

IMPERIAL

Scaling and structure of the momentum
and scalar fields within a turbulent
boundary layer perturbed by an effusion
film and its comparison to asymptotic
behaviour of pipe flow momentum

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Statement of Originality

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Abstract

The first part of this thesis presents a new scaling paradigm for the intermediate region of pressure-driven, fully developed turbulent boundary layers. The log law assumption is shown to be invalid for scaling the location of Reynolds shear stress maximum, which predicts $y_{m\tau}^+ \propto Re_\tau^\beta$ with $\beta = 0.5$. An outer peak in premultiplied production is observed to be further from the wall than the Reynolds shear stress peak, indicating that $\beta > 0.5$. Fitting to high Reynolds number turbulent pipe and channel data produces $\beta = 0.529 \pm 0.005$.

An intermediate velocity scale $\Delta U_m^+ \propto Re_\tau^{2\beta-1}$ is derived. The magnitude and location of the outer peak in streamwise velocity variance are shown to scale well with ΔU_m^+ . A second-order Taylor expansion about $y_{m\tau}^+$ is used to predict the location of the outer peak in premultiplied production, which has good agreement with high Reynolds number data.

The second part of this work presents a large-scale experimental study, where a canonical turbulent boundary layer is perturbed by an effusion film - applicable to gas turbine blade cooling. Lower temperature air is injected into the boundary layer through a staggered array of small holes. The injection-freestream velocity ratio (VR) is varied from $0.10 < VR < 0.37$. Three-component laser Doppler anemometry and oil film interferometry characterise the momentum field, and simultaneous x-wire and cold-wire characterise the joint momentum-scalar field.

The boundary layer scaling behaviour changes about a critical velocity ratio $VR_c \approx 0.20$, which is attributed to bulk film lift-off. Large spanwise differences in the temperature and momentum fields are observed. Turbulence is significantly enhanced, particularly in the outer region and the near-wall spanwise fluctuations. Neither the local equilibrium hypothesis or Reynolds' analogy are valid.

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Nomenclature

Acronyms

BL	Boundary layer
CFD	Computational fluid dynamics
CRVP	Counter-rotating vortex pair
DNS	Direct numerical simulation
FFT	Fast Fourier transform
FST	Freestream turbulence
LDA	Laser Doppler anemometry
LES	Large eddy simulation
LP	Long Pipe
LSMs	Large-scale motions
LUT	Look-up table
NSTAP	Nanoscale thermal anemometry probe
OFI	Oil film interferometry
PEG	Polyethylene glycol
PIV	Particle image velocimetry
PLA	Polylactic acid
PSD	Power spectral density
RANS	Reynolds-averaged Navier-Stokes
SP	Superpipe

TBL	Turbulent boundary layer	
TKE	Turbulence kinetic energy	m^2/s^2
ZPG	Zero pressure gradient	

Greek Symbols

α	Temperature coefficient	$^{\circ}\text{C}^{-1}$
β	Intermediate variable exponent	
Δ	Clauser-Rotta length scale	m
Δ	Difference	
δ	Shear layer thickness/ outer length scale	m
δ_1	Displacement thickness	m
δ_2	Momentum thickness	m
δ_ν	Viscous length scale, ν/u_τ	m
ϵ	Dissipation of turbulence kinetic energy	m^2/s^3
ϵ_θ	Destruction of temperature variance	$^{\circ}\text{C}^2/\text{s}$
η	Adiabatic effectiveness	
γ	Flow angle	$^{\circ}$, rad
κ	von Kármán constant	
Λ	LDA fringe spacing	m
λ	Wavelength	m
λ_f	OFI fringe wavelength	m
μ	Dynamic viscosity	$\text{Pa} \cdot \text{s}$
χ	Normalised intermediate layer thickness	
∇	Gradient operator	
ν	Kinematic viscosity	m^2/s
ϕ	Power spectral density function	

Π	Coles' wake parameter	
ψ	Oil film interferometry camera viewing angle	$^{\circ}$, rad
ρ	Density	kg/m ³
τ	Shear stress	Pa
Θ	Mean temperature	$^{\circ}$ C
θ	Fluctuating temperature	$^{\circ}$ C
$\varepsilon_m, \varepsilon_h$	Turbulent diffusivity of momentum, heat	m ² /s
φ	Angle between LDA beam pair	$^{\circ}$, rad
ϑ	Angle between LDA probes	$^{\circ}$, rad
Ξ	Log law indicator function	
ξ	Local normalised wall-normal Reynolds shear stress coordinate	

Roman Symbols

$\dot{m}$	Mass flow rate	kg/s
$\dot{q}$	Heat flux	W/m ²
C_f	Skin friction coefficient	
c_p	Specific heat capacity at constant pressure	J/kgK
c_v	Specific heat capacity at constant volume	J/kgK
D	Hole diameter	m
d	Diameter	m
DR	Density ratio	
E	Voltage	V
f	Frequency	s ⁻¹
H	Shape factor, δ_1/δ_2	
h	Heat transfer coefficient	W/m ² K
h	Height	m

I	Intensity	
J	Momentum flux ratio	
k	Turbulence kinetic energy	m^2/s^2
k_r	Roughness height	m
k_x	Streamwise wavenumber	m^{-1}
L	Turbulence length scale	m
l	Length	m
M	Mass flux ratio	
P	Mean pressure	Pa
P	Production of turbulence kinetic energy	m^2s^{-3}
p	Fluctuating pressure	Pa
Pr	Prandtl number	
Q	Mean absolute velocity	m/s
Re	Reynolds number	
Re_τ	Kármán number	
Ri	Richardson number	
S	Mean strain rate	s^{-1}
s	Fluctuating strain rate	s^{-1}
St	Stanton number	
t	Time	s
U, V, W	Mean streamwise, wall-normal, spanwise velocity	m/s
u, v, w	Fluctuating streamwise, wall-normal, spanwise velocity	m/s
u_τ	Friction velocity	m/s
VR	Velocity ratio	
x, y, z	Streamwise, wall-normal, spanwise coordinate	m

Y	Normalised outer coordinate, y/δ
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Superscripts

$*$	Instantaneous velocity
$+$	Non-dimensionalisation with viscous wall scaling

Subscripts

∞	Freestream/ mainstream conditions
aw	Adiabatic wall
CW	Cold-wire
e	Blowing/ injection conditions
i	Inner
$m\tau$	At the Reynolds shear stress maximum
o	Outer
oil	Oil for oil film interferometry
OP	At the outer peak in streamwise variance
w	At the wall

Other Symbols

ℓ	Prandtl's mixing length	m
$\mathcal{D}$	Molecular thermal diffusivity	m ² /s
$\mathcal{W}$	Coles' wake function	
e	Extra rate of strain	s ⁻¹
k	Thermal conductivity	W/mK
γ	Specific heat capacity ratio, c_p/c_v	
$—$	Time averaging	

Chapter 1

Introduction

Understanding the structure of turbulent boundary layers is not only intriguing from an analytical perspective, but also of major importance across a range of industrial applications. At any Reynolds number, there are small-scale motions near the wall which are directly affected by viscosity. The skin friction coefficient for a turbulent boundary layer consequently continues to decrease with Reynolds number, unlike the drag coefficient for separated flows (Morrison, 2010). This means that asymptotic boundary layer behaviour is very important, given that many real-world flows occur at very high Reynolds numbers (Gad-El-Hak & Bandyopadhyay, 1994).

Scaling behaviour and similarity laws form a crucial aspect of fluid mechanics due to their simplicity and predictive power. The simplest boundary layers with no perturbations are susceptible to Reynolds number effects only and therefore an understanding of Reynolds number similarity is paramount. The concept of scaling and similarity in boundary layers of a greater practical interest is still important, however. Examples of these boundary layers include those subjected to freestream turbulence, pressure gradients or a change in surface condition. Length and velocity scales that describe the nature of the perturbations still form an important part of the modelling process. The concept of similarity is also extended to thermal boundary layers in modelling heat transfer. Analogies between the transport of momentum and heat are often made to simplify calculation procedures and inform design choices.

The first part of this thesis investigates the fundamental structure of undistorted turbulent boundary layers, termed “canonical boundary layers”. A new scaling paradigm is presented for the intermediate layer. The second part presents an experimental study investigating the structure of a turbulent boundary layer which is perturbed by an effusion film. It represents an idealised form of the boundary layer on the surface of a turbine blade which is cooled through mass injection.

1.1 Governing equations for two-dimensional boundary layers

The continuity and streamwise momentum equations for a two-dimensional boundary layer ($\partial/\partial z = 0$) with Reynolds' averaging take the form

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \quad (1.1)$$

$$U \frac{\partial U}{\partial x} + V \frac{\partial V}{\partial y} = -\frac{1}{\rho} \frac{dP}{dx} - \frac{\partial \overline{uv}}{\partial y} - \frac{\partial \overline{u^2}}{\partial x} + \nu \nabla^2 U. \quad (1.2)$$

The boundary layer streamwise development length, L , is usually much larger than its thickness, δ , in the wall-normal direction. Therefore thin shear layer approximations can be applied, in which gradients in the streamwise direction are set equal to zero as they are much smaller than the wall-normal gradients. The Prandtl boundary layer approximation (Prandtl, 1904), $dP/dy = 0$, is also applied. With these assumptions, the wall-normal momentum equation integrates to

$$P + \rho \overline{v^2} = P_\infty(x). \quad (1.3)$$

With $\overline{v^2} = 0$ at the wall, the wall pressure equals the freestream pressure, which can be determined directly from Bernoulli's equation. The above assumptions are applied to equation 1.2, to obtain the streamwise boundary layer equation

$$U \frac{\partial U}{\partial x} + V \frac{\partial V}{\partial y} = -\frac{1}{\rho} \frac{dP_\infty}{dx} + \nu \frac{\partial^2 U}{\partial y^2} - \frac{\partial \overline{uv}}{\partial y}. \quad (1.4)$$

Equation 1.4 is identical to the equation obtained directly from the Navier-Stokes equations after applying the same assumptions, apart from the non-linear Reynolds shear stress gradient term $\partial \overline{uv}/\partial y$. Equations 1.1, 1.3, 1.4 form three equations, with four variables: U , V , P , $\overline{uv}$ to solve for. A transport equation for $\overline{uv}$ can be obtained, but introduces higher-order non-linear terms which will also require separate transport equations. This issue is the well-known turbulence closure problem, which can only be circumvented by empirical modelling of the non-linear terms. The simplest turbulence models therefore model $\overline{uv}$ directly. Higher order models can have the capability of being more accurate because of the potential to capture more dynamics of the turbulence, but lose physical relevance as the transport equations become more complicated, which also increases computational cost. Nevertheless, it is clear that the behaviour of the Reynolds shear stress term $\overline{uv}$ is crucial to understanding the structure of a turbulent boundary layer.

1.2 Turbulent boundary layer scaling

In undistorted turbulent boundary layers prescribed by equation 1.4, it can be shown through dimensional analysis that the flow is characterised by an inner, viscous-dominated length scale, $\delta_\nu = \nu/u_\tau$ and an outer, inertia-dominated length scale, δ . Here, ν is the kinematic viscosity of the fluid and u_τ is the friction velocity, defined through the wall shear stress and fluid density as $u_\tau = \sqrt{\tau_w/\rho}$. The ratio of the outer and inner length scales defines the characteristic Reynolds number for wall-bounded flows, $Re_\tau = u_\tau \delta/\nu$, often called the friction Reynolds number or Kármán number. In an unperturbed boundary layer, changes are isolated to Reynolds number effects only. Therefore, both the effect of Reynolds number and the limitations of Reynolds number similarity in a boundary layer are vital.

Prandtl (1933) formulated a scaling law for the inner region, termed the “law of the wall”, which takes the form

$$U^+ = f_i(y^+). \quad (1.5)$$

Equation 1.5 uses an inner-scaled wall-normal distance defined as $y^+ = yu_\tau/\nu$, and an inner-scaled streamwise velocity, $U^+ = U/u_\tau$. Stanton & Pannell (1914) and Von Kármán (1931) proposed the deficit scaling law for the outer region of the boundary layer as

$$U^+ - U_\infty^+ = f_o(y/\delta) = f_o(Y), \quad (1.6)$$

which was originally proposed as an empirical scaling law, but rigorously derived later using singular perturbation theory (Tennekes & Lumley, 1972; Panton, 2007).

The three widely researched ‘canonical’ layers that fit this description are the zero pressure gradient (ZPG) turbulent boundary layer (TBL), and fully developed pipe and channel flows. An outer scale commonly used for a ZPG TBL is δ_{99} - the boundary layer thickness at which the mean streamwise velocity is 99% of the freestream velocity ($U/U_\infty = 0.99$). However, there are other definitions such as the Clauser-Rotta length scale, $\Delta = \delta_1 U_\infty/u_\tau$, which removes the uncertainty in determining the precise location of the boundary layer edge. For a pipe and a channel, δ is strictly defined as the pipe radius and the channel half-height. In these cases the streamwise freestream velocity, U_∞ is defined as the pipe/channel centreline velocity. For consistency, the nomenclature uses δ to denote the outer length scale and y for the wall-normal or radial coordinate.

For sufficiently large scale separation ($Re_\tau \rightarrow \infty$), there will be a region in which $y^+ \gg 1$ and $Y \ll 1$ exists simultaneously, where the velocity profile can be described both in terms of inner and outer variables. The simplest form of matching assumes this common part to be constant, and the velocity gradients can be matched in this region, such that

$$\frac{dU}{dy} = \frac{u_\tau^2}{\nu} \frac{df_i}{dy^+} = \frac{u_\tau}{\delta} \frac{df_o}{dY}, \quad (1.7)$$

which through multiplication with y/u_τ yields

$$y^+ \frac{df_i}{dy^+} = Y \frac{df_o}{dY} = \frac{1}{\kappa}. \quad (1.8)$$

Integration of equation 1.8 gives a logarithmic overlap profile, termed the ‘log law’, expressed respectively in inner and outer variables as:

$$U^+ = \frac{1}{\kappa} \log y^+ + B_i, \quad (1.9)$$

$$U^+ - U_\infty^+ = \frac{1}{\kappa} \log Y + B_o. \quad (1.10)$$

In a rigorous asymptotic matching process, the limits $y^+ \rightarrow \infty$ and $Y \rightarrow 0$ can be simultaneously approached if

$$y^+ = c Re_\tau^\beta, \quad (1.11)$$

$$Y = c Re_\tau^{\beta-1}, \quad (1.12)$$

for any $0 < \beta < 1$ and $Re_\tau \rightarrow \infty$. This gives rise to an intermediate variable of the form

$$\mathcal{Y} = Y Re_\tau^\beta = y^+ Re_\tau^{\beta-1}. \quad (1.13)$$

The rigorous asymptotic matching analysis using intermediate variables was first accomplished by Isakson (1937) and Millikan (1938). The review by Panton (2007) contains further details. In a similar vein to equation 1.7, the logarithmic distribution is derived by assuming the common part is constant. If it is assumed to be a function of y , then a power law overlap is obtained (Millikan, 1938; Barenblatt, 1993). Another important note is that the derivation of the logarithmic overlap layer assumes u_τ to be the characteristic scale for both the inner and outer regions. While other velocity scales have been proposed (Zagarola & Smits, 1998; De Graaff & Eaton, 2000), there has not been overwhelming evidence to suggest that this is not a suitable choice.

Conditional to the log law matching argument is that separation between the two imposed inner and outer scales means that in the overlap region, y itself is the relevant scale. With a single length scale y and velocity scale u_τ , the logarithmic profiles are self-similar and κ is expected to be universal for a specific type of wall-bounded flow. For a fully rough surface in which the roughness height

$k_r \gg \delta_\nu$, the skin friction coefficient becomes independent of Reynolds number. In this scenario, k_r is the dominant imposed inner length scale, such that Millikan's matching arguments can be applied and complete self-similarity is recovered in the form

$$U^+ = \frac{1}{\kappa} \log \frac{y}{k_r} + B_k, \quad (1.14)$$

with $B_k \approx 8.48$. The fully rough regime is satisfied for $k_r^+ \gtrsim 70$ (Kays *et al.*, 2005).

Coles (1956) proposed a “law of the wake” to give a boundary layer composite profile of the form

$$U^+ = f_i(y^+) + \frac{\Pi(x)}{\kappa} \mathcal{W}\left(\frac{y}{\delta(x)}\right), \quad (1.15)$$

with Π and $\mathcal{W}$ defined as the wake parameter and wake function respectively. While the velocity profile for a laminar boundary can be expressed as an analytical function which is independent of x , the same cannot be achieved for a turbulent boundary layer due to the imposition of two length scales. Clauser (1956) defined an “equilibrium” turbulent boundary layer as one which exhibits similarity in the outer region, corresponding to a constant value for Π . In this instance, the boundary layer composite profile takes the form

$$U^+ = f_i(y^+) + \frac{\Pi}{\kappa} \mathcal{W}\left(\frac{y^+}{Re_\tau}\right). \quad (1.16)$$

Spalding (1961); Musker (1979); Chauhan *et al.* (2009) propose law of the wall functions $f_i(y^+)$. All of the functions approach $U^+ = y^+$ for $y^+ \rightarrow 0$ and asymptote to the log law (equation 1.9) for $y^+ \rightarrow \infty$. Empirical expressions for the wake function such as $\mathcal{W}(Y) = 1 - \cos \pi Y$ (Pope, 2000; Schlichting & Gersten, 2000) or the expression given by Chauhan *et al.* (2009) then form a complete composite boundary layer profile. Rodríguez-López *et al.* (2015) present a post-processing algorithm that simultaneously determines u_τ , κ , Π , δ , from boundary layer data, along with the error in the location of the first point in the boundary layer, Δy . The method uses the inner layer formulations of either Musker (1979) or Chauhan *et al.* (2009) and u_τ can be reliably inferred to within an uncertainty range of $\pm 0.6\%$.

1.3 Reynolds shear stress scaling

Through the same dimensional arguments previously discussed, the Reynolds shear stress is expressed as

$$-\overline{uv}^+ = g_i(y^+), \quad (1.17)$$

$$-\overline{uv}^+ = g_o(Y). \quad (1.18)$$

Obtaining functions g_i , g_o is more difficult than for the mean velocity due to the greater uncertainty associated with measuring $\overline{uv}$. For fully developed internal flows such as those in a pipe or channel, flow quantities are independent of x . The continuity equation then gives $\partial U/\partial x = \partial V/\partial y = 0$, and the streamwise momentum equation reduces to

$$0 = -\frac{1}{\rho} \frac{dP}{dx} + \nu \frac{\partial^2 U}{\partial y^2} - \frac{\partial \overline{uv}}{\partial y}, \quad (1.19)$$

with the pressure gradient along the pipe or channel being directly related to the wall shear stress through

$$\tau_w = \delta \frac{dP}{dx}. \quad (1.20)$$

Equation 1.20 can be substituted into equation 1.19. The result is integrated and wall scaling is applied to give

$$1 + \overline{uv}^+ = \frac{dU^+}{dy^+} + \frac{y^+}{Re_\tau}. \quad (1.21)$$

From equation 1.21 it is clear that the Reynolds shear stress distributions can be obtained from mean velocity measurements. This result, along with the fact that u_τ is obtained directly from measurements of the streamwise pressure gradient (with typical uncertainties of less than 1 %), makes pipe flow an attractive option in wall-bounded turbulence research. This is evidenced by the recent construction of high Reynolds number pipe facilities such as the Princeton Superpipe (Zagarola & Smits, 1998), CICLoPE Long Pipe (Talamelli *et al.*, 2009) and Hi-Reff facility (Furuichi *et al.*, 2015).

As $Y \rightarrow 1$ ($y^+ \rightarrow \infty$), $dU^+/dy^+ \rightarrow 0$. From inspection of equation 1.21, it is then clear that

$$g_o(Y) = 1 - Y. \quad (1.22)$$

For g_i , a commonly used function is Panton's stress model determined empirically by Panton (1997). The Taylor expansion of the model correctly follows $-\overline{uv}^+ \sim y^{+3}$ for $y^+ \rightarrow 0$ and accounts for the Reynolds number dependency of Reynolds shear stress peak maximum, τ_m^+ .

Assuming a logarithmic velocity distribution, such that $dU^+/dy^+ = 1/\kappa y^+$, equation 1.21 is used to derive the Reynolds shear stress maximum and its location, $y_{m\tau}^+$, respectively as

$$\tau_m^+ = 1 - \frac{2}{\sqrt{\kappa Re_\tau}} = 1 + \frac{a}{\sqrt{Re_\tau}}, \quad (1.23)$$

$$y_{m\tau}^+ = \sqrt{\frac{Re_\tau}{\kappa}} = c\sqrt{Re_\tau}. \quad (1.24)$$

The $\sqrt{Re_\tau}$ scaling in equation 1.24 corresponds to $\beta = 0.5$ in the intermediate variables of equations 1.11 - 1.13 and forms the basis of an intermediate layer. Equations 1.23 and 1.24 can more generally be rewritten as $\tau_m^+ - 1 = a/\sqrt{Re_\tau}$ and $y_{m\tau}^+ = c\sqrt{Re_\tau}$ respectively. This gives the values $a = -2\kappa^{-1/2}$ and

$$c = \kappa^{-1/2}.$$

Various values for c have been proposed in the literature and are summarised in table 1.1. Afzal (1982) and Antonia *et al.* (1992) noticed that the estimates in table 1.1 lead to lower estimates of the von Kármán constant from equation 1.24. They derive $\kappa \approx 0.3$ from the scaling of equation 1.24, whereas $\kappa \approx 0.4$ is observed from the mean velocity profiles. Afzal (1982) conjectures that the underestimation of κ from Reynolds shear stress scaling could be because the log law slopes with inner and outer scaling are unequal. Antonia *et al.* (1992) suggest that the location of the Reynolds shear stress peak is outside of the extent of the logarithmic region.

Table 1.1: Values reported in the literature of c for the location of the Reynolds shear stress peak.

Authors	c	Re_τ range
Long & Chen (1981)	1.89	150 - 8750
Afzal (1982)	1.85	150 - 8750
Sreenivasan & Sahay (1997)	1.8 ± 0.2	522 - 8750
Chin <i>et al.</i> (2014)	1.77	550 - 98180
Chen <i>et al.</i> (2019)	1.5	3000 - 98180
Klewicky (2021)	~ 1.62	1010 - 98180

Sreenivasan & Sahay (1997) further consider the scaling argument of equation 1.23 and determine a value of $a = -3.1 \pm 0.1$. Using these values for a and c in the first term of an expansion of the velocity gradient about $y_{m\tau}^+$, a value of $\kappa \approx 0.43$ was determined. However, it was calculated with a large range of uncertainty $0.31 < \kappa < 0.63$, due to uncertainties in the fitting. The same authors also observe that within the uncertainty limits for a and c , $a + 2c \equiv 0$ could still be valid as the logarithmic scaling of the Reynolds shear stress suggests.

Sreenivasan & Sahay (1997) observe that the Reynolds shear stress $\sqrt{Re_\tau}$ scaling does not agree well for very low Reynolds numbers ($Re_\tau < 500$). This was later confirmed by Chen *et al.* (2019), who derive a scaling transition for the Reynolds shear stress peak location in pipes and channels, from $y_{m\tau}^+ \propto Re_\tau^{1/3}$ for $Re_\tau \lesssim 3000$, to $y_{m\tau}^+ \propto Re_\tau^{1/2}$ for $Re_\tau \gtrsim 3000$. The low Reynolds number $Re_\tau^{1/3}$ scaling receives strong support from DNS data. The inclusion of low Reynolds number data in determining c for a $Re_\tau^{1/2}$ scaling could explain why early derived values of c are higher.

1.3.1 Reynolds shear stress modelling

The majority of RANS turbulence models use gradient-diffusion analogies to model turbulence quantities. Boussinesq (1877) proposed an expression for Reynolds shear stress of the form

$$-\rho \overline{uv} = \mu_t \frac{\partial U}{\partial y} \implies -\overline{uv} = \varepsilon_m \frac{\partial U}{\partial y}, \quad (1.25)$$

with μ_t termed the turbulent viscosity and ε_m the turbulent momentum diffusivity. Equation 1.25 is mathematically analogous to the stress-strain-rate relationship for a Newtonian fluid. The simplest model for ε_m is the algebraic Prandtl mixing length model, given as

$$\varepsilon_m = \ell^2 \left| \frac{\partial U}{\partial y} \right|, \quad (1.26)$$

such that

$$-\overline{uv} = \ell^2 \left| \frac{\partial U}{\partial y} \right| \frac{\partial U}{\partial y}. \quad (1.27)$$

Turbulence models such as $k - \epsilon$ (Jones & Launder, 1972), $k - \omega$ (Wilcox, 1998) use the turbulent momentum diffusivity in a balance equation for the turbulence kinetic energy $k = \frac{1}{2} (\overline{u^2} + \overline{v^2} + \overline{w^2})$. The k -equation is written as

$$\underbrace{U_j \frac{\partial k}{\partial x_j}}_{\text{advection}} = \underbrace{-\overline{u_i u_j} S_{ij}}_{\text{production}} - \underbrace{\frac{\partial}{\partial x_j} \left(\frac{1}{\rho} \overline{u_j p} + \overline{k u_j} - 2\nu \overline{u_i s_{ij}} \right)}_{\text{transport}} - \underbrace{2\nu \overline{s_{ij} s_{ij}}}_{\text{dissipation}}, \quad (1.28)$$

where the mean strain rate is defined as $S_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$, and s_{ij} is the fluctuating rate of strain.

In a turbulent boundary layer that conforms to Prandtl's boundary layer equation, the k -equation reduces to

$$0 = -\overline{uv} \frac{\partial U}{\partial y} - \frac{\partial}{\partial y} \left(\frac{1}{\rho} \overline{vp} + \overline{kv} \right) - \epsilon. \quad (1.29)$$

In the logarithmic region, also termed the constant shear stress region:

$$\partial U / \partial y = \frac{u_\tau}{\kappa y}, \quad \overline{uv} \approx u_\tau^2, \quad \overline{v^2} \approx \text{constant}, \quad (1.30)$$

as predicted in the attached eddy model (Marusic & Monty, 2019). Equation 1.29 then reduces to a balance between production and dissipation

$$P = -\overline{uv} \frac{\partial U}{\partial y} = \frac{u_\tau^3}{\kappa y} = \epsilon, \quad (1.31)$$

called the local equilibrium assumption. A consequence of local equilibrium is that through substitution of the relationships of equation 1.30 into equation 1.27, the mixing length has an analytical form of $\ell = \kappa y$. This is valid in the logarithmic region of a turbulent boundary layer. The Van Driest (1956) mixing length formula uses a damping function so that it is valid from the wall to the end of the overlap region, given as

$$\ell = \kappa y \left(1 - e^{-y/A} \right), \quad (1.32)$$

typically with $A = 26 \delta_\nu$. Other inner-layer mixing length models are summarised by Schlichting & Gersten (2000).

1.4 Thermal boundary layers

The thermal boundary layer equation with Reynolds' averaging is given as

$$U \frac{\partial \Theta}{\partial x} + V \frac{\partial \Theta}{\partial y} = \mathcal{D} \frac{\partial^2 \Theta}{\partial y^2} - \frac{\partial \overline{v\theta}}{\partial y}, \quad (1.33)$$

where $\mathcal{D}$ is defined as the molecular thermal diffusivity. See Bejan & Kraus (2003); Lienhard IV & Lienhard V (2016) for derivations of the energy equation and the thermal boundary layer equation. In a similar manner to the Reynolds shear stress, the turbulent heat flux $\overline{v\theta}$ is modelled using the gradient diffusion hypothesis such that

$$-\overline{v\theta} = \varepsilon_h \frac{\partial \Theta}{\partial y}, \quad (1.34)$$

where ε_h is the turbulent thermal diffusivity.

1.4.1 The momentum-heat transfer analogy

Substituting equations 1.25 and 1.34 into equations 1.4 and 1.33 respectively (for a zero pressure gradient boundary layer), gives

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \frac{\partial}{\partial y} \left[(\nu + \varepsilon_m) \frac{\partial U}{\partial y} \right], \quad (1.35)$$

$$U \frac{\partial \Theta}{\partial x} + V \frac{\partial \Theta}{\partial y} = \frac{\partial}{\partial y} \left[(\mathcal{D} + \varepsilon_h) \frac{\partial \Theta}{\partial y} \right]. \quad (1.36)$$

It is observed that if the ratio for the molecular Prandtl number $Pr = \nu/\mathcal{D} = 1$ and the turbulent Prandtl number $Pr_t = \varepsilon_m/\varepsilon_h = 1$, then equations 1.35 and 1.36 are identical. An analogy between momentum and heat transfer is then inferred by taking the ratio between the surface heat flux ($\dot{q}_w$) and the wall shear stress, given as

$$\frac{\dot{q}_w}{\tau_w} = \frac{\mathcal{K} \partial \Theta / \partial y}{\mu \partial U / \partial y} = \frac{St}{C_f/2} \frac{c_p (\Theta_w - \Theta)}{U_\infty}, \quad (1.37)$$

where $\mathcal{K}$ is the fluid thermal conductivity. Equation 1.37 rearranges to

$$\frac{St}{C_f/2} = \frac{\partial [\Theta / (\Theta_w - \Theta)]}{\partial [U / U_\infty]}. \quad (1.38)$$

Equations 1.37 and 1.38 use the skin friction coefficient and the Stanton number, respectively defined as

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho U_\infty^2}, \quad (1.39)$$

$$St = \frac{h}{\rho c_p U_\infty}, \quad (1.40)$$

where h is the heat transfer coefficient. Equation 1.38 has used the previous assumption of $Pr = 1$, so that the temperature and velocity profiles become identical. In this case, equation 1.38 simplifies to

$$St = \frac{C_f}{2}, \quad (1.41)$$

the familiar, so-called ‘Reynolds analogy’. The quantity $2St/C_f$ is termed the Reynolds analogy factor and is a measure of departure from the Reynolds analogy. Reynolds (1874) was the first to infer an analogy between momentum and heat transfer, by observing that scalar mixing and heat transfer are due to the transport of momentum. The mathematical analogy of this is that $Pr_t = 1$, which will be therefore be referred to as Reynolds’ analogy henceforth. Equation 1.41 is only a consequence of Reynolds’ analogy, which uses the assumption that $Pr_t = 1$.

While momentum exchange is primarily through pressure forces acting on the fluid, no such analogous mechanism exists for the exchange of heat (Kays *et al.*, 2005). Pressure terms do not appear in the energy equation. In analogy to k , the budget for the halved temperature variance $k_\theta = \frac{1}{2} \overline{\theta^2}$ is

$$\underbrace{U_i \frac{\partial k_\theta}{\partial x_i}}_{\text{convection}} = \underbrace{-\overline{u_i \theta} \frac{\partial \Theta}{\partial x_i}}_{\text{production}} - \underbrace{\frac{\partial}{\partial x_i} \left(\overline{k_\theta u_i} - \mathcal{D} \frac{\partial k_\theta}{\partial x_i} \right)}_{\text{transport}} - \underbrace{\mathcal{D} \frac{\partial \theta}{\partial x_i} \frac{\partial \theta}{\partial x_i}}_{\text{destruction}}, \quad (1.42)$$

from which it is clear that there is no transport of temperature fluctuations through pressure. Guezenec *et al.* (1990) used DNS data with passive scalars to conclude that outside of the viscous wall region, the transport of momentum and heat are fundamentally different, even with molecular and turbulent Prandtl numbers of order one. Due to the role of pressure as discussed above, the turbulence mixes momentum more efficiently than it does heat, which is shown to be a poor marker for the vorticity field.

1.4.2 Thermal boundary layer scaling

Similarly to the momentum field, dimensional analysis of the thermal boundary layer gives

$$\frac{\Theta_w - \Theta}{\Theta_\tau} = \Theta^+ = f_i \theta(y^+, Pr), \quad (1.43)$$

with $\Theta_\tau = \dot{q}_w / (\rho c_p u_\tau)$, called the friction temperature. For $y^+ \rightarrow 0$, the temperature profile approaches the near-wall conduction limit $\Theta^+ = y^+ Pr$. The thermal boundary layer thickness is defined as δ_θ . For both $y^+ \gg 1$ and $y^+ Pr \gg 1$, simultaneously with $\delta_\theta \ll 1$, matching arguments give

$$\Theta^+ = \frac{1}{\kappa_\Theta} \log y^+ + B_\Theta. \quad (1.44)$$

The log law for Θ is often called the T -law. Canonical thermal boundary layers with either a constant wall temperature or a constant surface heat flux give $\kappa_\Theta \approx 0.48$ (Bradshaw & Huang, 1995; Kays *et al.*, 2005).

The thermal mixing length is defined as

$$\ell_\Theta = \frac{-\overline{v\theta}}{(\overline{uv})^{1/2} d\Theta/dy}. \quad (1.45)$$

The local equilibrium assumption in the overlap region of a thermal boundary layer gives $\ell_\Theta = \kappa_\Theta y$, from which the turbulent Prandtl number becomes $Pr_t = \kappa/\kappa_\Theta \approx 0.85$. CFD codes often prescribe a constant $Pr_t = 0.85$ across the whole layer (Ling *et al.*, 2016). While these scaling arguments work well in simple flow cases, they have been shown to break down in boundary layers with even small perturbations.

The velocity log law remains relatively unperturbed in boundary layers with more rapid changes in x , such as in the presence of pressure gradients (Harun *et al.*, 2013) or wall transpiration (Simpson *et al.*, 1969). Conversely, “the law of the wall for temperature breaks down spectacularly in flows where the law of the wall for velocity is still valid” (Bradshaw & Huang, 1995). This observation casts doubt on the logarithmic overlap law, due to the fact that the log law for velocity and the T -law are derived from similar dimensional arguments and should therefore be expected to behave similarly. The mixing length, which is closely related to the log law, predicts departures from it in cases where data shows it is valid. The T -law shows the opposite behaviour, with departures being much larger than what mixing length theory predicts (Bradshaw & Huang, 1995).

Chapter 2

Scaling in the intermediate region of turbulent pipe and channel flows

The intermediate scaling of the Reynolds shear stress peak (introduced in section 1.3) forms the basis of an intermediate layer. The significance of the intermediate layer has recently received considerable attention as a result of new high Reynolds number boundary layer experimental data. Tennekes (1968) was the first to derive an intermediate variable, with the aim of explaining departures from Reynolds number similarity by accounting for second-order effects. It scales as $Re_\tau^{1/3}$, derived using a “pressure velocity” $\rho u_p^3 = \nu dP/dx$, such that $u_\tau/u_p \propto Re_\tau^{1/3}$. Long & Chen (1981) originally proposed the “mesolayer” that forms the current accepted intermediate variable, with length scale $y^+ \propto Re_\tau^{1/2}$. Afzal (1982) developed a more rigorous argument using u_τ as the velocity scale and deriving two logarithmic overlap layers on either side of the intermediate layer, through an asymptotic analysis of the mean momentum equation for pipes and channels.

Wei *et al.* (2005) carried out a mean momentum balance of the Navier-Stokes equations and identify an intermediate layer as a region where there is a balance between turbulent inertia, viscous force and either the pressure gradient force in pipes or the mean advection in boundary layers. They find that the outer limit of this layer scales with $Re_\tau^{1/2}$. Vallikivi *et al.* (2015) find empirically that the location of the outer spectral peak in ZPG TBLs scales as $y_{OSP}^+ = 3.4 Re_\tau^{1/2}$. This location is similar to the centre of an intermediate layer empirically proposed by Diwan & Morrison (2021) as $y_m^+ = 3.5 Re_\tau^{1/2}$. Using a velocity scale of $u_m^+ = \overline{u^2}^+(y_m^+)$, Diwan & Morrison (2021) find logarithmic invariance in intermediate form, with a very good collapse of both the streamwise mean and variance velocity data.

2.1 The outer peak in streamwise variance

A prominent feature of wall-bounded turbulence at high Reynolds numbers is the outer peak in streamwise variance, $\overline{u^2}^+$. While the Reynolds number at which it emerges is not exactly known, data suggests it is at $Re_\tau \approx 2 \times 10^4$ (Hultmark *et al.*, 2013; Willert *et al.*, 2017). The outer peak had been

observed first in boundary layers (Fernholz & Finley, 1996), and later in pipe flows by Morrison *et al.* (2004) in the Princeton Superpipe. The true appearance of this peak in pipe flows was questioned due to the use of a hot-wire of length, $l = 0.25$ m (Hutchins *et al.*, 2009) at the largest Reynolds numbers. This produced a large non-dimensional sensor length ($l^+ = l u_\tau / \nu$) at high Reynolds numbers, causing significant spatial attenuation, the extent of which was acknowledged by (Morrison *et al.*, 2004). However recent studies (Hultmark *et al.*, 2012) in the same facility using a nanoscale thermal anemometry probe (NSTAP), and high-resolution PIV in the larger CICLoPE facility (Willert *et al.*, 2017), have shown the distinct appearance of the outer peak.

Various scaling laws have been proposed for the location and magnitude of the outer $\overline{u^2}^+$ peak and are presented in table 2.1. Some authors have associated the outer peak in streamwise variance to the intermediate layer. Morrison *et al.* (2004) first proposed by an empirical correlation for the location of $\overline{u_{OP}^2}^+$. The authors observe it is in a similar location to the Reynolds shear stress peak and scales similarly. Alfredsson *et al.* (2011) use a diagnostic plot of the streamwise turbulence intensity against the outer normalised mean velocity. It shows a second maximum in $\overline{u^2}$ at a constant value of the freestream velocity, indicating that this maximum should scale logarithmically if a logarithmic mean velocity profile is assumed. The derived wall-normal scaling using this approach is quoted in table 2.1. Hultmark *et al.* (2012) derive an empirical relationship with a scaling of $y_{OP}^+ \propto Re_\tau^{0.67}$, analogous to a critical layer that follows the same scaling in pipe flows (Mckeen & Sharma, 2010).

Pullin *et al.* (2013) use the logarithmic decay model of $\overline{u^2}^+$ and add a Reynolds number dependent correction term to account for the tapering off of the $\overline{u^2}^+$ profile to the outer peak. Using the y_{OP}^+ correlation as determined by Hultmark *et al.* (2012), they show a logarithmic increase in the outer peak of $2.82 + 0.42 \log Re_\tau$, which gives satisfactory agreement with the experimental Superpipe data of Hultmark *et al.* (2012). Better agreement was obtained using the classical intermediate scaling exponent of 0.5 for y_{OP}^+ , and the corresponding result is quoted in table 2.1. However, they retain the same coefficient as Hultmark *et al.* (2012) for y_{OP}^+ (0.23), which leads to a large under-prediction for the location of y_{OP}^+ .

Laval *et al.* (2017) use a similar methodology through modification of the Townsend-Perry logarithmic $\overline{u^2}^+$ decay model. The addition of a new range in the streamwise variance energy spectra produces a y_{OP}^+ scaling similar to Hultmark *et al.* (2012). Chen *et al.* (2018) introduce a mesolayer correction model to allow for the streamwise variance to increase in the intermediate region and therefore find y_{OP}^+ scales with the classical intermediate variable, and find a weak power law increase in $\overline{u_{OP}^2}^+$. Monkewitz (2022) uses channel DNS data and employs a two-term asymptotic expansion for $\overline{u^2}^+$ of the form $\{1, Re_\tau^{-1/4}\}$ to derive a composite profile for $\overline{u^2}^+$. Good agreement with experimental data for pipes and TBLs is observed. An “*ad hoc*” fit of the location of the outer peak was determined and is quoted in table 2.1.

Table 2.1: Scaling laws for the location, y_{OP}^+ , and magnitude of the outer peak in streamwise variance, $\overline{u_{OP}^2}^+$, from various authors.

Authors	Flow type	y_{OP}^+	$\overline{u_{OP}^2}^+$
Morrison <i>et al.</i> (2004)	Pipe	$1.8 Re_\tau^{0.52}$	-
Alfredsson <i>et al.</i> (2011)	TBL	$0.82 Re_\tau^{0.56}$	$0.00663 U_\infty^+$
Hultmark <i>et al.</i> (2012)	Pipe	$0.23 Re_\tau^{0.67}$	-
Pullin <i>et al.</i> (2013)	Pipe	$0.23 Re_\tau^{0.5}$	$0.63 \log Re_\tau + 0.33$
Vincenti <i>et al.</i> (2013)	TBL	$1.9 Re_\tau^{0.5}$	$2.2 \log Re_\tau - 2.28$
Laval <i>et al.</i> (2017)	Pipe, TBL	$\propto Re_\tau^{0.61}$	-
Chen <i>et al.</i> (2018)	Pipe	$1.1 Re_\tau^{0.5}$	$2.8 Re_\tau^{0.09}$
Monkewitz (2022)	Pipe, TBL	$1200 - 1900 Re_\tau^{-0.06}$	-

2.2 Implications of logarithmic scaling on the intermediate layer

Morrison (2007) states that the location of the Reynolds shear stress peak, which scales on intermediate variables, can be viewed as a lower bound to the log law. Above this, the effects of viscosity begin to become negligible. In a similar vein, Marusic *et al.* (2013) find that the start of the logarithmic layer scales with intermediate variables as $3Re_\tau^{1/2}$. This is in contrast to the findings of McKeon *et al.* (2004); Zagarola & Smits (1998) who suggest that the log law begins at $y^+ \approx 600$. Nevertheless, both of the proposed scalings for the start of the logarithmic layer invalidate the use of the logarithmic Reynolds shear stress scaling of equations 1.23 and 1.24 for $Re_\tau < 10^5$. This is due to the Reynolds shear stress peak appearing at a value of y^+ that is less than that identified by the start of the logarithmic layer, as suggested by Antonia *et al.* (1992).

The local equilibrium assumption with logarithmic scaling and $\overline{uv}^+ \approx 1$ produces

$$P^+ = \frac{1}{\kappa y^+}. \quad (2.1)$$

It has been hypothesised that the emergence of the outer peak in streamwise variance is due to a large region of ‘bulk’ production of turbulence kinetic energy in the overlap region (Marusic *et al.*, 2010; Hultmark *et al.*, 2012). This is visualised by plotting the production in premultiplied form ($y^+ P^+$) on semi-logarithmic axes, such that equal areas under the curve represent equal contributions to the global production. From equation 2.1, $y^+ P^+$ then assumes a constant value in the overlap region (Marusic *et al.*, 2010; Hultmark *et al.*, 2012).

It is apparent that a logarithmic velocity distribution should not be used to scale the Reynolds shear stress. Several authors place the location of the start of the log law above the location of the Reynolds shear stress peak for up to the highest Reynolds number experiments currently conducted.

Furthermore, the distribution of the premultiplied production when assuming a log law cannot scale the outer peak in streamwise variance. This is in contrast to the inner peak, which appears very close to the production peak. These aspects are explored in the following sections.

2.3 Datasets used for the analysis

An overview of the datasets used for the analysis is given in table 2.2, along with the symbols which will be used in the subsequent figures. Superpipe Pitot tube data (McKeon *et al.*, 2004) are shown as hollow symbols to highlight that they will not be used in the fitting procedures of subsequent plots. This is due to the relatively large uncertainty in the data near the wall ($y^+ < 1000$), associated with the corrections accounting for shear, blockage, and turbulence effects (Monkewitz, 2017). The peak in y^+P^+ in particular, is not well resolved at high Reynolds numbers for this dataset, where the proximity to the wall is small, but in viscous units, rather large.

The subsequent analysis focuses on scaling in the intermediate region for pipe and channel flows. The once-integrated mean momentum equation for a pipe/ channel is used to determine the Reynolds shear stress profiles and other derived quantities from the datasets. It is expressed in inner scaling as

$$(\tau^+ - 1) + \frac{dU^+}{dy^+} = -\frac{y^+}{Re_\tau}. \quad (2.2)$$

The Reynolds shear stress profiles are shown in figure 2.1. A third-order Savitzky-Golay filter was applied to experimental data to adequately resolve derivatives. The location of the Reynolds shear stress maximum, $y_{m\tau}^+$, was determined as the location at which $d\tau^+/dy^+ = 0$, which requires second derivatives in U^+ . The zero-crossing location was determined using cubic interpolation between points.

Table 2.2: The publication sources for the datasets used in this analysis, along with the boundary layer types and acquisition techniques, Reynolds number ranges, and symbols used in the subsequent plots. Abbreviations for legends are given in brackets.

Source	Type	Re_τ values	Symbol
McKeon <i>et al.</i> (2004)	Superpipe Pitot tube (SP Pitot)	3330 - 102200	
Hultmark <i>et al.</i> (2013)	Superpipe NSTAP (SP NSTAP)	3330 - 98180	
Willert <i>et al.</i> (2017)	Long Pipe PIV (LP PIV)	5380 - 39940	
Pirozzoli <i>et al.</i> (2021)	Pipe DNS	3030, 6020	
Yao <i>et al.</i> (2023)		5190	
Pirozzoli (2024)		12050	
Bernardini <i>et al.</i> (2014)		4080	
Lee & Moser (2015)		5190	
Kaneda & Yamamoto (2021)	Channel DNS	4000, 7990	
Hoyas <i>et al.</i> (2022)		10050	

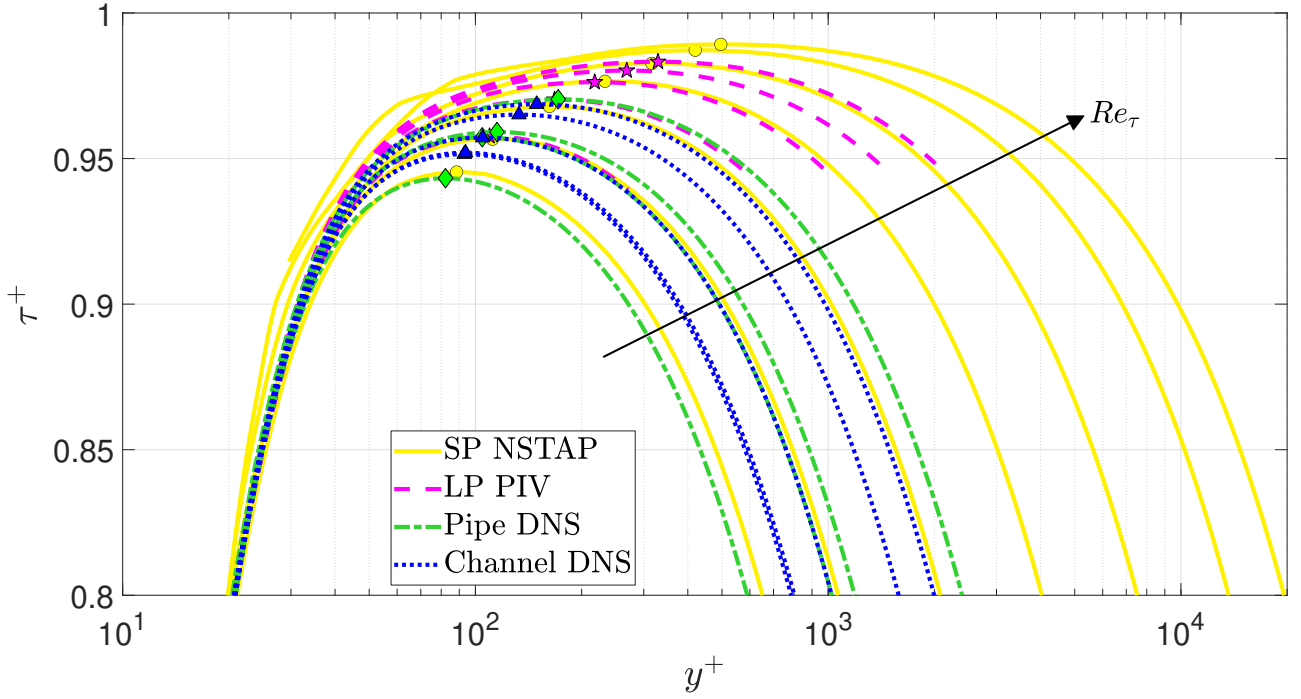


Figure 2.1: Reynolds shear stress profiles, with the peak highlighted with the symbols as described in table 2.2. The Reynolds numbers plotted are: Superpipe NSTAP = {3330, 5410, 10480, 20240, 37680, 68370, 98180}; Long Pipe PIV = {5380, 11710, 19200, 27980, 39940}; Pipe DNS = {3030, 5190, 6020, 12050}; Channel DNS = {4000, 4080, 5190, 7990, 10050} .

2.4 Motivation for reassessment of boundary layer intermediate scaling

Repeated from section 1.3, the equations for the magnitude and location of the Reynolds shear stress peak respectively are

$$\tau_m^+ = 1 + a Re_\tau^{-1/2}, \quad (2.3a)$$

$$y_{m\tau}^+ = c Re_\tau^{1/2}, \quad (2.3b)$$

such that, $a = -2\kappa^{-1/2}$, and $c = \kappa^{-1/2}$ for a logarithmic velocity distribution. Here it is clear that $a + 2c \equiv 0$. To scrutinise the validity of this relationship, the premultiplied production is investigated, written with the use of equation 2.2 as

$$y^+ P^+ = y^+ \tau^+ \frac{dU^+}{dy^+} = y^+ \tau^+ \left(1 - \tau^+ - \frac{y^+}{Re_\tau} \right). \quad (2.4)$$

Equation 2.4 is differentiated with respect to y^+ and evaluated at the location of maximum Reynolds shear stress using equations 2.3a and 2.3b. To leading order, this representation becomes

$$\left[\frac{d}{dy^+} (y^+ P^+) \right]_{y_{m\tau}^+} = -(a + 2c) Re_\tau^{-1/2}. \quad (2.5)$$

From figure 2.2, it is observed that the peak in Reynolds shear stress precedes the peak in premultiplied production. Hence $\left[\frac{d}{dy^+} (y^+ P^+) \right]_{y_{m\tau}^+} > 0$, which is realised through $a + 2c < 0$. This contradicts the result for the Reynolds shear stress scaling derived from assuming a logarithmic velocity distribution. The scaling of the peak in Reynolds shear stress is therefore explored in further detail in the following section.

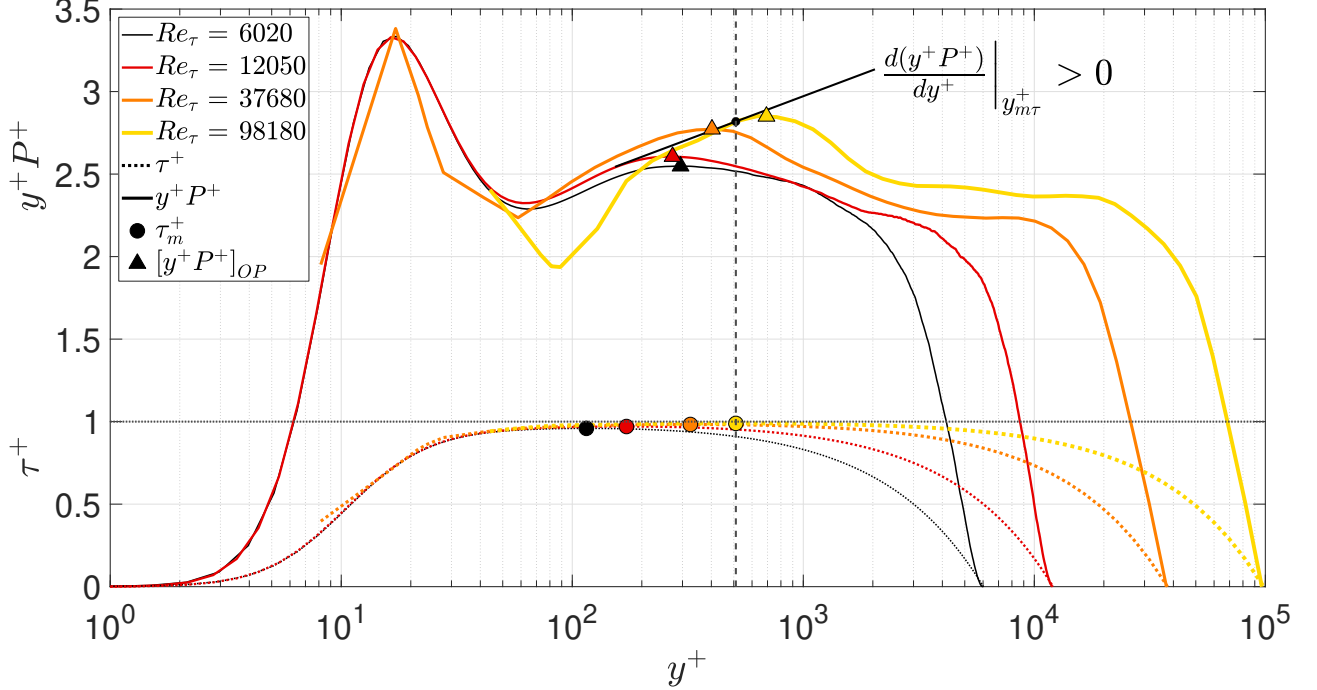


Figure 2.2: The Reynolds shear stress (dotted lines) and premultiplied production distributions (solid lines). Datasets used in ascending Reynolds number are Pirozzoli *et al.* (2021); Pirozzoli (2024); Hultmark *et al.* (2013). The corresponding locations of the peaks are marked with circles and triangles respectively.

2.5 Scaling in the intermediate region for pipes and channels

To study the asymptotic behaviour of the Reynolds shear stress, equation 2.2, evaluated at the Reynolds shear stress peak is written as

$$(\tau_m^+ - 1) + \left[\frac{dU^+}{dy^+} \right]_{y_{m\tau}^+} = -\frac{y_{m\tau}^+}{Re_\tau}. \quad (2.6)$$

The analysis makes the following assumptions:

1. The location of τ_m^+ scales as $y_{m\tau}^+ = c Re_\tau^\beta$, where c and β are unspecified constants. In this case, the value β can differ from the standard intermediate exponent of $\beta = 1/2$. The only constraint is $0 < \beta < 1$, such that the intermediate variable remains bounded in the limits $y^+ \rightarrow \infty$ and $y/\delta \rightarrow 0$ (Afzal, 1982; Sreenivasan & Sahay, 1997; Wei *et al.*, 2005).
2. The Reynolds shear stress, viscous stress, and pressure terms in equation 2.2 are of equal order in the intermediate region (Afzal, 1982; Wei *et al.*, 2005).

3. The variation of τ^+ around $y_{m\tau}^+$ over a distance of $\mathcal{O}(Re_\tau^\beta)$, i.e. $\propto y_{m\tau}^+$, can be approximated by the leading-order Taylor expansion in τ^+ (Sreenivasan & Sahay, 1997; Wei *et al.*, 2005).

The validity of these assumptions will be cross-checked in the analysis. With use of assumptions 1 and 2, the three terms in equation 2.6 can be written as:

$$\tau_m^+ - 1 = a Re_\tau^{\beta-1}, \quad (2.7a)$$

$$\left[\frac{dU^+}{dy^+} \right]_{y_{m\tau}^+} = b Re_\tau^{\beta-1}, \quad (2.7b)$$

$$y_{m\tau}^+ = c Re_\tau^\beta. \quad (2.7c)$$

The scaling constants a, b, c are a function of the chosen value of β , but the relationships between them can be determined in a form invariant of β , for the purpose of generality. The important relationships between the constants are defined as: $d_1 = c/a$, $d_2 = c/b$. Substituting the scaling relationships of 2.7 into equation 2.6, and dividing through by $c Re_\tau^{\beta-1}$ (for $Re_\tau \gg 0$), produces a momentum balance in the intermediate region, of the form

$$\frac{1}{d_1} + \frac{1}{d_2} = -1. \quad (2.8)$$

2.5.1 Premultiplied production scaling

As shown in figure 2.2, the peak in $y^+ P^+$ occurs further from the wall than the τ^+ peak but nonetheless quite close to it. The Reynolds shear stress in this region is approximated with a Taylor expansion about τ_m^+ (assumption 3), which is written as

$$\tau^+ = \tau_m^+ + \left[\frac{d^2 \tau^+}{dy^{+2}} \right]_{y_{m\tau}^+} \frac{(y^+ - y_{m\tau}^+)^2}{2} + H.O.T. \quad (2.9)$$

From equations 2.7a, 2.7c: $\tau_m^+ \propto Re_\tau^{\beta-1}$; and in the vicinity of $y_{m\tau}^+$, $(y^+ - y_{m\tau}^+) = \mathcal{O}(Re_\tau^\beta)$. For the first and second terms on the right-hand side of equation 2.9 to be of the same order, it follows that

$$\left[\frac{d^2 \tau^+}{dy^{+2}} \right]_{y_{m\tau}^+} = \tau_m^{+''} = q Re_\tau^{-(\beta+1)}, \quad (2.10)$$

where q is a constant of proportionality. A local variable for the wall-normal distance from $y_{m\tau}^+$ is introduced, expressed as

$$\xi = \frac{y^+ - y_{m\tau}^+}{y_{m\tau}^+}. \quad (2.11)$$

Substituting equations 2.7a, 2.7c, 2.10, 2.11 into equation 2.9 yields

$$\tau^+ - 1 = \left(a + \frac{qc^2}{2} \xi^2 \right) Re_\tau^{\beta-1}, \quad (2.12)$$

valid for wall-normal distances in which $(y^+ - y_{m\tau}^+) = O(Re_\tau^\beta)$. These distances can be expressed in inner scaling by using equations 2.7c and 2.11, written as

$$y^+ = (1 + \xi)cRe_\tau^\beta. \quad (2.13)$$

Equations 2.12 and 2.13 are used to form an equation for the premultiplied production. The derivative is set equal to zero and rearranged to give the location of the outer peak in premultiplied production

$$\xi_{mp,l} = \frac{-(2 + 1/d_1)}{2 + d_3}. \quad (2.14)$$

Here the relationship $d_3 = qc$ is introduced. The derivation for this is given in Appendix A. The observation that the outer peak premultiplied production is further from the wall than the Reynolds shear stress peak gives $\xi_{mp,l} > 0$ and $d_1 > -0.5$. Therefore the constraint $d_3 > -2$ must also be true. The magnitude of the outer peak in y^+P^+ (at (ξ_{mp}) to leading order, gives

$$y^+P_{mp}^+ = - \left[\left(a + \frac{qc^2}{2}\xi_{mp,l}^2 \right) + (1 + \xi_{mp,l})c \right] (1 + \xi_{mp,l})c Re_\tau^{2\beta-1}. \quad (2.15)$$

Given that a, c, q, ξ_{mp} are scaling constants, the standard intermediate variable exponent of $\beta = 0.5$ predicts the outer peak in premultiplied production will be constant/ Reynolds number invariant, since $2\beta - 1 = 0$. However, figure 2.2 suggests this peak increases in magnitude with increasing Reynolds number and therefore $\beta > 0.5$.

2.5.2 Determination of the scaling constants

The scaling relationships of equations 2.7a - 2.7c can be expressed in Re_τ^β form as

$$Re_\tau(\tau_m^+ - 1) = aRe_\tau^\beta = \frac{c}{d_1} Re_\tau^\beta, \quad (2.16a)$$

$$Re_\tau \left[\frac{dU^+}{dy^+} \right]_{y_{m\tau}^+} = bRe_\tau^\beta = \frac{c}{d_2} Re_\tau^\beta, \quad (2.16b)$$

$$y_{m\tau}^+ = cRe_\tau^\beta. \quad (2.16c)$$

Plotting $y_{m\tau}^+$ on the y-axis against $Re_\tau(\tau_m^+ - 1)$ and $Re_\tau[dU^+/dy^+]_{y_{m\tau}^+}$ will produce slopes with gradients d_1 and d_2 respectively, with no prior assumption required about the value of β . These are shown in figures 2.3a and 2.3b. The fits are forced through the origin, which is a necessary boundary condition if the terms are of equal order in this region (assumption 1). It is observed that $d_1 > -0.5$, which satisfies the empirical observation that the peak in Reynolds shear stress is closer to the wall than the outer peak in premultiplied production (and therefore $d(y^+P^+)/dy^+ > 0$ at $y_{m\tau}^+$). The mean momentum balance in the intermediate region (given by equation 2.8) is satisfied to within 0.3% through these

fits, and therefore validates assumption 1. The data points lie consistently below the line derived from assuming a logarithmic velocity profile, except for the high Reynolds number Superpipe Pitot tube data, which do not follow the line particularly well either.

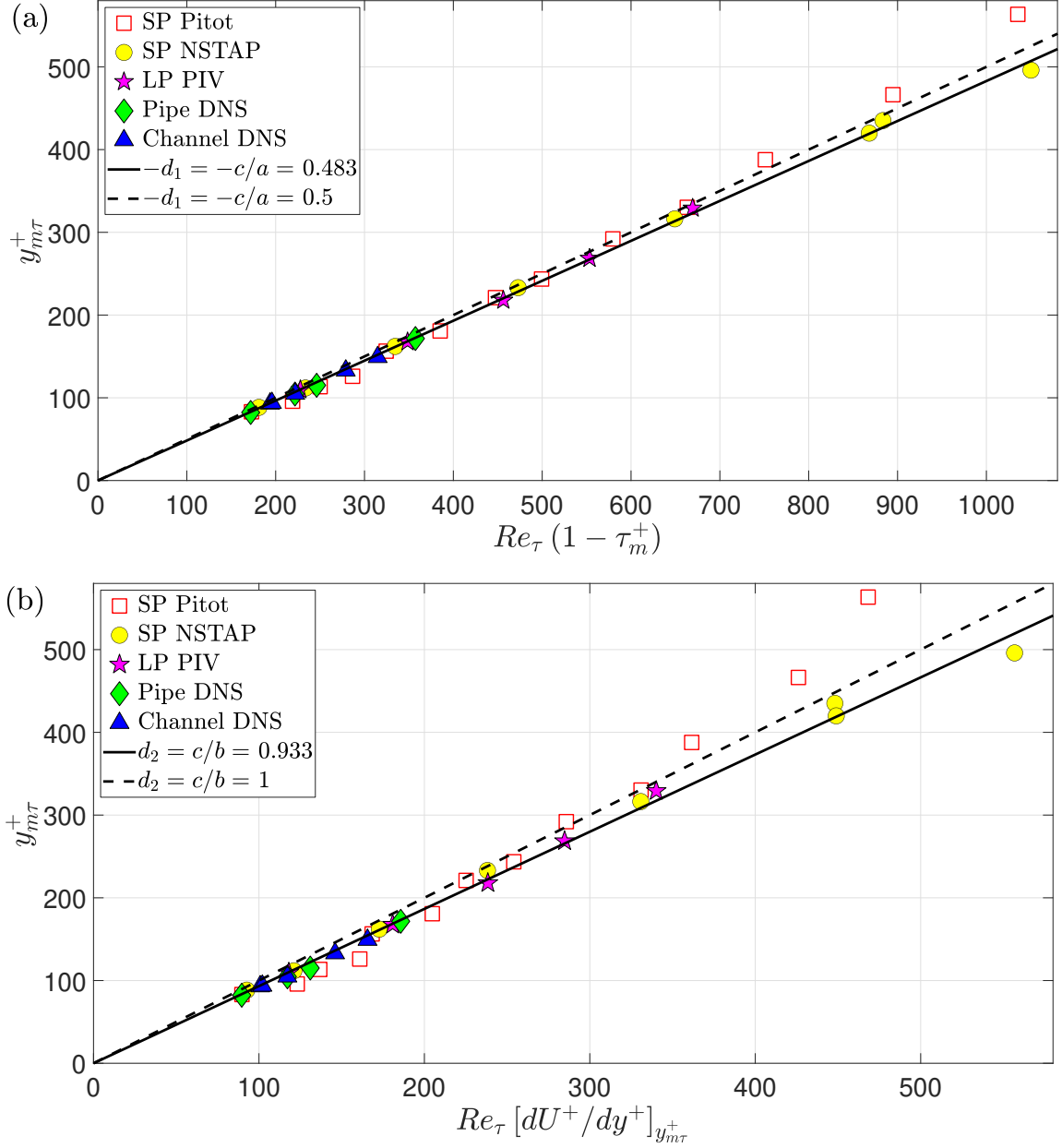


Figure 2.3: Plots to determine (a) d_1 , and (b) d_2 , with the least-squares linear regression fits (solid lines) and log law fits (dashed lines) shown.

To reduce bias towards a particular term in determining the value for β , the terms in equation 2.16 are combined with d_1 and d_2 , such that the quantities are expressed in the form $c Re_\tau^\beta$. The result, along with the least-squares regression fit is shown in figure 2.4. The intermediate scaling exponent was determined as $\beta = 0.529$, (the classical value is $\beta = 0.5$) but provides for a better fit to the data.

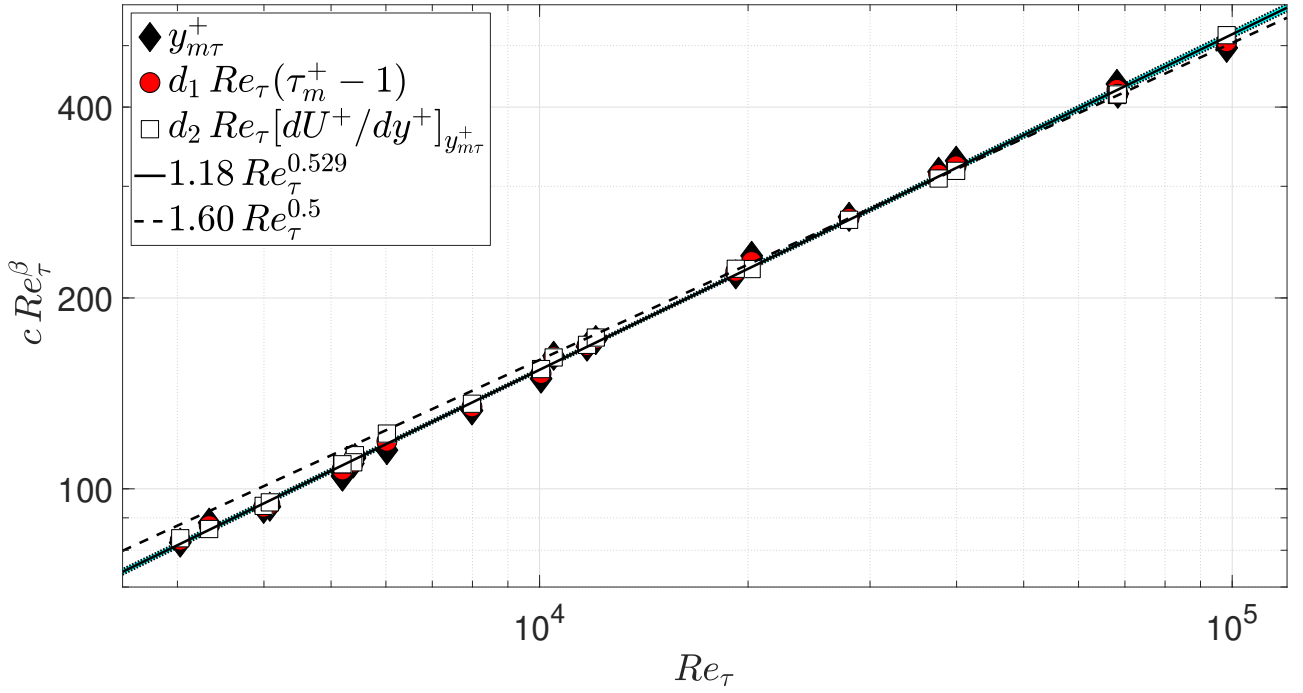


Figure 2.4: Log-log plot to determine the values of c and β through least-squares linear regression (solid line), compared against the best fit for $\beta = 0.5$ (dashed line). The legend shows the symbols for the plotted terms of equations 2.16a - 2.16c, rearranged into $c Re_\tau^\beta$ form. The blue band demarcated by dashed lines corresponds to the 95% confidence region for the fit.

In a similar style to the relationships given in 2.16, equation 2.10 can be expressed as

$$\frac{1}{\tau_m^{+''} Re_\tau} = \frac{1}{q} Re_\tau^\beta = \frac{c}{d_3} Re_\tau^\beta, \quad (2.17)$$

which when plotted on the x -axis against $y_{m\tau}^+$, produces a line with gradient d_3 . Figure 2.5a shows this fit, which gives $d_3 = -1.67$, and is consistent with the constraint that $d_3 > -2$. Figure 2.5b shows the corresponding fit given by equation 2.10 (solid line), where q was determined through $q = d_3/c$. Both plots show a reasonable fit to the data, given the uncertainty in calculating $\tau_m^{+''}$ (a third derivative of U^+), and therefore assumption 3 appears to be valid to within (the rather large) uncertainty.

It is noted that the fit in figure 2.5a uses only high Reynolds number data ($Re_\tau > 10^4$), due to an observed late scaling transition in $\tau_m^{+''}$, that can be seen in figure 2.5b. The scaling at moderate Reynolds numbers ($10^3 \lesssim Re_\tau \lesssim 10^4$) for $\tau_m^{+''}$ follows a $Re_\tau^{-5/3}$ scaling well (dashed line in figure 2.5b). The log law assumption gives $\tau_m^{+''} = -2\kappa^{1/2} Re_\tau^{-3/2}$. It is also worth noting that the $Re_\tau^{-5/3}$ scaling is different to the derived low Reynolds number scaling from the analysis of Chen et al. (2019). Here, it is assumed that $dU^+/dy^+ = 41/(\kappa y^{+2})$, such that $y_{m\tau}^+ \propto Re_\tau^{1/3}$, and $\tau_m^{+''} \propto Re_\tau^{-4/3}$, which has been verified to give a good fit to the data for $Re_\tau < 10^3$. At higher Reynolds numbers, the Superpipe Pitot tube data follows the $Re_\tau^{-(\beta+1)}$ slope well, although the values for $\tau_m^{+''}$ are offset from the values obtained from the Superpipe NSTAP and Long Pipe PIV data.

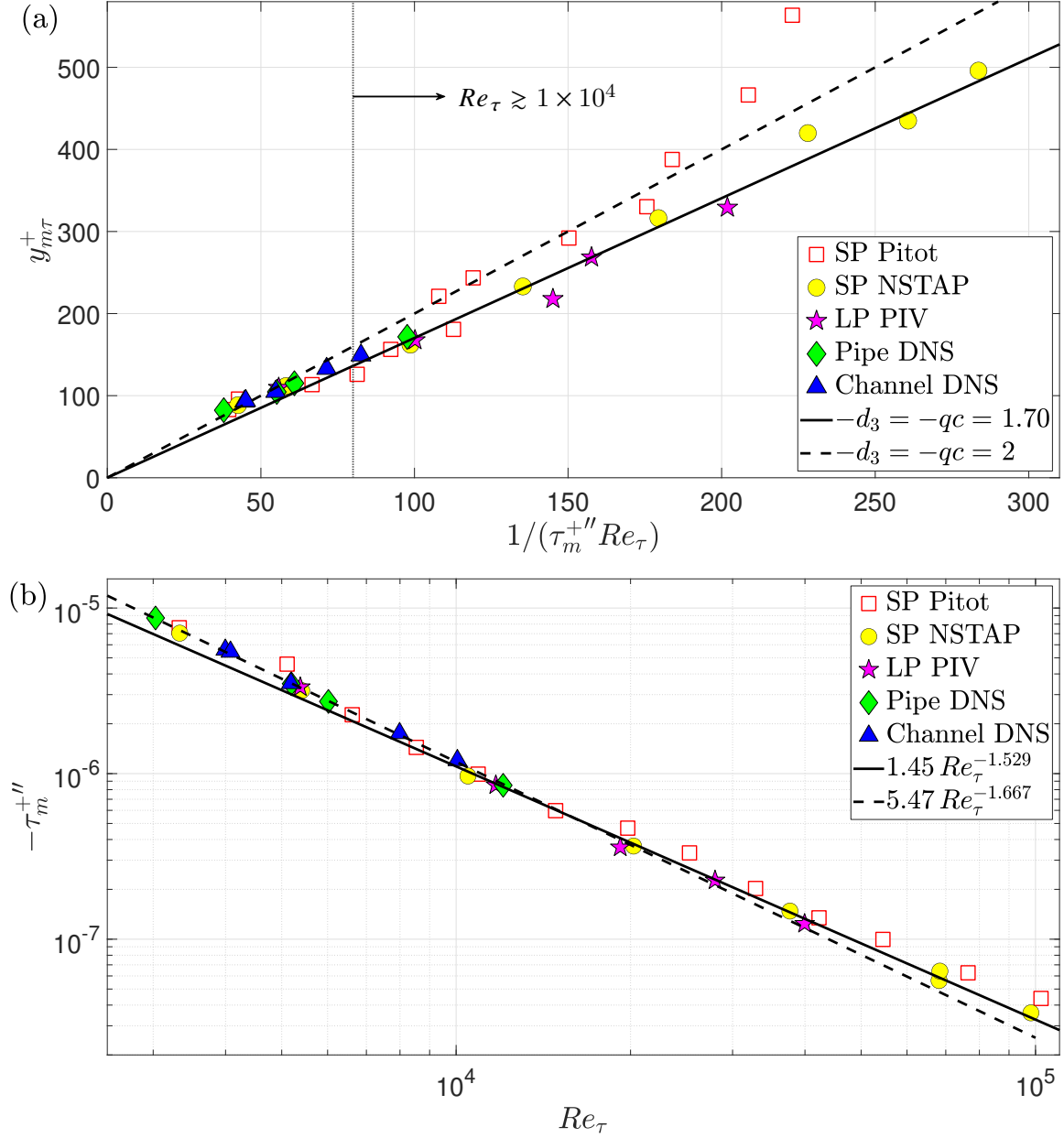


Figure 2.5: Plot to determine (a) d_3 , and (b) a log-log plot showing the scaling transition of $\tau_m^{+''}$ across the Reynolds number range.

2.5.3 Fitting of the individual terms and uncertainty estimates

Figures 2.6a - 2.6c show the fits to the data for the scaling relationships of equations 2.7a, 2.7c, using the values for the constants a, b, c , as derived in section 2.5.2 respectively. These fits are compared to the fits obtained from least-squares linear regression for each individual term, when β is fixed as the value obtained in this analysis, $\beta = 0.529$, and for the standard intermediate variable, $\beta = 0.5$. These results are summarised in table 2.3, along with the location of the outer peak in premultiplied production, $\xi_{mp,l}$.

It can be seen that for $\beta = 0.529$, the coefficients obtained from the β invariant plots and the

individual fits are very similar (less than 0.3%). This is except for q which shows greater differences, where there is the combined effect of the greater uncertainty in the determination of $\tau_m^{+''}$, and the absence of DNS data in the fitting and therefore less available data points. To reduce clutter, only the lines with the β invariant derived coefficients are shown in figures 2.6a - 2.6c. The curves for $\beta = 0.529$ give better agreement across the Reynolds number range than for when $\beta = 0.5$. Furthermore, when $\beta = 0.5$, $d_1 < -0.5$ which leads to a negative value of $\xi_{mp,l}$. However, the 95% confidence interval does allow for $d_1 > -0.5$ in this case.

The values obtained for a , c , when $\beta = 0.5$, are in reasonable agreement with the values reported in the literature. The value $a = -3.14$ lies within the uncertainty band of $a = -3.1 \pm 0.1$, as determined by Sreenivasan & Sahay (1997). The value $c = 1.60$ lies on the lower uncertainty bound of $c = 1.8 \pm 0.2$, as quoted by the same authors. It is however, between the values $c = 1.77$ and $c = 1.50$, which were determined by Chin *et al.* (2014) and Chen *et al.* (2019) respectively. It is also very close to the value determined by Klewicki (2021), $c \approx 1.62$, termed the Fife similarity parameter, ϕ , in that paper.

Table 2.3: Comparison between the scaling constants derived from the β invariant fits and the least-squares regression fits to the individual quantities, for $\beta = 0.5$ and $\beta = 0.529$. The percentage differences between the coefficients derived from the $\beta = 0.529$ fits are shown on row 4. Row 5 shows the 95% confidence intervals, evaluated as a percentage. The maximum value obtained value from individual and beta invariant fits is used.

	a	b	c	q	d_1	d_2	d_3	$\xi_{mp,l}$
Individual fit, $\beta = 0.5$	-3.14	1.64	1.60	-1.11	-0.509	0.978	-1.77	-0.070
Individual fit, $\beta = 0.529$	-2.44	1.27	1.17	-1.46	-0.481	0.925	-1.71	0.219
β invariant fit	-2.44	1.26	1.18	-1.45	-0.483	0.933	-1.70	0.252
% difference in coefficients	0.1	0.6	0.3	0.8	0.3	0.9	0.5	12.0
% uncertainty in coefficients	± 4.4	± 5.0	± 4.1	± 9.2	± 2.8	± 2.9	± 8.2	± 9.5

The uncertainty in β was calculated from the fit in figure 2.4, as $\beta = 0.529 \pm 0.005$. The uncertainty is the 95% confidence interval, determined from the linear regression fit, based on the coefficient of determination (R^2). The same method was used for the uncertainties quoted in table 2.3. To explore how realistic this range is, the value of β that gave the best fit for each scaling term in equations 2.7a - 2.7c was found. They were determined as $\beta_a = 0.529 \pm 0.008$, $\beta_b = 0.528 \pm 0.009$, $\beta_c = 0.520 \pm 0.012$, respectively. In each case, the uncertainty bands overlap with the value $\beta = 0.529$. A more tentative estimate places the likely value of β in the range $0.508 < \beta < 0.537$. The optimum value of β when using only channel DNS data (where the data is well resolved and does not require filtering to adequately calculate derivatives), gives $\beta = 0.526 \pm 0.007$.

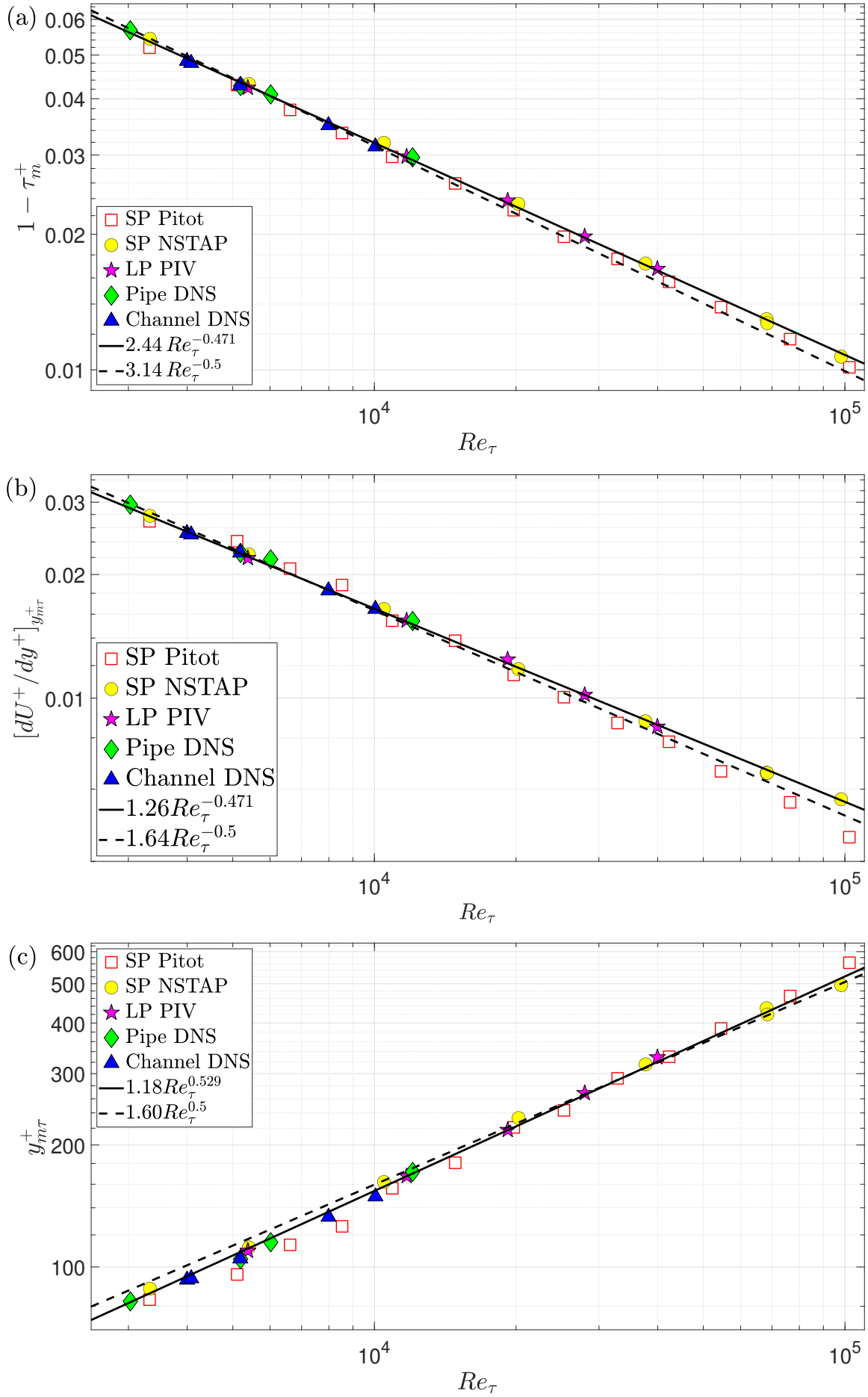


Figure 2.6: Scaling plots for (a) $\tau_m^+ - 1$, (b) $[dU^+/dy^+]_{y_m^+}$, and (c) y_m^+ , with a comparison of the best fits for $\beta = 0.529$ (solid lines), and $\beta = 0.5$ (dashed lines).

2.5.4 Logarithmic law indicator function at the Reynolds shear stress peak

A commonly used diagnostic function to determine the von Kármán constant is the premultiplied velocity gradient $y^+ dU^+/dy^+ = 1/\kappa$, termed the log law indicator function. The indicator function at the Reynolds shear stress peak is derived through the use of equations 2.7b and 2.7c as

$$\Xi_{m\tau} = y_{m\tau}^+ \left[\frac{dU^+}{dy^+} \right]_{y_{m\tau}^+} = bc Re_\tau^{2\beta-1}, \quad (2.18)$$

which is plotted in figure 2.7. An upward trend is observed with increasing Reynolds number, which invalidates universal logarithmic behaviour at the Reynolds shear stress peak. A mean value observed as approximately 2.6 would give $\kappa \approx 0.385$, which is consistent with the value for the “interior log law” (Monkewitz, 2017). However, the systematic Reynolds number dependence invalidates its universality. This point may be fairly non-controversial since, as discussed in section 2.5, there is now general agreement between authors that the ‘traditional’ logarithmic overlap layer starts beyond the location of the Reynolds shear stress peak at Reynolds numbers currently measurable.

Of greater contention is the fact that for $\beta = 0.5$, $\Xi_{m\tau}$ should also be constant (implying a log law). The use of $\beta = 0.5$ has continued to be used in Reynolds shear stress scaling, despite it being noted that when assuming a logarithmic velocity profile in this region, there are “serious problems with this approach” (Chin *et al.*, 2014). Figure 2.7 also shows the trend line of equation 2.18, calculated from the coefficients in table 2.3. The value $\beta = 0.529$ gives a reasonably good fit to the slope of the data. There is a systematic shift between the CICLoPE PIV/ Superpipe NSTAP experimental data and channel DNS data. However, the slopes are similar, which is the point of contention concerning the value of β . This fact merely reflects the wide range of values of the von Kármán constant reported in the literature for different datasets. The coefficient bc (responsible for the intercept in figure 2.7) bisects the data, which is likely to be a consequence of the global fitting procedures. Nevertheless, the Reynolds number dependence of $\Xi_{m\tau}$ provides good support for $\beta > 0.5$.

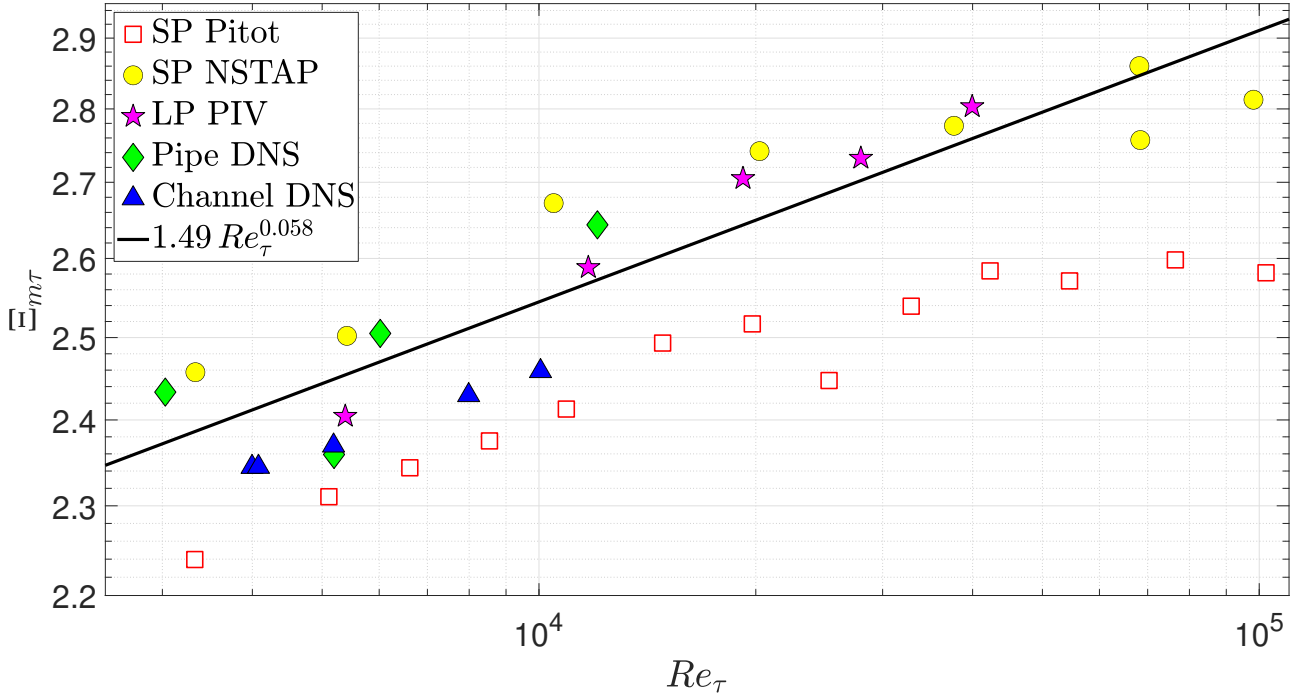


Figure 2.7: Inner-scaled premultiplied velocity gradient at the Reynolds shear stress peak (equation 2.18), as a function of Reynolds number.

2.6 Scaling of the mean momentum equation in the intermediate region

The once-integrated mean momentum equation (equation 2.2) is scaled in the intermediate region. The deficit in Reynolds shear stress is scaled as $(\tau - 1) = \Delta\tau_m$. A simple choice for this would be use of equation 2.7a, $\Delta\tau_m^+ = aRe_\tau^{\beta-1}$. For scaling dU/dy , a characteristic velocity difference ΔU_m , across a characteristic distance Δy_m , is used. In more quantitative terms, Δy_m can be viewed as the distance up to which assumption 3 is valid to within a certain error threshold. A natural assumption is that $\Delta y_m = \mathcal{O}(y_{m\tau})$, such that

$$\Delta y_m^+ = \chi y_{m\tau}^+ = \chi c Re_\tau^\beta, \quad (2.19)$$

for constant χ , which is a measure of the ‘thickness’ of the intermediate layer. Using these scaling variables, equation 2.2, is rewritten as

$$\frac{d(U^+/\Delta U_m^+)}{d(y^+/\Delta y_m^+)} \frac{\Delta U_m^+}{\Delta y_m^+} + \frac{(\tau^+ - 1)}{\Delta\tau_m^+} \Delta\tau_m^+ = -\frac{y^+}{y_{m\tau}^+} \frac{y_{m\tau}^+}{Re_\tau}, \quad (2.20)$$

which upon substitution of equation 2.19 and rearrangement, gives

$$\frac{d(U^+/\Delta U_m^+)}{d(y^+/\Delta y_m^+)} \frac{\Delta U_m^+}{(\chi c^2 Re_\tau^{2\beta-1})} + \frac{(\tau^+ - 1)}{\Delta\tau_m^+} \frac{\Delta\tau_m^+}{c Re_\tau^{\beta-1}} = -\frac{y^+}{y_{m\tau}^+}. \quad (2.21)$$

Inspection of equation 2.21 shows the emergence of a new velocity scale governing the intermediate region, $\Delta U_m^+ \propto Re_\tau^{2\beta-1}$. This velocity scale is only directly proportional to u_τ for $\beta = 0.5$, which has previously been shown to not adequately scale the intermediate region. To determine ΔU_m^+ , first Δy_m^+ must be chosen, such that $\Delta U_m^+ = U^+([1 + \chi/2] y_{m\tau}^+) - U^+([1 - \chi/2] y_{m\tau}^+)$. Figure 2.8 shows that assumption 3 is approximately valid to within the (rather arbitrary) threshold of $\pm 0.05\%$, for $|\xi| \leq 0.3$, corresponding to $\chi = 0.6$. Figure 2.9 shows that ΔU_m^+ agrees well with the $Re_\tau^{2\beta-1}$ scaling, for the selected value of $\chi = 0.6$, providing good support for this intermediate region velocity scale. The same systematic shifts across datasets in figure 2.7 are also observed here, and the resemblance between the two plots is striking. The line plotted in figure 2.9 is a least-squares fit to the data to illustrate the $Re_\tau^{2\beta-1}$ dependence. The coefficient is approximately $1.1\chi c^2$.

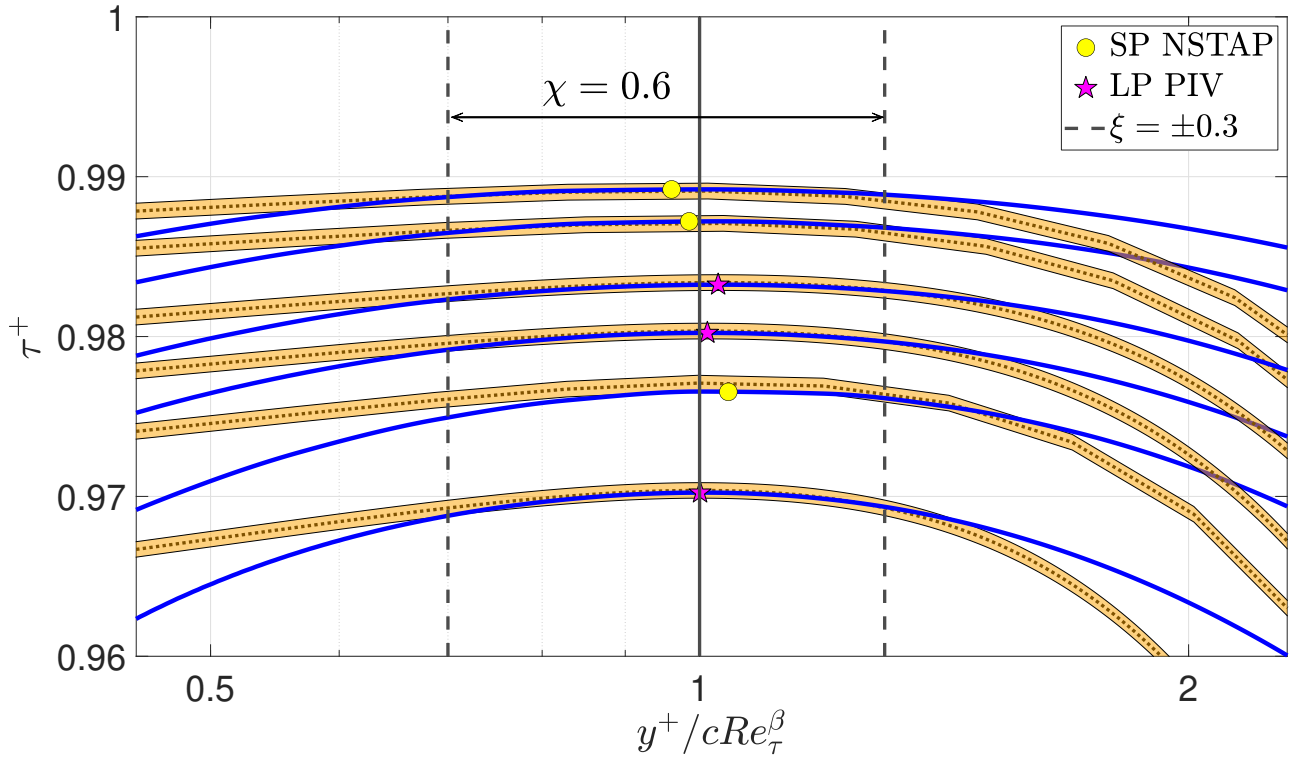


Figure 2.8: Reynolds shear stress profiles, as a function of the intermediate coordinate, for $Re_\tau > 2 \times 10^4$. Dotted lines indicate the prediction of equation 2.12, with a shaded region of magnitude $\pm 0.05\%$. The vertical solid line indicates the centre of the intermediate region, and vertical dashed lines demarcate the extent of the intermediate region.

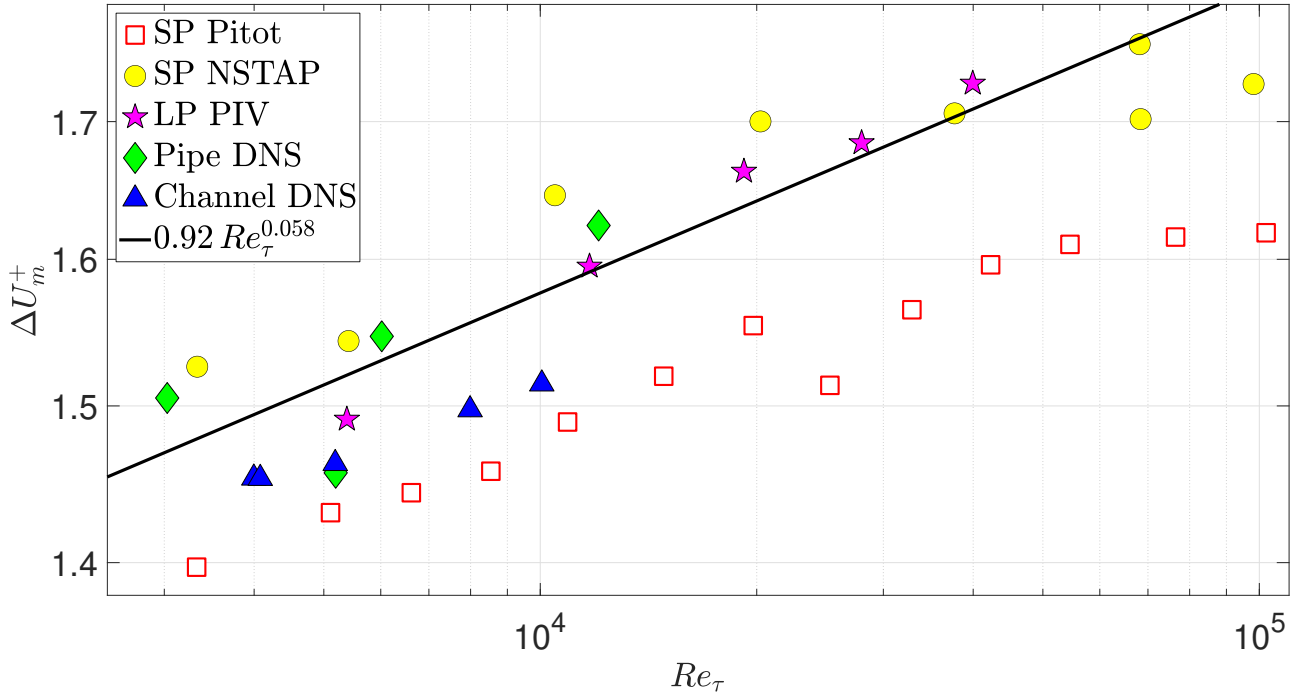


Figure 2.9: Variation of ΔU_m^+ with Reynolds number, with the least-squares regression fit for a $Re_\tau^{2\beta-1}$ scaling shown.

2.7 Scaling of the outer peak in premultiplied production

Figure 2.10 shows the comparison between the locations of the peaks in Reynolds shear stress and premultiplied production. The outer y^+P^+ peak location is defined as

$$y_{mp}^+ = (1 + \xi_{mp,l}) c Re_\tau^\beta. \quad (2.22)$$

At high Reynolds numbers ($Re_\tau > 2 \times 10^4$), the prediction of equation 2.22 is in good agreement with the data, given the uncertainty in determining this quantity. For $Re_\tau \lesssim 1 \times 10^4$, the peak is approximately constant, or in the case of channel DNS, exhibits a small Reynolds number dependence. In either case, the prediction of equation 2.22 is not adequate. However, the assumptions in deriving the prediction for $\xi_{mp,l}$ assume the terms in the Taylor expansion of equation 2.12 are of equal order. As figure 2.5b shows, $\tau_m^{+''}$ follows a different scaling for $Re_\tau < 1 \times 10^4$, which invalidates this assumption. While the reason for this scaling transition is currently unclear to the author, this observation is nevertheless consistent with the lack of agreement for the location of the peak in premultiplied production at these Reynolds numbers.

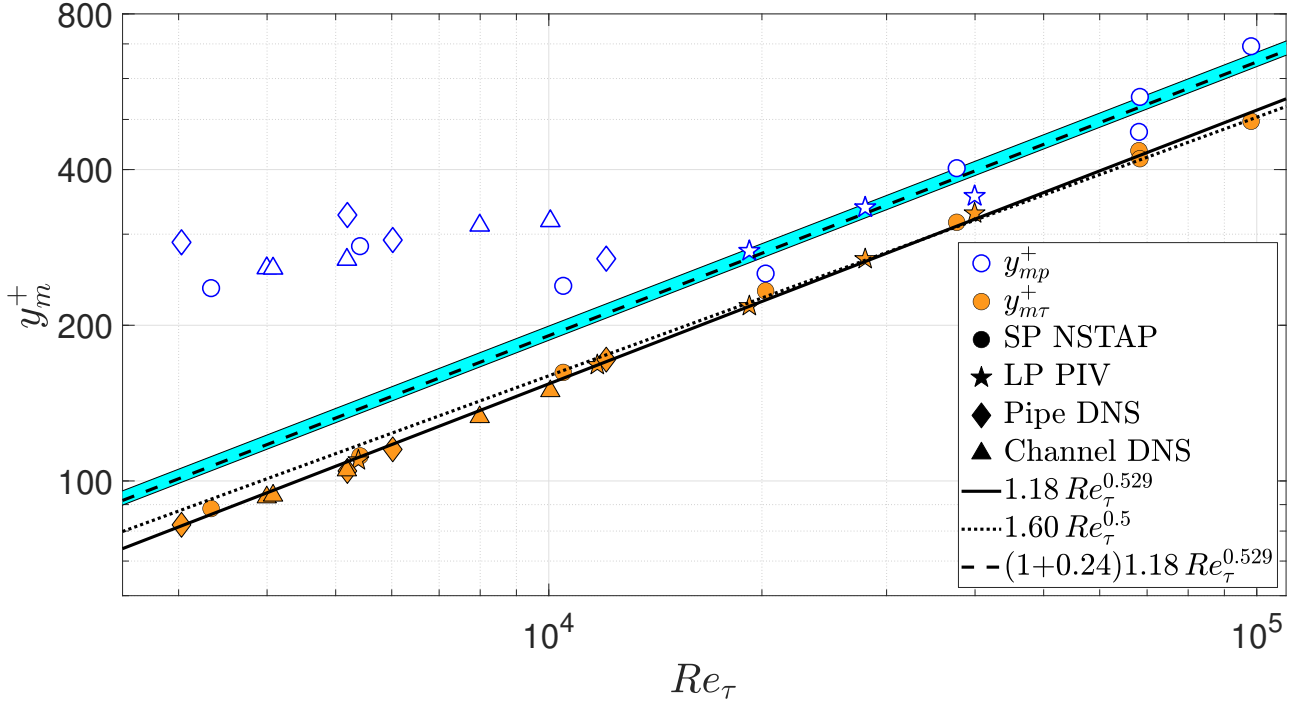


Figure 2.10: Reynolds number dependence of the locations of the peak in τ^+ and the outer peak in y^+P^+ , with comparisons to the respective fits of equations 2.7c and 2.22.

2.8 Scaling of the outer peak in streamwise variance

Figure 2.11a shows the location of the outer $\overline{u^2}^+$ peak, with the intermediate scaling relationship of the present work compared to scalings in literature (summarised in table 2.1). The line representing the current work was fitted to the data for $Re_\tau > 2 \times 10^4$. Little difference was observed between this and fitting to the data for $Re_\tau > 1 \times 10^4$. The calculated coefficient indicates that $y_{OP}^+ \approx 0.9 y_{m\tau}^+$. Despite this scaling providing a reasonable fit to the data, due to the lack of available high Reynolds number data and the uncertainty in determining $\overline{u_{OP}^2}^+$, one cannot conclusively remark that any particular scaling is better than the others. Similarly to the peak location of y^+P^+ at lower Reynolds numbers, the location of the outer ‘shoulder’ of $\overline{u^2}^+$ stays approximately constant in this Reynolds number range. This shoulder was determined as the location at which $d^2\overline{u^2}^+/d(\ln y^+)^2 = 0$ (Pirozzoli *et al.*, 2021).

Figure 2.11a shows the plot of the magnitude of the outer $\overline{u^2}^+$ peak, normalised with the new intermediate velocity scale ΔU_m^{+2} . The plot shows approximate constant behaviour, although the uncertainty in determining both $\overline{u_{OP}^2}^+$ and ΔU_m is non-negligible, but difficult to accurately quantify. The present scaling implies that $\overline{u_{OP}^2}^+ \propto Re_\tau^{4\beta-2} \approx Re_\tau^{0.12}$, which is similar to the scaling derived by Chen *et al.* (2019) (table 2.1), and comes with the theoretical support of the mean momentum equation. Similarly to figure 2.11b, the uncertainty in $\overline{u_{OP}^2}^+$ is too large and the Reynolds number range is too low to favour any particular scaling.

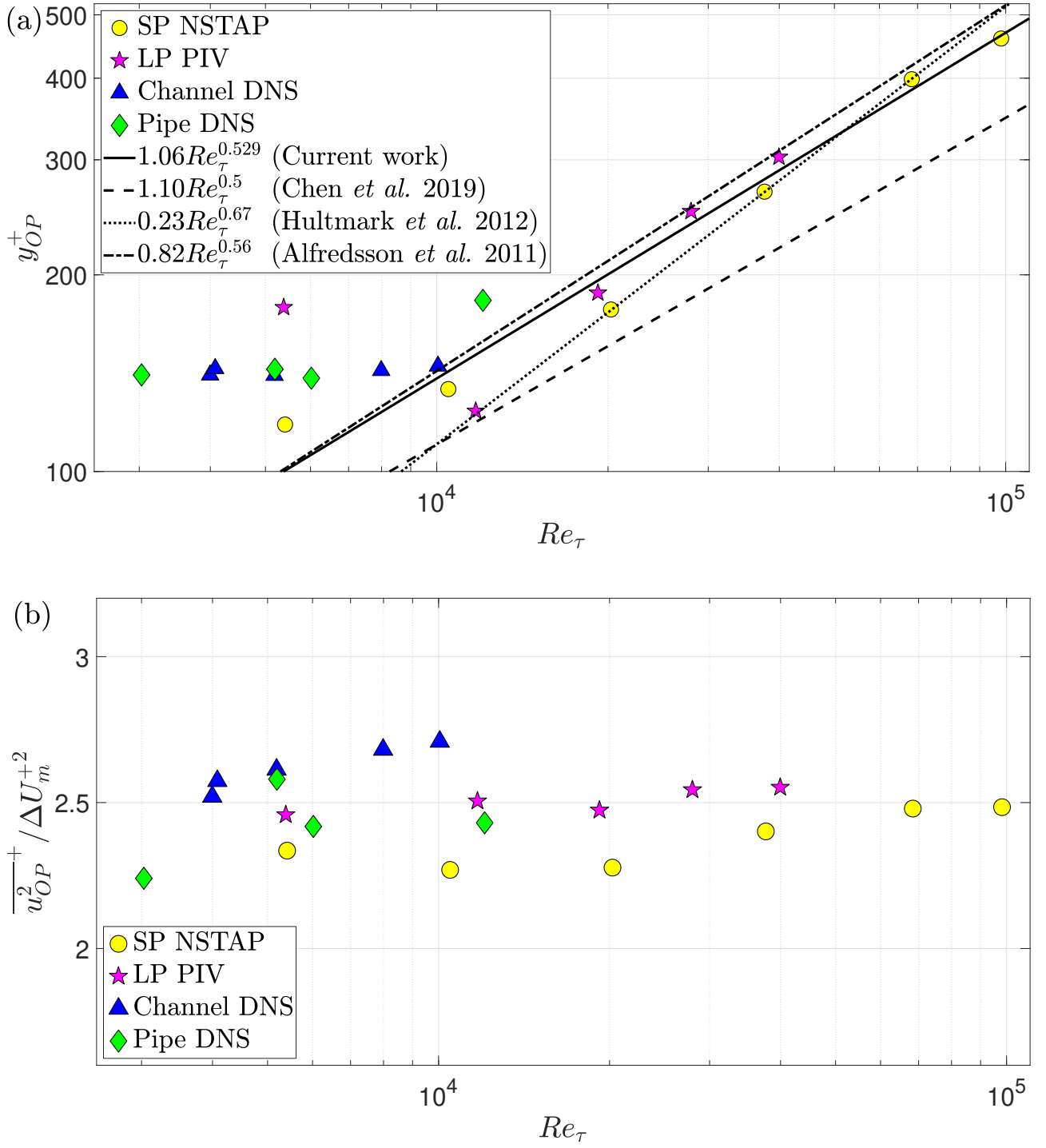


Figure 2.11: (a) comparison of scaling formulas for the location of the outer peak in streamwise variance. (b) the magnitude of the outer peak in streamwise variance, normalised by the intermediate velocity scale.

Chapter 3

Perturbed Boundary Layers

Whereas the structure of a canonical boundary layer can be adequately described through the use of two length scales, δ and ν/u_τ , a distorted boundary layer imposes extra length/ velocity scales which often require greater modelling complexity. The simplest type of boundary layer involving an extra length scale is a boundary layer with homogeneous (two-dimensional) roughness. The effects of the roughness can often be described through a single length scale, k_r , which is a measure of the roughness height. Regardless of the surface finish, roughness effects will become apparent in a turbulent boundary layer when the Reynolds number is sufficiently high, such that $k_r \gtrsim 5\delta_\nu$. In this regime, termed the transitionally rough regime, complete similarity cannot occur due to both the roughness height and viscous length being important. This is in contrast to the fully rough surfaces discussed in section 1.2, where the roughness elements comprise the inner layer and viscosity becomes unimportant (Schlichting & Gersten, 2000). A simple (and therefore useful) model describing the effects of roughness in the transitionally rough regime is

$$U^+ = \frac{1}{\kappa} \log y^+ + B_i - \Delta U^+(k_r^+), \quad (3.1)$$

with ΔU^+ being called the Hama roughness function and is an empirical function of k_r^+ (Colebrook, 1939; Schlichting & Gersten, 2000; Allen *et al.*, 2007). Fundamental studies have often used a sand-grain roughness (Nikuradse, 1933), which have been shown by Allen *et al.* (2007) to give different values for ΔU^+ to a honed surface finish, when k_r^+ is matched. Other rough surfaces generally take the form of a regular pattern of protruding elements, such as hemispherical bumps. These rough surfaces are termed technical roughness, and can be modelled with an “equivalent sand-grain roughness”, k_s . The equivalent sand-grain roughness then becomes a function of the element geometry, height, and pitch (Schlichting & Gersten, 2000). Equation 3.1 can then be used to describe the overlap region with correlations for ΔU^+ such as from Nikuradse (1933). While the simplest rough-wall cases exhibit similarity and can be modelled well with zero equation models, empirical correlations still have to be chosen with careful attention to the type of roughness.

Heterogeneous (three-dimensional) rough surfaces consisting of alternating rough and smooth strips produce different skin friction coefficient values for the same Reynolds number and roughness area, but for different strip widths (Frohnafel *et al.*, 2024). A particular application is in the turbines of axial turbomachines. Dust and combustion pollutant particles will erode or deposit on the blade/endwall surfaces, producing a heterogeneous surface roughness. The thermal boundary layer response is also drastically different to the momentum response in the presence of roughness. When the roughness length scale dominates over viscosity, a large contribution to wall shear stress is due to pressure drag from the roughness elements. Since there is no analogous heat transfer mechanism, the Reynolds analogy factor, $2St/C_f$, can be reduced by up to 50 % (Bons, 2005).

3.1 Classification of perturbed boundary layers

While there is no strict definition for a perturbed boundary layer, it can generally be described as a complex flow (Bradshaw, 1975). In these boundary layers, mixing length models generally do not adequately predict the flowfield. The one equation BFA model of Bradshaw *et al.* (1967) uses the k -equation (equation 1.28). Assuming that the stress ratio

$$a_1 = \frac{-\overline{uv}}{k}, \quad (3.2)$$

and the dissipation length scale

$$L_\epsilon = \frac{(-\overline{uv})^{3/2}}{\epsilon}, \quad (3.3)$$

are functions of y/δ , the TKE transport equation then becomes an equation for Reynolds shear stress. The BFA model has been shown to give better predictions than the $k - \epsilon$ model in boundary layers perturbed by a roughness strip of length 3δ (Andreopoulos & Wood, 1982). On the roughness strip, the wall shear stress increases rapidly and advection and turbulence transport of TKE become significant, indicating a breakdown of local equilibrium which invalidates the mixing length model (Smits & Wood, 1985). Bradshaw (1975) classifies two main scenarios in which a complex flow arises: through interacting shear layers, and through extra rates of strain - in addition to the simple shear rate, $\partial U/\partial y$.

3.2 Interacting shear layers

Examples of interacting shear layers are shown in figure 3.1. They involve boundary layers merging in a duct, boundary layer interactions in a corner, shear layers in a turbulent environment, and internal boundary layers. Shear layers in a turbulent environment are relevant to the boundary layer on compressor (moderate freestream turbulence) and turbine (very high freestream turbulence) blades in gas turbines. An internal layer forms when the boundary condition at one edge of the layer changes

suddenly, and characterises the extent of the outward propagation of the disturbance. Examples include step changes in roughness, step changes in wall heat flux, suction in boundary layer control and blowing in turbine blade cooling.

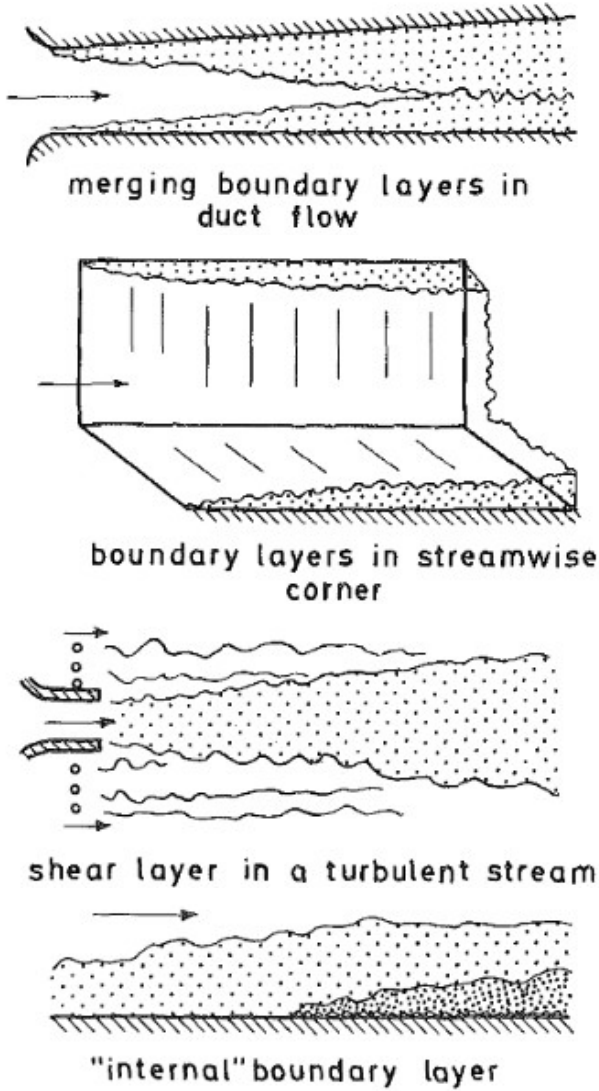


Figure 3.1: Interacting shear layers, taken from Bradshaw (1975).

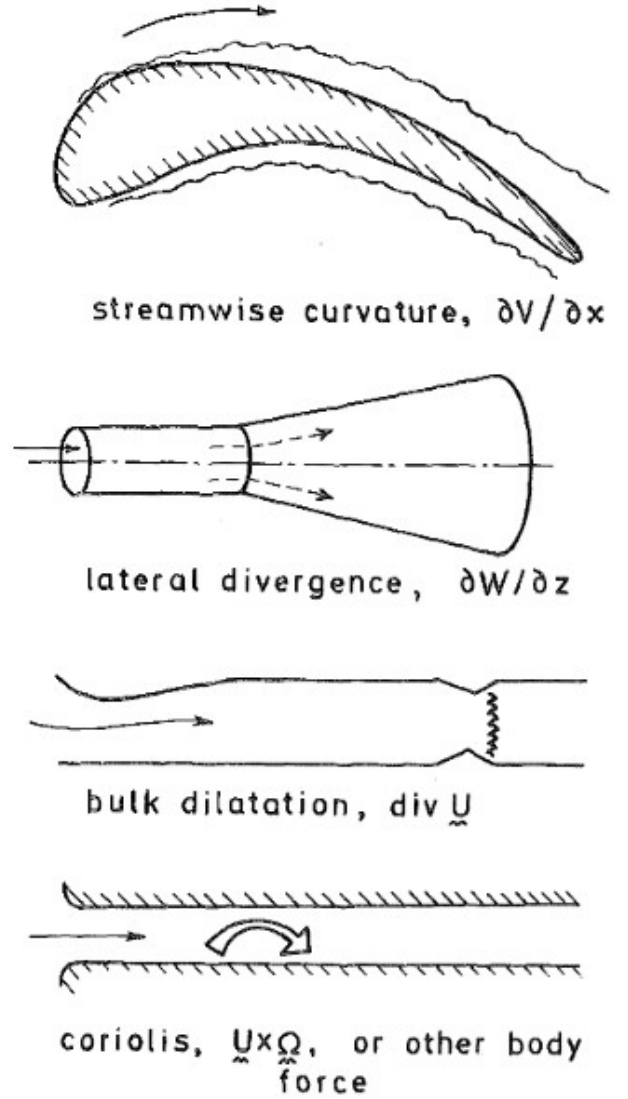


Figure 3.2: Extra rates of strain, taken from Bradshaw (1975).

3.2.1 Boundary layers perturbed by freestream turbulence

A canonical representation of a shear layer in a turbulent environment is the zero pressure gradient turbulent boundary layer exposed to freestream turbulence (FST). Hancock & Bradshaw (1989) conducted detailed measurements of the turbulence field, which resolved the budgets of turbulent shear stress and turbulence kinetic energy, as well as measuring the intermittency of the boundary layer interface with a cold-wire, through the use of a heated leading edge. They found a dependence of skin friction coefficient on both the freestream turbulence intensity and also the freestream turbulence length scale. Jooss *et al.* (2021) presents an up-to-date concise review of the literature.

In summary, the log law remains unperturbed, while the wake is suppressed and the skin friction

coefficient increases with increasing FST. Blair (1983) in one of few studies, calculated both the skin friction and heat transfer coefficients with varying FST and found that the Reynolds analogy factor increased with increasing FST. While this result was of debate in the mid-1900s (Bradshaw, 1975; Lienhard, 2020), the general consensus seems to be that the effect of freestream turbulence is to increase the heat transfer comparatively more than skin friction (Bons, 2005).

3.2.2 Boundary layer response to step changes in surface roughness and heat flux

Examples of boundary layers which develop a new internal layer are boundary layers subjected to a sudden change in surface condition such as roughness and/or heat flux (Smits & Wood, 1985). Antonia & Luxton (1971, 1972) carried out studies with large step changes in roughness, from smooth-to-rough ($S \rightarrow R$) and rough-to-smooth ($R \rightarrow S$). In the $S \rightarrow R$ case, there is a rapid rise in wall shear stress. The dissipation length scale significantly reduces, due to a breakdown of local equilibrium. The advection and diffusion terms become significant in the internal layer. The opposite effects are observed in the $R \rightarrow S$ case.

The results however indicate the non-linearity of the boundary layer response to perturbations. In the $S \rightarrow R$ case, self-similarity was reached after approximately 20δ , but was far from reached by 16δ in the $R \rightarrow S$ case. Here, the internal layer growth was much slower. Differences between cases are attributed to the importance of the transport term $\partial \overline{u^2 v} / \partial y$. In the $S \rightarrow R \rightarrow S$ impulse step change experiment of Andreopoulos & Wood (1982), recovery to self-similarity was not complete by 55δ .

Boundary layers in response to a step change in heat flux draw qualitatively similar conclusions. The results of Antonia *et al.* (1977) and Subramanian & Antonia (1981*b*) respectively suggest that the thermal internal layer grows faster for a cold-to-hot step change than a hot-to-cold step. In the cold-to-hot study (Antonia *et al.*, 1977), generally self-similarity occurred in the mean temperature profiles but not in the turbulent heat flux distributions. Consequently, large streamwise and wall-normal variations in Pr_t are observed, although the authors note the considerable uncertainty in calculating Pr_t .

Large variations in the thermal dissipation length scale

$$L_\theta = \frac{(\overline{v\theta})^2}{(-\overline{uv})^{1/2} \epsilon_\theta}$$

are observed, also caused by a breakdown of local thermal equilibrium. However, the thermal stress ratio

$$a_{1\theta} = \frac{\overline{v\theta}}{(-\overline{uv} \overline{\theta^2})^{1/2}} \quad (3.4)$$

remains approximately constant beyond $\delta \gtrsim 6$ downstream of the step change. The non-linear behaviour of the $\overline{v\theta}$ term is (in part) the cause for a spatially varying Pr_t and the differences in boundary layer response to the cold-to-hot and hot-to-cold step changes.

3.3 Extra rates of strain, e

In the derivation of the Prandtl boundary layer equations, it is assumed that the shear layer thickness is at least an order of magnitude, if not more, smaller than the shear layer development length ($\delta/x \ll 1$). Therefore strain rates in other directions can be neglected, which is the thin shear layer approximation. When extra rates of strain are introduced, the thin shear layer approximation may become invalid and the effect of the introduced extra strain rates can have a significant effect on the shear layer turbulence structure (Bradshaw, 1975). Boundary layers commonly experience extra rates of strain, e , such as:

- streamwise curvature ($\partial V/\partial x$), on the surface of compressor or turbine blades in a gas turbine;
- lateral divergence ($\partial W/\partial z$), in a diffuser of a vehicle or wind tunnel;
- bulk dilatation ($\nabla \cdot \mathbf{Q}$), in compressible flows such as a convergent-divergent nozzle;
- the Coriolis force ($\mathbf{Q} \times \boldsymbol{\Omega}$), or other body forces, as in a turbine rotor cascade.

These are illustrated in figure 3.2. The response of a turbulent boundary naturally depends on the magnitude of the extra strain rate and the distance over which it is applied. A useful measure of the magnitude of the extra strain rate is given by $e/(\partial U/\partial y)$. The effects of the extra strain rate magnitude are summarised in table 3.1. Modelling quantities such as the mixing length change by a factor F ($\ell = F \ell_0$), according to

$$F = 1 + A \frac{e}{\partial U/\partial y}, \quad (3.5)$$

where A is an empirical ‘constant’ of order 10. The significance of $A = \mathcal{O}(10)$ is that the effect of an extra rate of strain is approximately 10 times larger than the magnitude of the production terms they introduce into the Reynolds stress transport equations (Bradshaw, 1975). A change in $\partial U/\partial y$ in the absence of other rates of strain does not produce unexpectedly large changes to the turbulence structure. In this case, the thin shear layer approximation is not violated in any way (Castro & Bradshaw, 1976).

Table 3.1: The effect of extra rates of strain, reproduced from Bradshaw (1975).

Type of shear layer	Inequality	Effect of e on turbulence structure
Simple shear layer	$10 e/(\partial U/\partial y) \ll 1$	Negligible
Thin shear layer	$e/(\partial U/\partial y) \ll 1$	Possibly significant
Fairly thin shear layer	$10 e/(\partial U/\partial y) < 1$	Probably large

Bradshaw (1975) classifies the magnitude of the extra rate of strain against its effect on the thin shear layer approximation. The thin shear layer approximation generally holds if $e/(\partial U/\partial y) < 0.01$, and models such as equation 3.5 have been shown to work well. For $0.01 < e/(\partial U/\partial y) < 0.1$, the effect of the extra strain rate on the turbulence is large and the thin shear layer approximation is violated. Bradshaw (1973) gives a more comprehensive description. When $e/(\partial U/\partial y) > 0.05$, the strain rate is too large for equation 3.5 to give adequate predictions because it is linear (Bradshaw, 1975). In boundary layers with rapidly changing extra strain rates, the turbulence will depend on the history of the extra strain rate, so equation 3.5 is also not valid.

Reviews of experiments conducted which introduce extra rates of strain and the subsequent modelling strategies are given by Bradshaw (1975); Smits & Wood (1985); Morrison (2005). Of greater concern to this thesis is the boundary layer response to streamwise curvature, due to its presence on turbine stator and rotor blades. (Bradshaw, 1973) presents a still highly relevant review on this subject. Muck *et al.* (1985); Hoffmann *et al.* (1985) outline the effects due to convex and concave curvature respectively.

The boundary layer response is qualitatively and quantitatively different in the presence of concave and convex curvature of the same radius, despite having the same dimensional analysis. The Stanton number decreases for convex curvature and increases for concave curvature. The effects on the Reynolds analogy factor are generally smaller (Bradshaw, 1973), when compared to roughness perturbations for example. For convex curvature, turbulence is attenuated, particularly the Reynolds shear stresses in the outer region (Smits *et al.*, 1979). Turbulence is enhanced for concave curvature, along with the production of streamwise “Taylor-Görtler” vortices and significant alterations to the turbulent structure. Large changes to L_ϵ occur in both cases. It decreases for convex curvature and increases for concave curvature. The stress ratio a_1 is relatively constant for convex curvature, but can increase considerably for a similar change in concave curvature.

3.4 Jets in cross-flow

Applications which involve mass injection through discrete holes of diameter D , fall under a general category of jets in a cross-flow. However many of the scaling parameters for mass injection through slots or porous surfaces are the same as those characterising jets in cross-flow. This type of boundary layer can be classified as a secondary flow - a boundary layer that contains mean streamwise vorticity. Bradshaw (1987) presents a review of secondary flows. An important application of jets in cross-flow is in the cooling of turbine blades/endwalls and atmospheric re-entry spacecraft. A recent review of jets in cross-flow is given by Mahesh (2013).

The parameters that scale the injection effects are given as:

$$VR = \frac{U_e}{U_\infty}, \quad (3.6)$$

$$M = \frac{\rho_e U_e}{\rho_\infty U_\infty}, \quad (3.7)$$

$$J = \frac{\rho_e U_e^2}{\rho_\infty U_\infty^2}, \quad (3.8)$$

which are termed the velocity ratio, mass flux ratio (often called the blowing ratio) and momentum flux ratio respectively. Subscripts e and ∞ denote the the injection conditions at the surface and of the mainstream flow conditions respectively. U_e is generally defined as the velocity parallel to the axis/ angle of injection. Other important dimensionless parameters include the jet Reynolds number, $Re_j = U_e D / \nu$, and the ratio of a boundary layer characteristic length to the hole diameter. Two commonly used parameters are δ/D and δ_1/D , which use the boundary layer thickness and displacement thickness respectively.

The interactions between the jet and the cross-flow boundary layer cause the formation of many vortical structures. At high velocity ratios ($VR \gtrsim 1$), interactions are dominated by the formation of a counter-rotating vortex pair (CRVP). They have the effect of lifting the jet away from the wall and entraining fluid into the wall region beneath the jet. The far-field can often be seen to be dominated by the CRVP after the jet breaks down. Other vortical structures such as shear layer vortices, wake vortices and horseshoe vortices are also present. At low velocity ratios ($VR < 1$), the flowfield is dominated by hairpin vortices.

Bidan & Nikitopoulos (2013) investigated transverse jets (90° inclination) in cross-flow with a boundary layer size of $\delta/D = 0.63$ and $\delta_1/D = 0.172$. They identify a fully attached regime ($VR \lesssim 0.23$), a transitional regime ($0.23 \lesssim VR \lesssim 0.6$), and a detached regime ($VR \gtrsim 0.6$). It is likely that the parameters Re_j and δ/D will affect these threshold velocity ratios.

3.5 Turbulent boundary layers with transpiration

Boundary layers with transpiration involve wall-normal blowing (or suction) through a porous surface, usually at low velocity ratios of $VR < 0.01$. This perturbation falls under the internal layer description. A constant wall-normal velocity, V_e , is assumed so the boundary layer momentum equation becomes

$$U \frac{\partial U}{\partial x} + V_e \frac{\partial U}{\partial y} = \frac{\partial}{\partial y} \left[\nu \frac{\partial U}{\partial y} - \overline{uv} \right], \quad (3.9)$$

which reduces to

$$V_e \frac{\partial U}{\partial y} = \nu \frac{\partial^2 U}{\partial y^2} \quad (3.10)$$

in the viscous sublayer. Tennekes (1965) integrates equation 3.10 twice to derive the velocity profile in the near-wall region with transpiration, as

$$\frac{V_e U}{u_\tau^2} = e^{V_e y/\nu} - 1, \quad (3.11)$$

which yields a velocity scale of the form u_τ^2/V_e . The Taylor expansion of equation 3.11 is

$$U^+ = y^+ + \frac{1}{2} V_e^+ y^{+2} + H.O.T. \quad (3.12)$$

Tennekes (1965) states that equation 3.12 is valid for $0.04 < V_e^+ < 0.1$. For $V_e^+ < 0.04$, the traditional wall limiting behaviour $U^+ = y^+$ is a simpler approximation and just as accurate. Using the same velocity scale u_τ^2/V_e , a logarithmic overlap profile was derived, which agreed well with experimental data.

Kays *et al.* (2005) present a skin friction law of the form

$$C_f/2 = [C_f/2]_0 \frac{\log(1 + B_f)}{B_f}, \quad (3.13)$$

where $[C_f/2]_0$ is the no-transpiration skin friction coefficient evaluated at the same Reynolds number and $B_f = VR / [C_f/2] = V_e U_\infty / u_\tau^2$. The relationship therefore uses the same velocity scale as Tennekes (1965). Equation 3.13 agrees well with the blowing and suction experiments of Andersen *et al.* (1975) and Simpson *et al.* (1969) respectively. While the derivation assumes an equilibrium boundary layer (a constant B_f), equation 3.13 is found to work well when VR is kept constant, rather than B_f because the turbulence in the inner layer adjusts very quickly. The authors say that equation 3.13 is useful for $VR \lesssim 0.01$, as a “rule of thumb”. Beyond this limit, the boundary layer begins to be blown off the surface.

A variety of low-blowing transpiration studies have been carried out in different boundary layers such as flat plate TBLs (Mickley & Davis, 1957; Andersen *et al.*, 1975; Ferro, 2017), and Poiseuille flows (Avsarkisov *et al.*, 2014). Generally, analytical models such as the mixing length and appropriately scaled logarithmic laws match experimental data and simulations well. Modifications to the coefficient A in the Van Driest (1956) model (equation 1.32) account for changes in the boundary layer structure, which primarily reside in the sublayer and buffer region (Kays *et al.*, 2005).

More recently, direct numerical simulations (DNS) in a transpired turbulent boundary layer were carried out by Kametani & Fukagata (2011). The authors find that increasing the blowing transpiration velocity (for $VR < 0.01$) reduces skin friction and attribute this to a negative contribution from the mean advection term. Subsequent large eddy simulations (LES) at higher Reynolds numbers are presented in Kametani *et al.* (2015). Blowing causes an increase in amplitude of all normal Reynolds stresses and the shear stress, particularly in the outer region. Suction has the opposite effects. Experimental

transpiration cooling studies for hypersonic applications are reviewed in Langener *et al.* (2011) and a recent implementation is given in Naved *et al.* (2023).

Zangeneh *et al.* (2023) carried out DNS in a turbulent boundary layer with $Re_\tau \approx 1400$ at higher transpiration velocity ratios, reaching $VR = 0.0195$ and $V_e^+ = 0.63$. The authors present a modified turbulent viscosity and mixing length, incorporated into the k -equation. They find their model agrees well with the DNS. Of particular interest is the large near-wall increase in TKE and TKE production with blowing. The peaks increase by a factor of approximately 2.8 and 4.4 respectively at the highest velocity ratio. The peaks move further from the wall with increasing VR , which is attributed to a thickening of the sublayer. At the highest VR , local equilibrium is not established by $y^+ = 100$, despite the model using this assumption in the logarithmic region. Data for $y^+ > 100$ is not presented.

3.6 Turbine blade cooling in gas turbines

The thermal efficiency of an idealised Brayton cycle is expressed as

$$\eta_{th} = 1 - \frac{\Theta_4}{\Theta_3} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}}, \quad (3.14)$$

where Θ_4 , Θ_3 are the exhaust and combustion/ turbine entry temperatures respectively. r_p is the ratio of the compressor outlet pressure to the engine inlet pressure and γ is the specific heat capacity ratio, c_p/c_v . Increasing the pressure ratio in a gas turbine results in a larger thermal efficiency. Since Θ_3 , Θ_4 are coupled, increasing the turbine entry temperature does not directly result in a higher thermal efficiency, but it does allow for larger pressure ratios. A formal derivation of this result is given in appendix B. For this reason, considerable effort is put towards increasing the turbine entry temperature. Typical turbine inlet temperatures in modern aircraft engines exceed 1700°C , which is far above the blade material melting temperature ($\sim 1000^\circ\text{C}$) (Han, 2013).

Therefore the surfaces of turbine blades are cooled through film cooling. This involves injecting cooler air through discrete holes on the surface of the blade. Due to the large aerodynamic stresses (in stator and rotor blades) and centrifugal stresses (in only rotor blades), transpiration cooling cannot currently be implemented on turbine blades while maintaining their mechanical integrity. Typical mass flux ratios for gas turbine blades range from $0.5 < M < 2$ (Han, 2013).

The heat input to the idealised Brayton cycle through combustion is assumed to be isobaric. However, in reality, there is a pressure drop due to frictional and turbulent mixing losses, as well as a rise in the flow velocity. This allows cooler air to be bled from the final compressor stage and enter the film cooling holes at temperatures of $\sim 700^\circ\text{C}$ (Han, 2013). Roughly 3%–5% of compressor discharge air is used for turbine cooling, which reduces the total available engine work output and therefore the engine efficiency. This creates the challenging problem of sufficiently reducing heat transfer to the blade surface with the minimum amount of coolant usage.

3.6.1 Differences between film, effusion, and transpiration cooling

As described in section 3.5, transpiration cooling involves the addition of momentum at a lower temperature through a porous surface. The convective heat transfer through the surface is large, due to the large internal surface area. The resulting ‘film’ also remains fully attached to the surface. For these reasons, applications involving transpiration cooling require much lower mass flux ratios and produce higher cooling effectiveness values than with film cooling.

In film cooling, the holes are much larger and less densely spaced, producing jets in a cross-flow. The large velocity ratios mean the jets usually detach from the surface quickly, due to the generation of a CRVP. Hot gas is therefore entrained into the surface and cooling effectiveness values immediately downstream of the holes are low. Generally, the film undergoes large lateral expansion and reattaches to the surface, resulting in higher effectiveness values downstream. The process is aerodynamically inefficient, however. The complex flow behaviour and multiple boundary layer perturbations mean that surface heat transfer coefficients on the blade surface are not predicted accurately. This leads to excessive coolant use (Han, 2013).

The aim of next-generation turbine cooling systems is to introduce effusion cooling (Krewinkel, 2013; Xia *et al.*, 2024), occasionally referred to as quasi-transpiration cooling. As the second name suggests, it is an intermediate cooling method between film and transpiration cooling. The discrete holes are smaller and more densely spaced than with film cooling. The interactions between adjacent jets suppress vortex lift-off, so the jets remain attached to the surface for longer. Furthermore, the increased porosity allows for larger convective cooling and lower momentum flux ratios. Due to lower mechanical stresses but larger temperatures, effusion cooling technology is used on the walls of combustion liners.

3.6.2 Film/ effusion cooling parametrisation

The flow phenomena associated with gas turbine cooling are very complicated, and detailed flowfield measurements are very difficult. Significant boundary layer perturbations are pressure gradients, roughness, freestream turbulence, boundary layer corner/ endwall interactions, the generation of streamwise vorticity, surface curvature, and Coriolis forces. There are also interactions with a large range of coherent structures due to preceding blade wakes (Bradshaw, 1975) and unsteady phenomena such as clocking and surge (LaGraff *et al.*, 2006). Therefore research often focuses on parametric studies for the adiabatic effectiveness or film effectiveness, defined as

$$\eta = \frac{\Theta_{\infty} - \Theta_{aw}}{\Theta_{\infty} - \Theta_e}, \quad (3.15)$$

where Θ_{aw} is the temperature of the fluid immediately above the adiabatic wall and Θ_e is the fluid temperature at the exit of the holes. Heat transfer into the surface can only occur through conduction

and therefore Θ_{aw} is a measure of the driving potential for heat transfer (Bogard & Thole, 2006).

The parameter space for modelling flows with film/ effusion cooling is vast. Parameters that will have an impact on the film effectiveness are: hole size, hole shape, hole spacing (laterally and longitudinally), hole exit angle, mass flux ratio, density ratio, hole geometry, boundary layer thickness, FST intensity, Mach number and radiative heat transfer effects. Therefore an extensive part of film cooling research focuses on empirical correlations for η , as a function of the parameters discussed above. Goldstein (1971) gives a comprehensive review of film cooling studies and modelling strategies. More recent reviews include Bogard & Thole (2006) and Zhang *et al.* (2020). Han (2013) gives a general summary, reviewing all aspects of heat transfer in turbine blades, of which film cooling is a major component. Krewinkel (2013) reviews studies specifically pertaining to effusion cooling.

Correlations for film effectiveness and similarly derived effectiveness parameters are simple and useful for determining heat transfer into the blade surface, and spatial heat transfer distributions. However, they provide little insight into the flow physics which govern the observed heat transfer distributions. Hence, these are outside of the scope of this thesis which focuses on mean flow and turbulence measurements. Such studies will be reviewed in section 3.7.

3.6.3 Density ratio effects

Many blowing studies are isothermal so $VR \approx M \approx J$. However, in gas turbine blade cooling, typical density ratio values are $DR = \rho_e/\rho_\infty \approx 2$ (Han, 2013), so equations 3.6 - 3.8 cannot be simultaneously matched. Therefore the effect of density ratio deserves special attention. This is studied by observing differences in flow physics or cooling effectiveness when M is kept constant, but the ratio ρ_e/ρ_∞ is varied. Pedersen *et al.* (1977) used the heat-mass transfer analogy to investigate the effect of density ratio over a porous (transpiration) surface, and over a single row of film cooling holes. The film cooling holes were inclined at 35° to the streamwise direction. Through the porous strip, the density ratio was varied between $0.96 < DR < 4.17$ for given mass flux ratio values in the range $0.012 < M < 0.028$. The authors determine that the density ratio has no effect on cooling effectiveness over the porous strip, as the injected fluid is likely always attached to the surface.

Over the row of film cooling holes, little effect is also observed at low mass flux ratios of $M = 0.21, 0.52$. In these cases, the film is attached to the surface. At larger mass flux ratios of $M = 1.05, 1.96$, the effect of the density ratio is significant. Higher density ratios produce a larger film effectiveness for a given M , and this effect persists for up to $80D$ downstream of injection. To summarise, a larger density ratio helps keep the jets attached to the surface for a given M . When the film remains attached to the surface, DR has little effect. In these cases, a lower DR is actually preferred because the velocity ratio is larger. Baldauf *et al.* (2002) carried out a very large parametric study in which M was varied at two density ratios, $DR = 1.2, 1.8$. Similar conclusions could be drawn. The authors state that: “The flow structure, and subsequently, the resulting surface impact, is most closely related

to the velocity ratio". Ethridge *et al.* (2001) conducted experiments and compared film effectiveness on the suction side of a first-stage turbine vane. M was varied at density ratios $DR = 1.1, 1.6$. Similar conclusions to those previously stated are drawn. They find at high values of M (where the effect of DR becomes apparent), the film effectiveness distributions collapse well for different density ratios when scaled with J , rather than with M .

3.7 Film/ effusion cooling mean flowfield and turbulence measurements

The current section reviews literature which focuses on momentum and/or scalar field measurements in boundary layers which are perturbed by film or effusion injection. The resulting flowfield can be thought of as the interaction of multiple jets in cross-flow within the boundary layer. However, the interactions of jets in adjacent rows and columns are strong and cause large deviations in film effectiveness, compared to predictions using a linear superposition model (Murray *et al.*, 2018). Therefore these studies deserve particular attention. All studies were carried out over a flat plate in zero pressure gradient conditions, so this will not be explicitly stated each time. Upstream boundary layer parameters may have been fully reported in the literature, so Blasius TBL correlations are used here to provide an approximate measure of parameters δ/D , δ_1/D , for the sake of comparison. A summary of the experiments reviewed in this section is given in table 3.2.

Table 3.2: Comparison between film/ effusion cooling turbulence studies in the literature.

Authors	Measurements	δ/D	Hole inclination angle	Velocity ratios
Yavuzkurt <i>et al.</i> (1980 <i>a</i>)	Tri-axial hot-wire: $u^*, v^*, w^*, \partial/\partial y$	2.8	30°	0.4, 0.9
Kohli & Bogard (2005)	2c LDA and cold-wire: $u^*, v^*, \theta^*, \partial/\partial x, \partial/\partial y$	0.8	35°	0.38
Hasanuzzaman <i>et al.</i> (2020)	Stereoscopic PIV: $u^*, v^*, w^*, \partial/\partial y$	67	90°	0.01 - 0.06
Current work	3c LDA, x-wire and cold-wire: $u^*, v^*, w^*, \theta^*, \partial/\partial y$	13	30°	0.10 - 0.37

3.7.1 Yavuzkurt, Moffat and Kays (1980)

Yavuzkurt *et al.* (1980a), in one of the first detailed film cooling turbulence flowfield studies, used a tri-axial hot-wire to obtain three-component velocity measurements. The effusion plate had a staggered arrangement, consisting of 11 rows and 9 columns of holes, with diameters $D = 10.3$ mm and a hole spacing of $5D$ in both streamwise and spanwise directions. Holes were inclined at 30° to the streamwise direction. The upstream boundary layer had a Reynolds number $Re_L \approx 1.45 \times 10^6$. Using the Blasius TBL correlation, this gives $\delta/D \approx 2.8$.

Air was injected isothermally at velocity ratios $VR = 0.4, 0.9$. Crawford *et al.* (1976) observe minimum Stanton numbers for $M \approx 0.4$, and $M = 0.9$ is more representative of turbine blade mass flux ratios. Five streamwise locations were interrogated: two on the film cooling plate, one directly behind the plate and the final two locations were in the recovery region ($x/D \approx 17.5, 37D, 56D, 81.5D, 122D$). Five spanwise locations were also acquired, focusing on the regions directly behind a column of holes and in between adjacent columns of holes (where there is no direct effect from injection). It is worth explicitly noting that due to the use of a tri-axial wire, measurements were mostly restricted to the outer layer ($y/\delta > 0.1$).

The mean velocity profiles are taken from Yavuzkurt *et al.* (1980a) and shown in figure 3.3 for reference. Over the five streamwise measurement stations, the boundary layer thickens significantly. A 50 % increase is observed from the first to the third station, and a 75 % increase by the last. The growth of the internal layer is also large. Recovery of the turbulence kinetic energy is far from complete by the last measurement station at $x/\delta \approx 17$, using δ from the first measurement station. This is unsurprising and also typical of the perturbations discussed in previous sections. For $VR = 0.4$, mean streamwise velocity profiles show a large velocity deficit, relative to a ‘canonical’ case (shown with a $1/7^{\text{th}}$ power law). Large spanwise differences on the film cooling plate are observed and the largest velocity deficit is directly behind the holes.

For $VR = 0.9$, the perturbation to the mean profile in the outer region is less pronounced and there is less spanwise variation, likely due to the injection velocity being similar to the freestream value. There are instances however where the mean velocity exceeds the $1/7^{\text{th}}$ law velocity value, causing inflection points in the profile. This is not observed in the $VR = 0.4$ case. Separation of the jets is conjectured for $VR = 0.9$, but the flow remains fully attached at $VR = 0.4$. There is no evidence of logarithmic scaling for either velocity ratio in the blowing region, but it is observed in the recovery region. Recovery of the mean profile is distinctly faster in the $VR = 0.9$ case due to the lower outer layer velocity deficit. Spanwise flow homogeneity is achieved in both cases by the fourth streamwise measurement station.

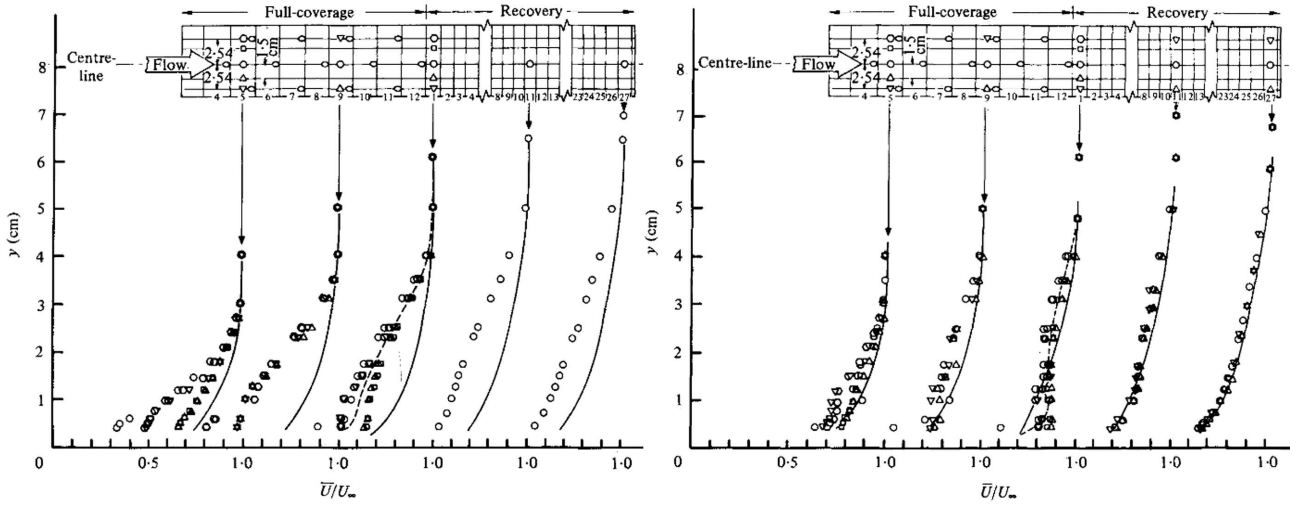


Figure 3.3: Mean velocity profiles for $VR = 0.4$ (left) and $VR = 0.9$ (right), taken from Yavuzkurt *et al.* (1980a).

The turbulence kinetic energy profiles for both velocity ratios have different structures. These are taken from Yavuzkurt *et al.* (1980a) and shown in figure 3.3 for reference. For $VR = 0.4$, there is a distinct large outer peak in TKE which is not observed in the $VR = 0.9$ case. Generally, the TKE values are much lower for $VR = 0.9$, except for in the region measured closest to the wall of $0.1 \lesssim y/\delta \lesssim 0.15$. It is therefore conjectured that the heat transfer mechanism is not dominated by turbulent mixing because higher Stanton numbers are observed at $VR = 0.9$. It is instead postulated to be due to jet detachment, which causes greater entrainment of (hotter) outer region fluid into the near-wall region. This hypothesis is supported by mean wall-normal velocity profiles which show large positive values of V on the centreline of the holes, but negative values in the region between two columns of holes. This phenomenon is much more pronounced at $VR = 0.9$.

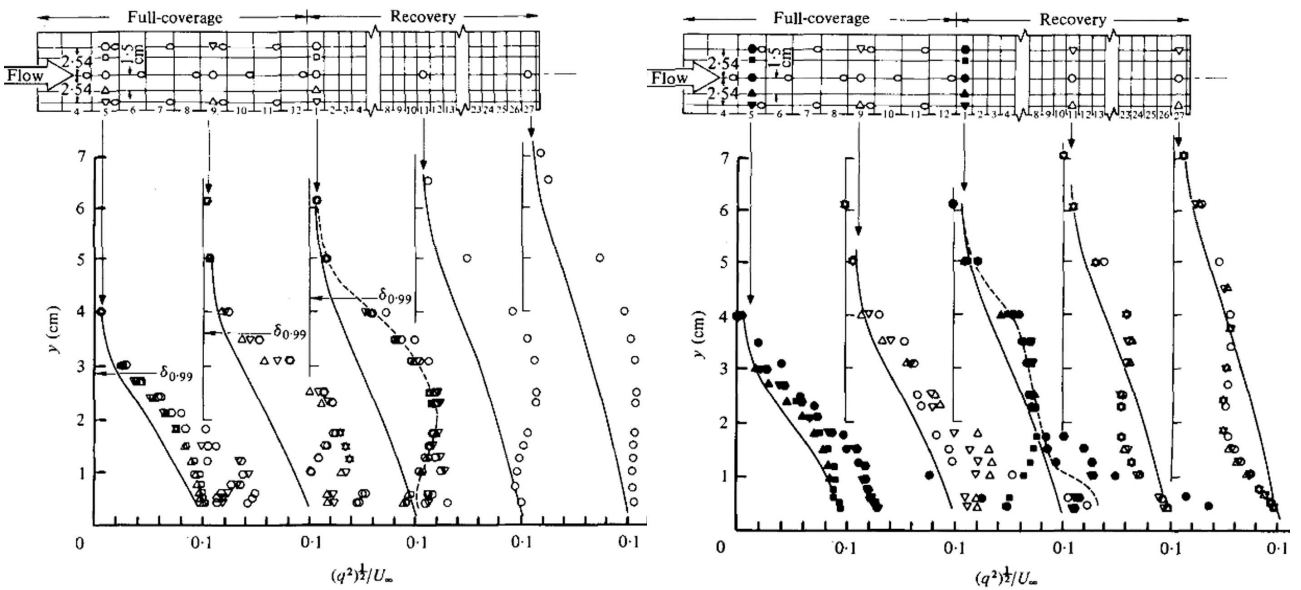


Figure 3.4: Turbulence kinetic energy profiles for $VR = 0.4$ (left) and $VR = 0.9$ (right), taken from Yavuzkurt *et al.* (1980a).

The authors find that the stress ratio $\overline{uv}/k$ collapses well for $VR = 0.4$ in the region $y/\delta > 0.5$, and falls below this value closer to the wall where the perturbation is stronger. Large scatter in the stress ratio is found at $VR = 0.9$. It is generally lower than the baseline TBL value, assumed to equal 0.3. Similar observations are made regarding the mixing length. Standard models give reasonable predictions at $VR = 0.4$, but are much poorer for $VR = 0.9$. This is due to the much lower velocity gradients, where the mixing length model is inadequate.

3.7.2 Kohli and Bogard (2005)

In one of the very scarce joint momentum and scalar transport studies, Kohli & Bogard (2005) took simultaneous laser Doppler anemometry (LDA) and cold-wire measurements behind a single row of film cooling holes. Measurements were taken over a polystyrene plate, assumed to be adiabatic. Hole diameters are 11.1 mm, with spanwise spacing of $3D$ and an inclination angle of 35° . Upstream TBL parameters are given as $\delta_1/D = 0.1$, $Re_2 = 420$, $H = 1.66$. For consistency with previous descriptions, this gives $\delta/D \approx 0.8$ using the Blasius TBL correlation (which was verified to accurate prediction of the quoted displacement thickness). Film cooling flow was injected with a mass flux ratio of $M = 0.4$ and density ratio $DR = 1.05$ through use of a liquid nitrogen cooling circuit.

Contour fields are presented, with streamwise and spanwise extents of $x/D = 10$ and $y/D = 2$ respectively. Results are presented on the jet/ hole centreline, although measurements at different spanwise planes are referred to. It is stated that $\overline{uv}$ dominates over $\overline{uw}$ and $\overline{vw}$ across the whole flowfield and particularly on the hole centreline, which was the reason why this spanwise location is thoroughly investigated. Two freestream turbulence intensities of 0.6% and 20% were tested.

In the low freestream turbulence case, the authors find that the largest values of the turbulent momentum and heat fluxes are at the interface between the injected fluid and freestream, where the shear rate is high. These values are approximately twice as high as the maximum values found in canonical momentum and thermal boundary layers. The authors conjecture that this causes the large dispersion of jets found in film cooling applications, which RANS models generally fail to predict. Values of the turbulent Prandtl number are generally at $Pr_t \approx 2$, but fall as low as 0.5. This highlights the inadequacy of Reynolds' analogy, but also turbulent diffusivity models in general for this flowfield.

The high freestream turbulence case shows a large increase in turbulent thermal transport compared to the low freestream turbulence case, but with lower correlation coefficients. This is likely due to higher mixing created by the freestream turbulence, but generally by smaller, less coherent structures. There are also no distinct maxima in turbulent fluxes on the jet interface line. The extent of the turbulent heat fluxes persist to much larger wall normal distances, due to a thickening of the internal layer resulting from the interaction of the two perturbations. The turbulent Prandtl number is observed to be much closer to 1, although this quantity is not plotted for either FST case.

3.7.3 Hasanuzzaman *et al.* (2020)

Hasanuzzaman *et al.* (2020) used stereoscopic particle image velocimetry (PIV) to measure the velocity statistics in a large canonical boundary layer, perturbed with isothermal effusion. It is noted that the study was motivated by the use of blowing for drag reduction purposes. The measurement region is approximately 1δ downstream of the end of the perturbation. The velocity ratios tested were $VR = 0, 0.01, 0.03, 0.06$ and the Reynolds number also varied between $2200 < Re_\tau < 5500$. Hole diameters are 3.6 mm, giving $\delta/D \approx 67$. Hole spacings are $4D$ and $2D$ in the streamwise and spanwise directions respectively, laid out in a staggered arrangement. Flow exits the holes normal to the wall.

Their results show that the Reynolds number effects are small compared to changes in velocity ratio. When the velocity ratio is varied, the same trends are qualitatively observed across all tested Reynolds numbers. Streamwise turbulence intensity and diagnostic plot, $\sqrt{u^2}/U$ vs U/U_∞ (Alfredsson & Örlü, 2010) profiles collapse well for a given velocity ratio at different Reynolds numbers.

As the velocity ratio is increased, the streamwise mean velocity profiles monotonically increase in velocity deficit, likely accompanied by a decrease in skin friction coefficient. Shape factor and momentum thickness also increase. Departures from logarithmic behaviour in the overlap region begin to become apparent at the higher velocity ratios. In this region, outer peaks in all of the Reynolds normal and shear stresses are observed, with their magnitude monotonically increasing with increasing VR . The streamwise turbulence intensity also increases in the overlap region and begins to show a distinct hump, although not a peak. The increase in turbulence is accompanied by an outer peak in the premultiplied shear production.

3.7.4 Previous effusion cooling experiments at Imperial College London: Basley, Gouder, Morrison (2022)

Basley *et al.* (2022) conducted isothermal measurements on a flat effusion panel with a staggered array of holes. Hole diameters were $D = 16$ mm, with a uniform spacing of $5D$ and an inclination angle of 30° from the streamwise direction. The same experimental rig is used in the research of this thesis and is therefore described thoroughly in Chapter 4. The geometry is shown in figure 3.5. The experiment was designed with the goal of high spatial and temporal resolution, targeting next-generation turbine blade designs with smaller holes. This gives $\delta_1/D \approx 1.8$ and $\delta/D \approx 12.5$.

Basley *et al.* (2022) acquired hot-wire measurements above an isothermal film at velocity ratios $VR = 0, 0.10, 0.17, 0.27, 0.37$. As VR is increased, the Reynolds number reduces from approximately $Re_\tau = 4700$ down to $Re_\tau = 2500$, due to a restriction on plenum mass flow rate and to achieve higher velocity ratios. As discussed in the previous section, Reynolds number effects seem to be unimportant compared to the effusion perturbation. The spanwise location selected is in between two columns of holes, at a streamwise distance of $37D$ from the beginning of the effusion plate. Hot-wire measurements reveal an

increase in streamwise turbulence intensity in the outer region ($y/D \gtrsim 2$, $y/\delta \gtrsim 0.16$), increasing with VR due to the additional effusion momentum penetrating further into the boundary layer.

While the momentum of the jets is higher for larger velocity ratios, the bulk of the injected fluid is pushed further into the boundary layer, where the mean velocity is also larger. Therefore the mean velocity profiles increase in velocity deficit compared to the upstream unperturbed case as VR increased, for $y/D \gtrsim 2$. The wall-normal location at which the mean streamwise velocity and turbulence intensity collapse back to the unperturbed case becomes larger with increasing VR , due to the perturbation strength and size of the internal layer increasing. For $VR = 0.37$ the perturbation affects the whole boundary layer.

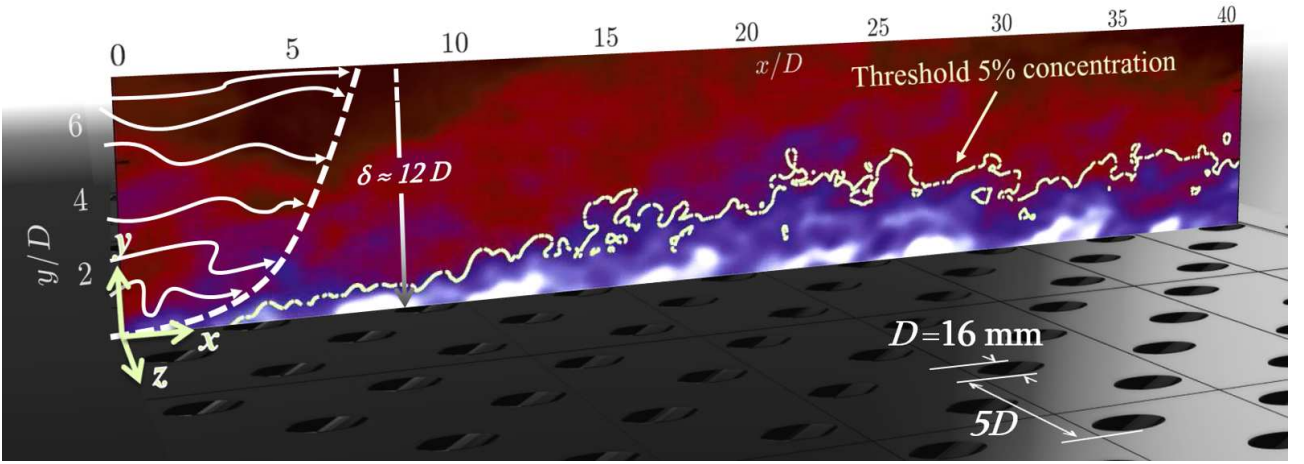


Figure 3.5: Graphic of the effusion panel with labels of the hole geometry. A snapshot of the streamwise velocity fluctuations superimposed with the concentration of injected fluid (yellow line denotes 5 % contour). The figure is reproduced from Basley *et al.* (2022).

Significant differences are observed in the near-wall region, which enables the characterisation of two flow regimes with a threshold at $VR \approx 0.17$. Below the threshold, low momentum diffusion of the film into the boundary layer takes place, in which the bulk of the film remains close to the surface. The velocity deficit, turbulence intensity, and skewness all monotonically increase with increasing velocity ratio in this regime. Above the threshold, the bulk of the film begins to lift from the surface. The mean velocity gradients are no longer monotonically decreasing and there is a higher momentum region of fluid pushed away from the wall. The velocity deficit in the near-wall region increases with increasing VR , up until the threshold where the trend is reversed and the deficit reduces.

In the near-wall region, turbulence intensity also increases in magnitude with VR , up until $VR = 0.17, 0.27$ where it is similar, before reducing at $VR = 0.37$. Beyond $VR = 0.17$ a distinct outer peak in turbulence intensity forms which moves further from the wall and decreases in magnitude with increasing VR . The measurements reflect similar observations to Yavuzkurt *et al.* (1980a), but with a lower VR threshold. One difference is that no significant thickening of the boundary layer is observed with mass injection in the study of Basley *et al.* (2022). These differences could be due to the large difference in δ/D between the two experiments.

Basley *et al.* (2022) also analysed joint momentum and scalar mixing by injecting polyethylene glycol (PEG) seeding particles into the effusion plenum, through the holes. The freestream was sparsely seeded due to seeding recirculation within the wind tunnel. This enabled planar PIV measurements in the (x, y) -plane, spanning the full streamwise extent of the perturbed region. The concentration, C , of scattered light intensity, I , was used as a scalar and as a surrogate for temperature. This allows the mixing of the film with the freestream to be tracked temporally and spatially using the definition

$$C(x, y, t) = \frac{I_{max} - I(x, y, t)}{I_{max} - I_{min}}, \quad (3.16)$$

where I_{max} and I_{min} correspond to the maximum (directly out of the effusion holes) and minimum (in the very sparsely seeded freestream) scattered light intensities respectively. A rendered snapshot of this experiment is shown in figure 3.5. The scalar concentration fields show similar trends in behaviour to the PIV and hot-wire results, with the bulk of the film starting to lift from the surface above the threshold velocity ratio.

The two-point amplitude modulation coefficient of small-scale turbulence by large-scale structures was measured at a streamwise reference location $x_0 = 28D$. It shows that beyond the critical velocity ratio, near-wall small-scale turbulence is no longer significantly amplified. A very intense region of modulation appears around $y \approx y_0 \approx D$ for $VR = 0.37$, corresponding to the deeper penetration of the film into the boundary layer.

Maps of the two-point correlation coefficient between the large-scale fluctuations (denoted with subscript L) of the streamwise velocity and scalar concentration are also presented, reproduced here in figure 3.6. They show that for $VR \lesssim 0.17$, there is a strong local anti-correlation in the near-wall region between the scalar concentration and momentum, because the film is carrying a velocity deficit into the boundary layer. The outer dynamics have very little impact on the film.

Above the threshold, the anti-correlation region moves up to $y/D \approx 2$ and is attenuated closer to the wall. This further suggests that the bulk of the film is being lifted away from the surface, since the anti-correlation represents recently injected, relatively unmixed fluid. However, there is a strong modulation of the large-scale velocity fluctuations from the outer layer on the near-wall region. This is seen in the bottom right corner of figure 3.6b. This suggests that at higher velocity ratios, the near-wall region is more susceptible to modulation by outer layer turbulence and that the surface is less protected.

3.7.5 Other notable studies

Kaszeta & Simon (2000) conducted three-component velocity measurements downstream of a row of eleven film cooling holes at velocity ratio $VR = 1$. Holes were 19 mm in diameter with a pitch of $3D$. Upstream conditions were measured as $Re_2 = 960$, $\delta_1/D = 0.094$, with a freestream turbulence

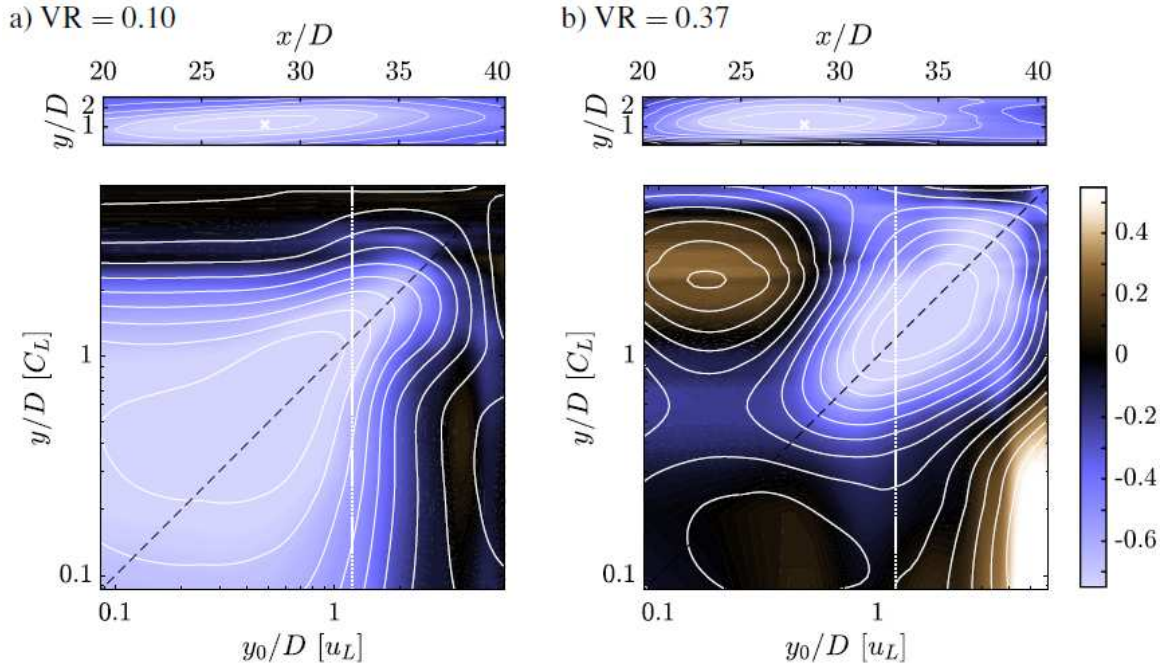


Figure 3.6: Correlation coefficient maps between large-scale fluctuations of velocity and large-scale fluctuations of scalar concentration for: a) $VR = 0.1$, b) $VR = 0.37$. (Top) single point correlation at reference location $x_0 = 28D$, $y_0 = 1.2D$, and (bottom) two-point correlation at $x_0 = 28D$. The figure is reproduced from Basley *et al.* (2022).

intensity of approximately 12%. The authors measured the turbulent momentum diffusivity in the wall-normal ($\varepsilon_{m,y}$) and spanwise ($\varepsilon_{m,z}$) directions. Larger transport in the spanwise direction than the wall-normal direction is observed. This leads to a large anisotropy in turbulent momentum diffusivity, with $1.5 < \varepsilon_{m,z}/\varepsilon_{m,y} < 2.5$. The distributions observed are too complex to be adequately represented with simple anisotropic models. Negative values of the turbulent momentum diffusivity were measured around the periphery of the jets.

Novel thermographic PIV studies in a film cooling flowfield were performed by Schreivogel *et al.* (2016). A single row of holes was used with diameters $D = 6$ mm, a pitch of $4.55D$ and an inclination angle of 30° . Boundary layer conditions were $Re_2 = 805$, giving $\delta/D = 1.83$. Elevated mainstream temperatures and cooler air fed through a liquid nitrogen cooling circuit gave $DR \approx 1.6$. The experiments focused on a high momentum flux ratio of $J = 8$, corresponding to $VR = 2.24$. The authors find that the streamwise heat flux $\overline{u\theta}$ is negative in both the upper and lower jet shear layer, which contradicts the gradient diffusion hypothesis in the upper shear layer.

3.8 A comment on the blowing scaling parameters

It is well known that wall shear stress and convective heat transfer at a surface are governed by the flow very close to the surface, ie. scale with the friction velocity u_τ which is frame invariant. However parameters such as M (equation 3.7) use the freestream and injection velocities as velocity scales, which appear not to be the correct velocity scale. It is unlikely that the turbulence scales on the injection

velocity U_e because it is not frame invariant (Morrison, 2010).

3.9 Motivation for experimental work

Despite the vast research into gas turbine blade cooling systems, only very few studies focus on the detailed characterisation of the velocity and temperature fields. The transport of heat and momentum inevitably determine the rate of heat transfer to the surface. Measurements of these fields will improve what is a currently poor understanding of the heat transfer mechanisms (Han, 2013) and highlight the current shortcomings of RANS models. The experiments presented in this thesis aim to build on the work of Basley *et al.* (2022). Specific attention will be directed towards detailed, three-component measurements of the velocity field extending down to the viscous sublayer and simultaneous momentum and scalar measurements, which are the most scarce in the literature.

Chapter 4

Experimental setup and methodology

4.1 The wind tunnel

Experiments were conducted in the lower section of the 10x5 wind tunnel. The test section has dimensions of 3 m x 1.5 m in width and height respectively, with a fetch of length 20 m. This enables the growth of very large boundary layers ($\delta_{99} > 0.2$ m), allowing for “big and slow” experiments of high spatial and temporal resolution. Assuming $\overline{v^2} \approx \overline{w^2}$, the unfiltered freestream turbulence intensity, $\sqrt{2k/3}/U_\infty$, was measured with an x-wire to be approximately 0.2%. The tunnel is equipped with a PID controlled chiller in the upper test section, which enables the freestream temperature to be maintained to within $\pm 0.15^\circ\text{C}$, independent of ambient temperature and Reynolds number.

4.2 Design of experiment

The experiment uses the same rig as Basley *et al.* (2022). It idealises the geometry of a turbine stage 1 stator blade with effusion cooling, through use of a flat porous plate. The plate consists of holes with diameters of $D = 16$ mm, in a staggered array with a pitch of $5D$. The holes are inclined upwards at 30° from the horizontal. This represents the geometry shown in figure 4.1, which is typical of previous effusion cooling studies (Murray *et al.*, 2018). The plate consists of 16 rows (in the streamwise direction) and 10 columns. Yavuzkurt *et al.* (1980a) observe spanwise uniformity with 9 columns of holes. The plate was tessellated together from small 3D printed tiles. Each tile was made using polylactic acid (PLA) and consists of 4 holes.

Compressed air was fed to a large plenum, before passing through the effusion holes. The plenum contained an array of mesh and porous plates, to ensure sufficient mixing inside the plenum, such that the air leaves the holes at a uniform velocity and temperature. The plates increased in hole-size and porosity closer to the effusion holes. The jet exit velocity (U_e) was measured with a Pitot tube parallel to the hole axis, under zero freestream velocity conditions. Variations in U_e of less than 10% were observed across the effusion panel. The largest differences in U_e were confined to the rows/ columns on

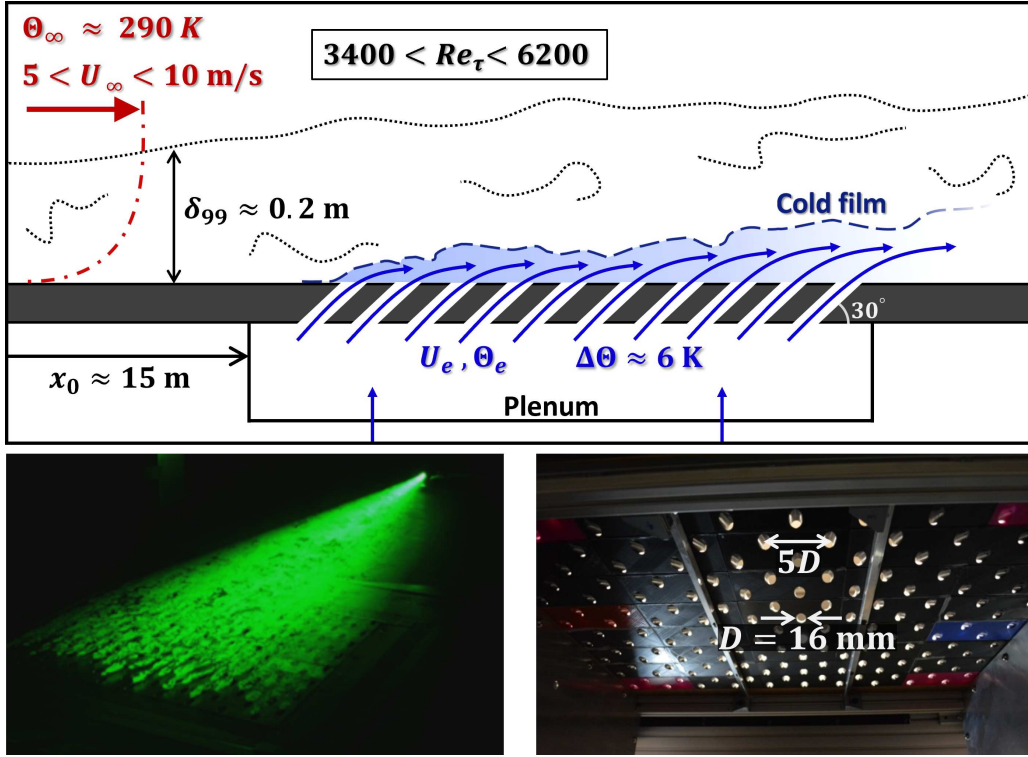


Figure 4.1: Diagram of the experimental setup (top). PIV flow visualisation of effusion through the plate (bottom left), and the inside of the plenum, with the effusion plate dimensions shown (bottom right).

the edges of the plate, where U_e was systematically lower. Away from the first two rows/ columns at the plate edges, uniformity to within 3% was observed. This variation is likely to be due to uncertainties in the Pitot tube angle and positioning, as the measured values for U_e were randomly distributed across the centre portion of the plate. Basley (2019) performed wall-normal single hot-wire measurements on different spanwise-symmetric planes and found any differences to lie within the uncertainty associated with the calibration procedure and the accuracy of determining the velocity ratio.

Cooler air is injected from the plenum, through the plate, into a large ‘canonical’ boundary layer with development length of approximately 15 m. The cooler injected fluid acts as a perturbation to the heat and momentum fields of the boundary layer, with known initial turbulence properties. Therefore, the heat and momentum exchange between the film and incoming flow can be isolated and interrogated. Turbulence intensities approaching 30% in the boundary layer near-wall region (Castro *et al.*, 2013), are similar to those measured at the combustor-turbine interface in gas turbines (10% – 35%) (Cha *et al.*, 2012). The turbulence length scales from the highly turbulent combustor-turbine interface ($l/D \approx 25$) are of similar magnitude to the large-scale motions (LSMs) in the incoming turbulent boundary layer. These generally have streamwise lengths of $2 - 3\delta_{99}$ (Kim & Adrian, 1999), where $\delta_{99}/D \approx 13$.

The ratio $\delta_1/D \approx 2$ is large compared to that observed in film cooling studies. Fraas *et al.* (2017) estimate $\delta_1/D \approx 0.1$ on the suction surface of turbine blades and $0.02 \lesssim \delta_1/D \lesssim 0.4$ on the pressure surface. However film cooling holes are generally of size $D = O(1\text{ mm})$, whereas effusion cooling holes

have smaller sizes of $D = O(0.1 \text{ mm})$. Therefore the experiment has a δ_1/D more similar to effusion cooling/ quasi transpiration cooling systems and explores a parameter space not currently found in the literature.

The temperature difference is sufficiently low ($\Delta\Theta < 10^\circ\text{C}$), such that the fluid can be treated as one of constant property and the momentum and energy equations can be decoupled. Furthermore, the Bulk Richardson number is very small ($Ri_\delta \approx 10^{-3}$), so buoyancy effects are negligible and temperature can be treated as a passive scalar. A schematic of the experiment is illustrated in figure 4.1.

The velocity ratio range for this study is $0.10 < VR < 0.37$ and measurements are also performed on the effusion panel under no-blowing conditions at $VR = 0$. In the context of turbine film cooling, these are low velocity ratios. However, the advantage of effusion cooling over film cooling is the requirement of less blowing to achieve the same film effectiveness (Krewinkel, 2013). Furthermore, Crawford *et al.* (1976) observe no further reduction in Stanton number for $VR > 0.4$. The average injection velocity, U_e , was calculated by measuring the volumetric flow rate, $\dot{m}/\rho$, of cooler air into the plenum, such that

$$U_e = \frac{4\dot{m}}{\rho n \pi D^2}, \quad (4.1)$$

where n represents the number of effusion holes. The volumetric flow rate was measured with IFM SD8500 compressed air meters.

For simultaneous momentum/ scalar field measurements, the freestream velocity varies from 5 m/s–10 m/s ($Re_\tau \approx 2300\text{--}4500$) to keep the experimental conditions consistent with previous studies (Basley *et al.*, 2022). These results are primarily discussed in Chapter 6. For further detailed momentum field characterisation (Chapter 5), the freestream velocity was kept constant at 5 m/s to fix the Reynolds number.

4.3 Effusion cooling system

Before entering the plenum, the air passes through a Ranque-Hilsch vortex tube, which cooled the air. A schematic of its operation is shown in figure 4.2. The mechanism by which the vortex tube produces a cold air stream is still uncertain and there are many different explanations in the literature, which are reviewed by Kaufmann (2022). However, a feasible mechanism is as follows: the pressure drop from the compressed air source causes large flow velocities to develop at the inlet of the swirl chamber. Total enthalpy is conserved, therefore assuming adiabatic expansion, the sensible enthalpy¹ (and therefore temperature) is reduced. After being passed through the radial swirler, the axial and tangential velocity gradients near the wall are very large. Through viscous transport, kinetic energy from the flow further from the wall is converted to sensible enthalpy, and the temperature of the fluid

¹A change in sensible enthalpy is associated with a change in heat/ temperature only. Enthalpy changes due to a phase change can also occur. In this case, the internal energy changes.

in the near wall region increases. Conversely, the total enthalpy is reduced away from the wall (at the core) through conservation of enthalpy and the temperature is therefore lower. The centre of the outlet at the hot end is blocked off, so the hotter and colder air streams are separated. For a thorough analysis of the mechanisms and relevant energy transport equations, see Kaufmann (2022).

The only reliable dimensionless parameters affecting the temperature difference from the vortex tube are the pressure ratio and the cold mass fraction (Kaufmann, 2022). The cold mass fraction can be varied through turning a valve at the hot end. With use of a pressure regulator connected to the compressed air outlet, the pressure ratio can also be adjusted. The outlet velocity and outlet temperature are different functions of pressure ratio and cold mass fraction, which enables semi-independent control of these quantities.

The maximum temperature difference between the inlet compressed air and the cool air inside the nozzle outlet is claimed to be $\Delta\Theta > 50^\circ\text{C}$ by the manufacturer (Meech). Measurements on the centre line, immediately downstream of the jet exit revealed a temperature difference of $\Delta\Theta \approx 45^\circ\text{C}$, at a pressure ratio of 6. However, large spanwise and streamwise temperature gradients were observed away from the centre line of the jet exit nozzle. A maximum of $\Delta\Theta_{max} \approx 15^\circ\text{C}$, at the exit of the effusion holes could be achieved, even with the insulation of the plenum and all compressed air pipes.

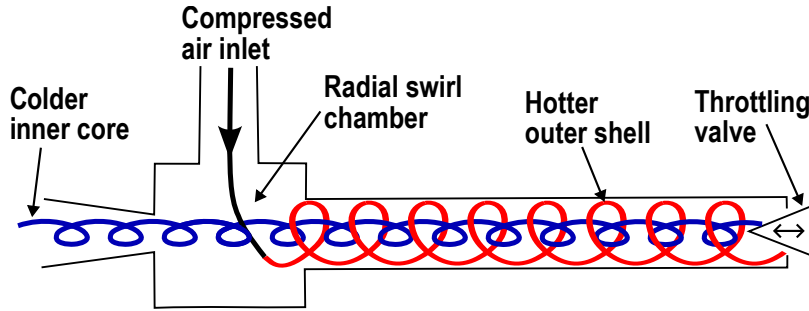


Figure 4.2: The Ranque-Hilsch Vortex tube schematic.

4.4 Measurement locations

The wall-normal boundary layer measurements were acquired with laser Doppler anemometry (LDA) and simultaneous x-wire and cold-wire. A review of these techniques and their experimental implementation are given in the subsequent sections 4.5 and 4.7 respectively. The boundary layer measurements were acquired at a streamwise distance of $x = 600\text{ mm} = 37.5D$ from the plate leading edge. Data at two spanwise locations were acquired: on the centreline of the holes ($z = 20\text{ mm} = 1.25D$), and between two columns of holes ($z = 0\text{ mm}$). Oil film interferometry measurements (reviewed in section 4.6) were taken behind the effusion panel, approximately $8D$ downstream of the wall-normal profiles. This was the nearest downstream location that allowed for optical access underneath the tunnel, for imaging of the oil film. See figure 4.3. To characterise the upstream boundary layer, wall-normal LDA

measurements were carried out at $x = -5D$.

The effusion temperature, Θ_e , was measured with Pt100 temperature sensors mounted within the effusion holes. This was used to determine the temperature difference and normalise temperature profiles. Their locations are shown in figure 4.3. Sensors were placed in the holes around the measurement location due to the larger influence on the measurements. The remaining sensors were scattered randomly around the plenum to obtain an approximate measure of the uniformity of the plenum temperature. This was generally found to be constant to within $\pm 0.2^\circ\text{C}$.

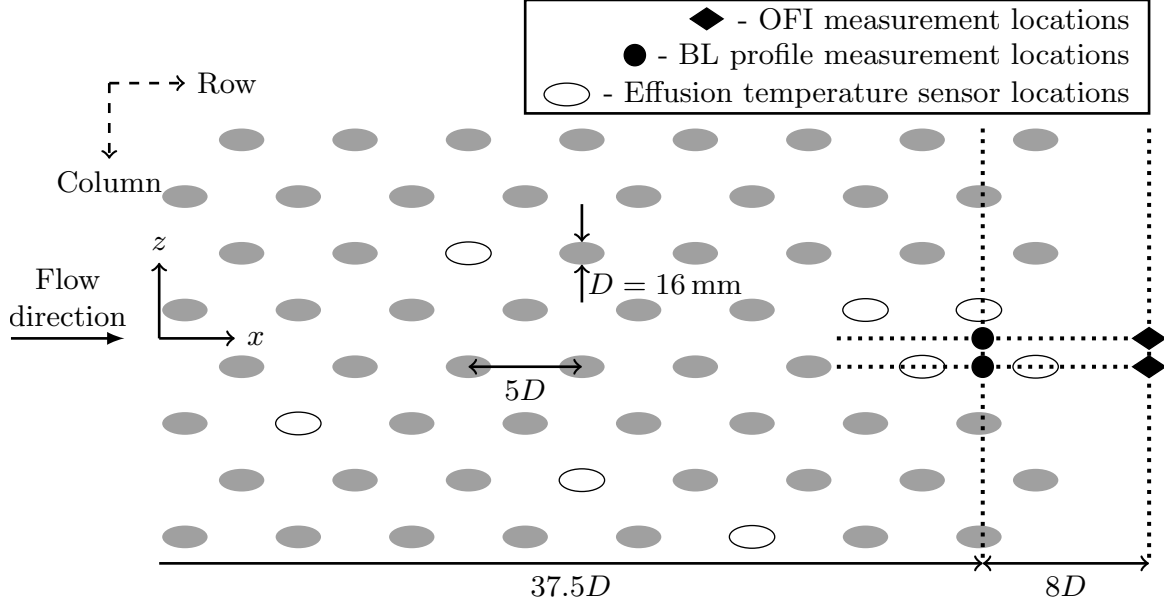


Figure 4.3: Planview diagram of the effusion panel indicating: the coordinate system, measurement locations for boundary layer traverses (black circles) and oil film interferometry measurements (black diamonds), and the locations of the Pt100 temperature sensors (white ellipses).

4.5 Laser Doppler anemometry

Laser Doppler anemometry (LDA) was used to measure wall-normal velocity profiles for a range of velocity ratios. LDA has the big advantages over hot-wire anemometry of having the capabilities to measure three velocity components in a smaller, overlapping measurement volume, and without the arduous calibration processes involved with using x-wires and tri-axial wires. Furthermore, it is non-intrusive, insensitive to temperature and there are no measurement restrictions with respect to flow angle constraints and reverse flow measurement. For example, an x-wire cannot detect reverse flow and can only resolve flow angles, γ , in the range $-45^\circ < \gamma < 45^\circ$. The advantage of LDA over particle image velocimetry (PIV) for this experiment is the ability to measure the whole boundary layer, with sufficient resolution in both the viscous sublayer and outer region. This enables the accurate determination of integral quantities such as the shape factor for example.

4.5.1 Theory

The working principle of LDA can be explained through two models: the more rigorous Doppler effect model or the more intuitive fringe model. For the sake of brevity, only the fringe model will be discussed here; the equations derived for the particle velocity are the same in both cases. Albrecht (2003) provides full derivations of both working principles. When two laser beams of wavelength λ , cross at a point with intersection angle φ , an interference fringe pattern of spatially varying light intensity is formed in the crossing volume, with fringe spacing determined through

$$\Lambda = \frac{\lambda}{2 \sin (\varphi/2)}, \quad (4.2)$$

as illustrated in figure 4.4a. When a seeding particle passes through the intersection point, known as the probe volume, the intensity of scattered light that is received by the detector will vary sinusoidally in a burst pattern similar to figure 4.4b. The Gaussian nature of the signal is symptomatic of the Gaussian cross-sectional light intensity distribution within the laser beam. The burst signal is high-pass filtered to remove the DC component. Then the time period $1/f$ is determined from a fast Fourier transform. Through use of equation 4.2, the particle velocity, normal to the fringes, is then calculated as

$$c^* = f \frac{\lambda}{2 \sin (\varphi/2)}. \quad (4.3)$$

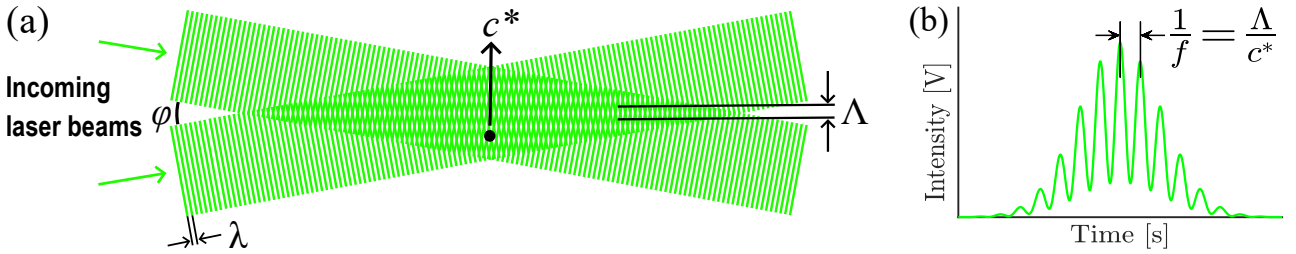


Figure 4.4: (a) LDA interference fringe pattern and (b) idealised typical burst signal.

4.5.2 Experimental implementation

A Dantec three-component LDA system was used, comprising two 85 mm probes, having a beam expander ratio of 2.98 and lenses with a focal length of 2000 mm. One probe has four beams and the other has two beams, as seen in figure 4.5. The flow was seeded with polyethylene glycol (PEG), producing particles with a mean diameter of $d_p \approx 1 \mu\text{m}$. The system was operated in coincidence mode, which means a seeding particle has to pass through all three measurement volumes to be registered by the software. This enables normal and shear stresses for the three components to be calculated. In the configuration shown in figure 4.5, the measurement volume is approximately spherical, with length $l = 0.2 \text{ mm}$, corresponding to a viscous length $l_{\text{max}}^+ \approx 2.5$. Due to the small probe volume, corrections to the turbulence moments such as those outlined by Durst *et al.* (1995) had little effect.

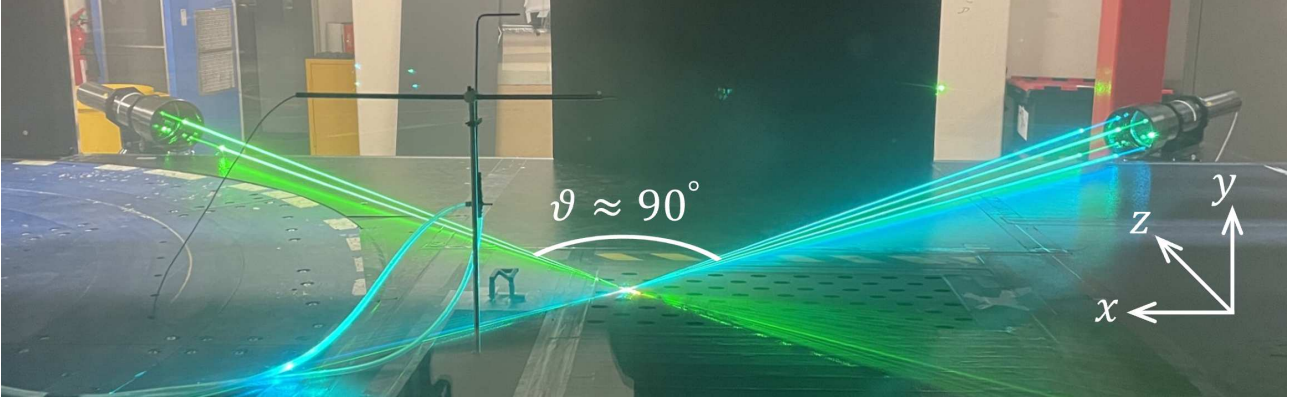


Figure 4.5: Three-component Dantec LDA setup used in the 10x5 wind tunnel.

In each set of beams, one passes through a Bragg cell, which shifts its frequency by $f_b = 40$ MHz. Interpreting this through the fringe model: it means one of the beams has a (very small) difference in wavelength, so the fringes advect forwards or backwards in time, which enables directional sensitivity of the velocity of the seeding particle. The maximum measurable negative velocity, beyond which directional ambiguity occurs is given by

$$c_{\min}^* = \frac{-\lambda f_b}{2 \sin(\vartheta/2)} \approx -360 \text{ m/s}$$

. Since particles pass randomly through the measurement volume, the data rate is not fixed. The acquired data rate is also a function of many variables such as the wall-normal distance, atmospheric conditions, and the volume of PEG inside the atomiser, which affects the seeding density. Obtaining high data rates with LDA in air flows is difficult due to the very small measurement volume (Tropea *et al.*, 2007). Typical data rates varied from: 50 Hz in the viscous sublayer, 1 kHz in the buffer/ overlap layer, and 4 kHz in the outer layer.

Since each pair of LDA beams measures a velocity component normal to the fringes at the measurement volume, the velocity components have to be transformed into the orthogonal velocity components u^*, v^*, w^* . This is done through a transformation matrix, T , which for a symmetrical three-component setup where the probe axes are parallel to the wall, takes the form

$$q^* = Tc^*, \quad (4.4a)$$

$$\begin{pmatrix} u^* \\ v^* \\ w^* \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{-1}{2 \cos(\vartheta/2)} & \frac{1}{2 \cos(\vartheta/2)} \\ 0 & \frac{1}{2 \sin(\vartheta/2)} & \frac{-1}{2 \sin(\vartheta/2)} \end{bmatrix} \begin{pmatrix} c_1^* \\ c_2^* \\ c_3^* \end{pmatrix}. \quad (4.4b)$$

For small angles between the probes, ϑ , the terms in row 3 of T in equation 4.4b become large, which causes large propagation of errors in w^* . Assuming errors to be stochastic, the spanwise mean velocity, W , will be unaffected but will take longer to converge. However, large errors in the variance of the

spanwise fluctuations will occur, given by

$$\overline{w^2}_{\text{meas}} = \frac{\overline{w^2}_{\text{true}}}{2 \sin^2(\vartheta/2)}. \quad (4.5)$$

Therefore the probes were symmetrically orientated to obtain $\vartheta \approx 90^\circ$, to minimise propagation error in $\overline{w^2}$ through the transformation matrix. This is shown in figure 4.5. Due to this large angle, the distance between the LDA probe bases was greater than 3 m. A custom mounting was therefore designed, which consisted of a 4 m long aluminium truss, with section dimensions of 30 cm x 30 cm. The truss structure makes the mounting platform lightweight (approximately 12 kg), but also very stiff due to the large cross-section and therefore large second moment of area.

In practice, the probe axes are inclined slightly to the wall to enable near-wall measurements and the geometry angles are difficult to measure accurately. Therefore to determine the coefficients in T , all the beams are aligned onto a fixed point on a camera. The lasers are traversed backwards by a known distance Δz . For each beam, the values of x and y required to get each beam onto the same point on the camera are recorded. Since the traverse is orientated with the tunnel orthogonal coordinate system, the vector of each laser beam with respect to the tunnel coordinate system is obtained and used to derive the transformation matrix.

In a given acquisition time, a larger amount of faster moving particles will pass through the probe volume than lower speed particles, resulting in a bias in the velocity moments. This means that LDA measurements require weighting to correct for this. For example, the n^{th} streamwise velocity moment is corrected using

$$\overline{u^n} = \left(\frac{\sum_{i=1}^N \overline{u_i^n} g_i}{\sum_{i=1}^N g_i} \right)^{1/n}, \quad (4.6)$$

where g_i is the weighting factor, usually given by the transit time (Tropea *et al.*, 2007). The transit time is the total time that the seeding particle resides in the probe volume and is inversely proportional to the particle velocity. It is a valid weighting factor when the seeding is spatially homogeneous (Tropea *et al.*, 2007). For weighting in three-component systems, the instantaneous particle velocity can be used and the weighting factor is given as

$$g_i = \frac{1}{\sqrt{u_i^{*2} + v_i^{*2} + w_i^{*2}}}.$$

Both methods gave indistinguishable results for the mean velocity profiles but the transit time weighting gave a slightly better fit for higher order moments of canonical boundary layer measurements, when compared to LES (Eitel-Amor *et al.*, 2014). Therefore transit time weighting was used to correct for velocity bias in measurements for this experiment.

4.6 Oil Film interferometry

Due to the significant perturbation to the boundary layer caused by effusion (Basley *et al.*, 2022), empirical skin friction correlations and logarithmic friction laws (Nikuradse, 1933; Schultz-Grunow, 1941; Clauser, 1956; Schlichting *et al.*, 1980; White, 2006), cannot be used with any reasonable degree of confidence. Furthermore, modifications to friction laws that incorporate transpiration are only reliable for $VR < 0.01$ (Kays *et al.*, 2005). Therefore, wall shear stress was measured in the experiment using oil film interferometry (OFI). At the time of the publishing this thesis, it is generally accepted to be the most accurate and reliable method for independent wall shear stress measurements in a wall bounded flow (Discetti & Ianiro, 2017).

The technique involves placing a drop/line of oil on the test section surface. The oil is thinned temporally and spatially due to wall shear stress. The oil is assumed to be sufficiently thin, such that it resides in the viscous sublayer, does not interfere with the flow field, and it follows the surface streamlines. The rate of oil thinning, $dh(x,t)/dt$, can be determined with the use of CCD/CMOS cameras and Fizeau interferometry (Tropea *et al.*, 2007). The wall shear stress is then calculated through use of the oil film equation, which takes many forms.

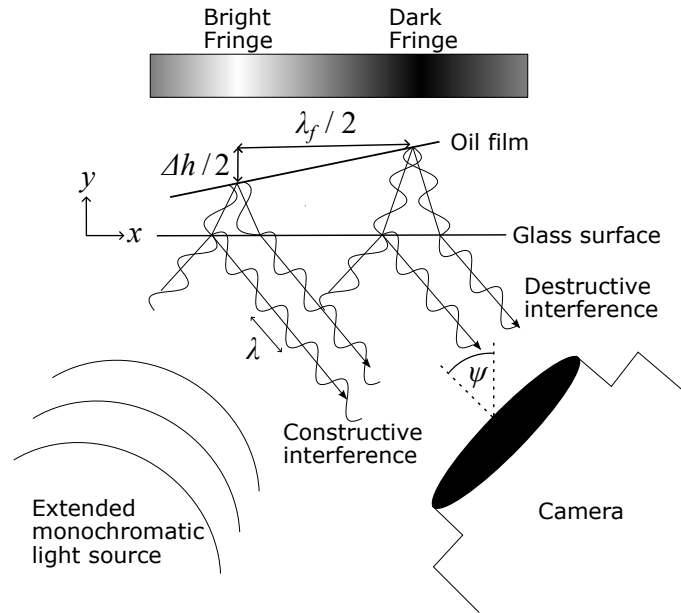


Figure 4.6: Diagram illustrating how the OFI interference pattern is formed in the experimental setup.

4.6.1 The oil film equation

Regardless of which implementation of the oil film equation is used, certain aspects of the fringe imaging skin friction technique remain the same. Figure 4.6 illustrates this process. An extended monochromatic light source of wavelength λ , is used to illuminate the thin oil film. Since $h = O(\lambda)$, light reflecting from the surface in contact with the oil and off the oil film top surface interfere to produce an alternating sequence of bright (constructive) and dark (destructive) bands of interference

‘fringes’. This happens when the additional distance travelled by the light rays through the film (the optical path difference) equals λ or $\lambda/2$, for bright or dark fringes respectively. The spacing between successive fringes corresponds to an extra optical path difference of λ , and therefore the fringes represent contours of constant oil height difference. Using interference optics (Fernholz *et al.*, 1996; Naughton & Sheplak, 2002), the oil height difference Δh , between consecutive fringes is expressed as

$$\Delta h = \frac{\lambda}{2\sqrt{n_{oil}^2 - n_a^2 \sin^2(\psi)}}, \quad (4.7)$$

where ψ is the viewing angle and n_{oil} and n_a are refractive indices of the oil and air respectively. In the literature, n_a is sometimes dropped from this equation, since $n_a \approx 1.022 \approx 1$.

To derive a form of the oil film equation that can be practically implemented, it is assumed that:

- the flow is two-dimensional, such that oil advection is in the streamwise direction only;
- viscous effects dominate over inertial effects ($Re_{oil} \rightarrow 0$);
- pressure gradients arising from surface tension effects due to oil surface curvature are negligible;
- the oil height is sufficiently small, $\mathcal{O}(\mu\text{m})$, such that the aerodynamic pressure gradient and body forces can be neglected;
- the wall shear stress is constant over the small interrogation region of the oil film.

Then the oil film equation becomes

$$\frac{\partial h}{\partial t} + \frac{\tau_w h}{\mu_{oil}} \frac{\partial h}{\partial x} = 0. \quad (4.8)$$

Naughton & Sheplak (2002) present a full detailed derivation of the oil film equation, which includes the terms that have been neglected here, and the order of magnitude analysis that justifies their exclusion. Their review also details many implementations of the oil film equation for different scenarios, including those which require analysis of a complex three-dimensional flow field. The solution to equation 4.8 is found through the method of characteristics as

$$x = \frac{\tau_w h t}{\mu_{oil}}. \quad (4.9)$$

The distance between fringes $x = \lambda_f$ (the fringe wavelength), corresponds to a constant height difference $h = \Delta h$. These are substituted into equation 4.9, which is differentiated with respect to time and rearranged to produce

$$\tau_w = \frac{\mu_{oil}}{\Delta h} \frac{d\lambda_f}{dt}. \quad (4.10)$$

Vinuesa *et al.* (2014) state that equation 4.10 is the most reliable and accurate way of determining wall shear stress, when λ_f is determined over several fringes, rather than through the spacing between

consecutive fringes. This method is therefore termed by the authors “the global wavelength method”. λ_f can be determined by using the Fourier transform or through least squares regression by fitting $I = I_{max} \sin([x + x_0]/\lambda_f)$ to the light intensity signals of the fringes.

4.6.2 OFI experimental implementation

Oil droplets were placed on flush-mounted low-iron glass panels immediately upstream and downstream of the effusion plate. The oil used was Sigma Aldrich silicone oil with a nominal kinematic viscosity² of 10 cSt. This gives $Re_{oil} = \mathcal{O}(10^{-10})$ for the flow conditions of this experiment. To minimise three-dimensional effects for shear stress measurements downstream of the effusion panel, small droplets relative to the hole sizes of diameters $d_{oil}/D \approx 0.25$ were used. The oil was illuminated by a low-pressure sodium lamp with wavelength $\lambda = 589.3$ nm. The light was placed in a tube light shade, behind a diffuser sheet, in an attempt to maximise light intensity homogeneously.

Images were captured with a Phantom Miro M310 camera, mounted below the wind tunnel and viewing through the glass at an angle of $\psi = 15^\circ$. This configuration was chosen to enable the camera to be mounted much closer to the oil droplets in a non-intrusive manner to maximise resolution. The glass thickness was 6 mm, and the coherence length of light waves from the sodium lamp is approximately 0.6 mm. Therefore, there will not be interference effects from light reflecting off the glass bottom surface. Whether the camera views the oil droplet from above or below the glass plate affects the fringe visibility only, which is a measure of the bright-dark fringe contrast. The differences were calculated to be small however.

The spatial domain was calibrated by placing a ruler in the field of view at the spanwise location where the oil droplets were placed. A calibration image was interpolated with the pixel number for every millimetre increment on the ruler in the streamwise direction. The linear regression slope determined the number of pixels per millimetre. The coefficient of determination, $R^2 > 0.99999$ for each calibration fit, suggesting that the camera distortion effects were small in the oil drop regions. Calibration images were taken before and after each set of runs and the average result was used in post-processing. The difference in calibration results on average was less than $\pm 0.15\%$.

In OFI measurements, the largest uncertainty in the determination of τ_w is from the uncertainty in measuring the oil kinematic viscosity, ν_{oil} (Tropea *et al.*, 2007; Discetti & Ianiro, 2017), which changes by approximately $2\%/^\circ\text{C}$. Zilliac (1996) gives a commonly used viscosity-temperature correction formula but the author associates uncertainties of approximately $0.2\%/^\circ\text{C}$ to its use. Therefore the oil kinematic viscosity was calibrated in-house with respect to temperature, Θ . A Huber CC-K6 constant temperature recirculating water bath was used, with a temperature stability of $\pm 0.02^\circ\text{C}$. A cylindrical vessel was built to contain the oil viscometer and was connected to the recirculating bath, to fully submerge the viscometer in a closed loop circuit of recirculating constant temperature water. The

²1 cSt = $1 \times 10^{-6} \text{ m}^2/\text{s}$

calibration setup is shown in figure 4.7a. A manufacturer-calibrated Canon Ubbelohde viscometer was used, which has a measurement uncertainty in ν_{oil} of $\pm 0.2\%$. For each temperature value, three to five calibration runs were performed and the standard deviation in ν_o across all repeat runs was found to lie within the quoted manufacturer viscometer uncertainty limits. The calibration law

$$\nu_{oil} = A e^{\alpha_{oil} \Theta}, \quad (4.11)$$

was fitted to the data, and the result is shown in figure 4.7b. Extrapolating equation 4.11 to $\Theta = 25^\circ\text{C}$ gives $\nu_{oil} = 9.808 \text{ cSt}$, which agrees with the manufacturer batch certificate value ($\nu_{oil} = 9.8 \text{ cSt}$) to within $\pm 0.1\%$. The uncertainty in the manufacturer value is not quoted, but the two values agree to within the calculated uncertainty from the in-house lab setup.

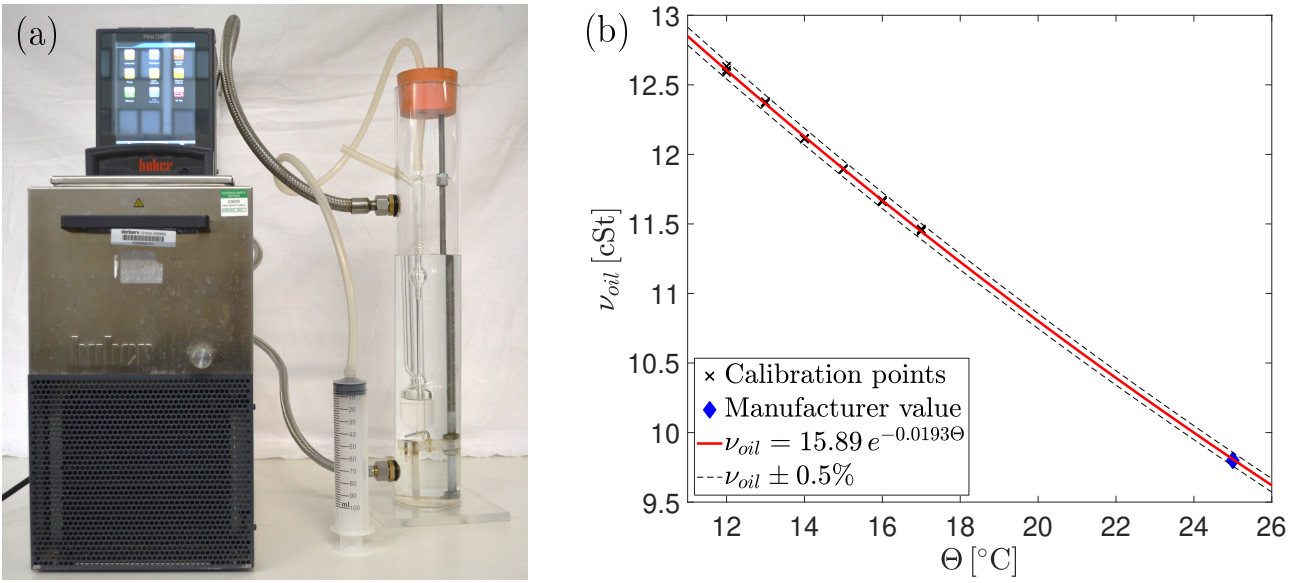


Figure 4.7: (a) Setup used to calibrate oil kinematic viscosity as a function of temperature. (b) Oil kinematic viscosity calibration fit as a function of temperature (red line), determined from the calibration points (black crosses), and compared with the manufacturer value (blue diamond).

The OFI measurements undertaken with blowing were isothermal, to reduce uncertainty in the oil/ wall temperature. Pt100 temperature sensors were placed in the exit holes of the effusion panel and one was placed in the freestream, local to the measurement location. The effusion temperature and freestream temperature were adjusted until they were within $\pm 0.05^\circ\text{C}$ of each other. The OFI measurements began about one hour after this, to ensure that the wall temperature within the wind tunnel had reached a steady state value. It was assumed that the oil/ wall temperature was the average of the freestream and effusion temperature. The Pt100 sensors were also calibrated within the water bath, to remove any systematic calibration differences.

For each flow condition, three to five OFI runs were performed to give a greater indication of the measurement uncertainty and enable outlier detection. In a given run, only one oil droplet could be placed due to the non two-dimensionality of the flow. In some runs, an accurate value of shear stress

could not be recovered due to excessive dust contamination, so a different number of points per flow condition are presented in the results.

4.6.3 Post-processing

The flow field with blowing is three-dimensional. However it was assumed to be sufficiently two-dimensional in the chosen measurement regions, where there should be a spanwise symmetry boundary condition. This was verified with three-component LDA measurements which showed a negligible mean spanwise velocity. Therefore the two-dimensional oil film equation was implemented. The global wavelength method, using a fast Fourier transform (FFT) to determine λ_f was used. Chauhan *et al.* (2010) present an analysis using the Hilbert transform to calculate fringe spacing, and conclude it is superior to the FFT in cases where the fringe intensity has large variations in amplitude and in noisy signals. However, in cases where it was quickly apparent that the fringes had been heavily contaminated (usually with dust), the run was abandoned due to the naturally higher uncertainty associated with determining the fringe spacing. In fringes with relatively low contamination, applying the FFT to a detrended signal with linear contrast adjustment seemed to give the most reliable results.

The interrogation regions were as close as possible to the centreline of the droplet and away from the leading edge of the droplet, where the oil surface curvature is low. Here, surface tension effects are negligible and equation 4.8 is valid. The result was the average of the shear stress values obtained from each line of pixels in the interrogation window. Temperature corrections for oil refractive index (Khodier, 2002) and density (Thibault & Poitras, 2017) were also implemented. The combined relative uncertainty in τ_w associated with these corrections is of the order of 1% of the kinematic viscosity uncertainty.

4.7 Simultaneous x-wire and cold-wire measurements

Measurement of turbulent heat fluxes and the turbulent Prandtl number require simultaneous estimates of u , v , θ using x-wire and cold-wire anemometry. The x-wire was orientated to measure the streamwise and wall-normal velocity components. With use of an x-wire, the minimum wall-normal distance ($y_{min}^+ \approx 18$) and the measurement volume for simultaneous measurements ($l_{max}^+ \approx 24$) are comparatively large. However, there is no issue of seeding deposition on the cold-wire, as with simultaneous LDA and cold-wire. Previous studies using this technique (Lemay *et al.*, 2003; Darisse *et al.*, 2015) have placed the cold-wire in ethanol to clean the wire after short acquisition periods. However, these experiments were performed in heated jets, where access to the wire is easy. For the current experiment, the wall proximity makes this very difficult. The ability to mount the x-wire and cold-wire on the same traverse made alignment easier and alignment uncertainties lower. Furthermore, the x-wire provides velocity spectra with a higher resolution and a larger bandwidth than LDA.

A Dantec 55P51 x-wire probe was used, although the platinum Wollaston wire was soldered in-house. A Dantec 55P31 platinum cold-wire of diameter $d_{CW} = 1 \mu\text{m}$ and length $l_{CW}/d_{CW} = 400$ was used, giving $l_{\text{max}}^+ \approx 9$. Therefore the effects of spatial attenuation are small (Xia *et al.*, 2022). However the temporal response is attenuated more significantly, with the corner frequency estimated to be $f_c \approx 3 - 4 \text{ kHz}$. The cold-wire response was compensated using the method outlined by Lemay *et al.* (2003), although other compensation strategies are given in the literature (Wroblewski & Eibeck, 1991; Arwatz *et al.*, 2013). Figure 4.8 shows the co-located wires used in the experiment and dimensions of the measurement volume.

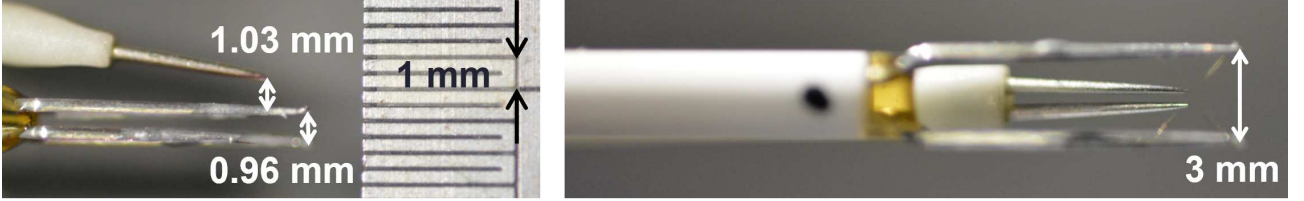


Figure 4.8: Close-up view of the simultaneous x-wire and cold-wire measurement probes.

The x-wire and cold-wire were connected to a Dantec Streamline Pro system, using 91C10 constant temperature anemometers and a 90C20 constant current anemometer respectively. For a comprehensive review of constant temperature and constant current anemometry, see Bruun (2002); Tropea *et al.* (2007). The x-wire was operated at an overheat ratio of 1.8. The cold-wire used a constant current of 0.2 mA, making the velocity sensitivity negligible. Data were acquired at 20 kHz through LabVIEW, using an NI USB-6229 with 16-bit resolution. Analogue measurements were low-pass filtered at a corner frequency of 10 kHz, before acquisition. The x-wires used the 91C10 low-pass filter and the cold-wire used an SRS SR560 amplifier and low-pass filter, connected at the output of the constant current anemometer. The time series of the x-wire voltages, E , were corrected in post-processing using the time series of the cold-wire temperature, θ_{CW}^* . The procedure outlined by Bruun (2002) was used, given as

$$E_{corr} = E \left(\frac{\Theta_w - \Theta_{cal}}{\Theta_w - \theta_{CW}^*} \right)^{0.5}, \quad (4.12)$$

where Θ_w is the wire temperatures of the x-wire and Θ_{cal} is the calibration temperature.

4.7.1 X-wire calibration system

There are two widely used x-wire calibration techniques: the effective angle method and the look-up table (LUT) method. Since the injection causes turbulence intensities to exceed 20 % over a large wall-normal range (Basley *et al.*, 2022), large instantaneous pitch angles, γ , were expected and the look-up table method was predominantly used. Lueptow *et al.* (2004); Tropea *et al.* (2007) comprehensively review the methodology behind the LUT calibration procedure. Burattini & Antonia (2005) compare both methods in decaying grid turbulence and the far field of a jet and deduce the look-up table is the

most reliable method.

The x-wire probe holder was mounted within a brass collar, which was connected to a long motorised pitching arm, with length of approximately 0.5 m. The arm protruded through a small slot in the tunnel floor, which enabled the motor and gearbox to remain underneath the tunnel and not disturb the flow-field. In the calibrations, the wires were traversed to the freestream and the pitch angle was varied for $-40^\circ \leq \gamma \leq 40^\circ$ in 5° increments. This was done for 10 absolute velocity values, with logarithmic spacing ranging from $0.05 U_\infty \leq Q \leq 1.05 U_\infty$. Note that when the wire is pitched, $Q = \sqrt{U^2 + V^2}$ is the velocity magnitude vector, which differs from the streamwise component along the axis of the probe. The voltage values E_1, E_2 were recorded and averaged for each combination of Q and γ . The resulting voltage calibration grid for each wire was interpolated with cubic splines. The interpolation increments corresponded to the digital voltage resolution from the data acquisition device. Therefore two look-up tables were generated for Q and γ . For any voltage pair E_1, E_2 , the look-up table produces a unique Q and γ , where

$$U = Q \cos \gamma, \quad (4.13a)$$

$$V = Q \sin \gamma. \quad (4.13b)$$

In the case that the voltage pair fell outside of the calibration domain, the effective angle method was used instead. A detailed outline of the effective angle method and its implementation is given by Wood (1980). Figure 4.9 shows an example of the lookup table implementation, to an acquired dataset at $VR = 0.10$. Near the wall, up to 4 % of points lied outside the calibration domain, possibly due to large spanwise fluctuations and/ or a wall-probe blockage effect. At $y^+ \approx 80$ and $y^+ \approx 200$, the number of points outside of the calibration domain decreased to less than 1 % and 0.1 % respectively.

4.7.2 Cold-wire calibration system

Due to the very large volume and therefore heat capacity of the 10x5 wind tunnel, calibrating cold-wires using the temperature controlled chiller unit takes a long time and is very inefficient. Furthermore, condensation at lower temperatures also causes issues. Other solutions require dedicated cooling systems such as the use of liquid nitrogen (Kohli & Bogard, 2005), which are expensive and not very feasible. Therefore a portable wind tunnel with variable inlet temperature was designed to enable accurate and repeatable cold-wire calibrations, which also enable in-situ calibrations.

Figure 4.10 shows an image of the calibration wind tunnel and a diagram of the main features respectively. The cool air is fed into the settling chamber from a Ranque-Hilsch vortex tube, through a manifold and multiple tubes, which improves the air uniformity within the settling chamber. The operation of the vortex tube is described in section 4.3. After expansion into the settling chamber, the air passes through a contraction into an acrylic tube with inner diameter, $D_1 = 152$ mm. Circular

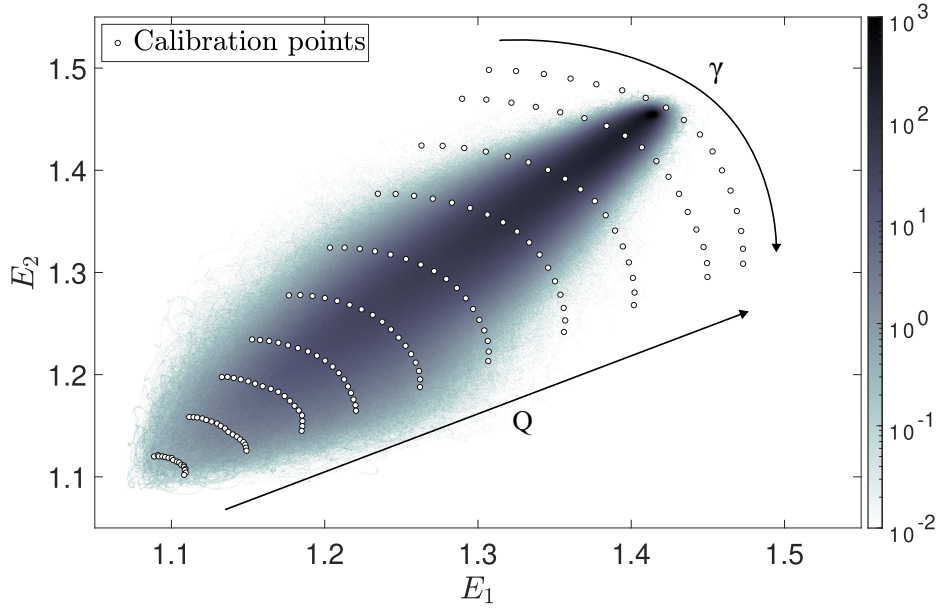


Figure 4.9: Colour map indicating the probability density of voltage pairs E_1, E_2 , acquired during a traverse for $VR = 0.10$, on the centreline of the holes ($z = 20$ mm). The calibration points are overlaid on the plot (white circles).

tubing was chosen due to its rigidity, relatively low cost, and to eliminate corner effects which can be particularly problematic within the contraction (Mehta, 1979). In the circular tubing the flow is straightened by two honeycomb sections of reducing porosity downstream, before passing through a screen and a contraction, with a contraction ratio of 4.46. The cold-wire is placed below a pre-calibrated Pt100 temperature sensor with a sensing length of 1 mm, placed within a circular section with inner diameter, $D_2 = 72$ mm. For cold-wire calibration, the portable wind tunnel was brought into the 10x5 test section and placed around the wires. The cold-wire was traversed up to below the Pt100 calibration sensor. This method ensured minimal disturbance to the x-wire and cold-wire during calibration.

Velocity profiles showed uniformity in the potential region to within $\pm 0.4\%$ of the mean velocity in this region, in both the vertical and horizontal directions, indicating that the flow is axisymmetric to within these bounds. This region extended to $y/R_2 = \pm 0.13$, which is much larger than the measurement region of the sensors used in calibration. At the calibration location, streamwise and vertical (radial assuming axisymmetry) turbulence intensities, $\sqrt{u^2}/U_c$ and $\sqrt{v^2}/U_c$, were measured with an x-wire at an acquisition frequency of 20 kHz for a duration of 180 s. They were found to be 0.6% and 0.5% respectively.

Mean temperature profiles were acquired simultaneously for 120 s at 10 Hz, with a rake of 8 calibrated Pt100 sensors, which ensured tunnel temperature drift would not affect the results as in a traverse scan. The two central measurements had a uniformity of $\pm 0.025^\circ\text{C}$, acquired at approximately $y/R_2 = \pm 0.13$. This is smaller than the calibration measurement area, so it is expected that the repeatability falls within these limits. The standard deviation of temperature fluctuations, $\sqrt{\theta^2}$, at the

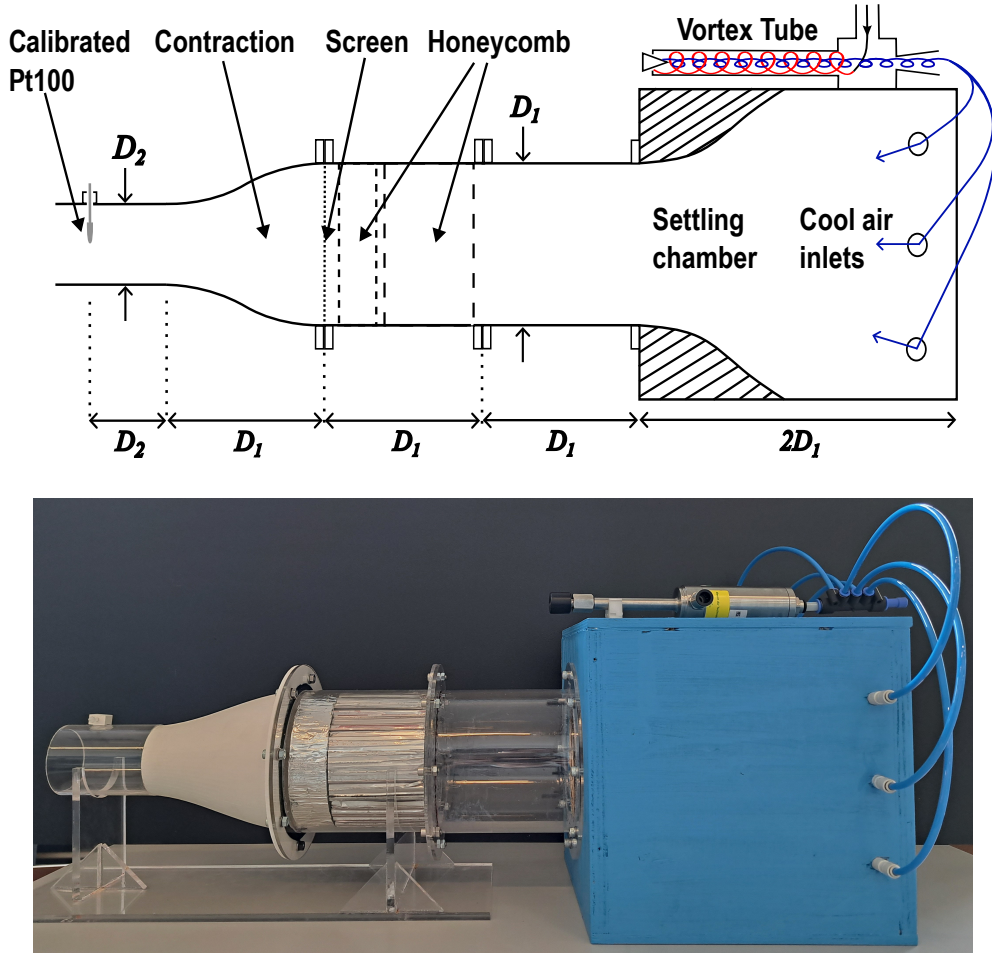


Figure 4.10: The portable calibration wind tunnel diagram (top), and an image of the device (bottom).

tunnel centreline, over short calibration runs (30 s) were found to be of the order of 0.001°C , measured with a $1\mu\text{m}$ diameter cold-wire, and acquisition rate of 20 kHz. This was observed to be invariant with respect to temperature difference, over a range of $\Delta\Theta \approx 10^\circ\text{C}$. Over a three-minute duration, $\sqrt{\theta^2} = 0.004^\circ\text{C}$, primarily caused by low-frequency mean temperature variations. Therefore the tunnel is capable of producing a very steady variable temperature flow.

A widely used equation relating the resistance of a platinum sensor to its temperature is given by

$$R = R_0 + R_0 \alpha \left[\Theta - \alpha_2 \left(\frac{\Theta}{100} - 1 \right) \left(\frac{\Theta}{100} \right) - \alpha_3 \left(\frac{\Theta}{100} - 1 \right) \left(\frac{\Theta}{100} \right)^3 \right], \quad (4.14)$$

which was initially determined by Callendar (1886) and refined by Van Dusen (1925) to include the third term in the square brackets, used only for $\Theta < 0^\circ\text{C}$. The values $\alpha, \alpha_2, \alpha_3$, are constants at approximately 0.0039, 1.5, 0.109 respectively. In the temperature range $-100 < \Theta [^\circ\text{C}] < 100$, the three terms in the square brackets of equation 4.14 are of order $\Theta, \Theta/10^2, \Theta^3/10^7 \sim \Theta/10^3$ respectively. Therefore a linear calibration law is widely used for Platinum temperature sensors for low temperature differences, given by

$$R = R_0 (1 + \alpha \Theta), \quad (4.15)$$

which is accurate to within $\pm 0.01^\circ\text{C}$ for the temperature range stated above. The temperature coefficient of resistivity, α , falls between $0.003850 < \alpha [^\circ\text{C}^{-1}] < 0.003926$, depending on the purity of the platinum. With an applied constant current, equation 4.15 can be written as a function of voltage.

Figure 4.11 shows the results for 6 cold-wire calibrations performed over a 3-day experiment. The coefficient of determination R^2 was greater than 0.9999 for each calibration, using a linear fit. Four to eight points were used during calibrations, depending on time constraints and taking advantage of the linear nature of the calibrations. Repeatability was found to be within $\pm 0.2^\circ\text{C}$. The majority of the discrepancy is caused by sensor drift, which is common in cold-wire sensors. Calibrations performed on the same day were found to lie within $\pm 0.05^\circ\text{C}$.

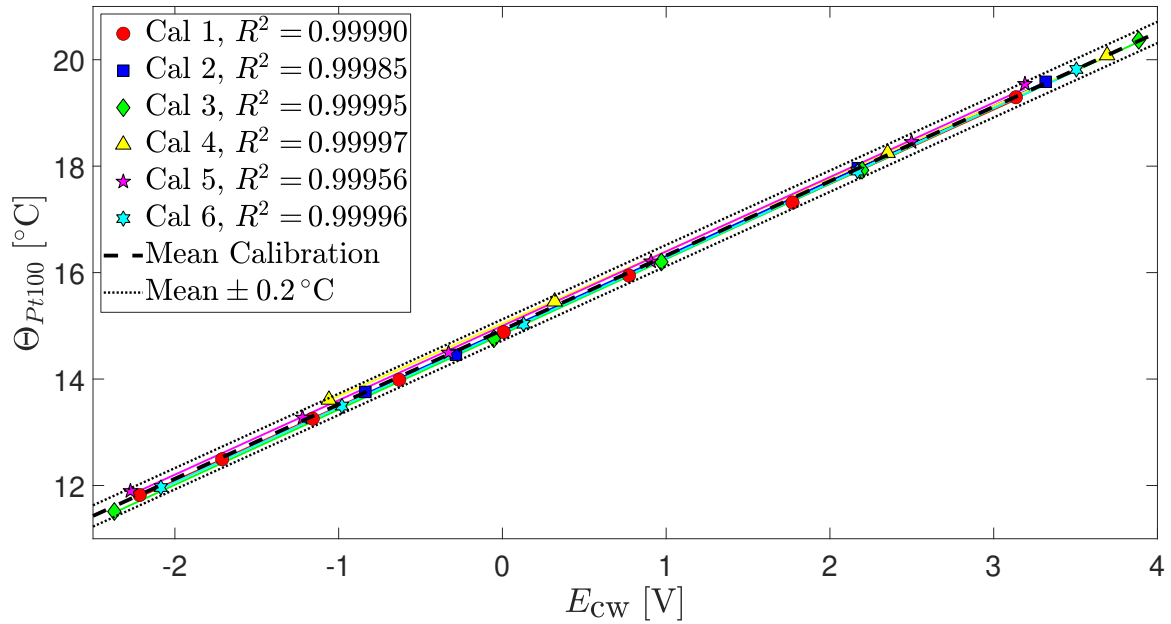


Figure 4.11: Cold-wire calibrations performed over a three-day experimental run.

Chapter 5

Momentum field characterisation

The following LDA velocity profiles were acquired at velocity ratios $VR = 0, 0.10, 0.17, 0.22, 0.27, 0.37$. Some velocity ratios may be omitted from the subsequent figures for the purpose of clarity only. In the $VR = 0$ case, the holes are left uncovered. While the net mass flux is zero, eruptions of fluid from the holes interact with the boundary non-linearly to produce a non-zero momentum flux. Therefore the holes can be perceived as a viscous roughness in the absence of blowing.

5.1 Mean velocity profiles

Figure 5.1 shows the mean streamwise velocity profiles. Profiles are normalised with respect to the friction velocity of the upstream unperturbed boundary layer, u_{τ_0} . The calculation of friction velocity and derived results are discussed in the next section (5.2). The profiles show that there is a much greater velocity deficit on the hole centreline (figure 5.1a) than between holes (figure 5.1b), particularly for $y/D < 1$. This is due to the more direct influence of the low momentum injected fluid on the hole centreline, and is consistent with the findings of Yavuzkurt *et al.* (1980a). The outer layer behaviour is the same for both spanwise locations and shows the same trends observed by Basley *et al.* (2022); an increasing velocity deficit relative to the upstream case with increasing VR .

Near-wall measurements reveal complex velocity distributions, unlike those observed by Yavuzkurt *et al.* (1980a). On the hole centreline, a clear velocity deficit extends down to the viscous sublayer with blowing, indicating a reduction in skin friction. The deficit increases with increasing VR , up until $VR \gtrsim 0.22$ where it remains approximately constant. For $y/D \gtrsim 0.1$, strong perturbations become apparent, which are more pronounced at higher velocity ratios. Two distinct plateaux begin to form by $VR = 0.37$.

Profiles between holes exhibit the same velocity ratio trends as the hot-wire results of Basley *et al.* (2022). Profiles in the viscous layer show small deviations from the unperturbed case but begin to have a pronounced velocity deficit at $y/D \approx 0.1$. The velocity deficit is largest at $VR \approx 0.17$. For $VR = 0.35$, the velocity deficit in the inner region has almost reduced to zero. Due to the large deficit in the outer

region, a plateau begins to form at $y/D \approx 1$, in a similar location to the second plateau for the hole centreline case.

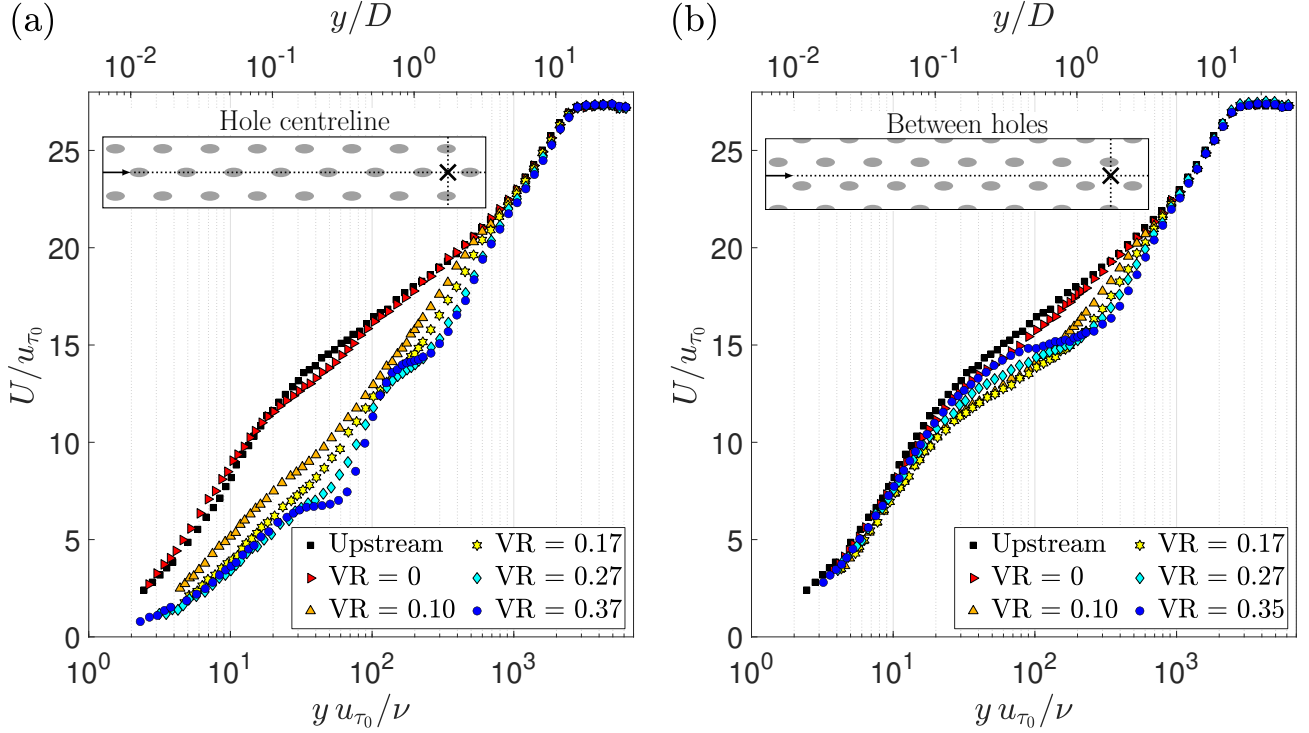


Figure 5.1: Mean streamwise velocity profiles on (a) the hole centreline, and (b) between holes, normalised by the upstream unperturbed boundary layer friction velocity u_{τ_0} .

Wall-normal mean velocity profiles are shown in figure 5.2. Similar trends to those that Yavuzkurt *et al.* (1980a) report are observed. A positive wall-normal velocity is observed across the whole boundary layer for every velocity ratio on the hole centreline. For $VR \gtrsim 0.17$, a region of downwash is observed between holes, which suggests the formation of streamwise vortices. From the current data, it is unclear however if this observation is due to the formation of counter-rotating vortex pairs (CRVPs), the legs of hairpin vortices, or other vortical structures that are characteristic of jets in cross-flow (Bidan & Nikitopoulos, 2013; Mahesh, 2013). No negative mean streamwise velocity is observed at any of the velocity ratios studied, which indicates the presence of attached jets at this streamwise location. Therefore it is unclear whether this phenomenon is directly linked to the CRVP lift-off mechanism observed in film cooling studies (Bidan & Nikitopoulos, 2013; Zhang *et al.*, 2020). For every case, a positive peak in V in the outer layer is observed, which becomes larger with increasing VR .

In the inner layer, the peaks in V occur at locations similar to the plateaux in U previously discussed. The wall-normal distance of the peaks in V increase with increasing VR , as the jets are pushed further into the boundary layer. The magnitude of the peaks display non-linear scaling with velocity ratio. On the hole centreline, the magnitude of the more dominant inner peak ($y/D \approx 0.2$) at $VR = 0.37$ is over twice that of the corresponding peak at $VR = 0.27$. The non-linear behaviour would be exacerbated when scaling with the local friction velocity u_{τ} .

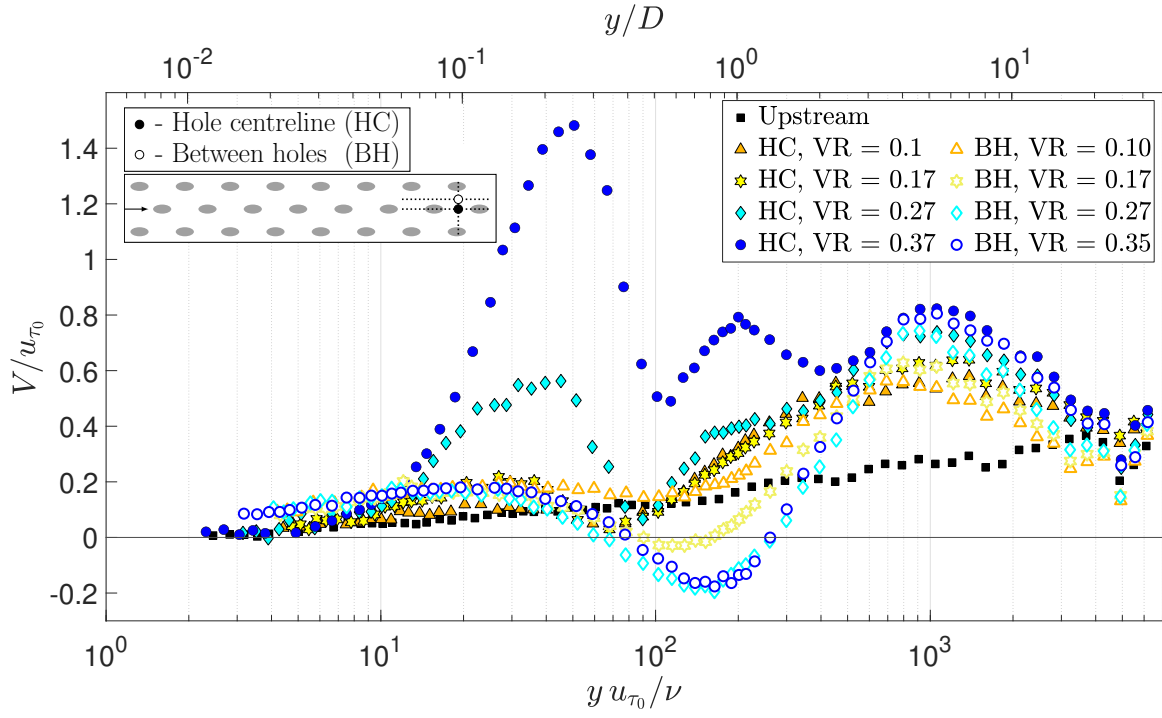


Figure 5.2: Mean wall-normal velocity profiles, normalised by the upstream unperturbed boundary layer friction velocity u_{τ_0} . Filled and open symbols correspond to spanwise locations on the hole centreline and between holes respectively (see inset in top the left-hand corner).

5.2 Skin friction coefficient

Estimates of skin friction coefficient were extracted from the LDA mean streamwise velocity profiles and also were obtained from OFI measurements. These techniques are discussed in sections 4.5 and 4.6 respectively. LDA velocity profiles were acquired on the effusion plate to keep consistent with previous work (Basley *et al.*, 2022). The OFI measurements, constrained by optical access, were acquired on a glass plate in the recovery region, $5D \approx 0.4\delta$ downstream of the end of the effusion plate and $8D$ downstream of the LDA measurement location. Therefore, the C_f values obtained between techniques cannot be expected to agree. However, trends in skin friction coefficient as a function of VR at the different spanwise locations can be compared.

LDA friction velocity estimates were obtained using a 5th order law of the wall expansion of the form

$$U = \frac{u_\tau^2}{\nu} (y - y_0) - \frac{A}{4} \frac{u_\tau^5}{\nu^4} (y - y_0)^4 - \frac{B}{5} \frac{u_\tau^6}{\nu^5} (y - y_0)^5, \quad (5.1)$$

with constants $A = 12.04 \times 10^{-4}$, $B = 0.73 \times 10^{-4}$ (Örlü *et al.*, 2010). The values u_τ and the wall-normal position of the first point, y_0 , were determined through least-squares regression fitting with equation 5.1. The coefficients A , B , were calculated from turbulent boundary layer simulation at $Re_2 \approx 4300$ (Schlatter *et al.*, 2009; Schlatter & Örlü, 2010). This is lower than the upstream turbulent boundary layer Reynolds number of $Re_2 \approx 7850$ for the present experiment. However, equation 5.1 was fitted for $y^+ \leq 9$, where inner scaling is essentially universal and Reynolds number effects are very small, at

least far smaller than the uncertainty associated with the fitting to the data points.

The boundary layers include the effects of blowing but a constant V_e is not implemented at the wall, so the transpiration law of the wall cannot be used. In figure 5.2, the near-wall distributions of V for all blowing cases on the hole centreline are very similar to that of the upstream case and appear to tend to $V = 0$ at the wall. Therefore it is likely that the traditional law of the wall is valid here. In the between holes cases where the direct effects of blowing are lower, the vertical velocity is systematically higher near the wall. It is unknown whether this is a physical phenomenon, a small systematic error arising from the transformation matrix used to determine the orthogonal velocity components from LDA measurements, or another source. For consistency and without reasonably better alternatives, the canonical law of the wall fit was applied in these cases too. The fitting range for equation 5.1 was restricted to $y^+ \leq 9$, a lower value than the range it is valid for ($y^+ \lesssim 14$). This was done to reduce the effect of any departures from canonical behaviour which are evident in figure 5.1. In this range, the wall-normal velocity is still comparable to that of the upstream case.

The results are shown in figure 5.3. They show a large reduction in skin friction coefficient for $VR > 0$ which is consistent with previous blowing studies (Kametani *et al.*, 2015). Both the LDA and OFI results show a greater reduction in C_f on the hole centreline than between holes, which is consistent with the behaviour of the streamwise mean velocity profiles in 5.1. In the between holes cases, C_f is constant to within experimental uncertainty for $VR > 0$. The LDA and OFI measurements agree well in a qualitative sense, except for a difference at $VR \approx 0.37$ between holes, where the LDA

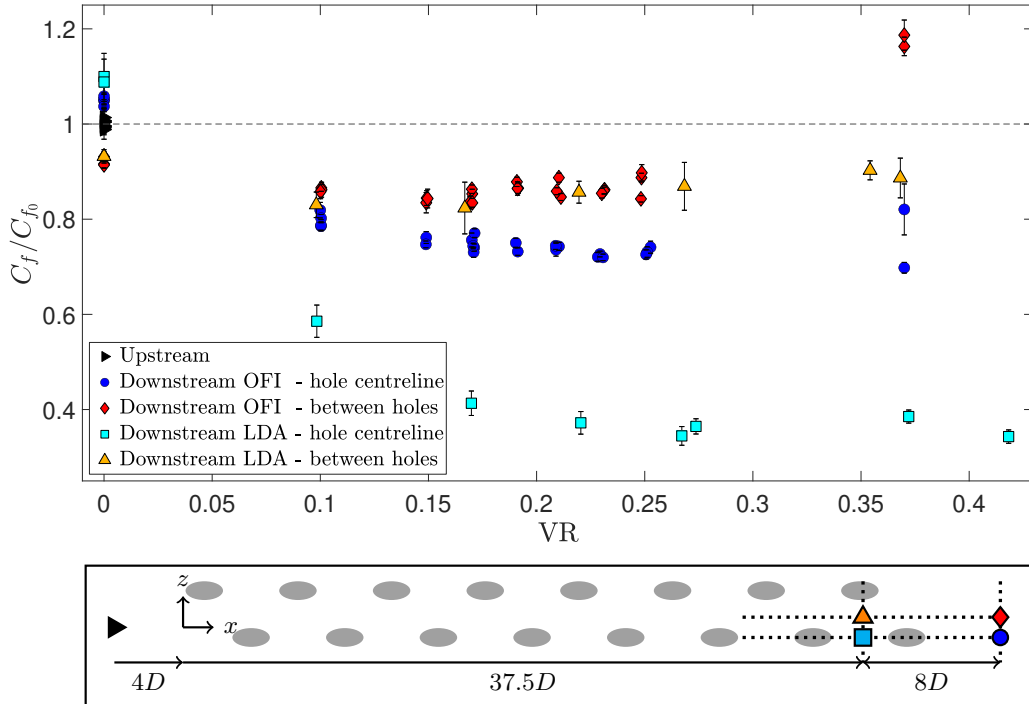


Figure 5.3: Skin friction coefficient results (C_f) from OFI and LDA law of the wall fits. Values are normalised by the upstream unperturbed boundary layer skin friction coefficient at the same Reynolds number (C_{f_0}). Error bars correspond to the 95 % confidence interval for each measurement. The diagram below the figure shows the symbols corresponding to each of the measurement stations.

results show a decrease in C_f relative to C_{f_0} but the OFI results show an increase. DNS results of Kong *et al.* (2001) show a large decrease in C_f in the immediate vicinity of injection through a slot and an overshoot above C_{f_0} at a streamwise distance of five slot widths downstream of injection. It is possible the boundary layer for $VR = 0.37$ has a faster recovery period, although more experimental data would be required to confirm this.

On the hole centreline, C_f decreases until $VR \approx 0.22$, after which it is approximately constant. The skin friction coefficient is much lower on the hole centreline at the location of LDA acquisition than at the OFI location. This is likely to be due to the measurements being directly behind a hole. At a distance $x = 5D$ behind the effusion plate where the OFI measurements were taken, the boundary layer has undergone partial recovery and the skin friction coefficient increases.

Figure 5.4 shows the mean velocity profiles normalised by the local friction velocity u_τ . In the between holes cases, the departure from canonical behaviour begins at $y^+ \approx 20$, and even earlier at $y^+ \approx 15$ on the hole centreline. From figure 5.1 it is seen that the outer layer remains relatively unperturbed. However from figure 5.4, the inner structure is heavily modified such that large departures from logarithmic overlap scaling are seen across all cases in which $VR \geq 0.10$.

When $VR = 0$ on the hole centreline, the profile exhibits characteristics of a roughness boundary layer. The non-dimensional roughness height was calculated using the formula from Schlichting *et al.* (1980) for technical roughness, derived from Colebrook (1939) as

$$k_r^+ = 3.4 \left(e^{0.4 \Delta U^+} - 1 \right), \quad (5.2)$$

which gives $k_r^+ \approx 2.3$. Therefore the influence of the holes relative to their size is small, with $k_r/D \approx 0.011$. The ratio k_r/D will depend highly on the pitch, which is large in the context of roughness for the present experiment ($p/D = 5$). Schlichting *et al.* (1980) reports reduction factors of equivalent sand grain roughness in the range 3.5 – 6.5, when the pitch spacing for different types of technical roughness is doubled.

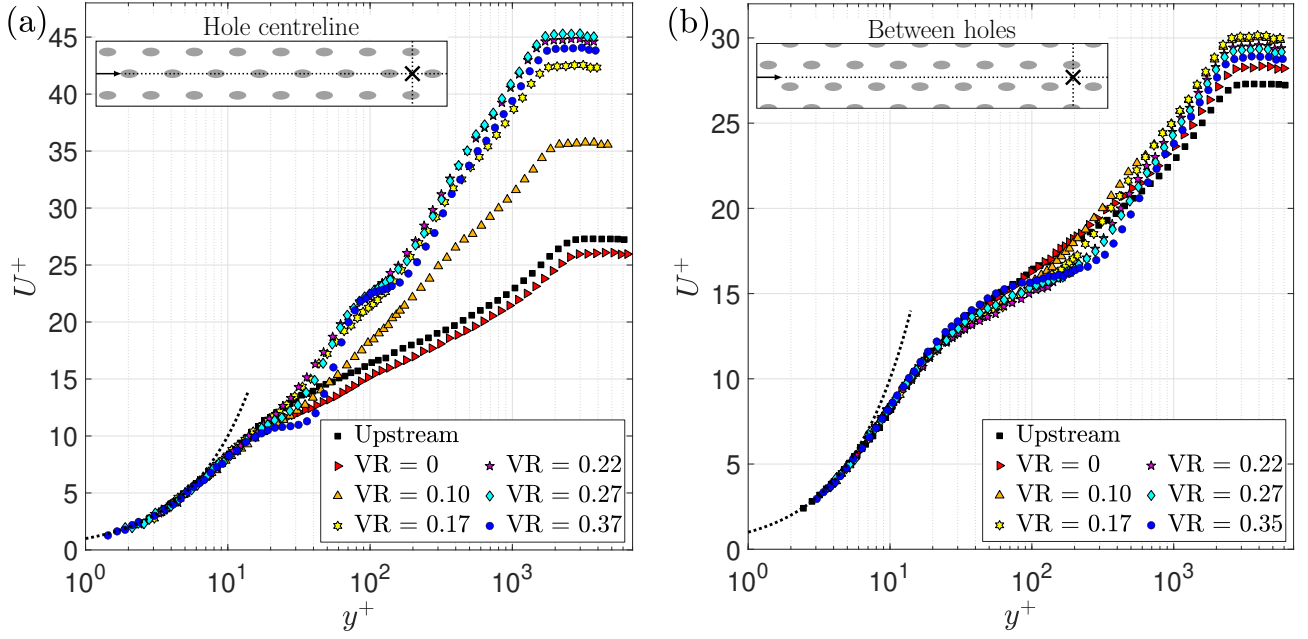


Figure 5.4: Mean streamwise velocity profiles on (a) the hole centreline, and (b) between holes, presented in inner scaling. Dotted lines correspond to $U^+ = y^+$.

5.3 Turbulent boundary layer validation

The upstream undistorted boundary layer LDA measurements are validated against LES at a comparable Reynolds number (Eitel-Amor *et al.*, 2014), and the results are shown in figure 5.5. The mean streamwise velocity profile is in very good agreement, which places confidence in the measurements and the friction velocity determination technique. The wall-normal velocity variance is in very good agreement, as well as the inner peak in streamwise velocity variance, although it is slightly over-predicted in the outer region. The spanwise velocity variance is over-predicted by approximately 10%. This is probably due to the transformation matrix errors discussed in section 4.5.2. Given that the LDA optical setup was very similar between cases (and therefore the transformation matrix), this systematic error should not affect the conclusions drawn when comparing blowing cases.

Skewness and flatness profiles are in very good agreement. There are small differences in the outer layer, which is likely to be due to the small Reynolds number difference. Small deviations are also observed in the viscous sublayer where the data rate is lower, so the results here may not be fully converged for the higher moments. The stress ratio $a_1 = \overline{uv}/k$ is also in good agreement. A constant value $a_1 = 0.3$ is often prescribed in models (Smits & Wood, 1985). Both datasets show that this is an overprediction, with $a_1 = 0.25$ giving a better estimate. There is also a small wall-normal dependence on a_1 in the overlap region. As with $\overline{w^2}$, Reynolds shear stress profiles are over-predicted in the current baseline dataset due to error propagation through the transformation matrix. These effects virtually cancel out when a_1 is determined.

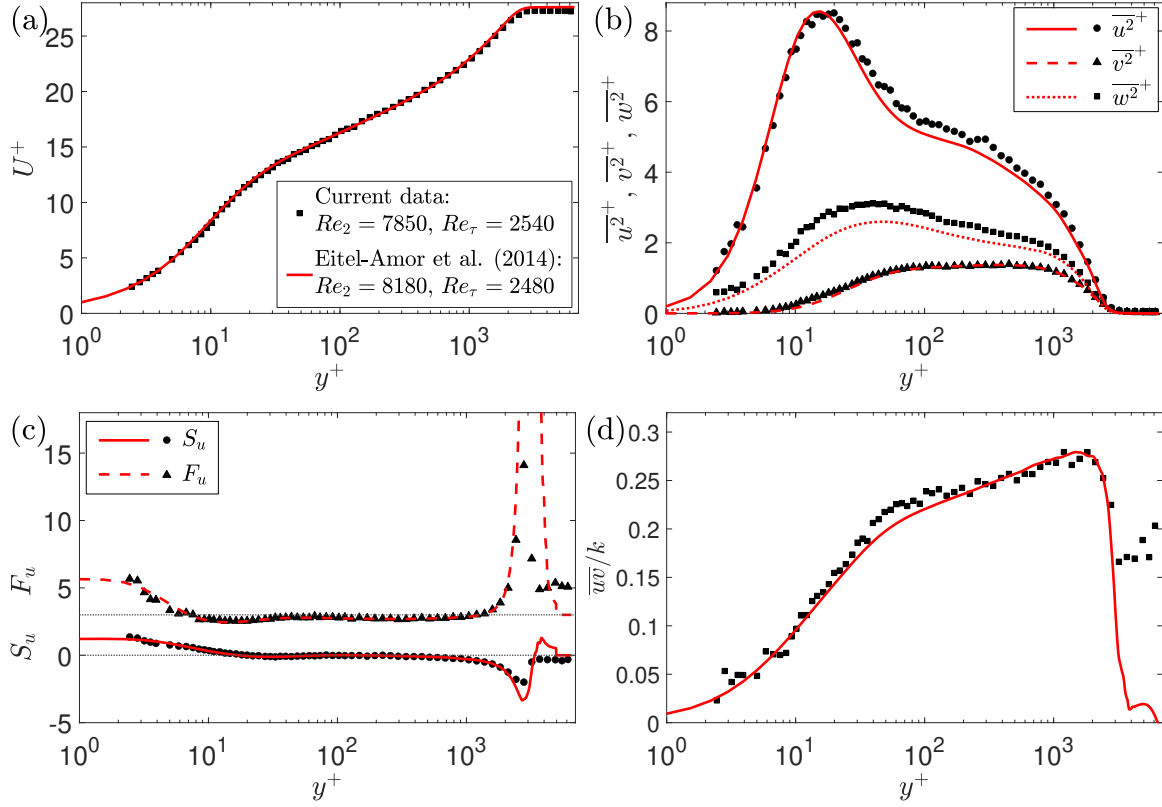


Figure 5.5: Comparison of the upstream undistorted boundary layer results from LDA measurements with LES results (Eitel-Amor *et al.*, 2014) at a similar Reynolds number.

5.4 Turbulence kinetic energy profiles

Figure 5.6 shows profiles of the turbulence kinetic energy. In the $VR = 0$ cases, the profiles resemble the $\overline{u^2}$ structure observed by Andreopoulos & Wood (1982) downstream of a short roughness trip. On the hole centreline, the inner peak is larger and displaced further from the wall. This is typical of the structure Andreopoulos & Wood (1982) observed in the more immediate recovery region downstream of the roughness strip. In the between holes case, the inner peak remains unchanged but a region of higher turbulence is observed beyond it. This is typical of further downstream in the recovery region, with the near-wall turbulence generated by the roughness being transported away from the wall. It is also consistent with the fact that the boundary layer is not directly affected by the perturbation from the holes in the location between them.

On the hole centreline (figure 5.6a), a very large inner peak forms which increases in magnitude and wall-normal distance with increasing VR . It could potentially be associated with the traditional inner peak being blown away from the surface. For the spanwise location between holes (figure 5.6b), the collapse of TKE in the inner layer is very good except for at $VR = 0.37$, where the TKE is lower. This could be associated with the lower amplitude modulation of small-scale turbulence by large structures in the near-wall region at higher velocity ratios (Basley *et al.*, 2022). Similarity to the traditional inner peak is found in the between holes cases, although it appears as a shoulder merging into a large outer

peak for all but the highest velocity ratio.

For both spanwise locations, there is a large increase of TKE in the outer region, consistent with previous blowing studies (Hasanuzzaman *et al.*, 2020; Basley *et al.*, 2022). On the hole centreline, the outer peak appears as a shoulder from the very large inner peak which has moved away from the wall. For $VR \gtrsim 0.17$, a distinct outer peak forms, although it is much smaller than the inner peak. Between holes, the outer peak is more pronounced due to the smaller turbulence in the inner region, but it is similar in magnitude in both spanwise cases. The outer peak moves away from the wall with increasing VR due to the increasing jet penetration. However, for $VR \gtrsim 0.22$, the peak begins to decrease in magnitude. Therefore traditional velocity scales of the form u_τ and u_τ^2/U_e will not scale this interaction. This is due to u_τ attaining a constant value for $VR \gtrsim 0.22$, while the injection velocity U_e keeps increasing.

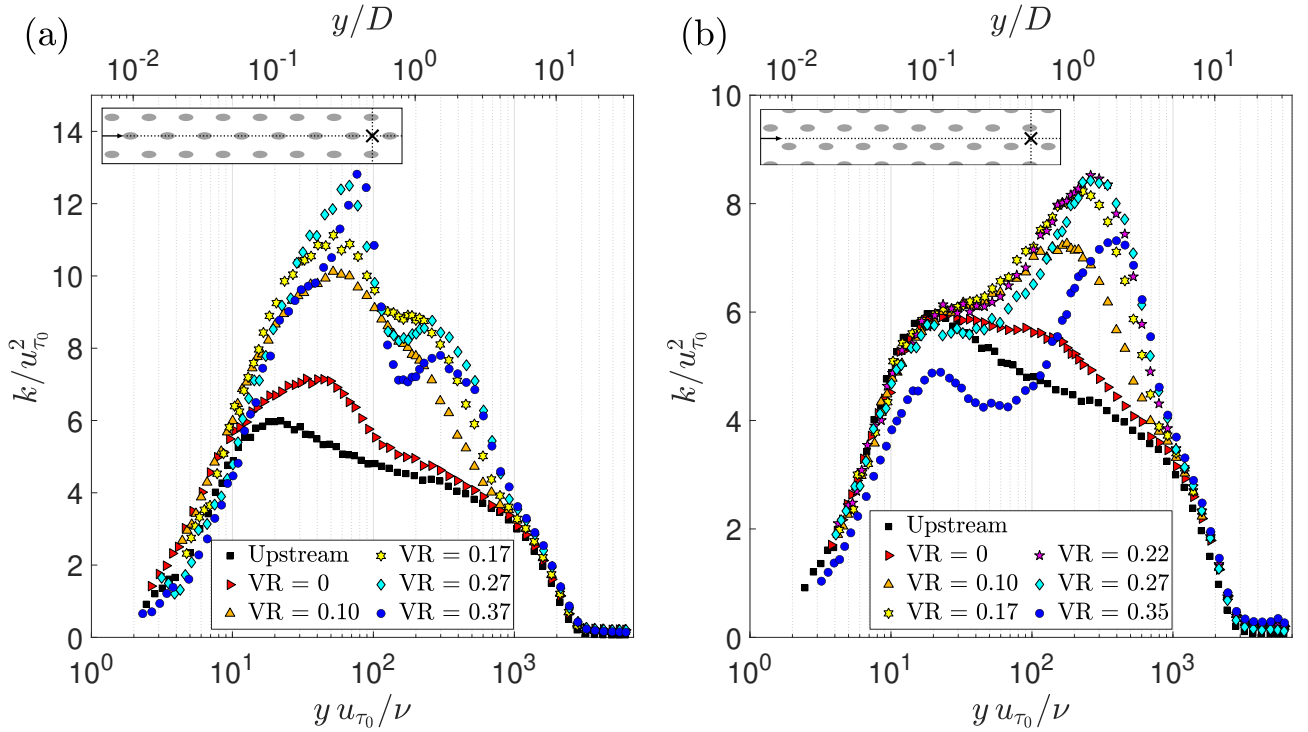


Figure 5.6: Turbulence kinetic energy profiles on (a) the hole centreline, and (b) between holes.

5.5 Production of turbulence kinetic energy

The dominant boundary layer production term $P = \overline{uv} \partial U / \partial y$ is plotted in premultiplied form in figure 5.7. While the streamwise velocity gradients do not show the y^{-1} decay that the log law predicts, the premultiplied form still allows clearer trends in the outer layer to be shown. The peaks in k correspond to peaks in the premultiplied production, which suggests the two-dimensional production term is still dominant in the turbulence production in the boundary layers with blowing. In the $VR = 0$ cases, a hump is observed due to an increase in the Reynolds shear stress related to the roughness perturbation.

For $0.22 \lesssim VR \lesssim 0.37$, the outer peak in premultiplied production at $y/D \approx 2$ is approximately

constant in magnitude, but reduces in width with increasing VR . Therefore the area under the peak is smaller which could explain why the magnitude of the outer peak in k reduces with increasing VR , in the range $0.22 \lesssim VR \lesssim 0.37$. In the near-wall region ($y/D \lesssim 0.1$) the premultiplied production with effusion is lower than the baseline case for all velocity ratios, particularly on the hole centreline. However, the TKE in this region is not attenuated, which implies different production terms could be responsible for the TKE observed near the wall.

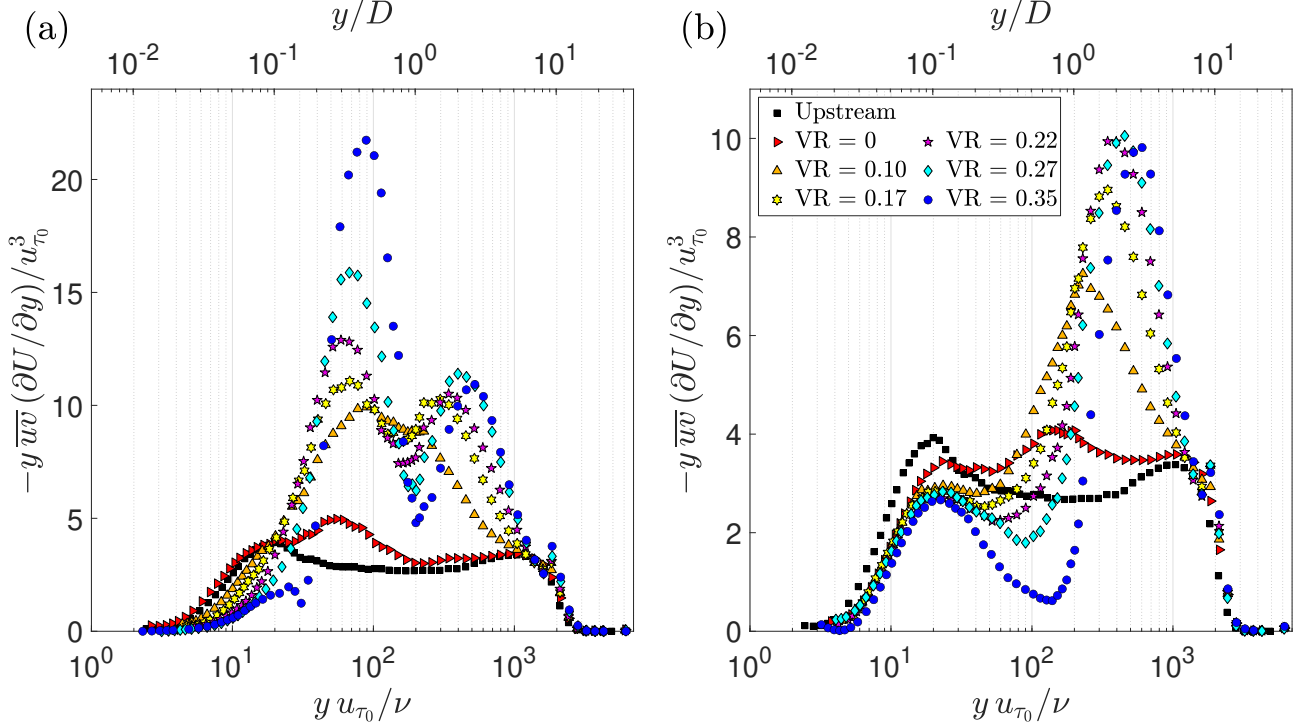


Figure 5.7: Premultiplied production profiles (a) on the hole centreline, and (b) between holes, normalised by the upstream unperturbed boundary layer friction velocity u_{τ_0} .

5.6 Assessment of local equilibrium

The crux of the mixing length model in the overlap region of a TBL is the assumption of local equilibrium. A direct assessment of the validity of this hypothesis requires measurements of the dissipation of TKE and of all production terms in a three-dimensional flowfield. However, an indirect assessment is made here by observing the magnitude of the wall-normal transport term in the k -equation (equation 1.28) relative to the dominant production term $P = \overline{uv} \partial U / \partial y$ of figure 5.7.

Figure 5.8 shows the turbulent transport term $\partial \overline{kv} / \partial y$ in premultiplied form. It assumes that the turbulent transport contribution is much larger than pressure and viscous transport. For $y u_{\tau_0} / \nu \gtrsim 30$, the transport term is significantly higher with blowing than in the upstream boundary layer. Its contribution to the energy budget in the overlap region is non-negligible, with values reaching up to 25% of the production term for the highest VR cases. Therefore significant departures from local equilibrium are expected in the overlap region, typical of other perturbed boundary layers (Smits

& Wood, 1985). The gradients in $\overline{k}v$ generally follow the gradients in k so diffusion models may be satisfactory, although the constant of proportionality is shown to vary significantly across the boundary layer. The transport term changes in sign rapidly, so bulk convection velocity models of the form $V_k = \overline{v}k/k$ are unlikely to be useful.

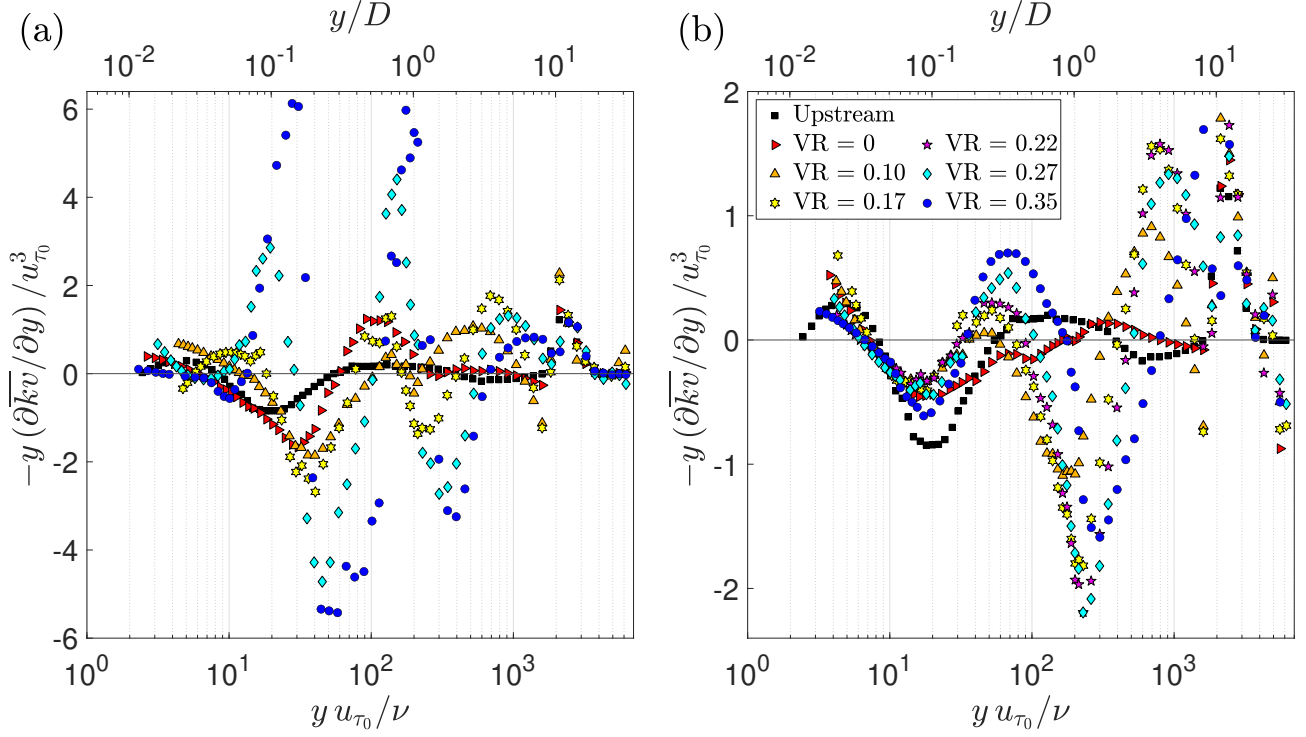


Figure 5.8: Premultiplied turbulent transport profiles (a) on the hole centreline, and (b) between holes (b), normalised by the upstream unperturbed boundary layer friction velocity u_{τ_0} .

5.7 Comparison between spanwise locations

Figures 5.9a and 5.9b show mean streamwise velocity and turbulence kinetic energy profiles respectively, with both spanwise locations plotted on the same graphs. The profiles for a given VR collapse at $y/D \approx 1.8$. This location has been termed the depth of penetration (Yavuzkurt *et al.*, 1980a), beyond which there is no direct effect of the upstream jet and the film assumes a homogeneous state. Other authors have used different metrics to describe the penetration distance (Ramsey & Goldstein, 1971; le Brocq *et al.*, 1973). Yavuzkurt *et al.* (1980a) observe a penetration depth of $y/D \approx 2$ for $VR = 0.4$. This exact value for a given VR is likely a function of δ/D , pitch p/D , and streamwise location x . The depth of penetration is different from the internal layer size δ_i , which is the wall-normal extent of the perturbation.

In the range $0.1 \lesssim y/D \lesssim 1$, figures 5.9a and 5.9b show respectively large differences in U and k across the two spanwise locations measured, for a given VR . This suggests that the advection and transport terms $W \partial k / \partial z$ and $\overline{uw} \partial U / \partial z$ in the TKE budget are non-negligible.

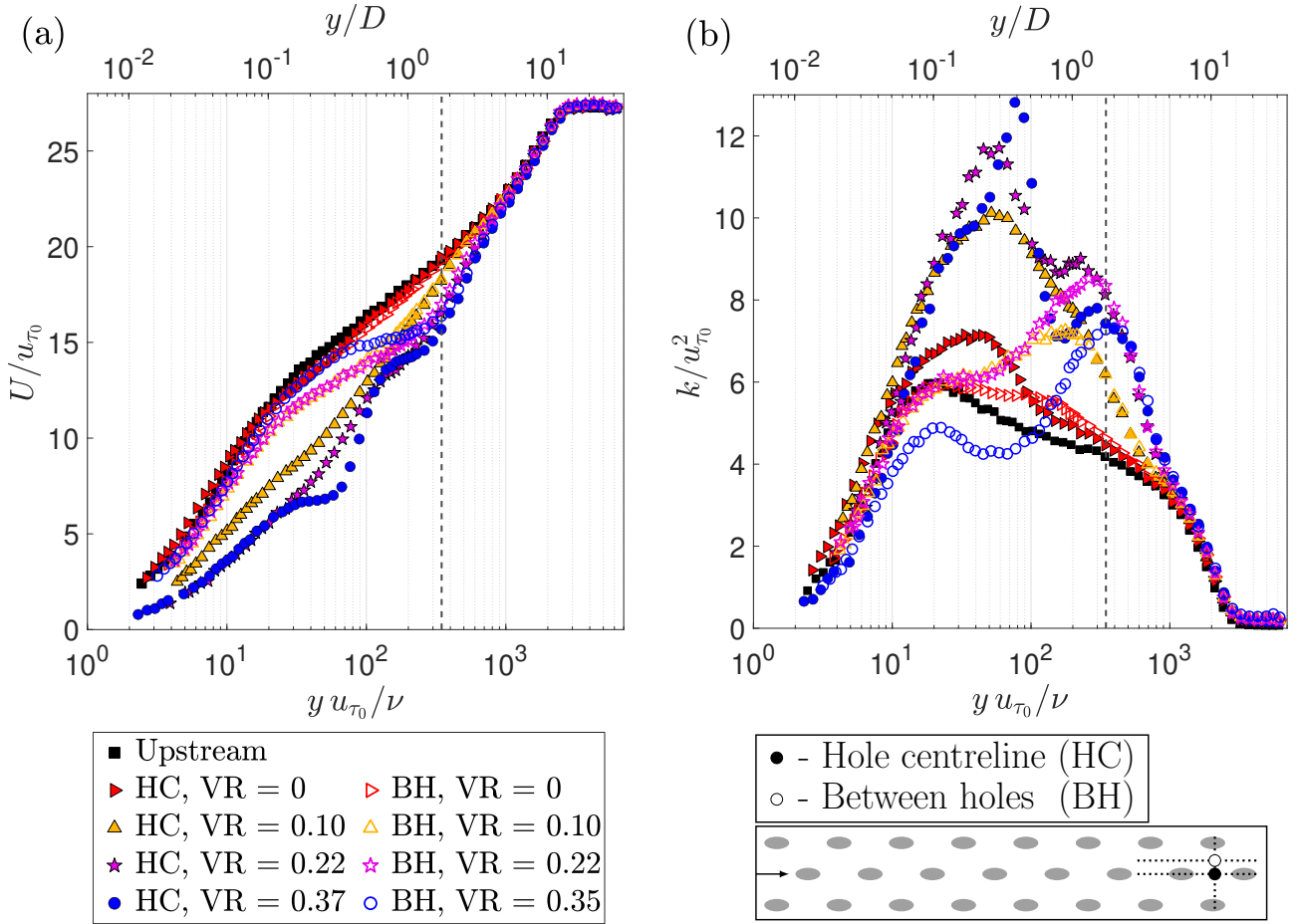


Figure 5.9: Comparison of (a) the mean streamwise velocity and (b) the turbulence kinetic energy profiles at the two measured spanwise locations. Filled and open symbols correspond to spanwise locations on the hole centreline and between holes respectively (see diagram in bottom right). Dashed lines correspond to $y/D = 1.8$

5.8 Distribution of turbulence kinetic energy

Figure 5.10 shows how the kinetic energy of the turbulence is distributed across the three components at different velocity ratios. On the hole centreline (figure 5.10a), there is a particularly large increase in $\overline{w^2}$ in the near-wall region with effusion. As VR increases, the near-wall spanwise velocity variance increases and the streamwise velocity variance is attenuated. The cross-over point is at $VR \approx 0.22$, where the near-wall streamwise and spanwise variances are similar in magnitude. By $VR = 0.37$, the near-wall turbulence is dominated by spanwise velocity fluctuations.

The reduction in streamwise variance is likely to correspond to the low production, $\overline{uv} \partial U / \partial y$ (figure 5.7a), observed in this region. Different production terms are responsible for the increase in turbulent spanwise fluctuations and may explain why RANS models generally underpredict the lateral spreading of the jets in film cooling applications (Murray *et al.*, 2018). The wall-normal location of the peak in $\overline{w^2}$ shows little VR dependence and remains local to the wall. Conversely, the peak in $\overline{u^2}$ moves away from the wall with increasing VR . The distribution of $\overline{v^2}$ is similar to that of $\overline{u^2}$ but lower in

magnitude. The peaks in $\overline{v^2}$ are at similar locations to those in $\overline{u^2}$ and k (figure 5.6a).

In the cases between holes (figure 5.10b), $\overline{u^2}$ reduces in the near-wall region as VR is increased. There is an increase in $\overline{w^2}$ near the wall but not as large as on the hole centreline. For $VR \gtrsim 0.22$, the spanwise velocity variance also begins to be attenuated near the wall. The collapse of the TKE profiles (figure 5.6b) for $y/D \lesssim 0.1$ at lower velocity ratios seems to be coincidental and due to similar levels of amplification and attenuation of the spanwise and streamwise velocity variances respectively.

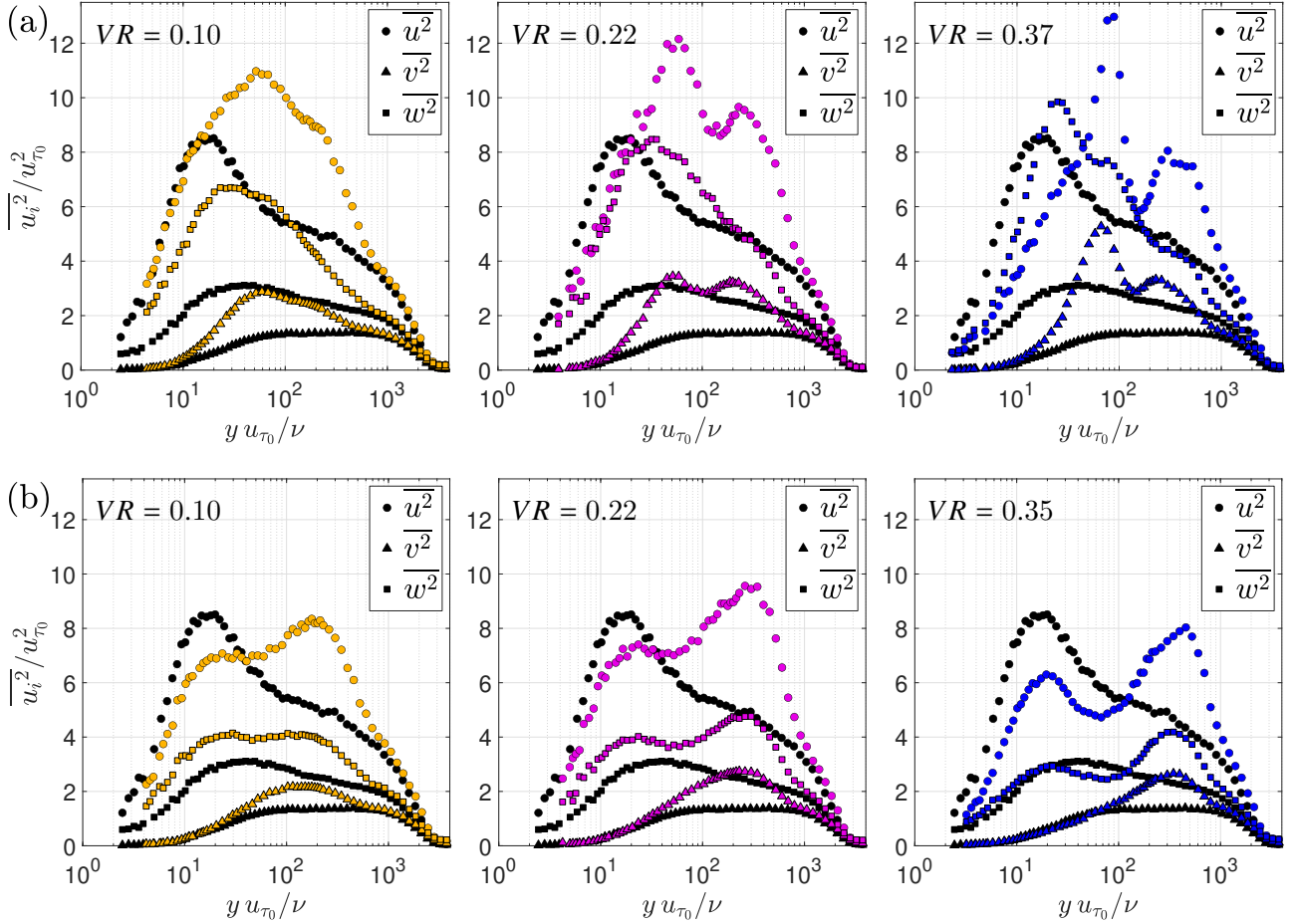


Figure 5.10: Turbulent momentum flux distributions in the three orthogonal directions x, y, z ($i = 1, 2, 3$): (a) on the hole centreline and (b) between holes. The velocity ratio is indicated in the top left-hand corner of each plot. Each plot shows the comparison with the baseline unperturbed case (black symbols).

5.9 Integral parameters

Figure 5.11a shows the variation of integral parameters as a function of velocity ratio. They show a clear change in behaviour about the observed critical velocity ratio of $0.17 < VR_c < 0.22$. Particularly pronounced in the between holes cases, for $VR \leq 0.17$ there is a steep increase in the displacement thickness and the momentum thickness with increasing VR , but for $VR \geq 0.22$ they show a mild increase. A similar trend is observed in the shape factor which reaches an approximately constant value

for $VR \geq 0.22$ in the between holes cases. On the hole centreline, the trends are much less pronounced but the relative increase in shape factor with increasing VR becomes smaller for $VR \geq 0.22$. The increase in shape factor with increasing velocity ratio resembles adverse pressure gradient boundary layers as the pressure gradient parameter increases (Monty *et al.*, 2011). The shape factor is larger on the hole centreline due to the direct influence of injection, which is seen to reduce the velocity close to the wall, such that the boundary layer becomes less ‘full’.

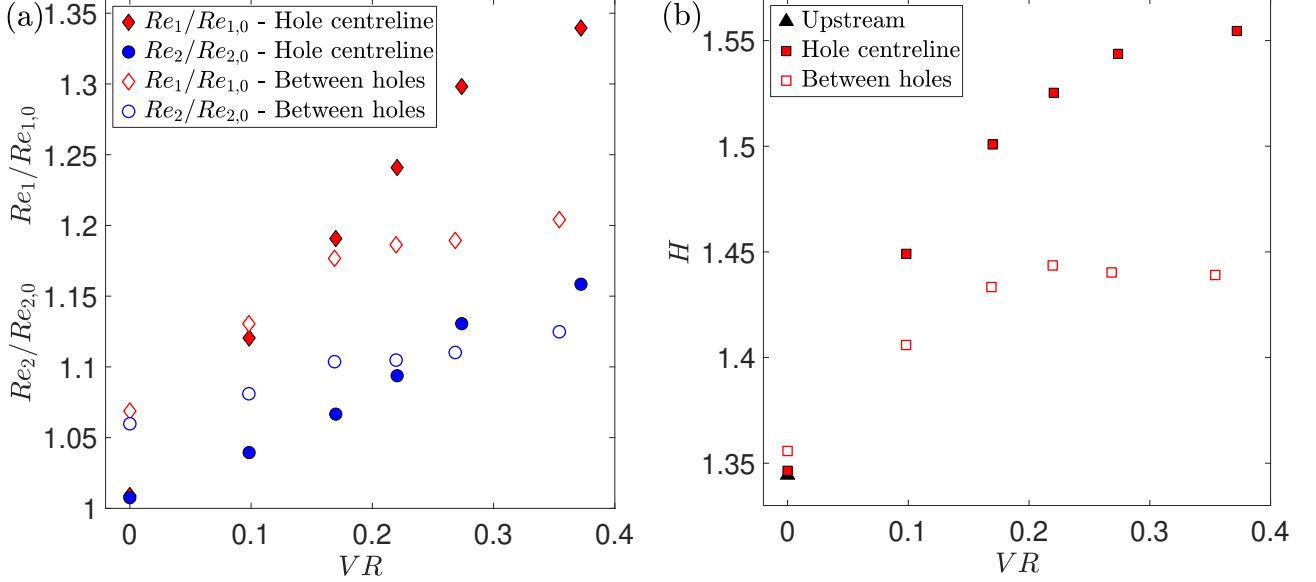


Figure 5.11: (a) Displacement thickness (Re_1) and momentum thickness (Re_2) Reynolds numbers, normalised by the respective unperturbed upstream values (denoted by the subscript 0), as a function of velocity ratio. (b) Shape factor for each case as a function of velocity ratio.

5.10 Higher order moments

The streamwise velocity skewness profiles on the hole centreline are shown in figure 5.12a. In the near-wall region, the skewness with a finite velocity ratio is larger than the baseline case, which indicates a greater prevalence of sweeps of higher momentum fluid further from the wall into the region closer to the wall. When normalised with the local friction velocity u_τ , an increase in skewness near the wall is still observed. At $VR = 0.37$, a complex distribution emerges. The peaks in S_u correspond to the plateaux observed in the mean streamwise velocity profile and the trough corresponds to the region of relatively high shear between the plateaux. The corresponding profiles between holes (figure 5.12b) show similar behaviour to the baseline case in the viscous sublayer. In the overlap region where $S_u \approx 0$ for a canonical TBL, the skewness is positive with blowing and becomes larger with increasing velocity ratio.

The streamwise velocity flatness factors are shown in figure 5.13. They also show complex distributions on the hole centreline (figure 5.13a). Similar to the skewness profiles, F_u is larger in the near-wall region for all blowing cases, which corresponds to a greater intermittency of the near-wall sweep events.

Due to the complex mean and fluctuating velocity distributions, more measurements are required to attribute physical descriptions to these distributions. In the between holes case (5.13b), the largest departures from canonical behaviour are observed in the overlap layer. Usually, a constant region of $F_u \approx 2.8$ is observed (Fernholz & Finley, 1996), as the current data for the upstream boundary layer also shows. For $VR = 0$, the same value is measured. For $VR > 0$, the flatness factor increases with increasing VR in this region, although a plateau is still observed.

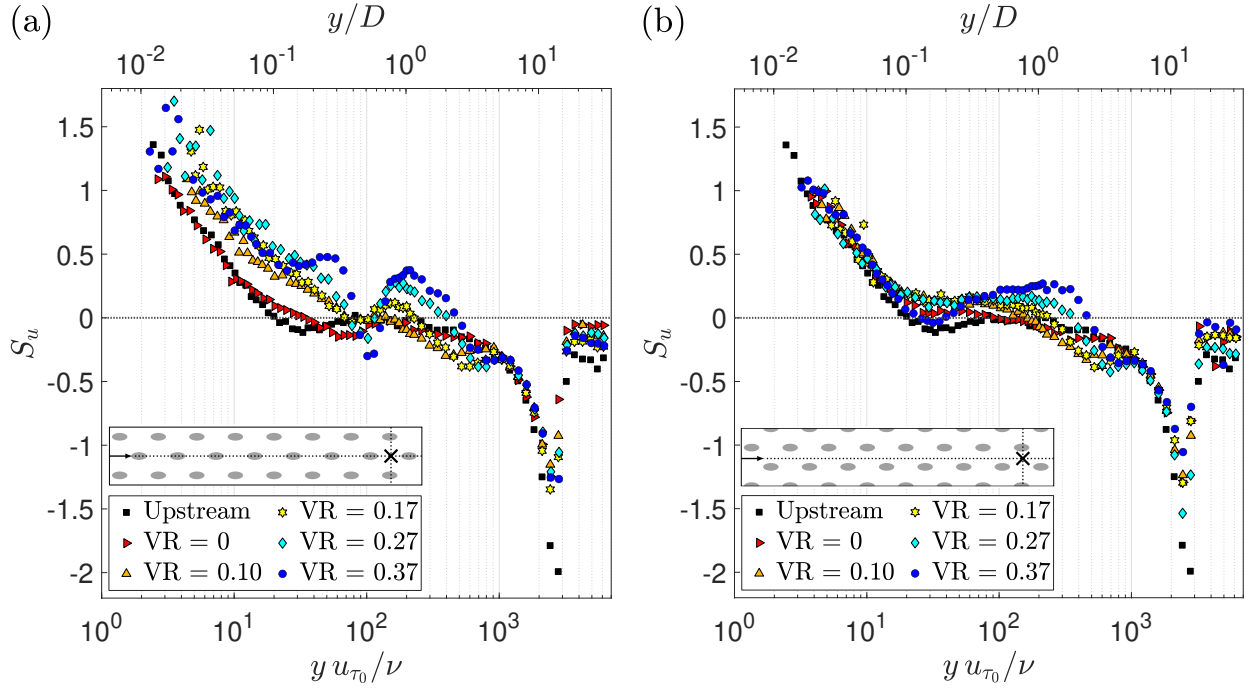


Figure 5.12: Streamwise velocity skewness profiles (a) on the hole centreline, and (b) between holes. The dotted line denotes the Gaussian value $S_u = 0$.

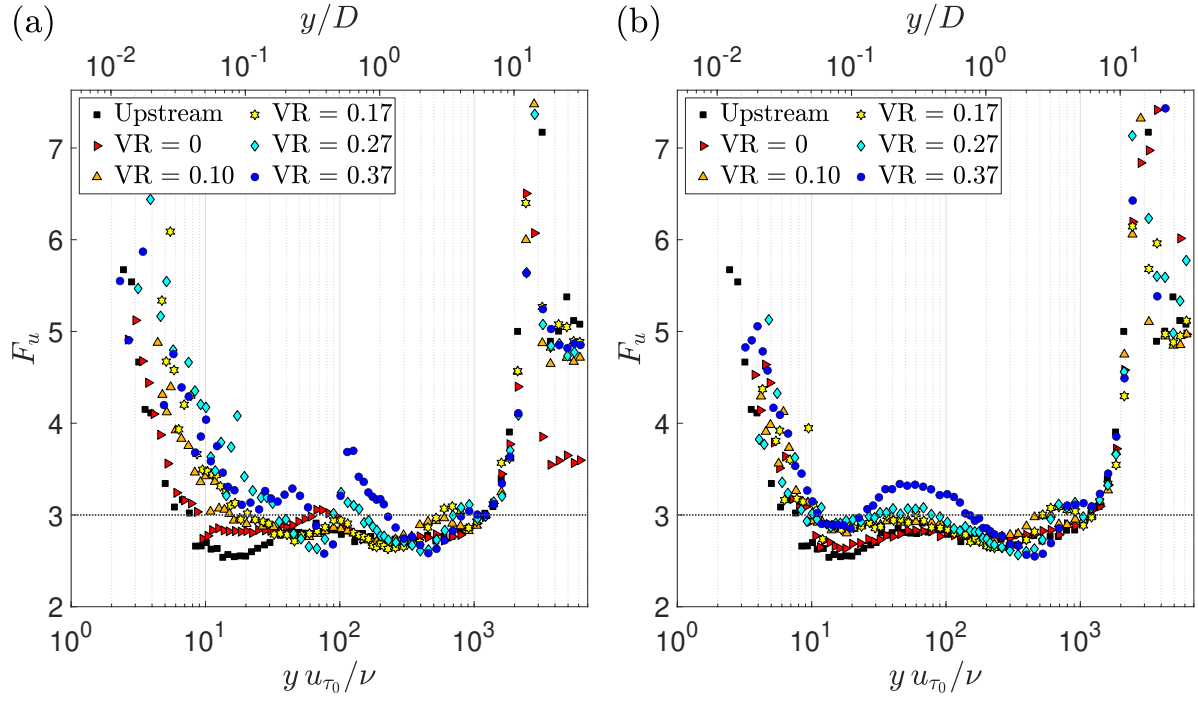


Figure 5.13: Streamwise velocity flatness profiles (a) on the hole centreline, and (b) between holes. The dotted line denotes the Gaussian value $F_u = 3$.

5.11 Spectra

Figure 5.14 shows premultiplied power spectral density (PSD) maps of the streamwise velocity fluctuations on the hole centreline. Streamwise wavelengths, λ_x , are calculated from the x-wire time series data with Taylor’s frozen turbulence hypothesis. They show large similarities to the premultiplied PSD maps of Basley *et al.* (2022), through which spatial correlations were calculated directly from PIV data. As Basley *et al.* (2022) reports, the spectral map at $VR = 0$ is characteristic of high Reynolds number boundary layers. An intense region of large-scale energy is found in the traditional overlap region, caused by roughness effects induced by the holes.

For $VR = 0.10$, turbulence is enhanced across a large range of scales and wall-normal distances. The intense small-scale turbulence in the near-wall region has moved further from the wall due to the injection. The spectral map of $VR = 0.17$ (not shown) shows similar characteristics to that of $VR = 0.10$. For $VR = 0.37$, large differences in the distribution of turbulence across scales is observed. The large-scale streamwise turbulence near the wall is significantly attenuated. The inner and outer regions of high turbulence energy become distinct from one another in the wall-normal direction, likely due to the plateaux in the velocity profile and corresponding low production. The streamwise wavelengths and the wall-normal distance (in outer scaling) of the intense regions of turbulence energy increases at $VR = 0.37$.

The power spectral density maps of the wall-normal turbulent fluctuations (figure 5.15) have the expected smaller streamwise wavelengths. The near-wall peak in $\overline{v^2}$ (figure 5.10) has an associated

maximum in the spectral map. The streamwise wavelengths of the wall-normal fluctuations scale with D . The energy is dominated by streamwise wavelengths of $\lambda_x \approx D$ for all measured velocity ratios.

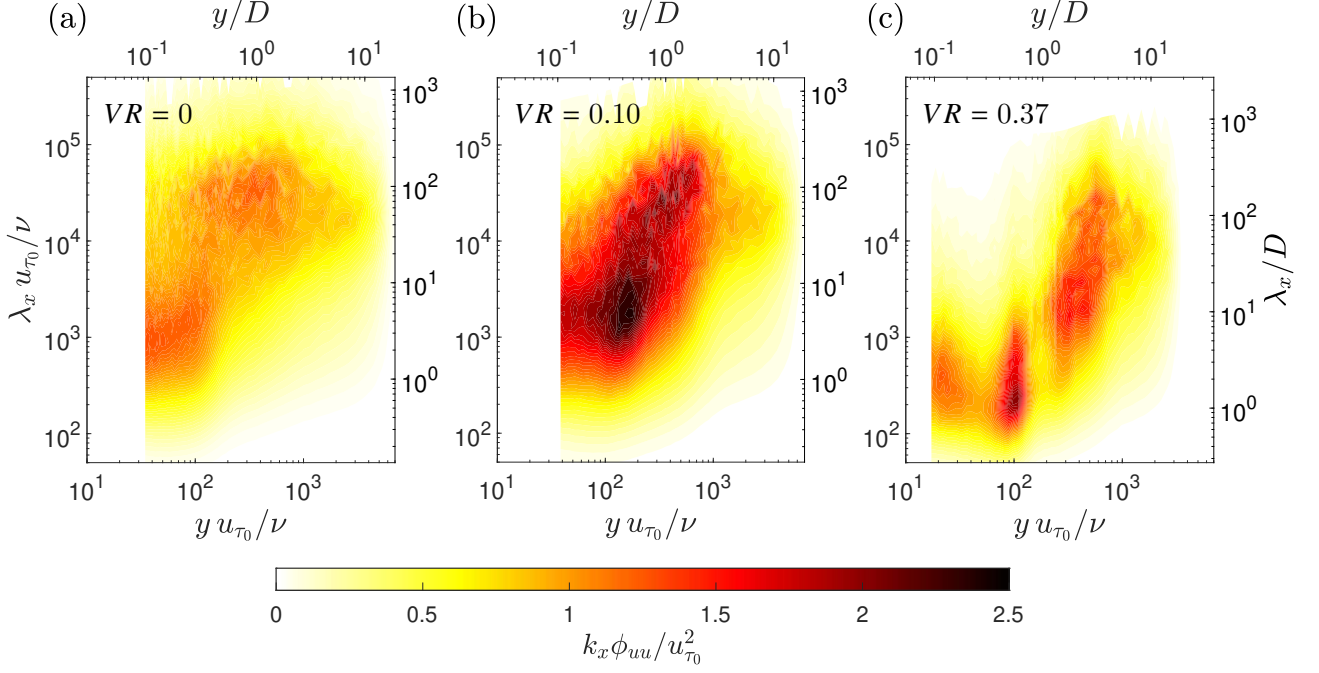


Figure 5.14: Premultiplied power spectral density maps of the streamwise velocity fluctuations on the hole centreline. The velocity ratio for each plot is shown in the top left-hand corner.

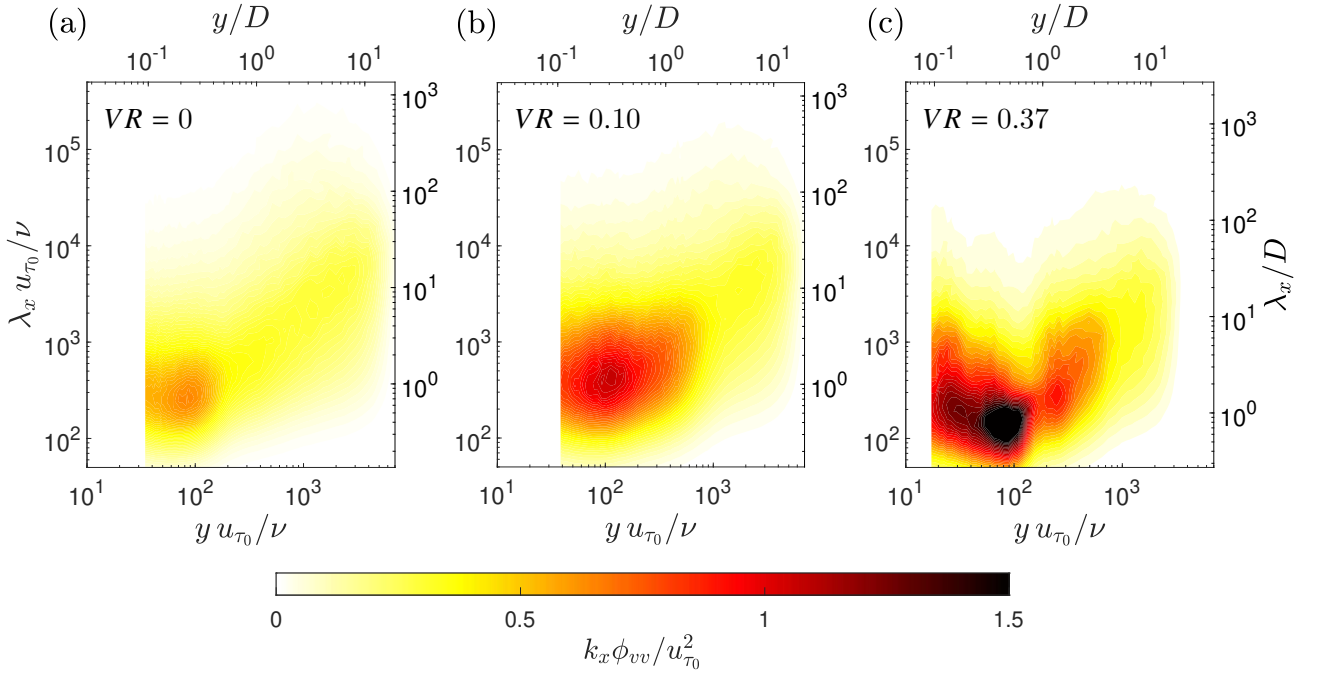


Figure 5.15: Premultiplied power spectral density maps of the wall-normal velocity fluctuations on the hole centreline. The velocity ratio for each plot is shown in the top left-hand corner.

Chapter 6

Temperature field characterisation

Inner scaling of the temperature profiles requires estimates of the friction temperature and therefore of the surface heat flux. The experiment was designed with a specific focus on momentum/ passive scalar mixing, using small temperature differences. The surface condition is neither constant temperature nor constant heat flux. Instead, it approximates an adiabatic wall with a 3D-printed porous PLA plate of thermal conductivity less than 0.13 W/mK (Islam *et al.*, 2023). Therefore accurate measurements of the surface temperature and surface heat flux are unfeasible. Consequently, the following results are presented in outer scaling - profiles are normalised by the thermal boundary layer thickness (δ_Θ) or hole diameter (D), and the effusion-freestream temperature difference ($\Delta\Theta = \Theta_\infty - \Theta_e$). The measurements presented in this section were obtained from simultaneous acquisition with an x-wire and adjacent cold-wire (see section 4.7). A comparison between LDA and x-wire results is given in Appendix C.

6.1 Mean temperature profiles

Mean temperature profiles are shown in figure 6.1. The near-wall normalised temperature reduces with increasing VR , which suggests a higher cooling effectiveness. Temperature gradients approach zero near the wall, particularly for the location between holes. This is typical of a thermal boundary layer after the removal of surface heating (Subramanian & Antonia, 1981*a*; Hattori *et al.*, 2012). On the hole centreline, temperature gradients become negative near the wall which is particularly pronounced at $VR = 0.37$, where the bulk of the film is lifted away from the surface. The lowest temperature is observed away from the wall at $y/D \approx 0.3$ on the hole centreline and at $y/D \approx 1$ between holes, coinciding with their plateaus in the mean streamwise velocity profiles. Ramsey & Goldstein (1971) associate the location of minimum temperature with the penetration of the jets into the boundary layer. For $VR \lesssim 0.17$, the bulk of the jet stays confined to the surface with little apparent penetration.

The large increase in the penetration of the film into the mainstream at $VR = 0.37$ is also accompanied by a large relative thickening of the thermal boundary layer. Compared to at $VR = 0.10$, the thermal boundary layer thickness is approximately 8% larger at $VR = 0.17$ and 53% larger at

$VR = 0.37$. As with the mean streamwise velocity profiles, the mean temperature profiles for a given VR collapse at $y/D \approx 1.8$, with a small velocity ratio dependence.

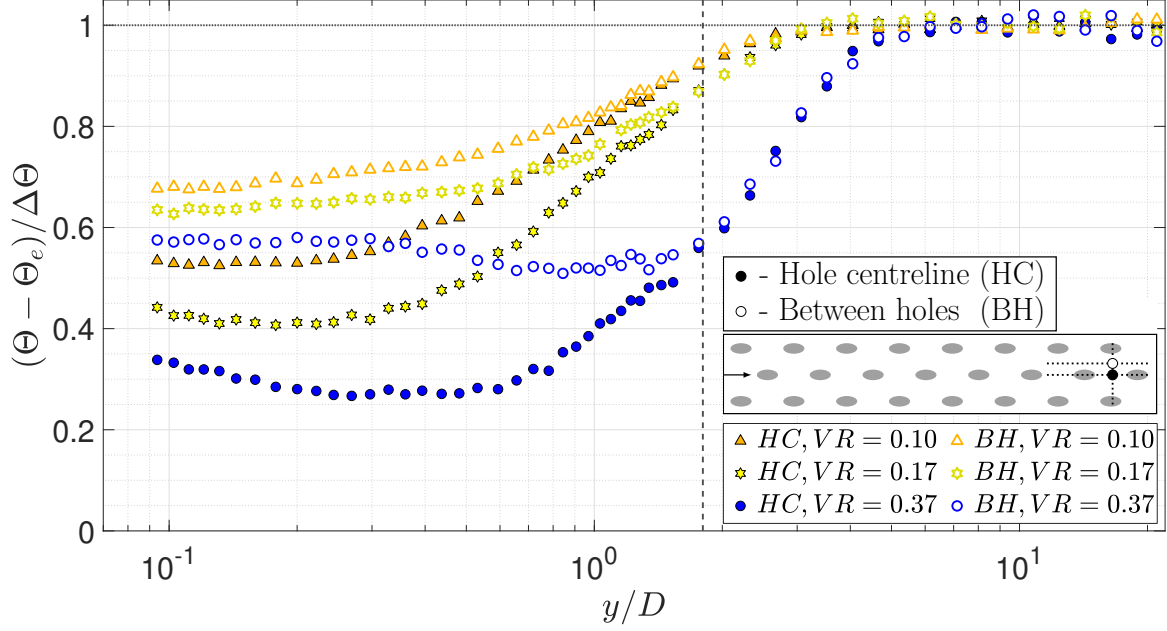


Figure 6.1: Mean temperature profiles, normalised by the effusion-freestream temperature difference. Filled and open symbols correspond to spanwise locations on the hole centreline and between holes respectively (see inset in bottom right). The vertical dashed line corresponds to the location $y/D = 1.8$.

6.2 Temperature variance profiles

Temperature variance profiles are shown in figure 6.2. The distributions resemble those of a turbulent boundary layer after a step decrease in surface heat flux (Antonia *et al.*, 1977), with a single broader peak displaced further from the wall than the traditional inner peak. They show little similarity to their respective TKE profiles (figure 5.6). The peaks in $\overline{\theta^2}$ occur at different locations to the peaks in k or $\overline{u^2}$, unlike in a canonical thermal/ momentum boundary layer (Subramanian & Antonia, 1981b), suggesting a lack of correspondence between the turbulent momentum and scalar fields.

For $VR \leq 0.17$, the peaks in $\overline{\theta^2}$ increase in magnitude and distance from the wall, as observed in the TKE profiles. For $VR = 0.37$, the distribution differs significantly. The peaks become broader and occur much further from the wall, even when scaled with δ_Θ . The temperature fluctuations are smaller on the hole centreline but enhanced between holes for $VR = 0.37$, when compared to $VR = 0.17$. This suggests that the thermal transport and lateral spreading of the film are significantly increased at $VR = 0.37$, despite the turbulence kinetic energy in this region being comparatively smaller. However, the near-wall temperature fluctuations are attenuated, similar to the attenuation in $\overline{u^2}$ in the near-wall region at higher velocity ratios. The temperature fluctuations on the hole centreline are significantly larger at lower velocity ratios due to the direct effects of the upstream lower temperature jets. For the cases between holes, the peak is shallow and broad, showing little resemblance to the distributions of the turbulent normal momentum flux components.

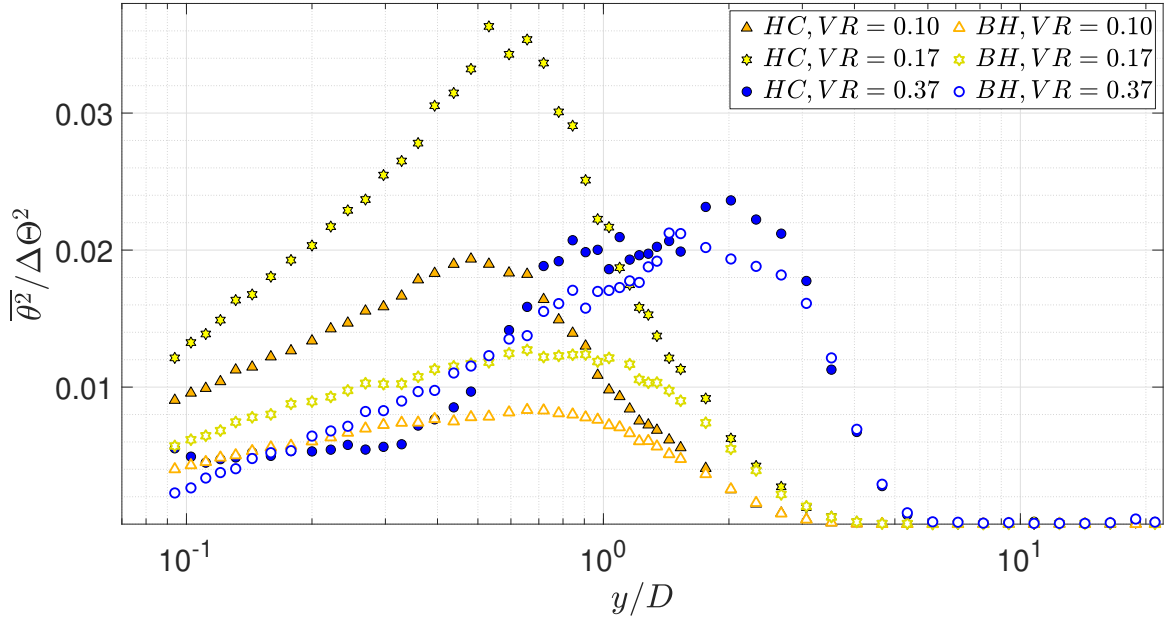


Figure 6.2: Profiles of the temperature variance normalised by the effusion-freestream temperature difference. Symbols are the same as in figure 6.1.

6.3 Turbulent heat flux distributions

The simultaneous measurement of turbulence quantities using multiple sensors requires them to be synchronised in time and occupy the same location in space. However sensors are generally displaced from one another, with a distance of approximately 1 mm between the x-wire and cold-wire for this experiment. To assess the capability of the x-wire and cold-wire probes to capture simultaneous measurements, the cold-wire was replaced with a single-hot-wire for the $VR = 0$ case on the hole centreline. The autocorrelation coefficient of the streamwise velocity fluctuations from the x-wire and the single hot-wire, R_{uu} , was measured and is shown in figure 6.3. This is used as a measure of simultaneity, where $R_{uu} = 1$ for true simultaneous measurements.

For $y/\delta > 0.05$ (corresponding to $y/D \gtrsim 0.8$), the correlation coefficient is greater than 0.9. Below this, R_{uu} decreases, probably due to the increased three-dimensionality of the flow and the larger contribution of small-scale motions to the velocity variance. Values in the outer layer exceed 0.95, as also measured by Kohli & Bogard (2005) with simultaneous LDA and single hot-wire. The authors used a separation distance of 0.2 mm, although the measurement location is not reported in their results.

Profiles of the turbulent heat flux $\overline{u\theta}$ are presented in figure 6.4a. Values are positive due to lower velocities closer to the wall in the boundary layer correlating with lower temperatures within the cooler film. The reverse also applies. The distributions are similar to those of $\overline{\theta^2}$ (figure 6.2). For $VR = 0.37$, values of $\overline{u\theta}$ approach zero, where the bulk of the film begins to lift away from the surface and where the temperature gradients are small or negative.

Profiles of $-\overline{v\theta}$ are shown in figure 6.4b. Values of $\overline{v\theta}$ are generally negative because an upwards (positive) velocity fluctuation carries colder fluid into a region with a larger mean temperature (causing

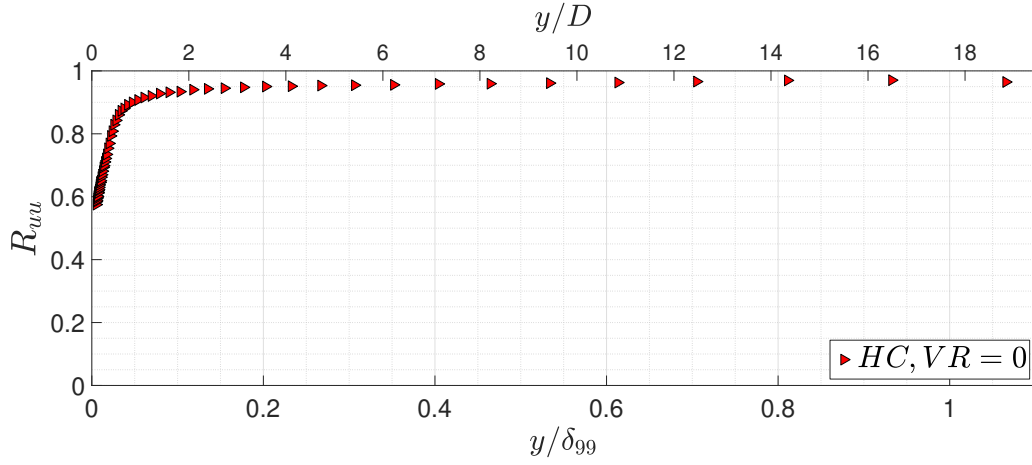


Figure 6.3: Autocorrelation coefficient of the streamwise velocity fluctuations from the x-wire and an adjacent single hot-wire, as a function of y/δ_{99} . The profile is taken at $VR = 0$ on the hole centreline.

a negative temperature fluctuation) and vice versa. The maxima in $-\overline{v\theta}$ correspond to the locations of the outer peaks in k , $\overline{u^2}$, $\overline{v^2}$ and are likely to be associated with the interface of the mixing layer between the film and the mainstream fluid.

Very near the wall, $-\overline{v\theta}$ approaches zero or becomes negative. Similar near-wall low values of $-\overline{v\theta}$ are reported in the film cooling study of Kohli & Bogard (2005). The authors attribute this to the jets still having a significant inclination angle near the wall, but away from the wall the jets flatten and become more parallel. Hattori *et al.* (2012) observe $-\overline{v\theta}$ approaching zero in the limit of $y^+ \rightarrow 0$ in a thermal boundary layer after the removal of wall heating. In this case, the temperature gradients near the wall become very small, as with the results of this experiment.

The gradient diffusion hypothesis assumes alignment between the turbulent heat flux vectors and the mean gradient vectors. The $VR = 0.37$ case on the hole centreline shows counter-gradient transport for $y/D < 0.4$. Here, the temperature gradient is negative (figure 6.1) but the $-\overline{v\theta}$ values are positive. A repeat experiment also showed the same phenomenon. This indicates that at higher velocity ratios the gradient diffusion hypothesis is not valid in the near-wall region.

Turbulent heat flux correlation coefficients $R_{u\theta}$ and $-R_{v\theta}$ are presented in figures 6.5a and 6.5b respectively. Maximum values of $R_{u\theta}$ exceed 0.6, which is higher than the maximum values of approximately 0.4 that Kohli & Bogard (2005) report with a mass flux ratio $M = 0.4$, with both low and high freestream turbulence. Maximum values of $-R_{v\theta}$ are approximately 0.4, but reach 0.5 for the $VR = 0.37$ cases. These values are similar to those that Kohli & Bogard (2005) report and also similar to the maximum values found in canonical thermal boundary layers (Chen & Blackwelder, 1978; Subramanian & Antonia, 1981a; Williams *et al.*, 2015). In canonical thermal boundary layers, the values remain approximately constant down to the wall. However, the present results show a large drop-off near the wall, where the temperature and velocity fluctuations are poorly correlated. In part, this is likely to be due to the low temperature gradients in this region. The location of the peak in the correlation coefficients scales well with the thermal boundary layer thickness.

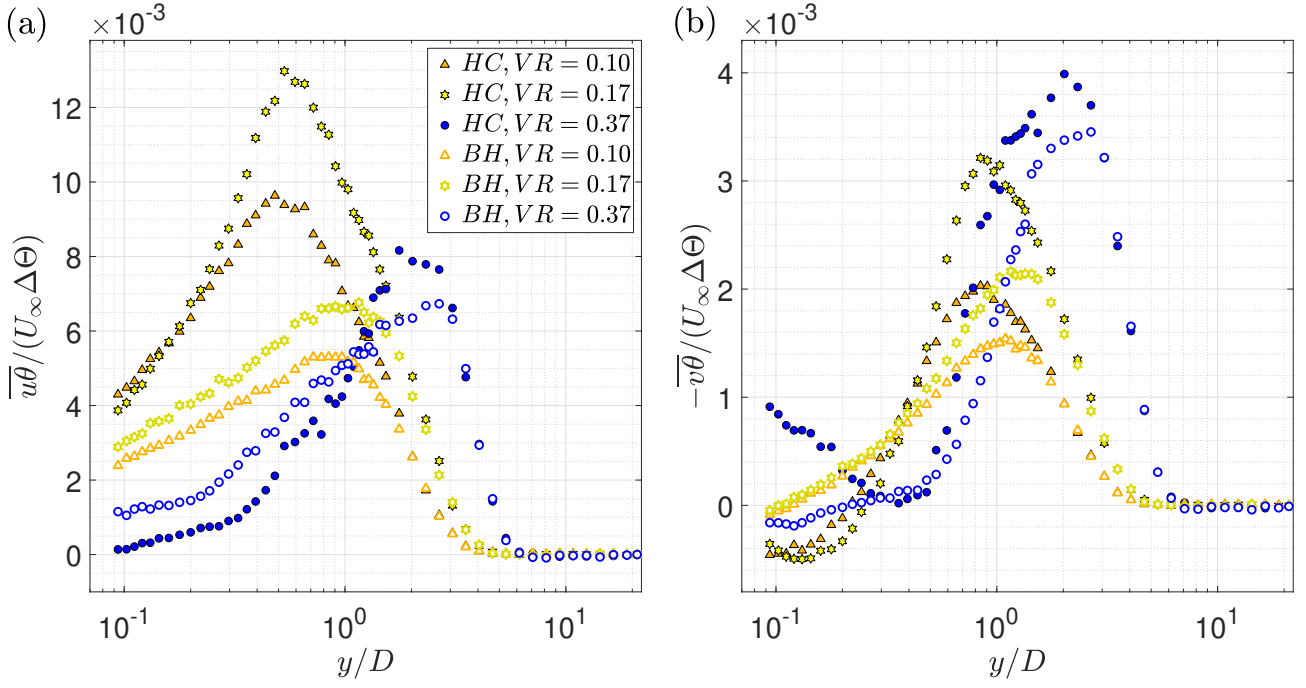


Figure 6.4: Turbulent heat flux distributions (a) $\overline{u\theta}$ and (b) $\overline{-v\theta}$, normalised with outer variables as a function of y/D .

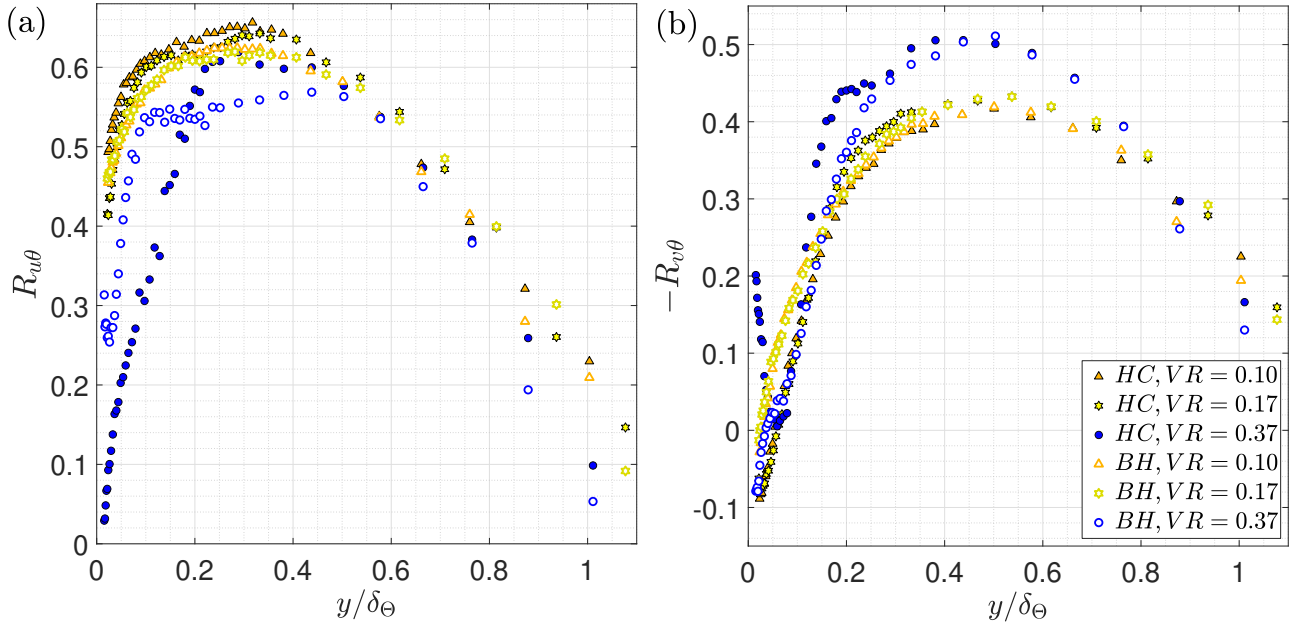


Figure 6.5: Turbulent heat flux correlation coefficient distributions (a) $R_{u\theta}$ and (b) $-R_{v\theta}$, as a function of y/δ_Θ .

6.4 Temperature variance budget

The thermal production ($\overline{v\theta}\partial\Theta/\partial y$) and transport ($\partial\overline{k_\theta v}/\partial y$) terms that feature in the k_θ -equation (equation 1.42) for a canonical thermal boundary layer are plotted in figures 6.6a and 6.6b respectively. It is assumed that the viscous transport term is negligible sufficiently far from the wall because the mean velocity is far larger than the molecular diffusion ($U \sim \mathcal{O}(1)$, $\mathcal{D} \sim \mathcal{O}(10^{-5})$). The wall-normal

turbulent diffusion term has also been conjectured to dominate thermal transport in film cooling flows (Kohli & Bogard, 2005).

The production distributions are similar to the temperature variance profiles. This suggests that the wall-normal gradient production term is still dominant in the scalar field, as also observed in the case of the momentum field. Turbulent thermal transport dominates over the thermal production near the wall in all cases. In the traditional overlap region, peaks in turbulent thermal transport are observed at the same location as the thermal production peaks. Their magnitudes are at least 20% of the maximum production in all cases, and up to 50% for the $VR = 0.10, 0.17$ cases on the hole centreline. This suggests an imbalance between production and destruction of the temperature fluctuations.

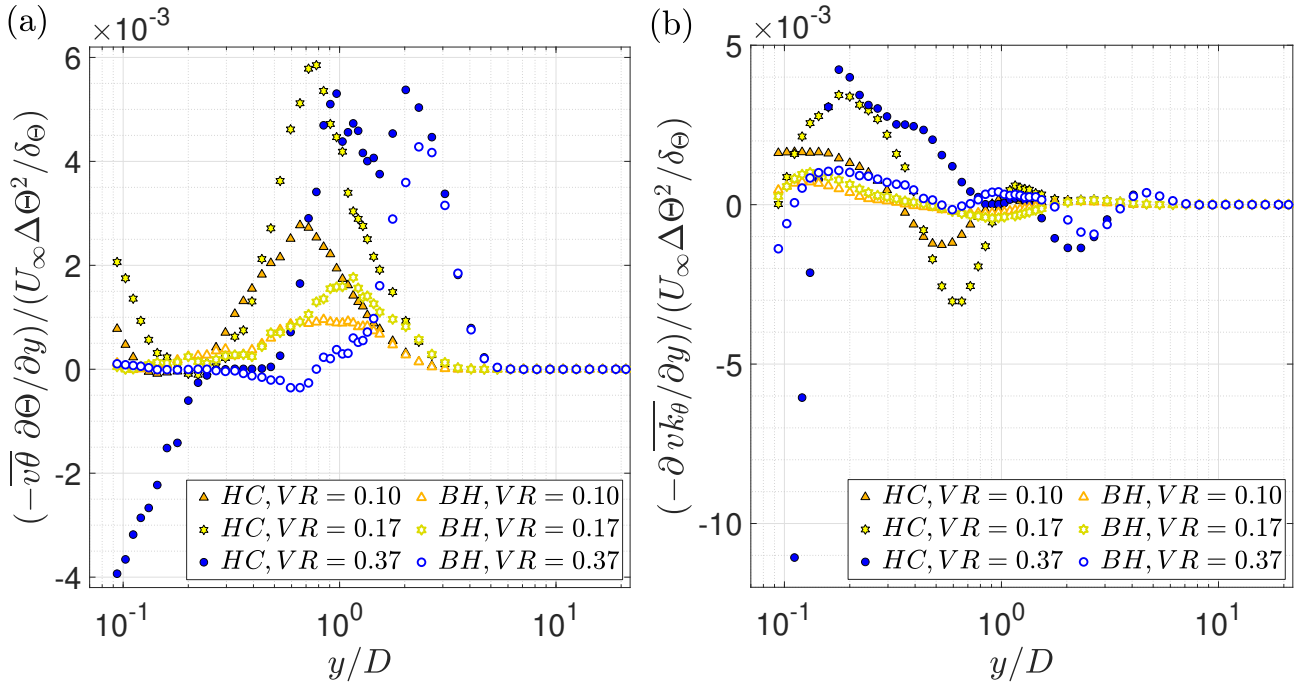


Figure 6.6: Distributions of (a) wall-normal thermal production and (b) turbulent transport, presented in outer scaling.

6.5 Higher order moments

The temperature skewness and flatness are presented in figures 6.7a and 6.7b respectively. Immediately noticeable is the increase in S_θ and F_θ at $VR = 0.37$, particularly on the hole centreline where it rises rapidly near the wall. This is consistent with the findings of Basley *et al.* (2022), who use the intensity of scattered light from seeding particles as a surrogate for temperature - at $VR = 0.37$, the near-wall scalar fluctuations are much more susceptible to modulation by large-scale outer structures. The near-wall sweep events and intermittency associated with them is therefore probably much larger at $VR = 0.37$. At lower velocity ratios, S_θ is positive on the hole centreline and negative between holes in the near-wall region. The distributions are approximately independent of VR for $VR < 0.17$.

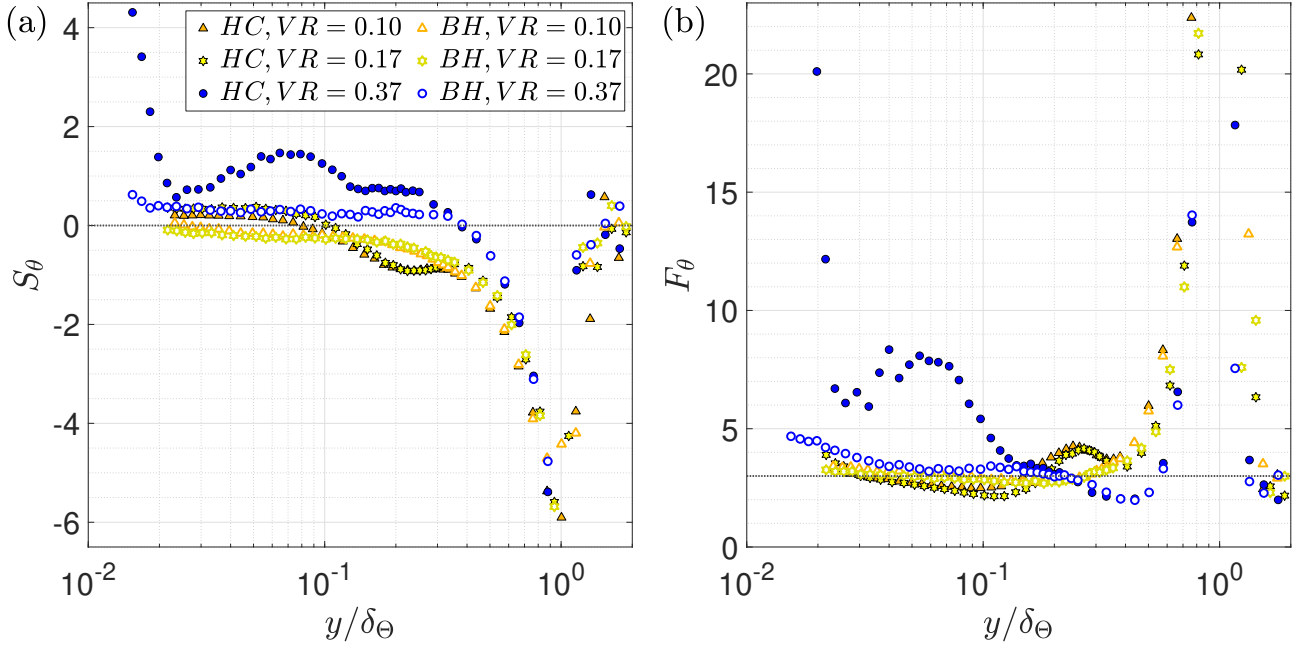


Figure 6.7: Profiles of (a) the temperature skewness and (b) temperature kurtosis.

6.6 Spectra

Premultiplied power spectral density maps of the temperature fluctuations on the hole centreline are shown in figure 6.8a. At lower velocity ratios, the peak in energy occurs at $y/\delta_\Theta \approx 0.1$ corresponding to streamwise wavelengths of $\lambda_x/D \approx 1 - 2$. For $VR = 0.10$ and $VR = 0.37$, the maximum energy is similar but in the $VR = 0.37$ case, the regions of largest energy occur at larger wall-normal distances, even when scaled with the thermal boundary layer thickness. The energy is distributed across larger streamwise wavelengths at $VR = 0.37$, due to the deeper penetration of the film into the boundary layer, with more interactions taking place between the cooler film and the mainstream boundary layer LSMs/ superstructures. Similar to the streamwise velocity fluctuations, the large-scale temperature fluctuations are also significantly attenuated near the wall at $VR = 0.37$.

There are significant differences in the distribution of energy between spanwise locations. Figure 6.8b shows the corresponding PSD maps for the spanwise location in between holes. In the near-wall region, there is a larger relative contribution to the turbulence from large-scale structures and the small-scale motions are attenuated.

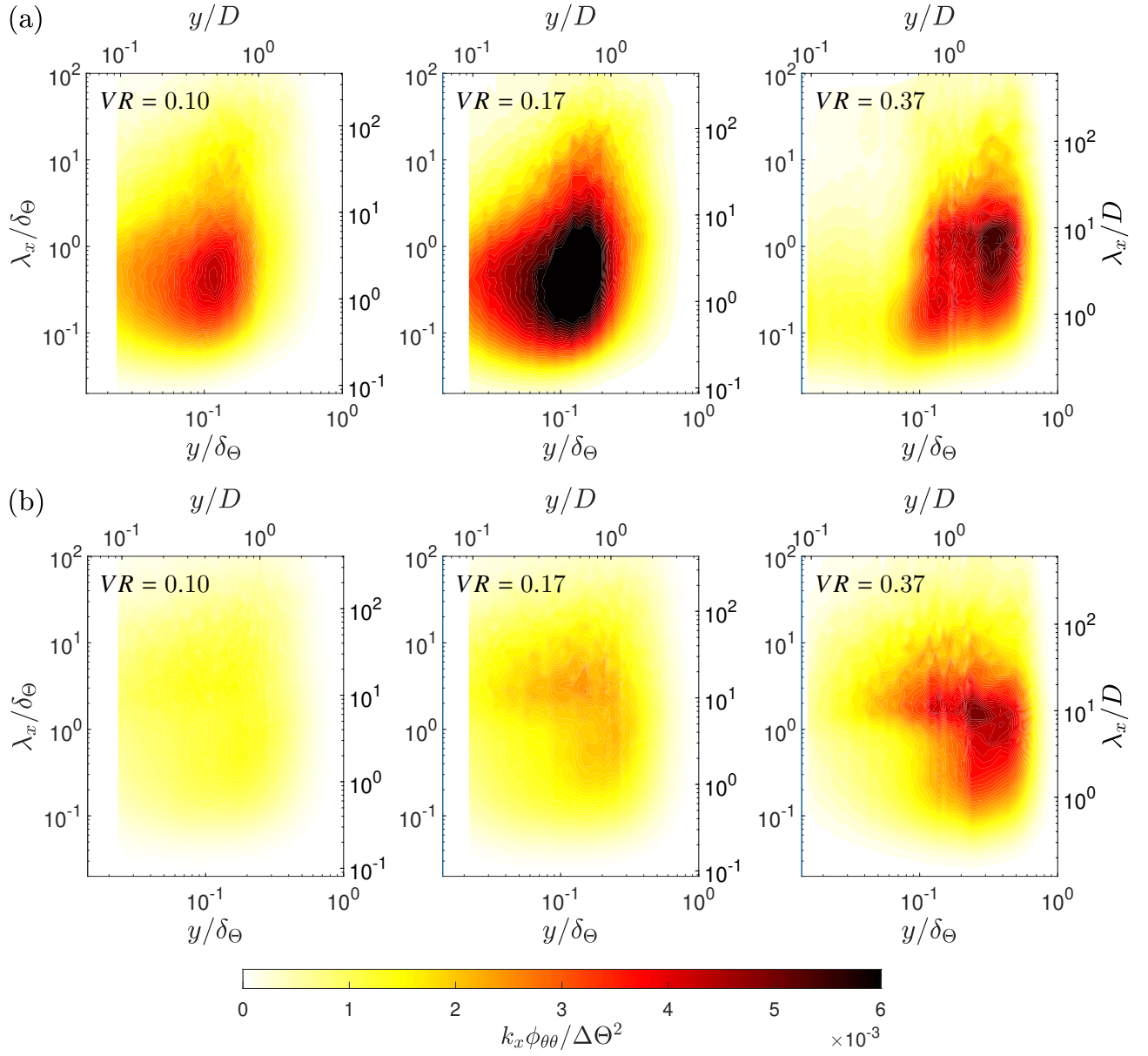


Figure 6.8: Premultiplied power spectral density maps of the temperature fluctuations: (a) on the hole centerline and (b) in between holes. The velocity ratio is indicated in the top left corner of each map.

Chapter 7

Modelling considerations

7.1 Mixing length model

The momentum and scalar mixing length distributions are shown in figures 7.1 and 7.2 respectively. The momentum mixing length in the viscous sublayer and buffer region is approximately the same as the unperturbed case and independent of velocity ratio and spanwise location. This suggests a relatively constant value of the Van Driest (1956) constant A (equation 1.32 is appropriate). This is in contrast to transpired turbulent boundary layers in which the sublayer is thickened and A is a function of V_e^+ (Kays *et al.*, 2005). The bulk of the injected momentum penetrates beyond the sublayer at these higher velocity ratios.

Departure from the unperturbed mixing length distribution occurs at $yu_{\tau_0}/\nu \approx 20$, $y/D \approx 0.1$ on the hole centreline and $yu_{\tau_0}/\nu \approx 40$, $y/D \approx 0.2$ between holes. Beyond this location, the gross failure of both the momentum and thermal mixing lengths to model any significant part of the boundary layer is apparent for all of the velocity ratios tested. This is unsurprising given the large turbulent transport of k and k_θ observed in figures 5.8 and 6.6b respectively. The thermal mixing length distributions presented show more severe departures across the whole boundary layer, including in the near-wall region.

Yavuzkurt *et al.* (1980*b*) propose a momentum mixing length model that is specific to film cooling in the recovery region, but state it could be adapted to the location between holes in the blowing region. Many of the aspects of their model are observed in figure 7.1b. The main difference is that region four in their model (the large peak in ℓ) seems to be before regions two and three in the present results. The large peak in ℓ corresponds to the plateau region in the mean streamwise velocity profile, where the mean shear rate is small. The only significant difference between the present experiment and that of Yavuzkurt *et al.* (1980*a,b*) is the ratio δ/D , so this parameter could scale the plateau region in the boundary layer.

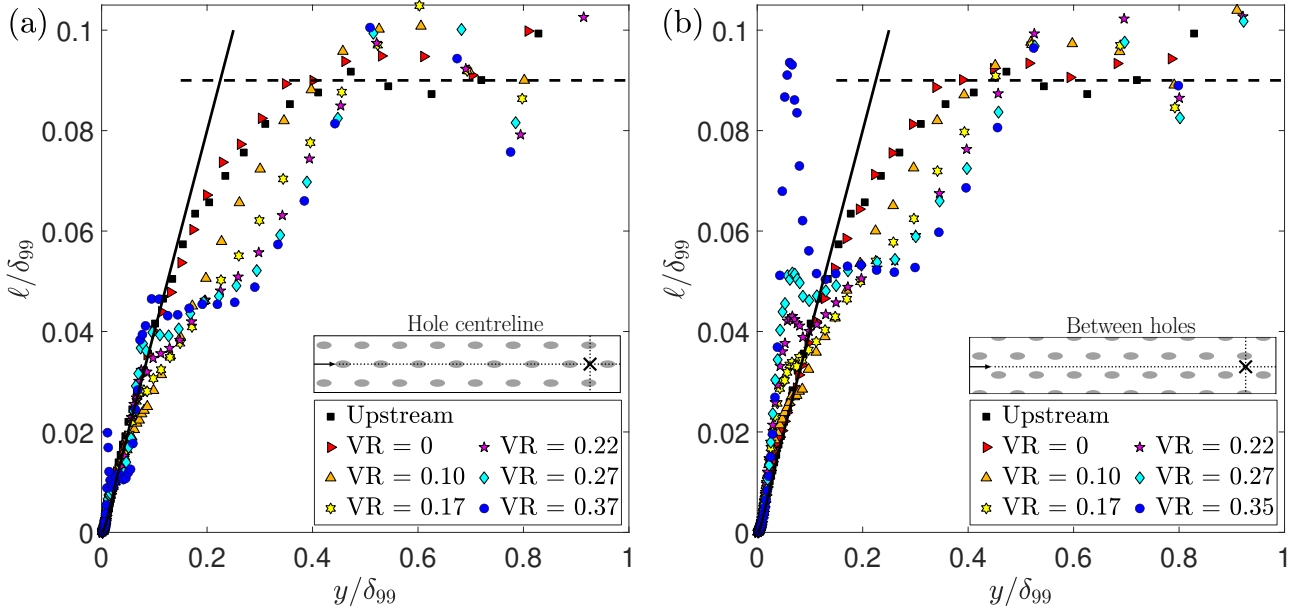


Figure 7.1: Distributions of Prandtl's mixing length (a) on the hole centreline, and (b) between holes, normalised by the boundary layer thickness. The solid line is the van Driest mixing length model (equation 1.32) with $\kappa = 0.41$ and the dashed line corresponds to $\ell = 0.09 \delta_{99}$ (Escudier, 1965).

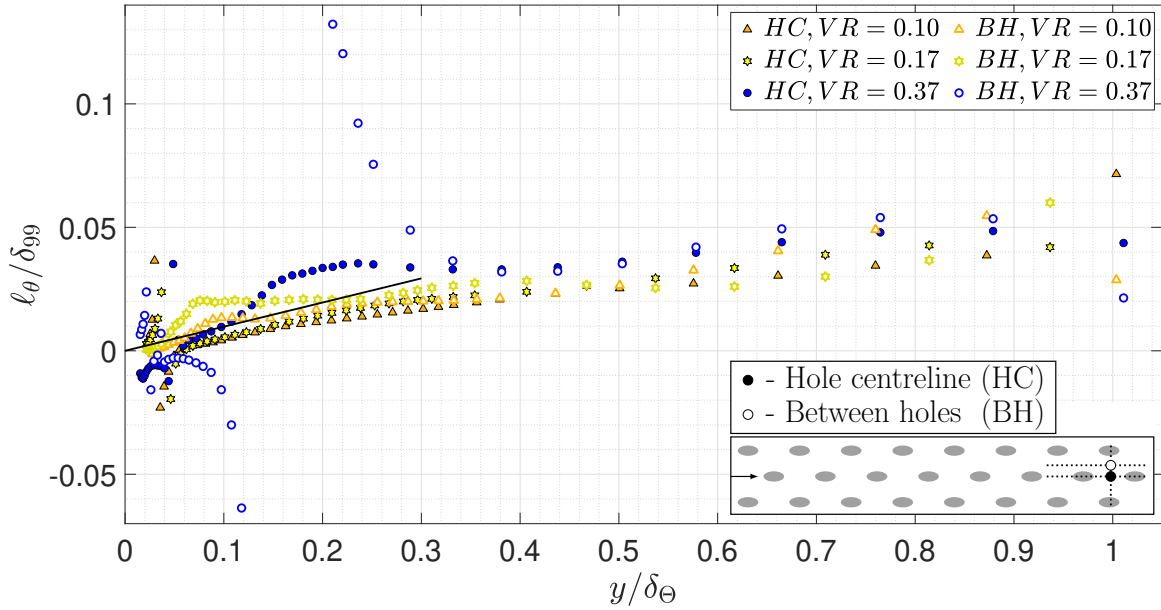


Figure 7.2: Distributions of the thermal mixing length normalised by the momentum boundary layer thickness d_{99} . The solid line denotes $\ell_\theta = \kappa_\theta y$ with $\kappa_\theta = 0.48$.

7.2 Structure parameters

Structure parameters a_1 (equation 3.2) and $a_{1\theta}$ (equation 3.4) are shown in figures 7.3 and 7.4 respectively. Here, the calculation for $a_{1\theta}$ uses $-\overline{v\theta}$ rather than $\overline{v\theta}$, due to the fact the surface is cooled rather than heated. The BFA model (Bradshaw *et al.*, 1967) assumes that they are universal functions of y/δ . They show significantly better behaviour than the mixing length. Both a_1 and $a_{1\theta}$ are approximately constant for $y/\delta > 0.2$. In this region, a_1 matches well with the canonical boundary layer values for

all blowing cases. Antonia *et al.* (1977) measure $a_{1\theta} \approx 0.62$ in a constant heat flux thermal boundary layer, which is similar to $a_{1\theta} \approx 0.6$, observed in the outer layer for $VR = 0.37$. For the lower velocity ratios, an approximate constant value of $a_{1\theta} \approx 0.5$ is measured.

Similarly to the mixing length, distributions of a_1 are unchanged in the sublayer and buffer layer relative to the canonical case. Near the wall between holes, $a_{1\theta}$ tends to zero due to low values of $-\overline{v\theta}$ corresponding to the low temperature gradients observed. However, the velocity gradients and $-\overline{uv}$ remain relatively larger. On the hole centreline for $VR \leq 0.17$, $a_{1\theta}$ becomes negative due to negative values of $-\overline{v\theta}$ and $d\Theta/dy$. At $VR = 0.37$, the distributions of $a_{1\theta}$ and a_1 vary rapidly, though in a similar manner.

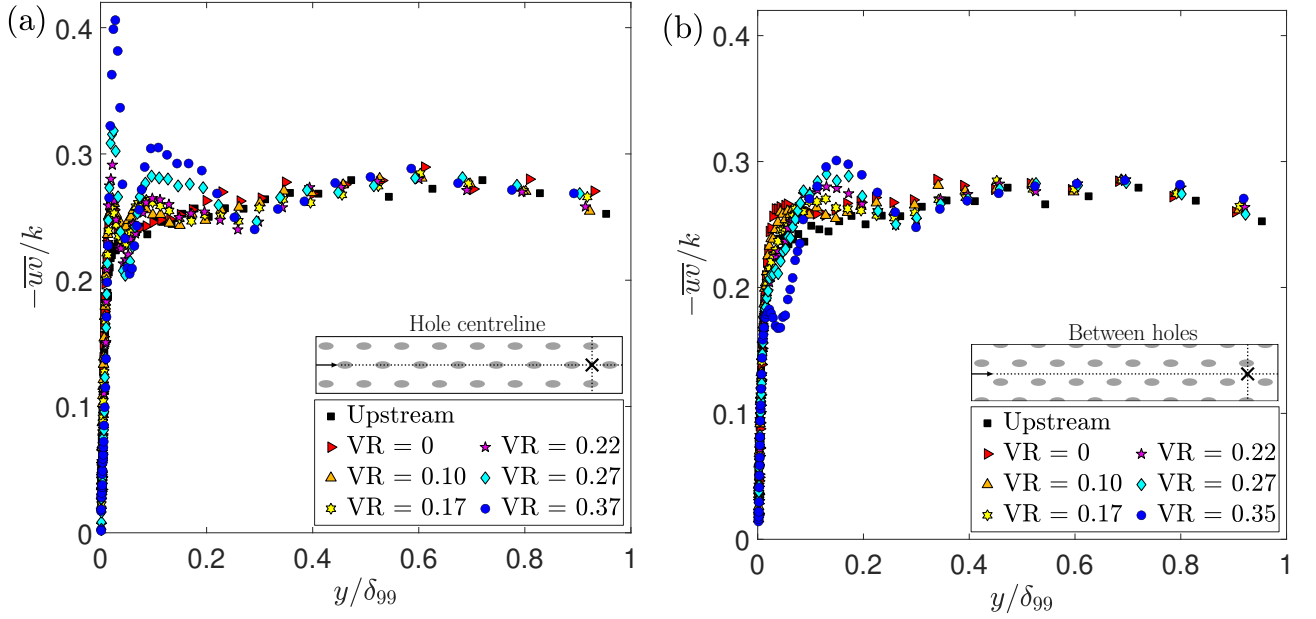


Figure 7.3: Distributions of the stress ratio a_1 (a) on the hole centreline, and (b) between holes.

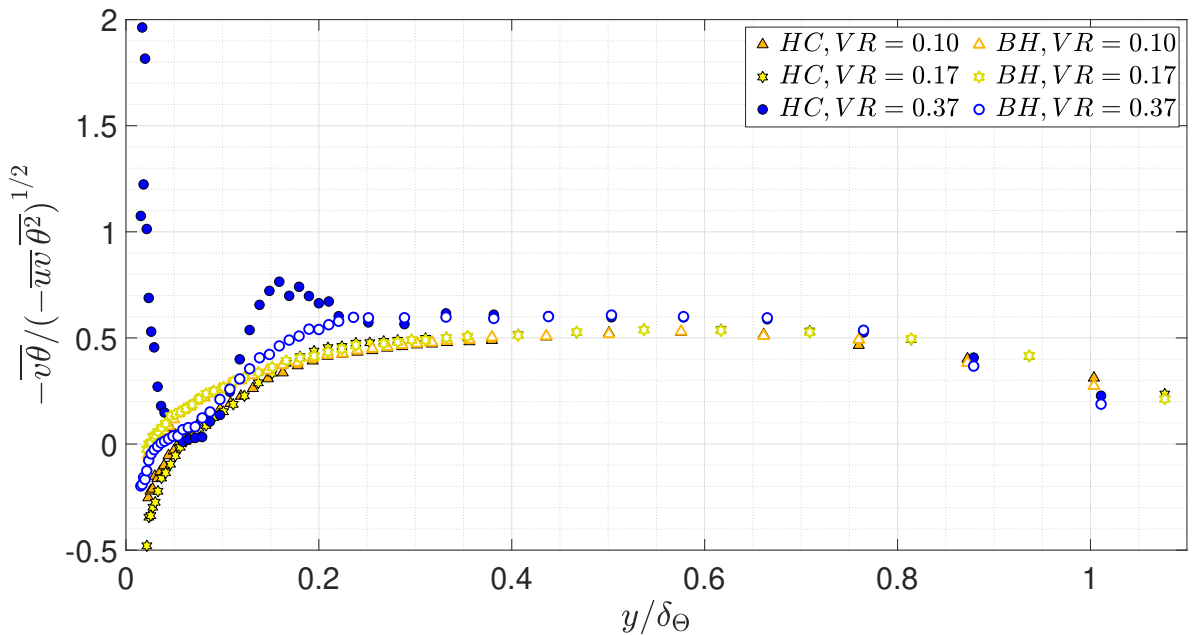


Figure 7.4: Distributions of the structure parameter $a_{1\theta}$

7.3 Turbulent Prandtl number

Figure 7.5 shows the distributions of the turbulent Prandtl number. Despite large variations in the momentum and thermal mixing lengths and significant departures from canonical behaviour, the turbulent Prandtl number in the outer region is relatively constant across all cases. Small departures from Reynolds' analogy are observed for $y/\delta_\Theta > 0.2$, with values measured in the range $1 < Pr_t < 1.4$. The scatter in this region is likely to be due to the large uncertainty associated with calculating Pr_t (Kays *et al.*, 2005).

For $y/\delta_\Theta < 0.1$, significant departures from Reynolds' analogy are observed with Pr_t varying significantly. For $VR = 0.37$ and at the lower velocity ratios on the hole centreline, regions of negative Pr_t are observed. Values of the turbulent momentum diffusivity generally remained positive, but regions of negative turbulent heat diffusivity contradict the gradient diffusion hypothesis and cause $Pr_t < 0$. In the $VR = 0.10, 0.17$ cases on the hole centreline and $VR = 0.37$ between holes, the regions of negative temperature gradient are small and the corresponding $-\overline{v\theta}$ values change sign and remain close to zero. Therefore it is difficult to distinguish whether the negative Pr_t values observed here are physical or due to experimental error, although the trends are consistent across the blowing cases.

However, for the $VR = 0.37$ hole centreline case, clear regions of counter gradient thermal transport are observed, leading to a pronounced region of negative Pr_t for $y/\delta_\Theta < 0.04$. In this case, as $d\Theta/dy \rightarrow 0$, $Pr_t \rightarrow \infty$, leading to the asymptotic behaviour of Pr_t about the minimum in the mean temperature profile. It is worth noting that very large measured values of Pr_t near this location fall outside the y-axis of figure 7.5, which has been truncated for clarity. DNS results of Kong *et al.* (2001) show the same asymptotic Pr_t behaviour downstream of blowing through a slot at $VR = 0.046$.

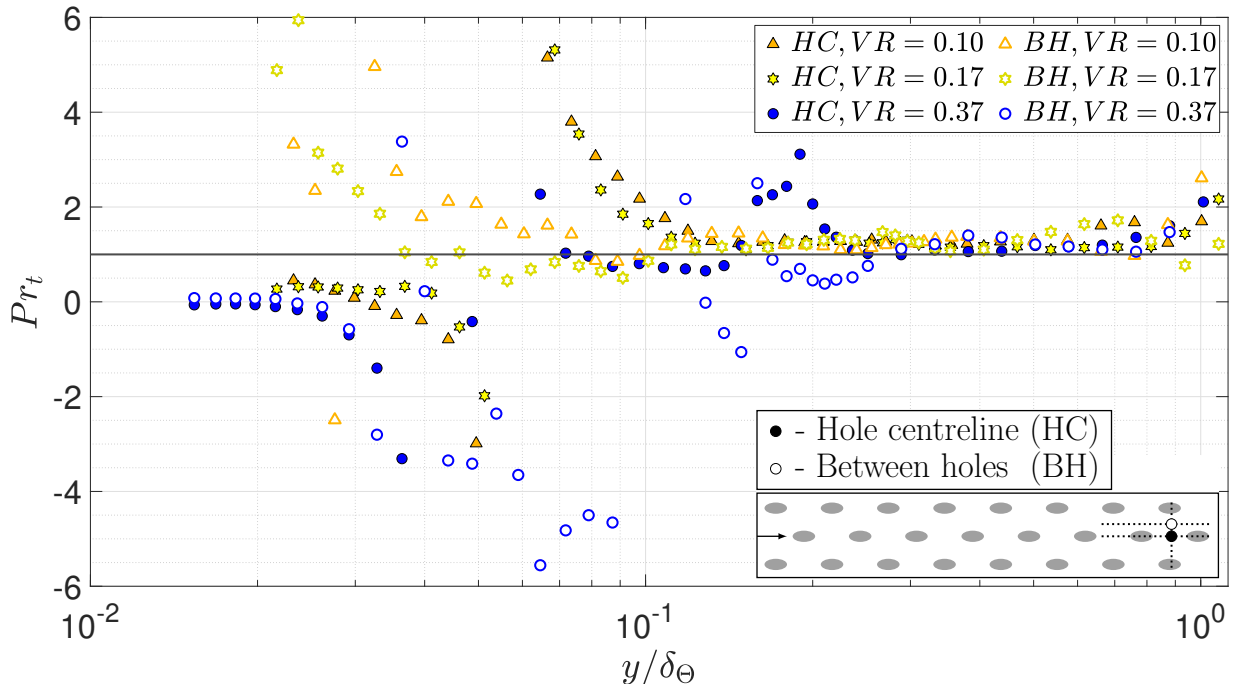


Figure 7.5: Distributions of the turbulent Prandtl number. The horizontal line corresponds to $Pr_t = 1$.

Conclusions

This thesis contains two distinct pieces of work, aligned only with the aim of improving understanding of the structure of turbulent boundary layers. The first part of the work focuses on the asymptotic scaling and structure of pressure-driven fully developed turbulent boundary layers in the intermediate region. It is motivated by the lack of a production mechanism to explain the emergence of an outer peak in streamwise variance at high Reynolds numbers. Furthermore, the observed TKE production distributions are inconsistent with a Reynolds shear stress scaling that assumes a logarithmic velocity profile. More specifically, an outer peak in the premultiplied production is observed that is further from the wall than the Reynolds shear stress maximum, but in the vicinity of it ($y_{mp}^+ > y_{m\tau}^+$). Logarithmic scaling indicates that they are at the same location.

In light of this, the scaling for the Reynolds shear stress maximum is generalised and determined empirically using high Reynolds number pipe and channel flow data available in the open literature. It is assumed that $y_{m\tau}^+ \propto Re_\tau^\beta$ for $0 < \beta < 1$, to satisfy the asymptotic behaviour of the intermediate variable. Through algebraic manipulation and empirical inspection of the condition $y_{mp}^+ > y_{m\tau}^+$, the constraint $\beta > 0.5$ is determined. Fitting to available data gives $\beta = 0.529 \pm 0.005$. Plots such as the premultiplied velocity gradient at the Reynolds shear stress maximum show an upward trend with increasing Reynolds number, which provides strong support for $\beta > 0.5$. It is also consistent with the view of several authors (Zagarola & Smits, 1998; McKeon *et al.*, 2004; Morrison, 2007; Marusic *et al.*, 2013) that the log law starts at a location beyond the $y_{m\tau}^+$ at Reynolds numbers currently obtainable.

A second-order Taylor expansion about the Reynolds shear stress maximum is used to derive a scaling law for the outer peak in premultiplied production. At low Reynolds numbers the outer peak in y^+P^+ is at an approximately constant location, as is the ‘shoulder’ in $\overline{u^2}^+$. Here, the y^+P^+ scaling did not work well. However, the prediction at high Reynolds numbers is good, where the location of outer peaks in y^+P^+ and $\overline{u^2}^+$ exhibit Reynolds number dependence. A velocity scale for the intermediate region is derived as a mean velocity difference across the intermediate layer of the form $\Delta U_m^+ \propto Re_\tau^{2\beta-1}$. The values ΔU_m^+ are shown to scale well with the previously determined value for β . The outer peak in streamwise variance is approximately constant across the Reynolds number range when scaled with the new intermediate velocity scale.

The second part of this thesis presents results from experiments focusing on the structure of a

turbulent boundary layer downstream of a perturbation from an effusion film. The motivation behind this work is the lack of experimental data in the literature that focuses on the flowfield dynamics and the turbulence structure of boundary layers in turbomachinery turbine blade cooling applications. The acquired data can be used to increase what is a currently poor understanding of heat transfer mechanisms in film cooling flows (Han, 2013) and validate CFD models. The geometry of a turbine blade with effusion cooling is idealised with injection of lower temperature air into a large turbulent boundary layer through a flat porous plate that consists of a staggered array of small holes. Wall-normal profiles of the velocity and temperature fields are measured at a fixed streamwise distance on the effusion plate. Two spanwise locations are interrogated: one on the centreline of the holes, and one between two columns of holes (where there is no immediate effect of injection).

Three-component velocity measurements are carried out with three-component laser Doppler anemometry. These are currently the most detailed and well-resolved boundary layer profiles in a film cooling-type flowfield to the author's knowledge. However, this comes at the cost of limited spatial information. Simultaneous temperature and velocity measurements are acquired with an x-wire and an adjacent cold-wire for turbulent heat flux measurements. Independent skin friction measurements are obtained with oil film interferometry.

A key result is the change in boundary layer scaling observed about a critical velocity ratio $VR_c \approx 0.2$, which is a much lower threshold than indicated by previous studies. The changes are summarised as follows:

- below VR_c , the skin friction coefficient reduces with increasing velocity ratio on the hole centreline, but assumes a constant value (to within experimental uncertainty) above VR_c .
- A region of negative wall-normal velocity is observed between holes beyond VR_c , with a corresponding region of enhanced positive wall-normal velocity on the hole centreline. This suggests the formation of streamwise vortices for $VR > VR_c$.
- A large outer peak in TKE develops associated with the mixing layer between the film and the mainstream boundary layer. The outer peak in TKE increases in magnitude with increasing VR for $VR < VR_c$, but begins to decrease for $VR > VR_c$.
- The spanwise velocity fluctuations begin to dominate the near-wall turbulence on the hole centreline for $VR > VR_c$.
- For $VR > VR_c$, the region of lowest temperature is observed away from the wall which suggests that the bulk of the film is lifted away from the surface beyond the critical velocity ratio.

The specific value of the critical velocity ratio is likely a function of δ/D . This ratio is much larger for the current experiment than that of previous studies, to enable closer representation with next-generation effusion/ quasi-transpiration cooling systems that will have smaller holes.

The results also show a large dissimilarity between the momentum and temperature fields, both in the mean and also in the turbulence structure. Turbulent transport of both TKE and temperature variance is large across the boundary layer, so the local equilibrium hypothesis is not valid. The “big and slow” experiment benefits from high-resolution data closer to the wall than what previous experiments are capable of achieving. Departures from canonical viscous scaling occur for $y^+ > 20$. In the overlap region, no evidence of logarithmic scaling is apparent and traditional modelling strategies are shown to be inadequate, particularly the mixing length model. In the outer region, structure parameters such as a_1 and $a_{1\theta}$ are approximately constant and similar to the unperturbed values. The turbulent Prandtl number assumes an approximately constant value $1 < Pr_t < 1.4$ in this region, although large departures from Reynolds’ analogy and counter-gradient transport of heat are observed near the wall.

Recommendations for future experimental work

The current results indicate that only Reynolds stress transport models may be capable of adequately predicting the flowfield, at least as far as RANS modelling is concerned. Therefore the momentum and scalar fields also need to be resolved in the streamwise and spanwise directions to calculate the full budgets for k and k_θ . Joint PIV and light intensity measurements of Basley *et al.* (2022) are capable of measuring turbulent flux gradients in the streamwise direction. Further experiments in the (y, z) -plane with stereoscopic PIV are recommended to measure spanwise gradients of k , W , $\overline{w^2}$, which are expected to be large. This experiment could also identify specific vortical structures in the boundary layer and their strengths at different velocity ratios, which would be useful in forming a basis for empirical modelling.

Spatial information of the temperature field in the x and z directions is important for understanding the streamwise and lateral dispersion of the jets. Simultaneous x-wire and cold-wire measurements in a grid would achieve this, although it would be cumbersome and have a limited spatial resolution. Simultaneous planar velocity and temperature measurements with thermographic PIV would be ideal. However, the technique often requires large temperature differences and hazardous fluorescent tracers, which are not suitable for large-scale facilities. Research by Chakravarthy (2021) indicates that DEHS particles commonly used in PIV experiments could be suitable for simultaneous measurement of temperature and velocity.

Finally, the simultaneous velocity/ scalar results with wall pressure measurements of Basley (personal communication, 2019) can be used to understand the influence of the pressure gradient on the dissimilarity between the temperature and momentum fields. Previous studies with slot injection (Kong *et al.*, 2001) have shown it is an important factor. It is probable that the highly three-dimensional flowfield of the current experiment produces more complex pressure distributions.

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Appendix A

Derivation of the location of the outer peak in premultiplied production

Equations 2.12 and 2.13 are repeated here from section 2.5.1 for readability:

$$\tau^+ - 1 = \left(a + \frac{qc^2}{2}\xi^2 \right) Re_\tau^{\beta-1}, \quad (\text{A.1})$$

$$y^+ = (1 + \xi)c Re_\tau^\beta. \quad (\text{A.2})$$

Through use of equations A.1 and A.2, the production P^+ in the vicinity of $y_{m\tau}^+$ is expressed as

$$\begin{aligned} P^+ &= \tau^+ \frac{dU^+}{dy^+} \equiv \tau^+ \left(1 - \tau^+ - \frac{y^+}{Re_\tau} \right), \\ &= \left[1 + \left(a + \frac{qc^2}{2}\xi^2 \right) Re_\tau^{\beta-1} \right] \left[- \left(a + \frac{qc^2}{2}\xi^2 \right) Re_\tau^{\beta-1} - (1 + \xi)c Re_\tau^{\beta-1} \right], \end{aligned} \quad (\text{A.3})$$

which is rearranged to give

$$P^+ = - \left[\left(a + \frac{qc^2}{2}\xi^2 \right) + (1 + \xi)c \right] Re_\tau^{\beta-1} - \left[\left(a + \frac{qc^2}{2}\xi^2 \right)^2 + \left(a + \frac{qc^2}{2}\xi^2 \right) (1 + \xi)c \right] Re_\tau^{2\beta-2}. \quad (\text{A.4})$$

Through multiplication of equations 2.13 and A.4, the locally-scaled premultiplied production is given by

$$y^+ P^+ = - \left[\left(a + \frac{qc^2}{2}\xi^2 \right) + (1 + \xi)c \right] (1 + \xi)c Re_\tau^{2\beta-1} + O(Re_\tau^{3\beta-2}), \quad (\text{A.5})$$

which is differentiated with respect to y^+ to give

$$\begin{aligned}\frac{d}{dy^+}(y^+P^+) &= \frac{1}{cRe_\tau^\beta} \frac{d}{d\xi}(y^+P^+) \\ &= -\frac{d}{d\xi} \left[\left(a + \frac{qc^2}{2}\xi^2 \right) (1 + \xi) + (1 + \xi)^2 c \right] Re_\tau^{\beta-1} + O(Re_\tau^{2\beta-2}).\end{aligned}\tag{A.6}$$

The location of the ‘outer’ peak in y^+P^+ (in the intermediate region), defined as $\xi = \xi_{mp}$, is derived through setting $\frac{d}{dy^+}(y^+P^+) = 0$. The leading order representation of this, $O(Re_\tau^{\beta-1})$, is therefore

$$\frac{d}{d\xi} \left[\frac{qc^2}{2}\xi^2(1 + \xi) + a(1 + \xi) + (1 + \xi)^2 c \right] = 0,\tag{A.7}$$

which upon differentiation, and substitution of $\xi = \xi_{mp}$, becomes

$$\frac{3}{2}qc^2 \xi_{mp}^2 + (2 + qc)c \xi_{mp} + (a + 2c) = 0.\tag{A.8}$$

Solving the quadratic of equation A.8, gives the solution of the location of the premultiplied production peak, relative to the Reynolds shear stress peak, as

$$\xi_{mp} = \frac{(2 + qc)}{3qc} \left[-1 \pm \sqrt{1 - \frac{6q(a + 2c)}{(2 + qc)^2}} \right].\tag{A.9}$$

The positive root in equation A.9 gives the solution to the outer y^+P^+ peak location, while the negative root corresponds to the trough in the cubic equation for the intermediate layer y^+P^+ approximation (equation A.5). Assuming the peak in premultiplied production is in the neighbourhood of the peak in Reynolds shear stress ($\xi_{mp} \rightarrow 0$), equation A.8 can be linearised to obtain

$$(2 + qc)c \xi_{mp,l} + (a + 2c) = 0,\tag{A.10}$$

such that

$$\xi_{mp,l} = \frac{-(a + 2c)}{c(2 + qc)}.\tag{A.11}$$

The same result is obtained by using the two-term Taylor expansion of the square root term in equation A.9 ($\sqrt{1-x} = 1 - x/2 - \dots$). Recalling the definition $d_1 = c/a$, and defining $d_3 = qc$, equation A.11 can be rewritten as

$$\xi_{mp,l} = \frac{-(2 + 1/d_1)}{2 + d_3},\tag{A.12}$$

which is the prediction for the location of the outer peak in premultiplied production used in Chapter 2.

Appendix B

Influence of turbine inlet temperature and pressure ratio on gas turbine efficiency and work output

The thermal efficiency of an idealised Brayton cycle is expressed as

$$\eta_{th} = \frac{1}{r_p^{(\gamma-1)/\gamma}}, \quad (\text{B.1})$$

where r_p is the pressure ratio P_2/P_1 , which is related to its corresponding temperature ratio through the isentropic relation

$$\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = \frac{\Theta_2}{\Theta_1}. \quad (\text{B.2})$$

As stated in section 3.6, increasing the pressure ratio therefore results in a higher thermal efficiency. To understand its relationship to the turbine entry temperature, the non-dimensional specific work output is used, given as

$$\frac{w_{net}}{c_p \Theta_1} = \left(\frac{\Theta_3}{\Theta_1} - \frac{\Theta_4}{\Theta_1}\right) - \left(\frac{\Theta_2}{\Theta_1} - 1\right). \quad (\text{B.3})$$

The optimum temperature Θ_2 to achieve the highest work output is obtained through $dw_{net}/d\Theta_2 = 0$. Applying this to equation B.3 yields

$$\Theta_{2,\max \text{ work}} = \sqrt{\Theta_1 \Theta_3}. \quad (\text{B.4})$$

Substituting equation B.4 into equation B.2, followed by some simple algebraic manipulation gives

$$\left[\frac{P_2}{P_1}\right]_{\max \text{ work}} = \left(\frac{\Theta_3}{\Theta_1}\right)^{\gamma/2(\gamma-1)}. \quad (\text{B.5})$$

Since Θ_1 is generally fixed as the atmospheric temperature, it is apparent that increasing Θ_3 leads to a larger optimum pressure ratio for maximum work output. In essence, a higher turbine entry temperature allows for greater pressure ratios, which leads to a larger thermal efficiency. It is for this reason that there is significant research interest in raising the turbine inlet temperature in aircraft engines. More detail can be found in Wang & Stiegel (2016).

Appendix C

Comparison between LDA and x-wire measurements

The LDA profiles were used to characterise the momentum field due to the capability of measuring the viscous sublayer, having a smaller measurement volume and being insensitive to temperature. However results such as the turbulent Prandtl number (presented in section 7.3) rely on x-wire data for the mean streamwise velocity and the Reynolds shear stresses. Therefore a comparison between the two techniques is presented here.

Experiments to validate x-wire measurements were performed with two-component LDA on the hole centreline for $VR = 0, 0.10, 0.37$. A comparison of the mean streamwise velocity profiles is presented in figure C.1. Overall the agreement between the techniques is good and the same perturbations to the velocity profiles are captured for each velocity ratio. The x-wire under-predicts the streamwise velocity relative to the LDA measurements in the near-wall region at $VR = 0.10$. Different x-wire temperature correction schemes were tested. This included the method outlined by Hultmark & Smits (2010) and varying the exponent in equation 4.12 (Tropea *et al.*, 2007), but neither accounted for the observed discrepancy.

Profiles of the turbulent momentum diffusivity are shown in figure C.2, which is required for determining Pr_t . Overall the trends observed between the two techniques for a given velocity ratio are consistent. The x-wire measurements slightly under-predict ϵ_m relative to the LDA measurements. In the outer layer, this discrepancy is approximately 10 % – 20 %. However, this difference still falls within the variability of $1 \leq Pr_t \leq 1.4$ that was measured in this region. The Reynolds shear stress magnitudes were generally lower for the x-wire measurement, which could be attributed to the larger measurement volume.

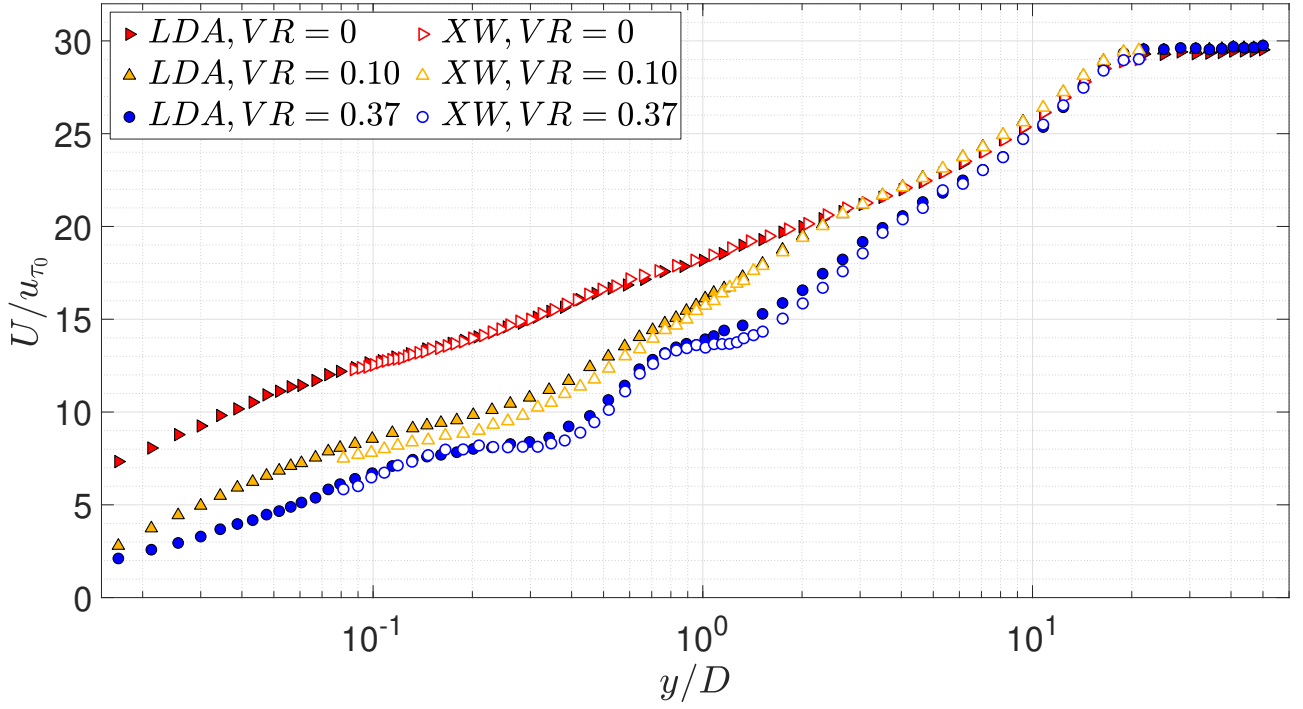


Figure C.1: Comparison of the mean streamwise velocity profiles acquired with two-component LDA (filled symbols) and with the x-wire (hollow symbols).

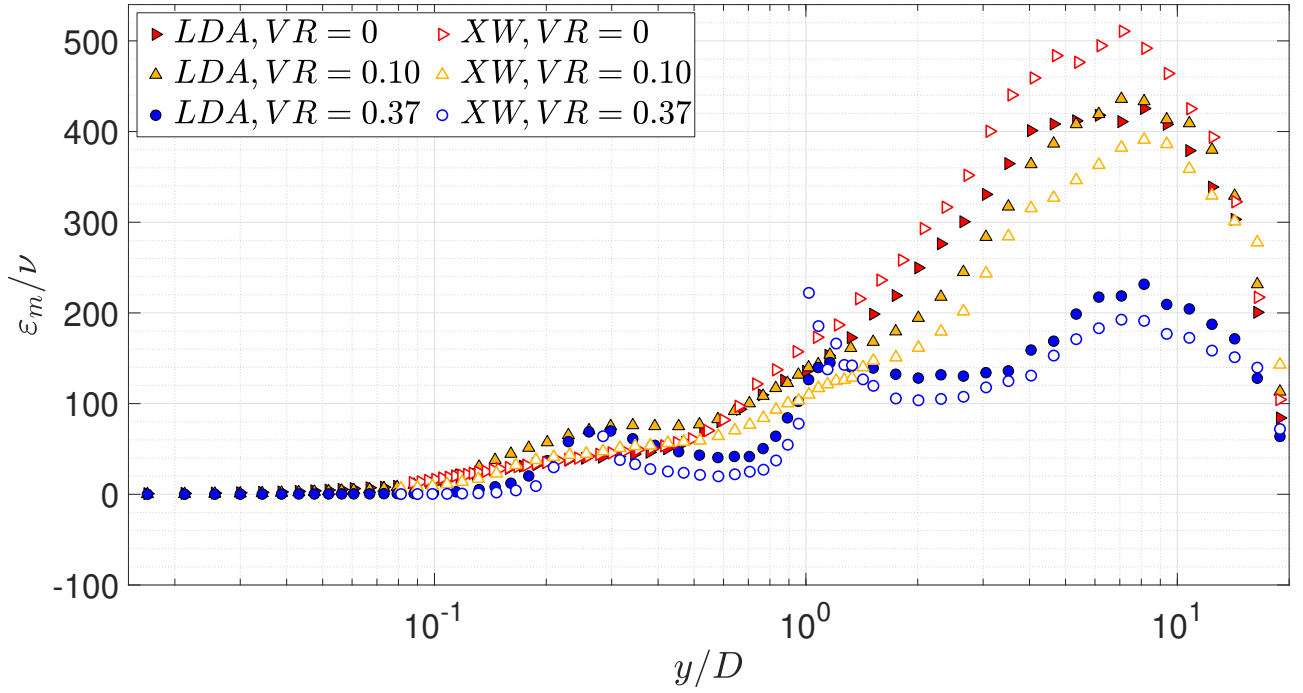


Figure C.2: Comparison of the turbulent momentum diffusivity profiles acquired with two-component LDA (filled symbols) and with the x-wire (hollow symbols).