$$\frac{\partial R_T}{\partial A_m} = 0 \quad (m = 1, \dots, NT) \quad (5.43)$$

and the following system of linear equations is obtained

$$\sum_{p=1}^N \sum_{q=1}^{NE} G_q \left[\sum_{i=1}^{NT} A_i Y_{qi} Y_{qm} + C_q Y_{qm} \right] = 0 \quad (5.44)$$

$$(m = 1, \dots, NT)$$

This system of equations, written in a matrix form, more suitable for computation, is expressed as

$$\begin{bmatrix} \sum_{p=1}^N \sum_{q=1}^{NE} G_q Y_{q1} Y_{q1} & \sum_{p=1}^N \sum_{q=1}^{NE} G_q Y_{q2} Y_{q1} & \dots & \sum_{p=1}^N \sum_{q=1}^{NE} G_q Y_{qNT} Y_{q1} \\ \sum_{p=1}^N \sum_{q=1}^{NE} G_q Y_{q2} Y_{q2} & \dots & \sum_{p=1}^N \sum_{q=1}^{NE} G_q Y_{qNT} Y_{q2} \\ \text{SYMMETRIC} & & & \\ \sum_{p=1}^N \sum_{q=1}^{NE} G_q Y_{qNT} Y_{qNT} & & & \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_{NT} \end{bmatrix} = - \begin{bmatrix} \sum_p \sum_q G_q C_q Y_{q1} \\ \sum_p \sum_q G_q C_q Y_{q2} \\ \vdots \\ \sum_p \sum_q G_q C_q Y_{qNT} \end{bmatrix}$$

The free parameters A_i are found by solving that system of equations by the subroutine SIMQ, which is a standard of the IBM System Subroutine.

By introducing the value of the free parameters, into equations (5.37 - 5.40) the stress field in the plastic deformation zone is found.

Finally the program generates a data file, where the upper bound pressure, the normal and shear stress distributions along the deformation

zone/mandrel interface and the parameters which define the position of that interface are stored. This data file will be used, later, as input data for the program GDME to be discussed in the next chapter.

The complete sequence of program STRESS is summarized in the form of a flow chart in Figure 5.2. A listing of the same program together with a variable name list is included in Appendix D.

5.4.6 Stress Field

The stress fields were calculated by the method of weighted residuals using the velocity field determined by the lower upper bound approach described previously.

The stress fields corresponding to the extrusion ratio of 3.5 and 2.65 for aluminium and 2.0 for copper are illustrated in Figures 5.3 to 5.8. For each of these extrusion ratios two cases are presented for different pulling forces. The values of the pressure indicated in the illustrations are those predicted by the lower upper bound approach.

As adopted for the equivalent stress distribution the stress distributions are represented by the ratio

$$\frac{\text{Stress Value}}{\text{Initial yield stress of the material}}$$

In the following discussion it is understood that the stresses are referred to their absolute value, unless the contrary is stated.

From the illustrations the effect of the drawing stress on the stress field is clear. When the pulling force, F , increases the extrusion pressure drops and all the stress field, with the exception of the shear stresses τ_{rz} , has clearly a lower value. In fact the values of τ_{rz} remain almost unchanged when two τ_{rz} distributions are compared for the same material and extrusion ratio but different

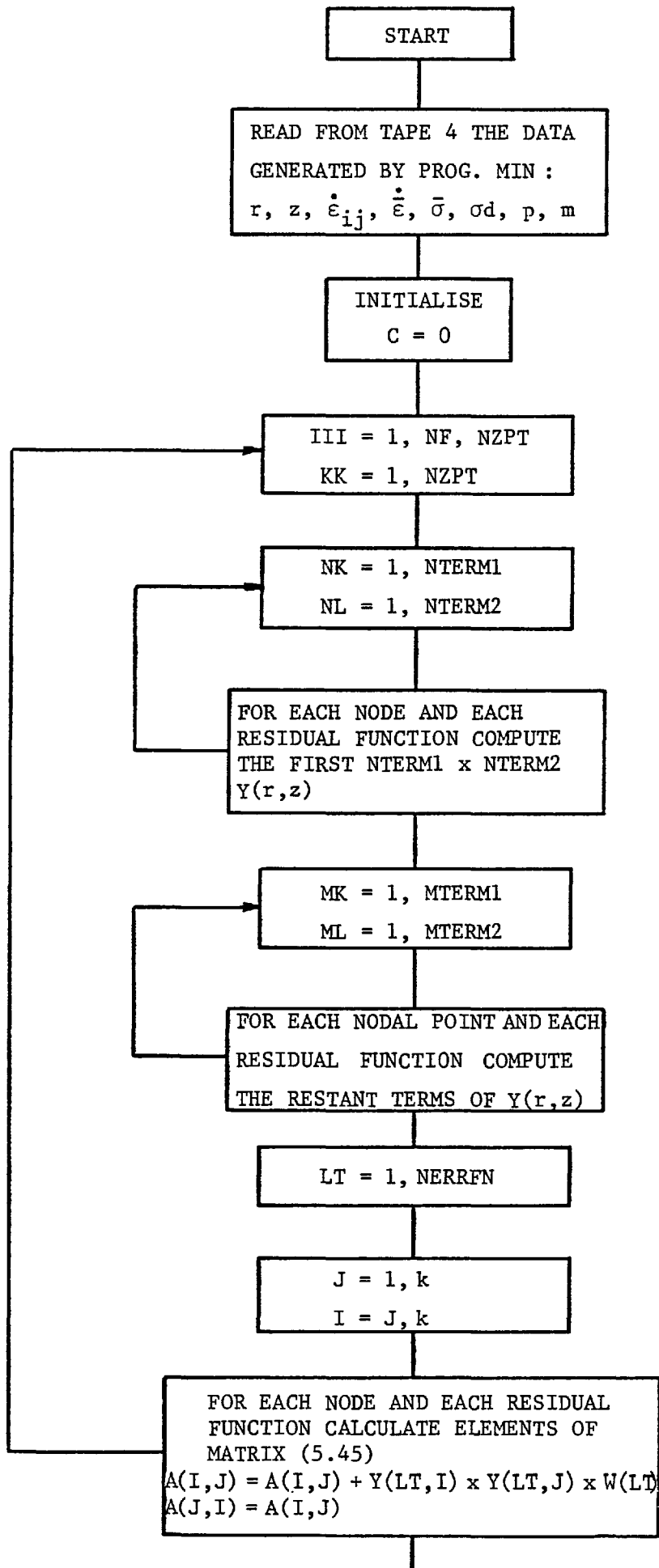


FIG. 5.2 - FLOW CHART OF PROGRAM STRESS

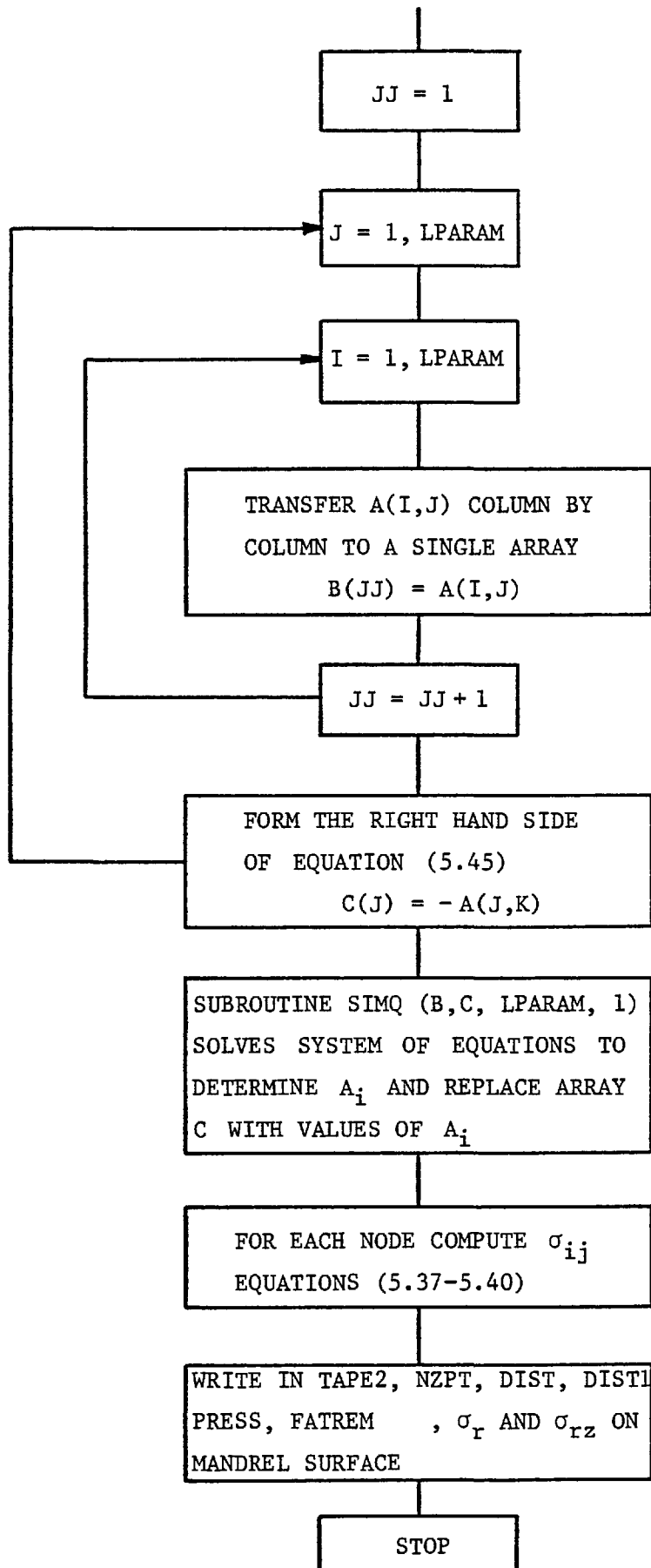


FIG. 5.2 (CONTD) - FLOW CHART OF PROGRAM STRESS

extrusion pressures. Although a variation on the friction force, F_f , along the billet/mandrel interface, was detected in the experiments (see Chapter 7) it does not alter significantly the shear stress distribution in the plastic zone. However along the mandrel/billet interface a small variation is noticed. For example, the curves in which the ratio $\frac{\tau_{rz}}{Y}$ is equal to -0.1 are more extended in Figure (5.3) than in Figure (5.4). The former corresponds to a larger friction force as expected.

Another aspect related to the shear stress distribution along the mandrel is that the weighted residual method gives the highest value at approximately half way along the plastic zone/mandrel interface, while the assumption of a constant friction factor leads to a maximum value at the exit, as the equivalent stress is at a maximum there.

As expected the shear stress changes sign between the mandrel and the die. The curve $\frac{\tau_{rz}}{Y} = 0$ is nearer the mandrel than the die.

The radial stress illustrations show in all cases a maximum value near the middle of the plastic zone/mandrel interface. Along the die the radial stresses decrease from the entrance towards the exit.

The radial and shear stress distribution play an important role in the mandrel design, as it will be discussed in the next chapter.

The illustrations show that along the mandrel surface the pressure is much higher than the shear stresses and, therefore, it will be the main factor to consider in the mandrel design for hydrostatic extrusion.

The hoop stress figures show that the maximum value is near the corner where the billet contacts the die. As for the radial stress distribution, the hoop stress distribution has a maximum along the billet/mandrel interface at the same location. Along the die the

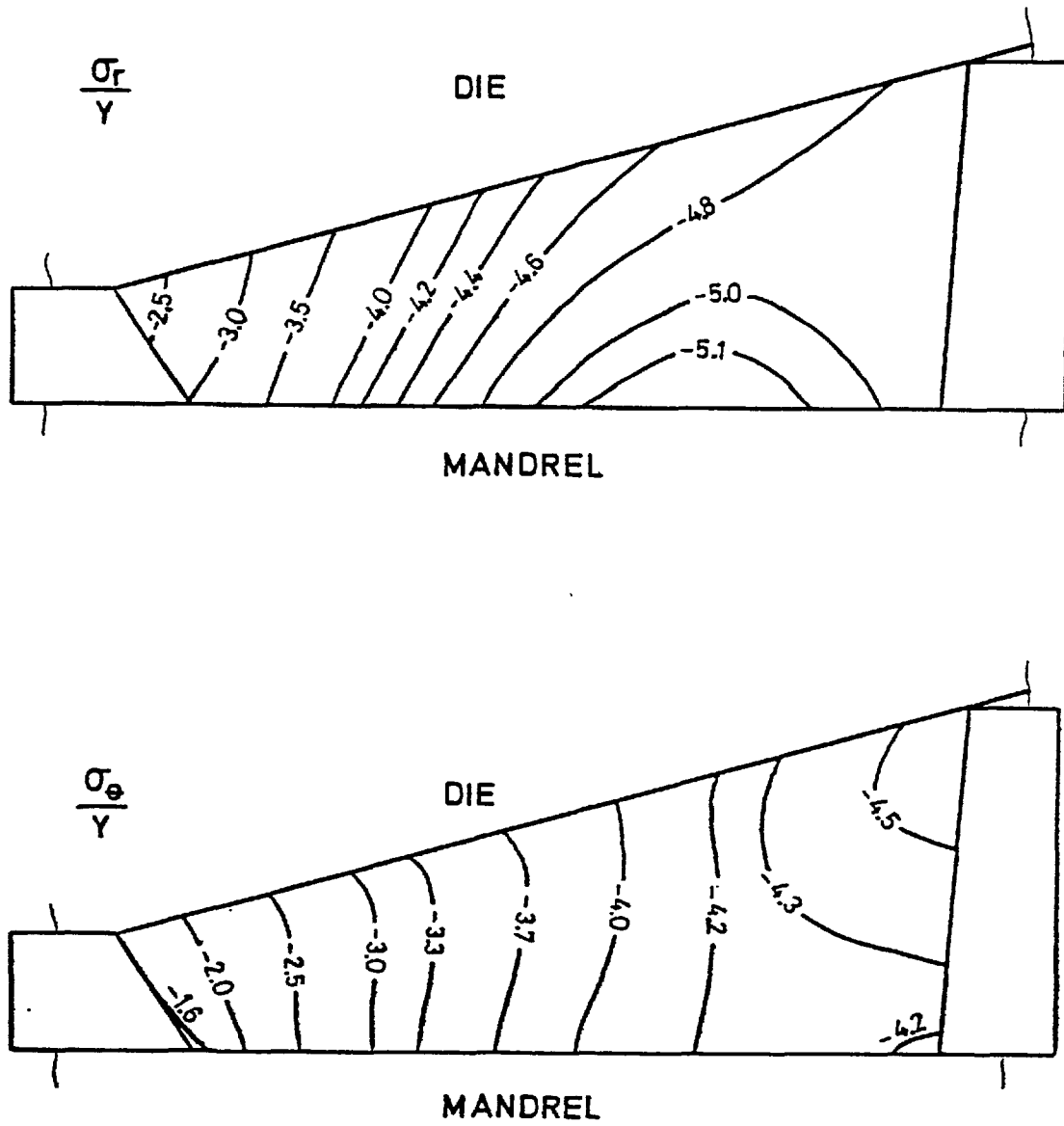


FIG. 5.3 - STRESS FIELD FOR ALUMINIUM TUBE EXTRUSION.

$$R = 3.5, \quad p = 192.8 \text{ MN/m}^2, \quad F = 3503 \text{ N}, \quad F_f = 4113 \text{ N}$$

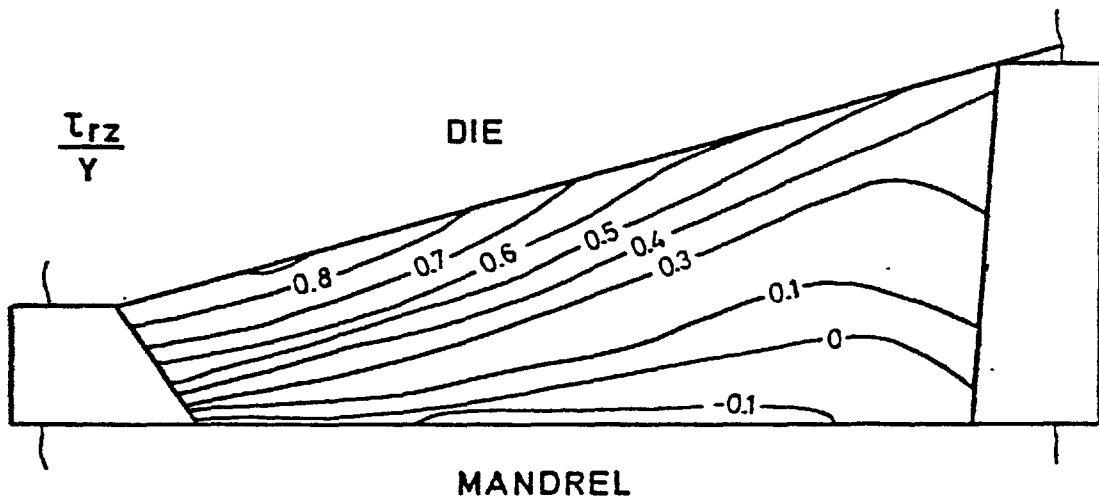
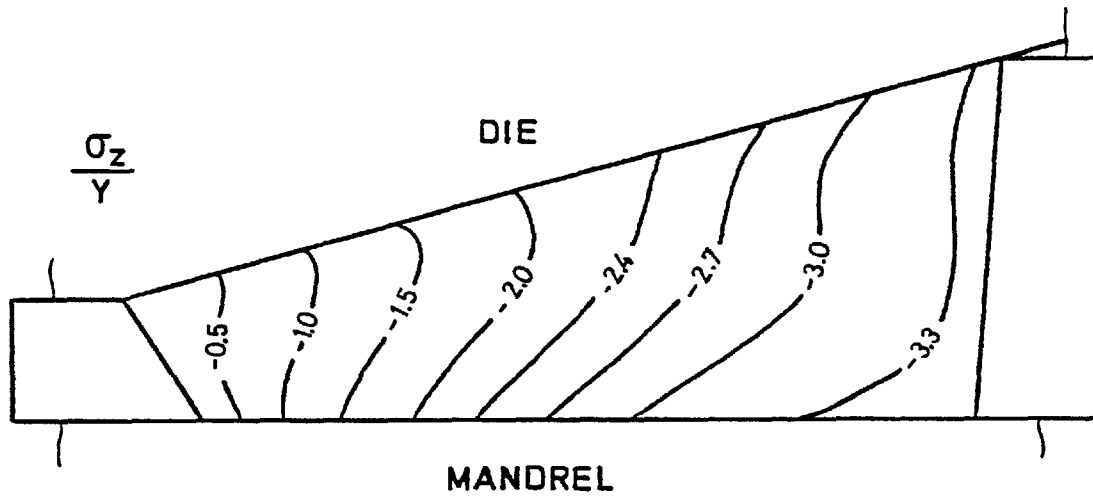


FIG. 5.3 (CONT.) - STRESS FIELD FOR ALUMINIUM TUBE EXTRUSION.

$$R = 3.5, \quad p = 192.8 \text{ MN/m}^2, \quad F = 3503 \text{ N}, \quad F_f = 4113 \text{ N}$$

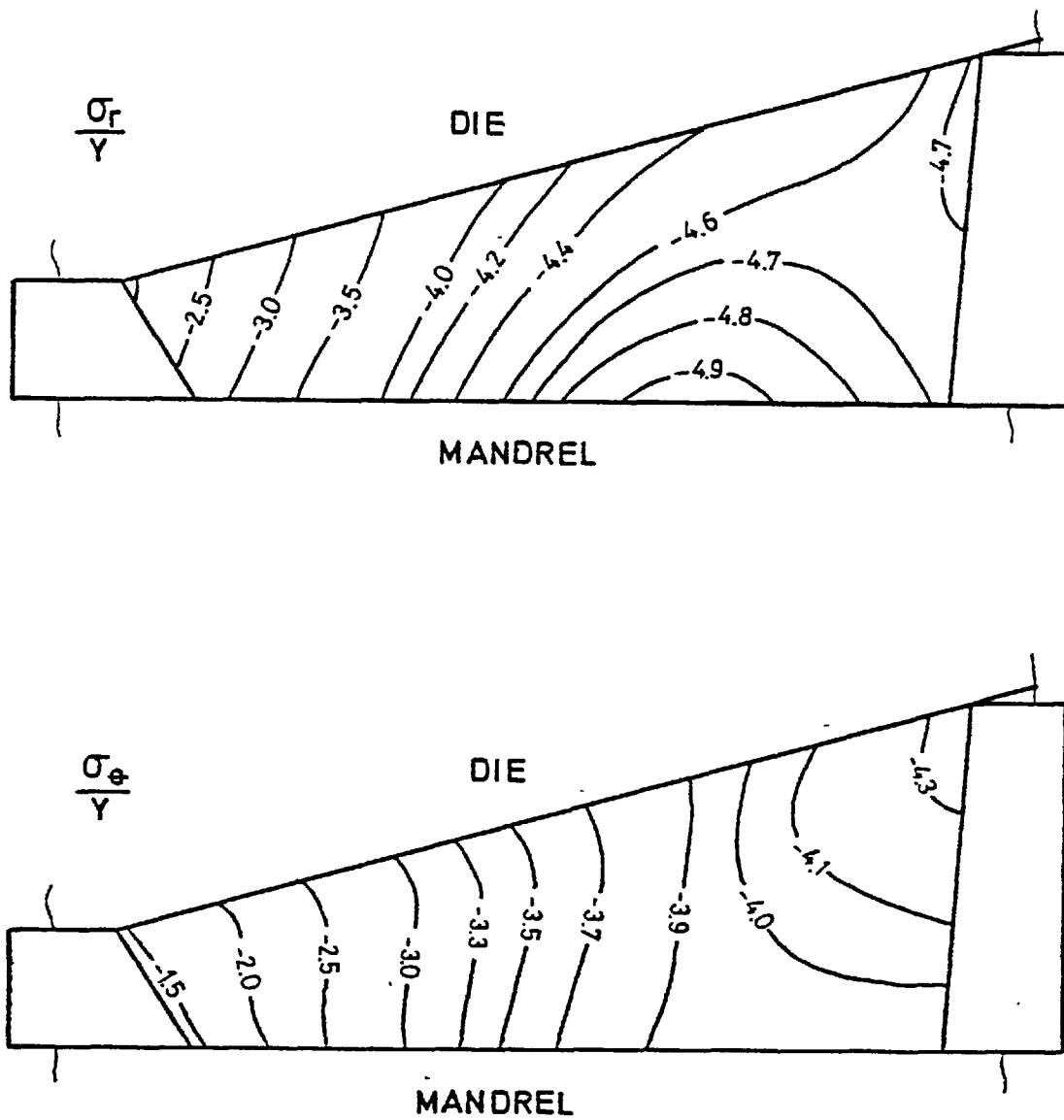


FIG. 5.4 - STRESS FIELD FOR ALUMINIUM TUBE EXTRUSION.

$$R = 3.5, \quad p = 178.4 \text{ MN/m}^2, \quad F = 6061 \text{ N}, \quad F_f = 3681 \text{ N}$$

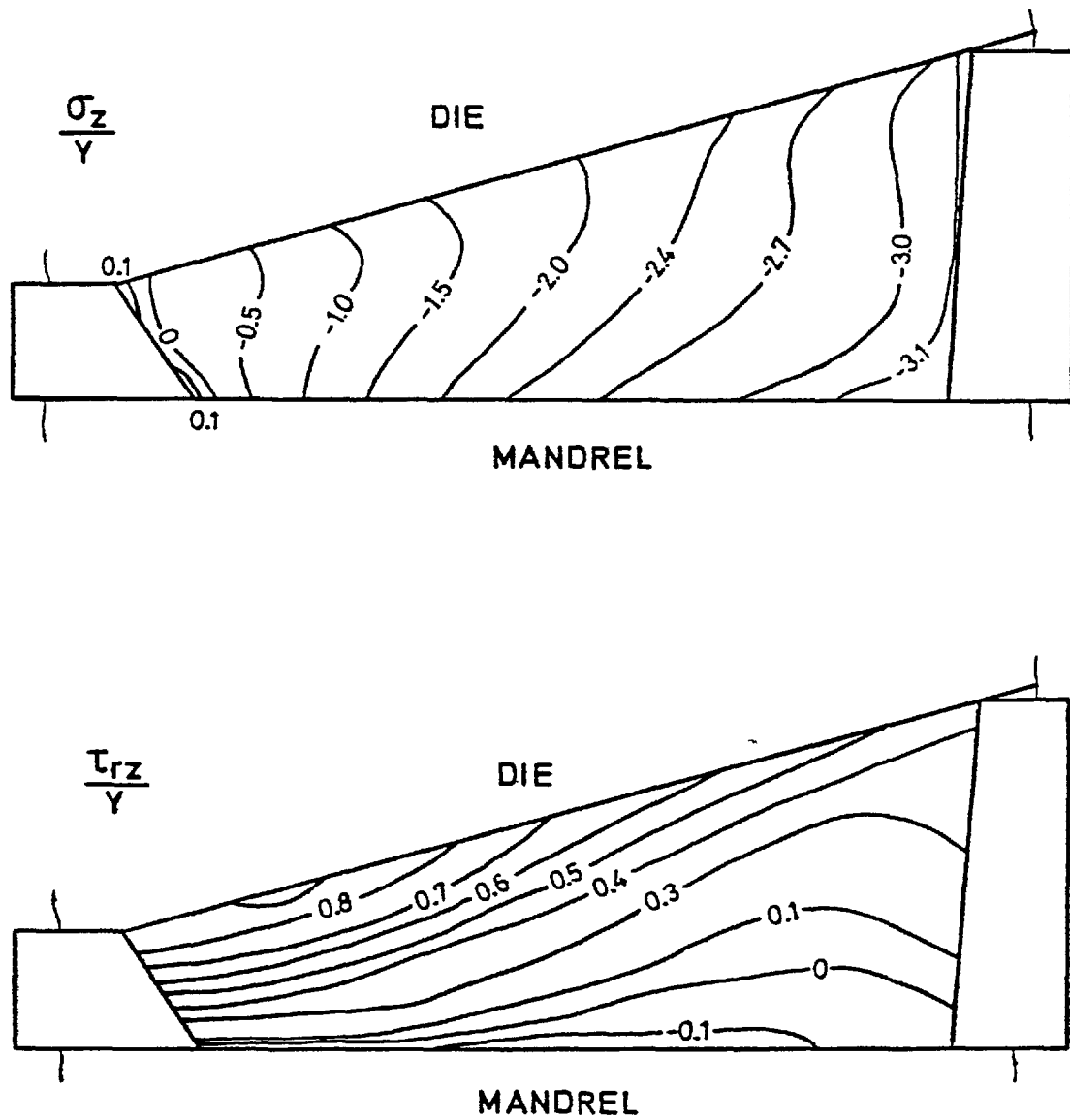


FIG. 5.4 (CONT.) - STRESS FIELD FOR ALUMINIUM TUBE EXTRUSION.

$R = 3.5$, $p = 178.4 \text{ MN/m}^2$, $F = 6061 \text{ N}$, $F_f = 3681 \text{ N}$

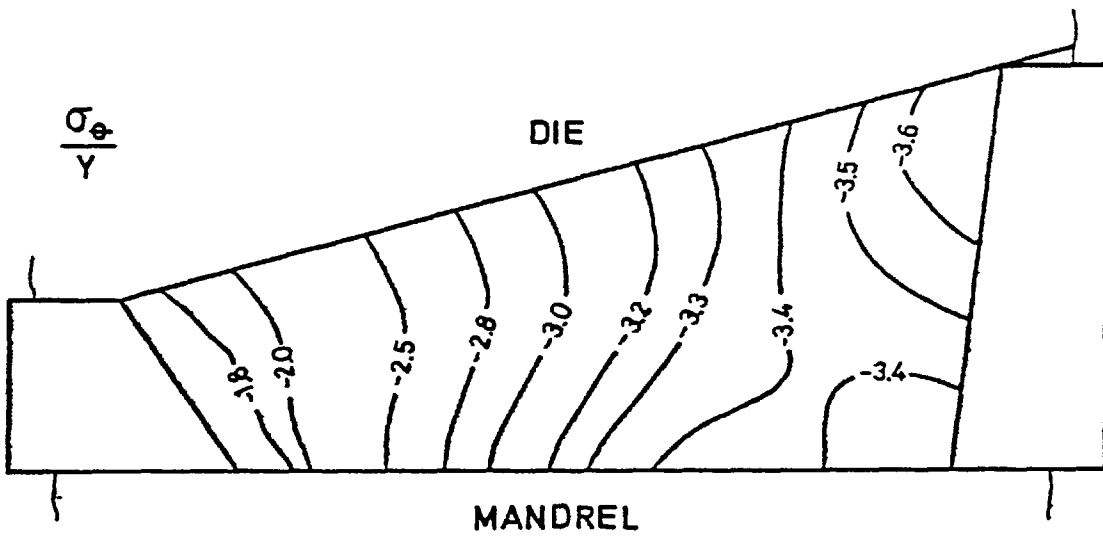
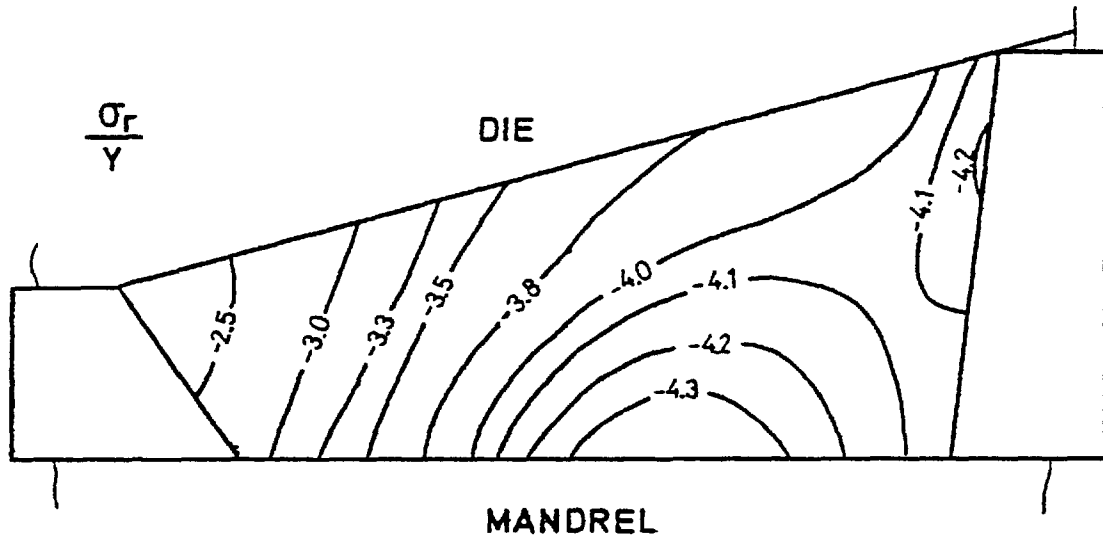


FIG. 5.5 - STRESS FIELD FOR ALUMINIUM TUBE EXTRUSION.

$$R = 2.65, \quad p = 143.6 \text{ MN/m}^2, \quad F = 3392 \text{ N}, \quad F_f = 3030 \text{ N}$$

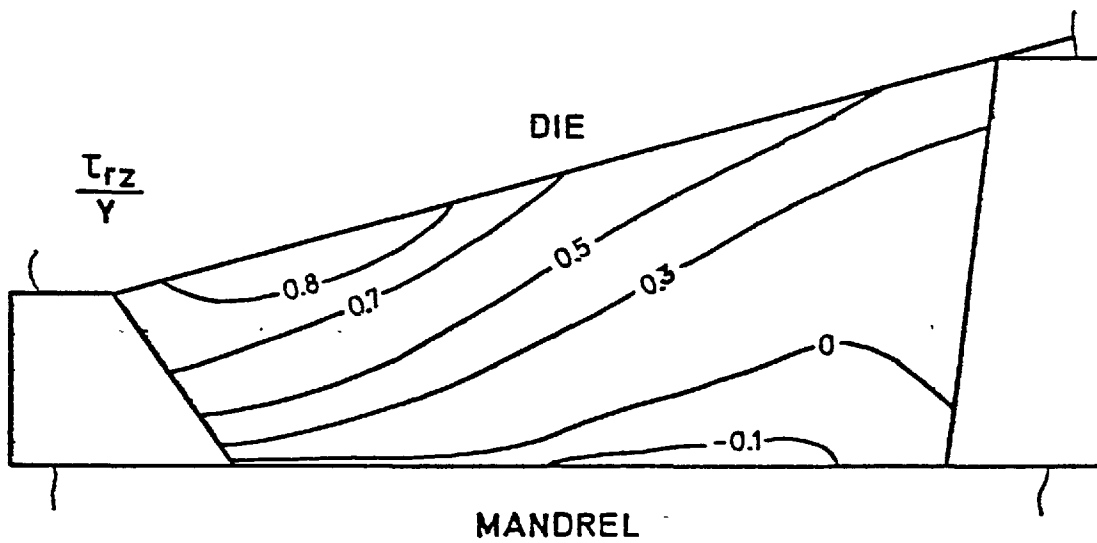
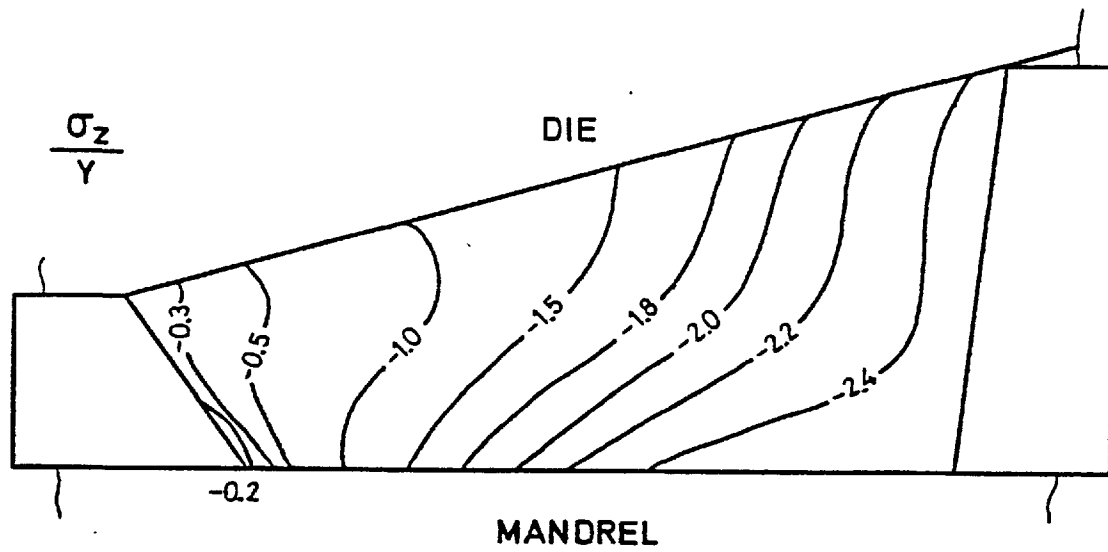


FIG. 5.5 (CONT.) - STRESS FIELD FOR ALUMINIUM TUBE EXTRUSION.

$R = 2.65$, $p = 143.6 \text{ MN/m}^2$, $F = 3392 \text{ N}$, $F_f = 3030 \text{ N}$

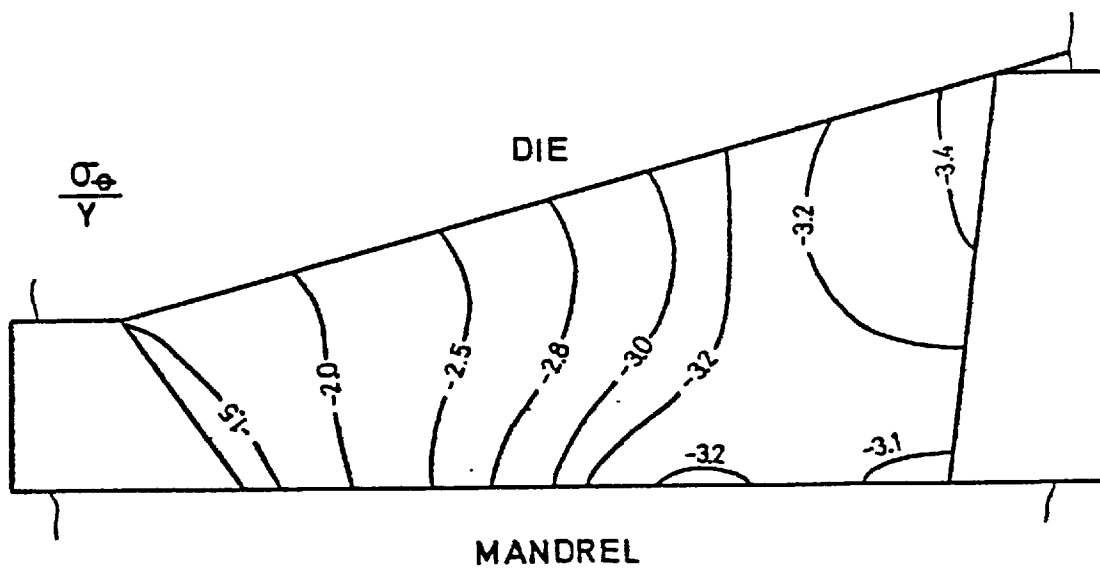
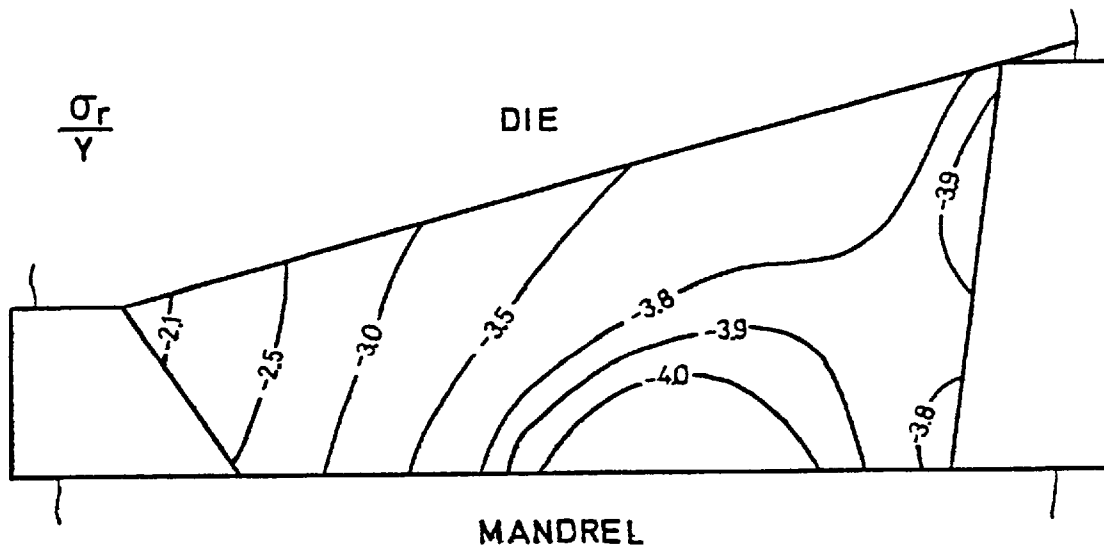


FIG. 5.6 - STRESS FIELD FOR ALUMINIUM TUBE EXTRUSION.

$$R = 2.65, \quad p = 129.4 \text{ MN/m}^2, \quad F = 6172 \text{ N}, \quad F_f = 2815 \text{ N}$$

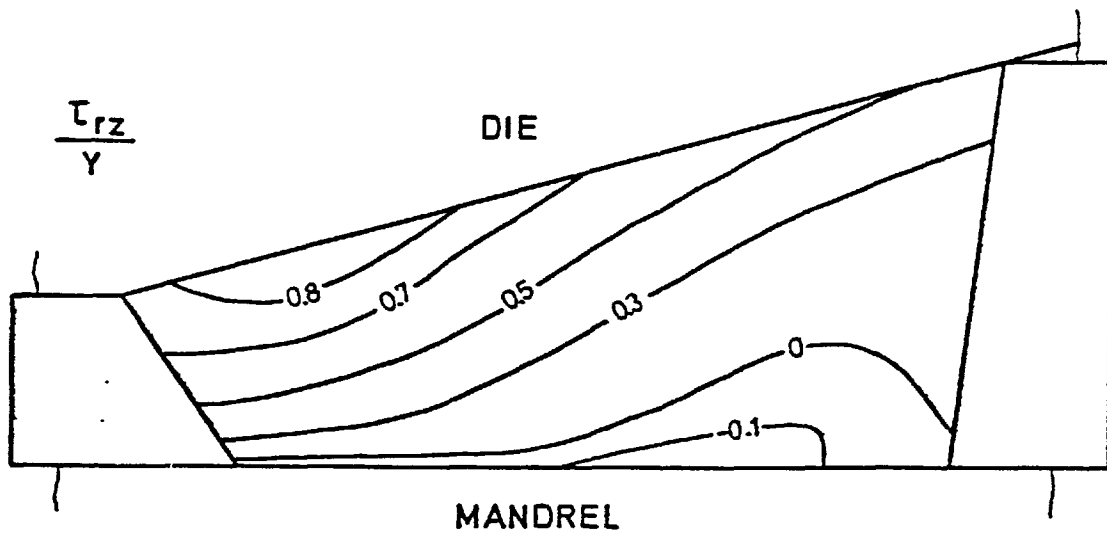
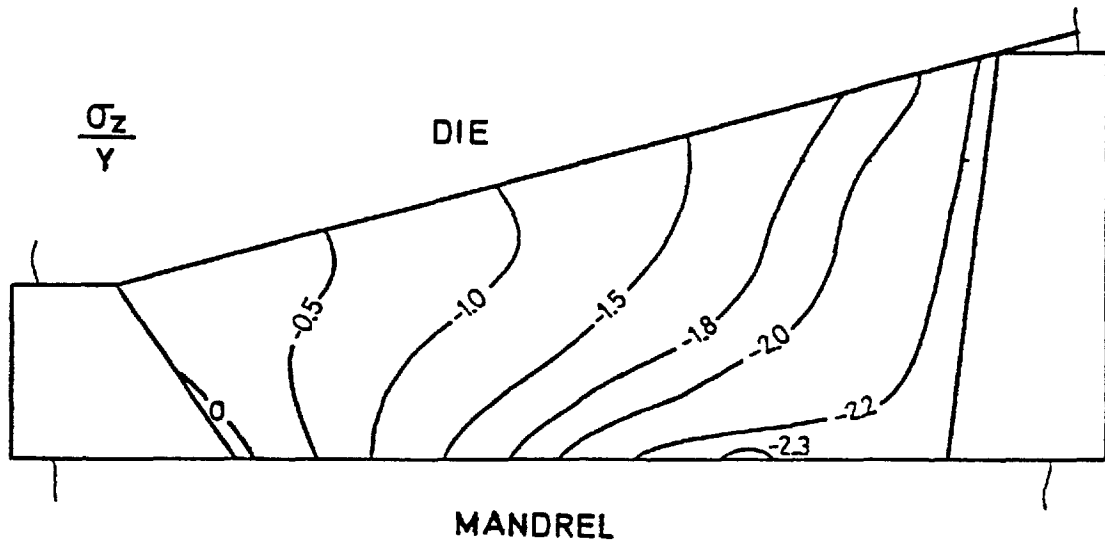


FIG. 5.6 (CONT.) - STRESS FIELD FOR ALUMINIUM TUBE EXTRUSION.

$R = 2.65$, $p = 129.4 \text{ MN/m}^2$, $F = 6172 \text{ N}$, $F_f = 2815 \text{ N}$

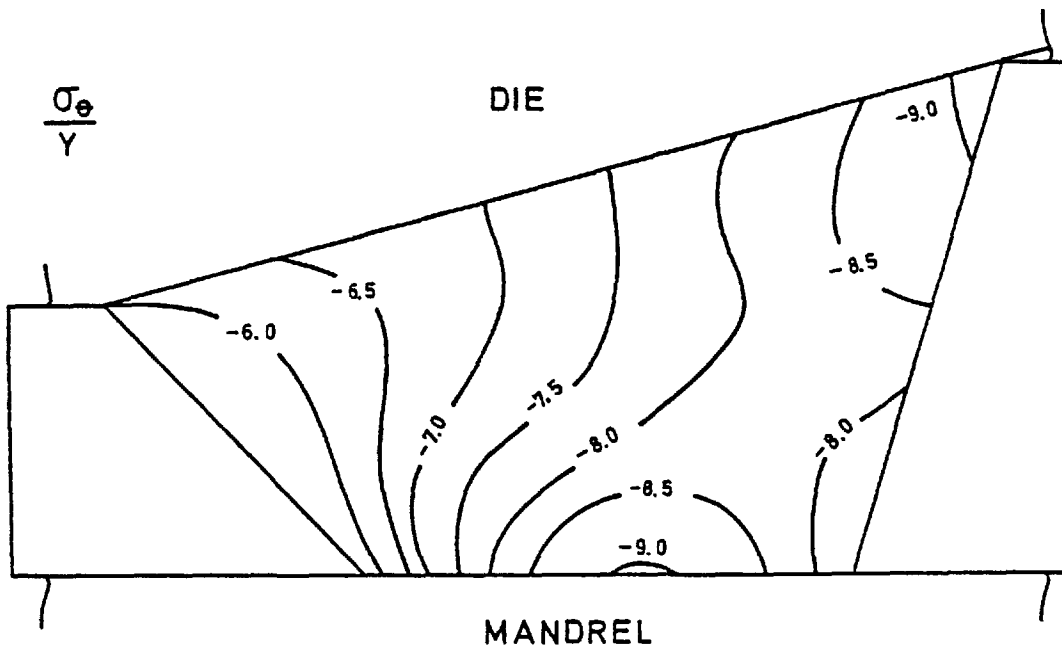
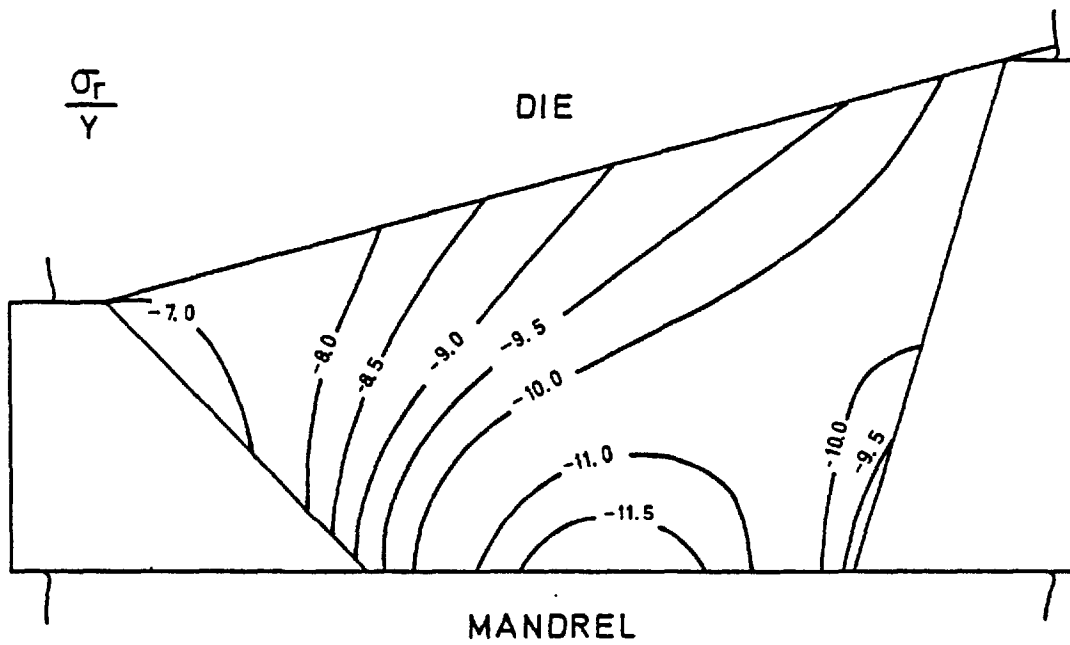


FIG. 5.7 - STRESS FIELD FOR COPPER TUBE EXTRUSION.

$R = 2.0$, $p = 287.7 \text{ MN/m}^2$, $F = 1501 \text{ N}$, $F_f = 3464 \text{ N}$

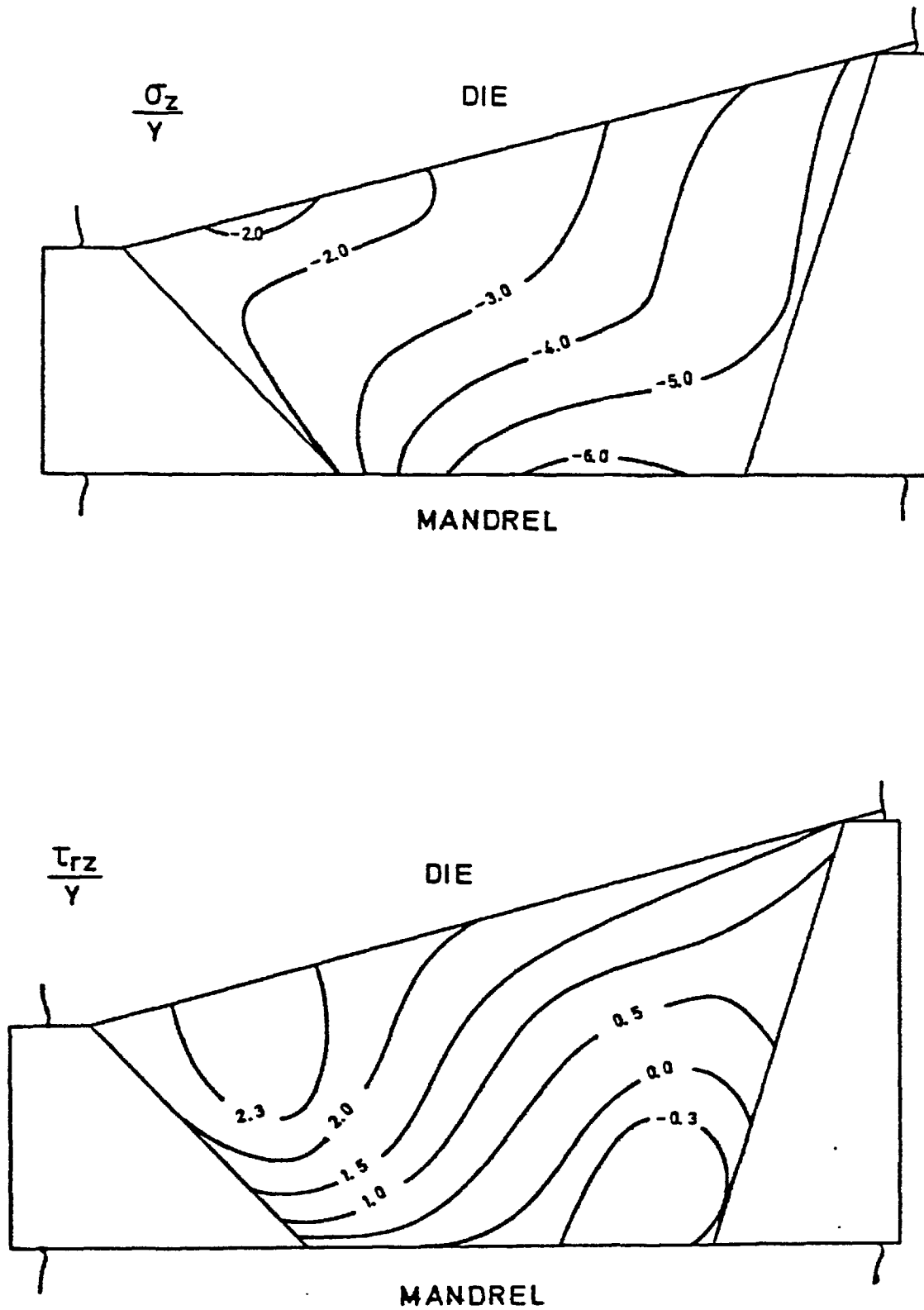


FIG. 5.7 (CONT.) - STRESS FIELD FOR COPPER TUBE EXTRUSION.

$$R = 2.0, \quad p = 287.7 \text{ MN/m}^2, \quad F = 1501 \text{ N}, \quad F_f = 3464 \text{ N}$$

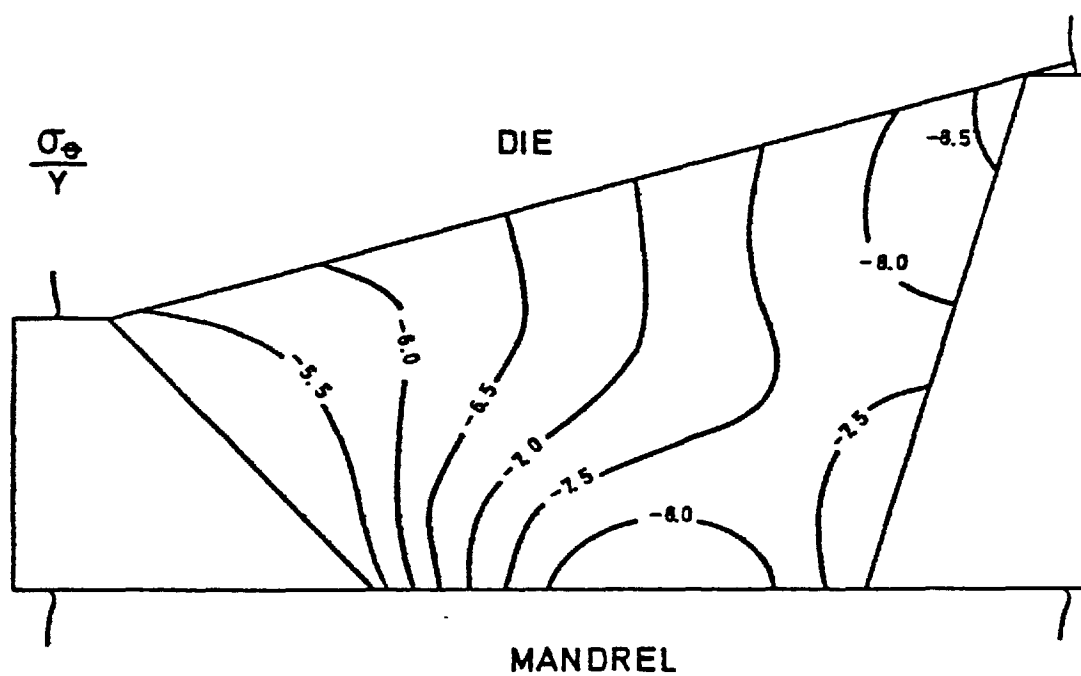
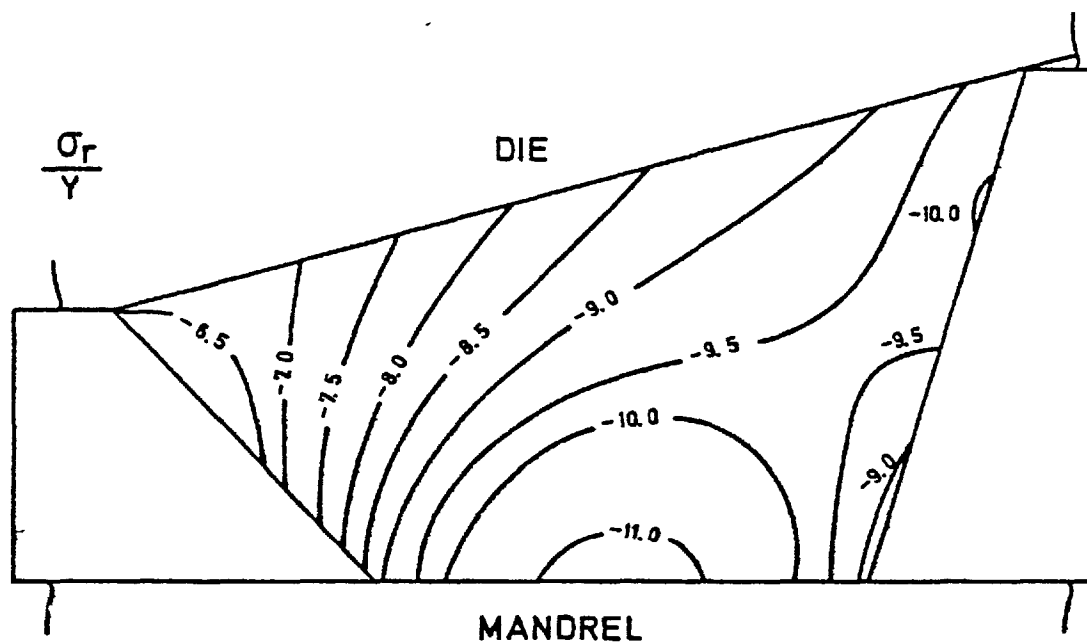


FIG. 5.8 - STRESS FIELD FOR COPPER TUBE EXTRUSION.

$R = 2.0$, $p = 263.7 \text{ MN/m}^2$, $F = 6394 \text{ N}$, $F_f = 3247 \text{ N}$

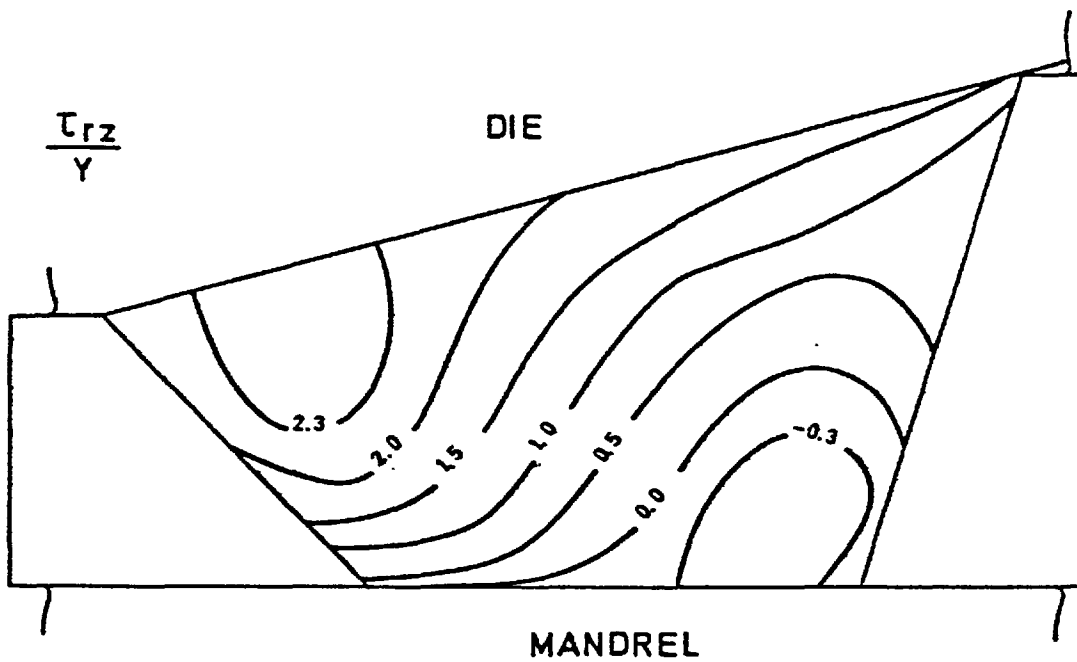
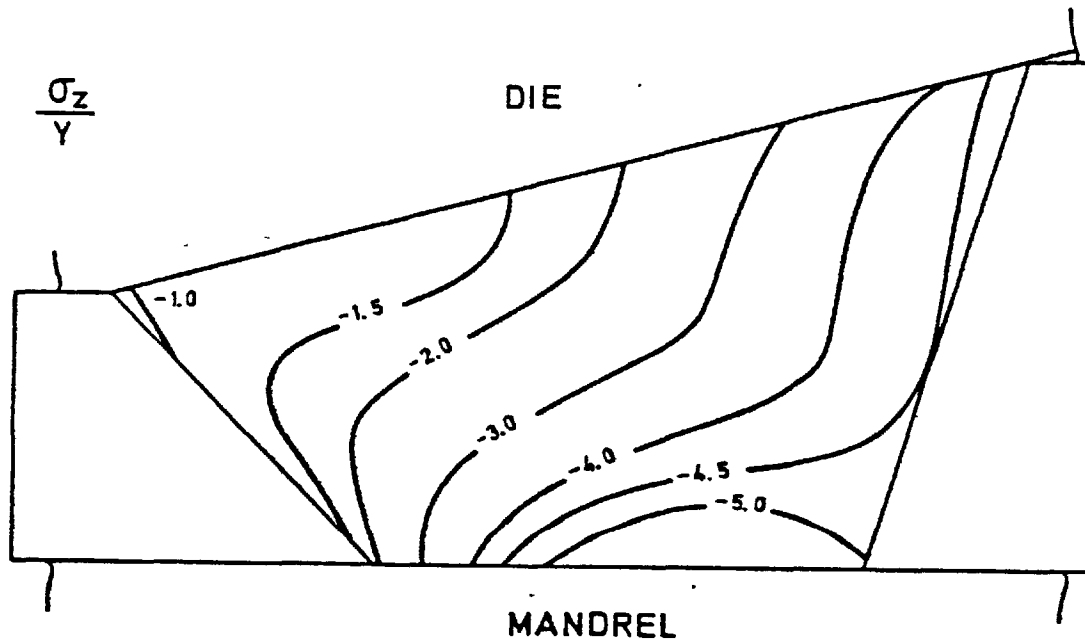


FIG. 5.8 (CONT.) - STRESS FIELD FOR COPPER TUBE EXTRUSION.

$R = 2.0$, $p = 263.7 \text{ MN/m}^2$, $F = 6394 \text{ N}$, $F_f = 3247 \text{ N}$

hoop stresses decrease from the entry to the exit boundaries.

The axial stress, σ_z , is the highest at the entrance and the lowest at the exit. Along the plastic/zone mandrel interface the axial stress has a maximum, more accentuated for the lower extrusion ratios.

The values of the axial stress at the entry boundary are in good agreement with the boundary conditions assumed. At the exit the boundary conditions are not so well satisfied. This is due to the fact that the weighting factor used for the exit boundary is low. It was found that at the points in the exit boundary, the value of the equivalent stress, calculated from the stress field, when the weighting factor was determined in accordance with expression (5.11), was much lower than the equivalent stress evaluated from the deviatoric stresses obtained from the upper bound solution. Because the boundary condition $\sigma_z = \sigma_d$ is only an approximation, it was decided to select a weighting factor which ensured that the difference between the equivalent stresses calculated by the two alternative methods was small.

The program STRESS enables also, the determination of the pressure distribution along the billet/die interface:

$$\sigma_{nd} = \left(\frac{\sigma_r + \sigma_z}{2} \right) - \frac{(\sigma_z - \sigma_r)}{2} \cos 2\alpha - \tau_{rz} \sin 2\alpha$$

where σ_{nd} represents the stress normal to the die surface.

Figures (5.9) and (5.10) show the pressure distribution along the die for the aluminium specimens extruded with an extrusion ratio of 3.5 and 2.65 respectively, while Figure (5.11) is related to the copper specimen with an extrusion ratio of 2.0. The points C and D in those Figures represent the die exit and entrance, as shown in Figure (5.1).

Figures (5.9) to (5.11) illustrate that the pressure is the highest at the die entrance and the lowest at the die exit. The pressure drops rather quickly near the die entrance, reaches a zone where the

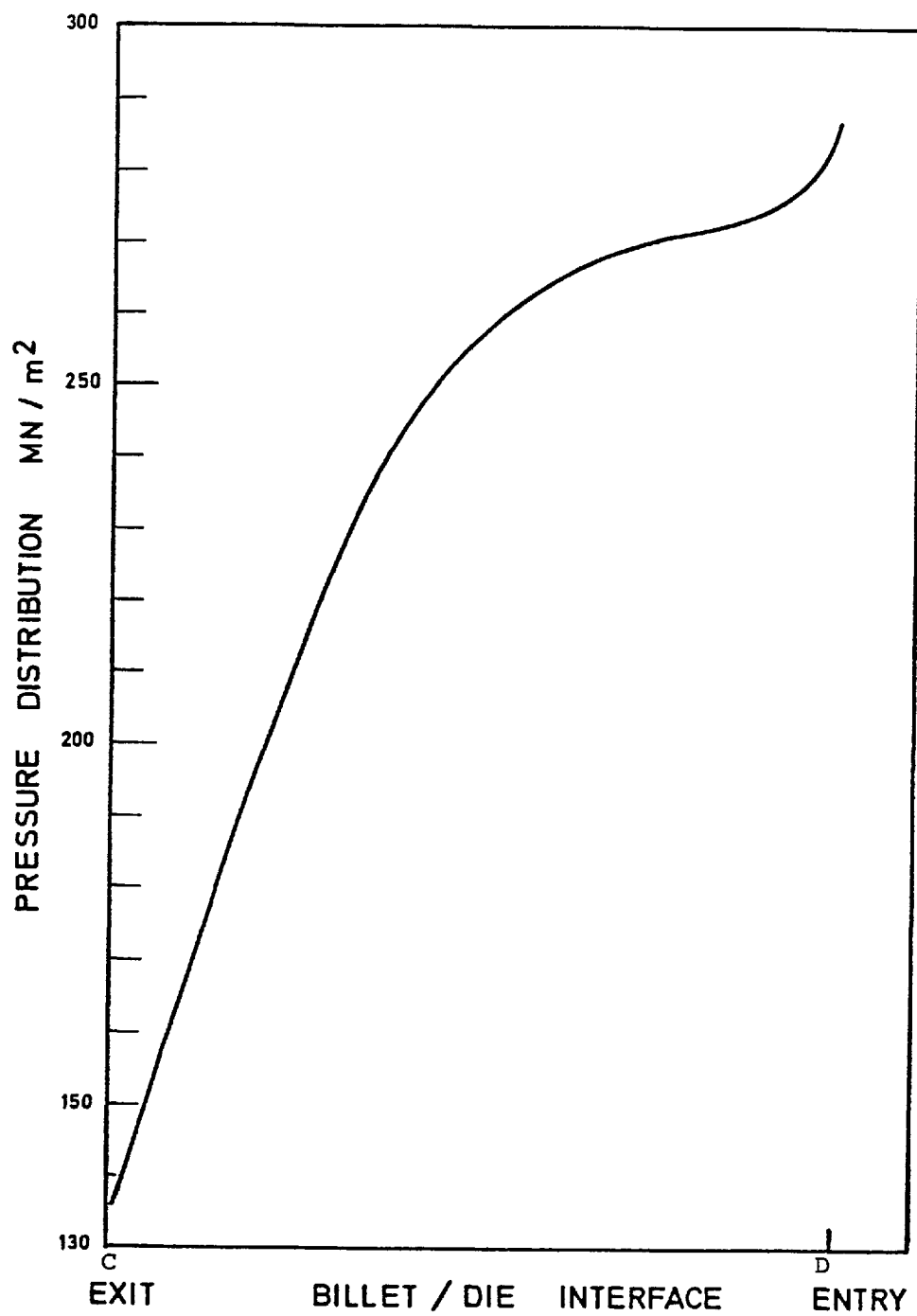


FIG. 5.9 - PRESSURE DISTRIBUTION ALONG THE DIE.
ALUMINIUM TUBE EXTRUSION.

$R = 3.5$ $p = 192.8 \text{ MN/m}^2$
 $F = 3503 \text{ N}$

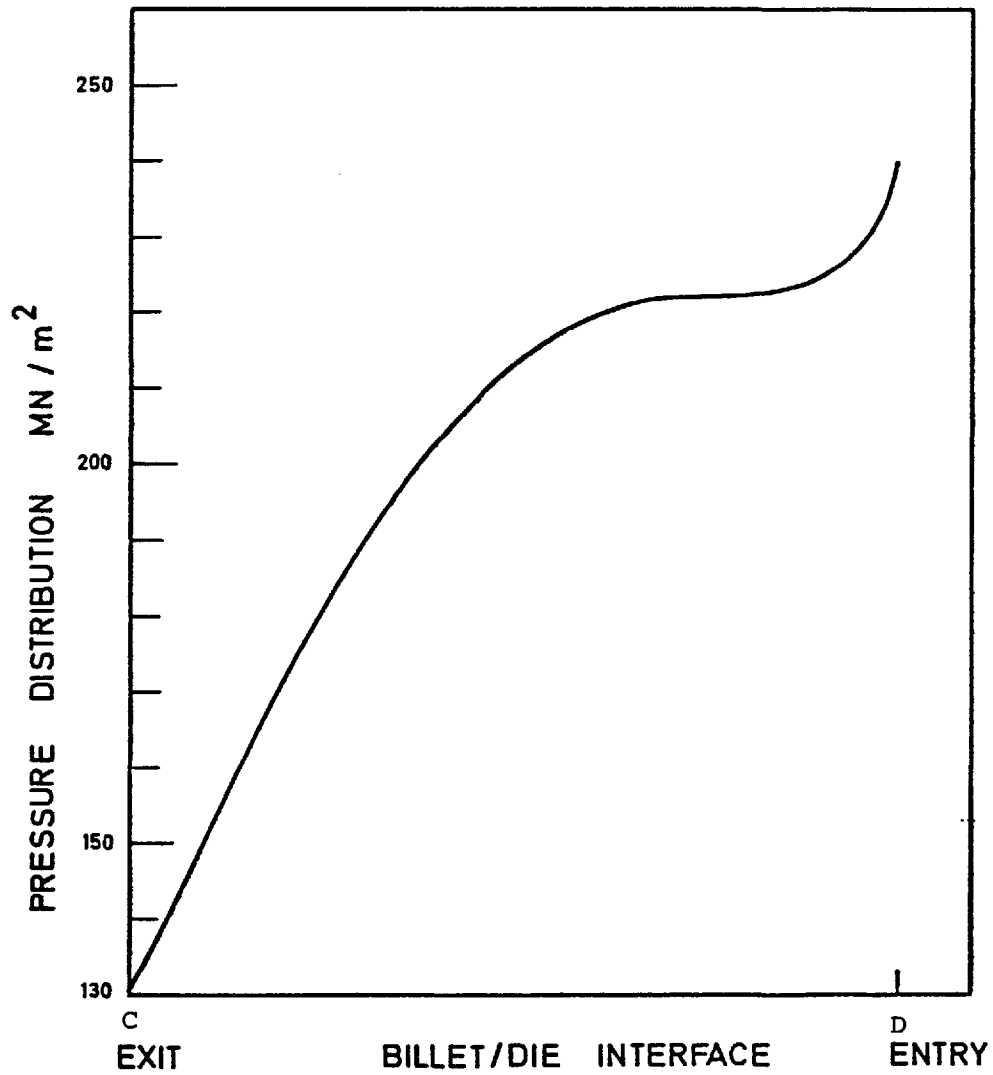


FIG. 5.10 - PRESSURE DISTRIBUTION ALONG THE DIE.
ALUMINIUM TUBE EXTRUSION.

$$\begin{aligned} R &= 2.65 & p &= 143.6 \text{ MN/m}^2 \\ F &= 3392.2 \text{ N} \end{aligned}$$

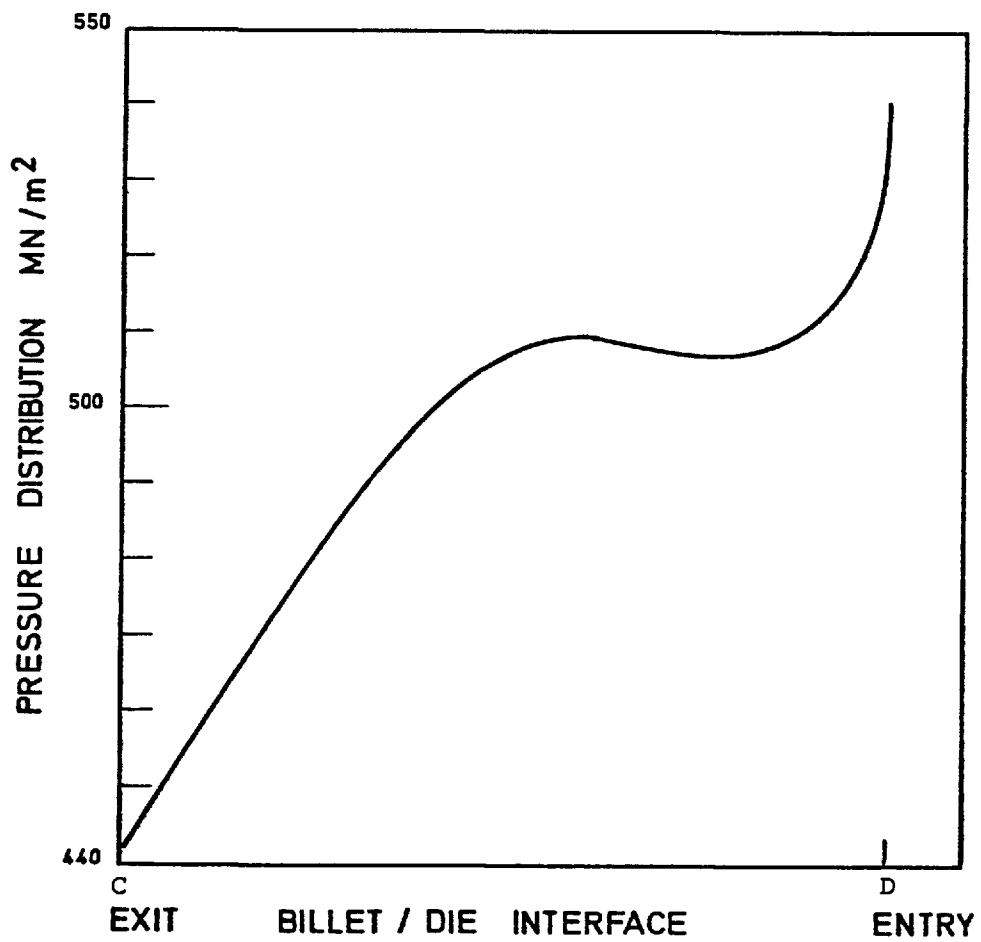


FIG. 5.11 - PRESSURE DISTRIBUTION ALONG THE DIE.
COPPER TUBE EXTRUSION.

$R = 2.0$ $p = 287.7 \text{ MN/m}^2$
 $F = 1501 \text{ N}$

pressure curve changes its curvature and falls again towards the exit.

The three distributions presented show practically the same pattern. The region where the curve curvature is changed can be considered as a platform for the extrusion ratio 2.65 and reduced to a point for the extrusion ratio of 3.5. In the case of copper the pressure after falling near the entrance, increases slightly to reach a maximum and drops again towards the exit.

The pressure distribution along the die for bar extrusion was solved by using the program STRESS and compared with that found, for the same conditions but by a different method, by Duffill, Hooke and Mellor (56). The comparison is presented in Figure 5.12.

DIE PRESSURE DISTRIBUTION CALCULATED BY

$$1 \text{ --- } q = \frac{P_z}{L_n R} \left(1 + L_n \frac{A}{A_2} \right)$$

$$p = 849.2 \text{ MN/m}^2 \text{ (55 ton/in}^2 \text{)}$$

2 --- POINT MATCHING TECHNIQUE

3 --- WEIGHTED RESIDUAL METHOD (PROGRAM STRESS)

upper bound pressure (PROGRAM MIN) = 852.3 MN/m² (55.2 ton/in²)

m = 0.06 ($\mu = 0.018$)

4 ---- WEIGHTED RESIDUAL METHOD (PROGRAM STRESS)

upper bound pressure (PROGRAM MIN) = 897 MN/m² (58.1 ton/in²)

m = 0.10 ($\mu = 0.032$)

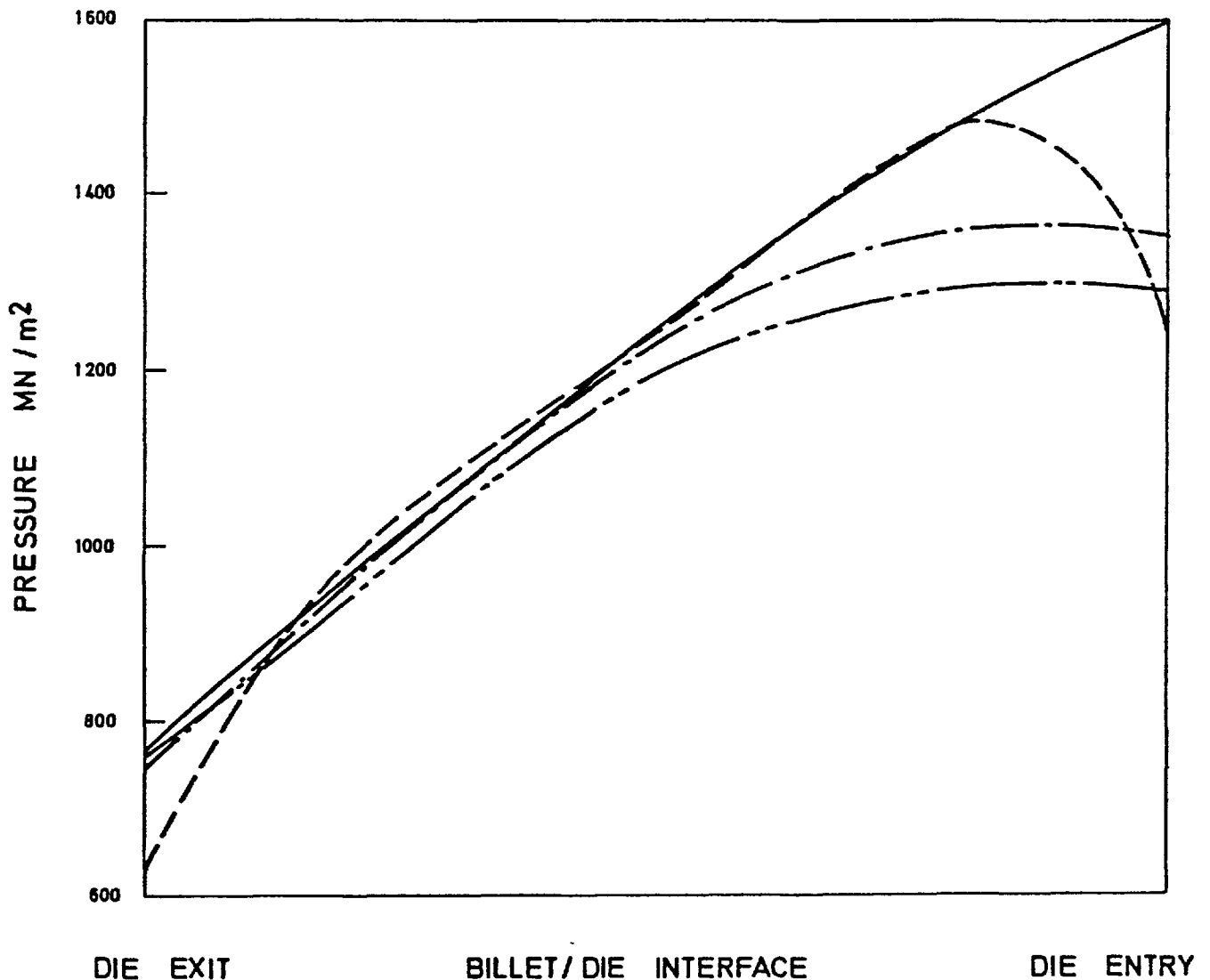


FIG. 5.12 - PRESSURE DISTRIBUTIONS ALONG THE DIE FOR BAR EXTRUSION

CHAPTER 6

ELASTIC STRESS ANALYSIS OF THE MANDREL FOR THE HYDROSTATIC EXTRUSION-DRAWING OF TUBES

6.1 Introduction

The stress analysis of the mandrel used in the experimental work is made, in the following sections, by means of the finite element method (145,146).

This technique consists of, in general terms, dividing a continuum by imaginary lines or surfaces into a finite number of elements. These elements are interconnected at a discrete number of nodal points situated on their boundaries. In the displacement formulation the displacements of the points within an element are related to its nodal displacements, which are the unknown parameters, by an assumed set of functions. So, the state of strain becomes defined and, together with the initial and boundary conditions and the material properties, determine the state of stress in the element. An elemental stiffness relationship is established by considering the equilibrium of the system of nodal forces, the boundary stresses and the distributed loads. The elements are then assembled and from the equilibrium conditions at the nodal points a set of equations containing the unknown nodal displacements is obtained. If this system of equations is solved the strains and stresses can be found for any material point.

6.2 The Finite Element Computer Program

The finite element computer program, called FRONT, utilised in the present analysis was developed at Imperial College by Silva (147)

and Pereira (80), from an earlier version, the FLAP 5 code (148) which uses the Gauss-Seidel iteration procedure.

The program is based on the outlined displacement approach and can be applied to axisymmetric cases. The elements used are the constant strain triangular elements with nodes situated at the vertices.

The solution routine in the FRONT code employs the Gaussain elimination technique. Programming details as well as a user's guide and a listing of the program are given in Reference (80).

The program enables the input data, the radial and vertical displacements of the nodes, the stresses at the elements and at the nodal points and the strains at the nodes to be printed out.

The nodal stresses are obtained by taking arithmetic means over surrounding elements. Therefore the stresses at the nodal points on the boundaries, surrounded by fewer elements, have a lower accuracy than the ones calculated at the nodes inside the structures. To obtain a more realistic approach in the regions whereby the boundary stresses are of interest a finer mesh should be prescribed. Also the boundary stresses can be evaluated by taking cross-plots and from the general trend of the stress distributions, extrapolated to the boundaries.

The hoop strains are calculated directly from the radial displacements at the nodes, and, therefore present a high accuracy even at the boundaries. The remaining strains at the nodes are calculated from the nodal stresses.

6.3 Data Generation Program

To use the finite element computer program it is necessary to supply it with data which defines the problem in study. For this purpose a data generation computer program called GDME, was produced

to provide information concerning the material properties, mesh, loading and displacement boundary conditions. This data is written in a computer file, which is, in turn, read by the finite element program.

6.3.1 Material Properties

In the present case, as no thermal effects are considered, only the values of the Young's Modulus, E , and the Poisson's ratio, ν , are supplied by the data file. The values adopted for the mandrel material are:

$$E = 2.06 \times 10^5 \text{ MN/m}^2$$

$$\nu = 0.3$$

6.3.2 Mesh

The elements, used in the finite element program, are described by their cross-section made by a plane containing the axis of symmetry of the mandrel. Figure 6.1 illustrates the mesh chosen for the case of aluminium extrusion, extrusion ratio of 2.65. In order to show the elements clearly the drawing scale in the r -direction is 2.5 greater than in the z -direction.

To improve the accuracy of the program the elements are smaller in the regions where larger gradients of stress are expected. It is known that when a thick hollow cylinder is subjected to an external constant pressure the gradients of the stresses are maximum in its bore. For this reason it is usual to choose a mesh which coarsens radially outwards from the bore. However, in the present case as the

FIG. 6.1 - FINITE ELEMENT MESH

mandrel is subjected to a pressure distribution, which varies along its length, with high gradients in the region of the plastic zone/mandrel interface, it was preferable to choose a very fine uniform mesh in the radial direction. In the z-direction the size of the elements varies according to the gradient of the pressure distribution, the higher the latter the smaller the former.

On the plastic zone/mandrel interface the mesh was prescribed in such a way that the relative position of the nodes is the same used in the previous computer programs MIN and STRESS. This arrangement enables the transference of the normal and shear stresses associated with these nodal points to the data generation program, to be made without losing any information.

6.3.3 Loading

The pressure and shear stress distributions along the mandrel surface, to be discussed in a further section, are applied to the nodes of the structure as follows:

If p_i , p_j and p_k are the external normal pressures acting on the nodes i , j and k respectively, the radial external "force" acting on node j to be introduced in the computation is

$$F_r = \pi R_m \left(\Delta z_{ij} \frac{p_i + p_j}{2} + \Delta z_{jk} \frac{p_j + p_k}{2} \right) \quad (6.1)$$

where R_m is the outside radius of the mandrel

Δz_{ij} is the distance between nodes i and j

and Δz_{jk} is the distance between nodes j and k

Similarly the axial external load acting on node j is

$$F_z = \pi R_m (\Delta z_{ij} \frac{\tau_i + \tau_j}{2} + \Delta z_{jk} \frac{\tau_j + \tau_k}{2}) \quad (6.2)$$

where τ_i , τ_j and τ_k are the shear stresses acting on nodes i, j and k respectively.

6.3.4 Displacement Boundary Conditions

As the top end of the mandrel rests on a support (figure 2.2), it is assumed that the displacements of the nodes on the mandrel/support interface are restricted in the z-direction. Therefore, only radial displacements of these nodes are allowed.

6.4 Results of the Finite Element Computer Program

6.4.1 Introduction

In what follows it is convenient to consider some cross-sections of the mandrel which are identified by the symbols S1 to S12 in Figure 6.2.

The cross-sections S1 to S11 are made by planes normal to the mandrel axis such that the plastic zone/mandrel interface is divided into ten equal surfaces. The cross section S12 is located in the region of fluid pressure far from the plastic deformation zone and the seal region.

In subsequent figures the symbols in them are related to Figure 6.2.

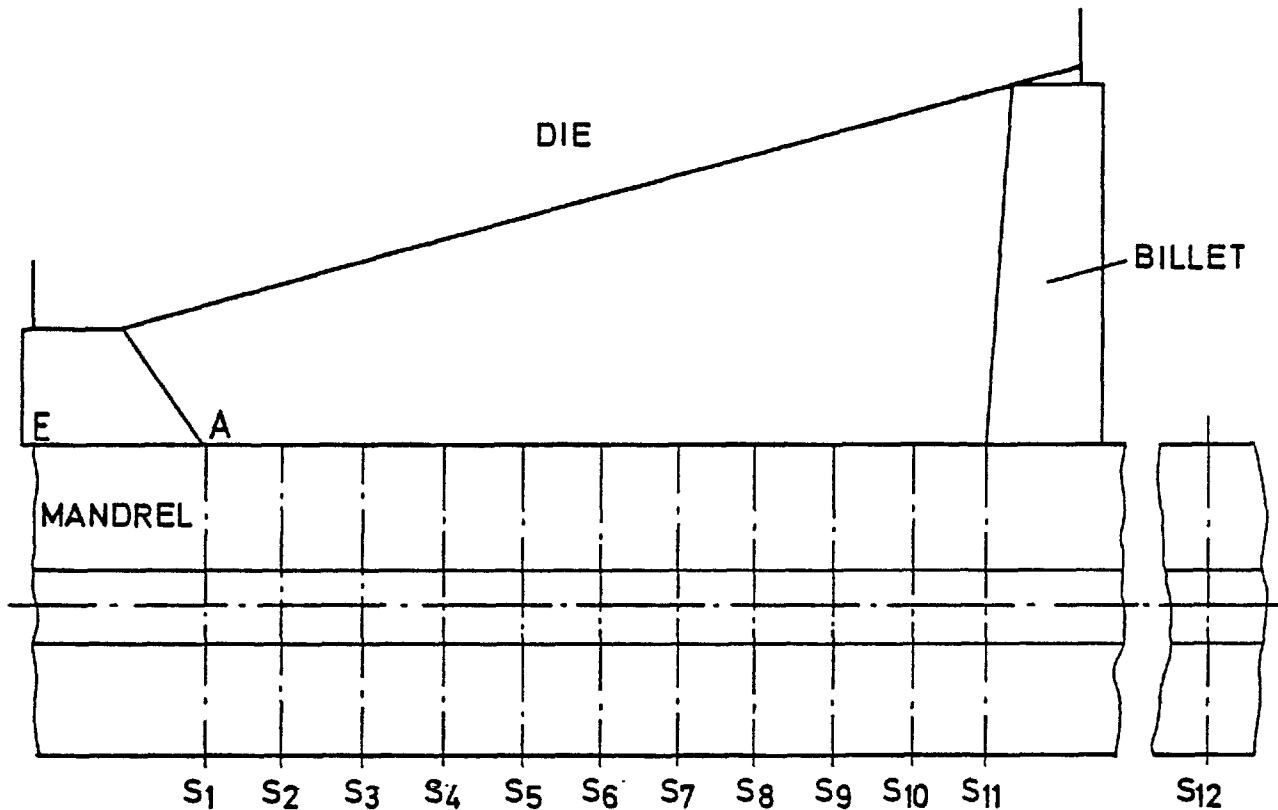


FIGURE 6.2 - RELATIVE POSITION OF THE MANDREL CROSS SECTIONS

The stress analysis of the mandrel is carried out for three selected cases of tube extrusion-drawing characterised as follows.

1) Aluminium

Extrusion Ratio, $R = 3.5$

Extrusion Pressure, $p = 192.2 \text{ MN/m}^2$

Drawing Force, $F = 3503 \text{ N}$

Friction Force, $F_f = 4113 \text{ N}$

2) Aluminium

Extrusion Ratio, $R = 2.65$

Extrusion Pressure, $p = 143.6 \text{ MN/m}^2$

Drawing Force, $F = 3392 \text{ N}$

Friction Force, $F_f = 3030 \text{ N}$

3) Copper

Extrusion Ratio, $R = 2.0$

Extrusion Pressure, $p = 287.7 \text{ MN/m}^2$

$$\text{Drawing Force} = F = 1501 \text{ N}$$

$$\text{Friction Force} = F_f = 3464 \text{ N}$$

The values of the pressure are those obtained by the upper bound approach while the drawing force and friction force on the billet/mandrel interface are represented by the experimentally measured values.

These cases will be identified, in the text of this chapter, by the material and extrusion ratio, the other parameters will be omitted.

As adopted in previous discussions the stresses and displacements will be referred to their absolute values while they will be affected by the corresponding sign in the graphical representations.

6.4.1.1 Pressure and Shear Stress Distributions along the Mandrel

Figures 6.3 to 6.5 show the pressure and shear stress distributions along the external surface of the mandrel for the three cases considered.

In the pressure distribution curves three stages are distinguished as follows:

1. Constant pressure distribution in the region between the seal section (Figure 2.2) and the section where the billet meets the mandrel. The pressure in this region was determined by the lower upper bound approach described in Chapter 4.
2. Pressure distribution at the plastic zone/mandrel interface. This distribution corresponds to the radial stress distribution on the interface obtained by means of the weighted residuals method, as shown in Chapter 5.
3. Pressure distribution at the product/mandrel interface. To

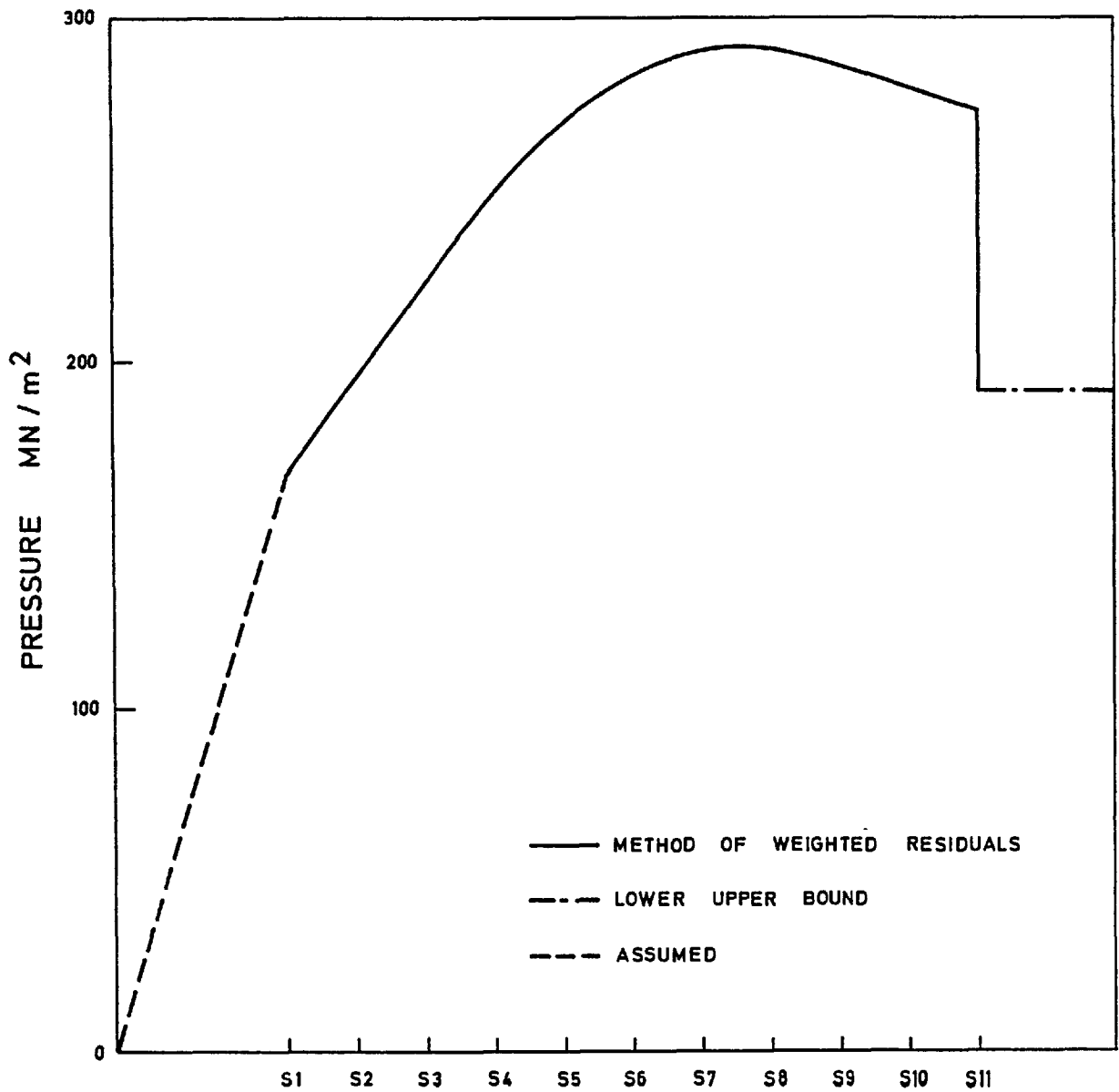
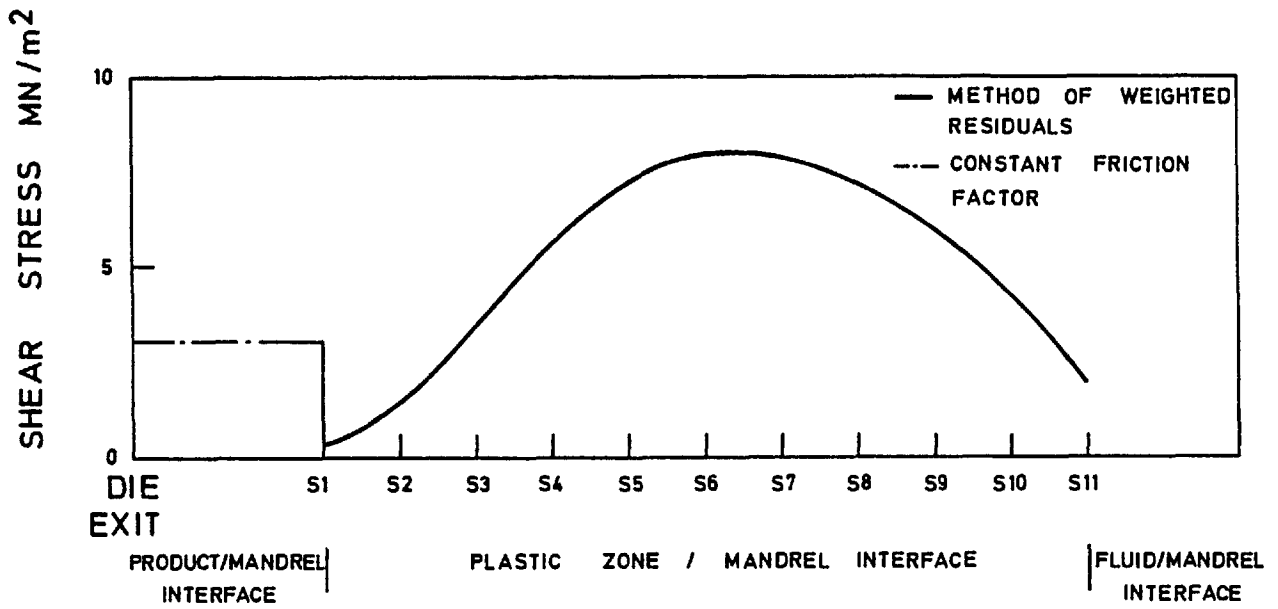


FIG. 6.3 - SHEAR STRESS AND PRESSURE DISTRIBUTIONS ALONG THE MANDREL.
ALUMINIUM EXTRUSION. $R = 3.5$

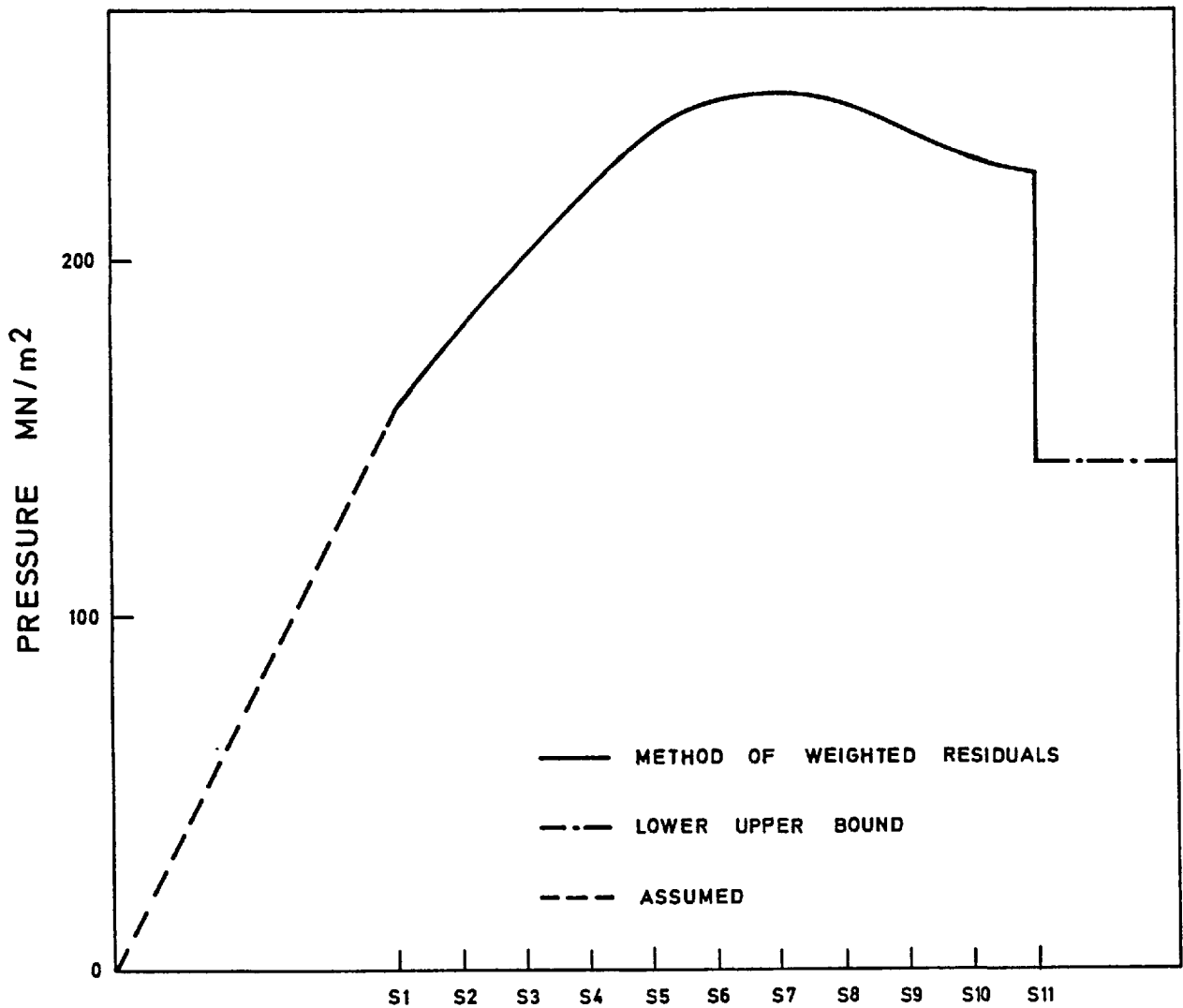
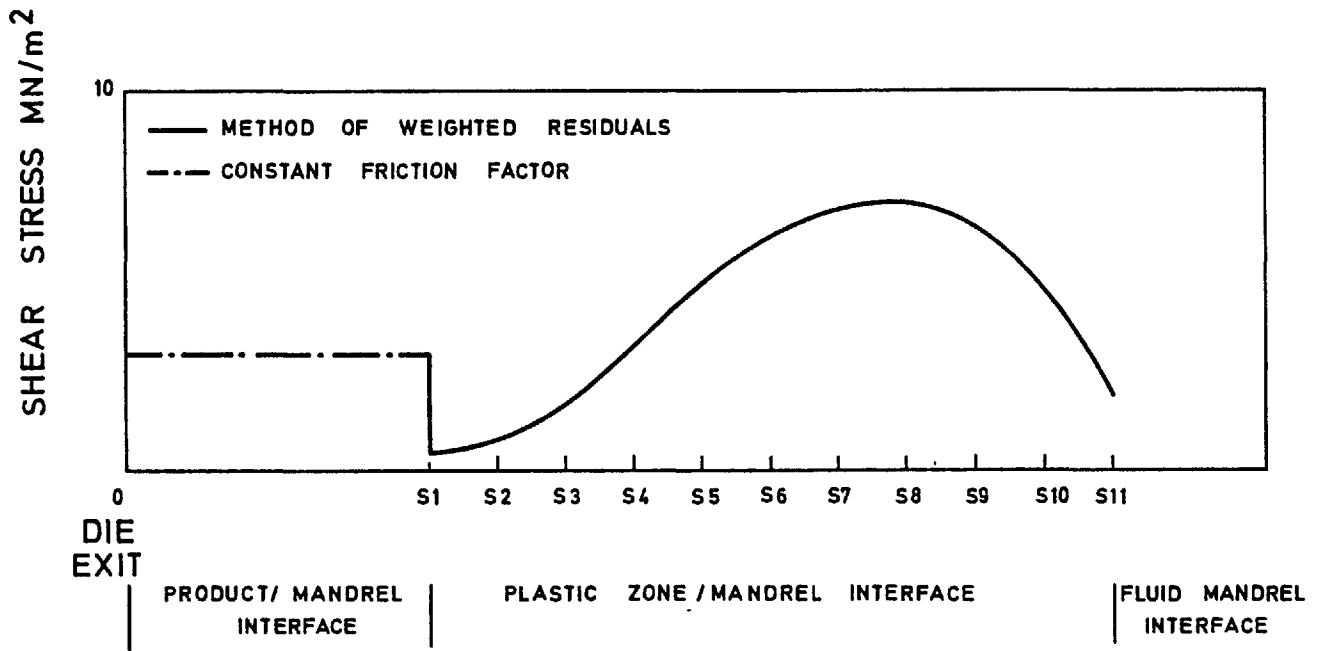


FIG. 6.4 - SHEAR STRESS AND PRESSURE DISTRIBUTIONS ALONG THE MANDREL.
ALUMINIUM EXTRUSION. $R = 2.65$

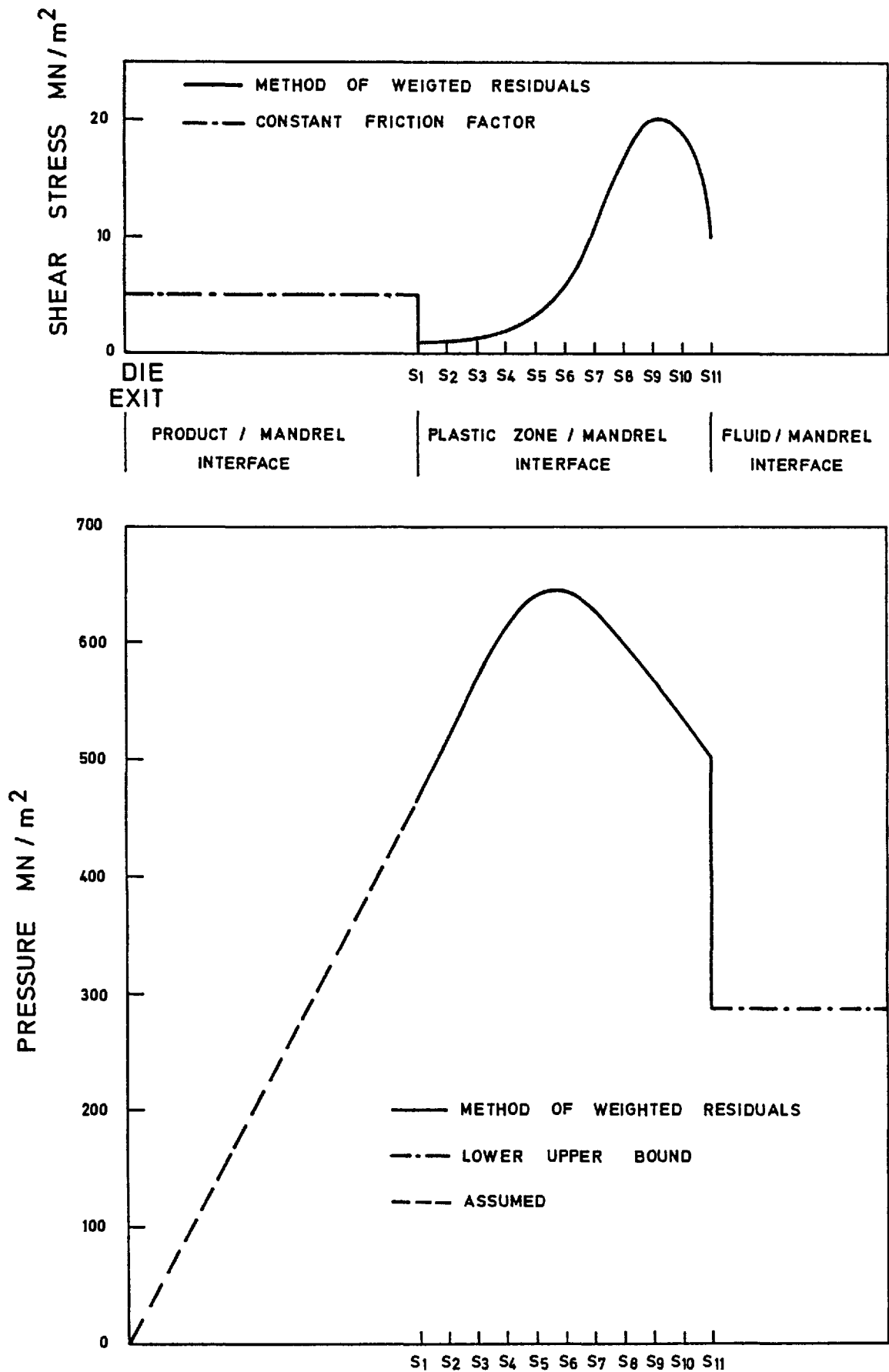


FIG. 6.5 - SHEAR STRESS AND PRESSURE DISTRIBUTIONS ALONG THE MANDREL.
COPPER EXTRUSION. $R = 2.0$

estimate this distribution an assumption has to be made, since the weighted residuals program only applies to the plastically deformed material. In this zone it is known that the pressure is zero at the points located in the die exit section (point E in Figure 6.2), and is equal to the calculated radial stress at the points situated on the plastic deformation zone exit boundary (point A in Figure 6.2).

The assumption is that the pressure varies linearly at the product/mandrel interface between those two values, as shown in Figures 6.3 to 6.5.

Also the shear stress distribution curves show three different phases:

1. The shear stress is zero at the fluid/mandrel interface.
2. Along the plastic deformation zone/mandrel interface the shear stress distribution is calculated by the weighted residuals method.
3. Along the product/mandrel interface the shear stress distribution results from the assumption of a constant friction factor as described in Chapter 4.

To assess the effect of these distributions on the elastic behaviour of the mandrel, the deformation as well as the stress distributions in some cross sections of the mandrel are discussed in the next sections.

6.4.2 Deformation of the Mandrel

To study the deformation of the mandrel when subjected to the loads illustrated in Figures 6.3 to 6.5, the radial displacements

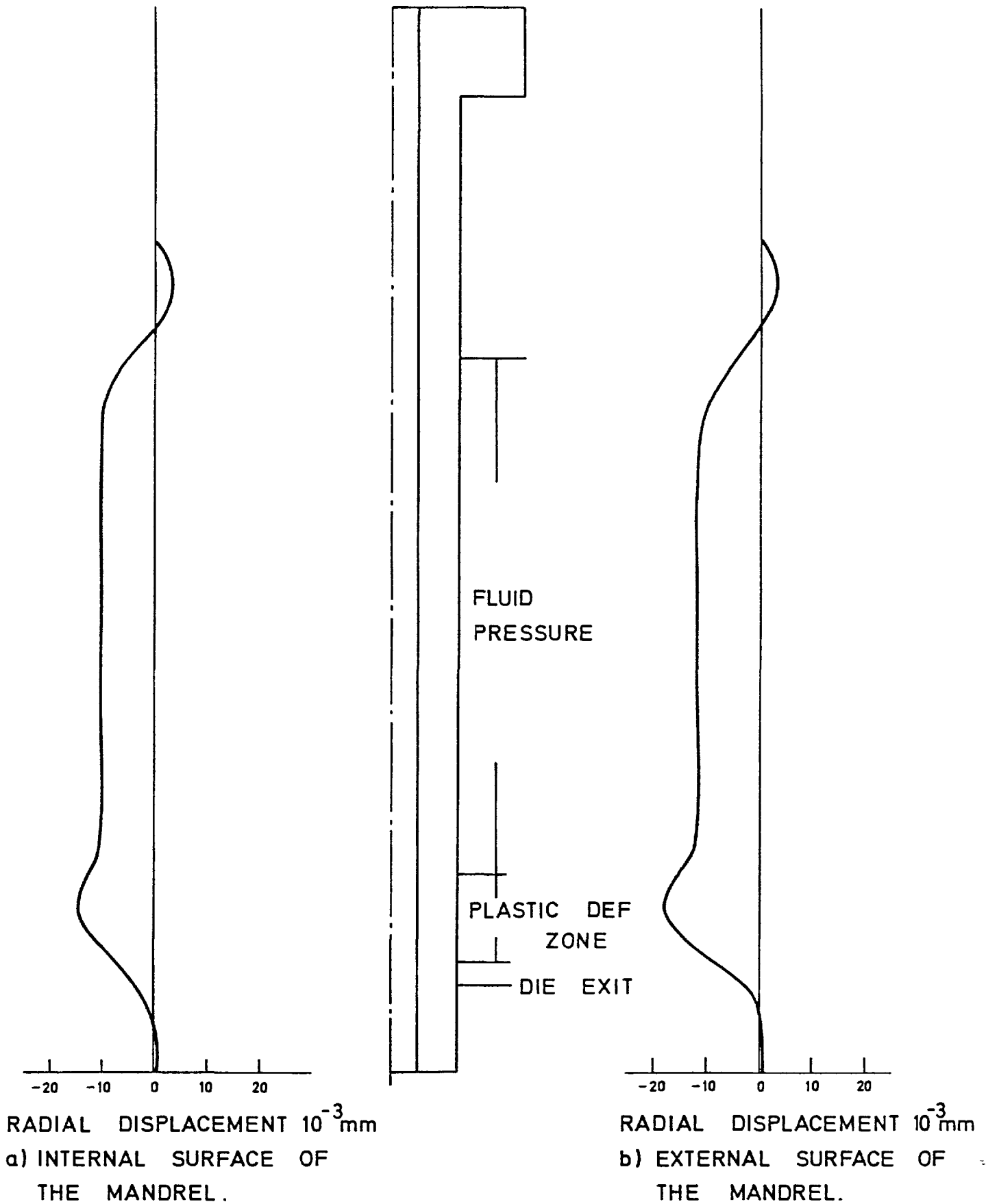


FIG. 6.6 - RADIAL DISPLACEMENTS OF THE INTERNAL AND EXTERNAL SURFACES OF THE MANDREL. ALUMINIUM EXTRUSION. $R = 3.5$

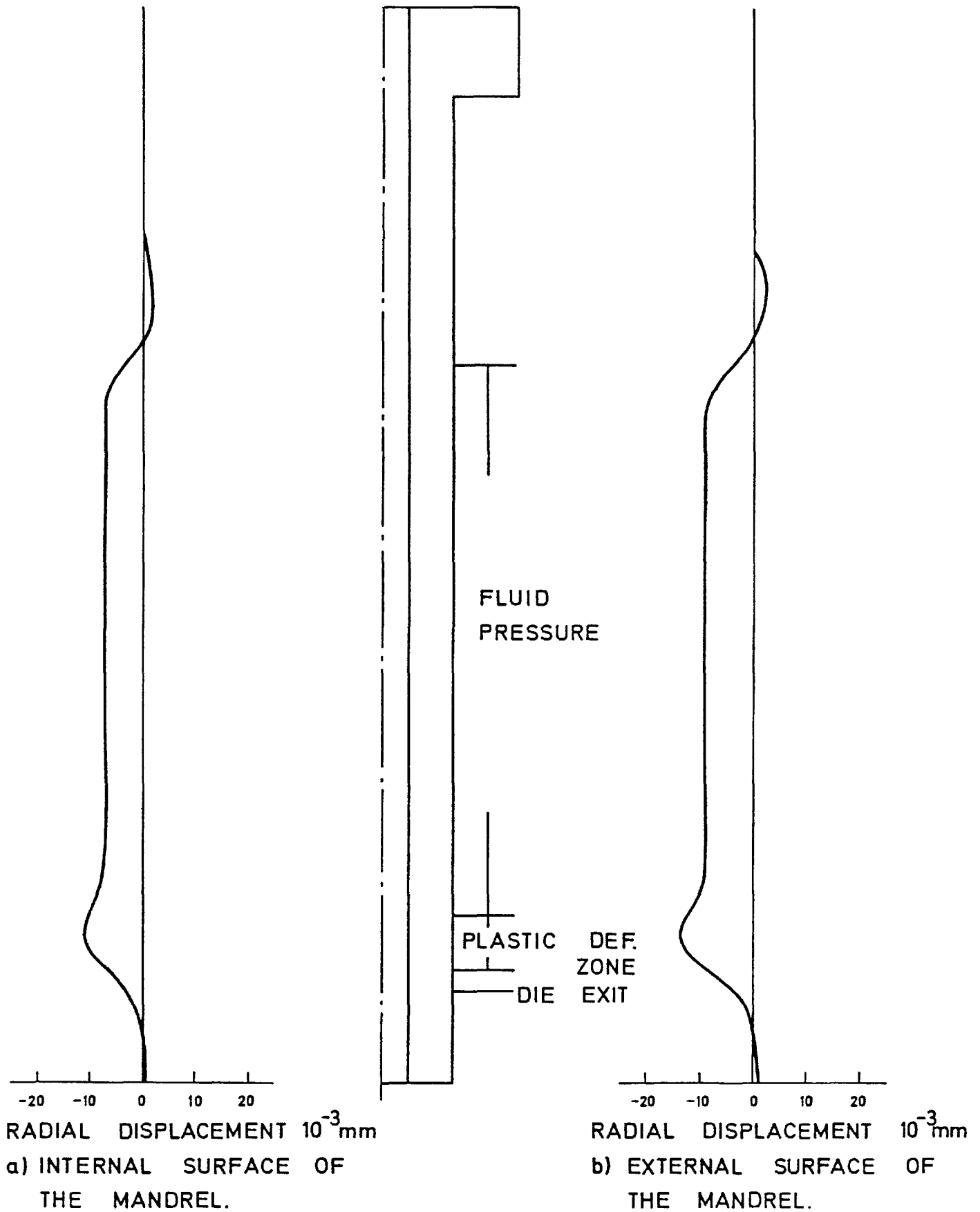


FIG. 6.7 - RADIAL DISPLACEMENTS OF THE INTERNAL AND EXTERNAL SURFACES OF THE MANDREL. ALUMINIUM EXTRUSION. $R = 2.65$

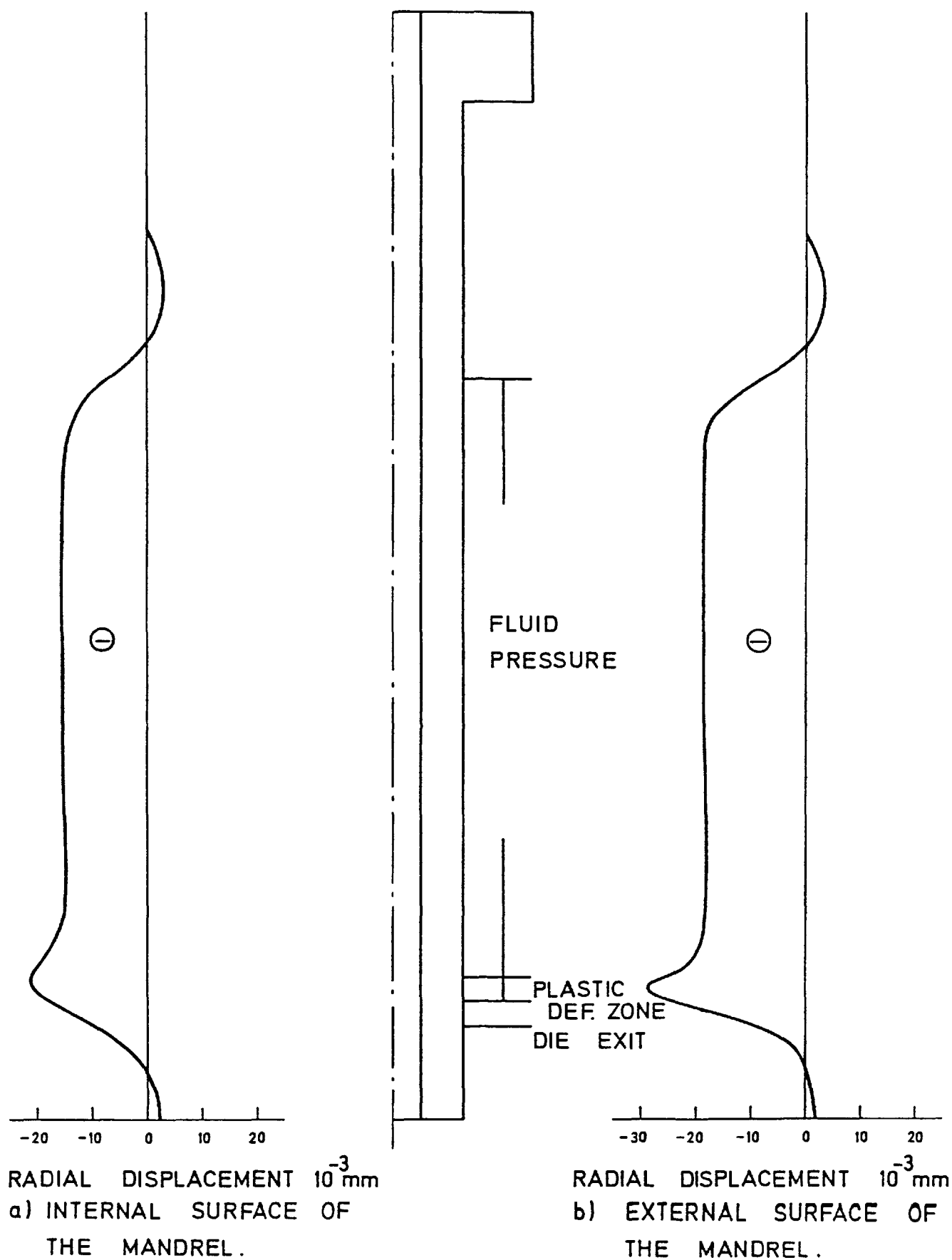


FIG. 6.8 - RADIAL DISPLACEMENTS OF THE INTERNAL AND EXTERNAL SURFACES OF THE MANDREL. COPPER EXTRUSION. $R = 2.0$

of its internal and external surfaces are analysed in this section. They are shown in Figures 6.6 to 6.8.

The vertical displacements of the mandrel cross sections are not represented, since the finite element analysis show that plane cross sections before the deformation remain plane during loading (the maximum variation of the vertical displacements of the points in the same cross section is 6.0×10^{-4} mm, for copper extrusion, extrusion ratio of 2.0).

Figures 6.6 to 6.8 show clearly that the highest values of the radial displacement occur in the region of the plastic zone/mandrel interface. The maximum of these values are for the copper extrusion, extrusion ratio of 2.0 and the minimum for aluminium, extrusion ratio of 2.65, as they correspond to the highest and lowest values of the pressure distributions.

As the pressure distribution at the plastic zone/mandrel interface for copper extrusion has a more pronounced maximum than the two others distributions, the radial displacement curves of the points on the internal and external surfaces, in that region, are sharper than the other two curves.

At the fluid/mandrel interface region the radial displacement curves have values practically constant, as the mandrel is subjected to a constant fluid pressure and a constant small axial stress due to the friction force. Near the seal the radial displacement changes sign and drops to zero.

6.4.3 Stress Fields

In order to supply the designer with information related to the behaviour of the mandrel, the stress fields in some of its cross sections are presented in this section.

One design condition that the mandrel has to satisfy is that it must remain elastic during the hydrostatic extrusion-drawing process. Therefore, according to the von Mises criterion, the equivalent stress at every point of the mandrel can not exceed the yield stress of the material from which the mandrel is made.

As, in the present case, the mandrel is hollow it is expected that the severest stressed points are in the mandrel bore. So the equivalent stress curves are plotted along the internal surface of the mandrel, for the three extrusion cases considered (Figures 6.9 to 6.11).

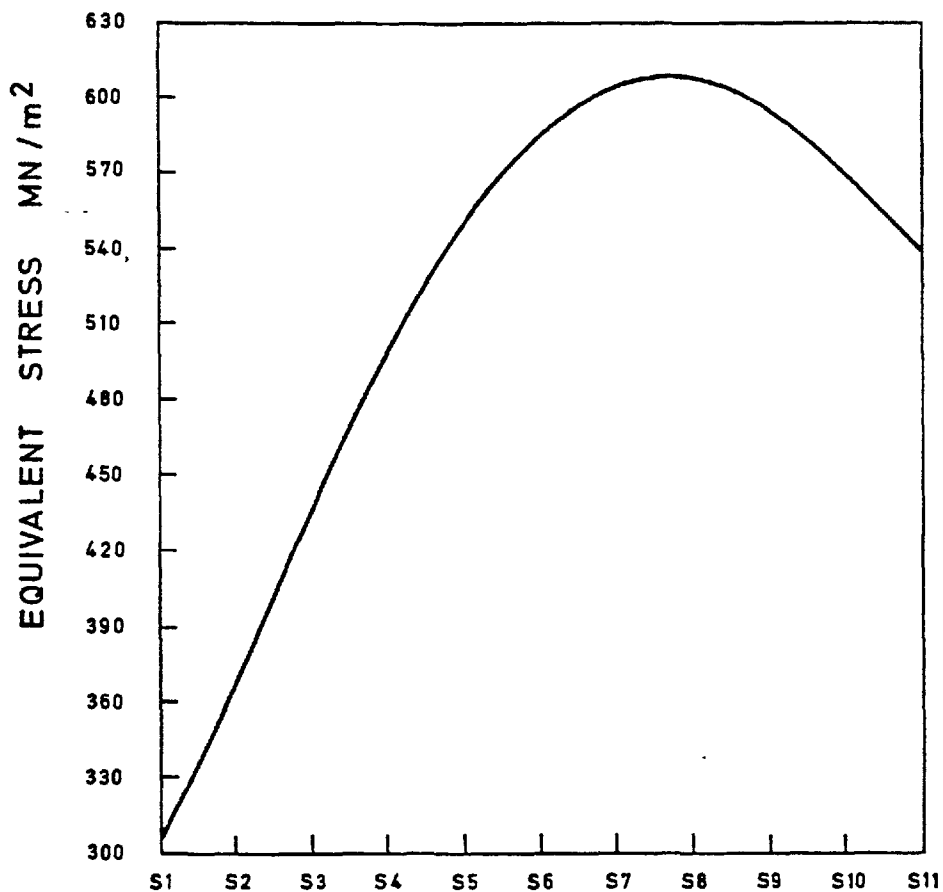


FIG. 6.9 - EQUIVALENT STRESS DISTRIBUTION ALONG THE INTERNAL SURFACE OF THE MANDREL. ALUMINIUM EXTRUSION.

$$R = 3.5$$

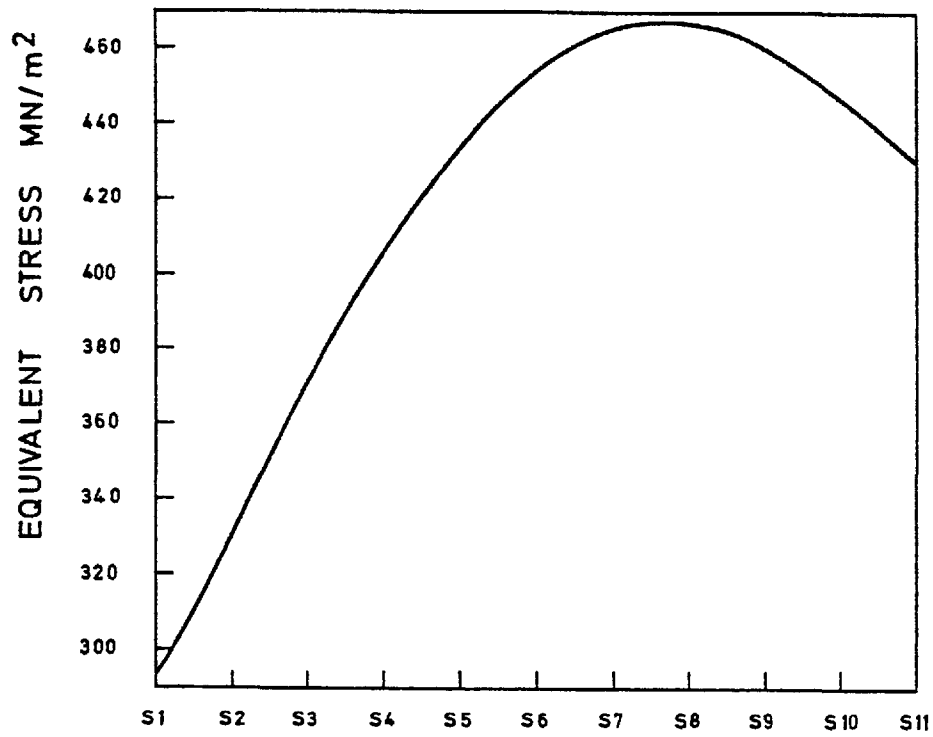


FIG. 6.10 - EQUIVALENT STRESS DISTRIBUTION ALONG THE INTERNAL SURFACE OF THE MANDREL. ALUMINIUM EXTRUSION.
R = 2.65

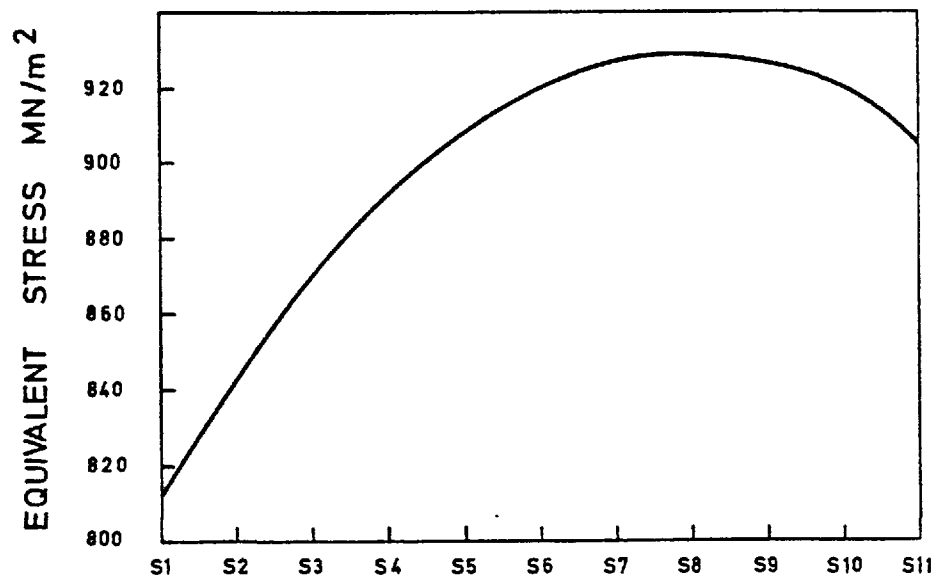


FIG. 6.11 - EQUIVALENT STRESS DISTRIBUTION ALONG THE INTERNAL SURFACE OF THE MANDREL. COPPER EXTRUSION.
R = 2.0

These figures show that at the internal surface of the mandrel the equivalent stress is maximum at the points situated in the cross section S8 or in between cross sections S7 and S8. For the case of aluminium extrusion, extrusion ratio 3.5 the maximum pressure acts on the same cross section where the maximum equivalent stress occurs. This does not apply to aluminium, extrusion ratio of 2.65 and copper, extrusion ratio of 2.0. In the latter case it is clear that the pressure is maximum at the cross sections S5 and S6. Referring to Figures 6.9 to 6.11, it should be noticed that the variation of the equivalent stress at the points in cross sections S5 to S11 is very small.

To clarify these comments the radial, hoop, axial, shear and equivalent stresses at several cross sections of the mandrel are illustrated in Figures 6.12 to 6.15.

The radial stresses are zero at the internal surface of the mandrel and increase in the radial direction to reach the value of the pressure at the external surface of the mandrel. Although the finite element calculations give a value greater than zero at the mandrel bore, due to a lower accuracy of the program at the boundaries, in the figures representing the radial stress, the curves are drawn to attain the value zero. Also, at the outside surface the value of the radial stress is within an error of 5% of the imposed pressure.

The hoop stress figures illustrate that the highest values are reached at the bore of the mandrel. However, for copper extrusion, in the region of the plastic zone/mandrel interface, the hoop stresses at the outside and bore of the mandrel are much closer to each other than for aluminium extrusions. In the same region, and for the cases considered, the values of the hoop stress at the bore vary little between cross sections S5 to S11.

Figures 6.12 to 6.14 show that the axial stresses change sign

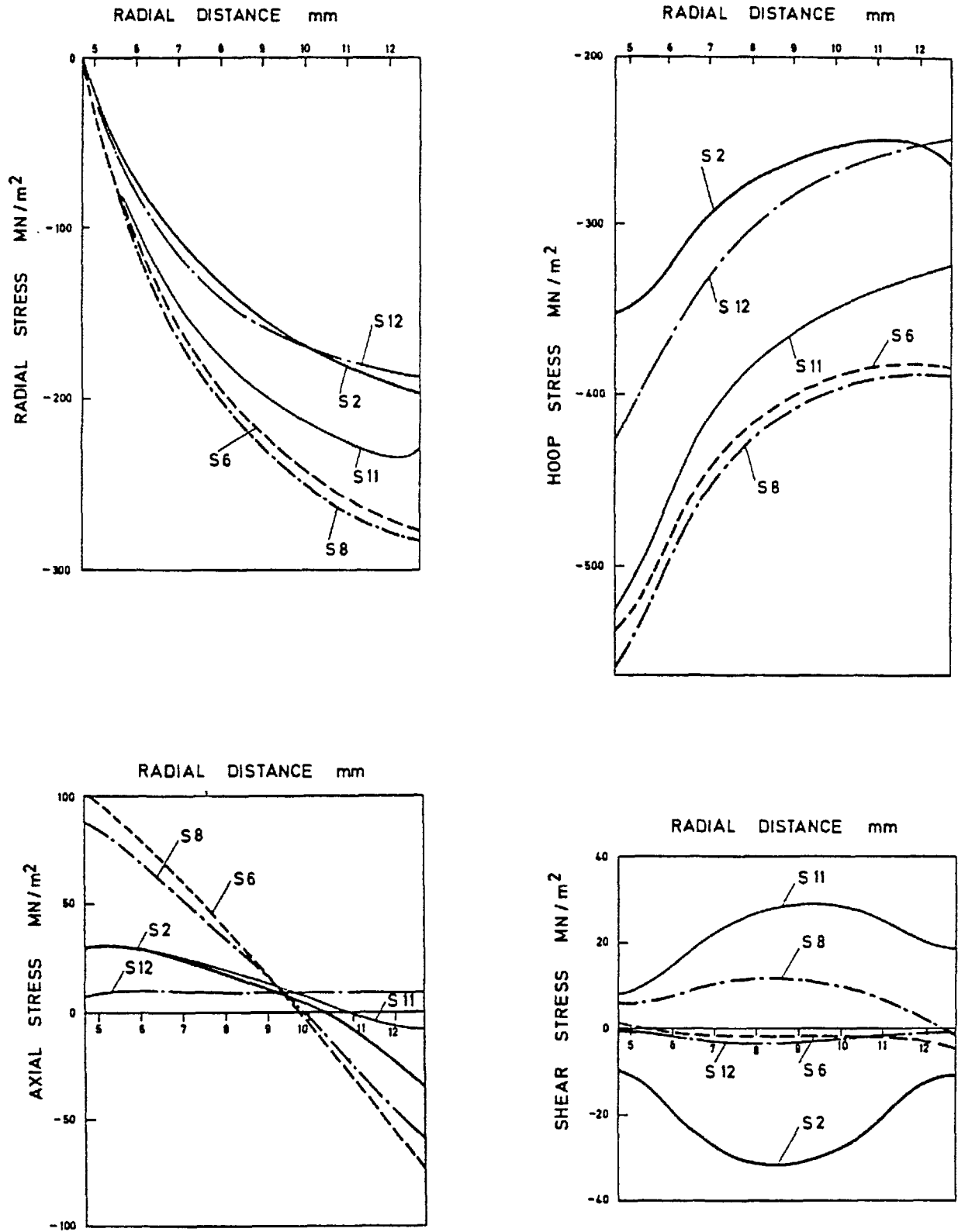


FIG. 6.12 - VARIATION OF σ_r , σ_θ , σ_z , τ_{rz} AT SEVERAL CROSS SECTIONS OF THE MANDREL IN THE R-DIRECTION. ALUMINIUM EXTRUSION. $R = 3.5$

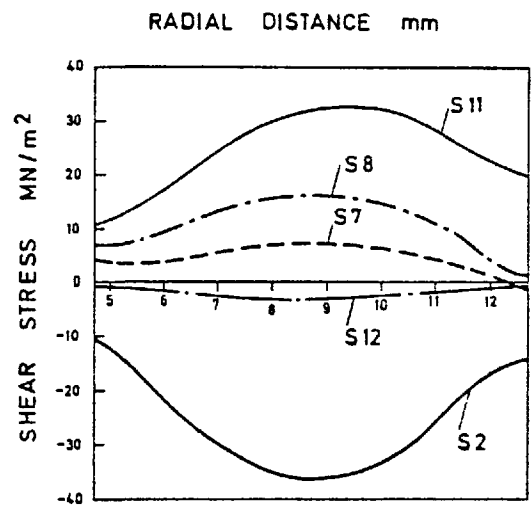
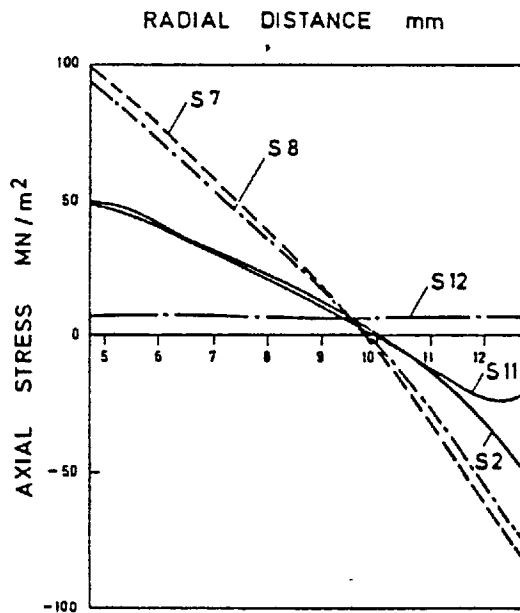
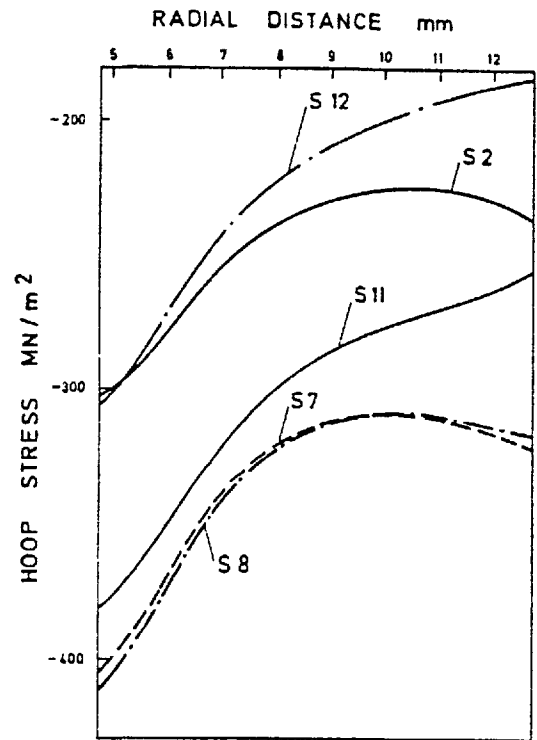
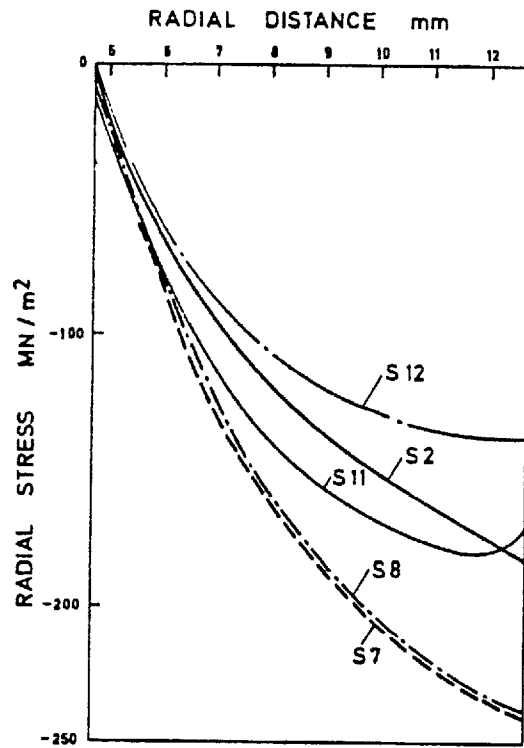


FIG. 6.13 - VARIATION OF σ_r , σ_θ , σ_z , τ_{rz} AT SEVERAL CROSS SECTIONS OF THE MANDREL IN THE R-DIRECTION. ALUMINIUM EXTRUSION. $R = 2.65$

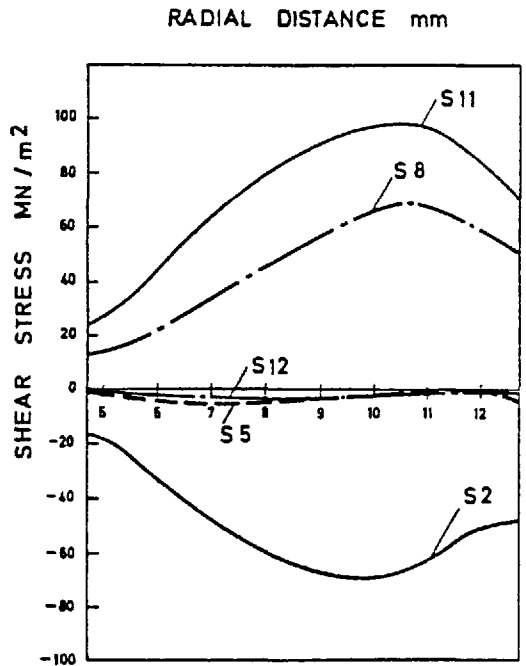
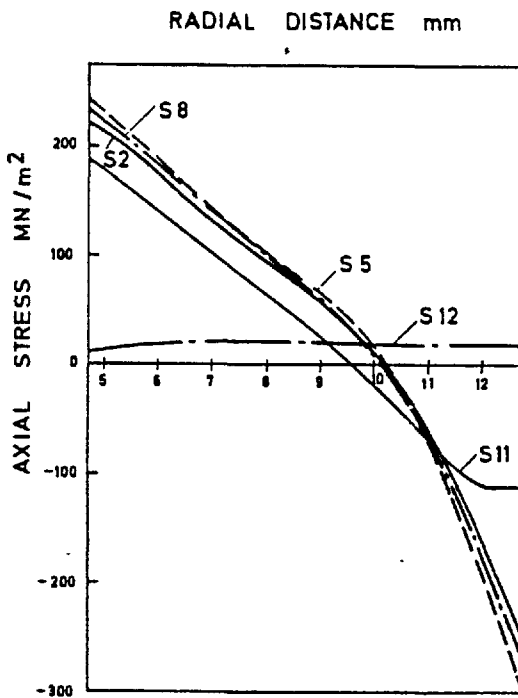
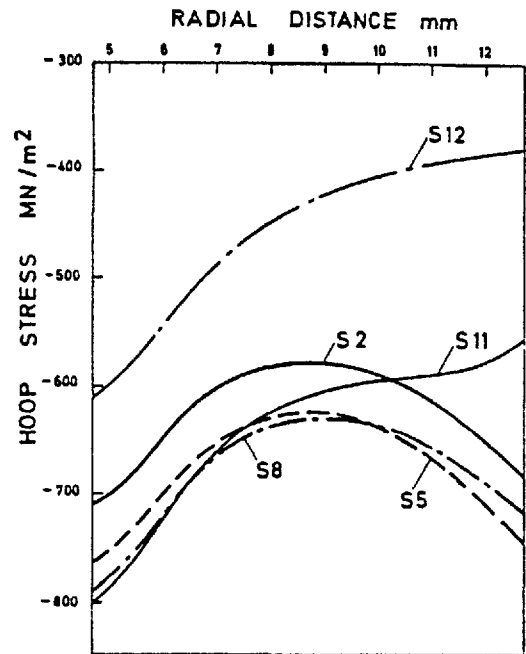
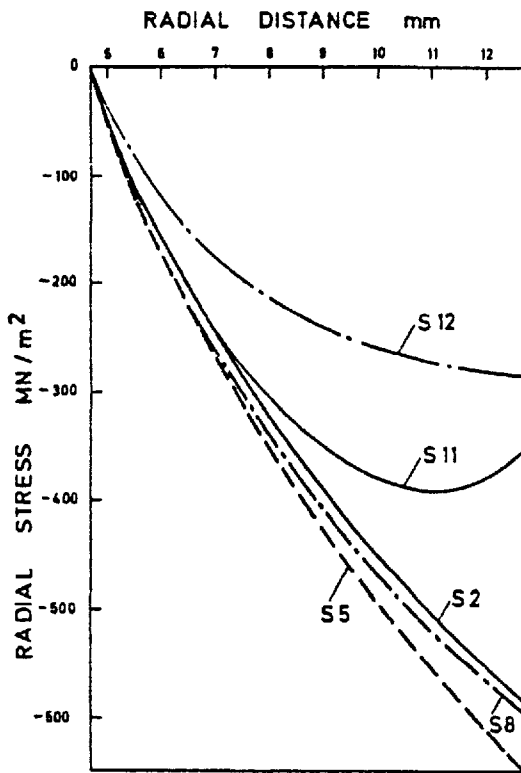


FIG. 6.14 - VARIATION OF σ_r , σ_θ , σ_z , τ_{rz} AT SEVERAL CROSS SECTIONS OF THE MANDREL IN THE R-DIRECTION. COPPER EXTRUSION. $R = 2.0$

in the radial direction, with the exception of cross section S12, they are tensile at the mandrel bore and compressive at the outside surface.

The values of the axial stress at the bore of the mandrel between cross sections S2 and S11 are much higher than the value obtained by

$$\frac{F_f}{A_m} \quad (6.3)$$

where F_f is the friction force on the billet/mandrel interface and A_m is the area of the cross section of the mandrel.

In cross section S12, which is far from the deformation zone, the axial stress is practically constant at the value given by (6.3), within the accuracy of the computation.

In any cross section $\int \sigma_z dA_m$ is equal to the friction force acting on the mandrel between that cross section and the exit.

Particularizing for the case of copper extrusion, the value of the axial stress calculated by expression (6.3) is approximately 8MN/m^2 which is, also, the value obtained by the finite element program at cross section S12. Figure 6.14 shows that between cross sections S2 and S11 the axial stress reaches as high as 250MN/m^2 at the mandrel bore and -300MN/m^2 at the external surface.

The explanation for these high values of the axial stress lies in the large deformation suffered by the mandrel in a narrow region between cross section S1 and S11. In fact, under the action of the normal stresses at the plastic deformation zone/mandrel interface the mandrel deforms, in this region, in such a way that the curvature produced at the fibres on the external and internal surfaces generates compressive stresses in the former and tensile stresses in the latter (Figures 6.6 to 6.8).

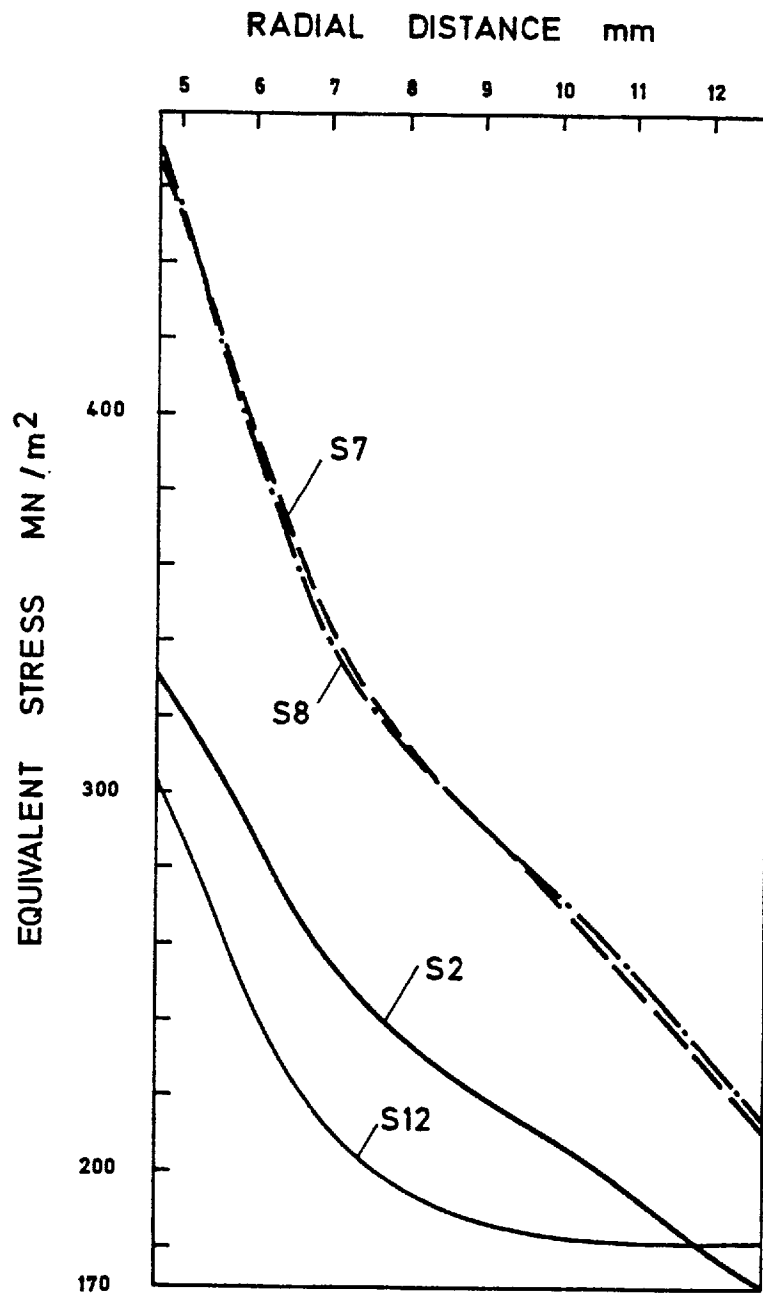
The values of the shear stresses τ_{rz} are low when compared with the other presented stresses. In any cross section the maximum value occurs at the points situated midway the limiting surfaces of the mandrel.

The variation of the equivalent stress along the radial direction in several cross sections of the mandrel is illustrated in Figure 6.15 for the three cases of extrusion considered. These illustrations confirm that the equivalent stress is much higher at the bore of the mandrel than at any other point. At the bore the equivalent stress is only a function of the hoop, σ_θ , and axial, σ_z , stresses as the radial, σ_r , and shear τ_{rz} stresses are zero. Therefore, the determination of the most stressed points depends on the values that σ_θ and σ_z take in the mandrel bore.

From the foregoing one can conclude that, although the tensile axial stresses in the mandrel, due to the friction force are small, the design criteria for the mandrel in hydrostatic extrusion should take into account not only the high compressive hoop stresses, but also the tensile axial stresses developed in the bore opposite the plastically deforming billet region.

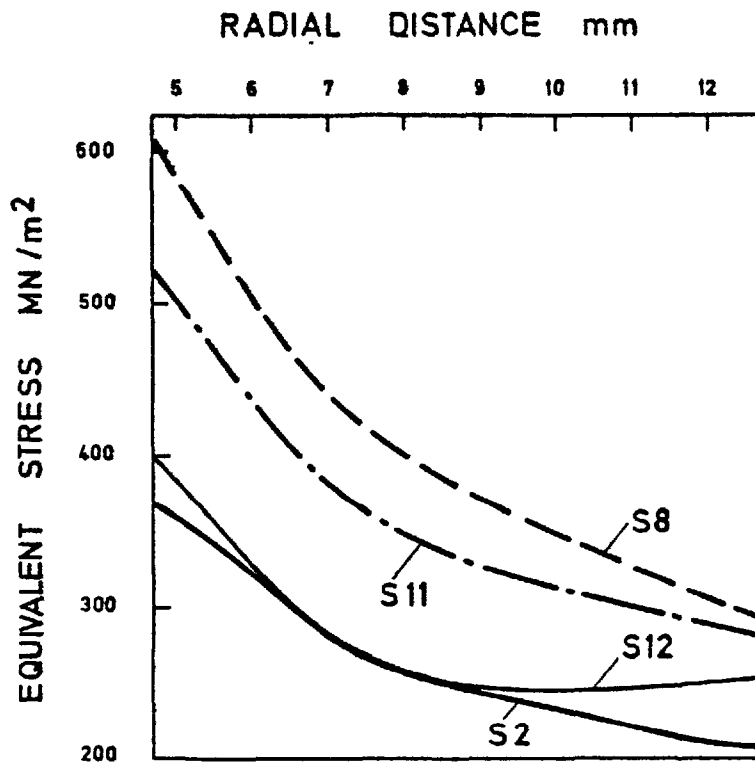
Obviously, in practice the mandrel is not hollow but a solid piece. Although some similarity exists between the hollow and solid mandrel the stress distributions in the latter should be assessed separately.

Figure 6.16 illustrates the variation of the axial, hoop, radial and shear stresses in various cross sections of a solid mandrel for copper extrusion at a ratio of 2.0. The pressure and shear stress distributions assumed at the mandrel surface are illustrated in Figure 6.3. The mandrel diameter is equal to the outside diameter of the hollow mandrel previously analysed.

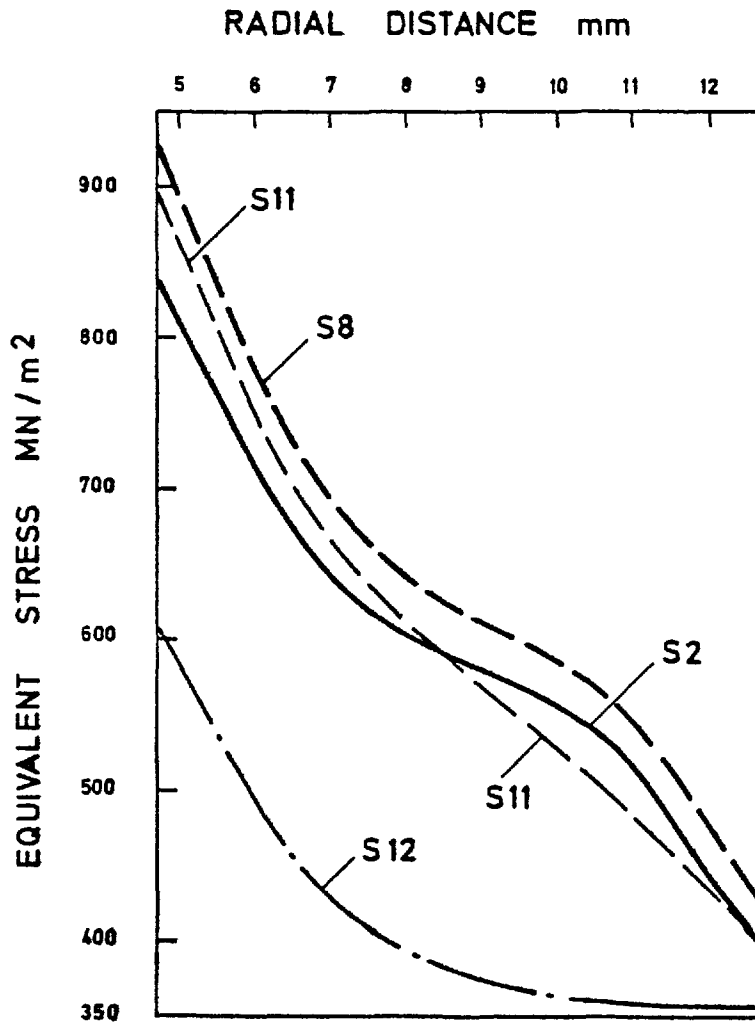


a) ALUMINIUM EXTRUSION, $R = 2.65$

FIG. 6.15 - VARIATION OF THE EQUIVALENT STRESS AT VARIOUS MANDREL CROSS SECTIONS. ALUMINIUM (a , b) AND COPPER (c) EXTRUSIONS AT SEVERAL RATIOS.



b) ALUMINIUM EXTRUSION, R = 3.5



c) COPPER EXTRUSION, R = 2.0

FIG. 6.15 (CONTD) - VARIATION OF THE EQUIVALENT STRESS AT VARIOUS MANDREL CROSS SECTIONS.

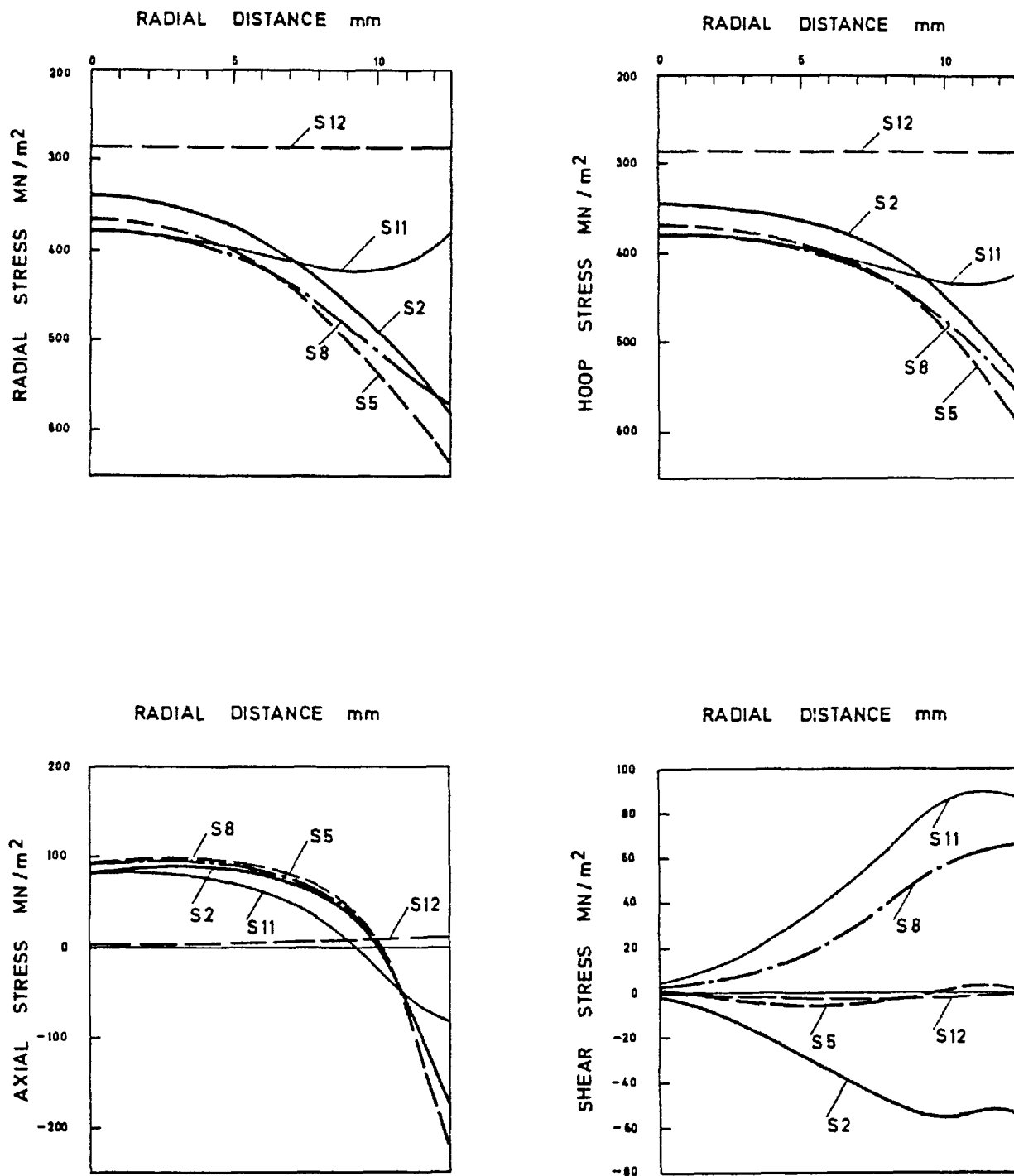


FIG. 6.16 - VARIATION OF σ_r , σ_θ , σ_z , AND τ_{rz} AT SEVERAL CROSS SECTIONS OF A SOLID MANDREL IN THE R-DIRECTION FOR COPPER EXTRUSION.

R = 2.0

CHAPTER 7

RESULTS. COMPARISON BETWEEN THEORY AND EXPERIMENTS.

CONCLUSIONS.

7.1 Introduction

In a previous chapter the experimental apparatus was described to measure extrusion pressure, drawing force, friction force along the billet/mandrel interface and the hoop and axial strains in the bore of the mandrel, during the hydrostatic extrusion-drawing of tubes.

Later, the theoretical solutions related to the plastic deformation of the billet and the elastic behaviour of the mandrel were discussed.

In this chapter the theoretical predictions will be compared with the experimental measurements in order to assess the degree of accuracy of the approximations used and the validity of the assumptions made.

7.2. Friction Force at the Billet-Mandrel Interface

The experimentally obtained variation of the friction force with the extrusion ratio and the extrusion pressure-drawing stress ratio is illustrated in Figure 7.1.a for aluminium extrusions and Figure 7.1.b for copper extrusions.

These illustrations show that the magnitude of the friction force measured in the hydrostatic extrusion-drawing experiments is small. In fact the maximum value of the friction force is 4100N for aluminium extrusion at a ratio of 3.5 and the lowest value is 1500N for copper extrusion at a ratio of 1.5. These low values of the friction force

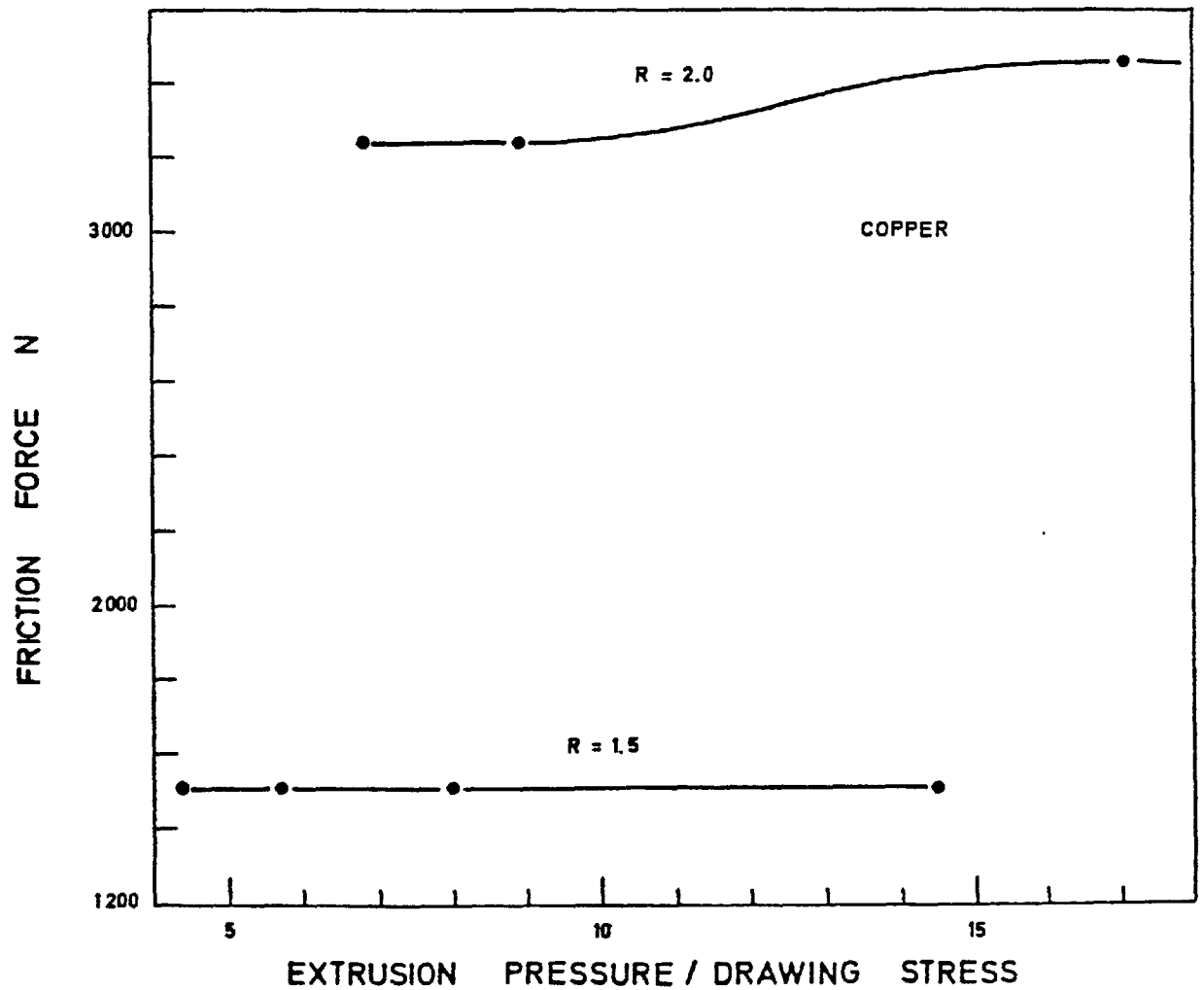
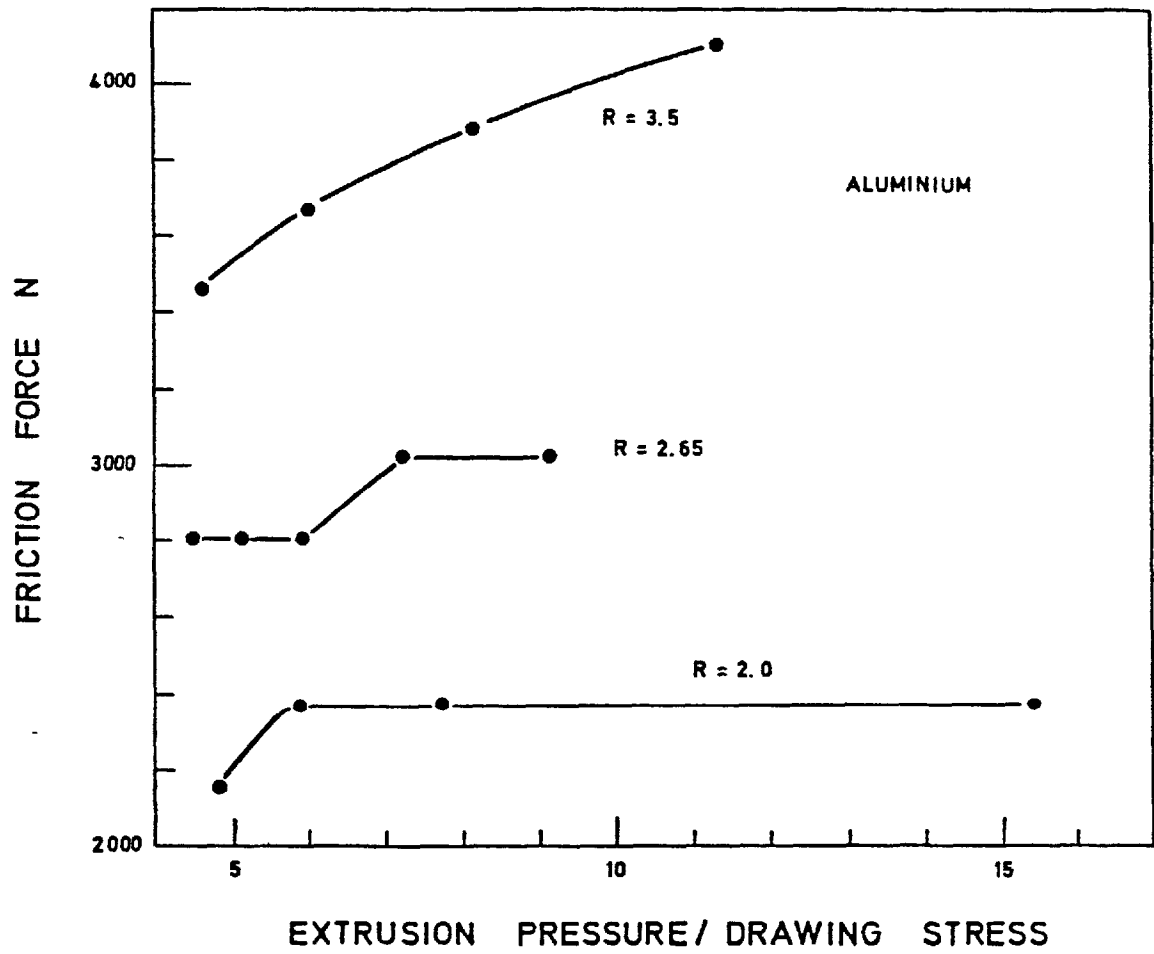


FIG. 7.1 - VARIATION OF THE FRICTION FORCE WITH THE EXTRUSION PRESSURE/DRAWING STRESS RATIO FOR ALUMINIUM AND

can be explained by the following reasons:

1. The small area of contact between mandrel and billet.

In the hydrostatic extrusion of tubes it is possible to reduce the area of contact to a very small value as a gap can be left between the undeformed part of the billet and the mandrel as shown in Section 2.2.1.2.7.

2. Lubrication between the plastically deforming material and the mandrel was provided by the high pressure fluid and the graphite coating of the billet.

The values of the calculated friction factor (Figure 7.2) are small (between 0.017 and 0.047) indicating that friction at the billet/mandrel interface is low.

Figure 7.1 also shows that the friction force increases with the extrusion ratio and, for a given extrusion ratio, with the extrusion pressure/drawing stress ratio. However, for copper extrusion at a ratio of 1.5 no variation in the friction force was detected. Also in other cases with the exception of aluminium extrusion at a ratio of 3.5 the curves of Figure 7.1 have regions in which the friction force is constant. The likely explanation is that the variation in friction was so small that the increase in load could not be detected by the present arrangement which is unsuitable to record differences less than 216N.

7.3 Extrusion Pressure - Drawing Stress Relationship

Figure 7.2 illustrates the comparison between the theoretical and the experimentally measured extrusion pressure-drawing stress relationship for aluminium and copper at several ratios.

The theoretical results were obtained by means of the lower upper bound approach as described in Chapter 4. The values of the

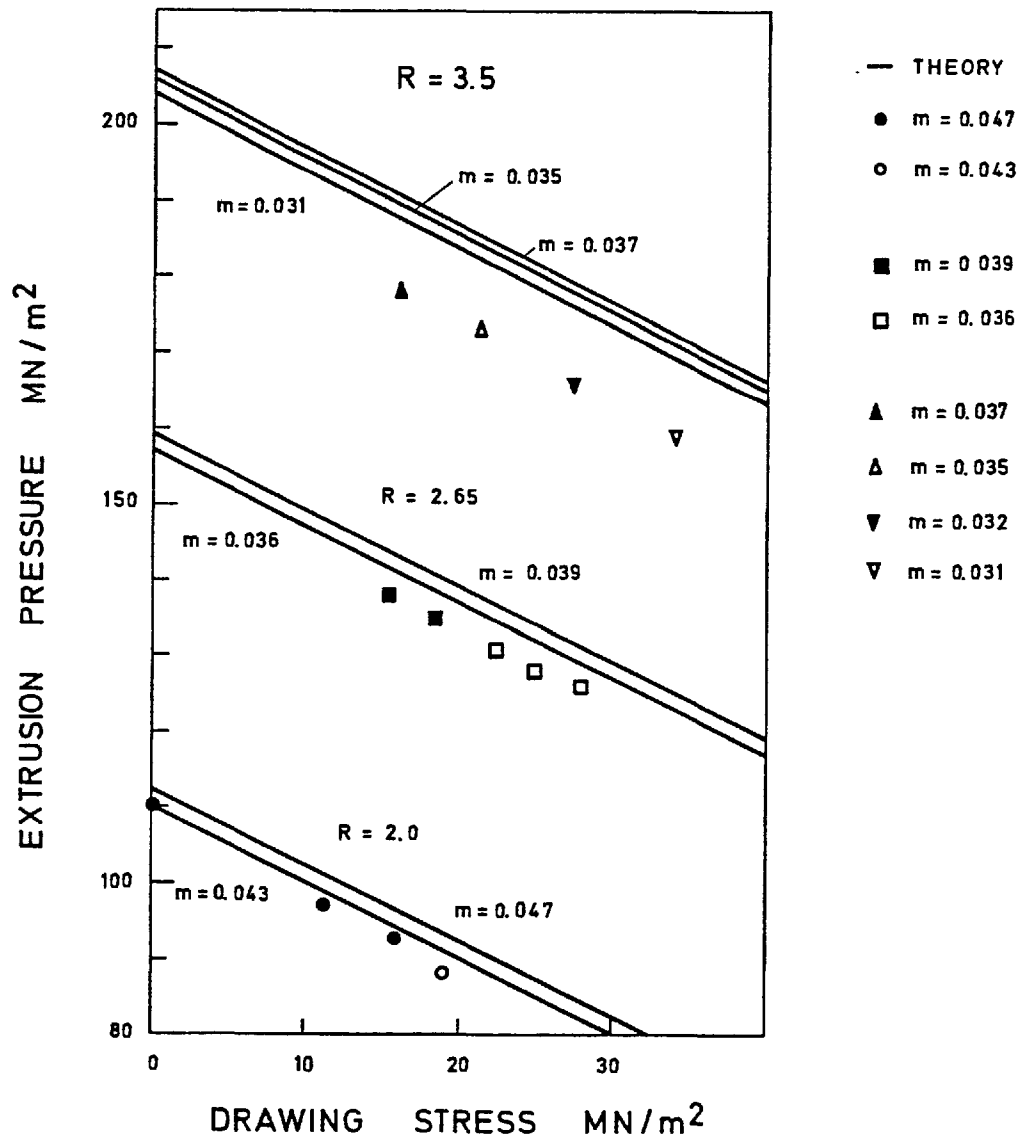


FIG. 7.2.a - EXTRUSION PRESSURE - DRAWING STRESS RELATIONSHIP
FOR ALUMINIUM EXTRUSIONS AT SEVERAL RATIOS.

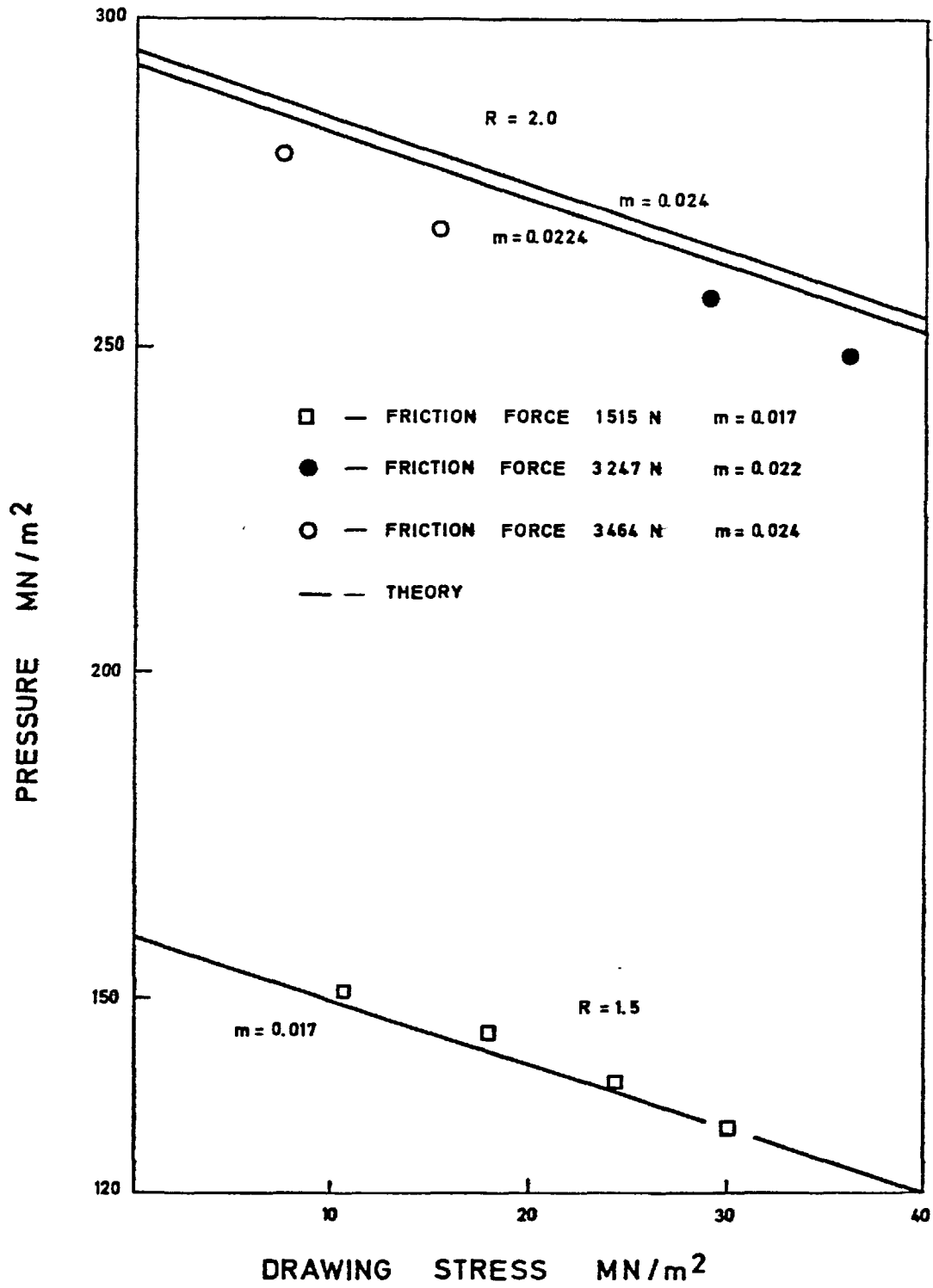


FIG. 7.2.b - EXTRUSION PRESSURE - DRAWING STRESS RELATIONSHIP
FOR COPPER EXTRUSIONS AT SEVERAL RATIOS.

friction factor, m , shown in Figure 7.2 were calculated by equation 4.50.

The experimental data show that the driving stress (the sum of the extrusion pressure and drawing stress) decreases with increasing drawing stress.

These conclusions drawn from the experiments in the hydrostatic extrusion-drawing of tubes confirm the earlier theoretical and experimental results (4,18) obtained for solid billets, that the requirement of energy is lower for a drawing biased than for an extrusion biased operation.

When the theoretical extrusion pressure is determined by an energy method and a constant friction factor is assumed, for a given extrusion ratio, the value of the driving stress is constant for all the theoretical extrusion pressure-drawing stress ratios. In Figure 7.2 the straight lines, with a constant value of m , lead to an extrusion pressure, in an extrusion process, equal to the drawing stress, in a drawing process.

The friction factor in the theoretical calculations is determined from the measured friction force for each extrusion pressure-drawing stress ratio, so it varies with this ratio according to the variation of the friction force. This is to say that the friction factor is not constant but varies with the extrusion pressure-drawing stress ratio.

Figure 7.2 indicates that in all cases for both materials, with the exception of copper extrusion at a ratio of 1.5, the predicted driving stresses are higher than those measured. The difference between the predicted and measured values is larger for the larger extrusion ratios. Thus, for aluminium extrusion at a ratio of 3.5, the error is 8.5% of the measured pressure, while at an extrusion ratio of 2.0 the error is within 6%.

Qualitatively similar results were obtained by Duffill and Mellor (148) and Osakada and Niimi (121) for the hydrostatic extrusion of solid billets.

7.4 Further Results from the Lower Upper Bound Approach. Experimental Verification

To obtain the lower upper bound extrusion pressure the total energy was minimized with respect to parameters z_A and z_B (Figure 7.3). These parameters define the plastically deforming region for the assumed conical entry and exit boundaries (see Chapter 4).

It is not possible to estimate the z -coordinate of the point A (Figure 7.3) from the billets used in the experiments, as no grid lines were inserted in them. However it is possible to estimate the z -coordinate of point B by

$$z_B = \frac{R_1 - R_0}{\tan \alpha} + C'B$$

where the length $C'B$ is measured on the billet after extrusion.

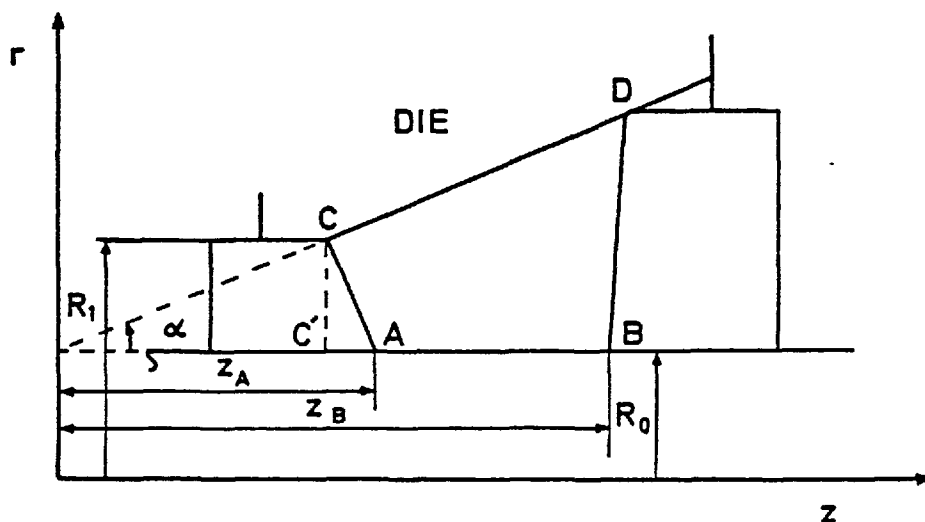


FIGURE 7.3

For this purpose some of the extruded specimens were split into two halves. The point B was easily identified in each half as a small gap had been left between the undeformed billet and the mandrel (see Section 2.2.1.2.7), enabling a clear distinction of the zone in contact with the mandrel. The point C is at the intersection of the conical surface in contact with the die and the product cylindrical surface. The point C' at the bore of the product has the same z-coordinate as C.

Tables I and II for copper and aluminium extrusions, respectively, summarize the theoretical and experimental results. The values of z_A and z_B show that, for a given extrusion ratio, the effect of friction on the plastic deformation zone is very small in the range of extrusion pressure-drawing stress ratios used in the experiments. Moreover, the variation of z_A and z_B in aluminium and copper extrusions at the same ratio is also small, justifying that the corresponding flow patterns shown in Figure 4.20, were represented by the same illustration.

7.5 Strain and Stress Distributions in the Bore of the Hollow Mandrel

In Chapter 6 the stress distributions in the mandrel were presented when subjected to the predicted pressure and shear stress distributions over its external surface. The stress analysis was carried out by means of a finite element program which also calculates the strains at the nodal points.

The aim of the present section is to compare the finite element analysis results with those obtained experimentally. In this way it is possible to assess the accuracy of the theoretical pressure and shear stress distributions on the outside surface of the mandrel, particularly those evaluated by the weighted residuals method. This purpose could be achieved by either comparing, at the mandrel bore, the predicted and measured strains or the predicted and experimentally

TABLE I (COPPER EXTRUSIONS)

R	$F^*(N)$	$\sigma MN/m^2$	$p MN/m^2$	$F_f^* N$	m	Z_A mm	Z_B mm	Z_B^* mm
1.5	2335	10.47	148.6	1515	0.0174	11.43	12.58	13.47
1.5	5338	23.93	135.2	1515	0.0174	11.43	12.58	—
1.5	6672	29.92	129.2	1515	0.0174	11.43	12.58	—
2.0	1501	6.73	287.7	3464	0.0239	11.70	16.03	17.17
2.0	6394	28.67	263.7	3247	0.0224	11.70	16.03	—
2.0	8062	36.15	256.3	3247	0.0224	11.70	16.03	—

TABLE II (ALUMINIUM EXTRUSIONS)

R	$F^* N$	$\sigma MN/m^2$	$p NN/m^2$	$F_f^* N$	m	Z_A mm	Z_B mm	Z_B^* mm
2.0	0	0	112.3	2381	0.047	11.76	16.13	—
2.0	2339	10.49	101.8	2381	0.047	11.76	16.13	—
2.0	4115	18.45	91.9	2164	0.044	11.76	16.13	17.27
2.65	3392	15.21	143.6	3030	0.039	11.44	21.22	—
2.65	4170	18.70	140.1	3030	0.039	11.44	21.22	21.88
2.65	5561	24.93	132.2	2815	0.036	11.44	21.22	—
2.65	6172	27.67	129.4	2815	0.036	11.44	21.22	—
3.5	3503	15.71	192.8	4113	0.037	11.24	27.45	27.87
3.5	4726	21.19	185.7	3896	0.035	11.24	27.45	—
3.5	6061	27.18	178.4	3681	0.033	11.25	27.37	—
3.5	7562	33.91	170.1	3464	0.031	11.25	27.34	—

* - experimentally measured quantities

R - extrusion ratio

p - upper bound pressure

F - drawing force

 F_f - friction force σ - drawing stress

m - friction factor

derived stresses. It was decided to compare strains for the following reasons:

1. The only error that affects the experimentally recorded strains is the experimental error, while the experimentally derived stresses are also affected by errors due to interpolation as will be shown in Section 7.5.2.
2. The accuracy of the theoretical hoop strains at the mandrel bore is higher than the theoretical stresses as discussed in Section 6.2.

7.5.1 Strain Distribution at the Bore of the Mandrel

Experimentally recorded and theoretically predicted hoop and axial strains along the mandrel bore are illustrated in Figures 7.4 to 7.19 for aluminium and copper at several extrusion ratios and various extrusion pressure-drawing stress ratios. The strain distributions are shown relative to the strain gauge positions in the mandrel bore. In those figures the scale of the top drawings has been chosen so that the position of the strain gauges is indicated in the bottom drawings. Both the hoop and axial strains indicate the combined effect of the fluid pressure, together with the normal stress and frictional drag applied by the deforming material to the hollow mandrel.

These illustrations show that the predicted and experimentally measured strains are in close agreement for the hoop and axial strains. However the error between the predicted and the experimental strains is greater for axial than for hoop strains, which can be explained by the less accurate theoretical axial strains because they are calculated from the nodal stresses as discussed in Section 6.2.

The total axial strains are large compared with those due to the frictional drag. For example the maximum axial load at an extrusion

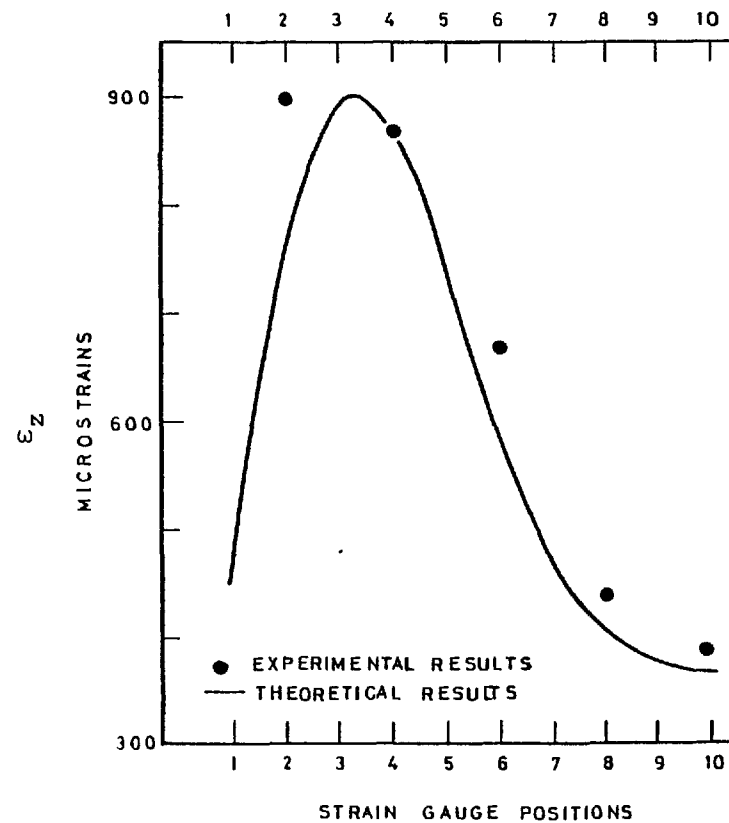
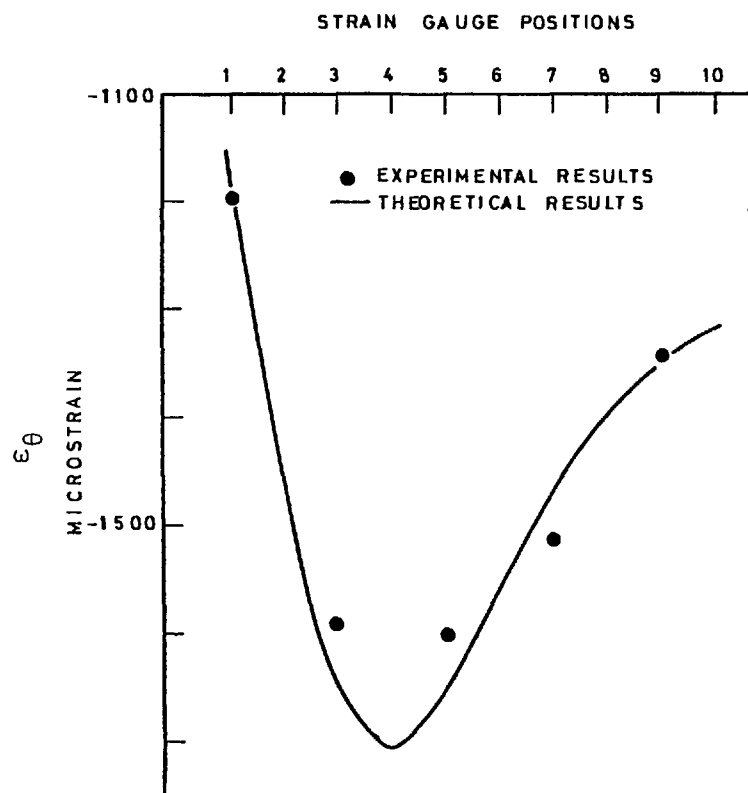
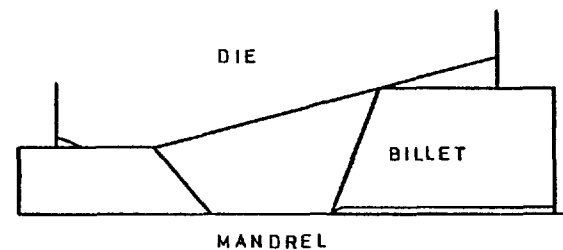
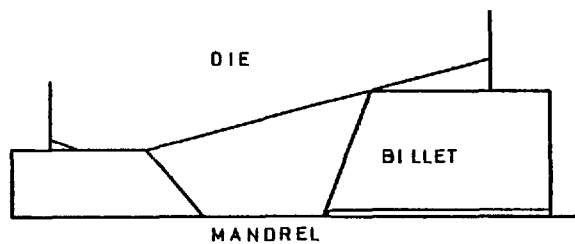


FIG. 7.4 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. ALUMINIUM EXTRUSION.

$$R = 2.0, \quad F = 0.0 \text{ N}, \quad p = 112.3 \text{ MN/m}^2$$

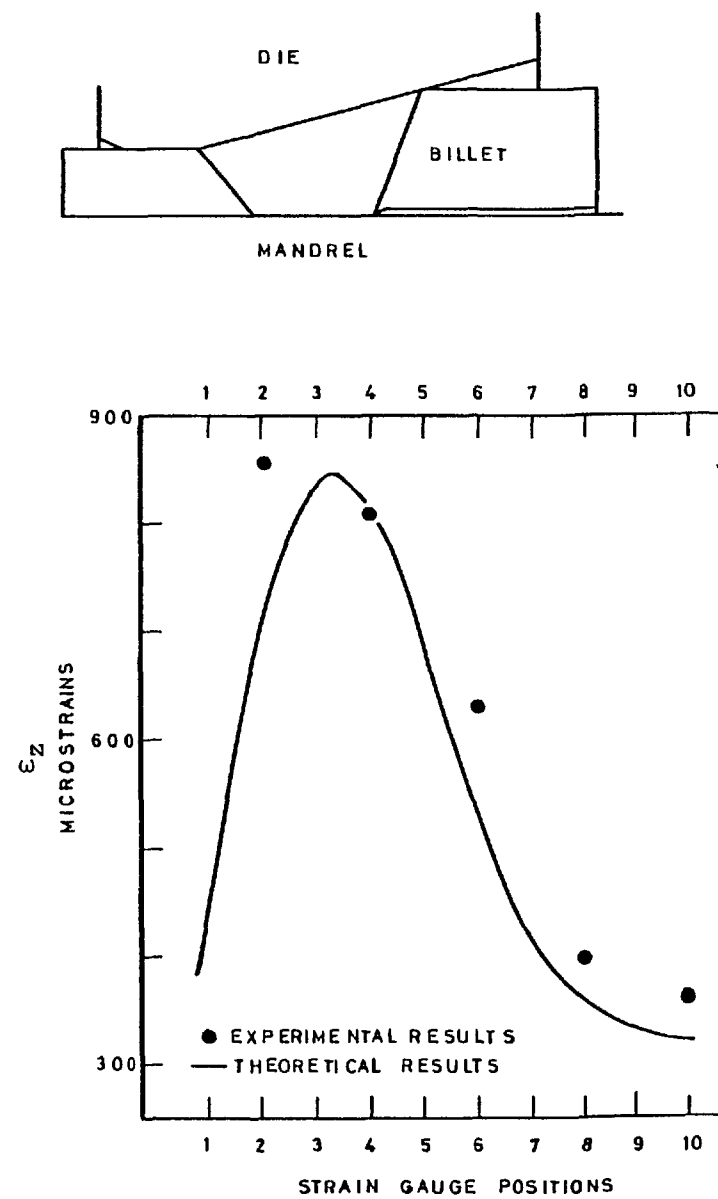
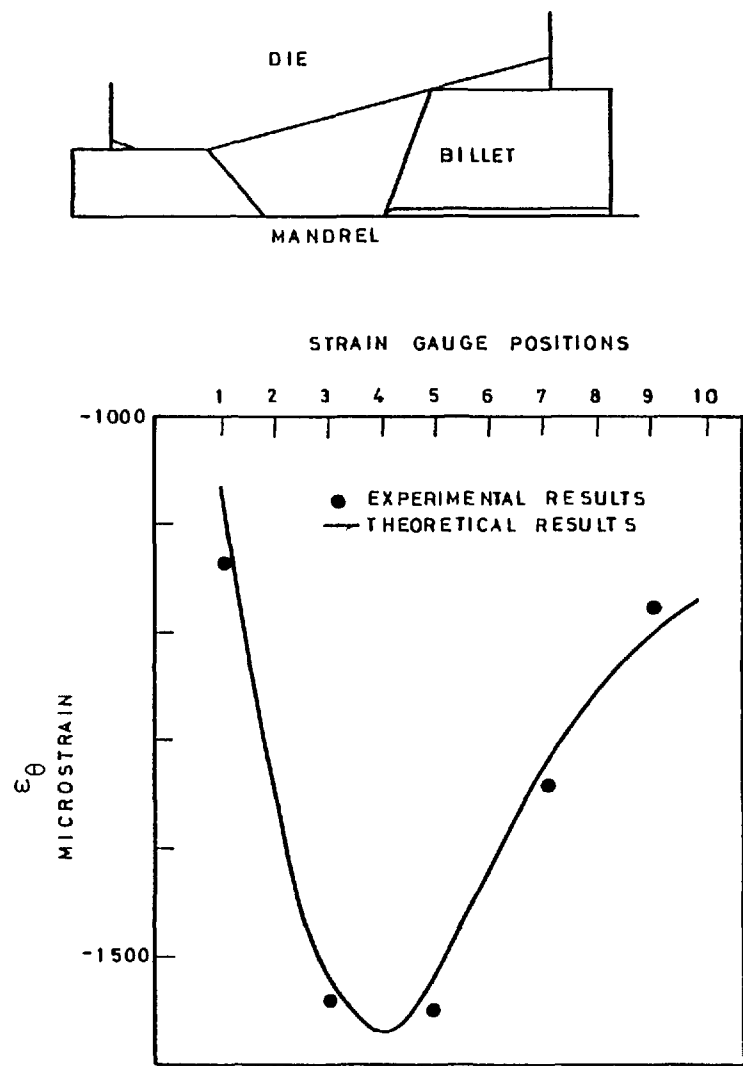


FIG. 7.5 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. ALUMINIUM EXTRUSION.

$$R = 2.0, \quad F = 2339 \text{ N}, \quad p = 101.8 \text{ MN/m}^2$$

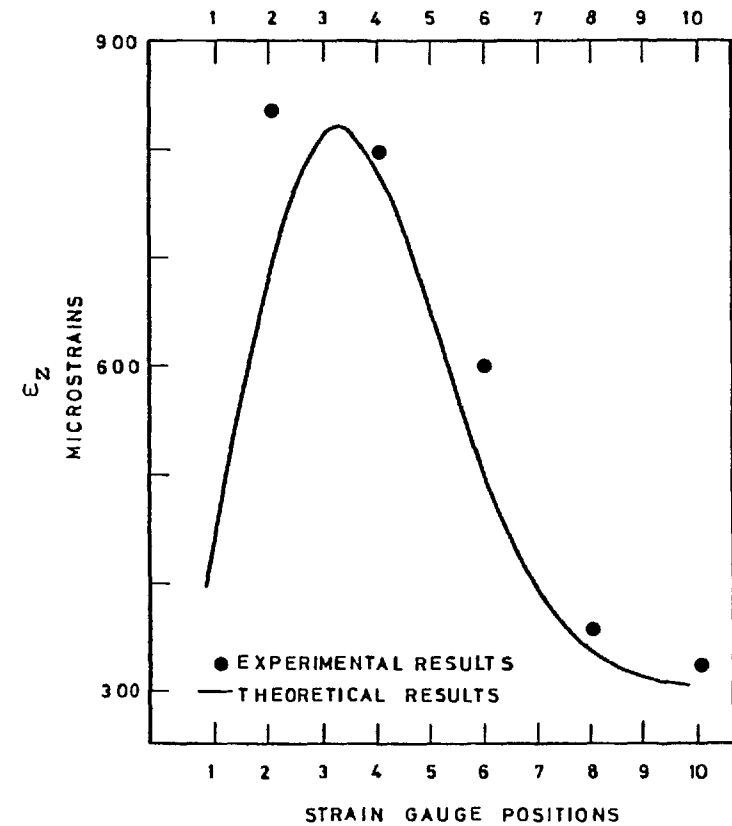
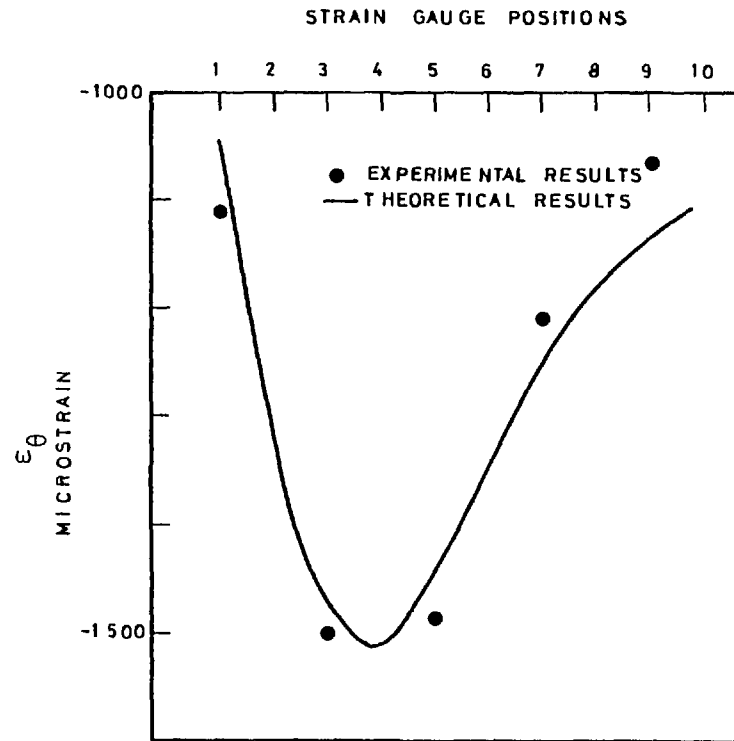
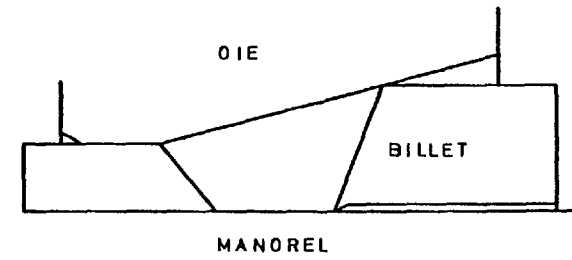
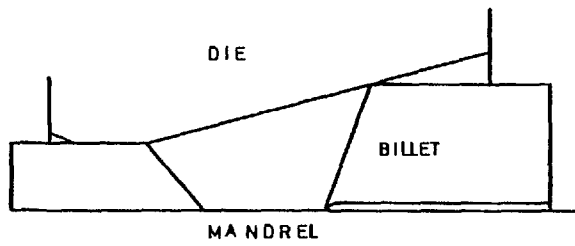


FIG. 7.6 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. ALUMINIUM EXTRUSION.

$$R = 2.0, \quad F = 4115 \text{ N}, \quad p = 91.9 \text{ MN/m}^2$$

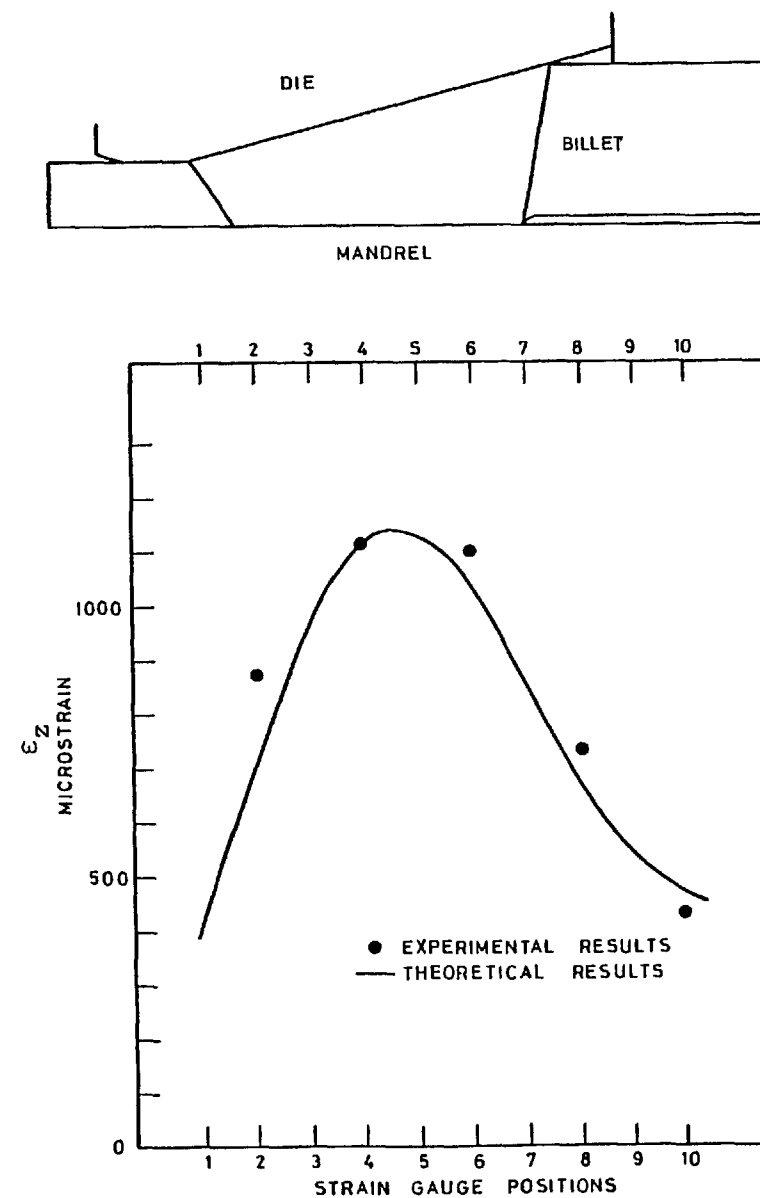
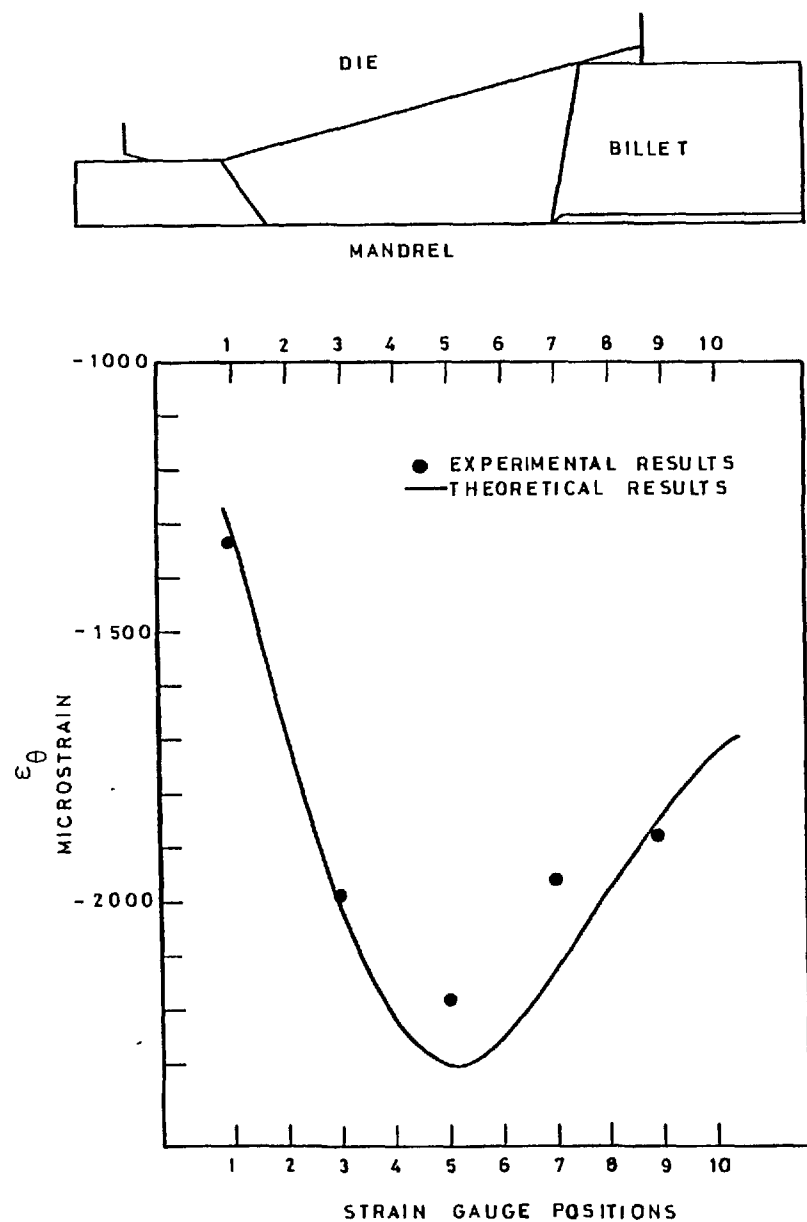


FIG. 7.7 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. ALUMINIUM EXTRUSION.

$$R = 2.65, \quad F = 3392 \text{ N}, \quad p = 143.6 \text{ MN/m}^2$$

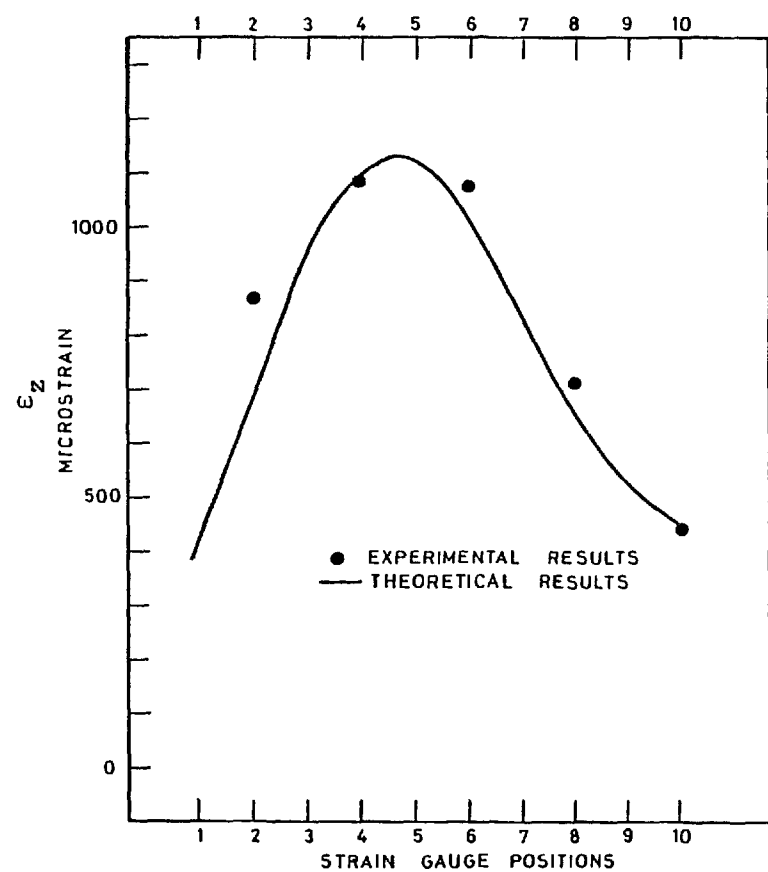
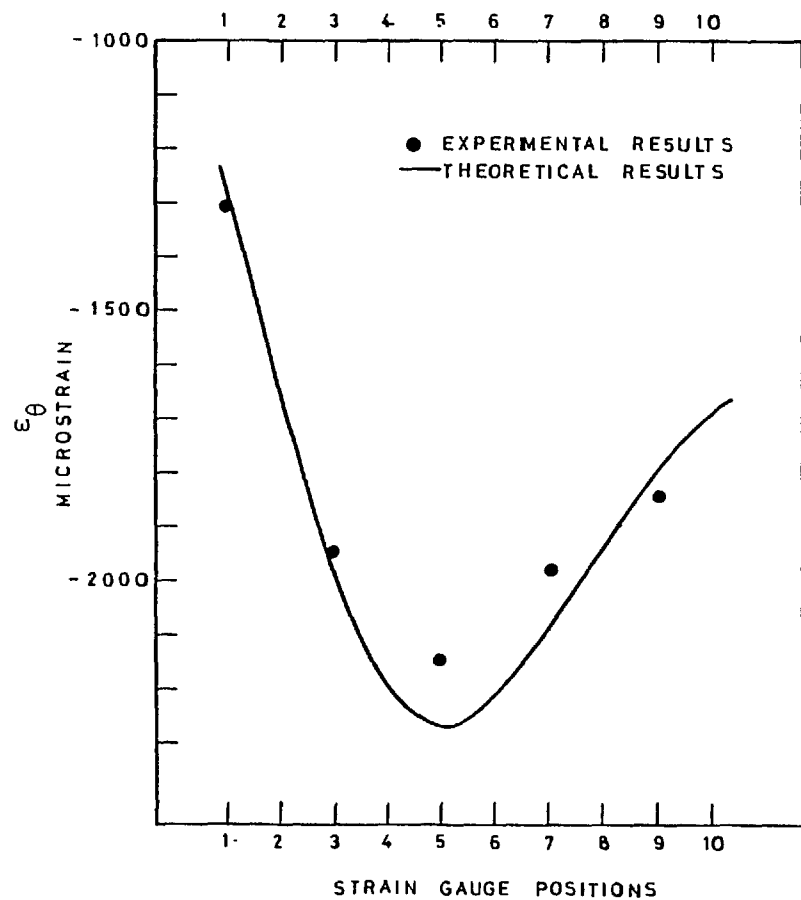
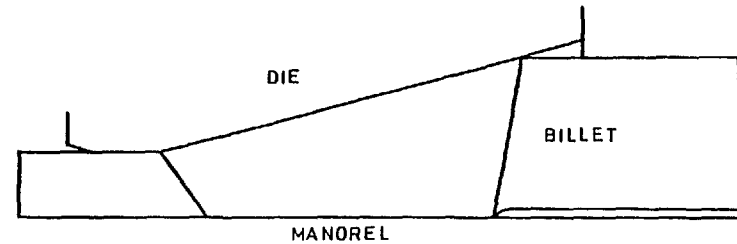
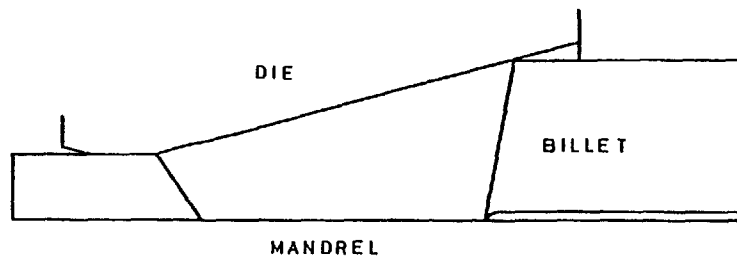


FIG. 7.8 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. ALUMINIUM EXTRUSION.

$$R = 2.65, \quad F = 4170 \text{ N}, \quad p = 140.1 \text{ MN/m}^2$$

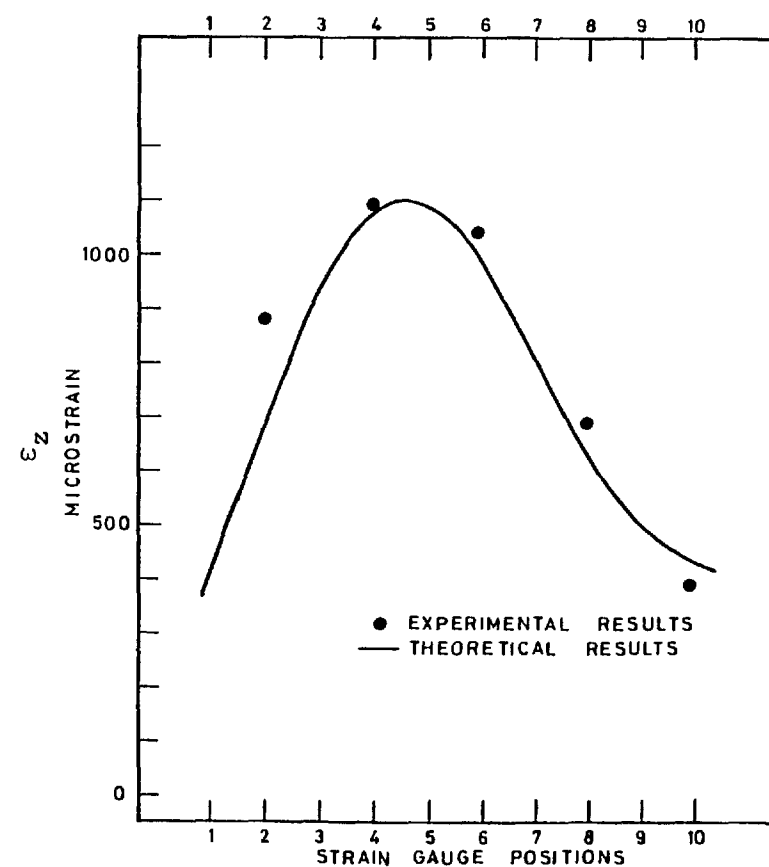
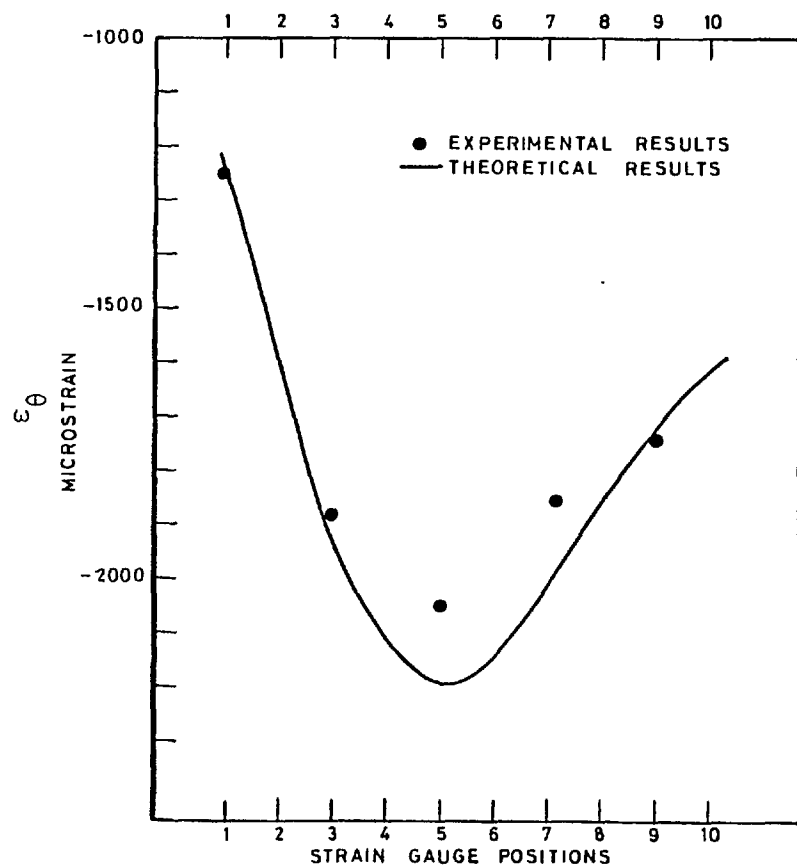
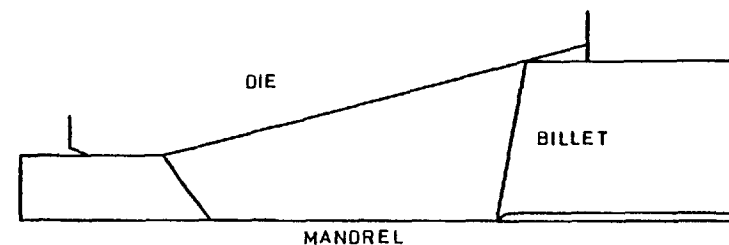
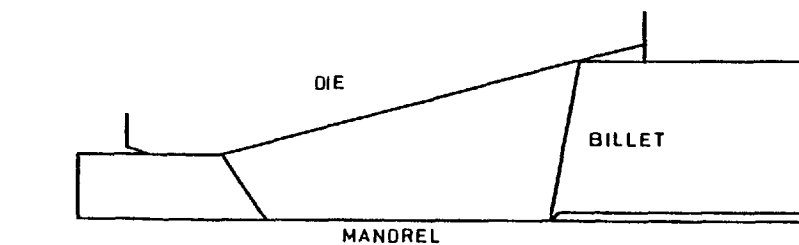


FIG. 7.9 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. ALUMINIUM EXTRUSION.

$$R = 2.65, \quad F = 5561 \text{ N}, \quad p = 132.2 \text{ MN/m}^2$$

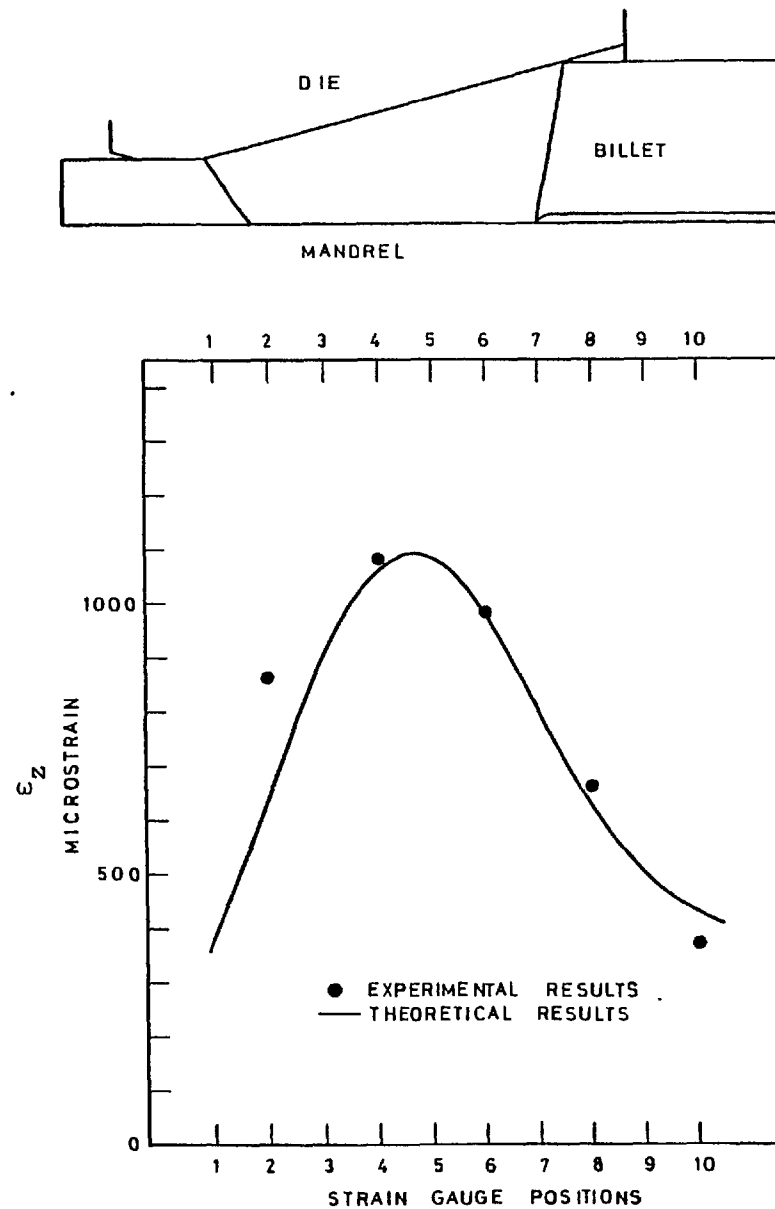
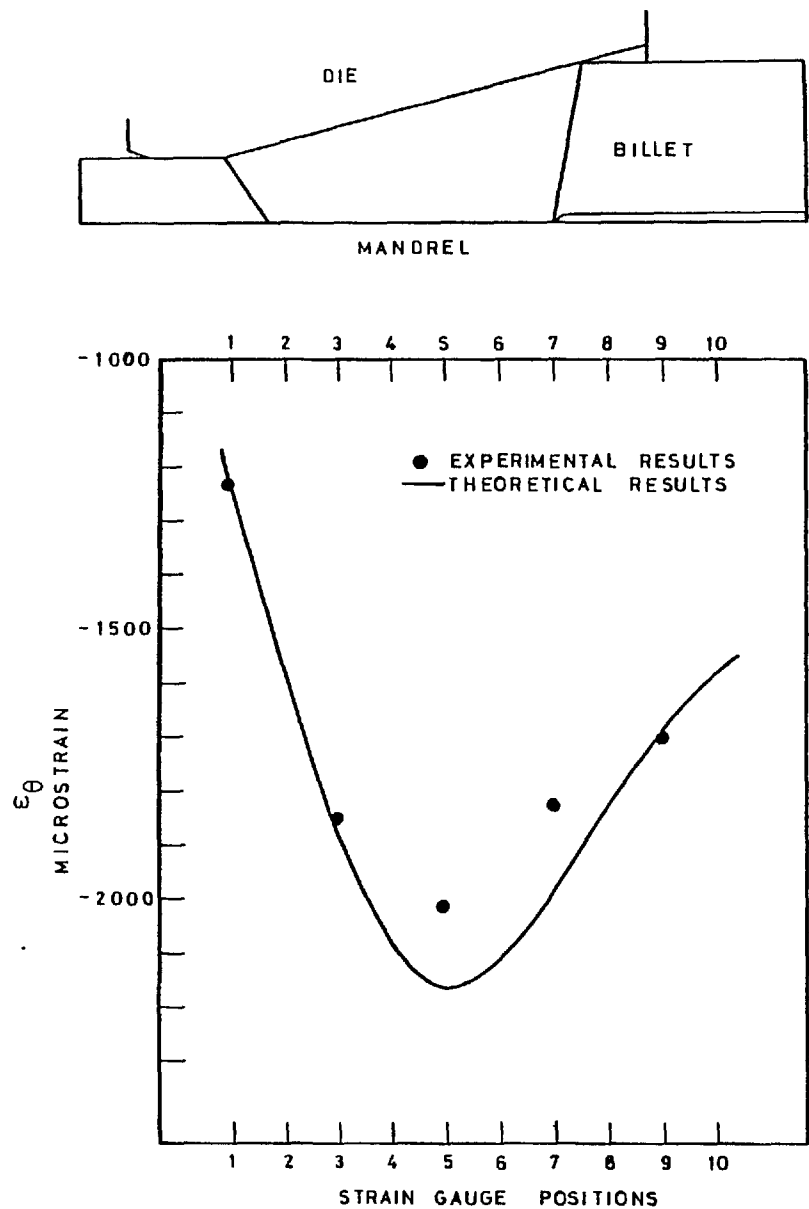


FIG. 7.10 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. ALUMINIUM EXTRUSION.

$$R = 2.65, \quad F = 6172 \text{ N}, \quad p = 129.4 \text{ MN/m}^2$$

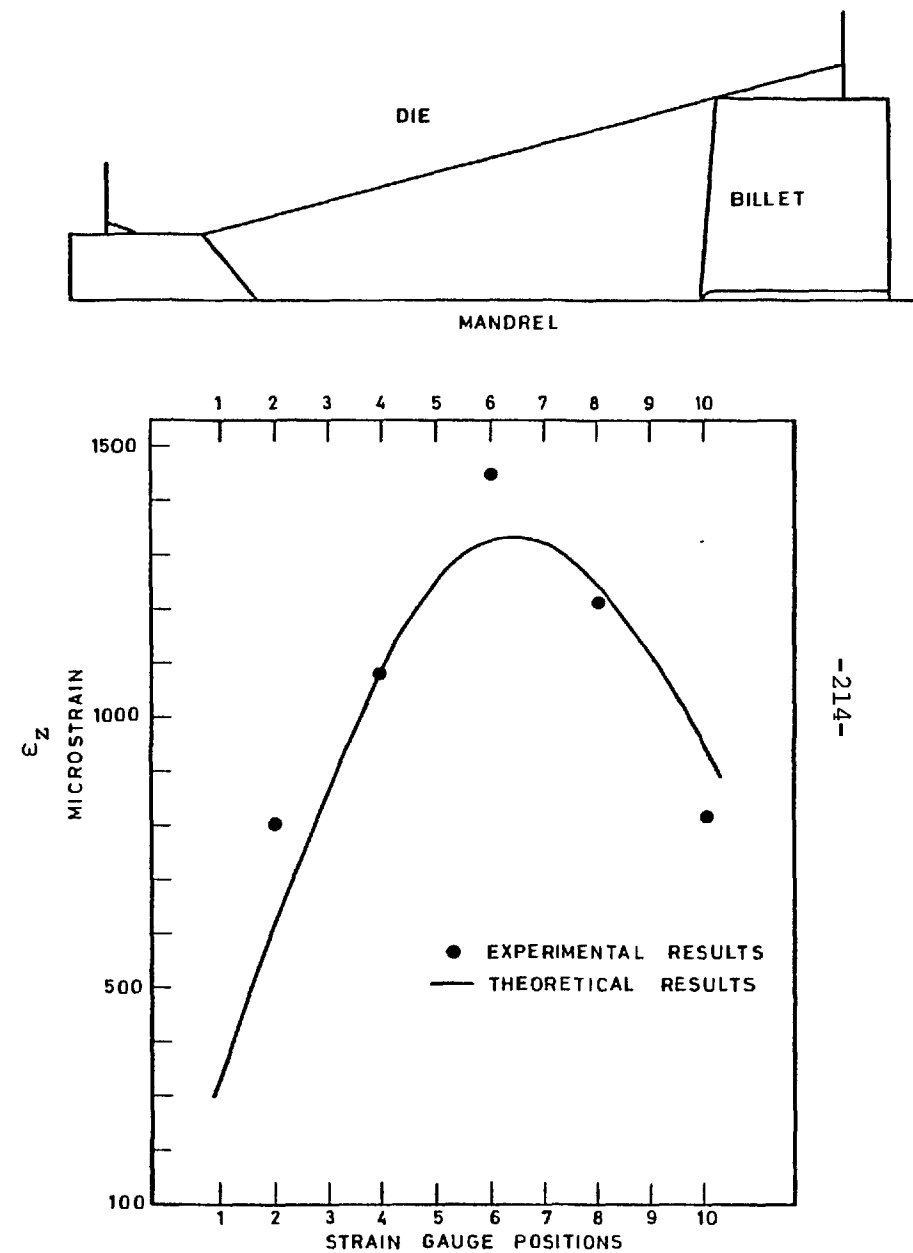
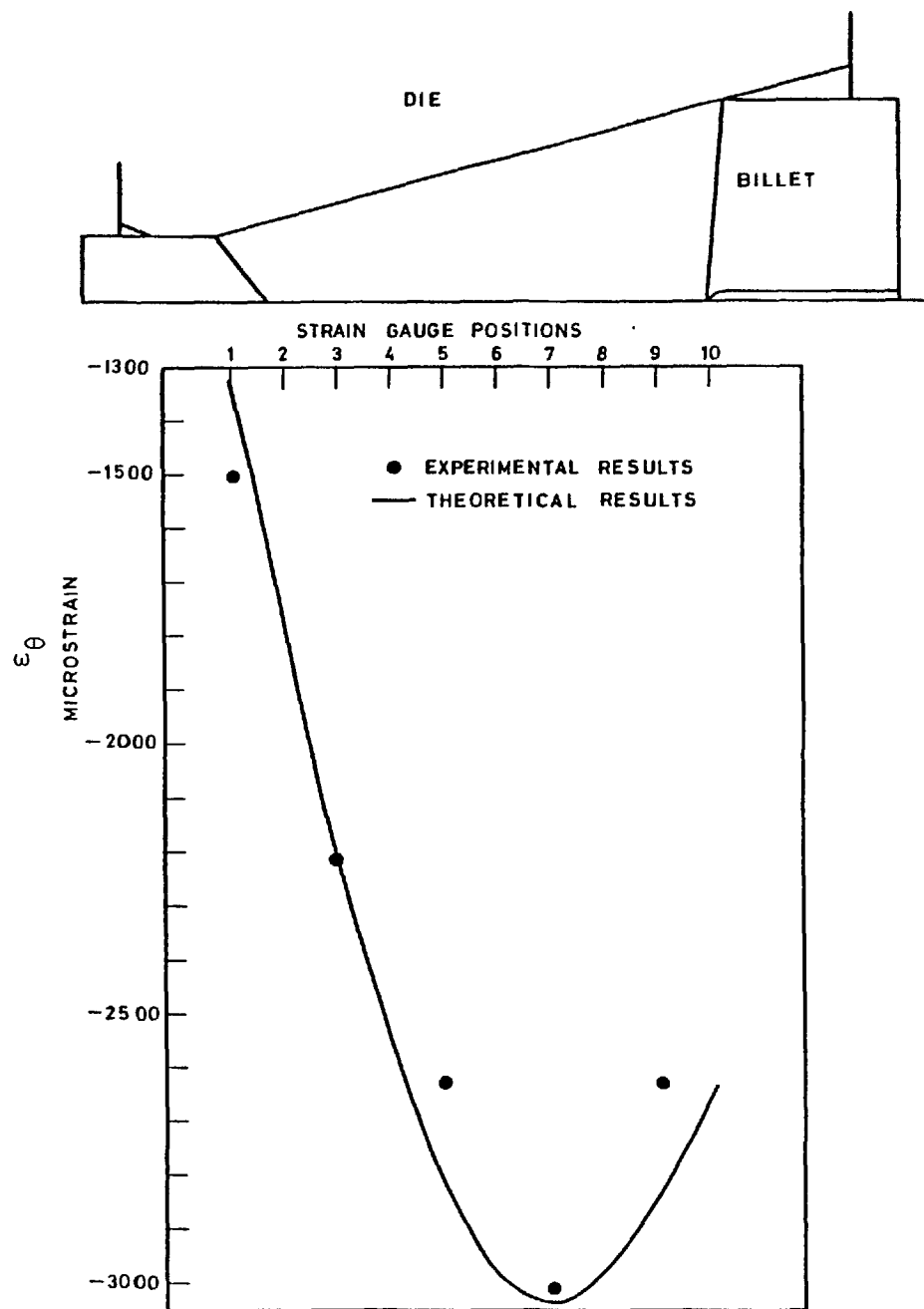


FIG. 7.11 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. ALUMINIUM EXTRUSION.

$$R = 3.5, \quad F = 3503 \text{ N}, \quad p = 192.8 \text{ MN/m}^2$$

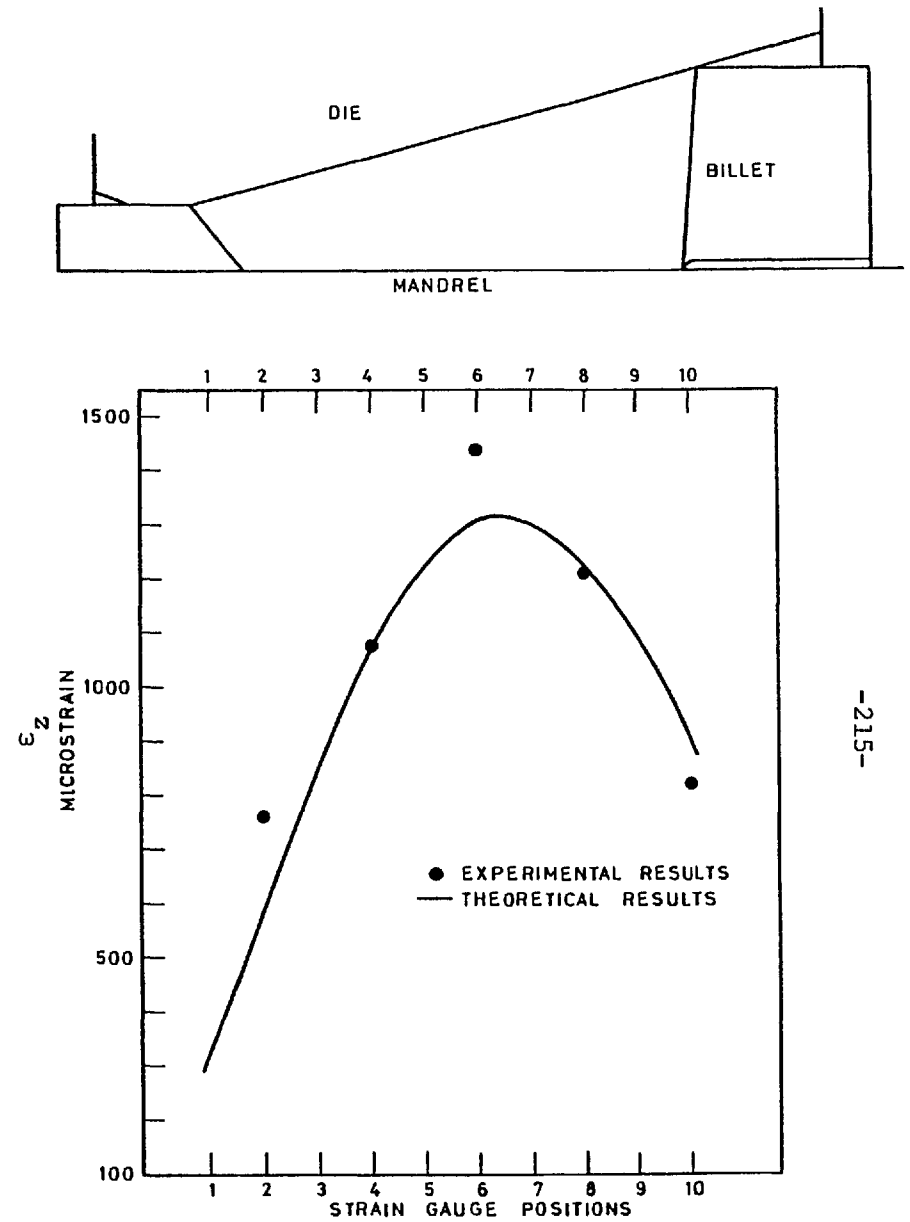
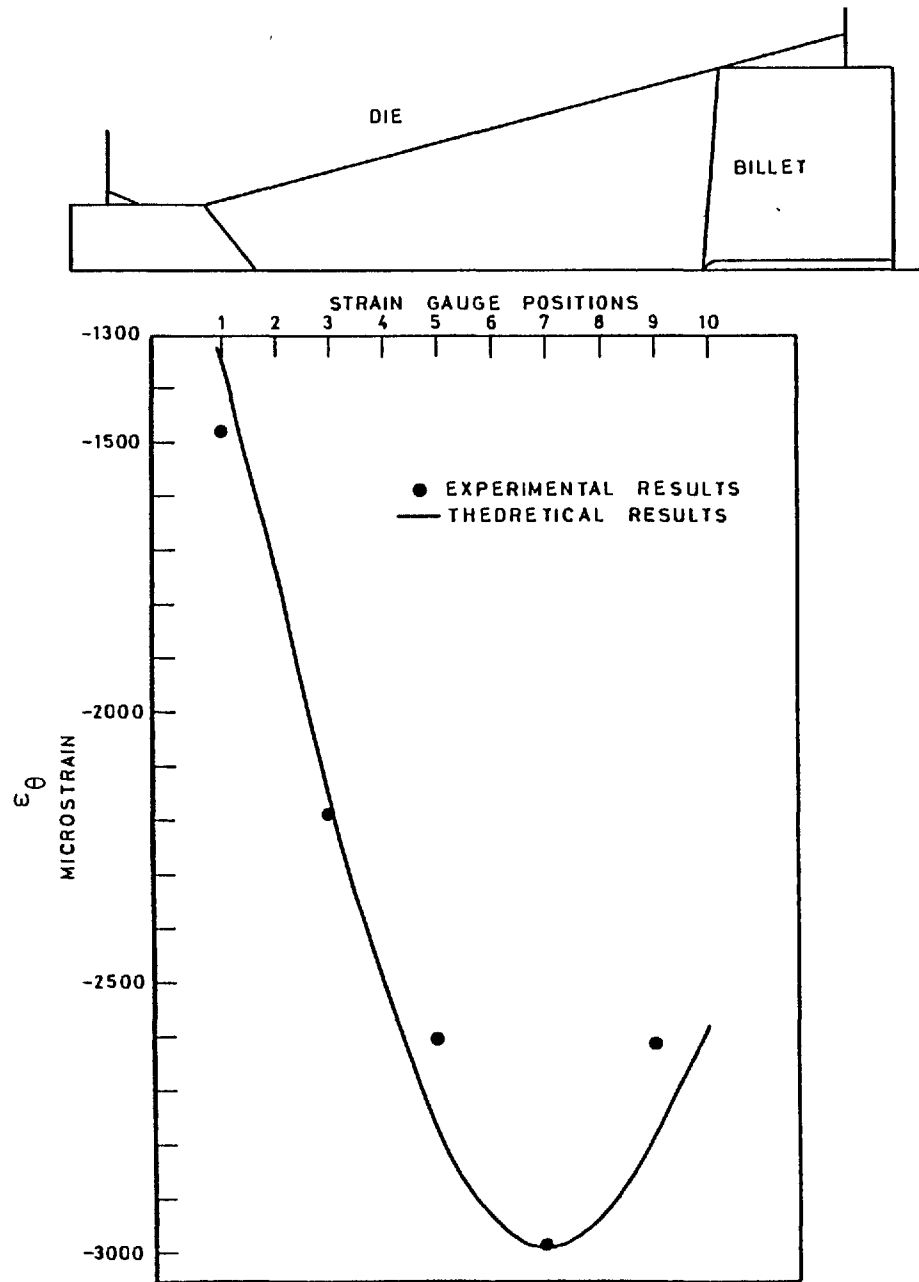


FIG. 7.12 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. ALUMINIUM EXTRUSION.

$$R = 3.5, \quad F = 4726 \text{ N}, \quad p = 185.7 \text{ MN/m}^2$$

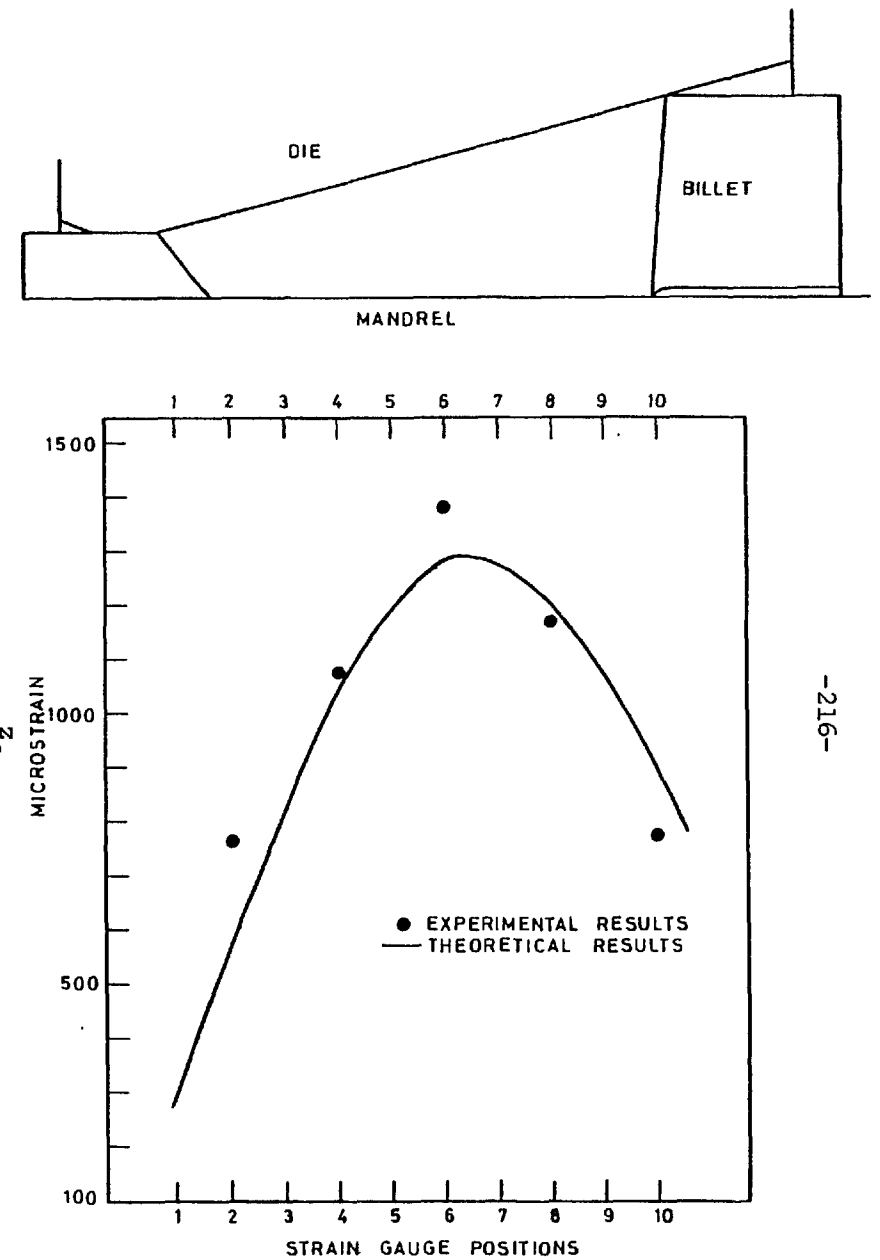
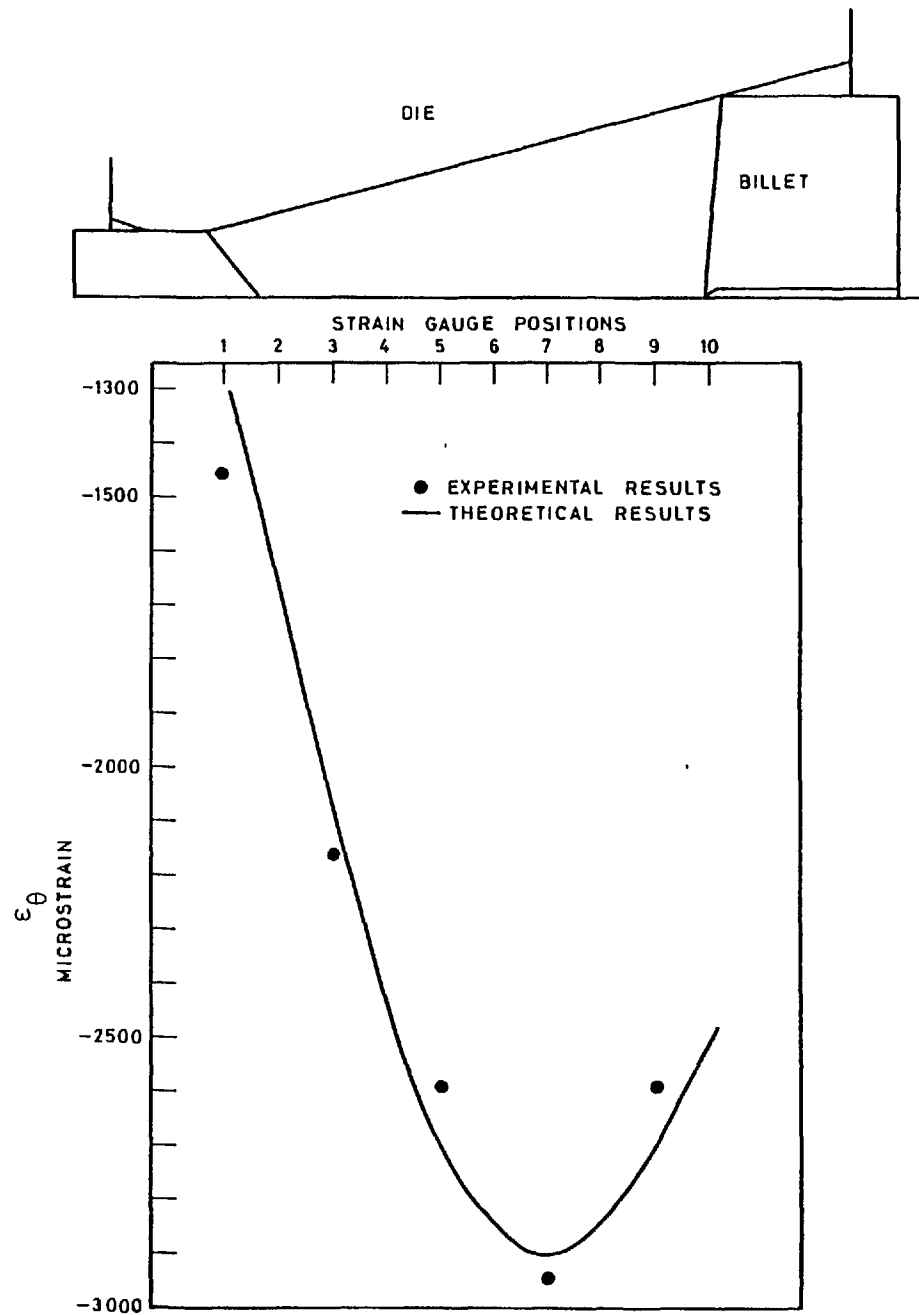


FIG. 7.13 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. ALUMINIUM EXTRUSION.

$$R = 3.5, \quad F = 6061 \text{ N}, \quad p = 178.4 \text{ MN/m}^2$$

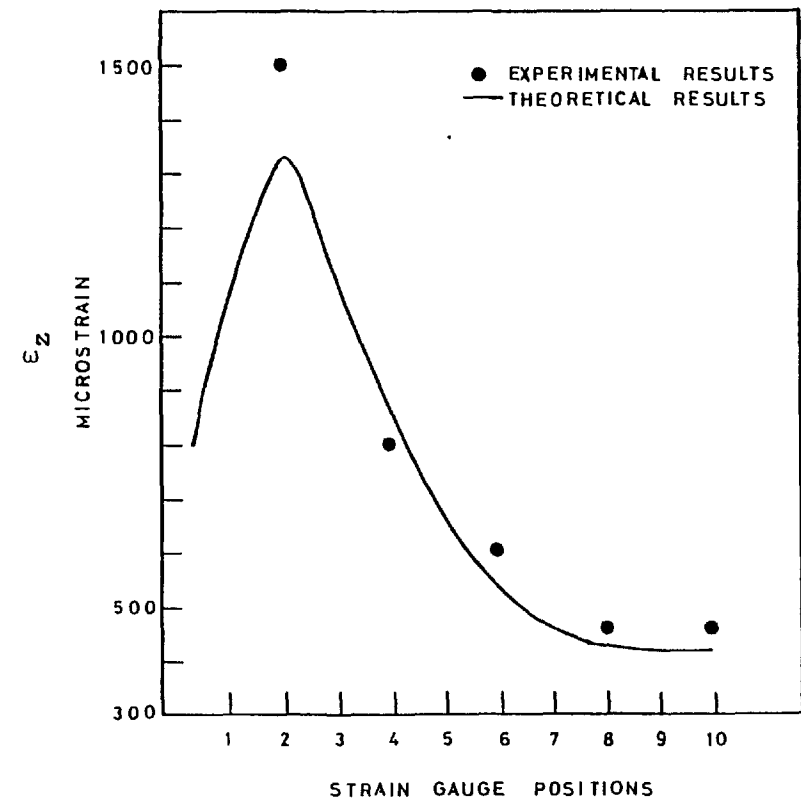
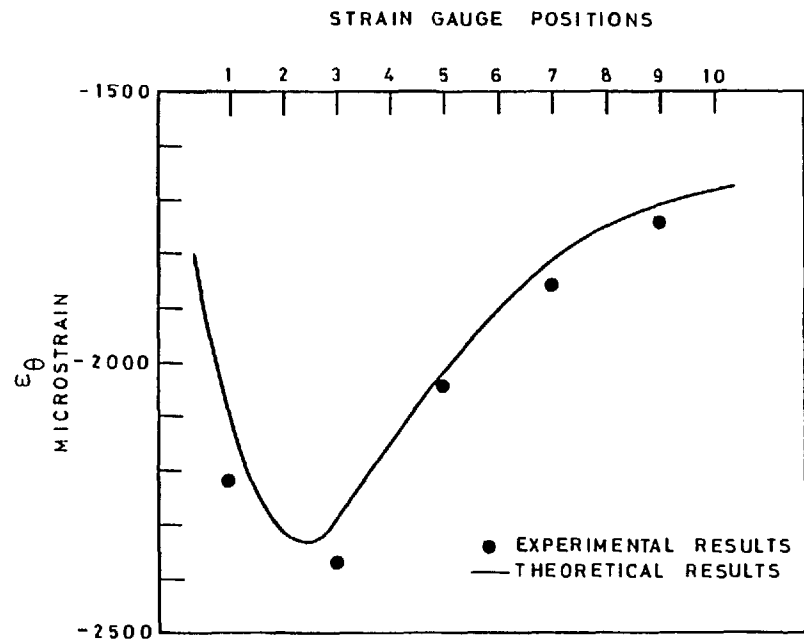
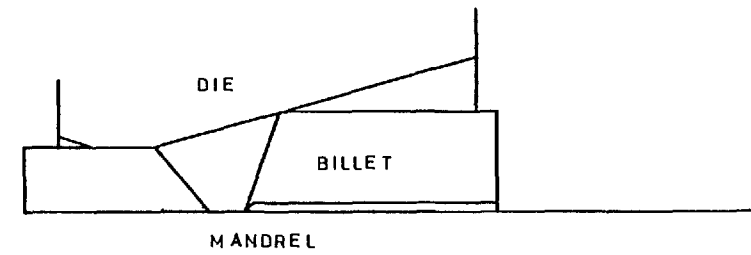
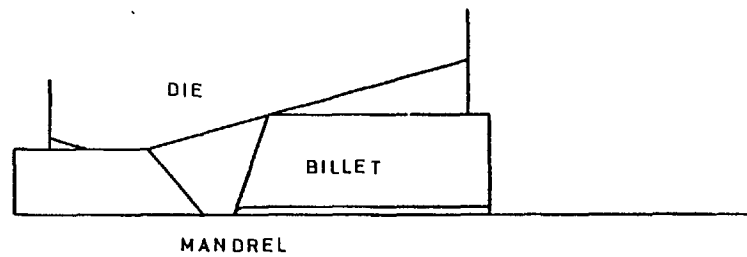


FIG. 7.14 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. COPPER EXTRUSION.

$$R = 1.5, \quad F = 2335 \text{ N}, \quad p = 148.6 \text{ MN/m}^2$$

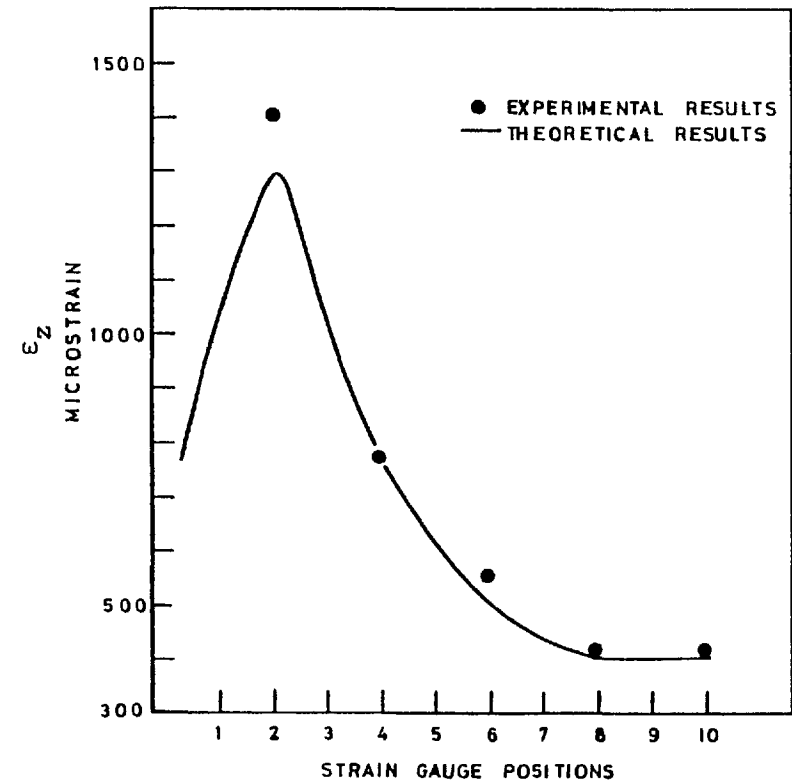
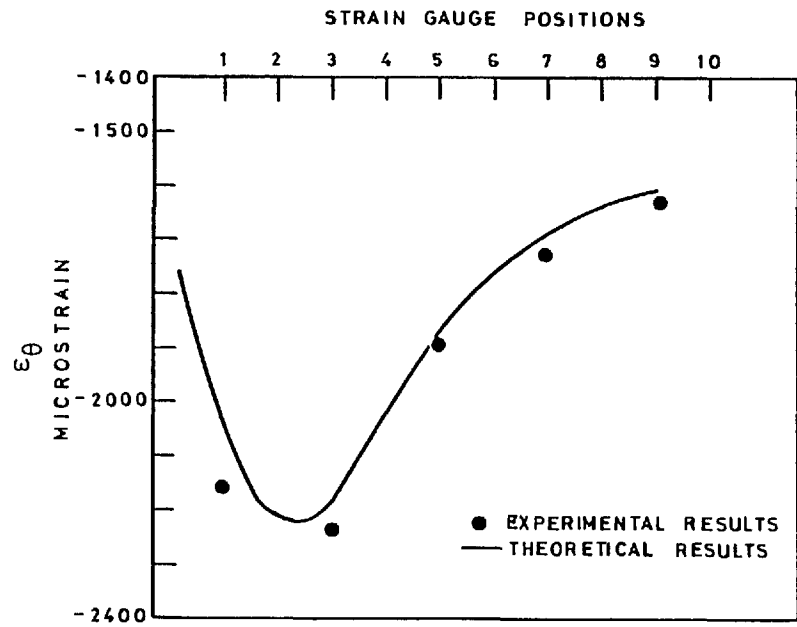
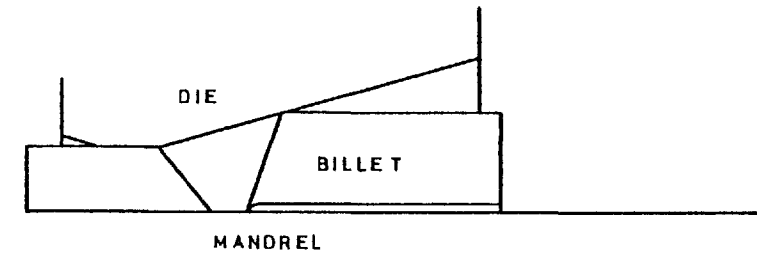
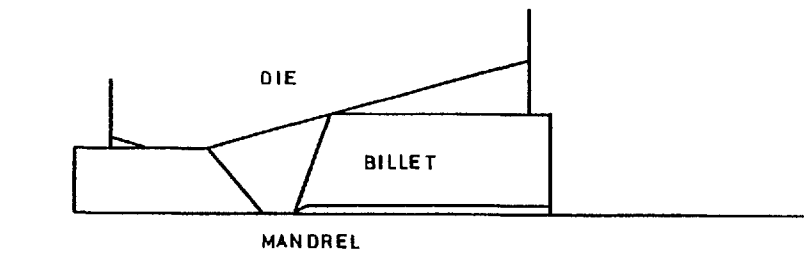


FIG. 7.15 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. COPPER EXTRUSION.

$$R = 1.5, \quad F = 5338 \text{ N}, \quad p = 135.2 \text{ MN/m}^2$$

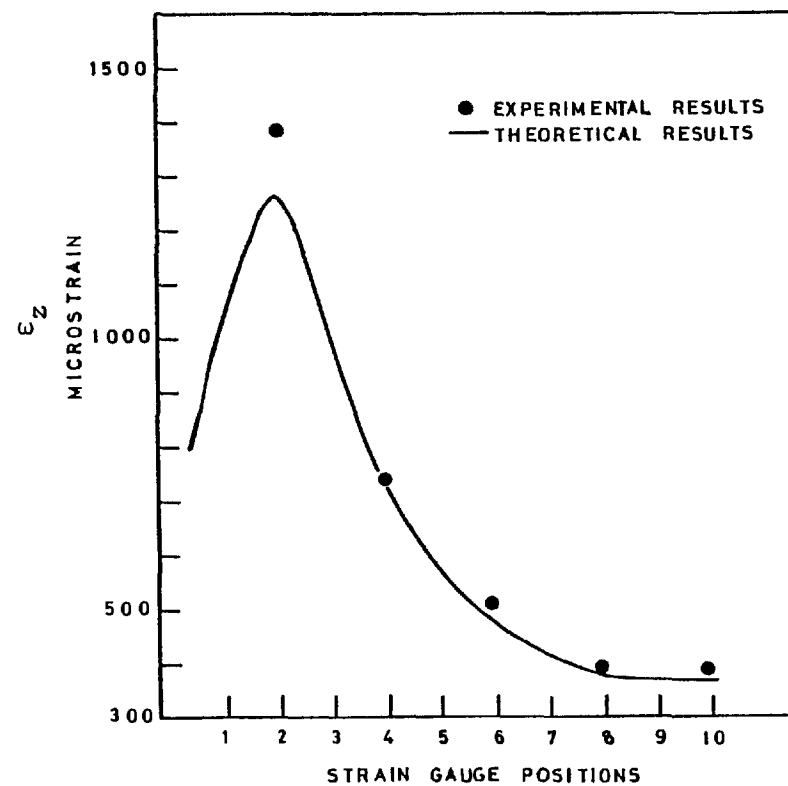
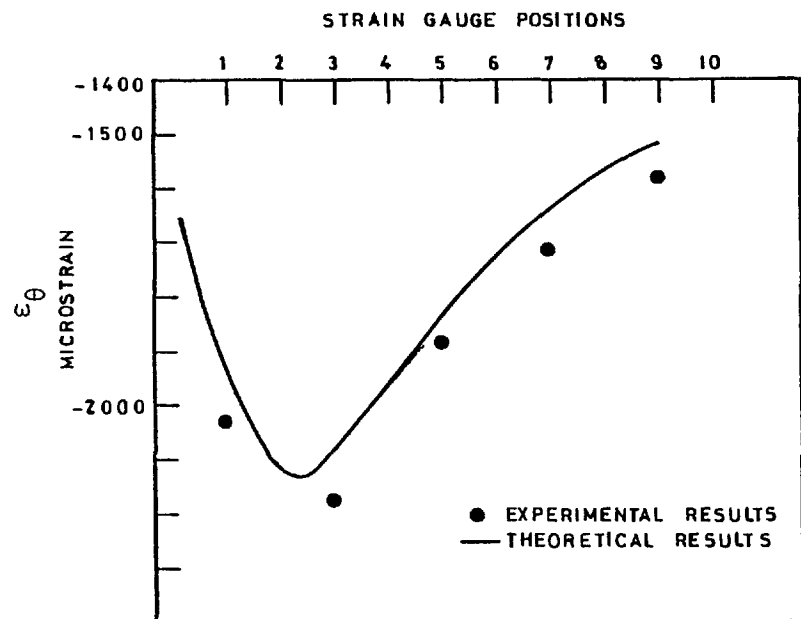
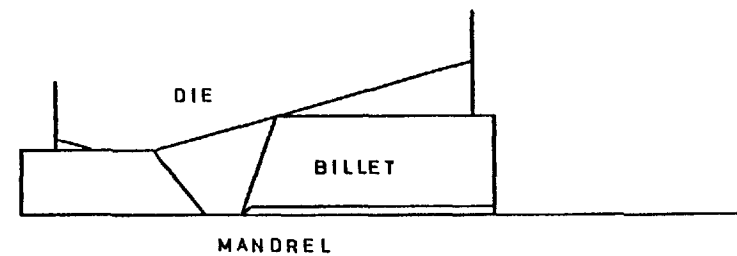
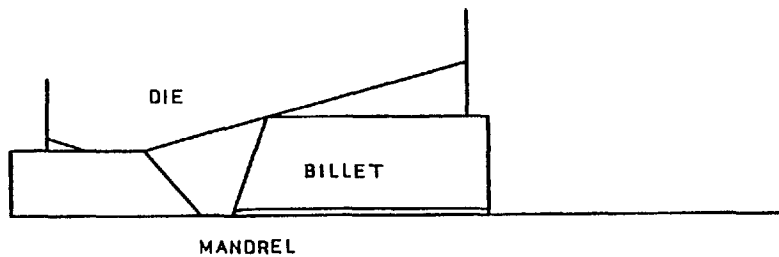


FIG. 7.16 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. COPPER EXTRUSION.

$$R = 1.5, \quad F = 6672 \text{ N}, \quad p = 129.2 \text{ MN/m}^2$$

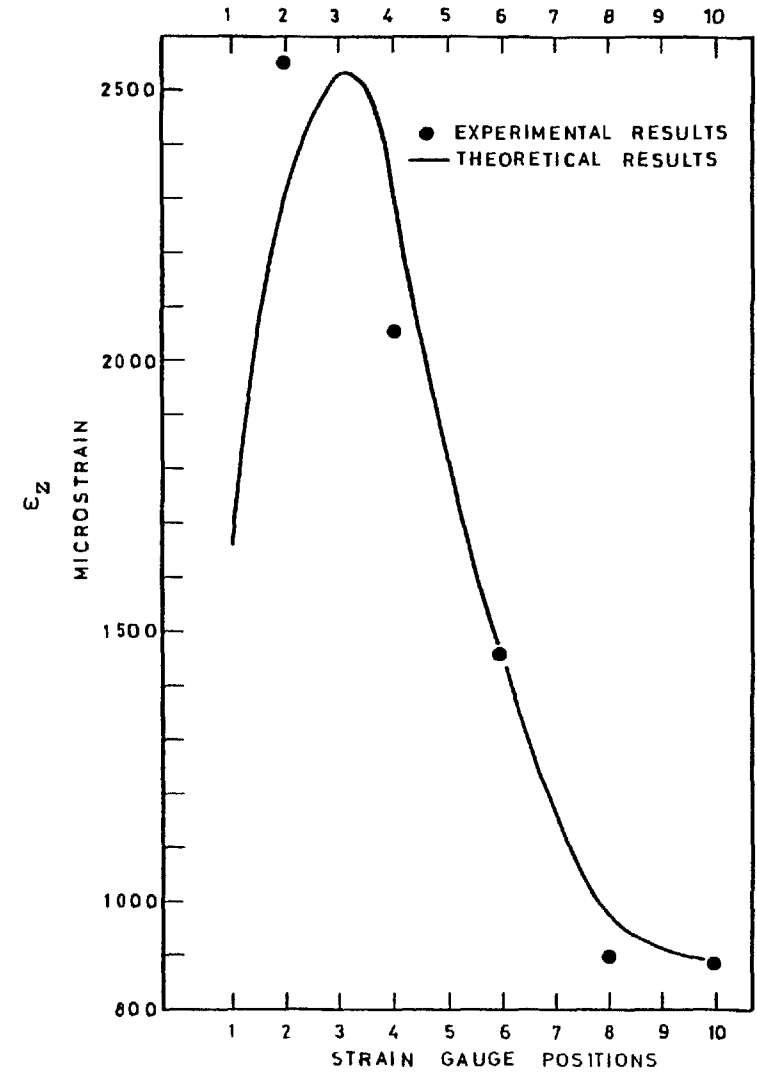
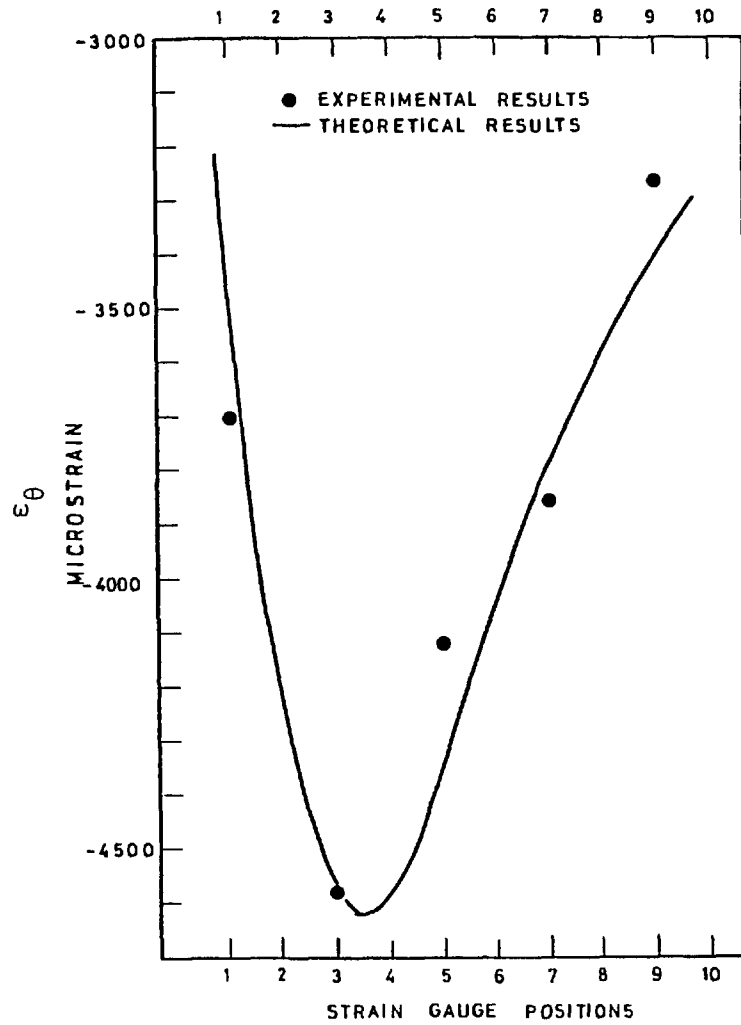
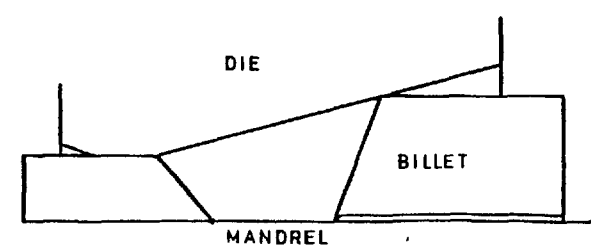
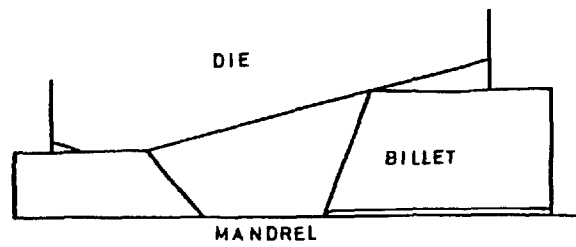


FIG. 7.17 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. COPPER EXTRUSION.

$$R = 2.0, \quad F = 1501 \text{ N}, \quad p = 287.7 \text{ MN/m}^2$$

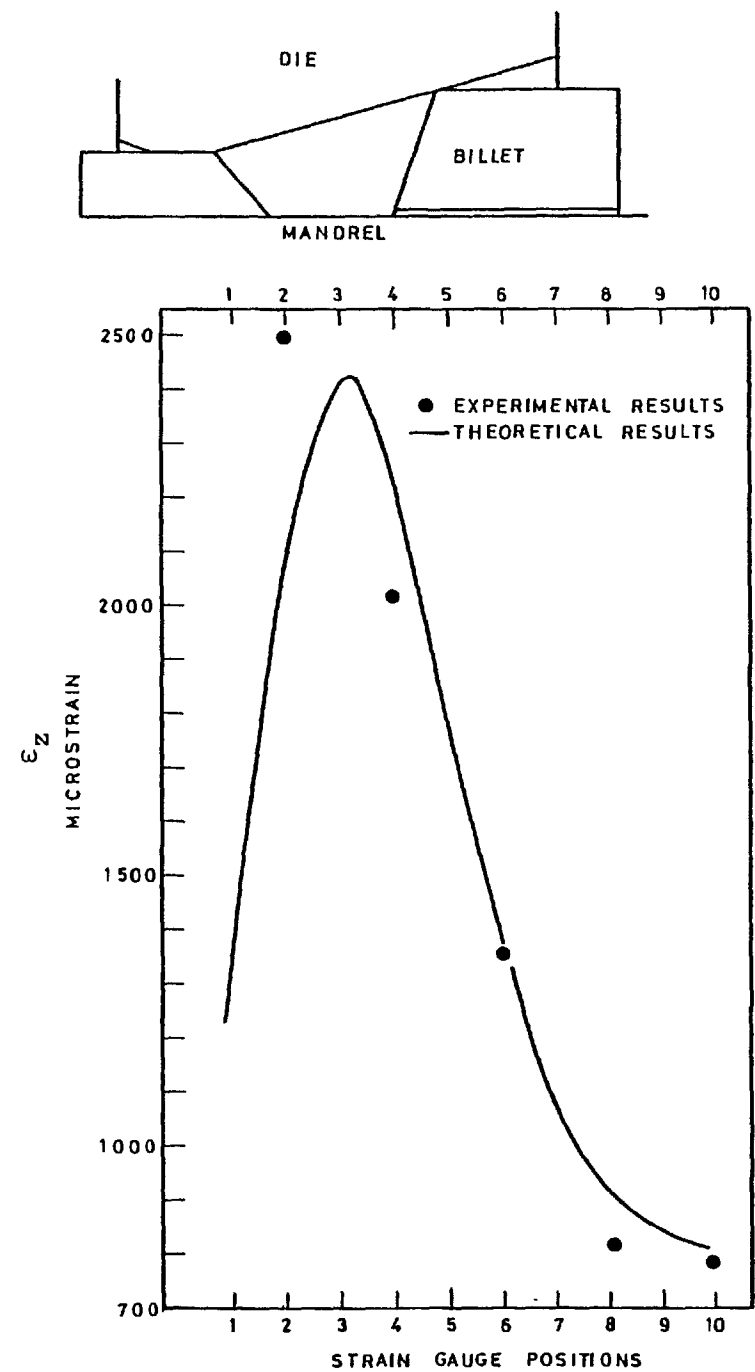
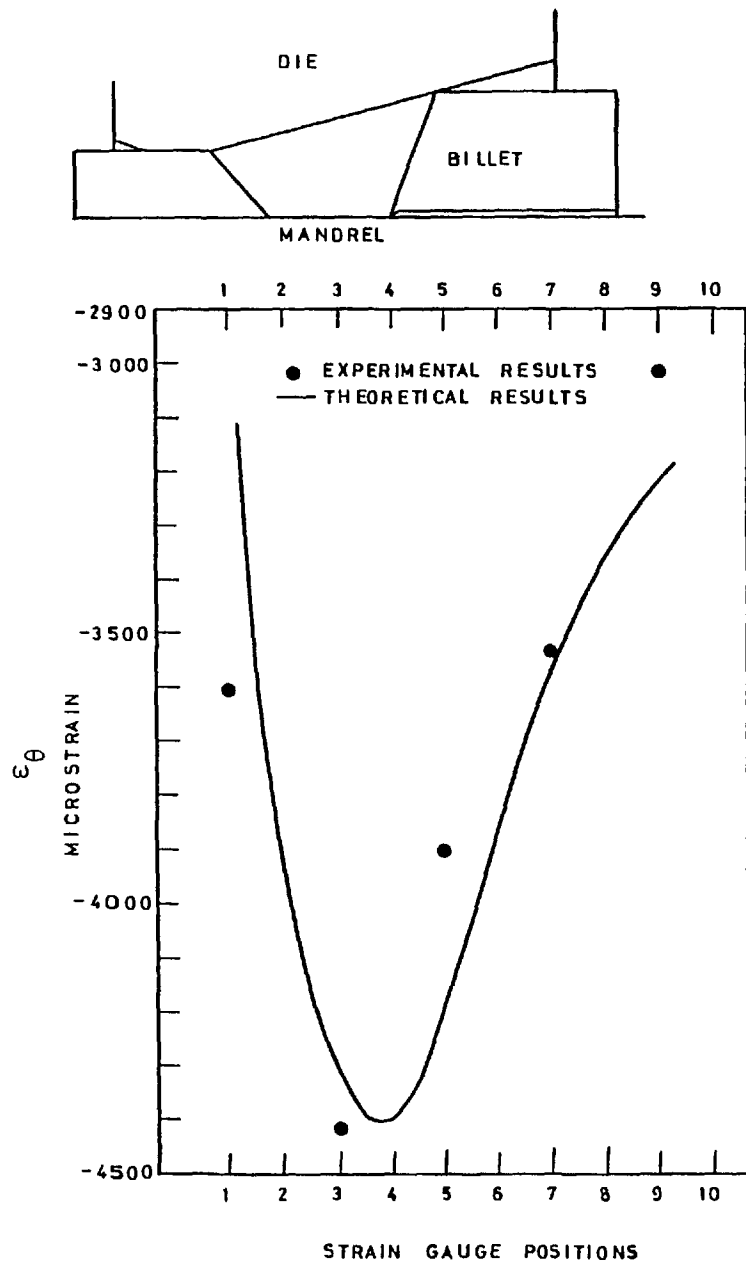


FIG. 7.18 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. COPPER EXTRUSION.

$$R = 2.0, \quad F = 6394 \text{ N}, \quad p = 263.7 \text{ MN/m}^2$$

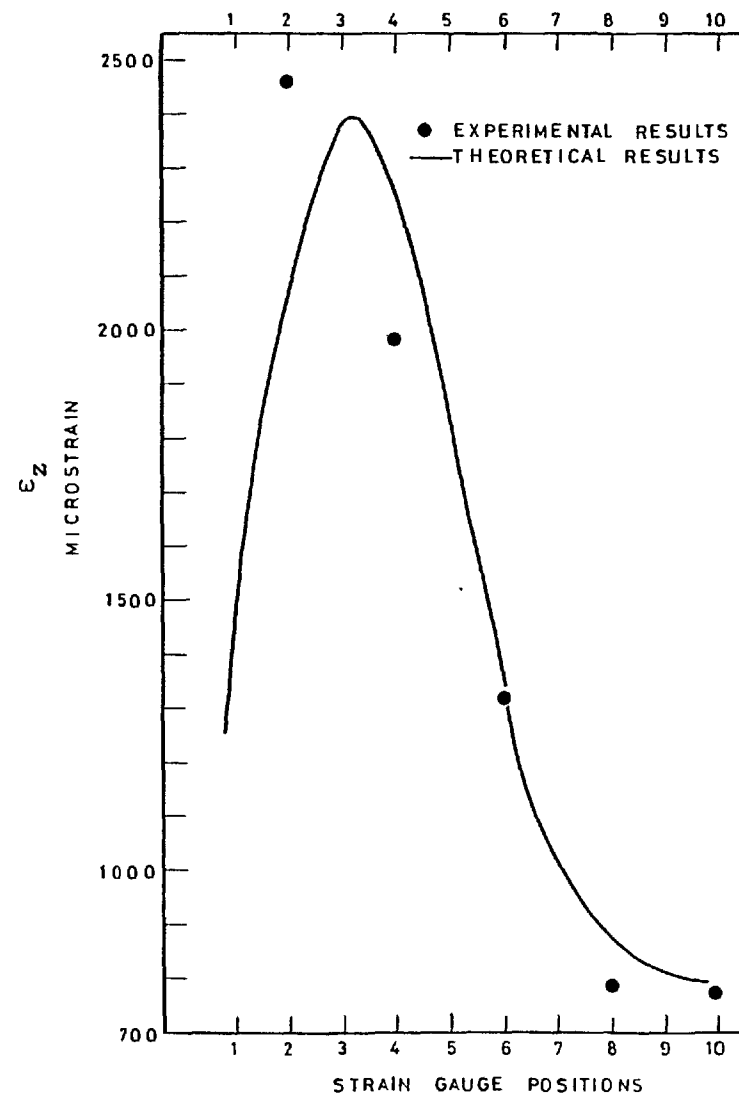
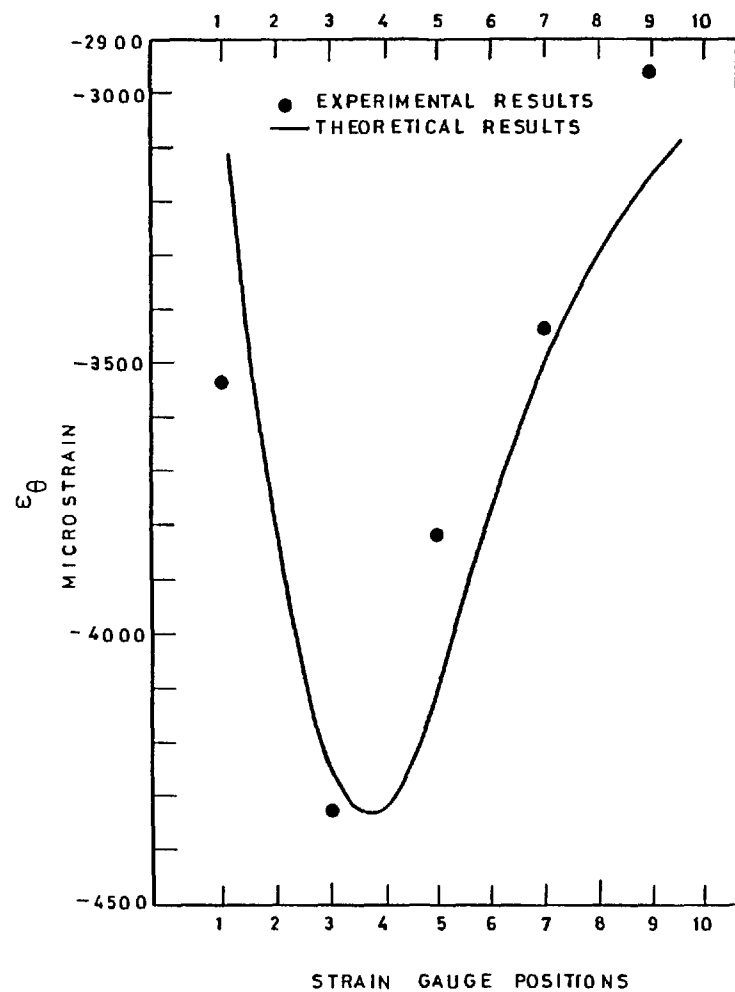
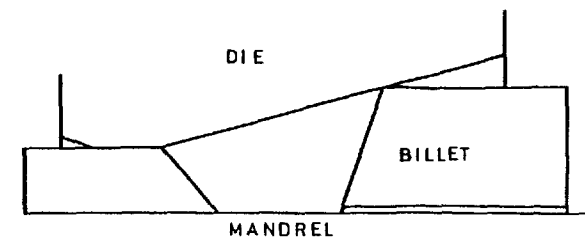
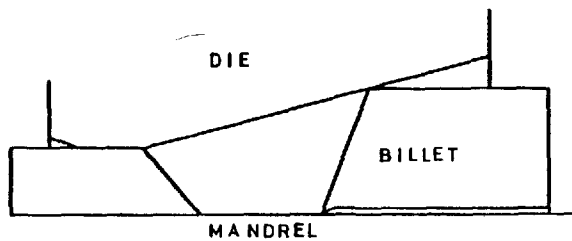


FIG. 7.19 - HOOP AND AXIAL STRAINS IN THE BORE OF THE MANDREL. COPPER EXTRUSION.

$$R = 2.0, \quad F = 8062 \text{ N}, \quad p = 256.3 \text{ MN/m}^2$$

of 3.5 for aluminium is 4100N (Figure 7.1) corresponding to an axial microstrain of 46, against the recorded maximum total axial strain of 1450 (Figure 7.11). The values for copper at an extrusion ratio of 2 are 39 and 2520 microstrain respectively. These figures indicate that when the design of the mandrel is considered, first of all the effect of pressure must be taken into account while that of the frictional drag is secondary, at least in hydrostatic extrusion/drawing.

Conditions could be quite different in conventional tube extrusion and to a lesser extent in drawing, since mandrel friction is likely to be significantly higher. Calculations indicate that, for the extreme case of sticking friction, for aluminium at an extrusion ratio of 3.5 the frictional drag on the mandrel would be over 100kN, corresponding to an axial microstrain of 1,100, assuming that friction is present only in the die zone as in the hydrostatic extrusion of tubes. For copper in extrusion or drawing at a ratio of 2, the frictional drag for sticking friction would be 112kN resulting in 1,200 axial microstrain on the mandrel. These strains are significant in relation to those shown in Figures 7.11 and 7.17. Although these calculated strains are for the extreme case of sticking friction they do indicate that in conventional tube extrusion or drawing, when friction is likely to be higher than in hydrostatic extrusion, frictional effects on mandrel design could not be neglected.

In conventional tube extrusion with a fixed mandrel not only the frictional shear stress is likely to be higher but the interfacial area between the billet and mandrel is much extended, adding to the axial tensile stress on the mandrel.

In the present case the maximum absolute value of the hoop and axial strains in the mandrel bore occurs nearer the entry boundary.

The experimental axial strains in Figures 7.4 to 7.6 and Figures 7.14 to 7.19 do not show a maximum, contrary to the theoretical results.

Because the deforming zone/mandrel interface is small (extrusion ratios of 1.5 and 2.0) the experimental strain distributions are drifted to the high pressure fluid side as no axial strain gauge was placed before the hoop strain gauge 1. However the corresponding hoop strain distributions in those illustrations show that in fact their absolute values have a maximum in a section within the deforming zone/mandrel interface as the nearest strain gauge to the product side is a hoop strain gauge.

The strain results confirm the conclusions drawn from the analysis of the stresses in the mandrel that the critical section for mandrel failure in hydrostatic extrusion/drawing is about midway along the mandrel/billet interface. On the other hand the maximum tensile stress in the mandrel due to frictional drag in conventional extrusion must be upstream near the billet/ram interface, which could alter the position and magnitude of the critical mandrel stress.

The experimental and theoretical results indicate that with increasing drawing stress the driving stress is reduced and the frictional and normal stresses at the deforming material/mandrel interface, thus the strains and stresses in the mandrel also become smaller.

From Figures 7.4 to 7.19 one can conclude that the experimental results confirm the theoretical predictions and, consequently, that the theoretical pressure and stress distributions at the billet/mandrel interface are close to the actual ones.

7.5.2 Stress Distribution at the Bore of the Mandrel

As mentioned earlier the stress distributions at the bore of the mandrel can be derived from the recorded strain distributions. In fact, as the radial stress is zero at the mandrel bore, the stress/strain equations for the axial σ_z and σ_θ stresses become

$$\sigma_{\theta} = \frac{E}{1 - \nu^2} (\epsilon_{\theta} + \nu \epsilon_z)$$

$$\sigma_z = \frac{E}{1 - \nu^2} (\epsilon_z + \nu \epsilon_{\theta})$$

where the axial, ϵ_z and hoop, ϵ_{θ} , strains can be determined from the experimental measurements. Unfortunately, the hoop and axial strains could not be measured at the same point, since the hoop and axial strain gauges are placed alternately along the mandrel bore.

Nevertheless, an approximate stress distribution can be obtained by interpolating the experimentally recorded strains. However, care should be taken as the gradients of strain and stress are large at the bore region opposite the deforming material. This is particularly important for the smallest extrusion ratio where there are only two strain gauges opposite the deforming region.

Figure 7.20 shows the hoop and axial stress distributions derived from the measured strains for aluminium at an extrusion ratio of 3.5. Clearly, the accuracy of these experimental stresses is reduced by the error due to interpolation, consequently a comparison between the theoretical and experimental strains is preferable.

7.6 Conclusions

1. The experimental results show that the friction force at the deformation zone/mandrel interface is small. It is higher for the larger extrusion ratios and, for the same ratio, it is greater for copper than for aluminium.
2. The friction force increases with increasing extrusion pressure/drawing stress ratio.
3. Both experimental and theoretical results indicate that the sum

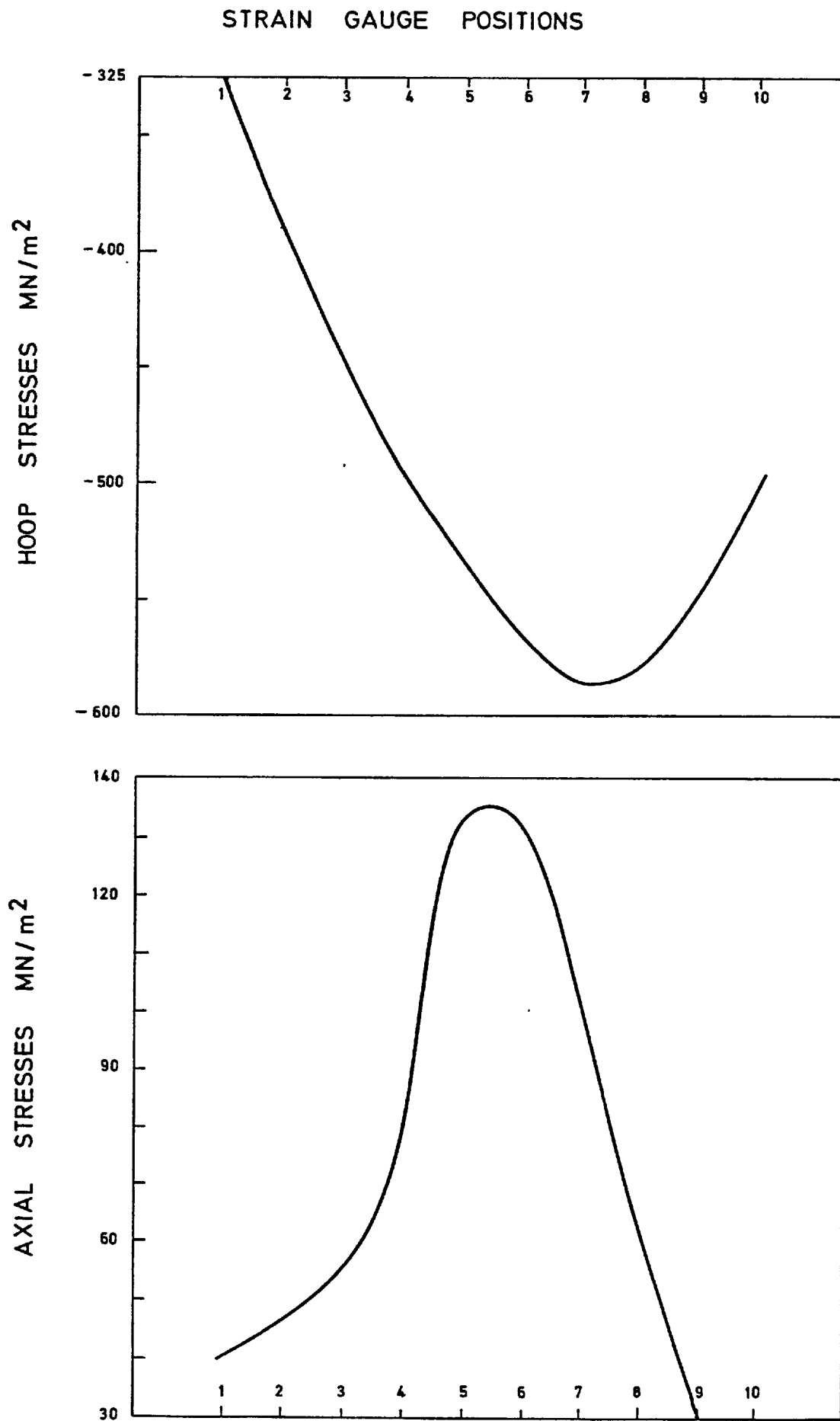


FIG. 7.20 .- HOOP AND AXIAL STRESSES AT THE MANDREL BORE

of the extrusion pressure and drawing stress decreases with increasing drawing stress.

4. By the application of a drawing stress not only the process speed can be controlled but also the radial, hoop and axial stresses in the mandrel become smaller.
5. From the lower upper bound approach the position of the conical boundaries was determined and the velocity, strain rate and strain fields derived. It was shown that the effect of friction on the velocity field was negligible in the hydrostatic extrusion-drawing of tubes, for the ratios considered in the experiments.
6. The use of the weighted residuals method combined with the velocity and strain rate fields allowed the assessment of the stress fields in the plastic deformation zone. The pressure distribution over the mandrel has a maximum about midway along the deformation zone/mandrel interface, while the shear stresses are small. On the other hand, the pressure distribution on the die has a maximum near the die entrance decreasing towards the exit.
7. The analysis of stresses in the mandrel was carried out by an elastic finite element program. This analysis has shown the existence of high compressive radial and hoop stresses and large tensile axial stresses in the mandrel at the deformation zone. These axial stresses are much larger than those due to the frictional drag alone, applied by the deforming material to the mandrel. Thus, in the hydrostatic extrusion of tubes, the critical mandrel stress is influenced mainly by the normal pressure on the mandrel.
8. The analysis of the experimental strain distributions confirm the results of theoretical analysis: the experimental and theoretically

predicted strain distributions on the mandrel are close to the actual ones.

9. . The method of weighted residuals is relatively simple and easy to handle.
10. To obtain the stress field in the deformation zone, using 25 terms in each estimate of the stress function, 50 seconds of central processing time and a memory of 25k were required on the CDC 6400 computer. Judging by this and by the good agreement between the theoretical and experimental results, it can be concluded that the method of weighted residuals used in combination with the strain rate field predicts the stress field in the plastically deforming region with good accuracy and requires only modest computing facilities.

7.7 Suggestions for Future Work

The experimental technique developed in the present work could be applied widely in the hot and cold extrusion and drawing of tubes. In extrusion the much larger billet/mandrel interface and greater friction could result in significantly different stress conditions in the mandrel.

In future work the hollow mandrel should have a greater internal diameter and larger length to ease the bonding of the strain gauges which posed a rather delicate task in the present case, restricted by the dimensions of the available high pressure apparatus.

It would be desirable to investigate the influence of process speed on the friction force and other parameters. In high speed extrusion and drawing the effect of strain rate and temperature on the stress field could be taken into account by updating the flow stress

of the deforming material.

The lower upper bound approach could be improved by developing more sophisticated optimization techniques using more parameters. If, for example, the optimization would be performed with respect to the coefficients of a higher degree polynomial the plastic zone boundaries and the upper bound pressure would become more accurate. However, care should be taken as the computation costs would increase with the number of parameters and a solution with a small number of parameters could be equally satisfactory. In the present work, where two parameters were used, the computer program to minimize the extrusion pressure required about 30 seconds of central processing time in the CDC 6400 machine when a mesh of 121 nodal points was employed. The required computing time was also a function of the accuracy of the initial estimate. Although the weighted residuals method in combination with a velocity field was satisfactory, it would be advantageous if the need to assume a velocity field was eliminated in the application of that method to plasticity problems.

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APPENDIX A

VELOCITY, STRAIN RATE AND STRAIN EXPRESSIONS FOR TUBE EXTRUSION

ASSUMING CONVERGING FLOW AND A SPHERICAL ENTRY BOUNDARY (FIG. A.1)

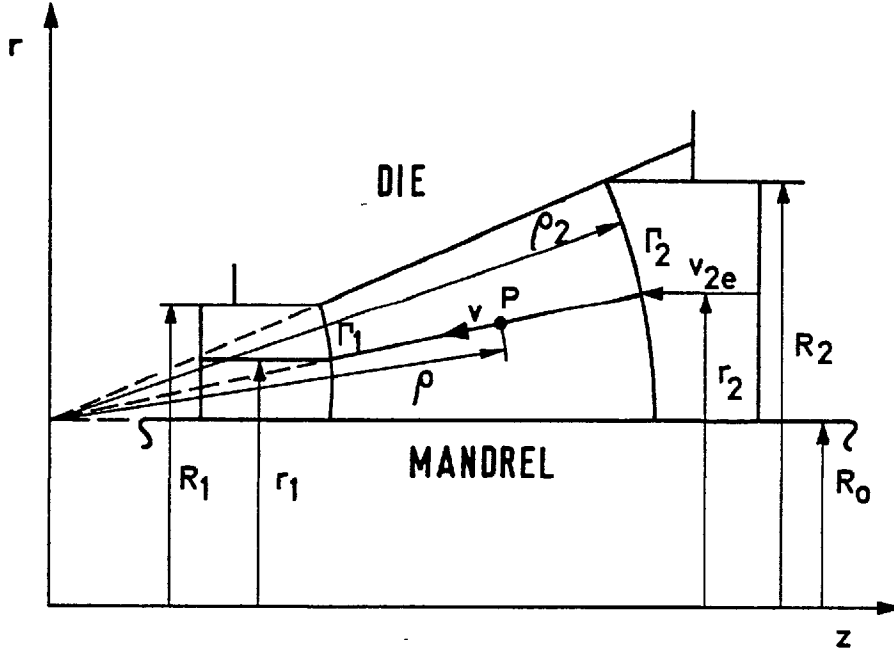


FIG. A.1

A.1 - Exit Boundary

The exit boundary is described by the following equation

$$z_1 = \frac{r_1 - R_0}{\sqrt{\frac{r_1^2 - R_0^2}{R} + R_0^2 - R_0}} \sqrt{\rho_2^2 - \frac{r_1^2 - R_0^2}{R} - 2R_0^2 + 2R_0} \sqrt{\frac{r_1^2 - R_0^2}{R} + R_0^2}$$

A.2 - Velocity Field

The velocity at a point P in the plastic deformation zone is

$$v = \frac{r_2 z \rho_2}{r \rho^2} v_{2e}$$

A.3 - Strain Rate Field

The strain rates can be obtained by the following expressions

$$\dot{\epsilon}_r = \left[\frac{\rho_2 z^2 (r - R_0)}{\rho^4 r_2} - \frac{r - R_0}{\rho r} - 2 \frac{(r - R_0)^2}{\rho^3} + \frac{z^2}{\rho^3} \right] v$$

$$\dot{\epsilon}_\theta = \frac{r - R_0}{r \rho} v$$

$$\dot{\epsilon}_z = \left[- \frac{\rho_2 z^2 (r - R_0)}{\rho^4 r_2} + \frac{1}{\rho} - 2 \frac{z^2}{\rho^3} + \frac{(r - R_0)^2}{\rho^3} \right] v$$

$$\dot{\epsilon}_{rz} = \left[\rho_2 z \frac{z^2 - (r - R_0)^2}{\rho^4 r_2} + \frac{r (r - R_0) - z^2}{\rho r z} - 6 \frac{z (r - R_0)}{\rho^3} \right] \frac{v}{2}$$

A.4 - Strain Field

The components of strain at the point P are found by applying equations (4.26). The integration of strain rates leads to

$$\epsilon_r = \left[\frac{1 - 3a^2}{a^2 + 1} + \frac{a\sqrt{a^2 + 1} \rho_2 + a^2 R_0}{a\sqrt{a^2 + 1} \rho_2 + (a^2 + 1) R_0} \right] \ln \frac{z}{z_2} - \ln \frac{r}{2}$$

$$\epsilon_\theta = \ln \frac{r}{r_2}$$

$$\epsilon_z = \left[\frac{2(a^2 - 1)}{a^2 + 1} + \frac{R_0}{a\sqrt{a^2 + 1} \rho_2 + (a^2 + 1) R_0} \right] \ln \frac{z}{z_2}$$

$$\epsilon_{rz} = \frac{1}{2} \left\{ \frac{a^3 - 7a}{a^2 + 1} + \frac{\sqrt{a^2 + 1} \rho_2 + 2aR_0}{a\sqrt{a^2 + 1} \rho_2 + (a^2 + 1) R_0} \right\} \ln \frac{z}{z_2} - \frac{1}{a} \ln \frac{r}{r_2}$$

where $a = \frac{r - R_0}{z}$

APPENDIX B

EXPRESSIONS TO CALCULATE THE AREA OF THE ELEMENTAL TRIANGULAR SECTION

AND THE CO-ORDINATES OF THE TRIANGLE CENTROID

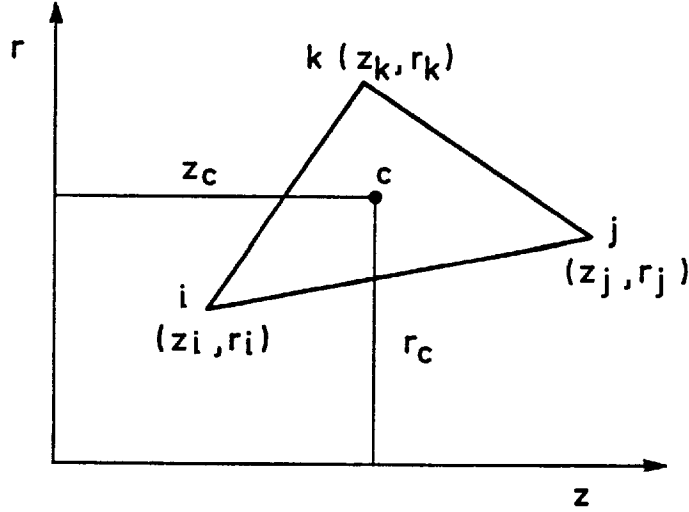


FIG. B.1

As shown in chapter 4 the mesh configuration (Fig. 4.17) is formed by triangular elements (Fig. B.1). For the sake of programming it is convenient to express the area of the element triangular as a function of the co-ordinates of its vertices (z_1, r_1)

$$\Delta A_e = \frac{1}{2} \begin{vmatrix} 1 & z_i & r_i \\ 1 & z_j & r_j \\ 1 & z_k & r_k \end{vmatrix}$$

The r and z co-ordinates of the triangle centroid are given respectively

by
$$r_c = \frac{r_i + r_j + r_k}{3}$$

and
$$z_c = \frac{z_i + z_j + z_k}{3}$$

APPENDIX C

COMPUTER PROGRAM "MIN"

The flow chart of this program is given in Chapter 4 Fig. 4.19 . The FORTRAN computer program variable names and the listing of the program are included in this appendix.

C.1 - Variable Name List

The program variable names are defined in alphabetic order in the following list.

AJ , AK	element dimensions
ALPHA	half die angle
ARC	angles ψ_i and $(\psi_i - \theta)$; see Fig. 4.15
AREA	areas of the elements
BJ , BK	element dimensions
CDR	r co-ordinate of the nodes
CDRM	r co-ordinate of the element centroids
CDZ	z co-ordinate of the nodes
DEXT	outside diameter of the mandrel
DF1DR	$\frac{\partial \psi_1}{\partial r}$
DF1DZ	$\frac{\partial \psi_1}{\partial z}$
DFI2DR	$\frac{\partial \psi_2}{\partial r}$
DFI2DZ	$\frac{\partial \psi_2}{\partial z}$
DIST	distance d ; see Fig. 4.15
DISTDR	$\frac{\partial d}{\partial r}$

DISTDZ	$\frac{\partial d}{\partial z}$
DIST1	distance d_1 ; see Fig. 4.15
DIST1DR	$\frac{\partial d_1}{\partial r}$
DIST1DZ	$\frac{\partial d_1}{\partial z}$
DLND	die land length
DRODR	$\frac{\partial \rho}{\partial r}$
DRODZ	$\frac{\partial \rho}{\partial z}$
DRO1DR	$\frac{\partial \rho_1}{\partial r}$
DRO1DZ	$\frac{\partial \rho_1}{\partial z}$
DR1DR	$\frac{\partial r_1}{\partial r}$
DR1DT	$\frac{\partial r_1}{\partial \theta}$
DR1DZ	$\frac{\partial r_1}{\partial z}$
DR2DR	$\frac{\partial r_2}{\partial r}$
DR2DT	$\frac{\partial r_2}{\partial \theta}$
DR2DZ	$\frac{\partial r_2}{\partial z}$
DTDR	$\frac{\partial \theta}{\partial r}$
DTDZ	$\frac{\partial \theta}{\partial z}$
DVDR	$\frac{\partial v}{\partial r}$
DVDZ	$\frac{\partial v}{\partial z}$

DVTI	}	tangencial velocity discontinuities
DVTL		
DV1DR		$\frac{\partial v_1}{\partial r}$
DV1DZ		$\frac{\partial v_1}{\partial z}$
DV2DR		$\frac{\partial v_2}{\partial r}$
DV2DZ		$\frac{\partial v_2}{\partial z}$
DZ1DR1		$\frac{dz_1}{dr_1}$
DZ1DT		$\frac{dz_1}{d\theta}$
DZ2DR2		$\frac{dz_2}{dr_2}$
DZ2DT		$\frac{dz_2}{d\theta}$
D2Z1DR		$\frac{d^2 z_1}{dr_1^2}$
D2Z2DR		$\frac{d^2 z_2}{dr_2^2}$
EFSTN		effective strain
EFSTR		effective strain rate
EXTR		extrusion ratio
FATR		measured friction force
FATRED		friction force at the die/product interface
FATREM		friction force at the mandrel/product interface
FATRP		friction force at the deformation zone/mandrel interface
FF		friction factor
FLST		flow stress
FORCE		drawing force
FXC		upper bound pressure

IFAIL	library subroutine parameter
ITER	counter
LW	library subroutine parameter
NEL	number of elements
NITER	number of iterations
NM	library subroutine parameter
NNP	number of nodes
NPI	indices of the first nodes of the elements
NPJ	indices of the second nodes of the elements
NPK	indices of the third nodes of the elements
NRPT	number of nodes along the exit or entry boundary
NZPT	number of nodes along the deformation zone/ /mandrel interface
PI	π
PWDEF	power of deformation
PWFB31	friction power dissipated at the deformation zone/mandrel interface
PWFB41	friction power dissipated at the deformation zone/die interface
PWRD11	redundant power at the exit boundary
PWRD21	redundant power at the entry boundary
RE	outside diameter of the billet
RLEN	length of the deformation zone/mandrel interface
RO	ρ ; see Fig. 4.15
RS	outside diameter of the product
SIGZO	axial stress acting at the exit boundary
STRNR	strain component ϵ_r
STRNRT	strain rates
STRNRZ	strain component ϵ_{rz}

STRNT	strain component ε_{θ}
STRNZ	strain component ε_z
TAGX1	slope of the exit boundary
TAGX2	slope of the entry boundary
TETA	angle between streamline and the z - axis
VEL	velocity at the nodes
VELR	r component of the velocity
VELZ	z component of the velocity
VF	velocity of the billet at the die entry
VLENC1	normal component of the velocity at the exit boundary in the rigid zone side
VLENC2	normal component of the velocity at the entry boundary in the rigid zone side
VLETC1	tangential component of the velocity at the exit boundary in the rigid zone side
VLETC2	tangential component of the velocity at the entry boundary in the rigid zone side
VLNC1	normal component of the velocity at the exit boundary in the deformation zone side
VLNC2	normal component of the velocity at the entry boundary in the deformation zone side
VLTC1	tangential component of the velocity at the exit boundary in the deformation zone side
VLTC2	tangential component of the velocity at the entry boundary in the deformation zone side
VOL	volume of the elements
VS	velocity of the product
W	library subroutine parameter
X	outside radius of the mandrel

C.2 - LISTING OF THE PROGRAM MIN

```
PROGRAM MIN, INPUT, OUTPUT, TAPE6=OUTPUT, TAPE5=INPUT, TAPE4)
C
C   THIS PROGRAM CALCULATED THE TOTAL ENERGY OF THE PLASTICALLY
C   DEFORMED ZONE ASSUMING CONICAL BOUNDARIES FOR THE ENTRY
C   AND EXIT. DATA IS IN DATMIN
C   MATERIAL : COPPER
C
C   DIMENSION M(30),NC(2)
C   COMMON NPPT,NC,NZPT,NZ,NP,DEXT,NNP,NX2,TAGX2,CTB2,ZX1,TAGX1,
1NF,ZE,PE,PS,X2,ALPHA,VF,CDP(400),CDZ(400),X,CTB1,NEL,FORCE,PI
2,EFSTR(400),EXTR,APC(4,30),TETA(30),AREA(722),CDPM(722),FATPE,
3VEL(400),VELZ(400),STNP1(4,400),PD(400),FATR,NC1,NZ2,FATP,FF
4,STNP1(400),STNP1(400),STNP2(400),STNP2(400),EFETH(400),NITEP
5,FLST(400),NPI(722),NPJ(722),NPK(722),PLEN,VC,CDF,SOT,ITEP,DLND
6,FATPEM,FATPED,IND,CBF,PPN,VLNC1(30),VLTC1(30),VLENC1(30),
7VLETC1(30),VLNC2(30),VLTC2(30),VLENC2(30),VLETC2(30)
C
C   READ(5,50) NP,NZ,NITEP,PE,PS,DEXT,DLND,ALPHA,VF,FORCE,FATR
50 FORMAT(3I5,8F8.0)
C   READ(5,51) NM,LM,IND,IFAIL
51 FORMAT(4I5)
C   READ(5,52) (XC(I),I=1,2)
52 FORMAT(4F5.0)
C
C   PI = 3.141592654
C   SOT = SOT(3.)
C   CDF = 2./SOT
C   NPPT = NP + 1
C   NZPT = NZ + 1
C   NNP = NPPT + NZPT
C   NF = NNP - NZPT + 1
C   PE2 = PE**2
C   PS2 = PS**2
C   X = DEXT / 2.
C   X2 = X**2
C   EXTR = (PE2 - X2) / (PS2 - X2)
C   ZE = (PE - X) / TAN(ALPHA)
C   Z2 = (PS - X) / TAN(ALPHA)
C   VC = EXTR * VF
C   ITEP = 0
C
C   WRITE(6,60)
60 FORMAT(/,30X,10HINPUT DATA,/)
C   WRITE(6,61) NP,NZ,NITEP,PE,PS,DEXT,ALPHA,VF,FORCE,
1   FATP,DLND,CBF,PPN
61 FORMAT(/,30X,7HNP      =,I5,/,30X,7HNZ      =,I5,
1/,30X,7HNITEP =,I5,/,30X,7HPE      =,E11.5,/,
2 30X,7HPS      =,E11.5,/,30X,7HDEXT =,E11.5,/,30X,
3 7HALPHA =,E11.5,/,30X,7HVF      =,E11.5,/,30X,7HFORCE =,
4 E11.5,/,30X,7HFATR =,E11.5,/,30X,7HDLND =,E11.5,/,
5 ,30X,7HCBF      =,E11.5,/,30X,7HPPN      =,E11.5,/,/)
```

```

      WRITE(6,63) ENTP
63  FORMAT(30X,17HENTRUCION RATIO =,F8.4,/)
C
      WRITE(6,62) (I,XC(I),I=1,2)
62  FORMAT(/,30X,14HINITIAL VALUES,/,30X,3HXC(,I1,2H)=,E20.14,/)
C
      WRITE(6,64)
64  FORMAT(/,30X,17HITERATION RESULTS)
C
      CALL E04CEF(NM,XC,FXC,W,LM,IFAIL)
C
      SIG20 = (FORCE - FATPE)/(PI*(R12 - X2)
      IF(IFAIL.NE.0) WRITE(6,1991) IFAIL
1991 FORMAT(/,30X,7HIFAIL =,I5,/)
      WRITE(6,1992) (I,XC(I),I=1,2)
1992 FORMAT(/,30X,3HXC(,I1,2H)=,E20.14)
      WRITE(6,1993) FXC,FATREM,FATRED,FATPE,FATPP,FF,CBF,SIG20
1993 FORMAT(/,30X,18HTHE PRESSURE IS ,E20.14,
1      /,30X,18HFATREM-----,E20.14,
2      /,30X,18HFATRED-----,E20.14,
3      /,30X,18HFATPE-----,E20.14,
4      /,30X,18HFATPP-----,E20.14,
5      /,30X,18HFRICITION FACTOR =,E20.14,
6      /,30X,18HCDEF OF FRICITION =,E20.14,
7      /,30X,18HSIG20 =,E20.14,/)
C
      WRITE(4,400) NCPT,NP,DEXT,PE,RS,ALPHA,VF,ENTP,ZC
400  FORMAT(2I3,6E12.6,/,E20.14)
      WRITE(4,402) (CDP(I),CDZ(I),I=1,NNP)
402  FORMAT(2E20.14)
      WRITE(4,403) (CTPNRT(K,M),K=1,4),EFSTR(M),M=1,NNP)
403  FORMAT(5E16.10)
      WRITE(4,404) (FLST(I),I=1,NNP)
404  FORMAT(E16.10)
      WRITE(4,405) FXC,SIG20,FF,FATREM
405  FORMAT(4E16.10)
C
      STOP
      END

```

SUBROUTINE FUNCT1(NM,NXC,FXC)

```

C      DIMENSION NXC(2)
      COMMON NRPT,CC,NRPT,NZ,NP,DEXT,HNP,CX2,TAGN2,CTB2,CX1,TAGN1,
1NF,CE,PE,PI,P2,ALPHA,VF,CDP(400),CDC(400),C,CTB1,HCL,FORCE,PI
2,EFCIR(400),ENTR,APC(4,30),TETA(30),AREA(722),CDPM(722),FATPE,
3VEL(400),VELC(400),CTRNP(4,400),PD(400),FATP,NC1,NC2,FATPP,FF
4,LTNP(400),LTNP1(400),LTNP2(400),LTNP2(400),EFCIN(400),NITER
5,FLIT(400),NPI(722),NPJ(722),NPK(722),PLEN,V1,CDF,COT,ITER,DLND
6,FATPEM,FATPED,IND,CBF,PRN,VLNC1(30),VLTC1(30),VLENC1(30),
7VLETC1(30),VLNC2(30),VLTC2(30),VLENC2(30),VLETC2(30)

```

NC1 = NXC(1)

NC2 = NXC(2)

CALL MESH

JJ = 0

DO 1 I=1,NR,NRPT

JJ = JJ + 1

L = I + NRPT - 1

COST3 = COS(TETA(JJ))

SENT3 = SIN(TETA(JJ))

TGT3 = TAN(TETA(JJ))

COST2 = COS(APC(2,JJ))

COST22 = COST2**2

SENT2 = SIN(APC(2,JJ))

TGT2 = TAN(APC(2,JJ))

COST1 = COS(APC(1,JJ))

COST12 = COST1**2

SENT1 = SIN(APC(1,JJ))

TGT1 = TAN(APC(1,JJ))

SEC2T3 = 1.0/(COST3**2)

APC(3,JJ) = APC(1,JJ) - TETA(JJ)

APC(4,JJ) = APC(2,JJ) - TETA(JJ)

COST4 = COS(APC(3,JJ))

SENT4 = SIN(APC(3,JJ))

COST42 = COST4**2

COST5 = COS(APC(4,JJ))

SENT5 = SIN(APC(4,JJ))

COST52 = COST5**2

CT1 = SENT1 + COST4

CT2 = COST1 + SENT4

CT3 = SENT2 + COST5

CT4 = COST2 + SENT5

DZ1DP1 = -TGT1

DZ2DP2 = -TGT2

DZ31DP = 0.0

DZ22DP = 0.0

V1 = -COS(T1) * COS(T4) * VC
V2 = -COS(T2) * COS(T5) * VF

DIST = SQRT((CDP(L)-CDP(I))**2+(CDZ(L)-CDZ(I))**2)
VK = CDP(I)*V1 + CDP(L)*V2
RO(I) = DIST*(VK-1.)
RO(L) = RO(I) + DIST

DP1DT = RO(I)*COS(T1)*COS(T4)
DP2DT = RO(L)*COS(T2)*COS(T5)
DC1DT = DC1DP1 + DP1DT
DC2DT = DC2DP2 + DP2DT

DO 1 J=1,NZPT
M = I + J - 1
DTDP = 1./.(DP1DT*(1.-TGT3*DC1DP1)+(CDZ(M)-CDZ(I))*
1/EOS(T3))
DTDC = -TGT3*DTDP
DP1DP = DP1DT + DTDP
DP1DC = DP1DT + DTDC
DP2DP = DP2DT + DTDP
DP2DC = DP2DT + DTDC
DC1DP = DC1DT + DTDP
DC1DC = DC1DT + DTDC
DC2DP = DC2DT + DTDP
DC2DC = DC2DT + DTDC

DFI1DP = -D2C1DP*DP1DP+COS(T12)
DFI1DC = -D2C1DP*DP1DC+COS(T12)
DFI2DP = -D2C2DP*DP2DP+COS(T22)
DFI2DC = -D2C2DP*DP2DC+COS(T22)

DF1DTP = DFI1DP - DTDP
DF1DT2 = DFI1DC - DTDC
DF2DTP = DFI2DP - DTDP
DF2DT2 = DFI2DC - DTDC

DV1DP = -(CT1*DFI1DP+CT2*DF1DTP) * VC / COS(T42)
DV1DC = -(CT1*DFI1DC+CT2*DF1DT2) * VC / COS(T42)
DV2DP = -(CT3*DFI2DP+CT4*DF2DTP) * VF / COS(T52)
DV2DC = -(CT3*DFI2DC+CT4*DF2DT2) * VF / COS(T52)

DIST1 = SQRT((CDP(M)-CDP(I))**2 + (CDZ(M)-CDZ(I))**2)
RO(M) = RO(I) + DIST1

```

C
C      VELOCITY FIELD
C
      VEL(M) = PO(I)*CDP(I)*V1/(PO(M)*CDP(M))
      VELP = VEL(M)*CENT3
      VELZ(M) = VEL(M)*COST3
C
      DISTDP = ((CDP(L)-CDP(I))*DP2DP-DP1DP) +
1      ((CDZ(L)-CDZ(I))*DZ2DP-DZ1DP)/DIST
      DISTDZ = ((CDP(L)-CDP(I))*DP2DZ-DP1DZ) +
1      ((CDZ(L)-CDZ(I))*DZ2DZ-DZ1DZ)/DIST

      DPO1DP1 = DP2DP/CDP(L) + DV2DP/V2 + DISTDP/DIST
      DPO1DP2 = DP1DP*V1 + DV1DP*CDP(I) - DP2DP*V2 - DV2DP*CDP(L)
      DPO1DP = (DPO1DP1 - DPO1DP2/(CDP(I)*V1 - CDP(L)*V2))*PO(I)

      DPO1DZ1 = DP2DZ/CDP(L) + DV2DZ/V2 + DISTDZ/DIST
      DPO1DZ2 = DP1DZ*V1 + DV1DZ*CDP(I) - DP2DZ*V2 - DV2DZ*CDP(L)
      DPO1DZ = (DPO1DZ1 - DPO1DZ2/(CDP(I)*V1 - CDP(L)*V2))*PO(I)

      IF(J.EQ.1) GO TO 2
      DIST1DP = ((CDP(M)-CDP(I))*DP1DP -
1      ((CDZ(M)-CDZ(I))*DZ1DP)/DIST1
      DIST1DZ = ((CDP(M)-CDP(I))*DP1DZ +
1      ((CDZ(M)-CDZ(I))*DZ1DZ)/DIST1
      GO TO 3
2  DIST2 = 18PT*((CDP(L)-CDP(M))*DP2+((CDZ(L)-CDZ(M))*DZ2)
      DIST2DP = ((CDP(L)-CDP(M))*DP2DP-1.) +
1      ((CDZ(L)-CDZ(M))*DZ2DP)/DIST2
      DIST2DZ = ((CDP(L)-CDP(M))*DP2DZ +
1      ((CDZ(L)-CDZ(M))*DZ2DZ-1.)/DIST2
      DIST1DP = DISTDP - DIST2DP
      DIST1DZ = DISTDZ - DIST2DZ

3  DPODP = DPO1DP + DIST1DP
      DPODZ = DPO1DZ + DIST1DZ

      DVDP1 = DPO1DP/PO(I) + DP1DP/CDP(I) + DV1DP/V1
      DVDP2 = DPODP/PO(M) + 1./CDP(M)
      DVDP = (DVDP1 - DVDP2)*VEL(M)

      DVDZ1 = DPO1DZ/PO(I) + DP1DZ/CDP(I) + DV1DZ/V1
      DVDZ2 = DPODZ/PO(M)
      DVDZ = (DVDZ1 - DVDZ2)*VEL(M)

```

```

C
C      STRAIN RATE
C
      STRNPT(1,M) = DVDZ*CENT3 + VELZ(M)*DTDP
      STRNPT(2,M) = VELP*CDP(M)
      STRNPT(3,M) = DVDZ*CDT3 - VELP*DTDC
      STRNPT(4,M) = .5*(DVDZ*CENT3 + VELZ(M)*DTDC +
1          DVDZ*CDT3 - VELP*DTDP)
C
      ST12 = STRNPT(1,M)**2
      ST22 = STRNPT(2,M)**2
      ST32 = STRNPT(3,M)**2
      ST42 = STRNPT(4,M)**2
      SMST = .5*(ST12 + ST22 + ST32)
C
      EFSTR(M) = COF * COPT/(SMST + ST42)
C
1  CONTINUE
C
      J = 0
      DO 6 I=1,NF,NZPT
      L = I + NZPT - 1
      J = J + 1
      VLTC2(J) = VEL(L) * SIN(ARC(4,J))
      VLET2(J) = -VF * SIN(ARC(2,J))
      VLNC2(J) = VEL(L) * COS(ARC(4,J))
      VLNC1(J) = VEL(L) * COS(ARC(3,J))
      VLTC1(J) = VEL(L) * SIN(ARC(3,J))
      VLET1(J) = -VF * SIN(ARC(1,J))
6  CONTINUE
C
      STRAINS
C
      DO 4 I=1,NMF
      STRNF(I) = 0.
      STRNT(I) = 0.
      STRNZ(I) = 0.
      STRNFC(I) = 0.
      EFSTN(I) = 0.
4  CONTINUE
C
      JJ = 0
      DO 5 I= NZPT,NMF,NZPT
      JJ = JJ + 1
      EFETE = (COF/2.) * ABS((VLET2(JJ) - VLTC2(JJ))/(VLNC2(JJ))
      EFETS = (COF/2.) * ABS((VLET1(JJ) - VLTC1(JJ))/(VLNC1(JJ))
      EFCTN(I) = EFETE
      DIRE2 = EFSTR(I) * VELZ(I)

```

```

DO 5 J = 2,NZPT
  I = I - J + 1
  DEL = (CDZ(I) - CDZ(K+1))
  DIPE1 = EFCTR(I) * VELZ(K)
  DIPE = (DIPE1 + DIPE2) / 2.
  STPE = DIPE * DEL
  EFCTN(K) = EFCTN(K+1) + STPE
  DIPE2 = DIPE1
  IF (J.EQ.NZPT) EFCTN(K) = EFCTN(K) + EFCTN
5 CONTINUE

C
C   FLOW STRESS (COPPER)
C
DO 7 I = 1,NHF
  EFC = EFCTN(I)
  FLCT(I) = 5.44988 + EFC*(121.327 + EFC*(-83.3341 + EFC*(-334.518
1    EFC * (796.206 + EFC*(-642.754 + EFC*180.616))))))
7 CONTINUE

C
C   THE POWER CALCULATIONS
C
C   POWER OF DEFORMATION
C
PMDDEF = 0.0
DO 8 L = 1,NEL
  I = NPI(L)
  J = NPJ(L)
  K = NPK(L)
  FINT = (FLCT(I)*EFCTR(I)+FLCT(J)*EFCTR(J)+FLCT(K)*EFCTR(K))/3.
  VOL = 2.*PI*AREA(L)*CDPM(L)
  PMDEF = PMDEF + FINT*VOL
8 CONTINUE

C
C
C
C   REDUNDANT POWER ON BOUNDARY 1
C
J = 1
PWRD11 = 0.0
DO 15 I=NZPT+1,NF,NZPT
  L = I - NZPT
  J = J + 1
  DELT = 2*DT*(CDP(I)-CDP(L))**2 + (CDZ(I)-CDZ(L))**2
  PADM = (CDP(I) + CDP(L))/2.
  SUP = 2.*PI*PADM*DELT
  DVTI = ABC(VLTC1(J) - VLTC1(J))
  DVTL = ABC(VLTC1(J-1)-VLTC1(J-1))
  CHERPI = FLCT(I) - 2*DT
  CHERPL = FLCT(L) - 2*DT
  FINT = (CHERPI*DVTI + CHERPL*DVTL) / 2.
  PWRD11 = PWRD11 + FINT*SUP
15 CONTINUE

```

REDUNDANT POWER ON BOUNDARY 2

```

J = 1
PMPD21 = 0.0
DO 17 I = 2,NZPT,NZPT
L = I - NZPT
J = J + 1
DELT = (CDP(I)-CDP(L))**2 + (CDZ(I)-CDZ(L))**2
PADM = (CDP(I) + CDP(L))/2.
SUP = 2.*PI*PADM*DELT
DVTI = ABC(VLTC2(J)-VLTC2(J))
DVTL = ABC(VLTC2(J-1)-VLTC2(J-1))
SHEAPI = FLCT(I) / 30T
SHEAPL = FLCT(L) / 30T
FINT = (SHEAPI*DVTI + SHEAPL*DVTL)/2.
PMPD21 = PMPD21 + FINT*SUP
17 CONTINUE

```

FRICTION POWER ON BOUNDARY 3

FRICTION FACTOR CALCULATION

```

SUMK = 0.0
DO 23 I=2,NZPT
L = I - 1
DELT = CDZ(I) - CDZ(L)
SHEAPI = FLCT(I)/30T
SHEAPL = FLCT(L)/30T
SHEAPM = (SHEAPI + SHEAPL)/2.
23 SUMK = SUMK + SHEAPM * DELT
SK1 = FLCT(I) / 30T
SK2 = FLCT(NF) / 30T

PMLT = CDZ(I) - C3
PLT = PMLT + DLND
TPX = 2. * PI * X
FF = FATP / (TPX*(SUMK + PLT *SK1))

FATPP = FF*TPX*SUMK
FATPEM = FF*TPX*SK1*PLT
FATPED = FF*2.*PI*PS*SK2*DLND
FATRE = FATPEM + FATPED

```

```

J = 1
PMFB31 = 0.0
PLOAD1 = 0.0
DO 19 I =2,NZPT
J = J + 1
L = I - 1
DELT = CDZ(I) - CDZ(L)
SUP = 2.*PI*X*DELT
DVTI = ABC(VEL(I))
DVTL = ABC(VEL(L))
SHEAPI = FLCT(I) / 30T
SHEAPL = FLCT(L) / 30T
SHEAPM = (SHEAPI + SHEAPL)/2.
FINT = (SHEAPI*DVTI + SHEAPL*DVTL)/2.
PLOAD1 = PLOAD1 + SHEAPM*SUP*FF
PMFB31 = PMFB31 + FINT*SUP*FF
19 CONTINUE

```

```

C      FRICTION POWER ON BOUNDARY 4
C
      J = 1
      PMFB41 = 0.0
      DO 20 I=NF + 1,NMF
      J = J + 1
      L = I -1
      DELT = (OPT*(CDP(I)-CDP(L))**2 + (CDZ(I) - CDZ(L))**2)
      RADM = (CDP(I) + CDP(L))/2.
      SUP = 2.*PI*RADM*DELT
      DVTI = ABS(VEL(I))
      DVTL = ABS(VEL(L))
      SHEAPI = FLCT(I) / 30T
      SHEAPL = FLCT(L) / 30T
      FINT = (SHEAPI*DVTI + SHEAPL*DVTL)/2.
      PMFB41 = PMFB41 + FINT*SUP*FF
20 CONTINUE
C
C      TOTAL POWER REQUIRED FOR EXTRUSION
C
      POWER1 = PMDEF + PMPD11 + PMPD21 + PMFB31 + PMFB41
C
C      POWER DUE TO THE PRESCRIBED FORCES
C
      PMPPF = (FORCE - FATRE)*VS
C
C      EXTRUSION PRESSURE
C
      FXC = (POWER1 - PMPPF)/(PI*(PE**2 - K**2)*VF)
C
      ITER = ITER + 1
      IPEST = ITER - ITER/NITER * NITER
      IF (IPEST.NE.0) RETURN
      WRITE(6,1999) NC1,NC2,FXC
1999 FORMAT( /,30X,7HXC(1) =,E20.14,/,30X,7HXC(2) =,E20.14,/,
1 30X,7HFXC    =,E20.14)
C
      RETURN
      END

```

SUBROUTINE MECH

```

COMMON NPPT,NC,NZPT,NZ,NP,DEXT,NMP,ZK2,TAGX2,CTB2,ZK1,TAGX1,
1NF,ZE,PE,PI,N2,ALPHA,VF,CDP(400),CDZ(400),N,CTB1,NEL,FORCE,PI
2,EFC(PI(400),EXTP,APC(4,30),TETA(30),AREA(722),CDPM(722),FATPE,
3VEL(400),VELZ(400),STPNPT(4,400),PD(400),FATP,NC1,NC2,FATPF,FF
4,STPNP(400),LTPNT(400),STPNZ(400),STPNR2(400),EFCSTN(400),NITER
5,FLC(PI(400),NPI(722),NPJ(722),NPK(722),PLEN,VS,COF,SGT,ITER,DLND
6,FATPEM,FATRED,IND,CBF,PRN,DUMMY(240)

```

C
C
C

```

CTB1 = NC1
PLEN = NC2
ZK1 = CTB1 * CC
ZK2 = ZK1 + PLEN
CTB2 = ZK2 * ZE
TAGZ1 = (CC - ZK1) / (PE - X)
TAGZ2 = (ZE - ZK2) / (PE - X)
TAGX1 = 1. - TAGZ1
TAGX2 = 1. - TAGZ2

```

C

```

DO 1 I = 1,NMP
  CDP(I) = 0.0
  CDZ(I) = 0.0
1 CONTINUE
  J = 0
  DO 2 I = NCPT, NMP, NCPT
    CDP(I) = X + (PE - X) / FLOAT(NP) * FLOAT(J)
    CDZ(I) = TAGZ2 * (CDP(I) - X) + ZK2
  2 J = J + 1
  DO 3 I=1,NF,NCPT
    CDP(I) = COPT((CDP(I+NZ) ** 2 - X2) / EXTP + X2)
  3 CDZ(I) = TAGZ1 * (CDP(I) - X) + ZK1
  DO 12 I = 1,NPPT
    DO 12 J = 2,NZPT - 1
      K = J + (I - 1) * NZPT
      L = 1 + (I - 1) * NZPT
      CDP(K) = CDP(L) + (CDP(L+NZ) - CDP(L)) / FLOAT(NZ) * FLOAT(J-1)
      CDZ(K) = CDZ(L) + (CDZ(L+NZ) - CDZ(L)) / FLOAT(NZ) * FLOAT(J-1)
  12 CONTINUE

```

```

C
C      COMPUTE THE ELEMENTS GEOMETRY
C
      NEL = NP*NE*2
      DO 14 I=1,NP
      DO 14 J=1,NE
      NID = J + (I-1)*NP
      M1 = NID*2-1
      M2 = M1 + 1
      K = J + (I-1)*NEPT
      NPI(M1) = K
      NPJ(M1) = K + NEPT + 1
      NPK(M1) = K + NEPT
      CDP(M1) = (CDP(NPI(M1))+CDP(NPJ(M1))+CDP(NPK(M1)))/3.
      AJ = -CDZ(NPK(M1))+CDZ(NPI(M1))
      AK = -CDZ(NPI(M1))+CDZ(NPJ(M1))
      BJ = CDP(NPK(M1))-CDP(NPI(M1))
      BK = CDP(NPI(M1))-CDP(NPJ(M1))
      APER(M1) = .5*(AK*BK-AJ*BK)
      IF(APER(M1).GT.0.0) GO TO 15
606  FORMAT(/,1X,14HELEMENT NUMBER,14,25H HAS NEGATIVE APER - STOP)
      STOP
15  NPI(M2) = K
      NPJ(M2) = K + 1
      NPK(M2) = K + 1 + NEPT
      CDP(M2) = (CDP(NPI(M2))+CDP(NPJ(M2))+CDP(NPK(M2)))/3.
      AJ = -CDZ(NPK(M2))+CDZ(NPI(M2))
      AK = -CDZ(NPI(M2))+CDZ(NPJ(M2))
      BJ = CDP(NPK(M2))-CDP(NPI(M2))
      BK = CDP(NPI(M2))-CDP(NPJ(M2))
      APER(M2) = .5*(AK*BK-AJ*BK)
      IF(APER(M2).GT.0.0) GO TO 14
607  FORMAT(/,1X,14HELEMENT NUMBER,14,25H HAS NEGATIVE APER - STOP)
      STOP
14  CONTINUE
C
C      NORMAL ARCS AT THE BOUNDARIES
C
      JJ = 0
      DO 4 I=1,NP*NEPT
      L = I + NEPT - 1
      JJ = JJ + 1
      TGT1 = TAGZ1
      TGT2 = TAGZ2
      TGT3 = (CDP(L)-CDP(1))/(CDZ(L)-CDZ(1))
      APC(1,JJ) = ATAN(-TGT1)
      APC(2,JJ) = ATAN(-TGT2)
4  TETA(JJ) = ATAN(TGT3)
C
      RETURN
      END

```

APPENDIX D

COMPUTER PROGRAM STRESS

The flow chart of the program STRESS is presented in Chapter 5, Fig. 5.2. . The listing of the program and the variable name list are included in this appendix. The subroutine SIMQ is a library subroutine and therefore is omitted from the listing of the program. This subroutine is a procedure for solving a system of equations which employs Gaussian elimination.

D.1 - Variable Name List

The FORTRAN computer program variable names are defined in alphabetic order in the following list.

A	coefficients of matrix (equation 5.44)
ALPHA	half die angle
AVFLW	average flow stress
AVPRS	average pressure
C	after call of SIMQ contains the estimates for free parameters
CBF	coefficient of friction
CDR	r co-ordinate of the nodes
CDZ	z co-ordinate of the nodes
DEXT	outside diameter of the mandrel
EFSTN	effective strain
EFSTR	effective strain rate
EQVDS	equivalent stress calculated from the deviatoric stress tensor
EQVS	equivalent stress calculated from the stress

	tensor
ERROR	error between the calculated deviatoric stresses and the deviatoric stresses obtained by the Reus - Prandtl equations
FATREM	friction force at the manfrel/product interface
FF	friction factor
FLST	flow stress
MPARAM	number of terms of the stress function ϕ_2 (eq. 5.36)
MIERM1	parameter m in stress function ϕ_2 (eq. 5.36)
MIERM2	parameter n in stress function ϕ_2 (eq. 5.36)
NERREN	number of error functions
NNP	number of nodal points
NPARAM	number of terms of the stress function ϕ_1 (eq. 5.35)
NTERM1	parameter k in stress function ϕ_1 (eq. 5.35)
NTERM2	parameter l in stress function ϕ_1 (eq. 5.35)
PRED	normal pressure at the die billet interface
PRESS	upper bound extrusion pressure
RE	outside radius of the mandrel
RS	outside radius of the product
SIGDR	deviatoric stresses obtained by the method of weighted residuals
SIGDT }	
SIGDZ	
SIGMAR	normal stresses
SIGMAT }	
SIGMAZ	

SIGMDR	}	
SIGMDRZ		deviatoric stresses calculated by the Reus
SIGMDT	}	Prandtl equations
SIGMDZ		
SIGMRZ		shear stresses calculated by the weighted residual method
SIGRA	}	
SIGRB	}	
SIGRZA		coefficients of free parameters
SIGTB	}	
SIGZA		
SIGZO		axial stress acting at the exit boundary
TRZ		shear stresses along the mandrel
VS		velocity of the product
W		weighting factors
X		outside radius of the mandrel
Y		residual functions

D.2 - LISTING OF THE PROGRAM STRESS

```

PROGRAM STRESS, DAT120, OUTPUT, SOLVE, TAPE5=DAT120, TAPE6=SOLVE
1, TAPE2, TAPE4

C
C   THIS PROGRAM IS TO BE USED WITH PROGRAMS VELD OF
C   MIN AND TRIAL. DATA IS IN DAT120 AND TAPE4.
C

COMMON NZPT, NP, DEXT, NNP, DS, SIG20,
1PE, P3, ALPHA, PDI(256), CDP(256), LDZ(256), STPNPT(4, 256),
2EFCTPT(256), EFCTN(256), TPZ(30), PPED(30), X, NF,
3NPAPAM, MPAPAM, LPAPAM, NTERM1, NTERM2, MTERM1, MTERM2, NEPPFN, FF,
4A(73, 73), C(73), FLCT(256), SIGMAP(256), SIGMAT(256), SIGMAZ(256),
5SIGMDP(256), FLW(256), Y(8, 73), M(8), EQVDC(256), SIGMDP(256),
6SIGMDT(256), SIGMDZ(256), SIGMDPZ(256), EQVDC(256), EXTP, SIGDP(256),
7SIGDT(256), SIGDZ(256), EPPDP(5, 256), PPEC
READ(4, 401) NZPT, NP, DEXT, NP, P3, ALPHA, V3, EXTP, DS
401 FORMAT(2I3, 6E12.6, /, E20.14)
READ(5, 501) NTERM1, NTERM2, NEPPFN, NTERM1, NTERM2, (W(I), I=1, NEPPFN)
501 FORMAT(5I4, 8F5.0)
NT = NP + 1
NNP = NZPT + (NR + 1)
NF = NNP - NZPT + 1
NC = NZPT - 1
MPAPAM = MTERM1 + MTERM2
NPAPAM = NTERM1 + NTERM2
LPAPAM = NPAPAM + MPAPAM
PDT = 10PT(3)
COF = 2. / PDT
READ(4, 402) CDP(I), CDZ(I), I=1, NNP)
402 FORMAT(2E20.14)
READ(4, 403) STPNPT(J, I), J=1, 4, EFCTPT(I), I=1, NNP)
403 FORMAT(5E16.10)
READ(4, 405) FLCT(I), I=1, NNP)
405 FORMAT(E16.10)
READ(4, 406) PPEC, SIG20, FF, FATREM
406 FORMAT(4E16.10)
DO 3 J=1, NNP
3 CONTINUE

C
C   DEVIATOPIC STRESS
C   DO 2 I=1, NNP
FLW(K) = (2./3.) + (FLST(K) / EFSTPT(K))

C
SIGMDP(K) = FLW(K) + STPNPT(1, K)
SIGMDT(K) = FLW(K) + STPNPT(2, K)
SIGMDZ(K) = FLW(K) + STPNPT(3, K)
SIGMDPZ(K) = FLW(K) + STPNPT(4, K)

C
SIG1 = SIGMDP(K) + SIGMDT(K)
SIG2 = SIGMDP(K) + SIGMDZ(K)
SIG3 = SIGMDT(K) + SIGMDZ(K)
SIG4 = SIGMDPZ(K) + SIGMDPZ(K)
VV = -(SIG1 + SIG2 + SIG3) + SIG4
EQVDC(K) = PDT + COPT(VV)

C
2 CONTINUE

```

```

C
      WRITE(6,1990) NZPT,NP,NTERM1,NTERM2,NEPPFN,MTERM1,MTERM2,
      1DENT,PE,PI,ALPHA,VC,PRESS,SIGZ0,FF
1990 FORMAT(1H1,/,1X,5X,7HNDPT =,I3,/,1X,5X,7HNMP =,I3,
      1X,5X,7HNTERM1=,I3,/,1X,5X,7HNTERM2=,I3,/,1X,5X,
      27HNEPPFN=,I3,/,1X,5X,7HMTERM1=,I3,/,1X,5X,7HMTERM2=,I3,
      3X,5X,7HDENT =,F7.3,/,1X,5X,7HPE =,F7.3,
      4X,5X,7HPI =,F7.3,/,1X,5X,7HALPHA =,F7.3,/,1X,5X,
      57HVC =,F7.3,/,1X,5X,7HPRESS =,F7.3,/,1X,5X,7HSIGZ0 =,
      6F7.3,/,1X,5X,7HFF.FACT=,F7.3,/)
      WRITE(6,1991) I,M,I,I=1,NEPPFN)
1991 FORMAT(5X,2HND,1I,4H) =,F7.3,/)
      WRITE(6,2000) ENTP
2000 FORMAT(1H1,/,1X,5X,17HENTRUCION PH10 =,E12.4,/,/)
C
C
      CALL MTRS
C
      DO 707 I = 1,NMP
      ERRDP(1,I) = (EQV2(I) - FLST(I)) * 100. / FLST(I)
      ERRDP(2,I) = (SIGMP2(I) - SIGMDP2(I)) * 100. / SIGMDP2(I)
      ERRDP(3,I) = (SIGDP(I) - SIGMDP(I)) * 100. / SIGMDP(I)
      ERRDP(4,I) = (SIGDT(I) - SIGMDT(I)) * 100. / SIGMDT(I)
      ERRDP(5,I) = (SIGDC(I) - SIGMDC(I)) * 100. / SIGMDC(I)
707 CONTINUE
      WRITE(6,5000)
5000 FORMAT(1H1,/,14X,107HND      FLSTRESS      EQTPRESS      ERRDP      SIG
      1MAP      SIGMAT      SIGMAZ      SIGMPZ      SIGMDP      ERRDP,/,
      WRITE(6,5001) I,FLST(I),EQV2(I),ERRDP(1,I),SIGMAP(I),SIGMAT(I),SIG
      1MAZ(I),SIGMPZ(I),SIGMDP2(I),ERRDP(2,I),I = 1,NMP)
5001 FORMAT(11X,15,9E12.4)
      WRITE(6,5002)
5002 FORMAT(1H1,/,14X,107HND      SIGDP      SIGMDP      ERRDP      SI
      1GDT      SIGMDT      ERRDP      SIGDC      SIGMDC      ERRDP,/,
      WRITE(6,5003) I,SIGDP(I),SIGMDP(I),ERRDP(3,I),SIGDT(I),SIGMDT(I),E
      1RRDP(4,I),SIGDC(I),SIGMDC(I),ERRDP(5,I),I = 1,NMP)
5003 FORMAT(11X,15,9E12.4)
C
      DO 4 I = 1,NMP
      FR = SIGMAP(I) / FLST(NZPT)
      FT = SIGMAT(I) / FLST(NZPT)
      FZ = SIGMAZ(I) / FLST(NZPT)
      FRZ = SIGMPZ(I) / FLST(NZPT)
      IF(I.EQ.1) WRITE(6,602)
602 FORMAT(/,1X,4HNODE,7X,2HFP,13X,2HFT,13X,2HFZ,13X,3HFRZ,/)
      WRITE(6,603) I,FR,FT,FZ,FRZ
603 FORMAT(1X,14,4F15.5)
      4 CONTINUE
      200 CONTINUE

```

```

C
  AVFLW = 0.0
  AVPRC = 0.0
  DO 708 I = 1,N2
    AVFLW = AVFLW + FLCT(I+1) + FLCT(I)
  708 AVPRC = AVPRC + SIGMAP(I+1) + SIGMAP(I)
    AVFLW = AVFLW / FLOAT(2*N2)
    AVPRC = AVPRC / FLOAT(2*N2)
    CBF = -FF*AVFLW + AVPRC*PDT
    WRITE(6,5004) SIGMAP(1),AVFLW,AVPRC,CBF
5004 FORMAT(//,10X,26H AVERAGE FLOW STRESS,      =,E12.4,
1         //,10X,26H AVERAGE FLOW STRESS,      =,E12.4,
2         //,10X,26H AVERAGE PRESSURE,         =,E12.4,
3         //,10X,26H COEF. OF FRICTION,         =,E12.4)
    DIST = CDZ(NCPT) - CDZ(1)
    DIST1 = CDZ(1) - CDZ(NF)
    WRITE(2,2220) NCPT,DIST,DIST1,PRESS,FHTREM
2220 FORMAT(15,3E20.14,/,E16.10)
    DO 709 I=1,NCPT
      TPC(I) = -FF*FLCT(I)*PDT
  709 WRITE(2,2221) SIGMAP(I),TPC(I)
2221 FORMAT(2E20.14)
    WRITE(6,5005) //,1,TPC(I),I=1,NCPT)
5005 FORMAT(//,10X,4H NODE,5X,32H SHEAR STRESS ALONG THE MANDREL
1         //,10X,14,E20.4,/)
    J = 0
    AVFLW = 0.0
    AVPRC = 0.0
    DO 710 I = NF,NNF
      J = J + 1
      PRD(J) = (SIGMAP(I) + SIGMAZ(I))/2. + (SIGMAZ(I) - SIGMAP(I))
1         // 2. * CDZ(2.*ALPHA) - SIGMAZ(I) * SIN(2.*ALPHA)
      IF(J.EQ.1) GO TO 5008
      AVFLW = AVFLW + FLCT(I) + FLCT(I-1)
      AVPRC = AVPRC + PRD(J) + PRD(J-1)
      GO TO 710
5008 WRITE(6,5006)
5006 FORMAT(//,20X,32H PRESSURE DISTRIBUTION ON THE DIE,/,
1         //,10X,4H NODE,10X,13H NORMAL STRESS,/)
  710 WRITE(6,5007) I,PRD(J)
5007 FORMAT(//,10X,14,E20.4)
    AVFLW = AVFLW / FLOAT(2*N2)
    AVPRC = AVPRC / FLOAT(2*N2)
    CBF = -FF*AVFLW + (AVPRC + PDT)
    WRITE(6,5009) AVFLW,AVPRC,CBF
5009 FORMAT(//,10X,26H AVERAGE FLOW STRESS,      =,E12.4
1         //,10X,26H AVERAGE PRESSURE,         =,E12.4
2         //,10X,26H COEF. OF FRICTION,         =,E12.4)
  STOP
END

```

```

SUBROUTINE MTPC
  DIMENSION B(5329)
  COMMON NCPT,NP,DEXT,NNP,20,SIG20,
  1PE,PC,ALPHA,PADI(256),CDP(256),CDZ(256),CTRPT(4,256),
  2EFCITRT(256),EFCITN(256),TPZ(30),PRED(30),N,NF,
  3NPAPAM,MPAPAM,LPAPAM,NTERM1,NTERM2,MTERM1,MTERM2,NERRFN,FF,
  4A(73,73),C(73),FLCT(256),SIGMAP(256),SIGMAT(256),SIGMAC(256),
  5SIGMPZ(256),FLM(256),Y(8,73),M(8),EQVDC(256),SIGMDP(256),
  6SIGMDT(256),SIGMDZ(256),SIGMDPZ(256),EUVG(256),ENTP,SIGDR(256),
  7SIGDT(256),SIGDC(256),EPRDP(5,256),PPEBY
  EQUIVALENCE (A(1,1),B(1))
  N = DEXT - 2.
  K = NPAPAM + MPAPAM + 1
  DO 100 J = 1,K
    C(J) = 0.0
    DO 100 I = 1,K
100  A(I,J) = 0.0
    DO 102 II = 1,NF,NCPT
      DO 102 KK = 1,NCPT
        N = II + KK - 1
        HP = FLCT(N)
        FLOE = FLM(N)
        P = CDP(N)
        Z = CDZ(N)
        DO 106 NK = 1,NTERM1
          DO 106 NL = 1,NTERM2
            INDM = NK - 1
            INDN = NL - 1
            NT = NL + (NK-1) * NTERM2
            SIGRA = FLOAT(INDN*(INDN-1)) * (P**2*INDM+2) * (Z**INDN-2)
            SIGZA = FLOAT((2*INDM+3)*(2*INDM+2)) * (P**2*INDM) * (Z**INDN)
            SIGRZA = -FLOAT((2*INDM+3) * INDN) * (P**2*INDM+1) * (Z**INDN-1)
            Y(1,NT) = (2./3.) * SIGRA - SIGZA/3.
            Y(2,NT) = -SIGRA/3. - SIGZA/3.
            Y(3,NT) = (2./3.) * SIGZA - SIGRA/3.
            Y(4,NT) = SIGRZA
            Y(5,NT) = 0.0
            Y(6,NT) = 0.0
            Y(7,NT) = 0.0
            Y(8,NT) = 0.0
            IF(KK.EQ.1) Y(5,NT) = SIGZA
            IF(KK.EQ.NCPT) Y(6,NT) = SIGZA
            IF(NL.EQ.NCPT) Y(7,NT) = SIGRZA
            IF(N.EQ.NF) Y(8,NT) = (SIGRA - SIGZA*(IN*2.*ALPHA)/2. +
1          SIGRZA*CDZ(2.*ALPHA)
106  CONTINUE

```

```

DO 107 MF = 1,MTERM1
DO 107 ML = 1,MTERM2
INDP = MF - 1
INDQ = ML - 1
NTT = NPAPAM + ML + (MF-1) * MTERM2
SIGPB = (P**2*INDP) * (Z**INDQ)
SIGTB = FLOAT(2*INDP+1) * (P**2*INDP) * (Z**INDQ)
Y(1,NTT) = -SIGTB*3. + (2.*3.) * SIGPB
Y(2,NTT) = (2.*3.) * SIGTB - SIGPB*3.
Y(3,NTT) = -SIGPB*3. - SIGTB*3.
Y(4,NTT) = 0.0
Y(5,NTT) = 0.0
Y(6,NTT) = 0.0
Y(7,NTT) = 0.0
Y(8,NTT) = 0.0
IF(N.LE.NF) Y(8,NTT) = SIGPB*2*IN(2.*ALPHA)*2.
107 CONTINUE
Y(1,K) = -FLOE * CTNPRT(1,N)
Y(2,K) = -FLOE * CTNPRT(2,N)
Y(3,K) = - FLOE * CTNPRT(3,N)
Y(4,K) = -FLOE* CTNPRT(4,N)
Y(5,K) = -SIGZU
Y(6,K) = PRECC
Y(7,K) = 0.0
Y(8,K) = 0.0
IF(N.LE.NCPT) Y(7,K) = FF*HP * COPT(3.)
IF(N.LE.NF) Y(8,K) = -FF*HP * COPT(3.)
DO 103 LT = 1,NPREFN
DO 101 J=1,K
DO 101 I=J,K
A(I,J) = A(I,J) + Y(LT,I) * Y(LT,J) * MLT
A(J,I) = A(I,J)
101 CONTINUE
103 CONTINUE
102 CONTINUE
JJ = 1
DO 105 J = 1,LPAPAM
DO 104 I=1,LPAPAM
B(JJ) = A(I,J)
JJ = JJ+ 1
104 CONTINUE
C(J) = - A(J,K)
105 CONTINUE
CALL SIMD(B,C,LPAPAM,1)
WRITE(6,610)
610 FORMAT(' ',20X,10HPARAMETERPC,/)
WRITE(6,611) (I,C(I),I=1,LPAPAM)
611 FORMAT(20X,2HC,12,3H) =,E15.8)

```

```

DO 132 I = 1,NHP
HP = FLST(I)
P = CDP(I)
Z = CDC(I)
SIGMAP(I)=0.0
SIGMAT(I)=0.0
SIGMAZ(I)=0.0
SIGMPZ(I)=0.0
DO 131 NH = 1,NTERM1
DO 131 NL = 1,NTERM2
INDM = NH - 1
INDN = NL - 1
IND = NL + (NH-1) * NTERM2
SIGPA = FLOAT(INDN*(INDN-1)) * (P**2+INDM+2) * (Z**2+INDN-2)
SIGCA = FLOAT(2*INDM+3)*(2*INDM+2) * (P**2+INDM) * (Z**2+INDN)
SIGPZA =-FLOAT(2*INDM+3) * INDN * (P**2+INDM+1) * (Z**2+INDN-1)
SIGMAP(I) = SIGMAP(I) + C(IND) * SIGPA
SIGMAZ(I) = SIGMAZ(I) + C(IND) * SIGCA
SIGMPZ(I) = SIGMPZ(I) + C(IND) * SIGPZA
131 CONTINUE
DO 130 MH = 1,MTERM1
DO 130 ML = 1,MTERM2
INDP = MH - 1
INDO = ML - 1
SIGPB = (P**2+INDP) * (Z**2+INDO)
SIGTB = FLOAT(2*INDP+1) * (P**2+INDP) * (Z**2+INDO)
INDD = ML + (MH-1) * MTERM2 + NHP*H
SIGMAP(I) = SIGMAP(I) + C(INDO) * SIGPB
SIGMAT(I) = SIGMAT(I) + C(INDO) * SIGTB
130 CONTINUE
SIGNED = (SIGMAP(I)+SIGMAT(I)+SIGMAZ(I)) / 3.
SIGDP(I) = SIGMAP(I) - SIGNED
SIGDT(I) = SIGMAT(I) - SIGNED
SIGDZ(I) = SIGMAZ(I) - SIGNED
DF1 = SIGMAP(I) - SIGMAT(I)
DF2 = SIGMAT(I) - SIGMAZ(I)
DF3 = SIGMAZ(I) - SIGMAP(I)
DF4 = SIGMPZ(I)
DF12 = DF1 * DF1
DF22 = DF2 * DF2
DF32 = DF3 * DF3
DF42 = DF4 * DF4
EQV(I) = SQRT(.5 * (DF12 + DF22 + DF32) + 3. * DF42)
132 CONTINUE
RETURN
END

```

APPENDIX E

STRAIN DISTRIBUTION AND FRICTION ALONG THE MANDREL IN THE HYDROSTATIC
EXTRUSION/DRAWING OF TUBES (Paper Presented at the 20th MTDR Conf.,
Birmingham, 1979)

STRAIN DISTRIBUTION AND FRICTION ALONG THE MANDREL IN THE HYDROSTATIC EXTRUSION-DRAWING OF TUBES

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Exhibition Road, London

SUMMARY

Experimental results are presented which provide new information for the design of fixed mandrels used in the extrusion and drawing of tubes.

A hydrostatic extrusion-drawing apparatus is described, together with instrumentation to record strains in the bore of the hollow mandrel and force due to friction along the deforming material-mandrel interface.

Aluminium and copper specimens were extruded into tubes, while the frictional load, the fluid pressure and the drawing force, together with the hoop and axial strains in the bore of the mandrel were recorded simultaneously.

Experimental frictional loads and strain distributions are given for various extrusion pressure-drawing stress ratios.

INTRODUCTION

In the hot and cold extrusion and drawing of tubes with a fixed mandrel, a matter of industrial significance is the design of the mandrel, which is subjected to normal and frictional shear stresses by the deforming material. While it is perfectly feasible to derive these stresses from theory, few attempts have been made so far to check the calculated results by experiments. In what follows the experimentally recorded mandrel strains and loads in the hydrostatic extrusion/drawing of tubes are presented, while the corresponding theoretical analysis and predictions will be the subject of a separate report later.

Although the widest application of a work of this nature would be in conventional hot and cold extrusion and in tube drawing, it was decided to study the hydrostatic extrusion/drawing of tubes in the first instance, because the process is eminently suitable to obtain experimental results for both the extrusion and drawing of tubes with a fixed mandrel, under a wide range of controlled conditions.

EXPERIMENTAL APPARATUS AND MATERIALS

A strain gauge method has been used for recording mandrel strains and loads, similar to the method applied by Lengyel and Musuva (1) to extrusion containers.

An outline drawing of the experimental apparatus is shown in Fig. 1. The extrusion fluid pressure in the container is generated by a pump-intensifier unit (not shown), while the prepared tag of the billet extending through the die is pulled by a device at a preselected speed.

Pulling force, together with strains and load on the mandrel were recorded simultaneously, while extrusion speed and pressure were kept constant.

Various components of the experimental apparatus have already been described in earlier publications: the container by Lengyel and Ashford (2), the pulling device by Dunn and Lengyel (3)

The pumps in the pump-intensifier unit were powered by electric motors and could deliver oil at the rate of $410 \text{ cm}^3/\text{min}$. at pressures up to 103.4 MN/m^2 ($15,000 \text{ lbf/in}^2$). They stopped automatically on attaining a preset pressure and held it constant, restarting when necessary to make up for losses. The pressure was further increased by an intensifier and measured by a manganin coil in the usual manner.

A Solartron LP 1029 Digital Data Recorder was used to record the output signals from this manganin coil and from all strain gauges at speeds of up to 10 samples per second. A high speed numeric printer connected on line with the data logger enabled the readings to be printed out onto a paper tape.

The thick walled cylindrical hollow mandrel was made from KEA 145 tool steel, heat treated to 52-54 Rockwell C hardness. It had a sufficient length so that the effect of seals on the pressure calibrations could be neglected. The method of calibrating the strain gauged mandrel under pressure was similar to that used by Lengyel and Musuva (1) for extrusion containers but will not be described here since the relevant results will form part of the second report.

HYDROSTATIC EXTRUSION-DRAWING OF TUBES

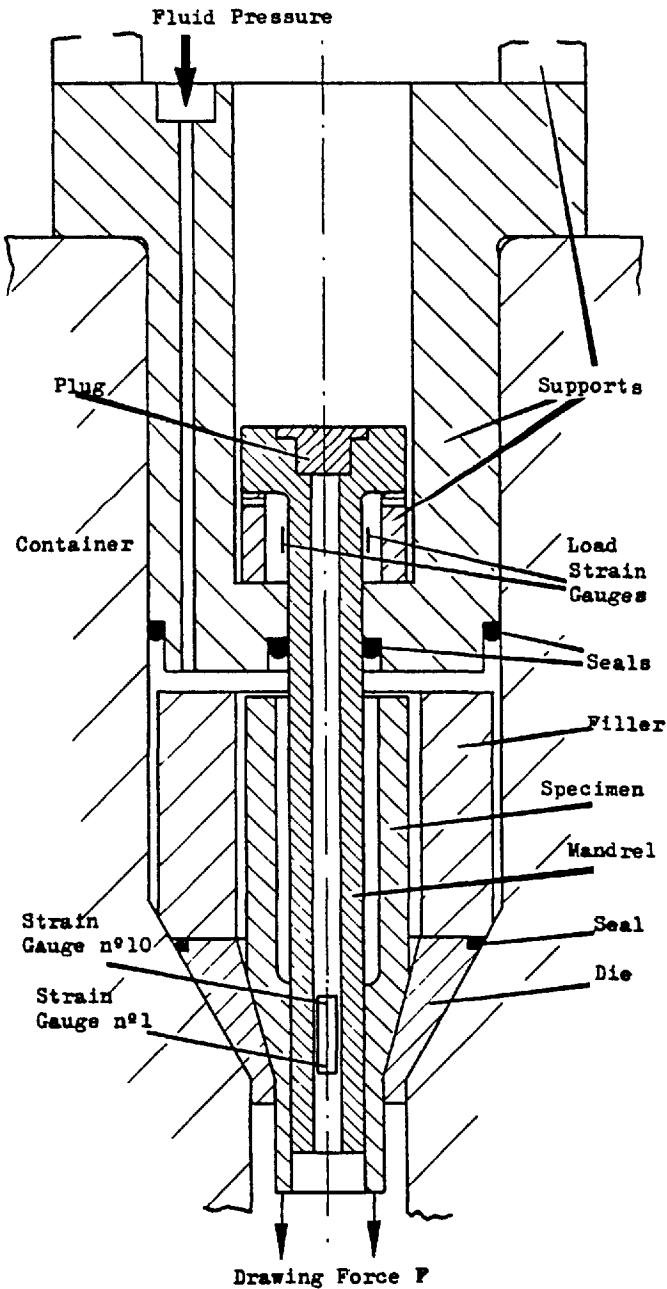


FIG. 1 EXPERIMENTAL ARRANGEMENT

Ten strain gauges, five to measure hoop strains five for axial strains, were placed alternately in the axial direction along the mandrel bore and bonded to it in such a way that the centre line of gauge No. 1 (a hoop strain gauge) was 20 mm from the product-end of the mandrel while the others were positioned in the manner illustrated in Fig. 1. Compensating strain gauges were bonded to a dummy mandrel.

The pulling force on the mandrel was measured by four axial and four hoop gauges bonded to the mandrel, away from the deformation zone, which could discriminate between load increments larger than 215 N. The velocity of pulling the product was kept constant throughout the experiments at 1.5 mm/s, which determined the speed of the process. All tests were carried out with a die of 30° included angle.

The extrusion fluid was a four to one mixture of castor oil and methyl alcohol.

Commercial purity aluminium and high conductivity copper billets were used in the experiments, with a useful length of 100 mm, outside diameter up to 41.148 mm and bore diameter of 26.416 mm. The extrusion ratios were 2, 2.65 and 3.5 for aluminium, 1.5 and 2 for copper. The billets were annealed, the aluminium for one hour at 300°C, the copper for two hours at 600°C. Before extrusion, the billets were coated with a graphite solution.

RESULTS AND DISCUSSION

The billets were machined and preformed, producing a narrow lip in the bore to assist sealing the fluid and prevent it escaping in the initial stage between the billet and mandrel.

First a small pulling force was applied to the tag on the billet-end to make an effective seal at the billet/mandrel and billet/die interfaces. Subsequently the extrusion fluid pressure was generated and raised to a predetermined level and kept constant. Then the pulling device was operated to apply the necessary drawing load to the tag so as to make up the total stress required for the process and pull the billet material through the die at a constant speed.

Fig. 2 shows the variation of the extrusion pressure with drawing stress with pressure shown negative. The slope of the lines is greater than unity in all cases, confirming earlier theoretical and experimental results (4, 5) that the sum of the absolute values of extrusion pressure and drawing stress decreases with increasing drawing stress.

Fig. 3 illustrates that the friction force along the mandrel decreases with increasing drawing stress, confirming the results in Fig. 2. The exception is copper when extruded at a ratio of 1.5, where the frictional load is constant. The likely explanation is that the increase in friction at larger extrusion pressures is so small that the resultant increase in load could not be detected by the present arrangement which is unsuitable to record differences less than 215 N.

Typical experimental results relating to mandrel strains are shown in Fig. 4, 5 and 6. Fig. 4a, 5a and 6a illustrate the hoop strain distribution in the mandrel relative to the strain gauge positions, while

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Fig. 4b, 5b and 6b show the axial strains, for aluminium at extrusion ratios 2 and 3.5 and for copper at an extrusion ratio of 2, with compressive hoop strains shown negative. Similar results have been obtained for extrusion ratios of 2.65 (aluminium) and 1.5 (copper) but these are omitted here for clarity. Both the hoop and axial strains indicate the combined effect of the normal pressure and frictional drag applied by the deforming material to the hollow mandrel.

The contribution of the frictional drag to the axial strain (and stress) is small. For example, the maximum axial load at an extrusion ratio of 3.5 for aluminium is 4,100 N (Fig. 3) corresponding to an axial microstrain of 46, against the recorded maximum total axial microstrain of 1,450 (Fig. 5b). The values for copper at an extrusion ratio of 2 are 39 and 2520 microstrain respectively. These figures indicate that, when the design of the mandrel is considered, first of all the effect of pressure must be taken into account while that of the frictional drag is secondary, at least in hydrostatic extrusion/drawing.

In conventional tube extrusion and to a lesser extent in drawing conditions could be quite different, since mandrel friction is likely to be significantly higher. Calculations indicate that for the extreme case of sticking friction, for

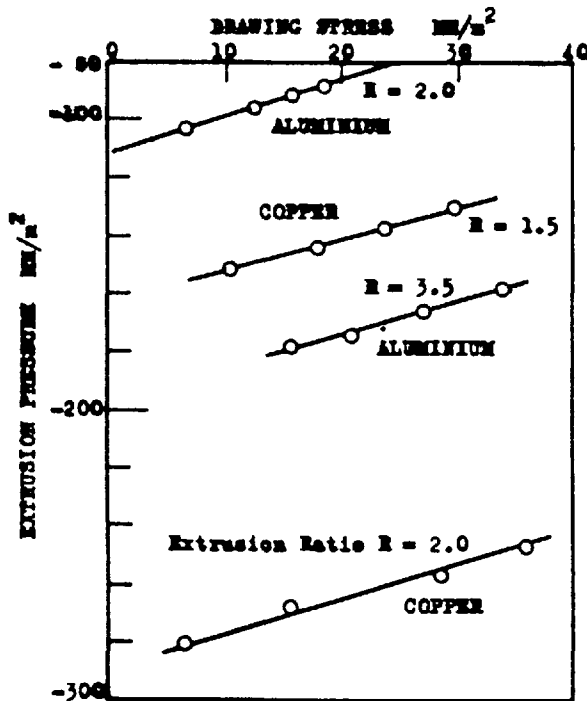


FIG. 2 EXTRUSION PRESSURE VERSUS DRAWING STRESS

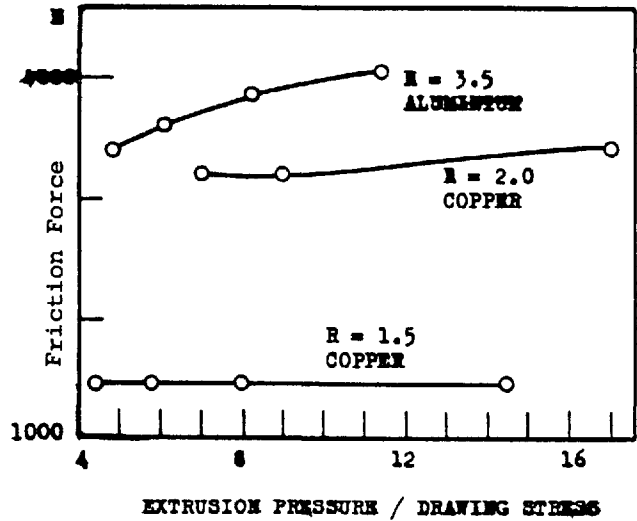


FIG. 3 FRICTION FORCE AGAINST EXTRUSION PRESSURE/DRAWING STRESS

aluminium at an extrusion ratio of 3.5 the frictional drag on the mandrel would be over 100 kN, corresponding to an axial microstrain of 1,100, assuming that friction is present only in the die zone as in the hydrostatic extrusion of tubes. For copper in extrusion or drawing at a ratio of 2, the frictional drag for sticking friction would be 112 kN resulting in 1,200 axial microstrain on the mandrel. These strains are significant in relation to those shown in Fig. 5b and 6b. Although these calculated strains are for the extreme case of sticking friction, they do indicate that in conventional tube extrusion or drawing, when friction is likely to be higher than in hydrostatic extrusion, frictional effects in mandrel design could not be neglected.

In conventional tube extrusion with a fixed mandrel not only the frictional shear stress is likely to be higher, but the interfacial area between the billet and mandrel is much extended, adding to the axial tensile stress on the mandrel.

In the present case, the strain data indicate that the maximum radial stress on the mandrel is in a section about midway along the mandrel/billet interface and it is this section which must be critical for mandrel failure. On the other hand, the maximum tensile stress in the mandrel due to frictional drag in conventional extrusion must be upstream near the billet/ram interface, which could alter the position and magnitude of the critical mandrel stress.

CONCLUSIONS

Hydrostatic extrusion/drawing experiments have been carried out on aluminium and copper specimens. The results show that,

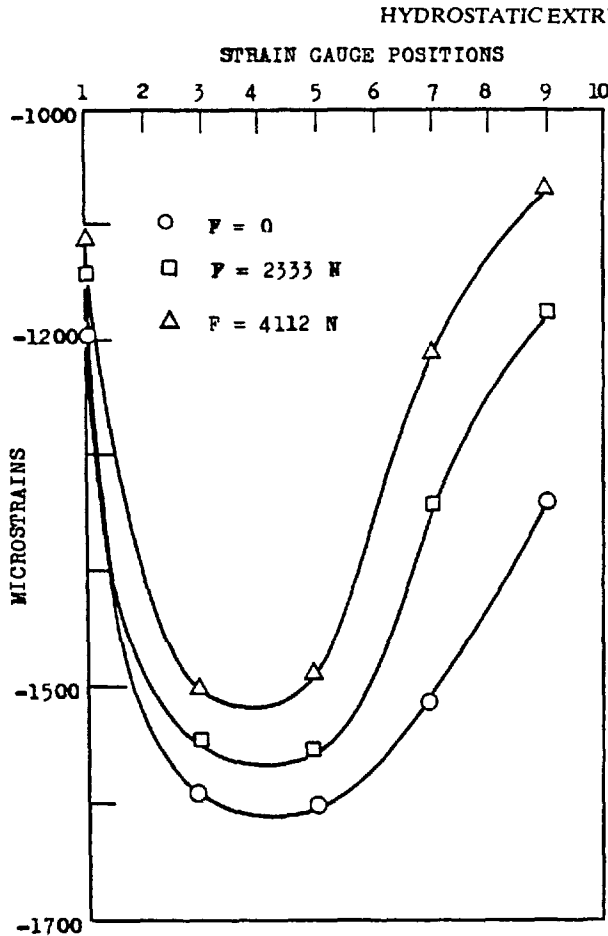


FIG. 4a MANDREL HOOP STRAINS,
EXTRUSION RATIO 2.0
ALUMINIUM

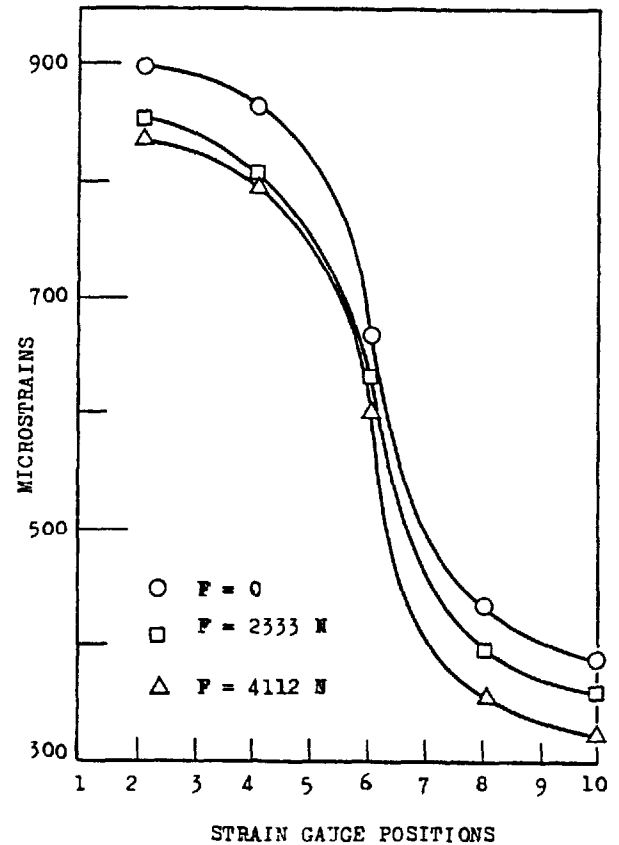


FIG. 4b MANDREL AXIAL STRAINS,
EXTRUSION RATIO 2.0 ALUMINIUM

by the application of a drawing stress not only the process speed can be controlled, but also the extrusion pressure, friction and the total stress required for the process are reduced. As a consequence, the radial, hoop and axial stresses in the mandrel also become smaller.

The experiments indicate that the tensile stress in the mandrel due to frictional drag at the deforming material/mandrel interface is relatively small so that the critical mandrel stress is influenced mainly by the normal pressure. While this could be approximately true also for tube drawing with a fixed mandrel under favourable lubrication conditions, in the extrusion of tubes (particularly in hot extrusion) the much larger billet/mandrel interface and greater friction could result in significantly different stress conditions, which need to be assessed separately.

This experimental work is part of a wider investigation in the development of design criteria for the fixed mandrel in the extrusion and drawing of tubes (6).

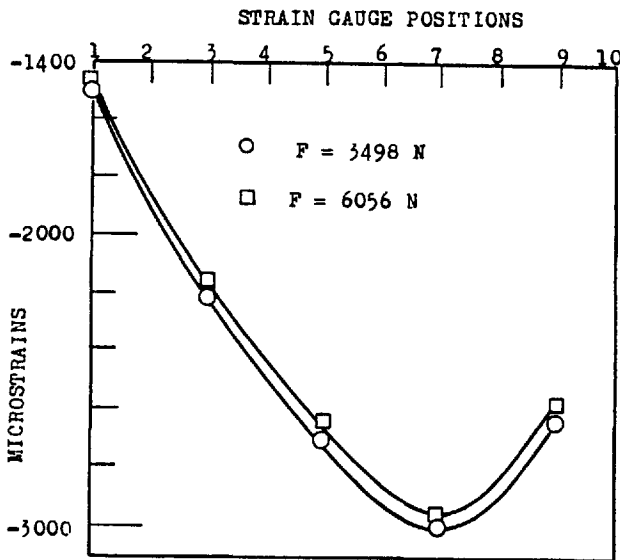
ACKNOWLEDGEMENTS

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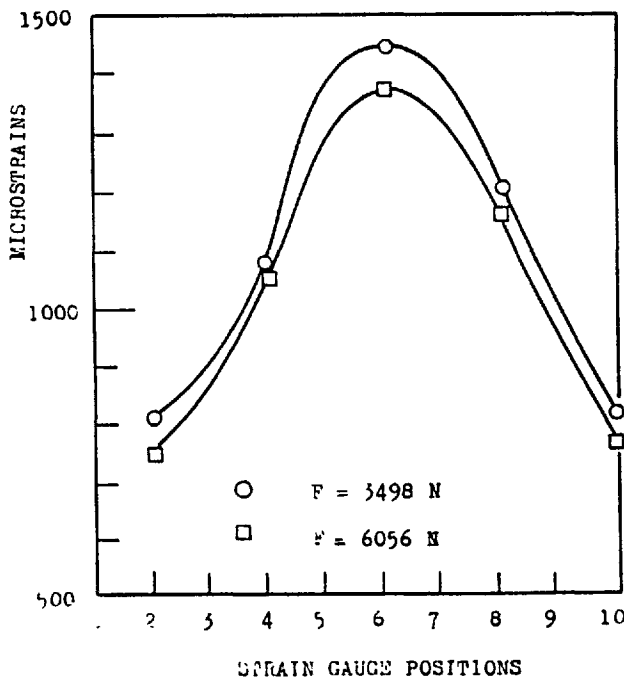
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a)



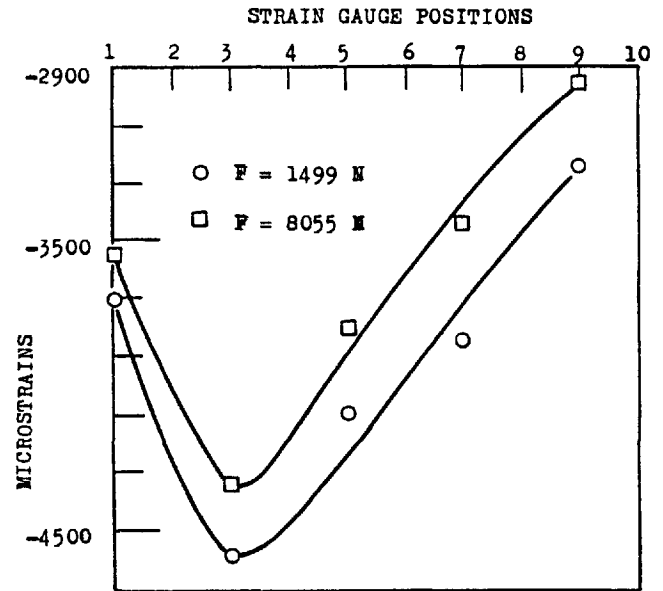
b)

FIG. 5 MANDREL STRAINS, EXTRUSION RATIO 3.5 ALUMINIUM

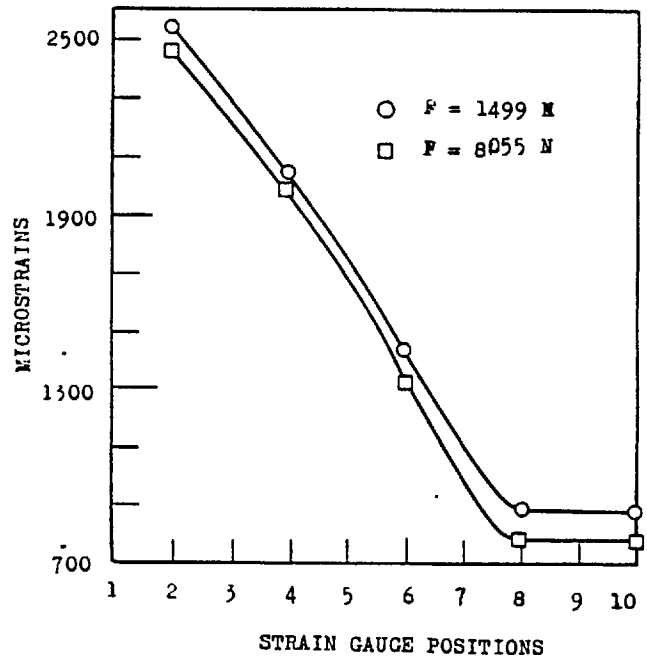
- a) Hoop Strains
- b) Axial Strains

Process, Proc. 9th MTDR Conference.

5. H. Ll. D. PUGH (1971) The Mechanical Behaviour of Materials under Pressure, Materials Science Series, London.



a)



b)

FIG. 6 MANDREL STRAINS, EXTRUSION RATIO 2.0 COPPER

- a) Hoop Strains
- b) Axial Strains

6. M.J.M.B. MARQUES (1979) Ph.D. Thesis to be presented.