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Development of a Two-Axis Cold Atom Interferometer System for Acceleration and Rotation Measurements

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Abstract

The use of atom interferometers within inertial navigation systems is very promising due to the potential for excellent bias stability and scale factor stability. This thesis presents the further development of a two axis atom interferometer system able to measure horizontal accelerations along two orthogonal axes and rotations along three orthogonal axes. An expression is derived for the phase of the interferometer in a moving vehicle near the surface of the rotating earth to better understand its use within an inertial navigation system.

Rubidium 87 atoms are collected in a side-arm 2D MOT and pushed into the ultra-high vacuum chamber where they are captured in a 3D MOT. The atoms are then cooled to few μK through optical molasses and prepared in a magnetically insensitive ground state with a narrow velocity distribution using a series of microwave and optical pulses. Using a $\pi/2 - \pi - \pi/2$ sequence of Raman pulses on an ensemble of atoms forms a Mach-Zehnder interferometer in which the atomic wavepackets from each arm interfere coherently at the output to measure accelerations or rotations. To improve the interferometer's sensitivity, the noise and fringe contrast are optimised, resolving fringes for an interferometer having a sequence of three laser pulses separated by more than 25ms. Further analysis shows the interferometer's shot-to-shot noise is $0.35\mu\text{g}$, almost an order of magnitude smaller than the MEMS accelerometer. Furthermore, this points to the possibility of achieving a sensitivity of 100ng at 1s by increasing the repetition rate to 10 shots per second. By reversing the direction and/or the relative phase of the Raman beams, the noise of the interferometer associated with acceleration and rotation averages down steadily as $t^{-1/2}$ over more than 10 hours. The intrinsic phase bias of the interferometer is found to be only few tens of milliradians with a drift of around a milliradian over 10 hours. For an interferometer with pulses separated by 25ms this corresponds to a drift of only 10ng, a thousand times better than the drift of the MEMS accelerometer.

Declaration

I declare that this thesis is my own work and all else is appropriately referenced.

Nicola Bilton

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Chapter 1

Introduction

1.1 Motivation

Inertial navigation is a technique by which position, velocity, attitude and heading can be dynamically calculated using accelerometers and gyroscopes. An inertial navigation system combines the integrated signals over time from multiple accelerometers and gyroscopes, usually three of each arranged orthogonally.

Inertial navigation systems (INS) have advanced since their inception in the 1940s. In early systems the sensors were mounted on a stable plate that was connected to the main platform by a series of motor-driven gimbals[2]. These systems were mechanically very complex and susceptible to thermal variation due to the heat dissipated by the motors. An alternative configuration was proposed in 1962[3] in which the sensors were mounted directly on the main platform. This was known as the “strapdown” configuration. The required dynamic range of gyroscopes in this configuration was about four orders of magnitude higher than in the gimbaled system, and a corresponding improvement in scale factor was also needed. Accelerometers were also required to have a better performance, including bandwidth, scale factor and bias accuracy, however this was less of a problem to overcome. Strapdown systems became a reality in the 1970s[2] after the development of the ring laser gyro (RLG) which was able to meet the dynamic range and scaling factor requirements. This is by far the most common configuration in modern

systems today[4].

Current state-of-the-art accelerometers and gyroscopes, such as microelectromechanical systems (MEMS) devices, can offer high precision measurements however may still have severe errors due to bias and bias drift[5]. Due to the double integration over time of the accelerometer output signal, bias or scale factor drift over time results in an error in position that increases with time. For applications over short timescales such errors may not be significant, however for longer term applications the use of some additional sensor is necessary to reduce such an error.

The advent of global navigation satellite systems (GNSS), such as the United States' Global Positioning System (GPS) in the 1980s, offered an external signal to aid the operation of inertial navigation systems by providing accurate periodic updates of position, allowing the internal errors caused by the inertial sensors to be corrected. Due to the highly accurate position information at relatively low cost offered across the entire globe, GPS has quickly become the primary signal used in augmented inertial navigation systems[6]. This technique of augmentation in inertial navigation systems allows much lower-performing inertial sensors to be used. Despite the extensive use of GNSS in augmented inertial navigational systems, it comes at the cost of its vulnerability to jamming or signal degradation in challenging environments such as underwater, indoors or deep valleys. Therefore, for particular applications or in the interest of having a resilient system, a completely stand-alone inertial navigation system that can provide high accuracy information for long periods of time would be advantageous.

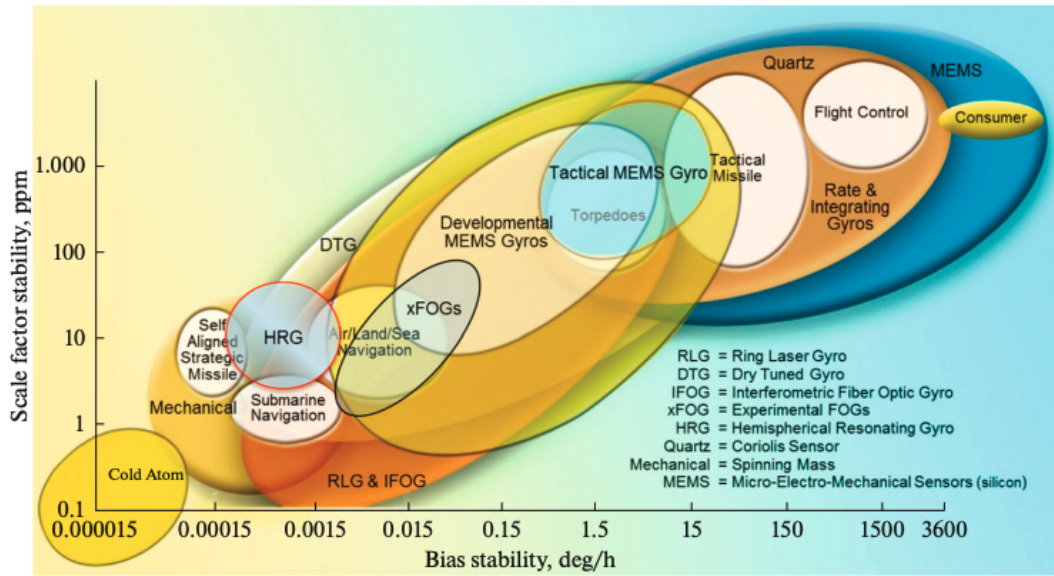
Table 1.1 shows how gyroscope and accelerometer bias stability performances are typically categorised in terms of application grade[4, 7]. In general, the lower the bias instability, the lower the error in position estimate for an inertial navigation system. Therefore low bias instability is critical for applications that require a high degree of accuracy over long periods of time. The standard definition of bias instability is the minimum point on the Allan deviation plot. As the bias instability is very hard to be calibrated or filtered out[8], it is a very important performance parameter to consider for an accelerometer or gyroscope in an inertial navigation system.

Application Grade	Gyro Bias Instability	Accelerometer Bias Instability
Consumer/Commercial	$>1 \text{ deg s}^{-1}$	$> 50 \text{ mg}$
Tactical	$\sim 1 \text{ deg h}^{-1}$	$\sim 1 \text{ mg}$
Navigation	$\sim 0.01 \text{ deg h}^{-1}$	$\sim 25 \mu\text{g}$
Strategic	$\sim 0.001 \text{ deg h}^{-1}$	$\sim 1 \mu\text{g}$

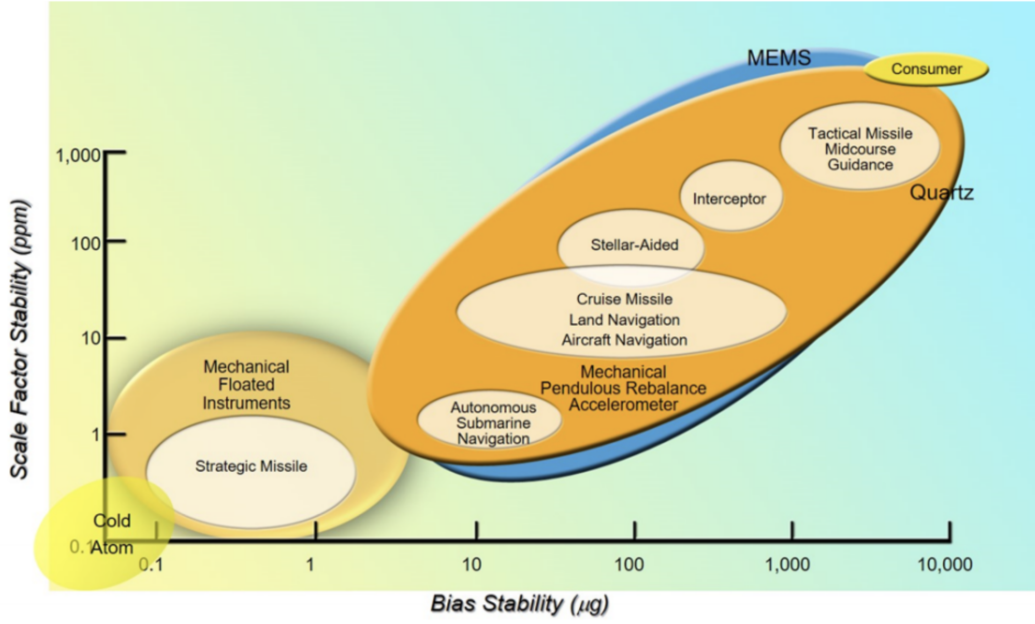
Table 1.1: Inertial sensor application grades and the corresponding required bias stability performance.

Other key performance parameters would need to be considered for a particular application. These include the short term sensitivity, scale factor stability, bias offset, temperature sensitivity, and shock and vibration sensitivity, but it is the bias and bias instability that limits the performance of existing high-end navigation systems.

Interferometry using cold atoms is a very promising approach to improving on existing inertial sensors because of the potential for excellent bias stability and scale factor stability as indicated in Figure 1.1. These figures, published in [9] are intended to give an overview of how accelerometer and gyroscope technologies are predicted to compare in the future. Cold atom interferometers are extremely sensitive to inertial forces and have an inherent stability due to the use of individual atoms as test masses within a very isolated environment.



(a)



(b)

Figure 1.1: The scale factor and bias stability achieved with different (a) gyroscope and (b) accelerometer technologies, with the expected performance of cold atom sensors overlaid. The stability requirements for various applications are also shown.

1.2 Cold Atom Interferometers

In an optical interferometer, a coherent light beam is split into two beams which travel along different paths before being recombined on a beam splitter. Because of interference, the power emerging in the output depends upon the difference of the phases accumulated along the two separate paths. By monitoring the output power, this phase difference is determined, allowing the interference to be a sensitive tool for measurements in many fields of science and engineering. Interferometry was invented in the late 19th century and used in the famous “Michelson-Morley Experiment” to disprove the existence of the proposed ether[10]. Initially interferometers used light as the coherent wave source, however the technique has since been extended to matter-waves, such as electrons[11, 12], neutrons[13, 14], atoms[15, 16, 17] and even molecules[18].

This thesis concerns an atom interferometer, in which the wave that is split and recombined is the atomic wavefunction, and the measurement being made is of inertial forces, i.e. the

acceleration and rotation of the atoms relative to the apparatus surrounding them. A matter-wave interferometer sensitive to inertial forces was first demonstrated in 1975 using neutrons[19]. Stimulated by the neutron experiment, several neutral atom interferometers were proposed in the 1980s[20, 21, 22, 23] to measure a variety physical effects. However a major obstacle in the development of atom interferometers was the difficulty of coherently manipulating the atomic wavefunction in a way analogous to optical beamsplitters and mirrors.

In 1991, Kasevich and Chu provided the groundwork for overcoming this issue using Stimulated Raman transitions[24]. These employ counter-propagating light beams to drive a two-photon electric dipole transition between two hyperfine sublevels of the electronic ground state. The light is sufficiently detuned from the intermediate excited state that there is a low probability of single-photon scattering and this ensures that the transition amplitude is largely phase coherent. Of course, a phase coherent transition can be driven by a microwave magnetic field, but the microwave photon has very little momentum. By contrast the Raman beams impart a two-photon recoil momentum when they drive the transition, and it is this recoil that allows the atomic wavefunction to be split onto two separate paths.

When the frequency difference between the two light beams is resonant with the hyperfine transition the wavefunction can evolve from state 1 to state 2 and back again: this is called Rabi oscillation. If the Raman pulse is applied for the right length of time - a so-called $\pi/2$ pulse - the atom is driven to a coherent superposition state with equal amplitudes for states 1 and 2, and these diverge along two separate paths because of the photon recoil. This $\pi/2$ pulse therefore acts as a beamsplitter for atoms. Alternatively, if we apply a Raman π pulse, the internal state amplitudes are interchanged (i.e. $1 \leftrightarrow 2$) and so are the momentum states, therefore a π pulse is analogous to a mirror.

Kasevich and Chu made the first measurement of gravity using an atom interferometer in 1992[15]. By applying a series of $\pi/2 - \pi - \pi/2$ pulses, they were able to create a Mach-Zehnder type interferometer with atoms. The phase accumulated by the atomic wavepacket was different on each of the two separated paths by an amount proportional to the gravitational acceleration g . By measuring the interference pattern in the probability of the two states at the output they

were able to determine g .

Since this first demonstration of a gravity measurement using an atom interferometer in 1992, there has been significant developments in the field of atom interferometry to measure inertial forces, with numerous high sensitivity gravimeters[25, 26, 27, 28], gravity gradiometers[29, 30, 31, 32] and gyroscopes[33, 34, 35, 36, 37] being demonstrated. The inherent high sensitivity and long term stability of atom interferometers have also made them a desirable tool for studies of fundamental physics and precision measurements, including measuring the fine structure[38] and the gravitational constants[39, 31] and even dark energy searches[40, 41].

1.3 Towards Cold Atom Inertial Navigation Systems

Navigation systems using classical sensors can exhibit broad bandwidths and high dynamic range, however they are limited at long times by the bias instability of the sensors. By contrast, cold atom based systems will likely be limited by low repetition rates and dynamic range but promise extreme long term stability.

The best of both worlds would be a hybrid system that employs both classical and quantum sensors. By monitoring correlations between the classical and quantum signals, the bias drift of the classical device can be tracked over time. A continuous high bandwidth hybrid inertial sensor is then achieved by subtracting this bias drift from the output of the classical sensor. The benefits and potential of a hybridised system have been demonstrated in recent work[42, 43, 44], primarily concentrating on incorporating classical accelerometers into cold atom gravimeter systems.

In addition to their application to inertial navigation, cold atoms also have very promising potential as ultra precise sensors in metrology, geophysics, civil engineering, and oil and mineral exploration[45, 46]. Due to the wide range of potential applications cold atom interferometry outside the laboratory, there has been increased focus recently on the development of more transportable accelerometers[47, 48, 49, 46, 50] and gravimeters[51, 52]. These devices are starting to operate outside the laboratory in noisy and challenging environments.

1.4 Previous results

The aim of the wider ongoing programme at Imperial College London is to develop a complete 6-axis (3-axis accelerometer and 3-axis rotation sensor) transportable hybrid inertial navigation sensor which combines the fast bandwidth and high sensitivity of classical sensors with the long term stability of the cold atom sensors. This wider work has involved developing three generations of systems; a horizontal one Raman axis system, a horizontal two Raman axis system and a three Raman axis system. The work presented in this thesis pertains to the two Raman axis second generation system.

The first atom interferometer developed within the group was presented in [53, 54], demonstrating sensitivity to accelerations on one horizontal axis. The system was successfully operated in a vibrating laboratory environment by combining the interferometer with a MEMS accelerometer. The total phase noise of the interferometer was found to be 134 mrad at 1s which corresponds to acceleration noise of $1.7\mu\text{g}$. This was over an order of magnitude more than the sensitivity goal of 100ng at 1s. This system was then successfully modified to operate outside a laboratory environment.

The demonstration of this one axis cold atom accelerometer system was closely followed by the design and assembly of a two axis system - the subject of this thesis. This system was also designed with a sensitivity goal of 100ng at 1s. Initial demonstration and characterisation of the interferometer fringes produced along both axes were described in [1]. The dominant noise source was the MEMS accelerometer with an acceleration noise of $7.3\mu\text{g}$ per shot followed by the detection noise of $3.5\mu\text{g}$ per shot. Sampling four times per second the total noise was $4.1\mu\text{g}$ at 1s.

The focus of my PhD has been to build on the work of the two axis accelerometer system presented in [1], with a particular focus on increasing the sensitivity (by improving the interferometer noise and contrast), characterising the bias of the two interferometers, and begin extending the two axis accelerometer system to measure rotations.

The interferometer noise was improved by reducing the detection noise to 3×10^{-3} per shot

(which corresponds to $0.3\mu g$ per shot for an interferometer time of 25ms and contrast of 20%), and reducing the acceleration noise of the MEMS accelerometer to $5\mu g$ per shot. The contrast of the two interferometers' fringes was increased to 22% and 18% by eliminating a residual magnetic field gradient. The stability of the two interferometers was also improved by implementing two techniques, known as k-reversal and π -flipping. Using both techniques, I further characterised the noise of the interferometer phase associated with acceleration and rotation, which averaged down at $t^{-1/2}$ over more than 10 hours. I also measured the bias of the two interferometers, finding this phase component intrinsic to the interferometer (i.e. not due to motion) drifts by a few milliradians over 10 hours.

Additionally, I derived an expression for the phase of the interferometer in a vehicle that is accelerating and rotating near the surface of the rotating earth to better understand how an atom interferometer would be used as part of an inertial navigation system.

1.5 Outline of Thesis

Chapter 2 provides an overview of the theory behind stimulated Raman transitions and how these are used to coherently manipulate the atoms. Following this in Chapter 3 I derive an expression for the phase of the interferometer in a vehicle that is accelerating and rotating near the surface of the rotating earth and describe how to measure the interferometer phase.

I detail the experimental apparatus in Chapter 4, including the custom made vacuum chamber, laser system and 2D and 3D magneto-optical traps. Chapter 5 describes the sequence of steps required to trap, cool, and launch the atoms and to prepare their internal state so that they are ready for the three pulse interferometer sequence.

Chapter 6 describes my investigation and optimisation of various noise sources in the interferometer and implementation of k-reversal and π -flipping techniques within the system. I then go on to measure the interferometer bias along both axes and study the stability of the interferometer phase.

In Chapter 7 I diagnose and correct the loss of contrast with increasing interferometer time T , which was described in [1] and I demonstrate the interferometer's sensitivity to rotation. Finally, in Chapter 8 I summarise the contents of this thesis.

Chapter 2

Stimulated Raman Transitions

Since the first proof of principle atom interferometer experiments in the early 1990s there has been considerable development in the field, resulting in various types of atom interferometer systems. This includes development of a four pulse “butterfly” configuration[55, 56], or an even larger number of pulses in systems employing the large momentum transfer (LMT) technique[57, 58, 59]. Bragg transitions have also become an alternative to stimulated Raman transitions to coherently manipulate atoms[60, 61].

In this experiment we use ^{87}Rb as the atomic source with a $\pi/2 - \pi - \pi/2$ Raman pulse interferometer sequence. My apparatus consists of two independent interferometers which measure two orthogonal components of horizontal acceleration, and rotations about three axes. The experimental apparatus design and assembly will be discussed in more detail later in Chapter 4.

2.1 Rubidium 87 Atomic Structure

Rubidium (Rb) is an alkali atom with two naturally occurring isotopes, ^{87}Rb and ^{85}Rb . Alkali atoms are often used in atom optic experiments due to their single valence electron in the 5s shell, which can be optically pumped on its closed cycling transition for cooling and trapping.

In addition, the ability to easily achieve an atomic vapour of alkali atoms and the range of readily available lasers at suitable wavelengths make these atoms an ideal candidate for such experiments.

The valence electron in ^{87}Rb is found in the $5S_{1/2}$ ground state. Transitions to the $5P$ state can be split into two lines; D1 ($5S_{1/2} \rightarrow 5P_{1/2}$) and D2 ($5S_{1/2} \rightarrow 5P_{3/2}$). The D2 transition is often used in experimental atomic physics due to its cycling transition for efficient cooling and trapping of ^{87}Rb . The key transitions within the hyperfine energy levels of the D2 line used in this experiment are indicated in Figure 2.1.

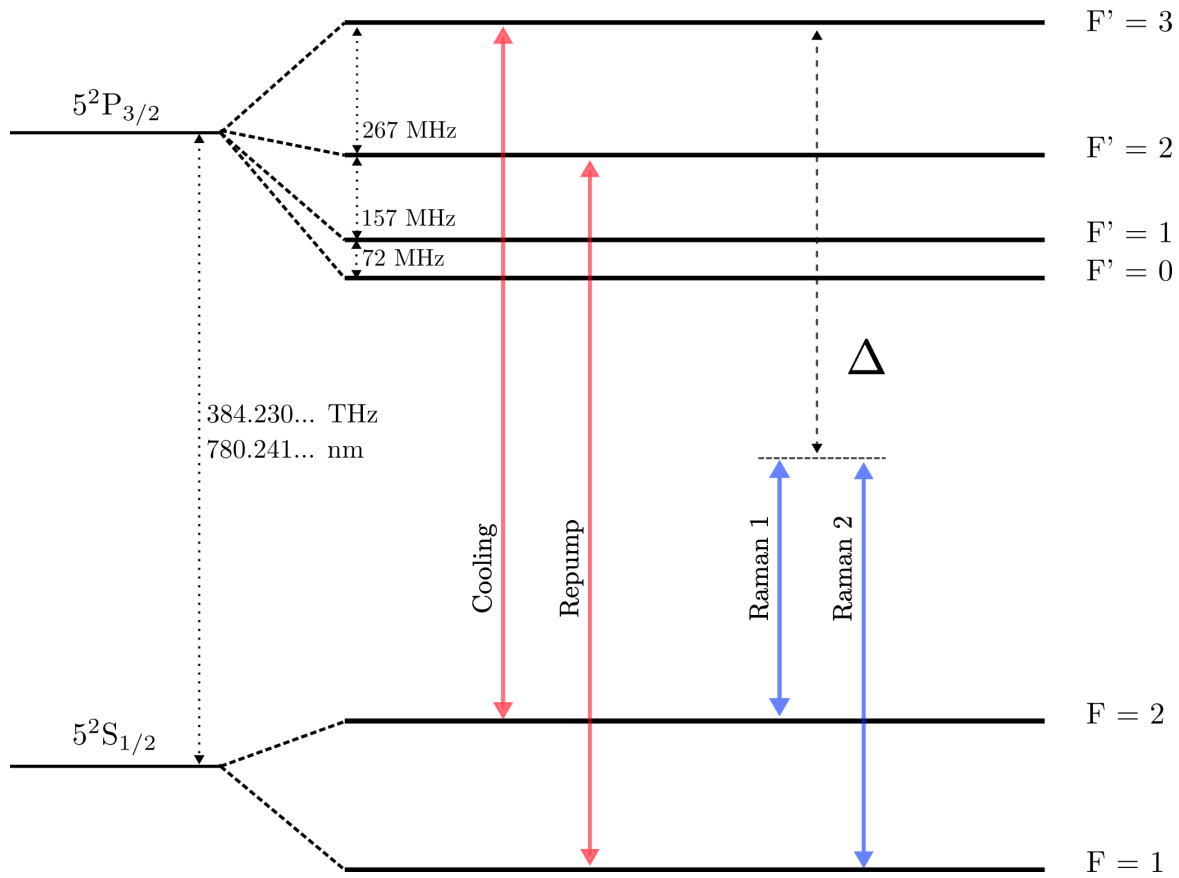


Figure 2.1: Hyperfine energy level structure of the D2 transition in Rb^{87} , and the corresponding cooling, repump and Raman transitions used in this experiment. The Zeeman sub-levels are not explicitly shown here.

The $5S_{1/2}$ ground state has two hyperfine levels, $F = 1$ and $F = 2$. The atom can be laser cooled by pumping atoms on the $F = 2$ to $F' = 3$ transition. However for efficient cooling to occur, the majority of atoms must be cycling on this cooling transition. This is achieved by the use of a repump light pumping any atoms out of the $F = 1$ dark state.

Both the $m_F = 0$ Zeeman sub-levels of the two hyperfine states $F = 1$ and $F = 2$ provide suitable states for stimulated Raman transitions for interferometry. These transitions are depicted in Figure 2.1, however the Zeeman sub-levels are not explicitly shown here. The choice for the particular $m_F = 0$ sub-levels is discussed in the following section.

2.2 Stimulated Raman Transitions in ^{87}Rb

The stimulated Raman transition is a very powerful technique used for the coherent manipulation of cold atoms and ions using lasers. It can be thought of as the simultaneous absorption and stimulated emission of photons by an atom. This process can be accompanied by a momentum transfer, allowing the entanglement of the internal state of an atom with its momentum state. This entanglement is key for this experiment as it allows different momentum states to be resolved at the output of the interferometer even when they are spatially unresolved, as discussed later in the next chapter.

The simplest form of a stimulated Raman transition occurs in the three-level Λ system, within which there are two long-lived ground states that are coupled via an upper intermediate state. In our system, we want to drive the microwave transition between the hyperfine states, $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$, in the $5S_{1/2}$ level of ^{87}Rb atoms. It is the insensitivity to magnetic fields of these hyperfine states which makes them an ideal choice due to the reduced perturbation of the energy levels in the presence of any stray fields. The three-level Λ configuration for our system, depicted in Figure 2.2, is driven by counter-propagating laser beams with angular frequencies ω_1 and ω_2 and wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively. The intermediate state is the $5P_{3/2}$ state. Assuming the atom starts in the lower hyperfine state $|5S_{1/2}, F = 1, m_F = 0\rangle$, the simultaneous absorption of a photon of frequency ω_1 and stimulated emission of frequency ω_2 drives the transition to the upper hyperfine state $|5S_{1/2}, F = 2, m_F = 0\rangle$.

Provided the population in the intermediate $5P_{3/2}$ state remains small, this level can be adiabatically eliminated using a series of approximations[24]. For simplicity let us label the lower

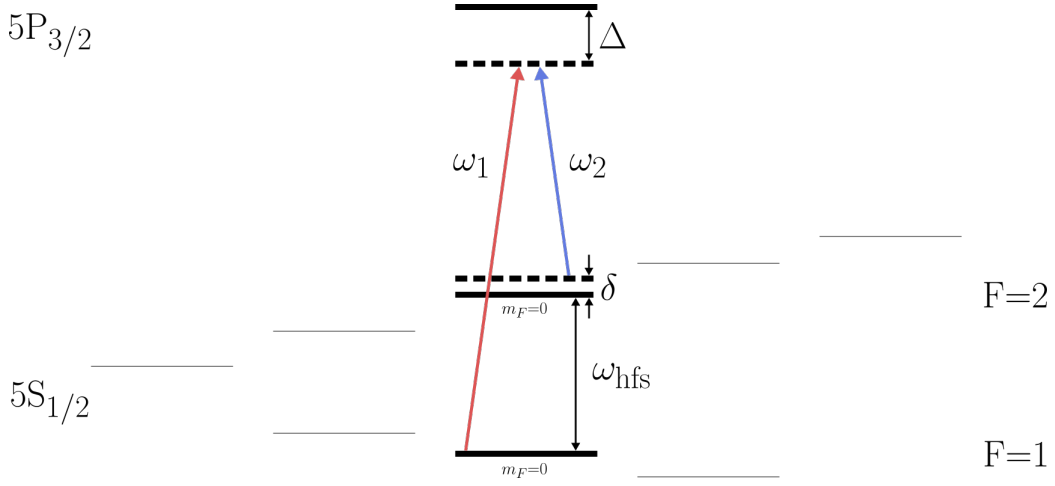


Figure 2.2: Energy level diagram of a three level stimulated Raman transition. The simultaneous absorption of ω_1 and stimulated emission of ω_2 coherently drives the atom from $|F=1, m_F=0\rangle$ to $|F=2, m_F=0\rangle$ via the intermediate state which is detuned by Δ from the D2 line. The stimulated Raman transition is on resonance when $\delta = 0$.

hyperfine state, $|5S_{1/2}, F=1, m_F=0\rangle$, as $|1\rangle$, the upper state, $|5S_{1/2}, F=2, m_F=0\rangle$, as $|2\rangle$, and the intermediate state as $|i\rangle$. The two laser light fields with wavevectors $\mathbf{k}_1$ and $\mathbf{k}_2$ can be approximated as the plane waves

$$\begin{aligned}\mathcal{E}_1 &= \mathbf{E}_1 \cos(\mathbf{k}_1 \cdot \mathbf{r} - \omega_1 t + \eta_1) \\ \mathcal{E}_2 &= \mathbf{E}_2 \cos(\mathbf{k}_2 \cdot \mathbf{r} - \omega_2 t + \eta_2)\end{aligned}\tag{2.1}$$

The two Rabi frequencies associated with the transitions $|1\rangle \longleftrightarrow |i\rangle$ and $|i\rangle \longleftrightarrow |2\rangle$ are defined as

$$\begin{aligned}\Omega_1 &= \frac{\langle 1 | \mathbf{d} \cdot \mathbf{E}_1 | i \rangle}{2\hbar} \\ \Omega_2 &= \frac{\langle 2 | \mathbf{d} \cdot \mathbf{E}_2 | i \rangle}{2\hbar},\end{aligned}\tag{2.2}$$

where $\mathbf{d}$ is the electric dipole moment operator. Let us define the hyperfine splitting as $\omega_{\text{hfs}} = \omega_e - \omega_g$. The Rabi frequencies we use are less than $6 \times 10^6 \text{ s}^{-1}$, whereas $\Delta \sim 7 \times 10^9 \text{ s}^{-1}$, $\omega_{\text{hfs}} \sim 4 \times 10^{10} \text{ s}^{-1}$ and $\omega_{1,2} \sim 4 \times 10^{15} \text{ s}^{-1}$ [62]. Therefore we are able to establish the frequency hierarchy: $\omega_{1,2} \gg \omega_{\text{hfs}} \gg \Delta \gg \delta, \Omega_{1,2}$. Under this hierarchy, the rotating wave and adiabatic approximations allow the intermediate state, $|i\rangle$, to be eliminated. The time evolution of the two level system can then be described by [24]

$$|\psi(t)\rangle = c_1(t)e^{-i\omega_g t} |1\rangle + c_2(t)e^{-i(\omega_e+\delta)t} |2\rangle \quad (2.3)$$

where the initial state vector at time t_0 is

$$\begin{bmatrix} c_1(t_0) \\ c_2(t_0) \end{bmatrix}.$$

The state vector after some time t is given by

$$\begin{bmatrix} c_1(t+t_0) \\ c_2(t+t_0) \end{bmatrix} = e^{i\delta t/2} \begin{bmatrix} \cos(\Omega t/2) - i\frac{\delta^-}{\Omega} \sin(\Omega t/2) & -i\frac{\Omega_{\text{eff}}}{\Omega} e^{i\phi} \sin(\Omega t/2) \\ -i\frac{\Omega_{\text{eff}}}{\Omega} e^{-i\phi} \sin(\Omega t/2) & \cos(\Omega t/2) + i\frac{\delta^-}{\Omega} \sin(\Omega t/2) \end{bmatrix} \begin{bmatrix} c_1(t_0) \\ c_2(t_0) \end{bmatrix}, \quad (2.4)$$

where

$$\begin{aligned} \delta &= \omega_R - \left(\omega_{\text{hfs}} + \mathbf{k}_{\text{eff}} \cdot \mathbf{v} + \frac{\hbar |\mathbf{k}_{\text{eff}}|^2}{2m} \right) \\ \delta^- &= \frac{\Omega_1^2}{\Delta} - \frac{\Omega_2^2}{\Delta} - \delta \\ \Omega_{\text{eff}} &= \frac{2\Omega_1\Omega_2}{\Delta} \\ \Omega &= \sqrt{\Omega_{\text{eff}}^2 + (\delta^-)^2} \\ \phi &= \phi_2 - \phi_1 \\ \omega_R &= \omega_2 - \omega_1. \end{aligned} \quad (2.5)$$

The effective wavevector is given by $\mathbf{k}_{\text{eff}} = \mathbf{k}_1 - \mathbf{k}_2$, $\mathbf{v}$ is the velocity of the atom and m is the atom mass. Therefore the third and fourth terms in δ are the Doppler shift and the recoil shift, respectively. We use $\phi_j = \mathbf{k}_j \cdot \mathbf{r} + \eta_j$ to describe the phase of the Raman laser light fields. The effective two photon Rabi frequency is given by Ω_{eff} , and the generalised Rabi frequency is given by Ω . The term $\frac{|\Omega_1|^2}{\Delta} - \frac{|\Omega_2|^2}{\Delta}$ is the AC Stark shift, which is dependent on the detuning, Δ , and the intensity of each beam. The effect of this light shift is undesirable but it can be cancelled out using an appropriate ratio of intensities of the two beams. The Raman resonance

condition, δ^- , is given by:

$$\delta^- = \frac{\Omega_1^2}{\Delta} - \frac{\Omega_2^2}{\Delta} - \left(\omega_R - \left(\omega_{\text{hfs}} + \mathbf{k}_{\text{eff}} \cdot \mathbf{v} + \frac{\hbar |\mathbf{k}_{\text{eff}}|^2}{2m} \right) \right) \quad (2.6)$$

Looking at Equation 2.4 we can see that if $\delta^- \neq 0$ we do not get a complete population inversion of the states, reducing the total number of atoms that participate in the interferometer. Assuming the Raman transition is exactly on resonance, i.e. $\delta^- = 0$, we get

$$\begin{bmatrix} c_1(t+t_0) \\ c_2(t+t_0) \end{bmatrix} = \begin{bmatrix} \cos(A/2) & -ie^{i\phi} \sin(A/2) \\ -ie^{-i\phi} \sin(A/2) & \cos(A/2) \end{bmatrix} \begin{bmatrix} c_1(t_0) \\ c_2(t_0) \end{bmatrix}, \quad (2.7)$$

where $A = \Omega t$ is known as the “pulse area”.

Starting with all atoms in state $|1\rangle$, such that $c_1(t_0) = 1$ and $c_2(t_0) = 0$, the probability of detecting the atom in state $|2\rangle$ after a Raman pulse of length t is given by

$$P_2(t) = |c_2(t+t_0)|^2 = \left| \frac{\Omega_{\text{eff}}}{\Omega} \right|^2 \sin^2 \left(\frac{\Omega t}{2} \right). \quad (2.8)$$

2.2.1 π and $\pi/2$ Pulses

Using the above equation for the state evolution of the system, we can demonstrate how these stimulated Raman transitions can be used to manipulate the atoms and act as either a beam-splitter or mirror. Let us first set the pulse area to $\Omega\tau = \pi/2$, which causes c_1 and c_2 to evolve in the following way over a time τ

$$c_1(\tau+t_0) = \frac{1}{\sqrt{2}}(c_1(t_0) - ie^{-i\phi}c_2(t_0)) \quad (2.9)$$

$$c_2(\tau+t_0) = \frac{1}{\sqrt{2}}(-ie^{i\phi}c_1(t_0) + c_2(t_0)) \quad (2.10)$$

If the atom starts with $c_1 = 1$ (or with $c_2 = 1$), the atom ends up after time τ having equal probability of being in either state. This $\pi/2$ pulse plays the role of a beamsplitter. In

comparison, setting the pulse area to $\Omega\tau = \pi$ causes the probability amplitudes to switch, acting as a mirror:

$$c_1(\tau + t_0) = -i(c_2(t_0)e^{i\phi}) \quad (2.11)$$

$$c_2(\tau + t_0) = -i(c_1(t_0)e^{-i\phi}) \quad (2.12)$$

2.2.2 Momentum Transfer in Stimulated Raman Transition

Consider an atom in state $|1, \mathbf{p}\rangle$, where $\mathbf{p}$ is its initial momentum. When this atom undergoes a stimulated Raman transition from state $|1\rangle$ to state $|2\rangle$ it also absorbs momentum $\hbar\mathbf{k}_1$ from one beam and loses momentum $\hbar\mathbf{k}_2$ to the other beam. The final state is therefore $|2, \mathbf{p} + \hbar\mathbf{k}_{\text{eff}}\rangle$, where $\mathbf{k}_{\text{eff}} = \mathbf{k}_1 - \mathbf{k}_2$. Thus the momentum of the atom is correlated with its internal state. In the case of the $\pi/2$ pulse, these two degrees of freedom become completely entangled.

As mentioned earlier, my interferometer uses counter-propagating Raman beams. It is possible to drive stimulated Raman transitions with co-propagating beams, however this configuration is not useful in my interferometer. That is because its sensitivity to acceleration and rotation is proportional to $\mathbf{k}_{\text{eff}}$, which is very small for co-propagating beams. This is discussed in more detail in Chapter 3.

2.2.3 Polarisation Requirement for Raman Transitions

The choice of polarisation of the two Raman lasers is extremely important to ensure the correct transition is being driven. In our interferometer, we want to drive the transition between the $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$ $5S_{1/2}$ states. This transition has no change in quantum magnetic number, $\Delta m_F = 0$, but a change in the total angular momentum quantum number of $\Delta F = \pm 1$. Because the Raman transition is driven by two electric dipoles, $\mathbf{d}_1$ and $\mathbf{d}_2$, each of rank 1, their combined effect can only be of rank 0, 1, or 2. The scalar combination is strictly forbidden to drive the transition because F must change. The tensor combination is forbidden to drive pure $J = 1/2 \leftrightarrow J = 1/2$ transitions but is weakly allowed because of the hyperfine

interaction. That leaves the cross-product $\mathbf{d}_1 \times \mathbf{d}_2$, and because we require $\Delta m_F = 0$, it is the z-component of this that drives the transition.

For linear polarisation, the two light fields must be perpendicular to each other, for example $E_1 \hat{\mathbf{x}}$ and $E_2 \hat{\mathbf{y}}$. To drive the transition with circularly polarised light beams, we require either circular polarisation σ^+/σ^+ or σ^-/σ^- . The combination of σ^+/σ^- light weakly drives $\Delta m_F = \pm 2$ transitions and so is not wanted in our experiment.

The two light fields are initially polarised and are delivered to the apparatus along the two principal axes of a single polarisation maintaining (PM) fibre. To achieve the desired polarisation, a quarter-wave plate at the output of the fibre converts the beam polarisations to σ^+/σ^- . These beams are then retro-reflected through a second quarter-wave plate such that the reflected beam polarisations are σ^-/σ^+ . In this configuration we have two pairs of counter-propagating beams that are able to drive the desired Raman transition (σ^+/σ^+ and σ^-/σ^-). These two pairs have equal and opposite effective wavevectors, given by $\mathbf{k}_{\text{eff}} = \pm(\mathbf{k}_1 - \mathbf{k}_2)$. In order to use just one of these two beam pairs we launch the atoms with velocity $\mathbf{v}$. The Doppler shift is upward for one pair and downward for the other, splitting the two Raman resonances by an amount $2\mathbf{k}_{\text{eff}} \cdot \mathbf{v}$. The detuning δ then allows us to select one resonance or the other.

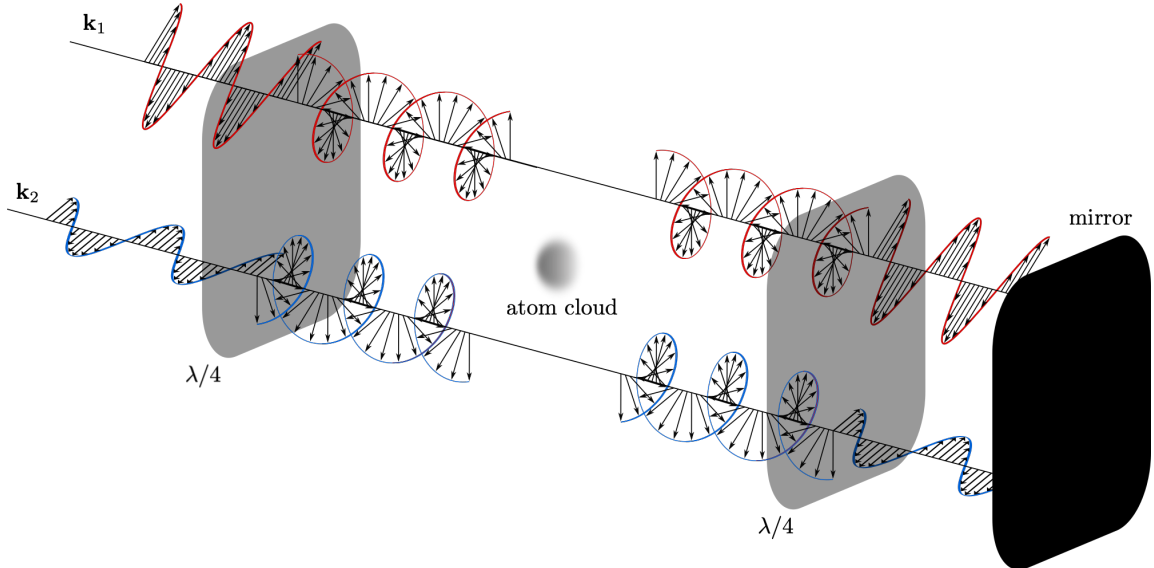


Figure 2.3: Retro-reflecting configuration used for the Raman beam to create counter-propagating Raman frequencies $\mathbf{k}_1$ and $\mathbf{k}_2$ with opposite helicity.

2.2.4 Raman Transitions within ^{87}Rb Atomic System

Light Shift

So far, we have imagined only a single intermediate P -state for the transition. However to calculate the light shift in ^{87}Rb we must sum over all the levels which the light fields couple. Figure 2.4 depicts the sublevels of $5P_{3/2}$, which together make the dominant contribution to the light shifts of the $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$ states. Both Raman laser frequencies, ω_1

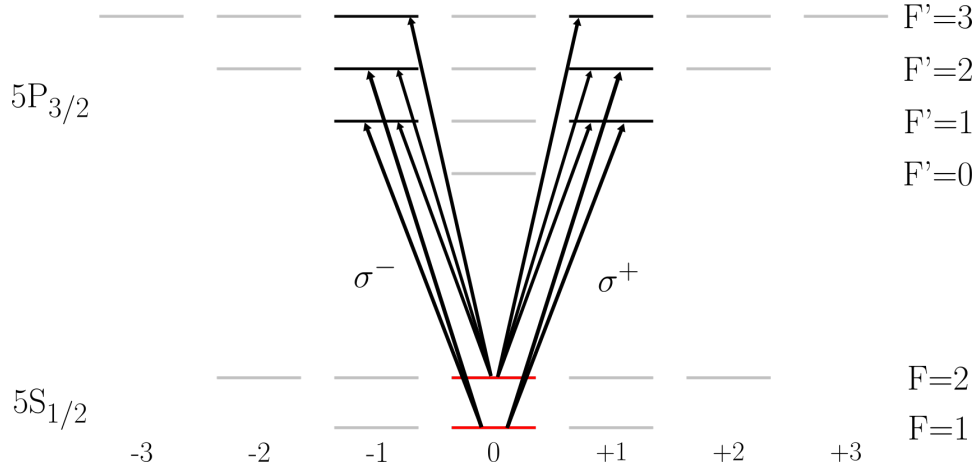


Figure 2.4: All D2 line transitions for σ^+ and σ^- polarised light which contribute to the light shift of the $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$ states (red). The detuning is not indicated in this diagram. There are two such sets of transitions corresponding to the detunings of ω_1 and ω_2 .

and ω_2 , drive both σ^+ and σ^- transitions because of the retro-reflected four beam arrangement.

Therefore the light shifts for the two Raman transition states are given by

$$\begin{aligned}\Lambda_{|1,0\rangle} &= \Lambda_{|1,0\rangle}^{\omega_1} + \Lambda_{|1,0\rangle}^{\omega_2} \\ \Lambda_{|2,0\rangle} &= \Lambda_{|2,0\rangle}^{\omega_1} + \Lambda_{|2,0\rangle}^{\omega_2}\end{aligned}\tag{2.13}$$

where $\Lambda_{|j\rangle}^{\omega_i}$ is the light shift of state j due to Raman frequency ω_i . The differential light shift of the two hyperfine states is then given by $\delta\Lambda = \Lambda_{|2,0\rangle} - \Lambda_{|1,0\rangle}$. As the light shift produced by each beam is proportional to the beam intensity there can be some ratio of the two intensities at which the two light shifts completely cancel each other out¹. This is an effective method to

¹Note this only works when $\Lambda_{|1,0\rangle}$ and $\Lambda_{|2,0\rangle}$ have the same sign.

set the differential light shift to zero across the entire Gaussian Raman light field, ensuring all atoms interacting with the light experience the same generalised Rabi frequency.

Effective Two-Photon Rabi Frequency

The whole $5P$ manifold must also be taken into account to accurately calculate the effective two photon Rabi frequency. Again assuming the two pairs of counter propagating Raman beams, with σ^+ and σ^- polarisation respectively, the possible paths that contribute to the Raman transition between these states are shown in Figure 2.5.

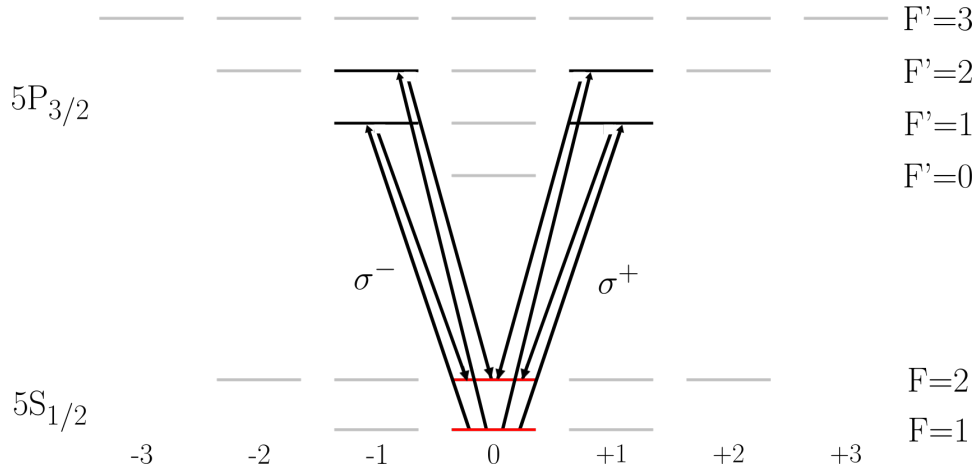


Figure 2.5: The possible transitions and corresponding intermediate states within the D2 line that are able to couple states $|1\rangle$ and $|2\rangle$ through the stimulated Raman transitions for both the σ^+ and σ^- polarised light pairs. The detuning is not indicated in this diagram, here we need only consider ω_1/ω_2 pairs that satisfy the two-photon resonance condition.

In this case, the effective two photon Rabi frequency is given by summing over all the individual Rabi frequencies with intermediate state $|i\rangle$ denoted by k

$$\Omega_{\text{eff}} = \sum_k \frac{2|\Omega_{1k}||\Omega_{2k}|}{\Delta_k} \quad (2.14)$$

where the individual Rabi frequencies are calculated using the Wigner-Eckart Theorem and the Clebsch-Gordan coefficients which describe the coupling between different sublevels through absorption or emission of a photon with some particular polarisation.

Chapter 3

Phase Shift

Using the $\frac{\pi}{2} - \pi - \frac{\pi}{2}$ sequence of Raman pulses on an ensemble of atoms, we are able to form a Mach-Zehnder interferometer in which the atomic wavepackets from each arm interfere coherently at the output. The resulting interference pattern is dependent on the difference in phases of the wavepackets travelling along the two arms. In order to be able to use an atom interferometer as part of an inertial navigation system, it is important to know how this phase difference is affected by the acceleration or rotation of the vehicle where the interferometer sits.

In this chapter we derive the phase terms for such a reference frame, and discuss how the phase shift of the interferometer is measured.

3.1 The Interferometer Phase

In the interferometer only a phase difference has physical meaning. The two types of phase difference discussed in the following are; (1) the phase difference between atomic wavepackets interfering at the output which is denoted by “interferometer phase”, and (2) the difference between the phase of a wavepacket at the input and its phase at the output of the interferometer, denoted by “wavepacket phase”.

The interferometer phase can be calculated by solving the Schrodinger equation with the ap-

proprate time dependent Hamiltonian. Results for this approach are well known [63, 64, 65]. Other equivalent approaches to calculate the interferometer phase also exist [66, 67]. The key assumption that makes this calculation possible is the semi-classical approximation which requires the higher-order position and momentum dependence of the Hamiltonian to be negligible at the size of the wavepacket in each arm of the interferometer. Under this approximation it can be shown that the expectation values of position and momentum of the wavepacket in each arm follow the classical trajectories (for more information see [65]). This approximation requires variations of the Hamiltonian to be small with respect to the size of the wavepacket, however it does not require the variations to be small with respect to the separation between interferometer arms. Therefore this approximation remains valid for large separations of interferometer arms provided the wavepackets in each arm are sufficiently narrow in position and momentum.

There are three contributions to the interferometer phase $\Delta\Phi$ [63]. These are: (1) Φ_{int} due to the interactions of the atoms with the light, which I call the interaction phase; (2) Φ_{prop} due to the propagation of the wavepackets; and (3) Φ_{sep} the phase difference due to the spatial separation of the two interfering wavepackets at the final pulse, known as the separation phase.

$$\Delta\Phi = \Phi_{\text{int}} + \Phi_{\text{prop}} + \Phi_{\text{sep}} \quad (3.1)$$

In Chapter 2 I showed how the interaction between the atom and the Raman light introduces a phase shift in the atomic wavefunction when the light drives a transition in the atom. This is accompanied by a momentum kick. Specifically (see Equation 2.7),

$$\begin{aligned} |1, \mathbf{p}\rangle &\rightarrow -i |2, \mathbf{p} + \hbar \mathbf{k}_{\text{eff}}\rangle e^{i\phi(\mathbf{r}, t)} \\ |2, \mathbf{p} + \hbar \mathbf{k}_{\text{eff}}\rangle &\rightarrow -i |1, \mathbf{p}\rangle e^{-i\phi(\mathbf{r}, t)} \end{aligned} \quad (3.2)$$

where $\mathbf{r}$ is the mean position of the wavepacket at the pulse time t , and ϕ is defined in Equation 2.5. Therefore the interaction contribution to the interferometer phase is

$$\Phi_{\text{int}} = \sum_j \pm \phi_L(\mathbf{r}_j, t_j) - \sum_i \pm \phi_L(\mathbf{r}_i, t_i) \quad (3.3)$$

where $(\mathbf{r}_j, t_j)$ and $(\mathbf{r}_i, t_i)$ are the atom-light interaction positions and times in the upper and lower arms respectively, and the plus (or minus) sign is selected for $1 \rightarrow 2$ (or $2 \rightarrow 1$) transitions.

The Feynman propagator for a particle travelling from point $(\mathbf{r}_a, t_a)$ to point $(\mathbf{r}_b, t_b)$ is $A \sum_{\Gamma} e^{iS_{\Gamma}/\hbar}$ where A is a normalisation, $\sum_{\Gamma}$ is the integral over all possible paths Γ , and S_{Γ} is the action along each path. In our case, $S_{\Gamma} \gg \hbar$ and we need only take the integral along the classical path [68] for which the action is $S^{\text{class}} = \int L dt$, where L is the Lagrangian evaluated on the classical path.

The propagation contribution to the interferometer phase is therefore

$$\Phi_{\text{prop}} = \frac{1}{\hbar} \int_{t_i}^{t_f} (L_{\text{upper}} - L_{\text{lower}}) dt. \quad (3.4)$$

Here t_i and t_f are the times of the first and last Raman pulses, while L_{upper} and L_{lower} are the values of the Lagrangian taken along the upper and lower paths. With constant acceleration and no rotation $\Phi_{\text{prop}} = 0$ [68].

The separation phase Φ_{sep} arises when the classical trajectories followed by the centre of each wavepacket do not overlap at the time of the last Raman pulse. In this case the centre of one packet interferes with the wing of the other, causing an additional phase shift of [63]

$$\Phi_{\text{sep}} = \frac{1}{\hbar} \bar{\mathbf{p}} \cdot \Delta \mathbf{r} \quad (3.5)$$

where $\bar{\mathbf{p}}$ is the average canonical momentum in the output state of interest, and $\Delta \mathbf{r} = \mathbf{r}_{\text{lower}} - \mathbf{r}_{\text{upper}}$ is the separation in space between the centres of the wavepackets from the lower and upper arms at time t_f . This phase also vanishes when there is no acceleration and no rotation since $\Delta \mathbf{r} = 0$ in that case.

I note that when the classical trajectories of the wavepackets do not overlap at the final beam-splitter, Φ_{sep} becomes frame dependent. If we transform to another reference frame moving at some relative velocity $\mathbf{v}$, Φ_{sep} increases by $m\mathbf{v} \cdot \mathbf{r}/\hbar$. However, the value of Φ does not change because there is an equal and opposite change in Φ_{prop} . I note further that the two output

states differ in their average canonical momentum by the recoil $\hbar k_{\text{eff}}/m$, so when $\Delta \mathbf{r} \neq 0$ they have different Φ_{sep} and it would seem that their fringes may not have the requisite relationship of $\sin^2(\Phi)$ and $\cos^2(\Phi)$. However, this same momentum difference produces a compensating change in Φ_{int} yielding an interferometer phase Φ that is the same for both ports.

3.2 Calculating the Phase Shift Terms for our System

Figure 3.1 is a position-time diagram for the $\frac{\pi}{2}$ - π - $\frac{\pi}{2}$ interferometer, drawn without any acceleration, where the solid and dashed lines represent state $|1\rangle$ state $|2\rangle$, respectively.

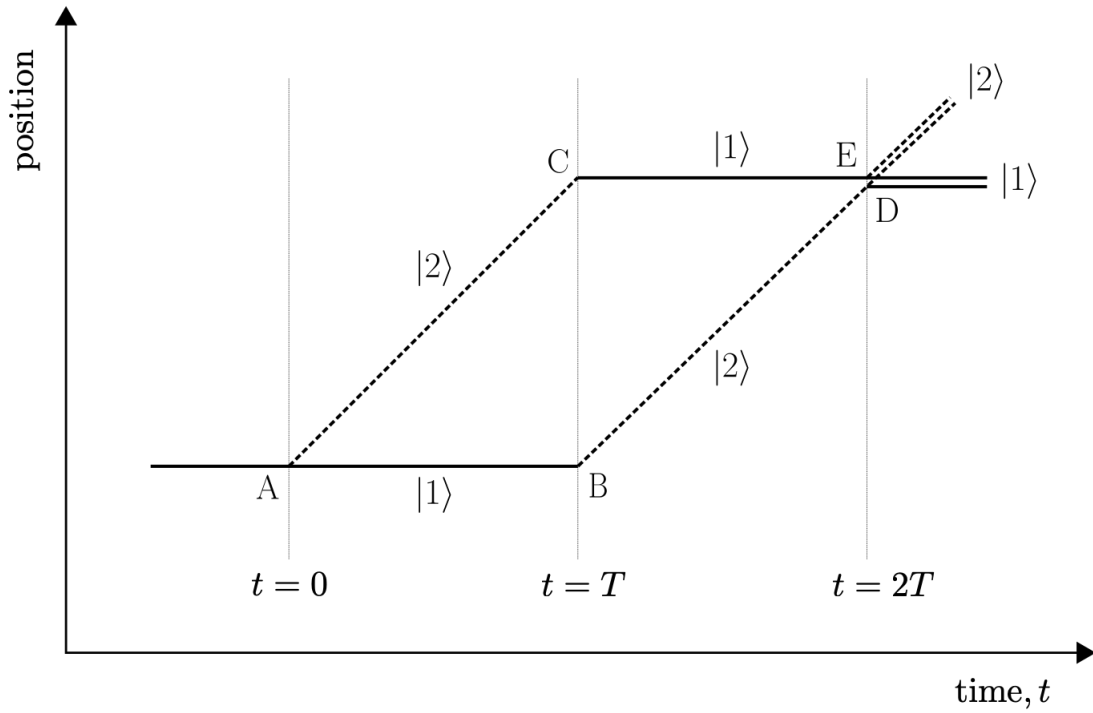


Figure 3.1: Position-time diagram of the interferometer sequence drawn without acceleration. Solid and dashed lines represent states $|1\rangle$ and $|2\rangle$. Vertical grey lines at times t_0 , $t_0 + T$ and $t_0 + 2T$ indicate the times at which the $\frac{\pi}{2}$, π and $\frac{\pi}{2}$ pulses are applied.

Taking the results from Section 3.1, we have the following expressions for the interferometer

phase contributions in output port $|1\rangle$:

$$\begin{aligned}\Phi_{\text{int}} &= \phi_L(0, r_A) - \phi_L(T, r_C) - \phi_L(T, r_B) + \phi_L(2T, r_D) \\ \Phi_{\text{prop}} &= \frac{1}{\hbar}(S_{AC} + S_{CE}) - (S_{AB} + S_{BD}) \\ \Phi_{\text{sep}} &= \frac{1}{2\hbar}(p_D + p_E) \cdot (r_D - r_E)\end{aligned}\tag{3.6}$$

where S_{ij} is the classical action along path between positions r_i and r_j , and $p_i = \frac{\partial L(r_i)}{\partial v_i}$ is the classical canonical momentum at position r_i after the final beamsplitter.

We want to evaluate these phases for an atom falling freely under gravity when it is interacting with Raman laser pulses whose optics are fixed to the vehicle that is accelerating and rotating. Let us call the frame of reference fixed to the vehicle K and let the position and velocity of an atom when viewed from that frame be $\mathbf{r}$ and $\mathbf{v}$. When viewed from an inertial frame K_0 that does not rotate and whose origin P_0 is at the centre of the earth, the Lagrangian that describes the freely falling atom is

$$L_0 = \frac{1}{2}m\mathbf{v}_0^2 + m\mathbf{g}_0 \cdot \mathbf{r}_0\tag{3.7}$$

where m is the mass of the atom, $\mathbf{v}_0$ is its velocity, $\mathbf{r}_0$ is its position and $\mathbf{g}_0$ is the acceleration due to gravity. If frame K (i.e. the vehicle) rotates with angular velocity $\mathbf{\Omega}$ around its own origin and has velocity $\mathbf{u}$ relative to K_0 , then the corresponding equation of motion viewed in K (see §39 of [69]) is

$$\frac{d\mathbf{v}}{dt} = \mathbf{r} \times \frac{d\mathbf{\Omega}}{dt} + 2\mathbf{v} \times \mathbf{\Omega} + \mathbf{\Omega} \times (\mathbf{r} \times \mathbf{\Omega}) + \left(-\frac{d\mathbf{u}}{dt} + \mathbf{g}_0 \right)\tag{3.8}$$

Because we are interested in navigating on the surface of the earth, it is convenient to write

$$\mathbf{u} = \mathbf{\Omega}_E \times \mathbf{R} + \mathbf{v}_a\tag{3.9}$$

where $\mathbf{\Omega}_E$ is the angular velocity of the earth seen from K_0 , $\mathbf{R}$ is the position vector in frame K_0 of a point fixed on the surface of the rotating earth, and $\mathbf{v}_a$ is the velocity of frame K relative to that point.

Similarly, we write

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_E + \boldsymbol{\omega} \quad (3.10)$$

where $\boldsymbol{\omega}$ is the angular velocity of frame K relative to that of the earth. Let us take $\boldsymbol{\Omega}_E$ to be constant and let us assume $|\boldsymbol{\omega}|$ is constant, giving $\frac{d\boldsymbol{\Omega}}{dt} = \frac{d\boldsymbol{\omega}}{dt} = \boldsymbol{\Omega}_E \times \boldsymbol{\omega}$ assuming that the direction of $\boldsymbol{\omega}$ rotates with the Earth as seems most natural. We recognise the second and third terms in Equation 3.8 as the Coriolis and centrifugal acceleration, respectively. The fourth term becomes

$$-\frac{d\mathbf{u}}{dt} + \mathbf{g}_0 = \mathbf{g}_0 - \boldsymbol{\Omega}_E \times (\boldsymbol{\Omega}_E \times \mathbf{R}) - \frac{d\mathbf{v}_a}{dt} \quad (3.11)$$

Assuming that the vehicle is close to the point $\mathbf{R}$, and the gravity gradient is small, the first two terms on the right are equal to $\mathbf{g}$, the gravitation acceleration as measured at the position of the vehicle because that automatically includes the centrifugal correction due to the earth's rotation. With these substitutions Equation 3.8 becomes

$$\frac{d\mathbf{v}}{dt} = \mathbf{r} \times (\boldsymbol{\Omega}_E \times \boldsymbol{\omega}) + 2\mathbf{v} \times \boldsymbol{\Omega} + \boldsymbol{\Omega} \times (\mathbf{r} \times \boldsymbol{\Omega}) + \mathbf{g} - \frac{d\mathbf{v}_a}{dt} \quad (3.12)$$

These approximations assume the change in latitude and longitude are small over the interferometer time, $2T$.

The following procedure is used to calculate the interferometer phase shift:

1. Analytic expressions for $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are determined by solving Equation 3.12. These expressions are then Taylor expanded about $t = 0$ to third order in t .
2. The interferometer is divided into path segments corresponding to the individual trajectories between each of the light pulses. The trajectory for each path is determined using the appropriate initial position and velocity. For the path directly after the first light pulse whose velocity remains unchanged (path AB in Figure 3.1), the initial conditions are:

$$\begin{aligned} \mathbf{r}_{AB}(0) &= \mathbf{r}_A = (x_A, y_A, z_A) \\ \mathbf{v}_{AB}(0) &= \mathbf{v}_A = (v_{x_A}, v_{y_A}, v_{z_A}) \end{aligned} \quad (3.13)$$

The initial position for path AC is also $\mathbf{r}_A$, however the initial velocity for this path has an additional recoil velocity $\hbar \mathbf{k}_{\text{eff}}/m$ due to the momentum imparted by the light. The initial conditions for subsequent paths of the atoms are then determined by the final position and velocity of the previous path, plus any recoil velocity due to the light-matter interaction.

3. The contributions Φ_{int} and Φ_{sep} are calculated by substituting the appropriate position and momentum values into Equations 3.6.
4. In order to determine Φ_{prop} , the position and velocity functions for each path are used to calculate the corresponding Lagrangian given by [69],

$$L = \frac{1}{2}m\mathbf{v}^2 + m\mathbf{v} \cdot \boldsymbol{\Omega} \times \mathbf{r} + \frac{1}{2}m(\boldsymbol{\Omega} \times \mathbf{r})^2 + m \left(\mathbf{g} - \frac{d\mathbf{v}_a}{dt} \right) \cdot \mathbf{r} \quad (3.14)$$

retaining only terms up to third order in t . The classical action for each path is then determined by integrating the Lagrangian along each path from the initial time, t_i , to the final time, $t_i + T$.

5. The interferometer phase is then the sum $\Delta\Phi = \Phi_{\text{int}} + \Phi_{\text{sep}} + \Phi_{\text{prop}}$.

3.3 Results

Using the method described in Section 3.2 I have calculated the contributions to the interferometer phase $\Delta\Phi$ as a power series in the time T between laser pulses, expanded out to order T^4 . These are listed in Table 3.1. The third column repeats the coefficient of each term and the last three columns provide a breakdown of the contributions to that coefficient. The first term is by far the largest phase shift to be produced by the the acceleration $\mathbf{a}$, which is (of course) intimately linked with the gravitation acceleration $\mathbf{g}$. Term number 2 is the main consequence of rotation, usually called the Sagnac phase. Term 3 is a correction to term 1, which comes from the centrifugal acceleration of the atom due to rotation of reference frame K about its origin. Term 4 is the leading cross-coupling term between acceleration and rotation: it is just

term 1 multiplied by the very small $2\Omega T$. Similarly, term 5 is the order ΩT correction to term 2. Term 6 is the recoil velocity correction to term 5. Finally, term 7 is the correction to term 1 of the second order in ΩT . Let us suppose that the vehicle rotates at 1mrads/s and the interferometer has $T = 25\text{ms}$. Then $(\Omega T)^2 < 1 \times 10^{-9}$ and the terms in Table 3.1 should capture the effect of acceleration a at that fractional accuracy. The phase due to this rotation should be correct at the level of 25nrad/s.

Term Number	Phase Term	Term Coefficient	Int	Prop	Sep
1	$-\mathbf{k}_{\text{eff}} \cdot (\mathbf{a} - \mathbf{g}) T^2$	-1	-1	0	0
2	$+2\mathbf{k}_{\text{eff}} \cdot \mathbf{v}_A \times \Omega T^2$	+2	+2	-2	+2
3	$-\mathbf{k}_{\text{eff}} \cdot \Omega \times (\Omega \times \mathbf{r}_A) T^2$	-1	-1	+2	-2
4	$-2\mathbf{k}_{\text{eff}} \cdot (\mathbf{a} - \mathbf{g}) \times \Omega T^3$	-2	-2	+4	-4
5	$+3\mathbf{k}_{\text{eff}} \cdot \Omega \times (\Omega \times \mathbf{v}_A) T^3$	+3	+3	0	0
6	$+\frac{3\hbar}{2m}\mathbf{k}_{\text{eff}} \cdot \Omega \times (\Omega \times \mathbf{k}_{\text{eff}}) T^3$	$+\frac{3}{2}$	0	$-\frac{1}{2}$	+2
7	$-\frac{7}{4}\mathbf{k}_{\text{eff}} \cdot \Omega \times (\Omega \times (\mathbf{a} - \mathbf{g})) T^4$	$-\frac{7}{4}$	$-\frac{7}{4}$	0	0

Table 3.1: The vector form of the phase shift terms and the corresponding individual contributions from the interaction, propagation and separation phases.

As discussed in Chapter 1, the purpose of this interferometer system is to measure the acceleration and rotation with accuracies of 100ng and $1\mu\text{rad s}^{-1}$. It should be possible to relate these to the interferometer phase with the required accuracy using just the terms presented in Table 3.1. These terms are implicit in other publications [63, 65, 68, 70, 71], but to my knowledge this is the first time they have been written in this compact vector form as well as for the reference frame of a moving vehicle on the surface of the rotating earth.

In order to consider the relative sizes of these term in more detail, I substitute typical values into Table 3.1. Let us now define the direction of the x, y and z axes of the vehicle reference frame at time $t = 0$. As illustrated in Figure 3.2, the z axis points directly away from the centre of the earth, and the x and y axes point due east and due north respectively. Therefore at time $t = 0$ we define $\Omega_E = (0, \Omega_{E_y}, \Omega_{E_z})$ and $\mathbf{R} = (0, 0, R_E)$. This gives $\mathbf{g} = (0, g_y, g_z) \approx (0, -\Omega_{E_y}\Omega_{E_z}R_E, g_{0_z} + \Omega_{E_y}^2 R_E)$. Let us assume that the Raman axis is aligned with one of the

principal axes x , y or z , and calculate the corresponding most significant phase terms. For each orientation we assume the atoms are launched with typical velocities of 10cm s^{-1} along the Raman axis, and 40cm s^{-1} along each perpendicular axis (so the interferometer will be sensitive to rotations). In addition to the initial velocities $(v_{x_A}, v_{y_A}, v_{z_A})$, the following values are used; $g_z = -9.81\text{m s}^{-2}$, $\mathbf{r} = (0, 0, 1\text{m})$, $R_E = 6.365 \times 10^6\text{ m}$, $\Omega_{E_y} = \cos(\theta_{\text{lat}}) 7.29 \times 10^{-5}\text{ rad/s}$, $\Omega_{E_z} = \sin(\theta_{\text{lat}}) 7.29 \times 10^{-5}\text{ rad/s}$ and $\theta_{\text{lat}} = 51^\circ$. For the local rotation we take $\boldsymbol{\omega} = (1\text{mrad s}^{-1}, 1\text{mrad s}^{-1}, 1\text{mrad s}^{-1})$, which greatly exceeds $\boldsymbol{\Omega}_E$ and therefore dominates the value of $\boldsymbol{\Omega}$. The magnitude of the effective Raman wavevector is $k_{\text{eff}} = \frac{4\pi}{780\text{nm}}$, an interferometer time of $T = 25\text{ms}$ is assumed between light pulses, and we take the acceleration to be $\mathbf{a} = (1\text{m s}^{-2}, 1\text{m s}^{-2}, 0)$.

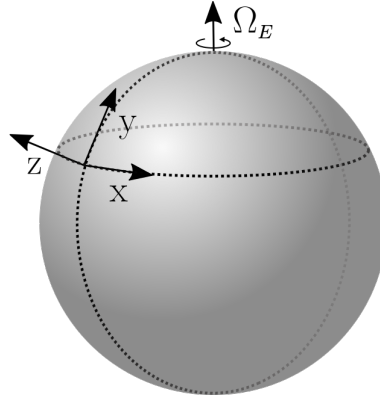


Figure 3.2: Orientation of the vehicle frame axes at time $t = 0$ relative to the surface of the earth.

The ten most significant terms for each configuration are listed in Table 3.2, 3.3 and 3.4. All the phase shift terms are given in rad. The 20 most significant terms for the interferometer oriented along each axis can be found in Appendix A.

Table 3.1 Term Number	Table 3.1 Phase Term	Phase Shift $\Delta\Phi$ (rad)
1	$-k_{\text{eff}}a_xT^2$	-10 069.2
2	$2k_{\text{eff}}v_{y_A}\omega_zT^2$	8.06
2	$-2k_{\text{eff}}v_{z_A}\omega_yT^2$	-8.06
4	$-2k_{\text{eff}}g_z\omega_yT^3$	+4.94
4	$-2k_xa_y\omega_zT^3$	-5.03×10^{-1}
2	$2k_{\text{eff}}v_{y_A}\Omega_{E_z}T^2$	4.56×10^{-1}
2	$-2k_{\text{eff}}v_{z_A}\Omega_{E_y}T^2$	-3.70×10^{-1}
4	$-2k_{\text{eff}}g_z\Omega_{E_y}T^3$	2.26×10^{-1}
4	$-2k_{\text{eff}}a_y\Omega_{E_z}T^3$	-2.85×10^{-2}
3	$-k_{\text{eff}}z_A\omega_x\omega_zT^2$	-1.01×10^{-2}

Table 3.2: The ten most significant terms for the total phase shift of the interferometer along the x axis, and the corresponding value in radians.

Table 3.1 Term Number	Table 3.1 Phase Term	Phase Shift $\Delta\Phi$ (rad)
1	$-k_{\text{eff}}a_yT^2$	-10 069.2
1	$k_{\text{eff}}g_yT^2$	-166.58
2	$-2k_{\text{eff}}v_{x_A}\omega_zT^2$	-8.06
2	$2k_{\text{eff}}v_{z_A}\omega_xT^2$	8.06
4	$2k_{\text{eff}}g_z\omega_xT^3$	-4.94
4	$2k_{\text{eff}}a_x\omega_zT^3$	5.03×10^{-1}
2	$-2k_{\text{eff}}v_{x_A}\Omega_{E_z}T^2$	-4.56×10^{-1}
2	$2k_{\text{eff}}a_x\Omega_{E_z}T^3$	2.85×10^{-2}
3	$-k_{\text{eff}}z_A\omega_y\omega_zT^2$	-1.01×10^{-2}
3	$-k_{\text{eff}}z_A\omega_y\Omega_{E_z}T^2$	5.70×10^{-4}

Table 3.3: The ten most significant terms for the total phase shift of the interferometer along the y axis, and the corresponding value in radians.

Table 3.1 Term Number	Table 3.1 Phase Term	Phase Shift $\Delta\Phi$ (rad)
1	$k_{\text{eff}}g_zT^2$	-98 778.9
2	$-2k_{\text{eff}}v_{yA}\omega_xT^2$	-8.06
2	$2k_{\text{eff}}v_{xA}\omega_yT^2$	8.06
4	$2k_{\text{eff}}a_y\omega_xT^3$	5.03×10^{-1}
4	$-2k_{\text{eff}}a_x\omega_yT^3$	-5.03×10^{-1}
2	$2k_{\text{eff}}v_{xA}\Omega_{E_y}T^2$	3.70×10^{-1}
4	$-2k_{\text{eff}}a_x\Omega_{E_y}T^3$	-2.31×10^{-2}
3	$k_{\text{eff}}z_A\omega_y^2T^2$	1.01×10^{-2}
3	$k_{\text{eff}}z_A\omega_x^2T^2$	1.01×10^{-2}
4	$-2k_{\text{eff}}g_y\omega_xT^3$	8.33×10^{-3}

Table 3.4: The ten most significant terms for the total phase shift of the interferometer along the z axis, and the corresponding value in radians.

As expected the dominant phase term along each axis is due to acceleration (term 1 from Table 3.1). For the assumed rotation rate, the rotation phase (term 2) is significantly lower than the acceleration term. Reliable separation of these two phase shifts would be required for inertial navigation. The 10 most significant phase terms when the Raman axis is aligned along any axis are derived from terms 1 to 4. Therefore for any alignment of the Raman axis we expect the most significant phase terms to be a combination of those listed in Tables 3.2, 3.3 and 3.4.

We see from Tables 3.2 - 3.4 that the largest contributions to the interferometer phase come from terms 1, 2, and 4 in Table 3.1. It is convenient therefore to approximate the interferometer phase as

$$\Delta\Phi = -\mathbf{k}_{\text{eff}} \cdot (\mathbf{a} - \mathbf{g}) T^2 + 2\mathbf{k}_{\text{eff}} \cdot \mathbf{v}_A \times \boldsymbol{\Omega} T^2 - 2\mathbf{k}_{\text{eff}} \cdot (\mathbf{a} - \mathbf{g}) \times \boldsymbol{\Omega} T^3 \quad (3.15)$$

For an interferometer with $T=25\text{ms}$, the first term shows that an acceleration noise of 100ng corresponds to noise in $\Delta\Phi$ of 10mrad . This same 10mrad corresponds to angular frequency noise of $1.2 \mu\text{rad s}^{-1}$ assuming the perpendicular velocity of 40cm s^{-1} .

3.4 Measuring the Phase Shift of the Interferometer

For an atom in some initial state $|\Psi(0)\rangle$, its state at the output of the interferometer is given by:

$$|\Psi(t)\rangle = U_{int} |\Psi(0)\rangle \quad (3.16)$$

where U_{int} is the operator for the total three pulse interferometer sequence $\pi/2 - \pi - \pi/2$. In the case when $|\Psi(0)\rangle = |1\rangle$, the probabilities of the atom being detected in either state, $|1\rangle$ or $|2\rangle$, at the output are:

$$P_1 = |\langle 1|\Psi(t)\rangle|^2 = \cos^2\left(\frac{\Delta\Phi}{2}\right) \quad (3.17) \quad P_2 = |\langle 2|\Psi(t)\rangle|^2 = \sin^2\left(\frac{\Delta\Phi}{2}\right) \quad (3.18)$$

Therefore the fringe pattern due to interference of the wavepackets from the two arms of the interferometer is observed in the probabilities of the two output states.

For an ensemble of atoms starting in state $|1\rangle$, inhomogeneity across the cloud is likely to cause some loss of fringe contrast of the overall interference pattern. We model this by writing the probability of state $|2\rangle$ as

$$P_2 = P_0 - \frac{C}{2} \cos \Delta\Phi \quad (3.19)$$

where P_0 is the probability offset and C is the contrast parameter. The interferometer is most sensitive to changes in acceleration and rotation when $|\frac{dP_2}{d\Delta\Phi}|$ is maximum. The (small) interference phase noise $\sigma_{\Delta\Phi}$ therefore corresponds to noise in P_2 given by

$$\sigma_{P_2} = \frac{C}{2} \sigma_{\Delta\Phi} \quad (3.20)$$

Assuming $\sigma_{\Delta\Phi} = 10 \text{ mrad}/\sqrt{\text{Hz}}$ (corresponding to $100 \text{ ng}/\sqrt{\text{Hz}}$) and taking a typical value of 40% for C , Equation 3.20 gives $\sigma_{P_2} = 2 \times 10^{-3}$. This sets a goal for the desired noise level on the steepest part of the fringe.

Chapter 4

Experimental Setup

4.1 Overview of Experiment

To perform the interferometer sequence and measure the output state probability, we require an ensemble of atoms which can be coherently manipulated. For this experiment we use a cloud of cold ^{87}Rb atoms. An ultra-high vacuum environment is required to reduce the likelihood of collisions of the ^{87}Rb atoms with any background gas. A pressure below 10^{-7} mbar is necessary for efficient trapping and cooling. However to preserve coherence of the state superposition created in the interferometer, we require a pressure below several 10^{-9} mbar.

The cloud of cold atoms is created by employing a 3D magneto-optical trap (MOT) at the centre of the vacuum chamber. This trap is loaded by an approximately collimated beam of atoms produced by a separate 2D MOT. The atoms are cooled to $\sim 200\mu\text{K}$ in the 3D MOT. The polarisation gradient cooling technique is then used to further cool the atoms well below the Doppler limit of $144\mu\text{K}$ [\[72\]](#), approaching temperatures of $\sim 8\mu\text{K}$. A series of state selection steps are subsequently applied, including optical and microwave pumping, and velocity selection, to produce a sufficiently cold cloud of atoms in the desired $|F = 1, m_F = 0\rangle$ starting state for interferometry. This atom cloud is launched vertically and horizontally, in a moving molasses, before then applying the $\frac{\pi}{2} - \pi - \frac{\pi}{2}$ sequence of Raman interferometer pulses. Finally, state selective fluorescence is used to measure the population of atoms in both output states.

This chapter describes the system that was designed and constructed to perform the atom preparation and interferometry sequence. The system comprises two orthogonal Raman beams denoted by Raman Axis 1 aligned north-south, and Raman Axis 2 aligned east-west as shown in Figure 4.1.

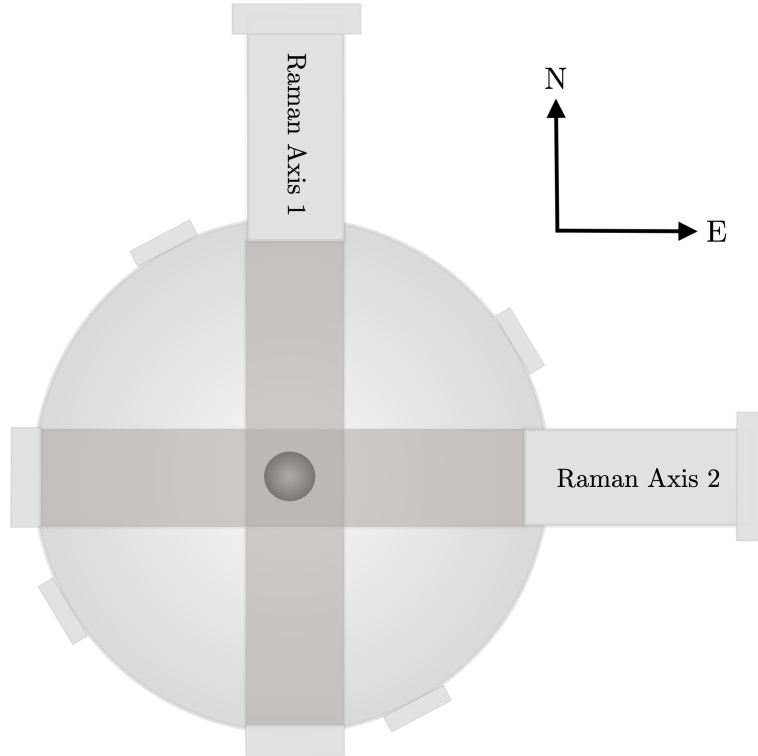


Figure 4.1: Orientation of the two Raman beams within the vacuum chamber system.

4.2 Vacuum Chamber

The vacuum chamber is a sphere truncated at the top and bottom, depicted in Figure 4.2. It has an inner radius of 225mm at the equator, and a height of 300mm between the top and bottom inner surfaces, with 3mm thick chamber walls. It is made from 316 stainless steel, chosen for its low magnetic permeability in order to minimise stray magnetic fields.

The chamber has a total of 34 standard knife edge flanges for attaching windows and other components, all sealed using copper gaskets. Four equally spaced DN63CF flanges are located around the equator. Two of which are used for the Raman fibre feedthroughs and collimating optics, and the remaining two for the retro-reflecting mirrors. There are 12 DN40CF flanges

also around the equator, spaced evenly between the larger DN63CF flanges. In addition, there are 16 DN40CF flanges positioned $\pm 19^\circ$ from the equator. Two DN160CF flanges are located on the top and bottom flat surfaces, allowing installation access for larger internal components.

The chamber is designed to provide optical access for two orthogonal sets of Raman beams, three mutually orthogonal MOT beams, and a detection beam, all of which must intersect at the centre of the chamber. Figure 4.2a is a cut through the equatorial plane, showing the Raman beams (blue), MOT beams (red) and detection fluorescence (yellow). The Raman light enters the chamber through a fibre feedthrough located along each Raman axis. This light is subsequently collimated, polarised and retro-reflected within the vacuum chamber. In comparison, the cooling MOT light is collimated and polarised externally and enters the vacuum chamber through anti-reflection (AR) coated windows attached to the vacuum chamber.

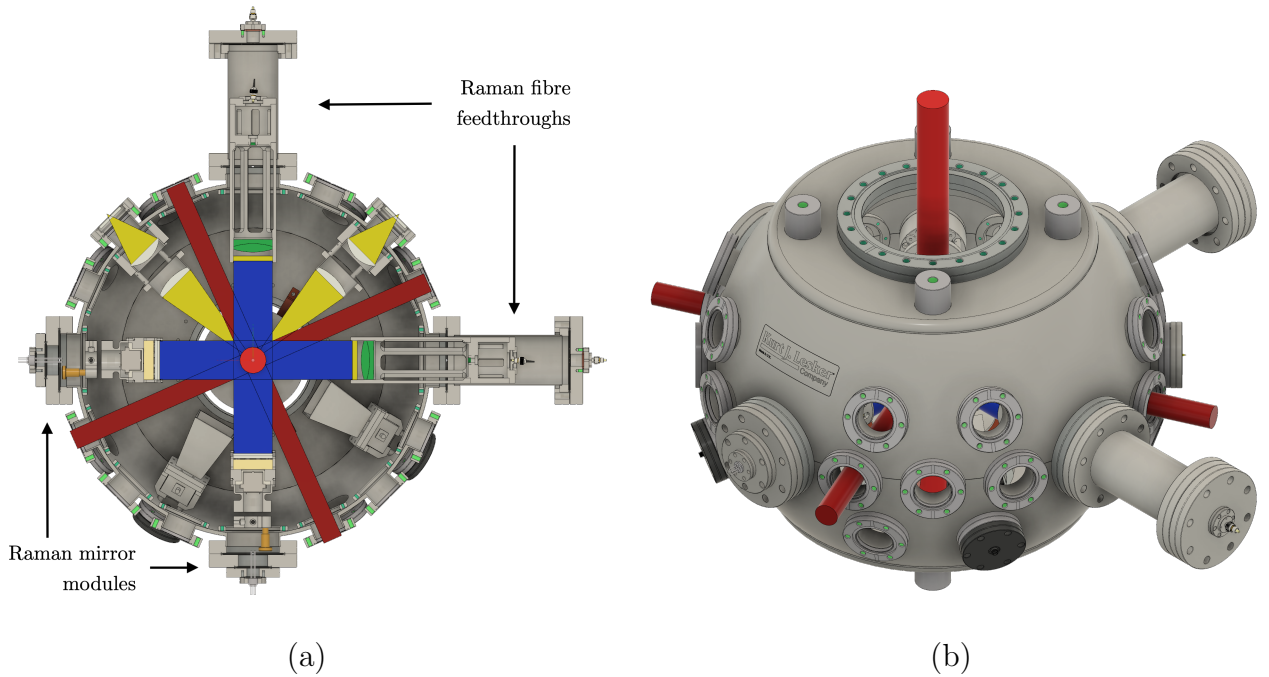


Figure 4.2: (a) Cut through image in the equatorial plane of the vacuum chamber, published in [1]. The two orthogonal Raman beams (blue) and equatorial cooling MOT beams (red) all intersect at the centre of the chamber. The fluorescence light (yellow) collected using the detection module (discussed in more detail later) is also shown. (b) External view of the vacuum chamber showing the relative positions of the vertical and horizontal cooling MOT beams (red) relative to chamber and Raman beam, published in [1].

To ensure the chamber reached a sufficiently low pressure, all components installed inside the chamber were first cleaned and baked separately. The cleaning process consisted of an ultra-

sonic bath and a subsequent series of acetone, methanol and isopropyl wipes. Once all internal components were installed and all ports sealed, the chamber was heated to approximately 100°C at a rate of 1°C/min. The chamber was then pumped using a turbo molecular pump (TMP) while its temperature was maintained at 100°C. The process of heating the chamber ensures any contaminants on the inner chamber walls evaporate and are subsequently pumped out. After two weeks of baking under the turbo molecular pump, the pressure reached 10^{-8} mbar. At which point an ion getter pump (SAES NEX Torr D100-5) was switched on and the chamber was then isolated from the TMP by closing an angle valve. A base pressure of 5×10^{-9} mbar was reached after a month of pumping.

4.3 Laser System

Figure 4.3 is an energy level diagram showing the transitions used in the experiment. The light required for all these transitions is provided by a purpose-built laser system developed by *M-Squared Lasers*. This laser system provides the two Raman frequencies, as well as the cooling, repump, state selection and detection frequencies. This high power laser system can output two frequencies at any particular time. The two frequencies are produced by two independent titanium-sapphire lasers, with the frequency difference controlled in a master-slave configuration. During a single experimental sequence the master laser frequency is changed by more than 1GHz. The master light provides all frequencies to drive transitions indicated by the red arrows, and the slave laser provides all frequencies to drive the transitions indicated by the blue arrows in Figure 4.3.

4.3.1 Requirements for the Raman Light

Aspects of the Raman light that need to be considered include the detuning, Δ , from the intermediate state (indicated in Figure 4.3), the intensity ratio of the two frequencies, and their relative phase stability needed to meet the desired accelerometer sensitivity of 100ng at 1 second.

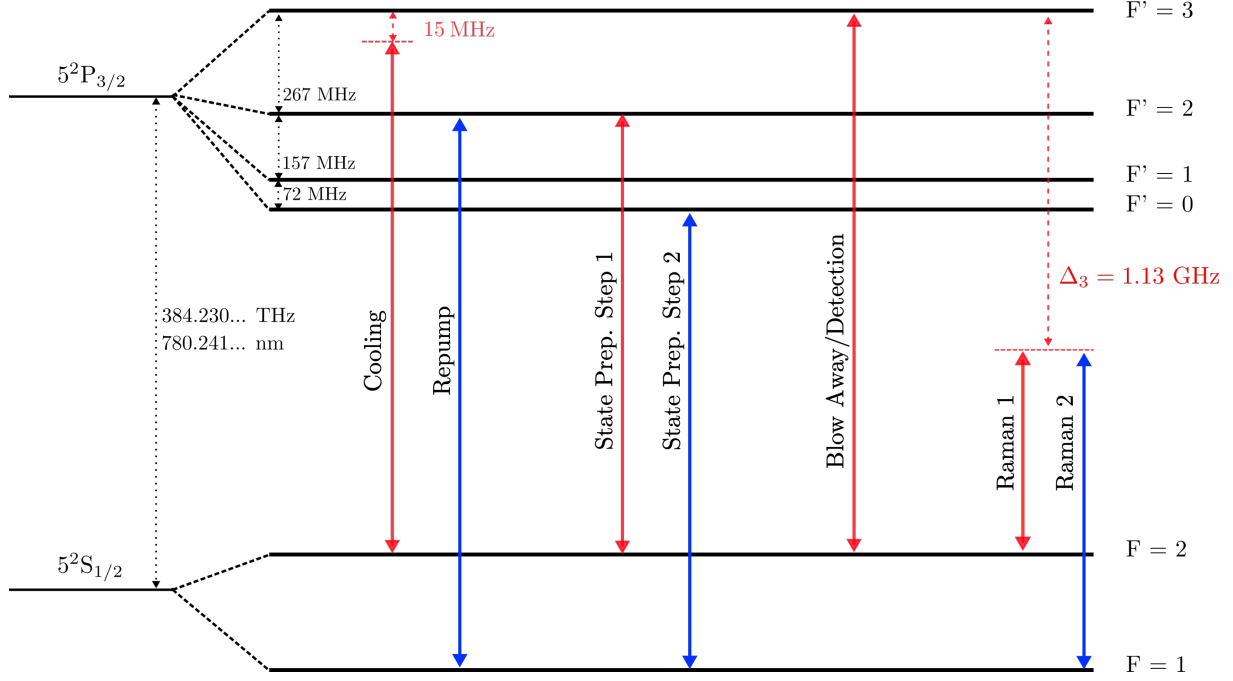


Figure 4.3: Energy level diagram of the D2 line in ^{87}Rb with the various transitions driven throughout a single sequence. All transitions indicated with a red arrow are driven by the master laser, and those indicated by a blue arrow are driven by the slave laser.

The detuning and intensity ratio are important for zeroing the differential light shift as discussed in Chapter 2. A detuning of 1.13GHz from the cycling transition with an intensity ratio of approximately 0.583 is chosen to cancel this light shift. The choice of these parameters is discussed in further detail in [1].

The final key design consideration is the phase stability of the beat note between the two Raman frequencies. The phase of the atomic coherence induced by a $\pi/2$ pulse at time t is $\mathbf{k}_{\text{eff}} \cdot \mathbf{r} + \phi_{\text{eff}}(\mathbf{r}, t)$, where ϕ_{eff} is the phase difference between the Raman lasers at the position $\mathbf{r}$ of the atom. Fluctuations of ϕ_{eff} therefore produce noise in the interferometer phase. To reach the desired sensitivity of 100ng with an interferometer having $T = 25\text{ms}$, we require the noise in the interferometer phase $\Delta\phi$ to be less than 10mrad.

4.3.2 MSquared Laser System Design

A schematic of the full laser system is depicted in Figure 4.4. The system contains two titanium-sapphire lasers pumped by 15W and 10W at 532nm, producing 5.5W and 2.8W of master and

slave light at 780nm, respectively.

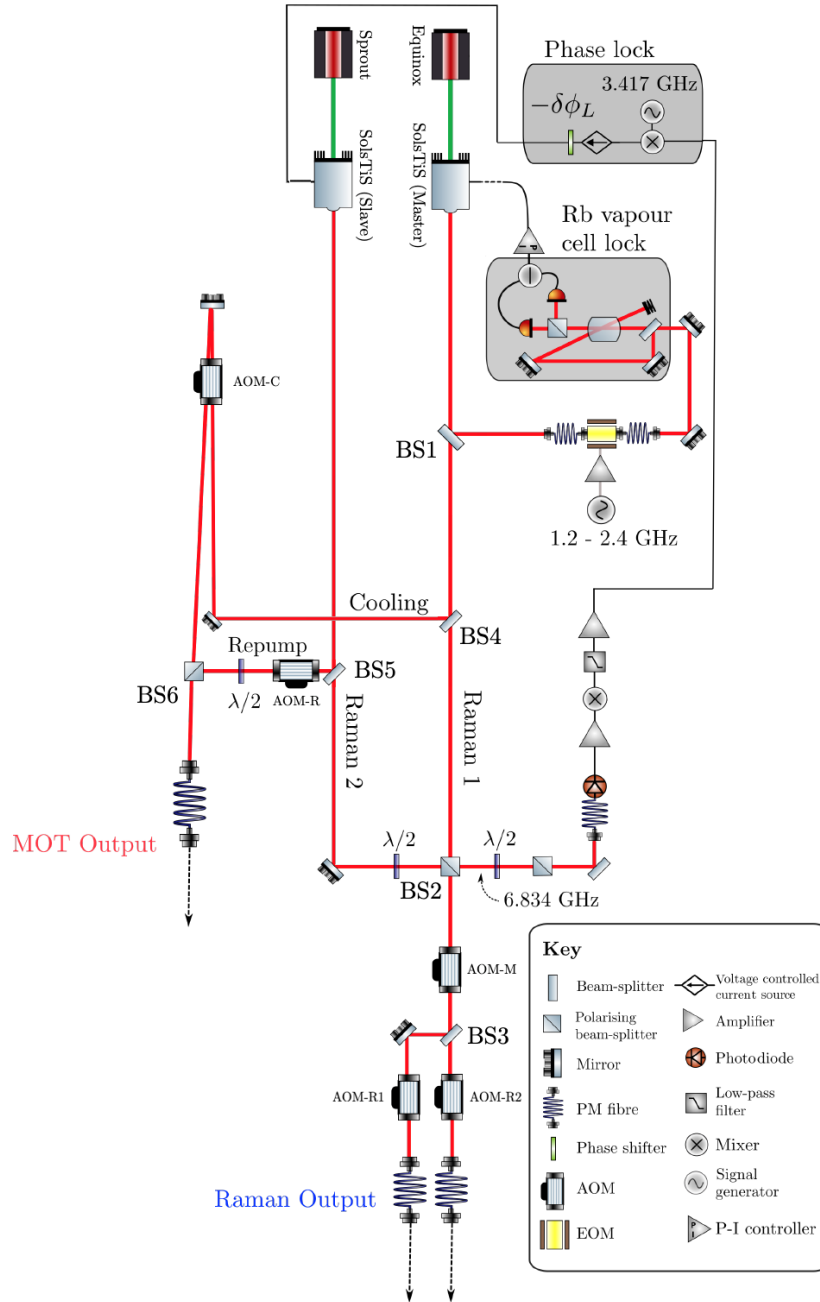


Figure 4.4: Laser system schematic. The master and slave titanium-sapphire lasers (MSquared SolS TiS) are pumped by an MSquared Equinox and Lighthouse Photonics DPSS Sprout laser, respectively. The master laser is locked to the ^{85}Rb $F = 3 \rightarrow F' = 3/4$ crossover transition at frequency offset f_{eom} within the vapour cell module. The slave laser is then phase locked to the master within the phase lock module, generating a phase locked dual-frequency output.

The master light is frequency locked to an atomic reference using a vapour cell lock. A small amount of the master light is picked off at BS1 and passes through an electro-optic modulator

(EOM), producing sidebands at $\pm f_{\text{eom}}$, before being sent into the vapour cell module. The blue sideband is frequency locked to the $^{85}\text{Rb } F = 3 \leftrightarrow F' = 3/4$ cross-over transition by saturation absorption spectroscopy within the vapour cell. The master laser output frequency can then be tuned by adjusting f_{eom} , which is supplied to the EOM by a voltage-controlled oscillator (VCO). This frequency is able to be tuned by more than 1GHz while maintaining its reference to the atomic transition.

The two lasers are then phase locked in a master and slave configuration through the phase lock module depicted in Figure 4.4. A small fraction of both the master and slave light is picked off at BS2 and sent into the phase lock module of the system. This light is incident on a fast photodiode which detects the beat note of the two frequencies. This signal is then amplified, divided by a factor of two, and mixed with a $f_{\text{PL}}/2 = 3.417\text{GHz}$ signal synthesised from a 100MHz reference oscillator. The mixed output signal is the phase lock error signal which is close to DC. This error signal is integrated and fed back to a piezo-actuated mirror and intracavity EOM inside the slave laser, creating a phase lock between the master and slave laser with an offset in frequency of precisely 6.834682GHz. The beat note signal is passed through an analog phase shifter (RF Lambda RVPT0408GBC) allowing adjustment of the relative phase using an applied control voltage. The phase noise in the Raman beat note was measured by *MSquared Lasers* and was found to be less than 11mrad for $1\text{ms} < T < 100\text{ms}$ [73].

The two beams are overlapped at polarising beamsplitter BS2 forming a high power beam consisting of both phase locked frequencies which are polarised orthogonally. This light is then passed through a single pass master acousto-optic modulator (AOM-M). This particular AOM acts as a global shutter for all output Raman light, as well as shaping the Raman pulses temporally. The light is then split equally at BS3, after which each beam passes through a second AOM (AOM-R1 or AOM-R2) before being coupled into two separate polarisation maintaining fibres. These second AOMs also serve as shutters for each individual Raman beam, providing light along only the desired axis. The output frequencies in each Raman fibre

are given by

$$\begin{aligned} f_{\text{master}} &= f_{\text{CO}} + f_{\text{eom}} + f_{\text{AOM-M}} + f_{\text{AOM-R1/R2}} \\ f_{\text{slave}} &= f_{\text{CO}} + f_{\text{eom}} + f_{\text{PL}} + f_{\text{AOM-M}} + f_{\text{AOM-R1/R2}} \end{aligned} \quad (4.1)$$

where f_{CO} is the cross-over transition frequency, $f_{\text{AOM-M}}$ is the frequency applied to AOM-M, and $f_{\text{AOM-R1/R2}}$ are the frequencies applied to AOM-R1/R2. There is 325mW of master and 185mW of slave light incident on Raman axis 1 fibre (corresponding to the fibre at the end of the AOM-R1 path), and 355mW of master and 230mW of slave light incident on Raman axis 2 fibre (corresponding to the fibre at the end of the AOM-R2 path).

After BS4, a fraction of the master light is directed through a double pass acousto-optic modulator (AOM-C). After BS5, a fraction of the slave light is sent through a single pass acousto-optic modulator (AOM-R). These beams are overlapped at beamsplitter BS6 before the orthogonally polarised beams are then coupled into the polarisation maintaining fibre. The two output frequencies of this fibre drive the cooling, repump, state selection, blowaway and detection transitions, given by

$$\begin{aligned} f_{\text{master}} &= f_{\text{CO}} + f_{\text{eom}} + 2f_{\text{AOM-C}} \\ f_{\text{slave}} &= f_{\text{CO}} + f_{\text{eom}} + f_{\text{PL}} + f_{\text{AOM-R}} \end{aligned} \quad (4.2)$$

where f_{CO} is the cross-over transition frequency, and $f_{\text{AOM-C}}$ and $f_{\text{AOM-R}}$ are the frequencies applied to AOM-C and AOM-R, respectively. This fibre output contains 990mW of master and 23mW of slave light. The output frequencies from this fibre are 135MHz red detuned from the transitions depicted in Figure 4.3 as the light is subsequently passed through fibre-coupled AOMs operating at 135MHZ, which is described in more detail in Section 4.3.4.

Frequency Output During Sequence

The output laser frequencies during the sequence are tuned by ramping the frequency supplied to the EOM, f_{eom} . In order to switch the laser between the various operating frequencies, the vapour cell lock is first disengaged, f_{eom} is ramped to the desired frequency, and then the vapour

cell lock is re-engaged. The various output laser frequency configurations and the corresponding frequencies applied to the EOM and AOMs are detailed in Table 4.1. The frequency applied to AOM-C is also adjusted during the sequence to output the desired frequency in the third fibre.

	MOT	Molasses	State Selection	Velocity Selection	Blow Away	Raman	Detection
f_{eom}	1.38GHz	1.38GHz ↓ 1.52GHz	1.61GHz	2.04GHz	1.38GHz - 15MHz	2.04GHz	1.38GHz - 15MHz
$f_{\text{AOM-C}}$	84MHz	84MHz	71MHz	-	84MHz	-	84MHz
$f_{\text{AOM-R}}$	-87MHz	-87MHz	-87MHz	-	-87MHz	-	-87MHz
$f_{\text{AOM-M}}$	-	-	-	-80MHz	-	-80MHz	-
$f_{\text{AOM-R1/R2}}$	-	-	-	-80MHz	-	-80MHz	-

Table 4.1: The frequencies supplied to the EOM and various AOMs during the different stages of the sequence. During the molasses stage, the frequency supplied to the EOM is ramped in order to cool the atoms further. During the blow away and detection stages f_{EOM} is decreased by 15MHz so that the master light is resonant.

4.3.3 Collimation and Polarisation of the Raman Fibre

The light beams at the two Raman frequencies, which are linearly polarised orthogonally within the PM fibres, must be collimated, circularly polarised and retro-reflected such that we have the desired σ^+/σ^+ or σ^-/σ^- counter propagating beam configuration. In order to reduce the intensity variation across the atom cloud while still achieving sufficient intensity with the present laser system, the Raman beam is designed to have a 40mm beam waist ($1/e$ field radius). To avoid unwanted phase shifts of the light as the atoms move across the beam its divergence is required to be less than $560\mu\text{rad}$. The optical system used to collimate and polarise the Raman beams is illustrated in Figure 4.5a, and the collimation module when mounted to the

chamber is depicted in Figure 4.5b. The light first passes through a doublet lens with focal length of -25mm and diameter of 6.25mm, followed by an achromat lens with a focal length of 150mm and diameter of 50mm. The distance between the fibre and doublet lens is 52.5mm, and the distance between the doublet lens and achromat lens is 122mm. This collimation system employs a Galilean configuration meaning there is no intermediate focal point. Finally, a $\lambda/4$ waveplate transforms the orthogonal linear polarisation into circular with opposite helicity. The optics are all housed within a stainless steel tube mounted inside the vacuum chamber.

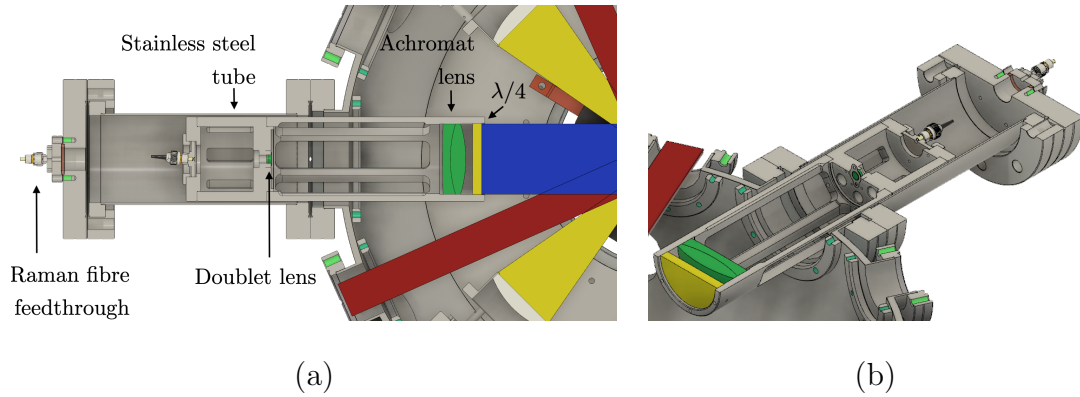


Figure 4.5: (a) Optical set-up used to collimate and circularly polarise the two linearly polarised frequency components emerging from the Raman fibre. The light passes through a doublet lens, achromat lens and $\lambda/4$ waveplate producing a circularly polarised, collimated beam with a waist of 40mm [1]. (b) The Raman collimator module mounted within the chamber containing the optics and fibre feedthrough [1].

Two identical Raman beam collimator systems have been assembled and are attached to the two PM fibres carrying the Raman light from the laser system. The collimators are arranged with their optic axes horizontal and perpendicular to each other. The beam waist is measured to be 41.5mm with a divergence $< 200\mu\text{rad}$ for both Raman beams. Further detail of the collimation method and analysis can be found in [1].

The retro-reflecting configuration provides the required counter-propagating $\sigma^\pm/\sigma^\pm$ polarisation, as discussed in Section 2.2.3. The retro-reflecting mirror module used to achieve this is shown in Figure 4.6. The module comprises another $\lambda/4$ waveplate placed in front of a mirror. After passing through this $\lambda/4$ waveplate twice, the helicity of the reflected light has the same handedness. The $\lambda/4$ waveplates in the collimation and retro-reflecting modules are identical, with a specified reflectance of $< 0.5\%$ at 780nm. The mirror reflectance is $> 98\%$ at 780nm.

The mounted optics are attached to a Radiant Dyes Laser and Accessories GmbH kinematic mount which has two mechanical actuators for fine adjustment of the mirror when in vacuum. A MEMS accelerometer is mounted directly to the back of the mirror as seen in Figure 4.6a. This mechanical sensor is used to measure the acceleration of the mirror with sufficient accuracy to identify the order number of the atom interferometer fringe. Technical details of the MEMS accelerometer will be discussed in more detail in Section 4.7.2. The mounting flange of the retro-reflecting module contains electrical feedthroughs which are used to apply the control voltages to the actuators, and to output the readout voltages from the MEMS accelerometer.

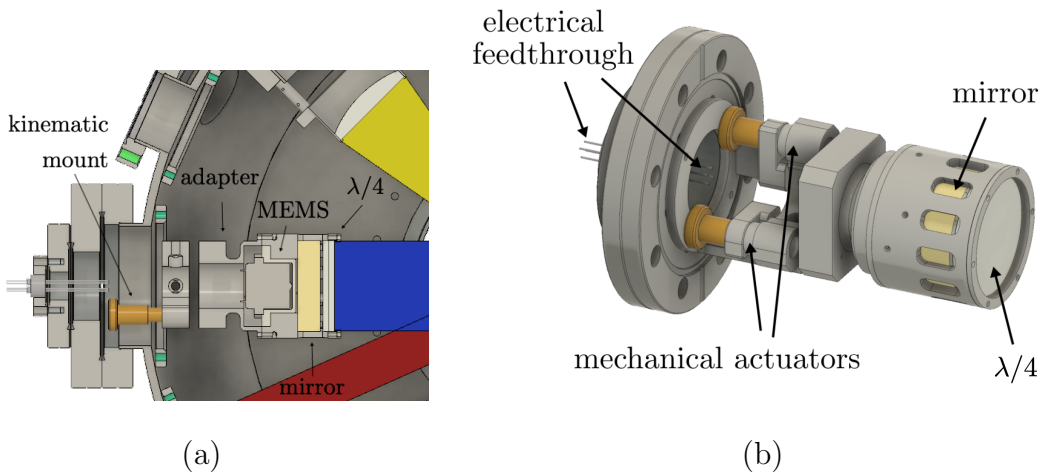


Figure 4.6: (a) Cut through image of the retro-reflecting module mounted within the chamber. The $\lambda/4$ waveplate, mirror and MEMS accelerometer are all indicated. (b) Full retro-reflecting module attached to the DN40 flange with the electrical feedthrough used for in-situ control of the kinematic mount and readout of the MEMS accelerometer.

4.3.4 MOT Fibre Output

The third fibre from the laser system, labeled “MOT output” in Figure 4.4, outputs orthogonally polarised beams at the two frequencies described by Equation 4.2. This light is used for cooling, state selection, blow away and detection. It is attached to a fibre distribution unit, where it is split into 9 separate fibres (six 3D MOT, one detection, one 2D MOT cooling and one 2D MOT push).

Fibre Distribution Unit

A schematic of the fibre distribution unit is shown in Figure 4.7. The light from the MOT output fibre is first collimated using a Schafer + Kirchoff 60SMS fibre collimator. This light is then passed through a $\lambda/4$ waveplate which compensates a slight birefringence we discovered in the fibre collimator. The orthogonally polarised master and slave beams are then separated into two arms by a polarising beamsplitter. The polarisation of the slave beam is rotated 90° by a $\lambda/2$ waveplate before it is recombined with the master beam at a 50:50 beamsplitter. This creates two arms of equal power in which the overlapped master and slave light beams are linearly polarised along the same axis. A sequence of beamsplitters and mirrors splits the light into nine independent beams. Each beam is then coupled into a Gooch & Housego 135MHz fibre AOM, driven by a voltage-controlled oscillator (VCO). A voltage-controlled attenuator (VCA) adjusts the RF power and hence the output power from each fibre AOM, allowing us to balance the six cooling beam powers. The chosen master and slave power outputs from each fibre AOM are detailed in Table 4.2. Because the master and slave beams are not perfectly overlapped, the relative efficiency of coupling into the output fibres varies from one output to another. Consequently there is quite a large variation across the nine outputs in the ratio of the master and slave powers.

Fibre Output	Master Power (mW)	Slave Power (mW)
X ⁺	19.4	1.1
X ⁻	19.4	0.4
Y ⁺	19.4	0.5
Y ⁻	19.4	0.8
Z ⁺	19.4	0.9
Z ⁻	19.4	0.6
2D MOT	100	2.2
Push	7	0.3
Detection	31.3	0.6

Table 4.2: Output master and slave powers for each fibre AOM from the distribution unit.

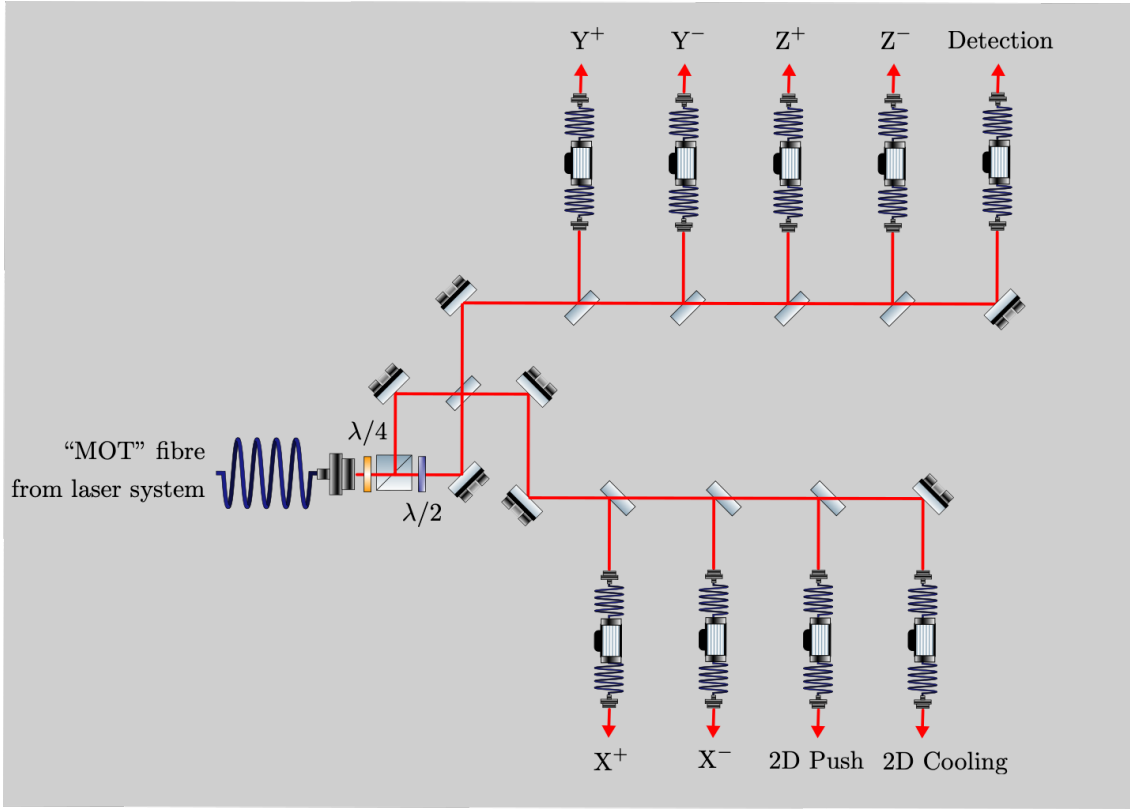


Figure 4.7: Fibre distribution module used to create the nine separate beams from the main MOT fibre. The polarisation of the repump light is rotated by 90° to overlap with the cooling light. Nine separate beams are created using a series of beamsplitters with each beam finally the coupled into a 135MHz fibre AOM. Individual RF signals are supplied to each AOM controlling its operation frequency.

MOT and Detection Beam Collimators

The detection and 3D MOT fibre outputs are all attached to modules which collimate and circularly polarise the light. Each collimator contains a fibre coupler plate, 150mm plano-convex lens, linear polariser and a zero order quarter-wave plate which generates a collimated beam with a waist of 12.7mm and either left or right handed circular polarisation. The linear polariser converts polarisation fluctuations into intensity fluctuations. This is desirable as the intensity can be directly measured using the detection system.

The 19.4mW of cooling light in each 3D MOT beam provides a sufficient intensity of $7.9\text{mW}/\text{cm}^2$ ($4.7I_{\text{sat}}$)[62]). The waveplates are set using a Thorlabs PAX1000 polarimeter which directly measures the polarisation state of each beam. To create a 3D MOT we require four horizontal beams with one helicity, and two vertical beams with the other helicity. The detection beam

is retro-reflected by a mirror placed on the opposite window, making two counter-propagating beams to ensure there is no average force pushing the atoms out of the detection region.

4.4 Atom Source using Magneto-Optical Traps

A MOT is a very useful technique in which atoms can be trapped and cooled along one cartesian axis by employing two red detuned and circularly polarised counter-propagating laser beams and a magnetic field gradient. The laser beams provide a damping force, while the magnetic field ensures the atom-light interaction is position dependent[74].

To understand how a MOT works we consider a simple system in which we have two energy levels, a ground level with total angular momentum, $F = 0$, and an excited level with $F'=1$, illustrated in Figure 4.8. The required magnetic field gradient is provided by a quadrupole field which has a zero at its centre. The Zeeman effect results in a position dependent splitting of the sublevels of the atom as it moves through the magnetic field. The red detuning of the counter-propagating laser beams is essential for cooling as the Doppler shift experienced by atoms travelling in the opposite direction brings the laser frequency closer to resonance, resulting in a velocity dependent force which always opposes motion of the atoms. The position dependent splitting of the Zeeman sub-levels then creates a position dependent restoring force, spatially confining the atoms to the trap. To demonstrate the trapping effect let us consider an atom on the right in Figure 4.8. Because the $m_F = -1$ sublevel is lower in energy than the $m_F = +1$, the left-going beam (polarised to drive $\Delta m_F = -1$ transitions) is closer to resonance and exerts a stronger radiation pressure than the right-going beam (polarised to drive $\Delta m_F = +1$). The atom is therefore pushed towards the centre. Similarly, an atom on the left (where the magnetic field is oppositely directed) will be more strongly pushed by the right-going, and again feels a force towards the centre.

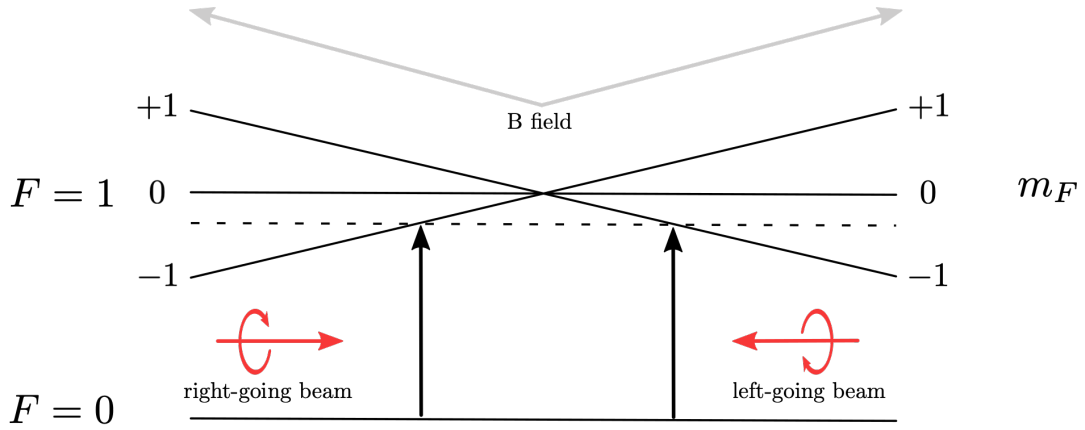


Figure 4.8: Illustration of a one dimensional MOT for an atom with a ground $F=0$ state and an excited $F'=1$ state. The magnetic field gradient due to the quadrupole field produces a linear Zeeman shift of the excited state's sub levels with position. The red detuned circularly polarised light and linearly varying magnetic field produces a restoring force on an atom which pushes it to the centre of the trap.

The combination of the counter-propagating, red-detuned and circularly polarised beams with the linearly varying magnetic field results in a velocity and position dependent force which is always directed towards the centre of the trap. Assuming the friction arises solely from the velocity dependence of the Doppler shift, the MOT force can be approximated by $F = -\alpha v - \beta x$ [72], where v and x are the velocity and position of the atom, respectively.

Extending this arrangement to three mutually orthogonal axes results in cooling and trapping of the atoms to some central volume. This configuration, known as the 3D MOT, is used in my experiment. It is possible to load a 3D MOT from a partial pressure of Rb vapour [75]. However to minimise the background pressure in the chamber and reduce the loading time of the MOT, we instead load from a collimated, high flux atomic beam created by an external 2D MOT [76, 77].

4.4.1 2D MOT

The 2D MOT is created in an external cell which is attached to the chamber at an off equator DN40CF flange. It cools and traps atoms along two orthogonal axes, creating a cloud of cold atoms which is elongated along the third axis, directed towards the centre of the 3D MOT.

The 2D MOT assembly is provided by *Cold Quanta Inc.* and consists of a glass cell and two pairs of rectangular quadrupole coils as depicted in Figure 4.9a. The coils produce a two dimensional quadrupole field with a line of zero magnetic field down the z axis. The coils are designed to provide a magnetic field gradient of 20G/cm/A along the x and y axes. The 30mm x 30mm x 44mm cell contains ^{87}Rb dispensers which release the Rb when heated by an applied current. On the chamber side of the cell is a 0.7mm diameter pinhole which allows the passage of the atomic beam into the main chamber. The small diameter of this pinhole isolates the UHV main chamber from the higher Rb pressure within the cell that is required to accumulate a large number of atoms in the 2D MOT. This pinhole also acts as a spatial filter, preventing atoms with a large transverse velocity from exiting the cell[76]. The two orthogonal pairs of counter-propagating beams in combination with the two-dimensional quadrupole field creates a “cooling volume” in which atoms experience a restoring force radially towards the line of zero magnetic field. Atoms entering this volume are cooled along the x and y axes, however the z axis “longitudinal” velocity remains unchanged, creating two oppositely travelling thin, collimated beams of atoms. The interaction time within the cooling volume must be long enough for an atom to be cooled sufficiently such that its trajectory passes through the pinhole. This interplay of finite cooling time and spatial filtering by the pinhole causes the longitudinal velocity distribution in the output beam to be skewed to much smaller velocities than the room temperature average [76].

In addition to the quadrupole 2D MOT coils, we also have two pairs of bias coils producing approximately homogeneous transverse magnetic fields. These bias coils are used for fine adjustment of the ambient field within the cell to shift the cooling volume of the 2D MOT. These coils are also rectangular and consist of 20 turns each. The atom cloud in the 2D MOT cell will shift across the pinhole aperture with a current of $\pm 0.3\text{A}$ through the bias coils. The flux

of the atomic beam leaving the 2D MOT can be optimised by adjusting the strength of the magnetic field produced by the bias coils.

A fifth beam is directed along the z axis towards the centre of the chamber. This beam, known as the push beam, provides a net momentum transfer to the atoms along the direction of propagation. As there is no magnetic field along this axis, the push beam addresses only a particular velocity class of atoms which is dependent on its detuning. The opposite cell wall partially reflects the push beam, which also results in some additional cooling along this axis [78].

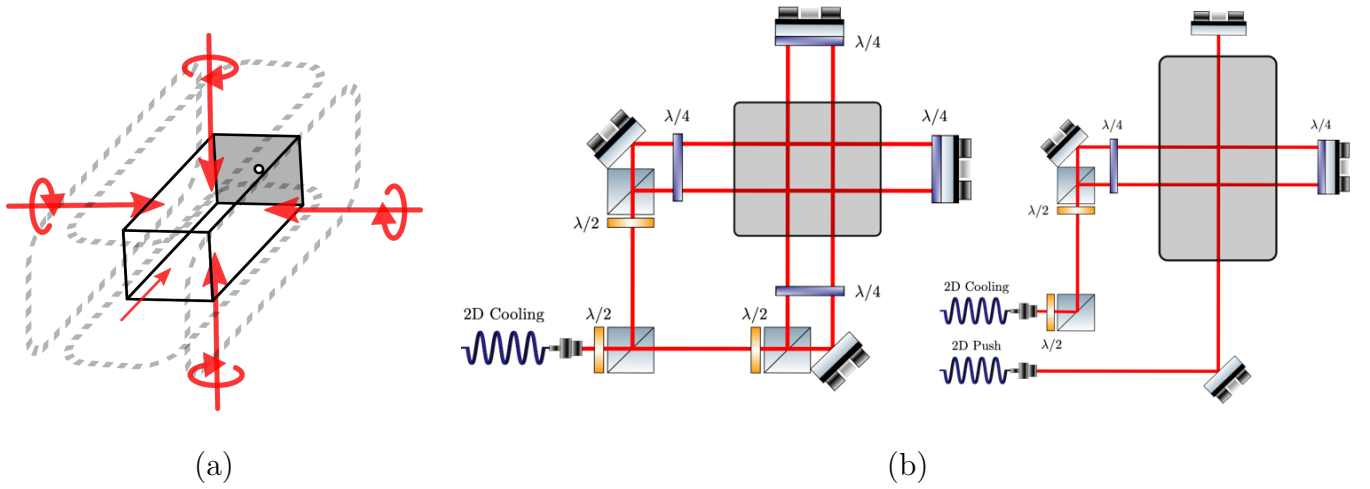


Figure 4.9: (a) Diagram of the 2D MOT cell and pair of quadrupole fields. The two orthogonal pairs of counter-propagating beams all drive either σ^+ or σ^- transitions leading to cooling and trapping along the x and y axes. The linearly polarised push beam is directed along the z axis towards the pinhole of the cell. (b) Schematic of the optical set-up used to generate the collimated cooling and push beams. All cooling light is supplied by the 2D MOT fibre, and the push beam light is supplied in a separate fibre.

As described in Section 4.3.4, the light for the 2D MOT is supplied in a single fibre, and another single fibre contains the push beam light. Figure 4.9b illustrates the optical set-up used to generate the required cooling and push beams using these two fibre inputs. The cooling light is collimated to a beam with a 9.5mm waist using two aspheric lenses. This light is then split into two arms, one for each cooling axis of the 2D MOT, by a polarising beamsplitter. Both arms contain 50mW of cooling light, with the power splitting controlled by a $\lambda/2$ waveplate directly in front of the polarising beamsplitter. The light in each arm is then passed through

a beamsplitter and prism mirror, in order to increase the cross-sectional area of the beam. A $\lambda/4$ waveplate placed directly after the beamsplitter and mirror, transforms the linearly polarised light into circularly polarised light. Since each beam after the second beamsplitter has approximately 25mW, we have close to $17I_{\text{sat}}$ of cooling light at the centre of the cell due to each individual beam. The incoming light is then retro-reflected on the opposite side of the cell by mirrors with integrated $\lambda/4$ waveplates such that the reflected light has the same helicity as the incoming. The 7mW push beam is collimated to a beam waist ($1/e^2$ intensity radius) of 0.75mm using a fixed focus collimator. A kinematic mount allows fine tuning of the alignment of the push beam relative to the 2D MOT cell and pinhole.

4.4.2 3D MOT

The stream of cooled atoms exiting the 2D MOT is then captured by the 3D MOT at the centre of the chamber and cooled along all three dimensions.

Each of the six 3D MOT beams is set to the same 19.4mW, with a detuning of -15MHz, ensuring the scattering force is balanced along each axis. This gives a total intensity provided by the six 3D MOT beams of $28.4I_{\text{sat}}$ at the centre of the trap. Such an intensity should be sufficient to operate in the regime where the number of atoms trapped in the MOT becomes independent of the laser power. Operating in this regime is desirable to minimise fluctuations in the number of atoms trapped in the 3D MOT due to intensity drift of each beam. The six MOT beam collimators are attached to the chamber allowing the light to enter through the AR coated windows. The vertical pair of MOT beams will be denoted by Z^+/Z^- , and the two horizontal pairs denoted by X^+/X^- , and Y^+/Y^- . The alignment of each beam was set such that light was coupled into the fibre of the opposite collimator to ensure a good overlap of the counter-propagating beams. All six beams intersect at the centre of the chamber.

The quadrupole magnetic field for the 3D MOT is produced by two coaxial coils mounted inside the chamber, with current running in opposite directions. To achieve a sufficiently dense and cold 3D MOT, a magnetic field gradient of approximately 15G/cm is desired[79, 54]. The coils are mounted inside the chamber in order to achieve such a magnetic field gradient without

requiring a very large current. A diagram of the coils is shown in Figure 4.10. The coils have an inner radius of 20mm, ensuring there is sufficient room for the vertical MOT beams to pass through, and an outer radius of 64mm. The distance between the inner faces of the top and bottom coil is 76mm. Each coil is attached to support legs which are in direct contact with the main body of the chamber allowing efficient removal by thermal conduction of heat generated by the current by thermal conduction. Both coils are wound using 1.1mm x 1.1mm rectangular copper wire coated in a UHV compatible Kapton film.

The coils are designed to provide the magnetic field gradient of 15G/cm across the central area of the vacuum chamber. The power dissipated in the coils as a function of current, and number of radial and axial turns was calculated for this particular field gradient. It was found that the power could be minimised to only 1.8W per coil when using 40 radial turns, 30 axial turns and a current of 543mA. The expected magnetic field gradient as a function of distance from the centre of the chamber for this particular configuration is shown in Figure 4.10b.

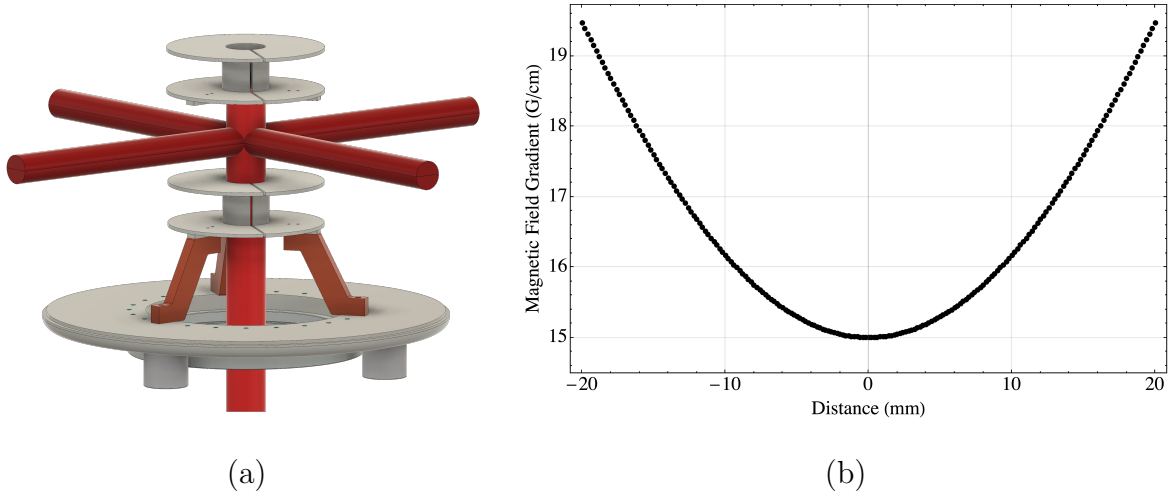


Figure 4.10: (a) The coaxial pair of coils used to provide the quadrupole field for the 3D MOT. The coil formers are attached to the chamber by three support legs, allowing efficient removal of heat (the legs attached to the top coil former are not shown here). The six cooling MOT beams are also shown, intersecting at the centre of the chamber. The black arrows indicate the direction of the quadrupole magnetic field[1]. (b) The magnetic field gradient produced by the quadrupole coils when using a current of 543mA, plotted as a function of distance from the centre of the chamber.

When the supply voltage driving the coils is instantaneously reduced to zero, the magnetic field $1/e$ decay time is measured by a Hall probe to be 14ms, which is comparable to the L/R natural

time constant of 11ms (for a measured resistance of 6.3Ω and an inductance of 60mH). The time taken for the field to drop to below 10% was 31ms. It was found that the magnetic field decay time could be reduced by running a reverse current through the coils for a short period of time before then ramping the current back up to zero. We want a short magnetic field decay time so that sub-Doppler cooling can become effective quickly. The optimum sequence was found to be linearly ramping the current to -1.3A over 0.1ms, holding the current at this value for 2.2ms and finally linearly ramping it back up over 7.2ms. Using this switch off sequence allowed the magnetic field to decay to below 10% within only 10ms.

4.5 Bias Magnetic Field Coils

Three orthogonal pairs of coils are mounted outside the chamber to provide a homogeneous magnetic field close to the centre of the vacuum chamber. This magnetic field is used to cancel the ambient magnetic field of the laboratory, and to provide a uniform bias field of known magnitude and direction which can be altered to suit the various different stages of the sequence. Two of the pairs produce fields, denoted by B_X and B_Y , in the equatorial plane orthogonally to each other, approximately 45° to the Raman axes. The third pair provides a vertical field, denoted by B_Z .

The four equatorial field coils have a radius of 15cm, and a separation of 44cm, and the vertical field coils have a radius of 10cm and a separation of 30cm. Each bias coil consists of 20 turns. Figure 4.11a shows the expected magnetic field produced by each coil pair at the centre of the chamber as a function of current supplied. Figure 4.11b shows the small magnetic field gradient produced by the coil pairs when the supplied current is set such that a 1G magnetic field is produced at the centre of the chamber.

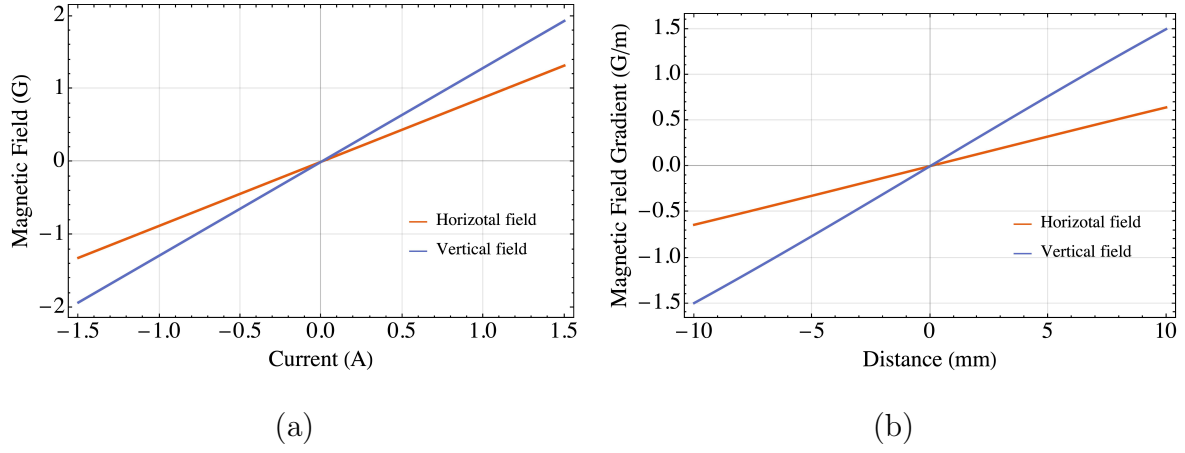


Figure 4.11: (a) The expected magnetic field produced by the vertical and horizontal coil pairs at the centre of the chamber as a function of applied current. (b) The expected magnetic field gradient along the axis of the coil pairs as a function of distance from the centre of the chamber. Here it is assumed that the current supplied produces a 1G magnetic field at the centre of the chamber.

4.6 Detection System

Following the atom preparation and interferometry pulse sequence we then want to measure the number of atoms in each hyperfine state to calculate the transition probability, and hence to infer the interferometer phase. We use state sensitive fluorescence, which involves measuring the number of scattered photons when light of known intensity and detuning is incident on the atoms. Using the known scattering rate, which is the rate at which a cycle of absorption followed by spontaneous emission occurs per atom, the number of atoms can be inferred. A photodiode is used to measure the scattered optical power which outputs a proportional current. This current is converted to a voltage using a low noise amplifier which is then measured using a National Instruments PXIe-4462 24-bit analog to digital acquisition card.

Four independent voltage measurements are used to calculate the transition probability. These are; (1) the scattered light signal while resonant with atoms in the $F = 2$ hyperfine level, V_2 , (2) the scattered light signal while resonant with atoms in both hyperfine states, V_{Tot} , (3) the background signal during the V_2 measurement, B_2 , and (4) the background signal during the V_{Tot} measurement, B_{Tot} . The probability ratio, P_2 , is then calculated using the background

corrected voltage values for the number of atoms in the $F = 2$ level, N_2 , and the total number of atoms N_{Tot} .

$$P_2 = \frac{N_2}{N_{\text{Tot}}} = \frac{V_2 - B_2}{V_{\text{Tot}} - B_{\text{Tot}}} \quad (4.3)$$

First, to measure V_2 we shine detection light that resonantly drives the $F = 2 \rightarrow F' = 3$ transition. Next, V_{Tot} is measured by also driving the $F = 1 \rightarrow F' = 2$ transition. All atoms are then blown away using a single resonant beam, which provides a non-zero net force that pushes the atoms out of the detection region. The background signals, B_2 and B_{Tot} , are then measured by switching on the corresponding light with no atoms in the detection region. The length of each detection period is the same. The DAQ card samples at 200kHz and the mean for each detection period is taken to give the final value for each individual measurement. Optical pumping during the detection period as well as atoms that do not participate in the interferometer but are still detected can lead to incorrect transition probability being measured. To overcome this we measure the appropriate scale factors to account for these effects.

The fluorescence is collected using a system of lenses that was designed in [1] and is shown in Figure 4.12. Lens 1 has a focal length of 125mm, and lens 2 has a focal length of 75mm. The indicated relative positions of the lenses, L_1 , L_2 and L_3 , are 140mm, 60mm and 59mm respectively. A fractional solid angle of 0.007 is collected by the lens 1, and the transverse magnification is 0.5. The collected light is incident on a Hamamatsu S2744-09 photodiode, which has a large active area of 20mm×10mm. This allows us to collect the fluorescence from a region 20mm wide and 40mm high in the object plane. The active material is Si with a spectral response of 0.58A/W at 780nm, and a typical dark current of 3nA (maximum rated dark current is 10nA). The output current from the photodiode is then amplified and converted to a voltage using a trans-impedance amplifier. A low noise Femto DLPCA-200 variable gain amplifier is used, with a gain of 10^3 to 10^{11} V/A and a bandwidth up to 500kHz. The photodiode is mounted outside the chamber as depicted in Figure 4.12b.

Two sets of these detection optics are mounted orthogonally within the chamber. Both sets are in the equatorial plane of the chamber at a 45° angle from the Raman beams. The detection optics are mounted in a stainless steel tube which is attached to the chamber internally at a

DN40CF flange.

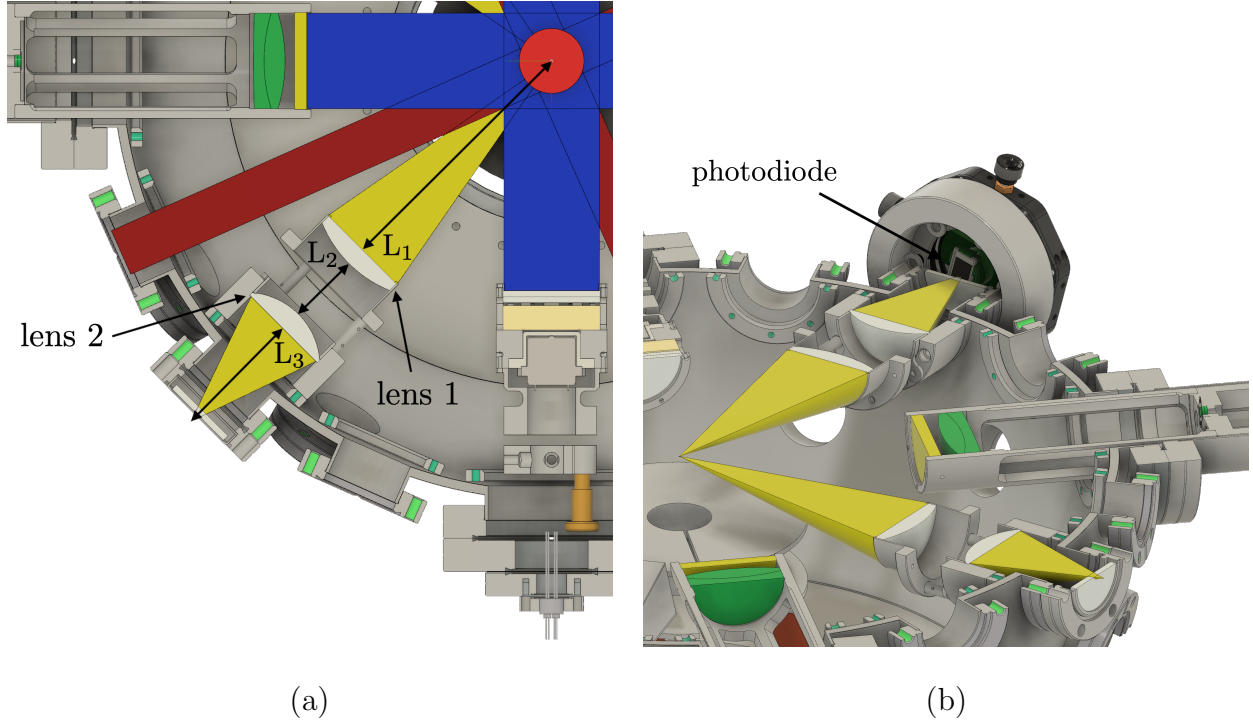


Figure 4.12: (a) Cut through image showing single set of detection optics in the equatorial plane collecting light (shown in yellow) from the centre of the chamber and imaging it onto the photodiode. Lens 1 is 140mm from the centre of the chamber, lens 2 is 59mm from the photodiode surface and the two lenses are separated by 60mm. (b) Cut through image of both sets of detection beams viewed from above. The light collected (yellow) is imaged onto the large surface area photodiode which is mounted to the chamber externally.

The dedicated detection light comprises two counter-propagating beams with opposite helicity. With the bias magnetic field parallel to the optic axis, these beams drive either pure σ^+ or σ^- transitions, quickly pumping the atoms onto the cycling transition used for laser-induced fluorescence detection. The detection beam is oriented at a 19° angle to the horizontal plane. As soon as the vertical position of the atoms is more than $\pm 12\text{mm}$ from the centre of the chamber they can no longer be detected as they fall outside of the detection beam, whose size is limited by the aperture of the chamber windows. Alternatively, using the vertical MOT beam pair allows the atoms to be detected over the larger vertical position range of $\pm 20\text{mm}$, limited by the height of the photodiode. However one MOT beam drives σ^+ transitions while the other drives σ^- transitions. This leads to considerable optical pumping of the atoms into a dark state and therefore a decrease in the fluorescence signal with time, which hinders accurate

measurement of the transition probability.

The absolute number of atoms can be inferred from the fluorescence signal by knowing the scattering rate, R_{sc} , for an atom is given by [72]

$$R_{\text{sc}} = \frac{\Gamma}{2} \frac{s}{1 + s + (2\delta/\Gamma)^2} \quad (4.4)$$

where Γ is the natural linewidth of the transition, s is the ratio of intensity of detection light and the saturation intensity, and δ is the detuning from resonance of the detection light. The photodiode voltage can be converted to the number of atoms using the following equation:

$$N_{\text{atom}} = \frac{V_{\text{Tot}} - B_{\text{Tot}}}{\nu \Omega \hbar \omega G_{\text{Tot}} R_{\text{sc}}} \quad (4.5)$$

where ν is the transmission efficiency of light through the collection optics, Ω is the fraction of photons collected by the optics, $\hbar \omega$ is the energy of a single photon and G_{Tot} is the total gain of the photodiode and amplifier module in V/W.

4.7 Other Components

4.7.1 Microwave Horns

The state preparation scheme used to prepare the atom cloud for interferometry (discussed in detail in Chapter 5) involves driving the $(5S_{1/2}, F = 1, m_F = 0) \rightarrow (5S_{1/2}, F = 2, m_F = 0)$ magnetic dipole transition at 6834.682MHz. The microwave pulse is launched into the chamber through a Pasternack PE9859/SF-10 horn waveguide. There are two of these mounted inside the chamber. Horn 1 is attached to a flange 19° above the equator, and horn 2 is attached to a flange 19° below the equator in the opposite direction such that the two horns are directed towards each other. The motivation of this is to provide some control over the standing wave pattern to ensure a strong microwave magnetic field at the site of the atoms.

The microwave fields are produced by two independent WindFreak SynthHD signal generators.

These triggerable devices output frequencies between 10MHz and 15GHz with 0.1Hz resolution. Both generators are referenced to the 100MHz clock used by the phase lock module within the laser system. They are connected to the PXI machine through USB connection enabling control of each RF output frequency and amplitude. The control circuit for the two microwave horn waveguides is shown in Figure 4.13. The output of each WindFreak is passed through a switch and amplifier. The switch provides 50dB of extra suppression when the microwave power is supposed to be off. This is important because the interferometer operates over tens of milliseconds and is therefore disturbed by even a weak resonant microwave field. The signal generator and switch are turned on and the output passes through a 22dB gain amplifier to drive the horn. A digital TTL signal is provided to both the WindFreak signal generator and switch. Separate TTL lines are used for the two horns for individual control.

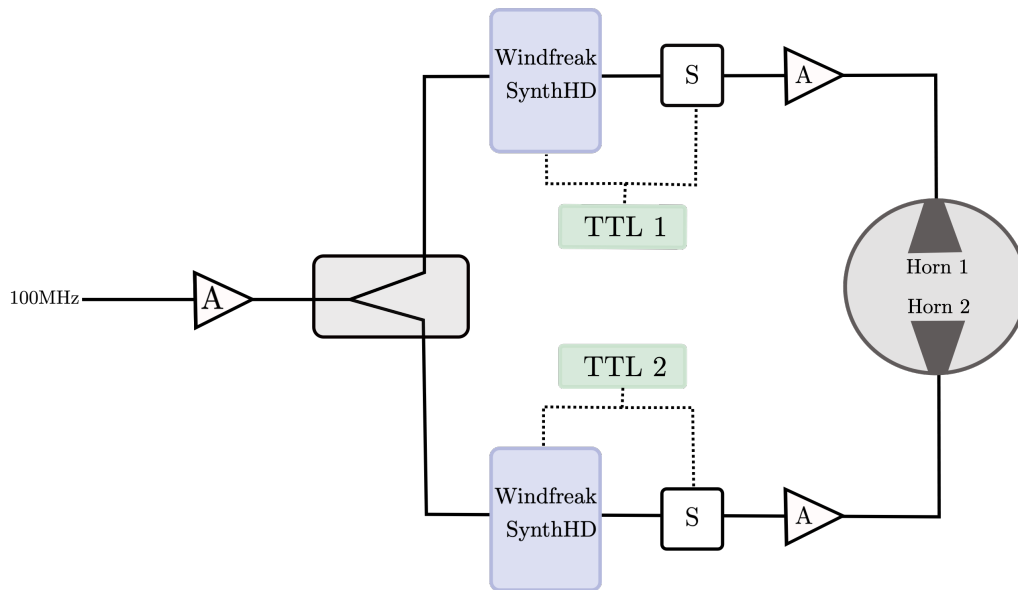


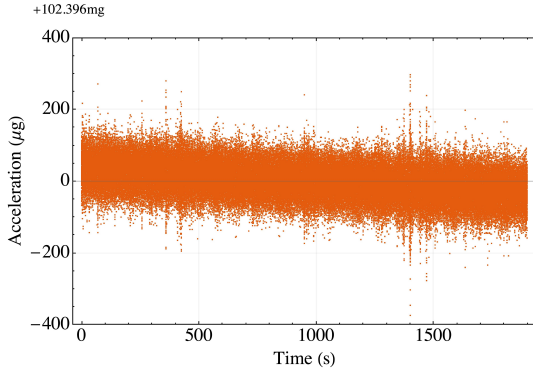
Figure 4.13: Control circuit for both WindFreak signal generators. Two separate TTL lines control the output of each microwave horn inside the chamber. Both WindFreak devices are referenced to the 100MHz clock used by the phase lock module within the laser system.

4.7.2 MEMS Accelerometer

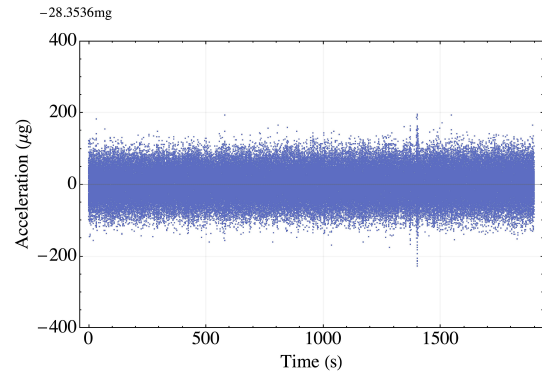
As mentioned in Section 4.3.3, a classical MEMS accelerometer is mounted directly behind the retro-reflecting mirror. The component of acceleration of this mirror projected along the Raman axis is what the interferometer phase shift measures. By monitoring this acceleration with the MEMS device it is possible to identify the order number of the interferometer fringe. Of course, this requires the MEMS accelerometer measurement to have an uncertainty much less than the fringe period $\frac{\lambda}{2T^2}$ of the interferometer. We use an Innalabs AI-Q-2010 quartz pendulum MEMS accelerometer which has a specified intrinsic noise level of $70\mu g$ for a bandwidth 0-500Hz (i.e. averaging over 1ms). This is comparable to the $64\mu g$ fringe period for a $2T = 50\text{ms}$ interferometer, and is much smaller than that when averaged over 50ms. Detailed operating principles of the MEMS device will not be discussed here, but can be found elsewhere [80]. The device outputs a current which is required to maintain the null position of an internal pendulum, and this current is in turn proportional to the acceleration. A temperature measurement is also output by the MEMS device which can be used to account for its temperature-dependent bias and scale factor. A printed circuit board connected directly to the vacuum feedthrough pins from each accelerometer contains a high precision resistor network manufactured by Vishay Foil Resistors. This printed circuit board eliminates unwanted additional noise that would be picked up by long connecting wires. Using a high a high precision resistance meter, we measured resistances of $6000.40 \pm 0.01 \Omega$ and $5999.71 \pm 0.02 \Omega$ for readout of the acceleration signal along Raman axes 1 and 2, respectively, and $2000.21 \pm 0.01 \Omega$ and $1999.90 \pm 0.02 \Omega$ for the temperature readout along Raman axes 1 and 2, respectively. The output acceleration and temperature voltages are measured using a 24-bit National Instruments analog to digital card.

With the MEMS devices mounted within the retro-reflecting module inside the chamber, the acceleration and temperature for each device was recorded simultaneously. Both devices were switched on and run initially for the recommended 30 minute “warm-up phase”. The temperature and acceleration outputs were then recorded after this. Both outputs were integrated over 20ms for a total measurement period of 2000s. These recordings were taken overnight to minimise ambient building vibrations. Figure 4.14 shows a time trace of the acceleration

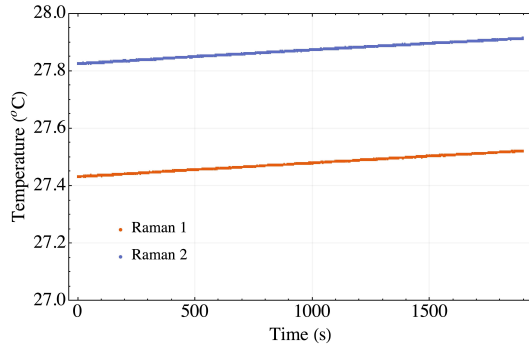
and temperature outputs, along with the corresponding Allan deviation of the temperature-corrected acceleration.



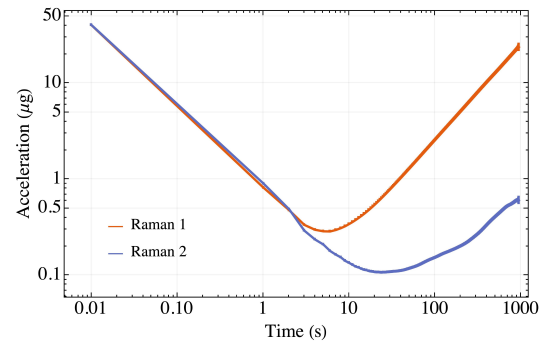
(a) Raman Axis 1 Acceleration



(b) Raman Axis 2 Acceleration



(c) Raman Axis 1 (red) and 2 (blue)



(d) Raman Axis 1 (red) and 2 (blue)

Temperature

Allan Deviation

Figure 4.14: Acceleration and temperature measurements from the MEMS devices along Raman axes 1 and 2. (a) Acceleration measured using the Raman axis 1 MEMS device. (b) Acceleration measured using the Raman axis 2 MEMS device. (c) Temperature measured using the Raman axis 1 and 2 MEMS devices. (d) Allan deviation of the temperature-corrected acceleration measurements along the Raman axes 1 and 2.

The noise of both MEMS devices at 20ms is $40\mu\text{g}$ (25Hz bandwidth) which is in agreement with the specified quiescent noise of $<70\mu\text{g}$ within the spectral band of 10-500Hz. Assuming the MEMS accelerometer integrates over the period of the interferometer ($20\text{ms} < 2T < 50\text{ms}$) the noise bandwidth will be $10\text{Hz} < \delta f < 25\text{Hz}$. Therefore the MEMS noise for each interferometer time will be a lot less than the interferometer fringe period.

The bias stability appears to be quite different between the two devices. This is evidenced by the differing times and acceleration values at which the two Allan deviation curves begin to

increase. The MEMS along Raman axis 1 is stable to $0.4\mu\text{g}$ at 5s, whereas the Raman axis 2 MEMS is stable to $0.1\mu\text{g}$ at 20s. The in-run stability for these devices is specified as $\leq 0.1\mu\text{g}$ at 20s to 100s after the 30 minute “warm-up phase”. The drift of the MEMS along Raman axis 2 is very close to this value, however the Raman axis 1 MEMS drift is much larger at $0.4\mu\text{g}$, and occurs at a much shorter time of 5s. It is difficult to determine whether the source of the drift is due to a larger than expected in-run stability of the device, or due to a slow tilt of the apparatus. A tilt of approximately $50\mu\text{rad}$ over 30 minutes would be sufficient to cause the instability observed in the Allan deviation plot for the MEMS along Raman axis 1. Nevertheless, as the sensitivity of both devices is sufficient to resolve the interferometer fringe in the bandwidth range corresponding to typical operation of the interferometer, the source of the drift is not investigated further.

4.7.3 Control System

The experiment is controlled using a National Instruments PXI system containing a PXIe 8880 embedded controller alongside several individual data acquisition (DAQ) NI card modules. These are all housed within a single chassis to synchronise the analogue and digital input/outputs. The sequencing of the laser is controlled by an independent MSquared DCS Ice Bloc: Digital Timing Controller and Synthesiser, which controls laser output frequency and power through a web-based interface. All other aspects of the experiment are controlled using the MOTMaster software developed in-house, which controls the output analogue and digital voltages generated by the NI cards. The two sequence control systems are synchronised at the start of each sequence and then run independently.

The graphical interface of the MOTMaster software allows us to easily build a sequence of precisely synchronised patterns operating the digital and analogue input/outputs. A MOTMaster sequence is composed of series of individual steps in which the state of each hardware channel is defined over some specified duration. MOTMaster converts this graphically represented information to hardware level commands which are then sent to the PXIe controller. At the end of each sequence, the data buffer from MOTMaster containing input analogue signals is

transferred to the data visualisation software, AxelHub. In addition to the data visualisation, AxelHub is also the main application to continuously measure the MEMS accelerometer during the interferometer section of each sequence. Both programs (MOTMaster and AxelHub) have been developed in C# using Microsoft Visual Studio and National Instruments NI-DAQ.

Chapter 5

Preparation of a cold atom ensemble for interferometry

The apparatus described in the previous chapter enables an ensemble of ^{87}Rb atoms to be captured, cooled sufficiently and prepared in the correct state ready for the three pulse interferometry sequence.

Figure 5.1 shows an overview of the steps that occur during the atom cooling and state preparation process. It begins by switching on the 3D magneto-optical trap (MOT) and the 2D MOT which loads it. After 200ms, the 2D MOT is switched off and the 3D MOT is left on for a further 10ms to allow all the atoms in the beam to arrive. The magnetic field of the 3D MOT is then switched off so that the MOT light can lower the temperature to $\sim 5\mu\text{K}$ by polarisation gradient molasses cooling. During this phase the rest frame of the optical molasses can be made to move in order to launch the atoms. A series of state preparation steps then prepare the atoms in the $F = 1, m_F = 0$ state ready for interferometry. The state preparation selects only a narrow band of velocities in order to have a small Doppler width for the Raman transition, as needed to maximise the fringe contrast. Efficient cooling of the atoms by the molasses is pertinent to maximise the number of atoms selected.

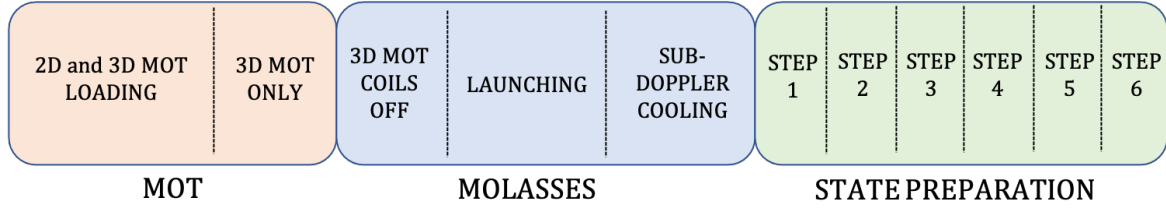


Figure 5.1: Overview of the magneto-optical trap (MOT), optical molasses and state preparation stages. After step 6 of the state preparation a cloud of atoms in the $F = 1, m_F = 0$ state with a narrow range of velocities along the Raman axis is ready for interferometry.

5.1 Optimisation of Doppler cooled atom cloud in 3D MOT

In the 3D MOT the atoms are trapped at the centre of the chamber and cooled along all three Cartesian axes. The temperature is typically a few hundred μK .

During the operation of the interferometer, there is an important trade off between the cycle rate (to increase the sensitivity at 1s) and the loading time of the MOT (to increase the number of trapped atoms). To optimise this trade off we maximise the rate at which atoms are captured by the 3D MOT. The loading of a MOT can be described by the following rate equation:

$$\frac{dN_{\text{MOT}}}{dt} = R - \beta N_{\text{MOT}}, \quad (5.1)$$

where N_{MOT} is the number of trapped atoms, and R is the rate of capturing atoms and β is the rate per atom of loss from the trap due to collisions with the background gas. The loss rate due to collisions between trapped atoms is negligible for the typical number densities in a MOT [81]. Assuming a constant loading rate R and no trapped atoms at the start of the loading time the number of atoms trapped in a MOT as a function of the time, t , is then given by:

$$N_{\text{MOT}}(t) = N_{\infty} (1 - e^{-\beta t}), \quad (5.2)$$

where $N_{\infty} = \frac{R}{\beta}$ is the steady state number of atoms trapped in the MOT. In our apparatus, the loading rate R depends on the flux of atoms from the external 2D MOT that enter the trap

volume with a longitudinal velocity less than the capture velocity of the 3D MOT [75]. As they are falling under gravity after exiting the 2D MOT cell, any atoms with a longitudinal velocity of less than $\sim 2\text{m/s}$ will fall below the 3D MOT trapping volume and will not be trapped. The next two sections describe how the parameters of the 2D and 3D MOTs are adjusted to achieve the highest loading rate of the 3D MOT. The temperature of the atom cloud in the 3D MOT is not investigated in great detail as further cooling in an optical molasses is necessary for interferometry.

The 2D and 3D MOT parameters were set to the values given in Table 5.1, with the 2D parameters set following guidance from Cold Quanta. With the 2D MOT running continuously, the number of atoms captured in the 3D MOT was measured as a function of time. It was found to load 7.7×10^8 atoms/s for short times (≤ 0.4 s), reaching a steady state $\sim 1.2 \times 10^9$ atoms after 4 s.

	2D MOT	3D MOT
Magnetic Field Gradient	37 G cm^{-1}	15 G cm^{-1}
Cooling Light Intensity	113.2 mW cm^{-2}	47.4 mW cm^{-2}
Cooling Light Detuning	-2.5Γ ($\Gamma = 2\pi \times 6.06 \times 10^6 \text{ s}^{-1}$)	-2.5Γ ($\Gamma = 2\pi \times 6.06 \times 10^6 \text{ s}^{-1}$)
Repump Light Intensity	2.8 mW cm^{-2}	1.2 mW cm^{-2}

Table 5.1: Initial parameter values for the 2D and 3D magneto-optical traps (MOTs). The 2D MOT parameters are set following operating guidance.

5.1.1 Optimisation of 2D MOT Parameters

To optimise the loading rate of the 3D MOT, we first maximise the 2D MOT output atom flux that is incident on the capture volume of the 3D MOT with a longitudinal velocity less than the capture velocity. This involves maximising the number of atoms captured in the 2D MOT cell, as well as investigating the effect of shifting the longitudinal velocity distribution using the push beam. The brightness of the 2D MOT is dependent on the detuning and intensity of the cooling light, as well as the magnetic field gradient of the quadrupole field [76]. Each

of the 2D MOT parameters was individually scanned while monitoring the number of atoms trapped in the 3D MOT after 200ms. The results for the frequency and power of the cooling light, magnetic field gradient and dispenser current scans are displayed in Figure 5.2.

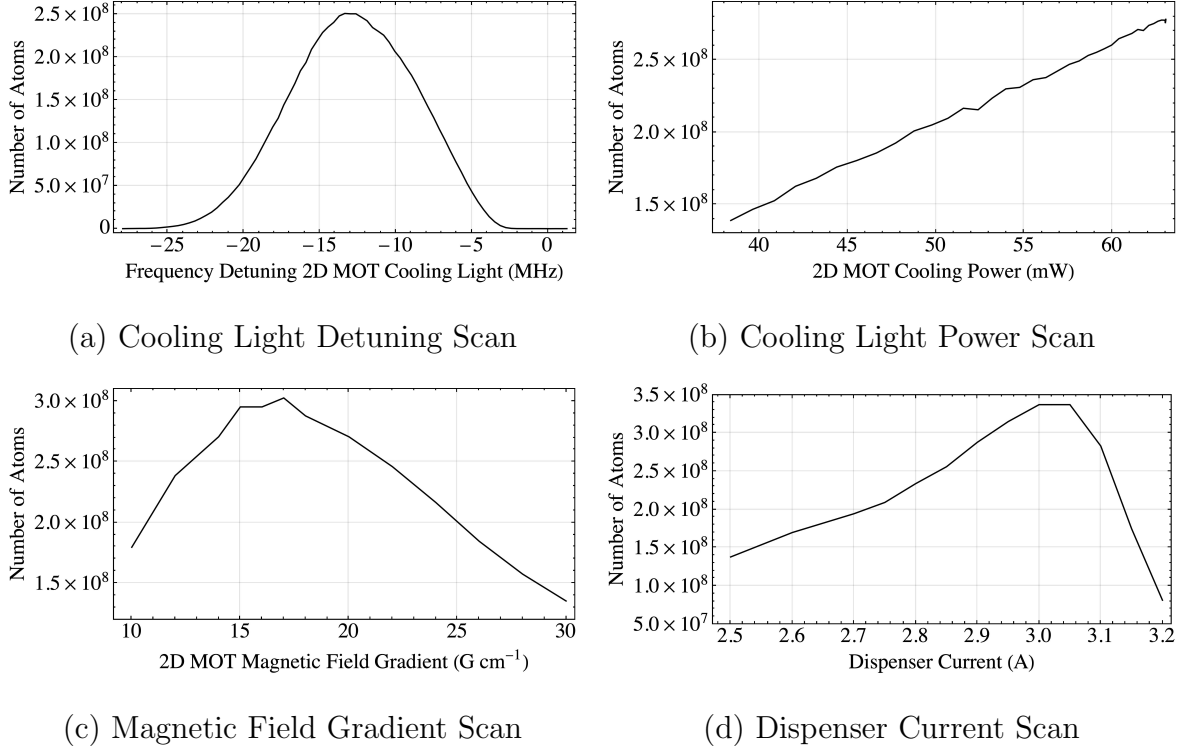


Figure 5.2: The number of atoms captured in the 3D MOT after a loading time of 200ms as a function of the 2D MOT parameters. (a) Varying the cooling light frequency with cooling power set to 63mW, magnetic field gradient of 37G cm^{-1} and dispenser current of 3A. (b) Varying the cooling light power with cooling frequency set to -13MHz, magnetic field gradient of 37G cm^{-1} and dispenser current of 3A. (c) Varying the 2D MOT magnetic field gradient with cooling power set to 63mW, cooling frequency set to -13MHz and dispenser current of 3A. (d) Varying the 2D MOT dispenser current with cooling power set to 63mW, cooling frequency set to -13MHz and 2D MOT magnetic field gradient 17G cm^{-1} .

It is evident that the 2D MOT output is not yet in the regime in which it becomes insensitive to cooling power, therefore the 2D MOT is operated using the maximum cooling power of 63mW. The optimum 2D MOT cooling light frequency is found to be -13 MHz red detuned, with a magnetic field gradient of 17G cm^{-1} and dispenser current of 3.0 A. This optimal magnetic field gradient is much lower than the 37G cm^{-1} recommended by the operating manual, and is similar to the field gradient chosen for the 3D MOT. As expected there is an optimal operating current for the ^{87}Rb dispenser. Above 3.0A the output flux begins to decrease because the mean free path of the Rb atoms becomes comparable to the width of the cooling volume. Using the

optimised settings for the 2D MOT, the effect of the push beam on the 3D MOT loading rate is then investigated with the results displayed in Figure 5.3.

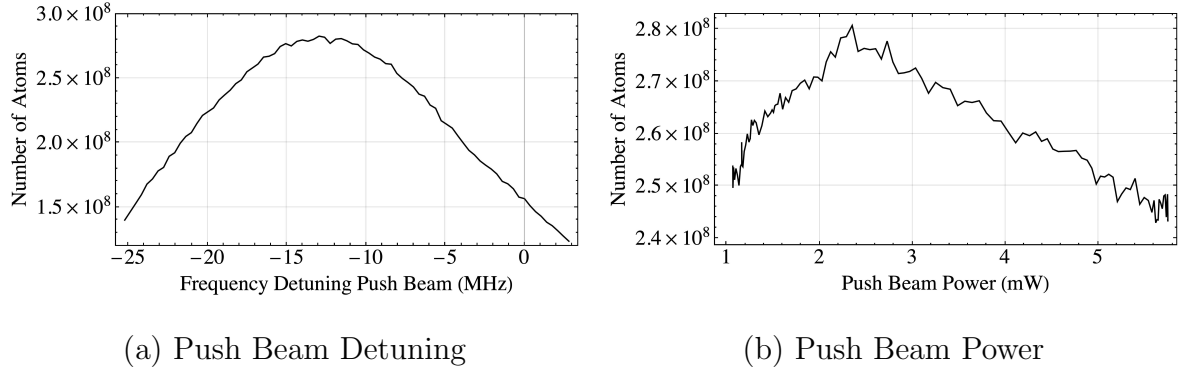


Figure 5.3: The number of atoms captured in the 3D MOT after a loading time of 200ms as a function of the 2D MOT push beam frequency (a), and power (b).

We see that the optimum detuning of the push beam is -13MHz which is very similar to the optimum cooling light detuning, suggesting the atoms are also cooled along this axis. The optimal push beam power is 2.4mW , with power above this disturbing the the 2D MOT and decreasing the capture rate. The corresponding intensity at the centre of the push beam is 2.7mW cm^{-2} .

5.1.2 Optimisation of 3D MOT Parameters

Using the optimal settings for the 2D MOT, with approximately 7.9 mW cm^{-2} of -15MHz detuned cooling light in each of the six 3D MOT beams, the magnetic field gradient of the 3D MOT is varied and the number of atoms captured after 200ms is measured, shown in Figure 5.4a. A magnetic field gradient of 11 Gcm^{-1} is found to optimal. Following this optimisation process of both the 2D and 3D MOT, the loading rate of the 3D MOT for short times ($\leq 0.4\text{ s}$) was increased from $7.7 \times 10^8\text{ atoms/s}$ to $2.5 \times 10^9\text{ atoms/s}$. Figure 5.4b shows the number of atoms captured in the 3D MOT when loading for 3s using the initial and optimised settings.

The magnetic quadrupole field is switched off in order to allow further cooling by optical molasses. If there is residual magnetic field, the atoms shoot off as a result of the differential

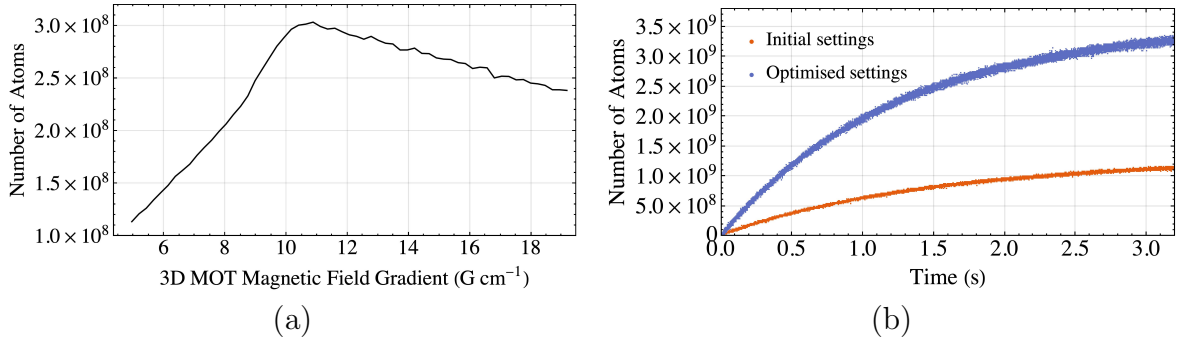


Figure 5.4: (a) The number of atoms captured in the 3D MOT after a loading time of 200ms as a function of the 3D MOT magnetic field gradient. (b) Loading curves of the 3D MOT using the initial (red), and optimised (blue) settings. Following the optimisation process the atom loading rate of the 3D MOT was increased from 7.7×10^8 atoms/s to 2.5×10^9 atoms/s.

radiation pressure caused by the Zeeman shift. By monitoring this movement, the three orthogonal magnetic bias fields were adjusted to cancel the ambient field. These cancelling fields generated by the three pairs of bias coils described earlier in Section 4.5 are $B_X = -0.17\text{G}$, $B_Y = -0.14\text{G}$ and $B_Z = -0.11\text{G}$.

The temperature of the atom cloud captured by the 3D MOT is measured by observing its rate of thermal expansion after switching off both the quadrupole field and the cooling light. The temperature is determined from the linear fit of the square of the atom cloud size, σ^2 , with the square of the time, t^2 , as described by

$$\sigma^2 = \sigma_0^2 + \frac{kT}{m}t^2, \quad (5.3)$$

where σ_0^2 is the initial size of the atom cloud at time $t = 0$, k is the Boltzmann constant, T is the temperature of the atom cloud and m is the mass of the atom. After switching off the cooling light and allowing the magnetic field to decay, the cloud was imaged at intervals of 1ms. The temperature was found to be approximately $300\mu\text{K}$, which is well below the temperature expected from Doppler cooling in this MOT, but still far above the temperature of $\leq 10\mu\text{K}$ desired for interferometry. That requires further sub-Doppler cooling.

5.2 Optimisation of Sub-Doppler Cooling

Polarisation gradient cooling is a mechanism by which atoms moving through a light field of rapidly varying polarisation can be cooled below the Doppler limit by optical pumping between differently light-shifted magnetic sublevels. The details of polarisation gradient cooling will not be discussed here, but can be found elsewhere [75, 82]. The final temperature, T , in an optical molasses is proportional to $I/|\delta|$ [72], where I is the cooling light intensity and δ is the detuning. In order to minimise the temperature, the cooling frequency detuning is ramped from -15MHz to -140MHz. This is achieved by sweeping the EOM in the Rb vapour cell lock loop described in Section 4.3. The repump frequency is kept on resonance with the $F = 1 \rightarrow F' = 2$ transition to ensure atoms do not fall into a dark state. Subsequently the intensity of the cooling and repump light in each beam is linearly ramped to zero by reducing the RF power to the cooling and repump AOMs in the laser system.

Optimisation of both ramps is achieved by empirically minimising the molasses temperature. After applying both ramps the cloud was imaged at intervals of 1ms to determine the temperature using Equation 5.3. Shown in Figure 5.5, we find that temperatures of $\sim 9\mu\text{K}$ for the vertical velocity component and $\sim 6\mu\text{K}$ for the horizontal components are achieved using a 0.1ms frequency ramp, followed by a waiting period of 0.8ms before ramping the intensities over 1ms. Approximately 20% of the atoms that are captured in the MOT are retained in the optical molasses. This retention rate is not particularly high, however the remaining number of atoms is sufficient.

5.3 Launching the Atoms

Recall from Section 2.2.3 that along each Raman axis there are two pairs of counter-propagating beams able to drive Raman transitions. These have opposite Doppler shifts, so when the atom cloud is launched with a sufficiently large velocity along the Raman axis, the two resonances are resolved. We also choose to launch the atoms vertically to prevent them from falling under

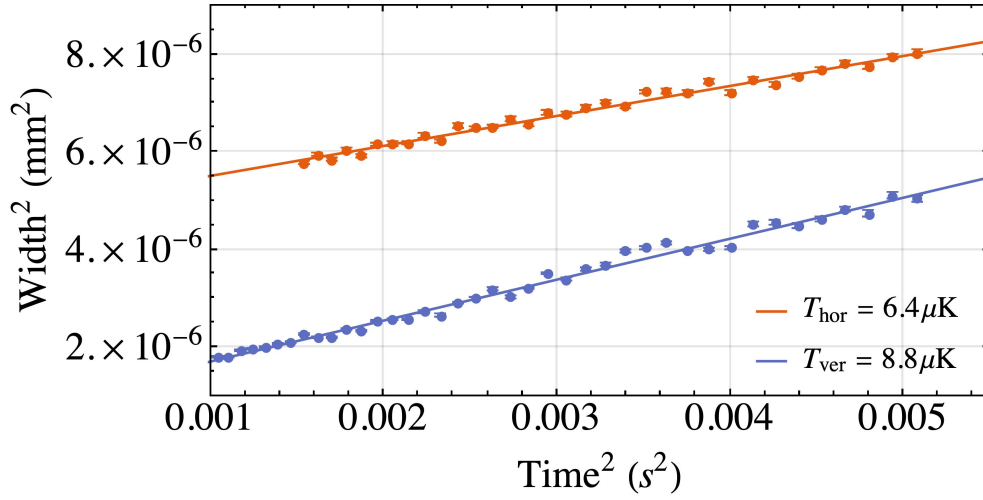


Figure 5.5: Ballistic expansion along the vertical and horizontal axes for the atom cloud after a frequency ramp of 0.1ms, followed by a waiting period of 0.8ms before then an intensity ramp of 1ms. Temperatures of $6.4\mu\text{K}$ and $8.8\mu\text{K}$ are estimated for the horizontal and vertical velocity components, respectively.

gravity too far from the optic axis. In addition, a velocity component that is perpendicular to the Raman axis allows the interferometer to sense rotations.

The atom cloud is launched along a counter-propagating pair of molasses beams by detuning one by $-\delta f$ while detuning the other by $+\delta f$. This results in the atoms being cooled to rest in the moving frame which has a velocity of $\delta v = \lambda \delta f$ along the axis of the two counter-propagating laser beams [83]. The choice of launch velocity along the three axes is discussed in the following.

5.3.1 Horizontal Velocity Component Along Raman Axis

The launch velocity along the Raman axis needs to be large compared with the thermal velocity so that the Raman resonance of one light beam pair is well resolved from that of the other as a result of the Doppler shift. Recall the Raman resonance condition is met when $\delta^- = 0$, and [24]

$$\delta^- = \left(\frac{\Omega_1^2}{\Delta} - \frac{\Omega_2^2}{\Delta} \right) - \omega_R + \omega_{\text{hfs}} + \mathbf{k}_{\text{eff}} \cdot \mathbf{v} + \frac{\hbar |\mathbf{k}_{\text{eff}}|^2}{2m} \quad (5.4)$$

where the first term is the light shift of the resonance, $\omega_R = \omega_2 - \omega_1$ is the beat note of the two Raman frequencies, ω_{hfs} is the unperturbed splitting of the hyperfine states, and the fourth

and fifth terms are the Doppler and recoil shifts, respectively. If we were to scan the beat note and look at the fraction of atoms transferred for a $6\mu\text{K}$ stationary cloud, we would expect to see the peaks for both pairs of counter-propagating beams centred on the same ω_R with a Doppler width of 149kHz . As we begin to launch the atoms along the Raman axis, the peaks for the two pairs of counter-propagating beams begin to shift in opposite directions because they have opposite values of k_{eff} . We must launch the atoms with sufficient velocity such that there is minimal overlap between the two resonances. The Raman beat note can then be tuned up or down such that the resonance condition ($\delta^- = 0$) is met for one beam pair or the other. We do not want a velocity that is too large as the atoms will have moved out of the detection region by the end of the sequence. A velocity of 8 cm s^{-1} along the Raman axis corresponds to a Doppler shift of $\pm 205\text{kHz}$ for the two counter-propagating beam pairs. The splitting between the two peaks is then 410kHz , which is much larger than the Doppler width given by a one-dimensional Maxwell-Boltzmann distribution as demonstrated in Figure 5.6. The figure also shows (in black) the lineshape for Raman transitions driven by co-propagating beams. This line has a negligible Doppler shift so it is centred on $\delta^- = 0$ and has a width given by the inverse of the pulse duration. This line is suppressed by our choice of circular polarisations of the Raman beams.

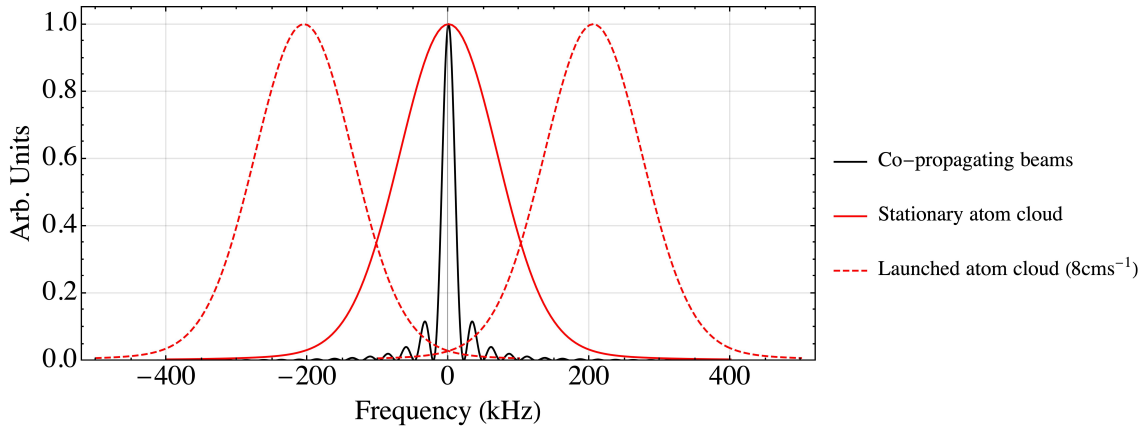


Figure 5.6: Expected lineshapes as a function of the detuning of the Raman beat note frequency for the co-propagating Raman beam pair (black), and counter-propagating Raman beam pair for $6\mu\text{K}$ atom clouds that are stationary (solid red), and launched with a velocity of 8 cm s^{-1} along the Raman axis (dashed red). The two counter-propagating Raman beam peaks are separated by 405kHz when the atoms are launched with a velocity of 8 cm s^{-1} .

5.3.2 Horizontal Velocity Component Perpendicular to Raman Axis

In Chapter 3 we found that the leading interferometer phase term sensitive to rotation is proportional to the velocity component orthogonal to both the Raman axis and the rotation axis. Therefore to measure rotation about the vertical axis we must launch the atoms horizontally and perpendicular to the Raman axis. We should make this velocity large to increase the interferometer's sensitivity to rotation, but it is limited by the size of the detection region that can be imaged onto the photodiode (described in Section 4.6). This limiting velocity component is shown as a function of interferometer $2T$ time in Figure 5.7a. Here we assume a horizontal detection region at the centre of the chamber of $\pm 8\text{mm}$, and a delay time of 35ms between launching the atoms and the first pulse of the interferometer (which is required to allow the laser to tune between the cooling, state selection and Raman frequencies). A horizontal perpendicular launch velocity of 8cm s^{-1} is chosen which allows $2T$ times of up to 50ms.

5.3.3 Vertical Velocity Component

Launching the atoms vertically to keep them close to the optic axis of the Raman beam during the interferometer stage of the sequence limits the variation in Rabi frequencies across the cloud of atoms in order to minimise the associated loss in fringe visibility. Figure 5.7b shows the expected trajectory for a vertical launch of 30cm s^{-1} as a function of time relative to the first interferometer pulse (35ms after launching the atoms). This launch allows the atoms the atoms to remain within 10mm of the optic axis for interferometer $2T$ times up to 50ms.

This vertical velocity component causes the interferometer to become sensitive to rotations about the horizontal axis perpendicular to the (horizontal) Raman axis. Therefore the Raman axis 2 interferometer, which is oriented along the east-west direction as shown in Figure 4.1, becomes sensitive to Ω_{E_y} , the component of the earth's rotation which points due north in the lab frame. The corresponding phase from rows 7 and 8 of Table 3.1 is $-2 k_{\text{eff}} \Omega_{E_y} T^2 (v_{z_A} + g_z T)$. Launching the atoms with a vertical velocity of 30cm s^{-1} gives $v_{z_A} = -4\text{cm s}^{-1}$ after the usual 35ms delay, then Ω_{E_y} gives an interferometer phase shift of 267mrad for an interferometer time

of $2T = 50\text{ms}$. This phase term can be eliminated by launching with a vertical velocity of 24.5 cm s^{-1} such that $v_{z_A} + g_z T = 0$.

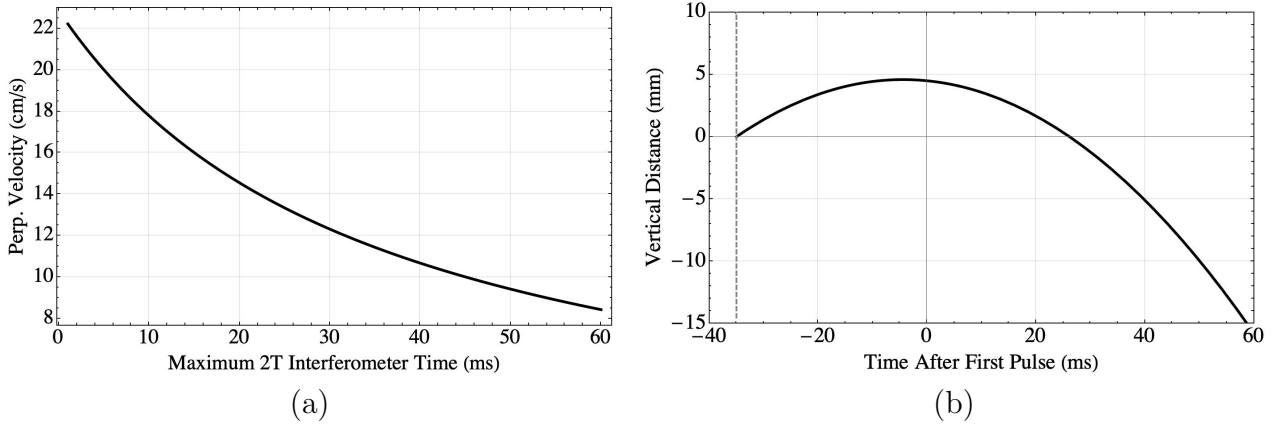


Figure 5.7: (a) The maximum horizontal perpendicular launch velocity for different interferometer T times, assuming a horizontal detection region of $\pm 8\text{mm}$ and a delay of 35ms between launching and the first Raman pulse. (b) Expected trajectory of an atom cloud launched vertically at 30cm s^{-1} . Time is relative to the first Raman pulse which occurs 35ms after launch.

5.3.4 Full Launching Configurations

Figure 5.8 is a schematic of the four horizontal MOT beams relative to the two Raman axes. To measure accelerations along Raman axis 1 we launch the atoms with velocity $\mathbf{v}_1^{\text{acc}}$ in order to resolve the two velocity-sensitive Raman resonances. To measure rotations about the vertical axis using the Raman axis 1, we launch the atoms with velocity $\mathbf{v}_1^{\text{rot}}$ (composed of two orthogonal components). Similarly, the required launch velocity when using Raman axis 2 is $\mathbf{v}_2^{\text{acc}}$ for measuring acceleration and $\mathbf{v}_2^{\text{rot}} = \mathbf{v}_2^{\text{acc}} + \mathbf{v}_2^\perp$ for rotation. I choose to operate with $\mathbf{v}_{1/2}^{\text{acc}} = \mathbf{v}_{1/2}^\perp = 8\text{cm s}^{-1}$.

Table 5.2 shows the frequency detuning required for each horizontal MOT beam in order to achieve the desired launch velocities. The vertical MOT beam detunings are also given.

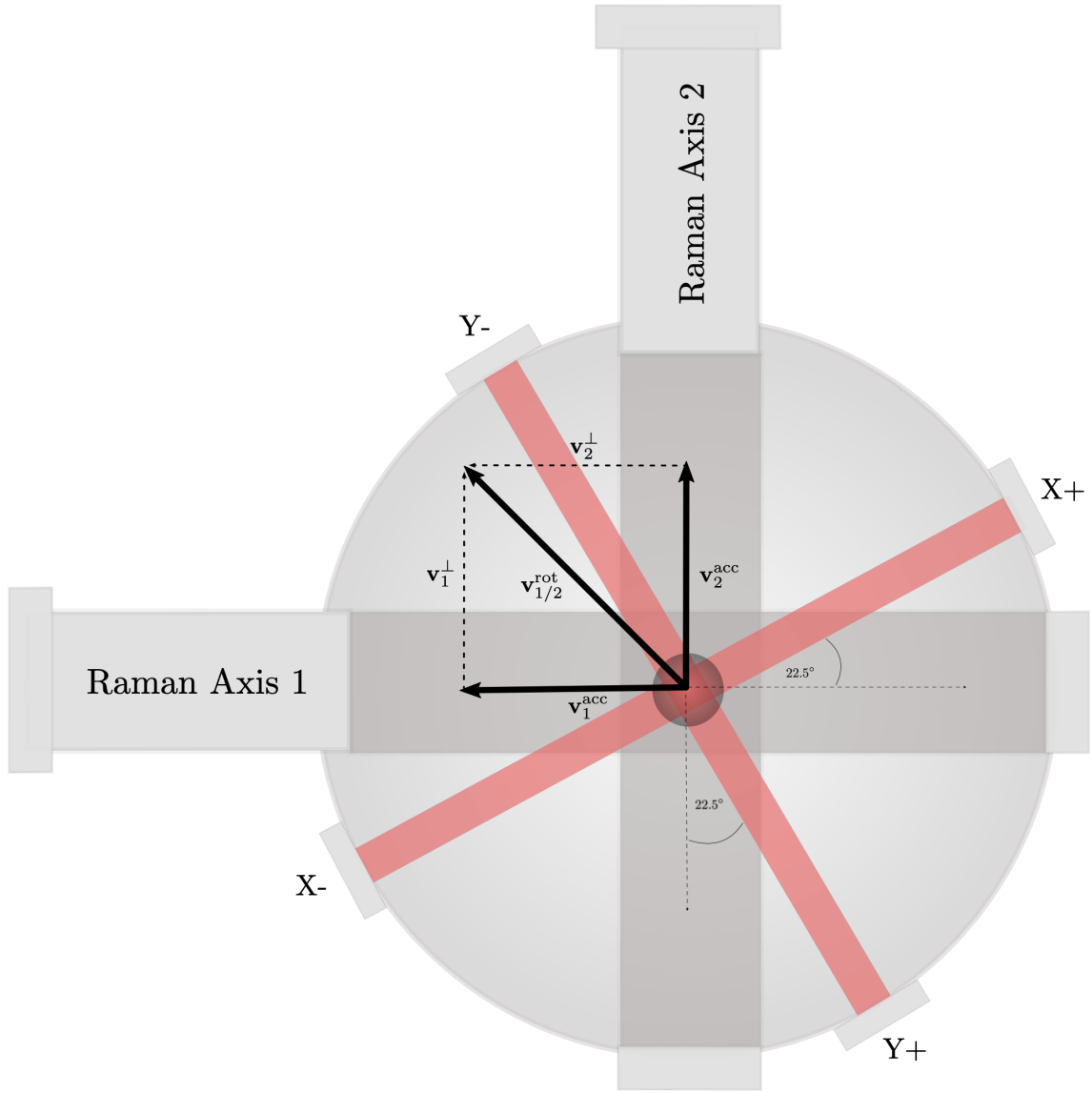


Figure 5.8: Acceleration and rotation measurement launching configurations relative to the Raman and MOT beams. Each MOT beam frequency is tuned either up or down to provide to desired velocity along each MOT beam axis (vertical not shown).

	$\delta f_{X\pm}$	$\delta f_{Y\pm}$	$\delta f_{Z\pm}$
$\mathbf{v}_1^{\text{acc}}$	$\pm \frac{v_1^{\text{acc}}}{\lambda} \cos(22.5^\circ)$	$\pm \frac{v_1^{\text{acc}}}{\lambda} \sin(22.5^\circ)$	$\pm \frac{v_{\text{vert}}}{\lambda}$
$\mathbf{v}_2^{\text{acc}}$	$\mp \frac{v_2^{\text{acc}}}{\lambda} \sin(22.5^\circ)$	$\pm \frac{v_2^{\text{acc}}}{\lambda} \cos(22.5^\circ)$	$\pm \frac{v_{\text{vert}}}{\lambda}$
$\mathbf{v}_{1/2}^{\text{rot}}$	$\pm \frac{\sqrt{2}v_{1/2}^{\perp}}{\lambda} \cos(67.5^\circ)$	$\pm \frac{\sqrt{2}v_{1/2}^{\perp}}{\lambda} \sin(67.5^\circ)$	$\pm \frac{v_{\text{vert}}}{\lambda}$

Table 5.2: Frequency shifts for the six MOT beams to launch the atoms in a moving molasses.

The frequencies of each of the six beams are linearly ramped over 1ms to allow enough time

for the velocity of the atoms to reach that of the moving optical molasses. As outlined in the sequence diagram in Figure 5.1, we launch the atoms before applying the frequency and intensity ramps because the molasses is much more viscous at this point, and therefore brings the atoms up to speed more quickly.

The atoms were initially launched horizontally along Raman axis 1 and vertically. The frequency detunings for the MOT beams were set using values $v_1^{\text{acc}} = 8 \text{ cm s}^{-1}$ and $v_{\text{vert}} = 27 \text{ cm s}^{-1}$. Approximately 30ms after launching the atoms, the cloud was imaged at 1ms intervals using two orthogonal CCD cameras which were both 45° to the Raman axes. The vertical and horizontal centre of mass position of the atom cloud at each time interval was then calculated for each camera. By measuring the horizontal position on the two orthogonal cameras we can use geometry to account for the angle between the cameras and determine the true trajectory along Raman axis 1. The measured position along Raman axis 1 as a function of time is plotted in Figure 5.9a (blue) along with the vertical trajectories measured on the two cameras (red). Using a least squares fit for the linear and parabolic trajectories, we measure the horizontal launch velocity along Raman axis 1 to be $7.34 \pm 0.07 \text{ cm s}^{-1}$ while the vertical launch velocities derived from the two cameras are $25.94 \pm 0.06 \text{ cm s}^{-1}$ and $26.34 \pm 0.04 \text{ cm s}^{-1}$. This horizontal launch velocity appears significantly lower than the target velocity of 8 cm s^{-1} , and vertical launch velocities differ by significantly more than their combined statistical error bar. These discrepancies are probably due to misalignment of the optic axes of the cameras, which cause systematic uncertainty.

This measurement was repeated when launching the atoms along Raman axis 2, again using frequency detunings in Table 5.2 corresponding to $v_2^{\text{acc}} = 8 \text{ cm s}^{-1}$ and $v_{\text{vert}} = 27 \text{ cm s}^{-1}$. The measured horizontal trajectory along Raman axis 2, and the vertical trajectories from each camera are plotted in Figure 5.9b. We measure a horizontal velocity of $v_2^{\text{acc}} = 8.06 \pm 0.14 \text{ cm s}^{-1}$, and a vertical velocities of $27.31 \pm 0.08 \text{ cm s}^{-1}$ and $26.56 \pm 0.05 \text{ cm s}^{-1}$. The horizontal launch velocity along Raman axis 2 agrees with the target velocity of 8 cm s^{-1} but again the two vertical velocity measurements differ significantly from each other, indicating a level of systematic uncertainty. Lastly we measure the launch velocities for $\mathbf{v}_{1/2}^{\text{rot}}$ using frequency detunings in Table 5.2 corresponding to $v_{\text{vert}} = 27 \text{ cm s}^{-1}$ and $v_{1/2}^\perp = 8 \text{ cm s}^{-1}$, resulting in a target velocity of $\mathbf{v}_{1/2}^{\text{rot}}$

$= 11.3 \text{ cm s}^{-1}$. The measured trajectories are plotted in Figure 5.9c. We measure a horizontal launch velocity of $\mathbf{v}_{1/2}^{\text{rot}} = 10.72 \pm 0.09 \text{ cm s}^{-1}$, and vertical launch velocities of $27.16 \pm 0.11 \text{ cm s}^{-1}$ and $27.02 \pm 0.09 \text{ cm s}^{-1}$. The horizontal launch velocity is slightly lower than the target velocity of 11.3 cm s^{-1} , however the vertical launch velocities both agree very well with each other and to the target launch velocity of 27 cm s^{-1} .

These measurements show that the launch method is working as expected, within a systematic uncertainty of about 10% in the horizontal velocity and 4% in the vertical velocity. A more stringent check using Raman spectroscopy is presented in Section 5.4.6.

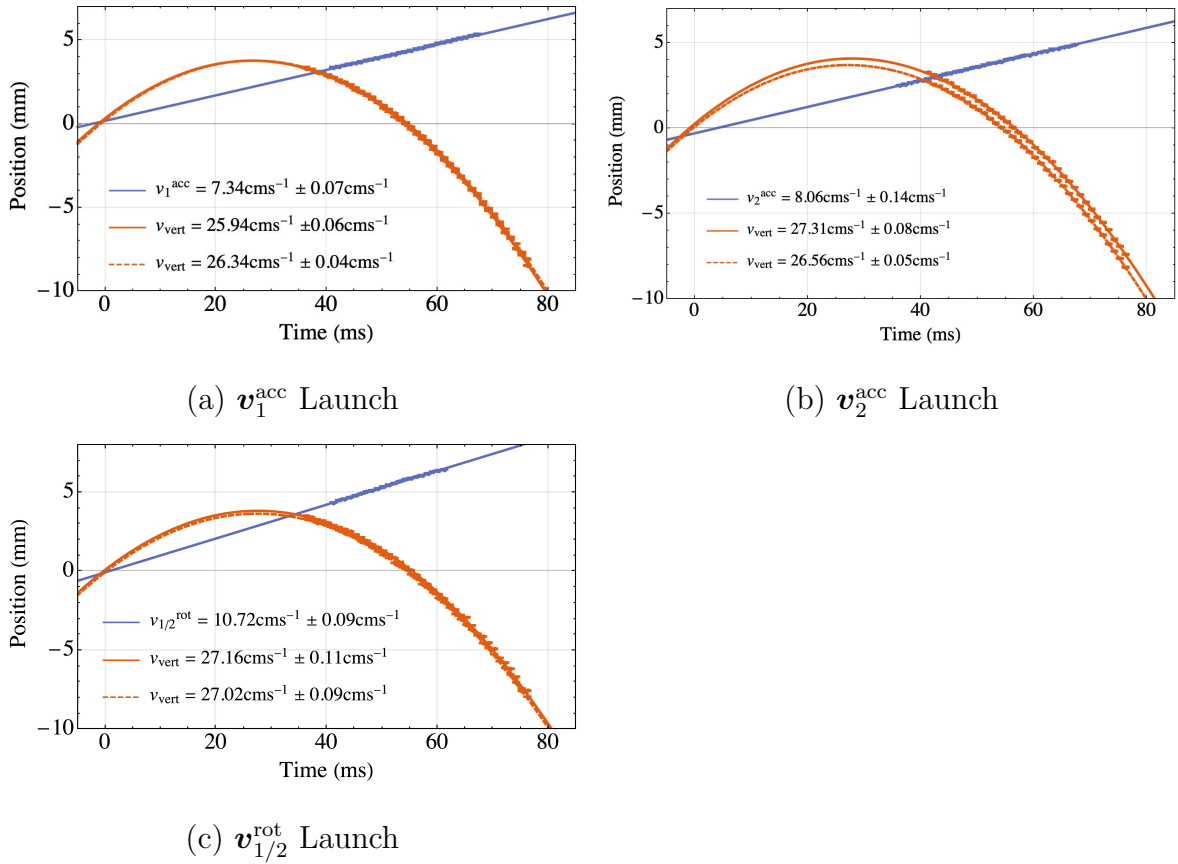


Figure 5.9: Measured vertical and horizontal atom trajectories using two orthogonal CCD cameras for the three different launching configurations; $\mathbf{v}_1^{\text{acc}}$, $\mathbf{v}_2^{\text{acc}}$ and $\mathbf{v}_{1/2}^{\text{rot}}$. The frequency detunings are given in Table 5.2 using values $v_{\text{vert}} = 27 \text{ cm s}^{-1}$, and $v_1^{\text{acc}} = v_2^{\text{acc}} = v_{1/2}^{\perp} = 8 \text{ cm s}^{-1}$.

5.3.5 Launching Stability

As a perpendicular velocity component induces a sensitivity to rotation for the interferometer phase, the velocity of the atoms must be stable. The stability of the launch velocity was measured by imaging the atom cloud 36ms after launch and looking at the stability of the cloud's centre of mass position. To decouple possible fluctuations in the initial position of the atom cloud from fluctuations in the launch velocity, the stability of the atom cloud position after 36ms was first recorded over a period of 30 minutes when it was not launched. The Allan deviation of the cloud's centre of mass 36ms after molasses cooling is shown in Figure 5.10a. We find that the horizontal and vertical position of the atom cloud is stable to within 0.5 pixels over 15 minutes. Using the CCD calibration of $36.4\mu\text{m}/\text{pixel}$ this corresponds to $\sim 18\mu\text{m}$.

The stability of the cloud's centre of mass position was then measured when launching the atom with velocity $\mathbf{v}_2^{\text{acc}}$, again for a period of 30 minutes. The corresponding Allan deviation plot for the cloud's centre of mass position 36ms after launch is shown in Figure 5.10b. We again find that the vertical and horizontal positions of the atom cloud are stable to within 0.5 pixels which corresponds to a spread in velocity of 0.05cm s^{-1} . Assuming a vertical launch velocity of 26.93cm s^{-1} (average of the two vertical velocities measured) this corresponds to a stability of $\sim 0.2\%$. Assuming a horizontal velocity of 8.06cm s^{-1} , this corresponds to a stability of $\sim 0.6\%$.

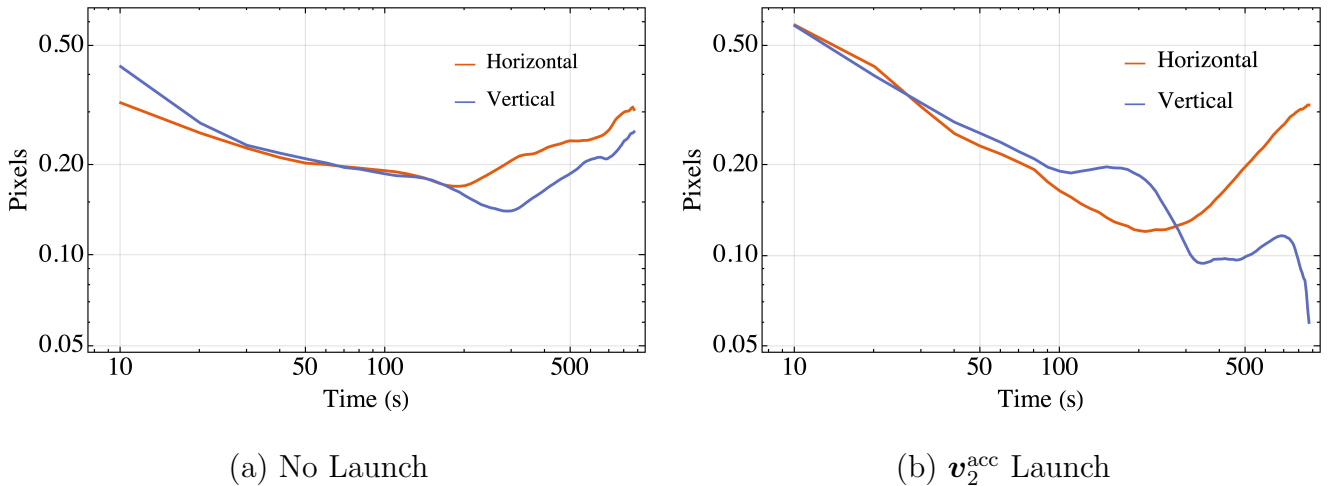


Figure 5.10: Allan deviation plots of the atom cloud's centre of mass measured with a CCD camera 36.27ms after molasses. (a) The atom cloud when there is no launching. (b) The atom cloud after launch along Raman axis 2 of 8.06cm s^{-1} and vertical launch of 26.93cm s^{-1} .

The launch stability for the other two launch configurations ($\mathbf{v}_1^{\text{acc}}$ and $\mathbf{v}_{1/2}^{\text{rot}}$) were also measured. All these measurements showed a 0.5 pixel noise level, which was practically the same whether or not the atoms were launched. This strongly suggests that the method is limited by vibration of the cameras, which are mounted on vertical posts attached to the grounded optical bench. We think therefore that the 0.2% (vertical) and 0.6% (horizontal) velocity noise levels are upper limits, rather than the real noise.

With the Raman axis oriented east-west, the vertical velocity makes the interferometer sensitive to the earth's rotation. Using an interferometer time of $2T = 50\text{ms}$, the upper limit of 0.2% uncertainty in the vertical velocity would result in a phase uncertainty of less than 1 mrad. This is well within the required phase noise level of 10mrad needed to reach the desired sensitivity of 100ng when operating the interferometer as an accelerometer.

However if the true instability of the horizontal launch velocity is 0.6%, it is not good enough to achieve the desired sensitivity of $1.2\mu\text{rads}^{-1}$ when operating the interferometer as a gyroscope. Future work should to focus on measuring the horizontal launch velocity stability with a more accurate method (such as Raman spectroscopy), and investigate ways in which the stability of the launch velocity can be improved, for example employing a beam intensity stabilisation system.

5.4 State Preparation into $F = 1, m_F = 0$

5.4.1 Outline

Now that we have a sufficiently cold cloud with enough atoms ($\sim 10^8 - 10^9$) we want to ensure they are in the required initial state for interferometry. After the cooling stage the majority of the atoms will be distributed across the $F = 2$ sub-levels, and so must be redistributed to maximise the population in the $F = 1, m_F = 0$ state to perform interferometry. As mentioned previously, the Raman transitions occur between the two hyperfine levels, $F = 1, m_F = 0$ and $F = 2, m_F = 0$, chosen due to their low sensitivity to magnetic fields. We choose to start in the

lower energy $F = 1, m_F = 0$ level as it is simple to blow away any atoms initially in the $F = 2$ sub-levels, as described below in step 6. The following steps provide an overview of the state preparation scheme implemented to maximise the population in the $F = 1, m_F = 0$ sublevel. A schematic of each step is shown in Figure 5.11. At each step a magnetic field is applied. For simplicity the Zeeman shifts in only the lower levels are shown here, however there is indeed splitting of the upper levels which has not been included. Here is a brief outline of the state preparation steps. This is followed in Section 5.4.2 by further details of the methods.

1. Optically pump the atoms into the $F = 1$ levels using light that is resonant with the $F = 2 \rightarrow F' = 2$ transition. As this optical transition has a natural linewidth of 6MHz [62], all the Zeeman sub-levels are excited.
2. Optically pump atoms out of the $F = 1, m_F = \pm 1$ levels and into the $F = 1, m_F = 0$ sublevel using σ^+ and σ^- polarised light that is resonant with the $F = 1 \rightarrow F' = 0$ transition. The circularly polarised light only excites the $m_F = \pm 1$ states.
3. A microwave pulse is then applied to drive all atoms, regardless of their velocity, from the $F = 1, m_F = 0$ state to the $F = 2, m_F = 0$ ground state. In this step, the microwave pulse lasts for around $150\mu\text{s}$, giving a resonance width of 6.5kHz. The Zeeman sub-levels are able to be fully resolved so we excite only the $F = 1, m_F = 0$ level, even with the imperfect microwave polarisation.
4. Atoms left behind in the $F = 1$ states are blown out of the interferometer region by the X+ MOT beam using only the repump light tuned close to resonance with the $F = 1 \rightarrow F' = 0$ transition. The bias field is either along Raman axis 1 or 2.
5. A Raman π -pulse is now applied to select a narrow velocity class of atoms in $F = 2, m_F = 0$ and transfer them back to the $F = 1, m_F = 0$ sublevel. A Blackman pulse of $100\mu\text{s}$ is used giving a linewidth of $\sim 24\text{kHz}$ FWHM. The magnetic field is 1G along the Raman axis, resulting in the Zeeman sublevels again being well resolved. The atoms selected in this way have a significantly reduced velocity spread along the Raman axis compared

with their initial thermal velocity spread. That is because their Doppler shift must lie within the narrow width of the Blackman resonance.

6. Finally, atoms left behind in the $F = 2$ states are blown out of the interferometer region using a single MOT beam using only the cooling light tuned close to resonance with the $F = 2 \rightarrow F' = 3$ transition.

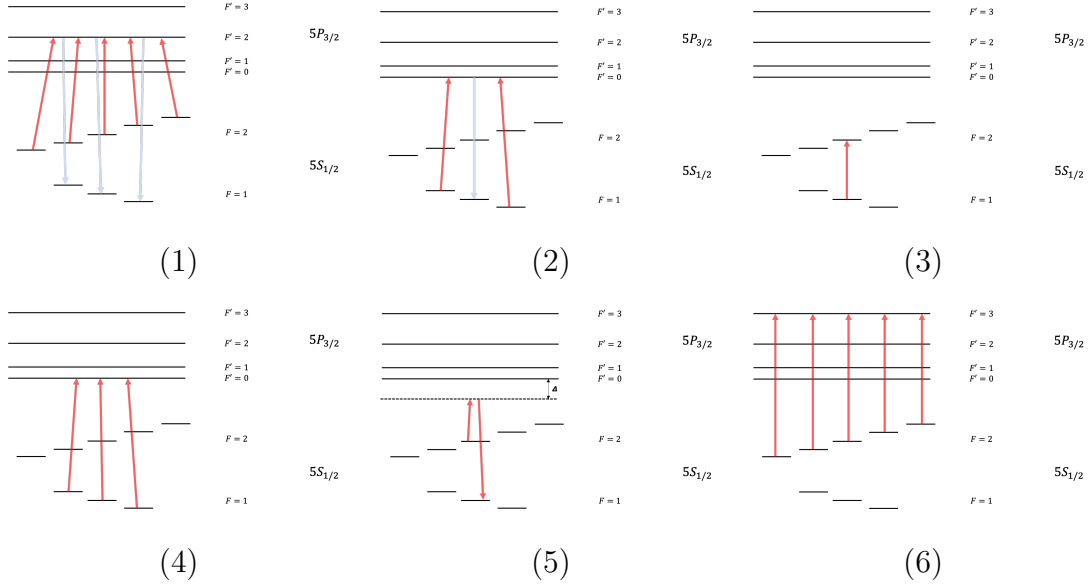


Figure 5.11: Schematic for each step in the state preparation scheme. The atoms start spread across the $F = 2$ sublevels. After the six steps we have a narrow spread of velocities in the $F = 1, m_F = 0$ sublevel.

5.4.2 Step 1: Optical Pumping into $F=1$

The first two steps of the state preparation scheme involve optically pumping most of the atoms from the $F = 2$ sub-levels to the $F = 1, m_F=0$ sub-level. After the molasses stage, we find almost all of the atoms are spread across the $F = 2$ sub-levels. Using the two vertical MOT beams, with the cooling light tuned to the $F = 2 \rightarrow F' = 2$ transition and no repump light, the atoms are optically pumped into the $F = 1$ sub-levels due to the spontaneous decay $F' = 2 \rightarrow F = 1$. Figure 5.12a shows the measured fraction of atoms cooled and launched in molasses that are transferred to $F = 1$ as a function of the duration of the pumping pulse. With a magnetic field of 0.4G along the vertical axis, the last of the atoms are slow to be

pumped, as shown by the green curve. That is because, whatever the polarisation of the light, there are always superpositions of the magnetic sublevels within $F = 2$ that are not excited - dark states. These are destabilised by precession of the angular momentum around the field at a rate determined by the Larmor frequency $\frac{1}{2}\mu_B B/\hbar$. With a larger field the dark states are removed more rapidly as shown by the blue and red curves. The stability of this optical pumping step was measured by looking at the fraction of atoms pumped into $F = 1$ from (shot to shot) over 1000s using a step 1 pulse of $15\mu\text{s}$ and a magnetic field of 1.9G. We find that the fraction of atoms prepared in $F = 1$ after the step 1 pulse is stable and reproducible over long times.

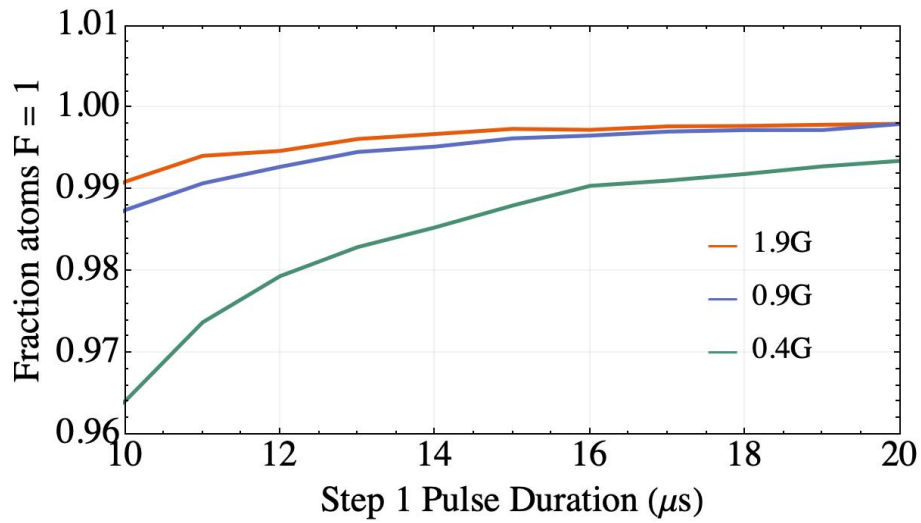


Figure 5.12: The fraction of cooled and launched atoms that are subsequently pumped into the $F = 1$ sublevels as a function of the step 1 pulse duration repeated for vertical magnetic field strengths of 0.4G, 0.9G and 1.9G.

5.4.3 Step 2: Optical Pumping of $F=1$ into $m_F=0$

The cooling light is then switched off and the repump light, tuned to the $F = 1 \rightarrow F' = 0$ transition, is switched on to drive σ^+ and σ^- transitions. As these transitions only excite the $m_F = \pm 1$ states, the atoms eventually fall into the $F = 1, m_F = 0$ dark state which is not destabilised by the magnetic field. To measure the fraction of atoms cooled and launched in the molasses that are pumped into this dark state we subsequently apply step 3 (indicated by (3) in Figure 5.11), a microwave π pulse which transfers the atoms from $F = 1, m_F = 0 \rightarrow F = 2, m_F = 0$. Optimisation of the microwave pulse is discussed in more detail in the next section.

Figure 5.13a shows the fraction of atoms detected in $F = 2$ after step 3, following a $15\mu\text{s}$ step 1 pulse at 1.9G and a step 2 pulse of varying duration. After a step 2 pulse of $30\mu\text{s}$ in a field of 1.9G, step 3 leaves 92% of the atoms in the $F = 2$ level as indicated by the red curve in Figure 5.13a. The blue curve shows little deterioration when the field is reduced to 0.9G but a much worse performance at 0.4G. Figure 5.13b shows the fraction of atoms detected in $F = 2$ after step 1 and step 2 only. We see that even without step 3 some atoms are being pumped by step 2 into the $F = 2$ states. That is due to the off-resonant excitation of states having $F' > 0$. However this population is less than 2% at $30\mu\text{s}$ and almost all of these scattered atoms are blown away in step 6.

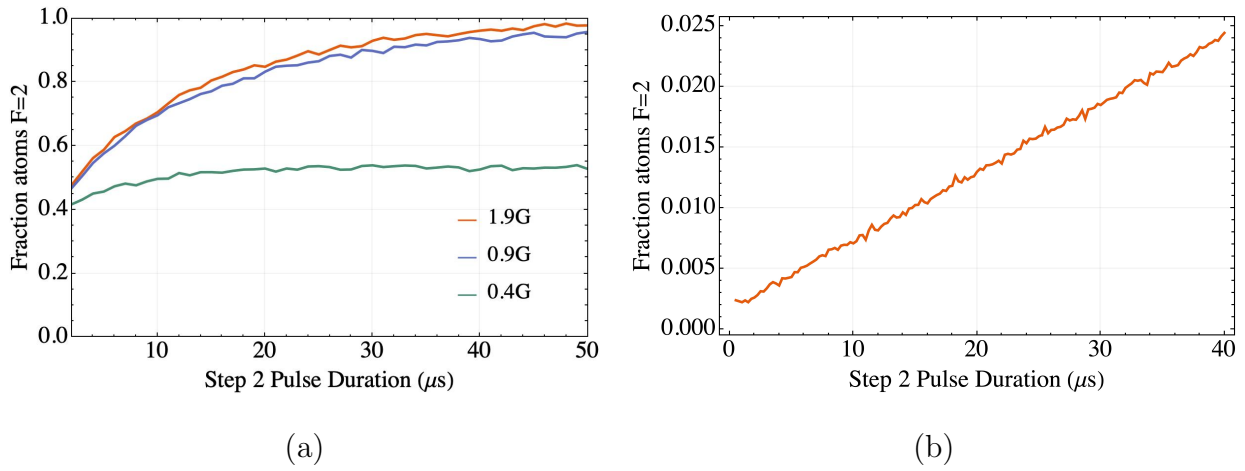


Figure 5.13: (a) The fraction of atoms cooled and launched in the molasses that are detected in $F = 2$ after step 3, following a $15\mu\text{s}$ step 1 at 1.9G and then step 2 with varying duration. Results are shown for three different magnetic fields in step 2. (b) The fraction of atoms cooled and launched in the molasses that are detected in $F = 2$ after step 2 as a function of step 2 pulse duration following a $15\mu\text{s}$ step 1.

5.4.4 Step 3: Microwave Pulse

Step 3, shown in Figure 5.11 (3), is the transfer from $F = 1, m_F = 0$ up to the $F = 2, m_F = 0$ in a magnetic field of 1.5G. We use the microwave horns described in Section 4.7.1 to drive this 6.8GHz $\Delta m_F = 0$ magnetic dipole transition with the static magnetic field rotated to be parallel to the magnetic field of the microwave horn. The axis of the static field is rotated over 2ms, so there is no danger of atoms being nonadiabatically lost from the $F = 1, m_F = 0$ sub-level. This is followed by a period of 1ms to allow any induced eddy currents to dissipate.

Figure 5.14 shows the Rabi oscillation of the $\Delta m_F = 0$ transition driven by the microwave pulse. The fraction of cooled and launched atoms that are driven to $F = 2$ by the microwave pulse after applying steps 1 and 2 is measured as a function of microwave pulse duration. It was found that the damping of the oscillation could be minimised by fine adjustments of each bias magnetic field component. This was achieved by setting the microwave pulse duration to the first minimum of the Rabi oscillation ($\sim 275\mu\text{s}$) and scanning the current in each field coil to minimise the transfer of atoms to $F = 2$. We think this optimisation may be finding the best direction for which the variation in Rabi frequencies across the atom cloud is minimised due to a residual magnetic field gradient. After optimising the magnetic field, the Rabi oscillation has a characteristic decay time of 10ms, with 92% of atoms detected in $F = 2$ after steps 1, 2 and 3. The remaining 8% of atoms in $F = 1$ are in the ± 1 sub-levels and therefore not addressed by step 3. A square pulse width of $150\mu\text{s}$ is used for the microwave pulse, giving a resonance width of 6.5kHz. The Zeeman splitting is of order MHz in the 1.5G magnetic field, so this ensures that we drive only the clock transition even though the microwave field is probably not exactly parallel to the static field.

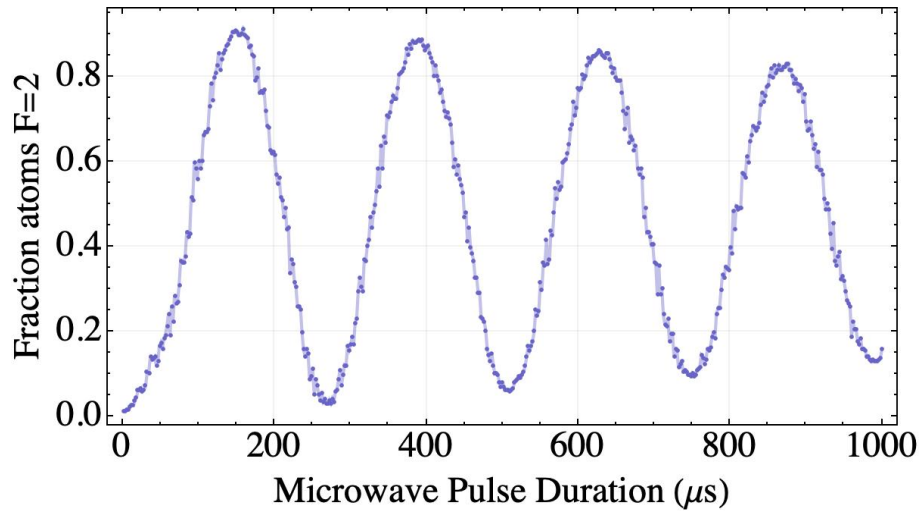


Figure 5.14: Rabi oscillation of the magnetic dipole clock transition driven by the microwave horn using the optimised bias magnetic field settings. The oscillation has a characteristic decay time of 10ms.

Figure 5.15 shows the fraction of atoms in $F = 2$ as a function of the step 3 microwave frequency with (blue curve) and without (red curve) step 2. The peaks are labelled according

to the various $m_F \rightarrow m_{F'}$ transitions being driven. The four outermost peaks of the red curve originate from the states $F = 1, m_F = \pm 1$. They are greatly reduced in the blue curve, while the three inner peaks, due to the $m_F = 0$ atoms are enhanced. This confirms that step 2 is indeed driving atoms from the $F = 1, m_F = \pm 1$ sub-levels to the $F = 1, m_F = 0$ sub-level. Indeed the central blue peak shows that after steps 1, 2 and 3, 90% of the atoms prepared in the molasses have been driven by the clock transition into state $F = 2, m_F = 0$. There is also a $\sim 2\%$ increase in the background $F = 2$ population after step 2 due to the off-resonant scattering. That leaves 8% of atoms in the $F = 1$ sublevels.

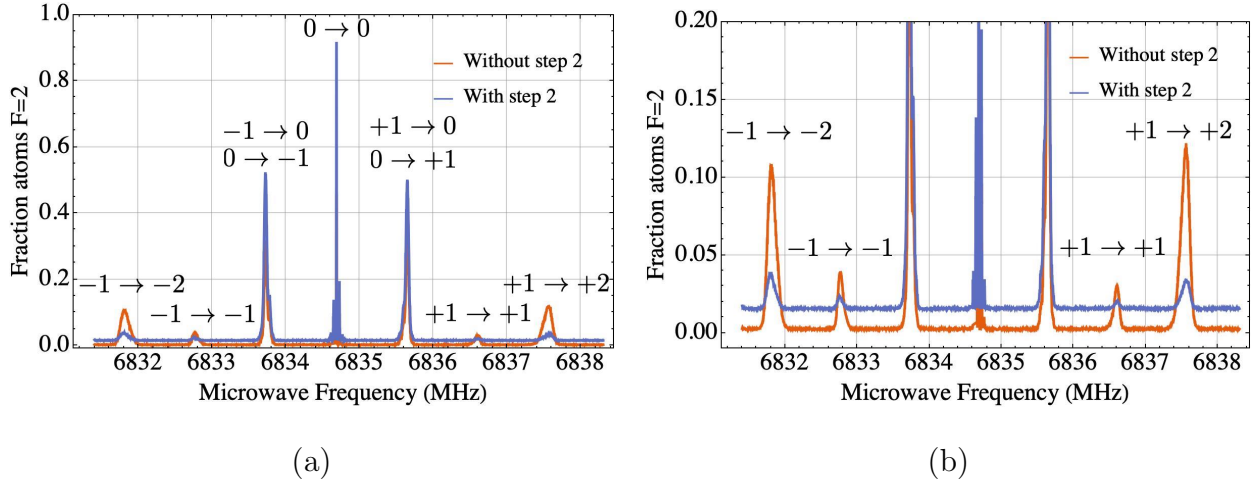


Figure 5.15: (a) The fraction of atoms in $F = 2$ after step 3 with (blue) and without (red) step 2. The large reduction in the four outer most peaks confirms step 2 is transferring most of the atoms from the $F = 1, m_F = \pm 1$ sublevels into the $F = 1, m_F = 0$ sublevel. (b) Magnified.

5.4.5 Step 4: Blow Away Pulse 1

Despite the high efficiencies of steps 1 to 3, there is still approximately 8% of atoms left in the $F = 1$ sub-levels after step 3. This atom population can be significant when compared with the number of atoms selected by step 5 to be used for interferometry. Atoms left in the $m_F = \pm 1$ sub-levels will not produce fringes but are simply spectators that reduce the apparent contrast. In addition to this, any atoms left in the $m_F = 0$ sub-level will participate in the interferometer but will have a much larger velocity distribution, and therefore much lower contrast, than the velocity selected atoms.

To avoid this, these atoms left in the $F = 1$ sub-levels are blown away by applying a single MOT beam with repump light tuned close to resonance with the $F = 1 \rightarrow F' = 0$ transition. It was found that using the X+ MOT beam was the most efficient to blow the remaining atoms in the $F = 1$ levels out of the interferometer region. The number of atoms remaining in $F = 1$ after this blow away pulse was measured by looking at the fraction $\frac{N_{1\text{rem}}}{N_{1\text{tot}}}$, where $N_{1\text{rem}}$ is the number of atoms in $F = 1$ after step 1, 2, 3 and blow away pulse 1 (step 4), and $N_{1\text{tot}}$ is the number of atoms in $F = 1$ after step 1 and 2. The percentage of atoms remaining in $F = 1$ as a function of step 4 pulse duration is plotted in Figure 5.16. Less than 0.6% of the number of atoms in $F = 1$ after step 1 and 2 remain in the $F = 1$ level after step 3 and a step 4 blow away pulse of 3ms. After loading the MOT for 200ms, we have approximately 2×10^8 atoms in the $F = 2, m_F = 0$ sub-level after step 4.

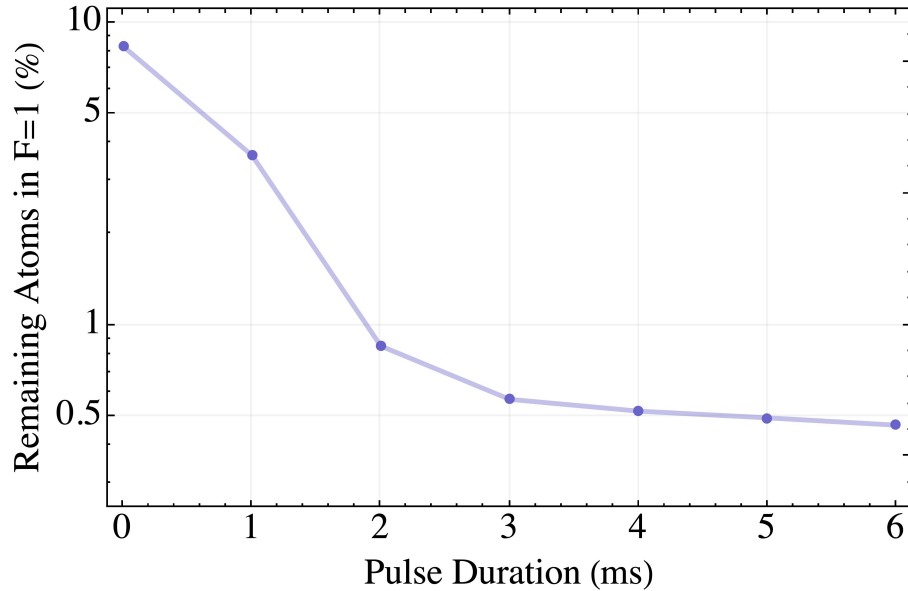


Figure 5.16: Fraction of atoms in $F = 1$ after step 1 and 2 that remain after applying step 3 and the blow away pulse in step 4.

5.4.6 Step 5: Velocity Selective Raman π Pulse

Our maximum laser intensity for driving the Raman transitions corresponds to a minimum π pulse time of order $40\mu\text{s}$, and hence to a maximum natural line width of order 25kHz. In our $6\mu\text{K}$ cloud the Doppler width is a much larger 150kHz and therefore the transition can only be driven very inefficiently, resulting in poor contrast of the interferometer fringes. To

achieve higher contrast, we use a long Raman π -pulse to drive a narrow velocity subgroup from $F = 2, m_F = 0$ back to $F = 1, m_F = 0$.

In preparation for this, we measure the Doppler shift distribution of the atom cloud by looking at the fraction of atoms transferred from $F = 1$ to $F = 2$ by a Raman π pulse as a function of the beat note of the two Raman frequencies, $f_R = \frac{\omega_R}{2\pi}$. The atoms are prepared in the $F = 1, m_F = 0$ state using step 1 and 2 before applying the Raman pulse. The spectra produced by the two Raman axes are shown in Figure 5.17.

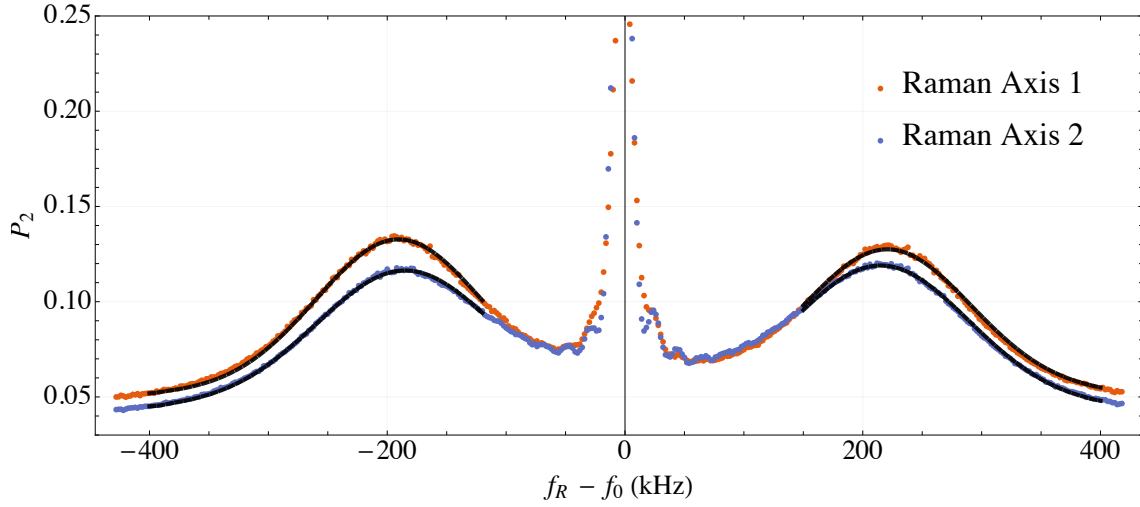


Figure 5.17: Measured fraction of atoms in $F = 2$ level after step 1, step 2 and a Raman π pulse as a function of the Raman beat note frequency, f_R relative to the clock transition frequency f_0 using Raman axis 1 (red) and Raman axis 2 (red). The fits for each left-hand and right-hand peak to Equation 5.5 are also shown in black.

As expected, we see two broad peaks on either side of a much narrower peak centred on $f_0 \sim 6834.682\text{MHz}$ in each spectrum. The two broad peaks correspond to the two counter-propagating Raman beam pairs driving Doppler-shifted and Doppler broadened transitions. The narrow central peak is from the co-propagating beam pair driving the velocity-insensitive transitions. The presence of the co-propagating beam pair arises from imperfect polarisation of the outgoing Raman beam. We fit the left and right peaks simultaneously to the Doppler-broadened line profiles for counter-propagating Raman transitions given by:

$$P_2(f_R) = P_0 + \int_{-\infty}^{+\infty} P_{\text{MB}}(v, v_R, T) \left(P_{\text{Raman}}(f_R, \Omega_{\text{eff}}(\text{LH}), v_R) + P_{\text{Raman}}(f_R, \Omega_{\text{eff}}(\text{RH}), v_R) \right) dv \quad (5.5)$$

where P_0 is a background, $P_{\text{MB}}(v, v_R, T)$ is the one dimensional Maxwell-Boltzmann distribution of velocities for an atom cloud of temperature T moving along the Raman beam at mean velocity v_R , and P_{Raman} is the transition probability, as given by Equation 2.8, for a square Raman pulse of duration τ , with effective Rabi frequency Ω_{eff} and detuning δ given by

$$\delta = 2\pi f_R - 2\pi f_0 - \mathbf{k}_{\text{eff}} \cdot \mathbf{v}_R - \frac{\hbar |k_{\text{eff}}|^2}{2m} \quad (5.6)$$

Here we assume the light shift is negligible as we find $f_R - f_0 \approx 0$ for the centre of the velocity-insensitive peaks in both spectra. The parameter values from the fits are summarised in Table 5.3, where we have left $T, v_R, P_0, \Omega_{\text{eff}}(\text{LH})$ and $\Omega_{\text{eff}}(\text{RH})$ as free parameters.

	Raman Axis 1	Raman Axis 2
v_R	$8.021 \pm 0.007 \text{ cm s}^{-1}$	$7.805 \pm 0.007 \text{ cm s}^{-1}$
T	$6.8178 \mu\text{K} \pm 0.0002 \mu\text{K}$	$7.5717 \mu\text{K} \pm 0.0002 \mu\text{K}$
$\Omega_{\text{eff}} (\text{LH})$	$70.98 \text{ krad s}^{-1} \pm 0.70 \text{ krad s}^{-1}$	$77.04 \text{ krad s}^{-1} \pm 0.34 \text{ krad s}^{-1}$
$\Omega_{\text{eff}} (\text{RH})$	$76.95 \text{ krad s}^{-1} \pm 0.50 \text{ krad s}^{-1}$	$74.05 \text{ krad s}^{-1} \pm 0.40 \text{ krad s}^{-1}$

Table 5.3: Parameters from the fit of Equation 5.5 to the counter-propagating Raman peaks in the spectrum using Raman axes 1 and 2.

The measured velocity along each Raman axis varies from value measured earlier in Section 5.3.4 using the CCD camera method. The velocity along Raman axis 1 now agrees very well with the target horizontal launch velocity of 8 cm s^{-1} . However the launch velocity along Raman axis 2 is now 0.2 cm s^{-1} lower than the target velocity. We think this may be a real discrepancy, perhaps due to an imbalance in the scattering force in the molasses beams. This would have to be fixed in a high precision gyroscope, but for the moment it is not important as we only seek to resolve the two velocity-sensitive Raman resonances.

For the $42 \mu\text{s}$ Raman pulse that was used, the Rabi frequency corresponding to a π pulse is 74.7 krad s^{-1} . This is very close to what we measure for the right-hand peak on Raman axis 2, with the left giving a slightly lower value. On Raman axis 1, one Rabi frequency is a few

percent high and the other a few percent low. In all cases I conclude that the Raman light is well set up.

The temperatures measured on the two Raman axes have astonishingly small error bars, which I am hard pressed to take seriously. Suffice it to say they are not greatly different from the horizontal temperature of $6.6\mu\text{K}$ measured in Section 5.2 using the thermal expansion method. As expected the Doppler widths of the left and right hand peaks are much larger than the $\sim 18\text{kHz}$ width of the Doppler-free central peak.

In order to design the velocity selection π -pulse that will be step 5, we first tune the beat note frequency to the maximum of either the left or right-hand peak of Figure 5.17. Next we look at the Rabi oscillation using a Blackman shaped Raman pulse whose profile is given by

$$\Omega(t) = \Omega_{\text{eff}} \left(\frac{21}{50} + \frac{1}{2} \cos \left(\frac{2\pi}{\tau} t \right) + \frac{2}{25} \cos \left(\frac{4\pi}{\tau} t \right) \right) \quad (5.7)$$

over the time interval $-\frac{\tau}{2} < t < +\frac{\tau}{2}$ and zero everywhere else. It is advantageous to use a Blackman pulse for velocity selection as this eliminates the side-lobes that are present in the line shape of a square pulse. This pulse needs to be long compared with the interferometer pulses so that the Doppler width of the selected velocity group will be narrow compared with the natural width of the interferometer pulses.

The atoms are again prepared in the $F = 1, m_F = 0$ state using steps 1 and 2. The Blackman pulse is then applied and the fraction of atoms transferred to the $F = 2$ level is measured as a function of the Blackman time τ , as plotted for both Raman axes in Figure 5.18a. These curves do not exhibit Rabi oscillation because of the broad velocity distribution in the $F = 1$ cloud. However we do see a peak around $95\mu\text{s}$ on each axis and this we find to be a good indicator of the π -pulse time for the atoms that are resonant. Therefore a Blackman time $\tau=95\mu\text{s}$ is chosen for the velocity selection pulse along each axis.

The numerically calculated resonance line width (FWHM) for a Blackman pulse is plotted in Figure 5.18b as a function of τ . We find that for a $95\mu\text{s}$ velocity selection pulse the natural line width is 24.2kHz which is narrow enough in comparison with the $\sim 25\text{kHz}$ natural line width of

the interferometer pulses, and of course much less than the $\sim 150\text{kHz}$ Doppler widths measured in Figure 5.17.

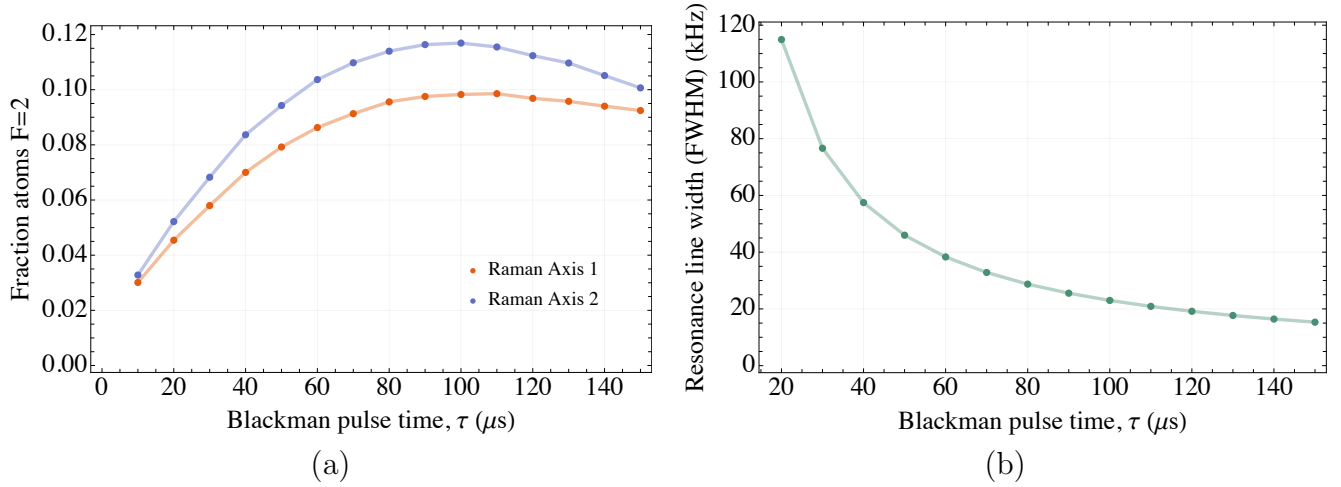


Figure 5.18: (a) The fraction of atoms in $F = 2$ driven by the Blackman shaped velocity selection pulse as a function of Blackman pulse time τ for both Raman axes. (b) Resonance line width (FWHM) for a Blackman shaped pulse as a function of pulse time τ .

Having chosen the velocity-selected pulse in this way, it is now used as step 5 to drive a narrow velocity group in the opposite direction, namely from $F = 2, m_F = 0$ down to $F = 1, m_F = 0$. This works well, as we shall see in Section 5.4.8, but we noticed that approximately 2% of all the atoms in $F = 2$ are driven down to $F = 1$, even when the beat note is tuned well away from the Raman resonance. This is due to off-resonant optical pumping via the $P_{3/2}$ states. The majority of these atoms will not participate in the interferometer either because they have $m_F = \pm 1$, or when $m_F = 0$, because their Doppler shift is too large for them to be addressed by the interferometer pulses. This 2% constitutes a significant background that leads to a loss in contrast of the interference fringes because only 10% of the atoms are velocity selected to participate in the interferometer.

The detection protocol of the apparatus measures the final number of atoms in $F = 2$ and the total number. The ratio of these is $P_{2\text{measured}}$. The denominator here includes the spectator atoms, whereas we are interested in knowing the transition probability, P_2 , for the clock transition, given by

$$P_2 = \alpha P_{2\text{measured}} \quad (5.8)$$

where α is a scale factor. We determined the scale factor by first measuring the fraction of atom of atoms in $F = 1$ after the velocity selection pulse when the frequency is off-resonance P_{off} then when the frequency is on resonance P_{res} (maximum of the counter-propagating peak in the Raman spectrum). The scale factor is given by

$$\alpha = \frac{P_{\text{res}}}{P_{\text{res}} - P_{\text{off}}} \quad (5.9)$$

This makes the good approximation that a negligible fraction of the atoms are off-resonantly pumped into $F = 1, m_F = 0$ state and then excited by interferometer pulses. We find that the scale factors for Raman axes 1 and 2 are 1.32 and 1.23, respectively. These scale factors are used for all results presented from here on when a velocity selection pulse has been applied.

5.4.7 Step 6: Blow Away Pulse 2

The final step in the state preparation scheme is to blow away all the $F = 2$ atoms remaining in the detection region. This is achieved using a single MOT beam with only the cooling light tuned close to resonance with the $F = 2 \rightarrow F' = 3$ transition. After a blow away pulse of 3ms, we reduce the remaining population of atoms in the $F = 2$ level from $\sim 90\%$ to $< 3\%$.

5.4.8 Final State of Atoms

After applying the full state preparation scheme we measure the Raman spectrum by applying a square Raman π pulse and scanning its beat note frequency. The measured spectrum along Raman axis 2 is shown in Figure 5.19. As expected we see a large reduction in Doppler width of the left and right peaks, now measured on Raman axes 1 and 2 to be $25.4 \pm 0.4 \text{ kHz}$ and $24.2 \pm 0.3 \text{ kHz}$. These agree well enough with the expected 24.2 kHz for a Blackman velocity selection pulse of $95 \mu\text{s}$, indicating that most atoms should now be addressed by the interferometer pulses. These Doppler widths equate to atom cloud temperatures along Raman axes 1 and 2 of $1.03 \pm 0.03 \mu\text{K}$ and $0.93 \pm 0.02 \mu\text{K}$.

After loading the 3D MOT for 200ms and applying the six state preparation steps, we have approximately 10^7 atoms in the desired $F = 1, m_F = 0$ state with a spread in Doppler width of $\sim 24\text{kHz}$ ready for interferometry.

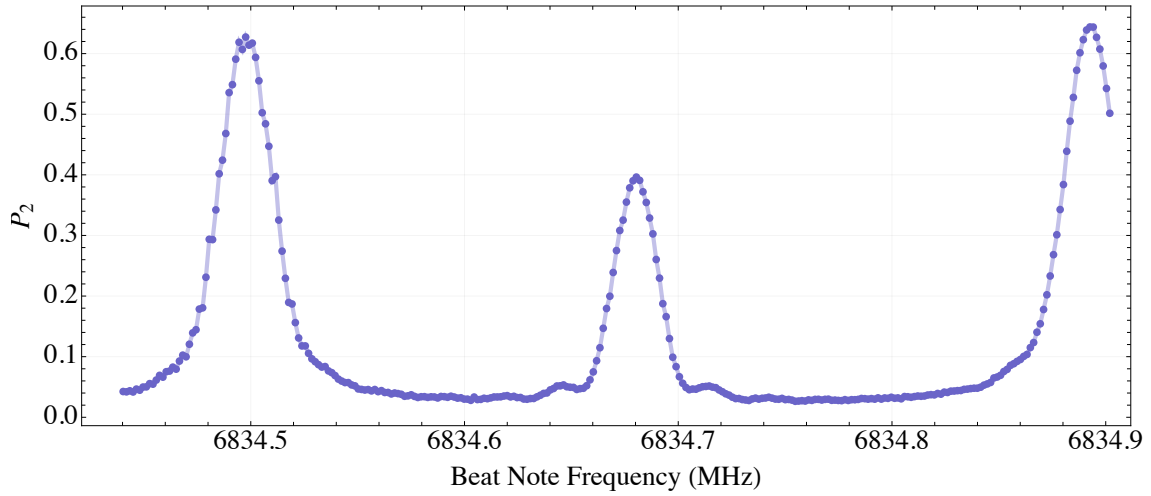


Figure 5.19: Measured Raman spectrum using square π pulse applied after the full state preparation scheme using Raman axis 2.

The stability of the full state preparation scheme was measured by looking from shot to shot over 1000s at the fraction of atoms detected in $F = 1$. The Allan deviation is plotted in Figure 5.20.

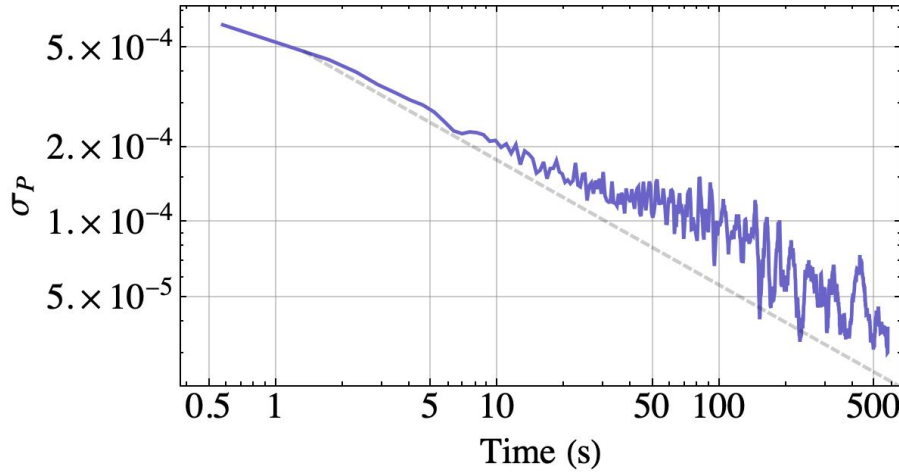


Figure 5.20: Allan deviation plot of the fraction of atoms prepared in $F = 1, m_F = 0$ after the full state preparation scheme. The noise appears to average down as $1/\sqrt{t}$, indicated by the grey dashed line.

The probability noise, σ_P , shot to shot is 6×10^{-4} . This is well below the required probability noise of 2×10^{-3} (discussed earlier in Section 3.4) to reach the short term sensitivity goal of

100ng, indicating we will not be limited fluctuations in the state preparation. As indicated by the grey dashed line, the noise in the fraction of atoms in $F = 1$ averages down as $1/\sqrt{t}$ out to 500s, as one would expect for white noise. At 500s the fractional uncertainty is $\sim 4 \times 10^{-5}$. There is no indication of an upturn in the Allan deviation plot, suggesting the efficiency of the state preparation does not drift over long times. This is important for achieving stability of the interferometer.

Chapter 6

Measuring the Interferometer Noise Sources

Following the state preparation scheme discussed in the previous chapter, we have approximately 10^7 atoms in the $F = 1, m_F = 0$ state ready for the three pulse interferometry sequence. Initial interference fringes demonstrated using this two axis system are presented in [1], which describes an interferometer of $T = 12.5$ ms with a contrast $C=30\%$ using 3×10^7 atoms. That interferometer has an acceleration noise of $8.1\mu\text{g}$ per shot. Sampling four times a second the total noise was $4.1\mu\text{g}$ at 1s which is much higher than the 100ng target.

In this chapter we focus on reducing the noise found in [1]. The total noise is due to three main sources; the detection noise, the quantum projection noise (QPN) and phase noise in the optical beat note. This may be due to noise in the phase lock of the Raman lasers or noise generated by vibration of the retro-reflecting mirror. When the fringe is plotted against the reading of the MEMS accelerometer, the noise in that reading also appears as interferometer phase noise.

6.1 Initial Fringes

To create the $\pi/2 - \pi - \pi/2$ interferometer we must first find the correct pulse duration for a π pulse. After full state preparation we therefore apply a single Raman pulse and measure the transition probability P_2 as a function of pulse duration. This initial Rabi oscillation is used to estimate the duration required to achieve a π pulse, τ_π . As with the velocity selection pulse, τ_π is estimated by the first peak in the Rabi oscillation. Setting the Raman pulse duration to this initial estimate of τ_π , we then measure the transition probability as a function of the chirp rate of the Raman beat note. A non-zero chirp rate may be required to ensure the Raman beat note frequency remains resonant throughout the interferometer sequence if there is some projection of gravity (or other acceleration) along the Raman axis. We find that the chirp rates which maximise the transition probability are 1.495MHz s^{-1} and 0.665MHz s^{-1} for Raman axes 1 and 2, as shown in Figure 6.1a. Using the optimised chirp rates we then remeasure the Rabi oscillations produced by the Raman pulse, shown in Figure 6.1b, to more accurately measure τ_π along each Raman axis. We find $\tau_\pi = 38\mu\text{s}$ for Raman axis 1, and $\tau_\pi = 43\mu\text{s}$ for Raman axis 2. To create the $\pi/2$ Raman pulses, we set the pulse duration to τ_π and halve the power.

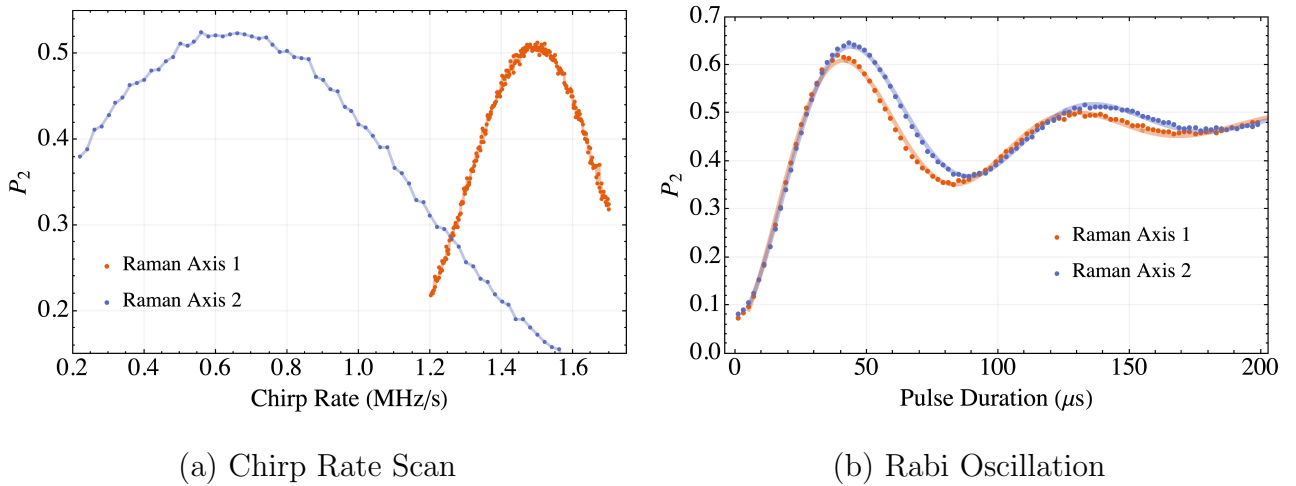


Figure 6.1: (a) Measured fraction of atoms in $F = 2$ as a function of chirp rate after single Raman pulse along Raman axis 1 (red) and 2 (blue). (b) Rabi oscillation for a single Raman pulse with optimised chirp rates along Raman axis 1 (red) and 2 (blue).

Next, we use the $\pi/2 - \pi - \pi/2$ sequence, with chirped pulses spaced by $T = 1\text{ms}$, adding a phase jump, η , to the beat note of the π pulse. Scanning η while measuring the final transition

probability, P_2 , produces the interferometer fringes shown in red in Figure 6.2. We then increase the T time to 3ms and 5ms, and repeat the scans to obtain the fringes shown in blue and green in Figure 6.2. Each fringe is then fit to the following function

$$P_2 = P_0 - \frac{C}{2} \cos(\Delta\Phi - 2\eta) \quad (6.1)$$

where $\Delta\Phi$, defined in Equation 3.15, is taken to be constant.

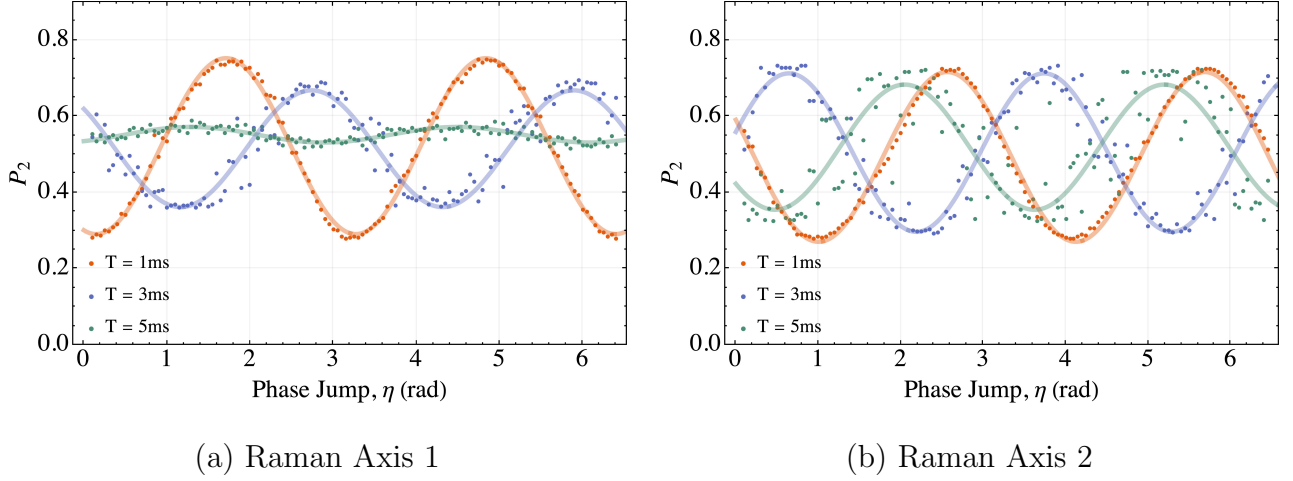


Figure 6.2: Interferometer fringes produced by scanning the phase jump η of the π pulse for $T=1, 3$ and 5 ms along Raman axis 1 and 2.

The fringe contrast for short $T < 2$ ms is very similar along both Raman axes. However we see a clear reduction in contrast with increasing T along Raman axis 1. The source of this decoherence along Raman axis 1 is investigated in Chapter 7.

6.2 Noise Sources

Analysis of the $T=12.5$ ms interferometer described in [1] quantifies each individual noise contribution. These were found to be as follows:

The quantum projection noise (QPN) on the transition probability is given by

$$\sigma_{P_{\text{QPN}}} = \sqrt{\frac{P_2(1 - P_2)}{N_{\text{Tot}}}} \quad (6.2)$$

Assuming the interferometer is operating at the fringe slope such that $P_2=0.5$, and with $N_{\text{Tot}} = 3 \times 10^7$, the QPN on the transition probability is 9.1×10^{-5} per shot. Recalling that acceleration a contributes $k_{\text{eff}} a T^2$ to the interferometer phase, we can convert noise in transition probability, σ_P , to uncertainty in acceleration, σ_a , by

$$\sigma_a = \frac{2 \sigma_P}{C k_{\text{eff}} T^2} \quad (6.3)$$

where C is the fringe contrast in Equation 6.1 and we have taken $\sin(\Delta\Phi - 2\eta) = \pm 1$. For the interferometer with $T=12.5\text{ms}$ and $C=30\%$, this gives acceleration uncertainty of 25ng per shot.

On the steepest part of the fringe, the noise in P_2 due to noise in $\Delta\Phi$ is just

$$\sigma_P = \frac{C \sigma_{\Delta\Phi}}{2} \quad (6.4)$$

In [1] the noise in $\Delta\Phi$ due to noise in the optical beat note was measured to be 31mrad per shot. Hence the corresponding noise in P_2 is $\sigma_{P_{\text{beat note}}} = 4.7 \times 10^{-3}$. Using Equation 6.3 this gives an acceleration uncertainty of $1.3\mu\text{g}$ per shot.

There is also uncertainty in P_2 arising from noise in the fluorescence detection (see Section 6.3). In [1] each of the four photodetector voltages, V_2 , V_{Tot} , B_2 and B_{Tot} , was averaged and read out at intervals of $5\mu\text{s}$ over a total of $300\mu\text{s}$. These 60 measurements were then averaged to produce the four average voltages, from which P_2 is determined. The noise in P_2 was measured in [1] to be 1.3×10^{-2} per shot, which corresponds to an uncertainty in the acceleration of $3.5\mu\text{g}$.

The interferometer measures the relative acceleration between the atoms and the mirror because the phase of the Raman beat note depends on the distance between the atoms and the mirror. Hence, the acceleration of the mirror is indistinguishable from that of the atoms as far as the interferometer is concerned. They can, however, be distinguished by means of the MEMS accelerometer, whose voltage V_{MEMS} is proportional to the acceleration of the mirror and hence to the part of the interferometer phase due to the acceleration of the mirror. In order to take out the mirror vibration noise it can therefore be convenient to plot the fringe against V_{MEMS} ,

expressed as an equivalent phase. In that case, the equivalent phase noise due to the MEMS accelerometer noise was measured to be 180mrad in [1], giving a transition probability noise of 2.7×10^{-2} , and an acceleration uncertainty of $7.3\mu\text{g}$ per shot. All these noise contributions described in [1] are summarised in Table 6.1. We see that the interferometer can be much more sensitive than the MEMS accelerometer, especially if the detector noise and phase fluctuation noise can be reduced, the ultimate limit being set by the quantum projection noise.

Noise Source	σ_a/shot	σ_P/shot
Quantum Projection σ_{QPN}	25ng	9.1×10^{-5}
Beat Note Phase Fluctuations	$1.3\mu\text{g}$	4.7×10^{-3}
Detection	$3.5\mu\text{g}$	1.3×10^{-2}
MEMS Accelerometer	$7.3\mu\text{g}$	2.7×10^{-2}

Table 6.1: Noise contributions found in [1] for the interferometer having $T=12.5\text{ms}$ and $C = 30\%$. Noises are given in terms of transition probability noise and acceleration noise per shot.

6.3 Detection Noise

Turning to my own research, I examine first the detection noise. The atoms are detected through laser induced fluorescence using the detection system outlined in Section 4.6, initially with a detection period of $300\mu\text{s}$ to replicate the detection noise observed in [1]. We expect contributions to the detection noise from the noise in the photodiode and transimpedance amplifier. In addition, and this was not considered in [1], we expect noise from fluctuations in the intensity and/or frequency of the detection light.

6.3.1 Initial Detection Noise

We first measure the detection noise due to the photodiode and transimpedance amplifier by running the detection sequence with no atoms and no detection light. This dark current noise,

averaged over the $300\mu\text{s}$ measurement period, is $24\mu\text{V}$. We now turn on the detection light, still without atoms, and the signals B_2 and B_{Tot} are $\sim 80\text{mV}$ as a result of light scattered by the apparatus into the photodiode. The noise in these signals is typically $300\mu\text{V}$, corresponding to a 0.4% fluctuation of the intensity averaged over $300\mu\text{s}$ bins. Next we add atoms prepared in the $F = 2$ state after molasses, and the resonance fluorescence increases the signal to $\sim 3\text{V}$, with a noise of 20mV , corresponding to 0.6% . As described in Section 4.6, there are two choices of beams for detection; the dedicated detection beam pair and the vertical MOT beam pair, both set to be resonant. A P_2 measurement is made by recording V_2 and V_{Tot} , then blowing the atoms away and recording B_2 and B_{Tot} . The red points in Figure 6.3 show a sequence of P_2 values, measured every ~ 1.3 seconds using the vertical beams. These points are of course close to 1, but have a standard deviation of 0.7% . The set of P_2 measurements using the dedicated detection beams, shown in blue in Figure 6.3, has a similar standard deviation of 0.6% . This noise is roughly half the 1.3% per shot reported in [1] but still much greater than one would expect from the measured intensity fluctuations. That is because the intensity of the detection light is four times the saturation intensity, and hence the 0.4% intensity fluctuations can only produce a 0.08% fluctuation in the scattering rate. Moreover, low frequency variations of intensity (below $\sim 1\text{kHz}$) do contribute to the noise in P_2 .

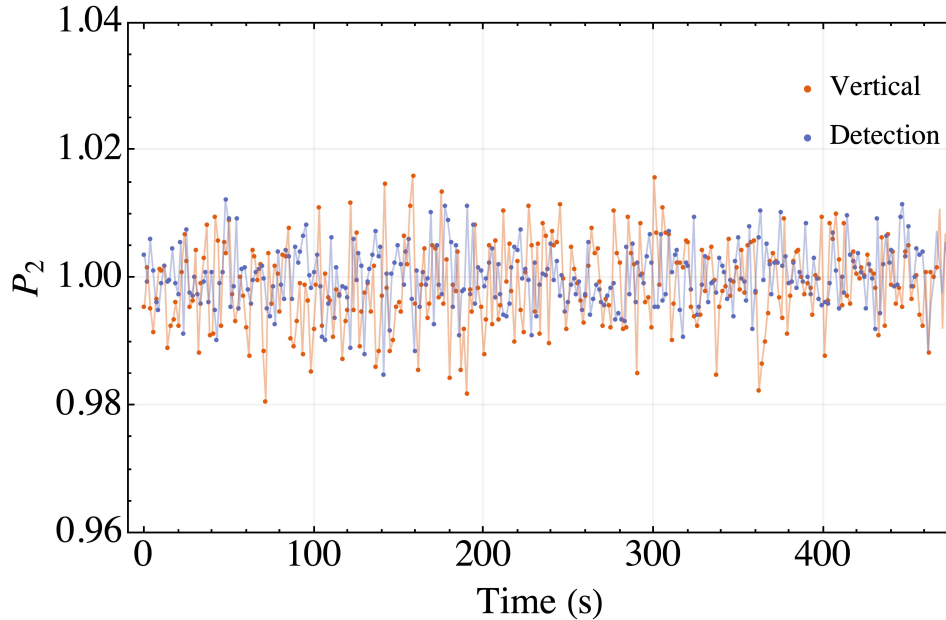


Figure 6.3: Time trace of the measured P_2 value when the detection period for each measurement (V_2 , V_{Tot} , B_2 and B_{Tot}) is $300\mu\text{s}$ when using the vertical (red) and detection (blue) beams.

Therefore we conclude that the noise in P_2 must be coming primarily from fluctuations in the frequency of the detection light. We can estimate the magnitude of these fluctuations by noting that in this particular measurement, where $P_2=1$ and the background is negligible, the noise in P_2 , σ_P , is just $\frac{\sqrt{2}\sigma_f}{V}$. Here $V = V_2 = V_{\text{Tot}}$ is the fluorescence signal and σ_f is the noise in the fluorescence due to frequency fluctuations between measuring V_2 and V_{Tot} . Taking $\sigma_P = 0.65\%$, we see that $\sigma_f=0.4\%$. With a saturation parameter of 4 it is straightforward to show that a detuning from resonance of 300kHz reduces the scattering rate by 0.4%, so we expect that this is the order of magnitude of the frequency variations.

6.3.2 Measuring the Frequency of Detection Light

The frequency of the detection light is measured using a Thorlabs SA200 Fabry-Perot cavity. Figure 6.4a shows the frequency fluctuations, δf , around the mean frequency over a time interval of 5ms. We see that the frequency does indeed fluctuate though not quite as much as the $\sim 300\text{kHz}$ we anticipated, so it is possible that some of this noise is real noise in P_2 caused by imperfect state preparation. There is a very clear oscillation with a period of $\sim 700\mu\text{s}$, so the $300\mu\text{s}$ integration time for each voltage measurement is particularly unfortunate. In an ideal world, we would have improved the laser system to remove this $\sim 1.3\text{kHz}$ frequency modulation, which corresponds to a natural resonance of the tweeter mirror, but the manufacturers were unable to do so without fundamental redesign of the system, so we needed to find a pragmatic solution. Still preparing the atoms in the $F = 2$ state, we measured the noise σ_P as a function of detection period, with the results plotted in Figure 6.4b. This shows that the detection period of $300\mu\text{s}$ almost maximises the noise, whereas extending it to $600\mu\text{s}$ reduces the noise to below 3×10^{-3} . We therefore now use a detection period of $600\mu\text{s}$, which is still short enough to produce little optical pumping during the V_2 phase of detection.

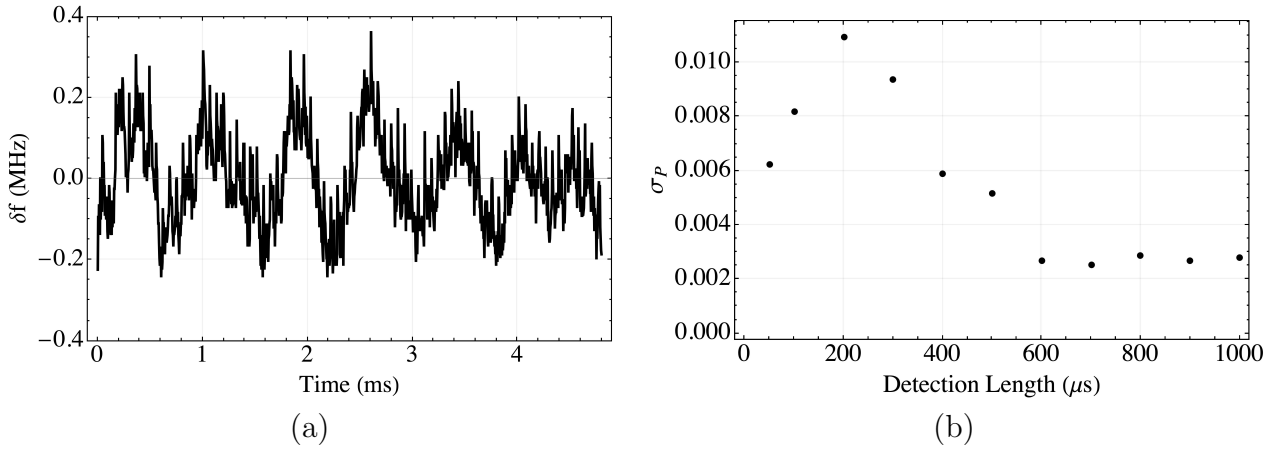


Figure 6.4: (a) Frequency fluctuations, δf , of the detection light measured on a Fabry-Perot cavity over 5ms. (b) Detection noise measured as a function of detection length for each of the V_2 , V_{Tot} , B_2 and B_{Tot} measurements.

6.3.3 Measuring the Detection Noise as a Function of N_2 and N_{Tot}

There can be circumstances in which the noise from scattered light and detector electronics, σ_{BG} , cannot be neglected in comparison with the noise in the fluorescence, σ_f . Writing the noise in V_{Tot} as $\sqrt{\sigma_f^2 + \sigma_{\text{BG}}^2}$ and the noise in V_2 as $\sqrt{P_2^2 \sigma_f^2 + \sigma_{\text{BG}}^2}$ (i.e. the noise that is uncorrelated over $600\mu s$), it is straightforward to show that

$$\sigma_P = \frac{1}{V_{\text{Tot}}} \sqrt{2 P_2^2 \sigma_f^2 + (1 + P_2^2) \sigma_{\text{BG}}^2} \quad (6.5)$$

With $P_2=1$ the fluorescence is bright enough that the second term under the square root can safely be neglected and we recover $\frac{\sqrt{2}\sigma_f}{V_{\text{Tot}}}$ from the previous section. Since both σ_f and V_{Tot} are proportional to the number of atoms, we expect σ_P to remain constant as that number is varied. The green curve in Figure 6.5 shows that this is indeed the case, and that the noise has been reduced by choosing the longer sampling time. With P_2 reduced to 0.5, we expect σ_P to drop to $\frac{1}{\sqrt{2}} \frac{\sigma_f}{V_{\text{Tot}}}$, which is half the size and still independent of the number of atoms. The blue curve in Figure 6.5 shows the noise measured with $P_2=0.5$. This noise is slightly more than expected and it rises a little when the atom number is very low, but there is qualitative agreement. The behaviour is quite different when the second term under the square root in Equation 6.5 is the dominant one. This is the case for the orange curve in Figure 6.5, where $P_2=0.01$, and Equation

6.5 gives the noise in P_2 as $\frac{\sigma_{BG}}{V_{Tot}}$. Because the background noise is constant we expect σ_P to vary inversely with the number of atoms, and that is what we see in the data. When operating the interferometer as an accelerometer we will normally choose to be near $P_2=0.5$. The blue curve in Figure 6.5 shows that for all atom numbers above $\sim 5 \times 10^6$, we expect the detection noise to be less than 3×10^{-3} per shot. This is significantly lower than the previously reported 1.3×10^{-2} in [1]. We attribute this reduction to the extended detection period of $600\mu s$. For an interferometer having $T=12.5ms$, that corresponds to an uncertainty in acceleration of $0.8\mu g$ per shot.

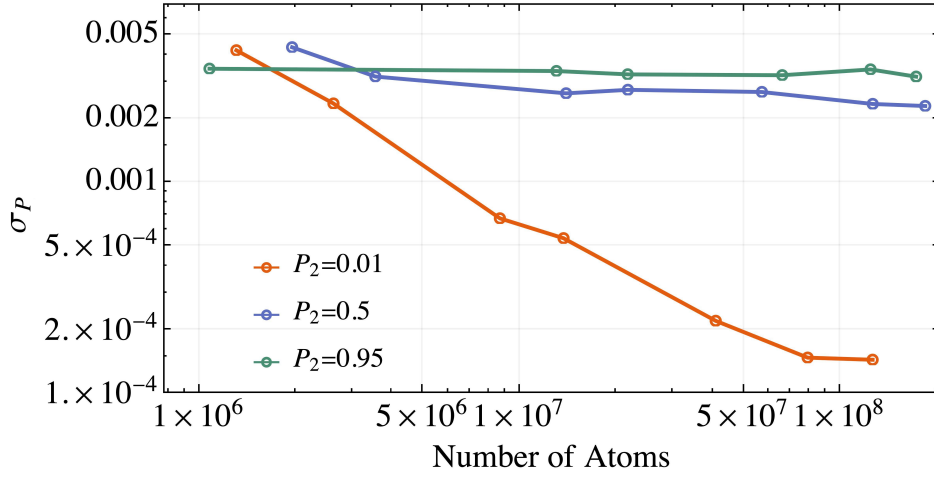


Figure 6.5: Measured detection noise on the transition probability, P_2 , as a function of the total number of atoms for P_2 values 0.95, 0.5 and 0.01.

6.4 Measurement of Beat Note Phase Noise

To measure the phase noise in the Raman beat note produced by the laser system, we employ a $\pi/2 - \pi/2$ Ramsey interferometer using the co-propagating beam pair, which is free of phase noise from vibration of the retro-reflection mirror. The resonance has a negligible Doppler shift so there is no need for velocity selection and all the atoms can be used. The atoms are prepared using the state preparation scheme outline in Chapter 5, except that a second microwave π pulse replaces the velocity selection pulse. The Ramsey interference fringe for $T=1ms$ shown in Figure 6.6a is produced by scanning a phase jump η added to the second Raman pulse. We then fix this phase at $\sim 3.4rad$ so that the interferometer is sitting on the side of a fringe where

it is maximally sensitive to changes in the phase of the beat note. There, we measure the noise in P_2 using the $600\mu\text{s}$ averaging time for each detector measurement. This noise is plotted in Figure 6.6b as a function of the number of atoms, which is varied by changing the 3D MOT loading time.

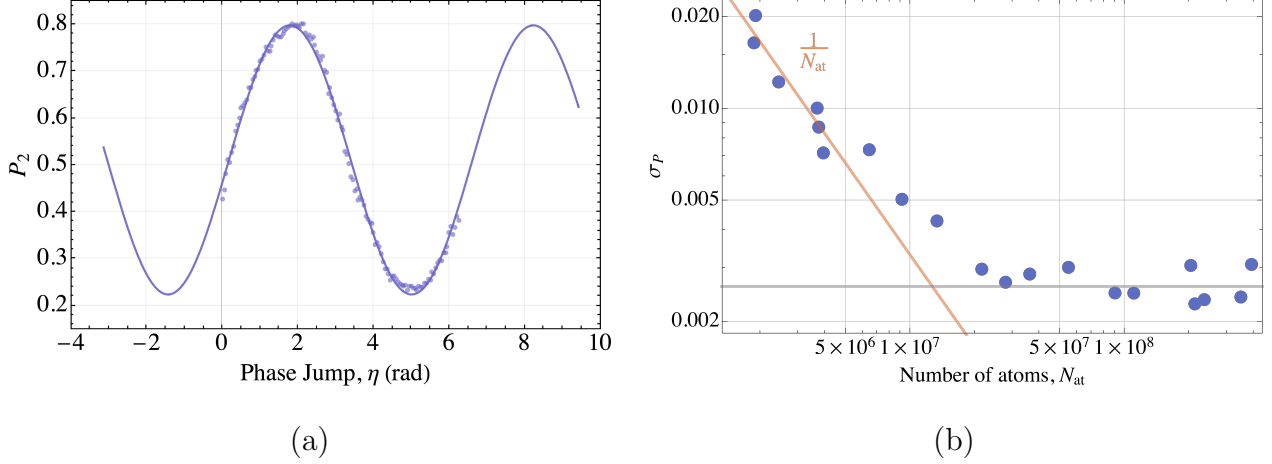


Figure 6.6: (a) Interference fringe produced when scanning a phase jump η added to the second pulse for the $T=1\text{ms}$ Ramsey interferometer. (b) Measured noise on the transition probability when the phase jump of the second Raman pulse is set to 3.4rad as a function of the number of atoms.

We find that the standard deviation σ_P reduces inversely with increasing number of atoms until it reaches approximately 2.6×10^{-3} and becomes independent of the number of atoms. This noise level is close to the detection noise measured in Section 6.3.3, so we cannot say from this data how much is due to phase noise and how much to detection noise. However, the two can be distinguished by their different dependencies on the phase jump η . At the peaks of the fringe, P_2 has no sensitivity to small phase noise but has maximum detector noise. By contrast, at $P_2=0.5$, there should be maximum sensitivity to phase noise and half the detector fluorescence noise. The data points in Figure 6.7 show the noise observed in P_2 as a function of the phase jump η applied to the second Raman pulse. The green curve shows the expected variation of the fluorescence noise (proportional to P_2), while the red curve shows the variation in Raman beat note phase noise (proportional to $\left| \frac{dP_2}{d\eta} \right|$). When these two are combined in quadrature we obtain the blue curve, which is a least-squares fit to the data. We conclude that

most of the noise measured in Figure 6.6b is coming from phase fluctuations. Since $\left| \frac{dP_2}{d\eta} \right| \simeq 0.3$ in Figure 6.6a, the σ_P of 2.6×10^{-3} corresponds to noise in the phase of the Raman beat note of a little below 10mrad. This is consistent with the measurement performed by *M-Squared* by monitoring the beat note directly on a photodiode, also mentioned in Chapter 4. The peak detection noise of 0.1% is roughly half what we measured in Figure 6.5. It is likely that this improvement comes from having better control over the reproducibility of the initial $F = 2$ fraction.

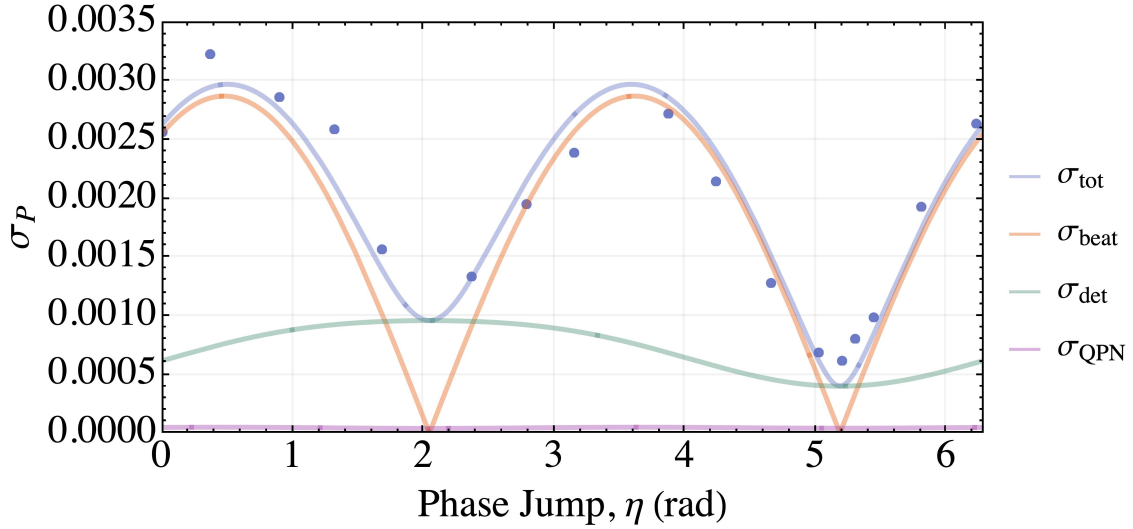


Figure 6.7: Transition probability noise measured as a function of phase jump η added of the second Raman pulse in the $T=1\text{ms}$ Ramsey interferometer with the corresponding total noise fit (blue), Raman beat note phase noise (red), detection noise (green) and quantum projection noise (purple).

We conclude that the phase difference of the beat note, sampled at two times separated by 1ms, is uncertain by $\sim 10\text{mrad}$. For the interferometer considered in Table 6.1, the pulses are separated by $T=12.5\text{ms}$. The experiment here does not probe that delay, but the measurements carried out by *M-Squared* looking directly at the beat note showed that the standard deviation of $\langle \phi(t) - \phi(t+T) \rangle$ is independent of T over the range $2\text{ms} < T < 24\text{ms}$. Assuming that this noise is random, we can associate an uncertainty of $\sigma_\phi = 10/\sqrt{2} \text{ mrad}$ or 7mrad with the phase at any time. In the the three pulse accelerometer, the interferometer phase is $\phi(0) - 2\phi(T) + \phi(2T)$, and the noise in this is $\sqrt{6}\sigma_\phi = 17\text{mrad}$. For the interferometer considered in Table 6.1, where $T = 12.5\text{ms}$, this level of interferometer phase noise corresponds to an uncertainty in the acceleration of 690ng , which is only half the $1.3\mu\text{g}$ cited from [1] in Table 6.1. However,

that number was inferred from the *M-Squared* beat note measurement in which there was some uncertainty about bandwidth, so I think this (almost) direct measurement of 690ng is more reliable.

6.5 Total Noise in Three Pulse Interferometer

The noise in the interferometer phase due to fluctuations, $d\phi(t)$ in the phase of the Raman beat note can be written as the integral over time [84]

$$\delta\Phi = \int g(t)d\phi(t) = \int g(t)\frac{d\phi}{dt}dt \quad (6.6)$$

where $g(t)$ is known as the sensitivity function. For the three pulse interferometer with pulse duration sequence of $\tau_{\pi/2} - \tau_{\pi} - \tau_{\pi/2}$, and under the approximation $\tau_{\pi} \ll T$, the sensitivity function can be simplified to [85]

$$g(t) = \begin{cases} -1 & 0 < t < T \\ +1 & T < t < 2T \\ 0 & \text{elsewhere} \end{cases} \quad (6.7)$$

Following reference [86], assuming the effective Rabi frequency Ω_{eff} is constant, we can relate the spectral power density of the beat note phase fluctuations, $S_{\phi}(\omega)$, to the variance of the interferometer phase $\sigma_{\Delta\Phi}^2$ through the following

$$\sigma_{\Delta\Phi}^2 = \int_{-\infty}^{\infty} |\omega G(\omega)|^2 S_{\phi}(\omega) d\omega \quad (6.8)$$

where $G(\omega)$ is the Fourier transform of $g(t)$. Looking at the transfer function $|\omega G(\omega)|^2$ we find, as one would expect, that the interferometer is insensitive to noise at low frequencies: for frequencies less than $1/(2T)$ the transfer function is approximately $f^4 T^4$. For increasing frequencies the transfer function oscillates with zeros at integer multiples of $1/T$. For frequencies larger than the Ω_{eff} , the transfer function acts as a low pass filter.

As the apparatus is mechanically connected to the floor of the laboratory, there are vibrations of the retro-reflecting mirror. A displacement, $\delta x(t)$, of the mirror along the direction of the Raman axis will shift the phase of the beat note by $d\phi(t) = 2k_2 dx(t)$. Substituting this into Equation 6.6, we see that the contribution to the interferometer phase resulting from mirror vibration is

$$\delta\Phi_{\text{vib}} = 2k_2 \int_0^{2T} g(t)v(t) dt \quad (6.9)$$

where $v(t) = \frac{dx}{dt}$. Integrating by parts, as in [85], this can be re-written

$$\delta\Phi_{\text{vib}} = 2k_2 \int_{-\infty}^{\infty} f(t)a(t) dt \quad (6.10)$$

where $a(t)$ is the acceleration of the distance between the atoms and the mirror, and

$$f(t) = - \int g(t')dt' = \begin{cases} t & 0 < t < T \\ 2T - t & T < t < 2T \\ 0 & \text{elsewhere} \end{cases} \quad (6.11)$$

Similar to the beat note phase, we can relate the power spectral density of the acceleration $S_{\text{acc}}(\omega)$, to the variance of the interferometer phase through a transfer function [86]

$$\sigma_{\Delta\Phi}^2 = (2k_2)^2 \int_{-\infty}^{\infty} |F(\omega)|^2 S_{\text{acc}}(\omega) d\omega \quad (6.12)$$

where $F(\omega)$ is the Fourier transform of the sensitivity function $f(t)$. Looking at the transfer function we find it is flat at low frequencies, with a bandwidth of $\sim 1/(2T)$ Hz. In the case of white acceleration noise S_{acc}^0 , Equation 6.12 gives

$$\sigma_{\Delta\Phi}^2 = \frac{8}{3} k_{\text{eff}}^2 T^3 S_{\text{acc}}^0 \quad (6.13)$$

Equation 6.13 shows that mirror vibrations, unlike the beat note phase noise, produces interferometer phase noise that increases with T .

Figure 6.8 shows the noise power spectral density of the mirror acceleration, as measured using

the MEMS accelerometer mounted on the back of the mirror (see Section 4.7.2).

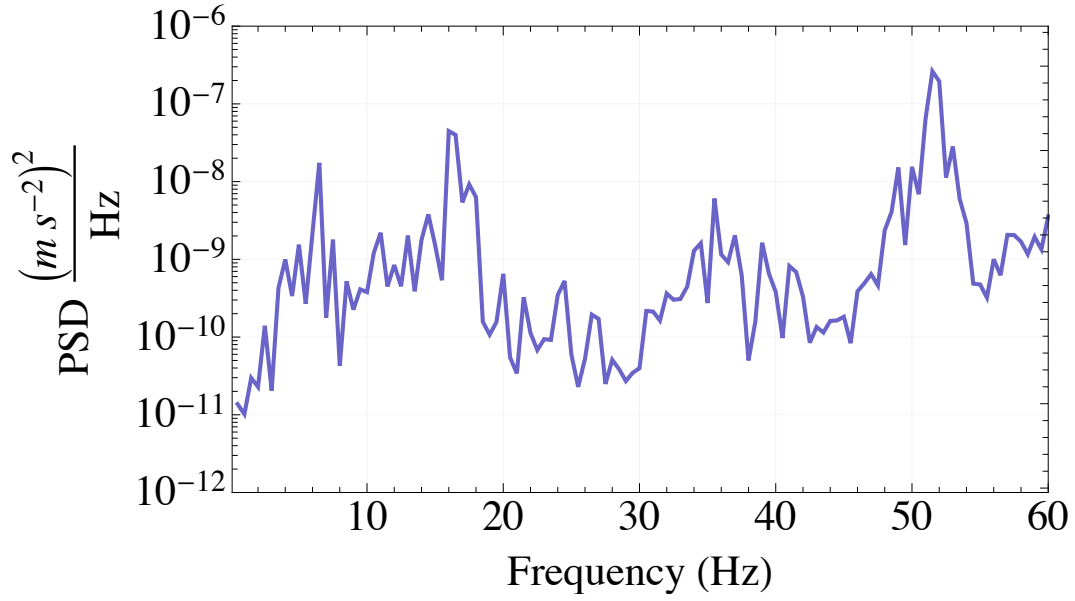


Figure 6.8: Noise power spectral density of the mirror acceleration measured using the MEMS accelerometer.

We see that it is not flat at all, so the integral in Equation 6.12 needs to be done numerically to compute the interferometer phase noise that the mirror vibration generates. The data points in Figure 6.9 show the values of $\sigma_{\Delta\Phi}$ calculated from the measured S_{acc} using Equation 6.12 for integer values of T between 1 and 20ms.

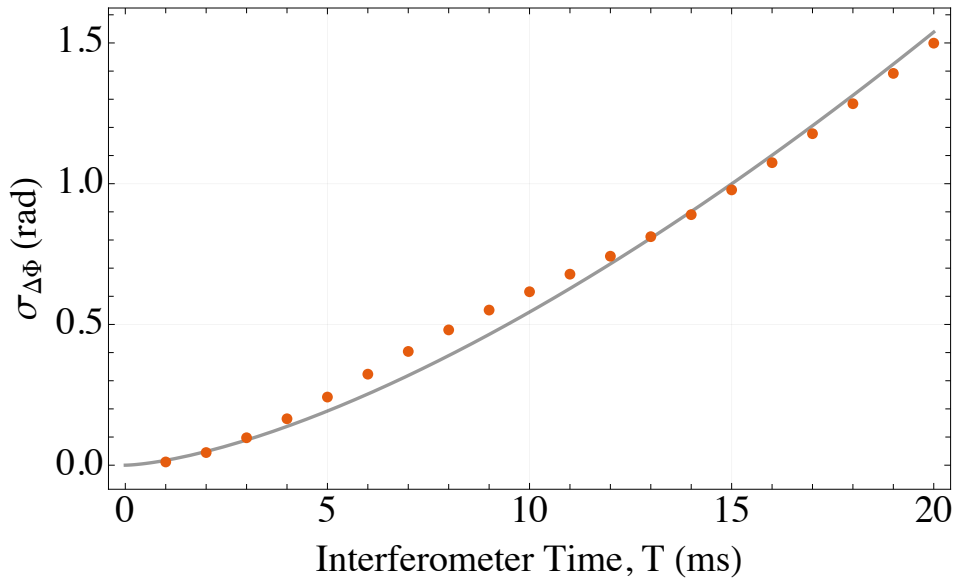


Figure 6.9: Interferometer phase noise generated by the mirror vibration calculated numerically from the noise power spectrum of the mirror measured using the MEMS accelerometer.

Although the noise spectrum is not flat, these points still fit well to the form $AT^{3/2}$. That is not altogether surprising because the interferometer phase, $\Delta\Phi$, is proportional to T^2 , so as long as the acceleration noise averages down as $T^{-1/2}$ we can expect $\sigma_{\Delta\Phi} \propto T^{3/2}$. The line in Figure 6.9 is a least squares fit to these points which gives $A=544\text{s}^{-3/2}$.

In order to measure the noise in $\Delta\Phi$, the interferometer was set to the point of maximum slope and we recorded the noise, σ_P in P_2 . The data points in Figure 6.10 shows $2\sigma_P/C$ as a function of T , where C is the contrast of the fringe.

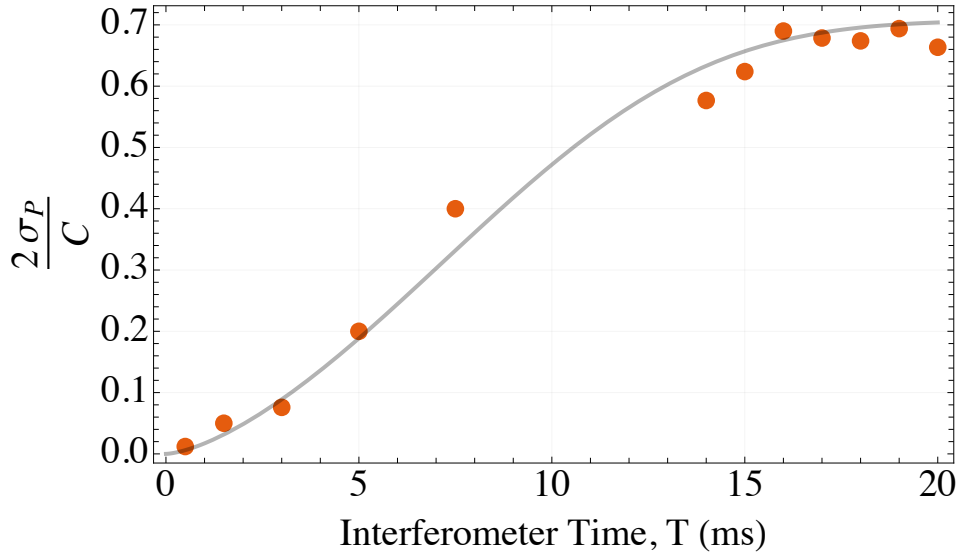


Figure 6.10: Noise σ_P in P_2 divided by fringe contrast C , plotted as a function of interferometer time T . Data points: measured when sitting on the steepest slope of the fringe. Solid line: Expected noise, as calculated from the output of the MEMS accelerometer. The flattening at large T simply reflects the fact that P_2 is bounded between $P_0 \pm C/2$.

If there is only interferometer phase noise, and that noise is small, then $2\sigma_P/C$ is the interferometer phase noise. However, for large phase fluctuations with a Gaussian distribution and standard deviation $\sigma_{\Delta\Phi}$, it is straightforward to show that

$$2\sigma_P/C = \sqrt{e^{-\sigma_{\Delta\Phi}^2} \sinh(\sigma_{\Delta\Phi}^2)} \quad (6.14)$$

The solid line in Figure 6.10 shows Equation 6.14 in which we have used the value $\sigma_{\Delta\Phi} = AT^{3/2}$ and $A = 544\text{s}^{-3/2}$, as calculated from the noise spectrum of the MEMS accelerometer. There is excellent agreement between the measured interferometer noise and the calculated noise. The

noise in σ_P flattens off at longer values of T for the simple reason that P_2 cannot fluctuate beyond the maximum and minimum of the fringe. With this amount of mirror vibration, we see in Figure 6.9 that an interferometer with $T = 15\text{ms}$ has an uncomfortably large phase noise of 1 rad, corresponding to an uncertainty in the acceleration measured by the interferometer of $28\mu\text{g}$.

In order to explore beyond the noise of mirror vibration, we use the output voltage of the MEMS accelerometer to give a continuous reading of the mirror acceleration, $a(t)$. Putting that into Equation 6.10, we compute the phase due to mirror vibrations, $\delta\Phi_{\text{vib}}$, for each interferometer measurement. Using $T = 15\text{ms}$, the red data points in Figure 6.11a show a plot of P_2 versus $\delta\Phi_{\text{vib}}$ as the fringe is swept out by the mirror vibration noise. There are 5,750 points taken over 15 hours. The solid red line is a fit to

$$P_2 = P_0 - \frac{C}{2} \cos(\delta\Phi_{\text{vib}} + \Delta\Phi). \quad (6.15)$$

with fitting parameters P_0 , C and $\Delta\Phi$. The blue data points are taken over the same 15 hours with the same interferometer, except that the direction of $\mathbf{k}_{\text{eff}}$ reversed by tuning to the Raman resonance with the opposite Doppler shift (see Section 6.6). This fringe is essentially the same, except that the mean operating point is now close to a minimum of the fringe. The residuals of the fit, δP , are plotted in Figure 6.11b as a function of time.

We see that the noise in δP is approximately 1%. There is also a slow drift within the range $\pm 1\%$, which is presumably a drift in P_0 .

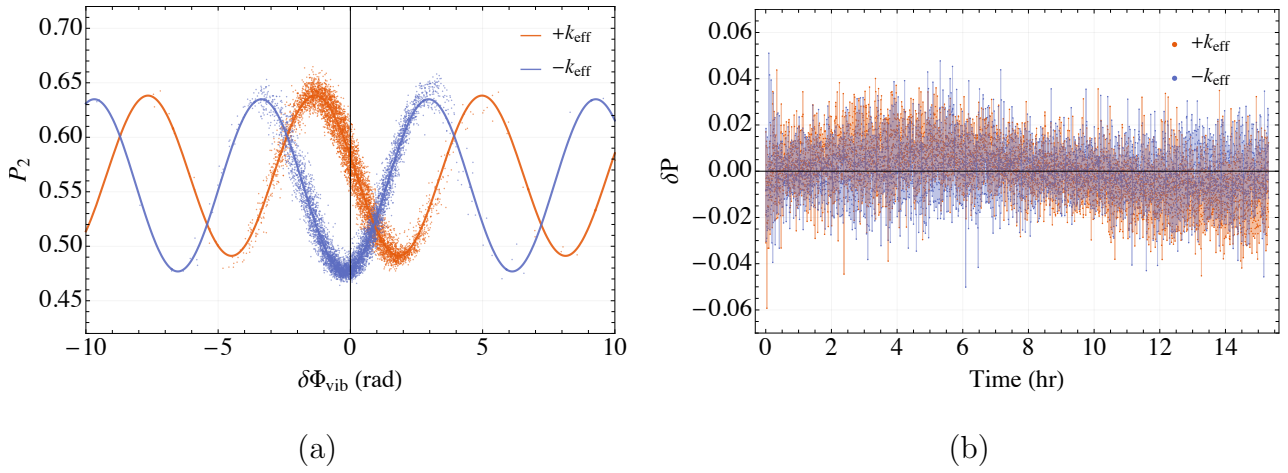


Figure 6.11: (a) Transition probability, P_2 , measured using an interferometer of $T=15\text{ms}$ and plotted against the interferometer phase shift due to mirror vibration. Red points: data measured over 15 hours with positive k_{eff} . Red line: fit to Equation 6.15. Blue points and line: the same as red except that the measurements are made with negative k_{eff} . (b) Residuals of the fits, δP , plotted against the time when each point was measured.

We distinguish the various different contributions to the noise by their dependence on $\delta\Phi_{\text{vib}}$, in much the same way as the noise analysis was performed on the Ramsey fringe. The residuals for $+k_{\text{eff}}$ plotted in Figure 6.11b are partitioned according to their value of $\delta\Phi_{\text{vib}}$, into bins of 50mrad width. Bins with less than 10 data points are discarded to avoid under-sampling. The standard deviation of the residuals in each bin, σ_P , is plotted in Figure 6.12a as a function of vibration phase, $\delta\Phi_{\text{vib}}$. This noise distribution is then fit to

$$\sigma_{\text{tot}}(\delta\Phi_{\text{vib}}) = \sqrt{\sigma_{\text{QPN}}^2 + \sigma_{\text{phase}}^2 + \sigma_{\text{det}}^2} \quad (6.16)$$

where

$$\sigma_{\text{QPN}} = \sqrt{\frac{P_2(\delta\Phi_{\text{vib}})(1 - P_2(\delta\Phi_{\text{vib}}))}{N_{\text{Tot}}}}, \quad (6.17)$$

$$\sigma_{\text{phase}} = \frac{C}{2} \sin(\delta\Phi_{\text{vib}} + \Delta\Phi) \sigma_{\Delta\Phi}, \quad (6.18)$$

$$\sigma_{\text{det}} = \alpha + \beta P_2(\delta\Phi_{\text{vib}}), \quad (6.19)$$

and $P_2(\delta\Phi_{\text{vib}})$ is given by the red fit to the fringe in Figure 6.11a. Here $\sigma_{\Delta\Phi}$ is the uncertainty in the interferometer phase, due to the combination of MEMS accelerometer noise and Raman beat note phase noise. For this interferometer we had $N_{\text{Tot}} = 7 \times 10^6$ atoms. We leave $\sigma_{\Delta\Phi}$, α and β

as free parameters. Figure 6.12a shows the fit for the total noise, σ_{tot} , and also the individual noise contributions. We find $\sigma_{\Delta\Phi}$ to be (196 ± 4) mrad for the $+\mathbf{k}_{\text{eff}}$ interferometer. Figure 6.12b shows the corresponding results for the $-\mathbf{k}_{\text{eff}}$ interferometer, which give $\sigma_{\Delta\Phi} = (154 \pm 6)$ mrad. I do not know why these disagree by more than the error bar returned by the fit, so I take (175 ± 20) mrad as a representative value. The two fits agree on the size of the detection noise. Table 6.2 summarises the results for the interferometer operating at maximum slope, with $C=16\%$ and with $T=15\text{ms}$.

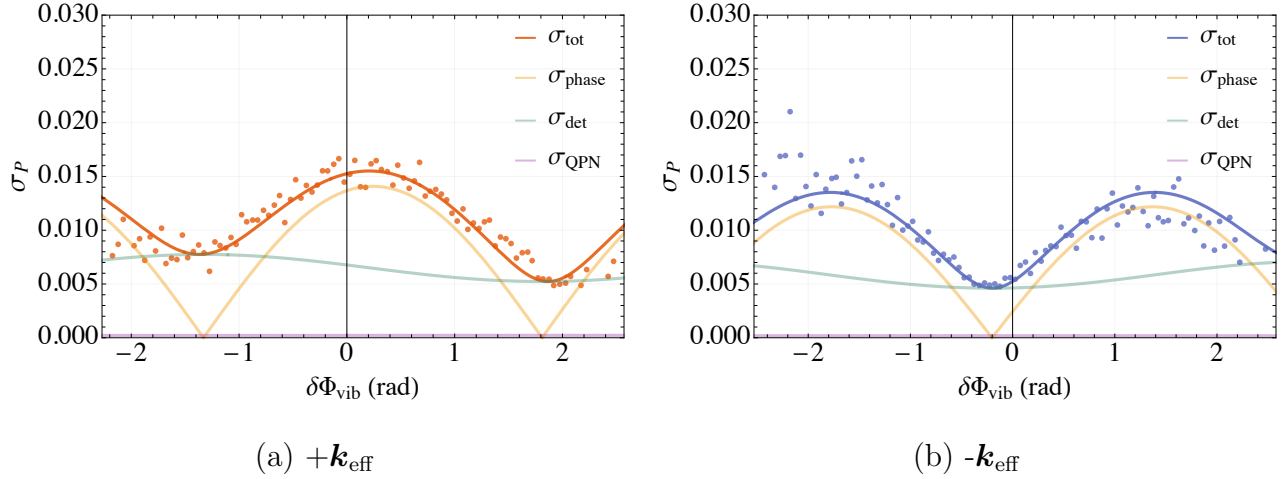


Figure 6.12: The standard deviation of the binned fit residuals as a function of $\delta\Phi_{\text{vib}}$ (see text), with the total noise function ((a) red and (b) blue), phase noise function (yellow), detection noise function (green) and quantum projection noise function (purple) fits.

Noise Source	σ_P/shot	$\sigma_{\Delta\Phi}/\text{shot}$	Equivalent σ_a/shot
Quantum Projection Noise	1.9×10^{-4}	-	54ng
Detection Noise	6.0×10^{-3}	-	$2.1\mu\text{g}$
Phase Noise (including MEMS accelerometer noise)	-	175 ± 20 mrad	$4.9 \pm 0.6 \mu\text{g}$

Table 6.2: Contributions from the individual noise sources for the $T=15\text{ms}$ interferometer with $C=16\%$. Equivalent acceleration noise is calculated assuming the fringe is at the point of greatest slope.

Knowing that the noise in the phase of the beat note contributes a negligible 17mrad to $\sigma_{\Delta\Phi}$ (see

Section 6.4), we conclude that the noise intrinsic to the MEMS accelerometer is approximately $5\mu\text{g}$. This is the noise in the bandwidth of the interferometer, which is $\sim 1/(2T) \simeq 30\text{Hz}$. The manufacturer of the MEMS accelerometer specifies a noise figure of $< 7\mu\text{g}$ within the spectral band of 0-10Hz. We see that theirs is quite a conservative specification. The $5\mu\text{g}$ per shot measured here is less than the $7.3\mu\text{g}$ found in [1]. We expect this is due to the use of a new printed circuit board described in Section 4.7.2 (in place of the circuit containing unshielded wires used in [1]). The measured detection noise of 6×10^{-3} per shot is twice the 3×10^{-3} shown in Figure 6.5. It is lower than the 1.3×10^{-2} reported in [1] because we have increased the detection period from $300\mu\text{s}$ to $600\mu\text{s}$.

6.6 Implementing k -Reversal

As described in Section 2.2.3, the retro-reflected laser beam configuration results in two counter-propagating pairs of frequencies which can drive the Raman transitions. These have equal and opposite wavevectors and therefore they have opposite Doppler shifts. The atoms are launched with sufficient velocity along the Raman beams that the frequency of the beat note can be tuned to select one pair or the other. The interferometer phase shifts due to acceleration or rotation are all proportional to $\mathbf{k}_{\text{eff}}$ and change sign when $\mathbf{k}_{\text{eff}}$ reversed. By contrast, various undesirable systematic phase shifts, for example from light shift or magnetic field gradient, are independent of the wavevector. These can therefore be eliminated by taking half the difference between the phases measured using the $\pm\mathbf{k}_{\text{eff}}$ interferometers.

In order to explore this idea, the red data points in Figure 6.11a were collected into groups of $N_{\text{dat}} = 15$ sequential points, each group covering a total time of approximately one minute. The points in a group spanned a wide enough range of $\delta\Phi_{\text{vib}}$ that they could be fitted using Equation 6.15 to determine the phase offset $\Delta\Phi(+k_{\text{eff}})_i$ and probability offset $P_0(+k_{\text{eff}})_i$ as measured in that i^{th} minute. The red points in Figure 6.13a show the sequence of phase offsets determined in that way over the whole 15 hours. The red points in Figure 6.13b show the sequence of probability offsets determined over the whole 15 hours. The blue points in Figures

6.13a and 6.13b show the phase and probability offsets determined in the same way from the dataset with $\mathbf{k}_{\text{eff}}$ reversed. The phase offset drifts by approximately 0.4 rad over the 15 hours. It has the same sign for both signs of $\mathbf{k}_{\text{eff}}$ so this is a slow systematic error, not an effect of motion. It is probably due to bias drift of the MEMS accelerometer or its readout electronics. We also see a drift in P_0 similar to that in Figure 6.11b for both $\pm\mathbf{k}_{\text{eff}}$ wavevectors.

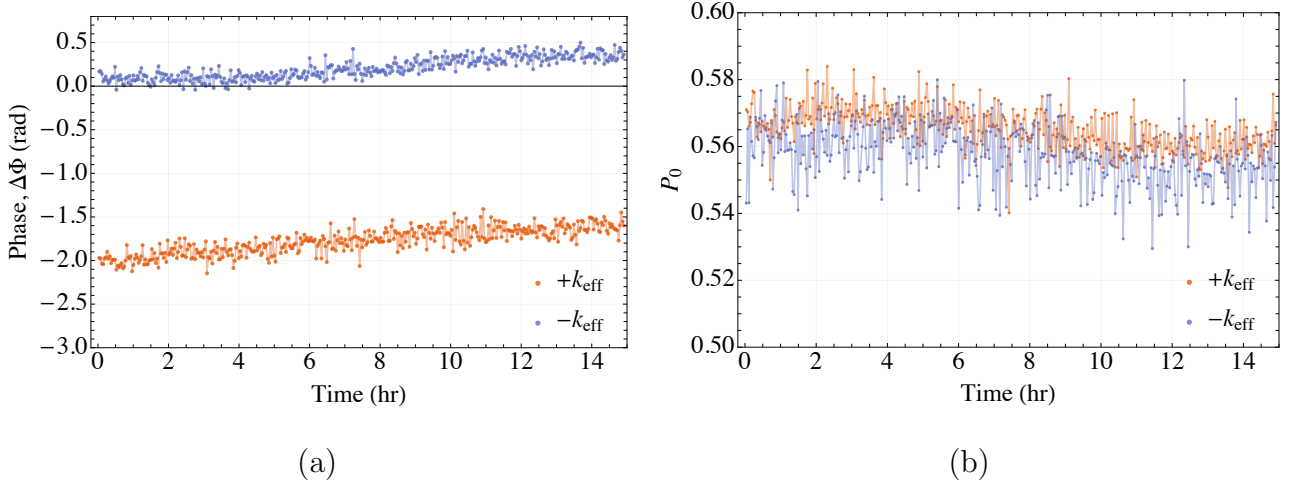


Figure 6.13: (a) The measured phase offsets, $\Delta\Phi$ for both $\pm\mathbf{k}_{\text{eff}}$ (red and blue) interferometers over the 15 hours. (b) The measured probability offsets P_0 for both $\pm\mathbf{k}_{\text{eff}}$ (red and blue) interferometers over the 15 hours.

Let us define the half sum and half difference of these phase offsets:

$$\Sigma_i = \frac{\Delta\Phi(+\mathbf{k}_{\text{eff}})_i + \Delta\Phi(-\mathbf{k}_{\text{eff}})_i}{2} \quad (6.20)$$

$$\Delta_i = \frac{\Delta\Phi(+\mathbf{k}_{\text{eff}})_i - \Delta\Phi(-\mathbf{k}_{\text{eff}})_i}{2} \quad (6.21)$$

Using $N_{\text{dat}} = 15$, the light green points in Figure 6.14a show the evolution of Σ , which reproduces the phase drift seen in Figure 6.13a. This is cancelled in the difference, Δ , as shown by the light yellow points in Figure 6.14a. By increasing N_{dat} to 60 points (plotted in dark green and yellow) we see the noise reduces.

The purple curve in Figure 6.14b is the mean fit uncertainty in $\Delta\Phi$ across the entire 15 hour data set, denoted by $\overline{\delta_{\Delta\Phi}}$, plotted as a function of N_{dat} . We see that $\overline{\delta_{\Delta\Phi}}$ decreases with increasing N_{dat} . Using a simulated data set, we find that the noise per shot in $\Delta\Phi$, $\sigma_{\Delta\Phi}$, is

obtained by multiplying $\overline{\delta_{\Delta\Phi}}$ by $\sqrt{N_{\text{dat}}}$. The blue curve in Figure 6.14b shows this noise per shot. It is relatively flat between N_{dat} values of 50 to 1000 and starts to increase for larger N_{dat} . This increase is due to the drift in the phase offset becoming comparable with or larger than the magnitude of the noise over the equivalent time for those large N_{dat} values.

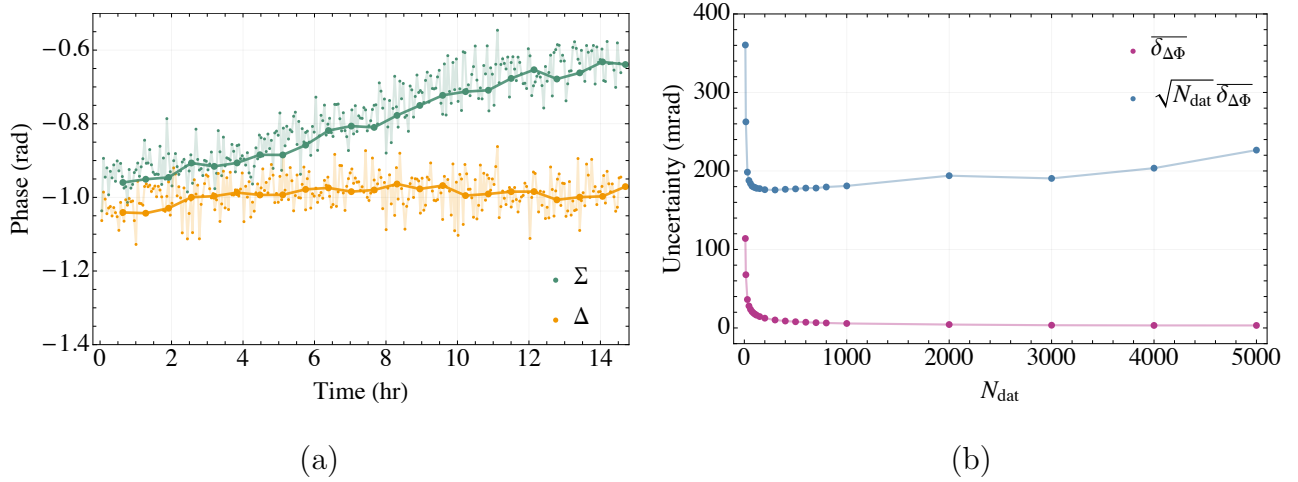


Figure 6.14: (a) The measured half sum, Σ , and half differences, Δ , of the interferometer phase offsets for $\pm \mathbf{k}_{\text{eff}}$ over the 15 hour period when $N_{\text{dat}} = 15$ (light curves) and $N_{\text{dat}} = 200$ (dark curves). (b) The fit uncertainty in $\Delta\Phi$ (purple), and the corresponding noise per shot (blue) as a function of the number of points N_{dat} fitted to the fringe.

Taking the mean value of $\sqrt{N_{\text{dat}}} \overline{\delta_{\Delta\Phi}}$ between N_{dat} values of 50 to 1000 I estimate that the noise in $\Delta\Phi$ is 179 mrad per shot. This corresponds very well to the total phase noise value of 175 ± 20 mrad per shot calculated in Section 6.5. This phase noise value includes contributions from errors in the acceleration measured by the MEMS accelerometer and noise in the Raman beat note phase. Recalling that $T = 15$ ms, this gives an upper limit of the noise contribution from the MEMS accelerometer reading of $5.0 \mu\text{g}$ per shot. Since the interferometer bandwidth is ~ 30 Hz this noise level is comfortably within the manufacturer's noise specification, which is $< 7 \mu\text{g}$ rms in the band 0.1-10 Hz.

I draw two conclusions from this section. (i) There is a small unwanted drift in the phase offset $\Delta\Phi(\pm \mathbf{k}_{\text{eff}})$ which can be eliminated using the k -reversal technique. (ii) When comparing the accelerations measured by the interferometer and the MEMS accelerometer, the dominant noise comes from the MEMS accelerometer.

6.7 Implementing π Flipping

The technique we call π flipping involves shifting the interferometer phase by π radians for every other shot, which we do by adding a shift of $\pi/2$ to the phase of the middle Raman pulse.

This gives two transition probability measurements

$$P_2(0) = P_0 - \frac{C}{2} \cos(\Delta\Phi) \quad (6.22)$$

$$P_2(\pi) = P_0 - \frac{C}{2} \cos(\Delta\Phi + \pi) \quad (6.23)$$

Taking the half difference of these two values gives

$$P'_2 = \frac{P_2(0) - P_2(\pi)}{2} = -\frac{C}{2} \cos(\Delta\Phi) \quad (6.24)$$

This removes the probability offset P_0 to the extent that it remains constant between the two measurements. Using a short interferometer of $T=0.5\text{ms}$, we adjust the phase of the second Raman pulse such that the fringe is at its steepest point, where $\Delta\Phi = \pi/2$. Due to the relatively small interferometer time T , the vibration phase is small and therefore the interferometer stays close to this point.

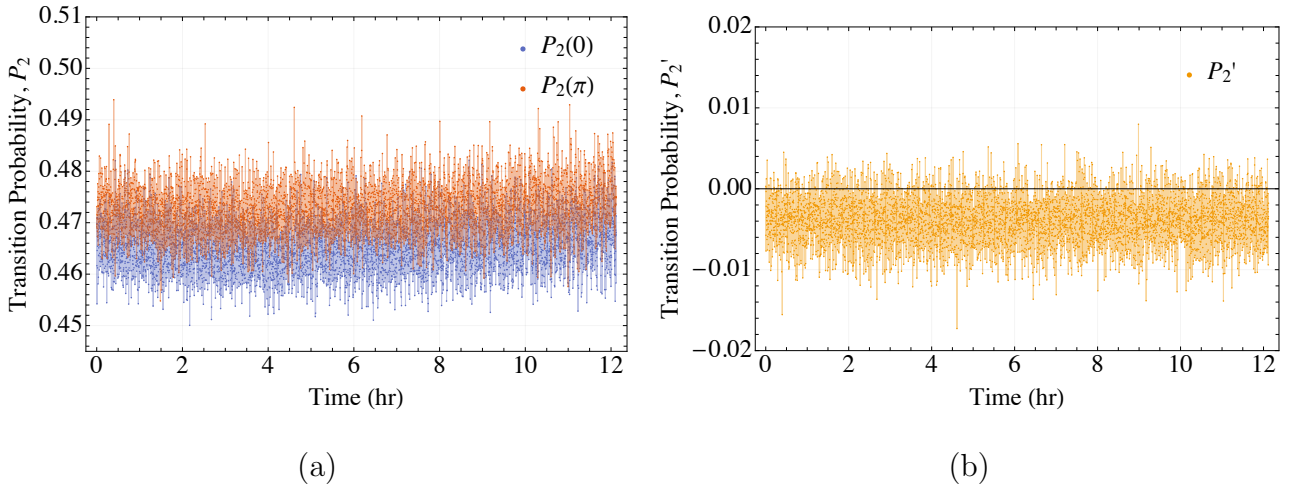


Figure 6.15: (a) Measured transition probability values $P_2(0)$ and $P_2(\pi)$ (see text) for a $T=0.5\text{ms}$ interferometer near the maximum fringe slope. (b) Transition probability P'_2 , defined as half the difference of $P_2(0)$ and $P_2(\pi)$.

Figure 6.15a shows the $P_2(0)$ and $P_2(\pi)$ measurements as a function of time. A very slow drift can be seen over similar a timescale to that of the drift of P_0 seen in Figure 6.13b. However the magnitude of the drift in P_0 is slightly smaller here. The standard deviations of $P_2(0)$ and $P_2(\pi)$ are 4.5×10^{-3} and 4.5×10^{-3} , respectively. Figure 6.15b shows the π flipped P'_2 as a function of time which has a standard deviation of 2.8×10^{-3} . Multiplying this by $\sqrt{2}$ gives the equivalent noise in the transition probability, P_2 of 4.0×10^{-3} . This is only slightly lower than the noise found for $P_2(0)$ and $P_2(\pi)$ because the drift is small compared with the noise. To further assess the stability of the interferometer, Figure 6.16 shows the Allan deviation as a function of the averaging time for the three quantities $P_2(0)$, $P_2(\pi)$ and P'_2 plotted in Figure 6.15.

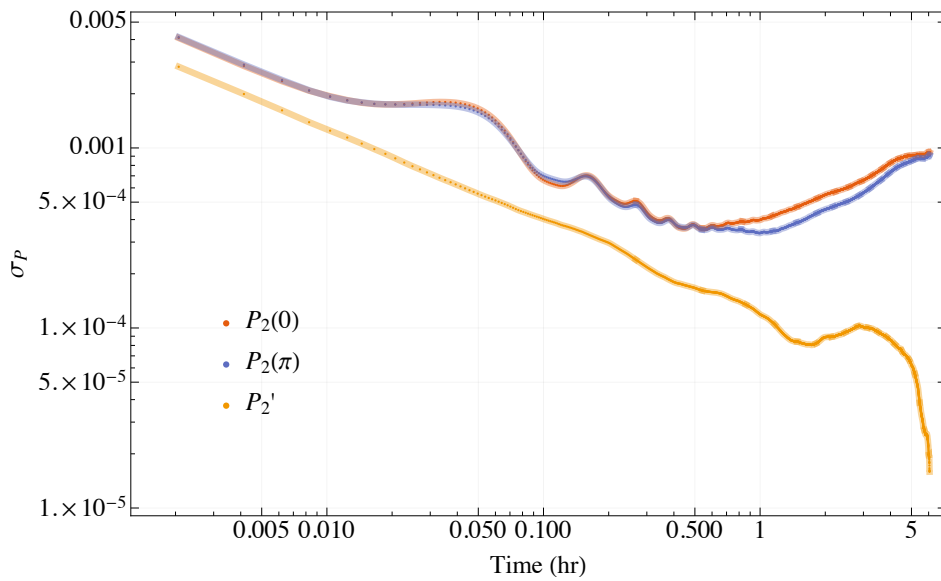


Figure 6.16: Allan deviation for a $T=0.5\text{ms}$ interferometer for measurements sitting at the fringe slope separated by π radians, $P_2(0)$ and $P_2(\pi)$, and the “ π -flipped” measurement P'_2 .

The Allan deviation curves are very similar for $P_2(0)$ and $P_2(\pi)$ as one would expect, however they both exhibit bumps that are characteristic of an oscillation in P_2 . Moreover, the curves turn upwards between 0.5 to 1 hour. By contrast, the Allan deviation for P'_2 , which sits lower by a factor of $1/\sqrt{2}$ as expected, has much less oscillation and does not turn up. We conclude therefore that π flipping does not reduce the short-term noise, but is effective in eliminating the drift in P_0 , improving the long term stability of the measured transition probability.

6.8 Drift of the Interferometer Phase

From Equation 6.24 we see that fluctuations, $\delta(\Delta\Phi)$, of the interferometer phase produce fluctuations of P'_2 , whose Allan deviation is shown by the orange curve in Figure 6.16. Since these fluctuations are small and the interferometer is operating on the steepest part of the fringe, $\delta(\Delta\Phi)$ is simply equal to $\delta(2P'_2/C)$. In order to learn more about the origin of these interferometer phase fluctuations the experiment was repeated, but this time with reversal of $\mathbf{k}_{\text{eff}}$ as well as π -flipping.

The green curve in Figure 6.17 shows the Allan deviation of $(P'_2(+\mathbf{k}_{\text{eff}}) - P'_2(-\mathbf{k}_{\text{eff}}))/C$, which is the part of the interferometer phase associated with acceleration and rotation. The noise here averages down steadily as $t^{-1/2}$ over 10 hours because we have chosen the very short interferometer time of $T=0.5\text{ms}$ and because the real motion is very steady, being just the rotation of the earth. By contrast, when P'_2 is averaged over both directions of $\mathbf{k}_{\text{eff}}$, the quantity $(P'_2(+\mathbf{k}_{\text{eff}}) + P'_2(-\mathbf{k}_{\text{eff}}))/C$ picks out the part of the interferometer phase that is not due to motion but instead intrinsic to the interferometer. The fluctuations of that are shown by the orange curve in Figure 6.17 and these are qualitatively similar to the orange curve in Figure 6.16.

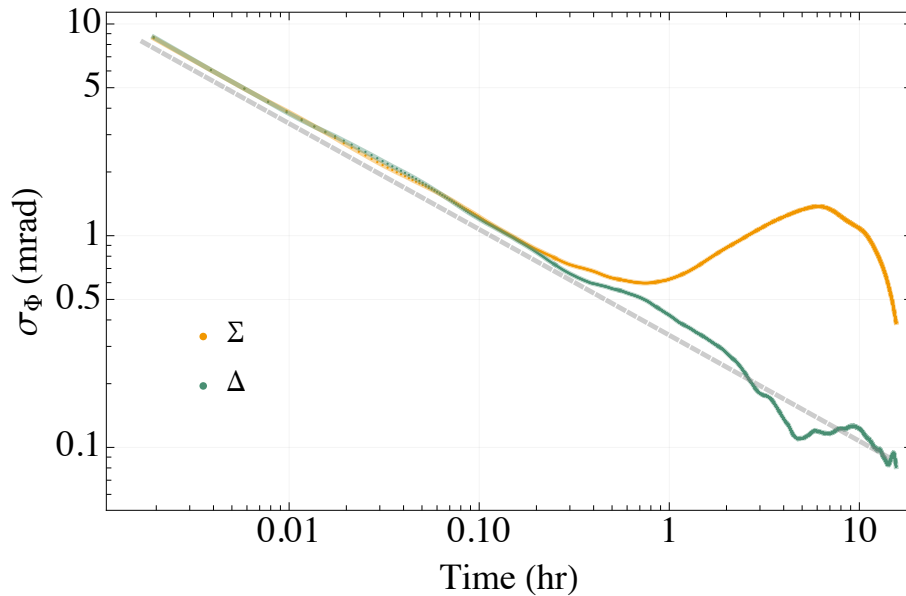


Figure 6.17: The Allan deviation for the half sum (yellow) and half difference (green) for the measured phases for the π -flipped $\pm\mathbf{k}_{\text{eff}}$ interferometers with $T=0.5\text{ms}$.

We see that over a few hours the interferometer phase has an intrinsic instability at the level

of a few milliradians. It seems likely therefore that there is an intrinsic interferometer phase offset that is not perfectly stable. In the next section we look for such a phase offset, which we call Φ_0 .

6.9 Measuring the Interferometer Bias

Equation 3.15 shows that the interferometer phase due to acceleration and rotation of the apparatus relative to the freely-falling atoms depends on T . In our case, the significant terms in this phase are proportional to T^2 , so let us write them as θT^2 . A phase can be added to this by means of a phase jump of η on the π pulse, as shown in Equation 6.1. Here, we investigate the possibility of a further phase offset, Φ_0 , which would seem to indicate acceleration or rotation even in the absence of motion. Such a zero-error is known in the field of accelerometry as a bias. Whereas real acceleration and rotation contribute to $\Delta\Phi$ in proportion to T^2 , a bias phase is constant, and this provides a way to identify it.

A fourth contribution to the interferometer phase can be made by chirping the detuning of the Raman beat note from resonance, defined as δ^- in Equation 2.5. Let us write the beat note frequency as $\omega_R(t) = \omega_R(0) + \omega'_R t$, where ω'_R is the chirp rate, then its phase is given by

$$\phi_R(t) = \int \omega_R(t) dt = \omega_R(0) t + \frac{1}{2} \omega'_R t^2. \quad (6.25)$$

From Equation 3.6, we know that the contribution to interferometer phase $\Delta\Phi$ from the phase of the beat note is

$$\Phi_R = \phi_R(0) - 2\phi_R(T) + \phi_R(2T) \quad (6.26)$$

where T is the time between interferometer pulses. Substituting Equation 6.25 into Equation 6.26, the part of the interferometer phase due to the chirp of the beat note frequency is $\omega'_R T^2$. Let us allow for the possibility that there is also a chirp in the hyperfine transition frequency,

ω_{hfs} . Then, taking both chirps together, the detuning contributes an interferometer phase of

$$\Phi_{\text{chirp}} = \omega'_R T^2 - \omega'_{\text{hfs}} T^2 \quad (6.27)$$

Collecting these phases together, the total interferometer phase is now given by

$$\Delta\Phi = (\theta + \omega'_R - \omega'_{\text{hfs}}) T^2 + \Phi_0 - 2\eta \quad (6.28)$$

The dots in Figure 6.18 show the transition probability P_2 , measured on axis 2, as a function of chirp rate ω'_R with $T = 4.00$ ms and $\eta = 0$. The frequency, ω_R , is set to be on resonance immediately after the atoms are launched. In addition to sweeping out the fringe pattern, we see that the scan of ω'_R modulates the fringe amplitude. With the perfect chirp, the Raman beat note remains on resonance as the atoms accelerate, but with a faster or slower chirp the Doppler shift moves the light away from resonance and the fringe height is reduced. In order to fit these points we adopt the empirical function

$$P_2 = \left[P_0 - \frac{C}{2} \cos(\phi_0 + \omega'_R T^2) \right] \exp - \frac{(\omega'_R - \omega'_0)^2}{2\sigma^2} + B. \quad (6.29)$$

Here, the expression in square brackets represents the fringe in the usual way, with ϕ_0 standing for $(\theta - \omega'_{\text{hfs}}) T^2 + \Phi_0$, in accordance with Equation 6.28. The Gaussian function represents the effect of detuning from resonance on either side of the ideal chirp ω'_0 . The solid line in Figure 6.18 shows this function after fitting to the data with P_0 , C , ϕ_0 , ω'_0 , σ and B as free parameters.

We see that this empirical fit describes the data well.

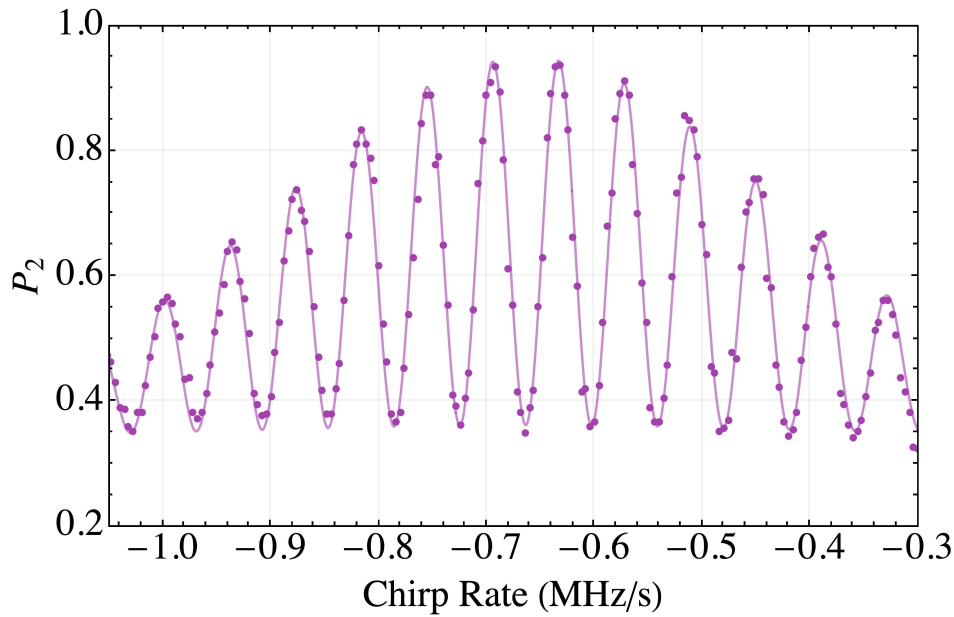


Figure 6.18: Transition probability P_2 as a function of chirp rate $\omega'_R/(2\pi)$ of the beat note. Dots: Measurements taken on Raman axis 2 with $+\mathbf{k}_{\text{eff}}$ and $T = 4\text{ms}$. Line: Fit using the empirical Equation 6.29.

In order to identify the bias on axis 2, Φ_0 , we record and fit the fringe in this way for six values of T ranging from 1 ms to 5 ms keeping η equal to zero. Figure 6.19 shows the interferometer phase, $\Delta\Phi = \phi_0 + \omega'_R T^2$, for each fit as a function of beat note chirp rate ω'_R . We see that the phases all converge at a “magic” chirp rate where the coefficient of T^2 in Equation 6.28 is zero, and $\Delta\Phi$ is equal to the bias, Φ_0 . The magic chirp rate is determined by finding the ω'_R value for which the standard deviation of the six $\Delta\Phi$ values is minimised. We find that the magic chirp rate for this interferometer is $-679.5 \text{ kHz/s} \pm 0.6 \text{ kHz/s}$, and that the bias is $44 \text{ mrad} \pm 21 \text{ mrad}$. On repeating the measurements with $\mathbf{k}_{\text{eff}}$ reversed, the bias is found to be $58 \text{ mrad} \pm 12 \text{ mrad}$. Taking the weighted mean of these two measurements, we conclude that the interferometer on axis 2 has a bias of $55 \text{ mrad} \pm 10 \text{ mrad}$. The cause of this bias is not known, but it does not seem surprising that a bias of this magnitude would drift by a few mrad over 10 hours, as found in Section 6.8.

According to Equation 6.28 the coefficient of T^2 vanishes when the chirp rate equals $(\omega'_{\text{hfs}} - \theta)$, and we know from Table 3.1 that θ reverses sign with the reversal of $\mathbf{k}_{\text{eff}}$. It is therefore instructive to compare the magic chirp rates in these two experiments. When $\mathbf{k}_{\text{eff}}$ was reversed, the magic chirp rate changed from $-679.5 \text{ kHz s}^{-1} \pm 0.6 \text{ kHz s}^{-1}$ to $+683.9 \text{ kHz s}^{-1} \pm 0.4 \text{ kHz s}^{-1}$.

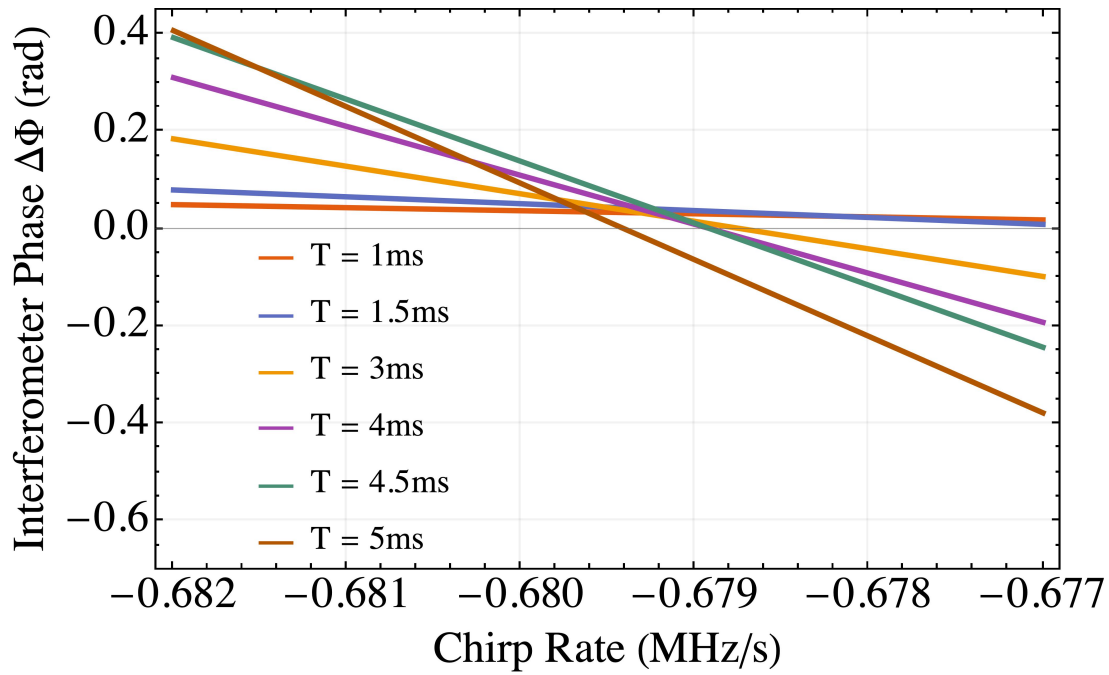


Figure 6.19: The total phase $\Delta\Phi$ as a function of chirp rate, $\omega'_R/2\pi$ of the beat note frequency for interferometer times T between 1ms and 5ms.

Half the difference of these gives $\theta = 682 \text{ kHz s}^{-1}$. We can understand this as the result of a gravitational acceleration, $a_2 = 0.266 \text{ m s}^{-2}$, along axis 2 (using $\theta = \mathbf{k}_{\text{eff}} a_2$). Such an acceleration would arise if axis 2 were tilted away from the horizontal by 1.6° , which is entirely possible.

Half the sum of the magic chirp rates gives a clock frequency chirp rate of $\omega'_{\text{hfs}}/(2\pi) = 2.2 \pm 0.4 \text{ kHz s}^{-1}$. At the time of the Raman pulses the vertical velocity is -23 cm s^{-1} , so this corresponds to a spatial gradient in the clock frequency of 96 Hz cm^{-1} . We investigate the cause of this gradient in the next chapter.

We turn now to Raman axis 1. Using the same method, the bias was found to be $57 \text{ mrad} \pm 68 \text{ mrad}$ with $+\mathbf{k}_{\text{eff}}$ and $77 \text{ mrad} \pm 48 \text{ mrad}$ with $\mathbf{k}_{\text{eff}}$ reversed, giving a weighted mean of $70 \text{ mrad} \pm 40 \text{ mrad}$. The uncertainty in the bias here is much larger than on axis 2 (because the visibility of the fringes is lower, as illustrated in Figure 6.2) but within the uncertainty, the two interferometers have similar bias.

The two magic chirp rates on axis 1 were $(1535.3 \pm 1.6) \text{ kHz s}^{-1}$ and $(-1488.6 \pm 1.1) \text{ kHz s}^{-1}$. The half difference corresponds to a tilt of axis 1 at 3.4° from the vertical. Although this is quite large, it is not implausible as no particular effort was ever made to level the axes. The half sum

corresponds to a chirp in the clock transition frequency of $23 \pm 1 \text{ kHz s}^{-1}$, which is ten times larger than we found on axis 2. In the next chapter we see that this much larger perturbation of the clock frequency is the reason that the fringes lose contrast more readily on axis 1 than on axis 2.

Chapter 7

Investigating the Inhomogeneity of the Clock Transition Frequency along Raman Axis 1

7.1 Measuring the Frequency Shift of the Clock Transition across the Raman Beam

At the end of Chapter 6, we found that the clock frequency has a chirp of 2.2kHz s^{-1} on axis 2 and 23kHz s^{-1} on axis 1. In this chapter, we identify the cause of these chirps and correct them. We start by making a direct measurement of the clock frequency as a function of the vertical position of the atoms.

The atoms are prepared in the $F = 1, m_F = 0$ sublevel by applying step 1 and 2 of the state preparation, then launched vertically with a velocity of 45cm s^{-1} . After a chosen time of flight, a $95\mu\text{s}$ -long Raman π pulse is applied with a beat note frequency close to the Doppler-free resonance frequency for the co-propagating beam pair. I then measure the fraction of atoms transferred to the upper $F = 2, m_F = 0$ sublevel as a function of this frequency in order to obtain a resonance curve. Figure 7.1a shows the resonance curve measured on Raman axis 1

92ms after launch, which returns the atoms to the initial launch position. We see that the resonance is shifted to approximately 7kHz below the unperturbed clock frequency, indicated by the 0kHz in the graph. This measurement is repeated at a variety of vertical positions by changing the flight time. The red points in Figure 7.1b show the variation of the clock frequency shift on axis 1 as a function of the position of the atoms. We see at once that this does not seem to be a light shift, because the Raman beam has a width (intensity FWHM) of 49mm, whereas this shift halves at +6mm and -18mm, corresponding to a FWHM of only 24mm. When the experiment is repeated on axis 2 we obtain the blue points shown in Figure 7.1b, which exhibit a much smaller variation of the shift than we see on axis 1. This is consistent with the much smaller chirp that was measured in Chapter 6.

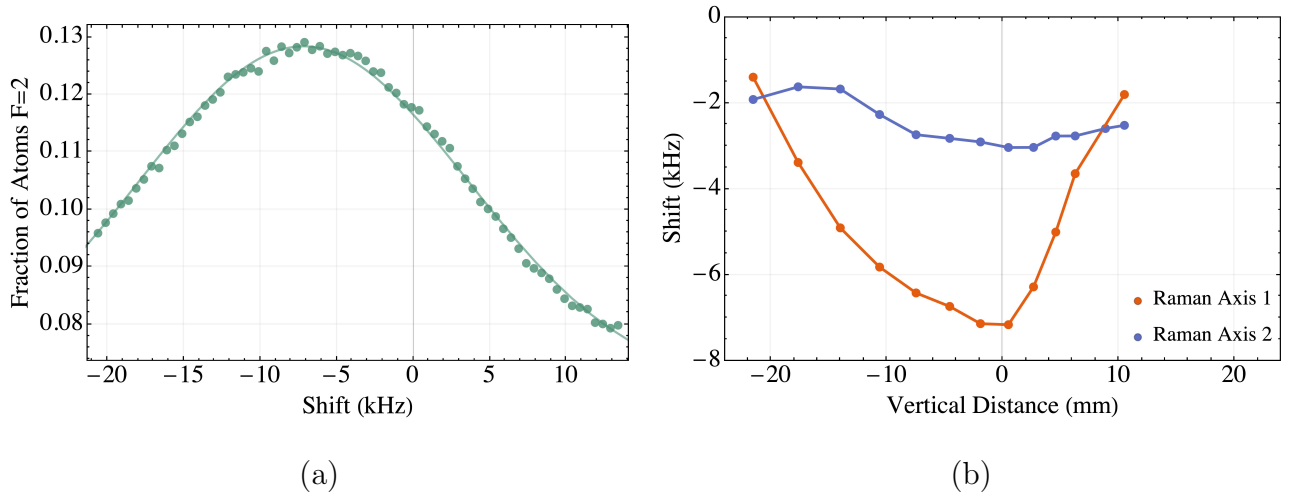


Figure 7.1: (a) Fraction of atoms transferred to the $F = 2$ level by the co-propagating beam pair along Raman axis 1 as a function of the difference between the Raman beat note frequency and the unperturbed clock transition frequency. (b) Shift of the clock transition frequency as a function of height of the atom cloud relative to the initial launch height.

In order to make a more direct proof that the chirp is not due to the light shift, we now change the magnitude of the light shift. As discussed in Chapter 2, we expect no light shift at all when the two frequency components have an intensity ratio of 0.58. However, the data in Figure 7.1 were taken with ratios of 0.44 and 0.53 on Raman axes 1 and 2 respectively, as measured by the light scattered into the detection photodiodes. These ratios are not equal, even though the two Raman axes are supplied from the same laser system, presumably because of different coupling efficiencies for each frequency into the two separate Raman PM fibres. After increasing

the power of the slave pump laser from 7.1W to 8.55W, the new intensity ratios are 0.54 and 0.63 for Raman axes 1 and 2 respectively. The frequency shift is then remeasured on axis 1, with the results shown in yellow in Figure 7.2a, together with the red curve reproduced from Figure 7.1b. We see that the shift at 0mm is reduced by a factor of 7, yet the range of shifts is hardly altered. We conclude that this change of detuning has merely altered the (almost) constant light shift and that the main cause of variation in the shift (and hence the chirp) is unrelated to the light shift. On axis 2 we see that the light shift has changed sign, as expected when changing from an intensity ratio 0.05 below the ideal value to 0.05 above, but there is no qualitative change in the variation of the shift.

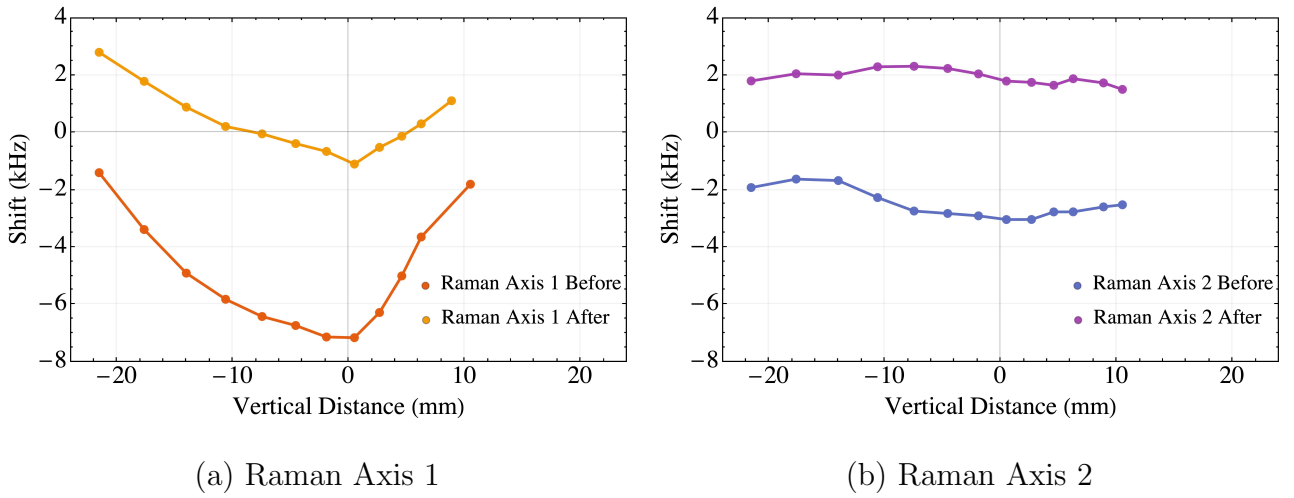


Figure 7.2: The measured frequency shift across the Raman axis 1 (a) and 2 (b) beams for vertical positions between +10mm and -20mm before (light) and after (dark) the intensity ratio adjustment.

We conclude that the inhomogeneity in the clock transition frequency, and hence the chirp ω'_{hfs} measured in the interferometer, is not due to the light shift.

7.2 Raman Spectra and Optimisation of the Retro-reflecting Mirror

After taking the narrow scans, such as Figure 7.1a, we looked at wider scans of the beat note frequency in order to see the Doppler shifted resonances on either side, driven by the counter-

propagating beams. The spectra taken on Raman axis 1 are plotted in Figure 7.3a, where we see that the peak heights depended strongly on the height of the atom cloud. At +10mm, the Doppler-free central peak is very small, while the Doppler-shifted peaks are not seen at all. This suggests that the atom cloud at the +10mm position sees only one Raman beam, which could be the case if the retro-reflection mirror on Raman axis 1 were misaligned. The same experiment on Raman axis 2 shows much less variation of the spectrum with height, as seen in Figure 7.3b.

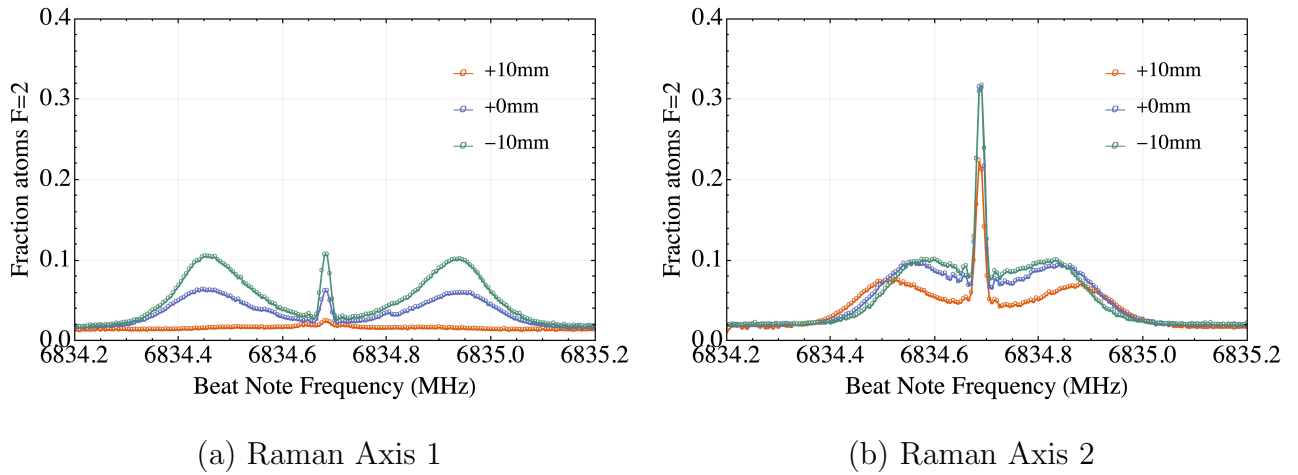


Figure 7.3: Raman spectra using the Raman axis 1 beam (a) and Raman axis 2 beam (b) when the vertical position of the atom cloud is +10mm (red), 0mm (blue) and -10mm (green).

We therefore adjusted the angle of the mirror on Raman axis 1 using the vertical piezo control. The mirror was adjusted in small increments whilst monitoring the Raman spectra at the +10mm, 0mm and -10mm vertical positions until the Doppler-shifted peaks at the heights ± 10 mm were approximately the same height. The final Raman spectra at the ± 10 mm vertical positions after this mirror adjustment are shown in Figure 7.4, together with the spectra shown in Figure 7.3a. One sees that tipping the retro-reflecting mirror has solved the problem.

After making this improvement in the alignment of the retro-reflecting mirror, we remeasured the inhomogeneity of the clock transition frequency and found no significant improvement in that.

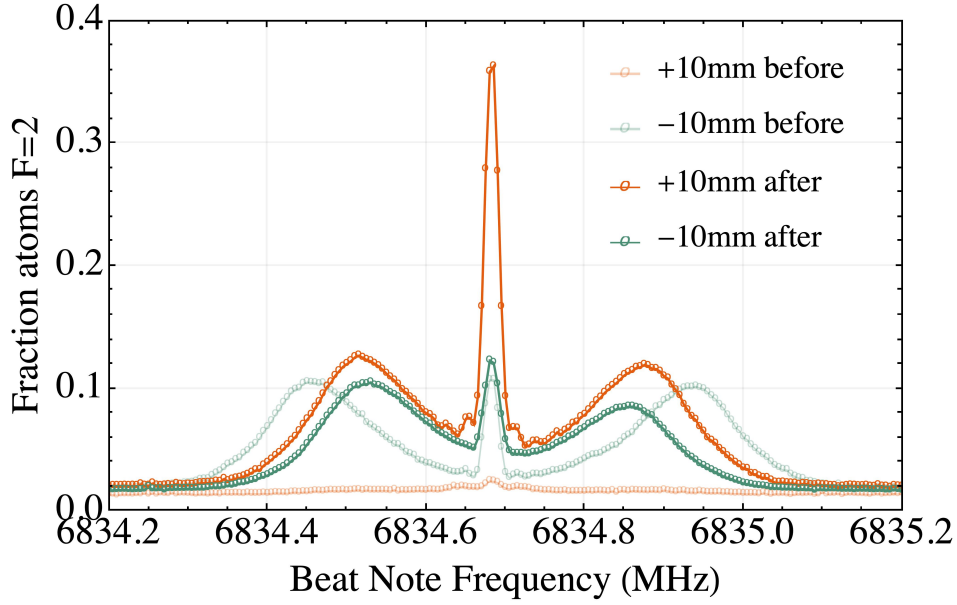


Figure 7.4: Raman spectra using Raman axis 1 at the ± 10 mm vertical positions before (light) and after (dark) the retro-reflecting mirror optimisation

7.3 Magnetic Field Gradient

Having found that the chirp of the clock frequency is not due to the light shift, we turn now to the possibility of a time-dependent Zeeman shift. To lowest order in the magnitude, B , of the magnetic field, this Zeeman shift is given by

$$f = \alpha B^2 \quad (7.1)$$

with $\alpha = 0.575$ kHz/G² (Table 6 in [62]). The chirp of the clock frequency resulting from the magnetic field is therefore

$$\omega'_{\text{hfs}} = 2\pi \alpha \frac{d}{dt} B^2 \quad (7.2)$$

In order to measure B directly, the atoms are prepared across the three m_F sublevels in $F = 1$ by applying step 1 of the state preparation scheme. The bias magnetic field is then switched from being vertical to pointing along the Raman axis 7ms after launch. After a further delay time, we then apply a microwave pulse of $150\mu\text{s}$ and measure the fraction of atoms transferred to the $F = 2$ level as a function of the microwave pulse frequency. The frequency difference

between the $m_F = \pm 1 \leftrightarrow m_F = 0$ peaks in the spectrum yields a value for $B(t)$ with the help of the Breit-Rabi formula (Equation 24 in [62]). The Zeeman shift is then calculated using Equation 7.1. The measurement is repeated for a variety of delay times and with the field configurations for both axes 1 and 2. Figure 7.5 shows the Zeeman shifts measured in this way as a function of the delay.

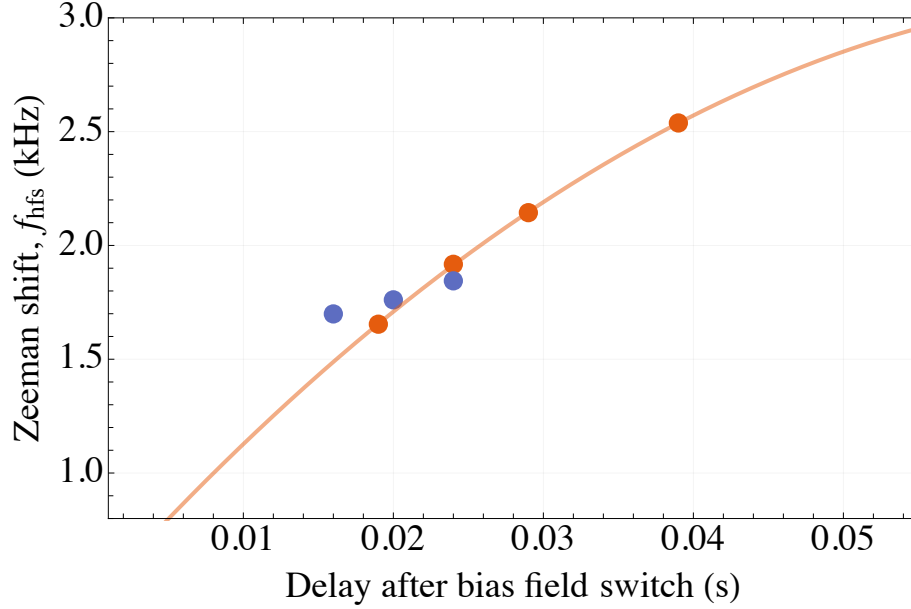


Figure 7.5: Second order Zeeman shift as a function of delay after switching. Red: Zeeman shift for axis 1. Blue: Zeeman shift for axis 2. Solid line is the quadratic least-squares fits to the axis 1 points.

In Chapter 6, the interferometry measurements on axis 1 gave a clock frequency chirp of 23 kHz s^{-1} at a flight time after launch of 51ms. Using the quadratic fit we extrapolate the data along axis 1 out to 44ms after switching the bias field (equivalent to 51ms after launch), and find its gradient at this point, to calculate a clock frequency chirp, is 25.4 kHz s^{-1} . The agreement of these two values provides strong evidence that the chirp measured in Chapter 6 is indeed due to a time-dependent Zeeman shift. On axis 2, there are not enough data points in Figure 7.5 to evaluate the chirp at 44ms, but it is clearly much smaller than on axis 1.

The derivative of the Zeeman shift may come either from a time-dependence of the magnetic field, or from motion of the atoms through a gradient of the field. The two contributions are

$$\omega'_{\text{hfs}} = 2\pi \alpha \left(\frac{\partial(B^2)}{\partial t} + \mathbf{v} \cdot \nabla (B^2) \right) \quad (7.3)$$

The velocity of the atoms varies over the trajectory because of gravity, but at 51 ms, the second term in Equation 7.3 would require a field gradient of $\sim 1 \text{ G cm}^{-1}$ to produce a clock frequency chirp of 25 kHz s^{-1} . This is so enormous that we can reasonably assume the first term in Equation 7.3 is the relevant one and that there is some transient settling of the field that is still in progress 44ms after switching onto axis 1. In order to check this we repeat the measurements of $B(t)$ but without turning on any field after switching to molasses. The purple data and fit in Figure 7.6 show the result. For comparison, red data points shows the field variation that was measured when the bias field was switched onto axis 1. A straight line fitted to the red data has a slope of 20 G s^{-1} , whereas the purple line has a slope of 0.05 G s^{-1} .

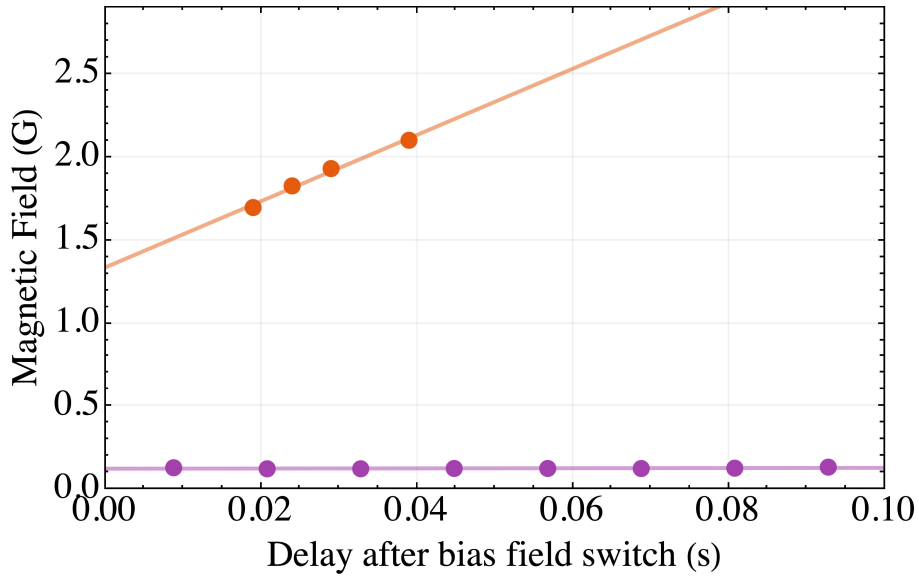


Figure 7.6: Magnetic field magnitude as a function of flight time. Red: $B(t)$ on axis 1 with interferometer fields switched on. Purple: $B(t)$ without switching on the interferometer fields.

This demonstrates that the chirp of the clock frequency is indeed a transient Zeeman shift associated with switching the interferometer fields on.

After reducing the size of the magnetic field along the Raman axis from $\sim 1.5 \text{ G}$ to $\sim 0.4 \text{ G}$ $B(t)$ was measured on both interferometer axes with the interferometer fields once again being switched on. The transients are shown in Figure 7.7, with the main field subtracted so that the very small variations can be seen. We find that the time-variation of the magnetic field in the interferometer has been reduced by at least a factor of 100 on both axes by reducing the magnitude of field produced by the bias coils.

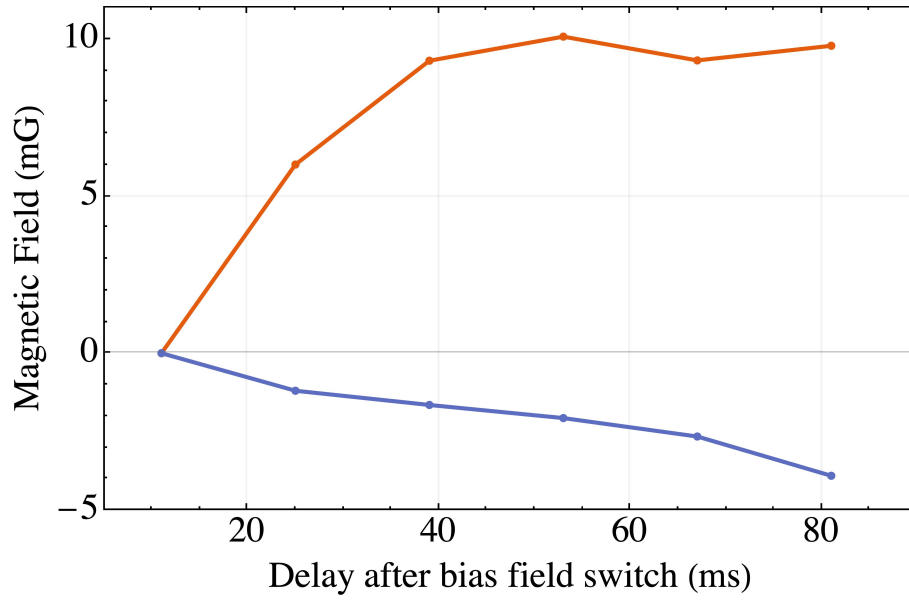


Figure 7.7: Residual field variation after fixing transient in interferometer fields. Red: axis 1. Blue: axis 2.

It appears the large gradient along axis 1 is caused by switching on a $\sim 1.5\text{G}$ bias field along that Raman axis which involves reversing the polarity of the current in one of the two horizontal bias coil pairs. After fixing the magnetic field transient, there was great improvement in the fringes on both axes, as shown in Figure 7.8. The pale points show data taken before fixing the transient, while the dark points show the fringes after fixing it. On axis 1 the contrast at $T=20\text{ms}$ increased from 1.2% to 22%, while on axis 2 at $T=25\text{ms}$ the contrast increased from 2.8% to 18%.

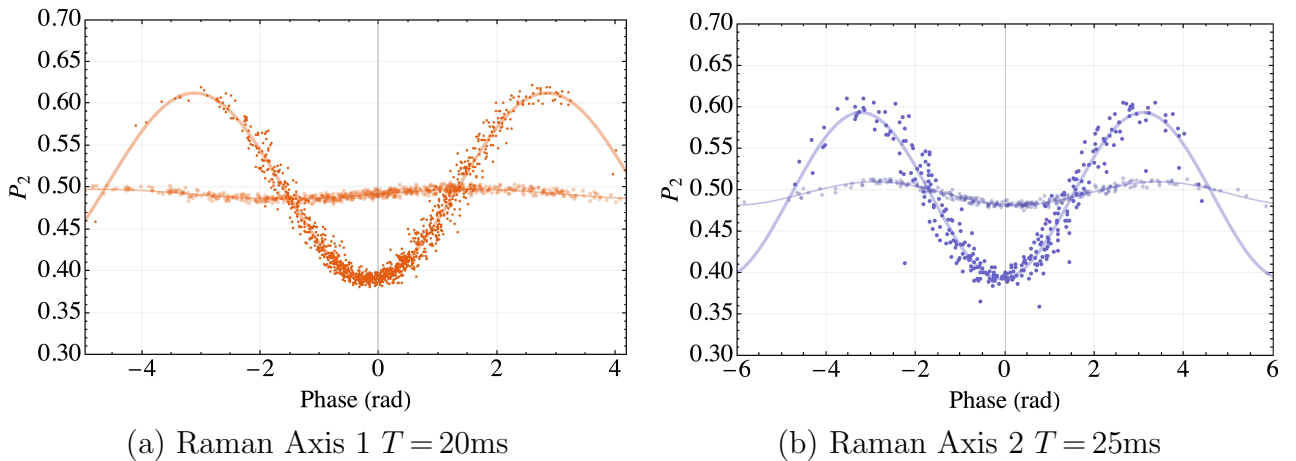


Figure 7.8: Interferometer fringes measured before (light points) and after (dark points) fixing the magnetic field transient. (a) Using Raman axis 1 with $T=20\text{ms}$. (b) Using Raman axis 2 with $T=25\text{ms}$.

We find that interferometer fringes with $T > 20\text{ms}$ can now be resolved along both Raman axes following the large reduction in the magnetic field transient.

After these measurements were made the apparatus underwent a general upgrade which involved rebuilding various components. Following this, the magnetic field transient was small both for 0.4 G bias and 1.5 G bias. We do not know the cause of the large transient, but it was fixed by the upgrade. Further investigation into this will be described by the next thesis on this experiment.

7.4 Interferometer's Sensitivity to Rotation

Now that we have reduced the magnetic field transient, we finally look to measure the interferometer's sensitivity to rotation. In Table 3.1 we saw that the leading terms in the interferometer phase are

$$\Delta\Phi = \mathbf{k}_{\text{eff}} \cdot (\mathbf{g} - \mathbf{a}) T^2 + 2\mathbf{k}_{\text{eff}} \cdot \mathbf{v}_A \times \boldsymbol{\Omega} T^2 \quad (7.4)$$

where $\mathbf{v}_A$ is the velocity of the atom at the time of the first interferometer pulse. Because the apparatus is mounted to a stationary optical bench, $\boldsymbol{\Omega}$ is the Earth's rotation. Using the XYZ coordinate system illustrated in Figure 3.2, $\boldsymbol{\Omega} = (0, \Omega_{E_y}, \Omega_{E_z})$. Raman axis 2 is aligned East-West, so $\mathbf{k}_{\text{eff}}$ on this axis lies along X ($+\mathbf{k}_{\text{eff}}$ pointing East and $-\mathbf{k}_{\text{eff}}$ pointing West). The atoms are launched both vertically and horizontally along the Raman axis, so $\mathbf{v}_A$ for axis 2 lies in the XZ plane of Figure 3.2. Therefore for $+\mathbf{k}_{\text{eff}}$

$$\Delta\Phi = \mathbf{k}_{\text{eff}} \cdot (\mathbf{g} - \mathbf{a}) T^2 - 2k_{\text{eff}} v_{z_A} \Omega_{E_y} T^2 + \Phi_0 \quad (7.5)$$

where v_{z_A} is the vertical component of $\mathbf{v}_A$ on axis 2 and we have included the interferometer bias, first introduced in Equation 6.28. Let us rewrite this phase shift in terms of a vertical launch velocity v_{l_z} , and the delay between launching the atoms and the first interferometer pulse, t_{delay}

$$\Delta\Phi = \mathbf{k}_{\text{eff}} \cdot (\mathbf{g} - \mathbf{a}) T^2 - 2k_{\text{eff}}(v_{l_z} - g t_{\text{delay}}) \Omega_{E_y} T^2 + \Phi_0 \quad (7.6)$$

The derivative with respect to t_{delay} suppresses the part due to acceleration:

$$\frac{d\Delta\Phi}{dt_{\text{delay}}} = 2k_{\text{eff}} g \Omega_{E_y} T^2 + \frac{d\Phi_0}{dt_{\text{delay}}} \quad (7.7)$$

We set out to measure this derivative on Raman axis 2 with $T = 25\text{ms}$ by measuring $\Delta\Phi$ at two delay times, t_{delay} , of 48ms and 54ms and with both wavevectors $\pm\mathbf{k}_{\text{eff}}$.

Due to a problem of instability of the laser phase lock, we were only able to perform 78 measurements at each particular delay time and wavevector. Figure 6.14 shows that the noise in the fit reduces with the number of data points, N_{dat} up to $N_{\text{dat}} = 1000$. Therefore we use all 78 data points in a single fit to get one phase value at each delay time with each wavevector.

With $+\mathbf{k}_{\text{eff}}$ the fits give interferometer phases of $92 \pm 26 \text{ mrad}$ at a delay of 48 ms and $47 \pm 28 \text{ mrad}$ at 54 ms. The corresponding slope is

$$2k_{\text{eff}} g \Omega_{E_y} T^2 + \frac{d\Phi_0}{dt_{\text{delay}}} = -7.54 \pm 6.4 \text{ rad s}^{-1} \quad (7.8)$$

With $-\mathbf{k}_{\text{eff}}$ the fits give interferometer phases of $280 \pm 40 \text{ mrad}$ at 48 ms and $82 \pm 29 \text{ mrad}$ at 54 ms, giving a slope of

$$-2k_{\text{eff}} g \Omega_{E_y} T^2 + \frac{d\Phi_0}{dt_{\text{delay}}} = -33 \pm 8 \text{ rad s}^{-1} \quad (7.9)$$

Taking half the sum of Equations 7.8 and 7.9 we see that the derivative of the bias with respect to the delay time is $-20 \pm 5 \text{ rad s}^{-1}$. That is something that will require further investigation by the next student. Taking half the difference in order to remove that offset, we find

$$2k_{\text{eff}} g \Omega_{E_y} T^2 = 12 \pm 5 \text{ rad s}^{-1} \quad (7.10)$$

or

$$\Omega_{E_y} = 61 \pm 25 \mu\text{rad s}^{-1} \quad (7.11)$$

This result is consistent with the known value of $\Omega_{E_y} = 46 \mu\text{rad}$ in London. However the $25 \mu\text{rad s}^{-1}$ noise in Equation 7.11 is roughly half the angular velocity Ω_{E_y} of the Earth in London. The rotation noise could presumably be reduced by increasing the separation between the two values of t_{delay} or, as with the acceleration measurements, by increasing the time T between interferometer pulses. For example by increasing the separation between the delay times to 30 ms, we expect the noise in the angular velocity to reduce to $\sim 11\%$ of the Earth's angular velocity in London at $5 \mu\text{rad s}^{-1}$.

This sensitivity is the result of having 78 points in the fit. If we want the noise per shot we need to multiply the fit uncertainty by $\sqrt{78}$, as outlined in Section 6.6. The four phase fits have an average uncertainty of 31 mrad which gives a phase noise per shot of 272 mrad. At $T=25\text{ms}$ this corresponds to an acceleration noise of $\sim 2.7 \mu\text{g}$ per shot, with each shot having a bandwidth of 20 Hz. This is similar to the result of Section 6.6, where we found an upper limit on the MEMS accelerometer noise of $5 \mu\text{g}$ in the larger bandwidth of 33 Hz.

Chapter 8

Conclusion

8.1 Summary

The aim of the wider ongoing project is to develop a 5-axis hybrid inertial navigation sensor (2-axis accelerometer and 3-axis rotation sensor) which benefits from the fast bandwidth and high sensitivity of classical sensors and the long term stability of cold atom based sensors.

We aim eventually to have an interferometer operating at $T=25\text{ms}$ with a phase noise $\sigma_{\Delta\Phi}$ better than 10 mrad at 1s. For such an instrument the corresponding acceleration noise is $\leq 100\text{ng}$ and, assuming a perpendicular launch velocity of 40 cm s^{-1} , this corresponds to an angular velocity noise of $1.2\mu\text{rad s}^{-1}$. The same level of acceleration noise results from transition probability noise σ_{P_2} on the steepest part of the fringe of 2×10^{-3} . Early stages of this work saw a one axis accelerometer developed and characterised [54, 53], followed by a two axis accelerometer system [1]. The first demonstration and characterisation of the interferometer fringes for the two axis system was described in [1], where an interferometer of $T=12.5\text{ms}$ had $C=30\%$ using 3×10^7 atoms. The dominant noise source was found to be from the MEMS accelerometer with an acceleration noise of $7.3\mu\text{g}$ per shot, followed by detection noise with an acceleration (probability) noise of $3.5\mu\text{g}$ (1.3×10^{-2}) per shot.

The focus of my PhD has been to further develop this two axis system by improving the

interferometer noise and contrast, characterising the bias of the interferometers and starting to use it to measure rotations.

In Chapter 3 we derived an expression for the phase of the interferometer in a vehicle moving near the surface of the rotating earth and discussed how the phase shift of the interferometer is measured. We found in Equation 3.15 that the total interferometer phase is well approximated by:

$$\Delta\Phi = -\mathbf{k}_{\text{eff}} \cdot (\mathbf{a} - \mathbf{g}) T^2 + 2\mathbf{k}_{\text{eff}} \cdot \mathbf{v}_A \times \boldsymbol{\Omega} T^2 - 2\mathbf{k}_{\text{eff}} \cdot (\mathbf{a} - \mathbf{g}) \times \boldsymbol{\Omega} T^3 \quad (8.1)$$

To my knowledge this is the first time that the interferometer phase has been written in this compact vector form as well as for this reference frame.

The experimental apparatus, including the *M-Squared* laser system is described in Chapter 4. This includes a new dedicated detection beam which I built to provide detection light beams having the polarisation required to suppress unwanted optical pumping described in [1].

The full sequence of state preparation steps to produce the final ensemble of atoms ready for the three pulse interferometer is outlined in Chapter 5. Rubidium-87 atoms collected in a 2D MOT side-arm were pushed over 200ms into a 3D MOT, formed by three orthogonal pairs of cooling beams which overlap at the centre of the chamber. In this way we were able to load over 10^8 atoms in 100ms cooled to around $300\mu\text{K}$. Next, the 3D MOT quadrupole magnetic field was rapidly switched off so that the temperature of the atoms could be further reduced through the mechanism known as polarisation gradient cooling. Optimisation of the frequency and intensity ramps of the cooling light resulted in an atom cloud with a vertical temperature of $9\mu\text{K}$ and horizontal temperature of $6\mu\text{K}$. Only about 20% of the atoms captured in the MOT were retained in the final ultra-cold sample. This is not particularly high and should be optimised by the next student.

During the optical molasses stage the atoms were also launched along the three counter-propagating beam axes by cooling the atoms to rest in a moving frame determined by the relative detuning of each counter-propagating laser beam. Three different horizontal launching configurations atoms were demonstrated which were selected depending on the desired opera-

tion of the interferometer: to measure acceleration along axis 1 or 2, or to measure rotation about the vertical axis. The atoms were also launched with a vertical velocity to ensure they remained close to the optic axis of the Raman beam during the interferometer stage, in order to limit the variation of Rabi frequencies experienced by the atoms. The stability of the launch velocity was measured, with an upper limit of 0.2% found for the vertical launch velocity and 0.6% for horizontal launch velocities. With the Raman axis oriented East-West, the uncertainty in vertical launch velocity combines with the Earth's rotation to produce an interferometer phase uncertainty of less than 1mrad for a $T = 25\text{ms}$ interferometer. However if the instability of the horizontal launch velocity is as large as the 0.6% upper limit, it is not good enough to achieve the desired sensitivity of $1.2\mu\text{rad s}^{-1}$ when operating the interferometer as a gyroscope. Future work should focus on measuring the horizontal launch velocity stability with a more accurate method (such as Raman spectroscopy). It may be necessary to investigate ways in which the stability of the launch velocity can be improved, for example employing a beam intensity stabilisation system.

Following the cooling stage, a series of optical and microwave pulses were employed to prepare an atomic ensemble of $\sim 10^7$ atoms with a narrow Doppler width of 24kHz in the $F = 1, m_F = 0$ state ready for interferometry. The shot-to-shot stability of this final number of atoms was found to be 6×10^{-4} , well below the maximum desired probability noise of 2×10^{-3} at 1s. The Allan deviation decreased as $t^{-1/2}$ out to 500s, showing that the efficiency of the state preparation was stable over long times - an important factor for achieving stability of the interferometer.

Chapter 6 describes the investigation and characterisation of the various noise sources in the interferometer. First looking at the detection noise we found the the frequency of the detection light was fluctuating by $\sim 200\text{kHz}$ with an oscillation period of around $700\mu\text{s}$. The large detection noise of 1.2×10^{-2} reported in [1] was successfully reduced to 3×10^{-3} by increasing the detection period to $600\mu\text{s}$. Next we looked at the noise in a $\pi/2 - \pi/2$ Ramsey interferometer using the co-propagating beam pair. Once the number of atoms was large enough, the noise was dominated by detection noise and by noise in the phase of the Raman beat note. The peaks and troughs of the Ramsey fringe are insensitive to the phase noise, whereas the steep sides of the fringe are sensitive to both noise sources. By measuring the noise across all parts of the

fringe, we determined that the shot-to-shot standard deviation of the Raman beat note phase was 7 mrad. Finally, the total noise in the three pulse interferometer was then analysed. We concluded the dominant noise source is the MEMS accelerometer at approximately $5\mu\text{g}$ in the bandwidth 0-30 Hz, which is quite a bit better than the manufacturer's specification of $< 7\mu\text{g}$ in the 0-10 Hz band. This is also less than the $7.3\mu\text{g}$ measured in [1] due to the use of a new printed circuit board for the readout electronics.

The second half of Chapter 6 describes how the interferometer was improved further by implementing k-reversal and π -flipping techniques. The k-reversal technique, which involves taking the phase half difference using the $\pm k_{\text{eff}}$ interferometers, allows undesirable systematic phase shifts that are independent of the wavevector (for example from light shift or magnetic field gradient) to be eliminated. A slow systematic drift in the phase offset of around 0.4 rad over 15 hours was observed in both signs of k_{eff} , likely due to bias drift of the MEMS accelerometer or readout electronics. Drift in the probability offset P_0 was also observed for both signs of k_{eff} . The interferometer phase noise per shot was found to be dominated by the MEMS accelerometer at $5\mu\text{g}$. Finally, by shifting the phase of the middle interferometer pulse by $\pi/2$ radians every other shot and taking the half difference of the two P_2 values, we could remove the dependence on the probability offset P_0 . We saw that this π -flipping technique does not reduce the short term noise but does improve the long term stability.

Using both k-reversal and π -flipping, we found that the noise of the interferometer associated with acceleration and rotation averaged down steadily as $t^{-1/2}$ over more than 10 hours. Similarly we saw that the component of the interferometer phase that is intrinsic to the interferometer, i.e. is not due to motion, varied over 10 hours by a few milliradians. This intrinsic phase bias was measured to be 70 ± 40 mrad along axis 1 and 55 ± 10 mrad along axis 2. Within the uncertainty both interferometers have a similar bias. The cause of this bias is not known however it is not surprising that bias of this magnitude would drift by a few mrad over 10 hours.

In [1], and at the start of my project, the contrast of the fringes at long T was much worse on Raman axis 1 than on axis 2. Measurement of the phase bias also revealed a large difference in

the chirp of the clock frequency between the two axes with 23kHz s^{-1} on axis 1 and 2.2kHz s^{-1} on axis 2. Investigation of the source of this concluded that it was due to a change in the magnetic field over time of 20G s^{-1} when the 1.5G bias field along Raman axis 2 was switched off. The source of this large derivative was not identified, however it was reduced by a factor of 100 by reducing the magnitude of the bias field. This produced great improvements in the contrast of both interferometers. On axis 1 the contrast at $T=20\text{ms}$ increased from 1.2% to 22%, while on axis 2 at $T=25\text{ms}$ the contrast increased from 2.8% to 18%. For the first time we were able to resolve interferometer fringes with $T > 20\text{ms}$ along both Raman axes.

Finally we measured the interferometer's sensitivity to rotation using the derivative $\frac{d\Delta\Phi}{dt_{\text{delay}}}$ on Raman axis 2 to determine the angular velocity component Ω_{E_y} of the Earth. The uncertainty in this measurement was $25\mu\text{rad s}^{-1}$, roughly half Ω_{E_y} in London. However this measurement was not taken with optimal conditions to best demonstrate the interferometer's sensitivity to rotation. I estimate this result could very easily be improved to achieve a sensitivity of $\sim 5\mu\text{rad s}^{-1}$ by increasing the separation between the two delay times from 6ms to 30ms.

8.2 Outlook

A number of major improvements have been made to the interferometer as a result of my PhD research. However, there are still several areas in which improvements are suggested to increase the sensitivity of the interferometer as both an accelerometer and rotation sensor. The first would be to accurately determine the stability of the horizontal launch when operating the interferometer as a rotation sensor. I suggest first to measure the launch stability using a more accurate method (such as Raman spectroscopy) before then (if necessary) investigating ways in which the stability could be improved, for example employing a beam intensity stabilisation system or replacing the voltage controlled oscillators (VCO) with more stable and accurate direct digital synthesisers (DDS). The second area is further improving the detection noise, which could be achieved through removing the frequency fluctuations within the detection light or employing a beam intensity stabilisation system (as already suggested to improve the

launch velocity stability). And finally the cycling rate of the experiments is currently limited to 1 s^{-1} by dead time in the software. Improvement to minimise this dead time could reasonably achieve cycling rates of up to 10 s^{-1} .

In addition to these suggested improvements to increase the system's sensitivity, an area which requires further investigation and characterisation is the phase bias of the two interferometers. This should include understanding the source of this bias and eliminating (if possible) its dependence on position or delay time within the interferometer. Finally, for this system to measure any rotations (beyond the Earth's rotation) the system must be moved onto a platform which can rotate.

While there is still room for the interferometer to be further improved, my work here has demonstrated that it is already clearly superior to the MEMS accelerometer. The noise in the Raman beat note phase as determined in Section 6.4 is $\sigma_\phi=17\text{mrad}$ per shot. For an interferometer having $T=25\text{ms}$ this corresponds to $0.17\mu\text{g}$ per shot. The detection noise, as determined in Section 6.3, is $\sigma_P = 3 \times 10^{-3}$ per shot and quantum projection noise, as determined in Section 6.5, is $\sigma_P = 1.9 \times 10^{-4}$ per shot. For an interferometer having $T=25\text{ms}$ and contrast $C = 20\%$ these correspond respectively to $0.3\mu\text{g}$ and $0.02\mu\text{g}$ per shot. Adding these three noise values in quadrature gives $0.35\mu\text{g}$ per shot. This is to be compared with the MEMS accelerometer noise of $2.7\mu\text{g}$ per shot, as measured in Section 7.4. Assuming 10 shots per second in future, this is close to the design goal of 100ng at 1s .

Moreover, the key advantage of the interferometer as an accelerometer and rotation sensor is its long term stability. The MEMS accelerometer drift in Section 6.6 was $11\mu\text{g}$ over 15 hours (i.e. 400mrad with $T=15\text{ms}$). By contrast the interferometer bias was found in Section 6.9 to be only a few tens of milliradians, with a drift found in Section 6.8 of $\sim 1\text{mrad}$ over 10 hours. For an interferometer having $T=25\text{ms}$ this bias drift is only 10ng , which is a thousand times better than the drift of the MEMS accelerometer.

Appendix A

Phase Shift Terms

The 20 most significant terms for each configuration are listed in Table [A.1](#), [A.2](#) and [A.3](#). These results assume an acceleration of 1ms^{-2} along the Raman axis when it is aligned with the x and y axes. All phase shift terms are given in rad.

Table 3.1 Term Number	Table 3.1 Phase Term	Phase Shift ($a_x = 1\text{ms}^{-2}$)
1	$-k_x a_x T^2$	-10 069.2
2	$2k_x v_{y_A} \omega_z T^2$	8.06
2	$-2k_x v_{z_A} \omega_y T^2$	-8.06
4	$-2k_x g_z \omega_y T^3$	+4.94
4	$-2k_x a_y \omega_z T^3$	-5.03×10^{-1}
2	$2k_x v_{y_A} \Omega_{E_z} T^2$	4.56×10^{-1}
2	$-2k_x v_{z_A} \Omega_{E_y} T^2$	-3.70×10^{-1}
4	$-2k_x g_z \Omega_{E_y} T^3$	2.26×10^{-1}
4	$-2k_x a_y \Omega_{E_z} T^3$	-2.85×10^{-2}
3	$-k_x z_0 \omega_x \omega_z T^2$	-1.01×10^{-2}
4	$2k_x g_y \omega_z T^3$	-8.33×10^{-3}
3	$-k_x z_0 \omega_x \Omega_{E_z} T^2$	-5.70×10^{-4}
4	$2k_x g_y \Omega_{E_z} T^3$	-4.72×10^{-4}
5	$3k_x v_{z_A} \omega_x \omega_z T^3$	3.02×10^{-4}
5	$3k_x v_{y_A} \omega_x \omega_y T^3$	3.02×10^{-4}
7	$\frac{7}{4} k_x g_z \omega_x \omega_z T^4$	-1.08×10^{-4}
5	$-3k_x v_{x_A} \omega_z^2 T^3$	-7.55×10^{-5}
5	$-3k_x v_{x_A} \omega_y^2 T^3$	-7.55×10^{-5}
5	$3k_x v_{z_A} \omega_x \Omega_{E_z} T^3$	1.71×10^{-5}
5	$3k_x v_{y_A} \omega_x \Omega_{E_y} T^3$	1.39×10^{-5}

Table A.1: The 20 most significant terms for the total phase shift of the interferometer along the x axis, and the corresponding value in radians.

Table 3.1 Term Number	Table 3.1 Phase Term	Phase Shift ($a_y = 1\text{ms}^{-2}$)
1	$-k_y a_y T^2$	-10 069.2
1	$k_y g_y T^2$	-166.58
2	$-2k_y v_{xA} \omega_z T^2$	-8.06
2	$2k_y v_{zA} \omega_x T^2$	8.06
4	$2k_y g_z \omega_x T^3$	-4.94
4	$2k_y a_x \omega_z T^3$	5.03×10^{-1}
2	$-2k_y v_{xA} \Omega_{Ez} T^2$	-4.56×10^{-1}
4	$2k_y a_x \Omega_z T^3$	2.85×10^{-2}
3	$-k_y z_0 \omega_y \omega_z T^2$	-1.01×10^{-2}
3	$-k_y z_0 \omega_y \Omega_{Ez} T^2$	-5.70×10^{-4}
3	$-k_y z_0 \omega_z \Omega_{Ey} T^2$	-4.62×10^{-4}
5	$3k_y v_{zA} \omega_y \omega_z T^3$	3.02×10^{-4}
5	$3k_y v_{xA} \omega_x \omega_y T^3$	3.02×10^{-4}
7	$\frac{7}{4} k_y g_z \omega_y \omega_z T^4$	-1.08×10^{-4}
5	$-3k_y v_{yA} \omega_z^2 T^3$	-7.55×10^{-5}
5	$-3k_y v_{yA} \omega_x^2 T^3$	-7.55×10^{-5}
3	$-k_y z_0 \Omega_{Ey} \Omega_z T^2$	-2.62×10^{-5}
5	$3k_y v_{zA} \omega_y \Omega_{Ez} T^3$	1.71×10^{-5}
5	$3k_y v_{zA} \omega_z \Omega_{Ey} T^3$	1.39×10^{-5}
5	$3k_y v_{xA} \omega_x \Omega_{Ez} T^3$	1.39×10^{-5}

Table A.2: The 20 most significant terms for the total phase shift of the interferometer along the y axis, and the corresponding value in radians.

Table 3.1 Term Number	Table 3.1 Phase Term	Phase Shift ($g_z = -9.81\text{ms}^{-2}$)
1	$k_z g_z T^2$	-98 778.9
2	$-2k_z v_{y_A} \omega_x T^2$	-8.06
2	$2k_z v_{x_A} \omega_y T^2$	8.06
4	$2k_z a_y \omega_x T^3$	5.03×10^{-1}
4	$-2k_z a_x \omega_y T^3$	-5.03×10^{-1}
2	$2k_z v_{x_A} \Omega_{E_y} T^2$	3.70×10^{-1}
4	$-2k_z a_x \Omega_{E_y} T^3$	-2.31×10^{-2}
3	$k_z z_0 \omega_y^2 T^2$	1.01×10^{-2}
3	$k_z z_0 \omega_x^2 T^2$	1.01×10^{-2}
4	$-2k_z g_y \omega_x T^3$	8.33×10^{-3}
3	$2k_z z_0 \omega_y \Omega_{E_y} T^2$	9.24×10^{-4}
5	$3k_z v_{y_A} \omega_y \omega_z T^3$	3.02×10^{-4}
5	$3k_z v_{x_A} \omega_x \omega_z T^3$	3.02×10^{-4}
7	$-\frac{7}{4}k_z g_z \omega_y^2 T^4$	1.08×10^{-4}
7	$-\frac{7}{4}k_z g_z \omega_x^2 T^4$	1.08×10^{-4}
5	$-3k_z v_{z_A} \omega_y^2 T^3$	-7.55×10^{-5}
5	$-3k_z v_{z_A} \omega_x^2 T^3$	-7.55×10^{-5}
3	$k_z z_0 \Omega_{E_y}^2 T^2$	2.12×10^{-5}
5	$3k_z v_{y_A} \omega_y \Omega_{E_z} T^3$	1.71×10^{-5}
5	$3k_z v_{x_A} \omega_x \Omega_{E_z} T^3$	1.71×10^{-5}

Table A.3: The 20 most significant terms for the total phase shift of the interferometer along the z axis, and the corresponding value in radians.

Bibliography

- [1] S. de Silva. *A 2D Atom Interferometer Accelerometer in the Horizontal Plane*. PhD thesis, Imperial College London, 2021.
- [2] A. D. King. Inertial navigation - forty years of evolution. *GEC Review*, 13(3), 1998.
- [3] T. Wiener. *Theoretical analysis of gimballess inertial reference equipment using delta-modulated instruments*. PhD thesis, Massachusetts Institute of Technology, 1962.
- [4] N. Barbour. *Inertial Navigation Sensors*. Charles Stark Draper Laboratory Cambridge, 2010.
- [5] P. Logonne et al. Seis: Insight’s seismic experiment for internal structure of mars. *Space Science Reviews*, 215(12), 2019.
- [6] G. T. Schmidt and R. E. Phillips. *INS/GPS Integration Architectures*. Massachusetts Institute of Technology and Charles Stark Draper Laboratory Cambridge, 2010.
- [7] G. T. Schmidt. *INS/GPS Technology Trends*. Massachusetts Institute of Technology, 2010.
- [8] F. Gulmammadov. Analysis, modeling and compensation of bias drift in mems inertial sensors. *IEEE: 2009 4th International Conference on Recent Advances in Space Technologies*, pages 591–596, 2009.
- [9] G. Schmidt. Gps based navigation systems in difficult environments. *Gyroscopy and Navigation*, 10:41–53, 04 2019.
- [10] A. A. Michelson and E. W. Morley. On the relative motion of the earth and the luminiferous ether. *Americal Journal of Science*, XXXIV(23), 1887.

- [11] E. Krimmel, G. Mollenstedt, and W. Rothemund. Measurement of contact potential differences by electron interferometry. *Applied Physics Letters*, 5(10):209–210, 1964.
- [12] C. Davisson. The diffraction of electrons by a crystal of nickel. *Bell System Technical Journal*, 7(1):90–105, 1928.
- [13] H. Rauch. *Neutron interferometry : lessons in experimental quantum mechanics, wave-particle duality, and entanglement Second edition*. Oxford University Press, 2015.
- [14] A. Zeilinger, R. Gähler, C. G. Shull, W. Treimer, and W. Mampe. Single- and double-slit diffraction of neutrons. *Rev. Mod. Phys.*, 60:1067–1073, Oct 1988.
- [15] M. Kasevich and S. Chu. Measurement of the gravitation acceleration of an atom with a light-pulse atom interferometer. *Applied Physics B*, 54(5):321–332, 1992.
- [16] O. Carnal and J. Mlynek. Young’s double-slit experiment with atoms: A simple atom interferometer. *Phys. Rev. Lett.*, 66:2689–2692, May 1991.
- [17] D. W. Keith, C. R. Ekstrom, Q. A. Turchette, and D. E. Pritchard. An interferometer for atoms. *Phys. Rev. Lett.*, 66:2693–2696, May 1991.
- [18] S. Eibenberger, S. Gerlich, M. Arndt, M. Mayor, and J. Tüxen. Matter–wave interference of particles selected from a molecular library with masses exceeding 10000 amu. *Physical chemistry chemical physics*, 15:14696–14700, 2013.
- [19] R. Colella, A. W. Overhauser, and S. A. Werner. Observation of gravitationally induced quantum interference. *Phys. Rev. Lett.*, 34:1472–1474, Jun 1975.
- [20] C. Bordé, J. Sharma, P. Tourenç, and T. Damour. Theoretical approaches to laser spectroscopy in the presence of gravitational fields. *Phys. Rev. Lett.*, 44, 01 1983.
- [21] V. P. Chebotayev, B. Ya. Dubetsky, A. P. Kasantsev, and V. P. Yakovlev. Interference of atoms in separated optical fields. *J. Opt. Soc. Am. B*, 2(11):1791–1798, Nov 1985.
- [22] J. F. Clauser. Ultra-high sensitivity accelerometers and gyroscopes using neutral atom matter-wave interferometry. *Physica B+C*, 151(1):262–272, 1988.

- [23] C. J. Bordé. Atomic interferometry with internal state labelling. *Physics Letters A*, 140(1):10–12, 1989.
- [24] K. Moler, D. Weiss, M. Kasevich, and S. Chu. Theoretical analysis of velocity-selective raman transitions. *Physical Review A*, 45(1):342–348, 1992.
- [25] S. Dickerson, J. Hogan, A. Sugarbaker, D. Johnson, and M. Kasevich. Multiaxis inertial sensing with long-time point source atom interferometry. *Phys. Rev. Lett.*, 111:083001, Aug 2013.
- [26] A. Peters, K. Yeow, and S. Chu. Measurement of gravitational acceleration by dropping atoms. *Nature*, 400:849–852, 1999.
- [27] P. Gillot, O. Francis, A. Landragin, F. Pereira Dos Santos, and S. Merlet. Stability comparison of two absolute gravimeters: optical versus atomic interferometers. *Metrologia*, 51:5, 2014.
- [28] C. Freier, M. Hauth, V. Schkolnik, B. Leykauf, M. Schilling, H. Wziontek, H. Scherneck, J. Müller, and A. Peters. Mobile quantum gravity sensor with unprecedented stability. *Journal of Physics: Conference Series*, 723:012050, jun 2016.
- [29] M. J. Snadden, J. M. McGuirk, P. Bouyer, K. G. Haritos, and M. Kasevich. Measurement of the earth’s gravity gradient with an atom interferometer-based gravity gradiometer. *Phys. Rev. Lett.*, 81:971–974, Aug 1998.
- [30] J. M. McGuirk, G. T. Foster, J. B. Fixler, M. J. Snadden, and M. Kasevich. Sensitive absolute-gravity gradiometry using atom interferometry. *Phys. Rev. A*, 65:033608, Feb 2002.
- [31] Bertoldi, A., Lamporesi, G., Cacciapuoti, L., de Angelis, M., Fattori, M., Petelski, T., Peters, A., Prevedelli, M., Stuhler, J., and Tino, G. M. Atom interferometry gravity-gradiometer for the determination of the newtonian gravitational constant g . *Eur. Phys. J. D*, 40(2):271–279, 2006.

- [32] N. Yu, J. Kohel, J. Kellogg, and L. Maleki. Development of an atom-interferometer gravity gradiometer for gravity measurement from space. *Applied Physics B*, 84:647–652, 09 2006.
- [33] B. Canuel, F. Leduc, D. Holleville, A. Gauguet, J. Fils, A. Virdis, A. Clairon, N. Dimarcq, C. J. Borde', A. Landragin, and P. Bouyer. Six-axis inertial sensor using cold-atom interferometry. *Physical Review Letters*, 97(1), 2006.
- [34] T. Gustavson, A. Landragin, and M. Kasevich. Rotation sensing with a dual atom-interferometer sagnac gyroscope. *Classical and Quantum Gravity*, 17:2385, 06 2000.
- [35] D. S. Durfee, Y. K. Shaham, and M. A. Kasevich. Long-term stability of an area-reversible atom-interferometer sagnac gyroscope. *Phys. Rev. Lett.*, 97:240801, Dec 2006.
- [36] A. Gauguet, B. Canuel, T. Lévêque, W. Chaibi, and A. Landragin. Characterization and limits of a cold-atom sagnac interferometer. *Phys. Rev. A*, 80:063604, Dec 2009.
- [37] P. Berg, S. Abend, G. Tackmann, C. Schubert, E. Giese, W. P. Schleich, F. A. Narducci, W. Ertmer, and E. M. Rasel. Composite-light-pulse technique for high-precision atom interferometry. *Phys. Rev. Lett.*, 114:063002, Feb 2015.
- [38] R. Parker, C. Yu, W. Zhong, B. Estey, and H. Muller. Measurement of the fine-structure constant as a test of the standard model. *Science*, 360:191–195, 2018.
- [39] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, and G. Tino. Precision measurement of the newtonian gravitational constant using cold atoms. *Nature*, 510:518–523, 2014.
- [40] P. Hamilton, M. Jaffe, P. Haslinger, Q. Simmons, H. Müller, and J. Khoury. Atom-interferometry constraints on dark energy. *Science*, 349:849–851, 2015.
- [41] D. Sabulsky, I. Dutta, E. Hinds, B. Elder, C. Burrage, and E. Copeland. Experiment to detect dark energy forces using atom interferometry. *Phys. Rev. Lett.*, 123:061102, Aug 2019.
- [42] J. Lautier, L. Volodimer, T. Hardin, S. Merlet, M. Lours, F. Pereira Dos Santos, and A. Landragin. Hybridizing matter-wave and classical accelerometers. *arXiv: Atomic Physics*, 2014.

- [43] S. Merlet, J. Le Gouët, Q. Bodart, A. Clairon, A. Landragin, F. Pereira Dos Santos, and P. Rouchon. Operating an atom interferometer beyond its linear range. *Metrologia*, 46(1):87–94, jan 2009.
- [44] J. Lautier, L. Volodimer, T. Hardin, S. Merlet, F. Pereira dos Santos, and A. Landragin. Hybridizing matter-wave and classical accelerometers. *Applied Physics Letters*, 105, 09 2014.
- [45] S. Crawford, R. Shugayev, H. Paudel, P. Lu, M. Syamlal, P. Ohodnicki, B. Chorpening, R. Gentry, and Y. Duan. Quantum sensing for energy applications: Review and perspective. *Advanced Quantum Technologies*, 4(8):2100049, 2021.
- [46] K. Bongs, M. Holynski, J. Vovrosh, P. Bouyer, G. Condon, E. Rasel, C. Schubert, W. Schleich, and A. Roura. Taking atom interferometric quantum sensors from the laboratory to real-world applications. *Nature Reviews Physics*, 1, 10 2019.
- [47] B. Barrett, P. Gominet, E. Cantin, L. Antoni-Micollier, A. Bertoldi, B. Battelier, P. Bouyer, J. Lautier, and A. Landragin. Mobile and Remote Inertial Sensing with Atom Interferometers. *Proc. Int. Sch. Phys. Fermi*, 188:493–555, 2014.
- [48] A. Clairon, F. Lienhart, S. Boussen, A. Bresson, R. Nyman, G. Varoquaux, J. Clément, P. Bouyer, G. Santarelli, F. Pereira dos Santos, A. Landragin, and D. Chambon. I.c.e.: a transportable atomic inertial sensor for test in microgravity. *Applied physics. B, Lasers and optics*, 84, 2006.
- [49] R. Geiger, V. Ménoret, G. Stern, N. Zahzam, P. Cheinet, B. Battelier, A. Villing, F. Moron, M. Lours, Y. Bidel, A. Bresson, A. Landragin, and P. Bouyer. Detecting inertial effects with airborne matter-wave interferometry. *Nature Communications*, 2, 09 2011.
- [50] Carlos L. and Garrido A. Compact chip-scale guided cold atom gyrometers for inertial navigation: Enabling technologies and design study. *AVS quantum science*, 1:014702, 2019.
- [51] Y. Bidel, N. Zahzam, C. Blanchard, A. Bonnin, M. Cadoret, A. Bresson, D. Rouxel, and M. Lalancette. Absolute marine gravimetry with matter-wave interferometry. *Nature Communications*, 9, 02 2018.

- [52] Y. Bidel, N. Zahzam, A. Bresson, C. Blanchard, M. Cadoret, A. Vestergaard-Olesen, and R. Forsberg. Absolute airborne gravimetry with a cold atom sensor. *Journal of Geodesy*, 94:1–9, 2020.
- [53] J. Stammers. *An Atom Interferometer for Measuring Horizontal Accelerations*. PhD thesis, Imperial College London, 2018.
- [54] X. Cheng. *Building a sensitive atom interferometer for navigation*. PhD thesis, Imperial College London, 2019.
- [55] T. Lévêque. *Développement d'un gyromètre à atomes froids de haute sensibilité fondé sur une géométrie repliée*. PhD thesis, SYRTE - Observatoire de Paris, 02 2015.
- [56] R. Geiger, V. Ménéret, G. Stern, N. Zahzam, P. Cheinet, B. Battelier, A. Villing, F. Moron, M. Lours, Y. Bidel, A. Bresson, A. Landragin, and P. Bouyer. Detecting inertial effects with airborne matter-wave interferometry. *Nature Communications*, 2(1):474–474, 2011.
- [57] P. Berg, S. Abend, G. Tackmann, C. Schubert, E. Giese, W. Schleich, F. Narducci, W. Ertmer, and E. Rasel. Composite-light-pulse technique for high-precision atom interferometry. *Physical Review Letters*, 114(6):063002–063002, 2015.
- [58] J. McGuirk, M. Snadden, and M. Kasevich. Large area light-pulse atom interferometry. *Phys. Rev. Lett.*, 85(21):4498–4501, 2000.
- [59] S. Chiow, T. Kovachy, H. Chien, and M. Kasevich. $102\hbar k$ large area atom interferometers. *Phys. Rev. Lett.*, 107(13):130403–130403, 2011.
- [60] S. Herrmann, S. Chiow, H. Muller, and S. Chu. Atom interferometry using beam splitters based on multi-photon bragg diffraction: A tool for precision measurements. In *2008 Conference on Lasers and Electro-Optics and 2008 Conference on Quantum Electronics and Laser Science*, pages 1–2. IEEE, 2008.
- [61] P. Altin, M. Johnsson, V. Negnevitsky, G. Dennis, R. Anderson, J. Debs, S. Szigeti, K. Hardman, S. Bennetts, G. McDonald, L. Turner, J. Close, and N. Robins. Precision

- atomic gravimeter based on bragg diffraction. *New Journal of Physics*, 15(2):23009–19, 2013.
- [62] Daniel Adam Steck. *Rubidium 87 D Line Data*. 2003.
- [63] J. Hogan, D. S. Johnson, and M. Kasevich. Light-pulse atom interferometry. *arXiv: Atomic Physics*, 2008.
- [64] S. Dimopoulos, P. Graham, J. Hogan, and M. Kasevich. General relativistic effects in atom interferometry. *Physical Review D*, 2008.
- [65] C. Overstreet, P. Asenbaum, and M. Kasevich. Physically significant phase shifts in matter-wave interferometry. *American Journal of Physics*, 89(3):324–332, 2021.
- [66] B. Dubetsky and M. Kasevich. Atom interferometer as a selective sensor of rotation or gravity. *Physical Review A*, 74, 04 2006.
- [67] S. Kleinert, E. Kajari, A. Roura, and W. Schleich. Representation-free description of light-pulse atom interferometry including non-inertial effects. *Physics Reports*, 09 2015.
- [68] P. Storey and C. Cohen-Tannoudji. The Feynman path integral approach to atomic interferometry: A tutorial. *Journal of Physics II*, 4(11):1999–2027, 1994.
- [69] L. Landau and E. Lifshitz. *Mechanics, Third Edition: Volume 1 (Course of Theoretical Physics)*. Butterworth-Heinemann, January 1976.
- [70] G. Biedermann. *Gravity tests, differential accelerometry and interleaved clocks with cold atom interferometers*. PhD thesis, Stanford University, 2007.
- [71] K. Bongs, R. Launay, and M. Kasevich. High-order inertial phase shifts for time-domain atom interferometers. *Applied Physics B*, 84:599–602, 2002.
- [72] C. Foot. *Atomic physics*. Oxford master series on atomic, optical and laser physics. Oxford University Press, Oxford, 2005.
- [73] M Squared. *Raman/MOT Laser System for Atom Interferometry: Test Report*. M Squared, 2017.

- [74] K. Lindquist, M. Stephens, and C. Wieman. Experimental and theoretical study of the vapor-cell zeeman optical trap. *Phys. Rev. A*, 46:4082–4090, Oct 1992.
- [75] H. Metcalf and P. van der Straten. *Laser Cooling and Trapping of Neutral Atoms*. Springer New York, NY, 2007.
- [76] J. Schoser, A. Batär, R. Löw, V. Schweikhard, A. Grabowski, Yu. B. Ovchinnikov, and T. Pfau. Intense source of cold rb atoms from a pure two-dimensional magneto-optical trap. *Phys. Rev. A*, 66:023410, Aug 2002.
- [77] K. Dieckmann, R. Spreeuw, M. Weidemüller, and J. Walraven. Two-dimensional magneto-optical trap as a source of slow atoms. *Phys. Rev. A*, 58:3891–3895, Nov 1998.
- [78] ColdQuanta, Oxford, United Kingdom. *Atom Source 2D MOT Operating Manual*, 2018.
- [79] C. Townsend, N. Edwards, C. Cooper, K. Zetie, C. Foot, A. Steane, P. Szriftgiser, H. Perrin, and J. Dalibard. Phase-space density in the magneto-optical trap. *Phys. Rev. A*, 52:1423–1440, Aug 1995.
- [80] J. Beitia, A. Clifford, C. Fell, and P. Loisel. Quartz pendulous accelerometers for navigation and tactical grade systems. In *2015 DGON Inertial Sensors and Systems Symposium (ISS)*, pages 1–20, 2015.
- [81] M. Prentiss, A. Cable, J. Bjorkholm, S. Chu, E. Raab, and D. Pritchard. Atomic-density-dependent losses in an optical trap. *Opt. Lett.*, 13(6):452–454, Jun 1988.
- [82] J. Dalibard and C. Cohen-Tannoudji. Laser cooling below the doppler limit by polarization gradients: simple theoretical models. *J. Opt. Soc. Am. B*, 6(11):2023–2045, Nov 1989.
- [83] S. Ohshima, T. Kurosu, T. Ikegami, and Y. Nakadan. Cesium atomic fountain with two-dimensional moving molasses. *Japanese Journal of Applied Physics*, 34(Part 2, No. 9A):L1170–L1173, sep 1995.
- [84] G. Dick, J. Prestage, C. Greenhall, and L. Maleki. Local oscillator induced degradation of medium-term stability in passive atomic frequency standards. In *The 22nd Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting*, 06 1990.

- [85] B. Barrett, P. Gominet, E. Cantin, L. Antoni-Micollier, A. Bertoldi, B. Battelier, P. Bouyer, J. Lautier, and A. Landragin. Mobile and remote inertial sensing with atom interferometers. *Proceedings of the International School of Physics, "Enrico Fermi"*, 188:493, 10 2014.
- [86] P. Cheinet, B. Canuel, F. Pereira dos Santos, A. Gauguier, F. Yver-Leduc, and A. Landragin. Measurement of the sensitivity function in a time-domain atomic interferometer. *IEEE Transactions on Instrumentation and Measurement*, 57:1141 – 1148, 07 2008.