CONTRIBUTIONS TO ELEMENTARY PARTICLE PHYSICS
(Mass Scales & Polarisation Effects
in Grand Unified Theories)

by

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بسم الله الرحمن الرحيم

To my parents

TABLE OF CONTENTS

		Page
ABSTRACT		i
PREFACE		iii
CHAPTER I	INTRODUCTION	1
CHAPTER II	ELECTROWEAK INTERACTIONS	11
2.1	Introduction	11
2.2	The Salam - Weinberg theory	14
2.3	The physical parameters in the standard model	21
2.4	Extended electroweak theories	24
2.5	The SU(2) x U(1) x G models	26
2.6	Left-right symmetric theories	36
CHAPTER III	MULTI - NEUTRAL BOSON PROCESSES	45
3.1	Motivation	45
3.2	Notation and definitions	47
3.3	Differential cross section	50
3.4	Light lepton pair production	62
3.5	Final state polarisation effects	70
3.6	Deep-inelastic polarised e - N scattering	73
3.7	Summary	78

		Page
CHAPTER IV	GRAND UNIFIED THEORIES	81
4.1	Colour change	81
4.2	The basic idea of grand unification	84
4.3	The SU(5) model	87
CHAPTER V	RENORMALIZATION EFFECTS IN GRAND UNIFIED THEORIES	99
5.1	Motivation	99
5.2	Effective couplings	101
5.3	Effective couplings in grand unified theories	103
5.4	High order corrections	109
5.5	Conclusions	127
CHAPTER VI	INTERMEDIATE MASS SCALES	135
6.1	Motivation	135
6.2	The SU(16) model	137
6.3	$\sin^2\theta$ and effective couplings	139
6.4	Results	146
APPENDIX A		150
APPENDIX B		153
APPENDIX C		155
FOOTNOTES		157
REFERENCES		158

PREFACE

The work presented in this thesis was carried out in the Department of Theoretical Physics, Imperial College, London, between October 1978 and March 1981, under the supervision of Professor Abdus Salam, to whom I am deeply indebted for the guidance and encouragement he so generously provided throughout the period.

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ABSTRACT

This thesis is divided into two parts. In the first, some aspects of electroweak unified gauge theories are studied with particular view to the models which predict more than one neutral gauge vector boson. After a brief review of the "standard" model and "extended" electroweak models, attention is first focused on reactions in which many neutral gauge bosons are involved. A model independent analysis is then presented for multi-neutral boson-exchange processes. In this context, the effects of weak neutral current(s) on colliding beams of electrons and positrons are studied. Particular attention is paid to weak-electromagnetic interference effects which in turn play a significant role in distinguishing between different electroweak gauge models. This becomes clear when the results are applied to some electroweak gauge models. It is also shown that, from the experimental point of view polarised beams are especially useful for the processes in question.

Finally, in this part the asymmetry in the scattering of longitudinally polarised electrons by nucleons is considered in the context of electroweak theories that suggest more than one neutral gauge boson.

In part II, we first review the embedding of standard electroweak and strong interaction group in a larger gauge group in the framework of the grand unifying hypothesis. The renormalization effects up to the two-loop level are taken into account for evaluating the electroweak mixing angle and the grand unifying mass scales of some grand unified theories. The intermediate stages allowed in some grand unified theories are studied with a particular emphasis on the models in which the fermionic degrees of freedom are maximally gauged. It is then shown that apart from the already proposed methods for the determination of these intermediate stages, an accurate experimental value of the electroweak mixing angle also plays

a crucial role in determining the possible existence of such intermediate stages.

CHAPTER I. INTRODUCTION

Quantum gauge field theory seems to provide the appropriate framework for a dynamical theory of elementary particle interactions. The local phase invariance of the electrodynamics Lagrangian allows the construction of a renormilizable quantum field theory that agrees impressively well with experiment. This phase symmetry is referred to as a $U(1)_{EM}$ local symmetry, and it determines several significant features of elctromagnetics:

- a conserved electric charge exists,
- the photon is itself electrically neutral,
- charged particles do <u>not</u> change their identity when they emit or absorb a photon. That is, an electron that radiates a photon would <u>not</u> be changed into something else, it still remains an electron.

Local phase invariance was generalised to non-Abelian gauge groups (Yang - Mills - Shaw) /1,2/ that involve several generators T_a or charges in analogy with electric charge. Associated with each generator is a vector boson that couples to a current in a fashion similar to the photon-electromagnetic current coupling. In Yang-Mills theories, however, the gauge bosons may carry the charges and particles may change their identities when emitting or absorbing these gauge particles. For example, if an electron were to emit a charged boson, it could be transformed into a neutrino. The important result is that the weak and probably the strong interactions are described by Yang-Mills theories.

At first sight Yang-Mills theories appear to have the feature that all vector bosons are massless since explicit vector boson mass terms in the Lagrangian do not have the local symmetry; this is in analogy to electromagnetics, where the photon is massless because of

the local phase invariance of the Lagrangian and the electrical neutrality of the state with no particle (the vacuum). The implication is that the forces should be long range, just as the electric field around a static charge falls off as $1/r^2$. However, the fact that weak and strong interactions are short ranged would seem to obviate the physical relevance of Yang-Mills theories. It was then realized that a local symmetry of the Lagrangian does not have to be a symmetry of the vacuum. If some charge T_a is spread out throughout empty space (i.e., if the vacuum has a nonzero average value of some charge T_a), the vector boson will acquire an effective mass from its interaction with the vacuum charge. This phenomenon is called "spontaneous symmetry breaking." Actually, the vacuum charge does not really "break" the symmetry, but merely "hides" it, in the sense that some predictions differ from those of the case where the average vacuum charge is zero /3-7/.

The behaviour of electromagnetic radiation in a plasma provides a physical model of spontaneous symmetry breaking /8/. An electric charge is the source of the electromagnetic field, and each electric charge that is in motion can be described by the field "attached" to it. The field associated with a charge moving in empty space is that of the massless photon. However, if the electric charge is then inserted into plasma, then the electrons in the plasma rearrange themselves so as to screen the field of the electric charge. The field falls off more rapidly than it did in free space; it just corresponds to that of a massive photon. The photon in empty space has two spin degrees of freedom, whereas a massive photon has three degrees of freedom; the third degree corresponds to the plasma oscillation. Nevertheless, the Lagrangian of electromagnetics still has the local symmetry.

The vacuum charge associated with a spontaneously broken local symmetry is not observable. However, just as a photon did in plasma, the vector boson coupled to the current acquires a mass. Spontaneously

broken local symmetries imply massive vector bosons. A massless vector boson has two spin degrees of freedom, while a massive one has three; hence, one degree of freedom has to be taken from somewhere. If this third degree of freedom is obtained by removing from the theory a scalar field that appears in the Lagrangian (where the vector boson appear massless), then the spontaneous symmetry breaking is called the "Higgs-Kibble mechanism" /6,7,9/.

The vacuum can, in principle, carry an arbitrarily large average charge associated with a symmetry that is violated; consequently, the vector boson can have an arbitrary large mass (except that in present theories the mass should be below the range where quantum gravity effects are important). In a spontaneously broken gauge theory, the remains of the local symmetry are the short-range interactions of the massive vector bosons with the conserved currents. Very large boson masses imply very short-range interactions. This is just the kind of framework that is needed to describe the weak interactions.

By extending the local gauge invariance beyond the U(1) of electromagnetics, it then became possible to construct sensible models of weak and elctromagnetic interactions in which the weak bosons acquire large masses (of order 10^2 Gev) through the Higgs-Kibble mechanism (Weinberg, 1967; Salam, 1968) /10/. The experimental existence of the weak charged and neutral currents along with the electromagnetic current implies a local symmetry group at least as large as the SU(2) x U(1) gauge group (Glashow, 1959; Salam-Ward, 1964) /11/.

The formulation of Yang-Mills theory of the strong interactions is realized once the fundamental role of quarks and gluons is recognised. /F1/. The present candidate strong interaction theory, quantum chromodynamics /12/ (QCD) is conjectured to give an explanation of quarks and gluons. In this context the strong interaction gauge group, the colour group, must be at least as large as $SU(3)_C$. In QCD the $SU(3)_C$ is <u>not</u>

broken spontaneously and therefore the associated gauge bosons, the gluons, are massless. Unlike electromagnetics, the coupling of chromodynamics decreases as momenta increase. Correspondingly, quarks get closer and freer at shorter distances therefore the theory is "asymptotically free". The implication is that at large momenta the strong coupling is <u>not</u> really strong and it may be comparable with other interactions such as electroweak interactions. However, at large distances the interaction becomes so strong that quarks are all confined to construct hadrons /F1/.

Combining the two ideas of electroweak and strong interactions leads to a unified Yang-Mills theory of electroweak and strong interactions which must have a local gauge symmetry at least as large as $SU(3)_C \times [SU(2) \times U(1)]_{EW}$.

So far we have discussed the known interactions, but we have not mentioned the fundamental particles that make up the currents. A significant contribution to the currents comes from fundamental spin $\frac{1}{2}$ particles (fermions). Fermions may be divided into two categories, those that carry no $SU(3)_C$ (colour) charges are called leptons. The known spectrum of leptons includes the electron (e^-), the muon (μ^-), the (τ^-), and their neutrinos ν_e, ν_μ, (and ν_τ). Since they are colour neutral they do <u>not</u> interact strongly, but they do carry the $[SU(2) \times U(1)]_{EW}$(flavour) charges and consequently their electroweak interactions must be observed in the laboratory. The fundamental strongly interacting fermions who carry both flavour and colour charges are called quarks. They can exist in three colour states, r(red), y(yellow) and b(blue), and come in a number of different flavours, such as the u(up), d(down), s(strange) and c(charm) /F2/.

According to the above charge assignments, fermions are grouped into two different lepton and quark multiplets if the local gauge symmetry is assumed to be at least as large as the $SU(3)_C \times [SU(2) \times U(1)]_{EW}$

group. Such a theory is often called the "standard" unified theory of electroweak and strong interactions. However, apart from some defects, it has three independent coupling constants associated with the electroweak and strong interactions. These three couplings can be reduced to only one coupling if one embeds the symmetry structure $SU(3)_C \times [SU(2) \times U(1)]_{EW}$ into a larger gauge group G. It then turns out that leptons and quarks can be grouped in the same multiplet of the group G.

The grand idea of grouping leptons and quarks into the same multiplet(s) of a gauge group G is the basic assumption that goes under the structure of "grand" unified theories (Pati-Salam, 1973) /13/. In the simplest and minimal scheme of such theories not only leptons and quarks but also their antiparticles are grouped in the same multiplet of grand unified group G (Georgi-Glashow, 1974) /14/. If such a unification is naturally allowed, it would have to happen at high energies since only at high energies can the strong coupling be compared with the electroweak couplings. It turns out that when the grand unified group is "simple", all interactions are generated with only one gauge coupling at superhigh energies (of order 10^{15} Gev). Since leptons and quarks are allowed to be represented in the same multiplets of the unifying gauge group G, there will be interaction transforming quarks and leptons into each other and thus breaking both baryon and lepton number conservation. Consequently, the fact that ordinary matter seems pretty stable, that proton decay has <u>not</u> been seen, should <u>not</u> lead us to the conclusion that baryon and lepton conservation are fundamental conservation laws. To the accuracy with which they have been verified, baryon and lepton conservation can be explained as dynamical consequences of other symmetries, in the same way that strangeness conservation has been explained within QCD. However, superheavy particles can exist in grand unified theories, and these particles may have unusual SU(3) or $[SU(2) \times U(1)]_{EW}$ transformations, and in this case, there is no reason for baryon or

lepton number to be conserved.

This thesis is concerned with the above general notions of elementary particles. It is divided into two parts: electroweak gauge theories and grand unified theories.

In Chapter Two, the general aspects of electroweak "unified" gauge theories are discussed. We start with the Salam-Weinberg description of electroweak phenomena. The quanta of electroweak interactions and the associated currents realized through the spontaneous symmetry breaking phenomena are briefly but carefully analysed. The charges and masses of elementary particles and the free parameters of the theories are studied. We go on by considering the "extended" electroweak models, in fact, these are constructed by enlarging the gauge symmetry SU(2) x U(1) of the "standard" Salam-Weinberg model to SU(2) x U(1) x G symmetry under appropriate conditions. The case for which G = U(1) is discussed in some detail; at this stage we are more interested in the weak neutral current structure and the weak neutral gauge particle masses of the model. This procedure is repeated for a class of "left-right symmetric models" in which right-handed fields are introduced parallel to all left-handed fields. The characteristic feature of this class of models are studied by examining an $SU(2)_L$ x $SU(2)_R$ x U(1) gauge model and its extension to a larger $SU(2)_L$ x $SU(2)_R$ x $U(1)_L$ x $U(1)_R$ symmetry.

It is then argued in Chapter Three that the multi-gauge boson models based on the extended or the left-right symmetric models may explain the low energy behaviour of electroweak phenomena as the standard model. In particular, models with more than one neutral boson possess the interesting feature that the mass of the lightest neutral boson is smaller than that of the standard model. It is then interesting to look for a general formalism that would help to distinguish between different electroweak models. From the experimental point of view the effects of weak neutral current(s) in reaction $e^+e^- \longrightarrow f^+f^-$ are very

interesting and Chapter Three aims to study this reaction in detail from the theoretical point of view.

One of the main tasks of the e^+e^- machines (LEP, SLAC, ...) is the determination of the coupling of the fundamental fermion (leptons and quarks) to the weak neutral current(s). When the colliding e^+e^- beams are unpolarized, the measurement of the angular distribution of the reaction $e^+e^- \longrightarrow e^+e^-$ and $e^+e^- \longrightarrow f^+f^-$ allows the determination of the modula of the coupling constants. However, in this kind of experiments the relative signs of the couplings remain undetermined. To find out these signs one needs a true parity violation measurement, for example through the polarization effects.

Measurement of final fermion polarization can be done through a final state interaction of the fermion which involves a parity violation. In the Salam-Weinberg model, the final lepton polarization effects are proportional to $(\frac{1}{4} - \sin^2\theta_W)$; the present experimental prediction of $\sin^2\theta_W = 0.23$ therefore leads to rather small effects. So the measurement of the final lepton polarization is difficult and consequently, the maximal information would have to be extracted from the final state decay, in particular when the final state is a pair of leptons.

There have been many studies /F3/ on the $e^+e^- \longrightarrow f^+f^-$ reaction. There are, however, several points to develop. The production cross-section $e^+e^- \longrightarrow f^+f^-$ has already been analysed /15/ in the case of one photon and one weak neutral boson exchange. We present a general description of the $e^+e^- \longrightarrow f^+f^-$ process in which an arbitrary number of neutral bosons are involved. Both, initial and final fermion polarization effects are taken into account in our approach and it is applicable to any kind of light or heavy fermion at low or high energy. We also work out the differential cross-section and the asymmetry for the case in which the initial states are longitudinally polarized in $\mu^+\mu^-$

production. The results are then examined for three neutral bosons exchange processes.

The parity-violating helicity distribution for $\mu^+\mu^-$ pair production, assuming that the muon state are polarised, are computed.

Finally, we develop our procedure of multi-neutral boson exchange and apply it to deep inelastic polarized electron-nucleon scattering. Our treatment of deep-inelastic scattering is carried out in the framework of the quark-parton model.

Chapter Four begins by introducing the strong interactions and grand unified theories. After a brief review of these fields, the SU(5) grand unified theory is examined through which we show how strong and electroweak interactions are generated in this theory. By employing the minimal Higgs system needed for symmetry breaking in this model, the masses of gauge bosons and fermions and surviving scalars are worked out. We carry on by the study of the electroweak mixing angle that may be applied for any grand unified model. We also discuss the proton lifetime in the SU(5) model which is needed for the next chapter. The definition of physical paramenters and the structure of the SU(5) model in this chapter will fix our notations of Chapter Five and Six.

Chapter Five is devoted to renormalization effects in grand unified theories. We start by the definition of "effective" physical parameters, in particular, we are more interested in the effective gauge couplings in grand unified theories. The effects of superheavy gauge bosons in these theories and the low energy behaviour of gauge couplings are studied through the "decoupling theorem". The main purpose of our work is the study of effective gauge couplings, the electroweak mixing angle and the unifying mass scale in grand unified theories up to the two-loop level. In order to do such radiative corrections, the renormalization group equation technique is employed; it turns out that the resulting system of differential equations are coupled to each other.

Two rather different methods are suggested for solving the problem. In method I, we neglect the off-diagonal contributions to the relevant differential equations since their contributions are relatively small. In method II, an iterative technique is used to solve the equations.

For both methods we compute the effective couplings, $\sin^2\theta_W$ and the unifying mass scale M. It is then verified that the low energy value of $\sin^2\theta_W$ increases, while the unifying mass scale M decreases with respect to their corresponding one-loop results. The results are applied to the SU(5) model and it is shown that for this model the value of M does not change by very much and consequently, the proton lifetime may decrease by a factor of the order of 10 (with respect to one-loop results).

Finally, we summarize our results and compare them with the very recent developments in this field which have been reconsidered by other authors.

In Chapter Six, grand unified models with several intermediate mass scalars are examined and the work presented in Chapter Five is developed (up to one-loop approximation) for this type of models. This is done for the SU(16) "maximal" gauge theory, in which fermionic degrees of freedom are maximally gauged. When renormalization effects are taken into account for this model (for the case in which lepton number is regarded as the fourth colour quantum number) it turns out that two intermediate stages governed by the symmetries $G_3 = SU(8)_I \times SU(8)_{II} \times U(1)_F$ and $G_2 = SU(2)_L \times SU(2)_R \times SU(4)_C$ can naturally coexist if $\sin^2\theta(M_W) > 1/6 + 5/9\frac{\alpha(M_W)}{\alpha_s(M_W)}$. It is shown that these symmetries break down at a mass scale of the order of $M_X \simeq 10^4 - 10^5$ Gev. If neutral current phenomenology (or any other experiment) predicts $\sin^2\theta(M_W) > 0.206$, then quark-lepton unification and left-right symmetry simultaneously break down at $M_X \simeq 10^4$ Gev.

It is then argued that apart from proton decay, $n - \bar{n}$

oscillation and neutrinoless double β decay processes, an accurate experimental value of $\sin^2\theta(M_W)$ (to a 10^{-4} accuracy) plays a crucial role in determining the possible existence of such intermediate stages.

CHAPTER II. ELECTROWEAK INTERACTIONS

2.1 Introduction.

The development of our present ideas of weak and electromagnetic interactions can be traced a long way back into history. Unification of electricity and magnetism was established in the work of James Clark Maxwell in 1868. This theory of electric and magnetic fields is today recognised as the original gauge theory. A new force emerged from the discovery of radiative nuclei. This new "weak" force was then formulated by Fermi in 1936. He suggested a point-like four-fermion lepton-hadron interaction lacking any specific structure or dynamics. Yukawa introduced the idea of particle exchange as the dynamical origin of interactions.

Interactions can be characterized by their space-time transformation properties; scalar, pseudo-scalar, vector, axial-vector and tensorial interactions are naturally well defined. It was Sudarshan and Marshak /16/, Feynman and Gell-Mann /17/ and Sakurai /18/ who suggested that the weak interactions are of the V-A form, responsible for a large observed parity-violating effects observed in weak interactions. There existed serious problems with the point-like formulation of weak interactions introduced by Fermi. From the theoretical point of view, the non-renormalizability causes the phenomenological description ultimately to fail at high energies.

It was conjectured /10,11/ that the weak interactions would be mediated by a massive particle (the intermediate vector boson) which would dampen the high energy behaviour of weak processes. Since beta decay processes occurred with charge transfer, the vector boson was assumed to come in two signs W^+ and W^-. It was then natural to look for a neutral component of the weak interactions. An early suggestion that there would be a weak neutral current was published in 1957 by

Zel'dovich /19/. He suggested looking for weak-electromagnetic interference in polarized electron scattering and in optical rotation of the plane of linear polarization in heavy atoms.

In 1961 Glashow, pursuing the goal /11/ of unifying the electromagnetic and weak interactions, proposed a "partially symmetric" unified electroweak theory. By a "partial symmetry" he meant one which was broken only by explicit mass terms in the Lagrangian. In this scheme, the symmetry group was SU(2) x U(1) which was also pointed out /11/ as the correct symmetry of electroweak phenomena by Salam and Ward 1964.

This proposal still left unanswered the question of how the mass terms were to be treated, and also, in particular, the question of renormalizability. It was Weinberg /10/ and independently Salam /10/ who proposed that the spontaneous symmetry breaking mechanism could provide the resolution of these difficulties. They proposed that the vector bosons could acquire masses via the Higgs-Kibble mechanism /6,7/. In this view it is essential that SU(2) x U(1) symmetry be a local one the vector field being the quanta of the gauge fields so that, as it was hoped, enough of the gauge invariance might be "retained" after spontaneous symmetry breaking to render the theory renormalizable. Renormalizability was then proved /20/ by 't Hooft in 1971 /F_4/ .

In 1971 Glashow, Iliopolous and Miani (GIM) extended /21/ the theory to include couplings of quarks, and invented the charmed quark to explain the absence of strangeness changing neutral currents. These steps cleared up the many problems facing the unification of weak and electromagnetic interactions and left open the predictions of neutral currents for leptonic and non-strange hadronic processes. In 1974 neutral currents were first observed in inelastic neutrino scattering in the Gargamelle bubble chamber at CERN /22/. Parity violation in neutral currents was observed in 1978 in optical rotation of linearly polarized laser light in bismuth vapors at Novosibirsk /23/ and in inelastic

scattering of polarized electrons from hydrogen and deuterium at SLAC/23/.

2.2 The Salam - Weinberg Theory.

The Salam-Weinberg theory /10/ of electroweak interactions with the GIM mechanism /21/ is based on the local gauge group SU(2) x U(1). Associated with this symmetry are the generators T_a (a=1,2,3) (weak isospin) and $T_0=Y/2$ (weak hypercharge). The electric charge is defined as a linear combination of these generators

$$Q = T_3 + \tfrac{1}{2}Y. \qquad (2.2.1)$$

Associated with the local symmetry are four gauge fields, an isotriplet W_μ and an isoscalar B_μ. Fermion fields are incorporated into this model in the following way: first they are decomposed into right-handed and left-handed components

$$\psi = \psi_R + \psi_L \qquad (2.2.2)$$

where $\psi_R = \frac{1+\gamma_5}{2}\psi$ and $\psi_L \frac{1-\gamma_5}{2}\psi$, [the operators $\frac{1\pm\gamma_5}{2}$ project out the right and left-handed pieces of the fermion, respectively]. Then, right-handed fields are assigned to isosinglets, T=0, while left-handed fields transform as members of isodoublets, $T=\frac{1}{2}$ under weak isospin. [This asymmetric assignment leads to the parity violation observed in weak interactions]. Weak hypercharges are given so that the electric charge comes out right. The fundamental lepton and quark fields along with their weak isospin and hypercharge quantum numbers are

Leptons :

$$\left|\begin{matrix}\nu_e\\ e\end{matrix}\right|_L \quad \left|\begin{matrix}\nu_\mu\\ \mu\end{matrix}\right|_L \quad \left|\begin{matrix}\nu_\tau\\ \tau\end{matrix}\right|_L \qquad T=1/2\ ,\ Y=-1 \qquad (2.2.3a)$$

$$e_R \quad \mu_R \quad \tau_R \qquad T=0\ ,\ Y=-2 \qquad (2.2.3b)$$

Quarks :

$$\left|\begin{matrix}u\\ d'\end{matrix}\right|_L \quad \left|\begin{matrix}c\\ s'\end{matrix}\right|_L \quad \left|\begin{matrix}t\\ b'\end{matrix}\right|_L \qquad T=1/2\ ,\ Y=1/3 \qquad (2.2.4a)$$

$$u_R \quad c_R \quad t_R \qquad T=0\ ,\ Y=4/3 \qquad (2.2.4b)$$

$$d_R \quad s_R \quad b_R \qquad T=0\ ,\ Y=-2/3 \qquad (2.2.4c)$$

The primes in (2.2.4a) allow for quark mixing and CP violation phases through the K-M matrix /24/

$$\begin{vmatrix} d' \\ s' \\ b' \end{vmatrix} = \begin{vmatrix} C_1 & -S_1 C_3 & -S_1 S_3 \\ S_1 C_2 & C_1 C_2 C_3 - S_2 S_3 e^{i\delta} & C_1 C_2 S_3 + S_2 C_3 e^{i\delta} \\ S_1 S_2 & C_1 S_2 C_3 + C_2 S_3 e^{i\delta} & C_1 S_2 S_3 - C_2 S_3 e^{i\delta} \end{vmatrix} \qquad (2.2.5)$$

where $C_i = \cos\theta_i$ and $S_i = \sin\theta_i$ and the θ_i are generalised Cabibbo rotations. We have suppressed quark colour indices in the above discussion. Each quark comes in three colours as specified by QCD.

The $SU(2)_L \times U(1)_Y$ gauge invariant Lagrangian density which describes the gauge fields and their interactions with matter fields i.e. scalars and fermions are given by

$$L = -\frac{1}{4} W_\alpha^{\mu\nu} W_{\mu\nu}^\alpha - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + i \sum_f \bar{\psi}_f \gamma^\mu D_\mu \psi_f + \qquad (2.2.6)$$

$$+ D^\mu \phi^\dagger D_\mu \phi - V(\phi) + L_Y(\psi, \phi) .$$

where $\mu, \nu = 0,1,2,3$ are space-time indices, $\alpha = 1,2,3$, is an internal space index belonging to the SU(2) Lie group algebra with the structure constant C^{abc}, and

$$W_{\mu\nu}^\alpha = \partial_\mu W_\nu^\alpha - \partial_\nu W_\mu^\alpha + g C^{abc} W_\mu^b W_\nu^c , \qquad (2.2.7)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu . \qquad (2.2.8)$$

The covariant derivative D_μ is given by

$$D_\mu = \partial_\mu + i g T \vec{W}_\mu \cdot \vec{\tau} + i g' \frac{Y}{2} B_\mu \qquad (2.2.9)$$

with g and g' being the two independent gauge coupling constants associated with the SU(2) and U(1) groups, respectively. In (2.2.6) ψ_f

denotes any fermion multiplet (singlet or doublet) and the summation is over all fermion multiplets. The Lagrangian (2.2.6) contains a complex scalar doublet made up of a positively charged and neutral component i.e.

$$\phi = \begin{vmatrix} \phi^{+} \\ \phi^{o} \end{vmatrix} \tag{2.2.10}$$

with quantum numbers $T=\frac{1}{2}$ and $Y=1$. Its self interaction properties and its interactions with fermions are given by $V(\phi)$ and L_1, respectively. By means of the Higgs-Kibble mechanism, the scalar doublet ϕ develops a vacuum expectation value

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{vmatrix} 0 \\ v \end{vmatrix} \tag{2.2.11}$$

and the symmetry of the theory thereby breaks down to a local U(1) invariance generated by the electric charge operator Q in (2.2.1), i.e. the remaining U(1) gauge invariance is specified as that of electromagnetics. In this way, three gauge vector bosons acquire masss, while one (identified as the photon) remains massless. The resulting mass eigenstates and their masses are

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (W_{\mu}^{1} \mp i W_{\mu}^{2}) , \tag{2.2.12}$$

$$M_{W^{\pm}} = g v / 2 \tag{2.2.13}$$

while the physical neutral gauge boson Z_{μ} and the photon fields are

$$Z_{\mu}^{o} = W_{\mu}^{3} \cos\theta_{w} - B_{\mu} \sin\theta_{w} \tag{2.2.14}$$

$$A_{\mu} = W_{\mu}^{3} \sin\theta_{w} + B_{\mu} \sin\theta_{w} \tag{2.2.15}$$

with masses $M_{\gamma} = 0$ and

$$M_{Z^{o}} = \frac{1}{2} v (g^{2} + g'^{2})^{1/2} \tag{2.2.16}$$

where θ_W is the electroweak mixing angle (Weinberg angle) defined by

$$\tan \theta_W = g'/g. \tag{2.2.17}$$

The interpretation of A_{μ} as the photon is indeed plausible from the form

of D_μ written in terms of the physical fields

$$D_\mu = \partial_\mu + \frac{ie}{\sqrt{2}\sin\theta_w} W_\mu^+ (T_1 + iT_2) + \frac{ie}{\sqrt{2}\sin\theta_w} W_\mu^- (T_1 - iT_2)$$
$$+ ig\sin\theta_w \left(T_3 + \frac{Y}{2}\right) A_\mu +$$
$$+ i\left((g\cos\theta_w) T_3 - (g'\sin\theta_w)\frac{Y}{2}\right) Z_\mu^0 \qquad (2.2.18)$$

in which A_μ appears correctly coupled to the conventional charge operator given by (2.2.1), which is the unbroken residual symmetry. We also note that if this interpretation is correct physically, $g \sin\theta$ will be identified with the physical charge e, i.e.

$$g \sin\theta = e = g'\cos\theta. \qquad (2.2.19)$$

Thus

$$D_\mu = \partial_\mu + \frac{ie}{\sqrt{2}\sin\theta_w} W_\mu^+ (T_1 + iT_2) + \frac{ie}{\sqrt{2}\sin\theta_w} W_\mu^- (T_1 - iT_2)$$
$$+ ie\varphi A_\mu + ie'\varphi' Z_\mu^0 \qquad (2.2.20)$$

where

$$e' = e/\sin\theta_w \cos\theta_w \qquad (2.2.21)$$

is the strength of weak neutral charge defined through the operator

$$Q' = T_3\cos^2\theta - \tfrac{1}{2}Y\sin^2\theta_W \qquad (2.2.22)$$

which, in turn, by using Eq. (2.2.1) can be rewritten as

$$Q' = T_3 - Q\sin^2\theta_W . \qquad (2.2.23)$$

This basic formula relates the"neutral weak charge" Q' to the electromagnetic and weak isospin charges Q and T, respectively. Since the weak isospin charge is only left and electric charge is left and right, Eq. (2.2.23) can be read as

$$Q'_{L,R} = T_{3L} - \varphi_{L+R}\sin^2\theta_w \qquad (2.2.24)$$

and more clearly

$$Q'_L = T_{3L} - \varphi_L \sin^2\theta_w \qquad (2.2.25a)$$

and

$$\varphi'_R = 0 - \varphi_R \sin^2\theta_w . \qquad (2.2.25b)$$

For example, take the "up" quark. The weak isospin third component is T_{3L} (up) = $+\frac{1}{2}$, while the electric charge is $+2/3$; the result is

$$Q'_L(up) = \frac{1}{2} - \frac{2}{3}\sin^2\theta_w \qquad (2.2.26a)$$

and

$$\varphi'_R(up) = -\frac{2}{3}\sin^2\theta_w . \qquad (2.2.26b)$$

The values of the weak neutral charge for all known leptons and quarks are given in Table (2.2.1)

	spinors	T_L^3	φ_L	φ_R	φ_L'	φ_R'	φ_V	φ_A
Leptons	ν_e, ν_μ, ν_τ	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	$\frac{1}{4}$	$-\frac{1}{4}$
	e^-, μ^-, τ^-	$-\frac{1}{2}$	-1	-1	$-\frac{1}{2} + \sin^2\theta_w$	$\sin^2\theta_w$	$-\frac{1}{4} + \sin^2\theta_w$	$\frac{1}{4}$
Quarks	u, c, t	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{2} - \frac{2}{3}\sin^2\theta_w$	$-\frac{2}{3}\sin^2\theta_w$	$\frac{1}{4} - \frac{2}{3}\sin^2\theta_w$	$-\frac{1}{4}$
	d, s, b	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2} + \frac{1}{3}\sin^2\theta_w$	$\frac{1}{3}\sin^2\theta_w$	$-\frac{1}{4} + \frac{1}{3}\sin^2\theta_w$	$\frac{1}{4}$

Table (2.2.1). Values of charges of elementary fermion in the $SU(2)_L$ x $U(1)_{L+R}$ standard model.

By substituting D_μ from Eq. (2.2.20) in the fermionic kinetic part of the Lagrangian (2.2.6), we observe that four currents are realised. Two of them are charged left handed, $J^\pm$, they couple to the gauge fields $W^\pm$ with coupling strength e/sin Θ. While the other two currents are neutral, one is the familiar electromagnetic J^μ_{em} coupling to the photon with coupling strength e, and the other one is the "weak neutral current"

$$J^\mu_{NC} = J^\mu_{3L} - J^\mu_{em}\sin^2\theta_w \qquad (2.2.27)$$

that couples to the neutral gauge boson Z° with neutral weak coupling strength e' given by Eq. (2.2.21).

A further consequence of the Higgs-Kibble mechanism is that three of the four fields in (2.2.10) become the longitudinal components of the massive vector bosons, while one remains as a neutral physical scalar field. Its mass is determined through the self interaction terms [contained in the potential $V(\Phi)$ in the Lagrangian (2.2.6)]

$$V(\phi) = \frac{\lambda}{2}(\phi^{\dagger}\phi)^2 - \alpha\,\phi^{\dagger}\phi$$

$$\equiv \frac{\lambda}{2}\left(|\phi^{+}|^2+|\phi^{0}|^2\right)^2 - \alpha\left(|\phi^{+}|^2+|\phi^{0}|^2\right) \qquad (2.2.28)$$

by translating (shifting) the field $\phi(x)$ by its vacuum expectation value in order to do perturbation theory around small fields. One sets

$$\frac{h(x)}{\sqrt{2}} = \phi(x) - \frac{1}{\sqrt{2}}\begin{vmatrix} 0 \\ v \end{vmatrix} . \qquad (2.2.29)$$

Substituting for $\phi(x)$ in (2.2.28) from (2.2.29), one obtains a mass term $-\frac{1}{2}(\lambda v^2)h(x)$ which tells us that the neutral Higgs boson $h(x)$ has arbitrary mass

$$M_h = v\lambda^{1/2} . \qquad (2.2.30)$$

In the Salam-Weinberg model charged leptons and quarks acquire masses by coupling their left-handed and the right-handed components through a Higgs scalar isodoublet field (terms contained in L_Y in the Lagrangian (2.2.6)) which develops a vacuum expectation value. (Mass terms change chirality, so one must couple left to right in order to generate mass) As an example, a general Yukawa coupling of ϕ to (u,d) quarks is

$$-G_d\,\phi(x)\,\bar{d}_L d_R - G_u\,\phi(x)\,\bar{u}_L u_R + c.c. \qquad (2.2.31)$$

with no relation between G_d and G_u. In the case of spontaneous symmetry breaking $\phi(x) = \frac{v+h(x)}{\sqrt{2}}$ and apart from the Yukawa interaction between $h(x)$ and the fermions, a mass term is generated independently for each fermion f of the theory which has a right component. The mass term for a typical ψ_f = f is

$$-G_f\frac{v}{\sqrt{2}}\,\bar{f}_L f_R - G_f^{*}\frac{v}{\sqrt{2}}\,\bar{f}_R f_L . \qquad (2.2.32)$$

By a phase transformation on f_L (and/or f_R) one can always make G_f real positive, so that one finally finds

$$m_f = \frac{v}{\sqrt{2}}G_f . \qquad (2.2.33)$$

The fermion masses generated by this "Higgs" mechanism are totally arbitrary parameters; they are chosen to have the following values /24/

$$
\begin{aligned}
& m_e = 0.51 \ \text{Mev} \\
& m_\mu = 106 \ \text{Mev} \\
& m_\tau = 1784 \ \text{Mev} \\
& m_d \simeq 8 \ \text{Mev} \ , \quad m_u \simeq 3 \ \text{Mev} \\
& m_s \simeq 180 \ \text{Mev} \ , \quad m_c \simeq 1250 \ \text{Mev} \\
& m_b \simeq 4500 \ \text{Mev} \ , \quad m_t > 16000 \ \text{Mev} \ (?)
\end{aligned}
\tag{2.2.34}
$$

so as to agree with phenomenology /F_3/ .

2.3 The Physical Parameters in the Standard Model.

The Salam-Weinberg unified gauge theory of electroweak interactions, with three generations (families) of fermions and one Higgs doublet as was described in section 2.1, is sometimes referred to as the "standard model". It provides us a minimal scheme of the electromagnetic and weak phenomena in the framework of a gauge theory with spontaneous symmetry breaking. In such a scheme we are faced with seventeen, theoretically unpredicted, parameters. Nine of them are concerned with the fermion masses (or their nine couplings to the Higgs boson). Four parameters come through the unitary matrix (3 angles and one CP violating phase) which allow us to build the hadronic weak charged currents starting with the mass eigenstate quarks (see Eq. (2.2.31)). Two parameters λ and v (the parameter a can be absorbed in λ and v) involved with the Higgs mechanism; and finally the two gauge parameters g and g'. For these parameters, only one natural relation

$$\cos^2\theta_w = \frac{M_w^2}{M_Z^2} \qquad (2.3.1)$$

(obtained from Eqs. (2.2.13) and (2.2.16)) is given; hence, sixteen more relations are missing and the unknown parameters are, therefore, to be estimated through a phenomenological approach. A constraint on the parameter e is given by

$$e \equiv (4\pi\alpha)^{1/2} \qquad (2.3.2)$$

where $\alpha = (137.035987)^{-1}$, the fine structure constant, evaluated at zero momentum transfer as in Thomson scattering at $q^2 = 0$.

Another phenomenological constraint on the parameters of the theory comes from the muon decay rate which implies

$$\frac{G_F}{\sqrt{2}} \equiv \frac{g^2}{8M_w^2} = \frac{\pi\alpha}{2M_w^2 \sin^2\theta_w} \equiv \frac{1}{2v^2} \qquad (2.3.3)$$

where $G_F = 1.166400 \times 10^{-5}$ Gev^{-2} is the Fermi constant. From relation (3.2.3) one finds

$$M_W = \frac{37.320}{\sin \Theta_W} \text{ Gev.} \qquad (2.3.4a)$$

and Eq. (2.3.1) then gives

$$M_Z = \frac{74.640}{\sin 2\,\Theta_W} \text{ Gev.} \qquad (2.3.4b)$$

From high energy weak interaction phenomenology (charged and neutral weak currents, e-d polarization effects) the world average value of $\sin^2\Theta_W$ stands /25/ at $\sin^2\Theta_W = 0.230 \pm 0.020$ which gives

$$81.242 > M_W > 74.641 \text{ Gev} \qquad (2.3.5a)$$

and

$$93.810 > M > 83.977 \text{ Gev.} \qquad (2.3.5b)$$

The parameters involved with the Higgs-Kibble mechanism in the standard model are connected to experimental results through the quantity ρ which is defined as

$$\rho^2 = \frac{\text{rate of neutral currents}}{\text{rate of charged currents}} \equiv \left(\frac{(e/\sin\theta_w\cos\theta_w)}{(e/\sin\theta_w)}\right)^4 \left(\frac{(1/M_{z_o}^2)}{(1/M_w^2)}\right)^2 \qquad (2.3.6a)$$

or

$$\rho = \frac{M_w^2}{M_{z^o}^2 \cos^2\theta_w} . \qquad (2.3.6b)$$

It is theoretically predicted to be 1 (see Eq. (3.2.1)), while the experimental result is

$$\rho_{exp} = 0.98 \pm 0.05. \qquad (2.3.7)$$

The theory can accommodate more than n_g=3 generations of fermions, if they exist, even one can incorporate more than one Higgs doublet if it is necessary; but the number of parameters would rapidly increase in any case. There are, of course, some theoretical constraints

to be fulfilled in adding such extra fields to the theory. In particular for scalar fields, the uniqueness of the electromagnetic direction (charge conservation) implies that for any additional scalar system, all vacuum expectation values have to point in the same SU(2) direction, which impose constraints on the parameters of the Lagrangian.

The idea of Salam and Weinberg of electroweak interactions within the context of spontaneous symmetry breaking gauge theory has recently been extended in the physics of elementary particles in many different directions. A rather similar approach by enlarging the symmetry of $SU(2)_L \times U(1)_Y$ to other groups which is not yet ruled out (from phenomenological point of view) will be analysed in the next section.

2.4 Extended Electroweak Theories.

Extended elctroweak gauge theories are referred to as theories that are obtained by enlarging the gauge symmetry $SU(2)_L \times U(1)_Y$ of the standard model to $SU(2)_L \times U(1)_Y \times G$ symmetry where G may in principle be any simple group. They may in a certain sense also be regarded as a multi-boson version of the standard Salam-Weinberg theory of electroweak interactions. Depending on the symmetry breaking properties, there are, more precisely, two classes of such theories; in the first class the number of neutral bosons is equal to the number of charged ones, while in the second class the number of neutral bosons is larger than the number of charged ones.

The general structure of these theories based on $U(1) \times SU(2) \times G$ has first been analysed /29/ by Georgi and Weinberg, who pointed out that under appropriate conditions on the symmetry breaking, the low energy four fermion interaction for neutrino scattering is the same as in the standard model in spite of the presence of several weak neutral bosons in addition to a single charged $W^{\pm}$ as in the standard model. It was, then, shown /30/ that within such a framework one obtains "good" low energy behaviour also for a charged lepton and quark interaction, provided one assumes not only neutrinos, but also charged leptons and quarks to be classified as singlets under G. Moreover, not only several neutral bosons but also several charged ones may be present/31/.

There also exists a class of "left-right symmetric theories" in which right-handed fields, in particular, right-handed neutrinos are introduced parallel to all left-handed fields. In such schemes parity is spontaneously violated through the same mechanism which gives mass to gauge bosons of the theory /26/. The simplest case happens when the gauge symmetry $SU(2)_L \times U(1)_Y$ of the standard model is enlarged to the $SU(2)_L \times SU(2)_R \times U(1)$ symmetry /27/ that, in turn, can easily be extended

to models with the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ gauge symmetry/28/.

The motivation for the study of such electroweak models, that is achieved in the next two subsections, are as follows:

i) In both types of extended ideas discussed above, one can naturally choose the relevant parameters of the theory in such a way that the low energy behaviour of the standard model be fulfilled. It means that the standard model may not be a unique model that explains the nature of electroweak phenomena, in particular, when the high energy behaviour of this model has not yet been tested.

ii) One can verify that in the extended electroweak models based on the $SU(2)_L \times U(1)_Y \times G$ symmetry there exists a neutral boson whose mass is smaller than the Z_o mass of the standard model. While in the left-right symmetric models this can occur for some particular values of the free parameters of the specific model.

iii) The issue of neutrino masses has recently attracted considerable attention because of results coming from two different experiments /32/ and suggesting a non-zero mass for ν_e (in the range of 14 ev $\leq m_{\nu e} \leq$ 46 ev). The left-right symmetric models seem to explain this effect naturally.

iv) There are some grand unified theories that spontaneously break down to a low energy symmetry with an electroweak substructure that is governed by one of the above extended models. It is, therefore, interesting to analyse, both theoretically and phenomenologically, the low energy behaviour of such grand unified theories.

In the next two subsections we work out the neutral currents, the boson masses and analyse the low energy behaviour of two typical classes of extended electroweak model. This can, in turn, be useful for the study of some physical processes examined in the same approach in Chapter Three.

2.5 The $SU(2)_L \times U(1)_Y \times G$ Models

In this section we discuss a typical class of the extended electroweak models in which the gauge group is $SU(2)_L \times U(1)_Y \times G$ and in particular the simplest case when G=U(1) which is special interest in our procedure. In constructing these models, the following general assumptions are made:

1) The electric charge operator is defined to be

$$Q = T_L^3 + T^0 + \tilde{T}^0 \qquad (2.5.1)$$

where the generators T_L^3, $T^0 = \frac{1}{2}Y$ and $\tilde{T}^0 = \frac{1}{2}\tilde{Y}$ are associated with the weak isospin SU(2), hypercharge $U(1)_Y$ and a new hypercharge $\widetilde{U(1)}_{\tilde{Y}}$ group, respectively.

2) All matter fields (fermions and scalars) of the standard model transform in the usual way under the $SU(2)_L \times U(1)_Y$ part of the group $SU(2)_L \times U(1)_Y \times \widetilde{U(1)}_{\tilde{Y}}$. They are invariant under the new $\widetilde{U(1)}_{\tilde{Y}}$ group. [Neither fermions nor scalars of the standard model carry the quantum number of $\widetilde{U(1)}_{\tilde{Y}}$]

3) Apart from the usual isodoublet scalar field Φ of the standard model, an additional scalar filed $\tilde{\Phi}$, (being invariant under the $SU(2)_L$ group and transforming in a non-trivial way under $U(1)_Y \times \widetilde{U(1)}_{\tilde{Y}}$ is employed to break the symmetry of the theory down to a U(1) through the Higgs-Kibble mechanism. This is the U(1) which is identified as the electromagnetic $U(1)_{EM}$.

It follows from the above assumptions that the covariant derivative is given by

$$D_\mu = \partial_\mu + ig'B_\mu + i\tilde{g}'\tilde{B}_\mu + igW_\mu \qquad (2.5.2)$$

where $\tilde{B}_\mu = \tilde{B}_\mu \tilde{T}_0$ is the new gauge field associated with the new $\widetilde{U(1)}_{\tilde{Y}}$ with the new hypercharge quantum number $\frac{1}{2}\tilde{Y}$ and W_μ as the usual isotriplet of the $SU(2)_L$ group, while B_μ is an isoscalar belonging to the $U(1)_Y$ gauge group. Making use of the quantum number assignments of

assumptions (2) and (3), the quantum number of scalar fields Φ are identified as follows

$$T = \tfrac{1}{2}\ , \quad \tilde{Y} = 0 \quad \text{and} \quad \tfrac{1}{2}Y = \tfrac{1}{2} \tag{2.5.3}$$

while for the scalar field $\tilde{\Phi}$, we find

$$T_3 + \tfrac{1}{2}Y + \tfrac{1}{2}\tilde{Y} = Q\ . \tag{2.5.4}$$

Since this scalar field along with the doublet Φ is assumed to break the $SU(2)_L \times U(1)_Y \times \widetilde{U(1)}_{\tilde{Y}}$ symmetry down to the $U(1)_{EM}$, it then turns out (due to assumption (2) and (3)) that

$$y + \tilde{y} = 0 \tag{2.5.5}$$

where $\frac{1}{2}y$ and $\frac{1}{2}\tilde{y}$ are the eigenvalues of T^o and $\tilde{T}^o$. There is no additional assumption, neither any physical constraints, to specify the numerical values of the U(1) quantum numbers of $\tilde{\Phi}$, they remain basically unidentified real parameters in such kind of model and this is, of course, the first theoretical deviation from the standard S-W model of electroweak interactions. It is worth mentioning here the original source of the uncertainty of these quantum numbers in all extended electroweak models based on the standard model. This difficulty originates from the definition of the charge operator imposed on the structure of the standard model at the beginning. To clarify the point, let us recall how the standard model is constructed. As it was discussed in (2.2), the underlying symmetry of this model is $SU(2)_L \times U(1)_Y$.

Associated with this symmetry are the generators T_a(weak isospin) and $T_o = \frac{1}{2}Y$(weak hypercharge). In terms of these generators the electric charge operator Q is defined by relation (2.2.1), due to which, for any given fermion with specific electric charge, we can then evaluate the quantum number Y/2. For example the right-handed electron is an isosinglet (T_3=0) hence the quantum number Y_{e_R} is -2. The left-handed electron is a member of a weak isodoublet such that $T_3 = -\frac{1}{2}$ and consequently $Y_{e_L} = -1$. We observe that the hypercharge quantum number is an already unknown number that must be fixed by some observation, say the electric charge

of the particles.

A rather similar situation arises for the scalar fields which breaks the symmetry; it is argued that the null component of the scalar field must develop a non-zero vacuum expectation value due to the Higgs-Kibble mechanism, since the combination of T_3 and T_o given by Eq. (2.2.1) is to generate an exact $U(1)_{EM}$ symmetry after breaking the $SU(2)_L \times U(1)_Y$ symmetry, we find out that the hypercharge quantum number of the scalar field Φ is $T=\frac{1}{2}$. Note that, although all matter fields in the theory are representations of the weak $U(1)_Y$, we are unable to identify their associated hypercharge quantum number directly. Therefore, the definition of the hypercharge quantum number is itself a primary question (as far as electroweak gauge theories are concerned). Because of that there are several possibilities for hypercharge quantum number assignment that, in turn, leave possible choices for the scalar fields if we would enlarge the gauge group $SU(2)_L \times U(1)_Y$ to a larger one such as the $SU(2)_L \times U(1)_Y \times \widetilde{U(1)}_{\widetilde{Y}}$. This along with the charge quantization problem may become more clear when the symmetry of electroweak interactions is embedded in a larger "simple" or "semisimple" gauge group by applying the idea of grand unified theories. We will come back to this point in Chapter Four.

We now go back to the study of the case in which the gauge symmetry is the simplest one i.e. $SU(2)_L \times U(1)_Y \times \widetilde{U(1)}_{\widetilde{Y}}$ /30/.

Since we are more interested in comparing the neutral boson masses and the structure of the neutral currents with those of the standard model, we begin with the kinetic part of the Lagrangian containing the scalar fields Φ and $\widetilde{\Phi}$. This can easily be written down by making use of assumptions (1) and (2) and definition (2.5.2). We have

$$L_S = D_\mu \Phi^\dagger D^\mu \Phi + D_\mu \widetilde{\Phi}^\dagger D^\mu \Phi \qquad (2.5.6)$$

where according to Eq. (2.5.2)

$$D_\mu \Phi = (\partial_\mu + ig' \underline{B}_\mu + ig \underline{W}_\mu) \Phi \qquad (2.5.7)$$

and

$$D_\mu \tilde{\Phi} = (\partial_\mu + i g' \underline{B}_\mu + i \tilde{g}' \tilde{\underline{B}}_\mu) \tilde{\Phi} . \tag{2.5.8}$$

Let the scalar fields develop the following vacuum expectation values

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{vmatrix} 0 \\ v \end{vmatrix} , \tag{2.5.9}$$

$$\langle \tilde{\Phi} \rangle = \frac{1}{\sqrt{2}} \tilde{v} . \tag{2.5.10}$$

Then, by substituting these values in (2.5.6) one obtains

$$L_{mass} = \frac{v^2}{4} (g W_\mu^3 + g' B_\mu)^2 + g^2 \tilde{v}^2 (\tilde{g}' \tilde{B}_\mu + g' B_\mu)^2 + \frac{v^2}{4} g^2 W_+^\mu W_{+\mu}^+ . \tag{2.5.11}$$

By introducing a neutral column vector

$$\bar{W}_\mu \equiv \begin{vmatrix} B \\ W^3 \\ \tilde{B} \end{vmatrix} , \tag{2.5.12}$$

the neutral gauge boson mass terms in the Lagrangian can be written as

$$L^0_{mass} = \bar{W}_\mu^+ M^2 \bar{W}^\mu , \tag{2.5.13}$$

where the mass matrix M^2 is given by

$$M^2 = \begin{vmatrix} (\frac{M_1^2}{g^2} + \frac{M_2^2}{\tilde{g}^2}) \tilde{g}'^2 & -\frac{g'}{g} M_1^2 & -\frac{g'}{\tilde{g}'} M_2^2 \\ -\frac{g'}{g} M_1^2 & M_1^2 & 0 \\ -\frac{g'}{\tilde{g}'} M_2^2 & 0 & M_2^2 \end{vmatrix} , \tag{2.5.14}$$

with M_1 and M_2 being given by

$$M_1^2 = M_\pm^2 = \frac{1}{4} v^2 g^2 , \tag{2.5.15}$$

$$M_2^2 = g^2 \tilde{v}^2 \tilde{g}'^2 . \tag{2.5.16}$$

Note that in (2.5.11) M_1 is identical to the charged $W^{\pm}$ mass M_W.

The observable quanta are obtained by transforming the gauge fields (2.5.12) in such a way that the mass matrix becomes diagonal. Let the transformed field be

$$\bar{Z}_\mu = \begin{vmatrix} Z_0 \\ Z_1 \\ Z_2 \end{vmatrix}_\mu = R \begin{vmatrix} B \\ W^3 \\ \tilde{B} \end{vmatrix}_\mu \tag{2.5.17}$$

which obtains by acting on W_μ by an orthogonal three-dimensional transformation R. Substituting (2.5.17) in (2.5.13), we find

$$L^{\circ}_{mass} = \bar{Z}^{+}_{\mu} (R M^2 R^{+}) \bar{Z}^{\mu} . \tag{2.5.18}$$

Transformation R has to be determined such that $RM^2R^{\dagger}$ becomes diagonal. After diagonalizing the mass matrix, the eigenvalues, or the masses of the neutral gauge bosons are easily calculated. They are /30/ $m_0^2 = 0$, m_1^2 and m_2^2, while m_1^2 and m_2^2 satisfy

$$m_1^2 + m_2^2 = (1 + \frac{g'^2}{g^2}) M_1^2 + (1 + \frac{g'^2}{\tilde{g}'^2}) M_2^2 \tag{2.5.19}$$

$$m_1^2 \cdot m_2^2 = g'^2 (g^{-2} + \tilde{g}'^{-2} + g^{-2}) . \tag{2.5.20}$$

The fermionic part of the Lagrangian in this model may be written down as

$$L_f = i \bar{\psi}_L \not{D} \psi_L + i \bar{\psi}_R \not{D} \psi_R \tag{2.5.21}$$

in which $\not{D} = \gamma^{\mu} D_{\mu}$ is defined by Eq. (2.5.2) through assumptions (1-3). Substituting the covariant derivative and using these assumptions, the neutral current interaction is obtained as

$$L_{NC} = -\sum_{i=0,1,2} g^i J_i^{\mu} \bar{W}^i_{\mu} \tag{2.5.22}$$

where the currents

$$J_0^\mu \equiv J_Y^\mu = \bar{\psi}_{L(R)} \gamma^\mu \psi_{L(R)} \tag{2.5.23}$$

$$J_1^\mu \equiv J_{T_3}^\mu = \bar{\psi}_{L(R)} \gamma^\mu T^3 \psi_{L(R)} \tag{2.5.24}$$

$$J_2^\mu \equiv J_{\tilde{Y}}^\mu \tag{2.5.25}$$

are the weak hypercharge current, the third component of the weak isospin current and the $\widetilde{U(1)}_{\tilde{Y}}$ hypercharge current, respectively. Note that, since all fermions are assumed to have the quantum number $\tilde{y} = 0$, the fermions do not contribute to $J_{\tilde{Y}}^\mu$. By using Eqs. (2.5.23) and (2.5.18) one obtains

$$\begin{aligned} L_{NC} &= \sum_{j=0,1,2} \bar{w} j_\mu^j \left(\sum_{i=0,1,2} g_i R_{ji} J_i^\mu \right) \\ &= e A_\mu J_{em}^\mu + \sum_{j=1,2} f_j \left(J_3^\mu - k_j J_{em}^\mu \right) Z_j^\mu \end{aligned} \tag{2.5.26}$$

in which we have identified the electromagnetic current J_{em}^μ, due to the charge operator (2.5.1), as

$$J_{em}^\mu = J_3^\mu + J_Y^\mu + J_{\tilde{Y}}^\mu \tag{2.5.27}$$

where the coupling strength e and the constant k_j are given by

$$e = \left(g^{-2} + \tilde{g}^{-2} + g^2 \right)^{-1/2} \tag{2.5.28}$$

and

$$k_j = 1 - \frac{M_w^2}{m_j^2} \tag{2.5.29}$$

respectively. The coefficients f_j in (2.5.29) fulfill the sum rules

$$\sum_{j=1,2} \frac{f_j^2}{m_j^2} = \frac{g^2}{M_w^2} \tag{2.5.30}$$

and

$$\sum_{j=1,2} \frac{f_j^2}{m_j^4} = \frac{g^2 - e^2}{M_w^4} \tag{2.5.31}$$

which are obtained from the orthogonality of the R matrix. These sum rules, in fact, guarantee the correct behaviour of the four fermion interaction at low energy.

Let us now connect the parameters of the model with the measurable quantities which are chosen to be α, G_F and the electroweak mixing angle θ_W. By identifying A_μ in (2.5.27) as the photon the value of e given by (2.5.28) is then fixed. Thus

$$e \equiv (4\pi\alpha)^{1/2} . \tag{2.5.32}$$

By comparing the charged boson exchange four fermion interaction with the phenomenological local Fermi interaction, we have the well known relation

$$\frac{g^2}{8 M_W^2} \equiv \frac{G_F}{\sqrt{2}} . \tag{2.5.33}$$

To find the third connection, one may use the effective low-energy neutral current interaction. Eq. (2.5.26) can be used for obtaining the neutral current four fermion interaction

$$H_{NC} = -\frac{1}{2}\left[\frac{e^2}{q^2} \left(J^\mu_{em} \right)^2 + \sum_{j=1,2} f_j \frac{\left(J^\mu_{NC,j} \right)^2}{q^2 - m_j^2} \right] \tag{2.5.34}$$

in which we have identified

$$J_3^\mu - k_j J^\mu_{em} \equiv J^\mu_{NC,j} . \tag{2.5.35}$$

By using the sum rules (2.5.31) and (2.5.30), the effective low-energy ($q^2 \to 0$) neutral current can be cast into the form

$$H^{eff}_{NC} = -\frac{1}{2}\frac{e^2}{q^2}\left(J^\mu_{em} \right)^2 + \\ + \frac{g^2}{2 M_W^2}\left\{ \left[J_3^\mu - \frac{e^2}{q^2} J^\mu_{em} \right]^2 + C \left(J^\mu_{em} \right)^2 \right\} \tag{2.5.36}$$

where

$$C = \frac{1}{2}\left(\frac{M_w^4}{g^2}\right)^2 \sum_{j,k=1,2} \frac{f_j^2 f_k^2}{m_j^2 m_k^2}\left(\frac{1}{m_j^2} - \frac{1}{m_k^2}\right) . \qquad (2.5.37)$$

The first term in (2.5.36) is the usual electromagnetic one photon exchange contribution, while the second term describes the weak neutral current behaviour in the $q^2 \to 0$ limit. The coefficient e^2/g^2 of J^{μ}_{em} in the square bracket satisfies

$$e^2/g^2 \leq 1 \qquad (2.5.38)$$

(see Eq. (2.5.28)) and can be used to introduce the Weinberg angle θ_W

$$e^2/g^2 \equiv \sin^2\theta_W \qquad (2.5.39)$$

in agreement with the established convention. From Eqs. (2.5.39) and (2.5.33), we obtain the well known relation

$$M_w^2 = \frac{\pi\alpha}{\sqrt{2}\, G_F \sin^2\theta_w} . \qquad (2.5.40)$$

In this way the identities (2.5.32), (2.5.33) and (2.5.39) can fix the parameters g, g' and $\tilde{g}'$. While the neutral boson masses m_1 and m_2 remain as undetermined parameters. To find more information about these parameters, let us solve Eqs. (2.5.30) and (2.5.31) for f_1^2 and f_2^2. They are obtained in terms of the parameters m_1^2 and m_2^2 (by using relation (2.5.40)) as follows

$$f_1^2 = \frac{8 G_F}{\sqrt{2}}\, m_1^2\, \frac{m_2^2 \cos^2\theta_w - M_w^2}{m_2^2 - m_1^2}\left(\frac{m_1}{M_w}\right)^2 , \qquad (2.5.41)$$

and

$$f_2^2 = \frac{8 G_F}{\sqrt{2}}\, m_2^2\, \frac{m_1^2 \cos^2\theta_w - M_w^2}{m_1^2 - m_2^2}\left(\frac{m_2}{M_w}\right)^2 . \qquad (2.5.42)$$

As pointed out by many authors, from the positivity of f_1^2 and f_2^2 in Eqs. (2.5.41) and (2.5.42) one finds /29/

$$m_1^2 \geqslant \frac{M_W^2}{\cos^2\theta_W} \equiv M_{Z_o}^2 \quad , \tag{2.5.43}$$

and

$$m_1^2 \leqslant \frac{M_W^2}{\cos^2\theta_W} \equiv M_{Z_o}^2 \quad , \tag{2.5.44}$$

therefore

$$m_1^2 < M_{Z_o}^2 < m_2^2 \tag{2.5.45}$$

i.e., one of the neutral gauge bosons has to be lighter and the other ones heavier than the Z_o of the standard model of mass $M_{Z_o} = M_W^2/\cos^2\theta_W$.

In the standard model the coefficient C in (2.5.36) is absent, i.e. within extended gauge theories, C=0 implies the standard model. It was shown in Eq. (2.5.37) that the coefficient C depends on the two parameters m_1 and m_2. A simpler expression for C can be found by substituting in Eq. (2.5.37) for f_1^2 and f_2^2 from (2.5.41) and (2.5.42) and using (2.5.40) and (2.5.39). That is

$$C = \cos^4\theta_W \left(\frac{M_{Z^o}^2}{m_1^2} - 1\right)\left(1 - \frac{M_{Z^o}^2}{m_2^2}\right) . \tag{2.5.46}$$

The coefficient C may empirically be estimated through the data on $e^+e^- \to \mu^+\mu^-$ reaction. For instance, the low energy neutral current Hamiltonian (with e-μ universality) is conventionally written in the model independent form

$$\begin{aligned} H_{NC} = &- \frac{e^2}{2q^2}(J_{em}^{\mu})^2 + \\ &+ \frac{G_F}{\sqrt{2}}\Big\{ h_{VV}(\bar{e}\gamma^{\mu}e + \bar{\mu}\gamma^{\mu}\mu)(\bar{e}\gamma_{\mu}e + \bar{\mu}\gamma_{\mu}\mu) + \\ &+ 2h_{VA}(\bar{e}\gamma^{\mu}e + \bar{\mu}\gamma^{\mu}\mu)(\bar{e}\gamma_{\mu}\gamma_5 e + \bar{\mu}\gamma_{\mu}\gamma_5\mu) + \\ &+ h_{AA}(\bar{e}\gamma^{\mu}\gamma_5 e + \bar{\mu}\gamma^{\mu}\gamma_5\mu)(\bar{e}\gamma_{\mu}\gamma_5 e + \bar{\mu}\gamma_{\mu}\gamma_5\mu)\Big\} \end{aligned} \tag{2.5.47}$$

where the electromagnetic interaction has been included. Comparing with (2.5.36) and using (2.5.40) and (2.5.39) one finds /33/

$$h_{VV} = \frac{1}{4}(1 - 4\sin^2\theta_W)^2 + 4C \qquad (2.5.48a)$$

$$h_{VA} = \frac{1}{4}(1 - 4\sin^2\theta_W) \qquad (2.5.48b)$$

$$h_{AA} = \frac{1}{4} \,. \qquad (2.5.48c)$$

The QED cross-section for $e^+e^- \longrightarrow \mu^+\mu^-$, e^+e^- is then modified by an electroweak interference term, which is expressed in terms of h_{VV}, h_{VA} and h_{AA} and thus in terms of C and $\sin^2\theta_W$. A fit to experimental data yields $\sin^2\theta_W$ and C.

The JADE, MARK J and PLUTO groups at PETRA have analysed their data on $e^+e^- \longrightarrow \mu^+\mu^-$, e^+e^- in terms of h_{VV}, h_{VA} and h_{AA} at energies up to about 36 Gev and derived /34/ the following bounds (95% confidence level),

$$C \lesssim 0.033 \qquad \text{(JADE)} \,, \qquad (2.5.49a)$$

$$C \lesssim 0.027 \qquad \text{(MATK J)} \,, \qquad (2.5.49b)$$

$$C \lesssim 0.06 \qquad \text{(PLUTO)} \,. \qquad (2.5.49c)$$

2.6 Left - Right Symmetric Theories

a) The $SU(2)_L \times SU(2)_R \times U(1)_Y$ model

It is possible to preserve the successes of the $SU(2)_L \times U(1)_Y$ model and enlarging the gauge symmetry to a model based on the left-right symmetric group like $SU(2)_L \times SU(2)_R \times U(1)$. Parity violation occurs, then, via spontaneous break down of this symmetry. This class of models is of interest since neutrinons with non-zero mass can be classified in such models quite naturally. Moreover, there are arguments for left-right symmetric models coming from grand unification. For example SO(10) and SU(16) grand unified models may spontaneously break in many stages to this type of symmetry (see Chapter Six). Models based on the symmetry gauge group $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ have three distinguishing features /27/ :

1) There exist right-handed matter parallel to all left-handed matter. In particular, corresponding to the left-handed neutrinos (ν_L^e, ν_L^μ, ν_L^τ), there must exist the right-handed neutrinos (ν_R^e . ν_R^μ . ν_R^τ), distinct from the associated right-handed antineutrinos.

2) For every left-handed (V-A) current coupled to a set of gauge bosons $\underline{W}_L$, there exists within the theory a parallel right-handed (V+A) - current coupled to a distinct set of gauge bosons W_R with equal strength. Associated with the $SU(2)_L \times SU(2)_R$ substructures, there are two sets of charged gauge bosons $W_L^\pm$ and $W_R^\pm$ and two neutral ones W_L^3 and W_R^3.

3) The observed distinction between left and right in the charged current sector, i.e. the dominance of (V-A) over (V+A) in β - decay, μ decay and μ capture is brought about by the spontaneously - induced mass splitting between the charged W_R^+ and W_L^+; $(M_{W_R^+} > M_{W_L^+})$. Likewise, left-right asymmetry in the neutral current sector must arise by the spontaneously induced mass asymmetry between the neutral gauge

bosons. Such asymmetry in the charged as well as neutral sector must then disappear asymptotically for energies much bigger than $M_{W_R^+}$.

Due to the above general considerations, the weak interaction Lagrangian, in common with strong and electromagnetic interactions, is invariant under space reflection. Another point is that the observed parity non-invariance of low energy weak interactions in the class of models is attributed to the idea that the vacuum state of the theory is left-right asymmetric.

The electric charge operator in a model based on $SU(2)_L \times SU(2)_R \times U(1)_Y$ symmetry is defined as

$$Q = T_{3L} + T_{3R} + T_Y^{\circ} \tag{2.6.1}$$

where T_{3L} and T_{3R} are the third component of the $SU(2)_L$ and $SU(2)_R$ respectively and $T_Y^{\circ} = \frac{1}{2}Y$ generate the $U(1)_{L+R}$. Left-handed fermions transform as $SU(2)_L$ doublets, and $SU(2)_R$ singlets; while right-handed fermions transform as $SU(2)_L$ singlets and $SU(2)_R$ doublets. According to these transformations and the electric charge of the representations, leptons carry quantum number $Y = -1$ and quarks $Y = 1/3$, where $Y/2$ is assumed to be the hypercharge quantum number of $U(1)_{L+R}$ generated by T_Y°. It is required that the gauge invariant Lagrangian be invariant under the discrete symmetry $R \leftrightarrow L$. This constraint leads to the equality of the two $SU(2)_L$ and $SU(2)_R$ gauge couplings $g_L = g_R = g$ at the tree level. For the gauge symmetry of the theory, the covariant derivative is given by

$$D_\mu = \partial_\mu + i g_L \underline{W}_\mu^L + i g_R \underline{W}_\mu^R + i g' \underline{B}_\mu \tag{2.6.2}$$

and therefore the gauge invariant Lagrangian for $SU(2)_L \times SU(2)_R \times U(1)$ gauge group may be written as

$$\begin{aligned} L = & -\frac{1}{4} W^{a,L}_{\mu\nu} W^{\mu\nu}_{a,L} - \frac{1}{4} W^{a,R}_{\mu\nu} W^{\mu\nu}_{a,R} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + i \sum_f \bar{L}_f \not{D} L_f + i \sum_f \bar{R}_f \not{D} R_f + \sum (D_{\mu,i} \Phi_i)^\dagger (D_i^\mu \Phi_i) - V(\Phi_i) \\ & - (G_{ff'} \bar{L}_f \Phi R_{f'} + h.c.). \end{aligned} \tag{2.6.3}$$

where

$$\underline{W}_{\mu\nu}^{L(R)} = \partial_\mu \underline{W}_\nu^{L(R)} - \partial_\nu \underline{W}_\mu^{L(R)} - ig\left[\underline{W}_\mu^{L(R)}, W_\nu^{L(R)}\right] , \tag{2.6.4}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \; ; \tag{2.6.5}$$

$$L_\ell = \left|\begin{matrix}\nu_\ell \\ \ell\end{matrix}\right|_L \quad , \qquad L_q = \left|\begin{matrix}t_q \\ b_q\end{matrix}\right|_L \tag{2.6.6}$$

and

$$R_\ell = \left|\begin{matrix}\nu_\ell \\ \ell\end{matrix}\right|_R \quad , \qquad R_q = \left|\begin{matrix}t_q \\ b_q\end{matrix}\right|_R \tag{2.6.7}$$

such that the lepton index l runs over $l=e,\mu,\tau$ and the quark index takes the values $t_q=u,c,t$ and $b_q=d,s,b$.

In the Lagrangian (2.6.3) Φ_i are the appropriate scalar fields needed for breaking the symmetry which will be discussed later. And finally, the last term in the Lagrangian (2.6.3) is the Yukawa coupling term which generates the fermion masses. Note that the quantum numbers assignment implies

$$\underline{T}_L R_\ell = \underline{T}_R L_\ell = 0 \tag{2.6.8a}$$

and

$$\underline{T}_L R_q = \underline{T}_R L_q = 0 \; . \tag{2.6.8b}$$

To break the gauge symmetry and give mass to the gauge bosons at least three multiplets of scalars are needed /27/ . Thus, one introduces two doublets χ_L, χ_R and one (2x2) multiplets Φ, so that

$$\chi_L = \left|\begin{matrix}\chi^+ \\ \chi^\circ\end{matrix}\right| \tag{2.6.9a}$$

$$\chi_R = \left|\begin{matrix}\chi^+ \\ \chi_\circ\end{matrix}\right| \tag{2.6.9b}$$

and

$$\Phi = \left|\begin{matrix}\Phi_1^\circ & \Phi_1^+ \\ \Phi_2^- & \Phi_2^\circ\end{matrix}\right| \; . \tag{2.6.9c}$$

Choosing these scalars, the kinetic part of scalar field in the Lagrangian

$$D_{\mu,i}\,\Phi_i^{\dagger} D_i^{\mu}\phi_i \equiv D_\mu \chi_L^{\dagger} D^\mu \chi_L + D_\mu \chi_R^{\dagger} D^\mu \chi_R + \frac{1}{4} Tr\,(D_\mu \Phi^{\dagger} D^\mu \Phi + D_\mu \Phi'^{\dagger} D^\mu \Phi') \qquad (2.6.10)$$

where

$$\Phi' = \tau_2 \Phi^{*} \tau_2 \cdot \qquad (2.6.11)$$

Let the scalar fields develop the following vacuum expectation values

$$\langle \chi_L \rangle = \frac{1}{\sqrt{2}} \begin{vmatrix} 0 \\ \lambda_L \end{vmatrix} \qquad (2.6.12a)$$

$$\langle \chi_R \rangle = \frac{1}{\sqrt{2}} \begin{vmatrix} 0 \\ \lambda_R \end{vmatrix} \qquad (2.6.12b)$$

$$\langle \Phi \rangle = \begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix} \qquad (2.6.12c)$$

and

$$\langle \Phi' \rangle = \begin{vmatrix} k' & 0 \\ 0 & k' \end{vmatrix} \cdot \qquad (2.6.12d)$$

The neutral boson mass matrix and neutral current structure, then, obtain in the usual way /35/ . We find the neutral boson mass matrix as follows

$$M^2 = \begin{vmatrix} k^2+k'^2+\lambda_L^2 & -(k^2+k'^2) & -\frac{g'}{g}\lambda_L^2 \\ -(k^2+k'^2) & k^2+k'^2+\lambda_L^2 & -\frac{g'}{g}\lambda_R^2 \\ -\frac{g'}{g}\lambda_L^2 & -\frac{g'}{g}\lambda_R^2 & \frac{g'^2}{g^2}(\lambda_L^2+\lambda_R^2) \end{vmatrix} \frac{g^2}{8} \cdot \qquad (2.6.13)$$

After diagonalizing the mass matrix we find /35/

$$m_0^2 = m_\gamma^2 = 0 \qquad (2.6.14)$$

and

$$m^2_{1,2} \equiv m^2_{Z_1,Z_2} = \frac{g^2}{4}\left(k^2+k'^2+ \frac{\lambda_L^2+\lambda_R^2}{2}\right)^{1/2}\left\{ (1-\epsilon)\,\mathrm{Sec}^2\theta \right.$$

$$\left. +(1+\epsilon) \pm \left[\left(1+\epsilon-(1-\epsilon)\,\mathrm{Sec}^2\theta\right)^2 + 4\sigma^2(1+\mathrm{Tan}^2\theta)\right]^{1/2}\right\} \qquad (2.6.15)$$

where

$$\mathrm{Tan} = \frac{g'\sqrt{2}}{g} \qquad (2.6.16a)$$

$$\epsilon = \frac{k^2+k'^2}{k^2+k'^2+\frac{\lambda_L^2+\lambda_R^2}{2}} \qquad (2.6.16b)$$

and

$$\sigma = \left(\frac{\lambda_L^2-\lambda_R^2}{2}\right)\left(k^2+k'^2+\frac{\lambda_L^2+\lambda_R^2}{2}\right). \qquad (2.6.16c)$$

The physical fields are /35/

$$A_\mu = \frac{\sin\theta}{\sqrt{2}}\left(W^3_{\mu L}+W^3_{\mu R}\right)+B\cos\theta \qquad (2.6.17a)$$

$$Z^1_\mu = Z^A_\mu \cos\varphi - Z^V_\mu \sin\varphi \qquad (2.6.17b)$$

$$Z^2_\mu = Z^V_\mu \cos\varphi + Z^A_\mu \sin\varphi \qquad (2.6.17c)$$

where

$$Z^V_\mu = \frac{\cos\theta}{\sqrt{2}}\left(W^3_{\mu L}+W^3_{\mu R}\right) - B_\mu \sin\theta \qquad (2.6.18a)$$

$$Z^A_\mu = \frac{1}{\sqrt{2}}\left(W^3_{\mu L}-W^3_{\mu R}\right) \qquad (2.6.18b)$$

and

$$\mathrm{Tan}\,2\varphi = \frac{-2\sigma\,\mathrm{Sec}\,\theta}{(1-\epsilon)\,\mathrm{Sec}^2\theta-(1+\epsilon)}\,. \qquad (2.6.19)$$

Substituting W_μ^3s and B_μ in terms of the physical field in the fermion part of the Lagrangian, the neutral current Lagrangian is easily obtained as follows

$$L_{NC} = e A_\mu J^\mu_{em} + \frac{g}{\sqrt{2}} Z_{1\mu} J^\mu_{V-A} + \frac{g}{\sqrt{2}} Z_{2\mu} J^\mu_{V+A} \qquad (2.6.20)$$

where

$$J^\mu_{V-A} \equiv \bar{\psi}_f \gamma^\mu \left\{ \frac{\sin\varphi}{\cos\theta} \left(\frac{1}{2} - \varphi^{em} \sin^2\theta \right) - \frac{1}{2} \cos\varphi \gamma^5 \right\} \psi_f \qquad (2.6.21)$$

and

$$J^\mu_{V+A} \equiv \bar{\psi}_f \gamma^\mu \left\{ \frac{\cos\varphi}{\cos\theta} \left(-\frac{1}{2} + \varphi^{em} \sin^2\theta \right) + \frac{1}{2} \sin\varphi \gamma^5 \right\} \psi_f \,. \qquad (2.6.22)$$

An interesting case happens when $\varphi \to 0$. In this particular case, Eqs. (2.6.17b) and (2.6.17c) reduce to

$$Z_{1\mu} = Z^A_\mu \qquad (2.6.23a)$$

and

$$Z_{2\mu} = Z^V_\mu \qquad (2.6.23b)$$

and L_{NC} of (2.6.20) are simplified as follows

$$L_{NC} = e A_\mu J^\mu_{em} - \frac{g}{\sqrt{2}} \left(Z^V_\mu J^\mu_V + Z^A_\mu J^\mu_A \right) \qquad (2.6.24)$$

in which J^μ_V and J^μ_A correspond to the currents given in (2.6.21) and (2.6.22) in the limit of $\varphi \to 0$.

We observe that in this case the neutral currents conserve parity, while for a general value of φ parity is violated.

When $\varphi \to 0$ from Eqs. (2.6.16c) and (2.6.19) we find that $\lambda_L \to \lambda_R$; remembering so the values of λ_L and λ_R are vacuum expectation values of

left-handed and right-handed scalar doublets X_L and X_R, we conclude that parity violation arises as a result of spontaneous breaking of the gauge symmetry. Indeed, the same mechanism that gives rise to boson and fermion masses in the theory. In this case, the masses of the neutral vector bosons may be obtained from Eq. (2.6.15) as follows

$$m^2_{Z^A} = \frac{g^2}{2}(k^2+k'^2+\lambda^2)(1+\epsilon) \tag{2.6.25}$$

$$m^2_{Z^V} = \frac{g^2}{2}(k^2+k'^2+\lambda^2)(1-\epsilon)\sec^2\theta \tag{2.6.26}$$

where $\lambda_L = \lambda_R = \lambda$. One can show that

$$m^2_{W_L^\pm} = \frac{g^2}{2}(k^2+k'^2+\lambda^2)\,. \tag{2.6.27}$$

This along with Eq. (2.6.35) implies that

$$m^2_{Z^A} > m^2_{W_L^\pm}\,. \tag{2.6.28}$$

We have discussed the low energy behaviour of the model /35/ and it can be shown /36/ that it can reproduce the low energy result of the standard model by rearranging the free parameters of the model /37/, in particular when $\epsilon \to 0$ and $\sin^2\theta \to 2\sin^2\theta_W$.

b) The $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model

Another type of left-right symmetry model that can still accommodate the data is based on the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ gauge symmetry group . Gauge models with this gauge group possess one further neutral gauge boson. Such a structure results from colour flavour symmetric and left-right symmetric grand unified models. Moreover, it may be considered as a natural extention of the case (a). The models discussed in the literature /28,39,40) give also a mass smaller than M_{Z_o} in the standard model for the lightest of the three neutral bosons.

Associated with the subgroup $U(1)_L \times U(1)_R$ are two hypercharge quantum numbers $Y_L/2$ and $Y_R/2$ which are the eigenvalues of the generators T_L° and T_R° respectively. They contribute to the electric charge operator

$$Q = T_{3L} + T_{3R} + T_L^{\circ} + T_R^{\circ} \tag{2.6.29}$$

The typical generation of fermions are grouped into multiplets, characterised by $(T_{3L}, T_{3R}, Y_L, Y_R)$ quantum numbers:

$$\begin{aligned}
&\begin{vmatrix} \nu_e \\ e \end{vmatrix}_L , && (\tfrac{1}{2}, 0, -1, 0) \\
&\begin{vmatrix} \nu_e \\ e \end{vmatrix}_R , && (0, \tfrac{1}{2}, 0, -1) \\
&\begin{vmatrix} u \\ d \end{vmatrix}_L , && (\tfrac{1}{2}, 0, \tfrac{1}{3}, 0) \\
&\begin{vmatrix} u \\ d \end{vmatrix}_R , && (0, \tfrac{1}{2}, 0, \tfrac{1}{3})
\end{aligned} \tag{2.6.30}$$

The scalar fields X_L, X_R, Φ_1 and Φ_2 with the quantum number assignments $(\frac{1}{2},0,1,0)$, $(0,\frac{1}{2},0,1)$, $(0,0,1,-1)$ and $(\frac{1}{2},\frac{1}{2},0,0)$, respectively, develop the following vacuum expectation values:

$$\langle \chi_{L(R)} \rangle = \begin{vmatrix} 0 \\ \lambda_{L(R)} \end{vmatrix} , \quad \langle \Phi_1 \rangle = v \tag{2.6.31}$$

and

$$\langle \Phi_2 \rangle = \begin{vmatrix} k & 0 \\ 0 & k' \end{vmatrix} \tag{2.6.32}$$

to break the underlying symmetry of the theory to the $U(1)_{EM}$ generated by Q_λ of Eq. (2.6.29). This choice of the scalars and the restrictions

$$\varphi_\lambda \equiv \lambda_R^2/\lambda_L^1 \gg 10 \tag{2.6.33}$$

lead to an agreement with the SLAC data on parity violation in e–D scattering /28/.

The neutral sector contains, besides the photon three massive fields; one of them is a pure vector field Z^V_μ and the other two are pure axial fields ($Z^{A}_{\mu}1$, $Z^{A}_{\mu}2$), such that

$$Z^V = \frac{\cos\theta}{\sqrt{2}}(W_3^L + W_3^R) - \frac{\sin\theta}{\sqrt{2}}(B_L + B_R) \qquad (2.6.34a)$$

$$Z^{A_1} = \frac{1}{\sqrt{2}}(W_3^R - W_3^L) \qquad (2.6.34b)$$

$$Z^{A_2} = \frac{1}{\sqrt{2}}(B_L - B_R) \qquad (2.6.34c)$$

and here we have suppressed the space-time indices, and θ is defined by

$$\text{Tan}\,\theta = g/g' \qquad (2.6.35)$$

where $g = g_{L(R)}$ are the $SU(2)_{L(R)}$ and $g' = g'_{L(R)}$ are the $U(1)_{L(R)}$ coupling constants. The mass matrix contains the parameter Q_λ and the value $Q_\lambda \gg 10$, demanded by experiments, guarantees that the mass of the lightest neutral gauge boson is less than M_{Z_o} of the standard model. Masses of the heavier neutral gauge bosons depend weakly on the parameter Q_λ, so that the special choice of Q_λ has only a small influence in the energy range up to 180 Gev /41/.

CHAPTER III. MULTI - NEUTRAL BOSON PROCESSES

3.1 Motivation.

The Salam-Weinberg /10/ unified gauge theory of electroweak interaction, as it is now called the standard theory, has as its basis the two twin principles of local gauge invariance and spontaneous symmetry breaking. These principles are realised through the introduction of vector bosons and scalar (Higgs) particles, whose couplings to one another and to the elementary fermions are supposed to lie at the heart of all weak interaction phenomena.

At the presently available energies the experimental results, such as the neutrino, and the neutral current experiments /23/ are almost in agreement with the standard model of electroweak theory. However, as the experiments are not so accurate, the data does not allow to distinguish between the great variety of suggested electroweak gauge models. Only those have been excluded which conserve parity in electronic neutral current reactions. Decisive experimental tests are still missing: investigation of the high energy behaviour, in particular, production of the intermediate gauge bosons and the discovery of the Higgs scalars.

In order to distinguish between different models it will be necessary to perform many different experiments. Among such experiments, the effects of weak neutral current(s) or the neutral intermediate vector boson(s) in reactions $e^+e^- \to f^+f^-$ (with f being a charged light or heavy fermion) are important; in particular, when the particle beams are polarized.

The presence of weak neutral current(s) produce characteristic effects which would be absent or suppressed if the interactions were purely electromagnetic ones. For example, if the possible result of e^+e^- scattering is the production of a $\mu^+\mu^-$ pair, in the framework of

electroweak models, it can occur through a photon and neutral weak bosons. The interference effects between electromagnetic and weak interactions which is, indeed a direct result of the unification of these two forces, provides contribution to the scattering amplitude which plays a significant role in the parity violation question as well as corrections to the cross-section at present energies.

As we observed in sections (2.5) and (2.6), it is possible that the structure of the neutral current sector shows up to be more complicated than that of the standard model. For example a specific class of "extended gauge theories" of the type $SU(2)_L \times U(1)_Y \times G$ has the same low energy behaviour as the standard model but predicts more that one weak neutral current responsible for electroweak interaction. In view of determining the realistic gauge structure of possible models, it is of particular interest to compare the predictions of $e^+e^- \rightarrow f^+f^-$ reactions for different models and experimental measurements.

Alternative models with more than one neutral boson possess the interesting feature that the mass of the lightest neutral boson is smaller than that of the standard model. Consequntly, there exists a possibility to find resonance behaviour in cross sections below the Z^o mass. Furthermore, there are also other theoretical reasons for considering alternative models. Of particular interest are left-right symmetric modles with spontaneous breakdown of parity invariance. For instance, a neutrino mass different from zero, which is seriously discussed at present, seems to be more natural in such multi-boson models rather than in the standard model.

Our present knowledge permits a test of the low energy aspects of the multi-neutral boson theories. Current efforts are aimed at improving the precision of the low-energy tests and extending their range to reactions not yet explored. On the high energy side, new machines are in the making that will attempt to produce the weak quanta or detect their characteristic effects.

3.2 Notations and Definitions

In this section we set up a model independent analysis for determining the effective interaction Lagrangian, Feynman rules and the notation which we will use throughout our calculations.

Throughout the calculations the following notation and definitions are used:

a) ψ_f is an appropriate spinor which identifies a fermion (f)

b) The Dirac operators $(1+\gamma_5)/2$ and $(1-\gamma_5)/2$ project out the right- and left-handed pieces of the fermions, respectively

c) Q_f is the electric charge of the fermion (f)

d) g_V and g_A are the vector and axial neutral weak coupling constants, respectively

e) The chiral neutral weak coupling constants g_L and g_R are related to g_V and g_A by:

$$g_V = (g_R+g_L)/2 \tag{3.2.1a}$$

$$g_A = (g_R-g_L)/2 \,. \tag{3.2.1b}$$

Subject to the above notations, the neutral current interaction Lagrangian which is relevant for reactions

$$e^+ e^- \longrightarrow f^+ f^- \tag{3.2.2}$$

via neutral intermediate vector boson exchange may be written down as

$$\begin{aligned} L_{NC} &= \bar{\psi}_f \Gamma^0_{\mu f} \psi_f A^\mu + \bar{\psi}_f \Gamma^1_{\mu f} \psi_f Z_1^\mu + \bar{\psi}_f \Gamma^2_{\mu f} \psi_f Z_2^\mu + \cdots \\ &\equiv \sum_j \bar{\psi}_f \Gamma^j_{\mu f} \psi_f Z_j^\mu \,. \end{aligned} \tag{3.2.3}$$

In which we have introduced the compact notation

$$\Gamma^j_{\mu e} = \gamma_\mu (g^j_{Ve} - g^j_{Ae} \gamma_5) ; \quad g^0_{Ve} = -e , \; g^0_{Ae} = 0 \tag{3.2.4a}$$

$$\Gamma^j_{\mu f} = \gamma_\mu (g^j_{Vf} - g^j_{Af} \gamma_5) ; \quad g^0_{Vf} = e Q_f \quad g^0_{Af} = 0 \tag{3.2.4b}$$

where j=1,2,3,...n and γ_μ, (μ=0,1,2,3) is the Dirac matrices. In (3.2.3), the photon couples to a fermion (ψ_f) through the vector current

$$J_\mu^V = \bar{\psi}_f \Gamma_{\mu f}^{\circ} \psi_f \equiv e Q_f \bar{\psi}_f \gamma_\mu \psi_f \qquad (3.2.5)$$

with strength eQ_f and Q_f is the charge of the fermion f, in units of the proton charge 1. Similarly, the weak neutral intermediate vector boson, Z_j^μ, is defined to couple to a weak (V–A) neutral current

$$(J_\mu^j)_W = \bar{\psi}_f \Gamma_{\mu f}^{j} \psi_f \equiv g_{Vf}^{j} \bar{\psi}_f \gamma_\mu \psi_f - g_{Af}^{j} \bar{\psi}_f \gamma_\mu \gamma_5 \psi_f$$

$$= J_{\mu V}^{j} - J_{\mu A}^{j} \,. \qquad (3.2.6)$$

We now recall the Feynman rules which we use in our calculations. Our conventions are those of Ref. (36), and the Feynman rules, in analogy with the standard model, are summarized in Fig. (3.2.1) which, of course, correspond to the unitary gauge

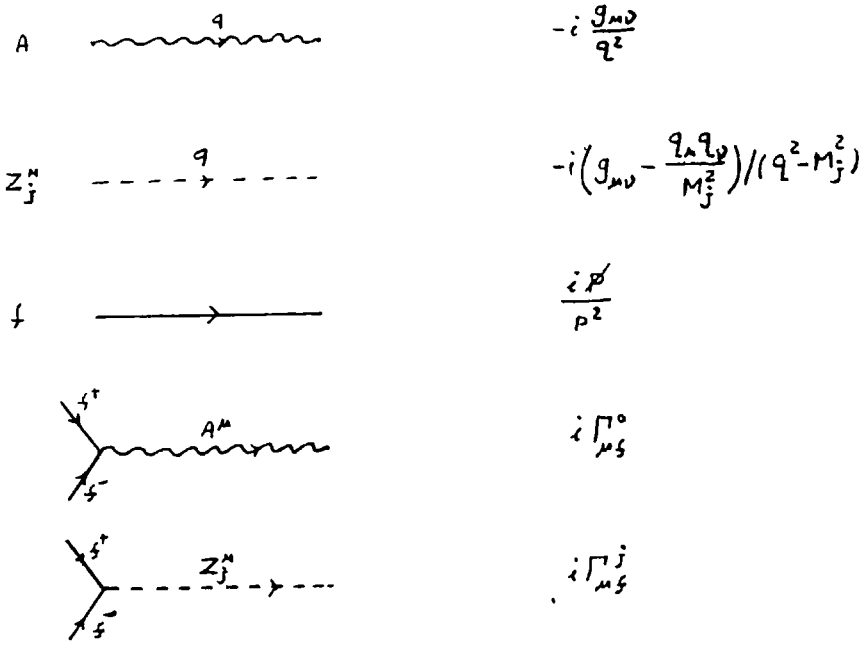


Fig. (3.2.1) Feynman rules, in analogy with the standard model.

The kinematics of the process $e^+e^- \longrightarrow f^+f^-$ is defined in the centre-of-mass frame for the electron positron system where (K_+, S_+) and (K_-, S_-) are the momentum and spin four-vectors of the incoming positron and electron, respectively, and the final state (f^-, f^+) momentum and spin four vector are denoted by (K'_+, S'_+) and (K'_-, S'_-). The variables of interest are (E) the energy of the e^+ and e^- beams and

$$q = K_+ + K_- = K'_+ + K'_- \quad . \tag{3.2.7}$$

By assuming that the collision takes place in the centre of mass system, in terms of electron and positron energy beams E, we have

$$K_+ = E(1,0,0,\beta) \tag{3.2.8a}$$

$$K_- = E(1,0,0,-\beta) \tag{3.2.8b}$$

where $E = K^\circ_+ = K^\circ_- = k'^\circ_+ = k'^\circ_-$

$$\beta = |\underline{k}_+|/E = |\underline{k}_-|/E$$

$$= \frac{(E^2 - m^2)^{1/2}}{E} \equiv (1 - \gamma^{-2})^{1/2} \quad . \tag{3.2.9}$$

The other parameters involved will be introduced whenever needed.

3.3 Differential Cross Section

The purpose of this section is to give a general expression for the cross section and the scattering amplitude of the reaction (3.2.2) in which an arbitrary number of neutral bosons are involved. The calculation is first presented for unpolarized beam and then both, initial and final state polarization effects are taken into account.

Expressions for the cross section are divided into two parts: first the invariant amplitude $\mathcal{M}$ which is a Lorentz scalar and in which physics lies, and second the phase space and kinetic factors. In terms of $\mathcal{M}$ the expression for the differential cross section is

$$d\sigma = \frac{1}{2}(2E)^{-2}\left|\mathcal{M}\right|^2(2\pi)^{-2}\frac{d^3k'_-}{2E}\cdot\frac{d^3k'_+}{2E}\times$$

$$\delta^4(k_+ + k_- - k'_+ - k'_-), \qquad (3.3.1)$$

in which we have adopted the notation of Ref. (36). The scattering amplitude $|\mathcal{M}|^2$ is computed by drawing all Feynman graphs for the process in question. According to our notation we have

$$\left|\mathcal{M}\right|^2 = \left|\quad \overset{q}{\underset{A}{\wedge}} \quad + \sum_j \quad \overset{q}{\underset{Z_j}{\wedge}} \quad\right|^2$$

Fig. (3.3.1) Diagrams contributing in the lowest order to the $e^+e^- \longrightarrow f^+f^-$ process.

Let us first consider the case in which the initial and final states are both unpolarized. Using the Lagrangian (3.2.3) we find

$$\left|\mathcal{M}\right|^2 = \frac{16\pi^2\alpha^2}{q^4}\left\{\frac{\mathcal{P}_s^2}{4}\mathrm{Tr}\left[(k_+ - m_e)\gamma_\mu(k_- + m_e)\gamma_\nu\right]\mathrm{Tr}\left[(k'_- - m_f)\gamma^\mu(k'_+ + m_f)\gamma^\nu\right] +\right.$$

$$+ 2\,\mathrm{Re}\,\frac{\varphi_f}{4}\sum_{k\geq 1}\frac{e^{-2}q^2}{q^2-M_k^2}\left(g^{\rho\nu}-\frac{q^\rho q^\nu}{M_k^2}\right)g^{\sigma\mu}\times$$

$$\mathrm{Tr}\left[(\not k_+ - m_e)\gamma_\mu(\not k_- + m_e)\Gamma_{\nu e}^{k}\right]\mathrm{Tr}\left[(\not k'_- - m_f)\gamma_\rho(\not k'_+ + m_f)\bar\Gamma_{\sigma f}^{k}\right] +$$

$$+\frac{1}{4}\sum_{j,k}\frac{e^{-2}q^2}{q^2-M_k^2}\left(g^{\rho\nu}-\frac{q^\rho q^\nu}{M_k^2}\right)\frac{e^{-2}q^2}{q^2-M_j^2}\left(g^{\sigma\mu}-\frac{q^\sigma q^\mu}{M_j^2}\right)\times$$

$$\mathrm{Tr}\left[(\not k_+ - m_e)\Gamma_{\mu e}^{j}(\not k_- + m_f)\bar\Gamma_{\nu e}^{k}\right]\mathrm{Tr}\left[(\not k'_+ - m_f)\Gamma_{\rho f}^{j}(\not k'_- + m_f)\bar\Gamma_{\sigma f}^{k}\right]\Big\}\,, \qquad (3.3.2)$$

where m_e and m_f are the electron and fermion masses respectively,

$\bar\Gamma \equiv \gamma^0\Gamma^\dagger\gamma^0$ and $\not k = \gamma^\mu k_\mu$.

Evaluating the traces in (3.3.2) can be carried out by applying the trace theorems in Appendix C. In particular (A.16c) is very useful in reducing the trace of five γ matrices, for example

$$\mathrm{Tr}\,\gamma_5\not k_+\gamma_\mu\not k_-\gamma_\nu = 4i\,\epsilon_{\alpha\mu\beta\nu}\,k_+^\alpha\,k_-^\beta\,.$$

Simplifying the result for relativistic energies $E \gg m$ and neglecting terms proportional to m^2, the purely electromagnetic term (i.e. the first term) is easily calculated. We find

$$|\mathcal{M}|^2_{EM} = \frac{16\pi^2\alpha^2}{q^4}\,\frac{\varphi_f^2}{4}\cdot 32\left((k_+\cdot k'_+)(k_-\cdot k'_-)+(k_+\cdot k'_-)(k_-\cdot k'_+)\right), \qquad (3.3.3)$$

while the electroweak term (i.e. the second term) which is the interference between electromagnetic and weak interactions is obtained as

$$|\mathcal{M}|^2_{EW} = \frac{128\pi^2\alpha^2}{q^4}\cdot\varphi_f\sum_{k\geq 1}2\mathrm{Re}\cdot\Big\{g_{ve}^{k}\,g_{vf}^{*k}\left((k_+\cdot k'_+)(k_-\cdot k'_-)+(k_+\cdot k'_-)(k_-\cdot k'_+)\right)+$$

$$g_{Ae}^{k}\,g_{Af}^{*k}\left((k_+\cdot k'_+)(k_-\cdot k'_-)-(k_-\cdot k'_-)(k_+\cdot k'_-)\right)\Big\}\frac{e^{-2}q^2}{q^2-M_k^2}\,. \qquad (3.3.4)$$

And finally the purely weak interaction scattering amplitude is given by

$$|\mathcal{M}|^2_W = \frac{128\pi^2\alpha^2}{q^2} \sum_{j,k} \Big\{ (g^j_{Ve} g^{*k}_{Ve} + g^j_{Ae} g^{*k}_{Ae})(g^j_{Vf} g^{*k}_{Vf} + g^j_{Af} g^{*k}_{Af}) \times$$

$$\big((k_+\cdot k'_+)(k_-\cdot k'_-) + (k_+\cdot k'_-)(k_-\cdot k'_+)\big) + \big((k_+\cdot k'_+)(k_-\cdot k'_-) - (k_-\cdot k'_+)(k_+\cdot k'_-)\big) \times$$

$$(g^j_{Ve} g^{*k}_{Ae} + g^j_{Ae} g^{*k}_{Ve})(g^j_{Vf} g^{*k}_{Af} + g^j_{Af} g^{*k}_{Vf}) \Big\} \frac{e^{-4} q^4}{(q^2 - M_k^2)(q^2 - M_j^2)} \quad . \qquad (3.3.5)$$

The total scattering amplitude is obtained by using Eqs. (3.3.5), (3.3.4) and (3.3.3); for Eq. (3.3.2), one finds

$$|\mathcal{M}|^2 = \frac{128\pi^2\alpha^2}{q^4} \big((k_+\cdot k'_+)^2 + (k_+\cdot k'_-)^2\big) \Big\{ \mathcal{P}_f^2 + \mathcal{P}_f \cdot \sum_{k\gg 1} 2\mathrm{Re} \Big(g^k_{Ve} g^{*k}_{Vf} +$$

$$\frac{(k_+\cdot k'_+)^2 - (k_+\cdot k'_-)^2}{(k_+\cdot k'_+)^2 + (k_+\cdot k'_-)^2} \; g^k_{Ae} g^{*k}_{Af} \Big) \frac{e^{-2} q^2}{q^2 - M_k^2} +$$

$$\sum_{j,k\gg} \Big((g^j_{Ve} g^{*k}_{Ve} + g^j_{Ae} g^{*k}_{Ae})(g^j_{Vf} g^{*k}_{Vf} + g^j_{Af} g^{*k}_{Af}) + \frac{(k_+\cdot k'_+)^2 - (k_+\cdot k'_-)^2}{(k_+\cdot k'_+)^2 + (k_+\cdot k'_-)^2} \times$$

$$(g^j_{Ve} g^{*k}_{Ae} + g^j_{Ae} g^{*k}_{Ve})(g^j_{Vf} g^{*k}_{Af} + g^j_{Af} g^{*k}_{Vf}) \Big) \frac{e^{-4} q^4}{(q^2 - M_k^2)(q^2 - M_j^2)} \Big\} . \qquad (3.3.6)$$

The quantity that can be compared with experimental results in this case (i.e. unpolarized bosons) is the value $R(q^2) = d\sigma/d\sigma_{QED}$ which is given by

$$R(q^2) = \mathcal{P}_f^2 + 2\mathrm{Re}.\ \mathcal{P}_f \sum_{k\gg 1} \frac{e^{-2} q^2}{q^2 - M_k^2} \; g^{*k}_{Ve} \; g^{*k}_{Vf} +$$

$$+ \sum_{j,k\gg 1} \frac{e^{-4} q^4}{(q^2 - M_j^2)(q^2 - M_k^2)} (g^j_{Ve} g^{*k}_{Ve} + g^j_{Ae} g^{*k}_{Ae})(g^j_{Vf} g^{*k}_{Vf} + g^j_{Af} g^{*k}_{Af}) . \qquad (3.3.7)$$

The last term in expression (3.3.7) is of the order of g^4, i.e. (weak coupling)^2 and hence very small. However the second term is a purely leptonic process and by using the (e–l) universality is proportional to $\sum_{k\gg 1} (g^k_{Ae})^2$ and $\sum_{k\gg 1} (g^k_{Ve})^2$. This may be compared with measurable quantities though it does <u>not</u> contain the $g_A g_V$ terms. When polarization effects are taken into account the interference between electromagnetic and weak interactions, (i.e. corresponding to the second term in 3.3.7) picks up terms

proportional to $g_V g_A$. It is, in fact, this term which makes the polarization effects so important in e^+e^- physics. To see this let us consider the case in which both the initial and final states are arbitrarily polarized.

If the initial and final states are polarized, the quantity $|m|^2$ in (3.3.1) is given by

$$
\begin{aligned}
|m|^2 = 16\pi^2\alpha^2 q^{-4}\Big\{ & \varphi_f^2\, Tr\Big[(\not k_- - m_e)\tfrac{1}{2}(1+\gamma_5 \not s_-)\gamma^\mu(\not k_+ + m_e)\tfrac{1}{2}(1+\gamma_5 \not s_+)\gamma^\nu\Big] \times \\
& \times Tr\Big[(\not k'_- - m_f)\tfrac{1}{2}(1+\gamma_5 \not s'_-)\gamma_\mu(\not k'_+ + m_f)\tfrac{1}{2}(1+\gamma_5 \not s'_+)\gamma_\nu\Big] + \\
& + 2\varphi_f \, Re. \sum_{k\gg 1} \frac{e^{-2}q^2}{q^2 - M_k^2}\left(g^{\rho\nu} - \frac{q^\rho q^\nu}{M_k^2}\right) g^{\sigma\mu} \times \\
& \times Tr\Big[(\not k_- - m_e)\tfrac{1}{2}(1+\gamma_5 \not s_-)\gamma_\mu(\not k_+ + m_e)\tfrac{1}{2}(1+\gamma_5 \not s_+)\Gamma^k_{\nu e}\Big] \times \\
& \times Tr\Big[(\not k'_- - m_f)\tfrac{1}{2}(1+\gamma_5 \not s'_-)\gamma_\rho(\not k'_+ + m_f)\tfrac{1}{2}(1+\gamma_5 \not s'_+)\bar\Gamma^k_{\sigma f}\Big] \\
& + \sum_{j,k\gg 1} \frac{e^{-2}q^2}{q^2 - M_k^2}\left(g^{\rho\nu} - \frac{q^\rho q^\nu}{M_k^2}\right)\frac{e^{-2}q^2}{q^2 - M_j^2}\left(g^{\sigma\mu} - \frac{q^\sigma q^\mu}{M_j^2}\right) \times \\
& \times Tr\Big[(\not k_- - m_e)\tfrac{1}{2}(1+\gamma_5 \not s_-)\Gamma^j_{\mu e}(\not k_+ + m_e)\tfrac{1}{2}(1+\gamma_5 \not s_+)\bar\Gamma^k_{\nu e}\Big] \times \\
& \times Tr\Big[(\not k'_- - m_f)\tfrac{1}{2}(1+\gamma_5 \not s'_-)\Gamma^j_{\rho f}(\not k'_+ + m_f)\tfrac{1}{2}(1+\gamma_5 \not s'_+)\bar\Gamma^k_{\sigma f}\Big]\Big\}.
\end{aligned}
\tag{3.3.8}
$$

The first term represents the purely electromagnetic effects that is identified as

$$
|m|^2_{EM} = \pi^2\alpha^2 q^{-4}\varphi_f^2 E^{\mu\nu}E'_{\mu\nu} \tag{3.3.9}
$$

and the value of $E^{\mu\nu}$ is obtained by noting that the product of the odd number of γ traces is zero. It then reads

$$
\begin{aligned}
E^{\mu\nu} = Tr\,(& \not k_-\gamma^\mu \not k_+\gamma^\nu - m_e \not k_-\gamma^\mu \not s_-\gamma^\nu\gamma_5 + \not k_- \not s_-\gamma^\mu \not k_+ \not s_+\gamma^\nu + \\
& m_e \not k_- \not s_-\gamma^\mu\gamma^\nu + m_e\gamma^\mu \not k_+ \not s_+\gamma^\nu\gamma_5 - m_e^2\gamma^\mu\gamma^\nu + \\
& - m_e^2 \not s_-\gamma^\mu \not s_+\gamma^\nu + m_e \not s_-\gamma^\mu \not k_+\gamma^\nu\gamma_5\,).
\end{aligned}
\tag{3.3.9a}
$$

The tensor $E^{\mu\nu}$ is easily computed by using the trace theorems of Appendix C, we find

$$
\begin{aligned}
E^{\mu\nu} = 4\Big(& k_-^\mu k_+^\nu - g^{\mu\nu} k_+\cdot k_- + k_+^\mu k_-^\nu + \\
& s_+\cdot s_- (k_+^\mu k_-^\nu + k_+^\nu k_-^\mu) - k_-\cdot s_+ (k_+^\mu k_-^\nu + k_+^\nu s_-^\mu) + \\
& k_+\cdot k_- (s_+^\mu s_-^\nu - g^{\mu\nu} s_+\cdot s_- + s_-^\mu s_+^\nu - m_e i\epsilon^{\alpha\mu\beta\nu} k_{+\alpha} k_{-\beta} + \\
& - k_+\cdot s_- (s_+^\mu k_-^\nu - s_+\cdot k_- g^{\mu\nu} + s_+^\nu k_-^\mu) \\
& + m_e i\epsilon^{\alpha\mu\beta\nu} s_{-\alpha} s_{+\beta} - m_e i\epsilon^{\alpha\mu\beta\nu} k_{-\alpha} s_{-\beta} + \\
& - m_e i\epsilon^{\alpha\mu\beta\nu} k_{+\alpha} s_{+\beta} - m_e^2 g^{\mu\nu} + \\
& - m_e^2 (s_-^\mu s_+^\nu - g^{\mu\nu} s_-\cdot s_+ + s_-^\nu s_+^\mu)\Big). \qquad (3.3.9b)
\end{aligned}
$$

The final state fermionic term, $E'_{\mu\nu}$, can be obtained in a similar manner, it turns out that the value of $E'_{\mu\nu}$ can be constructed by replacing the quantities of the initial state by the final state, i.e. the unprimed quantities are changed to primed ones. We will also apply this simple notation, whenever possible, in our next calculation.

The interference between weak and electromagnetic interactions is given in the second term of expression (3.3.8). That is

$$
\begin{aligned}
|m|^2_{EW} = {} & \pi^2\alpha^2 q^{-4} \mathcal{P}_f R_e \cdot \sum_{k\,l} \frac{e^{-2} q^2}{q^2 - M_k^2} \times \\
& \left(g^{\rho\nu} - \frac{q^\rho q^\nu}{M_k^2}\right) g^{\sigma\mu} E^{W,k}_{\mu\nu} E'^{W,k}_{\rho\sigma}. \qquad (3.3.10)
\end{aligned}
$$

After some computation and cancelling out the terms containing the trace of odd number of γ matrices, the value of $E^{W,k}_{\mu\nu}$ reads

$$
\begin{aligned}
E^{W,k}_{\mu\nu} = {} & g^k_{ve}\, Tr\big(\not k_+ \gamma_\mu \not k_- \gamma_\nu - \not k_+ \not s_+ \gamma_\mu \not k_- \not s_- \gamma_\nu - m_e \not k_+ \gamma_\mu \not s_- \gamma_\nu \gamma_5 \\
& - m_e \not s_+ \gamma_\mu \not k_- \gamma_\nu \gamma_5 - m \not k_+ \not s_+ \gamma_\mu \gamma_\nu - m_e \gamma_\mu \not k_- \not s_- \gamma_\nu \gamma_5 \\
& - m_e^2 \not s_+ \gamma_\mu \not s_- \gamma_\nu - m_e^2 \gamma_\mu \gamma_\nu\big) + \\
& g^k_{Ae}\, Tr\big(-\not k_+ \gamma_\mu \not k_- \gamma_\nu \gamma_5 - \not k_+ \not s_+ \gamma_\mu \not k_- \not s_- \gamma_\nu \gamma_5 - m_e \not k_+ \gamma_\mu \not s_+ \gamma_\nu +
\end{aligned}
$$

$$+ m_e \not{s}_+ \gamma_\mu \not{k}_- \gamma_\nu + m_e \not{k}_+ \not{s}_+ \gamma_\mu \gamma_\nu - m_e \gamma_\mu \not{k}_- \not{s}_- \gamma_\nu +$$

$$- m_e^2 \gamma_\mu \gamma_\nu + m_e^2 \not{s}_+ \gamma_\mu \not{s}_- \gamma_\nu \gamma_5) . \qquad (3.3.10a)$$

Using the trace theorems of Appendix C we find

$$\begin{aligned} E^{w.k}_{\mu\nu} = {} & 4 g^k_{ve} \Big(k_{+\mu} k_{-\nu} - g_{\mu\nu} k_+ \cdot k_- + k_{+\nu} k_{-\mu} - s_+ \cdot s_- (k_{+\mu} k_{-\nu} + \\ & k_{+\nu} k_{-\mu}) + k_- \cdot s_+ (k_{+\mu} s_{-\nu} + k_{+\nu} s_{-\mu}) + \\ & - k_+ \cdot k_- (s_{+\mu} s_{-\nu} - s_+ \cdot s_- g_{\mu\nu} + s_{+\nu} s_{-\mu}) \\ & + k_+ \cdot s_- (s_{+\mu} k_{-\nu} - k_+ \cdot k_- g_{\mu\nu} + s_{+\nu} k_{-\mu}) + \\ & m_e i \epsilon_{\alpha\mu\beta\nu} (k_+^\alpha s_-^\beta - s_+^\alpha k_-^\beta + k_+^\alpha s_+^\beta - k_-^\alpha s_-^\beta) \\ & - m_e^2 (s_{+\mu} s_{-\nu} - g_{\mu\nu} s_+ \cdot s_- + s_{+\nu} s_{-\mu} - g_{\mu\nu}) \Big) + \\ & 4 g^k_{Ae} \Big(- i \epsilon_{\alpha\mu\beta\nu} k_+^\alpha k_-^\beta + i \epsilon_{\alpha\beta\mu\gamma} k_+^\alpha s_-^\beta k_{+\mu} k_-^\gamma \\ & - i \epsilon_{\alpha\beta\mu\gamma} k_+^\alpha s_+^\beta k_{-\nu} s_-^\gamma + i \epsilon_{\alpha\nu\beta\gamma} k_+^\alpha s_-^\beta k_{+\mu} k_-^\gamma \\ & - i \epsilon_{\alpha\nu\beta\gamma} k_+^\alpha s_{+\mu} s_-^\beta k_-^\gamma - m_e (k_{+\mu} s_{-\nu} - g_{\mu\nu} k_+ \cdot s_- + k_{+\nu} s_{-\mu} \\ & - s_{+\mu} k_{-\nu} + g_{\mu\nu} s_+ \cdot k_- - s_{+\nu} k_{-\mu} + s_{-\mu} k_{-\nu} + s_{-\nu} k_{-\mu} - s_{+\mu} k_{+\nu} \\ & - s_{+\nu} k_{+\mu}) + m_e^2 (i \epsilon_{\alpha\mu\beta\nu} s_+^\alpha s_-^\beta - g_{\mu\nu}) \Big) , \end{aligned} \qquad (3.3.10b)$$

and by changing the electronic variables to the corresponding values of fermionic ones, the fermionic part contained in the tensor $E'^{w.k}_{\mu\nu}$ of Eq. (3.3.10a) can be easily built up from Eq. (3.3.10b).

Finally, the purely weak interaction scattering amplitude, the last terms in Eq. (3.3.8) can be written down in terms of the tensor $W^{jk}_{\mu\nu}$ in the following way

$$\begin{aligned} |\mathcal{M}|^2_w = {} & \pi^2 \alpha^2 q^{-4} \sum_{j,k \geq 1} \frac{e^{-2} q^2}{q^2 - M_j^2} \left(g^{\rho\nu} - \frac{q^\rho q^\nu}{M_j^2} \right) \times \\ & \frac{e^{-2} q^2}{q^2 - M_k^2} \left(g^{\sigma\mu} - \frac{q^\sigma q^\mu}{M_k^2} \right) W^{jk}_{\mu\nu} W'^{jk}_{\rho\sigma} \end{aligned} \qquad (3.3.11)$$

where $W^{jk}_{\mu\nu}$ is given by

$$W^{jk}_{\mu\nu} = (g^{j}_{Ve} g^{*k}_{Ve} + g^{j}_{Ae} g^{*k}_{Ae}) W^{1}_{\mu\nu} + (g^{j}_{Ve} g^{*k}_{Ae} + g^{j}_{Ae} g^{*k}_{Ve}) W^{2}_{\mu\nu} +$$

$$+ (g^{j}_{Ve} g^{*k}_{Ve} - g^{j}_{Ae} g^{*k}_{Ae}) W^{3}_{\mu\nu} + (g^{j}_{Ve} g^{*k}_{Ae} - g^{j}_{Ae} g^{*k}_{Ve}) W^{4}_{\mu\nu} \qquad (3.3.11a)$$

and

$$W'^{jk}_{\mu\nu} = (g^{*j}_{V\xi} g^{k}_{V\xi} + g^{j}_{A\xi} g^{k}_{A\xi}) W^{1}_{\mu\nu} + (g^{*j}_{V\xi} g^{k}_{A\xi} + g^{*j}_{A\xi} g^{k}_{V\xi}) W^{2}_{\mu\nu} +$$

$$+ (g^{*j}_{V\xi} g^{k}_{V\xi} - g^{*j}_{A\xi} g^{k}_{A\xi}) W^{3}_{\mu\nu} + (g^{*j}_{V\xi} g^{k}_{A\xi} - g^{*j}_{A\xi} g^{k}_{V\xi}) W^{4}_{\mu\nu} . \qquad (3.3.11b)$$

Applying the trace theorems of Appendix C for (3.3.11), the value of the W's are easily calculated as

$$W^{1}_{\mu\nu} = 4 \Big(k_{+\mu} k_{-\nu} - g_{\mu\nu} k_{+} \cdot k_{-} + k_{+\nu} k_{-\mu} + i m_e \epsilon_{\alpha\mu\beta\nu} (k^{\alpha}_{+} s^{\beta}_{-} - s^{\alpha}_{+} k^{\beta}_{-}) - (s_{+\mu} s_{-\nu} - g_{\mu\nu} s_{+} \cdot s_{-} + s_{+\nu} s_{-\mu}) m_e^2 \Big) , \qquad (3.3.11c)$$

$$W^{2}_{\mu\nu} = 4 \Big(i \epsilon_{\alpha\mu\beta\nu} (m_e^2 s^{\alpha}_{+} s^{\beta}_{-} - k^{\alpha}_{+} k^{\beta}_{-}) + m_e (k_{+\mu} s_{-\nu} - g_{\mu\nu} k_{+} \cdot s_{-} + k_{+\nu} k_{-\mu} + (s_{+\mu} k_{-\nu} - g_{\mu\nu} s_{+} \cdot k_{-} + s_{+\nu} s_{-\mu}) m_e^2 \Big) , \qquad (3.3.11d)$$

$$W^{3}_{\mu\nu} = 4 \Big(s_{+} \cdot s_{-} (k_{+\mu} k_{-\nu} + k_{+\nu} k_{-\mu}) - s_{+} \cdot k_{-} (k_{+\mu} s_{-\nu} + k_{+\nu} s_{-\mu}) - m_e^2 g_{\mu\nu} + k_{+} \cdot k_{-} (s_{+\mu} s_{-\nu} - s_{+} \cdot s_{-} g_{\mu\nu} + s_{+\nu} s_{-\mu}) - k_{+} \cdot s_{-} (s_{+\mu} k_{-\nu} - s_{+} \cdot k_{-} g_{\mu\nu} + s_{+\nu} k_{-\mu}) - i \epsilon_{\alpha\mu\beta\nu} (k^{\alpha}_{+} k^{\beta}_{+} + k^{\alpha}_{-} s^{\beta}_{-}) \Big) , \qquad (3.3.11e)$$

$$W^{4}_{\mu\nu} = 4 \Big(i \epsilon_{\alpha\mu\beta\gamma} k^{\alpha}_{+} s^{\beta}_{+} (s_{-\nu} k^{\gamma}_{-} - k_{-\nu} s^{\gamma}_{-}) + i \epsilon_{\alpha\nu\beta\gamma} s^{\beta}_{-} k^{\gamma}_{-} (k_{+\mu} s^{\alpha}_{+} - s_{+\mu} k^{\alpha}_{+}) + m_e (k_{+\mu} s_{+\nu} + k_{+\nu} s_{+\mu} + k_{-\mu} s_{-\nu} + k_{-\nu} s_{+\mu}) \Big) . \qquad (3.3.11f)$$

As was mentioned, the values of W' in Eq. (3.3.11b) can easily be constructed from the values of W through Eqs. (3.3.11c), (3.3.11d), (3.3.11e) and (3.3.11f).

The expression (3.3.9) may now be symbolically written as

$$|\mathcal{M}|^2 = |\mathcal{M}|^2_{EM} + |\mathcal{M}|^2_{EW} + |\mathcal{M}|^2_{W} \qquad (3.3.8)$$

with $|\mathcal{M}|^2_{EM}$, $|\mathcal{M}|^2_{EW}$ and $|\mathcal{M}|^2_{W}$ given by Eqs. (3.3.9), (3.3.10) and (3.3.11) respectively. This is quite general in the following sense:

i) It is applicable for the processes in which an arbitrary number of neutral bosons including the photon are exchanged.

ii) It can be applied for heavy and light fermions.

iii) It can be used for the processes in which the initial and final states are arbitrarily polarized.

There is also no restriction on the coupling g's hence it may be used for testing the (e–l) universality in purely leptonic reactions. Note also that if only the initial states are polarized and the final state polarization effects are <u>not</u> desired a factor of 4 must be taken into account.

To see these things more clearly, let us examine the case for which only the colliding e^+e^- beams are polarized and involve two neutral exchange particles, the photon and one weak neutral boson responsible for electroweak interactions. Such a case, obviously, is useful for those electroweak gauge models which predict only one massive weak neutral gauge boson as well as a massless photon. Now after, in addition to the kinematic variables given in Eqs. (3.2.8a) and (3.2.8b), we specify the spin and momentum four vectors $S_\pm$ and $K'_\pm$ in the centre of mass system as follows

$$S_+^\mu = P_+ (\beta\gamma \cos\theta_+, \sin\theta_+\cos\phi_+, \sin\theta_+\sin\phi_+, \cos\theta_+), \qquad (3.3.12a)$$

$$S_-^\mu = P_- (-\beta\gamma \cos\theta_-, \sin\theta_-\cos\phi_-, \sin\theta_-\sin\phi_-, \cos\theta_-), \qquad (3.3.12b)$$

$$k'^{\mu}_{+} = E(1, \beta' \sin\theta \cos\phi, \beta' \sin\theta \sin\phi, \beta' \cos\theta) , \qquad (3.3.12c)$$

$$k'^{\mu}_{-} = E(1, -\beta' \sin\theta \cos\phi, -\beta' \sin\theta \sin\phi, -\beta' \cos\theta) . \qquad (3.3.12d)$$

This definition of the coordinate system is shown graphically in Fig. (3.3.2).

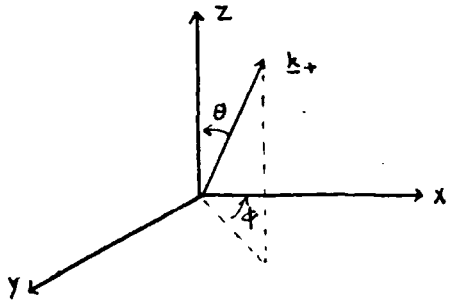


Fig. (3.3.2) Definition of coordinate system

In definition (3.3.12) the quantities β and γ have already been defined and $P_+(P_-)$ is the magnitude of the polarization of the positron (electron).

Evaluation of the scattering amplitude $|\mathcal{M}|^2$, in Eq. (3.3.8) can easily be carried out by substituting the kinematic variables of (3.3.12) and (3.2.8) for the corresponding quantities in (3.3.8). Inserting the resulting value of $|\mathcal{M}|^2$ in (3.3.1) and applying Appendix C one can then find the angular distribution for the reaction in question as

$$\begin{aligned}\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2}\Big\{ & (1+\cos^2\theta)(1+P_+P_-\cos\theta_+\cos\theta_-)F_1(q^2) + \\ & P_+P_-\sin^2\theta\sin\theta_+\sin\theta_-\cos(2\phi-\phi_+-\phi_-)F_2(q^2) + \\ & 2\cos\theta(1+P_+P_-\cos\theta_+\cos\theta_-)F_3(q^2) + \\ & (1+\cos^2\theta)(P_+\cos\theta_+ + P_-\cos\theta_-)F_4(q^2) + \\ & 2\cos\theta(P_+\cos\theta_+ + P_-\cos\theta_-)F_5(q^2) + \\ & -P_+P_-\sin^2\theta\sin\theta_+\sin\theta_-\sin(2\phi-\phi_+-\phi_-)F_6(q^2)\Big\} \end{aligned} \qquad (3.3.13)$$

in which we have neglected the terms of order m/E and the "form factors" F's are given by

$$F_1(q^2) = 1 + 2h(q^2)\,\mathrm{Re}(g^{V}_{Ve}\, g'_{Vs}) + h^2(q^2)(|g'_{Ve}|^2+|g'_{Ae}|^2)(|g'_{Vs}|^2+|g'_{As}|^2) , \qquad (3.3.14a)$$

$$F_2(q^2) = 1 + 2h(q^2)\,\mathrm{Re}\,(g'^{*}_{Ve}\, g'_{Vf}) + h^2(q^2)\,(|g'_{Ve}|^2 - |g'_{Ae}|^2)(|g'_{Vf}|^2 + |g'_{Af}|^2) \quad , \qquad (3.3.14b)$$

$$F_3(q^2) = 2h(q^2)\,\mathrm{Re}\,(g'^{*}_{Ae}\, g'_{Af}) + h^2(q^2)\,\mathrm{Re}\,(2g'^{*}_{Ve}\, g'_{Ae})\,\mathrm{Re}\,(2g'^{*}_{Vf}\, g'_{Af}) \quad , \qquad (3.3.14c)$$

$$F_4(q^2) = 2h(q^2)\,\mathrm{Re}\,(g'_{Vf}\, g'^{*}_{Ae}) + h^2(q^2)\,\mathrm{Re}\,(2g_{Ve}\, g'^{*}_{Ae})(|g'_{Vf}|^2 + |g'_{Af}|^2) \quad , \qquad (3.3.14d)$$

$$F_5(q^2) = 2h(q^2)\,\mathrm{Re}\,(g'^{*}_{Ve}\, g'_{Af}) + h^2(q^2)\,\mathrm{Re}\,(2g'_{Vf}\, g'^{*}_{Af})(|g'_{Ve}|^2 + |g'_{Ae}|^2) \quad , \qquad (3.3.14e)$$

$$F_6(q^2) = 2h(q^2)\,\mathrm{Im}\,(g'_{Vf}\, g'^{*}_{Ae}) + h^2(q^2)\,\mathrm{Im}\,(2g'_{Ve}\, g'_{Ae})(|g'_{Vf}|^2 + |g'_{Af}|^2) \quad , \qquad (3.3.14f)$$

where

$$h(q^2) = \frac{e^{-2}q^2}{q^2 - M_1^2} \quad . \qquad (3.3.15)$$

The angular distribution given by (3.3.13) is consistent with the result of Ref. (37) when the electron-positron beams are longitudinally polarized. We now summarize some physical interpretations resulting from Eqs. (3.3.13) and (3.3.14). From an observation of Eqs. (3.3.13) and (3.3.14), it is realised that if any measurement of angular distribution indicates a non-zero value for the form factors $F_3 \cdots F_6$ this would imply the existence of a weak neutral current. Secondly, if the experiments show a non-zero value for the coefficients F_4 and F_5 parity is not conserved in a world described by the associated electroweak theory which identifies the couplings g'_{Vf}, g'_{Ae}. Thirdly, if F_6 is non-zero time reversal invariance is violated. And finally, in the purely leptonic processes if the coefficient F_3 is positive (e-l) universality would be violated for energies $M_1^2 > q^2$. We point out that all these statements are correct when the effects of higher order (i.e. (weak)^2 terms) are neglected.

Similarly, if we neglect the terms of order m_e/E in the calculation of $|m|^2$ in Eq. (3.3.8), the angular distribution for arbitrarily polarized e^+e^- beams yield

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2}\beta' \left\{ \left((\beta'^{-1} + \beta' \cos^2\theta)(1 + P_+P_- \cos\theta_+ \cos\theta_-) \right) H_1(q^2) + \right.$$

$$+\Big((\beta'^{-1}+\beta')\cos\theta\,(P_+\cos\theta_+ + P_-\cos\theta_-)\Big)H_2(q^2)+$$

$$+\Big(\gamma'^{-2}(\beta'^{-1}+\beta'^{-1}P_+P_-\cos\theta_+\cos\theta_-)\Big)H_3(q^2)+$$

$$+\Big((\beta'^{-1}+\beta'\cos^2\theta)(P_+\cos\theta_+ + P_-\cos\theta_-)\Big)H_4(q^2)+$$

$$+\Big(2(1+P_+P_-\cos\theta_+\cos\theta_-)\cos\theta\Big)H_5(q^2)+$$

$$+\Big(\beta'P_+P_-\sin^2\theta\sin\theta_+\sin\theta_-\cos(2\phi-\phi_+-\phi_-)\Big)H_6(q^2)+$$

$$+\Big(\beta'P_+P_-\sin^2\theta\sin\theta_+\sin\theta_-\sin(2\phi-\phi_+-\phi_-)\Big)H_7(q^2)\Big\}\,, \qquad (3.3.16)$$

where the form factor H's are given by

$$H_1(q^2)\equiv\sum_{j,k}(g^j_{Ve}g^{*k}_{Ve}+g^j_{Ae}g^{*k}_{Ae})(g^{*j}_{Vf}g^k_{Vf}+g^{*j}_{Af}g^k_{Af})h_{jk}(q^2)\,, \qquad (3.3.17a)$$

$$H_2(q^2)\equiv\sum_{j,k}(g^j_{Ve}g^{*k}_{Ve}+g^{*j}_{Ae}g^k_{Ae})(g^{*j}_{Vf}g^k_{Af}+g^{*j}_{Af}g^k_{Vf})h_{jk}(q^2)\,, \qquad (3.3.17b)$$

$$H_3(q^2)\equiv\sum_{j,k}(g^j_{Ve}g^{*k}_{Ve}+g^{*j}_{Ae}g^k_{Ae})(g^{*j}_{Vf}g^k_{Vf}-g^{*j}_{Af}g^k_{Af})h_{jk}(q^2)\,, \qquad (3.3.17c)$$

$$H_4(q^2)\equiv\sum_{j,k}(g^j_{Ve}g^{*k}_{Ae}+g^{*j}_{Ae}g^k_{Ve})(g^{*j}_{Vf}g^k_{Vf}+g^{*j}_{Af}g^k_{Af})h_{jk}(q^2)\,, \qquad (3.3.17d)$$

$$H_5(q^2)\equiv\sum_{j,k}(g^j_{Ve}g^{*k}_{Ae}+g^j_{Ae}g^{*k}_{Ve})(g^{*j}_{Vf}g^k_{Af}+g^{*j}_{Af}g^k_{Vf})h_{jk}(q^2)\,, \qquad (3.3.17e)$$

$$H_6(q^2)\equiv\sum_{j,k}(g^j_{Ve}g^{*k}_{Ve}-g^j_{Ae}g^{*k}_{Ae})(g^{*j}_{Vf}g^k_{Vf}+g^{*j}_{Af}g^k_{Af})h_{jk}(q^2)\,, \qquad (3.3.17f)$$

$$H_7(q^2)\equiv\sum_{j,k}-i(g^j_{Ve}g^{*k}_{Ae}-g^j_{Ae}g^{*k}_{Ve})(g^j_{Vf}g^{*k}_{Vf}+g^j_{Af}g^{*k}_{Af})h_{jk}(q^2)\,, \qquad (3.3.17g)$$

and

$$h_{jk}=\frac{e^{-2q^2}}{q^2-M_j^2}\,\frac{e^{-2q^2}}{q^2-M_k^2}\,. \qquad (3.3.18)$$

From the above calculations it follows that

i) The form factor $H_3(q^2)$ is suppressed by a γ'^{-2} factor

(if $m_f^2 \ll E^2$ the expression (3.3.13) results for j,K=0,1, as expected)

ii) We note that, only the form factors $H_1(q^2)$, $H_3(q^2)$ and $H_5(q^2)$ can be compared with measurable quantities if electron-positron beams are unpolarized (this was pointed out in our previous argument in Eq.(3.3.6) for unpolarized beams). In order to observe parity violating terms, including the relative signs of g_V and g_A, longitudinally polarized beams essentially enhance this effect, as far as the coefficient $H_5(q^2)$ is concerned. We will discuss this point for purely leptonic pair production in the next section.

iii) Having both beams longitudinally polarized does not give more information than one beam longitudinally polarized and the other one unpolarized. Having $|\mathcal{M}|^2$ as it was given by (3.3.8) along with relations (3.3.9), (3.3.10) and (3.3.11) makes it, in principle, possible to compute the differential cross section, cross section and symmetry (up to some coupling constant coefficients) as soon as the kinematic variables are specified for the reaction $e^+e^- \longrightarrow f^+f^-$. We postpone these calculations until the next section, when we precisely decide which processes are phenomenologically interesting for the purpose of connecting experimental results with the corresponding ones of electroweak models.

3.4 Light Lepton pair production.

In this section light lepton pair production, in particular the reaction $e^+e^- \longrightarrow \mu^+\mu^-$ is analysed. We are interested in the case for which only the initial states are arbitrarily polarized.

The most important effects of multi-neutral boson processes in a typical reaction $e^+e^- \longrightarrow l^+l^-$ (with l being a lepton) come from the interference between weak and electrmagnetic interactions. In our procedure, these effects have been represented within the amplitude $|m|^2_{EW}$ of the expression (3.3.8). It is, in fact, these interference effects that play a crucial role both from an experimental and theoretical point of view. From the experimental point of view, the asymmetry caused by interference effects of electroweak interactions is a measurable quantity. This, in turn, provides the parity-violating effects in nature and consequently suggests a criterion for distinguishing between many different classes of Salam-Weinberg type electroweak models.

To make the relationships as simple as possible, we choose the coupling constants in (3.3.8) to be real, since this precisely coincides with the structure of the present electroweak gauge models.

The angular distribution for the reaction $e^+e^- \longrightarrow \mu^+\mu^-$ is easily computed through our previous calculations. That is

$$\begin{aligned}\frac{d\sigma}{d\Omega} = (4E)^{-2}\alpha^2 \sum_{j,k} \Big\{ & (1+P_+P_-\cos\theta_+\cos\theta_-)(1+\cos^2\theta)\times \\ & (g^j_{Ve}g^k_{Ve}+g^j_{Ae}g^k_{Ae})(g^j_{V\mu}g^k_{V\mu}+g^j_{A\mu}g^k_{A\mu}) + \\ & 2\cos\theta(g^j_{Ve}g^k_{Ae}+g^j_{Ae}g^k_{Ve})(g^j_{V\mu}g^k_{A\mu}+g^j_{A\mu}g^k_{V\mu})(P_+\cos\theta_+ + P_-\cos\theta_-) \\ & (P_+\cos\theta_+ + P_-\cos\theta_-)(1+\cos^2\theta)\times \\ & (g^j_{Ve}g^k_{Ve}+g^j_{Ae}g^k_{Ve})(g^j_{Ve}g^k_{V\mu}+g^j_{A\mu}g^k_{A\mu}) + \\ & 2\cos\theta(1+P_+P_-\cos\theta_+\cos\theta_-)(g^j_{Ve}g^k_{Ae}+g^j_{Ae}g^k_{Ae})\times\end{aligned}$$

$$(g^j_{V\mu} g^k_{A\mu} + g^j_{A\mu} g^k_{V\mu}) + P_+ P_- \sin^2\theta \sin\theta_+ \sin\theta_- \cos(2\phi - \phi_+ - \phi_-) \times$$

$$(g^j_{Ve} g^k_{Ve} - g^j_{Ae} g^k_{Ae})(g^j_{V\mu} g^k_{V\mu} + g^j_{A\mu} g^k_{A\mu}) \Big\} h_{jk} \qquad (3.4.1)$$

where

$$h_{jk}(q^2) = h_j(q^2)\, h_k(q^2) \qquad (3.4.2)$$

and

$$h_j(q^2) = \frac{e^{-2q^2}}{q^2 - M_j^2} \qquad j \geq 1 \qquad (3.4.3a)$$

$$h_o(q^2) = 1 \qquad (.4.3b)$$

In calculating the expression (3.4.1), terms of order m/E have been neglected.

If the hypothesis of (e-μ) universality is held i.e. $g^j_{Ae} = g^j_{A\mu} = g^j_A$ and $g^j_{Ve} = g^j_{V\mu} = g^j_V$ the angular distribution (3.4.1) becomes simpler; in particular for three boson exchange processes of reaction $e^+e^- \longrightarrow \mu^+\mu^-$ with arbitrary electron-positron beam polarizations, it reduces to the following form

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2}\left(m^2_{\gamma\gamma} + m^2_{Z_1Z_1} + m^2_{Z_2Z_2} + m^2_{\gamma Z_1} + m^2_{\gamma Z_2} + m^2_{Z_1Z_2}\right) \qquad (3.4.4)$$

in which we have separated the electromagnetic contribution $m^2_{\gamma\gamma}$:

$$m^2_{\gamma\gamma} = (1+\cos^2\theta)(1 + P_+P_-\cos\theta_+\cos\theta_-) + P_+P_-\sin^2\theta \times$$
$$\sin\theta_+ \sin\theta_- \cos(2\phi - \phi_+ - \phi_-). \qquad (3.4.5)$$

The purely neutral weak terms $m^2_{Z_jZ_k}$, contributing to the angular distribution, are:

$$m^2_{Z_jZ_j} = \Big\{ (1 + P_+P_-\cos\theta_+\cos\theta_-)(1+\cos^2\theta)(g^{j\,2}_V + g^{j\,2}_V)^2 +$$

$$2\cos\theta\,(4 g^j_V g^j_A)^2 +$$

$$+ P_+P_- \sin^2\theta \sin\theta_+ \sin\theta_- \cos(2\phi - \phi_+ - \phi_-)(g_V^{4j} - g_V^{4j})\Big\} \times$$

$$\frac{e^{-4}q^4}{(q^2 - M_j^2)^2} \qquad (3.4.6)$$

and

$$m^2_{Z_1Z_2} = 2\Big\{(1+P_+P_-\cos\theta_+\cos\theta_-)\Big((1+\cos^2\theta)(g_V^1 g_V^2 + g_A^1 g_A^2)^2 +$$

$$+ 2\cos\theta(g_V^1 g_A^2 + g_A^1 g_V^2)^2\Big) + (P_+\cos\theta_+ + P_-\cos\theta_-)\times$$

$$(1+\cos\theta)^2(g_V^1 g_A^2 + g_A^1 g_V^2)(g_V^1 g_V^2 + g_A^1 g_A^2) + P_+P_-\sin^2\theta\sin\theta_+\times$$

$$\sin\theta_-\cos(2\phi - \phi_+ - \phi_-)(g_V^1 g_V^2 - g_A^1 g_A^2)\Big\}\frac{e^{-4}q^4}{(q^2-M_1^2)(q^2-M_2^2)}. \qquad (3.4.7)$$

Finally, the terms containing interference between electromagnetic and weak amplitudes, responsible for parity-violation effects in $e^+e^- \longrightarrow \mu^+\mu^-$ are given by

$$m^2_{\gamma Z_j} = 2\Big\{(1+P_+P_-\cos\theta_+\cos\theta_-)\Big((1+\cos^2\theta)g_V^{2j} + 2\cos\theta\, g_A^{2j}\Big) +$$

$$(P_+\cos\theta_+ + P_-\cos\theta_-)(1+\cos\theta)^2 g_V^j g_A^j +$$

$$(P_+P_-\sin^2\theta\sin\theta_+\sin\theta_-\cos(2\phi - \phi_+ - \phi_-)g_V^{2j}\Big\}\times$$

$$\frac{e^{-2}q^2}{q^2 - M_j^2}. \qquad (3.4.8)$$

If the mass of weak neutral bosons are large with respect to q the lowest order of angular distribution is practically effective. In this limit formula (3.4.4) reduces to the following form

$$\frac{d\sigma}{d\Omega} \cong \frac{\alpha^2}{16E^2}\Big\{\Big((1+\cos^2\theta)(1+P_+P_-\cos\theta_+\cos\theta_-) + P_+P_-\sin^2\theta\sin\theta_+\sin\theta_-\times$$

$$\cos(2\phi - \phi_+ - \phi_-)\Big) +$$

$$\sum_{j=1,2} g_V^{2j}\Big((1+\cos^2\theta)(1+P_+P_-\cos\theta_+\cos\theta_- + P_+P_-\sin^2\theta\sin\theta_+\times$$

$$\sin\theta_-\cos(2\phi - \phi_+ - \phi_-)\Big)\frac{e^{-2}q^2}{q^2 - M_j^2} + \sum_{j=1,2} g_A^{2j}\Big(2\cos\theta\times$$

$$(1+P_+P_-\cos\theta_+\cos\theta_-)\frac{e^{-2}q^2}{q^2-M_j^2} + \sum_{j=1,2} g_A^j g_V^j \times$$

$$\left((P_+\cos\theta_+ + P_-\cos\theta_-)(1+\cos)^2\right)\frac{e^{-2}q^2}{q^2-M_j^2}\Bigg\} . \qquad (3.4.9)$$

For transversely polarized electron-positron beams Eq. (3.4.9) reads

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2}\Bigg\{\left(1+\cos^2\theta + P_+P_-\sin^2\theta\cos 2\theta\right) +$$

$$\sum_{j=1,2} g_V^{2j}\left(1+\cos^2\theta + P_+P_-\sin^2\theta\cos 2\phi\right)\frac{e^{-2}q^2}{q^2-M_j^2} +$$

$$\sum_{j=1,2} g_A^{2j}(2\cos\theta)\frac{e^{-2}q^2}{q^2-M_j^2}\Bigg\} \qquad (3.4.10)$$

and consequently when initial beams are transversely polarized the terms proportional to $g_V g_A$ are cancelled.

The total cross-section for the reaction $e^+e^- \longrightarrow \mu^+\mu^-$, via three bosons, can be easily computed by integrating Eq. (3.4.4). We find

$$\sigma_{e^+e^-\to\mu^+\mu^-} = \sigma_{\gamma\gamma} + \sigma_{Z_1Z_1} + \sigma_{Z_2Z_2} + \sigma_{\gamma Z_1} + \sigma_{\gamma Z_2} + \sigma_{Z_1Z_2} \qquad (3.4.11)$$

where

$$\sigma_{\gamma\gamma} = (1+P_+P_-\cos\theta_+\cos\theta_-)\frac{\pi\alpha^2}{3E^2} \equiv \sigma_{EM} , \qquad (3.4.12)$$

$$\sigma_{Z_jZ_j} = 2\sigma_{EM}\alpha^{-2}\left((g_V^{2j}+g_A^{2j})^2 + 2g_V^j g_A^j (g_V^{2j}+g_A^{2j})P(\theta_+,\theta_-)\right)\times$$

$$\frac{q^4}{(q^2-M_j^2)^2} \qquad (3.4.13)$$

and

$$\sigma_{Z_1 Z_2} = 2\,\sigma_{EM}\,\alpha^{-2}\Big(\big(g_V^1 g_V^2 + g_A^1 g_A^2\big)^2 + \big(g_V^1 g_A^2 + g_A^1 g_V^2\big)\times \big(g_V^1 g_V^2 + g_A^1 g_A^2\big)\Big)\frac{q^4}{(q^2-M_1^2)(q^2-M_2^2)}\,. \tag{3.4.14}$$

Finally the term $\sigma_{\gamma Z_j}$ is given by

$$\sigma_{\gamma Z_j} = 2\,\sigma_{EM}\,\alpha^{-1}\Big(g_V^{j\,2} + g_V^j g_A^j P(\theta_+,\theta_-)\Big)\frac{q^2}{q^2-M_j^2} \tag{3.4.15}$$

where j=1,2 and

$$P(\theta_+,\theta_-) = \frac{P_+\cos\theta_+ + P_-\cos\theta_-}{1 + P_+P_-\cos\theta_+\cos\theta_-}\,. \tag{3.4.16}$$

The Eq. (3.4.11) has been normalized with the well-known electromagnetic cross section σ_{EM}.

It is clear, from (3.4.14) how much the polarization factor $P(\theta_+,\theta_-)$, [in our parameterization and polarization] in general is significant in e^+e^- physics. In $e^+e^- \longrightarrow l^+ l^-$, [and particularly in our present example $e^+e^- \longrightarrow \mu^+\mu^-$], having transversely polarized electrons and positrons helps to enhance the vectorial coupling constants of the terms in (3.4.15), so that the total production cross-section in this particular case (i.e. $\theta_+ = \theta_- = \pi/2$, $\Phi_+ = 0$, $\Phi_- = \pi$) is obviosly independent of the beam polarization P. The low energy limit of the cross-section in this case is easily obtained (3.4.10); we find

$$\sigma_{e^+e^- \longrightarrow \mu^+\mu^-} = \sigma_{EM}\left(1 + \frac{2q^2}{\alpha}\left(\frac{g_V^{1\,2}}{M_1^2} + \frac{g_V^{2\,2}}{M_2^2}\right)\right). \tag{3.4.17}$$

We notice that if the electron-positron beam is transversely polarized, in the low energy limit, the contributions of axial coupling constants are suppressed. Whereas in the same limit, with both beams longitudinally polarized ($\theta_+ = \theta_- = 0$), the axial coupling constants are enhanced. This can directly be understood from the term given in Eq. (3.4.10) which

effectively changes the strength of parity violation in experiments.

Practically, the parity violation terms can be projected out by measuring the cross-section with both beams longitudinally polarized and then repeating the experiment with both polarization reversed ($\theta_+ = \theta_- = \pi$). In this way, the asymmetry

$$A(\theta) = \frac{(d\sigma/d\Omega)_{P(0,0)} - (d\sigma/d\Omega)_{P(\pi,\pi)}}{(d\sigma/d\Omega)_{P(0,0)} + (d\sigma/d\Omega)_{P(\pi,\pi)}} \tag{3.4.18}$$

is practically measured. Using Eq. (3.4.1) one can calculate this quantity for any purely leptonic process $e^+e^- \longrightarrow l^+l^-$; this quantity in our example for three vector boson exchange up to the lowest order electromagnetic and weak terms reduces to

$$A(\theta) = \frac{C_1(1+\cos^2\theta) + 2C_2\cos\theta}{2C_3\cos\theta + C_4(1+\cos^2\theta)} P(0,0) \tag{3.4.19}$$

where the coefficients C are given by

$$C_1 = C_2 = \frac{e^{-2} g_V' g_A'}{q^2 - M_1^2} q^2 + \frac{e^{-2} g_V^2 g_A'}{q^2 - M_2^2} q^2 \tag{3.4.20}$$

$$C_3 = \frac{e^{-2} g_A'^2 q^2}{q^2 - M_1^2} + \frac{e^{-2} g_A^2 q^2}{q^2 - M_2^2} \tag{3.4.21}$$

and

$$C_4 = 1 + \frac{e^{-2} g_V'^2 q^2}{q^2 - M_1^2} + \frac{e^{-2} g_V^2 q^2}{q^2 - M_2^2} \quad . \tag{3.4.22}$$

An alternative way of measuring the parity violation for the reaction $e^+e^- \longrightarrow \mu^+\mu^-$, (via one neutral gauge boson), has been discussed in /38/, that is, to measure the helicity of the outgoing muon. This can be investigated, in our approach, directly from relations (3.3.8), (3.3.1) and (3.3.10), with an appropriate parameterization, which will be done in

the next section.

Before closing the study of $e^+e^- \longrightarrow l^+l^-$, let us point out that the above discussion can be applied to Bhabha scattering of the $e^+e^- \longrightarrow e^+e^-$ reaction which is also interesting in the study of electroweak interaction. The diagram contributing, in the tree approximation, to $e^+e^- \longrightarrow e^+e^-$ reaction have been shown in Fig. (3.4.1).

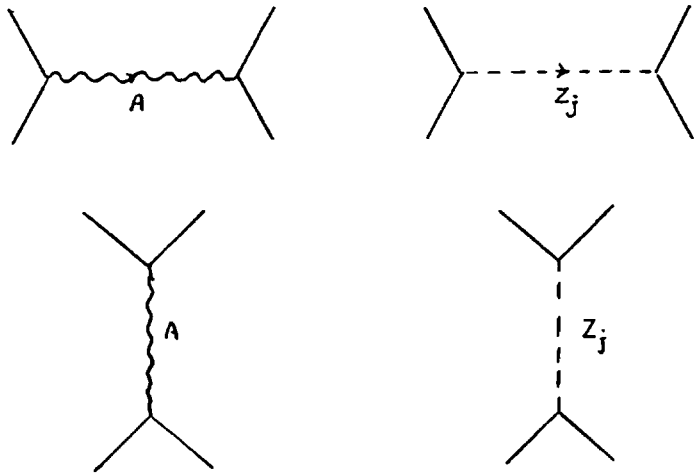


Fig. (3.4.1) Diagrams contributing in the lowest order to the Bhabha scattering.

In this case the invariant amplitude of Eq. (3.3.1) must be replaced by the following one

$$
\begin{aligned}
|\mathcal{M}|^2 = \Big| & \frac{e^2}{q^2}\, \bar{v}_e(k_+, s_+)\gamma_\mu u_e(k_-, s_-)\bar{u}_e(k'_-, s'_-)\gamma^\nu v_e(k'_+, s'_+) \\
& + \sum_{j=1} \Big\{ \bar{v}_e(k_+, s_+)\Gamma^j_{\rho e}\, u_e(k_-, s_-)\Big((g^{\rho\sigma} - \frac{q^\rho q^\sigma}{M_j^2})\frac{1}{q^2 - M_j^2}\Big) \\
& \bar{u}_e(k'_-, s'_-)\Gamma^j_{\sigma e}\, v_e(k'_+, s'_+)\Big\} - \frac{e^2}{q^2}\, \bar{v}_e(k_+, s_+)\gamma_\mu v(k'_+, s'_+)\bar{u}_e(k'_-, s'_-)\gamma^\nu \\
& u_e(k_-, s_-) - \sum_j \Big\{ \bar{v}_e(k_+, s_+)\Gamma^j_{\rho e}\, v(k'_+, s'_+)\Big((g^{\rho\sigma} - \frac{q'^\rho q'^\sigma}{M_j^2})\frac{1}{q^2 - M_j^2}\Big) \\
& \bar{u}_e(k'_-, s'_-)\Gamma^j_{\sigma e}\, u_e(k_-, s_-)\Big\}\Big|^2 ,
\end{aligned}
\tag{3.4.23}
$$

in which the same notation has been used as Eq. (3.3.1) and

$$q' = K - K' \tag{3.4.24}$$

We have not calculated the whole detail of computations of cross-section for this case, but having the relations (3.3.11), (3.3.10) and (3.3.9), the same method can be followed to evaluate the angular distribution and the asymmetry defined by (3.4.19) for both transversely and longitudinally polarised beams. The resulting asymmetry, in this case, seems to be approximately of the same order of magnitude as the $\mu^+\mu^-$ pair production.

3.5 Final state polarization effects.

In this section, we study the reaction $e^+e^- \rightarrow \mu^+\mu^-$ for the case in which the outgoing particles are polarized. If this effect is observable, it may be regarded as an alternative way for testing the electroweak phenomena and consequently another method for distinguishing between different electroweak models.

We work out a prity-violating helicity distribution for $\mu^+\mu^-$ pair production by assuming that one particle, for example the muon, is polarized. It is shown that the scattering amplitude, in particular the μ^- polarization term, picks up some contribution from the interference of electromagnetic and weak interactions. This suggests that in the reaction $e^+e^- \rightarrow \mu^+\mu^-$, the mean μ^- helicity

$$\langle h' \rangle_- = \frac{|m|^2_{h'=+1} - |m|^2_{h'=-1}}{|m|^2_{h'=+1} + |m|^2_{h'=-1}} \qquad (3.5.1)$$

could be compared with experimental results /38/.

The quantity $|m|^2$ has already been evaluated in section (3.3) and to complete its calculation, we only need to specify the kinematic variables. To do so, apart from the definitions (3.2.7), (3.2.8), (3.3.12c) and (3.3.12d), we take the electron and positron spins to be equal and opposite so that

$$S_+ = P_+(0,1,0,0) = -S_- \qquad (3.5.2)$$

and for outgoing muon

$$S'_- = h'(\beta'\gamma', \gamma' \sin\theta \cos\phi, \gamma' \sin\theta \sin\phi, \gamma' \cos\theta). \qquad (3.5.3)$$

Substituting for the kinematic variables in (3.3.8), (3.3.9), (3.3.10) and (3.3.11) and using Appendix C and taking the low energy limit for j=1,2 we find

$$|m|^2 = 16\pi^2\alpha^2 \left\{ \frac{1}{2}(1+\cos^2\theta - P^2\sin^2\theta\cos 2\phi) + \right.$$

$$\sum_{j=1,2} g_V^{j\,2}(1+\cos^2\theta - P^2\sin^2\theta\cos 2\phi)\frac{e^{-2}q^2}{q^2-M_j^2} +$$

$$-\sum_{j=1,2} h' g_V^j g_A^j \left(\beta'(1+\cos^2\theta - P^2\sin^2\theta\cos 2\phi) - 2\beta\cos\theta\right)\times$$

$$\left.\frac{e^{-2}q^2}{q^2-M_j^2} - \sum_{j=1,2} g_A^{j\,2}(2\beta\beta'\cos\theta)\frac{e^{-2}q^{-2}}{q^2-M_j^2}\right\}, \qquad (3.5.4)$$

where $P_+ = -P_- = P$. Eq. (3.5.4) indicates that the muon helicity h' has contribution from the $g_V g_A$ term. Applying Eq. (3.5.4) to (3.5.1) yields the helicity distribution for the μ^- muons. This is

$$\langle h'\rangle_- = \frac{((1+\cos\theta)^2 - P^2\sin^2\theta\cos 2\phi)H}{A(2\cos\theta) + V(1+\cos^2\theta - P\sin^2\theta\cos 2\phi} \qquad (3.5.5)$$

where

$$H \equiv -\sum_{j=1,2} g_A^j g_V^j \frac{e^{-2}q^2}{q^2-M_j^2}, \qquad (3.5.6)$$

$$A \equiv \sum_{j=1,2} g_A^{2j} \frac{e^{-2}q^2}{q^2-M_j^2}, \qquad (3.5.7)$$

and

$$V \equiv \frac{1}{2} + \sum_{j=1,2} g_V^{j\,2} \frac{e^{-2}q^2}{q^2-M_j^2}. \qquad (3.5.8)$$

The mean value of the muon helicity given by (3.5.5), up to coupling g's, is a model independent quantity and hence it can be used to distinguish between different electroweak models by comparing the coefficients H, A and V with the experimental ones.

Using expression (3.5.5) for one boson exchange processes one obtains /38/

$$\langle h' \rangle_{-} = - \frac{g_V g_A}{2\pi d} \frac{q^2}{M_{Z_0}^2} \left(1 + \frac{2\cos\theta}{1+\cos^2\theta - p^2 \sin^2\theta \cos 2\phi}\right) \qquad (3.5.9)$$

where $g_V' = g_V$, $g_A' = g_A$ and $M_1 = M_Z$. For p=0, it then reduces to the well-known result

$$\langle h' \rangle_{-} \sim \frac{(1+\cos\theta)^2}{1+\cos^2\theta} . \qquad (3.5.10)$$

3.6 Deep - inelastic polarized e - N scattering.

The observation of processes involving weak neutral currents leads one to expect parity-violating effects. Such effects would arise in lowest order of weak interactions from the interferences of the weak and electromagnetic amplitudes when either the initial or the final states is polarized (see Eqs. (3.4.19) and (3.5.5)). The dependence of the scattering cross section on these effects occur at a level of approximately $\sum_j G_j \frac{q^2}{M_j^2}$, where G_j is a function of coupling constants (see Eq. (2.4.15). In high energy experiments, where q^2 is typically of order a few Gev, the corresponding parity-violating effects, when only Z^0 is involved in the elctroweak processes, are of order a few times 10^{-5} (i.e. $q^2/M_{Z^0}^2 \approx G_F q^2/4\pi\alpha \approx 10^{-5}$). If there exist a few number of neutral weak bosons with masses around M_Zo their effects do <u>not</u> change the value of 10^{-5} by very much, since the strength of parity-violation ($\sum_j G_j q^2/M_j^2$) is an additive quantity in our procedure. Consequently, an accurate experiment is needed to determine the possible existence of the neutral gauge boson(s). The most accurate experiment that measures parity-violation effects through the asymmetry for polarized electron-nucleon scattering is now being undertaken at SLAC.

The study of weak-electromagnetic interference for deep-inelastic electron-nucleon scattering has been explored previously /39, 40, 41, 42/. However, here we want to develop this approach for multi-neutral boson processes. In what follows, we borrow the idea of Ref.(42) and adapt our method with the context of the usual quark-parton model /43/. The measurable quantity which is studied is the asymmetry in e-N scattering when the electron beam is longitudinally polarized.

According to our notation of sec. (3.2), the Lagrangian (3.2.3) can be written as

$$L_{NC} = \sum_j \left(g^j_{Rf}\, \bar{\psi}^R_f \Gamma_{\mu R}\, \psi^R_f Z^\mu_j + g^j_{Lf}\, \bar{\psi}^L_f \Gamma_{\mu L}\, \psi^L_f Z^\mu_j \right) \qquad (3.6.1)$$

where

$$\Gamma_{\mu R} = \frac{\gamma_\mu(1+\gamma_5)}{2} \qquad (3.6.2a)$$

and

$$\Gamma_{\mu L} = \frac{\gamma_\mu(1-\gamma_5)}{2} \,. \qquad (3.6.2b)$$

$\Gamma_{\mu R}$ and $\Gamma_{\mu L}$ have the same meaning as Eqs. (3.2.4), i.e. we treat $-i\Gamma_{\mu R}$ and $-i\Gamma_{\mu L}$ as vertices with the strength g_{Rf} and g_{Lf} respectively. When the fermion f is a constituent of hadron, it runs over the quark u,d,... which build up the hadron, in general.

We first calculate the differential cross-section $d\sigma_L^f$ and $d\sigma_R^f$ for the scattering of left and right polarized electrons off the constituents of nucleon in the framework of the standard quark-parton model. We then compute $d\sigma_{L(R)}$

$$d\sigma_{L(R)} = \sum_f u_f\, d\sigma^f_{L(R)} \qquad (3.6.3)$$

by assuming that the parton distribution function u_f is the same for left and right-handed processe. Since the calculations are done in the high energy and large momentum transfer regime, we neglect the electron and quark-parton masses. In this way, by using Eq. (3.6.3) and what we did in section (3.3), the scattering amplitude for right-handed electron on right-handed quark of type f is given by

$$|\mathcal{M}|^2_R = \left| \sum_j g^j_{Re}\, g^j_{Rf} \left(\bar{f}^R \Gamma_{\mu R} f^R \right) G^j_{\mu\nu} \left(\bar{e}^R \Gamma^\nu_R e^R \right) \right|^2 \qquad (3.6.4)$$

where j=0,1,2,... and

$$G_j^{\mu\nu}(q^2) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{M_j^2}\right)\frac{1}{q^2 - M_j^2} \equiv \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{M_j^2}\right) G_j(q^2) \qquad (3.6.5)$$

and

$$M_o = M_\gamma = 0 \,. \qquad (3.6.6)$$

Inserting $|\mathcal{M}|^2$ in Eq. (3.3.1), the differential cross-section is easily calculated. We find

$$d\sigma_R^f = \left|\sum_j g_{Re}^j \, g_{Rf}^j \, G_j(q^2)\right|^2 (k_+' \cdot k_-')(k_+ \cdot k_-) I \qquad (3.6.7)$$

where (K_+, K_-) and (K_+', K_-') have already been defined in (3.2.8). In a similar way the differential cross-section for right(left)-handed electron off left(right)-handed quark of type f are calculated. They are

$$d\sigma_{R(L)}^f = \left|\sum_j g_{R(L)e}^j \, g_{L(R)f}^j \, G_j(q^2)\right|^2 (k_+' \cdot k_-)(k_+ \cdot k_-') I \qquad (3.6.8)$$

and finally the differential cross-section for left-handed electron off left-handed quarks of type f becomes

$$d\sigma_L^f = \left|\sum_j g_{Le}^j \, g_{Lf}^j \, G_j(q^2)\right|^2 (k_+' \cdot k_-')(k_+ \cdot k_-) I \,, \qquad (3.6.9)$$

where I in Eqs. (3.6.7), (3.6.8) and (3.6.9) symbolically denotes the other factors.

Having the structure of differential cross-section of Eqs. (3.6.7) and (3.6.8) and (3.6.9), the asymmetry, A, characterized by

$$A(\theta) = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R} \qquad (3.6.10)$$

can be calculated. We find

$$A = \Big\{ \sum_f U_f \Big(\Big| \sum_j g^j_{Re}\, g^j_{Rf}\, G_j(q^2) \Big|^2 - \Big| \sum_j g^j_{Le}\, g^j_{Lf}\, G_j(q^2) \Big|^2 + \nu \Big| \sum_j g^j_{Re}\, g^j_{Lf}\, G_j(q^2) \Big|^2 - \nu \Big| \sum_j g^j_{Le}\, g^j_{Rf}\, G_j(q^2) \Big|^2 \Big) \Big\} \times$$

$$\Big\{ \sum_f U_f \Big(\Big| \sum_j g^j_{Re}\, g^j_{Rf}\, G_j(q^2) \Big|^2 + \Big| \sum_j g^j_{Le}\, g^j_{Lf}\, G_j(q^2) \Big|^2 + \nu \Big| \sum_j g^j_{Re}\, g^j_{Lf}\, G_j(q^2) \Big|^2 + \nu \Big| \sum_j g^j_{Le}\, g^j_{Rf}\, G_j(q^2) \Big|^2 \Big) \Big\}^{-1} \qquad (3.6.10\,b)$$

Eq. (3.6.11) is quite general and all parameters, except U and ν can be compared with the corresponding quantities given by electroweak gauge models which we will separately argue in the next sections. In order to evaluate U_f and ν we employ the quark-parton approach in which the "distribution function" U_f is usually interpreted as the probability density of finding a quark of type f with fractional longitudinal momentum

$$x = -\frac{q^2}{2 M_N \nu} \qquad (3.6.11)$$

where q is the momentum transfer to the nucleon N, M_N the mass of nucleon and

$$\nu = E_- - E_+ \qquad (3.6.12)$$

with E_- and E_+ being the energy of initial and final electron respectively. Defining

$$y = \frac{\nu}{E_-} \qquad (3.6.13)$$

after some calculation (see App. C) we find

$$\nu = (1-y)^2 \equiv \nu(y) \qquad (3.6.14)$$

It is now straightforward to compute the $A(x,y)$ in $e+N \longrightarrow e+$

hadron reaction. For the case in which one photon γ and two neutral bosons are involved, Eq. (3.6.10) reduces to

$$A(x,y) = \sum_{j=1,2} \left\{ \frac{q^2}{q^2 - M_j^2} \left(2 \sum_{f} u_f(x) \left[\left(g^0_{Re}\, g^0_{Rf}\, g^j_{Re}\, g^j_{Rf} \right. \right. \right. \right.$$

$$\left. - g^0_{Le}\, g^0_{Lf}\, g^j_{Le}\, g^j_{Lf} \right) + v(y) \left(g^0_{Re}\, g^0_{Lf}\, g^j_{Re}\, g^j_{Lf} - \right.$$

$$\left. \left. \left. g^0_{Le}\, g^0_{Rf}\, g^j_{Le}\, g^j_{Rf} \right) \right] \right) \left. \right\} \left\{ \sum_{f} u_f(x) \left[\left(g^0_{Re}\, g^0_{Rf} \right)^2 \right. \right.$$

$$\left. \left. + \left(g^0_{Le}\, g^0_{Lf} \right)^2 + v(y) \left(\left(g^0_{Re}\, g^0_{Lf} \right)^2 + \left(g^0_{Le}\, g^0_{Rf} \right)^2 \right) \right] \right\}^{-1}, \qquad (3.6.15)$$

in which we have kept the lowest order terms of order $q^2/(q^2 - M_j^2)$, in our calculation.

3.7 Summary.

We presented a general analysis for the multi-neutral boson exchange processes $e^+e^- \longrightarrow f^+f^-$. First, unpolarized beams were discussed which led to the relation (3.3.7) that might be useful for connecting the experimental results to the corresponding ones in electroweak models. Then we systematically considered processes in which initial electronic and final fermionic states were arbitrarily polarized. For this case the scattering amplitude given by Eq. (3.3.8) is also valid for any kind of light or heavy fermions at low or high energies. By neglecting the electron mass m_e a rather simple angular distribution formula was obtained in Eq. (3.3.16). The method was then specialized for the $e^+e^- \longrightarrow l^+l^-$ with l being any lepton; in particular the reaction $e^+e^- \longrightarrow \mu^+\mu^-$ which is more interesting in experiments was studied. The angular distribution, the cross-section and the asymmetry for μ pair production, and with arbitrary electronic beam polarization, were worked out. To clarify this method we have calculated the asymmetry for three vector boson exchange processes which was given by Eq. (3.4.20).

When initial electronic beams are transversely polarized for possibly measuring the final state polarization effects in section (5) we computed the mean value of the muon helicity and our results were summarized in Eq. (3.5.4).

Deep-inelastic processes can also be studied through our approach and we have calculated the asymmetry for polarized electron-nucleon scattering in section 6. In this case the results depend on the quark-parton model that has been shown in Eq. (3.6.15).

Our analysis of the $e^+e^- \longrightarrow f^+f^-$ reaction contain two types of parameters. The parameters which come into the calculations through the polarization effects such as θ_+, θ_-, ϕ_+ and ϕ_-, they must be fixed for any specific experiment. And the second types are the coupling constant g's

that can be compared with their corresponding values determined through the Lagrangian of the electroweak model for the tree graphs. For example, if parity is violated in Nature it may be observable in reaction $e^+e^- \longrightarrow \mu^+\mu^-$ through the asymmetry given by

$$A(\theta) = \frac{(1+\cos^2\theta)C_1 + 2C_2\cos\theta}{2C_3\cos\theta + (1+\cos^2\theta)C_4} P(0,0) \tag{3.7.1}$$

(see Eq. (3.4.21)). In this formula θ and P(0,0) is to be fixed by experiments and the coefficient C's depend on the couplings of neutral current(s) to the neutral gauge boson(s). For the standard model they correspond to the following values. (See Eqs. (3.4.21), (3.4.22), (3.4.23) and the Table (2.2.1))

$$C_1 = C_2 \simeq 2\zeta(4\sin^2\theta_W - 1) \tag{3.7.2}$$

$$C_3 \simeq 2\zeta \tag{3.7.3}$$

$$C_4 \simeq 1 + 2\zeta(4\sin^2\theta_W - 1) , \tag{3.7.4}$$

where we have used Eqs. (2.3.3) and (2.2.21), and

$$\zeta = \frac{q^2}{q^2 - M_{Z_0}^2} \frac{1}{4\sin^2 2\theta} \simeq \frac{q^2}{M_p^2}\left(\frac{G_F M_p^2}{8\pi\alpha\sqrt{2}}\right) = -0.425\times 10^{-4}\frac{q^2}{M_p^2} \tag{3.7.5}$$

with M_p being the proton mass and G_F is the Fermi constant

For the model based on the $SU(2)_L \times U(1)_Y \times \widetilde{U(1)}_{\tilde{Y}}$ gauge symmetry which was discussed in Chapter 2, the coefficients C are given in Eqs. (3.4.21), (3.4.22) and (3.4.23). The coupling constant g's can easily be calculated by comparing Eqs. (2.5.27) and (3.2.3). One obtains

$$g_A^j = -f_j \tag{3.7.6}$$

and

$$g_V^j = f_j \left\{ 4\left(1 - \frac{M_W^2}{M_j^2}\right) - 1 \right\} \qquad (3.7.7)$$

where $j=1,2$; f_1 and f_2 have been calculated in Eqs. (2.5.4) and (2.5.42). In Eqs. (3.7.6) and (3.7.7) the neutral boson masses M_1 and M_2 are specified by the corresponding parameters m_1 and m_2 introduced in Eqs. (2.5.20) and (2.5.21).

Measurement of parity-violating effects in purely leptonic interactions such as $e^+e^- \longrightarrow \mu^+\mu^-$ process is not only of general interest because it arises from interference between weak and electromagnetic amplitudes, but particularly because it involves the neutral currents coupling of the electron in a well defined manner. The exact vector and axial-vector nature of this coupling occupies a central role in determining the correct electroweak models.

CHAPTER IV. GRAND UNIFIED THEORIES

4.1 Colour charge

From what was described in part I, we learned that a gauge theory with spontaneously broken local gauge symmetry provides us with a Unified Electroweak Gauge Theory, in particular the Salam-Weinberg type of theory which

i) unifies electromagnetic and weak interactions

ii) gives rise to a renormalizable theory of weak interactions

iii) leads to the correct phenomenology of electroweak processes (at low energies).

On the other hand, attempts have recently been made to describe the strong interactions within the context of local gauge symmetry. After the unsuccessful idea of Yang and Mills in constructing such theory based on isospin symmetry, the discovery of the present candidate for a description of strong interactions, quantum chromodynamics /12/, involved a subtle interplay of theory and experiment.

"Deep inelastic" elctron-proton scattering /44/ which means that a large amount of momentum and energy is transferred from the electron to the proton was perhaps the first and deepest insight concerning the question of the constituents of protons and other hadrons. In this process the electromagnetic interaction probes the nucleon in a way that is most sensitive to its short distance structure, where it appears to be composed of elementary fermions. These appear to be quarks, the pointlike, fractionally charged fermions that had been conjectured to be the constituents of proton. The quark model /45/ has been very successful explaining the complicated pattern of hadronic states. However, quarks have never been seen in isolation, they seem to be confined in hadrons in an appropriate bound state with other quarks and interact rather weakly with each other.

Chromodynamics is supposed to give an explanation of this peculiar behavior /12/. It has survived many qualitative tests, but the only quantitative tests so far have examined its short distance structure.

Chromodynamics is a Yang-Mills theory based on the local gauge symmetry governed by the SU(3) group. The basic charged states are called colours; and the "chromodynamics" refers to the analog with electrodynamics. The SU(3) of colour (denoted by $SU(3)_C$) has eight generators, and therefore the strong interactions result from the complicated interaction of the eight vector bosons with the eight colour currents. The vector bosons, called "gluons",are electrically neutral and also carry no weak interaction charges, but they do carry colour charge. Each quark can exist in one of three colours, and is changed from one colour state to another upon the emission or absorption of a gluon. It is assumed that $SU(3_C)$ is not broken spontaneously and therefore the gluons are massless. If we accept the hypothesis that chromodynamics describes the strong interactions, then we must explain why it is impossible (or at least very difficult) to observe isolated colour charges, why the strong interaction is short ranged, and why the quarks do not interact strongly at short distances. Answers and conjectures about these questions are based on a property of the theory called "asymptoic freedom" /46/.

The strength or coupling constant for the vector boson-current interaction must be measured experimentally. The coupling constant depends on the momentum carried by the gluon, but its dependence on momentum is a prediction of the theory once it is known at one value of the momentum. This momentum dependence of the coupling constant is an important and nontrivial consequence of quantum field theory. (This is also true for the fine structure constant of electrodynamics /47/). The strong coupling α_S of chromodynamics decreases as momenta increase. Large momentum transfer processes correspond to the quarks being close together. Therefore, quarks become more free at shorter distances. This is what

is meant by asymptotic freedom. Correspondingly, as momenta are decreased, α_s grows and it is of order unity at a momentum scale of a few hundred Mev. This corresponds to the quarks being separated by a typical hadronic length scale of 10^{-13} cm. It is reasonable to conjecture that for longer distances the interaction becomes so strong that the colour charges are all confined inside hadrons. This also means that the gluons cannot escape and the strong force is short ranged. Over the last few years the problem of understanding the confinement of colour has absorbed much effort in theoretical physics. The experimental and theoretical support for chromodynamics is not yet as firm as that for the $SU(2)_L$ x $U(1)_Y$ theory of the electroweak interactions, but it is a viable candidate for a stong interaction theory. It is assumed here that the strong interactions are described by a local $SU(3)_C$.

In view of the present understanding of electroweak and strong interactions, we may, therefore, summarize the description of elementary particle interactions by saying it is a Yang-Mills theory based on a combined symmetry of strong and electroweak interactions i.e. G_{eW} x $SU(3)_C$, where $G_{eW} \supseteq SU(2)_L$ x $U(1)_Y$. In the minimal scheme when $G_{eW} = SU(2)_L$ x $U(1)_Y$ all interactions (but gravity) are described by $G_1 = SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$ symmetry.

In the next section we introduce grand unified theories for the embedding of G_1 in a larger group. In what follows, the minimal scheme of grand unified theories is briefly studied in section 3.

4.2 The basic idea of grand unification.

"Grand" unified gauge theories are based on the hypothesis that quarks and leptons are grouped in the same multiplet of a unifying group G /13/. This idea can be extended by saying not only (left-handed) quarks and leptons but their antiparticles may also group in the same multiplet of the unifying gauge group G /14/.

It then follows that any hypothetical Yang-Mills theory of electromagnetic, weak and strong interactions must have a local symmetry G at least as large as $G_{EW} \times SU(3)_C$

$$G \supseteq G_{EW} \times SU(3)_C \qquad (2.4.1)$$

where $G_{EW} = SU(2)_L \times U(1)_Y$. It is assumed that the photon couples to one of the G_{EW}currents and, of course, the corresponding U(1) is unbroken.

The simplest proposal that economizes the number of vector bosons and also minimizes the number of parameters is to enlarge G so that there is only one gauge coupling constant /14/ and so that all the elementary fermions (quarks and leptons) are contained in a few irreducible representation of G.

For simplicity, we begin by assuming that G is a Lie group, that G contains $SU(3)_C \times SU(2)_L \times U(1)_Y$, and that there is only one independent gauge coupling constant. Thus G must be "simple", [or semisimple in the form G x G (G simple) where some global reflection symmetry /13/ constrains the (unrenormalized) gauge coupling constants of the two simple factors to be equal. We do not consider this possibility here].

The basic hypothesis that the simple group G contains $G_1 = SU(3)_C \times SU(2)_L \times U(1)_Y$ implies that all interactions are generated by G, that quarks and leptons are represented in the same multiplet(s) of G implies a relation between the fermion charges "charge quantization". This is simply because the electric charge operator Q is a generator of G and

thus traceless; when it acts on any representation of G containing both quarks and leptons, it gives a relation between quark and lepton charges. For example, when G = SU(5), the constraint TrQ = 0 implies

$$-3\varphi_d + \varphi_{\bar{e}} = 0 \qquad (4.2.1)$$

[See Eqs. (4.3.3) and (4.3.36]. Eq. (4.2.1) not only relates the d quark and electron charges to each other, it also leads to

$$Q_p + Q_{e^-} = 0 \qquad (4.2.2)$$

which is regarded as the observation of charge quantization in Nature.

The unifying gauge group G has only one free gauge coupling constant g_G for all interactions, hence a relationship is expected to be found between the different "physical" coupling constants g_1, g_2 and g_3 associated with the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ subgroups respectively. They are equal when G governs the symmetry of the theory, however, the group G does not remain as an exact symmetry since only $G_o = SU(3)_C \times U(1)_{EM}$ is observed as an exact symmetry which, in turn, results from the spontaneous symmetry breaking of the $G_1 = SU(3)_C \times SU(2)_L \times U(1)_Y$ substructure. The breaking of G down to G_1 and then to G_o implies that the "physical couplings" (i.e. effective gauge coupling constants) g_1, g_2 and g_3 evolve differently with respect to energy. That is why, for different interactions they have different magnitudes /48/.

A further consequence of quarks and leptons being represented in the same multiplet(s) of G is a relationship between quark and lepton masses /50/. At the limit where G governs the symmetry of physical phenomena quark and lepton masses are more or less equal (modulo some possible Clabsch-Gordon coefficients). However, symmetry breaking properties modify these "symmetry relations" in an appropriate way i.e. making quarks heavier than leptons, mainly because quarks have strong interactions and leptons do not.

Grand unified theories, in general, group fermions into the same

multiplet(s) of the gauge group G. Since gauge theories contain vector bosons linking all fermion fields in a multiplet, there will naturally be some interactions changing quarks into leptons which violate baryon number (B) and lepton number (L) conservation. This is by far the most amazing consequence of grand unified theories and most certainly B and L violation play a central role in the possible inter-connection between the theories and cosmology. The gauge bosons causing the B and L violating interactions mediate proton decay; we will study this issue and the above statements in some detail in the following section.

The above genral arguments are almost valid in any grand unified model since they all result from the same idea of grouping leptons and quarks into the same multiplet of a unifying gauge group G. There exist a wide class of grand unified models currently being still considered in the literature such as SU(5) /14/, SO(10) /49/, E_6 /51/, and SU(16) /52/ models, for example. Each of these models has its own aspects in describing the nature of elementary physics and they are spontaneously broken down to the $G_o = SU(3)_C \times U(1)_{EM}$ exact symmetry in at least two stages via the Higgs-Kibble mechanism. In the next section we briefly consider the structure of the SU(5) model.

4.3 The SU(5) Model.

a) Fermions in the SU(5) model.

In the SU(5) grand unified theory /14/, each family of fermions consists of fifteen two-component spinors (left-handed and right-handed quarks each with $I_3 = \pm\frac{1}{2}$ and three colours, a left-handed and right-handed charged lepton and a left-handed neutrino). These are grouped into the representations $\underline{5}$ and $\underline{10}$ of SU(5) as follows

$$\underline{5} \equiv (\underline{2}, 1) + (1, \underline{3}) \qquad (4.3.1)$$

$$\downarrow \qquad\qquad \downarrow$$

$$(\nu_e, e) \qquad d$$

and

$$\underline{10} \equiv (1,1) + (2, \underline{3}) + (1, \underline{3}) . \qquad (4.3.2)$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\bar{e}_L \qquad (u,d)_L \qquad \bar{u}_L$$

In terms of particles, we may thus write (for electron family)

$$5 \equiv \psi_{\underline{5}} = \begin{vmatrix} d_1 \\ d_2 \\ d_3 \\ e \\ -\nu_e \end{vmatrix}_R \qquad (4.3.3)$$

and

$$\underline{10} \equiv \psi_{\underline{10}} = \begin{vmatrix} 0 & u & -u^2 & u_1 & d_1 \\ -u^3 & 0 & u^1 & u_2 & d_2 \\ u^2 & -u^1 & 0 & u_3 & d_3 \\ -u_1 & -\bar{u}_2 & -d_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{vmatrix}_L \frac{1}{\sqrt{2}} , \qquad (4.3.4)$$

where i = 1, 2, 3 (= r, y, b) denote colour indices and r, y and b refer to red, yellow and blue charges.

The triangle anomaly number of the representations $\bar{5}$ and 10 are A[5] = -1 and A[10] = 1 respectively /F5/. Thus the combination of 5 and 10 representations, for fermions in each family, is anomaly free, which guarantees the renormalizability of the theory.

The interaction of fermions with the SU(5) gauge boson A_μ can immediately be seen through the part of Lagrangian containing the kinetic term

$$L_{kinetic} = i\,\bar{\psi}_{\underline{5}}\,\not{D}\,\psi_{\underline{5}} + i\,\bar{\psi}_{\underline{10}}\,\not{D}\,\psi_{\underline{10}} \qquad (4.3.5)$$

where

$$\not{D} = \gamma^\mu(\partial_\mu + igA_\mu) \qquad (4.3.6)$$

and A_μ is the matrix representation of the SU(5) gauge fields with respect to the T^a (being the matrix representing t^a in the adjoint representation) such that

$$A_\mu = \sqrt{2}\sum_{a=1}^{24} A_\mu^a T_a \qquad (4.3.7)$$

or

$$A_\mu = \sqrt{2}\left\{ B_\mu T^B + \left|\begin{array}{ccc|cc} & & & X^r & Y^r \\ & \sum_1^8 G^a\frac{\lambda^a}{2} & & X^y & Y^y \\ & & & X^b & Y^b \\ \hline \bar{X}^r & \bar{X}^y & \bar{X}^b & & \\ \bar{Y}^r & \bar{Y}^y & \bar{Y}^b & W^i\frac{\tau^i}{2} & \end{array}\right| \right\}_\mu , \qquad (4.3.8)$$

where the B boson is coupled to the $U(1)_Y$ (hypercharge) generator

$$T_B = \sqrt{\frac{3}{5}} \left| \begin{array}{c|cc} -1/3 & 0 & 0 \\ & 0 & 0 \\ & 0 & 0 \\ \hline 0\ 0\ 0 & & \\ 0\ 0\ 0 & 1/2 & \end{array} \right| \equiv T_Y \qquad \text{diag.} \tag{4.3.9}$$

and the W bosons couple to the SU(2) generators

$$T_W^i = \left| \begin{array}{ccc|cc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & & \\ 0 & 0 & 0 & \multicolumn{2}{c}{\tau^i/2} \end{array} \right| \quad , \quad i = 1, 2, 3 \tag{4.3.10}$$

with τ being the Pauli matrices. The eight gluons G_μ^a are coupled to the $SU(3)_C$ generators

$$T_G^a = \left| \begin{array}{ccc|cc} & & & 0 & 0 \\ & \lambda^a/2 & & 0 & 0 \\ & & & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right| \qquad a = 1, 2, \ldots 8 \tag{4.3.11}$$

where $\lambda^{a,s}$ are the usual Gell-Mann matrices. Finally the gauge bosons X and Y are coupled to the generators belonging to the coset space $SU(5)/SU(3)_C \times SU(2)_L \times U(1)_Y$. The 12 generators belonging to this coset space are constructed from the requirement of orthonormality. They are similar to the Pauli matrices $\tau_1/2$ and $\tau_2/2$, which couple to W_1 and W_2 in SU(2). Substituting in Eq. (4.3.5) for ψ_5, ψ_{10} and A_μ, we find

$$L_{int.} = -\frac{3}{5} g B_\mu \sum_f \bar{f} \gamma^\mu \frac{Y_f}{2} f - g W_\mu \sum_f \bar{f}_L \gamma^\mu \frac{\tau}{2} f_L - g G_\mu^a \sum_q \bar{q} \gamma^\mu \frac{\lambda^a}{2} q$$

$$- \frac{g}{\sqrt{2}} \left\{ X_{\mu,i} \left(\bar{d}^i \gamma^\mu e^+ + \epsilon^{ijk} \bar{u}_j \gamma^\mu u_{k,L} \right) + \right.$$

$$\left. Y_{\mu,i} \left(\bar{\nu}_e \gamma^\mu d_L^i - \bar{u}^i \gamma^\mu e_L^+ + \epsilon^{ijk} \bar{u}_j \gamma^\mu d_{k,L} \right) + h.c \right\}. \tag{4.3.12}$$

b) Symmetry breaking in the SU(5) model.

The SU(5) model breaks down in two stages /50, 53/.

$$SU(5) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \longrightarrow SU(3)_C \times U(1)_Y \tag{4.3.13}$$

The minimal Higgs system needed for such symmetry breaking involves an adjoint $\underline{24} \equiv H$ representation of SU(5) for the "superstrong" breaking and a fundamental $\underline{5} \equiv h$, representation for the "weak" breaking. The part of the Lagrangian containing these scalars is

$$L(H,h) = D_\mu h^\dagger D^\mu h + \frac{1}{2} Tr\left((D_\mu H)^\dagger (D^\mu H)\right) + V(H,h) \tag{4.3.14}$$

where D_μ is the covariant derivative

$$D_\mu H = \partial_\mu H + ig A_\mu^a [T^a, H] \tag{4.3.15}$$

and the most general gauge invariant potential (up to a cubic term which is absent if we impose the symmetries $h \longrightarrow -h$ and $H \longrightarrow -H$) is

$$V(h,H) = -\frac{\mu_H^2}{2} Tr H^2 + \frac{\lambda_1}{4} (Tr H^2)^2 + \frac{\lambda_2}{4} Tr H^4 - \mu_h^2 h^\dagger h$$
$$+ \frac{\lambda}{2} (h^\dagger h)^2 + \frac{\alpha}{2} (h^\dagger h) Tr H^2 + \frac{\beta}{2} h^\dagger H^2 h . \tag{4.3.16}$$

Let us consider the case in which the Higgs fields do not couple to each other, then the ground state is obtained by minimizing the potential $V(h,H;\ \alpha = \beta = 0)$. The theory is spontaneously broken if the minimum of $V(h,H)$ (for particular field configurations h_0 and H_0) occur for positive values of v_h^2 and v_H^2. The minimum is obtained for

$$\langle h^\dagger h \rangle = v_h^2 = \frac{\mu_h^2}{\lambda} \tag{4.3.17}$$

and a field configuration H_0, which after a suitable SU(5) transformation reads /50/

$$\langle H \rangle = v_H \begin{vmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & 0 & \\ & & -\frac{1}{3} & & \\ & & & \frac{1}{2} & \\ & 0 & & & \frac{1}{2} \end{vmatrix} \equiv H_0 \tag{4.3.18}$$

where $v_{H_0} = 3\mu_H \left(\frac{15}{2}\lambda_1 + \frac{7}{2}\lambda_2\right)^{-1/2}$ with λ_2 being a positive parameter. In order for the SU(3) symmetry to be preserved by the second stage of symmetry breaking, the vector h_o in the 5 dimensional space has to point in a direction contained in the plane of the two last coordinates. Thus

$$h_o = \begin{vmatrix} 0 \\ 0 \\ 0 \\ h_4^o \\ h_5^o \end{vmatrix} . \qquad (4.3.19)$$

Then, an SU(2) transformation which leaves H_o unchanged allows one to write it as

$$h_o = v_h \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{vmatrix} \qquad (4.3.20)$$

where the parameter v_h is given in Eq. (4.3.17).

The gauge fields coupled to those Higgs fields which have non-vanishing expectation values acquire masses. In this way, the gauge boson mass term in relation (4.3.14) is identified as

$$\frac{1}{2} A_a^\mu M_{ab}^2 A_{\mu,b} = -\frac{g^2}{2} A_a^\mu A_{\mu,b} Tr\left([T^a, H_o][T^b, H_o]\right) \qquad (4.3.21)$$

with H_o given by Eq. (4.3.18).

The mass matrix thus reads

$$M_{a,b}^2 = g^2 Tr\left([T^a, H_o][T^b, H_o]\right). \qquad (4.3.22)$$

The 12 generators associated with the gluons, the W's and the B obviously commute with H_o. These gauge bosons thus remain massless, as they must do. The masses of the remaining gauge bosons X and Y are easily computed

from (4.3.22). They are

$$M_X^2 = M_Y^2 = \frac{25}{8} g^2 v_H^2 . \qquad (4.3.23)$$

In the same way as for the gauge fields, we may write the Higgs fields in the adjoint representation as a 5x5 matrix

$$H = \sqrt{2} \sum_{\alpha=1}^{24} H_\alpha T^\alpha , \qquad (4.3.24)$$

or in terms of the SU(3) x SU(2) x U(1) decomposition we find

$$H = (\sqrt{15}\, v_H + \sqrt{2}\, H_B) T_B + \sqrt{2} \left| \begin{array}{ccc|cc} & & & H_X & H_Y \\ & \sum_{\alpha=1}^{24} H_G^\alpha \frac{\lambda_\alpha}{2} & & H_X & H_Y \\ & & & H_X & H_Y \\ \hline \bar{H}_X & \bar{H}_X & \bar{H}_X & \multicolumn{2}{c}{\multirow{2}{*}{$\underline{H}_W \cdot \frac{\underline{\tau}}{2}$}} \\ \bar{H}_Y & \bar{H}_Y & \bar{H}_Y & & \end{array} \right| , \qquad (4.3.25)$$

in which all H fields have vanishing vacuum expectation values. They carry the same charge as the corresponding gauge bosons (see Eq. (4.3.8)) which also belong to the adjoint representation Such that the 8 coloured scalars $H_G^{\alpha,s}$, H_W^3 and H_B are electrically neutral, $H_W^\pm$ has charge ± 1 and the 3 complex H_X and 3 complex H_Y (colour triplets) have charge $-4/3$ and $-1/3$ respectively. The 6 colour triplet scalars H_X and H_Y remain massless in the Landau gauge. They can be "gauged away" (in the unitary gauge) and their 12 degrees of freedom are transferred to the 12 gauge bosons which become massive (see Eq. (4.3.23)). The 12 physical Higgs bosons H_G^α, H_W^i and H_B acquire masses which can be computed through the part of the Higgs potential (see Eq. (4.3.16)) quadratic in these fields. Using Eq. (4.3.25) one finds /50/

$$m_{H_G}^2 = \frac{5}{4}\lambda_2 v_H^2 \quad , \tag{4.3.26a}$$

$$m_{H_W}^2 = 5\lambda_2 v_H^2 \quad , \tag{4.3.26b}$$

$$m_{H_B}^2 = (15\lambda_1 + \frac{7}{3}\lambda_2) v_H^2 \quad . \tag{4.3.26c)}$$

Finally, we come to the prediction of the SU(5) model for the fermion masses generated by the second stage of symmetry breaking. The most general Yukawa couplings of the h Higgs bosons to the fermions take the following form /50/

$$L_Y = G v_h (\bar{\psi}_{\underline{5}})^\beta (\psi_{10})_{\alpha\beta} - \frac{1}{4}\tilde{G}\,\epsilon^{\alpha\beta\gamma\delta\zeta} h_\alpha (\psi_{\underline{10}})^T_{\beta\gamma} C^{-1} (\psi_{10})_{\delta\zeta}$$

$$+ \text{h.c.} \tag{4.3.27}$$

Since only h_5 develops a non-vanishing vacuum expectation value v_h , the first term is readily seen to yield the following mass term

$$L'_Y = G v_h (\bar{\psi}_{\underline{5}})^\beta (\psi_{\underline{10}})_{5\beta} + \text{h.c}$$

$$= -\frac{G v_h}{\sqrt{2}} \left(\sum_i \bar{d}^i_R d_{iL} + \bar{e}^+_R e^+_L \right) + \text{h.c} \quad . \tag{4.3.28}$$

The second fermion mass term of Eq. (4.3.27) is, (see Appendix A)

$$L^2_Y = -\frac{1}{4}\tilde{G} v_h \epsilon^{5\beta\gamma\delta\zeta} (\psi_{\underline{10}})^T_{\beta\gamma} C^{-1} (\psi_{\underline{10}})_{\delta\zeta} + \text{h.c}$$

$$= -\tilde{G} v_h \epsilon^{ijk} (\psi_{\underline{10}})^T_{ij} C^{-1} (\psi_{\underline{10}})_{k4} + \text{h.c.}, \tag{4.3.29}$$

where i, j, k = 1, 2, 3. Since $(\psi_{10})_{k4} \equiv \frac{1}{\sqrt{2}} u_{k,L}$ and

$$(\psi_{\underline{10}})_{ij} = \epsilon_{ijm} \frac{u^m_L}{\sqrt{2}} = \epsilon_{ijm} C \frac{(\bar{u}^m_R)^T}{\sqrt{2}} \tag{4.3.30}$$

we find /F6/

$$L_Y^2 = -\frac{\tilde{G} v_h}{2} \epsilon^{ijk} \epsilon_{ijm} \bar{U}_R^m U_{k,L} + h.c$$

$$= -\tilde{G} v_h \bar{U}_R^m U_{mL} + h.c. \qquad (4.3.31)$$

We observe that the second term gives only the U-quarks mass while the first term given by Eq. (4.3.28) contains both d and e masses. Moreover, it predicts that the down quark and the electron have the same mass. This degeneracy is a consequence of the SU(4) invariance of h_o. It is, however, broken by radiative corrections /56, 53/.

From the above brief review of the SU(5) grand unified theory, we observe what really happens when the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group is embedded in a larger gauge group G = SU(5). The relation (4.3.12), indeed contains all the information about any specific interaction of the SU(5) gauge bosons with matter. In particular, the first three terms contain the $SU(3)_C \times SU(2)_L \times U(1)_Y$ interactions in which the gauge bosons B, W and G couple to the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ generators respectively. By comparing the first term of relation (4.3.12) with (2.2.6) we observe that the SU(5) unification implies

$$g' = \sqrt{\frac{3}{5}}\, g \,, \qquad (4.3.32)$$

and consequently, due to the definition of the electroweak mixing angle (see Eq. 2.2.17) we obtain a prediction for this angle. Since

$$\tan\theta_w = g'/g \equiv \sqrt{3/5} \,, \qquad (4.3.33)$$

we find

$$\sin^2\theta_w = \frac{3}{8} \,. \qquad (4.3.34)$$

On the other hand the photon is supposed to be a combination of B and W_3, that is to say, the electron charge operator is accordingly defined as

$$Q = T_{3L} + CT_Y \tag{4.3.35}$$

in which Q is given by the $\bar{5}$ in relation (4.3.3) as

$$\varphi = \begin{vmatrix} -1/3 & & & & \\ & -1/3 & & & \\ & & -1/3 & & \\ & & & 1 & \\ & & & & 0 \end{vmatrix} . \tag{4.3.36}$$

This fixes the value of C in Eq (4.3.35), so that

$$C = Tr(\varphi T_Y) / Tr(T_r T_Y) . \tag{4.3.37}$$

It then follows from Eqs. (4.3.32) and (4.3.35) that

$$\sin^2\theta_w = Tr(T_{3L} T_{3L}) / Tr(\varphi\varphi) = (1+c^2)^{-1}, \tag{4.3.38}$$

or $\sin^2\theta_W = 3/8$.

We will later show that as long as the electric charge operator is defined by Eq. (4.3.35) , relation (4.3.38) holds for a large class of grand unified theories (up to the normalization coefficient C).

As a second consequence, the model also predicts that not only weak and electromagnetic interaction are governed by the same coupling constant g, but also that g is the srtong interaction coupling constant so that

$$\frac{\alpha}{\alpha_s} = \frac{e^2}{g^2} = \sin^2\theta_w . \tag{4.3.39}$$

Apart from $SU(3)_C \times SU(2)_L \times U(1)_Y$ interactions which are now related with each other (in the Lagrangian (4.3.12)), there are new interactions (the last term in the bracket of relation (4.3.12)) mediated by X and Y gauge bosons. We have graphically displayed their coupling with fermions in Fig. (4.3.1).

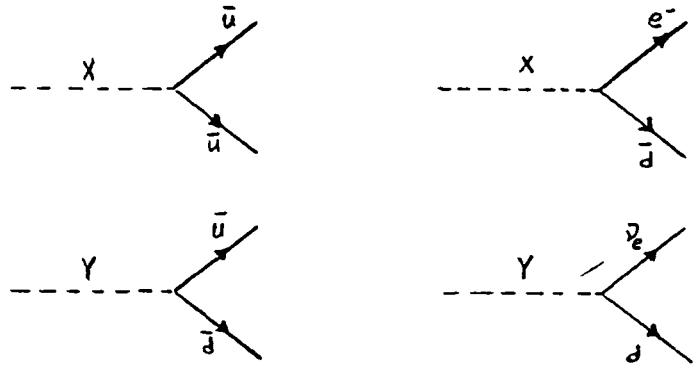


Fig. (4.3.1) The couplings of X and Y gauge bosons of the SU(5) model with fermions.

We observe [from the diagrams of Fig. (4.3.1) and the Lagrangian (4.3.12)] that the X's and Y's not only mediate transitions between quarks and leptons, but also that there are interactions in which X's or Y's can couple to two quarks (we will see later that it is indeed because of this dual role of the X's or Y's that proton decay is possible). The effective second order processes generated by diagrams of Fig. (4.3.1) are always of the type

$$\bar{\ell} \longleftrightarrow qqq\,. \tag{4.3.40}$$

For example we have processes such as

$$e^+ \longleftrightarrow uud, \quad e^+\bar{u} \longleftrightarrow ud, \ \ldots \tag{4.3.41a}$$

$$e^- \longleftrightarrow \bar{d}\bar{u}\bar{u} \quad e^-d \longleftrightarrow \bar{u}\bar{u}, \ \ldots \tag{4.3.41b}$$

$$\nu_e \longleftrightarrow \bar{u}\bar{d}\bar{d}. \quad \nu_e d \longleftrightarrow \bar{u}\bar{d}, \ \ldots \tag{4.3.41c}$$

where (...) denotes other possible interactions which may contribute to the above processes.

In the context of the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ unified gauge theory both baryon number (B) and lepton number (L) are conserved.

[There might be small baryon number violating corrections to the weak process of muon capture shown in Fig. (4.3.2).

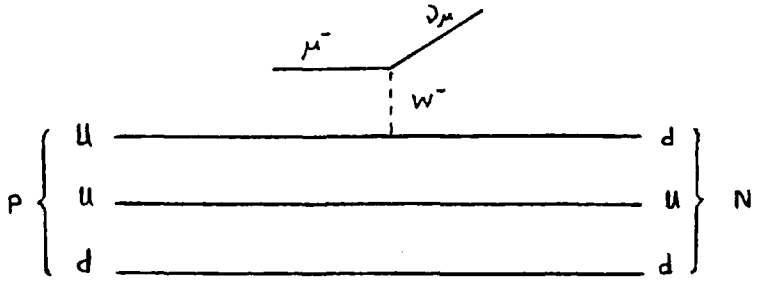


Fig. (4.3.2) A muon interacts with an up quark in a proton. The neutrino escapes and the proton in the nucleus is changed into a neutron.

However, in such flavour interactions the proton lifetime is predicted to be around 10^{134} Yr.] /54/. But the Lagrangian (4.3.12) violates both B and L conservation due to the interactions mediated by X's and Y's (see Eq. (4.3.41)). This is one of the peculiar results of embedding G_1 in a larger group like G = SU(5), and accordingly the proton can decay. Examples of diagrams contributing to the decay mode $p \longrightarrow e^+ \pi^0$ are shown in Fig. (4.3.3).

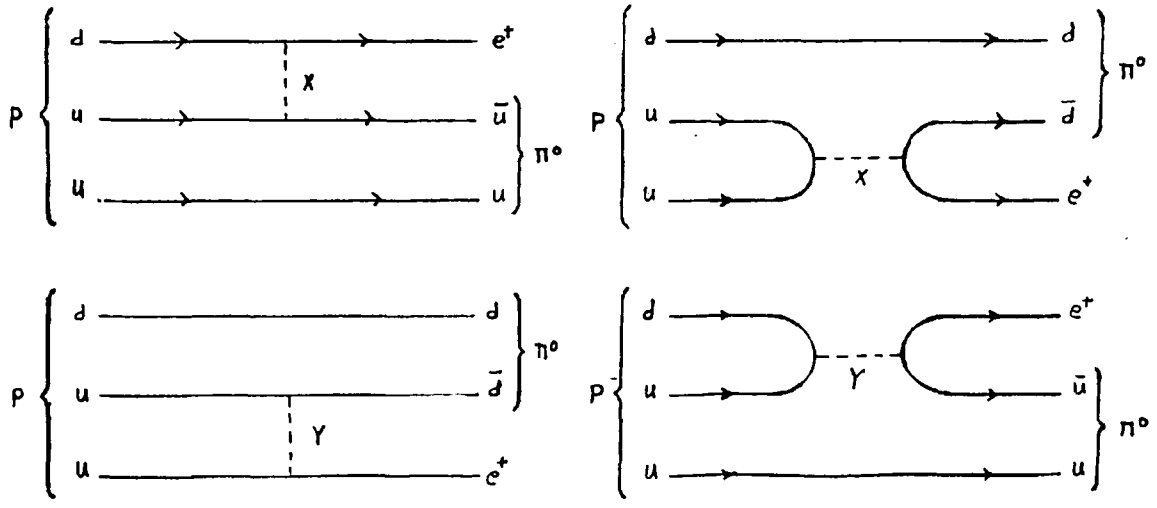


Fig. (4.3.3) Diagrams contributing to the decay $p \longrightarrow e^+ + \pi^0$

In each diagram two quarks fuse to give a lepton and an antiquark which pairs up with the quark inside the proton to make a pion. The proton decay rate for such processes are given by /50/

$$\Gamma(P \longrightarrow e^{+} + \pi^{0}) \simeq \frac{60}{\pi^{2} R^{3}} G_{GU}^{2} m_{q}^{2} \left(\frac{\alpha_{s}(\mu)}{\alpha_{G}(M)} \right)^{4/(11-2n_{g}/3)} \qquad (4.3.42)$$

where R is the proton radius,

$$\frac{G_{GU}}{\sqrt{2}} \equiv \frac{g_{G}^{2}}{8 M_{W}^{2}} = \frac{g_{G}^{2}}{8 M_{Y}^{2}} , \qquad (4.3.43)$$

m_q the quark mass, α_s strong coupling, $\alpha_G = g_G^2/4\pi$ and n_g is the number of generations. From Eq. (4.3.42) the proton lifetime is estimated to be

$$\tau_{p}(P \longrightarrow e^{+} + \pi^{0}) \simeq O(10^{3}-10^{4}) \frac{M_{X}^{4}}{M_{p}^{2}} \ \text{Yr.}, \qquad (4.3.44)$$

where M_p is the proton mass. On the other hand, the experimental bound on the proton lifetime is $\tau_p \geq 3 \times 10^{29}$ years. By comparing this result with Eq. (4.3.44) it turns out that M_X must be as large as 10^{14} Gev.

The requirement of such a large mass (in comparison with the masses of the order of 100 Gev belonging to the $SU(2)_L \times U(1)_Y$ substructure causes some difficulty in (many) grand unified theories. In general the principle underlying the calculation of M_X is the same as that which solves another problem of grand unified gauge theories. How can a theory with a simple gauge group and therefore only one coupling constant simultaneously describe the strong interactions, whose coupling constant is large, and the weak and electromagnetic interactions whose coupling constant is small? We look at these questions in the next section in which renormalization effects will be taken into account for grand unified theories.

CHAPTER V. RENORMALIZATION EFFECTS IN GRAND UNIFIED THEORIES

5.1 Motivation.

In section (2.3) we have discussed some aspects of electroweak gauge theories, especially we referred to the physical parameters of the standard model. This chapter aims to look at these parameters, in particular, the gauge couplings and the value of $\sin^2\theta_W$ in the framework of the grand unified theories.

One of the main motivations in extending the idea of electroweak unified theories (along with chromodynamics) to grand unified theories is that such theories might reduce the number and fix the values of these parameters. This expectation comes from the fact that the grand unified theory has only one gauge coupling (associated with the grand unifying group G) and hence many of the parameters would arise essentially as group theoretical constants.

In dealing with the question of these parameters, grand unified theories, in principle, are able to predict a definite value for $\sin^2\theta_W$. They also partially reduce the number of coupling constants and predict a relationship between quark and lepton masses. However, some of these predictions are totally inconsistent with the phenomenology we are familiar with. In particular, the known (i.e. the observed) values of the coupling constants of the strong and electroweak interactions are vastly different from and cannot be identified with the coupling constant of the grand unified gauge group. In the SU(5) model, for example, the value of $\sin^2\theta_W$ is calculated to be 3/8 (see Eq. 4.3.34)) while the experimental value of $\sin^2\theta_W$ is predicted to be around 0.23. The ratio of $\alpha/\alpha_s = u_s$ is experimentally evaluated to be : $u_s = 0.071$ (see Eq.5.4.41) while the theoretical prediction of u_s is apparently obtained from Eq. (4.3.39) as $u_s = 0.375$. A similar situation arises for the fermion masses; the down

quark and the electron due to Lagrangian (4.3.31) have the same mass but experimentally we find $m_d = 15\ m_e$ (see Eq. (2.2.34)).

This discrepancy of physical parameters can be understood if one notices that experiments do not in general observe bare quantities, rather they observe amplitudes which are dressed with all possible radiative corrections. When these higher order corrections are included the effective value of the physical parameters such as $\sin^2\theta_W$, coupling constants and fermion masses associated with a particular process differ from the corresponding undressed quantities. Such a dynamical mechanism, the renormalization effect, plays a fundamental role in grand unified theories.

In this chapter we study the radiative corrections to the values of coupling constants and $\sin^2\theta_W$ for higher order contributions to the beta function (up to two loop). The "effective" physical quantities, in particular the effective coupling constants are introduced in section 2. In section 3, these quantities are described in the framework of grand unified theories along with the "decoupling theorem", for the one loop beta function. Higher order corrections for the relevant quantities up to the two loop beta function are worked out in section 4 where we also introduce two rather different methods for handling the problem. The value of the grand unifying mass scale which plays an important role in distinguishing between different grand unified theories through comparison with experimental results are estimated. In this way the low energy behaviour of electroweak mixing angle is carefully connected to the measurable quantities. Our procedure is quite general in the sense that it can be applied to any unified gauge model that is broken spontaneously in two stages down to the low energy symmetry. The resulting relations are compared with both the already calculated one loop beta function and the recent experimental data. Finally section 5 ends this chapter by summing up the main points of our results. In this section we also compare our approach with those of other authors who have considered the same problem subsequently.

5.2 Effective couplings.

Effective coupling constants in any physical theory play a significant role in determining parameters of the theory by comparing these quantities with experimental results. Furthermore, they are in general different for different interactions and change as we vary the mass scale (or renormalization point) of the theory so that their behaviour is governed by a system of renormalization group equations. The basic idea underlying the renormalization group equations is as follows /46,47,55,56,57/.

A renormalizable field theory, in general, contains two types of parameters - masses or coupling constants with positive dimensions of mass (i.e. due to $M\bar{\psi}\psi$ or $\lambda\phi^3$ type terms in the Lagrangian) and dimensionless coupling constants (i.e. due to $\lambda\phi^4$ or $\bar{\psi}\gamma_\mu\psi A^\mu$ type terms in the Lagrangian). If one considers a Green's function for large and spacelike momenta (so as to exclude any Landau singularities), then one would expect that the mass terms in the Lagrangian could be neglected. In other words, the leading asymptotic behaviour of the Green's functions should be the same as would be calculated in a massless theory. This can be proved to any finite order in perturbation theory /57/.

The massless theory contains no dimensional parameters to set the scale of momenta. Therefore one might expect that the asymptotic behaviour of the amplitude would be determined by pure dimensional analysis [this is called canonical scaling]. However, this does not occur in practice, since the massless theory does contain a hidden dimensional parameter. This parameter, μ, must be introduced in order to perform the subtractions necessary to renormalize the theory and render it finite. Due to infrared singularities these subtractions, for the massless theory, must be performed off shell, say at some spacelike momentum $p^2=-\mu^2$. The subtractions then define the physical coupling constants and the scale of

the fields (which are determined by the wave-function renormalization constants). If we change the subtraction point the net effect is to change the value of the coupling constants and the scale of the fields. This fact is expressed by the renormalization group equations.

A change in the subtraction point, μ, is equivalent to a change in the scale of all momenta since μ is the only parameter that fixes the momentum scale. Therefore, one can use the renormalization group to relate Green's functions for one set of momenta and coupling constants to Green's function for a scaled set of momenta and different values of the coupling constants. To do this, one only needs to know the functions which determine the change in μ. In particular, the asymptotic form of the amplitudes (for large spacelike momenta) can be related to amplitudes at some fixed momentum and an effective coupling constant (which would be determined by performing the subtractions at asymptotic values of the momenta). The asymptotic value of this effective coupling constant will determine the ultraviolet behaviour of the theory and it is given by the zeros of a calculable function.

5.3 Effective couplings in grand unified theories.

As mentioned in Sec. (4.3), the most striking prediction of grand unified theories is that baryon number, in general, is not conserved. To ensure the observed level of proton stability, the X-bosons that mediate the relevant interactions must be at least as heavy as 10^{14} Gev. Since the energy scale of electroweak symmetry breaking is believed to be around 10^{2} Gev, there appears in grand unified theories widely different scales of symmetry breaking. One scale is around 10^{2} Gev and another scale about 10^{14} Gev. This hierarchy of symmetry breaking imposes severe constraints on the renormalized parameters in these theories. Once these constraints are satisfied, the effects of superheavy bosons on low (with respect to M) energy phenomena are expected to be very small, of the order of some inverse power of $M \sim 10^{14}$. This mechanism is called the decoupling theorem /58/. Let us briefly state the theorem, since it plays a very important role in determining the effective couplings and the values of other parameters in grand unified theories.

Consider a theory in which there are two types of fields namely light and heavy fields. We shall require that both the full theory and the reduced or light theory (that may be constructed from the full theory when the heavy fields are decoupled) are renormalizable. The decoupling theorem applies to processes with only light external particles in the limit that the heavy mass M becomes very large compared to both the light mass and the external momenta p^{2}. If in this limit the relevant light particle quantities calculated to O(1) from the full theory can also be calculated from the light theory, (after a suitable redefinition of the renormalization parameter), then the decoupling theorem is said to be applicable. The utility of the decoupling theorem is that physical quantities can often be calculated more simply from the light theory than from the full theory. The validity of this theorem has already been

established in many theories without spontaneous symmetry breaking /59/, for example for QED and QCD /59/. The main point of the argument in this theory is that, to the order of accuracy stated, the contribution of the heavy particles is only limited to the ultraviolet region.

In theories with spontaneous symmetry breaking, the situation is slightly different /60/. In particular, there are two new aspects which merit discussion:

i) How does the value of M become large?

The particle masses are generally proportinal to gv, where g is the coupling constant and v is the vacuum expectation value of scalar fields in the theory. This can allow the masses of the particles to get large by making either the appropriate g or v large. However, for the case in which the masses of the heavy Higgs fields are extremely large, the theory is non-renormalizable and the decoupling theorem is not valid /61/. We do not consider this approach in this note. What we shall be concerned with is the case where one or more vacuum expectation value becomes much larger than the other vacuum expectation values. This is the limit that usually applies to grand unified theories and in this case the decoupling theorem is valid /59/.

ii) The heavy mass effects.

A distinctive feature of the theories with spontaneously broken symmetry is that some coupling terms associated with the vertices pick up a term like $[\ln \frac{M}{\mu} + \text{constant}]$ as a factor which is not suppressed by a $1/M^2$ factor, as would be the case in theories without spontaneous symmetry breaking /62/.

We now go back to the main aim of this section to discuss the effective coupling constants in the framework of a grand unified theory. Our procedure is based on the renormalization group equations or more precisely, we study the renormalization effects in grand unified theories through the renormalization group equation method which in turn is

approximated by applying the decoupling theorem.

We assume that the gauge group G is spontaneously broken down to the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ model such that all vector bosons and scalar fields which are not part of pure QCD or the Salam-Weinberg model acquire about the same superheavy mass $M \gg M_W$ (M_W is the mass of the W-bosons belonging to the $SU(2) \times U(1)$ substructure). In the case of fermions, we assume that all fermions which are represented in the gauge group G have fairly light masses. Although our arguments are general and applicable to a large class of unified gauge theories, in order to be concrete, in this subsection the SU(5) model is mostly analysed.

Because there exist two very different mass scales, $\mu \sim M_W$ and $\mu \sim M_X$, the effective couplings $g_1(\mu)$, $g_2(\mu)$ and $g_3(\mu)$ associated with the subgroups U(1), SU(2) and SU(3), respectively evolve quite differently as μ varies from M_X to M_W, even though $g_3(\mu) = g_2(\mu) = g_1(\mu)/C$ [the value of C was given in Eq. (4.3.38)] for $\mu \gtrsim M_X$. Their behaviour below $\mu \sim M_X$ is governed by their respective effective beta functions which do not include contributions from superheavy particles (they decouple) and hence can differ appreciably. For any gauge coupling constant g_i in the theory we have /48/

$$\mu \frac{\partial}{\partial \mu} g_i = \beta_i(g_i) , \tag{5.3.1}$$

with β_i calculated in the effective field theory based on the G_i gauge group. If all $g_i(\mu)$ are small , then β_i depends only on g_i, with /48/

$$\beta_i(g(\mu)) = b_i g_i^3(\mu) + O(g^5) + \cdots \tag{5.3.2}$$

The one loop solution to Eq. (5.3.1) is then obtained as

$$g_i^{-2}(\mu) = -2 b_i \ln \mu + I_i \tag{5.3.3}$$

where b_i is given by /46/

$$b_i = -\frac{1}{16\pi^2}\left(\frac{11}{3} C_2(G) - \frac{4}{3}\sum_f T_f(R) - \frac{1}{6}\sum_s T_s(R)\right) , \qquad (5.3.4)$$

with $C_2(G)$ being the quadratic Casimir operator of the adjoint representation of the group G

$$C_2(G)\delta_{ad} = \sum_{b,c} C_{abc} C_{dbc} \qquad (5.3.5)$$

[e.g. $C_2(SU(N)) = N$]. The second and third terms of Eq. (5.3.4) denote the fermion and Higgs scalar contributions to b_i ($T(R)$ is defined as $T_r T_a T_b = T(R)\delta_{ab}$, where the T_a are group generators for fermions and scalars, respectively.) In order to determine the constant of integration I_C in Eq. (5.3.3) we impose the following boundary condition at $\mu \simeq M_X$

$$g_3(\mu) = g_2(\mu) = g_1(\mu) = g_G \qquad (5.3.6)$$

which is valid only in the symmetric limit where the breaking of G can be neglected. In this way Eq. (5.3.3) reduces to

$$g_i^{-2}(\mu) = g_G^{-2}(M) + 2b_i \ln\frac{M}{\mu} \quad , \quad i = 1,2,3 \qquad (5.3.7)$$

for $\mu \lesssim M$.

It is then argued /48/ that the gauge coupling constants measured in present experiments can be associated with the values of $g_i(\mu)$ when μ is some "ordinary" mass of the order of 10 Gev.

Using Eqs.(5.3.7), (4.3.39) and (5.3.4) one finds

$$\sin^2\theta = (1+3C^2)^{-1}\ (1+2C^2\frac{\alpha}{\alpha_s}) \qquad (5.3.8a)$$

and

$$\ln \frac{M}{\mu} = \frac{6\pi\alpha^{-1}}{11(1+3C^2)} \left(1 - (1+C^2)\alpha\alpha_s^{-1}\right) \qquad (5.3.8a)$$

where C is given by (4.3.27), $\alpha \equiv e^2/4\pi$, and $\alpha_s \equiv g_3^2/4\pi$.

From the above arguments, we learn that the relation (5.3.8a) predicts lower values for $\sin^2\theta_W$ than does Eq. (4.3.34). We also find out that in the context of grand unified theories, at energies well above the grand unifying mass scale $\mu \gtrsim M$ where the grand unification is manifest there is no distinction between strong and electroweak interactions as far as the effective couplings are concerned. All couplings have the same strength and the corresponding beta function respects the full symmetry of the grand unification group. At energies around $\mu \sim M$ superheavy particles of mass $M_X \sim M$ begin to decouple, and as the energy further decreases we observe the familiar symmetry of the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ theory. In other words, at energies well below M_X one can clearly distinguish colour gluons and electrweak gauge particles from the other gauge bosons. Correspondingly, the effective couplings associated with these different types of gauge bosons become different, and the β-function for each of them respects only a part of the full symmetry. When these couplings evolve in the energy scale from M to the ordinary energies, they become considerably different from the values at M. The effective coupling constant associated with colour gluons becomes much larger since its β-function respects the SU(3) symmetry, whereas the U(1) coupling becomes smaller [see Eqs. (5.3.7) and (5.3.4)].

From Eq. (5.3.8b) one can estimate the value of M_X, as promised in section (4.3), since the value of M_X is of the order of M. For reasonable values of the parameters α, α_s for low values of μ, [for example for $\alpha^{-1} = 137$, $\alpha_s = 0.2$ and $\mu \sim 10$ Gev], one obtains $M_X \sim M \simeq 2 \times 10^{16}$ Gev which can be comparable with the value of M_X predicted by Eq. (4.3.44).

Eqs. (5.3.8a) and (5.3.8b) which are the master results of Ref. (48) play a fundamental role for determining the low energy values electroweak mixing angle (θ) and the grand unifying mass scale M in all grand unified theories that spontaneously break, in one stage, down to the $G_1 = SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry. Because of the potential that the above procedure has in distinguishing between different specific grand unified models through the determination of the proton life-time as well as comparing the low energy value of $\sin^2\theta_W$ with present experiments of neutral current processes, in the next section we shall discuss some corrections to the above calculations.

5.4 Higher order corrections

The purpose of this section will be to establish the solution of the second order renormalization group equations for the effective gauge coupling constants of grand unified theories. The result is applied to the SU(5) model for calculating the low energy behaviour of the effective coupling constants as well as to the value of the electroweak mixing angle in this model. The proton life-time is then evaluated and the result is compared with the already obtained estimates of the one loop approximation.

At the one loop level, the diagrams contributing to the (strong) effective coupling g_3 have been shown in Fig. (4.5.1); [note that the diagrams involving X-gauge bosons and gluons are suppressed]

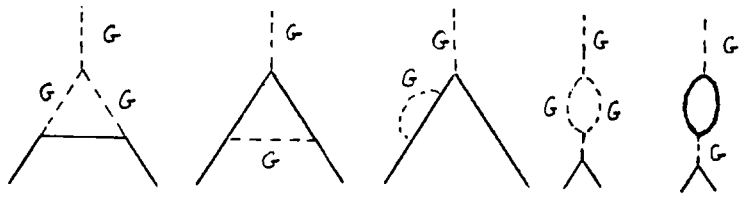


Fig. (5.4.1) Diagrams contributing to the g_3 coupling

Because of this the relation (5.3.1) at the first order (i.e. $\beta_i(g_i) = b_i g_i^3$ for i=1,2,3) reduces to three independent differential equations as far as the g's are concerned. These differential equations are then related to each other by a boundary condition

$$\alpha_1(M) = \alpha_2(M) = \alpha_3(M) \equiv \alpha_G(M) \qquad (5.4.1)$$

that in turn gives Eqs. (5.3.8a) and (5.3.8b) as the final results.

However, when higher orders in Eq. 5.3.2) are taken into account, because of the mixed diagrams like those displayed in Fig. (5.3.2)

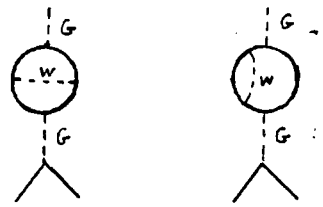


Fig. (5.4.2) Diagrams contributing to the off diagonal elements of b_{ij}

the beta function calculated in perturbation theory is in the following form

$$\beta_i(g) = b_i g_i^3(\mu) + \sum_{i,j} b_{ij} g_i^2(\mu) g_j^3(\mu) \qquad i,j = 1,2,3 \qquad (5.4.2)$$

where both the coefficients b_i and b_{ij} get contributions from the light particles of the theory. In other words, we assume that the "decoupling theorem" is also valid for the two loop renormalization effects. This leads us to consider the following system of differential equations

$$\mu \frac{d}{d\mu} g_i(\mu) = b_i g_i^3(\mu) + \sum_{j=1}^{3} b_{ij} g_i^2(\mu) g_j^3(\mu) \qquad (5.4.3)$$

where the coefficients b are to be calculated from the light particles contributing to the Feynman diagrams. The coefficients b_i are given by Eq. (5.3.4) while the b_{ij} have been already calculated for the Yang-Mills theory /46/. In our notation they correspond to the following matrix

$$(b_{ij}) = \left(\frac{1}{16\pi^2}\right)^2 \begin{vmatrix} \frac{19}{5} & \frac{3}{5} & \frac{44}{5} \\ \frac{1}{5} & \frac{11}{3} & 12 \\ \frac{11}{30} & \frac{3}{2} & -26 \end{vmatrix} \qquad (5.4.4)$$

in which we have neglected the effects of Higgs scalar fields contributing to the second stage of symmetry breaking; since due to our experience from the one loop calculations /68/ their effects are very small.

There does not exist a specific exact solution for the differential equation (5.4.3), hence one essentially needs to employ some

approximate method to find a suitable solution. Here we will examine two methods for finding the solutions for Eq. (5.4.3), it will then be seen that each solution has its own advantage to work with.

Method I

In this method we neglect the off diagonal contributions to Eq. (5.4.3). The reason why we are interested in making such an approximation is that the off diagonal contributions have relatively small effects (see Eq. (5.4.4)); thus we have

$$\mu \frac{d}{d\mu} g_i(\mu) = b_i g_i^3(\mu) + b_{ii} g_i^5(\mu) . \qquad (5.4.5)$$

This equation can be parametrised as follows

$$\frac{d\mu}{\mu} = \frac{dg_i(\mu)}{b_i g_i^3(\mu)} + \frac{b_{ii}^2 g_i(\mu) dg_i(\mu)}{b_i^2 (b_i + b_{ii} g_i^2(\mu))} - \frac{b_{ii} dg_i(\mu)}{b_i^2 g_i(\mu)} \qquad (5.4.6)$$

which leads to a solution for $\ln M/\mu$ as

$$\ln \frac{M}{\mu} = \frac{1}{2b_i} \left(g_i^{-2}(\mu) - g_G^{-2}(M) \right) + \frac{b_{ii}}{2b_i^2} \ln \frac{1 + b_i b_{ii}^{-1} g_G^{-2}(M)}{1 + b_i b_{ii}^{-1} g_i^{-2}(M)} \qquad (5.4.7)$$

where $\mu \sim M_W$ and the boundary condition which we imposed on this equation is

$$g_3(M) = g_2(M) = g_1(M) = g_G(M) \qquad (5.4.8)$$

Substituting $g_3^2/4\pi = \alpha_s$ in (5.4.7) we obtain the value of the unifying mass scale M up to the two loop beta function as follows

$$\frac{M}{M_W} = F(\alpha_s) \exp \left\{ \frac{1}{8\pi b_3} \left(\alpha_s^{-1}(M_W) - \alpha_G^{-1}(M) \right) \right\} \qquad (5.4.9)$$

where

$$F(\alpha_s) = \left(\frac{4\pi b_{33} + b_3 \alpha_G^{-1}(M)}{4\pi b_{33} + b_3 \alpha_s^{-1}(M_W)} \right)^{b_{33}/2b_3^2} \quad . \tag{5.4.10}$$

Comparing Eq. (5.4.9) with Eq. (5.3.8b) we observe that the two loop correction to M/M_W is obtained by multiplying the one loop result by $F(\alpha_s)$ given by Eq. (5.4.10). One can easily check that the quantity $F(\alpha_s)$ satisfies the following inequality

$$1 \geqslant F(\alpha_s) > 0 \tag{5.4.11}$$

and consequently the value of the grand unifying mass scale M could decrease when higher order corrections in perturbation theory are taken into account.

We have fixed the value of α_G [as that of the one loop result ($\alpha_G = 0.024$)] in Eq.(5.4.10) and worked out the quantity $F(\alpha_s)$ for some values of α_s. The results have been displayed in Table (5.4.1)

α_s	0.100	0.110	0.140	0.200
$F(\alpha_s)$	0.688	0.672	0.631	0.577

Table (5.4.1) Values for $F(\alpha_s)$ predicted by the SU(5) model for α_G=0.024.

We will come back to the above result for analysing our findings later in this section.

It is worthwhile pointing out that Eq. (5.4.5) for i=3 is indeed the QCD beta function and the solution associated with i=3 in Eq. (5.4.7) is therefore a solution for the QCD beta function. As far as Eq. (5.4.5) is concerned, unlike the well known results in the literature, in our procedure the solution given by Eq. (5.4.7) for i=3 is an exact solution of the QCD renormalization group equation. In fact we believe that there is no need to use approximation methods for solving the QCD beta function, it can easily be obtained if we only use the trick applied in Eq. (5.4.6). This technique can also be employed to solve Eq.(5.4.5) for higher order

corrections in QCD (i=3), for instance for the three loop beta function it can be easily seen that two additional logarithmic terms are added to Eq. (5.4.7). However, we do not consider this case in the present work.

To estimate the value of $\sin^2\theta_W$ we make use of Eq . (5.4.7), these can be written down as

$$g_1^{-2}(M) = g_G^{-2}(M) + 2b_1 \ln\left(F^{-1}(\alpha_1) \frac{M}{\mu} \right) , \qquad (5.4.12a)$$

$$g_2^{-2}(M) = g_G^{-2}(M) + 2b_2 \ln\left(F^{-1}(\alpha_2) \frac{M}{\mu} \right) , \qquad (5.4.12b)$$

and

$$g_3^{-2}(M) = g_G^{-2}(M) + 2b_3 \ln\left(F^{-1}(\alpha_3) \frac{M}{\mu} \right) , \qquad (5.4.12c)$$

where

$$F(\alpha_i) = \left(\frac{4\pi b_{ii} + b_i \alpha_G^{-1}(M)}{4\pi b_{ii} + b_i \alpha_i^{-1}(M)} \right)^{b_{ii}/2b_i^2} . \qquad (5.4.13)$$

From Eqs. (4.3.39) and (5.4.12b) we find

$$\sin^2\theta(M)_W = \frac{\alpha(M)}{\alpha_G} + 8\pi \alpha(M) b_2 \ln\left(F^{-1}(\alpha_2) \frac{M}{\mu} \right) . \qquad (5.4.14)$$

To find a relation comparable with Eq. (5.4.8a), let us first compute the value of $\ln \frac{M}{\mu}$ in terms of α_1, α_2 and α_3. By combining Eqs. (5.4.12a), (5.4.12b) and (5.4.12c) and using Eq. (4.3.39) one obtains

$$\alpha^{-1} - (1+c^2)\alpha_3 = 8\pi \left(b_1 c^2 + b_2 - (1+c^2) \right) \ln \frac{M}{\mu} + 2b_1 c^2 \ln F^{-1}(\alpha_1)$$

$$+ 2b_2 \ln F^{-1}(\alpha_2) - 2(1+c^2) b_3 \ln F^{-1}(\alpha_3) \qquad (5.4.15a)$$

or

$$\ln\frac{M}{\mu} = \frac{\alpha^{-1}(\mu)}{8\pi\left(b_1c^2+b_2-(1+c^2)b_3\right)}\left(1-(1+c^2)\frac{\alpha(\mu)}{\alpha_s(\mu)}\right)+$$

$$\frac{2b_1c^2\ln F(\alpha_1)+2b_2\ln F(\alpha_2)-2(1+c^2)b_3\ln F(\alpha_3)}{2\left(b_1c^2+b_2-(1+c^2)b_3\right)} \qquad (5.4.15b)$$

[The first term in Eq. (5.4.15) contributes to the one loop beta function and its form is exactly the same as the right hand side of Eq. (5.3.8b) as is expected. The second term contains the two loop beta function correction to the value of $\frac{M}{\mu}$. However, since $\alpha_1 = g_1^2/4\pi$ and $\alpha_2 = g_2^2/4\pi$ are not directly measured, Eq. (5.4.15) is not easily comparable with experimental results. The only information one obtains is from Eq. (5.4.9) which corresponds to the decreasing of $\frac{M}{\mu}$ for higher order corrections] Substituting in Eq. (5.4.14) for $\ln\frac{M}{\mu}$ from Eq. (5.4.15b) and using relation (5.4.12c) we find

$$\sin^2\theta(\mu)_w = \frac{b_2-b_3}{c^2b_1+b_2-(1+c^2)b_3} + \frac{(b_1-b_2)c^2}{c^2b_1+b_2-(1+c^2)b_3}\frac{\alpha(\mu)}{\alpha_s(\mu)}$$

$$+\frac{4\pi\alpha(\mu)c^2}{c^2b_1+b_2-(1+c^2)b_3}\ln H(\alpha_1,\alpha_2,\alpha_3) \qquad (5.4.16)$$

with H being related to the α's as follows

$$H = \left(\frac{4\pi b_{11}+b_1\alpha_G^{-1}}{4\pi b_{11}+b_1\alpha_1^{-1}(\mu)}\right)^{b_{11}(b_2-b_3)/b_1}\left(\frac{4\pi b_{22}+b_2\alpha_G^{-1}}{4\pi b_{22}+b_2\alpha_2^{-1}(\mu)}\right)^{b_{22}(b_3-b_1)/b_2}\times$$

$$\left(\frac{4\pi b_{33}+b_3\alpha_G^{-1}}{4\pi b_{33}+b_3\alpha_3^{-1}(\mu)}\right)^{b_{33}(b_1-b_2)/b_3} \qquad (5.4.17)$$

It can clearly be seen that the first two terms in Eq. (5.4.16) correspond to the first order corrections to the value of $\sin^2\theta(\mu)$ (see Eq. (5.3.8a)) while the third term contains the second order (i.e.two loop beta function)

correction to $\sin^2\theta(\mu)$. As was mentioned the value of α_2 and α_1 are not directly measured and hence unlike the first order corrections to $\sin^2\theta(\mu)$ (see Eq. (5.4.8a)). The theoretical prediction of $\sin^2\theta(\mu)$ for higher order (two loop beta function) corrections cannot directly be compared with the experimental results since the quantity H in Eq. (5.4.16) is a function of α_1 and α_2. To find out more information about $\sin^2\theta(\mu)$ we, therefore, need to know more about the H function and to do so we first substitute for the coefficients b in Eq. (5.4.17) from Eqs. (5.4.4) and (5.3.2), then we obtain

$$\ln H = \left(\frac{1}{16\pi^2}\right)^2 \ln\left\{ \left(\frac{13+14\pi\alpha_G^{-1}(M)}{13+14\pi\alpha_1^{-1}(M)} \right)^{572/121} \times \left(\frac{11-40\pi\alpha_G^{-1}(M)}{11-40\pi\alpha_2^{-1}(M)} \right)^{121/20} \times \left(\frac{19+80\pi\alpha_G^{-1}(M)}{19+80\pi\alpha_3^{-1}(M)} \right)^{209/60} \right\}, \tag{5.4.18}$$

where the effective couplings α_i are to be evaluated at some scale of the order of $\mu \sim M_W$. Relation (5.4.18) together with the fact that

$$\alpha_3(M) \gtrsim \alpha_2(M) \gtrsim \alpha_G \gtrsim \alpha_1(M) \tag{5.4.19}$$

implies

$$H \gtrsim 1 \,. \tag{5.4.20}$$

This means that the last term in Eq. (5.4.16) is always positive no matter what the values of α_i are, and consequently, it turns out that with respect to the one loop calculation the value of $\sin^2\theta(\mu)$, for higher corrections, increases up to some approximation which was used in this method (i.e. method I) by neglecting the off diagonal terms in Eq. (5.4.3).

We would like to emphasize that the above argument about the value of $\sin^2\theta(\mu)$ and the hierarchy strength $\frac{M}{M_W}$ in higher order corrections, is quite general in the sense that the value of $\sin^2\theta(\mu)$ seems to increase in any grand unified theory that is spontaneously broken to the $SU(3)_C \times U(1)_{EM}$ symmetry through the following symmetry breaking pattern:

$$G \xrightarrow{at\ \mu \sim M} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{at\ \mu \sim M_W} SU(3)_C \times U(1)_{EM} \qquad (5.4.21)$$

while the value of $\frac{M}{M_W}$ decreases for the second order corrections.

Clearly, the value of $\sin^2\theta(\mu)$ in Eq. (5.4.16) will change according to the value of C that of course depends on the group G which governs the symmetry of the specific grand unified theory. For $C^2 = 5/3$ [the SU(5) value] for example, we have

$$\sin^2\theta(M)_W = \frac{1}{6} + \frac{5}{9}\,\frac{\alpha(\mu)}{\alpha_s(\mu)} + \frac{10\,H\,\alpha(M)}{33}\ln H\ , \qquad (5.4.22)$$

where H is given by Eq. (5.4.12). [In deriving Eq. (5.4.22) we have used Eqs. (5.4.16) and (5.4.4)]

Method II

As was observed in method I, the logarithmic term appearing in Eq. (5.4.7) makes it difficult to give numerical values for the physical parameters. Therefore one is unable to compare these results with the corresponding experimental ones. The only advantage of such an analysis is that one clearly sees that the value of $\sin^2\theta(\mu)$ increases when higher order corrections are taken into account for Eq. (5.3.8a) while for the same order of corrections the ratio of the two mass scale or the strength of hierarchy, M/M_W, decreases up to an approximation in which the off diagonal terms in the beta function, arising from diagrams with a fermion loop inside a gauge boson line and a gauge boson from a different subgroup inside the fermion loop, are neglected. However, in " Method II " , we establish a different approach from that of method I that makes it possible to predict the numerical values of the physical parameters needed for comparison with the experimental data.

To start off, let us consider Eq. (5.4.3) that can be rewritten in the following form

$$\frac{dg_i(\mu)}{g_i^3(\mu)} = b_i \frac{d\mu}{\mu} + \sum_{j=1}^{3} b_{ij}\, g_j^2(\mu) \frac{d\mu}{\mu} . \qquad (5.4.23)$$

By integrating both sides of this equation we have

$$-\frac{1}{2g_i^2(\mu)} = b_i \ln\mu + I_i + I_C \qquad (5.4.24)$$

where I_C is the constant of integration and I_i is given by

$$I_i(\mu) = \sum_{j=1}^{3} b_{ij} \int g_j^2(\mu) \frac{d\mu}{\mu} . \qquad (5.4.25)$$

To specify the value of I_C we impose the boundary conditions which have already been used for the first order corrections, (see Eq. (5.3.61)). Thus from Eq. (5.4.24) we obtain

$$I_C = -\frac{1}{2g_i^2(M)} - b_i \ln M - I_i(M) \qquad (5.4.26)$$

where $I_i(M)$ is now the constant of integration which must be computed from Eq. (5.4.25). Substituting in (5.4.24) for I_C from Eq. (5.4.26) one finds

$$g_i^{-2}(\mu) = g_G^{-2}(M) + 2b_i \ln\frac{M}{\mu} - I_i(\mu) + I_i(M) . \qquad (5.4.27)$$

This can be written as

$$g_i^2(\mu) = \frac{1}{1+2b_i g_G^2 \ln\frac{M}{\mu}} \left(1 + g_G^2 \{\cdots\}_i + g_G^4 \{\cdots\}_i^2 + \cdots\right) , \qquad (5.4.28)$$

where

$$\{\cdots\}_i \equiv \frac{2\left(I_i(\mu) - I(M)\right)}{1+2b_i g_G^2 \ln\frac{M}{\mu}} . \qquad (5.4.29)$$

Eq. (5.4.28) is a general solution to the system of differential equations

(5.4.23) with the boundary condition (5.4.8). This solution, of course, depends on the integral (5.4.25). To compute this integral (5.4.25) we employ an iterative technique. Regarding Eq. (5.4.28) as a solution for $g_i(\mu)$, integral (5.4.25) becomes

$$I_i(\mu) = \sum_{j=1}^{3} b_{ij} \left\{ \int \frac{g_G^2}{1+2b_j g_G^2 \ln \frac{M}{\mu}} \frac{d\mu}{\mu} + g_G^4 \int \frac{\{\cdots\}_j}{1+2b_j g_G^2 \ln \frac{M}{\mu}} \frac{d\mu}{\mu} \right.$$

$$\left. + g_G^6 \int \frac{\{\cdots\}_j^2}{1+2b_j g_G^2 \ln \frac{M}{\mu}} \frac{d\mu}{\mu} + \cdots \right\}, \qquad (5.4.30)$$

and after calculation we obtain

$$I_i(\mu) = -\sum_{j=1}^{3} \frac{b_{ij}}{2b_i} \ln(1 + 2b_j g_G^2 \ln \frac{M}{\mu}) + I_i'(\mu) , \qquad (5.4.31)$$

where the first term is the result of the first integral of (5.4.30) and $I'(\mu)$ originates from the other terms and it is assumed that all other integrals exist and are well defined. By inserting $I_i(\mu)$ from Eq. (5.4.31) in Eq. (5.4.27) we find

$$g_i^{-2}(\mu) = g_G^{-2}(M) + 2b_i \ln \frac{M}{\mu} + \sum_{j=1}^{3} \frac{b_{ij}}{2b_i} \ln(1 + 2b_j g_G^2 \ln \frac{M}{\mu}) + F_i(\mu) \qquad (5.4.32)$$

where the function $F_i(\mu)$ satisfies a differential equation that can be obtained by substituting the value of $g_i(\mu)$ from Eq. (5.4.32) in Eq. (5.4.23) with a boundary condition

$$F_i(M) = 0 . \qquad (5.4.33)$$

The same method i.e. the iterative technique can then be applied to solve the resulting differential equation. It turns out that the effects of the term which is added to the solution (5.4.32) is too small , so that at order g_G^3 it is at most of the order of g_G^6. However, we neglect such terms in our calculation and keep the leading term only. In this way

Eq. (5.4.32) reads

$$g_i^{-2}(\mu) = g_G^{-2}(M) + 2b_i \ln\frac{M}{\mu} + \sum_{j=1}^{3} \frac{b_{ij}}{2b_i} \ln(1 + 2b_j g_G^2 \ln\frac{M}{\mu}). \quad (5.4.34)$$

Equation (5.4.34) is our final result for the effective coupling constant $g_i(\mu)$ as far as the two loop beta function is concerned. Since $g_i(M)$ is sufficiently small, we have neglected the renormalization effects of $g_G(M)$, in other words, we have neglected the renormalization effects in the grand unified field theory based on the gauge group G. That is why we sometimes refer to $g_G(M)$ as a constant of integration and to M as the unifying mass scale(and not the mass of the superheavy gauge bosons of the grand unified theory in question). It is worth noticing that in some theories for which there is degeneracy in masses of the superheavy gauge particles, the value of M in Eq. (5.4.34) may be regarded as the mass of superheavy particles. We will later discuss that for this case Eq. (5.4.34) seems to predict correct values for $g_i(\mu)$. However, practically we are faced with theories whose superheavy gauge particle masses differ from each other. In the simplest case, in the SU(5) model for example, there exist two sets of superheavy gauge bosons and their masses are slightly different from each other (due to the "weak" symmetry breaking of SU(3) x SU(2) x U(1) to SU(3) x U(1)). For these theories Eq. (5.4.34) may be used for determining the unifying mass scale M which is nearly of the order of the superheavy particle masses up to a renormalization that must be considered in the grand unified theory itself.

Having the μ dependence of the coupling constants in Eq. (5.4.34) it is, in principle, possible to compare them with their corresponding experimentally measured quantities. It is then due to this comparison that the parameters of the grand unified theories are specified. From the phenomenological point of view, the three measurable quantities

$$\alpha_s = g_3^2/4\pi \quad , \tag{5.4.35a}$$

$$\alpha = e^2/4\pi \quad , \tag{5.4.35b}$$

and

$$\sin^2\theta_W = g_2^2/e^2 \quad , \tag{5.4.38c}$$

are usually used for such purposes. The value of α_s (the QCD coupling constant) in (5.4.35a) comes from the analysis of scaling violations in electroproduction and from charmonium spectroscopy such that for $\mu_o \sim 3$ Gev

$$\alpha_s(\mu_o) = 0.320 \qquad \text{(electroproduction)} \tag{5.4.36a}$$

$$\alpha_s(\mu_o) = 0.190 \qquad \text{(charmonium)} \tag{5.4.36b}$$

while the value of α (the QED coupling constant) in (5.4.35b) is conventionally defined at zero momentum transfer as in Thomson scattering at $q^2=0$, so that

$$\alpha(o) = (137.135)^{-1} . \tag{5.4.37}$$

Finally, the third quantity, the electroweak mixing angle θ_W, is presently measured in neutral current experiments at a scale of the order of $\mu \sim M_W$.

We observe that the quantities which are to be used as input for Eq. (5.4.34) are measured at quite different scales. However, the physical parameters, the g's in Eq. (5.4.34), are to be evaluated at the same scale of the order of the typical masses at which the symmetry of $SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$ breaks down. This leads us to choose a suitable value for μ in Eq. (5.4.34) as a fixed point which we take to be M_W, the mass of the gauge bosons belonging to the SU(2) x U(1) substructure of the grand unified theories (we do not distinguish the masses of M_W and M_Z in our procedure). In this way, only the quantities α and α_s must be evaluated for $\mu=M_W$.

To estimate $\alpha(M_W)$ we use the argument of Ref. (63) in which $\alpha(M_W)$ is given by the following relation:

$$\alpha^{-1}(M_W) = \alpha^{-1}(0) - \frac{2}{3\pi} \sum_f Q_f^2 \ln \frac{M_W}{m_f} \quad , \tag{5.4.38}$$

where $\alpha(0)$ is given by (5.4.37) and the sum is over all fermions with quarks counted three times because of their colour (left and right-handed components are not treated separately). The m_f are fermion masses which are taken to be 1 Gev for the leptons and $m_c \simeq m_b/3 \simeq M_t/9 \simeq 1.5$ Gev along with $M_W = 85$ Gev. One finds /63/

$$\bar{\alpha}^{-1}(M_W) = \bar{\alpha}^{-1}(0) - (80/3) = 128 \, . \tag{5.4.39}$$

From now on we use the value of $\alpha(M_W)$ as given by Eq. (5.4.39) in our calculations though this is only a one loop beta function correction. This is valid since the effect of two loop beta function on Eq. (5.4.38) is very small /63/ and therefore negligible.

In order to calculate $\alpha_s(M_W)$ we use Eq. (5.4.7) in which by substituting for $\alpha_s(M_W)$ and $\alpha_s(\mu_o)$ the parameters M and $g_G(M)$ are cancelled out. Thus we find

$$\ln \frac{M_W}{\mu_o} = \frac{1}{8\pi b_3} \left(\alpha_s^{-1}(\mu_o) - \alpha_s^{-1}(M_W) \right) + \frac{b_{33}}{8\pi b_3^2} \ln \left(\frac{4\pi b_{33} + b_3 \alpha_s^{-1}(M_W)}{4\pi b_{33} + b_3 \alpha_s^{-1}(\mu_o)} \right) , \tag{5.4.40}$$

where b_3 and b_{33} are given by (5.3.4) and (5.4.4) respectively. Substituting for $\alpha_s^{-1}(\mu_o)$ from Eq. (5.4.36), we obtain

$$\alpha_s(M_W) = 0.142 \qquad \text{(electroproduction)} \tag{5.4.41a}$$

$$\alpha_s(M_W) = 0.115 \, . \qquad \text{(charmonium)} \tag{5.4.41b}$$

Our next task is to apply the values of $\alpha(M_W)$ and $\alpha_s(M_W)$ to Eq. (5.4.34) and then along with Eq. (5.4.38) we can estimate the values of M and $\sin^2\theta(M_W)$. To do so we choose the conventional combination of Eq. (5.4.34) for i=1,2 and 3 which gives the following result:

$$\ln \frac{M}{M_W} = \frac{6\pi}{11(1+3C^2)} \left\{ \frac{1}{\alpha(M_W)} - \frac{1+C^2}{\alpha_s(M_W)} - 4\pi \sum_{j=1}^{3} \frac{C^2 b_{1j} + b_{2j} - (1+C^2) b_{3j}}{b_j} \ln x_j \right\} , \tag{5.4.42}$$

where

$$X_j = 1 + 8\pi b_j \alpha_G \ln \frac{M}{M_w} \,. \tag{5.4.43}$$

To see the effects of the third term in Eq. (5.4.42) which is added to Eq. (5.3.8b) due to higher corrections let us give some estimates for it. We denote the last term of Eq. (5.4.42) as

$$\ln F(h, C, n_g) \equiv \frac{6\pi}{11(1+3C^2)} \left\{ -4\pi \sum_{j=1}^{3} \frac{C^2 b_{1j} + b_{2j} - (1+C^2) b_{3j}}{b_j} \ln X_j \right\}, \tag{5.4.44}$$

where $h=M/M_W$ determines the hierarchy strength of the considered theory and n_g is the number of gerations that is contained in the b's coefficients. Substituting for the b's from Eqs. (5.3.4) and Eq. (5.4.4) we find

$$\ln F(h, C, n_g) \equiv \frac{(110\pi)^{-1} C^{-2}}{(1+3C^2)} \ln X_1^{Y_1} \cdot X_2^{Y_2} X_3^{Y_3} \,, \tag{5.4.45}$$

where

$$Y_1 = \frac{25C^2 - 8}{16} \,, \tag{5.4.46a}$$

$$Y_2 = \frac{81C^2 - 25}{10} \,, \tag{5.4.46b}$$

$$Y_3 = \frac{43C^2 + 35}{7} \,, \tag{5.4.46c}$$

and

$$X_1 = 1 + \frac{2}{\pi} \alpha_G \ln \frac{M}{M_w} \,, \tag{5.4.47a}$$

$$X_2 = 1 - \frac{5}{3\pi} \alpha_G \ln \frac{M}{M_w} \,, \tag{5.4.47b}$$

$$X_3 = 1 - \frac{7}{3\pi} \alpha_G \ln \frac{M}{M_w} \,. \tag{5.4.47c}$$

From Eq. (5.4.47c) we learn that the solution given by (5.4.34) is

meaningful if

$$1 - \frac{\eta}{2\pi} \alpha_G \ln \frac{M}{M_w} > 0 \quad , \tag{5.4.48a}$$

which gives an upper bound on M/M_W , that is

$$\frac{M}{M_w} < \exp \frac{2\pi}{\eta} \alpha_G^{-1} \quad . \tag{5.4.48b}$$

This constraint provides us with an upper limit on the unifying mass scale M. Taking $\alpha_G = 0.024$ fixed by the one loop result one has $M/M_w \leqslant 1.748 \times 10^{16}$. Furthermore using Eqs. (5.4.46) and (5.4.47) along with Eq.(5.4.48b) we find that

$$1 \geqslant F(h,C,n_g) > 0 \quad . \tag{5.4.49}$$

We have worked out the values of $F(h,c,n_g)$ for reasonable values of α_G which are displayed in Table (5.4.2).

α_G	0.024	0.026	0.028	0.030
F	0.996	0.995	0.994	0.993

Table (5.4.2) Values of $F(h,c,n_g)$ for $n_g=3$, $c^2=5/3$ and $M \sim 6.293 \times 10^{14}$.

In analogy with Eq. (5.4.15b) that was obtained by "Method I", Eq. (5.4.42) can be rewritten in the following form

$$\frac{M}{M_w} = F(h,c,n_g) \exp \left\{ \frac{6\pi}{11(1+3c^2)} \left(\alpha^{-1}(M_w) - (1+c^2)\alpha_s^{-1}(M_w) \right) \right\}, \tag{5.4.50}$$

where $F(h,c,n_g)$ is given by (5.4.45). By using the values of $\alpha(M_W)=128$ and $\alpha_S(M_W)=0.142$ for Eq. (5.4.50) in the case of the SU(5) model, this yields

$$M = F(h,c,n_g) \times 2.81 \times 10^{15} \text{ Gev.} \tag{5.4.51}$$

while for the same value of $\alpha(M_W)$ but for $\alpha_S(M_W)=0.112$ Eq. (5.4.50) reads

$$M = F(h,c,n_g) \times 5.92 \times 10^{14} \text{ GeV}. \qquad (5.4.52)$$

We should perhaps point out that the exponential term in Eq. (5.4.50) is independent of the value of M. In fact, it corresponds to the value of M for the one loop beta function correction (see Eq. (5.3.8b)) and we have already shown that the function $F(h,c,n_g)$ satisfies the relation (5.4.49). This necessarily means that the value of M/M_W is smaller than this ratio when it is calculated up to the one loop level. In other words the higher order corrections (up to two loop) seem to decrease the value of M/M_W. For example if the theoretical results for M/M_W is to be compared with $\alpha_S(M_W) = 0.142$, then $M/M_W \leq 3.524 \times 10^{13}$ or if we take $\alpha_S(M_W) = 0.115$ then we obtain $M/M_W \leq 7.404 \times 10^{12}$ [see Eqs. (5.4.51) and (5.4.51)].

To see the low energy behaviour of the electroweak mixing angle θ_W we make use of Eq. (4.3.39) and Eq. (5.4.32) for i=1,2 along with Eq. (4.3.33) to find the following relation:

$$\sin^2\theta(M_W) = \frac{1}{1+c^2}\left\{1 - \frac{2c^2}{1+3c^2}\left(1-(1+c^2)\frac{\alpha(M_W)}{\alpha_s(M_W)}\right) - \frac{c^2(1+c^2)\alpha(M_W)}{1+3c^2}\sum_{j=1}^{3} Z_j \ln X_j\right\}, \qquad (5.4.51)$$

where

$$Z_j = \frac{(b_{1j} - 3b_{2j} + 2b_{3j})4\pi}{b_j} \qquad j = 1, 2, 3 \qquad (5.4.52)$$

and X_j is given by Eq. (5.4.43).

There are three distinguishable terms in Eq. (5.4.51). The first term contains $1/(1+c^2)$, which is purely a number. It originates from embedding the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ model in a grand unified theory based on a simple gauge group G (such as SU(5)). In the simplest example, when G = SU(5) the value of C is given by Eq. (4.3.38) and hence $1/(1+c^2) = 3/8$. The second term, which is symbolically denoted as $[\sin^2\theta(M_W)]^{(1)}$, reads

$$\left[\sin^2\theta(M_W)\right]^{(1)} = -2c^2(1+c^2)^{-1}(1+3c^2)^{-1}\left(1-(1+c^2)\alpha(M_W)\alpha_s^{-1}(M_W)\right), \qquad (5.4.53)$$

which comes from the one loop beta function correction to the gauge couplings of grand unified theories. As long as $\alpha(M_W)$ and $\alpha_s(M_W)$ are compared with their corresponding experimental values, $[\sin^2\theta(M_W)]^{(1)}$ for a specific theory, is a universal quantity [its value for $\bar{\alpha}^{-1}(M_W)=128$ and $\alpha_s(M_W)=0.115$ is $[\sin^2\theta(M_W)]^{(1)}=-0.168$ and for $\bar{\alpha}^{-1}(M_W)=128$ and $\alpha_s(M_W)=0.142$ is $[\sin^2\theta(M_W)]=-0.177$ (for $C^2=5/3$)]. Due to these two terms in Eq. (5.4.51), corrections to the value of $\sin^2\theta(M_W)^{(1)}$ up to one loop Feynman diagrams for $C^2=5/3$ is formulated as

$$\sin^2\theta(M_W)^{(1)} = \frac{1}{6} - \frac{5}{9}\frac{\alpha(M_W)}{\alpha_s(M_W)} \quad , \tag{5.4.54}$$

which corresponds to the results obtained in Ref. (48).

Finally, the last term in Eq. (5.4.51), which is symbolically denoted by $[\sin^2\theta(M_W)]^{(2)}$, reads

$$\left[\sin^2\theta(M_W)\right]^{(2)} = -\frac{C^2\alpha(M_W)}{1+3C^2}\sum Z_j \ln X_j \quad , \tag{5.4.55}$$

where Z_j and X_j are given by Eq. (5.4.52) and (5.4.43) respectively. Substituting for the b's in Eq. (5.4.52) and Eq. (5.4.43) from (5.3.4) and (5.3.4) and then using the results for Eq. (5.4.55) one obtains

$$\left[\sin^2\theta(M_W)\right]^{(2)} = -\frac{C^2}{1+3C^2}\frac{\alpha(M_W)}{10\pi}\ln X_1^{21/8}\cdot X_2^{31/20}\cdot X_3^{198/7} \quad , \tag{5.4.50}$$

where X_1, X_2 and X_3 are given in Eq. (5.4.47). The positivity constraint on X_2 implies that $M/M_W \lesssim 1.748 \times 10^{16}$ (as was discussed for Eq. (5.4.48a)). It then turns out that for any value of C we have

$$\left[\sin^2\theta(M_W)\right]^{(2)} \gtrsim 0 \quad . \tag{5.4.60}$$

It means that for any value of M satisfying relation (5.4.48a) the value of $\sin^2\theta$ in Eq. (5.4.51) increases with respect to the one loop correction.

For example for those models with $C^2=5/3$ from Eq. (5.4.51) we find that

$$\sin^2\theta(M_W) = \frac{1}{6} + \frac{5}{9}\frac{\alpha(M_W)}{\alpha_s(M_W)} - \frac{5\alpha(M_W)}{180\pi}\ln X_1^{21/8}\cdot X_2^{3/20}\cdot X_3^{198/9}$$

$$> \sin^2\theta(M_W)^{(1)}. \qquad (5.4.61)$$

(See also Eq. (5.4.54)).

We have displayed the value of $\sin^2\theta(M_W)$ and the unifying mass scale M for some values of $\alpha_s(M_W)$ in Table (5.4.3).

$\alpha_s(M_W)$	$\sin^2\theta(M_W)$	M	τ_p (Yr.)
0.11	0.209	0.586×10^{15}	9×10^{29}
0.12	0.205	1.059×10^{15}	5×10^{30}
0.13	0.203	1.701×10^{15}	2×10^{31}
0.14	0.202	1.800×10^{15}	4×10^{31}
0.15	0.200	2.001×10^{15}	7×10^{31}
0.16	0.198	2.402×10^{15}	1×10^{32}
0.17	0.197	2.491×10^{15}	2×10^{32}

Table (5.4.3) Values of $\sin^2\theta(M_W)$, M and τ_p predicted by the SU(5) model for a range of $\alpha_s(M_W)$.

The results presented in Table (5.4.3) have been worked out for the SU(5) model [for which $C^2=5/3$ and $n_g=3$] through Eqs. (5.4.61), (5.4.50), (5.4.15b) (5.4.40) and (5.4.44). The value of α_G has been estimated graphically by using Eqs. (4.3.61) and (5.4.5) for each value of α_s; it is predicted to be between 0.025 and 0.030.

There are now a number of comments to be made about the above procedure and results. These we present in section 5.

5.5 Conclusion.

a) We presented /67/ a general procedure for evaluating the low energy behaviour of the physical parameters of grand unified theories. In particular, we mainly analysed the question of gauge coupling constants and the electroweak mixing angle in the theories for which symmetry breaking occurs in two stages. It turns out that when the underlying symmetry group G is "simple" and the theory contains only one gauge coupling constant, the first stage of symmetry breaking takes place at energies around 10^{15} Gev (see Table (5.4.3)). At energies well above $M \sim 10^{15}$ Gev where the grand unification is manifest, there is no distinction between strong and electroweak interactions and all couplings have the same strength. We have used this as a boundary to our approach which in turn imposes a boundary condition on the relevant differential equations governing the behaviour of coupling constants. Although the coupling constant for a specific grand unified theory is the same for all interactions at energies above the unifying mass scale $M \sim 10^{15}$ Gev, it branches in the lower energy region in the space of effective coupling constants. In branching the coupling constant of grand unified theories, the "strong" spontaneous symmetry breaking seems to play the main role since at energies around M the superheavy gauge bosons acquire masses of the same order M and they begin to decouple such that any interaction then is governed by its own beta function. That is perhaps why the basic mechanism, in radiative correction in grand unified theories, is the "decoupling theorem" of heavy particle loops.

When one excludes the contributions of heavy particles from the relevant beta function for each interaction and applies the renormalization group technique for effective parameters, some very interesting information is obtained such as the order of unifying mass scale, the relations between effective couplings and particularly the value of the

electroweak mixing angle (see Eqs. (5.4.39), (5.4.50) and (5.4.61)). These findings are more or less consistent with the low energy behaviour of the theories in question.

b) We would like to point out that the fermionic threshold effects do <u>not</u> manifest themselves in the one loop corrections for $\sin^2\theta_W$ and the value of M/M_W. This can be seen from Eqs. (4.5.15b) and (4.5.16) in which the fermions contribute to the calculations through the coefficient b's. In Eq. (4.5.15b) these coefficients have been parameterized in the form $[b_1 C^2 + b_2 - (1+C^2) b_3]^{-1}$, where the b_i are given by

$$b_1 = -\frac{1}{16\pi^2}\left(-\frac{4}{3} n_g - \frac{1}{10} N_H\right) \tag{5.5.1a}$$

$$b_2 = -\frac{1}{16\pi^2}\left(\frac{11}{3} C_2(SU(2)) - \frac{4}{3} n_g - \frac{1}{6} N_H\right) \tag{5.5.1b}$$

$$b_3 = -\frac{1}{16\pi^2}\left(\frac{11}{3} C_2(SU(3)) - \frac{4}{3} n_g\right) \tag{5.5.1c}$$

with n_g being the number of fermion generations and N_H being the number of SU(2) Higgs isodoublets (see also Eq. (3.5.4)). Using the coefficient b's from (5.5.1) we find

$$\left(b_1 C^2 + b_2 - (1+C^2) b_3\right)^{-1} = \left(\frac{1}{16\pi^2}\left((1+C^2)\frac{11}{3} C_2(SU(3)) - \frac{11}{3} C_2(SU(3)) + N_H\left(\frac{1}{6} + \frac{C^2}{10}\right)\right)\right)^{-1} \tag{5.5.2}$$

which is indeed independent of n_g. A similar situation happens for the values of $b_2 - b_3$ and $(b_1 - b_2)$ in Eq. (5.4.16). It means that the fermionic threshold effects do not influence the value of M/M_W and $\sin^2\theta_W$ when the first order beta function is considered.

c) We analysed the two-loop effects on the beta function in order to study the higher order corrections to the coupling constants of the grand unified theories which in turn leads to a correction to the electroweak mixing angle as well as the unifying mass scale M. To obtain a solution to the resulting renormalization group equations, two rather

different methods were suggested. In method I we have neglected the off diagonal contributions to the relevant beta functions; this approximation seems to be reasonable since α_1 and α_2 are small and the off diagonal contributions have also relatively small effects. (See Eqs. (5.4.3) and (5.4.4)). In this case we found a solution for the hierarchy strength $h=M/M_W$ in terms of α_s and α_G which was given in Eq. (5.4.10). From Eq. (5.4.10) we find out that the ratio M/M_W decreases with respect to the one loop result by a functional factor $F(\alpha_s)\simeq\frac{1}{2}$ (see Table (5.4.1)). Following method I, we also argued that Eq. (5.4.7) for i=3 may be useful for calculating the value of α_s at the scale of $\mu\sim M_W$ by comparing our results with the corresponding experimental value of α_s. The value of $\alpha_s(M_W)$ was obtained in Eq. (5.4.41). We then studied the behaviour of the electroweak mixing angle θ_W and came to the conclusion that the low energy value of $\sin^2\theta(M_W)$ increases with respect to the one loop correction (see Eq. 5.4.16)). In method II, we established a solution for the system of differential equations (5.4.23) based on an iterative technique. Although the resulting relationships for the effective couplings are very complicated, Eqs. (5.4.50) and (5.4.51) give clear results for the behaviour of M/M_W and $\sin^2\theta(M_W)$ respectively. According to these Eqs. the value of M/M_W decreases while the low energy value of $\sin^2\theta(M_W)$ increases due to our approach which confirm the results of method I. In Eq. (5.4.50) the value of M/M_W decreases by the functional factor $F(h,C,n_g)$ satisfying the condition (5.4.49). The value of $F(h,C,n_g)$ depends on three indices h, C and n_g. We have studied the behaviour of $F(h,C,n_g)$ around the value of $h = 10^{14}$ by fixing $n_g=3$ and $C^2=5/3$ and found out that it does <u>not</u> have significant effects in our procedure. However, it can basically change the prediction of M/M_W through the other indices C and n_g. For theories with four generations, for example, the coefficients b_i must be recalculated through Eq. (3.5.4), and the coefficients b_{ij} of Eq. (5.4.4) must be replaced by

$$(b_{ij}) = \left(\frac{}{16\pi^2}\right)^2 \begin{vmatrix} \frac{76}{15} & \frac{12}{5} & \frac{176}{15} \\ \frac{4}{5} & 20 & 16 \\ \frac{44}{30} & 6 & -\frac{8}{3} \end{vmatrix} . \qquad (5.5.3)$$

We have checked that for theories with four generations and $C^2=5/3$ the condition (5.4.49) does <u>not</u> hold and consequently for models with four genrations the value of M/M_W increases with respect to the one loop correction.

d) Method II for $n_g=3$ does not change the value of M/M_W (predicted from the one loop correction) by very much. This can be understood from Table (5.4.2) in which the function $F(h,C,n_g)\sim 1$. But it does increase the value of $\sin^2\theta(M_W)$ by nearly a factor of 1.02% (see Eq.(5.4.61)).

e) The question of higher order corrections to the values of $\sin^2\theta_W$ and the grand unifying mass scale M was then reconsidered by other authors /64,65,66/.

In Ref. (64) a similar argument is given for higher order corrections to $\sin^2\theta_W$ and the unifying mass scale M in the SU(5) model. In this work the method which is chosen to handle the problem is the same as that of method I and it confirms our result that the low energy value of the electroweak mixing angle increases whereas the value of M decreases with respect to the one loop correction. The values of $M = 1.1 \times 10^{15}$ Gev, $\sin^2\theta(M_W) = 0.206$ are then suggested to be the best SU(5) model predictions. We have borrowed part of his result exhibited in Table (5.5.1) to compare with our result of Table (5.4.3). (Note the results in Table (5.4.3) have been worked out by using method II in our approach)

$\alpha_s(M_W)$	$\sin^2\theta(M_W)$	M	τ_p (Yr.)
0.11	0.214	2.6×10^{14}	$(1.0-19)\times10^{29}$
0.12	0.211	4.5×10^{14}	$(8.2-156)\times10^{29}$
0.13	0.208	7.2×10^{14}	$(5.4-102)\times10^{30}$
0.14	0.206	1.1×10^{15}	$(2.6-50)\times10^{31}$
0.15	0.204	1.5×10^{15}	$(1.0-19)\times10^{32}$
0.16	0.202	2.0×10^{15}	$(3.2-61)\times10^{32}$
0.17	0.199	2.6×10^{15}	$(9.1-174)\times10^{32}$

Table (5.5.1) Results borrowed from Ref. (64) for comparing with Table (5.4.3) for the SU(5) model.

A detailed discussion on the proton decay and higher order corrections in the SU(5) model has recently been presented /65/ by R.D. Ross. His approach is very close to method I but he considers threshold effects near the X superheavy gauge boson masses in the SU(5) model. The final result predicted for M in **this work is**

$$M = M_X \simeq 4.2 \times 10^{14} \text{ Gev}. \qquad (5.5.4)$$

Recently, Weinberg /67/ has suggested a new way of calculating M_X. This method consists of integrating out the heavy fields in the functional integral to obtain an effective $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant gauge theory. The details of this method were then worked out /66/ for the SU(5) model. The author of Ref. (66) constructs an effective SU(3) x SU(2) x U(1) invariant gauge theory by integrating out the superheavy fields from the SU(5) grand unified field theory. He then considers higher order corrections to the coupling constants of the effective SU(3) x SU(2) x U(1) gauge invariant theory and finds the following solutions for the g's

$$g_i^{-2}(\mu) = g_G^{-2}(M_i) + 2b_i \ln\frac{M_i}{\mu} + \sum_{j=1}^{3} \frac{b_{ij}}{2b_j} \ln\left(1+2b_j g_G^2(M_i)\ln\frac{M_i}{\mu}\right)$$
$$- \Lambda_i(M_i, \alpha_G) , \qquad (5.5.5)$$

where Λ_i is predicted to be

$$\Lambda_i(\mu, \alpha_G) = \lambda_i(\mu) + \lambda_i^{(2)}(\mu)\alpha_G(\mu) + \lambda_i^{(3)}\alpha_G^2(\mu) + \cdots , \qquad (5.5.6)$$

and the solution (5.5.5) which is the master relation of that work is obtained by using the boundary condition

$$g_i^{-2}(\mu) = g_G^{-2}(\mu) - \lambda_i(\mu) . \qquad (5.5.7)$$

The predictions of M_X and $\sin^2\theta(M_W)$ are the central subject of Ref. (66); they are given by

$$M_X = 1.50 \times 10^{15} \Lambda_4^{(2)} \ \text{Gev}, \qquad (5.5.8)$$

and

$$\sin^2\theta(\mu)_W = 0.176 + \frac{0.0042}{\alpha_s(\mu)} \pm 0.001 , \qquad (5.5.9)$$

where the factor $\Lambda_4^{(2)}$ varies from 0.1 to 0.7 and $\mu \sim M_W$. (Note that he uses the same notation as ours.)

Perhaps at this stage we should make some correspondence between our findings from method II and the above results which have been very recently obtained (independently) through the new idea of effective gauge theories /68/. First of all we note that the index i used for M in Eq. (5.5.5) originate from the fact that the renormalization effects have been taken into account for the value of g_G^2 in the original (grand unified) theory. (The boundary condition (5.5.7) also reflects this fact) Suppose the method II that was presented in our work is correct, then the solution (5.5.5) for

$$\lim_{\mu \to M} \lambda_i(\mu) = 0 \qquad (5.5.10)$$

must reduce to our solution (4.5.32). This correspondence is indeed possible since in this case we can identify the function F (μ) in Eq. (5.4.32) as

$$F_i(M) \equiv -\Lambda_i(M, \alpha_G(M)) . \qquad (5.5.11)$$

Note that under the limit (5.5.10) one can drop the index of M_i and the boundary condition (5.5.7) now reads

$$g_i(M) = g_G(M) \qquad (5.4.3)$$

which is exactly the same boundary condition we used in our approach. We observe that in this case the solutions (5.4.32) and (5.5.5) are identical, in other words, method II in our work is also confirmed in Ref. (66).

Eq. (5.5.8) corresponds to the relation (5.4.52) in our work. Since we have shown that for the SU(5) model $F(h, C, n_g) \sim 1$, our prediction for the grand unifying mass scale M coincides perfectly with the result of Ref. (66) for $\Lambda_4^{(2)} = 0.394$ (which is the mean value of $\Lambda_4^{(2)} = \frac{0.1 + 0.7}{2} = 0.4$).

Although our results are mostly reproduced in Ref. (66), Eq. (5.4.61) seems <u>not</u> to be in good agreement with Eq. (5.5.9). If we use the same value of parameters that have been used in Eq. (5.5.9) we find

$$\sin^2\theta(M_w) = 0.176 + \frac{0.0042}{\alpha_s(M_w)} + 0.003 . \qquad (5.5.12)$$

This value of $\sin^2\theta(M_W)$ not only quantitatively differs from that of Eq. (5.5.9) but also there is a difference in the sign of the third term.

The increasing of $\sin^2\theta(M_W)$ for higher order corrections in grand unified theories, in particular the SU(5) model was one of our sharp statements resulting from two different methods in section 4. We have been unable to include a minus sign in the third term of Eq. (5.5.12). Furthermore, as was mentioned, method I has been supported in Ref. (64).

f) In the next chapter we begin with the hierarchy problem in grand unified gauge theories. We will then discuss models with more than two stages of symmetry breaking and find the reasons why actually we need to

know the value of $\sin^2\theta(M_W)$ to the accuracy described above.

CHAPTER VI. INTERMEDIATE MASS SCALES

6.1 Motivation.

One of the most disturbing features of grand unified theories is the existence of two stages of spontaneous symmetry breaking characterised by radically different mass scales. More precisely, a large local gauge symmetry G is initially assumed in a Lagrangian, which is then "strongly" broken by a huge vacuum expectation value v_N down to a reduced local gauge symmetry G_1. This symmetry is further broken by another vacuum expectation value $v_h (\ll v_H)$ into the $G_o = SU(3)_C \times U(1)_{EM}$ symmetry. (See chapters 4 and 5). So far there has been no convincing natural explanation of the enormous ratio of the superheavy mass to ordinary mass which arises through such a hierarchy of symmetry breaking.

With regard to this difficulty, two questions may be posed. On the one hand one may ask how such drastically different scales can be naturally induced in the theory. To this we have no answer, and perhaps there is really no answer without further dynamical input or other assumptions. However, the question about the possible existence of intermediate symmetries between grand unifying group G and G_o which in turn permits several intermediate mass scales between 10^2 Gev and 10^{15} Gev can positively be answered. As far as gauge theories are concerned, many grand unified models (except the SU(5) model) can in principle break the grand unified symmetry down to the exact $G_o = SU(3)_C \times U(1)_{EM}$ symmetry with more than two stages. For example, a grand unified theory based on the SO(10) group, the following routes of symmetry breaking are allowed.

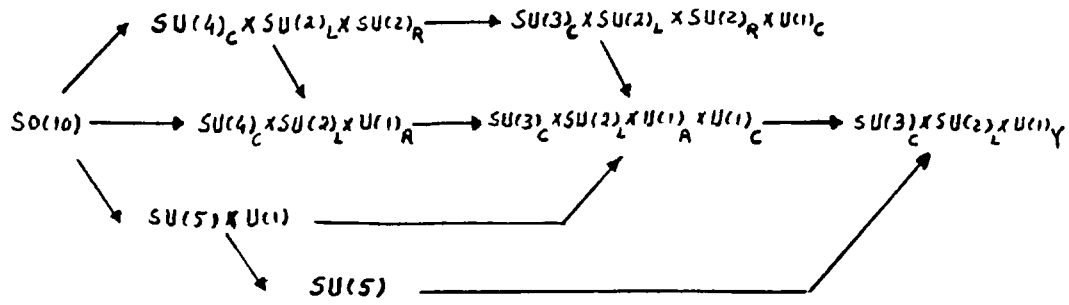


Fig. (6.1.1) The routes of symmetry breaking for the SO(10) model. The arrows show the possible symmetry breaking from SO(10) to $SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$ which then breaks to the exact $SU(3)_C$ x $U(1)_{EM}$ symmetry.

In this chapter, we study grand unified models for which more than two intermediate mass scales can naturally exist. Apart from the basic assumptions used in Chapter 5 for one loop beta function correction the main idea that goes under our approach in this chapter is the assumption that the $U(1)_Y$ (hypercharge) generator T_Y gets contributions from the other intermediate stage of symmetry breaking. We apply this idea for the "maximal" unified gauge theory based on SU(16). This chapter is organised as follows. In section 2 we briefly study the gauge boson structure of the SU(16) model. In section 3 we discuss the value of $\sin^2\theta_W$ and by taking into account renormalization effects, the values of the effective coupling constants and the mass scales of the model are computed. Finally in section 4 the results of our work are summarized.

6.2 The SU(16) model.

The "maximal" unified gauge theory provides us with an elegant description of quarks and leptons and their interactions /52/. In such theory with quarks and leptons in one multiplet of the gauge group G, a linear combination of bosonic, B, and leptonic, L, quantum numbers as well as the fermion number F = 3B + L are locally gauged. They are exact symmetries of the basic Lagrangian and their spontaneous violation arises when the associated particles acquire mass through spontaneously induced mixing between gauge particles carrying different B, L and F quantum numbers.

One of the interesting features of maximal gauging of symmetries is that several intermediate mass scales lying within the "grand plateau" between 10^2 and 10^{15} Gev can exist /52/. Here we wish to study the characteristic mass scales and the behaviour of the effective gauge coupling constants of these intermediate stages for which the underlying symmetry of the theory is the G = SU(16) gauge group.

In the SU(16) maximal unifying gauge theory /52/ of weak, electromagnetic and strong interactions, the fermions are grouped in a 16 dimensional representation. For the electron family, for example, the six left-handed quarks, u_v , u_y , u_b ; d_r , d_y , d_b and two leptons e and ν as well as their charge conjugate fields are represented within the same multiplet :

$$\psi = \{ u_r, u_y, u_b, \nu_e, d_r, d_y, d_b, e; d_r^c, d_y^c, d_y^c, e^+, u_r^c, u_y^c, u_b^c, \nu_e^c \}$$

$$\equiv \{ q, \ell ; q^c, \ell^c \} . \qquad (6.2.1)$$

There are various choices of intermediate symmtric stages, that may exist between G = SU(16) and the low energy $G_4 = SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry group discussed in detail in Ref. (52). Here we study the following route,

$$SU(16) \longrightarrow SU(8)_I \times SU(8)_{II} \times U(1)_F \longrightarrow SU(4)_C \times SU(2)_R \times SU(2)_L \longrightarrow$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y \longrightarrow SU(3)_C \times U(1)_{EM} \,. \qquad (6.2.2)$$

where the two SU(8)'s operate in the space of the eight fermions and the eight antifermions and F represents fermion number. Under $SU(8)_I \times SU(8)_{II} \times U(1)_F$ the 16-component objects ψ and $\bar{\psi}$ decompose as $\underline{16} = (8,1)_1 + (1,8)_{-1}$ and $16 = (8,1)_{-1} + (1,8)_{+1}$, respectively, while the 255 gauge particles of SU(16) decompose as : $255 = (1,1)_0 + (63,1)_0 + (1,63)_0 + (8,8)_{+2} + (8,8)_{-2}$ where the subscripts 0, ± 1, ± 2 denote the fermion number F. The $F = \pm 2$ gauge fields (coupling to the diquark, lepto-antiquark and dilepton currents) acquire masses at the first step of symmetry breaking. We specify the characteristic mass scale of this stage of symmetry breaking by M. The components with F=0 whose generators belong to the coset space $SU(8)_I \times SU(8)_{II} \times U(1)_F / SU(2)_R \times SU(2)_L \times SU(4)_C$ acquire masses of the order of M_F at the second step of symmetry breaking. The $SU(2)_R \times SU(2)_L \times SU(4)_C$ symmetry breaks down to the standard $SU(3)_C \times SU(2) \times U(1)_Y$ model at the mass scale M_X, the region where the X - leptoquark gauge particles become massive. The $G_1 = SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry does <u>not</u> remain exact, it breaks (at a mass scale of the order of $M_W \sim 10^2$ Gev) down to the $SU(3)_C \times U(1)_{em}$ symmetry (for details see Ref. (52)).

6.3 $\sin^2\theta_W$ and effective couplings.

In accordance with the electroweak substructure /10/ $G_{EW} = SU(2)_L \times U(1)_Y$, the unbroken generator, Q, (elecromagnetic charge operator) can be written as

$$Q = T_{3L} + CT_Y \tag{6.3.1}$$

in which C is a constant (that will be specified later), T_L and T_Y are the generators of G_{EW} arising at the fourth step of symmetry breaking. For the same reason, any generator of the group algebra in the fourth stage can be written as a linear combination of the $SU(2)_L \times SU(2)_R \times SU(4)_C$ generators that govern the symmetry of the third intermediate stage. Thus the hypercharge $U(1)_Y$ generator T_Y , for example, may be written as

$$T_Y = C_1 T_{3R} + C_2 T_{15} \tag{6.3.2}$$

where T_{3R} is the third component of $SU(2)_R$, T_{15} is the fifteenth ($SU(3)_C$ singlet) component of the SU(4) and the coefficient C's are easily calculated as follows:

$$C_1 = Tr(T_Y T_R)/ Tr(T_R T_R) , \tag{6.3.3}$$

$$C_2 = Tr(T_Y T_{15})/ Tr(T_{15} T_{15}) , \tag{6.3.4}$$

$$C = Tr(Q T_Y)/ Tr(T_Y T_Y) . \tag{6.3.5}$$

In this way, any generator of $SU(4)_C$, in particular those belonging to the centre of $SU(4)_C$, also may be regarded as a linear combination of the $SU(8)_I$ and $SU(8)_{II}$ generators,such that

$$T_C = C_I T_I + C_{II} T_{II} . \tag{6.3.6}$$

Note that the $SU(2)_L$ group originates from the second step of symmetry breaking and the generators T_C are a SU(2) singlet. Note also that

the $U(1)_F$ generator does <u>not</u> directly contribute to the electromagnetic charge operator, Q.

In a basis for which the electromagnetic charge operator Q is given by the 16 x 16 diagonal matrix

$$Q = (2/3, 2/3, 2/3, 0, -1/3, -1/3, -1/3, -1; 1/3, 1/3, 1/3, 1, -2/3, -2/3, -2/3, 0)_{diag.}, \quad (6.3.7)$$

and the $SU(2)_L$ generator T_{3L} (embedded in SU(16)) by

$$T_{3L} = \tfrac{1}{2}(1, 1, 1, 1, -1, -1, -1, -1; 0, 0, 0, 0, 0, 0, 0, 0)_{diag.} \quad (6.3.8)$$

the coefficient C's are: $C=(5/3)^{1/2}$, $C_1 = (3/5)^{1/2}$ and $C_2 = (2/5)^{1/2}$. Thus the charge operator reads

$$Q = T_{3L} + \sqrt{\tfrac{5}{3}}\, T_Y \ , \quad (6.3.1)$$

where T_Y is given by Eq. (6.3.1). This leads to the standard definition of the electroweak mixing angle as follows:

$$\sin^2\theta_w = e^2/g_{2L}^2 \ . \quad (6.3.9)$$

Gauge invariance and consistency with Eqs. (6.3.1) and (6.3.2) imply that the photon field is given by

$$A = e\left(\frac{W_{3L}}{g_{2L}} + \frac{B}{g'}\right) , \quad (6.3.10)$$

where $g' = g_1/C$ and

$$e^{-2} = g_{2L}^{-2} + g'^{-2} \ , \quad (6.3.11)$$

and B is identified as

$$B = g'\left(\frac{W_{3R}}{g_{2R}} + \sqrt{\tfrac{2}{3}}\,\frac{V_{15}}{g_4}\right) , \quad (6.3.12)$$

with V_{15} being the gauge boson corresponding to the fifteenth generator of $SU(4)_C$, g_{2R} and g_4 are the gauge coupling constants of $SU(2)_R$ and $SU(4)_C$, respectively. It turns out from Eqs. (6.3.1) and (6.3.2) that

$$g_1^{-2} = C_1^2 g_{2R}^{-2} + C_2^2 g_4^{-2} \,. \qquad (6.3.13)$$

Substituting in Eq. (6.3.9) for e from Eq. (6.3.12) and using relation (6.3.13) at the SU(16) symmetric limit we find

$$\sin^2\theta_W = (1+C^2)^{-1} \,. \qquad (6.3.14)$$

This, of course, coincides with the following definition of $\sin^2\theta_W$

$$\sin^2\theta_W = Tr(T_{3L} T_{3L}) / Tr(Q Q) = (1+C^2)^{-1} \,, \qquad (6.3.15)$$

which may be thought of as a direct consequence of definition (6.3.1).

We observe that in the maximal SU(16) gauge symmetry, the "canonical" electroweak mixing angle, θ, has the same value as in the minimal SU(5) unified gauge theory at the symmetric limit.

Having Eqs. (6.3.9), (6.3.11) and (6.3.13), it is now straightforward to take into account the renormalization effects for the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ coupling constants. The one loop solution to Eq. (5.3.1) yields the following results for g_1, g_2, and g_3:

$$g_1^{-2}(M_W) = g_G^{-2}(M) + 2b_3 \ln\frac{M}{M_F} + 2\left(\frac{3}{5}b_2 + \frac{2}{5}b_4\right)\ln\frac{M_F}{M_W} + 2b_1 \ln\frac{M_X}{M_W} \,, \qquad (6.3.15a)$$

$$g_2^{-2}(M_W) = g_G^{-2}(M) + 2b_3 \ln\frac{M}{M_F} + 2b_2 \ln\frac{M_F}{M_X} + 2b_2 \ln\frac{M_X}{M_W} \,, \qquad (6.3.15b)$$

$$g_3^{-2}(M_W) = g_G^{-2}(M) + 2b_3 \ln\frac{M}{M_F} + 2b_4 \ln\frac{M_F}{M_X} + 2b_3 \ln\frac{M_X}{M_W} \,. \qquad (6.3.15c)$$

In which M, M_F, M_X and M_W are the characteristic mass scales of the first, second, third and fourth step of symmetry breaking, respectively. And b_N are given by Eq. (5.3.4)

In what follows, we first show how the theory predicts the value of $\sin^2\theta(M_W)$, (at low energy), where the physical processes are governed by the $G_1 = SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry. To do so, we use equations

(6.3.11), (6.3.9), (6.3.13) and (6.3.15). We find

$$\sin^2\theta(M_W) = \frac{3(b_2-b_3)}{5b_1+3b_2-8b_3}\left(1+\frac{5(b_1-b_2)}{3(b_2-b_3)}\frac{\alpha(M_W)}{\alpha_s(M_W)}+\right.$$

$$\left.\frac{8\pi(b_2-b_4)(5b_1-3b_2-2b_3)\,\alpha(M_W)}{3(b_2-b_3)}\ln\frac{M_E}{M_X}\right), \tag{6.3.16a}$$

and

$$\sin^2\theta(M_W) = \frac{1}{2}-\frac{1}{3}\frac{\alpha(M_W)}{\alpha_s(M_W)}-\frac{20\pi}{3}\left((b_1-b_2)+\frac{2}{5}(b_2-b_3)\right)\alpha(M_W)\ln\frac{M_E}{M_X} \tag{6.3.16a}$$

[These two equations are valid for any unified gauge theory that spontaneously breaks through the route:

$$G \longrightarrow SU(8)\times SU(8)\times U(1) \longrightarrow SU(4)\times SU(2)\times SU(2) \longrightarrow SU(3)\times SU(2)\times U(1)$$

in which the unbroken U(1) generator is given by Eq. (6.3.1)] After using Eq. (5.3.4) for calculating the coefficients b, Eqs. (6.3.16) and (6.3.17) read

$$\sin^2\theta(M_W) = \frac{1}{6}+\frac{5}{9}\frac{\alpha(M_W)}{\alpha_s(M_W)}+\frac{88\alpha(M_W)}{9\pi}\ln\frac{M_E}{M_X}\,, \tag{6.3.16b}$$

and

$$\sin^2\theta(M_W) = \frac{1}{2}-\frac{1}{3}\frac{\alpha(M_W)}{\alpha_s(M_W)}-\frac{44\alpha(M_W)}{3\pi}\ln\frac{M_X}{M_W}\,. \tag{6.3.17b}$$

In deriving Eqs. (6.3.17b) and (6.3.16b) we have neglected the small effects of the Higgs scalars and [$\alpha(M_W) = e^2(M_W)/4\pi$ and $\alpha_s(M_W) = g^2(M_W)/4\pi$] Eqs. (6.3.17) and (6.3.16) play a significant role in determining the value of $\sin^2\theta(M_W)$; the information that is obtained from these equations is classified as follows:

a) The first immediate result derives from (6.3.16), that is

$$\sin^2\theta(M_W) > \frac{1}{6}+\frac{5}{9}\frac{\alpha(M_W)}{\alpha_s(M_W)} \tag{6.3.18}$$

in particular at $M_W \sim 80$ Gev for $\alpha(M_W) = 1/128$, $\alpha_s(M_W) = 0.11$ we find

$$\sin^2\theta(M_W) > 0.206.$$

Perhaps at this point we should make some correspondence between our result and the result of the "minimal" SU(5) gauge model of chapter 5. In the standard SU(5) model (for the same value of α and α_s) we have (see Eq. (5.3.3a))

$$\sin^2\theta(M_W) = \frac{1}{6} + \frac{5}{9}\frac{\alpha(M_W)}{\alpha_s(M_W)} = 0.206 . \qquad (6.3.19)$$

By comparing this equation with relations (6.3.16b) and (6.3.18), we observe that in the context of quark lepton-unification, when the baryon, lepton and , in general, fermion quantum numbers are gauged, the theoretical prediction of $\sin^2\theta(M_W)$ increases with respect to that of the SU(5) model. [Note that these quantum numbers are <u>not</u> locally gauged in the SU(5) model] This, in turn, implies the existence of at least two intermediate stages with two quite distinguishable mass scales M_F and M_X [if $\sin^2\theta(M_W) > 0.206$].

b) There is an upper limit on the value of M_X since from Eqs. (6.3.18) and (6.3.17a) we can obtain

$$M_X < M_W \exp\frac{2\pi}{33}\left((1+c^2)^{-1}\alpha^{-1}(M_W) - \alpha_s^{-1}(M_W)\right) . \qquad (6.3.20)$$

We have presented some values of $H_{32} = M_F/M_X$ and $H_{21} = M_X/M_W$ for $\alpha^{-1} \simeq 128$ and for $\alpha_s(M_W) \simeq 0.11$, (as suggested by charmonium spectroscopy) for a range of $\sin^2\theta(M_W)$ in Table (6.3.1). We have also calculated the value of M_F and M_X by assuming that $\alpha^{-1}(M_W) \simeq 128$ and $\alpha_s(M_W) \simeq 0.14$ suggested by electroproduction experiments for a range of $\sin^2\theta(M_W)$; this has been displayed in Table (6.3.2). It is worth noticing that for $\sin^2\theta(M_W)$ = 0.230, (the average value taken from neutral current experiments) we find $M_F \simeq 10^5$ Gev and $M_X \simeq 10^4$ Gev (see Tables (6.3.1) and (6.3.2)). We emphasize that the mass scales M_X and M_F are quite distinguishable, though they are close to each other. The reason is that the logarithmic terms in Eqs. (6.3.16b) and (6.3.17b) appear with opposite signs and because of

that the hierarchy strength H_{21} decreases as the effective value of $\sin^2\theta(M_W)$ ranges over 0.206 to 0.242, while for the same value of $\sin^2\theta(M_W)$ the strength of H_{32} increases.

c) If the neutral current (or any other) experiments at the scale of M_W($\sim$80 Gev) predict

$$\sin^2\theta(M_W) = \frac{1}{6} + \frac{5}{9}\frac{\alpha(M_W)}{\alpha_S(M_W)} \simeq 0.206 \qquad (6.3.19)$$

[which corresponds to the theoretical prediction of the SU(5) model], then the above calculations (see Eqs. (6.3.16) and (6.3.17)) show that if the SU(16) gauge model is to describe the physics of fermion, the two intermediate mass scales specified by M_F and M_X must be very close to each other i.e. $M_F \sim M_X$. In this case a lepton quark unified gauge theory emerges with a theoretical prediction of $\sin^2\theta(M_W)$ nearly equal to that of SU(5); so that lepton-quark unification, left-right symmetry and fermion number simultaneously break down at some mass scale of the order of $M_X \sim 10^5$ Gev. Since from Eqs. (6.3.16) and (6.3.17) we obtain

$$\sin^2\theta(M_W) \simeq \frac{1}{6} + \frac{5}{9}\frac{\alpha(M_W)}{\alpha_S(M_W)} \equiv \sin^2\theta(M_W)\Big|_{SU(5)} \quad , \qquad (6.3.21)$$

and

$$M_X \simeq M_W \exp\frac{2\pi\alpha^{-1}(M_W)}{33}\left(\sin^2\theta(M_W) - \alpha(M_W)\alpha_S^{-1}(M_W)\right) . \qquad (6.3.22)$$

Hence for $\alpha^{-1}(M_W) \simeq 128$ and $\alpha_S(M_W) \simeq 0.11$ we find: $\sin^2\theta(M_W) \simeq \sin^2\theta(M_W)\big|_{SU(5)}$ = 0.206 and $M_X \simeq 1.40 \times 10^5$ Gev, while for the same value of $\alpha(M_W)$ and $\alpha_S(M_W)$ $\simeq$ 0.14 we obtain $\sin^2\theta(M_W) \simeq \sin^2\theta(M_W)\big|_{SU(5)} = 0.194$ and $M_X \simeq 2.03 \times 10^5$ Gev.

However, any experiment that gives $\sin^2\theta(M_W) > 1/6 + 5/9\,\frac{\alpha(M_W)}{\alpha_S(M_W)}$, allows intermediate stages with underlying gauge symmetries $SU(8)_I \times SU(8)_{II} \times U(1)_F$ and $SU(2)_L \times SU(2)_R \times SU(4)_C$ to coexist such that $M_W < M_X < M_F < M$. It is indeed because of these two intermediate stages that the value of $\sin^2\theta(M_W)$ is pushed up in accordance with Eqs. (6.3.16) and (6.3.17).

d) As far as the coefficients b are concerned Eqs. (6.3.16) and (6.3.17) are independent of the number of fermions and the number of generations. Perhaps this means that fermionic threshold effects do not influence the above calculations [up to the one loop approximation]. It is also worthwhile pointing out that the mass scale M is cancelled out in Eqs. (6.3.16) and (6.3.17), it is not directly involved in these equations, rather it can be determined through the renormalization effects of the third and second intermediate stages where the symmetries are $G_2 = SU(2)_L \times SU(2)_R \times SU(4)_C$ and $G_3 = SU(8)_I \times SU(8)_{II} \times U(1)_F$, respectively.

6.4 Results.

Quark-lepton unification symmetry gauged in its maximal form, as was mentioned, permits, in general, several intermediate stages in the theory. In our example of SU(16), there are two such intermediate stages, between G = SU(16) and the $G_1 = SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetries, characterised by the mass scalars M_X and M_F as given by Eqs. (6.3.17) and (6.3.16). The determination of M_X and M_F depends on the experimental value of $\sin^2\theta(M_W)$ and an accurate value of $\sin^2\theta(M_W)$ determines whether intermediate stages with symmetries G_2 and G_3 can coexist (see Eqs. (6.3.16) and (6.3.17)). If the experimental prediction of $\sin^2\theta(M_W)$ coincides with the theoretical prediction of $\sin^2\theta(M_W)$ in the SU(5) model i.e. $\sin^2\theta(M_W) = 1/6 + 5/9\frac{\alpha(M_W)}{\alpha_s(M_W)}$ then in the SU(16) model, Eq. (6.3.16b) for $M_F \simeq M_X$ gives nearly the same value of $\sin^2\theta(M_W)$ as the SU(5) model. In this case Eq. (6.3.17b) says that in the context of the SU(16) model, at least two intermediate stages with the symmetries G_2 and G_3 very close to each other may exist; that break down to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry at a mass scale of the order of $M_X \sim 10^5$ Gev. On the other hand, it was shown (see (6.3.18) and (6.3.20)) that the two intermediate stages specified by the G_2 and G_3 broken symmetries can naturally coexist if the experimental value of $\sin^2\theta(M_W)$ is larger than 0.206 [for $\alpha_s = 0.11$ and $\alpha^{-1} = 128$]. This underlies the significant role that an accurate experimental value of $\sin^2\theta(M_W)$ would play in distinguishing between different grand unified models. Evidently, apart from proton decay, $n - \bar{n}$ oscillation and neutrinoless double β decay processes, an accurate experimental value of $\sin^2\theta(M_W)$ (to a 10^{-4} accuracy) plays a crucial role in determining allowed intermediate stages in a specific grand unified theory.

Finally, if such intermediate stages exist, it is interesting to study the connection between the effective value of the gauge coupling constants (and perhaps the value of $\sin^2\theta(M_W)$ if it is well defined) at

these intermediate stages for various physical processes allowed by them. In our procedure, we have used Eqs. (6.3.16), (6.3.13) and (6.3.11) to calculate $\sin^2\theta(M_X)$ and the effective value of $\alpha_C(M_X)$ ["colour coupling constant", $\alpha_C = g_4^2/4$] through renormalization effects for the G_3 invariant gauge theory. For $\alpha^{-1}(M_W) = 128$, $\alpha_s(M_W) = 0.11$ and $\sin^2\theta(M_W) = 0.230$ at $M_W \sim 80$ Gev we found $\alpha_C(M_X) = 0.041$ and $\sin^2\theta(M_X) = 0.251$ while for $\alpha^{-1}(M_W) = 128$ and $\alpha_s(M_W) = 0.14$ and $\sin^2\theta(M_W) = 0.230$ we obtained : $\alpha_C(M_X) = 0.040$ and $\sin^2\theta(M_X) = 0.252$ /F7/.

$\sin^2\theta(M_W)$	M_X/M_W	M_F/M_X	M_X	M_F
0.207	1.610×10^{3}	1.039	1.369×10^{5}	1.419×10^{5}
0.209	1.524×10^{3}	1.125	1.300×10^{5}	1.463×10^{5}
0.212	1.404×10^{3}	1.273	1.230×10^{5}	1.566×10^{5}
0.215	1.293×10^{3}	1.440	1.099×10^{5}	1.583×10^{5}
0.218	1.191×10^{3}	1.629	1.024×10^{5}	1.669×10^{5}
0.221	1.097×10^{3}	1.843	0.932×10^{5}	1.718×10^{5}
0.224	1.010×10^{3}	2.359	0.859×10^{5}	1.800×10^{5}
0.227	9.306×10^{2}	2.670	7.91×10^{4}	1.866×10^{5}
0.230	8.571×10^{2}	3.020	7.286×10^{4}	1.946×10^{5}
0.233	7.894×10^{2}	3.416	6.710×10^{4}	2.020×10^{5}
0.236	7.271×10^{2}	3.865	6.180×10^{4}	2.112×10^{5}
0.239	6.697×10^{2}	4.373	5.692×10^{4}	2.200×10^{5}

Table (6.3.1) Values for M_X and M_F predicted by the SU(16) maximal unified gauge symmetry for a range of $\sin^2\theta(M_W)$

$\sin^2\theta(M_W)$	M_X/M_W	M_F/M_X	M_X	M_F
0.207	1.85×10^{3}	1.467	1.573×10^{5}	2.308×10^{5}
0.209	1.751×10^{3}	1.593	1.489×10^{5}	2.372×10^{5}
0.212	1.613×10^{3}	1.802	1.371×10^{5}	2.471×10^{5}
0.215	1.486×10^{3}	2.039	1.263×10^{5}	2.576×10^{5}
0.218	1.369×10^{3}	2.307	1.163×10^{5}	2.683×10^{5}
0.221	1.260×10^{3}	2.610	1.071×10^{5}	2.795×10^{5}
0.224	1.161×10^{3}	2.953	0.987×10^{5}	2.914×10^{5}
0.227	1.069×10^{3}	3.341	0.909×10^{5}	3.037×10^{5}
0.230	0.985×10^{3}	3.779	8.373×10^{4}	3.165×10^{5}
0.233	9.07×10^{2}	4.276	7.712×10^{4}	3.296×10^{5}
0.236	8.356×10^{2}	4.837	7.103×10^{4}	3.434×10^{5}
0.239	7.696×10^{2}	5.472	6.5423×10^{4}	3.579×10^{5}
0.242	7.089×10^{2}	6.191	6.025×10^{4}	3.730×10^{5}

Table (6.3.2) Values for the mass scales M_X and M_F predicted by the SU(16) maximal unified gauge symmetry for a range of $\sin^2\theta(M_W)$

APPENDIX A

Our definitions concerning fermions and their representations in Dirac space coincide with Ref. (36). We recall a few properties of Dirac spinors and the γ matrices which are applied for our calculations. Let f be a fermion spinor. From

$$f_R = \frac{1+\gamma_5}{2} f \ , \tag{A.1}$$

follows

$$\bar{f}_R = \bar{f}\left(\frac{1-\gamma_5}{2}\right). \tag{A.2}$$

The charge conjugate of f is represented by the spinor

$$f^C = C(\bar{f})^T, \tag{A.3}$$

where C is the charge conjugation matrix. It is unitary and satisfies

$$C^\dagger \gamma_\mu C = -(\gamma^\mu)^T. \tag{A.4}$$

From these definitions, we obtain the following property for the charge conjugate $(f_R)^C$ of a right-handed fermion:

$$\begin{aligned}(f_R)^C &= C(\bar{f}_R)^T = C\,\frac{1-\gamma_5}{2}\,\bar{f}^T \\ &= \frac{1-\gamma_5}{2} C \bar{f}^T = \frac{1-\gamma_5}{2} f^C = (f^C)_L \ .\end{aligned} \tag{A.5}$$

Therefore, the conjugate of a right-handed fermion is the corresponding left-handed antifermion $(f^C)_L$. We choose the matrix C to be

$$C = \begin{vmatrix} 0 & -i\tau_2 \\ -i\tau_2 & 0 \end{vmatrix} = \left|\begin{array}{cc:cc} & & 0 & -1 \\ & 0 & & \\ \hdashline 0 & -1 & & \\ & & 0 & \\ 1 & 0 & & \end{array}\right| \ . \tag{A.6}$$

Thus, we have

$$C^T = C^\dagger = -C \ . \tag{A.7}$$

From (A.3) and (A.6) we find that the following relationships hold for two spinors f and f'

$$\bar{f}'^C f^C = -f'^T \bar{f}^T = +\bar{f} f' \ . \tag{A.8}$$

The second equality is due to the fact that fermion fields anticommute, and for the same reason

$$\bar{f}'^{C} \gamma^{\mu} f^{C} = - \bar{f} \gamma^{\mu} f' , \tag{A.9}$$

and

$$\bar{f}'^{C} \gamma^{\mu} \partial_{\mu} f^{C} = \bar{f} \gamma^{\mu} \partial_{\mu} f' . \tag{A.10}$$

When a fermion f carries an internal space index i and transforms according to a representation D of a Lie group as

$$f_i \longrightarrow D_{ij} f_j , \tag{A.11}$$

then its conjugate $(f_i)^C$ transforms according to D*. We denote it as f^i and thus

$$f^i = (f_i)^C \longrightarrow (D_{ij})^* f^j . \tag{A.12}$$

In our calculation, the γ matrices occur repeatedly without reference to their specific form in any particular representation. The rules of operation with these matrices are entirely given by the anticommutation relations

$$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2 g^{\mu\nu} , \quad (\mu,\nu = 0,1,2,3) \tag{A.13}$$

with $g_{\mu\nu}$ = (1, −1, −1, −1) diag. They are chosen in such a way that

$$\gamma^5 = i \gamma^0\gamma^1\gamma^2\gamma^3 = \gamma_5 . \tag{A.14}$$

From (A.13) and (A.14) the following theorems can easily be proven. (See Ref. (36)).

Theorem 1.

The trace of an odd number of γ matrices is zero.

Theorem 2.

$$Tr \not{a} \not{b} \equiv Tr \gamma^{\mu} a_{\mu} \gamma^{\nu} b_{\nu} = 4 a \cdot b . \tag{A.15}$$

Theorem 3.

$$Tr \gamma_5 = 0 , \tag{A.16a}$$

$$Tr \gamma_5 \not{a} \not{b} = 0 , \tag{A.16b}$$

$$Tr \gamma_5 \not{a} \not{b} \not{c} \not{d} = 4i \epsilon_{\alpha\beta\gamma\delta} a^{\alpha} b^{\beta} c^{\gamma} d^{\delta} , \tag{A.16c}$$

$$Tr \gamma_5 \gamma^{\lambda}\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}\gamma^{\tau} = -4i \left(\epsilon^{\lambda\mu\nu\rho} g^{\sigma\tau} - \epsilon^{\lambda\mu\nu\sigma} g^{\rho\tau} + \epsilon^{\lambda\mu\tau\sigma} g^{\rho\nu} - \epsilon^{\lambda\nu\tau\sigma} g^{\rho\mu} + \epsilon^{\mu\nu\tau\sigma} g^{\rho\lambda} \right) , \tag{A.16d}$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is +1 for $(\alpha, \beta, \gamma, \delta)$ an even permutation of (0,1,2,3); it is -1 for odd permutation; and is 0 if two indices are the same.

Theorem 4.

$$\gamma_\mu \not{a} \gamma^\mu = -2\not{a}\,, \tag{A.17a}$$

$$\gamma_\mu \not{a} \not{b} \gamma^\mu = 4\, a\cdot b\,, \tag{A.17b}$$

$$\gamma_\mu \not{a} \not{b} \not{c} \gamma^\mu = -2\not{c}\not{b}\not{a}\,, \tag{A.17c}$$

$$\gamma_\mu \not{a} \not{b} \not{c} \not{d} \gamma^\mu = 2(\not{d}\not{a}\not{b}\not{c} + \not{c}\not{b}\not{a}\not{d})\,. \tag{A.17d}$$

APPENDIX B

This appendix aims to recall some properties related to internal space, the space of charges, which is supposed to be a Lie group G with its algebraic structure $\mathcal{G}$ defined by the Lie bracket

$$[t^a, t^b] = i C^{abc} t^c \tag{B.1}$$

where t^a is the basis of the algebra and $C^{ab}{}_c$ are the structure constants which satisfy the Jacobi identity. Any element of G close to the identity can be written as

$$e - i \delta\lambda_a t^a \tag{B.2}$$

in which the $\delta\lambda_a$'s are the infinitesimal parameters of the group.

To each representation of G corresponds a representation of its algebra. Let T^a be the matrix representation t^a in the space of representation to which the vector ψ belongs. The infinitesimal transformation of ψ reads

$$\psi_j \longrightarrow \psi_j - i \sum_a \delta\lambda_a T^a_{jk} \psi_k \ , \tag{B.3}$$

or in matrix notation,

$$\psi \longrightarrow \psi - i \delta_a T^a \psi \ . \tag{B.4}$$

Here, $T = T^+$ and since T is a representation of t^a

$$[T^a, T^b] = i C^{abc} T^c \ . \tag{B.5}$$

With each element t^a of the algebra, the adjoint representation associates the linear transformation of t^b

$$t^b \longrightarrow -i[t^a, t^b] \ . \tag{B.6}$$

The adjoint representation of a simple algebra is irreducible (there is no subspace of the representation space which is invariant under all group transformations). Note that the matrix T^a associated with t^a in the adjoint representation satisfies by definition,

$$t^b \longrightarrow (T^a)^b{}_c \, t^c = -i[t^a, t^b] \ , \tag{B.7}$$

so

$$(T^a)^b{}_c = C^{abc} \ . \tag{B.8}$$

For the adjoint representation of a compact "simple" Lie algebra, one can find a basis, such that

$$Tr_{adj}(T^a T^b) = \delta^{ab} , \tag{B.9}$$

while for any irreducible representation (R) one finds,

$$Tr_R(T^a T^b) = C_2(R)\,\delta^{ab} . \tag{B.10}$$

For more details and review of the group theory, see Ref. (69) and references therein.

APPENDIX C

In this appendix, we present some details of the calculation needed for Chapter 3.

a) Calculation of terms like $Tr\, k_+ \not{S}_+ \gamma_\mu k_- \not{S}_- \gamma_\nu Tr\, k'_+ \gamma^\nu k'_- \gamma^\mu$.

In order to calculate such terms, we make use of theorems 2, 3 and 4 to find

$$Tr\, k_+ \not{S}_+ \gamma_\mu k_- \not{S}_- \gamma_\nu Tr\, k'_+ \gamma^\nu k'_- \gamma^\mu = 16\Big(S_+\cdot S_-(k_{+\mu} k_{-\nu} + k_{+\nu} k_{-\mu}) - S_+\cdot k_-(k_{+\mu} S_{-\nu} + k_{+\nu} S_{-\mu}) + k_+\cdot k_-(S_{+\mu} S_{-\nu} - S_+\cdot S_- g_{\mu\nu} + S_{+\nu} S_{-\mu}) - k_+\cdot S_-(S_{+\mu} k_{-\nu} - S_+\cdot k_- g_{\mu\nu} + S_{+\nu} k_{-\mu})\Big) \times \Big(k'^{\mu}_+ k'^{\nu}_- - g^{\mu\nu} k'_+\cdot k'_- + k'^{\nu}_+ k'^{\mu}_- \Big) \equiv 32\Big\{ (S_+\cdot S_-)\Big((k_+\cdot k'_+)(k_-\cdot k'_-) - (k_+\cdot k_-)(k'_+\cdot k'_-) + (k_+\cdot k'_-)(k_-\cdot k'_+)\Big) - (S_+\cdot k_-)\Big((k_+\cdot k'_+)(S_-\cdot k'_-) - (k_+\cdot S_-)(k'_+\cdot k'_-) + (k_+\cdot k'_-)(S_-\cdot k'_+)\Big) + S_+\cdot k'_+\Big((k_+\cdot k_-)(S_-\cdot k'_-) - (k_+\cdot S_-)(k_-\cdot k'_-)\Big) + S_+\cdot k'_-\Big((k'_+\cdot S_-)(k_+\cdot k_-) - (k_+\cdot S_-)(k'_+\cdot k_-)\Big)\Big\}. \tag{C.1}$$

b) Calculations of terms like $Tr\, \gamma_5 k_+ \not{S}_+ \gamma_\mu k_- \not{S}_- \gamma_\nu Tr\, \gamma_5 k'_+ \gamma^\nu k'_- \gamma^\mu$.

By making use of (A.16d) and (A.16c) we obtain

$$Tr\, \gamma_5 k_+ \not{S}_+ \gamma_\mu k_- \not{S}_- \gamma_\nu Tr\, \gamma_5 k'_+ \gamma^\nu k'_- \gamma^\mu = -16(i)^2\Big(k_+^\rho S_+^\lambda k_-^\sigma S_{-\nu} \epsilon_{\rho\lambda\mu\sigma} - k_+^\rho S_+^\lambda k_{-\nu} S_-^\tau \epsilon_{\rho\lambda\mu\tau} + k_{+\mu} S_+^\lambda k_-^\sigma S_-^\tau \epsilon_{\lambda\nu\tau\sigma} - k_+^\rho S_{+\mu} k_-^\sigma S_-^\tau \epsilon_{\rho\nu\tau\sigma}\Big) \times k'_{+\alpha} k'_{-\beta} \epsilon^{\alpha\nu\lambda\mu} = 16\Big\{ k_+^\rho S_+^\lambda k_-^\sigma S_{-\nu} k'^{\alpha}_+ k'^{\beta}_- \epsilon_{\rho\lambda\mu\sigma} \epsilon^{\alpha\nu\lambda\mu} - k_+^\rho S_+^\lambda k_{-\nu} S_-^\tau k'_{+\alpha} k'_{-\beta} \epsilon_{\rho\lambda\mu\tau} \epsilon^{\alpha\nu\lambda\mu} + k_{+\mu} S_+^\lambda k_-^\sigma S_-^\tau k'_{+\alpha} k'_{-\beta} \epsilon_{\lambda\nu\tau\sigma} \times \epsilon^{\alpha\nu\lambda\mu} - k_+^\rho S_{+\mu} k_-^\sigma S_-^\tau k'_{+\alpha} k'_{-\beta} \epsilon_{\rho\nu\tau\sigma} \epsilon^{\alpha\nu\lambda\mu} \Big\}$$

$$\equiv 32\Big\{ -(k_-\cdot k'_+)(k_+\cdot S_-)(S_+\cdot k'_-) + (S_+\cdot k'_+)(k_+\cdot S_-)(k_-\cdot k'_-) + (k_+\cdot k'_+)(S_+\cdot k_-)(S_-\cdot k'_-) - (S_-\cdot k'_+)(S_+\cdot k_-)(k_+\cdot k'_-) + (k_-\cdot k'_+)(S_+\cdot k'_-)(k_+\cdot S_-) - (S_+\cdot k'_+)(k_-\cdot k'_-)(S_-\cdot k_+) + -(k_+\cdot k'_+)(S_-\cdot k'_-)(S_+\cdot k_-) + (S_-\cdot k'_+)(k_+\cdot k'_-)(S_+\cdot k_-)\Big\} = 0 \tag{C.2}$$

c) Calculation of the traces in terms of the kinematic variables given by (3.2.9), (3.2.8) and (3.3.12).

i) $$Tr(\not k_+\gamma_\mu \not k_-\gamma_\nu + m_e\gamma_5 \not k_+\gamma_\mu \not s_-\gamma_\nu - m_e\gamma_5 \not s_+\gamma_\mu \not k_-\gamma_\nu - m_e^2 \not s_+\gamma_\mu \not s_-\gamma_\nu)\, Tr\, \not k'_+\gamma^\nu \not k'_-\gamma^\mu$$
$$= 32\Big\{(k_+\cdot k'_+)(k_-\cdot k'_-) + (k_+\cdot k'_-)(k'_+\cdot k_-)$$
$$- \frac{E^2}{\gamma^2}\big((s_+\cdot k'_+)(s_-\cdot k'_-) + (s_+\cdot k'_-)(k'_+\cdot s_-)\big)\Big\} \equiv 32\Big\{2E^4(1+\beta^2\beta'^2\cos^2\theta)$$
$$+\frac{2P_+P_-}{\gamma^2}E^4\Big(\gamma^2\cos\theta_+\cos\theta_-(\beta^2-\beta'^2\cos^2\theta) + \gamma\beta'\cos\theta(X_-\cos\theta_+ + X_-\cos\theta_-)\Big)$$
$$+\frac{P_+P_-E^4}{\gamma^2}X_+X_-\Big\}, \qquad (C.3)$$

where

$$X_\pm = \sin\theta_\pm\cos\phi_\pm - \sin\theta\,(\cos\phi + \sin\phi). \qquad (C.4)$$

ii) $$Tr(\not k_+\gamma_\mu \not k_-\gamma_\nu + m_e\gamma_5 \not k_+\gamma_\mu \not s_-\gamma_\nu - m_e\gamma_5 \not s_+\gamma_\mu \not k_-\gamma_\nu - m_e^2 \not s_+\gamma_\mu \not s_-\gamma_\nu)X$$
$$Tr\,\gamma_5 \not k'_+\gamma^\nu \not k'_-\gamma^\mu = \frac{32E}{\gamma}\Big\{(k_+\cdot k'_+)(s_-\cdot k'_-) - (k_+\cdot k'_-)(k'_+\cdot s_-) -$$
$$(s_+\cdot k'_+)(k_-\cdot k'_-) + (s_+\cdot k'_-)(k'_+\cdot k_-)\Big\} \equiv \frac{32E^4}{\gamma}\Big\{P_-(1-\beta\beta'\cos\theta)X$$
$$(-\beta\gamma\cos\theta_- + \beta'\gamma\cos\theta_-\cos\theta + \beta'X_-) - P_-(1+\beta\beta'\cos\theta)(-\beta\gamma\cos\theta_-$$
$$-\beta'\gamma\cos\theta_-\cos\theta - \beta'X_-) - P_+(1-\beta\beta'\cos\theta)(\beta\gamma\cos\theta_+ - \beta'\gamma\cos\theta_+$$
$$\cos\theta - \beta'X_+) + P_+(1+\beta\beta'\cos\theta)(\beta\gamma\cos\theta_+ + \beta'\gamma\cos\theta_+\cos\theta + \beta'X_+)\Big\} \qquad (C.5)$$

iii) $$Tr(\gamma_5 \not k_+\gamma_\mu \not k_-\gamma_\nu - m_e \not k_+\gamma_\mu \not s_+\gamma_\nu + m_e \not s_+\gamma_\mu \not k_-\gamma_\nu + m_e^2\gamma_5 \not s_+\gamma_\mu \not s_-\gamma_\nu)\, Tr\, \not k'_+\gamma^\nu \not k'_-\gamma^\mu$$
$$= \frac{32E}{\gamma}\Big\{(k_+\cdot k'_+)(s_-\cdot k'_-) + (k_+\cdot k'_-)(k'_+\cdot s_-) - (s_+\cdot k'_+)(k_-\cdot k'_-) - (s_+\cdot k'_-)(k'_+\cdot k_-)\Big\}$$
$$= \frac{32E^4}{\gamma}\Big\{P_-(1-\beta\beta'\cos\theta)(-\beta\gamma\cos\theta_- + \beta'\gamma\cos\theta_-\cos\theta + \beta'X_-) +$$
$$P_-(1+\beta\beta'\cos\theta)(-\beta\gamma\cos\theta_- - \beta'\gamma\cos\theta_-\cos\theta - \beta'X_-) -$$
$$P_+(1-\beta\beta'\cos\theta)(\beta\gamma\cos\theta_+ - \beta'\gamma\cos\theta_+\cos\theta - \beta'X_+) +$$
$$P_+(1+\beta\beta'\cos\theta)(\beta\gamma\cos\theta_+ + \beta'\gamma\cos\theta_+\cos\theta + \beta'X_+)\Big\} =$$
$$64E^4\beta\beta'\Big\{(\beta'^{-1} - \beta'\cos^2\theta)(P_+\cos\theta_+ + P_-\cos\theta_-) + \beta'\cos\theta\,(P_+X_- + P_-X_+)\frac{1}{\gamma}\Big\}$$
$$(C.6)$$

iv) $$Tr(-\gamma_5 \not k_+\gamma_\mu \not k_-\gamma_\nu - m_e \not k_+\gamma_\mu \not s_-\gamma_\nu + m_e \not s_+\gamma_\mu \not k_-\gamma_\nu + m_e^2\gamma_5 \not s_+\gamma_\mu \not s_-\gamma_\nu)\, Tr\,\gamma_5 \not k'_+\gamma_\mu \not s_-\gamma_\nu$$
$$= 64E^4\beta\beta'\Big\{\cos\theta(1 + P_+P_-\cos\theta_+\cos\theta_-) + \frac{P_+P_-}{\gamma^2}(\cos\theta_+X_- + \cos\theta_-X_+)\Big\}. \qquad (C.7)$$

FOOTNOTES.

F1 QCD was originally introduced by Han and Nambu /70/ in a version based on the unconfined and integrally charged quarks. This was developed by Pati and Salam (see Pati and Salam in Ref. (28)). Fractionally charged quarks were usually assigned parastatistics, as described by Greenberg /71/. Later it was shown by Fritzsch and Gell-Mann and Bardeen, Fritzsch and Gell-Mann /12,72/ that the concept of colour, with isolated particles restricted to colour singlets, had the same effect as parastatistics, with isolated particles restricted to bosons and fermions. For this aspect of QCD, see Ref. (12).

F2 We assume three e, μ, τ generations.

F3 For a review on e^+e^- physics, see Ref. (73).

F4 For more details of the historical developments of electroweak interactions, see Ref. (74).

F5 See, for example, J. Banks, H. Georgi, Phys. Rev. D14 (1976) 1159.

F6 We have neglected the small mixing angles;for the effects of these mixing angles, see Refs. (50,56).

F7 Details of the calculations will be given in a paper now in preparation.

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