

Highlights

Wave breaking energy dissipation in long-crested focused wave groups based on JONSWAP spectra

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- A new experimental study on wave breaking energy dissipation in JONSWAP focused waves is presented.
- Measurements using an accurate two-camera setup are used to reliably predict breaking characteristics from local wave geometry.
- An existing energy dissipation parameterisation is studied and further developed to be applicable for real ocean waves.
- An increase of approximately 85% in the proportionality constant is necessary to accurately compute the eddy viscosity for breaking energy losses in realistic oceans.

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Abstract

The present study is a new experimental investigation into energy dissipation due to wave breaking. Long-crested wave groups based on JONSWAP spectra are generated in a wave flume to estimate the energy loss from the wave packet due to the occurrence of single breaking events. These events range from gentle spilling to large plunging breakers. The fractional energy losses and breaking strength parameter are shown to be in good agreement with previous studies based on other spectral shapes.

Due to the significance of wave breaking in the evolution of large ocean waves, the dissipative model of Tian et al. (2010) [1] is examined and improved upon for JONSWAP spectra. Local wave geometry and breaking duration obtained from measurements using an accurate two-camera setup are employed to obtain new parameterisations, directly applicable to realistic ocean waves. An increase of approximately 85% in the proportionality constant needed to compute ν_{eddy} is deemed necessary such that energy losses from the new experimental wave groups are accurately captured. Furthermore, it is shown that the time and length scales of the breaking process are well correlated to the local steepness of the wave event, and new fits are obtained that considerably reduce the scatter of the original model.

As focused wave groups represent the most probable shape of large wave events in the open seas, the improved model can be implemented in computationally-efficient numerical potential flow solvers to simulate energy dissipation from breaking events in realistic waves.

Keywords: wave groups, breaking waves, energy dissipation, eddy viscosity

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1. Introduction

Energy dissipation due to wave breaking significantly contributes to several major interactions in the world's oceans and seas. According to [2] momentum and energy from waves is transferred to currents and near-surface turbulence by breaking. [3] note that the energy loss due to breaking is manifested as turbulence in the ocean surface layer and that some of this energy is also spent on submerging bubbles of air trapped in the overturning crest.

One of the first studies to quantify energy loss from wave breaking is the experimental work of [4], that employed wave groups generated through frequency focusing based on constant amplitude (Top Hat) spectra. Accounting for energy losses due to sidewalls and bottom friction within the wave flume, the reported losses relative to initial levels are 10% for spilling breakers all the way up to 25% for plungers. Later on, the work of [5], based on constant steepness spectra, reported up to 16% energy loss. Generating bimodal spectra and chirped wave groups, [6] reported around 17% energy loss. Further work on constant steepness spectra can be found in [2], [1], [7] and [8]. The work of [8] compared three different spectral shapes (specifically constant amplitude, constant steepness and Pierson-Moskowitz) and noted differences in the fractional energy losses due to breaking between spectral shapes. They reported that up to 40%, 35% and 25% of initial energy levels were lost by the constant amplitude, constant steepness and Pierson-Moskowitz spectra, respectively. Analysing the present literature, it can be concluded that there is a minimal number of publications investigating wave breaking energy dissipation in realistic sea-states represented by JONSWAP spectra.

From an engineering perspective, the effects of wave breaking on the short-term wave crest and height distributions were shown in [9] and [10]. They showed that for a steep sea-state where significant wave breaking occurs, it counteracts nonlinear amplifications. This effectively reduces the crest elevation back towards the Rayleigh distribution for small exceedance probabilities. As there is a need for long duration time-histories in order to determine statistically significant crest elevation distributions, from a numerical perspective this currently limits the approach to computationally-efficient potential flow solvers.

The work of [1] implemented energy dissipation into a pseudo-spectral potential flow solver, where the energy dissipation was empirically derived from wave groups based on constant steepness spectra. In a subsequent paper, [11] derived further empirical relationships to facilitate the applica-

tion of their dissipation model. A comparison between the [1, 11] model and the dissipative model described in [12] was performed by [13], who preferred the former approach. The choice was justified by the fact that the [1, 11] model best estimated energy dissipation as well as the overall spectral changes brought on by wave breaking. However, it was also noted by the authors that the model tends to apply viscous dissipation for too large a duration. In a subsequent study, [14] implemented the model in an open-source pseudo-spectral numerical wave tank and reported good agreement between the numerical simulations and the equivalent experimental work.

Acknowledging the above, [15] employed the model of [11] in the open-source high-order spectral (HOS) model of [16] to simulate extreme wave crest distributions in long-crested nonlinear sea-states. The results obtained based on JONSWAP spectra did not illustrate the collapse of the crest elevation below the second-order distribution of [17] for small exceedance probabilities in steep sea-states. This is in direct contrast to the findings of [18], [9] and [10]. In an attempt to explain the observed difference, [15] tested two different breaking detection methodologies: the original local geometric threshold of [1] as well as the kinematic criterion of [19]. No significant difference was noted in the results presented, so it must be concluded that a possible source of disagreement lies within the dissipation model of [11]. The most recent work of [20] also compared experimental measurements with numerical simulations employing the [11] model; they concluded that the empirical model did not dissipate sufficient energy for a given breaking wave event and recommended the need for further corrections to the model.

Considering the results reported in previous work, the present study has two key aims. First, it seeks to present energy dissipation characteristics brought on by wave breaking in focused wave groups generated by JONSWAP spectra. Second, it aims to validate and improve upon the energy dissipation model proposed by [1, 11]. To this end, a similar parameterisation is desired by studying both changes to the underlying spectrum and energy dissipation due to wave breaking. The present work differentiates itself by obtaining results applicable to realistic sea-states characterised by JONSWAP spectra. In order to implement an effective energy dissipation methodology that is applicable to potential flow solvers, a parameterisation based on measurements of the free surface profile needs to be devised. This will be based on experimental measurements recorded in a wave flume situated in the Hydrodynamics Laboratory at Imperial College London.

After this introductory section, this paper continues by describing the experimental investigation in Section 2. The methodology employed to estimate energy dissipation rates is then presented in Section 3 and the results

are shown to be in good agreement with previous studies. In order to further develop the model of [11], local wave geometry is examined in Section 4 and a refined eddy viscosity model is derived in Section 5. Section 6 draws the relevant conclusions of the present study and offers final recommendations.

2. Experimental Setup

2.1. Wave making facility

The experimental programme described in this study was undertaken within a 27.2 m long wave flume situated in the Hydrodynamics Laboratory at Imperial College London. The features and performance of this facility were detailed in studies such as [21]. The flume has a working depth of 0.7 m, a width of 0.3 m and has its side walls and bottom made out of glass which is supported by a steel frame. The layout of the steel frame is such that an uninterrupted view with a width of 1.6 m through the side glass panels is possible. The length scale of the experiments is 1:100, and using Froude scaling arguments, a time scale of 1:10 results. This is the same scaling using in studies such as [22] and [10].

At both ends of the wave flume there is a bottom-hinged flap-type wave paddle. The two individually controlled paddles are identical and able to produce wave components in the frequency range $0.4 \text{ Hz} \leq f \leq 3 \text{ Hz}$. As described in [23], the flume employs the paddles in a force feedback control loop, with wave components being generated at one end and absorbed at the other. The flap-type wave paddle employed as a downstream wave absorber is very effective at eliminating long period reflected waves (for which traditional passive absorbers perform poorly), with its effectiveness decreasing for short-waves (however, the present study also utilised an absorbing beach to minimise short-wave reflections as detailed in Section 3.4). Further assessment of the performance of the two paddle system is given in [21].

2.2. Instrumentation

2.2.1. Wave gauges

In the present study the water surface elevation time-history at a given location $\eta(t)$ was measured using drop-down, resistance-type wave gauges. These instruments cause negligible disturbance to the surface elevation of the incident wave field. A sample rate of 128 Hz was employed, the wave gauges were calibrated daily and studies such as [21] report an accuracy of the water surface elevation measurement of $\pm 0.5 \text{ mm}$.

115 2.2.2. Charge-coupled device camera

116 Complementary to the wave gauge measurements, the water surface ele-
117 vation was also recorded using charge-coupled device (CCD) cameras. Typ-
118 ically, the camera would be mounted on the side of the glass panel where the
119 desired wave event was to be observed. The camera was mounted slightly
120 higher than the still water level and at a slight angle to the glass panel,
121 resulting in the camera looking down onto the water surface. The aim of
122 the camera footage is to employ image edge detection algorithms to obtain
123 spatial profiles of the waves. In order to obtain a clear meniscus that sep-
124 arates the water fraction from air, a backlight illumination technique was
125 employed, where an LED light-sheet was used as the primary light source.
126 The choice of the LED light-sheet was motivated by the uniform lighting it
127 provides and because it uses a direct current power source which eliminates
128 flickering.

129 These grey-scale images from the camera are processed in a edge detec-
130 tion algorithm based on the Image Processing ToolboxTM from MATLAB[®].
131 To ensure a clear separation of the free surface and accurate readings, the
132 glass wall is cleaned daily to ensure no air bubble build up and limescale
133 deposits.

134 In order to recover accurate quantities from the edge detection algorithm,
135 the images need to be undistorted and an appropriate conversion between
136 pixels and physical length quantities needs to be established. This was
137 achieved using the CameraCalibrator application from the Computer Vision
138 ToolboxTM from MATLAB[®]. The method relies on a grid of square pixels
139 of known size to calibrate and compute the camera characteristics needed for
140 image undistortion. Evidently, the accuracy of the camera calibration is only
141 as good as the pattern used in the calibration process. For this purpose, a
142 22 by 15 squares intersections checkerboard with a square side size of 25 mm
143 was professionally plotted on an A2 size photo quality paper. In order to
144 ensure that the asymmetric checkerboard is flat, the high-accuracy photo
145 was installed in a rigid wooden photo frame. The asymmetry in x of the
146 pattern enables the clear identification of the origin of the checkerboard
147 regardless of the orientation of the frame.

148 A typical calibration consists of capturing a large number of images of
149 the checkerboard at different positions, orientations and angles so as to cover
150 the entire field of view of the camera. The calibration algorithm assumes
151 a pinhole camera model and uses the technique detailed in [24]. Using this
152 approach, a typical camera calibration used within the present study results
153 in a mean error between the detected points and reprojected points of 0.1

Table 1: Error estimates determined from camera calibration technique.

Direction	Min Error		Mean Error		Max Error	
	[mm]	[%]	[mm]	[%]	[mm]	[%]
X	0.000138	0.000551	0.033844	0.135377	0.149248	0.596994
Y	0.000176	0.000705	0.033505	0.134020	0.139906	0.559624

pixel.

Once the camera parameters are computed, they are used to remove the distortion from the images that are being processed by the edge detection algorithm. The translation of the detected pixels from pixel space to physical space is performed using the same high-accuracy checkerboard. A single frame where the checkerboard lies flat in the same plane as the glass panel is used to determine the position of the pixels. Tests with the reference frame situated in front and behind the glass showed no difference in the accuracy of the measurements and hence the reference frame was placed in front of the glass panel for convenience. An estimate of the expected errors obtained by comparing the predicted size of the checkerboard pattern as determined from camera with the known size of 25 mm is shown in Table 1; these are less than 0.6% or 0.15 mm. Similar estimates were obtained by checking against an object with known size placed in the frame, such as a ruler.

In order to synchronise the gauge measurements with the camera recordings, the trigger signal for the camera was the same one used to trigger the wave paddle. The frame rate for the recordings was set at 48.8 frames per second.

2.2.3. Camera and wave gauge intercomparison

While the errors expected from the camera calibration were estimated in Table 1, the accuracy of the edge detection algorithm and the camera measurements needs checking. In order to test the accuracy of the camera setup when combined with the edge detection algorithm, an intercomparison between wave gauge and camera was performed.

For this, a plunging breaker characterised by a JONSWAP spectrum with $\gamma=5$ was recorded using a wave gauge positioned in the breaking region of the flume. This is illustrated in Figure 1. Panel 1a shows the positioning of the wave gauge in quiescent conditions. This image was used to detect the pixel location from which the time-history of the free surface is determined. The subsequent set of figures illustrates the wave event as it evolves towards breaking. Additionally, the figures illustrate the behaviour

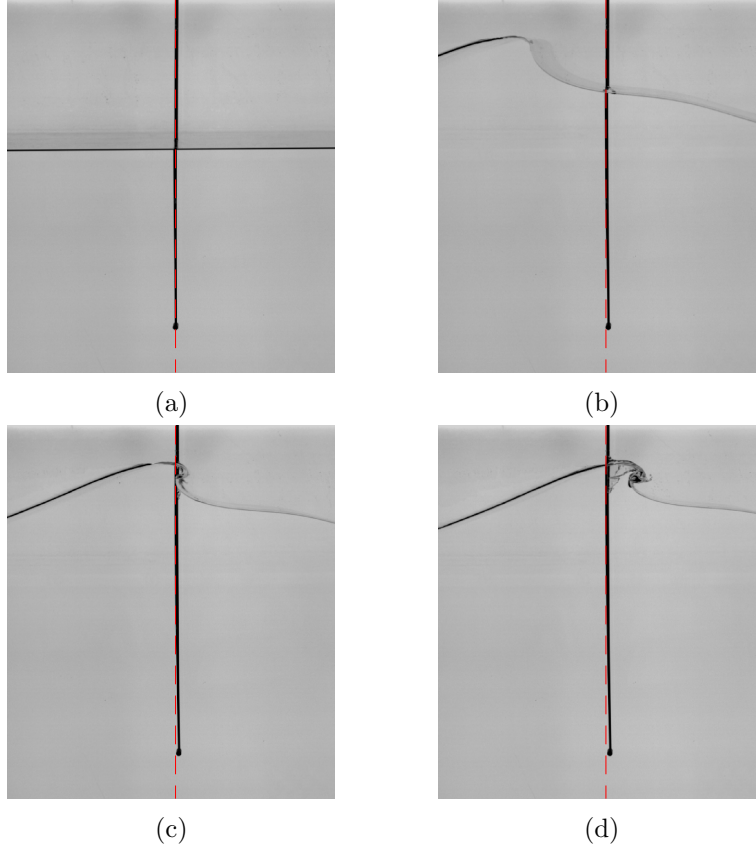
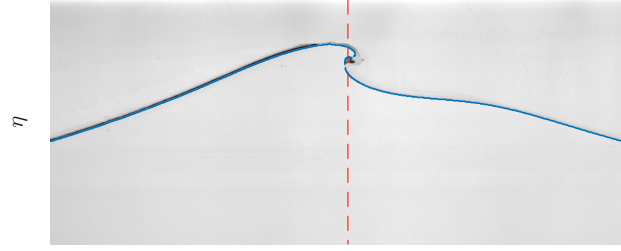


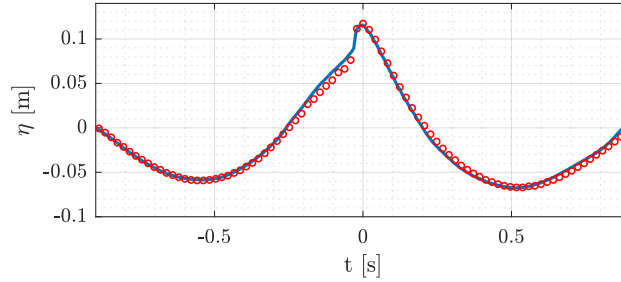
Figure 1: Example photographs of resistance-type wave gauge recording a plunging wave event with $A=90$ mm and $\gamma=5$. In all panels — — illustrates the position of the wave gauge as determined from panel (a).

of the slender drop-down wave gauge (analogous to a cantilevered beam) that slightly bends as the large wave event passes (this was also reported in [25]). This is best observed by referring to the original position of the gauge in quiescent conditions as shown by the red dashed line.

To complement Figure 1, Figure 2 illustrates the repeated case (with the wave gauge removed) together with the free surface profile extracted from the image using a Canny edge detection algorithm. In order to obtain the free surface elevation of the wave event, the coordinates of the SWL need to be obtained and used as reference. This is determined from the first frame capture by the CCD camera for a given wave group, such as the one shown in Figure 1a. The convention used within the present work is that the



(a) Free surface edge detection.



(b) Free surface time-history.

Figure 2: (a) The spatial free surface $\eta(x)$ as obtained from the edge detection algorithm together with (b) a comparison of the temporal free surface profiles $\eta(t)$ obtained from $\circ$ camera and — wave gauge. In panel (a) --- shows position of wave gauge as obtained from Figure 1a.

SWL is defined as the bottom edge of the concave meniscus formed between the water and the glass surface. Typically, within the present experimental setup, the thickness of the meniscus is around three to four pixels. The choice of picking between the bottom or top edge when detecting the free surface in the Canny algorithm results in a difference between the two levels of approximately 0.6 mm (note that the accuracy of the gauges is ± 0.5 mm). For reference, this represents approximately 0.5% of the elevation of a large wave event such as the one shown in Figure 2.

Figure 2b compares the free surface profile measured by the gauge positioned as shown in Figure 1 with the time-history recorded by the camera at the stationary gauge pixel identified from Figure 1a. Every image recorded by the camera offers a single point of data in Figure 2b. The very good agreement between the two measurement devices illustrates the fact that accurate results can be obtained using the camera. However, consistent with the findings of [25], the gauge tends to smooth out the wave profile compared to the camera measurements giving a slight difference between

the two profiles near the crest region. The slight movement of the gauge as the large wave passes (visible in Figure 1) might explain the slight discrepancies observed just before $t = 0$ s. Additionally, as observable in Figure 1d, there is a minimal disturbance in the wave profile due to the presence of the gauge (that is not present for the camera measurements), which might also contribute to the slight disagreement illustrated in Figure 2b. Lastly, it must be noted that this is one of the worst comparisons between the two instruments obtained in the present study, with the wave breaking exactly on the gauge causing the most deflection. More representative (better) comparisons are presented in Figure 12.

Considering all of the above, it must be made clear that these effects do not affect the present results in any way, as the energy dissipation estimates (based on gauge measurements) are obtained from wave gauges far away from the breaking event where no movement of the gauges was present.

2.3. Experimental Method

As the aim of this work is to study the breaking wave energy dissipation within JONSWAP sea states, wave breaking is generated using the frequency focusing technique. The NewWave formulation developed by [26], [27] and [28] is employed here as they represent the most probable shape of the largest waves in the ocean [29]. The benefits of adopting focused waves for the present study lies in the fact that they allow a precise control and identification of the linear underlying spectral components. The dispersive wave focusing technique was employed in the landmark study of [4] as well as [30], [31] and [32] among many others. Furthermore, both [33] and [34] showed that dispersive focusing is a likely mechanism for the formation of rogue waves.

2.3.1. Focused wave generation

The amplitudes of the frequency components are taken from the NewWave formulation of the JONSWAP spectrum with a peak period of $T_p = 1.2$ s. The frequencies of the spectral components lie in the range $26/64 \text{ Hz} \leq f \leq 3/T_p$. The input spectra to the wave paddle was discretised in 269 components resulting in a repeat time of 128 s. Compared to the generation of a random sea-state, the initial phasing of the components is not randomised, but clearly specified to create constructive interference at one point in space and time. Linear Irregular Wave Theory (LIWT) is employed to generate the linear target spectrum with the surface elevation at the given focusing

249 location x_b and focusing time t_b being:

$$\eta(x_b, t_b) = \sum_{n=1}^N a_n \cos(k_n x_b - \omega_n t_b + \epsilon_n), \quad (1)$$

250 where $N=269$, a_n is the amplitude of individual components taken from the
 251 NewWave formulation, the component wavenumber k_n and frequency ω_n are
 252 related through the linear dispersion equation and ϵ_n is calculated such that:

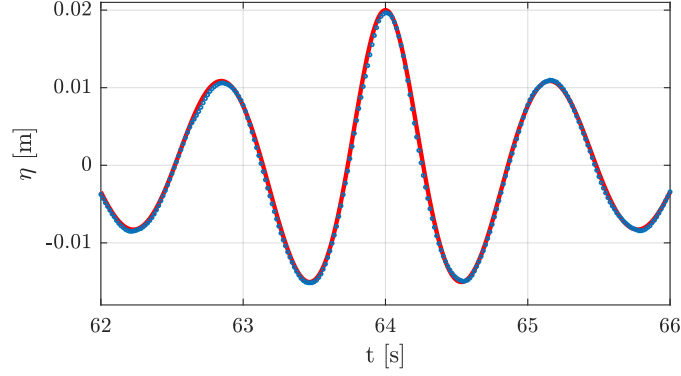
$$\cos(k_n x_b - \omega_n t_b + \epsilon_n) = 1. \quad (2)$$

253 The generation of the accurate amplitudes determined above is done
 254 through the Tank Transfer Function (TTF) that creates the demand signal
 255 for the wave paddle. This is based upon tank specific properties such as
 256 the water depth and the wave paddle geometry [35]. The validation of the
 257 generated wave cases and performance of the TTF is analysed by comparing
 258 the resulting free surface elevation and underlying spectrum (for a theory
 259 appropriate wave steepness) with analytical solutions such as LIWT and
 260 second-order wave theory of [36]. Figure 3 illustrates the free surface eleva-
 261 tion at the focal position and the underlying spectral properties for a linear
 262 focused wave with $A = \sum a_n = 20$ mm and a peak enhancement factor of
 263 4. The figure also includes the LIWT theoretical prediction in terms of free
 264 surface elevation and target spectral properties. Being a small amplitude
 265 focused wave event, there is very good agreement between experiments and
 266 LIWT. The only noticeable deviations lie in the phasing of the components
 267 ϵ_n that are close to the edges of input range, where the amplitudes can be
 268 observed to be essentially zero. Hence, the deviations from the target are
 269 due to noise in the processed signal.

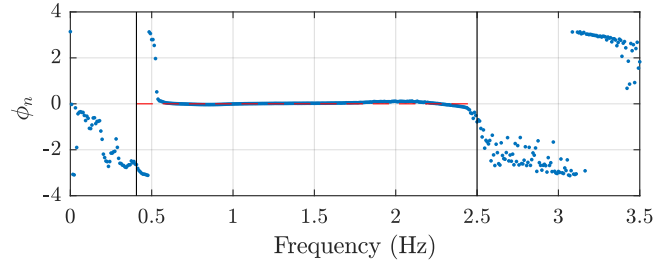
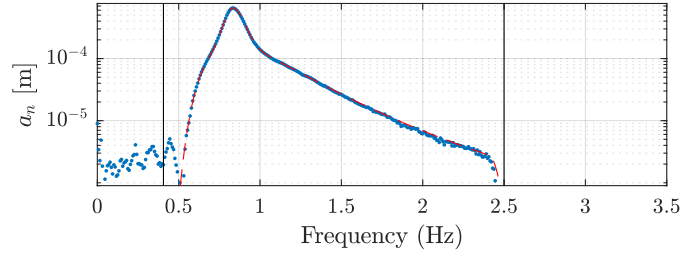
270 Further comparisons that validate the accurate generation of wave events
 271 within the experimental facility are shown in Figure 4. The figure concerns
 272 weakly-nonlinear wave events that can be accurately described using the
 273 second-order solution of [36]. Excellent agreement is again observed between
 274 the experimentally-generated waves and the theory for events with various
 275 spectral bandwidths.

276 2.3.2. Focused wave cases

277 For the purpose of parameterising the energy dissipation due to wave
 278 breaking through dispersive focusing, four different peak enhancement fac-
 279 tors γ for the NewWave spectra were studied ($\gamma= 2.5, 3.3, 4$ and 5). The
 280 values of Hasselmann et al (1973) of $\beta=1.25$, $\sigma = 0.07$ if $\omega \leq \omega_p$ and 0.09
 281 if $\omega_p < \omega$ (where ω_p is the frequency corresponding to the spectral peak)



(a) Free surface time-history.



(b) Spectral amplitude and phasing.

Figure 3: Comparison between (a) free surface elevation time-history from $\circ\circ\circ$ experimental results and — LIWT theoretical predictions and (b) spectral properties from $\dots$ experiments and -- linear target for a focal event with $\sum a_n = 20$ mm and $\gamma=4$. In both panels of (b) vertical black lines show input range.

282 were used for a given γ value. In terms of experimental procedure, for a
 283 given γ value the linear amplitude sum A was systematically increased until
 284 breaking initiation was observed. This was achieved by visual inspection of
 285 the wave profile and from studying the video footage. Once breaking was
 286 initiated A was increased further until a clear single spiller or plunger was
 287 observed. Only single breaking events per wave group were included in the

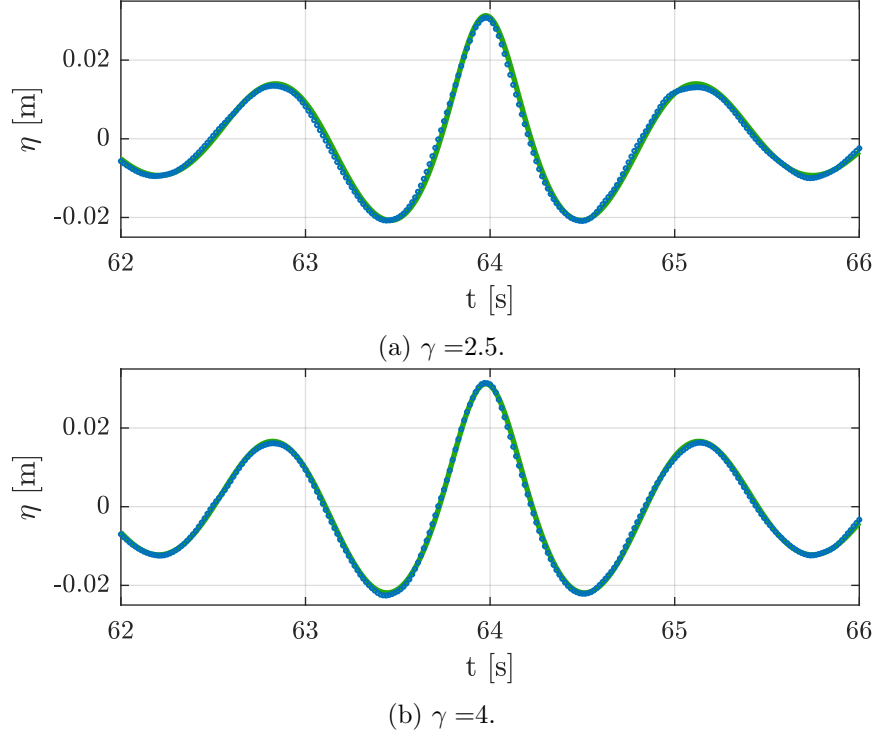


Figure 4: Comparison between free surface elevation time-history from ○○○ experimental results and — second-order wave theory of [36] predictions for focal events with $\sum a_n = 30$ mm and (a) $\gamma = 2.5$ and (b) $\gamma = 4$.

present study.

Given the peak period of the spectra is $T_p = 1.2$ s, the water depth of the flume sets the depth regime to $k_p d \approx 2.0$ (where k_p is the wavenumber corresponding to the frequency of the spectral peak), setting the study towards the deeper end of the intermediate range as in [9]. To take into account the change in spectral bandwidth given by altering γ , the global steepness of a given wave group is quantified through the measure Ak_c , where k_c is the equivalent weighted spectral wavenumber calculated as $k_c = \sum a_n k_n / \sum a_n$. Here, a_n and k_n are the amplitudes and wavenumbers of individual components. Using k_c the depth regime of all wave cases is set at $k_c d \approx 3.0$. The limiting case properties for all four γ values studied in the present work are given in Table 2. Consistent with the observations of [37], a spectrum characterised by more energy content in the high frequencies ($\gamma = 2.5$) is less stable and requires a smaller input amplitude sum A before breaking is initiated.

303 3. Energy Dissipation

304 3.1. Control volume approach

305 Previous work on wave breaking estimated the amount of energy dissi-
 306 pated by a specific wave group through a control volume approach [4, 2, 1].
 307 The depth-integrated total energy density $E(x, t)$ along a wave group is:

$$E(x, t) = \int_{-d}^{\eta} \frac{1}{2} \rho_w u_i^2 dz + \int_{-d}^{\eta} \rho_w g z dz + \frac{1}{2} \rho_w g d^2, \quad (3)$$

308 where ρ_w is the water density, g is the gravitational acceleration, d is the
 309 water depth, η is the free surface elevation, u_i are the underlying particle
 310 kinematics (where the subscript i represents all three Cartesian coordinates),
 311 z represents the vertical direction and the final term in the equation accounts
 312 for the potential energy of the still water level.

313 Analysing Equation (3) it can be said that to compute the total energy of
 314 a given wave group, measurements of the underlying particle kinematics u_i
 315 are needed. This would involve a complex setup in a laboratory environment.
 316 [4] employed linear wave theory to show that away from a focal point where
 317 the wave group is well dispersed, the total energy content of the wave group
 318 is proportional to the total potential energy. This result, accurate to second
 319 order, can be readily derived from small amplitude wave theory and shows
 320 that energy equipartitioning holds far away from the focal region. After
 321 depth integration the instantaneous total energy per unit area is obtained
 322 as:

$$E_t(x, t) = \rho_w g \eta^2(x, t). \quad (4)$$

323 It must be noted that both nonlinearity and wave breaking significantly
 324 violate the energy equipartitioning assumption, and hence energy estimates
 325 derived from free surface measurements using Equation (4) results in an
 326 underestimate of the total energy content in the fluid. This is avoided as
 327 much as possible through careful design of the experimental setup.

Table 2: Largest non-breaking experimentally-generated wave groups.

γ	$A = \sum a_n$ [mm]	Ak_p	Ak_c	$k_c d$	$k_p d$	$\tanh(k_c d)$
2.5	70	0.203	0.307	3.067	2.026	0.996
3.3	75	0.217	0.317	2.957		0.995
4	77.5	0.224	0.319	2.882		0.994
5	80	0.231	0.319	2.797		0.993

328 The energy transport or energy flux of the waves across a cross-section
 329 of the wave flume is related to the energy density via a characteristic group
 330 velocity of the wave group. At a given location in the flume the energy flux
 331 can be estimated as:

$$F(x, t) = \rho_w g C_g \eta^2(x, t), \quad (5)$$

332 where C_g is a characteristic group velocity that best describes the energy
 333 propagation. The control volume approach estimates the change in energy
 334 flux through it over a time duration $T: t_1 \rightarrow t_2$ using the following equation
 335 from [2]:

$$\Delta E_{tot} = \int_{t_1}^{t_2} \rho_w g C_{g1} \eta^2(x_1, t) dt - \int_{t_1}^{t_2} \rho_w g C_{g2} \eta^2(x_2, t) dt. \quad (6)$$

336 In Equation (6) the two terms refer to measurements recorded by wave
 337 gauges at the inlet (x_1) and outlet (x_2) of the control volume. In applying
 338 Equation (6), there are three important points to consider:

- 339 1. Design of **control volume**: this refers to the positioning of the two
 340 wave gauges within the wave flume that will act as the inlet and outlet
 341 of the control volume.
- 342 2. Choice of **characteristic group velocity** to best approximate the
 343 energy flux.
- 344 3. Selection of **time averaging window**: choosing the free surface mea-
 345 surement window to ensure quiescent conditions before and after the
 346 wave group arrives.

347 Each of the three key points noted above will be considered separately in
 348 the following subsections.

349 3.2. Wave gauge location

350 The key assumption that governs the positioning of the two wave gauges
 351 that act as boundaries to the control volume is energy equipartitioning.
 352 Essentially, the location of the first wave gauge (inlet of control volume)
 353 must be sufficiently far upstream enough of the breaking location such that
 354 the wave group is well dispersed and no significant nonlinear energy transfer
 355 has occurred between the spectral components. Similarly, the gauge that
 356 represents the outlet needs to be sufficiently far enough downstream of the
 357 breaking location. It has been shown in studies such as [38] that rapid

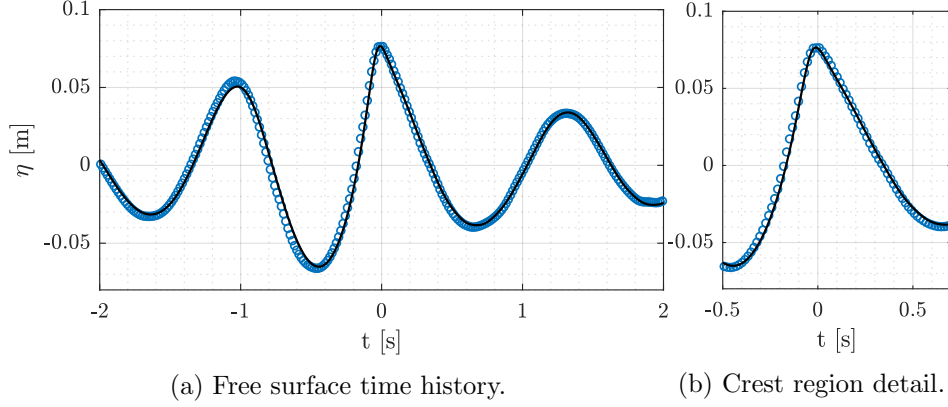


Figure 5: Comparison between free surface elevation time-history $\eta(t)$ predicted by — BEM and $\circ\circ\circ$ measured by wave gauge at theoretical focal position in the wave flume for the largest non-breaking case with $\gamma=3.3$. To facilitate comparison the focal time has been shifted to $t=0$ s. Wave group details given in Table 2.

spectral changes occur near the focal event, but after focusing (given that no breaking has occurred) the spectrum almost recovers to its initial shape.

To determine the position of the gauges within the wave flume, the multiple-flux boundary element model (BEM) of [39] was employed as a numerical wave tank. The experimental flume was recreated within the BEM model and the largest non-breaking wave case for $\gamma=3.3$ specified in Table 2 was numerically simulated. A spatial discretisation of $\Delta x=\Delta z=0.025$ m and temporal discretisations of $\Delta t=0.003$ s were used in the set-up of the numerical domain. This was shown to be sufficient for convergence in [40]. The computational domain was set as $-9.95\text{ m} \leq x \leq 17.55\text{ m}$ and $0\text{ m} \leq z \leq -0.7\text{ m}$ with focusing set to occur at $x=0$ m.

The comparison between the free surface elevation time-history measured in the flume using the wave gauge at the theoretical (linear) focal position and the free surface time-history predicted by the BEM model (at the equivalent position in the numerical domain) is shown in Figure 5. As can be observed, there is a very good agreement between the experimental measurements and the BEM simulations.

Having confirmed the accuracy of the BEM wave event, the depth and time integrated potential and kinetic energy per unit length of the wave group was computed at discrete locations x_i using the free surface elevation $\eta(x_i, t)$ and the particle kinematics obtained from the BEM model. The ratio between the kinetic energy and the potential energy along the wave flume determined from the BEM simulation is plotted in Figure 6. Consistent with

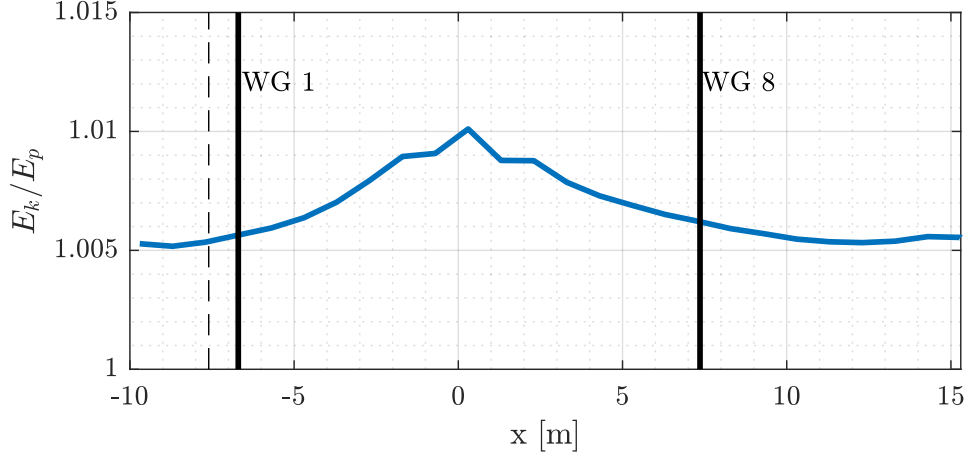


Figure 6: The spatial distribution of — the ratio between time- and depth-integrated kinetic (E_k) and potential (E_p) energy along the numerically simulated wave flume using the BEM. The thick vertical black lines show the location of the two wave gauges comprising the control volume. The vertical dashed black line marks the position $x = 3d$ from the wave generating paddle after which the influence of the evanescent modes becomes negligible.

the findings of [41], [42] and [43] (based on numerical results using the Gerris DNS solver) the assumption of energy equipartitioning holds except very close to the breaking region. It can be seen in Figure 6 that kinetic energy is the dominant form of energy near the focal region for the non-breaking wave case exemplified. [41] used an alternative boundary integral equation model to show that in breaking crests approximately 49% of the energy resides in the potential energy, with the surplus kinetic energy responsible for the formation of the breaker jet.

During the majority of the experimental programme, the free surface elevation along the flume was recorded using eight wave gauges. Figure 6 also illustrates the positioning of wave gauge 1 (WG1 – the inlet of the control volume) and wave gauge 8 (WG8 – the outlet of the control volume). The choice of the positioning of the gauges was such that the ratio between E_k and E_p was minimised.

In addition, the first wave gauge needs to be positioned far away enough from the generating wave paddle such that no spurious evanescent modes are present in the signal. [35] concluded that the influence of the evanescent modes is negligible if the wave measuring area starts after $x \geq 3d$, where d is the water depth. For the present wave flume, the dashed black vertical line in Figure 6 marks the location $x = 3d$ from the paddle.

401 The recommendations for the evanescent modes and the need to ensure
 402 energy equipartitioning governed the positioning of the first wave gauge
 403 *WG1*, whereas for wave gauge *WG8* there are two further considerations.
 404 First, within the wave flume, there are additional sources of energy dissipa-
 405 tion besides breaking, such as sidewall friction. Further details about how
 406 this was considered is detailed in Section 3.5. Second, the long time inte-
 407 gration of the measured wave gauge signal is used in computing the local
 408 energy. Reflections from the absorbing end of the flume might contami-
 409 nate the signal, especially for the gauges downstream of the focal location.
 410 Positioning *WG8* as far away as possible from the downstream end of the
 411 flume results in longer time needed for the reflections to reach the measuring
 412 position. Further considerations on this topic are given in Section 3.4.

413 3.3. Characteristic group velocity

414 As shown in Equation (6), the energy flux through the control volume
 415 is computed using a characteristic group velocity $C_g = \partial\omega/\partial k$. In the
 416 work of [2] it was shown that the wave group propagates along the flume
 417 with approximately a weighted average of the individual component group
 418 velocities. This was termed C_{gs} and is computed as:

$$C_{gs} = \frac{\sum C_{gn}(a_n)^2 \Delta f}{\sum (a_n)^2 \Delta f}, \quad (7)$$

419 where C_{gn} and a_n are the group velocity and amplitude of an individual
 420 wave component and Δf is the spectral discretisation. The amplitudes of
 421 the components are obtained from the underlying spectral information. This
 422 definition of the group velocity was also employed in the works of [44] and
 423 [1] to successfully estimate energy dissipation from breaking wave packets.
 424 In the numerical study of [42] the ratio between the energy flux and energy
 425 density was compared with estimates of C_{gs} . It was shown that indeed this
 426 weighted group velocity is a good estimate of the energy transport velocity
 427 along the wave group.

428 An important consideration regarding C_{gs} is its variation along the wave
 429 group, due to the underlying spectral changes as the group evolves down-
 430 stream. [2] reported no changes in the weighted group velocity between the
 431 two ends of their control volume. [1] computed C_{gs} using the data at the
 432 first gauge closest to the wave generator and used this value to estimate
 433 the energy losses. However, [1] reported an increase in C_{gs} downstream of
 434 the breaking position. The increase is due to the fact that wave breaking
 435 dissipates energy from the high-frequency range of the spectrum, which is

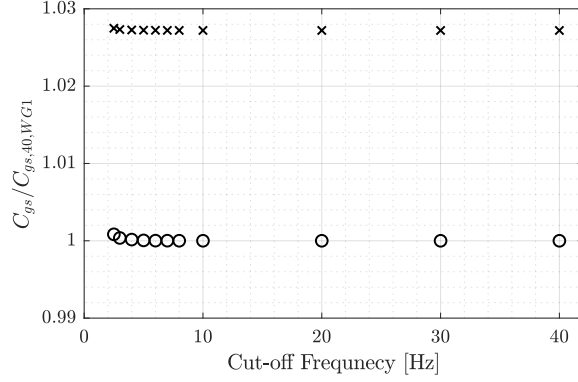


Figure 7: Typical example of weighted group velocity calculated from spectral information from $\circ$ WG1 and $\times$ WG8 against different cut-off frequencies. The normalisation is done using the value of C_{gs} at WG1 calculated using a cut-off frequency of 40 Hz.

characterised by components with slower group velocities. The need to account for the changes in the energy propagation velocity after breaking was discussed by [42]; they noted that if the increase in C_{gs} is not accounted for the errors in estimating wave breaking energy losses rose from 5% to 35%.

In the present work, given that C_{gs} is deemed a good estimate for the energy propagation velocity and to stay consistent with the works of [1], its value was calculated at WG1 and WG8 for all wave groups. Hence, in applying Equation (6), $C_{g1} = C_{gs,WG1}$ and $C_{g2} = C_{gs,WG8}$ are used. The amplitudes needed for the weighted calculations are obtained from Fourier transforms of the signal recorded at the gauges. To ensure convergence of the obtained values, different spectral cut-off frequencies are used, with an example result shown in Figure 7 for a wave group with a breaking event. The calculated group velocities at both gauges are normalised by the group velocity calculated at WG1 with a cut-off frequency of 40 Hz. The figure demonstrates two important points. First, an increase in the group velocity at WG8 is noticeable due to the loss of high-frequencies during the breaking process. Second, C_{gs} slightly reduces as more and more high-frequency energy is included and convergence is achieved rapidly with cut-off frequency. In the present study all characteristic group velocities are computed using a cut-off frequency of 10 Hz.

3.4. Time averaging window

In order to accurately obtain the total energy within a given wave group from the time integration of the free surface histories at WG1 and WG8,

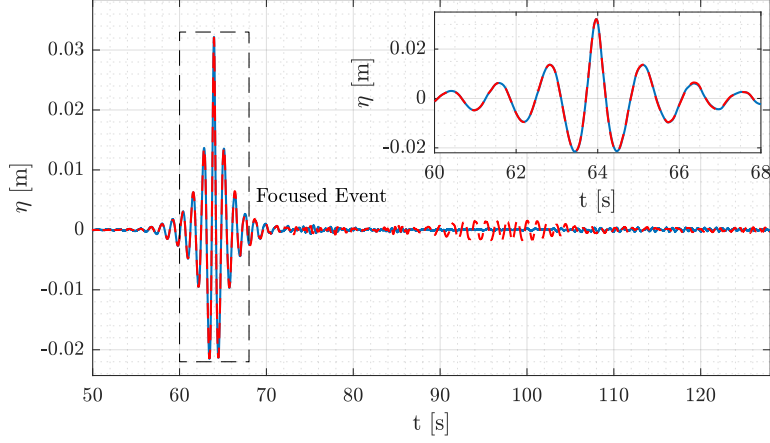


Figure 8: Free surface time-history $\eta(t)$ for a linear focused wave group with $\gamma=2.5$ recorded at the focal position in the wave flume with — — wave absorbing paddle and — wave absorbing paddle and parabolic beach. Figure inlay details focused wave group.

the measured signals need to be characterised by quiescent conditions before and after the arrival of the wave group.

Quiescent conditions are ensured at the beginning of the time signal by allowing sufficient time between two consecutive experimental runs so that the tank settles and no long wave components are present in the flume. In the present experimental programme, a settling time of 15 min was deemed sufficient to ensure quiescent conditions were re-established before the next run. During this time, all the acquired images from the CCD cameras were being written to hard drive.

After the wave group passes a measuring station, quiescent conditions are obtained by minimising the reflections from the downstream end of the flume. It was discussed in Section 2.1 that the flap-type wave paddle employed as a downstream wave absorber is very effective at eliminating long period reflected waves. However, in visually analysing the typical time-history obtained from the wave gauges, some higher frequency wave components were reflected back into the tank. An example is given for a small amplitude wave group recorded at the focal gauge in Figure 8, where the dashed red line shows the recorded signal when just the wave paddle was used as an active wave absorber. Immediately visible in the region $90 \text{ s} \leq t \leq 110 \text{ s}$ are high frequency waves reflected back from the wave paddle. While there is a very clear separation between the main focused wave group and the reflected packet for the focal gauge, gauges placed further downstream

481 had the two wave packets much closer together.

482 One solution would be moving the location of downstream gauges (such
483 as *WG8* that acts as the outlet of the control volume) further upstream.
484 However this results in an increased ratio between the total kinetic and po-
485 tential energy at that location. This is demonstrated in Figure 6, where by
486 moving the position of *WG8* back closer to $x = 0$ m, the ratio E_k/E_p in-
487 creases. This is undesirable and the decision was taken to install a parabolic
488 beach at the downstream end of the tank, which would further reduce re-
489 flections. After installing a parabolic beach, the resulting time history for
490 the same wave event recorded at the same position is illustrated in Figure
491 8 as the blue line. Analysing the region $90\text{ s} \leq t \leq 110\text{ s}$ it is immediately
492 evident that the passive absorbing beach effectively eliminates the reflected
493 short waves. In addition to this, the figure compares the two wave groups
494 and shows that the main wave group was not influenced by the reflected
495 components, showing great repeatability of the experimental runs.

496 While the beach seems to be effective at eliminating reflections by vi-
497 sual inspection of the time signals, to quantify its efficiency, a reflection
498 analysis was performed using regular waves. For this, an additional 8 wave
499 gauges were installed at the downstream end of the flume, with the last
500 gauge furthest downstream installed at 1.9 m from the beach, with the spac-
501 ing between subsequent gauges being $\Delta x = 0.2$ m. The reflection coefficient
502 as defined in [45] was calculated, where separation between the incoming
503 and reflected components of the signal was done up to second order. The
504 reflection coefficient was found to be less than 5%.

505 An example of the free surface time-histories recorded at the two gauges
506 used as a control volume is shown in Figure 9, illustrating the typical signal
507 that is being time integrated to determine energy levels. Even for the last
508 downstream gauge (*WG8*) it can be observed that no reflections are present
509 and a clear time window for integration is obtained. For each individual
510 gauge two repeats of the same wave case generated several days apart are
511 plotted. The excellent agreement between the two separate measurements
512 illustrates the high level of repeatability achievable with the current gener-
513 ation methodology, gauge calibration and wave facility. The repeatability
514 of the wave events is critically important for the results presented in this
515 paper.

516 3.5. Fractional energy loss

517 The aim of this section is to estimate wave breaking energy dissipation
518 for single breaking events. In the present wave generation facility, as the
519 wave group travels downstream from the generation end there will be some

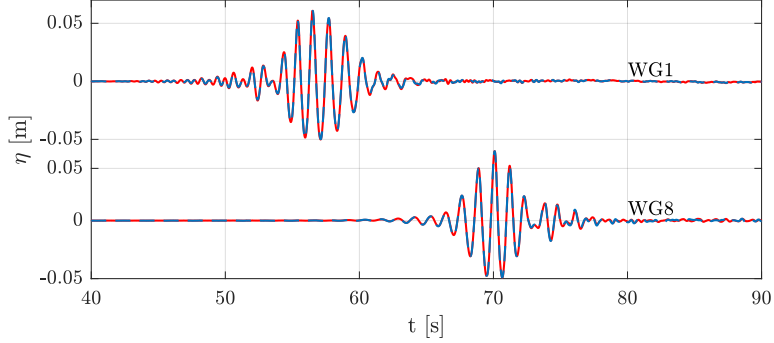


Figure 9: Free surface time-histories of the largest non-breaking wave group with $\gamma=3.3$ measured several days apart (—, —) at the gauges used as control volume ends (*WG1* and *WG8*). Details of wave case given in Table 2.

energy lost due to sidewall and bottom friction. As noted by [4, 2] and [44, 1] these losses might be significant and should be taken into account in order to clearly identify the energy loss exclusively due to breaking. As such, for a wave group where a breaking event is recorded, the total energy loss ΔE_{tot} calculated using Equation (6) can be divided into two parts:

$$\Delta E_{tot} = \Delta E + \Delta E_{nb}, \quad (8)$$

where ΔE is the energy dissipation exclusively due to breaking and ΔE_{nb} is due to non-breaking losses. Using the control volume approach, the total energy loss due to reasons other than breaking can be determined from the largest non-breaking wave group. This approach was adopted in both [4] and [2]. Further considerations on the dissipative effects of sidewalls on the wave group evolution are given in [46]. For the present work, the wave cases characterised in Table 2 were used to determine the non-breaking energy losses for each of the four peak enhancement factors employed here.

Figure 10 illustrates the results of the normalised total energy loss for all breaking cases generated in the present experimental programme against two measures of global steepness. The normalised total energy loss for an individual case was computed as:

$$\frac{\Delta E}{E_0} = \frac{(E_{tot, WG1} - E_{tot, WG8}) - (E_{nb, WG1} - E_{nb, WG8})}{E_0}, \quad (9)$$

where E_0 for the individual case represents the total time integrated energy as calculated from the first wave gauge E_{WG1} .

Figure 10 illustrates that the steepest cases, corresponding to plunging

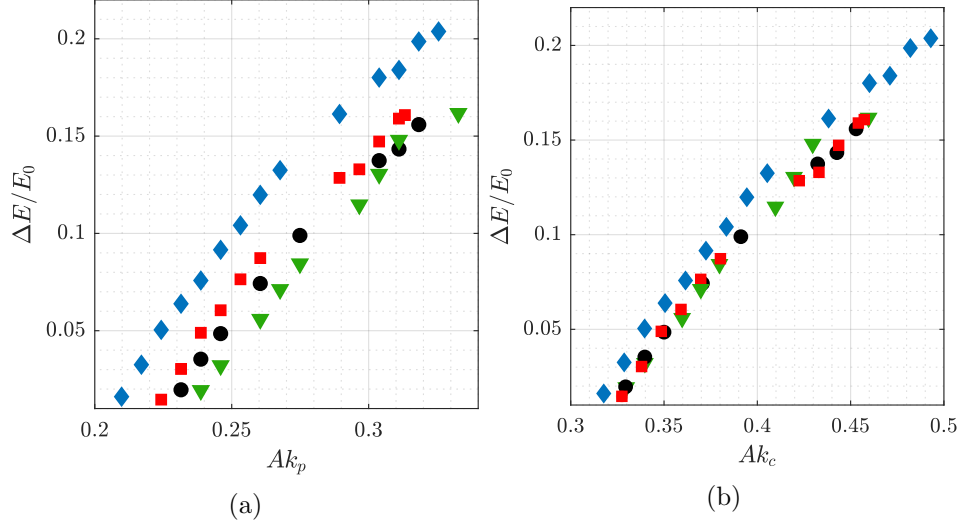


Figure 10: Normalised total energy dissipation due to wave breaking against global steepness (a) Ak_p and (b) Ak_c for wave cases with $\diamond \gamma=2.5$, $\blacksquare \gamma=3.3$, $\bullet \gamma=4$ and $\blacktriangledown \gamma=5$.

breakers, can dissipate between 16% and 20% of their initial energy. These results are consistent with [4] who reported that as much as 25% of the energy is lost for a large plunging breaker event generated through dispersive focusing using a constant amplitude spectrum. The experimental work of [37] reported that for a Pierson - Moskowitz spectrum (JONSWAP with $\gamma=1$) the group lost 20% of its initial energy. The newer study of [2], which employed spectra where each component has equal slope (constant steepness spectrum), 35% of the total energy entering the control volume was lost for the case with the strongest single breaker. Although [2] do not report the maximum loss exclusively due to breaking, an estimate can be obtained by analysing Figure 8 in [2] resulting in around 18% to 19% loss.

Analysing the data presented in the recent work of [1], it can be seen that most of the normalised fractional losses presented are under 20%, with a single case estimated as 25.1%. It must be noted that [1] employed constant steepness spectra for all wave cases. In addition to this, both [2] and [1] employed a constant group velocity C_{gs} across the evolution of their wave groups. In doing so, the total energy loss might be considered a slight overestimate as the increase in group velocity post breaking deems the constant C_{gs} values an underestimate. Further details on the importance of accounting for the change in C_{gs} are given in [42].

While overall the results presented here agree well with existing litera-

ture in terms of fractional losses, another important aspect to note is the greater energy loss as the global steepness increases for a given spectral bandwidth. This is shown in Figure 10a by using a global steepness based on the wavenumber of the peak spectral component (Ak_p). To take into account the effects due to changes in spectral bandwidth, Figure 10b employs the weighted wavenumber k_c and illustrates that broad-banded spectra dissipate slightly more energy compared to narrow banded spectra. Both [4] and [1] reported a very weak dependence on breaking fractional losses with spectral bandwidth. It must be noted that a direct comparison between present and past results is not straightforward, due to the different underlying spectral shapes. Similarly, for each spectral type there is a different definition of spectral bandwidth. As the present work focuses on JONSWAP spectra, further considerations on the effect of γ will be discussed in Section 4.

A final point to consider regarding the results presented in Figure 10 is the fact that all data points come from single wave breaking events. For certain input amplitude sums A , multiple breaking events were observed, which explains why in Figure 10 there seems to be some missing data (for example, at $Ak_p \approx 0.28$ in Figure 10a). Multiple breakers arise because at a certain value of A the crest before the main focal event reaches the breaking threshold and undergoes slight spilling, however, sufficient energy is not dissipated from the wave group and the next crest undergoes breaking as well. This results in two separate (subsequent) breaking (spilling) crests within the same wave packet. A further increase in A ensures that the crest before breaks more vigorously and dissipates sufficient energy so that only a single breaking event is obtained. The gaps in the data presented in Figure 10 correspond to the point of transition where breaking shifts to the previous crest (as observed in space - further upstream from the focal location). A similar behaviour of the wave group was noted in [37].

It was already discussed in Section 2.3.2 that a higher A was needed to generate breaking waves for large γ compared to the broad-banded spectra due to the more unstable nature of the spectra. A secondary effect brought on by reducing the bandwidth of the spectra is the persistent occurrence of multiple breaking events within the wave group after a certain A value. As the bandwidth is reduced the wave group tends more and more towards regular waves. As such, for a large value of A a typical $\gamma=5$ wave group contains several large crests that propagate in the flume and that undergo spilling upstream of the main focal position. This qualitative change in the waves in terms of their breaking behaviour with reducing bandwidth was also noted by [2]. For a fixed value of global steepness, a narrow-banded spectra

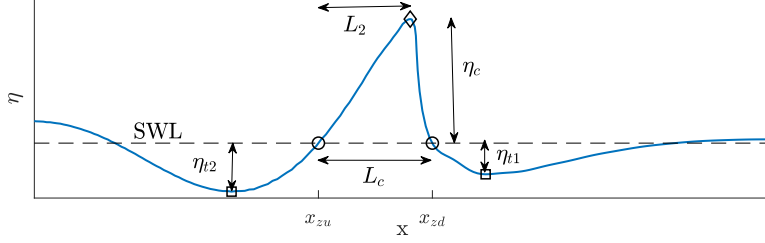


Figure 11: Schematic of local wave geometric parameters as defined in [11]

601 resulted in multiple breakers whereas a broad-banded spectra resulted in a
 602 clear single plunger. This explains the lack of cases included in the present
 603 data set for high Ak_c values for narrow banded spectra, observable in Figure
 604 10.

605 4. Pre- and Post-Breaking Wave Features

606 The use of the CCD camera to record the focused wave group allows
 607 the study of local geometric features. In the work of [11] and [42], the local
 608 features of the wave event at breaking were defined as shown in Figure 11.
 609 For the purpose of this section, the two zero crossing points adjacent to
 610 the breaking crest (black circles in Figure 11) and the troughs and crest
 611 elevations are determined using the video recordings at the point where the
 612 breaking crest exhibits a vertical front face. The front face is determined by
 613 visually inspecting the images.

614 4.1. Local crest length

615 Having established the accuracy of the calibrated camera and edge de-
 616 tection algorithm in Section 2.2, the wave breaking events were processed
 617 to determine the location of the right zero down-cross and left zero up-cross
 618 immediately adjacent to the crest as shown in Figure 11. For completeness,
 619 a set of results are presented in Figure 12, for wave cases with the same
 620 global steepness $Ak_c = 0.36$ and variable γ value. For each spectral band-
 621 width, the image captured just as the wave event achieves a vertical front
 622 face is illustrated and the agreement between the wave gauge and camera
 623 readings is presented. The edge detection results that identify the spatial
 624 free surface profile of the wave are overlaid on each image and the identified
 625 left and right zero crossings are also shown. As in [44], these locations of the
 626 down-cross x_{zd} and up-cross x_{zu} are used to compute $k_b = \pi/|x_{zu} - x_{zd}|$.
 627 Although the repeatability of the wave events has already been proven and

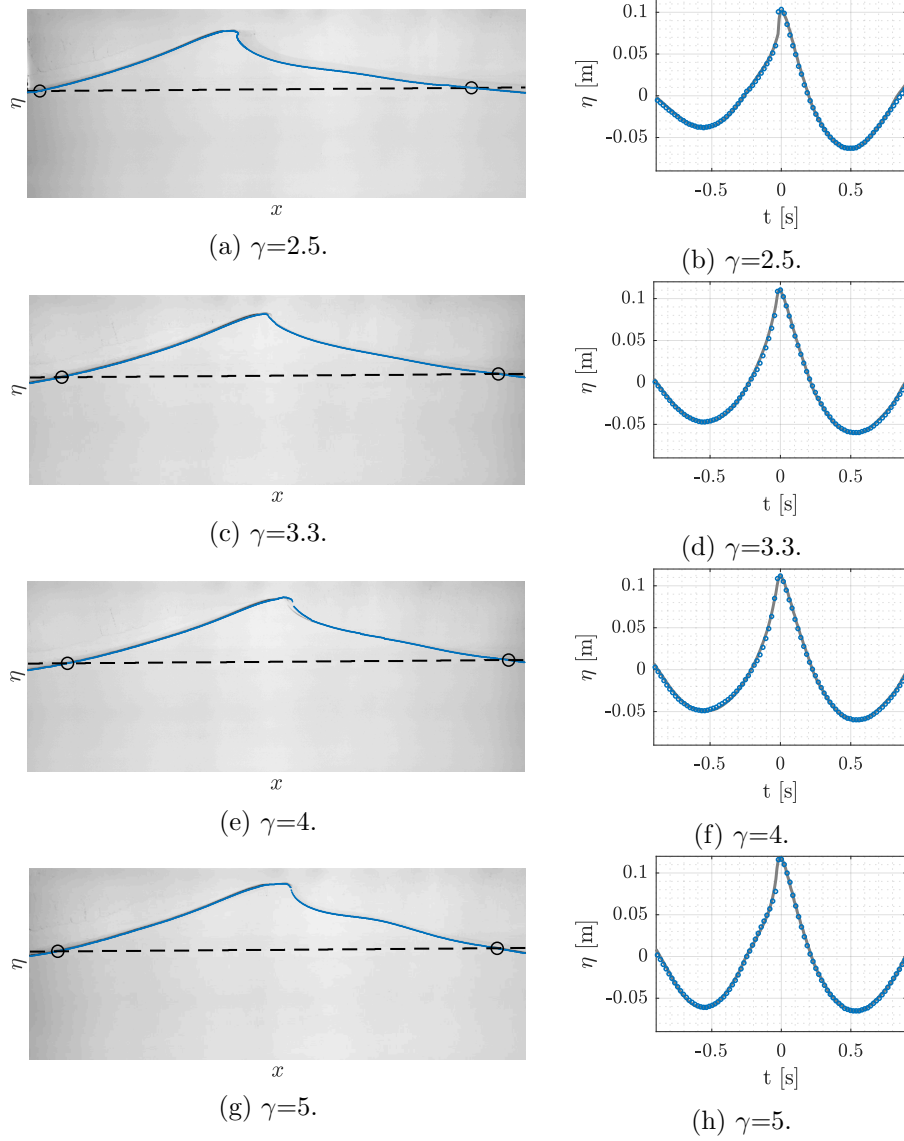


Figure 12: Results of processed images of breaking wave events with global steepness $Ak_c = 0.36$. Left hand panels show — edge detection algorithm results, — — SWL level and $\circ$ location of identified zero crossings. Right hand panels show free surface elevation $\eta(t)$ as obtained from $\circ\circ\circ$ camera and — wave gauge.

discussed, four repetitions of each event are used to obtain a characteristic k_b value. A typical coefficient of variation for the four repeats is 0.12%. The

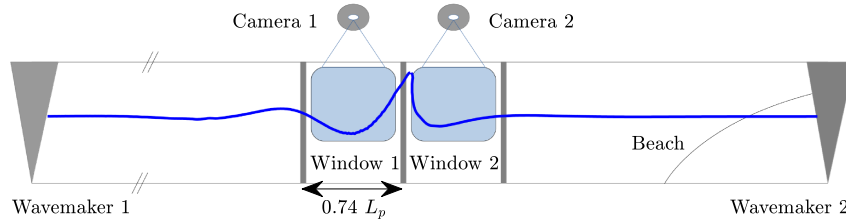


Figure 13: Graphical illustration of the two camera setup in the experimental facility. Drawing not to scale.

linear dispersion relationship is used to obtain ω_b for all wave cases. Using these definitions, [1] computed the breaking wave phase speed (based on linear dispersion) as $c_b = \omega_b/k_b$.

Analysing the right hand panels of Figure 12 where the free surface time-histories ($\eta(t)$) are plotted, it can be seen that as the bandwidth is reduced from Figure 12b to 12h (increase in γ), the asymmetry of the wave event reduces (the elevation of the two adjacent troughs become more similar). Furthermore, for the same global steepness, narrow-banded spectra generate larger crest elevations and a profile that tends more towards a regular wave. This further illustrates the facts stated in Section 3.5, where it became increasingly difficult to generate single breaking events for narrow-banded spectra within the present facility. It is well known that narrow-banded spectra tend to be subject to modulational instabilities which in turn generate recurring breaking events, as discussed in [47].

4.2. Local crest asymmetry

In the work of [11], the elevations of the crest as well as the two adjacent troughs were used to calculate a definition of the local steepness that takes into account the strong asymmetries observed in breaking events. The steepness was defined as:

$$S_b = k_b \frac{(2\eta_c + \eta_{t1} + \eta_{t2})}{4}, \quad (10)$$

where η_c , η_{t1} and η_{t2} are defined in Figure 11. The need to obtain the elevation of the troughs implies a fairly large viewing area of the camera (at least as wide as the trough-to-trough wavelength of the wave event). In the work of [44, 1, 11] experimental limitations resulted in obtaining the full spatial profile of the wave event by splitting the profile into multiple sub-sections that were then recorded by repositioning the camera. This

method obviously relies on the repeatability of the wave event and is a tedious process.

As noted in Section 2.1, due to the construction of the present wave flume, the maximum viewing width of the camera, free of obstructions, is approximately $0.74L_p$, where L_p is the wavelength corresponding to the peak spectral component. As such, it is not possible to obtain both crest elevation and troughs from a single viewing window. Hence, the approach adopted here is different from [11] in the sense that two identical synchronised cameras were used to capture images from two adjacent glass panels of the wave flume. A graphical illustration of the experimental setup using two synchronised cameras is shown in Figure 13. To achieve this, the location of the breaking event was shifted slightly forward so that the troughs of the wave event would fit in the two adjacent viewing windows.

The recorded images of the wave events at breaking initiation were employed to locate and compute the elevation of the left and right hand side troughs. An example is given in Figure 14. In order to locate the trough locations, the identified free surface elevation from the edge detection algorithm was converted into physical space and the minimum value subsequently taken.

All the geometric quantities determined in this section and Section 4.1 will be employed in future sections to study the relationship between pre- and post-breaking quantities as in [11].

4.3. Post-Breaking Wave Features

Post-breaking wave features refer to the time and length scales that are characteristic of the breaking process after wave breaking is initiated and active energy dissipation takes place, in the form of air entrainment and bubble plume generation. For example, the energy dissipation rate due to wave breaking can be estimated by assuming a constant rate that occurs over the entire duration of the breaking process. The numerical work of [48] showed that the energy dissipation rate for a given breaking event has a temporal variation and, as such, the constant rate offers an average estimate of the total energy dissipated.

In [49] and [50] active breaking is defined as the duration of the transfer of momentum from the surface waves to the fluid underneath. [51], using the experimental data of [4], showed that for a given wave group, active breaking is observed to last around one characteristic wave period T . [2] estimated the duration of breaking using a hydrophone and confirmed the findings of [4] regarding the duration of breaking. As noted by [52], the active breaking duration as measured by a hydrophone would encompass

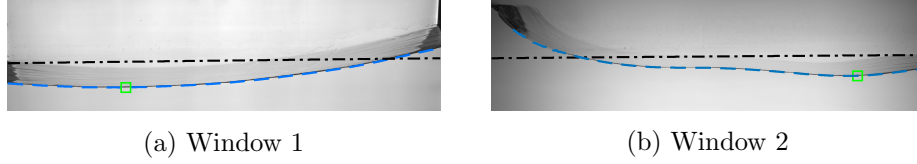


Figure 14: The location of □ left hand side (a) and right hand side (b) troughs as identified from — — free surface profile. In both figures · · · shows SWL. Example is given for a wave event with $\gamma=4$ and $A=107.5$ mm.

the acoustically active phase of wave breaking. This occurs as air bubbles are trapped by the overturning jet (in the case of plunging breakers) and these bubbles fragment inside the flow and are submerged. It is less clearly defined for spilling breakers. According to [52], this time period lasts around one second for laboratory breakers. Other studies such as [49] noted that the active wave breaking duration is approximated well by a characteristic wave period T_c taken from the central frequency of the spectrum.

This result was employed in the numerical studies of [42] and [43]. In [42] the breaking duration was estimated as $0.75T$. [43] employed an estimate of $1.2T$ and remarked at the difficulty in properly identifying the active breaking duration.

4.3.1. Active wave breaking duration t_{br}

Without relying on hydrophone measurements, [1, 11] estimated the breaking time scale t_{br} from the camera footage. This time scale was defined as the duration between the instant that the crest starts breaking (equivalent to the wave profile forming a vertical front) to when the disturbance to the water surface due to breaking is no longer significant (an example is given in Figure 3 of [1]). However, for large plunging breakers that form a clear jet, the equivalent hydrophone measurement would not include the time duration between the formation of the jet and the reconnection with the undisturbed free surface (as no significant sound is generated). Figure 4 of [51] compares the measurement taken by a hydrophone with a sequence of images illustrating the breaking process of an energetic spilling breaker. Hence, by employing the definition of the breaking time scale t_{br} used by [1], slightly higher breaking durations would result. This was also highlighted in the review paper of [53], which noted that the breaking times are greater in the work of [1] compared to the estimates of [2] (which relied on hydrophone measurements), but no explanation for the differences was offered.

For the present study, no hydrophone was available and only camera measurements are used. Consequently, as the present study aims to vali-

724 date and refine the eddy viscosity model of [11], an estimate of t_{br} will be
725 calculated in a consistent manner with [11]. As such, the start of t_{br} is the
726 instant that the crest develops a vertical face and the end of t_{br} is taken
727 when the advancing bore generated by the breaking event dies out and the
728 water surface is free of disturbances.

729 To this end, the start and end frames that would encompass the entire
730 breaking event would be noted. The duration of breaking is calculated using
731 the frame rate of the recorded images. A sequence of images recorded during
732 active breaking is shown in Figure 15. This shows an energetic spilling wave
733 event in which, after the crest develops a vertical face, the fluid slides down
734 the front face of the wave. Consequently, a turbulent bore is formed that
735 moves downstream. Hence, the start time is chosen as the instant that the
736 surface profile goes vertical and the end time is selected when the breaking
737 front (turbulent bore) dies out and is no longer seen to cause a disturbance
738 to the free surface profile (a similar end time was adopted in [48]).

739 An important point is that for larger wave events, active breaking takes
740 longer and is characterised by a larger length than can be recorded in a
741 single viewing window. The case shown in Figure 15 is an example of the
742 largest breaker for which clear visual confirmation of start and end frames for
743 breaking can be made with a single camera. In order to overcome this, the
744 two camera setup was once again employed, as shown in Figure 13. Hence,
745 breaking is initiated in the first window (by once again changing x_b) and
746 the end time clearly visible in the second window (Window 2 in Figure 13).
747 As the two cameras are triggered by the same signal and are synchronised,
748 a similar start and end frame approach can be adopted.

749 In order to check the measurements made with the two camera setup,
750 the cases for which accurate estimates could be made with only the single
751 camera setup were repeated; the measurements from both setups were in
752 good agreement. As noted in [11], this approach of visually selecting a start
753 and end frame based on recorded images is slightly subjective. Hence, to
754 eliminate bias, four repeats for each case were generated and an average
755 value was taken for t_{br} . Due to the highly repeatable nature of the wave
756 events, the most subjective choice for each run was the start and end frame.
757 Considering the current recording frame rate of 48.8 FPS, the coefficient of
758 variation between measurements made for the same case was approximately
759 5%.

760 As discussed earlier, this methodology of estimating the breaking du-
761 ration would potentially result in a slightly larger value compared to that
762 recorded by a hydrophone. For the present work, the choice of this break-
763 ing duration is motivated by several factors. First, as one of the aims of

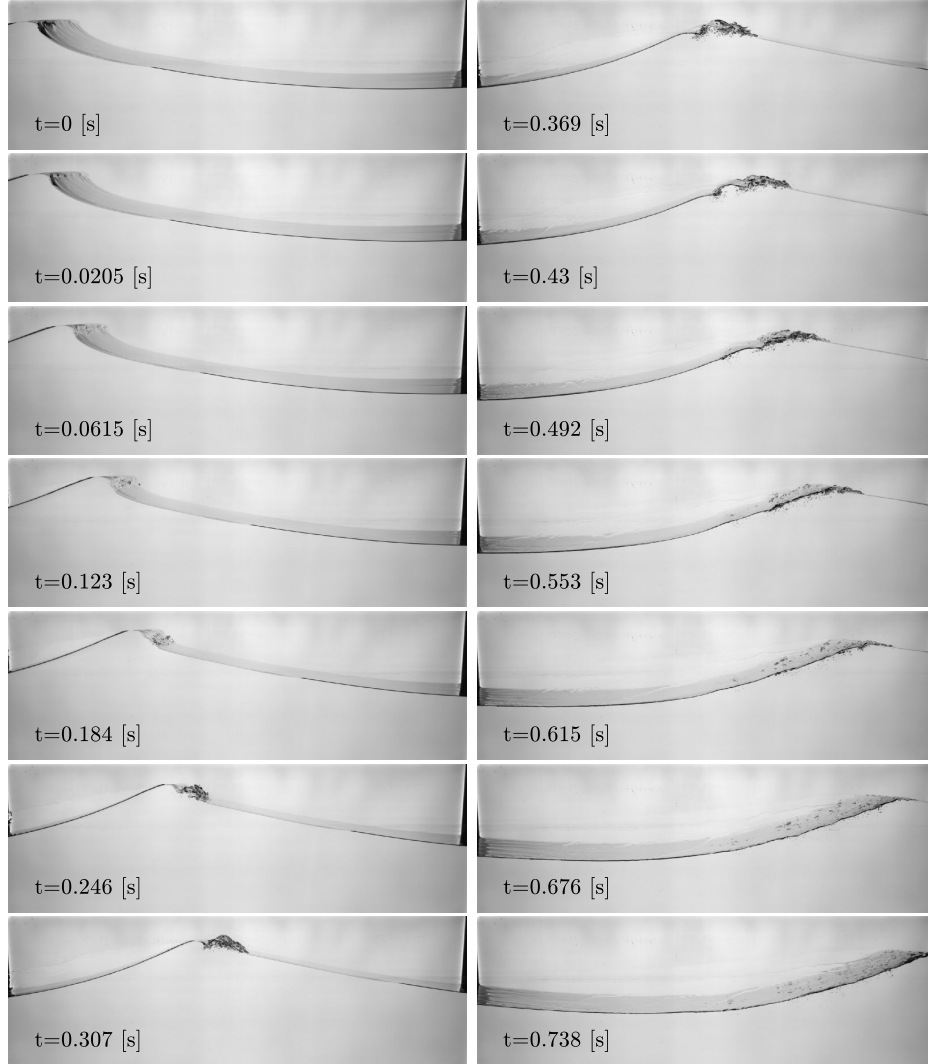


Figure 15: The frames recorded during breaking for an event with $A=87.5$ mm and $\gamma=3.3$. All frames shown were recorded in Window 1 by Camera 1 illustrated in Figure 13. The time recorded since breaking initiation is shown in the bottom left of each panel. These give an example of estimating the duration of breaking (t_{br}).

764 this work is to validate and improve the dissipation model of [11] for JON-
 765 SWAP spectra, a consistent methodology is obtained by using the present
 766 t_{br} estimates. Secondly, the parameterisation will be employed in numerical
 767 models based on a velocity potential formulation. Pseudo-spectral numerical
 768 methods that employ a velocity potential formulation cannot describe

769 a multi-valued free surface, and hence, to avoid numerical blow-up, the en-
 770 ergy dissipation model is employed just before a vertical crest is formed.
 771 Therefore, the parameterisation must be based on dissipation beginning at
 772 the vertical crest front. Finally, steep wave crests just prior to breaking and
 773 air entrainment might generate vorticity, which can contribute to energy
 774 dissipation [32].

775 The measured breaking time scales t_{br} for the present JONSWAP wave
 776 groups are plotted in Figure 16. Consistent with the findings of [2] and [1],
 777 the duration of the breaking event increases as the global steepness of the
 778 wave group increases (and more energy is pumped into the wave flume).
 779 Also, it can be seen that the duration of this time scale is the same order of
 780 magnitude as the breaking crest period ($T_b = 2\pi/\omega_b$).

781 While no dependence of t_{br} on the spectral bandwidth was reported in
 782 [1] or [42], the experimental work of [2] showed that for their constant steep-
 783 ness spectra there was a dependence of t_{br} on bandwidth. More specifically,
 784 Figure 10 of [2] illustrates that narrow-banded spectra generally result in a
 785 longer t_{br} compared to broad-banded spectra. In addition, [2] show that t_{br}
 786 tends to plateau for high global steepnesses. By analysing Figure 16 of the
 787 present work, it can be seen that the breaking duration for the broad-banded
 788 spectra $\gamma=2.5$ tends to plateau for a global steepness of $Ak_c > 0.4$. To a
 789 lesser extent, this behaviour is also noticed for $\gamma=3.3$, but no plateauing is
 790 observed for the narrow-banded spectra with $\gamma=5$. However, it must be once
 791 again noted that after a certain steepness, multiple spillers were observed
 792 within the $\gamma=5$ wave group and no further increase in steepness was possible
 793 due to the constraint of only examining single breaking events. The γ pa-
 794 rameter does not change the spectral bandwidth of the JONSWAP spectra
 795 as dramatically as, for example, [2] have altered the spectral bandwidth of
 796 their constant steepness spectra. Hence, for the JONSWAP spectra used in
 797 the present study, the bandwidth effect on t_{br} is reduced.

798 4.3.2. *Breaking horizontal length scale l_{br}*

799 Using the same approach as in the case of t_{br} , the characteristic hori-
 800 zontal length scale of the breaking event can be obtained. Essentially, l_{br}
 801 is the horizontal distance that the breaking front travels during t_{br} and is
 802 obtained from the undistorted images of the breaking event. The starting
 803 point is the position where the crest forms a vertical front and the end point
 804 is the leading edge of the turbulent bore that is generated by breaking. This
 805 methodology of computing l_{br} is similar to that of [1].

806 Similar to the issues encountered when estimating t_{br} for the larger
 807 breaking events, the two-camera setup was employed to obtain the length

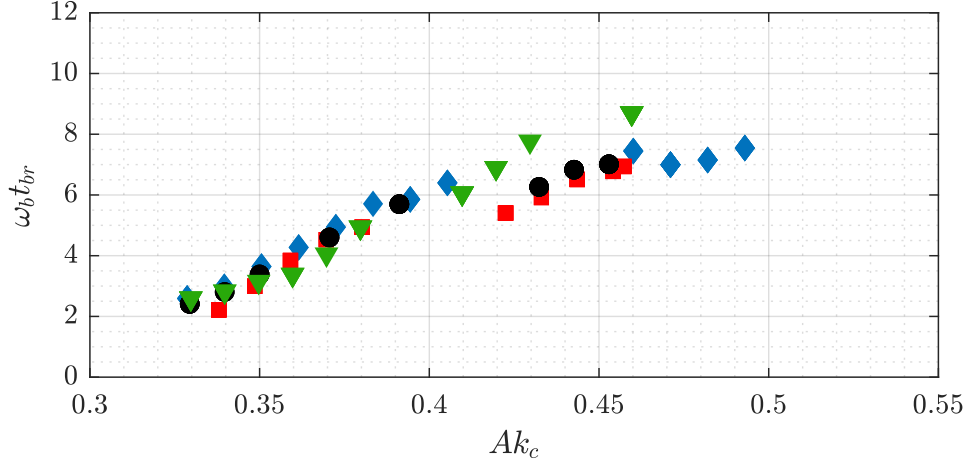


Figure 16: Active breaking time scale t_{br} normalised by the breaking crest frequency ω_b as calculated in Section 4.1 for wave cases with $\diamond \gamma=2.5$, $\square \gamma=3.3$, $\bullet \gamma=4$ and $\nabla \gamma=5$.

of breaking. This was again due to the limited width of a single viewing window which did not cover the entire extent of the breaking process. By using two adjacent windows, a method of merging the measurements from the two cameras was needed. This was achieved by using an object of known length (e.g. a ruler) that spanned both neighbouring glass windows. In the first window, the starting location of the object was determined, whereas in the second window, its end point was determined. This then allowed l_{br} to be measured using the start frame (Window 1) and end frame (Window 2).

The resulting horizontal breaking length scale for all four γ values employed in the present paper are plotted against the corresponding breaking duration in Figure 17a. Both breaking scales are normalised by the local breaking crest frequency ω_b and wavenumber k_b . As noted by [1], t_{br} is the time taken for the breaking wave front to travel the distance l_{br} and hence they describe the horizontal broken wave speed $u_{br} = l_{br}/t_{br}$. Generating waves based on energy focusing with a constant steepness spectrum, [1] determined the relationship $u_{br} = 0.836c_b$. In a latter study, [11] added further cases generated using the dispersive focusing technique (with constant steepness spectra) as well as wave cases with breaking triggered through modulational instability and found the relationship $u_{br} = c_b$; however they stated that this new observation was not well understood.

For the present experimental data it was found that $u_{br} = 0.854c_b$, which is closer to the value reported in [1]. The fit to the data is shown in Figure 17a as a thick dashed line (with an R^2 of 0.97), where the fit is constrained

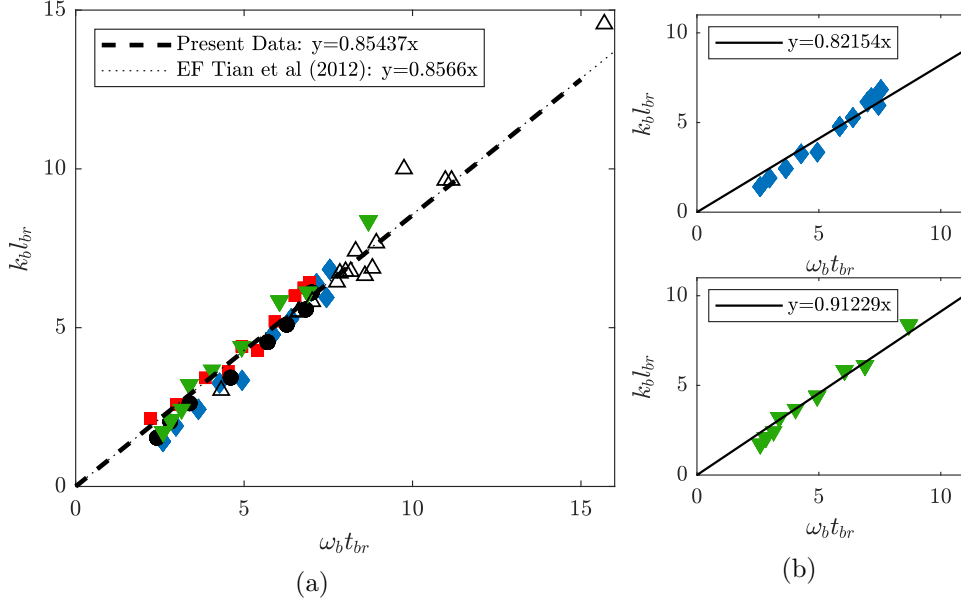


Figure 17: Relationship between active breaking time scale t_{br} (normalised by ω_b - characteristic breaking crest frequency) and breaking horizontal length scale l_{br} (normalised by k_b - characteristic breaking crest wavenumber) for (a) present dataset ($\diamond \gamma=2.5$, $\blacksquare \gamma=3.3$, $\bullet \gamma=4$ and $\blacktriangledown \gamma=5$) together with $\triangle$ energy focusing (EF) cases from [11] and (b) present dataset for spectra with $\gamma=2.5$ (top right) and $\gamma=5$ (bottom right). For all panels, the best fit line equations are given in the corresponding legend. Note that in panel (a) the lines of the two fitted equations almost overlap each other.

through the origin, as in [1, 11]. In an attempt to explain the differences in u_{br}/c_b ratios from [11] and the present study, a first step was to separate the data of [11] generated based on energy focusing from the modulational instability cases. Using a plot digitizer, the results extracted from [11] that consider just the energy focusing cases are plotted in Figure 17a as black open triangles. Based on this [11] subset, the relationship obtained is $u_{br} = 0.857c_b$ and is shown using a dotted line in Figure 17a. This result agrees very well with the present data for the JONSWAP spectra. Also note in this figure the slightly reduced breaking duration for the present JONSWAP results compared to the [1] estimates. It must be noted that [13] noted the need to slightly reduce the predicted breaking duration obtained from the [11] parameterisation, to improve the agreement between experimental and numerical predictions.

Acknowledging the above, it would seem that the inclusion of the data generated using modulational instability resulted in the one-to-one relation-

ship between u_{br} and c_b reported in [11]. [54] states that a clear distinguishing factor between the two wave generation mechanisms (energy focusing and side band growth due to modulational instability) is the spectral bandwidth. While the dispersive focusing technique employs a large number of components (269 for the present JONSWAP spectra), modulational instability is typically triggered when a single carrier wave interacts nonlinearly with perturbing waves, being a bandwidth limited effect.

The influence of the bandwidth on the relationship between the breaking scales and the breaking crest is explored in Figure 17b. The top panel focuses on the data characterised by the broadest spectra employed in the present study ($\gamma=2.5$) and shows that $u_{br} = 0.822c_b$. The bottom panels focuses on the data with the narrowest spectra ($\gamma=5$) and estimates $u_{br} = 0.912c_b$. Comparing the two panels, a reduction in the slope of the relationship is noticeable as the spectrum narrows. This small dependence of bandwidth helps explain the differences noted by [11]. The spectral shape might also affect this relationship, although this question is not pursued further in the present study.

4.3.3. *Breaking vertical length scale h_{br}*

In the work of [2] the local wave height for a breaking wave was used as a vertical length scale in the development of their energy dissipation rate model. Using video images of the breaking event, their length scale was defined as the vertical distance travelled by the toe of the breaking event until it impacts the clearly defined free surface underneath the breaking jet. It was shown that the trajectory of the breaking toe follows that of a free-falling object under gravity with an initial velocity.

The parametrisation of an eddy viscosity model in [1] employed this vertical length scale termed h_{br} , as it was shown in [2] that the energy dissipated by breaking scales with the falling crest height. It was shown qualitatively that a large falling breaking crest generates a large breaking penetration depth. [1] gave no clear definition of how h_{br} was obtained in their study. [11] obtained h_{br} from the video footage at the point where the breaking jet reconnects with the water surface beneath. While this can be clearly identified for a plunging wave event, it is less evident for spillers where no clear breaking jet is formed.

An example of the method employed to compute h_{br} for the present wave groups is shown in Figure 18. In Figure 18a a clear plunger is shown, with the falling crest height defined as the vertical extent of the overturning jet. Similarly, Figure 18b shows how h_{br} was defined and computed for an energetic spilling breaker. For clarity, only Figure 18b shows the detected

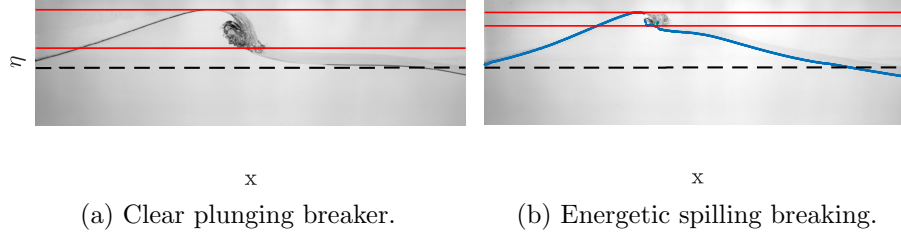


Figure 18: Definition of the falling crest height h_{br} as employed in the present study. In both panels: — — SWL and — — extent of crest height. In panel (b) — — shows a typical detected free surface profile $\eta(x)$ that was used to compute h_{br} .

free surface η that was used to calculate the vertical extent of breaking. While the top point can be clearly identified, the bottom point was manually picked for each individual breaking case. As in the case of the previous two breaking scales, four repetitions were used to determine an average value. In addition to this, for the smallest of spilling breakers a consistent h_{br} proved difficult to obtain and they were therefore eliminated from all results presented henceforth (usually corresponding to the first observed breaking wave in each γ set).

5. Eddy Viscosity Model Refinement

This section aims to validate the eddy viscosity model proposed by [1, 11] for use with wind-waves generated using JONSWAP ocean spectra.

5.1. Proportionality constant θ

The value of the constant eddy viscosity ν_{eddy} that characterises a given wave breaking event is estimated as:

$$\nu_{eddy} = \theta h_{br} l_{br} / t_{br}, \quad (11)$$

where h_{br} , l_{br} and t_{br} are the length and time scales of the breaking event as determined in Section 4.3 and θ is a constant of proportionality. For the wave groups in [1, 11] the proportionality constant was determined to be 0.02. This was obtained iteratively by matching the experimentally computed energy dissipation with the predictions obtained from a spectral numerical model. Latter on [55] employed a value of 0.1 in their numerical model in a study that compared numerical and experimental predictions regarding the spectral evolution due to wave breaking. The recent work of [14] employed a value of $\theta=0.02$ and obtained generally favourable results. The value of

906 the constant θ applicable to JONSWAP wave is estimated in this section
 907 for the present wave cases. Note that the values of ν_{eddy} are in the order of
 908 $1 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$.

909 Alternatively to Equation 11, [55] estimated the equivalent eddy viscosity
 910 for a given wave event as:

$$\nu_{eddy} = \beta b_b^{11/15} c_b / k_b, \quad (12)$$

911 where β is another proportionality coefficient, c_b and k_b are the breaking
 912 wave phase speed and wavenumber and b_b is the breaking wave strength.
 913 As shown in [2] the dimensionless breaking strength parameter is a propor-
 914 tionality constant determined from the energy dissipation rate of a given
 915 event. The energy dissipation rate ϵ is defined by:

$$\epsilon = \frac{b_b \rho_w C^5}{g}, \quad (13)$$

916 where C is a characteristic speed. In the present study, the energy dis-
 917 sipation rate is obtained from the experimental data based on the energy
 918 dissipated during a specific event (shown in Figure 10) and the duration of
 919 the breaking event t_{br} (shown in Figure 16) as $\epsilon = \Delta E / t_{br}$. [1] computed b_b
 920 by employing the breaking wave phase speed c_b in Equation (13):

$$b_b = \frac{(\Delta E / t_{br}) g}{\rho_w c_b^5}. \quad (14)$$

921 The results obtained for the present dataset are shown in Figure 19,
 922 where b_b values are plotted against measures of wave steepness. The value
 923 of the breaking strength parameter has seen considerable variations across
 924 literature, due to many approaches adopted in its computation. As a first
 925 step, Figure 19a plots the present b_b values against a measure of global steep-
 926 ness (determined from the input spectral information) $\sum a_n k_n$. Compari-
 927 son is made with two numerical datasets which adopted the same definition
 928 of c_b in computing b_b , however, both studies employed constant steepness
 929 spectra. From the figure, it can be observed that the breaking strength
 930 parameter is well correlated with the global wave steepness, showing an
 931 increase strength as more energy is present in a steeper wave event. A
 932 dependence of breaking strength on global steepness is a result that can
 933 be applied in wave-averaged models and hence the best fit through the
 934 present JONSWAP data is $b_b = 0.76(S - 0.2)^{5/2}$ (with an R^2 of 0.96),
 935 where $S = \sum a_n k_n$ and the exponent 5/2 agrees with the inertial scaling
 936 determined by [2]. The present values of b_b are consistent with [1] showing

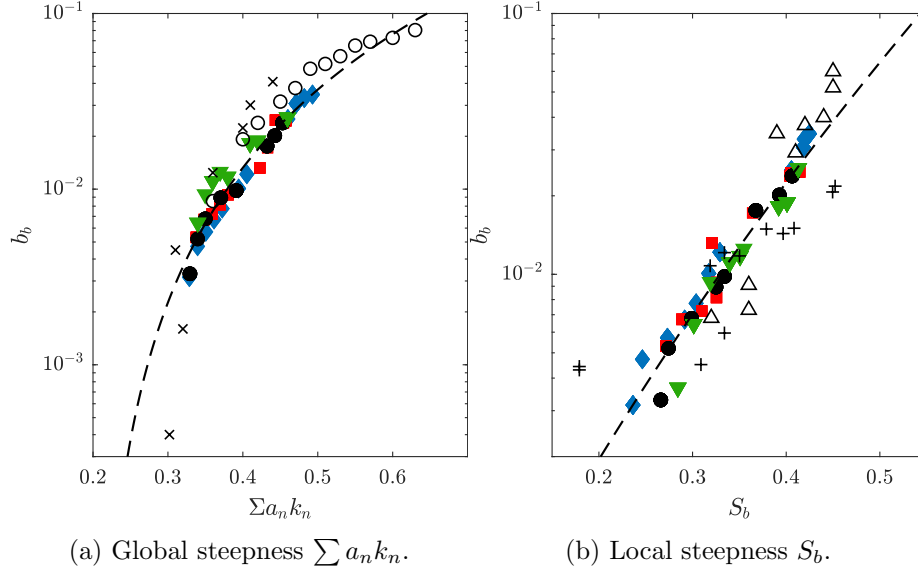


Figure 19: Breaking strength parameter b_b as a function of (a) global steepness $\sum a_n k_n$ and (b) local steepness S_b defined in Equation (10). In both panels the present JONSWAP data is shown with $\diamond \gamma=2.5$, $\blacksquare \gamma=3.3$, $\bullet \gamma=4$ and $\blacktriangledown \gamma=5$ and $- -$ shows best fit to JONSWAP data. In panel (a) comparison is made with data from $\circ$ [43] and $\times$ unidirectional dispersive focusing cases of [56]. In panel (b) comparison is made with $+$ [1] and $\triangle$ [42].

only an order of magnitude difference between the weakest and strongest breaker ($3.1 \times 10^{-3} \leq b_b \leq 34.6 \times 10^{-3}$). The most recent results of [56] report a range of $0.4 \times 10^{-3} \leq b_b \leq 40.9 \times 10^{-3}$ for the wave breaking events generated based on energy focusing with constant steepness spectra, which covers a range that is an order of magnitude larger than the JONSWAP results. In explaining the difference (besides the underlying spectral shape), it must be noted that [56] estimated the breaking duration as $t_{br} = 0.75T$, where T is a characteristic wave period. Breaking duration expressed as a fraction of T tends to increase with steepness (see Figure 16 or [2]).

In comparing the present dataset with other reported values, only breaking generated by energy focusing was considered. As noted by [54], wave breaking due to energy focusing generally results in higher values of the breaking strength parameter compared to modulated wave groups that specifically target the development of modulational instability, such as the ones in [47]. This is an important point to note when trying to understand the scatter of values reported in the literature for the breaking strength parameter [2, 57, 54].

Figure 19b adopts the local wave steepness S_b in presenting the break-

ing strength results. A good agreement is obtained with both results from [1] and [42] (for which values of S_b have been reported). The range of reported b_b values is also in good agreement. It must be noted again that [42] employed the same method of estimating breaking duration as [56]. Both studies compared here employed constant steepness spectra, further explaining possible differences.

Having established the range of b_b values for the present dataset, an estimate of the eddy viscosity can be obtained by employing Equation 12, where β was estimated as 0.13 in [1]. The use of this equation allows estimates of ν_{eddy} to be obtained directly from experimental measurements. Note that in [1] very similar estimates for ν_{eddy} were obtained by employing either Equation (11) or Equation (12). As such, for the present cases, this one-to-one relationship between the two methods of estimating ν_{eddy} will highlight if a correction to the constant is needed.

Hence, by equating (11) to (12) one can set the constant Λ as:

$$\Lambda \beta b_b^{11/15} c_b / k_b = \theta h_{br} l_{br} / t_{br}, \quad (15)$$

where $\beta=0.13$ and $\theta=0.02$. The normalised estimates from the two equations are plotted in Figure 20. As can be seen, the data for the JONSWAP spectra do not lie on the unity slope line ($\Lambda = 1$) but rather on $\Lambda=0.546$. Hence, a correction to the estimate of θ from [1] is needed to best describe the present dataset, resulting in the estimate of ν_{eddy} as:

$$\nu_{eddy} = 0.037 \frac{h_{br} l_{br}}{t_{br}}. \quad (16)$$

The resulting estimates for the present wave cases are obtained using Equation (16). The value of the eddy viscosity estimation increases with the slope of the wave event. The decision to correct Equation (11) and not (12) is driven by the desire to apply this result in potential flow solvers, where estimates of t_{br} , l_{br} and h_{br} can be obtained from the spatial geometry of individual wave events.

As noted by [1], applying Equation (16) within a numerical model requires the parameters of the breaking event to be known before breaking occurs. In this sense, it is not a predictive formula. To account for this and to facilitate the prediction of those parameters before breaking actually occurs, a relationship between the breaking parameters used in Equation (16) and the local geometry of the wave event at breaking initiation is needed. This is dealt with in the next section.

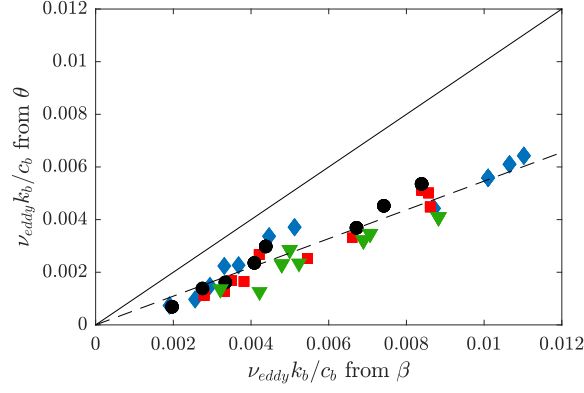


Figure 20: Normalised estimates of ν_{eddy} from Equation (12) (x-axis) against estimates from Equation (11)(y-axis) for wave cases with $\diamond \gamma=2.5$, $\square \gamma=3.3$, $\bullet \gamma=4$ and $\blacktriangledown \gamma=5$. According to [1] a one-to-one relationship (—) exists between Equation (12) and Equation (11). The slope (—) of the present data is 0.546.

5.2. Relating pre- and post-breaking characteristics

While the work of [1] determined the proportionality constant needed to estimate the eddy viscosity value for a given wave event, the model developed depends on time and length scales that are not available a priori to the breaking event. Hence, [11] attempted to relate certain geometric features of the wave event at breaking inception with the characteristic scales of the turbulent breaking process (i.e t_{br} , l_{br} and h_{br}). By establishing a relationship between these two sets of features, the breaking length scales needed to estimate ν_{eddy} can be inferred from the geometry of the wave event prior to breaking. The spatial wave profile is available (or can be obtained) in all phase-resolving numerical models employed in wind-wave modelling.

Many previous studies have employed a characteristic global steepness for a given wave event, which is computed based on spectral information. While studies such as [1] have shown (linear) correlations between local and global estimates of steepness, this relationship is highly dependent on the phasing of the underlying spectral components. In the context of a realistic random sea-state no consistent phase correlation is expected for a given large breaking wave event and hence parameters based on spectral information are not considered satisfactory. This prompted the decision to only use local geometric features obtained from the camera measurements to relate pre- and post-breaking wave characteristics. To keep consistent with the work of [11] the steepness S_b defined in Equation (10) will be employed.

The relationship between the local crest steepness S_b and the normalised

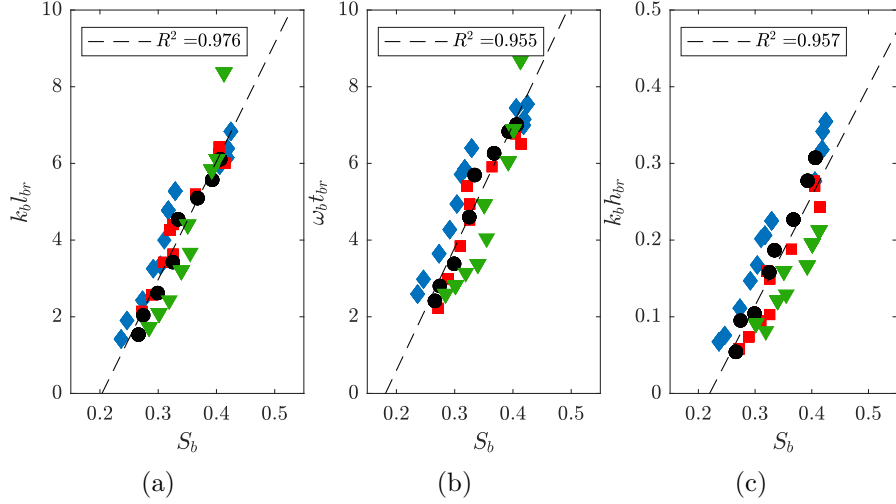


Figure 21: Relationship between local steepness S_b and normalised (a) breaking length scale $k_b l_{br}$ and (b) breaking time scale $\omega_b t_{br}$ and (c) normalised falling crest height $k_b h_{br}$ for wave cases with $\diamond \gamma=2.5$, $\square \gamma=3.3$, $\bullet \gamma=4$ and $\blacktriangledown \gamma=5$. In both panels $- -$ shows the linear best-fit to data with the R^2 value indicated in the panel legend.

1011 breaking horizontal length scale is shown in Figure 21a. Similarly, the rela-
 1012 tionship between the local crest steepness S_b and the normalised breaking
 1013 time scale is illustrated in Figure 21b. For both quantities, there is a clear
 1014 increase with the steepening of the wave event. The best-fit lines that de-
 1015 scribe the two relationships are:

$$l_{br} = (30.8S_b - 6.3)/k_b \quad (17)$$

1016 and

$$t_{br} = (32.0S_b - 5.8)/\omega_b. \quad (18)$$

1017 As can be observed from Figure 21 fairly accurate predictions of the
 1018 breaking time and horizontal length scales can be obtained by evaluating
 1019 the local steepness S_b for a given wave event.

1020 In the work of [11] considerable scatter was present when relating the
 1021 falling crest height h_{br} to the local crest steepness S_b . Exploiting the stronger
 1022 asymmetry of a plunging breaker compared to a spilling event, [11] related
 1023 the normalised falling crest height $k_b h_{br}$ to the local crest asymmetry $R_b =$
 1024 L_2/L_c . In the present study it was found that h_{br} is better predicted by S_b

1025 (as plotted in Figure 21c) rather than R_b , with the best fit being:

$$h_{br} = (1.43S_b - 0.31)/k_b. \quad (19)$$

1026 The comparison between the model developed by [11] and the present fits
 1027 is made in Table 3. Overall, the newly obtained empirical fits show reduced
 1028 scatter (especially for the falling crest height estimates h_{br} , where the fit
 1029 presented in [11] has an R^2 of 0.51 compared to 0.957 in the present study)
 1030 and are directly applicable to large wave events generated in JONSWAP sea-
 1031 states. In addition, the newly obtained slope coefficients for the relationship
 1032 between t_{br} , l_{br} and S_b are consistent with the findings of Figure 17 that
 1033 show a ratio of less than unity between u_{br} and c_b .

1034 6. Conclusions

1035 A new set of laboratory generated focused wave events was described and
 1036 studied in this paper in order to validate and improve upon existing models
 1037 of wave breaking energy dissipation. The model needs to be formulated
 1038 such that it can be readily employed in numerical wave models based on a
 1039 velocity potential formulation such as the one developed by [16]. Owing to
 1040 the success reported in studies such as [14] and [15], the model proposed in
 1041 [1, 11] was selected. The model was developed on an empirical basis, hence
 1042 its applicability to wind waves characterised by JONSWAP ocean spectra
 1043 was studied in the present work.

1044 To this end, NewWave type events with four different spectral band-
 1045 widths were generated in a wave flume in order to quantify the amount of
 1046 energy dissipated by single breaking events. The present study showed that
 1047 for JONSWAP spectra up to 20% of the initial wave packet energy is lost
 1048 during active breaking.

1049 It was shown that broad-banded spectra generate more asymmetric waves
 1050 that break at lower elevations due to increased energy in the high-frequencies.

Table 3: Comparison between fitted coefficients of [11] model and present results for JONSWAP spectra.

Equation	Present data	Tian et al. (2012) [11]
$l_{br}k_b = aS_b + b$	$a = 30.8, b = -6.3$	$a = 24.3, b = -1.5$
$t_{br}\omega_b = aS_b + b$	$a = 32.0, b = -5.8$	$a = 18.4, b = 1.4$
$h_{br}k_b = aS_b + b$	$a = 1.43, b = -0.31$	—

1051 Estimates of the time and length scales characteristic of the breaking wave
1052 front obtained from video footage were compared to characteristic wave
1053 properties before breaking. The relationship between the horizontal broken
1054 wave speed and local wave phase speed was shown to have a weak
1055 dependence on spectral bandwidth which might explain differences noted in
1056 previous studies.

1057 The reported values of the breaking strength parameter b_b have been
1058 shown to vary over a single order of magnitude for JONSWAP waves when
1059 local phase speeds and breaking durations are employed in calculations.
1060 Comparisons were made with studies based on different spectral shapes due
1061 to lack of existing JONSWAP spectra studies. Previously reported values
1062 from work using a consistent methodology to the present one showed good
1063 agreement with the JONSWAP data.

1064 The proportionality constant needed to determine the eddy viscosity
1065 term for a given wave event was updated by almost a factor of two from
1066 0.02 to 0.037 to correlate well with present experimental data, resulting in
1067 Equation (16). The empirical relationships between local wave geometry and
1068 post-breaking characteristics needed to use the [1, 11] model were updated
1069 based on the present data, which showed reduced scatter compared to initial
1070 proposals. The relationships are given in Table 3.

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