

Bubble Injection for Heat Transfer Enhancement: From Physics to Applications

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Abstract

This article presents a comprehensive review of recent advancements in bubble-induced heat transfer enhancement, with a primary focus on understanding the fundamental underlying physics. Accordingly, this review first highlights recent novel concepts and techniques developed to enhance heat transfer through bubble injection, followed by explaining the essential physical aspects of this development. It is attempted to clarify the impact of bubble injection on heat transfer by examining key mechanisms in two-phase bubbly flow. The factors that influence heat transfer and fluid flow, including mechanisms of bubble ascent, bubble breakage, and coalescence, as well as the impact of bubble size and shape, are examined. Furthermore, the review explores the use of bubble injection in different types of heat exchangers in addition to other applications, including solar collectors, hydrogen production, internal combustion engines, and energy storage systems. Furthermore, the article identifies current research gaps and existing challenges and suggests potential directions for future research in bubble-induced heat transfer enhancement.

Keywords: Bubble injection; Physics of bubbles; Heat transfer enhancement; Two-phase flow; Heat exchanger.

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Nomenclature

A	Surface area (m ²)	Δ	Difference (-)
A_i	Interfacial area (m ²)	θ	Angel of inclination (°)
AR	Bubble shape aspect ratio (-)	κ	Surface curvature (1/m)
B	Magnetic field intensity (T)	λ	Twist ratio (-)
C	Constant (-)	σ	Surface tension coefficient (N/m)
Ca	Capillary number (-)	μ	Dynamic viscosity (Pa. s)
C_D	Drag coefficient (-)	ν	Kinematic viscosity (m ² /s)
C_L	Lift coefficient (-)	π	Pi number (-)
C_{LF}	Lift coefficient due to lift (-)	ρ	Density (kg/m ³)
c_p	Specific heat capacity (J/kg. K)	σ	Surface tension coefficient (N/m)
C_{TD}	Turbulent dispersion coefficient (-)	τ	Stress tensor (N/m ²)
C_{VM}	Virtual mass coefficient (-)	φ_h	Two-phase heat transfer multiplier (-)
C_{WK}	Lift coefficient due to wake (-)	ω	Vorticity (-)
C_{WL}	Wall lubrication coefficient (-)	X	Martinelli parameter (-)
D	Diameter or distance (m)	Γ	Related to liquid-gas interface (-)
D_h	Hydraulic diameter (m)	ξ	Local scaling exponent (-)
Eo	Eötvös number (-)	Subscripts	
F	Force (N)	BI	Bubble-induced
Fr	Froude Number (-)	b	Bulk
f	Frequency (Hz)	bu	Bubble, gas, and air
f_f	Friction factor (-)	C	Coil
g	Gravity acceleration (m/s ²)	D	Drag
G	Mass velocity (kg/m ² .s)	eff	effective
Ga	Galilei number (-)	f	fluid
Gr	Grashof number (-)	g	Gas phase
h	Convection heat transfer coefficient (W/m ² . K)	H	Horizontal
J	Current density (A/m ²)	L	Lift
k	Thermal conductivity (W/m. K)	Liq or l	Liquid
$\dot{m}$	Mass flow rate (kg/s)	loc	local
Mo	Morton number (-)	m	Mean
n	Unit vector (-)	mod	modified
Nu	Nusselt number (-)	noz	Nozzle
N_{VM}	Virtual mass number (-)	o	Outer side
p	Pressure (Pa)	p	Helical coil pitch

Pe	Peclet Number (-)	rel	Relative
Pr	Prandtl number (-)	T	Related to the thermal boundary layer
q''	Heat flux (W/m ²)	sgs	Sub-grid scale
Q	Volumetric flow rate (m ³ /s)	sg	Related to superficial gas velocity
Ra	Rayleigh number (-)	sl	Related to superficial liquid velocity
Re	Reynolds number (-)	sh	Shell side
Sr	Dimensionless shear rate (-)	SI	Shear-induced
T	Temperature (°C or K)	sp	Single phase
Ta	Tadaki number (-)	t	Turbulence
t	Time (s)	TD	Turbulent dispersion
$\vec{u}$	Velocity vector (m/s)	tot	Total
U	Superficial velocity (m/s)	TP	Two-phase
V	Volume (m ³)	V	Vertical
w	z-component of velocity (m/s)	VM	Virtual mass
We	Weber number (-)	w	Water
x	Flow quality (-)	Superscripts	
y	Distance from the wall (m)	D	Drag
Greek symbols		hor	Horizontal
α	Mean void fraction (-)	L	Lift
α_d	Thermal diffusivity (m ² /s)	TD	Turbulent dispersion
β	Coefficient of thermal expansion (1/K)	ver	vertical
δ	Boundary layer thickness (m)	WL	Wall lubrication

1. Introduction and problem statement

1.1. Introduction

Bubbles and bubbly flows are governed by complex physical principles that have challenged scientists for decades. The unique properties and behaviors of bubbles arise from their interactions with various physical phenomena, including surface tension, buoyancy, pressure variations, thermodynamics, thermal effects, turbulence, capillary forces, and mass transfer. These principles govern the complex dynamics of bubble formation, growth, and movement. Bubbles can interact with their surroundings in numerous ways, such as coalescing with other bubbles, breaking into smaller bubbles, or dissolving into the surrounding fluid. External forces, such as electric and magnetic fields, can also influence bubble behavior. These

multifaceted interactions contribute to the versatile and complex nature of bubbles and bubbly flows.

A thorough understanding of bubble physics is essential for developing innovative approaches and methodologies that leverage their potential in various fields, such as heat transfer enhancement, acoustics, and fluid mechanics. By exploring the intricate world of bubbles, researchers can unlock new possibilities and applications, driving advancements in science and technology. One promising technique for enhancing heat transfer is bubble injection (non-boiling), which has gained significant interest in recent years. Generally, heat transfer enhancement methods can be classified into three categories: (i) passive, which does not require external forces; (ii) active, which employs external forces; and (iii) hybrid, which combines elements of both passive and active techniques. Fig. 1 provides an overview of the various available heat transfer enhancement techniques.

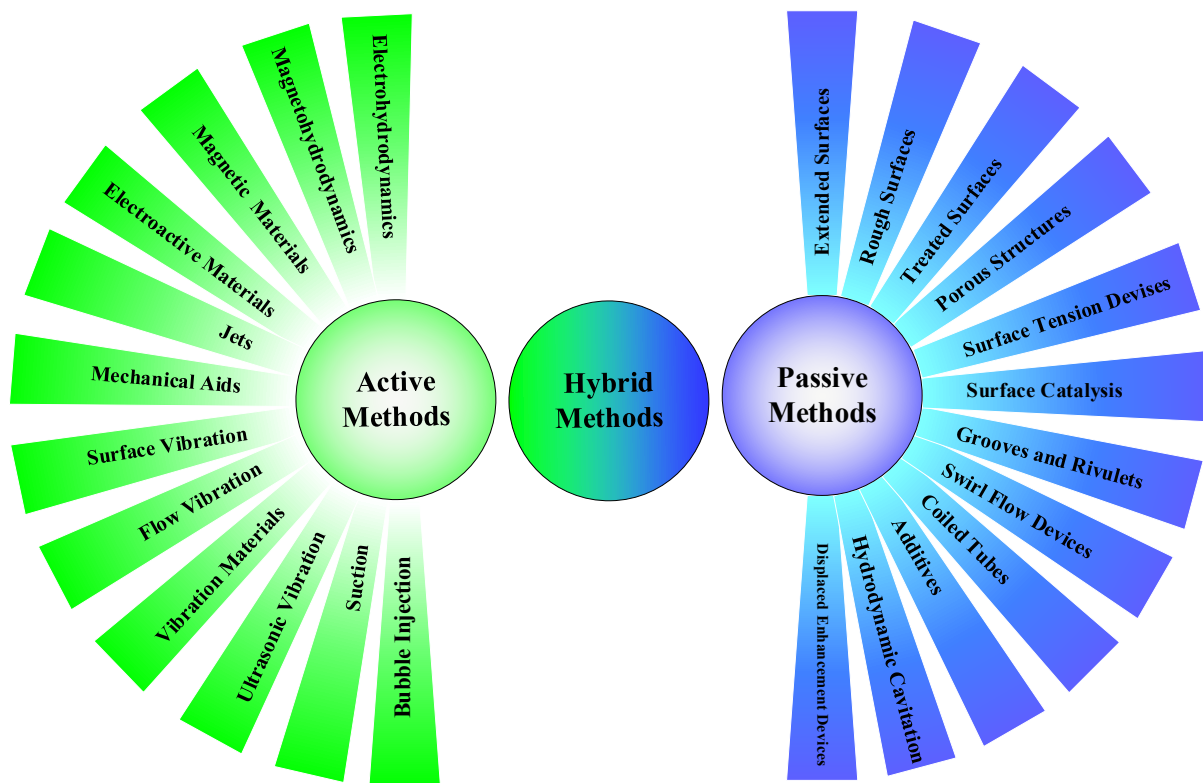


Fig. 1: Various heat transfer enhancement techniques available.

As one of the active approaches for heat transfer enhancement, bubble injection involves introducing gas bubbles into a continuous liquid phase, which creates a two-phase flow, leading to enhanced convective heat transfer. The presence of bubbles increases turbulence and surface area. This facilitates more efficient heat transfer without requiring additional mechanical equipment [1]. The bubble injection method has found applications in

various industries, including the mining industry [2], chemical processes [3], food industry [4-6], energy storage systems [7, 8], bioreactors [9, 10], petroleum (oil-gas transport) [11], and desalination systems [12]. Bubble injection contributes to a more uniform distribution of temperature and augments the heat removal process. Consequently, comprehending the principles of non-boiling bubble injection for heat transfer enhancement is crucial for the optimal design of heat exchangers. In the subsequent sections, a concise overview of methods for heat transfer enhancement and the objectives of this study are presented.

1.2. Aims and objectives

There are numerous review articles in the literature that discuss methods and techniques for enhancing heat transfer in various types of heat exchangers, such as plate heat exchangers and fin-and-tube heat exchangers. These enhancement methods include surface modifications, adding nanoparticles to the working fluid, and using tap inserts, among others. Such improved heat exchangers are utilized in diverse applications, particularly in solar energy systems like photovoltaic setups, where efficient thermal management is crucial for electricity generation. References [13-16] represent just a small selection of these review articles.

Among the various heat transfer enhancement techniques explored in the literature, bubble injection stands out as a promising method deserving more focused attention. Therefore, this review aims to provide a comprehensive examination of the mechanisms, benefits, challenges, and applications of bubble injection in enhancing heat transfer. The main research questions addressed in this review are summarized in Fig. 2, focusing on the physical mechanisms, applications, and future research directions of bubble injection.

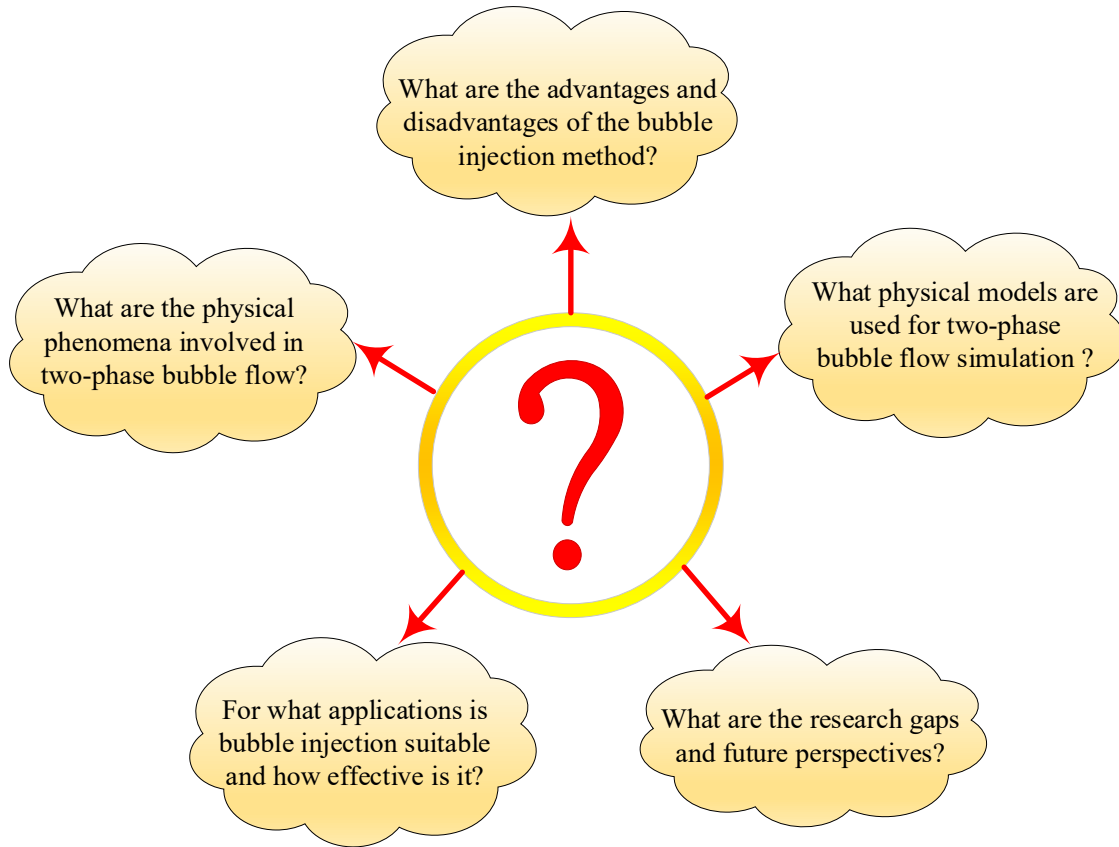


Fig. 2: Key questions addressed by the present review in the context of bubble injection for heat transfer enhancement.

The review is divided into the following main sections. The first section presents the problem statement and provides an overview of the heat transfer parameters involved in bubble injection. It also describes the dimensionless numbers frequently used in these studies, which are crucial for characterizing bubble behavior. These numbers help assess the effects of inertia, surface tension, buoyancy, and viscous forces, enabling comparisons across different flow regimes and conditions. Then, it examines the two-phase gas-liquid flows and recommends the suitable conditions for using the bubble injection method for the heat transfer enhancement. The next subsection briefly describes bubble generation methods and reviews correlations for bubble diameter found in the literature. The last part of this section discusses issues such as geometry, type of fluids, temperature range, and bubble size, which are essential for the bubble injection method of heat transfer enhancement.

The second and third sections focus on the numerical and experimental methods to study two-phase bubbly flows and heat transfer, and the fundamental physical mechanisms involved, respectively. The following section examines the two-phase heat transfer in simple geometries, followed by the section reviewing the application of the bubble injection method in heat exchangers and devices. The review focuses on applications of non-boiling bubble

injection, specifically on efficiency enhancement in various heat exchangers and thermal systems, including solar collectors and other devices. Recent research findings on the bubble injection method are presented, and the challenges in its applications are discussed. Additionally, new correlations based on experimental data for a commonly used heat exchanger incorporating bubble injection are provided. Finally, the review article identifies knowledge gaps and associated challenges and suggests potential future research directions to address these gaps in the field of bubble injection for heat transfer enhancement.

1.3. Basic concepts and general definitions

The heat transfer rate in two-phase flow depends on various factors, including bubble shape, diameter, flow properties, and velocity. This section provides an overview of the heat transfer mechanisms and the governing parameters in bubble injection flows. Essential parameters and dimensionless numbers are introduced using the example of bubble motion in a pipe or channel, as illustrated in Fig. 3.

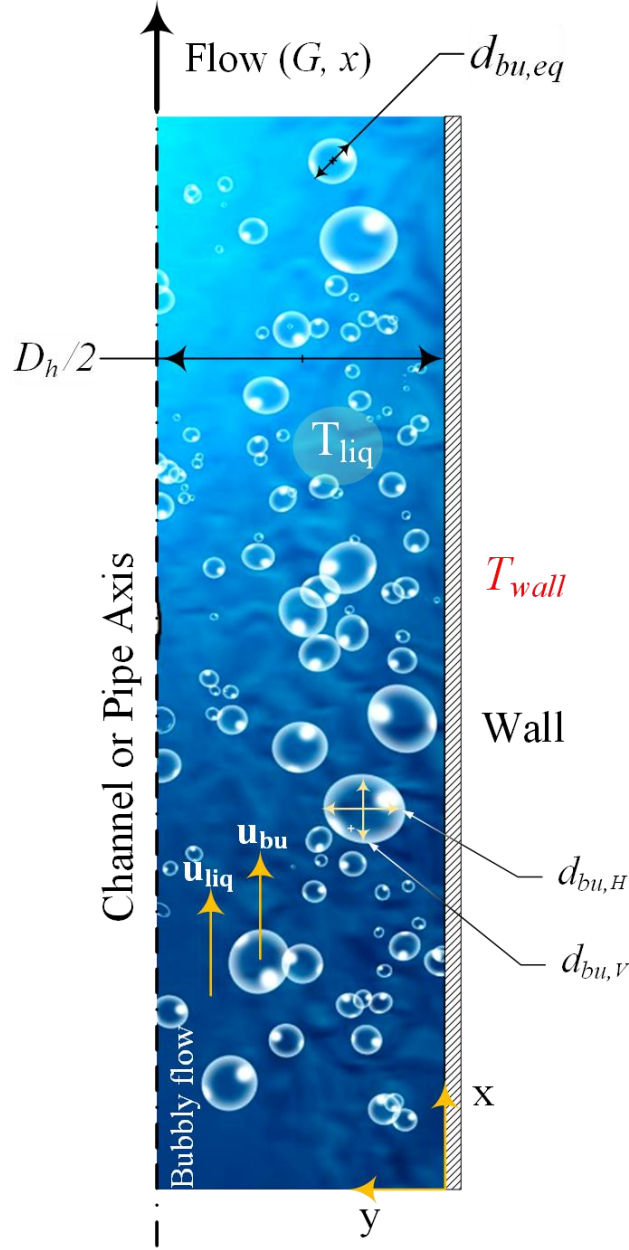


Fig. 3: Schematic view of a sample bubbly flow in a channel or pipe for parameter definition. Here, G and x are the total volumetric flow rate and quality of the flow (the ratio of bubble to liquid flow rate), d_{eq} is the equivalent bubble diameter, D_h is the hydraulic diameter of the pipe or channel, T_w and T_{liq} are the wall and liquid temperatures, and u_{liq} and u_{bu} are the velocities of liquid and bubble phases.

Here, definitions of various parameters used in two-phase flows are provided, including mass velocity, mean gas mass fraction, superficial velocity, drift velocity, and distribution function. The total mass velocity (G) and the flow quality or mean gas mass fraction (x) can be written as,

$$G = G_{liq} + G_{bu} = \rho_{liq}U_{liq} + \rho_{bu}U_{bu} = (1 - x)G + xG \quad (1)$$

$$x = \frac{\dot{m}_{bu}}{\dot{m}_{liq}} \quad (2)$$

The velocities that the phases would have if they flowed through the pipe or channel alone are known as the superficial-phase velocities. The liquid and gas superficial velocities are defined as [17],

$$U_{sl} = \frac{G(1 - x)}{\rho_{liq}} \quad (3)$$

$$U_{sg} = \frac{Gx}{\rho_{bu}} \quad (4)$$

A parameter that plays a critical role in describing two-phase flow is the void fraction (α). The void fraction is defined as the ratio of the local volume of the gas to that of the liquid and changes at different locations. The averaged void fraction in a specific cross-section of a pipe or channel is defined as the ratio of the bubble (or gas) cross-area to the total area ($\alpha = A_{bu}/(A_{bu} + A_{liq})$). A more precise void fraction prediction yields a more accurate heat transfer coefficient estimation in two-phase flow.

Many experimental correlations have been presented for the void fraction in different geometry and inclination angles [18]. Dong et al. [19] presented a drift-flux correlation model to describe the void fraction in upward two-phase flow in a pipe. The correlation for the average void fraction is as follows,

$$\alpha = \frac{U_{sg}}{C_0(U_{sg} + U_{sl}) + U_{dg}} \quad (5)$$

Here U_{sg} , U_{sl} , U_{dg} and C_0 are the superficial gas velocity (m/s), superficial liquid velocity (m/s), gas phase drift velocity (m/s), and distribution parameter (-), respectively. C_0 is defined as,

$$C_0 = C_\infty - (C_\infty - 1) \sqrt{\frac{\rho_g}{\rho_{liq}}} \quad (6)$$

where C_∞ is the asymptotic distribution parameter. Before presenting the correlation for this parameter, the non-dimensional superficial velocity and mixture volumetric velocity are defined,

$$U_{sg}^+ = U_{sg} / \left(\frac{\sigma g \Delta \rho}{\rho_{liq}^2} \right)^{1/4} \quad (7)$$

$$U^+ = U / \left(\frac{\sigma g \Delta \rho}{\rho_{liq}^2} \right)^{1/4}$$

Here, U represents the mixture volumetric velocity, which is the average velocity considering the volumetric contributions of both gas and liquid phases. Moreover, the parameters C_∞ and U_{dg} are determined by the following expressions,

$$C_{\infty} = (0.4\sin\theta + 0.8)\exp\left[\ln\left(\frac{1.8 - 0.7\sin\theta}{0.4\sin\theta + 0.8}\right)\left(\frac{U_{sg}^+/U^+}{0.9}\right)^{1.5}\right] \quad 0 \leq U_{sg}^+/U^+ < 0.9 \quad (8)$$

$$U_{dg} = \sqrt{2}\left(\frac{\sigma g \Delta \rho \sin\theta}{\rho_{liq}^2}\right)^{0.25} \quad 0 \leq U_{sg}^+/U^+ < 0.9 \quad (9)$$

$$C_{\infty} = \frac{(7.0\sin\theta - 0.8)U_{sg}^+}{U^+} + 0.9 - 7.0\sin\theta \quad 0.9 \leq U_{sg}^+/U^+ \leq 1 \quad (10)$$

$$U_{dg} = \sqrt{2}\left(\frac{\sigma g \Delta \rho \sin\theta}{\rho_{liq}^2}\right)^{0.25} \left(\frac{1 - U_{sg}^+/U^+}{0.1}\right) \quad 0.9 \leq U_{sg}^+/U^+ \leq 1 \quad (11)$$

where θ is the pipe inclination angle. In the literature, there are different ways to define the Reynolds number: the ratio of the inertia and viscous forces based on the mixture velocity and liquid viscosity. For example, the Reynolds number based on the equivalent bubble diameter is defined as [20],

$$Re_{bu} = \frac{\rho_{liq}|u_{bu} - u_{liq}|d_{eq}}{\mu_{liq}} \quad (12)$$

Therefore, if a bubble is injected into a quiescent fluid, the Reynolds number is given by,

$$Re_{bu} = \frac{\rho_{liq}u_{bu,y}d_{eq}}{\mu_{liq}} \quad (13)$$

where the $u_{bu,y}$ is the average bubble rise velocity, which is a function of gas void fraction as reported in the literature [21-24]. For example, Wallis [25] explored the bubble rise velocity for the bubble swarm in water, presenting the correlation of $u_{bu,y} \propto (1 - \alpha)$ for void fraction up to 30%. In another work, Colombet et al. [21] studied the bubble rising dynamics in homogeneous bubble swarm conditions at a high mean gas void fraction up to 30%. They introduced the new correlation for the bubble rising velocity as follows,

$$u_{bu,y} = 0.32[0.28 + 0.72\exp(-15\alpha)]^{0.5} \quad (14)$$

Several other correlations have been presented in the literature (e.g., Yan et al. [26] and Li et al. [27]). Similarly, the Reynolds number can be expressed using the mixture velocity, liquid viscosity (ν), and channel or pipe hydraulic diameter (D_h) [28],

$$Re_m = \frac{(G_{liq} + G_{bu})D_h}{\mu_{liq}} \quad (15)$$

Sometimes in literature, for the definition of Reynolds number, instead of using liquid viscosity in Eq.(15), the mixture viscosity has been used (See Awad and Muzychka [29]),

$$\mu_m = x\mu_{bu} + (1 - x)\mu_{liq} \quad (16)$$

In engineering applications, bubbly flow prediction is based on mean or bulk properties, such as mean temperature, mean flow rate, and pressure drop, which are mainly applicable to simplified one-dimensional analyses, such as bubbly pipe flow, where cross-sectional variations are often neglected [30]. However, more detailed studies and information are needed to describe bubbly flow behavior in more complex geometries and three-dimensional flows, such as bubble columns and heat exchangers. Bunner and Tryggvason [31] explored the effect of bubble deformation on the bubbly flow properties and observed a shift in flow behavior.

The shape and geometry of the bubble significantly influence heat transfer and flow properties. Additionally, bubble shape influences the detachment behavior of the bubble wake by changing the motion of the bubble and the forces acting on it, such as lift force. If the bubble is modeled as an oblate spheroid (shown in Fig. 3) with a horizontal diameter of (d_H) and a vertical diameter of (d_V), the bubble equivalent diameter and aspect ratio (AR) are defined as,

$$d_{eq} = (d_H^2 d_V)^{1/3} \quad (17)$$

$$AR = d_V/d_H \quad (18)$$

For many years, researchers have studied the relationship between the bubble shape and dimensionless numbers, also known as the regime map of bubble shape [32, 33]. Dimensionless numbers, such as Eötvös number ($Eo = \frac{g \Delta \rho d_{eq}^2}{\sigma}$), Morton number ($Mo = \frac{Eo^3}{Ga^4} = \frac{g \Delta \rho \mu_{liq}^4}{\rho_{liq}^2 \sigma^3}$), where Ga is Galilei Number, Weber number ($We = \frac{\rho_l U^2 d_{eq}^2}{\sigma}$), and Reynolds number ($Re = \frac{U d}{\nu}$) can be used to characterize bubble size for a given flow [34]. Therefore, in general, the bubble aspect ratio can be expressed as a function of these dimensionless numbers,

$$AR = f(Eo, Re, Mo, We) \quad (19)$$

So far, many empirical correlations have been presented for the bubble aspect ratio in different flow types and conditions. For instance, Rastello et al. [35] proposed a correlation based on the Weber number, while Okawa et al. [36] developed a correlation using the Eötvös number (Eo). Zhou et al. [37], however, presented a correlation based on We and Mo numbers. In addition, some studies use a dimensionless number, known as the Tadaki number ($Ta = Re Mo^{0.23}$) to estimate the bubble aspect ratio. This number was derived from a series of experiments conducted by Tadaki and Maeda in 1961 [38]. In some other studies the correlation between Eo and Re numbers are used to calculate the bubble aspect ratio [39]. Although these dimensionless numbers may qualitatively describe the bubble shape, more studies still need to

be dedicated to evaluating it. Many of the presented correlations are based on the behavior of a single bubble rising in quiescent liquids, whereas many industrial applications operate under conditions involving high gas volume fractions. Therefore, the bubble cannot be considered isolated since the high gas flow rate leads to a bubble swarm, increasing bubble agitation in both phases and affecting the bubbles' shape. The detailed information is presented in the "*bubble shape and diameter*" section (Sec 3.1.2.3).

The study of bubble dynamics, including their shape and behavior, is closely linked to heat and mass transfer processes in both single-phase and two-phase flows. The shape of bubbles affects their surface area, interaction with surrounding fluids, and the boundary layers where heat transfer occurs. As such, understanding the dimensionless parameters that govern heat transfer and those describing bubble shape is critical for many industrial applications.

The following section introduces the dimensionless numbers relevant to heat transfer processes in two-phase flows. It also reviews the heat transfer coefficient in single-phase flow as a foundational concept, which serves as a basis for understanding more complex heat transfer in bubbly flows. Typically, heat transport occurs in the boundary layer adjacent to the surface, and the local surface heat flux can be evaluated using Fourier's law [40],

$$\dot{q}_s = -k_{liq} \frac{\partial T}{\partial y} |_{y=0} \quad (20)$$

Here k_{liq} represents the fluid conductivity. As per the Newton's law of cooling, the heat flux, $\dot{q}_s$ can be expressed as,

$$\dot{q}_s = h(T_{wall} - T_{liq}) \quad (21)$$

where T_{wall} is the surface temperature. By combining Fourier's and Newton's laws, the heat transfer coefficient, h , is given as,

$$h = \frac{-k_{liq} \partial T / \partial y |_{y=0}}{T_{wall} - T_{liq}} \quad (22)$$

A significant dimensionless parameter for heat transfer is the Nusselt number (non-dimensional heat transfer coefficient), which is defined as,

$$Nu \equiv \frac{hL}{k_{liq}} \quad (23)$$

In fact, the Nusselt number is the slope of the nondimensional temperature gradient at the wall.

Besides the Reynolds number (Re), the other significant parameter that affects the heat transfer in the boundary layer is the Prandtl number (Pr). The Prandtl number, denoting the

ratio of momentum and thermal diffusivities, represents the fluid's capacity to conduct heat compared to transferring momentum effectively [41]. The Prandtl number is stated as,

$$Pr = \left(\frac{c_p \mu}{k} \right)_{liq} = \left(\frac{\nu}{\alpha_d} \right)_{liq} \quad (24)$$

The Prandtl number describes the ratio of momentum diffusivity to thermal diffusivity and provides insight into the relative thickness of the momentum and thermal boundary layers. The interplay between thermal and fluid dynamic properties becomes crucial in systems where convection plays a significant role, such as those involving buoyancy-driven flows.

The buoyancy force driving the bubble motion in bubble flows is closely linked to natural convection. The temperature difference between the bubble surface and the surrounding fluid can create additional density variations, further influencing the convection currents. These interactions are critical in many industrial processes, such as heat exchangers, chemical reactors, and boiling systems, where bubble-induced convection improves overall thermal performance. The Rayleigh number (Ra) is commonly used to quantify the heat transfer in such scenarios. It combines the effects of buoyancy, thermal diffusivity, and fluid properties, providing a dimensionless measure of the natural convection strength. [42]. The Rayleigh number is denoted as follows:

$$Ra_L = Gr_L Pr = \frac{g \beta (T_{hot} - T_{cold}) L^3}{\nu \alpha_d} \quad (25)$$

Here, L , α_d , and β are the characteristics length, thermal diffusivity of fluid, and thermal expansion coefficient of fluid, respectively. The *Grashof* number (Gr_L), is defined as the ratio of the buoyancy forces to viscous forces, providing a measure of the relative importance of these forces in driving fluid motion.

This review extensively uses dimensionless numbers to analyze and describe flow and heat transfer phenomena. Table 1 provides information about dimensionless parameters that describe single- and two-phase flow and heat transfer [40-44].

Table 1: Selected dimensionless numbers used in bubble-contained flows [40-44] (d in this table is the equivalent bubble diameter defined in Eq. 17)

Group	Definition	Interpretation
Mean void fraction (α)	$\alpha = \frac{V_g}{V_{tot}}$	The volume of the gaseous phase occupied in a channel (or pipe) divided by the channel's (or pipe) entire volume is known as the void fraction (α). Many correlations exist for different geometry and inclinations. Void fraction is defined locally and can vary across the flow.

Capillary number (Ca).	$Ca = \mu_l U / \sigma$	The impact of viscous stress at the gas-liquid interface on bubble deformation can be examined using the capillary number.
Eötvös number (Eo)	$Eo = \frac{g \Delta \rho d^2}{\sigma}, \Delta \rho = (\rho_l - \rho_g)$	The ratio of buoyancy to surface tension forces
Galilei Number (Ga)	$Ga = \frac{g^{1/2} d^{3/2}}{\mu_l}$	The ratio of the gravitational to the viscous force
Grashof number (Gr_L)	$Gr_L = \frac{g \beta (T_s - T_\infty) L^3}{\nu^2}$	Measure of the ratio of buoyancy forces to viscous forces
Lockhart–Martinelli parameter (X_{tt})	$X_{tt} = \left(\frac{1-x}{x} \right)^{0.9} \left(\frac{\rho_g}{\rho_l} \right)^{0.5} \left(\frac{\mu_l}{\mu_g} \right)^{0.1}$	Describes two-phase flow behavior in a fluid system, particularly in the context of gas-liquid mixtures (x is mixture quality ($\dot{m}_{bu}/\dot{m}_{total}$))
Morton number (Mo)	$Mo = \frac{Eo^3}{Ga^4} = \frac{g \Delta \rho \mu_l^4}{\rho_l^2 \sigma^3}$ $\Delta \rho = (\rho_l - \rho_g)$	This number, in addition to the Eo number, may be used to characterize the dynamic of a bubble because it depends on the fluid characteristics and not the size of the bubble, which is advantageous.
Nusselt number (Nu_L)	$Nu_L = \frac{hL}{k_f}$	The ratio of convection to pure conduction heat transfer
Peclet number (Pe_L)	$Pe_L = \frac{UL}{\alpha_d} = Re_L Pr$	The ratio of advection to conduction heat transfer
Prandtl number (Pr)	$Pr = \frac{c_p \mu}{k} = \frac{\nu}{\alpha_d}$	The ratio of the momentum and thermal diffusivities
Rayleigh number (Ra_L)	$Ra_L = Gr_L Pr = \frac{g \beta \Delta T L^3}{\nu \alpha_d}$	The ratio of buoyancy-driven natural convection effects to the combined influence of viscous forces and heat conduction in a fluid
Reynolds number (Re_d)	$Re_d = \frac{Ud}{\nu}$	Ratio of the inertia and viscous forces
Weber number (We)	$We = \frac{\rho_l U^2 d}{\sigma}$	The ratio of inertia to surface tension force

The heat transfer coefficient in two-phase flow depends not only on the parameters of the two-phase flow but also on the single-phase heat transfer coefficient, as will be discussed later. For example, for bubbly flow in a pipe, the Nu number is a function of void fraction (α), superficial liquid Re number (Re_{sl}), angle of inclination (θ), and single phase Nu number (Dong and Hibiki [45]). The heat transfer between bubbles and liquid can be overlooked because of the low heat capacity of air compared to liquid. Regarding single-phase flow and forced convection, depending on the nature of the flow, the Nusselt number is a function of Reynolds and Prandtl numbers, $Nu = f(Re_x, Pr)$, while in natural convection and single phase flow, $Nu = f(Ra, Pr, \theta)$ [42]. The Nusselt number correlations for some common geometries in single-phase forced convection flow (external and internal) are given in Table 2, and the additional correlations for other geometries can be found in [40-42]. As shown in the table, the dependence of the Nusselt number on the Reynolds and Prandtl numbers is evident across various flow regimes and geometries.

Table 2. Summary of Nusselt Number correlations in forced convection for different geometries [40-42]

Forced Convection			
Boundary layer flow (external flow)			
Geometry	Nusselt number correlation	Conditions	
Flat Plate	$0.332 Re_x^{1/2} Pr^{1/3}$	$Pr \geq 0.6$	Laminar, constant surface temperature
	$0.564 Pe_x^{1/2}$	$Pr \leq 0.05$ $Pr \geq 100$	
	$0.0296 Re_x^{4/5} Pr^{1/3}$	$Re_x \leq 10^8$ $0.6 \leq Pr \leq 60$	Turbulent, constant surface temperature
	$(0.037 Re_L^{4/5} - 871) Pr^{1/3}$	$Re_L \leq 10^8$ $0.6 \leq Pr \leq 60$ $Re_{x,c} = 5 \times 10^5$	Laminar and turbulent, constant surface temperature
	$0.453 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$	$Pr \geq 0.6$	Laminar, constant heat flux
	$0.0308 Re_x^{4/5} Pr^{\frac{1}{3}}$	$0.6 \leq Pr \leq 60$	Turbulent, constant heat flux
Circular Cylinder	$0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_D}{282000} \right)^{5/8} \right]^{4/5}$	$Re_D Pr \geq 0.2$, D is the diameter of the cylinder, laminar and turbulent flows.	
Sphere	$2 + (0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}) Pr^{0.4} \left(\frac{\mu}{\mu_s} \right)^{1/4}$	$0.71 \leq Pr \leq 380$ $3.5 \leq Re_D \leq 7.6 \times 10^4$, $1.0 \leq \left(\frac{\mu}{\mu_s} \right) \leq 3.2$, laminar and turbulent flows. D is the diameter of the sphere.	
Boundary layer flow(internal flow)			
Circular Tube	$\frac{hD}{k} = 3.66$	$T_s = \text{constant}$	Laminar fully developed $Re_{D,critical} \approx 2300$
	$\frac{hD}{k} = 4.36$	$\tilde{q}_s = \text{constant}$	
	$0.023 Re_D^{4/5} Pr^n$ for heating ($T_s > T_m$) $\rightarrow n = 0.4$ for cooling ($T_s < T_m$) $\rightarrow n = 0.3$	$0.6 \leq Pr \leq 160$ $Re_D \geq 10000$ $\frac{L}{D} \geq 10$	Turbulent fully developed D and L are the diameter and length of the pipe
	$0.027 Re_D^{4/5} Pr^{\frac{1}{3}} \left(\frac{\mu}{\mu_s} \right)^{0.14}$ for heating ($T_s > T_m$) $\rightarrow n = 0.4$ for cooling ($T_s < T_m$) $\rightarrow n = 0.3$	$0.7 \leq Pr \leq 16700$ $Re_D \geq 10000$ $\frac{L}{D} \geq 10$	
	$\frac{\left(\frac{f_f}{8} \right) (Re_D - 1000) Pr}{1 + 12.7 \left(\frac{f_f}{8} \right)^{1/2} (Pr^{2/3} - 1)}$	$0.5 \leq Pr \leq 2000$ $3000 \leq Re_D \leq 5 \times 10^6$ $\frac{L}{D} \geq 10$	

Table 3 provides a summary of Nusselt number correlations for single-phase natural convection flows across various geometries. These correlations are categorized based on boundary layer flow conditions (external or internal) and associated geometries, including vertical surfaces, inclined plates, horizontal plates, spheres, and enclosed cavities. It should be noted that the influence of the inclination angle is reflected in the Rayleigh number [40, 41].

Table 3: Summary of Nusselt Number correlations in natural convection flows[40, 41].

Natural Convection			
Boundary layer flow-external flow			
Geometry	Nusselt number correlation	Conditions	
Vertical Surface	$0.59Ra_L^{1/4}$	$10^4 \leq Ra_L \leq 10^9$	Laminar
	$0.1Ra_L^{1/3}$	$10^9 \leq Ra_L \leq 10^{13}$	Turbulent
	$0.68 + \frac{0.670Ra_L^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}}$	$Ra_L \leq 10^9$	
	$\left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$	Entire range of Ra_L	
Inclined Plate	$\left\{0.825 + \frac{0.387\cos\theta^{1/6}Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$	Entire range of Ra_L $0^\circ \leq \theta \leq 60^\circ$	
Horizontal Plate	$0.54Ra_L^{1/4}$	$10^4 \leq Ra_L \leq 10^7$ $Pr \geq 0.7$	Upper surface is a hot plate, or the lower surface is a cold Plate
	$0.15Ra_L^{1/3}$	$10^7 \leq Ra_L \leq 10^{11}$ <i>all Pr</i>	
	$0.52Ra_L^{1/5}$	$10^4 \leq Ra_L \leq 10^9$ $Pr \geq 0.7$	Lower Surface is a hot Plate, or the upper surface is a cold Plate
Horizontal Circular Cylinder	$\left\{0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$	$Ra_D \leq 10^{12}$	
Sphere	$2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$	$Ra_D \leq 10^{11}$ $Pr \geq 0.7$ Nu number is 2 if there is no fluid flow, which corresponds to pure conduction.	
Boundary layer flow-internal flow			
Vertical Parallel Plate Channels	$\frac{1}{24}Ra_S\left(\frac{S}{L}\right)\left\{1 - \exp\left[-\frac{35}{Ra_S(S/L)}\right]\right\}^{3/4}$	$10^{-1} \leq Ra_S\left(\frac{S}{L}\right) \leq 10^5$ Vertical channel. Symmetrically heated, isothermal plates. S and L are the parallel distance between the plate and the length of them, respectively.	
Inclined Channels	$0.645[Ra_S(S/L)]^{1/4}$	$Ra_S(S/L) \geq 200$ $0^\circ \leq \theta \leq 45^\circ$ S is the parallel distance.	
Rectangular cavity	$0.22\left(\frac{Pr}{0.2 + Pr}Ra\right)^{0.38}\left(\frac{H}{L}\right)^{-0.25}$	$Pr \leq 5$ $10^3 \leq Ra_L \leq 10^{10}$ $2 \leq H/L \leq 10$ Nu number highly depends on the cavity aspect ratio (H/L), where H and L are the cavity's height, width, and orientation angle.	

1.4. Two-phase gas-liquid flow patterns

Understanding flow patterns in two-phase systems is essential, as these patterns play a crucial role in determining heat and mass transfer behavior. Two-phase flows are generally classified according to flow patterns or regimes [46]. Various flow patterns can be observed in gas-liquid two-phase flows, depending on fluid properties, duct geometry, inclination angle,

and liquid/gas velocity ratio. Fig. 4 provides a comprehensive illustration of two-phase flow regimes in pipes of various orientations-vertical, horizontal, and inclined. These regimes are observed under different gas flow rates and are fundamental to understanding gas-liquid flow dynamics [47]. In Fig. 4. a, which shows a vertical pipe, the sequence of flow regimes begins with bubbly flow at low gas velocities. Here, the liquid acts as the continuous phase, with small bubbles of gas dispersed throughout the liquid. As the gas velocity increases, these bubbles coalesce into larger gas pockets, leading to the slug flow regime. In slug flow, elongated bubbles or "slugs" are separated by liquid segments that may contain smaller dispersed bubbles. Further increases in gas velocity lead to a chaotic and oscillatory pattern known as churn (or intermittent) flow, where the slugs lose stability, and liquid and gas mix irregularly. Finally, at even higher gas flow rates, the flow transitions to the annular flow regime, where the gas forms a continuous central core, and the liquid is pushed to the pipe walls, forming a thin film. In this regime, liquid droplets are often entrained in the gas flow. In Fig. 4-b, which illustrates the flow regimes in a horizontal pipe, the behavior of the two phases changes notably due to gravity, which causes the phases to separate more distinctly. At low gas flow rates, stratified flow is observed, with the gas phase occupying the top half of the pipe and the liquid flowing along the bottom. As the gas flow rate increases, larger gas bubbles begin to form, leading to slug flow in the horizontal pipe, where gas pockets push liquid slugs forward. With a further increase in gas velocity, the liquid at the bottom develops waves due to the shear forces at the interface, resulting in wavy flow. At high gas velocities, the annular flow regime occurs, where a liquid film coats the pipe walls, and the gas flows through the central region. This is similar to annular flow in vertical pipes, but the gravity-driven stratification influences the thickness of the liquid film along different parts of the pipe. Finally, Fig. 4-c demonstrates the flow regimes that occur in an inclined pipe, representing an intermediate case between vertical and horizontal configurations. In the inclined pipe, bubbly flow still involves discrete gas bubbles dispersed within the liquid, but the inclination causes the bubbles to migrate toward the upper side of the pipe preferentially. With increasing gas velocity, the slug flow regime appears, much like in vertical and horizontal pipes, but these slugs are influenced by both gravitational forces and the angle of inclination, making them less symmetric. As the gas velocity increases further, wavy flow can be seen, where waves form at the gas-liquid interface. In some cases, this can transition into a wavy annular flow, where the gas begins to dominate, and the liquid forms a wavy film along the pipe walls. Finally, at very high gas velocities, annular flow develops, where the liquid forms a thin layer along the pipe wall, similar to the behavior seen in both

vertical and horizontal pipes, though the film distribution may vary due to the inclined angle [48].

This comprehensive depiction in Fig. 4 shows how two-phase flow regimes vary with the pipe orientation and the gas injection rate, illustrating the different ways gas and liquid interact in different physical settings. Each regime presents unique characteristics that are significant for applications like oil and gas transportation, chemical processing, and heat exchangers, where understanding the behavior of multiphase flows is crucial for system design and optimization.

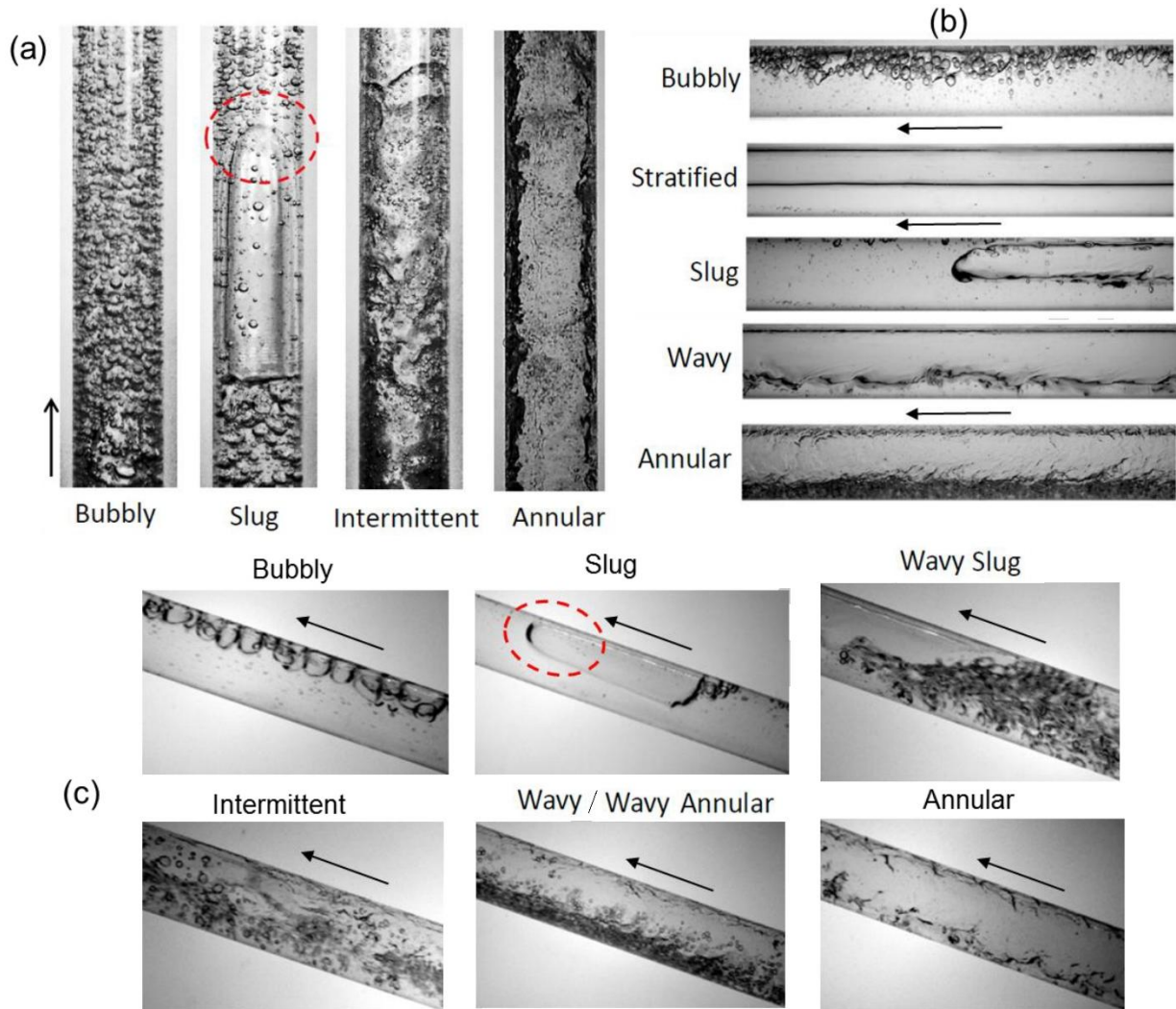


Fig. 4: Various two-phase flow regimes in (a) a vertical pipe, (b) a horizontal pipe, and (c) an inclined pipe [47]. The 12.7 mm ID polycarbonate was used for the flows in the picture with air- and water flow rates in the 0.001–0.2 kg/min and 1–10 kg/min, respectively (Figure is reprinted with permission from Elsevier).

Following Fig. 4, which provided an overview of the different flow regimes in vertical, horizontal, and inclined pipes, the focus now shifts to horizontal pipes. Fig. 5, adapted from Bhagwat and Ghajar [47], presents a flow regime map for a circular horizontal pipe. This map

depicts the relationship between superficial gas velocity (on the horizontal axis) and superficial liquid velocity (on the vertical axis), with different regions indicating distinct flow patterns. In this context, a flow regime called plug flow is also encountered. Plug flow occurs when the gas forms large, elongated bubbles or plugs that occupy most of the pipe's cross-section. These plugs are separated by pockets of liquid, creating a distinctive flow pattern that is less chaotic than churn flow but more dynamic than bubbly flow. Plug flow often transitions from slug flow under certain conditions, depending on the gas and liquid velocity.

Fig. 5 illustrates several key flow regimes, including bubbly flow, plug/slug flow, wavy flow, stratified flow, and annular flow:

- At lower gas velocities and higher liquid velocities, bubbly flow dominates. In this regime, small gas bubbles are distributed throughout the liquid.
- As gas velocity increases, the flow transitions to plug/slug flow, where larger bubbles (plugs) form, pushing through the liquid. The map shows that plug and slug flow share similar conditions and often overlap, which is why they are labelled together.
- With increasing gas velocity, wavy flow appears, followed by annular flow. In annular flow, the gas moves through the center of the pipe, while the liquid forms a thin film along the pipe walls.
- Stratified flow is also present, typically at low liquid velocities and moderate gas velocities. In this flow regime, the gas flows above a stable layer of liquid, creating a clear interface between the two phases.

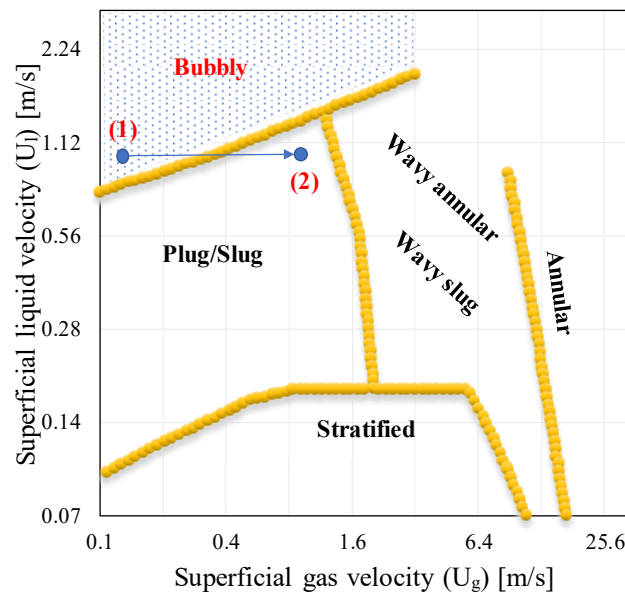


Fig. 5: Bubble flow regime map in a circular horizontal pipe. The 12.7 mm ID polycarbonate pipe was used, and the air and water flow rates are in the 0.001–0.2 kg/min range and 1–10 kg/min,

respectively (Figure is redrawn based on the original figure in Ref. [47], with permission from Elsevier)

Maintaining fluid flow in the bubbly regime is recommended for enhancing heat transfer. From a technical perspective, plug or slug flow can present challenges—such as large bubbles blocking the flow path (e.g., in plate heat exchangers or internal combustion engines) or causing a significant increase in pressure drop. Moreover, transitioning from the bubbly regime to slug or plug flow increases both the void fraction and bubble size. This transition can lead to the isolation of heat transfer surfaces from the fluid, ultimately reducing the heat transfer coefficient [49].

Several studies have highlighted the effectiveness of the bubbly regime in improving heat transfer. For example, Houston and Cornwell [50] showed that bubbly flow is the most effective regime for enhancing heat transfer. They suggested that the improvement in heat transfer comes from the turbulence generated by the motion of the bubbles. Bao et al. [51] indicated that increasing the gas flow rate and transition from bubbly flow to slug flow and annular flow causes sudden changes in pressure drop and a reduction in heat transfer coefficient. In another micro-channel study, Houshmand and Peles [52] showed that the thermal performance of the bubbly flow is better than the slug flow as the pressure drop is lower. Generally, the bubble motion contributes to increased mixing and agitation in continuous liquid flow, increasing the turbulence and directly affecting heat transfer enhancement.

This review article investigates the mechanism of heat transfer in bubbly flow. Additionally, for pipes, channels, and bubble columns, the flow patterns and regime maps for two-phase water-air flow are detailed in Refs [53-57]. The bubbly flow regime consists of discrete bubbles suspended in the liquid phase, with bubble sizes generally smaller compared to the characteristic dimensions of the flow geometry, such as pipe diameter or channel width [58, 59]. Due to its ability to enhance heat transfer, this flow regime is widely utilized in industries like petroleum, chemical, biochemical, and nuclear. The interplay between bubble coalescence and breakup significantly influences the flow characteristics in bubbly flow. These processes determine the bubble size distribution, ultimately affecting heat transfer efficiency in two-phase systems. Understanding and controlling these dynamics is crucial for optimizing the performance of industrial applications involving bubbly flow [60].

1.5. Bubble generation methods

Enhancing heat transfer through bubble injection in various applications heavily relies on the bubble sparger, which is essential for generating and distributing bubbles. The sparger's

significance lies in its ability to control bubble size distribution and manage flow regimes and transitions [61], which directly impacts the overall efficiency of bubble column reactors and heat exchangers. Although the performance of various spargers has been studied in some bubble column reactors [62-64], detailed research on the efficient design, operation, and impact of different spargers on heat exchanger performance remains limited.

There are three main categories when it comes to bubble generation methods in a bubble column: (1) orifice or perforated disc (Fig. 6-(a)), (2) porous sparger (Fig. 6-(b)), and (3) ejector or injectors (Fig. 6-(c and d)). For an orifice or perforated disc sparger, the gas exits through holes with diameters exceeding 1 mm, generating bubbles typically ranging from 3 to 6 mm in diameter [65]. Cylindrical or plate-based sparger heads with small nozzles or holes within linear, annular, or spiral-shaped pipes [66-68] are regarded as orifice categories (Fig. 6-(e)). A porous sparger is used to produce smaller bubble sizes. While it can produce a more uniform bubble distribution than an orifice one, the major drawback of this sparger is the clogging potential. Finally, injector spargers are used to produce small bubbles. For example, a needle sparger can create 0.5-1 mm bubble diameters.

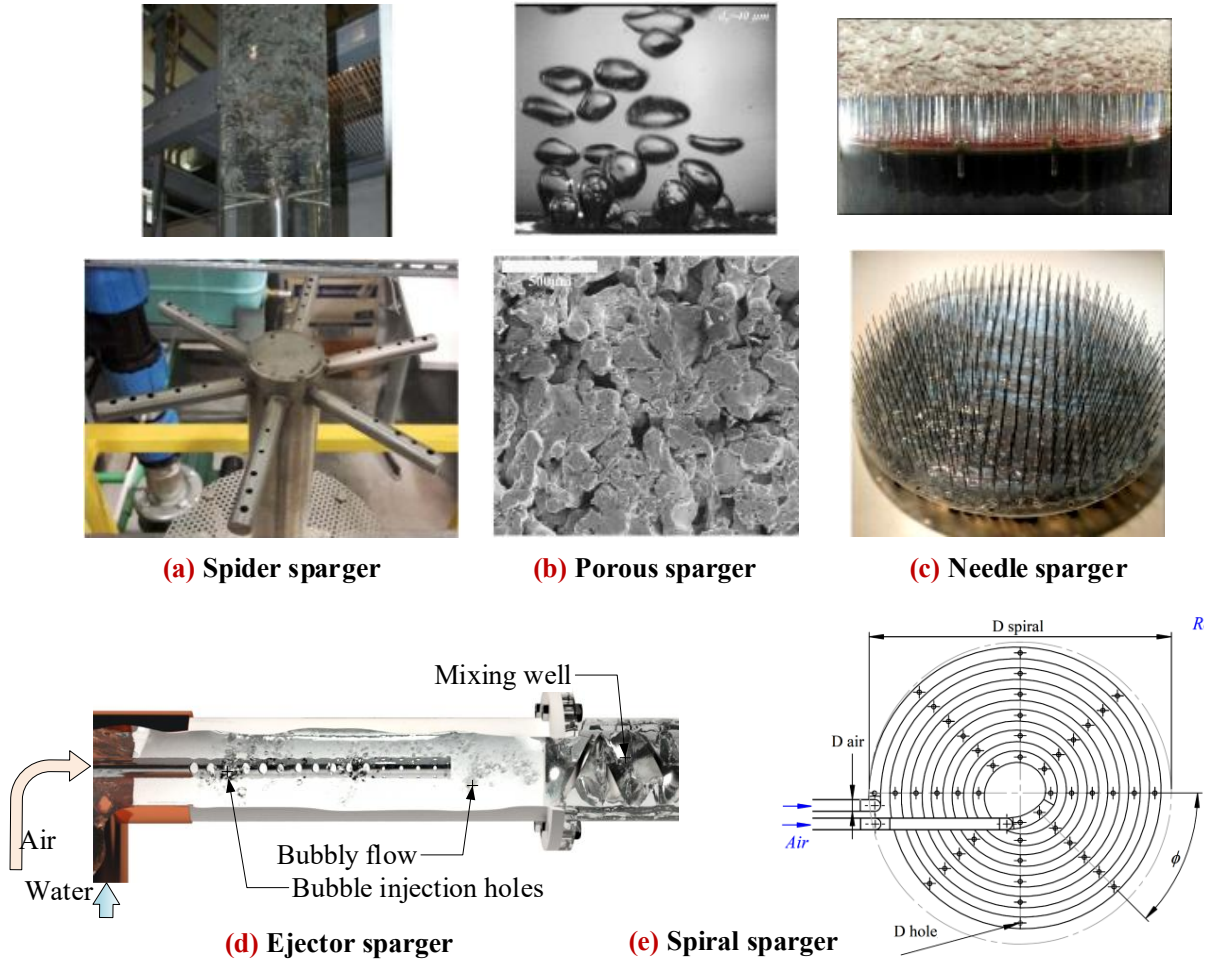


Fig. 6: Different types of spargers: (a) Spider sparger [61], which is classified into an orifice or perforated disc sparger, and it is suitable for generating bubbles in the size range of 3-6 mm [65]. (b) Porous sparger [69] can produce smaller bubbles than orifice ones, making the bubbles more uniform. (c) The needle sparger can be classified into the injector category [61], producing 0.5-1 mm bubble diameters. (d) Ejector sparger (this schematic sketched based on the ejector of Ghajar [17]). (e) Spiral sparger used by Zarei et al. [70] is an orifice category. (Figures a, b, c, and e are reprinted with permission from Elsevier).

In addition to the methods mentioned above, several other techniques are used for bubble generation. These include electrochemical processes, such as hydrogen production during water electrolysis [71, 72], which is used for microbubble generation (Fig. 7-(a)), and liquid-gas mixers like the venturi-type mixer (Fig. 7-(b)) [75].

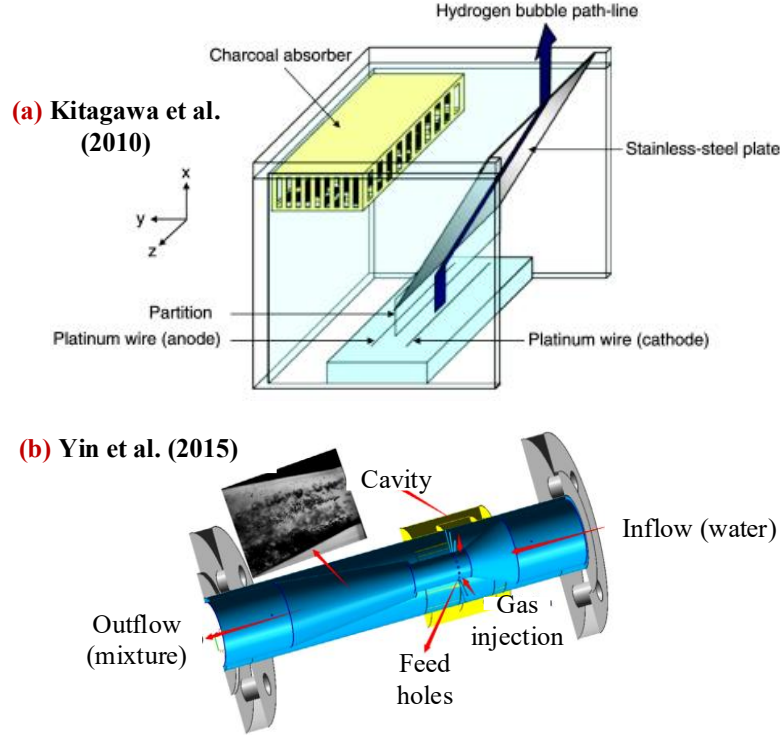


Fig. 7: (a) Micro-bubble generation using hydrogen production during water hydrolysis [72] uses platinum wire and platinum rod as a cathode and anode, respectively. The charcoal absorber is responsible for removing the generated chlorine bubbles. (b) the venturi-type mixer uses the compressor to inject gas into the venturi [73] (Figures are reprinted with permission from Elsevier).

In a study, Levitsky et al. [74] designed and introduced a new bubble generator, which can produce a bubble with controlled diameter using many hollow cylindrical bushings deployed at a 90° angle to one another in the flow. However, while these methods have not yet been applied to enhance heat transfer in heat exchangers, they present a promising avenue for future research.

1.6. Overview of Configurations and Fluids in Bubble Injection Research

Different configurations and fluid types play a critical role in the application of bubble injection for heat transfer enhancement. An overview of various configurations and fluid types used in bubble injection research is provided in Fig. 8. The configurations can be categorized into three groups: devices, heat exchangers, and simple structures.

The bubble injection technique is used in a variety of applications within the devices category, such as internal combustion engines and solar collectors. It is also applied to heat exchangers at multiple scales, from microscale applications in electronic devices [52], to small-scale experimental systems like refrigerators and cold thermal energy storage units [70], and up to large-scale industrial heat exchangers. Further study is required to successfully integrate this innovative method for enhancing heat transfer into practical industrial heat exchangers.

Fig. 8 also outlines the fluid types used in bubble injection research, categorized into gas and liquid phases. For the gas phase, common gases include air, nitrogen, hydrogen, helium, and pentane vapor. In the liquid phase, typical fluids are water, petroleum oils, ethylene glycol, and methanol. Since this study focuses on non-boiling bubble injection, the temperature must remain below the evaporation point of the liquid, which depends on the system's pressure conditions.

The primary goal of the bubble injection method is to enhance the heat transfer coefficient and thereby reduce thermal resistance. In shell and tube or double pipe heat exchangers, bubbles are injected into the side exhibiting higher thermal resistance or a lower heat transfer coefficient. Although bubble injection can be applied to both sides, several studies, including Moosavi et al. [75], have demonstrated that injecting bubbles into the shell side significantly enhances heat transfer. This effect is because the fluid flow inside the coil, due to its secondary flow, generally has a higher heat transfer coefficient and lower thermal resistance compared to the shell side.

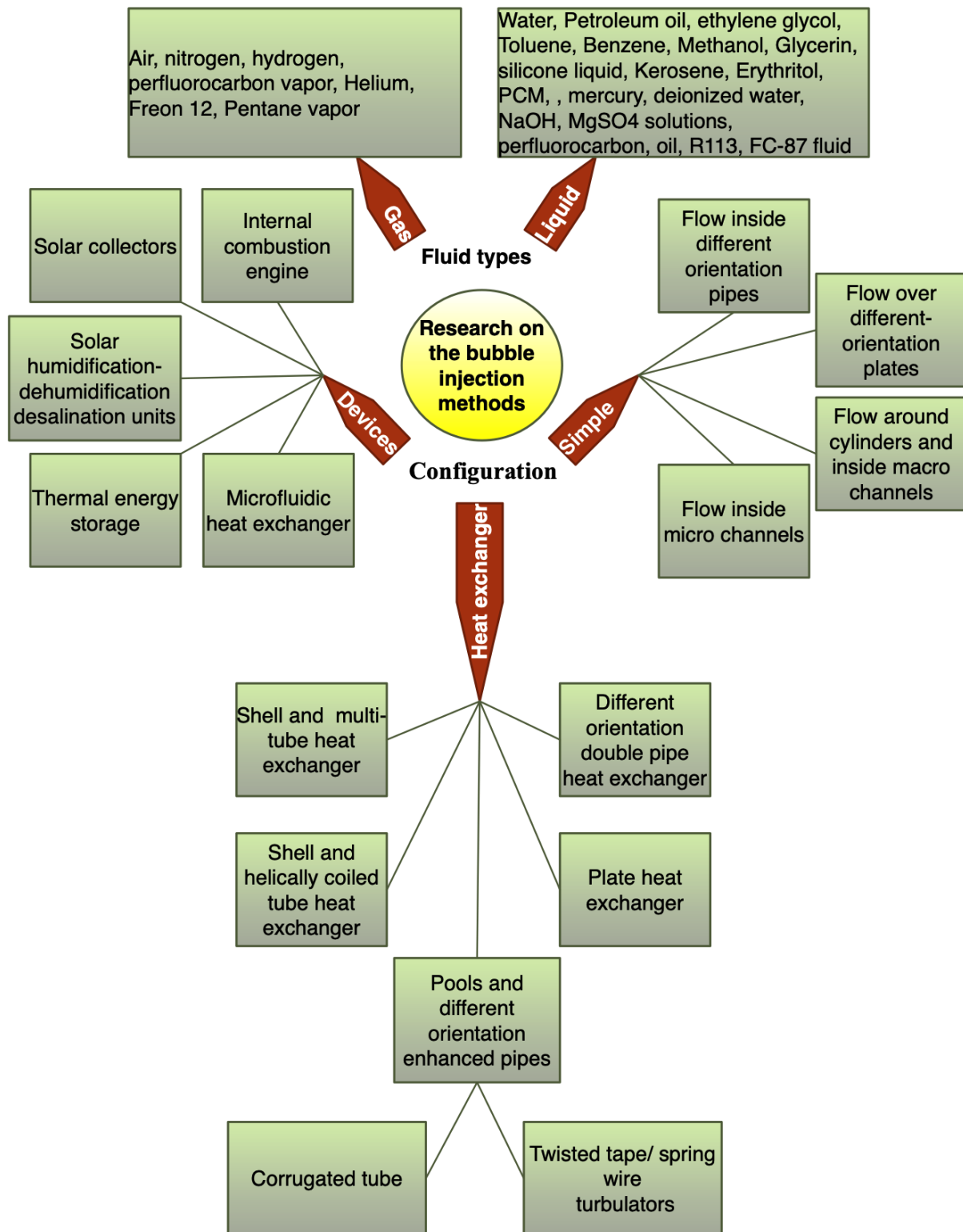


Fig. 8: Different configurations and fluids used in the literature for enhancing heat transfer via bubble injection.

The word cloud diagram for bubble injection applications in heat exchangers is shown in Fig. 9. This figure highlights that shell and tube, plate, and double pipe heat exchangers account for most bubble injection applications.

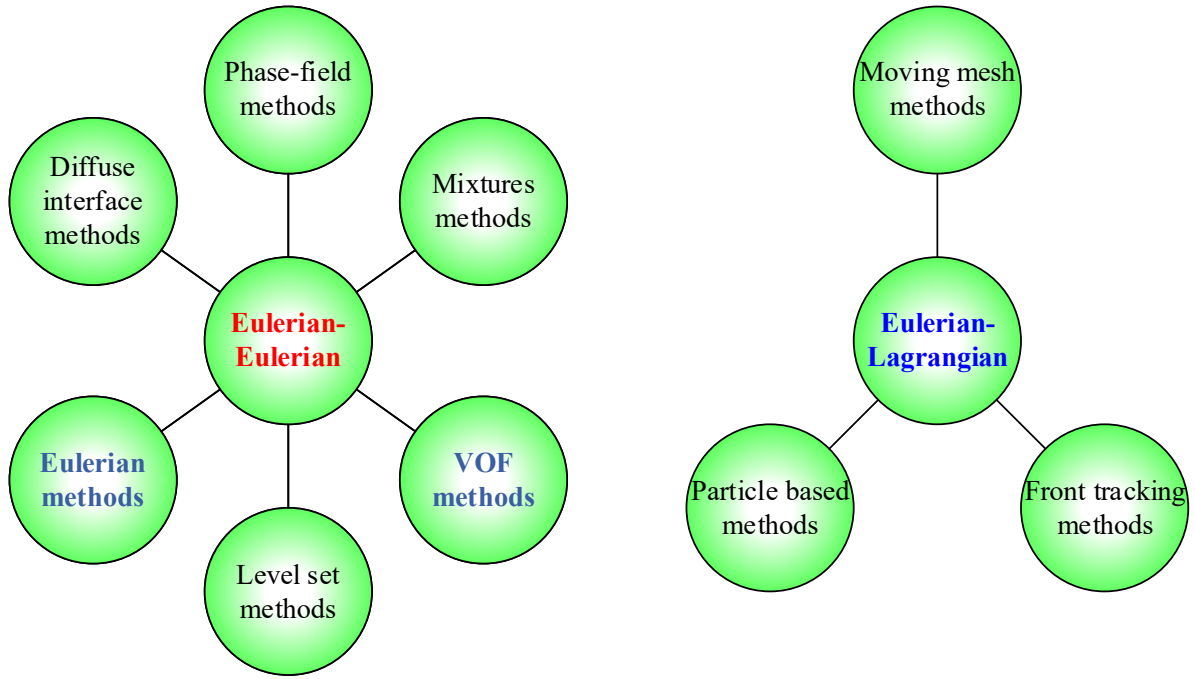


Fig. 10: Numerical methods for two-phase flow simulation [81].

In the Eulerian-Lagrangian framework [82-84], bubbles are considered a discrete phase, while base fluid, such as water, is considered a continuum phase. In models based on this framework, such as front-tracking, the position of the two-phase interface is determined by directly solving the equation of motion of the discrete phase [85]. On the other hand, the Eulerian-Eulerian approach considers both the liquid and gas phases as separate continuous phases. Although the Eulerian-Lagrangian models offer higher accuracy, they come at a higher simulation cost. Moreover, these models are unsuitable for addressing problems with significant interface changes. As a result, this review focuses on discussing Eulerian-Eulerian approaches. Between the Eulerian-Eulerian models, the most popular, versatile, and reliable approaches that can be effectively applied to two-phase bubbly flow are given in Fig. 11. They

have been used in the CFD tools such as Open FOAM, Flow-3D, and ANSYS-Fluent. These important methods are described in the present study.

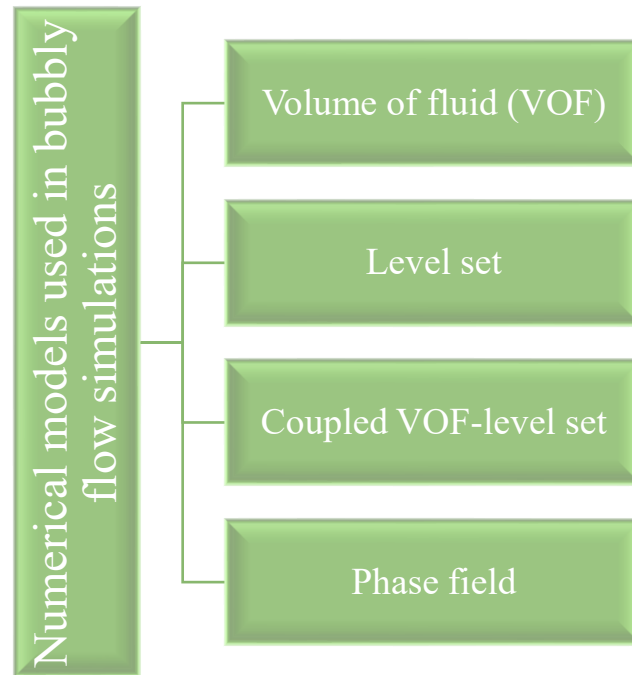


Fig. 11: Popular Eulerian-Eulerian models used for bubbly flow simulation [86].

2.1.2. Numerical analysis of two-phase bubbly flow with supercomputers

Experimental measurements of two-phase bubbly flow present several challenges, as discussed in “Sec 2.2”. However, advances in high-performance computing and supercomputers have facilitated the numerical examination of turbulent two-phase bubbly flows. Supercomputers are crucial for simulating bubbly flows because they possess the processing capacity to handle the complex interactions between bubbles and fluid dynamics [87]. Their high memory bandwidth and vast parallel processing capabilities allow researchers to perform large-scale Direct Numerical Simulations (DNS) of bubbly flows with remarkable detail and precision. These simulations track bubble motion, capture bubbles breakup and coalescence, and resolve turbulent fluctuations in the liquid phase. Numerous studies have demonstrated the effectiveness of supercomputers in simulating various features of bubble flows, from simpler single-phase systems to more complex multiphase ones [88].

A comprehensive physical understanding of turbulent bubbly flows can be achieved, primarily through DNS, which thoroughly resolves the fluid motion both inside and around each bubble. Although DNS is typically impractical for most real-world complex applications,

it provides valuable information and insights that can help improve turbulence closure models for small-scale simulations [87, 89]. The primary limitation remains the high computational cost; however, the DNS approach is regarded as a reliable data source alongside experimental methods [88, 90, 91].

As mentioned, direct numerical simulation of two-phase flows reveals significant details about complex flow phenomena [92, 93]. Table 4 summarizes the studies on implementing the DNS method to comprehend flow characteristics in bubbly flow. It is important to emphasize that the cost of running simulations with multiple bubbles rises as more grid points are needed to resolve each bubble and the flow surrounding them. The development of faster computers would make these kinds of simulations more and more possible [94]. Aside from a few simulations of drops breaking up in well-defined flows [95-97], not much work has been done on the bubble coalescence. Furthermore, as direct numerical simulations of multiphase flows become more common, there is a greater need to improve the theoretical framework for describing them. As a result, multiphase flow research will increasingly focus on simplifying simulation results into models that allow for more rapid predictions.

Table 4: Studies on DNS as well as their major findings [92].

Study	Study	Tracking Method	Remark
Feng et al. [98]	Studied the behavior of deformable bubbles in a vertical turbulent channel	The front-tracking method	This is the first study to prove that the DNS can effectively evaluate turbulence closure models.
Lu and Tryggvason [99]	Investigated turbulent bubbly flow in the upward direction in a vertical channel	The front-tracking method	deformability of the simulated bubbles had a significant impact on the void fraction profile
Dabiri et al. [100]	Investigated the impact of deformability of viscous bubbles on the flow rate in a bubble flow moving upwards in a vertical channel.	The front-tracking/finite-volume method	when the Eötvös number is between 2.0 and 3.0, a regime transition of vertical channel bubbly upward flow occurs.
Chekifi et al. [101]	Examined the breakup of air bubbles in flow-focusing configuration	The CLSVOF (coupled level set with volume of fluid) tracking method	As the flow velocity ratio increases, bubble size decreases sooner for low capillary numbers.
Bois [102]	DNS study of turbulent bubbly flow at the vertical channel at friction Reynolds number of 180.	front-tracking method	Results showed that the interfacial momentum closure is more significant than the turbulent closure hypothesis.
Cluzeau et al. [103]	DNS investigation of the Reynolds stresses in	front-tracking method	The interaction between bubble-induced fluctuations and single-phase turbulence was observed.

	turbulent bubbly flow in a duct at $Re=127$.		
Cluzeau et al. [104]	DNS investigation of forces in the two-phase turbulent bubbly flow in a channel.	front-tracking method	A new method for the lift force calculation was presented.
Innocenti et al. [105]	The DNS study of bubble-induced turbulence.	The volume of fluid (VOF) method	The kinetic energy spectra analysis of velocity fluctuations clearly showed the k^{-3} scaling.
Cluzeau et al. [106]	The DNS study of bubble-induced agitation in homogeneous bubbly flows	front-tracking method	There was a strong relation between wake-induced agitation and averaged wake fluctuations in homogenous bubbly flow.

2.1.3. Volume of fluid (VOF) method

The VOF method is a commonly used Eulerian approach for modeling multiphase flows. This method is particularly well-suited for simulating two-phase flows where a clear boundary separates the two phases. Examples of such flows include liquid-gas flows in various regimes, such as bubbly, slug, stratified, annular, and wavy flows. Another example of a two-phase flow that can be modeled using the VOF method is a flow with a free surface, where the interface between the air and liquid phases is not well-defined. In such cases, the VOF method effectively models the phasic flows and captures their interfaces. An indicator scalar function, denoted by α , is employed to determine the interface in the VOF method. This indicator represents the volume fraction of each phase in each cell [107]. In bubbly flows, the value of α for a specific cell is zero or one for the air or liquid phases in the cell, respectively. For the case where $0 < \alpha < 1$, the cell contains both air and liquid phases that indicate the interface passes through the cell.

While the VOF method offers advantages like straightforward implementation, low computational cost, and accurate mass conservation, making it attractive for simulating complex interfacial behavior and two-phase bubble flows, it suffers from a significant drawback: abrupt changes in volume fraction (α) can lead to non-physical outcomes at the interface [108, 109]. The accuracy of the VOF method can be enhanced by a more exact prediction of the surface tension and the interfacial curvature. To this end, some methods such as level set function [110], LS-based continuum surface force model [111], advanced curvature estimation approach based on piecewise linear interface calculation (PLIC)-VOF method [112] and other algorithms [113-116]. Fig. 12 summarizes key developments and studies aimed at improving the VOF method accuracy and efficiency.

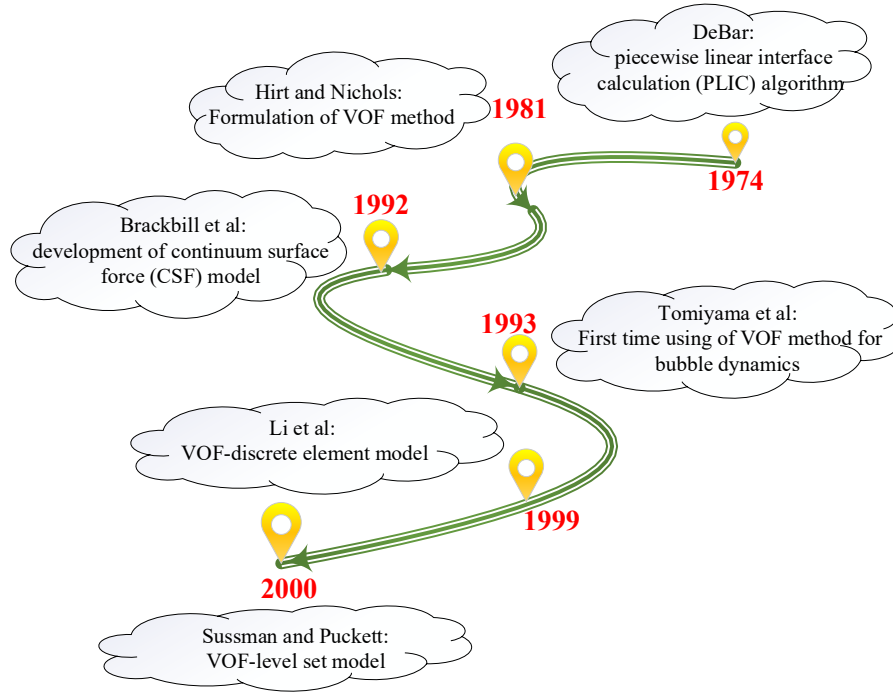


Fig. 12: Evolution of substantial developments in the VOF method [107, 110, 117-120] and [132].

Generally, the developments in the VOF method can be divided into two main categories, algebraic and geometric VOF, as shown in Fig. 13. The void fraction calculation is responsible for the difference between the two mentioned categories. In algebraic VOF, a function represents the void fraction, while in geometric VOF, it is represented by the geometry. The advantages and drawbacks of the two methods are listed in Fig. 13.

The algebraic-VOF can be separated into compressive and tangent hyperbola for interface capturing (THINC) schemes using a variety of flux calculation techniques. For the compressive schemes to compute the advection term, an artificial compressive term is required. A low Courant cell number is necessary for compressive schemes to obtain reliable results [121, 122]. Conversely, THINC schemes do not depend on the Courant number and compressive term. In this scheme, a hyperbolic-tangent profile is considered for the void fraction (α) in the cells of the interfacial area [123]. Although the computational cost for the THINC scheme is relatively lower than the compressive one [124], it is generally implemented into only the cartesian grids.

On the other hand, geometric-VOF uses geometrical methods to calculate the void fraction (α). The two main steps, reconstruction and interface advection, are required for geometric-VOF methods. Different methods have been developed for reconstruction schemes [125, 126], for instance, the gradient method, least-square VOF (LVIRA), and moment-of-fluid (MOF) interface reconstruction algorithms are examples of piecewise linear interface calculation (PLIC) techniques [127]. In the next step, the advection of the interface, the void

fraction is advected from the interfacial cell, which is calculated using geometrical approximation and can be traced using a Lagrangian trajectory. For a specific time interval, the two steps can be modelled by unsplit schemes in single step or split schemes in multiple step. Split schemes can be simply used than unsplit scheme, and generally suitable for structured meshes. Unsplit schemes are more accurate but owing to complex geometric reconstruction, the computational cost is high. Among split schemes, explicit Lagrangian schemes [128] and Eulerian implicit-Lagrangian explicit [129] are the most popular ones. Turning to unsplit schemes, conservative unsplit (CCU) schemes [127] or non-intersecting flux polyhedron advection (NIFPA) schemes [130] are among the most popular. The state-of-art review of the geometric-VOF method and its development is available in [131].

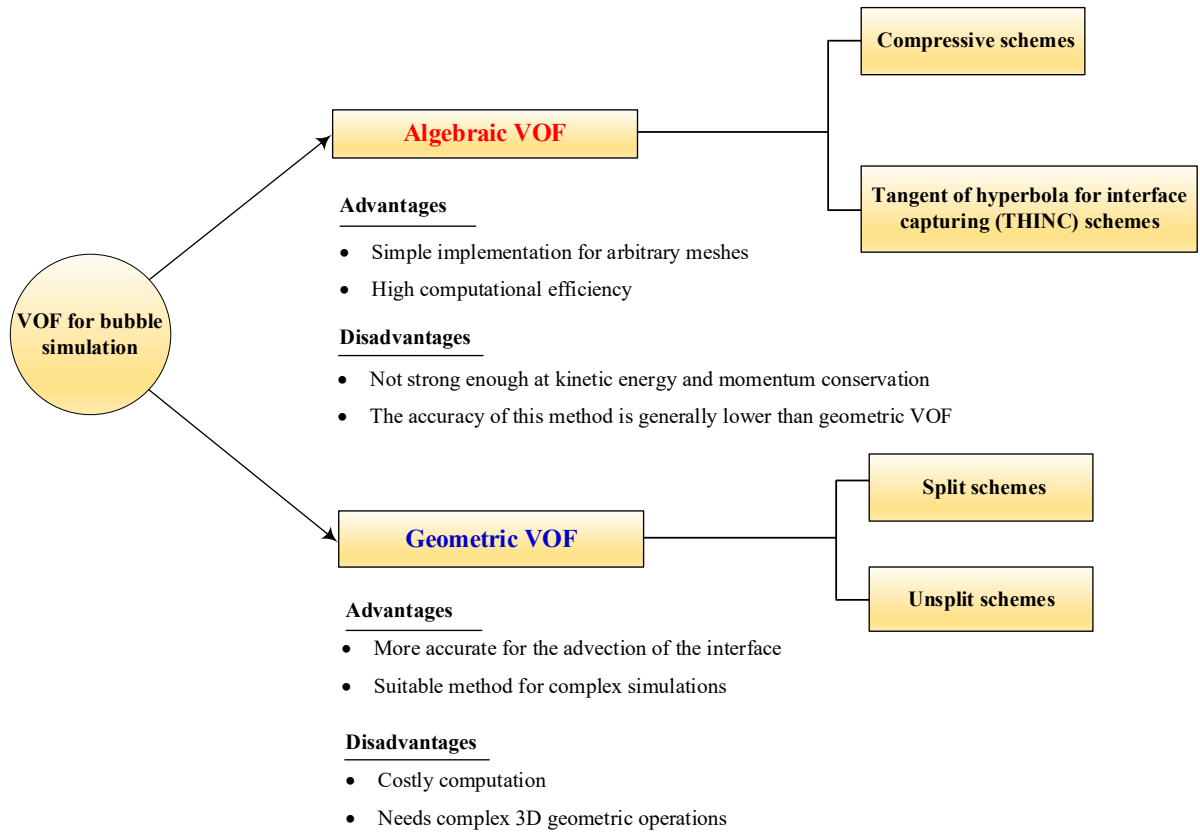


Fig. 13: Different VOF methods for bubbly flow simulation [132, 133].

In the VOF method, one set of governing equations is used, but the material properties change as the cells contain liquid, gas, or a fraction of each [107, 134]. The continuity equation for the incompressible phases is given as,

$$\nabla \cdot \vec{u} = 0 \quad (26)$$

The momentum equation for the liquid and gas phases is given by,

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p + \nabla \cdot [\mu (\nabla \vec{u} + \nabla \vec{u}^T)] + \mathbf{F}_\sigma + \rho \vec{g} \quad (27)$$

In this equation, u , p , and t represent the velocity vector, static pressure, and time [135]. F_σ denotes the surface tension. The volume fraction α is used to weigh the density ρ and viscosity μ based on the liquid phase and gas phase properties. That is,

$$\rho = \rho_l \alpha + \rho_g (1 - \alpha) \quad (28)$$

$$\mu = \mu_l \alpha + \mu_g (1 - \alpha) \quad (29)$$

In Eqs. (28) and (29), the subscripts g and l denote the gas and liquid phases, respectively. To calculate the surface tension F_σ , the following formula is used that considers this parameter as continuum surface force,

$$F_\sigma = \sigma \kappa \nabla \alpha \quad (30)$$

Where σ and κ are the surface tension coefficient and the local curvature at the interface. The mean interface curvature is estimated using,

$$\kappa = \nabla \cdot \hat{n} \quad (31)$$

where $\hat{n}$, the unit vector normal to the fluid-fluid interface is defined using the volume fraction,

$$\hat{n} = \frac{\nabla \alpha}{\|\nabla \alpha\|} \quad (32)$$

Elsayed et al. [136] introduced a new scheme, namely, the unit gradient of the signed distance function (SDF), $\Phi(X)$, to approximate the unit vector normal to the fluid-fluid interface as follows,

$$\hat{n} = \frac{\nabla \Phi}{\|\nabla \Phi\|} \quad (33)$$

They employed the least square method to calculate the gradient in the following formula. The Φ at a specific point X is evaluated using the following equation,

$$\Phi(X) = \text{sgn}(\alpha - 0.5) \cdot \|X - X_{ne}\| \quad (34)$$

where X_{ne} represents the nearest point on the interface to each cell center. The following equation is used to identify the location of the interface between two phases [109, 122],

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \vec{u}) + \nabla \cdot [(\alpha(1 - \alpha)) \vec{u}_c] = 0 \quad (35)$$

In Eq. (35), the third term is an additional artificial compressible term that is utilized to maintain the phase interface sharpness. The modeling velocity $\vec{u}_c$ is calculated by,

$$\vec{u}_c = c |\vec{u}| \frac{\nabla \alpha}{|\nabla \alpha|} \quad (36)$$

Where c represents the compression factor, which is equal to one. The energy equation is as follows,

$$\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\vec{u}(\rho E + p)) = (\nabla \cdot k \nabla T) + \dot{q} \quad (37)$$

Where E is the internal energy, k is the local average conductivity of the fluid, and $\dot{q}$ is the volumetric heat generation.

2.1.4. Level-Set and Phase field methods

Apart from the VOF method, two other important techniques for numerically modeling two-phase bubbly flow are the level set (LS) and phase field methods. Level set method utilizes a continuous LS function's zero-contour surface to depict the phase interface, which is determined by the signed distance function, representing the distance to the phase interface [137]. On the other hand, the phase field approach employs a chemical potential, which varies within a thin transitional layer, to provide the pressure of the interface. Basically, the physical sharp limit is substituted in phase-field models by a diffuse interface with a non-trivial thickness where the interfacial tension force and physical characteristics are dispersed gradually and continuously [138]. Table 5 summarises the functions of level-set and phase field approaches.

What would be worth mentioning is that level-set and VOF methods can be combined to produce a novel technique called the coupled level-set/volume-of-fluid (CLSVOF) approach, which can be utilized to gain higher accuracy in numerical modelling of bubbly two-phase flows. Many studies have reported satisfaction with the accuracy of geometry, even for complex flow [139]. Before computing the level-set function with the help of iterative geometric operation, one should first confirm the locations and shapes of the interface by rebuilding the interfaces. For interface reconstruction, the Piecewise Linear Interface Construction (PLIC) algorithm follows the approach that a normal vector and the volume fraction function c together determine a unique linear interface cutting the cell [140].

While the VOF, level set (LS), and phase field methods each offer distinct advantages and disadvantages for modeling two-phase bubbly flow (summarized in Fig. 14), the phase field approach generally presents a more advantageous option overall [141].

Table 5. Definition of Level-Set and Phase field functions [142-145].

Method	Main variable	Function	Properties
LS	$\phi(\vec{r}, t) = \begin{cases} -d(\vec{r}, \Gamma(t)) < 0 & \text{gas phase,} \\ 0 & \text{phase interface,} \\ d(\vec{r}, \Gamma(t)) > 0 & \text{liquid phase.} \end{cases}$ Γ is the gas-liquid interface.	$\frac{\partial \phi}{\partial t} + \vec{u} \cdot \nabla \phi = 0$	$\begin{cases} \rho_\epsilon(\phi) = \rho_d(1 - H_\epsilon(\phi)) + \rho_c H_\epsilon(\phi) \\ \mu_\epsilon(\phi) = \mu_d(1 - H_\epsilon(\phi)) + \mu_c H_\epsilon(\phi) \end{cases}$ $\rho_\epsilon(\phi)$ and $\mu_\epsilon(\phi)$ are density and viscosity in the proximity of the phase interface
Phase field	$C: [-1, 1] \rightarrow \begin{cases} C = 1 \rightarrow \text{continuous phase} \\ C = -1 \rightarrow \text{disperesed phase} \end{cases}$	$\begin{cases} \frac{\partial C}{\partial t} + u \cdot \nabla C = M_0 \nabla^2 C \\ \varphi = \frac{\delta F}{\delta C} = -\frac{3}{2} \sigma \epsilon \Delta C + 24 \frac{\sigma}{\epsilon} C(1 - C)(1 - 2C) \end{cases}$ $M_0 = \chi \epsilon^2$ (M_0 is Mobility) $\sigma = \frac{2\sqrt{2} K}{3 \epsilon}$ (σ is Interfacial tension) $F_\sigma = \phi \vec{\nabla} C$ (F_σ is surface Force)	$\begin{cases} \rho = \frac{(1 + \varphi)\rho_L + (1 - \varphi)\rho_G}{2} \\ \mu = \frac{(1 + \varphi)\mu_L + (1 - \varphi)\mu_G}{2} \end{cases} \quad (39)$ ρ and μ are averaged values of density and viscosity in the phase interface

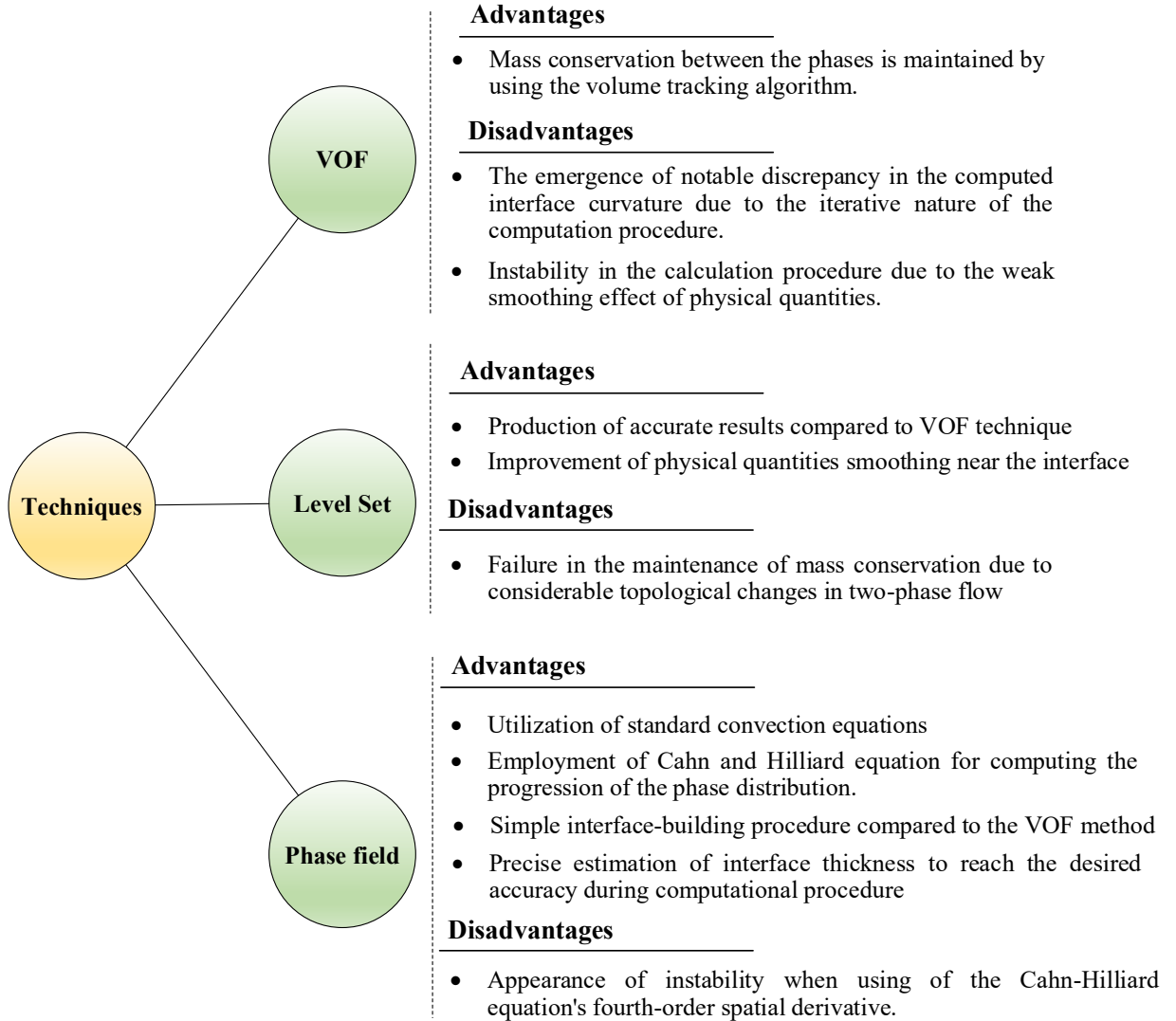


Fig. 14: Comparison between VOF, Level-Set, and Phase field methods [145-149].

2.1.5. Eulerian method

Another approach for simulating two-phase flows is the Eulerian (two-fluid) method. This method solves the mass, momentum, and energy conservation equations separately for each phase, and the interaction between the phases is included. Accordingly, the liquid and gas phases are treated as co-existing continua. The continuity equation for each phase can be written as [150-155],

$$\frac{\partial(\rho_l \alpha_l)}{\partial t} + \nabla \cdot (\rho_l \alpha_l \vec{u}_l) = \Gamma_{lg} \quad (40)$$

$$\frac{\partial \rho_g \alpha_g f_i}{\partial t} + \nabla \cdot (\rho_g \alpha_g \vec{u}_g f_i) = S_i - f_i \Gamma_{lg} \quad (41)$$

In the above, S_i is additional source term that accounts for the creation and destruction of bubbles due to coalescence and breakup, and f_i is the scalar variable of the dispersed phase. The term $f_i \Gamma_{lg}$ denotes the mass transfer because of the condensation redistributed for each of the discrete bubble classes. As $\alpha_g f_i = n_i v_i$, the void fraction of the vapor phase, α_g , and the scalar function, f_i , are connected to the number density of the discrete bubble i th class (n_i) (The volume corresponding to the bubble diameter is denoted by v_i). The size distribution of the dispersed phase is, therefore, defined by the scalar f_i . Γ_{gl} is the rate of mass transfer from the gas phase to the liquid phase and the relationship between Γ_{gl} and Γ_{lg} is $\Gamma_{gl} = -\Gamma_{lg}$. Furthermore, Γ_{lg} and Γ_{gl} are both set to zero for isothermal mode in a liquid-gas bubbly flow regime where no amount of heat and mass is transferred between the phases [156]. The momentum equations for the liquid and gas phases, respectively, are given as,

$$\begin{aligned} \frac{\partial \rho_l \alpha_l \vec{u}_l}{\partial t} + \nabla \cdot (\rho_l \alpha_l \vec{u}_l \vec{u}_l) = & -\alpha_l \nabla P + \alpha_l \rho_l \vec{g} + \nabla \cdot [\alpha_l \mu_l^e (\nabla \vec{u}_l + (\nabla \vec{u}_l)^T)] \\ & + (\Gamma_{lg} \vec{u}_g - \Gamma_{gl} \vec{u}_l) + F_{lg} \end{aligned} \quad (42)$$

$$\begin{aligned} \frac{\partial \rho_g \alpha_g \vec{u}_g}{\partial t} + \nabla \cdot (\rho_g \alpha_g \vec{u}_g \vec{u}_g) = & -\alpha_g \nabla P + \alpha_g \rho_g \vec{g} + \nabla \cdot [\alpha_g \mu_g^e (\nabla \vec{u}_g + (\nabla \vec{u}_g)^T)] \\ & + (\Gamma_{gl} \vec{u}_l - \Gamma_{lg} \vec{u}_g) + F_{gl} \end{aligned} \quad (43)$$

where, F_{lg} is the applied force from the liquid to the gas phase. Similar to the mass transfer rate, $F_{lg} = -F_{gl}$. The energy equations for liquid and vapor phases are as follows,

$$\frac{\partial \rho_l \alpha_l H_l}{\partial t} + \nabla \cdot (\rho_l \alpha_l \vec{u}_l H_l) = \nabla \cdot [\alpha_l \lambda_l^e \nabla T_l] + (\Gamma_{lg} H_g - \Gamma_{gl} H_l) \quad (44)$$

$$\frac{\partial \rho_g \alpha_g H_g}{\partial t} + \nabla \cdot (\rho_g \alpha_g \vec{u}_g H_g) = \nabla \cdot [\alpha_g \lambda_g^e \nabla T_g] + (\Gamma_{gl} H_l - \Gamma_{lg} H_g) \quad (45)$$

where F_{lg} is the total interfacial force, which is illustrated in the following section.

2.2. Experimental measurement techniques in two-phase bubbly flow

2.2.1. Overview

As heat transfer and fluid flow are inherently coupled, understanding the flow structure and velocity distribution within a two-phase, non-boiling, bubbly flow is crucial for comprehending the associated heat transfer mechanisms. Various techniques are commonly employed to measure the characteristics of two-phase bubbly flow, including:

- Photon attenuation technique (Makwana et al. [157])
- Fiber Optic Sensor (Tien et al. [158])
- Electromagnetic flow meter (Cha et al. [159])
- Turbine flowmeter (Uchiyama et al. [160])
- Ultra-fast MRI (Tayler et al. [161])
- Ultrasonic Doppler method (Zhang et al. [162])
- Particle Image Velocimetry (PIV) (El-Adawy et al. [163])

Among the various approaches listed above, Particle Image Velocimetry (PIV) and the Ultrasonic Doppler Method are discussed in greater detail below, as they are particularly effective for capturing detailed flow characteristics and bubble dynamics in two-phase bubbly flow regimes. These methods offer distinct advantages in terms of accuracy, spatial resolution, and applicability to different experimental setups, making them essential tools for studying the complex interactions between phases in such fluid flows.

2.2.2. Particle Image Velocimetry (PIV)

Particle Image Velocimetry (PIV) is a widely used technique for measuring velocity fields. It involves tracking the movement of tiny, neutrally buoyant particles within the flow to determine the velocity field.

In Fig. 15-(a), the PIV configuration for measuring the velocity field in single-phase flows is depicted schematically. A laser sheet is utilized to illuminate the test section's flow. Tracer particles are introduced into the flow, and they track the velocity of the fluid. The tracer particles are considered to be ideal when they meet the following criteria [164]:

- (1) They accurately follow the fluid movement.
- (2) The properties or the flow of the fluid are not affected by them.
- (3) No interaction takes place between them.

Any flow structure is only discernible upon the velocity field evaluation. The tracer particles are illuminated by two or more laser light pulses spaced at a specified time interval; as a result, the particle spacing is proportional to the local velocity vector. Once the highly

resolved PIV photos have been post-processed, the velocity field is retrieved from the image taken at an angle to the laser sheet [165] and [166]. But these days, PIV has advanced to the point where direct particle image capture is possible using electronic cameras ([167], [168], and [169]).

Various tracking techniques could be applied to the data processing. These comprise methods like particle tracking velocimetry [170], auto-correlation [171], and cross-correlation [172]. The auto-correlation method carries out the post-processing is on a single picture that has been double exposed to the flow, indicating that the particles illuminated by the first and second laser pulses are present in the same picture. However, one of the major drawbacks of the PIV algorithms utilizing autocorrelation and cross-correlation techniques is that they have a limited spatial resolution, and the size of the interrogation window primarily determines the limited spatial resolution. This disadvantage is significant, particularly now that the usage of digital photos has spread. Digitization is necessary since electronic camera resolution and image format are orders of magnitude lower than those of photographic medium [173]. Particle tracking velocimetry can be employed to achieve the best spatial resolution, which involves determining the displacements of individual tracers [174]. New algorithms based on pattern recognition have gained popularity in recent years; neural networks, fuzzy logic approaches, and genetic algorithms seem to offer the most potential for particle tracking [175, 176].

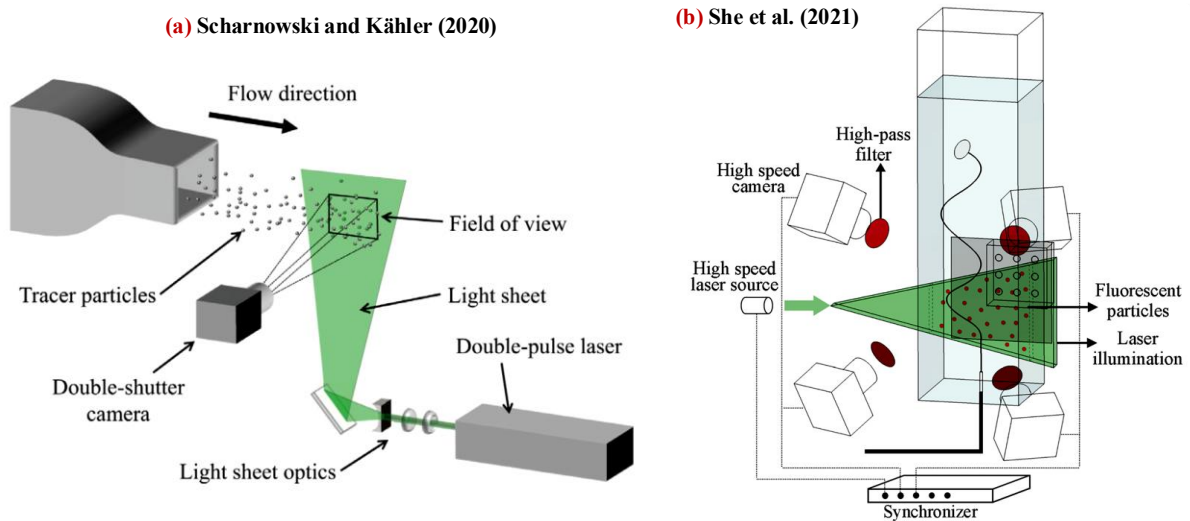


Fig. 15: Particle image velocimetry configuration: the measurement plane is illuminated by a laser sheet, which allows for the visualization of tiny tracer particles seeded into the flow. (a) In the case of single-phase flow, images of particles are captured by a CCD camera positioned at a right angle to the laser sheet [177]. (b) Different seeding techniques are used for each phase of two-phase flows: one type is used to scatter light from the laser source, and the other type produces light at a different wavelength. An additional camera with the proper filter can detect this emitted light. [178] (Figures a and b are reprinted with permission from Elsevier and AIP Publishing, respectively)

A specific number of seeded particles, typically at least 9–10, are present in each image's tiny sub-region. Subsequently, a spatial autocorrelation function of the region is computed [179],

$$R_{11}(\delta_x, \delta_y) = \lim_{\substack{x \rightarrow +\infty \\ y \rightarrow +\infty}} \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} F(x, y) F(x + \delta_x, y + \delta_y) dy \quad (46)$$

where $F(x, y)$ is an image's scalar property. R_{11} has two peaks: one at $(0, 0)$, where the image perfectly overlaps with itself, and another, smaller peak for $(\delta_{x^*}, \delta_{y^*})$, where two distinct pulses illuminate the particles with an overlap between them. Hence, δ_{x^*} and δ_{y^*} denote the elements of the most likely displacement of the particles within the interrogation window. With knowledge of the time interval between the laser pulses, this information can be used to compute the local velocity vector. This process is integrated over each subregion to extract the entire velocity field. A more modern and widely used technique involves taking distinct frames of the two images and performing a cross-correlation among the associated subregions.

The PIV approach can be simply applied to multiphase flow problems if the various phases are identified. There are many known techniques for separating the pictures produced by various phases. One solution to this issue is the combination of fluorescent tracers with an image acquisition setup containing two cameras [180] and [181]. As seen schematically in Fig. 15-(b), optical filters can distinguish the radiation scattered by fluorescent and neutral particles. In this setup, one camera records both phases, while the second camera—with an appropriate filter—captures images containing only the fluorescent seed particles. By subtracting the filtered images from the unfiltered ones, it becomes possible to isolate and analyze each phase separately.

The three possible routes that light scattered by particles illuminated by the laser sheet can take before reaching the camera are shown in Fig. 16:

- 1) The real particle images can be created by the first path (the direct path).
- 2) Light reflection at the interface of the gas/liquid creates the second path.
- 3) The third path is caused when the light beams are refracted twice: when the interface of gas/liquid interface is curved, a different place might be the interface of the refracted light, which is followed by a second refraction back into the first medium. The second phase also contains these virtual particles.

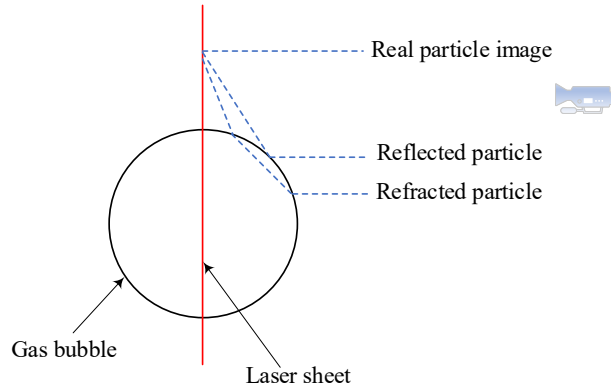


Fig. 16: Application of the PIV method in two-phase flows Creating simulated particle images near the boundaries of gas-liquid interfaces. Due to these optical phenomena, it seems necessary to employ an alternative approach to ascertain the gas/liquid interface, as the reliability of the PIV image itself is in question [164].

In an experimental and mathematical study, El-Adawy et al. [163] applied the magnetohydrodynamic (MHD) to analyze the motion of diesel/compressed natural gas (CNG) bubbly flow, following the addition of iron oxide nanoparticles (Fe_3O_4) to the liquid fuel in a horizontal pipe. The PIV and the high-speed video camera system (HSVC) were the two different approaches used for the experimental measurements. The PIV study's findings show that as the magnetic field intensity increased, the diesel/nanoparticle mixture's axial velocity profile shifted from a parabolic to a comparatively flat form.

Chen et al. [182] experimentally investigated the two-phase bubbly flow in a 180-degree elbow by developing a gas-liquid two-phase measurement technology called simultaneous Particle image velocimetry and pulsed shadowgraphy (PIV-PS). The schematic of PIV-PS technology is shown in Fig. 17. This novel method was used to investigate the two-phase features in a 180-degree elbow, achieved by connecting two cameras via a spectroscopy and optical filters. Furthermore, a method for post-processing images was suggested that relies on detecting bubble areas and estimating remaining particles. The findings demonstrate that the horizontal velocity on the elbow's centreline rises linearly as the liquid flow rates increase.

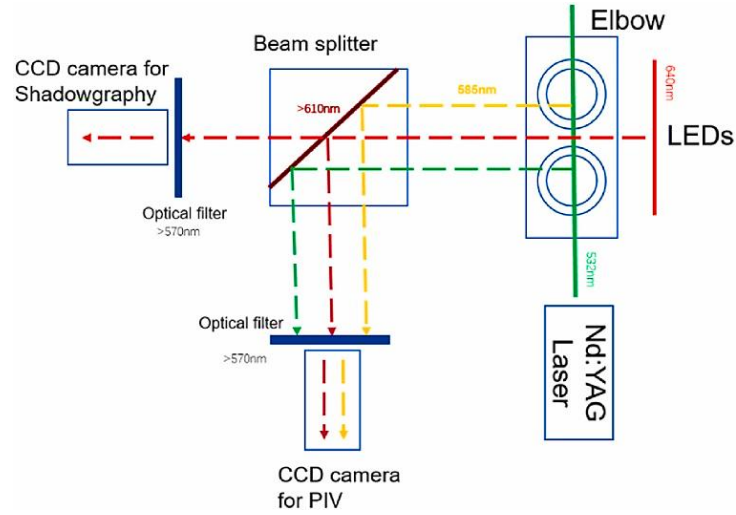


Fig. 17: PIV-PS principle. There are three light sources: LED light (640 nm), planar laser light (532 nm), and fluorescence (580 nm) produced by laser-activated fluorescent particles. These three lights simultaneously enter the beam splitter, where Dichroic and high pass filters separate them. The LED light passes through the dichroic filter, while the fluorescence reflects into the vertically arranged CCD camera [182].

2.2.3. Ultrasonic Doppler Method

Another effective approach for measuring the characteristics of two-phase bubbly flows is the ultrasonic Doppler method (UDM). This approach measures the instantaneous velocity profile along its measuring line utilizing pulsed echography of an ultrasonic beam utilizing an ultrasonic transducer (TDX). Ultrasonic Doppler Methods offer several advantages, including the ability to measure flow both spatially and temporally, applicability to opaque liquids such as liquid metals and magnetic fluids, and the capability to perform liquid line measurements. When UDM is applied to two-phase bubbly flows, ultrasonic pulses are reflected by seeding micro-particles in the liquid phase and at gas-liquid interfaces. Consequently, the velocity data captured by the Ultrasonic Velocity Profile (UVP) monitor includes information from both phases [162].

The UVP monitor and TDX are two critical components of ultrasonic Doppler measurement systems. The measured velocity profiles need to be separated into velocity information about each phase, i.e., liquid and gas, because ultrasonic pulses reflect on liquid-gas interfaces. A statistical method was used in [183] to calculate the liquid velocity information using the UDM for bubbly-flow measurements. The first stage in splitting the velocity data is to get the probability density function (PDF) from all recorded instantaneous velocities at each measuring position. In terms of an upward bubbly flow, the rising velocity of the bubbles is greater than the liquid velocity. Therefore, if a bubble crosses the measurement line, an instantaneous velocity profile shows a noticeable peak [184].

Ultrasonic pulses are regularly emitted at a pulse repetition frequency (PRF) along the measurement line, as shown in Fig. 18, to extract the Doppler shift from the carrier wave. The resulting collected echo signals are then demodulated. Every echo signal is often pre-processed using quadrature phase demodulation, which involves multiplying each signal by the cosine and sine components before filtering the result to remove the carrier wave frequency using a low-pass filter. The dotted line in Fig. 18 illustrates the resampling of each demodulated signal at the same delay period, corresponding to a specific location on the measuring line. The Doppler signal, which carries the Doppler shift frequency generated by the moving droplets in the associated sample volume, is then formed from the resampled data [185].

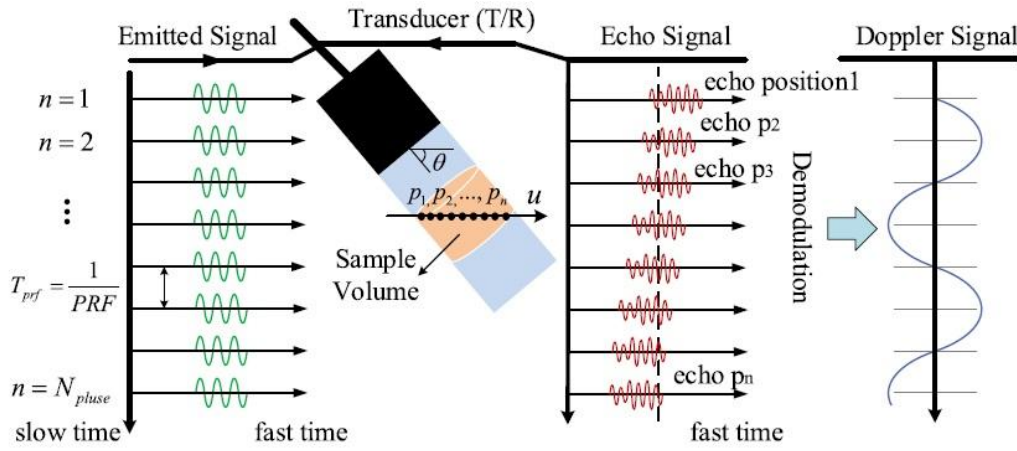


Fig. 18: Demodulating ultrasonic Doppler signals using the UDM method. The UDM was developed to address the challenge of extracting high-resolution Doppler shift information from a single emitted pulse by utilizing repeated pulse emissions and echo receptions [185] (Figure is reprinted with permission from Elsevier).

3. Fundamental Physical mechanisms

Bubble injection is a reliable technique for significantly enhancing heat transfer—by up to 100 times—compared to single-phase flow conditions (Deckwer [186]). Therefore, understanding the physical mechanisms involved in heat transport within bubbly flows is crucial. This knowledge can lead to developing more efficient designs and optimizing heat exchangers and similar devices. The impact of bubble injection on heat transfer is influenced by factors such as the geometry of the system, fluid flow rate, flow turbulence, bubble-induced agitation, bubble behavior, and void fraction. A comprehensive grasp of these factors is essential for optimizing bubble injection in heat transfer applications. The heat transfer mechanism in two-phase flow is quite complex and is coupled with the bubble motion and dispersion. Investigation of the dynamic of bubbles can help understand the two-phase bubbly

flow system and the associated heat transfer enhancement [187]. Therefore, the bubble behavior is first discussed, followed by the heat transfer mechanisms in these flows.

3.1. Bubble dynamic mechanisms

Before studying the behavior of the bubbly flow, the forces that affect bubble motion are reviewed first.

3.1.1. Forces affecting bubble motion

To better understand the two-phase bubbly flow regime, it is necessary to identify the influential forces acting on the bubbles and the interaction between phases. Solving the momentum equation (Eq.(47)) determines the interaction between the liquid and gas phases, including drag, lift, turbulent dispersion, wall lubrication, virtual mass, pressure, and Basset history forces, as illustrated in Fig. 19 [188-190]. That is,

$$\sum F_{l \rightarrow q} = F_{l \rightarrow g}^D + F_{l \rightarrow g}^L + F_{l \rightarrow g}^{TD} + F_{l \rightarrow g}^{WL} + F_{l \rightarrow g}^{VM} + F_{l \rightarrow g}^{Pressure} + F_{l \rightarrow g}^{Basset} \quad (47)$$

Where D, L, TD, WL, VM, Pressure, and Basset correspond to the drag, lift, turbulent dispersion, wall lubrication, virtual mass, pressure, and Basset history forces, respectively.

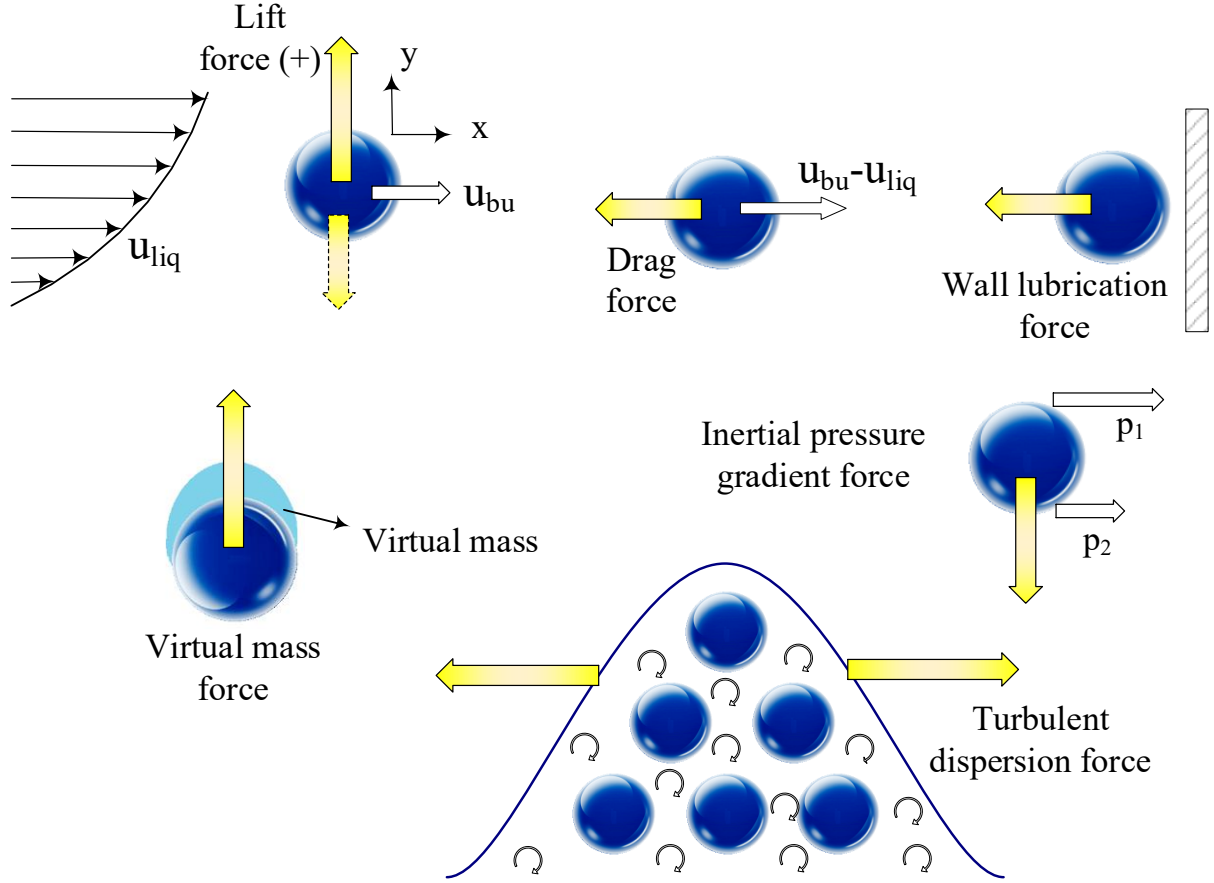


Fig. 19. Schematic of effective interphase forces in the bubble motion. The direction of the lift force is positive if $u_{liq} > u_{bu}$, and may be reversed due to the lift reversal phenomenon, which is explained more in [188-190].

3.1.1.1. Drag force

The drag force on the dispersed phase in a gas-liquid bubble column results from the interaction between the gas and liquid phases. This force, comprising both pressure and viscous drag components, is influenced by factors such as bubble shape, fluid properties, and the relative velocity between the phases. The drag force significantly affects the flow pattern, particularly the velocity fields of the gas and liquid phases, making it a critical factor in the dynamics of bubble columns. Therefore, accurate modeling and evaluation of the drag force are crucial in simulating gas-liquid bubble column flows [191, 192],

$$F_{l \rightarrow g}^D = -\frac{1}{2} \rho_{liq} C_D \|u_{bu} - u_{liq}\| (u_{bu} - u_{liq}) (A_i) \quad (48)$$

where u and A_i are the velocity and interfacial area, respectively. Due to the dependence of the drag coefficient on bubble size, shape, and flow field conditions, it can be defined as a function of bubble Reynolds number (Re), which characterizes the ratio of inertial forces to viscous forces, the Eötvös number (Eu), which represents the ratio of buoyancy to surface tension

forces, and the Morton number (Mo), which relates the effects of buoyancy force and viscosity force on turbulence [190].

Tomiyama et al. [193] presented a correlation for a single bubble under various conditions given by:

$$C_D = \text{Max} \left[\left[\frac{16}{Re_b} (1 + 0.15 Re_b^{0.687}), \frac{48}{Re_b} \right], \frac{8}{3} \frac{Eo}{Eo + 4} \right] \quad (49)$$

$$10^{-14} < Mo < 10^7, 10^{-2} < Eo < 10^3, 10^{-3} < Re_b < 10^6$$

Eq. (49) shows a non-linear relation between the drag coefficient and the dimensionless numbers. The literature shows a vast number of correlations for the bubble drag coefficient. For example, when it comes to bubble swarm (which is most common in the bubble injection method), Roghair et al. [194] suggested a correlation for drag coefficient given by:

$$C_{D,Swarm} = (1 - \alpha) C_{D\infty} \left(1 + \alpha \left(\frac{22}{0.4 + Eo} \right) \right) \quad (50)$$

$$0.13 < Eo < 4.83, 6.6 < Mo < 10.6, \alpha = 45\%$$

where $C_{D\infty}$ is calculated as follows:

$$C_{D\infty} = \sqrt{C_D(Eo)^2 + C_D(Re_b)^2} \quad (51)$$

Here $C_D(Eo)$ and $C_D(Re_b)$ are determined as:

$$C_D(Eo) = 4 Eo / (9.5 + Eo) \quad (52)$$

$$C_D(Re_b) = \frac{16}{Re_b} \left(1 + 2 / \left(1 + \frac{16}{Re_b} + \frac{3.315}{\sqrt{Re_b}} \right) \right) \quad (53)$$

Total drag coefficients are the result of pressure and viscous drag. The pressure differences across the bubble generate an additional drag that increases as the bubble's diameter or rising velocity increases. Chuang and Hibiki [195] and Khan et al. [188] provided a detailed review of the forces affecting bubble motion.

3.1.1.2. Lift force

In a shear flow, a bubble experiences a force from the surrounding liquid in a direction perpendicular to its relative motion. This force, commonly known as the shear-induced lift force [196], plays a crucial role in predicting the distribution of volume fractions in bubbly flows. Originating from inertial effects, the lift force is inherently complex to describe.

For the transverse migration of spherical bubbles, the shear lift force (the Saffman force) is first calculated [189, 192, 197]. The shear lift may be expressed as the cross-product of the slip velocity between phases and the local vorticity of the continuous phase. Nowadays, it is widely acknowledged that the lift force is positive (the positive direction is shown in Fig. 19) for a spherical bubble in laminar shear flows, regardless of the bubble Reynolds number [198]. However, as described in the following subsection (the lift reversal), the sign of lift can be reversed depending on the flow condition and bubble shape [199].

The shear-induced lift is given as [196]:

$$F_{l \rightarrow g}^L = -C_L \rho_{liq} \alpha_{bu} (u_{bu} - u_{liq}) \times (\nabla \times u_{liq}) \quad (54)$$

In this relation, C_L is the coefficient of lift force, u_{bu} is the velocity of the bubble, and u_{liq} is the velocity of the liquid phase.

At higher Reynolds numbers, the vorticity around the deformed bubble affects the wake behind the bubble, changing the lift force (vortex-induced lift force). As illustrated by Lighthill [200] and Auton [201], two vortices rotating counter-clockwise in the wake of the sphere produce lift force. The buoyancy force may be significantly less than the lift force generated by the shedding vortex [202]. Tomiyama et al. [203] assumed that the shear and vortex-induced forces are responsible for creating lift force, presenting the following correlation,

$$C_L = \begin{cases} \text{Min}[0.288 \tanh(0.121 Re_b, f(Eo_d))], & Eo_d < 4 \\ f(Eo_d) & , \quad 4 \leq Eo_d \leq 10.7 \\ -0.29 & , \quad Eo_d > 10.7 \end{cases} \quad (55)$$

$$f(Eo_d) = [0.00105 Eo_d^3 - 0.0159 Eo_d^2 - 0.0204 Eo_d + 0.474] \quad (56)$$

$$Eo_d = g(\rho_l - \rho_g) d_h^2 / \sigma, \quad d_h = d_b \sqrt[3]{1 + 0.163 Eo^{0.757}} \quad (57)$$

As it is clear, the sign of the lift coefficient may be negative, which refers to the reversal lift phenomenon and is discussed in the following section.

3.1.1.3. Reversal lift phenomenon

A critical factor that plays a key role in bubble lateral distribution is the lift reversal phenomenon, which significantly affects heat transfer in the flow. That is, the heat and mass transfer are profoundly affected by the direction of bubble migration in wall-bounded flows [204]. Thanks to experiments and numerical studies, it has been proven that the lift force for spherical bubbles and all ranges of Re number is positive [198], but there is another story for deformed bubble shape. The direction of the lift force is a function of bubble deformation [205], and an elliptical bubble experiences a negative lift force at a high Re number [206-208]. Adoura

et al. [209] showed that the vorticity produced at a non-spherical bubble accounts for lift reversal phenomena, which depended on the aspect ratio of the bubble.

According to [205], bubble lateral migration is significantly affected by its size. Smaller bubbles move towards the wall of the pipe, while larger bubbles migrate towards the center, mainly because of the wall-induced lift. In the case of an air-water system (at $T=25\text{ }^{\circ}\text{C}$ and $P=1\text{ atm}$), bubbles with a diameter equivalent to 1 to 5 mm are classified as small bubbles, while those exceeding 5 mm are considered large bubbles [210, 211]. To evaluate lateral bubble motion, Fig. 20-a illustrates the variation of the total transverse lift coefficient (C_T), which combines wake-induced and shear-induced lift coefficients as a function of bubble diameter in a pipe flow for an air-water system [205]. Accordingly, lateral bubble distribution has three potential regimes: migration to the wall, neutral, and migration to the core. Regarding wall regimes, C_T is controlled by viscous force and, in fact, bubble Reynolds number. Small bubbles rise in the viscous regime without shape oscillation and usually in a straight path and C_T has a positive value, resulting in the migration of bubbles toward the wall. In the core region, the C_T is influenced by both surface tension and gravitational forces. These effects can be characterized by the Eötvös number, which represents the ratio of buoyancy to surface tension forces. The bubble motion in this regime is not straight, experiencing a zigzag or spiral path and C_T changes its sign from positive to negative, meaning bubbles migrate toward the pipe center. The lateral migration of the bubble in the neutral regime is affected by various factors, such as bulk liquid turbulence.

As mentioned before, the lift reversal is directly related to bubble deformation and the bubble aspect ratio for a deformed bubble. So, the lift coefficient depends on three parameters: $Re (=2bU/\nu)$, dimensionless shear rate ($=\Omega D_{bu}/U_{rel}$), and bubble aspect ratio ($=b/a$), in which b and a are the horizontal and vertical axes of the bubble, U_{rel} is the velocity relative of the bubble to the liquid, Ω is the vorticity of a linear shear flow, and ν is the kinematic viscosity of fluid [209]. As shown in Fig. 20-b, the lift has been increased for a moderate aspect ratio (approximately 2), followed by a dramatic decrease and changing the sign for a higher aspect ratio, which depends on the bubble Reynold number [209]. The negative sign shows that the deformed bubble's direction is reversed compared to the spherical bubble. The lift force acting on a bubble is inherently complex and not fully comprehended. The results presented in this context are specific to experiments documented in the literature and should be regarded as such rather than being interpreted as universal laws.

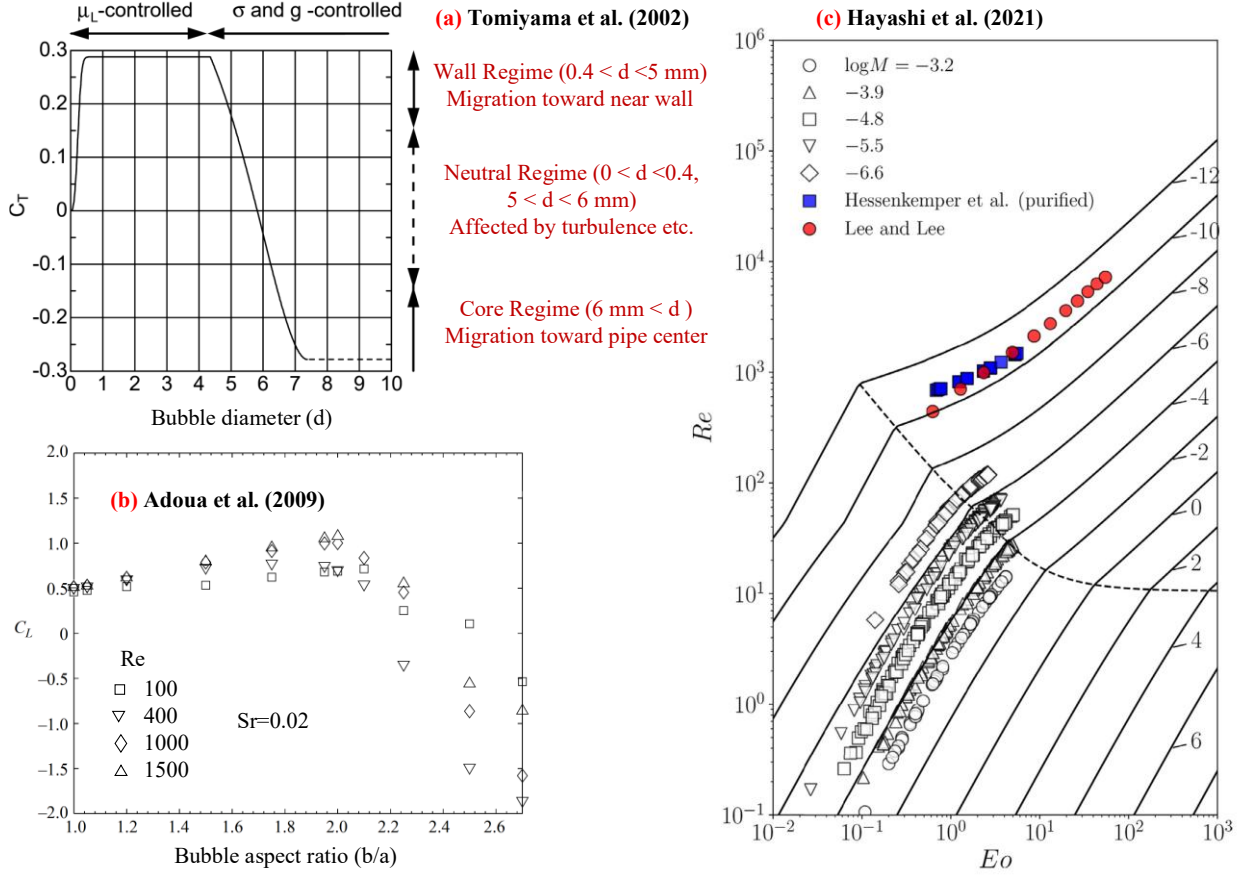


Fig. 20: (a) Traverse lift coefficient (C_T) and potential regimes for lateral bubble distribution in air-water systems. The lift coefficient is controlled by viscous force (Re) near the wall and by surface tension and gravity forces (Eo) in the pipe center [205]. (b) Variation of lift coefficient (C_L) as a function of the bubble aspect ratio, the lift coefficient becomes negative after a specific aspect ratio value, which depends on the Re number [209]. (c) Lift coefficient (C_L) map as a function of Re , Mo , and Eo numbers. The dashed line shows the transition from wall (viscous) to core regime [206] (Figures (a, c) and b are reprinted with permission from Elsevier and Cambridge University Press, respectively).

Hayashi et al. [206] collected the lift coefficient data of Tomiyama et al. [193], Lee and Lee [207], and Hessenkemper et al. [208], presenting a map as a function of Re , Mo , and Eo numbers, as shown in Fig. 20-c. The dashed line indicates the transition between the viscous and core regimes. They presented a correlation for evaluation of the lift coefficient in reversal cases as follows:

$$C_L = C_L^S(Re, Sr) - 0.078\omega_{max}^{*\infty}(AR) \left[\frac{8}{3} \frac{Eo}{Eo + 16(AR^2 - 1)/AR^{8/3}} \right] \quad (58)$$

Here AR , C_L^S , $\omega_{max}^{*\infty}$ are the bubble aspect ratio, lift coefficient for a spherical bubble [198], and dimensionless vorticity in infinite Reynolds number [212], which are given as,

$$C_L^S = \left[\frac{1}{2} \left(\frac{1 + 16/Re}{1 + 29/Re} \right) \right] + \left[\frac{6}{\pi^2} \frac{2.255}{\sqrt{SrRe} [1 + 0.2Re/Sr]^{3/2}} \right] \quad (59)$$

$$\omega_{max}^{*\infty} = \frac{2AR^{5/3}(AR^2 - 1)^{3/2}}{AR^2 \sec^{-1}(AR) - (AR^2 - 1)^{1/2}} \quad (60)$$

Various lift coefficients for different two-phase flow conditions can be found in Ref [188].

3.1.1.4. Turbulent dispersion force

The turbulent dispersion force describes the force produced by the interaction of the continuous phase turbulent eddies and the dispersed phase. This force critically affects the bubble volume fraction profile. The dispersion of bubbles from high-concentration regions to low-concentration regions is caused by this force [213]. While the dispersion force is a function of turbulence intensity, this force is proposed to be given as,

$$F_{l \rightarrow g}^{TD} \sim \nabla \alpha_{bu} \quad (61)$$

For example, for bubbly and slug flow, Lucas et al. [199] presented the following correlation,

$$F_{l \rightarrow g}^{TD} = -0.1 \rho_{liq} k_{liq} \nabla \alpha_{bu} - C_{TD} \rho_{liq} (Eo - 1) \nabla \alpha_{bu} \quad (62)$$

$$C_{TD} = \begin{cases} 0.0015, & Eo \geq 1 \\ 0, & Eo < 1 \end{cases} \quad (63)$$

Because the mass and momentum equation derivatives do not provide a natural model for the dispersion force, this force and also information on turbulence intensity need more investigation than other forces mentioned [214].

3.1.1.5. Wall lubrication force

The wall lubrication force in bubble injection flow is a force that acts on the bubble due to its interaction with the solid surface, and it arises due to the thin liquid layer between the bubble and the wall. The lubrication (squeeze film) builds pressure near the wall that pushes the bubble away from the wall. The wall lubrication force is particularly significant in low Reynolds number flows, where viscous forces dominate and strongly influence bubble behavior. In these cases, the wall lubrication force can be dominant, and neglecting it can lead to incorrect predictions of the bubble trajectory and deformation. Antal et al. [215] assumed laminar flow and a spherical bubble shape and proposed the following correlation for the wall lubrication force:

$$F_{l \rightarrow g}^{WL} = \frac{2 \rho_{liq} \alpha_{bu}}{D_{bu}} |V_{relative-wall}|^2 [C_{W1} + C_{W2}(D_{bu}/2y_W)] \hat{n} \quad (64)$$

where y_W and $\hat{n}$ are the bubble distance from the wall and the wall-normal direction, respectively. Here C_{W1} and C_{W2} are given as,

$$C_{W1} = -0.104 - 0.06|V_{relative-wall}| \quad (65)$$

$$C_{W2} = 0.147 \quad (66)$$

Based on experimental data, Tomiyama [216] modified the Antal et al. [215] model by considering the bubble distortion effect,

$$F_{l \rightarrow g}^{WL} = \frac{D_{bu} C_W \rho_{liq}}{2} |V_{relative-wall}|^2 \left(\frac{1}{y_{wall}^2} - \frac{1}{(D_h - y_{wall})^2} \right) \hat{n} \quad (67)$$

where D_h is the equivalent hydraulic diameter of the flow channel. In Eq. (67), C_W is calculated as,

$$C_W = \begin{cases} 0.47 & , \quad Eo_d < 1 \\ \exp(-0.933E_0^{0.179}) & , \quad 1 < E_0 < 5 \\ 0.00599E_0 - 0.0187 & , \quad 5 < E_0 < 33 \\ 0.179 & , \quad E_0 \geq 33 \end{cases} \quad (68)$$

The model of Tomiyama [216] is valid for $\log Mo = -2.8$. There are several alternative correlations for the lubrication force in the literature [195].

3.1.1.6. Virtual mass force

The virtual mass force arises due to the acceleration of bubbles in the liquid phase, which causes the surrounding liquid to be carried along with the bubble. This effect becomes more noticeable when there is a high-density difference between the primary and secondary phases. The virtual mass force can be calculated using the following equation [217],

$$F_{l \rightarrow g}^{VM} = -\rho_{liq} C_{VM} \left(\frac{D_{bu} V_{rel}}{Dt} - V_{rel} \cdot \nabla(u_{liq}) \right) \quad (69)$$

The parameters in the Eq. (69) are defined as,

$$C_{VM} = \begin{cases} \frac{1}{2} \alpha \frac{1+2\alpha}{1-\alpha} & , \quad \text{bubbly flow} \\ 5\alpha \left[0.66 + 0.34 \left(\frac{1 - \frac{D_{bu}}{L_{bu}}}{1 - \frac{D_{bu}}{3L_{bu}}} \right) \right] & , \quad \text{slug flow} \\ V_{rel} = u_{bu} - u_{liq} & \end{cases} \quad (70)$$

where L_b is the cylindrical length of the bubble.

3.1.1.7. Inertial pressure gradient force

If a spherical bubble is immersed in a liquid, the pressure gradient action on the bubble can be calculated by integrating the volume [218],

$$F_{l \rightarrow g}^{Pressure} = \int_v \nabla p dv = -(1/6\pi D_{bu}^3) \nabla p \quad (71)$$

The momentum equation can be used to calculate the ∇p , neglecting the viscous term outside of the boundary layer,

$$\nabla p = -\rho_{liq} \frac{Du_{liq}}{Dt} \quad (72)$$

Where the D/Dt in the material derivative. The inertial pressure gradient can be written in the final format as below:

$$F_{l \rightarrow g}^{Pressure} = \rho_{liq} \frac{Du_{liq}}{Dt} (1/6\pi D_{bu}^3) \quad (73)$$

3.1.1.8. Basset history force

The Basset history force is an unsteady viscous force resulting from relative velocity changing with time and delaying the boundary layer development. Although Basset's history force can be significant in unsteady flows, it is often negligible in bubbly flows where drag force predominates [219]. This force is given as [190, 191]:

$$F_{l \rightarrow g}^B = \frac{3}{2} D_{bu}^2 \sqrt{\pi \rho_{liq} \mu_{liq}} \int_0^t \left(\frac{Du_{liq}}{D\tau} - \frac{Du_{bu}}{D\tau} \right) \frac{d\tau}{\sqrt{t - \tau}} \quad (74)$$

Some correlations for different force types are available in [188, 220].

3.1.2. Bubble rising instability

3.1.2.1. Overview

Gas bubbles rising in a liquid are prevalent in both natural and industrial environments. The bubble-rising process significantly impacts the hydrodynamics of the continuous phase by introducing perturbations and generating wakes [221, 222]. In addition, the bubble deforms as it rises, which affects its wake and the hydrodynamic of the liquid phase [223]. Consequently, understanding bubble motion and path instability-which involve heat and momentum transfer between bubbles and the liquid-is crucial for optimizing thermal systems, including bubble-enhanced heat exchangers.

3.1.2.2. Rising instability due to the bubble wake and turbulence

For many years, scientists have been intrigued by the mechanism of bubbles rising and oscillating in still liquids due to their complex motion and dynamic wake structures. The bubble motion equation is influenced by density-driven effects, fluid resistance, and sideways pressure differences. For small bubbles, the trajectory during ascent is typically a straight line. However, the behavior changes for larger bubbles. Initially, large bubbles ascend in a straight line, but as their velocity increases, their motion becomes dominated by a zigzag or spiral path [178, 224, 225]. The reason is that the wake symmetry breaks when the bubble's speed approaches a critical value, creating vortex shedding. Because of the wake's instability, bubbles rise in a zigzag pattern. This phenomenon is known as path instability [86].

Bubble rise has been investigated extensively in the literature. The wake instability of a fixed spheroidal bubble was explored in-depth by Magnaudet and Mougin [212]. Although when the bubble ascends, its shape is approximately ellipsoidal. Experiments on bubble rise and wake formation conducted by Brücker [226] demonstrated a connection between the bubble's zigzag motion and the vortex shedding. In another study, Kure et al. [227] conducted a statistical study on the effect of hydrostatic pressure and bubble ascending instability. They suggested that decreased hydrostatic pressure and increased bubble equivalent diameter lead to path oscillation and zigzag motion. Zenit and Magnaudet [224] proved in their experimental study that the main reason for the bubble's path instability in the rising process was the bubble shape. It should be noted that while the bubble's wake determines the path instability in still water, the shape of bubble greatly influences the wake [228]. Veldhuis et al. [223] studied the relationship between the elliptical bubble shape and path instability when it rises.

She et al. [178] conducted an experimental investigation using tomographic particle image velocimetry (TPIV) with shadow image reconstruction and studied the bubble shape and wake flows. During the zigzag motions, they presented their 3-D results for the bubble wake and shape (aspect ratio). An ellipsoidal bubble with $d_{eq}=5.47$ mm, and $Re=1390$ was injected in a water tank (150 mm × 150 mm × 600 mm) via a nozzle with $d_{noz}=4.5$ mm. The dimension of the tank was large enough to disregard the wall effect. The 3D bubble trajectory and the corresponding bubble shape for the eight positions are shown in Figures 21-(a) and-(b), respectively. Points 2 and 6 denote the maximum lateral displacement and inversion points, while point 4 is an inflection point with zero lateral displacement. It can be seen from Fig. 21-(c) that the shape and its curvature during the ascending change during the process, and the curvature was related directly to lateral bubble movements. The shape oscillations of the bubble

clearly can be seen in Fig. 21-(d), which shows the variation of the bubble aspect ratio (defined as the ratio of the major bubble axis to the minor bubble axis) at different heights. She et al. [178] suggested that different vortex-shedding modes were the main reason for bubble-shape oscillations. The same trend was also reported by Brücker [226].

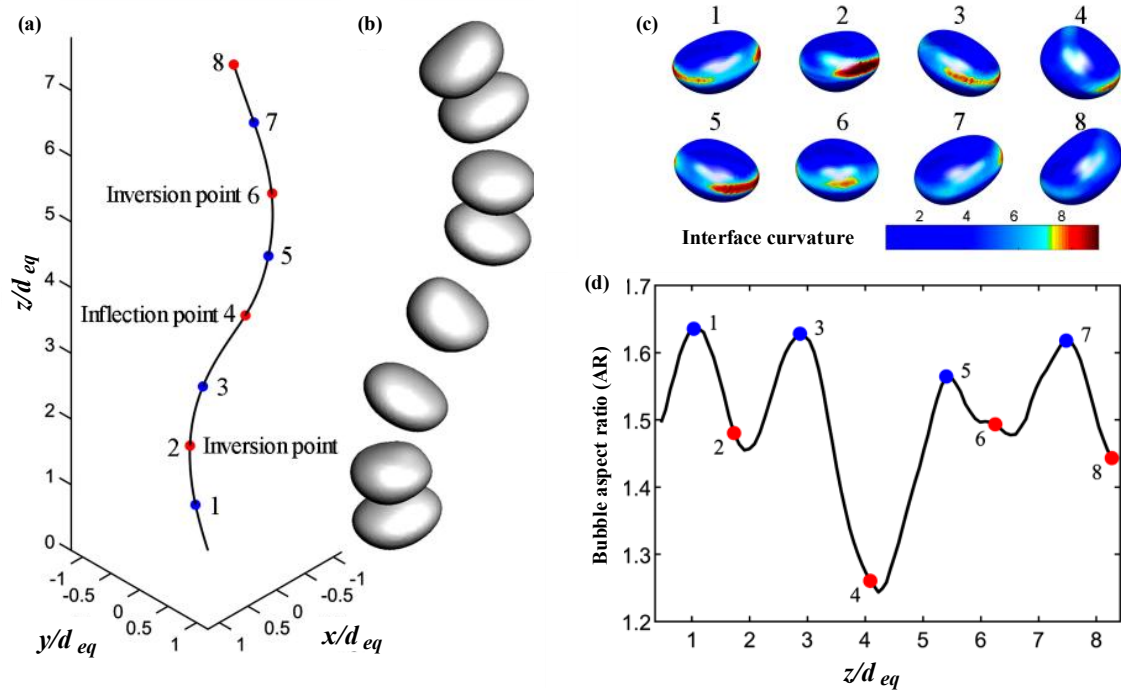


Fig. 21: (a) Zigzag path of the bubble in the course of ascending, and (b, c) bubble shape corresponds to 8 points illustrated in (a), and (d) bubble aspect ratio (AR), which is defined as the ratio of major to minor bubble axis, for She et al. [178] study. An ellipsoidal bubble with $d_{eq}=5.47$ mm, and $Re=1390$ injected in a water tank ($150\text{ mm}\times 150\text{ mm}\times 600\text{ mm}$) via a nozzle with $d_{noz}=4.5$ mm. The temperature was $25\text{ }^{\circ}\text{C}$, and the tank size was large enough to disregard the wall effect. (Figure is reprinted with permission from AIP Publishing).

The bubble wake structures, as reported by various studies, are shown in Fig. 22. Fig. 22-(a) shows the vortex structure for the experimental study of Lunde [229]. Fig. 22-(b) depicts the numerical vortex formation from Antepará et al. [230], while Fig. 22-(c) illustrates the vortex structure for point 8 in Fig. 21 (She et al. [178]), all of them for zigzag bubble ascending. As mentioned before, this figure shows that the hairpin vortex is not symmetrical. It is impressive that the PIV data, numerical simulation, and flow visualization results show similar hairpin vortex structures, while slight differences exist in dimensionless numbers (Re , Mo , Eo) for these different studies. The hairpin refers to structures whose legs are joined to the bottom side of the bubble. As shown in Fig. 22-(b), a counter-rotating hairpin vortex forms with a new head as the bubble's trajectory alters direction. The previous hairpin vortices dissipate downstream in the interim. This detail can be seen in Fig. 22-(b), where two hairpin vortices simultaneously form. Before the secondary hairpin vortex (SHV) is produced, close to the

inflection point in the center of the oscillated path, the primary hairpin vortex (PHV) forms, and then vortex dissipation occurs. The complete evolution of vortex shedding during the zigzag motion can be found in [178, 230]. Consequently, bubble rising and the related phenomena affect the fluid flow (and hence the heat transfer, which is discussed later in section 3.2).

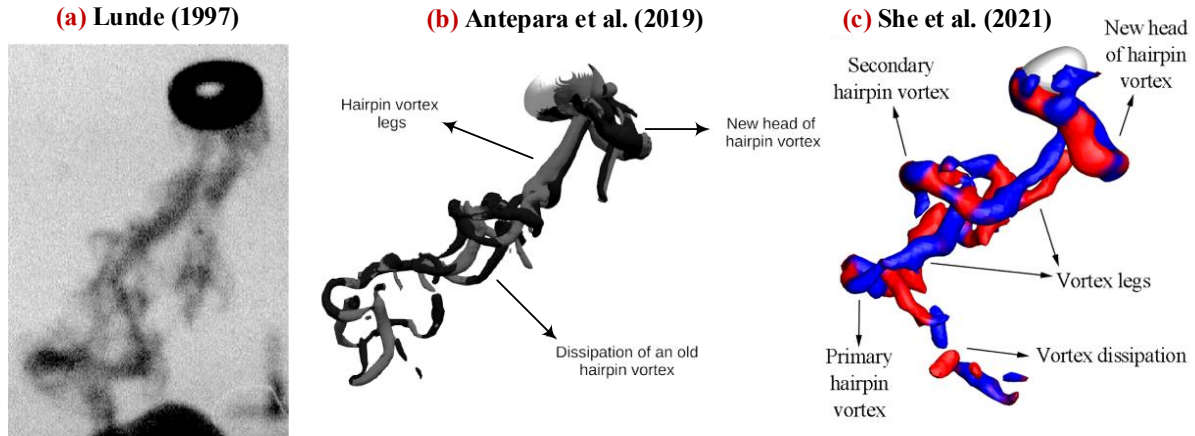


Fig. 22: Experimental bubble wake visualized by Lunde and Perkins [229] study at $Re=1500$. (b) Numerical vortex formation from Antepara et al. [230] at $Re=1106$, $Mo=2.5e-11$, and $Eo=3.6$. (c) The vortex structure for point 8 in Fig. 21 (She et al. [178]) at $Re=1390$, $Mo=1.6e-11$, and $Eo=4.06$ (colored by streamwise vorticity magnitude) (Figures a, b and c are reprinted with permission from ASME, Elsevier and AIP Publishing, respectively).

Studies showed that path instability is not only created by the wake of a bubble but also by the wake of the neighbouring bubbles [231]. Performing experimental and numerical studies, V'elez-Cordero et al. [232] discovered that during the ascending of two aligned bubbles, they steadily depart from one another due to the wake of the leading bubble. For side-by-side bubble rising, the critical Re number controls the repulsion and attraction among bubbles [233]. The surrounding vortices produce a hindering effect that prevents the bubbles from moving when their Reynolds numbers fall below a crucial threshold, making them repel one another. On the contrary, the venturi effect causes the bubbles to be pulled closer to each other at high Reynolds number (more than the critical threshold) when the distance between them falls below a particular threshold [234]. A similar study was done by Sanada et al. [231] to investigate the side-by-side rising of two bubbles. They found that when Re exceeds the critical value, the two bubbles tend to move closer and may potentially collide, which is discussed further in section 3.1.4. In addition, bubble interaction may impact induced turbulence, which is discussed more in the bubble mixing agitation in section 3.1.5. Consequently, it becomes essential to consider the impact of the flow surrounding the bubbles to completely understand the kinetic behavior of bubbles.

Compared to bubbles in still water, their dynamic behavior in turbulent flows is far more intricate. The mechanism of bubble motion in turbulent flow in the bubble mixing agitation is described in section 3.1.5. A bubble that encounters turbulence eddies is more likely to become entrapped, decreasing its ascent velocity [235, 236]. In addition, in turbulent flows, fluctuating velocity causes the bubble to undergo self-sustained oscillation and experience deformation [237]. In the case of homogeneous turbulence, Ravelet et al. [237] experimentally examined the rise of a bubble using the particle image velocimetry (PIV) method and a high-speed camera. Their findings indicated that the irregular fluctuations in velocity and direction resulting from the instability of the wake are similar to the situation when bubbles ascent in a non-axisymmetric manner in quiescent water.

Recently, it has been shown that, in the course of rising, a bubble experiences drag and lift force because of turbulence-induced distortions [235]. Additionally, significant turbulence may interfere with the bubble wake's creation, reducing the drag coefficient [235]. As mentioned before, the flow conditions surrounding a bubble substantially impact its lift coefficient. Shear flow can cause deformation of the bubbles and change their lift coefficient; these shifts can be positive or negative, depending on the size of the bubbles [205], as was discussed in the lift reversal section. It has been shown in uniform isotropic turbulent flow that the diffusion of millimeter-sized bubbles can be attributed to path instability, and the time scale of a vortex, which the bubbles encounter as they rise, determines the diffusion transition of bubbles [238]. According to Shim et al. [239], there were no appreciable variations in the terminal velocity of bubbles in isotropic turbulence. However, turbulence greatly affected the trajectory of bubbles. As the Reynolds number increased, the frequency and inclination of the bubbles' paths correspondingly decreased. Furthermore, the wake's intensity and coherence are reduced when turbulence is present [239, 240].

3.1.2.3. Bubble shape and diameter

Measuring bubble diameter before injecting it into heat exchangers and other thermal devices is essential, as it significantly influences chemical, biological, and manufacturing processes [241] and strongly affects fluid hydrodynamic properties [242].

Generally, bubble shape can be characterized using dimensionless groups such as the Re , We , Mo , and Eo numbers, which are influenced by the terminal velocity and fluid properties. For example, Grace [243] presented one of the most used charts for bubble shape as a function of dimensionless numbers (Re , Eo , and Mo) in quiescent viscous liquids by

investigating a lot of experimental data (Fig. 23). The characteristic length in the dimensionless numbers is D_{bu} , which is defined as the spherical bubble diameter with the same volume as a non-spherical bubble for different conditions. Each region in the map corresponds to a distinct stable bubble shape, ranging from nearly spherical forms at low Re and Eo values, through transitional shapes like ellipsoidal or wobbling, and eventually to more elongated or “skirted” geometries as Eo number increases. The solid lines drawn within the figure indicate the boundaries between these regimes, illustrating how changes in fluid properties, bubble size, and flow conditions can cause a bubble to shift from one shape category to another.

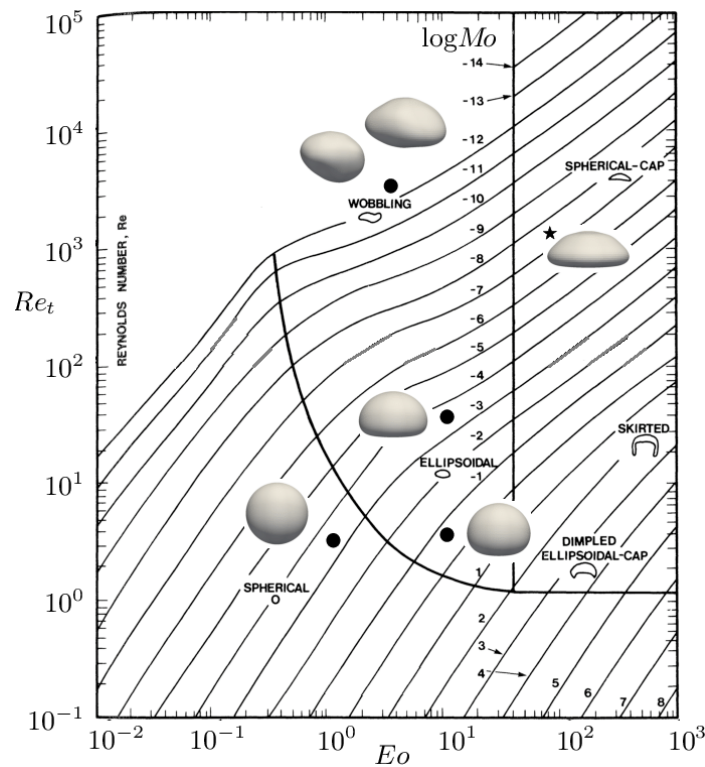


Fig. 23: Grace map for the different bubble shape regimes in viscous flow [243]. The solid line shows the border between different shapes (Figure is reprinted with permission from Taylor & Francis)

In a study, Tripathi et al. [203] carried out numerical simulations on the dynamics of an initially spherical bubble rising in quiescent liquid using a 3D geometry. They determined the different bubble behavior for different regimes based on the Galilei ($Ga = \sqrt[4]{Eo^3/Mo}$) and Eötvös (Eo) numbers. Fig. 24 displays the changes that occur in the shape of an initially spherical bubble for different regimes. The red dash-dotted line indicates $Mo = 10^{-3}$. It was reported that for low values of Eo and Ga numbers, i.e., the region I, the bubble remains azimuthal symmetric. In region II, where Ga numbers are low and Eo values are high, the shape of the bubble is like a skirt, while in region III (higher values of Ga and lower values of Eo)

the bubble path is either zigzag or spiral. As both Ga and Eo numbers increase, bubble breakage happens which leads to the formation of either satellite bubbles (region IV) or toroidal bubbles (region V).

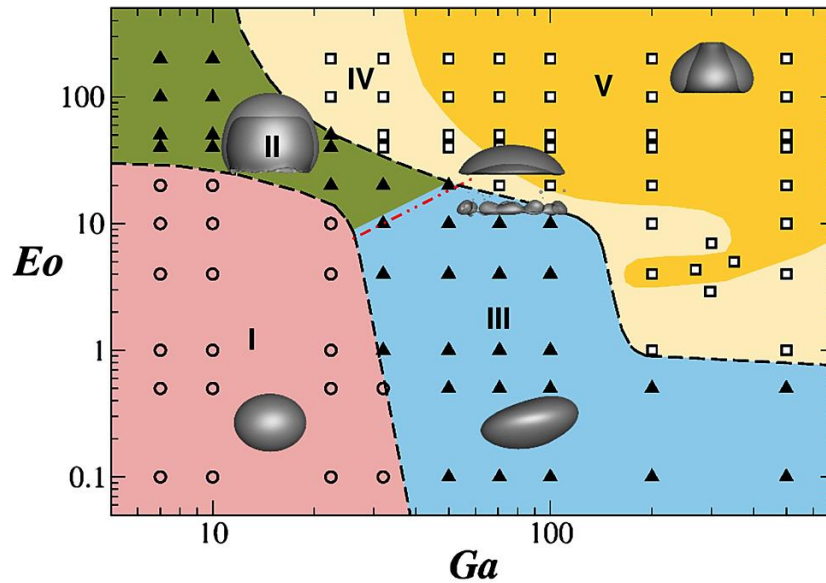


Fig. 24: Bubble rising behavior inside a fluid under standard conditions (Tripathi et al. [244] study). The red dash-dotted line indicates $Mo = 10^{-3}$. It can be seen that for low values of Eo and Ga number, the bubble shape will remain azimuthal symmetric (Figure is reprinted with permission from Springer Nature).

The spherical bubble diameter can be expressed as a function of the airflow rate and nozzle diameter [245]:

$$D_{bu} = a \times \left(\frac{4\dot{m}_{air}}{\mu_{air}\pi D_{noz}} \right)^b \times \left(\frac{16\rho_{air}Q_{air}}{\mu_{air}\sigma D_{noz}^3} \right)^c = a \times Re_{bu}^b \times We_{bu}^c \quad (75)$$

where D_{bu} , D_{noz} , and σ are bubble diameter, bubble injection nozzle diameter, and surface tension, respectively. In the above equation, the coefficients of a , b , and c are determined based on experiments and boundary conditions. Table 6 summarizes some existing relationships for bubble diameter size.

Table 6: Literature summary for bubble diameter correlations in gas-liquid injection systems.

Authors	Correlation	Conditions
Leibson et al. [246]	$D_{bu} = 0.18 D_{noz}^{1/2} Re_{bu}^{1/3}$	$Re_{bu} < 2100$
	$D_{bu} = 0.28 Re_{bu}^{-0.05}$	$Re_{bu} < 100,000$
	$D_{bu} = 1.56 Re_{bu}^{0.058} \left(\frac{\sigma D_{noz}^2}{\Delta\rho g} \right)^{0.25}$	$1 < Re_{bu} < 10$
Kumar et al. [247]	$D_{bu} = 0.32 Re_{bu}^{0.425} \left(\frac{\sigma D_{noz}^2}{\Delta\rho g} \right)^{0.25}$	$10 < Re_{bu} < 2100$
	$D_{bu} = 100 Re_{bu}^{-0.4} \left(\frac{\sigma D_{noz}^2}{\Delta\rho g} \right)^{0.25}$	$4000 < Re_{bu} < 70,000$

Moo-Young and Blanch [248]	$D_{bu} = 0.19 D_{noz}^{0.48} Re_{bu}^{0.32}$	$Re_{bu} < 2000$
Wilkinson et al. [249]	$\frac{g\rho_l D_{bu}^2}{\sigma} = 8.8 \left(\frac{U_{bu} \mu_{liq}}{\sigma} \right)^{-0.04} \left(\frac{\sigma^3 \rho_{liq}}{g \mu_{liq}^4} \right)^{-0.12} \left(\frac{\rho_{liq}}{\rho_{bu}} \right)^{0.22}$	-
Elgammal et al. [245]	$D_{bu} = 0.013 Re_{bu}^{1.045} We_{bu}^{-0.447}$	$69 < Re_{bu} < 686$

As shown in Table 6, the bubble diameter is more affected by the orifice size at low Re numbers, while at high Re numbers, the Re number is dominant. The bubble diameter size is a function of injection geometry and operational conditions for medium Re number [250, 251]. In Table 6, U_{bu} is the superficial gas velocity (air, for example). The listed studies show the relationship of the bubble diameter for a single nozzle injection in the tube.

As mentioned earlier, studies have proposed correlations for the aspect ratio of non-spherical bubbles based on various dimensionless parameters (see Eq. (19)). Besagni and Deen [252] presented a correlation for bubble aspect ratio based on a vast, broad spectrum of experimental data (more than 150000 data points). They stated that the general format for bubble aspect ratio can be determined as bellow, which is valid for an extensive range of flow types (single bubbles and dense bubbly flows), Mo number ($-10.8 < \text{Log}(\text{Mo}) < 2.3$),

$$AR = \frac{1}{[1 + 0.4Eo^{1.19}Re^{1.05}]^{0.07}} \quad (76)$$

This correlation was developed for both gravity- and pressure gradient-driven flows. In another study, Liu et al. [34] presented a highly accurate experimental correlation for bubble aspect ratio. Similar to Tian et al. [253], they stated that for high We number ($We > 12$), the buoyancy and inertial force mainly affect the bubble shape. However, viscous force determines the bubble shape for low We number ($We < 12$) and large Mo number ($Mo > 3$). The bubble shape is equally influenced by viscous and surface tension forces when $Mo < 3$ and $We < 12$. Building on this observation, the following correlations have been proposed:

$$AR = \begin{cases} \frac{1}{[1 + 8.9483(EoWe)]^{0.1572}}, & EoWe < 7 \\ 0.5263, & EoWe \geq 7 \end{cases} \quad (77)$$

$$AR = \begin{cases} \frac{1}{[1 + 0.0255(EoWe)]^{0.1572}}, & EoRe < 2300 \\ 0.5263, & EoWe \geq 2300 \end{cases} \quad (78)$$

Chen et al. [254] studied the geometry of a single bubble in a highly viscous flow. Gumulya et al. [255] studied the bubble shape in the wake region. Hence, when utilizing the bubble injection approach, the size and shape of the bubble can be adjusted according to the

desired application and geometry. On the other hand, bubble size can play an essential role in the mixing effect caused by bubble injection [71]. More discussion and literature review regarding the bubble aspect ratio effects can be found in [34, 252, 256, 257].

Due to the collision of bubbles and the nature of turbulence flow, the size of all bubbles is not identical in bubbly flows. Therefore, many researchers have studied the bubble size distribution in bubbly flows and the transition to slug flow. Sample size distributions of bubble diameter are shown in Fig. 25 and Fig. 26. The bubble size distributions are nearly normal (Gaussian) [258]. Grau and Heiskanen [259] investigated bubble distribution at different operating conditions (Fig. 25-a). Their results showed that air flow rate strongly affected bubble size distribution. It is observed that the size of the bubbles increases by increasing the air velocity. Also, the peak of bubble distribution is shifted toward a larger size with increasing the air flow rate, and the distribution width also increases.

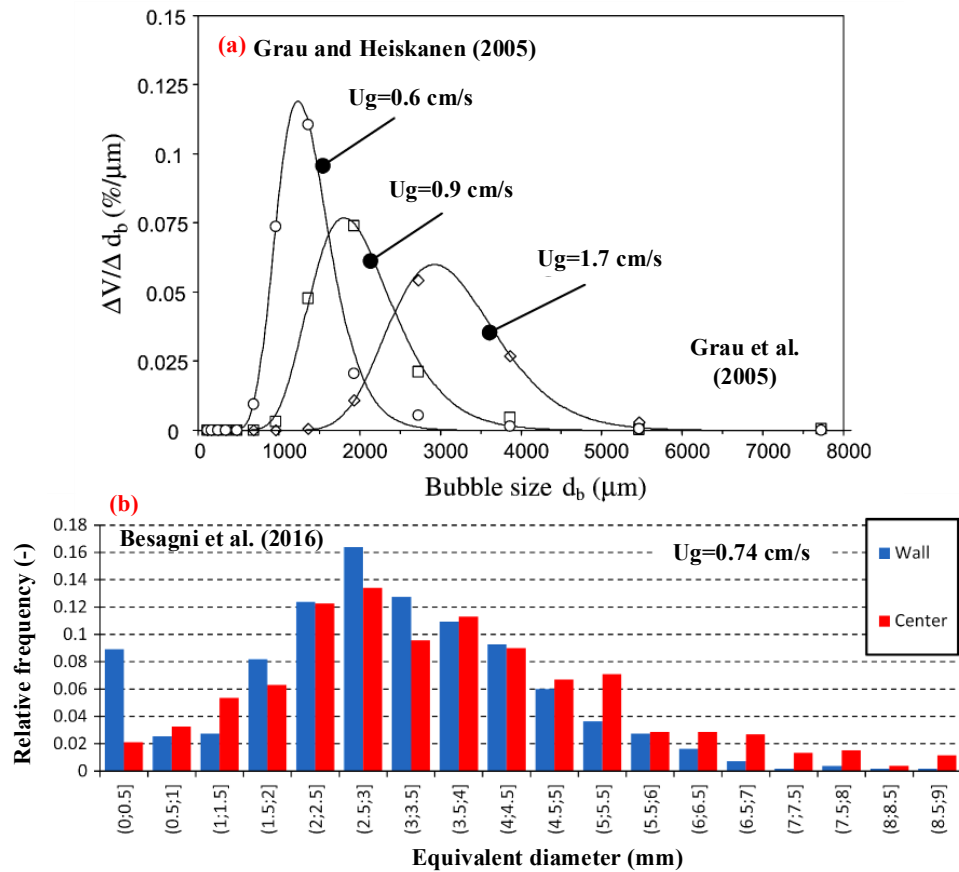


Fig. 25: (a) Effect of superficial gas velocity (0.6, 0.9, and 1.7 cm/s) on the bubble size distribution (Grau and Heiskanen [259]). The bubble size distribution is dramatically affected by the airflow rate. (b) bubble size distribution in the center and near the wall of a large bubble column (Besagni et al. [260]). The population of the small bubble is high near the wall because of the lift reversal phenomenon (Figures is reprinted with permission from Elsevier).

Besagni et al. [260] conducted an experimental study on bubble distribution in a large air-water bubble column using image analysis and optical probes. Fig. 25-b shows their bubble

size distributions in the center and near the wall of the column for air velocity $U_g=0.74$ cm/s. It is seen that the population of small bubbles is higher near the wall than the center, which is due to the lift reversal phenomenon.

Kazakis et al. [69] experimented on bubble distribution in a bubble column equipped with a porous sparger. They studied the effect of sparger characteristics and continuous phase properties on the bubble distribution. In Fig. 26-a, the effect of different pore diameters (40 and 100 μm) on bubble distribution is illustrated for a fixed airflow rate of 10 ml/s. Fig. 26-b shows the effect of different liquid properties on the bubble distribution. It is seen that the bubble distribution strongly depends on porous sparger pore size and liquid properties. For example, reducing the surface tension force using different liquids (water and Isobutanol 2.25% and 0.75%) can shift the peak of generated bubbles to small diameters. Also, using a sparger with larger pore diameters generates larger diameter bubbles.

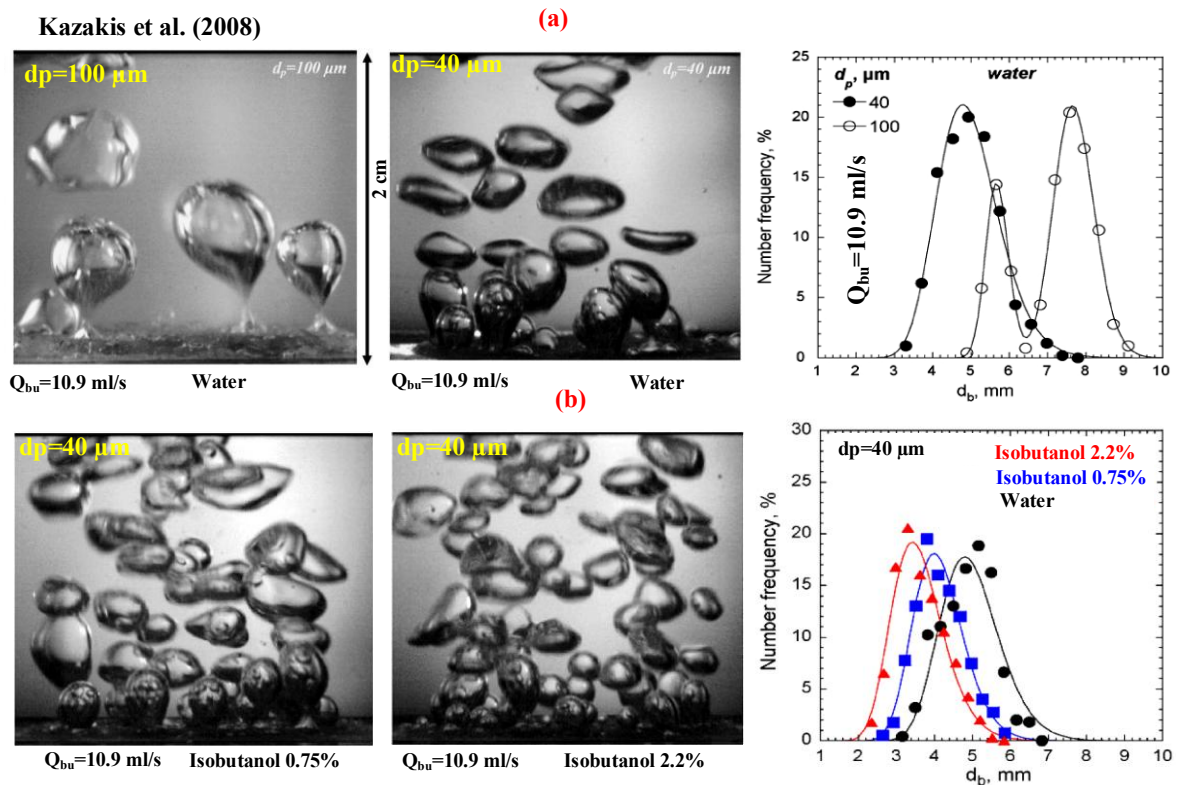


Fig. 26: (a) Effect of different pore diameters (40 and 100 μm) of porous spargers on bubble distribution in a bubble column. Water is a continuous fluid, and the bubble flow rate is 10.9 ml/s. (b) effect of different liquid properties (Isobutanol 2.2%, Isobutanol 0.75%, and water) on the bubble distribution in a bubble column. The distribution peak is different for each fluid due to changes in surface tension force (Kazakis et al. [69]). The surface tension of water, Isobutanol 2.2%, and Isobutanol 0.75% are 72 (mN/m), 49 (mN/m), and 60 (mN/m), respectively (at 25 $^{\circ}\text{C}$). (Figures are reprinted with permission from Elsevier).

3.1.3. Bubble breakage

3.1.3.1. Overview

Bubble breakage is important as it affects the fluid flow, mixing, and heat transfer rate. The properties of the liquid surrounding the bubble control the bubble breakdown process [261]. The main elements affecting the bubble breakage process include fluid dynamics, transport processes, and interfacial interaction. The bubble breakage mechanisms are categorized into four groups: turbulence, viscous shear, shearing-off process, and interfacial instability [262, 263].

3.1.3.2. Breakage due to turbulence

Much research has been done to describe the concepts relating to breakage and coalescence and their functional mechanisms. Most of these studies agree that the breakage and coalescence of bubbles occur due to turbulence. In some of these studies, the bubble breakage is described as a competition of turbulence stress as destabilizing and surface tension as stabilizing forces. In this way, if the ratio of the turbulence stress force to the surface tension (Weber number) exceeds a critical value ($We_{critical}$), the bubble breaks up [264-266]. In this scenario, bubbles break up due to collisions with eddies or turbulent pressure fluctuations along their surface. These interactions cause the bubbles to deviate from their spherical shape. When the oscillations in the surrounding fluid or the collisions between bubbles and eddies reach a certain amplitude, they create instabilities on the bubble's surface. This instability causes the bubble to stretch in a specific direction, eventually fragmenting into two or more smaller bubbles [262].

Another group of researchers described the bubble breakage mechanism based on the competition of turbulence kinetic energy and surface energy in the stability of the bubble. Hence, bubble breakage occurs if the energy of turbulent eddies is greater than the increase in surface energy caused by bubble breakage [267, 268].

In a study, Li and Kang [269] conducted numerical and experimental investigations involving successively rising bubbles in a rectangular bubble column for carbon dioxide absorption in nano-absorbents (Methanol with various concentrations of alumina nanoparticles). They concluded that the bubble breakage occurred due to an eddy collision and the coalescence of two bubbles with a small surface tension force. Likewise, they found that the coalescence and breakage can improve the mass transfer coefficient by generating polydisperse bubble swarms (Fig. 27). Table 7 Provides a detailed review of some studies performed on the bubble breakage due to turbulence.

Researchers have identified the Weber number as a critical criterion for determining whether a bubble will break up, based on its degree of deformation,

$$We = \frac{C \rho_{liq} \epsilon^{2/3} d_{bu,0}^{5/3}}{\sigma} \quad (79)$$

where ϵ is the average turbulent dissipation rate, and C is the parameter linked to Re , found empirically between 2 and 2.2 [270]. Previous experimental studies have found that the critical value of the Weber number for breakage ranges between 1 and 5 [271].

Table 7: Studies on bubble breakage due to turbulence and their major findings.

Author	Study	Remark
Zednikova et al. [272]	Numerical and experimental investigations to evaluate the collision process between a bubble and a vortex ring.	Bubble breakage is influenced by both the bubble size and the Weber number. During the interaction between a single bubble and a vortex ring, the vortex's behavior is altered by the bubble's presence.
Sridhar and Katz [273]	An experimental investigation to study how a large vortex ring and a bubble interact.	Under specific combinations of bubble diameter and vortex intensity, bubbles can alter a ring's vortex nucleus. But as the bubble exits the ring, the vortex returns to its initial state.
Foroushan and Jakobsen [274]	Investigation of the effect of multimodal excitation on bubble interface oscillations.	A decrease in the surface tension led to an increase in the amplitude of the interface oscillation.

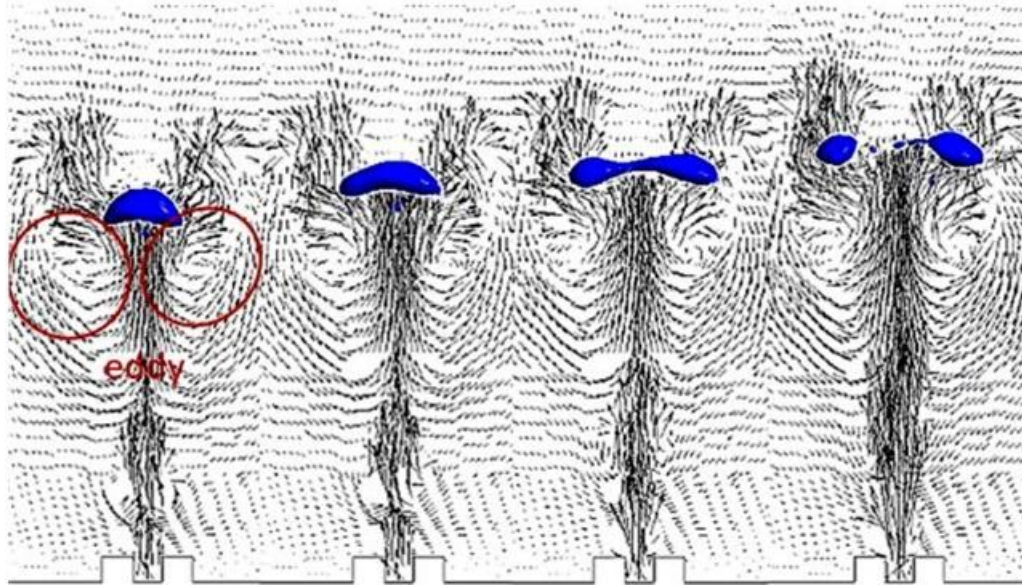


Fig. 27: Bubble breakage due to turbulent eddy collision (Li and Kang study [269]). Asymmetrical or peripheral breakage happens when the bubble collides with an eddy the same size or smaller as the bubble. The CO₂ bubble was used in the experiment with a 2.4e-6 (kg/s) flow rate in the Methanol with alumina nanoparticles. (Figure is reprinted with permission from Elsevier).

3.1.3.3. Viscous shear breakage

Deformation of the bubbles and the resulting breakage occurs when the viscous shear forces bring about a velocity gradient around the interface [275]. The wake effect also leads to the appearance of shear stresses. The larger part of a bubble may undergo necking, elongation, and bubble surface indentation outside a wake region, mainly due to shear stress across the wake boundary. The particle first turns into two lumps, then breaks into two roughly similar daughters [78, 276].

A balance exists between external viscous stresses and surface tension forces in the viscous shear-induced breakage. A dimensionless number known as the Capillary number ($Ca = \mu V / \sigma$) is considered to express this force balance. Grace [277] proposed that the critical capillary number depends not only on the type of flow but also on the gas-liquid viscosity ratio. Tucker and Moldenaers [278] noted that bubble breakage occurs when the viscosity ratio is significant, and the capillary number slightly exceeds its critical value. However, breakage begins at the bubble's tip when the viscosity ratio is significantly less than 1. For viscous stress-controlled breakage processes, previous investigations also demonstrated that there is a relationship between the critical capillary number and the Reynolds number, expressed as, $Ca_c \sim 1/Re$, under critical flow conditions with a Re number of up to 100 [276]. Table 8 presents the different mechanisms of bubble breakage induced by viscous shear stress [279].

Table 8: Different categories of viscous shear stress-induced bubble breakage mechanism.

Category	Condition
Capillary instability	Bubble breakage is controlled when $Ca > 2Ca_c$.
End-pinching	Causes the bubble to break up when $Ca < 2Ca_c$.
Necking mode	When $Ca \sim Ca_c$, the bubble splits into two sub-bubbles of equal size, often accompanied by the formation of smaller satellite bubbles.

3.1.3.4. Breakage due to shearing-off process

When a large bubble (e.g., a slug bubble) rises within a bubble column, and its horizontal diameter reaches approximately 9 to 11 cm with an equivalent diameter of about 5 to 6 cm, the bubble's trailing edge breaks up, producing tiny bubbles [280]. This shearing-off process, known as erosive breakage, refers to the generation of numerous small bubbles from a larger one. To determine bubble shearing-off in flows significantly influenced by viscosity, the balance between surface tension at the bubble's cap skirt and the shear force must be considered [262]. As the relative velocity increases, the bubble skirt becomes unsteady. This instability leads to skirt shedding, creating numerous small bubbles at the rim.

However, if the liquid viscosity is comparatively low, the interfacial viscous shear in gas-liquid flow is negligible. In this case, the mechanism arises due to the absence of a net liquid flow near the interface [281]. More specifically, in low-viscosity liquids—such as air-water systems—the steady movement of gas inside the bubble caps contributes to this effect. A noticeable change in speed at the gas-liquid boundary allows gas to seep into the liquid layer, generating tiny bubbles [60, 262, 281].

3.1.3.5. Breakage due to interfacial instability

All the bubble breakage mechanisms rely on flow characteristics in the continuous phase. Nevertheless, experimental studies have proved that interfacial instabilities can cause bubble breakage even if a net flow is not present in the continuous phase. This usually happens during certain conditions, such as the rise of bubbles in a liquid. These types of instabilities consist of Raleigh-Taylor and Kelvin-Helmholtz instabilities [282]. Both of these instabilities are discussed in detail by Sharma et al. [283].

3.1.4. Bubble coalescence

The bubble coalescence process modifies the flow dynamics by altering bubble size and distribution, which in turn affects turbulence, mixing, and heat transfer efficiency [284].

During the transport of bubble clusters, collisions frequently occur, creating the potential for bubbles to merge. When two or more neighboring bubbles collide, they may coalesce [192]. Kamp et al. [285] conducted a comprehensive study of bubble coalescence and reviewed the available experimental techniques and numerical methods. Here, a quick review of the sequence of coalescence is presented. Additional information can be found in [285-288]. When the bubbles approach each other, a film of liquid forms between them, with a thickness of several microns. In the next stage, when the film thickness reaches a critical value, the rupture of the liquid film takes place. Eventually, following the film rupture, the bubble coalescence takes place. The bubbles merge if the drainage period is shorter than the contact time. Coalescence can be done in two ways: complete and partial. Daughter bubbles are formed in partial coalescence, as described experimentally by Zhang et al. [289].

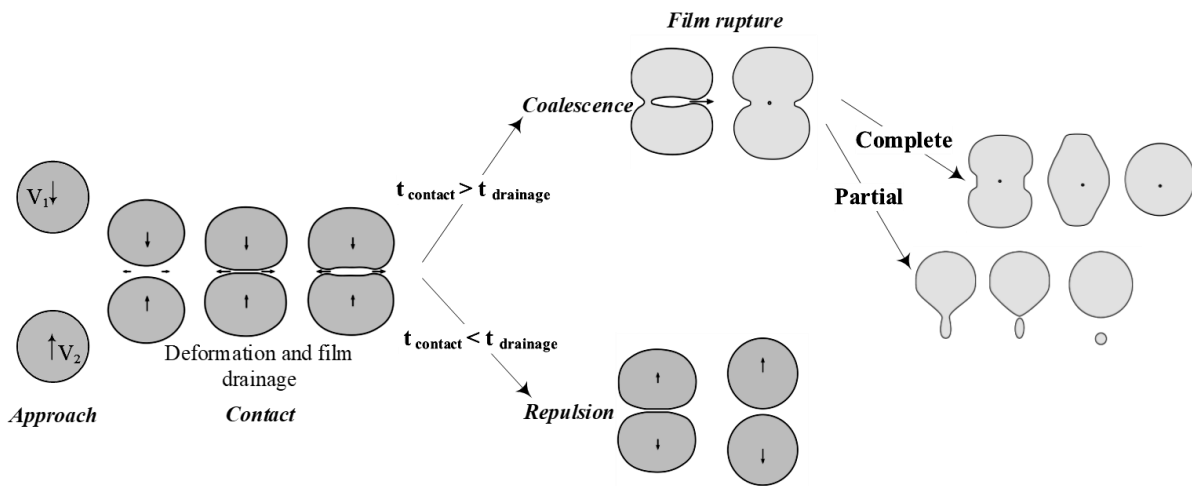


Fig. 28: Schematic of the coalescence process for two bubbles [285](Figure is reprinted with permission from De Gruyter).

The bubble coalescence process has two mechanisms:

- (1) Bubble coalescence resulting from turbulence eddies [290]: Bubble coalescence by turbulence eddies refers to the process where the collision and merging of bubbles occur due to the action of turbulent eddies in the surrounding liquid. As the eddies move and swirl, they could bring two or more bubbles into proximity, leading to their coalescence [291]. It is reasonable to assume that the turbulence is far less complex than the random bubble collisions it induces.
- (2) Bubble coalescence resulting from wake entrainment [292]: Bubble coalescence due to wake entrainment refers to the process by which two bubbles merge as a liquid flow carries them. When a bubble rises in a fluid, it creates a wake behind it, an area of lower pressure and

velocity. If another bubble is present in this wake, it can be entrained or carried along by the liquid flow and merge with the rising bubble. The wake can be produced when bubbles from different rise velocities encounter each other, which may coalesce, leading to bubble size and shape changes. The speed at which the bubbles rise can be affected by buoyancy, viscosity, and surface tension, which can cause bubbles to rise at different rates.

When bubbles interact, they may experience both repulsive and attractive forces. Wake attraction takes place when a wake flow reduces pressure in front of the bubble in the wake region. A repulsive force acts on the leader when it becomes smaller than the bubble in the wake region. Simultaneously, lift forces make both bubbles rise but move horizontally in opposite directions, preventing a collision. For the case of smaller bubble in the wake, only an attractive force is present, causing the bubbles to make contact [293].

Fig. 29 presents the experimental and numerical results of bubble coalescence driven by wake entrainment [269]. As depicted in the first stage, the following bubble comes near the wake region created by the leading bubble. The following bubble then accelerates until a thin interface layer separates the two bubbles. Simultaneously, the following bubble alters its shape from round to more like a bullet because it moves quickly towards the leading bubble and eventually collides with it, losing its original shape. In the next step, the thin film between the bubbles gets even thinner until it breaks completely, which leads to bubble coalescence [269].

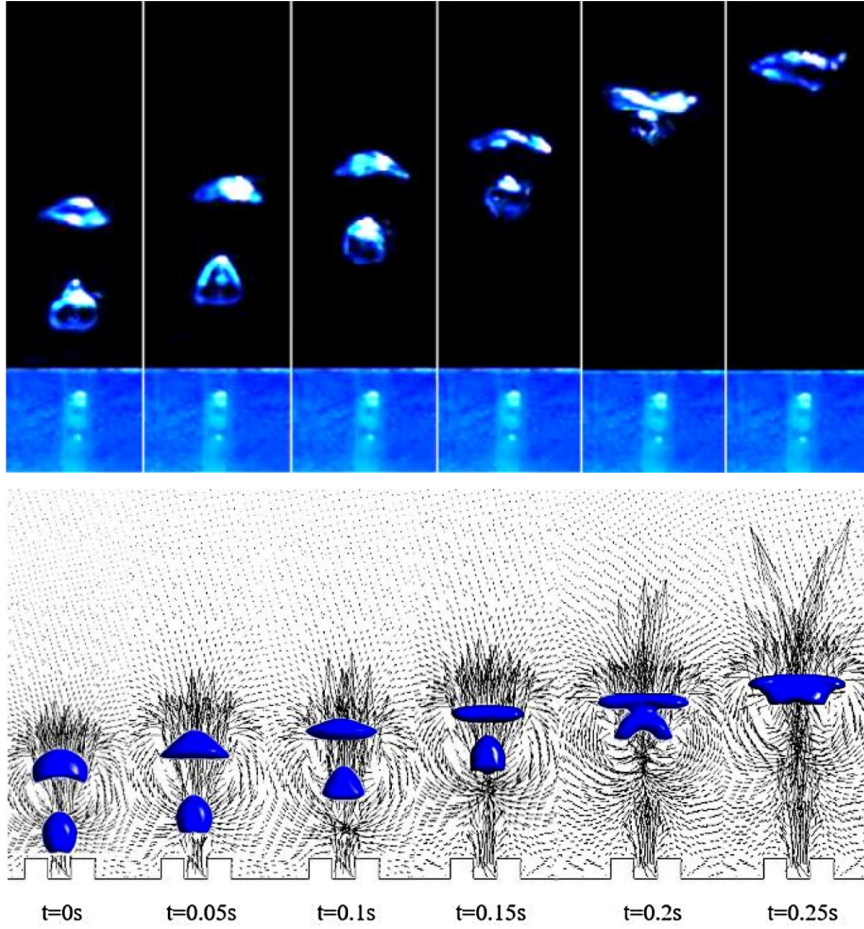


Fig. 29: The experimental and numerical results for the bubble coalescence process due to the wake entrainment [269]. The bubble column was rectangular with 200 mm width $\times$ 200 mm height $\times$ 5 mm depth dimensions, in which CO₂ sparged into methanol+ 0.01% Al₂O₃ with a mass flow rate of 0.4 Lit/min and utilizing mass flow controller (MFC) ($Re_{bu}=256$, $f=20$ Hz, $d_{eq, bu}=5.35$ mm). The Population Balance Model (PBM) was used for CFD simulation based on the Eulerian–Eulerian approach [294]. The process of bubble coalescence can be seen clearly, and the numerical model agrees with experimental results (Figures are reprinted with permission from Elsevier).

Balcázar et al. [295] investigated the coalescence of a bubble in buoyancy-driven flow. Fig. 30 presents their numerical results for the velocity field and pressure contours during the bubble coalescence process for $Mo=10^{-9}$, $Eo=4$, $\rho_l/\rho_{gas}=\mu_l/\mu_{gas}=100$. It is seen that the pressure difference between the top and bottom of the upper bubble creates an artificial lift for another one in the wake region. In addition, the figure indicates that at $t^* = 4.4$, the generated horizontal velocity component leads to the bubbles' alignment. When the two bubbles are close enough, their surface is deformed, creating a neck in between. In the end, the created larger bubble rises in the wobbling regime.

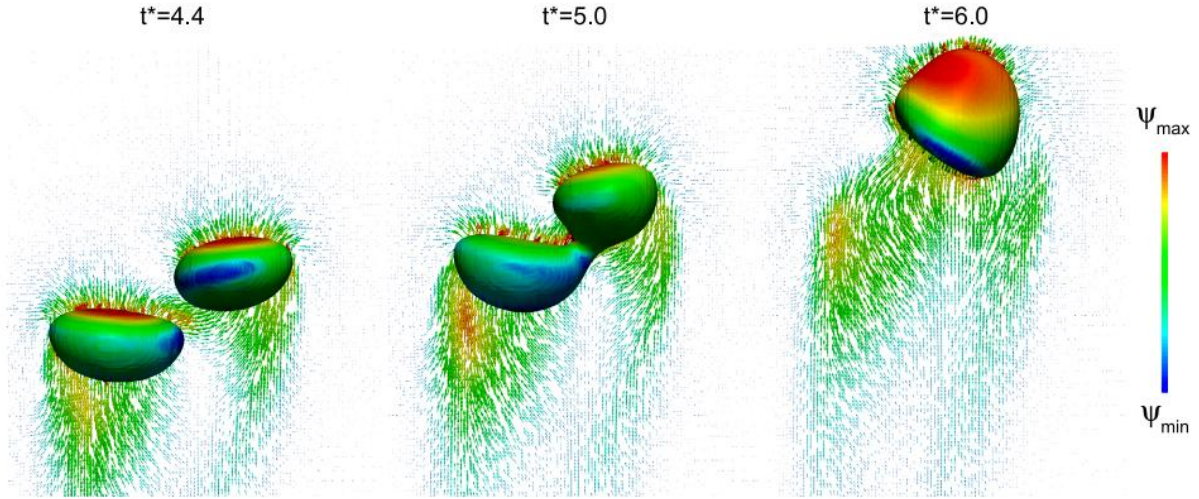


Fig. 30: The velocity field on the plane and pressure contour on the bubble surface in the course of the bubble coalescence process due to the wake entrainment ($Mo=10^{-9}$, $Eo=4$, $\rho_l/\rho_{gas}=\mu_l/\mu_{gas}=100$) [295]. The velocity field is strongly affected by the coalescence process. The pressure difference between the top and bottom of the bubble creates a wake, which causes a lift force for the downstream bubble, which is the main reason for coalescence because of wake entrainment (here $\psi \in \{p, \|v\|\}$, where $\|v\|$ is velocity magnitude) (Figure is reprinted with permission from Elsevier).

Among the many factors influencing bubble interactions, the relative bubble velocity, the orientation between the two bubbles [108, 296], and the initial bubble distance are critical in determining whether a pair of bubbles will coalesce, repel, or bounce [132]. These factors play a key role in deciding whether two equal-sized, laterally aligned bubbles will attract or repel each other. Depending on these conditions, individual bubbles may rise independently, merge through coalescence, or bounce off each other without merging.

3.1.5. Bubble mixing agitation

As mentioned earlier, bubble rising in a liquid, wake-induced path instability, bubble breakup, and coalescence influence the surrounding flow, creating disturbances known as bubble-induced agitation (or turbulence) in the liquid. This agitation, in turn, affects bubble dynamics. Thus, bubble-induced agitation is a key characteristic of bubbly flows. Understanding turbulent bubbly flows requires a comprehensive analysis of the agitation generated by a population of bubbles moving freely within a fluid. In recent decades, numerous numerical simulations and experimental studies have been conducted on homogeneous swarms of rising bubbles of fixed size, without bubble breakage or coalescence, to evaluate the statistical characteristics of bubble-induced agitation (BIA). This section examines the mixing effects caused by the wakes generated behind the bubbles [297]. Bubble motion and agitation increase the mixing effect, enhancing heat transfer. Understanding bubble mixing agitation

principles is important to optimize process parameters and improve overall performance [298]. For example, according to Godin et al. [299], bubble agitation is an effective and novel way to start the crystallization of highly viscous undercooled Xylitol (a kind of PCM material). That is why it is essential to focus on the mechanisms, underlying physical processes, and the modeling of bubble-induced agitation. The concept of bubble-induced agitation in turbulent flow has been thoroughly investigated by Risso et al. [297] and Cluzeau et al. [106].

Generally, the total fluctuations in bubbly flow can be divided into two major parts: shear-induced turbulence and bubble-induced agitation [103, 106, 297, 300-304] (Fig. 31). The shear-induced turbulence results from the average shear generated within a fluid in areas near walls. It can be said that it is turbulence without bubbles or single-phase turbulence [305]. In inhomogeneous or heterogeneous bubbly flow at high void fractions, the bubbles tend to form clusters resulting from hydrodynamic interactions between bubbles [306], as shown in Fig. 32. Shear-induced turbulence can be generated in the regime. When the bubbles are sufficiently large, their rising creates turbulence in the liquid, known as buoyancy-driven turbulence (BDT) [297, 302]. In some references (such as [307]), it is called mean cluster-induced turbulence fluctuations. In addition, mean [106] cluster-induced turbulence can be regarded as bubble-induced agitation since all the fluctuations result from bubble shear generated.

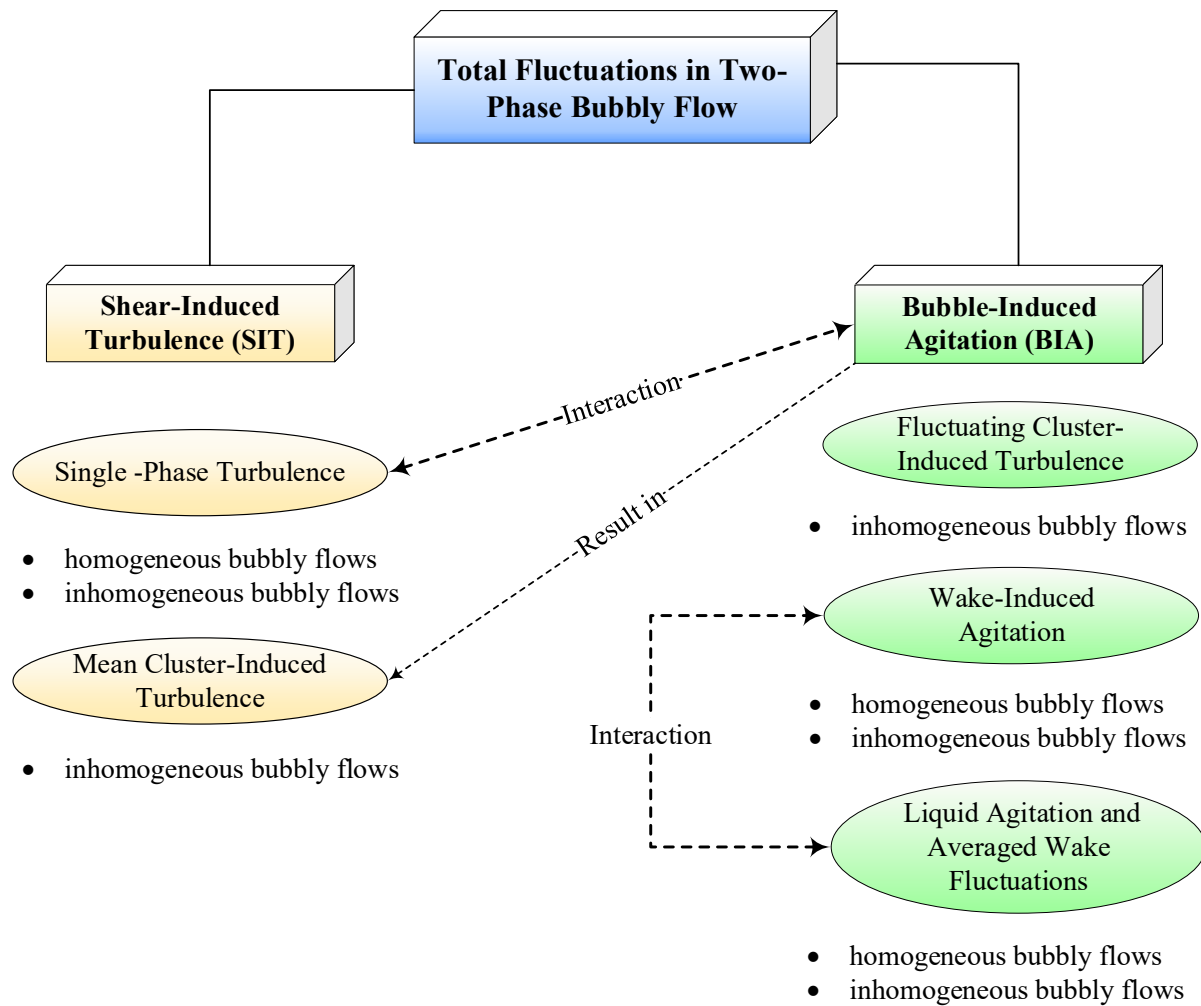
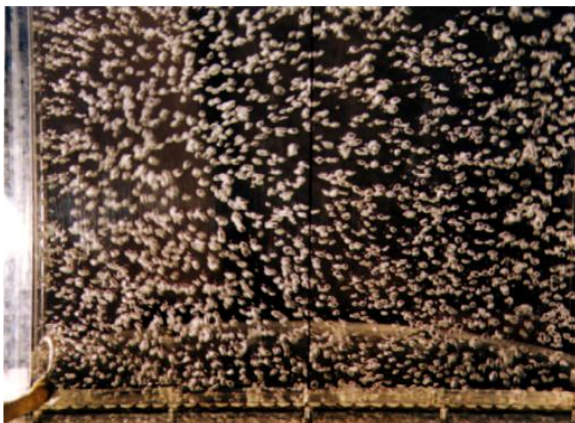


Fig. 31: bubble agitation and total fluctuation mechanism in bubbly flow [103, 106, 297, 300-304]. Bubble agitation depends on the flow pattern, so some factors are applied if the flow pattern is heterogeneous.

(a) Homogeneous bubbly flow



(b) Inhomogeneous bubbly flow

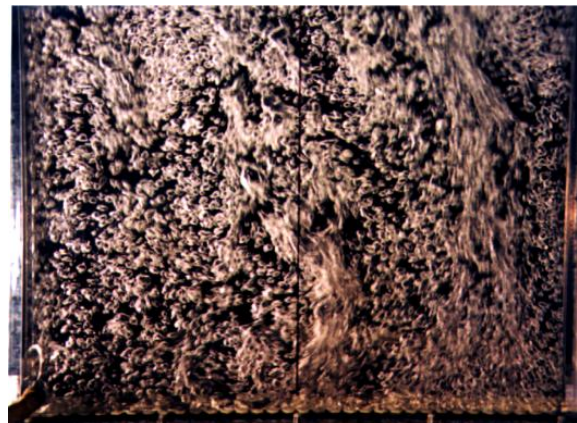


Fig. 32: bubbly flow pattern for (a) homogenous regime and (b) heterogeneous regime [303]. As shown in (a), the bubble rising can be observed due to hydrodynamic interactions of bubbles in a heterogenous pattern, resulting in gross circulation and buoyancy-driven turbulence (BDT) structure and forming bubble clusters (Figure is reprinted with permission from De Gruyter).

Bubble-induced agitation contributes to flow fluctuations due to the presence of bubbles, but these fluctuations are distinct from turbulence. Unlike turbulence, bubble-induced agitation is not associated with shear forces (as depicted in Fig. 31). It should be mentioned that there is an interaction between bubble-induced agitation and single-phase turbulence, as revealed by [308]. In heterogenous bubbly flow, bubble-induced agitation can be caused by fluctuating cluster-induced turbulence in high void fraction cases. In homogenous bubbly flow, the bubble-induced agitation can be resolved into two main parts [309]: spatial fluctuations (which is a result of liquid agitation around bubbles such as averaged wake fluctuations) and time fluctuations (which is a result of wake-induced agitation) [310]. An example of this decomposition of total fluctuation is shown in Fig. 33. Cluzeau et al. [106] investigated the nonlinear interactions between wake-induced agitation and averaged wake fluctuations.

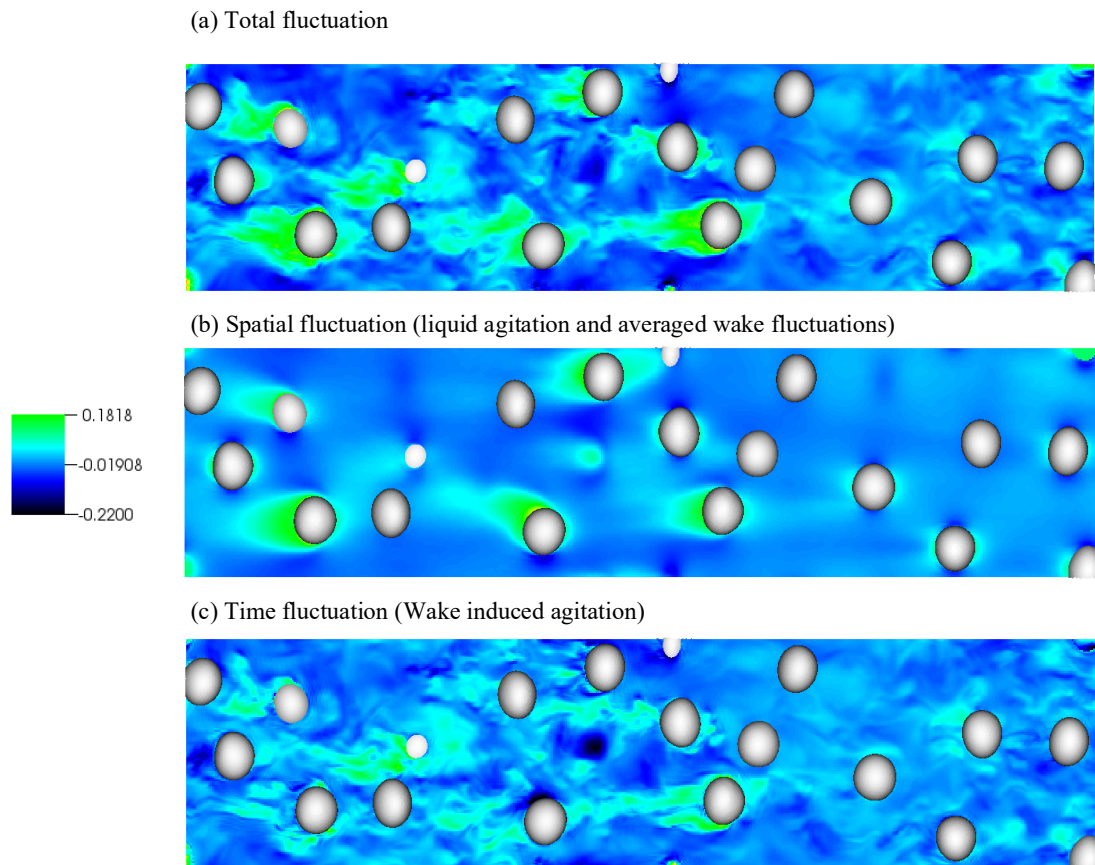


Fig. 33: The DNS result for total velocity fluctuations of bubbly flow at $Re_b=483$ [106], (a) The total fluctuation, (b) spatial fluctuation (due to liquid agitation and averaged wake fluctuation), and (c) time fluctuation (due to wake-induced agitation). The liquid velocity fluctuations can be separated into two different components, spatial and time fluctuations, in a frame of reference that is fixed to the bubbles [311] (Figure is reprinted with permission from American Physical Society).

Risso [297] investigated the bubble agitation for a homogeneous bubble swarm and the relation between different mechanisms in more detail, as shown schematically in Fig. 34. According to this study, different bubble agitation is categorized based on the driving motion. For example, when gas bubbles are injected at the base of a tank filled with initially quiescent liquid, buoyancy is the sole factor driving motion. So, buoyancy leads to upward bubble movement, resulting in velocity fluctuations in the adjacent liquid (bubble-induced agitation), which is not the sole reason for agitation. In fact, bubble path instability [202] and the fluctuations induced by the wake, as mentioned in the bubble rising section, significantly contribute to bubble agitation.

On the other hand, an external pressure gradient can drive motion to liquid, generating near-wall turbulence at high Re numbers (shear-induced turbulence), which induces the motion to bubbles with a velocity different from the fluid velocity [297]. In conclusion, regarding the bubbles, the wake-induced path instability is primarily responsible for agitation. It should be noted that wake instabilities can generate fluctuation, such as von Kármán alleys, which is not necessarily turbulent, and hence, the agitation is more resealable than turbulence for this kind of fluctuation. There are two different contributions to the liquid. The first one is associated with the anisotropic flow disturbances primarily created in a vertical direction close to the bubbles. The second one, ultimately responsible for horizontal fluctuations, is the nearly isotropic turbulence caused by the flow instability through a population of bubbles. Interaction between shear-induced turbulence and bubble-induced agitation is one of the important challenges. More detailed information on the mechanisms is depicted in Fig. 34.

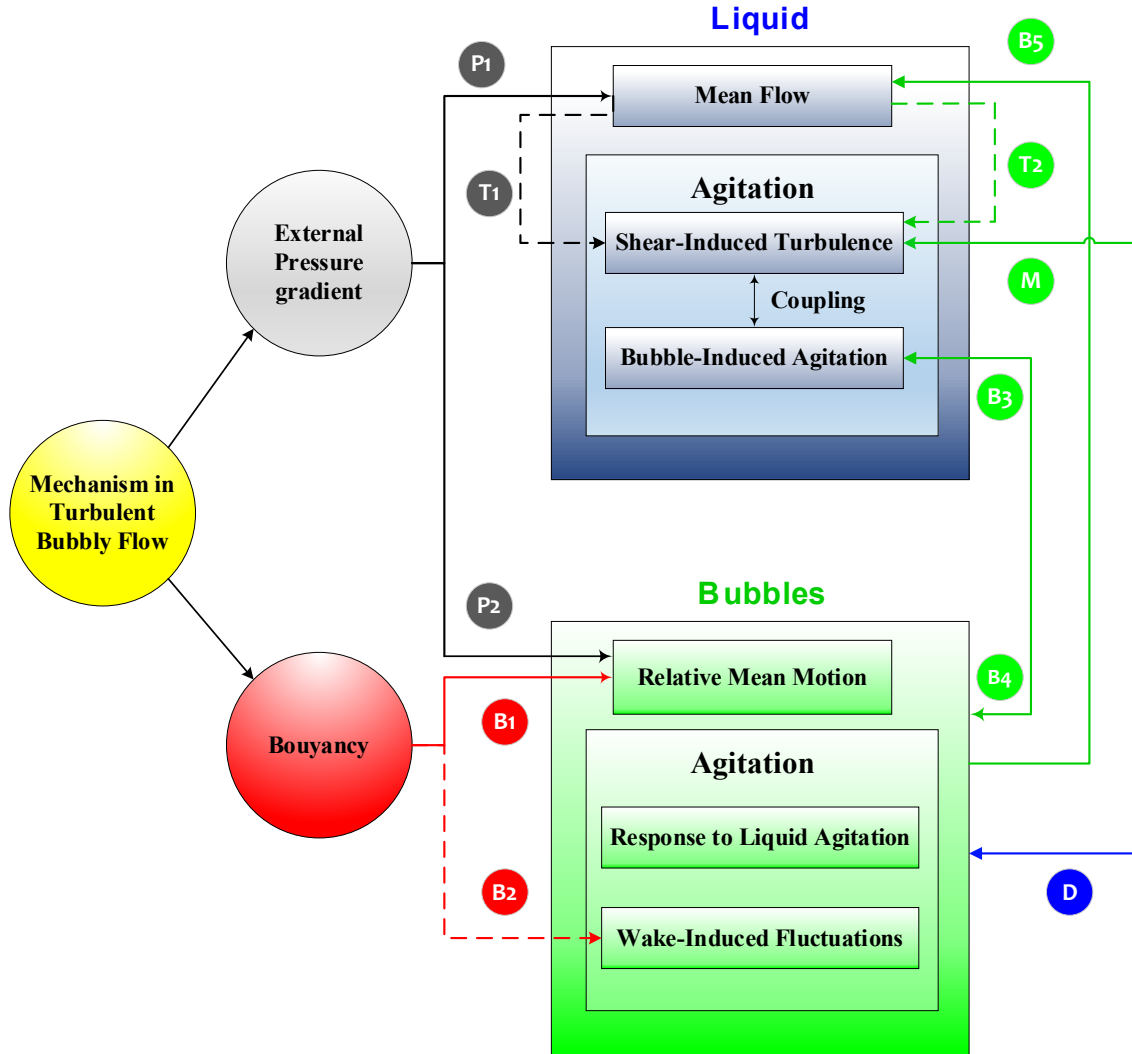


Fig. 34: The physical processes of bubble agitation in a high Re number homogeneous bubbly flow. The two potential sources of motion are buoyancy (red) and an applied external pressure gradient (black). The pressure gradient induces a mean flow (P1) in the liquid (blue), which gives rise to near-wall turbulence (T1) when the Reynolds number is sufficiently high. In parallel, the pressure gradient induces a mean motion (P2) in the dispersed phase (green). The bubbles move randomly due to the shear-induced turbulence (D), which modifies the liquid turbulence (M). The bubbles rise due to buoyancy (B1). The surrounding liquid experiences velocity changes due to the flow disturbances caused by bubble-rising motions (B3), affecting the dispersed phase (B4). Objects ascending or descending exhibit oscillatory trajectories when their wakes become unsteady, contributing significantly to the agitation, especially for millimeter-sized bubbles in water (B2). Buoyancy may also generate mean liquid flows (B5) through an inhomogeneity of the bubble concentration. These motions cause large liquid velocity gradients, leading to turbulence when the concentration rises (T2) [297].

In homogeneous bubble swarms with high Reynolds numbers, a specific mechanism works to prevent the formation of bubble clusters. Previous studies, such as Riboux et al. [312] and Esmaceli and Tryggvason [313], reported that high cluster bubbles cannot be observed for the Re number higher than about 100. To explain the reason for this observation, Risso [297] suggested that the primary mechanism in a high Re number bubble swarm is self-induced

bubble motion caused by wake instability. In conclusion, according to studies, homogeneous swarms of bubbles rising at large Re do not demonstrate any noticeable clustering.

3.2. Heat transfer mechanisms in two-phase bubbly flow

3.2.1. Overview

The heat transfer mechanism in two-phase bubbly flows is closely related to the dynamics of bubble motion described in the previous section. This section explores in detail how bubble motion contributes to enhancing heat transfer. While the investigation of heat transfer in single-phase laminar and turbulent flow has been well-examined in previous studies, the numerical examination of heat transfer in two-phase bubbly flow remains relatively rare compared to experimental research. The reason is primarily attributed to the significant computational demand of simulating 3D turbulent two-phase bubbly flow, necessitating a significant computer infrastructure. In addition, further exploration requires understanding the impact of bubble breakage and coalescence on heat transfer in two-phase bubbly flow.

3.2.2. General physical mechanisms

The thermal boundary layer thickness adjusts the rate of heat transfer in single-phase flow, as discussed in the problem statement section. However, when bubbles are injected into a liquid, bubble-induced mixing, agitation, and turbulence dominate the heat transfer rate. As described in the following sections, the bubble injection takes place in natural or forced convection and laminar or turbulent flows. As earlier experimental studies demonstrated, bubble injection improves mixing and generates turbulence, enhancing heat transfer. Considering the dynamic of bubble motion and literature review, we presented schematically the underlying reasons for heat transfer improvement because of the bubble injection in Fig. 35. However, all the factors mentioned in Fig. 35 are coupled. For instance, the shape of a bubble influences the liquid flow in its vicinity, and the altered flow dynamically modifies the bubble shape. A deformed bubble, for instance, ascends in a zigzag trajectory accompanied by non-symmetrical vortex shedding. Throughout this ascent, instabilities arise, prompting alterations in the bubble's trajectory, or the reversal of lift influences a change in the bubble's direction that disturbs the thermal boundary layer. Hence, assessing the individual impact of each factor on heat transfer becomes inherently challenging due to the interconnected nature of these factors. Overall, the effects of bubble injection on heat transfer are complex and depend on various factors, such as the system's geometry, the fluid's flow rate, and the bubbles' size and concentration.

For example, in laminar flow, bubble size can play an essential role in the mixing effect caused by bubble injection [71]. Kitagawa et al. [71] stated that nanobubble does not enhance the mixing effect. However, microbubbles ($d=0.04$ mm) can improve the mixing effect to a certain extent in small dimensions due to the increase of bubble swarms near the wall. However, increasing the microbubbles' diameter for a fixed airflow rate decreases the mixing effect. The reason is that with the rise of the bubble diameter, the number of bubbles decreases, and the effects of the bubble wake region decrease, as well [71]. Finally, sub-millimeter ($d=0.5$ mm) and millimeter bubbles improve the mixing effect because of the bubbles' wake region and their collision [314].

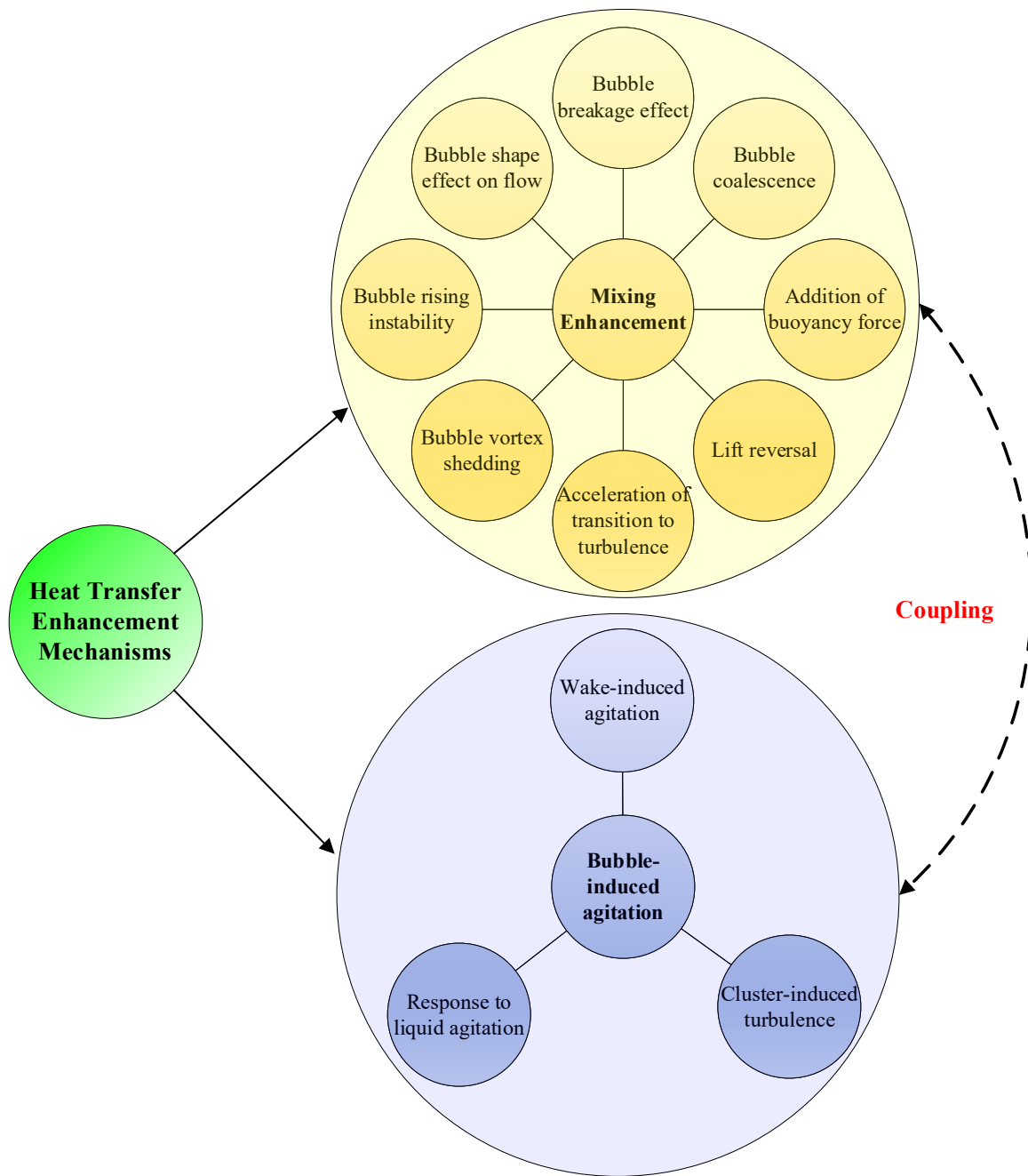


Fig. 35: Heat transfer enhancement mechanisms due to bubble injection in liquids.

Generally, bubble injection in laminar flow increases mixing and accelerates the transition of flow from laminar to turbulent. Similarly, Deen and Kuipers [315] demonstrated that the heat transfer between a heated wall and a quiescent liquid can be augmented by a small number of high Reynolds number bubbles ascending in the liquid. Utilizing bubble injection can increase the buoyancy effect in fluid and enhance heat transfer [316]. Bubble injection can create a more homogeneous distribution of temperature [317] and promote effective heat extraction [318].

3.2.3. Heat transfer in turbulent bubbly flow

As previously mentioned, the study of heat transfer in two-phase bubbly flows is inherently complex. In turbulent flows, bubble injection generates additional bubble-induced turbulence, which combines with shear-induced turbulence to enhance mixing and heat transfer. It is worth noting that the primary application of bubble injection in heat exchangers takes place in turbulent flow conditions. In Fig. 22, we reviewed the effect of bubble shape and bubble path (zigzag) on the vortex shedding and hairpin vortex structures. Panda et al. [319] conducted numerical investigations on the heat transfer characteristics of a single bubble rising near a hot wall, focusing on how bubble shape influences heat transfer from the wall. Additionally, they studied the behavior of a swarm of bubbles rising in a dense, turbulent bubbly flow within an upward channel. For the single bubble case, they found that the Nusselt number was higher for a wobbling bubble compared to an ellipsoidal one, highlighting the significant impact of bubble shape on heat transfer efficiency (see Fig. 36-(a)). Furthermore, as depicted in Fig. 36-(a), the Nusselt number for the wobbling bubble exhibits fluctuations, whereas the ellipsoidal bubble exhibits a plateau trend over time. These results unequivocally indicate the influence of vortex shedding on the wobbling bubble (or in a zigzag trajectory), as illustrated in Fig. 22. The wake of spherical and ellipsoidal bubbles remains symmetrical; however, for wobbling bubbles, the presence of intricate nonsymmetric hairpin structures disrupts the thermal boundary layer, leading to enhanced heat transfer. As asserted by Panda et al. [319], the primary factor contributing to increased heat transfer is the thinning of the thermal boundary layer, particularly pronounced in laminar boundary layers.

To gain a more comprehensive understanding of the heat transfer mechanisms in bubbly flows near a wall, it is essential to examine the wake structure of the bubbles. In this regard, Jeong and Park [222] and Lee and Park [320] conducted experimental investigations into the wake structure of an oscillating rising bubble close to a wall. As illustrated in Fig. 36-(b), a conceptual diagram depicts the thinning of the liquid layer between the wall and the rising bubble in a zigzag trajectory (Jeong and Park [222]). Consequently, a temperature gradient perpendicular to the wall forms across this slender film, increasing heat flux in this direction. The mechanism of heat transfer during the impingement of a bubble onto a surface is further elucidated in the subsequent subsection.

Furthermore, apart from thinning the liquid layer, a robust mixing occurs near the wall due to vortex structures behind the rising bubble, as shown in Fig. 36-(c) (Lee and Park [320]). In fact, the influence of the wall on bubble motion depended on the initial distance between the bubble and the wall. More Information can be found in the Lee and Park [320] study. Using

the front-tracking model, Bhuvankar and Dabiri [321] numerically investigated the effect of a single bubble rising on the heat flux from the wall in shear flow. As per their simulations, heat transfer in the upstream regions of the bubble demonstrates an increase, while in the downstream regions, it shows a decrease. The flow reversal close to the wall due to a bubble is responsible for both the increase and decrease in heat transfer.

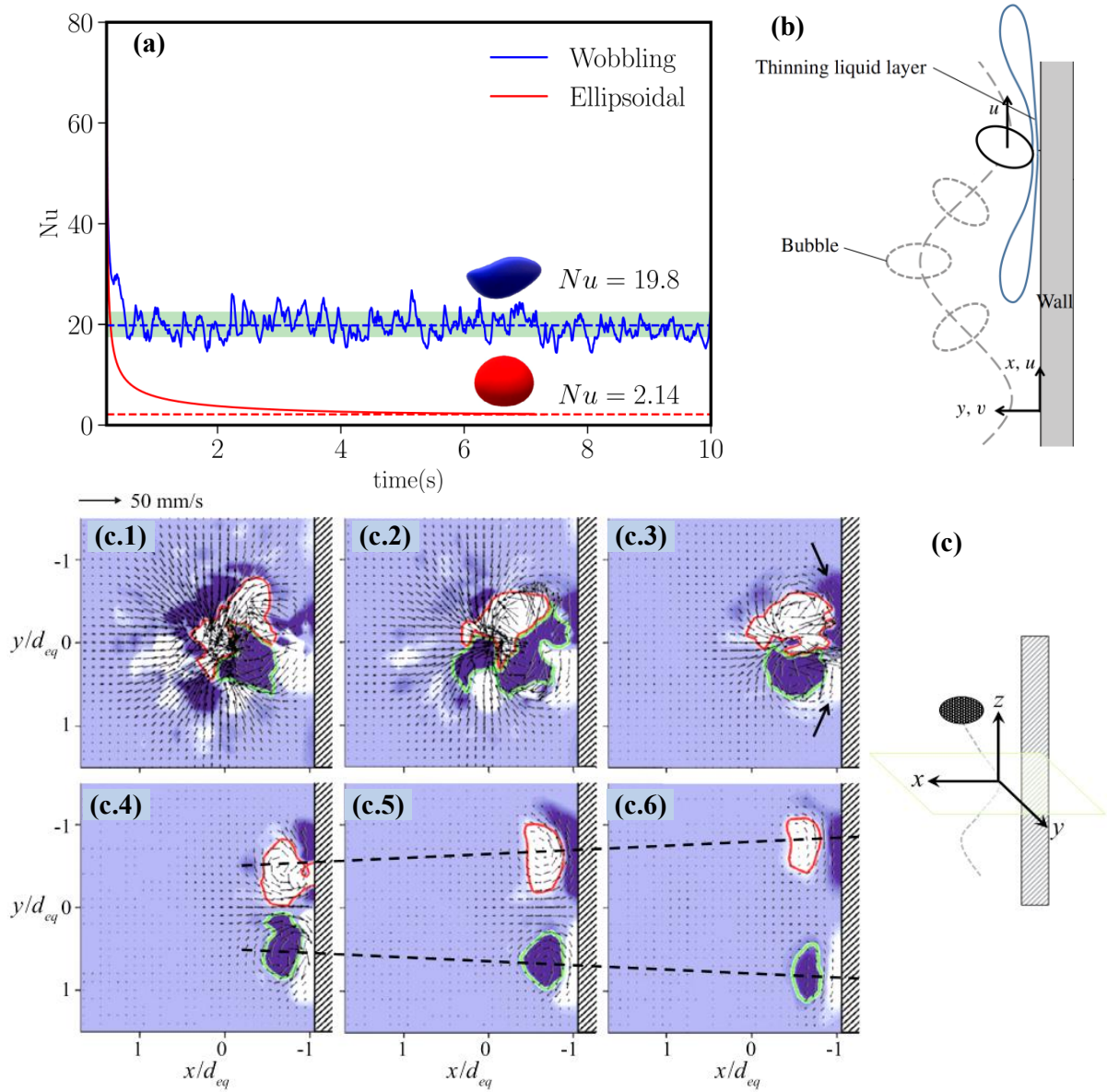


Fig. 36: (a) Instantaneous trend of Nu number for two different bubble shapes near a hot wall: wobbling and ellipsoidal (Panda et al. [319]). The equivalent diameter of the bubble in two cases was 0.01 (m). Air and water are used as the dispersal and continuation phases, respectively. The Wall temperature was 393 K, while the initial water temperature was 293 K. (b) Conceptual diagram showing the thin liquid layer between the wall and the bubble [222]. (c) the streamwise contour of vorticity and wake structure of a zigzag bubble rising in the proximity of a vertical wall in different times (or locations, c.1 to c.6 corresponds to a location of $z=-0.5, -0.8, -1.4, -1.9, -2.1$ and -2.4). The iso-vorticity contours are shown by solid lines with $\omega_z = 0.147$ (red) and -0.147

(green) (Lee and Park [320]). (Figures (a, c) and b are reprinted with permission from Elsevier and Cambridge University Press, respectively).

Panda et al. [319] also investigated the heat transfer of swam bubbles by varying the gas injection percentages to produce wobbling bubbles within a void fraction range of 2% to 20% in a dense bubbly flow. With increasing the gas fraction from 2 to 10%, the wall-averaged Nusselt number was remarkably increased. Subsequently, as the gas fraction increased from 10% to 20%, the Nusselt number gradually approached a plateau. Notably, it was identified that the Nusselt number exhibited a scaling correlation with the gas percentage, $Nu \propto \alpha^{0.25}$. In another study, Gvozdić et al. [322] experimentally investigated the heat transfer in turbulent homogeneous bubbly flow. The bubble was injected at $Re=200$ with a gas void fraction between 0%-5%, and the Ra number was in a range of 4.0×10^9 - 1.2×10^{11} (indicative of turbulent flow). The surprising result was that the average Nusselt number is approximately independent of the Rayleigh number (Ra) and depends solely on the gas void fraction, following the scaling law $Nu \propto \alpha^{0.45}$. This result is primarily attributed to the presence of two distinct sources of heat transfer in homogeneous bubbly flow: natural convection, driven by the horizontal temperature differential, and bubble-induced diffusion, which plays a predominant role in heat transfer. This scaling, $Nu \propto \alpha^n$ (which shows that Nu number remains almost constant by increasing the Ra number), suggests that the temperature in bubbly flow acts like a passive scalar and that the heat transfer is not dominated by the boundary layer along the walls.

Alméras et al. [323] demonstrated that heat transfer efficiency at low void fractions is primarily governed by bubble-induced mixing. They observed that passive scalar transport in homogeneous bubble swarms at high Péclet numbers behaves as a diffusive process. When heat transfer is treated as a diffusive process, the Nusselt number is defined as the ratio of the effective diffusivity induced by the bubble swarm to the thermal diffusivity. Indeed, Alméras et al. [323] showed that the diffusion terms (D) can be defined as $D \propto u' \Lambda$ for a low gas void fraction. Here, Λ is the integral Lagrangian length scale ($\Lambda \simeq D_{bu}/C_D$, which C_D is the drag coefficient of a single bubble) and u' is the standard deviation of the velocity fluctuations (Riboux et al. [312]). The only term in the diffusivity, which relies on the volume fraction of gas phase, is u' , and this dependency can be written as $u' \propto V_0 \alpha^{0.4}$ (where V_0 is the single bubble rising velocity) as presented by Risso and Ellingsen [324]. That was the conclusion of Gvozdić et al. [322] in their study, demonstrating the diffusivity (D), or the Nu number, given as $D \propto \alpha^{0.45}$ extending the model of Alméras et al. [323].

One of the most significant advantages of bubble injection, rendering it valuable for industrial applications, is its capability to generate a more homogeneous temperature distribution over a surface, thanks to mixing produced by bubbles. The numerical study by Dabiri et al. [325] demonstrated a decrease in wall temperature fluctuations attributed to the increased mixing induced by bubbles (Fig. 37-a,b,c). The rise of nearly spherical bubbles near the wall induces turbulence (velocity fluctuations) in the viscous sublayer region, consequently enhancing the heat transfer rate. Despite the lack of proximity to the wall, deformable bubbles increase stirring, elevating the Nusselt number. The Nu number for nearly spherical bubbles was higher than for deformable bubbles. The mixing due to a swarm of wobbling bubble injection near the wall can be seen in Fig. 37-(d) (Panda et al. [319] study). As anticipated, the bubble-induced agitation enhances the mixing of cold and hot fluid, increasing the heat transfer rate.

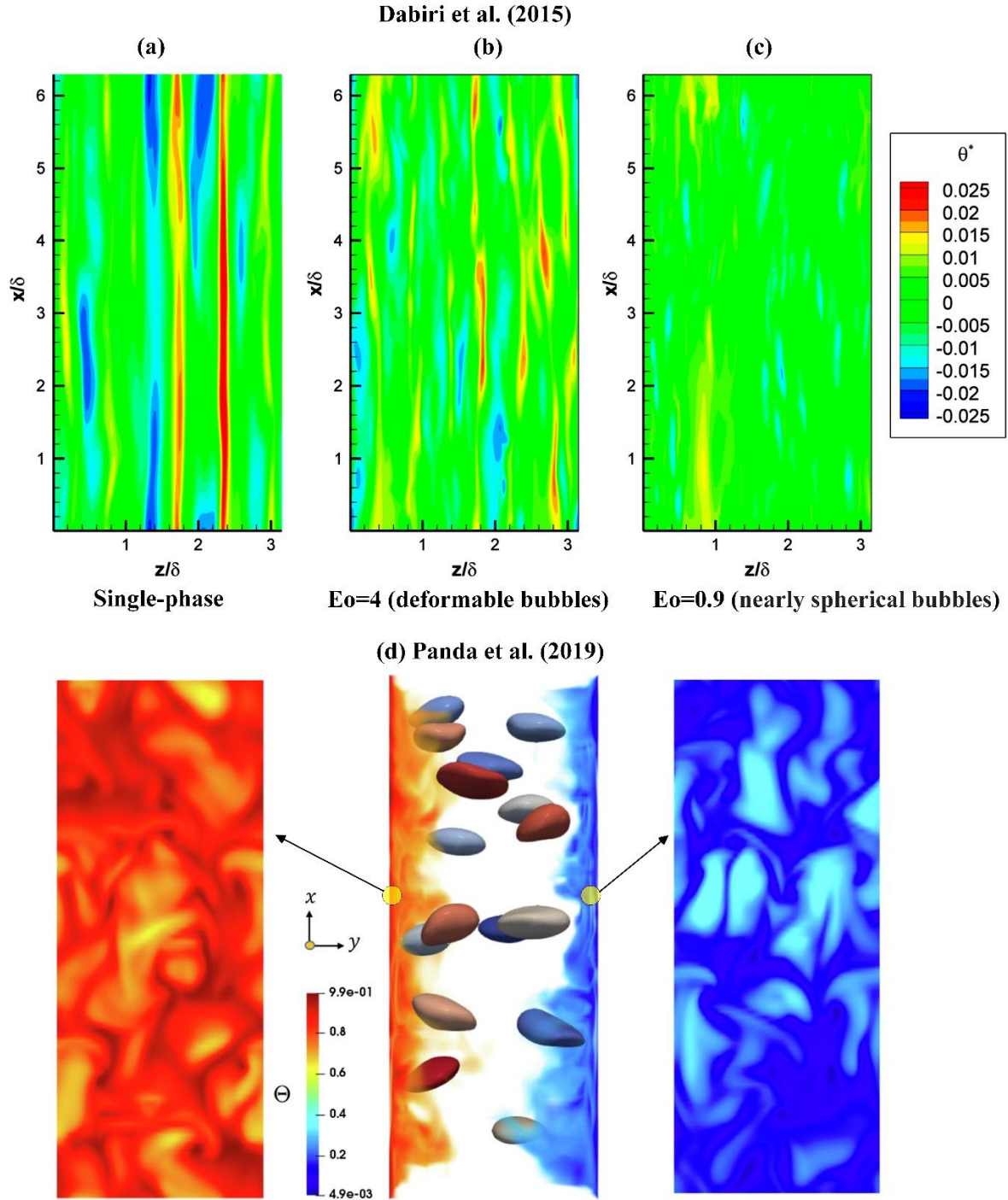


Fig. 37: Temperature fluctuation over the wall (Dabiri et al. [325] work): (a) single-phase flow ($Nu=47.7$), (b) deformable bubbles and $Eo=4$ ($Nu=60.7$), and (c) nearly spherical bubble with $Eo=0.9$ ($Nu=77.3$). The Re and Pr of the liquid phase were 5600 and 7, respectively. The rise of nearly spherical bubbles near the wall induces turbulence in the viscous layer, reducing the conduction region's size adjacent to the wall and consequently improving the heat transfer rate. (d) the dimensionless temperature in a swarm of wobbling bubbles in an upward channel flow ($Mo=2.62e-11$, $Eo=2.17$, $Pr=1$). The mixing has been shown in the half grid size away ($dy/2=6.25e-5$) from the hot and cold walls. The bubble equivalent diameter is 0.002 m (Panda et al. [319]). (Figures are reprinted with permission from Elsevier)

A universality scaling is observed in the temperature and velocity fluctuations in fully developed turbulent flows and associated spectra [326]. As extensively investigated both

experimentally and numerically by researchers, and also based on the Kolmogorov–Obukhov–Corrsin hypothesis ([327-329]), in the inertial-convective range, a scalar's spectrum exhibits universal scaling with a $-5/3$ scaling ($E_\theta(k) \propto k^{-5/3}$), for large values of Reynolds and Peclet numbers. Here, θ and k are the passive scalar and wavenumber [330].

The use of bubble injection in turbulent flows introduces additional complexity to the problem, potentially distorting the scaling law and affecting its universality [331-333]. Lance and Bataille [304] investigated the emergence of the k^{-3} scaling law for the bubble-induced flow in the energy spectrum (velocity fluctuation spectrum). They suggested that balancing viscous dissipation and energy production due to the bubble rising leads to the k^{-3} scaling. Fig. 38 shows the velocity fluctuations spectra for homogenous turbulent bubbly flow [297, 312].

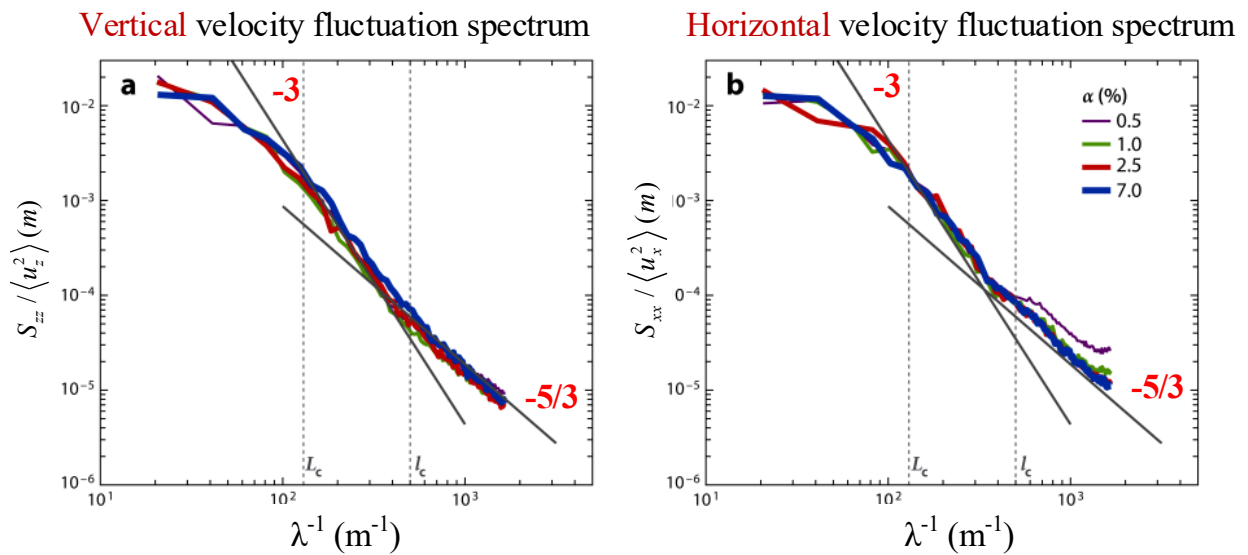


Fig. 38: Velocity fluctuations spectra: (a) vertical and (b) horizontal of a homogeneous swarm bubbly flow with 2.5 (mm) bubbles diameter rising in water. ([297, 312]) for various ranges of bubble void fractions. λ ($2\pi/k$) is the wavelength. The dashed line depicts the area of k^{-3} scaling between 2 mm and 7.7 mm, which shows the bubble-induced scaling in turbulent flow, followed by classical turbulent inertia scaling of $k^{-5/3}$. There is not much difference between the horizontal and vertical fluctuation spectra. (Figure is reprinted with permission from Annual Reviews).

The dashed line in Fig. 38 depicts the area of k^{-3} scaling between 2 mm and 7.7 mm, which shows the bubble-induced scaling in turbulent flow, followed by classical turbulent inertia scaling of $k^{-5/3}$. A negligible difference exists between the spectra of horizontal and vertical velocity fluctuations. It is apparent that the k^{-3} scaling is consistently observed across the range of void fractions up to 7%

Furthermore, beyond the alteration in scaling observed in velocity fluctuation spectra, a recent investigation by Dung et al. [334] revealed the presence of k^{-3} scaling in thermal spectra in the bubble-induced turbulent flows. They injected large bubbles ($Re=O(10^2)$) at the

void fraction in the range of 0-5%. They look at the frequencies that mark the beginning of the -3 scaling of the scalar spectra. The $-5/3$ scaling lessens as α rises from 0% to 5.2%, and a -3 scaling follows when α reaches a sufficient size; Fig. 39-a illustrates this phenomenon. By applying the parameterization, the onset frequency f_t can be determined as follows,

$$P_\theta(f) = \frac{\left(f/f_L\right)^{\zeta_b}}{\left[1 + \left(f/f_t\right)^2\right]^{(\xi_b - \xi_a)/2}} \quad (80)$$

The $P_\theta(f)$ is the power spectral densities [334], which depicts a change in scaling throughout. In this case, as shown in Fig. 39, f_t is the transition frequency from the scaling f^{ζ_b} (for $f \ll f_t$) to f^{ζ_a} (for $f \gg f_t$) with scaling exponents ζ_b and ζ_a , respectively. f_L is a fitting parameter that represents the curve height. Here, the ζ_b , which is known by the Kolmogorov–Obukhov value, is set to be $-5/3$ and ζ_a is -3 , which are consistent with the exponent of the velocity spectra in high-Re bubbly flows and reflect the observed limiting cases instead of trying to fit these exponents [335] and [297]. They also investigate the local logarithmic slope $\xi(f)$ of the scalar power spectra to identify the range that primarily encompasses the $-5/3$ and -3 scaling.

$$\xi(f) = \frac{d \log_{10} P_\theta(f)}{d \log_{10} f} \quad (81)$$

Every spectrum has a local logarithmic slope that can be estimated at a different frequency. This is done by using a moving fit of a straight line in logarithmic space centered at a frequency with a window size of a one-quarter decade of the frequency. Fig. 39-b displays the $\xi(f)$ results for various α . Fig. 39-b illustrates that, for $\alpha = 2.2\%$, $-5/3$ scaling begins to emerge at 1.2 Hz and ends at 15 Hz. Consequently, for all cases, a rational range covering the change from $-5/3$ to -3 scaling is 1.2–15 Hz. Further information can be found in [334]. In conclusion, bubble injection changes the universality in turbulent flow spectra, scaling from $-5/3$ to -3 , for temperature and velocity fluctuations.

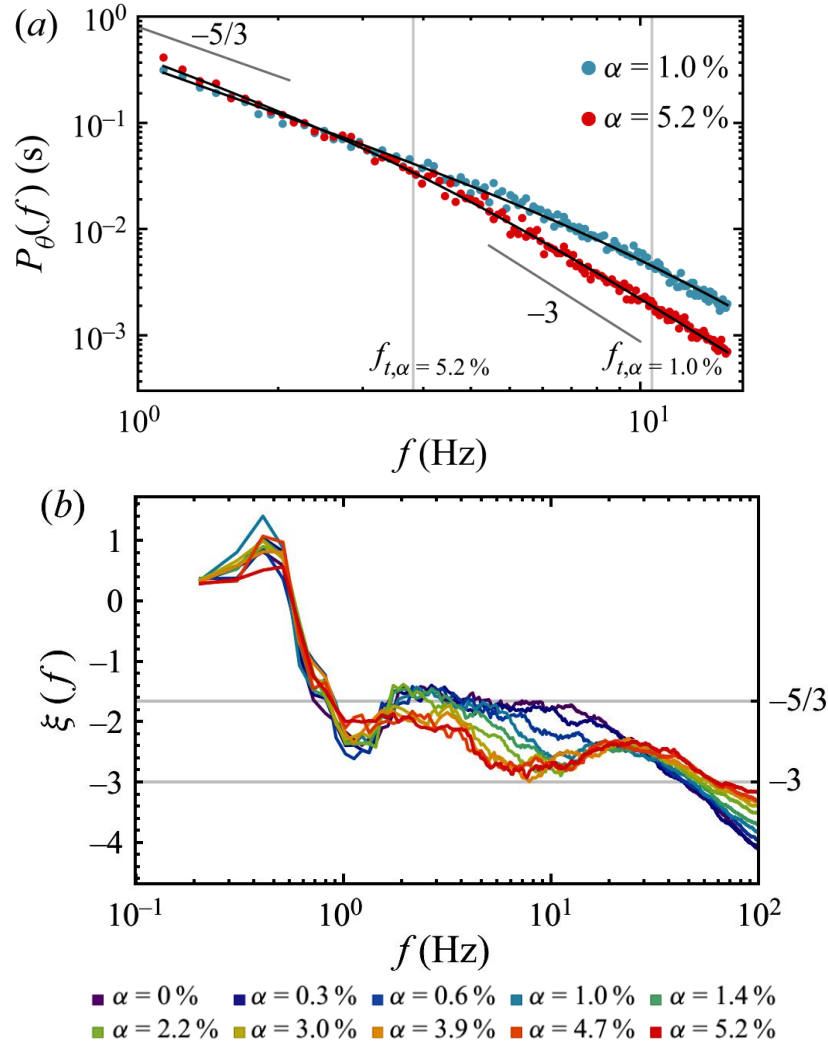


Fig. 39: (a) The power spectral densities $P_\theta(f)$ regarding the fluctuation of temperature (dots) for two different gas void fractions: 1% and 5.2%, (b) The local scaling exponent $\xi(f)$ at different gas void fractions. The gray lines in both show the scaling behavior of -3 and $-5/3$ scaling Dung et al. [334] (Figures is reprinted with permission from Cambridge University Press)

3.2.4. Mechanism of heat transfer in bubble impingement

Bubble impingement and contact with the heat transfer surface are inevitable processes in a heat exchanger. Therefore, it is crucial to understand the physical mechanisms governing heat transfer during the impingement phase. Donoghue et al. [336] performed an experimental study investigating the heat transfer mechanism in bubble impingement. Findings are shown in Fig. 40 for a single bubble discharged from an opening 30 mm away from the heated surface via a nozzle with a 1 mm inner diameter, initially at a temperature of 51°C.

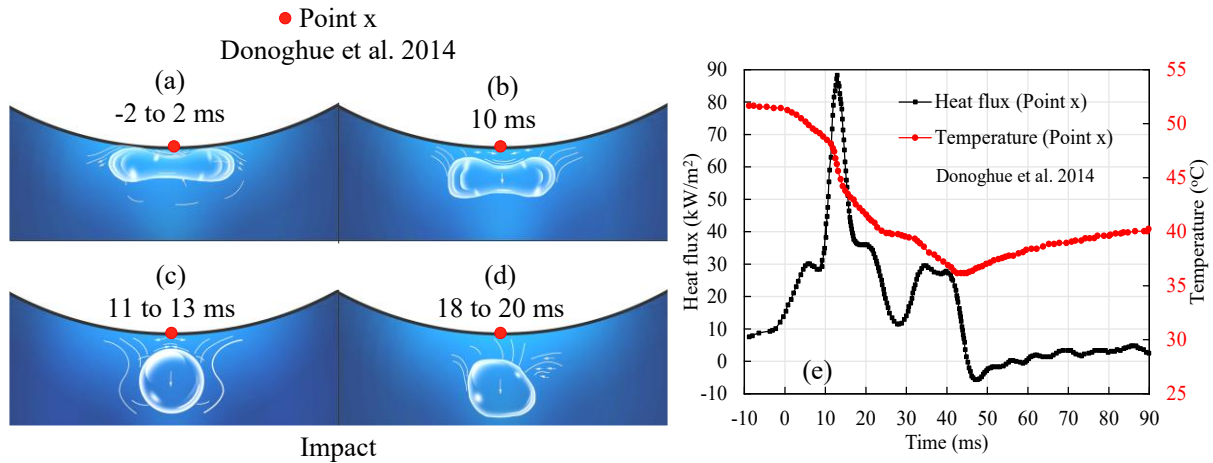


Fig. 40: Fluid flow in the course of bubble impingement process (Donoghue et al. [336] study): (a) impact (-2 to 2 ms), (b) retraction process from surface (10 ms), (c) dimple inversion stage (11–13 ms) (d) rebound of the bubble from the surface and how the fluid moves during this phase (18–20 ms), and (e) the temperature and heat flux of the red point (x) during the impact. The initial temperature on the point was 51 °C. The bubble aspect ratio changes during the process, impacting the flow and heat transfer rate.

Fig. 40-(a) to (d) displays the collision of a bubble with a solid surface before and after the impact. Also, Fig. 40-(e) shows the graph of heat flux and temperature of point x. The bubble attains a maximum terminal velocity of 333 mm/s before contacting the solid surface. Before the impact ($t < 0$), the heat flux has increased slowly, and then it experiences a relatively sharp increase up to the relative extremum point. Then, during the collision, the heat flux increases strongly to reach its maximum value as the bubble rebounds from the surface.

Fig. 40-(a) portrays the bubble in its pre-impact state. As the bubble approaches the surface, a phenomenon known as cupping transpires, leading to liquid trapping between the bubble and the solid surface. Consequently, the thickness of the boundary layer is reduced, leading to an initial peak in the heat flux diagram. Fig. 40-(b)&(c) illustrate the bubble bounce to the surface and rebounds from it, which occurs multiple times. In this process, the bubble recovers its spherical shape, which results in the rapid movement of cold liquid between the bubble and the surface and causes a sharp increase and peak of heat flux. It is important that the temperature profile also changes according to the heat flux profile depicted in Fig. 40-(d). The surface temperature decreases as the heat flux increases due to fluid convection heat transfer escalation. Fig. 40-(e) demonstrates the rebound of the bubble from the surface and how the fluid moves during this phase (18–28 ms). This fluid movement is because of the bubble's wake during the initial contact with the surface and rebounding from it. During this phase, the mixing of the hot and cold fluid results in an increase in the heat flux. This process is repeated during the second bubble impact, leading to another peak in the heat flux diagram.

Finally, the heat flux diagram experiences minor fluctuations caused by the wake disturbances created by the bubble after rebounding. More details about this process are given in [319]. In conclusion, the temperature at point x decreases from approximately 52 to 40°C degrees, highlighting the positive impact of bubble injection on the surface in enhancing heat transfer.

4. Bubble injection method for simple geometry: plate and cylinder

4.1. Inclined, vertical, and horizontal plates using bubble injection

The previous section focused on the general heat transfer mechanisms in bubbly flows. However, this section examines cases where bubbles are injected into the fluid near flat plates, including inclined, vertical, and horizontal configurations. These scenarios have been extensively studied to understand how bubble injection affects heat transfer in specific geometries [321]. In 1971, Kenning et al. [337] investigated the heat transfer coefficient of water-containing micro-bubbles on a vertical plate (upward flow direction). The significant result obtained from this study was the independence of the heat transfer coefficient from the thermocapillary effect. Also, the maximum percentage of increase in the heat transfer coefficient was 50%, dependent upon the velocity of water flow and bubbles (Table 9).

Natural convection in mercury on a flat plate undergoing nitrogen bubble injection was studied by Tokuhiro et al. [338]. The results demonstrated that in a low heat flux region when the flow is still laminar, the utilization of a bubble could remarkably increase the Nu number up to 25%. In another study, Tokuhiro et al. [339] studied natural convection by applying a transverse magnetic field with an intensity ranging from $B=0.07-0.5$ T. Details about their work are presented in Table 9.

To investigate the convection heat transfer improvement on a horizontal plate due to the bubble injection in an impingement liquid jet, Zumbrunnen et al. [340] designed an experiment. In their experimental procedure, boiling did not occur since the surface temperature was lower than the boiling temperature of the liquid (Fig. 41-a). A nozzle with a diameter of 2.03 mm was utilized to combine water with different air fluxes (Table 9). Compared to the non-bubble injection mode, it was found that (Fig. 41-b) the Nu number was enhanced by 2.2 times at the stagnation point on the surface as the flow quality ($\dot{m}_{bu}/\dot{m}_{tot}$) increased. Several other studies were performed on heat transfer augmentation in liquid impingement jets under bubble injection [341-347]. The explanation of different studies and their significant findings are listed in Table 9.

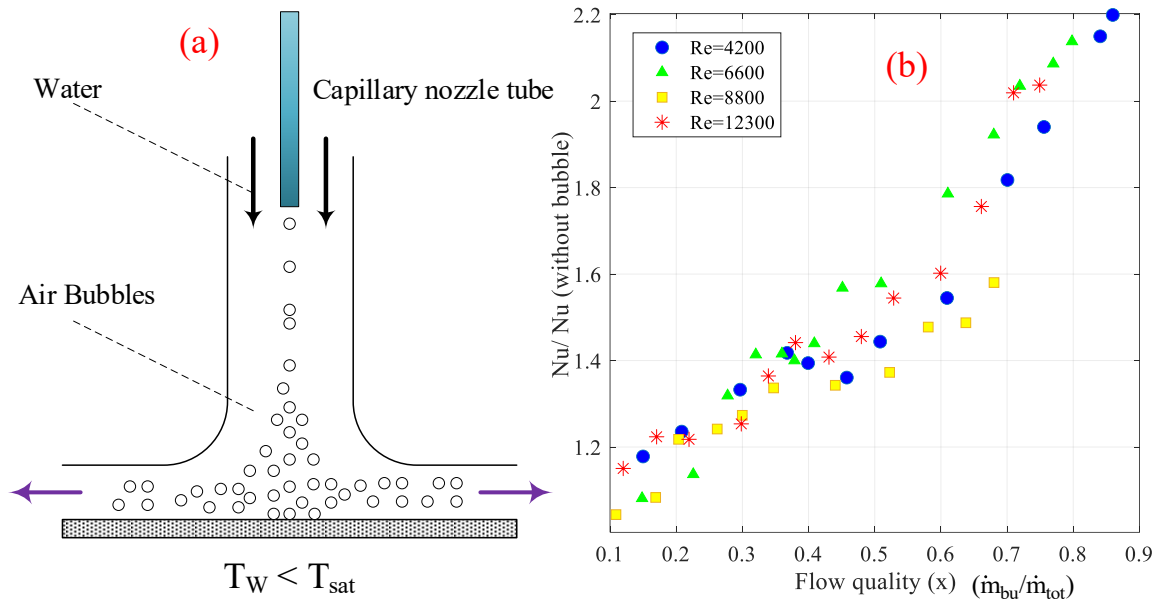


Fig. 41: (a) Schematic diagram of the test section used by Zumbrunnen et al. [340], and (b) variation of Nu number with quality. As the flow quality (x) increased, the Nu number at the surface stagnation point was enhanced by 2.2 times compared to single-phase flow (Figure is reprinted with permission from ASME).

In recent years, there has been a growing interest in studying the effect of sliding air bubbles on enhancing surface heat transfer. For instance, Donnelly et al. [348] investigated this problem on an inclined surface with an angle of 30° for different sizes of bubbles over time. A schematic of the experimental setup is shown in Fig. 42-a. They also studied the effect of fluid flow on heat transfer. They demonstrated that the local and average values of the heat transfer coefficient increased up to 8 times compared to the mode in which no bubble is injected. The reason for this enhancement was the combination of the effects of two critical phenomena: first, the forced convection that happens due to the motion of the bubble inside the fluid (Fig. 42-b), followed by the formation of vortices within the wake zone (Fig. 42-c). The explanation and findings of other related research projects undertaken recently are listed in Table 9 [349-351].

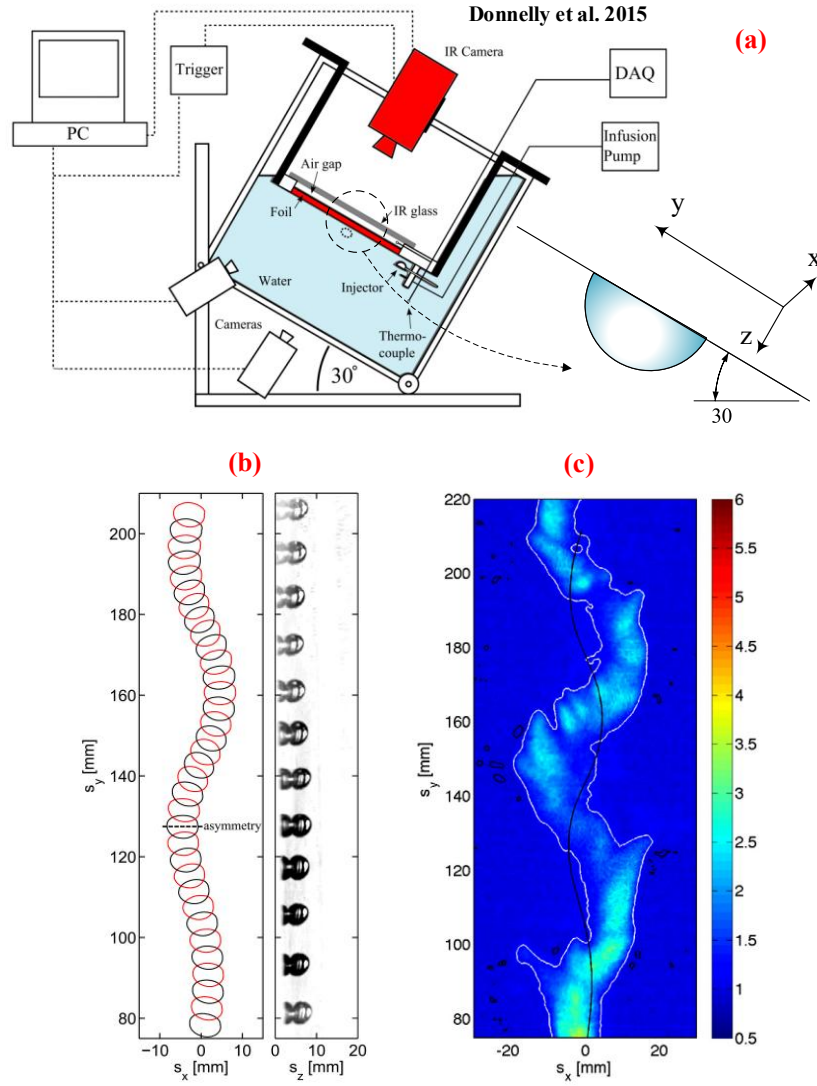


Fig. 42: (a) Experiment apparatus used by Donnelly et al. [348], (b) bubble shape and path variations, and (c) heat transfer coefficient contour ($Dbu = 5.76$ mm, $t = 1$ s). Compared to the non-bubble injection mode, the local and average values of the heat transfer coefficient rose by up to eight times due to bubble mixing agitation followed by the formation of vortices within the wake zone (Figures is reprinted with permission from Elsevier)

One of the vital applications of the bubble injection method is improving the natural convection heat transfer process. Kitagawa and Murai [71] studied the effect of injecting microbubble on the natural convection heat transfer of water on a vertical surface. The schematic of their experimental apparatus is illustrated in Fig. 43-a in which a hot surface with a constant heat flux of 1480 W/m^2 , an electrolyzer for hydrogen bubble production, a DC power supply, and a water-cooled heat exchanger represent its significant parts. For bubble generation in two different volumetric flow rates of 30 and $42 \text{ mm}^3/\text{s}$, a platinum wire with lengths and diameters of 12 and 20 mm was utilized as a cathode. The schematic of the platinum wire and the image of the bubbles near the wall for the volumetric flow rate of $42 \text{ mm}^3/\text{s}$ are depicted in Fig. 43-b&c. Results proved that micro-bubbles could dramatically increase the heat transfer

coefficient compared to their absence. Fig. 43-d shows the heat transfer coefficient along the surface length for different volumetric fluxes of injection. The heat transfer coefficient was improved by two times. The influence of the bubble diameter on the mixing effect was also investigated in this work.

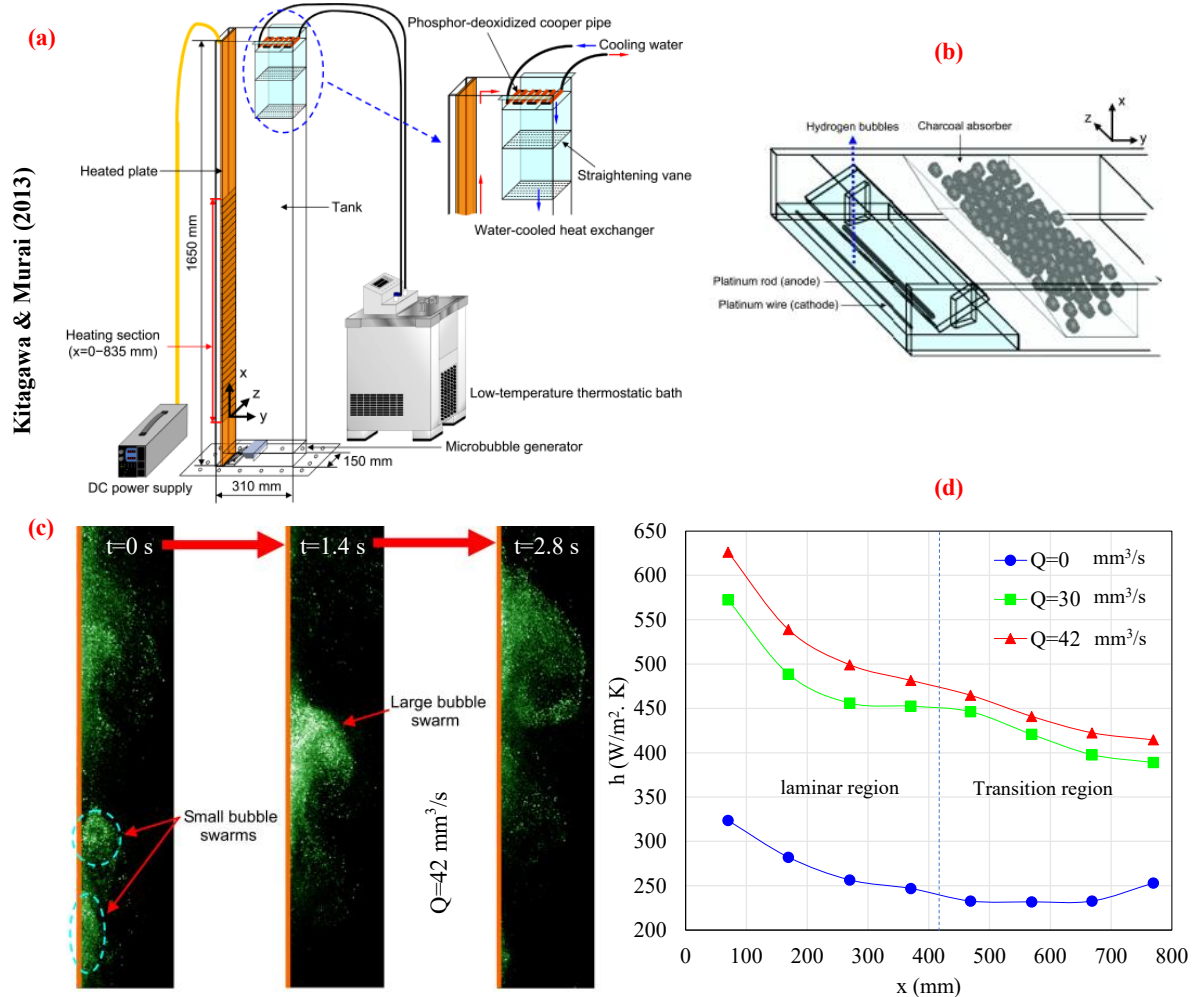


Fig. 43: Schematic of (a) Experimental apparatus, (b) bubble generator, (c) bubble images near the heated wall, and (d) convection heat transfer coefficient along the plate (Kitagawa and Murai [71] study). The heat transfer coefficient was improved two times compared to the non-bubble injection mode (Figure is reprinted with permission from Elsevier).

Kitagawa et al. [352] examined the impact of incorporating a flow-guiding plate parallel to a heated vertical surface in a water medium with bubble injection. This investigation analyzed the interplay between two phenomena: the reduction in heat exchange caused by vortices and the augmentation of heat transport facilitated by flow parallel to the wall. The study revealed that while bubble injection can triple the heat transfer, introducing the flow-guiding plate leads to an additional 17% improvement in heat transfer.

Table 9 summarizes similar studies done on the bubble injection method under different flow regimes (e.g., natural convection and mixed convection) and different bubble sizes (e.g.,

sub-millimeter and millimeter) and shows the significant results obtained from each study for heat transfer coefficient or Nu number separately.

Table 9: Summary of research on bubble injection over simple plate geometries.

Authors	Geometry	Working fluids	Bubble injection descriptions	Conditions	Remark
Kenning et al. [337]	Vertical plate	Air-water	Micro-bubbles at different flow rates were injected from the bottom to investigate heat transfer enhancement.	$U_{bu}=[0.011,0.022,0.044]$ (m/s) $U_w=[0.6,1.2,1.8]$ (m/s) $D_{bu}=1.5$ (mm)	The h increased by 50% and was independent of the thermocapillary effect.
Tokuhiro et al. [338]	Vertical plate	Nitrogen-mercury	Nitrogen bubbles at different flow rates improved the natural convection process.	$Q_{bu}=[0.9-9.2]$ (cm ³ /s) $q=[370-16000]$ (W/m ²)	The Nu number enhancement was between 20 to 25%.
Tokuhiro et al. [339]	Vertical plate	Nitrogen-mercury	Similar to the study of Tokuhiro et al. [338] but with the effect of the presence of a magnetic field with different intensities.	$Q_{bu}=[0.9-9.2]$ (cm ³ /s) $q=[370-16000]$ (W/m ²) $B=[0.07-0.5]$ (T)	The Nu number was reduced in the presence of a bubble and magnetic field enhancement.
Zumbrunnen et al. [340]	Horizontal plate	Air-water	The bubble was injected into an impingement liquid jet at different flow rates through a capillary tube to identify its thermal effect on a hot horizontal plate.	$U_{bu}=[0-1.176]\times 10^{-3}$ (m/s) $U_w=[1.82-9.97]\times 10^{-4}$ (m/s) $Re_{bu}=[3.7-21.0]\times 10^3$ (-) Flow quality (x)= [0.0-0.86] (-)	The Nu number increased by 220% compared to the non-bubble injection mode.
Friedrich et al. [341]	Horizontal plate	Air-water	Investigation of the hydrodynamic and thermal performance of a jet impingement under bubble injection at different flow rates.	Flow quality (x)= [0.0-0.9] (-) $Re_w=[3.3-4.329]\times 10^3$ (-)	The maximum Nu number was in the flow quality (x) of 0.8, and a correlation was introduced for the Nu number at the stagnation point. The Nu number was enhanced up to 71% at the stagnation point.
Glaspell et al. [342]	Horizontal plate	Air-water	Investigation of the hydrodynamic and thermal performance of a jet impingement under bubble injection at different flow rates and closer distances between the nozzle and the plate.	Flow quality (x)= [0.3-0.8] (-) $Re_w=[3.0-5.0]\times 10^3$ (-) nozzle-to-surface distance = [0.02-0.51] (-)	The Nu number increased by 1.8 times at distances near the plate, and a new correlation for the Nu number was proposed.
Trainer et al. [343]	Horizontal plate	Air-water	The heat transfer and fluid flow were investigated for an air-assisted impinging jet at different bubble and water flow rates and the effect of nozzle geometry dimensions on the Nu number.	$q=[15.0-57.0]$ (W/cm ²) $Re_w=[7.5-15.0]\times 10^3$ (-) $Re_{bu}=[0.0-5.9]\times 10^3$ (-) $We_L=[150.0-650.0]$ (-)	The Nu number increased by 2.6 times compared to the non-bubble injection mode at the stagnation point, and a correlation for calculating the Nu number was presented.
Kowsary et al. [344]	Horizontal plate	Air-water	A two-dimensional investigation was conducted to determine the heat transfer coefficient distribution in a bubbly impinging jet on a horizontal plate.	$Re_{bu}=[0.0-5.9]\times 10^3$ $T_w=[60-70-80-90]$ (°C)	It increased by 128% in the Nu number compared to the non-bubble injection mode.
Pakhomov and Terekhov [345]	Horizontal plate	Air-water	They numerically investigated the heat transfer and fluid flow in a turbulent bubbly jet impingement at different flow rates and sizes of the bubble.	$U_{bu}=[0.25-1.0]$ (m/s) $Re_w=[0.8-6.0]\times 10^4$ (-) $T_w=T_{bu}=15$ (°C) Flow quality (x)= [0.0-0.1] (-) $D_{bu}=[0.0-5.0]$ (mm)	The h was enhanced up to 50%. Wall friction was enhanced by up to 40%.
Friedrich et al. [346]	Horizontal plate	Air-water	Investigation of the effect of bubble injection at different flow rates on the circular hydraulic jump and the Nu number in an air-assistant circular water jet impinging was done.	Flow quality (x)= [0.0-0.9] (-) $Re_w=[3.3-4.329]\times 10^3$ (-)	The maximum rate of increase in the hydraulic jump radius value occurred at the flow quality of 0.8. A new correlation for the Nu number was found based on the hydraulic jump radius.
Choo and kim [347]	Horizontal plate	Air-water	Heat transfer and two-phase impinging jets at different flow rates of bubble injection were studied.	Flow quality (x)= [0.0-1.0] (-)	The maximum rate of increase in the Nu number occurred at the flow quality between 0.2 and 0.3 in the bubble flow regime. Also, a novel correlation was suggested for the Nu number.

Donnelly et al. [348]	Inclined plate	Air-water	The effect of a sliding bubble was investigated at different sizes of the bubble on the h on an inclined plate.	$D_{bu}=[4.75-9.14]$ (mm) time (t)=[1.0-16.0] (s) surface angle= 30 (°)	The local and average h increased by 8 and 2 times compared to the non-bubble injection case.
Maeng and Park [349]	Vertical plate	Air-water	Heat transfer analysis near a hot vertical wall under the effect of a sliding bubble was done.	$D_{bu}=[2.0,2.8]$ (mm) $Re_{bu}=[220,750]$ (-) $We=[0.77,3.2]$ (-) $Eo=[0.16,1.2]$ (-)	The Nu number was enhanced by 4.8 times compared to the non-bubble injection mode.
Donnelly et al. [350]	Inclined plate	Air-water	Surface heat transfer was investigated under a sliding bubble at different plate angles.	surface angle= 10, 20, 30 (°) $D_{bu}=6$ (mm)	The Nu number was enhanced by 4.8 times compared to the non-bubble injection mode. Increasing the surface angle enhanced the heat transfer coefficient since the bubble velocity increased.
Bayazit et al. [351]	Inclined plate	Perfluorocarbon and its vapor	The effect of sliding bubbles on heat transfer improvement was examined.	surface angle= 12 (°) $U_{bu}=[0-20.0]$ (cm/s) $q'=1600$ (W/m ²)	The local h was improved.
Kitagawa and Murai [71]	Vertical plate	Hydrogen-water	Natural convection heat transfer enhancement was studied under microbubble injection at different flow rates and diameters on a vertical plate.	$T_w=22$ (°C) $Q_{bu}=30, 42$ (mm ³ /s) $q''=1480$ (W/m ²) $D_{bu}=41, 71$ (μm)	The natural convection heat transfer coefficient increased by two times compared to the non-bubble injection mode.
Kitagawa et al. [353]	Vertical plate	Hydrogen-water	Laminar natural convection heat transfer was studied under sub-millimeter-bubble injection on a flat plate.	$T_w=20$ (°C) $Q_{bu}=34, 45, 57$ (mm ³ /s) $q''=533, 947, 1480$ (W/m ²)	The heat transfer enhancement rate was between 1.35 and 1.85. As the heat flux increased, the heat transfer coefficient reduced.
Kitagawa et al. [314]	Vertical plate	Hydrogen-water	The effects of sub-millimetre-bubble size on laminar natural convection on a flat plate were investigated.	$T_w=20$ (°C) $Q_{bu}=33, 45, 56$ (mm ³ /s) $q''=533, 947, 1480$ (W/m ²) $D_{bu}=0.3, 0.51, 0.7$ (mm)	The heat transfer enhancement rate was between 1.3 and 2.2. At a constant volumetric flow rate of the bubble, h due to the advection effect increased as the bubble diameter decreased.
Kitagawa et al. [354]	Vertical plate	Hydrogen-water	laminar mixed-convection heat transfer was studied under sub-millimeter-bubble injection on a flat plate.	$T_w=20$ (°C) $Re_w=100, 150, 200$ (°C) $Q_{bu}=33, 34, 35$ (mm ³ /s) $q''=1790$ (W/m ²)	Compared to the non-bubble injection mode, the heat transfer enhancement rate was proposed to be between 1.24 and 1.38. Increasing the Reynolds number reduced the heat transfer enhancement ratio.
Kitagawa et al. [355]	Vertical plate	Hydrogen-water	Turbulent natural convection heat transfer was studied under sub-millimeter-bubble injection on a flat plate.	$T_w=20$ (°C) $Q_{bu}=33, 56$ (mm ³ /s) $q''=530, 950, 1480$ (W/m ²)	Heat transfer increased by 1.3 times compared to the non-bubble injection mode. In a turbulent regime, heat transfer enhancement was due to the rotation of eddies and the increase in velocity gradient near the wall.
Kitagawa et al. [356]	Vertical plate	Hydrogen-water	Transition to turbulence in natural convection heat transfer was investigated under sub-millimetre-bubble injection on a flat plate.	$T_w=20$ (°C) $Q_{bu}=56$ (mm ³ /s) $q''=530, 950, 1480$ (W/m ²) Injection position=1.6,3.6 (mm)	Heat transfer increased by 2.12 times compared to the non-bubble injection mode. Transition to a turbulent regime depended heavily on the shape and size of the bubble-water flow region.
Kitagawa et al. [357]	Inclined plate	Air-water	The effects of surface angle and millimeter-sized bubble injection on water natural convection heat transfer were studied.	surface angle= 0, 20, 40 (°) $T_w=22$ (°C) $Q_{bu}=0.1$ (LPM) $q''=1480$ (W/m ²) $D_{bu}=2.3-2.8$ (mm) $D_{noz}=0.36$ (mm)	Compared to the non-bubble injection mode, heat transfer enhancement was in the range of 2.4-5.5. As the plate angle increased, the Nu number increased due to the enhancement of unsteady vortices.

Kitagawa et al.[352]	Vertical heated plate with a guiding flow parallel plate	Air-water	The effect of bubble injection on the natural convection heat transfer was studied experimentally between a heated vertical plate and a flow-guiding plate in water. Air bubble injection was done using a peristaltic pump through nine stainless steel needles.	Plates distance= 20, 30 (mm) $T_w=22$ (°C) $q''=530, 1480$ (W/m ²) $D_{bu}=2.4$ (mm) $Q_{bu}=0.1$ (LPM) Injection position= 10 (mm) from heated plate	The flow guiding plate accounts for 17% of heat transfer enhancement compared with the case of the flow plate without guiding.
Khodadadi et al. [358]	Inclined plate	Not mentioned	The influence of bubble injection on natural convection heat transfer was investigated through numerical simulations employing the Volume of Fluid (VOF) method solver within the Open FOAM package. The initiation of bubble injection, initially devoid of any initial velocity, was achieved using the set Fields utility at a distance of $\delta=0.0125$ m from the inclined hot wall.	surface angle= 30, 45, 60, 90 (°) contact angle= 0 to 120 (°) $D_{bu}=20$ [Single bubble], 15.8 [Bubble pair] (mm) $q = 79$ W	The peak Nusselt number is attained at $\theta=60$, with an accompanying average Nusselt number increase of 26% in the bubble pair mode and 18% in the single bubble mode, both with the same bubble volume. Notably, the Nusselt number experiences a significant surge of over 92% in the intermittent contact regime. It's worth highlighting that the Nusselt number is notably influenced by the surface angle and the Bond number, making them two of the most impactful parameters in this regard.

In conclusion, the research in this section yields several significant findings. In laminar flow, bubble injection enhances the force convection effect by increasing mixing, resulting in enhanced heat transfer. This enhancement is more pronounced in laminar flow compared to transient flow. In transition flow, bubble injection promotes increased mixing and facilitates the transition to turbulent flow. For natural convection, the size of the bubble plays a crucial role in the mixing effect induced by bubble injection, with micro-bubble injections having a more pronounced effect. Additionally, in turbulent flow, it was observed that the Nusselt number is higher for a bubble exhibiting a wobbling shape than for an ellipsoidal one. Under turbulent natural convection, the average Nu number obeys the scaling law $Nu \propto \alpha^n$, that is, approximately independent of Ra. The two main heat-transfer mechanisms account for this scaling in homogeneous bubbly flow: bubble-induced diffusion, which serves as the primary heat-transfer mechanism, and natural convection, driven by the horizontal temperature differential. The distribution of temperatures can become more uniform with bubble injection. In addition, it was revealed that inhomogeneous bubbly injection results in increased heat transmission for smaller gas volume fractions ($\alpha < 4\%$) due to the induction of large-scale circulation. Conversely, when $\alpha > 4\%$, homogeneous injection proves more effective, with the rise in SIT (shear-induced turbulence) contribution being dominant.

4.2. Cylinders using bubble injection

This part discusses the influence of bubble injection on the heat transfer process around cylinders. Few studies have been focused on this research topic, and the available ones are mainly dedicated to bubble injection in heat exchangers with cylindrical tubes, including double pipe or shell and tube heat exchangers, which are discussed in the upcoming sections.

To better understand the heat transfer enhancement around a cylinder with bubble injection, it is rational to first check the flow regime all over the cylinder as it affects the Nu number on both the front and the backside of the cylinder. Murai et al. [359] designed an experimental setup to observe the air-water two-phase flow passing through a coaxial confined double rectangular chamber in two dimensions. They fabricated a narrow channel using two parallel plates with a distance of 10 mm (Fig. 44-a). In total, 492 nozzles with a diameter of 0.3 mm were used to inject bubbles. The authors observed that the injection of bubbles led to an increase in the wall friction stress, resulting in a decrease in the fluid velocity. According to Fig. 44-b, not only do the bubbles cause agitation in the flow, but they are also separated after hitting the cylinder. The bubble path was divided into two stages: the horizontal movement of a bubble followed by an upward motion. The sudden change in the bubble motion is basically because of the wake velocity of the liquid phase behind the cylinder, which is in a downward direction and opposite to the velocity vector of the bubbles. So, it can be concluded that bubble separation is a function of the cylinder's cross-section. Based on the flow regime around the cylinder, the changes in the heat flux and the resulting Nu number are higher on the front than on the backside. The numerical and experimental findings of the work by Nguyen et al. [360] also show similar trends, as can be seen in Fig. 44-c, where the bubbles are separated from both sides and are not directly transferred to the back wall of the cylinder after hitting it. This topic is also associated with a numerical study done by Zhu et al. [361] on the influence of the injection velocity of a pair of air jets on the vortex shedding at a low Reynolds number.

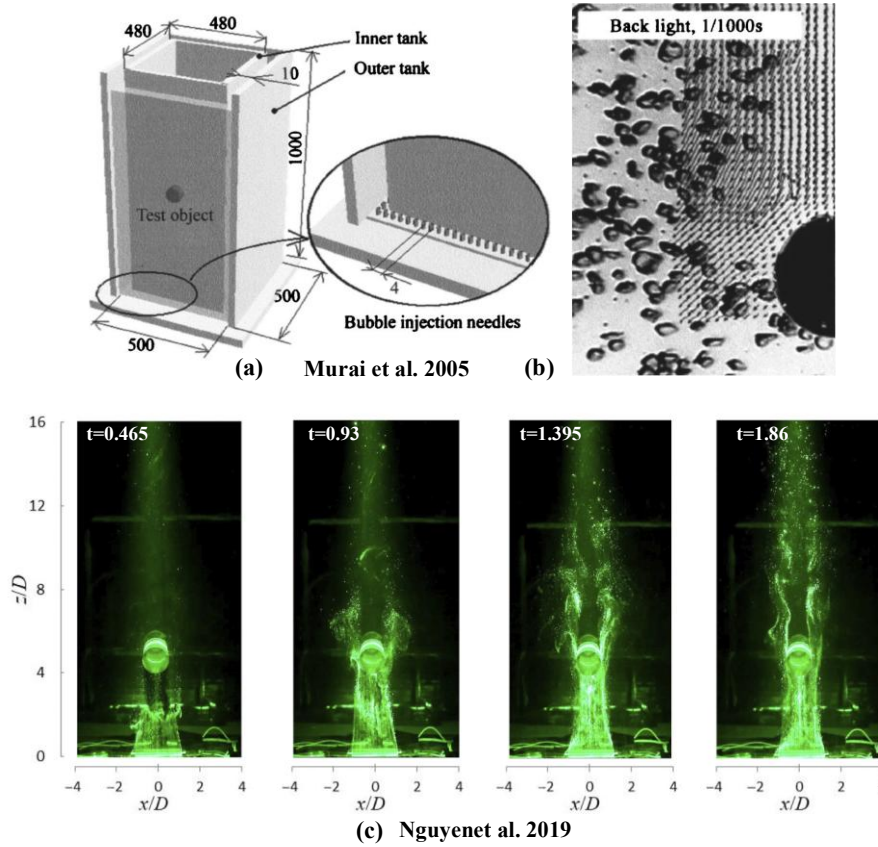


Fig. 44: (a) Experimental apparatus, (b) air-water flow pattern around a circular cylinder (Murai et al. [359]), and (c) air-water flow pattern around a circular cylinder during times (Nguyen et al. [360]). The variations in heat flux and the ensuing Nu number are greater on the front than the rear of the cylinder due to the flow regime surrounding it (Figures (a, b) and c are reprinted with permission from ASME and Emerald Publishing, respectively).

In 2005, a research was conducted by Atmane and Murray [362] to find the impact of bubble injection on the flow and the heat flux around a cylinder. As predicted, the results reflected that the amount of heat flux and the resulting heat transfer in the front side of the cylinder ($\theta=0^\circ$) reached the maximum value (Fig. 45). Then, the heat flux decreased until $\theta=60^\circ$, after which it reached a relative peak at $\theta=110^\circ$, and then the whole process repeats once again from the beginning. The underlying reason for the peak heat flux value in the front side of the cylinder was described by Murai et al. [359]. The second peak is due to the separation of bubbles and the resulting turbulence, and the third peak is only because of the wake region turbulence. Furthermore, the Nu number was enhanced by 4.5 times in comparison to the non-injected mode at the angle of zero and at the time of the bubble collision (Table 10).

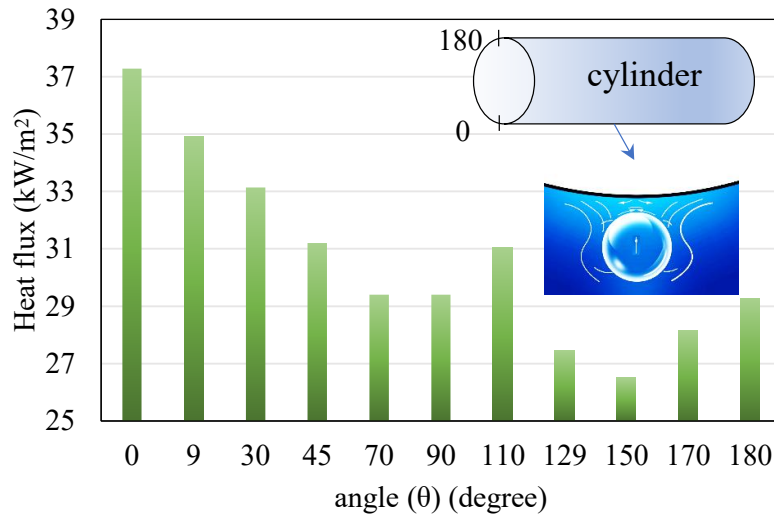


Fig. 45: Heat flux variation as a function of cylinder angle during bubble injection [362]. Heat transfer in the front side of the cylinder ($\theta=0^\circ$) reached the maximum value. The turbulence that results from the bubbles separating is responsible for the second peak, while the wake region turbulence alone is the origin of the third peak.

An experimental study was implemented by Tow and Leinhard [363] to investigate the heat transfer from a horizontal cylinder in a shallow bubble column (Fig. 46-a). The effect of fluid depth, cylinder height from the sparger, and bubble velocity on the heat transfer rate was examined. The findings confirmed that the height of the cylinder from the sparger had an influential impact on the heat transfer coefficient compared to other parameters, and the optimized value for this study was calculated to be 3 cm.

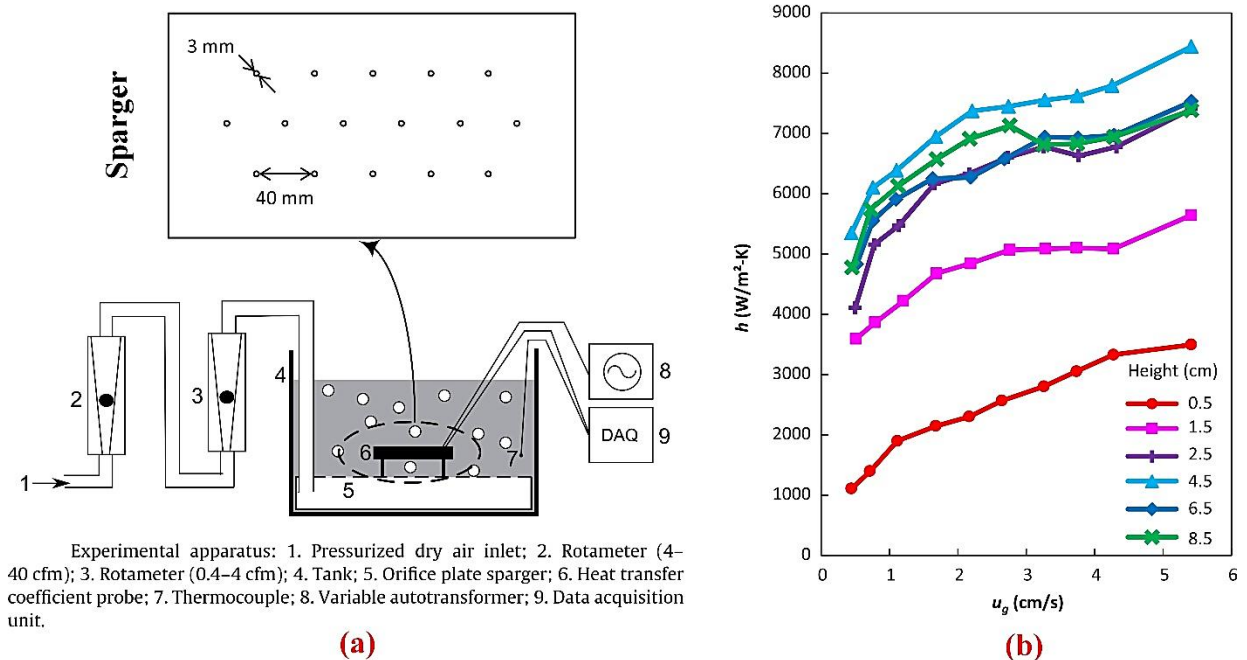


Fig. 46: (a) Experimental apparatus, and (b) heat transfer coefficient as a function of the gas velocity of Tow and Leinhard [363] study. It was shown that as air velocity rose, the heat transfer coefficient improved at all heights. (Figures are reprinted with permission from Emerald Publishing).

The heat transfer coefficient was shown to improve across all heights as air velocity increased. For example, increasing the gas velocity from 0.4 to 5.4 cm/s increased the heat transfer coefficient from 1099 to 3492 W/m².K, leading to an enhancement rate of 217%. This growth rate went down to 57% when the height of the cylinder was 4.5cm (Fig. 46-b). Hamed and He [364] worked on the heat transfer from a cylinder in crossflow under single-phase and multiphase (water-air-oil) flows. Results also indicated that using the oil phase significantly influenced the heat transfer coefficient by up to 70% relative to the single-phase case.

The icy cylinder in two-phase flows has also been a topic of interest for researchers. Yin-Chao [365] investigated the air-water flow heat transfer around an icy cylinder to find the Nu number. They conducted 171 experiments using different injection fluxes, different diameters of injection nozzle, and an icy cylinder with different dimensions. In the end, they found that the Nu number can be calculated based on the bubble injection Reynolds number and the nozzle diameter as follows:

$$Nu = 630(D_{noz})^{0.4}(Re_{bu})^{0.391} \quad (82)$$

A similar study was also carried out by Yen [366]. Likewise, some researchers studied the heat and mass transfer from a cylinder due to the air-water spray mixture and reported improving the heat transfer process, such as Refs. [367, 368].

Table 10: Summary of research on bubble injection in simple cylinder geometries.

Authors	Geometry	Working fluids	Bubble injection descriptions	Conditions	Remark
Yen [365]	Ice cylinder	Air-water	The air-water two-phase flow was used at different bubble injection flow rates and injection nozzle diameters to investigate the Nu number changes.	$D_{noz} = [0.152, 0.406, 0.838]$ (mm) $Q_{bu} = [0.739-9.91] \times 10^{-7}$ (m ³ /s) $Re_{bu} = [8.0-1000.0]$ (-) $T_w = 4$ (°C)	The Nu number was proportional to the nozzle diameter and the bubble injection, and an empirical correlation was introduced for the Nu number.
Atmane and Murray [362]	Simple cylinder	Air-water	Bubble injection on a hot cylinder submerged in water was done at a constant flow rate to calculate heat flux and temperature at different points of the copper cylinder.	$D_{bu} = 4$ (mm) $U_{bu} = 28$ (cm/s) $D_{noz} = 0.3$ (mm) $N_{noz} = 9$ (-) $\theta = [0-180]$ (°) $[T_w, T_{cyl}] = [11, 17.6]$ °C	The maximum value of the heat flux at $\theta = 0^\circ$ was 37.5 (kW/m ²). At the zero and bubble collision moment angle, the Nu number increase was 4.5 times higher than in the non-bubble injection mode.
Tow and Leinhard [363]	Simple cylinder	Air-water	The bubble was injected inside a shallow bubble column at different flow rates using a sparger with 16 cavities.	$U_{bu} = [0.4-5.4]$ (cm/s) $D_{noz} = 3$ (mm) $H_{cyl} = [0.5-8.5]$ (cm)	The effect of the cylinder height from the sparger on heat transfer was greater than that of other parameters, and the optimal value achieved was 3cm. A 217% increase in the h was reported when the velocity increased from 0.4 to 5.4 (cm/s).
Hamed and He [364]	Simple cylinder	Air-water Oil-water Air-oil Air-oil-water	The average h on a cross-flow cylinder was investigated for single-phase and two-phase flows. The cylinder was heated uniformly.	$Re_w = [2442-9749]$ (-) $Re_{oil} = [179-717]$ (-) Flow quality (x) = [0-34] (%) $q' = 8500, 7100$ (W/m ²)	Using bubbles in the oil phase increased the heat transfer coefficient by 70%.

5. Bubble injection method for pipes and channels

5.1. Overview

Inclined pipes can experience a variety of flow patterns, including bubbly, intermittent, stratified, and annular flow, contingent upon the specific flow conditions. The intricate flow characteristics of upward no-phase-change two-phase flow in inclined pipes are caused by the substantial interaction between the gas and liquid phases [369]. In recent years, there has been an increasing interest in the thermal properties of no-phase-change two-phase flow in inclined pipes. Due to its importance in many applications, the bubble injection method for pipes and channels is presented in this section.

5.2. Vertical, horizontal, and inclined pipes using bubble injection

Bubble injection can be applied mainly for (a) pressure drop reduction and (b) heat transfer rate enhancement. Each of these applications can be effective in different problems. In the first strategy, the amount of pressure drop can be strikingly affected by the bubble injection. Some bubbles move to the near-wall region when bubbles are injected inside the pipe. Therefore, the effect of bubble injection on pressure drop reduction is that it reduces the surface tension between the fluid and the solid surface [370]. For instance, in a study by Nouri et al. [371], it was found that for upward flow, when the Reynolds numbers ranges from 15,000 to 30,000 and void fraction values are between 0 and 9%, pressure drop in a vertical pipe can be reduced by up to 35% at the maximum void fraction and minimum Reynolds number. In a study by Samaroo et al. [372], air bubble injection and subcooled flow boiling were experimentally investigated, and the results were presented in terms of time-averaged velocity profiles and fluctuations, as well as air bubble data. This kind of research is essential for understanding the complex fluid dynamics and heat transfer mechanisms involved in the two-phase flow systems and developing models that can accurately predict their behavior. The results of this study can provide valuable insights into the design and optimization of various engineering systems, such as heat exchangers, reactors, and cooling systems, where two-phase flow is prevalent.

Calculating the heat transfer coefficient inside a pipe with bubble injection can be challenging due to the involvement of various parameters, such as the angle of the pipe, the velocity ratio of two phases, the Martinelli parameter, and the flow pattern. The studies in which heat transfer inside pipes with different horizontal, vertical, and inclined orientations is investigated are outlined in Table 11. According to these works, the mass flow rate and two-phase flow regime influence the heat transfer variation more strongly than the pipe angle effect.

Table 11: Summary of research on bubble injection in vertical, horizontal, and inclined pipes.

Authors	Geometry	Working fluids	Bubble injection descriptions	Conditions	Remark
King [373]	Horizontal pipe	Air-water	Experimental investigation of the effects of Re_w and Re_{bu} on flow behavior and two-phase flow heat transfer of non-isothermal water and air in the slug flow pattern.	$\frac{U_{bu}}{U_w} = [1.2- 6.49] (-)$ $Re_w = [22500-11.9 \times 10^4] (-)$ $Re_{bu} = [1570-8.28 \times 10^4] (-)$ $Pr_w = 3.4 (-)$	A correlation for h was developed based on void fraction and two-phase multiplier.
Groothuis and Hendaal [374]	Vertical pipe	Air-water	The superficial velocity effects were investigated for different fluids and air flow rates on h and pressure drop parameters.	$U_{bu} = [31.00- 48.00] (m/s)$ $U_w = [0.20- 0.80] (m/s)$ $M = [2000- 8000] (kg/m^2s)$ $\frac{V_{bu}}{V_w} = [60-155] (-)$	The maximum value of h happened at a high air-liquid flow rate of up to 24%.
		Air-oil		$U_{bu} = [15.10- 21.00] (m/s)$ $U_{oil} = [0.27- 0.52] (m/s)$ $M = [2200- 4200] (kg/m^2s)$ $\frac{V_{bu}}{V_{oil}} = [40,55] (-)$	
Knott et al. [375]	Vertical pipe	Petroleum oil-Nitrogen gas	Experimental investigation of the effects of different ranges of oil and nitrogen Reynolds numbers on h and development of a correlation for heat transfer in the bubbly flow.	$Re_{liq} = [6.7- 162] (-)$ $Re_{bu} = [126- 3920] (-)$ $\frac{U_{bu}}{U_{liq}} = [0.1- 0.4] (-)$ $\frac{\dot{m}_{bu}}{\dot{m}_{liq}} = [1.57 \times 10^{-3}- 1.19] (-)$	The liquid-gas heat transfer coefficient in the bubbly flow regime was higher than that of the single-phase flow in the same fluid velocity.
Kudirka et al. [376]	Vertical pipe	Air-water	Investigation of the effect of lower gas-liquids ratio on hydrodynamical aspects of two-phase heat transfer accompanied by air bubble injection.	$\frac{U_{bu}}{U_w} = [0.16-75] (-)$ $Re_w = [5.5-49.5] \times 10^4 (-)$ $\frac{\dot{m}_{bu}}{\dot{m}_w} = [1.92 \times 10^{-4}- 0.1427] (-)$	Heat transfer coefficients of air-ethylene glycol were much lower than that of the air-water flow due to the higher viscosity of ethylene glycol. In annular flow, the dependence of the two-phase heat transfer coefficient on mass velocity was greater at higher flow rates. In the same liquid velocity, the transition between flow patterns happened at a lower U_{bu} for air-ethylene glycol compared to the air-water.
		Air-ethylene glycol		$\frac{U_{bu}}{U_{liq}} = [0.250- 67.0] (-)$ $Re_{liq} = [380-1700] (-)$ $\frac{\dot{m}_{bu}}{\dot{m}_{liq}} = [0-0.11] (-)$	
Ueda and Hanaoka [377]	Vertical pipe	Air-water	Investigation of liquid flow rate, Void fraction, and liquid Prandtl number was done on the heat transfer coefficient of slug and annular flows.	$Pr_w = [4-160] (-)$ $\frac{U_{bu}}{U_w} = [4-50] (-)$ $\frac{\dot{m}_{bu}}{\dot{m}_w} = [9.4 \times 10^{-4}- 0.059] (-)$ $\alpha = [0-0.5] (-)$	For lower liquid Pr numbers, two-phase h values were affected slightly by the void fraction changes during slug and annular flows. A correlation for h is introduced as a function of the Re and liquid Pr numbers (based on the average liquid velocity).

Dorresteyn [378]	Vertical pipe	Air-oil	Experimental investigation of the effect of gas and liquid velocities on the forced convection heat transfer in both upward and downward directions of bubbly, slug, and annular flows.	$\frac{U_{bu}}{U_{oil}} = [0.004- 4500] (-)$ $Re_{oil} = [300-66000] (-)$ $U_{bu} = [0.01-100.00] (m/s)$ $U_{oil} = [0.02- 4.64] (m/s)$	Heat transfer enhancement was 15 times bigger than that of the single-phase flow. Generally, the h was improved as the velocity increased. However, a dramatic decrease occurs in the heat transfer coefficient during the transition from slug flow to annular flow. When the flow pattern changes from bubbly to slug flow, the two-phase heat transfer coefficient becomes lower than that of the single-phase case.
Beggs and Brill [379]	Inclined pipe	Air-water	Study of the effects of liquid and gas flow rates, pipe diameter, and inclination angle on liquid holdup and pressure drop.	$\theta = [-90- 90](^{\circ})$ $\dot{m}_w = [0-30] (gal/min)$ $\Delta P = [0-0.800] (psi/ft)$	An increase in the pipe inclination led to the rise and slight change of void fraction in two-phase flow with $0^{\circ} < \theta < 20^{\circ}$ and $20^{\circ} < \theta < 90^{\circ}$, respectively.
Ravipudi [380]	Vertical pipe	Air-water/ Toluene/ Benzene/ Methanol	A vertical steam condenser investigated the effect of gas-liquid properties on the two-phase heat transfer coefficient.	$\frac{U_{bu}}{U_{liq}} = [1- 90] (-)$ $Re_{liq} = [8554-89626] (-)$ $Re_{bu} = [3562-82532] (-)$	A correlation was predicted to calculate the Nu number.
Aggour [381]	Vertical pipe	Air-water	The gas density effect was studied on the average and local heat transfer coefficient values, frictional pressure drop, and flow patterns.	$\dot{m}_w = [267-8996] (lbm/hr)$ $\dot{m}_{bu} = [0.084-200.17] (lbm/hr)$ $U_{bu} = [0.240-315] (m/s)$ $\frac{U_{bu}}{U_w} = [0.020- 305.8] (-)$ $Re_{bu} = [63.3-1.49 \times 10^5] (-)$ $\rho_{bu} = [0.0799-0.3276] (lbm/ft^3)$ $Pr_{bu} = 0.709 (-)$	Flow patterns were strikingly dependent on the superficial velocities of both phases and slightly on the gas density. Pressure drop and h are dependent upon flow patterns. Correlations for local and average h were developed and authentic for bubbly, slug, forth, annular, bubbly-slug, and slug-annular flows.
		Helium-water		$\dot{m}_w = [267-8996] (lbm/hr)$ $\dot{m}_{bu} = [0.020-33.7] (lbm/hr)$ $U_{bu} = [0.423-483.6] (m/s)$ $\frac{U_{bu}}{U_w} = [0.042-470] (-)$ $Re_{bu} = [13.95- 2.3 \times 10^4] (-)$ $\rho_{bu} = [0.0116- 0.0375] (lbm/ft^3)$ $Pr_{bu} = [0.6908- 0.691] (-)$	
		Freon 12 -water		$\dot{m}_w = [267,3599] (lbm/hr)$ $\dot{m}_{bu} = [0.837,206.6] (lbm/hr)$ $U_{bu} = [0.515, 117.7] (m/s)$ $\frac{U_{bu}}{U_w} = [0.035, 114] (-)$ $Re_{bu} = [1894, 2.09 \times 10^5] (-)$ $\rho_{bu} = [0.3431-0.6034] (lbm/ft^3)$	

				$Pr_{bu} = 0.769$ (-)	
Vijay et al. [382]	Vertical pipe	Air-water	The effect of liquid viscosity and Prandtl number on the local h and upward no-phase-change two-phase flow performance was investigated.	$Re_w = [250-12.6 \times 10^4]$ (-) $Pr_w = [5.0-8.3]$ (-) $\frac{U_{bu}}{U_w} = [0.03-6700]$ (-)	A correlation for h was developed based on frictional pressure drop. The proposed correlation was invalid for slug flow and flow transition from slug to annular (with pressure drops below 1000 pa/m). Flow patterns depended on liquid and gas flow rates and the viscosity of the liquid.
		Air-Glycerin		$Re_{liq} = [0.8-21]$ (-) $Pr_{liq} = [5800-7125]$ (-) $\frac{U_{bu}}{U_{liq}} = [0.05-330]$ (-)	
		Air-Glycerin-water		$Re_{liq} = [8.0-4500]$ (-) $Pr_{liq} = [180-238]$ (-) $\frac{U_{bu}}{U_{liq}} = [0.01-6700]$ (-)	
Kago et al. [383]	Horizontal Pipe	Air-water	The effects of liquid and gas velocities and liquid viscosity on flow properties and two-phase heat transfer coefficient were examined in plug and slug flows.	$Re_w = [3 \times 10^2 - 3 \times 10^5]$ (-) $U_{bu} = [0.2-10]$ (m/s)	A predicted correlation shows the mixing length effect on the two-phase h.
Rezkallah and Sims [384]	Vertical pipe	Air-water	The effect of surface tension reduction was studied on average parameters of forced convection h, frictional pressure drop, and flow patterns.	$U_w = [0.56-10.6]$ (m/s) $U_{bu} = [0.329-129]$ (m/s) $Re_w = [845-1.27 \times 10^5]$ (-) $Re_{bu} = [318-1.35 \times 10^5]$ (-) $Pr_w = [5.4-7.2]$ (-) $W_w = [0.525-1.8 \times 10^4]$ (-)	The average heat transfer coefficient increased with the gas and water flow rate, and this enhancement was higher for a lower liquid flow rate. The average h in silicone was different from other liquids at higher gas flow rates in a way that enhancement occurred after a maximum decrease. In $Re_{liq} > 2000$, the Nu number for Air-silicone was 10-40% higher than other compounds.
		Air- Glycerine-water		$U_{liq} = [0.022-7.38]$ (m/s) $U_{bu} = [0.058-105]$ (m/s) $Re_{liq} = [42-1.3 \times 10^4]$ (-) $Re_{bu} = [68-12 \times 10^5]$ (-) $Pr_{liq} = [42.4-65.8]$ (-) $W_{liq} = [0.095-1.02 \times 10^4]$ (-)	
		Air-silicone liquid		$U_{liq} = [0.022-9.20]$ (m/s) $U_{bu} = [0.052-111]$ (m/s) $Re_{liq} = [51-2.07 \times 10^4]$ (-) $Re_{bu} = [52-1.15 \times 10^5]$ (-) $Pr_{liq} = [61.0-72.3]$ (-) $W_{liq} = [0.073-1.27 \times 10^4]$ (-)	
Hetsroni et al. [385]	upward inclined pipe	Air-water	Local heat transfer coefficients and flow parameters were investigated in a pipe wall under non-uniform heat flux	$\theta = [2, 5]$ (°) $U_{bu} = [0.02-0.39]$ (m/s)	In slightly inclined conditions, the heat transfer rate increased dramatically.

			and transient conditions at different superficial velocities of water and air.	$U_w = [0.4-1.4]$ (m/s) $Fr_{bu} = [0.03-0.57]$ (-) $Fr_w = [0.059-2.0]$ (-)	Heat transfer generally depends on the liquid velocity.
Mosyak and Hetsroni [386]	horizontal and inclined pipes	Air-water	They considered different superficial velocities of water and air to experimentally investigate the wall temperature distribution in a pipe under uniform heating conditions.	horizontal pipe $q'' = [0.7-19]$ (kW/m ²) $U_w = [0.62-0.104]$ (m/s) $U_{bu} = [0.02-0.3]$ (m/s) $Fr_w = [0.9-2.0]$ (-) $Fr_{bu} = [0.03-0.43]$ (-) inclined pipe $\beta = 5$ (°) $q'' = [27-53]$ (kW/m ²) $U_w = [0.4-0.1.4]$ (m/s) $U_{bu} = [0.02-0.39]$ (m/s) $Fr_w = [0.59-2.0]$ (-) $Fr_{bu} = [0.03-0.57]$ (-)	Circumferential changes on top of the wall that lead to drying happen in a horizontal tube. In $0.9 < Fr < 1.3$, the horizontal tube contains a complex non-isothermal behavior. When the FR number reaches 2, this behavior becomes approximately isothermal.
Trimble et al. [387]	horizontal and inclined pipes	Air-water	An experimental investigation was done for heat transfer behavioral patterns in slug flow.	$\theta = [0, 2, 5]$ (°) $Re_w = [5155-10958]$ (-) $Re_{bu} = [804-3729]$ (-)	Heat transfer enhancement rates in $\theta = 2^\circ$ and $\theta = 5^\circ$ were 10% and 20%, respectively, compared to the horizontal pipe.
Durant [388]	Horizontal and slightly upward-inclined pipe	Air-water	This project focused on the applications and calculating heat transfer on a two-phase flow at horizontal and inclined positions.	$\theta = [0, 5, 7]$ (°) $Re_w = [7510-27816]$ (-) $Re_{bu} = [1043-2994]$ (-) $U_w = [0.100-0.26]$ (m/s) $U_{bu} = [15.0-20.3]$ (m/s)	A correlation was developed to calculate the Nu number for air-water two-phase flow in annular flow.
Hetsroni et al. [389]	Inclined pipe	Air-water	Analysis of flow regime and heat transfer in two inclined pipes with different internal diameters under different liquid and superficial air velocities in bubbly and slug flows.	$\theta = 8$ (°) $Re_w = [2.86-8.36] \times 10^3$ (-) $Re_{bu} = [5.85-6.40] \times 10^4$ (-) $U_w = [0.005-0.099]$ (m/s) $U_{bu} = [20.0-36.0]$ (m/s)	The heat transfer coefficient was about 2.5 to 8 times higher than the air single-phase flow.
Ghajar [390]	vertical, horizontal, and inclined pipes	Air-water	A robust heat transfer correlation was introduced that contains most of the fluid compounds, flow patterns, flow regimes, and the direction of pipes (vertical, inclined, and horizontal).	$\theta = [0, 2, 5, 7]$ (°) $Re_w = [5.00-17] \times 10^3$ (-) $Re_{bu} = [549-4.83 \times 10^4]$ (-) $U_w = [0.171-0.582]$ (m/s) $U_{bu} = [0.331-29.1]$ (m/s)	In slug flow, as θ increases from 0° to 2° , from 0° to 5° , and from 2° to 5° , the maximum percentage of increase in h was 48%, 85%, and 32%, respectively. In annular flow, as θ increases from 0° to 2° , from 0° to 7° , and from 5° to 7° , the maximum percentage of increase in h was 33%, 35%, and 9%, respectively.
Malhotra [391]	Horizontal and slightly inclined pipe	Air-water	Numerical and experimental investigation of average heat transfer coefficient and slug flow pattern in single-phase and two-phase flows at different pipe inclinations.	$\theta = [0, 2, 5, 7]$ (°) $Re_w = [3161-28631]$ (-) $Re_{bu} = [6.34 \times 10^2-6634]$ (-)	A correlation was introduced to predict h in horizontal and inclined pipe angles.

				$U_w = [0.113-2.03] \text{ (m/s)}$ $U_{bu} = [0.325-3.40] \text{ (m/s)}$	The $\theta = 5^\circ$ was a limiting case for total average h enhancement in the slug flow pattern. In $\theta = 0^\circ$, $\theta = 2^\circ$, and $\theta = 5^\circ$ cases, the h values were in the ranges of 521-5602, 898-4904, and 1159-4659 W/m²K respectively.
Ghajar and Tang [392]	Horizontal and slightly inclined pipe	Air-water	The effect of air and water Reynolds numbers was investigated on the average value of the two-phase heat transfer coefficient for different flow regimes.	$\theta = [0, 2, 5, 7] \text{ (}^\circ\text{)}$ $q'' = [1800-10900] \text{ (W/m}^2\text{)}$ $Re_w = [740-26000] \text{ (-)}$ $Re_{bu} = [560-48000] \text{ (-)}$	The total average two-phase h increased by 16% and 12% for the values θ increasing from 0° to 2° and from 2° to 5° . However, a change was not achieved in the total average value of the two-phase h when θ surpassed 5° .
Vaze and Banerjee [393]	Inclined pipe	Air-water	An experimental investigation was performed for the pipe angles on flow properties and heat transfer.	$\theta = [0, 2, 5, 7] \text{ (}^\circ\text{)}$ $Re_w = [500-10000] \text{ (-)}$ $Re_{bu} = [500-30000] \text{ (-)}$	A correlation was developed for the Nu number based on parameters such as Reynolds number, Pr number, and pipe inclination for different patterns. The inclination angle considerably influenced the heat transfer rate in all cases except for wavy/annular and annular flows.
Liu and Cheng [394]	Inclined smooth circular pipe	Air-water	An experimental investigation was conducted for the polyacrylamide effect on air-water two-phase flow heat transfer.	$\theta = 8 \text{ (}^\circ\text{)}$ $Re_w = [3.17-8.90] \times 10^4 \text{ (-)}$ $Re_{bu} = [4415-12.147] \text{ (-)}$ $U_w = [0.520-1.46] \text{ (m/s)}$ $U_{bu} = [1.79-6.54] \text{ (m/s)}$	The maximum values of the two-phase h increased by 80.8%.
Kalapatapu [395]	Horizontal and near horizontal upward inclined	Air-water	An experimental investigation was performed for local and average convection h for different flow patterns in horizontal and near-horizontal upward-inclined two-phase flow.	$\theta = [0, 5, 10, 20] \text{ (}^\circ\text{)}$ $Re_w = [1.87-12] \times 10^3 \text{ (-)}$ $Re_{bu} = [274-19400] \text{ (-)}$ $U_w = [0.15-0.96] \text{ (m/s)}$ $U_{bu} = [0.340-24.2] \text{ (m/s)}$	The two-phase heat transfer coefficient increased from 0° to 5° and 10° and then decreased to 20°. The maximum value of the average h increased to $8000 \frac{W}{m^2K}$.
Bhagwat and Ghajar [47]	Upward inclined pipes	Air-water	An investigation was conducted for the effects of gas and liquid mass flow rates on void fraction, total pressure drop, and heat transfer coefficient.	$\theta = [0, 10, 20, 30, 45, 60, 75, 90] \text{ (}^\circ\text{)}$ $Re_w = [1.88-16.9] \times 10^3 \text{ (-)}$ $Re_{bu} = [84-1.56 \times 10^4] \text{ (-)}$ $U_w = [0.15-1.35] \text{ (m/s)}$ $U_{bu} = [0.1-18.7] \text{ (m/s)}$	Increasing the pipe inclination affected the void fraction, total pressure drop, and heat transfer coefficient at lower gas and liquid flow rates.
Dong and Hibiki [396]	Horizontal pipe	Gas-liquid	Comparison of two-phase heat transfer in horizontal pipes between previous studies.	-	Based on Reynolds and Chilton-Colburn analogies, a semi-theoretical heat transfer correlation was introduced for two-phase flow in horizontal pipes. A precise prediction of the heat transfer coefficient was made using an annular flow.
Dong and Hibiki [397]	Vertical pipe	Gas-liquid	Comparison of two-phase heat transfer in vertical pipes between previous studies.	-	A new correlation was provided to predict vertical pipes' two-phase heat transfer coefficient under different conditions, such as upward and downward flows, developing and fully

					developed flows, and laminar and turbulent flows.
Kim et al. [53]	Horizontal pipes	Air-water	Bubble injection was done with plug, slug, bubbly, and annular flow patterns to predict the Nu number.	$Re_w = [820 - 26000]$ $Re_{bu} = [560 - 48000]$ $q'' = [3000-10600] \text{ (W/m}^2\text{)}$	A correlation is introduced to predict h with an 11.7% standard deviation and 5.5% mean deviation. 144% increase in h in $Re_w = 5000$ was observed.
Dong et al. [398]	Horizontal pipe	Air-oil	An experimental investigation was conducted for gas and liquid flow rates, pressure drop, void fraction, and slug frequency on horizontal pipes' heat transfer and temperature distribution.	$U_{oil} = [0.19-0.8] \text{ (m/s)}$ $U_{bu} = [2.0-5.9] \text{ (m/s)}$	The heat transfer coefficient was enhanced by 150% and 300% for air-water and air-oil two-phase mixtures.
Bae et al. [399]	Vertical pipe	Air-Kerosene	Heat transfer in a bubble column was experimentally studied with a dual gas distribution (microbubble generator).	$U_{bu} = [0.0075-0.1] \text{ (m/s)}$ $D_{pipe} = 0.097 \text{ (m)}$ $H_{pipe} = 1.8 \text{ (m)}$ $D_{noz,1} = 1 \text{ (mm)}$ $D_{noz,2} = [1.7, 2, 2.5, 3.46] \text{ (mm)}$	The dual gas distributor could increase the heat transfer coefficient by 25%.

5.2.1. Heat transfer coefficient

Many correlations have been proposed to calculate the non-boiling two-phase heat transfer coefficient inside a pipe. These correlations are categorized based on the pipe's horizontal, vertical, or inclined orientations. Table 12 provides a list of these correlations. The results which have been highlighted might not be valid for a great deal of the Reynolds numbers. Heat transfer between bubbles and liquid can be disregarded because air has a lower heat capacity than liquid. Having collected numerous experimental data, Dong et al. [45] conducted a comprehensive study to find a suitable correlation for two-phase non-boiling Nu numbers for a wide range of parameters. According to the findings, the two-phase heat transfer coefficient can be calculated as follows,

$$h_{TP} = \phi_h h_{SP} \quad (83)$$

where h_{TP} and h_{SP} are the two-phase and single-phase heat transfer coefficients, respectively. Here, ϕ_h is the modification factor that can be found based on the empirical results. As discussed in the previous section on the concepts of heat transfer in pipes, the heat transfer coefficient in two-phase flow is dependent upon the parameter ϕ_h and void fraction α in addition to all other important single-phase flow parameters. Based on the results described in Dong et al. [45] study, a correlation can be described to calculate ϕ_h as a function of pipe angle in laminar and turbulent flows as,

$$\text{Laminar flow } (Re_f \leq 2300, \theta < 20): \quad \phi_h = (1 - \alpha)^{0.257 + 0.0820 \sin \theta} \left(1 + \frac{4.27 + 0.757 \sin \theta}{X^{0.697 - 0.288 \sin \theta}} \right) \quad (84)$$

$$\text{Laminar flow } (Re_f \leq 2300, \theta \geq 20): \quad \phi_h = (1 - \alpha)^{0.257 + 0.0820 \sin \theta} \left(1 + \frac{4.46 + 0.186 \sin \theta}{X^{0.697 - 0.288 \sin \theta}} \right) \quad (85)$$

$$\text{Turbulent flow } (Re_f > 2300, \theta < 20): \quad \phi_h = (1 - \alpha)^{-0.194 + 0.174 \sin \theta} \left(1 + \frac{0.687 + 3.34 \sin \theta}{X^{0.700 - 0.192 \sin \theta}} \right) \quad (86)$$

$$\text{Turbulent flow } (Re_f > 2300, \theta \geq 20): \quad \phi_h = (1 - \alpha)^{-0.194 + 0.174 \sin \theta} \left(1 + \frac{1.45 + 1.11 \sin \theta}{X^{0.700 - 0.192 \sin \theta}} \right) \quad (87)$$

$$\text{Transition region } (Re_f = [2000-2300]): \quad \phi_h = \phi_{h,laminar}(1 - \omega) + \omega \phi_{h,turbulent} \quad (88)$$

In this equation, α is the void fraction. X (Martinelli parameter) and ω can be calculated using the equations below,

$$1/\rho_m = (1 - x)/\rho_m + x/\rho_{bu} \quad (89)$$

$$X = \left[\frac{\left(\frac{dp}{dz} \right)_{F,f}}{\left(\frac{dp}{dz} \right)_{F,g}} \right]^{0.5} \quad (90)$$

$$\omega = (\log Re_f - \log 2000)/(\log 2300 - \log 2000) \quad (91)$$

Table 12: Two-phase heat transfer coefficient correlations.

Authors	Geometry	Two-phase correlations	Remark	Purpose of study	Flow regime
King [373]	Horizontal pipe	$\frac{\bar{h}_{TP}}{\bar{h}_{liq}} = \frac{(1 - \alpha)^{-0.52}}{1 + 0.025 Re_{bu}^{0.5}} (\Phi_f^2)^{0.32}$, $\Phi_f^2 = \left(\left(\frac{\Delta P}{\Delta L} \right)_{TP} / \left(\frac{\Delta P}{\Delta L} \right)_{liq} \right)$	-	Investigation of pressure drop and heat transfer	Slug
Groothuis and Hendal [374]	Vertical pipe	$Nu_{TP} = 0.029 Re_{TP}^{0.87} Pr^{1/3} \left(\frac{\mu_b}{\mu_{wall}} \right)^{0.14}$	Water-air	Heat transfer measurement	General
		$Nu_{TP} = 2.6 Re_{TP}^{0.39} Pr^{1/3} \left(\frac{\mu_b}{\mu_{wall}} \right)^{0.14}$	Gas-oil-air		
Knott et al. [375]	Vertical pipe	$\frac{\bar{h}_{TP}}{\bar{h}_{liq}} = (1 + \frac{U_{bu}}{U_{liq}})^{1/3}$	-	Experimental study of heat transfer	Bubbly
Kudirka et al. [376]	Vertical pipe	$Nu_{TP} = 125 \left(\frac{U_{bu}}{U_{liq}} \right)^{1/8} \left(\frac{\mu_{bu}}{\mu_{liq}} \right)^{0.6} Re_{liq}^{0.25} Pr_{liq}^{1/3} \left(\frac{\mu_b}{\mu_{wall}} \right)^{0.14}$	-	Investigation of heat transfer in two-phase flow	Bubbly Slug Froth
Ueda and Hanaoka [377]	Vertical pipe	$Nu_{TP} = 0.075; Re_m^{0.60} \frac{Pr_{liq}}{1 + 0.035(Pr_{liq} - 1)}$ $Re_m = \frac{\rho_{liq} U_{liq} D}{\mu_{liq}} + \frac{\rho_{bu} U_{bu} D}{\mu_{bu}}$	-	Heat transfer analysis of gas-liquid mixtures flow upwards	Slug Annular
Dorresteyn [378]	Vertical pipe	$\frac{\bar{h}_{TP}}{\bar{h}_{liq}} = (1 - \alpha)^{-1/3}$	Laminar flow	Experimental investigation of heat transfer in upward and downward two-phase flow in tubes	Bubbly Slug Annular
		$\frac{\bar{h}_{TP}}{\bar{h}_{liq}} = (1 - \alpha)^{-0.8}$	Turbulent flow		
Ravipudi [380]	Vertical pipe	$Nu_{TP} = 0.56 \left(\frac{U_{bu}}{U_{liq}} \right)^{0.3} \left(\frac{\mu_{bu}}{\mu_{liq}} \right)^{0.2} Re_{liq}^{0.6} Pr^{1/3} \left(\frac{\mu_b}{\mu_{wall}} \right)^{0.14}$	-	Investigation of the influence of mass	Froth

		C=3.89	m=0.124	n=-1.376	p=0.0136	q=0.1335	$\theta = 0^\circ$	$\theta = 2^\circ$	$\theta = 5^\circ$	$\theta = 7^\circ$	$Re_w = [4693-7660]$ (-)	Slug/bubbly/annular
							r=0	r=0.787	r=0.856	r=0.996	$Re_{bu} = [7182-23473]$ (-)	
Kalapatapu [395]	Horizontal /inclined pipe	$\frac{h_{TP}}{h_{liq}} = (\Phi_f^2)^{0.39}$					-	Non-boiling heat transfer in gas-liquid two-phase flow			Bubbly Slug Annular	
Dong and Hibiki [396]	Horizontal pipe	$\frac{h_{TP}}{h_{liq}} = (1 - \alpha)^{-0.194}(1 + 0.687 X^{-0.700})$					Re>2300	Heat transfer correlation for two-component two-phase			Slug Annular	
		$\frac{h_{TP}}{h_{liq}} = (1 - \alpha)^{0.257}(1 + 4.27 X^{-0.697})$					Re≤2300					
Dong and Hibiki [397]	Vertical pipe	$\frac{Nu_{TP}}{Nu_{liq}} = \left(\frac{h_{TP}}{h_{liq}}\right) = (1 - \alpha)^{-0.940}$					Re≤2300	Development of a correlation of heat transfer coefficient for two-component two-phase			Bubbly Slug Churn Annular	
		$\frac{Nu_{TP}}{Nu_{liq}} = \left(\frac{h_{TP}}{h_{liq}}\right) = (1 - \alpha)^{-0.756}$					Re>2300					
Dong et al. [398]	Horizontal pipe	$\frac{Nu_{TP}}{Nu_{liq}} = \left(\frac{h_{TP}}{h_L}\right) = (1 - \alpha)^{0.669} \left(\left(\frac{\Delta P}{\Delta L}\right)_{TP} / \left(\frac{\Delta P}{\Delta L}\right)_{liq}\right)^2$					Re≤2300	Experimental study on non-boiling heat transfer			Slug	
		$\frac{Nu_{TP}}{Nu_{liq}} = \left(\frac{h_{TP}}{h_{liq}}\right) = (1 - \alpha)^{1.437} \left(\left(\frac{\Delta P}{\Delta L}\right)_{TP} / \left(\frac{\Delta P}{\Delta L}\right)_{liq}\right)^2$					Re>2300					
Kim et al. [53]	Horizontal pipes	$h_{TP} = \alpha h_{liq} \left\{ +0.7 \left[\left(\frac{x}{1-x}\right)^{0.08} \left(\frac{\alpha}{1-\alpha}\right)^{0.06} \left(\frac{Pr_{bu}}{Pr_{liq}}\right)^{0.03} \left(\frac{\mu_{bu}}{\mu_{liq}}\right)^{-0.14} \right] \right\}$					-	Introduction of a general heat transfer correlation for non-boiling gas-liquid flow with different flow patterns			General	
Fedotkin and Zarudnev[401]	Horizontal Pipe	$Nu_{TP} = 0.0182; Re_m^{0.882} Pr^{0.43} \left(\frac{Pr_b}{Pr_{wall}}\right)^{0.25}$ $Re_m = \frac{\rho_{liq} U_{liq} D}{\mu_{liq}} + \frac{\rho_{bu} U_{bu} D}{\mu_{bu}}$					-	Development of a correlation of experimental data on local heat transfer in heating of air-liquid mixtures in pipes			General	
Martin and Sims [402]	Horizontal Pipe	$\frac{h_{TP}}{h_{liq}} = 1 + 0.64 \sqrt{\frac{U_{bu}}{U_{liq}}}$					-	Forced convection heat transfer to water with air injection			Bubbly Slug Annular	
Chu and Jones [403]	Vertical pipe	$Nu_{TP} = 0.43(Re_{TP})^{0.55}(Pr_{liq})^{1/3} \left(\frac{\mu_b}{\mu_{wall}}\right)^{0.14} \left(\frac{P_{atm}}{P}\right)^{0.17}$					-	Study of convective heat transfer coefficient in upward and downward, non-boiling flows			Bubbly Slug Froth/annular	
Davis and David [404]	Horizontal /Vertical Pipe	$Nu_{TP} = 0.060 \left(\frac{\rho_{liq}}{\rho_{bu}}\right)^{0.28} \left(\frac{DG_t x}{\mu_{liq}}\right)^{0.87} (Pr_{liq})^{0.4}$					-	Development of a correlation in two-phase gas-liquid convection heat transfer			Annular Mist/annular	
Dusseau [405]	Vertical pipe	$Nu_{TP} = 0.029(Re_{TP})^{0.87}(Pr_{liq})^{0.4}$					-	Overall heat transfer coefficients analysis			Froth	
Elamvaluthi and Srinivas [406]	Vertical pipe	$Nu_{TP} = 0.5 \left(\frac{\mu_{bu}}{\mu_{liq}}\right)^{1/4} (Re_{TP})^{0.7}(Pr_{liq})^{1/3} \left(\frac{\mu_b}{\mu_{wall}}\right)^{0.14}$					-	Two-phase heat transfer analysis in two-component vertical flows			Bubbly Slug	
Hughmark [407]	Horizontal Pipe	$Nu_{TP} = 1.75(1 - \alpha)^{-1/2} \left(\frac{\dot{m}_{liq} C_{liq}}{(1 - \alpha) K_{liq} L}\right)^{1/3} \left(\frac{\mu_b}{\mu_{wall}}\right)^{0.14}$					-	Holdup and heat transfer in gas-liquid flow			Slug	
Oliver and Wright [408]	Horizontal Pipe	$Nu_{TP} = Nu_{liq} \left(\frac{1.2}{(1 - \alpha)^{0.36}} - \frac{0.2}{(1 - \alpha)}\right)$					-	Pressure drop and heat transfer analyses in gas-liquid flow			Slug	
Serizawa et al. [204]	Vertical pipe	$\frac{\bar{h}_{TP}}{\bar{h}_{liq}} = 1 + 462X^{-1.27}$					-	Investigation of turbulence structure of air-water flow			bubbly	
Khoze et al. [409]	Vertical pipe	$Nu_{TP} = 0.26(Re_{bu})^{0.2}(Re_{liq})^{0.55}(Pr_{liq})^{0.4}$					-	Heat and mass transfer analyses in rising two-phase flows			Annular	

Shah [410]	Horizontal /Vertical Pipe	$\frac{h_{TP}}{h_{liq}} = (1 + \frac{U_{bu}}{U_{liq}})^{1/4}$	-	Development of a general correlation for heat transfer during film condensation	Bubbly Slug Froth Mist Froth/annular
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As previously discussed, the Nusselt number for two-phase bubbly flow in natural convection is primarily a function of the void fraction ($Nu \propto \alpha^n$). Moreover, it becomes independent of Ra number. However, the Nu number for forced convection bubbly flow in tubes is a function of the void fraction, Martinelli parameter, and inclination angle. In contrast, as detailed in the problem statement section, the single-phase heat transfer coefficient for a circular tube is generally a function of Reynolds (Re) and Prandtl (Pr) numbers.

Flow visualizations at various pipe inclinations show that the change in pipe angle influences the multiple flow patterns and, hence, changes in the flow pattern structure. Fig. 47 displays the flow pattern maps with the transition boundaries between different flow patterns and pipe inclinations. The liquid and gas superficial velocities (U_{sl} and U_{sg}) are used as the coordinate system to display these flow pattern maps. Fig. 47 shows that the change in pipe inclination has a negligible effect on the transition between inertia-driven flows, such as intermittent/wavy and annular flows [47]. Dong et al. [45] also suggested a flow regime map, as shown in Fig. 48, based on the data collected by Bhagwat and Ghajar [47]. The lines clearly show the transition between different flow patterns. The flow patterns for the angles of 0° , 30° , 60° , and 90° are shown by the red dashed line, blue dotted line, pink short-dashed line, and black solid line, respectively.

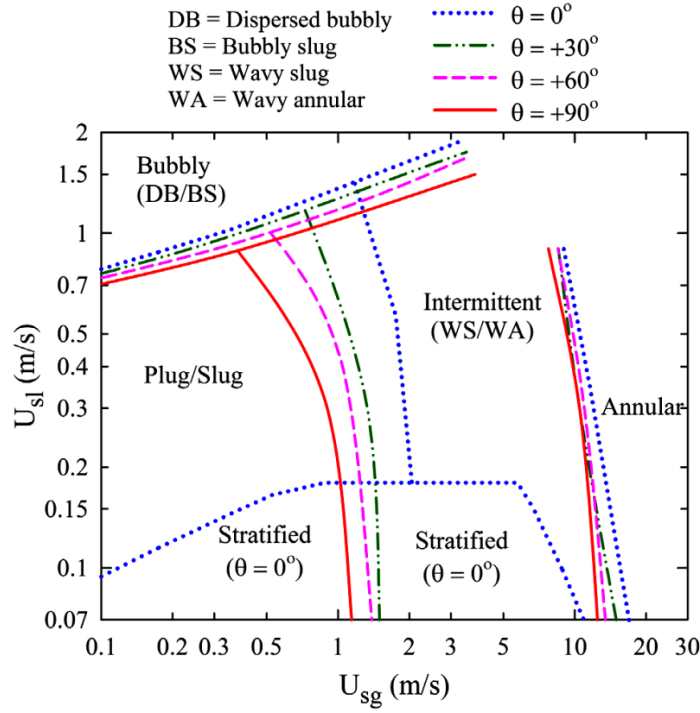


Fig. 47: Flow pattern maps for different upward pipe inclinations, where U_{sl} and U_{sg} are the liquid and gas superficial velocities. A slight change in the incline of the pipe has a marginal impact on the shift between inertia-driven flow types, such as intermittent or wavy flows and annular flows [47] (Figure is reprinted with permission from Elsevier).

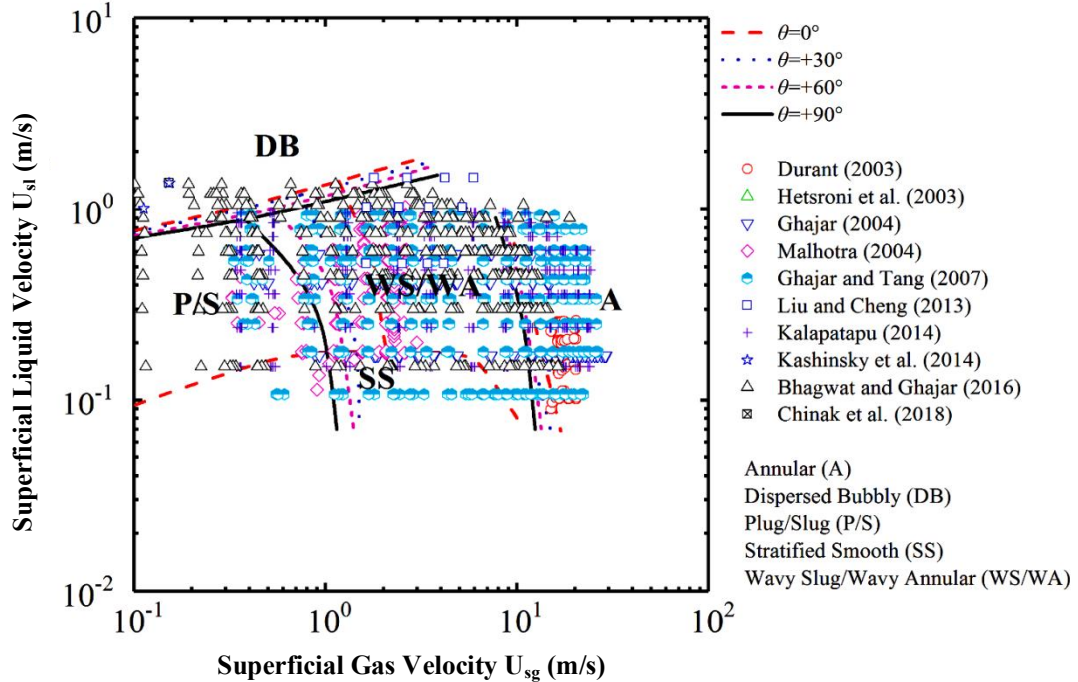


Fig. 48: The flow-regime map of the collected database in the flow-regime map presented by Dong et al. [45]. The collected data covers upward laminar and turbulent flows in inclined pipes, with varying pipe inclination angles (Figure is reprinted with permission from Elsevier).

5.3. Channels using bubble injection

Bubble injection in channels is a widely used technique for enhancing heat transfer in various engineering applications, including evaporators, condensers, and heat exchangers.

The experiments conducted by Houston and Cornwell [50] aimed to investigate the effect of air injection on heat transfer enhancement around a horizontal cylinder placed in a channel. To create a bubbly flow around the cylinder, they injected bubbles through holes located at the lowest point of the cylinder. They used R113 refrigerant, which was introduced into the channel at a pressure and temperature below the saturation state to prevent its evaporation on the cylinder's surface (Table 13). The adjustment of air injection rates by Houston and Cornwell [50] was aimed at creating a bubbly flow around the experimental cylinder. The results showed that air injection positively impacted the enhancement rate of heat transfer in both cooling and heating conditions. The study also revealed that the bubbly-flow turbulence mechanism was much more effective in heat transfer than the evaporation of bubbles, especially at low heat fluxes. This observation suggests that the turbulence created by the bubbly flow played a crucial role in enhancing heat transfer, leading to a higher heat transfer coefficient.

Thorncroft and Klausner [411] recognized the significance of bulk turbulent enhancement on heat transfer improvement. Accordingly, they analyzed the air injection to the FC-87 fluid flow on a DC-powered nichrome heating strip in a vertical channel with a square cross-section. They compared the results obtained for fluid moving upward and downward directions with each other. They found that the bubble growth and sliding represented a significant fraction (about 52%) of the total heat transfer. Moreover, for the bubble injection condition, the heat transfer coefficient increased remarkably compared to the single-phase case by 3.5 times.

With the rapid advancement of electronic devices and the increasing need for effective thermal management, numerous innovative microscale heat transfer enhancement techniques have emerged. These include impinging jets, shear-driven flows, segmented flows, extended surfaces[52], and bubble injection methods.

Lim et al. [412] studied the two-phase flow passing through circular micro-tubes for both adiabatic and heat transfer modes (Table 13). The results showed that increasing the gas velocity causes a gradual change in the flow pattern, leading to the flow transition from bubbly to slug. Nevertheless, as opposed to macro-channels, where stratified flow usually forms, such flow does not take place in microchannels to represent the dominance of the surface tension effect relative to the gravitational effects. They demonstrated that the optimal heat transfer

condition is created by keeping the bubble size near the channel diameter and the void fraction at about 10%. Under this circumstance, the maximum thermal performance can be achieved with an increase of 176% in the Nusselt number and less than 27% in the pressure drop. The thermal performance of the bubbly flow is better than the slug flow as the pressure drop is lower for the earlier pattern [52].

Betz and Attinger [413] acknowledged that using bubbles in microchannels effectively enhances heat transfer. However, they also noted that this approach is not without its challenges. For example, if water is used, high saturation temperatures can occur. Additionally, using refrigerants requires pumps that are resistant to chemicals and can increase the required pumping power.

To address these challenges, Betz and Attinger [413] proposed periodically injecting non-condensable bubbles into an array of microchannels filled with water to increase heat transfer experimentally. This approach allowed for the control of the size and frequency of the bubbles injected, which in turn helped to mitigate the challenges associated with using bubbles in microchannels. The study demonstrated that periodic bubble injection effectively improved heat transfer in microchannels. They observed that when bubbles are injected into microchannels, the length of the bubbles is generally longer than the channel diameter, and the flow becomes segmented. For mass fluxes less than $2000 \text{ kg/m}^2\text{s}$, they found that the Nusselt number of the segmented flow is significantly higher than that of single-phase flow (up to 100%). However, the enhancement in the Nusselt number decreases rapidly for mass fluxes higher than $2500 \text{ kg/m}^2\text{s}$. They also found that increasing flow rates increases the Nu number of the two-phase flow. However, the enhancement due to the segmented flow decreases.

These findings have significant implications for developing microchannel heat transfer systems, as they suggest that the periodic injection of bubbles can be a practical and effective method for enhancing heat transfer in microchannels. This approach could lead to the development of more efficient and effective microchannel heat transfer systems, which could be used in various engineering applications, including cooling electronics, micro-reactors, and microchemical systems.

Similarly, the influence of air injection on the local heat transfer coefficient of the water flow passing through a microchannel was investigated by Houshmand and Peles [52] (Fig. 49-a). They believed that the boundary layer description could be a suitable criterion for comparing the results reported in studies for the heat transfer systems in micro or macro scales. Hence, various heaters and resistance temperature detectors (RTDs) were used in different parts of a channel (according to Fig. 49-b) to investigate how the bubbles and the boundary layer interact

with each other. The enhancement factor (E) indicates the relative difference between the local heat transfer coefficient of bubbly flow and single-phase flow when examining the effect of bubble injection. In case 1, all the heaters were connected in series, but in case 2, the first three heaters were off, and only the fourth heater was on. For case 1, it was observed that there is a slight difference between the two-phase and single-phase heat transfer at the low liquid flow rate that follows the slug flow pattern. Increasing the liquid flow rate changes the flow pattern, leading to a transition to the bubbly pattern, and E varies in a way that it first increases and then decreases. Regarding case 2, E starts from negative values and increases by 50% as the liquid flow rate increases. A comparison between the results of these two cases shows that the increase factor in the first scenario is much more impressive due to the higher thickness of the thermal boundary layer. As a result, it can be stated that heat transfer enhancement in the developing region with a thicker boundary layer is more impressive and can even lead to an increase of up to 100% in the heat transfer coefficient compared to the single-phase flow, depending on liquid and gas flow rates as well as the longitudinal position (Fig. 49-c).

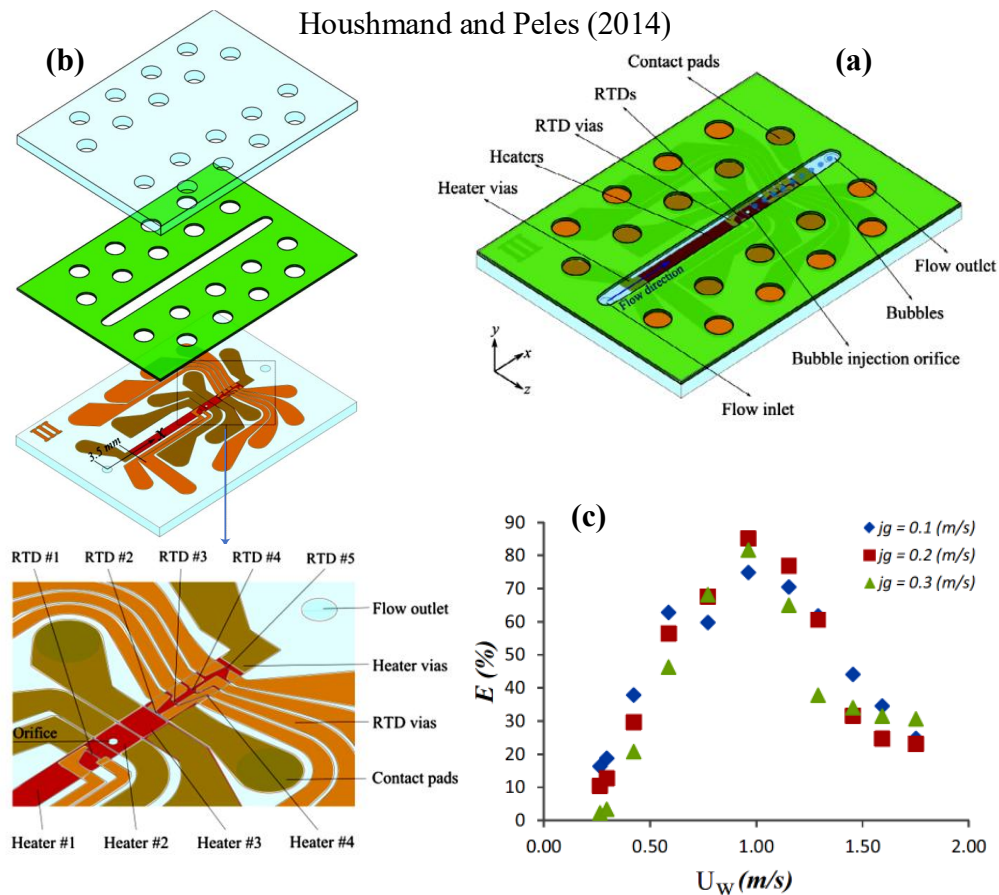


Fig. 49: (a) Schematic view of the micro-device, (b) the microdevice details, and (c) heat transfer enhancement of Houshmand and Peles [52] study. Enhancing heat transfer in the developing region with a thicker boundary layer yields more significant improvement. This enhancement can result in up to a 100% increase in the heat transfer coefficient compared to single-phase flow. The

extent of improvement depends on factors such as liquid and gas flow rates and the longitudinal position(Figure is reprinted with permission from Elsevier).

Table 13: Summary of research on bubble injection in channels

Authors	Geometry	Working fluids	Bubble injection description	Conditions	Remark
Houston and Cornwell [50]	Channel with a circular cylinder	Air-R113	The bubbles are mixed with R113 refrigerant at pressures and temperatures lower than the saturation point to prevent boiling, with varying mixing ratios. The resulting mixture is injected into a channel containing transverse cylinders that are heated by heaters.	$Q_{bu}=[0-5]$ (LPM) $T_{R113,in}=34$ (°C) Mass flux $_{R113}=95.1$ (kg/m ² . s) $q''=[-30-15]$ (kW/m ²)	The bubbly flow was found to be one of the prominent regimes that increased heat transfer. Bubble injection increased the heat transfer coefficient by 240% compared to the case without bubble injection.
Thornicroft and Klausner [411]	Vertical heated channel	Air- FC87	This research aimed to focus on the effective mechanisms for increasing heat transfer in the presence of a sliding air bubble in a channel. Compressed air entered the test section from the channel inlet.	$Q_{bu}=[0-0.050]$ (LPM) $D_{noz}=100$ (μm) $q''=1.41$ (kW/m ²)	The mechanism for increasing the heat transfer rate is the increase of bulk turbulent transport due to bubble injection. The heat transfer coefficient increased up to 3.5 times compared to the single-phase mode.
Lim et al. [412]	Circular micro-tube	DI (de-ionized) water- Nitrogen gas (N ₂)	The flow and heat transfer were investigated in a microtube under bubble injection. Air injection was conducted from the center of the tube and parallel to the flow.	Hydrodynamically fully developed flow before entering the heating zone $D_{tube}=[300,500]$ (μm) For internal diameter of 300 (μm): $U_{Liq}=[0.12-0.71]$ (m/s) $U_{bu}=[0.12-1.41]$ (m/s) For internal diameter of 500 (μm): $U_{Liq}=[0.08-0.42]$ (m/s) $U_{bu}=[0.04-0.64]$ (m/s)	The Nusselt number increased by 176% compared to the single-phase case, while the pressure drop increased by only 27%. The optimum void fraction is 10%.
Betz and Attinger [413]	Array of microchannels	Air-water	To increase the heat transfer rate in a microchannel, the air is mixed with the fluid in a T joint and injected periodically into the channel.	$q_{wall}=40$ (W) mass velocity=[300-3000] (kg/m ² s)	Heat transfer increases occur only for flow rates in the segmented flow range. The heat transfer rate increases up to 100% in this case.
Chang et al. [414]	U-bend channel	Air-water	The effect of the aspect ratio of a U-bent rectangular channel on the thermal performance of the channel is studied. Two horizontal and vertical configurations are considered, and intermittent slug and slug-annular flow are studied.	Aspect ratio= [0.33,0.83,1] (-) $Re_w=[1500-10000]$ (-) Flow quality (x)= [0-0.024] (-)	A correlation is introduced based on the aspect ratio of the channel and the flow quality.
Bao et al. [51]	Narrow conduit	Air–water	To study the pressure drop and heat transfer in the test section, air and water are mixed in a T joint prior to the entrance of the test section.	$U_w=[0.08-0.5]$ (m/s) $U_{bu}=[0.1-50]$ (m/s) $q''=[3-58]$ (kW/m ²) $Re_w=[150-1400]$ (-) $Re_{bu}=[70-11110]$ (-) $X=[0.1-8.2]$ (-)	Increasing the gas flow rate and transition from bubbly flow to slug flow and annular flow causes sudden changes in pressure drop and less heat transfer coefficient. The heat transfer coefficient increases up to 5 times.
Hetsroni et al. [415]	Parallel triangular micro-channels	Air-deionized water	Air and water are mixed in a Y-shaped mixer and are injected into the test section to study the heat transfer and pressure drop.	$Re_{bu}=[4.7-270]$ (-) $Re_w=[4-56]$ (-) $U_{bu}=[0.5-28.6]$ (m/s) $U_w=[0.015-0.244]$ (m/s) Hydraulic diameter= 130 (μm)	The connection between the different flow regimes and heat transfer coefficient in microchannels is investigated. A correlation is presented to correlate the Nusselt number and Reynolds number.
Houshmand and Peles [52]	Horizontal microchannel	Air-water	The effect of the bubble injection downstream of a microchannel is studied to investigate the impact of the boundary layer on heat transfer.	$U_{bu}=[0.1,0.2,0.3]$ (m/s) $U_w=[0.25-1.75]$ (m/s) $Re_w=[125-825]$ (-) Channel depth=210 (μm) Channel width=1.5 (mm) Channel length=22 (mm) $D_{noz}=250$ (μm)	100% increase of convection heat transfer coefficient compared to single-phase flow.
Chang and Huang [416]	Furrowed narrow channel	Air–water	Air is mixed with water by four porous cylindrical ceramic rods and injected into a channel with skewed wall waves and ribs to study the fluid flow and heat	$Re_w=[800-6000]$ (-) Flow quality (x)=[0-0.05] (-) $U_{bu}=[0.18-2.97]$ (m/s) $U_w=[0.061-0.45]$ (m/s) $T_{wall}=80$ (°C) quasi-steady state	The maximum increase of the average Nusselt number under the air injection condition is 235% compared to single-phase flow. A correlation is presented to calculate the Nusselt number.

			transfer. The channel is heated with constant heat flux.		
Kashinsky et al. [417]	Flat inclined channel	Air-water	An experimental study measures wall shear stress and heat transfer in a rectangular channel with different orientations.	$\theta = [0-90] (^{\circ})$ $Re_w = [3200-25000] (-)$ $Re_{bu} = [25-193] (-)$ $U_w = [0.176-3.17] (m/s)$ $U_{bu} = [0.0195-0.153] (m/s)$	The maximum wall shear stress and heat transfer are observed when the channel inclination angle is between 20° and 50° . Under these conditions, the Nusselt number can increase by up to 300% compared to the no-injection mode.
Chinak et al. [418]	Flat inclined channel	Air-water	Experimental and numerical studies are conducted to investigate the effect of bubble gas-liquid flow and angle of channel inclination on the heat transfer and wall shear stress of a straight inclined channel.	$\theta = [0-90] (^{\circ})$ $Re_w = [5000-25000] (-)$ $Re_{bu} = [39.6, 198] (-)$ $U_w = [0.275, 1.37] (m/s)$ $U_{bu} = [0.0305, 0.153] (m/s)$	The maximum wall shear stress and heat transfer are observed at a channel inclination angle between 30° and 50° . In inclined two-phase bubble flow, the maximum increases are 15% for wall shear stress and 30% for heat transfer. The maximum increases for inclination angles near the horizontal position are 10% for wall shear stress and 25% for heat transfer, respectively.
Chandan et al. [419]	Horizontal heater horizontal cooler (HHHC)	Air	Investigating the impact of bubble injection on the vertical leg's top, middle, and bottom.	Heat input = $[100-600] (W)$ $Q_{bu} = [40, 80, 120] (cc/min)$ HHHC loop = $(L_t/D=298)$	The maximum heat transfer fluid flow rate was attained by employing the lowest heat input (heat Input = $100 W$) and the highest air injection rate ($Q_{bu} = 120 cc/min$) into the bottom of the vertical leg. This resulted in an 8.9-fold increase compared to a scenario without bubble injection.

As a summary of Table 13, the findings indicate that elevating gas velocity induces a gradual shift in the flow pattern, causing a transition from bubbly to slug flow. Studies revealed that maintaining bubble size close to the channel diameter establishes optimal heat transfer conditions. Furthermore, the Nusselt number in two-phase flow in micro-channels rises with increased flow rates, with a diminishing enhancement attributed to segmented flow. The flow becomes segmented when the bubbles' length exceeds the channel's diameter.

6. Bubble-enhanced heat exchangers

In the previous section, the application of bubble injection was investigated in simple geometries such as pipes, cylinders, and plates. However, bubble injection is not limited to such simple configurations. It is also essential to consider the use of bubble injection in different types of heat exchangers that are commonly used in various engineering industries, such as cooling, heating, power generation, and renewable applications, as shown in Fig. 50. Moreover, bubble injection has been studied in internal combustion engines and solar collectors, which are critical areas of research. These applications have been investigated in various research studies, and their findings are discussed in this section.

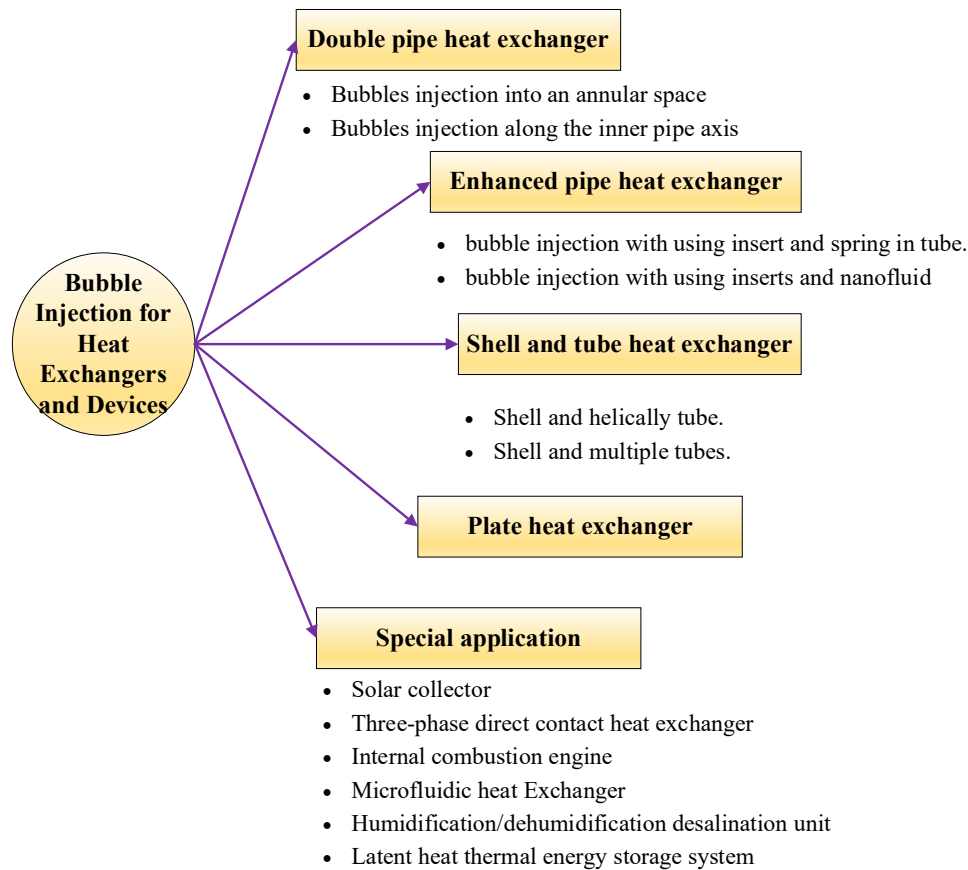


Fig. 50: An overview of the application of bubble injection in various heat exchangers and devices.

6.1. Double pipe heat exchanger using bubble injection

A double-pipe heat exchanger is a device that transfers heat between two fluids that flow through the inner and outer pipes, respectively. The fluids are usually at different temperatures, and the heat is exchanged through the pipe walls from the hotter to the cooler fluid. Because of their versatility and ease of production, double-pipe heat exchangers have become popular in various industrial and engineering applications [420].

In this field, Dizaji et al. [421] examined the effects of bubble injection on a horizontal double-pipe heat exchanger. Two different methods of bubble injection were utilized: the first involved injecting bubbles along the inner pipe axis, while the second involved injecting bubbles into an annular space using the same injection pipe. Results indicated that the enhancement in Nusselt number increased by 1.9 to 35% compared to the mode without bubble injection. The most significant increase was observed when bubbles were injected into the annular space.

According to Fig. 51, in experimental work, the heat transfer and exergy analyses of a double-pipe heat exchanger were carried out by Heyhat et al. [68]. They used three straight, internal, and external paths to inject bubbles. The internal and external paths were selected for

bubble injection in a vertical direction. They mainly focused on the effects of changes in air injection flow rate on the heat transfer rate and exergy analyses. The results of this study indicated that the increase in exergy destruction varies from 2 to 226% compared to the non-bubble injection case. However, the enhancement rate of NTU changes from 6 to 180%, and the total heat transfer coefficient improvement varies from 10 to 150%. So, it is necessary to optimize the bubble injection method by conducting more research.

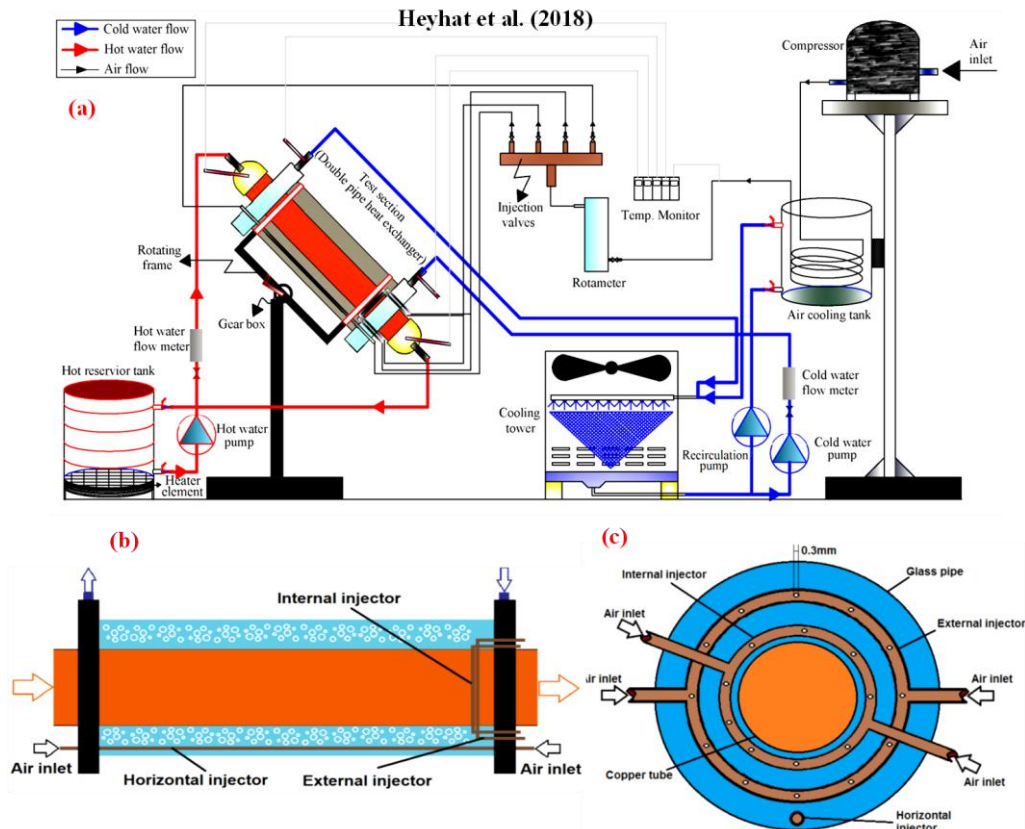


Fig. 51: (a) Experimental set-up, (b) test section, and (c) bubble injection geometry (Heyhat et al. [68] study). The enhancement of NTU fluctuates between 6% and 180%, while the total heat transfer coefficient improvement ranges from 10% to 150% (Figure is reprinted with permission from Elsevier).

In another study, Pourhedayat et al. [422] investigated the heat transfer enhancement in a vertical double-pipe heat exchanger. An annular pipe made up of plastic that contained holes for the air passage was used to inject the bubbles. The diameters of the created holes are 0.3 and 0.8 mm (Table 14). They mentioned that the maximum enhancement of the Nusselt number is 57% compared to the non-bubble injection mode and is related to the case where $n = 36$ and $d = 0.3$ mm. According to these findings, using more bubble holes with smaller diameters for the vertical case is much more effective and improves the heat transfer coefficient. Sinaga et al. [423] study on the bubble injection in a horizontal double-pipe heat exchanger also reveals that bubble injection to an internal flow brings about exergy destruction inside the heat

exchanger. They combined air and water inside a mixer from the double-pipe heat exchanger and then sent it to the heat exchanger through the internal pipe (hot flow). Based on the results, the maximum percentage of increase for heat transfer coefficient and NTU was 33% and 38%, respectively.

Likewise, heat transfer can be improved by combining bubble injection methods with nanofluids. Generally, nanoparticles improve the fluid thermophysical properties [424]. In this regard, Jafarzad et al. [425] combined the air bubble injection with nanofluid inside a double-pipe heat exchanger to investigate the heat transfer from the heat exchanger. They found that the energy efficiency of the heat exchanger increased by 192.5%. Nonetheless, conducting both exergy and thermo-economic analyses is recommended for a more comprehensive analysis to determine the optimal number of nanoparticles in the fluid within the bubbles.

Table 14: Summary of research on air injection in double-pipe heat exchangers.

Authors	Geometry	Working fluid	Bubble injection description	Condition	remark
Dizaji et al. [421]	Horizontal double-pipe heat exchanger	Air-water	Bubble injection in the inner and outer tubes of the heat exchanger.	$\dot{m}_{bu}=0.098 \times 10^{-3}$ (kg/s) $\dot{m}_{w,cold}=0.083$ (kg/s) $\dot{m}_{w,hot}=[0.0831-0.2495]$ (kg/s) $N_{hole}=40$ (-) $D_{noz}=[0.3, 0.7]$ (mm) $T_{in,cold}=25$ (°C) $T_{in,hot}=40$ (°C)	The Nusselt number increases up to 35% compared to the no-injection case.
Heyhat et al. [68]	Inclined double-pipe heat exchanger	Air-water	Horizontal, internal, and external bubble injection is conducted to study the heat transfer in different air and water flow rates.	$\theta=[0,45,90]$ (°) $D_{noz}=0.3$ (mm) $Q_{bu}=[100,150,200]$ (LPM) $Q_{w,hot}=[4-6]$ (LPM) $Q_{w,cold}=[2-3.5]$ (LPM)	The overall heat transfer coefficient and NTU increase by 10.3-149.5% and 5.9%-177.3%, respectively.
Pourhedayat et al. [422]	Vertical double-pipe heat exchanger	Air-water	Bubble injection in the outer tube of the heat exchanger to study the heat transfer and exergy of the heat exchanger.	$Re_{w,hot}=5500$ (-) $Re_{w,cold}=[5000-16000]$ (-) $Q_{bu}=5$ (LPM) $D_{noz}=[0.3, 0.8]$ (mm) $N_{noz}=[6, 36]$ (-)	Nusselt number increases up to 57%.
Sinaga et al. [423]	Horizontal double-pipe heat exchanger	Air-water	Bubble and water are mixed in an external mixer and then fed to the heat exchanger	$Q_{bu}=[1-5]$ (LPM) $Q_{w,hot}=[3-6]$ (LPM) $Q_{w,cold}=2$ (LPM) $\alpha=[0.14-0.62]$ (-)	Maximum 33% and 38 % increase in heat transfer coefficient and NTU, respectively
Jafarzad et al. [425]	Vertical double-pipe heat exchanger	Air-water-SiO ₂ nanoparticles	Bubble injection in the outer tube of the heat exchanger	$Q_{bu}=[0-300]$ (LPM) $Q_{w,hot}=[1, 2, 3, 4]$ (LPM) $Q_{w,cold}=[1, 2, 3, 4]$ (LPM) $\phi_{nano}=[0,0.1,0.5]$ (%)	192% increase in the energy efficiency of the heat exchanger 57.3% decrease in exergy destruction
Jie et al. [426]	Horizontal double-tube heat exchanger (DTHEX)	Air/water or air/CuO nanoparticles - water	Examining the impact of V-cut twisted tape turbulator, bubble injection, and nanofluid on the hydrothermal performance of DTHEX.	$Re=[7200-14500]$ $Q_{bu}=[2-6]$ (LPM)	Combining all three methods improves effectiveness 2.36 times, compared to a plain tube, while employing them individually increases effectiveness by up to 81%.

6.2. Enhanced pipe and pool using bubble injection

The thermal performance of pipes can be enhanced by introducing bubbles, either independently or in conjunction with other techniques, such as incorporating inserts into the

pipe or utilizing nanofluids. Celata et al. [427] explored the feasibility of injecting gas into liquid flows in a vertically insulated channel heated by an electric current, focusing on low-speed and mixed-convection flows. The study also examined the heat transfer coefficient for both injected and non-injected flows under varying gas injection conditions. The study's findings indicated that a lower gas injection rate resulted in a ten-fold increase in the heat transfer coefficient for mixed-convection upward heated flow and up to a 40% increase for forced-convection flow. Additionally, the researchers developed a correlation that linked the heat transfer coefficient under air injection conditions with that of a single-phase case (Eq.(92)). They finally observed that the gas injection technique is more effective at low Reynolds numbers.

$$h_{2\phi} = 2176h_{1\phi}^{0.11} \left(1 + 0.19 \log \frac{U_{Bu}}{U_w} \right), \quad (450 \leq h_{1\phi} \leq 6500) \quad (92)$$

The combined utilization of active and passive methods to enhance heat transfer has recently gained much attention. Cao et al. [428] studied the thermal properties of air-water two-phase flow and flow pattern passing through a vertical (upward) annularly corrugated tube. They highlighted that the flow patterns transit earlier in this case than the smooth tube, and the bubbles created inside it were smaller than the smooth ones. The same result was achieved for horizontal pipe arrangement by Ayed et al. [429]. The swirling flow formed at the corrugated sections causes pressure and velocity fluctuations in the liquid phase, eventually decreasing the size of the bubbles in the corrugated tube, which plays a vital role in heat transfer enhancement. In Fig. 52, heat transfer enhancement was compared between this case and without bubble injection mode. As the volumetric flow rate ratio enhances, heat transfer coefficient increases by 1.16 times. The major reason a decrease happens in some parts is that slugs get bigger and the contact between fluid and wall lessens. The details for these two studies are available in Table 15.

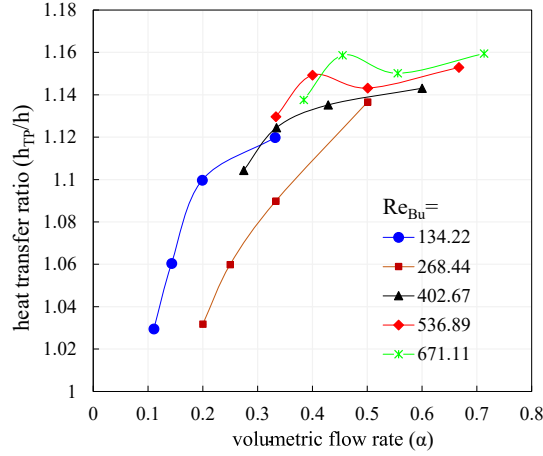


Fig. 52: The heat transfer ratio (two-phase per single-phase) of Cao et al. [428]. As the ratio of volumetric flow rates rises, the heat transfer coefficient experiences a 1.16-fold increase. The swirling motion generated in the corrugated sections induces fluctuations in pressure and velocity within the liquid phase. This ultimately reduces bubble size within the corrugated tube, resulting in the enhancement of heat transfer.

The simultaneous effect of twisted tape turbulators and bubble injection on the heat transfer improvement was investigated for the fluid flow inside a vertical and horizontal pipe under constant heat flux by Moria [430] and Azizi et al. [431], respectively. The utilization of twisted tape turbulators in different pitches and twist ratios (λ) were mentioned in [430] and [431] in turn. Rotational and swirling flows due to the creation of shear stresses in the existence of twisted tape turbulators cause many differences in flow structure compared to that in the smooth tube. For example, in a two-phase flow passing through a pipe without insertion, increasing Re_{bu} , leads to an increase in the length of the slugs. However, for the flow inside a pipe containing a twisted tape turbulator, as the flow rate of each phase rises, the number of bubbles increases, and their size becomes smaller (Fig. 53-a).

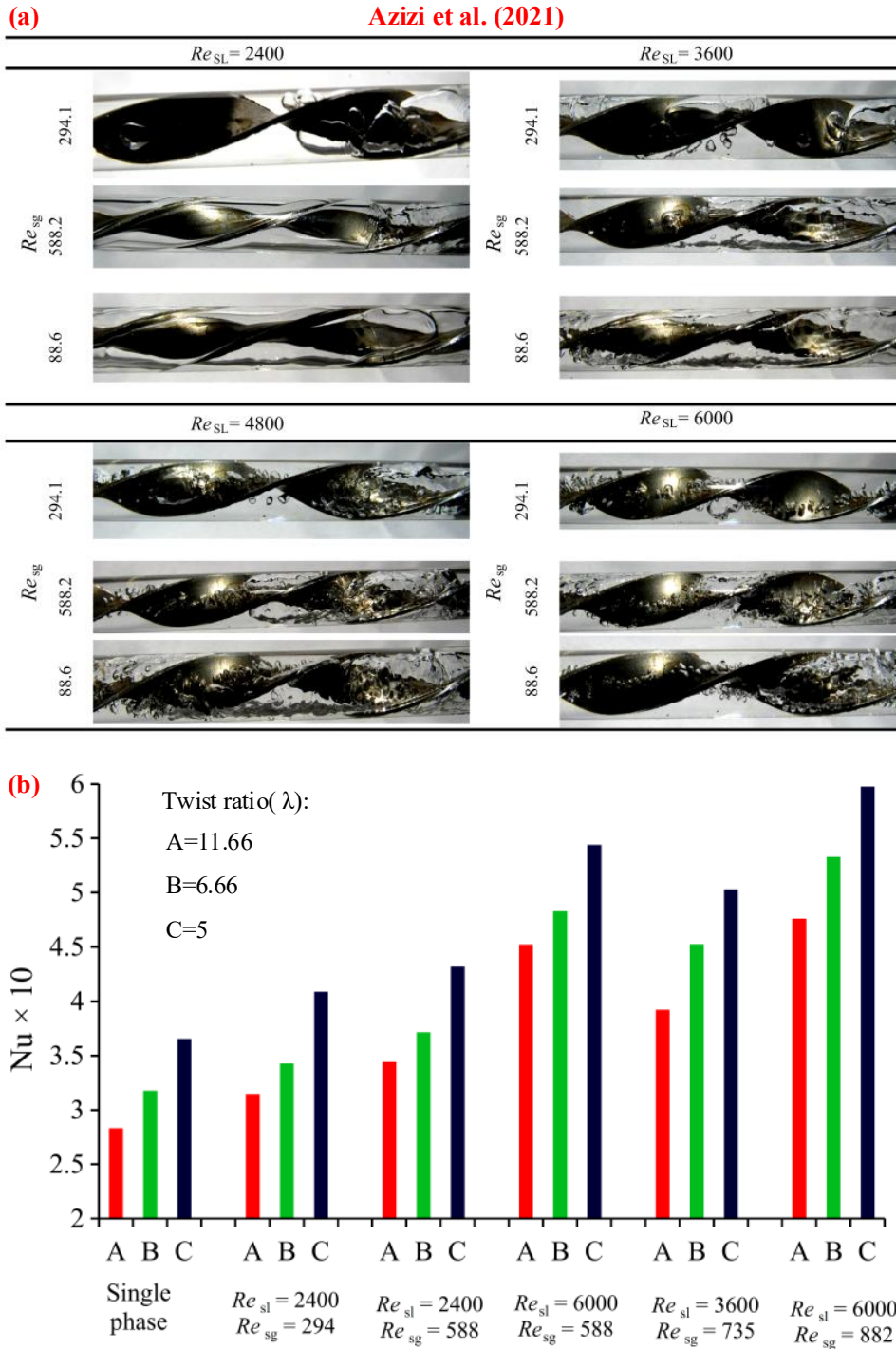


Fig. 53: (a) The flow patterns and (b) the Nusselt number for different cases of Azizi et al. [431] study. In a pipe with a twisted tape turbulator, an escalation in the flow rate for each phase leads to an increase in bubbles and a simultaneous reduction in their size. Bubble injection into a horizontal pipe equipped with a twisted tape turbulator resulted in a rise of up to 35% in the Nusselt number and an increase of 81% in pressure drop. (Figure is reprinted with permission from Springer Nature).

High turbulence in such flows can lead to increased friction losses, but it can also enhance heat transfer by promoting greater interaction between hot and cold fluid masses (Fig. 53-b). Compared to the single-phase flow, air injection to the vertical pipe containing a twisted tape

turbulator can increase the heat transfer coefficient and pressure drop by 30.3% and 62%, respectively. Air injection to the horizontal pipe containing a twisted tape turbulator causes the Nusselt number and pressure drop to increase up to 35% and 81%, respectively [431].

In another work, Zhou et al. [432] investigated the simultaneous effects of gas injection as an active approach and spring wire turbulator as a passive technique on the flow pattern, pressure drop, Nusselt number, and dimensionless exergy destruction of a flow inside a smooth vertical pipe in an upward direction. Results proved that the pitch variation influences the induced rotational flow, leading to a considerable change in the flow pattern (Fig. 54-a). The pressure drop rate and heat transfer coefficient increase dramatically as the turbulator pitch decreases and the water and air flow rate increase (Fig. 54-b). At a higher water flow rate, the effect of the airflow rate on the pressure drop gets higher since the collisions between the elements of air and water intensify. The maximum heat transfer coefficient occurs at the minimum pitch and the maximum liquid and gas flow rate.

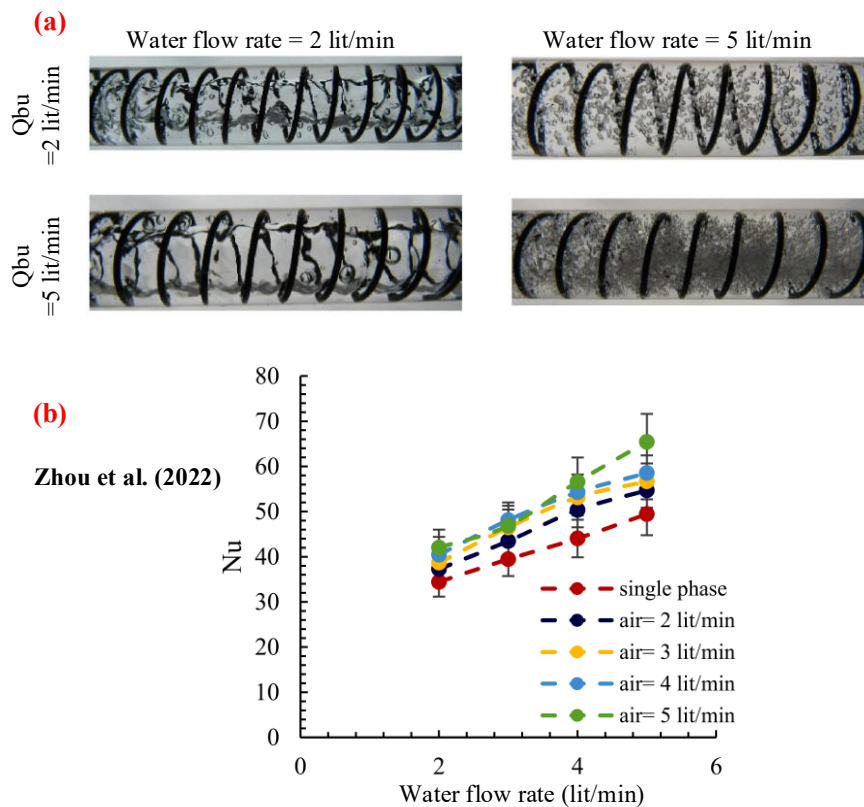


Fig. 54: (a) The flow patterns and (b) the Nusselt number as a function of the airflow rate of Zhou et al. [432] study. The pitch fluctuation influences the generated rotational flow, significantly altering the flow pattern. The greatest heat transfer coefficient is achieved at the smallest pitch and highest liquid and gas flow rate (Figure is reprinted with permission from Elsevier).

Bull et al. [433, 434] utilized a tank with high aspect ratio to explore the heat transfer from a liquid pool while the heat was being generated volumetrically, its surfaces were being

cooled, and tiny bubbles were injected. Heat transfer coefficient values were reported as a function of power density, axial position, and U_{bu} , and a correlation was developed for the heat transfer coefficient. In an experimental study, Bull et al. [434] used Magnesium sulfate as a surrogate for uranium sulfate salt in an aqueous solution. They found that gas injection led to a significant increase in heat transfer, higher than natural convection. Heat transfer increases at a higher gas injection rate so that when $U_{bu} = 0.05$ cm/s, the average heat transfer coefficient increases by 40%, compared to the single-phase condition. However, higher U_{bu} values increase the average heat transfer coefficient by 20-30%.

Table 15: Summary of research on bubble injection in enhanced pipe and pool.

Authors	Geometry	Working fluids	Bubble injection descriptions	Conditions	Remark
Bull et al. [433]	Pool	Air-water	The heat transfer from a liquid pool in which heat is generated, its surfaces are cooled, and tiny bubbles are injected into it was investigated.	Power density=[115-400] (W/L) U_{bu} =[0.0-0.3] (cm/s) Q_{bu} =[0.0-7.0] (LPM) D_{noz} =0.3 (cm)	The heat transfer coefficient increases by 239% compared to the case without bubble injection. A correlation is introduced to calculate the Nusselt number.
Bull et al. [434]	Pool	Air-MgSO ₄ solutions	Similar to Bull et al. [433]	power densities= 200, 400 (W/L) U_{bu} =[0.0-0.3] (cm/s) Q_{bu} =[0.0-7.0] (LPM) D_{noz} =0.3 (cm)	The heat transfer coefficient increased to 1.81 times compared to the case without bubble injection, and a new relationship for the Nusselt number was obtained.
Cao et al. [428]	Vertical corrugated tube	Air-water	The fluid flow and heat transfer of bubble-water flow in a vertical corrugated tube are investigated and compared with a simple tube case.	Q_{bu} =[1.0-5.0] (LPM) Q_w =[2.0-8.0] (LPM) q'' =800 (W) Re_w =[3417.82-13671.31] (-) Re_{bu} =[134.22-671.11] (-)	The heat transfer coefficient increased to 1.16 times, and the pressure drop increased to 1.8 times compared to the case without bubble injection.
Moria [430]	Vertical tube with twisted tape	Air-water	Combining twisted tape turbulators and air injection increases the heat transfer in a vertical tube.	Q_w =[2.0-8.0] (LPM) Q_{bu} =[1.0-6.0] (LPM) Tape pitch=4.5, 6, 10.5 (cm) D_{tube} =11 (mm)	Injection of the air into a tube containing twisted tape turbulators increases the heat transfer coefficient by up to 30.3% and pressure drop by up to 62% compared to the case without air injection.
Ayed et al. [429]	horizontal corrugated tube	Air-water	Various fluid flow and bubble-water heat transfer regimes in a horizontal corrugated tube were investigated and compared with a simple tube.	U_w =[0.42-1.69] (m/s) U_{bu} =[0.21-1.06] (m/s) q'' =740 (W)	The results showed that transition occurs earlier in a corrugated tube. Nusselt number increased to 1.2 times, and pressure dropped up to 4.69 times compared to the condition without injection.
Azizi et al. [431]	Horizontal tube with twisted tape	Air- water	Heat transfer in a horizontal tube was investigated using the combination of bubble injection and twisted tape turbulator.	Re_{bu} =[147.07-882.3] (-) Re_w =[2400-6000] (-) Twist ratio=[5.0-11.66] (-)	The maximum Nusselt number and pressure drop increase were about 35% and 81%, respectively.
Zhou et al. [435]	Vertical tube with the spring wire	Air-water	The entropy generation and heat transfer in a horizontal pipe were investigated using the combination of bubble injection and spring turbulator methods.	wire pitch=[3.0-10.0] (mm) Q_w =[2.0-5.0] (LPM) Q_{bu} =[2.0-5.0] (LPM) D_{tube} =11 mm q'' =1000 (W)	Air bubble injection increased the Nusselt number by 40% and dimensionless exergy destruction by 36%.
Fair et al. [436]	Pool-sparged contactor	Air-water	The heat transfer coefficient, temperature distribution, and fluid behavior of sparged contactors with different diameters and at different bubble flow rates were investigated.	U_{bu} =[0.0-0.15] (ft/sec) D_{pool} =18, 42 (in) Q_w =[10.0-13.0] (gal/min)	The heat transfer coefficient increased up to 4.5 times compared to the case without bubble injection. A correlation was presented to calculate the Nusselt number as a function of the air injection flow rate.
Celata et al. [427]	Vertical heated tube	Air-water	The heat transfer enhancement in a tube with hot walls and bubble injection is investigated in three different geometries and flow rates.	$T_{w,in}$ =[15.0-108.0] (°C) $\dot{m}_w$ =[80.0-850.0] (kg/h) $\dot{m}_{bu}$ =[40.0-1800.0] (g/h) q'' =[4.8-178] (kW/m ²)	The heat transfer coefficient was increased up to 10 times, and a correlation was presented to calculate it.

Hart [437]	Pool-vertical vessel	Air-water/glycol	An experiment is conducted to calculate the heat transfer coefficient in a vertical vessel by injecting bubbles at different flow rates for two fluids, water and glycol.	$D_{noz}=0.25$ (in) $U_{bu}=[0.001-0.067]$ (ft/sec) $q''=3000$ (Btu/hr. ft ²) $T_{w,in}=160$ (°F) $T_{glycol, in}=160$ (°F)	A correlation was presented to calculate the heat transfer coefficient as a function of the bubble speed and the thermophysical properties of the fluid.
Hetsroni et al. [438]	Horizontal heated tube	Air–water	This research investigates the heat transfer in intermittent air-water flow in a horizontal pipe.	$q''=[3000-20000]$ (W/m ²) $U_w=[0.62,0.87,1.4]$ (m/s) $U_{bu}=[0.02-0.03]$ (m/s) $Fr_w=[0.9-2.0]$ (-) $Fr_{bu}=[0.03-0.43]$ (-)	The overall heat transfer coefficient increased by 83%. In low Re_{bu} numbers, which indicate an intermittent flow regime, the local heat transfer coefficient can be even more than 4 orders lower than single-phase flow.
Zimmerman et al. [439]	Horizontal tube	Air–water	This research aims to investigate the heat transfer coefficient and its relationship with two-phase flow regimes.	$U_w=[24.2-41.5]$ (m/s) $U_{bu}=[0.02-0.09]$ (m/s) $T_{w,in}=[19.6-24]$ (°C) $q''=[6260-22600]$ (W/m ²) $D_{in}=25$ (mm)	The local heat transfer coefficient in the lower part of the tube can be up to 3 times the value corresponding to the upper part of the tube.
Hetsroni and Rozenblit [440]	Vertical heated circular tube	Air–water	This research aims to study the temperature patterns on a hot wall. Air is continuously introduced into a two-phase mixing chamber through a funnel with a porous layer. The flow regime is a hydrodynamically developed bubble or slug flow.	$q''=[1.9-23.5]$ (kW/m ²) $Re_w=[6800-63000]$ (-) $Fr_w=[0.094-0.87]$ (-) $U_w=[0.08-0.74]$ (m/s) $U_{bu}=[0-0.195]$ (m/s) $D_{in}=120$ (μm)	The heat transfer coefficient for the bubble flow increased up to 8 times compared to the single-phase mode. A correlation is provided to calculate the heat transfer coefficient for slug flow using the Peclet and Froude numbers.
Safrangian et al. [441]	Vertical tube with spring wire and twisted tape	Air-water	Investigating the effect of spring wire and twisted tape on the thermal and exergetic performance of two-phase flow.	$\alpha=[0.11-0.77]$ Twist ratio=[2.2,4.4,6.6]	The Nusselt number exhibits a 17% increase when using spring wire compared to the twisted tape turbulator. The utilization of spring wire led to an 11% increase in exergy efficiency when contrasted with the use of twisted tape.

In conclusion, incorporating additional methods alongside bubble injection, such as utilizing inserts and nanofluids, could further enhance heat transfer. Nevertheless, it is essential to consider the impact of the increased pressure drop, leading to a subsequent rise in power consumption. As a recommendation for future research, it is necessary to investigate the economic analysis of this particular heat exchanger design.

6.3. Shell and tube exchangers

6.3.1. Shell and helically coiled tube heat exchanger using bubble injection

Among the available heat exchangers, helically coiled tube heat exchangers have gained much attention due to their high heat transfer coefficient, compact structure, and ease of production [442, 443]. Although these categories of heat exchangers have positive roles in heat transfer improvement, the curvature of coils creates a centrifugal force that significantly increases the pressure drop on the coil side [444]. This section is dedicated to the review of the studies that have been done on heat transfer improvement using bubble injection into the shell or coil side in helically coiled tubes heat exchanger.

6.3.1.1. Bubble injection into the coil side

Prior to exploring the heat transfer effects of bubble injection in the coil side of a heat exchanger, it is important to note that research has demonstrated that the presence of a bubble inside the coil can decrease the friction factor. This decrease in friction factor can ultimately result in a reduction in pump power consumption. Saffari et al. [444] experimentally investigated the bubble injection into the coil using a cylindrical apparatus. Their findings indicated that the presence of the bubble inside the coil resulted in a 25% decrease in friction factor. Khorasani et al. [445] designed an experimental setup (Fig. 55) to evaluate the thermal performance of a helical coil heat exchanger undergoing bubble injection. Considering the second law of thermodynamics, the bubble injection effect at various void fractions (α) on NTU, entropy generation, and pressure drop on the coil side was examined. Results determined that increasing the void fraction from 0.14 to 0.33 caused the NTU (NTU stands for "Number of Transfer Units" and is a dimensionless parameter used in heat transfer analysis. It is a measure of the effectiveness of a heat exchanger in transferring heat from one fluid to another) to increase from 0.395 to 0.423 (Table 16).

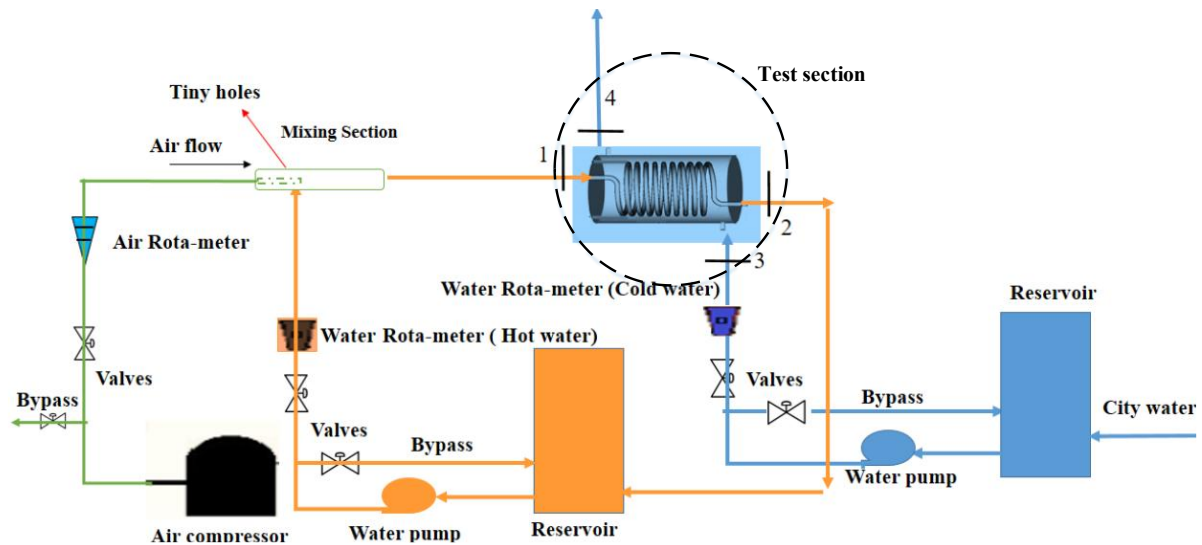


Fig. 55: Experimental arrangement and test section used in the Khorasani et al. [445] study. There was an increase in the NTU from 0.395 to 0.423 when the void fraction was increased from 0.14 to 0.33 (Figure is reprinted with permission from Elsevier).

In another work, Rostami et al. [446] experimentally investigated a helically corrugated tube's thermal and exergy behavior with bubble injection under a constant heat flux. The effect of liquid and air Reynolds numbers on parameters like pressure drop, average convective heat transfer coefficient, and exergy destruction in a non-boiling two-phase flow were studied. It was reported that bubble injection could lead to an improvement of up to 19% in NTU. In another study, Moradi et al. [447] conducted experimental work investigating the effects of

bubble injection inside a helical coil. The general schematic of this study is shown in Fig. 56. This experiment was done at five different water and air mass flow rates (Table 16). The findings proved that as air and water superficial Dean (De) numbers increased, the convective heat transfer coefficient increased by 35%.

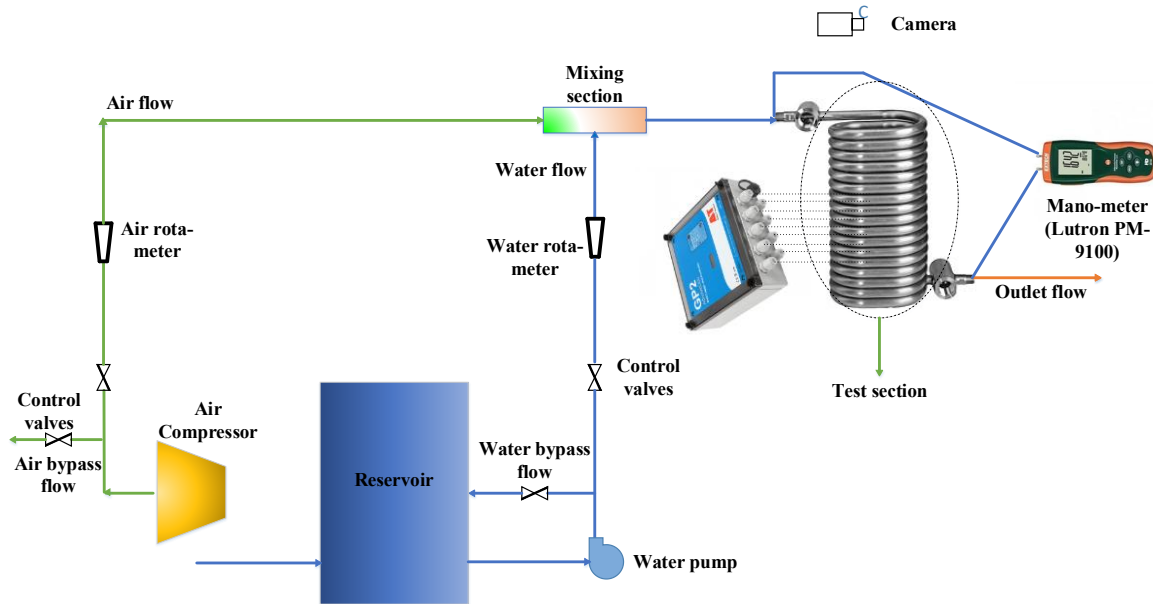


Fig. 56: Experimental arrangement and test section used in the Moradi et al. [447] study. A 35% increase in the convective heat transfer coefficient was observed (Figure is reprinted with permission from Elsevier).

Similarly, Abdzadeh et al. [448] investigated the thermal efficiency of air bubble injections on a horizontal helical tube in an experiment. Details and findings of this work are reported in Table 16. According to these studies, however, it can be concluded that no independent research has been done to propose a relationship for calculating the Nusselt number in a helical coil under bubble injection.

6.3.1.2. Bubble injection into the shell side

In a helical coil heat exchanger, the flow in the shell side is associated with higher thermal resistance due to the large volume of fluid that requires heating or cooling. Injecting bubbles into the shell side is a practical way to decrease thermal resistance and enhance heat transfer. Bubbles introduce additional turbulence and mixing in the fluid, leading to better contact between the fluid and the heat exchanger wall and disrupting the thermal boundary layer.

Furthermore, the curved geometry of the coil side generates a pressure gradient that propels the flow radially through centrifugal force, inducing secondary flows. These secondary flows enhance the fluid mixing and increase the rate of heat transfer. Hence, there is a lower

requirement for an external booster, such as bubble injection, on the helical side compared to the shell side.

Dizaji et al. [449] utilized an aquarium air pump to inject bubbles with a constant airflow rate inside the shell side of a shell and tube heat exchanger to examine its thermal performance experimentally (Fig. 57). As the bubble injection flow rate increased NTU increased by 4.2 times compared to the non-bubble injection mode. The bubble injection effect on NTU is more significant for the counter-flow case because the outlet hot water location was near the injection source, providing better mixing conditions for the thermal boundary layer. Additionally, the effectiveness of the bubble injection mode increased from 1.36 to 2.44 compared to the case without bubble injection.

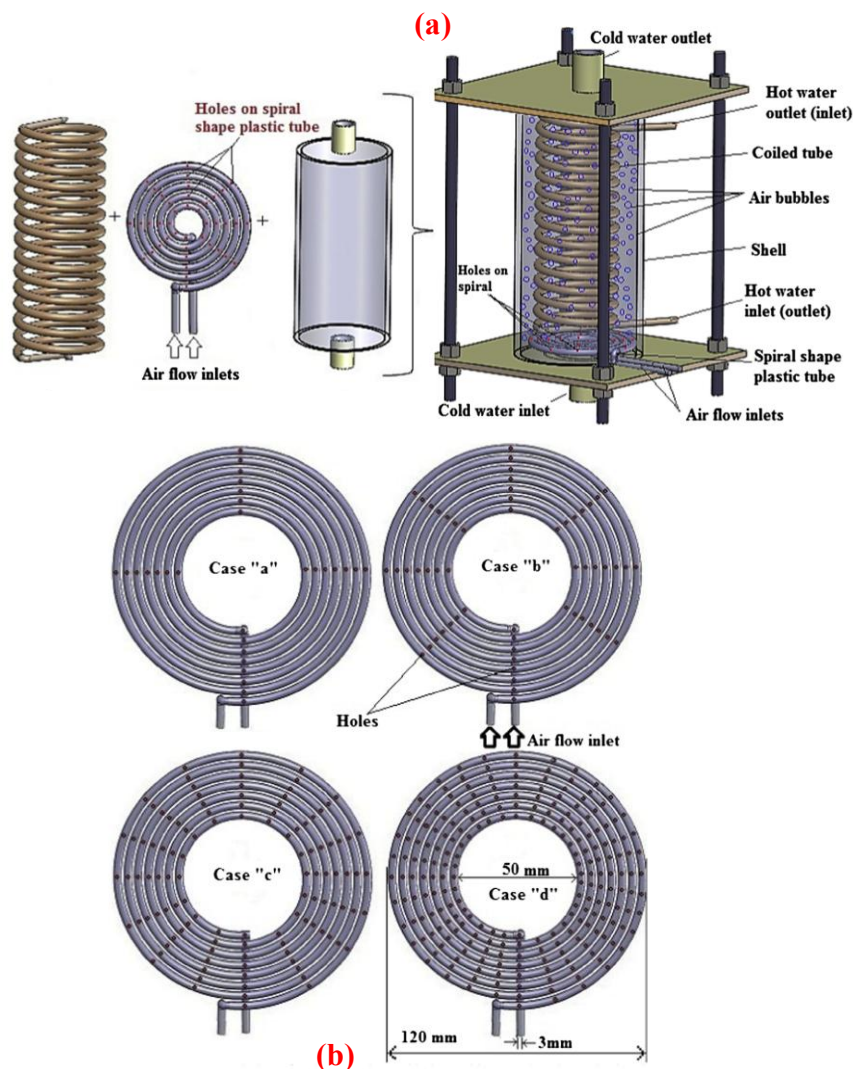


Fig. 57: (a) Test section and (b) different bubble injection geometries of Dizaji et al. [449] study. Compared to the non-bubble injection mode, NTU increased by 4.2 times as the bubble injection flow rate increased. The bubble injection effect on NTU was more significant for the counter-flow scenario (Figure is reprinted with permission from Elsevier).

In another work, Panahi [67] utilized an experimental setup similar to [449] to study the effectiveness and Nusselt number of a shell and helically coiled tube heat exchanger under bubble injection. It was determined that the Nusselt number in the bubble injection mode increased by 328% compared to the non-injected mode. Then, Khorasani and Dadvand [450] employed the same experiment conducted by Dizaji et al. [449] to inject bubbles inside the shell side. Subsequently, an investigation was undertaken by Baqir et al. [442] to optimize the operational conditions of a shell and helically coiled tube heat exchanger. An aquarium air compressor was used to supply the air required for the porous sparger to produce tiny bubbles with an average diameter of $100\ \mu\text{m}$. It was mentioned that the injection of tiny bubbles in the shell side gave rise to the considerable improvement of NTU (Table 16). In a research similar to this, a compressor was utilized by Ghashim et al. [451] to inject bubbles to the shell side of a shell and helically coiled tube heat exchanger and to study heat transfer and pressure drop in turbulent regime experimentally. Results reported an increase in the Nu number accompanied by a decrease in the friction factor as the bubble injection Re number increased. At the bubble injection flow rate of 3.5 LPM, the Nu number increased from 65 to 127, and the friction factor decreased from 0.3 to 0.25.

Zhou et al. [452] used the same method adopted by Khorasani and Dadvand [450] to experimentally investigate the injection of bubbles with a diameter of 3 mm into the shell side. Another experimental study on a compression refrigeration cycle was implemented by Zarei et al. [70] to study the bubble injection effects in the performance of an evaporator (Fig. 58). The impact of different air bubble injections was investigated in four different geometries. The findings indicated that the Nusselt number increased by 4.5 times compared to the non-bubble injection mode. This enhancement was a strong function of bubble injection geometry and flow rate. This study obtained the optimal point for the geometry and injection flow. Other results showed a direct relationship between Nu and COP, and COP increased by 123% compared to the case in which bubbles are not injected. Eventually, an experimental correlation was proposed to calculate the shell side Nusselt number. The explanation of this research is shown in Table 16.

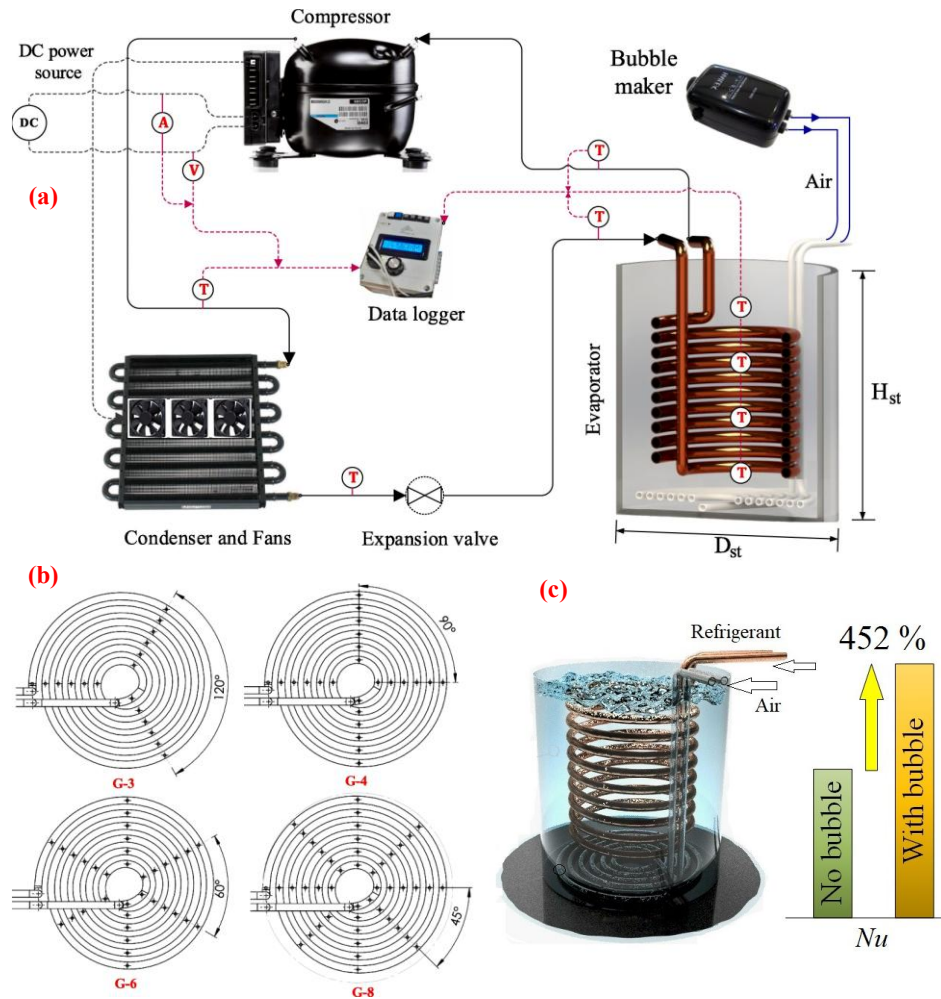


Fig. 58: (a) Experimental arrangement and test section, (b) bubble injection geometries, and (c) Nu results of Zarei et al. [70] study. The Nusselt number increased by 4.5 times. The geometry and flow rate of the bubble injection had a significant impact on this improvement. There was a direct correlation between Nu and COP, with COP increasing by 123% compared to the non-bubble injection mode (Figure is reprinted with permission from Elsevier).

6.3.1.3. Bubble injection into the shell and coiled sides

The utilization of the bubble injection technique to both the shell and tube sides of a heat exchanger has also been a topic of interest. Applying an injection technique similar to Dizaji et al. [449] work, Moosavi et al. [75] used a spiral tube to inject airflow into the shell and coil sides and to study the thermal performance of a heat exchanger experimentally. This experiment was conducted in two main modes of air injection: to the shell side, with the water flow rate ranging from 1-5 LPM, and to the coil side, with a constant water flow rate of 1 LPM. The results obtained from air injection into the shell side showed that the heat transfer rate increased dramatically to 187%. Besides, the findings for air injection to the coil side demonstrated that this injection method slightly affected the heat transfer improvement compared to the injection to the shell side. Furthermore, the findings for pressure drop due to

the bubble injection showed that an increase in the pressure drop for the case where the bubble was injected to the coil side was more significant than that of the shell side.

Table 16: Summary of research on bubble injection into the shell and helically tube heat exchangers.

Bubble injection into the coil side					
Authors	Geometry	Working fluids	Bubble injection descriptions	Conditions	Remark
Khorasani et al. [445]	Horizontal shell and coiled tube heat exchanger	Air-water	The effect of air injection is investigated with different flow rates on the NTU, entropy generation, and pressure drop on the coil side.	$Q_{w, shell}=3$ (LPM) $Q_{w, coil}=[3.0-6.0]$ (LPM) $Q_{bu}=[1.0-5.0]$ (LPM) $\alpha=[0.142-0.625]$ (-)	Air injection enhances NTU by 36% for 3 (LPM) of hot water flow rate.
Rostami et al. [446]	Horizontal four-start helically corrugated heat exchange	Air-water	The fluid and coil side air Reynolds number effect is investigated on the convective heat transfer coefficient, pressure drop, and exergy destruction.	$Re_{w, coil}=[4500-14500]$ (-) $Re_{bu, coil}=[95-500]$ (-)	Air injection enhanced heat transfer and exergy destruction by up to 19% and 32%, respectively.
Moradi et al. [447]	Vertical helically coiled tube	Air-water	The thermal and exergy characteristics of a helical coil with different flow rates and Dean numbers were studied.	$Q_{w, coil}=[2.0-8.0]$ (LPM) $Q_{bu}=[1.0-5.0]$ (LPM) $\alpha=[0.11-0.55]$ (-) $De_{w, coil}=[1000-4800]$ (-) $De_{bu, coil}=[45-235]$ (-)	The maximum heat transfer coefficient and entropy generation increase were 35% and 26%, respectively.
Abdzadeh et al. [448]	Horizontal helical tube	Air-water	Experimentally investigated the effects of air injection with different flow rates into a horizontal helical tube.	$Q_{w, coil}=[2.0-8.0]$ (LPM) $Q_{bu}=[1.0-5.0]$ (LPM) $\alpha=[0.07-0.75]$ (-)	Maximum heat transfer coefficient and entropy generation increases were 20% and 33%, respectively.
Bubble injection into the shell side					
Dizaji et al. [449]	Vertical shell and coiled tube heat exchanger	Air-water	An experiment was conducted on the air injection into the shell side of a heat exchanger. Parallel and counter-flow configurations are considered to investigate NTU, dimensionless exergy loss, and effectiveness.	$\dot{m}_{w, coil}=0.06249$ (kg/s) $\dot{m}_{w, shell}=[0.831-0.2495]$ (kg/s) $\dot{m}_{bu}=0.098 \times 10^{-3}$ (kg/s)	With an increase in air injection flow rate from 0.083 kg/s to 0.12 kg/s, the NTU, dimensionless exergy loss, and effectiveness rise by 180%, 110%, and 79.4%, respectively.
Panahi [67]	Vertical shell and coiled tube heat exchanger	Air-water	Similar to Dizaji et al. [449]	$\dot{m}_{bu}=0.098 \times 10^{-3}$ (kg/s) $Re_{w, shell}=[1000-4000]$ (-) $Re_{w, coil}=26500$ (-)	Increasing the Reynolds number from 1300 to 3900 increases the Nusselt number and the effectiveness by 50-328% and 53-127%, respectively.
Khorasani and Dadvand [450]	Horizontal helical shell and coiled tube heat exchanger	Air-water	Similar to Dizaji et al. [449]	$Q_{w, coil}=1$ (LPM) $Q_{w, shell}=[1.0-5.0]$ (LPM) $Q_{bu, shell}=[1.0-5.0]$ (LPM)	230% enhancement of NTU by increasing air injection flow rate.
Baqir et al. [442]	Vertical helical coil tube heat exchanger	Air-water	An experiment was conducted for different coil side temperatures and constant shell side temperatures. The effect of water and the airflow rate on NTU and its effectiveness were investigated.	$Q_{w, shell}=[2.0-10.0]$ (LPM) $Q_{bu, shell}=[0.0-10.0]$ (LPM) $\Delta T = 20, 30, 36$ (°C)	The most effective air injection flow rate is 6 LPM. The maximum enhancement in NTU and effectiveness is 1.93 and 0.83, respectively.
Ghashim et al. [451]	Vertical multi-helical coil heat exchanger	Air-water	An experimental investigation was performed for heat transfer and pressure drop in turbulent flow on the shell side.	$Re_{w, shell}=[9000-50000]$ (-) $\dot{m}_{w, coil}=0.0331$ (kg/s) $Q_{bu, shell}=[1.5-3.5]$ (LPM)	The maximum increase in Nusselt number was 126%.
Hasan et al. [453]	Vertical helically coiled tube heat exchanger	Air-water	A sparger is used for air injection into the shell side, and the temperature difference between the inlet and outlet of the hot flow is measured.	$Q_{w, shell}=[4.0-8.0]$ (LPM) $Q_{w, coil}=[1.0-2.0]$ (LPM) $Q_{bu}=[0.5-2.0]$ (LPM)	The maximum decrease in hot fluid temperature happened at the beginning of the air injection (air injection flow rate was 0.5 LPM).
Hasan et al. [454]	Vertically coiled tube heat exchanger	Air-water	Similar to Hasan et al. [453]	$Q_{w, shell}=[2.0-8.0]$ (LPM) $Q_{w, coil}=[1.0-2.0]$ (LPM) $Q_{bu}=[0.5-2.0]$ (LPM)	The heat transfer rate improved by increasing the temperature distribution along the heat exchanger and the size of the bubbles.
Zhou et al. [452]	Horizontal shell and	Air-water	Similar to Khorasani and Dadvand [450]	$Q_{w, coil}=1$ (LPM) $Q_{w, shell}=[1.0-5.0]$ (LPM)	The maximum increase in NTU is 430%.

	helically coiled tube exchanger			$Q_{bu, shell}=[1.0-5.0]$ (LPM)	The economic benefit of counter-flow was 13% more compared to parallel flow.
Zarei et al. [70]	Helical coil heat exchanger	Air-water	Air is injected into the evaporator of a helical coil of a compression refrigeration cycle. The thermal performance of the system is investigated.	$Q_{w, shell}=0$ (LPM) $Q_{bu, shell}=[3.0-11.0]$ (LPM)	The optimum air injection flow rate is 9 LPM, and the maximum increase in COP and Nusselt number is 123% and 423%, respectively.
Al-darraj et al. [455]	Vertical shell and coiled tube heat exchanger	Air-oil-water	Air injection into the shell side was conducted in a heat exchanger equipped with three novel baffle designs: the single-segmental baffle (SSB), disc and ring baffle (DRB), and disk and ring with circular holes baffle (DRCHB).	$\dot{m}_{hot, oil, tube}=0.23$ (kg/s) $\dot{m}_{cold, w, shell}=[0.15-0.23]$ (kg/s) $Q_{bu}=(3.33, 6.66, \text{ and } 10.00) \times 10^{-5}$ (m^3/s)	At $Q_{bu}=10.00 \times 10^{-5}$ (m^3/s), DRCHB showed the most effective baffle design, increasing heat transfer from 202% to 231%. This experimental study aimed to assess the hydrothermal performance of the heat exchanger and compare it to a scenario without air injection. A numerical analysis of the baffle designs was also performed to investigate their impact on temperature distribution.
Bubble injection into the coil and shell sides					
Moosavi et al. [75]	Vertical shell and coiled tube heat exchanger	Air-water	Under air injection into the shell and coil side, the thermal and exergy performance was experimentally investigated.	$Q_{w, shell}=[1.0-5.0]$ (LPM) $Q_{w, coil}=[1.0-5.0]$ (LPM) $Q_{bu}=[1.0-5.0]$ (LPM)	Maximum enhancement in heat transfer (187%) occurs with 5 LPM of air injection flow rate. Air injection into the shell side is more effective than air injection on the coil side.

As a summary of Table 16, injecting bubbles to the side in which heat transfer coefficient is lower or thermal resistance is higher gets more effective. Studies have indicated that although bubbles can be injected on either side, injecting the bubble on the shell side significantly improves heat transfer. Furthermore, because of its secondary flow and lower thermal resistance, the fluid flow inside the coil has a greater heat transfer coefficient than the shell side. Other geometries can be included in this argument.

6.3.1.4. Nusselt number correlation

Despite the importance of the bubble injection method in geometry, such as shell and helically coiled heat exchangers, a correlation has not yet been developed to predict the Nusselt number under such conditions. The primary purpose of this section is to collect the available experimental data from the literature and present a correlation for the Nusselt number in this geometry under bubble injection mode. The Nu number can be defined as follows,

$$Nu_{Dc} = hD_C/k_f \quad (93)$$

where D_C , h and k_f represent the helical coil diameter, heat transfer coefficient outside the coil, and fluid thermal conductivity, respectively. The Reynolds number for the shell side (Re_{sh}) and the bubble injection Reynolds number (Re_{bu}) can be calculated as follows,

$$Re_{sh} = \frac{4\dot{m}_f}{\mu_f \pi D_h} \quad (94)$$

$$Re_{bu} = \frac{4\dot{m}_{air}}{\mu_{air}\pi D_{air}} \quad (95)$$

where D_{air} is the hole diameter of bubble injection, and D_h represents the hydraulic diameter of the shell. The equation for hydraulic shell diameter is given below [67],

$$D_h = \frac{D^2 - \pi D_c d_o^2 p^{-1}}{D^2 + \pi D_c d_o p^{-1}} \quad (96)$$

where D , d_o , and p are shell diameter, the outer diameter of the coil, and coil pitch, respectively. Some research projects evaluated the Nusselt number, while others studied the overall heat transfer coefficient. The latter approach is used in this work based on the available data to calculate the Nusselt number. Table 17 summarizes the related studies and ranges for their results.

Table 17: Ranges of parameters in different experimental studies for Nu number in shell and coiled tube heat exchangers with bubble injection.

Experiments	Number of data	Deviation	Range of parameters		
			Re_{sh}	Re_{bu}	Nu based on D_c
Zarei et al. [70]	20	4.46 %	0	[84.1-822.1]	[5.1-133.3]
Hasan et al. [454]	28	3 %	[1740.7-6962.6]	[4.8-18.2]	[60.4-104.1]
Zhou et al. [452]	50	11.58 %	[3653.2-18759.3]	[60.6-302.8]	[671.2-2392.9]
Panahi [67]	40	11.78 %	[1000-3000]	[100.6-805.1]	[128.1-557.3]
Moosavi et al. [75]	25	8.37	[13.9-69.4]	[20.0-100.0]	[119.1-647.6]

Using experimental data, a correlation can be presented as,

$$Nu_{mo} = C_1 \exp(C_2 Re_{mo}^{C_6}) + C_3 \exp(C_4 Re_{mo}^{C_7}) + C_5 \quad (97)$$

$$Nu_{mo} = \left(\frac{Nu}{Re_{mo}} \right)^{0.05} \quad (98)$$

$$Re_{mo} = \frac{Re_{bu}}{Re_{sh} + 1} \quad (99)$$

The coefficients for this correlation are given in Table 18 for different ranges of modified Re numbers.

Table 18: Coefficients for proposed Nu number correlation (Eq.(97))

Conditions	C_1	C_2	C_3	C_4	C_5	C_6	C_7
$Re_{mo} < 0.5$	0.1283	-10.28	0.2731	-184.9	1.345	2	2
$0.5 \ll Re_{mo} < 7$	-0.467	0.1513	0.1004	0.294	1.741	1	1
$7 \ll Re_{mo} < 617$	0.8887	-1.76e-4	3.58e-4	5.74e-3	0	1	1

To evaluate and validate this correlation, we can refer to Fig. 59. It is clear that the maximum error for this correlation is 10%, a good agreement between this correlation and experimental data.

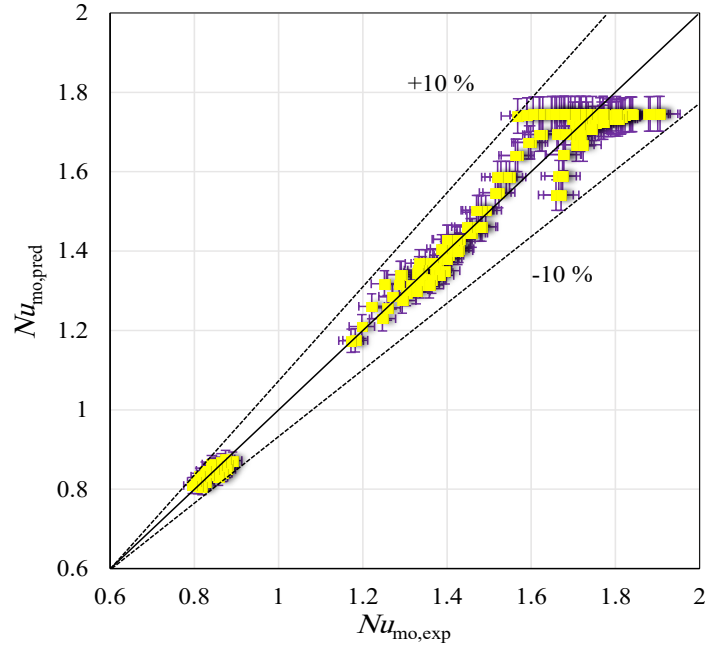


Fig. 59: Comparison of the experimental data of Table 17 with Eq. (97) predictions.

6.3.2. Shell and multi-tube heat exchanger using bubble injection

A shell and multi-tube heat exchanger is a widely used type of heat exchanger that operates by passing a hot fluid through the tubes and a cooler fluid over the shell's exterior surface. The fluids flow oppositely to facilitate efficient heat transfer, with the heat energy transferred from the hot fluid to the cooler fluid via convection.

The use of shell and multi-tube heat exchangers is prevalent across various industrial sectors, including chemical processing, heating and cooling [456], petroleum refining, and power generation, owing to their ability to facilitate efficient heat transfer between fluids while minimizing overall energy consumption. The performance of these heat exchangers can be improved by introducing various enhancement techniques, such as injecting bubbles on the shell or tube sides. Furthermore, bubble injection can minimize fouling in the heat exchanger and increase its thermal performance, making it an attractive solution for industries requiring high heat transfer levels.

Marzouk et al. [457] studied the utilization of the bubble injection technique on the heat transfer improvement rate in a shell and multi-tube heat exchanger. Fig. 60 illustrates the schematic of the shell and multiple-tube heat exchanger utilized in their study. Rods were used as inserts inside the tube to enhance heat transfer further. Combining these inserts and bubble injection interacted with the fluid flow passing through the heat exchanger tubes, increasing the heat transfer rate.

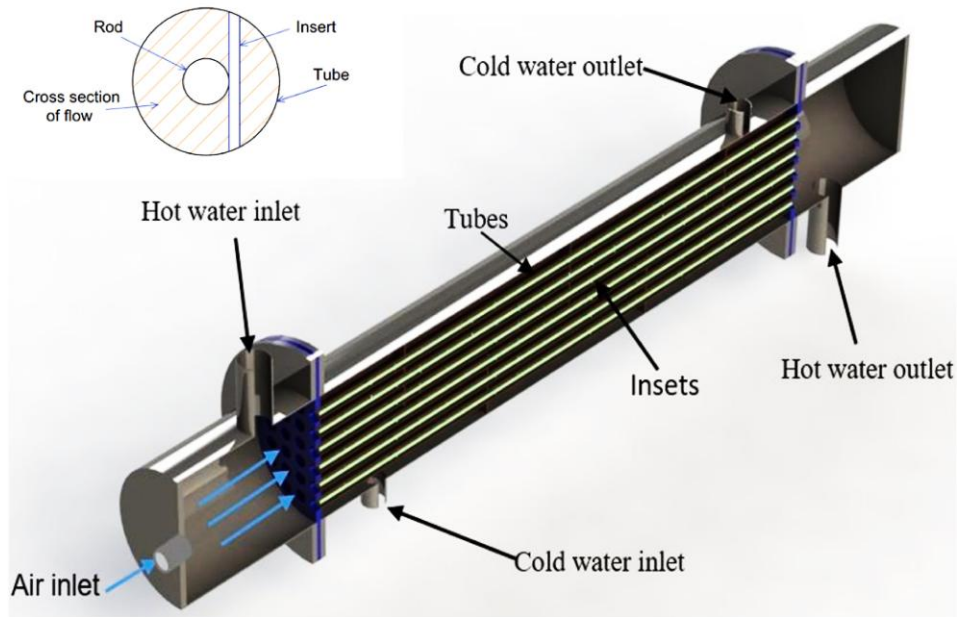


Fig. 60: Schematic view of the heat exchanger with bubble injection and insert in tubes of Marzouk et al. [457]. The shell diameter and tube length were 205 mm and 1000 mm, respectively. Using inserts on the tube side is more effective and increases heat transfer by up to 180% (Figure is reprinted with permission from Elsevier).

The study's findings demonstrate that, in the absence of inserts within the tube path, implementing the bubble injection technique can result in a remarkable 36.5% increase in rate of heat transfer. On the other hand, bubble injection can be applied to the shell side of the heat exchanger. Two methods have been proposed for bubble injection in the shell in the studies carried out in the literature. The first method of bubble injection is in the shell inlet area (injection parallel to the fluid flow). The injection is perpendicular to the direction of fluid flow and from the shell wall side in the following method. This can be done using air injection tubes on the shell wall [458].

In two similar studies conducted by El-Said et al. [458] and Subesh et al. [459], thermo-hydraulic analysis of a shell and multiple tube heat exchanger was performed when subjected to bubble injection. The studies highlight that introducing bubbles on the shell side of the heat exchanger is a more effective approach than introducing bubbles inside the tubes. The maximum heat transfer enhancement recorded in El-Said et al.'s study was 77.3%, while Subesh et al. reported a maximum heat transfer enhancement of 88.3%. Furthermore, the cross arrangement increased the heat exchanger efficiency compared to the parallel case. However, the massive pressure drop is still the main stumbling block. In an experimental work, Sokhal et al. [460] studied the influence of three different bubble injection methods. The first method was an injection in the tube inlet area, and the subsequent one was an injection in the shell inlet

zone. The last technique was an all-around injection in the tube. They showed that the all-around injection method in the tube leads to the maximum rate of heat transfer improvement, which was reported to be 82% for this study. Also, similar to the findings achieved from previous studies, the bubble injection method on the shell side is more effective than on the tube side. However, due to the wide range of applications of this type of heat exchanger in industries, it is necessary to analyze its performance under bubble injection much more comprehensively.

Table 19: Summary of research using air injection in shell and multiple tube heat exchangers.

Authors	Geometry	Working fluid	Bubble injection description	Condition	remark
Marzouk et al. [457]	Shell and multiple tube heat exchanger	Air-water	Bubble injection in the tube side and rod inserts are used to investigate the thermo-hydraulic performance of the heat exchanger.	$Q_{w,hot}=[14, 15, 16, 17,18]$ (LPM) $Q_{w,cold}=18$ (LPM) $Q_{bu}=[0,1, 2, 3]$ (LPM) $T_{hot,in}=90$ (°C) $T_{cold,in}=27$ (°C)	Using air injection on the tube side can increase the heat transfer rate to 36.5%. Using inserts on the tube side is more effective and increases heat transfer by up to 180%.
El-Said et al. [458]	Shell and multiple tube heat exchanger	Air-water	Bubble injection in the shell side in cross and parallel configuration is used to investigate the thermo-hydraulic performance of the heat exchanger.	$Q_{w,shell}=[12-21]$ (LPM) $Q_{w,tube}=23$ (LPM) $Q_{bu}=[0-5]$ (LPM) $T_{hot,in}=68.5$ (°C) $T_{cold,in}=33$ (°C)	Maximum 57.8% and 77.3% increase in heat transfer rate for parallel and crossflow air injection. The overall heat transfer coefficient increases by 131-176%.
Subesh et al. [459]	Shell and multiple tube heat exchanger	Air-water	Bubble injection in the shell side in the cross and parallel configuration is used to investigate the thermo-hydraulic performance of the heat exchanger.	$Q_{w,shell}=[10-20]$ (LPM) $Q_{w,tube}=1$ LPM) $Q_{bu}=[0-6]$ (LPM) $D_{noz}=0.6$ (mm)	Maximum 65.1% and 88.3% increase in heat transfer rate for parallel and crossflow air injection. The overall heat transfer coefficient increases by 110-192%.
Sokhal et al. [460]	Shell and multiple tube heat exchanger	Air-water	Bubble injection into the shell side, tube inlet, and throughout the tube is used to investigate the thermo-hydraulic performance of the heat exchanger.	$Re_w=[4000-22000]$ (-) $Q_{bu}=3.5$ (LPM) $T_w=[40-70]$ (°C)	Air injection throughout the tube results in the highest increase in heat transfer rate (82%)

6.4. Plate heat exchangers

Plate heat exchangers are a type of heat exchanger that is widely used in various industrial processes due to their compactness, high efficiency, and ease of use. A schematic of the plate heat exchanger is shown in Fig. 61. Plate heat exchangers find applications in various industries, including chemical and food processing, shipping, cooling, and heating. They are also utilized in building heating, ventilation, and air conditioning (HVAC) systems to transfer heat between air and water streams. These heat exchangers consist of thin, corrugated plates arranged in parallel and separated by a small gap.

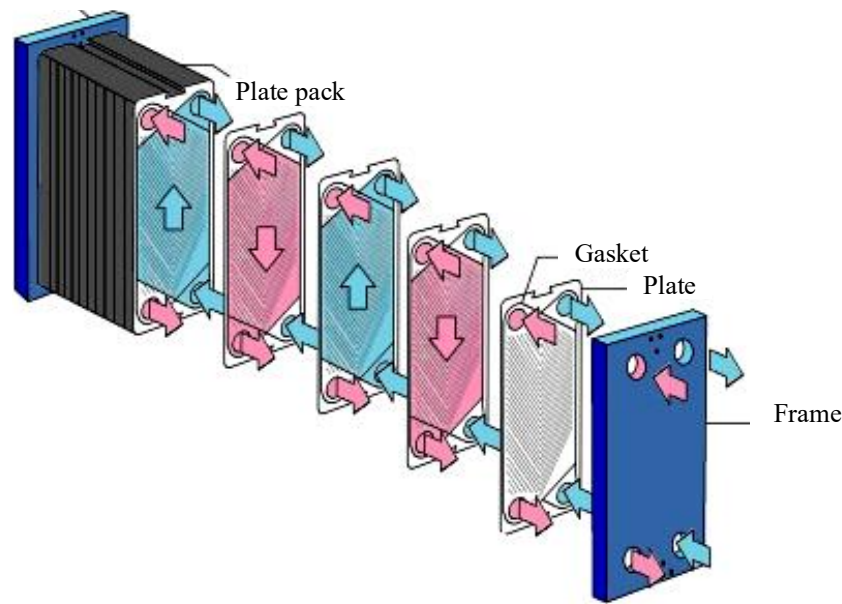


Fig. 61: Schematic of a plate heat exchanger [461] (Figure is reprinted with permission from Elsevier).

Corrugated channels are a popular method of enhancing heat transfer in plate heat exchangers. Corrugations can augment heat transfer within an appropriate range of Reynolds numbers by inducing fluid recirculation and swirl flow [462]. This recirculation also aids in reducing the fouling rate, thus extending the lifespan of the heat exchanger. In a study by Nilpueng and Wongwises [463], the pressure drop and flow behavior of air-water two-phase flow passing through a plate heat exchanger were examined to determine the flow pattern. Furthermore, there has been significant interest in using gas injection into the liquid to enhance heat transfer in this type of heat exchanger. From a heat transfer analysis perspective, Vlasogiannis et al. [464] investigated the use of an air-water mixture as the cold flow in a chevron-type corrugated plate heat exchanger. They analyzed the captured images to generate a flow regime map (refer to Table 9). The study concluded that, under all conditions, the heat transfer coefficient for two-phase flow surpassed that of single-phase water flow, increasing by a factor of 3.2.

Chang and Huang [416] conducted an investigation on air-water flow within a furrowed channel featuring ribs and skewed wall waves. Through image analysis, they determined that the flow regime is contingent upon both the liquid Reynolds number and the air-to-water (AW) mass flow ratio. Utilizing the drift flux model, they also developed a correlation for the channel-wise averaged void fractions (α) within a wavy-ribbed channel. Due to the generation of eddy flow within the wavy-ribbed channel and its influence on bubble shape, the corresponding value of the Nusselt number is higher compared to two-phase flow passing

through a smooth tube. Both the air-to-water (AW) ratio and the liquid Reynolds number (Re_{liq}) impact the local Nusselt number. As these parameters increase, air bubbles serve as mixers, thereby augmenting the Nusselt number. At a fixed Re number, increasing AW causes the $\overline{Nu}/\overline{Nu}_{AW=0}$ ratio to rise, but at a fixed AW, this ratio decreases as the Re number increases.

The proposed correlation is as follows,

$$\frac{\overline{Nu}}{\overline{Nu}_{AW=0}} = a_1 + a_2 \exp \left[\left(-\frac{a_3}{10000} \right) Re_{liq} \right] \quad (100)$$

$$a_1 = 1 + 5.56AW + 2052.8AW^2 \quad (101)$$

$$a_2 = 218.28AW \quad (102)$$

$$a_3 = 5.56 + 77.84AW + 17727AW^2 \quad (103)$$

Since bubble injection leads to both increased pressure drop and enhanced heat transfer, an index called the thermal performance factor (TPF) can be used to evaluate its effectiveness. The TPF is defined as the ratio of the average Nusselt number in the bubble-injection scenario to that in the single-phase case, normalized by the cubic root of the corresponding friction factor ratio. The value of TPF was found in the ranges of 1.6–2 and 4.12–6.43 for the laminar and turbulence reference cases, respectively. The airflow injection is effective when the value of TPF is greater than one. Given the importance and wide range of applications of plate heat exchangers, few studies have been conducted on utilizing bubble injection to enhance their efficiency. Therefore, it is crucial to conduct further research to optimize this type of heat exchanger and increase its overall efficiency. Table 20 summarizes the important results of studies on bubble injection in plate heat exchangers.

Table 20: Summary of research on bubble injection in plate heat exchangers.

Authors	Geometry	Working fluids	Problem description	Conditions	Remark
Vlasogiannis et al. [464]	Corrugated plate heat exchanger	Air-water	Air and water are blended at a T-joint with varying flow rates before being introduced into a plate heat exchanger to investigate heat transfer and flow regimes.	$U_{bu}=[0.3-10]$ (m/s) $U_w=[0.01-0.25]$ (m/s) Corrugation height=2.4 (mm) Corrugation wavelength= 10 (mm) $T_{w,hot}=40$ (°C) $T_{w,cold}=25$ (°C)	The heat transfer increased up to 3.2 times compared to the no-injection mode.
Marouf et al. [465]	Plate heat exchanger	Air-water	Air is combined with a cold liquid stream at a T-joint before being injected into a plate heat exchanger.	$Q_{bu}=[2.5-14]$ (LPM) $Q_w=[4.66-14.66]$ (LPM)	NTU and the effectiveness of the heat exchanger are enhanced by 12.4 and 14.6 %, respectively.
Marouf and Fouad [466]	Plate heat exchanger	Air-water	The study focused on the energetic and exergetic performance analysis and optimization of submillimeter air bubble injection into both cold and hot flows within a plate heat exchanger.	$T_{w,cold}=50$ (°C) $Q_{bu}=[2.5-14]$ (LPM) $Q_{w,hot}=[4.67-14.67]$ (LPM) $Q_{w,cold}=4.84$ (LPM) Number of plates=10 (-)	NTU and the effectiveness of the heat exchanger are enhanced by 59 and 18.6 %, respectively.
Marouf et al. [467]	Plate heat exchangers	Air-water	An experimental comparison between injecting bubbles into cold and hot water streams to	$Q_{bu}=[150-840]$ (LPH) $Q_{w,cold}=290$ (LPH) $Q_{w,hot}=[280-880]$ (LPH)	Compared to single-phase flow, there was a 0.6% increase in maximum efficiency, a 2.68 times

			assess their impact on energy, exergy, and economic performances.	$T_{W,hot,i}=50\text{ (}^{\circ}\text{C)}$	increase in the number of heat transfer units, and a 2.45 times reduction in maximum entropy generation.
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In summary, while some studies have explored the impact of bubble injection into plate heat exchangers, it is crucial to note, as highlighted in the channel section, that segmented flow may counteract heat transfer enhancement by insulating the surface from the fluid. Therefore, a well-designed bubble sparger with a controlled bubble diameter is essential for optimizing heat transfer in such systems.

6.5. Special and other applications using bubble injection

Besides its application in heat exchangers, the bubble injection method has found widespread use in enhancing thermal performance in various fields. These applications range from improving solar collector thermal efficiency [468] to enhancing the performance of internal combustion engines [469]. This section provides an overview of some of the studies conducted in these areas.

In 1988, Baliño et al. [470] conducted an experiment where they utilized a cylinder as an electrolyzer for the production of hydrogen bubbles, through which cross flow passed. Since it is notoriously difficult to calculate the vapor volumetric flow in a nucleate boiling flow, the electrolyzer was used for hydrogen bubble generation. The general design of the experimental setup includes a test section, separator tank, flow control valve, and pump. Sodium hydroxide (NaOH) solution was considered as the fluid, and a hollow cylinder with a 5 cm length and 1.2 mm outer diameter was used in the test section. In this experiment, the effects of heat flux (q''), the temperature difference of the cylinder wall (ΔT_W), fluid bulk temperature (T_B), and flow velocity, as well as the current density (J_E), were investigated (Table 21).

Results demonstrated that at a constant temperature difference between the fluid and the cylinder, attainable heat flux increased as the bubble generation rate increased. Since the wall temperature was always lower than the fluid's boiling temperature, this enhancement has nothing to do with the fluid's boiling, and it only happens due to the hydrogen bubble and its impact on the flow. For example, cylinder heat flux changed from 250 to 350 kW/m² when $\Delta T=40\text{ }^{\circ}\text{C}$. Furthermore, it was seen that the heat transfer coefficient (h) increased as the bubble generation rose. For instance, increasing ΔT from 30 to 50 C at $J_E=2\times 10^4\text{ A/m}^2$, caused the heat transfer coefficient to vary from 8 to 10 kW/m².

In another study, Mahood et al. [471] utilized the bubble injection method to improve the performance of a three-phase direct contact heat exchanger in a power generation cycle

with a low-grade energy heat source. In this experiment, a Perspex tube with a 70 cm length and 4 cm diameter was considered as the test section. Bubble injection was conducted using a sparger with a total diameter of 36 cm and 42 cavities, each with a diameter of 0.5 mm. The experimental findings showed that the bubble mass flow rate strongly affected the efficiency and cost reduction of the heat exchanger. The cost of this heat exchanger with the same cross-section decreased by 30 times in comparison to the corresponding shell and tube heat exchanger. Also, the bubble mass flow rate increase gave rise to the enhancement of the heat exchanger efficiency. For instance, at the bubble's initial temperature of 47.5 C, heat exchanger efficiency increased from approximately 25% to 75%.

Later, Zavaragh et al. [469] investigated the influence of bubble injection on the cooling circuit of an internal combustion engine (gasoline engine) to realize whether this method can effectively reduce the amount of fuel consumption and exhaust emission (Fig. 62). They attempted to use a bubble at three distinct flow rates of 10, 20, and 30 LPM (by volume) and the constant pressure of 1.6 and 1.75 bar. The findings proved that the bubble injection caused the fuel consumption to decrease between 0.13-5.69% during the experimental period of 900 s. Furthermore, regarding environmental pollutants, bubble injection decreased the NO dispersion between 1.58-198.43%. Technically, the time required for the engine to warm up decreased between 5.09-33.33% after using bubble injection.

The bubble injection method can also be widely used in renewable energy systems. For example, Khwayyir et al. [468] investigated the effect of bubble injection in a flat plate solar collector on its efficiency and Nusselt number. Results demonstrated that using the bubble injection increased the collector efficiency dramatically, so this enhancement was 16.5%, 69.2%, and 68.8% for volumetric flow rates of 1, 1.5, and 2 LPM, respectively. Additionally, the flow rate variation in the above range causes the Nusselt number to increase from 3 to 7, 9, and 11.5, respectively. These increases indicate a growth rate of 133-283% in this case compared to the non-injected mode. The same researchers also published another similar research paper in Ref. [472], which is listed in Table 21.

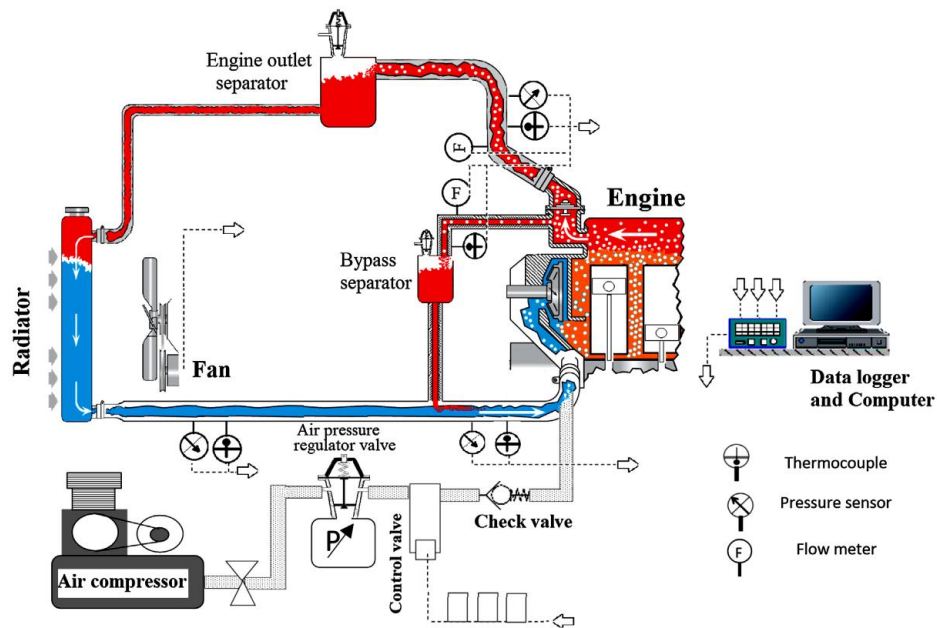


Fig. 62: Schematic layout of the experimental setup of Zavaragh et al. [469] study. The bubble injection caused the fuel consumption to decrease between 0.13-5.69% (Figure is reprinted with permission from Elsevier).

In another work, Vasilev et al. [473] studied the impact of bubble injection on a microfluidic heat exchanger. Fig. 63 depicts the test section of this research in which the microchannel consisted of six square channels with sides of 1 cm, and the channel length was 55 mm. In this experiment, seven gas fractions of bubble ranging from 4-18% were injected to study the Nusselt number, heat transfer coefficient, and pressure drop.

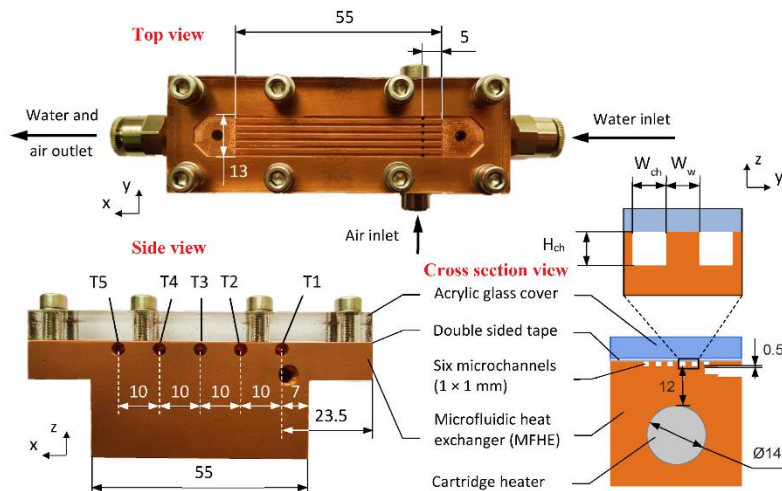


Fig. 63: Schematic layout of test section (microfluidic heat exchanger) of Vasilev et al. [473]. When the bubble's diameter matches the microchannel's diameter, the ideal gas percentage for heat transmission occurs at 12% (Figure is reprinted with permission from Elsevier).

Results of Vasilev et al. [473] showed that the optimal gas fraction in heat transfer happens at 12% when the diameter of the bubble corresponds to the diameter of the microchannel. Also, as opposed to the single-phase case, bubble injection enhanced the Nusselt

number by 1.54 times and decreased thermal resistance by 1.67 times. These researchers also conducted another similar study in [474] for different ranges of gas fractions, which is available in Table 21 in detail.

The bubble injection technique has also recently been applied to energy storage systems. Choi et al. [475] investigated the effect of bubble injection on the improvement of charging performance of a latent heat thermal energy storage (LHTES) whose body was made up of transparent polycarbonate with specific dimensions (Fig. 64-a&b). Using bubble injection reduced the charging time by 53%. Also, it increased the energy storage rate and the average velocity of the PCM liquid phase by 2.2 and 6.4 times, respectively. Choi et al. [8] conducted another research similar to the previous one with a significant focus on the discharge time of the PCM. They conducted an experimental investigation [476] to explore heat transfer enhancement in phase change materials during the charging process. This enhancement resulted from the combined utilization of nanoparticles and bubble injection. A computational fluid dynamics (CFD) simulation was employed in situations with high nanoparticle concentrations, as visualizing the cloudy liquid PCM was challenging. Their findings demonstrated that the concurrent application of nanoparticles and bubble-induced flow in PCM led to improvements in conductive and convective heat transfer, fostering a uniform particle concentration distribution. The details are reported in Table 21.

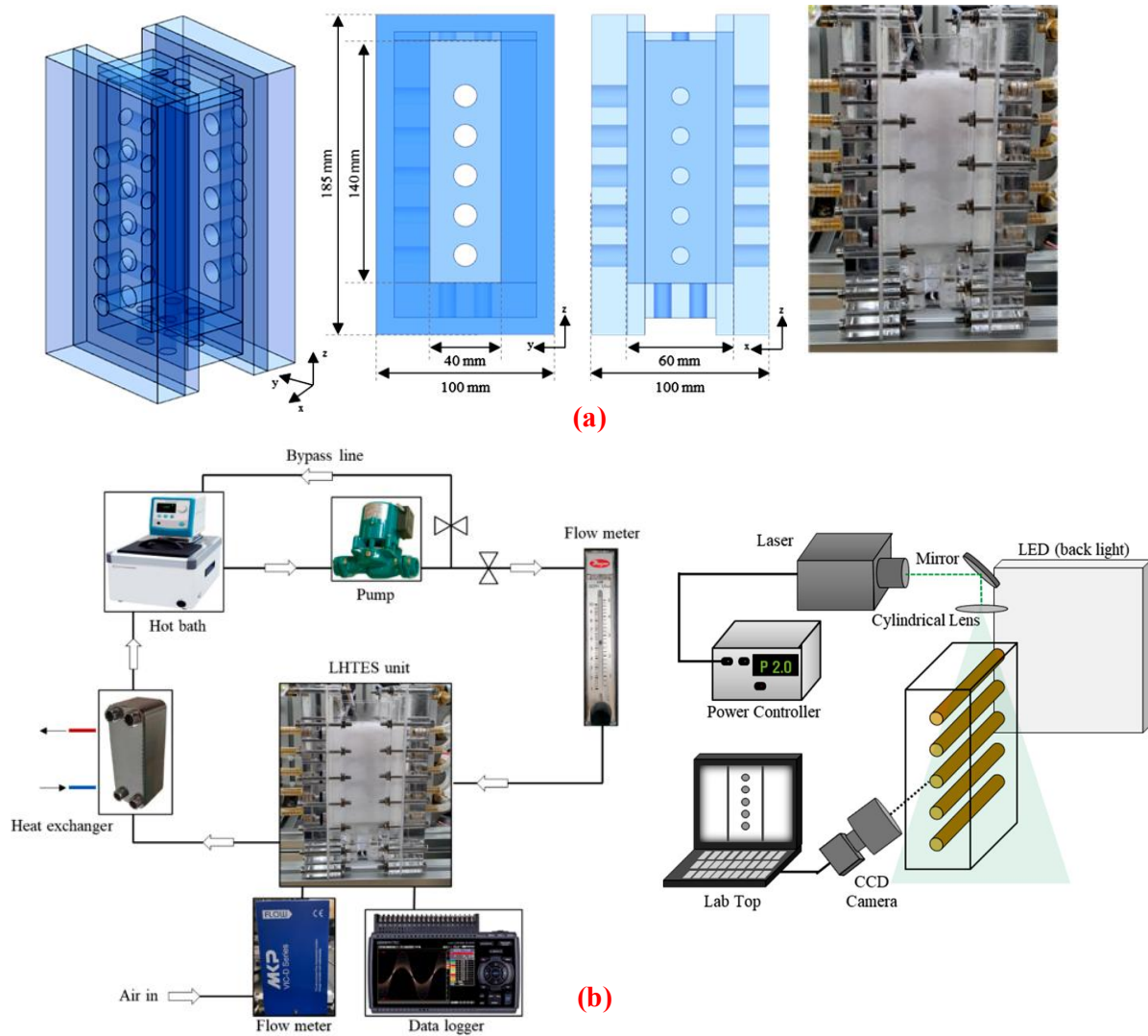


Fig. 64: (a) Test section and (b) experimental arrangement used by Choi et al. [8, 475]. These studies showed that the charging time was shortened by 53% with bubble injection, and the energy storage rate and average liquid phase velocity of PCM were enhanced by 2.2 and 6.4 times, respectively (Figure is reprinted with permission from Elsevier).

Looking at other research papers regarding the use of bubbles in the energy storage system, Yang et al. [7] carried out a complex experiment to investigate the impact of bubble injection in subcooled liquid erythritol to improve the nucleation of erythritol crystals process and the resulting supercooling temperature reduction. They found that using the bubble could decrease the subcooled temperature by 5 degrees, which was a remarkable result compared to other methods presented, such as ultrasonication. Also, the latent heat of crystallization increased by 50% due to the bubble injection effect. They demonstrated that using the bubble was a highly efficient and affordable method that would be convenient for energy storage applications.

Another study was done using bubble injection into the PCM, which is applied and integrated with a photovoltaic/thermal panel [477]. The study compares three cases of PCM with bubble injection (PCM-WB), PCM without bubble injection (PCM-WOB), and water-filled PVT unit. The bubble injection case exhibits a more uniform temperature distribution, resulting in a decrease in the average temperature on the PV surface, thereby enhancing the overall electrical and thermal energy generation of the PVT panel.

The bubble injection technique has also been employed in humidification and dehumidification (HDH) desalination systems. Aref et al. [478] investigated the improvement rate of such a system equipped with a heat pipe solar collector under bubble injection. This research used a novel combination of the closed-loop heat pipe and bubble basin HDH. This project aimed to study the effect of parameters such as solar radiation and the mass flow rate of bubbles on the desalination system performance. The study demonstrated that increasing the mass flow rate of bubbles from 0.002 to 0.005 kg/s improved the overall efficiency of the system from 15% to 57%. Additionally, Cui et al. [479] reviewed several topics related to improving the performance of membrane processes in the context of bubble injection applications.

Table 21: Summary of research on bubble injection in special applications.

Authors	Geometry	Working fluids	Bubble injection descriptions	Conditions	Remark
Baliño et al. [470]	A circular cylinder in crossflow	NaOH- Hydrogen	A cylinder is used as an electrolyzer to produce hydrogen bubbles in the flow. The DC control controls the bubble production intensity.	$q''=[0-600]$ (W/m ²) $J_E=[0-2]$ (A/m ²) $T_w-T_B=[0-70]$ (°C) $U_{bu}=[6.4-10]$ (cm/s)	Attainable heat flux increases as the hydrogen generation rate increases. The heat transfer coefficient is a function of the fluid temperature, cylinder temperature, and current density. Increasing the bubble generation rate increases the heat transfer coefficient.
Mahood et al. [471]	Three-phase direct contact heat exchanger	Pentane vapor- Water	Pentane vapor is injected into a three-phase direct contact heat exchanger through a sparger. Different flow rates and temperatures affect the heat exchanger efficiency, and costs are studied.	$\dot{m}_w=[0.056-0.0381]$ (kg/min) $\frac{\dot{m}_{bu}}{\dot{m}_w}=[0.039-0.066]$ (-) $T_{bu}=[40,43.5,47.5]$ (°C)	An increase in heat exchanger efficiency is a function of the bubble flow rate and a weak function of the inlet temperature. By injecting bubbles with 47.5 °C, the effectiveness of the heat exchanger increases from 25% to 75%. The heat exchanger cost is reduced to 30 times compared to the base shell and tube heat exchanger.
Zavaragh et al. [469]	Internal combustion engine	Air-Water	A compressor injects air into a diesel engine at constant pressure and different flow rates. The engine speed was also variable in the test.	$Q_{bu}=[10-30]$ (LPM) $P=[1.75,1.6]$ (bar) $RPM=[1500,2500]$ (rpm)	Fuel consumption decreased by 5.69%. NO emission decreased up to 198.43%. Engine warm-up time decreased by 33.33%.
Khwayyir et al. [468]	Flat-plate solar collector	Air-Water	Air is injected into the pipes of a Flat-plate solar collector with three different flow rates, and	$Q_{bu}=[0,1,1.5,2]$ (LPM) Variable environmental conditions	Heat exchanger efficiency increases up to 69.2% for the flow rate of 1.5 LPM.

			the panel efficiency is compared to the base case (natural convection in the tubes). Furthermore, the performance of the collector is evaluated during the day.		Nusselt number increases up to 283% for the flow rate of 2 (LPM).
Mohammed et al. [472]	Flat-plate solar collector	Air-Water	Like Khwayyir [468], the heat transfer in the pipes is forced convection.	$Q_{bu} = [0, 1.5, 2, 2.5]$ (LPM) Variable environmental conditions	Thermal efficiency increases to 5% compared to the case without bubble injection (both cases are forced convection).
Vasilev et al. [473]	Microfluidic heat exchanger	Air-Water	The bubble is injected into a microchannel with constant heat flux boundary conditions to study the heat transfer, pressure drop, and thermal resistance. This research focuses on the gas fraction in the two-phase flow. The results are compared to the single-phase flow.	Gas fraction = [4 to 18] (%) $[Re_w, Re_{bu}] = [100, 1500]$ (-) $[U_w, U_{bu}] = [0.1, 1.5]$ (m/s)	The optimum gas fraction is 12%. The Nusselt number increases up to 1.54 times compared to the single-phase flow. The thermal resistance decreased by 1.67 times compared to the single-phase flow.
Vasilev et al. [474]	Microfluidic heat exchanger	Air-Water	Similar to Vasilev et al. [473]	Gas fraction = [10, 20, 30] (%) $[Re_w, Re_{bu}] = [100, 1250]$ (-)	The Nusselt number increases up to 1.67 times compared to the single-phase flow. The thermal resistance decreased by 1.7 times compared to the single-phase mode.
Choi et al. [475]	Latent heat thermal energy storage system	Air-PCM	The bubble is injected at a fixed flow rate in a tank containing PCM and five copper tubes of hot water with a fixed flow rate (from 4 holes with 1mm diameter). The study investigates the effect of bubble injection on PCM charging time in a Latent Heat Energy Storage System.	$Q_{bu} = 0.1$ (LPM) $Q_w = [0.2, 0.4, 0.6]$ (LPM) $T_w = 75$ (°C)	The charging time was reduced by 53% compared to the case without bubble injection. The energy storage rate increases up to 218%, and the PCM liquid phase velocity increases by 638% compared to the case without bubble injection.
Choi et al. [8]	Latent heat thermal energy storage system	Air-PCM	Similar to Choi et al. [475], this research focuses on studying the effect of bubbles on discharge time.	$Q_{bu} = 0.1$ (LPM) $Q_w = [0.2, 0.4]$ (LPM) $T_w = [5.0, 15.0]$ (°C)	The discharge time decreases by 29%. The bubble injection required energy is only 1.42% of the total released energy.
Choi et al. [476]	Latent heat thermal energy storage system	Air-PCM - TiO ₂ nanoparticles n-octadecane (CH ₃ (CH ₂) ₁₆ CH ₃) as the PCM	A predetermined mass of TiO ₂ nanoparticles is added to a melted PCM and sufficiently agitated. Oleic acid is used as the dispersant.	$D_{noz} = 1$ (mm) $D_{bu} = 2 - 3$ (mm) $\dot{m}_{bu} = 1.4 \times 10^{-3}$ (kg/s) $m_{PCM} = 0.56$ (kg) average nanoparticle size = 20 (nm) nanoparticle concentration = [0.5 - 5.0] (wt%)	The inclusion of nanoparticles alone led to a 1.09-fold enhancement in average charging power, whereas the implementation of bubble-driven flow alone increased it by a factor of 1.27. When both bubble-driven flow and nanoparticles were combined, there was a 1.41-fold increase in average charging power.
Yang et al. [7]	Medium-temperature thermal energy storage	Nitrogen-Erythritol	Nitrogen bubbles are injected to decrease the subcooling temperature of erythritol with different flow rates to investigate the effect of bubble diameter.	$Q_{bu} = [0, 0.02, 0.06, 0.1]$ (LPM) $D_{bu} = [0.5, 1, 3]$ (mm) $\Delta T_{subcool} = [8-108]$ (°C)	The subcool temperature decreases by 5°C. The latent heat of crystallization increases up to 50%.

Choi et al. [477]	Integrated PVT-PCM system	Air- water or PCM n-docosane as the PCM	The effect of bubble injection on the thermal panel of a PVT field with PCM on temperature distribution and energy generation is investigated and compared with PCM without bubble injection and in traditional water field thermal panel cases. The experiment was conducted in two phases for a total of 300 minutes, half for charging and half for discharging.	average irradiation= 800 W/m ² maximum temperature of the PV panels is below 65 (°C) $\dot{m}_{air} = 9.15 \times 10^{-3}$ kg/s $D_{noz} = 1$ mm $D_{bu} = [3 - 4]$ (mm) $T_{HTF,initial} = 28$ (°C)	In the case of bubble injection, the maximum temperature rise at the back surface of the PV panel witnessed a reduction of 48%. The total energy efficiency saw a notable increase, with a 22% improvement compared to the PCM-WOB case and a 33% enhancement when contrasted with the water-filled cases.
Aref et al. [478]	Solar humidification/ dehumidification desalination unit	Air-Water	The bubble is injected into a solar HDH desalination system basin to increase the heat transfer rate.	$Q_{bu} = [2,3,4,5] \times 10^{-3}$ (kg/s) $T_{bu} = [24,31,38]$ (°C) Variable environmental conditions.	Increasing the bubble injection flow rate increases the system's overall efficiency from 15% to 57%.
Liu et al. [480]	Ice thermal storage based on micro heat pipe arrays (MHPA-ITS)	Water/ice Ethylene glycol solution as HTF	The study investigates the thermal characteristics of the MHPA-ITS device utilizing bubble-driven flow to enhance the convection heat transfer.	$M_{water} = 6.5$ kg $D_{noz} = 0.5$ mm $Q_{bu} = [2, 4, 6, 8, 10]$ (LPM)	The operation strategy of turning on bubble flow during the discharging process (because of a 20% reduction of the discharging time and a 31.07% enhancement of the discharging power) and turning it off during the charging process (because of a 19.46% enhancement of charging time and a 17.91% reduction of charging power) is recommended.

7. Conclusions, research gaps, and future perspective

The review analyzed the physics behind the injection of bubbles into liquids to enhance heat transfer in thermal systems, particularly in various heat exchangers. It aimed to provide a comprehensive understanding of the complex physical phenomena involved in bubble-induced heat transfer enhancement, including bubble coalescence, bubble breakup, and bubble-induced turbulence, as well as heat and mass transfer between the bubbles and liquid and the effects of various parameters such as bubble size, injection rate, and liquid properties. This review identified knowledge gaps and future research directions to improve the efficiency and applicability of bubble-induced heat transfer enhancement in various industrial and engineering applications by critically examining the current state of research in this field. The most important notes of this review can be summarized as follows:

- The thermal boundary layer regulates heat transfer in single-phase flow. However, the heat transfer rate in turbulent flow is controlled by bubble-induced mixing, agitation, and turbulence when bubbles are injected into a liquid.

- Bubbly flow is the most effective flow regime for enhancing heat transfer through bubble injection.
- The effectiveness of heat transfer through bubble injection decreases when transitioning from bubbly flow to slug flow. Slug flow can increase pressure drop and reduce the heat transfer coefficient, making it less suitable for enhancing heat transfer.
- The heat exchanger inclination angle can influence the flow regime, but the impact of the injection flow rate on the Nusselt number is more significant.
- The presence of bubbles was found to promote the formation of thin liquid films on heat transfer surfaces, significantly reducing thermal resistance and improving heat transfer efficiency.
- At high flow rates and in flow regimes such as slug and annular regimes, bubble injection can isolate the surface from the flow, reducing the heat transfer coefficient and increasing pressure drop.
- Bubble injection has a greater impact on laminar flow and free convection flow than in a turbulent flow regime. Nonetheless, it still has a significant effect on the latter. In turbulent natural convection ($Ra \geq 10^9$), the Nu number is mainly a function of void fraction and independent of Ra number ($Nu \propto \alpha^n$).
- The studies reveal the emergence of the k^{-3} scaling in the energy and velocity fluctuation spectrum for bubble-induced flow. This scaling differs from the universal scaling observed in high Reynolds turbulent flow, which follows a $-5/3$ scaling (Kolmogorov–Obukhov–Corrsin theory).
- The shape and diameter of bubbles significantly influence the heat transfer rate. Deformed bubbles have a more pronounced effect on mixing and heat transfer compared to ellipsoidal bubbles, primarily due to the enhanced agitation and vortex shedding they induce, especially near the walls. Bubble size is also a critical factor in improving both mixing and heat transfer, driven by lateral bubble motion caused by lift force reversal and the wake generated by the bubble. The lift reversal phenomenon is particularly important, as it not only impacts heat transfer within the flow but also plays a key role in the lateral distribution of bubbles.
- Bubble injection can enhance the efficiency of various systems beyond heat exchangers, such as solar collectors, hydrogen production processes, energy storage, and internal combustion engines. Bubble injection offers a notable advantage, making it valuable for industrial applications. Thanks to the mixing produced by bubbles, its

capability to generate a more homogeneous temperature distribution over a surface is one of its most significant benefits.

- An empirical Nusselt number correlation for shell and helically coiled tubes under bubble injection was established based on experimental data from various studies (see Eq. (97)).

Fig. 65 provides an overview of the physical phenomena and significant parameters crucial in the bubble injection process. Here is a short description of each parameter.

- Liquid Properties:

Effect: Viscosity and surface tension fundamentally influence bubble formation, stability, and dynamics, consequently affecting heat and mass transfer rate.

- Bubble-Induced Turbulence:

Effect: Turbulence generated by bubbles can significantly enhance mixing and interaction between phases, directly impacting heat and mass transfer rates.

- Bubble Size:

Effect: The size of the bubbles determines the surface area available for heat and mass transfer. For a fixed airflow rate, smaller bubbles provide a larger surface area-to-volume ratio, hence a higher heat transfer rate.

- Injection Rate:

Effect: The rate at which bubbles are injected can influence the density and interaction rates of bubbles within the fluid. Therefore, the injection rate impacts the efficiency of the transfer processes.

- Bubble Breakup:

Effect: bubble breakup typically improves heat transfer by increasing surface area, turbulence, and mixing efficiency.

- Geometric Factors:

Effect: The geometry of the system affects flow patterns and contact times. While important, its impact is more indirect compared to direct bubble dynamics.

- Bubble Coalescence:

Effect: Bubble coalescence can increase heat transfer by enhancing turbulence and mixing or decrease it by reducing surface area and small-scale agitation. Its effect depends on bubble size, flow regime, and wall proximity.

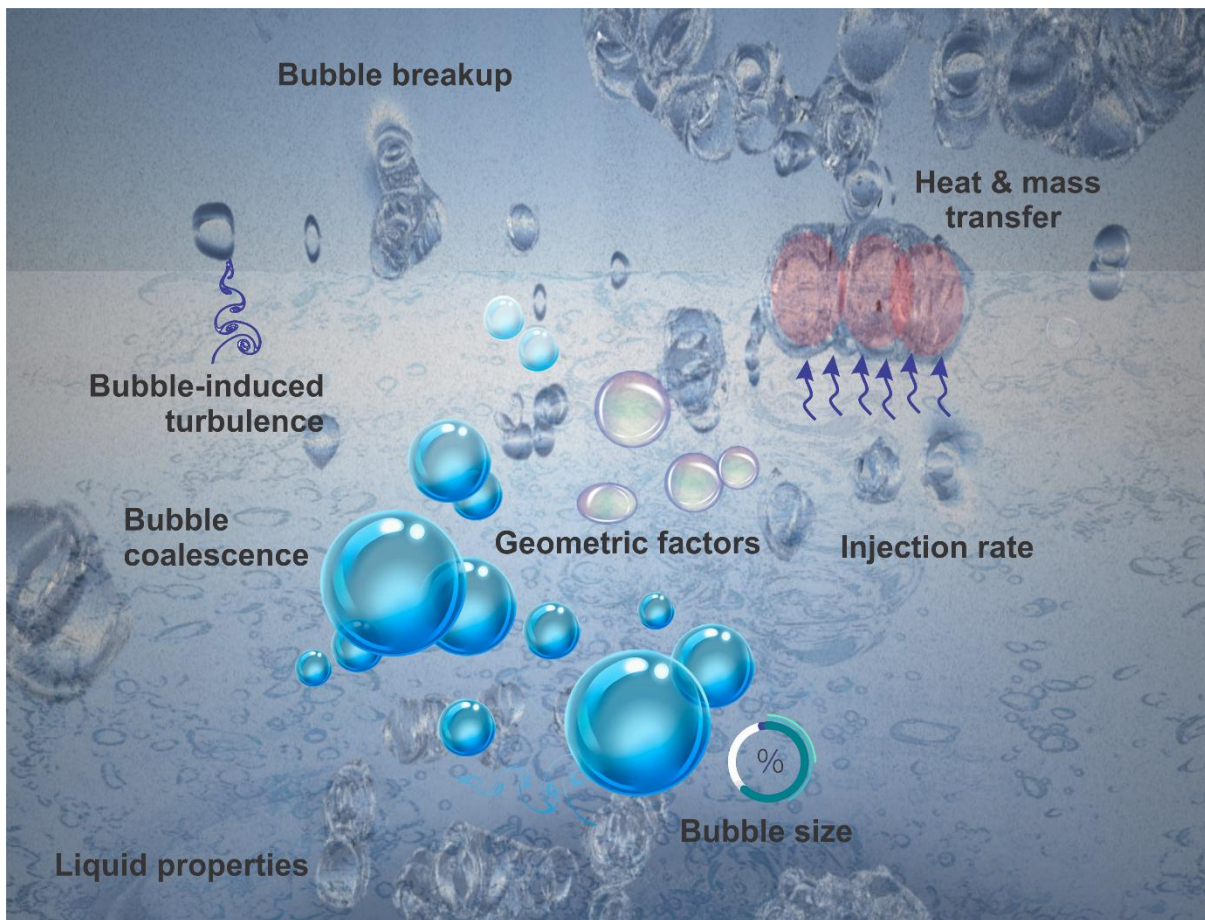


Fig. 65: Illustration of physical phenomena and important parameters involved in the bubble injection process.

Some of the research gaps and suggestions for future works are listed below:

Bubble generation method and bubble sparger: Bubble diameter and the generation method are paramount in enhancing heat transfer rate and, hence, in heat exchanger design. While the performance of various spargers has been studied in certain bubble columns, a noticeable research gap exists in the effective design of diverse bubble spargers for optimizing the efficiency of heat exchangers. So, developing new high-performance bubble spargers that control the bubble diameter and frequency is recommended. Further investigation is also needed regarding the location of bubble injection and the injection angle.

Lack of experimental studies on most common heat exchangers: Limited studies have explored using bubble injection to enhance the efficiencies of most used heat exchangers, such

as plate heat exchangers, double pipe heat exchangers, solar collectors, etc. Therefore, it is crucial to undertake additional research to optimize these heat exchangers and elevate their overall efficiency.

High-resolution measurements in turbulent bubbly flow: Reviews of available studies showed that collecting high-resolution data in turbulent bubbly flows is challenging. For example, the thermistors employed in certain experiments could not resolve the higher frequencies inherent in bubble-induced turbulence. In bubbly flows, the dissipative scales are usually smaller than the resolution of the available experimental methods.

Limited understanding of bubble formation and interactions: Although bubble injection has been extensively studied, there is still an incomplete understanding of the precise mechanisms of bubble formation, how they vary across different fluids and injection methods, and how bubbles interact with each other or surfaces. Therefore, future research could focus on elucidating these mechanisms to optimize bubble production and stability.

Bubble collapse effect: Bubble collapse can generate high-energy shockwaves that can damage the thermal system, but little is known about its influence on bubble injection systems. Future research could explore the potential of bubble collapse in enhancing or disrupting bubble injection systems.

Coupled processes: Bubble injection can be coupled with other processes, such as chemical reactions, and creates complex flows, or at the same time, it can be used with other passive/active techniques for heat transfer enhancement. Such complex flows could be investigated in future works.

Economic analyses: In summary, both bubble injection and the integration of supplementary methods, like the use of inserts and nanofluids, have the potential to enhance heat transfer. However, it is crucial to weigh this against the impact of an increased pressure drop, which could result in higher power consumption. Therefore, exploring the economic aspects of heat exchanger design is imperative.

Advancing CFD techniques for heat exchanger modeling:

There are CFD techniques for simulating two-phase bubbly flow, such as VOF, Level-Set, Phase field, and coupling of VOF and Level-Set. While heat transfer has been extensively

explored in single-phase flows, the numerical examination of heat transfer in two-phase bubbly flow remains relatively scarce. For instance, no numerical simulation has been conducted on shell and helically coiled tube heat exchangers with bubble injection. This scarcity is mainly attributed to the challenging nature of simulating 3D turbulent two-phase bubbly flows, which requires an extensive computer infrastructure with high computational capabilities. This research gap represents one of the most significant challenges in the existing literature. Future developments in CFD techniques for two-phase bubbly flow should focus on enhanced interface capturing, coupled Multiphysics models, and high-fidelity turbulence models to improve accuracy and efficiency. Efficient computational algorithms and robust validation frameworks using experimental data are essential. Integrating machine learning and developing user-friendly software tools would further advance the field.

Machine learning for bubble size control: Machine learning is a hot topic of research that can be employed for more precise control of bubble size, flow rate, and injection location. Future research efforts in these areas are recommended.

Surfactant effects: While surfactants can reduce surface tension and stabilize bubbles, their effects on the behavior and stability of bubbles in injection systems are not yet fully understood. Further research could explore the potential of various surfactants in optimizing bubble injection systems.

Non-Newtonian fluids: Although many industrial fluids are non-Newtonian, little is known about how bubble injection affects their behavior. Future research could explore the potential of bubble injection in optimizing non-Newtonian fluid systems such as paints and slurries.

Applications in renewable energy systems: Bubble injection has the potential to be applied in various renewable energy systems, including solar energy systems. Further research could be conducted to explore the practical applications of bubble injection in these systems.

Lack of standardized testing methods: The lack of standardized testing methods for evaluating bubble injection systems creates challenges in comparing results across different studies. Therefore, future research could focus on developing standardized testing approaches to enhance consistency and repeatability across studies.

Experimental visualization challenges: Dealing with bubbly flows poses certain challenges in measurement. Visualization approaches, such as high-speed imaging or particle tracking,

can offer valuable insights into the complex behavior of bubble injection systems. However, when employing Particle Image Velocimetry (PIV) for bubbly flows, there are issues with the strong absorption of penetrating laser light sheets due to scattering on the bubbles' surfaces, particularly at higher void fractions. Another challenge in bubbly flows with higher void fractions is the increased likelihood of overlapping bubble images. Further research could be done to develop new, non-invasive monitoring methods that enable real-time monitoring of bubble injection processes.

Scaling up: Bubble injection is typically utilized on a small scale, which may not accurately reflect the system's behavior at larger scales. Further research should focus on scaling up bubble injection systems and investigating the challenges associated with implementing bubble injection at larger scales to meet the demands of industrial or commercial applications.

Fig. 66 presents the challenges and gaps in bubble injection research, as discussed above.

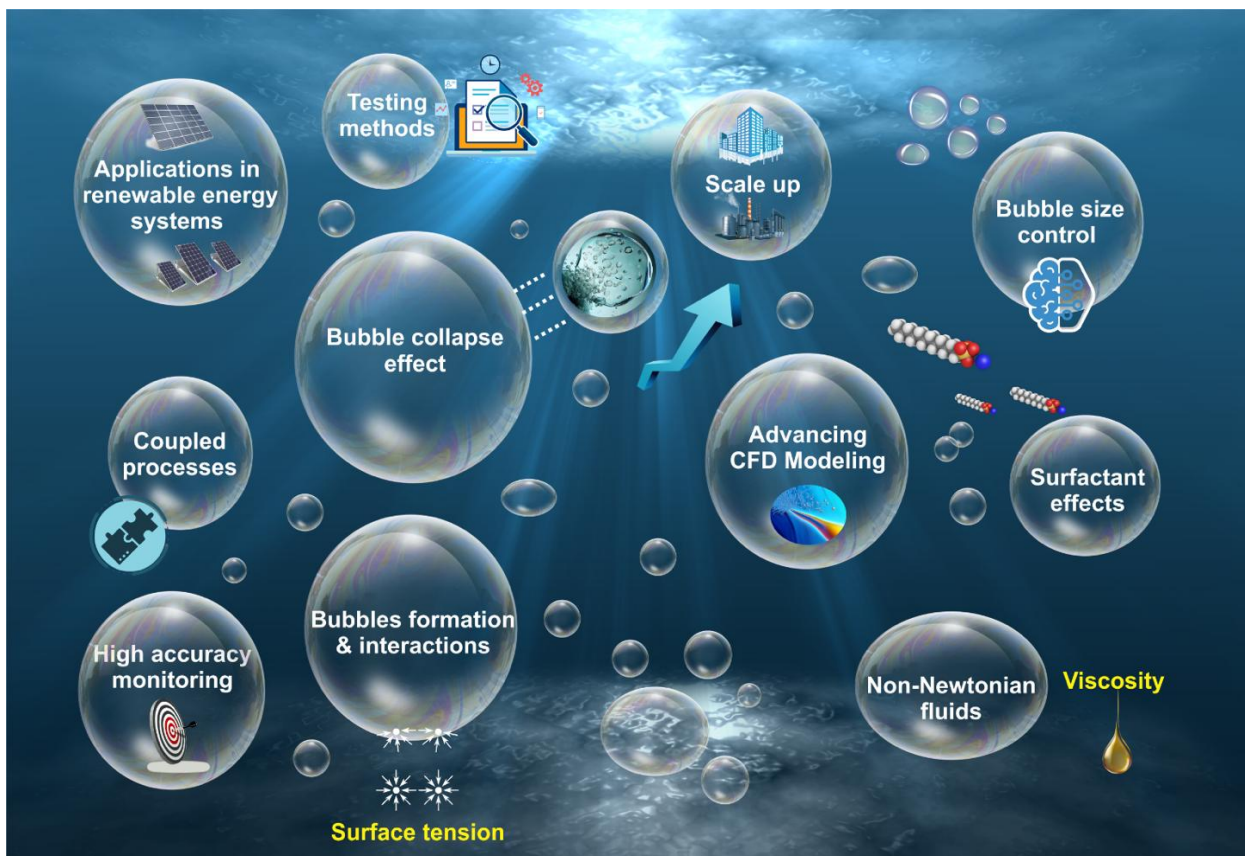


Fig. 66: Challenges and research gaps in bubble injection research.

Finally, focusing on industrial applications, Fig. 67 summarizes the key challenges of using bubble injection in heat exchangers and devices. These include difficulties in determining heat

transfer coefficients, issues with segmented flow reducing performance, challenges in bubble generation methods, and equipment reliability and maintenance. Practical barriers, such as applying the method to complex systems like combustion engines and the lack of economic analyses, are also highlighted. Overcoming these challenges is crucial for improving the efficiency and practicality of bubble injection in industrial systems.

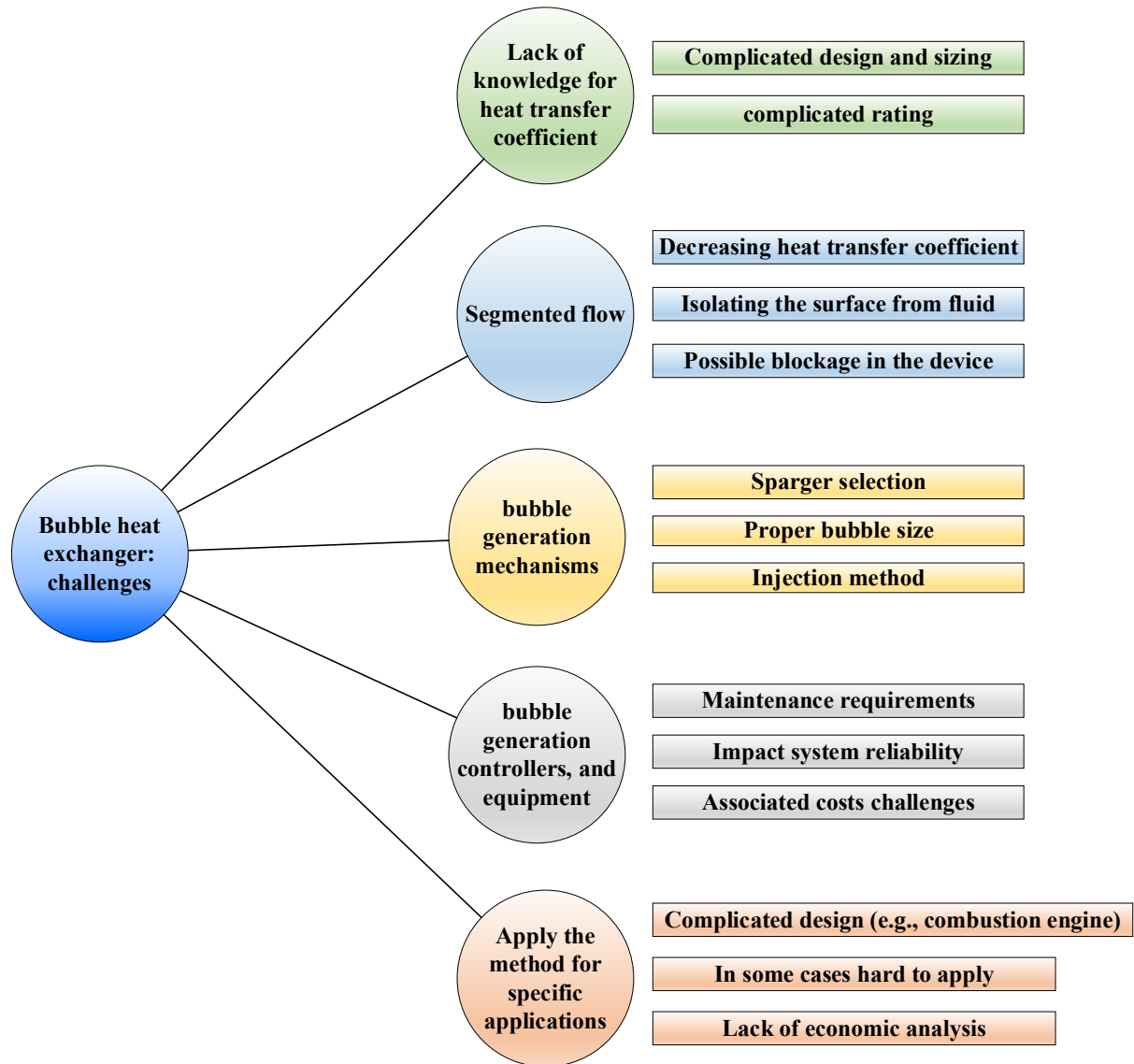


Fig. 68: Main challenges related to the bubble injection in heat exchangers and devices.

In conclusion, this paper aimed to enhance the understanding of bubble dynamics, focusing on their formation, interactions, and impact on heat transfer. By addressing key aspects of bubble behavior, this study provides a foundation for future investigations to overcome existing challenges and expand the potential applications of bubble injection.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGPT in order to improve the writing and English of some sections. After using this tool/service, the authors reviewed and edited the content as needed and they take full responsibility for the content of the publication.

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