

Vibration behaviours of single/multi-debonded curved composite sandwich structures

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Abstract

Sandwich structures, which are light but have high strength, are extensively used in automobile, marine and aircraft structures. However, core/facesheet debonding can occur and reduce the stiffness of the structures and hence result in the failure of the structures, whilst affecting the vibration behaviours. Hammer impact tests and finite element simulations were conducted on flat and curved sandwich structures, which were composed of carbon fibre reinforced polymer (CFRP) facesheets and epoxy cores, to analyse how the vibration behaviours were affected by debonding and other factors. The 15°- and 30°-curved structures showed only slight differences in the natural frequencies of bending and torsional modes. However, significant difference in the lateral modes were found as the mode shape merged with torsional movement and hence the natural frequencies increase. Debonding resulted in the reduction in the natural frequencies, an 80 mm debonded region reduced the natural frequency of the flat and 30°-curved structures by 57.0% and 56.6%, respectively. Change of natural frequencies can make a structure resonate and lead to failure, therefore how the debonded regions affect the vibration of sandwich structures is important for applications.

Keywords: Sandwich structure, Vibration, Curvature, Debonded region, CFRP

1. Introduction

Sandwich structures, which are manufactured by adhering two thin but strong facesheets to a core, are lightweight but have high stiffness and resistance to flexural loading, have

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25 been widely used in shipbuilding, automotive and aerospace industries. Light materials, e.g.
26 honeycomb panels or polymers are often chosen as the core; strong materials which have
27 high stiffness, such as carbon fibre reinforced polymer (CFRP) are used as the facesheets
28 [1]. The adhesive layers between the facesheets and the core of sandwich structures have a
29 significant influence on the stiffness and failure of sandwich structures [2], as this is where
30 the early-stage damage often initiates.

31 Thermosetting epoxy polymers are widely used as the matrices of the FRPs because of
32 their good thermal properties, chemical resistance and low creep [3]. Modifiers such as silica
33 nanoparticles (SNPs) [4, 5], graphene [6, 7] or carbon nanotubes (CNTs) [5–7] have been
34 used to modify the matrices of FRPs. The particles should be small enough to flow between
35 the fibres if manufacturing by resin infusion methods [8]. The tensile modulus and thermal
36 conductivity of the CFRP can be increased by 11% and 45% respectively by adding 4 wt%
37 graphene into the matrix [9]. The fatigue life can be extended by 3 to 4 times by adding 10
38 wt% of SNPs into the matrix epoxy [10].

39 Vibration is an important issue for the safety of practical applications as damage and fail-
40 ure of the structures can be induced by resonance. Various approaches to predict the natural
41 frequencies and deformation mode shapes of sandwich structures have been developed us-
42 ing finite element analysis and other techniques such as the finite strip method [11–13]. The
43 vibration behaviours can be significantly affected by the geometry and the materials of the
44 sandwich structures [14]. The damping properties can be increased because the loss factor
45 is raised by the increase in the thickness of the core. As continuous-fibre reinforced poly-
46 mers are anisotropic materials, changing the facesheet stacking orientations of the sandwich
47 structures changes the stiffness of the structure and hence the natural frequencies are altered.

48 Bonding the facesheets and the core together firmly is important to prevent debonding of
49 sandwich structures. Debonding can occur because of manufacturing defects and in-service
50 damage, e.g. unexpected impact from external objects like stones, can lead to crack propa-
51 gation, reduction in the stiffness and strength, and subsequent failure of the structures. The
52 bending rigidity of sandwich structures decreases when the delamination length increases
53 [15].

54 Debonding can also have a significant influence on the vibration behaviours of sandwich
55 structures. The size and location of debonded regions, shape and boundary conditions of the
56 structures and the vibration modes can affect the vibration behaviours of debonded structures
57 [16–18]. The natural frequencies of sandwich structures decrease more when the length of
58 the debonded region increases [15, 19]. The natural frequencies of the beam can decrease fur-
59 ther if multiple debonded regions are located at the same place [19]. The damping increases
60 as the damage area increases [20]. The curvature of the structures affects the stiffness of the
61 structures and hence results in the change of natural frequencies [17, 18]. Large displacement
62 and failure of structures can happen because resonance can be excited at lower natural fre-
63 quencies more easily when the structures are in service. Therefore lower natural frequencies
64 need to be avoided to prevent failure of structures.

65 Vibration of sandwich structures is critical for practical applications to avoid resonance
66 and failure of the structures. In the present work, the results of hammer impact tests and finite
67 element simulation are compared. This work investigates the vibration behaviours of flat and
68 curved sandwich structures with single or double debonded regions of different lengths and
69 arrangements. The effects of facesheet CFRPs stacking orientations and the material change,
70 which was achieved by adding modifiers into the epoxy polymer used for the core and the
71 matrix of the facesheets, are also discussed. The effects of these factors when combined have
72 been rarely considered in the literature. In this work, how the natural frequencies of sandwich
73 structures are affected by these factors are presented and discussed. The aim is to investigate
74 the effects of these factors on the vibration behaviours of sandwich structures.

75 **2. Materials and specimen manufacturing**

76 The epoxy system used was a diglycidyl ether of bis-phenol A epoxy resin, Araldite LY
77 556 [21] from Huntsman, UK, and an accelerated methylhexahydrophthalic acid anhydride,
78 Albidur HE 600 [22], supplied by Evonik, Germany. The fibre used was the unidirectional
79 (UD) carbon fibre fabric UT-C300 [23] manufactured by Gurit and supplied by Marineware,
80 UK. To achieve material change, two modifiers were used: Graphene nanoplatelets, xGnP

81 M-25, supplied by XG Sciences, USA and silica nanoparticles (SNPs), supplied as a master-
82 batch, Nanopox F400 [24] from Evonik, Germany.

83 The manufacturing process of the sandwich structures followed the reported process
84 [25, 26]: To manufacture the core, the required amounts of the epoxy resin, hardener and
85 modifiers were calculated and weighed. The graphene, as it increased the viscosity of the
86 resin significantly, was mixed with the epoxy using a 3 roll mill Exakt 80E from Exakt, Ger-
87 many to achieve a better degree of dispersion. The SNPs were mixed with the epoxy using
88 an overhead stirrer RZR 2010 from Heidolph, Germany. The mixture was then degassed,
89 poured into the flat or curved moulds and cured in the oven at 90 °C for 1 hour and post-cured
90 at 160 °C for 2 hours. To manufacture the facesheet CFRP, the required orientation angles
91 and layers of UT-C300 were cut. Resin infusion under flexible tooling (RIFT) was then used
92 to manufacture the laminate. The resin formulation, preparation and the RIFT curing cy-
93 cle were the same as for manufacturing the core epoxy. The manufactured core epoxy and
94 facesheet CFRP were cut to the desired sizes using a table saw. The core and the facesheets
95 were bonded together using a two-component epoxy adhesive, Araldite 2014-1 [27] from
96 Huntsman, Switzerland. The stacked sandwich structures were heated in an oven at 50 °C for
97 2 hours to cure the adhesive. The curvature angles of the sandwich structures were flat, 15°
98 and 30°. Other dimensions were: 200 mm (midplane) × 25 mm × 8.6 mm (length × width ×
99 thickness). The thickness of the core and the facesheets were 6 mm and 1.3 mm respectively.

100 To form debonded regions within the sandwich structures, polytetrafluoroethylene (PTFE)
101 films, which were about 20 µm thick, with the required width were placed at the desired
102 places between the facesheets and the cores when bonding them. After curing, the PTFE
103 films were pulled out.

104 3. Experimental

105 The mechanical properties of the materials used in this work are reported elsewhere [25,
106 26]. The values were averaged and used in the finite element (FE) simulations, see Table 1.

107 Hammer impact tests were conducted to find the natural frequencies and the mode shapes.
108 The hammer and the accelerometer were from PCB Piezotronics and had sensitivities of 22.5

109 mV/N [28] and 0.981 mV/m/s^2 (calibrated, standard value was 1.02 mV/m/s^2 [29]) respec-
110 tively. To reduce the noise level, a soft cap was fitted onto the hammer tip. The portable
111 signal acquisition card SignalCalc Ace and the analysis software SignalCalc 240 version
112 5.0.346 used were from Data Physics Corporation, USA. Fast Fourier transform (FFT) anal-
113 ysis was used to transfer the time-domain signals to frequency-domain responses, to obtain
114 the amplitude and phase corresponding to frequency.

115 The boundary conditions were free-free: the samples were hung horizontally with two
116 strings. To measure the frequency responses of the sandwich structures, the accelerometer
117 was adhered at a fixed position and the hammer was tapped at different points. Each point
118 was tapped by the hammer at least twice. The coherence of the response was checked, only
119 when the coherence of the frequency ranges at the peaks of amplitude were good would the
120 results be taken into averaging and recorded.

121 **4. Finite element simulation**

122 Three-dimensional finite element (FE) simulations were performed using Abaqus 6.14
123 [30] from Dassault Systemes, France. The geometries of the models are shown in Fig. 1. The
124 flat models were developed in Abaqus [25, 26], while the curved models were constructed in
125 SOLIDWORKS 2015 sp4.0 [31] from Dassault Systemes, France, and then transferred into
126 Abaqus. The models were composed of a core and two 4-layer CFRP facesheets, using 8-
127 node brick elements, C3D8. The material properties are shown in Table 1. The effect of fibre
128 direction of each layer of the facesheets in the real structures was simulated by assigning a
129 “material orientation” to each layer according to the facesheet stacking orientations ($[0^\circ]_4$,
130 $[90^\circ]_4$ and $[+45^\circ/-45^\circ]_s$).

131 The core and facesheets were adhered using “tie constraints” at the bonded faces (i.e.
132 excluding the debonded regions). To prevent overlapping of the facesheets and the core at
133 the debonded regions in the simulations, “symmetric surface to surface contact” was applied
134 at the interfaces of the debonded regions. To achieve the free-free boundary conditions, the
135 models were assigned with no constraint. The natural frequencies and their corresponding
136 mode shapes of the structures were calculated in Abaqus using the Lanczos method. The

frequency range was set as 100 to 10000 Hz, with the range of 0 to 100 Hz blocked out to avoid rigid body motions of the structures.

A convergence study was undertaken on the intact, flat sandwich model to find the optimum mesh density. The accepted mesh used 60000 elements (20000 for each facesheet and 20000 for the core) for good convergence and saving of computational time, and was used for the subsequent simulations of other sandwich structures.

5. Results and discussion

5.1. Introduction

Hammer impact tests and finite element simulations using Abaqus [30] were conducted on the flat and curved sandwich structures to analyse the vibration behaviours of the structures. The effects of the debonding length (for single debond), facesheet CFRP stacking orientation and material change (affected by using different modifiers in the resin and matrix) on the vibration behaviours of 15°-curved and 30°-curved sandwich structures are presented and discussed in this section. The natural frequencies are normalized to the natural frequency for the same mode of the non-debonded flat structures [25, 26].

5.2. Bending stiffness matrices of composite structures

Since natural frequencies are positively correlated to the structure's stiffness, the bending stiffness matrices, D , of the flat and intact sandwich structures using the unmodified resin can be calculated [25, 26] according to the mechanics of composite materials [32]:

$$D_0 = \begin{bmatrix} 4373.14 & 97.56 & 0 \\ 97.56 & 292.20 & 0 \\ 0 & 0 & 175.22 \end{bmatrix} (\text{GPa}\cdot\text{mm}^3) \quad (1)$$

where the subscript “0” means using $[0^\circ]_4$ facesheets. The bending stiffness matrices of the structures using $[90^\circ]_4$ or $[+45^\circ/-45^\circ]_s$ facesheets are as follows:

$$D_{90} = \begin{bmatrix} 292.20 & 97.56 & 0 \\ 97.56 & 4373.14 & 0 \\ 0 & 0 & 175.22 \end{bmatrix} (\text{GPa}\cdot\text{mm}^3) \quad (2)$$

158

$$D_{\pm 45} = \begin{bmatrix} 1390.33 & 1039.89 & 8.00 \\ 1039.89 & 1390.33 & 8.00 \\ 8.00 & 8.00 & 1117.56 \end{bmatrix} (\text{GPa}\cdot\text{mm}^3) \quad (3)$$

159 According to Equations (1) to (3), it is clear that D_0 has the highest value of d_{11} of 4373.14
 160 $\text{GPa}\cdot\text{mm}^3$, which means that the structure using $[0^\circ]_4$ facesheets has the highest bending stiff-
 161 ness. This will result in the highest natural frequencies of the bending modes of the sandwich
 162 structures using $[0^\circ]_4$ facesheets. While for D_{90} , the value of d_{11} is 292.20 $\text{GPa}\cdot\text{mm}^3$, which
 163 is the lowest within the three types of structures and hence results in the lowest bending
 164 stiffness of the structure using $[90^\circ]_4$ facesheets. This will result in the lowest natural fre-
 165 quencies of the bending modes of the sandwich structures. For $D_{\pm 45}$, it has the highest value
 166 of d_{33} of 1117.56 $\text{GPa}\cdot\text{mm}^3$, which means that the structure using $[+45^\circ/-45^\circ]_s$ facesheets has
 167 the highest torsional stiffness and hence will result in the highest natural frequencies of the
 168 torsion modes.

169 5.3. Comparisons of simulation and experimental results

170 The simulated and experimental natural frequencies and mode shapes of the first five
 171 modes of the intact, 30° -curved sandwich structures using the unmodified epoxy resin, $[0^\circ]_4$
 172 and $[90^\circ]_4$ facesheets are shown in Table 2, those of the 80-mm-debonded structures are
 173 shown in Table 3. The differences between the simulation and experimental results are gen-
 174 erally less than the published values of typically about 10%. In addition, the simulation and
 175 experimental frequencies show the same trends, so it is considered that the agreement is good.

176 5.4. Effects of curvature on the intact sandwich structures

177 The simulated mode shapes of the 30° -curved sandwich structure using $[0^\circ]_4$ facesheets
 178 are shown in Fig. 2. The relative natural frequencies of the curved sandwich structures
 179 compared to the intact, flat structures are shown in Fig. 3. The natural frequencies of the
 180 bending modes decrease when the sandwich structures become more curved, i.e. from flat to
 181 30° -curved. For the structures using the 3 types of facesheets, the reductions in the natural
 182 frequencies of bending modes are very close. According to the mechanics of materials [33],

183 when a beam structure becomes curved, the neutral axis moves down towards the curvature
184 centres. Also, it is easier to bend the structure, which means that the bending stiffness of
185 the structure decreases, so the natural frequencies of bending modes decrease. However, the
186 reductions are small compared to the effects of a debonded region on the structures (discussed
187 in Section 5.5).

188 The effects of curvature on the natural frequencies of the torsion modes are shown in
189 Figs. 3d to 3f. Although the natural frequency of the 2nd torsion mode of the structure using
190 the $[0^\circ]_4$ facesheets increases, there is generally a decreasing trend in the natural frequencies
191 of the torsion modes when increasing the curvature when the 1st torsion mode and the other
192 facesheet stacking orientations are considered. The natural frequencies of the torsional modes
193 of the structure using the $[+45^\circ/-45^\circ]_s$ facesheets decrease the most significantly. The reason
194 for the greatest reduction being caused by the $[+45^\circ/-45^\circ]_s$ facesheets is that increasing the
195 curvature results in a greater reduction in the torsional stiffness of the structure using the
196 $[+45^\circ/-45^\circ]_s$ facesheets, therefore the natural frequencies of the torsion modes of the structure
197 decrease the most.

198 The bending and torsional modes of the curved structures are independent of each other,
199 see Figure 2. However, the lateral modes are significantly coupled with torsional movement.
200 The trend in the natural frequencies of the lateral modes is opposite to that of the torsion
201 modes: there is generally an increasing trend in the natural frequencies of the lateral modes
202 with increasing curvature. The natural frequencies for the structure using the $[+45^\circ/-45^\circ]_s$
203 facesheets increase the most significantly. Since the structures are curved, the mode shapes of
204 the lateral modes become combined with torsional movement rather than having pure lateral
205 movement (see Figs. 2c and 2f), which makes the structure stiffer and hence the natural
206 frequencies increase. The natural frequencies of the torsion-lateral modes of the structure
207 using the $[+45^\circ/-45^\circ]_s$ facesheets increase the most because $[+45^\circ/-45^\circ]_s$ provides the most
208 torsional stiffness to the structures.

209 According to Fig. 3, it is hard to distinguish whether a flat beam structure has become
210 slightly curved or not in practical applications by measuring the resonant frequencies of the
211 bending modes since the difference is not significant. However, measuring the resonant fre-

quencies of the lateral modes has the potential to detect whether a structure has become curved since the difference in the lateral modes is more significant.

5.5. Effects of debonding length

The effects of curvature and a central debonded region of different lengths on the natural frequencies of the first three bending modes are shown in Fig. 4. Increasing the length of the debonded region reduces the stiffness of the structures, reducing the natural frequencies. For the flat structure using the $[0^\circ]_4$ facesheets and the unmodified resin, when there is an 80-mm debonded region, the natural frequencies of the first three bending modes decrease by 6.3%, 57.0% and 37.5%, respectively; while for the 30° -curved structure with the same configuration, the frequencies of the first three bending modes decrease by 6.9%, 56.6% and 37%, respectively. Therefore, the curvature angle of 30° increases the reduction in the natural frequency of the 1st bending mode by 0.6% but mitigates the reductions in the natural frequencies of the 2nd and 3rd bending modes by 0.4% and 0.5%. The reason for the lesser reductions is that the curved structure is less stiff than the flat structure, as discussed in Section 5.4. Although the natural frequencies drop due to the debonded region, the frequencies drop less for the less-stiff structures (i.e. the curved structures) than for the stiff structure (i.e. the flat structure). However, compared to the effect of debonding length, the effect of curvature on the natural frequencies of the bending modes of the debonded sandwich structures is relatively small, as can be seen in Fig. 4.

The effects of curvature on the natural frequencies of the torsion and lateral modes of the single-debonded sandwich structures are shown in Fig. 5. Although for the same debonding length, increasing the curvature results in an increase in the natural frequencies of the 2nd torsion mode of the structures using the $[0^\circ]_4$ facesheets (see Fig. 5d), for all the three types of facesheet stacking orientations, the natural frequencies of torsion modes drop when increasing the debonding length.

For the lateral mode, the natural frequencies are not significantly affected by the horizontal debonded region since the debonded region has less effect on the stiffness in the lateral direction. However, for the curved structures, it is noted that the reduction in the natural fre-

quencies of the lateral mode is more significant than for the flat structure. For the structures using the $[0^\circ]_4$ facesheets, when there is an 80-mm debond, the frequency drops by only 0.9% for the flat structure, while it drops by 2.0% for the 30° -curved structure, see Fig. 5g. This is because the torsion-lateral mode shape caused by increasing the curvature has torsional movement rather than pure lateral movement, as discussed in Section 5.4. Since the torsional movement is involved in the mode shape, the reduction in the torsional stiffness caused by the debonded region decreases the natural frequency of the torsion-lateral mode.

To detect damage and bending deformation in the flat structures, since the curvature has less effect on the natural frequencies of bending modes, measuring the resonant frequencies of both the 2nd bending mode (for identifying damage) and lateral modes (for identifying bending deformation), then comparing with those of the intact, flat structure, would be more useful than just measuring the resonant frequency of the 2nd bending mode.

5.6. Effects of CFRP stacking orientation

Comparing the same bending modes (same colour/row) in Fig. 4, it is clear that structures using the $[0^\circ]_4$ facesheets have the greatest frequency reductions for the flat and curved structures. Although the $[0^\circ]_4$ stacking provides the highest flexural stiffness to the sandwich structure, the stiffness also decreases the most when a debonded region exists in the structure. Therefore the natural frequencies of the bending modes of the structures using the $[0^\circ]_4$ facesheets drop the most significantly [25, 26]. The same effect is also found in the torsion modes, the structures using the $[+45^\circ/-45^\circ]_s$ facesheets have the most significant drops within the same torsion mode and curvature (see the torsion modes, red and pink bars, in Fig. 5). This is because the $[+45^\circ/-45^\circ]_s$ facesheet provides the highest torsional stiffness, but the stiffness also decreases the most when the debonded region exists, which results in the greatest reductions in the natural frequencies of the torsion modes. It is also noted that the effect of facesheet stacking orientation is more significant than that of the curvature.

5.7. Effects of double debonding

The effects of curvature and double debonded regions with different configurations on the natural frequencies of the sandwich structures are shown in Figs. 6 and 7. It is clear that

different curvatures show the same trend for the same mode under different debonding conditions. Also, as discussed before, the structures using the $[0^\circ]_4$ facesheets have the greatest reductions in the natural frequencies of bending modes, see Fig. 6; while the structures using the $[+45^\circ/-45^\circ]_s$ facesheets have the greatest reductions in the natural frequencies of torsion modes, see Fig. 7.

5.7.1. Effects of Increasing the Separation (Δ) between Two Debonds

The changes of the natural frequencies do not follow a certain pattern when increasing the distance between the two 40-mm debonded regions, i.e. $\Delta 0$ (central 80-mm debond), $\Delta 20$, and $\Delta 40$ (see Figs. 1b to 1d), since the positions of the debonds and of the vibration nodes also affect the natural frequencies. The positions of the vibration nodes of the intact curved structures can be seen in Fig. 2. For the 1st bending mode, increasing the separation between the two debonds makes them just overlap ($\Delta 20$) or fully overlap the nodes ($\Delta 40$) and hence allows relative movement of the facesheets and cores as they are not bonded together at the nodes, which reduces the stiffness more, and hence the natural frequency of the 1st bending mode decreases more significantly. For the 2nd bending modes, the two debonds move away from the central vibration node and do not overlap the other two nodes when the separation increases. Since the facesheet and the core are bonded together at the nodes, relative movement is not allowed. Therefore, the reduction in the natural frequency of the 2nd bending mode is smaller than for the 80-mm-debonded structure.

According to Fig. 6, the curvature also causes differences in the natural frequencies when increasing the distance between the two debonds. For condition $\Delta 40$, increasing the curvature slightly decreases the natural frequencies of the 1st and 2nd bending modes. However, the effect is small when compared to the effects of separation length.

5.7.2. Effects of the Positions of Two Debonds

The configurations of the structures having two debonded regions on the same side with a separation of 20 mm ($\Delta 20$), at diagonal (D) or symmetric (S) locations can be seen in Figs. 1c, 1e and 1f. According to Fig. 6, like the flat structures, the natural frequencies of the lower bending modes of the curved structures with debonding condition $\Delta 20$ and D are very close.

Therefore, it is hard to distinguish these two debonding conditions by only measuring the resonant frequencies in practical applications if detection of damage is required.

Comparing condition S to conditions $\Delta 20$ and D, for the flat and curved structures using the $[0^\circ]_4$ facesheets, it is clear that the debonded regions of condition S result in a greater reduction in the natural frequencies of bending modes, see Figs. 6a and 6d. This is because of a greater reduction in the stiffness caused by two debonded regions being located at the same places for the upper and lower facesheets [19].

For the curved structures, the debonded regions of condition S make the structure asymmetric, which affects the mode shapes and the node positions (see Fig. 8). Comparing Fig. 8b to Fig. 2a, the left node of the 1st bending mode moves to the right and is overlapped by the debonded regions, while the right node has no significant movement. For the 2nd bending mode, the left node (in Fig. 8d compared to Fig. 2d) has little movement, the central node moves to the left and is overlapped by the debonded regions, and the right node moves to the right. For the 1st torsion mode, the node moves to the left and becomes overlapped by the debonded regions (in Fig. 8c compared to Fig. 2b). As discussed before, relative movement of the facesheets and the core at the vibration nodes occurs when the debonded regions cover the vibration nodes, which results in a greater reduction in the natural frequencies.

Comparing the natural frequencies of the flat and curved sandwich structures having debonded regions of condition $\Delta 20$, D and S, although increasing the curvature to 30° further decreases the natural frequencies of the bending modes of the symmetrically-debonded structures, the effect is not as significant as the reduction in the stiffness caused by two debonded regions being located at the same places (i.e. condition S).

5.8. Effects of material change

To achieve changes of materials, silica nanoparticles and graphene were added into the epoxy for the core and the facesheet CFRPs.

The curved sandwich structures used in this study are not highly curved (i.e. the ratio of curvature radius to thickness is larger than five [33]), so the $\sqrt{E_{1,\text{flex}}/\rho_{\text{sand}}}$ values of the flat sandwich structures can also be applied to the 15° - and 30° -curved sandwich structures,

where $E_{1,\text{flex}}$ is the flexural modulus and ρ_{sand} is the effective density of the sandwich structures using the different modifiers. $E_{1,\text{flex}}$ was calculated according to the mechanics of composite materials [32]; ρ_{sand} was calculated using the rule of mixtures [32].

Graphs of the natural frequencies of the bending modes of the curved structures versus the $\sqrt{E_{1,\text{flex}}/\rho_{\text{sand}}}$ are shown in Fig. 9, and show linear relationships. The values of the gradient m_f , which describes the sensitivity of the natural frequency to the change of materials, of the 1st bending mode of the flat, 15°- and 30°-curved sandwich structures using the unmodified resin and $[0^\circ]_4$ facesheets are 6.11×10^3 , 6.10×10^3 and 6.05×10^3 Hz/(GPa/kg/m³)^{0.5}, respectively; those of the 2nd bending mode are 12.81×10^3 Hz/(GPa/kg/m³)^{0.5} for the flat and 15°-curved structures and 12.75×10^3 Hz/(GPa/kg/m³)^{0.5} for the 30°-curved structure. Therefore, the 2nd bending mode is more sensitive to material change than the 1st bending mode. Moreover, increasing the curvature decreases m_f , which indicates that increasing the curvature makes the natural frequencies of the bending modes less sensitive to material changes. This is because increasing the curvature reduces the bending stiffness and hence reduces the natural frequencies of bending modes, see Section 5.4, which makes the natural frequencies of the bending modes of the curved structures less sensitive to material change. By combining the data for all the three types of facesheet stacking, the plots still show the same linearity for the same mode of the flat or curved structures, see Fig. 10. This indicates that the natural frequencies of bending modes are proportional to the square root of the structure's flexural modulus ($E_{1,\text{flex}}$), which corresponds to the principles of vibration mechanics of beams [34].

5.8.1. Effects of Material Change on the Single-debonded Sandwich Structures

According to Fig. 11, increasing the debonding length reduces the gradient m_f , which indicates that the natural frequencies become less sensitive to material change. For example, the m_f values of the 2nd bending mode of the same structures decrease from 12.81×10^3 and 12.75×10^3 Hz/(GPa/kg/m³)^{0.5} to 4.14×10^3 and 4.18×10^3 Hz/(GPa/kg/m³)^{0.5}, respectively.

5.8.2. Effects of Material Change on the Double-debonded Sandwich Structures

The m_f of the double-debonded, flat and curved sandwich structures are shown in Fig. 11. Increasing the separation between the two debonded regions ($\Delta 0$ (80 mm), $\Delta 20$, and $\Delta 40$, see Fig. 11) reduces the m_f of the 1st bending mode while it increases that of the 2nd bending mode. The reason for the different trends is the relative locations of the vibration nodes and the debonded regions, see Section 5.7.1. For the sandwich structures with two debonded regions of configuration $\Delta 20$, D and S (see Fig. 11), the m_f values are close for the 1st and 2nd bending modes of $\Delta 20$ and D, this results from the close natural frequencies of the two configurations. For condition S, the values of m_f drop more than for conditions $\Delta 20$ and D, this is because of the greater reduction in the stiffness and natural frequencies caused by the debonding configuration, which significantly reduces the effects of material change on the natural frequencies and hence decreases the m_f values.

6. Conclusions

Delamination in sandwich structures can lead to the reduction in the stiffness and strength, which affects the vibration behaviours. This work has discussed the effects of the structure's curvature, single and double face/core debonding, facesheet stacking orientation and material (achieved by adding modifiers into the resin). Generally, the results of the curved structures show the same trends as the flat structures for the same factors.

Increasing the curvature of the intact sandwich structures reduces the natural frequencies of the bending and torsion modes, since the stiffness decreases. For the lateral modes, there is an increase in the natural frequencies when increasing the curvature, since the mode shapes are merged with torsional movement and hence the stiffness increases.

For the single-debonded structures, increasing the curvature angle slightly mitigates the reduction of the same mode. The curved structure is less stiff than the flat, therefore the frequencies drop less for the less-stiff structures (i.e. curved structures) than for the stiff structure (i.e. flat structure). For the lateral mode, the reduction in the torsional stiffness caused by the debond decreases the natural frequency of the torsion-lateral mode more significantly than for the pure lateral mode of the flat structure.

378 The reduction in natural frequencies of bending/torsion modes is more significant for the
379 structure using $[0^\circ]_4/[+45^\circ/-45^\circ]_s$ facesheets. The highest flexural/torsional stiffness of the
380 sandwich structures results in the highest bending/torsion natural frequencies. However, the
381 stiffness also decreases the most when there is a debonded region.

382 For the double-debonded sandwich structures, increasing the separation between the two
383 debonds moves the vibration nodes and hence results in a different pattern in the change of
384 the natural frequencies. On the other hand, condition S results in a greater reduction in the
385 natural frequencies of bending modes since the stiffness decreases more as debonded regions
386 are at the same locations for the both upper and lower facesheets.

387 The gradient m_f describes the the sensitivity of the natural frequency to the change of
388 materials. Increasing the curvature decreases m_f , indicating that the natural frequencies of
389 the bending modes become less sensitive to material changes, as the bending stiffness of the
390 structures reduces.

391 The most significant effects of the curvature exist in the lateral modes, since the mode
392 shapes of the curved structures involve torsional movement and hence affect the natural fre-
393 quencies. A longer debond or two debonds with configuration S also result in a greater
394 reduction in the natural frequencies. This work has shown how the natural frequencies of
395 sandwich structures are affected by curvature and debonding, enabling significant decreases,
396 which may lead to catastrophic damage due to resonance, to be avoided in structural applica-
397 tions

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405 **Appendix A. Supplementary data**

406 The raw and processed data required to reproduce these findings cannot be shared at this
407 time as the data also forms part of an ongoing study.

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Table 1

Materials and mechanical properties [25, 26]

	Bulk epoxy			CFRP of 0° lay-up						
	ρ (kg/m ³)	E (GPa)	ν	ρ (kg/m ³)	E_1 (GPa)	E_2 (GPa)	ν_{12}	ν_{23}	G_{12} (GPa)	G_{23} (GPa)
unmodified	1154	2.8	0.35	1557	122.6	6.7	0.33	0.48	4.5	2.3
5 wt% SNPs	1163	3.3	0.37	1527	139.3	6.5	0.37	0.58	4.0	2.1
10 wt% SNPs	1174	3.4	0.36	1557	132.9	7.4	0.34	0.51	3.6	2.4
15 wt% SNPs	1249	3.6	0.36	1550	127.8	7.0	0.36	0.55	3.7	2.2
0.5 wt% graphene	1105	2.9	0.36	1525	131.0	6.1	0.35	0.53	3.4	2.0
1.0 wt% graphene	1155	2.9	0.39	1566	120.3	6.7	0.36	0.55	4.1	2.2

Note: $E_2 = E_3$, $\nu_{12} = \nu_{13}$, $G_{12} = G_{13}$

Table 2

Natural frequencies and mode shapes of intact, 30°-curved structure using unmodified resin

Facesheet stacking	Mode	FE (Hz)	Exp. (SD) (Hz)	Difference (%)	Shape
[0°] ₄	1	1628.7	1560.2 (25.5)	-4.2	1st bending
	2	2057.9	1994.8 (43.8)	-3.1	1st torsion
	3	3132.7	2935.2 (15.9)	-6.3	1st lateral*
	4	3869.5	3776.4 (64.5)	-2.4	2nd bending
	5	4551.5	4476.3 (47.3)	-1.7	2nd torsion
[90°] ₄	1	447.9	459.1 (4.7)	2.5	1st bending
	2	1217.6	1263.8 (13.1)	3.8	2nd bending
	3	1220.0	1265.1 (24.2)	3.7	1st lateral*
	4	2004.7	1968.1 (19.9)	-1.8	1st torsion
	5	2329.9	2424.4 (18.8)	4.1	3rd bending

*: combined with torsion

Table 3

Natural frequencies and mode shapes of 80-mm-debonded, 30°-curved structure using unmodified resin

Facesheet stacking	Mode	FE (Hz)	Exp. (Hz)	Difference (%)	Shape
[0°] ₄	1	1532.0	959.0	-37.4	1st bending
	2	1658.0	1593.4	-3.9	1st torsion
	3	1687.4	1660.9	-1.6	2nd bending
	4	2926.8	2288.7	-21.8	1st lateral*
	5	4061.2	4077.7	0.4	3rd bending
[90°] ₄	1	444.8	365.2	-17.9	1st bending
	2	985.8	921.1	-6.6	2nd bending
	3	1204.1	1124.6	-6.6	1st lateral*
	4	1505.1	1472.7	-2.2	1st torsion
	5	2081.6	2261.7	8.7	3rd bending

*: combined with torsion

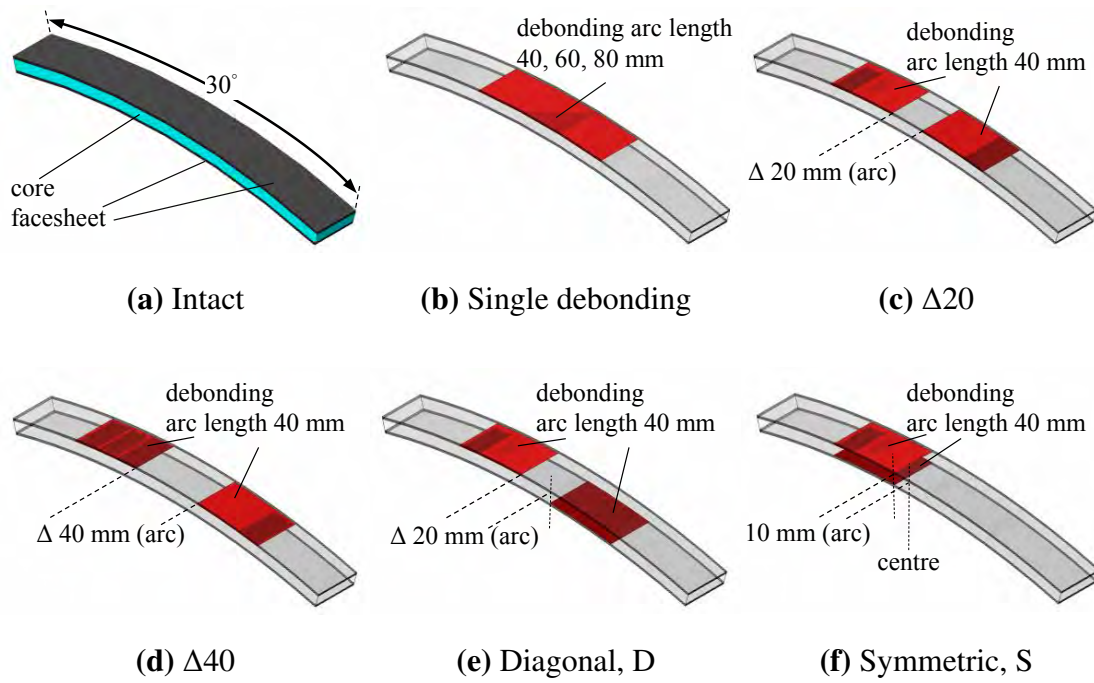
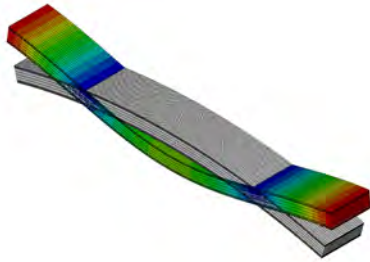
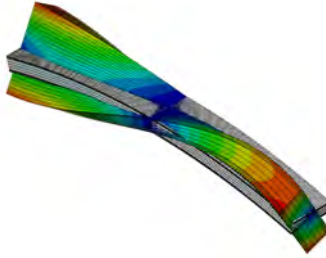


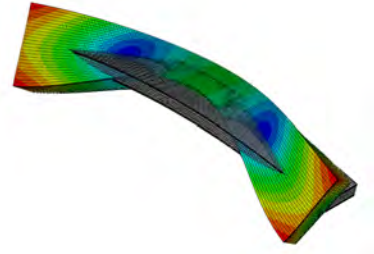
Fig. 1. Finite element models of 30°-curved sandwich structures with locations of debonded regions (shown in red). The same debonded regions were used for the flat and 15°-curved structures.



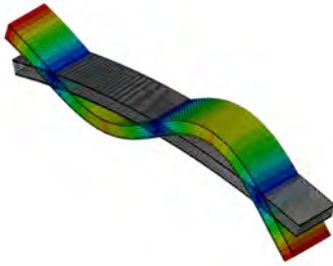
(a) mode 1: 1st bending



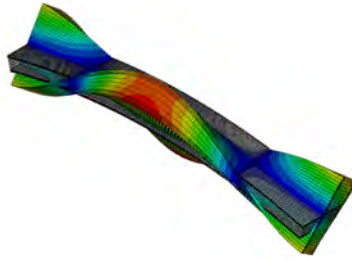
(b) mode 2: 1st torsion



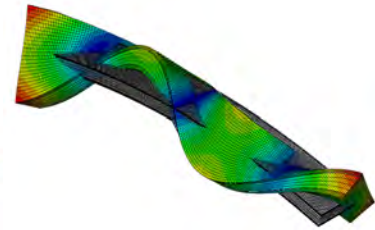
(c) mode 3: 1st lateral (combine with torsion)



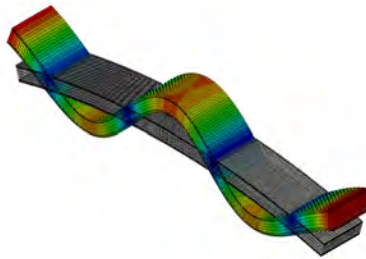
(d) mode 4: 2nd bending



(e) mode 5: 2nd torsion

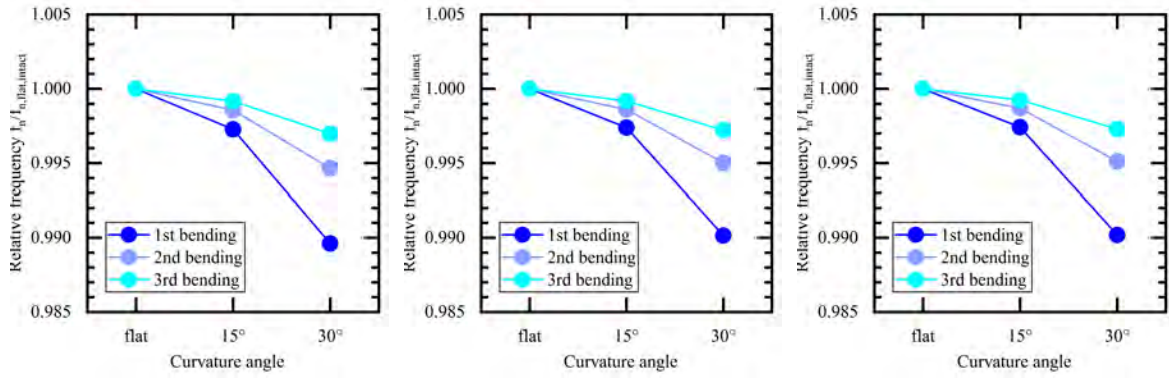


(f) mode 6: 2nd lateral (combine with torsion)

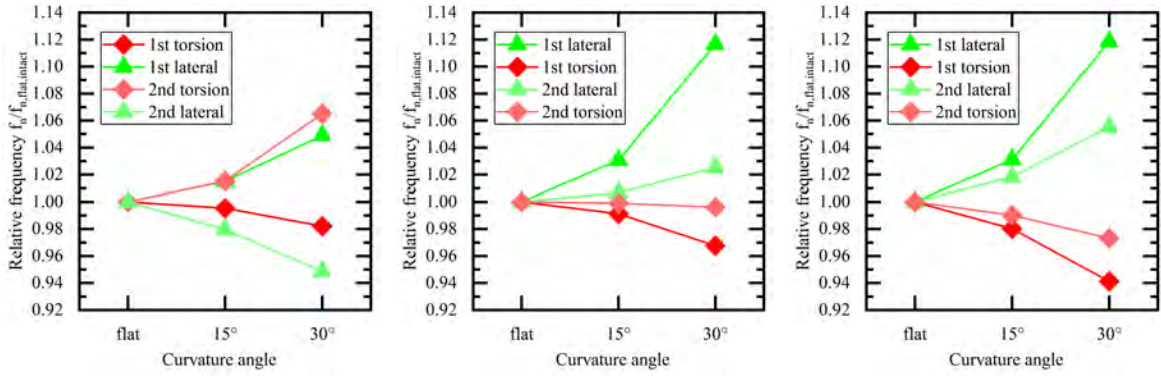


(g) mode 7: 3rd bending

Fig. 2. Simulation mode shapes of 30°-curved, intact structure using unmodified resin and $[0^\circ]_4$ facesheets



(a) $[0^\circ]_4$ facesheets, bending (b) $[90^\circ]_4$ facesheets, bending (c) $[+45^\circ/-45^\circ]_s$ facesheets, bending



(d) $[0^\circ]_4$ facesheets, torsion & lateral (e) $[90^\circ]_4$ facesheets, torsion & lateral (f) $[+45^\circ/-45^\circ]_s$ facesheets, torsion & lateral

Fig. 3. Relative natural frequencies of intact, flat and curved sandwich structures (simulation results)

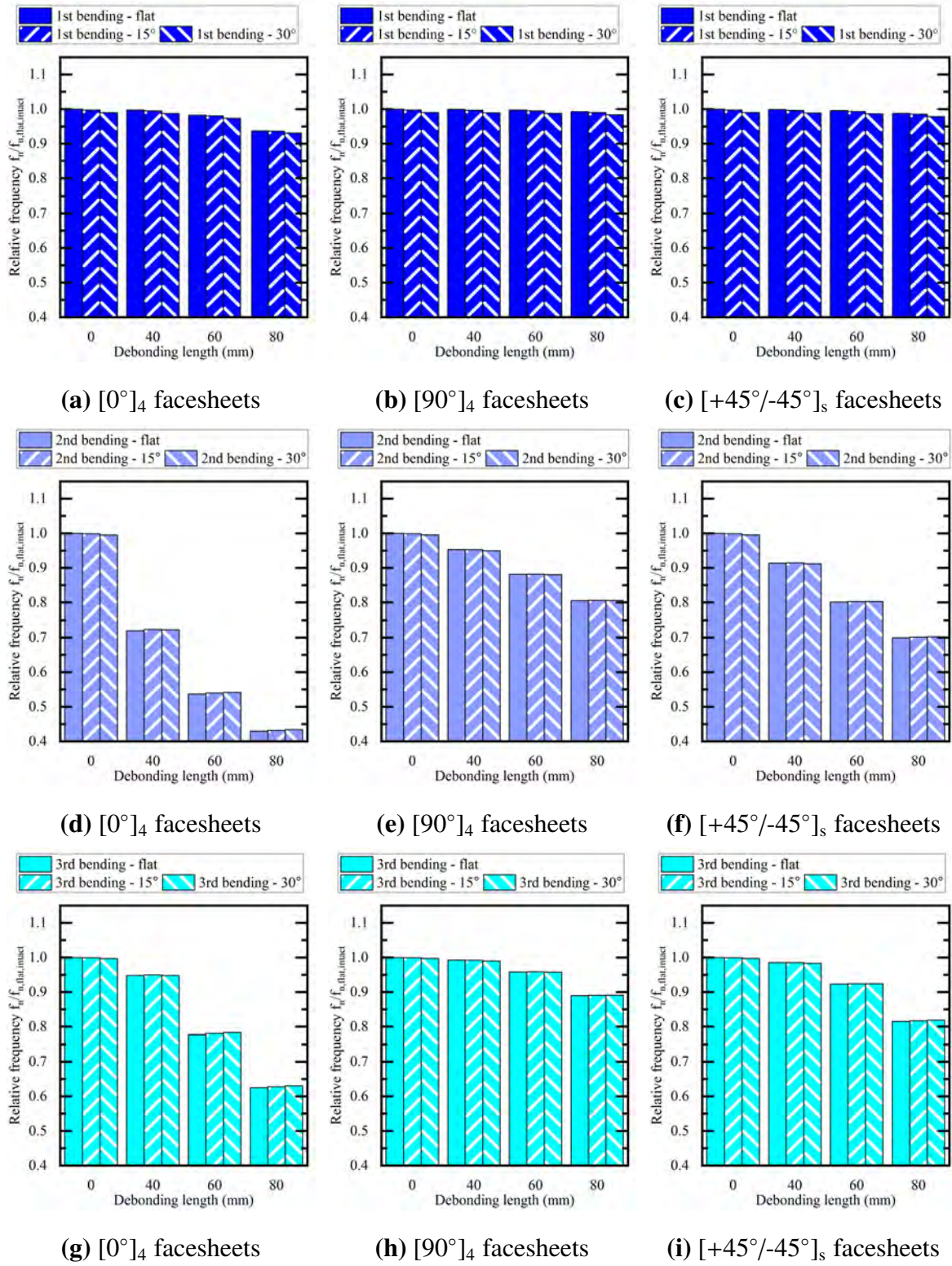


Fig. 4. Relative natural frequencies of bending modes affected by a central debonding and curvature (simulation results)

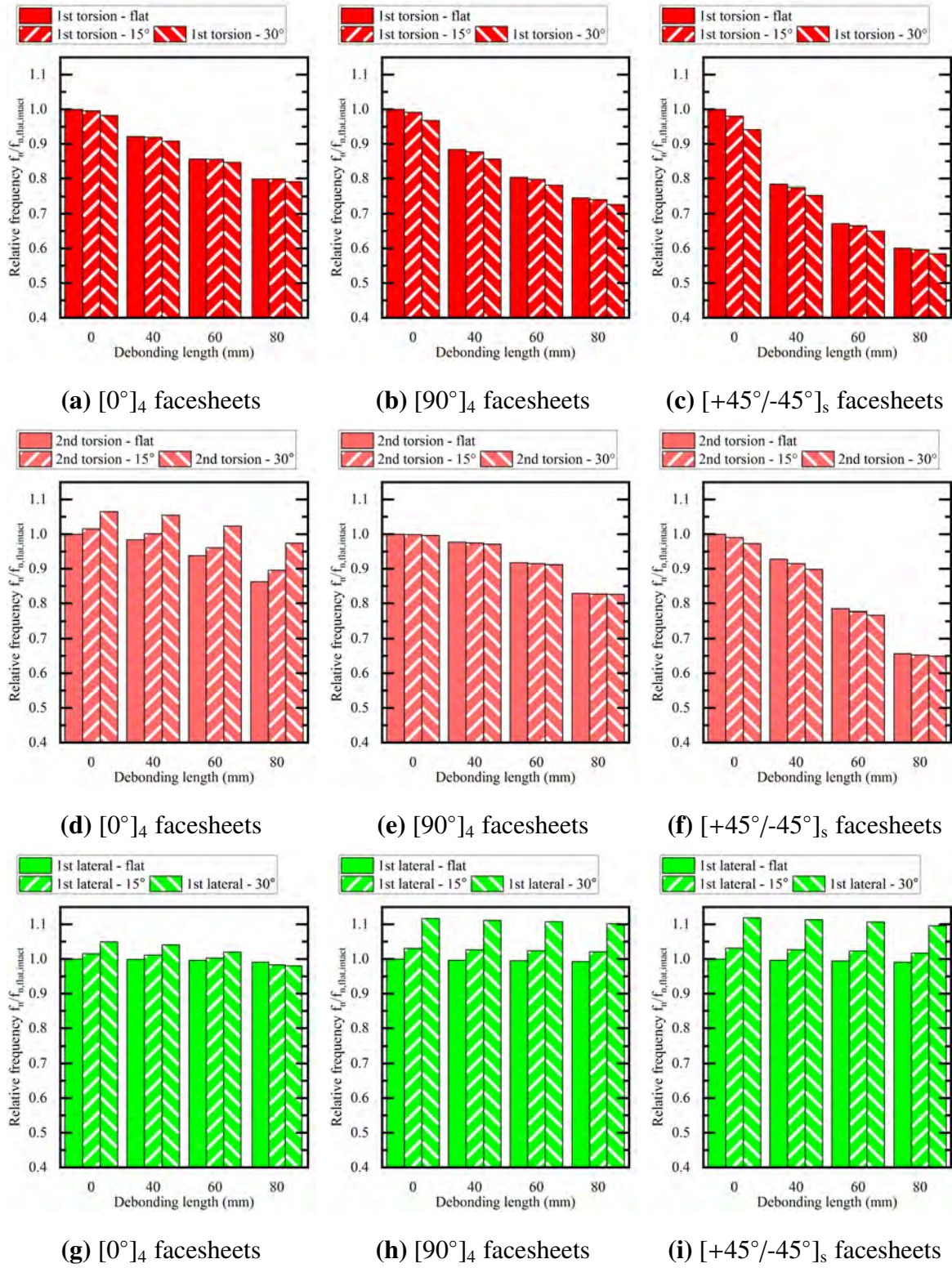


Fig. 5. Relative natural frequencies of torsion and lateral modes affected by a central debonding and curvature (simulation results)

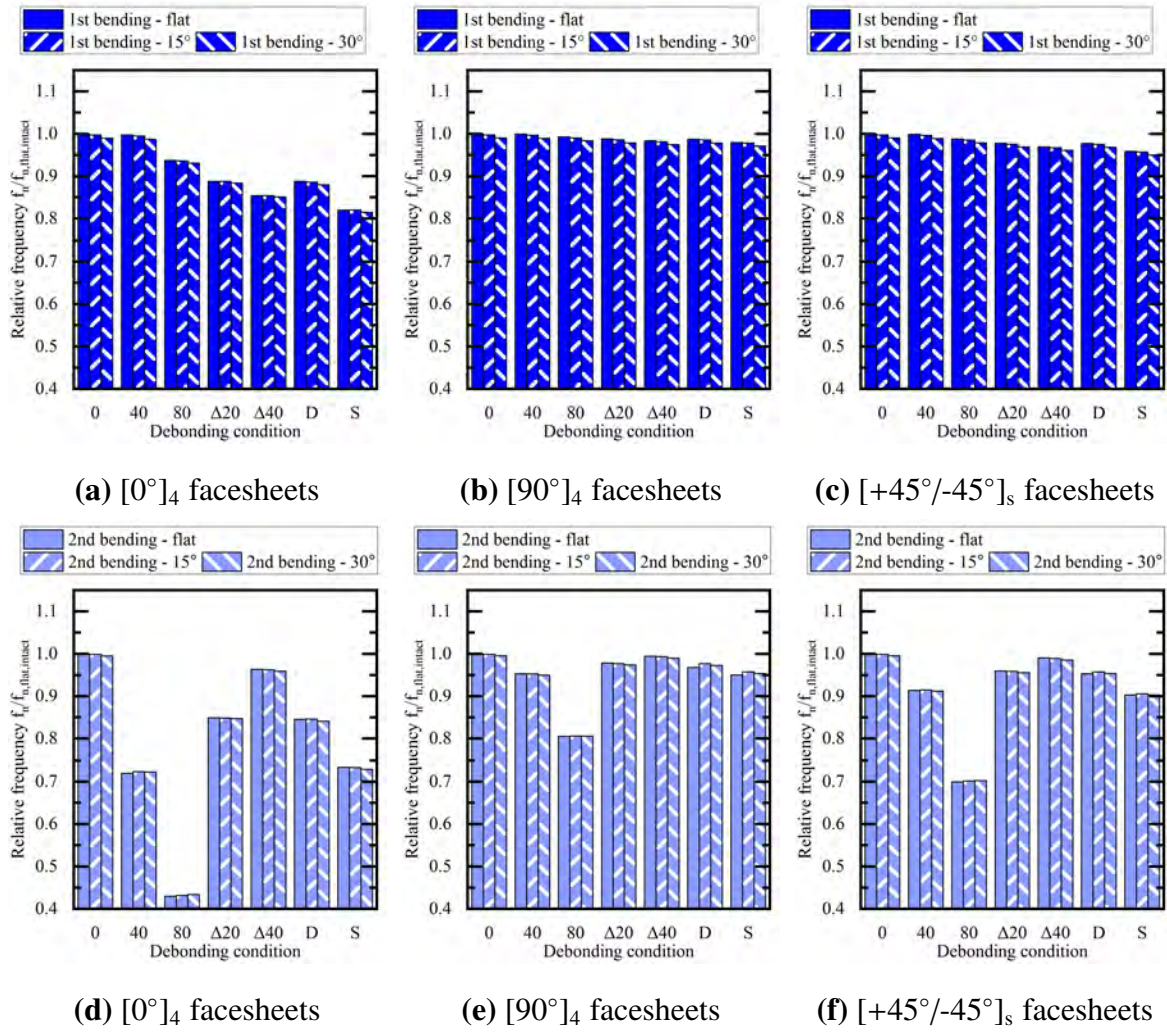


Fig. 6. Relative natural frequencies of bending modes affected by double debonding and curvature (simulation results)

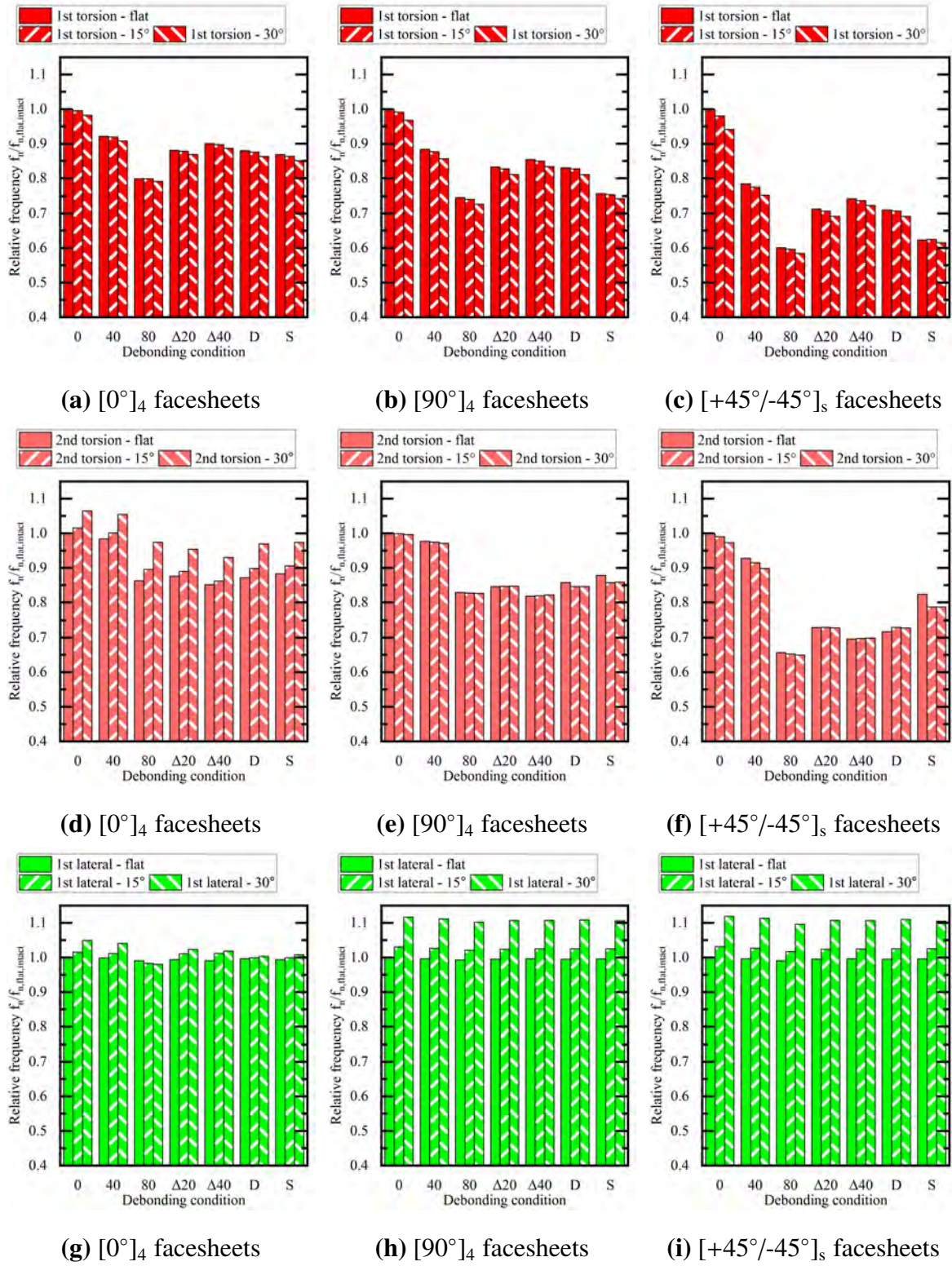


Fig. 7. Relative natural frequencies of torsion and lateral modes affected by double debonding and curvature (simulation results)

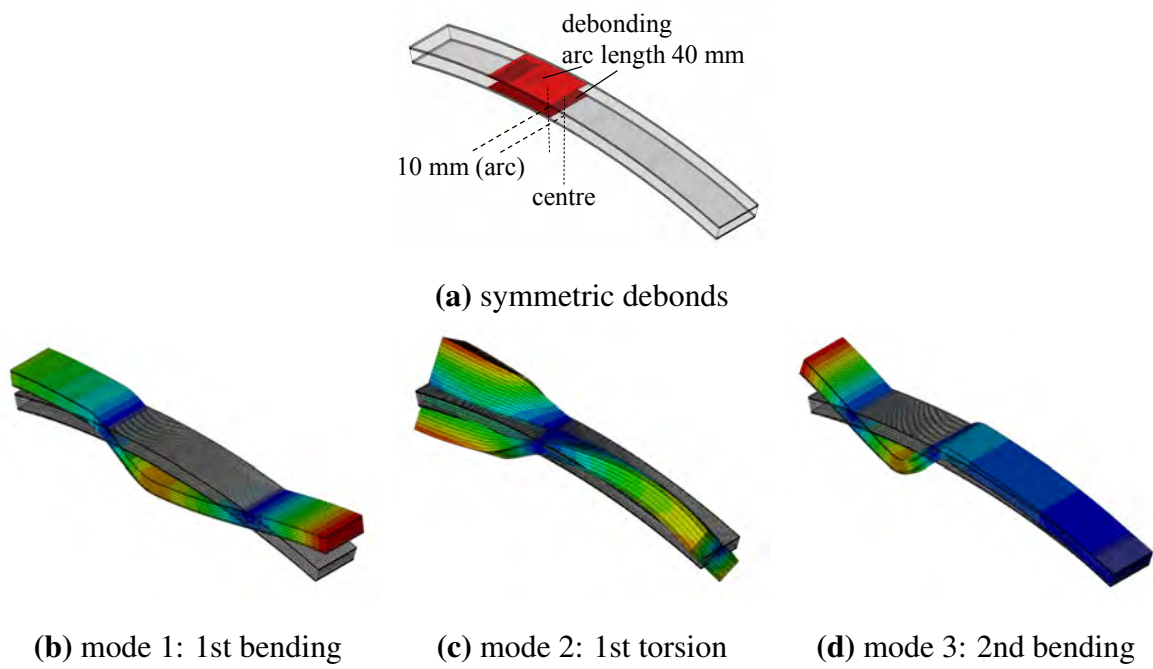
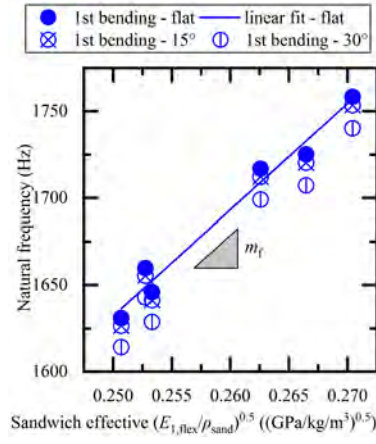
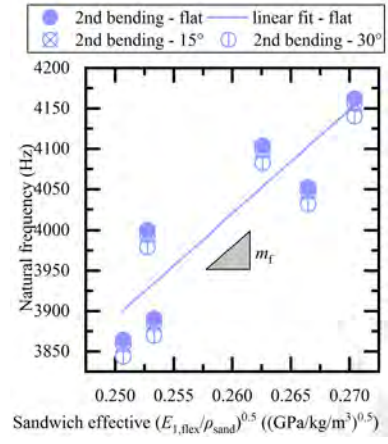


Fig. 8. Simulation mode shapes of 30°-curved, S-debonded structure using unmodified resin and $[0^\circ]_4$ stacking



(a) 1st bending



(b) 2nd bending

Fig. 9. Influence of material change on the natural frequencies of bending modes of intact, flat or curved structures, $[0^\circ]_4$ facesheets (simulation results)

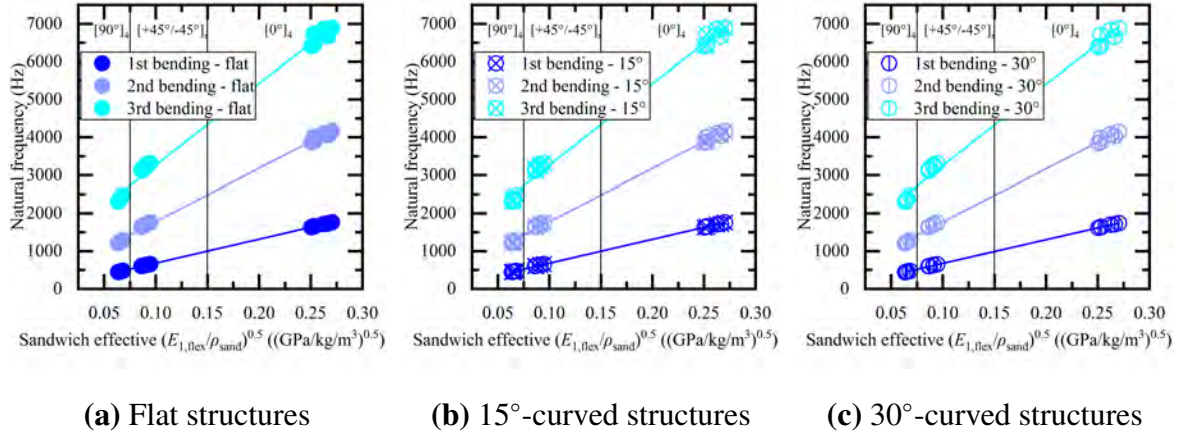
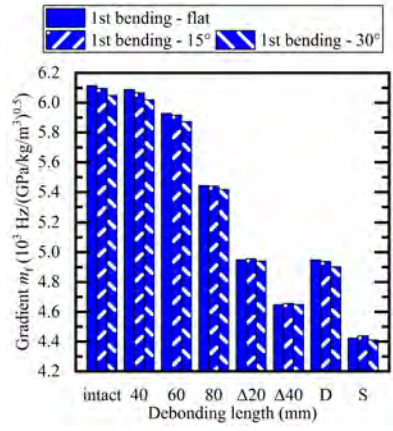
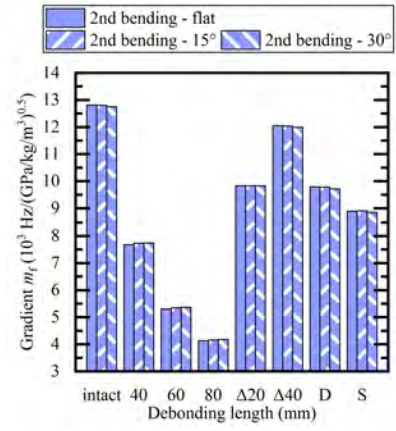


Fig. 10. Influence of material change on the natural frequencies of bending modes of intact, flat or curved structures, all facesheets (simulation results)



(a) 1st bending



(b) 2nd bending

Fig. 11. Gradient m_f of single- and double-debonded, flat or curved structures, $[0^\circ]_4$ facesheets (simulation results)