

ACTIVE VARIABLE GEOMETRY SUSPENSION FOR CARS

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Abstract

This thesis investigates the characteristics and performance of a new type of active suspension for cars through modelling, simulation, control design and experimental testing. The Series Active Variable Geometry Suspension (SAVGS) [Evangelou et al., 2011] concept is first put in context by reviewing the history and current trends in automotive suspensions. Its potential is then critically evaluated and work is carried out to maximise its performance for various suspension functions.

A multi-model multi-software modelling and simulation approach is followed throughout the thesis in order to cross-check and substantiate simulation results in the absence of experimental data. The simpler linear models are used to inform the selection of suitable parameter sets for the case studies, to synthesise control systems and to qualitatively validate the more complex, nonlinear multi-body models. The latter are developed as a platform to virtually test the system and its control algorithms. When possible, these tests are based on standard open-loop test manoeuvres and on standardised external disturbances.

The SAVGS-retrofitted suspension displays a very nonlinear behaviour, which is at the same time a liability and an opportunity from the point of view of control. Nevertheless, different linear control techniques are effectively applied to improve various suspension functions: PIDs are applied to the lower frequency suspension functions such as mitigation of chassis attitude motions, and the $\mathcal{H}_\infty$ framework is applied to the higher frequency suspension functions such as comfort and road holding enhancement. In all cases, a cascade control approach is employed, and mechanisms are implemented to ensure that physical and design actuator constraints are always respected.

This thesis also covers the design and construction of a quarter-car experimental test rig facility. Step-by-step recommendations for its refinement as well as a testing plan are also outlined.

To my mother. To my father.

The work presented in this thesis has been conducted by the author. Part of it has already been published and can be found in:

- C. Arana, S.A. Evangelou, and D. Dini, “Series Active Variable Geometry Suspension application to chassis attitude control,” *IEEE/ASME Trans. Mechatronics*, vol. 21, no. 1, pp. 518–530, Feb 2016.
- C. Arana, S.A. Evangelou, and D. Dini, “Series active variable geometry suspension for road vehicles,” *IEEE/ASME Trans. Mechatronics*, vol. 20, no. 1, pp. 361–372, Feb 2015.
- C. Cheng, S.A. Evangelou, C. Arana, and D. Dini, “Active Variable Geometry Suspension robust control for improved vehicle ride comfort and road holding,” In *American Control Conference (ACC)*, pages 3440–3446, Jul 2015.
- C. Arana, S.A. Evangelou, and D. Dini, “Car attitude control by series mechatronic suspension,” In *IFAC World Congress*, vol. 19, no. 1, pp. 10688–10693, Aug 2014.
- C. Arana, S.A. Evangelou, and D. Dini, “Pitch angle reduction for cars under acceleration and braking by active variable geometry suspension,” In *51st IEEE Conference on Decision and Control (CDC)*, pages 4390–4395, Dec 2012.

All content that is the result of the work of other people has been appropriately referenced and acknowledged throughout the text.

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List of Abbreviations

ABC	Active Body Control, page 32
ABS	Anti-lock braking system, page 30
AC	Alternating Current, page 91
ADAS	Advanced Driver Assistance Systems, page 34
AGCS	Active Geometry Control Suspension, page 33
CDC	Continuous Damping Control, page 32
CL	Closed-loop, page 188
DC	Direct Current, page 90
DOF	Degrees Of Freedom, page 43
ECU	Electronic Control Unit, page 29
EDC	Electronic Damper Control, page 31
EMF	Electromotive force, page 93
ER	Electrorheological, page 38
ESC	Electronic Stability Control, page 59
ESP	Electronic Stability Program, page 59
FC	Full-car, page 99
FCL	Full-car Linear, page 66
GBX	Gearbox, page 76

GT	Grand Tourer, page 100
HC	Hub Carrier, page 86
HSS	High-speed shaft, page 94
ICE	Internal Combustion Engine, page 86
IEC	International Electrotechnical Committee, page 90
IL	Inner Loop, page 183
IO	Input-Output, page 183
ISO	International Organization for Standardization, page 49
LFT	Linear Fractional Transformation, page 181
LMI	Linear Matrix Inequality, page 56
LQG	Linear Quadratic Gaussian control, page 48
LQR	Linear Quadratic Regulator, page 52
LSS	Low-speed shaft, page 96
LW	Lower Wishbone, page 86
MBC	Magic Body Control, page 32
MPC	Model Predictive Control, page 52
MR	Magnetorheological, page 39
NHTSA	National Highway Traffic Safety Administration, page 49
NVH	Noise, Vibration and Harshness, page 37
OCD	Overturning Couple Distribution, page 135
OEM	Original Equipment Manufacturer, page 34
OL	Open-loop, page 188
PDU	Power Distribution Unit, page 91
PID	Proportional-Integral-Derivative, page 30

PIV	Proportional position loop, Integral and proportional Velocity loop, page 123
PMSM	Permanent Magnet Synchronous Machine, page 26
PSD	Power Spectral Density, page 165
PWM	Pulse Width Modulation, page 115
QC	Quarter-car, page 85
QCL	Quarter-car Linear, page 64
SAE	Society of Automotive Engineers, page 49
SAVGS	Series Active Variable Geometry Suspension, page 23
SD	Spring-damper, page 23
SF	Scaling Factor, page 141
SL	Single-link, page 24
SMC	Sliding Mode Control, page 55
SUV	Sport Utility Vehicle, page 32
SVM	Space Vector Modulation, page 114
TACS	Toyota Active Control Suspension, page 30
TEMS	Toyota Electronic Modulated Suspension, page 29
UW	Upper Wishbone, page 86
WH	Wheel, page 86

Chapter 1

Introduction

Active suspensions were introduced in mass-produced passenger cars in the 1980s [Aburaya et al., 1990], but only recently have they become widely available. According to [MarketsandMarkets, 2014], the automotive suspension market will be worth \$66.2 bn by 2018, and most of the premium/luxury vehicles will be equipped with active or semi-active suspensions.

Although hydraulic actuators have dominated in this industry so far, increasing electrification levels in modern automobiles and the pursue of the all-electric vehicle, devoid of hydraulic systems, is reversing this trend and fostering the development of new electromechanical solutions. One such solution, namely the patented Series Active Variable Geometry Suspension (SAVGS) concept, is the research focus of this project. The original idea had already been conceived at Imperial College London [Evangelou et al., 2011] with the hope that it would 1) fulfil the suspension functions better than passive/semi-active solutions (e.g. improved comfort and attitude control), and 2) avoid or reduce the inherent disadvantages of conventional active suspensions (such as weight and energy consumption).

Section 1.1 introduces the SAVGS concept and sets the scene for the research aims and thesis outline that are presented in Section 1.2 and Section 1.3, respectively.

1.1 Series Active Variable Geometry Suspension

The concept of a variable geometry suspension is already well-established in the literature [Sharp, 1998]. It implies that the actuation force is applied to the passive spring-damper (SD) perpendicularly to the main suspension force. The SAVGS combines this principle with a more direct contribution to the main suspension force. As a consequence, the SAVGS preserves some of the better traits of variable geometry suspensions, such as fail safe operation, whilst increasing control authority.

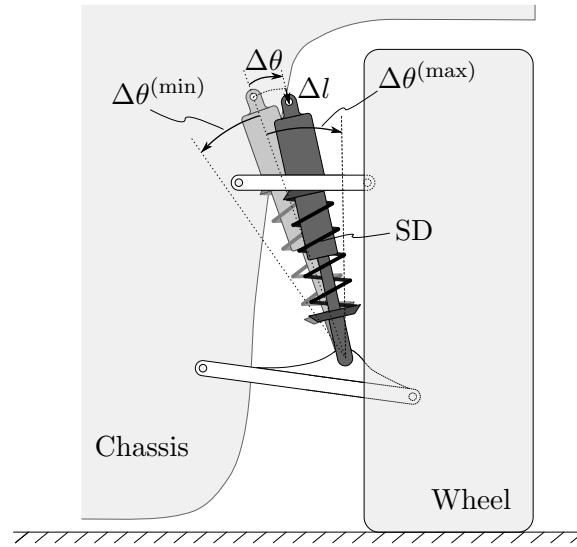


Fig. 1.1: Application of the SAVGS concept to a conventional double-wishbone suspension. The original SD position is shown in light grey and the modified configuration in dark grey. For clarity, the actuator and mechanism connecting the upper end of the SD to the chassis are not shown in the figure.

Its main advantages with respect to alternative active suspensions include:

- Negligible increment of unsprung mass (actuator can be attached to the chassis)
- Fail-safe operation in case of power loss or actuator blockage
- Potentially low power and energy consumption
- Fully electromechanical
- Reliance on readily available technology

The SAVGS maintains all elements of a passive or semi-active suspension and introduces a device between one of the end eyes of the SD and its adjacent body. This device, which acts in series with the SD and comprises an electro-mechanical actuator and a mechanism, is able to control the position of the end eye, thus modifying the orientation and elongation of the SD. The mechanisms considered are required to settle to a stable equilibrium position when the actuation force is removed, thus ensuring fail safe operation in case of power loss.

The concept is illustrated in Fig. 1.1 for a double-wishbone suspension. Whatever the actuator technology and mechanism topology considered, the SAVGS actions affect 1) the SD force due to the change Δl they produce in its length, and 2) the *effective* properties of the SD due to the induced rotation $\Delta\theta$. Two possible embodiments for this concept are presented in Fig. 1.2. In both cases, a rotary actuator attached to the chassis drives a simple mechanism to move the upper end of the SD with respect to the car body. In the duo-link case, the mechanism comprises two mechanical links whose motions are coupled, for example, through a set of gears. The single-link (SL) variant, which is studied throughout the thesis, relies on a single mechanical link that can be rotated about its point of connection to the chassis.

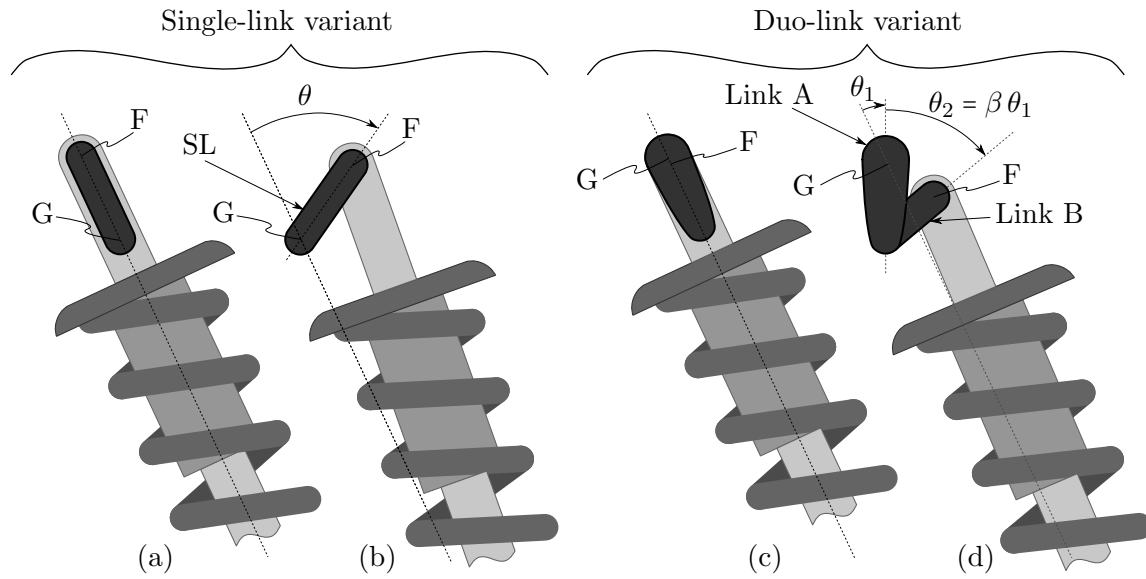


Fig. 1.2: Single-link and duo-link variants of the SAVGS concept. The mechanical links in these SAVGS variants, SL, Link A and Link B, connect point G in the chassis to point F in the upper end eye of the SD through revolute joints. When no actuation torque is applied between the chassis and the SL or Link A, the system remains in equilibrium in positions (a) or (c). Alternative configurations, such as (b) and (d), can be reached by applying a torque about point G.

1.2 Research aims

The main research aims for this project can be summarised as follows:

- To study the single-link variant of the SAVGS and to develop virtual and experimental tools to better understand its behaviour, advantages and weaknesses.
- To investigate the interdependence between the vehicle properties and the requirements for the main SAVGS components to enable system dimensioning.
- To develop control strategies to govern the SAVGS response and to evaluate which suspension functions can be effectively improved with this technology.

1.3 Thesis outline

Background information regarding the history and development of active suspension systems is provided in Chapter 2 together with a summary of current trends and an overview of competing technologies. A review of the academic literature covering modelling, analysis and control topics related to active suspensions is also included.

Chapter 3 is the backbone of this thesis and describes the modelling and validation approaches on which the remainder of the work relies. The selection of software packages is justified and related to the use of vehicle and actuator models of varying levels of complexity. Moreover, the

parameter sets considered for two case studies developed in subsequent chapters are introduced and explained. Some of the work presented in this chapter has already been published in [Arana et al., 2012, 2015a].

Chapter 4 deals with the control of electromechanical actuators. Basic concepts related to the control of Permanent Magnet Synchronous Machines (PMSM) are described first to ensure that the chapter is self-contained. The cascade control strategy derived for the purposes of this work is presented next, with particular emphasis on the implementation of mechanisms to enforce that physical and design constraints, such as torque and power limitations, are respected at all times. An earlier version of these control strategies has already been described in [Arana et al., 2015a; Cheng et al., 2015].

The application of the SAVGS to the control of chassis attitude motions is investigated in detail in Chapter 5. A single-link pre-dimensioning algorithm based on steady-state performance requirements is proposed, and the dependencies between the vehicle properties and the SAVGS requirements are analysed. Moreover, a multi-objective control scheme is presented and extensive simulation results for standard open-loop test manoeuvres are shown to support the dimensioning and control design approaches proposed. The material presented in this chapter corresponds to a large extent to the work published in [Arana et al., 2015b] and to a lesser extent in [Arana et al., 2014a, 2015a].

The potential of the SAVGS for comfort and road holding enhancement is explored in Chapter 6. To enable the use of linear control techniques, the chapter commences studying the dynamics of the SAVGS and its nonlinearities, and comparing the suitability of two different linear vehicle models. $\mathcal{H}_\infty$ control is applied to synthesise a control system that tackles comfort and road holding objectives simultaneously and the tuning process is discussed in detail. Thorough simulation results are presented to characterise the behaviour of the system and to quantify control performance in various scenarios. Additional supporting material, not strictly necessary for the flow of the chapter, is included in the appendices. This work will be submitted for publication in the near future.

Chapter 7 deals with the design and construction of an experimental quarter-car test rig facility. Starting with the initial objectives, requirements and constraints, the chapter proceeds to review the academic literature for examples of similar test rigs prior to discussing the main design questions that arose in this process. The final mechanical design is presented in a step-by-step format to clarify the function of each component and the electrical layout is briefly discussed. The existing rig is critically evaluated against the initial objectives and areas for potential improvement are identified.

Finally, Chapter 8 summarises the main conclusions that can be drawn from the work presented in this thesis and outlines the main areas for further development.

Chapter 2

Literature review

The development of a new type of active suspension system for road vehicles involves many disciplines that range from mechanical engineering (kinematics, dynamics, vehicle dynamics, strength analysis, CAD etc.) to electrical (electric motor and drive modelling) and control engineering. Given the broad scope of the project, it has been preferred to provide a concise rather than a detailed account of recent developments in all associated fields.

This chapter presents the technical and commercial background needed to understand the history and state of the art of active suspensions. In Section 2.1, the role of suspension systems in the automotive industry is described and a chronological account of key milestones is provided. Present and future trends are also identified and discussed. Section 2.2 briefly explains the functions to be fulfilled by suspension systems and leads to the categorisation and comparison of passive, semi-active and active suspensions in Section 2.3. This is followed by two more technical sections, 2.4 and 2.5, in which analysis and control aspects are reviewed. The chapter concludes with a brief summary, linking this background information with the work presented in the remainder of the thesis. When appropriate, more specific analyses of published material are performed in subsequent chapters.

2.1 Brief historical overview

This section provides the relevant historical and commercial context for the development of the SAVGS. A chronological summary of the main milestones in the production and commercialisation of suspension systems is broadly outlined, and current and future trends are discussed. Technical details are kept to a minimum, but those mentioned are covered in the following sections of this chapter.



Fig. 2.1: Benz Patent-Motorwagen (left), a two-seater, three-wheeled vehicle built in 1886, is regarded as the first automobile. Its suspension comprised elliptic springs in the rear axle. In 1906, the Brush Two-Seatby Runabout (right) was the first car equipped with coild springs and shock absorbers. Photographs from [Daimler AG, 2015a; Wikipedia, 2015a].

2.1.1 1880's to 1930's: the early days

Suspension systems for cars have been in use since the first automobile, Benz Patent-Motorwagen (Fig. 2.1), was built by Karl and Bertha Benz in 1886. During the first four decades, most vehicles were equipped with some variant of leaf springs.

Coil springs and shock absorbers were first used in automobiles in 1906 in the Brush Two-Seatby Runabout (see Fig. 2.1), but did not reach mass-produced vehicles until 1934 when General Motors, Chrysler and others installed them in independent front suspensions [Schultz, 1985]. This coincides in time with the beginning of the systematic study of suspension systems in combination with other aspects of vehicle dynamics, such as steering and stability [Crolla, 1996]. For a detailed account of the development of vehicle dynamics since the late 19th century the reader is referred to the comprehensive historical review written by Segel [Segel, 1993].

2.1.2 1950's to 1980's: refinement of passive suspensions and dawn of active suspensions

It was in the 1950's when the first studies on active and semi-active suspensions appeared. These stemmed from vibration isolation problems faced by the aerospace industry, such as helicopter rotor isolation and isolation of fighter plane pilots from aircraft vibrations, as well as from the growing interest in optimisation in control theory in the 1960's [Karnopp et al., 1974]. Around that time, innovative suspension concepts were developed and commercialised on both sides of the Atlantic. Citroën introduced its hydropneumatic suspension in 1954 on the rear axle of the Traction Avant, and in both axles of the Citroën DS in 1955 (see Fig.2.2). The system offered self-levelling capabilities, driver-variable ride height and outstanding ride comfort — something that was of particular value for the post-war French roads [Wikipedia, 2015b]. Within three years, General Motors presented its air suspension that offered similar characteristics, and introduced it in its luxurious Cadillac Eldorado Brougham in 1957 (see

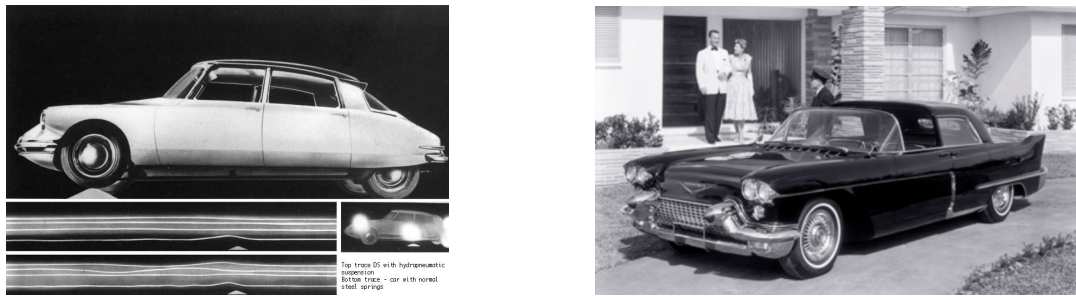


Fig. 2.2: Citroën DS (left) and Cadillac Eldorado Brougham (right) were the first automobiles to incorporate hydropneumatic and air suspensions, respectively. They offered self-levelling capabilities as well as improved comfort, and are important milestones in the history of suspension systems. Photographs from [Marsh, 2011; Durmisevich, 2015].

Fig. 2.2), the most expensive American car of its time [Durmisevich, 2015]. These were the first steps towards active suspensions, and have become important milestones in the history of suspension systems.

2.1.3 1980's and 1990's: a technology full of potential

Notwithstanding extensive research ever since, and despite its use in military applications for several decades [Cao et al., 2011], active and semi-active suspensions did not reach the mass automotive market until the 1980's, when economic and reliable servo valves, sensors and electronic control units (ECUs) with sufficient performance became available [Aburaya et al., 1990]. This period was characterised by rapid developments in active technologies in Japan and Europe, accompanied by grandiose promises of the end of the compromise era in suspension design.

The Lotus 91 (see Fig. 2.3) was the first non-prototype car to be fitted with an active suspension. It competed in the 1982's Formula 1 season, but despite its revolutionary suspension and good performance on very fast tracks thanks to its active height control, it finished the season in fifth position.

In February 1983, Toyota introduced the world's first road car equipped with a semi-active suspension. The Toyota Soarer featured the in-house developed Toyota Electronic Modulated Suspension (TEMS). It operated in three different modes (auto, sport and normal), and achieved 10 to 30% reductions in body motions (squat, nose-dive, roll) with respect to the conventional suspension [Yokoya et al., 1984].

In May 1983, Lotus fitted its active suspension to a Lotus Esprit Turbo prototype and presented it to the press [Marquez, 2005]. This was a hydraulic, high-bandwidth fully active suspension system that demonstrated extremely good body attitude control capabilities (see Top Gear 1983, series 11, episode 1). However, it would not be marketed.

Just three years after Lotus' presentation, Toyota upgraded the Soarer's TEMS to include not

only variable damping, but variable spring rate and height levelling through the use of air springs. Stiffness, damping and ride height could be automatically switched between three different settings depending on the operating conditions. The suspension's geometry was co-designed to provide higher anti-dive and anti-squat characteristics and thus allow for lower natural frequencies and more comfortable ride [Tanahashi et al., 1987].

In 1987, BMW was the first European manufacturer to introduce semi-active damping (to its 635 CSi automobile). In the meantime, Lotus had perfected its Formula 1 active suspension, and in the season of 1987, Ayrton Senna drove the Lotus 99T to its first victories.

A good example of the publicity surrounding the technology in the late 1980's can be found in a 1987 piece from the New York Times, and reproduced in [Hrovat, 1993], regarding the Lotus Esprit prototype:

Test drivers usually emerge from the car with their imagination in overdrive. “The greatest single advance in car engineering since the war,” the British magazine *Car* declared on the cover of a recent issue. *Car*'s editor, Steve Cropley, wrote that one could take the benefits of all other modern automobile developments, “add them up and double the total — and you might come somewhere near the degree to which full active suspension improves a car”. Doug Nye, a contributing editor of the American magazine *Road & Track*, said he was initially skeptical, but “in fact it is amazing”. He said, “I’ve been writing about cars for 24 years and honestly found it the most impressive thing I’ve ever tried”.

The 1989 Toyota Celica (see Fig. 2.3) was the world's first road car to be fitted with a (narrow-bandwidth) fully active suspension [Aburaya et al., 1990], dubbed Toyota TACS (Toyota Active Control Suspension). Using hydraulic actuators it achieved pitch and roll reductions of 66% and 75% for acceleration levels of up to 7 m/s^2 , and reduction in body vibrations at frequencies of up to 7 Hz (particularly at 1 and 2 Hz). In terms of control, PID (Proportional-Integral-Derivative) schemes were used, in some cases helped by feedforward factors. Soon after, Toyota integrated the control for the TACS, the four-wheel steering system and the anti-lock braking system (ABS), providing slightly better performance than when controlling each system independently [Yokoya et al., 1990].

Also in 1989, Citroën presented the XM, the first to upgrade the hydropneumatic suspension introduced with the DS. The new suspension, named Hydractive, maintained the self-levelling and height adjustment features, but included several sensors to switch an extra suspension sphere in or out of the circuit, thus altering the spring and damping rates [Automobile Association, 1990]. This meant that a smooth and supple ride was possible under most circumstances, and roll could still be very well under control when cornering. The system, however, was not very reliable and secondary ride (harshness) was not as good as expected [Wikipedia, 2015d].

Two years later, in 1991, Nissan followed suit and marketed the Q45 equipped with a hydraulic



Fig. 2.3: Lotus 91 (left) and Toyota Celica 1989 (right) were the first automobiles (Formula 1 and road car) to incorporate fully active suspensions. None of them was an immediate success, but they ushered in the golden age of active suspensions. Photographs from [Wikipedia, 2015c; Toyota, 2012].

active system (narrow-bandwidth). It featured active bouncing control based on the Skyhook strategy (see Section 2.5.1), a frequency-dependent damping, and active control of roll, dive and squat [Aoyama et al., 1990]. In [Iijima et al., 1993], Nissan engineers stated that “Suspension development efforts today have moved on to focus primarily on active suspensions, which promise to provide dramatic improvements in performance over their passive counterparts”.

Formula 1 seasons in 1992 and 1993 were clearly dominated and comfortably won by Williams and its active suspension systems. However, electronic aids (including active suspension) were banned in 1994 due to alleged risks associated with the increased cornering speeds. Sadly, it was in the following, electronic aids-free season of 1994, when Ayrton Senna died driving the successor of Williams’ FW15C.

Other systems followed in the 1990’s, including Citroën’s Activa technology (evolution of Hydractive that added active anti-roll) in 1994, but in general the interest in the fully active technologies cooled down significantly. Proof of this is that in 1994, Ford decided to stop its active suspension development programme after spending \$1.5m/year since 1987 in a team of 8 people [Keebler, 1995]. They had started working on hydraulic systems, but also considered permanent magnet motors connected to ball-screw mechanisms [Tseng and Hrovat, 2015]. Despite good performance results obtained with their test vehicle (a 1989 Ford Thunderbird), the programme was stopped because, as Ford engineers phrase it in [Shuttlewood et al., 1993], “It appears that the full active suspensions are unfavourable to the motor industry for installation in mass-produced cars on the grounds that they are expensive and consume a significant amount of energy” [Keebler, 1995]. Along the same line, engineers working on the development of a new version of BMW’s semi-active solution named Electronic Damper Control (EDC) concluded their paper [Hennecke et al., 1990] stating “For there is one thing that we suspension engineers should never lose sight of: the customer has to be able to afford our products before he can assess their quality”.

2.1.4 Late 1990's to 2015: active and semi-active options become widespread

Active and particularly semi-active suspensions have been adopted by an ever increasing number of vehicle classes and manufacturers in the last two decades, in part driven by lower costs, and in part by higher consumer expectations. Mercedes trucks received ZF's Continuous Damping Control (CDC) in 1998, and its high end sedans were equipped with the (narrow-bandwidth) Active Body Control (ABC) system in 1999. Using hydraulic actuators, ABC acts at frequencies of up to 5 Hz [Tseng and Hrovat, 2015] and focuses mainly on reducing chassis attitude motions.

In 2007 approximately 20% of roadster drivers were prepared to pay around 1700 euros for a semi-active suspension [Acock, 2007]. Even in the C-segment, some vehicles have been offering semi-active suspension for a while. It's the case of the Alfa Romeo Mito, equipped with Magneti Marelli Synaptic Damping Control, or the Opel Astra with FlexRide option (an extension of ZF's CDC). Premium motorbikes were getting ready for semi-active dampers a few years ago [Spelta et al., 2010], and these can already be found in some premium models of BMW and Ducati (see Fig. 2.4).

Thus, although fully active systems remain rare, simpler, less capable but more economical solutions have proliferated. This is the case, for example, of active roll control systems, which have become very popular for Sport Utility Vehicles (SUV). They can make the SUVs feel more "car-like" when on the road [Parsons et al., 2000] and they are not as costly as fully active solutions. Some examples are Land Rover's Active Cornering Enhancement [Parsons et al., 2000] introduced in 1998, BMW's Active Stabilizer Bar System presented in 2001, Toyota's Kinetic suspension (initially developed by Kinetic Pty Ltd and Tenneco Automotive) commercialised in 2003 on Lexus SUVs, Mercedes' Active Curve System, or ZF's Active Roll Stabilisation. In general these systems rely on having an anti-roll bar with controllable stiffness. This is achieved, for example, by splitting a conventional anti-roll bar into two and connecting both ends through a hydraulic actuator. The actuator can then be asked to remove all relative motion between both sides, to let them rotate freely, or to do anything in between. Thus, the system can provide tight roll control when driving on road (both axles are rolling in the same direction), and disengage the anti-roll bar in off-road situations when maximum articulation is needed (axles rolling in opposite directions). Moreover, this system can be easily combined with either air springs and/or semi-active dampers, as done in Mercedes' SUVs.

Regarding fully active suspensions, the latest development corresponds to Mercedes. Although it's ABC system received mixed initial reviews for its performance [Cross, 1999; Teschler et al., 2000], it has remained one of the long-standing technologies and is part of Mercedes' recently introduced Magic Body Control (MBC), arguably the most advanced suspension available today (see Fig. 2.4). MBC combines ABC with a road surface scan (stereo camera) to provide preview information to the control system. This leads to a significant enhancement in comfort,

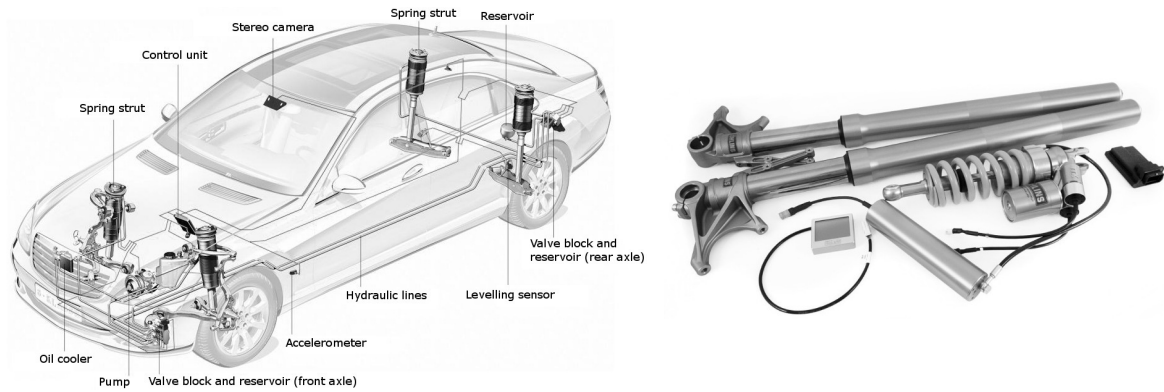


Fig. 2.4: Mercedes' Magic Body Control (left) and Öhlins semi-active suspension for Ducati Multistrada (right). Two examples of high-end active and semi-active technologies currently available for road cars and motorbikes. Images adapted from [Jordan, 2013; Öhlins, 2015].

particularly when driving over speed bumps and other similar road obstacles.

2.1.5 Current and future trends

Despite many positive developments, the future of fully active suspensions remains unclear. Semi-active systems are already affordable to a wide range of customers, but fully active solutions cling to high-end, luxury vehicles. Although they provide better performance than passive and semi-active alternatives and ameliorate the dichotomy between comfort and handling, actuator prices, maintenance costs, complexity and packaging issues still restrain their widespread implementation. These challenges are exemplified by Bose's electromagnetic active suspension (see Fig. 2.5), whose research began in 1980. In 1990, the company hoped to roll out the system within two years [Jennings, 2004]. Then in 2004 they performed the first public demonstration of the system, which attracted a great deal of acclaim, but 35 years after research begun, commercialisation remains elusive.

Apart from the conventional active suspensions (i.e. hydraulic full active suspension, hydropneumatic, hydraulic anti-roll, semi-active dampers, air springs), more unusual active alternatives are also being developed. For example, Audi is working on an active system that is able to adjust toe and camber wheel angles independently from the main suspension [Anon., 2010], Hyundai is developing a similar concept dubbed Active Geometry Control Suspension (AGCS), and active seats are receiving attention as they may provide increased comfort and reduced fatigue for drivers at a fraction of the cost of full body active suspensions [Anon., 2009].

Some of these technological solutions are designed in-house by the car manufacturers themselves (Citroën, Toyota, Mercedes etc.), but there seems to be a growing trend towards external development and testing. In this direction, TRW offers scalable active suspensions combined with active roll control [Kimberley, 2005], MagneRide semi-active suspension and ArvinMeritor

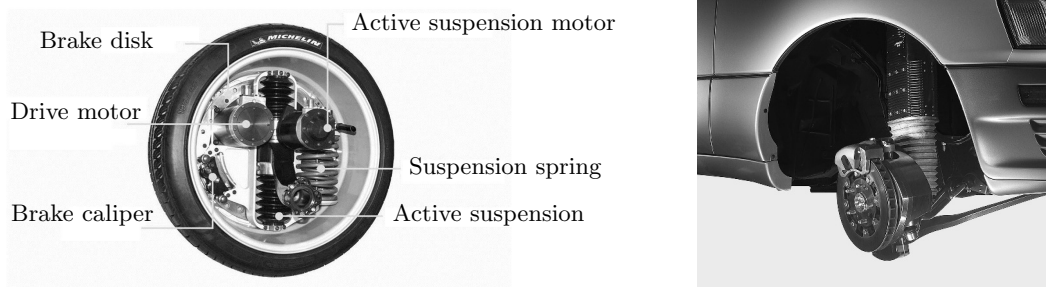


Fig. 2.5: Michelin's Active Wheel (left) and Bose's linear electromagnetic actuator (right) are two examples of the risk that tier one suppliers are taking in order to position themselves as leaders in the active suspension market. Bose's active suspension is also a symbol of the difficulties that the industry faces in the pursuit of the ultimate suspension system. Images adapted from [Cabande, 2012; Howard, 2011].

active air suspension solutions have been adopted by several first class manufacturers [Anon., 2007], ZF supplies Mercedes, Opel and BMW among others, and centres with testing facilities are specifically targeting active suspension developers [Anon., 2006].

Michelin, the tyre manufacturer, has developed an active suspension that fits within the wheel itself, and comprises not only an active suspension but also an electric drive motor, brakes etc. This is dubbed the Active Wheel (see Fig. 2.5). Siemens is following the same approach with its eCorner, which also includes the steering actuator within the wheel itself. These systems are paramount examples of two trends in the automotive industry: i) the large extent to which OEMs (Original Equipment Manufacturer) are outsourcing the design and development processes of an ever increasing number of key subsystems, and ii) the pursuit of the fully electric vehicle in which no hydraulic systems remain.

At the moment, the prevalence of software in modern automobiles is creating new opportunities for active suspensions. Car makers can release software upgrades to correct bugs and improve reliability, but also to add new features to existing systems. For example, Tesla recently upgraded the software for its Model S. On the one hand the upgrade prevents the vehicle's air suspension from lowering the chassis at high speeds to improve reliabilityⁱ, and on the other hand it allows for automatic, location-dependent height controlⁱⁱ.

In the near future, autonomous driving is likely to bring about larger demand for active suspensions: as occupants become less aware of their environment and more focused on other tasks (e.g. working/reading), they will expect higher degrees of comfort. Moreover, the proliferation of Advanced Driver Assistance Systems (ADAS) will bring their cost down, as the number of additional sensors required by the active suspension control will be reduced.

ⁱRide height was originally reduced to diminish aerodynamic drag, but following two incidents in which the car caught fire after hitting road debris, the car maker has decided to temporarily remove this feature [Teslarati Network, 2013].

ⁱⁱThe vehicle now remembers where the driver previously selected high ride heights (e.g. driveway), and based on GPS readings automatically rises the car in those locations thereafter [Korosec, 2014].

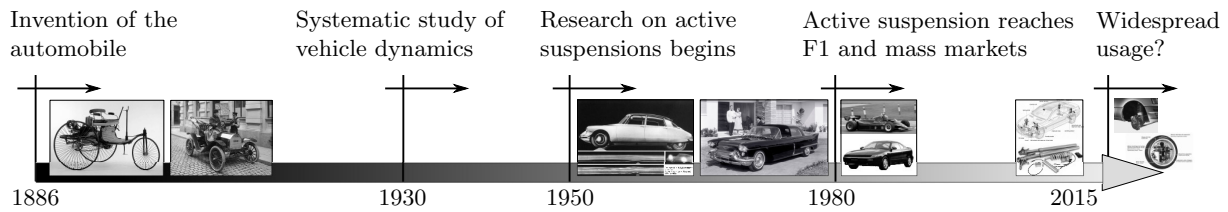


Fig. 2.6: Timeline with the main milestones in the development and commercialisation of suspension systems. Will active suspensions become widespread in the near future?

2.1.6 Summary

For 130 years, passive suspensions have dominated the automotive industry. The field of vehicle dynamics was in its infancy until the 1930's, and only basic suspension implementations were available. In the 1930's, coils springs and dampers became widespread and in the following decades passive suspensions matured. Research on active suspensions began in the 1950's and the first precursors (hydropneumatic and air suspensions) to active suspensions were commercialised. It was finally in the 1980's when developments in electronics brought the era of real active and semi-active suspensions. Expectations were high, and some manufacturers started an arms race towards the definitive suspension system. However, it became clear early in the 1990's that cost was a large barrier for customers, and many car makers decided to stick to passive or to intermediate solutions (e.g. semi-active, anti-roll, interconnected). In the 21st century, these intermediate alternatives have become widespread not only in the high-end models, but also in the more affordable ones. The timeline in Fig. 2.6 captures the most important events described so far.

According to [MarketsandMarkets, 2014], the automotive suspension market will be worth \$66.2bn by 2018, and most of the premium/luxury vehicles will be equipped with active or semi-active suspensions. Advances in sensors and actuators as well as synergies with other intelligent systems on-board are reducing the cost of fully active suspensions and expanding their features and potential performance. Moreover, OEMs are no longer limited to developing their own systems, but they can now partner with several tier one suppliers that have experience in this area. Yet, many OEMs are still reluctant, as they are well aware of the fact that active suspensions, despite their many benefits, do also have significant drawbacks.

New technological solutions, particularly electromagnetic, that help bridge the gap between the performance of fully active suspensions and the simplicity, robustness and low cost of passive suspensions, do have an important role to play in the coming decades. The SAVGS is a promising candidate for becoming one of such solutions.

2.2 Suspension functions

The suspension system connects the wheels of the vehicle to the chassis. Its main functions, which have to be fulfilled within the available working space, are [Elbeheiry et al., 1995]:

- to support the vehicle chassis,
- to keep the rider's comfort within allowable limits,
- to control chassis and wheel attitudes,
- to maintain yaw and roll stability, and
- to ensure good road holding by minimizing the vertical force variation in the tyre-road interaction.

As shown in Section 2.4.3, these are conflicting objectives for the suspension system and therefore a compromise is always needed. Engineers strive to *a)* find suitable objectives for the particular vehicle class of interest, *b)* expand the suspension's performance in line with the chosen target compromise, and *c)* do so within volume, mass, energy, cost and reliability constraints.

Suspension systems for mainstream passenger cars may prioritise driver and passenger comfort, sports cars may prioritise stability, tyre grip (road holding and wheel attitude changes) and aerodynamics (limited heave and pitch body motions), and heavy vehicles may prioritise stability, load protection and road friendliness (limited vertical tyre forces).

2.3 Classification of suspension systems

To fulfil these functions, a large number of topologies and geometries have been developed. Suspension systems can be classified according to two main criteria: geometric layout and nature of the force-producing elements (see Fig. 2.7).

According to their geometric layout, there are three main suspension types: rigid axles, semi-rigid axles and independent suspensions.

- Rigid axles: wheels on the same axle (either front or rear axle) are rigidly connected to one another, and therefore have coupled motions. Nowadays this type of suspension can only be found in driven rear axles of relatively heavy vehicles.
- Semi-rigid axles: similar to the rigid axles, although in this case a certain degree of relative motion between the wheels of the same axle is allowed thanks to the presence of some central elastic component.
- Independent suspensions: they comprise a collection of articulating joints and rigid elements that connect the vehicle's chassis to the wheel carriers. The motion of each wheel is completely independent from the others.

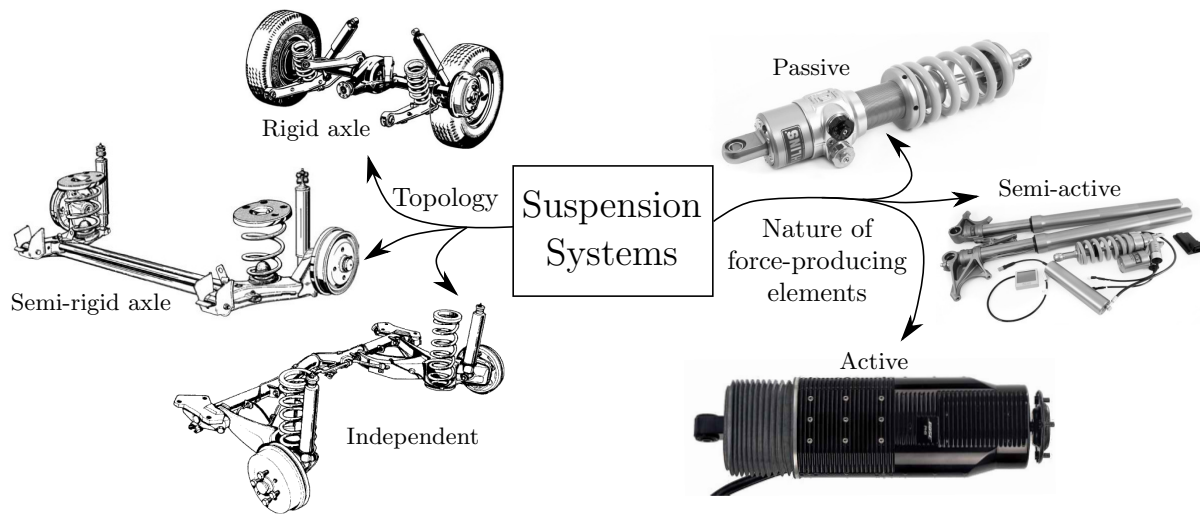


Fig. 2.7: Broad classification of suspension systems according to mechanical layout and nature of the force-producing elements. Images from [Dixon, 2009; Öhlins, 2015; Howard, 2011].

There are various sub-classes within these three main categories. A comprehensive graphical compendium can be found in [Heißing and Ersoy, 2010].

According to the nature of the force-producing elements, there are once again three main suspension types: passive, active and semi-active suspensions.

2.3.1 Passive suspensions

The force-producing elements in passive suspensions are typically coil springs and pneumatic or hydraulic dampers (also known as shock absorbers), although there exist more advanced solutions incorporating tuned mass absorbers, inerters (also known as J-dampers in Formula 1) [Smith, 2002], interconnected hydraulic systems that can decouple modes of motion of the vehicle [Smith and Walker, 2005] and others.

Springs and dampers are usually connected to the chassis and suspension arms through bushings. These may reduce the overall performance of the system at frequencies below the wheel's hop frequency [Ryba, 1974], but are necessary in order to avoid NVH (Noise Vibration and Harshness) issues at higher frequencies.

The selection of appropriate stiffness and damping values has been a recursive problem since the beginning of the development of vehicle dynamics. The reason for this is that the optimum values for road holding, ride comfort and working space are different, and therefore a compromise must always be reached.

Fundamental conflicts arise between comfort and working space [Sharp and Crolla, 1987] and between comfort and chassis attitude control [Sharp and Hassan, 1986b]. Low spring rates are

needed to improve ride quality, but can give rise to large body motions and excessive wheel displacements. These lead to constant hitting of the suspension bump and rebound stops, moment at which the suspension performance drops dramatically as it effectively becomes very stiff and lightly damped [Sharp and Hassan, 1986b].

Regarding damping, early analyses [Thompson, 1969] carried out with analogue computers already highlighted the fact that the optimum damping value for ride comfort depends heavily on several factors (road profile, forward speed, dynamic properties of the seat etc.), whereas the optimum damping value for improving road holding is quite insensitive to these and is usually significantly higher than that required to optimise ride comfort. The difference between these two optimum values also depends on the stiffness of the spring: relatively small difference for stiff suspensions, but large for soft suspensions [Sharp and Hassan, 1986b]. This problem is also fuelled by the necessity to provide damping for the unsprung mass through the main suspensions due to the low tyre damping values [Karnopp, 1983]. It is also more difficult to tackle in the case of large unsprung masses (above one fifth of the sprung mass) [Ryba, 1974].

Asymmetrical damper characteristics, with higher damping during the rebound stage than during the compression stage, have been widely used for many decades. The consequence is an improved response to transient events such as pot-holes and bumps, that leads to better road holding capabilities.

Passive suspensions were already considered a mature technology in the 1980's, and significant improvements in performance were not expected unless changes in principle were developed [Sharp and Crolla, 1987]. Nowadays, the expected improvements have to do more with the use of new materials and advances in damper design [Xue et al., 2011].

Manufacturers are heading towards semi-active and active solutions in order to achieve a better compromise between the conflicting objectives of vehicle's suspensions. The possibility of controlling the vertical suspension motion has at least three potential advantages apart from the obvious increase in comfort: *a)* remove/reduce roll and pitch motions, *b)* control the balance of load transfer between front and rear axles during cornering and *c)* improve the dynamic tyre load control. From these, the second one has the potential to allow the vehicle to have optimum handling until the limit of stability [Crolla, 1996].

2.3.2 Semi-active suspensions

Semi-active suspensions typically include the same force-producing elements than passive suspensions, with the main difference being that the damping coefficient of the shock absorber can be actively adjusted by a control unit.

Controllable dampers achieve changes in damping coefficient through valve cross section control or by using smart fluids. In the latter case, viscosity of electrorheological (ER) or magnetorhe-

ological (MR) fluids can be rapidly and accurately modified by applying electric or magnetic fieldsⁱⁱⁱ. Both technologies are already mature and commercially available (e.g. ZF’s CDC relies on valve control, and BWI’s Magneride on MR fluids).

Semi-active suspensions can “tune” the shock absorber, but this is still subject to passivity constraints [Smith and Walker, 2000] and cannot *introduce* any force — it can only react to damper compression/extension. The passivity constraint may be expressed as

$$\int_{-\infty}^T F(t) v(t) dt \leq 0, \quad (2.1)$$

where F is the force produced by the damper (positive when opposing compression), and v is any admissible damper speed (positive in extension).

2.3.3 Active suspensions

Active suspensions are not restricted by Eq. (2.1), and therefore are a more flexible and powerful solution, particularly when the available working space is limited [Sharp and Hassan, 1986a]. A brief comparison between semi-active and active suspension in terms of features not available in passive suspensions is presented in Table 2.1.

Table 2.1: Comparison of semi-active and active suspension capabilities. Functions not available are marked with ✕. Symbols ✓ and ✓✓ are used to denote good and excellent performance, respectively.

Feature	Semi-active	Active
Self-levelling and height adjustment	✕	✓✓
Pitch and roll mitigation	✓	✓✓
Pitch and roll control for improved aerodynamics/lean into curves	✕	✓✓
Handling enhancement	✓	✓✓
Comfort enhancement	✓✓	✓✓
Road friendliness	✓✓	✓✓

Active suspensions typically include the same force-producing elements as passive suspensions, *and* some kind of actuator either in series or in parallel with the spring and damper. Depending on the control bandwidth, active suspensions are further classified as slow (low/narrow-bandwidth) active suspensions if their bandwidth falls below the wheel’s hop frequency [Sharp and Crolla, 1987], and as high/broad-bandwidth active suspensions otherwise [Elbeheiry et al., 1995].

ⁱⁱⁱMR dampers seem to have an edge over their ER counterparts, as they require much lower voltages.

Narrow-bandwidth active suspensions

Slow active suspensions generally have the actuator installed in series with the passive spring, and they can be further classified as rigid^{iv} or soft depending on their behaviour at frequencies above their control bandwidth. The fundamental dynamic response in narrow-bandwidth active suspensions is determined by the passive elements, with the active components performing only some low frequency activity such as levelling [Goodall, 1997].

Koch et al. [2010] argues that an actuator with 3 Hz bandwidth performs very well in lightly damped suspensions, with results comparable to those achievable with broad-bandwidth actuators. This coincides with previous estimates by Sharp and Crolla [1987] regarding attitude control of the chassis, and Hrovat [1997] concludes that actuators with 5 Hz bandwidth are already capable of improving suspension performance to a level not far from that of an infinitely fast actuator. Indeed, Sharp and Hassan [1987] suggests that active solutions with 3 Hz bandwidth and variable damping are one of the most promising arrangements.

Broad-bandwidth active suspensions

Broad-bandwidth active suspensions generally have the actuator installed in parallel with the passive spring. There is some discrepancy in the literature regarding the minimum bandwidth for which the performance is equivalent to that of an infinite bandwidth system. The suggested values range from 25 Hz [Koch et al., 2010] to 50 Hz [Smith, 1995], but obviously depend on the kind of problems that are tackled by the suspension.

Overall, car makers seem to prefer slow active suspensions over their broad-bandwidth counterparts due to the lower cost, complexity and possibly energy consumption [Tseng and Hrovat, 2015], despite the fact that usually they have to carry the full weight of the vehicle (actuator in series with the spring) [Crolla, 1996].

Actuators

Regarding the practical implementation of active suspensions, hydraulic and electromagnetic actuators are the most widely considered. Their main characteristics are explored in Gysen et al. [2010] and summarised in Table 2.2. Although it is not difficult to find advocates of both technologies in the literature, at the moment only hydraulic solutions have reached the mass market [Xue et al., 2011].

^{iv} Actuators of the rigid type are better treated as displacement producers instead of as force producers [Sharp and Crolla, 1987].

Table 2.2: Comparison of hydraulic and electromagnetic actuators

Actuator	Strengths	Weaknesses
Hydraulic	Very high force density	Low efficiency
	Ease of control	Relatively slow response
	Ease of design	Possible environmental pollution (leaks)
	Commercial availability	Heavy
	Reliability	Bulky
Electromagnetic	High efficiency	Low force density
	Fast response	High current and/or voltage requirements
	Regenerative capabilities	Cost
	Accurate force control	Packaging

2.3.4 Active variable geometry suspensions

It has been appreciated for several decades [Sharp and Hassan, 1986b] that it would be desirable to have a suspension system in which the spring and damper had variable stiffness and damping characteristics. As mentioned in Section 2.3.2, variable damping shock absorbers that can be controlled in real time are a mature technology. On the other hand, variable stiffness springs, of which air springs represent the most common implementation, still suffer from some significant drawbacks, such as heavy auxiliary components and slow response.

Active variable geometry suspensions may achieve the same effect, i.e. to have a variable and controllable *effective* stiffness and damping characteristics, despite relying on conventional springs and dampers. By changing the leverage ratio between the passive force-producing elements and the road wheels, they are thought to have the potential to represent a lighter, cheaper and less power demanding alternative than conventional active suspensions [Sharp, 1998].

A direct implementation of this concept may be found in [Anubi et al., 2013; Anubi and Crane, 2013], where the upper end of the spring-damper is connected to a linear actuator that controls its position along a horizontal line that is perpendicular to the longitudinal axis of the chassis. Thus, the desired change in the installation ratios (i.e. effective stiffness and damping) is produced. A more involved arrangement is used by the Delft Active Suspension (DAS) [Knaap, 1989] and subsequent developments [Evers et al., 2011b,a] in order to change the orientation of the spring despite having a fixed connection to the chassis. This implementation is more focused on secondary suspensions for commercial vehicles. Another example of the application of the active variable geometry concept to road vehicles is described in [Evangelou, 2010]. In this case, the idea is implemented in the rear suspension of a high performance motorbike, and successful reduction of weave oscillations is achieved.

The concept of active variable geometry also refers to arrangements in which the focus is on improving the directional response of the vehicle, and/or on reducing the roll angle of the chassis, through the modification of the suspension kinematics. For example, the roll centre height is

controlled in [Lee and Han, 2001; Goodarzi et al., 2011], the toe angle in [Lee et al., 2008] and the camber angle in [Nemeth and Gaspar, 2011a,b]. These solutions have little or no capacity to improve the suspension performance in terms of comfort and tyre deflections, and therefore are not as versatile as previous examples.

2.4 Study of suspension systems

There is a long way from conception to actual implementation of an active suspension in a car. A framework commonly adopted in this process is the so called V-model-development scheme [Isermann, 2008], which guides the engineer throughout the selection of functional requirements, system specifications, system design, modelling and simulation, component design, prototype building, component testing, system integration (hardware and software), system testing, field testing and production.

In this section, an overview of some key elements in this process are discussed. Some of the questions that are tackled are: how are suspension systems usually modelled for control synthesis and analysis? How can the performance of an active suspension be quantified, and what level of performance can be expected? Are there any relevant standards in this field?

2.4.1 Modelling

Models needed to study active suspensions range from very specific component models (e.g. of hydraulic and electromechanical actuators), and simplified vertical dynamics models (e.g. simplified quarter-car and full-vehicle models), to complex full vehicle models capable of reproducing realistic driving conditions and of capturing the effect of the active suspension on the directional response of the vehicle. Some models put the emphasis on loading conditions and associated mechanical stresses (mainly for mechanical dimensioning and reliability studies), whereas others put it on the dynamic response (mainly for control design and system analysis).

Here the focus is on the models that are most frequently found in the literature for control synthesis and dynamic analysis of suspension systems. These are either simplified models of the vertical dynamics of the car (quarter-car, half-car and full-car linear models), or multi-body nonlinear models that can capture all relevant vehicle dynamics.

Vehicle dynamics

In order to choose the appropriate model for each study, it is important to be familiar with the broader theory behind vehicle dynamics. Vehicle dynamics encompasses everything from steering to yaw and roll stability, road-tyre interactions, chassis motions, handling, road holding,

passenger comfort etc. Its development has been key to allow the design of safer and better performing vehicles, and although it has been studied for many decades it still receives significant attention from the research community.

Vehicle dynamics is not covered in any detail in this chapter as it is a very broad topic. Instead, explanations are provided throughout the thesis whenever necessary. Here, some references are given to guide the interested reader towards relevant material. A brief summary of the nomenclature most commonly used in the area of vehicle dynamics can be found in [Sayers, 1996], and should probably be the starting point for anyone unfamiliar with the topic. More comprehensive lists of terms are provided in [ISO 8855:2011] and [SAE J670e]. Once these basic concepts are understood, the reader is referred to good review articles such as [Segel, 1993; Crolla, 1996; Cao et al., 2011] and books [Gillespie, 1992; Milliken and Milliken, 1995; Jazar, 2008; Dixon, 2009].

Quarter-car model

Used to study the bounce or heave mode, it is a simplified linear model of a quarter of a car with either 1 (very rare nowadays) or 2 degrees of freedom (DOF), namely vertical displacement of the sprung and unsprung masses (see Fig. 2.8). It neglects the kinematics involved in wheel travel, yet it captures ride dynamics reasonably well [Crolla, 1996] and offers a simple representation that is ideal for the design of control systems for active suspensions^v. It is by far the most employed model in the literature due to the fact that *a*) it requires only a few parameters, *b*) it is (typically) a single input system and *c*) it illuminates the relationships between design variables and suspension performance [Sharp and Crolla, 1987].

Half-car model

It is an extension of the 1 or 2 DOF quarter-car models that includes pitching or rolling dynamics of the chassis (see Fig. 2.8). In its most common form, it has 4DOF: vertical displacement of two wheels and chassis heave and pitch (θ) motions. It is not as commonly used as the other models because it is more complex than the quarter-car, but fails to capture the full range of chassis motions provided by the full-car model.

^vA recent study has claimed that it tends to over-predict the sprung mass acceleration [Maher and Young, 2011]. However, this study was based on unrealistic parameters (5 to 10 times smaller mass values than usual and ratio of sprung to unsprung mass of just 3, when it normally ranges from 5 to 10) and therefore its conclusions should be taken with a pinch of salt.

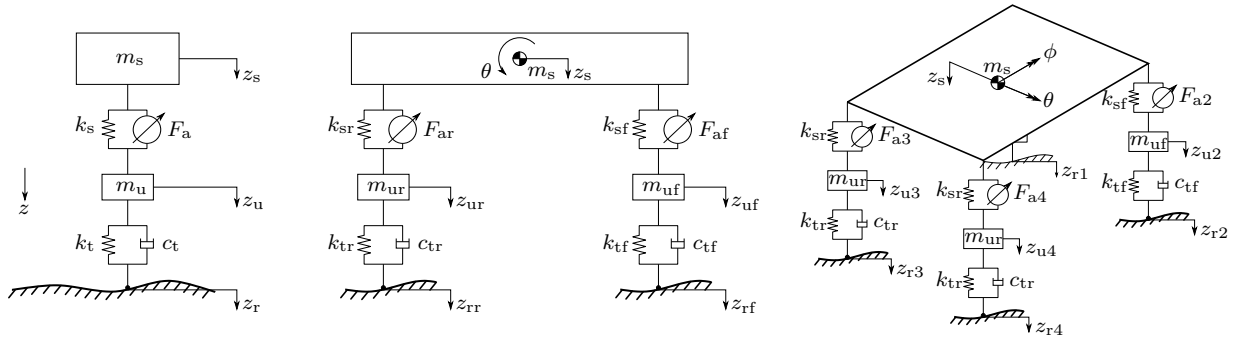


Fig. 2.8: Simplified 2DOF quarter-car (left), 4DOF half-car (centre) and 7DOF full-car (right) models commonly found in the literature to study the vertical dynamics of the vehicle. The particular case of an actuator in parallel with a passive spring is shown in this figure.

Full-car model

It is the extension of the half-car model to include rolling dynamics of the chassis (see Fig. 2.8), and has 7DOF: vertical displacement of the four wheels and chassis heave, pitch and roll (ϕ) motions. It is generally used for the design of body attitude control algorithms.

Multibody models

Much more complex than the previous models, multibody representations of the car can accommodate all kinematic effects and nonlinearities in the study. Not so popular within the academic community, but very much so within industry [Cao et al., 2011], they may be too complex for effective suspension design but represent the best option for virtual testing of the proposed solutions.

Individual subsystems that may be incorporated within the multibody models are *a)* aerodynamics, *b)* driver (steering wheel control [Sharp et al., 2000; Sharp, 2005; Thommyppillai et al., 2009] and human body dynamics [Kruczek and Stribrsky, 2004]), *c)* seat characteristics [Elbeheiry et al., 1996], *d)* steering system, *e)* propulsion and braking systems, *f)* suspension system, *g)* tyres (in plane dynamic properties [Zegelaar and Pacejka, 1996], footprint types [Captain et al., 1979; Mucka and Gagnon, 2015] and shear forces [Besselink et al., 2010] with various degrees of accuracy [Ammon, 2005]), and *h)* road profiles (stochastic roads and singular events) [Elbeheiry et al., 1995; Dixon, 2009].

Multibody models allow to simulate all driving conditions and to analyse not only vertical dynamics (relevant for comfort), but also directional response (handling). They are suitable for the study of the four so called *handling regimes* of interest [Dixon, 1988]:

- Kinematic handling: speed is very low, there are no dynamic effects and the sideslip angle is negligible.

- Primary handling: linear relationship between the understeer angle and the lateral acceleration. This regime typically includes lateral accelerations of up to 0.3 g [Radt and Pacejka, 1963]. The linear regime assumes small steering inputs, linear tyre behaviour with slip and camber angles, smooth flat roads and lateral tyre forces independent of small changes in vertical loads [Hegazy et al., 1999].
- Secondary handling: nonlinear relationships.
- Final handling: limit case, typically reaching 0.8 g of lateral acceleration.

Software

Although some model manipulations and control synthesis may be performed by hand, most researchers rely on computer software for system analysis, control synthesis, and virtual testing.

For quarter-, half- and full-car models, MATLAB/Simulink seems to be the preferred choice. For multibody models, specific software, which can be either numeric (e.g. ADAMS) or algebraic (e.g. VehicleSim), is generally used.

The numeric alternatives are the industry standard, powerful for complex systems, with pre- and post-processors linked to CAD packages and with good animation capabilities. Yet they are very expensive, require significant computer power and are not so convenient for investigating design issues. An option that is becoming quite popular among researchers is SimMechanics [Wood and Kennedy, 2003]. Its main advantage is that it is integrated within the MATLAB/Simulink environment.

On the other hand, algebraic alternatives are not as expensive, more efficient in terms of computational requirements and equally powerful for complex systems. On the down side, they are less user-friendly and less developed than their numeric counterparts.

2.4.2 Performance evaluation

The quality of the dynamic performance of a vehicle can be measured through subjective evaluation (driver ratings) or through objective metrics [Data and Frigerio, 2002; Johnston, 2010]. The following factors need to be quantified in order to allow the engineer to compare different suspension designs/control strategies for active suspensions.

- Ride comfort: it is usually quantified through the ISO weighted RMS vertical acceleration of the sprung mass. It is sensible to include the effect of pitch and roll accelerations [Williams and Haddad, 1997] in the index, but this is frequently ignored in the literature.
- Directional response (handling): as pointed out by Gillespie [1992], *handling* is a loosely used term meant to imply the responsiveness of a vehicle to driver input, or the ease of

control. Complex metrics such as the maximum achievable velocity in a slalom circuit [Aburaya et al., 1990] are used to evaluate it. Handling is therefore a property of the closed-loop system formed by the driver and the vehicle. To characterise the vehicle only, open-loop behaviour has to be analysed. The vehicle response to specific steering and throttle pedal inputs is more precisely defined as *directional response*. The understeer gradient^{vi} and the final understeer number^{vii} are some of the standard variables used to evaluate the responsiveness and stability of vehicles.

- Road holding (or how well the tyre grips the road): it is usually characterised through the standard deviation of the wheel load, although the range of frequencies that should be considered is not completely clear. A commonly used range is 0.25 to 15 Hz [Sharp and Hassan, 1986b].
- Working space: it may be evaluated through the RMS value of the variation of wheel-to-body displacement from the mean value [Sharp and Hassan, 1986a].
- Attitude control: it may be measured through peak or RMS values of pitch and roll angles (or some combination of the two). It is generally compared under steady-state conditions.
- Actuator requirements: force/displacement/velocity and power requirements can be evaluated through their peak, RMS and idling values. In the case of power consumption, the average value is also relevant.

Typically, all or some of these individual performance parameters are lumped together into a final performance index through the application of different weighting factors (which may be frequency dependent). The selection of these weights is not standardised, as each car maker and driver prioritises suspension functions differently. Thus, most published work considers only a small subset of these factors (typically one or two), and combines them through a simple, intuition-based selection of weights.

2.4.3 Performance limits

Active suspensions are subject to some fundamental limitations due to the fact that the force can only be introduced between the chassis and the wheel^{viii}. In linear system terms, the limitations are seen in a lack of adjustability of terms in the numerators of various transfer functions [Karnopp, 1986].

Referring to the quarter-car model shown in Fig. 2.8 and neglecting the damping of the tyre,

^{vi}Slope of the graph of the front steer angle versus lateral acceleration in a steady-state constant radius test [Bedner et al., 2007].

^{vii} $N_u = (A_r/A_f) - 1$, where A_f and A_r are the lateral acceleration capabilities of the front and rear axles [Dixon, 1988]. It is zero for a notional final neutral vehicle, positive for final understeer and negative for final oversteer.

^{viii}Some researchers are looking at alternatives such as the use of actively controlled aerodynamic surfaces in order to avoid this limitation [Corno et al., 2014].

Newton's second law applied to the z direction leads to:

$$m_s \ddot{z}_s + m_u \ddot{z}_u - k_t (z_r - z_u) = 0, \quad (2.2)$$

where m_s is the sprung mass, m_u the unsprung mass, $\ddot{z}_s$ the vertical acceleration of the sprung mass, $\ddot{z}_u$ the vertical acceleration of the unsprung mass, k_t the radial stiffness of the tyre and z_r and z_u the vertical coordinates of the road profile and the unsprung mass, respectively. Equation (2.2) is denominated the basic invariant equation in [Hedrick and Butsuen, 1990], as it is satisfied irrespective of the suspension arrangement (e.g. passive, semi-active, active etc.) and the suspension forces.

The Laplace transform of (2.2) assuming zero initial conditions is:

$$[m_s z_s(s) + m_u z_u(s)] s^2 - k_t [z_r(s) - z_u(s)] = 0, \quad (2.3)$$

and the transfer functions from vertical road velocity to sprung mass acceleration, $H_A(s)$, from vertical road velocity to suspension deflection, $H_{SD}(s)$, and from vertical road velocity to tire deflection, $H_{TD}(s)$:

$$H_A := \frac{\ddot{z}_s}{\dot{z}_r} = \frac{s z_s(s)}{z_r(s)}, \quad H_{SD} := \frac{z_u - z_s}{\dot{z}_r} = \frac{z_u(s) - z_s(s)}{s z_r(s)}, \quad H_{TD} := \frac{z_r - z_u}{\dot{z}_r} = \frac{z_r(s) - z_u(s)}{s z_r(s)}, \quad (2.4)$$

are commonly found in the literature as measures of ride comfort, working space requirements and road holding, respectively [Williams, 1997]. Substituting the transfer functions in (2.4) into (2.3) and manipulating the resulting expression, it becomes clear that these transfer functions are not independent:

$$[k_t + s^2 (m_s + m_u)] H_A + s^2 (k_t + s^2 m_u) H_{SD} - k_t s = 0 \quad (2.5)$$

$$m_s H_A - (k_t + m_u s^2) H_{TD}(s) + m_u s = 0 \quad (2.6)$$

Equations (2.5) and (2.6) highlight that once one of the three transfer functions in (2.4) has been fixed, the other two are automatically determined. As a consequence, and irrespective of the type, power and bandwidth of the active suspension, there must always be a compromise between comfort, road holding and working space requirements. Setting $s = j\omega$ in (2.5) and (2.6), it is also clear that there are three frequencies, ω_1 to ω_3 , at which some of the transfer functions are fixed and independent of the suspension forces (invariant points):

$$\begin{aligned} \omega_1 &:= \sqrt{\frac{k_t}{m_u}} &\Rightarrow H_A(j\omega_1) &= -j \left(\frac{\sqrt{k_t m_u}}{m_s} \right), \\ \omega_2 &:= \sqrt{\frac{k_t}{m_u + m_s}} &\Rightarrow H_{SD}(j\omega_2) &= -j \left(\frac{(m_u + m_s)^{3/2}}{m_s \sqrt{k_t}} \right), \\ \omega_3 &:= 0 &\Rightarrow H_{TD}(j\omega_3) &= H_A(j\omega_3) = 0. \end{aligned} \quad (2.7)$$

There is an invariant point in the road velocity to body acceleration transfer function at the wheel hop frequency $\omega_1 = \sqrt{k_t/m_u}$. This means that the suspension system cannot achieve much improvement in comfort at frequencies close to the wheel's hop (typically in the 10-15 Hz range). There is another invariant point in the road velocity to suspension deflection transfer function at the frequency at which the suspension locks, $\omega_2 = \sqrt{k_t/(m_u + m_s)}$. The third invariant point is in the road velocity to tire deflection transfer function, at $\omega_3 = 0$, where the transfer function is zero. The interested reader is referred to [Yue et al., 1988; Hedrick and Butsuen, 1990] for a more detailed discussion about the performance trade-offs as well as about the constraints on the low- and high-frequency asymptotes for H_A , H_{SD} and H_{TD} .

Considerable work has been carried out in order to identify the maximum performance that could be expected from an active suspension, both under ideal conditions and with various, more realistic, sets of assumptions. Particularly interesting are the studies performed with 1 and 2 DOF quarter-car models by Hrovat and summarised in [Hrovat, 1993]. Based on LQ-optimal control, the cost function to be minimised was defined as a weighted sum of the squares of sprung mass acceleration, tyre deflection and suspension deflection. All possible values for these weights were studied and carpet plots were generated. Some conclusions that were drawn from these studies follow:

- LQ-optimal performance for stochastic road inputs cannot be achieved with passive devices for any combination of weights (i.e. whatever the performance priorities, a passive suspension can never match the performance of the optimal active suspension).
- The full advantage of active suspensions stems from possible adaptive tuning (or gain scheduling) of controller parameters.
- Achievable performance in the degenerate case of zero unsprung mass (i.e. 1 DOF) is drastically better than with conventional sprung to unsprung mass ratios.
- Fast (1-3s settling time) self-levelling can be achieved by integrating it in the LQR synthesis process.

Other interesting but more specific studies have been published in recent years. For example, Smith [1995] investigates the maximum achievable performance of active suspensions in the quarter-car context for typical choices of measured outputs, Türkay and Akçay [2008] look at the aforementioned performance constraints from the point of view of active suspension control by applying LQG methodologies and the interpolation approach, Bedner et al. [2007] explore trade-offs between handling stability and responsiveness, Vahidi and Eskandarian [2002] investigate the impact of road preview uncertainties on achievable performance, and Pires et al. [2013] look at design trade-offs for energy regeneration applications.

2.4.4 Common practice and standards

Several standards and technical reports attempt to provide a common set of definitions, methodologies and guidelines to all engineers working on the vehicle dynamics field.

Nomenclature and reference frames

There are two widely used standards, one by the International Organization for Standardization (ISO) [ISO 8855:2011], and one by the Society of Automotive Engineers (SAE) [SAE J670e], that provide basic definitions. Unfortunately, they differ in some fundamental aspects such as the orientation of the reference frames: ISO's reference frames have their vertical axes pointing upwards, whereas SAE's have them pointing downwards. This simple difference can cause lots of confusion if publications do not state which convention they are using.

Testing of handling and directional response

The most relevant manoeuvres used to test handling and directional response are:

- closed-loop manoeuvres to assess handling, such as obstacle avoidance [ISO 3888-2:2011] and J-turn manoeuvres [Bedner et al., 2007],
- open-loop steady state manoeuvres such as the steady state circular test [ISO 4138:2004],
- open-loop transient directional manoeuvres such as step steering input, sinusoidal input (one period), random input, pulse input and continuous sinusoidal input [ISO 7401:2011],
- open-loop transient manoeuvres to test rollover propensity such as NHTSA's (National Highway Traffic Safety Administration) fishhook manoeuvre, and
- open-loop test manoeuvres to test the vehicle's response when braking in a turn [ISO 7975:2006].

A very informative report classifying the various types of manoeuvres and the kind of handling and stability information that can be extracted from them is provided in [Bedner et al., 2007].

Human response to vibration

Human response to vibrations is also covered by the literature as well as by ISO standards^{ix}, and it is generally evaluated through RMS acceleration metrics.

^{ix}Fundamental vocabulary is provided in [ISO 5805:1997], human sensitivity is discussed in [ISO 2631-1:1997] and guidelines regarding measurement instrumentation are given in [ISO 8041:2005].

The sensitivity of the human body is different for vertical (maximum in the 4-8 Hz range) and horizontal (maximum in the 1-2 Hz range) vibrations [Kruczek and Stribrsky, 2004], and depends on the body's position, weight and other factors. Some recent studies have focused on understanding sensitivity to individual excitations, such as roll motion [Buma et al., 2008], but it is extremely rare to find publications on suspension systems that go into so much detail.

[ISO 2631-1:1997] recommends the use of frequency dependent weights, although some studies have shown equally good (or even slightly better) correlation between weighted and un-weighted RMS vertical acceleration metrics and subjective passenger comfort ratings [Smith CC, 1978]. Thus, most research papers consider the simpler un-weighted RMS vertical acceleration metric. [ISO 2631-1:1997] also recommends that analyses evaluating human perception and comfort should be extended up to 80 Hz, but most published research focuses on the 0-20 Hz range [Cao et al., 2011].

2.5 Control of active and semi-active suspension systems

The aim of this section is to provide an overview of the main control strategies that have been investigated for active suspension systems. It is unavoidable that many approaches, although novel and innovative, will be missing from this summary; and that those that have been surveyed will not be analysed in great depth, but relevant references are provided throughout the section to guide the interested reader to detailed analyses and state of the art reviews.

Whatever the control technique to be applied, there are two ways of tackling the synthesis problem. In the first one, the dynamics of the vehicle and the actuator are taken into account in the same model. In the second one, actuator dynamics are initially neglected and the control output is assumed to be the actual force/displacement/velocity. Then, a secondary control loop that governs the actuator to track the desired force/displacement/velocity reference is designed independently. In this section, the emphasis is on the outer level control loops irrespective of whether actuator dynamics were considered or not.

Although control laws for active and semi-active suspensions are necessarily different due to passivity constraints of semi-active actuators, there has been a tendency to transfer/adapt control strategies from one environment to the other, and the underlying philosophies are in many cases quite similar. Thus, the section starts reviewing some aspects of semi-active suspension control, and then moves on to the control of fully active suspensions of different types.

2.5.1 Control of semi-active suspensions

A myriad of control strategies for semi-active suspensions have been developed in recent years, but most of them are variations of the Skyhook damping proposed by Karnopp et al. [1974] and,

to a lesser extent, of the Groundhook damping proposed by Novak and Valasek [1996].

Skyhook control

Skyhook control aims to improve ride comfort by making the actively controlled force-producing elements emulate a damper connected between the sprung mass and the inertial reference (the sky), as shown in Fig. 2.9. This simple yet effective model free control algorithm can achieve a better combination of resonance damping and high frequency isolation than a conventional spring-damper system.

The logic behind the original algorithm is as follows: when the sprung mass is moving upwards (downwards), the suspension should try to oppose that motion by providing a downwards (upwards) force. If the wheel is also moving downwards (upwards), or upwards (downwards) but at a slower rate than the sprung mass, then it is possible to generate a force with the desired direction and a high damping coefficient is desirable. If, on the other hand, the wheel is moving upwards (downwards) at a higher rate than the sprung mass, then it is not possible to generate a force with the desired direction and the best that can be done is to minimise the damping coefficient and thus the force. For a semi-active damper with two states, $c_{\min}$ and $c_{\max}$, this is realised through the following law:

$$c = \begin{cases} c_{\min}, & \text{if } \dot{z}_s (\dot{z}_u - \dot{z}_s) \geq 0 \\ c_{\max}, & \text{if } \dot{z}_s (\dot{z}_u - \dot{z}_s) < 0 \end{cases} \quad (2.8)$$

Although Skyhook control was originally envisioned for semi-active dampers with two states, it has been successfully applied to semi-active dampers with continuously varying damping coefficient as well as to fully active suspensions.

Groundhook control and other Skyhook offshoots

Groundhook control [Valasek and Novak, 1996], which suits heavy vehicles particularly well, is conceptually very similar to Skyhook control. It aims to reduce dynamic vertical tyre deflections by making the actively controlled force-producing elements emulate a damper acting between the unsprung mass and the road surface (ground). As a consequence, road holding is enhanced and road damage reduced.

Groundhook and Skyhook control can be combined (see Fig. 2.9) to provide consistent improvements in comfort and road holding with respect to passive arrangements. However, Groundhook's performance suffers when large discrete events, such as bumps, are encountered. The extended Groundhook control aims to extend the performance gains to situations in which tyre deformation is unavoidable (see [Valasek et al., 1997] for details).

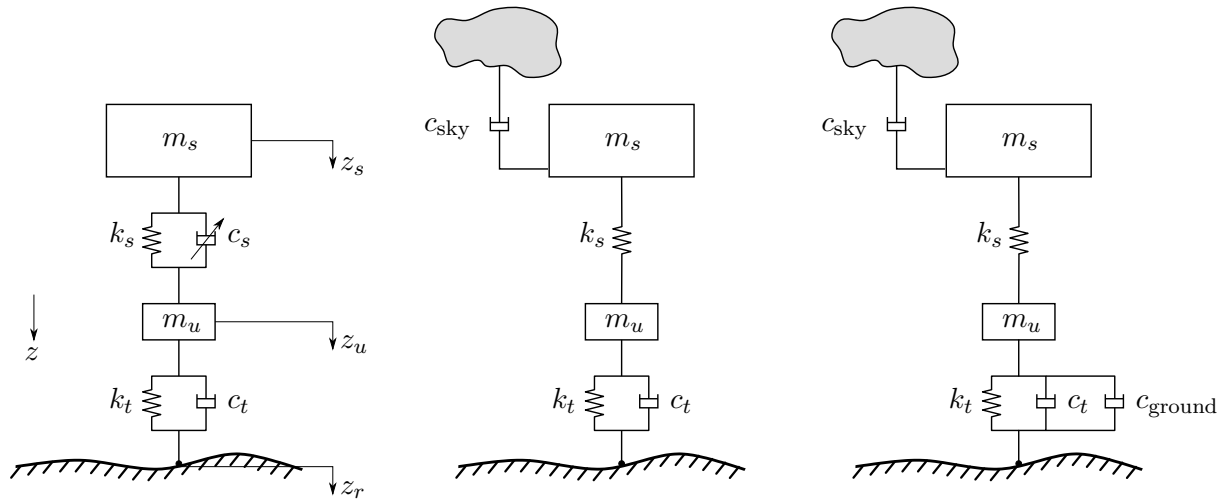


Fig. 2.9: Actual semi-active suspension system (left), ideal Skyhook concept (centre) and ideal Skyhook-Groundhook concept (right).

Many other offshoots from Skyhook control have appeared in the last few decades. Some examples are the acceleration driven damper control, the power driven damper and the mixed Skyhook-acceleration driven damper. A brief summary of these, together with some references, can be found in [Poussot-Vassal et al., 2012].

Other topics

Optimality of active suspension schemes Optimal control laws for unconstrained active suspensions have been widely investigated and it is very common to find examples in the literature where these are directly transferred to the semi-active case by simply passing the control command through a limiter. But does the saturated version of the optimal control law (clipped-optimal) remain optimal for the semi-active case? Giorgetti et al. [2006] attempted to answer this question by applying model predictive control (MPC) with different control horizons to the semi-active case, and comparing the results with those obtained with the clipped version of Linear Quadratic Regulators (LQR) derived for the unconstrained active case. The authors concluded that although the hybrid MPC matches the clipped-optimal control when the control horizon is equal to 1, significant differences appear when the control horizon is extended.

Measurements Many control schemes including the Skyhook control, require the measurement of the absolute velocity of the sprung mass^x. As this is not always possible in practice due

^xAs shown in Eq. (2.8), what is generally needed is the sign of the product between the absolute velocity of the sprung mass and the suspension deflection velocity. In [Bolandhemmat et al., 2010], it was found that most of the time this coincides with the sign of the product between the suspension deflection and the suspension deflection rate, which are easier to measure. Unfortunately, some loss of performance when using these alternative values is inevitable.

to the characteristics of accelerometers commonly used in automotive applications, observers need to be implemented^{xi} [Bolandhemmat et al., 2010].

Tuning Most control strategies have a switching nature and usually require an extensive trial and error tuning process [Bolandhemmat et al., 2010]. This has led to the publication of countless studies dealing with the semi-active control problem, and highlights the need for more systematic methodologies to evaluate the relative performance of each controller. Some attempts in this directions can be found in [Savaresi et al., 2003; Poussot-Vassal et al., 2012].

2.5.2 Control of active suspensions

Skyhook control and its offshoots have also been extensively applied to fully active suspensions, particularly in the prototyping and commercialisation stages due to their ease of implementation and effectiveness. But hydraulic or electro-mechanical actuators are no longer limited by passivity constraints and therefore there is much more freedom in the control synthesis phase. Numerous alternative and more advanced approaches have been investigated and reported in the literature, and a brief account of these is provided in this subsection.

Firstly, the concept of modal control, used among others in the Lotus Esprit prototype [Baker, 1984], is concisely discussed. Then, advances in the application of optimal control, robust control and intelligent control to the active suspension problem are covered. Finally some related topics, such as road surface preview and gain scheduling, are briefly discussed.

Modal control

In a vehicle equipped with a conventional passive suspension system, pitch and heave, and roll and warp modes are highly coupled. This implies that small pitch and roll motions can only come at the expense of a harsh ride. Moreover, roll-warp coupling shapes to a great extent the directional response of the vehicle: the roll-warp coupling stiffness determines the roll moment distribution between the front and rear axles, which in turn governs the longitudinal distribution of lateral forces and therefore the under/over-steering response of the car.

An active suspension can, if effectively controlled, decouple these modes to a great extent. By applying a coordinate transformation to the standard 7 DOF full-vehicle linear model, the variables at each of the corners of the vehicle can be combined into new ones that represent the modes of vibration of the sprung mass and of the suspension deflection. The new model still has 7 DOF, namely body heave, pitch and roll, and suspension heave, pitch, roll and warp, but

^{xi}Typical measurements with which the observers are fed include sprung and unsprung mass accelerations and suspension deflection [Rajamani and Hedrick, 1995].

its structure facilitates the synthesis of control laws that tackle vibrations along different modes independently.

In [Malek and Hedrick, 1986], an active hydraulic interconnected suspension was used to achieve almost complete decoupling between the pitch and heave modes. The authors report that the conventional vehicle considered in their study had two body modes in the heave-pitch plane, with amplitudes of the eigenvectors corresponding to 100% heave and between 70 and 80% pitch. The decoupled vehicle, on the other hand, had eigenvector amplitudes of 100% heave and 1% pitch in one mode, and 100% pitch and 4% heave in the other.

According to Milliken [1988], Lotus used modal control in their prototype vehicle showcased in the 1980's. The aim of the Lotus control was to make the vehicle soft in heave and rigid in pitch and roll. This would maintain acceptable comfort levels whilst improving handling by ensuring small changes in body attitude during aggressive manoeuvring. Thompson and Davis [1991] gave some additional technical hints regarding the operation of the Lotus control. They explained that the desired actuator displacements for each corner of the vehicle were computed from the demanded pitch, roll and heave body motions, and that these displacements could be effectively tracked by the hydraulic actuators through a simple inner control loop.

Williams and Haddad [1997] blended modal control with the Skyhook damper concept in order to improve comfort whilst maintaining the capability to affect the roll moment distribution. Over time, modal control has not only been combined with other control techniques such as $\mathcal{H}_\infty$ control [Hayakawa et al., 1999; Wang et al., 2001], but it has also been used in the study and development of passive interconnected suspensions [Smith and Walker, 2005; Shao, 2014].

Optimal control

The active suspension control problem is well suited for optimal control techniques.

LQR and LQG frameworks are a very good fit for the conventional linear quarter-car models with either 1 DOF or 2 DOF: suspension performance may be naturally expressed through a quadratic index and external road disturbances are well represented by a white noise process. Some authors have highlighted the difficulties in the implementation of these controllers in real vehicles (e.g. [Williams and Haddad, 1997] point out that these difficulties arise because a real car is a large and uncertain system with many unmodelled dynamics, multiple force paths from unsprung to sprung mass and a high reliance on rubber bushings for isolation and misalignment), but their role in expanding the understanding of design compromises and achievable performance should not be underestimated. To the knowledge of the author, the review paper by Hrovat [1997] is the best summary of relevant LQR and LQG studies and the conclusions that can be drawn from them^{xii} — some of which have already been mentioned in Section 2.4.2.

^{xii} Although this is quite an old review paper, it is worth noting that the theory matured long time ago and that

MPC has also attracted attention in the context of active suspensions [Chen and Scherer, 2004]. One of the advantages of the MPC framework is its ability to explicitly account for physical and design constraints such as wheel travel, actuator force/displacement/velocity, and power/energy consumption limits [Göhrle et al., 2012]. Thus, the performance index no longer needs to penalise constrained variables (e.g. a large suspension deflection is not undesirable per se as long as it remains within the allowable limits), and the functional to be minimised can capture the real control aims more precisely. Moreover, robustness considerations may also be included in this framework [Chen and Scherer, 2004].

Two major implementation drawbacks that were preventing MPC from being more pervasive in suspension applications, namely computational burden and the need for road surface information, are being overcome thanks to increasing computing power, new image processing techniques and remote sensing technologies such as Lidar [Schindler, 2009]. Furthermore, experimental implementations, like the one reported in [Göhrle et al., 2014] for an Audi car equipped with narrow-bandwidth actuators, show promising results and will surely lead to further research and developments.

Robust control

Most control techniques reported in the literature require a linear model of the plant. Generally speaking, linear approximations of real suspension systems are easily derived, of contained size, and reasonably accurate. However, there are potentially substantial nonlinearities even in the vicinity of the trim state (e.g. asymmetric bound/rebound damper force-speed relationships), unmodelled compliances (e.g. of the rubber bushings connecting the suspension links to the chassis), measurement noise, and parameter changes (e.g. payload or tyre pressure), that can significantly degrade the performance and even the stability of the closed loop system.

As a consequence, linear and nonlinear robust control techniques have been extensively investigated in the context of active suspensions during the last 25 years. The most widely reported are $\mathcal{H}_\infty$ control and Sliding Mode Control (SMC).

During the late 1980's and 1990's, just after the commercialisation of their first semi-active and active suspensions systems, $\mathcal{H}_\infty$ was applied by researchers at Toyota to a quarter-car test rig [Yamashita et al., 1990] and an experimental vehicle [Yamashita et al., 1994; Hayakawa et al., 1999] equipped with hydraulic actuators. Some of the topics discussed in these papers include the comparison of full state feedback versus output feedback and of constant weights versus frequency-dependent weights, the impact of changes in sprung mass and actuator characteristics, or the trade-off between performance and robustness. In their last paper, mode decoupling techniques as described in [Malek and Hedrick, 1986] were applied to synthesise independent

most of the fundamental questions that could be addressed with these techniques were already answered two or three decades ago.

controllers for the pitch-heave and roll-warp systems. It was found that the $\mathcal{H}_\infty$ control was capable of improving ride comfort and drivability at the same time, as well as of providing sufficient robustness against various parameter changes and uncertainties [Hayakawa et al., 1999]. This was the case even when using a reduced set of measurement signals consisting of body heave, pitch and roll, and suspension deflections.

Subsequent investigations by other researchers considered variations in either the types of uncertainties, actuator dynamics (if any), sensor signals, input disturbances, control objectives, or mathematical methods used to synthesise the controller.

For example, Wang et al. [2001] attempted to build on the work from [Hayakawa et al., 1999] and applied Linear Matrix Inequalities (LMI) to synthesise the control. As in other works, results were good in terms of comfort but tyre deflections were not explicitly considered and ended up being significantly larger than in the passive suspension case.

The problem was presented with an interesting twist by Lauwerys et al. [2005]. An experimental quarter-car arrangement was used to identify the linear black-box model to be considered for control synthesis. Frequency domain identification techniques were used, and the resulting linear model was combined with frequency dependent output multiplicative plant uncertainty to account for the discrepancies between the real system and the approximated linear model. $\mathcal{H}_\infty$ and μ -synthesis controllers were derived and tested with positive results.

One of the most recent interesting examples in the literature can be found in [van der Sande et al., 2013], where a novel high bandwidth linear electromagnetic actuator was installed in an experimental quarter-car test rig. $\mathcal{H}_\infty$ and μ -synthesis controllers were synthesised and tested, although in this case a sweep was performed on the control priorities (i.e. eleven controllers with different levels of priority assigned to comfort and road holding were designed). Good comfort or tyre deflection improvements were achieved independently, although it is not clear from the results reported whether both aspects were effectively tackled at the same time.

Some authors argue that linear methods, even if robust, are not well suited for the active suspension control problem. Thus, apart from $\mathcal{H}_\infty$ techniques, nonlinear control has also been explored in the literature. SMC, which is a nonlinear control method characterised by a suite of feedback control laws and a switching function, is one of the most widely investigated.

Kim and Ro [1998] identified hardening springs, quadratic damping forces and tyre lift-off as nonlinearities of sufficient significance as to prompt the investigation of Sliding Mode Control methods. In their paper, the authors represented the force-displacement and force-velocity relationships of spring and damper through polynomials of third and second order, respectively, and tyre lift-off was modelled through a piecewise linear function. Their reference model was based on Skyhook damping, but instead of using a 2 DOF quarter-car reference model, they used a 1 DOF quarter-car in which the road disturbance corresponded to the displacement of the real unsprung mass. The authors performed this simplification to remove the need for actual

road information, which was considered to be too difficult to measure and therefore could not be fed to the controller. They compared SMC with an adaptive self-tuning control, and showed that both perform well when subject to moderate sinusoidal inputs, but SMC performed better when the input was more demanding (i.e. road step or road bump).

In many publications dealing with SMC, the focus is on actuator nonlinearities and on improving the force tracking characteristics of the inner loop. In [Chen and Huang, 2006], SMC was successfully applied to control a hydraulic actuator with some time-varying uncertainties with unknown bounds. Alleyne et al. [1993] also considered hydraulic actuators, but had to correct their initial control design because the SMC scheme was exciting unmodelled dynamics of the actuator and causing excessive high frequency overshoots^{xiii}. In their following study, Alleyne and Hedrick [1995] combined SMC control with parameter identification techniques and an adaptive scheme in order to improve tracking accuracy whilst avoiding large gains.

In terms of the type of sliding surface, most publications define it as a function of the error between the reference Skyhook damper and the actual suspension force. There are exceptions such as [Yoshimura et al., 2001], where the sliding surface was obtained by using LQ theory, or [Sam et al., 2004], where it was based on proportional-integral sliding mode control.

Intelligent methodologies

SMC, MPC, $\mathcal{H}_\infty$, and other control techniques have been extensively applied in combination with intelligent methodologies such as fuzzy logic, artificial neural networks and evolutionary computing for over two decades [Cao et al., 2008]. These soft computing techniques are good at dealing with imprecision and approximate reasoning (fuzzy logic), learning and curve fitting (neural networks) and global optimisation based on natural selection and genetics.

Some companies engaged in this development from early on (e.g. Ford [Hampo and Marko, 1991]), and some quarter-car test rigs have been used to test these new controllers (e.g. [Huang and Lin, 2007]), but most studies have remained in the realm of simulation-based testing.

Related topics

Preview The use of information about the road profile ahead of the axle that is being controlled (preview) in order to improve performance [Thompson and Pearce, 2001], particularly regarding road holding and working space requirements [ElMadany, 2012] as well as energy usage [Hac, 1990] has been widely investigated since the 1960's [Bender and Paul, 1967].

Sensors capable of sensing the road ahead were not initially available, so research was purely theoretical for some decades; the focus was on identifying potential benefits of such a feature. It

^{xiii}Similar chattering problems are commonly reported [AL-Holou et al., 2002; Cao et al., 2008].

was found that even small amounts of preview could induce significant potential improvements: Hrovat [1997] reports 50 ms leading to ~30% reduction in tyre deflection. Even without road sensors, this preview time could be achieved for the control of the actuators in the rear axle based on measurements from the front axle. Typical values of forward speed and wheelbase (e.g. 120 km/h and 2.75 m) would lead to preview times of $82.5 \text{ ms} > 1.5 \times 50 \text{ ms}$. Thus, front wheel preview (also known as wheelbase preview) can lead to more effective heave and pitch control.

Still, handling enhancement and bump stop avoidance require sensing of the road surface ahead of both axles and this prompted the development of new sensors. In the 1980's, some research in this area was published. For example, Sugawara et al. [1985] described work carried out at Nissan in order to develop a road preview sensor that used super sonic waves. Although not apt for detecting road characteristics far ahead of the vehicle, the super sonic road sensor was capable of measuring vehicle height above the ground; this, combined with frequency analysis techniques allowed them to identify road conditions (smooth, rough, heaving) and to adjust suspension properties accordingly (three damping levels). More capable sensors based on laser technology have been tested in recent years by car manufacturers such as Mercedes [Schindler, 2009] and Toyota [Yamamoto et al., 2014] with promising results.

Daimler AG's Magic Body Control introduced in 2013 [Daimler AG, 2015b] has been the first active suspension system to include road preview in production vehicles. Based on stereo-vision, the system is capable (in good visibility conditions) of reconstructing the road surface up to 15 m ahead of the vehicle, at forward speeds of up to 130 km/h [Tseng and Hrovat, 2015].

Based on these recent developments, and on the fast improvement of sensor and image processing techniques spurred by the pursue of the autonomous vehicle, the future of preview systems for active suspensions seems bright in the medium and long term.

Active roll control and other attitude control systems Most published work on active suspensions considers road irregularities to be the main disturbance whose influence has to be minimised. However, comfort and handling can also be significantly improved by limiting pitching and rolling motions of the body due to load transfer caused by acceleration/braking/cornering manoeuvres.

These cases are typically the focus of studies involving the so-called slow active suspensions, and although there are cases in which fuzzy logic, $\mathcal{H}_\infty$, LQR, SMC, or other advanced techniques have been used, simple controllers seem to provide a good balance between performance and complexity. For example, proportional or proportional-derivative feedback control gains for hydraulic anti-roll systems proved to be good enough in [Sharp and Pan, 1992; Darling et al., 1992; Shuttlewood et al., 1993].

Adaptive control and gain scheduling Vehicle and actuator parameters will change during normal operation and this may compromise the performance and even the stability of the active suspension. As discussed earlier, linear robust control techniques attempt to cover the effect of these changes, but alternative approaches such as adaptive control have also been the subject of intensive research. From adaptation to road conditions [Hac, 1987], to adaptation to changes in payload, inertia, and actuator properties [Sun et al., 2015], there is a large number of publications dealing with adaptive active suspension control during the last three decades.

The particular case of gain scheduling with respect to road characteristics is one of the most beneficial and popular schemes. In some cases, it is assumed that the road profile can be measured [Fialho and Balas, 2002; Hiraoka et al., 2014], whilst in others, such as in [Tran and Hrovat, 1993], road characteristics are reconstructed from suspension stroke measurements.

Integrated control Active suspensions perform a distinct function in road vehicles. However, there is some degree of overlap with other ADAS such as ESP (Electronic Stability Program), and potential synergies with others such as ABS. Thus, control systems need to integrate these subsystems to provide the best possible performance.

This was appreciated long time ago [Milliken, 1988], and as early as in 1990 Toyota showed that integrating active suspension control with four wheel steering control could lead to ~5% increase in maximum slalom speed as compared to the case with individually designed controllers. Nowadays, efforts continue in this direction. For example, Heo et al. [2015] report progress made for Hyundai Motor Company regarding the integrated control of ESC (Electronic Stability Control), four wheel drive and active roll control to improve high speed cornering performance.

2.6 Conclusions

This chapter has reviewed the state of the art of active suspensions to identify potential opportunities for innovative solutions such as the SAVGS. Furthermore, main functions, classifications, analysis tools and control algorithms for active suspensions have been discussed.

Key conclusions that can be drawn from the reviewed material are:

- Despite significant performance advantages, excessive cost, weight, complexity and energy consumption of current solutions render fully active suspensions unattractive but for the high-end/luxury categories.
- Autonomous driving, proliferation of ADAS and higher electrification levels are likely to foster new developments in active suspensions and particularly in electromagnetic active suspensions.

- Semi-active suspensions outperform their passive counterparts but cannot deliver the new and innovative features that characterise active suspensions.
- Narrow-bandwidth active suspensions (like the SAVGS) with control bandwidth between 3 and 5 Hz have the potential to offer performance improvements comparable to high-bandwidth active suspensions, whilst reducing complexity, cost and NVH issues.
- Although simple linear models like the 2 DOF quarter-car model can be used for control synthesis, high-fidelity models are required to properly assess the performance of suspension systems.
- Actuator dynamics can have a significant effect on the suspension's response and should be ideally considered in simulation models.
- Research on a non-conventional suspension topology such as the SAVGS requires the use of polyvalent multibody software capable of integrating multi-domain dynamics.
- In order to test the effect of active suspensions on the car's directional response, open-loop test manoeuvres should be simulated if complex human modelling is to be avoided.
- Simple control techniques may be used for attitude control but more advanced approaches should be considered for comfort enhancement and in general for multi-objective control.
- MPC and $\mathcal{H}_\infty$ techniques are the best performing, well tested control approaches reported in the literature thus far.
- The full potential of an active suspension can only be exploited when the road profile ahead of the vehicle is previewed, and the control algorithm is integrated with other ADAS on board.

Chapter 3

Modelling, simulation and parameter selection

Modelling and simulation are key to understand, design and test an active suspension system and its control schemes, and are the backbone of this project. As no initial models, parameter values or experimental data were available at the beginning of this work, it has been necessary to proceed with care, to develop models with various degrees of accuracy and to implement them in different software environments to substantiate the results presented.

This chapter covers the software packages used, the vehicle and actuator models developed and the checks performed to validate the simulation results obtained with them. A summary of the main parameter values selected for the two vehicles that have been studied throughout the thesis is also provided.

3.1 Computer software

Three types of models have been developed to study the SAVGS and its influence on the dynamic response of the vehicle. These are described in Section 3.2 and Section 3.3, but are listed here to justify the selection of software packages.

- Type I models: simplified linear models for the vertical and lateral dynamics of the vehicle (equipped with a passive suspension),
- Type II models: nonlinear, multi-body model of one corner of the vehicle (equipped with a passive or SAVGS-retrofitted suspension),
- Type III models: nonlinear, multi-body, full-vehicle model (equipped with a passive or SAVGS-retrofitted suspension).

Two software packages, namely AutoSim and MATLAB + SimMechanics/Simulink, have been used to implement and simulate these dynamic models.

3.1.1 MATLAB + SimMechanics/Simulink

MATLAB is ubiquitously used by engineers and academics from many disciplines and therefore needs no introduction. It has been extensively used in this project:

- to automate validation and post-processing tasks for all models and vehicle subsystems,
- to implement type I models, and
- to synthesise controllers for comfort enhancement.

MATLAB's accompanying environments, SimMechanics [Wood and Kennedy, 2003] and Simulink, are convenient tools for the modelling of multi-body systems and control systems, respectively, and have been selected in this thesis for their functionalities, ease of use and full integration with MATLAB. SimMechanics has been employed to validate results obtained with AutoSim for type II models, whereas Simulink has been mainly utilised to design several control loops needed to operate the SAVGS.

3.1.2 AutoSim

AutoSim is a computer language designed to automatically generate computationally efficient simulation programs for mechanical systems composed of multiple rigid bodies [Sayers, 1993]. It originated in the University of Michigan Transportation Research Institute, and was the precursor of today's CarSim [Mechanical Simulation Corporation, 2015].

AutoSim models need to be coded by the user in a Lisp-based language, and are processed by AutoSim to produce C, Fortran or CMEX programs containing the resulting equations of motion. AutoSim is also able to linearise the equations of motion of a system and write MATLAB-compatible code containing its state-space representation.

Although not user-friendly, AutoSim grants the user access to the definition of bodies, forces, auxiliary variables etc. at a fundamental level — a feature that has been key to make the integration of a non-conventional suspension topology with the differential equations of the SAVGS actuators possible. The equations of motion are derived following Kane's method [Kane and Levinson, 1980] and the resulting computer programs run faster than equivalent SimMechanics models (one-corner models have been seen to run 5 to 40 times faster in AutoSim).

In this project AutoSim has been used to represent type II and type III models, as well as to obtain linearised versions of type II models.

3.2 Vehicle models

As already mentioned in Section 3.1, the vehicle has been modelled with three different levels of detail:

- Simplified linear models capturing the vertical and lateral dynamics of the vehicle (equipped with passive suspension only) have been used to guide the development and qualitatively validate more complex models. These models are covered in Section 3.2.2 and Section 3.2.3, respectively, and are referred to as type I models for short.
- More complex, multi-body quarter-corner models (with passive and SAVGS-retrofitted suspensions) have been used in comfort-based studies, as well as to guide the selection of suspension parameters. These are briefly described in Section 3.2.4 and are referred to as type II models for short.
- Finally, multi-body full-vehicle models (with passive and SAVGS-retrofitted suspensions) have been employed to assess the SAVGS influence on the vehicle's dynamic response, and to derive attitude control schemes. These models are referred to as type III models for short, and their subsystems are described in Section 3.2.5.

Before describing the models, the reference frames used are listed in Section 3.2.1. Throughout the rest of the section the focus is on the underlying physics and modelling assumptions, and only a few remarks about the actual implementation in AutoSim and/or MATLAB + SimMechanics/Simulink are made.

3.2.1 Reference frames

All reference frames considered are right-hand orthogonal axis systems.

The vehicle axis system has been defined according to [SAE J670e] and attached to the vehicle's centre of mass. When moving steadily in a straight line on a level road, the x -axis is substantially horizontal and pointing forward, the y -axis points to the driver's right, and the z -axis points downward, as shown in Fig. 3.1.

When studying one-corner models of the vehicle, the axes have been renamed to x' , y' and z' to differentiate them from those of the vehicle axis system. The origin, O' , is attached to the sprung mass, the z' -axis points downward and the y' -axis is parallel to the ground and points towards the wheel.

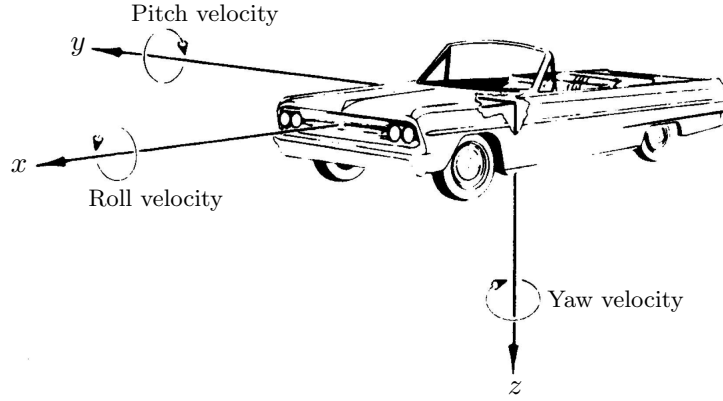


Fig. 3.1: Vehicle axis system as defined in [SAE J670e].

3.2.2 Hand-derived linear models for the study of vertical vehicle dynamics

Linear quarter-car model (QCL)

This is the classical 2DOF linear model used to study vehicle suspensions in the context of vertical dynamics. As described in Section 2.4.1, it neglects suspension kinematics and therefore it is not valid for a proper analysis of the SAVGS. However, it is a good starting point in order to 1) understand the behaviour of the passive suspension, 2) assess the validity of the vehicle parameters used for the case studies, and 3) cross-check the results obtained with the more complex multi-body models.

The equations of motion for the quarter-car model (see Fig. 3.2) can be found by applying Newton's second law to the sprung and unsprung masses:

$$\begin{aligned} m_s \ddot{z}_s &= k_{eq} (z_u - z_s) + c_{eq} (\dot{z}_u - \dot{z}_s) + F_{zs} \\ m_u \ddot{z}_u &= -[k_{eq} (z_u - z_s) + c_{eq} (\dot{z}_u - \dot{z}_s)] + k_t (z_r - z_u) + c_t (\dot{z}_r - \dot{z}_u), \end{aligned} \quad (3.1)$$

where subscripts s and u refer to the sprung and unsprung masses, k_{eq} and c_{eq} are the equivalent suspension stiffness and damping coefficients, subscripts t and r refer to the tyre and road, respectively, and F_{zs} is an external vertical force that can represent longitudinal and/or lateral load transfers due to horizontal vehicle accelerations.

Defining the following state variables: $x_1 = \dot{z}_s$, $x_2 = \dot{z}_u$, $x_3 = \Delta l_s = z_u - z_s$ and $x_4 = \Delta l_t = z_r - z_u$, considering the road velocity and external force acting on the sprung mass as external disturbances, $u_1 = \dot{z}_r$, $u_2 = F_{zs}$, and defining the sprung mass acceleration, suspension deflection and tyre deflection as outputs, $y_1 = \ddot{z}_s$, $y_2 = \Delta l_s$ and $y_3 = \Delta l_t$, the model for the quarter-car can

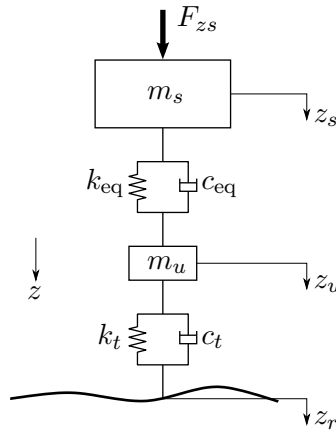


Fig. 3.2: Linear quarter-car model for the study of vertical dynamics.

be expressed in state-space form as

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u}, \\ \mathbf{y} &= \mathbf{C} \mathbf{x} + \mathbf{D} \mathbf{u},\end{aligned}\tag{3.2}$$

with

$$\left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & \mathbf{D} \end{array} \right] = \left[\begin{array}{cccc|cc} -\frac{c_{eq}}{m_s} & \frac{c_{eq}}{m_s} & \frac{k_{eq}}{m_s} & 0 & 0 & \frac{1}{m_s} \\ \frac{c_{eq}}{m_u} & -\frac{c_{eq}+c_t}{m_u} & -\frac{k_{eq}}{m_u} & \frac{k_t}{m_u} & \frac{c_t}{m_u} & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ \hline -\frac{c_{eq}}{m_s} & \frac{c_{eq}}{m_s} & \frac{k_{eq}}{m_s} & 0 & 0 & \frac{1}{m_s} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array} \right].\tag{3.3}$$

It is also convenient to obtain analytic expressions for the transfer functions of the system. These can be derived from the state-space matrices through:

$$H(s) = \mathbf{C} (s \mathbf{I} - \mathbf{A})^{-1} \mathbf{B} + \mathbf{D}.\tag{3.4}$$

In particular, and defining the total mass in the system as $m = m_s + m_u$, the transfer functions from road velocity to sprung mass acceleration, suspension deflection, and tyre deflection, are

commonly used in the literature to study passive and active suspension systems:

$$\begin{aligned}
 H_{\dot{z}_r \rightarrow \ddot{z}_s}(s) &= \frac{s [c_{eq} c_t s^2 + (c_{eq} k_t + c_t k_{eq}) s + k_{eq} k_t]}{d(s)}, \\
 H_{\dot{z}_r \rightarrow \Delta l_s}(s) &= \frac{s [c_t m_s s + k_t m_s]}{d(s)}, \\
 H_{\dot{z}_r \rightarrow \Delta l_t}(s) &= \frac{s [m_s m_u s^2 + c_{eq} m s + k_{eq} m]}{d(s)},
 \end{aligned} \tag{3.5}$$

with the denominator

$$d(s) = m_s m_u s^4 + (c_{eq} m + c_t m_s) s^3 + (c_{eq} c_t + k_{eq} m + k_t m_s) s^2 + (c_{eq} k_t + c_t k_{eq}) s + k_{eq} k_t. \tag{3.6}$$

Linear full-car model (FCL)

This is an expanded version of the linear quarter-car model that can be used to calculate heave, pitch and roll natural frequencies as well as to synthesise attitude control schemes. As shown in Fig. 3.3, the chassis is defined as a rigid body with 3 DOF (vertical displacement of the CM and pitching and rolling rotations), each wheel has 1 DOF (vertical displacement), vertical road displacements are independently defined for each corner, and two external actions are applied to the chassis (exogenous pitching and rolling moments).

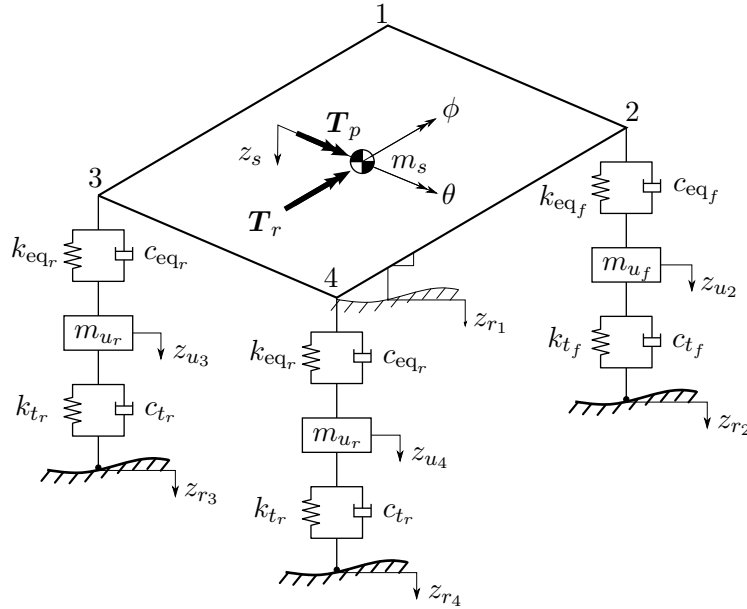


Fig. 3.3: Linear full-car model for the study of vertical dynamics.

In order to determine the suspension forces acting between the sprung and unsprung masses, four points are defined in the chassis, each of them lying in the same vertical line as the corresponding

unsprung mass. Points 1 to 4 correspond to the front left, front right, rear left and rear right corners, respectively.

The linearised equations for the positions and speeds of these points are:

$$\begin{aligned} z_1 &= z_{\text{CM}} - a_f \theta - (t_f/2) \phi, & \dot{z}_1 &= \dot{z}_{\text{CM}} - a_f \dot{\theta} - (t_f/2) \dot{\phi}, \\ z_2 &= z_{\text{CM}} - a_f \theta + (t_f/2) \phi, & \dot{z}_2 &= \dot{z}_{\text{CM}} - a_f \dot{\theta} + (t_f/2) \dot{\phi}, \\ z_3 &= z_{\text{CM}} + a_r \theta - (t_r/2) \phi, & \dot{z}_3 &= \dot{z}_{\text{CM}} + a_r \dot{\theta} - (t_r/2) \dot{\phi}, \\ z_4 &= z_{\text{CM}} + a_r \theta + (t_r/2) \phi, & \dot{z}_4 &= \dot{z}_{\text{CM}} + a_r \dot{\theta} + (t_r/2) \dot{\phi}, \end{aligned} \quad (3.7)$$

where rolling and pitching angles are denoted by ϕ and θ , subscripts f and r refer to the front and rear axles, a_f and a_r are the longitudinal distances between each axle and the centre of mass, CM, and t_f and t_r are the track widths (i.e. lateral distance between the points of contact of the tyres in each axle).

The suspension deflections and deflection rates can therefore be calculated through:

$$\begin{aligned} \Delta l_{s_1} &= z_{u_1} - z_1 = z_{u_1} - z_{\text{CM}} + a_f \theta + (t_f/2) \phi, & \dot{\Delta} l_{s_1} &= \dot{z}_{u_1} - \dot{z}_{\text{CM}} + a_f \dot{\theta} + (t_f/2) \dot{\phi} \\ \Delta l_{s_2} &= z_{u_2} - z_2 = z_{u_2} - z_{\text{CM}} + a_f \theta - (t_f/2) \phi, & \dot{\Delta} l_{s_2} &= \dot{z}_{u_2} - \dot{z}_{\text{CM}} + a_f \dot{\theta} - (t_f/2) \dot{\phi} \\ \Delta l_{s_3} &= z_{u_3} - z_3 = z_{u_3} - z_{\text{CM}} - a_r \theta + (t_r/2) \phi, & \dot{\Delta} l_{s_3} &= \dot{z}_{u_3} - \dot{z}_{\text{CM}} - a_r \dot{\theta} + (t_r/2) \dot{\phi} \\ \Delta l_{s_4} &= z_{u_4} - z_4 = z_{u_4} - z_{\text{CM}} - a_r \theta - (t_r/2) \phi, & \dot{\Delta} l_{s_4} &= \dot{z}_{u_4} - \dot{z}_{\text{CM}} - a_r \dot{\theta} - (t_r/2) \dot{\phi} \end{aligned} \quad (3.8)$$

Tyre deflections and deflection rates are simply $\Delta l_{t_i} = z_{r_i} - z_{u_i}$ and $\dot{\Delta} l_{t_i} = \dot{z}_{r_i} - \dot{z}_{u_i}$, respectively. The suspension forces produced by the spring-damper (SD) and acting on the sprung mass (reacted by the unsprung masses) as well as the tyre forces acting on the unsprung masses are given by:

$$\begin{aligned} F_{\text{SD}1} &= k_{\text{eq}_f} \Delta l_{s_1} + c_{\text{eq}_f} \dot{\Delta} l_{s_1}, & F_{t_1} &= k_{t_f} \Delta l_{t_1} + c_{t_f} (\dot{z}_{r_1} - \dot{z}_{u_1}) \\ F_{\text{SD}2} &= k_{\text{eq}_f} \Delta l_{s_2} + c_{\text{eq}_f} \dot{\Delta} l_{s_2}, & F_{t_2} &= k_{t_f} \Delta l_{t_2} + c_{t_f} (\dot{z}_{r_2} - \dot{z}_{u_2}) \\ F_{\text{SD}3} &= k_{\text{eq}_r} \Delta l_{s_3} + c_{\text{eq}_r} \dot{\Delta} l_{s_3}, & F_{t_3} &= k_{t_r} \Delta l_{t_3} + c_{t_r} (\dot{z}_{r_3} - \dot{z}_{u_3}) \\ F_{\text{SD}4} &= k_{\text{eq}_r} \Delta l_{s_4} + c_{\text{eq}_r} \dot{\Delta} l_{s_4}, & F_{t_4} &= k_{t_r} \Delta l_{t_4} + c_{t_r} (\dot{z}_{r_4} - \dot{z}_{u_4}) \end{aligned} \quad (3.9)$$

The equations of motion are obtained by applying Newton's second law to the chassis and unsprung masses, and Euler's second law to the chassis. Considering that the vehicle's axes coincide with its principal axes of inertia, the following 15 state variables:

$$\mathbf{x}^T = [\dot{z}_{\text{CM}}, \dot{\theta}, \dot{\phi}, \dot{z}_{u_1}, \dot{z}_{u_2}, \dot{z}_{u_3}, \dot{z}_{u_4}, \Delta l_{s_1}, \Delta l_{s_2}, \Delta l_{s_3}, \Delta l_{s_4}, \Delta l_{t_1}, \Delta l_{t_2}, \Delta l_{t_3}, \Delta l_{t_4}], \quad (3.10)$$

have been preferred over a reduced (and minimal) selection of 14 state variables, such as $\mathbf{x}^T = [\dot{z}_{\text{CM}}, \dot{\theta}, \dot{\phi}, \dot{z}_{u_1}, \dot{z}_{u_2}, \dot{z}_{u_3}, \dot{z}_{u_4}, z_{\text{CM}}, \theta, \phi, z_{u_1}, z_{u_2}, z_{u_3}, z_{u_4}]$, because they lead to simpler expressions for the equations of motion. Thus, the equations of motion can be conveniently expressed as:

$$\ddot{z}_{\text{CM}} = \frac{\sum F_{\text{SD}_i}}{m_s} = \frac{1}{m_s} \left(-2(c_{\text{eq}_f} + c_{\text{eq}_r})\dot{z}_{\text{CM}} + 2(c_{\text{eq}_f}a_f - c_{\text{eq}_r}a_r)\dot{\theta} \right. \\ \left. + c_{\text{eq}_f}\dot{z}_{u_1} + c_{\text{eq}_f}\dot{z}_{u_2} + c_{\text{eq}_r}\dot{z}_{u_3} + c_{\text{eq}_r}\dot{z}_{u_4} \right. \\ \left. + k_{\text{eq}_f}\Delta l_{s_1} + k_{\text{eq}_f}\Delta l_{s_2} + k_{\text{eq}_r}\Delta l_{s_3} + k_{\text{eq}_r}\Delta l_{s_4} \right) \quad (3.11)$$

$$\ddot{\theta} = \frac{\sum M_y}{I_{yy}} = \frac{1}{I_{yy}} \left(-a_f(F_{\text{SD}_1} + F_{\text{SD}_2}) + a_r(F_{\text{SD}_3} + F_{\text{SD}_4}) + T_p \right) \\ = \frac{1}{I_{yy}} \left(2(c_{\text{eq}_f}a_f - c_{\text{eq}_r}a_r)\dot{z}_{\text{CM}} - 2(c_{\text{eq}_f}a_f^2 + c_{\text{eq}_r}a_r^2)\dot{\theta} \right. \\ \left. - a_fc_{\text{eq}_f}\dot{z}_{u_1} - a_fc_{\text{eq}_f}\dot{z}_{u_2} + a_rc_{\text{eq}_r}\dot{z}_{u_3} + a_rc_{\text{eq}_r}\dot{z}_{u_4} \right. \\ \left. - a_fk_{\text{eq}_f}\Delta l_{s_1} - a_fk_{\text{eq}_f}\Delta l_{s_2} + a_rk_{\text{eq}_r}\Delta l_{s_3} + a_rk_{\text{eq}_r}\Delta l_{s_4} + T_p \right) \quad (3.12)$$

$$\ddot{\phi} = \frac{\sum M_x}{I_{xx}} = \frac{1}{I_{xx}} \left(\frac{t_f}{2}(-F_{\text{SD}_1} + F_{\text{SD}_2}) + \frac{t_r}{2}(-F_{\text{SD}_3} + F_{\text{SD}_4}) + T_r \right) \\ = \frac{1}{I_{xx}} \left(-\frac{1}{2}(c_{\text{eq}_f}t_f^2 + c_{\text{eq}_r}t_r^2)\dot{\phi} \right. \\ \left. - \frac{t_f}{2}c_{\text{eq}_f}\dot{z}_{u_1} + \frac{t_f}{2}c_{\text{eq}_f}\dot{z}_{u_2} - \frac{t_r}{2}c_{\text{eq}_r}\dot{z}_{u_3} + \frac{t_r}{2}c_{\text{eq}_r}\dot{z}_{u_4} \right. \\ \left. - \frac{t_f}{2}k_{\text{eq}_f}\Delta l_{s_1} + \frac{t_f}{2}k_{\text{eq}_f}\Delta l_{s_2} - \frac{t_r}{2}k_{\text{eq}_r}\Delta l_{s_3} + \frac{t_r}{2}k_{\text{eq}_r}\Delta l_{s_4} + T_r \right) \quad (3.13)$$

$$\ddot{z}_{u_1} = \frac{F_{t_1} - F_{\text{SD}_1}}{m_{u_f}} = \frac{1}{m_{u_f}} \left(c_{\text{eq}_f}\dot{z}_{\text{CM}} - c_{\text{eq}_f}a_f\dot{\theta} - c_{\text{eq}_f}(t_f/2)\dot{\phi} \right. \\ \left. - (c_{t_f} + c_{\text{eq}_f})\dot{z}_{u_1} - k_{\text{eq}_f}\Delta l_{s_1} + k_{t_f}\Delta l_{t_1} + c_{t_f}\dot{z}_{r_1} \right) \quad (3.14)$$

$$\ddot{z}_{u_2} = \frac{F_{t_2} - F_{\text{SD}_2}}{m_{u_f}} = \frac{1}{m_{u_f}} \left(c_{\text{eq}_f}\dot{z}_{\text{CM}} - c_{\text{eq}_f}a_f\dot{\theta} + c_{\text{eq}_f}(t_f/2)\dot{\phi} \right. \\ \left. - (c_{t_f} + c_{\text{eq}_f})\dot{z}_{u_2} - k_{\text{eq}_f}\Delta l_{s_2} + k_{t_f}\Delta l_{t_2} + c_{t_f}\dot{z}_{r_2} \right) \quad (3.15)$$

$$\ddot{z}_{u_3} = \frac{F_{t_3} - F_{\text{SD}_3}}{m_{u_r}} = \frac{1}{m_{u_r}} \left(c_{\text{eq}_r}\dot{z}_{\text{CM}} + c_{\text{eq}_r}a_r\dot{\theta} - c_{\text{eq}_r}(t_r/2)\dot{\phi} \right. \\ \left. - (c_{t_r} + c_{\text{eq}_r})\dot{z}_{u_3} - k_{\text{eq}_r}\Delta l_{s_3} + k_{t_r}\Delta l_{t_3} + c_{t_r}\dot{z}_{r_3} \right) \quad (3.16)$$

$$\ddot{z}_{u_4} = \frac{F_{t_4} - F_{\text{SD}_4}}{m_{u_r}} = \frac{1}{m_{u_r}} \left(c_{\text{eq}_r}\dot{z}_{\text{CM}} + c_{\text{eq}_r}a_r\dot{\theta} + c_{\text{eq}_r}(t_r/2)\dot{\phi} \right. \\ \left. - (c_{t_r} + c_{\text{eq}_r})\dot{z}_{u_4} - k_{\text{eq}_r}\Delta l_{s_4} + k_{t_r}\Delta l_{t_4} + c_{t_r}\dot{z}_{r_4} \right) \quad (3.17)$$

Choosing the output variables as

$$\mathbf{y}^T = [\ddot{z}_{\text{CM}}, \ddot{\theta}, \ddot{\phi}, \Delta l_{s_1}, \Delta l_{s_2}, \Delta l_{s_3}, \Delta l_{s_4}, \Delta l_{t_1}, \Delta l_{t_2}, \Delta l_{t_3}, \Delta l_{t_4}], \quad (3.18)$$

and the inputs as the time derivatives of the vertical road profiles and the exogenous pitching

and rolling torques:

$$\mathbf{u}^T = [\dot{z}_{r1}, \dot{z}_{r2}, \dot{z}_{r3}, \dot{z}_{r4}, T_p, T_r], \quad (3.19)$$

the system matrices, $A \in \mathbb{R}^{15 \times 15}$, $B \in \mathbb{R}^{15 \times 6}$, $C \in \mathbb{R}^{11 \times 15}$ and $D \in \mathbb{R}^{11 \times 6}$, have the following structure:

$$A = \begin{bmatrix} * & * & 0 & * & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & 0 & * & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & * & 0 & 0 & 0 & * & 0 & 0 & 0 \\ * & * & * & 0 & * & 0 & 0 & 0 & * & 0 & 0 & 0 & * & 0 & 0 \\ * & * & * & 0 & 0 & * & 0 & 0 & 0 & * & 0 & 0 & 0 & * & 0 \\ * & * & * & 0 & 0 & 0 & * & 0 & 0 & 0 & * & 0 & 0 & 0 & * \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \end{bmatrix} \quad (3.20)$$

$$C = \begin{bmatrix} * & * & 0 & * & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & 0 & * & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.21)$$

The non-zero elements in the matrix A are:

$$\begin{aligned} a_{1,1} &= -2(c_{eq_f} + c_{eq_r})/m_s; \quad a_{1,2} = 2(c_{eq_f} a_f - c_{eq_r} a_r)/m_s; \quad a_{1,4} = a_{1,5} = c_{eq_f}/m_s; \quad a_{1,6} = a_{1,7} = c_{eq_r}/m_s; \\ a_{1,8} &= a_{1,9} = k_{eq_f}/m_s; \quad a_{1,10} = a_{1,11} = k_{eq_r}/m_s; \\ a_{2,1} &= 2(a_f c_{eq_f} - a_r c_{eq_r})/I_{yy}; \quad a_{2,2} = -2(a_f^2 c_{eq_f} + a_r^2 c_{eq_r})/I_{yy}; \quad a_{2,4} = a_{2,5} = -a_f c_{eq_f}/I_{yy}; \quad a_{2,6} = a_{2,7} = \\ &= a_r c_{eq_r}/I_{yy}; \quad a_{2,8} = a_{2,9} = -a_f k_{eq_f}/I_{yy}; \quad a_{2,10} = a_{2,11} = a_r k_{eq_r}/I_{yy}; \\ a_{3,3} &= -(t_f^2 c_{eq_f} + t_r^2 c_{eq_r})/(2I_{xx}); \quad a_{3,4} = -a_{3,5} = -t_f c_{eq_f}/(2I_{xx}); \quad a_{3,6} = -a_{3,7} = -t_r c_{eq_r}/(2I_{xx}); \end{aligned}$$

$$\begin{aligned}
a_{3,8} &= -a_{3,9} = -t_f k_{eq_f} / (2I_{xx}); \quad a_{3,10} = -a_{3,11} = -t_r k_{eq_r} / (2I_{xx}); \\
a_{4,1} &= a_{5,1} = c_{eq_f} / m_{u_f}; \quad a_{4,2} = a_{5,2} = -a_f c_{eq_f} / m_{u_f}; \quad a_{4,3} = -a_{5,3} = -t_f c_{eq_f} / (2m_{u_f}); \quad a_{4,4} = a_{5,5} = \\
&= -(c_{eq_f} + c_{t_f}) / m_{u_f}; \quad a_{4,8} = a_{5,9} = -k_{eq_f} / m_{u_f}; \quad a_{4,12} = a_{5,13} = k_{t_f} / m_{u_f}; \\
a_{6,1} &= a_{7,1} = c_{eq_r} / m_{u_r}; \quad a_{6,2} = a_{7,2} = a_r c_{eq_r} / m_{u_r}; \quad a_{6,3} = -a_{7,3} = -t_r c_{eq_r} / (2m_{u_r}); \quad a_{6,6} = a_{7,7} = \\
&= -(c_{eq_r} + c_{t_r}) / m_{u_r}; \quad a_{6,10} = a_{7,11} = -k_{eq_r} / m_{u_r}; \quad a_{6,14} = a_{7,15} = k_{t_r} / m_{u_r}; \\
a_{8,1} &= a_{9,1} = a_{10,1} = a_{11,1} = -1; \quad a_{8,2} = a_{9,2} = a_f; \quad a_{10,2} = a_{11,2} = -a_r; \quad a_{8,3} = -a_{9,3} = t_f / 2; \quad a_{10,3} = \\
&= -a_{11,3} = t_r / 2; \quad a_{8,4} = a_{9,5} = a_{10,6} = a_{11,7} = 1; \\
a_{12,4} &= a_{13,5} = a_{14,6} = a_{15,7} = -1.
\end{aligned}$$

The non-zero elements in the matrix B are:

$$\begin{aligned}
b_{2,5} &= 1/I_{yy}, \quad b_{3,6} = 1/I_{xx}, \quad b_{4,1} = b_{5,2} = c_{t_f} / m_{u_f}, \quad b_{6,3} = b_{7,4} = c_{t_r} / m_{u_r}, \\
b_{12,1} &= b_{13,2} = b_{14,3} = b_{15,4} = 1.
\end{aligned}$$

The first three rows of C are equal to the first three rows of A . The remaining non-zero elements in the matrix C are:

$$c_{4,8} = c_{5,9} = c_{6,10} = c_{7,11} = c_{8,12} = c_{9,13} = c_{10,14} = c_{11,15} = 1.$$

Finally, the non-zero elements in the matrix D are:

$$d_{2,5} = 1/I_{yy}, \quad d_{3,6} = 1/I_{xx}.$$

3.2.3 Hand-derived linear bicycle model for the study of lateral dynamics

The bicycle model (BM) is the simplest model to study a car's lateral dynamics. It is a linear, time invariant representation of the vehicle, with both wheels in the same axle lumped together at the axle's centre. It can provide useful information regarding the vehicle's behaviour in the linear range (up to $\sim 0.3g$). As it was highlighted by Xia and Willis [1995], it may produce results that are comparable to those obtained with more detailed models that include nonlinear suspension and steering systems, and it may be used to assess the impact of the main vehicle characteristics on the lateral dynamics response.

Although this model does not include the suspension system, and therefore it does not allow to study the SAVGS, it is a valuable tool to understand the vehicle's lateral response at low lateral acceleration levels. It has proved to be particularly useful in the selection of the main vehicle parameters for the case studies and to cross-check the results obtained with the more complex, multi-body model employed throughout the thesis. Response times to step-steering inputs, as well as steering gain, understeering gradient, and natural frequency and damping ratio in yaw, are some of the relevant vehicle characteristics that can be predicted with it.

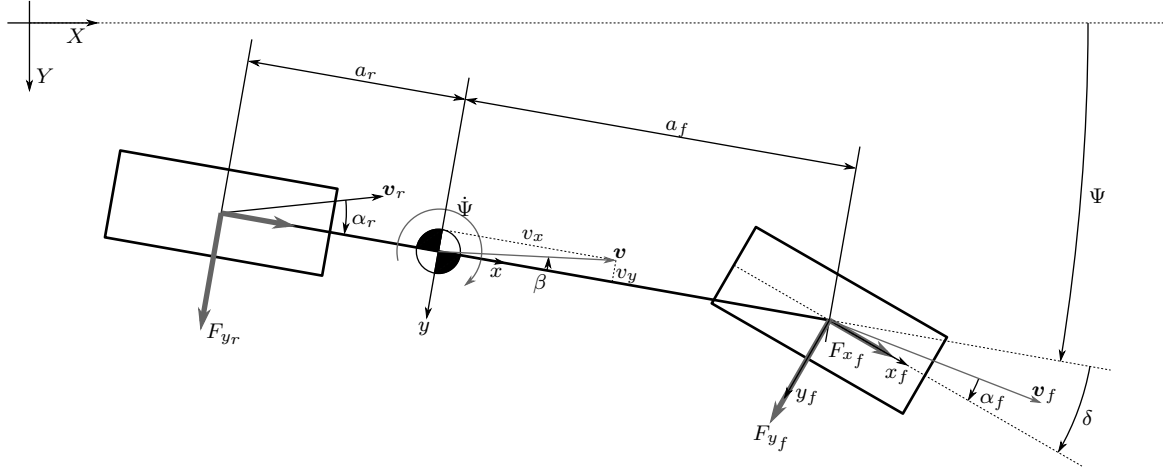


Fig. 3.4: Linear bicycle model.

The equations of motion for the single-track vehicle shown in Fig. 3.4 are:

$$\begin{aligned}\Sigma F_x &= m a_x = m (\dot{v}_x - v_y \dot{\Psi}) \\ \Sigma F_y &= m a_y = m (v_x \dot{\Psi} + \dot{v}_y) \\ \Sigma M_z &= I_z \ddot{\Psi}\end{aligned}\tag{3.22}$$

The summation of forces and moments acting on the vehicle (dropping the symbol Σ) can be expressed as:

$$\begin{aligned}F_x &= F_{x_f} \cos(\delta) + F_{x_r} - F_{y_f} \sin(\delta) \approx F_{x_f} + F_{x_r} - F_{y_f} \delta, \\ F_y &= F_{x_f} \sin(\delta) + F_{y_f} \cos(\delta) + F_{y_r} \approx F_{y_f} + F_{y_r}, \\ M_z &= [F_{x_f} \sin(\delta) + F_{y_f} \cos(\delta)] a_f - F_{y_r} a_r \approx F_{y_f} a_f - F_{y_r} a_r,\end{aligned}\tag{3.23}$$

where the final simplification is achieved by assuming a small steering angle (i.e. $\cos(\delta) \approx 1$ and $\sin(\delta) \approx \delta$), and considering that the product $F_{x_f} \delta$ is much smaller than the lateral forces.

The first equation of motion, (3.22), becomes decoupled when the forward speed is constant (i.e. $\dot{v}_x = 0$). The remaining two equations of motion can be represented in the standard state space form with $\mathbf{x} = [v_y, \dot{\Psi}]^T$ as the state vector, $\mathbf{y} = [v_y, \dot{\Psi}]^T$ as the output, $u = \delta$ as the control input, and the system matrices given by:

$$A = \begin{bmatrix} \frac{C_\beta}{m v_x} & \frac{C_r}{m} - v_x \\ \frac{D_\beta}{I_z v_x} & \frac{D_r}{I_z} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{C_\delta}{m} \\ \frac{D_\delta}{I_z} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\tag{3.24}$$

where coefficients C_r to D_δ represent the lateral force and yawing moment sensitivities to yaw

rate, slip angle and steering input, respectively:

$$\begin{aligned} F_y &= F_y(\dot{\Psi}, \beta, \delta) \approx \frac{\partial F_y}{\partial \dot{\Psi}} \dot{\Psi} + \frac{\partial F_y}{\partial \beta} \beta + \frac{\partial F_y}{\partial \delta} \delta = C_r \dot{\Psi} + C_\beta \beta + C_\delta \delta \\ M_z &= M_z(\dot{\Psi}, \beta, \delta) \approx \frac{\partial M_z}{\partial \dot{\Psi}} \dot{\Psi} + \frac{\partial M_z}{\partial \beta} \beta + \frac{\partial M_z}{\partial \delta} \delta = D_r \dot{\Psi} + D_\beta \beta + D_\delta \delta \end{aligned} \quad (3.25)$$

The physical significance of these partial derivatives is described in [Milliken and Milliken, 1995, Chapter 5], and they can be computed from the vehicle properties through:

$$\begin{aligned} C_r &= \frac{1}{v_x} (a_r C_{\alpha r} - a_f C_{\alpha f}), \quad C_\beta = -(C_{\alpha f} + C_{\alpha r}), \quad C_\delta = C_{\alpha f}, \\ D_r &= -\frac{1}{v_x} (a_f^2 C_{\alpha f} + a_r^2 C_{\alpha r}), \quad D_\beta = a_r C_{\alpha r} - a_f C_{\alpha f}, \quad D_\delta = a_f C_{\alpha f}, \end{aligned} \quad (3.26)$$

where $C_{\alpha f}$ and $C_{\alpha r}$ are the front and rear cornering stiffnesses, respectively.

Steady-state conditions

Some important characteristics of the vehicle's response in steady-state conditions can be approximated from the bicycle model. The curvature response S_ρ , sideslip response S_β , yaw rate response S_r , and lateral acceleration response S_a , are some widely used parameters and can be computed as indicated by Jazar [2008]. The expressions needed to calculate them are shown here for completeness:

$$\begin{aligned} S_\rho &= \frac{\rho}{\delta} = \frac{1}{R\delta} = \frac{C_\beta D_\delta - C_\delta D_\beta}{v_x (C_r D_\beta - C_\beta D_r - m v_x D_\beta)}, \quad S_r = \frac{\dot{\Psi}}{\delta} = v_x S_\rho, \\ S_\beta &= \frac{\beta}{\delta} = \frac{D_\delta (C_r - m v_x) - C_\delta D_r}{C_\beta D_r - D_\beta (C_r - m v_x)}, \quad S_a = \frac{v_x^2/R}{\delta} = v_x S_r. \end{aligned} \quad (3.27)$$

The understeer gradient, η , which is another key parameter, can be approximated by

$$\eta = -\frac{m g}{l} \left(\frac{a_f C_{\alpha f} - a_r C_{\alpha r}}{C_{\alpha f} C_{\alpha r}} \right) = \frac{F_{z_f}}{C_{\alpha f}} - \frac{F_{z_r}}{C_{\alpha r}}, \quad (3.28)$$

where F_{z_f} and F_{z_r} are the axle loads at rest. For an oversteered car $\eta < 0$ and the critical speed above which its motion becomes unstable is $v_{\text{crit}} = \sqrt{\frac{g l}{-\eta}}$. For an understeered car $\eta > 0$ and its characteristic speed at which S_r reaches its maximum is $v_{\text{char}} = \sqrt{\frac{g l}{\eta}}$. More details regarding the physical significance of these parameters can be found in [Gillespie, 1992; Milliken and Milliken, 1995; Jazar, 2008].

3.2.4 Quarter-car multi-body model

This model is an extension of the QCL model in which the actual suspension geometry and nonlinear damper characteristics are included. It is much simpler than the full-vehicle multi-body model that is described in Section 3.2.5, and it has proven to be a useful tool for the study of the SAVGS and the influence it can have on the vertical dynamic of the car.

Although the SAVGS concept may be adapted to work with various types of suspension systems, the vehicle classes targeted in this thesis, namely high performance sports cars and SUVs, tend to rely on double-wishbone or multi-link arrangements. These offer very good wheel attitude control throughout the suspension travel, and the SAVGS is particularly well suited for them as the spring-damper (SD) compression and rotation it produces are not necessarily translated into wheel camber, and therefore lateral force, changes.

For simplicity, double-wishbone suspensions, whose model is described next, have been considered throughout this thesis.

Suspension system (double-wishbone)

The main passive suspension characteristics and modelling assumptions considered are:

- The passive suspension system comprises two rigid wishbones, a rigid wheel spindle, a vertically compliant tyre, a suspension spring and a suspension damper.
- The wheel motion is planar and remains constrained to the local $y'z'$ plane.
- All the unsprung mass is lumped at the wheel centre, H.
- All joints are ideal revolutes (i.e. rubber bushings are not included).
- The suspension spring and damper act in parallel between points E and F, and are assembled together to form a single unit, referred to as spring-damper (SD), that can only produce forces along its axis.
- The suspension spring is linear, and the suspension damper is fully non-linear.
- The tyre is represented by a linear vertical spring and a linear vertical damper that act in parallel between the road and point I.

A schematic of the passive and retrofitted double-wishbone suspensions is shown in Fig. 3.5. In the nominal configuration, points A, B, C, D, E, G, H and I in the retrofitted suspension remain in the same position as in the passive case. The SD, however, may need to be shortened/extended with respect to the passive case depending on the desired single-link length and nominal angle.

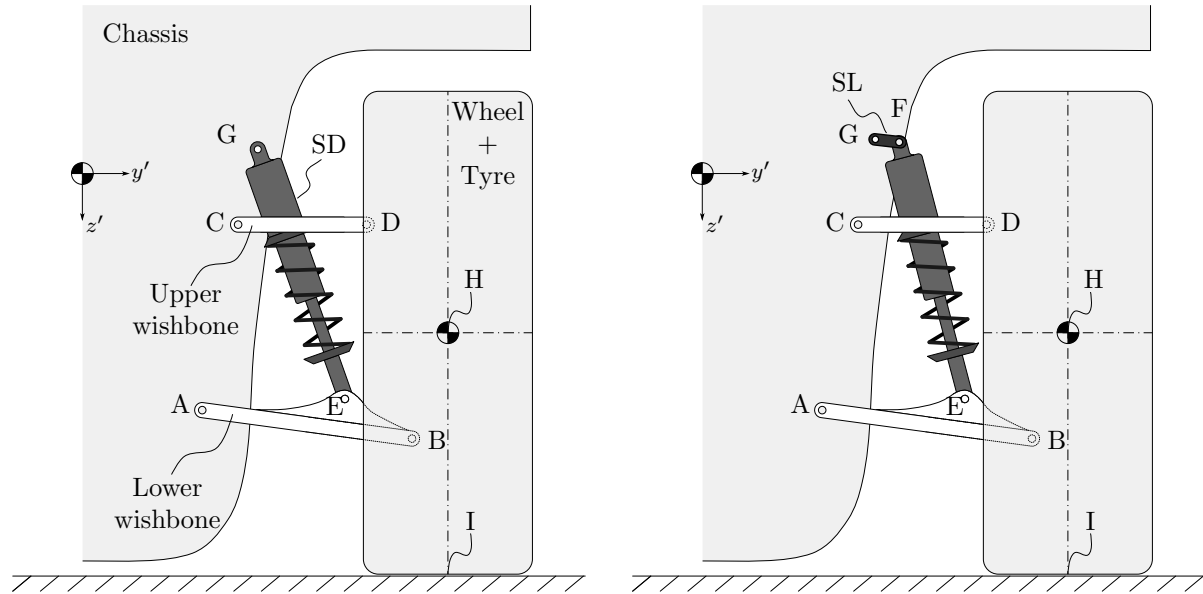


Fig. 3.5: Original double-wishbone suspension (left) and SAVGS-retrofitted suspension (right).

3.2.5 Full-vehicle multi-body model

This high fidelity model has been used to synthesise attitude control schemes and to analyse the effect of the SAVGS on the attitude motions and directional response of the vehicles.

The model includes a 6 DOF chassis, the same suspension system as the quarter-car multi-body model, and incorporates aerodynamic forces, steering system, powertrain, braking system, tyres and virtual drivers. These subsystems are described next.

Suspension system

The basic definitions are equal to those given in Section 3.2.4.

Given that the SAVGS renders the anti-roll bars unnecessary, and that the active and passive suspensions studied should be exactly the same except for the presence of the SAVGS in order to obtain meaningful comparisons, neither the passive nor the active suspensions studied include anti-roll bars.

Aerodynamics

Aerodynamic forces and moments depend on the air density, on the square of the air stream velocity and on the aerodynamic properties of the vehicle. Full-scale wind tunnel testing is the most accurate method to estimate the aerodynamic properties of the vehicle. Different wind tunnel configurations are possible and a variety of sensor arrangements are commonly used. The

interested reader is referred to [Hucho, 2013] for a description of the most common ones. In one such arrangement, each wheel of the vehicle is positioned on a separate platform that is instrumented to obtain the force components in three different directions (x , y and z in the reference frame of the vehicle). These force components can be subsequently converted to any combination of forces and moments, typically x , y and z forces acting on the centre of mass of the vehicle as well as yawing, pitching and rolling moments. In this thesis, aerodynamic forces and moments are considered to be independent of the level and attitude angles of the chassis, and their contribution to the lateral forces as well as to the rolling and yawing moments acting on the chassis are neglected.

The drag force, F_{drag} , is assumed to act longitudinally on the chassis at a point located at ground level, whereas the lift (down) forces F_{d_f} and F_{d_r} are considered to act vertically on the chassis at the midpoint of the front and rear axles, respectively. Modelled lift (down) forces account for the total lift (down) force as well as for the aerodynamic pitching moment. As there is no anti-pitch suspension geometry, the aerodynamic drag force acting on the chassis at the ground level does not produce any pitching moment on the chassis.

The classic formulae used to calculate the aerodynamic forces are:

$$F_{\text{drag}} = \frac{1}{2} \rho A C_{\text{drag}} v^2, \quad F_{\text{d}_f} = \frac{1}{2} \rho A C_{\text{d}_f} v^2, \quad F_{\text{d}_r} = \frac{1}{2} \rho A C_{\text{d}_r} v^2, \quad (3.29)$$

where ρ is the air density, A is the frontal area of the vehicle, C_{drag} is the drag coefficient, C_{d_f} and C_{d_r} are the front and rear downforce coefficients and v is the forward speed of the vehicle (assuming no relative speed between the air and the ground). Although in practice the drag and downforce coefficients depend on the air stream velocity, they are considered to be constant in this thesis as this effect is not critical (at moderate vehicle speeds) for the operation of the suspension system.

Steering

For simplicity, a vertical kingpin axis (i.e. zero caster and kingpin inclination angles), zero scrub radius and a small (positive) mechanical trail are considered in the nominal configuration. Moreover, “parallel steer” has been used as, according to Milliken and Milliken [1995] and other authors, it offers a good compromise between low and high lateral acceleration cornering performance.

Position control as defined in ISO 8855:2011 is applied to the steering wheel at all times. This means that the steering wheel inertia becomes irrelevant for the purposes of this thesis, and can therefore be safely neglected in the multi-body model.

The rotation of the steering wheel is transmitted to the steering pinion through a compliant steering column of torsional stiffness k_{sc} . A small damping torque is considered to act between

the chassis and the steering pinion, opposing their relative rotation.

The rotation of the pinion with respect to the chassis (scaled by a fixed steering ratio) is translated as a spring-like torque acting on the front wheels and making them rotate with respect to the kingpin axes. Finally, a damping torque that opposes the wheel rotation and provides some additional stability to the steering system is included.

Powertrain

Two different powertrain architectures have been considered in this thesis: 1) rear longitudinal engine with rear wheel drive (sports car), and 2) front longitudinal engine with four-wheel drive (high performance SUV).

Engine The only engine characteristics of relevance for this thesis are its maximum torque output as a function of engine speed, and its inertia. The maximum torque output has been approximated as a third order polynomial of the engine speed, and the engine inertia has been assumed to be constant.

Gearbox A multi-speed gearbox connects the engine to the propeller shaft. For simplicity, the clutch has not been modelled, and thus the gear shift process cannot be performed in a step-wise manner as that would lead to excessive (infinite) inertial torques in the powertrain. Instead, a linear transition between gear ratios is considered.

In AutoSim, the engine and propeller shaft bodies are connected through kinematic constraints (provided with commands `add-position-constraint` and `add-speed-constraint`), and the gear ratio, G_{gbx} , is selected by the virtual driver in charge of the longitudinal dynamics of the car. As the driver reacts to the longitudinal motion of the vehicle, which in turn depends on the selected gear, an algebraic loop arises. One way of braking this algebraic loop is to filter the gear ratio to be used in the kinematic constraints with a (fast) first order lag filter. This is a simple yet efficient solution that has been implemented in AutoSim through an auxiliary state variable.

Differential The difference between left and right traction forces can be related to the action of a differential with even torque split through:

$$F_{x_i} - F_{x_o} = -(\omega_i - \omega_o) \frac{c_{\text{diff}}}{R_{\text{wh}}}, \quad (3.30)$$

where c_{diff} is a torsional damping coefficient, R_{wh} is the wheel radius, ω is the wheel angular speed, F_x is the longitudinal force, and subscripts *i* and *o* refer to the inner and outer wheels, respectively.

The special cases of an open differential and a locked differential correspond to $c_{\text{diff}} = 0$ and c_{diff} arbitrarily large, respectively. Open differentials offer poor traction characteristics when one wheel is largely unloaded or in contact with a low friction surface. A locked differential, on the other hand, offers good off-road and racing performanceⁱ at the expense of tyre wear. As a consequence, various types of locking and active differentials and torque vectoring techniques are frequently used in modern high performance vehicles to improve traction and handling under a wide range of operating conditions.

Complex traction situations are not studied in this thesis and therefore a limited slip differential with even torque split and small internal losses (c_{diff} small but larger than zero) is considered to provide a realistic framework without adding excessive complexity.

Lastly, the differential contains some gearing, the so called *final drive ratio*, G_{fdr} . This is introduced in the propeller shaft to crown wheel mesh, and increases the total gear ratio of the powertrain, from engine to wheels, to:

$$G_{\text{pwt}} = G_{\text{gbx}} G_{\text{fdr}} \quad (3.31)$$

In the rear-wheel drive vehicle model, one differential is installed in the rear axle. In the four-wheel drive vehicle model, one differential is installed in each axle.

Braking system

The dynamics in the braking system are neglected and therefore braking commands from the virtual drivers (brake pedal position) are immediately translated into braking torques applied from the wheel spindles to the wheels. Brake torques applied are saturated to ensure that $M_{\text{B}} \in [0, M_{\text{Bmax}}]$.

Two types of braking manoeuvres are simulated in this thesis: 1) light to moderate braking manoeuvres in which the tyres remain in the low-slip region before the peak of longitudinal force ($|a_x| < 0.8g$), and 2) emergency braking manoeuvres in which the tyres are required to provide the maximum possible braking force.

In [Heiing and Ersoy, 2010], the ideal brake force distribution is calculated for each axle from geometric and weight distribution data. Converting the brake forces to brake moments applied to each wheel, the optimum brake torque ratio can be obtained as a function of the deceleration rate, $|a_x|$, through

$$T_{\text{Bdist}} = \frac{T_{\text{B}_f}}{T_{\text{B}_r}} = \frac{F_{\text{B}_f} R_{\text{wh}_f}}{F_{\text{B}_r} R_{\text{wh}_r}} = \left(\frac{l}{b_f - h_{\text{CM}}|a_x|/g} - 1 \right) \frac{R_{\text{wh}_f}}{R_{\text{wh}_r}}, \quad (3.32)$$

ⁱSee Perantoni and Limebeer [2014] for a sensitivity analysis studying the effect of c_{diff} on the optimum lap time achievable by a Formula 1 car.

where T_{B_f} and T_{B_r} are the front and rear braking torques, F_{B_f} and F_{B_r} are the front and rear braking forces, b_f is the distance from the centre of mass to the front axle, and R_{wh_f} and R_{wh_r} are the front and rear wheel radii.

For deceleration rates of up to $0.8g$ it is required by law [Heißing and Ersoy, 2010] that the front wheels begin to skid before the rear wheels in order to avoid unstable handling behaviour. Thus, a fixed brake torque distribution ratio, $T_{B_{dist}}$, higher than the highest optimum value in the $(0, 0.8g)$ deceleration rate range, is implemented. This is considered to be a simple and appropriate implementation for the first type of manoeuvres.

For the second and more demanding type of braking events, an ideal brake control system is used to ensure that each individual wheel operates at (or close to) the longitudinal slip value that leads to the peak longitudinal force.

Tyres

Steady state characteristics A general description of the tyre shear forces and self-aligning moment as a function of the vertical load, camber angle, longitudinal and lateral slips is quite complicated. Such static tyre models are in most cases functions of the form

$$[F_x, F_y, M_z] = f(F_z, \kappa, \alpha, \gamma, \Psi), \quad (3.33)$$

where the calculated values are the tyre shear forces, namely longitudinal force (F_x), lateral force (F_y) and aligning moment (M_z), and the input values are the tyre load F_z , slip ratio (κ), slip angle (α), camber angle (γ) and a set of parameters capable of representing the tyre behaviour for a given road and weather conditions (Ψ).

The *Magic Formulae* originally proposed by Bakker et al. [1987] and frequently updated by TU-Delft and TNO-Delft in the so called *Delft-Tyre* description [Besselink et al., 2010], is arguably the state-of-the-art for tyre characterisations of the form given in (3.33). The evaluation of all the parameters involved in such a description requires extensive tyre testing, which is expensive and, as pointed out by Sharp [2004], even extravagant in some cases. The idea that any given tyre running over the road in general circumstances can be characterised accurately by one set of measurements is not a good one. In this thesis, as in many vehicle dynamics problems, the aim is to capture the generic behaviour of a representative tyre, not to match a particular set of experimental measurements precisely.

Therefore, a less complex formulation based on similarity and normalisation ideas has been chosen. This model, which was proposed in [Sharp and Bettella, 2003a] and subsequently refined in [Sharp and Bettella, 2003b; Sharp, 2004], is able to provide the full spectrum of shear force and aligning moment characteristics for a tyre from a very modest set of experimental data. This is particularly convenient because even when experimental values are available (rarely),

they are typically limited to the lateral tyre forces and the aligning moments as functions of the slip angle for various load levels, but ignore the longitudinal forces or the tyre characteristics under combined slip conditions as most experimental rigs are not equipped with driving and/or braking systems.

The basic assumptions behind the similarity or normalisation ideas used in [Sharp, 2004] are: 1) the sideforce/longitudinal force/aligning moment are the same function of slip angle/longitudinal slip/slip angle for any load; 2) sideslip and camber angles can be combined to give an effective slip angle, which in turn can be combined with the longitudinal slip ratio to give a combined slip parameter; 3) combined slip forces are the same function of combined slip for any load; and 4) the force-slip relationships for pure longitudinal force, pure sideforce and combined forces are all the same.

The only notable difference between the equations used in this thesis and the ones given in [Sharp, 2004] is that the term $\frac{F_{ync}}{F_{y0}}$ in the expression used to calculate the aligning moment,

$$M_z = \left(\frac{F_{ync}}{F_{y0}} \right)^2 M_{z0} + \gamma (0.05 F_z - 0.15 F_x), \quad (3.34)$$

is assumed to be equal to one at all times in order to avoid a singularity that has been seen to occur under very specific conditions.

Transient response The previous model deals satisfactorily with the tyre behaviour under steady-state conditions, but ignores the dynamic properties of the tyre and therefore is not capable of predicting the tyre response during transients. There are a number of models of varying complexity that deal with this issue, but if experimental results are scarce for steady-state conditions, they are even more so for transients. Thus, a simple and very widely used approach that considers the tyre forces to transition to the steady-state values as a first order lag system has been chosen in this thesis [Pacejka, 2006].

The implementation is based on the semi-non-linear model described in [Pacejka, 2006, Chapter 7], which is a single contact point transient tyre model that is able to accurately capture the tyre's response as it was demonstrated by Higuchi and Pacejka [1997] even for large slip and camber conditions.

The model studies the lateral and longitudinal tyre deflections and their effect on the generation of shear forces and aligning moment. It considers that the sideslip angle and the longitudinal slip ratio develop with time (i.e. distance travelled by the tyre) as a first order lag system, and that the relaxation lengths (which may be different for longitudinal slip and sideslip) are the parameters that control this lag.

Defining the longitudinal and lateral tyre deformations as ϵ_x and ϵ_y , respectively, their evolution

in time can be calculated through

$$\begin{aligned}\frac{d\epsilon_x}{dt} &= |V_x| \left(\kappa - \frac{\epsilon_x}{\sigma_\kappa} \right), \\ \frac{d\epsilon_{y_\alpha}}{dt} &= |V_x| \left(\alpha - \frac{\epsilon_{y_\alpha}}{\sigma_a} \right), \\ \frac{d\epsilon_{y_\gamma}}{dt} &= |V_x| \left(\gamma \frac{C_{F\gamma}}{C_{F\alpha}} - \frac{\epsilon_{y_\gamma}}{\sigma_a} \right),\end{aligned}\tag{3.35}$$

where ϵ_{y_α} and ϵ_{y_γ} capture the contributions of the sideslip angle and the camber angle to the lateral tyre deformation, σ_κ and σ_a are the longitudinal and lateral relaxation lengths, V_x is the projection of the wheel's centre speed on the x -axis of the tyre's reference frame, and $C_{F\alpha}$ and $C_{F\gamma}$ are the cornering and camber thrust stiffnesses, respectively.

From these, the filtered slip variables, κ' , α' and γ' , are computed through

$$\begin{aligned}\kappa' &= \frac{\epsilon_x}{\sigma_\kappa}, \\ \alpha' &= \arctan \left(\frac{\epsilon_{y_\alpha}}{\sigma_a} \right), \\ \gamma' &= \frac{C_{F\alpha}}{C_{F\gamma}} \frac{\epsilon_{y_\gamma}}{\sigma_a}.\end{aligned}\tag{3.36}$$

Finally, the dynamic tyre forces are obtained by feeding the steady-state tyre model with the filtered slip variables:

$$\begin{aligned}F_x &= F_x(F_z, \kappa', \alpha', \gamma'), \\ F_y &= F_y(F_z, \kappa', \alpha', \gamma'), \\ M_z &= M_z(F_z, \kappa', \alpha', \gamma, \gamma'),\end{aligned}\tag{3.37}$$

where F_x and F_y depend on the filtered slip variables only, but M_z maintains some dependency on the unfiltered camber angle as suggested by Higuchi and Pacejka [1997].

The relaxation lengths depend on the vertical tyre load and on the slips: they increase with tyre load and decrease with slip. Although there are ways of incorporating these variations into the modelⁱⁱ, the use of constant relaxation lengths is common practice. Generally speaking, the lateral relaxation length is of the order of magnitude of the tyre radius, whereas the longitudinal relaxation length is equal to half the contact patch length.

As described by Clover and Bernard [1998], the longitudinal dynamics tyre model may lead to high amplitude oscillations at low speeds if relatively large longitudinal relaxation length values are used. Although it is reported that this phenomenon has been detected in experimental tests,

ⁱⁱSee for example Rill [2011], where an analytical method to estimate the relaxation lengths as functions of tyre loads and slip angles based purely on the tyre's steady-state characteristics and the tyre's longitudinal and lateral stiffnesses and damping coefficients is proposed.

the model seems to overestimate it. For that reason, Pacejka [2006] proposes the addition of an artificial damping coefficient that modifies the filtered longitudinal slip at low speeds, and the addition of an upper limit to the allowable tyre deformation, ϵ_x . However, the extent of these corrections and the speeds at which to apply them in general, are not clear.

Based on experimental values reported in the literature (see [Zegelaar and Pacejka, 1996]), it seems reasonable to expect the longitudinal relaxation length to be between 5 and 10 times smaller than the rolling radius. For very stiff tyres like the ones equipped in high performance sports cars and SUVs (considered in this thesis), the contact patch and therefore the longitudinal relaxation length is likely to be even smaller.

Given 1) the scarcity of experimental values, particularly for high performance tyres, 2) the potential problems associated with the addition of the longitudinal tyre dynamics, and 3) the small longitudinal relaxation lengths that are expected for the types of tyres considered in this thesis, it does not seem worthy to implement the longitudinal tyre dynamics at this stage. Therefore, in this thesis the longitudinal tyre forces are generated according to the classical, steady-state tyre characteristics, and κ is used instead of κ' in all cases.

Virtual drivers

The aim of the simulations carried out in this thesis is to perform an objective comparison between the response of a vehicle equipped with a conventional passive suspension, and that of the same vehicle retrofitted with the SAVGS.

The driver, the environment and the vehicle constitute a closed-loop system. Each of these elements is complex, and their interaction very difficult to studyⁱⁱⁱ. Moreover, the more detail and richness in the driver, environment and interaction models, the more difficult it becomes to assess the net effect of the new active suspension on the vehicle characteristics. For that reason, many of the manoeuvres studied in this thesis are open-loop manoeuvres performed on perfectly defined environments. That is, the driver inputs (gas and brake pedal positions, steering wheel angle and gear selection) are predefined and applied in exactly the same way to the vehicles equipped with the passive and active suspensions.

In some cases, though, it is not practical to consider open-loop control. When trying to study the SAVGS performance under steady-state longitudinal and/or lateral acceleration levels, the same driver inputs should not be applied to the vehicles equipped with the passive and the active suspensions: as the vehicle's response depends on the longitudinal and lateral load transfers (i.e. tyre loads), and these in turn depend on the suspension characteristics, equal driver actions lead

ⁱⁱⁱThe interested reader is referred to two comprehensive review papers: one by Macadam [2003] on the human driver and associated topics, and another one by Plöchl and Edelmann [2007] on the types of driver models used in automobile dynamic simulations as a function of the main focus of the study at hand: focus on the vehicle, on the driver, on the combined system, or on the environment and traffic.

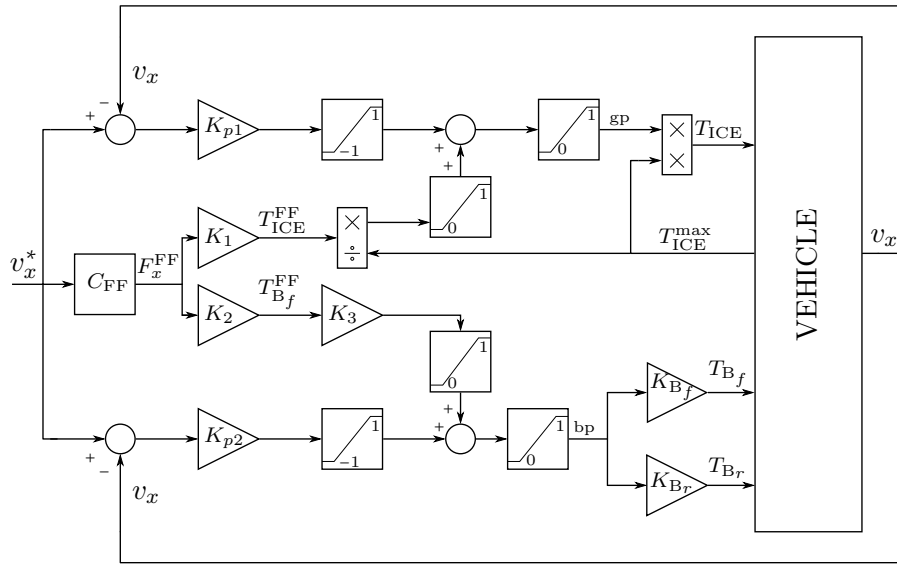


Fig. 3.6: Forward speed control for closed-loop operation.

to different longitudinal and/or lateral acceleration levels for the passive and active suspension vehicles, rendering the intended comparison impossible. For these cases, independent closed-loop control for the longitudinal (gas pedal position, brake pedal position and gear selection) and lateral (steering wheel angle) is implemented instead.

Closed-loop longitudinal speed control Driver inputs affecting the longitudinal speed of the vehicle are: 1) gas pedal position, 2) brake pedal position, and 3) gear selection. The gas and brake pedal positions are defined in the $[0,1]$ range, and they act as scaling factors applied to the maximum available engine and brake torques, respectively. The gear selection determines the gearbox' transmission ratio.

When operating under open-loop control, these inputs are read by AutoSim from external tables. When operating under closed-loop control, they are determined by a virtual driver whose aim is to track a certain forward speed reference, v_x^* . This closed-loop driver does not aim to be a realistic representation of a human driver, but rather to behave as an almost perfect robotic driver that provides fast and accurate forward speed tracking capabilities.

As shown in Fig. 3.6, the engine and brake control comprises a feedforward term that determines the required longitudinal force based on a simplified model of the vehicle, and feedback terms that correct the output of the feedforward block to ensure accurate velocity tracking despite the influence of external disturbances and/or changes in the vehicle's response. The feedforward contribution to the gas and brake pedal positions must be in the $[0,1]$ range, whereas the feedback contributions are allowed to be in the $[-1,1]$ range. Feedforward and feedback terms are added together and subsequently saturated so that the gas and brake pedal positions remain in the $[0,1]$ range.

The required longitudinal force, F_x^{FF} , predicted by the feedforward control C_{FF} , is calculated based on 1) the expected aerodynamic drag force, 2) the expected inertial forces due to the rate of change of the reference speed, and 3) the estimated losses in the powertrain. That is:

$$F_x^{\text{FF}} = F_{x_{\text{drag}}}^{\text{FF}} + F_{x_{\text{acc}}}^{\text{FF}} + F_{x_{\text{pwt}}}^{\text{FF}}, \quad (3.38)$$

with the individual contributions to the predicted force being:

$$\begin{aligned} F_{x_{\text{drag}}}^{\text{FF}} &= \left(\frac{1}{2} \rho A C_{\text{drag}} \right) v_x^{*2}, \\ F_{x_{\text{acc}}}^{\text{FF}} &= \left(m + \frac{I_{\text{ax}_f}}{R_{\text{wh}_f}^2} + \frac{I_{\text{ax}_r}}{R_{\text{wh}_r}^2} \right) \frac{dv_x^*}{dt}, \\ F_{x_{\text{pwt}}}^{\text{FF}} &= \left\{ \left[\frac{\sum (c_i G_i)}{R_{\text{wh}}} \right]_f + \left[\frac{\sum (c_i G_i)}{R_{\text{wh}}} \right]_r \right\} v_x^*. \end{aligned} \quad (3.39)$$

In (3.39), m refers to the total mass of the vehicle, I_{ax} represents the total inertia of the elements (wheels, differential, propeller shaft, engine) attached to an axle (as seen at the axle), R_{wh} is the wheel radius, c_i represents the viscous damping coefficient associated with each rotating element linked to an axle, and G_i is the total gear ratio from that element to the linked axle.

If R_{wh_d} is the radius of the driving wheels, then the expected engine torque requirement calculated by the feedforward control can be obtained as:

$$T_{\text{ICE}}^{\text{FF}} = \left(\frac{R_{\text{wh}_d}}{G_{\text{pwt}}} \right) F_x^{\text{FF}} = K_1 F_x^{\text{FF}}. \quad (3.40)$$

Considering that the same braking torque is applied to the two front wheels, and that a scaled braking torque (determined by the braking torque distribution factor introduced previously in the description of the braking system) is applied to the two rear wheels, the expected braking torque per wheel in the front axle is:

$$T_{\text{B}_f}^{\text{FF}} = \left[-\frac{R_{\text{wh}_f} R_{\text{wh}_r} T_{\text{Bdist}}}{2(R_{\text{wh}_f} + T_{\text{Bdist}} R_{\text{wh}_r})} \right] F_x^{\text{FF}} = K_2 F_x^{\text{FF}}. \quad (3.41)$$

This torque command is scaled down by $K_3 = \frac{1}{T_{\text{B}_f}^{\text{max}}}$ in order to add it meaningfully, i.e. with comparable magnitude, to the brake pedal command produced by the feedback gain K_{p2} .

Finally, the engine torque, T_{ICE} , is obtained by multiplying the gas pedal position, gp , by the maximum available engine torque, $T_{\text{ICE}}^{\text{max}}$ (which depends on the engine speed as mentioned previously in the engine description), and the front and rear braking torques, T_{B_f} and T_{B_r} , by multiplying the brake pedal position, bp , by $K_{\text{B}_f} = K_3^{-1} = T_{\text{B}_f}^{\text{max}}$ and $K_{\text{B}_r} = (T_{\text{Bdist}} K_3)^{-1}$, respectively.

It is worth mentioning that some feedback terms that may reduce the gas pedal command (not shown in Fig. 3.6 for readability) are put in place in order to avoid 1) exceeding the maximum engine speed, or 2) loosing traction due to excessive wheel slip. Similarly, the brake pedal command is also corrected to prevent the vehicle from coming to a standstill or reversing.

Closed-loop brake control for emergency braking manoeuvres In order to get some additional information about the behaviour of the SAVGS under extreme deceleration rates, emergency braking manoeuvres are simulated. The brake control previously described is not practical because it leads to wheel lock and it does not allow the vehicle to reach deceleration rates above $\sim 0.9g$. In order to achieve higher deceleration rates, some sort of anti-lock scheme must be used.

In practice, an ABS system maintains the tyre slip close to the maximum of the longitudinal force vs. slip curve by modulating the brake cylinder pressure. This leads to rapid brake pressure variations, which couple with the longitudinal tyre characteristics and leads to a complex and oscillatory response. This behaviour cannot be captured with conventional static tyre models (valid up to ~ 10 Hz) nor with simple transient models based on the relaxation length concept (valid up to ~ 30 Hz). It requires the implementation of more advanced tyre models such as the rigid ring model, as argued by Jansen et al. [1999]. The level of detail required for the ABS model and its control, as well as for the longitudinal tyre properties, although worth considering for the final stages of development and validation of the SAVGS, is deemed to be excessive for this thesis.

A simplified approach with an ideal anti-lock system is considered more appropriate instead. The anti-lock control is presumed to have access to the longitudinal slip of each tyre, α , as well as to the longitudinal slip that would generate the peak longitudinal force for the actual operating conditions, α_p . Using a PI control scheme, the brake torque applied to each wheel can be effectively modulated to minimise the difference $\alpha_p - \alpha$ and therefore to produce (almost) optimal braking forces.

The tyre and anti-lock models proposed here are a simplification of the complex dynamics that exist in reality. Yet they offer a practical alternative (at this stage) to simulate and evaluate the SAVGS potential for chassis attitude control under high deceleration rates.

Closed-loop control of the steering-wheel In this thesis position control (open-loop) of the steering wheel is appropriate in most cases. There is, however, one standard manoeuvre of interest for which closed-loop control is convenient. This is ISO 4138:2004, method 1, a manoeuvre in which the vehicle needs to remain in a constant radius path despite changes in forward speed. A control for this manoeuvre is implemented based on the yaw rate control proposed by Ackermann [1990].

During steady-state cornering in a circular path of radius R , and assuming that the slip angle of the vehicle is small, the tangential speed, v_t , is very similar to the forward speed, v_x . The yaw rate is therefore related to the forward speed and the path radius through:

$$\dot{\Psi} = \frac{v_t}{R} \approx \frac{v_x}{R}. \quad (3.42)$$

According to (3.42), the desired path radius, R^* , can be converted into a yaw rate reference, $\dot{\Psi}^*$. The virtual driver that tracks this reference determines the angular speed at which to rotate the steering wheel, $\dot{\delta}_{sw}$, from the yaw rate error and its rate of change:

$$\dot{\delta}_{sw} = K_p (\dot{\Psi}^* - \dot{\Psi}) + K_d \frac{d(\dot{\Psi}^* - \dot{\Psi})}{dt}. \quad (3.43)$$

Summary

The resulting multi-body tree structure, as defined in AutoSim, is shown in Fig. 3.7. Commands from the virtual drivers are shown with diamond-head arrows and tyre and braking forces as thick black arrows. Dotted lines represent kinematic constraints between bodies, and labels x , y and z are used to indicate the relative rotation that is allowed between a body and its parent body.

3.2.6 Parameter conversion from multi-body to linear suspension models

Significant differences exist between the linear and multi-body representations of the quarter-car:

- In the multi-body representation of a quarter of the car (QC), the chassis (sprung mass) is allowed to move vertically, whilst the wishbones and wheel carrier form a one-degree-of-freedom closed kinematic loop attached to the chassis. The wheel can thus travel along the z' - and y' -axis with respect to the sprung mass, and also rotate about the x' -axis. During the wheel's motion with respect to the chassis, the spring-damper is not only compressed/extended, but also rotated. As a consequence, the *effectiveness* of its action varies. Finally, the forces arising from the tyre-road interaction, which have relevant components in the z' and y' directions, act upon a point that also moves vertically and laterally with respect to the chassis (point I).
- In the linear quarter-car model (QCL), on the other hand, both the sprung and unsprung masses are constrained to move vertically without rotation, and the equivalent spring-damper opposes their relative motion by introducing a vertical force between them. Lateral tyre forces are neglected, and the vertical tyre force is considered to act along the same line as the SD force.

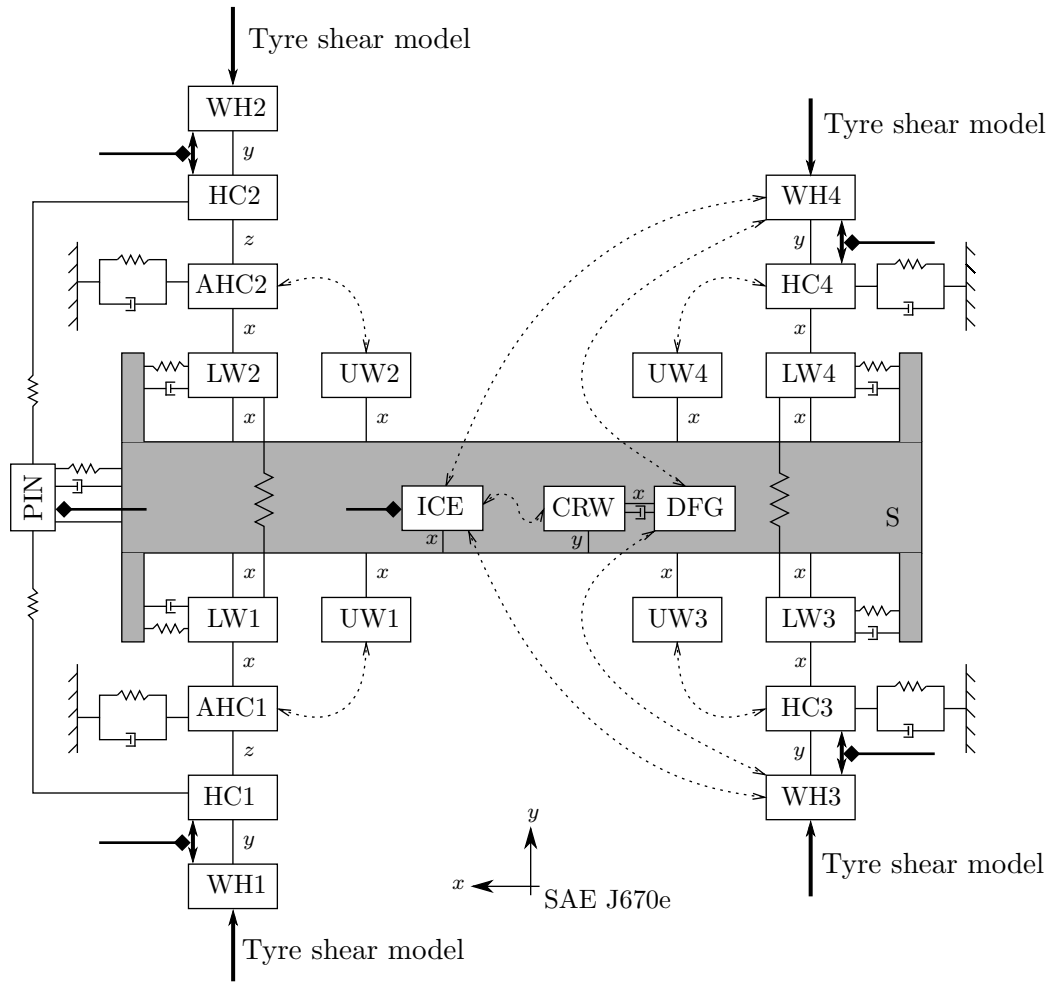


Fig. 3.7: Tree structure of the vehicle model. The chassis, S, has six degrees of freedom with respect to the inertial reference frame. All other bodies are defined with rotational degrees of freedom (DOF) only. LW refers to lower wishbones, UW to upper wishbones, AHC and HC to hub carriers, WH to wheels, PIN to steering pinion, ICE to internal combustion engine, CRW to the crown wheel in the differential and DFG to the gears in the differential.

Due to the simplifications made in the QCL model, it is clear that the equivalent SD should not be assigned the same properties (i.e. stiffness and damping coefficient) as the real component.

One option to calculate the equivalent parameters to be used in the QCL would be to solve an optimisation problem in which the distance (to be defined) between the behaviour of the QC and QCL is minimised in some way. This approach is followed for example in [Kim and Ro, 2000], where the multi-body representation of the quarter-car is initially linearised with ADAMS and then simplified through order reduction techniques (classical dominant mode and balance realisation) to a four-state model. The parameters for the two-mass quarter-car model are computed by matching the coefficients in the transfer functions from actuator force to suspension deflection for the reduced-order four-state and two-mass models.

Although this approach seems to provide a good match based on some simple time-domain simulations, it does not provide much information in terms of the influence that the suspension geometry has on the equivalent values to be used. A comparison between the frequency responses of the models is not performed, and results for inputs coming from the road and/or from load transfer are not considered at all. Thus, the optimisation for the MIMO case may not be as straight forward as it seems from the analysis performed by Kim and Ro [2000]. Moreover, the equivalent parameters obtained with this method are far from intuitive: the equivalent sprung and unsprung masses as well as the equivalent tyre stiffness are far from their real values (+60%, +200%, +80%), whereas the equivalent suspension stiffness and damping coefficient remain practically unchanged in comparison.

An alternatively method to compute the suspension properties to be used in the QCL model based on energy considerations is proposed here. The idea is that each equivalent component should have similar energy flows from/to it than its counterpart in the multi-body model.

Sprung mass The sprung masses in the QC, m_s , and QCL models, m_s^* , move solely on the vertical direction. Considering that both experience the same motion, that is, that $z_s^*(t) = z_s(t)$, then the kinetic and potential energy of the two bodies are equal if, and only if, their masses are equal. Thus, $m_s^* = m_s$.

Spring-damper The energy stored in the suspension spring and the energy dissipated in the suspension damper depend exclusively on the relative motion between the sprung and unsprung masses. Thus, for the derivation of its equivalent properties, it is possible and convenient to consider the sprung mass to be fixed, and the unsprung mass to move with the wishbones.

The rate of change of the energy stored in the real and equivalent suspension springs, $\frac{dE_k}{dt}$ and $\frac{dE_k^*}{dt}$, must be equal. Considering that the vertical motion of the equivalent unsprung mass is equal to the vertical motion of point H (see 3.5), i.e. $z_u = z_H$, then:

$$\begin{aligned} \frac{dE_k}{dt} &= \frac{d}{dt} \left(\frac{1}{2} k (l_{SD} - l_{SD0})^2 \right) = k (l_{SD} - l_{SD0}) \frac{dl_{SD}}{dt} = -F_k \frac{dl_{SD}}{dz_u} \frac{dz_u}{dt} = -F_k R_{SD} \frac{dz_u}{dt}, \\ \frac{dE_k^*}{dt} &= \frac{d}{dt} \left(\frac{1}{2} k^* (l_{SD}^* - l_{SD0}^*)^2 \right) = k^* (l_{SD}^* - l_{SD0}^*) \frac{dl_{SD}^*}{dt} = -F_k^* \frac{dz_u}{dt}, \end{aligned} \quad (3.44)$$

where k and k^* are the real and equivalent spring stiffnesses, l_{SD} , l_{SD}^* , l_{SD0} and l_{SD0}^* are the actual and unloaded spring lengths (real and equivalent), F_k and F_k^* are the real and equivalent spring forces, and the installation ratio for the spring-damper unit, R_{SD} , is:

$$R_{SD} = \frac{dl_{SD}}{dz_H}. \quad (3.45)$$

Thus, for $\frac{dE_k}{dt} = \frac{dE_k^*}{dt}$ to hold, the forces exerted by the real and equivalent springs must be related

through:

$$F_k^* = F_k R_{SD}. \quad (3.46)$$

Furthermore, the stiffness of the equivalent suspension spring must satisfy:

$$k^* = -\frac{dF_k^*}{dl_{SD}^*} = -\frac{dF_k}{dl_{SD}^*} R_{SD} - F_k \frac{dR_{SD}}{dl_{SD}^*} = -\frac{dF_k}{dl_{SD}} R_{SD}^2 - F_k \frac{dR_{SD}}{dz_u} = k R_{SD}^2 - F_{SD} \frac{dR_{SD}}{dz_H} \quad (3.47)$$

Similarly, the rate of energy dissipation in the real and equivalent linear dampers must be equal:

$$P_{dis} = c \left(\frac{dl_{SD}}{dt} \right)^2 = P_{dis}^* = c^* \left(\frac{dl_{SD}^*}{dt} \right)^2. \quad (3.48)$$

As $\frac{dl_{SD}}{dt} = \frac{dl_{SD}}{dz_H} \frac{dz_H}{dt} = R_{SD} \frac{dz_H}{dt}$, and $\frac{dl_k^*}{dt} = \frac{dz_u}{dt} = \frac{dz_H}{dt}$, then the equivalent damping coefficient to be used in the QCL model is:

$$c^* = c R_{SD}^2. \quad (3.49)$$

Unsprung mass For convenience, let z and y refer to the displacement of point H in the local $x'y'z'$ reference frame. The kinetic and potential energy of the unsprung mass, T and V , can then be calculated through:

$$\begin{aligned} T &= \frac{1}{2} m_u (\dot{z}^2 + \dot{y}^2) + \frac{1}{2} I_{xx} \omega_x^2, & T^* &= \frac{1}{2} m_u^* \dot{z}^2 \\ V &= -m_u g z, & V^* &= -m_u^* g z \end{aligned} \quad (3.50)$$

Expressing the inertia of the wheel assembly as $I = m_u i^2$, with i its radius of gyration about the x' -axis, the rate of change of energy for the real and equivalent unsprung masses are:

$$\begin{aligned} \frac{dE_u}{dt} &= \frac{d(T+V)}{dt} = m_u (\dot{z} \ddot{z} + \dot{y} \ddot{y} - g \dot{z} + i^2 \omega_x \dot{\omega}_x) \\ \frac{dE_u^*}{dt} &= \frac{d(T^*+V^*)}{dt} = m_u^* (\dot{z} \ddot{z} - g \dot{z}) \end{aligned} \quad (3.51)$$

The lateral and rotational velocities and accelerations of the wheel can be approximated from the vertical velocity and acceleration of the wheel through:

$$\begin{aligned} \dot{y} &= \beta_1 \dot{z}, & \ddot{y} &= \beta_1' \dot{z}^2 + \beta_1 \ddot{z}, \\ \omega_x &= \beta_2 \dot{z}, & \dot{\omega}_x &= \beta_2' \dot{z}^2 + \beta_2 \ddot{z}, \end{aligned} \quad (3.52)$$

where suitable expressions for $\beta_1 = f(z)$, $\beta_2 = g(z)$, $\beta_1' = \frac{d\beta_1}{dz}$ and $\beta_2' = \frac{d\beta_2}{dz}$ can be obtained from a kinematic analysis of the suspension geometry performed with AutoSim.

The ratio between the real and the equivalent unsprung masses, $\gamma = m_u^*/m_u$, may then be calculated as:

$$\gamma = 1 + \frac{[\beta_1^2 + (\beta_2 i)^2] \ddot{z} + [\beta_1 \beta_1' + \beta_2 \beta_2' i^2] \dot{z}^2}{\ddot{z} - g} \quad (3.53)$$

For the suspension geometries considered in this thesis, γ remains very close to unity except for downward acceleration values approaching gravity. To justify that these deviations from unity are not particularly relevant, the relative error between the RMS of $\frac{dE_u}{dt}$ and $\frac{dE_u^*}{dt}$ when γ is set to 1 is shown in Fig. 3.8 for a typical suspension geometry and harmonic wheel-to-chassis motions of varying frequencies and vertical amplitudes. The error remains within acceptable levels, and therefore $\gamma = 1$ and $m_u^* = m_u$ are sensible assumptions for the simplified QCL and FCL models.

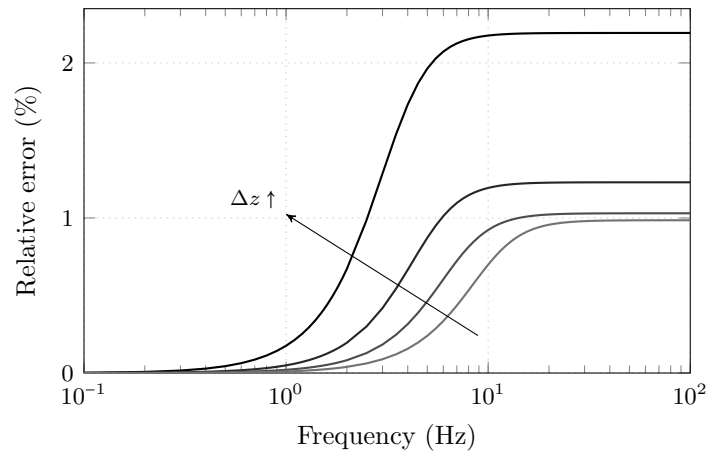


Fig. 3.8: Relative error in the rate of change of energy stored in the real and equivalent unsprung masses when $\gamma = 1$. Results correspond to a typical suspension subject to harmonic wheel-to-chassis motions of varying frequencies and displacement amplitudes $\Delta z = 0.5, 1, 2, 4$ cm.

3.2.7 Road models

A flat, smooth road has been considered for attitude control studies. For comfort enhancement analyses, three types of road irregularities have been implemented in the vehicle models:

- Discrete events, such as speed bumps, have been used as demanding tests for the SAVGS.
- Harmonic road profiles have been used to understand the SAVGS capabilities as a function of frequency.
- Random road profiles according to [ISO 8608:1995] and representative of typical road qualities have been used to assess the typical comfort improvements that could be achieved with the SAVGS.

These models are described in more detail in Appendix A.

3.3 SAVGS model

Actuator models were not commonly included in active suspension design and control studies until the mid-90's [Alleyne and Hedrick, 1995]. Nowadays, it is still common practice to neglect or simplify actuator dynamics and limitations in control-focused publications. Moreover, actuator limitations, when included, are often modelled as fixed-value limits on the actuation force, independent of the actual operating conditions (e.g. [Sun et al., 2013]).

In this thesis the focus is on assessing the capabilities of the SAVGS, on quantifying its power and energy requirements, and on identifying potential control issues. Therefore it is essential to model the main actuator dynamics, power losses and limitations, and to integrate these with the full-vehicle multi-body model and high-level SAVGS control schemes.

In each corner of the car, the actuator in charge of controlling the SL position comprises an electric motor and a gearbox. The models used to represent these components is described in Section 3.3.1 and Section 3.3.2, and their integration within the multi-body models is discussed in Section 3.3.3. Finally, a summary of power flows and losses is provided in Section 3.3.4.

3.3.1 Electric motor model

A permanent magnet synchronous motor (PMSM) has been the actuator of choice due to its high power density, low inertia, excellent performance in servo applications and high IP ratings^{iv} [Hughes and Drury, 2013].

Although PMSMs would benefit from the high DC voltages available in electric and hybrid vehicles, it is also possible to utilise them in vehicles with conventional powertrain architectures by using appropriate power electronics. The electrical arrangement considered in this thesis is shown in Fig. 3.9, where the typical range for battery voltage is $V_{\text{low}}=24\text{--}48\text{ V}$, and a convenient range for the high DC voltage is $V_{\text{high}}=160\text{--}600\text{ V}$.

The equations that capture the PMSM dynamics are generally expressed in the so called dq frame instead of in the reference frame of the three-phase abc wye-connected stator. The transformation from one frame to the other is not unique but, unfortunately, many authors forget to mention the details of the specific transformation that they used. Here, the transformation applied following [Krause et al., 2002] is explained first and then the dynamic equations compatible with it are presented.

^{iv}The International Electrotechnical Committee (IEC) standards for motor enclosures take the form of a two-digit classification number prefixed by the letters IP. The first digit corresponds to the level of protection against ingress of solid particles and the second against ingress of water. Typical PMSMs offer IP65 or IP66 ratings, which correspond to complete protection against dust and protection against powerful water jets.

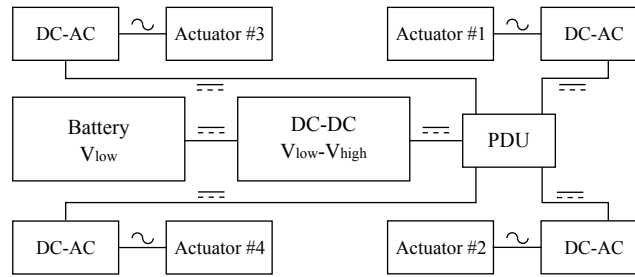


Fig. 3.9: Electrical arrangement for vehicles with conventional powertrain architectures. All electrical power flows are bidirectional. Starting from (to) the battery, a DC-DC converter steps-up (steps-down) the voltage to (from) that of the high DC voltage bus. A power distribution unit (PDU) interconnects the high DC voltage bus with the four DC-AC converters that drive the servo motors.

Frame transformation

First, a general frame transformation is described. This is followed by some comments on its application to the PMSM case and on its physical interpretation.

Generic frame transformation Denoting an electric variable in the PMSM windings (voltage or current) as f , the transformation from the real phase axes, abc , to a stationary reference frame $\alpha\beta 0$ with the α -axis aligned with the a -axis is defined as:

$$\mathbf{f}^{\alpha\beta 0} = \begin{bmatrix} f_\alpha(t) \\ f_\beta(t) \\ f_0(t) \end{bmatrix} = \mathbf{K}_{abc}^{\alpha\beta 0} \mathbf{f}^{abc} = k_{tr} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} f_a(t) \\ f_b(t) \\ f_c(t) \end{bmatrix}, \quad (3.54)$$

where the transformation constant, k_{tr} , may be freely chosen, and superindices refer to the reference frame in which a given vector is expressed. The zero axis is perpendicular to the $\alpha\beta$ plane and the zero component can be ignored for balanced three-phase systems (as in the case covered here).

Selection of the transformation constant Although k_{tr} may be arbitrarily chosen, there are two specific values that are generally used for convenience, as they lead to a transformation that is either amplitude invariant or power invariant. Magnitude invariant means that

$$|f_a| = |f_b| = |f_c| = \sqrt{f_\alpha(t)^2 + f_\beta(t)^2} \quad (3.55)$$

and power invariant means that

$$P = v_a(t) i_a(t) + v_b(t) i_b(t) + v_c(t) i_c(t) = v_\alpha(t) i_\alpha(t) + v_\beta(t) i_\beta(t). \quad (3.56)$$

If the voltages and current constitute a balanced set of 3-phase voltages and currents of amplitudes M_v , and M_i , then

$$\mathbf{v}^{abc} = \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} = M_v \begin{bmatrix} \sin(\omega t + \varphi) \\ \sin(\omega t + \varphi - \frac{2\pi}{3}) \\ \sin(\omega t + \varphi + \frac{2\pi}{3}) \end{bmatrix}, \quad \mathbf{i}^{abc} = \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} = M_i \begin{bmatrix} \sin(\omega t + \theta) \\ \sin(\omega t + \theta - \frac{2\pi}{3}) \\ \sin(\omega t + \theta + \frac{2\pi}{3}) \end{bmatrix}, \quad (3.57)$$

and the voltage and current vectors in the stationary $\alpha\beta$ reference frame are:

$$\mathbf{v}^{\alpha\beta} = \begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = k_{tr} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} M_v \begin{bmatrix} \sin(\omega t + \varphi) \\ \sin(\omega t + \varphi - \frac{2\pi}{3}) \\ \sin(\omega t + \varphi + \frac{2\pi}{3}) \end{bmatrix} = \frac{3}{2} k_{tr} M_v \begin{bmatrix} \sin(\omega t + \varphi) \\ \sin(\omega t + \varphi - \frac{\pi}{2}) \end{bmatrix} \quad (3.58)$$

$$\mathbf{i}^{\alpha\beta} = \begin{bmatrix} i_\alpha(t) \\ i_\beta(t) \end{bmatrix} = k_{tr} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} M_i \begin{bmatrix} \sin(\omega t + \theta) \\ \sin(\omega t + \theta - \frac{2\pi}{3}) \\ \sin(\omega t + \theta + \frac{2\pi}{3}) \end{bmatrix} = \frac{3}{2} k_{tr} M_i \begin{bmatrix} \sin(\omega t + \theta) \\ \sin(\omega t + \theta - \frac{\pi}{2}) \end{bmatrix} \quad (3.59)$$

The lengths of these vectors are:

$$\|\mathbf{v}^{\alpha\beta}\| = \sqrt{v_\alpha(t)^2 + v_\beta(t)^2} = \frac{3}{2} k_{tr} M_v, \quad \|\mathbf{i}^{\alpha\beta}\| = \sqrt{i_\alpha(t)^2 + i_\beta(t)^2} = \frac{3}{2} k_{tr} M_i, \quad (3.60)$$

whereas the electrical power is,

$$\begin{aligned} P &= v_a(t) i_a(t) + v_b(t) i_b(t) + v_c(t) i_c(t) = \frac{3}{2} M_i M_v \cos(\varphi - \theta) \\ &= v_\alpha(t) i_\alpha(t) + v_\beta(t) i_\beta(t) = \left(\frac{3}{2}\right)^2 k_{tr}^2 M_i M_v \cos(\varphi - \theta). \end{aligned} \quad (3.61)$$

From the previous analysis it is clear that k_{tr} must be equal to $\frac{2}{3}$ for the transformation to be amplitude invariant, and equal to $\sqrt{\frac{2}{3}}$ for it to be power invariant.

In this thesis, the amplitude invariant transformation is used because 1) it is a more wide spread nomenclature and 2) it simplifies the calculation of $\alpha\beta$ currents from real life measurements when the direct axis current is controlled to be zero^v [Wallmark, 2004].

Application to the PMSM and physical interpretation So far, currents and voltages have been transformed into vectors that rotate at an angular frequency equal to the electrical frequency around the origin of a generic, static, 2D reference frame denoted $\alpha\beta$.

There is an alternative and more convenient interpretation of these results in the context of the control of permanent magnet synchronous machines: instead of considering the voltage (current) vector as rotating around the zero axis (perpendicular to the plane) of a stationary and arbitrary

^vThis is the control strategy used in this thesis, described in Chapter 4.

reference frame $\alpha\beta$, it can be considered as a static vector in a rotating reference frame, dq , that rotates at the electrical frequency about the zero axis of the $\alpha\beta 0$ frame. Aligning this rotating reference frame with the magnetic axes of the rotor magnets (Park's transformation) has two significant advantages:

- the flux linkage and output torque can be independently controlled through the d (or direct) and q (or quadrature) components of the current, respectively;
- mutual inductances become constant.

Conventional PMSM model Considering a surface mounted PMSM (i.e. with no saliency and therefore equal dq inductances, $L_d = L_q$), the motor dynamics in the rotor-fixed reference frame assuming:

- no magnetic saturation,
- sinusoidal back electromotive force (back EMF), and
- negligible eddy current and hysteresis losses,

are given by [Pillay and Krishnan, 1988]:

$$\begin{aligned}\frac{di_d}{dt} &= \frac{1}{L_s} (v_d - R_s i_d + \omega_e \lambda_q), \\ \frac{di_q}{dt} &= \frac{1}{L_s} (v_q - R_s i_q - \omega_e \lambda_d),\end{aligned}\tag{3.62}$$

where $R_s = \frac{R_{\text{line-line}}}{2}$ and $L_s = L_d = L_q = \frac{L_{\text{line-line}}}{2}$ are the dq phase resistances and inductances, $\omega_e = n_{pp} \omega_r$ is the electrical frequency, n_{pp} is the number of pole pairs, ω_r is the angular speed of the rotor, $\lambda_d = L_s i_d + K_e$ and $\lambda_q = L_s i_q$ are the stator flux linkages, and K_e is the back EMF constant.

Core losses

Core losses due to hysteresis and eddy currents are significant ($\sim 20\%$) at speeds close to and above the rated speed [Monajemy, 2000], and can be included through an iron loss resistance, R_i , in the equivalent dq circuits [Morimoto et al., 1994]. As shown in Fig. 3.10, this implies that the dq currents are split into magnetizing components, i_{d_m} and i_{q_m} , and undesirable iron components, i_{d_i} and i_{q_i} .

The dynamic equations, generalised from those provided by Urasaki et al. [2003], become:

$$\begin{aligned}\frac{di_{d_m}}{dt} &= \frac{1}{\Gamma L_s} (v_d - R_s i_{d_m} + \Gamma \omega_e \lambda_{q_m}), \\ \frac{di_{q_m}}{dt} &= \frac{1}{\Gamma L_s} (v_q - R_s i_{q_m} - \Gamma \omega_e \lambda_{d_m}),\end{aligned}\tag{3.63}$$

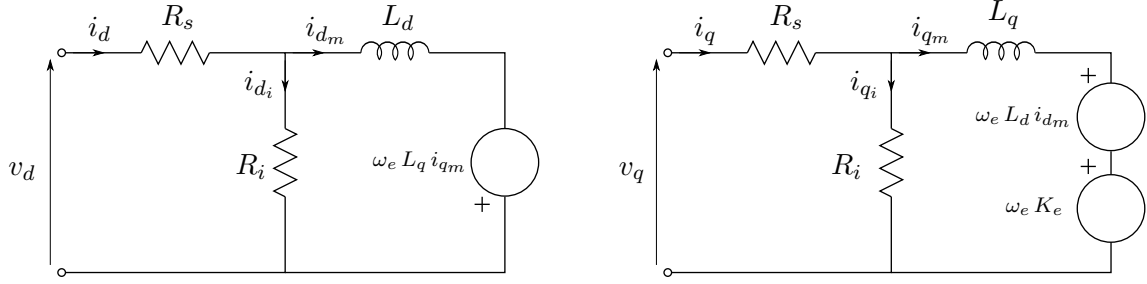


Fig. 3.10: Equivalent d -axis (left) and q -axis (right) circuits.

where $\Gamma = 1 + \frac{R_s}{R_i} \approx 1$, $\lambda_{d_m} = L_s i_{d_m} + K_e$ and $\lambda_{q_m} = L_s i_{q_m}$ are the stator flux linkages.

In this formulation, dq currents are related to their magnetizing and iron components through:

$$\begin{aligned} i_d &= i_{d_m} + i_{d_i} = i_{d_m} + \frac{1}{R_i} \left(\frac{d\lambda_{d_m}}{dt} - \omega_e \lambda_{q_m} \right), \\ i_q &= i_{q_m} + i_{q_i} = i_{q_m} + \frac{1}{R_i} \left(\omega_e \lambda_{d_m} + \frac{d\lambda_{q_m}}{dt} \right). \end{aligned} \quad (3.64)$$

Finally, the electromagnetic torque, T_{em} , and the output torque applied to the high-speed shaft (HSS) of the gearbox, T_{HSS} , are calculated according to:

$$\begin{aligned} T_{em} &= \frac{3}{2} n_{pp} K_t i_{q_m}, \\ T_{HSS} &= T_{em} + T_m = T_{em} - \frac{\omega_r}{|\omega_r|} (T_f + c_w |\omega_r|), \end{aligned} \quad (3.65)$$

where T_f is the frictional torque, c_w the viscous damping coefficient (windage), T_m the torque due to mechanical losses in the PMSM, $K_t = \lambda_{af} - K_s$ the torque constant, and K_s a degrading coefficient to account for the stray load loss^{vi} [Urasaki et al., 2003].

The introduction of this degrading stray load loss factor also affects the definition of the back EMF constant, K_e , which becomes

$$K_e = \begin{cases} \lambda_{af}, & \text{in motoring mode} \\ \lambda_{af} - K_s, & \text{in generating mode} \end{cases} \quad (3.66)$$

in order to ensure that the power balances are satisfied in both modes of operation. For numerical reasons, the term $\frac{\omega_r}{|\omega_r|}$ in (3.65) is substituted with $\frac{\omega_r}{|\omega_r|} \min\left(1, \frac{|\omega_r|}{\omega_{th}}\right)$, where ω_{th} is a small angular velocity.

^{vi}The stray load loss coefficient, K_s , is not a conventional efficiency coefficient as it leads to a reduction in HSS torque irrespective of the mode of operation [Urasaki et al., 2003].

Limiting steady-state torque-speed envelope

There are physical constraints that limit the region of torque-speed points in which a PMSM can be operated:

- The bearings supporting the rotor should not be operated above a certain speed.
- There is a maximum torque that the rotor can withstand due to fatigue and ultimate strength considerations.
- There are maximum power, current and voltage values at which the PMSM and its drive can be operated. These arise, among others, from electrical insulation and thermal dissipation characteristics.

Simple measures for the mechanical limits are generally provided by PMSM manufacturers. Power, current and voltage limits are also provided, but the translation from these values to limiting torque-speed combinations depends on the control strategy with which the DC-AC converter drives the PMSM. Details regarding this calculation are provided in Chapter 4.

3.3.2 Gearbox model

Electric motors in the desired range of power values (approximately 0.5 to 2.5 kW), offer a torque-speed compromise that is too skewed towards high speed applications. That means that, for the same weight/volume/price, a properly selected combination of PMSM and gearbox is able to drive the SL much more effectively than a PMSM alone.

There are several gearing technologies from which to choose, each of them providing distinct advantages. For example, magnetic gearboxes provide contactless torque transfer, are inherently protected against torque overloads, and can be designed to have a controllable transmission ratio. However, and despite recent advances in their application to high bandwidth motion control [Montague et al., 2012], magnetic gearboxes are still unable to compete with mechanical gearboxes in demanding servo applications and are not commercially mature [Tlali et al., 2014].

Among the mechanical alternatives, namely parallel shaft, bevel gears, epicyclic and harmonic gearboxes, epicyclic gearboxes have been preferred: they provide coaxial shafting and are more compact, rigid and quiet than parallel shaft and bevel gear arrangements, and they are more efficient and reliable than harmonic gearheads. Moreover, epicyclic gearboxes (planetary, solar or star, but typically planetary) are available in a wide range of sizes and qualities and are the de facto industry standard for demanding servo applications.

High precision epicyclic gearboxes, like the ones selected for this project, have great torsional stiffness and low backlash. A simplified gearbox model that neglects these aspects has been used thus far. In the model considered, the high speed shaft (HSS) of the gearbox is rigidly

coupled to the rotor of the PMSM, and the low speed shaft (LSS) is rigidly connected to the single-link. The angular speed of the HSS and the LSS are assumed to be related through a constant transmission ratio, G . That is:

$$\omega_{\text{HSS}} = G \omega_{\text{LSS}}. \quad (3.67)$$

Power losses within the gearbox occur in the rolling bearings, gears (churning and meshing losses) and seals, and they do not increase significantly with increasing torques [Fernandes et al., 2015]. Most manufacturers, however, provide a single efficiency value, η_{gbx} , typically corresponding to the rated output torque. For simplicity, this constant efficiency has been considered so far in the gearbox model, and the resulting relationship between the LSS and HSS torques is:

$$T_{\text{LSS}} = \eta_{\text{gbx}}^M G T_{\text{HSS}}, \quad (3.68)$$

where the mode of operation, M , is set to 1 (-1) when the PMSM is operating as motor (generator). AutoSim does not support variable exponents, and therefore a simplification of (3.68) is used instead in this software:

$$T_{\text{LSS}} = [1 - M (1 - \eta_{\text{gbx}})] G T_{\text{HSS}}. \quad (3.69)$$

The relative error between (3.68) and (3.69) remains below 1% for typical efficiencies $\eta_{\text{gbx}} \geq 0.9$.

For numerical reasons M is made to vary linearly from +1 to -1 for power values below a certain threshold, P_{th} , as given by:

$$M = \frac{\omega_{\text{HSS}} T_{\text{HSS}}}{|\omega_{\text{HSS}} T_{\text{HSS}}|} \cdot \min\left(1, \frac{|\omega_{\text{HSS}} T_{\text{HSS}}|}{P_{\text{th}}}\right). \quad (3.70)$$

3.3.3 Single-link model

No bodies have been included in the AutoSim and SimMechanics models to represent the motor or gearbox. Instead, the necessary algebraic and differential equations have been manually specified, and the inertia of the PMSM rotor and gearbox have been lumped into a single body representing the single-link. Thus, the equivalent SAVGS inertia assigned to the SL body, $I_{xx}^{(\text{SAVGS})}$, is:

$$I_{xx}^{(\text{SAVGS})} = I_{xx}^{(\text{SL})} + G^2 \left(I_{xx}^{(\text{PMSM})} + I_{xx}^{(\text{gbx})} \right), \quad (3.71)$$

where $I_{xx}^{(\text{SL})}$ is the actual inertia of the SL, $I_{xx}^{(\text{PMSM})}$ is the inertia of the PMSM's rotor, and $I_{xx}^{(\text{gbx})}$ is the gearbox' inertia referred to the HSS.

3.3.4 Power flows within the SAVGS

The power flows and losses included in the SAVGS model are depicted in Fig. 3.11 and summarised in Table 3.1. There are small voltage drops in the bridge converters due to switching and conducting losses (modelled through the constant efficiencies $\eta_{\text{DC-DC}}$ and $\eta_{\text{DC-AC}}$), variable copper, core, stray and mechanical losses in the PMSM, and mechanical losses in the gearbox (fixed efficiency).

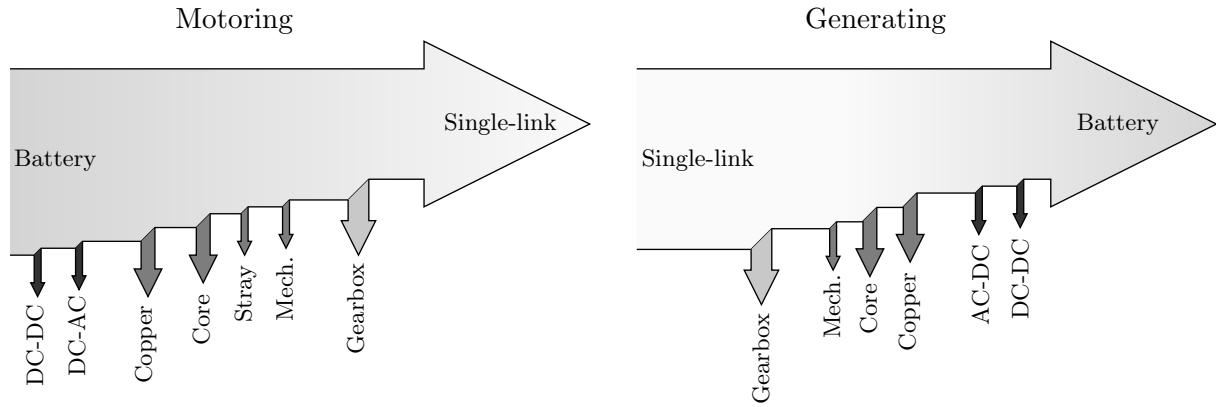


Fig. 3.11: Power flows from the battery to the single-link whilst in driving mode (left) and from the single-link to the battery whilst in regenerating mode (right). Losses in the bi-directional bridge converters, in the PMSM, and in the gearbox are included in the SAVGS model.

The starting point for the calculation of the power flows within the actuator is the PMSM. Applying an amplitude invariant transformation from the frame of the three-phase abc wye-connected stator of the PMSM to the $dq\theta$ frame fixed to the rotor, which has been explained in Section 3.3.1, the power from the DC-AC converter to the PMSM is given by:

$$P_{\text{PMSM}} = \frac{3}{2} (v_d i_d + v_q i_q). \quad (3.72)$$

Power flows from the DC-DC converter to the DC-AC converter, $P_{\text{DC-AC}}$, and from the battery to the DC-DC converter, $P_{\text{DC-DC}}$, can be subsequently computed through:

$$P_{\text{DC-AC}} = \eta_{\text{DC-AC}}^{-M} P_{\text{PMSM}}, \quad (3.73a)$$

$$P_{\text{DC-DC}} = \eta_{\text{DC-DC}}^{-M} P_{\text{DC-AC}}, \quad (3.73b)$$

where the implementation in AutoSim needs to be modified in a similar way than (3.68).

Finally, the mechanical power in the HSS and LSS are:

$$P_{\text{HSS}} = T_{\text{HSS}} \omega_{\text{HSS}}, \quad (3.74a)$$

$$P_{\text{LSS}} = \eta_{\text{gbx}}^M P_{\text{HSS}}. \quad (3.74b)$$

Table 3.1: Power losses captured by the actuator model

Component	Source	Expression
DC-DC converter	Switching & conducting losses	$ P_{\text{DC-DC}} - P_{\text{DC-AC}} $
DC-AC converter	Switching & conducting losses	$ P_{\text{DC-AC}} - P_{\text{PMSM}} $
PMSM	Copper (resistive)	$1.5 R_s (i_d^2 + i_q^2)$
	Core (eddy current & hysteresis)	$1.5 R_i (i_{di}^2 + i_{qi}^2)$
	Stray	$0.5 [1 + \text{sign}(i_{qm} \omega_e)] 1.5 K_s i_{qm} \omega_e$
	Mechanical (friction & windage)	$-T_m \omega_r$
Gearbox	Roller bearings, gears, seals	$ T_{\text{LSS}} \omega_{\text{LSS}} - T_{\text{HSS}} \omega_{\text{HSS}} $

3.4 Model validation

Although it is desirable to validate simulation results experimentally, substantial resources are needed to ensure that a test track, a vehicle equipped with a prototyped version of the SAVGS, a driver, and all necessary sensors are available for testing. Such an investment could follow in later stages of development of the SAVGS but has not been accessible during this project.

Even if working exclusively with quarter-car multi-body models, their validation using the quarter-car test rig that has been built as part of this project (see Chapter 7) requires extensive testing and has been left as future work. As a consequence, model validation has been performed on a theoretical basis thus far.

As indicated by Sharp and Limebeer [2001], the first step in the theoretical validation process is to ensure that the models built are the same as the ones conceived by the analyst. Positions of all points, orientations of all bodies and initial forces and moments have been checked and confirmed to be as intended with the help of purpose-built auxiliary MATLAB scripts. Moreover, models with various levels of accuracy have been implemented in different software packages to debug the more complex and error-prone models and to substantiate simulation results.

A summary of comparisons performed is shown in Fig. 3.12, where comparisons (a) to (h) correspond to:

- (a) Quarter-car linear (QCL) and multi-body (QC) models have been used to obtain and compare the linear and nonlinear frequency responses from road velocity to sprung mass acceleration, suspension deflection and tyre deflection. This has helped to confirm that the selection of equivalent parameters for the QCL model is correct, and that the system's natural frequencies, damping ratios etc. are as expected.
- (b) Results produced with the quarter-car linear (QCL) and full-car linear (FCL) models have been compared to confirm that there are no mistakes in the FCL matrices.

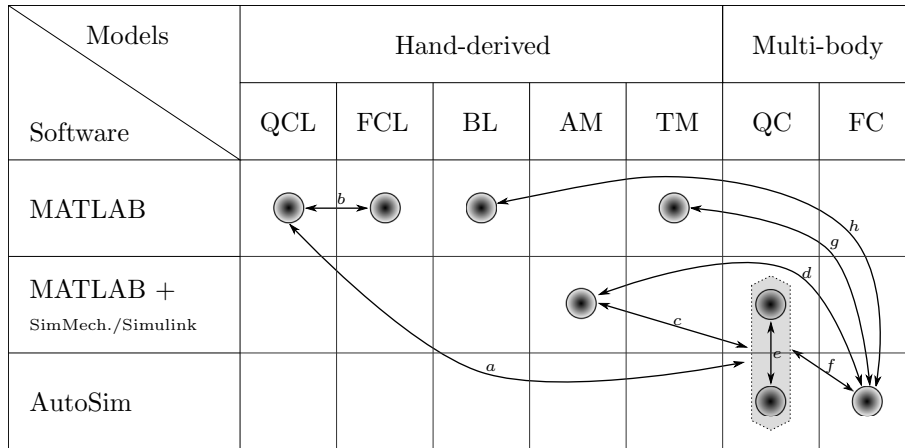


Fig. 3.12: Model and software comparisons, a to h, performed to substantiate simulation results and to guide the development of multi-body models. QCL stands for quarter-car linear, FCL for full-car linear, BL for bicycle linear, AM for actuator model, TM for tyre model, QC for quarter-car and FC for full-car.

- (c), (d) The actuator and a simple controller have been connected to an inertial load and modelled in Simulink and AutoSim to confirm that the implementation of electrical equations in AutoSim (the same that would be used in the quarter-car and full-car multi-body models) is correct.
- (e) Quarter-car multi-body (QC) models have been modelled in AutoSim and SimMechanics and it has been confirmed that, when sufficiently small time steps are used, simulation results produced with them are virtually identical.
- (f) Quarter-car (QC) and full-car (FC) multi-body models have been used to validate initial equilibrium conditions.
- (g) The tyre model (TM) has been implemented in MATLAB and fed with longitudinal slip, lateral slip, camber angle etc. obtained from FC simulations. The predicted shear forces have been confirmed to be equal to those read from the FC model.
- (h) The linear bicycle model (BL) has been used to qualitatively validate the step-steer response of the FC at low lateral acceleration levels.

The second step in the validation process aims to prove that the multi-body model built is simulated with sufficient accuracy. A power balance is proposed by Limebeer et al. [2001] and a similar approach is followed here: power and energy balances are performed at the end of each time step in the simulation.

The general expressions for the power error, P_{err} , at any given time, t^* , and for the energy error, E_{err} , from the beginning of the simulation at $t = 0$ up until t^* , are:

$$P_{\text{err}} = P_{\text{in}} - P_{\text{out}} - \frac{dE_{\text{sys}}}{dt} \quad (3.75a)$$

$$E_{\text{err}} = \int_0^{t^*} P_{\text{in}} dt - \int_0^{t^*} P_{\text{out}} dt - (E_{\text{sys}}(t^*) - E_{\text{sys}}(0)) \quad (3.75b)$$

These equations capture the power and energy:

- provided by the internal combustion engine,
- provided by the driver through the steering wheel,
- provided by the battery,
- dissipated due to aerodynamic forces,
- dissipated due to tire slip forces,
- dissipated due to damping forces in the suspension struts, viscous differential, tyres (due to radial damping), and steering column,
- dissipated in the wheel brakes,
- dissipated in the SAVGS actuators: in the bridge converters, PMSMs and gearboxes,
- stored in the system in the form of:
 - kinetic energy of all bodies,
 - gravitational potential energy of all bodies,
 - elastic potential energy stored in the suspension springs, tyres (radial), steering column, and steering rack, and
 - energy stored in the magnetic fields of the PMSMs (inductors).

Thanks to the quality of the models produced by SimMechanics and AutoSim, power and energy imbalances can be made negligibly small at all times by selecting appropriate integrators and sufficiently small time steps (of the order of $10\ \mu\text{s}$).

3.5 Parameter selection

3.5.1 Vehicle subsystems

As discussed in Chapter 2, active suspensions are expensive systems available (as an option) only in premium vehicles. In this thesis, two vehicles representative of the Grand Tourer (GT) and full size Sports Utility Vehicle (SUV) classes have been considered in order to explore the SAVGS capabilities. Representative models within these classes are Ferrari F430 and Mercedes GL AMG 63 (see Fig. 3.13).

The purpose of the models used is not to mimic the exact behaviour of any particular car, but to capture, at least approximately, the working conditions that could be encountered by the SAVGS if it was to be used in real life. To find such a set of suitable parameters is not a straightforward task, as vehicle parameters reported in the literature or online are generally scarce, incomplete, and even unreliable. This section summarises the main parameter values used for the GT and SUV, and briefly outlines the process followed to select them.



Fig. 3.13: Ferrari F430 (left) and Mercedes GL AMG 63 (right) are representative of the high performance sports car and full size SUV vehicle classes.

Weights and dimensions

Masses, inertias and lengths are fundamental properties that determine the dynamic response of a vehicle. A summary of the values considered is provided in Table 3.2.

For the GT, wheelbase, l , front and rear tracks, t_f and t_r , and weight distribution, w_f and w_r , were obtained from the official specifications of Ferrari F430. Total mass, m , has been considered to be the official kerb mass for Ferrari F430, plus an additional 75 kg to account for the driver's mass. The height of the centre of mass, h_{CM} , as well as the unsprung masses, m_{u_f} and m_{u_r} , were estimated from those reported for a Lamborghini Murcielago in [Cruden, 2015], and chassis' inertias, I_{xx} , I_{yy} and I_{zz} , were downscaled from those provided by Heydinger et al. [1999] for a Ford Mustang GT.

For the SUV, most values were obtained through private communications with an OEM, and correspond to the fully laden configuration. Inertia values, however, were downscaled from those reported in [Heydinger et al., 1999] for Volvo XC90.

Table 3.2: Values for chassis parameters

Parameter	Units	GT	SUV
l	m	2.6	3.076
t_f	m	1.669	1.677
t_r	m	1.615	1.696
h_{CM}	m	0.424	0.710
w_f	%	43	50
w_r	%	57	50
m	kg	1525	2950
m_{u_f}	kg	47.5	62.5
m_{u_r}	kg	52.5	62.5
I_{xx}	kg m ²	300	625
I_{yy}	kg m ²	1500	3500
I_{zz}	kg m ²	1500	3500

Suspension

The suspension system shapes, to a great extent, the driving experience that a car offers. Manufacturers' know-how in this area, developed over decades, is a key asset that is kept confidential. As a consequence, detailed information regarding suspension geometry and properties is not publicly available, and many assumptions are needed to feed the model with a sensible set of parameters.

In this thesis, the main suspension properties for the SUV were obtained from an OEM through private communication, and geometric and secondary parameters were selected to be compatible with these.

In the GT case, some values were derived from a second-hand Ferrari F430 rear suspension assembly that was purchased for the construction of the experimental rig. In particular, wishbone dimensions and coordinates of the wheel spindle joints^{vii} were estimated from measurements performed on the suspension assembly, and spring stiffnesses and damping force versus speed curves (see Fig. 3.14) were provided by [Aurok Ltd, 2015] for Öhlins TTX36IL (front) and Öhlins TTX40 (rear).

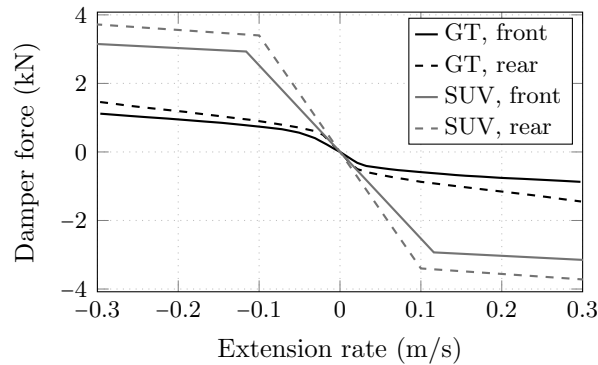


Fig. 3.14: Nonlinear damper force versus damper extension rate.

Other properties were either estimated based on comments found in the literature, or approximated from pictures available online. According to Milliken and Milliken [1995], 1.5 deg/g and 3.0 deg/g are typical roll angle versus lateral acceleration gradients for racing cars and very high performance sports cars, respectively. A modern European GT, like Ferrari F430, is probably very close to 1.5 deg/g when fitted with anti-roll bars. Based on this, roll centre heights and installation ratios for the GT were adjusted to produce a total chassis roll angle of approximately 2.75 deg/g (without anti-roll bars). Wishbone angles were chosen to get roll centre heights close to 180 mm above the ground (as in the SUV case), and spring-damper angles were selected to get installation ratios of 0.60 at the front and 0.56 at the rear. Tyres' radial stiffnesses

^{vii}Based on drawings of Ferrari F430's predecessor, Ferrari 360, wishbone dimensions and spindle connections were assumed to be equal for the front and rear axles of the car.

for the GT were taken from [Cruden, 2015] for a Lamborghini Murcielago. The value considered, 275 N/mm, is slightly lower than the 290 N/mm given by an OEM for the SUV. Tyres' radial damping coefficients have been assumed to be 300 Ns/m, as used in [Thommyppillai, 2010].

Table. 3.3 shows the natural frequencies obtained with these parameters plugged into the linear full-vehicle model for vertical dynamics described in Section 3.2.2. As expected, natural frequencies for the GT are slightly higher than for the SUV, and overall the values obtained seem to be in line with comments found in the literature. For example, Milliken and Milliken [1995] report that typical heave natural frequencies for passenger cars are in the 1-1.5 Hz range and that values above 2 Hz are considered to be harsh.

Table 3.3: Natural frequencies

Mode	Units	GT	SUV
Heave	Hz	1.7	1.3
Pitch	Hz	2.0	1.7
Roll	Hz	2.9	2.3
Wheel hop (front)	Hz	12.1	11.1
Wheel hop (rear)	Hz	12.5	11.3

Another important parameter affecting lateral dynamics is the camber angle. An OEM reported zero nominal camber at the front and rear axles of the SUV, whilst Heißing and Ersoy [2010] states that typical values are between -2 and 2 deg, and that sports cars are closer to the lower limit, generally with more negative values at the back. With this in mind, camber angles of -0.7 deg at the front and -1 deg at the rear axle have been selected for the GT.

Aerodynamics

Although aerodynamic characteristics are not essential for the studies carried out in this thesis, they may be relevant in future analyses and therefore some comments are provided here to justify the selection of values (see Table 3.4 for a summary).

Air density, ρ , has been assumed to be the standard for sea level at 15 °C. Cross sectional areas, A , for the GT and SUV have been estimated from drawings/pictures of Ferrari F430 and Mercedes GL AMG 63, respectively. The aerodynamic drag coefficient, C_{drag} , can be found online for Mercedes GL. For the GT, a similar value to the one reported online for McLaren F1 has been selected. Aerodynamic downforces for the front and rear axles of Ferrari F430 can be found online for a forward speed of 300 km/h, and front and rear downforce coefficients have been found by plugging these values into (3.29). Downforces for the SUV have been assumed to be half of those found for the GT.

Table 3.4: Values for aerodynamic parameters

Parameter	Units	GT	SUV
ρ	kg/m ³	1.225	1.225
A	m ²	1.859	3.404
C_{drag}	-	0.330	0.380
C_{d_f}	-	0.170	0.046
C_{d_r}	-	0.196	0.053

Steering

The steering system is relevant in terms of handling, but not so much in terms of load transfer characteristics. Few parameters were available for the vehicle classes of interest, and therefore most values had to be estimated based on comments found in the literature.

According to Milliken and Milliken [1995], steering ratios (from steering wheel rotation to road wheel rotation) for racing cars range from 20 (slow) to less than 10 (extremely fast). A value of 12 has been selected for the GT, whereas a value of 13 was obtained for the SUV through private communication with an OEM.

Wheel's xx and zz inertias have been scaled linearly with mass and quadratically with radius from the values provided in [Hegazy et al., 1999]. The mechanical trail (caster trail), according to Heißing and Ersoy [2010], is typically 0 when there is no power steering, and between 10 and 40 mm when there is power steering. A value of 19.2 mm, as considered in [Thommyppillai, 2010], has been used for both the GT and SUV.

Values reported in the literature for the steering column and steering rack equivalent stiffnesses, as well as for the equivalent viscous damping coefficient in the steering system, differ greatly. Values have been roughly estimated to match steering-wheel torques provided in [Ciarla et al., 2012; Fankem and Mller, 2014; Shin et al., 2014] for step-steer manoeuvres, and as a compromise between values reported in [Hegazy et al., 1999; Sharp and Granger, 2003; Ciarla et al., 2012; Fankem and Mller, 2014; Shin et al., 2014].

Powertrain

Longitudinal load transfers (front to rear) are limited by the forward acceleration performance of the vehicle, and the powertrain contributes to this.

Maximum torque and power values versus engine speed have been approximated from the official specifications for Ferrari F430 and Mercedes GL AMG 63 and are shown in Fig. 3.15. These parameters, together with other less relevant ones such as the viscous damping coefficient used to model losses in the propeller shaft to differential connection, have been tuned so that the

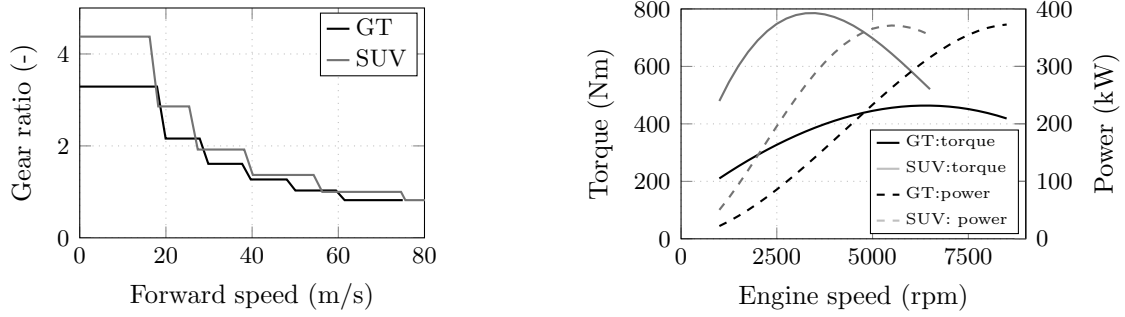


Fig. 3.15: Gearbox' transmission ratios as a function of forward vehicle speed (left) and engine's maximum torque and power as a function of engine speed (right).

official acceleration values^{viii} are approximately matched by simulation results.

Gearbox' transmission ratios have been taken from the official specifications for Ferrari F430 and Mercedes GL AMG 63, and stored as piece-wise linear functions (see Fig. 3.15) that depend on the forward speed of the vehicle. Final drive ratios, G_{FDR} , are provided by official data for Ferrari F430 and Mercedes GL AMG 63.

The powertrain's inertia, which is dominated by the engine's inertia, is not critical for the work presented here and has therefore been roughly estimated. For the GT, the inertia per kW has been assumed to be 20% of that of the engine described in [Crolla, 2009], and the SUV's inertia has been taken to be two times larger than that of the GT. Wheel's radii and yy inertias have been estimated for the following tyre codes^{ix}:

- GT, front: 225/35 R19
- GT, rear: 285/35 R19
- SUV, front and rear: 295/40 R21

Braking system

Longitudinal load transfers (rear to front) are limited by the braking performance of the car.

The ideal front-to-rear braking ratios for longitudinal deceleration rates of 1 to 8 m/s², calculated according to (3.32), range from 0.74 to 1.174 for the GT, and from 1 to 2.208 for the SUV. According to Heißing and Ersoy [2010], the front wheels must saturate before the rear ones in this range of longitudinal decelerations. Thus, T_{Bdist} has been selected as 1.25 > 1.174 and 2.5 > 2.208, respectively.

Maximum braking torques have been chosen to be significantly larger than the maximum ex-

^{viii}Time to accelerate from 0 to 100 km/h and times to cover 400 m and 1000 m from a standing start.

^{ix}The first 3-digit number is the width of the tyre in mm. The second 2-digit number is the aspect ratio of sidewall height to tyre width, as a percentage. R stands for radial construction, and is followed by the internal diameter in inches.

pected requirements for deceleration rates of 1.2 g, to ensure that the maximum deceleration rate is determined by the tyre's properties.

Tyres

Most forces acting on a vehicle originate from complex tyre-road interactions, and therefore the tyre's characteristics play a key role in determining the vehicle's motion and associated load transfers.

Most tyre models require large sets of parameters (e.g. over 20 in the model used in this thesis). Unfortunately, tyre properties are rarely shared publicly, and when they are, they appear generally as graphs and/or simplified values that convey only a limited amount of information.

In this thesis, high performance tyres are considered. The base set of parameters has been selected to correspond to the Corvette tyre described in [Sharp and Bettella, 2003b]. Some simple values, such as cornering stiffnesses for two vertical load levels, were obtained for the SUV through private communication with an OEM. Based on these values, as well as on step-steer responses and on comments found in the literature, the base set of parameters has been updated for both vehicles.

Some modifications performed on the GT values have been:

- Peak friction coefficient when vertical tyre load tends to zero is chosen to be close to 1.2 for the longitudinal and lateral directions. These are made to decrease linearly with load in both directions, but faster in the lateral direction.
- Cornering stiffness for the front tyres is derived from two pairs of data of the form $[F_z, C_\alpha]$ retrieved from experimental data plotted in [Milliken and Milliken, 1995] for a stock car tyre. For the rear tyres, the cornering stiffness at the two vertical load values considered is assumed to be 15% larger. This reduces the reaction time to a step-steer significantly.
- From [Pacejka, 2006], it seems reasonable to expect the longitudinal stiffness to be around 1.5 times the cornering stiffness. Moreover, according to [Zegelaar, 1998], the longitudinal slip stiffness increases more than proportionally with tyre load, whereas the lateral slip stiffness increases less than proportionally with tyre load. A ratio between longitudinal and cornering stiffness has been chosen to be approximately 1 at 4 kN load, and approximately 2 at 12 kN load.
- Self-aligning torque sensitivity to slip angle is kept at roughly 1/50 of the cornering stiffness. This seems sensible as it implies a pneumatic trail of about 2 cm that increases with load. According to [Pacejka, 2006], the pneumatic trail is about 1/4 of the contact patch length, which in turn is reported to be of the order of 7 to 14 cm in [Zegelaar and Pacejka, 1996]. With the proposed values, the pneumatic trail starts at around 2 cm, and increases slowly with load (this should be compatible with the very stiff tyres considered here).

- Some plots in [Pacejka, 2006] suggest peak longitudinal forces are reached at longitudinal slips of 0.07. The longitudinal slip for peak longitudinal force is assumed to increase slightly with load, ranging from about 0.07 to 0.08. This is similar to the behaviour of the Corvette tyre, but the values are lower.
- The lateral slip for peak lateral force is taken from the values provided for the SUV by an OEM (through private communication).
- The ratio between lateral force sensitivity to camber angle (camber thrust) and to sideslip is tuned to be similar to that of the Corvette tyre.
- Lateral relaxation lengths are assumed constant and equal to the wheel radius. This is compatible with [Pacejka, 2006], although the relaxation length would increase with increasing tyre load, and decrease with increasing sideslip.

3.5.2 SAVGS components

Selection and sizing of SAVGS components for a real application should be performed according to a wide range of functional requirements and design constraints. Amongst many other requirements, the SAVGS may be asked to:

- compensate body roll motions so that a certain steady-state roll angle versus lateral acceleration characteristic is achieved
- compensate body pitch motions so that a certain steady-state pitch angle versus longitudinal acceleration characteristic is achieved
- compensate transient body roll motions so that roll angle overshoot in step-steer manoeuvres remains below a certain threshold
- provide a certain reduction in the transmissibility from road disturbances to body accelerations
- respect packaging, weight and power limitations
- be able to operate for a certain number of kilometres in a wide range of conditions, including large temperature gradients and sustained exposures to water, dirt, shock and vibration

The exact requirements for the SAVGS may differ greatly between manufacturers and vehicle classes, and it is not the aim of this thesis to derive a realistic and complete set of specifications for each vehicle considered. Instead, the focus is on studying the SAVGS and on identifying its capabilities, strengths and weaknesses, and therefore a simple SAVGS design approach has been followed:

- The same actuators had to be installed in the front and rear axles of each vehicle.
- Total SAVGS mass had to remain below 35 kg and 90 kg for the GT and SUV, respectively (2.3% and 3.1% of vehicle weight).

- The system had to provide significant roll and pitch mitigation under steady-state conditions, peak pitch angles during hard braking manoeuvres had to remain below those obtained with the passive suspension, and roll overshoot during step-steer manoeuvres had to remain below 20%.

The simple design process followed to select the SAVGS components is depicted in Fig. 3.16. SL lengths were selected first based on rough^x steady-state pitch and roll mitigation requirements. Then, compatible actuators that satisfied transient pitch and roll mitigation specifications and remained within the weight limits were chosen from commercial catalogues.

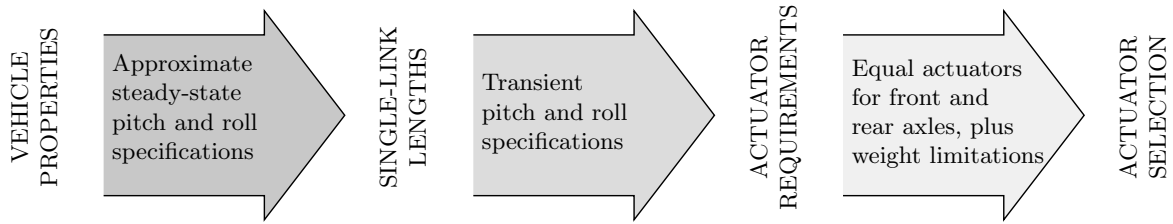


Fig. 3.16: Simple design process used to dimension and select SAVGS components.

Actuator selection

GT actuator A Kollmorgen AKM33H PMSM, to be used in the experimental test rig, was available at the beginning of this project. The UltraTRUE size 75 series of gearboxes from Danaher Motion was found to be compatible with the existing PMSM and to offer very high torque densities. These are high-quality helical planetary gearboxes with one or two stages and transmission ratios per stage ranging from 4 to 10. The model with transmission ratio of 40, UT075-040, was chosen based on the resulting torque-speed output characteristics for the combined PMSM plus gearbox actuator: it provides sufficiently high output speeds and the largest continuous and intermittent output torques (see Fig. 3.17). These high torques enable the use of longer single-links and lead to better overall SAVGS performance.

Although AKM33H and UT075-040 are independently designed components, they work well together. As it can be seen in Fig. 3.18, the limiting envelope originating from the gearbox (fatigue strength → continuous torque limit; ultimate strength → intermittent torque limit; bearings and lubrication → maximum angular speed) is reasonably similar to the one originating from the PMSM (thermal limits → continuous torque limit; current limits and ultimate strength → intermittent torque limit; voltage limit and bearing characteristics → maximum angular speed). This ensures that both components operate close to their limits, and that none of them is unnecessarily oversized.

^xA more systematic SL sizing procedure, based on steady-state performance requirements, is proposed in Section 5.1.2.

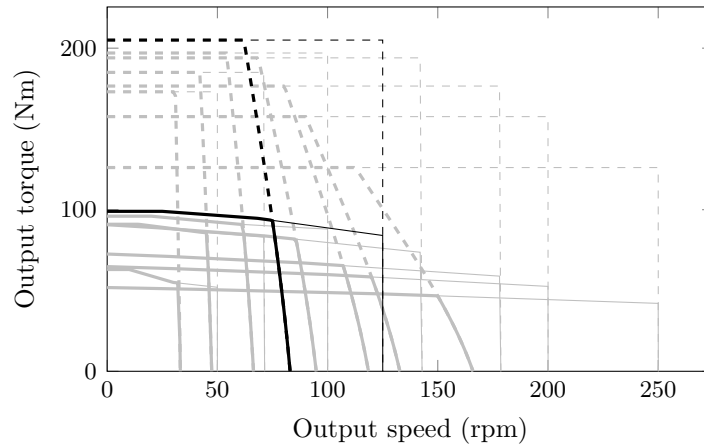


Fig. 3.17: Output steady-state torque-speed envelopes for AKM33H combined with eight different Danaher UT075 gearboxes with transmission ratios ranging from 20 to 100. Limits for continuous operation (solid) and intermittent operation (dashed) are shown for $V_{\text{high}}=160$ V (thick) and $V_{\text{high}}=320$ V (thin). The envelopes corresponding to AKM33H combined with the selected model, namely UT075-040, are shown in black.

SUV actuator An all-in-one Wittenstein actuator with properties linearly interpolated with mass from those of existing off-the-shelf models has been considered. In this case, it is clear that both the motor and gearbox have been designed to work together — the limiting steady-state torque-speed envelopes originating from the gearbox and the PMSM are remarkably similar (see Fig. 3.18). These actuators are also clearly superior in terms of packaging to the combination of PMSM and gearbox used for the GT (see Fig. 3.19).

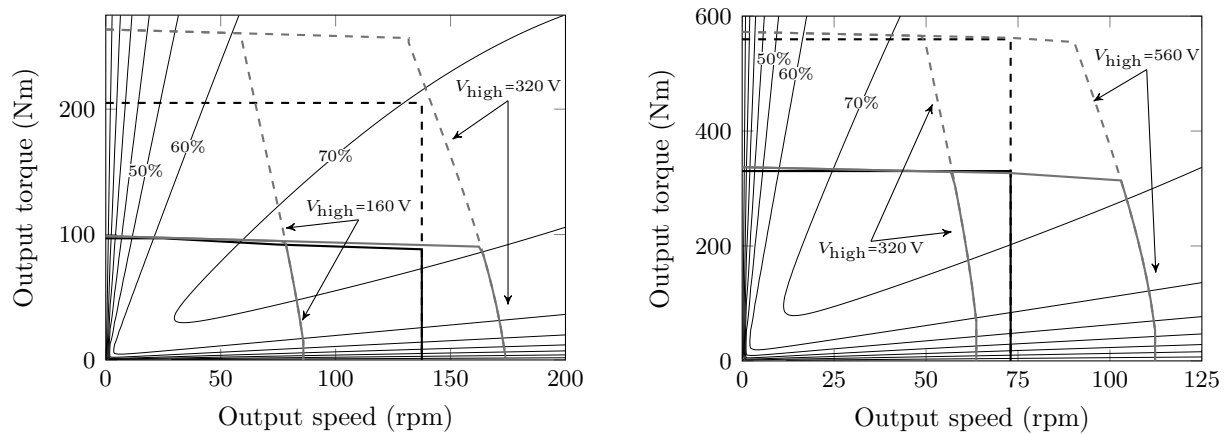


Fig. 3.18: Output steady-state torque-speed envelopes for the GT (left) and SUV (right) actuators for two different V_{high} voltages. Limits originating from the gearbox are shown in black, and from the PMSM in grey. Limits for continuous operation (solid) and intermittent operation (dashed) are shown for both the gearbox (black) and PMSM (grey). Total efficiency contours, from DC-AC converter to SL motion, are also shown (thin, black).

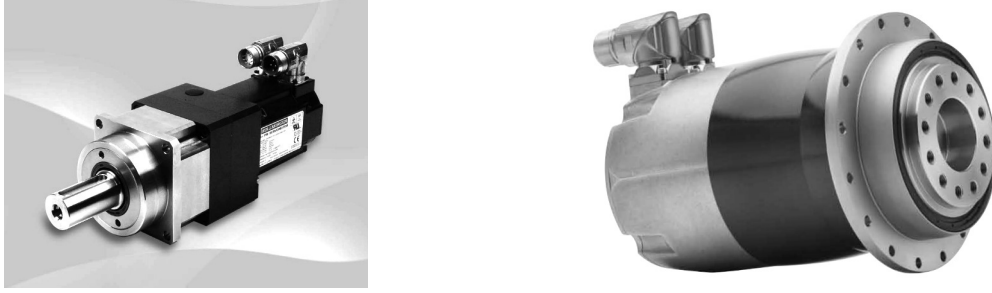


Fig. 3.19: Typical appearance of an actuator comprised of two independently designed off-the-shelf components (left: AKM PMSM and Danaher Gearbox) and of an all-in-one off-the-shelf actuator (right: Wittenstein TPM+ High Torque servo actuator).

Main parameter values

The main properties of the SL, gearbox, PMSM and converters selected for the GT and SUV are summarised in Table 3.5.

Table 3.5: SAVGS properties

Component	Parameter	Units	GT	SUV
SAVGS	Single-link lengths (front/rear)	mm	15.0/11.0	25.5/22.0
	SL equivalent inertia (front & rear)	kg m ²	0.16	1.42
	Actuator mass (front & rear)	kg	6	12
Gearbox	Gear ratio	-	40	66
	Efficiency	%	90	92
	Max. acceleration torque	Nm	205	560
	Static output torque	Nm	97	330
	Max. speed	rpm	137.5	73.0
PMSM	Static friction	Nm	0.026	0.070
	Winding (damping) coefficient	Nm s/rad	0.000038	0.000540
	Number of pole pairs	-	4	6
	dq resistance	Ω	0.91	1.02
	dq inductance	mH	2.05	2.75
	Flux linkage due to the rotor magnets	mV s/rad	61.8	67.5
	Stray losses coefficient	mV s/rad	4.6	4.7
	Core resistance	Ω	250	682
	Max. cont. phase current amplitude	A	8.0	9.8
	Max. peak phase current amplitude	A	21.2	16.7
Converters	Max. DC voltage	V	320	560
	DC-DC converter efficiency	%	95	95
	DC-AC converter efficiency	%	98	98

3.6 Conclusions

This chapter has described the models developed as part of this thesis, the software packages used to implement and simulate them, the parameter sets chosen for the case studies presented in subsequent chapters, and the checks and comparisons performed to substantiate the validity of the results obtained.

- Different software packages, each with its own strengths, have been used:
 - AutoSim produces very efficient code and can cope with the SAVGS-retrofitted full-vehicle multi-body model. However, it is not user-friendly and model debugging is difficult and time-consuming.
 - MATLAB and MATLAB + SimMechanics/Simulink produce slower codes but offer advanced features that are useful for the control synthesis, validation and post-processing stages.
 - Results obtained with both software packages have been cross-checked.
- Vehicle models of increasing complexity have been developed and results obtained with them have been qualitatively compared as a form of cross-check.
 - Linear quarter-car, linear full-car and linear bicycle models have been used to gain basic understanding of the system's response. The linear quarter-car model has also been used for the synthesis of comfort-enhancing controllers.
 - The full-vehicle multi-body model has been used to synthesise attitude control schemes and to analyse the effect of the SAVGS on the attitude motions and directional response of the vehicles. This model can be further refined and may be a valuable tool for future studies dealing with active suspensions and other ADAS.
- Parameter sets for two case studies involving a GT and an SUV have been selected/derived from information publicly available, the academic literature, actual measurements and private communication with an OEM.
- Additional aspects that may need to be modelled in the future:
 - Rotational compliance of SL, gearbox and PMSM
 - Backlash in the gearbox
 - Thermal PMSM model (some properties, such as λ_{af} , may change significantly with temperature [Krishnan, 2010])
 - Additional suspension features (e.g. geometric anti-dive/anti-squat, toe-in/out etc.)
 - Additional suspension elements, such as rubber bushings
 - Alternative suspension topologies, such as multi-link
 - More advanced tyre models that include rigid ring dynamics, contact patch transients and obstacle enveloping features (e.g. the SWIFT model [Besselink et al., 2010])

Chapter 4

Actuator Control

Control systems for the SAVGS have been developed following a multi-level approach. High level control schemes determine suitable torque or position references for each individual actuator based on the operating conditions of the vehicle and on some performance objectives. These references are then fed to controllers at the actuator level that are in charge of tracking them whilst ensuring that physical and design constraints are respected at all times.

The high level control schemes proposed to reduce chassis attitude motions caused by horizontal accelerations and to improve comfort in the presence of road irregularities are covered in Chapter 5 and Chapter 6, respectively. But first, this chapter presents and discusses the actuator-level controllers that are used in all subsequent studies.

Relevant background information on the control of PMSMs is covered first in Section 4.1. In Section 4.2, a procedure to convert all actuator constraints to the output torque-speed plane is derived. The proposed current and position control loops, which comprise PID loops tailored to ensure that operational constraints are satisfied, are discussed in Section 4.3 and Section 4.4, and some example results are shown to highlight important aspects of the control system's response. Finally, conclusions from the work performed are drawn in Section 4.5.

4.1 Basics of PMSM control

Before discussing the specific controllers proposed for the actuator level, it is opportune to provide a brief overview of aspects of the control of DC-AC converters and PMSMs.

Given the electrical arrangement shown in Fig. 3.9, this section focuses on the control of a single actuator assuming that torque or position references are provided by the higher level controllers. The innermost control loops at the actuator level, namely the bridge converter and current control loops, are shown in Fig. 4.1.

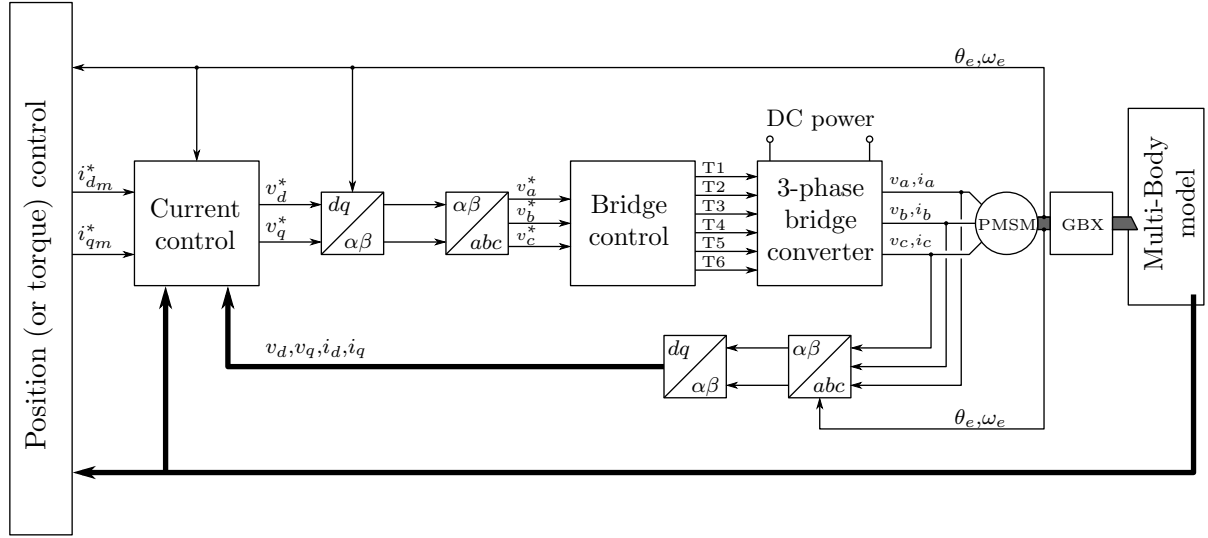


Fig. 4.1: Innermost actuator control loops.

The position (or torque) controllers are assumed to provide current references in the dq frame, i_{dm}^* and i_{qm}^* . These are fed to a current control loop that generates voltage references v_d^* and v_q^* based on measurements from the PMSM, the SL and possibly other components in the vehicle. These references are then converted to the abc reference frame associated with the stator windings of the PMSM in a two-stage process: from the rotating dq reference frame to a static $\alpha\beta$ frame, and from the static 2-phase $\alpha\beta$ to the static 3-phase abc frame through the inverse of (3.54).

The voltage references v_a^* , v_b^* and v_c^* , are then fed to the controller of the bidirectional DC-AC converter, which is a 3-phase bridge converter that operates between a constant (or slowly changing) DC voltage supply and a star-connected PMSM with floating neutral (i.e. $i_a(t) + i_b(t) + i_c(t) = 0$).

4.1.1 Control of 3-phase bridge converters

As shown in Fig. 4.2, the DC-AC converter comprises six bidirectional switches where the two switches per phase leg cannot conduct at the same time and are therefore operated in complimentary fashion.

There are two main modulation techniques that are generally used to control this type of converters: carrier-based modulation and space-vector modulation (SVM). The fundamental principle of both techniques is to control the average value of the output voltage of the converter on a cycle-to-cycle basis [Miller, 2008].

- Carrier-based modulation

A low frequency modulation signal (typically a sinusoid) is compared to a high frequency

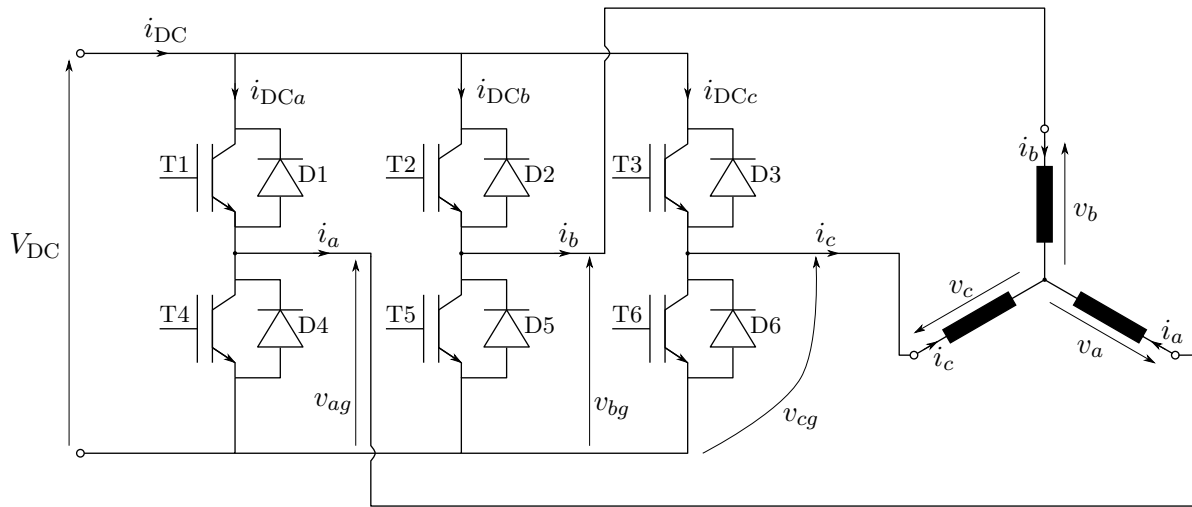


Fig. 4.2: 3-phase bridge converter and wye-connected PMSM with floating neutral.

carrier signal (typically triangular or sawtooth). The result of such comparison is used as control signal for the switches in the converter. Some of the most common techniques within this category are pulse-width modulation (PWM), sine-triangle modulation and third harmonic injection [Krause et al., 2002]. PWM may be realised through analogue or digital electronics. Digital implementation has the advantages of higher stability (no drift, offsets or ageing effects), precision (noise immunity) and flexibility (software customisation), but requires significant computing power.

- Space vector modulation

The main idea behind SVM is to transform the time domain output voltages (or currents) of the converter into a single space vector that rotates around the stationary $\alpha\beta$ plane at the desired electrical frequency, $\omega = \omega_e$. SVM is more convenient than PWM for digital control as it simplifies the calculation procedure. It generates voltage signals that are devoid of low frequency harmonics, it provides high voltage amplitude capability and it is particularly well-suited for the control of synchronous machines in dq frame variables.

Based on these reasons, SVM has been considered throughout the thesisⁱ. In particular, SVM in the linear region (i.e. without over-modulation) has been used: the voltage reference is constrained to remain within the circle inscribed in the hexagon shown in Fig. 4.3, and the maximum line to neutral voltage that can be generated is $V_{\max} = V_{\text{DC}}/\sqrt{3}$.

The basic operation of SVM is as follows:

1. The desired dq voltage references, v_d^* and v_q^* , are combined to form the voltage reference $\mathbf{V}^*$, ensuring that $\|\mathbf{V}^*\| \leq V_{\max}$.

ⁱThis choice is supported by the fact that the servo drive bought for the experimental test rig is actually controlled by SVM.

2. $\mathbf{V}^*$ is projected to the abc frame making use of the electrical angle, θ_e .
3. The sector containing $\mathbf{V}^*$ is identified.
4. The switching sequence and the amount of time spent in each state are calculated (further details may be found in [Krause et al., 2002]). Table 4.1 lists the switching characteristics associated with space vectors 0 to 7.

Switching frequencies in commercial DC-AC drives are much higher than other dynamics in the system (e.g. 16 kHz in the Kollmorgen S306 servo drive bought for the experimental rig), and for that reason the average-value model of the machine and purely sinusoidal voltage signals have been considered. Moreover, actual voltages applied to the PMSM windings have been assumed to be equal to the reference voltages fed to the DC-AC converter: $v_d = v_d^*$, $v_q = v_q^*$.

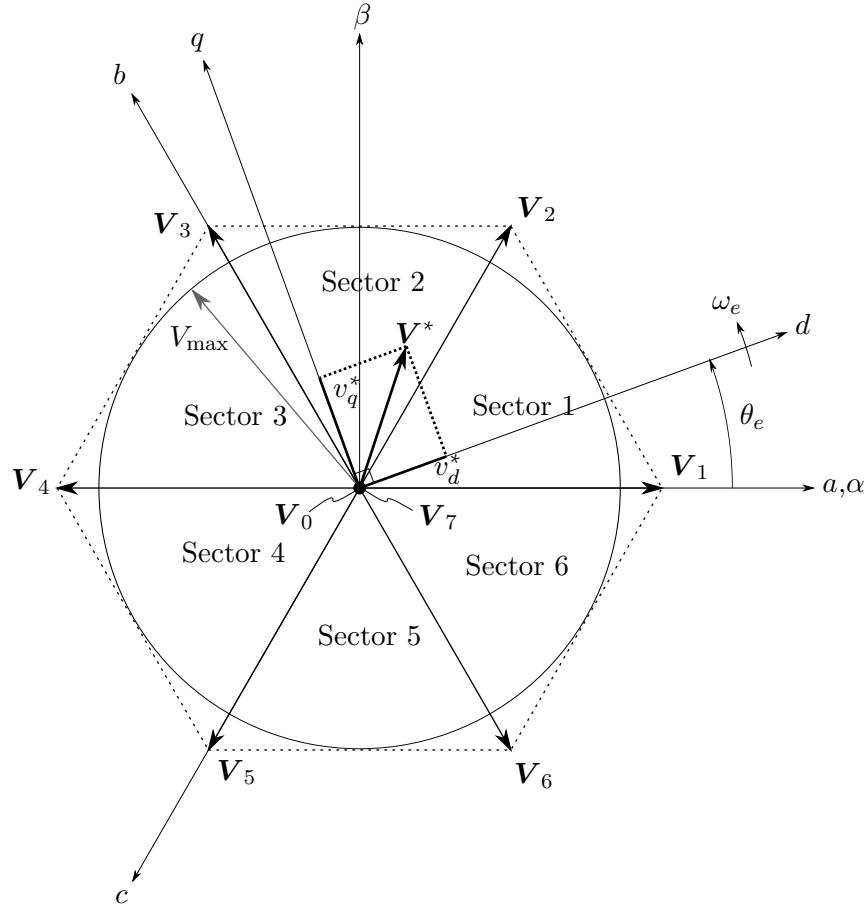


Fig. 4.3: Diagram illustrating space vector modulation. Switching commands for transistors T1 to T6 are computed on a cycle-to-cycle basis. In each cycle, switching commands are based on four states: the first and last ones are V_0 and V_7 , whereas the second and third are the two vectors forming the sector in which V^* lies (V_2 and V_3 as shown in the diagram). More details regarding switching times and other aspects of SVM can be found in [Krause et al., 2002].

Table 4.1: Switching states and voltage space vectors in SVM

Space-vector	$T1/\overline{T4}$	$T2/\overline{T5}$	$T3/\overline{T6}$	v_{ag}	v_{bg}	v_{cg}	v_a	v_b	v_c
$\mathbf{V}_0$	0	0	0	0	0	0	0	0	0
$\mathbf{V}_1$	1	0	0	V_{DC}	0	0	$2/3 V_{DC}$	0	0
$\mathbf{V}_2$	1	1	0	V_{DC}	V_{DC}	0	$2/3 V_{DC}$	$2/3 V_{DC}$	0
$\mathbf{V}_3$	0	1	0	0	V_{DC}	0	0	$2/3 V_{DC}$	0
$\mathbf{V}_4$	0	1	1	0	V_{DC}	V_{DC}	0	$2/3 V_{DC}$	$2/3 V_{DC}$
$\mathbf{V}_5$	0	0	1	0	0	V_{DC}	0	0	$2/3 V_{DC}$
$\mathbf{V}_6$	1	0	1	V_{DC}	0	V_{DC}	$2/3 V_{DC}$	0	$2/3 V_{DC}$
$\mathbf{V}_7$	1	1	1	V_{DC}	V_{DC}	V_{DC}	$2/3 V_{DC}$	$2/3 V_{DC}$	$2/3 V_{DC}$

4.1.2 Selection of dq current references

Irrespective of the actual control strategy used, there are numerous well established approaches for the calculation of d and q current references for PMSMs. For example, Krishnan [2010] covers the following cases in detail:

- Constant torque angle control, or zero direct current control: this is the industry standard, easy to implement but with limited high speed performance as it does not include any flux weakening logic.
- Unity power factor: low voltage requirements and extended constant torque region.
- Constant mutual flux linkages control: smooth integration of control at low speeds and flux weakening at high speeds.
- Angle control of air gap flux and current phasors: suitable for sensorless operation.
- Optimum torque per unit current control: optimum utilisation of PMSM and drive.
- Constant power loss control: maximum torque-speed envelope taking into account thermal robustness.
- Maximum efficiency control: maximum efficiency and improved reliability.

In the specific case of the SAVGS, overall system performance is limited mainly by the maximum torque that the actuator can deliver at low speeds. Thus, output speed maximisation is not particularly interesting and flux weakening approaches are not needed. Sensorless operation is not a requirement neither, and energy efficiency, although important, is not critical.

For these reasons, the standard zero direct current control has been chosen. It is a straightforward approach that provides near optimum torque per ampere and an efficiency that is comparable to loss minimisation control strategies [Cavallaro et al., 2005]. The approach involves setting $i_{dm}^* = 0$ at all times, and controlling i_{qm}^* to achieve the desired output torque or single-link motion.

4.2 Computation of steady-state torque-speed envelopes

Physical and design constraints for all actuator components need to be taken into account in the PMSM control. The PMSM is subject to current, voltage, power and speed constraints, whereas the gearbox and single-link are subject to torque and speed constraints. The strategies implemented in the PMSM control to ensure that these limitations are not violated and therefore that the actuator can be operated safely and reliably are discussed in Section 4.3 and Section 4.4.

In this section, however, the aim is to map all physical and design limitations to the output torque-speed plane. This is useful because:

- Output torque-speed characteristics determine to a great extent the effect that the SAVGS can have on the response of the car.
- Limitations arising from different components can be compared (useful for component matching and selection, as already shown in Fig. 3.17 and Fig. 3.18).
- Torque-speed operating points obtained from simulations can be superimposed on the predicted envelopes and compared as a way of assessing the effectiveness of the proposed control schemes. If the control is too conservative, torque-speed operating points will not cover the permitted region fully (i.e. will remain close to the origin) and therefore the actuator capabilities will not be wholly utilised. If, on the other hand, the control is too aggressive, there will be excursions outside the permitted torque-speed region and results obtained will not be realistic.

PMSM limitations

The PMSM is subject to current, voltage, power and speed constraints.

- Speed constraints are hard constraints that arise from the bearing characteristics. They can be expressed as:

$$|\omega_{\text{HSS}}| < \omega_{\text{max}} \quad (4.1)$$

- Current and voltage limits are also hard constraints, although the limiting values may be reduced by the higher level controllers in order to improve reliability, account for variations in available DC voltage etc. These restrictions are of the form:

$$I = \sqrt{i_d^2 + i_q^2} \leq I_{\text{max}}, \quad (4.2a)$$

$$V = \sqrt{v_d^2 + v_q^2} \leq V_{\text{max}} = \frac{V_{\text{DC}}}{\sqrt{3}}, \quad (4.2b)$$

- Power constraints are typically soft constraints imposed by higher level control schemes in order to limit energy consumption. These are constraints of the form:

$$-P_{\min} \leq P = \frac{3}{2} (v_d i_d + v_q i_q) \leq P_{\max}, \quad (4.3)$$

where typically $P_{\max} \leq P_{\min}$ to allow for higher power flows when regenerating.

The boundaries resulting from these constraints can be transferred to the single-link side (i.e. low-speed side of the gearbox) and superimposed on the same plot, as shown in Fig. 4.4. The permitted region is defined as the set $S \subset \mathbb{R}^2$ (typically non-convex due to the power limitations) that contains the origin and is not intersected by any of the aforementioned boundaries.

For the specific case considered in this thesis, namely constant torque angle control, the steady-state torque-speed envelopes are obtained by substituting $i_{dm} = 0$ and $\frac{di_{dm}}{dt} = \frac{di_{qm}}{dt} = 0$ into the PMSM equations (3.63) and (3.64). This leads to:

$$\begin{aligned} v_d &= -\Gamma L_s \omega_e i_{qm}, & v_q &= \Gamma K_e \omega_e + R_s i_{qm}, \\ i_d &= -\frac{L_s}{R_i} \omega_e i_{qm}, & i_q &= \frac{K_e}{R_i} \omega_e + i_{qm}. \end{aligned} \quad (4.4)$$

At low to intermediate speeds, voltage constraints remain inactive but current constraints of the bridge converter and PMSM limit the maximum torque that can be generated. At high speedsⁱⁱ, voltage constraints become active and limit the maximum torque that can be generated at any given speed.

Substituting (4.4) into (4.2) leads to voltage and current constraint equations of the form:

$$a_2 i_{qm}^2 + a_1 i_{qm} + a_0 = 0. \quad (4.5)$$

where the coefficients for the current constraints are given in (4.6a) and for the voltage constraints in (4.6b).

$$a_2 = 1 + \frac{L_s^2}{R_i^2} \omega_e^2, \quad a_1 = \frac{2 K_e}{R_i} \omega_e, \quad a_0 = \frac{K_e^2}{R_i^2} \omega_e^2 - I_{\max}^2 \quad (4.6a)$$

$$a_2 = R_s^2 + \Gamma^2 L_s^2 \omega_e^2, \quad a_1 = 2 R_s \Gamma K_e \omega_e, \quad a_0 = \Gamma^2 K_e^2 \omega_e^2 - V_{\max}^2 \quad (4.6b)$$

Finally, power flows from/to the bridge converter can be computed as a function of angular

ⁱⁱThe base speed is defined as the speed at which both current and voltage constraints are active simultaneously. With the $i_{dm} = 0$ control used in this thesis, current constraints are active only at and below base speed, and voltage constraints are active only at and above base speed.

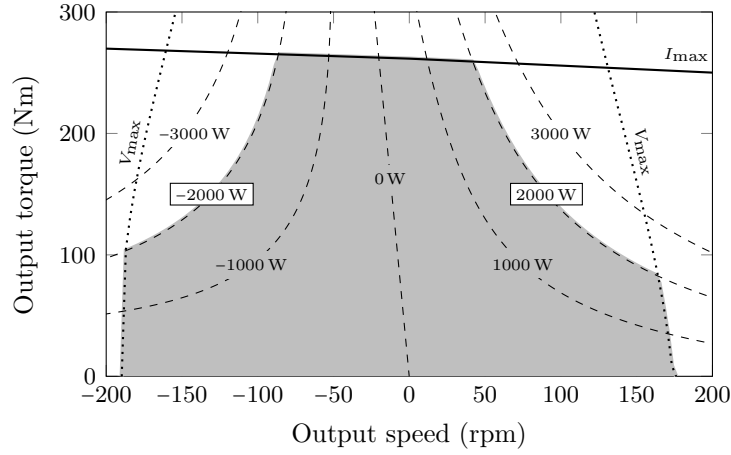


Fig. 4.4: Example output (i.e. on the single-link side) steady-state torque-speed envelopes arising from current (solid), voltage (dotted) and power (dashed) limitations of PMSM and bridge converter. As an example, the permitted region for power flows to/from the bridge converter equal to or less than 2000 W is shaded in grey. For clarity, results shown correspond to positive output torques only and vertical lines representing speed constraints have been omitted.

speed and current by substituting (4.4) into (3.72):

$$P = \frac{3}{2} \left[\frac{\Gamma L_s^2}{R_i} \omega_e^2 i_{q_m}^2 + R_s i_{q_m}^2 + \frac{\Gamma K_e^2}{R_i} \omega_e^2 + \left(\frac{R_s}{R_i} + \Gamma \right) K_e \omega_e i_{q_m} \right] \quad (4.7)$$

Once the power flows have been calculated for all pairs $\omega_e - i_{q_m}$ of interest, power isolines can be obtained, mapped to the output $T_{LSS} - \omega_{LSS}$ plane through (3.65), (3.67) and (3.68), and plotted in the same graph as the previously derived envelopes.

Gearbox

The gearbox is subject to torque and speed hard constraints that can be easily transferred to the output $T_{LSS} - \omega_{LSS}$ plane through (3.67) and (3.68). Typical values provided by manufacturers correspond to ultimate or low-cycle torque limits, also referred to as peak torque limit, and fatigue or high-cycle limits, also referred to as continuous limits. The peak torque provided in the official specifications is a single output torque value, whereas the continuous torque limits depend on the angular speed because they are computed on the basis of a certain number of hours of operation (typically 20000 h for servo applications) at constant speed, whilst the actual fatigue life of the components depends on the number of cycles (not hours). Typical peak and continuous torque-speed envelopes for an off-the-shelf planetary gearbox are shown in Fig. 4.5.

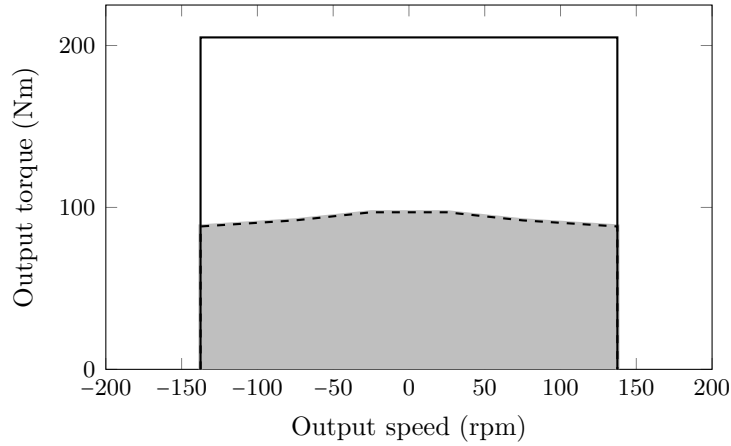


Fig. 4.5: Output torque-speed envelopes for the gearbox are shown in solid (peak) and dashed (continuous) black lines. The permitted region for continuous operation is shaded in grey.

4.3 Inner loop control: current reference tracking

The current control loop proposed here (shown in Fig. 4.6) comprises two proportional integral controllers with anti-windup. Mechanisms are also put in place to ensure that power and voltage constraints are respected at all times.

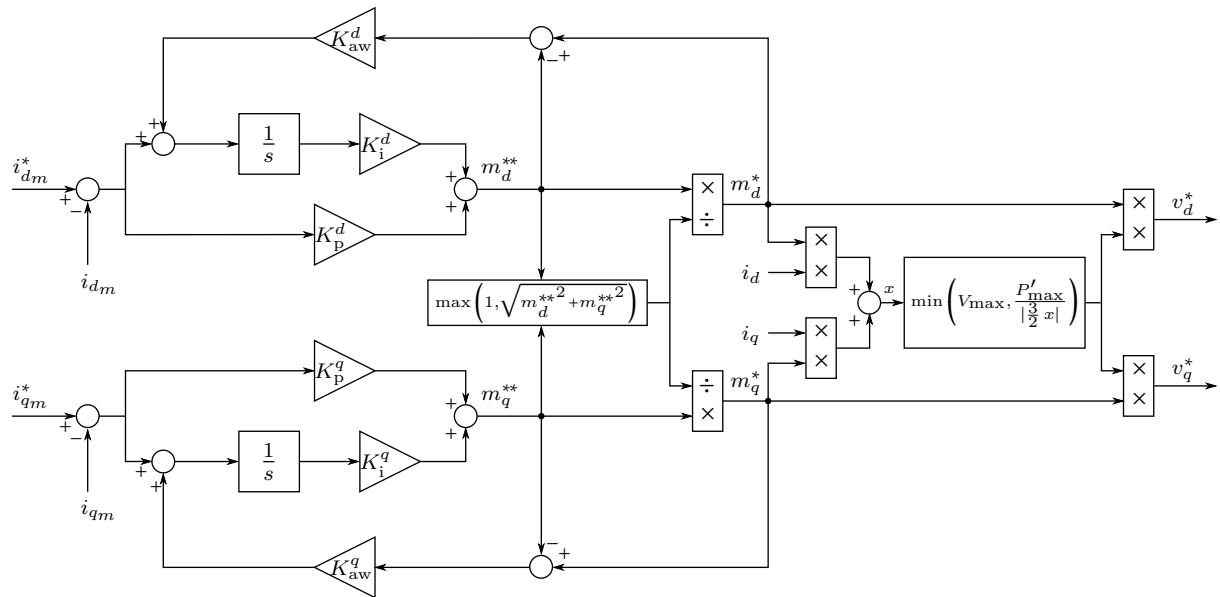


Fig. 4.6: Current control loop. Inputs from the outer level control loops are the reference magnetising currents: i_{dm}^* and i_{qm}^* . Inputs from the PMSM are the dq currents and their magnetising components: i_d , i_q , i_{dm} and i_{qm} . Outputs to the DC-AC converter are the dq voltage references: v_d^* and v_q^* . Proportional, integral and anti-windup gains are denoted by subscripts p, i and aw. dq components are denoted by subscripts or superscripts d and q .

In this scheme it is useful to work with modulation indexes as intermediate variables. The

reference dq modulation indexes are defined as the ratios between the reference dq voltages and the maximum available voltage. That is:

$$m_d^* = v_d^*/V_{\max}, \quad m_q^* = v_q^*/V_{\max}. \quad (4.8)$$

Current errors are fed to the PI controllers that generate provisional values for the reference modulation indexes, m_d^{**} and m_q^{**} . In a first step, these provisional references are scaled down to ensure that the bridge converter is not forced to operate outside its linear region. Substituting (4.8) into (4.2b) leads to the condition that the modulation indexes must satisfy:

$$V = \sqrt{v_d^2 + v_q^2} = V_{\max} \sqrt{m_d^{*2} + m_q^{*2}} \leq V_{\max} \Rightarrow \sqrt{m_d^{*2} + m_q^{*2}} \leq 1. \quad (4.9)$$

By dividing m_d^{**} and m_q^{**} by the maximum between 1 and the magnitude of the provisional modulation index, $\sqrt{m_d^{**2} + m_q^{**2}}$, condition (4.9) is imposed evenly, reducing both the d and q reference voltages whenever necessary.

Power limitations do also need to be taken into account at this stage. Given that power flows depend on the dq voltages and currents, and that voltage references are assumed to be immediately delivered by the bridge converter, the best place to implement power constraints is just before the bridge converter input. The power flow between the bridge converter and the PMSM that would result from the use of (4.8) can be estimated by plugging (4.8) into (3.72):

$$P^* = \frac{3}{2} V_{\max} (m_d^* i_d + m_q^* i_q). \quad (4.10)$$

If $P^* < -P_{\min} < 0$ or $P^* > P_{\max} > 0$, then the conversion from the reference modulation indexes to the reference voltages given in (4.8) needs to be modified to ensure that the power limits are respected. This is done through:

$$v_d^* = m_d^* V'_{\max}, \quad v_q^* = m_q^* V'_{\max}, \quad (4.11)$$

where the corrected value $V'_{\max}$ is calculated as

$$V'_{\max} = \min \left(V_{\max}, \frac{P'_{\max}}{\frac{3}{2} |m_d^* i_d + m_q^* i_q|} \right), \quad (4.12)$$

with the maximum allowable power flow, $P'_{\max}$, computed based on the mode of operation of the PMSM, M , through

$$P'_{\max} = \frac{P_{\max} + P_{\min}}{2} + M \left(\frac{P_{\max} - P_{\min}}{2} \right). \quad (4.13)$$

4.4 Outer loop control: position reference tracking

In initial preliminary studies [Arana et al., 2012], current references for the PMSM were generated directly by the controller in charge of the attitude motions of the chassis. The resulting control scheme comprised only two closed loops: a current control loop similar to the one already described in Section 4.3 and a pitch control loop. Despite its apparent simplicity, the control system contained several adaptive blocks and the tuning process was not straightforward.

In subsequent analyses, such as [Arana et al., 2015a], this control was refined and an additional closed loop, corresponding to the position control of the single-link, was introduced. This enabled the use of larger SL angles without compromising the stability of the system. Moreover, the implementation of position control opened up the possibility of operating the SAVGS from a default SL position different from the passive equilibrium configuration. Overall, the addition of a position control between the attitude control loop and the current control loop led to significant improvements in the SAVGS performance.

The position control introduced in [Arana et al., 2015a] was refined further in evaluation studies performed for an OEM [Arana et al., 2014b]. In particular, 1) both positive and negative current references, i_{qm}^* , were allowed, thus improving position tracking performance and enabling the selection of reference angles equal to or even larger than 180 deg, and 2) the feedback paths used to limit the rotor speed were made dependent on the position error so that large position errors led to large allowable speeds and small errors to reduced speed limits. This modification led to much smoother operation and significant overall performance improvements.

The position control loops described so far relied on classic PID controllers. Although they provided good performance for the tasks at hand and proved to be sufficiently robust to parameter changes, a final refinement that improved slightly the performance but most importantly simplified the tuning process was implemented for the simulation studies presented in [Arana et al., 2015b]. This controller is based on the PIV (Proportional position loop, Integral and proportional Velocity loop) structure introduced (but barely explained) in a white paper by Parker Hannifin [ParkerMotion, 2010].

The PIV structure may be understood as part of the PID family. Let $R(s)$ correspond to the reference command, $X(s)$ to the variable being controlled, $U(s)$ to the control output, and consider the following expression for the control output:

$$U(s) = \left[K_p^f R(s) - K_p^b X(s) \right] + \frac{1}{s} \left[K_i^f R(s) - K_i^b X(s) \right] + s \left[K_d^f R(s) - K_d^b X(s) \right], \quad (4.14)$$

where K_p , K_i and K_d are the proportional, integral and derivative gains, and superscripts f and b refer to the term to which these are applied: f refers to forward and means that the gain is applied to the input reference command, whereas b refers to backward and means that the

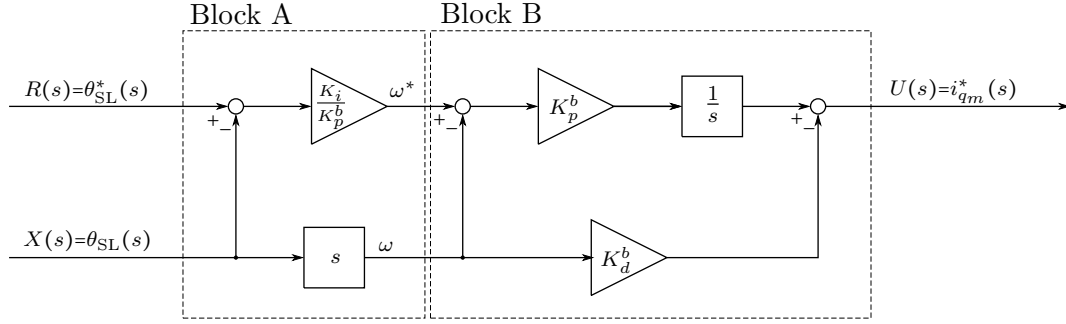


Fig. 4.7: PIV control scheme. More detailed information regarding the implementation of Block A and Block B is provided in Fig. 4.8 and Fig. 4.9, respectively.

gain is applied to the signal being controlled (and fed back). In a conventional PID controller, forward and backward gains are equal and the control output becomes a function of the tracking error, its integral and its time derivative. In the PIV controller, on the other hand, $K_p^f = K_d^f = 0$ and $K_i^f = K_i^b = K_i$. The control output then simplifies to the expression:

$$U(s) = \left[-K_p^b X(s) \right] + \frac{K_i}{s} \left[R(s) - X(s) \right] + s \left[-K_d^b X(s) \right]. \quad (4.15)$$

The application of this scheme to the SL position control problem is shown in Fig. 4.7.

A good starting point for the tuning of control gains is proposed in [Parkermotion, 2010] based on the system's inertia, expected mechanical losses, desired control bandwidth and damping ratio.

4.4.1 Description of position and velocity control loops

The proposed position and velocity loops are shown in Fig. 4.8 and Fig. 4.9, respectively. The position reference for the SL, θ_{SL}^* , is received from some higher level controller and compared with the actual SL position. The position error is then scaled by K_p to form a velocity reference for the HSS. Before feeding it to the velocity loop, this reference is 1) saturated to maintain it within the allowable limits and thus avoid requesting too much from subsequent controllers, and 2) further scaled down, if the actual speed has exceeded the allowable limits, in order to return the actuator to the permitted region.

The velocity loop shown in Fig. 4.9 combines integral action on the velocity error with proportional action on the actual high-speed shaft velocity. An output gain, $C1 = \frac{1}{(3/2)n_{pp}K_t}$, converts the control output from a torque reference to a preliminary current reference, i_{qm}^{**} . As in the position loop, this preliminary reference is saturated to facilitate keeping the actuator within its permitted operating conditions. The actual limits used for this dynamic saturation, $i_{qm}^{(\max)}$ and $i_{qm}^{(\min)}$, are obtained as a function of the motor speed, its mode of operation and the mechanical

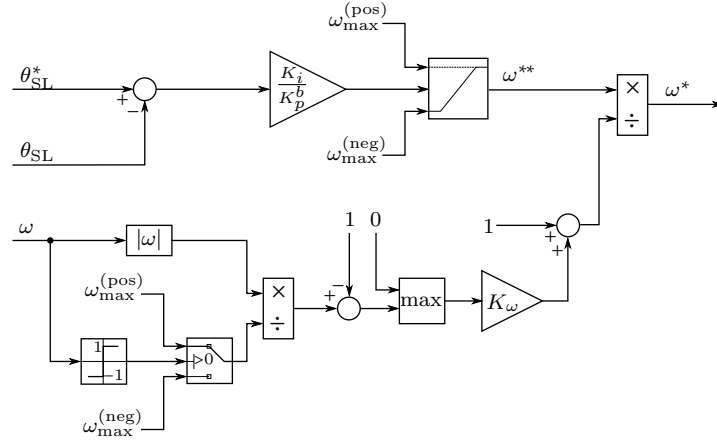


Fig. 4.8: Position control loop (corresponds to Block A in Fig. 4.7). The position reference, θ_{SL}^* , comes from higher level controllers. The output from the position control loop is a reference speed that remains within the allowable limits and that is corrected, when necessary, to ensure that the actuator returns to permitted speeds as soon as it exceeds some prescribed limits, $\omega_{max}^{(pos)}$ and $\omega_{max}^{(neg)}$.

losses in the PMSM through a function $\mathbf{g}$ that outputs:

$$\begin{aligned} i_{q_m}^{(max)} &= CF \cdot \min \left({}^{(A)}i_{q_m}^{(max)}, {}^{(B)}i_{q_m}^{(max)}, {}^{(C)}i_{q_m}^{(max)} \right), \\ i_{q_m}^{(min)} &= -\alpha_{min} i_{q_m}^{(max)}, \end{aligned} \quad (4.16)$$

where $CF \lesssim 1$ is a correction factor and $0 \leq \alpha_{min} \leq 1$ adjusts the minimum currents and therefore torques (positive torques push down the SL and wheel assembly, negative torques tend to pull the wheel assembly upwards) as a function of the maximum ones. If $\alpha_{min} = 0$, the control prevents the actuator from generating negative torques. This eliminates the effects of backlash altogether and provides very stable responses, but is overly conservative. If $\alpha_{min} = 1$, the control is aggressive and rapid changes in torque direction can occur. Intermediate values can be used to modulate the conservatism/aggressiveness of the controller.

The three current values in (4.16), ${}^{(A)}i_{q_m}^{(max)}$, ${}^{(B)}i_{q_m}^{(max)}$ and ${}^{(C)}i_{q_m}^{(max)}$, aim to predict the maximum allowable current based on torque limitations arising from:

- Power constraints: limiting currents obtained through a simplified computation of electrical power flow to/from the PMSM based on the electromechanical output power and on the mode of operation. Thus:

$${}^{(A)}i_{q_m}^{(max)} = (1 - M_a M) \frac{P'_{max}}{|\omega_{HSS}|} \frac{1}{(3/2) n_{pp} K_t}, \quad (4.17)$$

where M_a is a small positive coefficient to account for the fact that $P_{em} = T_{em} \omega_{HSS} = (3/2) n_{pp} K_t i_{q_m} \omega_{HSS}$ is the electromechanical power and not the power flow to/from the PMSM. Thus, $(1 - M_a M) \lesssim 1$ when motoring, and $(1 - M_a M) \gtrsim 1$ when regenerating.

- Current constraints: limiting i_{q_m} values are computed solving $i_d^2 + i_q^2 = I_{max}^2$ for i_{q_m} in the

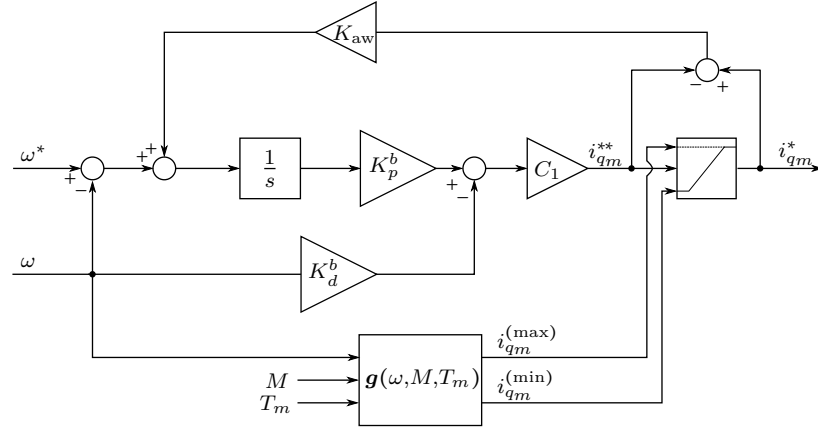


Fig. 4.9: Velocity control loop (corresponds to Block B in Fig. 4.7). Input from the outer level control loops is the reference PMSM speed: ω^* . Inputs from the PMSM are the rotor speed, mode of operation and mechanical losses torque: ω , M and T_m . Output to the current control loop is the reference magnetising q current: i_{qm}^* .

general case in which i_{dm} , $\frac{di_{dm}}{dt}$ and $\frac{di_{qm}}{dt}$ are allowed to be different from zero. The maximum allowable current is then obtained by solving the resulting second order polynomial $a_2 i_{qm}^2 + a_1 i_{qm} + a_0 = 0$ through:

$$^{(B)}i_{qm}^{(\max)} = \max \left(0, \frac{-a_1 + \sqrt{a_1^2 - 4 a_2 a_0}}{2 a_2} \right), \quad (4.18)$$

where coefficients a_0 , a_1 and a_2 are:

$$\begin{aligned} a_2 &= 1 + \left(\frac{\omega_e L_s}{R_i} \right)^2, \\ a_1 &= \frac{2}{R_i} \left(\omega_e K_e + L_s \frac{di_{qm}}{dt} - \frac{\omega_e L_s^2}{R_i} \frac{di_{dm}}{dt} \right), \\ a_0 &= i_{dm}^2 \left[1 + \left(\frac{\omega_e L_s}{R_i} \right)^2 \right] + i_{dm} \left[2 \frac{L_s}{R_i} \frac{di_{dm}}{dt} + 2 \frac{\omega_e L_s}{R_i^2} \left(\omega_e K_e + L_s \frac{di_{qm}}{dt} \right) \right] \\ &\quad + \left(\frac{L_s}{R_i} \right)^2 \left[\left(\frac{di_{dm}}{dt} \right)^2 + \left(\frac{di_{qm}}{dt} \right)^2 \right] + \frac{\omega_e K_e}{R_i} \left(\frac{\omega_e K_e}{R_i} + 2 \frac{L_s}{R_i} \frac{di_{qm}}{dt} \right) - I_{\max}^2. \end{aligned} \quad (4.19)$$

- Gearbox torque limits: limiting currents can be obtained from (3.65) and (3.68):

$$^{(C)}i_{qm}^{(\max)} = \frac{T_{em}^{(\max)}}{3/2 n_{pp} K_t} = \frac{T_{HSS}^{(\max)} + M |T_m|}{3/2 n_{pp} K_t} = \frac{T_{LSS}^{(\max)} \eta_{gbx}^{-M} G^{-1} + M |T_m|}{3/2 n_{pp} K_t} \quad (4.20)$$

4.4.2 Example simulation results

Although details of actuator operation during attitude control and comfort enhancement simulations are shown in Chapter 5 and Chapter 6, respectively, some example results are presented in this section to illustrate some aspects of the control action.

Results have been obtained with the multi-body quarter-car model and the parameter set corresponding to a rear corner of the GT. Results corresponding to two different simulations are shown: 1) position reference changes with no external disturbances, and 2) constant position reference with external disturbance. In both cases, peak current and torque actuator constraints have been imposed, power limits of 1500 W for motoring and 3000 W for regenerating have been used, and negative torques have been allowed. The set of control gains used is shown in Table 4.2, together with equivalent values for the SUV actuators.

Table 4.2: Typical control gains for GT and SUV actuators

Loop	Parameter	Units	GT	SUV
Position	K_i/K_p^b	s^{-1}	4488	3366
	K_ω	s/rad	5	5
Velocity	K_p^b	Nm/rad	5596	10107
	K_d^b	Nm s/rad	0.89	2.14
	K_{aw}	rad/s/A	5	5
	CF	-	0.99	0.99
	α_{min}	-	1	1
	M_a	-	0.075	0.075
Current	K_p^d	A^{-1}	0.3	0.3
	K_p^q	A^{-1}	0.3	0.3
	K_i^d	$A^{-1} s^{-1}$	200	200
	K_i^q	$A^{-1} s^{-1}$	200	200
	K_{aw}^d	s^{-1}	12	12
	K_{aw}^q	s^{-1}	12	12

Changes in reference position, no external disturbances

Fig. 4.10 and Fig. 4.11 correspond to a simulation in which the system is in equilibrium and, without any external disturbance, is subject to a series of step-like changes in the reference SL position. Fig. 4.10 shows some relevant time series and Fig. 4.11 the corresponding output torque-speed operating points for the actuator.

The initial configuration has the SL at 90 deg with respect to the line EG (see Fig. 3.5) and the amplitude of the reference changes is 60 deg. As it can be seen in the top plot in Fig. 4.10, position tracking is fast (~ 0.1 s to settle to the new reference) and well damped in both directions of motion. Moreover, zero steady-state error is achieved.

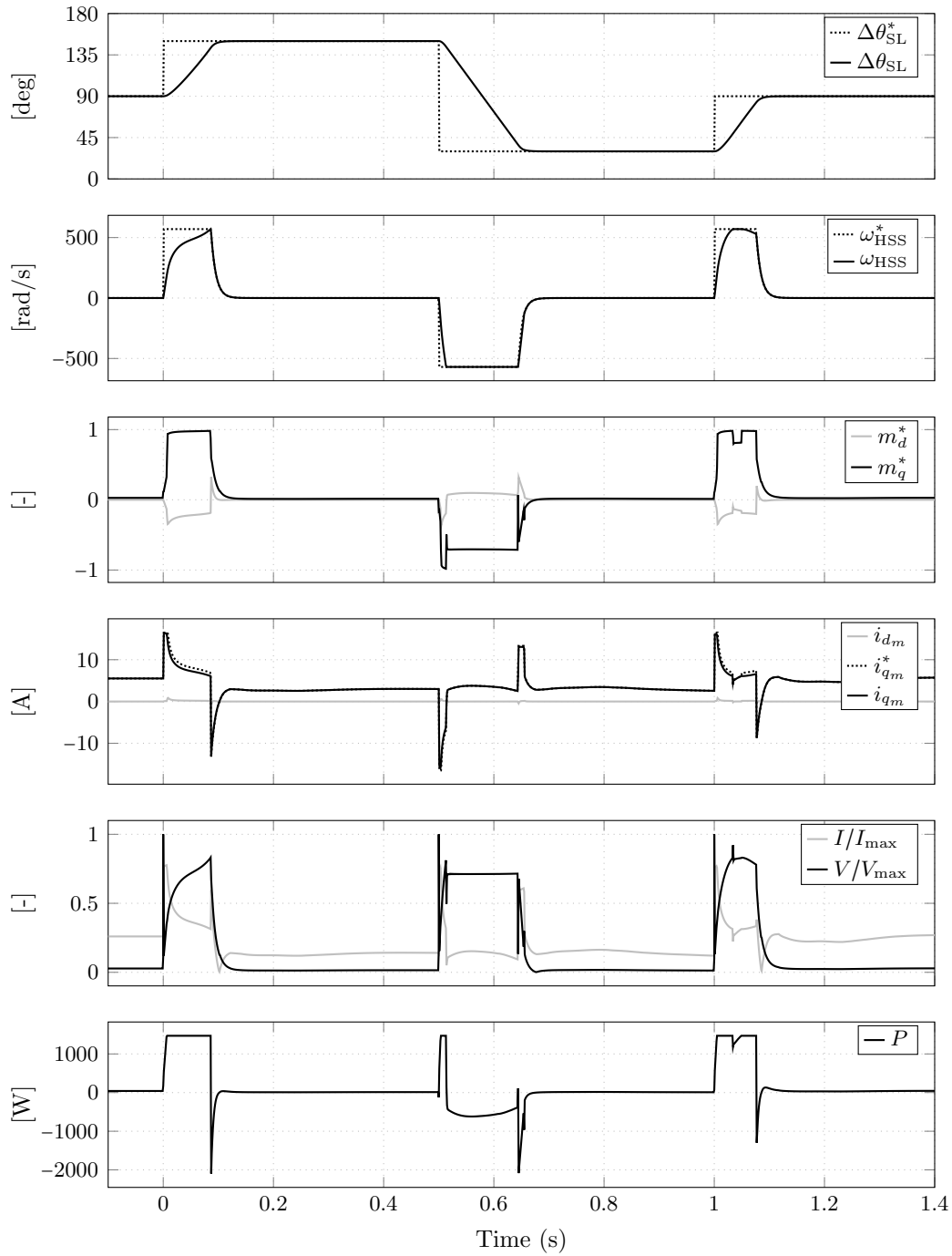


Fig. 4.10: Results obtained with the multi-body model of one rear corner of the GT for a series of step-like changes in reference SL position. Starting from the top: the first plot shows the reference and actual SL angles, measured with respect to the EG line in the equilibrium configuration (see Fig. 3.5); the second plot depicts the reference and actual angular speeds of the HSS; the reference modulation indexes are presented in the third plot; individual current amplitudes are shown in the fourth plot; normalised current and voltage amplitudes are shown in the fifth plot; electrical power flows to the PMSM are shown in the sixth plot.

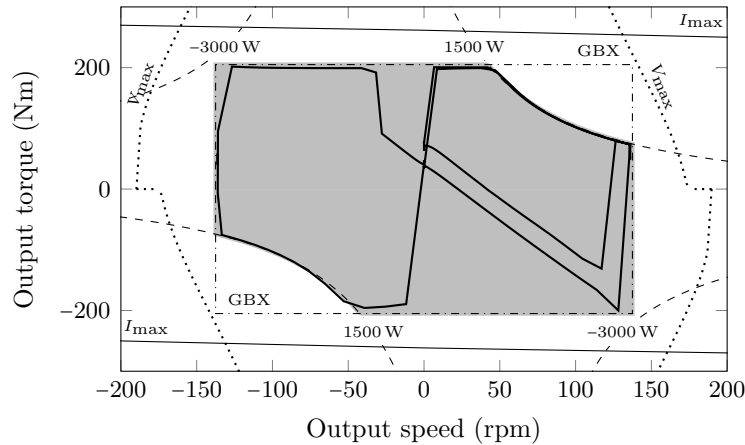


Fig. 4.11: Output torque-speed operating points (black, thick) corresponding to the results shown in Fig. 4.10. Current (solid), voltage (dotted) and power (dashed) limits arising from the PMSM, as well as torque limits arising from the gearbox (dash-dotted) are shown. The permitted area is shaded in grey.

The reference velocity command resulting from the position loop is shown together with the actual PMSM speed in the second plot. Changes in speed reference are sharp because of the step-like nature and high amplitude of the changes in position reference, but this is not representative of less demanding scenarios. As expected, the SL can be accelerated faster in the negative direction as that means letting the sprung mass fall rather than pushing it up.

The dq modulation indexes and magnetising currents are shown in the third and fourth plots. Modulation indexes saturate only during the hard positive acceleration phases. i_{d_m} is effectively kept small at all times, and $i_{q_m}^*$ tracking is very fast and accurate.

Current and voltage magnitudes, normalised by their limiting values, are presented in the fifth plot. Current and voltage constraints are respected at all times and therefore these traces remain within the 0-1 band. Current grows during accelerations to provide the required torque, whereas voltage is high when accelerating as well as when sustaining high speeds.

Electrical power flows to the PMSM are shown in the sixth and last plot. Power consumption is minimal whilst maintaining constant angles, it is equal to the maximum motoring power during positive accelerations and sustained positive speeds, and it approaches the regenerating power limit during negative accelerations.

Fig. 4.11 highlights that the proposed control system utilises the actuator capabilities fully: the operating torque-speed points approach all boundaries without exceeding them. In this case, the actuator is allowed to use its peak current values and is fed by the bridge converter from a 320 V DC voltage source. As a consequence, the gearbox becomes the limiting factor in the whole speed range. There are small excursions outside the power limits when motoring, but this is only due to the fact that power isolines shown in the figure were obtained for steady-state conditions (power flows do remain within the specified values at all times).

Constant reference position, external force disturbance

Fig. 4.12 corresponds to a simulation in which the system is in equilibrium and, without any other external disturbances or reference changes, it is subject to a series of step-like changes in the external vertical force applied to the sprung mass. The amplitude of these changes is equal to 50% of the vertical tyre force in equilibrium conditions.

In the top plot, the external force and the resulting tyre force are shown together. The tyre loses contact with the road surface around half way through the stage with negative (i.e. upwards) external force, and it hits the surface again approximately 0.2 s later. As expected, significant oscillations in tyre forces are present throughout the 2 s shown in the plots.

The second plot demonstrates the good disturbance rejection behaviour of the proposed actuator control, as the motion of the SL is not noticeable despite the significant and sudden force changes occurring in the system.

As the SL remains static, magnitudes of d currents and modulation indexes are negligible (shown in the third and fourth plots), and a small q modulation index (below 0.06) is sufficient to provide almost perfect q current tracking.

The last two plots corroborate that, when the SL does not move, low voltages, currents and powers are required to compensate for significant and fast-changing external conditions.

4.5 Conclusions

This chapter has covered the actuator control problem in the context of the SAVGS. Starting from the basics of bridge converter control and the selection of direct and quadrature current references, the chapter has explained the specific current, velocity and position control loops implemented in this thesis, paying particular attention to the physical and design constraints that need to be considered.

Some conclusions that can be drawn from this chapter are:

- Space vector modulation is the most appropriate control strategy for the 3-phase bridge converter. It facilitates the control in the dq frame, is devoid of damaging low frequency harmonics and leads to high maximum voltage amplitudes.
- Zero direct current control, also known as fixed torque angle control, is a simple yet efficient strategy well suited for the SAVGS problem.
- It is useful to convert all physical and design constraints (i.e. voltage, current, power, torque and speed constraints) to the output torque-speed plane. The resulting torque-speed envelopes can inform the component selection process and can be used in plausibility

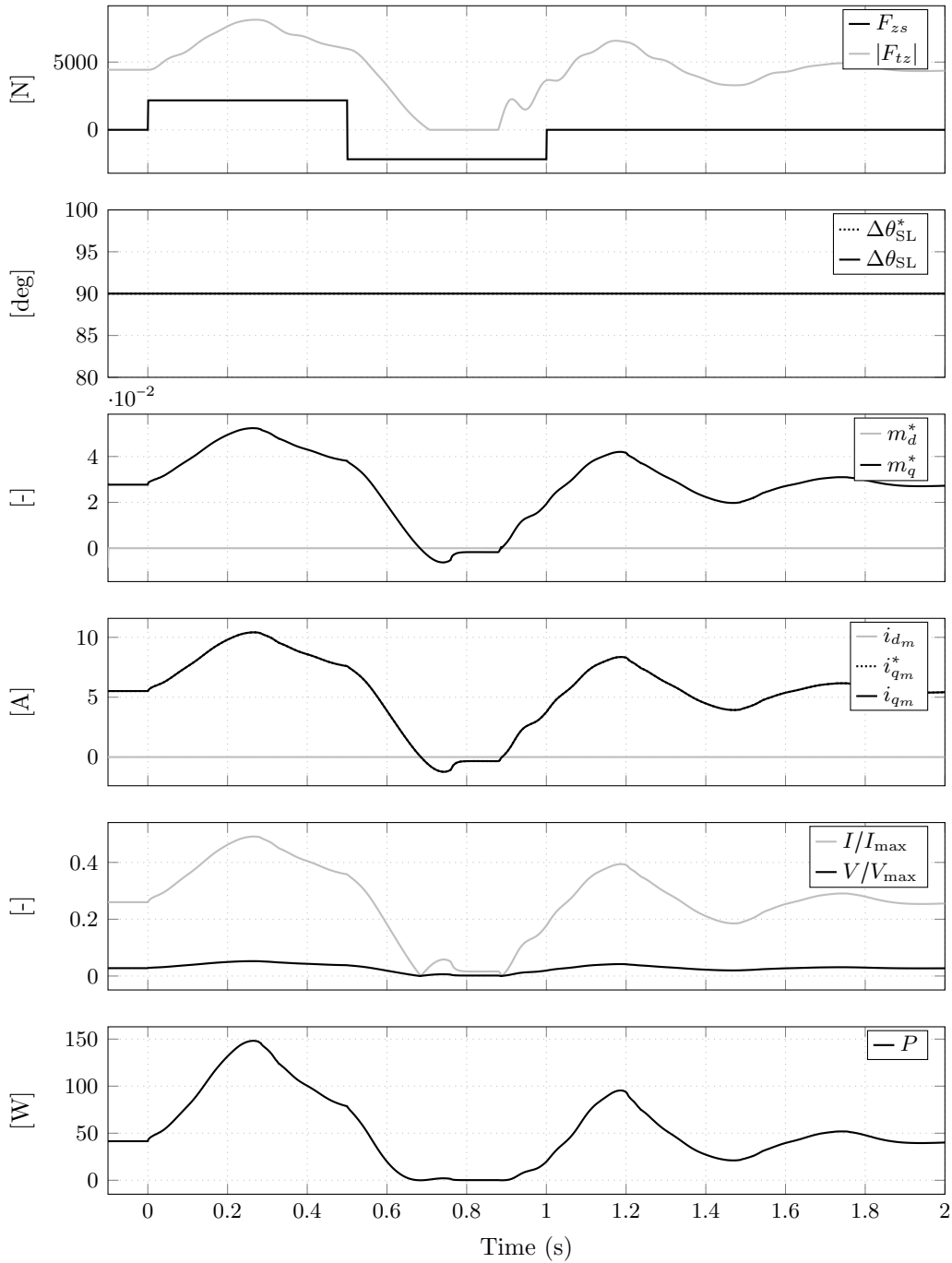


Fig. 4.12: Results obtained with the multi-body model of one rear corner of the GT for a series of step-like changes in the external vertical force acting upon the sprung mass. The amplitude of these changes is equal to half the vertical tyre force in the equilibrium configuration. Starting from the top: the first plot shows the external force applied to the sprung mass, F_{zs} , together with the resulting vertical tyre force, F_{tz} ; the second plot depicts the reference and actual SL angles (as in Fig. 4.10); the reference modulation indexes are presented in the third plot; individual current amplitudes are shown in the fourth plot; normalised current and voltage amplitudes are shown in the fifth plot; and electrical power flows to the PMSM are shown in the sixth plot.

checks for simulation results. A procedure to obtain these envelopes for steady-state conditions has been proposed and explained in detail.

- Position control of the SL is achieved through a three-level control comprising position, velocity and current loops that rely on PID controllers. Within each level, expected operational limits are taken into account when generating references for subsequent loops. This has been shown to be effective and computationally affordable.
- Alternative feedback mechanisms have been tried to enforce operational constraints, but they proved to be computationally expensive and unreliable if used without adequate reference limits, and superfluous when adequate reference limits are in place.
- The proposed control scheme offers good position tracking, well damped response, good disturbance rejection, and zero steady-state error.
- The conservatism/aggressiveness of the proposed control can be altered in different ways: constraints may be artificially reduced, gains may be adjusted or correction factors may be tuned. For example, negative torques may be ruled out in order to avoid potential backlash issues, and different motoring and regenerating power limits can be implemented.
- As future work, backlash and rotational compliances would be two of the main areas with scope for improvement and the work surveyed in [Nordin and Gutman, 2002] would be a good starting point. Also, work towards the elimination of the current control loops from the models could reduce the computational cost of the simulations. However, alternative ways to impose voltage and electrical power constraints would need to be developed.

Chapter 5

SAVGS application to attitude control

Keeping the attitude of the chassis within allowable limits is important to 1) avoid degrading the quality of the ride, 2) avoid increasing the load transfer during acceleration/braking and turning manoeuvres, 3) avoid excessive changes in the camber angle of the wheels, and 4) maintain the desired level of aerodynamic forces on the vehicle.

This chapter, based on the work already published in [Arana et al., 2015b], studies the capability of the SAVGS to effectively control chassis attitude motions. Firstly, the system is studied for steady-state conditions. Mathematical equations are derived based on the principle of virtual work and used to perform a sensitivity analysis that sheds light on the dependencies that exist between the properties of the passive suspension, the vehicle and the SAVGS. Based on this, a pre-dimensioning algorithm for the single-link based on steady-state performance objectives is proposed. Secondly, a general multi-objective control scheme is introduced and the case of heave, pitch, roll and overturning couple distribution control is covered in detail. Finally, a comprehensive set of manoeuvres is employed to (virtually) test the performance of the SAVGS with the proposed control.

5.1 Steady-state analysis

5.1.1 Steady-state mathematical model

In this section, a steady-state mathematical model is developed to study the equilibrium of forces in a vehicle retrofitted with the SAVGS. This model is subsequently used in Section 5.1.2 to derive a pre-dimensioning algorithm based on steady-state performance objectives. It also

provides valuable insights into the behaviour of the SAVGS, which are incorporated in the control scheme presented in Section 5.2.

In a first step, the tyre forces are estimated for the vehicle and operating conditions of interest by applying Newton's second law and Euler's second law to the whole car. Then it is assessed whether the SAVGS is able to maintain an equilibrium of forces in all corners of the vehicle by applying the principle of virtual work.

The mathematical model is fed with the nominal properties of the car, passive suspension and SAVGS, and with a set of operating conditions, including the forward speed, payload, longitudinal and lateral accelerations, and overturning couple and lateral force distributions.

Tyre forces

Vertical forces are estimated as follows. Referring to Fig. 5.1 and letting F_{tz} be the vertical tyre force at any given corner:

$$F_{tz} = F_{tz}^{(\text{ne})} + \Delta F_{tz}^{(\text{pl})} + \Delta F_{tz}^{(\text{ae})} + \Delta F_{tz}^{(a_x)} + \Delta F_{tz}^{(a_y)}, \quad (5.1)$$

where $F_{tz}^{(\text{ne})}$ is the vertical tyre force in the nominal configuration, $\Delta F_{tz}^{(\text{pl})}$ represents the vertical tyre force increment due to a change in payload (sprung mass), $\Delta F_{tz}^{(\text{ae})}$ is the vertical tyre force increment due to a change in aerodynamic downforce, $\Delta F_{tz}^{(a_x)}$ is the vertical tyre force increment due to the longitudinal acceleration, and $\Delta F_{tz}^{(a_y)}$ is the vertical tyre force increment due to the lateral acceleration.

For the equilibrium of forces in the z direction to hold for the whole vehicle as well as for each axle independently, the following must be satisfied:

$$\begin{aligned} {}^1\Delta F_{tz}^{(a_x)} &= {}^2\Delta F_{tz}^{(a_x)} = -{}^3\Delta F_{tz}^{(a_x)} = -{}^4\Delta F_{tz}^{(a_x)}, \\ {}^1\Delta F_{tz}^{(a_y)} &= -{}^2\Delta F_{tz}^{(a_y)}, \quad {}^3\Delta F_{tz}^{(a_y)} = -{}^4\Delta F_{tz}^{(a_y)} \end{aligned} \quad (5.2)$$

where the left superscripts refer to the corner number, with 1 to 4 corresponding to the front left, front right, rear left and rear right corners, respectively. The balance of pitching moments, $\sum M_y = 0$, leads to:

$${}^1\Delta F_{tz}^{(a_x)} = \left[\frac{m_s h_{\text{CMC}} + 2(m_{u_f} R_{\text{wh}_f} + m_{u_r} R_{\text{wh}_r})}{2(b_f + b_r)} \right] a_x \quad (5.3)$$

The front and rear unsprung mass values, m_{u_f} and m_{u_r} , correspond to the mass that is not supported by the spring in each corner of the vehicle (generally tyre, wheel carrier, brake disk, brake pad, caliper, and some portion of the wishbones); the sprung mass, m_s , represents the remainder of the mass of the car; and h_{CMC} is the height above the ground of the center of mass

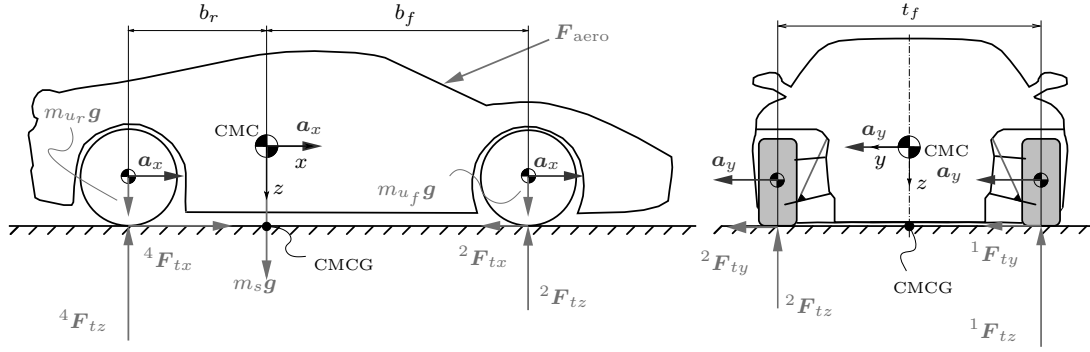


Fig. 5.1: External forces acting on the vehicle in the $x-z$ (left) and $y-z$ (right) planes during acceleration and cornering. Forces acting on the right (left) and front (right) wheels are shown for the $a_x, a_y > 0$ case.

of the chassis (CMC).

The overturning couple due to the lateral acceleration, $M_{x,CMCG}$, can be calculated through (5.4). As it can be seen in Fig. 5.1, this torque is compensated by a lateral load transfer from the inner to the outer wheel in each axle. The percentage of overturning couple compensated by each axle has a significant effect on the directional response of the vehicle, as the tyre properties are non-linear functions of the tyre loads. In a vehicle equipped with a passive suspension system, the contribution of each axle is predetermined by its geometry (i.e. roll center height) and its roll stiffness (i.e. spring stiffness, installation ratio and track width). If equipped with the SAVGS, the front (or rear) SAVGS actuators can be commanded to compensate for a given percentage of the total overturning couple as long as the single-links have not reached their limiting positions. Defining the Overturning Couple Distribution (OCD) factor, σ , as the ratio of overturning couple compensation provided by the rear axle, the front and rear lateral load transfer terms can be obtained from (5.4).

$$\begin{aligned}
 M_{x,CMCG} &= - \left[m_s h_{CMC} + 2 \left(m_{u_f} R_{wh_f} + m_{u_r} R_{wh_r} \right) \right] a_y, \\
 {}^1 \Delta F_{tz}^{(a_y)} &= \left(\frac{1 - \sigma}{t_f} \right) M_{x,CMCG}, \\
 {}^3 \Delta F_{tz}^{(a_y)} &= \left(\frac{\sigma}{t_r} \right) M_{x,CMCG}.
 \end{aligned} \tag{5.4}$$

Finally, the remaining terms in the right hand side of (5.1) can be calculated for each wheel through:

$$\begin{aligned}
 {}^1 F_{tz}^{(ne)} = {}^2 F_{tz}^{(ne)} &= \frac{1}{2} \left[\left(\frac{m_s^{(ne)} b_r}{b_f + b_r} + 2m_{u_f} \right) g + \frac{1}{2} \rho A C_{df} \left(v_x^{(ne)} \right)^2 \right], \\
 {}^3 F_{tz}^{(ne)} = {}^4 F_{tz}^{(ne)} &= \frac{1}{2} \left[\left(\frac{m_s^{(ne)} b_f}{b_f + b_r} + 2m_{u_r} \right) g + \frac{1}{2} \rho A C_{dr} \left(v_x^{(ne)} \right)^2 \right],
 \end{aligned} \tag{5.5}$$

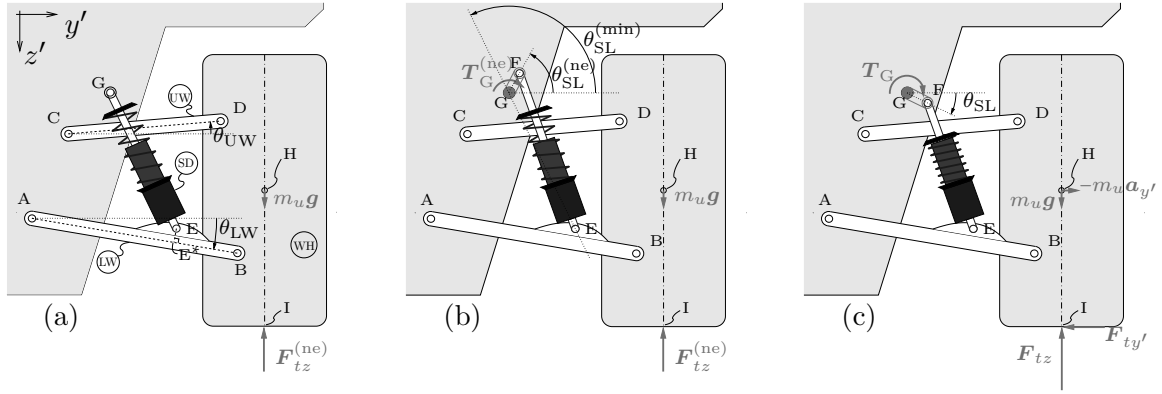


Fig. 5.2: Nominal equilibrium for the passive suspension (a) and for the same system retrofitted with the SL variant of the SAVGS (b). A dynamic equilibrium is shown in (c), where the SAVGS compensates for the increased tyre load and ensures that the wishbones remain in their desired positions. Angles are measured around the x' -axis with respect to the y' -axis, and tyre forces are negative as drawn. Points A, C and G are fixed to the chassis; points B, D, H and I are fixed to the wheel; points E and F are fixed to the spring-damper unit; and point E* is fixed to the lower wishbone.

$${}^1\Delta F_{tz}^{(pl)} = {}^2\Delta F_{tz}^{(pl)} = \frac{1}{2} \left[\left(m_s - m_s^{(ne)} \right) \frac{b_r}{b_f + b_r} g \right], \quad (5.6)$$

$${}^3\Delta F_{tz}^{(pl)} = {}^4\Delta F_{tz}^{(pl)} = \frac{1}{2} \left[\left(m_s - m_s^{(ne)} \right) \frac{b_f}{b_f + b_r} g \right],$$

$${}^1\Delta F_{tz}^{(ae)} = {}^2\Delta F_{tz}^{(ae)} = \frac{1}{2} \left[(1/2) \rho A C_{df} \left(v_x^2 - (v_x^{(ne)})^2 \right) \right], \quad (5.7)$$

$${}^3\Delta F_{tz}^{(ae)} = {}^4\Delta F_{tz}^{(ae)} = \frac{1}{2} \left[(1/2) \rho A C_{dr} \left(v_x^2 - (v_x^{(ne)})^2 \right) \right],$$

where ρ is the air density, A is the frontal area of the vehicle, C_{df} and C_{dr} are the aerodynamic downforce coefficients for the front and rear axlesⁱ, v_x is the longitudinal speed of the vehicleⁱⁱ, and g is the gravitational acceleration.

The lateral forces are estimated as follows: 1) defining ζ as the share of lateral tyre force provided by each wheel within the axle, and 2) combining the equations resulting from the lateral force and yawing moment equilibriums (assuming that the contribution of longitudinal forces is small with respect to that of the lateral forces). This leads to (5.8), where F_{ty} are the lateral tyre

ⁱThese coefficients, which for simplicity are assumed to be independent of the relative air speed, include the effect of the aerodynamic pitching moment.

ⁱⁱIt is assumed that the relative air speed is equal to the forward speed of the vehicle.

forces and $j = 1$ if $a_y \geq 0$, and $j = 2$ otherwise.

$$\begin{aligned}\zeta_f &= \frac{(3-j)F_{ty}}{jF_{ty}}, \quad \zeta_r = \frac{(5-j)F_{ty}}{(2+j)F_{ty}} \\ jF_{ty} &= \frac{1}{1+\zeta_f} \left(m_s \frac{b_r}{b_f+b_r} + 2m_{uf} \right) a_y \\ (2+j)F_{ty} &= \frac{1}{1+\zeta_r} \left(m_s \frac{b_f}{b_f+b_r} + 2m_{ur} \right) a_y\end{aligned}\tag{5.8}$$

The use of ζ as a factor that is independent of tyre properties, sideslip angles etc. is useful to explore a wide range of lateral force distributions representative of different operating conditions and friction coefficients in each corner of the car.

Corner equilibria

A set of equations to assess whether the SAVGS is able to maintain the equilibrium of forces in all corners of the vehicle is derived next. The double-wishbone arrangement under consideration is shown in Fig. 5.2. The passive and active configurations for the nominal equilibrium (nominal payload and forward speed, no longitudinal or lateral acceleration) are shown in Fig. 5.2-a and Fig. 5.2-b, respectively. The dynamic equilibrium shown in Fig. 5.2-c corresponds to the general case in which a steady-state is reached for a given payload, forward speed, overturning couple and lateral force distributions, and longitudinal and lateral acceleration values.

Using the local reference frame, $x'y'z'$, for each corner of the vehicle, the equations can be obtained for a “generic” corner and applied to all others without further modifications. As described in Section 3.2.1, the correspondence between the local reference frames, $x'y'z'$, and the global reference frame, xyz [SAE J670e], is:

- front left and rear left (corners 1 & 3): $x' = -x$, $y' = -y$,
- front right and rear right (corner 2 & 4): $x' = x$, $y' = y$,

and $z' = z$ in all cases. Rotation angles are defined about the local x' with respect to the local y' axis.

The vehicle’s properties, operating conditions and tyre forces as well as the SAVGS configuration (i.e. SL lengths and angular positions) are assumed to be known and fixed in this analysis. Looking at each corner independently, the first step is to calculate the spring force, F_{SD} , required to maintain the wishbones and wheel in the desired positions. This force is then used to determine the torque that the SAVGS actuator needs to apply to the SL.

Spring-damper force applying the principle of virtual work to a corner of the car, and considering a static chassis, the wishbones and wheel being allowed to move with respect to the

chassis in the $y' - z$ plane, the vertical and lateral tyre forces, the inertial forces due to gravity and lateral acceleration, and the spring force acting on the lower wishbone, gives:

$$\delta W = F_{tz} \delta z_I + F_{ty'} \delta y'_I + m_u (g \delta z_H - a_{y'} \delta y'_H) + F_{SD} \delta l_{SD} = 0, \quad (5.9)$$

where the dashed symbol is used to indicate coordinates in the local reference frame. The spring force is considered to be positive when the spring is compressed with respect to its unloaded condition (i.e. $l_{SD} \leq l_{SD0}$), and is given by (5.10),

$$F_{SD} = k \left[l_{SD0} \left(l_{SL}, \theta_{SL}^{(ne)} \right) - l_{SD} (l_{SL}, \theta_{SL}) \right]. \quad (5.10)$$

A general expression for the SD length can be obtained from the geometrical analysis of Fig. 5.2:

$$l_{SD} = \overline{FE} = \sqrt{\overline{FE}_{y'}^2 + \overline{FE}_z^2} = \sqrt{l_{SL}^2 + a_1 l_{SL} + a_0}, \quad (5.11)$$

with a_0 being a constant independent of the SAVGS configuration, and a_0 and a_1 given by

$$\begin{aligned} a_0 &= \alpha_0 + \alpha_1 c_1 + \alpha_2 s_1, \\ a_1 &= \alpha_3 c_2 + \alpha_4 s_2 + \alpha_5 c_{12} + \alpha_6 s_{12}, \end{aligned} \quad (5.12)$$

where $c_1 = \cos(\theta_{LW})$, $s_1 = \sin(\theta_{LW})$, $c_2 = \cos(\theta_{SL})$, $s_2 = \sin(\theta_{SL})$, $c_{12} = \cos(\theta_{LW} - \theta_{SL})$, $s_{12} = \sin(\theta_{LW} - \theta_{SL})$, and constants α_0 to α_6 , which depend only on the passive suspension geometry, are given in (5.13). Subscripts y' and z indicate projections in the y' and z directions, respectively.

$$\begin{aligned} \alpha_0 &= \overline{GA}^2 + \overline{AE}^{*2} + \overline{EE}^{*2}, \quad \alpha_1 = 2 \left(\overline{GA}_z \cdot \overline{EE}^* - \overline{GA}_{y'} \cdot \overline{AE}^* \right), \\ \alpha_2 &= -2 \left(\overline{GA}_z \cdot \overline{AE}^* + \overline{GA}_{y'} \cdot \overline{EE}^* \right), \quad \alpha_3 = 2 \overline{GA}_{y'}, \\ \alpha_4 &= 2 \overline{GA}_z, \quad \alpha_5 = -2 \overline{AE}^*, \quad \alpha_6 = -2 \overline{EE}^*. \end{aligned} \quad (5.13)$$

Finally, a usable expression to assess if the corner is in equilibrium is obtained by dividing (5.9) by δz_I with $\delta z_I \rightarrow 0$:

$$F_{tz} + F_{ty'} \frac{dy'_I}{dz_I} + m_u \left(g \frac{dz_H}{dz_I} - a_{y'} \frac{dy'_H}{dz_I} \right) + F_{SD} \frac{dl_{SD}}{dz_I} = 0. \quad (5.14)$$

The tyre forces, F_{tz} and $F_{ty'}$, can be calculated as a function of the vehicle characteristics (i.e. m_s , m_{uf} , m_{ur} , b_f , b_r , t_f , t_r), the OCD, σ , the share of lateral tyre forces within each axle, ζ_f and ζ_r , and the operating conditions, v_x , a_x and a_y , through equations given in Section 5.1.1. Terms $\frac{dy'_I}{dz_I}$, $\frac{dz_H}{dz_I}$, and $\frac{dy'_H}{dz_I}$ depend solely on the suspension geometry (i.e. wishbone lengths and angles, wheel radius and camber angle). F_{SD} is given by (5.10), and the spring deformation rate

with respect to the vertical displacement of point I can be split into two factors:

$$\frac{dl_{SD}}{dz_I} = \frac{dl_{SD}}{d\theta_{LW}} \frac{d\theta_{LW}}{dz_I}. \quad (5.15)$$

The second term, $\frac{d\theta_{LW}}{dz_I}$, depends solely on the passive suspension arrangement and can be treated as a parameter, whereas the first one depends on the SL length and its angle:

$$\frac{dl_{SD}}{d\theta_{LW}} = \frac{1}{2l_{SD}} \left[-\alpha_1 s_1 + \alpha_2 c_1 - l_{SL} (\alpha_5 s_{12} - \alpha_6 c_{12}) \right]. \quad (5.16)$$

Actuation torque applying once again the principle of virtual work, but this time focusing on the SL alone, gives:

$$\delta W = T_G \delta\theta_{SL} + F_{SD} \delta l_{SD} = 0. \quad (5.17)$$

A usable expression that relates the actuation torque to the SD force and the geometrical arrangement is obtained by dividing (5.17) by $\delta\theta_{SL}$ and taking the limit $\delta\theta_{SL} \rightarrow 0$:

$$T_G = -F_{SD} \frac{dl_{SD}}{d\theta_{SL}}, \quad (5.18)$$

with $\frac{dl_{SD}}{d\theta_{SL}}$ given by:

$$\frac{dl_{SD}}{d\theta_{SL}} = \frac{l_{SL}}{2l_{SD}} (-\alpha_3 s_2 + \alpha_4 c_2 + \alpha_5 s_{12} - \alpha_6 c_{12}). \quad (5.19)$$

5.1.2 SL pre-dimensioning based on steady-state performance requirements

An important design criterion for active suspension systems dealing with chassis attitude motions is the maximum roll (pitch) angle allowable for a given level of steady-state lateral (longitudinal) acceleration.

The aim in this section is to derive a pre-dimensioning algorithm that computes the shortest SL lengths and associated nominal angles that satisfy the steady-state performance objectives. By minimizing the SL lengths, the required actuation torques are minimised and the surplus torques available at any given time to rotate the single-links are maximised. As a consequence, the actuators are less likely to saturate and the control design can be simplified. The dynamic response of the SAVGS can thus be improved through proper SL dimensioning.

Parameter sensitivity analysis

A parameter sensitivity analysis is performed to cast some light on the dependencies that exist between the vehicle, the passive suspension and the SAVGS properties.

Once all vehicle and SAVGS parameters have been fixed, the maximum longitudinal and lateral acceleration values for which full (heave, suspension pitch and roll) or partial (suspension pitch and roll) attitude control is achievable can be calculated by making use of the equations derived in Section 5.1.1. This combination of longitudinal and lateral accelerations will be referred to as the reachable envelope.

In order to compute this envelope, all combinations of longitudinal and lateral acceleration must be considered, and for each of them, it must be determined whether the vehicle can be in equilibrium with the full and partial attitude controls in place. The equilibrium is possible for a given set of operating conditions if, and only if, the SAVGS is able to maintain the force balances in all corners of the car. Thus, in each corner, $f(\theta_{\text{SL}})$ as given in (5.20), must equal zero for some value $\theta_{\text{SL}} \in [\theta_{\text{SL}}^{(\min)}, \theta_{\text{SL}}^{(\max)}]$.

$$f(\theta_{\text{SL}}) = F_{tz} + F_{ty'} \frac{dy'_I}{dz_I} + m_u \left(g \frac{dz_H}{dz_I} - a_{y'} \frac{dy'_H}{dz_I} \right) + F_{\text{SD}}(\theta_{\text{SL}}) \frac{dl_{\text{SD}}(\theta_{\text{SL}})}{dz_I} = 0 \quad (5.20)$$

Thanks to the intermediate value theorem, it is known that if $\exists a, b \in [\theta_{\text{SL}}^{(\min)}, \theta_{\text{SL}}^{(\max)}]$ so that

$$f(a) \cdot f(b) \leq 0, \quad (5.21)$$

there is at least one SL angle in $[\theta_{\text{SL}}^{(\min)}, \theta_{\text{SL}}^{(\max)}]$ for which (5.20) holds and hence for which the equilibrium is possible.

For reasonably sized single-links (i.e. $\theta_{\text{SL}} \leq \overline{\text{EG}}/10$), the effect of the SAVGS on the installation ratio, which is captured in the term $\frac{dl_{\text{SD}}}{dz_I}$, is modest. Thus, once the suspension geometry and tyre loads have been fixed, the evolution of f with θ_{SL} closely follows that of F_{SD} . As the compression of the SD is minimum and maximum at $\theta_{\text{SL}}^{(\min)}$ and $\theta_{\text{SL}}^{(\max)} = \theta_{\text{SL}}^{(\min)} + \pi$, respectively, it can be seen that $\min(f) \approx f(\theta_{\text{SL}}^{(\min)})$ and $\max(f) \approx f(\theta_{\text{SL}}^{(\max)})$. Therefore the sufficient and necessary condition (5.21) can be replaced with

$$f(\theta_{\text{SL}}^{(\min)}) \cdot f(\theta_{\text{SL}}^{(\max)}) \leq 0. \quad (5.22)$$

$\theta_{\text{SL}}^{(\min)}$ and $\theta_{\text{SL}}^{(\max)}$ are obtained from the geometrical analysis of the suspension arrangement, and (5.22) can be evaluated during the calculation procedure at low computational expense.

The procedure followed to calculate the reachable envelopes is outlined in Fig. 5.3, and some of the key ideas behind it are: 1) the envelopes are symmetric with respect to the a_x axis, and therefore only positive values for the lateral acceleration are considered; 2) if the SAVGS is not

able to maintain the equilibrium for a given pair (a_x^*, a_y^*) , then it is not able to maintain it for $(a_x^*, \gamma a_y^*)$ if $\gamma \geq 1$; 3) if the SAVGS is able to maintain the equilibrium for (a_x^*, a_y^*) , then it is also able to maintain it for $(a_x^*, \gamma a_y^*)$ if $\gamma \leq 1$; 4) if the SAVGS is able to maintain the equilibrium for (a_x^*, a_y^*) for the full attitude control case, then it is also able to maintain it for the partial attitude control case; 5) tyre forces change with chassis height. The algorithm sweeps the longitudinal acceleration from a starting value $a_x^{(\min)}$, to a final value $a_x^{(\max)}$ in fixed Δa_x steps. For each a_x , a_y is set to zero and increased in variable Δa_y steps until the maximum lateral acceleration for which full attitude control is possible is within a certain tolerance, tol_1 . The partial attitude control case is considered next. It starts from the maximum lateral acceleration value found for the full attitude control case, $a_y^{(f)}$, and allows for changes in chassis height. Fig. 5.4 complements Fig. 5.3 and clarifies the approach followed in this case.

The concept of the reachable envelope always applies to the whole vehicle. However, the SAVGS requirements at the front and rear axles can be studied separately by assuming that one of the axles is fitted with very long SLs, thus reducing the number of variables involved and facilitating the interpretation of results. This is the case because the axle equipped with very long SLs will never be the limiting factor when calculating the reachable $a_x - a_y$ points, and therefore the resulting envelope depends exclusively on the characteristics of the other axle. From now on, the reachable envelope for the front (rear) axle will be used to refer to the $a_x - a_y$ points for which the SAVGS is able to provide full attitude control when the rear (front) axle comprises very long SLs.

Fig. 5.5 shows the reachable envelopes for the front and rear axles of the GT. Longitudinal and lateral acceleration values have been normalised by the peak values reachable for some reference configuration: $\hat{a}_x = a_x/a_x^{(\text{ref,peak})}$, $\hat{a}_y = a_y/a_y^{(\text{ref,peak})}$. SL length, nominal SL angle, OCD, distribution of lateral forces, spring stiffness and sprung mass have been swept by multiplying their nominal values (denoted by superscript $^{\text{ref}}$) with a scaling factor, SF.

The influence of SL lengths is shown in the top row of Fig. 5.5. Changing the SL length scales the size of the reachable envelope almost linearly, but it does not affect either its shape nor its location in the $a_x - a_y$ plane. Increasing the SL lengths is very beneficial in terms of the attitude control capabilities of the SAVGS, but unfortunately this improvement comes at the expense of higher actuation torques for any given set of operating conditions.

The effect of the nominal SL angle is presented in the second row of Fig. 5.5. Altering the nominal SL angle affects the location of the reachable envelope in the $a_x - a_y$ plane, but not its size nor its shape. Increasing the nominal angle improves the behaviour of the rear axle during braking, and of the front axle during acceleration. Moreover, roll correction under pure lateral acceleration is maximum for nominal angles equal or close to $\pi/2$.

The impact of the OCD is displayed in the third row of Fig. 5.5. As expected, this parameter does not affect the behaviour under pure longitudinal acceleration. It scales the peak reachable lateral

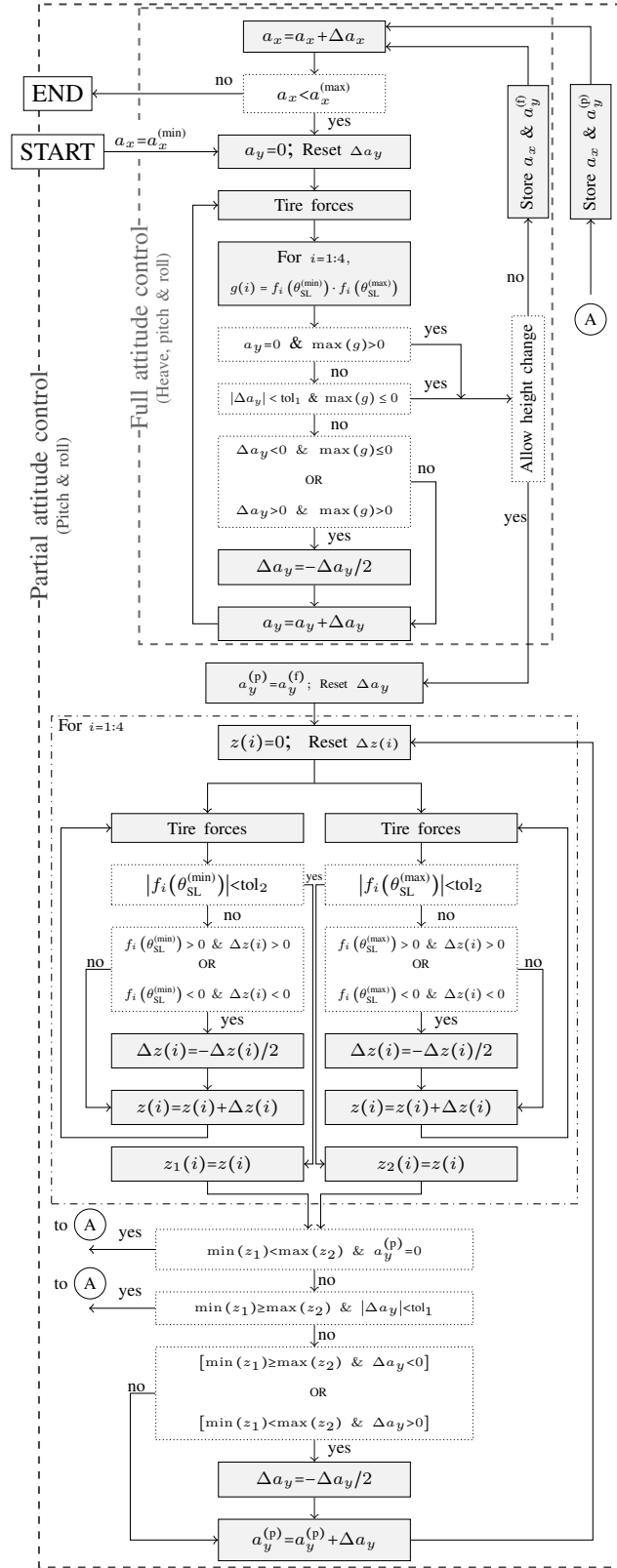


Fig. 5.3: Flowchart to determine the reachable envelope for the full (f) and partial (p) attitude control approaches. z is the chassis height change ($z > 0 \rightarrow$ chassis is lowered), and i refers to the corner number.

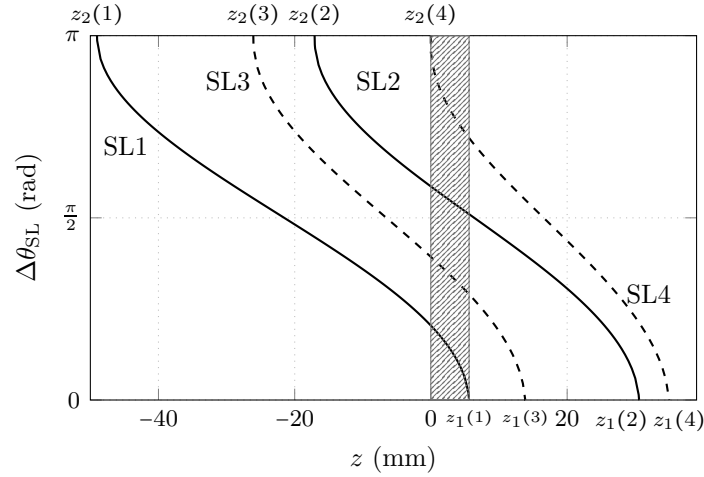


Fig. 5.4: Example plot of $f = 0$ lines as a function of SL angles ($\Delta\theta_{\text{SL}} = \theta_{\text{SL}} - \theta_{\text{SL}}^{(\min)}$) and chassis height change. An equilibrium is possible, if and only if, $\exists z$ for which all corners can be in equilibrium (shaded area in this figure). Thus, $\min(z_1)$ must be greater or equal than $\max(z_2)$.

acceleration without distorting the envelope in the a_x direction. The rear (front) axle benefits from lower (higher) σ values. The selection of σ is mainly determined by handling considerations, and therefore it is likely to be an input parameter when designing the SAVGS.

The importance of the lateral force distribution between the inner and outer wheels is apparent from the fourth row of Fig. 5.5. A lower ζ value implies that the split of lateral force is more uneven, with the outer wheels generating a higher proportion of the total lateral force. Changing ζ moves the upper vertex of the reachable envelope in the a_x direction by Δ . As ζ decreases, the upper vertex moves toward regions with higher tyre loads. In practice, ζ is likely to vary widely depending on the operating conditions and on the road characteristics under each tyre. For a given variation of ζ , Δ also depends on the suspension geometry through dy'_1/dz_1 (see (5.20)). The greater this term, the greater Δ will be. Generally speaking, dy'_1/dz_1 is larger in vehicles with roll centers far from the road surface, as this tends to imply a larger coupling of lateral and vertical relative displacement between the wheel and the chassis. Depending on the level of certainty required from the predicted $a_x - a_y$ envelopes, a different range of ζ values should be considered when dimensioning the single-links. The predicted envelope should be obtained as the intersection (which is still triangular) of those calculated with all ζ values of interest.

The last two factors considered in the sensitivity analysis are the spring stiffness and the payload (bottom rows of Fig. 5.5). The size of the reachable envelope is scaled linearly with the spring stiffness, but its position and shape remain unchanged. Increasing the spring stiffness is beneficial, and its effect is similar to that of lengthening the SL. Furthermore, although increasing the SL lengths implies greater demands from the actuators, stiffening the spring does not. This may have a negative impact on comfort or other suspension functions, but in terms of attitude control, the stiffer the better.

Changes in payload, on the other hand, affect the size and the location of the reachable envelope in the $a_x - a_y$ plane. As expected, a higher payload implies a smaller reachable envelope and a drift towards regions with lower tyre loads.

Pre-dimensioning algorithm

A successful retrofit of a vehicle with the SAVGS requires to take into account many factors and to reach design trade-offs in different areas. The pre-dimensioning procedure outlined in this section fulfills an important function: to provide a sensible estimate for the SL lengths and nominal angles with which to start the inherently iterative design process.

The procedure takes as inputs the properties of the vehicle and a target $a_x - a_y$ area in which full attitude control is required, and it provides as outputs the smallest SL lengths and associated nominal angles that ensure that the reachable envelope contains the target envelope. The solution is obtained using a strategy that takes into account the physics of the system under consideration, and that exploits the fact that each axle can be dimensioned independently.

As shown in Section 5.1.2, neither the SL lengths nor the nominal angles have a major impact on the shape of the reachable area. Knowing this, a first reachable envelope is calculated assuming small SL lengths (e.g. equal to 1% of $\overline{EG}$), and nominal angles equal to $\pi/2$. Depending on the relative size and aspect ratios of the target and reachable areas, it is determined whether the reachable envelope is potentially able (i.e. large enough) to contain the target envelope, too big, or of the right size. If it's size is appropriate, then the nominal SL angle is updated in steps so that the reachable envelope moves towards the desired position. If it is not, then the SL length is increased/reduced in steps, until it's size matches the desired one. Once one of the design variables (either length or nominal angle) has been updated and is deemed to be appropriate, the other one is corrected if necessary, and the reachable envelope is recalculated. This process is repeated until the reachable envelope contains the target envelope, and the distance between the reachable envelope and one or more of the vertexes of the target envelope is within a given tolerance.

Fig. 5.6 shows the final envelopes for an example vehicle and two different sets of steady-state performance requirements. On the left hand side, a target envelope with a narrow base is shown. The reachable envelopes are wider and therefore the peak lateral acceleration for which full attitude control is required becomes the limiting factor. On the right hand side, a wider target envelope is considered. In this case, the requirements in terms of lateral and longitudinal acceleration determine the SL lengths.

As expected, the reachable envelope for the partial attitude control approach is, in all cases, equal or larger than that obtained for the full attitude control approach and lies somewhere in between the reachable envelopes for each axle under full attitude control conditions.

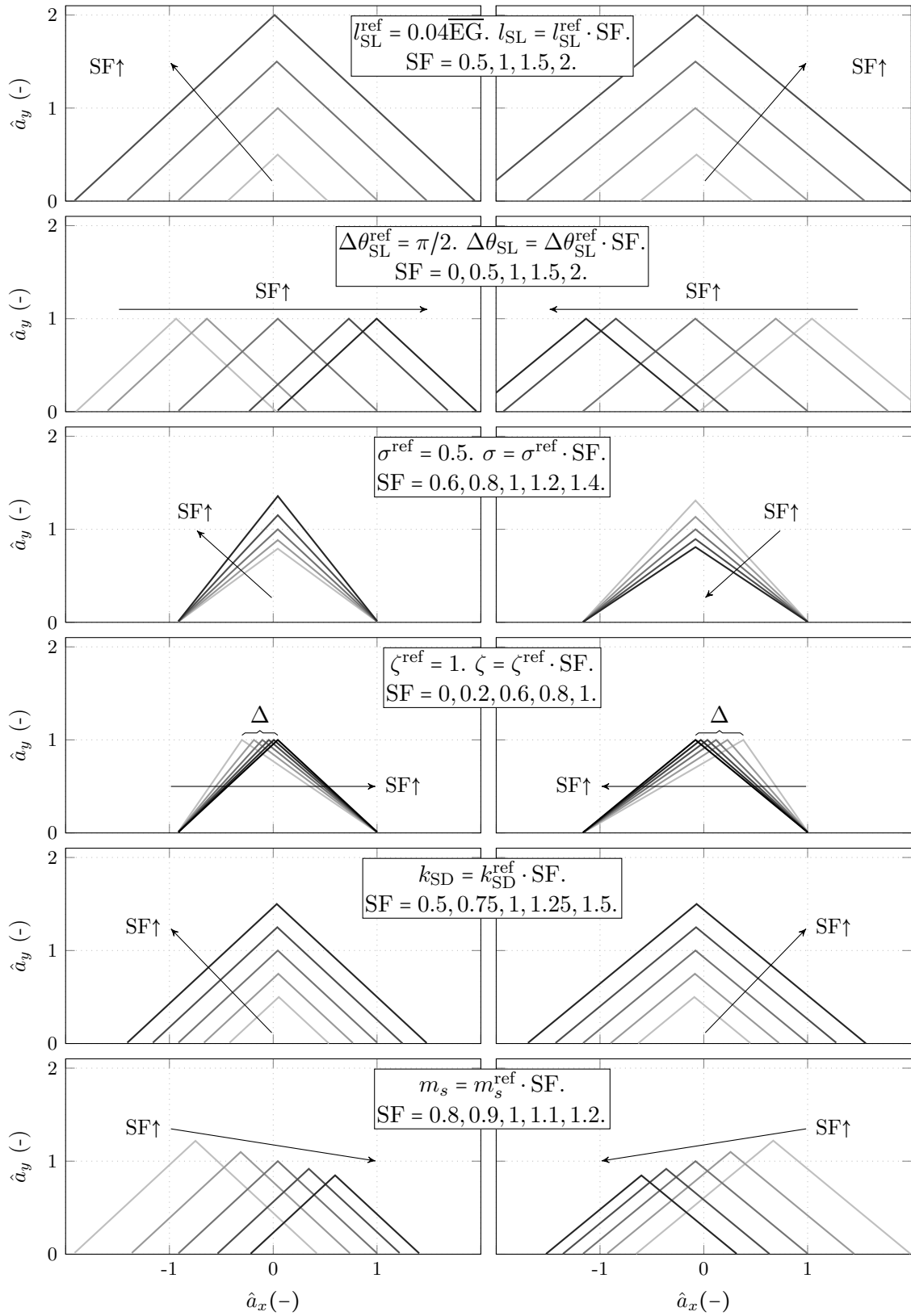


Fig. 5.5: Reachable envelopes for the front (left) and rear (right) axes as a function of SL lengths, nominal SL angles, OCD, lateral force distribution, spring stiffness, and sprung mass. Acceleration values have been normalised by the peak longitudinal and lateral accelerations reachable with the reference parameter values. Reference parameter values, denoted by ^{ref} , have been multiplied by a scaling factor, SF.

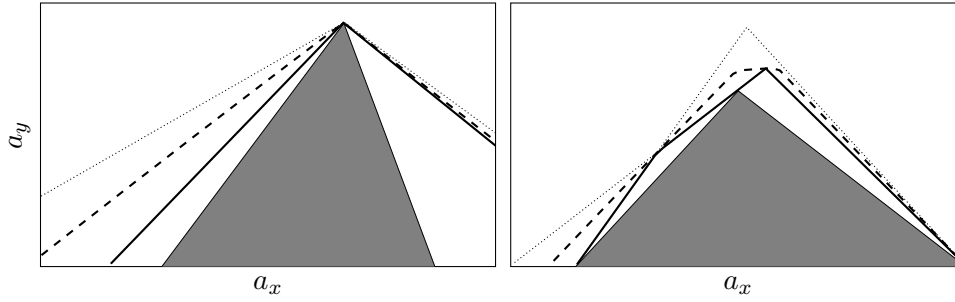


Fig. 5.6: Target (shaded gray) and reachable envelopes for the full attitude (solid black) and partial attitude (dashed black) control approaches for two different sets of steady-state performance objectives. Full control triangles for individual axes shown as dotted lines.

5.2 Attitude control scheme

The mathematical model needed to study the SAVGS capabilities when tackling changes in chassis attitude has been established in Section 5.1.1 for the steady-state case and the prediction of this model has been used to size the SLs for certain performance (Section 5.1.2). In this section the full-vehicle multi-body model with full dynamics is used to design control algorithms that can drive the SAVGS with the desired performance under general conditions. A general multi-objective control scheme is proposed first, and it is then specialised for the control of chassis attitude motions and OCD and subsequently tested by simulation in Section 5.3. The realisation of the desired control performance might generally involve some iteration between the dimensioning process and the control system development.

A general control scheme for the SAVGS, suitable to tackle N control objectives simultaneously, is shown in Fig. 5.7. Measurements taken from the vehicle are conditioned and fed to a higher level logic that also receives inputs from the driver, the environment and other intelligent systems on-board. This system determines the control objectives for the SAVGS and sets the priorities (i.e. relative gains) for each of them. The outer loop subsequently calculates the SL position reference, θ_{SL}^* , for each corner of the vehicle based on the measured/estimated signals and the control objectives provided by the higher level logic. The actuator units, which comprise the actuator (PMSM + gearbox) and the position control scheme (P.C.) in each corner of the vehicle as detailed in Chapter 4, try to follow the aforementioned references whilst respecting all physical and design constraints (voltage, current, torque, speed and power limits).

Each controller in the outer loops calculates a position increment for the corresponding actuator. These increments are combined and added to the actual SL position to form the (unsaturated) position reference. This ensures zero steady-state tracking error for realistic control demands, as the reference angle equals the actual angle when all tracking errors are zero.

The position references should not grow indefinitely and they are saturated so that $\theta_{SL}^* \in [\theta_{SL}^{(\min)}, \theta_{SL}^{(\max)}]$. Moreover, (5.20) indicates that the initial and final 20-30 deg (out of 180 deg)

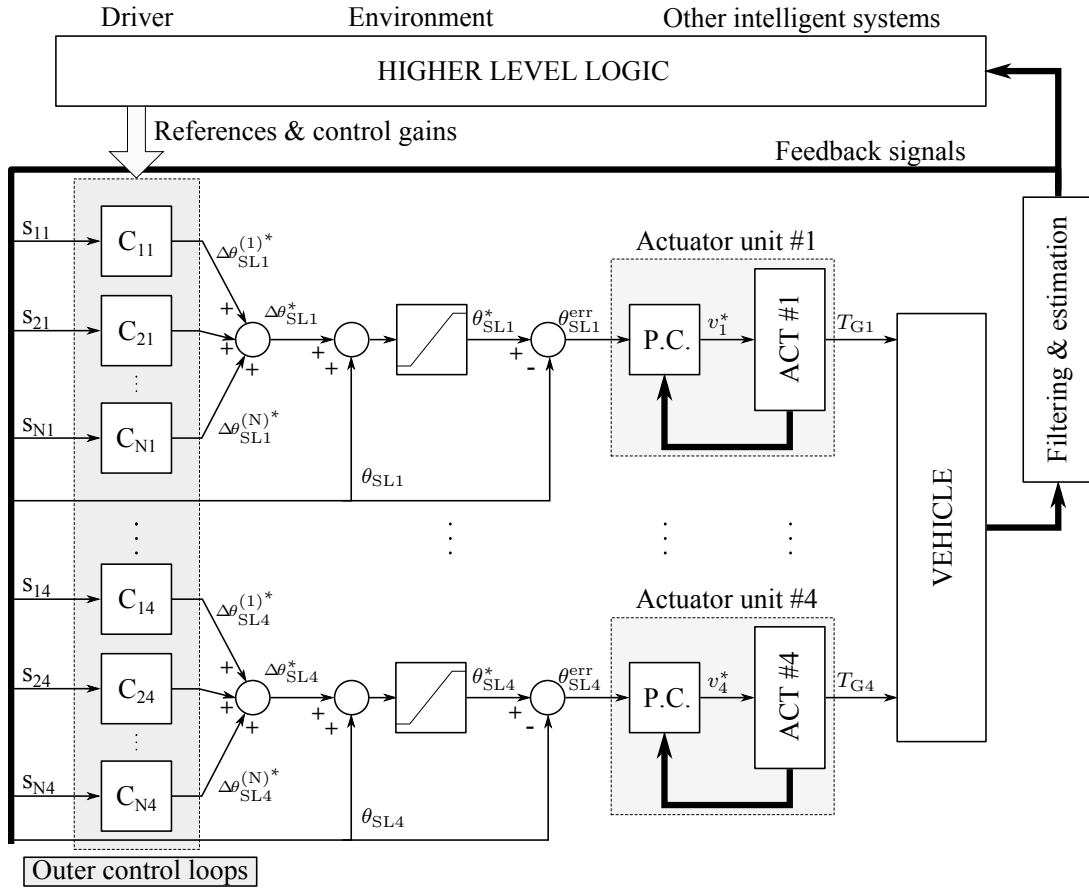


Fig. 5.7: General control scheme for the SAVGS.

of SL rotation barely provide any additional suspension force changes. Thus, the actuators may waste precious time when trying to rotate to the very limits of the allowable range, and hence the proposed control limits the range to $[\theta_{SL}^{(\min)} + \Delta\theta_{SL}^{(\min)}, \theta_{SL}^{(\max)} - \Delta\theta_{SL}^{(\max)}]$, where $\Delta\theta_{SL}^{(\min)}$ and $\Delta\theta_{SL}^{(\max)}$ may be constants or functions of the operating conditions.

The general scheme shown in Fig. 5.7 is tailored in this chapter for the control of chassis attitude motions and of the OCD. For corners $i = 1, \dots, 4$, C_{1i} deals with vertical motions (heave) of CMC, C_{2i} with pitch rotations of the chassis, C_{3i} with roll rotations of the chassis, and C_{4i} with the distribution of the overturning couple between the front and rear axles. C_{1i} indirectly ensures that $\theta_{SLi} = \theta_{SLi}^{(ne)}$ under nominal conditions, and therefore that the reachable envelope (see Fig. 5.5) is centered at the desired a_x value. The controllers used to generate the results shown in Section 5.3 are manually tuned PIDs acting on the first time derivatives of the input signals. These provide good performance and robustness characteristics for the attitude control problem.

Human comfort and perception are subjective and complex topics and the selection of suitable references for the attitude controllers is not a trivial task [Buma et al., 2008]. Here, constant zero

references are selected for pitch and roll motions, and a constant reference equal to the nominal height of CMC is considered for the heave controller. This provides a consistent framework for the study of the capabilities of the SAVGS.

Low-pass filtering the input signals, s_{1i} to s_{4i} , is of paramount importance in the present simulation studies to uncouple the fast dynamics of the four actuators and to stabilise the model numerically (a time constant of 0.005 s is appropriate in this case). It enables the additive blending of control commands proposed here, and avoids numerical issues when selecting similar gains for both axes and/or for different control objectives. That is, it provides the necessary actuator-to-actuator and objective-to-objective decoupling needed to ensure smooth operation of the attitude control scheme.

Outputs from the vertical (heave) motion, pitch, roll and OCD control loops are given in (5.23) to (5.26), where all control gains, K , are positive. Actuators operate in pairs due to the geometric and loading symmetry of the attitude control problem considered, but this would not be the case if the controller was aiming to tackle additional disturbances such as road irregularities. For the OCD case, the control approach is as follows: first, the total roll moment, M_{roll} , generated by the vertical tyre loads is estimated; then, the roll moment to be compensated by each axle is computed as a function of the total roll moment and the reference share to be provided by the rear axle, σ^* . That is, $M_{\text{roll},f}^* = (1 - \sigma^*) M_{\text{roll}}$ and $M_{\text{roll},r}^* = \sigma^* M_{\text{roll}}$. The tracking errors are defined as $e_f = M_{\text{roll},f}^* - M_{\text{roll},f}$ and $e_r = M_{\text{roll},r}^* - M_{\text{roll},r}$.

$$\begin{aligned}\Delta\theta_{\text{SL1}}^{(1)} &= \Delta\theta_{\text{SL2}}^{(1)} = +K_{\text{p},f}^{(1)} \dot{z}_{\text{CMC}} + K_{\text{i},f}^{(1)} \Delta z_{\text{CMC}} + K_{\text{d},f}^{(1)} \ddot{z}_{\text{CMC}} \\ \Delta\theta_{\text{SL3}}^{(1)} &= \Delta\theta_{\text{SL4}}^{(1)} = +K_{\text{p},r}^{(1)} \dot{z}_{\text{CMC}} + K_{\text{i},r}^{(1)} \Delta z_{\text{CMC}} + K_{\text{d},r}^{(1)} \ddot{z}_{\text{CMC}}\end{aligned}\quad (5.23)$$

$$\begin{aligned}\Delta\theta_{\text{SL1}}^{(2)} &= \Delta\theta_{\text{SL2}}^{(2)} = -K_{\text{p},f}^{(2)} \dot{\theta} - K_{\text{i},f}^{(2)} \theta - K_{\text{d},f}^{(2)} \ddot{\theta} \\ \Delta\theta_{\text{SL3}}^{(2)} &= \Delta\theta_{\text{SL4}}^{(2)} = +K_{\text{p},r}^{(2)} \dot{\theta} + K_{\text{i},r}^{(2)} \theta + K_{\text{d},r}^{(2)} \ddot{\theta}\end{aligned}\quad (5.24)$$

$$\begin{aligned}\Delta\theta_{\text{SL1}}^{(3)} &= -\Delta\theta_{\text{SL2}}^{(3)} = -K_{\text{p},f}^{(3)} \dot{\phi} - K_{\text{i},f}^{(3)} \phi - K_{\text{d},f}^{(3)} \ddot{\phi} \\ \Delta\theta_{\text{SL3}}^{(3)} &= -\Delta\theta_{\text{SL4}}^{(3)} = -K_{\text{p},r}^{(3)} \dot{\phi} - K_{\text{i},r}^{(3)} \phi - K_{\text{d},r}^{(3)} \ddot{\phi}\end{aligned}\quad (5.25)$$

$$\begin{aligned}\Delta\theta_{\text{SL1}}^{(4)} &= -\Delta\theta_{\text{SL2}}^{(4)} = +K_{\text{p},f}^{(4)} \dot{e}_f + K_{\text{i},f}^{(4)} e_f + K_{\text{d},f}^{(4)} \ddot{e}_f \\ \Delta\theta_{\text{SL3}}^{(4)} &= -\Delta\theta_{\text{SL4}}^{(4)} = +K_{\text{p},r}^{(4)} \dot{e}_r + K_{\text{i},r}^{(4)} e_r + K_{\text{d},r}^{(4)} \ddot{e}_r\end{aligned}\quad (5.26)$$

5.3 Simulation results

In this section, the effect of the SAVGS on the dynamic response of a vehicle is investigated through the simulation of five standard open-loop manoeuvres and of two longitudinal acceler-

ation manoeuvres. In order to understand the influence of the SAVGS, the dynamic response of the vehicles retrofitted with the SAVGS is compared with that of the original passive configurations.

The proposed control system is very flexible regarding the selection of control priorities. Here, most results are obtained with a controller that prioritises roll mitigation and does not explicitly act upon the OCD (this will be referred to as C-RM). Where appropriate, sets of gains that prioritise OCD control (this will be referred to as C-OCD) or pitch mitigation (C-PM) are also considered and the differences discussed. Any intermediate configuration is possible, and the results presented should be interpreted as some of the boundaries of the response of the car rather than as the results of the best compromise control. Actual control gains used are given in Table 5.1, and the control parameter that regulates the use of negative torques, $\alpha_{\min}$ as explained in Chapter 4, has been set to 0.25.

Table 5.1: Attitude control gains for the GT and SUV

Controller	Control aim	Eq. #	Axle	P	I	D
C-RM	Heave	(5.23)	F&R	0.1	2	1
	Pitch	(5.24)	F&R	1	5	0.04
	Roll	(5.25)	F&R	5	25	0.04
	OCD	(5.26)	F&R	0	0	0
C-OCD	Heave	(5.23)	F&R	0.01	0.2	0.1
	Pitch	(5.24)	F&R	0.1	0.5	0.004
	Roll	(5.25)	F&R	0.5	2.5	0.004
	OCD	(5.26)	F&R	0	0.0001	0
C-PM	Heave	(5.23)	F&R	0.1	2	1
	Pitch	(5.24)	F&R	1	5	0.04
	Roll	(5.25)	F&R	0.2	1	0.04
	OCD	(5.26)	F&R	0	0	0

5.3.1 Steady-state cornering

This open-loop test method is simulated to assess the potential of the SAVGS for roll mitigation and for the alteration of the directional response of the car.

Vehicles are driven at increasing forward speeds in a circular path of 100m radius, as defined in [ISO 4138:2004]. Firstly, the SAVGS potential for attitude control is analyzed by setting the control gains associated with the OCD to zero (C-RM). Fig. 5.8 shows that the roll angle is completely neutralised up to lateral accelerations of approximately 6 and 4 m/s², and that roll reductions remain above ~50% for the expected range of lateral accelerations for each vehicle.

The effect of the SAVGS on the directional response can be clearly appreciated through the

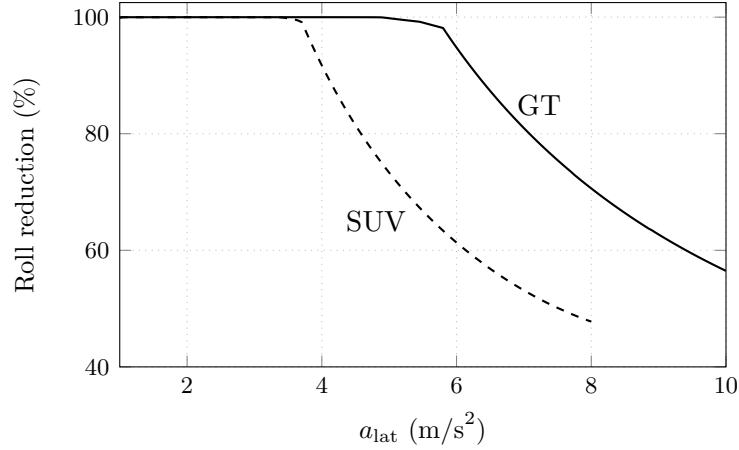


Fig. 5.8: Roll reduction with respect to the original passive vehicles during steady-state cornering.

standard plots of yaw velocity gain (yaw velocity, $\dot{\Psi}$, over steering wheel angle, δ) vs. forward velocity shown in Fig. 5.9. The response with the passive suspensions (dotted) is very similar to that obtained with the SAVGS and C-RM (solid). For the GT, the active suspension leads to slightly more understeering at high speeds, whereas for the SUV the drop in yaw velocity gain is pushed to higher speeds.

Results are also presented for an alternative set of control gains corresponding to almost exclusive focus on the control of the OCD (dashed). In this case (C-OCD), roll angles are similar to those of the passive arrangements, but the directional responses can be greatly affected by the SAVGS. Cases shown correspond to $\sigma^* = 0$ (increased understeering) and $\sigma^* = 1$ (increased oversteering).

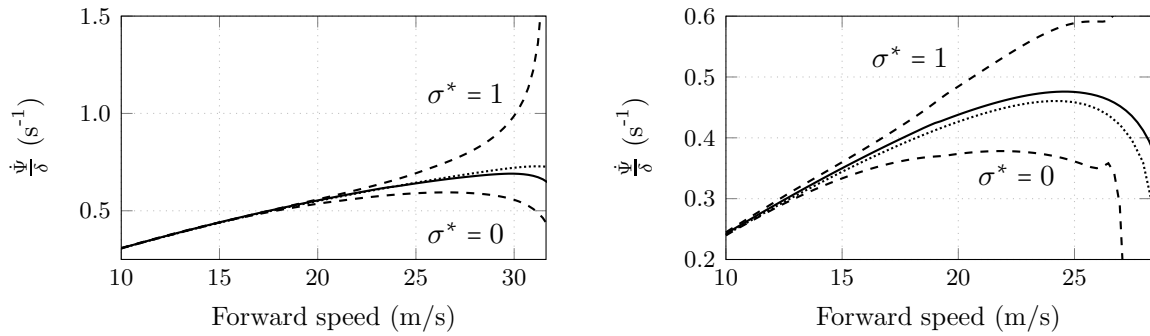


Fig. 5.9: Yaw velocity gain as a function of forward speed for the GT (left) and SUV (right) while driving in a circle of 100 m radius.

5.3.2 Step steer

This is a transient open-loop manoeuvre useful to compare the attitude control capabilities of the SAVGS during rapid transients with those obtained under steady-state conditions.

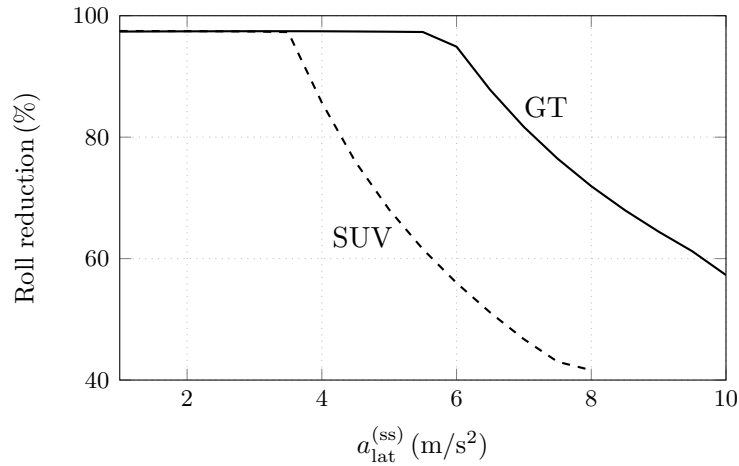


Fig. 5.10: Reduction in peak roll angle during step-steer.

A sudden steering wheel input is applied when driving in a straight line at 100 km/h as defined in [ISO 7401:2011]. The steering wheel angle is increased at a constant rate of 500 deg/s from 0 to $\delta^{(ss)}$, where $\delta^{(ss)}$ is the steering wheel angle needed by the vehicles equipped with passive suspensions to maintain a certain level of steady-state lateral acceleration, $a_{lat}^{(ss)}$, when driving at a constant forward speed of 100 km/h.

The reduction in peak roll angle achieved thanks to the SAVGS and C-RM is presented in Fig. 5.10. Comparing this with Fig. 5.8, it becomes clear that the attitude control provided by the SAVGS is almost as good as in the steady-state case.

5.3.3 Continuous sinusoid steer

This lateral transient open-loop test method is useful to understand the performance of the SAVGS at various frequencies and to assess its power demands when pressed to its limits.

A continuous steering-wheel sinusoid is applied when driving in a straight line at 100 km/h as defined in [ISO 7401:2011]. The amplitude of the steering-wheel sinusoid is $\delta^{(ss)}$, as defined in Section 5.3.2. Steering frequencies from 0.2 Hz to 3 Hz in 0.2 Hz steps are applied and results obtained with the passive and active suspensions compared. For each case, 10 full steering cycles are simulated and results are processed from the last 5 cycles to remove initial transients. The ratio of the RMS roll angle obtained with the active suspension and C-RM over the one computed with the passive suspension is shown in Fig. 5.11. This ratio remains below 10% in most cases and rises to 20% (GT) and 55% (SUV) at 1 Hz and large steering-wheel amplitudes. The transition between the low and high ratio areas is approximately linear and goes from 4 and 6 m/s² at 0.5 Hz for the SUV and GT, respectively (similar to the corner frequencies for steady-state cornering shown in Fig. 5.8), to ~8 m/s² at 2 Hz. The average power consumed by the SAVGS, considering losses in the power electronics and actuators, is shown in the same

figures as contour lines. In the worst case, the average power consumption barely exceeds 500 W and 3 kW for the GT and SUV, respectively.

To check if the actuators are requested to operate above safe levels, the worst (in the sense of leading to the largest proportion of time working above torque limits suitable for continuous operation) torque histogram obtained from the SUV actuators is presented in Fig. 5.12. This corresponds to a steering frequency of 1 Hz and a steering amplitude leading to $a_{\text{lat}}^{(\text{ss})} = 8 \text{ m/s}^2$. Even in such a demanding event, the output torque remains below the limit for continuous operation during 75% of the time, and torques close to the peak limit are reached during less than 1% of the time.

5.3.4 Fishhook

The fishhook [US Department of Transportation, 2013] is an open-loop manoeuvre test procedure used by the National Highway Traffic Safety Administration (NHTSA) of the US Department of Transportation, under the New Car Assessment Program (NCAP) to evaluate light vehicle dynamic rollover propensity. Here it has been simulated to understand the influence of the SAVGS in the roll propensity of the SUV (the GT is not prone to rollover).

The test procedure comprises two stages. In the first one, a slow-turning manoeuvre is performed to determine the steering-wheel angle required to reach a lateral acceleration of 0.3 g, $\delta_{0.3g}$, at a forward speed of 50 mph. In the second stage, the vehicle is driven in a straight line at a certain MES (Manoeuvre Entrance Speed). Then the throttle pedal is released, and the steering-wheel is suddenly (720 deg/s) rotated up to $\delta_{\text{fh}} = 6.5 \delta_{0.3g}$. When the roll rate drops below 1.5 deg/s, the steering input is reversed (again at 720 deg/s) to $-\delta_{\text{fh}}$, maintained there for 3 s and then reverted to zero in a 2 s period. The first manoeuvre is performed with MES=35 mph. Depending on the results obtained, the manoeuvre is repeated with MES=40, 45, 47.5 and 50 mph.

The passive SUV survives the fishhook with MES=35 mph (although it displays two-wheel lift-off), but rolls over with MES=40 mph. The SUV retrofitted with the SAVGS, on the other hand, manages to avoid rollover even at MES=50 mph.

Results for MES=50 mph are presented in Fig. 5.13 for the passive and active (C-RM) configurations. The applied steering input is included in the top plot to help understand all other results. In the second and third plots, the lateral acceleration and roll angles are shown. The delay of the lateral acceleration with respect to the steering input is significantly smaller for the vehicle retrofitted with the SAVGS. In terms of roll angle, the SAVGS keeps it below 5 deg in the first steering phase, and it peaks at around 10 deg in the second. Two-wheel lift happens at time t_1 , but contact with the ground is regained at time t_2 and the vehicle remains stable until the end of the manoeuvre. The vehicle equipped with the passive suspension, on the other hand, displays two-wheel lift at the point indicated by •, and eventually rolls over.

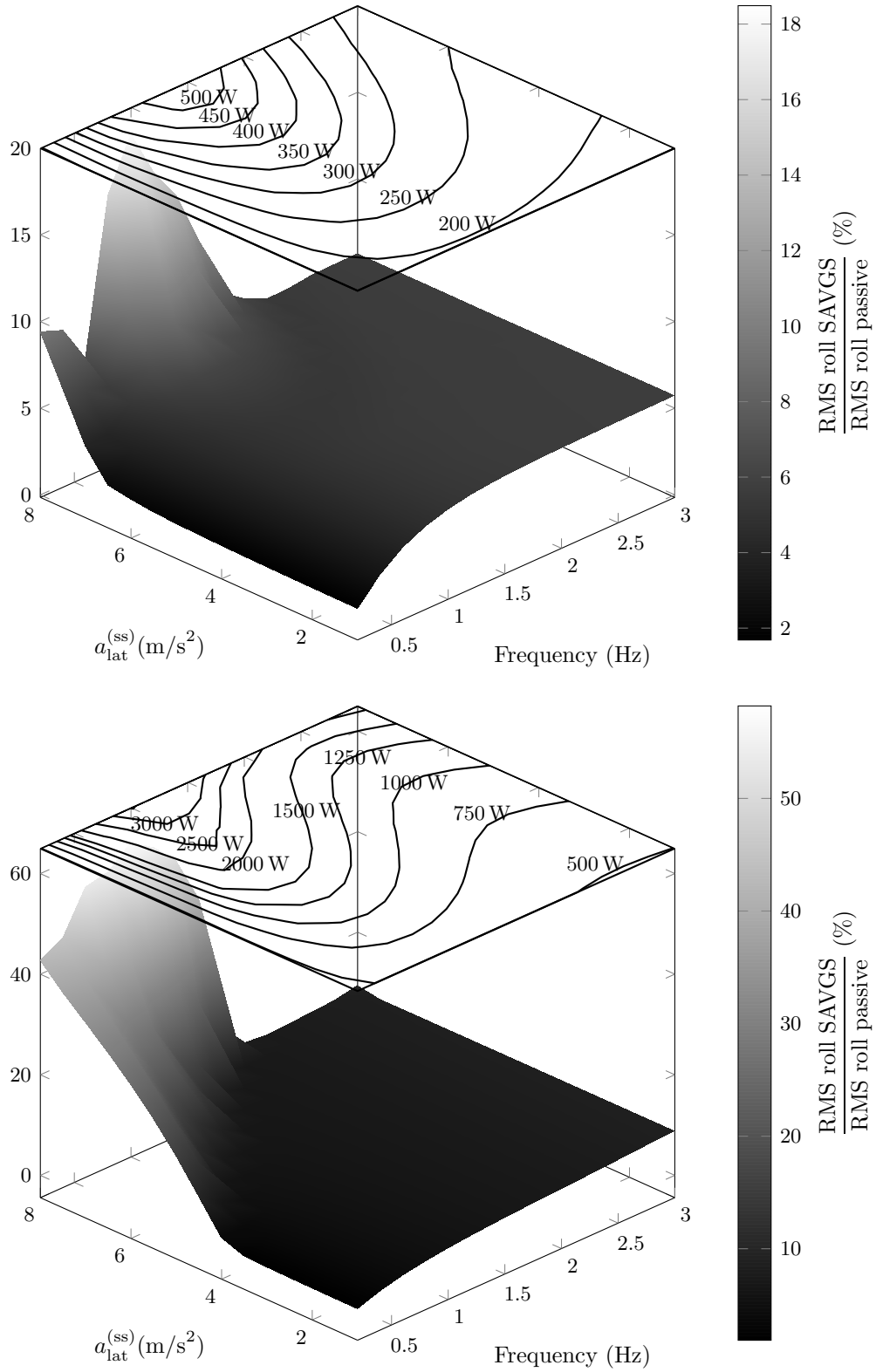


Fig. 5.11: Ratio of RMS roll angles obtained with the active and passive suspensions of the GT (top) and SUV (bottom) during continuous sinusoid steering-wheel input manoeuvres. Average power consumption contours are also shown.

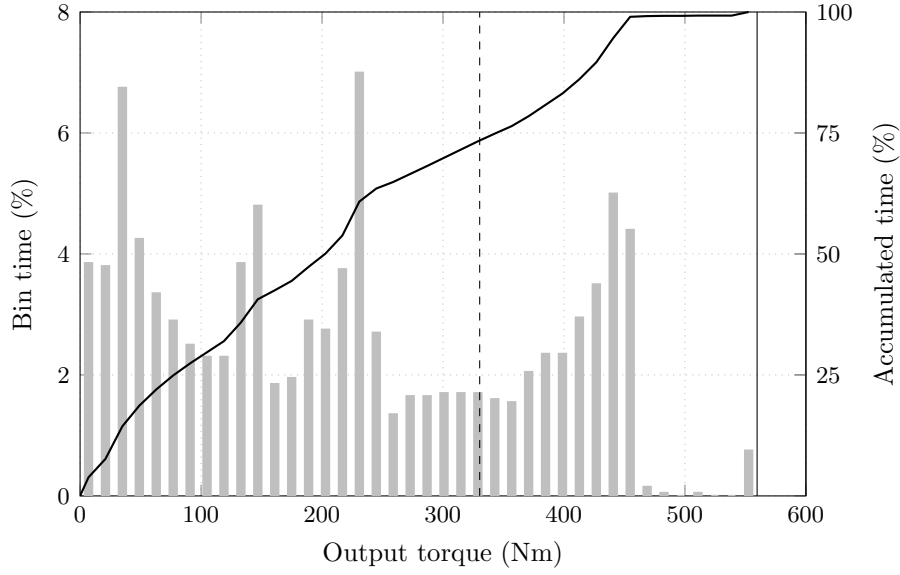


Fig. 5.12: Worst output torque histogram obtained from the SUV actuators. Grey bars indicate the percentage of time for which individual torque levels (bins) are outputted, and the solid black line represents the percentage of time for which equal or lower torques are outputted. The two thin vertical lines correspond to the actuator limits for continuous (dashed) and peak (solid) operation.

The fourth plot in Fig. 5.13 shows the angles rotated by the single-links with respect to $\theta_{SL}^{(\min)}$. Front and rear actuators react equally fast, and their motion is smooth throughout the allowable range of operation. Finally, the bottom plot corresponds to the power consumed by the SAVGS. For most of the simulation, the power consumption is negligible. This is due to the fact that the single-links are very close to being aligned with the spring-damper unit, and therefore the required torques and currents are very small. Power flows grow when the actuators are moving, but consumption is well under control in all cases and does not exceed 7 kW.

5.3.5 Braking in a turn

This manoeuvre is simulated to study the different requirements arising from the pitch and roll control loops. As defined in [ISO 7975:2006], the car is initially driven in a circular path of 100 m radius at a constant lateral acceleration of 5 m/s^2 . The steering wheel is fixed and brakes applied so that the vehicle slows down at a constant deceleration rate of 5 m/s^2 .

Results obtained with the SUV are shown in Fig. 5.14. Three cases are included: passive suspension, SAVGS with roll focus (C-RM), and SAVGS with pitch focus (C-PM).

Before the brakes are applied at $t = 0$, both controllers cope equally well with roll demands and provide significant correction with respect to the passive suspension. Behaviour differs as soon as deceleration begins because the combined requirements arising from pitch and roll mitigation exceed the SAVGS potential for full attitude control. In the case with roll focus, pitch is improved

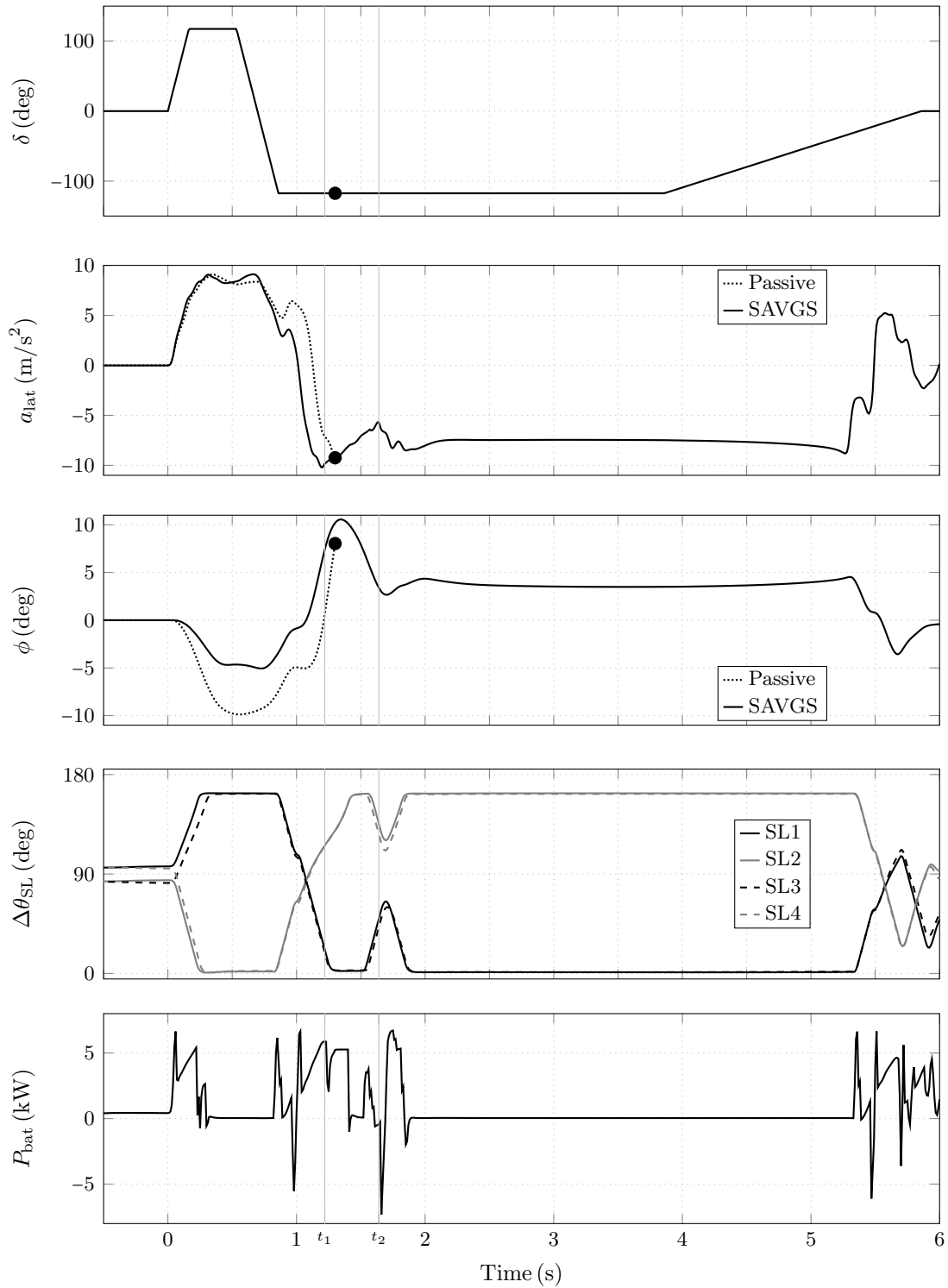


Fig. 5.13: Results for the passive and active configurations of the SUV during the fishhook manoeuvre (MES=50 mph). The vehicle equipped with the passive suspension displays two-wheel lift at the point indicated by $\bullet$, and eventually rolls over. The SAVGS-retrofitted vehicle also displays two-wheel lift at time t_1 , but regains contact with the ground at time t_2 and remains stable throughout the manoeuvre.

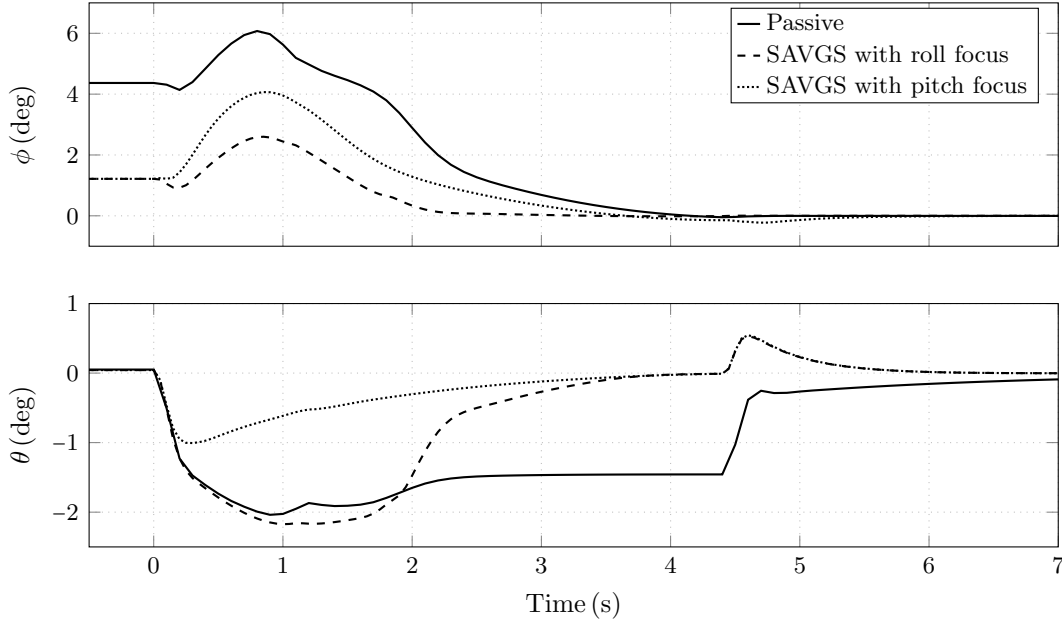


Fig. 5.14: SUV results for ISO 7975 brake in turn manoeuvre.

with respect to the passive case only when the demands arising from roll mitigation drop below the point of complete SL saturation at roughly $t = 2$ s. In the case with pitch focus, the SAVGS capabilities are transferred from roll to pitch control immediately after the application of the brake pedal. Roll is still significantly reduced with respect to the passive case, but a fraction of the SAVGS potential is allocated to pitch mitigation and the resulting behaviour is more balanced. Regarding heave motions, the variation of CMC height above ground is 24 mm in the passive case, and 33 mm and 10 mm in the cases with roll and pitch focus, respectively.

Overall, this manoeuvre shows how pitch correction requires a higher share of the SAVGS potential per degree of attitude angle reduction, and how different compromises can be easily reached by simply adjusting the relative gains between the pitch and roll control loops.

Finally, the power and energy imbalances computed through (3.75) are shown in Fig. 6.32 to demonstrate that the numerical accuracy achieved in these simulations is appropriate. Power and energy imbalances are 3 to 6 orders of magnitude smaller than power and energy flows in the model, and therefore numerical accuracy is deemed to be satisfactory.

5.3.6 Pure longitudinal acceleration and braking

Two sets of manoeuvres have been simulated to study the potential of the SAVGS for pitch mitigation.

In the first one, a hard acceleration phase from 0 to 100 km/h in 6.5 s is followed by a 2 s constant speed period and an emergency stop. The second set of simulations deals with a more common

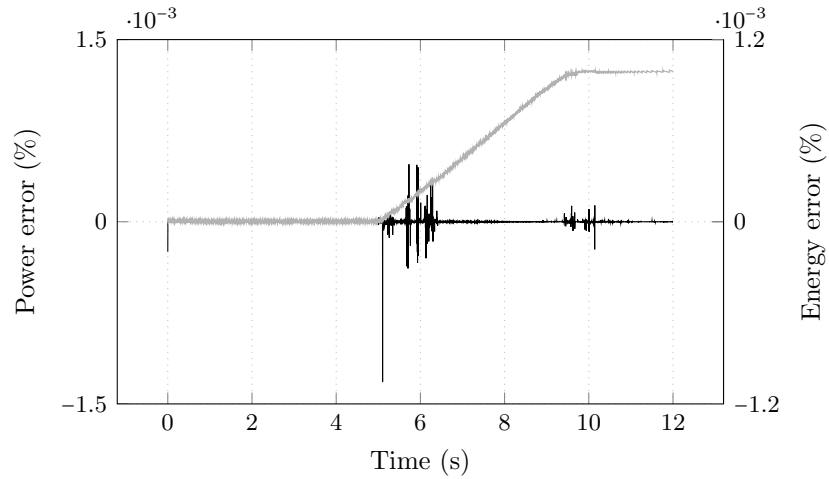


Fig. 5.15: Power (black) and energy (grey) errors for the SUV with controller C-PM during the brake in turn manoeuvre. Errors peak at approximately 5 W and 5 J and have been normalised with respect to the maximum power dissipation and the total energy dissipation in the manoeuvre, respectively.

event: joining/exiting a highway from/to a low speed lane. The velocity profiles simulated include an acceleration phase from a typical urban environment speed of 50 km/h, to 120 km/h in T seconds, followed by 5 s at constant velocity and a final exit from the highway, decelerating back to 50 km/h in T seconds. The acceleration/deceleration phases have been made to last from $T=3$ s to $T=10$ s, with constant acceleration/deceleration rates varying from 2 to 6 m/s². The reference forward speed profiles for both sets of simulations are shown in Fig. 5.16.

Neither the GT nor the SUV models incorporate any anti-dive or anti-squat geometric effects and therefore these manoeuvres are quite demanding for the SAVGS. All results are obtained with control gains corresponding to (C-PM). In the GT case, torque and current limits are made to transition (smoothly) from their continuous limits to their peak limits at forward accelerations around 11 m/s². This implies that continuous limits are active except during extreme events such as an emergency stop. In the SUV case, peak limits need to be allowed at all times in order to achieve a similar level of correction. Of course, this would not be the case in a lightly laden configuration.

Simulation results for manoeuvre 1

Time responses for both vehicles are shown in Fig. 5.17. In the top row, the reference and actual speed profiles are compared. Both the GT and SUV follow the acceleration profile closely, and the resulting deceleration rates remain within the 1.05 g to 1.25 g band during the emergency stop. Braking distances are 34.8 and 37.2 m, respectively.

Pitching angles, shown in the second row, are 1.5 to 2 times larger for the passive SUV than for the passive GT. The SAVGS maintains the vehicles practically flat during the acceleration phase

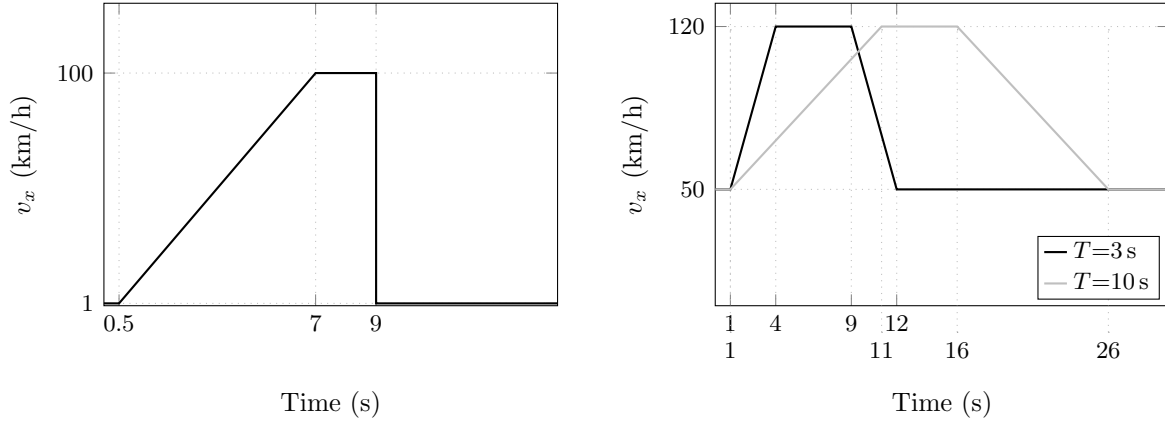


Fig. 5.16: Reference forward speed profiles for the first (left) and second (right) longitudinal acceleration/deceleration manoeuvres. In the right hand side, traces are shown only for the most ($T=3$ s) and least ($T=10$ s) demanding cases.

and achieves significant (approximately 50%) correction during the emergency stop.

In the third row, single-link angles with respect to $\theta_{\text{SL}}^{(\min)}$ are shown. Behaviour is similar for both cars but during the acceleration phase the actuators in the SUV make use of a greater proportion of the available range of motion. During the emergency stop, all single-links are kept at their limiting positions.

Finally, power and energy flows from the battery are shown in the fourth row. Power flows peak at approximately 1.5 kW and 6.5 kW and average 150 W and 410 W for the GT and SUV, respectively.

The output torque-speed operating points for the actuators are plotted in Fig. 5.18 alongside the actuator envelopes for continuous and peak operation. In the GT case, the output torque remains within the continuous envelope except for a small excursion during the emergency stop. In the SUV case, on the other hand, the operating points spread across the full permitted region and torques exceed the continuous limits more frequently (bear in mind that $\alpha_{\min}$, as defined in (4.16), has been set to 0.25).

Simulation results for manoeuvre 2

Pitch angles normalised by the maximum (in absolute value) pitch angle for the passive case are shown in Fig. 5.19. The relative improvement provided by the SAVGS is similar in both vehicles and ranges from approximately 60% for the most demanding case and 80–90% for the least demanding ones.

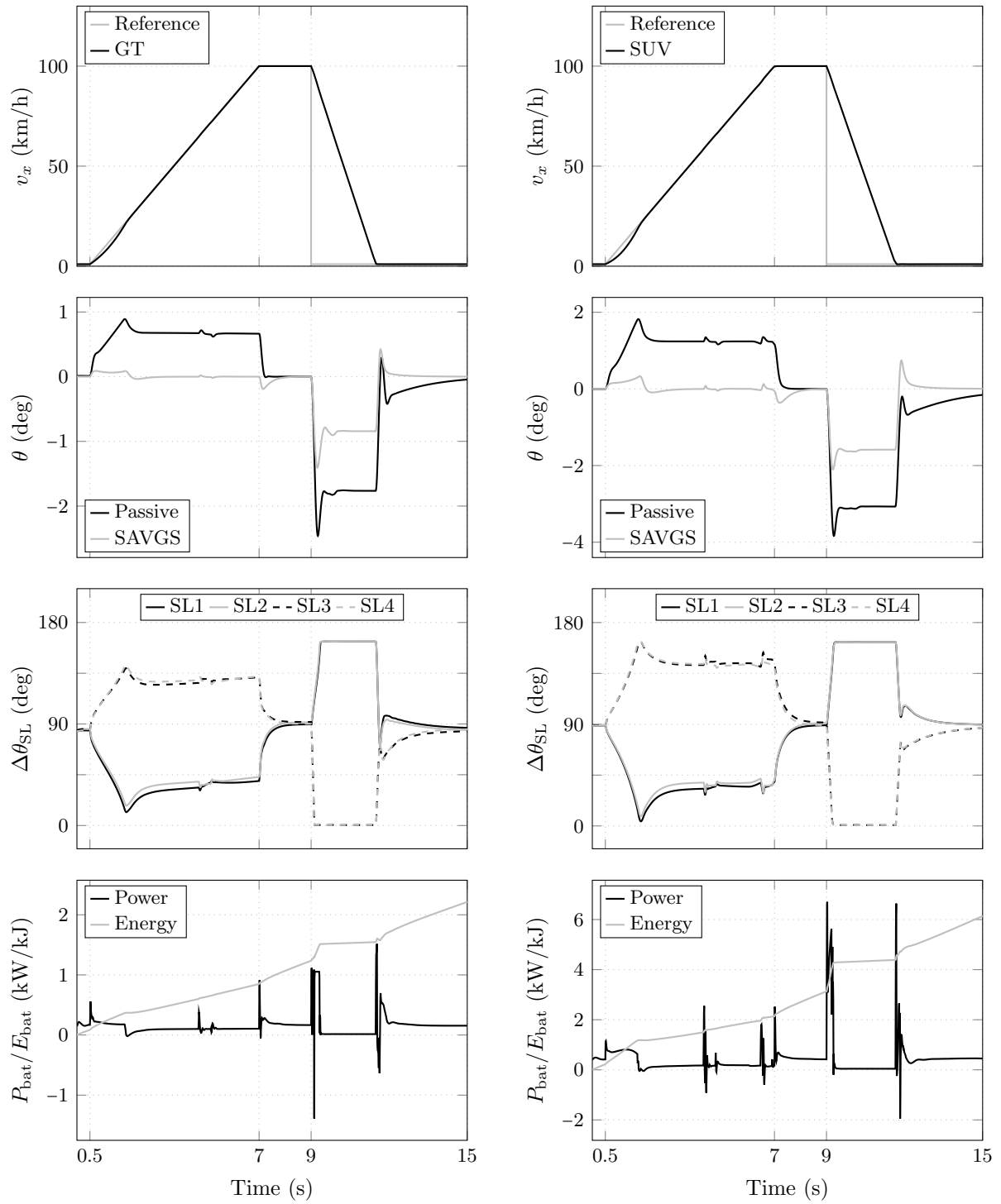


Fig. 5.17: Simulation results for the GT (left) and SUV (right) for the first longitudinal acceleration/deceleration manoeuvre.

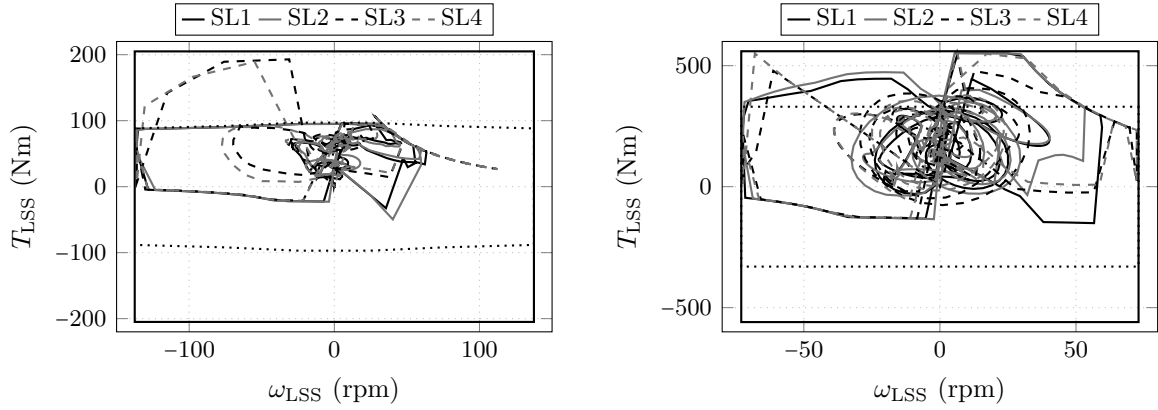


Fig. 5.18: Output torque-speed operating points for the GT (left) and SUV (right) during the first longitudinal acceleration/deceleration manoeuvre. Limiting envelopes for continuous (solid) and intermittent (dotted) operation are also shown for reference.

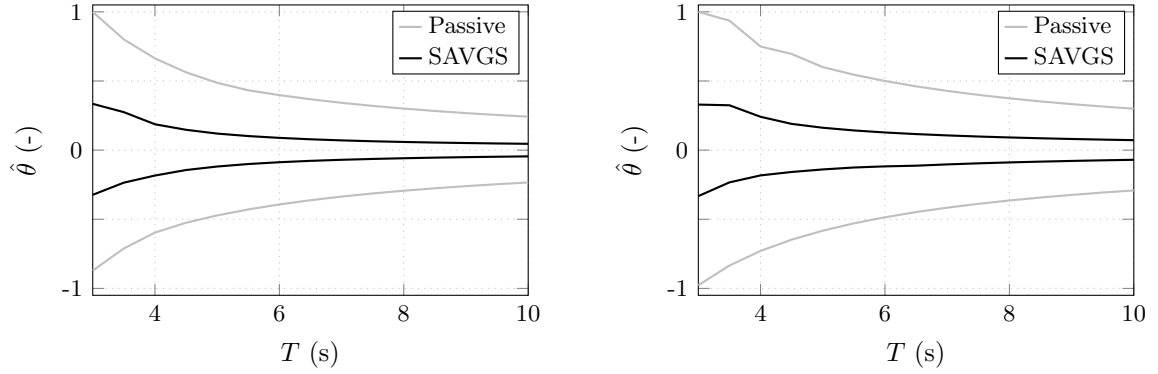


Fig. 5.19: Normalised pitch ratios vs. acceleration/deceleration time for the GT (left) and SUV (right) during the second set of longitudinal acceleration/deceleration manoeuvres.

5.4 Conclusions

This chapter has tackled the control of chassis attitude motions with the SAVGS. Starting with the derivation of steady-state equations for a generic corner of the vehicle, a sensitivity analysis has been proposed as a tool to help understand the relationships between the vehicle and SAVGS properties and to inform the development of a pre-dimensioning algorithm based on steady-state performance requirements. A generic multi-objective control scheme has also been proposed and thoroughly tested through the simulation of multiple standard open-loop test manoeuvres. Results obtained for the GT and SUV show that the SAVGS is capable of achieving significant reductions in chassis attitude motions with sensibly sized off-the-shelf actuators, both in steady-state and during transients.

Other conclusions that can be drawn from this chapter are:

- Maximum longitudinal and lateral acceleration values for which full attitude control is possible depend, among others, on:

- The length of the SL: the longer the SL the larger the reachable $a_x - a_y$ area but the larger the actuation torques required for any given operating condition.
- The nominal SL angle: changes in the nominal SL angle move the reachable $a_x - a_y$ area for each axle along the a_x axis.
- The desired fore-aft distribution of overturning couple: the maximum lateral acceleration for which the nominal suspension geometry can be maintained in each axle increases as the axle's share of overturning couple compensation decreases.
- The distribution of lateral force generation between the tyres within each axle: for each axle, the area of the reachable $a_x - a_y$ surface is not affected, but its shape is distorted for uneven force distributions. Considering both axles, an even share of lateral forces leads to the maximum reachable $a_x - a_y$ area.
- The stiffness of the passive springs: the stiffer the springs the larger the reachable $a_x - a_y$ surfaces. Higher stiffnesses have no detrimental effect on the actuation torque requirements.
- The payload: the reachable $a_x - a_y$ area is very sensitive to changes in sprung mass.
- The proposed attitude control scheme:
 - Works well in combination with the actuator control scheme proposed in Chapter 4.
 - Leads to zero steady-state tracking error and has been shown to respond well during transients.
 - Is robust to vehicle and actuator parameter changes, as the same gains produce very good results with both the GT and SUV parameter sets.
 - Can be easily tuned to prioritise different control objectives.
- Achievable levels of roll and pitch correction for the GT and SUV with sensibly sized off-the-shelf actuators range from 40% in the worst case scenario to over 90% for less demanding situations.
- If desired, the directional response of the car can be influenced by the SAVGS.
- Power and energy consumption levels are low and can be adjusted by changing the power constraints in the actuator control loops. With the gains proposed in this chapter, power flows peak at approximately 1.5 kW and 6.5 kW for the GT and SUV, respectively.

Chapter 6

SAVGS application to comfort and road holding enhancement

The previous chapter explored the potential of the SAVGS for attitude control and showed that significant reductions in pitch and roll motions are achievable when maintaining the original passive suspension components. A direct conclusion that can be drawn from these results is that, if the SAVGS was instead paired with a softer and less damped SD than that of the passive suspension, the retrofitted vehicle could match the original levels of pitch and roll motions thanks to the SAVGS actions, whilst providing a better ride quality thanks to the enhanced filtering properties of the new SD.

This chapter analyses the possibility of improving comfort and road holding through appropriate control of the SAVGS rather than through changes in the passive components. Maintaining the SD and suspension kinematics of the benchmark passive suspension, $\mathcal{H}_\infty$ techniques are applied to a linear model representing one corner of the vehicle.

Section 6.1 assesses the suitability of two different quarter-car linear models for control synthesis: an automatically generated AutoSim model and a simplified hand-derived model. The application of $\mathcal{H}_\infty$ control to the SAVGS problem in one corner of the car is described in Section 6.2. Results obtained with the synthesised controller for a variety of scenarios are presented and discussed in Section 6.3, first for the quarter-car and then for the full-car nonlinear models. Finally, Section 6.4 summarises the key findings from this study and outlines the main aspects to be investigated in future work.

6.1 Linearised models

Although the nonlinear multi-body quarter-car model is an appropriate representation of the vehicle for virtual testing the SAVGS and assessing its performance in the context of comfort and road holding improvement, it is not the most convenient model for control synthesis.

A linearised model, if sufficiently accurate for the range of operating conditions of interest, enables the application of a wider range of control techniques. It is also a better tool for studying the input-output relationships of the system in the frequency domain, and to analyse how these are affected by the vehicle and SAVGS parameters.

In this section, details regarding the linearisation performed with AutoSim and the accuracy of the resulting model are provided in Section 6.1.1. To better understand the behaviour of the system, Bode magnitude plots and their dependency on the SL angle are discussed in Section 6.1.2, and results for the rear suspension systems of the GT and SUV vehicles are used to highlight the limitations of the AutoSim model. Finally, an alternative linearisation procedure is proposed in Section 6.1.3 and its advantages and disadvantages with respect to the AutoSim approach are analysed. Subsequent sections present simulation results to support the use of this hand-derived model for the synthesis of $\mathcal{H}_\infty$ controllers.

6.1.1 AutoSim linearisation and accuracy of linear models

Automatic linearisation performed by AutoSim

The nonlinear quarter-car multi-body model can be automatically linearised by AutoSim if some minor modifications are included in the model definition.

If the kinematical and dynamical equations of the multi-body system are given by

$$\dot{\mathbf{q}} = \mathbf{s}, \quad \dot{\mathbf{u}} = \mathbf{f}, \quad (6.1)$$

where $\mathbf{q}$ and $\mathbf{u}$ are vectors of the generalised coordinates and speeds, $\mathbf{s}$ and $\mathbf{f}$ are vectors of known functions of the state variables, $\mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \mathbf{u} \end{bmatrix}$, and time, then the equations of motions are linearised and put into the form

$$\begin{aligned} \delta \dot{\mathbf{x}} &= \mathbf{A} \delta \mathbf{x} + \mathbf{B} \delta \mathbf{u}, \\ \mathbf{y} &= \mathbf{C} \delta \mathbf{x} + \mathbf{D} \delta \mathbf{u}, \end{aligned} \quad (6.2)$$

by taking partial derivatives of the coefficients in the arrays $\mathbf{s}$ and $\mathbf{f}$. This is done symbolically and the resulting equations are valid for any trim point.

Omission of lateral tyre forces

In this chapter, as in most suspension studies that focus on comfort and road holding, the vehicle is assumed to be moving over an uneven road in a straight line and at a constant forward speed. Under these conditions, lateral tyre forces are typically neglected to reduce model complexity. However, and depending on the suspension geometry, the wheel may undergo significant lateral and rotational motions as it displaces vertically with respect to the sprung mass. These, in turn, lead to non-zero side slip angles and changes in camber that can produce significant variations in lateral tyre forces that can affect the vertical dynamics of the system.

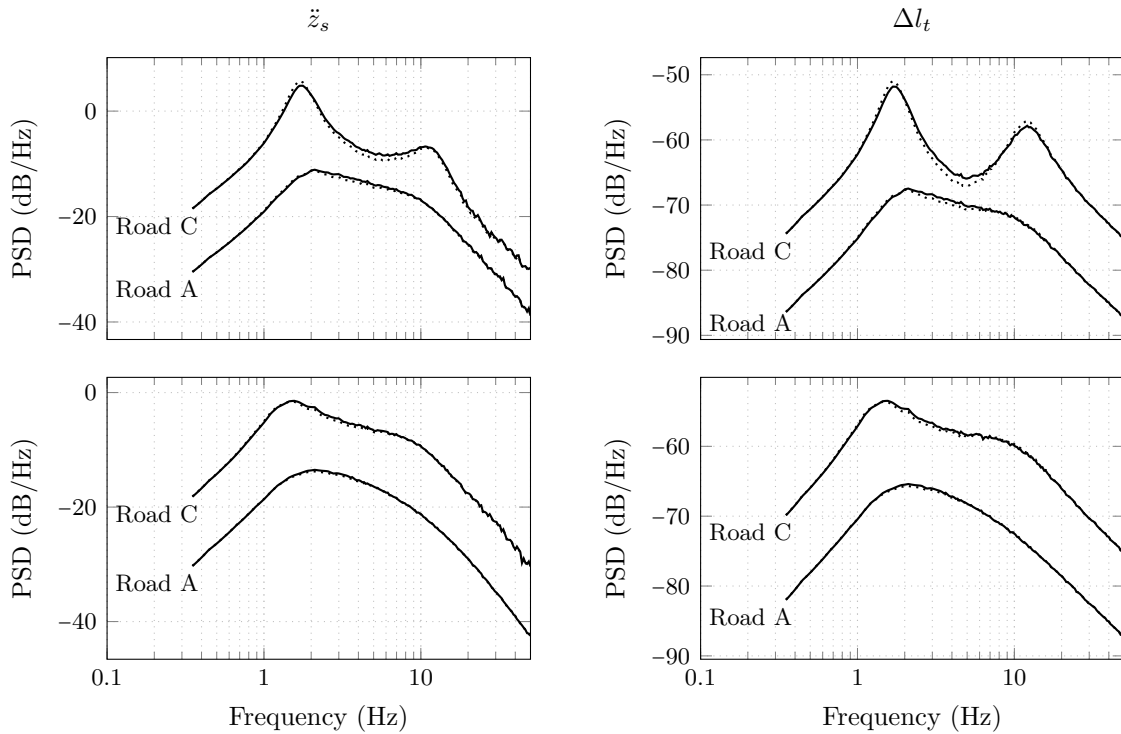


Fig. 6.1: Power spectral density of sprung mass acceleration (left) and dynamic tyre deflection (right) for a rear corner of the GT (top row) and of the SUV (bottom row). Values were obtained with the nonlinear multi-body quarter-car model that includes shear tyre forces (solid) and neglects them (dotted) for a good quality road (road A) and a poor quality road (road C).

In this subsection, the effect of the lateral tyre forces for the rear suspension geometries of the GT and SUV is analysed in order to justify their omission from the linearised models. Their effect has been assessed by simulating the nonlinear multi-body quarter-car models, with and without lateral tyre forces, over two typical random road profilesⁱ at a fixed forward speed of 100 km/h and with constant longitudinal tyre slip. The resulting power spectral densities (PSD) have been compared and used to quantify the influence of the lateral tyre forces.

Simulations have been performed for 10 km long road segments and PSDs have been obtained

ⁱSee Appendix A for more details on the road characteristics.

using MATLAB's implementation of Welch's power spectral density estimate. In this process, time series (sampled at 400 Hz) have been split into segments corresponding to 250 m long road sections with 60% overlap, and filtered with Hanning windowsⁱⁱ. Thus, sufficient frequency resolution is achieved in the range of frequencies of interest (roughly 0.3 Hz to 50 Hz) and noise content is significantly reduced. Power values outputted by MATLAB's `pwelch` function are converted to dB/Hz through $10 \log_{10}(\cdot)$ and therefore considering a *power* density reference of $1 \text{ m}^2/\text{s}^4/\text{Hz}$ for the sprung mass acceleration and of $1 \text{ m}^2/\text{Hz}$ for the dynamic tyre deflection.

Results are shown in Fig. 6.1 for the two outputs relevant to comfort and road holding assessment (i.e. vertical body acceleration, $\ddot{z}_s$, and dynamic tyre deflection, Δl_t). The match is good for both vehicles, with minor differences occurring only in the frequency range between the sprung and unsprung mass resonant frequencies. The maximum difference corresponds to the GT case for the PSD of dynamic tyre deflection: the difference reaches 1.2 dB/Hz at approximately 5 Hz. Differences in the SUV case are even smaller due to its zero nominal camber angle and lesser lateral motions of the point of contact.

Discrepancies between the results produced by the models with and without lateral tyre forces depend on the level of road disturbance (greater for rougher roads as this implies larger lateral wheel motions and camber changes and therefore lateral tyre forces) as well as on the vehicle forward speed (smaller for larger forward vehicle speeds as the side slip angle and therefore lateral tyre force for any given level of lateral displacement decreases with forward speed). Thus, although it may not be appropriate to state that lateral tyre forces are negligible in general, it seems reasonable to omit them for the operating conditions and suspension geometries considered in this chapter.

Comparison between linear and nonlinear models

Although the linearised models produced by AutoSim are accurate for *small* perturbations from any given trim point, significant discrepancies with respect to the nonlinear models may arise for realistic disturbance levels if the linear damping coefficient is selected incorrectly. This is explored in Fig. 6.2 and Fig. 6.3, where PSDs obtained with the linear and nonlinear models for the same road profiles as previously discussed are shown. In the linear model case results have been obtained with damping coefficients ranging from the maximum slope in the damping force vs. damper extension speedⁱⁱⁱ, denoted by c_0 , and 10% of it. That is, the actual damping coefficient, c , has been multiplied by a scaling factor, SF, so that $c \in [0.1 c_0, c_0]$.

A few conclusions can be drawn from these figures:

ⁱⁱAn overview of PSD related computations and signal processing concepts may be found in [Heinzel et al., 2002].

ⁱⁱⁱSee damping force vs. damper extension speed for front and rear GT and SUV dampers in Fig. 3.14.

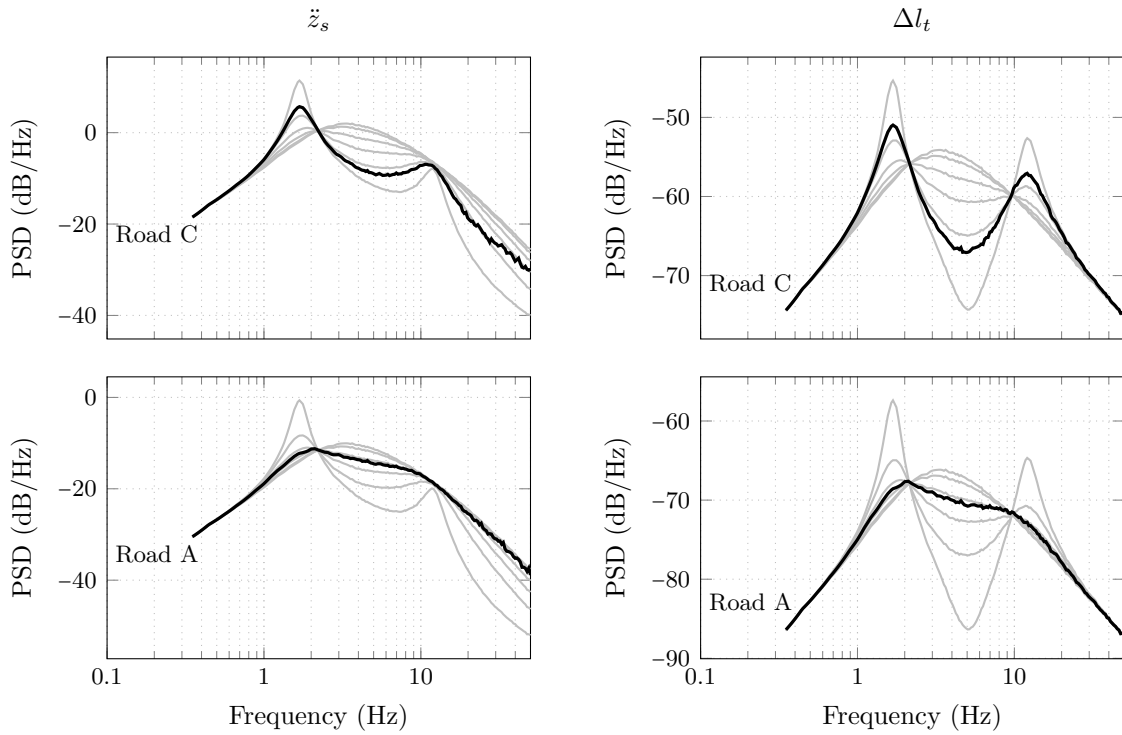


Fig. 6.2: Power spectral density of sprung mass acceleration (left) and dynamic tyre deflection (right) for a rear corner of the GT for a good quality road (road A) and a poor quality road (road C). PSDs obtained with the nonlinear (black) and linear AutoSim models with various damping coefficients (grey). Damping coefficients are scaled with respect to the damping coefficient for zero damper extension speed, c_0 , by 0.1, 0.3, 0.5, 0.7, 0.9 and 1.

- The damping coefficient in the linear model that provides the best fit to the nonlinear PSD depends on the road quality and excitation frequency.
- A damping coefficient capable of producing a good match throughout the range of frequencies of interest is more likely to exist for good quality roads.
- A damping coefficient capable of producing a good match throughout the range of frequencies of interest is more likely to exist for the SUV than for the GT.
- The effect of changing the damping coefficient is greater in the dynamic tyre deflection than in the sprung mass acceleration.

These conclusions are also supported by results presented in Fig. 6.4, where the mean square error between the predictions of the linear and nonlinear models has been computed for all roads and outputs and plotted against the damping scaling factor, SF. Lower damping coefficients, compatible with larger damper extension rates, are needed to provide a good match for rougher roads. Furthermore, nonlinearities in the GT damper manifest themselves at these levels of road disturbances, whereas the SUV damper remains closer to linear, with negligible discrepancies between linear and nonlinear predictions for roads A and B and a damping coefficient $c = c_0$.

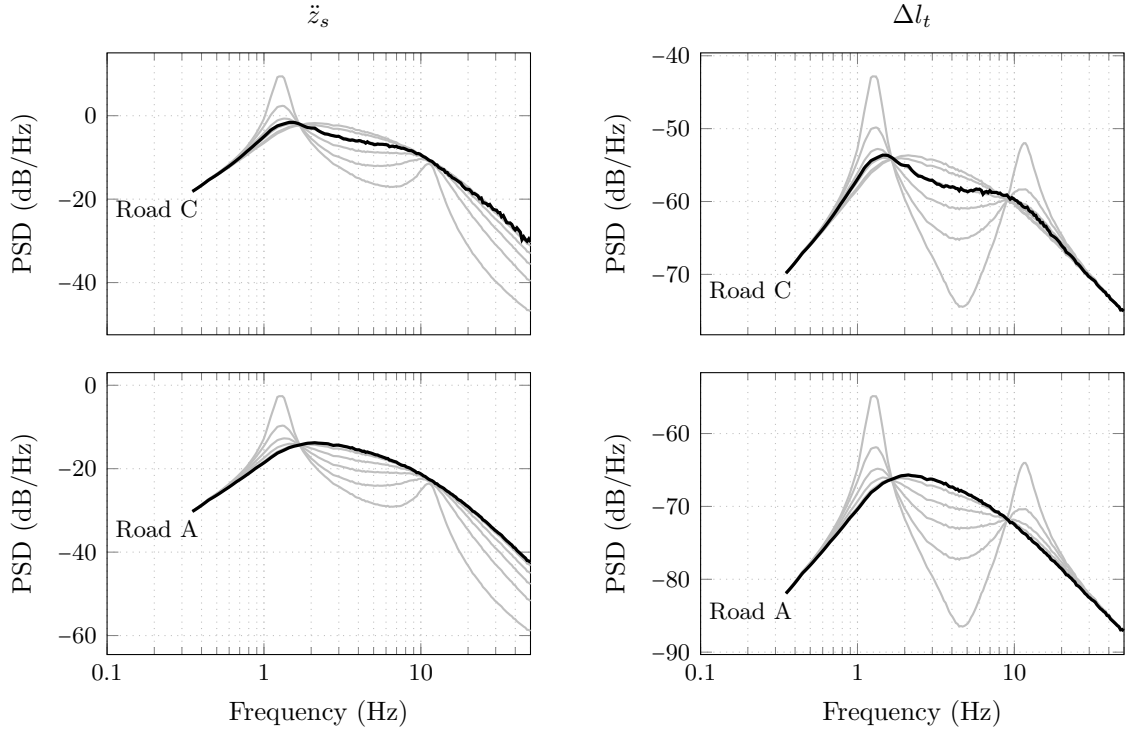


Fig. 6.3: Power spectral density of sprung mass acceleration (left) and dynamic tyre deflection (right) for a rear corner of the SUV for a good quality road (road A) and a poor quality road (road C). PSDs obtained with the nonlinear (black) and linear AutoSim models with various damping coefficients (grey). Damping coefficients are scaled with respect to the damping coefficient for zero damper extension speed, c_0 , by 0.1, 0.3, 0.5, 0.7, 0.9 and 1.

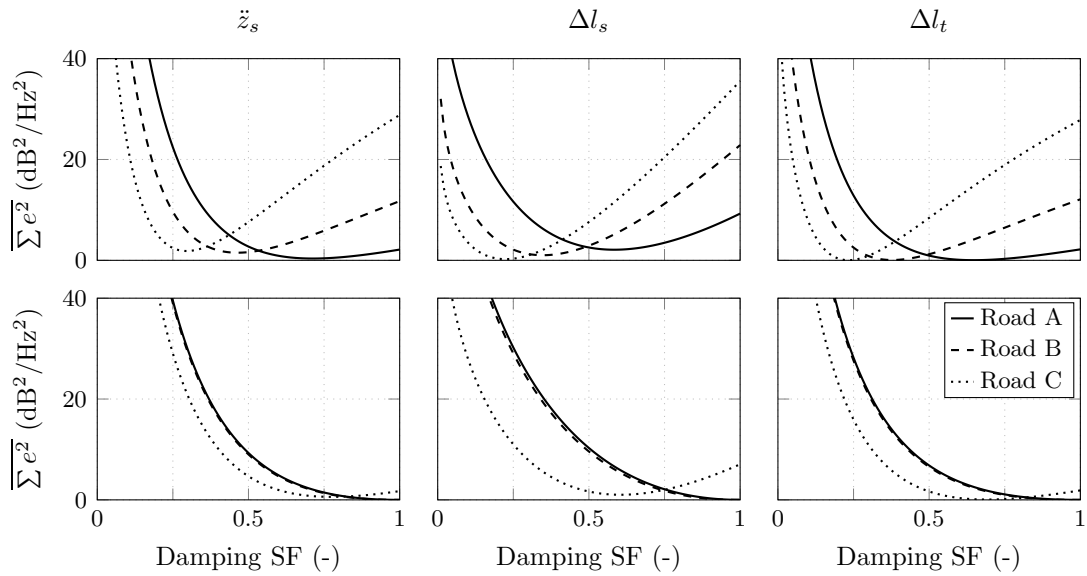


Fig. 6.4: Mean square error between the linear and nonlinear predictions for the GT (top) and SUV (bottom) for three standard road qualities (A to C). The three outputs considered are the sprung mass acceleration (left), dynamic suspension deflection (centre) and dynamic tyre deflection (right).

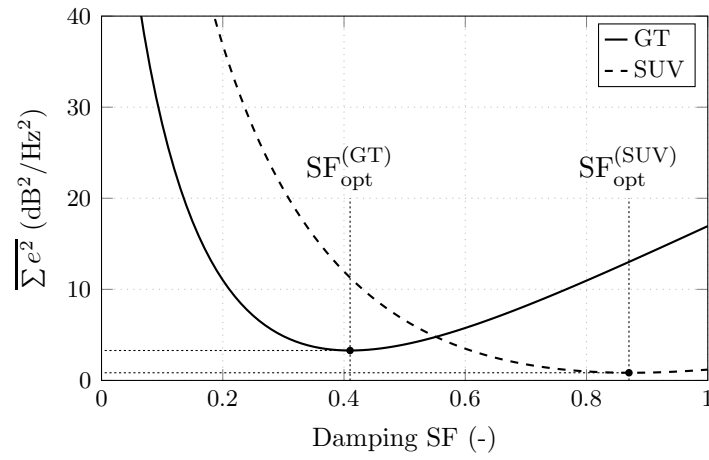


Fig. 6.5: Mean square error between the linear and nonlinear predictions for the GT and SUV against damper scaling factor. Values shown were computed from the three standard road qualities (A to C) and the three outputs of interest. The optimum scaling factors are $SF_{opt}^{(GT)} = 0.41$ and $SF_{opt}^{(SUV)} = 0.87$.

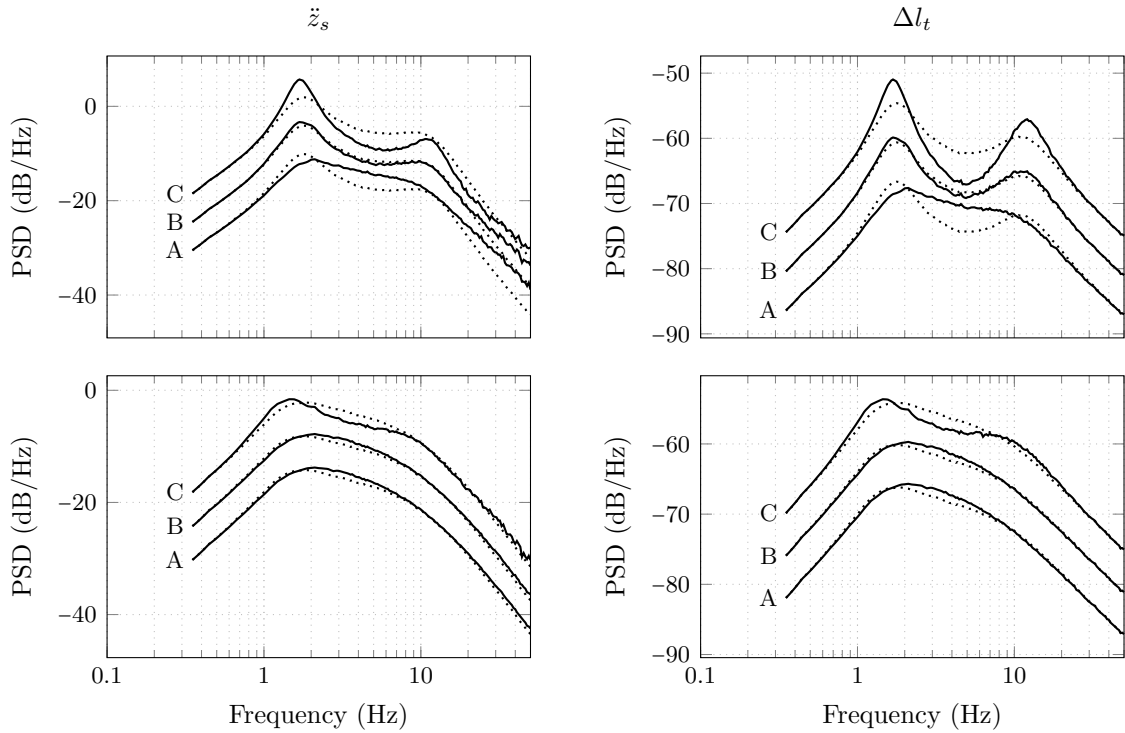


Fig. 6.6: PSDs of sprung mass acceleration (left) and dynamic tyre deflection (right) for a rear corner of the GT (top) and SUV (bottom) for road qualities A to C. PSDs obtained with the nonlinear (solid) and linear (dotted) AutoSim models with the *optimum* damping coefficients.

Fig. 6.5 shows the total error added for roads A to C and the three outputs already mentioned. Although the calculation of the *optimum* damping coefficient would require a rigorous definition of *error* between the linear and nonlinear models, the addition of squared errors for the road classes and outputs considered is chosen here as a simple approach to identify a sensible damping coefficient value to use as reference in subsequent sections. Fig. 6.6 compares the PSDs obtained for the nonlinear model and for the linear model with the best damping coefficients selected from Fig. 6.5. The match is very good for road B and acceptable for the others although slightly underdamped for the good road and slightly overdamped for the rough road.

The previous results suggest that the linearised models generated by AutoSim capture the main system dynamics reasonably accurately. For vehicles with very nonlinear damper characteristics, such as the ones considered in this thesis, the selection of the linear damping coefficient is critical in order to ensure that the linear model produces realistic predictions for the expected level of external disturbances (e.g. forward vehicle speed and road quality).

The next section explores the Bode magnitude responses of the rear suspensions of the GT and SUV, and discusses the quality of the predictions when the system is operated far from the trim state for which the model was linearised.

6.1.2 Frequency response and effect of SL angle

In this section, the input-output frequency responses of the system are calculated first for its nominal configuration and then for the whole range of angular SL positions.

For these comparisons the three inputs considered are the rate of change of the road height (commonly preferred in the literature over road height to represent road irregularities), the external load acting on the sprung mass (to represent longitudinal and lateral load transfers due to horizontal vehicle accelerations) and the angular speed of the SL (the control action). The outputs considered are the ones typically reported in the literature, namely acceleration of the sprung mass (related to comfort), dynamic suspension deflection (relevant to identify working space, i.e. suspension travel, requirements) and dynamic tyre deflection (related to road holding).

The input-output Bode magnitude responses for $\theta_{SL} = \theta_{SL}^{(ne)} = \theta_{SL}^{(min)} + \pi/2$ are shown in Fig. 6.7 for the rear suspensions of the GT and SUV with scaling factors for the damping coefficient c_0 of SF=0.41 and SF=0.87, respectively. With these damping coefficients, the GT displays the typical double-hump response to road irregularities, whereas the SUV displays only one peak. As expected, low- and high-frequency slopes are the same for both vehicles. Apart from constant offsets, differences between them do only occur in the 1 to 20 Hz frequency range. Particularly interesting are the results obtained for the case with SL angular speed as input:

- Slopes at low frequencies are +20 dB/decade for the sprung mass acceleration and dynamic

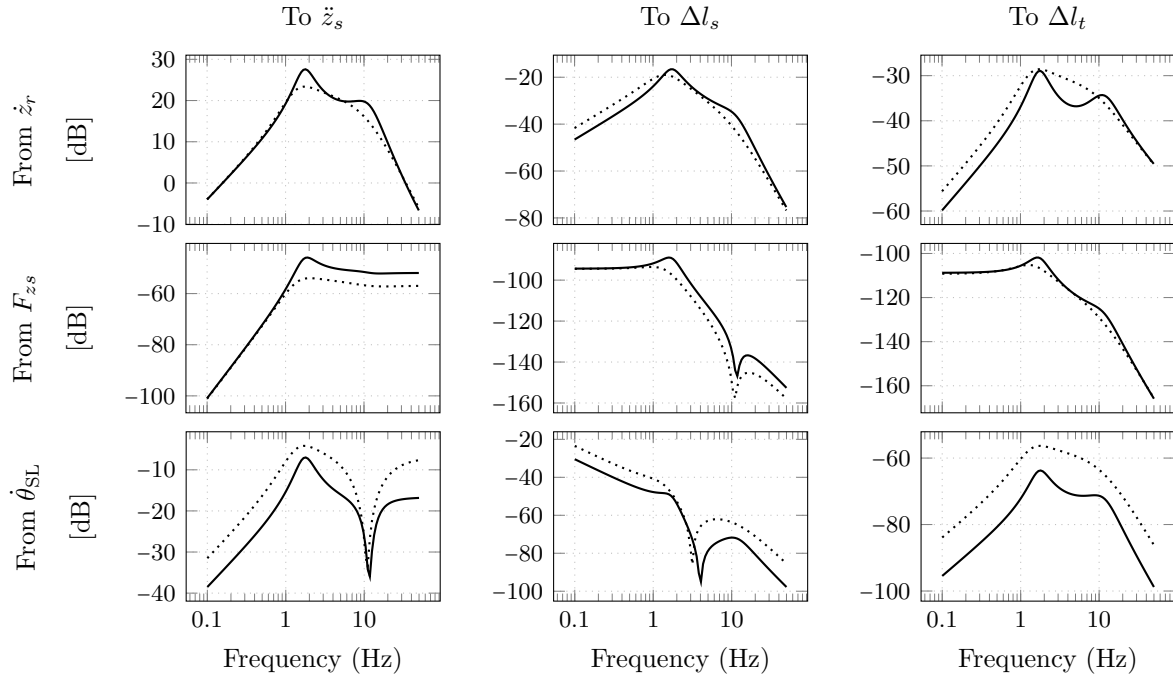


Fig. 6.7: Magnitude Bode plots from vertical road speed (top), external vertical force acting on the sprung mass (middle) and SL angular speed (bottom) to sprung mass acceleration (left), dynamic suspension deflection (centre) and dynamic tyre deflection (right) for the rear suspensions of the GT (black) and SUV (grey). Results correspond to a nominal SL angle offset $\Delta\theta_{SL}^{(ne)}$.

tyre deflection outputs (pure differentiator), and -20 dB/decade for the dynamic suspension deflection (pure integrator). This confirms the intuition that, at low frequencies, the SAVGS will not be able to improve comfort or road holding, but will be capable of providing chassis height and attitude control.

- At high frequencies, magnitudes drop at a rate of -40 dB/decade for the dynamic suspension and tyre deflections, but remain constant for the sprung mass acceleration. This implies that at high frequencies, although the SAVGS will not be able to enhance road holding, it could potentially improve comfort without excessive working space requirements (if the actuator was responsive enough).
- The longer SL and larger damping coefficient in the SUV results in consistently higher gains in the SUV case despite its heavier masses.
- The existence of three invariant points in the transfer functions from vertical road speed to sprung mass acceleration, suspension deflection and tyre deflection, which has already been discussed in Section 2.4.3, implies that there are three frequencies at which the gains from vertical road velocity to sprung mass acceleration, suspension deflection and tyre deflection are independent from the SAVGS action (interpolation conditions). This is clearly visible in the third row in Fig. 6.7:
 - In the transfer function from $\dot{\theta}_{SL}$ to $\ddot{z}_s$, there is a zero at the wheel-hop frequency (~ 10 Hz), which corresponds to ω_1 in Section 2.4.3. This implies that, irrespective

of how fast the actuator is, the SAVGS will not be able to provide much comfort improvement at frequencies close to the wheel-hop frequency.

- In the transfer function from $\dot{\theta}_{SL}$ to Δl_s , there is a zero at the rattle-space frequency, at approximately 3.5 Hz, which corresponds to ω_2 in Section 2.4.3.
- In the transfer function from $\dot{\theta}_{SL}$ to Δl_t , the gain tends to zero as $\omega \rightarrow \omega_3 = 0$.

A key element of the SAVGS control is the nominal SL angle. In Chapter 5, the effect of the nominal SL angle on the attitude control capabilities of the SAVGS for steady-state conditions was analysed, and the results shown confirmed the intuitive idea that a nominal angle close to $\theta_{SL}^{(ne)} \approx \theta_{SL}^{(min)} + \pi/2$ provides the best attitude control performance. In order to describe the influence of the nominal SL angle on the dynamic response of the SAVGS, the Bode magnitude plots from SL speed to sprung mass acceleration, dynamic suspension and tyre deflections as a function of nominal SL angle are shown in Fig. 6.8. From these:

- As in the attitude control case, a nominal angle offset of approximately 90 deg leads to the maximum gain.
- The shape of the Bode plot remains largely unchanged by the nominal angle offset.
- The magnitude of the Bode plot is very sensitive to the nominal angle offset except in the region around $\Delta\theta_{SL}^{(ne)} = 90$ deg.
- The SAVGS becomes ineffective at nominal angle offsets close to 0 and 180 deg. At low frequencies, damping forces are negligible and the SAVGS action is materialised mainly through changes in spring force. In this case, the minima can be found through (5.20) and for the suspension geometries under consideration happen to be slightly below 0 and 180 deg (approximately 2-5 deg). At high frequencies, damping forces dominate and these are zero at exactly 0 and 180 deg. If the contour plots are checked in detail, it can be seen that the zeroes shift from slightly below 0 and 180 deg at low frequencies (below 1 Hz), to 0 and 180 deg at high frequencies (above the wheel-hop frequency).

6.1.3 Linear model for control synthesis

So far, it has been shown that linear models obtained with AutoSim can be a good representation of the nonlinear model as long as a sensible damping coefficient is used and external disturbances are not excessively large. But is this model valid for a wide enough range of operating conditions? Is it the best choice for control synthesis?

Suitability of the AutoSim linear model

According to the results shown in Section 6.1.2, the nominal SL angle offset has a remarkable effect upon the ability of the SAVGS to affect the dynamic response of the vehicle. Although the SAVGS may be asked to operate at around the nominal offset angle by default, the attitude

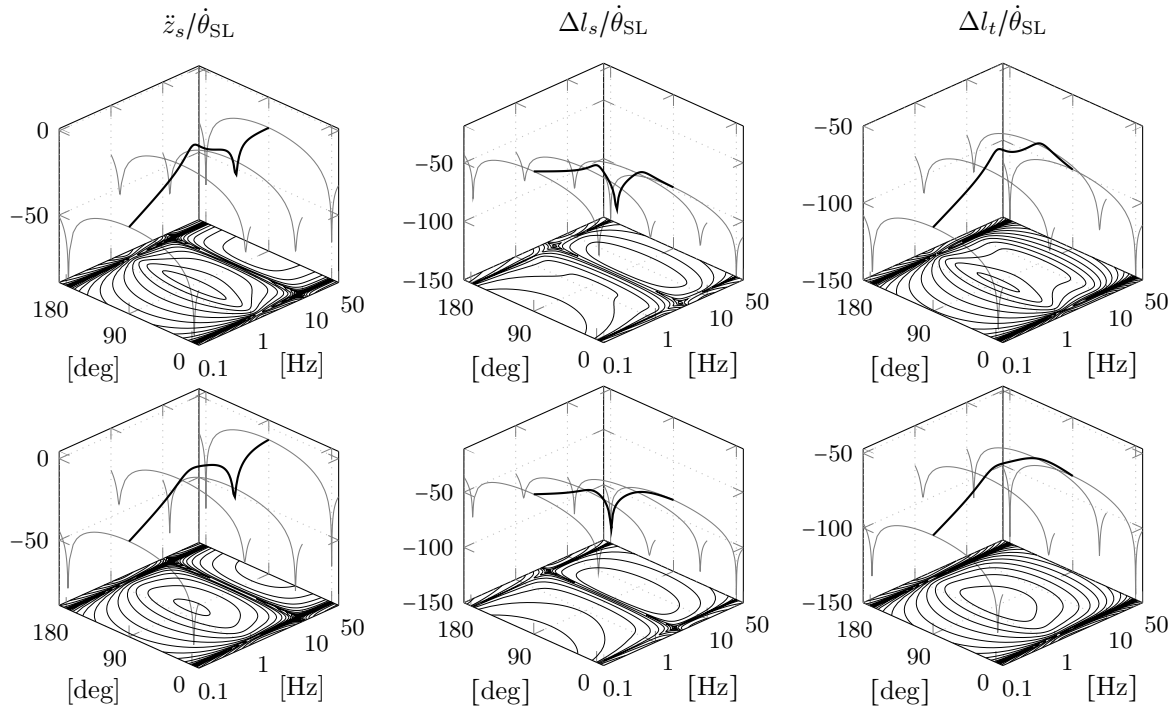


Fig. 6.8: Magnitude Bode plots, in dB, from SL speed to sprung mass acceleration (left), dynamic suspension deflection (centre) and dynamic tyre deflection (right) as a function of frequency and nominal SL angle offset, $\Delta\theta_{\text{SL}}^{(\text{ne})}$. Results are shown for a rear corner of the GT (top) and SUV (bottom). Thick black lines correspond to the $\Delta\theta_{\text{SL}}^{(\text{ne})} = 90^\circ$ case. Slices at $f = 0.1, 1, 10, 50$ Hz as well as magnitude contours are also shown in the 3D figures.

control loop may require significant SL rotations in order to keep appropriate chassis height and attitude, and also the comfort and road holding control loops may ask for large SL rotations when driving over rough roads or demanding individual obstacles such as speed bumps. With this in mind, it is important to establish whether the AutoSim model linearised at the nominal configuration is a good representation of the system in these circumstances, and therefore whether a control system synthesised for it will be effective in a wide range of operating conditions.

Fig. 6.9 and Fig.6.10 try precisely to answer this question. To generate them, the first step is to linearise the GT and SUV quarter-car AutoSim models for a trim state corresponding to the nominal configuration (straight line driving at 100 km/h, nominal payload, nominal single-link angle, nominal suspension geometry). These models will be referred to from now on as nominal AutoSim linearised models. Considering no external loads, new equilibria are also found for SL rotations with respect to the nominal configuration, $\theta_{\text{SL}} = \theta_{\text{SL}}^{(\text{ne})} + \Delta\theta_{\text{SL}}$, and the AutoSim models are linearised for the new trim states. In these new equilibria, suspension deflections and chassis heights differ from those of the nominal configurations. These models will be referred to from now on as updated AutoSim linearised models.

Bode magnitude plots for the new equilibria conditions have been computed with both the

nominal and updated AutoSim models. A good match between these Bode plots would indicate that the nominal linear AutoSim models are able to capture the system dynamics in the new operating conditions reasonably well.

Fig. 6.9 and Fig.6.10 show the absolute value of the magnitude difference between the transfer functions with the nominal and updated linearised models, divided by the magnitude of the transfer functions for the updated linearised models (and expressed as a percentage). From these figures, it becomes clear that the gains from road velocity and external load to all outputs are quite insensitive to SL angle. With short SLs the changes in installation ratio, and therefore in effective spring rate and damping coefficient, are small and can be safely neglected when synthesising a controller. On the other hand, the nominal linear AutoSim model fails to accurately estimate gains from SL speed to all outputs for SL rotations with respect to the nominal configuration larger than ± 30 deg.

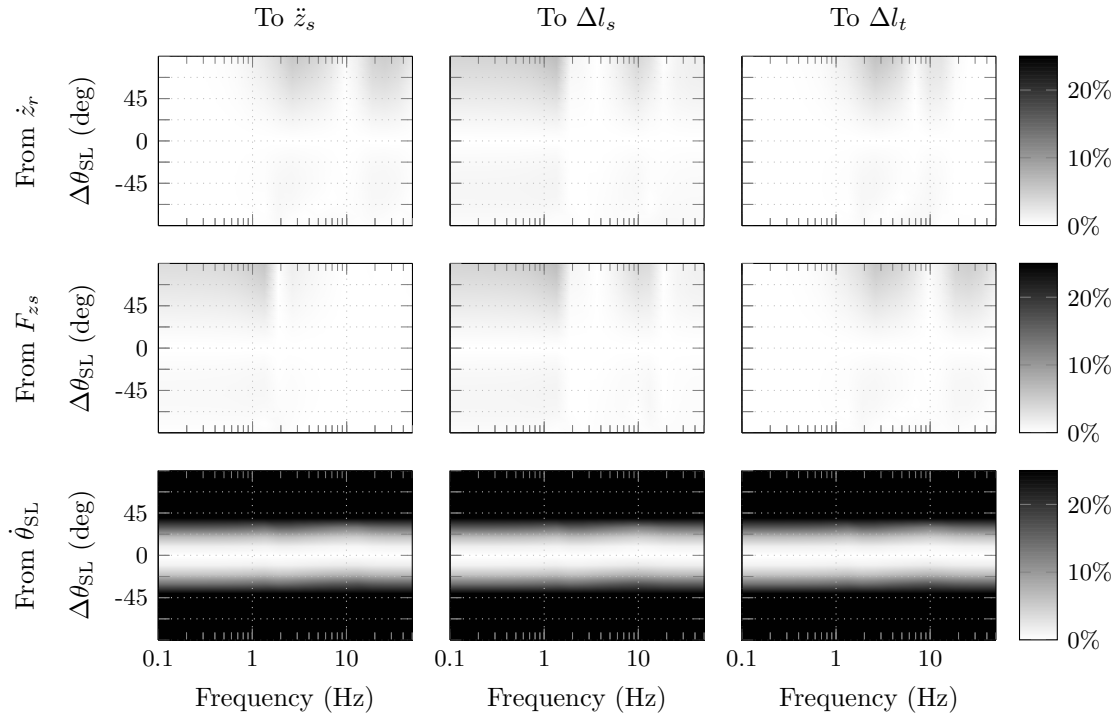


Fig. 6.9: Relative error (in absolute value) between the Bode magnitudes obtained with the AutoSim model linearised at the trim state corresponding to $\theta_{SL} = \theta_{SL}^{(ne)}$, and the AutoSim model linearised at the trim state corresponding to $\theta_{SL} = \theta_{SL}^{(ne)} + \Delta\theta_{SL}$. In both cases $\theta_{SL}^{(ne)} = \theta_{SL}^{(min)} + 90$ deg. Results correspond to the rear suspension of the GT.

Although different controllers could be synthesised for different ranges of operating conditions (gain scheduling), this is not ideal and therefore the following section derives a simplified but more robust to SL position changes model.

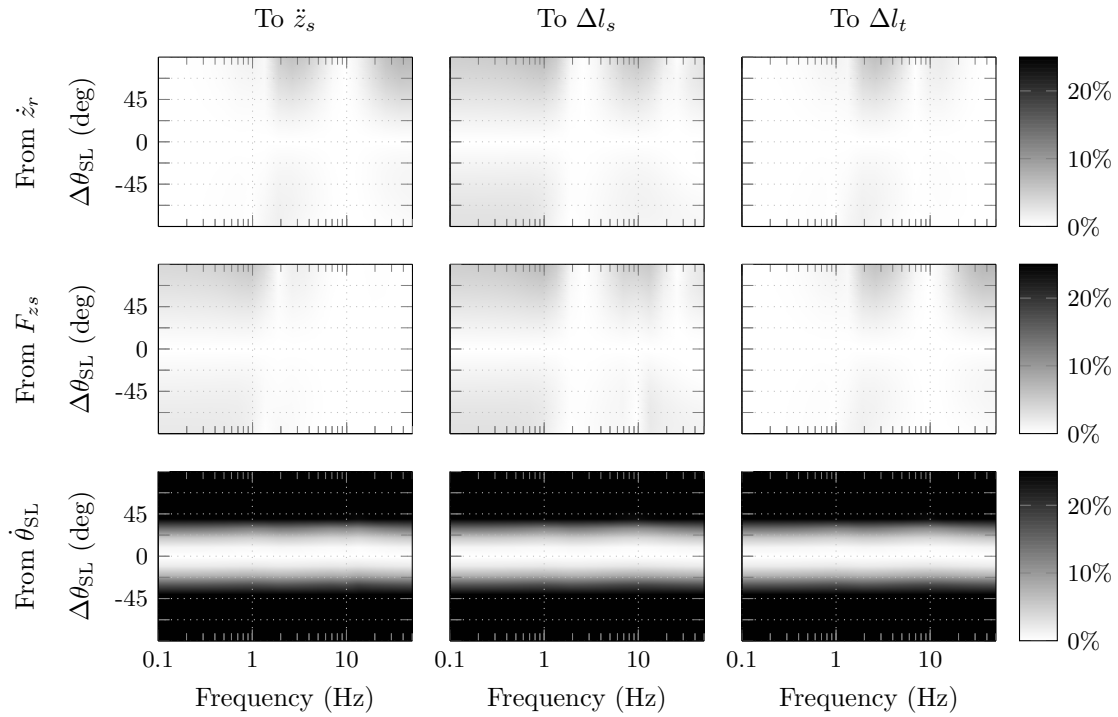


Fig. 6.10: Relative error (in absolute value) between the Bode magnitudes obtained with the AutoSim model linearised at the trim state corresponding to $\theta_{SL} = \theta_{SL}^{(ne)}$, and the AutoSim model linearised at the trim state corresponding to $\theta_{SL} = \theta_{SL}^{(ne)} + \Delta\theta_{SL}$. In both cases $\theta_{SL}^{(ne)} = \theta_{SL}^{(min)} + 90$ deg. Results correspond to the rear suspension of the SUV.

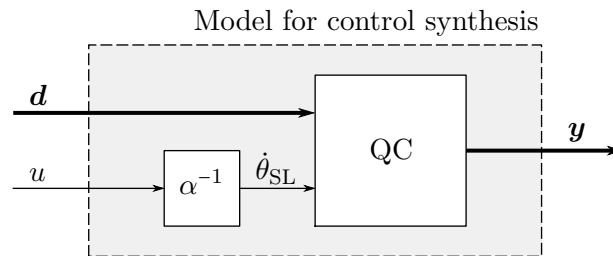


Fig. 6.11: A quarter-car model that removes the strong dependency on SL angle is desirable to synthesise controllers that are suitable for wider ranges of SL angles. $\mathbf{d}$ is the vector of exogenous disturbances, $\mathbf{y}$ is the vector of measurements and u is the command generated by the control system.

Hand-derived linear model

Ideally, it should be possible to derive a simple model that managed to remove the main geometric nonlinearity associated with the SL rotation. This model, shown in Fig. 6.11, would not take $\dot{\theta}_{SL}$ as control input, but instead a new signal, u , that would make the input-output relationships from u to the measurements, $\mathbf{y}$, independent of the actual SL angle. This would be achieved by extracting the nonlinearities from the quarter-car model, lumping them in an external block α^{-1} , and including it in the control input channel so that $\dot{\theta}_{SL} = \alpha^{-1} u$.

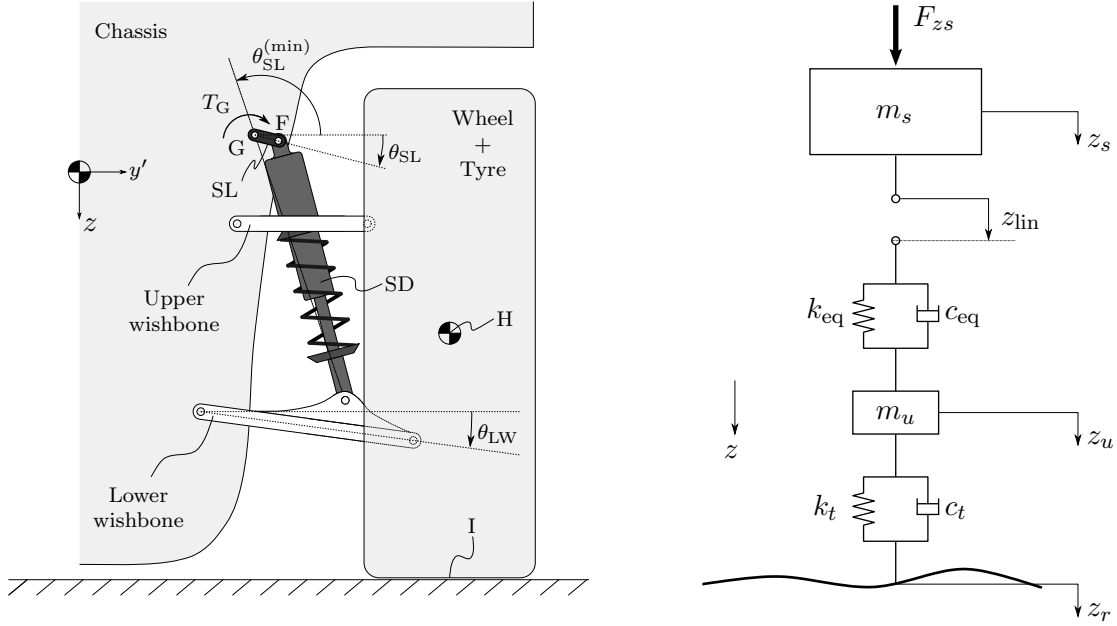


Fig. 6.12: SAVGS-retrofitted double-wishbone suspension (left) and representation of the equivalent hand-derived model (right).

This model is an extension of the linear quarter-car model and has been derived based on energy considerations, in a similar way to how the equivalent parameters for the passive linear quarter-car model were calculated from the real quarter-car parameters in Section 3.2.6. The new model needs to account for the effect of the SL. As the SL appears in series with the passive SD, its influence in the system dynamics is more easily incorporated in the equations of motion by treating it as a displacement input rather than as a force/torque input. Thus, the suspension deflection (i.e. the distance between the sprung and unsprung masses) may now be split into two terms: the deflection of the equivalent SD, and the displacement of an equivalent linear actuator, z_{lin} , that accounts for the contribution of the SL. This is shown in Fig. 6.12.

The power provided by the SAVGS to the rest of the system through the SL must be matched by the power provided by the equivalent linear actuator to the rest of the simplified quarter-car model. That is:

$$T_G \dot{\theta}_{SL} = F_{SD}^* \dot{z}_{lin} \quad (6.3)$$

The SAVGS torque can be obtained by applying the principle of virtual work to the single-link alone, as in (5.18):

$$T_G = -F_{SD} \frac{dl_{SD}}{d\theta_{SL}} \quad (6.4)$$

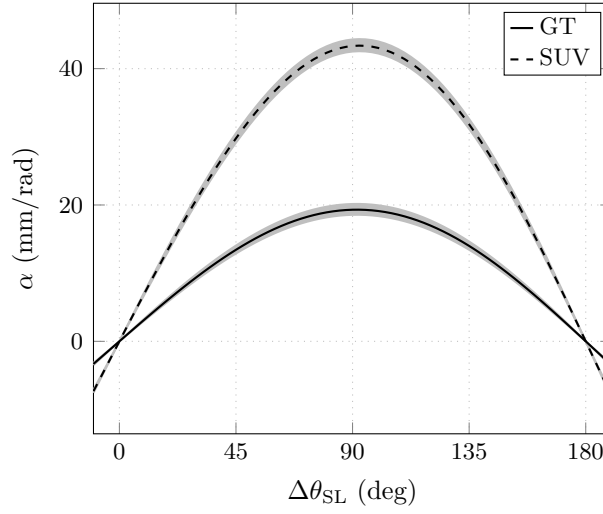


Fig. 6.13: Evolution of α as a function of SL angle, $\Delta\theta_{\text{SL}} = \theta_{\text{SL}} - \theta_{\text{SL}}^{(\min)}$ for the rear suspensions of the GT and the SUV. Grey bands indicate the variations of α with suspension travel for maximum wheel displacements of ± 50 mm with respect to the nominal equilibrium position.

Substituting (6.4) into (6.3), and remembering from (3.46) that $F_{\text{SD}}^* = R_{\text{SD}} F_{\text{SD}}$:

$$\dot{z}_{\text{lin}} = -\frac{1}{R_{\text{SD}}} \frac{dl_{\text{SD}}}{d\theta_{\text{SL}}} \dot{\theta}_{\text{SL}} = \alpha \dot{\theta}_{\text{SL}}, \quad (6.5)$$

where $\frac{dl_{\text{SD}}}{d\theta_{\text{SL}}}$ can be computed from (5.19), and the installation ratio of the spring-damper is:

$$R_{\text{SD}} = \frac{dl_{\text{SD}}}{dz_{\text{H}}} = \frac{dl_{\text{SD}}}{d\theta_{\text{LW}}} \frac{d\theta_{\text{LW}}}{dz_{\text{H}}}. \quad (6.6)$$

The term $\frac{dl_{\text{SD}}}{d\theta_{\text{LW}}}$ is given in (5.16), whereas $\frac{d\theta_{\text{LW}}}{dz_{\text{H}}}$ depends solely on the passive suspension geometry and can be obtained from a kinematic analysis of the passive suspension.

The parameter that accounts for the geometric nonlinearities of the SL, α , is therefore given by

$$\alpha = -\frac{1}{R_{\text{SD}}} \frac{dl_{\text{SD}}}{d\theta_{\text{SL}}}, \quad (6.7)$$

and it is a function of wheel travel and SL angle. Fig. 6.13 shows that its evolution with SL rotation for the rear suspensions of the GT and SUV is quite similar. In both cases it is shaped like a negative parabola that crosses zero at $\Delta\theta_{\text{SL}} = 0$ deg and $\Delta\theta_{\text{SL}} = 180$ deg, when the SL is aligned with the SD, and that reaches approximately 19 mm/rad and 43 mm/rad for the GT and SUV, respectively, at or close to $\Delta\theta_{\text{SL}} = 90$ deg. Furthermore, the influence of suspension travel, Δz_{H} , on α is shown for $\Delta z_{\text{H}} \in [-50, 50]$ mm through grey bands around the nominal case (i.e. $\Delta z_{\text{H}} = 0$ mm). This effect is reasonably small, with a band half-width of approximately 4% with respect to the nominal $\alpha(\Delta z_{\text{H}} = 0)$ case for the GT and 2% for the SUV.

Considering the following state variables and system inputs/outputs:

$$\mathbf{x}^T = [\dot{z}_s, \dot{z}_u, \Delta l_s, \Delta l_t, z_{\text{lin}}], \quad (6.8)$$

$$\mathbf{u}^T = [\dot{z}_r, F_{zs}, \dot{z}_{\text{lin}}], \quad (6.9)$$

$$\mathbf{y}^T = [\ddot{z}_s, \Delta l_s, \Delta l_t], \quad (6.10)$$

the resulting state-space representation for the new model is given by:

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right] = \left[\begin{array}{ccccc|ccc} -\frac{c_{\text{eq}}}{m_s} & \frac{c_{\text{eq}}}{m_s} & \frac{k_{\text{eq}}}{m_s} & 0 & -\frac{k_{\text{eq}}}{m_s} & 0 & \frac{1}{m_s} & -\frac{c_{\text{eq}}}{m_s} \\ \frac{c_{\text{eq}}}{m_u} & -\frac{c_{\text{eq}}+c_t}{m_u} & -\frac{k_{\text{eq}}}{m_u} & \frac{k_t}{m_u} & \frac{k_{\text{eq}}}{m_u} & \frac{c_t}{m_u} & 0 & \frac{c_{\text{eq}}}{m_u} \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline -\frac{c_{\text{eq}}}{m_s} & \frac{c_{\text{eq}}}{m_s} & \frac{k_{\text{eq}}}{m_s} & 0 & -\frac{k_{\text{eq}}}{m_s} & 0 & \frac{1}{m_s} & -\frac{c_{\text{eq}}}{m_s} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{array} \right]. \quad (6.11)$$

The equivalent properties of the SD, k_{eq} and c_{eq} , may be computed as in the passive case through (3.47) and (3.49), although the final expressions are a bit more elaborate because now they also depend on the SL length and angular position:

$$\frac{dR_{\text{SD}}}{dz_{\text{H}}} = \frac{d}{dz_{\text{H}}} \left(\frac{dl_{\text{SD}}}{dz_{\text{H}}} \right) = \frac{d}{dz_{\text{H}}} \left(\frac{dl_{\text{SD}}}{d\theta_{\text{LW}}} \frac{d\theta_{\text{LW}}}{dz_{\text{H}}} \right) = \frac{d}{dz_{\text{H}}} \left(\frac{dl_{\text{SD}}}{d\theta_{\text{LW}}} \right) \frac{d\theta_{\text{LW}}}{dz_{\text{H}}} + \frac{dl_{\text{SD}}}{d\theta_{\text{LW}}} \frac{d^2\theta_{\text{LW}}}{dz_{\text{H}}^2} \quad (6.12)$$

$$\begin{aligned} \frac{d}{dz_{\text{H}}} \left(\frac{dl_{\text{SD}}}{d\theta_{\text{LW}}} \right) &= \frac{1}{2l_{\text{SD}}^2} \left[l_{\text{SD}} \frac{d\theta_{\text{LW}}}{dz_{\text{H}}} \left(-\alpha_1 c_1 - \alpha_2 s_1 - l_{\text{SL}} (\alpha_5 c_{12} + \alpha_6 s_{12}) \right) \dots \right. \\ &\quad \left. - R_{\text{SD}} \left(-\alpha_1 s_1 + \alpha_2 c_1 - l_{\text{SL}} (\alpha_5 s_{12} - \alpha_6 c_{12}) \right) \right] \end{aligned} \quad (6.13)$$

The accuracy of the predictions obtained with this simple model are compared for the rear suspensions of the GT and SUV with those provided by the updated linearised AutoSim models in Fig. 6.14 and Fig. 6.15, respectively. The only difference between these figures and figures 6.9 and 6.10 is that the model tested is the simple hand-derived linear model with parameters corresponding to $\Delta\theta_{\text{SL}}^{(\text{ne})} = 90^\circ$ instead of the nominal linearised AutoSim model. For easier comparison, the same colour scales and axes as in previous figures have been used.

The differences with respect to the nominal linear AutoSim model are evident. First of all, the quality of the approximation is slightly worse for inputs $\dot{z}_r$ and F_{zs} , as the simple model does not capture the exact system dynamics for the $\Delta\theta_{\text{SL}} = 0$ case. Yet, these errors remain below 10% in most areas and are considered to be acceptable. Similarly, the quality of the

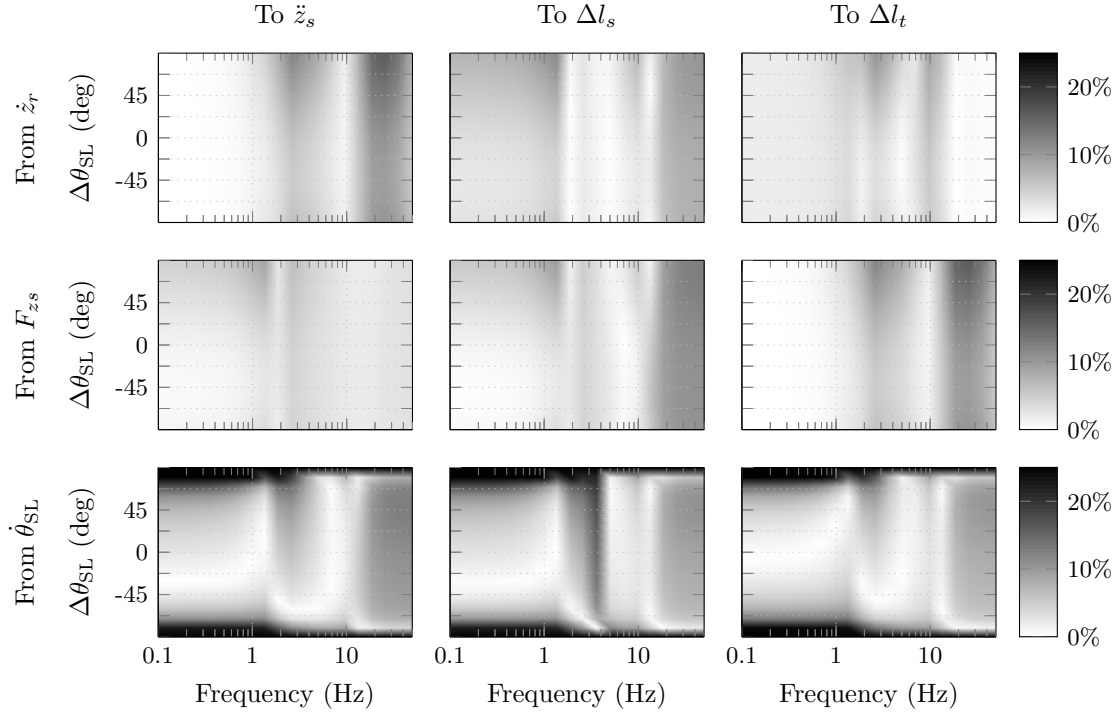


Fig. 6.14: Relative error (in absolute value) between the Bode magnitudes obtained with the hand-derived model linearised at the trim state corresponding to $\theta_{SL} = \theta_{SL}^{(ne)}$, and the AutoSim model linearised at the trim state corresponding to $\theta_{SL} = \theta_{SL}^{(ne)} + \Delta\theta_{SL}$. In both cases $\theta_{SL}^{(ne)} = \theta_{SL}^{(min)} + 90$ deg. Results correspond to the rear suspension of the GT.

approximation deteriorates slightly for input $\dot{\theta}_{SL}$ and small SL rotations. However, the range of SL rotations for which an acceptable match is achieved is greatly increased from approximately ± 30 deg to ± 70 deg. This means that the hand-derived linear model is able to represent the system accurately in the whole range of desirable SL rotations, which in general are not limited to $[\theta_{SL}^{(min)}, \theta_{SL}^{(max)}]$ but to $[\theta_{SL}^{(min)} + \Delta\theta_{SL}^{(min)}, \theta_{SL}^{(max)} - \Delta\theta_{SL}^{(max)}]$, with $\Delta\theta_{SL}^{(min)}$ and $\Delta\theta_{SL}^{(max)}$ typically in the 0 to 30 deg range as already discussed in Section 5.2.

In short, the proposed model:

- Is of smaller size than the linear AutoSim model: $A \in \mathbb{R}^{5 \times 5}$ instead of $A \in \mathbb{R}^{8 \times 8}$.
- It approximates the system dynamics reasonably well for the whole range of SL rotations.
- As opposed to the linear AutoSim model, it can be easily extended to represent a linear full-vehicle model equipped with the SAVGS. The hand-derived full-car model is provided in Appendix B.

Although the choice is not clear-cut, the simpler hand-derived linear model has been preferred over the linearised AutoSim models for the design of comfort and road holding enhancing control algorithms due to its reduced size and improved robustness to SL rotations.

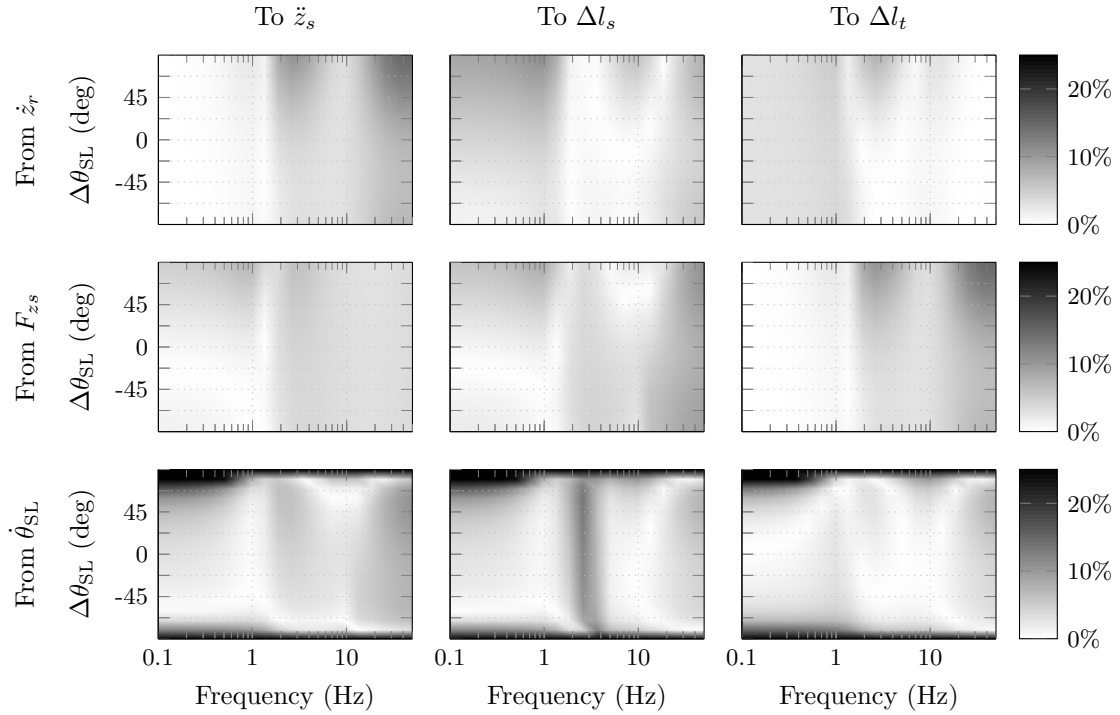


Fig. 6.15: Relative error (in absolute value) between the Bode magnitudes obtained with the hand-derived model linearised at the trim state corresponding to $\theta_{SL} = \theta_{SL}^{(ne)}$, and the AutoSim model linearised at the trim state corresponding to $\theta_{SL} = \theta_{SL}^{(ne)} + \Delta\theta_{SL}$. In both cases $\theta_{SL}^{(ne)} = \theta_{SL}^{(min)} + 90$ deg. Results correspond to the rear suspension of the SUV.

6.2 $\mathcal{H}_\infty$ control synthesis framework

The generalised regulator considered for $\mathcal{H}_\infty$ control synthesis is shown in Fig. 6.16, where:

- w represents the exogenous disturbances acting on the system,
- z represents the variables that are to be controlled,
- y are the measurement signals made available to the controller, and
- u are the control actions.

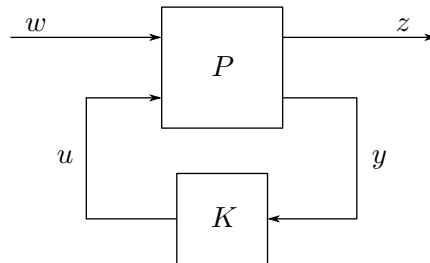


Fig. 6.16: Schematic of the generalised regulator.

The overall transfer function from w to z for the interconnection shown in Fig. 6.16 corresponds

to a lower Linear Fractional Transformation (LFT)[Doyle et al., 1991]:

$$R_{zw} = \mathcal{F}_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \quad (6.14)$$

The aim of the controller K designed with $\mathcal{H}_\infty$ techniques is to find an admissible controller^{iv} that satisfies the infinity norm objective [Green and Limebeer, 1995]:

$$\|R_{zw}\|_\infty < \gamma. \quad (6.15)$$

That is, the maximum gain from w to z in the whole frequency range, which is given by:

$$\|R_{zw}\|_\infty = \sup_{\omega} \bar{\sigma}(R_{zw}(j\omega)), \quad (6.16)$$

with $\bar{\sigma}(R_{zw}(j\omega))$ representing the maximum singular value of R_{zw} at any given frequency ω , needs to be bounded by a suitably small real number γ .

The $\mathcal{H}_\infty$ controller, K , is synthesised according to standard methods (see for example [Green and Limebeer, 1995; Zhou and Doyle, 1998]). Here it suffices to say that it can be obtained using MATLAB's `hinfsyn`^v function.

The following subsections cover the specific disturbances, control objectives, measurements and control actions considered in this chapter.

6.2.1 Exogenous disturbances, w

Three external inputs have been considered for the control synthesis: rate of change of road height, vertical load transfer and exogenous position command for the SL^{vi}.

The rate of change of road height, $\dot{z}_r$, which is an input to the hand-derived quarter-car model, represents the main exogenous disturbance acting on the suspension system. Individual road bumps, potholes and general road unevenness can introduce large amounts of energy in the system by exciting the vertical motion of the wheel. Significant changes in vertical tyre force as well as suspension deflection and sprung mass acceleration arise as this energy is transmitted from the wheel to the chassis. Road disturbances can introduce energy in a wide range of frequencies. Typically, spatial frequencies in the 0.01 to 10 cycles/m range are the most relevant ones, as reported by [ISO 8608:1995].

^{iv}An admissible controller will be causal, linear, time-invariant, finite-dimensional and will internally stabilise P .

^vMATLAB's function `hinfsyn` calculates the $\mathcal{H}_\infty$ controller solving two Riccati formulae described in [Glover and Doyle, 1988; Doyle et al., 1989]. In their derivation, it is assumed that the plant has been simplified through loop-shifting, and the resulting controller $\tilde{K}$ can be reverse-transformed to obtain the controller for the original problem [Safonov et al., 1989]. All these transformations are performed automatically by MATLAB.

^{vi}Measurement errors were also included as external disturbances in the synthesis phase. However, suitable results and robustness against measurement noise was achieved even when not including these signals in the control synthesis stage, and therefore they were finally dropped from the design scheme.

Load transfers arising from longitudinal and/or lateral vehicle accelerations are captured in the model by an external vertical force acting on the sprung mass, F_{zs} . This external force leads to a vertical motion of the sprung mass, which in turn results in suspension deflections and changes in vertical tyre forces. Typically, pitch and roll natural frequencies are in the 1 to 3 Hz range, and therefore it is expected that most input energy will be concentrated in this frequency band.

Finally, SL position commands provided by higher level controllers are also considered as exogenous inputs. This may represent the angle from which the SAVGS is asked to operate (this will be referred to as $\theta_{\text{SL}}^{(e)}$ when dealing with SL angles, and as $z_{\text{lin}}^{(e)}$ when dealing with equivalent linear displacements). For example, at low speeds the SAVGS may be required to increase ground clearance and to compensate for road and load transfer disturbances by operating in the proximities of an equilibrium position corresponding to a large SL angle.

Thus, the $p=3$ input variables are weighted versions of the physical (denoted by tilde) signals:

$$\tilde{\mathbf{w}}^T = [z_{\text{lin}}^{(e)}, \dot{z}_r, F_{zs}]. \quad (6.17)$$

6.2.2 Control objectives and constraints, z

In this chapter, the two main control objectives considered are 1) to improve ride comfort and 2) to increase tyre grip. These two aspects of suspension performance are commonly quantified in the quarter-car context through the vertical acceleration of the sprung mass (comfort), and vertical tyre force variations or dynamic tyre deflections (grip). As already discussed in Section 2.4.3, these objectives are not independent. There are some invariant points and slope constraints (or interpolation conditions) in the transfer functions of interest, and once one of the transfer functions from road velocity to either sprung mass acceleration or tyre deflection is determined, the other one is automatically fixed. Here, both objectives have been maintained in the synthesis phase to facilitate shaping their relative importance in different frequency regions through the application of individual frequency weights.

A third control objective included in the synthesis scheme is the tracking of exogenous SL position commands. As explained before, it is convenient to be able to change the SL position from which the SAVGS operates, and this can be done by including a tracking metric as one of the controlled variables in z .

The performance of the suspension system deteriorates significantly if the bump stops are hit frequently and therefore it is important to ensure that maximum suspension travel is maintained within allowable limits. As most active suspension studies consider actuators with large strokes that act in parallel with the SD, suspension deflection is one of the variables that typically need to be minimised by control systems. In the SAVGS case, however, the short SL acts in series

with the SD and has a limited capacity to increase suspension travel beyond allowable bounds. This control objective has been dropped from the synthesis stage, and simulation results shown in Section 6.3 demonstrate that this is an appropriate design choice.

Finally, the aforementioned objectives need to be achieved with sensible levels of actuation effort. Actuation torque and power cannot be penalised in this linear framework, but angular speed and maximum rotation angle of the SL can be. In the hand-derived linear model this is achieved indirectly by penalising z_{lin} and $\dot{z}_{\text{lin}}$, respectively.

Thus, the $m=5$ output variables are weighted versions of the physical signals:

$$\tilde{\mathbf{z}}^T = [z_{\text{lin}}^{(e)} - z_{\text{lin}}, \ddot{z}_s, \Delta l_t, z_{\text{lin}}, \dot{z}_{\text{lin}}]. \quad (6.18)$$

6.2.3 Measurements, y

There is a very large number of possible input-output (IO) sets, i.e. combinations of measurement signals and control actions, for the active suspension problem^{vii}. Here, signals made available to the controller have been chosen based on the expected availability of sensors and are in line with most active suspension studies that can be found in the literature. In particular, two uniaxial accelerometers (one for the vertical acceleration of the sprung mass and one for the vertical acceleration of the wheel centre), one LVDT linear position sensor or potentiometer (to measure suspension deflection) and one encoder (to measure angular position of SL) have been considered, and the signals assumed to be fed to the controller in the control synthesis stage are:

$$\mathbf{y}^T = [\dot{\Delta} l_s, \ddot{z}_u, \ddot{z}_s, z_{\text{lin}}^{(e)} - z_{\text{lin}}]. \quad (6.19)$$

6.2.4 Control action, u

The control output is the reference speed for the equivalent linear actuator:

$$u = \dot{z}_{\text{lin}}^*. \quad (6.20)$$

As the control command, $\dot{z}_{\text{lin}}^*$, cannot be perfectly tracked by the actuators in each corner of the vehicle, a transfer function, P_{IL} , that represents the dynamics of the actuator and its inner control loops (IL) has been introduced in the $\mathcal{H}_\infty$ control synthesis framework (IL stands for

^{vii}For example, Van De Wal et al. [1998] took a half-vehicle model and attempted to find all IO sets for which a controller existed that could achieve a certain level of robust performance. Despite considering 3577 candidate IO sets, the procedure failed to provide reliable results. Moreover, it was not possible to identify the types of problems for which this kind of selection method could be effectively applied.

Inner Loop):

$$\dot{z}_{\text{lin}} = P_{\text{IL}} \dot{z}_{\text{lin}}^*. \quad (6.21)$$

Unfortunately, the system response is very nonlinear and therefore it is not possible to capture the real system dynamics in a single linear transfer function P_{IL} .

The tracking error problem can be particularly acute for rapidly changing speed commands, for situations in which the SL is far from the 90 deg nominal offset position and for cases in which the actuator is limited by the continuous torque and current constraints. Fig. 6.17 highlights this fact and shows the relationship, in the frequency domain, between $\dot{z}_{\text{lin}}^*$ and $\dot{z}_{\text{lin}}$ for the undisturbed quarter-car model and various operating conditions.

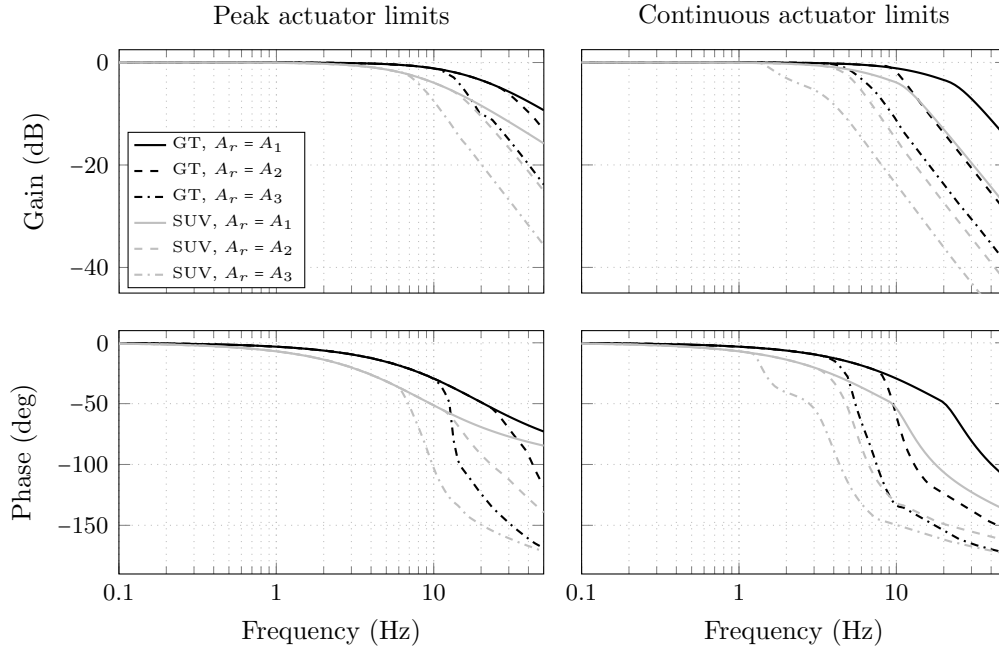


Fig. 6.17: Magnitude gain and phase delay from $\dot{z}_{\text{lin}}^*$ to $\dot{z}_{\text{lin}}$ for a rear corner of the GT (black) and SUV (grey) subject to peak (left column) and continuous (right column) current and torque limitations. Results are shown for three levels of relative amplitude $A_r = |\dot{z}_{\text{lin}}^*|/|\dot{z}_{\text{lin}}^{(\max)}|$: $A_1 = 0.01$, $A_2 = 0.06$ and $A_3 = 0.2$ which correspond approximately to SL peak-to-peak reference angular motions of 1, 7 and 22.5 deg.

For small z_{lin} amplitudes and peak actuator limits, the actuator and its control loops can be accurately represented by a first order low-pass filter. This is the assumption in this chapter, and the resulting transfer functions are:

$$\text{GT: } P_{\text{IL}} = \frac{1}{s/(2\pi 17.8) + 1}, \quad \text{SUV: } P_{\text{IL}} = \frac{1}{s/(2\pi 8.1) + 1} \quad (6.22)$$

However, as z_{lin} amplitudes increase, the corner frequency is reduced, the falling slope of the

magnitude plot at high frequencies becomes steeper, and, overall, the response becomes more erratic (particularly with continuous torque and current limits). Appendix C outlines how this uncertainty could be incorporated in the synthesis framework.

6.2.5 System interconnection

The specific interconnection considered for control synthesis is shown in Fig. 6.18, where nomenclature and signal labelling is compatible with what has been described so far. The measurement signal corresponding to the tracking error for the exogenous position command is filtered by M to ensure zero steady-state tracking error is achieved (as suggested by Zhou and Doyle [1998]). Otherwise, the performance weight W_{z_1} could not include integral action, as this would imply that $s = 0$ would be an uncontrollable pole of the feedback system. Controller $\hat{K}$ is obtained using `hinfsyn`, and the final controller, K , is computed by incorporating the effect of M , as shown in Fig. 6.18.

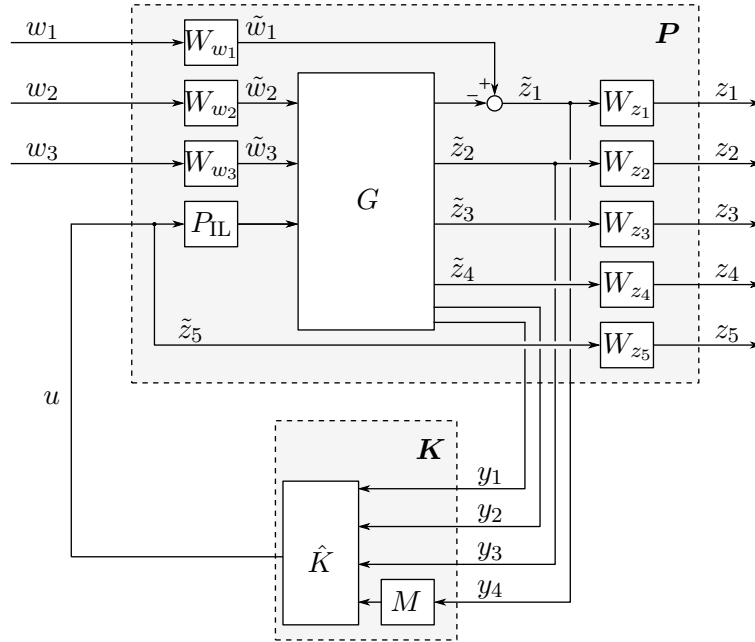


Fig. 6.18: Interconnection for controller synthesis.

Input and output weights, W_w and W_z , are block diagonal matrices with diagonal terms W_{w_1} to W_{w_3} and W_{z_1} to W_{z_5} , that are selected to scale physical quantities (denoted by tilde) and to shape the relative importance between signals in different frequency regions:

$$\begin{aligned}\tilde{w} &= W_w w, \\ z &= W_z \tilde{z}.\end{aligned}\tag{6.23}$$

6.2.6 Selection of frequency weights

Although there are some general guidelines regarding the selection of weights in the context of $\mathcal{H}_\infty$ control synthesis (see for example the attempts presented in [Lundström et al., 1991] and [Beaven et al., 1996]), it becomes clear from the numerous publications dealing with active suspension control that there is not an obvious or unique procedure to select them. Moreover, the final controller is very sensitive to the frequency weights, making the tuning process time consuming [dos Santos et al., 2012].

The final weights selected for the rear suspension of the GT are given by (6.24):

$$W_{w_1} = 0.02, \quad W_{w_2} = 0.25, \quad W_{w_3} = 1000, \quad (6.24)$$

where the DC gains are related to the maximum expected input values. The in-loop block M has been selected according to (6.25):

$$M = \frac{s/(2\pi \cdot 1) + 1}{s}, \quad (6.25)$$

and the output weights are:

$$\begin{aligned} W_{z_1} &= \left(\frac{1}{0.0035} \right) \frac{1}{s}, & W_{z_2} &= \left(\frac{1}{0.9} \right) \frac{1}{s/(2\pi \cdot 10) + 1}, & W_{z_3} &= \left(\frac{1}{0.005} \right) \frac{1}{s/(2\pi \cdot 5) + 1}, \\ W_{z_4} &= 1, & W_{z_5} &= \left(\frac{1}{0.1} \right) \frac{s/(2\pi \cdot 15) + 1}{s/(2\pi \cdot 100) + 1}, \end{aligned} \quad (6.26)$$

where the inverse of the DC gain are related to the maximum expected output values. The weight for exogenous SL position command tracking is a pure integrator, for sprung mass acceleration and dynamic tyre deflections are first order low-pass filters with cut-off frequencies of 10 Hz and 5 Hz, respectively, and the penalty for the control effort $\dot{z}_{\text{lin}}$ increases from 10 at frequencies below 15 Hz to ~ 65 at frequencies above 100 Hz. The penalty for the control effort z_{lin} is practically irrelevant in this case, but has been left here for completeness.

The resulting controller has 12 states, which have been subsequently reduced to 11 without any significant loss of accuracy through MATLAB's `reduce` function with a multiplicative error tolerance of 10^{-5} . The following paragraphs summarise the main aspects that have been taken into account when choosing these weights.

High order frequency weights allow more flexibility in shaping the response of the system, but they also lead to higher order controllers [Bibel and Malyevac, 1992] and complicate the tuning process. In this work, the weight selection process began considering scalar weights for all signals, and frequency-dependency was added carefully when it made sense from the physical and control design points of view. In the end, good performance has been obtained with a mix of scalar and first order frequency weights.

The condition number for the original plant is poor due to the difference in physical units (e.g. an external force of 1 N acting on the sprung mass has an almost unnoticeable effect on the quarter-car, whereas a rate of change of road height of 1 m/s is an extremely demanding disturbance). Given that the $\mathcal{H}_\infty$ controller aims to reduce the maximum gain of the closed loop system augmented with the frequency weights, it is important to ensure that the relevant input-output paths dominate in the frequency ranges of interest (e.g. exogenous position command tracking should dominate at low frequencies but should be negligible at high frequencies). To be able to achieve this, the open-loop and closed-loop singular values and their associated input-output directions have been used to inform the weight tuning process. Given the $m \times p$ transfer matrix for the plant, R_{zw} , the singular value decomposition at any given frequency ω is given by:

$$R_{zw}(j\omega) = U(j\omega) \Sigma(j\omega) V(j\omega)^H, \quad (6.27)$$

where U and V are square matrices of size $m \times m$ and $p \times p$ whose columns have unit $\mathcal{L}_2$ norm and are the singular output and input vectors, H denotes the conjugate transpose, and Σ is an $m \times p$ diagonal matrix with $r = \min(m, p)$ nonzero, positive singular values $\sigma_1 \geq \sigma_2 \geq \dots \sigma_r > 0$ and $\sigma_q = 0 \ \forall q > r$. The singular values represent the gains of the system, and the input and output singular vectors are the associated input and output directions. For example, $\sigma_1(j\omega) = \bar{\sigma}(j\omega)$ is the largest gain in the system at the frequency ω , the first column of $V(j\omega)$ is the input direction with the largest amplification at that frequency and the first column of $U(j\omega)$ is the output direction in which the inputs are most effective.

The singular values and condition number, $\kappa(j\omega) = \bar{\sigma}(j\omega)/\underline{\sigma}(j\omega)$, are shown in Fig. 6.19 for the open-loop system representing the rear suspension of the GT, with and without frequency weights. The associated input and output directions for the open-loop system with and without weights, and for the closed-loop weighted system are presented in Fig. 6.20, and the singular values for the weighted open-loop and closed-loop systems are compared in Fig. 6.21.

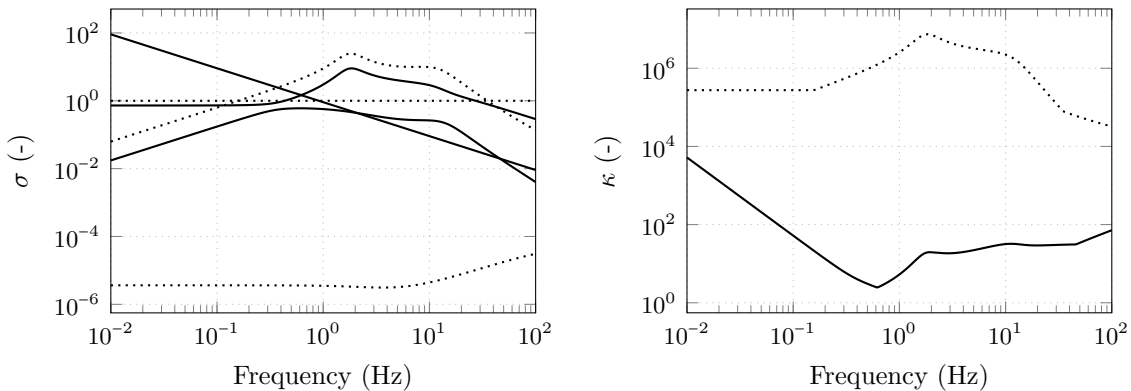


Fig. 6.19: Singular values and condition number for the open-loop system without (dotted) and with (solid) frequency weights.

A few relevant aspects highlighted by these figures are:

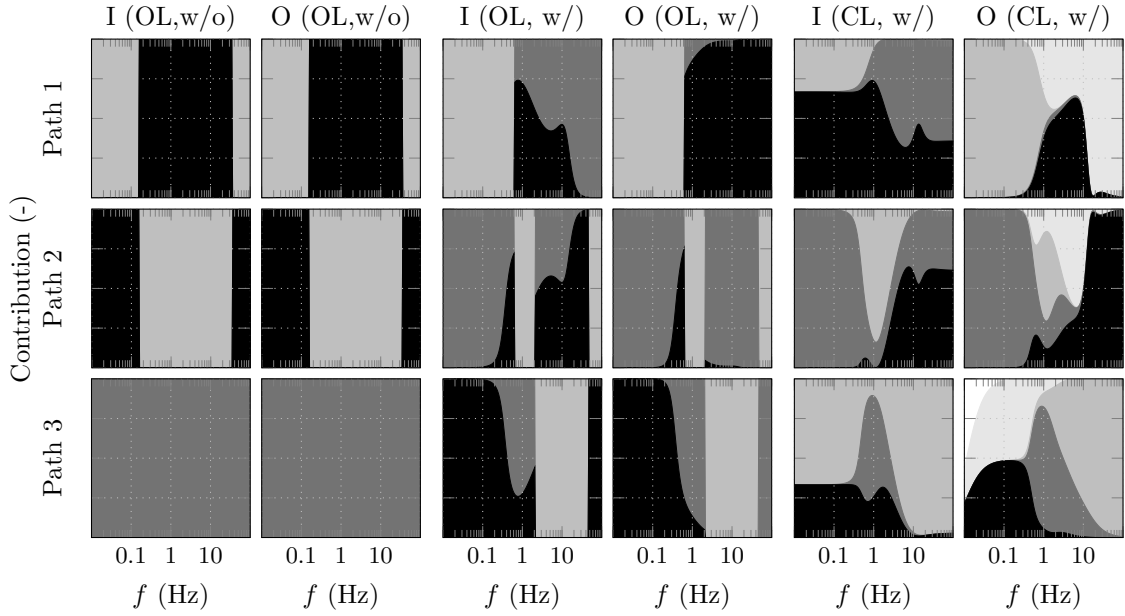


Fig. 6.20: Singular vector components: contributions to the input (I) and output (O) singular vectors for the open-loop (OL) and closed-loop (CL) systems with (w/) and without (w/o) frequency weights. The vertical distance shown in a particular colour at any given frequency represents the proportion of the input (or output) vector length due to a particular input (or output) component. From dark to light the input components are rate of change of vertical road profile, external vertical force acting on the sprung mass and reference actuator position, and the output components are sprung mass acceleration, tyre deflection, tracking error, and control efforts $\dot{z}_{lin}$ and z_{lin} . Paths 1 to 3 are associated with singular values $\sigma_1(j\omega)$ to $\sigma_3(j\omega)$ shown in Fig. 6.19 and Fig. 6.21. At any given frequency, $\sigma_1(j\omega)$ is the largest singular value and path 1 (top row) the associated input-output path.

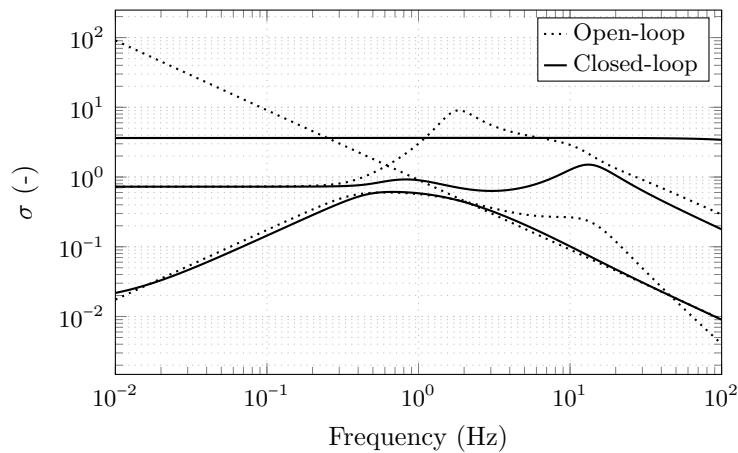


Fig. 6.21: Singular values for the weighted system.

- In the 0.1 to 100 Hz range, the weighted open-loop system has condition numbers that are 3 to 5 orders of magnitude smaller than those of the unscaled open-loop system (Fig. 6.19).
- In the unscaled open-loop system, the path from exogenous SL position command to tracking error dominates at low and high frequencies, and road velocity to sprung mass acceleration dominates in the mid-frequency range. Gains of the path involving the external vertical force and dynamic tyre deflection are negligible in comparison throughout the frequency range (Fig. 6.19, columns 1 and 2 in Fig. 6.20).
- In the weighted open-loop system, on the other hand, exogenous SL position command and tracking error dominate only at frequencies below 0.6 Hz. In the 0.7 to 30 Hz range, the main input direction transitions from 75% vertical road velocity and 25% external force, to 100% external force, and the main output direction from 75% sprung mass acceleration and 25% tyre deflection to 100% sprung mass acceleration (see Fig. 6.20, first row, columns 3 and 4). What happens at frequencies above 10 Hz is not that relevant in this case, as the penalty on $\dot{z}_{\text{lin}}$ will ensure that the SAVGS does not attempt to do much at or above the wheel hop natural frequency. Moreover, although these proportions may not match exactly what would be desirable in principle, they are reasonably close considering that only scalar and first order frequency weights were used. Simulation results validate this choice.
- The closed-loop system displays a much more interlinked response. The maximum singular value remains practically constant in the frequency range of interest, but the input-output path associated with it varies significantly. Generally speaking, sprung mass acceleration, exogenous SL position command tracking error and SL speed are the dominant outputs. Tyre deflection also has some significance, but z_{lin} appears only at low frequencies in the path associated with the minimum singular value (see Fig. 6.20, columns 5 and 6).

The Bode magnitude plot for the synthesised controller is shown in Fig. 6.22. As desired, the relative importance of the gain for exogenous SL position tracking error is largest at low frequencies. All gains roll off at approximately 10 Hz except the one for unsprung mass acceleration, which only does so above 100 Hz.

Finally, the predicted Bode magnitude plots for the open-loop and closed-loop unscaled systems are compared in Fig. 6.23. These results confirm the weight selection, as they indicate that the control objectives will be, at least in principle, properly tackled by the synthesised controller. To summarise:

- No steady-state error for exogenous SL position tracking
- Significant reductions in sprung mass acceleration and dynamic tyre deflection for road disturbances below the wheel hop frequency (~ 10 Hz)
- Significant reductions in sprung mass acceleration and dynamic tyre deflection for load transfers below the pitch and roll natural frequencies (~ 3 Hz)

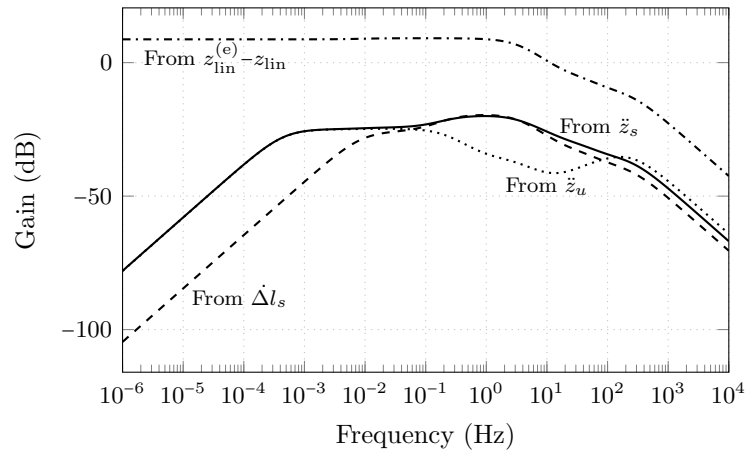
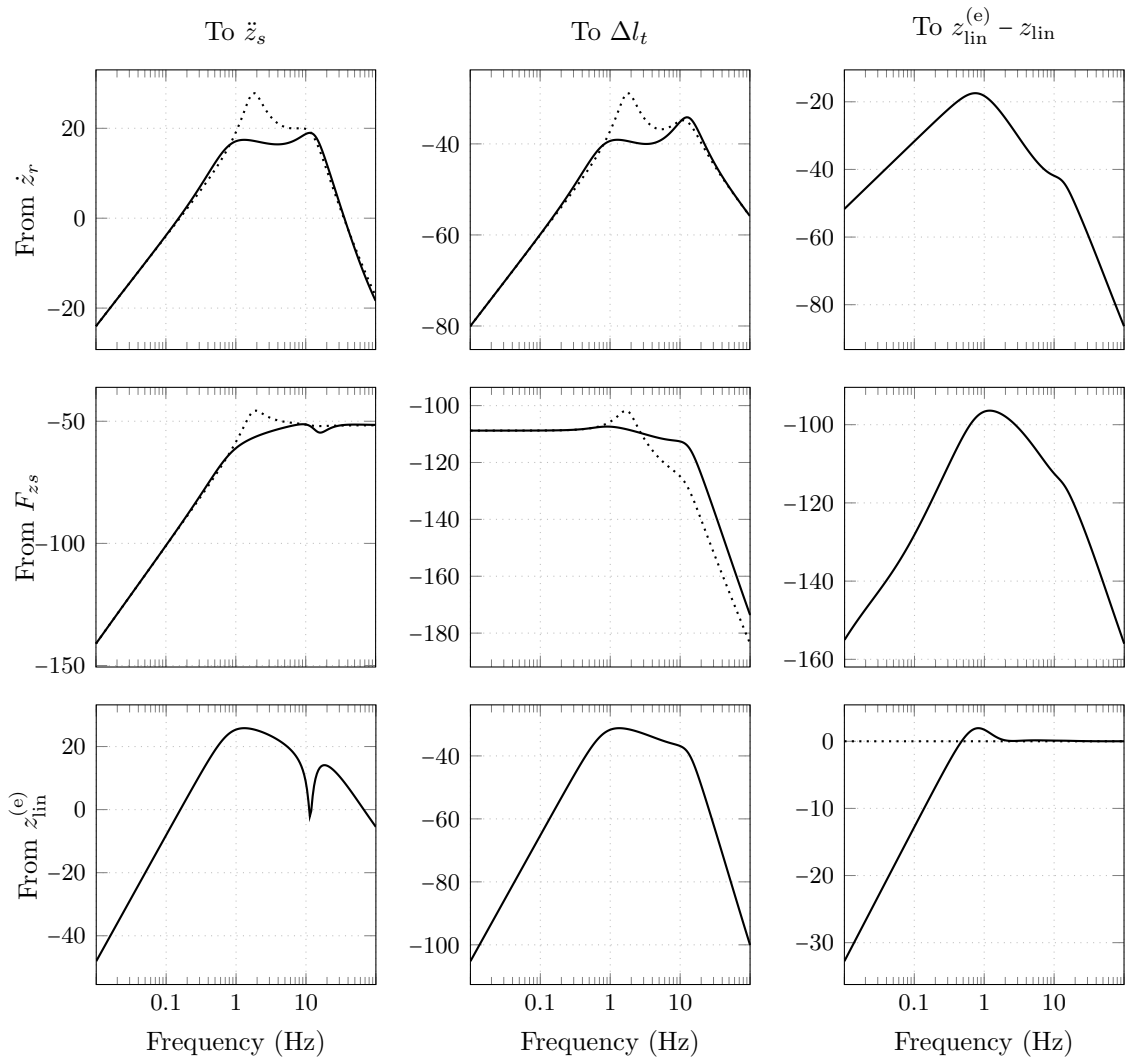
Fig. 6.22: Bode diagram for controller K .

Fig. 6.23: Comparison of open-loop and closed-loop Bode magnitude plots.

Although all figures so far refer to the rear suspension of the GT, the same weighting strategy leads to similar results for the front suspension as well as for the SUV. The suitability of the proposed control system is discussed based on the simulation results obtained with the nonlinear quarter-car and full-car AutoSim models and presented in Section 6.3.

6.3 Simulation results

Once a controller has been synthesised using the simplified hand-derived linear model, extensive testing has been performed with the nonlinear quarter-car model in order to assess the performance of the SAVGS as well as the quality of the control. Results presented in this section cover three main scenarios:

- Driving at low speed (20 km/h) over a large and smooth road bump (sinusoidal profile, 5 cm high and 2 m long)
- Sudden but smooth vertical load transfer (50% of nominal tyre load) when cruising at 100 km/h
- Driving at 100 km/h for 10 km over three random road profiles representative of very good to poor roads (classes A to C in [ISO 8608:1995])

But first, it is worth noting that the results presented have been obtained corrupting all measurements with additive Gaussian white noise and subsequently filtering them with first order low-pass filters with a cut-off frequency of 200 Hz. Noise levels have been estimated based on the official specifications of real sensors and it has been assumed that actual noise values are within the official specifications for 99.7% of the time. Table 6.1 provides the standard deviations considered for each measurement signal. As already mentioned, the synthesised controller

Table 6.1: Standard deviation for Gaussian additive measurement noise signals.

Signal	Units	Standard deviation
$\ddot{z}_s$	m/s ²	0.15/3
$\ddot{z}_u$	m/s ²	0.30/3
Δl_s	m	0.001/3
$\theta_{SL}^{(e)} - \theta_{SL}$	rad	0.01/3

cope well with these levels of noise and therefore it was deemed superfluous to include additive noise as additional disturbances in the synthesis stage.

Another difference with respect to the interconnection used for control synthesis is that, for convenience, the exogenous position command for the actuator and the tracking error have been computed based on SL angular positions rather than on the equivalent linear displacement. The

actual tracking error measure has thus been obtained through:

$$y_4 = \frac{1}{s/(2\pi 200) + 1} \left(\frac{z_{\text{lin}}^{(\max)}}{\pi/2} \right) \left[\theta_{\text{SL}}^{(e)} - (\theta_{\text{SL}} + N) \right], \quad (6.28)$$

where N represents the Gaussian noise and the factor $\frac{z_{\text{lin}}^{(\max)}}{\pi/2}$ has been introduced to approximately scale the tracking error from SL angle to equivalent actuator displacement. Simulation results confirm that this approximated conversion is appropriate.

Finally, the output of the controller, which represents the reference linear speed for the equivalent linear actuator, $\dot{z}_{\text{lin}}^*$, has been converted to reference angular speed for the SL through

$$\dot{\theta}_{\text{SL}}^* = \alpha^{-1} \dot{z}_{\text{lin}}^*, \quad (6.29)$$

where the conversion factor α , given in (6.7), is computed on-line from the measured SL position and suspension travel (noisy and low-pass filtered as all other measurements). Moreover, and in order to avoid excessively large $\dot{\theta}_{\text{SL}}^*$ values when the SL is aligned with the SD, α has been saturated to ensure that it does not fall below 0.001 m/rad.

6.3.1 Quarter-car simulation results

All graphs presented in this section correspond to the rear suspension of the GT, but tabular summaries are also provided for the front suspension.

Individual events: smoothed bump and load transfer

Fig. 6.24 shows the time series for the smoothed road bump event. The exogenous position command has been chosen in this case as 120 deg with respect to the $\overline{\text{EG}}$ line in the nominal equilibrium, in order to illustrate:

- That it may be convenient to control the SL from different positions depending on the forward speed of the vehicle. For example, a lower chassis may reduce aerodynamic drag at high forward speeds, whereas an increased ground clearance at low speeds may be beneficial in order to reduce the chances of hitting high obstacles.
- That the proposed controller synthesised with the simplified hand-derived model is actually able to operate far from the default 90 deg position.
- That for high obstacles, a large exogenous command angle allows for more effective use of the range of motion of the SL.

The first three graphs in Fig. 6.24 compare the evolution of sprung mass acceleration, vertical tyre force and suspension deflection between the benchmark passive suspension and the SAVGS-

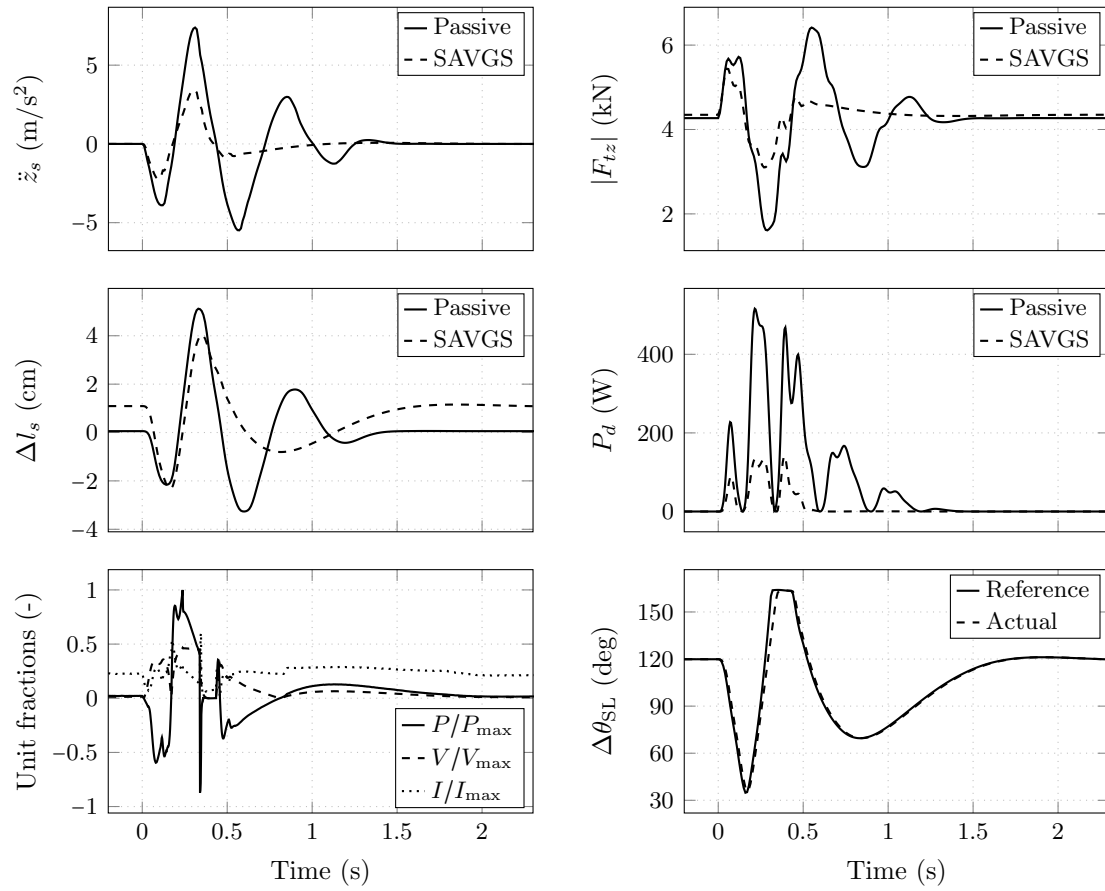


Fig. 6.24: Simulation results for the rear suspension of the GT with peak actuator limits and a maximum power consumption of 1.5 kW when driving over a 5 cm high and 2 m long speed bump at 20 km/h.

retrofitted quarter-car. Very significant improvements are achieved simultaneously in all these variables, which is remarkable considering the severity of the bump. To give some context to these results, van der Sande et al. [2013] reported results for another road bump, only 3 cm high, that led to sprung mass accelerations of the passive vehicle of approximately 2.5 m/s^2 (as opposed to the 7.4 m/s^2 reached in the case considered here). In their case, the active suspension reduced sprung mass acceleration by 53%, but suspension deflection and tyre force were damaged. In the SAVGS case, improvements in terms of RMS values are approximately 63% for sprung mass acceleration and 61% for vertical tyre force. Moreover, maximum suspension travel is reduced by 21% with respect to the passive case. A whole summary of RMS and peak ratios with respect to the passive system are provided in Table 6.2 for peak and continuous actuator limits for both axles of the GT.

The fourth plot depicts the power dissipated in the passive damper. In the passive suspension, 500 W are exceeded whereas in the SAVGS case the peak value remains below 150 W. The low levels of power dissipation indicate that there is potential for effective energy regeneration. Moreover, the reduced power dissipation (and therefore heat generation) in the passive dampers

may affect the design choices made in their development.

The fifth plot shows that power, voltage and current constraints have been respected at all times. Finally, the sixth plot shows the reference and actual SL angular positions. Starting from the constant 120 deg exogenous position command, the motion of the SL is smooth throughout. A minimum angle of approximately 30 deg is reached, close to the point where additional rotations would only have minor effects on the suspension forces. After the bump absorption, the SL angle needs to increase quickly and its allowable range of motion is actually insufficient, as it is demonstrated by the saturation of the position reference at approximately 160 deg. Finally, once the bump is over and vertical tyre forces and sprung mass acceleration are small, the SL returns to the exogenous position command of 120 deg without activating any undesirable dynamics.

Fig. 6.25 shows similar time series but for one cycle of a sinusoidal load transfer, continuous actuator limits and a maximum power consumption of 500 W. The external force peaks at 50% of the nominal vertical tyre force and the application period is 0.4 s. That is, $F_{zs} = 2132 \sin(2\pi 2.5 t)$ for $t \in [0, 0.4]$. A summary of RMS and peak ratios with respect to the passive suspension vehicle are provided for continuous and peak actuator limits in Table 6.2.

Results are as good as for the road bump regarding sprung mass acceleration, and slightly worse (but still better than the passive) in terms of vertical tyre force variations and suspension travel. The low power and torque limits imposed in this case lead to much slower SL motion and higher reliance on power dissipation in the passive dampers.

Output torque-speed operating points for the road bump and load transfer simulations are presented for completeness in Fig. 6.26.

Table 6.2: Ratio of vertical acceleration, tyre force and suspension travel obtained with the SAVGS, over those obtained with the passive suspension during the road bump and load transfer events. Ratios have been computed for root mean square (RMS) and peak values and are shown for peak and continuous (in brackets) actuator limits.

Parameter	Measure	Road bump		Load transfer	
		Front	Rear	Front	Rear
$\ddot{z}_s$	RMS	0.347 (0.472)	0.371 (0.546)	0.367 (0.387)	0.358 (0.545)
	Peak	0.373 (0.537)	0.464 (0.670)	0.369 (0.373)	0.380 (0.641)
$F_{tz} - F_{tz}^{(ne)}$	RMS	0.399 (0.576)	0.391 (0.592)	0.690 (0.695)	0.664 (0.762)
	Peak	0.526 (0.635)	0.470 (0.736)	0.790 (0.867)	0.805 (0.940)
Δl_s	Peak	0.796 (0.595)	0.789 (0.612)	0.777 (0.805)	0.742 (0.735)

Random road profiles

Three 10 km long road sections have been generated and used to test the SAVGS in typical uneven road surface conditions corresponding to a good highway (road A), an average quality

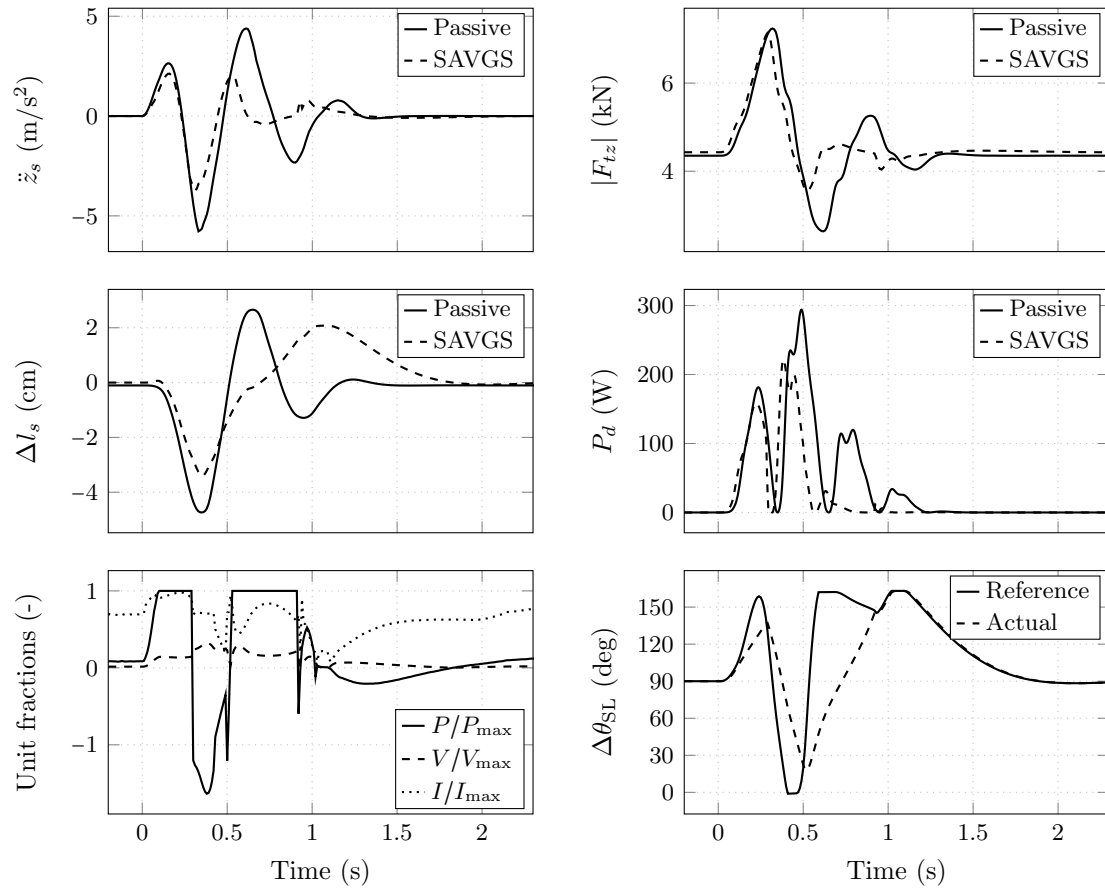


Fig. 6.25: Simulation results for the the rear suspension of the GT with continuous actuator limits and a maximum power consumption of 500 W during a load transfer event driving at 100 km/h.

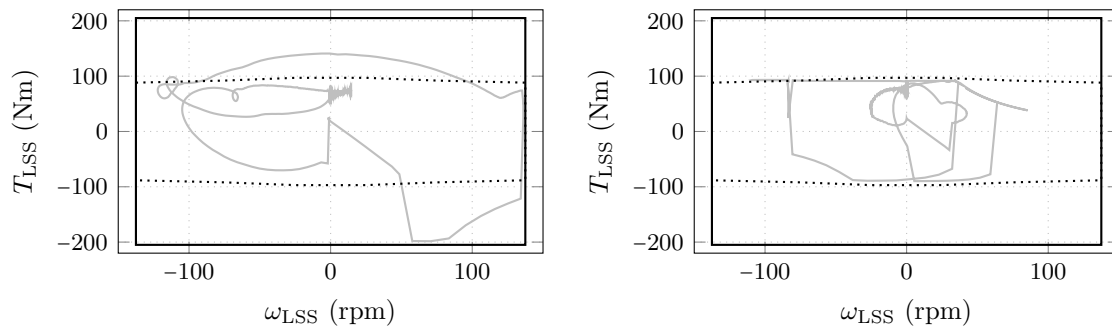


Fig. 6.26: Output torque-speed operating points for the road bump (left) and load transfer (right) events. The peak and continuous actuator limits are shown as solid and dotted black lines.

road (road B) and a poor quality road (road C). More details regarding the road characteristics and the generation of the random road profiles is provided in Appendix A. All simulations have been performed assuming a constant forward vehicle speed of 100 km/h. Continuous actuator limits and a maximum power consumption of 500 W have been considered, as it would be unrealistic to allow peak limits in such a commonplace scenario.

Fig. 6.27 and Fig. 6.28 show 5 s long samples of the simulation results for roads A and C, respectively. The PSDs for sprung mass acceleration and vertical tyre force are presented in Fig. 6.29 and Table 6.3 summarises the main results. Some key remarks are:

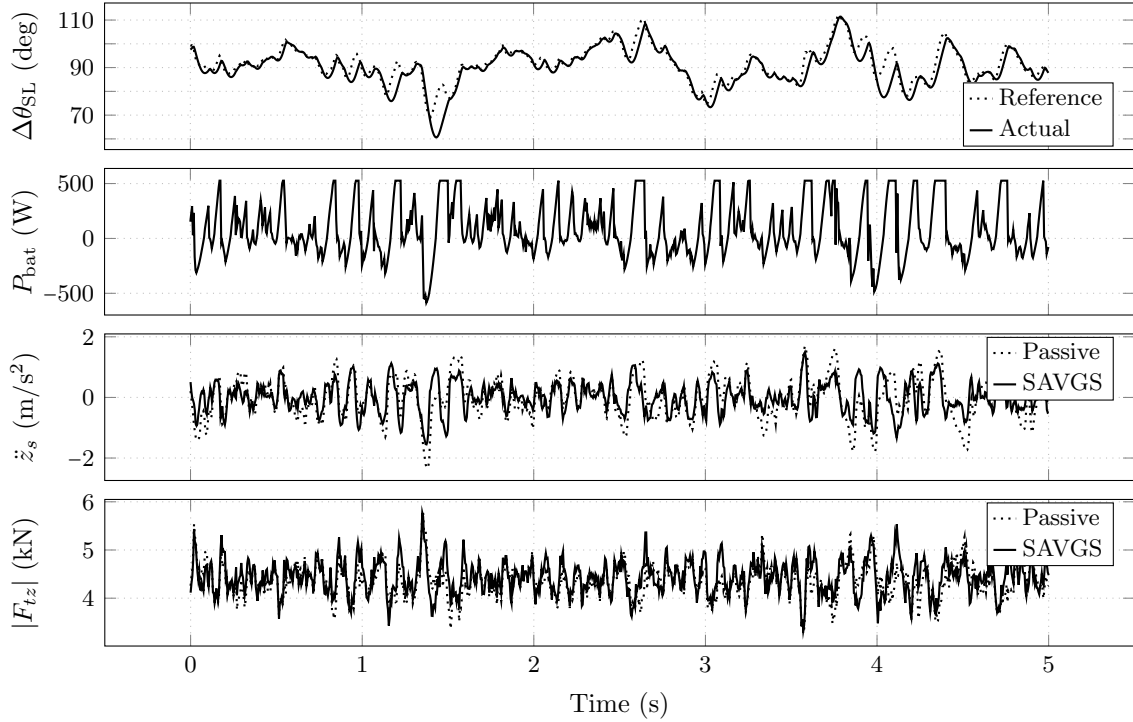


Fig. 6.27: Time domain simulation results for the rear suspension of the GT driving over road A. SL angle, power consumption of the SAVGS, sprung mass acceleration and vertical tyre force traces are shown for a 5 s segment.

- The main peak in the PSDs of sprung mass acceleration and vertical tyre force is significantly reduced for the three road classes.
- The RMS of sprung mass acceleration, which is a good metric for assessing ride comfort, is consistently improved for the three road classes by 15 to 27%.
- In good quality roads, vertical tyre forces experience a larger amplification at the wheel hop frequency than in the passive system, counterbalancing the improvements achieved at lower frequencies. However, tyre force variations are still reasonably small in these roads (peak-to-peak values of approximately 3 kN in road A, as opposed to 8 kN in road C), and therefore some degradation is unlikely to be relevant.
- In the poor quality road, the tyre frequently leaves the ground with the passive suspension.

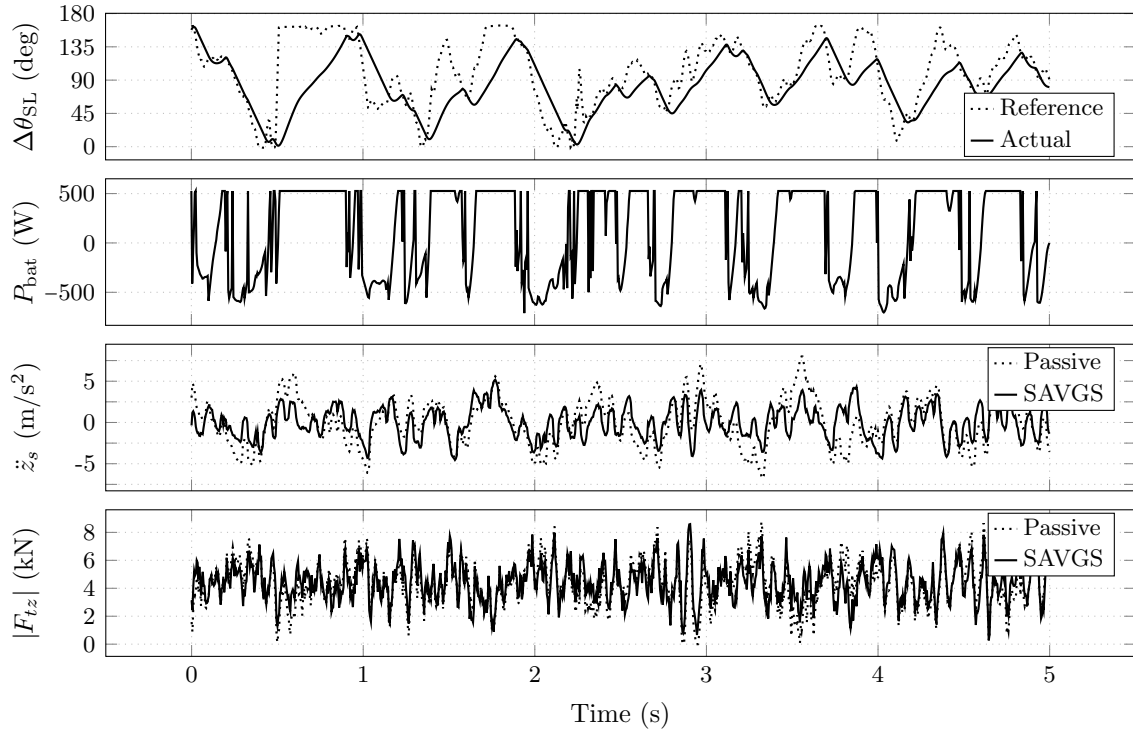


Fig. 6.28: Time domain simulation results for the rear suspension of the GT driving over road C. SL angle, power consumption of the SAVGS, sprung mass acceleration and vertical tyre force traces are shown for a 5 s segment.

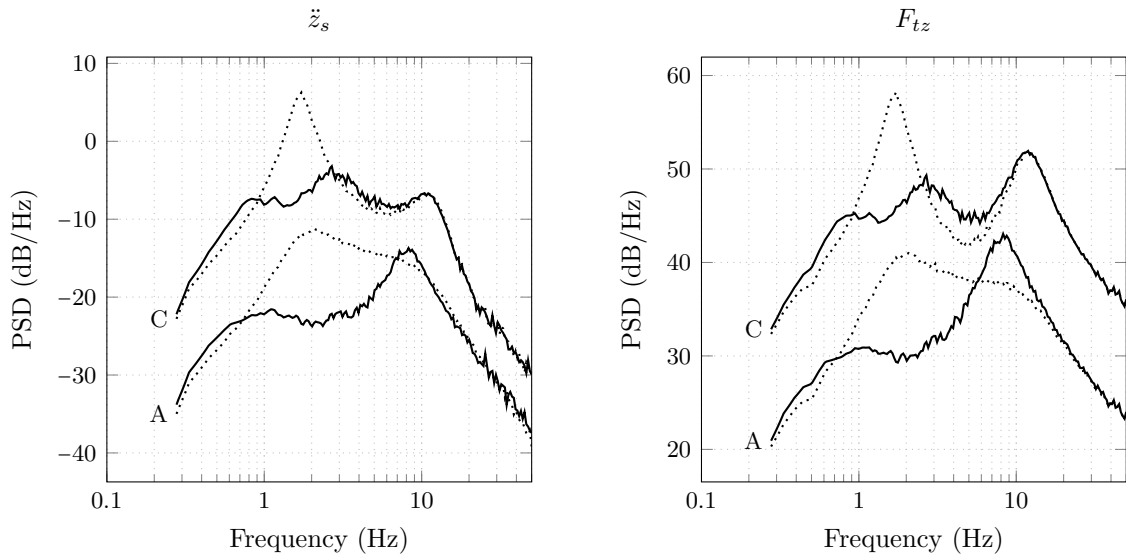


Fig. 6.29: Sprung mass acceleration and vertical tyre force PSDs for the rear suspension of the GT and road classes A and C. Results obtained for the passive (dotted) and SAVGS-retrofitted (solid) suspensions.

The SAVGS significantly reduces the likelihood of this event.

- In some instances, the SL is unable to maintain the required angle with the continuous actuator limits, and a non-negligible deviation arises. This is visible in Fig. 6.27 at approximately $t=1.3$ s, when the actuator cannot prevent the SL from being pushed an additional 10 deg towards the passive equilibrium.
- In the rough road, the SL consistently reaches the maximum and minimum allowable angles. The controller may be a bit over-aggressive for the continuous actuator limits, as SL position tracking is not as good as in previous cases.
- Average power consumption is low in all cases and increases with road roughness. A summary is provided in Table 6.4.

Table 6.3: Ratio of vertical acceleration, tyre force and suspension travel obtained with the SAVGS, over those obtained with the passive suspension for random roads A to C. Ratios have been computed for root mean square (RMS) and peak values and are shown for continuous actuator limits.

Parameter	Measure	Road A		Road B		Road C	
		Front	Rear	Front	Rear	Front	Rear
$\ddot{z}_s$	RMS	0.725	0.781	0.842	0.841	0.798	0.768
	Peak	0.764	0.838	0.802	0.854	0.801	0.787
$F_{tz} - F_{tz}^{(ne)}$	RMS	1.119	1.095	1.108	1.049	0.996	0.942
	Peak	1.129	1.102	1.101	1.106	0.834	0.784
Δl_s	Peak	1.302	1.265	0.986	1.120	1.139	0.920

Table 6.4: Average power consumption from the battery for a rear actuator of the GT and roads A to C.

Parameter	Units	Road A		Road B		Road C	
		Front	Rear	Front	Rear	Front	Rear
$\overline{P}_{bat}$	W	66	89	104	130	133	141

These results were obtained considering a constant exogenous position command of 90 deg with respect to the $\overline{EG}$ line in the nominal equilibrium. This is the position at which the SAVGS has the greatest influence on the suspension response, but also the most demanding for the actuator. As the continuous torque limits considered here are quite stringent, the capability of the actuator to track the position commands provided by the $\mathcal{H}_\infty$ controller suffers. In such conditions, it could be beneficial to operate the SL from a different, less demanding position.

Focusing on the smooth road case (road A), Fig. 6.30 depicts the influence of the constant exogenous position command on the performance and energy consumption of the SAVGS. The three metrics considered, namely sprung mass acceleration, tyre deflection and average power consumption, reach a minimum at or close to the 52.5 deg case. For that configuration, RMS values of a_z and F_{tz} are 0.698 and 1.002 times those obtained with the passive system (as opposed to 0.781 and 1.095 for the 90 deg case), and the average power consumption is 73 W (as opposed to 89 W for the 90 deg case).

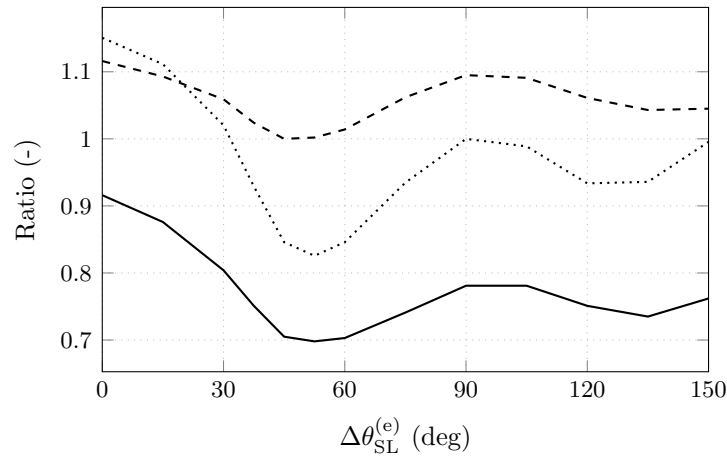


Fig. 6.30: Influence of constant exogenous position command on SAVGS performance on road A. RMS values for vertical acceleration (solid) and tyre forces (dashed) have been normalised with respect to the results obtained with the passive suspension, whereas average power consumption (dotted) has been normalised with respect to the nominal 90 deg case.

The evolution of these metrics with the exogenous position command depends on the level of road disturbances as well as on the capabilities of the actuator. A detailed analysis of this aspect of the SAVGS control is not presented here, but the previous discussion suggests that a higher level control could improve the system performance and reduce its power consumption by adjusting the exogenous position command based on the level of road irregularities, forward vehicle speed, actuator constraints etc.

6.3.2 Full-car simulation results

Previous results have shown that the SAVGS has the potential to significantly improve comfort and road holding, and that a single $\mathcal{H}_\infty$ controller can effectively tackle both objectives subject to road bumps, load transfers and irregular roads, irrespective of whether continuous or peak actuator limits are used.

In this section, the same controllers, which were synthesised for the quarter-car hand-derived model, are tested in the full-car nonlinear model. Of course, this is only a starting point in the process of designing suitable controllers for the full-car. Results presented are nevertheless of value as they can be understood as lower bounds for SAVGS performance.

Fig. 6.31 presents relevant results for the vehicle driving at 20 km/h over the smoothed bump road already described in the previous section. Here it is assumed that the road profile under the left and right wheels is the same, and that the rear wheels drive over the same points as the front ones but with some delay determined by the wheelbase (i.e. distance between the front and rear wheels, which for the GT is 2.6 m). Results are very similar to what was achieved with the quarter-car models, thus highlighting that negative front-to-rear interactions do not occur

in this particular case. SL motions are smooth and improvements are very significant:

- Comfort: vertical acceleration of the centre of mass of the vehicle (-62.4% in RMS) as well as pitch acceleration (-58.5% in RMS) are greatly reduced.
- Road holding: RMS dynamic vertical tyre forces are reduced by 60.7% at the front and by 54.8% at the rear.
- Pitch angle and chassis height: improved ground clearance as the CM reaches a minimum height of 433 mm as opposed to 421 mm in the passive case, and maximum pitch angle deviation is -1.28 deg as opposed to +1.54 deg in the passive case.
- Peak power consumption from the battery remains below 4 kW although each of the four actuators is allowed to consume 1.5 kW from the high-voltage DC bus.

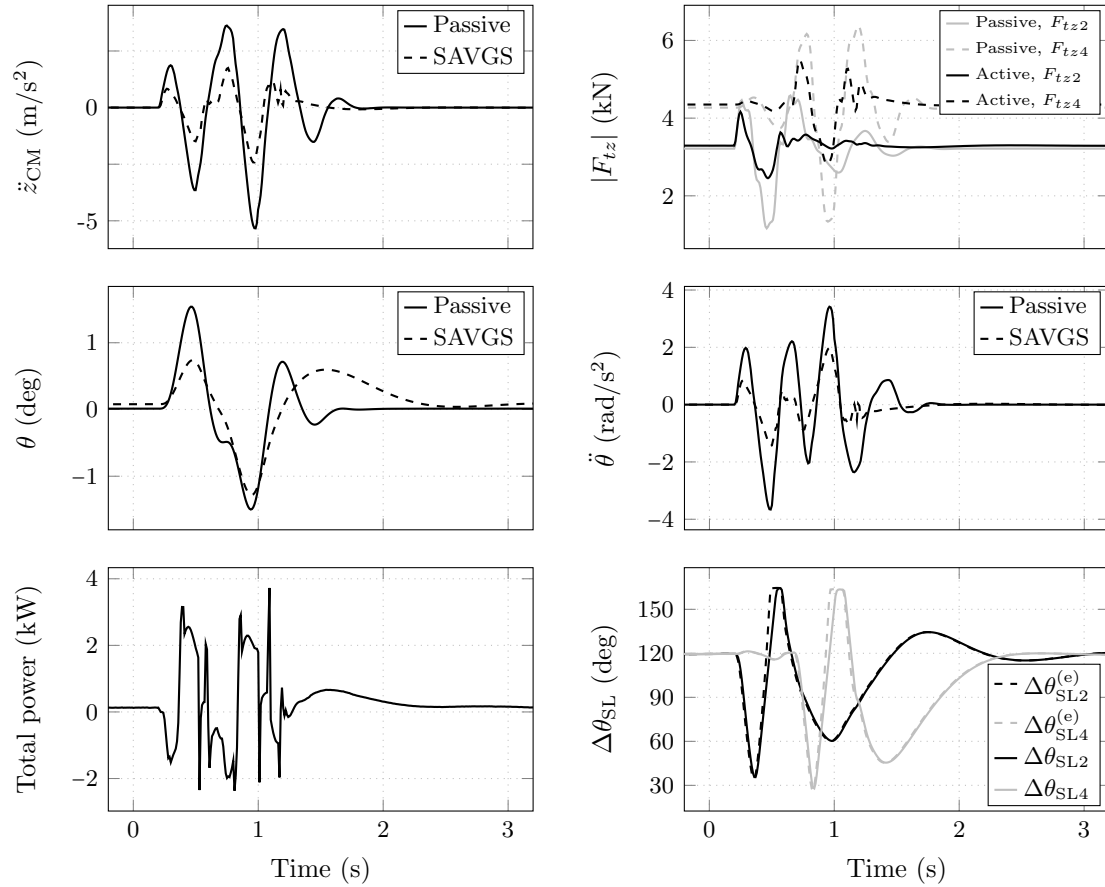


Fig. 6.31: Full-car GT simulation results for the road bump and peak actuator limits.

The full-car was also tested in the 10 km long roads used in the previous section. A summary of the main results is provided in Table 6.5. The main conclusions from these are:

- Reduction in vertical acceleration of the centre of mass of the car is similar to the reduction in sprung mass acceleration in the quarter-car studies.
- Pitch acceleration is reduced in all cases, particularly in the good quality road (-52%).

- Variations in vertical tyre forces are improved in the rear axle at the expense of those at the front axle.

Table 6.5: Results for the full-car GT model and random roads A to C. Ratio of vertical acceleration of the centre of mass, pitch acceleration, and front and rear tyre force variations obtained with the SAVGS, over those obtained with the passive suspension. Ratios have been computed for root mean square (RMS) and peak values and correspond to the continuous actuator limits.

Parameter	Measure	Road A	Road B	Road C
$\ddot{z}_s$	RMS	0.792	0.865	0.786
	Peak	0.916	0.870	0.835
$\ddot{\theta}$	RMS	0.480	0.689	0.921
	Peak	0.698	0.986	0.979
$F_{tz2} - F_{tz2}^{(ne)}$	RMS	1.169	1.250	1.063
	Peak	1.216	1.133	0.929
$F_{tz4} - F_{tz4}^{(ne)}$	RMS	0.897	0.991	0.957
	Peak	0.919	1.127	0.827

Finally, the power and energy imbalances computed through (3.75) for the nonlinear full-car simulation of the GT cruising on road A are shown in Fig. 6.32. Results demonstrate that the numerical accuracy of the simulations is satisfactory, as power and energy imbalances are 3 to 6 orders of magnitude smaller than power and energy flows in the model.

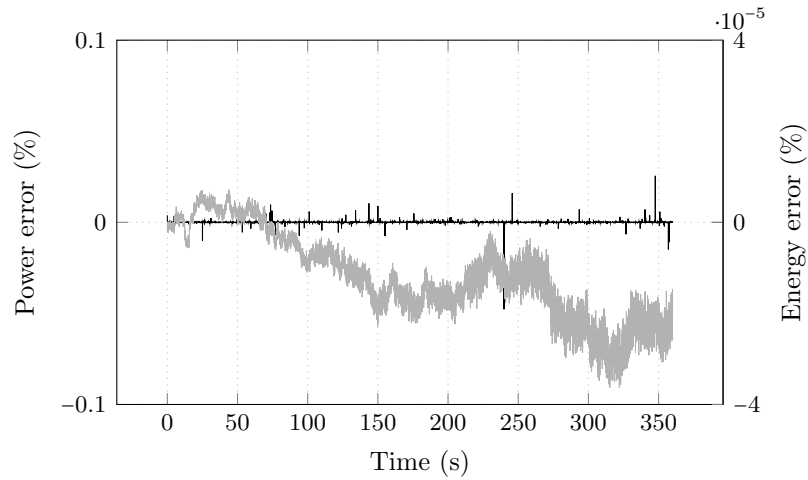


Fig. 6.32: Power (black) and energy (grey) errors for the nonlinear full-car simulation. Errors peak at approximately 10 W and 1 J and have been normalised by the maximum input power and the total input energy in the manoeuvre, respectively.

6.4 Conclusions

This chapter has explored the potential of the SAVGS for comfort and road holding enhancement, with particular focus on the development of a suitable controller in charge of generating SL

position commands for the position control loop in one corner of the car. Aspects of quarter-car modelling and linearisation were discussed and a simplified hand-derived linear model was derived to remove the strong dependency that exists on SL angle, thus enabling the use of a single controller for the whole range of SL motions. An $\mathcal{H}_\infty$ controller was synthesised, and the weight selection and tuning processes were covered in detail. In particular, magnitude Bode plots together with singular values and input-output singular vectors were used to inform the tuning process. Finally, extensive simulation results obtained with high fidelity quarter-car and full-car nonlinear models were presented and discussed.

The main conclusions that can be drawn from this work, as well as some suggestions for future work, are:

- Lateral tyre forces can be safely neglected, at least for mid to high speeds and suspension arrangements with low to moderately high roll centres.
- Nonlinear dampers have a large impact on the system dynamics and display very different responses depending on the level of external disturbances. This complicates the use of a single linear vehicle model for a wide range of operating conditions.
- A simplified linear quarter-car model that includes the SAVGS action through an equivalent linear displacement actuator was derived. This model has several advantages over linearised versions of multi-body models: 1) it significantly extends the range of SL angles for which the model is appropriate, 2) it requires less states, and 3) it can be easily expanded to the full-car case.
- The $\mathcal{H}_\infty$ framework is appropriate for the synthesis of controllers that tackle comfort and road holding and the proposed controllers are sufficiently robust against measurement noise despite these disturbances being omitted during the synthesis phase.
- The proposed controllers for the GT lead to simultaneous and significant improvements in comfort and road holding related metrics when subject to road bumps, load transfers and general road unevenness of various severities. The robustness of the proposed control is highlighted by the fact that these improvements are achieved with both continuous and peak actuator limits and for different constant exogenous position commands.
- The controllers obtained for the quarter-car models have been tested in the full-car nonlinear model and represent a good reference point for developing full-car specific controllers.
- The ability of the proposed control to follow different exogenous position commands, especially at low frequencies, may be utilised to blend this high frequency control with the body attitude control proposed in Chapter 5.
- For the SUV in fully laden configuration driving over roads A to C, actuators limited by the continuous torque and current constraints are not capable of tracking SL angles. This highlights the limitations of the technology for some vehicle classes.
- Further improvements in performance may be possible through more extensive frequency weight tuning. In particular, it should be studied whether including tyre forces instead

of tyre deflections as controlled variables could have a beneficial effect (the effect of tyre damping is neglected when working with tyre deflections).

- Parametric uncertainty in damping coefficient as well as multiplicative output uncertainty for the actuator and inner-loop dynamics block were considered and MATLAB's `dksyn` function for μ -synthesis was used. However, results obtained after spending a similar tuning time as with the $\mathcal{H}_\infty$ method discussed here were not better than the ones presented in this chapter. Additional information about these attempts is provided in Appendix C.

Chapter 7

Experimental rig

Experimental testing is key to validate the SAVGS concept and to identify potential problems. Hence, the development of a test rig was one of the objectives for this PhD.

Although presented at the end of this thesis, the task had to be undertaken in the early stages of the project and within the framework of collaboration with Imperial Innovations. Building such a rig involved aspects of mechanical and electrical system design, component selection, time and budget management, risk assessment, machining, assembling etc.

This chapter describes the final rig design and highlight some aspects of its development.

7.1 Starting point

This section aims to put the test rig design and its construction in context. First, the original objectives set for the rig development in the first months of the PhD are summarised in Section 7.1.1. Section 7.1.2 then lists all components initially available. This is relevant because it determined many subsequent design decisions. Finally, the functional requirements for the rig, together with all relevant design constraints are outlined in Section 7.1.3.

7.1.1 Objectives

The initial objectives for the development of the experimental test rig were summarised as:

- To have a platform to show and explain the AVGS concept to potential customers
- To demonstrate that the motion of the system can be effectively controlled with a reasonably sized actuator
- To identify modelling inaccuracies, assembling problems and other issues
- To compare how actuators of different capabilities perform

- To pave the way for a possible future implementation on a real vehicle (i.e. to learn how to implement the control algorithms in a real system etc.)

7.1.2 Available components

At the beginning of the PhD, some components were already available for the construction of the test rig. These were:

- 3 $\times$ Kollmorgen PMSMs of different sizes (AKM23F, AKM24F, AKM33H), 3 $\times$ Danaher SERVOSTAR S30661-NA servo drives and corresponding power and feedback cables.
- 2 $\times$ Öhlins spring-damper units: 1 $\times$ TTX40 NH04 and 1 $\times$ TTX36IL.
- National Instruments CompactRIO system comprising: 1 $\times$ CompactRIO reconfigurable chassis (cRIO-9114), 1 $\times$ real time controller (cRIO-9022), 1 $\times$ power supply for CompactRIO (PS-15), 1 $\times$ NI 9219 (universal analogue input, 24-bit, 4-channels), 1 $\times$ NI 9234 (acquisition module for high-accuracy audio frequency measurements), 1 $\times$ NI 9237 (4-channel simultaneous bridge module), 1 $\times$ NI 9402 (bidirectional digital I/O module).

7.1.3 Requirements and constraints

Apart from the functional requirements that arise from the objectives previously outlined, the design, manufacture and assembly of the rig was subject to the following constraints:

- Time: the proof of concept rig had to be built at the beginning of the PhD so that it could be used to show the SAVGS concept to potential customers.
- Volume: the rig and all accessory components had to fit in a 1.5 m by 1.5 m area. No connection to the ground or walls was possible.
- Budget: there was a limited budget that had to be shared between purchases (i.e. material and components) and externalised work (i.e. machining). Suppliers already approved by the college had to be used unless strictly necessary.
- Location: the rig would need to be moved back and forth between the Tribology and Dynamics labs due to building refurbishment works.
- Assembly: it had to be possible to assemble the whole rig, safely, by a single person.

7.2 Test rig design

Due to time constraints, the experimental rig had to be designed whilst basic analysis and simulations studies were still ongoing and many parameters were unknown. First, a broad mechanical design was proposed based on the functional requirements and constraints for the rig, as well as on ideas gathered from published work. Then, the specific mechanical components

were designed and/or selected, machined or purchased, and assembled. Finally, the electrical layout was detailed, components purchased, and the whole system installed and tested.

7.2.1 Quarter-car experimental rigs in the literature

Before proceeding with the actual rig design, it was important to review previous work on similar test beds. A graphical summary is outlined in Table 7.1.

Table 7.1: Selection of publications dealing with quarter-car test rigs

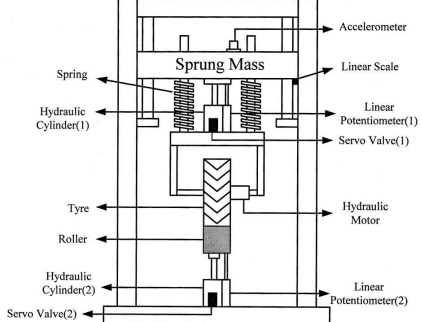
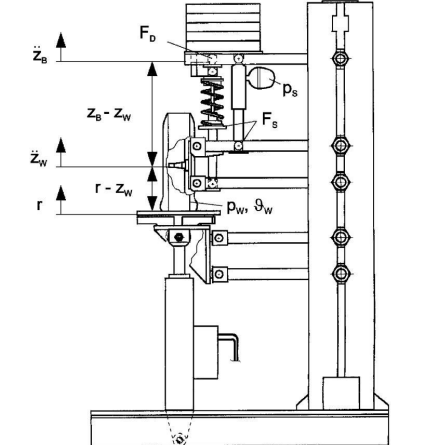
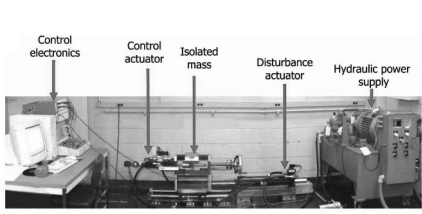
Description	Image
Image from [Huang and Lin, 2003]. The test rig comprises two hydraulic actuators: one for the active suspension and one to emulate the road excitation. Motion is purely vertical and enforced through two parallel guides.	
Image from [Fischer and Isermann, 2004]. The test rig comprises a variable suspension damper and a hydraulic actuator to emulate the road excitation. As in the previous case, motion is purely vertical and aspects such as changes in the installation ratio cannot be investigated.	
Image from [Zhang and Alleyne, 2005]. A horizontal 1 DOF system is used to test different active suspension control strategies. Two hydraulic actuators are used: one to disturb the system and one to improve the mass isolation.	

Image from [Kou and Fang, 2007].

As in previous cases, the active suspension comprises a hydraulic actuator and vertical motion is enforced through two parallel guides. Road excitation, on the other hand, is provided by a permanent magnet electric motor coupled to an excentric circular cam.



Image from [Kruczek et al., 2009].

Road excitation is provided through a hydraulic actuator. The active suspension comprises a linear electric motor. Although the suspension geometry is respected in this case and motion is not purely vertical, the location of the electric motor outside the wheel is clearly unrealistic.



Image from [Priyandoko et al., 2009].

Pneumatic actuators are employed to generate the road disturbance and suspension action. Details are given regarding the road excitations generated: two smooth but far from harmonic signals of 1.5 Hz with 3 cm amplitude, and 0.8 Hz with 1.2 cm amplitude.

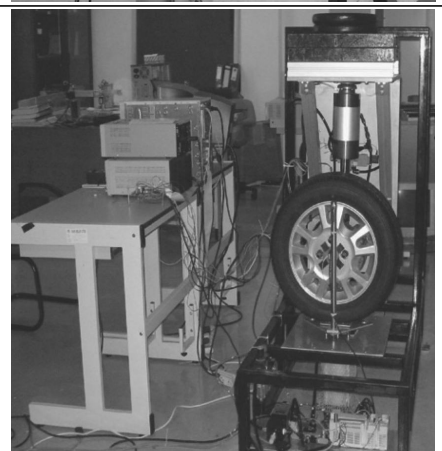


Image from [Lee and Kim, 2010].

Road disturbance (sinusoid of 3.5 Hz and 3 cm of amplitude) is emulated through an eccentric cam system. The active suspension actuator is a linear electric motor capable of generating forces below 30 N, and the whole system is a scaled-down version of a realistic quarter-car. Unfortunately, parameter values were not selected based on any appropriate similitude concepts (e.g. sprung and unsprung masses are almost equal and the wheel stiffness is ten times lower than that of the suspension spring).

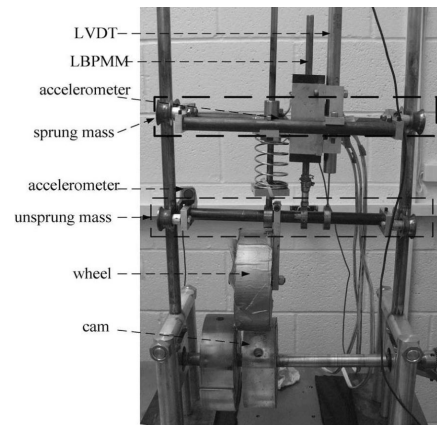


Image from [Maher and Young, 2011].

Suspension kinematics are included in this rig designed to test MacPherson struts. A passive strut was tested and road excitation was provided through a cam system driven by electric motors. Information regarding the amplitude and frequencies of excitations was not provided. Parameter values, as in the previous case, are scaled down version of a real quarter car. Although they seem more realistic, no details of the similitude approach followed are given, and ratios of sprung mass over unsprung mass ~ 3 , and wheel stiffness over spring stiffness >15 , are outside the typical ranges.

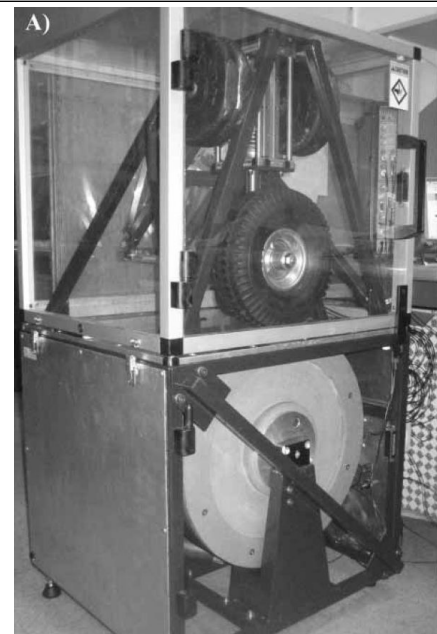
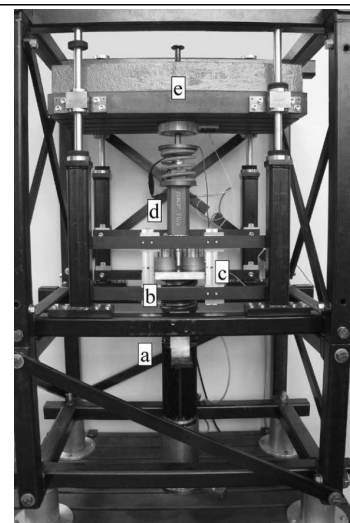


Image from [Gysen et al., 2011].

In this test bed, both sprung and unsprung masses are constrained to vertical motions by linear bearings. The tyre is replaced by a coil spring and a brushless permanent-magnet actuator is used to provide active suspension action. A hydraulic actuator is used to introduce road disturbances representative of smooth and rough random road profiles as described in [ISO 8608:1995], and a reasonably good match between simulation and actual disturbances is achieved for frequencies up to 30 Hz.



7.2.2 Test rig layout

The initial step in the rig design was to brainstorm potential configurations and to shortlist the best two or three of these. Making use of preliminary simulation results obtained with the full-car multi-body model, loading conditions were roughly estimated and employed to detail the shortlisted designs up to a point at which it was possible to select/dimension the key components, estimate costs, identify the main advantages and drawbacks of each alternative and finally select the best design.

It was decided early on that the external excitation system could be left for future refinements of the rig if budget/time were too tight to include it in the first design. Welding and other permanent unions should be avoided as much as possible in order to keep the weight of individual components below 20 kg (this was thought to be the value that would allow one person to move, machine and assemble all components safely). Also, the rig would not only need to respect the 1.5 m by 1.5 m footprint limit, but its height would need to remain below 2 m to ensure that it could be moved in and out of the labs as requested — a plausible way of moving the rig around the building would need to be envisaged.

The two shortlisted designs were 1) a conventional quarter-car arrangement with suspension kinematics and the potential to add road excitation at a later stage, and 2) a novel arrangement in which the sprung mass would be emulated by an inerter and other two-port mechanical components.

Conventional quarter-car arrangement

The rig would comprise 1) a frame or supporting structure, 2) a vertical guiding system on which the sprung mass would move vertically, 3) a sprung mass weighting between 300 kg and 400 kg, 4) SAVGS actuator and SL attached to the sprung mass, 5) suspension system representative of the sports car category, 6) wheel assembly and tyre resting on the frame, 7) possibility to install an excitation system that would emulate road disturbances.

Novel quarter-car with emulated sprung mass

The difference with respect to the conventional quarter-car arrangement is that the sprung mass would be much lighter and would incorporate two-port mechanical elements that would emulate the dynamic response of a heavy mass.

The mass emulation could be achieved by using a properly sized inerter [Smith, 2002], probably of the rack-and-pinion type described in [Papageorgiou et al., 2009] and shown in Fig. 7.1. However, although an inerter would be able to reproduce the inertial forces arising from the vertical accelerations of the sprung mass, it would fail to introduce the effect of the weight of

the sprung mass. For that, a component that produces a constant force between the sprung mass and the frame (i.e. independent of their relative displacement) would be needed. Three options were considered for this:

- One or more constant-force mechanisms such as the one described in [Boyle et al., 2003] and shown in Fig. 7.1. Potentially economical and effective, but very time consuming (design, construction and validation).
- Constant-force springs like the ones shown in Fig. 7.1. Inexpensive and light but relatively difficult to package and unreliable for the range of frequencies of interest.
- One of the small PMSMs (AKM23F or AKM24F) available at the beginning of the project, combined with a rack-and-pinion mechanism. The PMSM would be able to provide a constant force and to emulate load transfers like the ones occurring when braking/cornering.

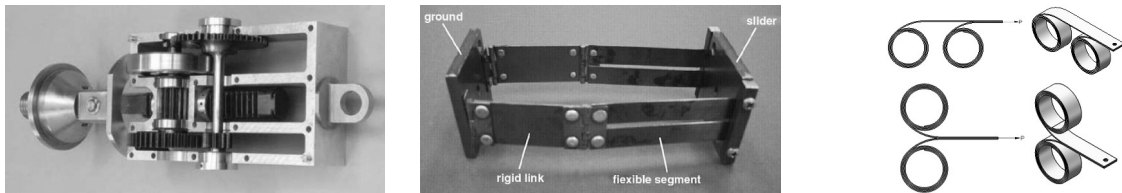


Fig. 7.1: Rack-and-pinion inerter (left) from [Papageorgiou et al., 2009]. Constant-force mechanism (centre) from [Boyle et al., 2003]. Tandem and back-to-back arrangements of constant-force springs (right) from <http://www.vulcanspring.com/conforce-constant-force-springs>.

Comparison

The design with the emulated sprung mass would be more compact and much lighter than the conventional quarter-car arrangement, making it easier to handle and transport. Moreover, it would be a novel approach to active suspension testing, not yet reported in the literature and therefore more challenging and stimulating. In the case of using the small PMSM, it would even solve the problem of introducing disturbances in the rig.

Unfortunately, time, reliability and cost are important disadvantages associated with each of the three variants proposed to materialise the concept. Moreover, the use of sprung mass emulation would hinder the appeal of the rig as a proof of concept: testing would need to be much more extensive in order to convince potential customers of the plausibility of the results.

For these reasons, the conventional quarter-car arrangement was finally chosen. Specific aspects of mechanical and electrical design are briefly discussed in Section 7.2.3 and Section 7.2.4, respectively.

7.2.3 Mechanical systems

In this section, the mechanical structure of the test rig is described and the function of individual components is briefly explained. Hand calculations and conservative safety factors were used throughout the design process, leaving detailed finite element analyses as future work. Loading conditions were estimated based on initial results obtained for emergency braking manoeuvres simulated with the parameter set of the GT.

The proposed design is flexible enough to accommodate changes, reinforcements etc. and is shown in Fig. 7.2 and Fig. 7.3 for illustrative purposes.

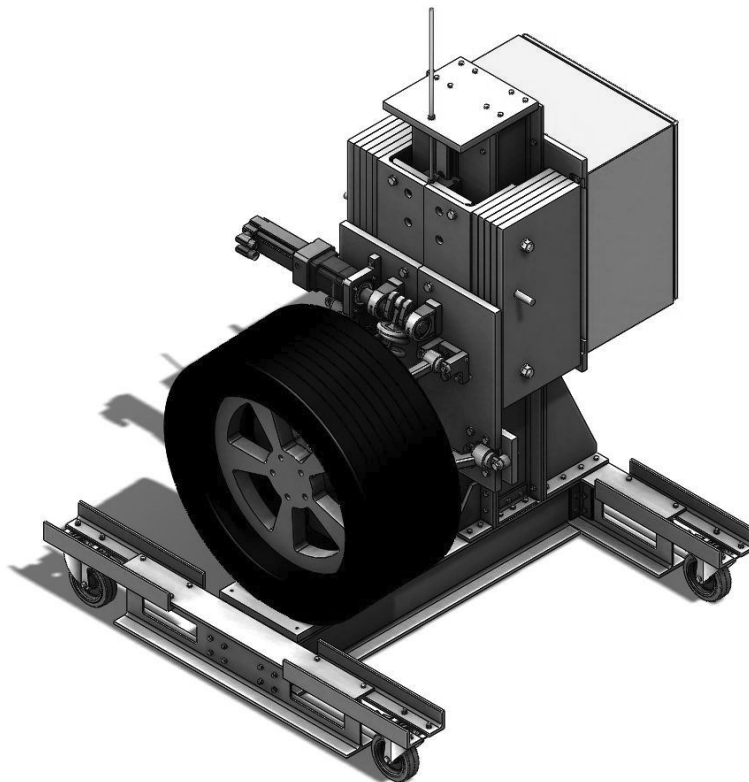


Fig. 7.2: Isometric view of the final test rig design.

Frame

An isometric view of the test rig frame is shown in Fig. 7.4. Longitudinal, lateral and vertical universal I beams made of structural steel S275 form the main body of the frame and are connected to each other through short L beams and M6 bolts. Two triangular substructures are employed to reinforce and stiffen the connection between the longitudinal and vertical beams and a 20 mm thick steel plate connects the two longitudinal beams and serves as support for the tyre. The lateral beams are ready to accommodate the forks of standard forklifts in case the rig

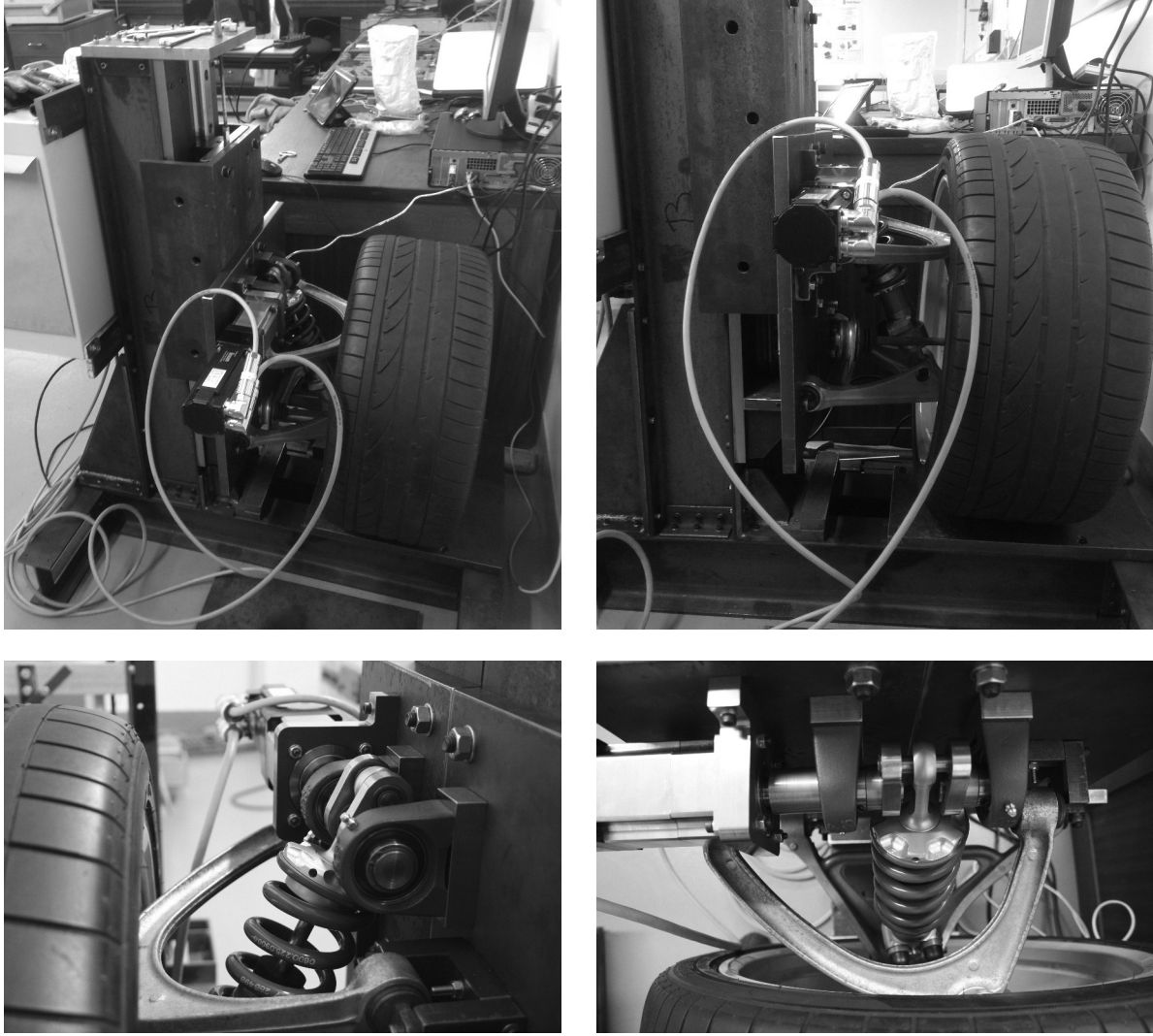


Fig. 7.3: General (top left) and lateral (top right) views of the test rig. Lateral (bottom left) and top (bottom right) views of the assembled SAVGS.

needs to be lifted. Total length, width and height are 1093 mm, 750 mm, 1252 mm.

All material was sourced from Ravenace Metals Ltd and machined to the required accuracy in a milling machine in the research workshop in the Mechanical Engineering Department.

Guiding system for the sprung mass

The vertical guiding system (see Fig. 7.5) has to withstand all loads acting on the sprung mass that are not colinear with the guides. Forces and moments to be considered are those transferred by the wishbones and spring-damper and the inertial forces acting on the centre of mass of the sprung mass. To minimise forces along the x' axis and moments about the z' and y' axes, the SD and the centre of mass of the sprung mass are designed to lie in the $y'z'$ plane.

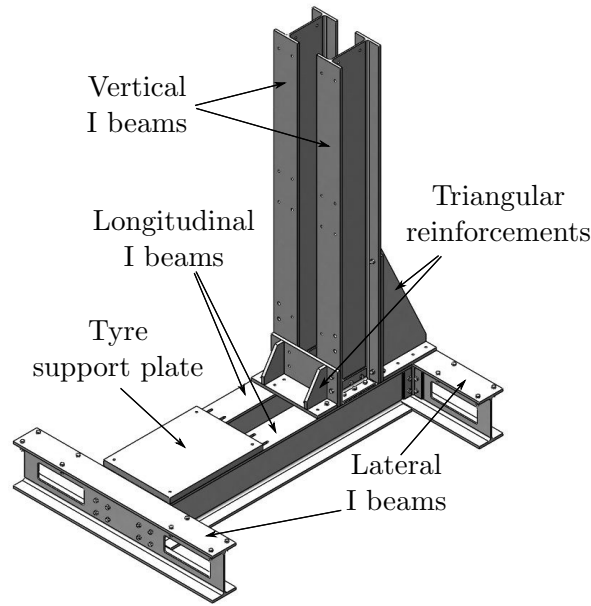


Fig. 7.4: Isometric view of the test rig frame. The local reference frame for the quarter car has the z' axis pointing downward and the y' axis aligned with the longitudinal beams, pointing from the vertical beams towards the tyre support plate.

The guiding system selected comprises a single linear profile rail guide in order to avoid infeasibly small requirements in terms of assembling tolerances. The rail is connected to the vertical beam profiles of the frame through two 20 mm thick steel plates and M12 boltsⁱ. Two colinear carriages, rigidly connected by two support plates of 20 mm and 5 mm thickness that will support the sprung mass, provide sufficient strength and stiffness, particularly to forces along the y' direction and moments about the x' axis (i.e. the main loads).

SKF rail and carriages were supplied by Orbic Bearings Ltd. Steel plates are made of structural steel S275 and were supplied by Ravenace Metals Ltd. Machining was performed in a milling machine in the research workshop in the ME Department.

Sprung mass

Two L150x150x15 profiles are fixed to the supporting plates attached to the carriages and are used to hold lateral weights in the form of rectangular 500x250x20 mm steel plates (see Fig. 7.6). Weight is added symmetrically with respect to the $y'z'$ plane, and a rigid connection between the L profiles and the lateral weights is achieved through the use of M16 grade 12.9 bolts preloaded to 80-90% of their yield limit. Two additional 500x250x20 mm plates, to which the SAVGS and wishbones will be connected, are attached to the front of the L profiles.

ⁱIf a single but longer plate had been used instead, the machining and assembling process would have been much simpler. Unfortunately, the 20 kg/component weight restriction did not allow it and although some shims were used, the process was not straightforward.

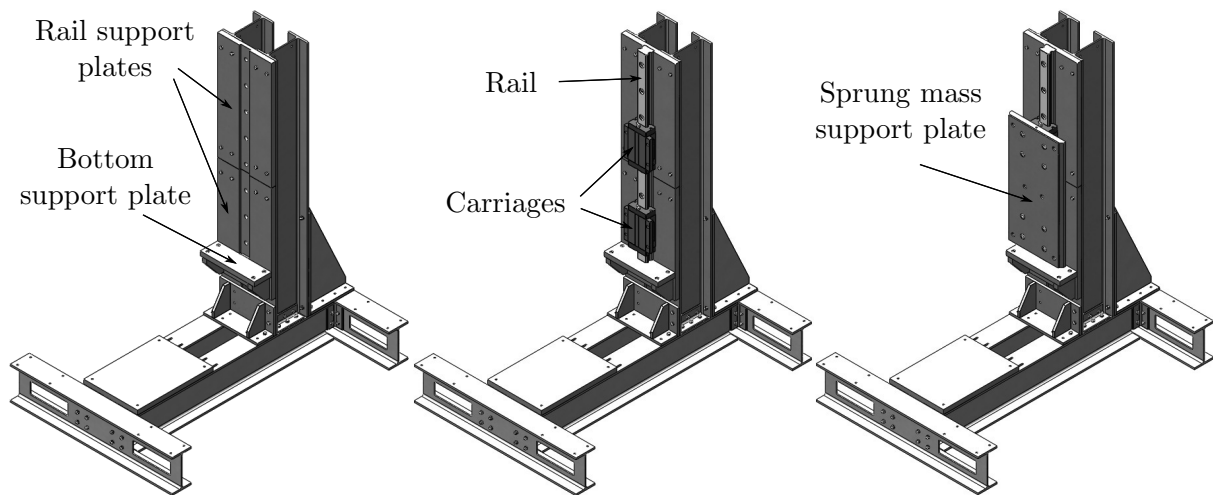


Fig. 7.5: Isometric view of the test rig frame and guiding system.

All components are made of structural steel S275 and supplied by Ravenace Metals Ltd. Machining was performed in a milling machine in the research workshop in the ME Department.

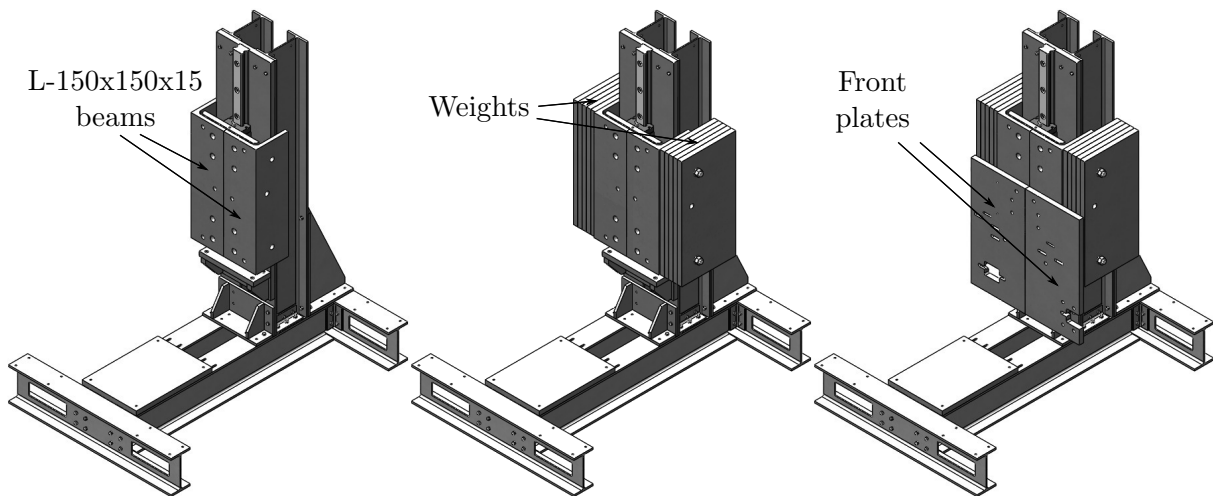


Fig. 7.6: Isometric view of the test rig frame, guiding system and sprung mass.

SAVGS assembly

The largest of the three PMSMs available is the most appropriate for the experimental rig and a suitable gearbox that maximises the output torque-speed characteristics of the actuator was selected (as described in Section 3.5.2) and purchased together with an adaptor to connect it to the PMSM. The gearbox is fixed to the front plates of the sprung mass through a flanged attachment plate (see Fig. 7.7). The design also comprises an extension of the gearbox' output

shaft, two symmetrically positioned bearings, two arms and a cylindrical mechanical link to which the spring-damper is connected.

The UltraTRUE Danaher gearbox was supplied by Heason Technology and the SKF bearings by Orbic Bearings Ltd. Other elements were machined (milling machine, lathe and CNC) in the research workshop in the ME Department from S355 steel supplied by Ravenace Metals Ltd.

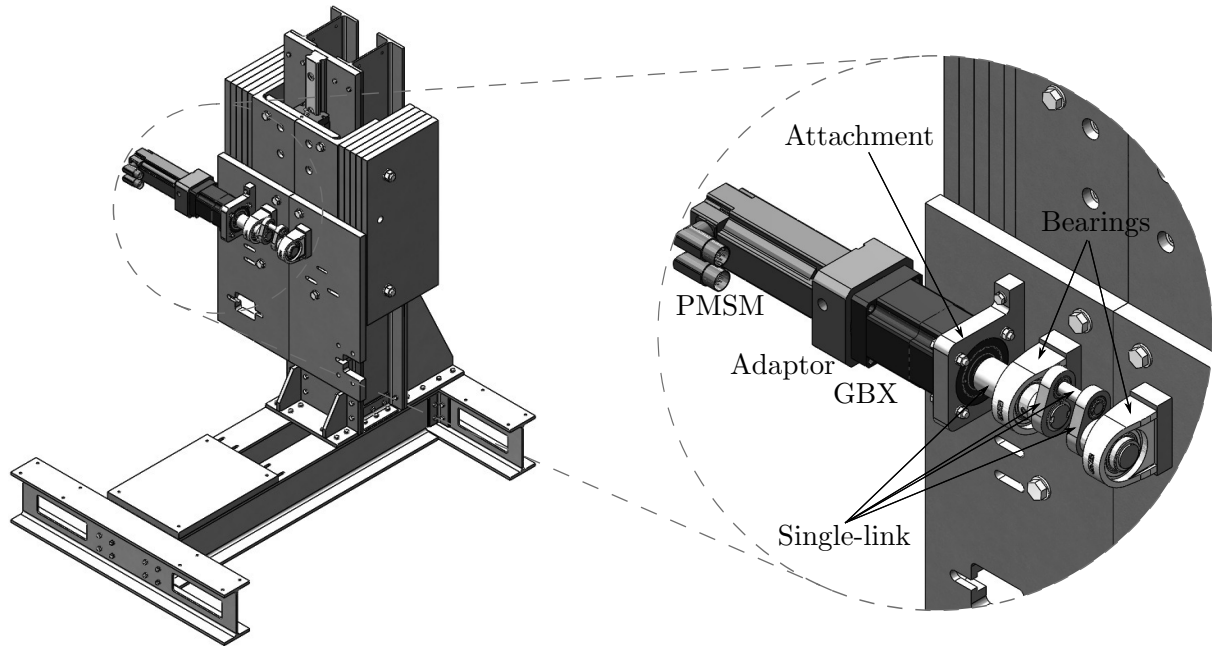


Fig. 7.7: Isometric view of the test rig frame, guiding system, sprung mass and SAVGS.

Suspension and wheel assemblies

The SAVGS alters the angle of the SD when operated and therefore a suspension assembly with its full kinematics is needed in order to assess this aspect of its behaviour. Ideally, a suspension system representative of the high performance sports car class should be used. The problem, however, was that such components were difficult to find and extremely expensive (i.e. most of the budget would have been needed just to purchase a suspension and wheel assembly from an official supplier). A scrapyard was visited looking for appropriate components, and the possibility of designing and manufacturing the suspension was considered. Finally, a second hand rear suspension from a Ferrari F430 challenge car was found on eBay in very good condition and at an acceptable price. The system, shown in Fig. 7.8, included top and lower wishbones with bushings, wheel bearing, hub carrier, tie rod, road wheel and 5 wheel bolts. It is exactly what was needed and it perfectly complements the Öhlins SD units and SAVGS actuator.

The suspension was purchased from A Stone Vehicle Management Ltd and long end eyes for the

SD units were purchased from Aurok Ltd in order to accommodate the 30 mm long SL.



Fig. 7.8: Top and lower wishbones, bushings, wheel bearing, hub carrier and wheel bolts (left), rim and tyre (centre) and tie rod (right).

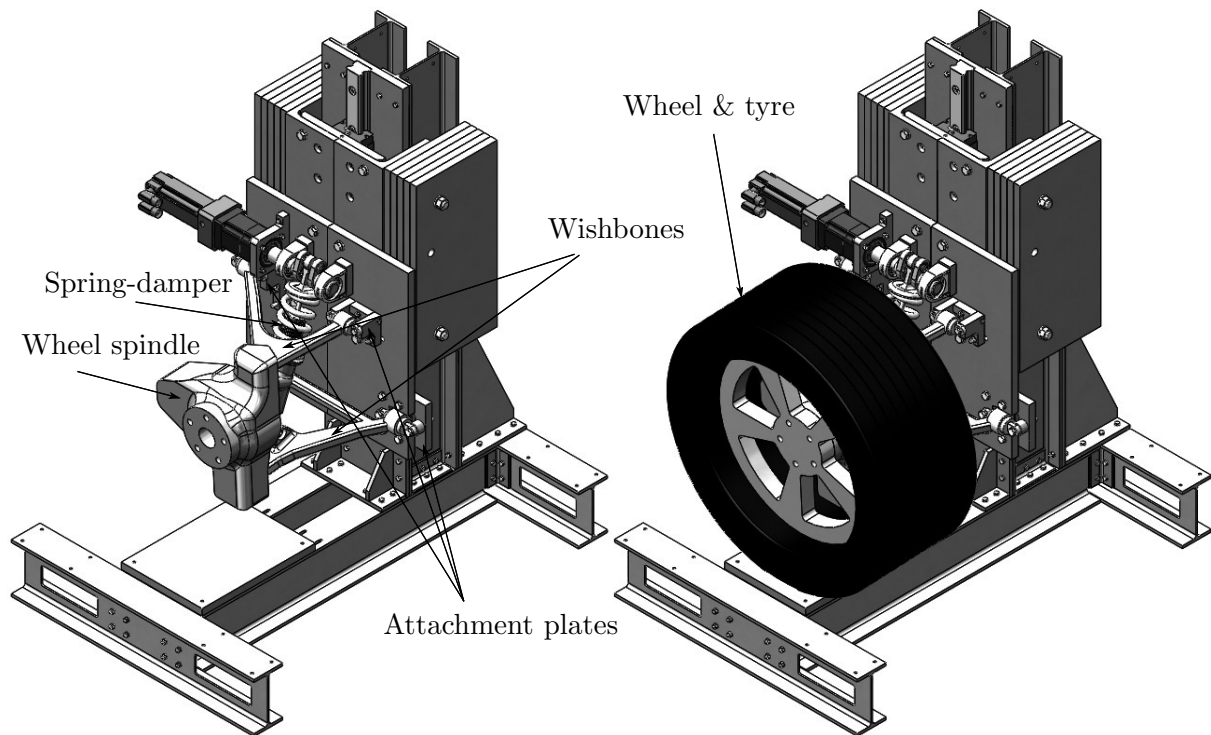


Fig. 7.9: Isometric view of the test rig frame, guiding system, sprung mass, SAVGS, suspension and wheel assemblies.

Auxiliary components

Some additional components, shown in Fig. 7.10, were added to help in the assembling process and to facilitate moving the rig. First of all, a 20 mm thick top plate and an M8 threaded stud were installed to enable easy assembly and disassembly of the sprung mass: once the stud is fixed to the front steel plate attached to the carriages the sprung mass can be easily lifted or lowered by rotating a long and lubricated nut against the top plate. The sprung mass can thus

be positioned at the right height for the connection of the wishbones. Also, a cylindrical rod with a diameter of 16 mm is used to load the lateral weights onto the sprung mass and to hold them in place until the M16 bolts are tightened. A teflon sheet was laid between the tyre and the tyre support plate to reduce friction. Finally, the lateral beams were extended to accommodate castor wheels with which to move the rig by hand.

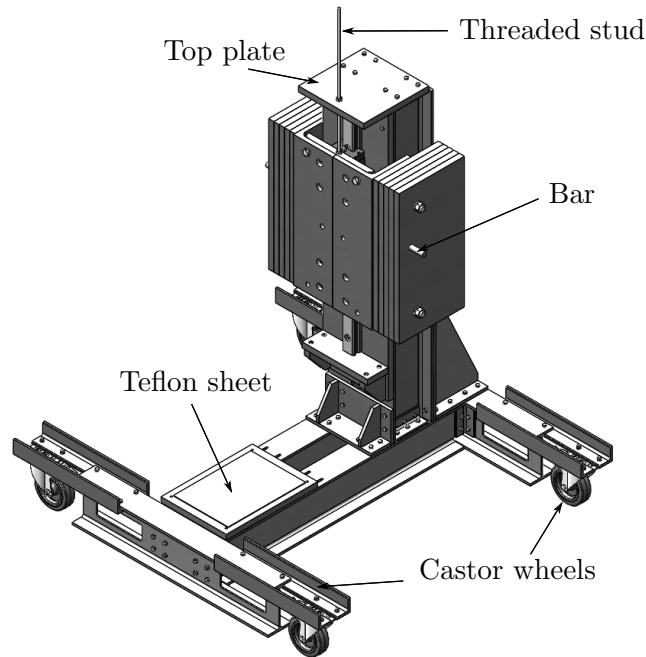


Fig. 7.10: Isometric view of the test rig frame, guiding system, sprung mass and auxiliary mechanical components.

7.2.4 Electrical systems

Electrical components needed to operate and control the PMSM were purchased from RS and Farnell and include: 1) a 500x500x300 mm steel cabinet with a gland plate in the base and a zinc plated mounting plate to which all other components would be fixed, 2) emergency stop, isolator, contactor and fuses, 3) 3 m long 13 A leads, 4) DC power supply for the servo drive, 5) DIN rails, 6) NI 9263 analogue output module to provide position, velocity or current references to the servo drive, 7) NI 9411 digital input module to read encoder emulation signals from the servo drive, 8) multiple adaptors, glands, wires etc.

With these components, the PMSM can be controlled in two ways:

- Through S30661-NA software: motion profiles can be defined and loaded into the servo drive for execution. This is perfect to run some preliminary tests.
- Through NI LabView software: closed-loop position or velocity control. This is needed to implement comfort enhancing controllers and test the SAVGS performance.

The wiring diagram is shown in Fig. 7.11.

7.3 Proof of concept testing

Initial testing of the rig was performed by running motion paths of different amplitudes and frequencies through the servo drive software. The aim was to confirm that the rig could be operated as intended and to identify problems and design flaws. Some of the important insights gained from these initial runs include:

- There is enough clearance between the wishbones and the SD as to avoid interference even with a large SL, large SL rotations and large suspension deflections.
- As the wheel is not rotating in this test rig, lateral motion of the tyre over a high friction surface such as a steel plate leads to very significant frictional forces and undesired stick-slip. The introduction of a low friction component between the tyre and the tyre support plate, such as the teflon sheet described in Section 7.2.3, reduces lateral forces and avoids stick-slip.
- The initial SL design is flawed because the torque is applied to the SL from one side alone and this leads to asymmetrical force transfer to the supporting bearings, large deflections and potential damage to the gearbox. This is particularly important when the SL is operated from a nominal configuration different from the passive equilibrium configuration.

Although not fully tested as part of this project, an excitation system to emulate road disturbances of different frequencies was broadly designed and components were identified and purchased. These are described in Section 7.4.

7.4 Excitation system

Hydraulic and linear electromagnetic actuators were considered, but a scaled-up version of the SAVGS actuator, capable of driving a mechanical cam and to displace the tyre support plate with the desired amplitudes and frequencies, was finally chosen. The main advantages of the proposed system are the minimal modifications needed to install it in the test rig and the ease of manufacture, assembly and use.

Component selection was based on simulations performed with a customised one corner AutoSim model and various parameter sets to cover for the main parameter uncertainties. It is expected that the selected actuator will work well with an eccentric cam and a flat follower and that it will be capable of providing road motions with amplitudes of up to 28mm peak to peak at frequencies of up to 3Hz (well above the natural frequency of the sprung mass) for continuous

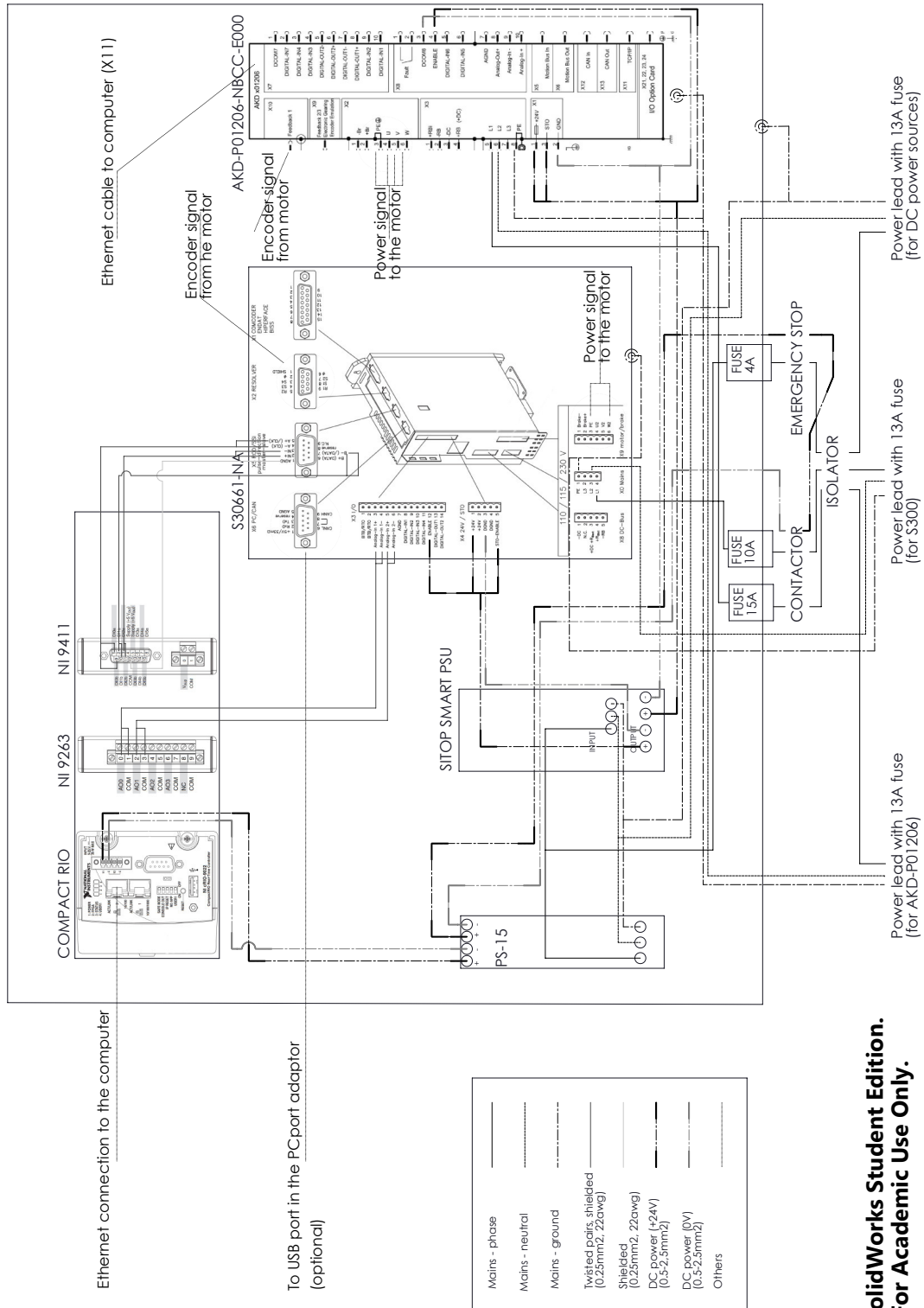


Fig. 7.11: Wiring diagram for all electrical components within the cabinet, including the AKD servo drive for operation of the excitation system.

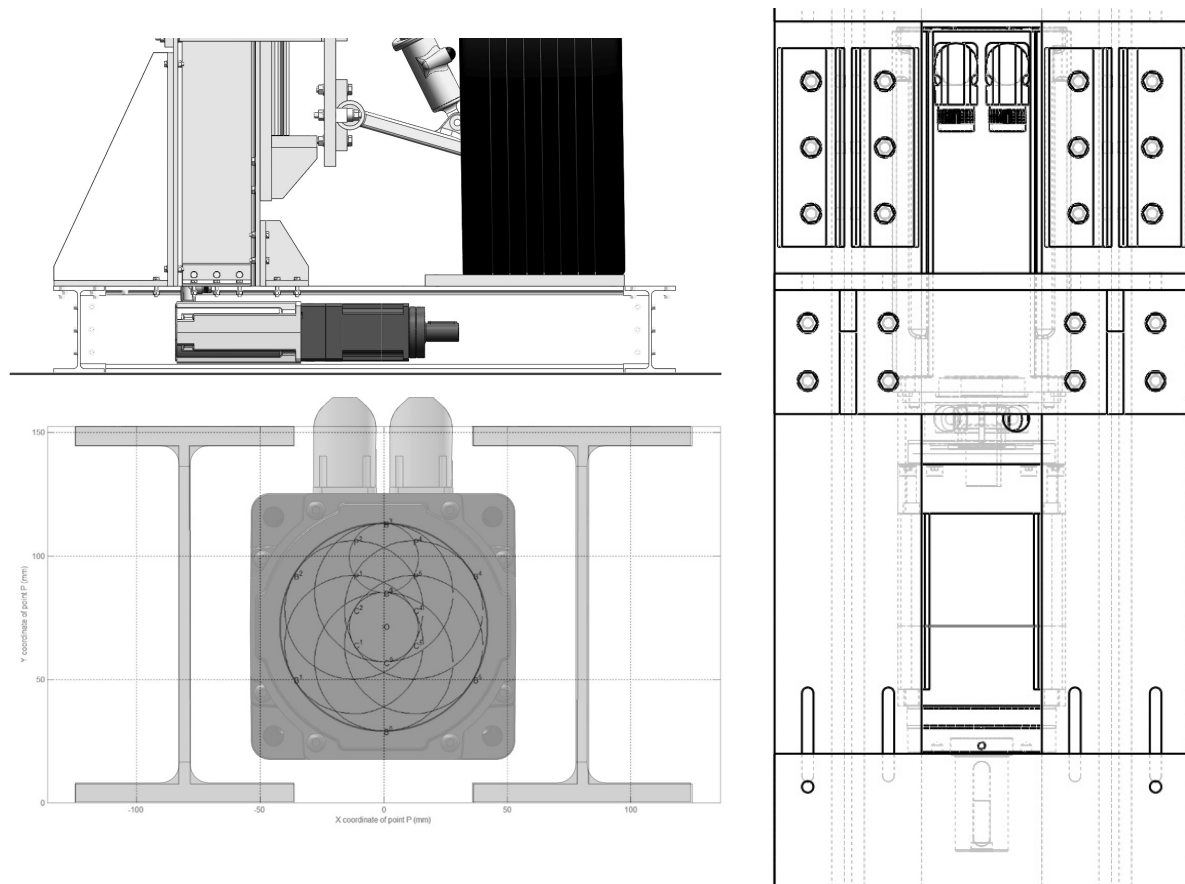


Fig. 7.12: Proposed actuator location for the excitation system. Side view (top left), front view (bottom left) and top view (right). The circle in the bottom left drawing represents the area swept by the proposed eccentric cam capable of providing harmonic motion to the tyre support plate with 28 mm peak-to-peak amplitude.

operation, and above 5 Hz for intermittent operation. Moreover, different cam profiles and sizes could be used to increase the range of excitation frequencies further.

In the proposed design, the actuator is located between the two longitudinal I beams, as shown in Fig. 7.12. The installation of the cam system and the longitudinal guides for the tyre support plate would only require minor machining on the existing longitudinal I beams and tyre support plate.

Although not fully tested as part of this project, the servo drive and other electrical components required were purchased and installed in the rig cabinet. Through preliminary tests, it was verified that the actuator could actually be driven as intended from the test rig computer via the servo drive software without integration with the CompactRIO.

7.5 Conclusions

Although the experimental rig is still work in progress, significant advances towards its development have been made as part of this project. A brief assessment of the original objectives defined for the test rig construction, as shown in Section 7.1.1, is presented in Table 7.2.

Table 7.2: Assessment of initial objectives for the test rig development.

Objective	Assessment
To have a platform to show and explain the AVGS concept to potential customers.	Achieved. Within budget and in a sensible time frame.
To demonstrate that the motion of the system can be effectively controlled with a reasonably sized actuator.	Achieved.
To identify modelling inaccuracies, assembling problems and other issues.	Partially achieved. Model validation is still pending and requires extensive testing.
To compare how actuators of different capabilities perform.	Not attempted so far, although the rig is ready for it: a different suspension arrangement and/or actuator can be connected by simply replacing the two front steel plates. Moreover, the sprung mass can be easily adjusted by adding/removing steel plates (weights) from the sides of the sprung mass.
To pave the way for a possible future implementation on a real vehicle (i.e. to learn how to implement the control algorithms in a real system etc.).	Partially achieved. Component selection and wiring needed to perform open-loop and closed-loop position control of the SAVGS has already been done.

The design and construction of the test rig was done when it was believed that the SAVGS would normally be operated from the passive equilibrium position. It was also expected that long single-links with limited rotations would be used. Loading conditions and parameter values were largely unknown and many assumptions had to be made.

Based on the latest analyses and simulation results, the initial SAVGS design will have to be refined to include a shorter SL and to ensure effective and safe operation at nominal angles different from that corresponding to the passive equilibrium position. Also, the design of the excitation system needs to be finalised in order to start proper testing of the SAVGS.

Overall, and despite a few pending items, this project has laid solid foundations for the experimental testing of the SAVGS.

Chapter 8

Conclusions and future work

This project was set out with the aim of studying and developing further the recently introduced Series Active Variable Geometry (SAVGS) concept. To this end, work on the design, modelling, control synthesis and virtual and experimental testing of the single-link variant of the SAVGS has been carried out. A summary of the progress made in each of these areas, together with the main conclusions drawn from them, has already been provided at the end of each chapter and is not repeated here. Instead, Section 8.1 distils the research carried out as part of this project into key take-home messages and Section 8.2 reflects on the open research threads, thus identifying areas for future work.

8.1 Conclusions

8.1.1 SAVGS technology

- There is currently a window of opportunity for the introduction of new and innovative electromechanical active suspension systems and the Series Active Variable Geometry Suspension concept ticks (almost) all the boxes.
- The technology is better suited for:
 - Independent suspensions in which the wheel camber is not directly coupled to the orientation of the spring-damper (e.g. double-wishbone and multi-link, not MacPherson).
 - On-road, relatively light vehicles with small payload changes and stiff/highly damped passive suspensions (e.g. sports cars, not SUVs).
- Although conventionally categorised as a *slow* active suspension because it acts in series with the passive spring-damper, simulation results have shown that the SAVGS has the ability not only to reduce the attitude motions of the chassis but also to provide a safer

and more comfortable ride. This is in agreement with predictions from leading researchers in the field who identified the large potential of active suspensions with control bandwidths in the 3-5 Hz band [Sharp and Hassan, 1987; Hrovat, 1997]ⁱ.

8.1.2 Vehicle and SAVGS modelling and simulation

- In the absence of experimental tests, power and energy balance computations and a multi-model, multi-software approach become important tools to cross-check simulation results and increase the confidence in them.
- Although rarely found together in the academic literature, both simple linear models and complex nonlinear models have an important role to play in the study, design and control of active suspensions.

8.1.3 SAVGS control

- The control synthesis problem can be simplified by using a cascade control approach that splits the problem into 1) generating a suitable position reference for each actuator based on measurements performed at the vehicle level and 2) independently tracking the aforementioned position reference in each corner of the car.
- The single-link introduces a significant nonlinearity in the suspension system. This is both a liability and an advantage in terms of control: on the one hand it complicates the application of linear control theory but on the other it enables the fail-safe use of controllers with different compromises between performance and power/energy consumption.
- Despite these nonlinearities, linear controllers that offer good performance and robustness against parameter changes, measurement noise and model uncertainties, can be designed for the single-link variant. In particular, PIDs and $\mathcal{H}_\infty$ controllers have been successfully applied to low- and high-frequency suspension functions, respectively.

8.2 Future work

8.2.1 SAVGS technology

- Standard off-the-shelf actuators are appropriate but significant improvements should be expected if custom designs were considered instead. Purpose-built actuators should:
 - Combine the PMSM, gearbox and single-link in a single actuator unit in order to increase stiffness and simplify packaging

ⁱThis is particularly true when considering that the SAVGS concept could be combined with a semi-active damper.

- Prioritise the maximisation of continuous output torque density
- Be protected against water, dirt and shock and be able to operate in a wide range of ambient temperatures
- Although briefly considered during this project, more work should be carried out to evaluate the possibility of using a soft but highly preloaded linear or torsional spring to reduce the torque that the actuator needs to provide in the nominal equilibrium configuration.
- Realistic load time histories should be obtained for the suspension system and fatigue and general durability analyses should be performed to ensure that the proposed system satisfies all reliability requirements.

8.2.2 Vehicle and SAVGS modelling, simulation and testing

- Torsional compliances and individual inertias in the actuator and single-link, as well as backlash in the gearbox, should be included in the models.
- A thermal model for the actuator should be derived and the whole range of operating temperatures should be considered in simulation studies.
- Work on the experimental quarter-car test rig should continue. Some milestones could be:
 - To refine the SL design and to implement the excitation system.
 - To develop a finite element model of the test rig in order to identify all natural frequencies of the structure. If any of these lay in the range of frequencies of interest (approximately 0.5 to 15 Hz), the rig should be retrofitted to move them away from that range of frequencies or at least to increase the damping ratio for those natural modes of vibration.
 - To improve the multi-body model of the rig by ensuring that all relevant nonlinearities of the passive components are taken into account. By exciting the passive suspension in a wide range of frequencies and with varying amplitude levels, the nonlinearities in the system (e.g. in the bushings, suspension damper, guiding system etc.) could be activated, characterised and included in the multi-body model.
 - To validate the actuator model by testing the actuator when connected to a dummy, simple load (e.g. inertial). The actuator should be driven at varying frequencies and with different torque amplitudes, and tests should be performed at different temperatures.
 - To validate the whole test rig model by comparing experimental and simulation results corresponding to simple SAVGS control strategies.
 - To assess the SAVGS performance by implementing advanced control strategies for the SAVGS and comparing the responses of the passive and the active system.

8.2.3 SAVGS control

- The linear model provided in Appendix B should be a good starting point to develop $\mathcal{H}_\infty$ controllers for the full car. When doing so, mitigation of attitude motions and comfort and road holding enhancement could be considered simultaneously as control objectives.
- Alternatively, a strategy to blend the attitude control proposed in Chapter 5 with the comfort and road holding enhancement control derived in Chapter 6 could be explored.
- Some form of gain scheduling should be implemented to fully exploit the SAVGS potential. The scheduling variables could be, for example, the level of road unevenness, the forward speed of the vehicle, the driver actions and/or the single-link angles.
- Feedforward control could be implemented to reduce reaction times by sensing the driver actions.
- If road information is expected to be available, MPC or preview techniques could also be investigated.
- Model uncertainties and parameter changes should be formally included in the control synthesis, for example through the μ -synthesis framework.

References

- T. Aburaya, M. Kawanishi, H. Kondo, and T. Hamada. Development of an electronic control system for active suspension. In *29th IEEE Conference on Decision and Control (CDC)*, volume 4, pages 2220–2225, 1990.
- J. Ackermann. Robust car steering by yaw rate control. In *29th IEEE Conference on Decision and Control (CDC)*, volume 4, pages 2033–2034, 1990.
- N. AL-Holou, T. Lahdhiri, D. S. Joo, J. Weaver, and F. Al-Abbas. Sliding mode neural network inference fuzzy logic control for active suspension systems. *IEEE Transactions on Fuzzy Systems*, 10(2):234–246, Apr 2002.
- A. Alleyne and J. K. Hedrick. Nonlinear adaptive control of active suspensions. *IEEE Transactions on Control Systems Technology*, 3(1):94–101, Mar. 1995.
- A. Alleyne, P. D. Neuhaus, and J. K. Hedrick. Application of nonlinear control theory to electronically controlled suspensions. *Vehicle System Dynamics*, 22(5-6):309–320, 1993. doi: 10.1080/00423119308969033.
- D. Ammon. Vehicle dynamics analysis tasks and related tyre simulation challenges. *Vehicle System Dynamics*, 43(sup1):30–47, 2005. doi: 10.1080/00423110500141003.
- Anon. Objects of interest. *Automotive Design & Production*, 118(8):74, 2006. ISSN 15368823.
- Anon. Arvinmeritor firms up suspension links in Europe. *Automotive Engineer*, 32(5):53, 2007. ISSN 03076490.
- Anon. Smart seats could receive active suspension systems. *Automotive Engineer*, 34(2):48, 2009. ISSN 03076490.
- Anon. Audi engineers set up new suspension system. *Automotive Engineer*, 35(10):49, 2010. ISSN 03076490.
- O. M. Anubi and C. D. Crane. A new active variable stiffness suspension system using a nonlinear energy sink-based controller. *Vehicle System Dynamics*, 51(10):1588–1602, 2013. doi: 10.1080/00423114.2013.815358.
- O. M. Anubi, D. R. Patel, and C. D. Crane III. A new variable stiffness suspension system: passive case. *Mechanical Sciences*, February 2013.
- Y. Aoyama, K. Kawabata, S. Hasegawa, Y. Kobari, M. Sato, and E. Tsuruta. Development of the full active suspension by Nissan. SAE Technical Paper 901747, Society of Automotive Engineers, 1990.
- C. Arana, S. A. Evangelou, and D. Dini. Pitch angle reduction for cars under acceleration and braking by active variable geometry suspension. In *51st IEEE Conference on Decision and Control (CDC)*, pages 4390–4395, 2012.
- C. Arana, S. A. Evangelou, and D. Dini. Car attitude control by series mechatronic suspension. In *IFAC World Congress*, volume 19, pages 10688–10693, 2014a.

- C. Arana, S. A. Evangelou, and D. Dini. Series active variable suspension - application to roll control. Technical report, Imperial College London, January 2014b. Internal report.
- C. Arana, S. Evangelou, and D. Dini. Series active variable geometry suspension for road vehicles. *IEEE/ASME Trans. Mechatronics*, 20(1):361–372, Feb 2015a.
- C. Arana, S. Evangelou, and D. Dini. Series active variable geometry suspension application to chassis attitude control. *IEEE/ASME Trans. Mechatronics*, 2015b. doi: 10.1109/TMECH.2015.2446412.
- R. Aucock. Future shocks. *Automotive Engineer*, 32(6):27–28, 2007. ISSN 03076490.
- Aurok Ltd. Aurok limited, 2015. URL <http://www.aurok.com/>.
- Automobile Association. Citroën XM 2.0Si. *Car Test*, 1990. R9018.
- A. Baker. Lotus active suspension. *Automotive Engineer*, 9:56–57, 1984.
- E. Bakker, H. B. Pacejka, and L. Nyborg. Tyre modelling for use in vehicle dynamics studies. *SAE paper*, 870421, 1987.
- R. Beaven, M. Wright, and D. Seaward. Weighting function selection in the $\mathcal{H}_\infty$ design process. *Control Engineering Practice*, 4(5):625 – 633, 1996.
- E. Bedner, D. Fulk, and A. Hac. Exploring the trade-off of handling stability and responsiveness with advanced control systems. Technical Report 2007-01-0812, Society of Automotive Engineers, 400 Commonwealth Dr, Warrendale, PA, 15096, USA, 2007.
- E. K. Bender and I. L. Paul. Analysis of optimum and preview control of active vehicle suspensions. Technical Report Rept Dsr-76109-6, Massachusetts Institute of Technology, 1967.
- I. J. Besselink, A. J. Schmeitz, and H. B. Pacejka. An improved magic formula/swift tyre model that can handle inflation pressure changes. *Vehicle System Dynamics*, 48(sup1):337–352, 2010.
- J. E. Bibel and D. S. Malyevac. Guidelines for the selection of weighting functions for H-infinity control. DTIC document, Weapons Systems Department, Naval Surface Warfare Center, Dahlgren Division, Virginia, 1992.
- H. Bolandhemmat, C. M. Clark, and F. Golnaraghi. Development of a systematic and practical methodology for the design of vehicles semi-active suspension control system. *Vehicle System Dynamics*, 48(5):567–585, 2010. doi: 10.1080/00423110902971809.
- C. Boyle, L. L. Howell, S. P. Magleby, and M. S. Evans. Dynamic modeling of compliant constant-force compression mechanisms. *Mechanism and Machine Theory*, 38(12):1469–1487, 2003.
- S. Buma, H. Kajino, T. Takahashi, and S. Doi. Consideration of a human dynamic characteristic and performance evaluation of an electric active suspension. In *IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM)*, pages 1030–1036, July 2008.
- G. Cabande. Elec’trike, 2012. URL <https://grabcad.com/library/elec-trike>.
- D. Cao, X. Song, and M. Ahmadian. Editors perspectives: road vehicle suspension design, dynamics, and control. *Vehicle System Dynamics*, 49(1-2):3–28, 2011. doi: 10.1080/00423114.2010.532223.
- J. Cao, H. Liu, P. Li, and D. J. Brown. State of the art in vehicle active suspension adaptive control systems based on intelligent methodologies. *IEEE Transactions on Intelligent Transportation Systems*, 9(3):392–405, sept. 2008. ISSN 1524-9050. doi: 10.1109/TITS.2008.928244.

- K. M. Captain, A. B. Boghani, and D. N. Wormley. Analytical tire models for dynamic vehicle simulation. *Vehicle System Dynamics*, 8(1):1–32, 1979. doi: 10.1080/00423117908968566.
- C. Cavallaro, A. Di Tommaso, R. Miceli, A. Raciti, G. Galluzzo, and M. Trapanese. Efficiency enhancement of permanent-magnet synchronous motor drives by online loss minimization approaches. *IEEE Trans. Ind. Electron.*, 52(4):1153–1160, 2005. ISSN 0278-0046. doi: 10.1109/TIE.2005.851595.
- H. Chen and C. Scherer. An LMI based model predictive control scheme with guaranteed $\mathcal{H}_\infty$ performance and its application to active suspension. In *American Control Conference (ACC)*, volume 2, pages 1487–1492, June 2004.
- P.-C. Chen and A.-C. Huang. Adaptive sliding control of active suspension systems with uncertain hydraulic actuator dynamics. *Vehicle System Dynamics*, 44(5):357–368, 2006. doi: 10.1080/00423110600621664.
- C. Cheng, S. Evangelou, C. Arana, and D. Dini. Active variable geometry suspension robust control for improved vehicle ride comfort and road holding. In *American Control Conference (ACC)*, pages 3440–3446, July 2015.
- V. Ciarla, V. Cahouet, C. de Wit, and F. Quaine. Genesis of booster curves in electric power assistance steering systems. In *15th International IEEE Conference on Intelligent Transportation Systems (ITSC)*, pages 1345–1350, Sept 2012.
- C. Clover and J. Bernard. Longitudinal tire dynamics. *Vehicle System Dynamics*, 29(4):231–260, 1998.
- M. Corno, S. Bottelli, M. Tanelli, C. Spelta, and S. M. Savaresi. Active control of aerodynamic surfaces for ride control in sport vehicles. In *19th World Congress of the International Federation of Automatic Control*, volume 19, pages 7553–7558. IFAC (International Federation of Automatic Control), 2014.
- D. Crolla. *Automotive Engineering e-Mega Reference*. Butterworth Heinemann. Elsevier Science, 2009. ISBN 9781856175784.
- D. A. Crolla. Vehicle dynamics - theory into practice. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 210(2):83–94, 1996. ISSN 09544070.
- J. Cross. Farewell to rock and roll. *Automotive Engineer*, 24(7):42, 1999. ISSN 03076490.
- Cruden. Racer: the real deal, 2015. URL <http://www.racer.nl/>.
- Daimler AG. The birth of the automobile, 2015a. URL <http://www.daimler.com/dccom/>.
- Daimler AG. Magic body control, 2015b. URL http://techcenter.mercedes-benz.com/en/magic_body_control/detail.html.
- J. Darling, R. Dorey, and T. Ross-Martin. A low cost active anti-roll suspension for passenger cars. *Journal of dynamic systems, measurement, and control*, 114(4):599–605, 1992.
- S. Data and F. Frigerio. Objective evaluation of handling quality. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 216(4):297–305, 2002.
- J. Dixon. *Suspension geometry and computation*. Wiley Online Library, 2009.
- J. C. Dixon. Linear and non-linear steady state vehicle handling. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 202(3):173–186, 1988. doi: 10.1243/PIME.PROC.1988.202.171.02.

- F. L. M. dos Santos, J. A. de Bonfim Gripp, A. Adade Filho, and L. C. S. Góes. Comparison of robust control techniques for use in active suspension systems. In *ABCM Symposium Series in Mechatronics*, volume 5, 2012.
- J. Doyle, K. Glover, P. Khargonekar, and B. Francis. State-space solutions to standard $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems. *IEEE Transactions on Automatic Control*, 34(8):831–847, Aug 1989.
- J. Doyle, A. Packard, and K. Zhou. Review of LFTs, LMIs, and μ . In *30th IEEE Conference on Decision and Control (CDC)*, volume 2, pages 1227–1232, Dec 1991.
- G. Durmisevich. 1957 & 1958 Cadillac Eldorado Brougham, 2015. URL https://history.gmheritagecenter.com/wiki/index.php/1957_%26_58_Cadillac_Eldorado_Brougham.
- M. E. Elbeheiry, D. Karnopp, M. E. Elaraby, and A. M. Abdelraaouf. Advanced ground vehicle suspension systems - A classified bibliography. *Vehicle System Dynamics*, 24(3):231–258, 1995. doi: 10.1080/00423119508969089.
- M. E. Elbeheiry, D. C. Karnopp, M. E. Elaraby, and A. M. Abdelraaouf. Suboptimal control design of active and passive suspensions based on a full car model. *Vehicle System Dynamics*, 26(3):197–222, 1996. doi: 10.1080/00423119608969309.
- M. M. ElMadany. Control and evaluation of slow-active suspensions with preview for a full car. *Mathematical Problems in Engineering*, 2012:Article ID 375080, 2012. doi: doi:10.1155/2012/375080.
- S. A. Evangelou. Control of motorcycles by variable geometry rear suspension. In *IEEE International Conference on Control Applications (CCA)*, pages 148–154, sept 2010. doi: 10.1109/CCA.2010.5611082.
- S. A. Evangelou, D. Dini, O. De Meerschman, A. Tocatlían, C. Kneip, and C. Palas. Variable-geometry suspension apparatus and vehicle comprising such apparatus, July 2011. US9026309B2.
- W.-J. Evers, I. Besselink, A. Teerhuis, and H. Nijmeijer. On the achievable performance using variable geometry active secondary suspension systems in commercial vehicles. *Vehicle System Dynamics*, 49(10):1553–1573, 2011a. doi: 10.1080/00423114.2010.527996.
- W.-J. Evers, A. Teerhuis, A. van der Knaap, I. Besselink, and H. Nijmeijer. The electromechanical low-power active suspension: Modeling, control, and prototype testing. *Journal of Dynamic Systems, Measurement, and Control*, 133(4):041008, 2011b. doi: 10.1115/1.4003278.
- S. Fankem and S. Mller. A new model to compute the desired steering torque for steer-by-wire vehicles and driving simulators. *Vehicle System Dynamics*, 52(sup1):251–271, 2014.
- C. M. Fernandes, P. M. Marques, R. C. Martins, and J. H. Seabra. Gearbox power loss. Part III: Application to a parallel axis and a planetary gearbox. *Tribology International*, 88(0):317–326, 2015.
- I. Fialho and G. Balas. Road adaptive active suspension design using linear parameter-varying gain-scheduling. *IEEE Transactions on Control Systems Technology*, 10(1):43–54, jan 2002. ISSN 1063-6536. doi: 10.1109/87.974337.
- D. Fischer and R. Isermann. Mechatronic semi-active and active vehicle suspensions. *Control Engineering Practice*, 12(11):1353–1367, 2004.
- T. Gillespie. *Fundamentals of Vehicle Dynamics*. Society of Automotive Engineers, 1992. ISBN 9781560911999.
- N. Giorgetti, A. Bemporad, H. E. Tseng, and D. Hrovat. Hybrid model predictive control application towards optimal semi-active suspension. *International Journal of Control*, 79(5):521–533, 2006. doi: 10.1080/00207170600593901.

- K. Glover and J. C. Doyle. State-space formulae for all stabilizing controllers that satisfy an $\mathcal{H}_\infty$ -norm bound and relations to risk sensitivity. *Systems & Control Letters*, 11(3):167 – 172, 1988.
- C. Göhrle, A. Wagner, A. Schindler, and O. Sawodny. Active suspension controller using MPC based on a full-car model with preview information. In *American Control Conference (ACC)*, pages 497–502, Jun. 2012.
- C. Göhrle, A. Schindler, A. Wagner, and O. Sawodny. Design and vehicle implementation of preview active suspension controllers. *IEEE Transactions on Control Systems Technology*, 22(3):1135–1142, May 2014.
- R. Goodall. Active railway suspensions: Implementation status and technological trends. *Vehicle System Dynamics*, 28(2-3):87–117, 1997. doi: 10.1080/00423119708969351.
- A. Goodarzi, E. Oloomi, and E. Esmailzadeh. Design and analysis of an intelligent controller for active geometry suspension systems. *Vehicle System Dynamics*, 49(1-2):333–359, 2011. doi: 10.1080/00423110903401913.
- M. Green and D. Limebeer. *Linear robust control*. Prentice-Hall information and system sciences series. Prentice Hall, 1995. ISBN 9780131022782.
- B. Gysen, T. van der Sande, J. Paulides, and E. Lomonova. Efficiency of a regenerative direct-drive electromagnetic active suspension. *IEEE Transactions on Vehicular Technology*, 60(4):1384–1393, May 2011.
- B. L. J. Gysen, J. J. H. Paulides, J. L. G. Janssen, and E. A. Lomonova. Active electromagnetic suspension system for improved vehicle dynamics. *Vehicular Technology, IEEE Transactions on*, 59(3):1156–1163, Mar. 2010. ISSN 0018-9545. doi: 10.1109/TVT.2009.2038706.
- A. Hac. Adaptive control of vehicle suspension. *Vehicle System Dynamics*, 16(2):57–74, 1987. doi: 10.1080/00423118708968870.
- A. Hac. Optimal linear preview control of active vehicle suspension. In *29th IEEE Conference on Decision and Control (CDC)*, volume 5, pages 2779–2784, Dec. 1990.
- R. Hampo and K. Marko. Neural network architectures for active suspension control. In *IJCNN-91-Seattle International Joint Conference on Neural Networks*, volume ii, pages 765–770, Jul. 1991.
- K. Hayakawa, K. Matsumoto, M. Yamashita, Y. Suzuki, K. Fujimori, and H. Kimura. Robust $\mathcal{H}_\infty$ -output feedback control of decoupled automobile active suspension systems. *IEEE Transactions on Automatic Control*, 44(2):392–396, Feb. 1999.
- J. K. Hedrick and T. Butsuen. Invariant properties of automotive suspensions. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 204(1):21–27, 1990.
- S. Hegazy, H. Rahnejat, and K. Hussain. Multi-body dynamics in full-vehicle handling analysis. *Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics*, 213(1):19–31, 1999.
- G. Heinzel, A. Rüdiger, and R. Schilling. Spectrum and spectral density estimation by the discrete fourier transform (DFT), including a comprehensive list of window functions and some new flat-top windows. Technical report, MPI für Gravitationsphysik/Laser Interferometry & Gravitational Wave Astronomy, MPI für Gravitationsphysik/Teilinstitut Hannover, 2002. URL <http://pubman.mpdl.mpg.de/pubman/item/escidoc:152164:1/component/escidoc:152163/395068.pdf>.
- B. Heißing and M. Ersoy. *Chassis Handbook: Fundamentals, Driving Dynamics, Components, Mechatronics, Perspectives*. Atz/Mtz-Fachbuch. Vieweg+Teubner, 2010. ISBN 9783834809940.

- D. Hennecke, P. Baier, B. Jordan, and E. Walek. EDC III - The new variable damper system for BMW's top models - A further development of our adaptive, frequency-dependent damper control. In *SAE Technical Paper*. SAE International, Mar. 1990. doi: 10.4271/900662. URL <http://dx.doi.org/10.4271/900662>.
- H. Heo, E. Joa, K. Yi, and K. Kim. Integrated chassis control for enhancement of high speed cornering performance. *SAE Int. J. Commer. Veh.*, 8:102–109, 04 2015. doi: 10.4271/2015-01-1568. URL <http://dx.doi.org/10.4271/2015-01-1568>.
- G. J. Heydinger, R. A. Bixel, W. R. Garrott, M. Pyne, J. G. Howe, and D. A. Guenther. Measured vehicle inertial parameters - NHTSAs data through November 1998. In *SAE Technical Paper*. SAE International, Mar. 1999.
- A. Higuchi and H. B. Pacejka. The relaxation length concept at large wheel slip and camber. *Vehicle System Dynamics*, 27(sup1):50–64, 1997.
- S. Hiraoka, T. Suzuki, and M. Takahashi. Gain-scheduling $\mathcal{H}_\infty$ control to improve ride comfort and driving stability of vehicle active suspension. In *10th International Conference on Motion and Vibration Control (MOVIC)*, Tokyo, Japan, 2014.
- B. Howard. Bose active suspension Lexus LS400, 2011. URL <http://www.extremetech.com/extreme/97177-bose-active-suspension-moves-toward-market/attachment/bose-active-suspension-lexus-ls400>.
- D. Hrovat. Applications of optimal control to advanced automotive suspension design. *Journal of Dynamic Systems, Measurement, and Control*, 115(2B):328–342, 1993.
- D. Hrovat. Survey of advanced suspension developments and related optimal control applications. *Automatica*, 33(10):1781–1817, 1997. ISSN 0005-1098. doi: 10.1016/S0005-1098(97)00101-5.
- S. Huang and W. Lin. Adaptive fuzzy controller with sliding surface for vehicle suspension control. *IEEE Transactions on Fuzzy Systems*, 11(4):550–559, Aug. 2003.
- S.-J. Huang and W.-C. Lin. A neural network based sliding mode controller for active vehicle suspension. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 221(11):1381–1397, 2007. doi: 10.1243/09544070JAUTO242.
- W. H. Hucho. *Aerodynamics of Road Vehicles: From Fluid Mechanics to Vehicle Engineering*. Elsevier Science, 2013. ISBN 9781483102078.
- A. Hughes and B. Drury. *Electric Motors and Drives: Fundamentals, Types and Applications*. Elsevier Science, 2013. ISBN 9780080993683.
- T. Iijima, Y. Akatsu, K. Takahashi, and H. Murakami. Development of a hydraulic active suspension. In *SAE Technical Paper*. SAE International, 11 1993. doi: 10.4271/931971. URL <http://dx.doi.org/10.4271/931971>.
- R. Isermann. Mechatronic systems - Innovative products with embedded control. *Control Engineering Practice*, 16(1):14 – 29, 2008.
- ISO 2631-1:1997. Mechanical vibration and shock - Evaluation of human exposure to whole-body vibration - General Requirements, 1997.
- ISO 3888-2:2011. Passenger cars - Test track for a severe lane-change manoeuvre - Obstacle avoidance, 2011.
- ISO 4138:2004. Passenger cars - Steady-state circular driving behaviour - Open-loop test methods, 2004.

- ISO 5805:1997. Mechanical vibration and shock - Human exposure - Vocabulary, 1997.
- ISO 7401:2011. Road vehicles - Lateral transient response test methods - Open-loop test methods, 2011.
- ISO 7975:2006. Passenger cars - Braking in a turn - Open-loop test method, 2006.
- ISO 8041:2005. Human response to vibration - Measuring instrumentation, 2005.
- ISO 8608:1995. Mechanical vibration - Road surface profiles - reporting of measured data, 1995.
- ISO 8855:2011. Road vehicles - Vehicle dynamics and road-holding ability - Vocabulary, 2011.
- S. T. Jansen, P. W. Zegelaar, and H. B. Pacejka. The influence of in-plane tyre dynamics on abs braking of a quarter vehicle model. *Vehicle System Dynamics*, 32(2-3):249–261, 1999.
- R. Jazar. *Vehicle dynamics: theory and applications*. Springer, 2008. ISBN 9780387742434.
- J. Jennings. A Bose Suspension? *Automobile*, 2004. URL http://www.automobilemag.com/features/news/0410_bose_suspension/.
- M. J. Johnston. Development and evaluation of vehicle suspension tuning metrics. Master’s thesis, University of Windsor, 2010.
- M. Jordan. Vision accomplished. Die neue S-Klasse., 2013. URL <http://blog.mercedes-benz-passion.com/wp-content/uploads/magicflower3.jpg>.
- T. R. Kane and D. A. Levinson. Formulation of equations of motion for complex spacecraft. *Journal of Guidance, Control, and Dynamics*, 3(2):99–112, Mar. 1980.
- D. Karnopp. Active damping in road vehicle suspension systems. *Vehicle System Dynamics*, 12(6):291–311, 1983. doi: 10.1080/00423118308968758.
- D. Karnopp. Theoretical limitations in active vehicle suspensions. *Vehicle System Dynamics*, 15(1):41–54, 1986. doi: 10.1080/00423118608968839.
- D. Karnopp, M. J. Crosby, and R. A. Harwood. Vibration control using semi-active force generators. *Journal of Engineering for Industry*, 96(2):619, 1974. ISSN 00220817. doi: 10.1115/1.3438373.
- J. Keebler. Ford kills active-suspension program: costly components shelved for now, 1995. URL <http://www.autonews.com/article/19950123/ANA/501230771/ford-kills-active-suspension-program:costly-components-shelved-for>.
- C. Kim and P. I. Ro. A sliding mode controller for vehicle active suspension systems with non-linearities. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 212(2):79–92, 1998. doi: 10.1243/0954407981525812.
- C. Kim and P. I. Ro. Reduced-order modelling and parameter estimation for a quarter-car suspension system. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 214(8):851–864, 2000.
- W. Kimberley. Keeping control. *Automotive Engineer*, 30(3):48, 2005. ISSN 03076490.
- V. D. Knaap. *Design of a Low Power Anti-Roll/Pitch System for a Passenger Car*. PhD thesis, Vehicle Research Laboratory, Delft University of Technology, 1989.

- G. Koch, O. Fritsch, and B. Lohmann. Potential of low bandwidth active suspension control with continuously variable damper. *Control Engineering Practice*, 18(11):1251–1262, 2010. ISSN 0967-0661.
- K. Korosec. Potholes and Tesla’s Model S: Never the twain shall meet, 2014. URL <http://fortune.com/2014/09/19/tesla-model-s-suspension-upgrade/>.
- F. Kou and Z. Fang. An experimental investigation into the design of vehicle fuzzy active suspension. In *IEEE International Conference on Automation and Logistics*, pages 959–963, aug. 2007.
- P. Krause, O. Wasynczuk, and S. Sudhoff. *Analysis of Electric Machinery and Drive Systems*. Wiley-IEEE Press, 2002. ISBN 9780470544167.
- R. Krishnan. *Permanent Magnet Synchronous and Brushless DC Motor Drives*. CRC Press/Taylor & Francis, 2010. ISBN 9781420014235.
- A. Kruczek and A. Stribrsky. A full-car model for active suspension - Some practical aspects. In *IEEE International Conference on Mechatronics*, pages 41–45, june 2004. doi: 10.1109/ICMECH.2004.1364409.
- A. Kruczek, A. Stribrsky, J. Honcu, and M. Hlinovsky. Controller choice for car active suspension. *International Journal of Mechanics*, 3:61–68, 2009.
- C. Lauwerys, J. Swevers, and P. Sas. Robust linear control of an active suspension on a quarter car test-rig. *Control Engineering Practice*, 13(5):577–586, 2005. ISSN 0967-0661.
- S. Lee and W. Kim. Active suspension control with direct-drive tubular linear brushless permanent-magnet motor. *IEEE Transactions on Control Systems Technology*, 18(4):859–870, july 2010.
- U. Lee and C. Han. A suspension system with a variable roll centre for the improvement of vehicle handling characteristics. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 215(6):677–696, 2001. doi: 10.1243/0954407011528266.
- U. K. Lee, S. H. Lee, C. S. Han, K. Hedrick, and A. Catal. Active geometry control suspension system for the enhancement of vehicle stability. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 222(6):979–988, 2008. doi: 10.1243/09544070JAUTO693.
- D. Limebeer, R. Sharp, and S. Evangelou. The stability of motorcycles under acceleration and braking. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 215(9):1095–1109, 2001.
- P. Lundström, S. Skogstad, and Z. Wang. Weight selection for H-infinity and mu-control methods - Insights and examples from process control. In *Symposium on Robust Control System Design Using H-infinity and Related Methods*, Cambridge, UK, 1991.
- C. C. Macadam. Understanding and modeling the human driver. *Vehicle System Dynamics*, 40(1-3):101–134, 2003.
- D. Maher and P. Young. An insight into linear quarter car model accuracy. *Vehicle System Dynamics*, 49(3):463–480, 2011.
- K. Malek and J. Hedrick. Decoupled active suspension design for improved automotive ride quality/handling performance. *Vehicle System Dynamics*, 15(sup1):383–398, 1986. doi: 10.1080/00423118608969149.
- MarketsandMarkets. Automotive suspension systems market worth \$66.2 billion by 2018, 2014. URL <http://www.marketsandmarkets.com/PressReleases/automobile-suspension-systems.asp>. Accessed: 2014-12-22.

- K.-F. Marquez. Brief lotus history, 2005. URL <http://www.espritfactfile.com/Misc.html>.
- J. Marsh. Citroën hydraulic system, 2011. URL <http://www.citroenet.org.uk/miscellaneous/hydraulics/hydraulics-1.html>.
- Mechanical Simulation Corporation. Carsim, 2015. URL <http://www.carsim.com/>.
- S. K. T. Miller. *Analysis of Three-phase Rectifiers with AC-side Switches and Interleaved Three-phase Voltage-source Converters*. PhD thesis, Rensselaer Polytechnic Institute, 2008.
- W. F. Milliken. Active suspension. In *SAE Technical Paper*. SAE International, Apr. 1988. 880799.
- W. F. Milliken and D. L. Milliken. *Race car vehicle dynamics*. Society of Automotive Engineers, 1995. ISBN 1-56091-526-9.
- R. Monajemy. *Control strategies and parameter compensation for permanent magnet synchronous motor drives*. PhD thesis, Virginia Polytechnic Institute and State University, 2000.
- R. Montague, C. Bingham, and K. Atallah. Servo control of magnetic gears. *IEEE/ASME Trans. Mechatronics*, 17(2):269–278, 2012.
- S. Morimoto, Y. Tong, Y. Takeda, and T. Hirasaka. Loss minimization control of permanent magnet synchronous motor drives. *IEEE Transactions on Industrial Electronics*, 41(5):511–517, 1994.
- P. Mucka and L. Gagnon. Influence of tyreroad contact model on vehicle vibration response. *Vehicle System Dynamics*, 0(0):1–20, 2015. doi: 10.1080/00423114.2015.1041992.
- B. Nemeth and P. Gaspar. Enhancement of vehicle stability based on variable geometry suspension and robust LPV control. In *IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM)*, pages 253–258, 2011a. doi: 10.1109/AIM.2011.6027020.
- B. Nemeth and P. Gaspar. Integration of control design and variable geometry suspension construction for vehicle stability enhancement. In *50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)*, pages 7452–7457, 2011b. doi: 10.1109/CDC.2011.6160441.
- M. Nordin and P.-O. Gutman. Controlling mechanical systems with backlash — A survey. *Automatica*, 38(10): 1633 – 1649, 2002.
- M. Novak and M. Valasek. A new concept of semi-active control of trucks suspension. In *Proc. of AVEC*, volume 96, pages 141–151, 1996.
- Öhlins. Ducati multistrada mechatronics, 2015. URL <http://www.ohlins.com/product/ducati-multistrada-mechatronics/>.
- H. Pacejka. *Tyre and Vehicle Dynamics*. Automotive engineering. Butterworth-Heinemann, 2006. ISBN 9780750669184.
- C. Papageorgiou, N. E. Houghton, and M. C. Smith. Experimental testing and analysis of inerter devices. *Journal of Dynamic Systems, Measurement, and Control*, 131(1):011001, 2009.
- Parkermotion. Fundamentals of servo motion control. Technical report, Parker Hannifin - Electromechanical Automation Division, 2010. URL <http://www.compumotor.com/whitepages/ServoFundamentals.pdf>.
- K. G. R. Parsons, M. Pask, and W. Burdock. The development of ACE for Discovery II. In *SAE Technical Paper*. SAE International, Mar. 2000.

- G. Perantoni and D. J. Limebeer. Optimal control for a Formula One car with variable parameters. *Vehicle System Dynamics*, 52(5):653–678, 2014.
- P. Pillay and R. Krishnan. Modeling of permanent magnet motor drives. *IEEE Transactions on Industrial Electronics*, 35(4):537–541, Nov. 1988.
- L. Pires, M. Smith, N. Houghton, and R. McMahon. Design trade-offs for energy regeneration and control in vehicle suspensions. *International Journal of Control*, 0(0):1–13, 2013. doi: 10.1080/00207179.2013.830197.
- M. Plöchl and J. Edelmann. Driver models in automobile dynamics application. *Vehicle System Dynamics*, 45(7-8):699–741, 2007.
- C. Poussot-Vassal, C. Spelta, O. Sename, S. Savaresi, and L. Dugard. Survey and performance evaluation on some automotive semi-active suspension control methods: A comparative study on a single-corner model. *Annual Reviews in Control*, 36(1):148 – 160, 2012. ISSN 1367-5788. doi: 10.1016/j.arcontrol.2012.03.011.
- G. Priyandoko, M. Mailah, and H. Jamaluddin. Vehicle active suspension system using skyhook adaptive neuro active force control. *Mechanical Systems and Signal Processing*, 23(3):855 – 868, 2009.
- H. S. Radt and H. B. Pacejka. Analysis of the steady-state turning behavior of an automobile. In *Proceedings of the Symposium on Control of Braking and cornering*, London, 1963. Institution of Mechanical Engineers.
- R. Rajamani and J. K. Hedrick. Adaptive observers for active automotive suspensions: theory and experiment. *IEEE Transactions on Control Systems Technology*, 3(1):86–93, Mar. 1995. ISSN 1063-6536.
- G. Rill. *Road Vehicle Dynamics: Fundamentals and Modeling*. Ground vehicle engineering series. Taylor & Francis, 2011. ISBN 9781439838983.
- D. Ryba. Improvements in dynamic characteristics of automobile suspension systems part 1: Two-mass systems. *Vehicle System Dynamics*, 3(1):17–46, 1974. doi: 10.1080/00423117408968445.
- SAE J670e. Vehicle dynamics terminology, 1976.
- M. G. Safonov, D. J. N. Limebeer, and R. Y. Chiang. Simplifying the $\mathcal{H}_\infty$ theory via loop-shifting, matrix-pencil and descriptor concepts. *International Journal of Control*, 50(6):2467–2488, 1989.
- Y. M. Sam, J. H. S. Osman, and M. R. A. Ghani. A class of proportional-integral sliding mode control with application to active suspension system. *Systems & Control Letters*, 51(34):217 – 223, 2004. ISSN 0167-6911.
- S. M. Savaresi, E. Silani, S. Bittanti, and N. Porciani. On performance evaluation methods and control strategies for semi-active suspension systems. In *42nd IEEE Conference on Decision and Control*, pages 2264–2269, Dec. 2003.
- M. Sayers. Autosim. *Vehicle System Dynamics*, 22(sup1):53–56, 1993.
- M. Sayers. Standard terminology for vehicle dynamics simulations, 1996. The University of Michigan Transportation Research Institute.
- A. Schindler. *Neue Konzeption und erstmalige Realisierung eines aktiven Fahrwerks mit Preview-Strategie*. PhD thesis, Universitätsverlag Karlsruhe, Karlsruhe, 2009.
- M. Schultz. Suspension: a century of progress. *Popular Mechanics*, 162(8), 1985.

- L. Segel. An overview of developments in road-vehicle dynamics: past, present and future. In *Proceedings of IMechE conference on vehicle ride and handling*, pages 1–12, London, 1993. IMechE, Mechanical Engineering Publications. C466/052.
- X. Shao. Modeling and model analysis of a full-car fitted with an anti-pitch anti-roll hydraulically interconnected suspension. Technical report, SAE, Detroit, MI, United states, 2014.
- R. Sharp and D. Limebeer. A motorcycle model for stability and control analysis. *Multibody System Dynamics*, 6(2):123–142, 2001. ISSN 1384-5640.
- R. S. Sharp. Variable geometry active suspension for cars. *Computing Control Engineering Journal*, 9(5):217–222, oct 1998. ISSN 0956-3385. doi: 10.1049/ccej:19980505.
- R. S. Sharp. Testing and improving a tyre shear force computation algorithm. *Vehicle System Dynamics*, 41(3):223–247, 2004.
- R. S. Sharp. Driver steering control and a new perspective on car handling qualities. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 219(10):1041–1051, Oct. 2005.
- R. S. Sharp and M. Bettella. Tyre shear force and moment descriptions by normalisation of parameters and the magic formula. *Vehicle System Dynamics*, 39(1):27–56, 2003a.
- R. S. Sharp and M. Bettella. On the construction of a general numerical tyre shear force model from limited data. *Proceedings of the institution of mechanical engineers, Part D - Journal of automobile engineering*, 217(D3):165–172, 2003b.
- R. S. Sharp and D. A. Crolla. Road vehicle suspension system design - A review. *Vehicle System Dynamics*, 16(3):167–192, 1987. doi: 10.1080/00423118708968877.
- R. S. Sharp and R. Granger. On car steering torques at parking speeds. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 217(2):87–96, 2003.
- R. S. Sharp and S. A. Hassan. The relative performance capabilities of passive, active and semi-active car suspension systems. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 200(3):219–228, July 1986a.
- R. S. Sharp and S. A. Hassan. An evaluation of passive automotive suspension systems with variable stiffness and damping parameters. *Vehicle System Dynamics*, 15(6):335–350, 1986b. doi: 10.1080/00423118608968859.
- R. S. Sharp and S. A. Hassan. On the performance capabilities of active automobile suspension systems of limited bandwidth. *Vehicle System Dynamics*, 16(4):213–225, 1987. doi: 10.1080/00423118708968879.
- R. S. Sharp and D. Pan. On active roll control for automobiles. *Vehicle System Dynamics*, 20(sup1):566–583, 1992.
- R. S. Sharp, D. Casanova, and P. Symonds. A mathematical model for driver steering control, with design, tuning and performance results. *Vehicle System Dynamics*, 33(5):289–326, 2000.
- M. C. Shin, S. H. Kim, G. H. Cho, and C. N. Chu. Development of a steering-system model considering viscous friction and its verification. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 228(2):144–163, 2014.
- D. W. Shuttlewood, D. A. Crolla, R. S. Sharp, and I. L. Crawford. Active roll control for passenger cars. *Vehicle System Dynamics*, 22(5-6):383–396, 1993.

- M. C. Smith. Achievable dynamic response for automotive active suspensions. *Vehicle System Dynamics*, 24(1): 1–33, 1995.
- M. C. Smith. Synthesis of mechanical networks: the inerter. *IEEE Transactions on Automatic Control*, 47(10): 1648 – 1662, Oct. 2002.
- M. C. Smith and G. W. Walker. Performance limitations and constraints for active and passive suspensions: a mechanical multi-port approach. *Vehicle System Dynamics*, 33(3):137–168, 2000.
- M. C. Smith and G. W. Walker. Interconnected vehicle suspension. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 219(3):295–307, 2005. doi: 10.1243/095440705X6578.
- H. A. Smith CC, McGehee DY. The prediction of passenger riding comfort from acceleration data. *ASME. J. Dyn. Sys., Meas., Control*, 1:34–41, 1978.
- C. Spelta, S. M. Savaresi, and L. Fabbri. Experimental analysis of a motorcycle semi-active rear suspension. *Control Engineering Practice*, 18(11):1239–1250, 2010. ISSN 0967-0661. doi: 10.1016/j.conengprac.2010.02.006.
- F. Sugawara, H. Kobayashi, T. Kakimoto, Y. Shiraishi, and Y. Tateishi. Electronically controlled shock absorber system used as a road sensor which utilizes super sonic waves. In *SAE Technical Paper*. SAE International, 10 1985. doi: 10.4271/851652.
- W. Sun, H. Gao, and O. Kaynak. Adaptive backstepping control for active suspension systems with hard constraints. *IEEE/ASME Trans. Mechatronics*, 18(3):1072–1079, 2013.
- W. Sun, H. Pan, and H. Gao. Filter-based adaptive vibration control for active vehicle suspensions with electro-hydraulic actuators. *IEEE Transactions on Vehicular Technology*, 0(0):1–1, 2015. ISSN 0018-9545. doi: 10.1109/TVT.2015.2437455.
- H. Tanahashi, K. Shindo, T. Nogami, and T. Oonuma. Toyota electronic modulated air suspension for the 1986 soarer. In *SAE Technical Paper*. SAE International, Feb. 1987.
- L. Teschler, S. J. Mraz, and S. L. Koucky. CL500 Mercedes - No squat, no dive. *Machine Design*, 72(12):160, 2000. ISSN 00249114.
- Teslarati Network. Tesla Model S firmware 5.8 discreetly removes feature to address fires?, 2013. URL <http://www.teslarati.com/tesla-model-s-firmware-5-8/>.
- M. Thommyppillai. *Optimal path-tracking of virtual race-cars using gain-scheduled preview control*. PhD thesis, Imperial College London, 2010.
- M. Thommyppillai, S. A. Evangelou, and R. S. Sharp. Advances in the development of a virtual car driver. *Multibody System Dynamics*, 22(3):245–267, 2009. ISSN 1384-5640. doi: 10.1007/s11044-009-9164-4.
- A. G. Thompson. Optimum damping in a randomly excited non-linear suspension. *Proceedings of the Institution of Mechanical Engineers: Automobile Division*, 184(1):169–184, 1969.
- A. G. Thompson and B. R. Davis. A technical note on the lotus suspension patents. *Vehicle System Dynamics*, 20(6):381–383, 1991. doi: 10.1080/00423119108968998.
- A. G. Thompson and C. E. M. Pearce. Performance index for a preview active suspension applied to a quarter-car model. *Vehicle System Dynamics*, 35(1):55–66, 2001. doi: 10.1076/vesd.35.1.55.5616.
- P. Tlali, R.-J. Wang, and S. Gerber. Magnetic gear technologies: A review. In *International Conference on Electrical Machines*, pages 544–550, Sep. 2014.

- Toyota. 75 years of toyota - vehicle lineage, 2012. URL http://www.toyota-global.com/company/history_of_toyota/75years/vehicle_lineage/car/id60010093/index.html.
- M. N. Tran and D. Hrovat. Application of gain scheduling to design of active suspensions. In *32nd IEEE Conference on Decision and Control (CDC)*, volume 2, pages 1030–1035, Dec. 1993.
- H. E. Tseng and D. Hrovat. State of the art survey: active and semi-active suspension control. *Vehicle System Dynamics*, 0(0):1–29, 2015.
- S. Türkay and H. Akçay. Aspects of achievable performance for quarter-car active suspensions. *Journal of Sound and Vibration*, 311(12):440 – 460, 2008.
- N. Urasaki, T. Senjyu, and K. Uezato. Investigation of influences of various losses on electromagnetic torque for surface-mounted permanent magnet synchronous motors. *IEEE Transactions on Power Electronics*, 18(1): 131–139, Jan. 2003.
- US Department of Transportation. Laboratory test procedure for dynamic rollover: The fishhook maneuver test procedure. Technical report, National Highway Traffic Safety Administration, 1200 New Jersey Avenue SE Washington, DC 20590, 2013.
- A. Vahidi and A. Eskandarian. Influence of preview uncertainties in the preview control of vehicle suspensions. *Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics*, 216(4):295–301, 2002. doi: 10.1243/146441902320992383.
- M. Valasek and M. Novak. Ground hook for semi-active damping of truck’s suspension. In *Proc. of CTU Workshop*, volume 96, pages 467–468, 1996.
- M. Valasek, M. Novak, Z. Sika, and O. Vaculin. Extended ground-hook - new concept of semi-active control of truck’s suspension. *Vehicle System Dynamics*, 27(5-6):289–303, 1997. doi: 10.1080/00423119708969333.
- M. Van De Wal, P. Philips, and B. De Jager. Actuator and sensor selection for an active vehicle suspension aimed at robust performance. *International Journal of Control*, 70(5):703–720, 1998.
- T. van der Sande, B. Gysen, I. Besselink, J. Paulides, E. Lomonova, and H. Nijmeijer. Robust control of an electromagnetic active suspension system: Simulations and measurements. *Mechatronics*, 23(2):204 – 212, 2013.
- O. Wallmark. *On Control of Permanent-Magnet Synchronous Motors in Hybrid-Electric Vehicle Applications*. PhD thesis, Chalmers University of Technology, 2004.
- J. Wang, D. Wilson, and G. Halikias. $\mathcal{H}_\infty$ robust-performance control of decoupled active suspension systems based on LMI method. In *American Control Conference (ACC)*, volume 4, pages 2658–2663, 2001.
- Wikipedia. Brush motor company, 2015a. URL http://en.wikipedia.org/wiki/Brush_Motor_Car_Company.
- Wikipedia. Hydropneumatic suspension, June 2015b. URL https://en.wikipedia.org/wiki/Hydropneumatic_suspension.
- Wikipedia. Lotus 91, 2015c. URL http://fr.wikipedia.org/wiki/Lotus_91.
- Wikipedia. Citroën XM, 2015d. URL http://en.wikipedia.org/wiki/Citro%C3%ABn_XM.
- D. E. Williams and W. M. Haddad. Active suspension control to improve vehicle ride and handling. *Vehicle System Dynamics*, 28(1):1–24, 1997.

- R. A. Williams. Automotive active suspensions part 1: Basic principles. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 211(6):415–426, 1997. doi: 10.1243/0954407971526551.
- G. D. Wood and D. C. Kennedy. Simulating mechanical systems in simulink with simmechanics. Technical report, The Mathworks, 2003.
- X. Xia and J. N. Willis. The effects of tire cornering stiffness on vehicle linear handling performance. In *SAE Technical Paper*. SAE International, 02 1995.
- X. D. Xue, K. W. E. Cheng, Z. Zhang, J. K. Lin, D. H. Wang, Y. J. Bao, M. K. Wong, and N. Cheung. Study of art of automotive active suspensions. In *International Conference on Power Electronics Systems and Applications (PESA)*, pages 1–7, June 2011. doi: 10.1109/PESA.2011.5982958.
- A. Yamamoto, H. Sugai, R. Kanda, and S. Buma. Preview ride comfort control for electric active suspension (eActive3). Technical report, SAE, Detroit, MI, United states, 2014.
- M. Yamashita, K. Fujimori, C. Uhlik, R. Kawatani, and H. Kimura. $\mathcal{H}_\infty$ control of an automotive active suspension. In *29th IEEE Conference on Decision and Control*, volume 4, pages 2244–2250, Dec. 1990.
- M. Yamashita, K. Fujimori, K. Hayakawa, and H. Kimura. Application of $\mathcal{H}_\infty$ control to active suspension systems. *Automatica*, 30(11):1717–1729, 1994. ISSN 0005-1098. doi: 10.1016/0005-1098(94)90074-4.
- Y. Yokoya, K. Asami, T. Hamajima, and N. Nakashima. Toyota electronic modulated suspension (TEMS) system for the 1983 Soarer. In *SAE Technical Paper*. SAE International, Feb. 1984.
- Y. Yokoya, R. Kizu, H. Kawaguchi, K. Ohashi, and H. Ohno. Integrated control system between active control suspension and four wheel steering for the 1989 Celica. In *SAE Technical Paper*. SAE International, Sep. 1990.
- T. Yoshimura, A. Kume, M. Kurimoto, and J. Hino. Construction of an active suspension system of a quarter car model using the concept of sliding mode control. *Journal of Sound and Vibration*, 239(2):187–199, 2001.
- C. Yue, T. Butsuen, and J. K. Hedrick. Alternative control laws for automotive active suspensions. In *American Control Conference (ACC)*, pages 2373–2378, Jun. 1988.
- P. W. A. Zegelaar. *The dynamic response of tyres to brake torque variations and road unevennesses*. PhD thesis, Delft University of Technology, 1998.
- P. W. A. Zegelaar and H. B. Pacejka. The in-plane dynamics of tyres on uneven roads. *Vehicle System Dynamics*, 25(sup1):714–730, 1996.
- Y. Zhang and A. Alleyne. A practical and effective approach to active suspension control. *Vehicle System Dynamics*, 43(5):305–330, 2005. doi: 10.1080/00423110412331282869.
- K. Zhou and J. Doyle. *Essentials of Robust Control*. Prentice Hall Modular Series for Eng. Prentice Hall, 1998. ISBN 9780135258330.

Appendix A

Road profiles

Three types of road profiles have been used to study the behaviour of the SAVGS:

- Random profiles representative of typical roads, which are a realistic benchmark for continuous operation of the active suspension.
- Harmonic road profiles, which are useful for design purposes as they provide clear information regarding the behaviour of the SAVGS at different frequencies.
- Discrete events, such as road bumps and steps, which are a good representation of obstacles that are likely to be encountered when the vehicle travels at low speeds.

A.1 Random road profiles

According to [ISO 8608:1995], the power spectral density (PSD) of typical road profiles may be approximated through

$$G_d(n) = G_d(n_0) \left(\frac{n}{n_0} \right)^{-w}, \quad (\text{A.1})$$

where n is the spatial frequency in cycles per metre, $n_0 = 0.1$ cycles/m is the reference spatial frequency, and w is a constant that is typically equal to 2. [ISO 8608:1995] also provides numeric values representative of typical road qualities. The reference value, $G_d(n_0)$, for these road profiles is given by

$$G_d(n_0) = 10^{-6} \cdot 2^{2k}, \quad (\text{A.2})$$

where $k = 2, 3, \dots, 9$ corresponds to road classes A to H, sorted in ascending order of roughness.

When measuring road profiles, only values corresponding to spatial frequencies in the range

$[n_{\min}, n_{\max}]$ shall be reported. The lower end, $n_{\min}$, is given as 0.01 cycles/m for on-road vehicles, and 0.05 cycles/m for off-road vehicles. The higher end is determined by the enveloping effect of the tyre. Although this effect depends on the actual size and construction of the tyre, a value of 10 cycles/m is recommended for general on-road measurements. The fact that [ISO 8608:1995] limits the spatial frequency of interest to $[n_{\min}, n_{\max}]$ implies that what happens outside this range is not relevant for suspension studies.

There are two approaches to generate random road profiles from the PSD functions provided in [ISO 8608:1995]. The first one corresponds to the integration of a random white noise signal that represents the change of road height with respect to the longitudinal coordinate. In practice, the contribution at very low frequencies is not desirable for suspension studies and therefore a coloured noise source that limits the average power at low frequencies is used instead. This approach was followed in [Cheng et al., 2015] in combination with a geometric filter that emulated the enveloping effect of the tyre (treating it as a rigid ring).

In the second approach, the road profile may be obtained as the addition of a series of harmonic signals with varying amplitudes and spatial frequencies. This method is very simple and provides absolute control regarding the power that the road profile contains at each spatial frequency. The random road height, h , can thus be obtained as a function of the longitudinal coordinate, x , as:

$$h(x) = \sum_{n_i=n_{\min}}^{n_{\max}} A_i \cos(2\pi n_i x + \phi_i), \quad (\text{A.3})$$

where A_i is the amplitude corresponding to the spatial frequency n_i , and ϕ_i is a uniformly distributed random number in the range $(0, 2\pi)$.

Bearing in mind that $G_d(n)$ represents the power spectral *density*, we have

$$G_d(n) = G_d(n_0) \left(\frac{n}{n_0} \right)^{-w} = \lim_{\Delta n \rightarrow 0} \frac{\Psi(n)^2}{\Delta n}, \quad (\text{A.4})$$

with $\Psi(n)^2$ the average power of the signal at a given spatial frequency, and Δn the spatial frequency step used to generate the random road profile. Knowing that the average power contained in a harmonic signal is simply its RMS value squared (i.e. $\Psi_i^2 = A_i^2/2$), we may find the required amplitude for each of the harmonic signals considered in (A.3). For a finite but small spatial frequency step Δn , this can be approximated through:

$$A_i = \sqrt{2 \Delta n G_d(n_i)} = 2^{(k+1/2)} 10^{-3} \sqrt{\Delta n} \left(\frac{n_0}{n_i} \right). \quad (\text{A.5})$$

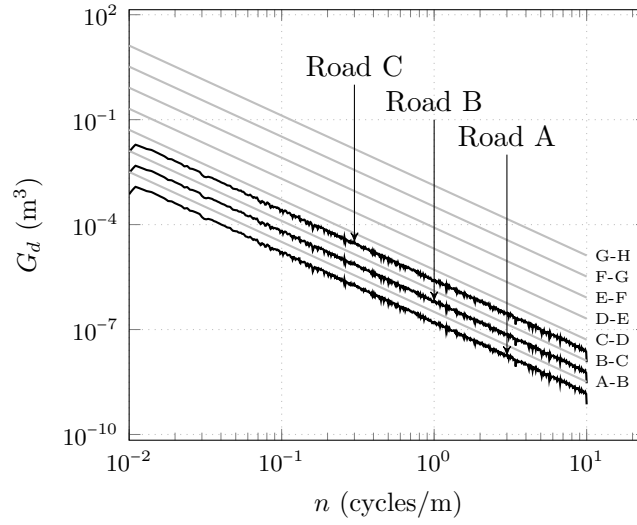


Fig. A.1: Displacement power spectral density, G_d , as a function of spatial frequency n . Divisions between road classes A to H as defined in [ISO 8608:1995] are shown in grey. Roads profiles A to C used in Chapter 6 are shown in black.

A.1.1 Generation of random road profiles

The number of signals to be used in (A.3) is $N = \frac{n_{\max} - n_{\min}}{\Delta n} + 1$, and the width of the frequency band should be $\Delta n \leq 1/L$, where L is the length of the random road segment to be generated. Thus, the spatial frequencies to be considered are $n_i = n_{\min} + (i - 1) \Delta n$ for $i = 1, \dots, N$.

The time domain signal to be used with the one-corner models can be obtained from (A.3) by mapping the longitudinal coordinate to time. Assuming a constant forward speed $v_x = V$:

$$h(t) = h(x = V t), \quad (\text{A.6})$$

and the relationship between spatial (n) and temporal (f) frequencies is:

$$f = n V. \quad (\text{A.7})$$

Finally, the random signal needs to be modified in order to ensure that the first value is not far from zero. Subtracting the initial value from the whole signal would introduce an undesired DC component. Instead, the first zero crossing is found, and all points preceding it are removed.

The PSDs for the random road profiles considered in Chapter 6 are shown in Fig. A.1.

A.2 Harmonic road profiles

Harmonic excitations at specific frequencies are very useful for understanding the system's behaviour. They allow for direct validation of the predicted frequency responses obtained with the linear models, and to assess the level of performance improvement, actuator saturation, etc. as a function of frequency and amplitude. These road profiles were used in the control tuning process, but no results have been included in Chapter 6.

Although any frequency and amplitude can be tested, it makes sense to use values that can be related to the characteristics of typical roads. In order to obtain meaningful road amplitude values, two approaches have been followed:

- To match the average power provided by the random road in a relatively wide spatial frequency band centered at the frequency of interest.
- To match the average power provided by the random road in the frequency range from the frequency of interest to $n = n_2$, where n_2 may be set to $n_{\max}$ or made to tend to infinity.

For the first approach (A.5) gives the required amplitudes. These are inversely proportional to the spatial frequency, and directly proportional to the square root of the frequency band width. Once the band width has been chosen, this is equivalent to using the same *velocity*ⁱ amplitude for all frequencies.

For the second approach, we can see that the average power provided by the random road in the $[n, n_2]$ range, $P_{\text{ave}}(n, n_2)$, is the integral of the PSD within that interval:

$$P_{\text{ave}}(n, n_2) = \int_n^{n_2} G_d(n) \, dn = G_d(n_0) n_0^2 \left(\frac{n_2 - n}{n n_2} \right). \quad (\text{A.8})$$

Equating this to the average power contained in a single harmonic signal at the desired frequency, the required amplitude $A(n, n_2)$ can be computed through (A.9). In this case, the amplitude is inversely proportional to the square root of the spatial frequency.

$$A(n, n_2) = 2^{(k+1/2)} 10^{-4} \sqrt{\frac{n_2 - n}{n n_2}} \stackrel{n_2 \rightarrow \infty}{\approx} 2^{(k+1/2)} 10^{-4} \frac{1}{\sqrt{n}}. \quad (\text{A.9})$$

A.3 Discrete events

Two types of discrete events have been used:

- Bump: smoothed profile that could represent a speed bump.
- Step: sharp step down that could represent the event of driving down from a sidewalk.

ⁱUnderstood as the rate of change of road height with respect to the longitudinal coordinate.

Appendix B

Full-car hand-derived model for control synthesis

As in the quarter-car case, the implementation of the SL rotation through equivalent linear displacement actuators requires the definition of an additional state and an additional input for each corner. Taking $\mathbf{x}_p$ and $\mathbf{u}_p$ as the vectors with state variables and inputs for the passive full-vehicle model given in (3.10) and (3.19), respectively, the vectors with state variables and inputs for the active case are simply:

$$\begin{aligned}\mathbf{x}^T &= [\mathbf{x}_p^T, z_{\text{lin}_1}, z_{\text{lin}_2}, z_{\text{lin}_3}, z_{\text{lin}_4}] \\ \mathbf{u}^T &= [\mathbf{u}_p^T, \dot{z}_{\text{lin}_1}, \dot{z}_{\text{lin}_2}, \dot{z}_{\text{lin}_3}, \dot{z}_{\text{lin}_4}]\end{aligned}\tag{B.1}$$

The new terms in the $A \in \mathbb{R}^{19 \times 19}$ and $B \in \mathbb{R}^{19 \times 10}$ state matrices are:

Matrix A

$$\begin{aligned}a_{i,15+j} &= -a_{i,7+j}, & \text{for } i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4 \\ a_{3+i,15+i} &= -a_{3+i,7+i}, & \text{for } i = 1, 2, 3, 4 \\ a_{15+i,j} &= 0, & \text{for } i = 1, 2, 3, 4 \text{ and } j = 1, 2, \dots, 19\end{aligned}\tag{B.2}$$

Matrix B

$$\begin{aligned}
 b_{1,7} = b_{1,8} &= -\frac{c_f}{m_s}, & b_{1,9} = b_{1,10} &= -\frac{c_r}{m_s} \\
 b_{2,7} = b_{2,8} &= a_f \frac{c_f}{I_{yy}}, & b_{2,9} = b_{2,10} &= -a_r \frac{c_r}{I_{yy}} \\
 b_{3,7} = -b_{3,8} &= t_f \frac{c_f}{2I_{xx}}, & b_{3,9} = -b_{3,10} &= t_r \frac{c_r}{2I_{xx}} \\
 b_{4,7} = b_{5,8} &= \frac{c_f}{m_{u_f}}, & b_{6,9} = b_{7,10} &= \frac{c_r}{m_{u_r}} \\
 b_{16,7} = b_{17,8} &= 1, & b_{18,9} = b_{19,10} &= 1
 \end{aligned} \tag{B.3}$$

Once the output variables have been selected, the new C and D matrices can be found in a similar way.

Appendix C

Uncertainties to be considered in the control synthesis

Two types of uncertainties have been included in the control synthesis stage:

- Parametric uncertainty in the damping coefficient of the passive SD
- Unstructured uncertainty to capture changes in the actuator and actuator control dynamics

C.1 Uncertain damping coefficient

The importance of the nonlinearity in the damper force vs. damper extension rate characteristic has already been highlighted in Section 6.1.1. The variability in damping coefficient can be incorporated in MATLAB by defining the damping coefficient as an uncertain real parameter. This can be done through function `ureal`:

```
c = ureal('c',cnom,'Percentage',100*[(cmin-cnomo)/cnomo,(cmax-cnomo)/cnomo]);
```

`cnom` could correspond to the value that provides the *best* match of the nonlinear frequency response for the expected range of disturbance levels. `cmin` and `cmax` would correspond to the minimum and maximum slopes in the damper force vs. damper extension curves.

C.2 Uncertain inner loop dynamics

As already discussed in Section 6.2, the control command, $\dot{z}_{lin}^*$, cannot be perfectly tracked by the actuators in each corner of the vehicle and the tracking dynamics are very nonlinear. One way of incorporating the uncertainty in the system response to the synthesis framework could

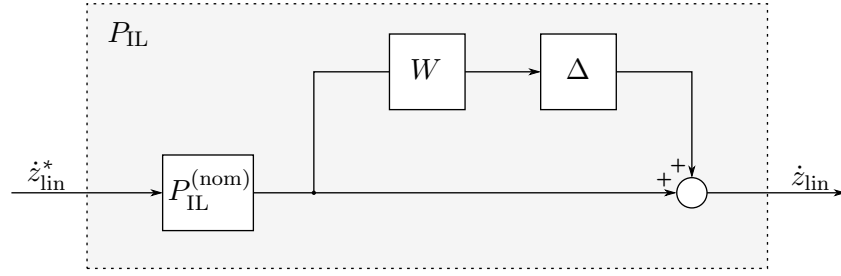


Fig. C.1: Output multiplicative uncertainty for the inner-loop dynamics block.

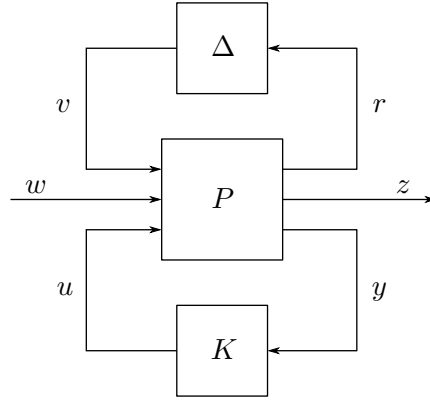


Fig. C.2: General framework for analysis and control synthesis.

be through an output multiplicative uncertainty:

$$P_{\text{IL}} = (1 + \Delta W) P_{\text{IL}}^{(\text{nom})}, \quad (\text{C.1})$$

with Δ and W stable transfer functions, and $\|\Delta\|_{\infty} \leq 1$. This is shown in Fig. C.1.

Useful MATLAB commands to approximate a range of system responses, such as the ones shown in Fig. 6.17, by a multiplicative uncertainty model can be found in [Zhou and Doyle, 1998].

C.3 μ -synthesis

The uncertainties mentioned in the previous sections can be included in the control synthesis framework (see Fig. C.2). MATLAB's `dksyn` function applies the D-K algorithm (an approximation to μ -synthesis control design) to synthesise a robust controller that minimises the structured singular value, μ_{Δ} , of $M = \mathcal{F}_l(P, K)$ [Zhou and Doyle, 1998]:

$$\mu_{\Delta(M)} := \frac{1}{\min[\bar{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(I - M\Delta) = 0]}, \quad (\text{C.2})$$

with $\mathbf{\Delta}$ representing block diagonal matrices.

When tested in the nonlinear multi-body models, controllers synthesised with **dksyn** were not found to provide better performance than those obtained with **hinfosyn** and the nominal plant model. However, there is scope for improvement as more time could be spent fine tuning frequency weights and interacting with the D-K algorithm.