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An Airline Fleet Planning Model

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SYNOPSIS

The purpose of this project is to provide airline corporate planners with an additional model to assist them in their difficult task of aircraft fleet planning. An analysis of the airline planning process is first embarked upon in order to identify its fundamental elements. These are then examined in greater detail to provide a framework around which the performance of the many models produced by operations researchers can be compared with the requirements of the airline fleet planners.

The model which evolved is in two parts. The first part, the Master Program, analyses the airline's operation over the whole of the planning period and takes into account all the financial constraints which the planner wishes to satisfy. It ensures continuity in fleet composition and does not retire the existing fleet before it is economical to do so. This model uses a Dynamic Programming procedure which maximises an objective function subject to the above constraints.

Linked to this program is a one year assignment algorithm which allocates aircraft to routes such that the demand, frequency, and technical constraints are satisfied and operating profit maximised. To ensure that the model is economical to use, in terms of computer processing time, the algorithm is based on a heuristic optimisation procedure.

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INTRODUCTION

The predicted or desired increase in traffic and the wish of airlines to meet future demand in order to maintain, or increase, their share of the market impose on them a growth rate that is beginning to cause difficulties. Aircraft prices are increasing and thus the airlines have to contend with substantial investments at the very time when their profit margins are low and the money markets strained. They have to choose new aircraft to be used for ten years, or more, along with old equipment which was bought at different dates and must be gradually retired.

Many of the models developed so far to assist airline management in their difficult task of planning future fleet compositions [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11] generally involve an analysis of one year's operation with the objective of maximising profits or minimising costs, such that the demand, frequency and technical constraints are satisfied. Unfortunately this approach is too simple for several reasons. These can be classified under two general headings: 'Long term profitability' and 'Financial constraints'.

Long Term Profitability

Which aircraft types to acquire is only one of the major decisions to be taken by management; others are how many and when, and which retirement policy should be followed. Furthermore, management has to decide whether to lease the aircraft from a financial firm, buy the aircraft outright, or follow a policy which includes both alternatives.

Many of the models so far developed have no safeguard against the possibility of the replacement of the existing fleet by a newer one when the updated difference between revenues and operating costs will not, in the long run, cover the additional investment for new equipment.

Financial Constraints

The fundamental financial constraint is that the capital available for investment in any one year is limited. This capital will depend on the accumulated profits over the planning period, the increase in long term capital, depreciation and repayment schedules, and on other capital commitments such as additional hangars and maintenance facilities.

Furthermore, the general economic climate, the financial position of the airline industry, and the tight money market, has made it very difficult, and expensive for many of the world's commercial airlines to obtain the long-term loans to pay for the new generation of aircraft. In 1969 Swissair had great difficulty selling convertible debentures on the Eurobond market. More recently, Pan American announced in its 1972 annual report that its new credit agreement with its bankers would carry more restrictive terms and higher interest rates than the line of credit arrangement it replaced. Besides facing greater restriction on the purchase of new equipment, it also had stiffer requirements on the maintenance of minimum financial ratios, and a minimum tangible net worth of \$350 m. The essential change in the agreement was that the lenders drew-up the conditions. In the past, the airline prepared the agreement and offered it to lenders on a take-it-or-leave-it basis.

It is, therefore, unrealistic to develop a model which solely tries to maximise some economic function with complete disregard of the financial constraints. This, unfortunately, is the common characteristic of the models developed to consider the effect of time, with the exception of the one developed by Peyrelevade and Gervais [5].

The model described in this thesis, consists of a Master-Program and a Sub-Program. The Master-Program, described in Chapter 2, analyses the whole of the planning period and takes into account all the financial constraints which the planner wishes to satisfy. It ensures continuity in fleet composition and does not retire the existing fleet before it is economical to do so. The model uses a Dynamic Programming

procedure which maximises an objective function, subject to the above constraints. In doing so, it will 'suggest' at regular intervals a fleet composition, calculate the fixed cost, which includes repayments to the manufacturers, and call-up the Sub-Program. This will test the feasibility of that fleet and calculate the associated operating profit.

The Sub-Program, described in Chapter 3, is a one year assignment algorithm which allocates aircraft to routes such that the demand, frequency, and technical constraints are satisfied and operating profits maximised. The great number of times that the Sub-Program is called-up has dictated the need for a fast heuristic method which will 'maximise' profits by finding a near-optimal solution.

The report begins with a description of the airline planning process in order to construct a framework on which the analysis can be based.

CHAPTER 1

THE AIRLINE PLANNING PROCESS

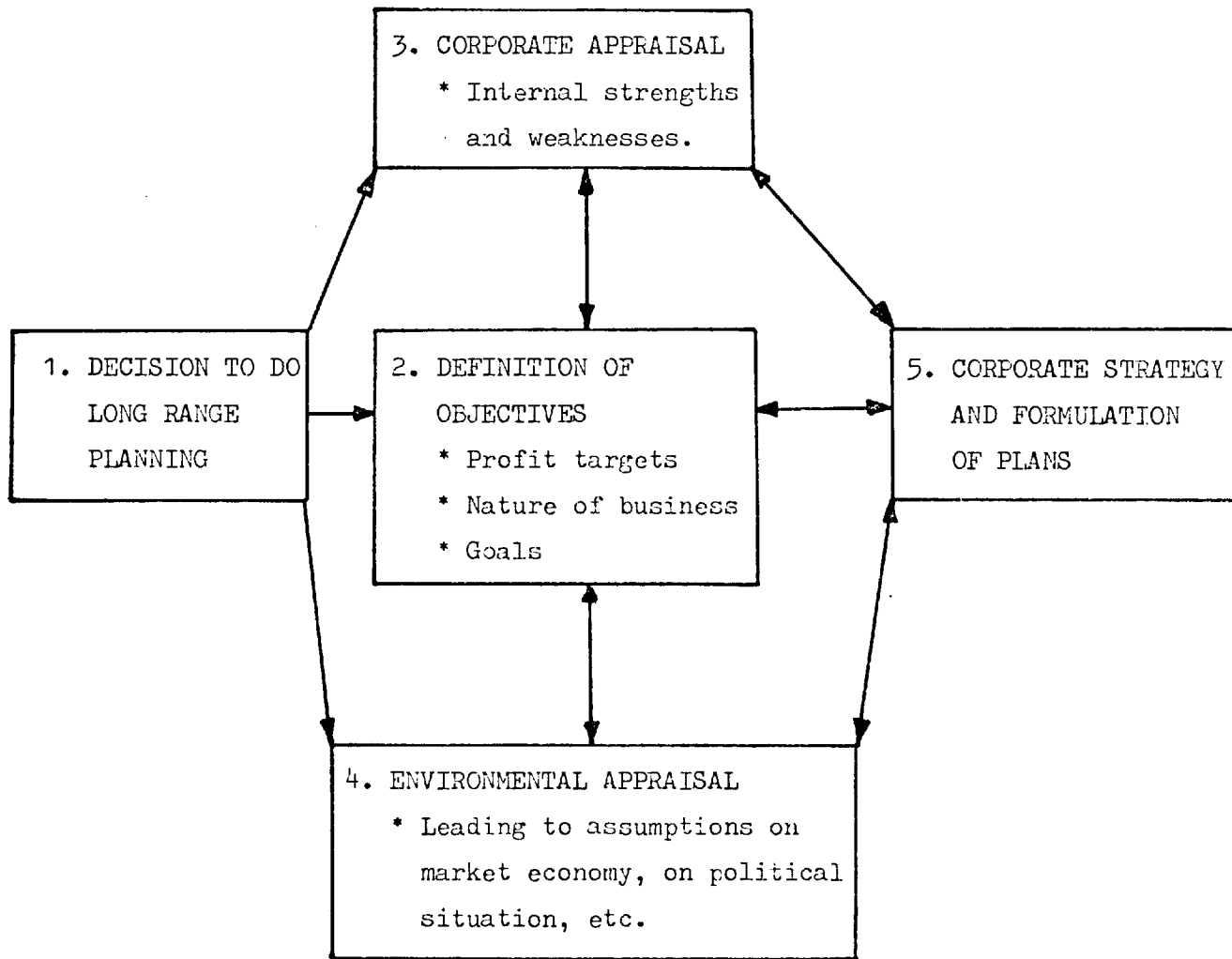
The development of an aircraft procurement/retirement model is based on the philosophy that it should be one of the many tools which can be used by airline management in their function of Corporate Planning. It is therefore necessary to examine the planning process in order to construct a framework around which the design specifications of the model can evolve, and also to pin-point the important variables through an analysis of their basic characteristics.

The Corporate Planning process, in general, begins with the definition of the firm's objectives and culminates in a strategy and a plan. During the process the firm's strengths and weaknesses are examined and assumptions are made about the operating environment. It is not a simple 'Beginning, middle and end' process, but an iterative one in which a re-examination of the various functions and constraints is repeated several times (See Fig. 1.1).

They are, however, very few industries in which this simple planning process can be translated into actual practice with the ease with which it was devised - the air transport industry is no exception. The airline planning process is intricate due mainly to the diversity of the objectives and to the great number of variables involved. It will, nevertheless, be useful to work through a simplified version of the planning process in order to become familiar with the main issues.

The initial phase of the process is the definition of the objectives which the plan has to satisfy. There are several types of objectives; some are essentially broad in character such as 'diversification into the Inclusive Tour Market', or if the airline is state controlled 'to maximise earnings of foreign exchange'; others are more specific such as 'maintaining net worth above \$350 m'.

FIG. 1.1 THE PLANNING PROCESS [12]



The second phase can be adequately described as 'Market Research'. Fundamental to this activity are the assumptions made. These relate not only to the markets to be served, such as their rate of growth and the extent of competition, but also to the possibility of changes in the political structure in different parts of the world and to the expected level of economic prosperity in the various countries. The political, legal and commercial controls, along with the airline's resources (actual or planned) will determine the share of the potential market forecast which the airline can achieve (see Fig. 1.2). This is, in effect, the airline's required 'Production'.

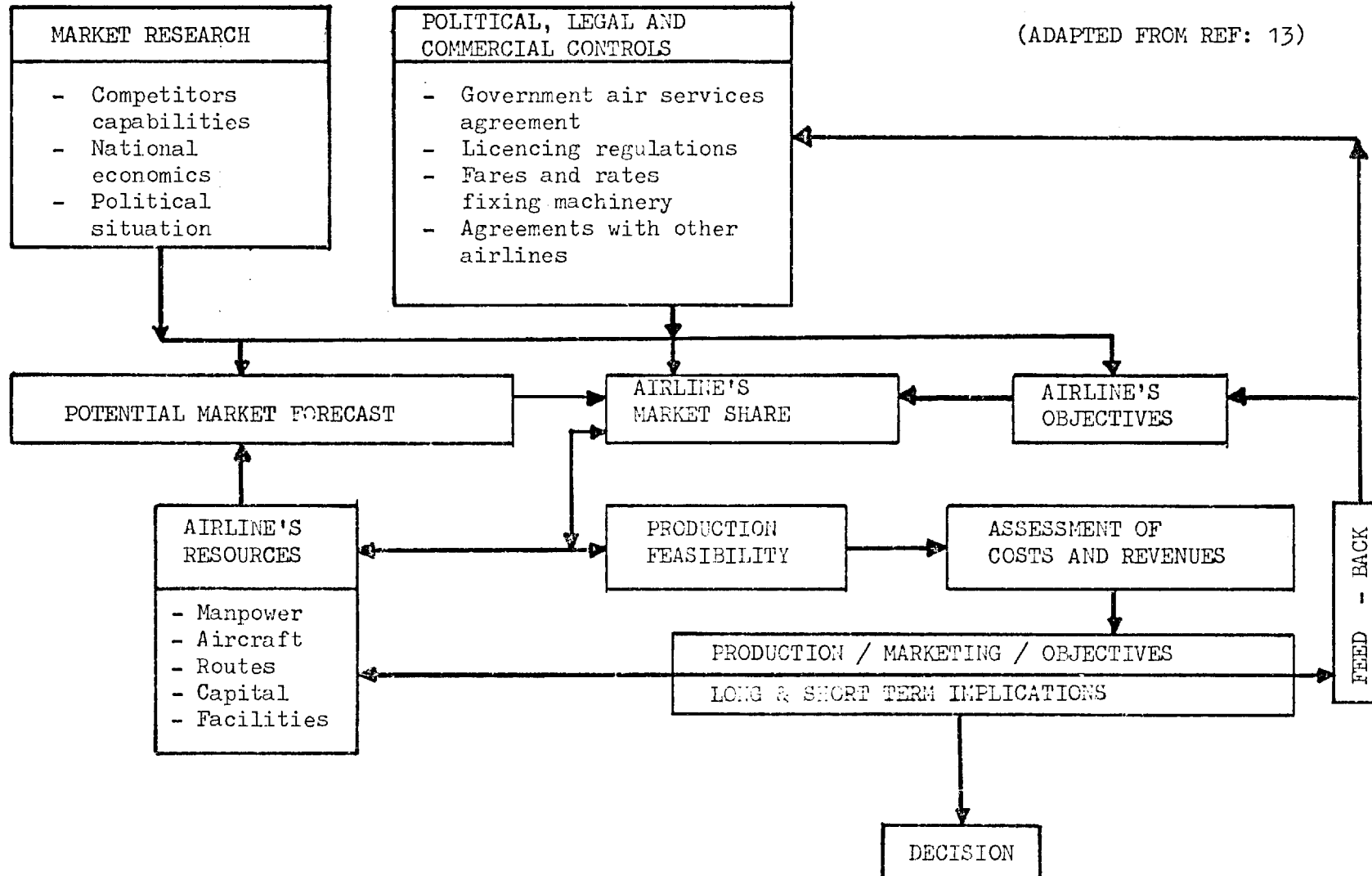
The airline has resources: aircraft, routes, capital, facilities and manpower. The Production Feasibility stage, which is basically the matching of a set of aircraft/route combinations to the demand pattern, tests to see whether these resources are enough to meet the requirements of the 'Production Schedule'. The analysis dictates the additions to the fleet that are required; and these, in turn, determine the additional spares, maintenance facilities, ramp equipment and manpower that are required. The analysis also determines the revenues and direct operating costs associated with the operation.

The operational financial results are then aggregated with the forecasts of the expenditure on the additional property and equipment and the additions to working capital, and are summarised in statements such as Profit and Loss, and Sources and Uses of Funds. The long and short-term implications of these results will lead to a reappraisal of the objectives and other decisions taken previously. The whole process is repeated until a satisfactory state of affairs is achieved.

One point clearly emerges at the end of this short description. The number of times that the process must be repeated in order to evaluate different marketing policies under a different set of assumptions about the socio-economic factors, is not small. Therefore, a model which is robust over a range of possible future states is of more use in Corporate Planning than one which has impeccable performance in just one.

FIG 1.2 AIRLINE PLANNING

(ADAPTED FROM REF: 13)



This initial description of Corporate Planning indicates the importance of the 'Definition of Objectives' to the process. In fact this is the most important problem to be resolved in the design of any model, and is now discussed in detail.

1.1. AIRLINE OBJECTIVES

'The great divide on aviation policy is between those who think that airlines are for airlines and the governments which sponsor them, and those who think that airlines are for customers'.

J.E.D. Williams 1968 [14]

The concept of civil air transport as a service is of course quite fundamental to the whole business. Most airlines would claim that its sole raison d'être was to serve the public; they might even go on to claim that any lack of profits observable in its finances was due entirely to the necessity of providing services needed by the public, but incapable of being provided on a profitable basis. But a service industry is still an industry; the airlines, state-owned or independent, are in business to make a commercial success of it. In that sense airlines are for airlines and it is simply confusing to pretend that they are not.

J.E.D. Williams's remark is based on the experience, which certain operators had had, that the constraints which kept them from traditional scheduled service operations forced them to innovations which arose from a study of customer needs. In the case of these airlines, being 'for the customer' was the best way of being 'for the airline'. One thing is certain; an airline that thinks of itself as 'for customers' will not be so for very long unless it is also just a little 'for itself'. In other words, unless capital is remunerated adequately and unless new capital can be attracted for the future the airline will not be able to go on serving the customer.

The ownership of airlines varies very much throughout the world; airlines range from completely private enterprises to those which are wholly state-owned, with many variations in between.

Some wholly state-owned airlines are very much instruments of government policy, 'carrying the flag' with cost not being the overriding factor, and allowed to suffer no competition from other airlines registered in the state. This may be for reasons of prestige, or in order to earn, or save, foreign exchange - a matter of real importance to any government over the years.

More typical of the wholly or partially state-owned international operator is one that, though it may have a monopoly or near-monopoly of the scheduled operations flown under its national flag, is at the same time expected to pay its way. Examples of major airlines in this situation are SAS, KLM and Swissair. The same is also true of the British State Corporation, British Airways. Nor is this situation confined to the international scene. British Airways in its domestic operations, Trans Australian Airways within Australia, and others, meet the competition of private enterprise and are expected to be profitable.

The problem arises when criteria - other than the strictly economic - are used in making judgments as to whether services should be operated or not. The most obvious case of this kind are seen in what are called the social air services in Great Britain. Similar circumstances exist in the U.S.A. and in Australia, where communities also exist whose demand for air transport, in money terms, cannot support air services.

If a decision is made to provide air services in these cases, then this is a political decision, whoever makes it. However, if the political and management decisions are kept separate, then a state-owned airline can be put on exactly the same footing as the others; all that is required is

- (a) The airline management must be told that it is to operate a commercial concern; and
- (b) If the state requires services to be operated that a commercial undertaking would reject, these particular services should be subject of special payment of compensation for the losses involved. In this way the needs of the community in question are assessed politically and met out of general taxation, and not by random cross-subsidisation from profitable services.

These are precisely the recommendations of the Edwards Committee on British Civil Air Transport i.e.,

'.... The primary long-term objective should be to satisfy the individual customer at the lowest price consistent with an economic return on investment ... short-term policy must, however, reflect the country's urgent balance of payment problem.

'.... If unprofitable services are required for reasons of public policy, ... and the support required goes beyond commercial justification, the finance should come from taxation and not from other airline customers.

'.... Support for the British aircraft manufacturing industry though important, should not be regarded as a prime objective of the civil air transport industry. If for balance of payments or other reasons civil airlines are required to act against their commercial judgment, it is the Government's duty to protect them from any economic consequences there may be'.

1.1.1 Objectives in Fleet Planning

The firm is a complex organisation of 'pressure groups', each with its own desires, which are often in conflict. Dissatisfaction on behalf of any of the parties will represent failure and the performance of management will be questioned. If one accepts 'survival', both of the company and of the management, as the major objective in fleet planning, then it is necessary for the airlines to act in a manner which will ensure both short-term and long term survival.

Since the fundamental function of an airline is the provision of a competitive air service at an economic return on investment, the most important management activities are those of finance and product planning.

The role of the product planning department is to determine the appropriate fleet mix for each year of the planning period which will satisfy its objectives. It does this by assigning aircraft to routes so that the expected demand is carried with a reasonable quality of service. On the other hand, the finance department has the task of working out a financing plan for the proposed investments that is not prohibitive in cost and is compatible with the various constraints that may be imposed on the airline by its lenders and shareholders. It is this department which in the end, accepts or rejects an investment project depending on whether it can finance it or not.

The Aircraft Procurement/Retirement model has been developed specifically to resolve the sometimes conflicting objectives of the various parties. The criteria and objectives of the finance and product planning departments, listed below, are examined in greater detail in Chapters 2 (The Multi-Period Model) and 3 (The One-year Model) respectively.

CRITERIA [11]

Maximisation of Profit

This can be expressed as either Return on Investment or Net Present Worth. This reflects the need for the airline to 'pay its way'.

Growth in Turnover

This criterion goes side by side with prestige and stability of the airline's position as regards competition and is an assurance of its long-term development.

Quality of Service

This quality, which is difficult to quantify, contributes to the airline's reputation and guarantees its medium and long-term markets. In general, this can be stated as the use of modern aircraft and adequate and well distributed frequencies.

Fleet Standardisation

This promotes convenience in maintenance, operation, and staff training; and finally cuts down unit costs.

CONSTRAINTS [11]

Meeting passenger and freight demand

The forecast demand can either reflect a constant market share or a new marketing policy (new aircraft, higher frequencies, etc. - additional constraints) to increase the market share. The operating schedule has to meet this demand, subject to a maximum permissible load factor on each sector.

Schedules constraint

The optimum allocation of aircraft is calculated on the basis that each aircraft type can achieve a given utilisation rate.

Fleet Constraints

An airline has old aircraft that can still be used and often cannot be sold. These aircraft must be used if new aircraft do not offer sufficient advantages to eliminate them. Furthermore, the expansion of a new fleet is constrained by the manufacturers' production capabilities, and by the fact that the number of new types of aircraft should not decrease in the initial years.

Technical constraints

Some aircraft are excluded by their performance from certain routes.

Financial Requirements

The fundamental financial constraint is that the capital available in any one year for investment in new aircraft is limited. Others include the financial ratios which must be maintained to satisfy the airline's creditors, and the growth in dividends to satisfy shareholders.

CHAPTER 2

THE MULTI-PERIOD PROBLEM

2.1 INTRODUCTION

The production planning department will usually present the finance department with a list of possible fleets for each year of the planning period. Each of these fleets will satisfy the demand and quality of service constraints and any other objective of the production department. Associated with each of these fleets is the generated operating profit (Revenue minus variable operating cost) and the function of the finance department is to find the optimal aircraft procurement/retirement policy subject to the constraints which affect its decision.

The Cash Constraint and the Objective Function

Suppose that there is an initial decision to have the fleet $\underline{F}_{k1}$ in the first year of the planning period, then the principal financial equation is:

$$R_{k1} - V_{k1} - FC_{k1} - P_{k1} + B_1 - B_0(1+i) - I_1 + I_0(1+j) + C_1 = 0 \quad 2.1$$

Where

$\underline{F}_{kt}$ is the k^{th} fleet composition suggested by the production planning department for possible use in year t .

R_{kt} is the revenue generated in year t should the K^{th} suggested fleet be in operation during that year; and is a function of $\underline{F}_{kt}$.

V_{kt} is the variable operating cost associated with the K^{th} fleet in year t and is a function of $\underline{F}_{kt}$.

FC_{kt} is the cash fixed operating cost, such as insurance and overheads but excluding depreciation, associated with the k^{th} fleet in year t , and is a function of $\underline{F}_{kt}$.

$$TC_{kt} = FC_{kt} + V_{kt}$$

P_{kt} is the repayments to the manufacturers in year t with respect to the present and past procurement/retirement policy and is, therefore, a function of $(\underline{F}_{kt}, \underline{F}_{a\ t-1}, \dots, \underline{F}_{b0})$

EQ_t is the addition in year t to the long term capital of the airline, raised through the issue of equity.

DV_t is the dividends to shareholder in year t , and is a management decision.

DB_t is the addition to long term funds in year t , raised through the issue of debentures.

RDB_t is the repayment of part of the debenture issue.

IDB_t is the interest payment in year t required to service the debenture portion of the long term capital.

$$C_t = EQ_t + DB_t - RDB_t - IDB_t - DV_t$$

B_t is the amount of funds borrowed for short term use in year t .

i is the rate of interest paid on the short term debt.

I_t is the amount of funds which can be invested in short term securities in year t .

j is the return from funds invested in short term securities.

The above equation states that the cash outflows is equal to the cash inflows, and assumes that the aircraft procurement/retirement policy taken in previous years is known. By re-arranging the above equation we get:

$$I_1 - B_1 = R_{k1} - V_{k1} - FC_{k1} - P_{k1} - B_0(1+i) + I_0(1+j) + C_1 \quad 2.2$$

Should the right-hand-side of equation 2.2 be negative then the airline cannot invest but will need to borrow in the short term. If the airline's objective is to maximise Net Terminal Value at the end of year N, then the problem transforms into one of finding the procurement/retirement policy which maximises $(I_N - B_N)$ subject to N constraints of the type shown in equation 2.1.

2.2 THE LIQUIDITY CONSTRAINT

The flow of funds during an operating cycle is only in parts made up of straight cash transactions. Accounts receivable, pre-paid expenses and accounts payable are some of the other forms in which revenues and costs will occur. This has two important implications. The first is that the principal financial constraint, given by equation 2.1 is no longer accurate; and the second is that the important relationship between fixed and working capital (current assets minus current liabilities) cannot be ignored. Inadequate working capital may mean that the fixed assets purchased from permanent capital cannot be utilised effectively. This will not only lead to disastrous liquidation or bankruptcy in the long term, but will in the short-term lead to loss of good-will and the restriction of credit from suppliers.

To determine the adequate level of working capital is not a simple exercise, but planners can be guided by the information on a Balance Sheet. Ratios can be calculated and, although "ideal" ratios are difficult to establish, those pre-determined and/or ascertained from actual figures can be extremely useful.

The two ratios most commonly used by financial managers are the current ratio (current assets/current liabilities) and the liquid ratio (liquid assets/current liabilities). The former is the one used in this thesis, although the latter can be easily introduced.

Both current assets and current liabilities can be sub-divided into two parts. The first of these is dependent on the airline's level of operation and is, therefore, a function of revenue and total operating cost; whilst the second depends on various items such as management policy of repayment of long-term debt, the airline's cash and profitability position and lastly the taxation environment.

Nomenclature

$VCA_t (R_t, TC_t)$ = Accounts receivable
 + Pre-paid expenses
 + Materials and supplies
 + Cash

CA_t Current assets = $I_t + VCA_t (R_t, TC_t)$

TX_t deferred income tax

$VCL_t (R_t, TC_t)$ = Accounts payable
 + Un-earned transportation revenue*

* When a passenger takes a trip which involves the use of more than one airline, he usually pays the price of the whole trip to the airline he flies first. So although that airline is paid for the whole trip, it can only eventually keep the revenue appropriate to the part of the trip flown by it.

CL_t Current liabilities = $B_t + RDB_{t+1} + TX_t + VCL_t (R_t, TC_t)$

$B_{\max}$ is the maximum level of the short term debt.

CR is the minimum permissible current ratio.

The Cash Constraint Re-Stated

The cash inflows are:-

$$IN_t = R_{kt} - VCA_t + EQ_t + DB_t + CA_{t-1} + (I_{t-1} \cdot j)$$

and the cash outflows are:-

$$OUT_t = TC_{kt} - VCL_t + DV_t + IDB_t + CL_{t-1} + (B_{t-1} \cdot i) + P_{kt} ;$$

and now three equations state the cash constraint for the year t, and these include the liquidity constraint:

$$I_t - B_t = IN_t - OUT_t \quad 2.3$$

$$B_t \leq B_{\max} \quad 2.4$$

$$(VCA_t + I_t)/CL_t \geq CR \quad 2.5$$

Since the flow of funds is not made up of straight cash transactions, the objective function must reflect this. It is, therefore, no longer adequate to state this as $I_N - B_N$, but it must include the other elements of current assets and current liabilities, i.e.

$$CA_N - CL_N$$

2.3 OTHER FINANCIAL CONSTRAINTS

A firm is judged on its performance by many financial ratios and in evaluating future investments financial managers would like to see the more important ratios predicted to within certain limits of the acceptable norms for the whole of the planning period.

Since the principle motivating force in the normal commercial business is profit, the most important ratios are those which relate to profit and more specifically to profit efficiency. The term used to describe this measure of efficiency is "the rate of return on capital employed".

"Capital employed" is used to describe the investment made in a business. Unfortunately, there is no generally accepted definition of the term, and the main three are:-

1. Gross Capital Employed
(Fixed assets + Current assets)
2. Net Capital Employed
(Fixed assets + Current assets - Current liabilities)
3. Proprietors' Net Capital Employed
(Total assets - Current liabilities - Long term debt - "Other outside funds")

The first of these is used in the model and, as before, the others could be easily accommodated at a later stage, if required.

The main variables which make up the Income and Earnings Statement (Profit) and the Balance Sheet are shown in Tables 2.1 and 2.2, respectively. Some of these need to be described in a little more detail.

Depreciation

Two depreciation schedules relating to aircraft, property and equipment are used in the model. The first determines the amount which must be deducted from revenue to obtain the Earnings Statement and is a straight line depreciation over a number of years to a residual value. The second is used to calculate the taxation element for a given year and assumes an initial allowance followed by a series of yearly allowances.

Interest on Loans

This quantity is made up of the interest paid on short-term borrowing plus that paid to service the long term debt, plus the interest element of the repayments to the manufacturers minus the return on short-term investments.

Notes Payable on One Year

The value of this variable for a given year t is the quantity of long term borrowed funds, e.g. debentures, which are scheduled to be repaid in the year $t+1$. This quantity is, of course, deducted from the total long term debt.

TABLE 2.1 INCOME AND EARNING STATEMENT

(1)	Total revenue	
(2)	Variable cost	
(3)	Fleet Insurance	
(4)	Indirect Operating Cost	
(5)	Operating Profit	$\{ (1) - [(2) + (3) + (4)] \}$
(6)	Aircraft Depreciation	
(7)	Property and Equipment Depreciation	
(8)	Profit on sale of Aircraft	
(9)	Interest on Loans	
(10)	Total Income	$\{ (5) - [(6) + (7) - (8) + (9)] \}$
(11)	Provision for Taxation	
(12)	Dividends	
(13)	Additions to Earnings	$\{ (10) - [(11) + (12)] \}$

TABLE 2.2 BALANCE SHEET

Fixed Assets

Aircraft at cost

less depreciation

Property and Equipment at Cost

less depreciation

Current Assets

Short term Investments

Others

Other Assets

Stock holders' Equity

Value of Stock

Retained Earnings

Long Term Debt

Current Liabilities

Notes Payable in 1 year

Short term debt

Provisions for Taxation

Others

Other Liabilities

2.3.1 General Evaluation Constraints

In addition to the financial constraints described above, others could also be included at a later stage to meet the requirements of the user of the model. Some of the more important ones are now stated:-

- a) Efficiency Ratio =
$$\frac{\text{Sales}}{\text{tangible operating assets}}$$
- b) Rate of Return on operating assets =
$$\frac{\text{Operating Income}}{\text{Operating Assets}}$$
- c) Rate of Return (before taxes) on investors equity =
$$\frac{\text{Income before taxes and interest}}{\text{Long term debt and stockholders' equity}}$$

2.4 AIRCRAFT PROCUREMENT/RETIREMENT CONSTRAINTS

When an airline proposes to modernise its equipment it already has in mind, in broad terms, a procurement/retirement schedule. The purpose of the model is to check the feasibility of the schedule and to make the necessary adjustments so that the operation can be performed in an optimal manner.

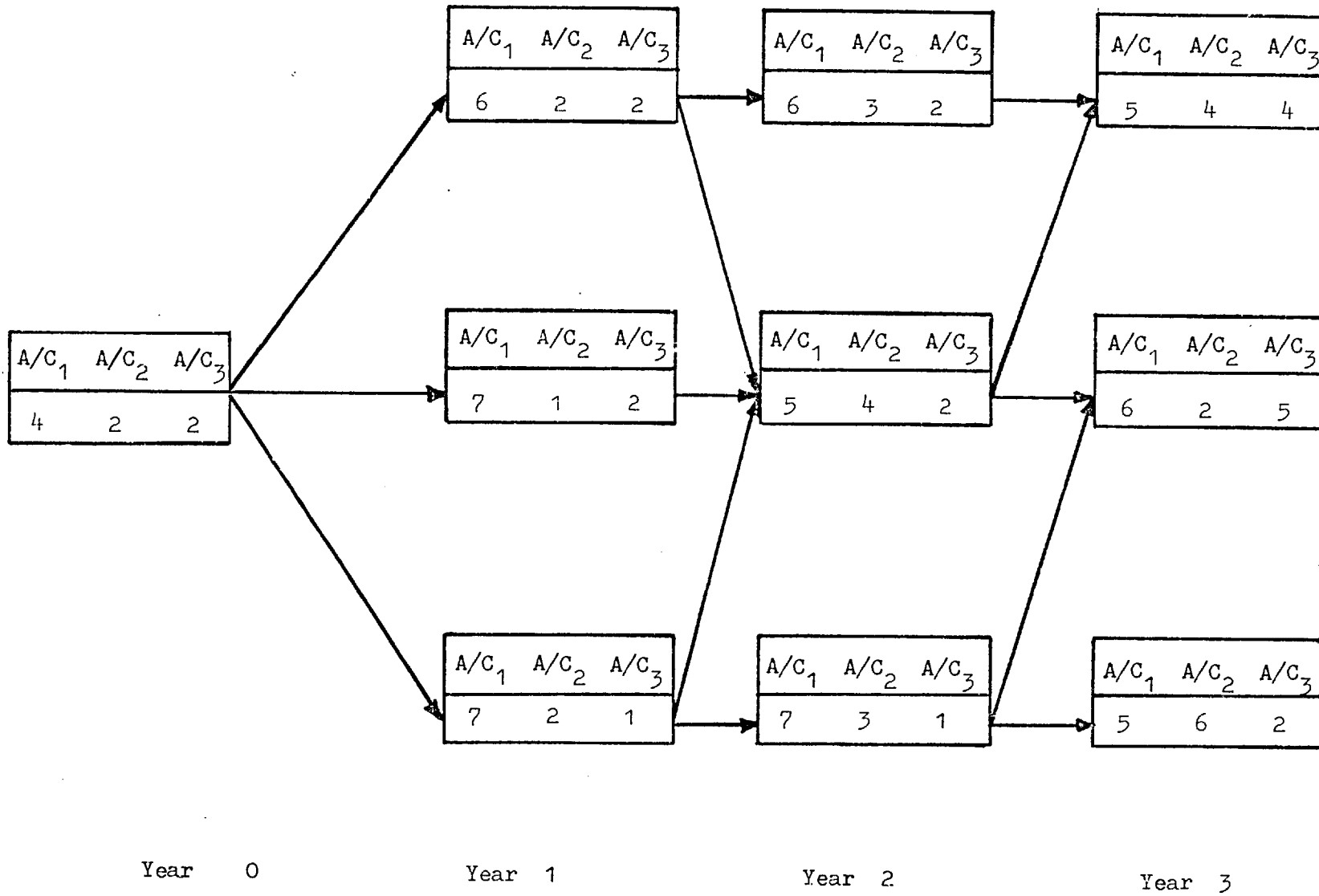
In the transition period crews and other personnel will have to be trained and additional facilities will have to be purchased to ensure a smooth change-over. Associated with this process are various cost items which would be difficult to enumerate and extremely arduous to introduce effectively to the model; for example aircraft which are purchased then sold only to be re-bought at a later stage. Therefore, in order to guard against such oscillations, a procurement/retirement constraint is introduced which ensures that the number of new aircraft types cannot decrease in the early years of the planning period and that the number of old types cannot increase. A facility is also available which limits the number of old aircraft in a given year to a specific value.

2.5 DYNAMIC PROGRAMMING SOLUTION TO THE MULTI-PERIOD PROBLEM

Consider the diagram shown in Fig 2.1, which depicts the list of alternative fleets which satisfy the objectives of the production planning department, and from which the finance department must choose one for each year of the planning period such that the aircraft procurement/retirement policy which evolves is an optimal one. The problem is a classical example of sequential decision making and is very similar to the shortest path problem discussed in Appendix A (Introduction to dynamic programming). As such, the technique of Dynamic Programming, which has been specifically developed for optimising multi-stage processes, is adopted as the logical solution to this problem. This is not the only justification for the use of this technique, as opposed to other mathematical approaches: it has the additional virtue of being able to cope with integer and nonlinear variables very easily.

Although the aircraft procurement/retirement problem and that of the shortest path are very similar, there are some differences. By the very nature of the problem the possible states form a sequential state-transition diagram, and there is therefore no need to test for sequentiality. Furthermore, there is no pre-specified "end-state" and, of course, the optimisation process involves a maximum rather than a minimum. However, the most important difference is in the number of state variables which can exist at each stage. In the shortest path problem the number was quite small; on the other hand, the number of fleets which will satisfy the requirements of the product planning department is very large. Hence, to apply the unmodified Dynamic Programming algorithm of Appendix A to solve the multi-period problem would be very expensive both in terms of computer processing time and in terms of computer storage space. It is therefore necessary to adopt the "Coarse Grid" [15] approach, which is an approximate method that overcomes the problem.

FIG 2.1 LIST OF POSSIBLE FLEETS



2.5.1 The Coarse Grid Approach To Dynamic Programming

An initial solution to the multi-period optimisation problem is obtained by finding the optimal procurement/retirement policy through a coarse grid of possible state variables. A "Fine Grid" is then generated around the previous optimal path and a new policy found. The process is repeated until two successive optimal paths are the same.

The risk involved in using a coarse grid is that a very sharp absolute maximum may be missed and instead a flat relative maximum is obtained (Fig 2.2). The possibility of having to cope with such drastic situations is, in practice, of course highly unlikely, especially in fleet planning.

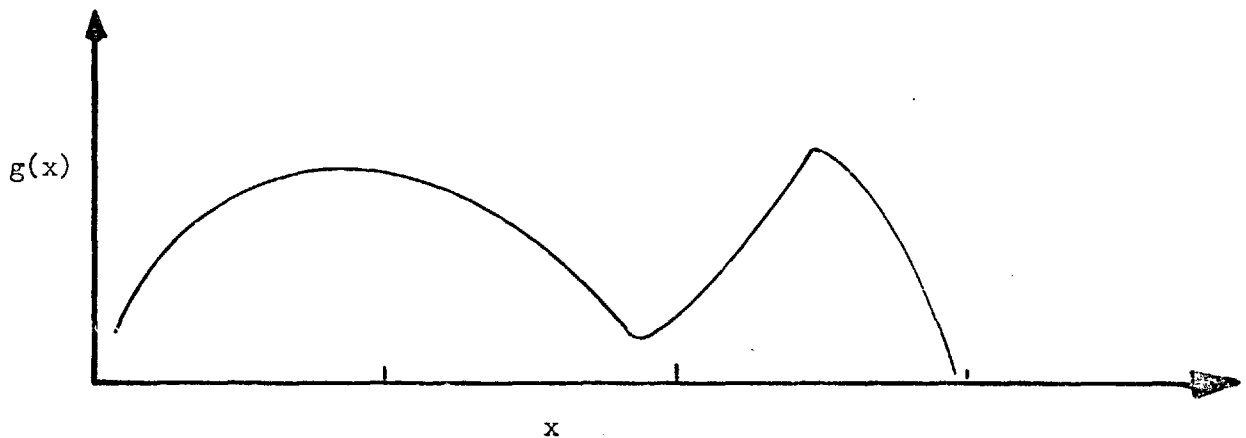


FIG 2.2

Generation of the Coarse Grid

The coarse grid of possible fleets is generated by the following procedures:-

- Step 1 Find the maximum number of aircraft which will be available for use in each year of the planning period. This maximum depends on the delivery programme of the manufacturers and on the airlines own retirement policy.

- Step 2 For each year of the planning period three groups of fleets are generated subject to the constraints of step 1. In each case each of the aircraft types are set in turn to a chosen level whilst the other aircraft types are set to the level which maximises operating profit.
- Step 2.1 The first set of fleets in each of the planning years is made up of the combinations of aircraft which maximises the total operating profit - no aircraft is set to a specific number of units.
- Step 2.2 The second set of fleets is made up of aircraft combinations which exclude each aircraft in turn.
- Step 2.3 The third set of state variables is made up of aircraft combinations for which the number of each aircraft type in turn is set to half that determined in the corresponding first set.
- Step 3 For each year eliminate the duplicate fleets if any.

2.5.2 Generation of Fine Grid

Once the optimal path has been found, a fine grid is generated by setting each of the aircraft types in turn to the "optimum" number of aircraft, then to that number ± 1 and then to that number ± 2 for each year of the planning period, whilst the other aircraft types are set to the level which maximises operating profit.

In generating a fine grid around the previous "optimal" path, the correct theoretical approach is to create a set of fleets consisting of all possible combinations of, say, (Optimum number $\pm 1, 2$). If, for example, the previous optimal fleet for a given year was 4 B747 and 3 B727 then the fine grid for that year would be that shown in Table 2.3. Although a search through such a list would be theoretically correct, it would be highly inefficient.

On the one hand most of the smaller fleets are unlikely to meet the requirements of the product planning department, whilst on the other hand, most of the large fleets would result in a very low utilisation rate of very expensive aircraft.

B747	5	6	6	6	6	5	5	5	5	5	4	4	4*	4	4	3
B727	5	4	3	2	1	5	4	3	2	1	5	4	3*	2	1	5
B747	3	3	3	3	2	2	2	2	2	1	1	1	1	1		
B727	4	3	2	1	5	4	3	2	1	5	4	3	2	1		

It is quite feasible for the above example to search through the above list and to consider in the optimisation process only those fleets for which an increase in one aircraft type is accompanied by a corresponding decrease in the other. This is certainly the most efficient fine grid for a two aircraft problem. However, to perform the same analysis for three four or five aircraft types would, be a highly complicated exercise.

In the above example it might be that the 4 B747s are an unsatisfactory investment for that year in terms of an efficient use of capital in the long term, and that the optimal fleet is one with 3 B747s and 5 B727s. It is clear that the B747 decreases in "efficiency" as the number of aircraft increases from its optimal value of 3. Therefore, if the number of B747s is set to three and the number of B727s is not restricted, then by using the traffic allocation program, the minimum number of B727s which are required to operate with the B747s, such that the demand constraint is satisfied, will be determined.

In general the need to deviate from the previous optimal policy arises because it suggests the purchase of either too few of an efficient aircraft type or of too many of an inefficient type. By increasing and decreasing the number of each aircraft type in turn and letting the others take their "natural" associated values a simple method has been devised which concentrated on the main function of the need to create a fine grid.

2.5.3 Manufacturers' Delivery Constraints

The fleet planning exercise is limited in the number of possible policies by the manufacturers' production and delivery schedules. In the first instance the number of units of a particular aircraft type which can be delivered in any one year can be limited: this simple constraint has been included in the model. The constraints which arise from delivery delays are more complicated and have not been included, but are discussed in detail.

Delivery Delays

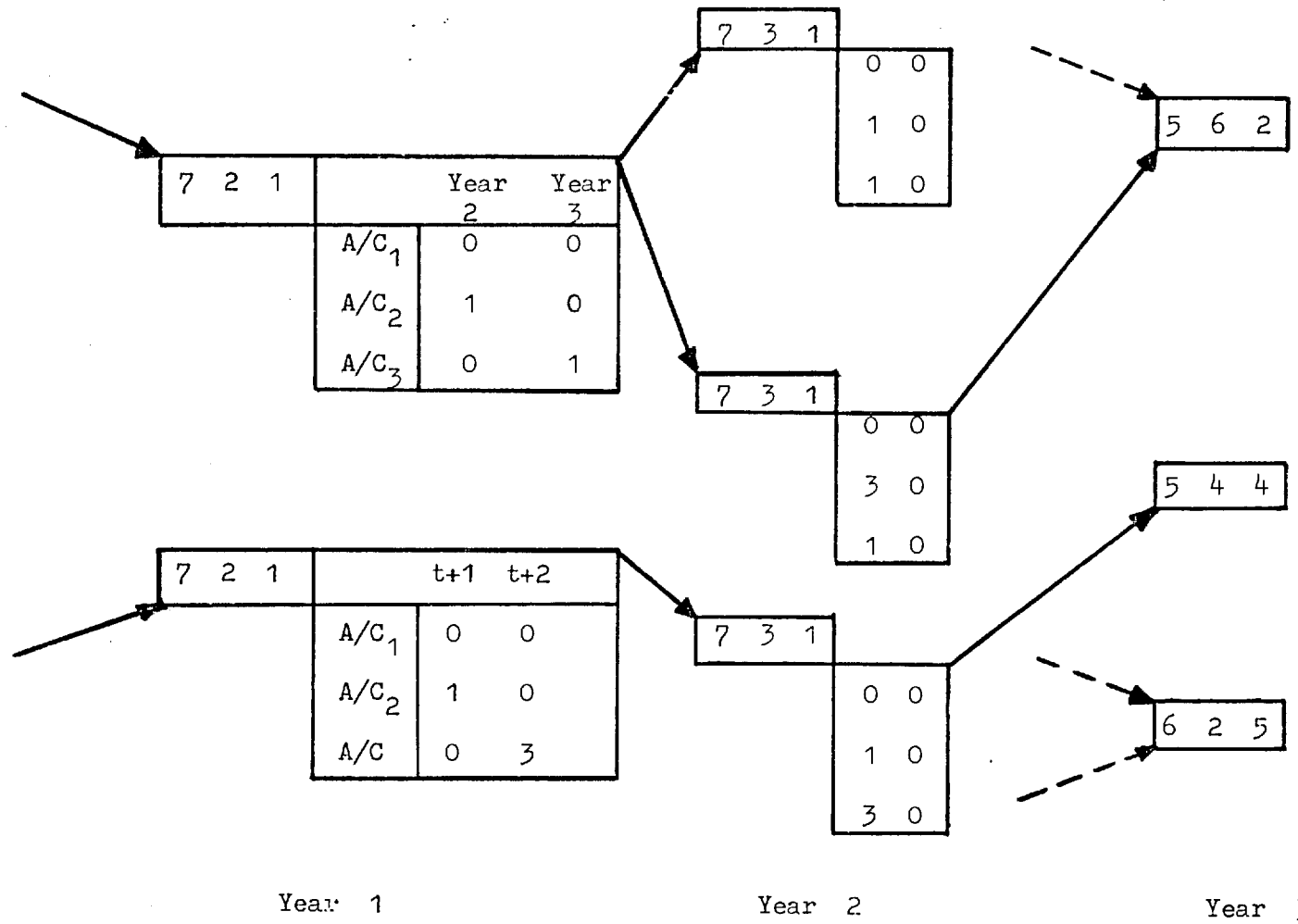
In many cases there is a significant time-lag between the order of an aircraft by an airline and its delivery by the manufacturer. By itself, this is not a particularly difficult situation to simulate in the model. However, if pre-delivery payments accompany such a situation, and they generally do, then the problem becomes more intricate.

Associated with the list of possible fleets, shown in Fig. 2.1, are delivery delays of 1 and 2 years for aircraft types 2 and 3 respectively. As a result aircraft must be ordered and pre-delivery payments made for transition to be possible from one year to the next. If the optimal policy includes the fleet (7, 2, 1) in Year 1, then at least one unit of A/C_2 must have been ordered in Year 1 for delivery in Year 2, and, also at least one unit of A/C_3 order for delivery in Year 3.

By taking account of the delivery delays, the list of possible fleets can be transformed into the grid of possible state variables, part of which is shown in Fig. 2.3. Using this grid, the Dynamic Programming Algorithm will produce the optimal policy.

Consider the third state variable for Year 2 shown in that diagram. The only possible transition to year 3 is to the fleet (5, 4, 4), since 1 A/C_2 and 3 A/C_3 are scheduled for delivery in that year. Since the 3 A/C_3 must have been ordered in year 1, to allow for the 2 year delay, then the transition to Year 2, could only come from the second state variable shown for year 1.

FIG 2.3 LIST OF POSSIBLE STATE VARIABLES



2.6 FLEET PLANNING EXAMPLE

In this section the theoretical analysis developed in this chapter is tested using a computer based model and data provided from several sources.

The airline has in the year 1969, 5 Boeing 707s, 4 Trident 3Bs, and 4 Trident 2Es. The question is which aircraft procurement/retirement policy should it adopt if it is trying to maximise working capital (current assets minus current liabilities) at the end of the planning period. The aircraft characteristics, supplied in parts by Hawker Siddeley Aviation, are shown in Figs 2.4, 2.5, and 2.6. The aircraft purchase and resale values are realistic but cannot be quoted outside the context of this example.

The Balance Sheet for 1969 (Fig 2.7), which states the initial state of the financial position of the airline, is based on Eastern Air Lines' financial accounts. This is also the case for the relationship of accounts receivable, pre-paid expenses, materials and supplies, cash, accounts payable and unearned transportation to total revenue and total operating cost; i.e $VCA_t(R_t, TC_t)$ and $VCL_t(R_t, TC_t)$. The values of the other financial information, shown in Fig 2.8, are purely for the purpose of this example, as are the following financial variables:-

Minimum Current Ratio	1.0
Minimum Return on Capital Employed	0.5
Tax Rate	40.0%
<u>Depreciation Allowance (Tax)</u>	
Initial allowance	45.0%
Yearly allowance	25.0%
<u>Depreciation Allowance (Accounts)</u>	
10 years to 20% residual value	8.0%

Since the purpose of this computer model is to test the Coarse Grid Dynamic Programming approach to the problem of fleet planning, the traffic and network analysis used is not the one discussed in Chapter 3.

Nevertheless, although a simplified traffic allocation model is used, in order to reduce the running of the test runs, it is an efficient and realistic model which assigns aircraft to non-stop routes.

The coarse grid generated by the model is shown in Fig 2.9, as are all the feasible paths and the optimal strategy. This led to the fine grid shown in Fig 2.10. For the five year optimisation problem, the optimal paths through the coarse and fine grids are the same, and the summary tables of the solution are shown in Figs 2.11, 2.12, 2.13, and 2.14.

However, had the optimisation been over two years, the optimal strategy would have included the fleet compositions of 5 B707s, 5 HS3Bs and 5 HS2Es in 1970 and 5 B707s, 6 HS3Bs and 5 HS2Es in 1971. The reason for this is that, although the optimal fleets for those two years are (2,5,5) and (2,6,5) in terms of operating profit, the tax bill in the first year is greater and as a result the working capital is reduced. This is because the resale value of the B707s is greater than the depreciated value. In this case three sets of fine grids would have been generated starting from the coarse grid to produce the final optimal solution.

FIG. 2.4

BOEING 707 INFORMATION
 =====

MAXIMUM PAYLOAD 151 PASSENGERS

MAXIMUM RANGE 3000 MILES

COST EQUATION $102000.0 + (131.6 \times \text{ESAD})$

BLOCK TIME EQUATION $0.66720 + (0.00424 \times \text{ESAD})$

EQUALISED REPAYMENT SCHEDULE

PAYMENT ON SIGNATURE (PERCENT OF PRICE) 10.00

YEARLY REPAYMENTS (PERCENT OF PRICE) 13.30

INTEREST RATE PERCENT 8.25

NUMBER OF YEARLY REPAYMENTS 10

NUMBER OF PAYMENTS IN ONE YEAR 4

YEAR	1970	1971	1972	1973	1974
PRICE OF AIRCRAFT (\$M)	9.500	9.750	10.000	10.000	10.000
RESALE VALUE (\$M)	7.25	6.35	5.45	4.55	3.65
MAXIMUM DELIVERY	5	3	2	0	0
PROCUREMENT/RETIREMENT CODE	1	1	1	1	1
RETIREMENT POLICY (NO OF A/C)	5	3	3	2	2
MAXIMUM AIRCRAFT UTILISATION	3000.	3000.	3000.	3000.	3000.
INSURANCE PREMIUM (PERCENT)	3.	3.	3.	3.	3.

FIG. 2.5

TRIDENT 3R INFORMATION
=====

MAXIMUM PAYLOAD 132 PASSENGERS

MAXIMUM RANGE 1350 MILES

COST EQUATION $73600.0 + (105.2 \times \text{ESAD})$

BLOCK TIME EQUATION $0.82580 + (0.00412 \times \text{ESAD})$

EQUALISED REPAYMENT SCHEDULE

PAYMENT ON SIGNATURE (PERCENT OF PRICE) 10.00

YEARLY REPAYMENTS (PERCENT OF PRICE) 12.38

INTEREST RATE PERCENT 6.50

NUMBER OF YEARLY REPAYMENTS 10

NUMBER OF PAYMENTS IN ONE YEAR 2

YEAR	1970	1971	1972	1973	1974
PRICE OF AIRCRAFT (\$M)	5.860	7.120	7.400	7.710	8.010
RESALE VALUE (\$M)	5.80	6.12	6.40	6.71	7.01
MAXIMUM DELIVERY	100	100	100	100	100
PROCUREMENT/RETIREMENT CODE	3	3	3	3	3
RETIREMENT POLICY (NO OF A/C)	100	100	100	100	100
MAXIMUM AIRCRAFT UTILISATION	3000.	3000.	3000.	3000.	3000.
INSURANCE PREMIUM (PERCENT)	3.	3.	3.	3.	3.

FIG. 2.6

TRIDENT 20 INFORMATION
=====

MAXIMUM PAYLOAD 99 PASSENGERS

MAXIMUM RANGE 2200 MILES

COST EQUATION $82300.0 + (90.1 \times \text{ESAD})$

BLOCK TIME EQUATION $0.76560 + (0.00402 \times \text{ESAD})$

EQUALISED REPAYMENT SCHEDULE

PAYMENT ON SIGNATURE (PERCENT OF PRICE) 10.00

YEARLY REPAYMENTS (PERCENT OF PRICE) 12.38

INTEREST RATE PERCENT 6.50

NUMBER OF YEARLY REPAYMENTS 10

NUMBER OF PAYMENTS IN ONE YEAR 2

YEAR	1970	1971	1972	1973	1974
PRICE OF AIRCRAFT (£M)	5.860	6.090	6.340	6.690	6.850
RESALE VALUE (£M)	4.86	5.09	5.34	5.69	5.85
MAXIMUM DELIVERY	100	100	100	100	100
PROCUREMENT/RETIREMENT CODE	3	3	3	3	3
RETIREMENT POLICY (NO OF A/C)	100	100	100	100	100
MAXIMUM AIRCRAFT UTILISATION	3000.	3000.	3000.	3000.	3000.
INSURANCE PREMIUM (PERCENT)	3.	3.	3.	3.	3.

FIG. 2.7

BALANCE SHEET FOR 1969 (\$M)

=====				=====			
FIXED ASSETS				STOCKHOLDERS EQUITY			
=====				=====			
AIRCRAFT	88.75	I	50.00	VALUE OF STOCK			
DEPRECIATION	10.98	I	25.00	RETAINED EARNINGS			
PROPERTY AND EQUIPMENT	110.00	I	97.35	LONG TERM DEBT			
DEPRECIATION	7.00	I		=====			
CURRENT ASSETS		I		CURRENT LIABILITIES			
=====		I		=====			
SHORT TERM INVESTMENTS	7.00	I	10.00	NOTES PAYABLE IN ONE YEAR			
OTHERS	26.58	I	0.00	SHORT TERM DEBT			
		I	15.00	TAX PROVISIONS			
		I	4.00	OTHERS			
OTHER ASSETS	12.00	I	15.00	OTHER LIABILITIES			
		I					
		I					
=====				=====			
	216.35	I	216.35				
		I					
=====				=====			

FIG. 2.8 FINANCIAL INFORMATION FOR PLANNING PERIOD
=====

	1970	1971	1972	1973	1974
LIABILITIES =====					
INCREASE IN EQUITY (\$M)	50.00	0.00	0.00	0.00	0.00
INCREASE IN LONG TERM DEBT (\$M)	0.00	100.00	0.00	0.00	0.00
REPAYMENT OF LONG TERM DEBT (\$M)	10.00	10.00	12.00	10.00	10.00
INTEREST TO SERVICE LONG TERM DEBT (\$M)	9.00	19.00	18.00	17.00	15.00
DIVIDENDS (\$M)	4.00	4.00	4.00	4.00	4.00
MAXIMUM LEVEL OF SHORT TERM DEBT (\$M)	20.00	20.00	20.00	20.00	20.00
COST OF SHORT TERM BORROWING (PERCENT)	15.00	15.00	15.00	15.00	
ASSETS =====					
INCREASE IN PROPERTY AND EQUIPMENT (\$M)	40.00	0.00	0.00	0.00	0.00
DEPRECIATION ON PROPERTY AND EQUIPMENT (\$M)	7.00	7.00	7.00	7.00	7.00
TAX ALLOWANCE ON PROPERTY AND EQUIPMENT (\$M)	25.00	7.00	7.00	7.00	7.00
RETURN FROM SHORT TERM SECURITIES (PERCENT)	10.00	10.00	10.00	10.00	

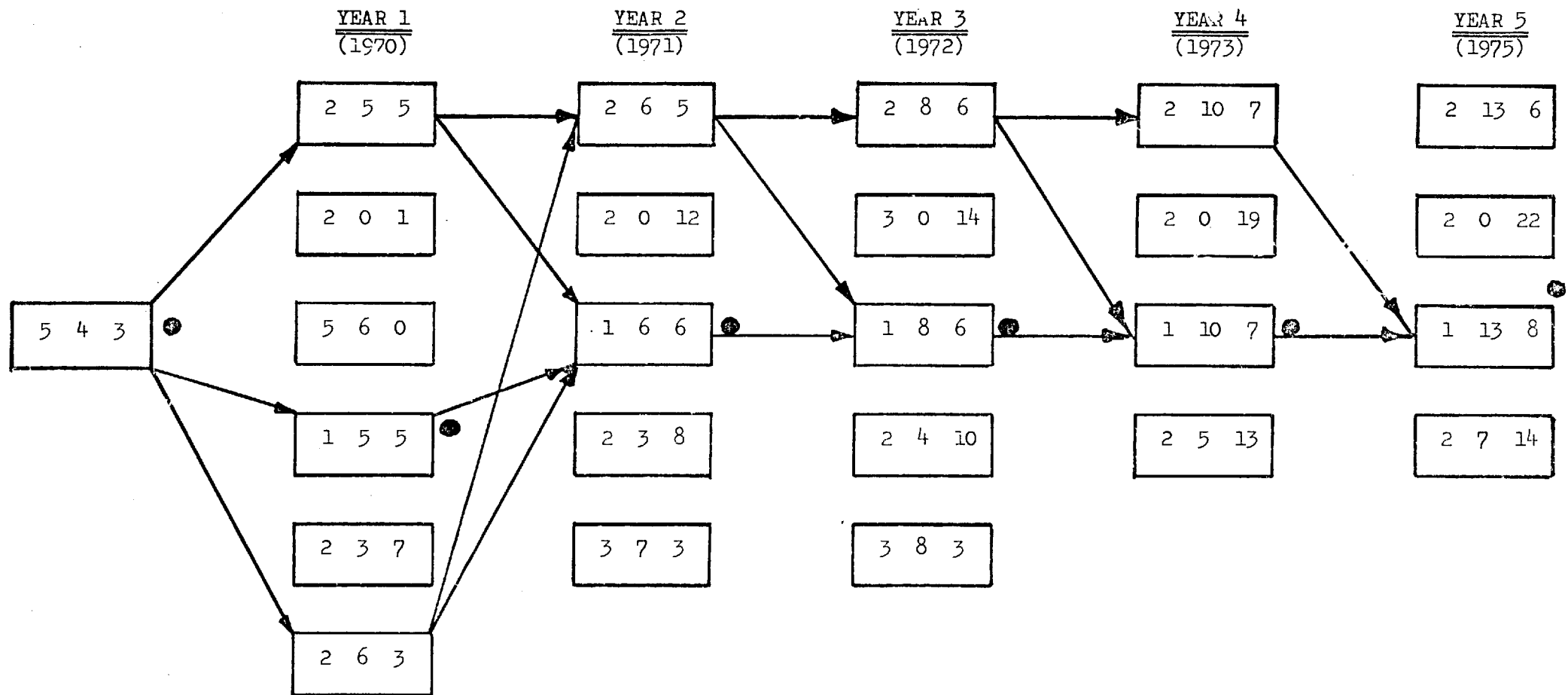


FIG 2.9 FLEET PLANNING EXERCISE

FEASIBLE PATHS

OPTIMAL PATH

B707	HS3B	HS2E
------	------	------

<u>YEAR 1</u> (1970)	<u>YEAR 2</u> (1971)	<u>YEAR 3</u> (1972)	<u>YEAR 4</u> (1973)	<u>YEAR 5</u> (1974)
1 5 5	1 6 6	1 8 6	1 10 7	1 13 8
2 0 11	2 0 12	3 0 14	2 0 19	2 0 22
5 6 0	2 6 5	2 8 6	2 10 7	2 13 6
2 5 5	2 3 8	2 4 10	2 5 13	2 7 14
2 3 7	3 7 3	3 8 3	2 11 7	2 14 6
2 6 3	2 7 5	2 9 6	2 10 8	2 13 9
2 6 5	2 6 7	2 8 7	3 10 4	3 13 6
2 5 6	3 6 5	3 8 6	2 12 7	2 15 6
3 5 5	2 8 5	2 10 6	2 10 9	2 13 10
2 7 5	2 6 8	2 8 8	2 9 8	2 12 7
2 5 7	2 5 7	2 7 7	2 10 6	2 13 7
2 4 6	2 4 7	2 8 5	2 8 9	2 1 9
2 6 4	2 7 4	2 6 8	2 10 5	
		3 8 4		

FIG 2.10 FINE GRID GENERATED

FIG. 2.11 PROPOSED FLEETS

=====

YEAR	1970	1971	1972	1973	1974
AIRCRAFT OPERATING					
BOEING 707	1	1	1	1	1
TRIDENT 38	5	6	8	10	13
TRIDENT 2E	5	6	4	7	8
BOUGHT/SOLD					
BOEING 707	-4	0	0	0	0
TRIDENT 38	1	1	2	2	3
TRIDENT 2E	2	1	0	1	1

FIG. 2.12 INCOME AND EARNINGS STATEMENT
 =====

YEAR	1970	1971	1972	1973	1974
TOTAL REVENUE	121.08	137.15	157.41	186.05	224.76
VARIABLE COSTS	21.27	24.53	28.91	35.14	43.34
FLEET INSURANCE	2.12	2.52	2.97	3.63	4.55
INDIRECT OPERATING COST	51.48	36.59	43.10	52.68	65.58
OPERATING PROFIT	66.21	73.51	82.44	94.79	110.79
A/C DEPRECIATION	5.67	6.72	7.91	9.68	12.15
PROPTY + EQUIPM DEPPCTN	7.00	7.00	7.00	7.00	7.00
PROFIT ON SALE OF A/C	-3.14	0.00	0.00	0.00	0.00
INTEREST ON LOANS	11.51	13.36	5.54	1.26	-4.41
TOTAL INCOME	39.09	41.62	61.99	76.85	96.05
PROVISIONS FOR TAXATION	10.72	14.02	22.42	27.66	34.28
DIVIDENDS	4.00	4.00	4.00	4.00	4.00
ADDITIONS TO EARNINGS	24.37	23.60	35.57	45.19	57.77

FIG. 2.13

BALANCE SHEET

YEAR	1970	1971	1972	1973	1974
STOCKHOLDERS EQUITY					

VALUE OF STOCK	110.00	110.00	110.00	110.00	110.00
RETAINED EARNINGS	49.37	72.78	108.35	153.54	211.36
LONG TERM DEBT	68.77	163.63	160.70	162.95	161.10

CURRENT LIABILITIES					

NOTES PAYABLE IN 1 YEAR	10.00	12.00	10.00	10.00	20.00
SHORT TERM DEBT	0.00	0.00	0.00	0.00	0.00
PROVISIONS FOR TAXATION	10.72	14.02	22.42	27.66	34.28
OTHERS	12.11	13.89	16.16	19.40	23.85
OTHER LIABILITIES	15.00	15.00	15.00	15.00	15.00

TOTAL LIABILITIES	275.97	401.32	442.63	498.54	575.54

FIXED ASSETS					

AIRCRAFT ASSETS	70.83	84.04	98.84	120.95	151.83
DEPRECIATION	12.29	12.01	26.92	36.99	48.74
PROPERTY AND EQUIPMENT	150.00	150.00	150.00	150.00	150.00
DEPRECIATION	14.00	21.00	28.00	35.00	42.00
CURRENT ASSETS					

SHORT-TERM INVESTMENTS	44.66	167.04	204.07	248.29	305.02
OTHERS	24.77	28.24	32.64	38.89	46.43
OTHER ASSETS	12.00	12.00	12.00	12.00	12.00

TOTAL ASSETS	275.97	401.32	442.63	498.54	575.54
=====					

FIG. 2.14 SOURCES AND USES OF FUNDS STATEMENT

YEAR	1970	1971	1972	1973	1974
SOURCES OF FUNDS					
OPERATING PROFIT	66.21	73.51	82.44	94.79	110.79
SALE OF AIRCRAFT	29.00	0.00	0.00	0.00	0.00
INCREASE IN L.T DEBT	-28.58	94.86	-2.93	2.25	-1.85
INCREASE IN EQUITY	50.00	0.00	0.00	0.00	0.00
USES OF FUNDS					
INVESTMENT ON AIRCRAFT	18.58	13.21	14.80	22.11	30.82
PROPERTY AND EQUIPMENT	40.00	0.00	0.00	0.00	0.00
INTEREST ON DEBT	11.31	18.36	5.54	1.26	-6.45
DIVIDENDS	4.00	4.00	4.00	4.00	4.00
WORKING CAPITAL (INCRS)	32.01	118.78	32.75	42.00	44.19
PROVISIONS FOR TAXATION	10.72	14.02	22.42	27.66	34.23
OTHER ASSETS	0.00	0.00	0.00	-0.00	0.00

CHAPTER 3

THE ONE YEAR PROBLEM

The number of aircraft required by an airline to perform its operations is governed by the objectives of the marketing department (i.e. to provide a high quality service which offers enough capacity to meet the expected demand), and by the needs of the maintenance/engineering department to service the aircraft when appropriate. Therefore, the function of the production planning department, in its relationship to the total planning process, is to determine sets of possible fleets for each year of the planning period which will satisfy the above constraints.

In this chapter, the decision making process of product planning is analysed in detail and the planning model developed is described. The main variables associated with this process and their relationship to the model are discussed, as is the problem of testing for fleet feasibility and the finding of the associated profit.

3.1 THE MAIN VARIABLE IN PRODUCT PLANNING

TABLE 3.1 FACTORS AFFECTING GROWTH [16]

Stimulants	Deterrents
Economic Boom Creation of tourist resort Development of industry Improvement or new airport New aircraft New routes Lower fares	Economic depression New or better competition Decline of industry Political restriction Congestion Poor corporate image High fares

3.1.1 Demand Forecast

The successful airline is built on good planning, and good planning is dependent on good forecasting. Hence, the fundamental importance of traffic forecasting.

Traffic forecasting is a very complicated exercise, and is not simply a projection of historical trends. A projection can be useful as a forecast base provided that all market influences in the future will continue to develop as in the past. But in many cases there will be over-riding market factors which will elevate or depress the extrapolated historical trend line. Typical environmental factors which can alter the trend of history are shown in Table 3.1 and are listed under 'Stimulants' or 'Deterrents' depending on whether they increase or decrease traffic growth. Thus, in most cases, it is the job of the forecaster to combine, in an intelligent manner, reliable historical traffic data with economic, political and sociological factors which are relevant during the forecast period. Furthermore, he must anticipate not only the airline's proposed change in equipment, network and schedules, but also those of its competitors.

It is clear that although the forecasts based on environmental factors will be affected by the output of the model (equipment and schedules) it is unwise to let the model perform the up-dating, since the precise effects depend on the reaction of the competitors in an extremely oligopolistic market. Hence it is proposed that the 'complete forecast' should be an input to the model.

Nevertheless, the model has been designed to include an option which will generate extra traffic when the quality of service offered is increased, simply to show that the model is flexible enough to allow modifications to be included to take account of such factors.

3.1.2 Revenue rate

The level of fares clearly affect the demand forecast (Table 3.1) and therefore one could presume that it could be left to the model to calculate the level of fares and the resulting demand such that the overall solution is optimal. The main objection to this proposal is that pricing is controlled externally in the scheduled sector, which will continue to dominate the industry. The actions of airlines operating international scheduled services are governed by bi-lateral Air Services Agreements made between the sovereign governments, which either as a matter of practice or specifically in the agreement, invariably accept the rate-fixing machinery of the International Air Transport Association (IATA).

This body, formed in 1945, is the trade association of the international scheduled air carriers, with almost complete membership, except most - but not all - airlines of the Communist states. It performs many functions, but the one that matters is its activity, through what are called Traffic Conferences, in setting tariffs. These embrace the levels of rates and fares and all related provisions; in particular relating to the standards that may be offered for the agreed fares. Agreements reached in these conferences cover every international fare and rate used by members. They have to be reached unanimously and they are binding.

It is more difficult to outline the general picture of the regulatory systems of domestic air transport operations. Fundamentally, the situation that emerges is the same as that for international carriage. Air transport is rarely free to go after markets at will, or to charge the fares it likes.

Furthermore, as far as the charter sector is concerned, a great deal of it is based on Inclusive Tours, where the level of demand is influenced by the total price of the package, and therefore the price of such service is also controlled externally.

As for the charter industry in general, it seems very likely that this will also be regulated in the near future so as to stabilise the industry. The argument put forward to this end by Stephen Wheatcroft, who sat on the Edwards committee [17], is - 'What is needed is a general acceptance by governments that we cannot continue to treat the regulation of scheduled and charter operations as two entirely separate questions New definitions of scheduled and charter would make it possible for the provisions of bi-lateral air agreements to regulate both types of services in a co-ordinated fashion It would seem to me desirable that charter pricing should be brought within the jurisdiction of IATA conferences'. [18]

In view of the above discussion, it is sensible to leave revenue rate as an input. This must, however, be an average revenue rate, as each sector is made up of passengers flying at different rates - youth, advanced booking, other promotional and 'normal'.

3.1.3 Load Factor

The principle product of an airline is extremely perishable; an unused seat-mile is forever beyond recovery. Thus, the motive for selling most, if not all, of the available capacity is clear. But there are reasons for not operating at very high average load factors (No. of passengers on aircraft/ No. of seats on aircraft) in a scheduled operation. All these reasons are due to the variability of demand for service by the potential passengers.

The mid-curve on Fig. 3.1 is the average demand and the width of the band represents the variability of the demand. The demand is those travellers seeking services. When the demand is equal to or less than the number of seats, the demand become passengers - but when the demand is greater, then there will be some unserved demand.

Competition and regulation generally insist that the airline industry provide almost enough seats for potential passengers during peak periods. Even without competition or regulation the airline would normally try to provide enough seats for all but a relatively few passengers during peak periods.

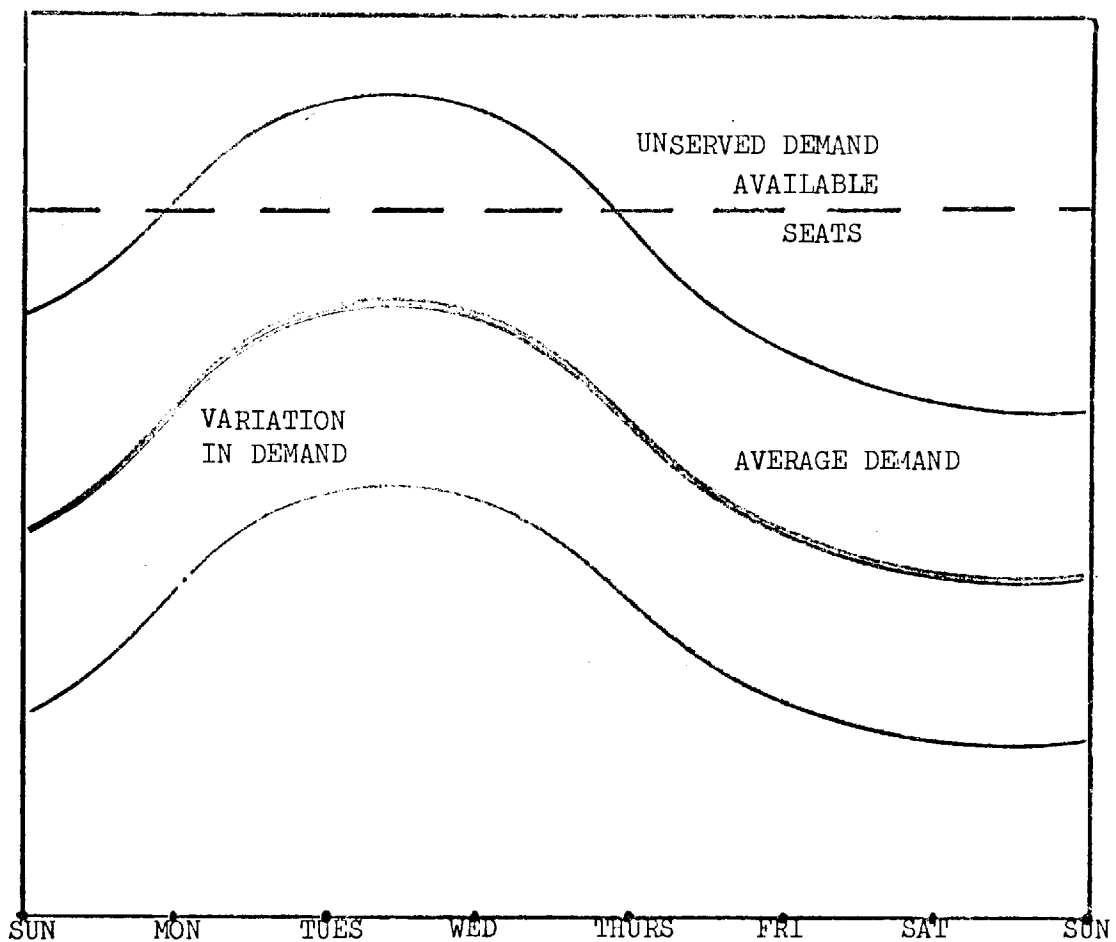


FIG. 3.1 VARIATION IN DEMAND

because to provide fewer seats can result in losses in revenue which are almost entirely profits. Since the demand is variable but mainly takes on intermediate values, it is convenient and meaningful to talk about average load factors. In general, for a scheduled service the more variable the demand the lower will be the average load factor.

The maximum permissible average load factor which is required to minimise the 'turnaway risk' - i.e the risk of not satisfying potential passengers - depends on the nature of the demand. Variability as was stated earlier, is important; but also important is the relative proportion of the business, personal and tourist passengers which go to make-up that demand. Furthermore, if a route is served by many flights it is more likely to support a higher maximum load factor than if it were not.

Although it is theoretically possible to construct a formula which incorporates all these factors such that the maximum permissible load factor on each sector is calculated within the model [19], the knowledge on the relationship between traffic, frequency, capacity and load factor is such that any formula which results would not be acceptable to many. It is felt that this parameter is best evaluated by an experienced analyst and used as an input to the model.

3.1.4 Scheduling and Utilisation

If the routes and frequencies have been determined, then there are only three important factors to consider in attempting to produce an optimum schedule. These are aircraft utilisation, timing for maximum revenue generation and maintenance requirements. From a preliminary survey of the problem it is usually possible to decide which method of maintenance will be used and to specify the demand on aircraft time. Scheduling is then a straightforward issue between the need for high utilisation of modern equipment and the need to time services according to when the customers wish to travel.

There are two main problems in devising a schedule. The first is that whilst traffic specialists can confidently specify the times when the customers wish to travel they are rarely able to estimate the loss of revenue caused by deviations from them. The problem is further complicated on flights with multiple sectors by the conflicting preferences of passengers on different sectors.

However, the need for building a time-table is only relevant to the short term planning of a scheduled airline. It is, therefore, unnecessary to enter into the realm of demand sensitivity to flight times when one is analysing demand forecasts for five years ahead. The usual method adopted for airline corporate planning is to specify a conservative estimate of average aircraft utilisation, which will allow enough flexibility to build a schedule when the time comes.

In general, the utilisation capability of an aircraft is controlled by traffic demand and/or maintenance requirements. Unless the demand is so low

so as to allow the aircraft to be idle for six or more hours a day, the limiting factor is maintenance requirements. Thus, aircraft potential utilisation is a function of:-

- Daily maintenance inspection time
- Periodic maintenance time
- Time on ground for unloading, loading and refuelling.

The Lockheed Corporation have derived a formula for potential utilisation which is [20] :

$$U_p = R_t \cdot \left\{ \frac{24 - T_{dm}}{T_s/T_b + R_{pm} + 1} - \frac{T_b}{2} \right\}$$

where U_p is the potential utilisation per day
 T_{dm} time for daily maintenance
 R_{pm} ratio of periodic maintenance time to utilisation
 T_s time per stop for loading, unloading and refuelling
 T_b average block time (see 3.2.5)
 R_t is the 'Realisation Factor' which takes account of other factors which can reduce the actual availability of the fleet.

Typical annual values of average utilisation, for $T_{dm} = 2$, $R_{pm} = 0.25$ and $R_t = 0.8$, are shown in Figs. 3.2 and 3.3.

Of course, each airline will have its own set of figures, or they might even have only one - independent of block time. The philosophy, however, is the same: an experienced analyst will estimate the numbers and these will be an input to the model.

FIG. 3.2 AIRCRAFT UTILISATION -
MEDIUM/LONG HAUL OPERATION

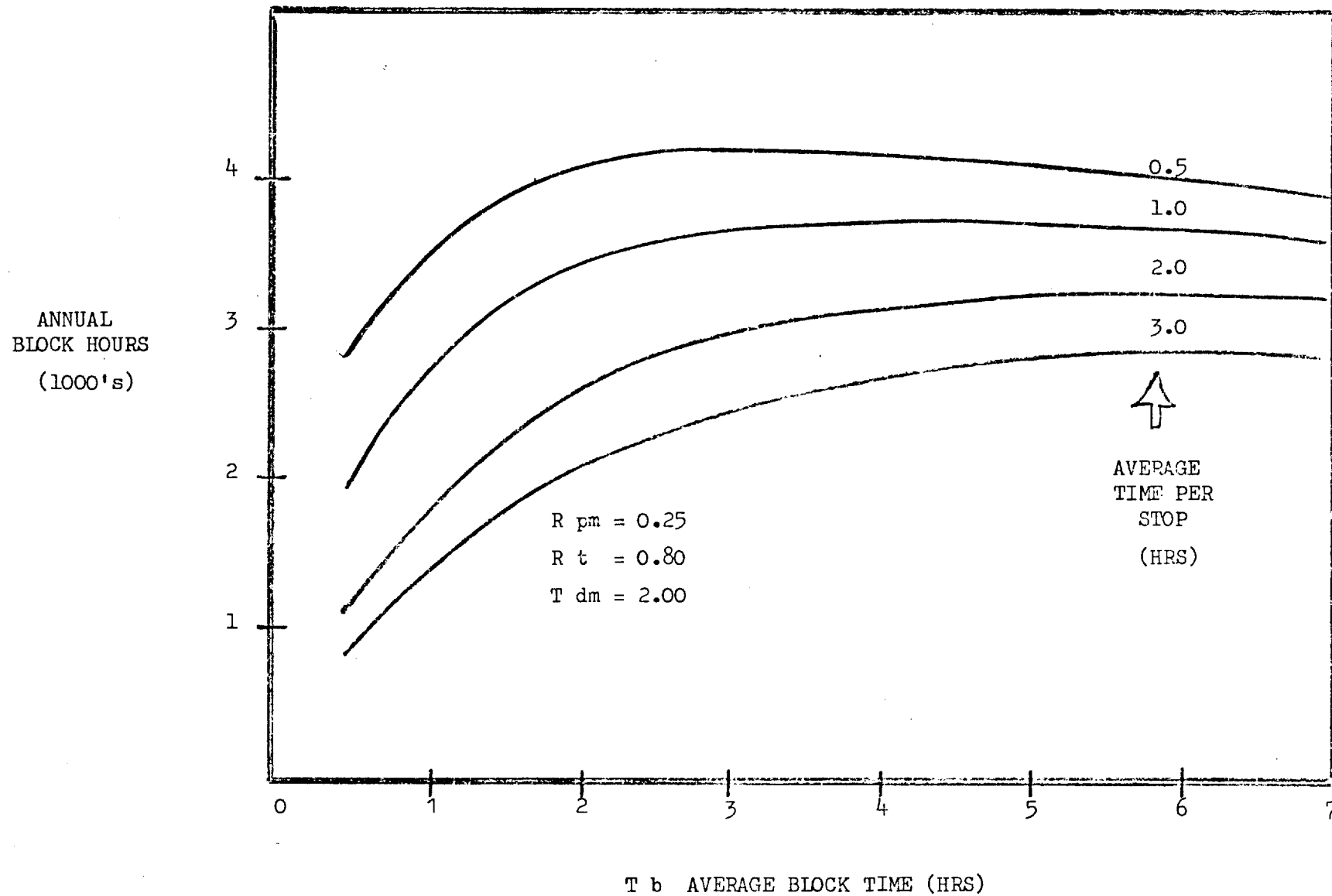
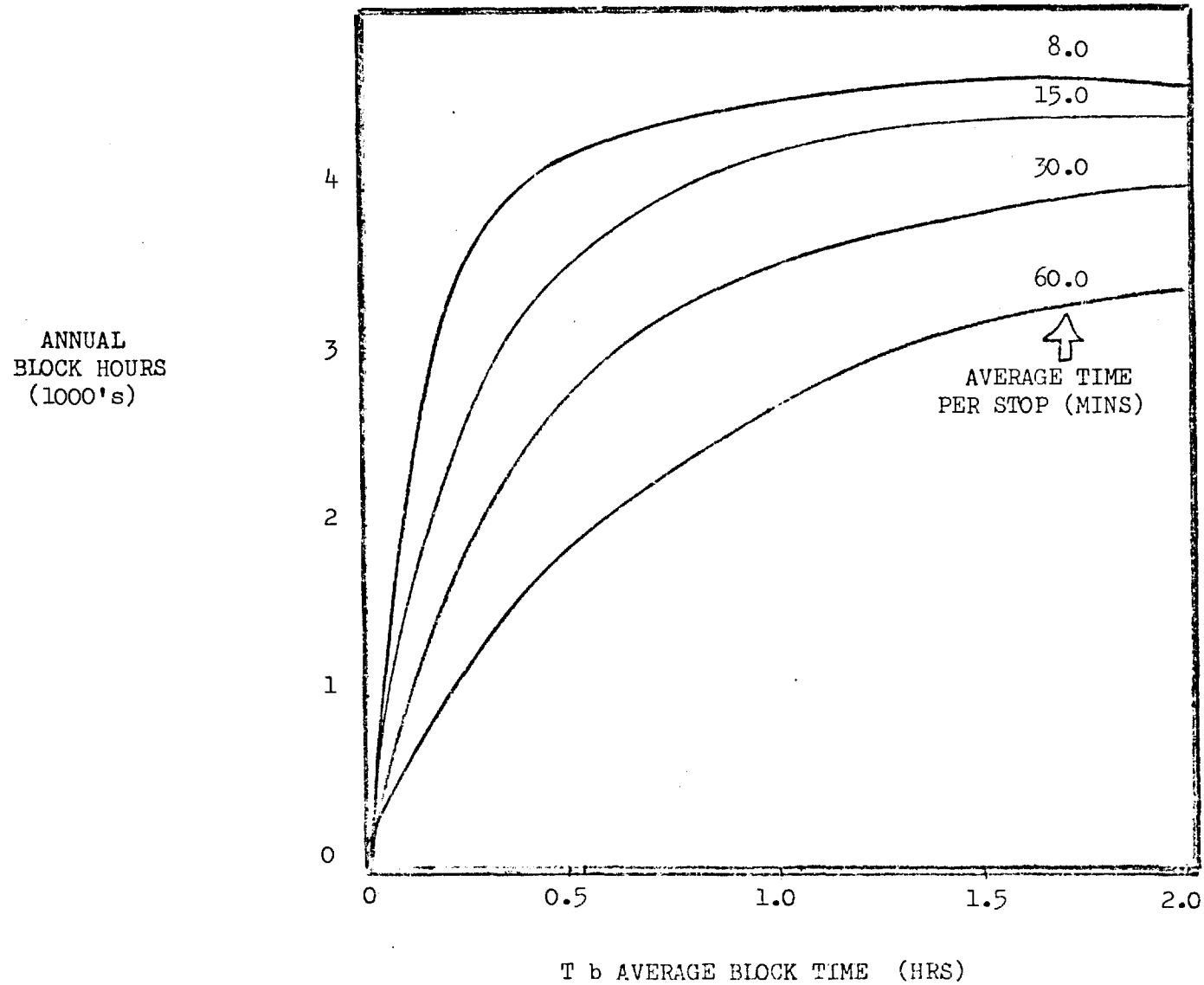


FIG 3.3 AIRCRAFT UTILISATION -
SHORT/MEDIUM HAUL OPERATION



3.2 SYMBOLS USED FOR ONE-YEAR PROBLEM

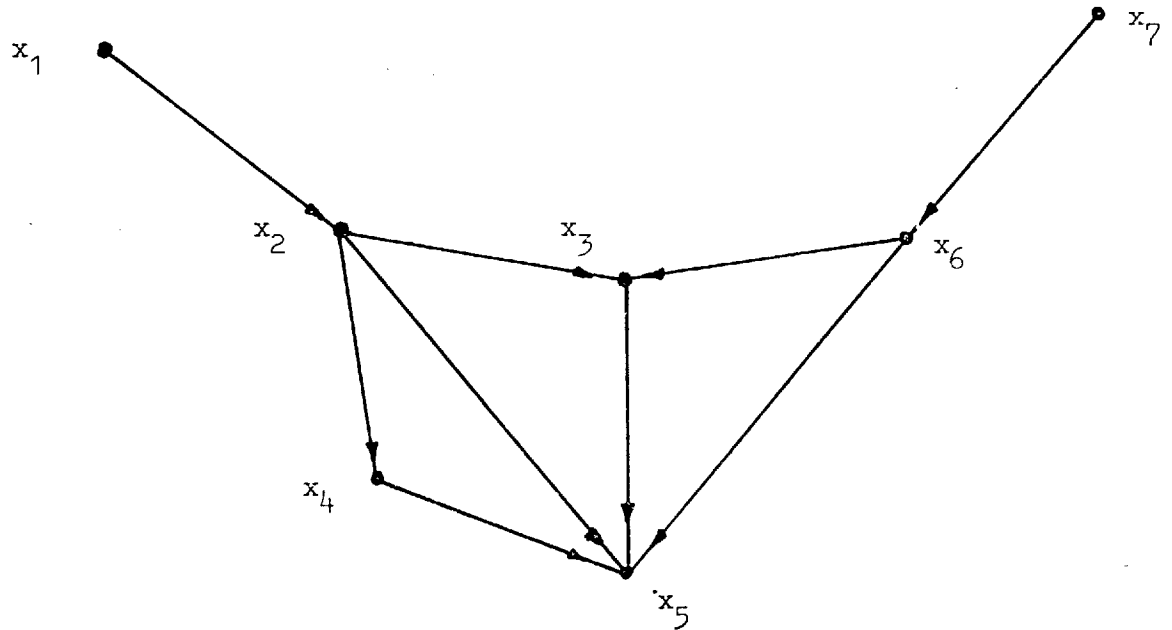


FIG. 3.4 A NETWORK

3.2.1 Network Symbols

A network	is defined by
<u>X</u>	the set of cities which the airline serves, i.e. $\{x_1, x_2, x_3, \dots\}$ and by
<u>A</u>	the set of city-pairs, $\{i,j\}$, i.e. $\{(x_1 - x_2), (x_1 - x_3), (x_1 - x_4), \dots\}$
A leg	is the portion of a route, $[pq]$ between consecutive cities. For a route with m intermediate stops there are $(m+1)$ legs.
A network	can also be defined by
<u>R</u>	a set of routes representing specific paths on the map

- $\underline{R}_{ij}$ is a sub-set of $\underline{R}$ of routes with routings through cities i and j , where a routing is a sub-route or portion of route r .
- $\underline{R}_{ijm}^*$ is the sub-set of $\underline{R}_{ij}$ of routes which start at city i and ends at city j with exactly m intermediate stops.
- f is a flight, i.e the assignment of an aircraft to a specific route.
- $\underline{F}$ is a set of flights representing specific assignments of aircraft to routes of the set $\underline{R}$. Thus $\underline{F}$ represents in effect the airlines planned route structure.

3.2.2. Aircraft Symbols

- a is an aircraft type.
- FL_a is the number of aircraft of type a in the airline's proposed fleet.
- C_a is the seating capacity of aircraft type a .
- C_{aij} is the effective seating capacity of aircraft type a when flying along the leg $[ij]$. If there is no loss in aircraft performance when flying this leg, then $C_{aij} = C_a$.

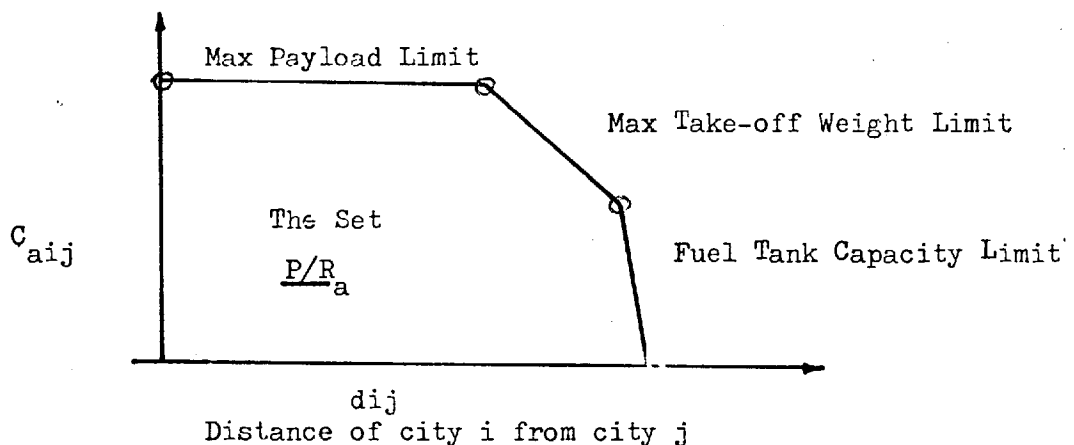


FIG 3.5 PAYLOAD/RANGE DIAGRAM

$\underline{P/R}_a$ is the set of points below the Payload/Range curve of aircraft type a (Fig. 3.5). The circled points are all that is required to describe the diagram.

3.2.3. Traffic and Quality of Service Symbols.

D_{ij} is the base demand forecast of passengers wanting to fly with the airline along the city-pair $\{ij\}$. This forecast does not include the extra demand which could be generated through an increase in the quality of service.

$LF_{\max ij}$ is the maximum permissible load factor on city-pair $\{ij\}$.

$N_{\min ij}$ is the minimum number of flights required to fly along the city-pair $\{ij\}$ so that the airline can maintain its competitive position.

$N^*_{\min ij}$ is the minimum number of direct flights (or least number of stops, if a direct flight is infeasible) required to fly along city-pair $\{ij\}$.

$\underline{R}_a^*$ is a set of routes along which aircraft type a cannot fly for competitive or political reasons.

ΔD_{ij} is the extra demand generated as a result of an increase in the quality of service above the required minimum. Δ is an operator and is a function of C_{aij} and n_{ij} , where

n_{ij} is the assigned number of flights flying the city-pair $\{ij\}$ - n_{ija} is the equivalent number of flights by aircraft type a.

n^*_{ij} is the assigned number of 'direct' flights to the city-pair $\{ij\}$.

P_{ijf}	is the demand on city-pair $\{ij\}$ allocated to the flight f .
P_{pqf}^*	is the number of passengers allocated to fly along leg $[pq]$ of flight f .
LF_{pqf}^*	is the assigned load factor along the leg $[pq]$ of flight f .

3.2.4 Revenue and Cost Symbols

DOC_f	is the marginal direct operating cost of operating the flight f .
Y_{ij}	is the expected average net yield, or revenue rate, for passengers flying the city-pair $\{ij\}$.
OP_f	is the expected operating profit of flight f :
$OP_f = \left\{ \left(\sum_{ij \in f} Y_{ij} \cdot P_{ijf} \right) - DOC \right\}$	

3.2.5 Time Symbols

t_{pqa}	is the block time for the flight along the leg $[pq]$ by aircraft type a .
Block Time	is the time for take-off, acceleration to climb speed, climb, cruise, descent, deceleration to circuit speed, circuit and landing, plus a time allowance for ground manoeuvres.
U_a	is the maximum permissible utilisation for aircraft a .

3.3. THE TRAFFIC ALLOCATION PROBLEM

In its attempt to define a set of permissible fleets for each year of the planning period, the product planning department will perform the exercise of 'suggesting' a fleet and then testing its feasibility. This entails the allocation of aircraft to routes such that the demand, quality of service, and technical constraints are satisfied. Since the object of the exercise is not simply to test for feasibility but also to generate the

associated revenues and costs for use in the total planning process, the procedure becomes one of finding the optimal traffic allocation for each feasible fleet. The problem can now be stated as:-

Maximise the total operating profit, i.e.

$$\text{MAX} \quad \left\{ \sum_{f \in \underline{F}} \text{OP}_f \right\}$$

such that

- 1) The basic demand forecast is satisfied;

$$\sum_{f \in \underline{F}} P_{ijf} \geq D_{ij} \quad \forall \{ij\} \in \underline{A}$$

- 2) The allocated passengers do not exceed the maximum possible generated demand;

$$\sum_{f \in \underline{F}} P_{ijf} \leq D_{ij} + \text{Max } \Delta D_{ij} \quad \forall \{ij\} \in \underline{A}$$

- 3) The number of flights allocated to each city-pair is greater than the minimum required frequency;

$$\sum_a n_{ija} \geq N_{\min ij} \quad \forall \{ij\} \in \underline{A}$$

- 4) The number of direct flights must be greater than the minimum;

$$\sum_a n_{ija}^* \geq N_{\min ij}^* \quad \forall \{ij\} \in \underline{A}$$

- 5) The maximum permissible load factor constraint must not be violated;

$$\left\{ \left[\text{LF}_{\max pq} \cdot \sum_a (C_{apq} \cdot n_{pqa}^*) \right] - \sum_a P_{pqa}^* \right\} \geq 0 \quad \forall [pq] \in \underline{P}_{pq}^*$$

6) The fleet availability must not be exceeded;

$$\sum_{\forall [pq] \in \underline{P}_{pq}} (t_{pqa} \cdot n_{pqa}) \leq U_a \cdot FL_a \quad \forall a$$

7) There must not be any infeasible aircraft/route combinations;

$$\underline{R}_a \not\subset \underline{F} \quad \forall a$$

8) All allocations must lie within the boundaries of the payload/range diagram;

$$\left[d_{pq}, (P_{pqa}^* / n_{pqa}^*) \right] \in \underline{P/R}_a \quad \forall [pq] \in \underline{P}_{pq}, \forall a$$

The problem has been studied extensively [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. Most of the various approaches devised to solve it (complicated heuristics or linear programs) suffer from the same drawback; they were designed explicitly to solve the traffic allocation problem with little or no concern to the relationship of the solution of this problem to that of total fleet planning. As a result they tend to be very inflexible and extremely expensive in terms of computer processing time, with the result that they must be used sparingly within the fleet planning process.

At the other end of the spectrum, models have been developed around the fleet planning problem and have a very simple traffic allocation segment. So simple, in fact, that the quality of the solution - and sometimes its feasibility - is questionable.

The fundamental philosophy of the design specification of the algorithm developed to solve the traffic allocation problem is that it should have intimate interaction with the fleet planning model. This led to the need of a heuristic algorithm which is modular in structure to allow it to operate at different levels of complexity. At its lowest level it should be extremely fast and provide an approximate solution; at its highest level it should be very accurate and reasonably fast.

The algorithm described in the remainder of this chapter is just that. At the core is a very fast Standard Allocation Algorithm, whilst at the periphery are 'perturbations' which when applied to the initial approximate solution 'guide' the total allocation towards the global optimum. Its relationship to the Dynamic Programming Fleet Planning model is discussed in detail in Chapter 4.

3.4 INPUTS TO THE TRAFFIC ALLOCATION ALGORITHM

The traffic allocation algorithm described in this report uses a considerable amount of data as input. This includes sector information such as demand forecast and distance; aircraft characteristics such as cruising speed and number of seats; and station information such as landing fees and handling charges. These are now discussed in some detail.

3.4.1 Sector Information

The algorithm assigns aircraft and passengers to routes which form a radial structure. An airline has a radial route structure if it performs its main operations from one home base, and all its aircraft flight cycles start and end at this base. As a result it tends to be most competitive on those sectors (city-pairs) which link a city to the home base, i.e. A_{ij} (i or j = h). These sectors, which carry traffic into and out of the home base, are the airline's main source of revenue and, in general, will be those sectors on which the quality service constraints, such as minimum frequency and maximum permissible load factors, will be imposed. These will be referred to as the 'strong sectors'.

The airline cannot usually compete, for political or commercial reasons, on the other sectors, A_{ij} ($i, j \neq h$), but can get a portion of the demand, ΔD_{ij} , simply by operating a service. There are two reasons for operating on these 'weak sectors'. The first relates to the range limitations of aircraft. An aircraft might not be able to fly a sector non-stop even without a payload. Thus, these sectors passengers would have to be routes via a third city, and hence the creation of services along 'weak sectors'. The second reason is that some 'strong sectors' are not particularly 'strong', in that the potential demand is not large enough to warrant non-stop services.

Therefore, by creating a multi-stop route which links these sectors, the extra revenue generated by flying the 'weak sectors' might transform this segment of the operations into a more profitable one.

In performing the allocation the algorithm assumes that the outward sector demand is equal to the return demands, i.e. $D_{ij} = D_{ji}$ and $\Delta D_{ij} = \Delta D_{ji}$, and that if D_{ij} is carried on a flight f_a , then D_{ji} will be carried on the inverse of f_a , i.e. f_a^{-1} (where, say $f_a = i - k - j$ and $f_a^{-1} = j - k - i$). This is a reasonable assumption to make since the airline sells the majority of its tickets as return tickets. This assumption means that the values of D_{ij} , $LF_{\max ij}$, $N_{\min ij}$, etc. are the average of those specified for the outward and return trip.

The assumption that $D_{ij} = D_{ji}$ does not mean that the direction of flow of the demand along the sector is arbitrary. The direction of the sectors will determine the final route structure, and the importance of these directions will become more obvious when the route generator is discussed. It is sufficient at this stage to say that the one way demand must flow in the direction of the shortest path connecting the cities and must be in accordance with either a radial flow towards a sink, i.e. the airline's base, or with a radial flow out of a source. The only arbitrary decision is whether the flow is a sink or source flow.

The only sector information not discussed so far is the sector distance. This is an average Equivalent Still Air Distance (E.S.A.D.), which reflects the force of the winds which are likely to be encountered when flying the sector.

Finally, it should be pointed out that the freight demand has been ignored. Its inclusion in the model can be easily achieved, as its volume, for most airlines, is not large enough to warrant a change in the proposed allocation algorithm.

3.4.2 Aircraft characteristics

The following information is read in for each aircraft type:-

- Seating capacity
- Engineering costs (costs per flight and hourly costs)
- Payload/Range Diagram (the four points which describe the diagram.
- Block time equation $\{\gamma + \beta \text{ (ESAD)}\}$
- Block fuel equation $\{\alpha + \delta \text{ (ESAD)}\}$

The first two items are fairly straight forward. The engineering costs are made up of two main costs, those associated with each take-off and landing, such as the maintenance of tyres, flaps and undercarriage, and those associated with the length of the flight, such as maintenance of the powerplant and instruments. The last three items, however, need to be discussed in more detail.

The block fuel, or total fuel burnt, is certainly a function of the distance flown. However, for two equal stage distances the values of block fuel will, in general, be different. This is because block fuel is also a function of the initial take-off weight required. This initial weight may not be the same for an aircraft flying two stage distances for several reasons. The first is that the total payload carried on the respective stages may be different; the second is that the level of fuel reserves may not be the same, because the distances of the diversion airfields from the respective destinations are not the same; and finally the third reason is that the physical characteristics of the two departure airfields are not similar in terms of runway length, height above sea level and mean temperature, and hence a different amount of fuel burnt during the take-off segment.

An analysis of aircraft route performance of different stage distances, from airports of different diversion distances, for various aircraft, showed that the sample correlation coefficient for Equivalent Still Air Distance and block fuel was in the region 0.99 ($\rho = 1.0$ means a perfect correlation). The correlation coefficient for E.S.A.D. and block time was also in the region of 0.99. It can, therefore, be assumed that the above equations give a good representation of block fuel and block time.

The reason for including payload/range diagram in the algorithm is clear. The allocation process must be allowed to allocate an aircraft to route even though the aircraft cannot carry a full payload when flying one of the legs, for capacity payload capability may not be required. What is less clear, is how the model can use a standard payload/range diagram when its dimensions are sensitive to the level of reserves and the maximum permissible take-off weight. In general, many of the allocations will not be suspect in their relationship to the diagram but for those that are, an additional input facility is available in the model:-

- Specific aircraft/sector information (i.e. maximum effective seating capacity, actual block time and actual fuel burnt).

This facility also accommodates combinations which greatly diverge from the estimates of block fuel and block time given by the equations. This will be true for sectors whose departure airfields are extremely high and hot, and for sectors whose diversion airfields are very distant. It can also be used as an indirect method to input the aircraft/route combinations which are unacceptable for political and commercial reasons.

3.4.3 Station Information

For each city the following information is read in:-

- Price of fuel
- Landing fees for each aircraft type
- Handling charges for each aircraft type.

3.5 THE ROUTE GENERATOR

The object of the route generator is to create a set of routes $\underline{R}$, each route having no more than three intermediate stops, i.e. connecting no more than five cities, and each being in accordance with a sink radial flow. A sink radial flow means that each route must end at the airline's home base; x_5 for the network shown in Fig. 3.6. For example the routes $x_1 - x_2 - x_5$ and $x_2 - x_3 - x_5$ would be part of the set $\underline{R}$; the routes $x_1 - x_2 - x_3$ and $x_1 - x_4$ would not.

The set of routes is made up of the paths connecting

$$x_i \text{ to } \underline{G}^1(x_i) = \Gamma^1 x_i$$

$$\text{to } \underline{G}^2(x_i) = \Gamma^2 x_i$$

$$\text{to } \underline{G}^3(x_i) = \Gamma^3 x_i$$

$$\text{and to } \underline{G}^4(x_i) = \Gamma^4 x_i$$

for all $x_i \in \underline{X}$, where Γ is the correspondence operator.
For the example shown in Fig. 3.6,

$$\underline{G}^1(x_1) = \Gamma x_1 = \{x_2, x_3, x_4, x_5\}$$

$$\text{where } \{ (x_1 x_2), (x_1 x_3), (x_1 x_4), (x_1 x_5) \} \in \underline{A}$$

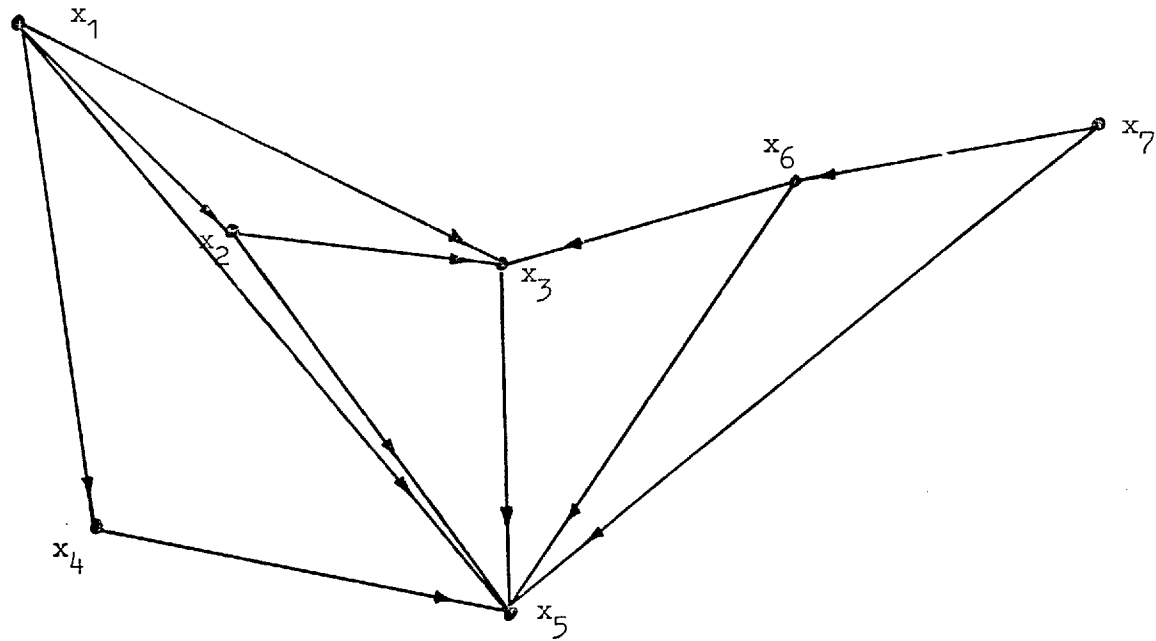


FIG. 3.6 GRAPHICAL REPRESENTATION OF THE SET A

The correspondence set is found by a simple search technique through the set of sectors A.

So far the above set of routes are those with routings connecting five cities or less. Before this set can be transferred to the set R, all routings which do not terminate at the airline's base must be eliminated. Not all the routes that remain will be suitable for transfer into R; some will have leg stage distances which are so great that such routes would be meaningless. A maximum distance of 4990 miles has been included in the program, and any routes which has a leg distance greater than this is also eliminated. The remaining routes now constitute the set R'.

The next stage of the route generator searches through the set R' to find all the subsets R'_{ij} ($\forall \{ij\} \in \underline{A}$). These subsets store the possible routes on which a given sector demand can be carried.

The final phase of the route generator searches through all $\underline{R}'_{ij}$ checking that the number of intermediate stops on the routes in that set for the particular city-pair is not greater than the maximum permissible. Those routes which violate this constraint are eliminated from $\underline{R}'$ and $\underline{R}'_{ij}$ to finally give $\underline{R}$ and $\underline{R}_{ij}$.

3.6 THE STANDARD ALLOCATION ALGORITHM

The fundamental philosophy of the traffic allocation algorithm is that a heuristic approach should be used to give an initial solution to the problem followed by modifications to that solution which will guide the total allocation towards the global optimum. The Standard Allocation Algorithm produces the initial solution and is the centre of all the modifications - which are discussed in 3.7, 3.8, 3.9, and 3.10.

The algorithm is as follows:-

STEP 1 Choose the next city-pair whose passengers must be assigned to an aircraft/route combination.

STEP 1.1 Find the set of city-pairs for which the base demand is not equal to zero.

$$\text{i.e. } \underline{A}^1 = \{ \underline{ij} \} \in \underline{A}$$

$$\text{where } D_{ij} \neq 0$$

STEP 1.2 Find the sub-set of $\underline{A}^1$ for which the minimum number of flights required must be direct.

$$\text{i.e. } \underline{A}^2 = \{ \underline{ij} \} \in \underline{A}^1$$

$$\text{where } N_{\min ij} = N^*_{\min ij}.$$

$$\text{If } |\underline{A}^2| = 0$$

$$\text{then } \underline{A}^2 = \underline{A}^1$$

STEP 1.3 Find the sub-set of $\underline{A}^2$ of city-pairs with the minimum number of possible routings.

$$\text{i.e. } \underline{A}^3 = \{ \underline{ij} \} \in \underline{A}^2$$

$$\text{where } |\underline{P}_{ij}| = \min$$

STEP 1.4 Find the city-pair in $\underline{A}^3$ which has the least base demand.

$$\text{i.e. } i^*j^* \in \underline{A}^3$$

$$\text{where } D_{i^*j^*} = \min$$

STEP 2 Allocate passengers from the chosen city-pair i^*j^* to the aircraft/route combination which generates the greatest operating profit.

$$\text{i.e. } \text{MAX } OP_f$$

$$(a, \underline{r})$$

$$\text{where } OP_f = \sum_{ij} [Y_{ij} \cdot P_{ijf}] - DOC_f$$

$$\underline{r} \in \underline{R}_{i^*j^*}$$

$$i, j \in \underline{r}$$

STEP 2.1 For each aircraft/route combination find P_{ijf} for all $i, j \in R_{i^*j^*}$

$$P_{ijf} \leq \frac{D_{ij}}{N_{\min ij}} + \Delta D_{ij}$$

subject to

$$\left\{ \left[LF_{\max pq} \cdot \sum_a (c_{apq} \cdot n_{pqa}) \right] - \sum_a P_{pqa}^* \right\} \geq 0$$

Should the load factor constraint be violated, the 'Priority Loading' rule (see 3.6.1) is used.

STEP 2.2 For each aircraft/route combination calculate DOC_f

In calculating $OP_f(a, r)$ the revenue from the one-way demand is calculated, then doubled to give total revenue. Therefore, DOC_f must be the cost for the total flight cycle.

Total Variable Cost = $\sum$ Leg Costs + $\sum$ Engineering costs

Leg Costs = Fuel costs + Landing Fees + Handling charges

$$\text{Fuel costs} = \sum_{\substack{i=1,n \\ j=2,n+1}} \{(\text{fuel}_{ij})(\text{price}_i + \text{price}_j)\}$$

$$\text{Landing fees} = \sum_{\substack{i=1,n \\ j=2,n+1}} \{(\text{landing}_i + \text{landing}_j)\}$$

$$\text{Handling charges} = \sum_{\substack{i=1,n \\ j=2,n+1}} \{(\text{handling}_i + \text{handling}_j)\}$$

Engineering costs = Flight Costs + Hourly Costs

$$\text{Flight Costs} = \sum_{i=1,n} \{2.0 (\text{flight}_a)\}$$

$$\text{Time} = \sum_{\substack{i=1,n \\ j=2,n+1}} \{2.0 (\text{time}_{ija})\}$$

$$\text{Hourly costs} = \{(\sum \text{Time}) (\text{hourly}_a)\}$$

STEP 3 Store the allocation which corresponds to MAX CP_f(a,r)

STEP 4 Update traffic information and the availability of each aircraft type.

If the frequency constraint for a sector (city-pair) has been satisfied but not the demand one, then increase the minimum frequency by one.

STEP 5 Eliminate any redundant routes from the set $\underline{R}$ and its sub-set $\underline{R}_{ij}$

STEP 6 Repeat STEPS 1 to 5 until

either A) the demand and frequency requirements have been satisfied,
or B) all the aircraft in the proposed fleet have been utilised
 to the maximum.

3.6.1 Discussion on the Standard Algorithm

The fundamental philosophy, of both the Standard Algorithm and the modifications to it, is based on the fact that certain sector passengers are more difficult to accommodate in an optimal allocation process: in general, it will be those sectors with the most constraints. By dealing with these first, the algorithm is allowed greater flexibility in the allocation of the other sector passengers.

Consider the network shown in Fig. 3.7. As was explained earlier, one of the reasons for creating a multi-stop route, is to link the sectors which do not warrant non-stop flights. Now it is quite feasible that the relative magnitude of the sector traffic is

$$D_{14} , D_{34} \neq 0$$

$$D_{14} \ll D_{34}$$

$$D_{24} , \Delta D_{12} , \Delta D_{13} \dots\dots\dots = 0$$

and that

$$N_{\min 14} = N_{\min 34} = 1$$

$$N_{\min 24} = \dots\dots\dots = 0$$

and also that

$$\{ (Y_{14} \cdot D_{14} + Y_{34} \cdot D_{34}) - \text{DOC}_{1234} \} \leq \{ Y_{34} \cdot D_{34} - \text{DOC}_{34} \}$$

If the next sector to be allocated were {3-4}, and the passengers flying the chosen sector are allocated to the aircraft/route combination which yields the greatest profit (see STEP 2), then the resulting assignment would be D_{34} to the route 3-4 and D_{14} to the route 1-4. This is clearly inferior to the allocation of both D_{14} and D_{34} to the route 1-2-3-4, which would result from choosing the sector {1-4} as the next sector; this is the reasoning behind the heuristic rule of STEP 1.3.

The STEP 1.4 is really an extension of the reasoning in STEP 1.3. For the example shown in Fig. 3.8, the latter does not indicate the 'weaker' sector, hence the need for an additional step. The reason this rule does not make the previous one, 1.3, redundant is that there might be a situation where one of the normally 'weak' sectors is in fact strong and thus require specific flights. If, in the above examples D_{12} and $N_{\min 12} \neq 0$, then clearly it is important to choose the sector {1-2} before the others.

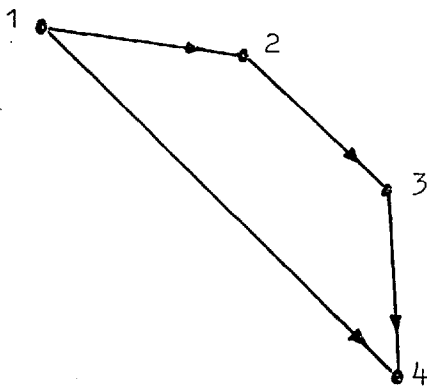
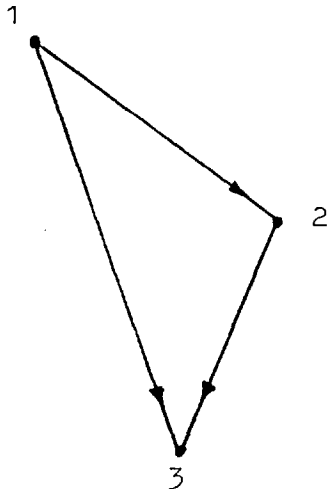


FIG. 3.7

$$\underline{R} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & \\ 3 & 4 & & \\ 1 & 4 & & \end{bmatrix}$$

FIG. 3.8



$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & \\ 2 & 3 & \end{bmatrix}$$

$$D_{13}, D_{23}, \neq 0$$

$$D_{13} \ll D_{23}$$

$$D_{12}, \Delta D_{12}, \Delta D_{13} \dots \dots \dots = 0$$

$$N_{\min 13} = N_{\min 23} = 1$$

PRIORITY LOADING RULE

In general, the desired P_{ijf} is equal to $\{(D_{ij}/N_{\min ij}) + \Delta D_{ij}\}$, so that the total demand is allocated equally to each of the flights flying the sector $\{ij\}$. However, because in many cases the maximum leg load factor constraint will not allow the desired allocation for all sectors on the route, a 'Priority Loading' rule is required.

Consider the traffic allocation problem shown in Fig. 3.9. If passengers are to be 'loaded' onto route 1-2-3-4-, then it seems sensible to start the loading process with those passengers flying out of City 1 and finish with those flying the sector $\{3-4\}$. The resulting allocation (Table 3.2) shows that only the passengers assigned to the sector $\{3-4\}$ are not at the desired level - this is because of the limiting load factor on the leg $[3-4]$. This means that another flight will have to be provided, in the second iteration through the algorithm, for the unsatisfied passengers on sector $\{3-4\}$, i.e. $D_{34} = 8$: this flight will clearly be along the route 3-4. If the sector passengers of $\{1-4\}$ had been loaded last instead of those of $\{3-4\}$ then the 8 unsatisfied passengers on sector $\{1-4\}$ would have to be allocated to a flight along the route 1-2-4. the second solution is nearly always inferior to the first, since

$$\{DOC_{34} - DOC_{124} + Y_{12} \cdot \Delta D_{12} + Y_{14} \cdot \Delta D_{14} + Y_{24} \cdot \Delta D_{24}\} \text{ is rarely positive.}$$

This analysis leads to the first definition of the Priority Loading rule, viz:

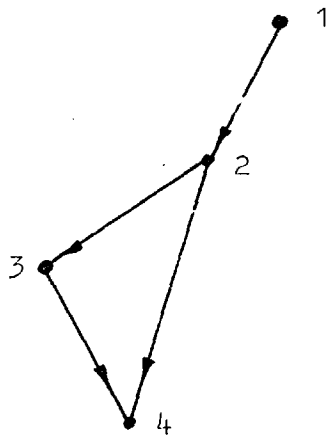
- Sector passengers are loaded in descending order of distance from the airline's base.

If the above priority rule is maintained when the example is modified slightly by making $D_{14} = 56$ and $N_{\min 14} = 2$, then the allocation to the route 1-2-3-4- will be that shown in Table 3.2 and $D_{14} = 28$ and $D_{34} = 8$ will be the unsatisfied demand. These will have to be assigned to another flight along the same route. If, however, there was some justification in loading the passengers on sector $\{1-4\}$ last, the allocation on the route 1-2-3-4 would be that shown in Table 3.3 and $D_{14} = 36$ would be the unsatisfied demand. This demand would be allocated to a flight on either the route 1-2-4 or the route 1-2-3-4, depending on whether

$$\{DOC_{1234} - DOC_{124} - Y_{13} \cdot \Delta D_{13} - Y_{23} \cdot \Delta D_{23} - Y_{34} \cdot \Delta D_{34}\} \text{ is positive or negative.}$$

If the demand is allocated to the first of these routes, then the modified priority loading rule has allowed the algorithm to find a more 'optimal' allocation.

FIG. 3.9



$\underline{R} =$

1	2	3	4
1	2	4	
2	3	4	
2	4		
3	4		

$C_a = 91$

Sector	D_{ij}	$N_{\min ij}$	ΔD_{ij}	$LF_{\max ij}$
12	0	-	5	-
13	0	-	2	-
14	28	1	0	61.0%
23	0	-	2	-
24	15	1	0	61.0%
34	25	1	0	66.0%

TABLE 3.2

Loading order	Sector	P_{ijf}		
		Leg 1-2	Leg 2-3	Leg 3-4
1	12	5	-	-
2	13	2	2	-
3	14	28	28	28
4	23	-	2	-
5	24	-	15	15
6	34	-	-	17
Total Leg Passengers		35	47	60
Leg Load Factor		38.4	51.6	66.0

The foregoing analysis is not the only justification for the new order in the loading procedure. The following conceptual analysis will also lead to the same conclusion.

The demand on city-pair {1-4} must be allocated to at least two different flights. If the desired level of allocation is not achieved on the first, then the unsatisfied demand will, more likely than not, be accommodated on the second flight with no cost penalty to the total allocation. This, however, is not true for the other sectors. If the desired level of allocation is not achieved on the first flight, then the allocation of the unsatisfied demand is very likely to include a cost penalty.

TABLE 3.3

Loading order	Sector	P_{ijf}		
		Leg 1-2	Leg 2-3	Leg 3-4
1	12	5	-	-
2	13	2	2	-
6	14	20	20	20
3	23	-	2	-
4	24	-	15	15
5	34	-	-	25
Total Leg Passengers		27	39	60
Leg Load Factor		29.7	42.9	66.0

The modified priority loading rule is therefore:-

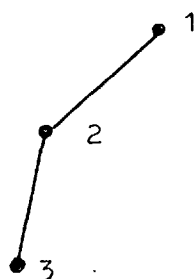
Sector passengers are loading in ascending order of unsatisfied flights. For each level of number of unsatisfied flights, the sector passengers are loaded in descending order of distance from the airline's base.

The number of passengers loaded from each city-pair is given by:

$$\text{MIN} \left\{ \begin{array}{l} D_{ij} + \Delta D_{ij} ; \\ \frac{\quad}{N_{\min ij}} \end{array} \right. \quad \left. \begin{array}{l} \text{the maximum number of passengers} \\ \text{as specified by the load factor} \\ \text{constraints.} \end{array} \right\}$$

The maximum load factor constraint is in fact a combination of two constraints. The first is the maximum effective seating capacity on the legs on which the sectors lie, and the second is the maximum load factor on each leg - this is the minimum of the maximum permissible sector load factor appropriate to that leg. For example (see Fig. 3.10), the maximum number of passengers on leg [1-2] is 35 (70 x 0.5). The only load factor appropriate to the leg [2-3] is that permissible on sector 2-3. For although the sector passengers of {1-3} fly along this leg, their turnaway risk is unaffected by the load factor on this leg. Hence, the maximum number of passengers permissible on leg [2-3] is 70 (100 x 0.7).

FIG. 3.10



ij	C _{aij}	LF _{max ij}
1-2	70	60%
1-3	-	50%
2-3	100	70%

REDUNDANT ROUTES

The tableaux shown in Table 3.4 might arise after several iterations through the algorithm to find the optimal allocation to a network similar to that shown in Fig. 3.9. It is unlikely that the algorithm will choose routes which start at City 1 to carry the unsatisfied passengers of sectors {2-4} and {3-4}, unless of course D_{ij} is very large. Therefore, in order to reduce the running time of the algorithm, a facility is available to eliminate routes which are unlikely to be chosen in future iterations; i.e. for the above example, the routes 1-2-3-4 and 1-2-4. This stage can be by-passed should the extra generated traffic be large enough.

TABLE 3.4

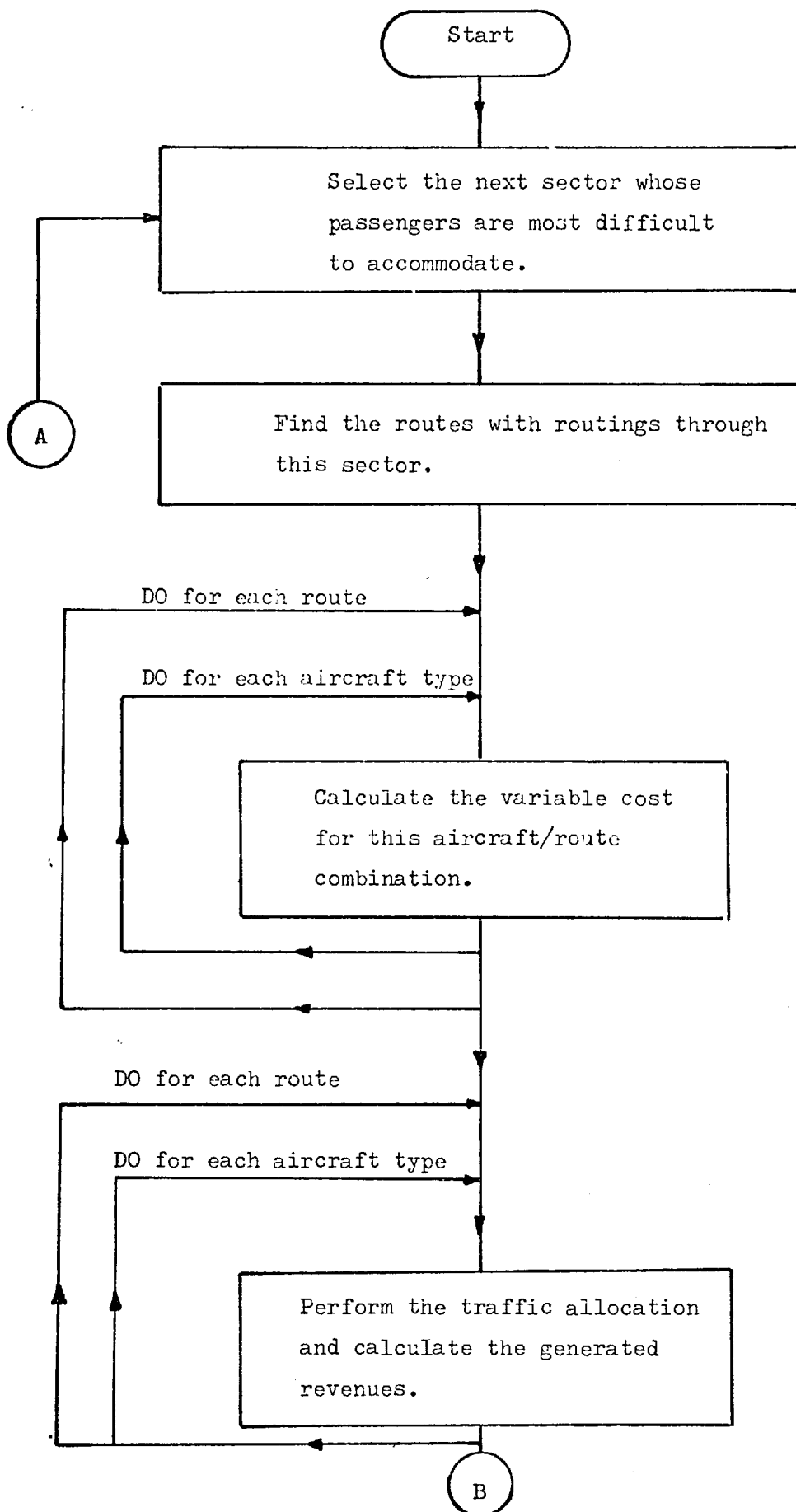
Sector	D_{ij}	$N_{\min ij}$	ΔD_{ij}
12	0	-	5
13	0	-	2
14	0	0	0
23	0	-	2
24	15	1	0
34	25	1	0

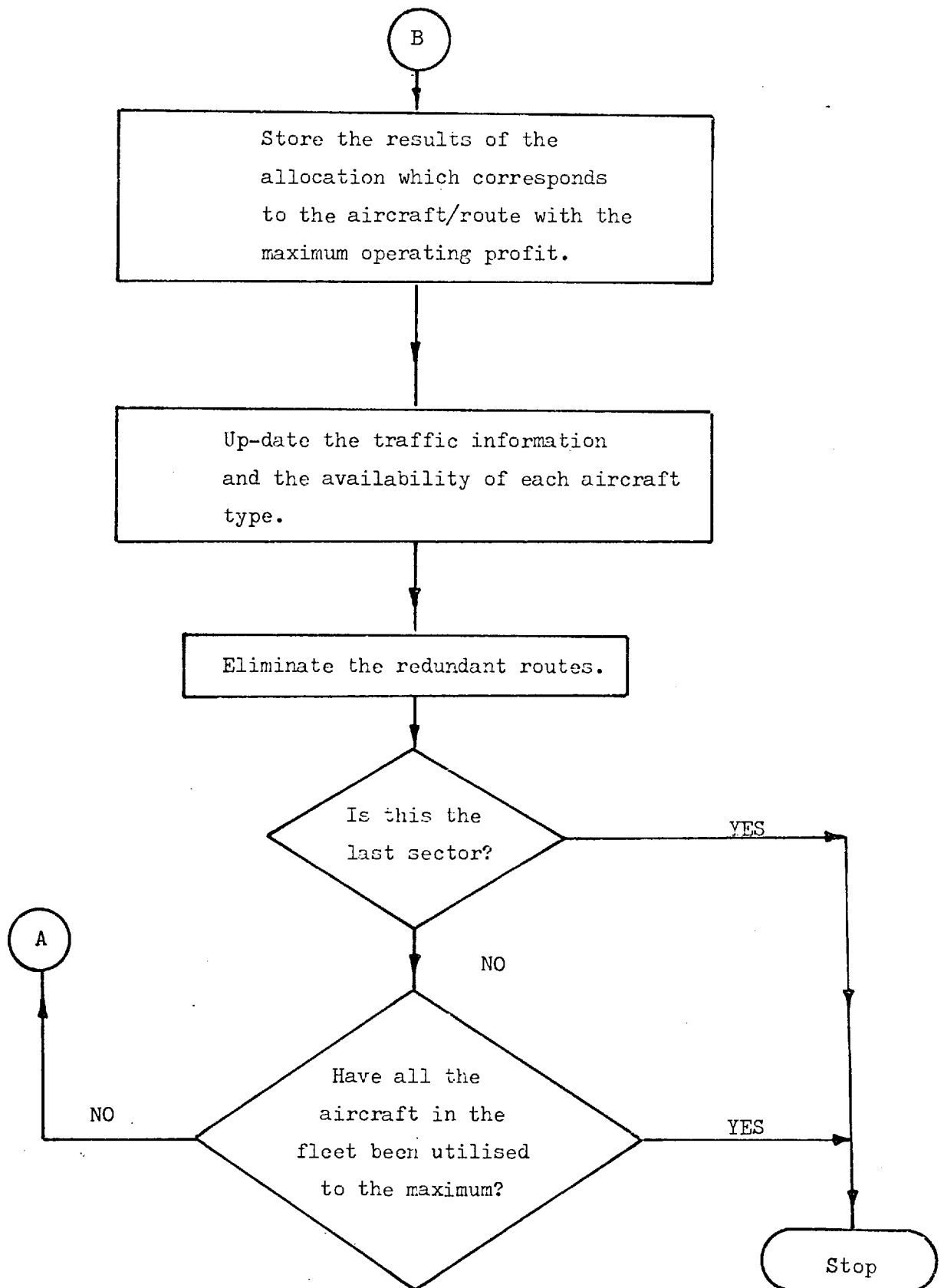
The Standard Traffic Allocation Algorithm, like all other heuristic algorithms, cannot search for the 'global optimum' explicitly, but does optimise 'locally' at each iteration in order to 'guide' the solution towards the global optimum. Inherent in such heuristic methods is the optimisation of a particular iteration irrespective of the consequences on future iterations. As was stated earlier, the solution from a heuristic algorithm has to be modified through further heuristic rules in an attempt to guide the solution towards the 'global optimum'. In the traffic allocation problem, the inefficient allocations arise from four problem areas:-

1. the characteristics of the aircraft types;
 2. the characteristics of the generated routes;
 3. the allocation of the desired level of passengers;
- and
4. the limited availability of some of the aircraft.

The modifications to the Standard Algorithm devised to overcome these difficulties are discussed in the next four sections, whilst the standard Algorithm is shown diagrammatically in Fig. 3.11.

FIG. 3.11 THE STANDARD ALGORITHM





3.7 THE CHARACTERISTICS OF THE AIRCRAFT TYPES

An airline has to choose a fleet of mixed aircraft types because, in most cases, one aircraft does not offer the complete flexibility required to operate a heterogeneous route structure efficiently. In general, one aircraft type may be best suited to operate on high density long-haul routes, whilst another is best suited to low density medium-range routes. The problem which arises because of these varying characteristics is that, given a set of routes and the associated demand and frequency requirements, is it more profitable to use the aircraft which will satisfy these requirements exactly or to use smaller aircraft which will yield extra revenue derived from an increase in the quality of service due to the higher level of frequency offered.

In the analysis which follows, three aircraft types are considered and their characteristics are shown in Table 3.5. The cost equation gives the cost of one flight per week operated for fifty-two weeks in the year.

Aircraft Type	Aircraft Seating Capacity (Seats)	Aircraft Range (N.M)	Aircraft Cost Equation $\$ = a + b \text{ (E.S.A.D.)}$	
			Intercept (a)	Gradient (b)
1	151	3500	102,000	131.6
2	132	1330	73,600	103.2
3	99	2200	62,300	90.1

TABLE 3.5 AIRCRAFT CHARACTERISTICS

In the discussion on the standard traffic allocation algorithm, it was stated that at each iteration the desired level of passengers flying the sector $\{ij\}$ to be allocated to a specific flight is given by $\{(D_{ij}/N_{\min ij}) + \Delta D_{ij}\}$, and that the only reason this value is not attained is because the capacity constraint has been reached. Hence, if ΔD_{ij} is a function of $(n_{ij} - N_{\min ij})$, where n_{ij} is the number of flights offered on the sector $\{ij\}$, it is clear that if D_{ij} can be allocated to $N_{\min ij}$ flights, then the algorithm will never reach the stage where it can specifically consider the effects of increasing the frequency on this city-pair. Consider the example shown in Fig. 3.12. Also included in the information for this traffic allocation problem is the formula

$$\frac{D'_{ij}}{D_{ij}} = \left\{ 1 + 0.2 \left[\frac{n'_{ij} - 1}{N_{ij}} \right] \right\}$$

which gives the extra demand generated as a result of an increase in the number of flights above the minimum required.

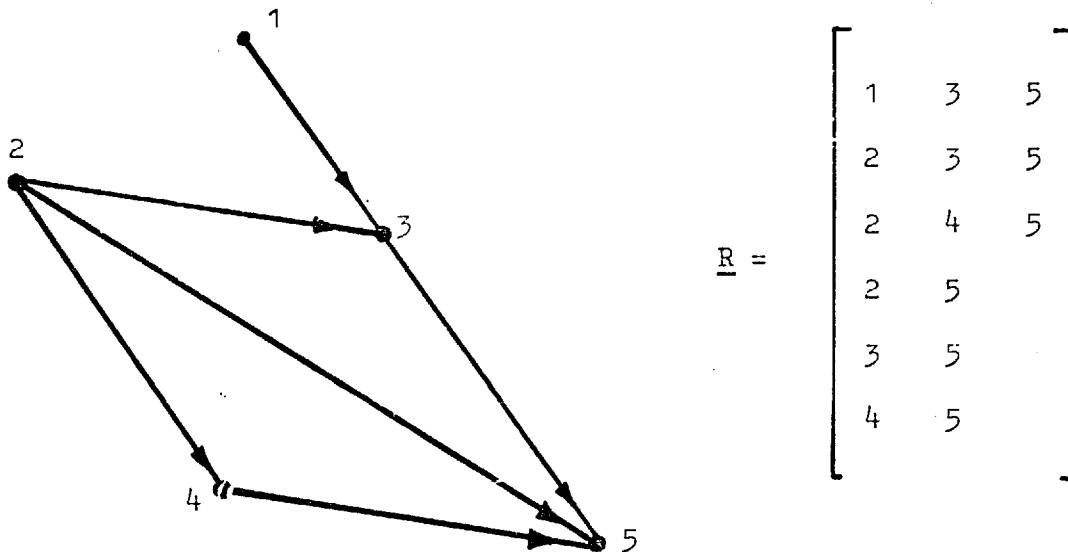


FIG. 3.12 PROBLEM OF AIRCRAFT CHARACTERISTICS

Sector	E.S.A.D.	Yearly Demand D_{ij}	Min Freq (weekly) $N_{\min ij}$	Revenue Rate Y_{ij}	Maximum Load Factor $LF_{\max ij}$
1 - 5	5000	3706	2	233.40	57%
2 - 5	1828	11519	5	233.40	57%
3 - 5	1624	8037	5	216.38	57%
4 - 5	1532	1500	2	199.48	57%
1 - 3	314	-	-	-	-
2 - 3	236	-	-	-	-
2 - 4	392	-	-	-	-

FIG. 3.12 PROBLEM OF AIRCRAFT CHARACTERISTICS

Using this information an experience analyst produced the allocation shown in Table 3.7, which shows that his solution includes extra passengers due to an increase in the number of flights offered on sectors {2-5} and {3-5}. Whilst producing this solution, the analyst was aware from the start of the possible gains which would accrue as a result of assigning the smaller aircraft to increase the level of frequency. However, the algorithm cannot 'see' further than each iteration, and as a result did not allocate the smaller aircraft to produce the extra flights; its allocation is shown in Table 3.6.

The importance of transforming the solution in Table 3.6 to that in Table 3.7 is clear - an increase in operating profit of \$211,985. What, perhaps, is not so obvious is the method of achieving this transformation. A complete enumeration of the possible combinations or a branch and bound search, would be theoretically correct, but would be completely out of place in an algorithm which is supposed to be fast. The method devised to analyse the effects of increases in the frequencies offered is the introduction of 'perturbations' to the standard allocation algorithm, followed by a comparison of the profitability of the 'perturbed solution' to that of the 'standard solution'.

A 'perturbation', in the context of heuristic rules formulated to improve the solution of the traffic allocation problem, is defined as a reduction in the number of aircraft/route combinations which the standard algorithm can choose in its allocations. The clue to finding the appropriate perturbation for the problem at hand, lies in the respective assignment of aircraft to the route 2-3-5 of the two solutions, shown in Tables 3.6 and 3.7. The total cost of the four flights by aircraft 3 assigned to this route in the second solution is less than the equivalent three flights by aircraft 1 in the first. Further analysis show that it will only be for comparatively high levels of frequencies that $(n+x)$ flights by a smaller aircraft type will be cheaper to operate than (n) flights by a larger type. Furthermore, it will only be for those sectors with comparatively high levels of minimum frequencies that the extra generated traffic will be large enough to generate a significant increase in revenue, since only these sectors will have a large enough base demand to create the necessary level of extra traffic. It, therefore, appears that, if the solution is going to be improved by increasing the number of flights offered in a given sub-network*, then, this will occur when the smaller aircraft types are assigned to those sectors with the highest frequencies.

TABLE 3.6

Route	Aircraft Type	Number of Flights	Revenue (Total)	Costs (Total)	Allocation			
					Leg 1		Leg 2	
					P^*_{pqf}	LF^*_{pq}	P^*_{pqf}	LF^*_{pq}
1-3-5	3	2	1333226	598400	3706	36.0	5870	57.0
2-4-5	3	2	1319177	596000	4370	42.4	5870	57.0
2-3-5	1	3	2939375	1347300	7149	30.3	13022	55.3
			5591778	2541700				

* A sub-network is defined as a set of sectors and routes which can be analysed independently of the rest of the airline's network.

TABLE 3.7

Route	Aircraft Type	Number of Flights	Revenue (Total)	Costs (Total)	Allocation			
					Leg 1		Leg 2	
					P* pqf	LF* pq	P* pqf	LF* pq
1-3-5	3	2	1308559	598400	3706	36.0	5756	56.0
2-4-5	3	2	1292550	596000	4256	41.3	5756	56.0
2-3-5	3	4	2579629	1168800	5222	25.4	11511	56.0
2-5	3	1	671725	227200	2878	56.0		
			5852463	2590400				

A 'Gamma Perturbation' restricts the choice of aircraft that can be assigned to the routes with routings through the sector with the greatest minimum frequency. Initially it restricts the allocation process by allowing only one aircraft type to be assigned to the first ($N_{\min ij}$) flights through this sector; the unassigned passengers, should there be any, can, however, be allocated to any aircraft type. To allow for some form of sensitivity analysis, the process is repeated for ($N_{\min ij} + 1$) flights; if there is no improvement in the solution then the analysis is terminated. However, should there be an increase in the profitability, the process is repeated for ($N_{\min ij} + 2$), ($N_{\min ij} + 3$), ... flights, until there is no further gain. The complete Gamma Perturbation process includes an analysis for each aircraft type and for each of the sectors with the greatest minimum frequency.

A substitution of a small aircraft for a larger one is bound to raise the level frequency, and this may create a paradox between the underlying philosophy of the priority 'loading rule' in the standard algorithm and its application. For the above example, consider the demand and frequency constraints that remain to be satisfied after four iterations;

Sector	D_{ij}	$N_{\min ij}$
2 - 5	7149	3
3 - 5	5874	3

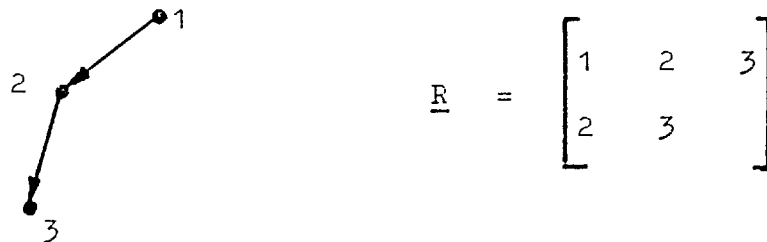
the gamma perturbation has been applied to sector $\{2-5\}$ with aircraft type 3 being the gamma aircraft for 6 flights. If the route 2-3-5 is being analysed, the 'priority loading' rule will dictate that sector $\{2-5\}$ be loaded first. However, the purpose of the gamma perturbation is to allocate more than three flights to this sector in the remaining iterations. Therefore, if the application and the philosophy of the 'priority loading' rule are not to contradict, sector $\{2-5\}$ must be loaded last. Thus, when a gamma perturbation is in operation, the 'priority loading' rule is modified such that the gamma sector is at the end of the 'loading' procedure. Using the modified 'loading' rule and the above gamma parameters, the perturbed allocation of aircraft to routes is the same as the analyst's.

The gamma perturbation is suitable for most, but not for all the members, of the family of problems which suffer from inefficient allocations due to the characteristics of the aircraft types. The example shown in Fig. 3.12 and that shown in Fig. 3.13 have the same fundamental problem, but their characteristics are such that two different approaches are required if the solutions, as given by the standard algorithm, are to be improved.

Unlike the previous example, the extra revenue generated from an increase in the quality of service is zero for all the sectors in this example, and, hence, there is no gain in allocating extra flights to these sectors. It, therefore, appears that the strategy of allocating D_{ij} passengers to $N_{\min ij}$ flights, which is incorporated in the standard algorithm, is correct. However, after one iteration through the algorithm, the demand and frequency constraints are $D_{23} = 16638$ and $N_{\min 23} = 3$, and therefore, with the largest aircraft having an effective seating capacity of only 5252 seats ($151 \times 0.67 \times 52$), the remaining passengers will be allocated to at least four flights.

The algorithm, unfortunately, cannot 'see' that far ahead, and will assume that it will require only three iterations to produce the desired allocation. The resulting allocation, which was achieved by maximising profit at each of the five iterations is shown in Table 3.8; the 'optimal' allocation is shown in Table 3.9.

FIG. 3.13



Sector	E.S.A.D.	D_{ij}	$N_{\min ij}$	V_{ij}	$LF'_{\max ij}$
1-2	532	-	-	-	-
1-3	5000	3690	1	146.96	67%
2-3	930	18200	4	124.72	67%

The comparison of the two solutions clearly indicates the importance of the average system load factor to the quality of the final allocation. In its attempts to maximise profits in the first four iterations, the algorithm was indirectly trying to find a solution with the highest average system load factor. Unfortunately, the same problem of 'local optimisation' occurs once again when the remaining 853 passengers have to be allocated, and therefore, the Delta perturbations have been designed specifically to find the solution with the 'maximum' average load factor for sub-networks with characteristics similar to those in this example.

TABLE 3.8

Route	Aircraft Type	Number of Flights	Costs (Total)	Allocation			
				Leg 1		Leg 2	
				P* pqf	LF* pq	P* pqf	LF* pq
1-2-3	1	1	396,500	3690	47.0%	5252	67.0%
2-3	1	3	672,300	15785	67.0%		
2-3	3	1	145,900	853	16.6%		
			1,214,700				

TABLE 3.9

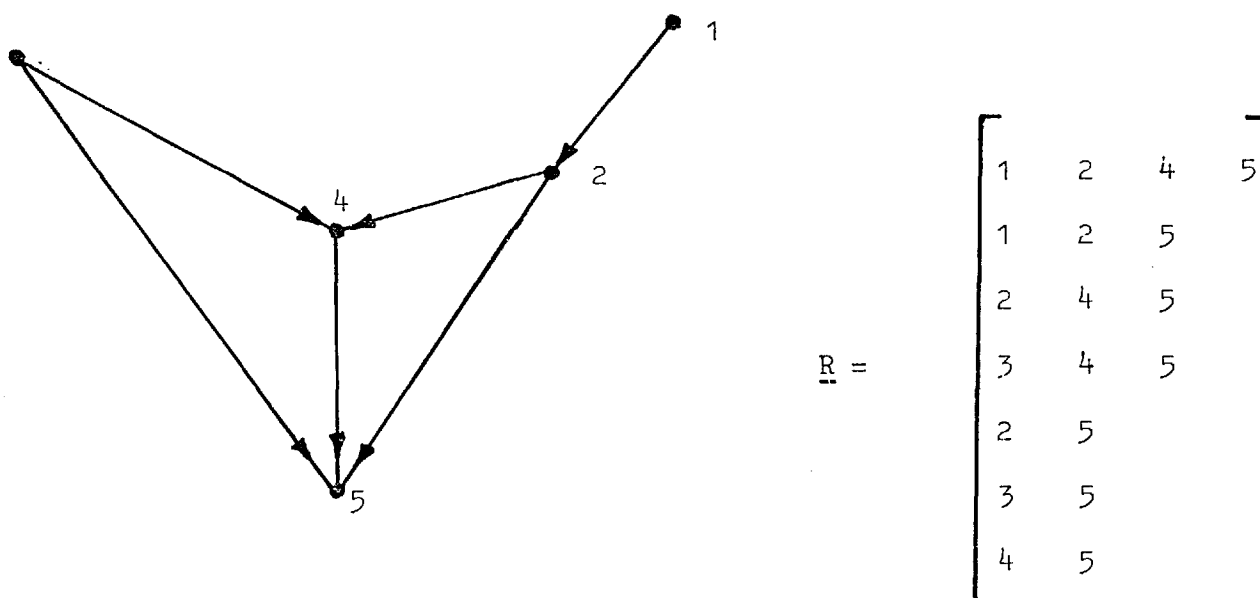
Route	Aircraft Type	Number of Flights	Costs (Total)	Allocation			
				Leg 1		Leg 2	
				P* pqf	LF* pq	P* pqf	LF* pq
1-2-3	1	1	396,500	3690	47.0%	5252	67.0%
2-3	2	3	507,600	13800	67.0%		
2-3	3	1	145,900	2838	55.0%		
			1,050,000				

The assignment of more than one aircraft type to a non-stop route, and the allocation of more than the minimum number of flights to the sector on this route, are the distinguishing characteristics of the sub-network in this example. The delta perturbations attempt to raise the average load factor by finding the 'optimal' allocation of the traffic previously allocated to this non-stop route. The method devised involves the 'loading' of only one aircraft type (the Delta Aircraft) to capacity. At each iteration, should the remaining passengers not constitute a capacity payload for the Delta Aircraft, then these are allocated to the appropriate aircraft. The complete Delta analysis is the application of one perturbation for each aircraft type.

3.8 THE CHARACTERISTICS OF THE GENERATED ROUTES

The previous discussion has emphasised the inadequacy of using an unmodified heuristic method. Because it adheres rigidly to the rules and optimises at a local level, it will frequently produce a sub-optimal solution. The method adopted to rectify this situation was to define the problem explicitly and then to devise a series of perturbations to the standard algorithm in order to allocate the passengers efficiently. The same approach is used in the following discussion on the sub-optimal solutions which arise because of the characteristics of the generated routes.

The routes are generated using the sector information initially read-in. These not only determine the routes which are eligible for assignment, but also the order in which the allocation occurs. These factors can, for certain sub-networks cause difficulties in the search for optimal solution.



Sector	E.S.A.D.	D _{ij}	N _{min ij}	Y _{ij}	LF _{max ij}
1-2	540	-	-	-	-
1-5	5000	1100	1	261.08	0.61
2-4	532	-	-	-	-
2-5	1272	3698	2	146.96	0.62
3-4	704	-	-	-	-
3-5	1645	4437	2	246.46	0.62
4-5	930	25790	7	124.72	0.67

FIG. 3.14 PROBLEM OF CHARACTERISTICS OF GENERATED ROUTES

In performing the allocation of passengers to the sub-network shown in Fig. 3.14, the standard algorithm will allocate in its first iteration the sector passengers D_{15} to one of two routes: route 1-2-4-5 or 1-2-5. The profits which accrue from each of these routes is \$ 338,340 and \$ 273,422 respectively with the consequence that the algorithm will allocate these passengers to the first of these options. Unfortunately, the total solution (shown Table 3.10) which follows as a result of allocating these passengers to the longer route is sub-optimal, since the solution (shown in Table 3.11), which is a consequence of allocating the passengers D_{15} to the route 1-2-5, is more profitable by \$ 277,482.

TABLE 3.10

Route	Aircraft Type	Number of Flights	Profit (Total)	Allocation					
				Leg 1		Leg 2		Leg 3	
				PAX	LF	PAX	LF	PAY	LF
1-2-4-5	2	1	338,340	1109	16.2%	2985	43.1%	5599	67.0%
2-4-5	2	1	316,503	1849	26.9%	4599	67.0%		
3-4-5	2	2	1,055,536	4437	32.3%	9199	67.0%		
4-5	1	3	1,298,473	15785	67.0%				
4-5	3	1	- 39,822	852	16.6%				
			2,969,030						

TABLE 3.11

Route	Aircraft Type	Number of Flights	Profit (Total)	Allocation			
				Leg 1		Leg 2	
				PAX	LF	PAX	LF
1-2-5	3	1	273,432	1109	21.5%	2958	57.5%
2-4-5	2	1	316,504	1849	26.9%	4599	67.0%
3-4-5	2	2	1,055,536	4437	32.3%	9199	67.0%
4-5	2	4	1,601,050	18278	66.5%		
			3,246,512				

The sub-optimality which arises through the algorithm choosing the longer routes is analogous to the situation where it chose the larger aircraft. In the latter case the optimal solution was reached through the application of Gamma Perturbations which substituted smaller aircraft on the sectors with the highest frequencies. The Alpha Perturbations will substitute routes with shorter routings through the sector which had previously been assigned the longest routing. The complete Alpha Perturbation process is shown diagrammatically in Fig. 3.15.

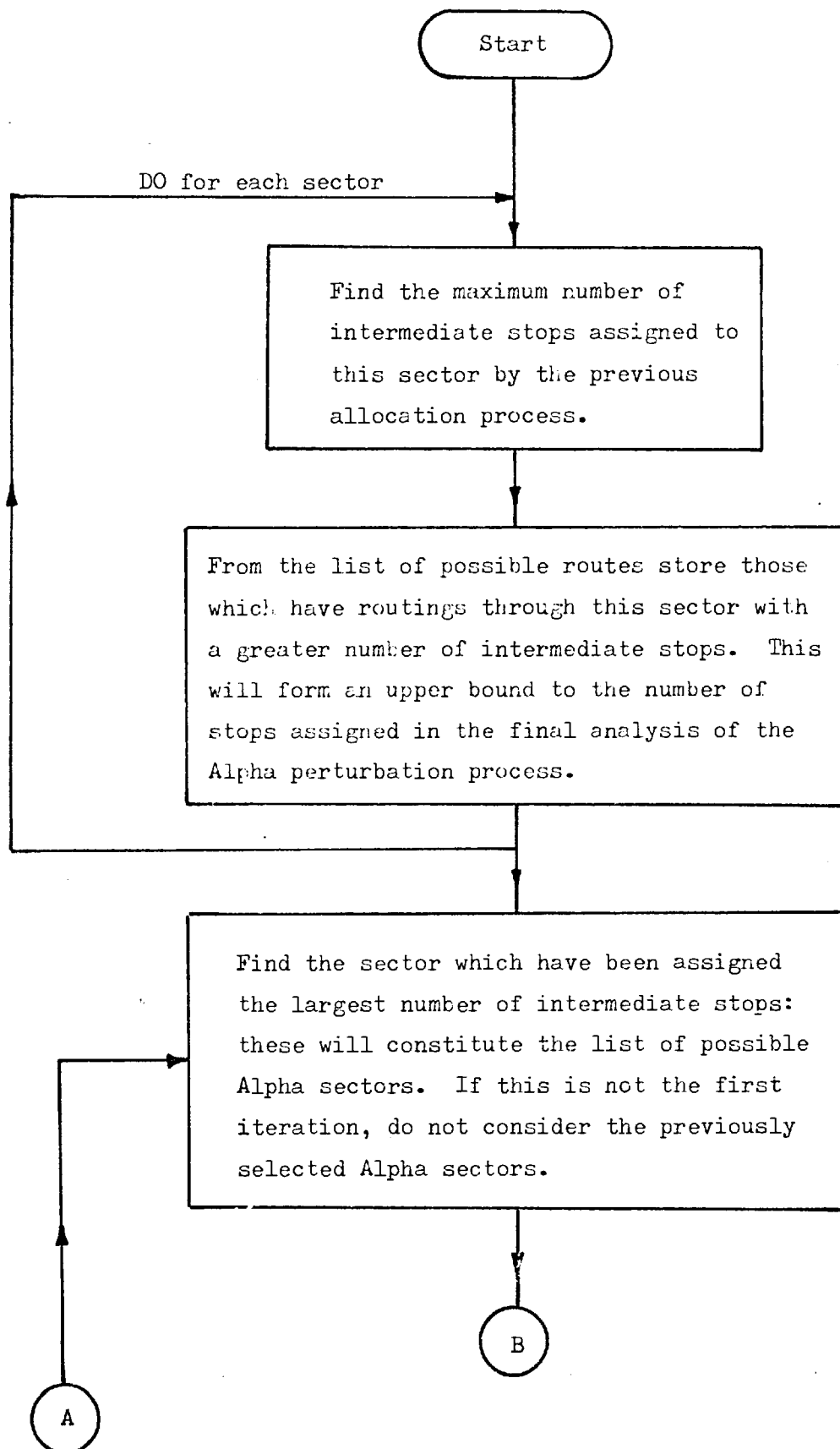
3.9 THE ALLOCATION OF THE DESIRED LEVEL OF PASSENGERS

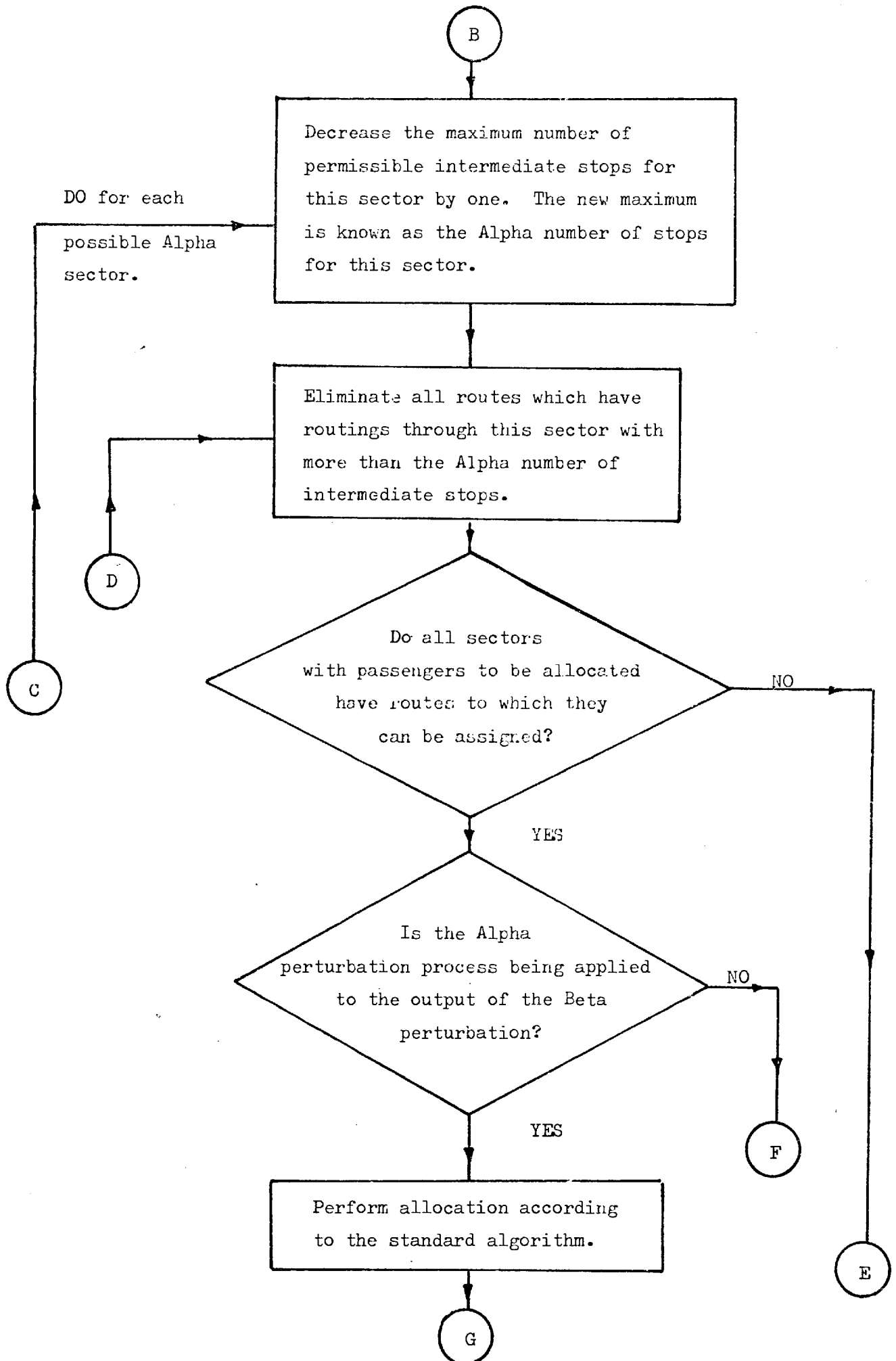
The Gamma perturbation was specifically designed to raise the level of frequency so as to benefit from the possible gains which accrue as a result of a higher level of service. However, in many cases the assignment of more than the minimum number of flights to a sector is not the sign of an efficient allocation, but the complete opposite. These inefficient allocations are the result of the algorithm trying to achieve the desired level of passenger allocations; viz.

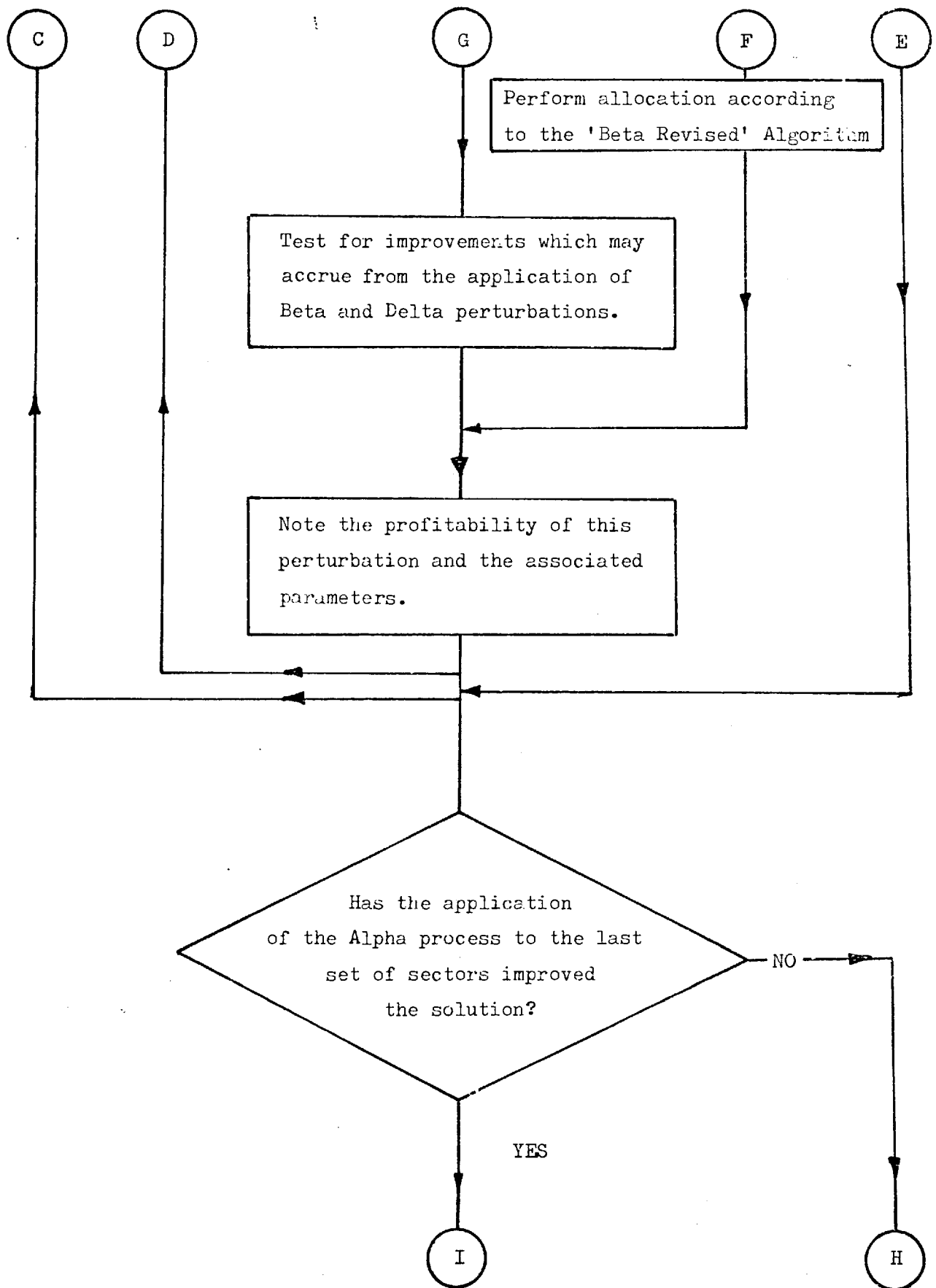
$$\text{Min } \left\{ \frac{D_{ij}}{N_{\text{min } ij}} + \Delta D_{ij} ; P_{ij} \text{ specified by the load factor constraint.} \right\}$$

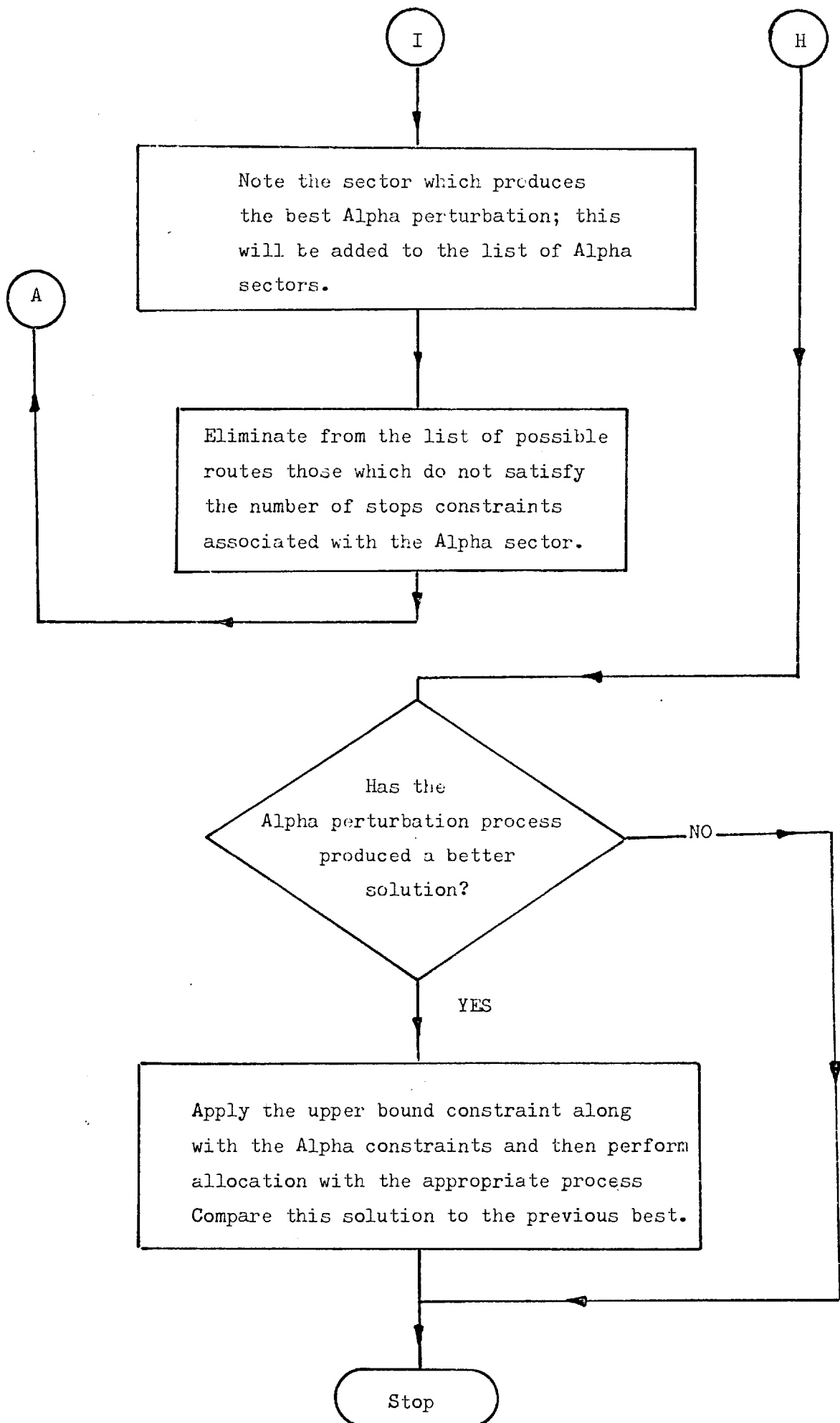
An analysis of many sub-networks which suffered from the over-assignment of aircraft revealed that two different approaches are required to resolve all but a very few members of this family of problems. The Delta perturbation attempts to find the 'optimal' solution by transferring the passengers on the problem sector from one set of aircraft to another. It does, however, restrict the re-allocation of passengers to those flying the sector on the non-stop route. The Beta perturbation, on the other hand, attempt to resolve the problem by transferring the excess passengers to other flights on the sub-network.

FIG. 3.15 THE ALPHA PERTUREATION PROCESS









The example shown in Fig. 3.14, discussed in the previous chapter, is modified slightly by making D_{45} equal to 25990. The application of the same Alpha perturbation produces the output shown in Table 3.12. Clearly, this solution is not optimal. However, the over-assignment of flights to the sector 4-5 can be easily resolved simply by transferring passengers from one flight to another. The redundant flight is obviously the one by aircraft type 3 on the route 4-5, since it carries the least number of passengers. By transferring the 79 passengers from this flight to the one on route 2-4-5, and then transferring the excess 79 passengers on sector 2-5 to the flight on route 1-2-5, an allocation is achieved (Table 3.13) which is \$146,073 more profitable.

The complete Beta Perturbation shown diagrammatically in Fig. 3.16 is a 'two-optimal' re-allocation process. When all the appropriate passengers have been aggregated, the allocations are performed according to the 'revised' algorithm. The algorithm needs to be revised because the original criteria used to determine 'the next most difficult sector' is not appropriate for the Beta analysis. By the very nature of the problem it is the Beta sector which is difficult to accommodate in an optimal allocation. Therefore, the criteria is revised by choosing the Beta sector as the 'next sector' when one more flight is required to satisfy its minimum frequency constraint. In order to maintain the philosophy of the 'next most difficult sector', the 'priority loading' rule is also modified at this critical condition so that the Beta sector is 'loaded' before the others.

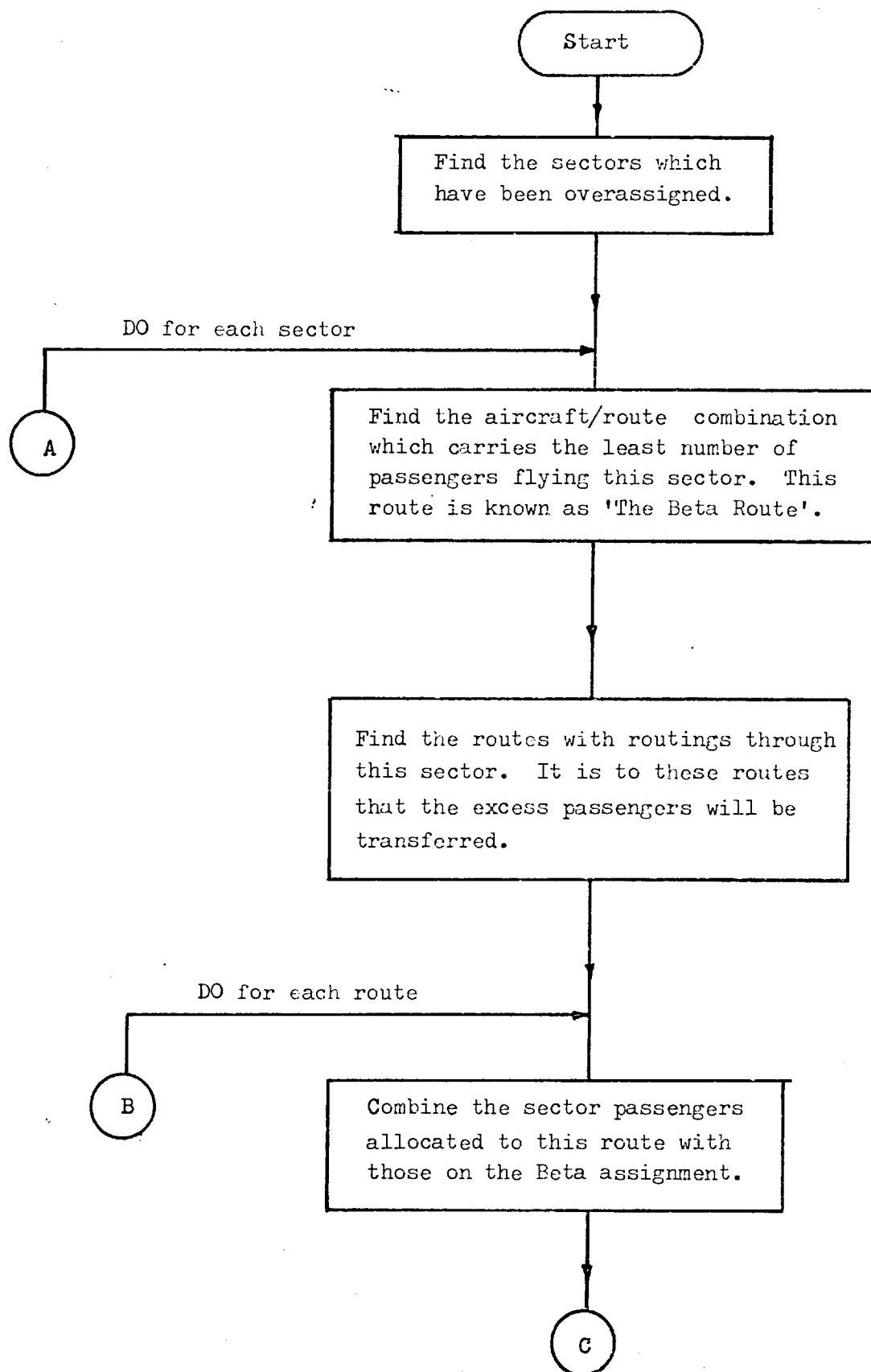
TABLE 3.12

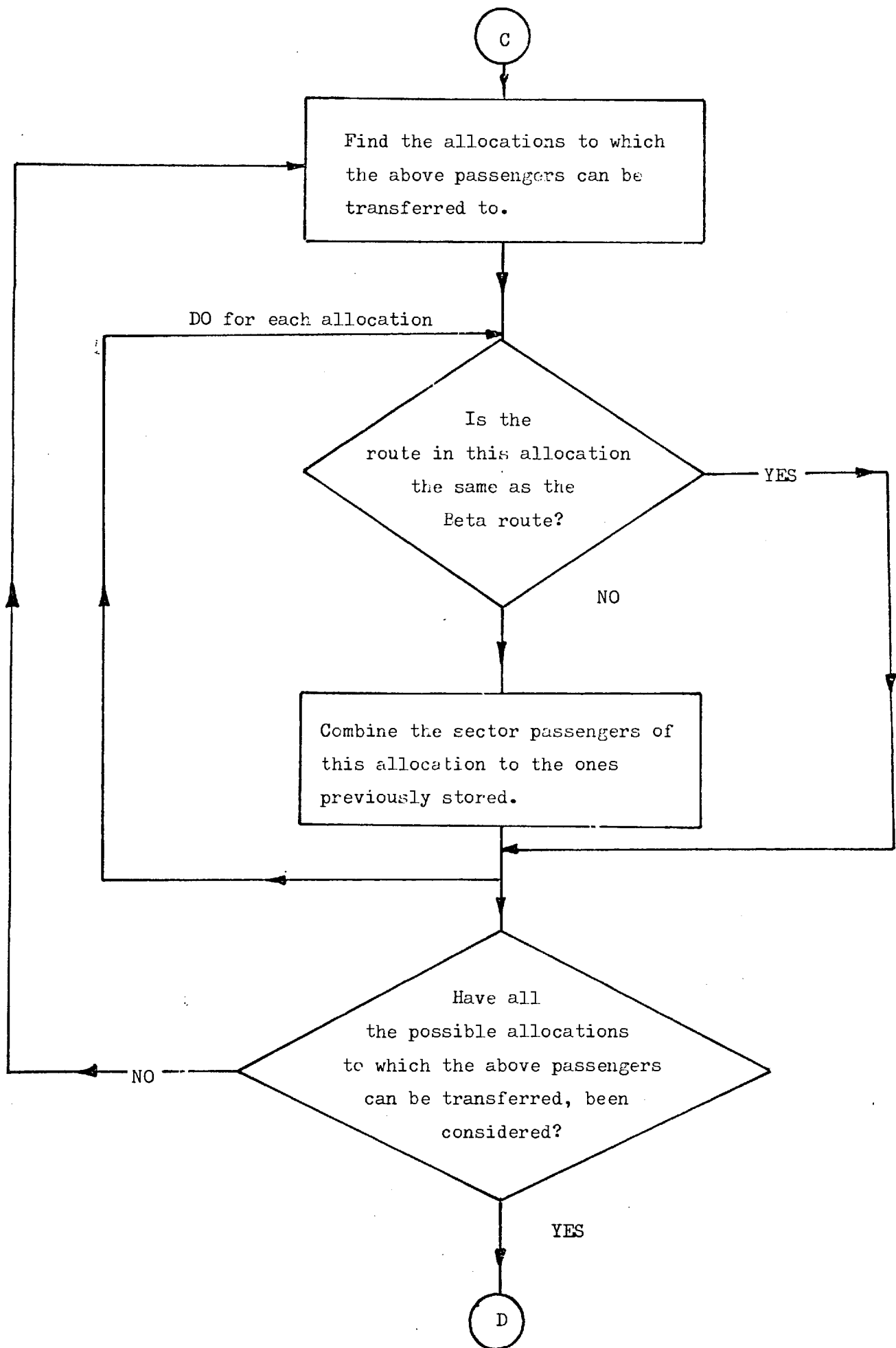
Route	Aircraft Type	Number of Flights	Profit (Total)	Allocation			
				Leg 1		Leg 2	
				P* _{pqf}	LF* _{pq}	P* _{pqf}	LF* _{pq}
1-2-5	3	1	273,432	1109	21.5%	2958	57.5%
2-4-5	2	1	316,504	1849	26.9%	4599	67.0%
3-4-5	2	2	1,055,536	4437	32.3%	9199	67.0%
4-5	2	4	1,616,143	18299	67.0%		
4-5	3	1	136,231	79	1.5%		
			3,125,384				

TABLE 3.13

Route	Aircraft Type	Number of Flights	Profit (Total)	Allocation			
				Leg 1		Leg 2	
				P* _{pqf}	LF* _{pq}	P* _{pqf}	LF* _{pq}
1-2-5	3	1	285,032	1109	21.5%	3037	59.0%
2-4-5	2	1	314,746	1770	25.8%	4599	67.0%
3-4-5	2	2	1,055,536	4437	32.3%	9199	67.0%
4-5	2	4	1,616,113	18399	67.0%		
			3,271,457				

FIG 3.16 THE BETA PERTURBATION PROCESS





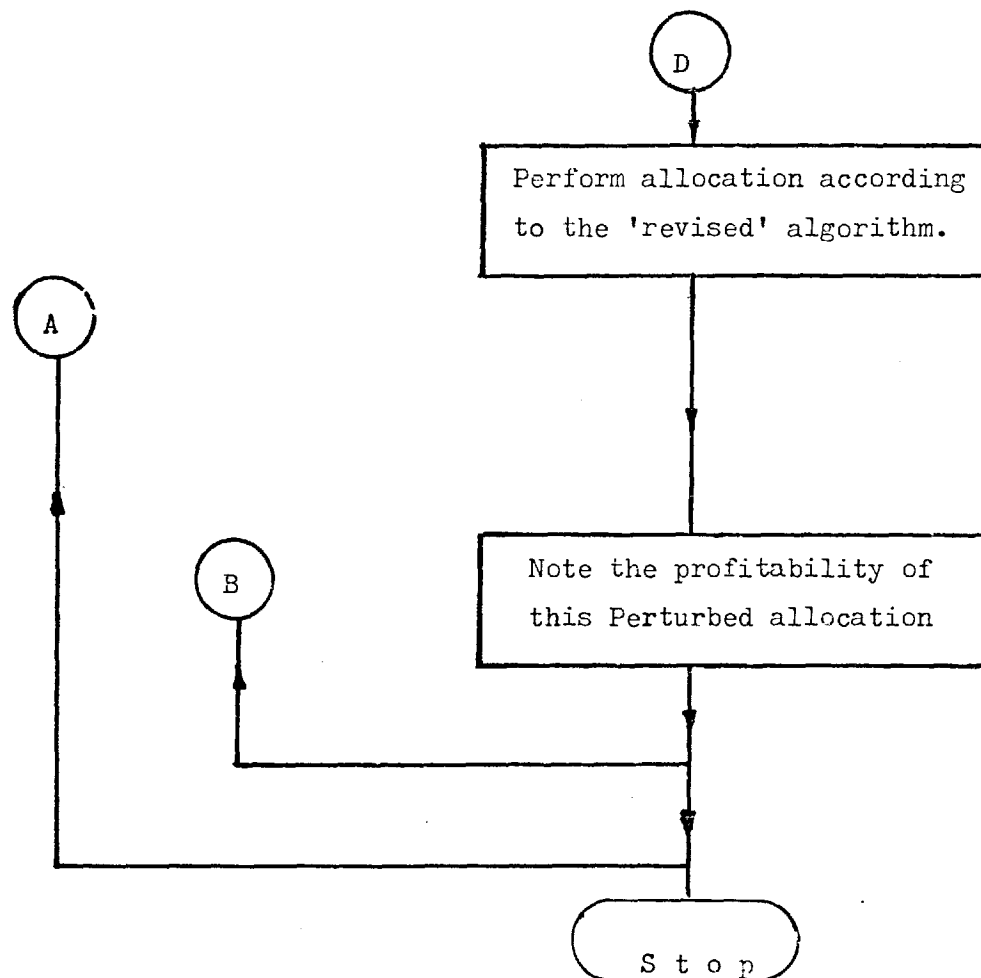


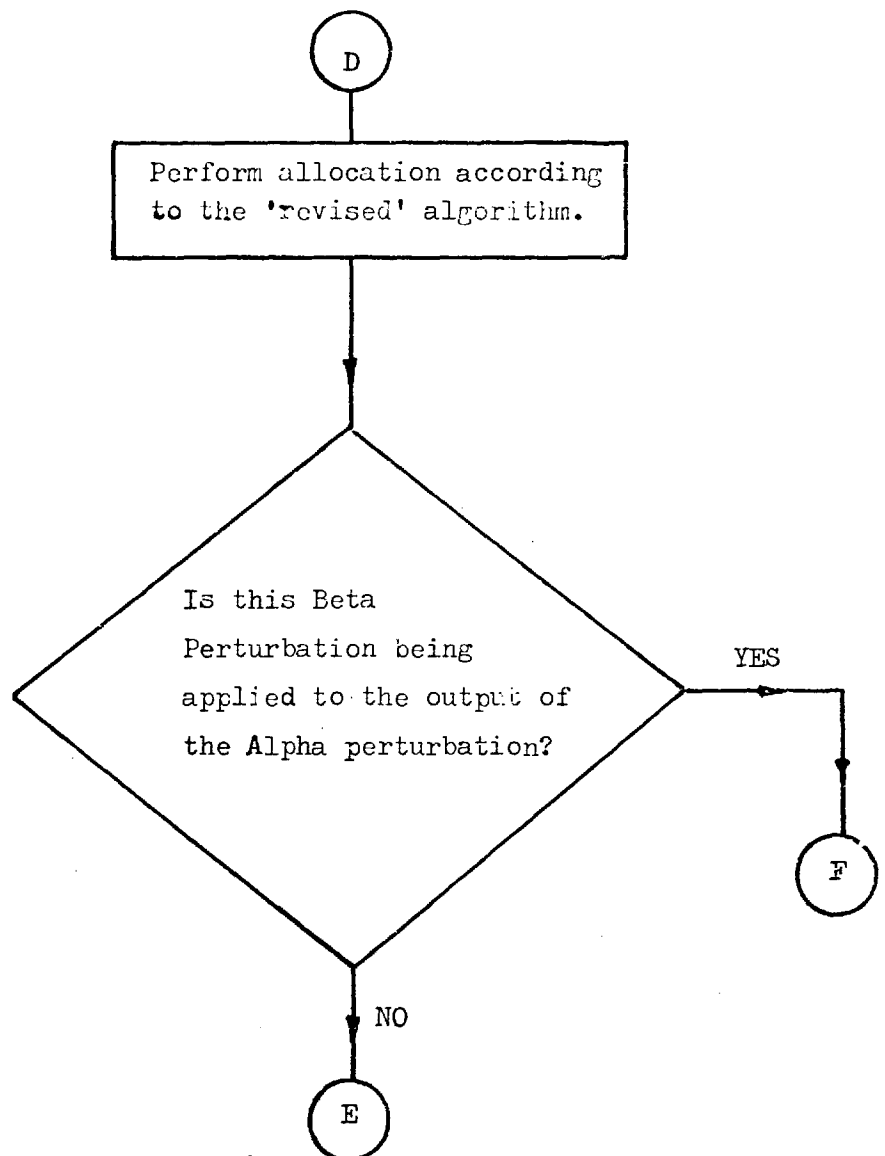
FIG. 3.16 THE BETA PERTURBATION PROCESS

The Beta perturbation is the transfer of passengers from the Beta flight to flights with routings through the Beta sector, followed by the re-allocations of the 'dislodged' passengers, if any, from the latter flights. The exclusion from the perturbation process of those allocations which have passengers assigned to the Beta route but not to the Beta aircraft/route combination, needs some explanation. In general, the Beta re-allocation passengers are the residue of 'optimal allocations' to the Beta route, and as such, there is no possibility of an effective transfer of passengers between these sets of allocations.

The exclusion of the above allocation serves a second purpose. Coupled with the inclusion in the re-allocation process of the aircraft/route combinations to which non-Beta sector passengers can be transferred, it, indirectly, allows the problem sector to be considered before the others at an early and appropriate stage; earlier and more appropriate than could

have been achieved with the 'revised algorithm' operating on its own. The reason being that the frequency constraint of the problem sector has been reduced to a level lower than those of the other sectors.

The Beta perturbation is applied to the output of the Standard allocation and to that of the Alpha perturbation (it is not applied to that of the Gamma perturbation since this would be counter-productive), as is the Delta perturbation. It, therefore, appears to be like the latter a perturbation of minor adjustments as opposed to fundamental ones of the Alpha and Gamma types. Unfortunately, this is not the case. The 'two-optimal' analysis is an important concept in many of the distribution and ware-house location heuristic models, and is therefore a major perturbation which requires itself, in some cases, minor adjustments; these come from the application of Alpha perturbations to the output of the 'two-optimal' re-allocation process. The tail-end of the Beta perturbation process is therefore modified, as shown in Fig. 3.17.



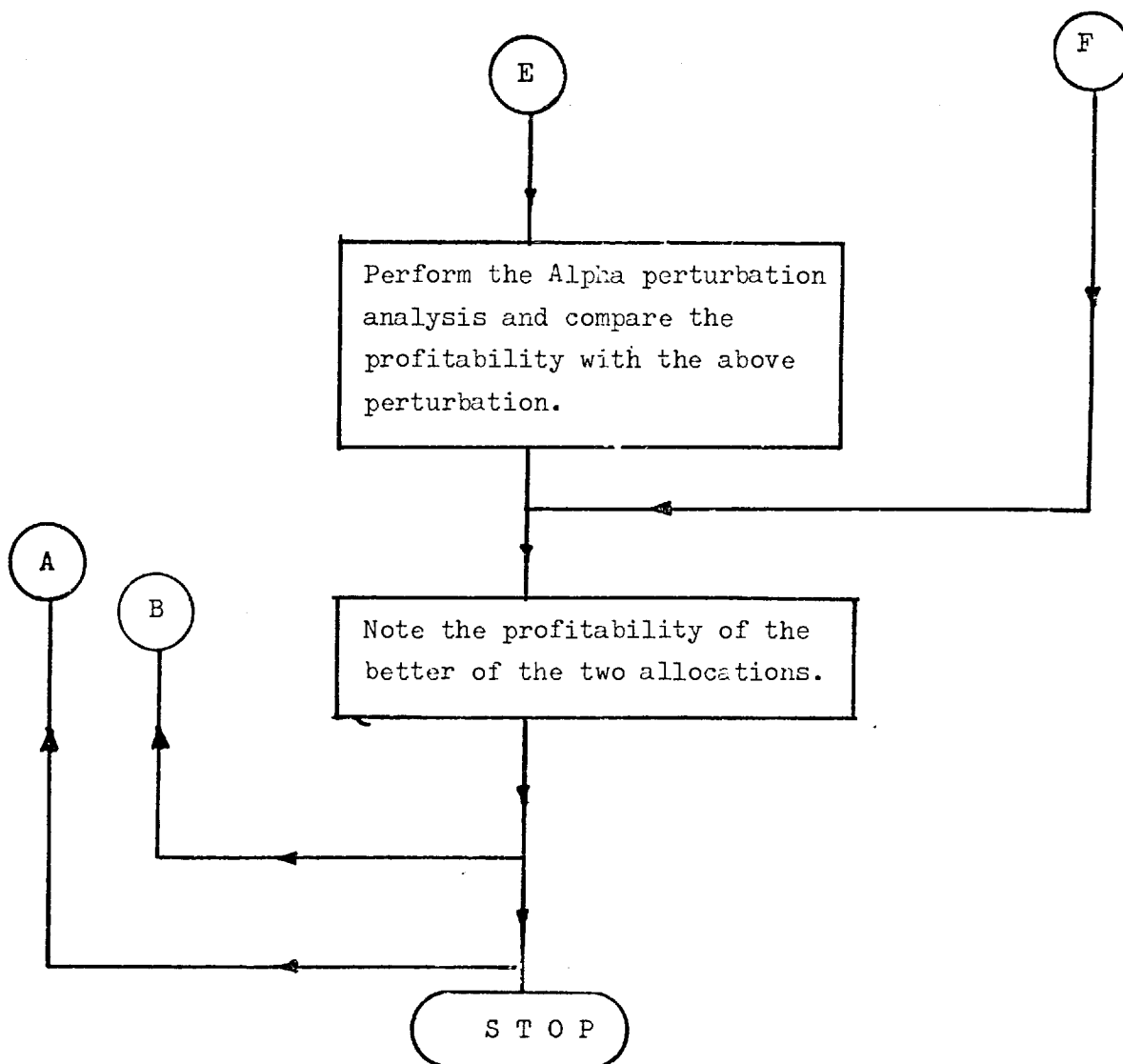


FIG. 3.17 THE BETA PERTURBATION PROCESS

3.10 THE LIMITED AVAILABILITY OF THE PROPOSED FLEET

The function of the traffic allocation algorithm is to assign aircraft to routes with the underlying objective of generating the maximum profit. To this purpose, the algorithm assigns at each iteration the aircraft which yields the greatest profit, where 'profit' is defined as:- total revenue minus total operating cost. In conditions where the availability of an aircraft exceeds the claims on it, there is no 'opportunity cost' associated with the employment of the aircraft, and the only cost involved is the operating cost, which includes such items as handling charges, landing fees and fuel costs. However, when the demand for an aircraft type exceeds its availability, the costing procedure must include the 'opportunity cost' associated with the employment of the aircraft if the objective of maximising profits is to be achieved.

The concept of 'opportunity costs' is simple and will be familiar to the reader, viz; the cost of using a resource in a particular operation is the benefit foregone (or opportunity lost) by not using it in its best alternative use. Consider the following example: sub-network A requires 2,000 hours of aircraft type 1 to generate £4,000 profit above the solution which does not use this aircraft, whilst sub-network B generated £3,000 more profit by employing the same aircraft for the same number of hours; these are the only sub-networks which can use this aircraft efficiently. If the total availability of the aircraft is 6,000 hours, then the opportunity cost of the hours employed by both A and B is zero - since there is no alternative use for these hours. However, should the availability be reduced to 2,000 hours, then the opportunity cost of the 2,000 hours used by A is £3,000 and that of the 2,000 hours used by B is £4,000.

As a result of this discussion it is clear that an operation will be justified in having a claim on the scarce resource up to the point where its marginal opportunity cost becomes greater than its marginal profitability. In the above example, for instance, if the profits are a linear function of time, then sub-network A will be justified in claiming all the 2,000 hours, since its marginal profitability is £2.0 per hour for 2,000 whilst its marginal opportunity cost is only £1.5 per hour. Unfortunately, although it is relatively simple to devise an optimal strategy for the distribution of the limited hours of an aircraft between sub-networks which have linear profit functions, most of the functions which the algorithm will have to analyse will be non-linear.

In general, whether the functions are convex, concave or linear, there exists one opportunity cost, which when applied to the traffic allocation algorithm, will resolve the problem of conflicting demands on the limited availability of a aircraft. Therefore, the fundamental strategy adopted to achieve the optimal allocation of an aircraft is to find this 'No Conflict' opportunity cost.

The method devised to find this 'No Conflict' cost is one of successive approximations. Initially, the actual availability is noted, and then at each iteration the availability of the aircraft is set to a very large number of hours and the opportunity cost adjusted so that the demand on the aircraft and its availability converge. The process is repeated until the two are within a given limit. Consider the sub-networks A, B₂, C and D which have the profitability functions shown in Fig. 3.18 and an aircraft which has a maximum utilisation of 5,000 hours: what is the 'No Conflict' opportunity cost for this example? The first iteration set the opportunity cost to zero with the result that the demand on the aircraft exceeds its availability by 3,000 hours (see Fig. 3.19). The second iteration set the opportunity cost to the average profitability achieved in the first iteration, with the result that the availability exceed the demand by nearly 2,000 hours. The third iteration, which set the opportunity cost to the intercept of the line connecting the two previous points on the graph, showed an excess capacity of nearly 1,000 hours. The fourth iteration resolved the conflict by setting the opportunity cost to \$0.35 per hour.

The method of successive approximation has three drawbacks, which relate to:-

- 1) computer processing time;
- 2) critical discontinuities; and
- 3) the interdependence of the aircraft
 'excess hours - opportunity cost' curves

These are now discussed in detail.

FIG. 3.18

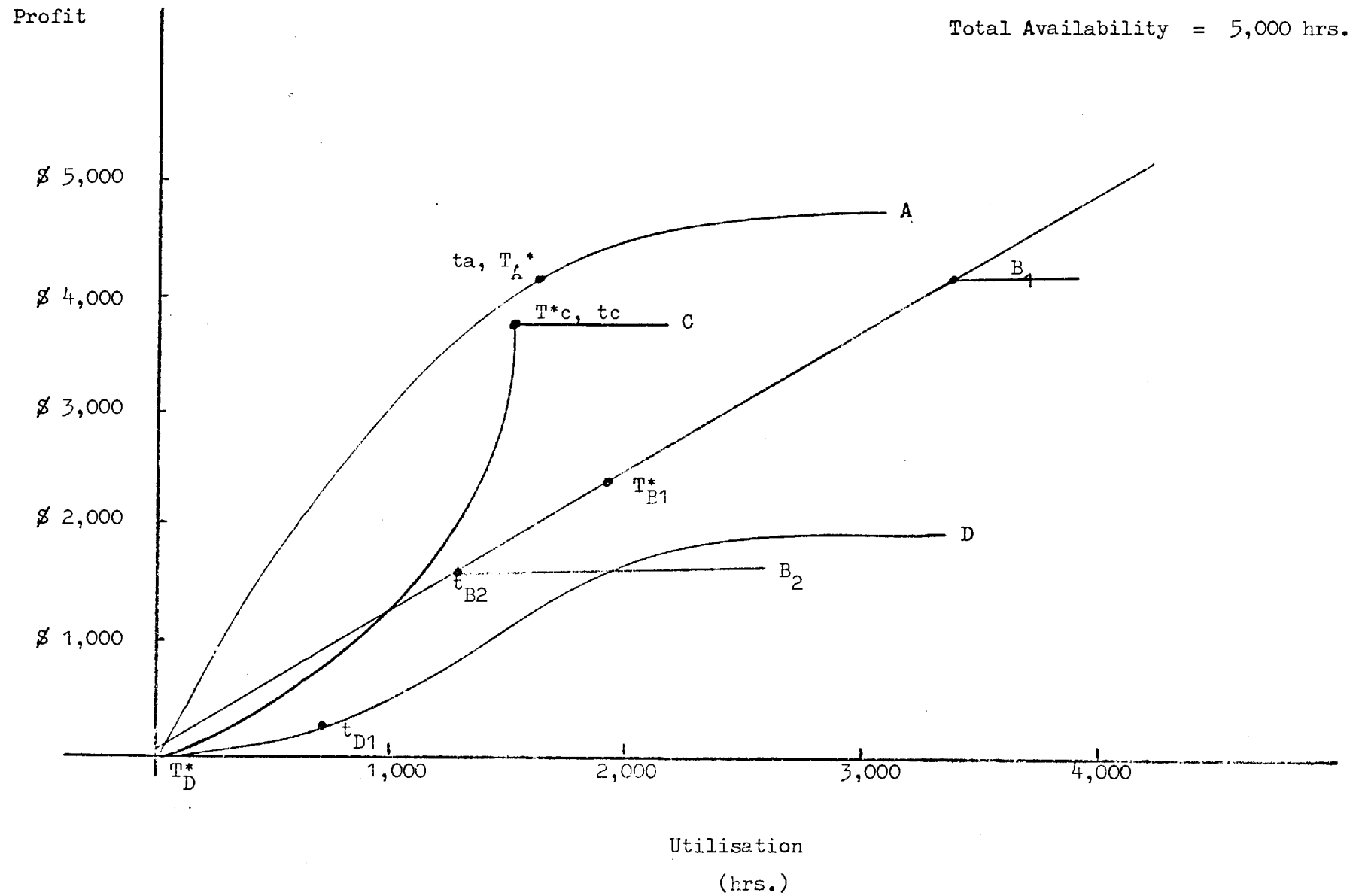
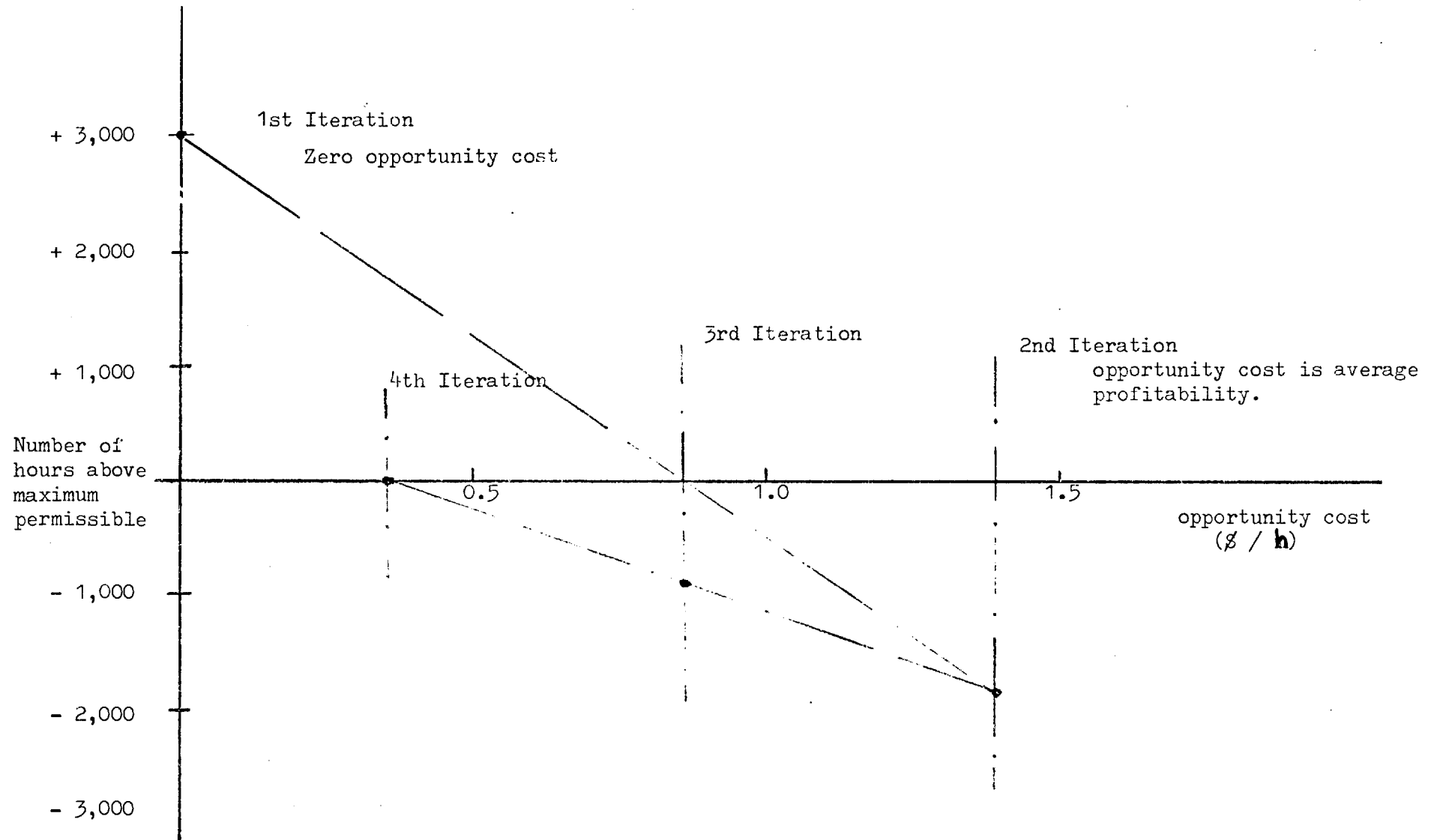


FIG. 3.19



Computer Processing Time

Depending on the nature of the sub-networks and on the tightness of the aircraft availability constraints, the method of successive approximations can be very expensive in terms of computer processing time. Unfortunately, this is an inherent feature of the method and very little can be done about it. Nevertheless, the running time has been reduced by making a suspect assumption (suspect in view of the occasional interdependence of the opportunity cost curves) that if a sub-network requires the same number of hours of the scarce aircraft at the most recent excess capacity and excess demand opportunity costs, then this sub-network is not considered in the next iteration. Furthermore, the sub-networks which do not utilise this aircraft type in the excess demand state are also not considered.

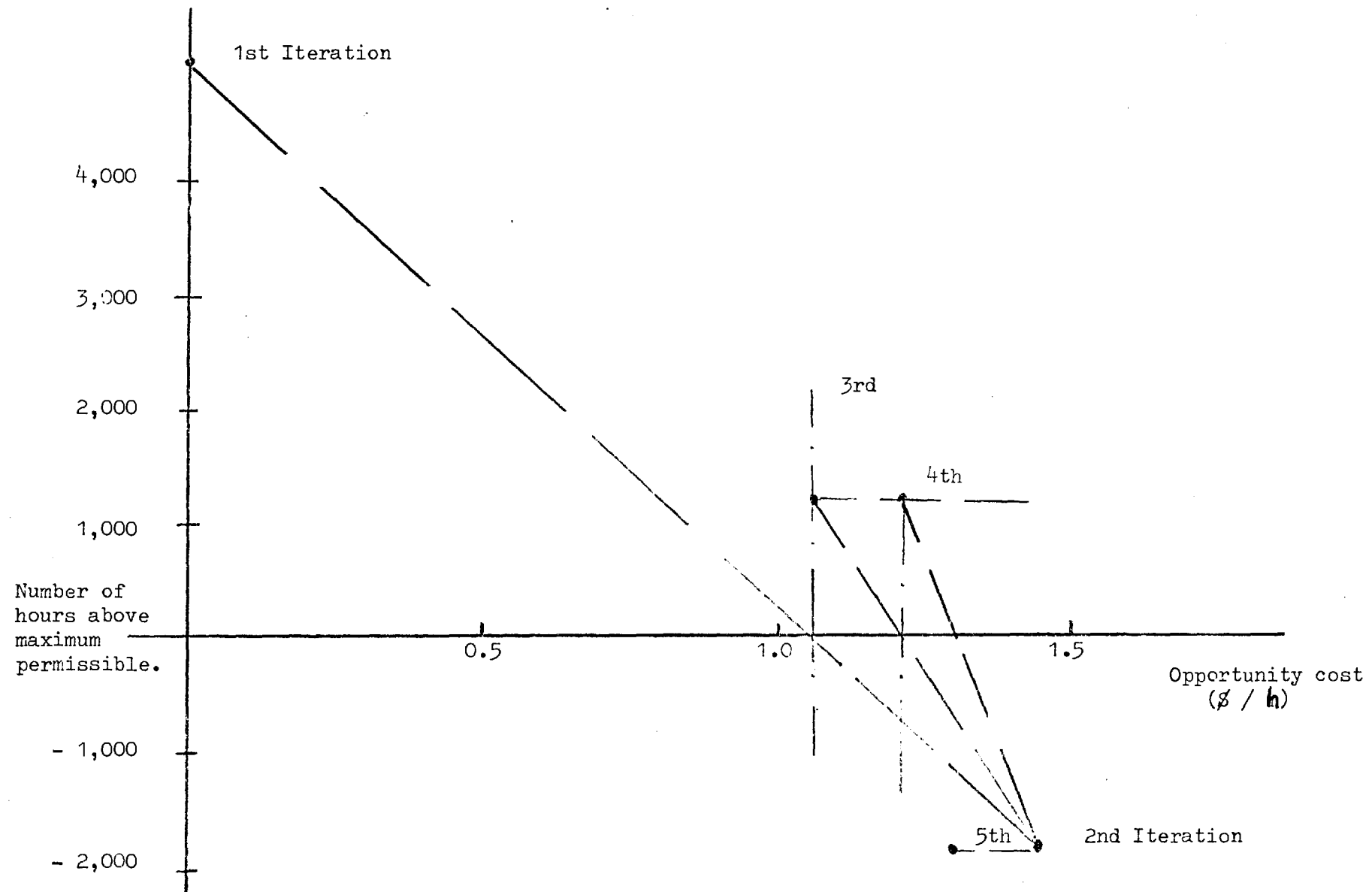
These eliminations will greatly reduce the running time of the model, but may not lead to a near-optimal solution. The author felt that this was an acceptable trade-off in view of the difficulty of finding the optimum in the case of high interdependence in the opportunity cost curves.

Critical Discontinuity

The method has a second, and very important, drawback: it cannot cope with critical discontinuities on the 'excess hours - opportunity cost' curve. For instance, if the profit function of sub-network b is altered from B_2 to B_1 (Fig. 3.18), then the attempt to find the 'No Conflict' cost comes to an abrupt end on the fourth iteration (Fig 3.20). The reason for this is the discontinuity of the curve at \$ 1.26 per hour, which arises because sub-network B has a linear profit function. Since a marginal increase in the cost creates an excess capacity of 2,000 hours whilst a marginal decrease creates an excess demand of 1,300, a 'No Conflict' opportunity cost does not exist.

To allow the method to continue when a discontinuity occurs, critical or otherwise, an additional opportunity cost is generated at each iteration; this is the intercept of the line joining the most recent excess demand and excess capacity points (see Fig. 3.20). The one chosen of the two as the next opportunity cost is the one closest to that of the previous iteration.

FIG. 3.2C



To resolve the problem of a critical discontinuity, the opportunity cost is set to the latest excess capacity value whilst the availability of the aircraft is set to its actual maximum. This revision to the method is made operative when the Critical Counter becomes five - the Critical Counter is augmented by one unit everytime the resulting hours allocated is equal to the latest excess capacity or excess demand.

THE INTERDEPENDENCE OF THE AIRCRAFT 'EXCESS HOURS - OPPORTUNITY COST' CURVES

If the excess demand on an aircraft type is easily absorbed by the other types, then there is no interdependence of the cost curves and the above method will easily resolve the 'No Conflict' problem. If, however, the transfer of the excess hours allocated to the scarce aircraft is to an aircraft which also has a tight utilisation constraint, then an interaction will occur in the processes which find the 'No Conflict' opportunity costs. This can easily lead to unstable oscillations about the maximum utilisation level of both aircraft types. This is a difficult problem to overcome by this method, and the best that can be achieved in the circumstances is to use the best feasible solution, should one exist.

The process is defined to be in an unstable state when the successive iterations have produced an aircraft cost curve which has performed two complete oscillations. If to this stage no feasible solution has been reached, then the problem is defined as infeasible and the allocations of the last iterations are printed.

In the general method of successive approximations the choice of the second point can artificially lead the system into a state of interdependence. Since the choice is to a certain extent arbitrary (in the above example the average relative profitability was chosen, whilst in the model it is $0.1 \times (1 + (\text{Excess hours}/\text{maximum hours})) \times \text{Average Profitability}$), it is quite feasible that the resulting allocations will force a second aircraft to be over-utilised and, thus, to join in the process of finding the 'No Conflict' opportunity cost. To overcome this problem, a facility has been introduced which disregards most of the associated results when a scarce aircraft is being under-utilised for the first time as a result of the application of a high opportunity cost. The only purpose of that iteration then becomes to generate another point on the graph of that particular aircraft and for no other.

3.11 TRAFFIC ALLOCATION EXAMPLE

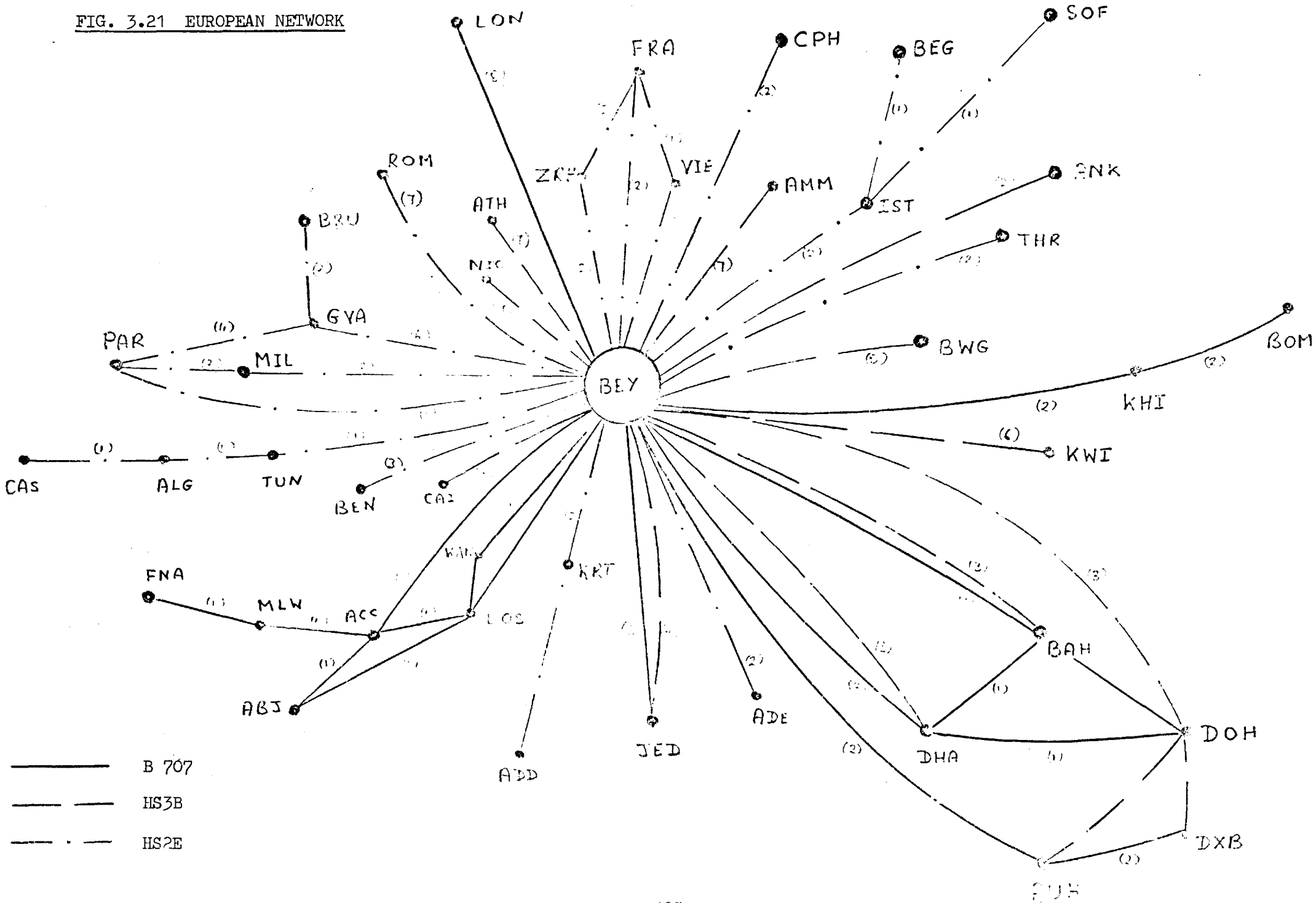
The program, written in Fortran, was tested using data provided by Hawker Siddeley Aviation for a typical 'European Airline'. The network shown in Fig. 3.21 consists of 69 city-pairs with potential passengers and covers 43 cities: it is the solution produced by an experienced traffic analyst.

The program was run on a CDC-6400 and the 'optimal' solution provided by the algorithm, in less than 2 minutes (43K words), was more profitable than the above solution by over \$ 0.5 m. Closer examination revealed that the analyst had applied certain constraints which were not applied in the test run. Had these been included, the program would have produced the solution shown in Fig. 3.21, but in only one minute.

The analyst restricted certain routes to have flights by B707 as opposed to the more efficient (in the context of this problem) HS2E: the exact constraint was to include sixth-freedom traffic by continuing the B707 flights on LON - BEY to DHA, BAH, DXB, AUH, DOH and JED beyond BEY. Had this been included in the program, the total utilisation required for use by the HS2E would have been considerably reduced and the need for the opportunity cost analysis would have been eliminated. However, the analyst admitted that he would not have applied these constraints had he been aware of the difference in profitability.

The following pages show the difference in the two allocations for one sub-network - CPH, FRA, VIE, ZRH and BEY. Fig. 3.22 shows part of the input traffic data, whilst Fig. 3.24 shows part of the list of 82 generated routes. Figs. 3.25, 3.26, and 3.27 show the computer output of the traffic allocations to the above sub-network.

FIG. 3.21 EUROPEAN NETWORK



[illegible]

SECTOR	I	AVERAGE	I	AVERAGE	I	MINIMUM	I	MAXIMUM	I	AVERAGE	I	FIFTH	I	EXTRA	I	EXTRA	I	MINIMUM	I	MAXIMUM
		I STILL AIR		I ONE WAY		I FREQUENCY		I LOAD		I REVENUE		I FREEDOM		I TRAFFIC		I TRAFFIC		I DIRECT		I STOPS
		I DISTANCE		I DEMAND		I		I FACTOR		I RATE		I PASSENG		I --CAPCTY		I --FREQUENCY		I FLIGHTS		I
LON BEY	I	2022	I	36475	I	8	I	.63	I	126.84	I	0	I	0	I	0	I	0	I	5
GVA BEY	I	1624	I	8037	I	5	I	.57	I	108.19	I	0	I	8	I	8037	I	0	I	5
PAR BEY	I	1828	I	11519	I	5	I	.57	I	116.70	I	0	I	8	I	11519	I	0	I	5
BRU BEY	I	5000	I	3705	I	2	I	.57	I	116.70	I	0	I	8	I	3706	I	0	I	5
ATH BEY	I	637	I	23517	I	7	I	.57	I	52.98	I	0	I	0	I	0	I	0	I	5
ROM BEY	I	1330	I	17309	I	7	I	.57	I	93.65	I	0	I	8	I	17309	I	0	I	5
VIE BEY	I	1438	I	1307	I	1	I	.51	I	108.19	I	0	I	8	I	1307	I	0	I	5
FRA BEY	I	1850	I	8001	I	5	I	.55	I	116.70	I	0	I	8	I	8001	I	0	I	5
CPH BEY	I	1958	I	4445	I	2	I	.52	I	140.47	I	0	I	8	I	4445	I	0	I	5
ZRH BEY	I	1613	I	2480	I	2	I	.52	I	108.19	I	0	I	8	I	2480	I	0	I	5
MIL BEY	I	1532	I	1500	I	2	I	.57	I	99.74	I	0	I	8	I	1500	I	0	I	5
IST BEY	I	547	I	2003	I	2	I	.62	I	44.89	I	0	I	6	I	2003	I	0	I	5
BEG BEY	I	1037	I	1618	I	1	I	.61	I	80.31	I	0	I	6	I	1618	I	0	I	5
SOF BEY	I	857	I	1107	I	1	I	.61	I	67.16	I	0	I	6	I	1107	I	0	I	5
ANK BEY	I	402	I	5213	I	2	I	.62	I	31.96	I	0	I	0	I	0	I	0	I	5

FIG. 3.23

AIRCRAFT CHARACTERISTICS

	B707	HS3B	HS2E
AIRCRAFT SEATS	151	132	99
HOURLY COSTS	312.0	270.5	251.2
FLIGHT COSTS	226.3	159.0	139.1
MAXIMUM UTILISATION	12000	12000	12000
BLOCK FUEL ***			
HIGH SPEED-- INTERCEPT	2013	3503	2651
-- GRADIENT	28.509	20.609	17.181
BLOCK TIME ***			
HIGH SPEED-- INTERCEPT	.333600	.412900	.382800
-- GRADIENT	.00212	.00206	.00201

FIG. 3.24

GENERATED ROUTES =====

I=====I=====I=====I=====I=====I=====I												
I	ROUTE	I	CITY 1	I	CITY 2	I	CITY 3	I	CITY 4	I	CITY 5	I
I	NUMBER	I		I		I		I		I		I
I=====I=====I=====I=====I=====I=====I												
I	1	I	LON	I	BEY	I		I		I		I
I	2	I	GVA	I	BEY	I		I		I		I
I	3	I	PAR	I	BEY	I		I		I		I
I	4	I	PAR	I	GVA	I	BEY	I		I		I
I	5	I	PAR	I	MIL	I	BEY	I		I		I
I	6	I	BRU	I	GVA	I	BEY	I		I		I
I	7	I	ATH	I	BEY	I		I		I		I
I	8	I	ROM	I	BEY	I		I		I		I
I	9	I	VIE	I	BEY	I		I		I		I
I	10	I	FRA	I	BEY	I		I		I		I
I	11	I	FRA	I	ZRH	I	BEY	I		I		I
I	12	I	FRA	I	VIE	I	BEY	I		I		I
I	13	I	CPH	I	BEY	I		I		I		I
I	14	I	CPH	I	FRA	I	VIE	I	BEY	I		I
I	15	I	CPH	I	FRA	I	BEY	I		I		I
I	16	I	CPH	I	FRA	I	ZRH	I	BEY	I		I
I	17	I	ZRH	I	BEY	I		I		I		I
I	18	I	MIL	I	BEY	I		I		I		I
I	19	I	IST	I	BEY	I		I		I		I
I	20	I	BEG	I	BEY	I		I		I		I
I	21	I	BEG	I	IST	I	BEY	I		I		I
I	22	I	SOF	I	BEY	I		I		I		I
I	23	I	SOF	I	IST	I	BEY	I		I		I
I	24	I	ANK	I	BEY	I		I		I		I

FIG. 3.25

ROUTE-----FRA -VIE -BEY AIRCRAFT TYPE -----HS2E
=====

FREQUENCY ----- 1 UTILISATION 446.3
=====

OPERATIONAL INFORMATION
=====

TOTAL REVENUE	590663.3

FUEL COSTS	77456.1
HANDLING CHARGES	17139.2
LANDING FEES	46987.2
ENGINEERING COSTS	141038.0

TOTAL VARIABLE COST	282620.4

OPERATING PROFIT	308042.8
*****	*****

SECTOR	SECTOR	LEG 1	LEG 2
	REVENUES	FRA VIE	VIE BEY
FRA VIE	0.0	0	0
FRA BEY	307854.6	1319	1319
VIE BEY	282806.7	0	1307
LEG PASSENGERS		1319	2626
LEG LOAD FACTOR		25.6	51.0

AIRCRAFT TYPE -----HS2E

FREQUENCY ----- 2 UTILISATION 900.5

OPERATIONAL INFORMATION

TOTAL REVENUE	1207647.4
---------------	-----------

FUEL COSTS	156339.1
------------	----------

HANDLING CHARGES	34278.4
------------------	---------

LANDING FEES	93974.4
--------------	---------

ENGINEERING COSTS	284071.4
-------------------	----------

TOTAL VARIABLE COST	568663.3
---------------------	----------

OPERATING PROFIT	638984.1
------------------	----------

SECTOR	SECTOR	LEG 1	LEG 2
	REVENUES	FRA ZRH	ZRH BEY
FRA ZRH	0.0	0	0
FRA BEY	671025.0	2875	2875
ZRH BEY	536622.4	0	2480
LEG PASSENGERS		2875	5355
LEG LOAD FACTOR		27.9	52.0

FIG. 3.27

ROUTE-----CPH -FRA -BEY

AIRCRAFT TYPE -----B707

FREQUENCY ----- 2 UTILISATION 1128.6

OPERATIONAL INFORMATION

TOTAL REVENUE 2137332.1

FUEL COSTS 297876.7

HANDLING CHARGES 77792.3

LANDING FEES 173596.8

ENGINEERING COSTS 446271.9

TOTAL VARIABLE COST 995537.5

OPERATING PROFIT 1141794.5

SECTOR	SECTOR	LEG 1	LEG 2
	REVENUES	CPH FRA	FRA BEY
CPH FRA	0.0	0	0
CPH BEY	1248778.3	4445	4445
FRA BEY	888553.8	0	3807
LEG PASSENGERS		4445	8252
LEG LOAD FACTOR		28.3	52.5

CHAPTER 4

CONCLUSION

The problem of airline fleet planning has been studied in this project and two computer based models have been developed to test the validity of the theoretical analysis in this thesis.

The Master Program was used to test the validity of the Course Grid Dynamic Programming approach to the problem of finding the optimal aircraft procurement/retirement policy. Using financial data based on Eastern Air Lines' financial accounts and aircraft information supplied by Hawker Siddeley Aviation, a test example was devised which would test fully the Master Program. When optimising over a period of five years, the model reduced the number of inefficient aircraft to the absolute minimum and increased the number of units of the efficient aircraft types. However, when optimising over the shorter period of two years, the objective of maximising working capital was achieved by increasing the number of units of the efficient aircraft types, so as to reduce operating cost; and retaining the inefficient type for no other purpose than not having to pay the tax bill associated with selling equipment at a price higher than its depreciated value.

The traffic allocation model was tested using data provided by Hawker Siddeley Aviation for a typical "European Airline". The program produced a near-optimal solution which was more profitable than that produced by an experienced analyst by more than £ 0.5 m. Closer examination revealed that the analyst had applied certain a priori restrictions (not included in the program) in order to generate more revenue. However, the analyst admitted that he would not have applied these restrictions had he been aware of the resultant £ 0.5 m difference in profit. Even so, had these constraints been included, the program would have produced exactly the same solution as that produced by the analyst (in several days) but in only one minute.

Although the models have proved to be individually successful, the traffic allocation and fleet planning programs have not been linked and operated together. The reason for not performing the link-up between the two models is due to the core size of the former program. The traffic allocation model has been successfully developed to its conclusion, which is to perform a complicated allocation exercise over a network efficiently. As such, it is a very valuable package for use in traffic analysis. Unfortunately its size, 35-40 K words on the CDC 6400, and the time it takes to solve a very large problem (up to 5 mins) means that it should be used sparingly in a fleet planning exercise.

To overcome this problem, it is suggested that the Standard Allocation Algorithm (without the perturbation sections) be used as the routine which tests fleet feasibility and generates operating profit in the total fleet planning model. Inaccuracies are bound to occur, but the time required to identify the possible optimal policies will be very significantly reduced. Having identified these fleets, the complete allocation model, with the perturbation analysis, can then be used to generate a more accurate set of operating revenues and costs for a restricted set of fleets and these can then be an input for a second time to the financial planning model.

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APPENDIX A
PRINCIPLES OF DYNAMIC PROGRAMMING

Let

$$R(x_1, x_2, \dots, x_N) = g_1(x_1) + g_2(x_2) + \dots + g_N(x_N)$$

be a general linear or non-linear function of the N variables $x_1, \dots, x_N$, which can be decomposed into N functions $g_i(x_i)$, each one involving only one variable x_i .

Then the problem of maximising

$$R(x_1, x_2, \dots, x_N)$$

subject to the constraints

$$\sum_{i=1}^N x_i = B, \text{ where } B \text{ is a constant}$$

and $x_i \geq 0$

becomes a typical "dynamic" allocation process which is readily solved by Dynamic Programming. By permitting, and indeed requiring, the allocations to be made one at a time, a "time-like" property is introduced which is "dynamic" in character.

The maximum value of $R(x_1, x_2, \dots, x_N)$ over the specified region depends on B and N ; this dependence is made specific by introducing the sequence of functions $f_N(B)$ defined by

$$f_N(B) = \text{Max}_{\{x_i\}} R(x_1, x_2, \dots, x_N)$$

for $N = 1, 2, \dots$ and $B \geq 0$

and $x_i \geq 0$

$$\sum_{i=1}^N x_i = B, \text{ as above}$$

$i = 1$

The function $f_N(B)$ is then the optimal return from an allocation of the quantity of resources to N activities. In two particular cases the elements of the sequence $f_N(B)$ assume very simple values. It is

clear that $f_N(0) = 0$, provided that $g_i(0) = 0$ for each i , and that $f_1(B) = g_1(B)$ for $B \geq 0$.

To obtain a recurrence relation connecting $f_N(B)$ and $f_{N-1}(B)$ for arbitrary N and B , allow x_n to be the allocation made to the N^{th} activity - $0 \leq x_n \leq B$: it follows that the remaining quantity of resources $B - x_n$ will be used to obtain a maximum return from the remaining $N-1$ activities. Since this optimal return for the $N-1$ activities starting with the quantity $B - x_n$ is by definition $f_{N-1}(B - x_n)$, it follows that the initial allocation of x_n to the N^{th} activity results in a total return of $g_N(x_n) + f_{N-1}(B - x_n)$ from the N activity process. Therefore, the basic functional equation is

$$f_N(B) = \text{Max} \left\{ g_N(x_n) + f_{N-1}(B - x_n) \right\}$$

$$0 \leq x_n \leq B$$

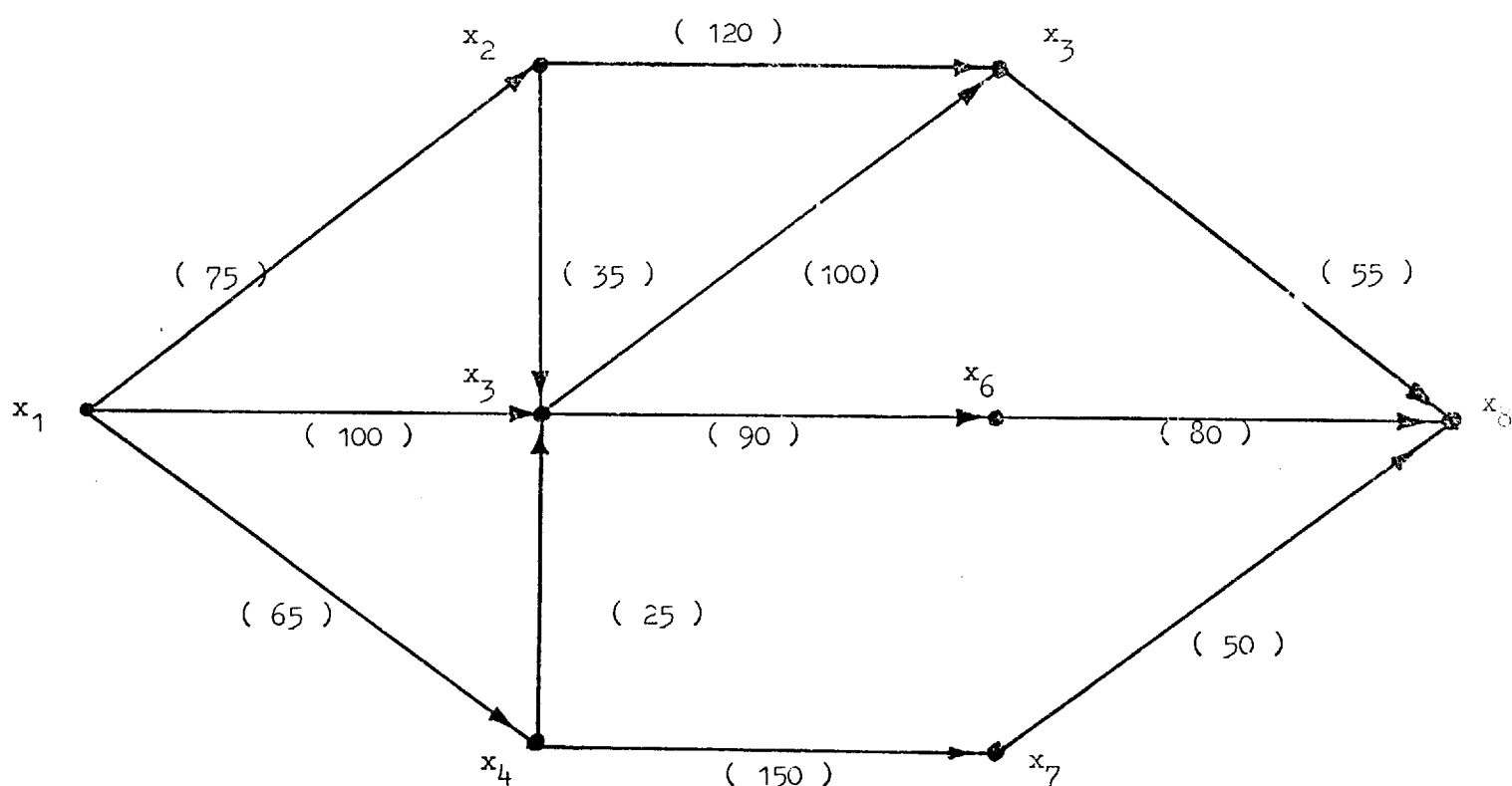
In deriving the above relation, a very general technique called "The Principle of Optimality" [15] was used. This states that an optimal policy had the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

A.1 GENERAL DYNAMIC PROGRAMMING - THE FORWARD ALGORITHM

A multi-stage process is described by the values which the state variables can take at each stage and by the permissible "paths" which link these variables from one stage to the next. A typical multi-stage decision process is the "shortest path" problem, shown in Fig. A.1, where a car which can average 40 mph has to travel from City X_1 to City X_8 in the shortest possible time; the permissible paths are the feasible routes and the state variables are the cities.

A.1.1. The State Transition Diagram

The set of possible values of the state variable X_{n+1} , which can result from a transition from X_n is defined by $\bigcup_{n+1} X_n$ where $\bigcup$ is a correspondance



(d) is the distance between two cities in miles

FIG. A.1 A "SHORTEST PATH" PROBLEM

operator; and therefore:-

$$X_{n+1} \in \bigcap_{n+1} X_n$$

For the above example the possible states which can result from a transition from City X_1 are the cities X_2 , X_3 and X_4 . Since Dynamic Programming is a technique used for optimising multi-stage systems, it is of crucial importance to indentify correctly the possible states associated with each stage. The only certain way of doing this for a complicated state-transition diagram, is through the application of the correspondance operator.

It appears that in the example the cities X_2 , X_3 and X_4 constitute the possible states of the second stage. However, the application of the operator Γ^{-1} , the inverse of the correspondence operator, suggests that the possible states of the first stage are X_1 , X_2 and X_4 . There is obviously a contradiction in this argument since X_2 and X_4 cannot be possible states of both the first and second stages. Therefore, to be correct, the possible states must be

$$x_n \left\{ \Gamma_n x_{n-1} \cap \Gamma_{n+1}^{-1} x_{n+1} \right\}$$

However, this does not guarantee that the state diagram will be sequential, i.e. that $X_{n+1} \in \Gamma_n X_n$ and that $X_n \in \Gamma_{n+1}^{-1} X_{n+1}$ and it is, therefore, sometimes necessary to introduce dummy states. By introducing the dummy cities Y_3 , Y_5 and Y_7 , it is possible to produce the sequential state transition diagram (see Fig. A.2) for the above problem

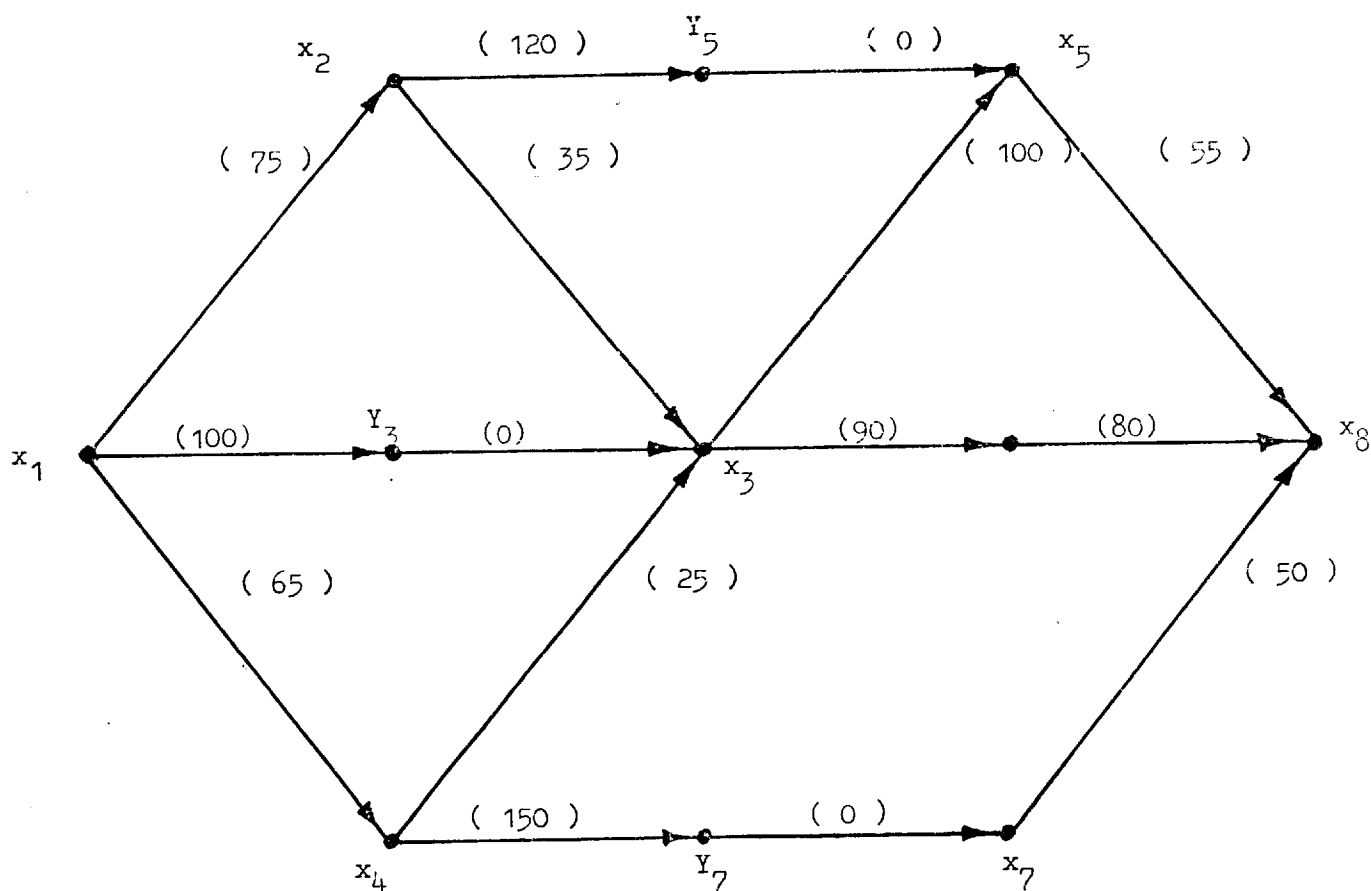


FIG. A.2 SEQUENTIAL STATE TRANSITION DIAGRAM

A.1.2 The Policy

The system is controlled by a policy, which assigns specific values to the state variables for each stage of the process i.e

$$\pi_{0,n} (x_0, x_n) = (x_0, x_1, x_2 \dots x_n);$$

and the quantitative implication of this policy is equal to the sum of the returns of going from one stage to the next i.e

$$S(\pi) = \sum_{n=1}^N V_n(x_{n-1}, x_n)$$

The objective of the Dynamic Programming Algorithm is to find the optimal policy, π^* , which maximises (or minimises) $S(\pi)$.

A.1.3 The Forward Algorithm

STEP 1

Given the initial state of the system, x_0 , find the possible states of the first stage and calculate the return for each of these transitions.

i.e a) Find $x_1 \in X_1(x_0, \xi) = x_0$, where $X_n(x_0, \xi)$ is the set of all possible states of the system at stage n starting x_0 .

b) Calculate $v(x_0, x_1) = f_{0,1}(x_0, x_1)$ where $f_{0,N}(x_0, x_N)$ is the optimal return of the transition from state variable x_0 to state variable x_N .

e.g $f_{0,1}(x_1, x_2) = 75$

$$f_{0,1}(x_1, x_3) = 100$$

$$f_{0,1}(x_1, x_4) = 65$$

STEP n

Calculate the optimal return to each of the possible state variables of the n^{th} stage.

$$\text{i.e. } f_{0,n}(x_0, x_n) = \text{OPT} \left\{ f_{0,n-1}(x_0, x_{n-1}) + v_n(x_{n-1}, x_n) \right\}$$

AND FOR THE ABOVE EAMPLE

STEP 2

$$f_{0,2}(x_1, y_5) = \text{Min} \left\{ (75 + 120), \infty, \infty \right\} = 195$$

$$\begin{array}{lll} \text{path } x_1 - x_2 - y_5 & x_1 - y_3 - y_5 & x_1 - x_4 - y_5 \\ & \text{infeasible} & \text{infeasible} \end{array}$$

$$f_{0,2}(x_1, x_3) = \text{Min} \left\{ (75 + 35), (100+0), (65+25) \right\} = 90$$

$$f_{0,2}(x_1, y_7) = \text{Min} \left\{ (65+150), \infty, \infty \right\} = 215$$

STEP 3

$$f_{0,3}(x_1, x_5) = \text{Min} \left\{ (195+0), (90 + 100), \infty \right\} = 190$$

$$f_{0,3}(x_1, x_6) = \text{Min} \left\{ (90+90), \infty, \infty, \right\} = 180$$

$$f_{0,3}(x_1, x_7) = \text{Min} \left\{ (215+0), \infty, \infty, \right\} = 215$$

STEP 4

$$f_{0,4}(x_1, x_8) = \text{Min} \left\{ (190 + 55), (180+80), (215+50) \right\}$$

$$\text{paths } x_1 - x_4 - x_3 - x_5 - x_8 \quad x_1 - x_4 - x_5 - x_6 - x_8 \quad x_1 - x_4 - x_7 - x_7 - x_8$$

$$f_{0,4}(x_1, x_8) = 245$$

Therefore the car can make the journey in 6 hours by following route $x_1 - x_4 - x_5 - x_6 - x_3$.

APPENDIX B

THE AIRLINE FLEET PLANNING PROGRAM

This appendix describes the inputs required by the airline fleet planning model; its listing is available at Imperial College. To facilitate the user in his understanding of the routines, there is a brief description of all the routines and of the main variables used by them. The program is simply documented with comment cards (see Appendix C 6.0 subroutine, AIRRUT for a sample listing), and for some of the routines flow charts are shown (Main Program and subroutine FEAZIB).

Inputs to the program

		<u>FORMAT</u>
Card 1	year in which the oldest aircraft in the airline's fleet was bought	I4
	First year in the planning period	I4
	Final year in the planning period	I4
Card 2	Terminator	A4

Aircraft Information

11 cards are required for each aircraft type (Max No. of A/C 5)

Card 1	Aircraft code	A4
	Maximum payload	I3
	Maximum range	I4
	Aircraft name	2A6
Card 2	Intercept of Cost Equation	F.8.1
	Gradient of Cost Equation	F.5.1
	Intercept of block time equation	F.7.5
	Gradient of block time equation	F.7.5

		<u>FORMAT</u>
Card 3	Initial down-payment (% of price)	F.5.2
	Interest charged by the manufacturer	F5.2
	Number of years in which the payments must be made to the manufacturers	I2
	The number of payments in one year	I2
Card 4	The purchase price of the aircraft for each year in the pre-planning period and for each year of the planning period (in millions)	10F6.3
Card 5	The aircraft resale value for each year of the planning period (millions)	20F6.3
Card 6	The maximum number of aircraft which can be delivered to the airline for each year of the planning period	20I4
Card 7	The number of aircraft bought in each year of the pre-planning period	20I2
Card 8	The aircraft "order code" for each year of the planning period	20I2
Card 9	The maximum number of aircraft which the airline can have in its fleet for each year of the planning period	20I4
Card 10	The maximum utilisation per aircraft in each year of the planning period	20F5.0
Card 11	The insurance premium (%) for the aircraft type for each year of the planning period	20F3.0
Card 11A +1	Terminator	A4

Balance sheet for year prior to the planning period

Two cards one for the assets side of the balance sheet and one for the liabilities side, to read in the relevant information. The amounts are in million units of the currency.

Card 1	Gross aircraft assets	F7.2
	Total aircraft depreciation	F7.2
	Gross assets in property and equipment	F7.2
	Total property and equipment depreciation	F7.2
	Funds invested in short-term securities	F7.2
	Other current assets	F7.2
	Other assets	F7.2
	Total assets	F7.2
Card 2	Airline's Equity	F7.2
	Total airline earnings	F7.2
	Airline's long term debt	F7.2
	Notes payable in 1 year	F7.2
	Short term debt	F7.2
	Provision for taxation	F7.2
	Other current liabilities	F7.2
	Other liabilities	F7.2
	Total liabilities	F7.2

Financial information for each year of the planning period

All information, unless otherwise stated, should be in millions of the currency.

Card 1	Planned Increase in Equity	12F6.2
Card 2	Planned Increase in long term debt.	12F6.2
Card 3	Planned reduction in long term debt.	12F6.2
Card 4	Interest payments to service long term debt.	12F6.2
Card 5	Planned dividends to shareholders	12F6.2

FORMAT

Card 6	Maximum level of short term debt	12F6.2
Card 7	Cost of short term debt (%)	12F6.2
Card 8	Planned expenditure on property and equipment	12F6.2
Card 9	Depreciation on property and equipment	12F6.2
Card 10	Tax allowance on property and equipment	12F6.2
Card 11	Return from short term securities (%)	12F6.2

Traffic Allocation Information

The traffic allocation information presently read-in is specific to the simplified allocation algorithm used to test the Dynamic Programming algorithm. When the link-up between the appropriate programs is achieved, this section of the input data will need to be modified.

Card 1	Sector distance	I4
	Sector passengers (Base value)	I8
	Sector yield (Base value)	F5.2
Card 2	Passenger growth rate for each year in the planning period	15F7.2
Card 3	Revenue yield growth rate for each year in the planning period	15F7.2
Card 3N+1	9999	I4

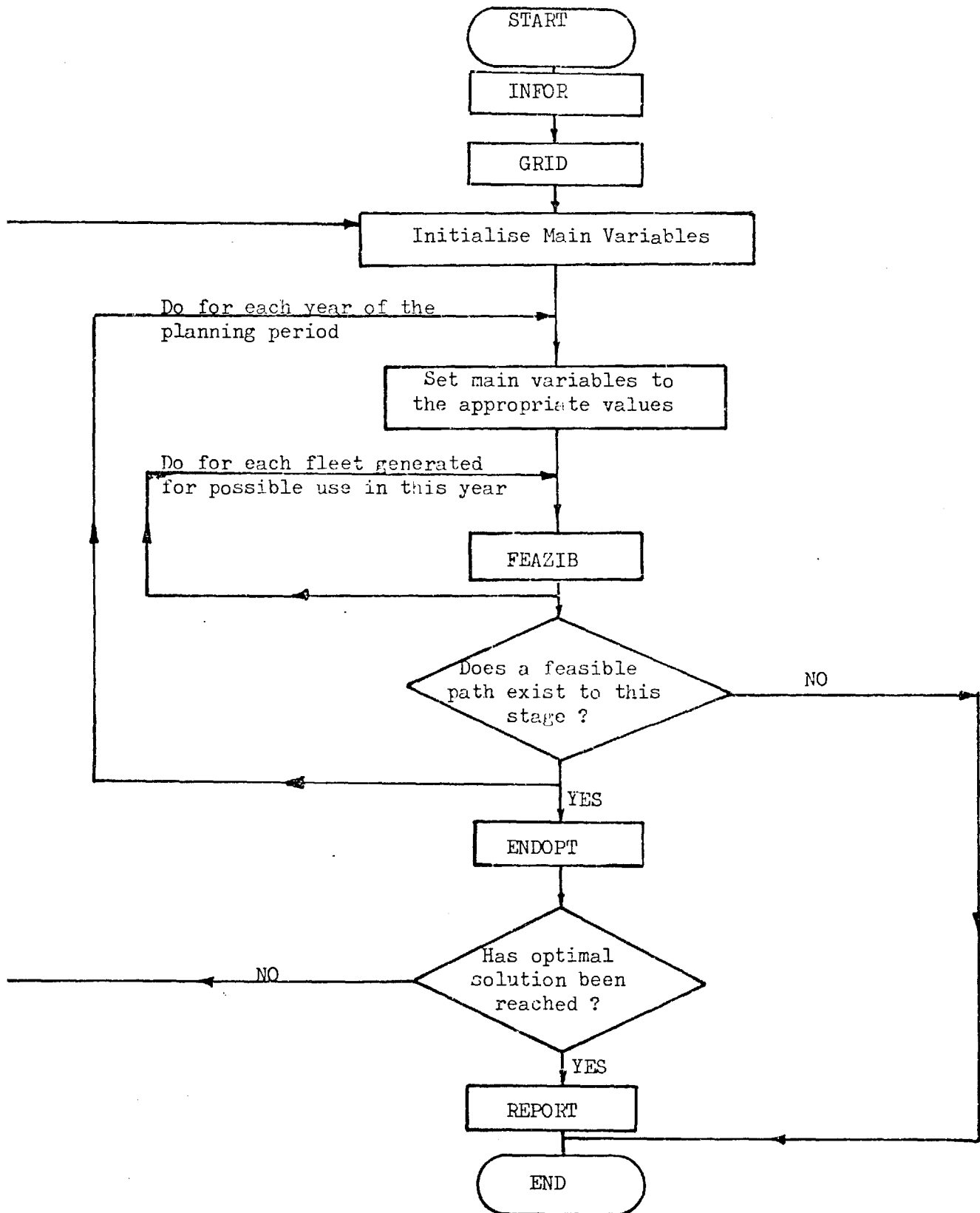
1. MAIN ROUTINE

This routine is the main program of the Fleet Planning Model.

1.1 Listing of Main Variables

PLNYRS	is the number of years in the planning period.
ACTYPE	is the number of aircraft types in the fleet planning exercise.
BSTORD (A,T)	stores the most recent "Optimal" plan. This will be updated by the D.P. procedure until it is the same for two consecutive iterations.
GRID	is the sub routine which generates the initial course grid.
INFOR	is the sub routine which reads in the necessary information.
NUMFLT (T)	is the number of fleets generated for year T.
NMSIGM (T)	is the number of feasible fleets in year T.
FEAZIB	is the sub routine which test for feasibility of transitions from the previous stage to the appropriate state variable.
ENDOPT	is the sub routine which finds the "optimal" plan for the present iteration and compares it with the previous one. If necessary it will generate a fine grid.
STOPDP	is an indicator. When it is equal to 1, the fleet planning process has been completed.

1.2. Flowchart of Main Program



2. SUBROUTINE INFOR

This subroutine reads in the information to be used in devising the "Optimal" airline corporate plan.

2.1. Listing of Main Variables

YROLD	is the year in which the oldest aircraft in the fleet was bought.
YRFRST	is the first year in the planning period.
YRLAST	is the final year in the planning period.
PREYRS	is the number of years prior to the planning period in which the old aircraft were bought.
PLNTYS	is the number of years in the planning period.
ACCODE (A)	is the aircraft code for type A.
ACNAME (A)	is the name of aircraft type A.
PAYLOD (A)	is the number of seats in aircraft A.
RANGE (A)	is the maximum range of aircraft A.
COSTA (A)	is the cost per flight cycle.
COSTB (A)	is the hourly cost.
TIMEA (A)	These constitutes the block times equation
TIMEB (A)	

INTPAY (A) is the initial payment made on the purchase of a new aircraft, where

PMNYRS (A) is the number of years where the outstanding debt must be paid, and

NUMPAY (A) is the number of payments in a year.

PAYINT (A) is the interest rate charged by the manufacturer

YRLYPM (A) is the total yearly repayment.

PURCHS (A,T) is the purchase price of aircraft type A in year T.

RESALE (A,T) is the resale value

MAXBUY (A,T) is the maximum number of aircraft type A which can be delivered by the manufacturer in year T.

ORDERZ (A,T) is the number of aircraft ordered in the pre-planning period in year T.

ORDCDE (A,T) is the order code for aircraft A in year T.
 1 means that the number of aircraft cannot increase from one year to the next
 2 means that the number of aircraft cannot decrease from one year to the next and
 3 means that the number of aircraft in any one year is free from the above restrictions.

MAXHLD (A,T) is the maximum number of aircraft type A which can be in the fleet in year T

MAXUTL (A,T) is the maximum utilisation per aircraft.

INSRNC (,T) is the insurance premium per aircraft.

The following information relates to the Balance Sheet for the year prior to the planning period.

AIRPLN is the total assets in aircraft.

AIRDEP is the total aircraft depreciation to date.

EQUIPM is the total assets in equipment and property and

EQUPEP is the associated depreciation

CPLNTI is the funds invested in short-term securities

CURASS is the other funds invested in current assets

OTHASS Are the other assets held by the airline.

EQUITI the total equity of the airline

EARNIN is the accumulated retained earnings.

LTDEBT is the long term debt of the airline.

NOTESP is the long term debt scheduled for repayment in the first year of the planning period.

CPBORI is the capital borrowed for use in the short-term

TAXPRI is the present provision for taxation

CURLIA is the other current liabilities of the company

EQUITY (T)	is the planned increase of long term capital through the issue of equity.
FUNDS (T)	is the planned increase of long term capital through the issue of debentures.
PAYDBT (T)	is the scheduled repayment of long term debt.
DBTINT (T)	is the interest required in year T to service the long term debt.
DIVIDN (T)	are the planned dividends to shareholders in Year T.
MAXDET (T)	is the maximum level of short term debt in Year T.
IPRCNT (T)	is the cost of short-term borrowing (Percent).
PROPTY (T)	is the planned capital expenditure on property and equipment, and
DEPRTY (T)	is the associated depreciation for the total assets in property and equipment.
TAXPRP (T)	is the tax allowance for the assets in property and equipment.
JPRCNT (T)	is the return from short-term securities (percent)

3. SUBROUTINE GRID

This subroutine generates the initial "coarse grid" of possible fleets which will be used by the Dynamic Programming process to formulate the initial "optimal" policy.

3.1. Listing of Main Variables

ACTYPE	is the number of aircraft types considered in the corporate plan.
PREYRS	is the number of years prior to the planning period in which the old aircraft were bought.
PLNYRS	is the number of years in the planning period.
MAXAC (K,T)	is the maximum number of aircraft in year T which can be considered as a result of the manufacturers delivery constraint and of the airline retirement policy.
FLEETS (K,A,T)	is the K^{th} fleet generated for year T.
MAXBUY (A,T)	is the maximum number of aircraft type A which can be delivered by the manufacturer in year T.
ORDERZ (A,T)	is the number of aircraft ordered in the pre-planning period in year T.
MAXHLD (A,T)	is the maximum number of aircraft type A which can be in the fleet in year T, and is a result of the airlines' retirement policy

TRAFIK (T)	generates the traffic levels appropriate to year T. This traffic will be used by the fleet feasibility testing routine.
HEURIS	is an indicator which states which heuristic rule is generating the fleet.
MAXUTL (A,T)	is the maximum aircraft utilisation.
ALOCTN	is the subroutine which tests for fleet feasibility and calculates the associated:
REVNU (K,T)	Revenue and
VARCST (K,T)	operating variable cost
NUMFIT (T)	is the number of fleets generated for year T.

4. SUBROUTINE FEAZIB

This subroutine tests the feasibility of a given fleet in terms of the financial constraints. In doing so it considers the best aircraft procurement/retirement policy to the given state variable.

4.1. Listing of Main Variables

ACTYPE	is the number of aircraft types considered in the corporate plan.
PREYRS	is the number of years prior to the planning period in which the old aircraft were bought.
PLNYRS	is the number of years in the planning period.
EQUITY (T)	is the planned increase of long term capital through the issue of equity.
FUNDS (T)	is the planned increase of long term capital through the issue of debentures.
PAYDBT (T)	is the scheduled repayment of long term debt.
DBTINT (T)	is the interest required in year T to service the long term debt.
DIVIDN (T)	are the planned dividends to shareholders in year T.
PROPTY (T)	is the planned capital expenditure on property and equipment.
IPRCNT (T)	is the cost of short term borrowing.

JPRCNT (T)	is the return from short term securities
REVNU (K,T)	is the revenue associated with the K th fleet generated in year T and
VARCST (K,T)	is the operating variable cost.
SIGMAT (I,J,T)	Stores the optimal path to a particular state variables and
ORDER 1 (A,T)	are the associated order levels
OPTORD (A,T)	
OPTIMA	is the routine which uses the volumes stored in SIGMAT (I,J,T) to find the optimal path.
DEPREC	uses the values stored in ORDER 1 (A,T) to calculate to depreciation, tax allowance and the next assets.
INSURE	Calculates the insurance cost for the proposed fleet.
TRADE	is the operating trading profit.
INDRKT	is the indirect operating cost.
ORDERS	Calculates the revenue from the sale of aircraft and the resulting outstanding debt. It also calculates the payments which must be made to the manufacturers and the interest portion of those payments.

CAPBOR (I,J)	are the short term debt and the
CAPLNT (I,J)	investment in short term securities in the
	previous year associated with the optimal
	path to the given state variable.

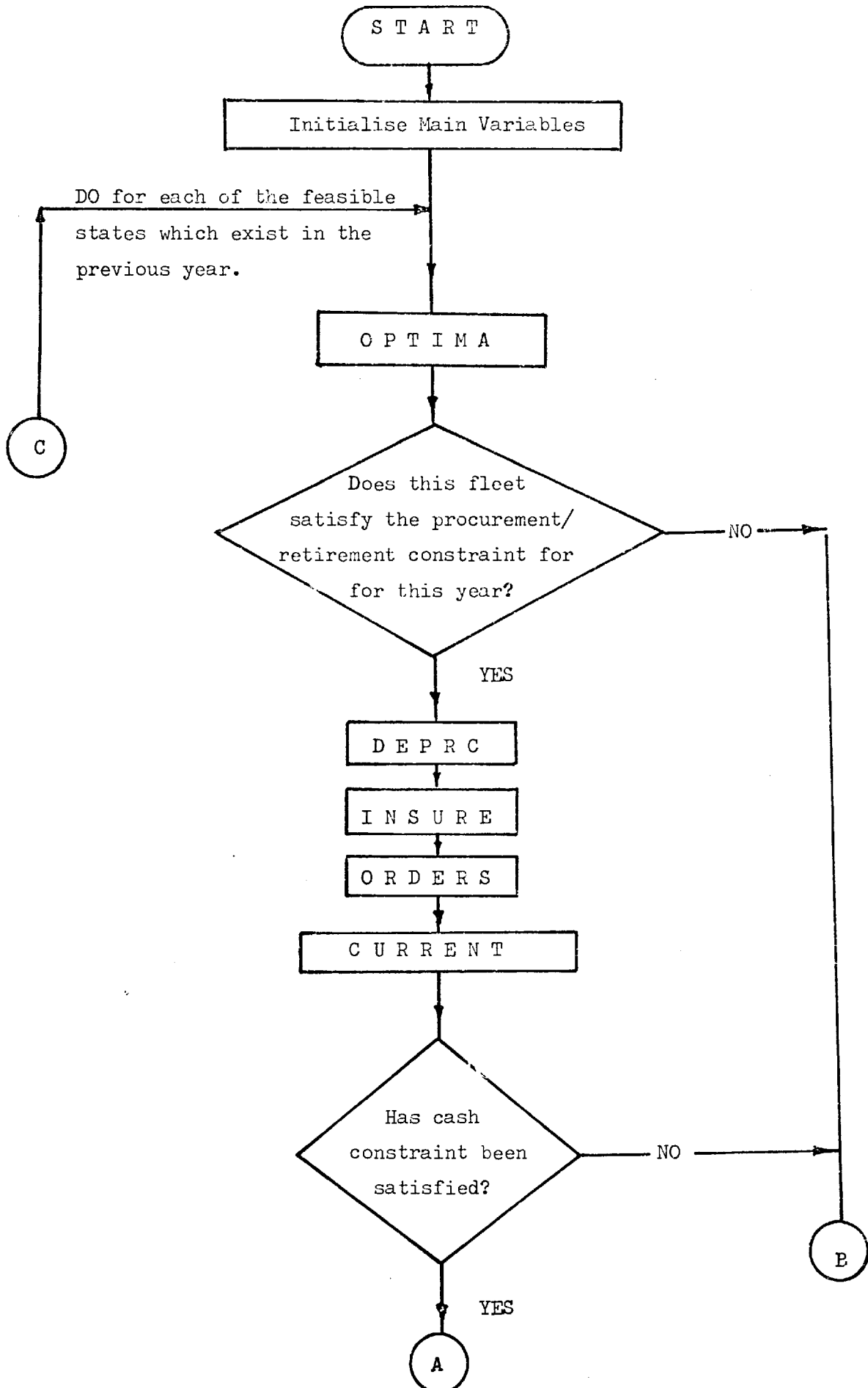
CURRENT	is the routine which calculates the current
	assets and current liabilities.

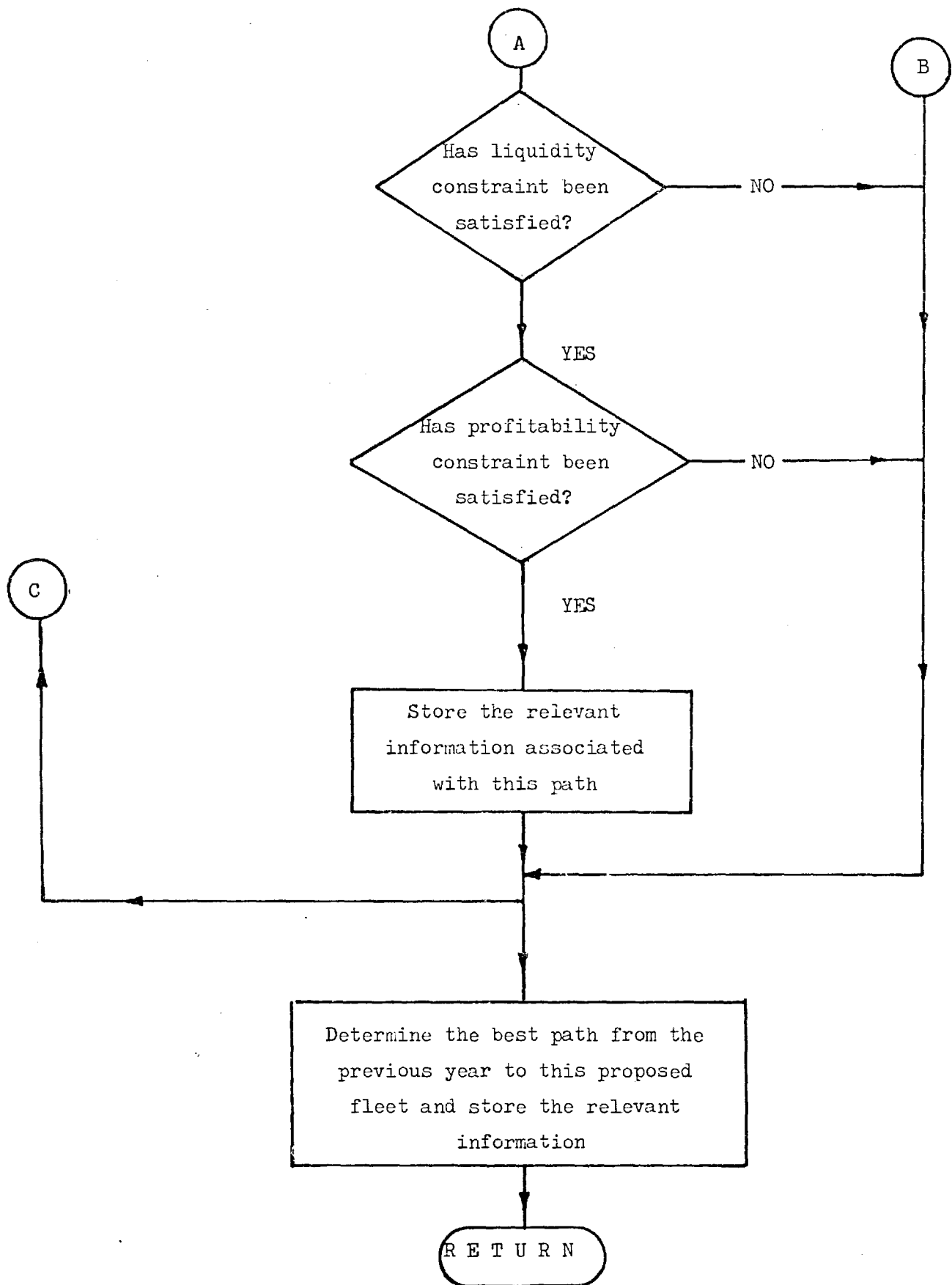
MAXDBT	is the maximum quantity of funds which
	can be borrowed in the short-term without
	violating the current ratio.

The following information is stored when appropriate and represents the main parameters of the latest optimal policy.

GROSS (T)	is gross aircraft assets
ASSETS (T)	is net assets in aircraft
DEPRCN (T)	is the aircraft depreciation for year T
ADJUST (T)	is the balancing charge on the sale of aircraft
ACSALE (T)	is the revenue from the sale of aircraft.
STINVS (T)	is the investment in short term securities.
CRASTS (T)	are the other current assets held by the airline.
STDEBT (T)	is the short term debt of the airline.
TAXPRV (T)	is the provision for taxation.
CRLIBS (T)	are the current liabilities
FLTINS	is cost of insuring the fleet.
INDRCT (T)	is the indirect operating cost.
PROFIT (T)	is the trading profit.
LOANS (T)	is the total interest payment for the year.
INCOME (T)	is the income for the year T.
INVEST (1)	is the investment in aircraft assets for the year T.
DEBTLT (T)	is the associated level of long term debt.

4.2 Flowchart for subroutine FEAZIE





5. SUBROUTINE OPTIMA

This subroutine finds the characteristics of the optimal path to a given state variable.

5.1 Listing of Main Variables

ACTYPE	is the number of aircraft types considered in the corporate plan.
PREYRS	is the number of years prior to the planning period in which the old aircraft were bought.
PLNYRS	is the number of years in the planning period.
ORDERZ (A,T)	is the number of aircraft ordered in the pre-planning period year T.
SIGMAT (I,J,T)	stores the optimal path to a particular state variable, and
OPTORD (A,T)	stores the optimal procurement/retirement policy to the given stage
ORDER 1 (A,T)	this stores the years in which the currently held aircraft were ordered for the optimal policy.
FLEET (K,A,T)	is the K^{th} fleet generated for the T^{th} year.

6. SUBROUTINE DEPREC

This subroutine calculates the depreciation allowances and the balancing charges/allowances on the sale of aircraft for use in calculating the Income and the Taxation.

6.1 Listing of Main Variables

ACTYPE	is the number of aircraft types considered in the corporate plan.
ORDER 2 (A,T)	this stores the year in which the currently held aircraft were ordered for the optimal policy.
SOLD	is the number of units of the aircraft type considered, sold in the relevant year.
BOTYRI	is the number of units of the aircraft type considered, bought in the relevant year.
PURCHS (A,T)	is the purchase price of aircraft type A in year T.
INTALW	is the initial allowance for use in the calculation of the taxation elements and
YRSALW	is the following yearly allowance.
DPCHRG	is the depreciation charge per annum expressed as a percentage, as are the above two.
BKVALU	is the book value of the aircraft for use in the taxation calculation.
ASSITS	is the book value of the aircraft for use in the income calculation.
BLCHRG	is the balancing charge/allowances for use in the taxation calculation.

ADJUST	is the balancing charge/allowance for use in the income calculation.
RESALE (A,T)	is the resale value of aircraft type A in Year T.
CAPALW	is the capital allowance for use in taxation calculations.
GROSS	is the total value of the aircraft held prior to depreciation.
DEPRCN	is the depreciation charge for use in the income calculation.

7. SUBROUTINE ORDERS

This subroutine calculates the information associated with the purchase and sale of aircraft.

7.1 Listing of Main Variables

IYEAR	is the year of the present calculation.
PAYMNT	is the total repayment to the manufacturers in IYEAR, and
INTRST	is the interest portion of that payment.
ACSALE	is the revenue from the sale of aircraft and
OUTND	is the outstanding debt on those aircraft.
INTPAY (A)	is the initial payment when an aircraft of type A is bought, and
PMNYRS (A)	is the number of years over which the outstanding debt must be repaid.
NUMPAY (A)	is the number of payments in a year and
PAYINT (A)	is the interest charged.
PAIMNI	is the subroutine which calculates the main variables associated with the purchase and sale of a given aircraft type.

8. SUBROUTINE PAIMNI

This subroutine calculates the repayments to the manufacturer assuming an equalised repayment schedule. It also calculates the debt outstanding on the aircraft which have been sold.

8.1 Listing of Main Variables

PLANE	is the aircraft type for which this calculation is being applied to.
PRCNTI	is the interest rate charged by the manufacturer.
ORDERI (A,T)	is the number of aircraft type A bought in year T.
INTPAI	is the initial payment on acquisition of the aircraft.
PURCHS (A,T)	is the purchase price of aircraft type A in year T.
PAYMNT	is the total repayment for the year being analysed.
INTRST	is the interest portion of the repayment. This will be used for calculating profit.
OUSTND	is the outstanding debt on the aircraft sold.

The next two subroutines (TRAFIK and ACOCTN) are associated with the simplified traffic allocation model. In the model which will be used in actual practice these will have to be altered.

11. SUBROUTINE ENDOPT

This subroutine finds the 'optimal' policy of the most recent iteration through the Dynamic Programming Algorithm and compares it with that of the previous one. If they are the same the indicator which stops the D.P. process is activated. If they are different, a finer grid is generated so that the process can be repeated.

11.1 Listing of Main Variables

ACTYPE	is the number of aircraft types considered in the corporate plan.
PREYRS	is the number of years prior to the planning period in which the old aircraft were bought.
PLNYRS	is the number of years in the planning period.
CAPBOR (I,J)	are the short-term debt and
CAPINT (I,J)	the investment in short-term securities of the final year.
CRASST (I,J)	are the current assets and liabilities associated
CRLIAB (I,J)	with the final year.
OPTIMA	finds the characteristics of the optimal path.
OPTORD (A,T)	is the 'optimal' policy of the most recent iteration and,
BSTORD (A,T)	is the 'optimal' policy of the previous iteration.
ACHELD (A,T)	is the 'optimal' fleet suggested by the most recent iteration.

NUMFLT (T) is the number of fleets generated for the T^{th} stage.

FLEETS (K,A,T) is the K^{th} fleet generated in year T.

REVNU (K,T) is the revenue associated with the K^{th} fleet generated in year T and,

VARCST (K,T) is the associated variable cost.

FLTGEN is the fine grid generator.

12. SUBROUTINE FLTGEN

This subroutine generates a 'finer grid' through which the Dynamic programming procedure will find the 'optimal' path.

12.1 Listing of Main Variables

ACTPYE	is the number of aircraft types considered in the corporate plan.
PREYRS	is the number of years prior to the planning period in which the old aircraft were bought.
PLNYRS	is the number of years in the planning period.
MAXAC (A,T)	is the maximum number of aircraft in year T which can be considered as a result of the manufacturers delivery constraint and of the airlines retirement policy.
FLEETS (K,A,T)	is the K^{th} fleet generated for year T.
MAXBUY (A,T)	is the maximum number of aircraft type A which can be delivered by the manufacturer in year T.
ORDERZ (A,T)	is the number of aircraft ordered in the pre-planning period in year T.
MAXHLD (A,T)	is the maximum number of aircraft type A which can be in the fleet in year T, and is a result of the airlines retirement policy.
TRAFIK (T)	generated the traffic levels appropriate to year T. This traffic will be used by the fleet feasibility testing routine.

HEURIS	is an indicator which states which heuristic rule is generating the fleet.
MAXUTL (A,T)	is the maximum aircraft utilisation.
ALOCTN	is the subroutine which tests for fleet feasibility and calculates the associated.
REVNU (K,T)	Revenue and
VARCST (K,T)	Operating variable cost.
NUMFLT (T)	is the number of fleets generated for year T.

APPENDIX C

THE TRAFFIC ALLOCATION PROGRAM

This appendix describes the inputs required by the traffic allocation program; its listing is available at Imperial College. To facilitate the user in his understanding of the routines, there is a brief description of all the routines and of the main variables used by them. The program is amply documented with comment cards (6.0 Subroutine AIRRUT) for a sample listing, and for the more complicated routines flowcharts are shown (Main Program, Subroutine AIRRUT, ALOCTN and PERTBN): flowcharts for the perturbation routines are in the main text and are shown in Figs. 3.15 and 3.16.

Input to the Traffic Allocation Program FORMAT

Card 1	Terminator	A3
Card 2	Description of Season	3A6
	Number of weeks season	I2
<u>Traffic Information</u>		
2 Cards for each sector (Max Sectors 100)		
Card 1	Origin	A3
	Destination	A3
	Sector Distance	I4
	Average one way Demand	I5
	Minimum Frequency	I2
	Maximum Load Factor	F3.2
	Revenue Rate	F6.2
Card 2	Fifth Freedom Traffic	F6.2
	Extra Generated Traffic (capacity)	F6.2
	Extra Generated Traffic (frequency)	F6.1
	Minimum Number of Direct lights	I2
	Maximum Number of Stops	I2
Card 2N + 1	Terminator	A3

FORMAT
==

Aircraft Information

2 Cards for each aircraft type (Max 5 A/C)

Card 1	Aircraft Code	A4
	Number of Seats	I3
	Hourly Cost	F6.1
Cost per	Cost per Flight Cycle	F6.1
	Maximum Utilisation per A/C	F5.0
Card 2	Intercept of Block Fuel Equation	F10.4
	Gradient of Block Fuel Equation	F10.4
	Intercept of Block Time Equation	F10.6
	Gradient of Block Time Equation	F10.6
Card 2N + 1	Terminator	A3
Card 2N + 2	Number of each aircraft type	5 I3
1 Card for each pay load/range diagram		
	(PAYLOAD (I), RANGE (I) = 1,4)	4 (I3, 14)

Station Information

1 Card for each city (Max 50 Cities)

Card 1	City	A3
	Price of Fuel	F10.9
	and for each aircraft type:-	
	Handling Charges	F5.1
	Landing Fees	F5.1
Card N + 1	Terminator	A3

Deviation from Standard Calculation

1 Card for each sector/aircraft combination

(Max 20 Combinations)

Card 1	Origin	A3
	Destination	A3
	Aircraft Code	A4
	Effective Seating Capacity	I3
	Block Fuel	F7.1
	Block Time	F4.2
Card N + 1	Terminator	A3

1.0 SUBROUTINE SASSON

This is the master program of the airline traffic allocation algorithm.

1.1 Listing of main variables

TRAFIK	this subroutine reads in and stores the traffic information.
CSTINF	this subroutine reads in and stores the; aircraft characteristics.
NETWRK	this subroutine generates the list of feasible routes and calculates the number of routes through each of the sectors.
ACTYPE	is the number of aircraft types considered in the allocation process.
NUMBAC (A)	is the number of aircraft of type A.
MAXUTZ (A)	is the number availability of aircraft type A.
MAXUTL (A)	is the remaining number of hours which can be assigned to aircraft type A.
ROUTOS	determines the "best most difficult sector" NXTSEC.
COSTS	is the subroutine which costs the aircraft/routes which will be considered in the allocation that follows.
AIRRUT	performs the traffic allocation according to the Standard Algorithm.

IALFA	is the indicator associated with AIRRUT: when it is equal to zero the allocations are stored in the primary storage space.
STORE	stores the output of each allocation
UPINFO	updates the information of the traffic primary storage space at the end of each allocation.
REPOSB	is the subroutine which deletes NORUTE from the set of feasible routes and re-calculates the number of routes through each of the appropriate sectors.
ZRODMD	is an indicator: when it is equal to one only, the sectors which have base demand passengers are considered in subroutine REPOSB.
SUBNTK	this subroutine identifies the sectors which go to make up each of the airline's net-work where
NUMSUB	is the number of sub-networks.
SUBIND(S)	is the reallocation index of sub-network S. When this index is equal to 0 it means that the passengers of this sub-network must not be reallocation in this iteration of the "No conflict" opportunity cost calculations.
ALOFLT	finds the storage locations in the appropriate storage space (primary or secondary) of the aircraft/route combinations assigned to the given sub-network.

ZROINF	re-sets the auxilliary traffic storage space to zero.
SETINF	aggregates the traffic and allocation information associated with a given number of routes previously assigned to the sub-network. This information is stored in the auxilliary traffic storage space.
RESET	this subroutine re-sets the primary traffic and allocation information to the appropriate values aggregated and stored in the auxilliary storage space.
UPSTRE	this subroutine deletes from the primary allocation storage space those associated with the given sub-network and derived in the previous iteration of the "No conflict" opportunity cost calculation.
ALOCTN	performs the various perturbation analysis for the given sub-networks.
PROFIT (P)	is the profit associated with each perturbation analysis.
PERTBN	this subroutine updates the primary allocation storage space with the new "perturbed" allocations for the given sub-network.
INFIZ	is the infeasibility index. When this counter is equal to ACTYPE then the solution is infeasible.

FEZPRF is the profitability of the best feasible solution encountered in the process of finding the optimal ones where,

FEZOPT (A) is the opportunity cost of each aircraft type associated with the above solution.

OPTIMA identified the aircraft which constitute a scarce resource and then for each of these aircraft attempts to find the "No conflict" opportunity cost.

OPTMUM is the optimality index. When this counter is equal to ACTPYE then the solution has reached its optimal state.

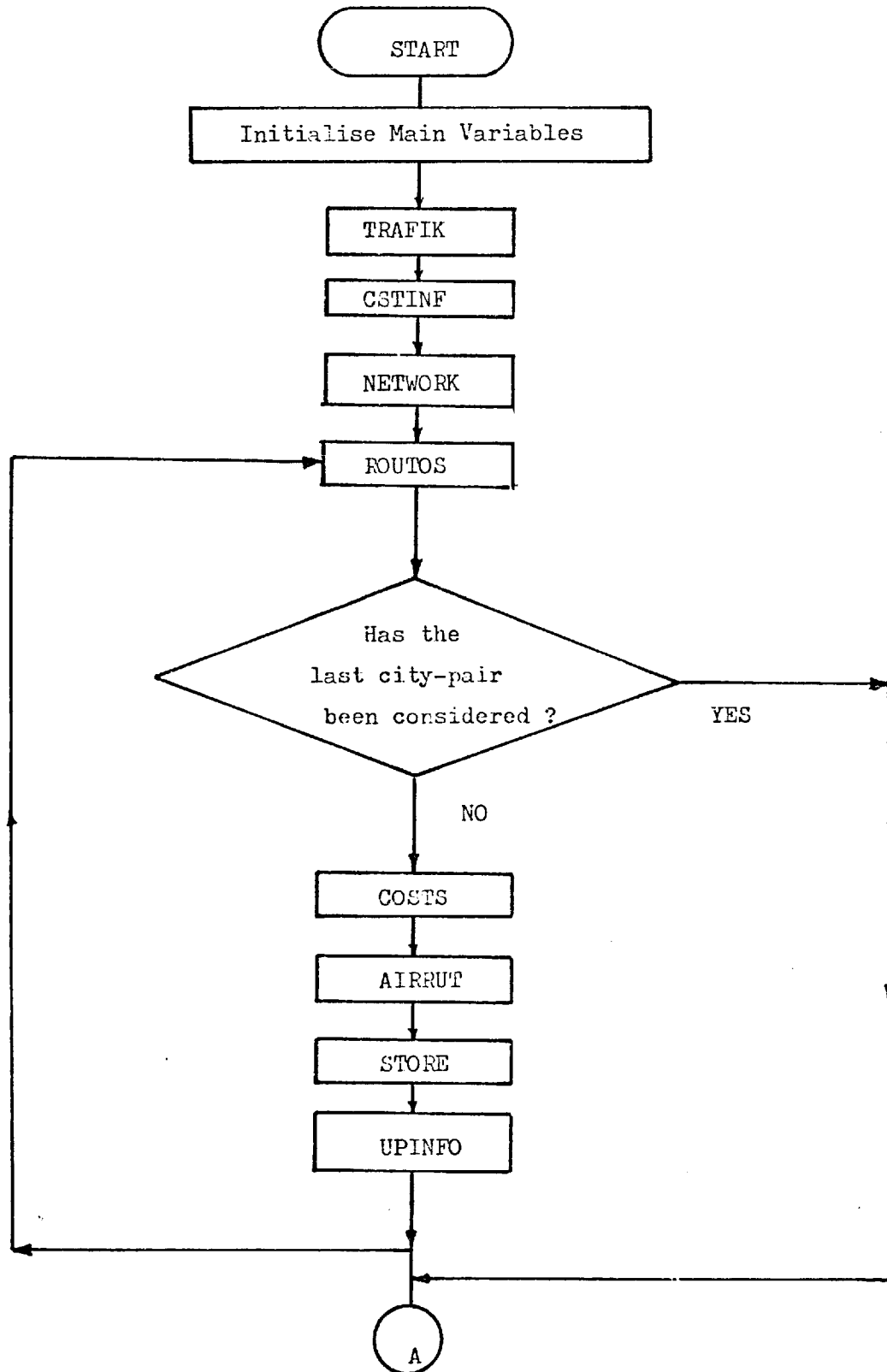
REALOK determines the sub-networks which should be re-allocated in the next iteration of the "No conflict" opportunity cost calculation.

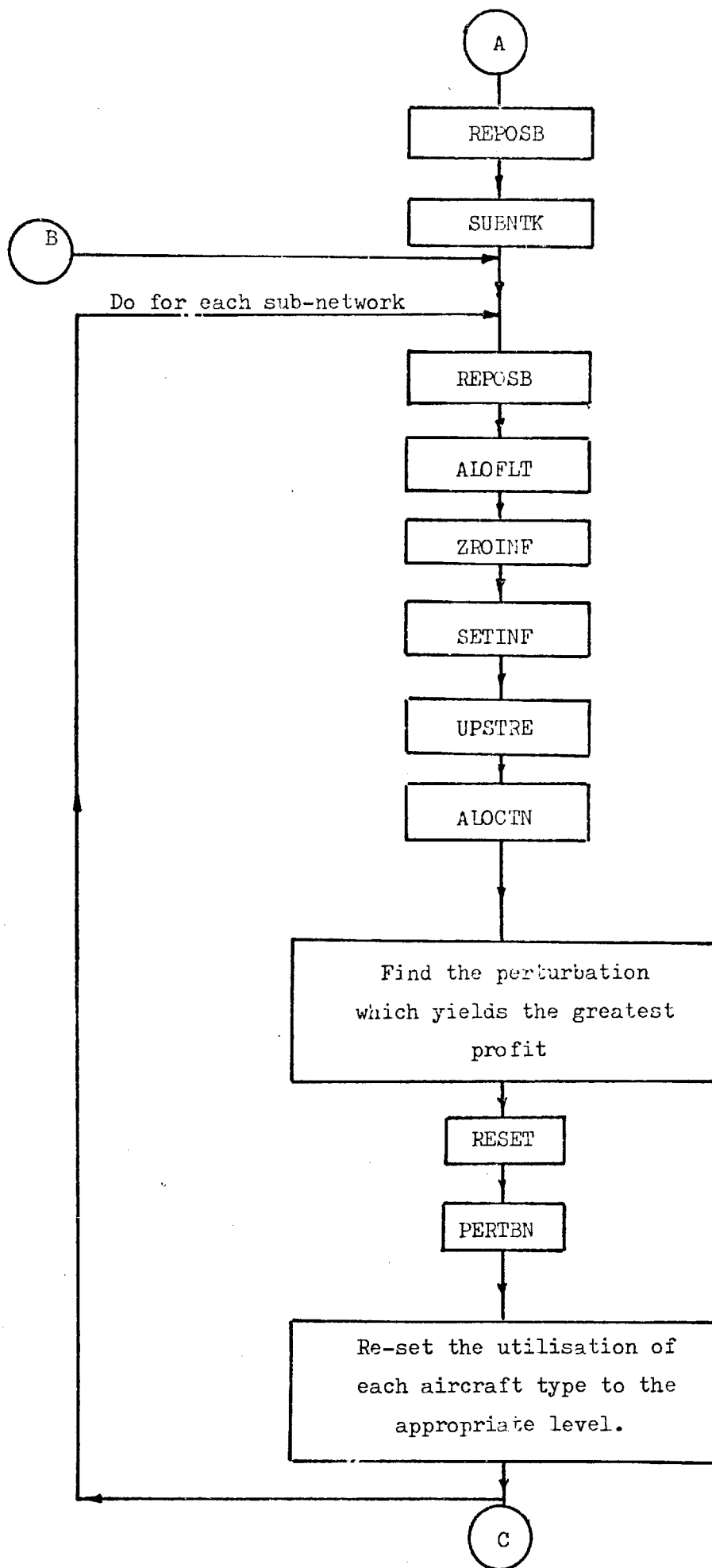
ACUTLZ(A) is the actual maximum availability of aircraft type A.

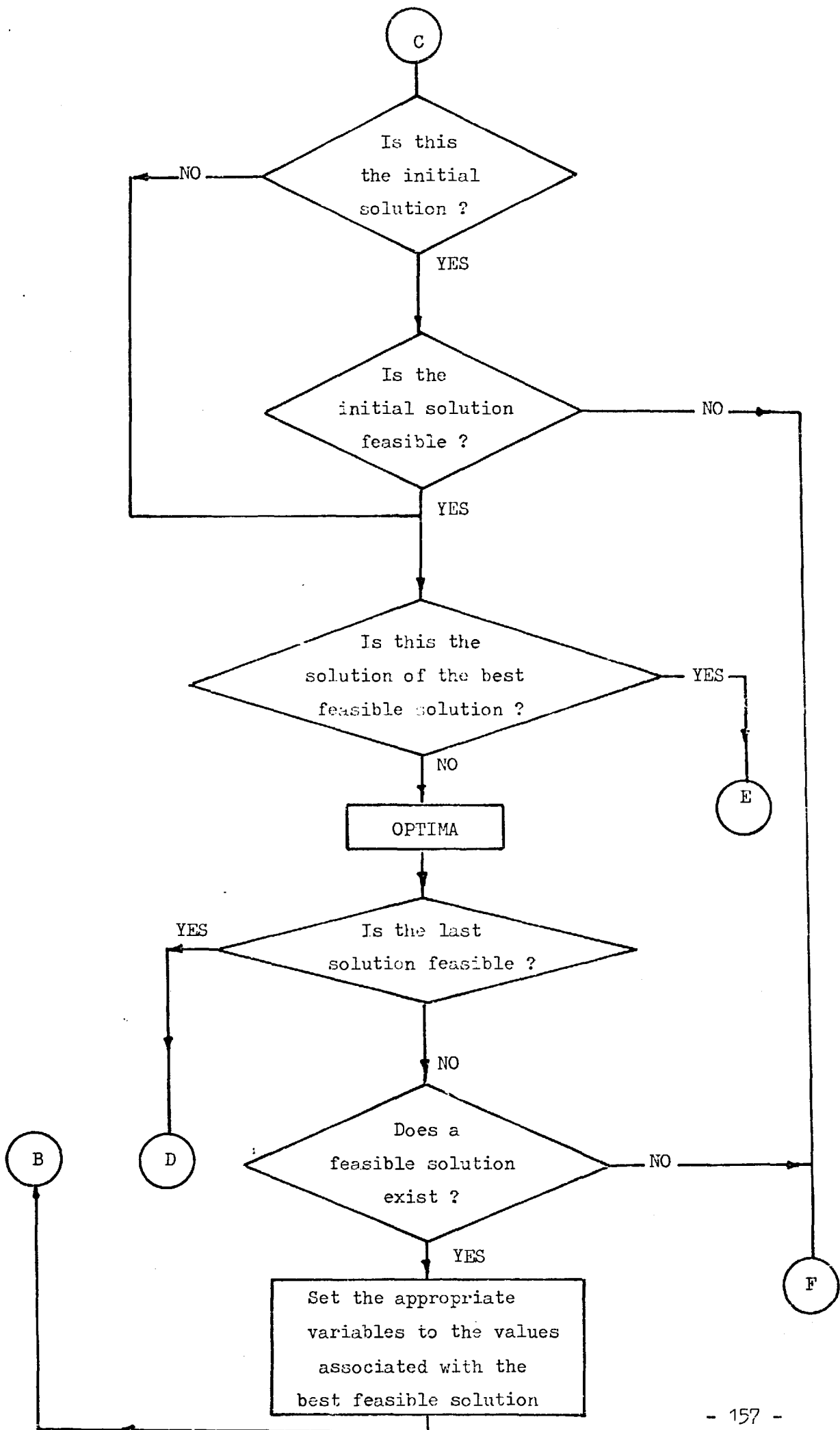
ACUTZ(A) is the number of hours which represents the most recent excess demand/capacity on aircraft type A.

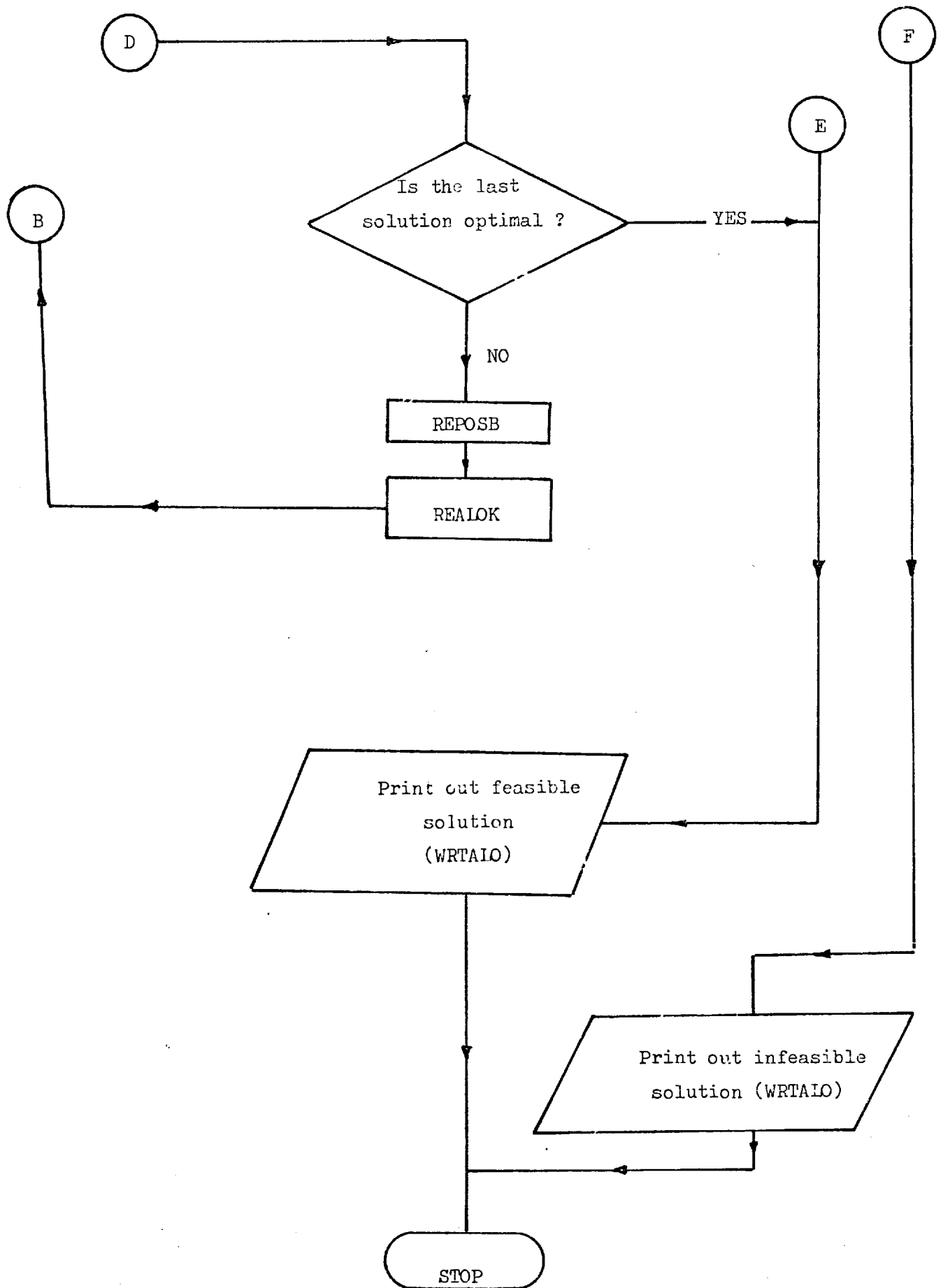
WRTALO is the report generation, which prints out the allocations stores in the primary storage space.

1.2 MAIN PROGRAM FLOWCHART









2.0 The Subroutine TRAFIK

This subroutine reads in and stores the traffic information.

2.1 Listing of Main Variables.

CAPTRF(I)	is D_{ij}^t due to an increase in the quality of service as a result of an increase in the capacity offered.
DELFRQ(I)	is the remaining number of flights that must be assigned to the I^{th} sector.
DEMAND(I)	is the unsatisfied base demand on the I^{th} sector, and is still to be allocated to flights.
DIRECT(I)	is equal to $N_{min\ ij}^*$, and is the minimum number of 'direct' flights required on the I^{th} sector.
DLDRCT(I)	is the remaining number of direct flights that must be assigned to the I^{th} sector.
ESADST(I)	is d_{ij} , the stage distance of the I^{th} sector.
FIFTH(I)	is ΔD_{ij}^t due to Fifth Freedom rights.
FRQTRF(I)	is ΔD_{ij}^t due to an increase in the number of flights on the I^{th} sector above the minimum required.
MXSTOP(I)	is the maximum number of intermediate stops for passengers flying the I^{th} sector.
MINFRQ(I)	is equal to $N_{min\ ij}$, and is the minimum number of flights required on the I^{th} sector.
MXLOAD(I)	is the maximum permissible load factor on the I^{th} sector; i.e. $LF_{max\ ij}$.

NUMORG	is the number of cities served by the airline; i.e. $ Q $.
NUMSEC	is the number of sectors read in; i.e. $ A $.
ORIGIN(I) DESTIN(I)	these define the I^{th} sector in the set A .
SEASON	this is an alpha-numeric variable which describes the period being analysed.
TOWN(I)	stores the alpha-numeric code of the I^{th} city in the set Q .
YIELD(I)	is the average revenue rate from passengers flying the I^{th} sector; i.e. Y_{ij} .

3.0 The Subroutine NETWORK

This subroutine generates a set of feasible routes, connecting no more than five cities, from the previously inputted sector information. It also searches through this set, $\underline{R}$, to find the subsets, $\underline{R}_{ij}$ ($\forall \{ij\} \in \underline{A}$), of routes which contain routings along the sector $\{ij\}$.

3.1 Listing of Main Variables.

ESADST(I)	is the stage distance of the I^{th} sector.
CITYA	is the starting point of a sector.
CITYB	is the terminating point of a sector.
CRSREF	is the location of the sector CITYA - CITYB in the set $\underline{A}$.
TRANSF	is a subroutine which finds CRSREF for the sector CITYA - CITYB.
MAX	is the maximum leg distance. Any route which has a leg distance greater than MAX will not be stored.
MATRAX(J)	is the number of sectors starting from city j; i.e. $ \underline{G}(J) $.
MXSTOP(I)	is the number of maximum intermediate stops on the assigned routings for this sector.
NO ROOT (K)	stores the routes that cannot be assigned, as a result of the MXSTOP(I) constraint.

MATRIX (I,J,1)	is the I^{th} terminating city in $\underline{G}(J)$.
MATRIX (I,J,2)	stores the location of sector $J-\underline{G}(J,I)$ in the set $\underline{A}$.
ROUTES	is the set, $\underline{R}$, of radial routes connecting five cities or less and terminating at the airline's base.
RTAVBL	is the number of routes stored in $\underline{R}$.
TERMIN	is the airline's base.
POINTA	is the number of legs on a given route.
POINTB	is the number of cities linked by a given route.
POINTS	is a subroutine which finds POINTA and POINTB for a given route.
POSBRT (J,I)	is the I^{th} possible route which contains a routing along the J^{th} sector.
NMPOSB(J)	is the number of routes with routings along the J^{th} sector.

4.0 The Subroutine CSTINF

This subroutine reads in all the aircraft characteristics and all the station information which will be used to calculate the cost of a flight.

4.1 Listing of Main Variables

ACTYPE	is the number of different aircraft types.
ACCODE(I)	is the code of the I^{th} aircraft type.
ACSEAT(I)	is the number of seats in the I^{th} aircraft.
HOURLY(I)	is the hourly cost of the I^{th} aircraft.
FLIGHT(I)	is the flight cost.
MAXUTL(I)	is the maximum permissible utilisation for the I^{th} aircraft type.
HSFULA(I)	is the intercept of the block fuel equation.
HSFULB(I)	is the gradient of the block fuel equation.
HSTIMA(I)	is the intercept of the block time equation.
HSTIMB(I)	is the gradient of the block time equation.
NUMBAC(I)	is the number of aircraft of type I.
NUMORG	is the number of cities served by the airline.
FULPRC(J)	is the price of fuel at city J.
HANDLG(I,J)	is the landing charge at city J for the I^{th} aircraft.

LANDNG(I,J)	is the landing fee at city J for the I th aircraft.
EXPSHN(K,1)	is the location in the set <u>A</u> of the sector in the K th sector/aircraft combination which deviates from the standard payload range diagram.
EXPSHN(K,2)	is the aircraft in the K th sector/aircraft deviation.
EXPSHN(K,3)	is the effective seating capacity of the K th sector/aircraft deviation.
EXPSHO(K,1)	is the block fuel for the K th deviation.
EXPSHO(K,2)	is the block time.
NUMEXP	is the number of combinations which deviate from the standard payload-range diagram.

5.0 The Subroutine ROUTOS

This subroutine eliminates any redundant routes from the sets of possible routes and also find the next sector to be allocated in the standard allocation algorithm.

5.1 Listing of Main Variables

REDUNT	is an indicator. When it is equal to 1, the redundant routes, as defined in the main text, are to be eliminated.
NUMSEC	is the number of sectors read in.
DEMAND(I)	is the unsatisfied base demand on the I^{th} sector, and is still to be allocated to flights.
NMPOSB(I)	is the number of routes which contain routings along the I^{th} sector.
POSBRT(I,K)	are the routes along the I^{th} sector.
ROUTES	is the set of all routes.
MINPOS	is the minimum of the up-dated NMPOSB(I).
MINPAX	is the minimum number of passengers on a sector which has MINPOS possible routes.
NXTSEC	is the location in the set <u>A</u> of the chosen sector pq.
RTVBLE	is the number of possible routes along the sector pq.
DELFRQ(I)	is the remaining number of flights that must be assigned to the I^{th} sector.
DLDRCT(I)	is the remaining number of 'direct' flights that must be assigned to the I^{th} sector.

MINLEG

is the number of possible 'direct' routes along a given sector.

6.0 The Subroutine AIRRUT

This subroutine finds the 'best' aircraft/route combination to allocate passengers from the chosen sector.

6.1 Listing of the Main Variables

In view of the complexity of this subroutine, the variables have been listed in the order in which they appear in the program listing. This should make it easier to understand this subroutine.

NUMPHY	is the number of aircraft/route combinations to which passengers from the chosen sector, pq, can be allocated.
ACTYPE	is the number of different aircraft types.
RTVBLE	is the number of possible routes which contain routings along the sector pq; i.e. $\frac{R}{pq}$. If the flights are to be 'direct' then RTVBLE is equal to $\frac{R}{pq} (\text{min } m)$.
NXTSEC	is the location in <u>A</u> of the chosen sector pq.
POSBRT(I,K)	is the K^{th} possible route which contains a routing along the I^{th} sector.
PHY(K)	is the profitability of the K^{th} aircraft/route combination; i.e. $OP_f(a, \underline{r})$.
PHYI(K,1)	is the route in the K^{th} combination; i.e. <u>r</u> .
PHYI(K,2)	is the aircraft in the K^{th} combination; i.e. a.
ROUTE	is the route in the combination with the maximum PHY.
AIRCAF	is the aircraft in the combination with the maximum PHY.

PRVIOS	is an indicator. When it is equal to 1, it means that this aircraft/route combination has previously been assigned.
FLYTE(K,1) FLYGHT(K,1)	is the route in the K^{th} previously assigned aircraft/route combination.
FLYTE(K,2) FLYGHT(K,2)	is the aircraft in the K^{th} previously assigned aircraft/route combination.
LTERTN JTERTN	is the location of an aircraft/route combination in the stored information of previously assigned flights.
PRVFLT	is the number of previously assigned flights using the above aircraft/route combination.
PRVTRF(S)	is the number of passengers flying the S^{th} sector of the route who have previously been allocated to flights using this aircraft/route combination.
SECTRS	is the number of sectors on the route.
TABLOX(S,L)	is an indicator. When it is equal to 1, it means that the passengers flying on sector S of the route must also fly along the leg L.
TABLO(S,L)	stores the passenger demand on sector S allocated to leg L of the route.
DEMAND(I)	is the unsatisfied base demand of the I^{th} sector in the set <u>A</u> .
DMANDX(S)	is the unsatisfied base demand of the S^{th} sector on the route.
DELFRQ(I)	is the number of unsatisfied flights on the I^{th} sector in the set <u>A</u> .

DLFRQX(S) is the unsatisfied number of flights on the S^{th} sector on the route.

YIELD(I) is the average revenue rate from passengers on the I^{th} sector in the set A.

REVNU(S,1) is the average revenue rate from passengers flying the S^{th} sector on the route.

MXLOAD(I) is the maximum permissible load factor on the I^{th} sector in the set A.

LOADX(S) is the maximum permissible load factor on the S^{th} sector on the route.

MAXLOD(L) is the maximum permissible leg load factor. This is calculated from the most critical LOADX(S) applicable to the leg L.

POINTA is the number of legs on a given route.

POINTB is the number of cities which a given route links.

POINTS is a subroutine which finds POINTA and POINTB for a given route.

CITYA is the starting point of a sector.

CITYB is the terminating point of a sector.

CRSREF is the location of the sector CITYA-CITYB in the set A.

FREQNX(K,2) is the number of sectors on the route which have FREQNX(K,1) unsatisfied flights.

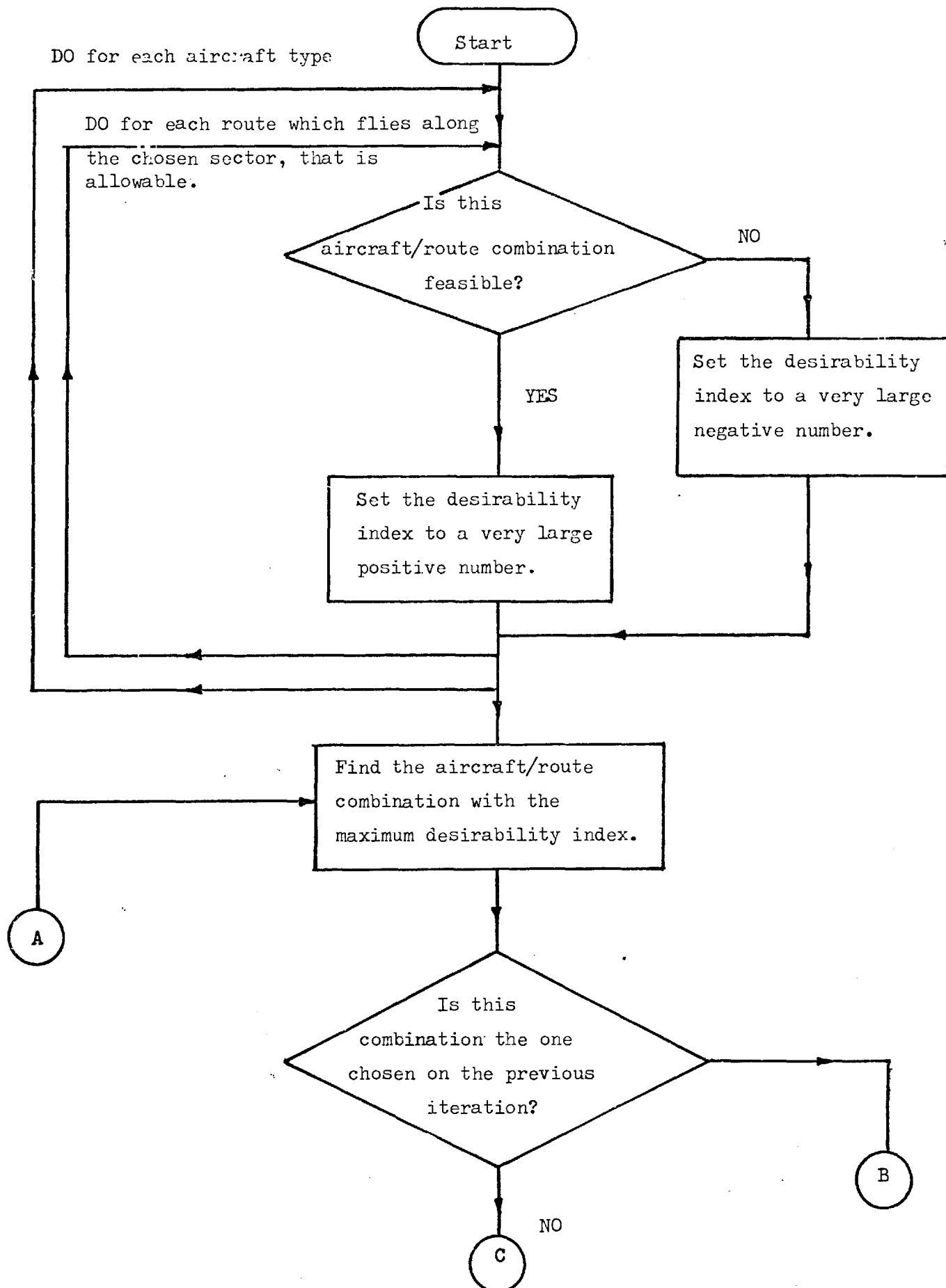
FREQNC(K,J) is the Jth sector which has FREQNX(K,1) unsatisfied flights.

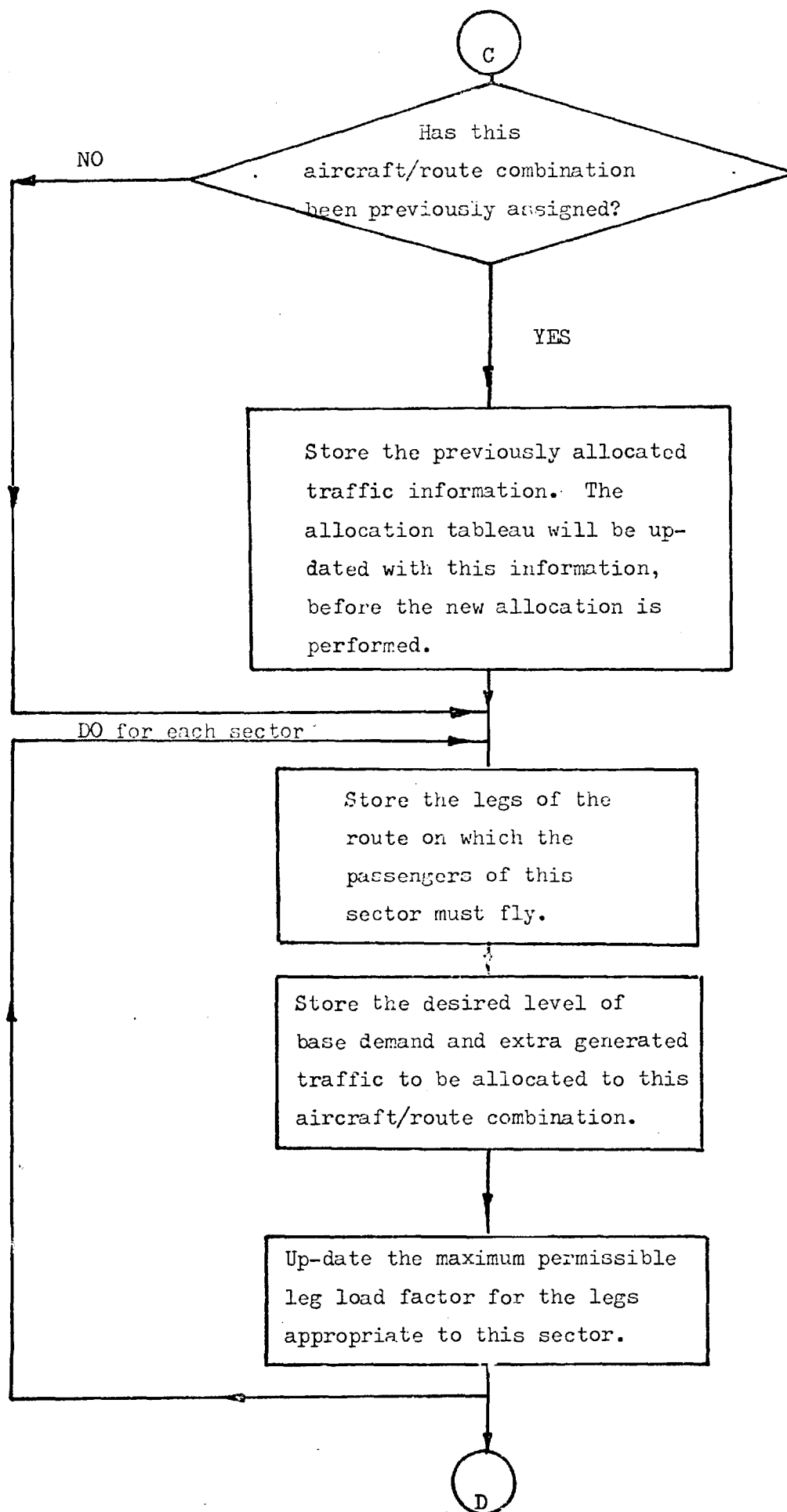
NUMFRQ is the number of different FREQNX(K,1).

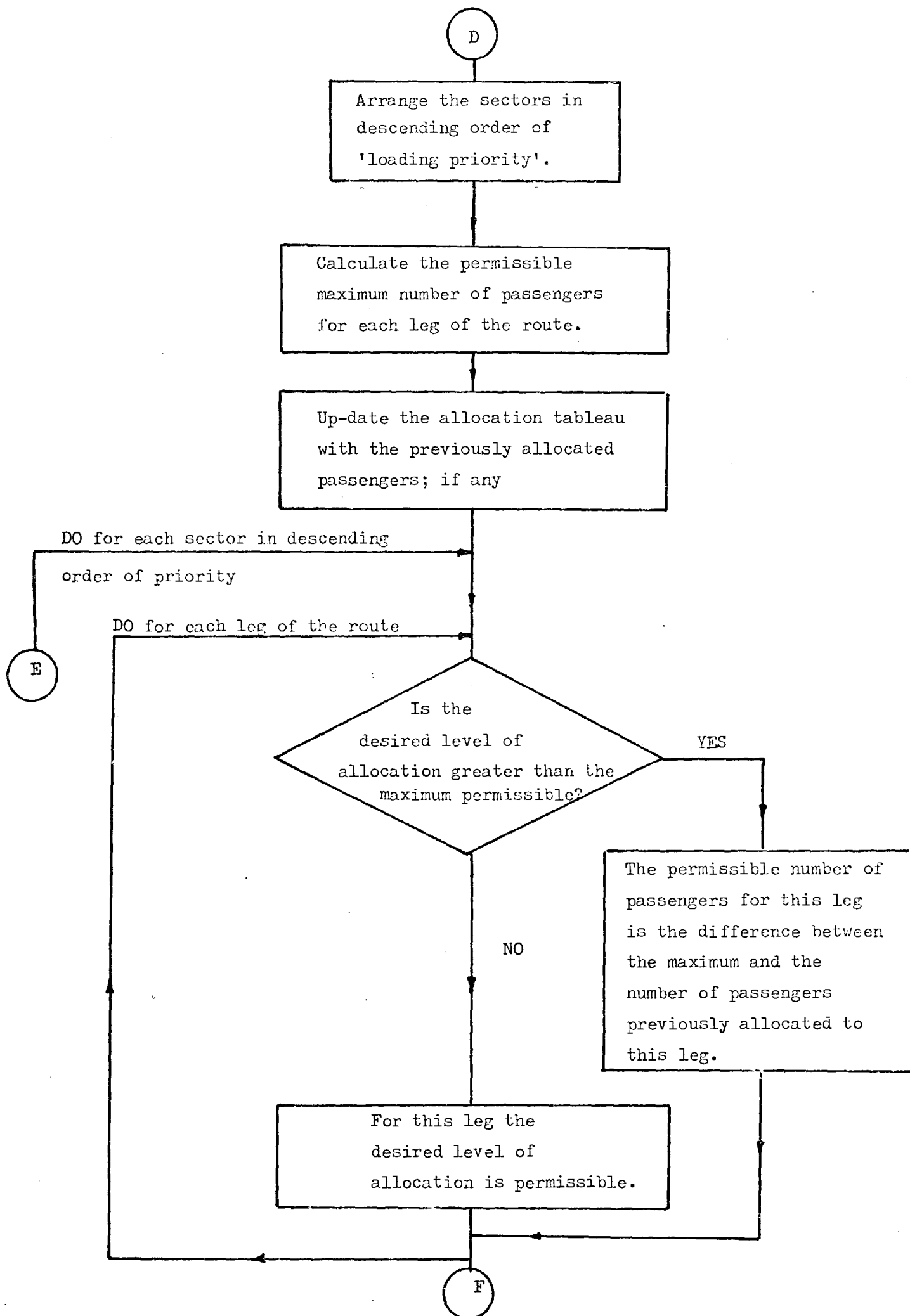
SECFST	is an indicator which states whether or not the sector FSTSEC should be inserted at the head of the 'priority loading' list.
GMAPRT	is an indicator which states whether or not the sector GMADMY should be inserted at the tail of the 'priority loading' list.
MAXPAX(L)	is the maximum number of passengers allowed to fly the leg L.
MXSEAT(L)	is the maximum effective seating capacity of a given aircraft on the leg L.
FREQCY	is the number of unsatisfied flight for the route sector whose passengers are being allocated.
SECTR	is the route sector whose passengers are being allocated.
PASENG	is the number of passengers on the route sector SECTR who are eligible for allocation on the given aircraft/route combination.
PAX(L)	is the number of passengers which can be allocated from the sector SECTR to the L th leg of the flight.
PASGRS(S,1)	is the number of passengers from sector S who are allocated to the given flight.
PASGRS(S,2)	is the number of extra generated passengers allocated to the given flight from sector S.
LEGLF(L)	is the load factor on the L th leg of the flight.
TOTREV	is the total revenue from the passengers allocated to the given flight.

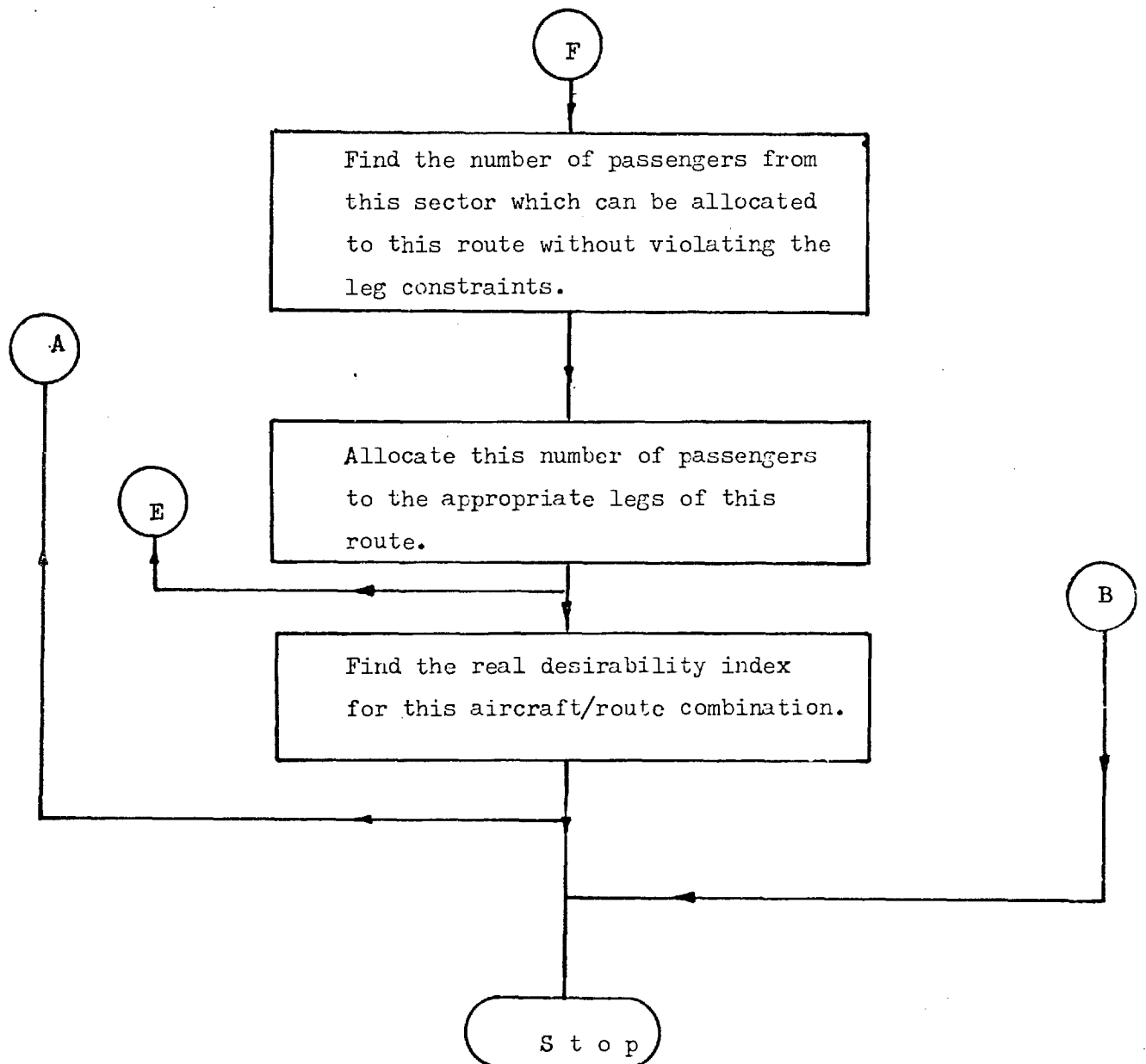
REVNU(S,2)	is the revenue from the passengers on the S th sector allocated to the given flight.
ACCSTX(K,1)	is the aircraft in the K th cost calculation performed in the subroutine COSTS.
ACCSTX(K,2)	is the route in the K th cost calculation.
UTILIS	is the total time consumed by the aircraft in performing the given flight.
XFUEL	is the total fuel cost for the given flight.
XLAND	is the total landing fees for this flight.
XHAND	is the total landing charges for this flight.
XENGR	is the total engineering cost of this flight.
OPTUNY (A)	is the opportunity cost per hour associated with aircraft type A.
POFT	is the 'profit' associated with the aircraft/route combination being analysed.

6.2 Flow Chart for Subroutine AIRRUT









```

SUBROUTINE AIRRUT(ISCND,IREALO,NUMWKS,POFT,K,GMAPRX,GMASIC,SICFST,
1FSTSIC,OPTUNY)

```

```

C=====I
C                                     I
C                                     I
C THIS SUBROUTINE FINDS THE BEST AIRCRAFT/ROUTE COMBINATION I
C WHICH FLIES ALONG THE DESIRED SECTOR I
C                                     I
C=====I
C
C

```

```

INTEGER GMAPRT,GMAPRX,GMASEC,GMASIC,GMADMY
INTEGER ROUTES,POSBRT,RTVBLE
INTEGER PASGRS,ROUTE,AIRCAF
INTEGER TRAFIC,FLYGHT
INTEGER TRAFYC,FLYTE
INTEGER DEMAND,DELFRQ,ACTFRQ,ACFLTS
INTEGER DMANDX(10),DLFRQX(10)
INTEGER ACCSTX,SICFST,FSTSIC,SECFST,FSTSEC,FSTDY
INTEGER ACCODE,ACSEAT,ACTYPE
INTEGER PHY(30,2),AUX1,TABLOX(10,4),TABLO(10,4),PAENG,PAX(4)
INTEGER PRVLOS,PRVRF(10),PRVFLT,EXDMND(10),EXTRA
INTEGER CITYA,CITYB,POINTA,POINTB,CRSREF
INTEGER FREQNC(10,10),FREQNX(10,2),FREQCY,SECTR,SECTRS
REAL LOADX(10),MAXLOD(4)
REAL K(6,2)
REAL LEGLF,LODFCT,LUDEFCT,MXLOAD,LANFES
DIMENSION PHY(30),MAXPAX(4),MXSEAT(4),OPTUNY(5)
COMMON TRAFIC(90,10,3),LEG(90,4),LODFCT(90,4),FLYGHT(90,4),CASH(90
1,14),UTLSTN(90),ITERTN,JTERTN
COMMON/SG2/DEMAND(100),MINFRQ(100),YIELD(100),MXLOAD(100),DELFRQ(1
100),ACTFRQ(100),ACFLTS(100)
COMMON/SG4/ROUTES(100,5),NMPOSB(100),POSBRT(100,8),RTVBLE,NXTSEC
COMMON/SG8/TRAFYC(15,10,3),LEG1(15,4),LUDEFCT(15,4),FLYTE(15,4),CAS
1H1(15,14),UTLZTN(15),KTERTN,LTERTN
COMMON/SG10/ACCODE(5),ACSEAT(5),HOURLY(5),FLIGHT(5),MAXUTL(5),NUMB

```

```

1AC(5),ACUTLZ(5),ACTYPE
COMMON/SG12/ACCSTX(30,2),TIME(30),FUEL(30),HANLNG(30),LANFES(30),E
1NGCST(30)
COMMON/SG13/UTILIS,XFUEL,XHAND,XLAND,XENGR,REVNU(10,2),PASGRS(10,2
1),LEGPAX(4),LEGLF(4),ROUTE,AIRCAF

```

```

GMAPRT=GMAPRX
GMASEC=GMASIC
FSTDNY=FSTSIC
SECFST=SICFST

```

```

PHY(RT,AC) IS A MEASURE OF AIRCRAFT/ROUTE DESIRABILITY

```

```

PHY(RT,AC)=TOTAL REVENUE - VARIABLE COST - OPPORTUNITY COST

```

```

'NXTSEC' IS THE SECTOR WITH THE GREATEST DEGREE OF DIFICULTY

```

```

NUMPHY=0

```

```

DO 4000 IBA=1,ACTYPE

```

```

DO 4001 IBB=1,RTVBLE

```

```

NUMPHY=NUMPHY+1
PHY(Y(NUMPHY,1)=POSBRT(NXTSEC,IBB)
PHY(Y(NUMPHY,2)=IBA

```

```

C      PHY(NUMPHY)=0.1E20
C
C      CHECK TO SEE THAT THIS AIRCRAFT/ROUTE COMBINATION IS
C      NOT INFEASIBLE
C
C      ADUMY=0.1E10
C      IF(FUEL(NUMPHY).LT.ADUMY) GO TO 4001
C      PHY(NUMPHY)=-0.1E20
C
C
C      4001 CONTINUE
C
C
C      4000 CONTINUE
C
C
C      FIND AIRCRAFT/ROUTE COMBINATION WITH MAXIMUM PHY
C
C
C
C      MAXRTE=0
C      ROUTE=0
C      AIRCAF=0
C
C
C
C      4002 JADUMY=NUMPHY-1
C      4003 IF(JADUMY.EQ.0) GO TO 4006
C      JBDUMY=1
C
C
C
C      DO 4004 IBA=1,JADUMY
C
C
C      JCDUMY=IBA+1
C
C
C      IF(PHY(IBA).GE.PHY(JCDUMY)) GO TO 4004

```

```

C
C
DO 4005 IBB=1,2
AUXI=PHY(Y(IBA,IBB)
PHY(Y(IBA,IBB)=PHY(Y(JCDUMY,IBB)
PHY(Y(JCDUMY,IBB)=AUXI
4005 CONTINUE
C
C
AUX=PHY(IBA)
PHY(IBA)=PHY(JCDUMY)
PHY(JCDUMY)=AUX
C
C
JBDUMY=IBA
C
C
4004 CONTINUE
C
C
JADUMY=JBDUMY-1
GO TO 4003
C
C
C
C
MAXRTE=1 MEANS THAT THIS IS A CHECK TO SEE IF THIS COMBINATION
IS STILL THE ONE WITH THE MAXIMUM PHY
C
C
MAXRTE=0 MEANS THAT THIS IS THE FIRST RUN
C
C
C
4006 IF(MAXRTE.EQ.0) GO TO 4007
C
C
THE PROPOSED 'BEST' AIRCRAFT/ROUTE COMBINATION IS AIRCAF/ROUTE
C
IF(PHY(Y(1,1).NE.ROUTE) GO TO 4007
IF(PHY(Y(1,2).EQ.AIRCAF) GO TO 4250
4007 ROUTE=PHY(Y(1,1)
AIRCAF=PHY(Y(1,2)

```



```

C      IF(FLYTE(IBA,2).NE.AIRCAF) GO TO 4011
C
C      DO 4012 IBB=1,10
C      PRVTRF(IBM)=TRAFYC(IBA,IBB,1)
4012  CONTINUE
C
C      PRVLOS=1
C      PRVFLT=FLYTE(IBA,3)
C      LTERTN=IBA
C      GO TO 4020
C
C
4011  CONTINUE
C      GO TO 4020
C
C
C
4015  IF(ITERTN.EQ.0) GO TO 4020
C
C
C      DO 4016 IBA=1,ITERTN
C
C
C      IF(FLYGHT(IBA,1).NE.ROUTE) GO TO 4016
C      IF(FLYGHT(IBA,2).NE.AIRCAF) GO TO 4016
C
C
C      DO 4017 IBB=1,10
C      PRVTRF(IBM)=TRAFIC(IBA,IBB,1)
4017  CONTINUE
C
C
C      PRVLOS=1
C      PRVFLT=FLYGHT(IBA,3)
C      JTERTN=IBA
C      GO TO 4020
C
C

```

4016 CONTINUE

```
C
C
C=====I
C
C
C
C      INITIALISE THE ALLOCATION OF PASSENGERS TO SECTORS OF THIS ROUTE I
C
C      --- POINTA IS THE NUMBER OF LEGS ON THIS ROUTE I
C      --- POINTB IS THE NUMBER OF CITIES ON THIS ROUTE I
C
C=====I
C
C
C
```

4020 CALL POINTS(POINTA,POINTB,ROUTE)

```
JADUMY=0
GMADMY=0
FSTSEC=0
DO 4008 IBA=1,10
DO 4009 IBB=1,4
TABLO(IBA,IBB)=0
TABLOX(IBA,IBB)=0
```

4009 CONTINUE

```
DMANDX(IBA)=0
DLFRQX(IBA)=0
LOADX(IBA)=0.0
REVNU(IBA,1)=0.0
REVNU(IBA,2)=0.0
PASGRS(IBA,1)=0
PASGRS(IBA,2)=0
EXDMND(IBA)=0
```

4008 CONTINUE

```
DO 4010 IBA=1,4
LEGPAX(IBA)=0
LEGLF(IBA)=0.0
```

4010 CONTINUE

```
C
C
```

```

C      DO 4021 IBA=1,POINTA
C
C      CITYA=ROUTES(ROUTE,IBA)
C      JBDUMY=IBA+1
C      MAXLOD(IBA)=1.0
C
C      DO 4022 IBB=JBDUMY,POINTB
C
C      CITYB=ROUTES(ROUTE,IBB)
C      CALL TRANSF(CITYA,CITYB,CRSREF)
C      JADUMY=JADUMY+1
C
C      JDUMY=IBA
C      IDUMY=IBB-1
C
C      TABLOX(Y,Z)=1 INDICATES THAT THE PASSENGERS FLYING ON SECTOR Y
C      MUST FLY ALONG LEG Z
C
C      DO 4023 IBC=JDUMY,IDUMY
C      TABLOX(JADUMY,IBC)=1
4023 CONTINUE
C
C      IF(CRSREF.EQ.0) GO TO 4022
C
C      CHECK TO SEE IF THE ALLOCATION PROCESS IS TO BE REVISED
C      BY ALLOCATING PASSENGERS TO A SPECIFIC SECTOR
C      AT THE START OF THE PROCESS
C
C      IF(SECFST.EQ.0) GO TO 4027
C      IF(CRSREF.NE.FSTDUMY) GO TO 4027

```

```

      FSTSEC=JADUMY
C
C
C   CHECK TO SEE IF THE ALLOCATION PROCESS IS TO BE REVISED
C   BY ALLOCATING PASSENGERS TO A SPECIFIC SECTOR
C   AT THE END OF THE PROCESS
C
C
C 4027 IF(GMAPRT.EQ.0) GO TO 4026
      IF(CRSREF.NE.GMASEC) GO TO 4026
      GMADMY=JADUMY
C
C
C   DMANDX(I) IS THE BASE DEMAND LEFT FOR SECTOR I
C
C 4026 DMANDX(JADUMY)=DEMAND(CRSREF)
C
C
C   FIND THE EXTRA TRAFIC WHICH ARISES OUT OF FIFTH FREEDOM ETC
C
C
C   CALL EXTFIF(CRSREF,EXTRA)
      EXDMND(JADUMY)=EXDMND(JADUMY)+EXTRA
C
C
C   FIND THE TRAFFIC GENERATED DUE TO AN INCREASE IN THE
C   FREQUENCY OFFERED
C
C
C   CALL EXTFRQ(CRSREF,EXTRA,K,MINFRQ,ACFLTS)
      EXDMND(JADUMY)=EXDMND(JADUMY)+EXTRA
C
C
C
      REVNU(JADUMY,1)=YIELD(CRSREF)
      DLFRQX(JADUMY)=DELFRQ(CRSREF)
      IF(MXLOAD(CRSREF).LE.0.1) GO TO 4022
      LOADX(JADUMY)=MXLOAD(CRSREF)
      IF(PRVIGS.EQ.0) GO TO 4024

```

```

      IF (PRVTRF(JADUMY).NE.0) GO TO 4025
4024 IF (DMANDX(JADUMY).EQ.0) GO TO 4022
4025 IF (MAXLOD(IBA).LE.LOADX(JADUMY)) GO TO 4022
      MAXLOD(IBA)=LOADX(JADUMY)

```

```

C
C
4022 CONTINUE

```

```

C
C
4021 CONTINUE

```

```

      SECTRS=JADUMY

```

```

      STORE THE MAXIMUM NUMBER OF PASSENGERS FOR EACH LEG

```

```

      CALL SEATS(AIRCAF,MXSEAT,ACSEAT)

```

```

      DO 4041 IBA=1,POINTA
      MAXPAX(IBA)=((MAXLOD(IBA)*(FLOAT(MXSEAT(IBA)*NUMWKS)))+1)*(PRVFLT+
11)

```

```

4041 CONTINUE

```

```

      TABLO(I,J) STORES THE PASSENGERS ALLOCATED TO LEG J FROM SECTOR I

```

```

      LEGPAX(J),J=1,4 STORES THE NUMBER OF PASSENGERS FLYING ON LEG J

```

```

      LEGLF(J),J=1,4 STORES THE LEG LOAD FACTOR

```

```

      IF PREVIOUS SIMILAR FLIGHTS EXIST --- UPDATE TABLO(I(J)) AND
      LEGPAX(J)

```

```

C
C
C      IF(PRVIOS.EQ.0) GO TO 4099
C
C
C      DO 4050 IBA=1,SECTRS
C      DO 4051 IBB=1,POINTA
C      TABLO(IBA,IBB)=PRVTRF(IBA)*TABLOX(IBA,IBB)
C      LEGPAX(IBB)=LEGPAX(IBB)+TABLO(IBA,IBB)
4051 CONTINUE
4050 CONTINUE
C
C
C
C=====I
C                                     I
C                                     I
C      THE SECTOR PASSENGERS WILL BE LOADED IN ASCENDING ORDER OF      I
C      UNSATISFIED FREQUENCY                                             I
C                                     I
C      FOR SECTORS WHICH HAVE EQUAL FREQUENCIES,THE SECTOR WHOSE       I
C      ORIGIN IS MORE EXTREME WILL BE LOADED FIRST                     I
C                                     I
C      ADJUST THE PRIORITY LOADING IF A REVISED PROCESS IS IN OPERATION I
C                                     I
C=====I
C
C
C
C      4099 DO 4030 IBA=1,10
C          DO 4031 IBB=1,10
C          FREQNC(IBA,IBB)=0
C      4031 CONTINUE
C          DO 4032 IBB=1,2
C          FREQNX(IBA,IBB)=0
C      4032 CONTINUE
C      4030 CONTINUE
C
C

```

```

C      JBDUMY=SECTRS
      IADUMY=0
      NUMFRQ=1
C
C      DO 4035 IBA=1,50
C
C      IBB=IBA-1
      NUMFRQ=NUMFRQ+IADUMY
      IADUMY=0
      JCDUMY=0
C
C      DO 4036 IBC=1,SECTRS
C
C      IF(DLFRQX(IBC).NE.IBB) GO TO 4036
      IF(IBC.NE.0) GO TO 4037
      IF(EXOMND(IBC).EQ.0) GO TO 4038
C
C      4037 JCDUMY=JCDUMY+1
      FREQNX(NUMFRQ,1)=IBB
      FREQNX(NUMFRQ,2)=JCDUMY
      FREQNC(NUMFRQ,JCDUMY)=IBC
      IADUMY=1
C
C      4038 JBDUMY=JBDUMY-1
      IF(JBDUMY.EQ.0) GO TO 4040
C
C      4036 CONTINUE
C
C      4035 CONTINUE
C
C
C

```

C CHECK TO SEE IF ONE SECTOR MUST BE ALLOCATED BEFORE THE OTHERS

C
C

4040 IF(SECFST.EQ.0) GO TO 4300
IF(FSTSEC.EQ.0) GO TO 4300

C
C
C
C
C

DELETE THE PREVIOUS PRIORITY LOADING OF THIS SECTOR

DO 4310 IBA=1,NUMFRQ
JADUMY=FREQNX(IBA,2)
DO 4315 IBB=1,JADUMY
IF(FREQNC(IBA,IBB).NE.FSTSEC) GO TO 4315
IF(IEB.EQ.JADUMY) GO TO 4322
DO 4320 IBC=IBB,JADUMY
IBD=IBC+1
FREQNC(IBA,IBC)=FREQNC(IBA,IBD)

4320 CONTINUE

4322 FREQNC(IBA,JADUMY)=0
FREQNX(IBA,2)=JADUMY-1
GO TO 4325

4315 CONTINUE

4310 CONTINUE

C
C
C
C
C

INSERT THE SECTOR AT THE HEAD OF THE PRIORITY LOADING LIST

4325 JADUMY=FREQNX(1,2)
IF(JADUMY.EQ.0) GO TO 4335
DO 4330 IBA=1,JADUMY
KXDUMY=JADUMY-IBA+2
KYDUMY=KXDUMY-1
FREQNC(1,KXDUMY)=FREQNC(1,KYDUMY)
4330 CONTINUE
4335 FREQNC(1,1)=FSTSEC
FREQNX(1,2)=JADUMY+1

C
C

```

C
C      CHECK TO SEE IF ONE OF THE SECTORS MUST BE ALLOCATED LAST
C
C
C
C
4300 IF(GMAPRT.EQ.0) GO TO 4100
      IF(GMADMY.EQ.0) GO TO 4100
      DO 4045 IBA=1,NUMFRQ
        JADUMY=FREQNX(IBA,2)
        DO 4046 IBB=1,JADUMY
          IF(FREQNC(IBA,IBB).NE.GMADMY) GO TO 4046
          DO 4048 IBC=IBB,JADUMY
            IBD=IBC+1
            FREQNC(IBA,IBC)=FREQNC(IBA,IBD)
4048 CONTINUE
          FREQNC(IBA,JADUMY)=GMADMY
          GO TO 4100
4046 CONTINUE
4045 CONTINUE
C
C
C=====I
C                                     I
C                                     I
C                                     I
C                                     I
C                                     I
C                                     I
C                                     I
C=====I
C
C
C      4100 DO 4200 IBA=1,NUMFRQ
C
C
C      JADUMY=FREQNX(IBA,2)
C      IF(JADUMY.EQ.0) GO TO 4200
C
C
C      DO 4210 IBB=1,JADUMY

```

```

C
C
    SECTR=FREQNC(IBA,IBB)
    FREQCY=DLFRQX(SECTR)
    PASENG=0
    IF(FREQCY.EQ.0) GO TO 4101
    PASENG=DMANDX(SECTR)/FREQCY
4101 PASENG=PASENG+EXDMND(SECTR)
C
C
    FIND THE MAXIMUM NUMBER OF PASSENGERS FLYING THIS SECTOR WHICH
    CAN BE ALLOCATED TO EACH LEG OF THIS FLIGHT
C
C
    DO 4105 IBC=1,POINTA
C
C
    JCDUMY=(LEGPAX(IBC)+PASENG)*TABLOX(SECTR,IBC)
    IF(MAXPAX(IBC).GT.JCDUMY) GO TO 4106
    PAX(IBC)=MAXPAX(IBC)-LEGPAX(IBC)
    GO TO 4105
C
C
4106 PAX(IBC)=PASENG
C
C
4105 CONTINUE
C
C
    FIND THE MINIMUM OF THE MAXIMUM
C
C
    I.E. SATISFY THE CONSTRAINTS FOR ALL THE LEGS OF THE FLIGHT
C
C
    PASGRS(SECTR,1)=8000000
C
C
    DO 4110 IBC=1,POINTA
    IF(PASGRS(SECTR,1).LE.PAX(IBC)) GO TO 4110

```

```

      PASGRS(SECTR,1)=PAX(IBC)
4110 CONTINUE
C
C
C   ALLOCATE THE PASSENGERS TO THE APPROPRIATE LEGS OF THE FLIGHT
C
C
      DO 4115 IBC=1,POINTA
      IDUMY=(PASGRS(SECTR,1)*TABLOX(SECTR,IBC))
      TABLO(SECTR,IBC)=TABLO(SECTR,IBC)+IDUMY
      LEGPAX(IBC)=LEGPAX(IBC)+IDUMY
4115 CONTINUE
C
C
      IF(PASGRS(SECTR,1).EQ.0) GO TO 4210
C
C
C   FIND THE MIX BETWEEN BASE DEMAND AND EXTRA DEMAND OF THE
C   PASSENGERS ALLOCATED
C
C
      IDUMY=PASGRS(SECTR,1)-EXDMND(SECTR)
      IF(IDUMY.GT.0) GO TO 4120
      PASGRS(SECTR,2)=PASGRS(SECTR,1)
      GO TO 4210
4120 PASGRS(SECTR,2)=EXDMND(SECTR)
C
C
C
4210 CONTINUE
C
C
C
4200 CONTINUE
C
C
C   CALCULATE THE LOAD FACTOR OF EACH LEG
C
C

```

```

C
C
ADUMY=FLOAT(ACSEAT(AIRCAF)*NUMWKS)
IDUMY=PRVFLT+1
DO 4220 IBA=1,POINTA
LEGLF(IBA)=(((FLOAT(LEGPAX(IBA))/ADUMY)*100.0)/IDUMY)
4220 CONTINUE
C
C
C=====I
C                                     I
C                                     I
C   FIND REAL PHY AND GO BACK TO SEE IF THIS IS STILL THE MAXIMUM   I
C                                     I
C   MULTIPLY REVENUES BY TWO SO AS TO GET THE TWO-WAY REVNUES       I
C                                     I
C=====I
C
C
TOTREV=0.0
DO 4225 IBA=1,10
REVNU(IBA,2)=REVNU(IBA,1)*(2.0*PASGRS(IBA,1))
TOTREV=TOTREV+REVNU(IBA,2)
4225 CONTINUE
C
C
C   FIND THE BLOCK TIME AND VARIABLE COST APPLICABLE TO THIS
C   AIRCRAFT/ROUTE COMBINATION
C
C
DO 4230 IBA=1,NUMPHY
IF(ACCSTX(IBA,1).NE.AIRCAF) GO TO 4230
IF(ACCSTX(IBA,2).NE.ROUTE) GO TO 4230
JADUMY=IBA
GO TO 4231
4230 CONTINUE
C
C
C   CALCULATE THE TOTAL VARIABLE COST AND UTILISATION FOR THE

```

```

C      WHOLE SEASON
C
C
C 4231 UTILIS=TIME(JADUMY)*NUMWKS
      XFUEL=FUEL(JADUMY)*NUMWKS
      XHAND=HANLNG(JADUMY)*NUMWKS
      XLAND=LANFES(JADUMY)*NUMWKS
      XENGR=ENGCST(JADUMY)*NUMWKS
      TVCST=XFUEL+XHAND+XLAND+XENGR
C
C
C      INCLUDE THE OPPORTUNITY COST IN THE CALCULATION OF
C      THE DESIRABILITY INDEX PHY
C
      OPYCST=OPTUNY(AIRCAF)*UTILIS
      POFT=TOTREV-TVCST-OPYCST
      PHY(1)=POFT
      MAXRTE=1
      GO TO 4002
C
C
C 4250 RETURN
      END

```

7.0 The Subroutine COSTS

This subroutine calculates the costs and block times for the aircraft/route combinations being analysed in the subroutine AIRRUT.

7.1 Listing of Main Variables

ACTYPE	is the number of different aircraft types.
RTVBLE	is the number of possible routes along the chosen sector pq.
ACCSTX(I,1)	is the aircraft in the I th aircraft/route combination.
ACCSTX(I,2)	is the route in the I th aircraft/route combination.
TIME(I)	is the block time for the return trip for the I th combination.
FUEL(I)	is the cost of fuel for the return trip for the I th combination.
HANLNG(I)	is the total handling charges.
LANFES(I)	is the total landing fees.
ENGCSST(I)	is the total engineering cost.
INDEX	is an indicator. If it is equal to 0, then there are no legs which deviate from the standard payload-range diagram; if it is equal to 2 then the aircraft/route combination being considered is infeasible.
NOFLY	tests to see if this aircraft/route combination is feasible and it also up-dates the values of block time and fuel costs for those sectors which deviate from the standard payload-range diagrams.

LEGY(K)

stores the legs which deviate from the standard
payload-range diagram.

8.0 The Subroutine NOFLY

This subroutine checks the feasibility of an aircraft/route combination. Should the combination be feasible, the subroutine compensates for any deviation from the standard payload range diagram of the aircraft.

8.1 Listing of Main Variables

PAYLOD(A,I,J)	are the points on the standard payload range diagram of aircraft A.
ROUTEX	the route in the combination being analysed.
AIRCAF	the aircraft in the combination being analysed.
EXPSHN(K,1)	is the location in the set <u>A</u> of the sector in the K^{th} sector/aircraft combination which deviates from the standard payload range diagram.
EXPSHN(K,2)	is the aircraft in the K^{th} deviation.
EXPSHN(K,3)	is the effective seating capacity of the aircraft in the K^{th} deviation.
EXPSHO(K,1)	is the block fuel for the K^{th} deviation.
EXPSHO(K,2)	is the block time.
NUMEXP	is the number of combinations which deviate from the standard payload range diagram.
INDEX	is an indicator. If it is equal to 0, then there are no legs that deviate from the standard payload range diagram; if it is equal to 2, then the aircraft/route combination being considered is infeasible.
LEGGY(L)	stores the legs which deviate from the standard diagram.

9.0 Subroutine UPINFO

This subroutine updates the basic information of demand and frequency and aircraft availability.

9.1 Listing of Main Variables

ACFLTS(I)	is the number of flights assigned to the I th sector.
ACTFRQ(I)	is the number of flights assigned to the I th sector which carries base demand as opposed to extra generated traffic.
DEMAND(I)	is the remaining number of passengers who wish to fly the I th sector.
DIRECT(I)	is the maximum number of direct flights that must be assigned to the I th sector.
DLDRCT(I)	is the remaining number of 'direct' flights that must be assigned to the I th sector.
DELFRQ(I)	is the remaining number of flights that must be assigned to the I th sector.
MAXUTL(A)	is the remaining number of hours of aircraft type A available for allocation.
ROUTE AIRCAF	is the aircraft/route combination to which the appropriate traffic of the 'next sector' and the associated traffic from other sectors have been allocated.

The next four subroutines, (EXTRQ, EXTRIF, TITZ, and SEATS), are specific to the data used to develop the model. When the system is introduced as a package, these four routines will have to be modified to suit the user.

14.0 Subroutine SHORT

This subroutine finds the routes which have the shortest routing through the sector CRSREF.

14.1 Listing of Main Variables

ORIGINS(S)	are the origin and destination of the sector S.
DESTIN(S)	
NMPOSB(S)	is the number of routes with routings through the sector S.
POSBRT(S,I)	is the I^{th} route with a routing through the sector S.
ROUTES (R,I)	is the I^{th} city on the R^{th} route in the set of feasible routes.
POINTS (L,C,R)	is the subroutine which finds the number of legs (L) and the number of cities on the R^{th} route.
MAX LEG	is the number of legs connecting the city pair on the sector S on the route which has the routing with the least number of intermediate stops for that sector.
MINLEG	is the number of routes with the minimum number of intermediate stops for that sector.

15.0 Subroutine REPOSB

This subroutine re-calculates the number of possible routes passing through each of the sectors. If the indicator ZRODMD is equal to 1, then only the sectors which have a base demand will be considered.

15.1 Listing of Main Variables

INORUT	is the number of routes which must not be considered in the summation process.
NORUTE(K)	is the set of routes which must not be considered as a result of the application of an Alpha perturbation.
DEMAND(S)	is the basic demand forecast for sector S.
NMPOSB(S)	is the number of routes with routings through the S th sector.
POSBRT(S,R)	is the R th route with a routing through the S th sector.

16.0 Subroutine SUBNTK

This subroutine defines the airlines various sub-networks.

16.1 Listing of Main Variables

SET(S)	is the set of sectors which are linked to the sector S through the routes which have routings through S.
SETS	is the subroutine which finds the elements of SET.
NUMSUB	is the number of sub-networks in the airlines network.
SUBNET(K,S)	is the set of sectors which define the K th sub-network.
ELMNTS(K)	is the number of sectors which constitute the K th sub-network.
SUBIND(K)	are variables associated with each sub-network and will
SUBUTL(K,L)	be used to calculate the 'No-conflict' opportunity cost. These are defined in "Subroutine REALOK".

17.0 Subroutine SETS

The subroutine finds the set of sectors which lie on the routes with routings through the chosen sector. This subroutine is used by SUBNTK to define the airline's sub-networks.

17.1 Listing of Main Variables

NMPOSB(S)	is the number of routes with routings through the sector S.
POSBRT(S,I)	is the I th route with a routing through the sector S.
ROUTES(R,I)	is the I th city on the R th route in the set of possible routes.
POINTS(L,C,R)	is the subroutine which finds the number of legs (L) and the number of cities (C) on the R th route.
TRANSF(A,B,S)	is the sub-routine which finds the shortage location of the input traffic information associated with the sector made up of the city pair A-B.
SET(S)	is the set of sectors which are linked to the sector S through the routes which have routings through S.

18.0 Subroutine ALOFLT

For a given sub-network this subroutine finds the storage location of the aircraft/route combinations assigned to this sub-network.

18.1 Listing of Main Variables

NUMAC(K)	is the number of aircraft types which have been assigned by the algorithm to the route ROOTES (K), where
ROOTES(K)	is the K^{th} route assigned to this sub-network, and
NMROOT	is the number of different routes assigned by the algorithm to this sub-network.
ACALOK(K,I)	is the location of the I^{th} aircraft assigned to the routes ROOTES(K) in the allocation storage space.
FLYGHT(J,I)	is the assigned route in the J^{th} allocation storage space.

19.0 Subroutine SETINF

This subroutine aggregates the traffic and allocation information associated with a given number of routes previously assigned to this sub-network. It then re-sets the primary storage space of traffic and sector information appropriately and then stores this information along with other important parameters associated with the previous allocation in an auxilliary storage to facilitate the running of several perturbations.

19.1 Listing of Main Variables

NUMAC(L)	These are the parameters which identify the storage
ROOTES(L)	locations associated with this sub-network.
ACALOK(L,M)	These are described in detail in 18.1.
KBA, KBB	define the routes whose assigned traffic information must be aggregated.
ACFITS(S)	is the number of flights assigned to the S^{th} sector.
ACTFRQ(S)	is the number of flights assigned to the S^{th} sector which carry based demand as opposed to extra generated traffic.
DEMAND(S)	is the remaining number of base demand passengers who wish to fly the S^{th} sector.
DELFREQ(S)	is the remaining number of flights that must be assigned to the S^{th} sector.
MINFRQ(S)	is the minimum number of flights that must be assigned to the S^{th} sector.
DIRECT(S)	is the minimum number of 'direct' flights that must be assigned to the S^{th} sector.
DLDRCT(S)	is the remaining number of 'direct' flights that must be assigned to the S^{th} sector.

TRAFIC(K,I,1)	stores the number of passengers allocated to the I th sector associated with the aircraft/route combination stored in the K th storage location.
TRAFIC(K,I,2)	is the number of extra generated passengers allocated to the I th sector associated with the aircraft/route combination stored in the K th storage location.
TRAFIC(K,I,3)	stores the number of flights which carry base demand passengers.
FLYGHIT(K,3)	is the number of flights assigned to the K th aircraft/route combination.
FLYGHIT(K,2)	is the aircraft of the K th aircraft/route combination.
UTLSTN(K)	is the number of hours of the appropriate aircraft type utilised by the K th aircraft/route combination.
SHORT	is a subroutine which finds the number of routes (MINLEG) which have routings through the sector SECTR with the minimum number of intermediate stops.
ELMNTS(N)	is the number of sectors which constitute the N th sub-network.
SUBNET(N,SK)	is the set of sectors which define the N th sub-network.
DLUTLX(A)	is the number of hours of aircraft type A assigned to the routes considered; in the aggregation.
NMSECX	is the number of sectors associated with this sub-network whose information is held in the auxilliary storage space.
SCTORX(J)	is the J th sector in the auxilliary storage space.

MXSTPS(J) is the maximum number of intermediate stops associated with the sector J and the routes considered in this aggregation. These routes were assigned by the previous allocation process.

DMANDX(J) is the number of passengers who wish to fly the sector J and must be re-allocated by the perturbation process.

ACFLTX(J,I) is the number of flight assigned to the Jth sector in the previous allocation.

ACFLTX(J,2) is the number of flights assigned to the Jth sector by the previous allocation process after taking account of the flights which will be re-allocated.

DLFRQX(J,1) is the number of flights assigned to the Jth sector by the previous allocation process, which carry base demand passengers.

DLFRQX(J,2) is the minimum number of flights which must be allocated to sector J, after taking into account those flights previously allocated but not included in the aggregation.

DRCTX(J,1) this parameter indirectly gives the number of 'direct' flights assigned to the sector J by the previous allocation process.

DRCTX(J,2) is the minimum number of 'direct' flights which must be allocated to sector J, after taking into account those flights not included in the aggregation.

20.0 Subroutine UPSTRE

This subroutine deletes from the primary storage the allocations associated with the sub-network defined by: NMROOT, NUMAC, etc.

20.1 Listing of Main Variables

NMROOT	These are the parameters which identify the storage
NUMAC(L)	locations associated with this sub-network. These
ROOTES(L)	are described in detail in 18.1.
ACALOK(L,M)	
ITERTN	is the number of aircraft/route combinations whose allocations are stored in the primary allocation storage space.
TRAFIC(K,J,I)	stores the passengers allocated to sector J for the K^{th} aircraft/route combination.
TRAFIC(K,J,2)	stores the non-base demand allocated to sector J e.g. fifth freedom passengers.
TRAFIC(K,J,3)	stores the number of flights which carry base demand passengers.
CASH(K,J)	is the revenue earned from the passengers on sector J.
LEG(K,I)	stores the number of passengers flying the I^{th} leg.
LODFCT(K,I)	stores the leg load factor.
CASH(K,A)	A = 11 Fuel costs
	A = 12 Handling charges
	A = 13 Landing fees
	A = 14 Engineering costs

FLYGHT(K,B) B = 1 Route number
 B = 2 aircraft type
 B = 3 frequency of this aircraft/route combination
 B = 4 the number of legs on this route

UTLSTN(K) is the number of hours of aircraft type FLYGHT(K,2)
 utilised by the Kth aircraft/route combination.

21.0 Subroutine ALOCTN

The subroutine is the 'perturbed traffic' allocation algorithm. It applies each of the perturbation analysis to the required sub-network.

21.1 Listing of Main Variables

ZROFLY	re-sets the secondary allocation storage space to zero.
ROUTOS	determines the 'next most difficult sector' NXTSEC.
COSTS	is the subroutine which costs the aircraft/route which will be considered in the allocation that follows:
AIRRUT	performs the traffic allocation according to the standard Algorithm.
STORE	stores the output of each allocation.
UPINFO	updates the information of the traffic primary storage space at the end of each allocation.
PROFIT(P)	stores the profit associated with each perturbation analysis. P = 1 Standard Algorithm P = 2 Alpha Perturbation P = 3 Beta Perturbation P = 4 Gamma Perturbation P = 5 Delta Perturbation
REPOSB	is the subroutine which deletes NORUTE from the set of feasible routes and re-calculates the number of routes through each of the appropriate sectors.

ZRODMD is an indicator. When it is equal to one, only the sectors which have base demand passengers are considered in subroutine REPOSB.

ALOFIT finds the storage locations in the appropriate storage space (primary or secondary) of the aircraft/route combinations assigned to the sub-network SUB.

ZROINF re-sets the auxilliary traffic storage space to zero.

SETINF aggregates the traffic and allocation information associated with a given number of routes previously assigned to the sub-network. This information is stored in the auxilliary traffic storage space.

RESET this subroutine re-sets the primary traffic and allocation information to the appropriate values aggregated and stored in the auxilliary storage space.

DELTA performs the Delta analysis on the sub-network.

DLTPRT(P.K.I) stores the main parameters of the successful Delta perturbations associated with the P^{th} perturbation.

I = 4 route number
I = 3 sector
I = 2 aircraft type
I = 1 storage index

NMDLTA(P) is the number of Delta perturbations associated with the P^{th} perturbation.

BETAH performs the Beta analysis on the sub-network.

BTASCT(P) is the sector whose passengers need re-allocation to produce the best improvement over the previous allocation using the Beta perturbation.

NMBETA(P) is the number of routes previously allocated to this sub-network, which were aggregated to give the improved solution, where

BETAI(P.M.) is that set of routes, and

BTAPRB(P.M.) is the number of aircraft types allocated to the route BETAI(P.M.) which were considered in the allocation;

BTAPRT(P.M.N) is the location of the N^{th} aircraft assigned to the route BETAI(P.M.) in the allocation storage space.

ALFPRT(P.N.) is the set of routes which were eliminated to produce an improved solution using Alpha perturbation, and

NUMALF(P) is the number of such routes.

ALPHA performs the Alpha analysis on the sub-network.

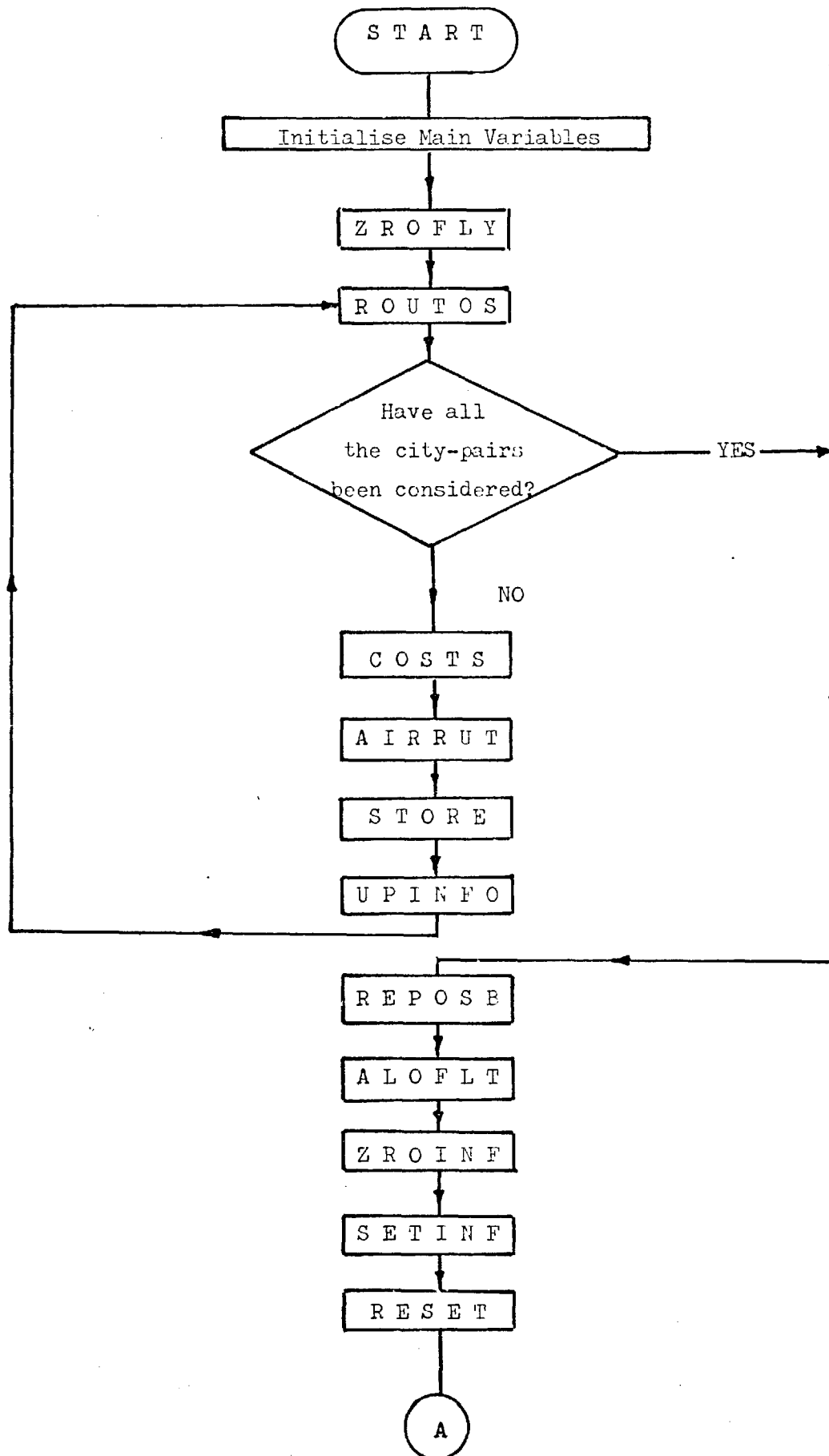
NOIETA is an indicator associated with the Beta perturbation process. When it is equal to 1 it triggers the analysis appropriate to the output of the Alpha perturbation.

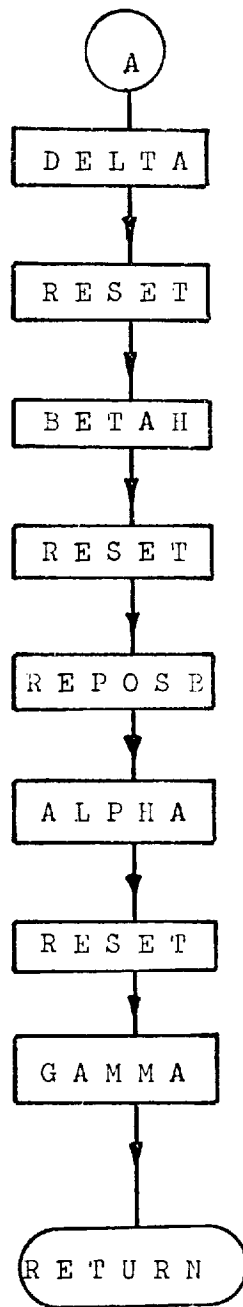
GAMMA performs the Gamma analysis on the sub-network.

GMAPRT(P.I) stores the relevant parameters which despite the best Gamma perturbation.

I = 1 sector
I = 2 aircraft type
I = 3 is the number of flights which must be provided by the Gamma aircraft on the Gamma sector.

21.2 Flow chart for subroutine ALOCTN





22.0 Subroutine ALPHA

Inherent in the Standard Algorithm is the assignment of long multi-leg routes. This, in many cases, is not profitable when analysing the total sub-network. This subroutine studies the effects of a decrease in the permissible number of intermediate stops.

22.1 Listing of Main Variables

ACTYPE	is the number of aircraft types considered in the allocation process.
NMSECT	is the number of sectors stored in the auxilliary storage space.
SECTOR(S)	are the set of sectors in the auxilliary storage space and are the only ones involved in the re-allocation process.
MXSTPS(S)	is the maximum number of intermediate stops assigned to the S^{th} sector in the previous allocation.
DEMAND(I)	is the remaining number of passengers who wish to fly the I^{th} sector. This is part of the primary traffic storage space.
NMPOSB(I)	is the number of routes in the list of possible routes which have routings through the I^{th} sector.
POSBRT(I,J)	is the J^{th} route with a routing through the I^{th} sector.
POINTS(A,B,R)	is the subroutine which finds the number of legs, A and the number of cities, B, on the route R.
ORIGIN(I) DESTIN(I)	these identify the two cities which make up the I^{th} sector.
ROUTES(R,K)	is the K^{th} city on the R^{th} route: ROUTES is the set of possible routes.

STOPS	is the number of stops on a particular route.
BOUND(K)	is the set of routes which form the upper bound in the Alpha perturbation process, and
KROUTE	is the number of routes which form the upper bound.
MAXI(K)	is the set of sectors which have been assigned the maximum number of intermediate stops (MAX), and
NUMMAX	is the number of such sectors.
ALFSKT(K)	is the set of Alpha sectors which when applied to the Alpha perturbation in conjunction with ALFPRT produces a solution better than the previous one, and
ALFSCT	is the number of such sectors.
MAXY	is the Alpha number of stops operating.
NOROOT(N)	is the set of routes which must not be included in the operating Alpha re-allocation process.
REPOSB	is the sub-routine which deletes NOROOT from the set of feasible routes and re-calculates the number of routes through each of the appropriate sectors.
INDEX	is an indicator: when it equals 2 it triggers the upper bound analysis.
NOBTA	is an indicator, when it equals 0 it triggers the Alpha analysis appropriate to the output of the Beta perturbation.
NOSTOR	is an indicator; when it equals 1 the output of the allocation process is not stored.
BETAM	is the subroutine which performs the traffic allocation according to the 'Beta Revised' algorithm.

ZROFLY	re-sets the secondary allocation storage space to zero.
ROUTOS	determines the 'next most difficult' sector.
COSTS	is the subroutine which costs the aircraft/routes which will be considered in the allocation that follows:-
AIRRUT	performs the traffic allocation according to the standard Algorithm but 'revised' by the following parameters.
1) GMADMI	is an indicator which states whether or not the sector GMADMZ should be inserted at the tail of the 'priority loading' list.
2) SECFST	is an indicator which states whether or not the sector FSTSEC should be inserted at the head 'priority loading' list. In this subroutine both these indicators are inoperative.
STORE	stores the output of each allocation.
UPINFO	updates the information of the traffic primary storage space at the end of each allocation.
BETAH	performs the Beta perturbation analysis on the output.
BTAPRF	is the resulting increase in profitability and
NMBETA	
BTASCT	
BTAPRB(A)	are the relevant parameters which describe the best
BETAI(A)	Beta perturbation.
BTAPRT(A,B)	
DELTA	performs the Delta perturbation analysis on the output.
DLTPRY	is the resulting increase in profitability and

NMDLTA	are the relevant parameters which describe the best
DLTPRT(A,B)	Delta perturbation.
ALFPRT(N)	is the set of routes which were eliminated to produce the best Alpha perturbation, and
NUMALF	is the number of such routes.

23.0 Subroutine DELTA

The subroutine searches through the allocations of a sub-network to find routes which have been assigned more than one aircraft type. It then proceeds to check if the solution is improved by assigning only one aircraft type to the route.

23.1 Listing of Main Variables

NORUTE(N)	is the set of routes which must not be included in the allocation of aircraft to routes, and
INORUT	is the number of such routes.
REPOSB	is the subroutine which deletes NORUTE from the set of feasible routes and re-calculates the number of routes through each of the appropriate sectors.
ZRODMD	is an indicator: when it is equal to one only the sectors which have base demand passengers are considered in subroutine REPOSE.
ALOFLT	finds the storage location in the appropriate storage space (primary or secondary) of the aircraft/route combinations assigned to the sub-network SUB.
NUMAC(K)	is the number of aircraft types which have been assigned to the route ROOTES(K), where
ROOTES(K)	is the K^{th} route assigned to the sub-network, and
NMROOT	is the number of such routes.
ACALOK(K,1)	is the location of the I^{th} aircraft assigned to the route ROOTES(K) in the allocation storage space.
ZROINF	re-sets the auxilliary traffic storage space to zero.

SETINF	aggregates the traffic and allocation information associated with a given number of routes previously assigned to the sub-network. This information is stored in an auxilliary traffic storage space.
RESET	this subroutine re-sets the primiaary traffic that aggregated and stored in the auxilliary storage space.
DMANDX(J)	is the number of passengers who wish to fly the sector J. These were aggregated by SETINF.
DELTAH	performs the allocation of the passengers aggregated: the 'Delta Revised' algorithm is used.
CHKPRT	checks the profitability of the perturbed allocation with the previous one.
NMDLTA	is the number of non-stop routes which have been transformed into a more profitable operation through the application of a Delta perturbation.
DLTPRT(K,1)	stores the main parameters of the successful Delta perturbations. I = 4 route number I = 3 sector I = 2 aircraft type I = 1 storage index

24.0 Subroutine DELTAH

This subroutine performs the required allocation for the Delta perturbations.

24.1 Listing of Main Variables

SECFST	is an indicator which states whether or not the sector FSTSEC should be inserted at the head of the 'priority loading' list.
GMADM1	is an indicator which states whether or not the sector GMADM2 should be inserted at the tail of the 'priority loading' list.
ACSEAT(A)	is the seating capacity of aircraft type A.
MXLOAD(S)	is the maximum permissible load factor on sector S.
DEMAND(S)	is remaining number of base demand passengers who wish to fly to S th sector.
NMFLTS	is the number of flights that must be assigned aircraft type AIRCRF.
NOSTOR	is an indicator: when it is equal to 1 the output of the allocation process is not stored.
ROUTOS	These are the main subroutines in the traffic allocation algorithm (See 22.1).
COSTS	
AIRRUT	
STORE	
UPINFO	

25.0 Subroutine BETAH

The Beta analysis tries to re-allocate passengers whose previous allocation is questionable in the context of an optimal solution.

25.1 Listing of Main Variables

REPOSB	is the subroutine which deletes the set NORUTE from the set of feasible routes and re-calculates the number of routes through each of the appropriate sectors.
ALOFLT	finds the storage location in the appropriate storage space of the aircraft/route combinations assigned to the sub-network SUB.
NUMAC(K)	is the number of aircraft types assigned to the route ROOTES(K), where
ROOTES(K)	is the K^{th} route assigned to the sub-network and
NMROOT	is the number of such routes.
ACALOK(K,I)	is the location of the I^{th} aircraft assigned to the route ROOTES(K) in the allocation storage space.
ZROINF	re-sets the auxilliary traffic storage space to zero.
SETINF	aggregates the traffic and allocations information associated with a given number of routes previously assigned to the sub-network. This information is stored in the auxilliary traffic storage space.
RESET	this subroutine re-sets the primary traffic and allocation information with that aggregated and stored in the auxilliary storage space.
SECTOR(J)	is the J^{th} sector in the auxilliary storage space, and

NMSECT is the number of such sectors.

NMPOSB(S) is the number of routes in the list of possible routes which have routings through the S^{th} sector.

POSBRT(S,K) is the K^{th} route with a routing through the S^{th} sector.

NUMBTA is the number of sectors whose passengers require re-allocation to produce a better solution, and

ALOKTN(L,2) is the L^{th} of the above sector, where

ALOKTN(L,1) is the location in the allocation storage space of the aircraft/route combination which carries the least base demand passengers flying the sector ALOKTN(L,2)

TRAFIC(K,I,1) stores the passengers allocated to sector I for the K^{th} aircraft/route combination.

TRAFIC(K,I,2) stores the non-base demand allocated to sector I e.g. fifth freedom passengers.

BETA combines the appropriate aircraft/route combinations and re-allocates the passengers with the 'Beta Revised' Algorithm.

BTASCT is the sector whose passengers need re-allocation to produce the best improvement over the previous allocation using the Beta perturbation.

NMBETA is the number of routes previously allocated to this sub-network which were aggregated to give the improved solution, where

RUUTES(M) is that set of routes, and is the number of aircraft types allocated to the route RUUTES(M) which were considered in the aggregation.

ACALUK(MN) is the location of the N^{th} aircraft assigned to the route RUUTES(M) in the allocation storage space.

BTAPRX is the improvement in profitability over the previous allocation which accrues from the best Beta perturbation.

The following variables store the parameters which describe the best Beta perturbation.

BTAPRB	NONAC
BETA1	RUUTES
BTAPRT	ACALUK

ALFPRT(N) is the set of routes which were eliminated to produce the improved solution using Alpha perturbations and

NUMALF is the number of such routes.

26.0 Subroutine BETA

This subroutine combines the appropriate aircraft/route combination and performs the relevant re-allocations to see if an effective transfer of passengers can be achieved.

26.1 Listing of Main Variables

SECTR	is the sector which has been assigned more than the minimum number of flights and
ROUTEX	is the route which carries the least number of Beta passengers. These passengers will be combined with those on the route:
RTBETA	the perturbation route, to see if an effective transfer of passengers can be achieved.
NOSTOR	is an indicator when it is equal to 1, the output of the allocations are not stored.
NMROOT	
NUMAC(K)	These are the parameters which identify the storage locations associated with the sub-network.
ROOTES(K)	
ACALOK(K,I)	
NMRUUT	There are the parameters which identify the storage locations of the aircraft/route combinations which are aggregated for this particular Beta perturbation.
NOMAC(J)	
RUUTES(J,I)	
POINTS(A,B,R)	is the subroutine which finds the number of legs, A, and the number of cities B, on the R th route.

ROUTES(R,I)	is the I th city on the R th route in the set of feasible routes.
TRANSF(A,B,S)	is the subroutine which finds the storage location of the input traffic information associated with the sector made up of the city-pair A-B.
CHKPRT	checks the profitability of the perturbed allocation with the previous one.
BETAM	performs the traffic allocation according to the 'Beta Revised' algorithm.
NOBETA	is an indicator: when it equals 1 it triggers the analysis appropriate to the output of the Alpha perturbation.
ALPHA	performs the Alpha perturbation analysis on the output of the Beta perturbation.

27.0 Subroutine BETAM

This subroutine performs the allocation of the Beta perturbation using the 'Beta Revised' algorithm. The revision is to choose the Beta sector as the 'next sector' to be analysed when it has one unsatisfied flight.

27.1 Listing of Main Variables

SECFST	is an indicator which states whether or not the sector FSTSEC should be inserted at the head of the 'priority loading' list.
GMADM1	is an indicator which states whether or not the sector GMADM2 should be inserted at the tail of the 'priority loading' list.
NOSTOR	is an indicator: when it is equal to 1 the output of the allocation is not stored.
ROUTOS	
COSTS	
AIRRUT	
STORE	
UPINFO	

These are the main subroutines in the traffic allocation algorithm (See 22.1).

28.0 Subroutine GAMMA

This subroutine analysis the effects of increasing frequencies in this sub-network in terms of extra revenue generated from an increase in the quality of service. The increase in frequency is obtained by substituting smaller aircraft for the larger ones in the initial allocation. Since benefits from increases in frequencies are less likely to be on those sectors which have small frequencies, the first stage of the subroutine finds the sectors with the highest frequencies.

28.1 Listing of Main Variables

ACTYPE	is the number of aircraft types considered in the allocation process.
NMSECT	is the number of sectors stored in the auxilliary traffic storage space.
SECTOR(J)	are the set of sectors in the auxilliary storage space and are the only ones involved in the re-allocation process.
DEMAND(S)	is the remaining number of passengers who wish to fly the S th sector. This is part of the primary traffic storage space.
FRQTRF(S)	is a parameter which will determine the magnitude of the extra generated traffic which results from an increase in frequency.
MINFRQ(S)	is the minimum number of flights that must be assigned to the S th sector.
MAXI(I)	is the set of sectors in the subroutine which have the maximum minimum number of flights, and
NUMMAX	is the number of such sectors.

REPOSB	is the subroutine which deletes the set NORUTE from the set of feasible routes and re-calculates the number of routes through each of the appropriate sectors.
MINFLT	is the number of flights which must be provided by the Gamma aircraft on the Gamma sector.
GAMMAH	performs the allocation using the 'Gamma Revised' algorithm.
DELTA	performs the Delta analysis on the output of the Gamma perturbation.
GMAPRT(K)	stores the relevant parameters which describe the best Gamma perturbation K = 1 sector K = 2 aircraft type K = 3 MINFLT
NMDLTA DLTPRT(I,J)	are the parameters which describe the associated Delta perturbation (See 31.1).

29.0 Subroutine GAMMAH

The subroutine performs the traffic allocation according to the 'Gamma Revised' algorithm. The revision is to allocate MINFLT flights of aircraft type AIRCRF to the Gamma sector SECTR.

29.1 Listing of Main Variables

SECFST	is an indicator which states whether or not the sector FSTSEC should be inserted at the head of the 'priority loading' list.
GMADMA	is an indicator which states whether or not the sector GMADM2 should be inserted at the tail of the 'priority loading' list. In this subroutine these variables are in the parameters of the subroutine AIRRUT as 1 and SECTR.
ZROFLY	re-sets the secondary allocation storage space to zero.
ROUTOS	
COSTS	These are the main subroutines in the traffic allocation algorithm (See 22.1).
AIRRUT	
STORE	
UPINFO	
ACFLTS(S)	is the number of flights allocated to the sector S.
NMPOSB(S)	is the number of routes with routings through the sector S.
POSBRT(S,I)	is the I th route with a routing through the sector S.

NXTSEC is the next sector to be allocated by the algorithm.

RTVBLE is the number of routes which have routings through the sector NXTSEC.

30.0 Subroutine PERTBN

This subroutine updates the primary allocation storage space with the new 'perturbed' allocations of the relevant sub-network.

30.1 Listing of Main Variables

PERTBX	is the perturbation appropriate to the sub-network SUB.
ROUTOS	
COSTS	These are the main subroutines in the traffic
AIRRUT	allocation algorithm (See 22.1).
STORE	
UPINFO	
ALFPRT(P,N)	is the set of routes which were eliminated to produce an improved solution using Alpha perturbations and
NUMALF(P)	is the number of such routes
REPOSB	is the subroutine which deletes NORUTE from the set of feasible routes and re-calculates the number of routes through each of the appropriate sectors.
ZROFLY	re-sets the secondary allocation storage space to zero.
IALFA	is an indicator associated with AIRRUT when it is equal to one, the allocations are stored in the secondary storage space.
GAMMAH	performs the allocation using the 'Gamma Revised' algorithm.
MINFLT	is the number of flights which must be provided by the Gamma aircraft GMAPRT (4.2) on the Gamma sector GMAPRT (4.1).

RESTRE updates the primary allocation storage space with the appropriate allocations stored in the secondary storage space.

DLTPRT(P.K.I) stores the main parameters of the successful Delta perturbations associated with the P^{th} perturbation.

I = 4 route number
I = 3 sector
I = 2 aircraft type
I = 1 storage index

RISTRE this subroutine deletes the unperturbed allocations from the secondary storage space leaving the perturbed elements in an ordered set.

ZROINF re-sets the auxilliary traffic storage space to zero.

SETINF aggregates the traffic allocation information associated with a given number of routes previously assigned to the sub-network. This information is stored in the auxilliary traffic storage space.

DELTAH performs the allocation of the passengers aggregated. The 'Delta Revised' algorithm is used.

BTASCT(P) is the sector whose passengers need re-allocation to produce the best improvement over the previous allocation using the Beta perturbation.

NMBETA(P) is the number of routes previously allocated to this sub-network, which were aggregated to give the imposed solution, where

BETA 1 (P,M) is that set of routes and

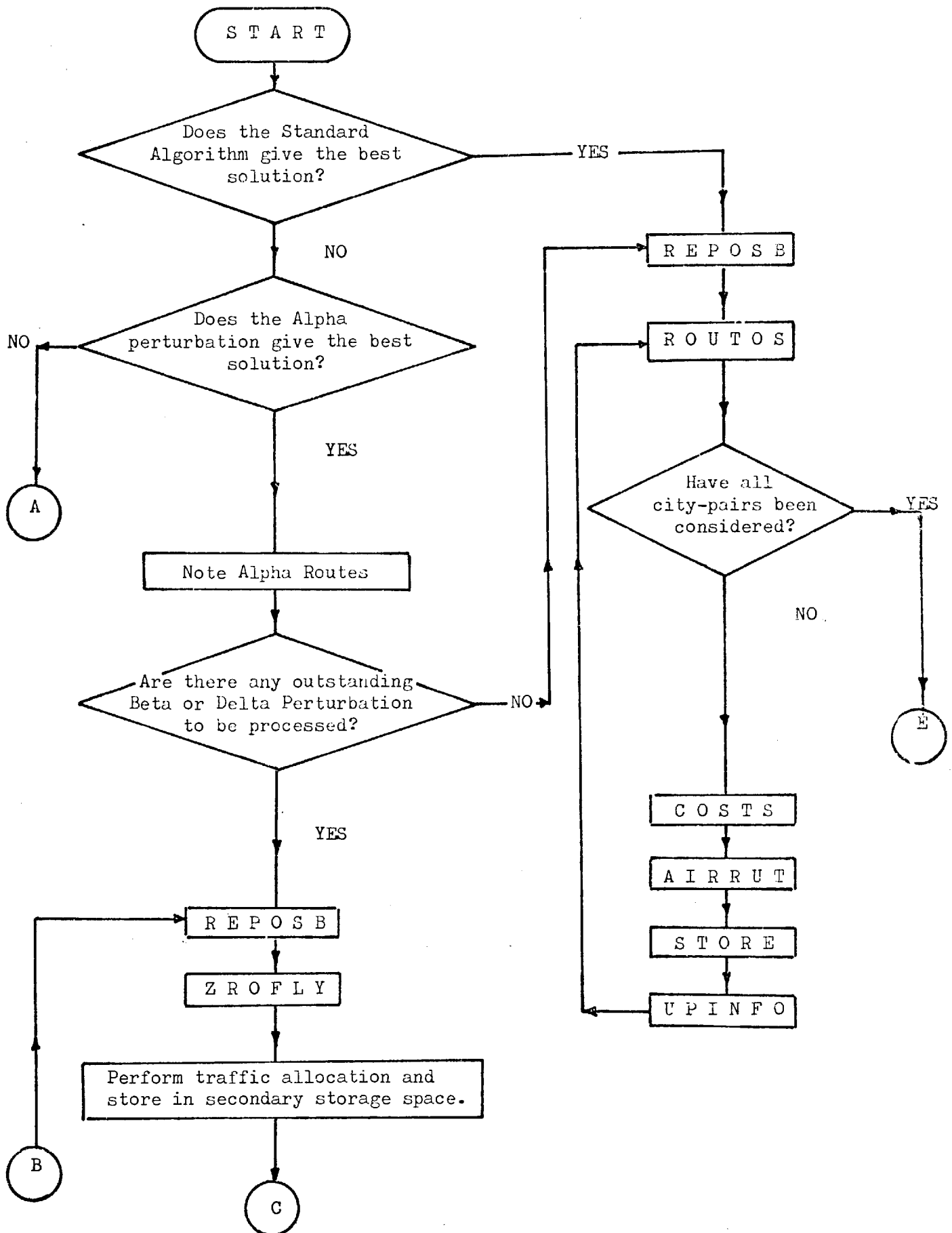
BTAPRB(P,M) is the number of aircraft types allocated to the route BETA 1 (P,M) which were considered in the allocation.

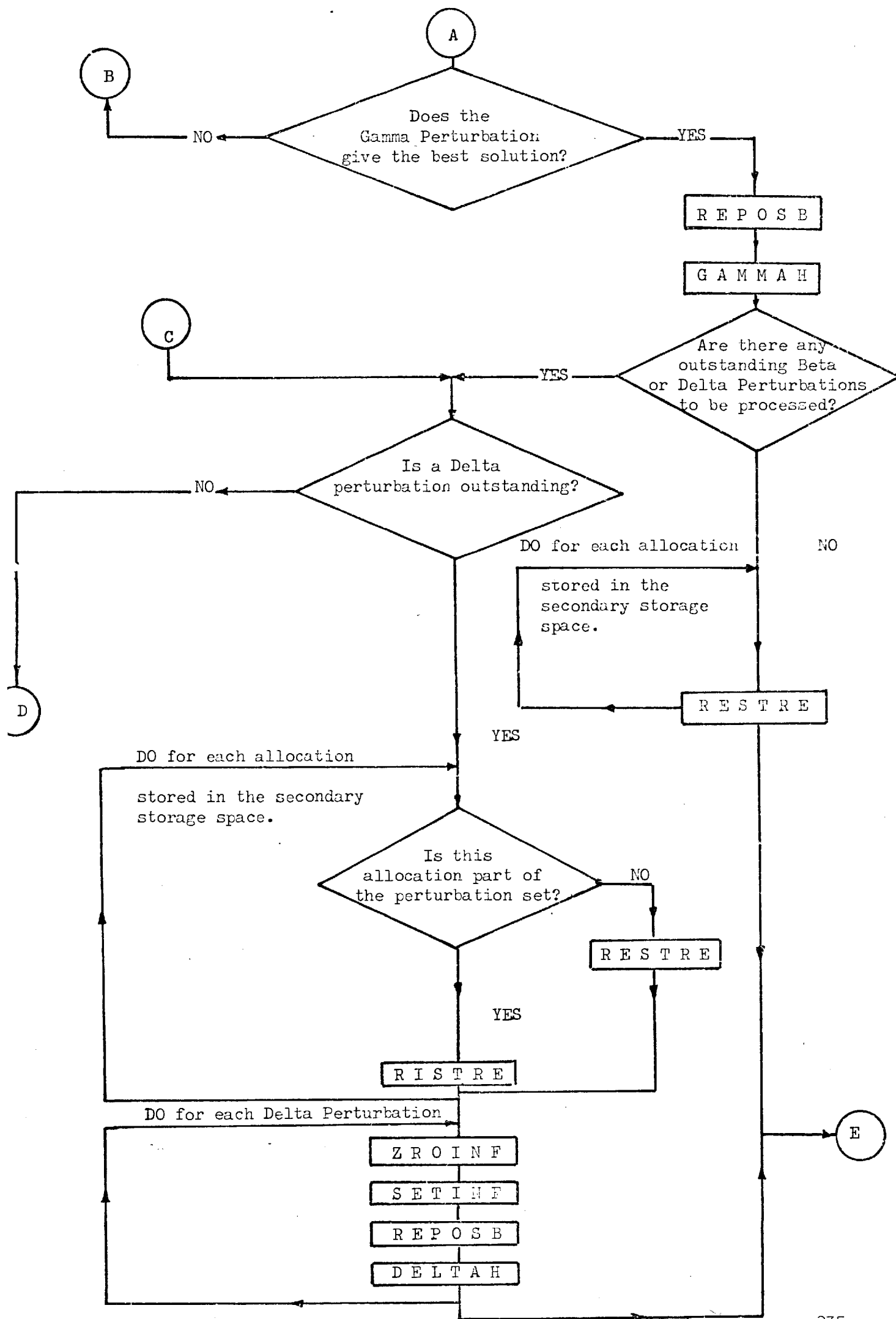
BTAPRT(P,M,N) is the location of the Nth aircraft assigned to the route BETA1 (P,M) in the allocation storage space.

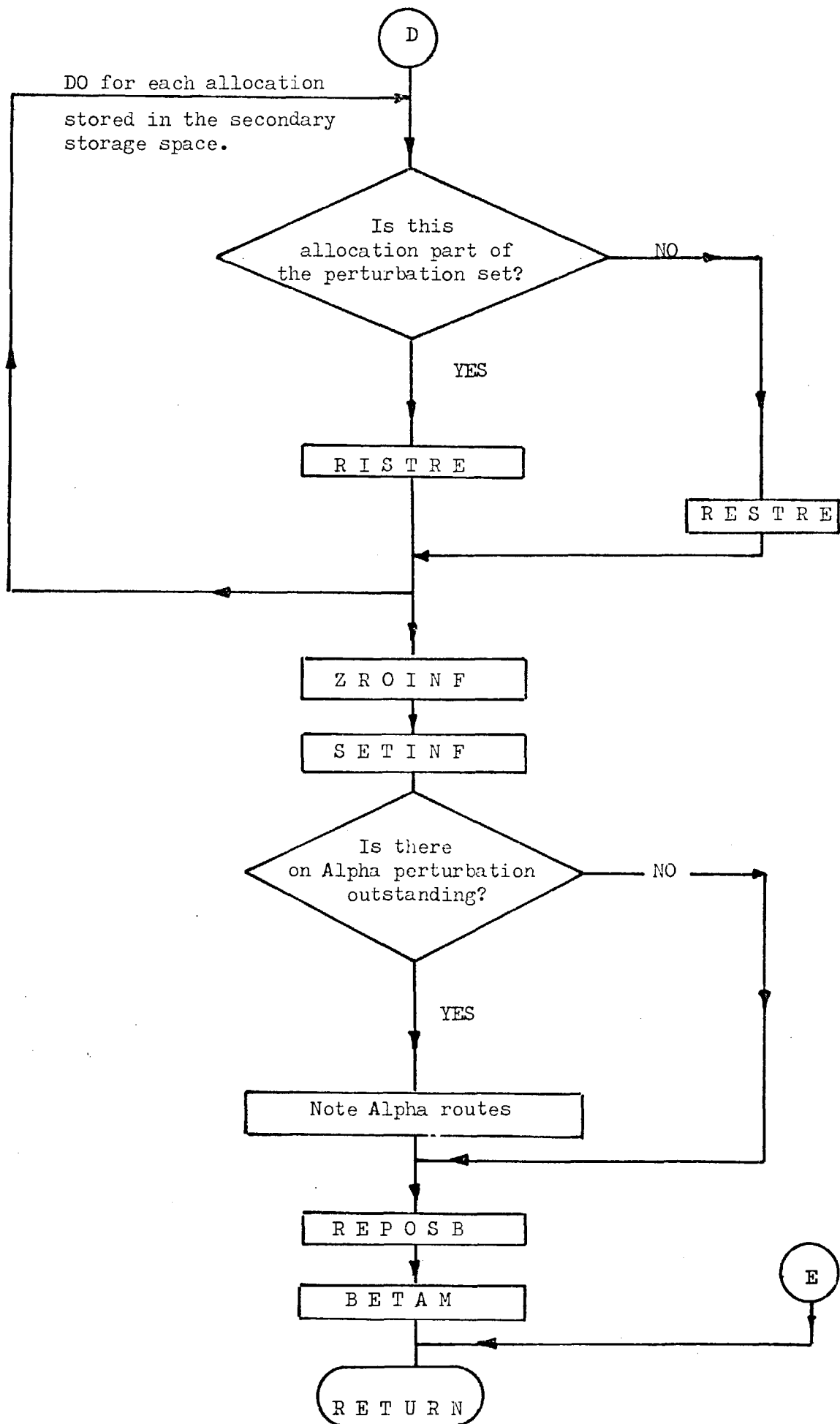
BETAM performs the traffic allocation according to the 'Beta Revised Algorithm'.

NOSTOR is an indicator - when it is equal to 1 the output of the allocation is not stored.

30.2 Flowchart for subroutine PERTEN







31.0 Subroutine OPTIMA

This subroutine identified the aircraft which constitute a scarce resource in the traffic allocation problem. For each of these aircraft, it then attempts to find the 'No conflict' opportunity cost.

31.1 Listing of Main Variables

ACTYPE	is the number of aircraft types considered in the allocation process.
UTZ(A)	is the number of hours which represents the most recent excess demand/capacity on aircraft type A.
OPTUNY(A)	is the most recent opportunity cost associated with aircraft type A.
ACUTLZ(A)	is the actual maximum availability of aircraft type A.
EXTHRS(A)	is the number of hours which represents the most recent excess demand on aircraft A associated with the opportunity cost EXTOPP(A).
SLKHRS(A)	is the number of hours which represents the most recent excess capacity on aircraft A associated with the opportunity cost SLKOPP(A).
DLHRS2(A)	is the number of hours which represents the most recent excess demand/capacity on aircraft A associated with the opportunity cost DLOPP2(A).
AVERAGE	is the subroutine which calculates the average profitability per hour achieved by a given aircraft type.
MAXUTZ(A)	is the maximum number of hours which can be assigned to aircraft type A by the allocation process.

ACINDX(A) is an index which represents the state which has been reached in the search for the 'No conflict' opportunity cost of aircraft A.

- = 1 is the first iteration
- = 2 the aircraft is a scarce resource
- = 3 the scarce aircraft is on a discontinuity of the 'excess hours - opportunity cost' curve.
- = 4 the aircraft is under-utilised.

INFIZ is the infeasibility index when this counter is equal to ACTYPE then the solution is infeasible.

NMOSCL(A) stores the number of oscillations performed by the 'excess hours - opportunity cost' curve of aircraft type A.

OPTUN1 is the new value of the 'No conflict' opportunity cost as calculated by the intercept on the opportunity cost axis of the line joining the two most recent points on the graph.

OPTUN2 is the new value of the 'No conflict' operating cost as calculated by the intercept on the cost axis of the line joining the two most recent points above and below this axis.

OPTMUM is the optimality index - when this counter is equal to ACTYPE then the solution has reached its optimal state.

FEAZB(A) is the infeasibility index associated with an aircraft on a discontinuity of the opportunity cost curve.

FEZPRF is the profitability of the best feasible solution encountered in the process of finding the optimal one, where,

FEZOPT(A) is the opportunity cost of each aircraft type associated with that solution.

ITERTN is the number of aircraft/route combinations whose allocations are stored in the primary storage space, and

FLYGHT(I,2) is the aircraft of the I^{th} aircraft/route combinations and

UTLSTN(I) is the number of hours used by the I^{th} allocation of aircraft type FLYGHT(I,2)

CHKPRT is the subroutine which calculates the profitability of a give set of aircraft/routes.

32.0 Subroutine REALOK

This subroutine determines the sub-networks which should be re-allocated in the next iteration of the 'No conflict' opportunity cost calculation.

32.1 List of Main Variables

NUMSUB	is the number of sub-networks in the allocation process.
ACTYPE	is the number of aircraft types considered in the allocation process.
ALOFLT	finds the storage location in the appropriate storage space (primary or secondary) of the aircraft/route combinations assigned to the given sub-network.
NUMAC(K)	is the number of aircraft types which have been assigned to the routes ROOTES(K) where
ROOTES(K)	is the K^{th} route assigned to the sub-network, and
NMROOT	is the number of such routes
ACALOK(K,I)	is the location of the I^{th} aircraft assigned to the route ROOTES(K) in the allocation storage space.
FLYGHT(L,2)	is the aircraft of the L^{th} aircraft/route combination stored in the primary storage space, and
UTLSTN(L)	is the number of hours used by the L^{th} allocation of aircraft type FLYGHT(L,2)

EXTHRS(A) is the number of hours which represents the most recent excess demand on aircraft A.

ACUTZ(A) is the number of hours which represents the most recent excess demand/capacity on aircraft type A.

SUBUTL(S,A) is the most recent number of hours by aircraft type A allocated to the sub-network S, when aircraft A is being under-utilised.

SBXUTL(S,A) is the most recent number of hours by aircraft type A allocated to the sub-network S when aircraft B is over-utilised.

SUBIND(S) is the re-allocation index of sub-network S. When this index is equal to 0 it means that the passengers of this sub-network must not be re-allocated in the next iteration of the 'No conflict' opportunity cost calculation.