

AN EXAMINATION OF THE PERFORMANCE OF
PRESSURE-EXCHANGER EQUALISERS AND DIVIDERS

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1. ABSTRACT

Definitions of "divider" and "equaliser" precede statements of the objectives of the theoretical and experimental pressure-exchanger performance investigations. Various pressure exchanger divider and equaliser arrangements are described, including the particular cycles chosen for further study. Criteria of the performances of dividers and equalisers are deduced and methods of presenting these are established.

The theoretical treatment is divided into three parts; in the first an approximate method is used to determine the range of pressure ratios over which operation of pressure-exchanger dividers and equalisers is possible; it also focuses attention on fundamental performance limitations. The second portion of the analysis predicts divider and equaliser performances by a fairly simple procedure. The final part makes performance predictions using the method-of-characteristics applied to unsteady flow, account being taken of leakage and frictional effects.

A description of the pressure-exchanger apparatus precedes that of the experiments. Particular attention is given to factors limiting the scope of the experiments and to the anticipated accuracy of the experimental results.

An analysis of the experimental results draws attention to noticeable trends therein. A comparison of actual and predicted performances enables most features of the actual performances to be accounted for.

Suggestions have been made for some practical applications for pressure-exchanger dividers and equalisers; these take account of the performance characteristics revealed by theory and experiment. Recommendations have been made for ways in which the programme of research could be extended.

It has been concluded that single stage pressure-exchanger dividers and equalisers are limited to efficient operation at low pressure ratios only.

It has also been concluded that performances can be fairly accurately predicted by the method of characteristics, provided leakage and frictional effects are taken into account.

CONTENTS LIST

<u>SECTION</u>	<u>PAGE No.</u>
TITLE	1.
1. ABSTRACT	2.
CONTENTS LIST	4.
ACKNOWLEDGEMENTS	9.
2. INTRODUCTION	10.
2.1 Definitions of the Terms; Pressure Divider and Pressure Equaliser.	10.
2.2 Previous Work.	10.
2.3 Objectives of the Investigation.	11.
2.3.1 Main Objectives.	11.
2.3.2 Subsidiary Aims.	11.
2.3.3 Limitations of Scope.	11.
2.4 Pressure-Exchanger Divider & Equaliser Cycles.	12.
2.4.1 Single Ended Dividers.	12.
2.4.2 Single Ended Equalisers.	14.
2.4.3 Double Ended Dividers.	15.
2.4.4 Double Ended Equalisers.	15.
2.4.5 Open Ended Dividers & Equalisers.	16.
2.4.6 Single Cell Dividers & Equalisers.	17.
2.5 Compounding of Dividers & Equalisers.	18.
2.5.1 Compounding Without Feedback.	18.
2.5.2 Compounding With Feedback.	19.
2.6 Potential Commercial Applications.	20.
2.6.1 Dividers.	20.
2.6.2 Equalisers.	20.
3. PRESENTATION OF PERFORMANCE	22.
3.1 Meaning & Criteria of the Performance of Dividers & Equalisers.	22.

3.2	Dividers.	22.
3.2.1	Significance of Mass Ratio & Product Efficiency.	22.
3.2.2	Criteria of Cooling Effect.	24.
3.2.3	Presentation of Performance	25.
3.3	Equalisers.	26.
3.3.1	Significance of Enthalpy Ratio & Product Efficiency	26.
3.3.2	Presentation of Performance	27.
4.	THEORY	28.
4.1	Outline of Theoretical Approach	28.
4.2	Prediction of Hypothetical Isentropic Performance.	28.
4.2.1	Divider.	29.
4.2.2	Equaliser.	30.
4.3	Simple Theory for Predicting Pressure Exchanger Performance Inclusive of Irreversible Effects.	31.
4.3.1	Divider	34.
4.3.2	Equaliser.	36.
4.4	Application of the Method-of-Characteristics to the Study of Dividers & Equalisers.	37.
4.4.1	Scope of the Characteristic Calculations.	37.
4.4.2	Fundamental Assumptions	37.
4.4.3	The Adaptation & Extension of Method-of-Characteristics Procedures.	38.
4.4.4	Calculation Procedure.	50.
5.	APPARATUS	53.
5.1	Specification for a Suitable Test Rig.	53.
5.2	Description of the Power Jets Machine.	53.
5.3	General Arrangement of the Rig.	54.
5.3.1	Mechanical Construction.	55.
5.3.2	Special Features.	55.
5.4	Instrumentation.	56.
5.4.1	Brief Description of the Arrangement.	56.
5.4.2	Constructional Features.	57.
5.4.3	Averaging Manometers.	58.
5.5	Summary of the Constructional Work.	59.
6.	OUTLINE OF EXPERIMENTAL PROCEDURE.	60.
6.1	Divider Tests	60.

6.2 Equaliser Tests.	61.
7. LIMITATIONS OF THE EXPERIMENTS.	63.
7.1 Performance Limitations of the Rig.	63.
7.1.1 Limitations Due to Restricted Mass Flow.	63.
7.1.2 Limitations Due to Pressure Losses In Ducts.	64.
7.1.3 Effects Arising from Asymmetric Flow in Ports.	66.
7.2 Limitations Arising from Shortage of Time.	68.
7.2.1 Range of Machine Variables for Divider Tests.	68.
7.2.2 Range of Machine Variables for Equaliser Tests.	69.
7.3 Accuracy of Experimental Results.	69.
8. ANALYSIS OF EXPERIMENTAL RESULTS.	72.
8.1 Introduction.	72.
8.2 Divider.	73.
8.2.1 Effect of Port Geometry on Pressure-Boosting Performance.	74.
8.2.2 Effect of Rotor Speed on Pressure-Boosting Performance.	74.
8.2.3 Effect of Port Geometry on Cooling Cycle Performance.	75.
8.2.4 Effect of Rotor Speed on Cooling Cycle Performance.	76.
8.3 Equaliser.	77.
8.3.1 Effect of Port Geometry on Equaliser Performance.	77.
8.3.2 Effect of Rotor Speed on Equaliser Performance.	78.
9. COMPARISON OF THEORETICAL & EXPERIMENTAL PERFORMANCES.	80.
9.1 Dividers	80.
9.1.1 Comparison of Actual with Ideal Isentropic Performances	80.
9.1.2 Comparison of Actual Performance with that Predicted from Simplified Theory, Taking Account of Irreversibilities.	81.
9.1.3 Comparison of Actual Performance with the Results of Characteristic Studies	82.
9.2 Equalisers.	87.
9.2.1 Comparison of Actual with Ideal Isentropic Performance.	87.

9.2.2	Comparison of Actual Performance with that predicted from Simplified Theory, Taking Account of Irreversibilities.	88.
9.2.3	Comparison of Actual Performance with the Results of Characteristics Studies.	89.
10.	DISCUSSION & RECOMMENDATIONS.	94.
10.1	Properties & Potentialities of Pressure-Exchanger Dividers & Equalisers.	94.
10.1.1	Dividers.	96.
10.1.2	Equalisers.	97.
10.2	Commercial Applications.	98.
10.2.1	Dividers.	98.
10.2.2	Equalisers.	100.
10.3	Recommendations for Further Work	101.
10.3.1	Modifications to the Test Rig.	101.
10.3.2	Programme for Divider.	102.
10.3.3	Programme for Equaliser.	103.
11.	CONCLUSIONS	104.
12.	GRAPHICAL PRESENTATION OF RESULTS.	108.
12.1	Results of Theoretical Studies.	108.
12.2	Results of Experimental Investigations.	126.
13.	DIAGRAMS & ILLUSTRATIONS	154.
14.	REFERENCES	159.
15.	APPENDICES	161.
15.1	Appendix I; Notation	161.
15.2	Appendix II; Significance of Product Efficiency & Criteria of Performance	168.
15.3	Appendix III; Prediction of the Performance of Isentropic Pressure-Exchanger Dividers & Equalisers.	174.
15.4	Appendix IV; Simplified Theoretical Treatment.	179.

15.5	Appendix V;	The Adaptation of the Method-of-Characteristics to the Study of Divider & Equaliser Cycles.	190.
15.6	Appendix VI;	A Study of Flow in the Ports of Pressure Exchangers.	211.
15.7	Appendix VII;	Averaging Manometers.	224.
15.8	Appendix VIII;	Justification for the Use of Average Static Pressures.	226.

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2. INTRODUCTION

2.1 DEFINITIONS OF THE TERMS; PRESSURE DIVIDER & PRESSURE EQUALISER

For the purposes of this thesis the terms pressure divider and pressure equaliser are defined as follows:

Pressure divider: a machine taking in a single stream of gas or vapour and discharging two separate streams, one at a higher and the other at a lower stagnation pressure than that of the ingoing stream. The work needed to compress the fluid emerging from the high pressure outlet is obtained from expansion of the remainder of the flow.

Pressure equaliser: a machine taking in two streams, of (not necessarily identical) compressible fluid, one stream having a higher stagnation pressure than the other; the fluids are discharged from a common outlet with a stagnation pressure between those of the ingoing streams. The work required to compress the low pressure fluid is obtained from expansion of the high pressure stream.

Pressure dividers may find application in cooling or pressure-boosting plant, pressure equalisers for gas pumping duties, etc.

2.2 PREVIOUS WORK

So far as is known to the author, prior to the work described in this thesis, no pressure-exchangers have been built working on pressure divider or pressure equaliser cycles. Pressure-exchanger dividers and equalisers have however been described in patent literature notably by Jendrassik (8) and Spalding (14). Some theoretical work has been carried out by Spalding (15) in which the isentropic performances of some basic pressure-exchanger divider and equaliser arrangements have been studied.

The work described herein was an attempt to obtain experimental performance data from a pressure-exchanger divider and a pressure-exchanger equaliser (henceforth frequently abbreviated to "divider" and "equaliser"), and to compare these data with the results of theoretical studies.

2.3 OBJECTIVES OF THE INVESTIGATION.

2.3.1 Main Objectives

The main objectives were:

- (a) to investigate the performance of dividers and equalisers.
- (b) to compare experimental and theoretical performance studies.
- (c) to determine the limits (if any) of the performance regimes of dividers and equalisers.
- (d) to draw conclusion from the investigations.

2.3.2 Subsidiary Aims

The following objectives, subordinate to those listed in 2.3.1 were kept in mind:

- (a) the need to find some suitable commercial uses for dividers and equalisers.
- (b) to observe and note any special features of pressure-exchanger dividers and equalisers which may make them outstandingly suitable for use in special circumstances.
- (c) in the light of experience to note any improvements which could be made to cycles or structures.

2.3.3 Limitations of Scope

In order to reduce the quantity of work to manageable proportions, only one divider and one equaliser cycle were investigated out of a whole range of possibilities. A variety of possible arrangements is briefly reviewed in section 2.4., the actual cycles studied being those illustrated on Fig.2.4-1 and Fig.2.4-4. The choice of cycles for

inclusion in the investigation was based upon likely commercial potential, and also their suitability for the apparatus available for the experiments.

Throughout the whole series of experiments the working fluid was restricted to air at approximately room temperature.

2.4 PRESSURE-EXCHANGER DIVIDER & EQUALISER CYCLES

This section is intended to be a brief survey of some possible cycles. The isentropic performances of these are illustrated by $a - u$ diagrams, each being accompanied by an appropriate $t - x$ illustration. The foregoing and other symbols are defined in Appendix I.

2.4.1 Single Ended Dividers:

Single ended dividers are herein defined as dividers in which fluid enters the rotor at one end only. Some possible arrangements are shown in the following diagrams; u is taken as positive when flow is from left to right;

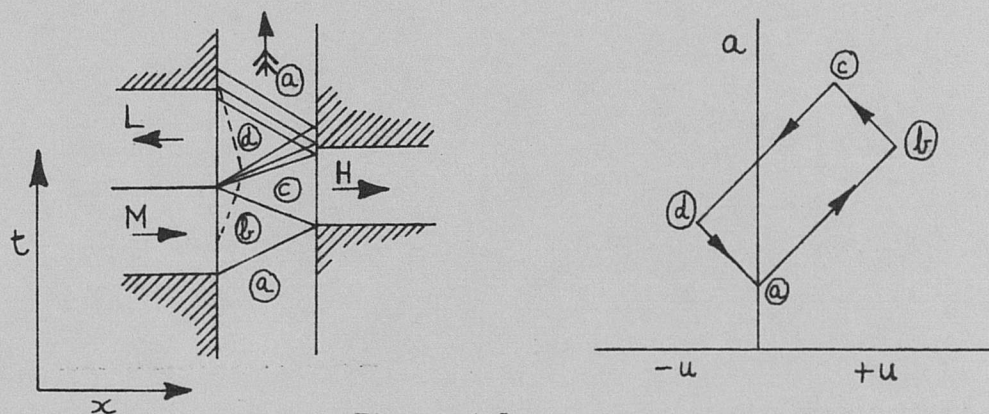


Fig. 2.4-1.

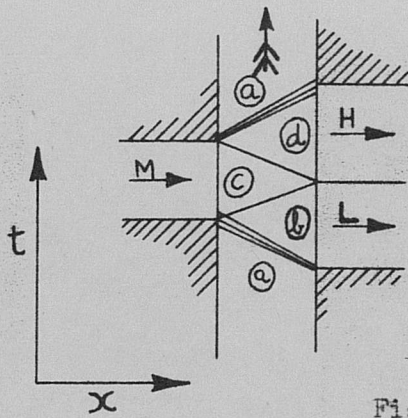


Fig. 2.4-2

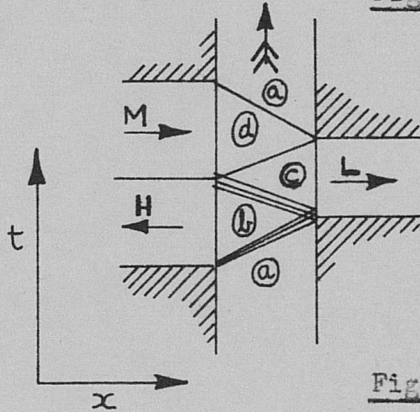


Fig. 2.4-3

The cycle illustrated by Fig. 2.4-1, that chosen for study in the divider performance investigation, is fairly suitable for applications in cooling plant where attention is focussed on the condition of the low pressure stream. A good expansion efficiency may be anticipated due to the improbability of discharging from the low pressure port fluid which has remained in the rotor for more than a part of one cycle; this point is illustrated by a dotted line on the $t-x$ chart.

Fig. 2.4-2 shows an arrangement having the advantage of using generally weaker waves for specified pressure ratios than the cycle shown on Fig. 2.4-1. The major disadvantage is that both the high and low pressure streams have to work their way from one end of the rotor to the other before being discharged.

A good compression efficiency may perhaps be expected from the

layout shown in Fig. 2.4-3; for prescribed pressure ratios the waves are generally stronger than those for the cycle shown on Fig. 2.4-2. The $a-u$ diagrams for Fig. 2.4-1, 2 & 3 are drawn to the same scale for cycles having identical port pressures in each case.

2.4.2 Single Ended Equalisers:

A single ended equaliser is defined as an equaliser discharging fluid at one end of the rotor only. Again u is taken as positive when flow is from left to right on the $t-x$ diagram.

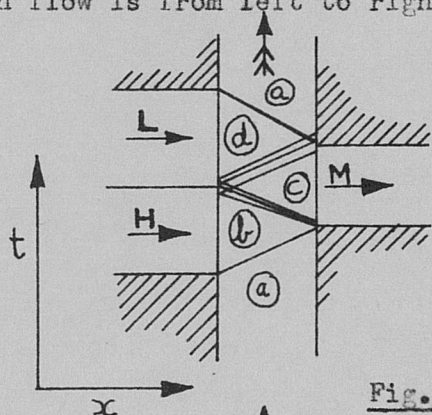


Fig. 2.4-4

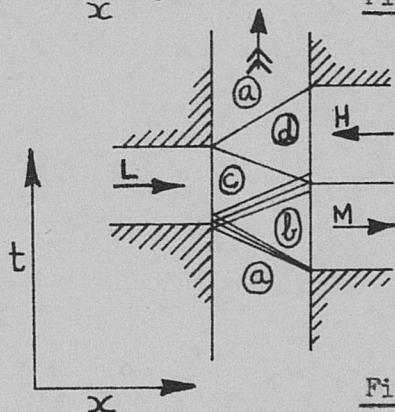
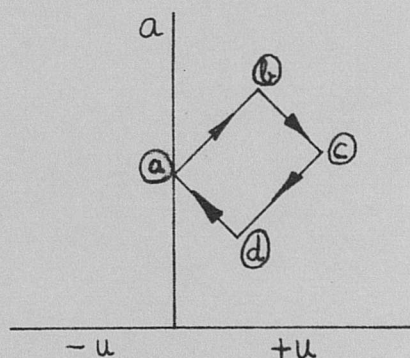


Fig. 2.4-5

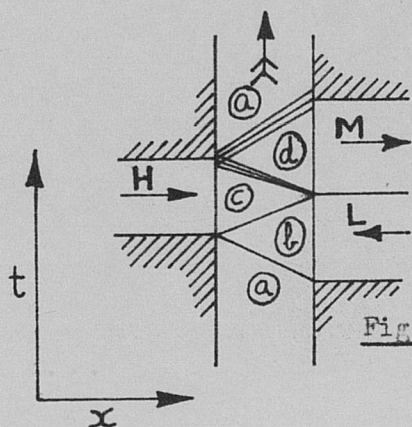
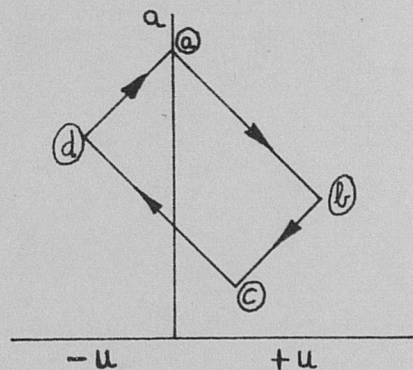
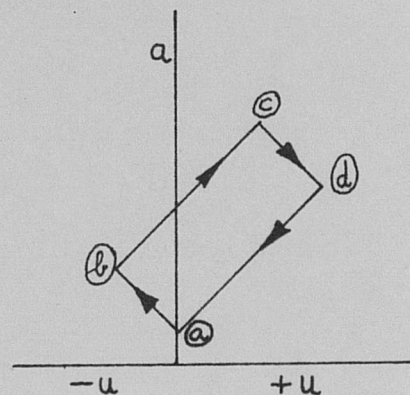


Fig. 2.4-6



The arrangement shown in Fig. 2.4-4, that chosen for inclusion in the equaliser performance investigation, has the advantage that, for specified pressure ratios, it requires generally weaker pressure waves than either of the other two schemes. An attempt has been made to illustrate this point by drawing the $a-u$ diagrams for Fig. 2.4-4, 5 & 6 to the same scale for cycles having the same port pressures in each case.

2.4.3 Double Ended Dividers.

Double ended dividers are here defined as dividers in which fluid enters the rotor at both ends of the cells. Generally double ended cycles allow better use to be made of the available space on the stator end plates by accommodating more ports than can be incorporated in single ended machines.

If double ended arrangements are made using any of the cycles shown in Fig. 2.4-1, 2 & 3 a common result is obtained as shown in Fig. 2.4-7 on the $t-x$ and $a-u$ planes.

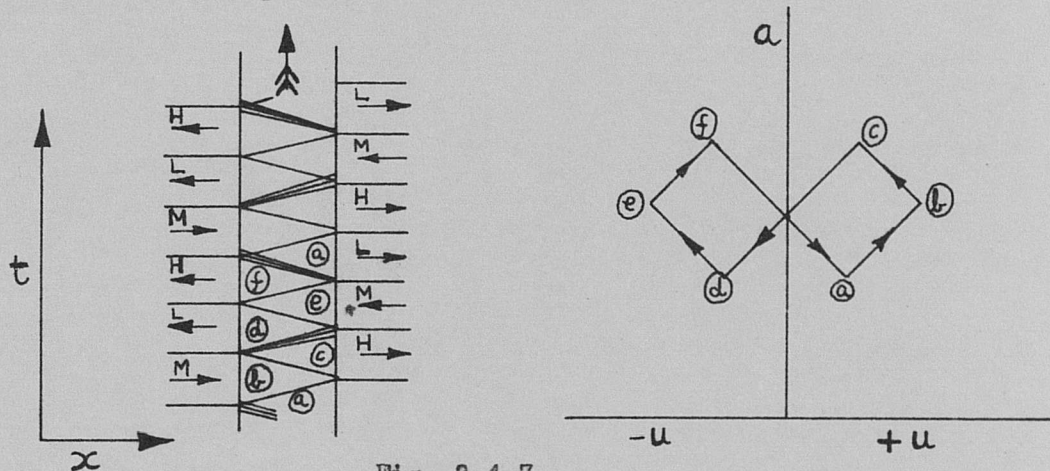


Fig. 2.4-7

2.4.4 Double Ended Equalisers

Double ended equalisers are defined as equalisers discharging medium pressure fluid from both ends of the cells.

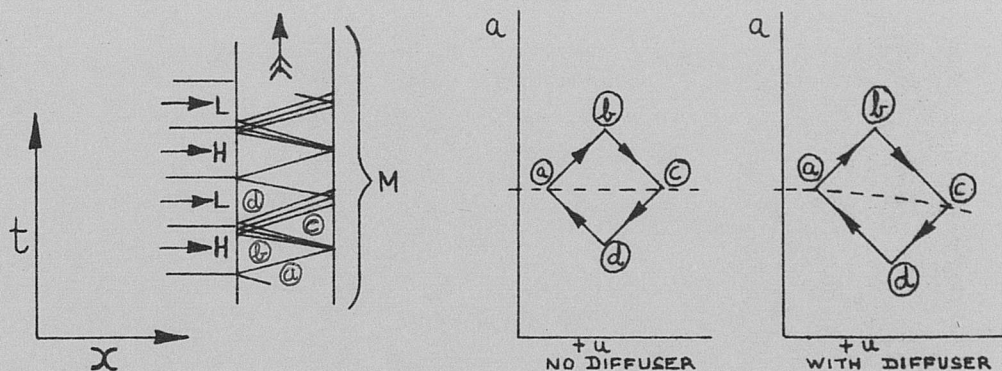


Fig. 2.4-10.

The main disadvantage of open ended dividers and equalisers is revealed by the α - u diagrams; on these the points (a) and (c) must lie on the dotted curves. The dotted curves are the steady flow boundary curves for fluid passing into or out of the cells. Two α - u diagrams are shown on Fig. 2.4-10, the left hand one for cells having no diffusing action, the one on the right for cells incorporating diffusing passages.

2.4.6 Single Cell Dividers and Equalisers:

Pressure-exchanger construction can often be simplified by replacing the orthodox multi-celled rotor by a single tubular cell. If this is done suitable valve gear must be provided at one or both ends of the tube to replace the normal valve action of orthodox machines resulting from relative motion between cells and stator. An example of a single cell pressure-exchanger, featuring internal combustion, was the German Schmidt tube flying-bomb engine of the Second World War.

A particular simplification can be obtained in the construction of dividers and equalisers by combining the single cell concept with the open ended cycles described in the previous section. The appropriate t - x and α - u diagrams are similar to those shown in section 2.4.5.

An associated programme of research has been carried out at

Imperial College to investigate the performance of open ended single cell dividers and equalisers. This work is reported by Barham (2).

2.5 COMPOUNDING OF DIVIDERS AND EQUALISERS

Dividers and equalisers may be compounded in order to make operation possible over greater pressure ratios than might otherwise be the case. There appear to be at least two distinct methods of compounding; one involving feed-back circuits, the other, a more simple method suggested by Spalding (14) avoids this complication.

2.5.1 Compounding Without Feedback

Fig. 2.5-1 shows in diagrammatic form a divider with three stages of compression. The dotted curves on the $q-u$ plane are intended to represent the expansion of fluid passing from the high pressure outlet of one stage through the inlet nozzle of the succeeding stage. Dividers can also be conceived with compounded expansion as well as or instead of multi-stage compression.

Fig. 2.5-2 shows an equaliser compounded for multi-stage compression of the low pressure stream. Equalisers can be arranged to provide multi-stage expansion of the high pressure flow in addition to or instead of the scheme shown in the diagram.

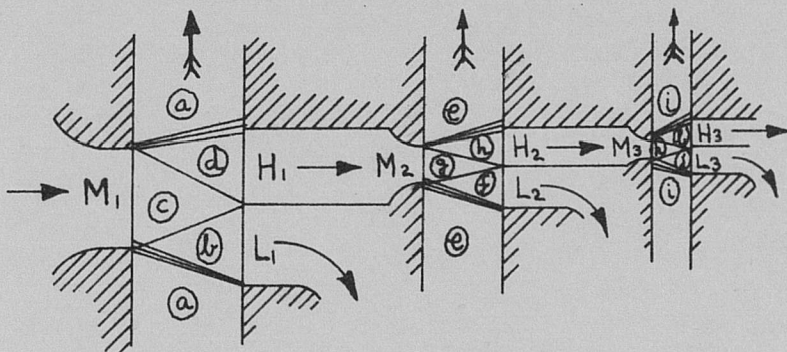
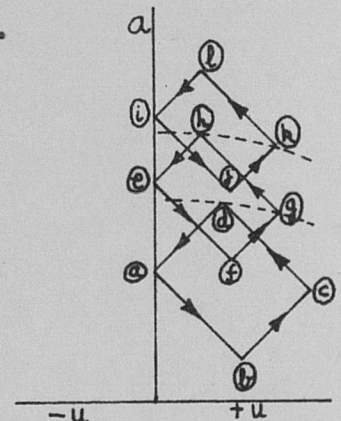


Fig. 2.5-1



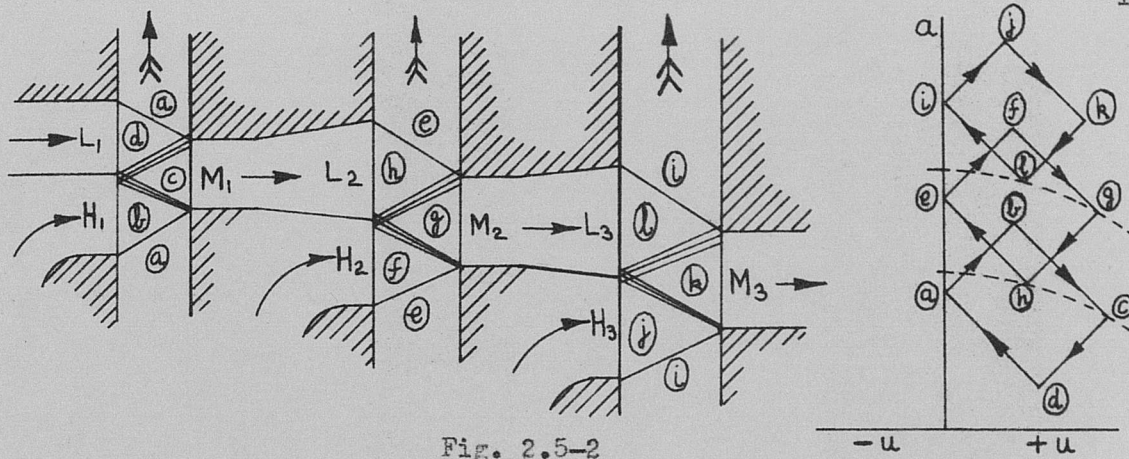


Fig. 2.5-2

For many applications, a disadvantage of designs of the kind shown on Fig. 2.5-1 & 2 is the multiple pressures at which fluid is removed from, or supplied to, the machines. In the divider arrangement shown in Fig. 2.5-1 low pressure fluid is discharged at pressures P_{L1} , P_{L2} & P_{L3} . In the equaliser shown in Fig. 2.5-2 the high pressure supply is admitted at pressures P_{H1} , P_{H2} & P_{H3} .

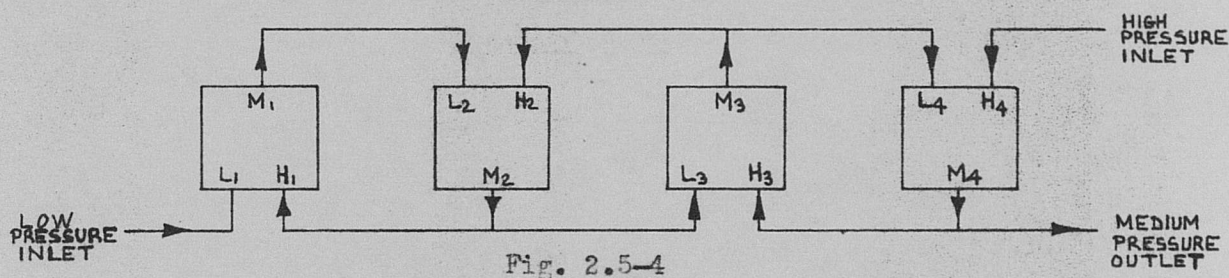
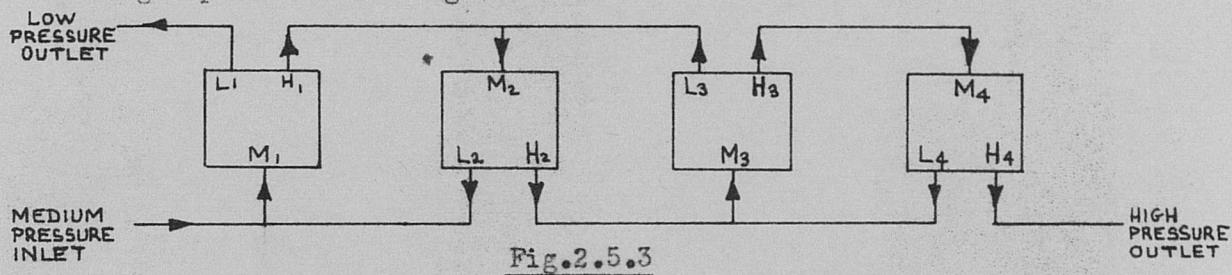
2.5.2 Compounding With Feedback;

The prime disadvantages of the system of compounding described in 2.5.1 can be eliminated by incorporating feedback or recirculating ducts; these in turn have the disadvantage of adding engineering complexity. Fig. 2.5-3 is a block diagram of a divider arrangement featuring four stage compression; Fig. 2.5-4 is a scheme for an equaliser arranged for compounded compression of the low pressure stream. It is also possible to arrange for multi-stage expansion instead of, or in addition to, multi-stage compression in both dividers and equalisers.

In real machines it would probably be necessary to install inter-coolers in the feedback circuits in order to counter some of the effects of irreversibilities.

The compounded machines described in this and the preceding sub-

sections may be designed to incorporate more than one stage in a single pressure-exchanger unit.



2.6 POTENTIAL COMMERCIAL APPLICATIONS

This sub-section seeks to review briefly some of the suggested commercial applications for dividers and equalisers.

2.6.1 Dividers

Divider applications appear to fall into two broad categories;

- (1) Applications where attention is focussed on the cooling potential of the low pressure stream. These included the cooling of high speed aircraft by the use of ram air after expansion in dividers. Another suggestion incorporates a divider as an "expansion engine" in a vapour compression refrigerator.
- (2) Applications in which the high pressure stream constitutes the useful output. These include pressure boosting operations for air or gas compression plant, also the pressure boosting of the output of steam boilers.

2.6.2 Equalisers

Applications include uses as forced or induced draught fans for steam boilers, etc. In such applications the driving fluid would often be steam expanding through a comparatively large pressure ratio, the driven fluid air or products of combustion. Another idea is to use

equalisers as thrust augmenters for aircraft jet-propulsion units.

Equalisers may, in general, be thought of as potential replacements for ejectors working with compressible fluids. In principle equalisers should be more efficient than ejectors because the former operate on cycles which are in essence reversible.

3. PRESENTATION OF PERFORMANCE

3.1 MEANING & CRITERIA OF THE PERFORMANCE OF DIVIDERS & EQUALISERS:

Knowledge of the performance of dividers or equalisers is taken to mean, for prescribed geometry and rotor speed, knowledge of the pressure, temperature, mass-flow and physical properties of the fluid passing through the high, medium and low pressure ports. This information, preferably expressed in dimensionless form, may be compared with corresponding data for idealised machines.

Due to the mode of operation of dividers and equalisers the appropriate idealised cycles are isentropic. The choice of isentropic cycles as criteria with which to make comparisons is a particularly convenient one because the isentropic performances are readily predictable using simple theory, also because collations made in this way allow the performances of pressure exchangers to be compared easily with those of orthodox machines.

It is the purpose of the following sub-sections, 3.2 and 3.3, to explain in more detail the choice and significance of the criteria of performance and to justify the chosen methods of presentation.

3.2 DIVIDERS:

3.2.1 Significance of Mass Ratio & Product Efficiency:

For specified port pressures the ratio of the mass flow passing through the outlets of an adiabatic divider is, if expressed as a fraction of the corresponding ratio for an isentropic machine, a direct measure of the product of the machine's isentropic efficiencies of compression and expansion:

$$\frac{\dot{m}_H}{\dot{m}_L} \bigg/ \frac{\dot{m}_H^*}{\dot{m}_L^*} = \eta_{CHS} \eta_{EIS} \equiv \eta_{PIS} \quad 3.2-1$$

Proof of this and other expressions appearing in section 3 are

given in Appendix II. A mass ratio, the physical significance of which is more easily appreciated than that given in 3.2-1, is the ratio of the high pressure mass flow divided by the total ingoing mass flow; this has been given the symbol β , i.e.:

$$\frac{\dot{m}_H}{\dot{m}_H + \dot{m}_L} = \frac{\dot{m}_H}{\dot{m}_M} \equiv \beta \quad 3.2-2.$$

Where β^* refers to an isentropic machine it follows from 3.2-1 & 3.2-2 that:

$$\frac{\beta}{1-\beta} \bigg/ \frac{\beta^*}{1-\beta^*} = \eta_{PIS} \quad 3.2-3$$

Expressing β^* in terms of the stagnation pressures in the high, medium and low pressure ports:

$$\frac{\beta^*}{1-\beta^*} = \frac{\left[1 - (P_{L0}/P_{M0})^{\frac{\gamma-1}{\gamma}}\right]}{\left[(P_{H0}/P_{L0})^{\frac{\gamma-1}{\gamma}} - 1\right]} \quad 3.2-4.$$

Equations 3.2-3 and 3.2-4 show that if the mass ratio of a divider is known for specified stagnation pressures, then the isentropic product efficiency η_{PIS} is known implicitly. Hence the mass ratio is an important criterion of cycle performance. The mass ratio is particularly suitable for display on machine performance curves in the form given in equation 3.2-2.

Another criterion, which may be shown on performance presentation curves, is either isentropic compression or expansion efficiency; knowledge of one means that the other is known implicitly, provided β is also known. Knowledge of η_{EIS} or η_{CIS} and β implies that the cycle temperature ratios are known.

Specification of a mass flow, preferably dimensionless, in either the high, medium, or low pressure ports links the size of a machine of known geometry with the mass flow it can handle. Added meaning can be given to the dimensionless mass flow by expressing it as a Mach number

in the appropriate high, medium or low pressure ports. It then has some significance as a measure of the strength of the pressure waves in the cycle without losing any value as a mass flow criterion, and is at the same time a record of the dimensionless velocity through the ports.

3.2.2 Criteria Of Cooling Effect:

When dividers are to be used as coolers it is particularly important to find appropriate criteria with which to express their cooling effect.

For a machine having an idealised performance which is isentropic, a criterion of cooling effectiveness is the isentropic expansion efficiency η_{EIS} . This has the advantage that, being one of the two alternatives suggested in section 3.2.1 as a desirable way of expressing cycle temperature ratios, the cooling effectiveness of a potentially isentropic divider can be included in the general performance presentation. Knowledge of η_{EIS} may only be adequate as an assessment of cooling performance when the sole aim is to produce the lowest possible temperature regardless of the fraction of the ingoing fluid which is discharged at that state. Even under these circumstances a somewhat clearer picture can be obtained by weighting η_{EIS} so that it becomes an expression for the dimensionless stagnation temperature drop Θ :

$$\Theta \equiv \frac{T_{M0} - T_{L0}}{T_{M0}} = \eta_{EIS} \left[1 - \left(\frac{P_{L0}}{P_{M0}} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad 3.2-5$$

Where the thermal capacity of the low pressure stream is a more important factor than the mere maximum temperature drop attainable, a more appropriate criterion has been given the symbol Ψ and is defined;

$$\Psi \equiv \frac{C_P \dot{m}_L (T_{M0} - T_{L0})}{C_P \dot{m}_M T_{M0}} = (1-\beta) \eta_{EIS} \left[1 - \left(\frac{P_{L0}}{P_{M0}} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad 3.2-6$$

If Ψ is expressed as a fraction of the corresponding value for an isentropic machine:

$$\frac{\Psi}{\Psi^*} = \eta_{EIS} \left(\frac{1-\beta}{1-\beta^*} \right) \quad 3.2-7.$$

If it is remembered that $\beta \rightarrow 0$ when $\eta_{EIS} \rightarrow 0$, then it can be seen from 3.2-7 that $\frac{\Psi}{\Psi^*}$ can exceed unity, and approaches a maximum value as $\eta_{CIS} \rightarrow 0$ and $\eta_{EIS} \rightarrow 1.0$. Hence it may be possible in a real machine to achieve a greater cooling effect than can be obtained from an isentropic cycle. Equation 3.2-5 indicates that it would not in principle be possible for an irreversible machine to produce a lower minimum temperature than an isentropic one. Orthodox refrigerator-cycle coefficient of performance is of little use as a criterion of divider-cooler performance, the expression involves some details of the performance of the compressor feeding the divider.

3.2.3 Presentation Of Performance

The method chosen for presenting the performance of dividers displays curves of constant β , curves of constant axial component of medium pressure port Mach No. M_M , contours of constant η_{EIS} and sometimes also of constant η_{PIS} on a plane the ordinate of which represents the stagnation pressure ratio P_{H0}/P_{M0} , and the abscissa the stagnation expansion ratio P_{L0}/P_{M0} .

This method has the advantage of not only displaying the performance in terms of the appropriate pressure ratios, but also lines of constant overall pressure ratio P_{H0}/P_{L0} are simply inclined straight lines crossing the P_{H0}/P_{M0} versus P_{L0}/P_{M0} plane.

For cases where the cooling effectiveness is of particular interest curves of constant Ψ and constant Φ are also displayed.

3.3 EQUALISERS

3.3.1 Significance Of Enthalpy Ratio & Product Efficiency

The ratio of the absolute stagnation enthalpies of the fluids entering the high and low pressure intakes of an adiabatic equaliser may be expressed as a function of the isentropic compression and expansion efficiencies, and corresponding pressure ratios:

$$S \equiv \frac{C_{PL} \dot{m}_L T_{L0}}{C_{PH} \dot{m}_H T_{H0}} = \eta_{CIS} \eta_{EIS} \frac{[1 - (P_{H0}/P_{M0})^{\frac{\gamma-1}{\gamma}}]}{[(P_{M0}/P_{L0})^{\frac{\gamma-1}{\gamma}} - 1]} \quad 3.3-1$$

Defining S^* as the enthalpy ratio of an isentropic equaliser:

$$S^* = \frac{[1 - (P_{H0}/P_{M0})^{\frac{\gamma-1}{\gamma}}]}{[(P_{M0}/P_{L0})^{\frac{\gamma-1}{\gamma}} - 1]} \quad 3.3-2$$

And thus from 3.3-1 and 3.3-2;

$$\frac{S}{S^*} = \eta_{PIS} \quad 3.3-3$$

where η_{PIS} is the product of the isentropic compression and expansion efficiencies.

From equations 3.3-1 and 3.3-2 it can be seen that enthalpy ratio rather than simple mass ratio is the more useful criterion for expressing equaliser performance data. The reason for this stems from the introduction of two intake flows having independent temperatures. Enthalpy ratio allows these temperatures to be taken into account, hence it is possible to express enthalpy ratio as a unique function of pressure ratio for an isentropic machine as in equation 3.3-2.

The design of equaliser studied herein discharged a sensibly mixed medium pressure stream, thus the experimental assessment of the isentropic compression and expansion efficiencies from temperature measurement presented serious difficulties. The separate isentropic efficiencies η_{CIS} and η_{EIS} were not therefore regarded as useful

performance criteria.

As for the divider case, specification of a port Mach number serves as combined dimensionless port velocity, mass flow and wave-strength-indication criterion.

3.3.2 Presentation of Performance:

Equaliser performance was displayed as families of curves of constant S , curves of constant axial component of high pressure port Mach No. M_H and sometimes also contours of constant η_{PIS} , all presented on a plane on the ordinate of which is marked the stagnation pressure ratio P_{Lo}/P_{Mo} , and on the abscissa the stagnation pressure ratio P_{Ho}/P_{Mo} .

Lines of constant overall pressure ratio P_{Ho}/P_{Mo} are simply inclined straight lines crossing the P_{Lo}/P_{Mo} versus P_{Ho}/P_{Mo} plane. The method of presentation is therefore generally similar to that chosen for the divider case.

4. THEORY

4.1 OUTLINE OF THEORETICAL APPROACH

Theoretical performance analyses have been undertaken at three levels.

- (1) A simple analysis using $a-u$ diagrams and isentropic performance relations in combination to establish complete, hypothetical, isentropic performance characteristics for dividers and equalisers.
- (2) A slightly more involved procedure especially developed to predict pressure-exchanger performance from available experimental results for basic pressure-exchanger cell filling and cell emptying processes. This theory makes allowances for cycle irreversibilities; although inherently less precise, it is much more easily and more quickly applied than the method-of-characteristics.
- (3) Method-of-characteristics analyses of divider and equaliser performances. These analyses have been so arranged that the relative effects on performance of cell width, leakage, and friction become apparent.

The characteristic calculations are particularly rewarding as they allow wave events within the machine to be studied, as well as permitting predictions to be made of port geometry for optimum performance.

The three theoretical methods will be dealt with in the foregoing order.

4.2 PREDICTION OF HYPOTHETICAL ISENTROPIC PERFORMANCES:

The isentropic performances predicted by the methods described in the following two sub-sections are essentially hypothetical. The use of $a-u$ diagrams implies that isentropic pressure wave cycles should be conceivable in principle if irreversibility effects were absent. In fact finite steep

isentropic compression and expansion waves, as implied by the application of α - u diagrams to p.e. cycles, are not generally possible for well established reasons, hence the qualification "hypothetical" appearing in the title of this section.

The means used to locate the constant β^* (divider) and constant S^* (equaliser) curves on the $\frac{P_{Ho}}{P_{Mo}} - \frac{P_{Lo}}{P_{Mo}}$ plane are correct for any isentropic divider or equaliser cycle, pressure exchanger or otherwise.

The calculations serve to reveal some of the inherent limitations of p.e. dividers and equalisers shorn of the complexities of reality.

4.2.1 Divider

The hypothetical isentropic performance of a pressure-exchanger divider was predicted from the results of two independent sets of calculations. The mass ratio β^* was evaluated for chosen $\frac{P_{Ho}}{P_{Mo}}$ and $\frac{P_{Lo}}{P_{Mo}}$ from equation 3.2-4. The Mach numbers in the ports of the isentropic pressure-exchanger divider were established from the geometry of appropriate α - u diagrams. The procedure is explained in more detail in Appendix III section (a), wherein will be found derivations of the required expressions.

For reasons given in Appendix III it was deduced that the flow Mach number in the LP outlet port could not exceed unity. It was also deduced that, for a given value of $\frac{P_{Lo}}{P_{Mo}}$, $\frac{P_{Ho}}{P_{Mo}}$ was a maximum when both the medium and low pressure port Mach numbers were each unity. These factors served to impose a limit on the range of pressure ratios for which simple pressure-exchanger dividers are in essence suitable.

Isentropic divider performance characteristics on the $\frac{P_{Ho}}{P_{Mo}} - \frac{P_{Lo}}{P_{Mo}}$ plane are shown in Fig. 12.1-1. Contours of constant supersonic high pressure and/or medium pressure port Mach numbers have been omitted from

the diagram as such contours would be superimposed on the subsonic region enclosed by the $M_M=1$ line and the P_{H0}/P_{M0} and P_{L0}/P_{M0} axes.

The constant β^* curves have been extended, with solid lines, to illustrate the performance of any isentropic divider-like device over a greater range of pressure ratios than lie within the pressure-exchanger operating regime.

Fig.12.1-2 shows the cooling performance of an isentropic divider cycle. The $M_M=1$ contour marking the boundary, to the right of which lies the hypothetical isentropic pressure-exchanger divider performance regime, has been added to the diagram.

4.2.2 Equaliser

The hypothetical isentropic performance of a pressure-exchanger equaliser was predicted by a similar procedure to that outlined for dividers in the previous section. Details of the procedure, as well as derivations of the expressions required, are given in Appendix III, section (b).

From the analysis it was deduced that the Mach number in a medium pressure outlet port could not in general exceed unity. As a consequence of this and of the geometry of the $Q-U$ diagrams the maximum Mach numbers in the HP and LP inlets were also restricted to unity. The Mach number limitations impose restrictions on the range of pressure ratios for which pressure-exchanger equalisers are suited.

The idea of circumventing the foregoing restrictions and working with greater pressure ratios for the particular case $M_H=M_M$, $M_L=0$, by means of making M_H supersonic can be shown to be impractical when the effects of expansion wave fanning are taken into account by the full method-of-characteristics.

Fig.12.1-10 shows the performance characteristics, on the $\frac{P_{L0}}{P_{M0}} - \frac{P_{H0}}{P_{M0}}$ plane, of a hypothetical isentropic pressure-exchanger equaliser cycle. The entire performance regime lies within a roughly triangular region bounded by the $\frac{P_{L0}}{P_{M0}}$ and $\frac{P_{H0}}{P_{M0}}$ axes and the $M_M=1$ contour. The curves of constant S^* have been extended over a larger area than that covered by the isentropic pressure-exchanger equaliser performance curves.

4.3 SIMPLE THEORY FOR PREDICTING PRESSURE EXCHANGER PERFORMANCE INCLUSIVE OF IRREVERSIBLE EFFECTS:

A simple theory was developed for making rapid, approximate pressure-exchanger cycle performance predictions. The method was based on the following procedure involving four main steps; only the final step needs to be repeated during the course of cycle calculations.

- (1) The development of a generalised form of energy equation applicable to pressure-exchanger processes involving (nominally) only a single finite pressure wave.
- (2) Determination from experimental cell filling and cell emptying data, used in conjunction with the generalised form of the energy equation, of static pressure ratio (ϕ) versus static temperature ratio (τ) relations for compression and expansion waves.
- (3) Establishment, with the aid of the generalised energy equation and τ versus ϕ relations, of static pressure ratio versus Mach number curve families for compression and expansion wave processes.
- (4) Pressure exchanger cycle calculations performed with the assistance of the $\tau - \phi$ curves referred to in (2) and the $\phi - M$ curve families mentioned in (3). Details of the particular cycle

calculation procedures for dividers and equalisers are given in section 4.3.1 and 4.3.2 respectively.

The fundamental assumptions are:

- (a) fluid states and velocities throughout each cell are taken as uniform at the commencement and completion of each single wave process.
- (b) there is no composition or temperature discontinuity at the interface between fluid entering during a filling process and the original contents of the cells.
- (c) velocity distribution assumed to be uniform across each port.

NB. restriction (b) may be relaxed and (a) amended accordingly at the price of more tedious cycle calculations.

The following assumptions are implied as consequences of the three fundamental assumptions already quoted:

- (d) end effects due to wave reflection at boundaries are absent; this is a consequence of assumption (a).
- (e) pressure waves are infinitely thin in the vicinity of boundaries.

This assumption arises from the consideration that the flow velocity near a solid boundary must be zero in a direction normal to the boundary, or the normal velocity must be uniform (e) if the wave overlaps a port. Since end effects are ruled-out (d) neither of these situations is compatible with a gradual change of normal velocity across a thick wave.

Details will now be given of the four steps in the theoretical procedure.

Step (1): Applying the energy equation to the boundaries of either of the regions shown in Fig.4.3-1

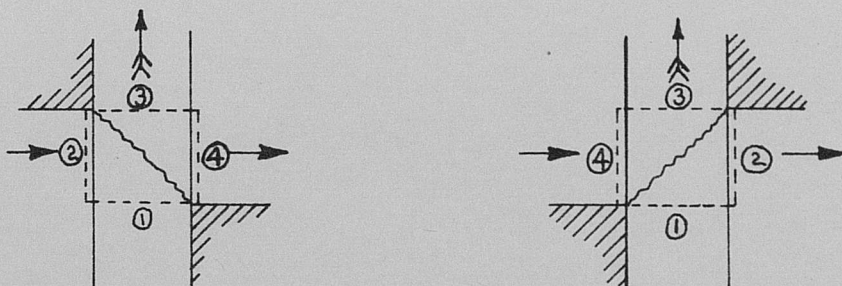


Fig. 4.3-1

leads, provided use is made of the foregoing assumptions, to the following common expression:

$$\frac{(\gamma-1) \left[1 - \frac{P_4}{P_2} \right] - \dot{Q}}{\gamma \left\{ \left[1 + \left(\frac{\gamma-1}{2} \right) M_2^2 \right] - \left[1 + \left(\frac{\gamma-1}{2} \right) M_4^2 \right] \right\}} = \frac{\left[\frac{P_4 T_2}{P_2 T_4} - 1 \right]}{\left[1 - \frac{P_4}{P_2} \sqrt{\frac{T_2}{T_4}} \frac{M_4}{M_2} \right]} + 1 \quad 4.3-1$$

A derivation of equation 4.3-1 is given in Appendix IV.

When $M_2=0$, 4.3-1 reduces to:

$$1 = \gamma \left[1 + \left(\frac{\gamma-1}{2} \right) M_4^2 \right] \frac{T_4}{T_2} - (\gamma-1) \frac{P_4}{P_2} - \dot{Q} \quad 4.3-2$$

Equation 4.3-2 applies to the first part of simple cell emptying (L.H. sketch) or the first part of simple cell filling (R.H. sketch).

When $M_4=0$, 4.3-1 becomes:

$$\frac{P_4 T_2}{P_2 T_4} \left[\gamma \left(\frac{\gamma-1}{2} \right) M_2^2 + \gamma - \frac{T_4}{T_2} \right] = (\gamma-1) - \dot{Q} \quad 4.3-3$$

Equation 4.3-3 is applicable to the latter part of simple cell filling (L.H. sketch) or the latter part of simple cell emptying (R.H. sketch).

Step (2): The adiabatic forms ($\dot{Q}=0$) of equations 4.3-2 and 4.3-3 were used in conjunction with experimental data and the well known isentropic relationship linking flow Mach number with static stagnation pressure ratio to enable the $\Upsilon-\sigma$ relationships shown on Fig. 15.4-1 to be deduced. The experimental cell filling and cell emptying data were similar to those shown on Fig. 4 of reference (16), with the addition of Mach number information.

Step (3): The curve families shown in Fig. 15.4-2 to 15.4-5 inclusive were constructed from evaluation of equation 4.3-1, use being made of the

appropriate τ - σ relation on Fig.15.4-1. The choice of the particular τ - σ relation in the evaluation of 4.3-1 was governed by the nature of the wave process under consideration; whether it was akin to the first or second part of filling, or first or second part of an emptying process. The τ - σ relation used in its construction, and the process to which it applies are written on each curve family.

Curves Fig.15.4-1 to 15.4-5 inclusive form the basic tools needed for cycle calculations, within the limits of the previously stated assumptions (a) to (e), when air is used as the working fluid.

Step (4) is described for dividers and equalisers respectively in subsections 4.3.1 and 4.3.2.

4.3.1 Dividers:

When applying the general theory to particular cycle calculations it is sometimes helpful to draw pressure ratio - Mach number (σ - M) and temperature ratio - Mach number (τ - M) diagrams as well as a t - x diagram. These have been drawn for a typical divider cycle in Fig.4.3-2.

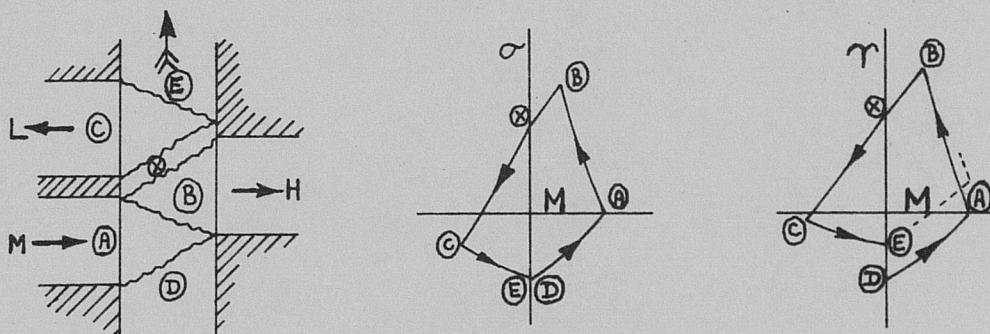


Fig.4.3-2

Because the model's waves are infinitely thin (at least at the boundaries) and because considerations based on the prevention of direct leakage between MP inlet and LP outlet call for the provision of a finite land between these ports, expansion waves RX and XC were treated as distinct

waves. The space between them enclosed stationary fluid at state X.

It may be noticed that the Υ -M diagram has been drawn in such a way that the temperature of fluid at state E is apparently greater than that of the fluid initially in the cell at state D. For a leak free adiabatic machine the actual temperature and pressure of fluid in the cells at the commencement of a wave cycle will clearly be those prevailing at the completion of the previous cycle. In the model this state of affairs is incompatible with assumption (a) of section 4.3. This complication was resolved by assuming (correctly) that $\sigma_D = \sigma_E$, and that Υ_D and Υ_E were as shown on the Υ -M diagram. These assumptions imply that, taking into account the constancy of cell volume, the internal energy of the contents of a cell at state E is equal to that of a cell at state D.

Compared with the real situation, the model therefore discharges too great a mass flow at too low a temperature from the ~~region of the~~ HP port. ~~to the left of the demarcation line shown on the sketch.~~ On an energy balance basis the model is similar to the real situation except for quite small second order effects. The nature of these is explained in more detail in the section on the method-of-characteristics.

By determining the mass ratio β from the relative mass flow in the medium and low pressure ports, it was possible to predict the correct mass ratio corresponding to the real circumstances. The appropriate expression for β , the derivation of which is given in Appendix IV, is:

$$\beta = 1 + \frac{T_D}{T_E} \left[\frac{\frac{P_X T_E}{P_E T_X} - 1}{\frac{P_B T_D}{P_D T_B} \left\{ 1 - \frac{\sqrt{\frac{T_D}{T_A}} M_B}{\frac{T_A}{T_B} M_A} \right\} - 1} \right] \quad 4.3-4$$

Pressure-exchanger divider performance characteristics were therefore deduced from appropriate use of Fig.15.4-1 to 15.4-5, equation 4.3-4 and

Bernoulli's equation for compressible flow. The range of pressure ratios over which predictions could be made were restricted by the limits of the experimental cell filling and cell emptying data; these in turn restricted the range of the γ - α relationships. Fig.12.1-3 shows pressure-exchanger divider performance characteristics predicted by the foregoing method.

4.3.2 Equaliser:

The predictions of pressure-exchanger equaliser performance were carried out by a generally similar procedure to that used in the previous section. The appropriate t - x , α - M , and γ - M diagrams are shown on Fig.4.3-3.

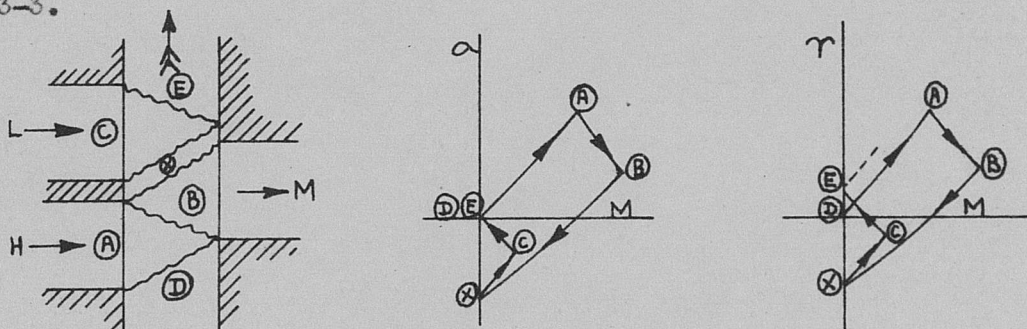


Fig. 4.3-3

As for the divider case, fluid at state X has been specified as stagnant; this is consistent with the assumption of infinitely thin waves and a land of finite width between the HP and LP ports. Examination of equaliser-cycle wave diagrams produced by the method-of-characteristics shows that the supposition of stagnant fluid at state X is not, for cycles involving strong waves, far removed from the much more complex situation occurring in reality.

The difference between γ_D and γ_E occurs for exactly similar reasons to those described in section 4.3.1. The expression for $\bar{S}$ is given by:

$$\xi = \frac{T_{co}}{T_{ao}} \cdot \frac{T_D}{T_E} \cdot \left[\frac{1 - \frac{P_x T_E}{P_E T_x}}{\frac{P_B T_D}{P_D T_B} \left\{ \frac{1 - \sqrt{\frac{T_B}{T_A}} \frac{M_B}{M_A}}{1 - \frac{P_B}{P_A} \sqrt{\frac{T_A}{T_B}} \frac{M_B}{M_A}} \right\} - 1} \right] \quad 4.3-5$$

Appropriate use of Fig.15.4-1 to 15.4-5, equation 4.3-5 and the well known relations resulting from application of the energy equation to isentropic steady flow processes, allowed pressure-exchanger equaliser performance characteristics to be determined. These are displayed on Fig.12.1-11.

4.4 APPLICATION OF THE METHOD-OF-CHARACTERISTICS TO THE STUDY OF DIVIDERS AND EQUALISERS:

4.4.1 Scope of the Characteristic Calculations:

The aims of the characteristic calculations and their ramifications were to:

- (1) predict optimum performances and corresponding port geometries rather than attempt to reproduce, by theoretical means, the experimental performance results.
- (2) produce wave diagrams from which an understanding of cycle wave actions could be obtained.
- (3) provide an indication of the relative importance of the effects on performance of:
 - (a) wave events
 - (b) cell width
 - (c) leakage
 - (d) entry, exit and cell wall friction losses.

4.4.2 Fundamental Assumptions:

The following assumptions apply to all the characteristic calculations for finite and infinitely narrow cells.

- (1) Residual pressure waves trapped in the cells near the completion of each cycle were assumed to decay adiabatically and allow the contents of the cells to become quiescent before the commencement of the succeeding cycle.
- (2) Cell partitions were assumed to be infinitely thin. In the case of finite cell width calculations, cell partition thickness was taken into account by a correction procedure applied after completing the characteristic calculations.
- (3) Total head pressure was assumed to be uniform across inlet ports. The total head pressure in a port, before allowing for frictional effects, was taken to be that corresponding to steady isentropic flow when the cell end is fully exposed to the port.

This assumption is consistent with homentropic flow from the reservoirs supplying the inlet ports.

- (4) Static pressure was assumed to be uniform across an outlet port at a valve, before allowing for frictional effects, equal to that prevailing in the cell ends with unchoked flow when these are in full communication with the port. This assumption is consistent with the anticipated behaviour of discharge ports connected to large reservoirs; it is also consistent with streamlines remaining parallel in outlet ducts. The more complicated circumstances likely to prevail in practice are discussed in Appendix VI.

4.4.3 The Adaptation and Extension of Method-of-Characteristic Procedures:

The adaptation of the method-of-characteristics to the study of divider and equaliser cycles involved the following steps.

- (1) Development of a means for allowing the method-of-characteristics

applied to one-dimensional, unsteady, isentropic flow to be used for the study of complete wave cycles involving non-isentropic processes.

- (2) Establishment of a procedure to enable the Spalding - Jenny method (16) of catering for finite cell width effects to be generalised. The procedure allowed Jenny's boundary chart to be used for any appropriate application regardless of the reference (or datum) conditions of the wave diagrams.
- (3) Development of a means for taking into account the finite width of the cell partitions when correlating the flow conditions in the cell ends with the apparent steady flow Mach numbers in the ports.
- (4) Derivation of a means for taking leakage effects into account. The leakage theory was contrived to be applicable to otherwise complete characteristic calculations carried out independently of leakage.
- (5) Development of a method for modifying calculations, otherwise complete, to take account of viscosity effects.

The fundamentals of the method-of-characteristics for 1D unsteady flow are described by Jenny (9). The application of characteristics to pressure-exchangers are fully explained by Spalding (16, 17), hence no attempt will be made here to cover this aspect of the theoretical treatment.

Steps (1) to (5) will be described as these have not, to the author's knowledge, been reported elsewhere.

Step (1)

Complications arise when the method of characteristics for isentropic unsteady flow is used to predict the performance of wave cycles involving an increase of entropy due to the decay of residual waves trapped in the cells

during the latter part of each wave cycle. Several ways of avoiding these difficulties are listed.

- (a) To assume that the residual waves do not decay; this has the disadvantage of requiring several wave cycles to be drawn for each case to satisfy cycle end conditions. The calculations are tedious, and probably not particularly accurate, as at least some wave decay will occur during a cycle.
- (b) To allow for the difference in entropy between new gas entering the cells and the original cell contents by applying isentropic characteristic theory in each zone of uniform entropy. This is a complicated process requiring the simultaneous application of characteristics in the two, or more, regions.
- (c) To use the method-of-characteristics adapted to allow for irreversible effects, and employed in conjunction with (b). This is probably the most accurate possible procedure, but it is also the most complicated.
- (d) Use isentropic characteristic theory in conjunction with the assumption that the fluid left in the cells at the completion of a wave cycle may be regarded as replaced by fresh fluid, at the same pressure, having an entropy the same as that of the fluids in the ports of the machine.

Method (d) was adopted for the Simple Theory and was described in section 4.3.1; it was also used for all characteristic calculations. It has the advantage of being the most easily applied of the schemes listed.

The method can be expected to lead to accurate performance predictions provided only small wave effects would have arisen from the temperature discontinuities in the cells if those had not been ignored. The accompanying

diagram, Fig.4.4-1, shows two typical divider cycles. In the left hand sketch all the fluid in the cell at the commencement of the cycle is discharged from the HP port. In the right hand diagram some fluid has been shown recirculating. ABC and its extension DEF are the demarcation lines marking boundaries of regions of different temperatures.

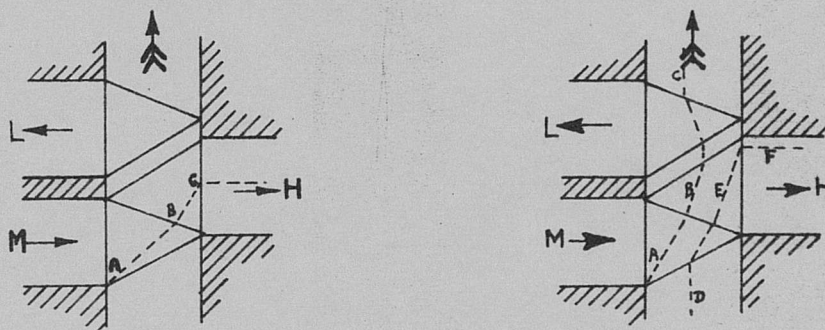


Fig. 4.4-1

Over the maximum range of the calculations covered here, the difference in absolute temperature of the gases on either side of a demarcation line is not more than about 10%. There is therefore in the worst case only a difference of 5%, or thereabouts, in the sonic velocities of the gases on either side of a demarcation line.

The waves reflected at the interfaces between the regions of different temperatures are thus very weak, and are about 2% of the strength of incident waves. A transmitted wave is about 98% of the strength of its incident wave when the incident wave is travelling in the colder gas. These results were obtained from evaluation of equations (7) and (8) of Appendix IX of a report by Spalding (17).

It can thus be seen that in fact very little error is likely from the kind of approximation first mentioned in section 4.3.1, and referred to here as proposal (d).

As described in section 4.3.1, initial and final cell pressures and hence internal energies are assumed to be equal. These conditions are specified

if the following dimensionless energy equations are satisfied.

For dividers:

$$\int_1^2 M_{OZM} \alpha_{ZM} \left(\frac{P_{ZM}}{P_c} \right) d\tau - \int_1^2 M_{SL}^+ \alpha_L \frac{P_L}{P_c} \left[1 + \frac{\gamma-1}{2} \left(\frac{M_{SL}^+}{\phi} \right)^2 \right] d\tau - \int_1^2 M_{SH}^+ \alpha_H \frac{P_H}{P_c} \left[1 + \frac{\gamma-1}{2} \left(\frac{M_{SH}^+}{\phi} \right)^2 \right] d\tau = 0 \quad 4.4-1$$

For equalisers:

$$\int_1^2 M_{OZH} \alpha_{ZH} \left(\frac{P_{ZH}}{P_c} \right) d\tau + \int_1^2 M_{OZL} \alpha_{ZL} \left(\frac{P_{ZL}}{P_c} \right) d\tau - \int_1^2 M_{SM}^+ \alpha_M \frac{P_M}{P_c} \left[1 + \frac{\gamma-1}{2} \left(\frac{M_{SM}^+}{\phi} \right)^2 \right] d\tau = 0 \quad 4.4-2$$

Each integral expression in these equations represents the sum of the enthalpy and kinetic energy flowing into or out of the high, medium or low pressure ports of the cycle. A derivation of equations 4.4-1 and 4.4-2 is given in Appendix V Part 1. In practice, each wave diagram was adjusted iteratively to satisfy equation 4.4-1 or 4.4-2.

For reasons explained in section 4.3.1, the expression for β was written in such a way that direct reference to the flow in the HP port was avoided, the appropriate expression is:

$$\beta = 1 - \frac{\int_1^2 M_{SL}^+ \alpha_L \left(\frac{P_L}{P_c} \right)^{\frac{\gamma}{\gamma-1}} d\tau}{\int_1^2 M_{OZM} \alpha_{ZM} \left(\frac{P_{ZM}}{P_c} \right)^{\frac{\gamma}{\gamma-1}} d\tau} \quad 4.4-3$$

For equalisers the expression for stagnation enthalpy ratio S is:

$$S = \frac{\int_1^2 M_{OZL} \alpha_{ZL} \left(\frac{P_{ZL}}{P_c} \right) d\tau}{\int_1^2 M_{OZH} \alpha_{ZH} \left(\frac{P_{ZH}}{P_c} \right) d\tau} \quad 4.4-4$$

The derivation of these expressions is also given in Appendix V Part 1.

Step (2)

Orthodox methods of coping with finite cell width effects are not easily applicable to pressure exchanger cycle calculations involving filling and emptying processes at various pressures.

The Jenny-Spalding methods can be applied individually to all the opening and closing processes included in a complete cycle. This however

leads to complications due to the use of several different reference states on the same wave diagram. This difficulty can be circumvented by specially constructing filling and emptying boundary curves similar to Jenny's Fig. 23 (16), but in this case all are referred to an arbitrary (common) reference state used for the construction of the wave diagram. A state diagram, with superimposed Jenny boundary curves redrawn in the way described, is shown in Figure 4.4-2.

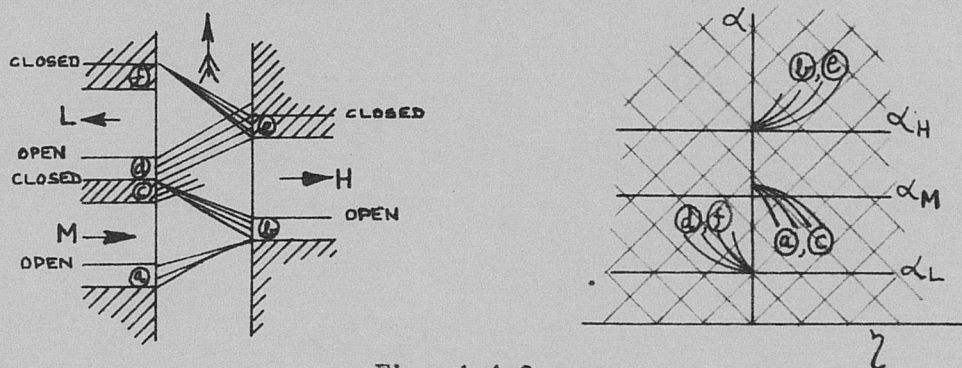


Fig. 4.4-2

In practice the idea shown in the illustration is tedious to apply, as it is necessary to reconstruct some or all of the boundary curves, when calculations are repeated for different pressure ratios. This disadvantage can be eliminated by using a lattice state diagram in conjunction with a generalised procedure employing Jenny's boundary curves. An additional chart, termed a scaling diagram, can be used as a link to connect the orthodox state, wave and Jenny diagrams.

The generalised procedure uses Jenny's original boundary curve equations and makes use of the following relations for filling and emptying processes.

For filling processes:

$$\frac{u}{a_z} = \frac{\lambda}{\alpha_z} \quad 4.4-5$$

$$\frac{a}{a_z} = \frac{\alpha}{\alpha_z} \quad 4.4-6$$

$$\text{where: } \eta \equiv \frac{u}{a_{\text{REF}}}, \quad \alpha \equiv \frac{a}{a_{\text{REF}}}, \quad \alpha_z \equiv \frac{a_z}{a_{\text{REF}}} \quad 4.4-7$$

and a_z = sonic velocity at stagnation conditions in a filling port

a_{REF} = sonic velocity at the reference conditions to which the state and wave diagrams are referred.

For cell emptying

$$\frac{u}{a_{os}} = \frac{\eta}{\alpha_{os}} \quad 4.4-8$$

$$\frac{a}{a_{os}} = \frac{\alpha}{\alpha_{os}} \quad 4.4-9$$

$$\text{where: } \eta \equiv \frac{u}{a_{\text{REF}}}, \quad \alpha \equiv \frac{a}{a_{\text{REF}}}, \quad \alpha_{os} \equiv \frac{a_{os}}{a_{\text{REF}}} \quad 4.4-10$$

and a_{os} = sonic velocity at static pressure P_s after isentropic expansion, or compression, from cycle inlet conditions.

As a consequence of the assumptions 4.4.2, (3) & (4), α_z and α_{os} are constant in an inlet or outlet port respectively.

The L.H. sides of equations 4.4-5, 4.4-6, 4.4-8 and 4.4-9 are the dimensionless coordinates of Jenny's boundary curves. Use of equations 4.4-5 to 4.4-10 inclusive allows Jenny's Fig.23 to be used as boundary curves for wave diagrams constructed with any a_{REF} .

Jenny's boundary equations are, for the sake of completeness, listed in Appendix V Part 1. Fig.15.5-2 is a scaling diagram suitable for linking a lattice state diagram, such as that by Spalding (17), with Jenny's boundary curves.

Step (3)

Isentropic relationships were established between conditions in the cell ends and the steady flow assumed to occur in the ducts of the machine. The relations take into account the effects of the thickness of the cell partitions.

The results were obtained from application of the 1st law, energy and continuity equations to fluid leaving the cells under non-steady flow conditions and flowing steadily in the ports and ducts. The relationship for a filling process is:

$$\bar{M} = K \left\{ 1 + \left(\frac{\gamma-1}{2} \right) \bar{M}^2 \right\}^{\frac{\gamma+1}{2(\gamma-1)}} \frac{\int_1^2 M_{oz} d\tau}{\tau_p} \quad 4.4-11$$

For cell emptying:

$$\bar{M} = K \left\{ \frac{1 + \left(\frac{\gamma-1}{2} \right) \bar{M}^2}{1 + \left(\frac{\gamma-1}{2} \right) M_{AV}^2} \right\}^{\frac{\gamma+1}{2(\gamma-1)}} M_{AV} \left(\frac{\tau_T}{\tau_p} \right) \quad 4.4-12$$

In both cases $\bar{M}$ is the duct steady-flow Mach number. K is the projected free flow area through the cells communicating with a port, divided by the port area; K is therefore a parameter dependent on cell partition thickness. Equations 4.4-11 and 4.4-12 are derived in Appendix V Part II.

Step (4):

A theory already existed (18) for predicting the effect of leakage in pressure-exchangers having large lands or blank regions between sets of ports. In that theory no special consideration was given to leakage occurring around the edges of the ports, etc. This was justifiable as the ports were considered to occupy a relatively small proportion of the total stator end face area.

In pressure-exchangers of the types being considered here most of the area of the stator end faces is occupied with ports to ensure the greatest possible throughput for a given size of machine. A theory has therefore been developed for predicting the effects of leakage occurring in the vicinity of the ports of pressure-exchangers. The new theory used in conjunction with the earlier method should allow leakage to be fairly fully studied in all conceivable designs of orthodox machines.

The theory to be outlined here involves the creation of a simplified

leakage model of the cycle being studied. The first of the following sketches, Fig.4.4-3, purports to illustrate the real leakage situation in either a pressure exchanger divider or equaliser. Fig.4.4-4 shows the simplified leakage model for these cycles.

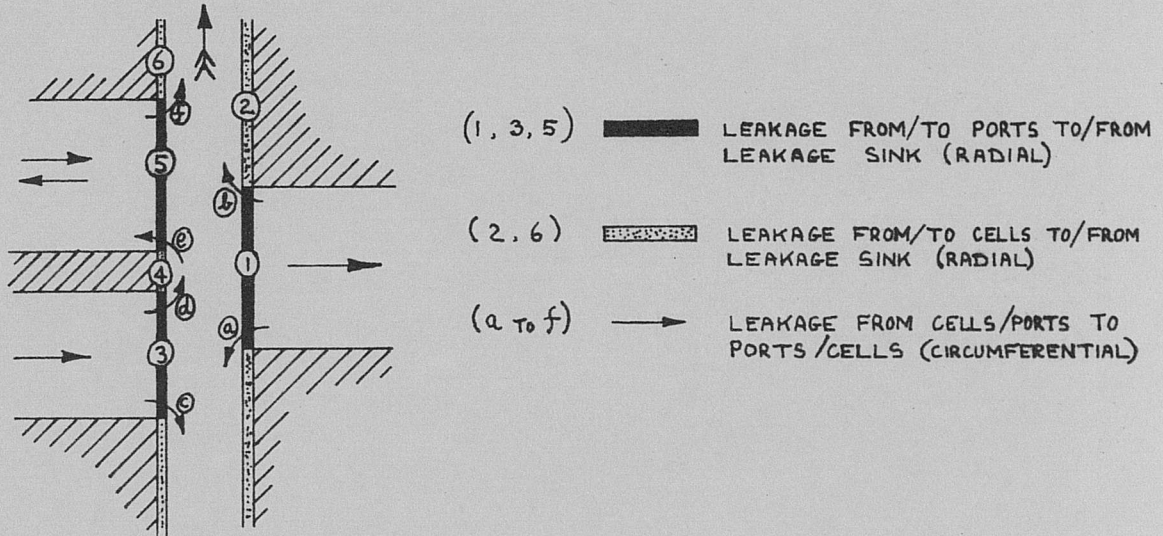
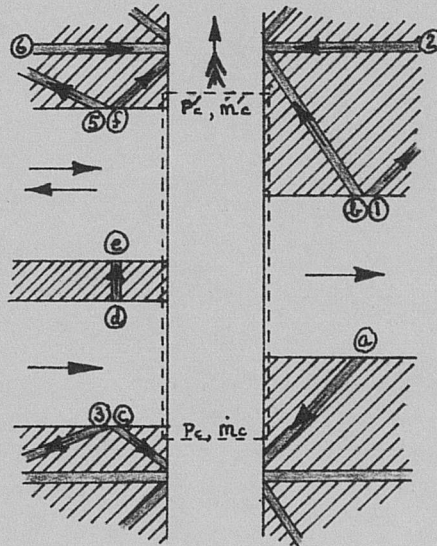


Fig. 4.4-3.

The leakage model corresponding to the situation in the foregoing sketch is:



NB. The number and letter code corresponds to that shown on the diagram of the real situation.

Fig. 4.4-4.

The dotted boundary in the second sketch marks the perimeter of the region within which wave diagram calculations were applied. The leakage model has been so arranged that all leakage flows pass around the outside of the dotted boundary.

The assumptions made to simplify the leakage computations based on the model were:

- (1) for the purposes of computing leakage quantities the cell pressures adjacent to the blanked-off parts of the endplates were taken to have the uniform value P_c .
- (2) the duct static pressure distributions in the vicinity of the leaks in the HP, MP and LP ducts were assumed to be uniform. Leakage was taken to be a function of the static pressure ratio across a leakage channel.
- (3) flows in the leakage channels were assumed to be intrinsically isentropic, viscosity and vena-contracta effects being allowed for by the use of suitable flow coefficients. Justification for this approach is given by Kearton & Keh (10).

Because leakage effects were relatively small compared with wave effects, the approximations inherent in the leakage model, and in the list of simplifying assumptions, were not deemed likely to introduce serious error into the overall performance predictions. An indication of the difficulties in making accurate leakage predictions with more simple geometries than those considered here is given by Zabriskie & Sternlight (22).

The leakage effects were considered to modify the performance predicted without taking leakage into account in two ways.

- (I) Due to the nett leakage into (or out of) the cells between adjacent wave cycles, the ratio $\frac{P'_c}{P_c}$, normally set to unity

when interpolating from wave diagrams, had in general a value slightly different to unity. This was determined from the requirement that the cell pressures at the commencement of each cycle were identical when leakage was allowed for.

- (II) Due to leakage from the ducts into the leakage sink, and between ducts and cells, the mass flows and mass ratio were not in general the same as those predicted without taking account of leakage.

The value of P'_c/P_c is computed from

$$\left[1 - \frac{P'_c}{P_c}\right] = \gamma \left\{ \gamma_c \sum \frac{L P_L}{K P_c} \alpha_l C \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{P_x}{P_L} \right)^{\frac{2}{\gamma}} - \left(\frac{P_x}{P_L} \right)^{\frac{\gamma+1}{\gamma}} \right]} + \sum \frac{\gamma L}{\lambda} \alpha_l C_L \sqrt{\frac{F}{n\gamma} \left[\left(\frac{P_L}{P_c} \right)^2 - \left(\frac{P_L}{P_c} \right)^2 \right]} \right\} \quad 4.4-13$$

The ratio of the sum of the mass flows leaking into a duct divided by the mass flow carried in the cells is given by:

$$\frac{\sum \dot{m}_l}{\dot{m}_c} = \left\{ \gamma_c \sum \frac{L P_L}{K P_c} \frac{\alpha_c^2}{\alpha_l} C \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{P_x}{P_L} \right)^{\frac{2}{\gamma}} - \left(\frac{P_x}{P_L} \right)^{\frac{\gamma+1}{\gamma}} \right]} + \sum \frac{\gamma L}{\lambda} \frac{\alpha_c^2}{\alpha_l} C_L \sqrt{\frac{F}{n\gamma} \left[\left(\frac{P_L}{P_c} \right)^2 - \left(\frac{P_L}{P_c} \right)^2 \right]} \right\} \quad 4.4-14$$

Data for the modification of leak-free wave diagram calculations were obtained from fresh interpolation procedures to suit the new values of P'_c/P_c . Alternatively finite difference methods may be used, but these appear to offer no special advantages. Derivations of equations 4.4-13 and 4.4-14 are given in Appendix V Part III, together with details of the application of the leakage theory to divider and equaliser cycles.

Step (5)

Viscous effects, those arising from cell wall friction and cell entry and exit losses, were taken into account by an approximate procedure. The method was designed to be applied as a correction to characteristic calculations already modified to allow for leakage. The justification for this technique and its inherent approximations lay in the comparative

insignificance of frictional effects over the range of pressure ratios for which characteristic calculations were made.

The method, which uses steady flow friction data, involved the following procedure.

- (I) The evaluation, with the aid of a suitable reference source (19) of the steady flow pressure loss due to wall friction in one cell length.
- (II) Obtainment, from a suitable reference (20) of the steady flow pressure losses at cell entry and exit.
- (III) The addition of half the wall friction loss (I) to either cell entry or cell exit losses, the results were expressed as multiples of the dynamic head corresponding to the flow velocity in the cells.
- (IV) Frictional effects were taken into account by increasing inlet, and decreasing outlet total head pressures by the appropriate amounts specified in (III).

The factor of one half mentioned in (III) is to take account of the division of the flow losses between an inlet and an outlet port during a scavenging process. For simple cell filling and emptying processes the "half" allows for the average position of the wave fronts, and hence velocity fronts, lying half way along the length of a cell.

The foregoing procedure is an attempt to establish a simple way of approximating the real circumstances; the method has the virtue of being very easy to apply. A model of the viscosity correction procedure applied to a divider is shown in Fig.4.4-5; the fictitious throttles in the inlets and outlets accommodate the pressure losses associated with cell wall friction, entry and exit losses.

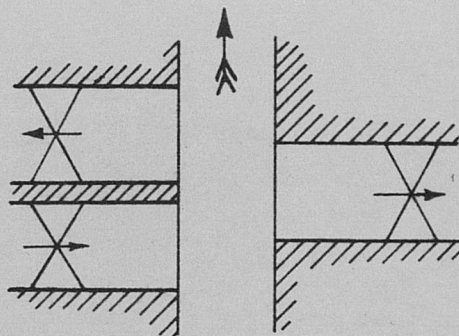


Fig.4.4-5

A derivation of the equations used in the application of the method to dividers and equalisers is given in Appendix V Part IV. More sophisticated and more laborious ways of taking viscous effects into account are given in references (9) (11) and (12).

4.4.4 Calculation Procedure

The application of the method of characteristics to satisfy the objectives specified in section 4.4.1 involved carrying out the following procedures.

- (1) A "learning" process using, for the sake of simplicity, infinitely narrow cell characteristic calculations. The calculations were intended to establish general rules to indicate at what points in the cycle the divider HP and LP outlets and equaliser MP outlet and LP inlet should be opened and closed for best performance. The conclusions were that in all cases the ports should be opened as soon as possible, and closed as late as possible, consistent with the avoidance of flow reversal.

The maximum period for which divider MP and equaliser HP inlets were permitted to remain open was governed by wave events. No study was made of the effects of closing these ports earlier or later than dictated by the arrival of reflected waves.

- (2) The construction of optimum performance characteristics on the P_{Ho}/P_{Mo} versus P_{Lo}/P_{Mo} planes from wave diagrams drawn for machines with infinitely narrow cells. Fig.12.1-4 shows the divider results, Fig.12.1-12 the equaliser performance.
- (3) Construction of optimum performance characteristics on the P_{Ho}/P_{Mo} versus P_{Lo}/P_{Mo} planes from wave diagrams drawn for finite cell widths, but omitting friction and leakage effects. Fig.12.1-5 and Fig.12.1-13 show the divider and equaliser performances respectively.
- (4) Construction, from the basis of the wave diagrams referred to in (3), of optimum performance characteristics on the P_{Ho}/P_{Mo} versus P_{Ho}/P_{Mo} plane taking account of leakage, but omitting viscosity effects. Fig.12.1-6 shows the divider performance, Fig.12.1-14 the equaliser results.
- (5) Modification of the optimum performance characteristics referred to in (4) to take account of cell wall-friction, entry and exit losses. The modified performance characteristics for dividers and equalisers are shown on Fig.12.1-7 and Fig.12.1-15 respectively.
- (6) The construction of contours of constant port geometry, on the P_{Ho}/P_{Mo} versus P_{Lo}/P_{Mo} planes for the performance curves shown on Fig.12.1-7 and Fig.12.1-15. The geometry contours for dividers are displayed on Fig.12.1-8, and for equalisers on Fig.12.1-16; it may be noticed that cell width is not constant on either figure. This is because the wave diagrams were, for convenience, constructed with Q_H as the reference sonic velocity for both divider and equaliser with

$\frac{t_c a_H}{\lambda} = 0.4$ in both cases. Modification of the results to suit a_{M0} and a_{H0} as the respective divider and equaliser reference states led to the geometries illustrated.

A specially made device designed to assist with the construction of wave diagrams is illustrated in Fig.13-1; it allowed some of the more tedious operations involved in the construction of wave diagrams to be eliminated. In essence the machine enabled a line of known tangent to be drawn on a wave diagram without recourse to the counting of squares, etc.

5. APPARATUS

5.1 SPECIFICATION FOR A SUITABLE TEST RIG:

In section 3.1 it was explained that knowledge of the performance of a divider or equaliser of specified geometry implied knowledge of the rotor speed and of the pressure, temperature, mass-flow and physical properties of fluid passing through the high, medium and low pressure ports. Accordingly the provision of facilities to be incorporated in a rig for experimentally determining the performance of single ended pressure-exchanger dividers or equalisers should arrange for the following features.

- (a) Provision of a pressure-exchanger with suitably arranged ports in which rotor speed and also the relative width of all ports can be varied easily. In addition means should ideally be provided for varying the relative phasing of the sets of ports on opposite ends of the stator.
- (b) Provision of instrumentation for allowing measurements to be made of the mass-flows, pressures, temperatures and also physical properties, if these are not otherwise known, of the fluids passing through the high, medium and low pressure ports.
- (c) Provision of suitable resevoirs of high, medium and low pressure fluid connected to the pressure-exchanger, and means for varying the relative pressures in the resevoirs.
- (d) Provision of a facility to allow the function of the rig to be readily changed from divider to equaliser or vice-versa.

5.2 DESCRIPTION OF THE POWER JETS MACHINE

The pressure-exchanger used for the experiments was designed by the sponsors of the project, Power Jets (Research & Development) Ltd., and was

specially conceived for tests on single ended divider and equaliser cycles. The design centred around a rotor of orthodox pressure-exchanger type; this item was identical to those used in some other Power Jets pressure-exchanger rigs. The rotor, which contained thirty axially disposed cells, was 11 inches in length and 8 inches overall diameter. Provision was made to allow for relative angular movement of the stator end faces, and also for altering the relative port sizes by means of built-in variable width ducts at one end of the machine and by changeable end plates at the other. The rotor was driven at a nominal speed of 6000 RPM by a variable speed d.c. motor via a toothed belt drive and countershaft; the electric drive ensured that the operator was always in control of rotor speed.

In order to make best use of the stator end plates three complete sets of high, medium and low pressure ports were provided; six ports were situated at one end of the machine, the remaining three at the opposite end.

Fig.13-2, a close-up photograph of the pressure-exchanger, illustrates some of the described features.

From the foregoing description it can be seen that the Power Jets pressure-exchanger fulfils the requirements set out in (a) of 5.1. The following sections seek to describe the attendant equipment accompanying the pressure-exchanger, and to show that the rig as a whole satisfied all the basic requirements laid down in 5.1.

5.3 GENERAL ARRANGEMENT OF THE RIG.

The pressure-exchanger described in 5.2 was incorporated in a rig, a photograph of which constitutes Fig.13-3. The pressure-exchanger occupies the lower central part of the picture; the main instrumentation panel can be seen attached to the wall behind the rig. Some of the ducts for leading

the working fluid, air, to and from the machine are also visible. The rig circuits are shown in diagrammatic form on Fig.13-4.

From an examination of Fig.13-4 it is apparent that by confining tests to a divider arrangement of the type shown in Fig.2.4-1 and a equaliser set-up similar to that shown in Fig.2.4-4, the same rig could easily be made to function as a divider or as an equaliser. The only structural change needed to convert from divider to equaliser operation being replacement of the divider low pressure outlet duct by a low pressure metering intake drawing air from the surroundings.

Compressed air driving fluid, at approximately ambient temperature, for both divider and equaliser operation is supplied by the City and Guilds College compressor plant; the maximum output being approximately 4 LB/sec. at pressures up to 5 atmospheres absolute.

5.3.1 Mechanical Construction

The rig's pressurized duct work was made from 6" diameter steam-pipe, the remaining ducts were constructed from light weight tubing of the same diameter. The throttle on the compressed air supply line and also the back pressure control on the divider high pressure outlet (equaliser medium pressure outlet) were 6" bore gate valves.

To prevent straining the pressure-exchanger ball duct work was flexibly connected to the machine by rubber-canvas sleeves. Separation of the pressurized flexible joints was prevented by tie-bars.

The upper face of a welded steel frame of channel and angle sections formed a support for the pressure-exchanger. The rotor driving motor was mounted inside the lower part of the frame-work.

5.3.2 Special Features:

A removable conical mesh type filter was fitted in the compressed air supply duct, as close to the pressure-exchanger as possible, in order to prevent particles of rust, etc. which became detached from the internal surface of the compressed air supply system, finding their way into the machine. This item was added to the original installation in the light of running experience.

A device, specially conceived by Power Jets for the particular rig described here, was a rotatable collecting duct. The duct allowed the three separate streams discharged from the movable stator end plate to be gathered together and passed into a single rigid outlet pipe. The axis of the rotatable portion was coincident with the axis of the pressure-exchanger rotor, and was so designed that rephasing of the pressure-exchanger ports could be carried out whilst the rig was operating. The fixed portion of the collector duct also served as a support for the bearings of a countershaft coaxial with the rotor, and forming part of the transmission system connecting the rotor with the driving motor further down in the frame. The collector duct appears on the lower right hand portion of Fig.13-3.

5.4 INSTRUMENTATION

5.4.1. Brief Description Of The Arrangement:

In essence the instrumentation, which was complete for only one of the three sets of ports in the pressure-exchanger, comprised the following.

- (a) An orifice meter for recording the compressed air mass flow entering the rig.
- (b) An orifice meter in the divider low pressure outlet duct, and for equaliser operation, a bell mouth air meter on the low

pressure intake.

- (c) Temperature measurement in all ducts, except the equaliser low pressure intake, by mercury - in - glass thermometers in stagnation sheaths. The equaliser low pressure flow stagnation temperature was measured in the surroundings near the intake by a mercury - in - glass thermometer.
- (d) Measurement of the mean static pressures in the high, medium and low pressure ports of the machine by a special type of averaging manometer. This design of instrument is described more fully in 5.4.3.
- (e) Removable yaw-meter pitot tubes for installation in any of the tappings in the high, medium and low pressure ports.
- (f) Symmetry devices to ensure that flow conditions were similar in ports of the same kind in each of the three sets of ports with which the pressure-exchanger was endowed.
- (g) Facilities for recording the pressures in cells passing one of the blanked-off portions of the movable stator end plate.
- (h) Instrument panels mounting all manometers and pressure gauges connected to the various meters, tappings, etc. referred to in the foregoing list.

Some of the preceding items were only feasible because the pressure-exchanger was equipped with numerous connections in all ports and in the blanked-off regions of the movable stator end plate. The machine was also fitted with graduated scales allowing the port phasing and width of the variable ducts to be readily deduced.

5.4.2 Constructional Features

All tubing connecting tappings and manometers etc. was of the

transparent plastic kind with the exception of short lengths of neoprene tubing in places where pinch cocks were used. The transparent plastic tubing had the advantages of being durable and easy to install; it also permitted inspection of the inside of the tubing to ensure that condensation, etc. had not accumulated therein.

A necessary precaution when using thermo-plastic tubing is to ensure that it does not become heated; as a safeguard to prevent heat conduction from the pressure-exchanger softening the plastic, the pressure pick-ups have been arranged to serve as thermal insulators. This has been achieved by constructing the connectors in such a way that a few inches of steel tube are interposed between the plastic tube and pressure-exchanger casing.

In order to prevent fluid discharged by accidental over-pressurisation of the manometers from fouling the lines, etc, traps were fitted to all manometers which seemed likely to suffer in this way.

In an effort to prevent the occurrence of manometer over-pressurisation particularly likely whilst starting and stopping the plant, some manometers are equipped with short circuiting devices. These consist of short lengths of neoprene tubing connecting the tops of the two limbs of each manometer. For normal use of the manometers the neoprene tubes are closed-off by pinch cocks; the pinch cocks are slackened when starting and stopping the plant.

5.4.3 Averaging Manometers:

Because of the general non-uniformity of total head pressure in the ports of the pressure-exchanger, an experimental procedure was adopted which involved measurement of static pressures in the ports. It was found that the static pressures were themselves not uniform across the

width of each port; in order to minimize effects arising from this special averaging manometers were developed. More detailed justification for the experimental technique involving static pressure measurements, also an explanation of the observed static pressure gradients appears in section 7.1.3.

A diagrammatic illustration of an averaging manometer appears in Appendix VII together with a proof that such instruments do work.

Three averaging manometers were used on the pressure-exchanger rig for recording static pressures in the high, medium and low pressure ports. Each manometer was designed to record an arithmetic average pressure and was equipped with three stand pipes. The manometers were provided with adjustable graduated scales so arranged that absolute pressures could be read from them.

The author believes that averaging manometers of the kind described in Appendix VII are original.

5.5 SUMMARY OF THE CONSTRUCTIONAL WORK:

The entire rig including the instrumentation, but with the notable exception of the pressure-exchanger and collector duct mounted on their supporting frame containing the driving motor, was designed, or designed and manufactured at Imperial College. Contractors handled the assembly of the heavy 6" duct work, and the wiring-up of the driving motor.

6. OUTLINE OF EXPERIMENTAL PROCEDURE

6.1 DIVIDER TESTS:

Most of the divider tests were aimed at establishing the performance characteristics, at fixed geometry, for presentation on the $P_{H0}/P_{M0} - P_{L0}/P_{M0}$ plane.

Because of the use of averaging manometers, directly recording absolute static pressures in the ports, it was found convenient to gather data whilst keeping one of the port static pressure ratios constant, and allowing the others to vary over the maximum possible range. Purely for ease of operation of the rig it was found to be best to maintain P_L/P_M constant. To enable complete performance characteristics to be plotted experimental data were obtained for several discrete values of P_L/P_M , thereby covering the greatest possible range of performance.

The whole experimental procedure can be visualized more easily by examining some of the performance characteristics deduced from the experiments, Fig. 12.2-1 to 5, and by imagining the ordinates and abscissae to be static rather than total head pressure ratios; indeed this is only a moderate distortion of the truth since the Mach numbers are fairly low. It can be seen that for a specified value of P_L/P_M the whole accessible performance range can be covered by varying P_H/P_M between unity and an upper limit governed by the location of the $\beta = 0$ curve. By taking other sufficiently low values of P_L/P_M a situation will arise in which the maximum value of P_H/P_M is limited not by the $\beta = 0$ curve, but by the quantity of driving fluid available. This and other performance limitations are discussed in more detail in section 7.

The information gathered from the divider tests was processed on the University of London Mercury digital computer.

A few more specialised tests were carried out which departed from the foregoing procedure. These experiments were mostly concerned with investigating flow in the ports of the machine, etc. Details of the methods used are not described as the tests involved orthodox steady flow experimental techniques.

6.2 EQUALISER TESTS

As for the divider tests, the majority of experiments were devoted to the establishment of performance characteristics with a fixed geometry.

The general procedure adopted for the divider experiments, maintaining one of the port static pressure ratios constant whilst allowing the others to vary, was used for the equaliser tests. In this case it was found most convenient to work with parameters of constant overall pressure ratio P_H/P_L whilst varying the other ratios, P_H/P_M and P_L/P_M accordingly.

For all observations, except those with zero low pressure flow, the high and medium pressure stream were adjusted by means of the gate valves provided, the low pressure channel remaining un-impeded. For the limiting case of zero low pressure flow, the low pressure intake was blanked-off by means of a dismountable air-tight cover plate. This technique avoided the disadvantages inherent with the incorporation of a throttle valve in the low pressure inlet duct, but at the same time permitted positive cutting-off of the low pressure flow.

Again as for the divider tests, experimental data was processed

on the University of London digital computer.

Some of the more significant fixed geometry equaliser performance characteristics deduced from the experiments are shown in Fig.12.2-16 to 21. Factors affecting the range of the experiments are discussed in the next section.

7. LIMITATIONS OF THE EXPERIMENTS

7.1 PERFORMANCE LIMITATIONS OF THE RIG

In this section an explanation is given for the major factors limiting the performance of the rig. Reasons for the particular shapes of the performance characteristics obtained from the experiments are given in sections (8) and (9).

7.1.1 Limitations Due To Restricted Mass Flow:

Mention has already been made in section 6.1 of the effect on the divider experiments of the limited quantity of compressed air available. Examination of the divider performance characteristics plotted on the $P_{H0}/P_{M0} - P_{L0}/P_{M0}$ plane, for example Fig. 12.2-1 to 12.2-5, reveal that the lesser valued constant β curves terminate at their left hand ends in the approximate neighbourhood of the line denoting that the axial component of the medium pressure inlet Mach number M_M is equal to 0.3. Bearing in mind that for constant P_{M0} , T_{M0} and rotor speed, lines of constant M_M are also lines of constant input mass flow, the significance of the ends of the β characteristics lying roughly parallel to curves of constant M_M becomes more clear.

In practice, because of the low pressure exhaust arrangements of the rig, P_{L0} never rises very much above a pressure of 1 atmosphere absolute. This means that, in the experimental apparatus, a reduction of P_{L0}/P_{M0} is therefore largely obtained by increasing P_{M0} . Hence when $M_M = 0.3$ corresponds to approximately the maximum compressed air output close to the $\beta = 0$ characteristic, then it will not be possible to achieve $M_M = 0.3$ at lower values of P_{H0}/P_{M0} because there will be insufficient compressed air to boost P_{M0} to the required value. The

experimentally observed facts do indeed square with the foregoing explanation.

In the case of the equaliser experiments mass flow limitations are most easily explained if it is remembered that, because the low pressure stream is fed by the atmosphere, P_{Lo} is a sensibly constant pressure. Thus horizontal lines on the experimental performance characteristics Fig. 12.2-16 to 12.2-21 are lines of approximately constant P_{Mo} , P_{Mo} being equal to 1 atmosphere when P_{Lo}/P_{Mo} is unity. Hence P_{Ho} increases from left to right along constant P_{Lo}/P_{Mo} lines. A reduction of P_{Lo}/P_{Mo} for constant P_{Ho}/P_{Mo} also means that P_{Ho} must increase because P_{Lo} is nearly constant.

Noting the location of the constant M_H curves, and remembering T_{Ho} is approximately constant, it can be seen that the flow velocity through the HP ports increases as P_{Lo}/P_{Mo} is reduced whilst maintaining constant P_{Ho}/P_{Mo} . Continuity considerations indicate therefore that the maximum available compressor mass flow should lead to a truncation of the constant S characteristics on a curve rising from left to right, intersecting the $S = 0$ characteristic and the $P_{Lo}/P_{Mo} = 1$ axis at its extremities.

Experimental evidence shows that the foregoing considerations only partially account for the form of the possible operating regime of the equaliser; other factors are explained in the next sub-section.

7.1.2 Limitations Due to Pressure Losses In Ducts

The effects of pressure losses in the duct-work on the divider experiments were:

- (a) to increase P_{Lo} to a value greater than the pressure in the low pressure sink; this effect was aggravated by the presence

of an orifice meter in the low pressure line.

- (b) to prevent the introduction of a scheme intended to increase the operating range by recirculating high pressure fluid back into the medium pressure supply duct.

Pressure losses in the high pressure duct were of no real significance because this duct was throttled over the range of test conditions of greatest practical interest.

For equaliser operation pressure losses in the medium pressure outlet duct were of considerable importance when working with P_{Lo}/P_{Mo} fairly close to unity, the medium pressure throttle valve then being wide open. Under these circumstances the closeness with which the operating regime could approach $P_{Lo}/P_{Mo} = 1$ was indeed governed by the losses in the medium pressure duct and also, to a much lesser extent, by the pressure losses in the short low pressure intake.

The following diagram, Fig.7.1-1, presents a qualitative picture of the performance limitations imposed by the factors described in this and the preceding sub-sections. The regions explorable in the experiments are shown cross-hatched:

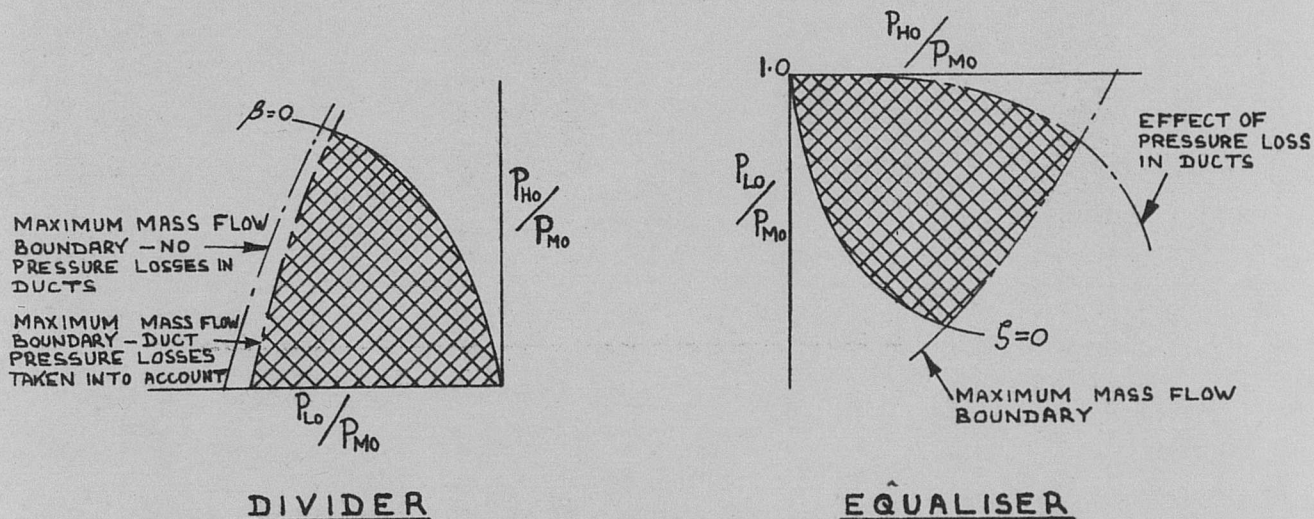


Fig. 7.1-1

It may appear that the pressure loss restrictions on the performance of the rig could have been relaxed by fitting larger duct-work and venturi-meters in place of orifice, etc. The 6" ducts used on the rig were, however, about the largest permissible within the compass of space and cost limitations.

7.1.3 Effects Arising From Asymmetric Flow In Ports

During some of the early preliminary tests circumferential pressure gradients were discovered in the ports of the machine. Experiments showed that these gradients could account for differences of 10% or so in the magnitudes of the absolute static pressures on the opposite sides of a port. Fig.15.6-1 shows the static pressure gradients recorded during a test conducted on the divider arrangement.

From Fig.15.6-1 it can be seen that the effects of the static pressure gradients could not justifiably be regarded as insignificant compared with the pressure ratios over which the machine worked. Pitot traverses to obtain the total head pressure distributions found undulating profiles superimposed on the background set by the smoother static pressure distributions. It was noted that non-uniform total head distributions prevailed in inlet as well as outlet ports.

A theoretical and experimental study of the causes of the pressure distributions in the ports indicated that the static pressure gradients were in large measure due to small or non-existent fairing surfaces on the port walls. Larger fairing surfaces would serve to guide fluid smoothly into or out of the cells. Such surfaces would only be correct for one set of gas velocities at any particular rotational speed. Fairing surfaces were mostly omitted from the original design of the Power Jets machine because they would have added considerable complexity to the

variable width ducts. The special study of pressure-exchanger port flow is summarised in Appendix VI; a by-product of the investigation was the development of a theoretical method for the design of pressure-exchanger ducts.

Due to the rather complicated geometry arising from the incorporation of variable ports, etc. it was not practical to take representative single point pressure measurements well away from the ports. Such a procedure would have involved estimation of the pressure losses under all operating conditions in the channels between the ports and the pressure measuring stations. It was therefore decided that the relevant pressure measurements should be made in the ports of the machine.

Since total head profiles could in principle be subject to "sudden jumps" or discontinuities due, for instance, to shock or expansion waves running out of the cells and into the ports, it was decided to base all port pressure measurements on assessment of static pressures. Static pressure profiles should at least be continuous across each port; this was indeed confirmed by experiment as can be seen from Fig.15.6-1.

To make the performance of the machine meaningful it was necessary to quote only one total head (or corresponding static) pressure in each kind of port. Accordingly the special averaging manometers, the principle of which is described in Appendix VII, were constructed in order to determine the arithmetic average static pressure in each type of port. The average static pressures, together with the mass flows passing through the ports of known area, were used to deduce corresponding total head pressures. Justification for this procedure is given in Appendix VIII.

7.2 LIMITATIONS ARISING FROM SHORTAGE OF TIME:

Due to the limited time available for the experimental work the range of machine variables investigated was somewhat small. Descriptions of the regimes of machine variables, defining machine geometry and rotor speed, is given for dividers and equalisers in 7.2.1 and 7.2.2 respectively.

7.2.1 Range of Machine Variables For Divider Tests:

The design of the Power Jets machine was such that the phasing of the HP outlet relative to the MP and LP ports, HP, MP and LP port widths, and rotor speed could all be regarded as machine variables. In practice a change of HP port width requires partial rebuilding of the machine and therefore was the least frequently changed machine variable.

For the tests described in this report there was indeed no time to carry out tests with more than one particular value of HP port width, namely 48° . Because of the design of the machine, the sum of the MP plus LP port widths was always constant at 90° . Deleting HP port width from the list of machine variables, and bearing in mind that the MP and LP port widths are interdependent, the effective number of independent machine variables was reduced to three: port phasing, MP or LP port width and rotor speed.

A complete set of tests was carried out at one speed for various combinations of port phasing and relative port widths, each test determining machine performance on the $\frac{P_{H0}}{P_{M0}} - \frac{P_{L0}}{P_{M0}}$ plane. A second set of tests was undertaken over a range of speeds for a fixed geometry. A diagrammatic interpretation of the range of machine variables studies is given in Fig.7.2-1.

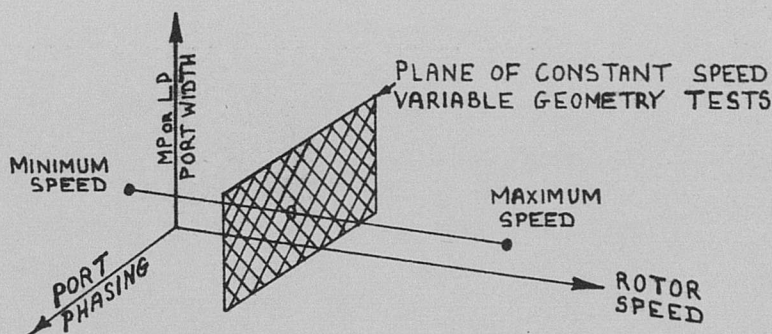


Fig. 7.2-1

7.2.2 Range of Machine Variables For Equaliser Tests

Remembering that the interchange of the functions of various ports between divider and equaliser operation is as indicated on Fig. 2.4-1 and 2.4-4, the relevant machine variables for the equaliser tests were port phasing, the width of the HP or LP inlet, and rotor speed; the width of the medium pressure outlet port was maintained constant throughout the series of tests. The general pattern of the tests was similar to that already described for the divider case. The range of machine variables investigated is illustrated in diagrammatic form in Fig. 7.2-2.

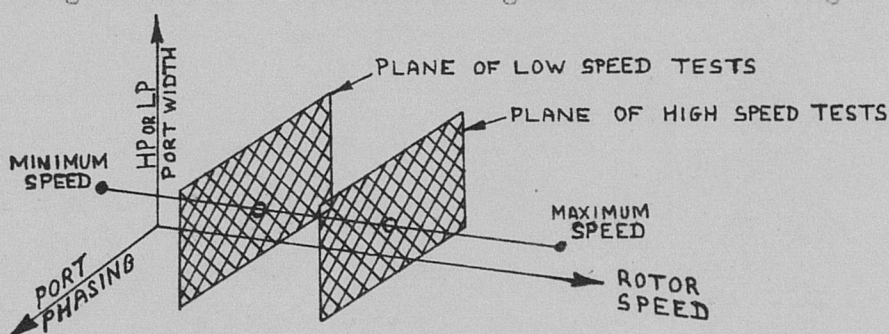


Fig. 7.2-2

7.3 ACCURACY OF EXPERIMENTAL RESULTS:

The accuracy with which the experimental results represented the true performance of the pressure-exchanger can, for convenience, be considered as a function of three factors.

- (a) The accuracy with which the machine variables were known.
- (b) The accuracy with which the experimental readings represented the true conditions inside the machine.
- (c) The overall effect on (b) of the data processing procedures leading to the final graphical performance presentations.

Making a separate study of (a), (b) and (c):

- (a) The three independent machine variables should be known to within the following approximate limits:

variable width port:	0.25%
port phasing:	0.5%
rotor speed:	0.5%

The first two items are based on a total angular error of about 0.1° degree in each case. The assessment of the speed measurement accuracy is quoted from data supplied by the tachometer manufacturer.

- (b) The accuracy of the flow measuring devices was derived from British Standard data (4). The accuracy of the interpreted mass flow and pressure measurements was obtained by making allowances for instrument and other errors. The accuracy of the mass and pressure ratios, etc. was obtained by application of probability theory (21).

The table shows the results of the error analysis:

ITEM	APPROXIMATE MAGNITUDE OF ERROR	MAXIMUM OR MOST PROBABLE ERROR
Mass flow measuring devices.	± 1.1 to 1.7%	Most probable
Interpreted Mass flows.	$\pm 2.5\%$	" "
Measurement of absolute pressure in ports.	$\pm 2.0\%$	Maximum
Ratios of total head pressures in ports.	$\pm 3.0\%$	Most probable
Divider mass ratio.	$\delta\beta = \pm 0.04$	" "
Equaliser enthalpy ratio.	$\pm 4.0\%$	" "

When β or S are zero the tabulated errors for these quantities cease to apply because the $\beta = 0$ and $S = 0$ characteristics were located by pressure measurements only.

- (c) Because an electronic digital computer, working internally to the equivalent of eight places of decimals, was used to process the experimental data, the calculation processes appeared to introduce no noticeable random errors.

An intermediate graphical procedure, used to enable the computed experimental points to be presented in the required form, allowed a smoothing process to be introduced. The random errors detailed in (b) should therefore emerge with reduced effect on the final performance presentations.

8. ANALYSIS OF EXPERIMENTAL RESULTS

8.1 INTRODUCTION:

The aim of this section is to describe the presentation of the experimental results and to indicate the main points of interest; in short to give the reader a general picture of the experimental findings. Section 9, a comparison of experimental and theoretical performance characteristics, includes explanations of various features better understood when the experimental results are studied in conjunction with the theoretical performance analyses.

The section is divided into two; the first part deals with experiments conducted on the divider arrangement, the second portion covers the experiments on the equaliser cycle.

In order to generalise the experimental results and to assist their comparison with theoretical performance predictions, rotor speed, port widths, phasing angles, etc. have been expressed in dimensionless units. The dimensioned forms directly applicable to the Power Jets machine are also displayed on the presentations of the experimental performance results.

Because the divider MP and equaliser HP inlet stagnation temperatures were maintained at a unique value ($\pm 3\%$) throughout the whole series of experiments, it was therefore convenient to use that temperature as the basis for the (constant) reference sonic velocity in the dimensionless forms for rotor speed and port geometry. Dimensionless rotor speed, M_{RO} , is defined by:

$$M_{RO} \equiv \frac{V_{MP}}{a_{REF}} \quad \left\{ \begin{array}{l} \text{Where ; } a_{REF} \equiv a_{MO} \text{ for dividers} \\ a_{REF} \equiv a_{HO} \text{ for equalisers} \end{array} \right.$$

NB. for all experiments $a_{MO} \approx a_{HO} \approx 1155 \text{ ft/sec.}$

Port widths, phasing angle, cell width, etc. are expressed in terms of dimensionless time τ . A τ value of unity is the dimensionless time taken for an acoustic wave to travel the length of a cell at the stagnation sonic velocity α_{REF} . A port width in terms of τ is therefore the dimensionless time required for a point on the rotor to pass the port opening; this is clearly a function of rotor speed. Another consequence of the constancy of the reference temperature was that dimensioned speed ($N \text{ REV./MIN}$) could be used as a performance parameter instead of the $N/\sqrt{T}$ form.

8.2 DIVIDER

Because each test was carried out with a fixed geometry and at a constant speed, each set of results was plotted on the P_{Ho}/P_{Mo} versus P_{Lo}/P_{Mo} plane. Pictures of the effects on performance of changes of geometry or speed were therefore obtained by cross-plotting from the P_{Ho}/P_{Mo} versus P_{Lo}/P_{Mo} presentations.

An attempt was made to establish the optimum performance of the machine working as a pressure booster. The evidence presented here, in conjunction with conclusions drawn from a less exact performance exploration over a larger regime, convinced the author that the quoted experimental results encompassed the optimum pressure boosting performance. The cooling cycle performance was determined for the speeds, and some of the geometries, studied in the pressure boosting performance investigation.

The effects of changes of geometry and changes of speed on the experimental results are discussed separately. Pressure boosting and cooling cycle functions are also dealt with under separate sub-headings.

8.2.1 Effect of Port Geometry On Pressure-Boosting Performance

The effect of changes of τ_ϕ on performance when port width ratio and rotor speed were maintained constant is shown in Fig. 12.2-6 and 12.2-7. The criterion of merit on the diagrams, which were each drawn for a specified overall pressure ratio P_{H0}/P_{L0} , is the maximum P_{H0}/P_{M0} attainable with a given value of β . It is therefore clear that the best performance was obtained with $\tau_M = \tau_L = 1.57$. The optimum phasing showed a tendency to decrease as P_{H0}/P_{L0} was increased, optimum τ_ϕ being about 0.94 when $P_{H0}/P_{L0} = 1.3$, and 0.84 when $P_{H0}/P_{L0} = 1.6$. A tendency for τ_ϕ to increase as β decreased was also slightly in evidence. The absence of any great sensitivity in the effect on performance of small changes in either port widths or phasing is noteworthy.

Fig. 12.2-1, 2, 3 show fixed geometry performance characteristics, for $M_{R0} = 0.147$, with medium and low pressure port widths of $\tau_M = \tau_L = 1.57$. Fig. 12.2-2, 3 show the divider cycle performances at the extremities of the $\tau_M = 1.57$ curves in Fig. 12.2-6, 7. Fig. 12.2-1 shows the best (fixed geometry) divider pressure booster performance obtained from the experiments; the phasing $\tau_\phi = 0.84$. Fig. 12.2-1 has, in addition to the usual contours of β , M_M and η_{EIS} , lines of constant isentropic product efficiency η_{PIS} . These were evaluated from equation 15.2-9 of Appendix II.

8.2.2 Effect of Rotor Speed On Pressure-Boosting Performance

Fig.12.2-8 and 12.2-9 present, over a range of rotor speeds, the pressure boosting performance of the machine at what was found to be the optimum (dimensioned) geometry at $M_{RO}=0.147$ (6000 rev/min), i.e. phasing angle = 24° , MP = LP port width = 45° , HP port width = 48° . Because port widths, etc. expressed in terms of dimensionless time are directly proportional to the angle subtended, and inversely proportional to rotor speed; the dimensionless geometry applicable to Fig.12.2-8 and 12.2-9 is not independent of rotor speed: curves of dimensionless geometrical properties are superimposed on Fig.12.2-8

From the curves on Fig.12.2-8, 9 it can be seen that the performance alters but little between $M_{RO}=0.135$ and $M_{RO}=0.147$, and worsens as the speed is raised above or reduced below these limits.

At $M_{RO}=0.098$ the performance has depreciated to such an extent that the pressure rise, for a given value of β , was only about half that obtained when $M_{RO}=0.147$. This point is clarified by comparing Fig.12.2-4, the performance on the P_{Ho}/P_{Mo} versus P_{Lo}/P_{Mo} plane at $M_{RO}=0.0978$, with Fig.12.2-1. Fig.12.2-5 shows the results of a test conducted at $M_{RO}=0.1591$, the maximum speed attainable from the Power Jets machine working as a pressure-exchanger divider. The absence of sharp peaks on any of the curves displayed on Fig.12.2-8, 9 is worthy of note.

8.2.3 Effect Of Port Geometry On Cooling Cycle Performance

Fig.12.2-13 shows, for $M_{RO}=0.147$, the dimensionless temperature drop Θ of the low pressure stream, against phasing γ_ϕ , for the range of port widths and phasings studied in the pressure boosting performance investigations. The merit of each curve is determined by the magnitude of P_{Lo}/P_{Mo} needed to establish a given Θ , the largest value of

P_{Lo}/P_{Mo} being the best. From Fig.12.2-13 it can be seen that the best phasing was roughly $\tau_{\phi} = 0.87$; the best port width of those investigated ranged from $\tau_M = 1.50$ for $\Theta = 0.01$, to $\tau_M = 1.64$ for $\Theta = 0.06$.

It was concluded that a reasonable compromise for the optimum geometry for the best Θ at $M_{Ro} = 0.147$ involved $\tau_M = \tau_L = 1.57$ ports, and a port phasing between $\tau_{\phi} = 0.80$ and $\tau_{\phi} = 0.94$. It was noted that better performances may be obtainable for the higher values of Θ with MP inlet ports wider than $\tau_M = 1.64$.

8.2.4 Effect Of Rotor Speed on Cooling Cycle Performance

Fig.12.2-14 shows contours of constant Θ , for the optimum (compromise) cooling cycle (and pressure boosting) geometry for $M_{Ro} = 0.147$, on a plane the ordinate and abscissa of which represent P_{Lo}/P_{Mo} and rotor speed respectively. The relationships between dimensionless geometry and rotor speed is the same as that shown on Fig.12.2-8.

Fig.12.2-15 illustrates the corresponding constant Ψ curves for the same geometry; Ψ , another cooling cycle performance criterion is defined in equation 3.2-6. From both Fig.12.2-14 and 12.2-15 it can be seen that the optimum rotor speed is given by $M_{Ro} \approx 0.12$, the exact apex not being sharply defined on any of the curves.

It is pointed out that there is no indication that the optimum (dimensioned) geometry at $M_{Ro} = 0.147$ is necessarily also the optimum at $M_{Ro} = 0.12$; so it may well be that the best possible cooling cycle performance may be better than indicated on either Fig.12.2-14 or 12.2-15.

Fig.12.2-10, 11, 12 show, for the optimum (dimensioned) geometry at $M_{Ro} = 0.147$, Θ and Ψ contours on the P_{Ho}/P_{Mo} versus P_{Lo}/P_{Mo} plane

at M_{Ro} values of 0.1225, 0.0978 and 0.1591 respectively. Fig.12.2-11 and 12.2-12 therefore show the cooling performances at the extremities of the curves displayed on Fig.12.2-14 and 12.2-15. Fig.12.2-10 illustrates the best complete cooling cycle performance recorded during the experiments.

8.3 EQUALISER:

The results of each complete test to determine equaliser performance characteristics with fixed geometry at a constant speed, were presented on the P_{Lo}/P_{Mo} versus P_{Ho}/P_{Mo} plane. As for the divider, the effects on performance of changes of port geometry or rotor speed were determined by cross-plotting from the fixed geometry, constant speed, experimental results.

8.3.1 Effect Of Port Geometry On Equaliser Performances

Fig.12.2-22 and 12.2-23 show the effect on performance of variation of phasing τ_ϕ , when the overall pressure ratio P_{Ho}/P_{Lo} and rotor speed are maintained constant. For Fig.12.2-22 $P_{Ho}/P_{Lo} = 1.3$, for Fig.12.2-23 $P_{Ho}/P_{Lo} = 1.6$; for both cases the rotor speed corresponded to $M_{Ro} = 0.1346$. Separate curves are shown on each diagram for HP port widths ranging from $\tau_H = 1.48$ to $\tau_H = 1.94$. The best geometrical arrangement for any prescribed S and of course overall pressure ratio is that which produced the lowest P_{Lo}/P_{Mo} .

From an examination of Fig.12.2-22 it can be seen that the best port width is $\tau_H = 1.94$ with $\tau_\phi = 0.95$, when $S = 0$, changing to $\tau_H = 1.79$ with $\tau_\phi = 1.00$ when $S = 1.0$. Fig.12.2-23 shows that the best performance is obtained when $\tau_H = 1.94$ with $\tau_\phi = 0.95$ at $S = 0$, changing to $\tau_H = 1.48$ with $\tau_\phi = 0.70$ at $S = 1$.

It was concluded, from Fig.12.2-22, 23, that, for $M_{R0}=0.1346$, very little loss of performance was incurred over the range of pressure ratios explorable, if the geometry were fixed with an HP port width given by $\tau_H=1.79$, when $\tau_\phi=0.91$. The performance with this geometry for, $M_{R0} = 0.1346$, is shown on the $\frac{P_{Lo}}{P_{Mo}}$ versus $\frac{P_{Ho}}{P_{Mo}}$ plane in Fig.12.2-16. This was the best performance, at fixed speed and geometry, revealed by the equaliser tests. Contours of product efficiency η_{pis} have been added to the diagram; it may be noticed that the maximum efficiency occurs fairly close to the $\frac{P_{Lo}}{P_{Mo}} = \frac{P_{Ho}}{P_{Mo}} = 1$ origin. Fig.12.2-17, 18 show the performance characteristics near the ends of the $\tau_H=1.79$ curves on Fig.12.2-22, 23.

Fig.12.2-24 and 12.2-25 illustrate similar information to that displayed on Fig.12.2-22, 23, but for the lower speed $M_{R0}=0.1225$. Again the general conclusion was drawn that, without much loss of performance, the geometry could be fixed with the same port widths and phasing as for the $M_{R0}=0.1346$ case. In this case the dimensionless HP port width and phasing are therefore $\tau_H = 1.96$ and $\tau_\phi = 1.00$. For $\frac{P_{Ho}}{P_{Lo}} = 1.3$, the best HP port width was $\tau_H = 1.96$ the corresponding optimum phasing ranging from $\tau_H = 0.96$ when $S = 0$, to $\tau_\phi = 1.09$ when $S = 1.0$. From Fig.12.2-25 it can be seen that the best performance is obtained, when $\frac{P_{Ho}}{P_{Lo}} = 1.6$, with an HP port width of $\tau_H = 1.96$, and $\tau_\phi = 0.90$ when $S = 0$, changing to $\tau_H = 1.80$ with $\tau_\phi = 0.85$ when $S = 1.0$.

8.3.2 Effect Of Rotor Speed On Equaliser Performance:

Fig.12.2-26 and 12.2-27 show the performance of the equaliser arrangement over a range of rotor speeds between $M_{R0}=0.10$ and $M_{R0}=0.15$,

the latter being the highest speed attainable with the Power Jets machine working as an equaliser. The overall pressure ratios applicable to Fig.12.2-26 and 12.2-27 are $P_{H0}/P_{L0} = 1.3$ and $P_{H0}/P_{L0} = 1.6$ respectively; both diagrams were constructed for a fixed geometrical arrangement giving optimum performance in the $M_{R0} = 0.1225$ to 0.1346 speed range, i.e. HP port width = 47° , LP port width = 43° , MP port width = 48° , with a phasing angle of 24° . The corresponding dimensionless form of this geometry has been plotted, over the appropriate speed range on Fig.12.2-26.

From Fig.12.2-26 and 12.2-27 it can be seen that the best rotor speed is approximately $M_{R0} = 0.13$; there is a slight tendency, noticeable in both diagrams, for the optimum speed to increase slightly as S increases.

Fig.12.2-20 and 12.2-21 show the performances, on the P_{H0}/P_{M0} versus P_{H0}/P_{M0} plane, at the low and high speed extremities respectively of the curves shown on Fig.12.2-26 and 12.2-27. Fig.12.2-19 and 12.2-16 display the performance characteristics for $M_{R0} = 0.1225$ and $M_{R0} = 0.1346$; these contribute the best results appearing on Fig.12.2-26 and 12.2-27.

9. COMPARISON OF THEORETICAL AND EXPERIMENTAL PERFORMANCES

An objective of this section, additional to that implicit in the title, is to provide explanations of the performance characteristics revealed by the experiments. The section is sub-divided into two main parts, one dealing with dividers, the other with equalisers. Each of the main sub-sections is further divided into three portions; each of these covers a comparison of the experimental results with a stage of the three level theoretical treatment explained in 4.1.

9.1 DIVIDERS

9.1.1 Comparison of Actual with Ideal Isentropic Performances

Comparison of the ideal isentropic pressure boosting performance shown on Fig.12.1-1 with its best experimentally obtained counterpart (Fig.12.2-1) shows a considerable difference between the real and the ideal performance characteristics. Possibly the most noticeable feature concerns the shape and position of the constant mass ratio curves. For the ideal performance these become steeper as they approach the left hand side of the diagram; the converse is true of the experimental results. The zero mass ratio curve lying on the right hand ordinate of the ideal performance characteristics has been curved over to the left in the experimental case; the other constant β curves were also similarly deflected from their ideal locations.

The right hand ends of the constant Mach number curves on the experimental result were raised-up, the contours consequently being steepened, in relation to their location on the ideal performance characteristics. The whole impression on comparing the ideal and actual performances was therefore that the factors omitted from the theoretical

treatment combined to have an effect which could perhaps be simulated by the application of an (approximately) conformal transformation to the ideal results. No attempt was made to establish such a transformation function as it would not necessarily help to provide physical reasons for the observed performance characteristics.

Comparison of the ideal cooling cycle performance shown on Fig.12.1-2 with the best experimental result, displayed on Fig.12.2-10, shows that Θ is greater for the isentropic cycle as indeed would be expected. If Θ^* had been drawn in contour form on Fig.12.1-2 it would have appeared as a series of vertical lines; because 2_{EIS} is everywhere unity, Θ^* is a function of P_0/P_{M0} only.

Comparing the location of the Ψ contours on the experimental and theoretical performance curves, it can be seen that the experimental result is slightly superior to that predicted by theory. This possibility and the reason for it were discussed in 3.2.2.

9.1.2 Comparison of Actual Performance with that Predicted from Simplified Theory Taking Account of Irreversibilities.

A comparison of the **best** experimentally obtained pressure boosting performance shown on Fig.12.2-1 with the performance illustrated on Fig.12.1-3, predicted by the procedure described in section 4.3 and subsection 4.3.1, shows that the theoretical performance underestimates the results obtained in practice. The constant β lines are noticeably less curved on the predicted performance, but their general shapes are very similar to those of the experimental results. As a consequence of the pessimistic estimates of β for specified pressure ratios, the predicted product efficiency was inferior to the experimental value. The constant Mach number contours were slightly less steep

on the theoretical results; this effect appears to be consistent with the general flattening-out of the β curves noticed on the predicted performance.

The simplified theory, despite the short-comings springing from its over simplifications, produced an estimate of performance having generally the same characteristics as the actual result. The discrepancy between theory and practice may have been largely due to the use of the only basic cell filling and cell emptying data available, for cell widths of $\tau_c \approx 0.54$, instead of data relevant to the effective cell width of $\tau_c \approx 0.36$ of the experiment producing the result shown on Fig.12.2-1

The simplified theory, bearing in mind the difference in cell width previously mentioned, appears to show that there is a reasonable correlation between earlier pressure-exchanger work and the results of the divider tests. Consequently it seems likely that the simplified theory may be of use in the speedy prediction of pressure-exchanger cycle performance from libraries of basic data, thereby avoiding laborious characteristic calculations. The simplified procedure does not itself provide detailed physical explanations for the form of the experimental performance characteristics beyond those implicit in the agreement between theory and experiment; such explanations can be obtained from method-of-characteristics studies.

9.1.3 Comparison of Actual Performance with the Results of Characteristic Studies:

A comparison of the isentropic mass ratio curves on Fig.12.1-1 with the results, shown on Fig.12.1-4, of a characteristic study of the (hypothetical) performance of a pressure-exchanger divider with infinitely narrow cells showed that, by taking wave effects into account, the

constant mass ratio curves are transformed into shapes qualitatively resembling those obtained by experiment. The curves constituting Fig.12.1-4 are all drawn from optimised wave diagram calculations; as described in section 4.4.4, the optimisation procedure studied the virtues of delaying the closure of the HP and LP ports until flow reversal occurs.

The results of the port-timing studies showed that the best divider cycle performance was obtained when these ports were kept open as long as possible. The gains were sufficiently great to outweigh the increased difficulty of diffusing the resultant non-uniform HP and LP flows. The effect of closing the MP inlet, at points other than that indicated as appropriate by the arrival of a reflected shock from the HP end, was not investigated; this would have involved the study of cycles outside the scope of the work reported here.

An examination of the wave diagrams for infinitely narrow cells, or if preferred, for cells of finite width capable of opening or closing with infinite rapidity, led to the conclusion that the divergence of the mass ratio contours from their ideal isentropic locations was due to two effects. One of these was a consequence of the pressure waves not propagating at relative speeds which would make an isentropic cycle possible. Closely allied to this and superimposed upon it, was the apparently more serious effect arising from the fanning-out of expansion waves, with consequent wave interaction, during the LP emptying process. Wave fanning and interaction, which rapidly become more significant as the LP Mach number increases, makes it necessary to reduce P_{L0} below the value pertinent to an isentropic cycle in order that sufficient fluid may be withdrawn from the LP port to maintain operation of the

machine. The foregoing factors account for the consequent curling-over of the β characteristics which is such a significant feature of Fig.12.1-4.

Fig.12.1-5 displays the constant mass ratio curves, predicted from wave diagram calculations, taking account of finite cell width. A comparison of Fig.12.1-4 with Fig.12.1-5 shows that the effect of finite cell width is to accentuate the tendencies already apparent in Fig.12.1-4; the curling over of the constant β curves is more pronounced, performance is generally poorer. The wave diagrams providing the data for Fig.12.1-5 were constructed for the maximum possible HP and LP port widths consistent with the avoidance of flow reversal; these limitations correspond to the optimum conditions previously described.

The wave phenomena follow the basic pattern described for infinitely narrow cells with additional complications arising from the finite time required to fully expose or cover up the ends of the cells. The central part of Fig.12.1-9 shows, on a reduced scale, the contours displayed on Fig.12.1-5; surrounding the mass ratio curves are eight of the wave diagrams contributing information for the construction of Fig.12.1-5. The wave diagrams display the following common features:

- (1) formation of an initial shock by coalescence of weak compression waves as cells are gradually exposed to the MP port.
- (2) generation of a reflected shock closely pursued by an expansion wave formed by the gradual communication of cells with the HP port, or to an observer in the cells, by the opening of the HP port.

- (3) formation of an expansion wave by closure of the MP port; the expansion wave is followed and partially overtaken by the shock reflected from the point of MP cut-off. The shock is itself pursued by an expansion wave generated by the opening of the LP port.
- (4) generation of a compression wave by the closure of the MP port.
- (5) reflection of a shock wave from the closed end of the cell; this shock wave therefore runs towards the LP outlet behind the compression wave referred to in (4).
- (6) arrival and reflection of waves (4) and (5) in the LP port.
On arrival each wave accelerates the flow in the port, the flow subsequently decelerating as reflected expansion waves, (initially) generated during MP port closure or LP port opening arrive at the LP port.
- (7) closure of the LP port to prevent flow reversal.

For diagrams (b), (c) and (e), wave (4) is of zero strength; on (f) (4) is sufficiently strong to coalesce and form a shock wave.

A comparison of Fig.12.1-5 with the best experimentally obtained performance, shown in Fig.12.2-1, indicates that the theoretical performance has similar general characteristics to those obtained in practice. Quantitatively the predicted result is wholly optimistic because it takes no account of leakage or viscous losses.

Fig.12.1-6 illustrates the predicted performance when allowances are made for leakage and finite cell width effects. Leakage affects the entire range of pressure ratios for which performance predictions were made; the most serious effects are concentrated in the mid-portion of the operating region. Leakage is of some importance at low

pressure ratios because, although leakage flows are small, so also are the mass flows, hence leakage can still be significant; the converse argument implies that leakage also denigrates the performance at high pressure ratios.

Fig.12.1-7 shows the performance with finite width cells still further modified to take account of friction as well as leakage. Friction losses have their greatest effect at high Mach numbers; this consequently causes the constant β contours to droop as they approach the LH side of the diagram.

A comparison of the predicted performance shown on Fig.12.1-7 with the best experimental result (Fig.12.2-1) reveals a considerable measure of agreement between the two, although the theoretical treatment produces a slight underestimate of the experimental result. The best experimentally obtained value of $2_{pis} \approx 0.6$ agrees with the predicted value; however, the predicted region of maximum efficiency lies nearer to the $\frac{P_{Ho}}{P_{Mo}} = \frac{P_{Lo}}{P_{Mo}} = 1$ origin of the plane of the divider performance presentation. Close to $\frac{P_{Ho}}{P_{Mo}} = \frac{P_{Lo}}{P_{Mo}} = 1$, the predicted performance will be optimistic because churning losses were not taken into account. Churning losses are those associated with fluid remaining in the rotor for a sufficient period of time to be carried through several cycles prior to discharge from the machine.

A comparison of the fixed geometry of the experimental result shown on Fig.12.2-1 with the predicted geometry for optimised performance (Fig.12.1-8) shows that the average theoretical HP and LP port widths are each about 10% wider than the optimum experimental values, these being the maximum port widths possible with the experimental apparatus. The mean predicted HP port width is about 7% less than the experimental

value; according to theory, the width of the HP port should, for optimum performances vary from $\gamma_H = 1.1$ to $\gamma_H = 1.65$ over the range of the experiments. A typical phasing from the theoretical geometry overestimates its experimental counterpart by about 25%. This discrepancy is likely to be a consequence of the too narrow MP and LP ports and the over-wide HP ports apparently used on the experimental apparatus.

It was concluded that the method-of-characteristics calculations, modified to allow for leakage and friction, enabled a moderately accurate prediction to be made of divider cycle performance. Additionally the characteristic calculations provided explanations for the shape of the divider performance contours; they also gave indications of the geometry required for optimum performance and of cycle performance potential. The omission from the theory for reasons of simplicity, of internal heat transfer and of any consideration of the effect of residual waves generated by earlier cycles may account for the predicted performance slightly underestimating the actual result.

9.2 EQUALISERS

9.2.1 Comparison of Actual with Ideal Isentropic Performance

Comparison of the ideal isentropic performance appearing on Fig.12.1-10 with the best experimentally obtained equaliser performance illustrated on Fig.12.2-16 shows that the real performance departs ever increasingly from the ideal as P_{H0}/P_{M0} increases. The curving over of the constant S^* characteristics of the ideal performance is very much accentuated in the actual result. The $S=0$ curve, lying on the $P_{H0}/P_{M0}=1$ ordinate for the ideal result, bends away from the axis and follows the

profile of the other S lines in the experimentally obtained performance.

The experimental constant Mach number contours are also deflected towards the abscissa relative to their ideal locations. Conforming to the impression given in the divider case, described in sub-section 9.1.1, the apparent effect of all the losses ignored in the calculation of the ideal performance is to apply a roughly conformal transformation to the ideal performance characteristics.

9.2.2 Comparison of Actual Performance with that Predicted from Simplified Theory Taking Account of Irreversibilities

A comparison of the best experimental equaliser performance (Fig.12.1-16) with the theoretical performance shown in Fig.12.1-11, predicted by the method explained in section 4.3 and sub-section 4.3.2, shows that the predicted performance generally underestimated the experimental result. The exception was for low values of P_{t0}/P_{M0} with small S ; for these conditions the theory anticipated a better result than was obtained in practice; this was a consequence of the experimental LP port being over wide for these operating conditions. Justification for the latter statement is given in sub-section 9.2.3 where the results of characteristic studies are discussed. The constant Mach number contours follow the trends apparent on the experimental results; relative to their ideal shape, the contours curve towards the P_{t0}/P_{M0} axis.

The general underestimate of equaliser performance given by the simplified theory may, as anticipated for the divider case, be a consequence of using basic data relevant to cells wider than those of the experimental apparatus. The appropriate cell widths are $\gamma_c \approx 0.54$.

and $\gamma_c \approx 0.39$ for the theory's input data and the experimental equaliser respectively.

9.2.3 Comparison of Actual Performance with the Results of Characteristic Studies

The constant stagnation enthalpy ratio curves on Fig.12.1-12, constructed from the results of optimised wave diagram calculations for a hypothetical cycle with infinitely narrow cells, curve-over yielding minima completely absent from their ideal isentropic counterparts; the latter are asymptotic to a horizontal line drawn through $P_{Lo}/P_{Mo} = 0$. From a study of the performance of equalisers with infinitely narrow cells it was discovered that the most efficient utilisation of wave effects was obtained when the MP outlet and LP inlet were both kept open for as long as possible consistent with the avoidance of flow reversal. This consideration therefore governed the opening point of the LP port and, because the cut-off of the HP port was dictated by the arrival of an expansion wave, it automatically fixed the width of the land between the HP and LP inlets. The study was not extended to investigate the effect of advancing or retarding the point of closure of the HP port relative to the arrival of the expansion fan.

An examination of the wave diagrams shows that the departure of the constant S contours on Fig.12.1-12 from their ideal isentropic forms was mainly due to the fanning of expansion waves; this phenomenon to some extent upset the MP outlet process and seriously affected the LP filling event. These effects become more serious as MP Mach number increases and also as P_{Lo}/P_{Mo} decreases. These factors therefore account for the curving-over of the S characteristics in the manner shown on Fig.12.1-12.

Fig.12.1-13 shows the $\bar{S}$ characteristics for an equaliser with finite width cells; as for the contours on Fig.12.1-12, the corresponding wave diagrams were drawn for the optimum conditions of maximum possible MP and HP port widths. A comparison of Fig.12.1-12 and Fig.12.1-13 shows similar general performance on both diagrams, the most striking feature being the movement of the $\bar{S}$ contours upwards, with a bias to the right, on Fig.12.1-13. This implies that for finite width cells, greater driving pressure ratios ($\frac{P_{Ho}}{P_{Mo}}$) are required to maintain given $\frac{P_{Lo}}{P_{Mo}}$ with specified $\bar{S}$.

The wave events with finite width cells follow the pattern for infinitely narrow cell cycles with the superposition of effects stemming from the finite time required to cover or uncover the end of a cell. Fig.12.1-17 shows, at its centre, a copy of Fig.12.1-13 drawn to a reduced scale. Surrounding the family of $\bar{S}$ contours are reproductions of eight of the contributing wave diagrams, these illustrate the following common features:

- (1) generation of a shock wave by coalescence of compression waves formed during the exposure of cells to the HP port.
- (2) reflection of the initial shock followed by the generation of an expansion wave as cells communicate with the MP outlet; the expansion wave pursues, and partly overtakes the reflected shock as they travel towards the HP inlet.
- (3) formation of an expansion wave by gradual closure of the HP inlet. This is followed by reflection of the shock wave approaching the HP port; the shock is in turn pursued by the reflection of the expansion wave, originated by the opening of the MP port, from the land between the HP and LP inlets.

The LP inlet is opened as soon as inflow can be assured.

- (4) formation of a shock due to closure of the MP outlet port.
- (5) reflection of the shock wave travelling from the HP inlet end

Usually this wave reflects as an expansion at the MP outlet port.

- (6) closure of the LP inlet, with resultant generation of expansion waves, in sufficient time to prevent the shock (4) from causing flow reversal in the port; shock (4) is usually about to be, or is being, overtaken by wave (5) when it reaches the LP inlet end of the cells.

A comparison of Fig.12.1-14 with Fig.12.1-13 shows the effect of taking leakage into account in addition to finite cell width effects. From Fig.12.1-14 it can be seen that leakage has the effect of reducing the P_{Lo}/P_{Mo} obtainable for given S with a specified P_{Ho}/P_{Mo} ; it also deflects the minima in the constant S curves towards the right.

Fig.12.1-15 shows the performance of an equaliser when account is taken of leakage and friction in a cycle using finite width cells. A comparison of Fig.12.1-15 and Fig.12.1-14 shows that the effect of friction is to still further depress cycle performance, the effects being most pronounced in regions where the flow Mach numbers are greatest. The curling-over of the S contours is also accentuated by friction, a minimum P_{Lo}/P_{Mo} being clearly discernible for each S contour.

Comparison of Fig.12.1-15 with the best experimental result on Fig.12.2-16 shows good agreement between the theoretical and experimental locations of the S contours and hence also the γ_{PIS} contours, over a large portion of the region covered by the experiments. Actual and predicted performances diverge for small S ; the theoretical

method anticipated a better result than was obtained in practice. The clue to the reason for this discrepancy and also for the poor correlation between the experimental and theoretical locations of the constant Mach number contours, comes from a comparison of the geometries of the optimum theoretical and experimental performances.

The geometry corresponding to the predicted optimum performance on Fig.12.1-15 is displayed in Fig.12.1-16. From a comparison of real and theoretical port widths it can be seen that the width of the LP port on the experimental apparatus is two to three times that predicted for optimum performance with small values of S . The width of the MP outlet port on the experimental apparatus is approximately equal to the average optimum predicted width over the range of the experiments. The width of the HP inlet port exceeds by about 5% an average of the predicted value for the range of pressures investigated. The port phasing found to give best experimental results is about 10% less than the predicted optimum. The average of the predicted width of the optimum land between the HP and LP inlets was found to be about 50% greater than that of its counterpart on the experimental equaliser.

It was concluded that the method-of-characteristics, modified to take account of friction and leakage, allowed a fairly accurate estimate to be made of equaliser performance in regions where the optimum geometry coincided with that used for the experiments. Discrepancies between the theoretical and experimental performances were probably the consequence of geometric differences between the actual and theoretical machines. The experimental apparatus was not capable of suiting the whole of the wide range of geometries encompassed in the theoretical treatment. The characteristics calculations also allowed an under-

standing to be obtained of the wave mechanism in the apparatus.

The theory enabled the relative significance of the parts played by basic wave events, finite cell width, leakage and friction to be studied.

10. DISCUSSION & RECOMMENDATIONS

10.1 PROPERTIES & POTENTIALITIES OF PRESSURE-EXCHANGER DIVIDERS & EQUALISERS.

Certain common properties may be ascribed to both dividers and equalisers. Some of these emerge as both direct and indirect revelations of the work described in this thesis.

Probably the most important and restrictive feature, affecting the scope of the commercial uses to which dividers and equalisers may be put, is the lowness of the pressure ratios over which efficient operation of single stage machines is possible. The types of compounding shown in Fig.2.5-1 & 2 of section 2.5.1 and Fig.2.5-3 & 4 of section 2.5.2 cannot be regarded as wholly satisfactory means for making the operation of dividers and equalisers possible over large pressure ratios. The reasons for this are discussed separately for dividers and equalisers in the next sub-sections.

Although not within the scope of the work covered by this report, mention may be made of a proposal to introduce cyclically variable volume cells into the concept of a dynamic pressure-exchanger. Such a machine can, for instance, result from the combination of the principles of an orthodox pressure-exchanger with those of a sliding-vane rotary machine. Variable volume cell pressure-exchangers should be capable of coping with greater pressure ratios than would be feasible with either ordinary constant-volume-cell pressure-exchangers or sliding-vane machines making no use of wave effects. Such pressure-exchangers, which are the subject of current patent action, may receive or produce shaft work; because of this, and as a result of the greater pressure ratios acceptable, these

machines offer more design scope than orthodox pressure-exchangers at the price of greater mechanical complexity.

Most pressure-exchangers can be, and so far have been, designed to run at much lower rotational speeds than turbo-machines. This implies that simple robust construction is usually adequate, and that extreme precautions to prevent overspeeding may well be unnecessary, even in situations where maintenance standards are low.

Because all the functions of a pressure-exchanger divider or equaliser cycle are performed within a single rotor, the rotor cell partitions etc. tend to settle at roughly the mean cycle temperature. This therefore implies that higher maximum cycle temperatures are often permissible in a pressure-exchanger than, for instance, in a comparable turbo-compressor. *

Due to the low relative velocities of the gasses entering, passing through and leaving the cells of pressure-exchangers, their erosion resistance should be much better than that of most turbo-machines. Some experimental evidence, so far unpublished, lends weight to this view. This therefore suggests the suitability of pressure-exchangers for use with dirty gases or wet vapours.

A pressure exchanger without silencers can be very noisy. Measurements of the noise produced by the Power Jets machine showed that a fundamental note was emitted having an intensity of 116 to 120 decibels and a frequency equal to the product of the number of cells in the rotor multiplied by the number of revolutions of the rotor per unit time. The recorded frequency agreed with observations by Berchtold and Gull (3). The wave analyser also showed the existence of a number of harmonic frequencies. The harmonics may have been the result of the

fundamental disturbance not being truly sinusoidal the Fourier analysis revealing the contributions of other frequencies.

The features of pressure-exchangers particularly relevant to dividers and equalisers are discussed separately in sub-sections 10.1.1 and 10.1.2 respectively.

10.1.1 Dividers:

Because a pressure-exchanger divider takes in only one stream of medium pressure fluid, some of which is compressed and the remainder expanded prior to leaving the machine, the mean cycle temperature is not likely to be much lower than the temperature of the ingoing fluid. Indeed if the HP outflow is shut-off the mean gas temperature in the cells may well be roughly equal to that of the ingoing fluid. It is therefore concluded that the inherent advantage of pressure-exchangers of being able to withstand high maximum cycle temperatures is fairly marginal for a divider arrangement.

The system of compounding shown in Fig.2.5-1 fails to make best use of the medium pressure ingoing flow entering the first stage, if the low pressure stagnation pressures are not all equal and of as low a value as possible. The equality of the stagnation pressures of the emergent low pressure streams is not however consistent with the restriction of flow Mach numbers to reasonably small values and with efficient low pressure expansion in the second and subsequent stages. It therefore appears that manifestations of the scheme proposed in Fig.2.5-1 will involve separate LP flows of different stagnation pressures, as is indeed indicated on the $Q-U$ diagram forming the right hand part of Fig.2.5-1. Similar arguments suggest that compounded expansion of the LP stream gives rise to multiple HP outlet streams also

having different stagnation pressures.

The type of compounding shown in Fig.2.5-3 avoids intermediate pressure outlets, but only at the expense of increased complexity in the ducting connecting the various stages. To prevent irreversible effects propagating throughout the system it is desirable to fit an intercooler adjacent to the pressure-exchanger on each medium pressure inlet duct. A similar arrangement to that shown in Fig.2.5-3 can be devised to cater for multi-stage expansion of the LP stream; such a device could obviously be used in combination with the circuit shown on Fig.2.5-3 to provide multi-stage compression and expansion. More than one stage of a multi-stage divider can be accommodated in a rotor; a coaxial arrangement offers a convenient way to incorporate sets of cells of different volumes in a single rotor.

10.1.2 Equaliser:

In an equaliser, advantage may be taken of the high maximum cycle temperature permissible, without causing overheating of the structure, when high and low temperature fluid streams both pass through the same rotor. Due to the effects of internal heat transfer the author would expect a better product efficiency with a cold driving fluid and a hot driven fluid than for the converse possibility.

Internal heat transfer has not been taken into account in the theoretical calculations described in this report because:

(a) nowhere are the temperature differences in the cycle appreciable.

(b) little data seems to exist on unsteady flow heat transfer

The omission of heat transfer considerations does not appear to have had any serious effects on the method of characteristics performance

predictions judging by the measure of agreement between them and reality.

The methods of compounding illustrated in Fig.2.5-2 and Fig.2.5-4 suffer from similar disadvantages to those described in detail when considering the corresponding divider compounding schemes. The arrangement in Fig.2.5-2 involves multiple HP inlets; an alternative circuit to cater for multi-stage expansion of the HP flow requires intermediate low pressure intakes. The circuit shown in Fig.2.5-4 for multi-stage compression of the LP flow, or its opposite (not illustrated) providing a facility for compounded HP expansion; involves complicated duct work. As for the corresponding divider schemes, it is preferable to install an intercooler at each intermediate inlet.

10.2 COMMERCIAL APPLICATIONS:

The commercial applications suggested in this section are primarily intended for single stage dividers and equalisers and are therefore essentially low pressure ratio uses.

10.2.1 Dividers:

Perhaps the most obvious applications for dividers are those involving pressure boosting. Good uses of this kind may provide for the recovery, by recompression, of fluid otherwise going to waste. Such an application could, for instance, involve the recompression, and subsequent recirculation of process steam emerging for example from a steam cooking process carried out at a pressure sensibly greater than that prevailing in the low pressure sink into which all the steam leaving the process would otherwise flow. In such applications, dividers would clearly be useful in helping to conserve process fluid; indeed if the fluid was a wet vapour, or was particle contaminated, pressure-

exchanger dividers may be uniquely suitable as erosion problems may prevent the use of turbo-machines.

Another field of application for dividers focusses attention on the cooling properties of the low pressure stream. An example, making good use of pressure-exchanger erosion resistance potentialities, involves the employment of a divider as an expansion engine in a vapour compression refrigerator. The right hand part of Fig.10.2-1 shows such an arrangement; the cycle is shown on the adjacent temperature entropy diagram.

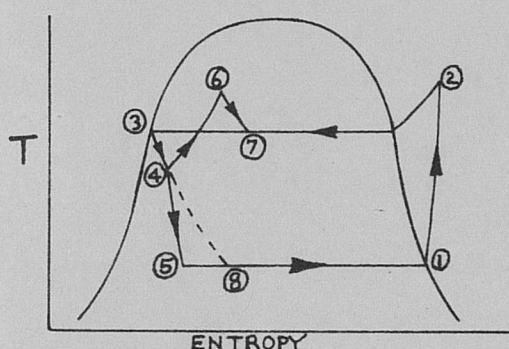
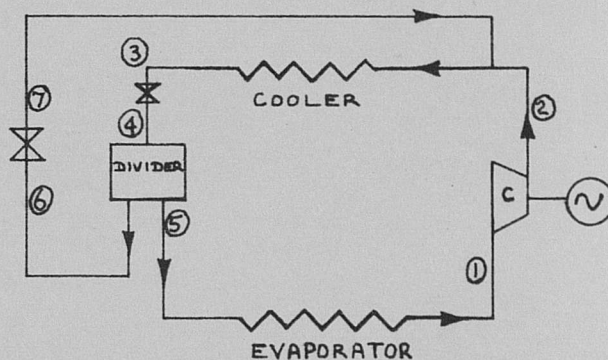


Fig. 10.2-1



Point 5 on the temperature entropy sketch shows the state of the fluid entering the evaporator when the divider is fitted; point 8 is the corresponding state point when a simple throttle replaces the divider. The slight throttling from 3 to 4 is merely to ensure that vapour rather than liquid passes into the pressure-exchanger.

Less ambitious uses for divider cooling devices are as replacements for Hilsch vortex tubes in laboratory refrigeration applications. From what evidence the author can find, (13), (6) Hilsch tubes generally have rather low isentropic expansion efficiencies, usually less than 50%. Provided the supply pressure ratio is suitable, it may therefore be economic to employ a divider; if need be the same machine could also be used as a pressure booster, a function not possible with a Hilsch tube.

It may be noticed, in the foregoing, that some of the potential applications suggested in section 2.6.1, for instance the cooling of high speed aircraft, have been omitted. This is because the actual characteristics of dividers revealed by the experiments, etc. show that the machines are not well suited for these uses.

10.2.2 Equaliser:

Pressure-exchanger equalisers may find application in vacuum stills and similar equipment. For such uses their erosion resistance should be an important advantage giving them a longer life than would be obtainable from corresponding turbo-compressor units. From the experimental and theoretical performance results for equalisers it appears that, at suitable pressure ratios, they give much better performances than are possible with ejectors (7), (5). The use of an equaliser in a still should therefore allow driving fluid to be conserved in comparison with the consumption when an ejector is used. Alternatively it should cheapen the apparatus due to lower replacement costs when the comparison is between a pressure-exchanger and a turbo-compressor.

Another use involves the replacement by an equaliser of a throttle through which steam or some other fluid passes on its way to, say, a steam cleaning process. By directing steam leaving the process into the low pressure intake of the equaliser, compressing and then recirculating it through the process, a saving is made in the quantity of fresh steam required. Whilst an ejector could be used instead of the proposed equaliser, for favourable pressure ratios, the saving in HP fluid should be greater with an equaliser owing to its superior performance.

Although a much more difficult application, an equaliser could be

used as a thrust augmenter for an aircraft jet propulsion unit.

An elementary scheme could make use of the normal efflux of a turbo-jet as the driving fluid, the driven fluid being air entrained from the atmosphere. Quantative calculations for such an arrangement showed that a simple-jet gas generator was not likely to produce the best results because the driving pressure ratio was greater than that desirable for optimum functioning of the pressure-exchanger. A more promising application involves the use of an equaliser thrust augmenter for which the driving fluid is the efflux of an afterburning, by-pass jet engine. The indications are that, for such a design, the pressure-exchanger could at least double the thrust otherwise obtainable.

10.3 RECOMMENDATIONS FOR FURTHER WORK:

10.3.1 Modifications To The Test Rig:

Whilst the experimental apparatus has performed reliably, and the author is well pleased with its behaviour throughout the series of tests, a few improvements suggest themselves in the light of experience with the equipment.

Probably a slight betterment in performance, certainly a removal of doubt about their effects, would result from elimination of the static pressure gradients observed in the ports of the machine. The gradients are referred to in section 7.1.3; an approximate analysis is given in Appendix VI.

Another improvement would be to provide an increase in the range of dimensionless port widths explorable; notably to allow the low pressure port width to be varied over a much wider range than at present and to enable the width of the land between the MP and LP ports to be altered. An increase in the dimensionless port widths, could perhaps

be achieved most easily by reducing the running speed and replacing the existing rotor by one having a greater number of cells so that the dimensionless cell width is not increased despite the reduction of speed.

Suggested programmes for extending the work on divider and equaliser cycles are studied in sections 10.3.2 and 10.3.3 respectively.

10.3.2 Programme for Divider:

It is suggested that further work on pressure-exchanger dividers could be carried out with the objects of:

- (1) studying the effect of changing the width of the high pressure outlet port, also the width of the land between the MP and LP ports.
- (2) extending the tests to cover a greater range of pressure ratios, adjusting the variable geometry appropriately, and testing of the pressure boosting performance of the arrangement shown in Fig.2.4-2 section 2.4.1.
- (3) making a more detailed study of cooling cycle performance, including searching for optima, and traversing the LP outlet ports to find the coldest fluid, etc.
- (4) extending the research to cover working fluids other than dry air; the choice of fluids being indicated by the requirements of potential commercial applications.
- (5) programming the method-of-characteristics calculations for digital computation. The computer calculations should take account of leakage and friction, preferably by more exact methods than were used for the manual procedure, and extend the range of pressure ratios covered by the theoretical treatment. Ideally the programme should be capable of

determining the optimum geometry for any operating condition. It would also be useful if it could determine the performance with a specified geometry, thereby indicating the price of fixed geometry in terms of loss of performance.

10.3.3 Programme For Equaliser:

The equaliser work may perhaps be extended to:

- (1) study the effect of varying the width of the MP outlet port, also the width of the land between the high and low pressure inlets.
- (2) extend the range of pressure ratios covered by the experiments.
- (3) carry out tests with HP and LP streams with appreciably differing stagnation temperatures.
- (4) conduct tests with working fluids other than dry air; also possibly to establish the performance using HP and LP fluids of different compositions. The choice of possibilities for investigation under (3) or (4) could be decided from likely commercial requirements.
- (5) programme the characteristic calculations as for the divider case, additionally taking account of internal heat transfer and the possibility of using driving and driven fluids of differing temperatures and compositions.

11. CONCLUSIONS

The conclusions have been divided into three parts (a), (b) and (c) where:

- (a) states conclusions drawn from the experiments.
- (b) reports conclusions deduced from theoretical predictions of the optimum performance characteristics of single stage divider pressure boosters and equalisers.
- (c) states conclusions obtained from theoretical studies of the practicability of utilising non-dynamic processes, supersonic flows and compounding.

(a) Conclusions drawn from the experiments.

The experimental results for both the divider pressure booster and equaliser cycles showed that the regions of greatest product efficiency were located at values of $\frac{P_{ho}}{P_{Mo}}$ and $\frac{P_{Lo}}{P_{Mo}}$ fairly close to unity.

These maxima, which were roughly comparable with values of product efficiency obtainable from turbo-machines, were approximately $\eta_{PIS} = 0.6$ and $\eta_{PIS} = 0.7$ for the divider and equaliser respectively. There was no real indication in any of the divider or equaliser experiments that the regions of maximum efficiency could be moved appreciably by changing any of the experimental variables.

Experimental results showed that optimised performances were not, in general, particularly sensitive to small changes in speed and geometry. It was also observed that the settings for the best divider cooling cycle performance did not coincide with those for optimum pressure boosting performance. The best observed isentropic

expansion efficiency η_{EIS} was approximately 0.75.

- (b) Conclusions drawn from the results of theoretical predictions of the performance of single stage dividers and equalisers.

Predictions of idealised isentropic performances, obtained from $Q - U$ diagrams and the energy equation applied to isentropic divider and equaliser cycles, allowed basic cycle characteristics to be studied in simplified form. The technique, which was very easy to apply, also allowed the effects of fundamental performance limitations to be examined without the superposition of other factors included in more sophisticated theoretical analyses.

The simplified method for predicting pressure-exchanger performance introduced in section 4.3 underestimated actual divider pressure booster and equaliser performances. This was probably the consequence of applying the only experimental cell filling and emptying data available, for cell widths of $\tau_c \approx 0.54$, to calculations requiring data for $\tau_c \approx 0.4$. The method did predict divider and equaliser performance characteristics of generally correct form.

Method-of-characteristics calculations indicated that optimum divider pressure booster performance was obtained when the HP and LP outlets were made as wide as possible consistent with the avoidance of flow reversal in these ports. The equaliser performed best when the MP outlet and LP inlet were both of maximum width. For both divider and equaliser the widths of the MP and HP inlets respectively were limited by the arrival of reflected waves.

Characteristics calculations for optimum geometry with finite width cells, taking account of friction and leakage, produced results which in

general agreed fairly closely with the experiments. It was noteworthy that the regions of maximum product efficiency were located near the $\frac{P_{H0}}{P_{M0}} = \frac{P_{L0}}{P_{M0}} = 1$ origin of the planes on which the performances were presented; this also was consistent with the results of the experiments. The main discrepancy between theoretical and experimental performances occurred in regions where the geometry of the experimental apparatus diverged appreciably from that indicated as optimum in the theoretical analyses. This was particularly the case for the equaliser at small values of ξ .

The characteristics study showed that the geometries corresponding to optimum performances over the ranges of pressure ratios investigated were not constant for the divider or equaliser. In some cases the variation amounted to about 40% of the maximum value of the phasing or port width concerned.

The characteristics treatment indicated that the major causes of the departure of the actual performances from the isentropic ideals were, in order of decreasing importance:

- (1) the basic wave effects taken into account when performance was predicted by the method of characteristics applied to infinitely narrow cells.
- (II) the effect of finite cell width on wave events.
- (III) cell wall friction, entry and exit losses.
- (IV) leakage.

For pressure ratios very close to unity (III) and (IV) should be interchanged.

- (c) More general theoretical considerations on the uses of non-dynamic processes, supersonic flows and compounding led to the

following conclusions.

Pressure-exchanger divider pressure boosters and equalisers depend entirely upon dynamic (wave) effects for their functioning; in this respect dividers and equalisers differ from some other pressure-exchanger cycles.

It was concluded that it was not practical to make use of supersonic flows in pressure-exchangers of the types studied. All indications were that the most efficient operation of both dividers and equalisers occurred with low Mach number sub-sonic flows.

Compounded divider and equaliser cycles are conceivable, thereby indicating the possibility of operating with greater pressure ratios than are practical with single stage machines. Each of the two methods of compounding, introduced in section 2.5 and discussed in section 10.1, suffer from certain disadvantages which may be serious in some applications.

12. GRAPHICAL PRESENTATION OF RESULTS

12.1 RESULTS OF THEORETICAL STUDIES

PREDICTED DIVIDER ISENTROPIC PRESSURE-BOOSTING PERFORMANCE

$$(\gamma = 1.4)$$

β^* _____
 M_H - - - - -
 M_M _____
 M_L - - - - -

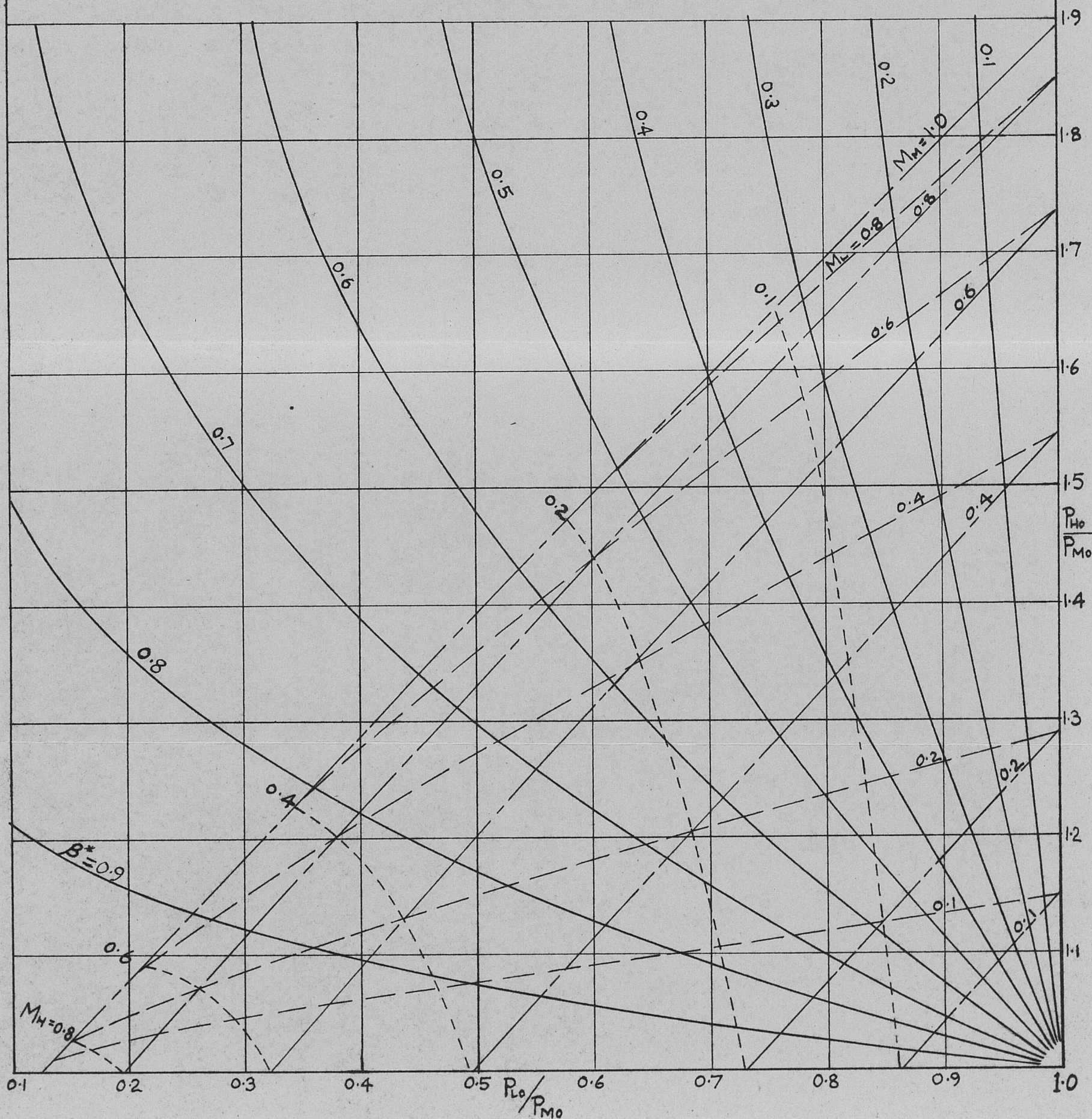
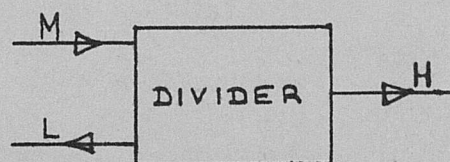
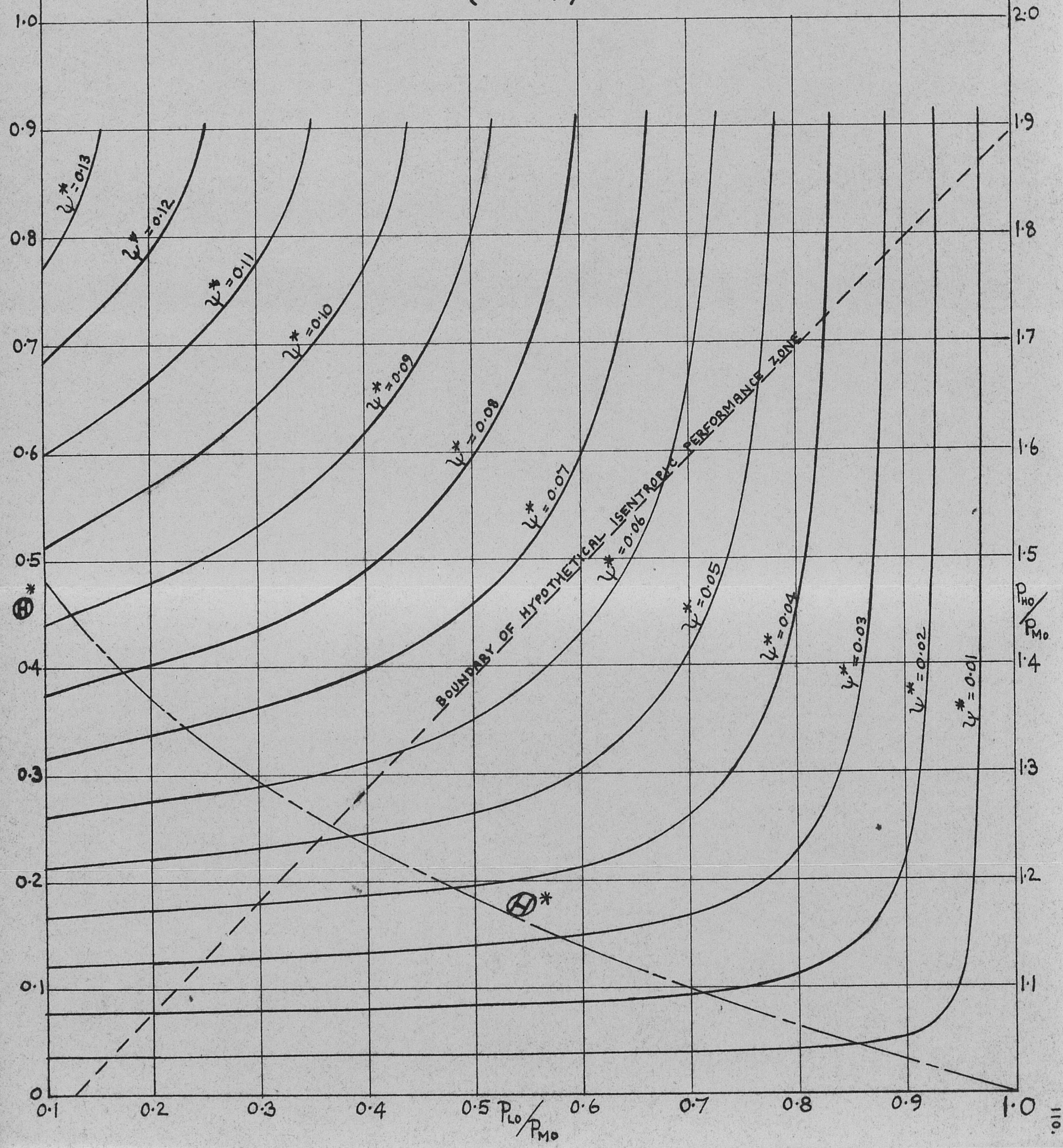


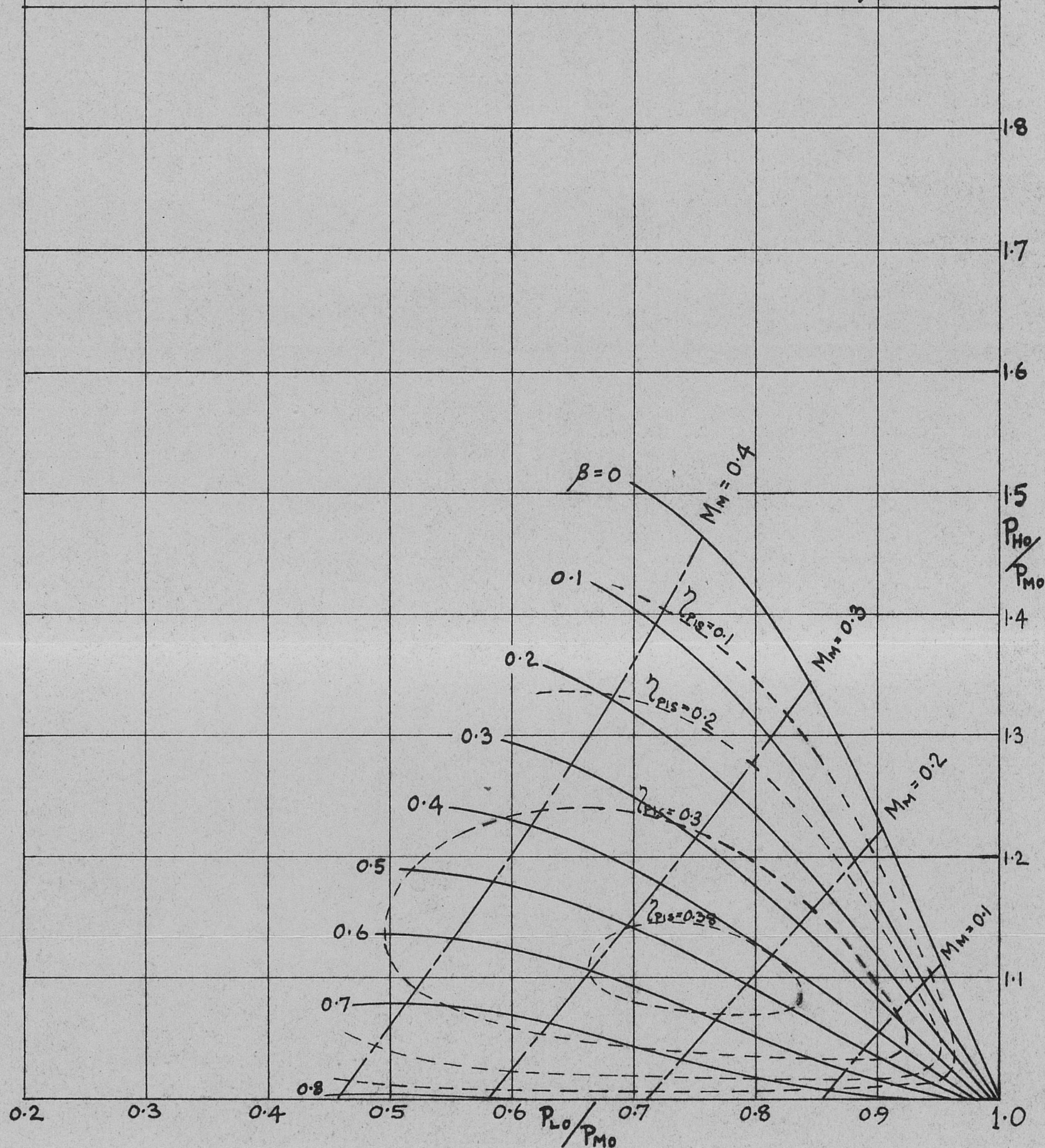
FIG. 12-1-2

PREDICTED DIVIDER ISENTROPIC COOLING PERFORMANCE
($\gamma = 1.4$)



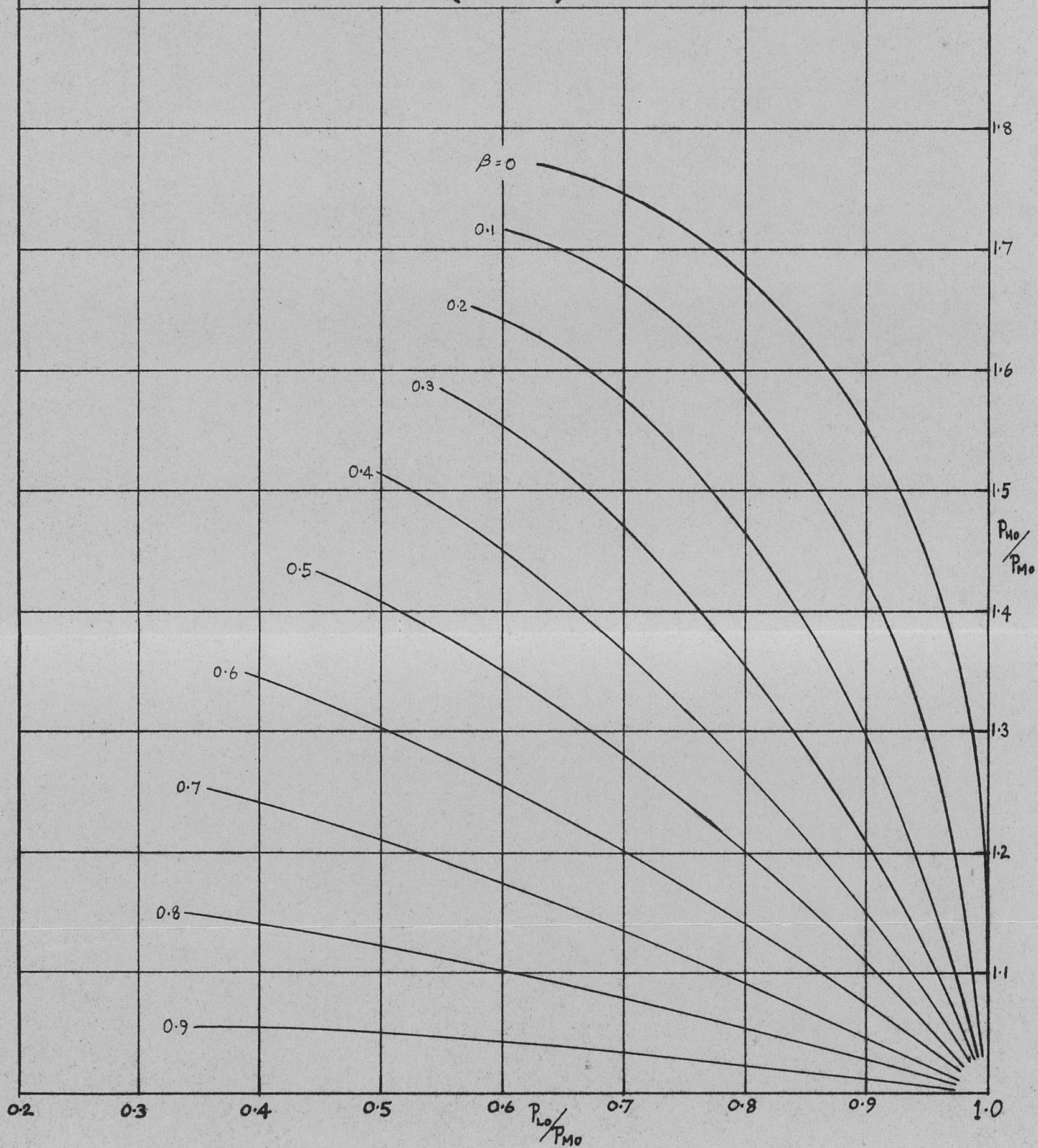
PREDICTED DIVIDER PRESSURE-BOOSTING PERFORMANCE
WITH IRREVERSIBILITIES TAKEN INTO ACCOUNT
(SIMPLIFIED THEORETICAL TREATMENT, $\gamma_c \approx 0.54, \gamma = 1.4$)

FIG. 12.1-3



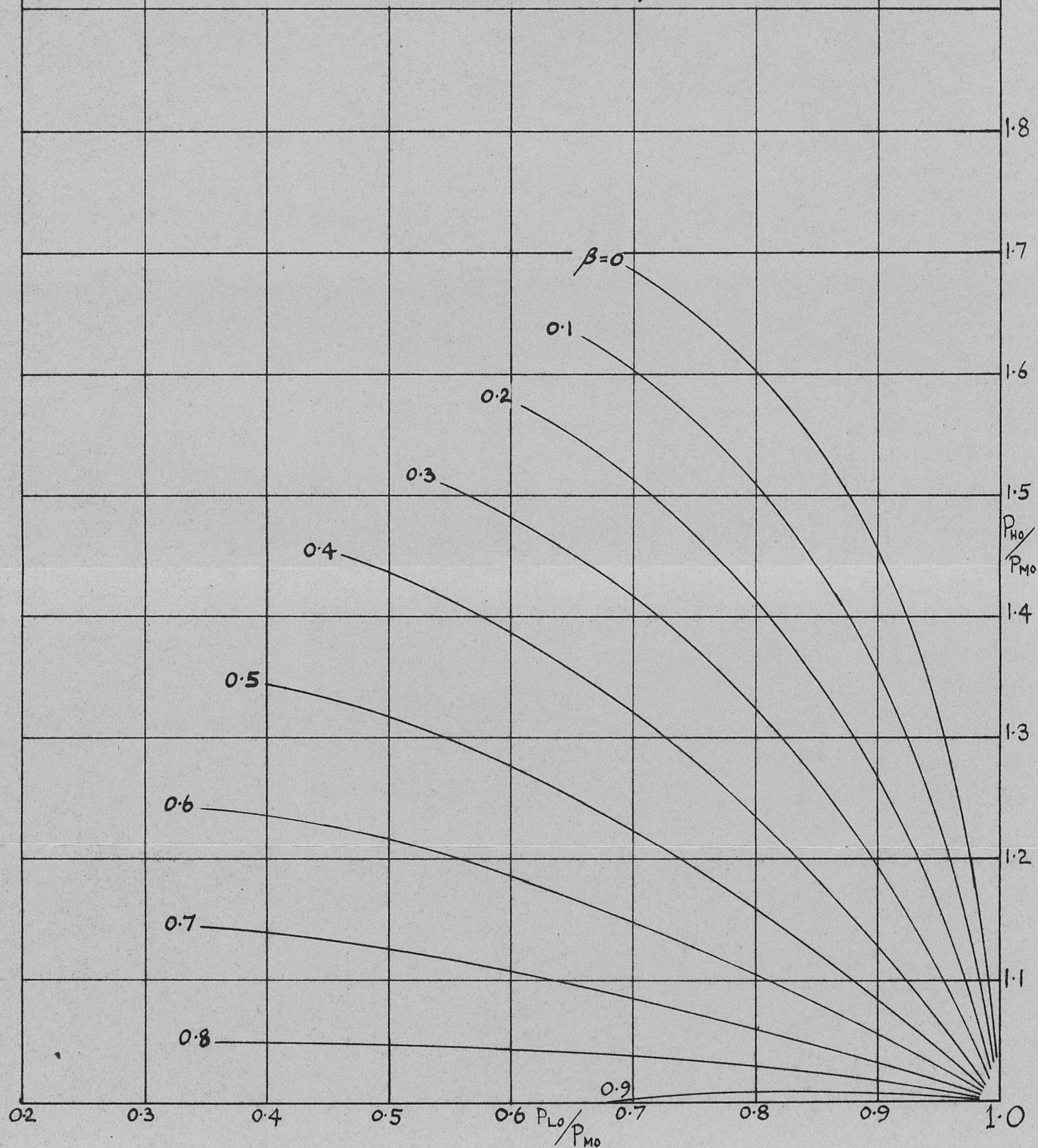
PREDICTED DIVIDER PRESSURE-BOOSTING PERFORMANCE
WITH INFINITELY NARROW CELLS
($\gamma = 1.4$)

FIG. 12.1-4



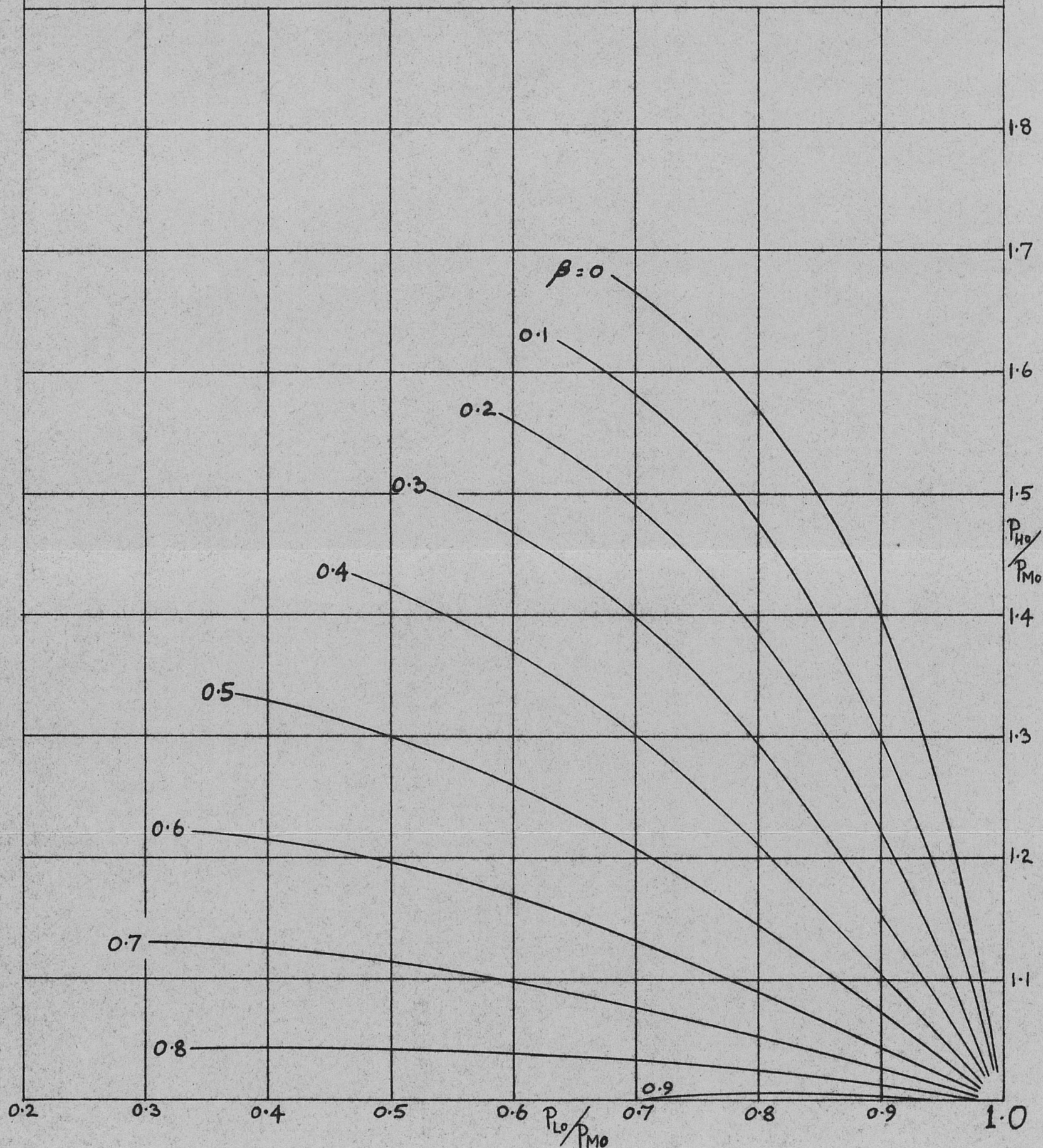
PREDICTED DIVIDER PRESSURE - BOOSTING PERFORMANCE
WITH FINITE WIDTH CELLS
($\tau_c = 0.4 \frac{q_{M0}}{q_H}$, $\gamma = 1.4$)

FIG.12-1-5



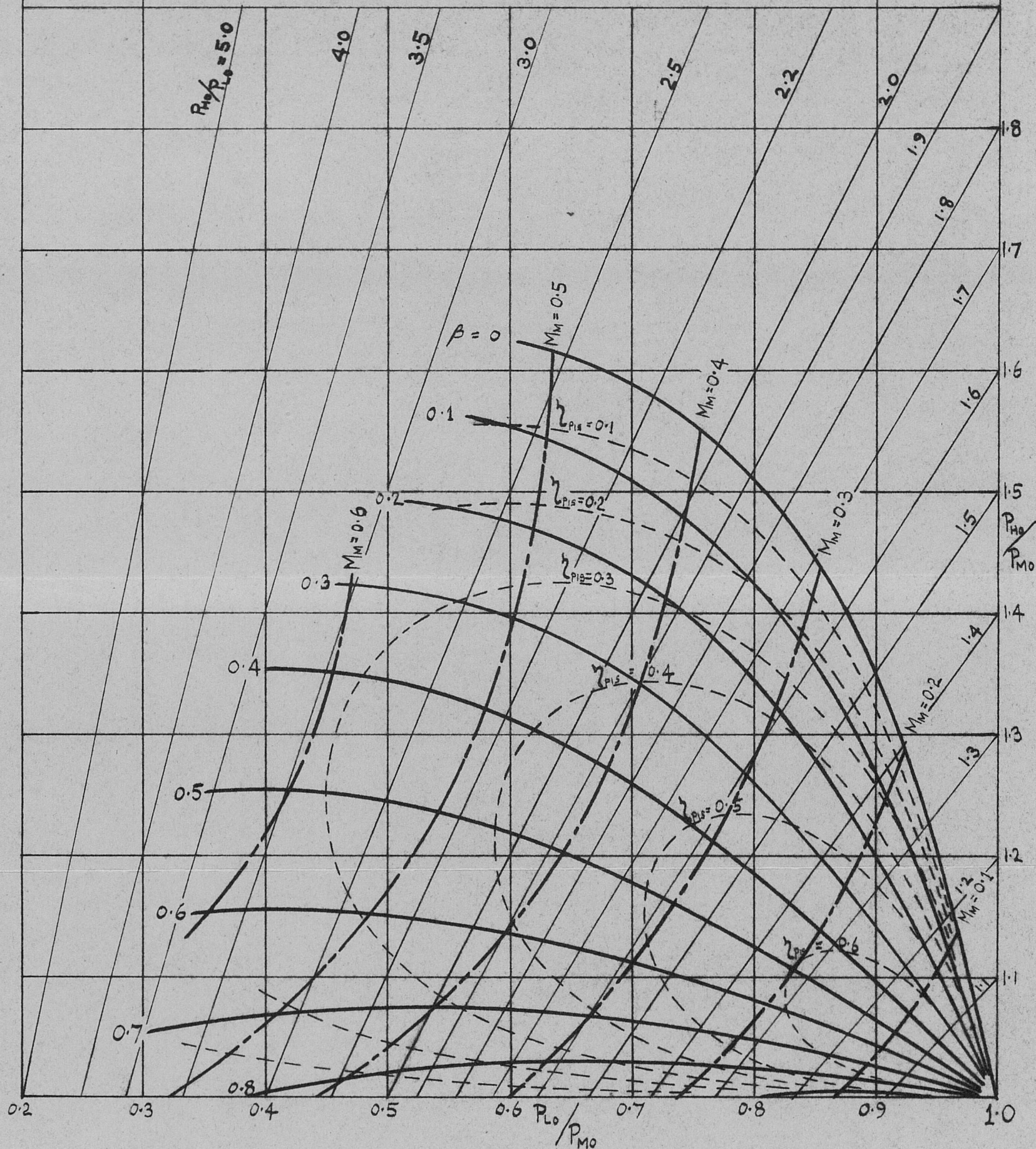
PREDICTED DIVIDER PRESSURE—BOOSTING PERFORMANCE
WITH FINITE WIDTH CELLS; LEAKAGE TAKEN INTO ACCOUNT
($\gamma_c = 0.4 \frac{a_{M0}}{a_H}$, $\gamma = 1.4$)

FIG. 12.1-6



PREDICTED DIVIDER PRESSURE-BOOSTING PERFORMANCE
WITH FINITE WIDTH CELLS; LEAKAGE AND FRICTIONAL
EFFECTS TAKEN INTO ACCOUNT ($\gamma_c = 0.4 \frac{Q_{M0}}{Q_H}$, $\gamma = 1.4$)

FIG. 12.1-7



PREDICTED GEOMETRIC PROPERTIES FOR THE PERFORMANCE CHARACTERISTICS SHOWN ON FIG.12.1-7

NB. AT $P_{H0}/P_{M0} = P_{L0}/P_{M0} = 1$:

$$0.8 \leq \gamma_H \leq 1.6,$$

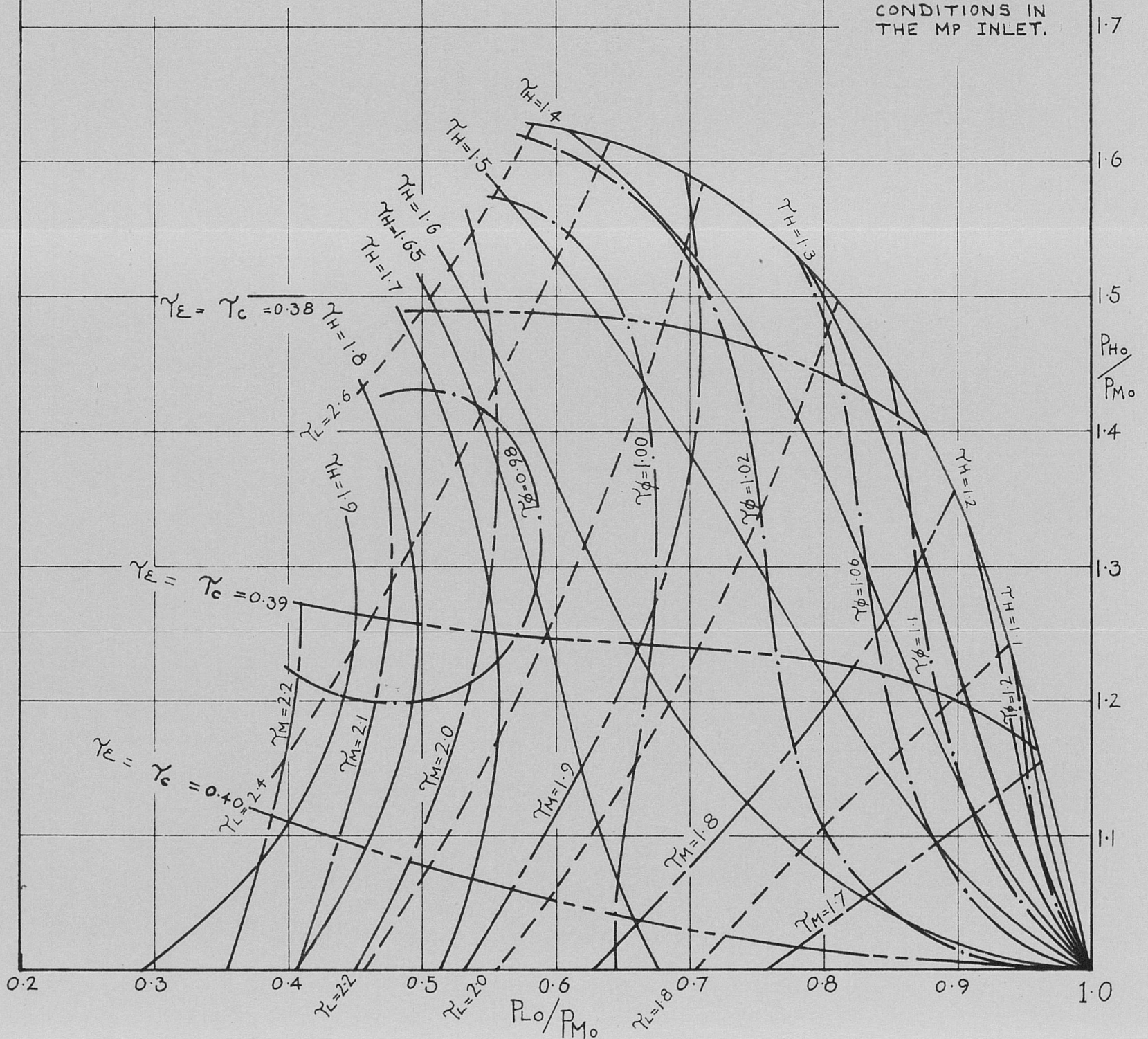
$$\gamma_L = 1.6, \gamma_M = 1.6,$$

$$1.0 \leq \gamma_\phi \leq 1.4,$$

$$\gamma_E = \gamma_C = 0.4.$$

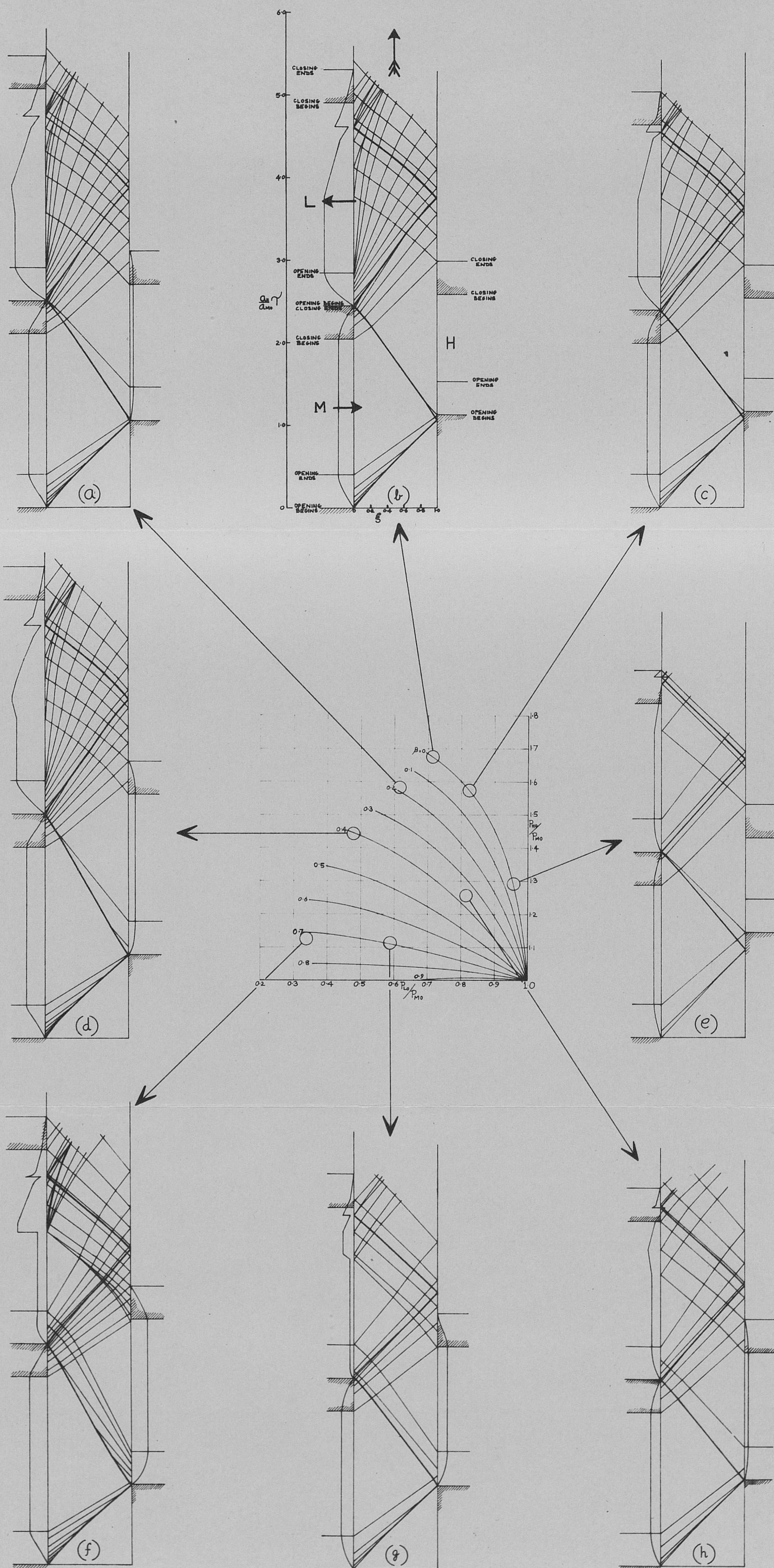
$$\gamma = \frac{t a_{M0}}{l}$$

WHERE: l = LENGTH OF A CELL.
 a_{M0} = VELOCITY OF SOUND
AT STAGNATION
CONDITIONS IN
THE MP INLET.



DIVIDER WAVE DIAGRAMS, FINITE CELL WIDTH, ($\gamma_c = 0.4 \frac{Q_{M0}}{Q_H}, \gamma = 1.4$)

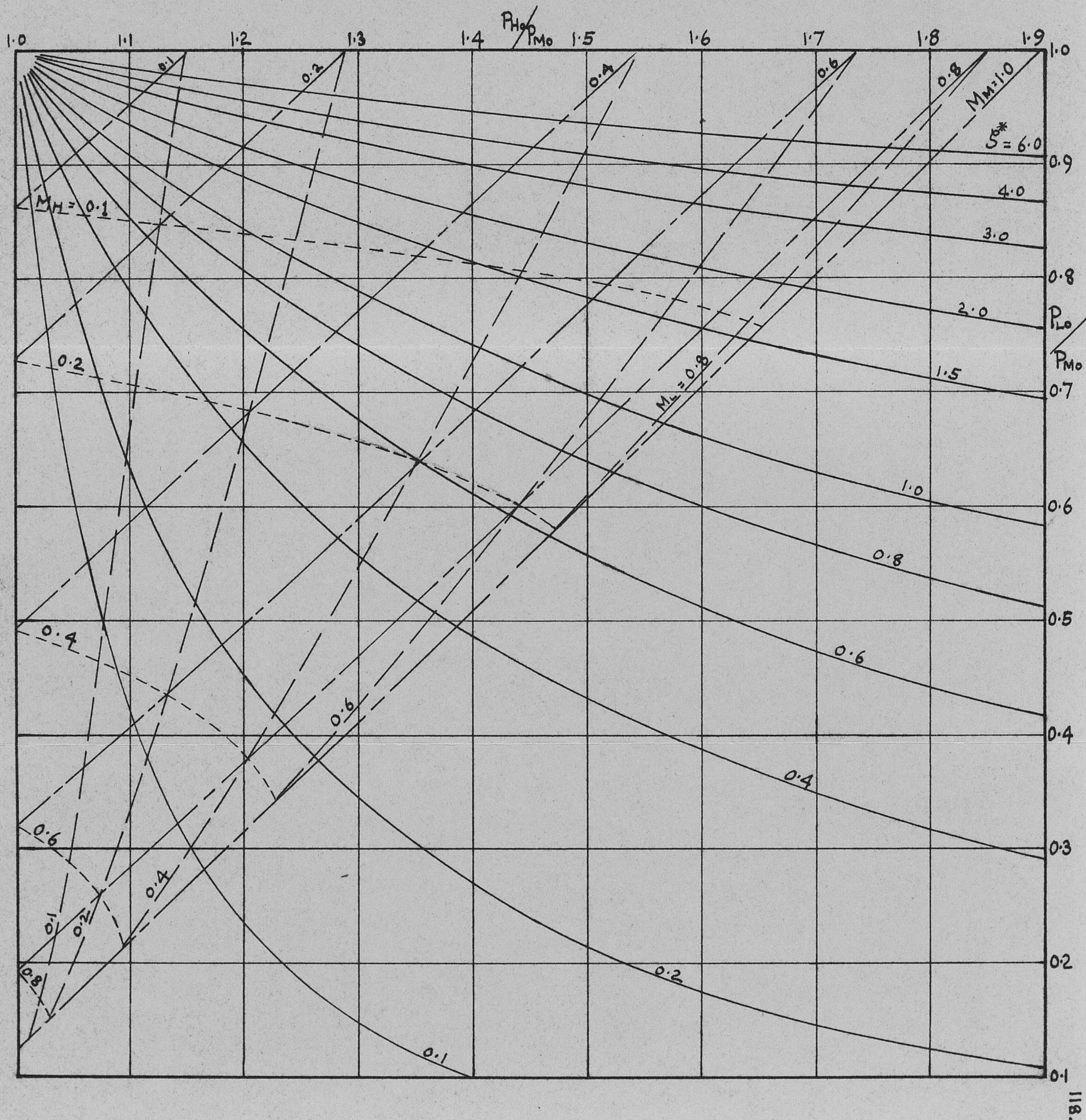
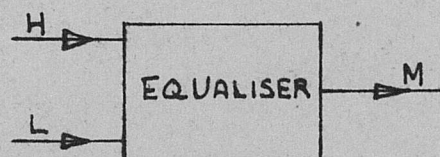
NB. A SMALL SCALE REPRODUCTION OF FIG.12.1-5 FORMS THE CENTRAL PART OF THE ILLUSTRATION. INFLOW AND OUTFLOW VELOCITY PROFILES ARE SKETCHED ON THE WAVE DIAGRAMS.



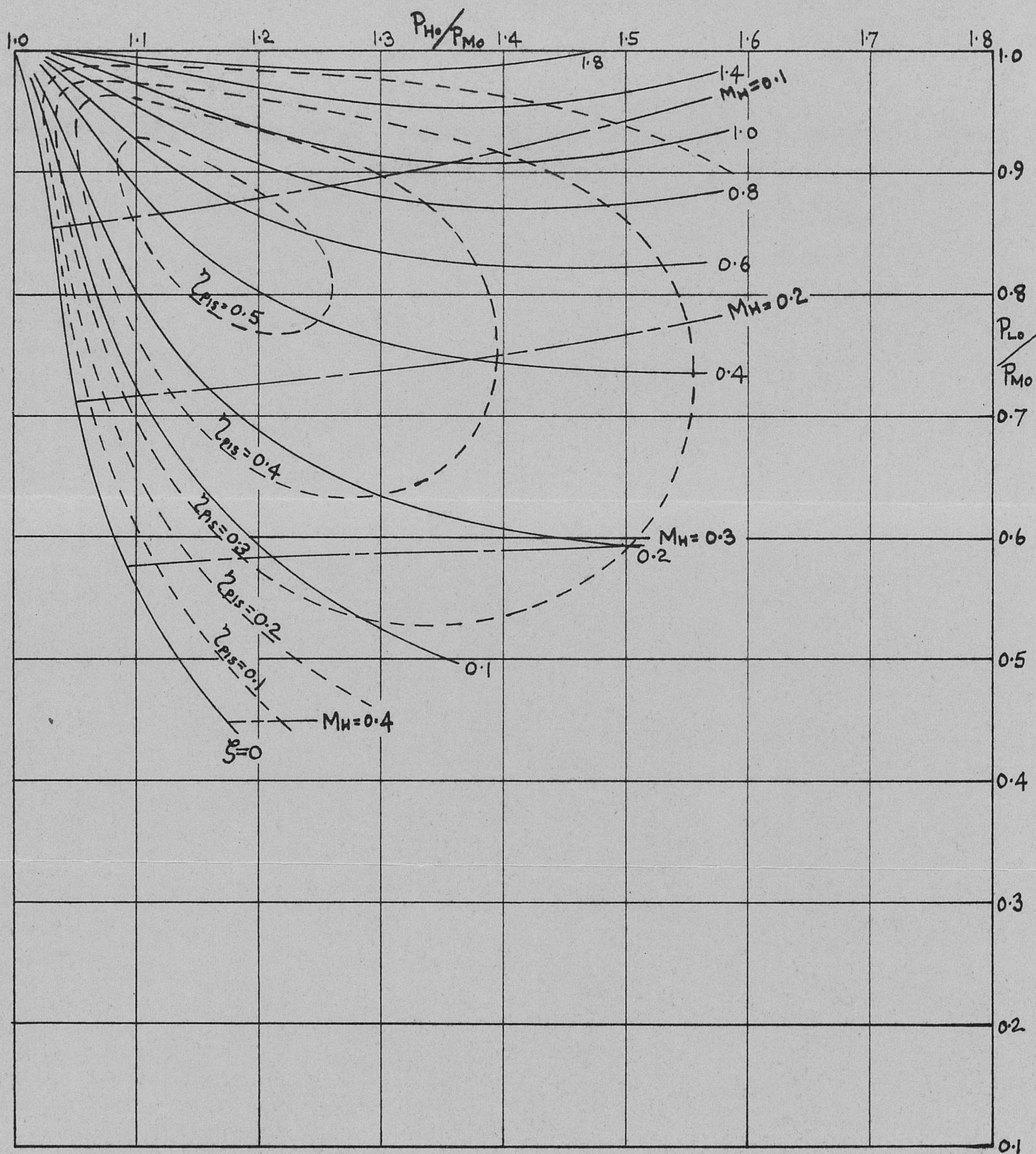
PREDICTED EQUALISER ISENTROPIC PERFORMANCE

$$(\gamma = 1.4)$$

S^*
 M_H
 M_M
 M_L

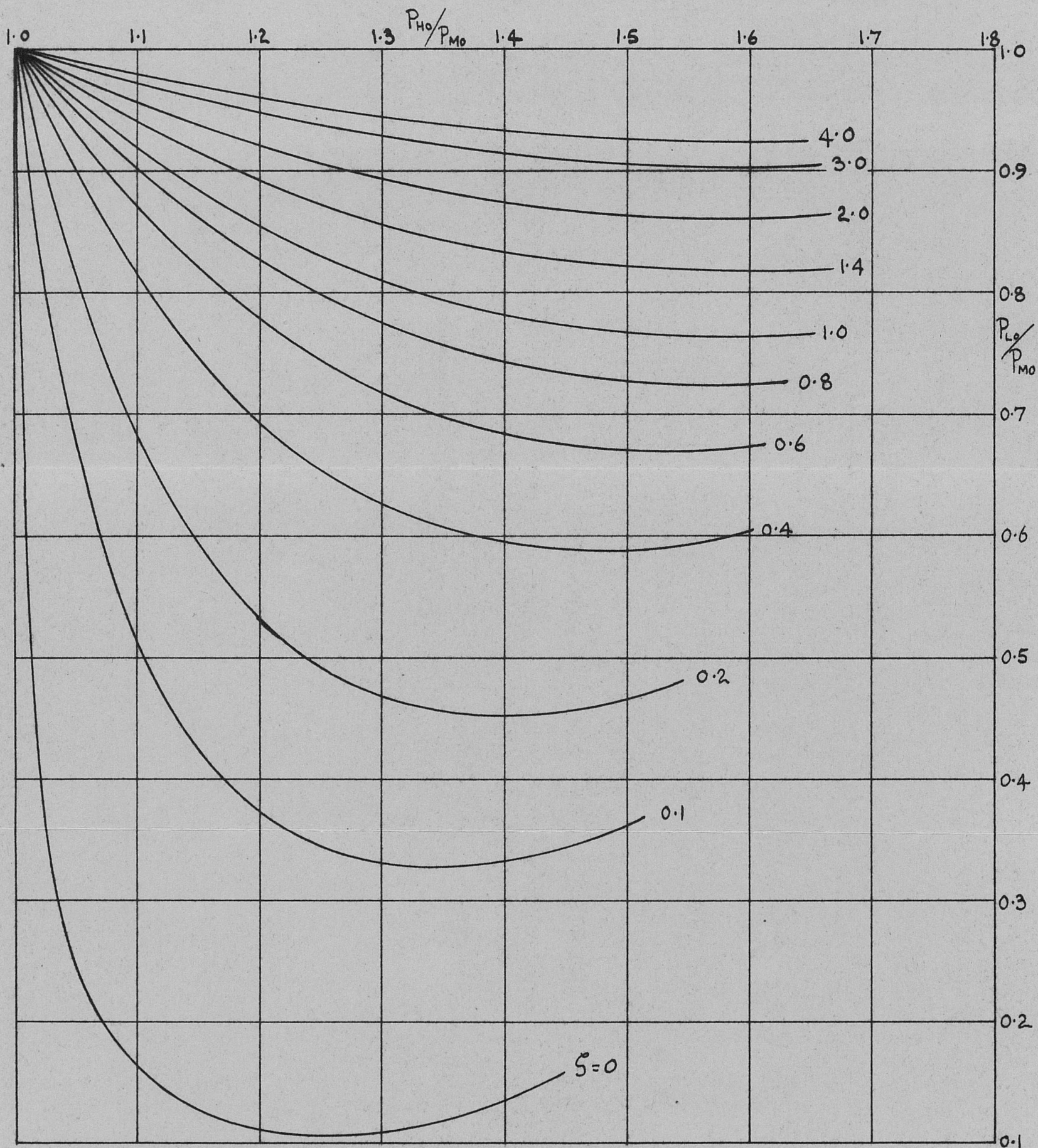


PREDICTED EQUALISER PERFORMANCE WITH
IRREVERSIBILITIES TAKEN INTO ACCOUNT
(SIMPLIFIED THEORETICAL TREATMENT, $\gamma_c \approx 0.54$)
($\gamma = 1.4$)



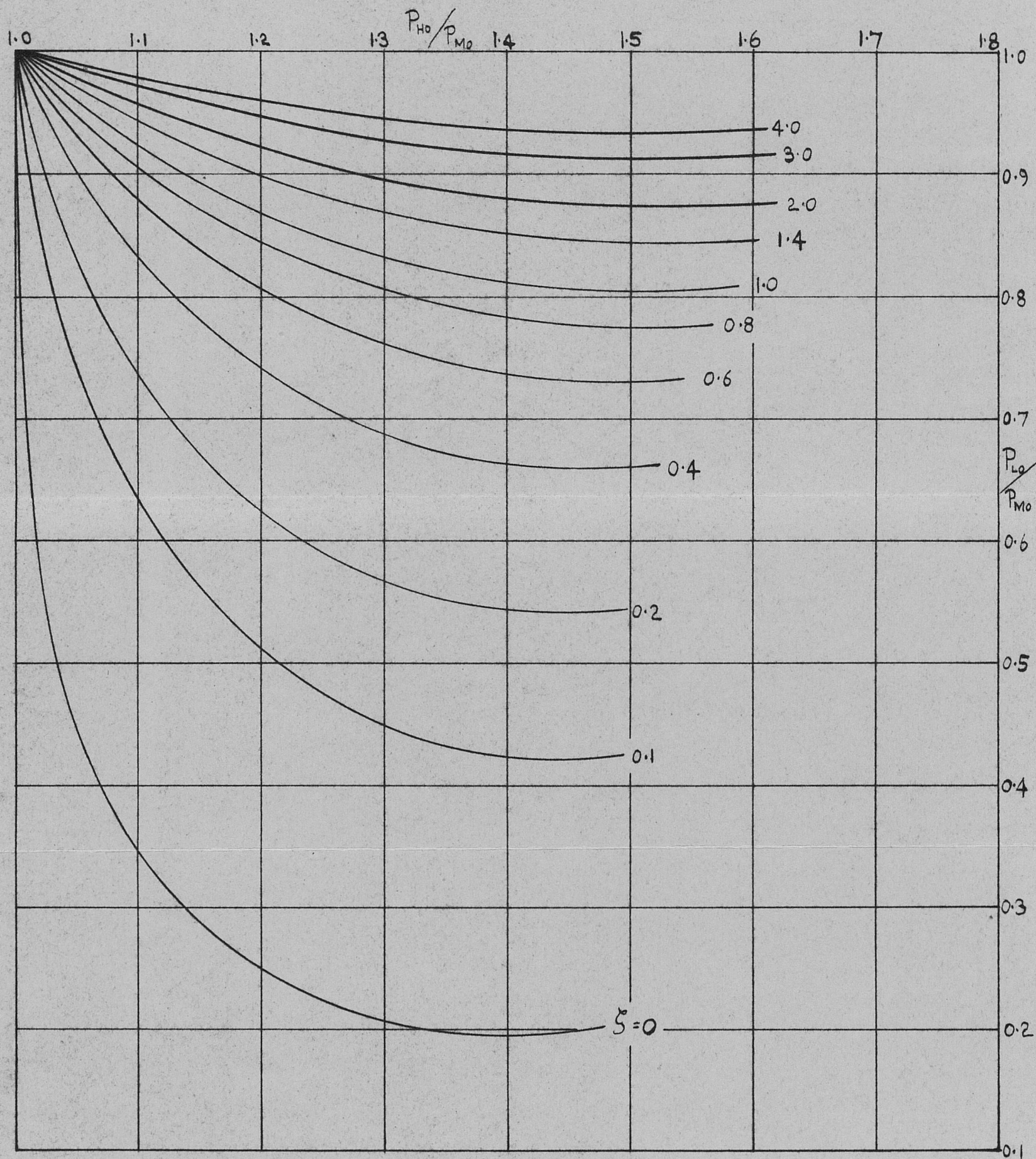
PREDICTED EQUALISER PERFORMANCE WITH
INFINITELY NARROW CELLS

$$(\gamma = 1.4)$$



PREDICTED EQUALISER PERFORMANCE WITH
FINITE WIDTH CELLS

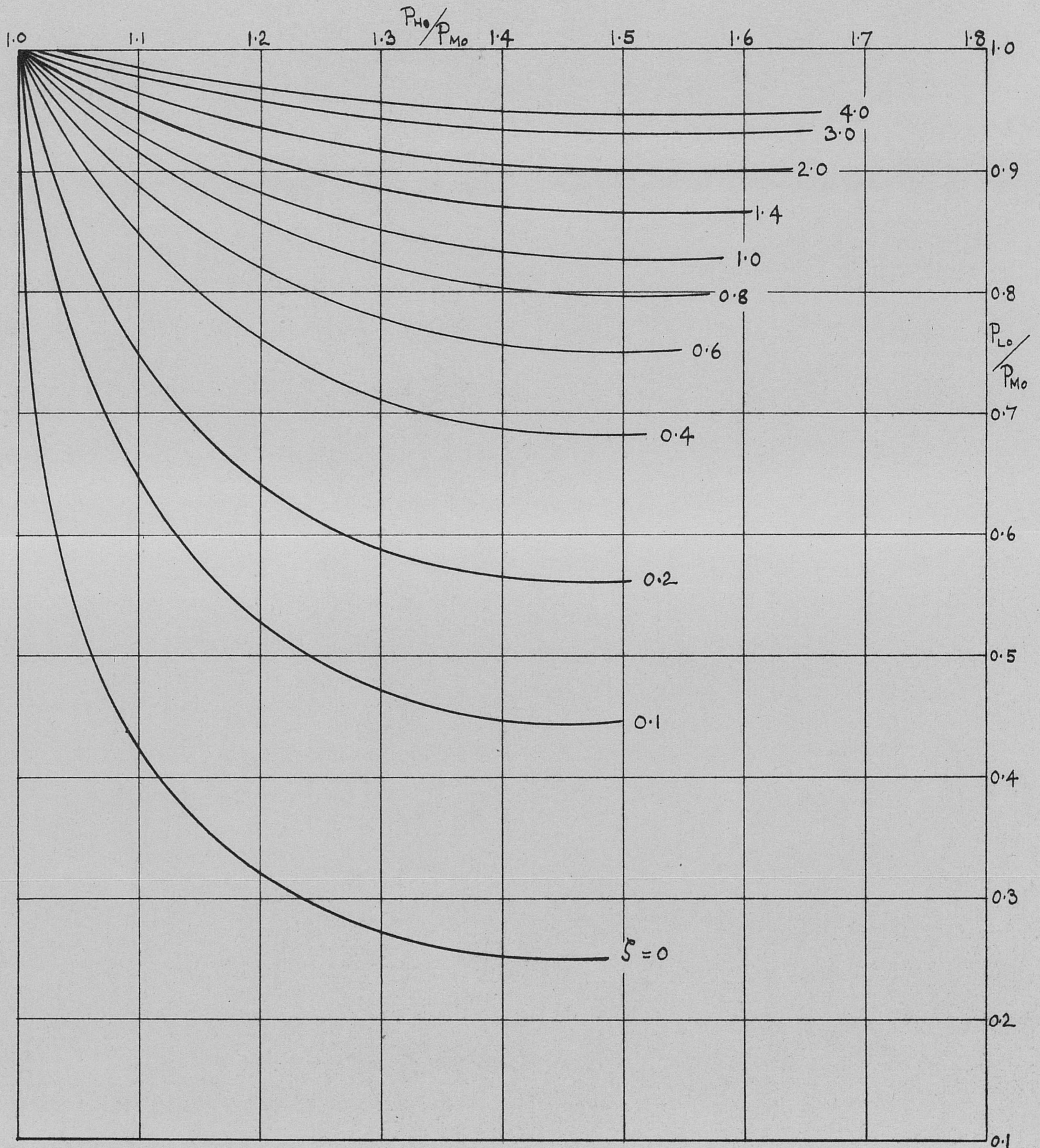
$$\left(\gamma_c = 0.4 \frac{a_{H0}}{a_H}, \gamma = 1.4 \right)$$



PREDICTED EQUALISER PERFORMANCE WITH
FINITE WIDTH CELLS; LEAKAGE TAKEN INTO
ACCOUNT

FIG.12.1-14

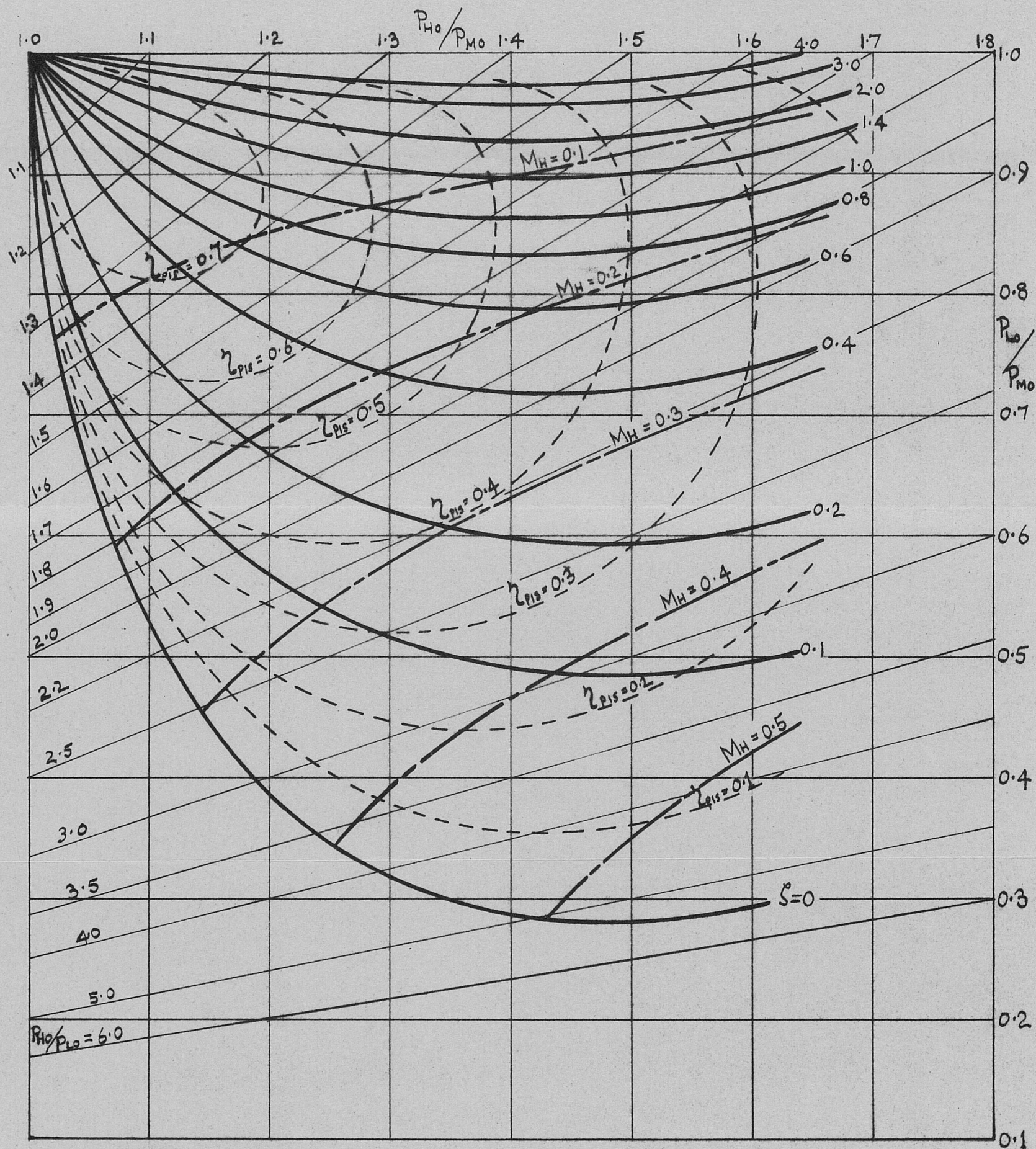
$$\left(\gamma_c = 0.4 \frac{a_{H0}}{a_H}, \gamma = 1.4 \right)$$



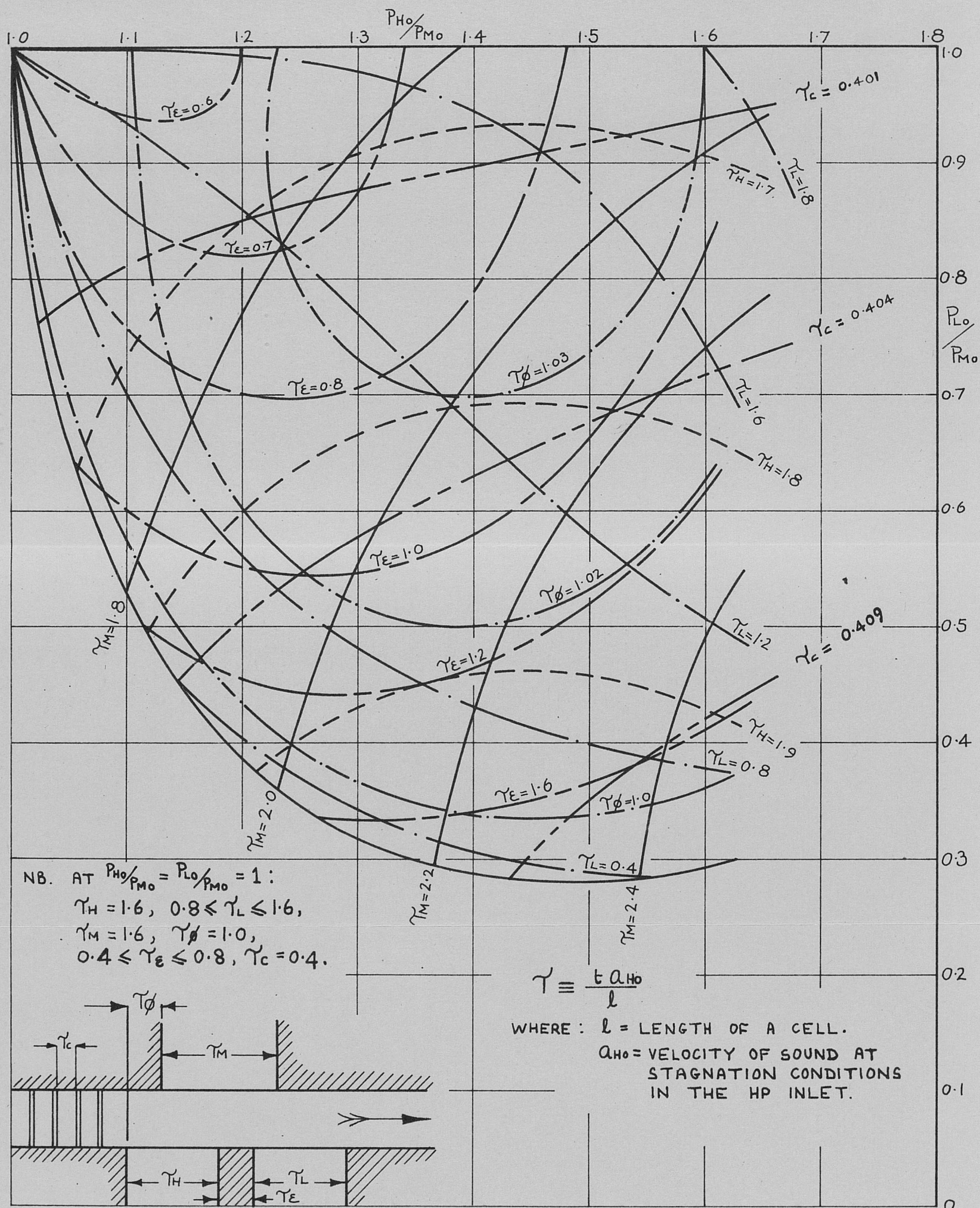
PREDICTED EQUALISER PERFORMANCE WITH
FINITE WIDTH CELLS; LEAKAGE AND FRICTIONAL
EFFECTS TAKEN INTO ACCOUNT

FIG. 12.1-15

$$\left(\gamma_c = 0.4 \frac{a_{H0}}{a_H}, \quad \gamma = 1.4 \right)$$

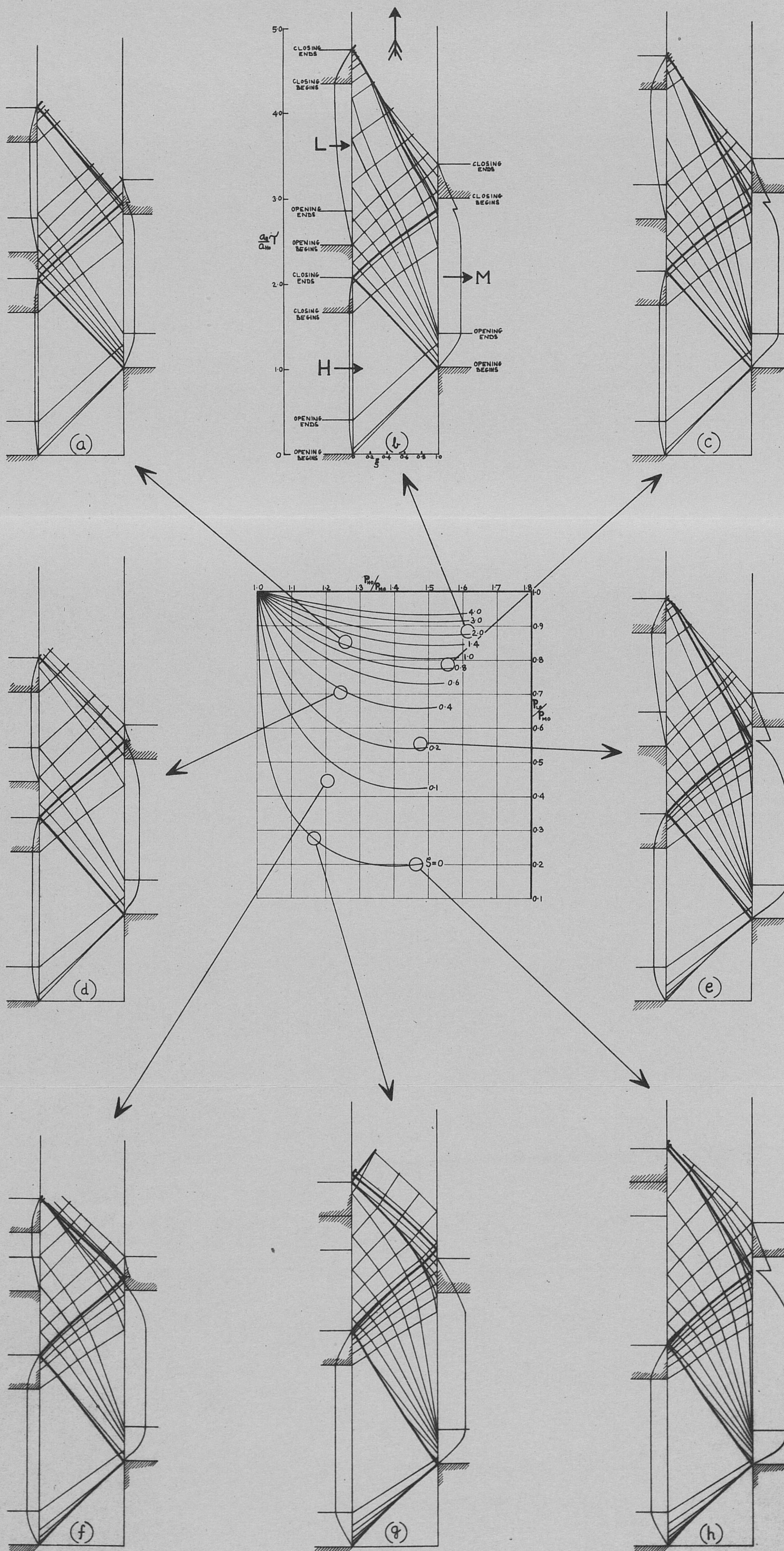


PREDICTED GEOMETRIC PROPERTIES FOR THE
PERFORMANCE CHARACTERISTICS SHOWN ON FIG. 12.1-15



EQUALISER WAVE DIAGRAMS, FINITE CELL WIDTH, ($\gamma_c=0.4 \frac{Q_{mo}}{Q_H}, \gamma=1.4$)

NB. A SMALL SCALE REPRODUCTION OF FIG.12:1-13 FORMS THE CENTRAL PART OF THE ILLUSTRATION. INFLOW AND OUTFLOW VELOCITY PROFILES ARE SKETCHED ON THE WAVE DIAGRAMS.



12.2

RESULTS OF EXPERIMENTAL INVESTIGATIONS

FIG. 12-2-1

EXPERIMENTALLY OBTAINED DIVIDER PRESSURE-BOOSTING PERFORMANCE

(FIXED GEOMETRY, CONSTANT SPEED, $T_{M0}=308^\circ\text{K}$, $P_{00}\approx 1\text{ATM.}$)

ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	6000 REV./MIN.	$M_{R0} = 0.1470$
HP PORT WIDTH	48°	$\gamma_H = 1.67$
LP " "	45°	$\gamma_L = 1.57$
MP " "	45°	$\gamma_M = 1.57$
PHASING	24°	$\gamma_\theta = 0.835$
LAND WIDTH	12°	$\gamma_E = 0.417$
CELL " "	10.4°	$\gamma_C = 0.360$

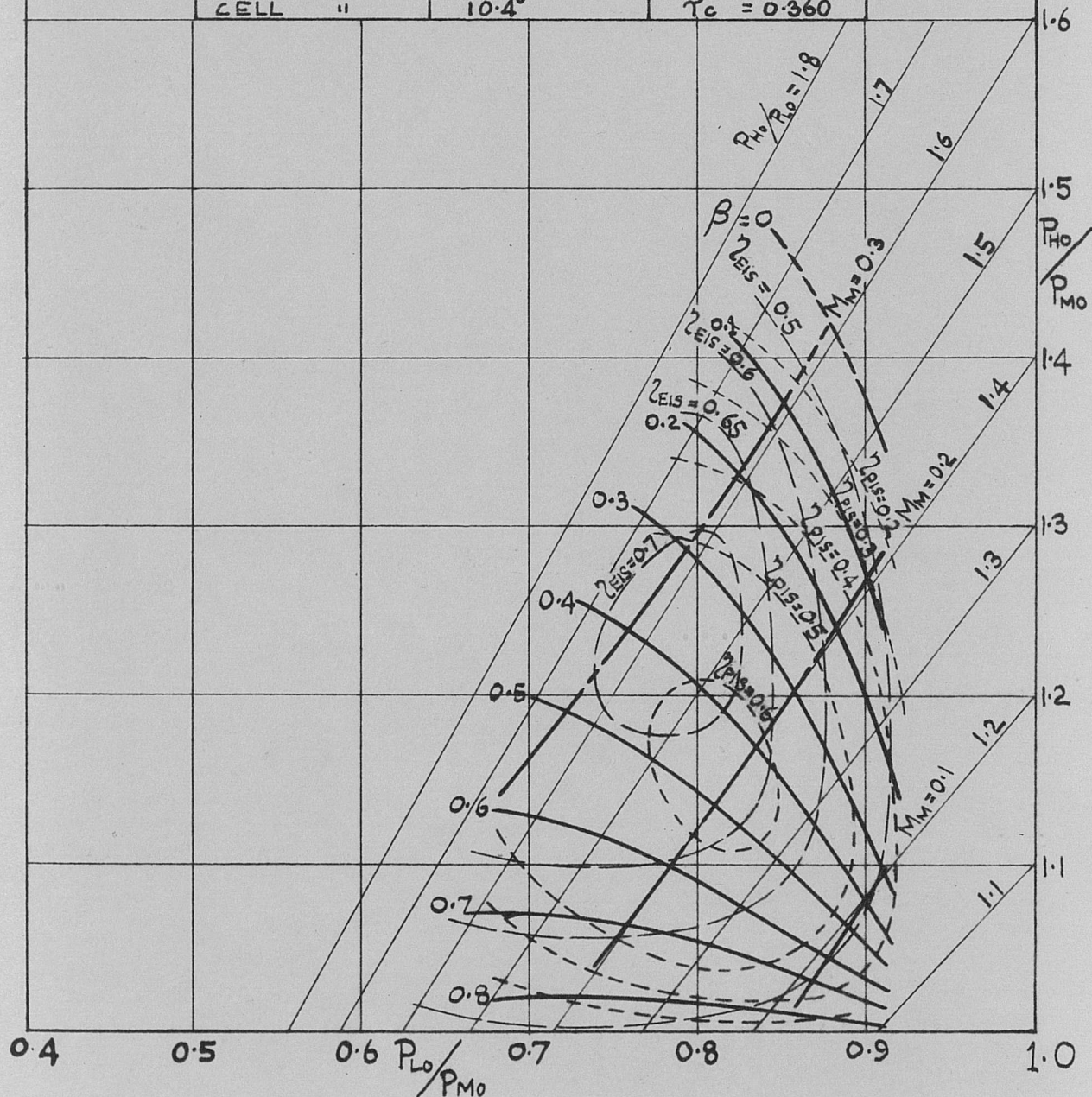


FIG.12-2-2
 EXPERIMENTALLY OBTAINED DIVIDER
 PRESSURE-BOOSTING PERFORMANCE
 (FIXED GEOMETRY, CONSTANT SPEED, $T_{M0}=308^{\circ}\text{K}$, $P_{L0}\approx 1\text{ATM.}$)

ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	6000 REV./MIN.	$M_{R0} = 0.1470$
HP PORT WIDTH	48°	$\gamma_H = 1.67$
LP " "	45°	$\gamma_L = 1.57$
MP " "	45°	$\gamma_M = 1.57$
PHASING	19°	$\tau_{\theta} = 0.661$
LAND WIDTH	12°	$\tau_E = 0.417$
CELL " "	10.4°	$\tau_C = 0.360$

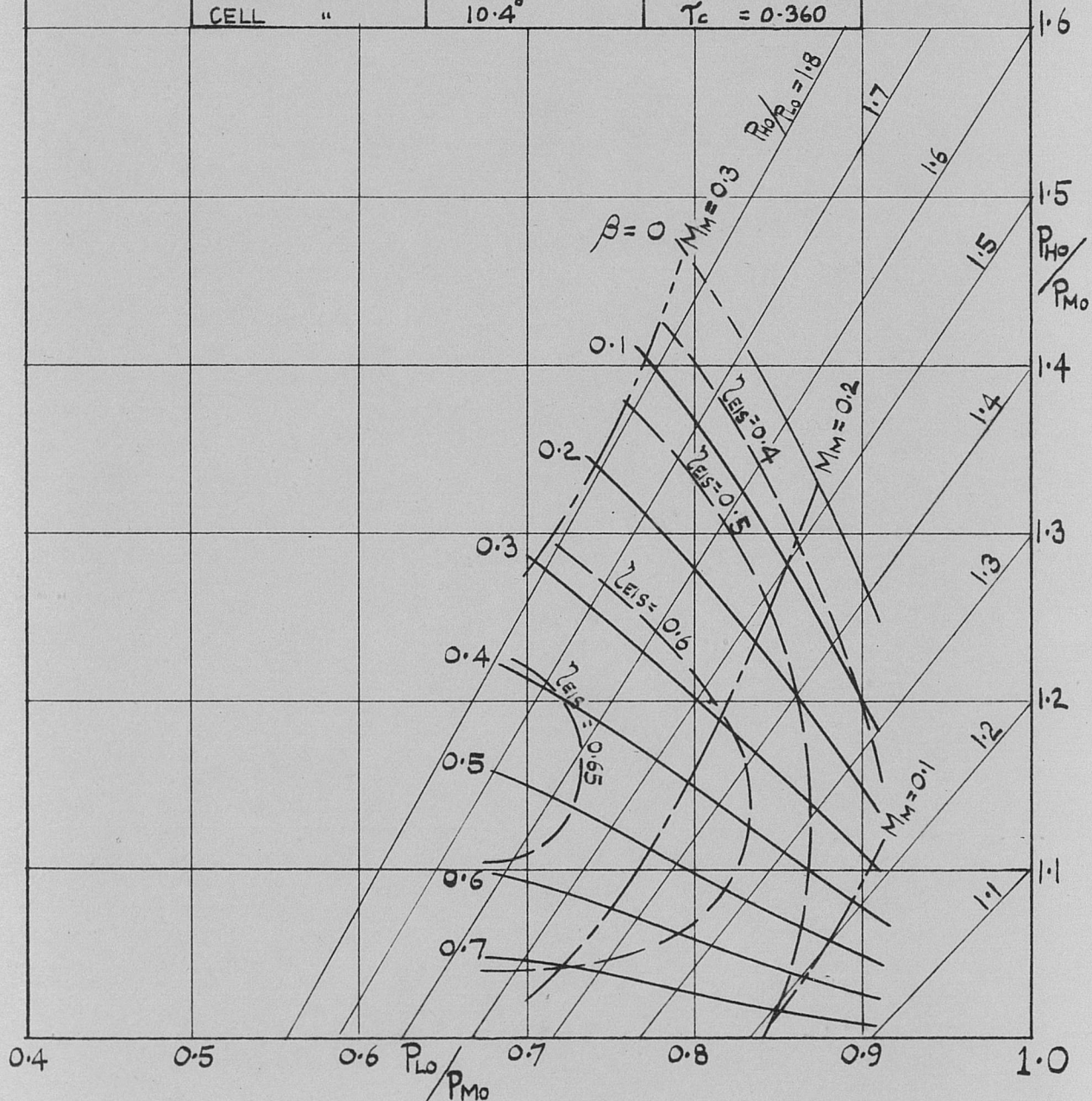


FIG. 12.2-3

EXPERIMENTALLY OBTAINED DIVIDER PRESSURE-BOOSTING PERFORMANCE

(FIXED GEOMETRY, CONSTANT SPEED, $T_{M0} = 308^\circ\text{K}$, $R_0 \approx 1\text{ATM.}$)

ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	6000 REV./MIN.	$M_{R0} = 0.1470$
HP PORT WIDTH	48°	$\gamma_H = 1.67$
LP " "	45°	$\gamma_L = 1.57$
MP " "	45°	$\gamma_M = 1.57$
PHASING	29°	$\gamma_\phi = 1.011$
LAND WIDTH	12°	$\gamma_E = 0.417$
CELL " "	10.4°	$\gamma_C = 0.360$

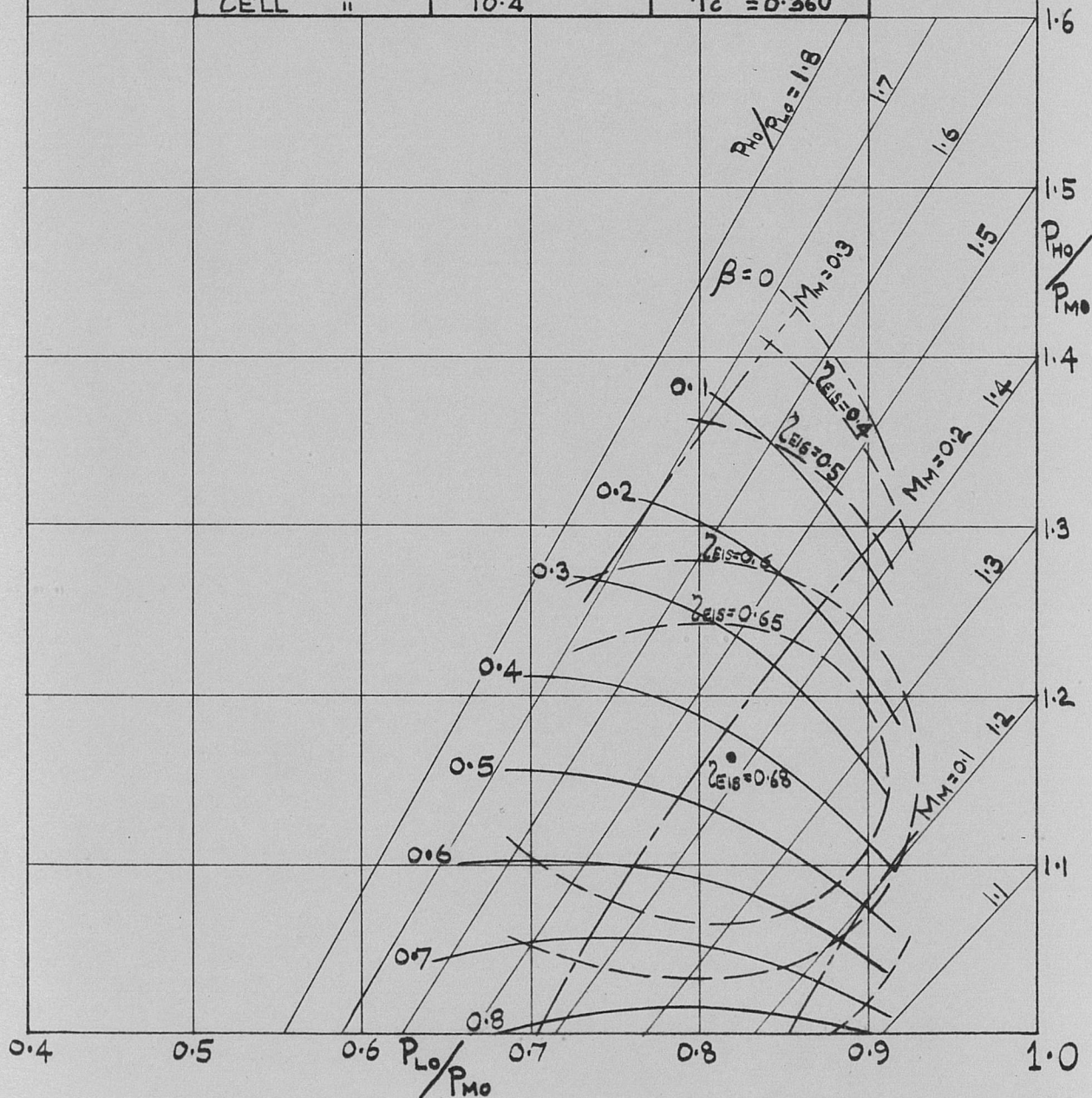


FIG. 12.2-4

EXPERIMENTALLY OBTAINED DIVIDER PRESSURE-BOOSTING PERFORMANCE

(FIXED GEOMETRY, CONSTANT SPEED, $T_{M0}=308^\circ\text{K}$, $P_{L0}\approx 1\text{ATM.}$)

ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	4000 REV./MIN.	$M_{R0} = 0.0987$
HP PORT WIDTH	48°	$\gamma_H = 2.51$
LP " "	45°	$\gamma_L = 2.35$
MP " "	45°	$\gamma_M = 2.35$
PHASING	24°	$\gamma_D = 1.26$
LAND WIDTH	12°	$\gamma_E = 0.626$
CELL " "	10.4°	$\gamma_C = 0.540$

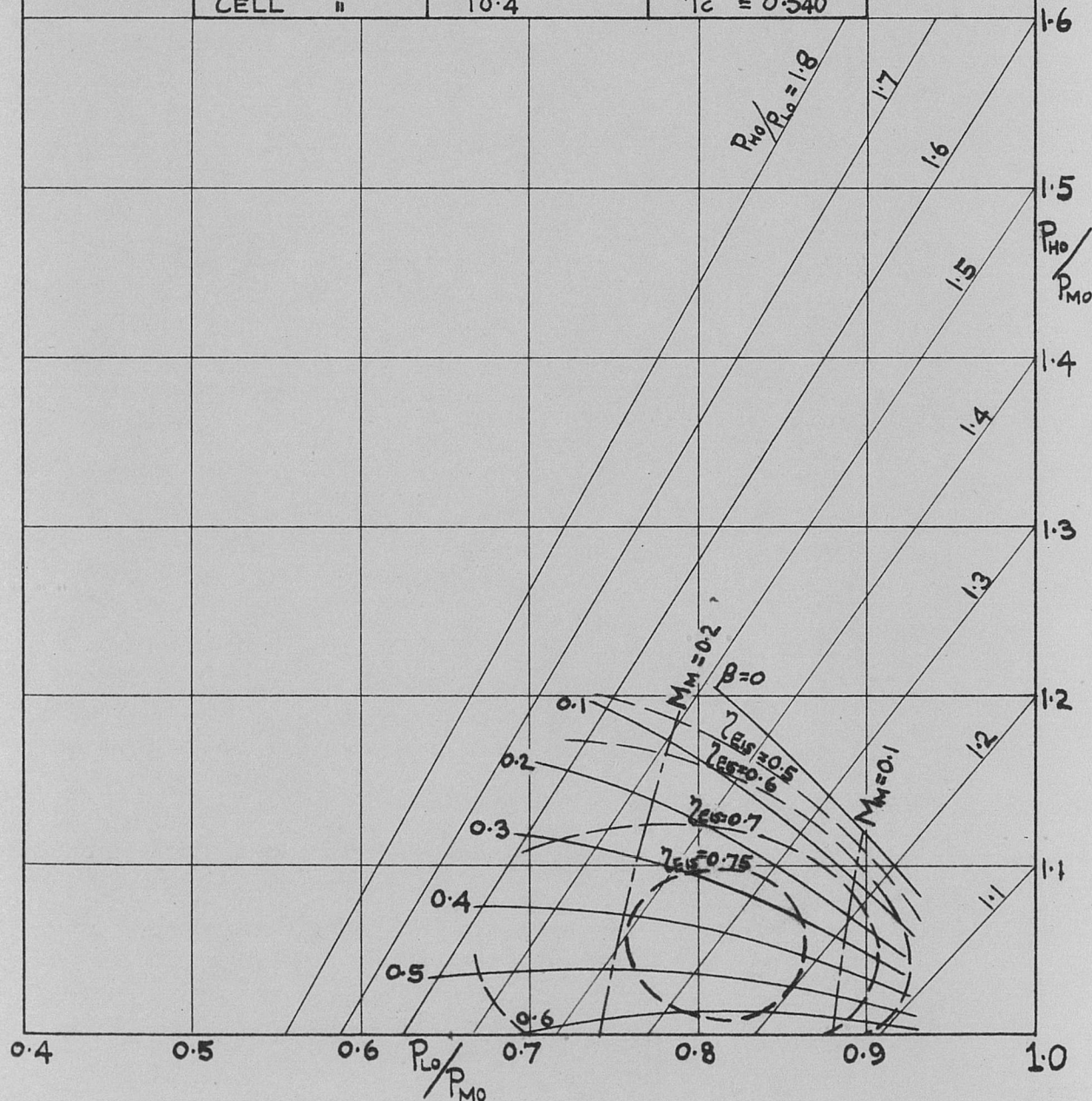
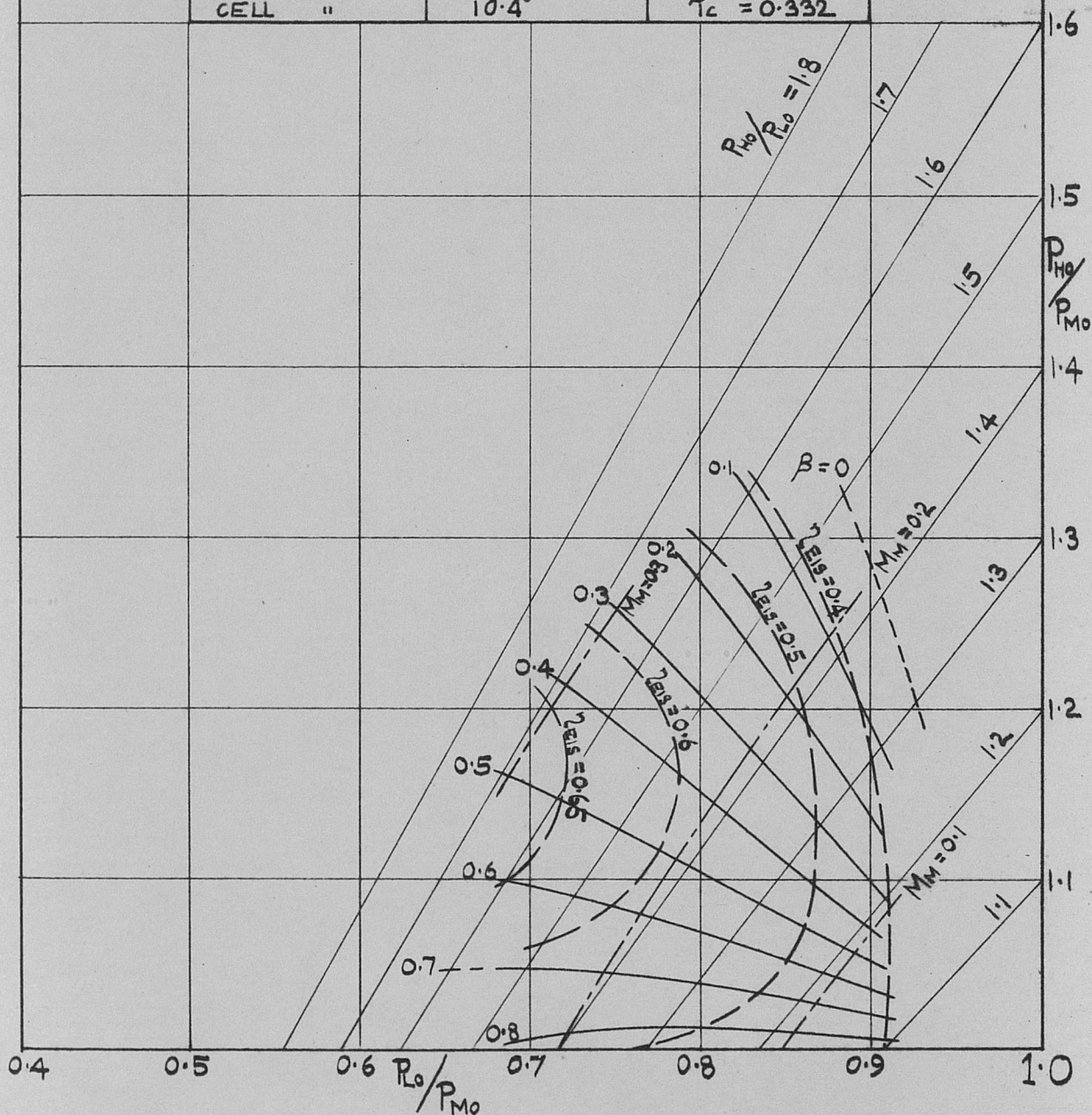


FIG. 12.2-5

EXPERIMENTALLY OBTAINED DIVIDER PRESSURE-BOOSTING PERFORMANCE

(FIXED GEOMETRY, CONSTANT SPEED, $T_{M0}=308^{\circ}\text{K}$, $P_{L0}\approx 1\text{ATM.}$)

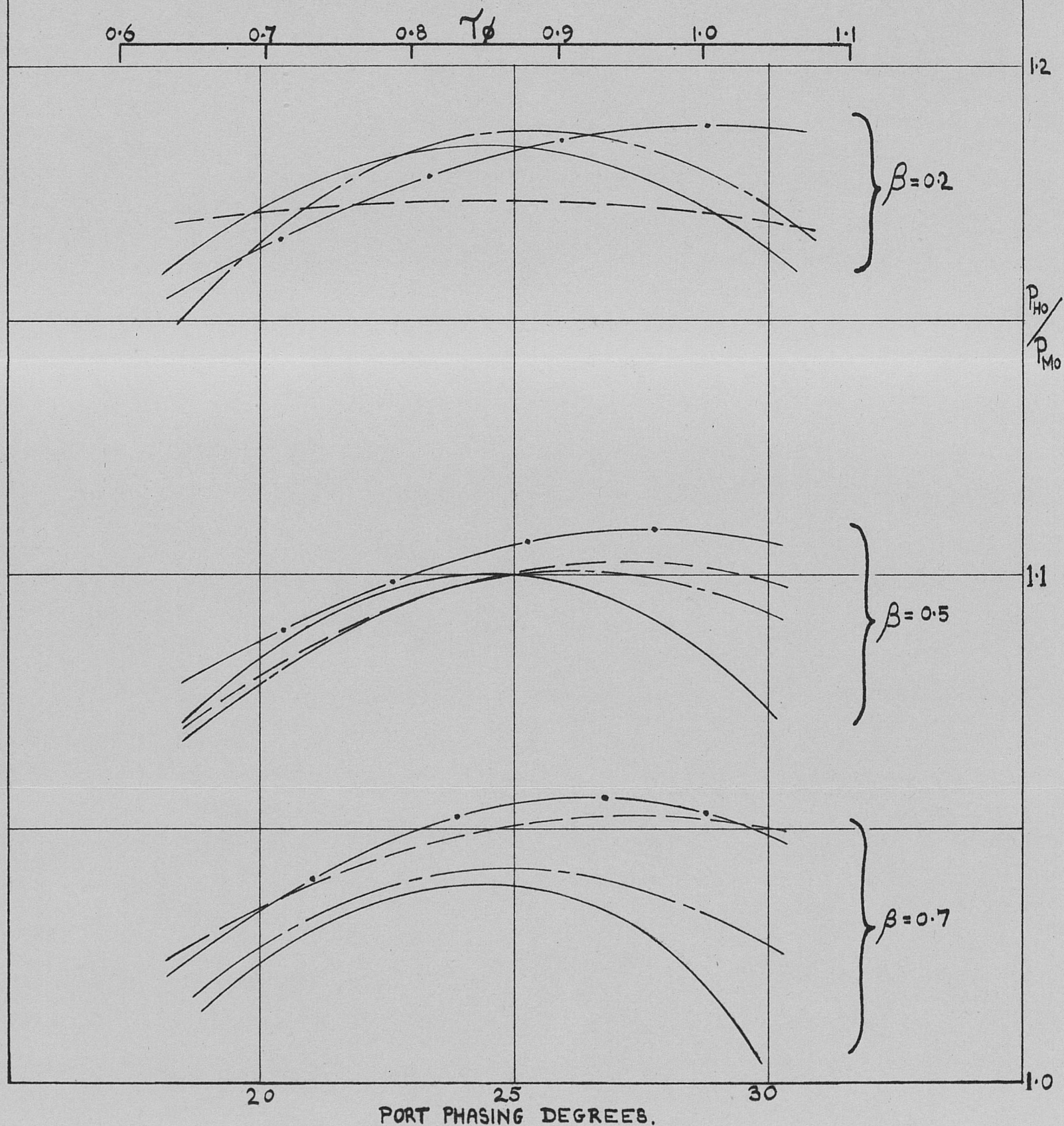
ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	6500 REV./MIN.	$M_{R0}=0.1591$
HP PORT WIDTH	48°	$\gamma_H = 1.54$
LP " "	45°	$\gamma_L = 1.45$
MP " "	45°	$\gamma_M = 1.45$
PHASING	24°	$\gamma_{\phi} = 0.77$
LAND WIDTH	12°	$\gamma_E = 0.386$
CELL " "	10.4°	$\gamma_C = 0.332$



VARIATION OF EXPERIMENTALLY OBTAINED DIVIDER PRESSURE-BOOSTING PERFORMANCE WITH PORT WIDTH AND PHASING

$$\left[P_{H0}/P_{L0} = 1.3, M_{R0} = 0.1470 (6000 \text{ REV./MIN.}), \gamma_H = 1.67 (48^\circ), \gamma_L = 0.417 (12^\circ), \right. \\ \left. \gamma_C = 0.360 (10.4^\circ), T_{M0} = 308^\circ\text{K}, P_{L0} \approx 1 \text{ ATM.} \right]$$

—————	WIDTH OF MP PORT = 41° , $\gamma_M = 1.43$, $\gamma_L = 1.71$.
-----	" " " " = 43° , $\gamma_M = 1.50$, $\gamma_L = 1.64$.
—•—•—•—	" " " " = 45° , $\gamma_M = 1.57$, $\gamma_L = 1.57$.
-----	" " " " = 47° , $\gamma_M = 1.64$, $\gamma_L = 1.50$.



VARIATION OF EXPERIMENTALLY OBTAINED DIVIDER PRESSURE-BOOSTING PERFORMANCE WITH PORT WIDTH AND PHASING

$$\left[\begin{array}{l} P_{H0}/P_{L0} = 1.6, \quad M_{R0} = 0.1470 \text{ (6000 REV./MIN.)}, \quad \gamma_H = 1.67 \text{ (48}^\circ\text{)}, \quad \gamma_E = 0.417 \text{ (12}^\circ\text{)}, \\ \gamma_C = 0.360 \text{ (10.4}^\circ\text{)}, \quad T_{M0} = 308^\circ\text{K}, \quad P_{L0} \approx 1 \text{ ATM.} \end{array} \right]$$

—————	WIDTH OF MP PORT = 41°	, $\gamma_M = 1.43$	, $\gamma_L = 1.71$
- - - - -	" " " " = 43°	, $\gamma_M = 1.50$	, $\gamma_L = 1.64$
— · — · —	" " " " = 45°	, $\gamma_M = 1.57$	, $\gamma_L = 1.57$
- - - - -	" " " " = 47°	, $\gamma_M = 1.64$	, $\gamma_L = 1.50$

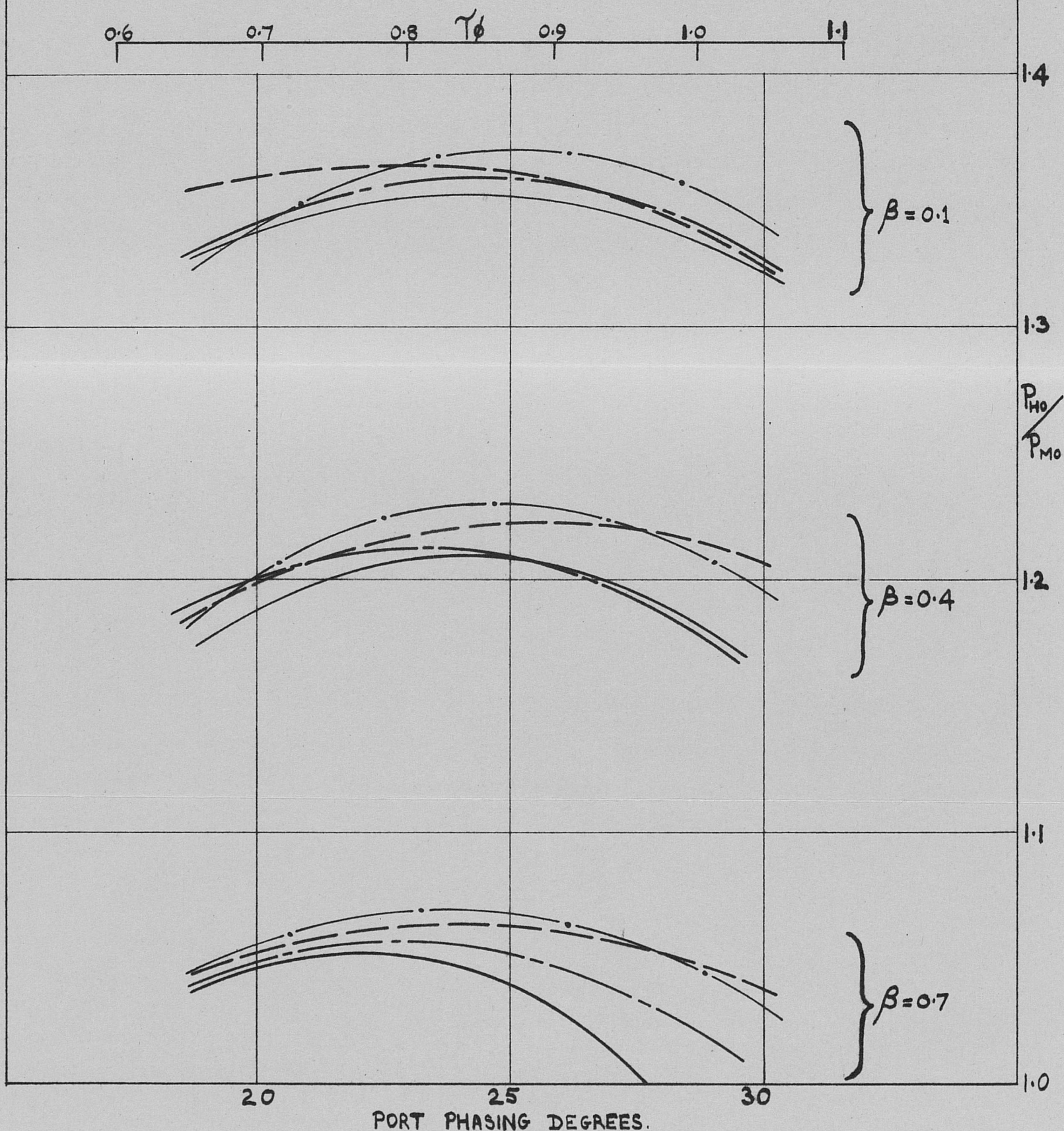


FIG. 12.2-8

VARIATION OF EXPERIMENTALLY OBTAINED DIVIDER PRESSURE-BOOSTING PERFORMANCE WITH ROTOR SPEED. $P_{H0}/P_{L0} = 1.3$, HP PORT WIDTH = 48° , LP PORT WIDTH = 45° , MP PORT WIDTH = 45° , PHASING = 24° , LAND WIDTH = 12° , CELL WIDTH = 10.4° , $T_{M0} = 308^\circ\text{K}$, $P_{L0} \approx 1\text{ATM}$.

(NB. THE MARKED POINTS WERE OBTAINED FROM THE PRESENTATION, ON THE P_{H0}/P_{M0} VERSUS P_{L0}/P_{M0} PLANE, OF THE APPROPRIATE INDEPENDENT TEST RESULTS.)

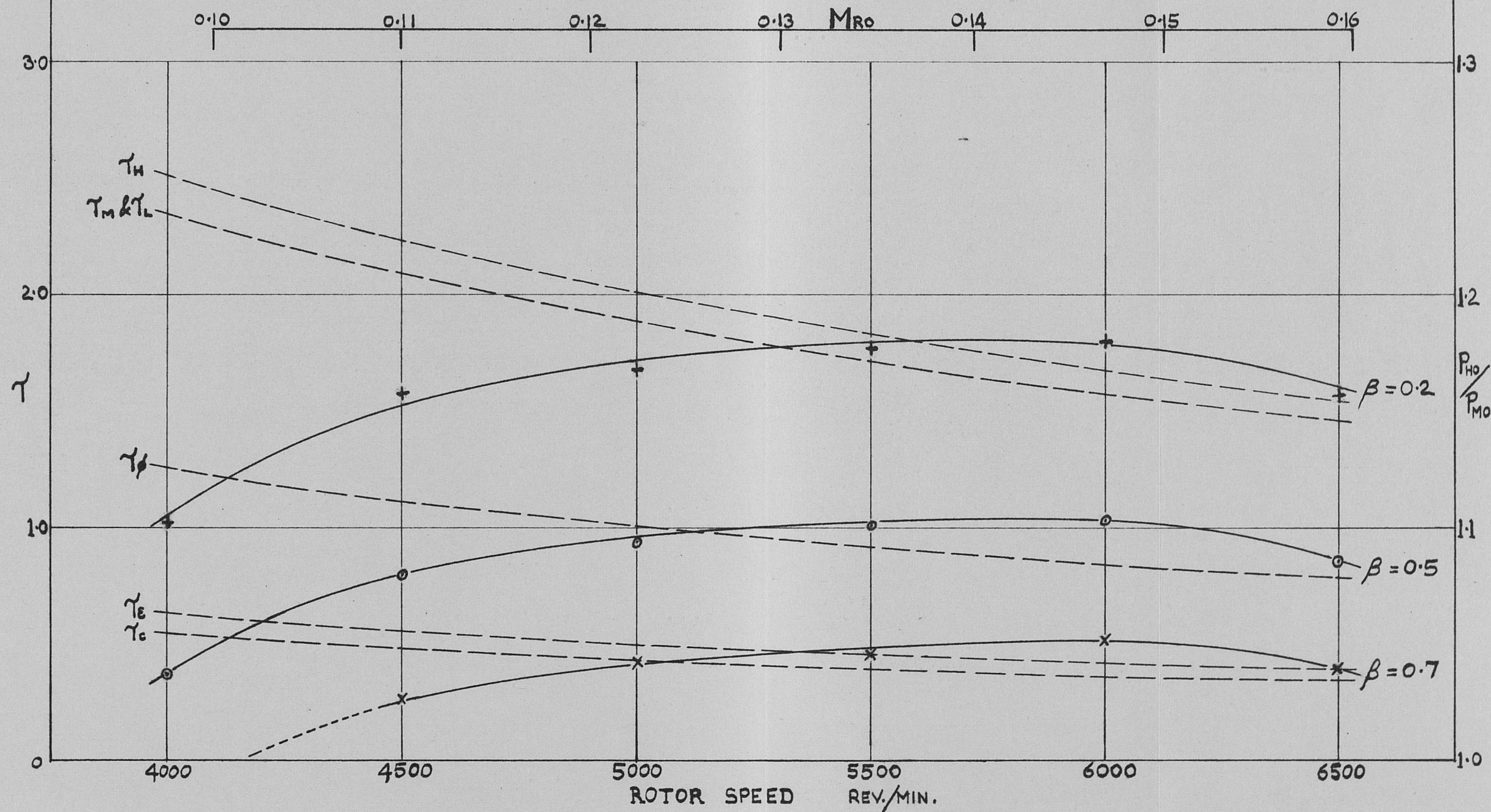


FIG. 12-2-9

VARIATION OF EXPERIMENTALLY OBTAINED DIVIDER PRESSURE-BOOSTING PERFORMANCE WITH ROTOR SPEED. $P_{H0}/P_{L0} = 1.6$, HP PORT WIDTH = 48° , LP PORT WIDTH = 45° , MP PORT WIDTH = 45° , PHASING = 2.4° , LAND WIDTH = 12° , CELL WIDTH = 10.4° , $T_{M0} = 308^\circ\text{K}$, $R_0 \approx 1\text{ATM}$.

(NB. THE MARKED POINTS WERE OBTAINED FROM THE PRESENTATION, ON THE P_{H0}/P_{M0} VERSUS P_{L0}/P_{M0} PLANE, OF THE APPROPRIATE INDEPENDENT TEST RESULTS.)

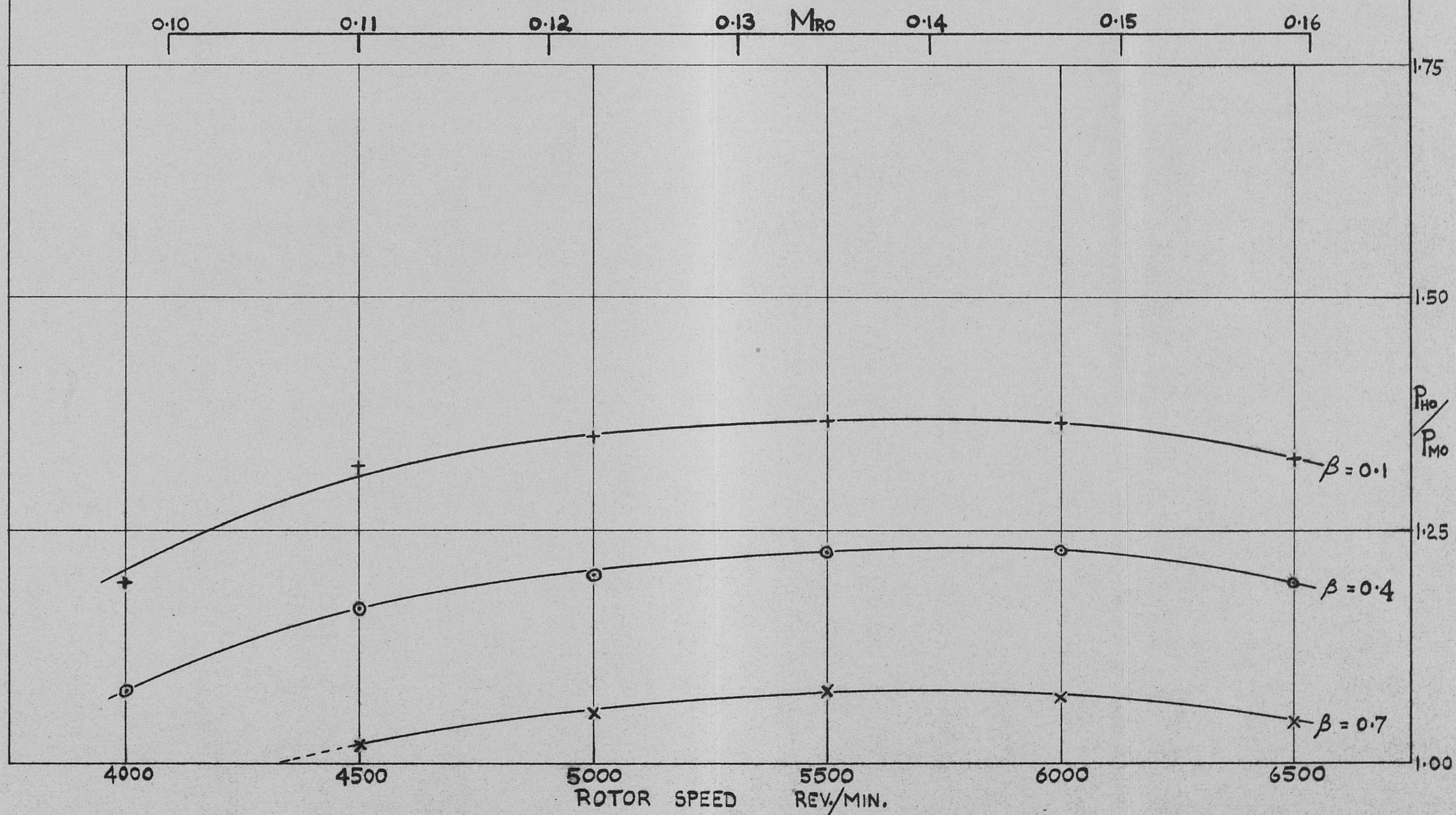


FIG. 12-2-10

EXPERIMENTALLY OBTAINED DIVIDER COOLING PERFORMANCE

(FIXED GEOMETRY, CONSTANT SPEED, $T_{M0} = 308^\circ\text{K}$, $P_{L0} \approx 1\text{ATM.}$)

ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	5000 REV./MIN.	$Ma_0 = 0.1225$
HP PORT WIDTH	48°	$\tau_H = 2.01$
LP " "	45°	$\tau_L = 1.88$
MP " "	45°	$\tau_M = 1.88$
PHASING	24°	$\tau_\phi = 1.00$
LAND WIDTH	12°	$\tau_\lambda = 0.501$
CELL "	10.4°	$\tau_c = 0.431$

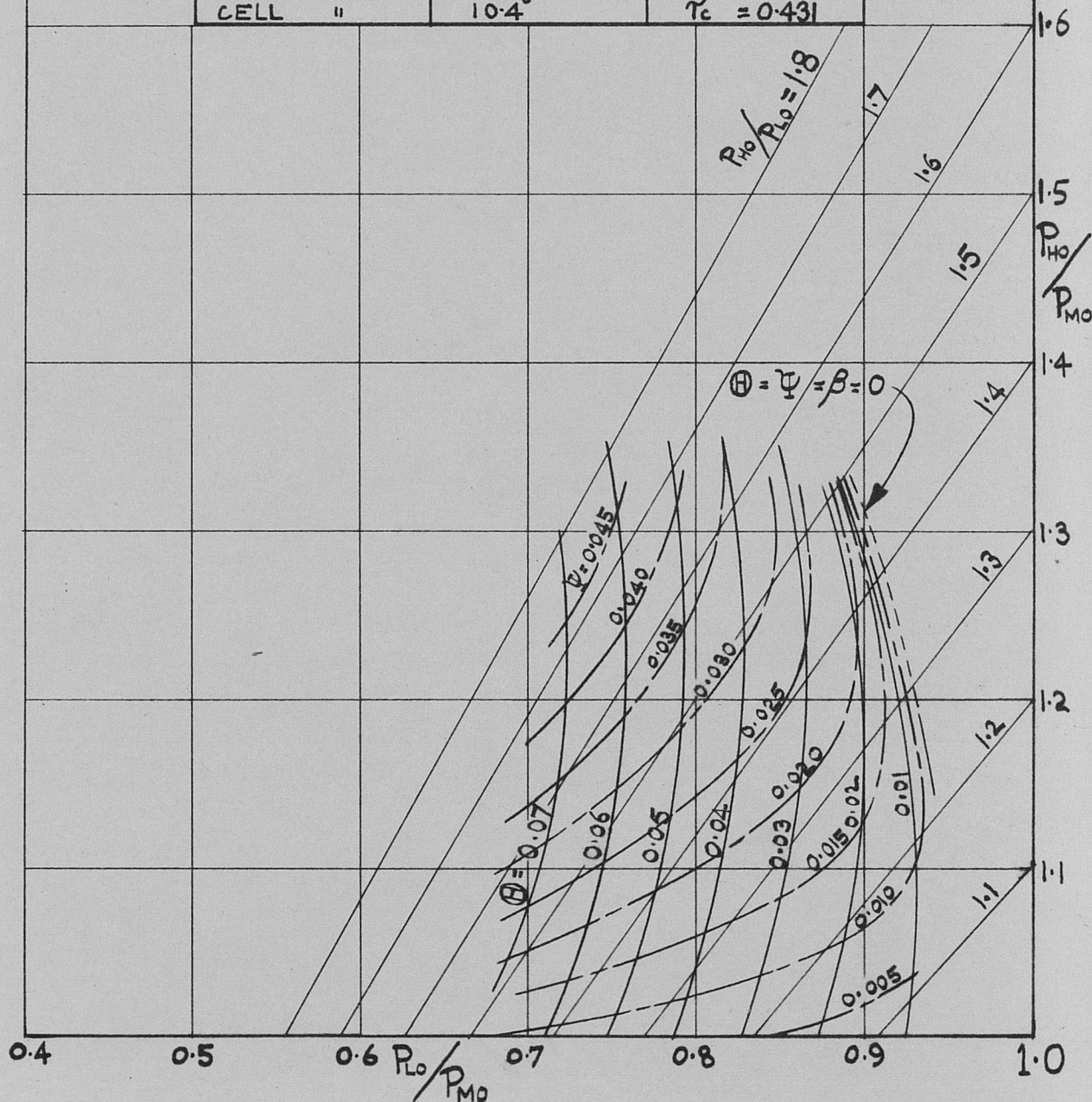


FIG.12-2-11

EXPERIMENTALLY OBTAINED DIVIDER COOLING PERFORMANCE

(FIXED GEOMETRY, CONSTANT SPEED, $T_{M0}=308^{\circ}\text{K}$, $P_{L0}\approx 1\text{ATM}$.)

ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	4000 REV./MIN.	$MR_0 = 0.0978$
HP PORT WIDTH	48°	$T_H = 2.51$
LP " "	45°	$T_L = 2.35$
MP " "	45°	$T_M = 2.35$
PHASING	24°	$T_{\phi} = 1.26$
LAND WIDTH	12°	$T_E = 0.626$
CELL "	10.4°	$T_C = 0.540$

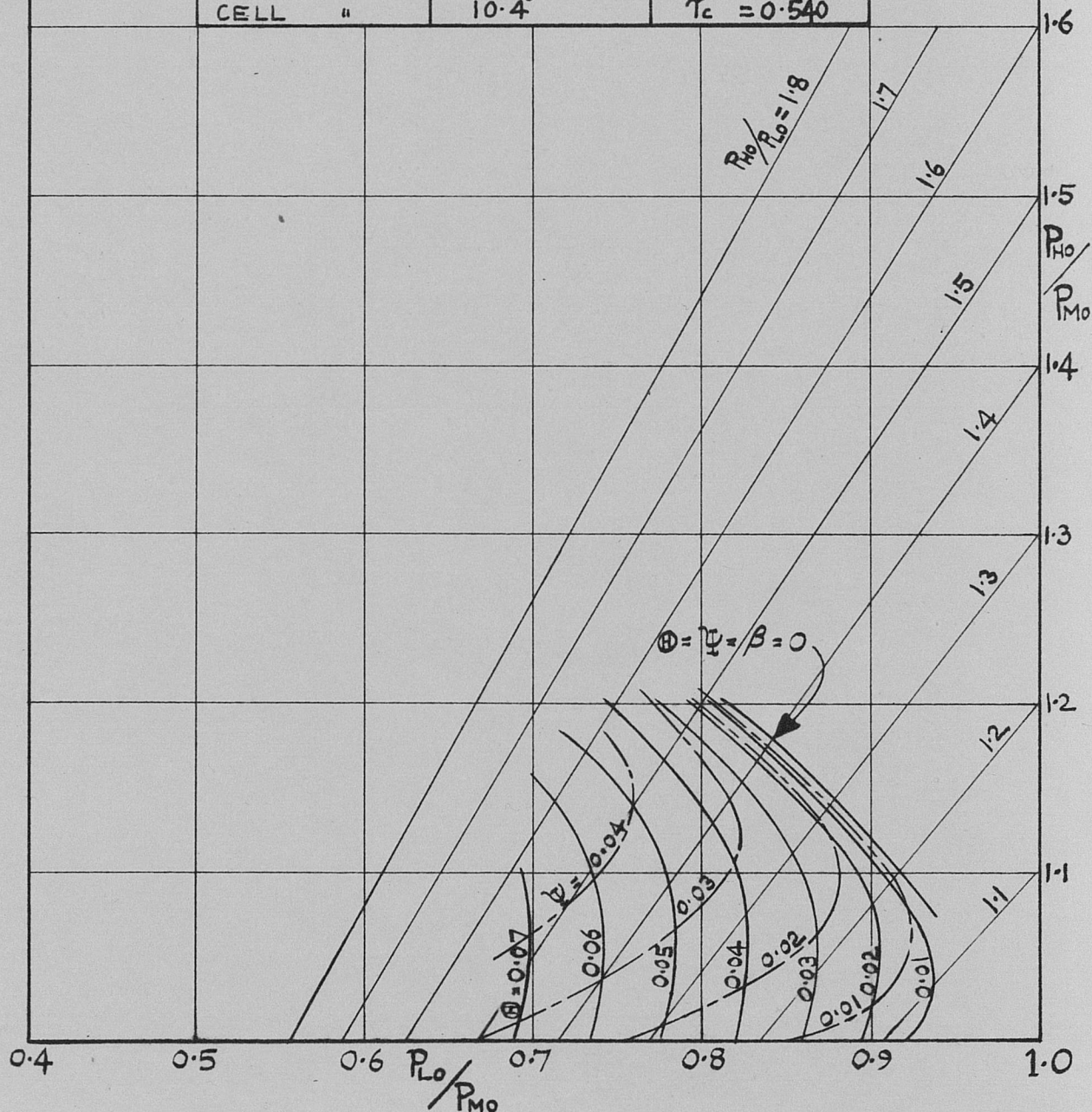
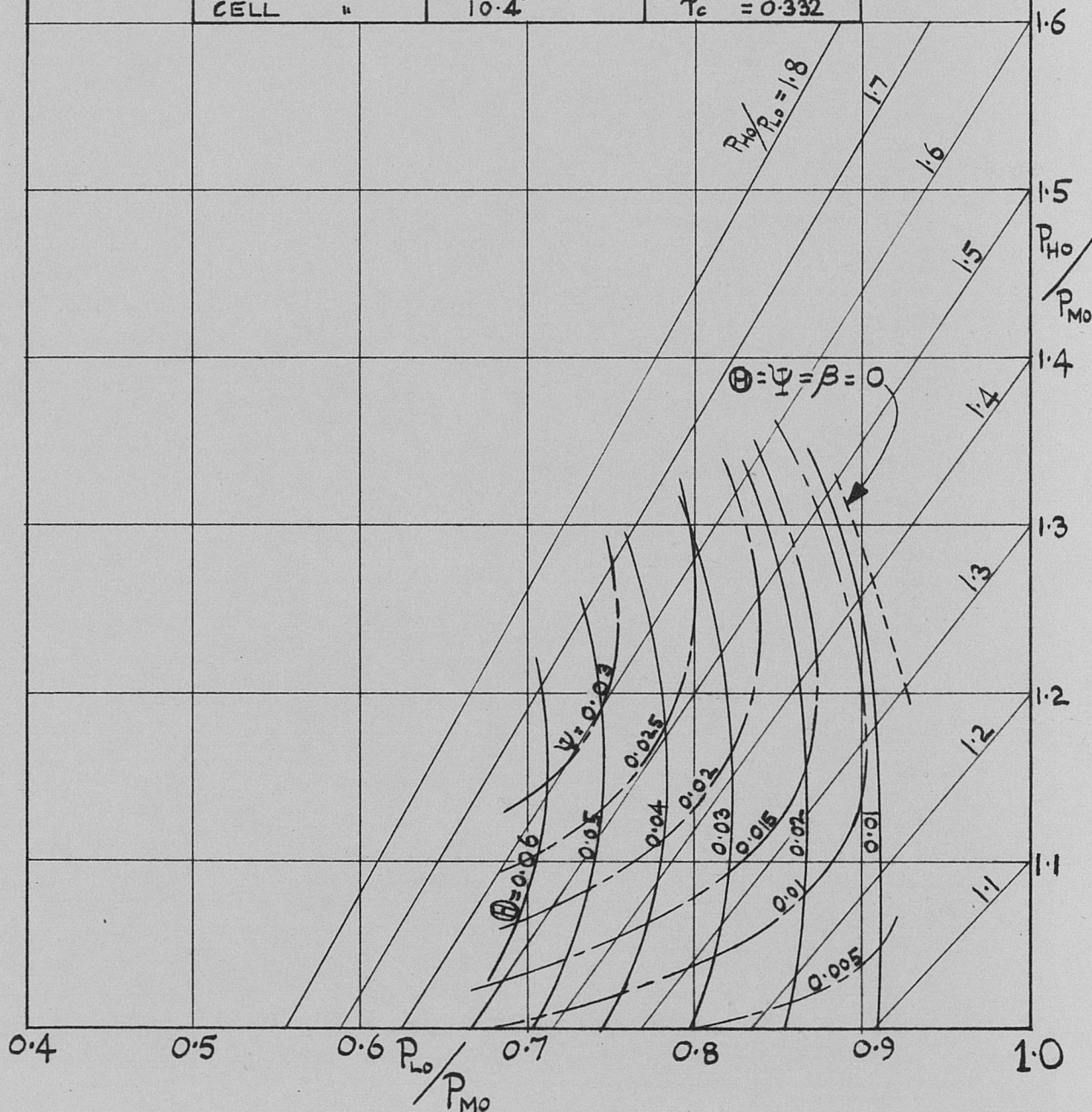


FIG. 12-2-12

EXPERIMENTALLY OBTAINED DIVIDER COOLING PERFORMANCE

(FIXED GEOMETRY, CONSTANT SPEED, $T_{M0} = 308^\circ\text{K}$, $P_{L0} \approx 1\text{ATM.}$)

ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	6500 REV./MIN.	$M_{R0} = 0.1591$
HP PORT WIDTH	48°	$\gamma_H = 1.54$
LP " "	45°	$\gamma_L = 1.45$
MP " "	45°	$\gamma_M = 1.45$
PHASING	24°	$\gamma_\phi = 0.77$
LAND WIDTH	12°	$\gamma_E = 0.386$
CELL "	10.4°	$\gamma_c = 0.332$



VARIATION OF EXPERIMENTALLY OBTAINED DIVIDER COOLING PERFORMANCE WITH PORT WIDTH AND PHASING

$[M_{R0} = 0.1470 \text{ (6000 REV./MIN.)}, \gamma_H = 1.67 (48^\circ), \gamma_E = 0.417 (12^\circ), \gamma_C = 0.360 (10.4^\circ),$
 $T_{M0} = 308^\circ\text{K}, P_{L0} \approx 1 \text{ ATM.}]$

—————	WIDTH OF MP PORT = 41°	, $\gamma_M = 1.43$	, $\gamma_L = 1.71$.
-----	" " " "	= 43°	, $\gamma_M = 1.50$, $\gamma_L = 1.64$.
— . — . — .	" " " "	= 45°	, $\gamma_M = 1.57$, $\gamma_L = 1.57$.
-----	" " " "	= 47°	, $\gamma_M = 1.64$, $\gamma_L = 1.50$.

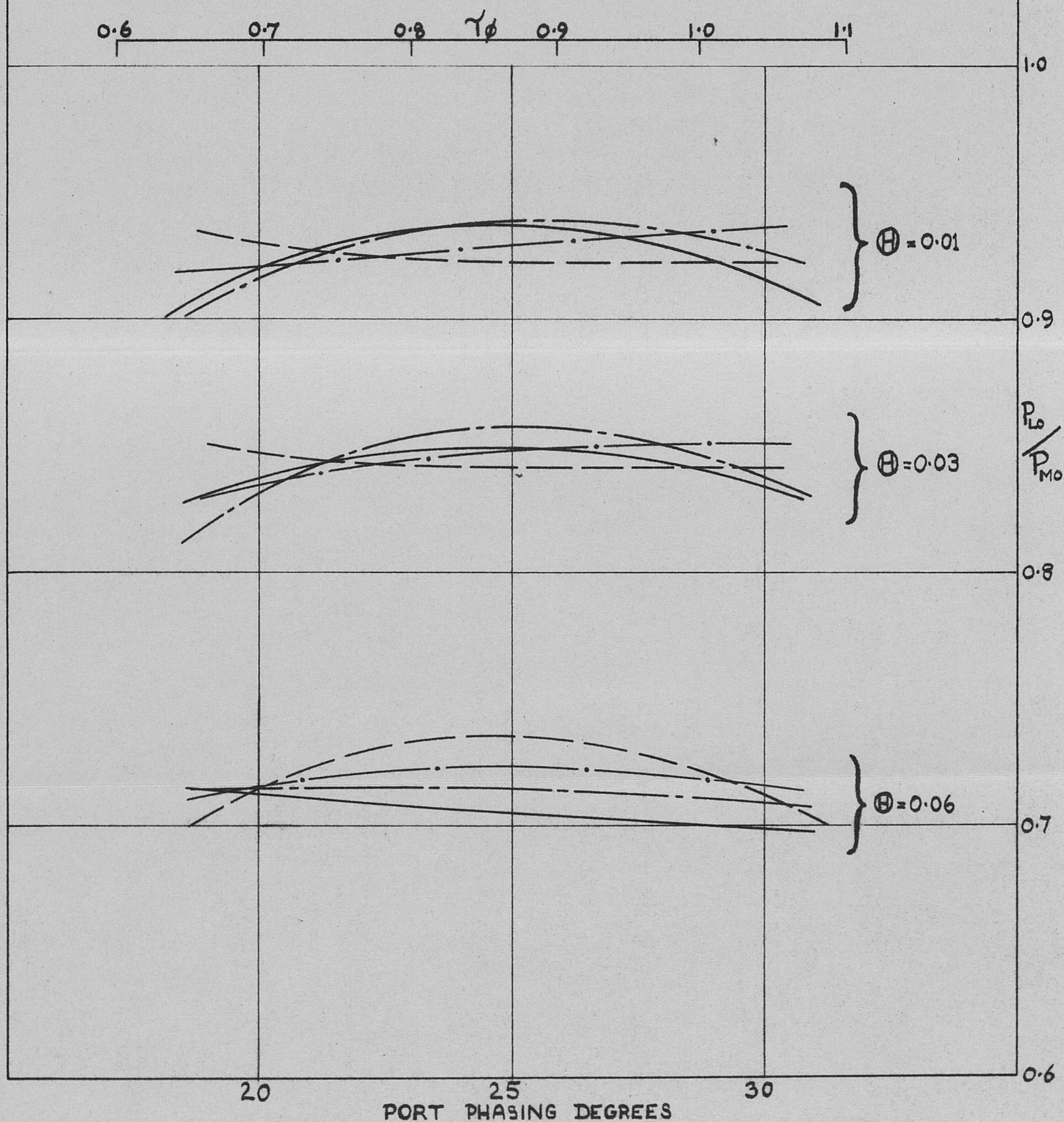


FIG. 12.2-14

VARIATION OF EXPERIMENTALLY OBTAINED DIVIDER COOLING PERFORMANCE WITH ROTOR SPEED

PARAMETERS OF Θ , HP PORT WIDTH = 48° , LP PORT WIDTH = 45° , MP PORT WIDTH = 45° , PHASING = 24° , LAND WIDTH = 12° , CELL WIDTH = 10.4° ,
 $T_{m0} = 308^\circ\text{K}$, $P_{L0} \approx 1\text{ATM}$.

(NB. THE MARKED POINTS WERE OBTAINED FROM THE PRESENTATION ON THE P_{H0}/P_{M0} VERSUS P_{L0}/P_{M0} PLANE OF THE APPROPRIATE INDEPENDENT TEST RESULTS.)

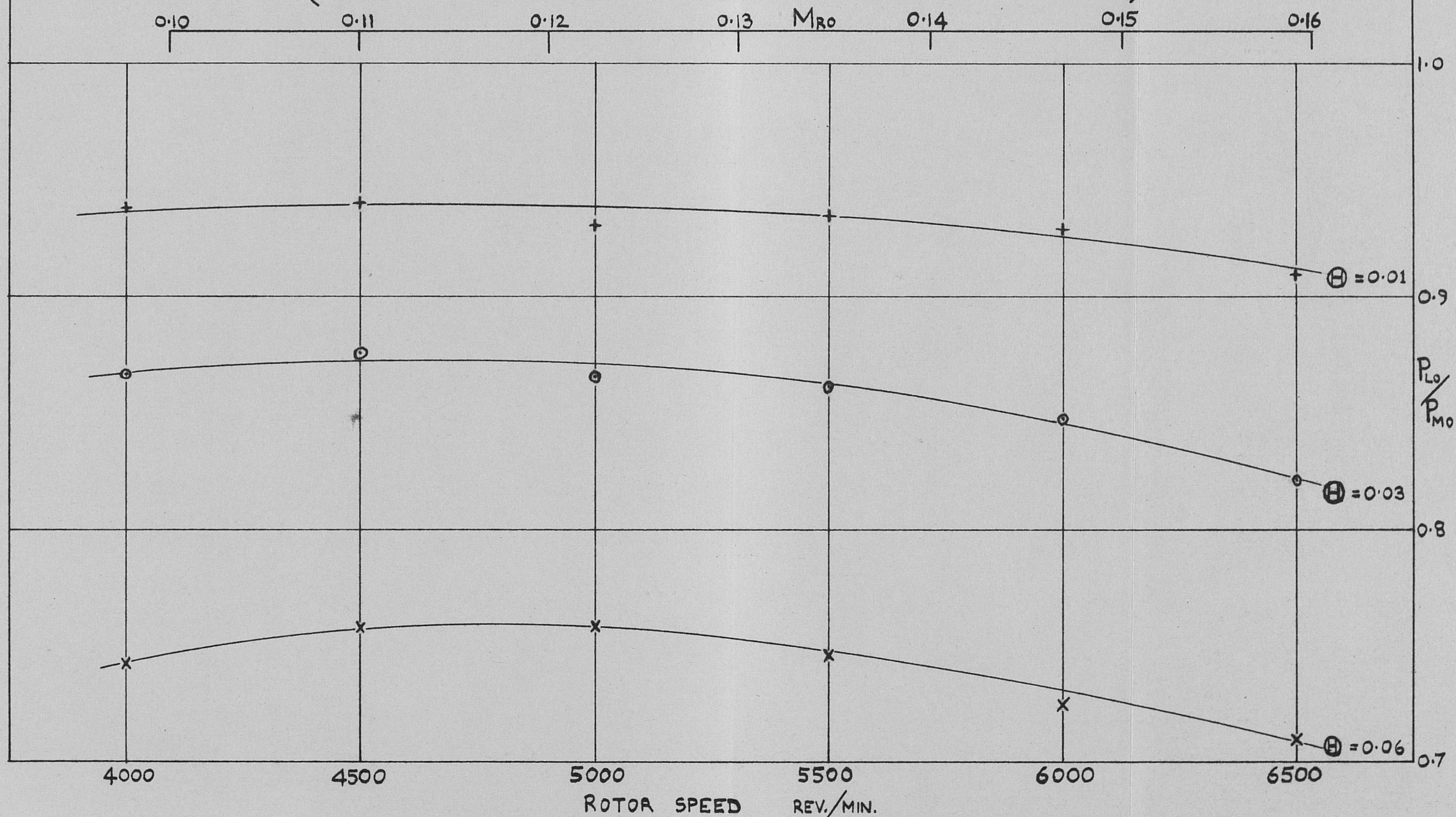


FIG. 12-2-15

VARIATION OF EXPERIMENTALLY OBTAINED DIVIDER COOLING PERFORMANCE WITH ROTOR SPEED

PARAMETERS OF Ψ , HP PORT WIDTH = 48° , LP PORT WIDTH = 45° , MP PORT WIDTH = 45° , PHASING = 24° , LAND WIDTH = 12° , CELL WIDTH = 10.4° ,
 $T_{M0} = 308^\circ\text{K}$, $P_{L0} \approx 1\text{ATM}$.

(NB. THE MARKED POINTS WERE OBTAINED FROM THE PRESENTATION, ON THE P_{H0}/P_{M0} VERSUS P_{L0}/P_{M0} PLANE, OF THE APPROPRIATE INDEPENDENT TEST RESULTS.)

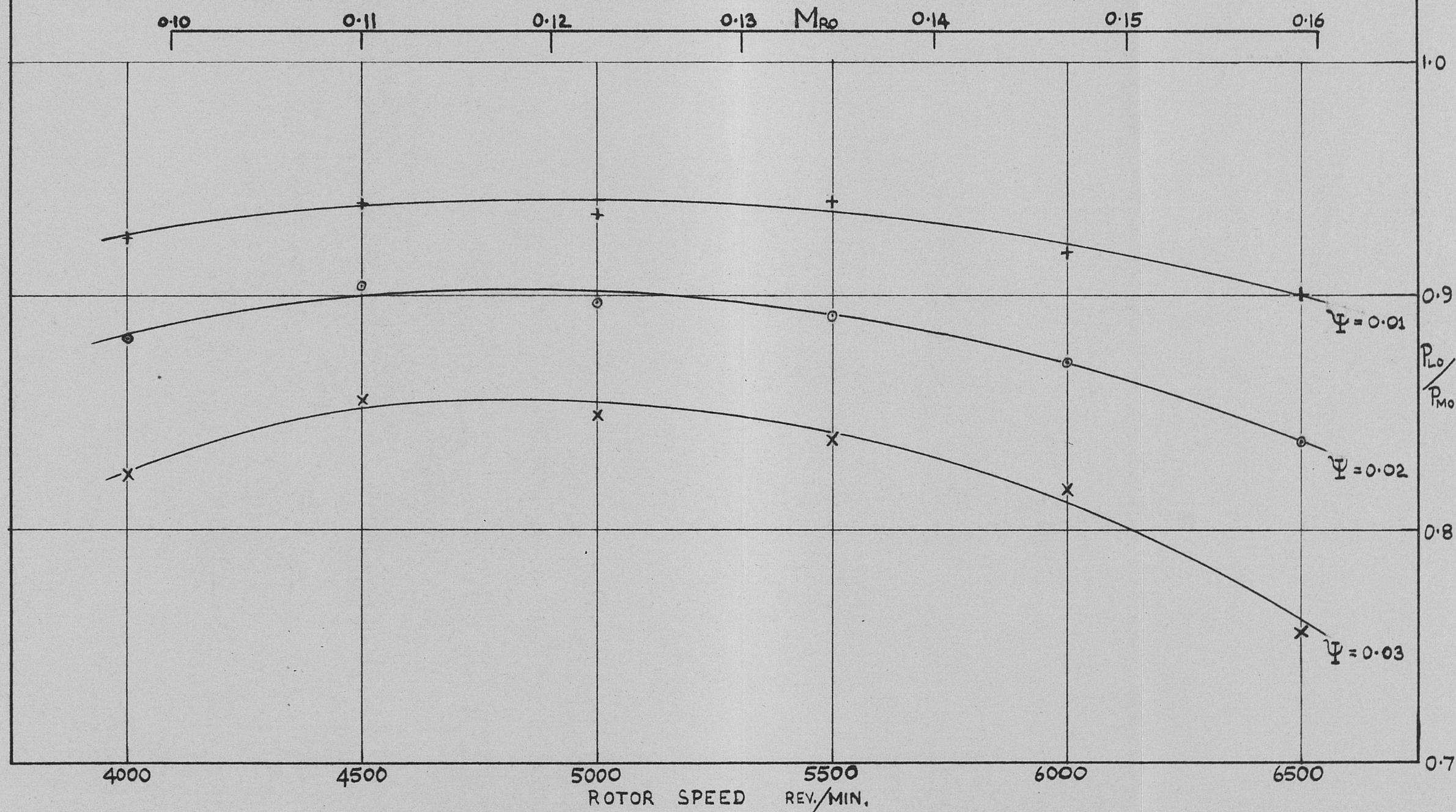


FIG.12-2-16

EXPERIMENTALLY OBTAINED EQUALISER PERFORMANCE

(FIXED GEOMETRY, CONSTANT SPEED, $T_{H0} = 308^\circ\text{K}$, $P_{L0} \approx 1\text{ATM}$, $T_{L0}/T_{H0} \approx 0.95$)

ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	5500 REV./MIN.	$M_{R0} = 0.1346$
HP PORT WIDTH	47°	$\gamma_H = 1.79$
LP " "	43°	$\gamma_L = 1.63$
MP " "	48°	$\gamma_M = 1.83$
PHASING	24°	$\gamma_\phi = 0.91$
LAND WIDTH	12°	$\gamma_z = 0.456$
CELL " "	10.4°	$\gamma_c = 0.393$

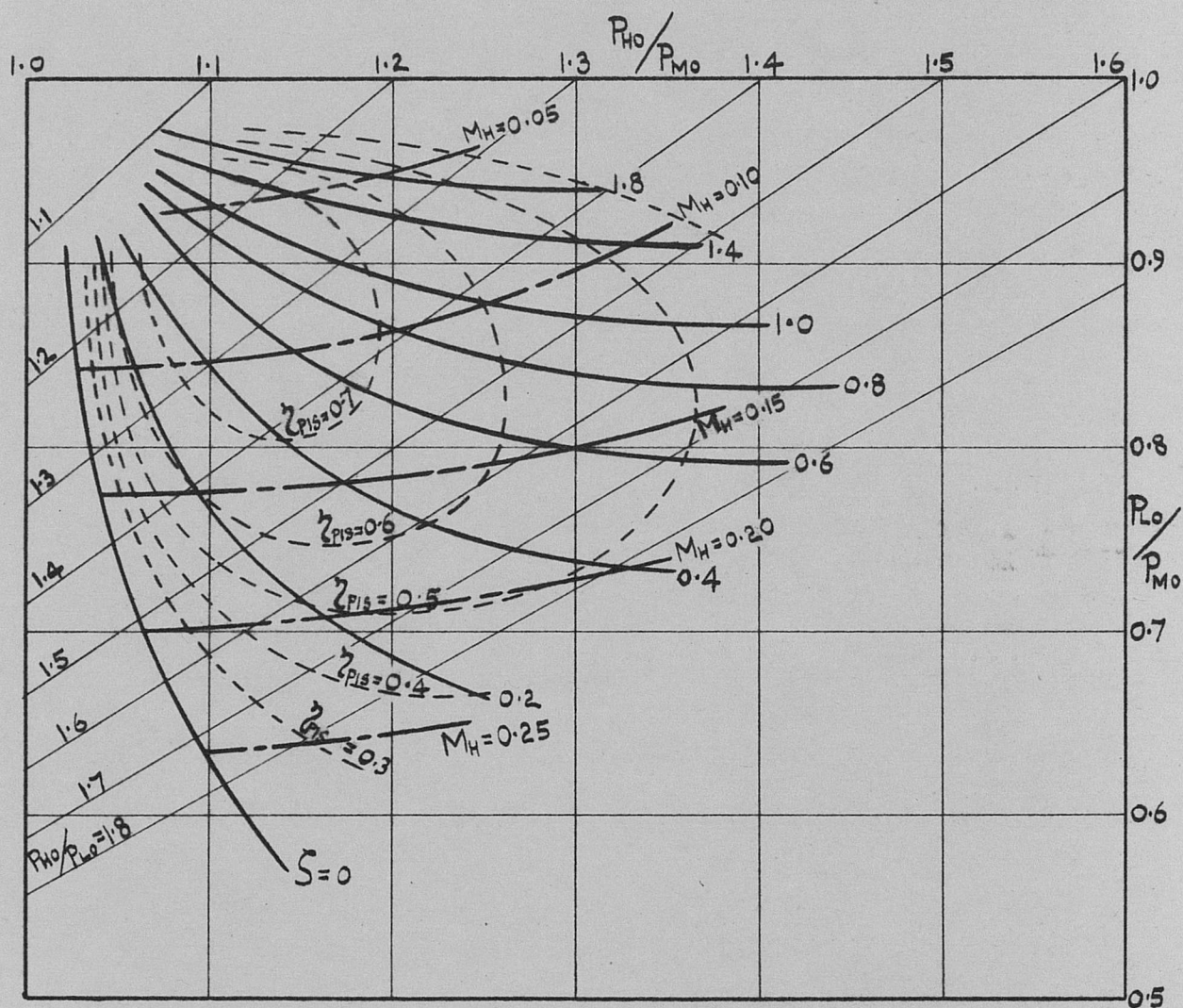


FIG. 12.2-17

EXPERIMENTALLY OBTAINED EQUALISER PERFORMANCE
 (FIXED GEOMETRY, CONSTANT SPEED, $T_{H0} = 308^\circ\text{K}$, $P_{L0} \approx 1\text{ATM}$, $T_{L0}/T_{H0} \approx 0.95$)

ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	5500 REV./MIN.	$M_{R0} = 0.1346$
HP PORT WIDTH	47°	$\gamma_H = 1.79$
LP " "	43°	$\gamma_L = 1.63$
MP " "	48°	$\gamma_M = 1.83$
PHASING	18°	$\gamma_\phi = 0.68$
LAND WIDTH	12°	$\gamma_L = 0.456$
CELL " "	10.4°	$\gamma_c = 0.393$

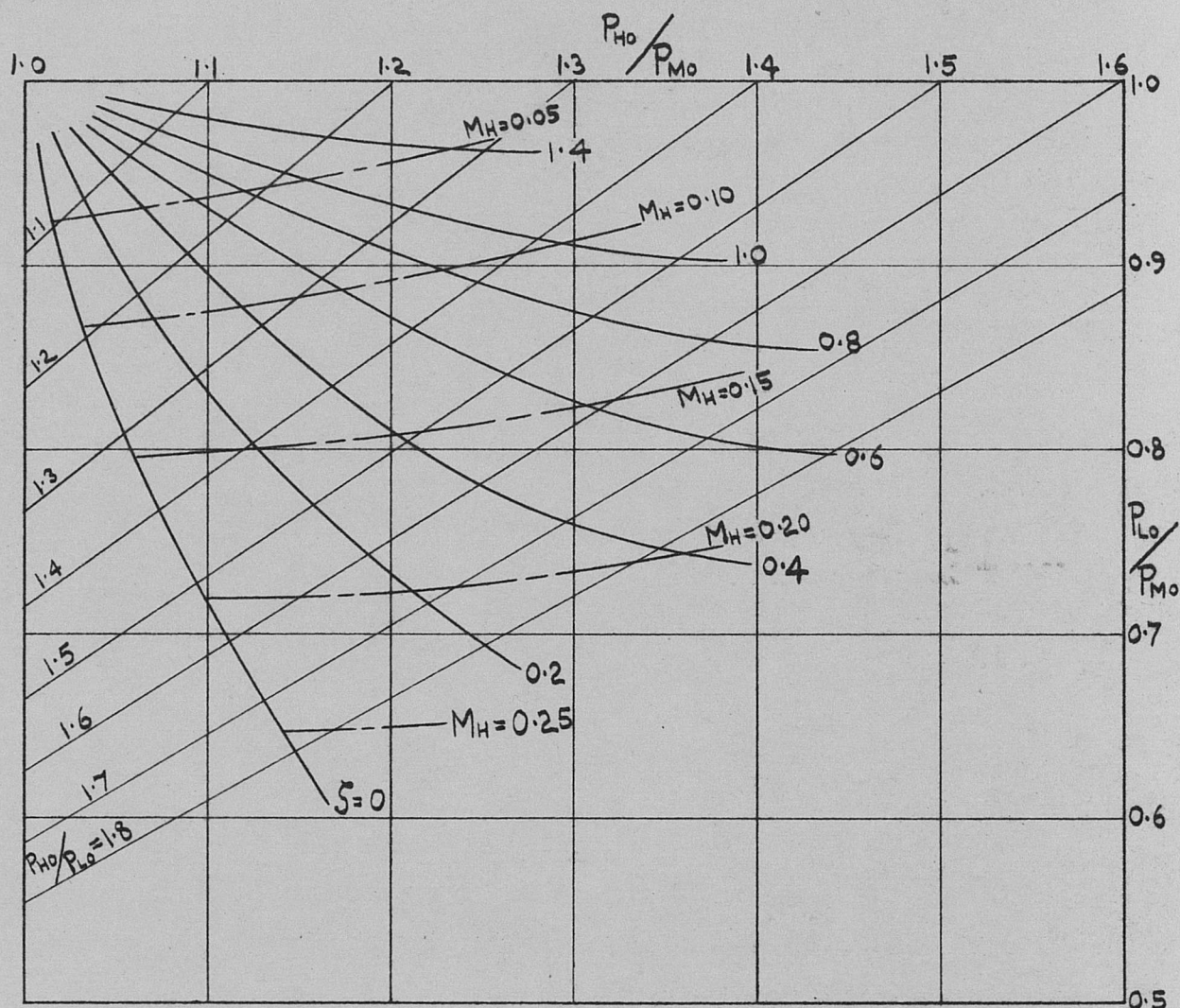


FIG. 12.2-18

EXPERIMENTALLY OBTAINED EQUALISER PERFORMANCE
(FIXED GEOMETRY, CONSTANT SPEED, $T_{H0} = 308^\circ\text{K}$, $P_{L0} \approx 1\text{ATM}$, $T_{L0}/T_{H0} \approx 0.95$)

ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	5500 REV./MIN	$M_{R0} = 0.1346$
HP PORT WIDTH	47°	$\gamma_H = 1.79$
LP " "	43°	$\gamma_L = 1.63$
MP " "	48°	$\gamma_M = 1.83$
PHASING	29°	$\gamma_\theta = 1.10$
LAND WIDTH	12°	$\gamma_E = 0.456$
CELL " "	10.4°	$\gamma_c = 0.393$

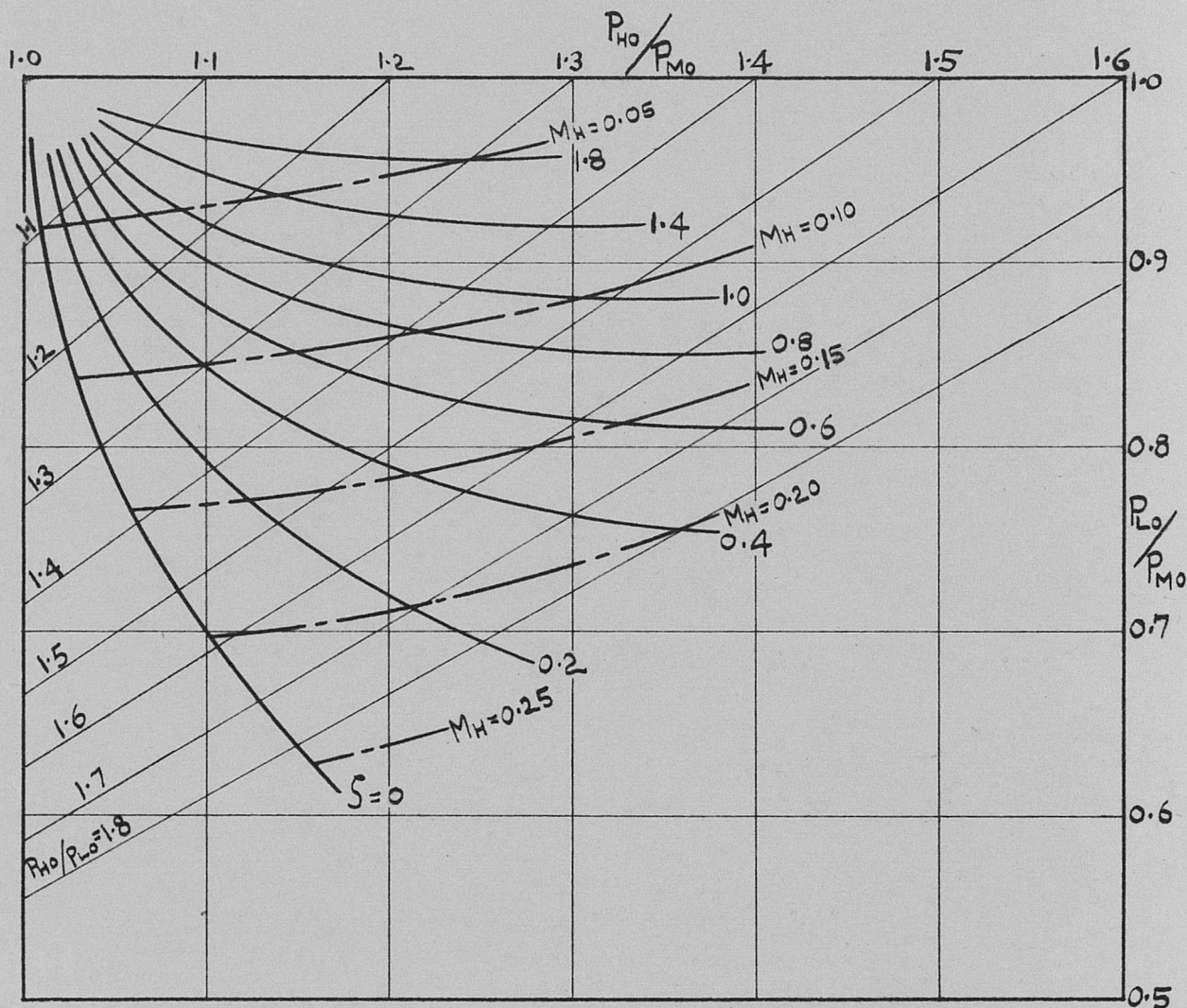


FIG. 12-2-19

EXPERIMENTALLY OBTAINED EQUALISER PERFORMANCE
(FIXED GEOMETRY, CONSTANT SPEED, $T_{H0} = 308^\circ\text{K}$, $P_{L0} \approx 1\text{ATM}$, $\frac{T_{L0}}{T_{H0}} \approx 0.95$)

ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	5000 REV./MIN.	$MR_0 = 0.1225$
HP PORT WIDTH	47°	$\gamma_H = 1.96$
LP " "	43°	$\gamma_L = 1.80$
MP " "	48°	$\gamma_M = 2.01$
PHASING	24°	$\gamma_\theta = 1.00$
LAND WIDTH	12°	$\gamma_E = 0.501$
CELL " "	10.4°	$\gamma_C = 0.431$

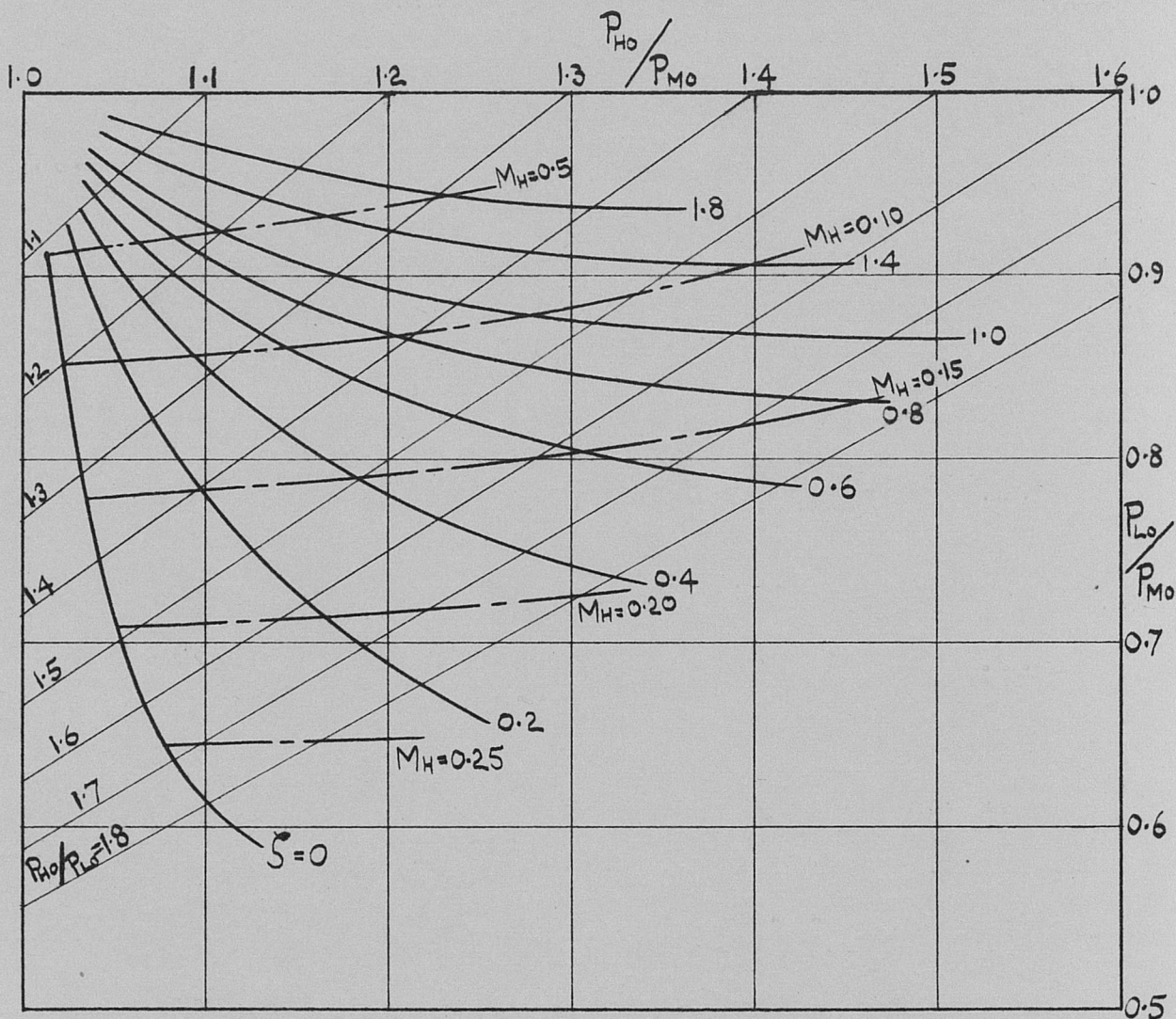


FIG. 12.2-20

EXPERIMENTALLY OBTAINED EQUALISER PERFORMANCE
 (FIXED GEOMETRY, CONSTANT SPEED, $T_{H0} = 308^\circ\text{K}$, $P_{L0} \approx 1\text{ATM}$, $\frac{T_{L0}}{T_{H0}} \approx 0.95$)

ROTOR SPEED AND GEOMETRIC DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	4000 REV./MIN.	$M_{R0} = 0.0978$
HP PORT WIDTH	47°	$\gamma_H = 2.46$
LP " "	43°	$\gamma_L = 2.25$
MP " "	48°	$\gamma_M = 2.51$
PHASING	24°	$\gamma_\phi = 1.26$
LAND WIDTH	12°	$\gamma_\varepsilon = 0.626$
CELL " "	10.4°	$\gamma_c = 0.540$

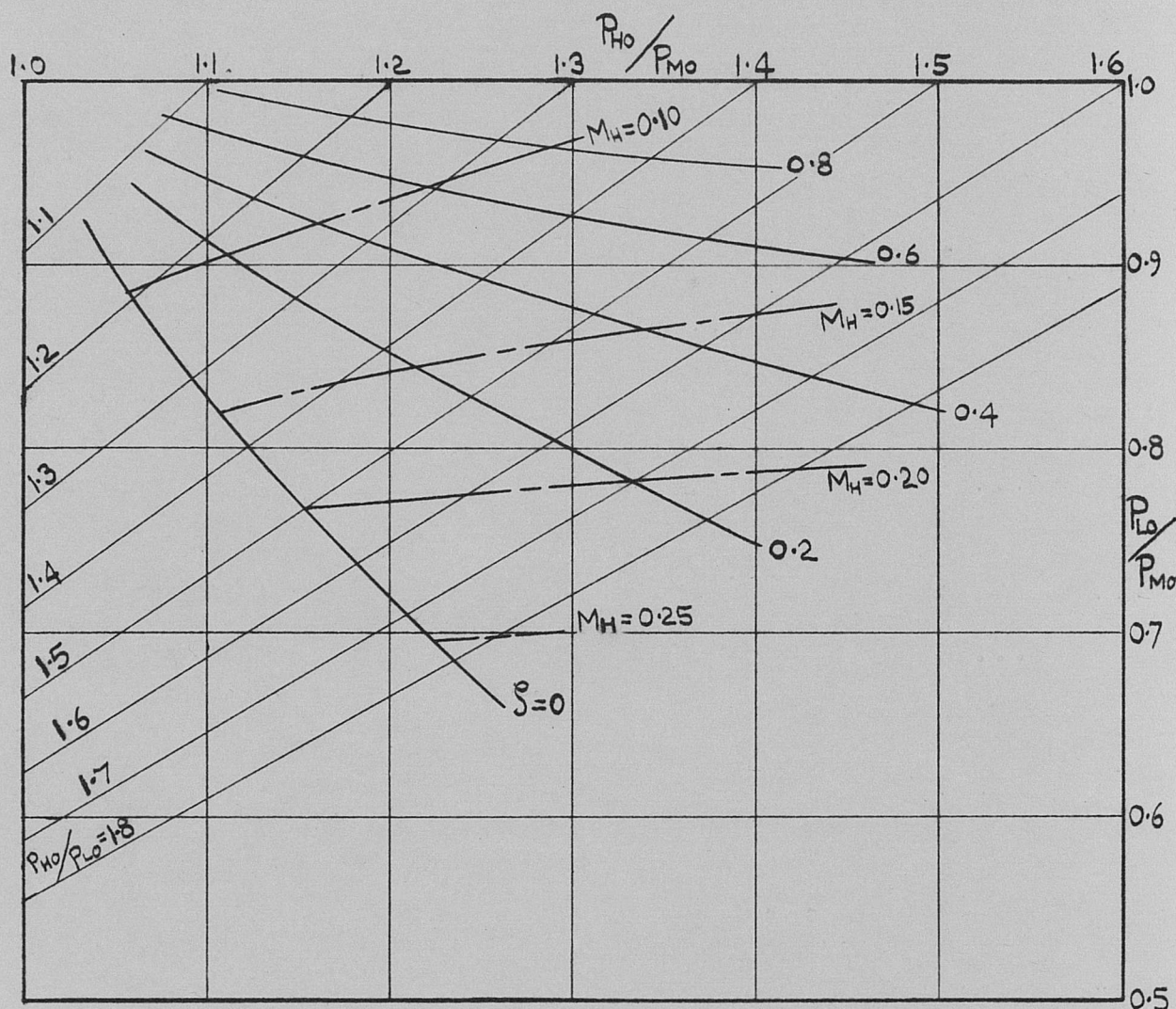
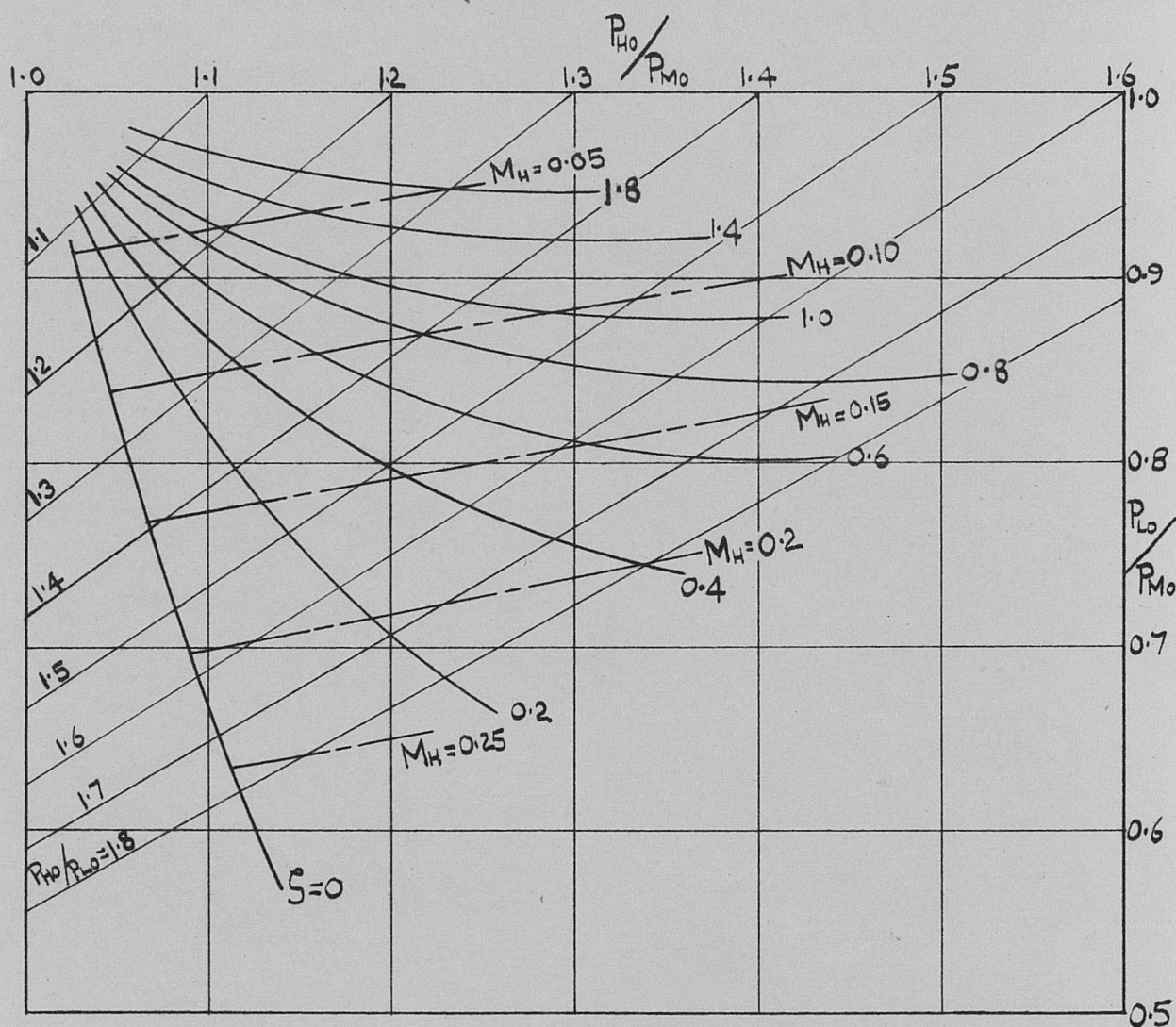


FIG. 12-2-21

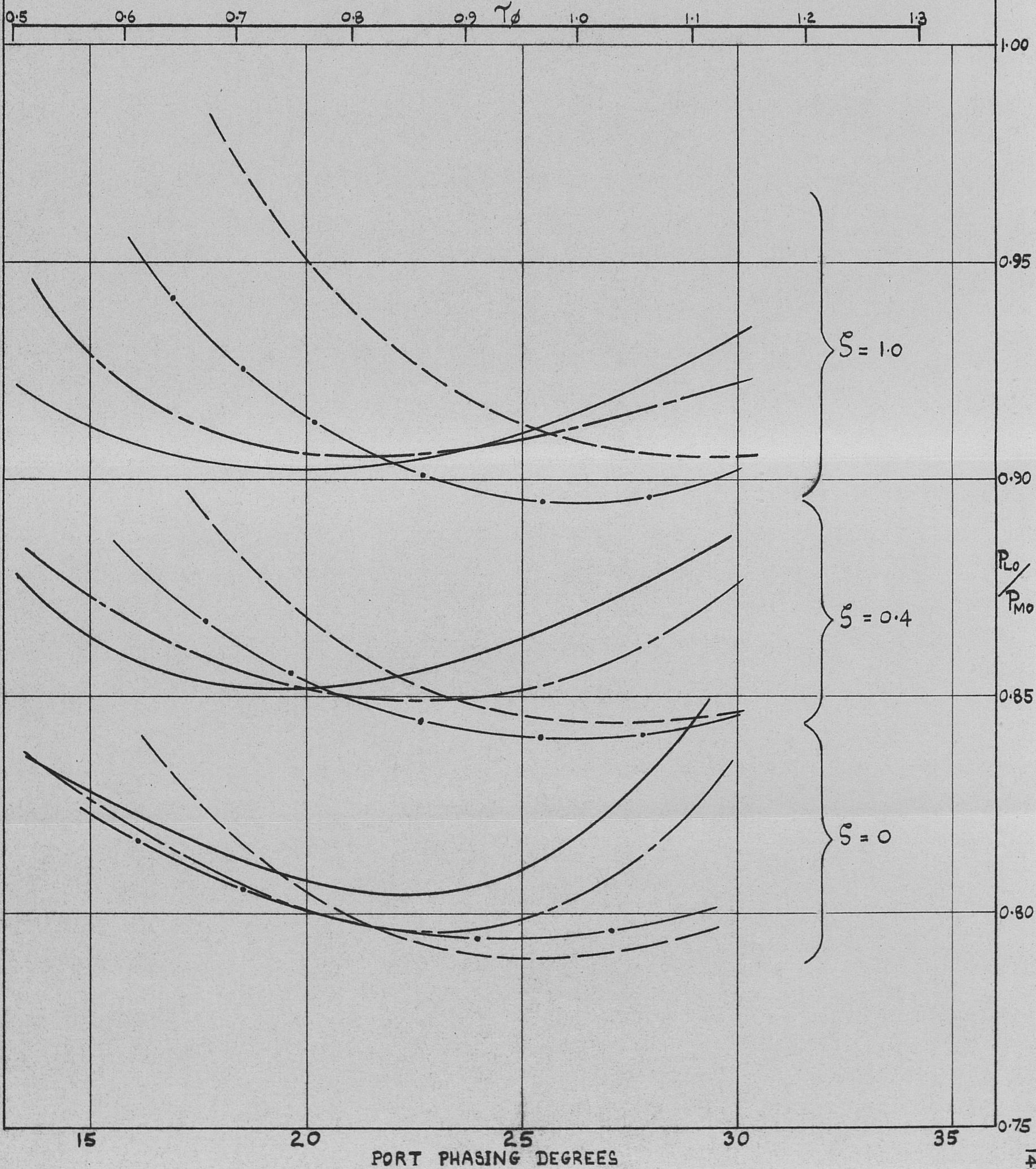
EXPERIMENTALLY OBTAINED EQUALISER PERFORMANCE
 (FIXED GEOMETRY, CONSTANT SPEED, $T_{H0} = 308^\circ\text{K}$, $P_{L0} \approx 1\text{atm}$, $\frac{T_{L0}}{T_{H0}} \approx 0.95$)

ROTOR SPEED AND GEOMETRICAL DATA		
	DIMENSIONED	DIMENSIONLESS
ROTOR SPEED	6000 REV./MIN.	$M_{R0} = 0.1470$
HP PORT WIDTH	47°	$\gamma_H = 1.64$
LP " "	43°	$\gamma_L = 1.50$
MP " "	48°	$\gamma_M = 1.67$
PHASING	24°	$\gamma_\phi = 0.835$
LAND WIDTH	12°	$\gamma_\Sigma = 0.417$
CELL " "	10.4°	$\gamma_c = 0.360$



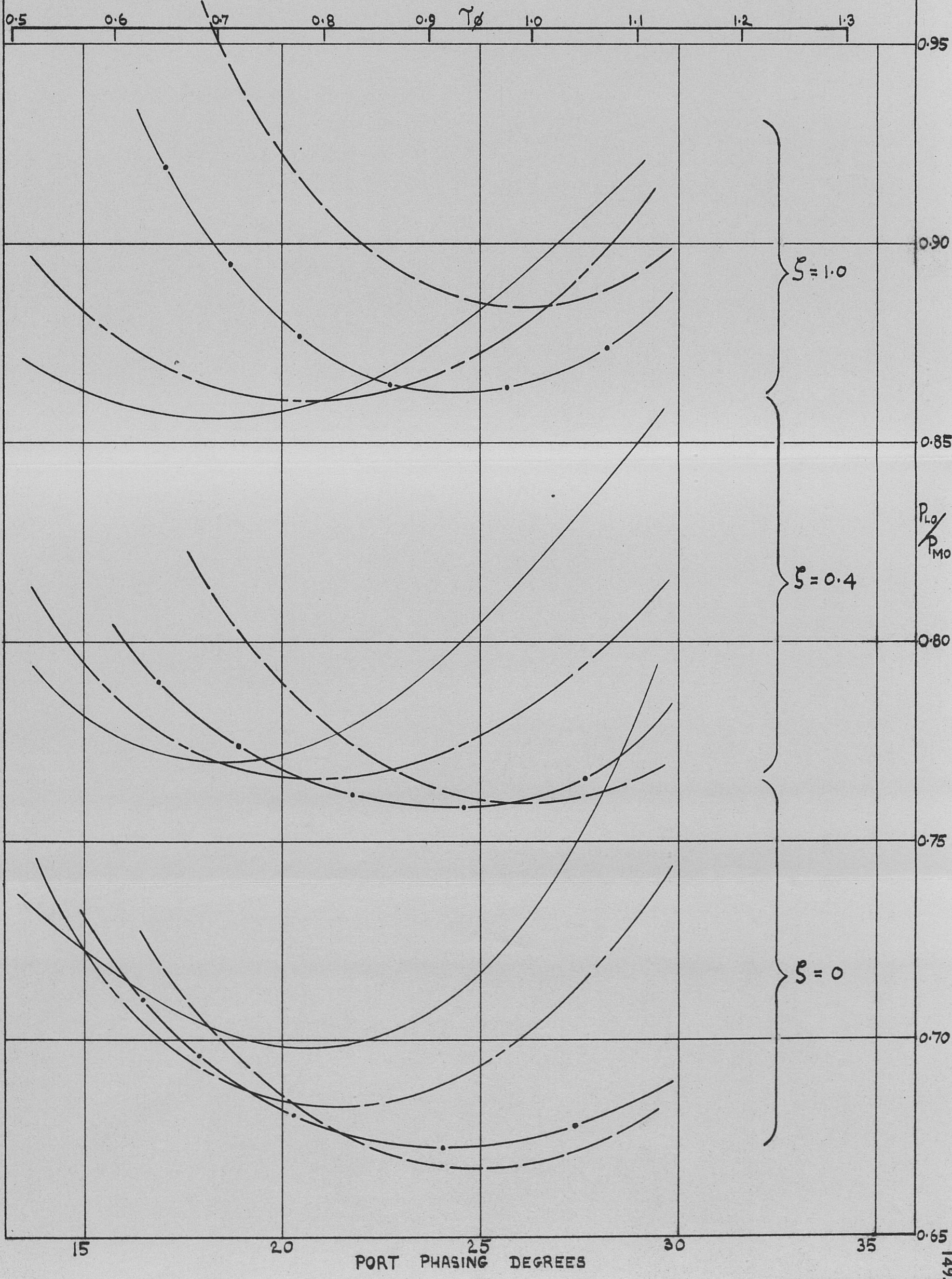
VARIATION OF EXPERIMENTALLY OBTAINED EQUALISER PERFORMANCE WITH PORT WIDTH AND PHASING. $P_{H0}/P_{L0} = 1.3$, $M_{R0} = 0.1346$ (5500 REV./MIN.), $\gamma_M = 1.83$ (48°), $\gamma_E = 0.456$ (12°), $\gamma_C = 0.393$ (10.4°), $T_{H0} = 306^\circ K$, $P_{L0} \approx 1 \text{ ATM.}$

—————	WIDTH OF HP PORT	= 39°	$\gamma_H = 1.48$	$\gamma_L = 1.94$
- - - - -	"	"	$\gamma_H = 1.63$	$\gamma_L = 1.79$
— • —	"	"	$\gamma_H = 1.79$	$\gamma_L = 1.63$
- - - - -	"	"	$\gamma_H = 1.94$	$\gamma_L = 1.48$



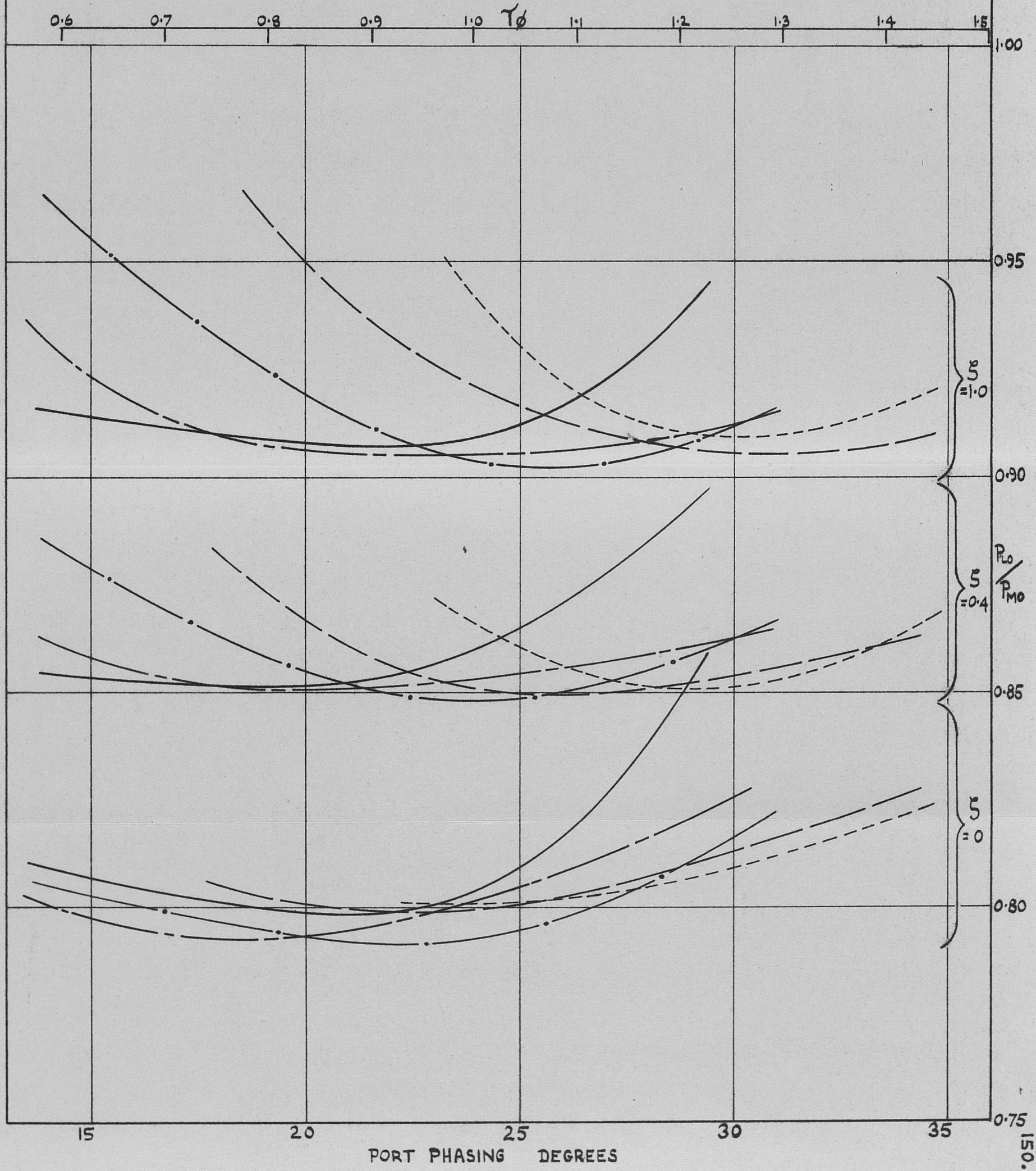
VARIATION OF EXPERIMENTALLY OBTAINED EQUALISER PERFORMANCE WITH PORT WIDTH AND PHASING. $P_{H0}/P_{L0}=1.6$, $M_{R0}=0.1346$ (5500 REV./MIN.), $\tau_M=1.83$ (48°), $\tau_E=0.456$ (12°), $\tau_C=0.393$ (10.4°), $T_{H0}=308^\circ K$, $P_{L0}\approx 1 \text{ ATM.}$

—————	WIDTH OF HP PORT = 39°	$\tau_H = 1.48$	$\tau_L = 1.94$
-----	" " " " = 43°	$\tau_H = 1.63$	$\tau_L = 1.79$
—•—•—	" " " " = 47°	$\tau_H = 1.79$	$\tau_L = 1.63$
- - - - -	" " " " = 51°	$\tau_H = 1.94$	$\tau_L = 1.48$



VARIATION OF EXPERIMENTALLY OBTAINED EQUALISER PERFORMANCE WITH PORT WIDTH AND PHASING. $P_{H0}/P_{L0} = 1.3$, $M_{R0} = 0.1225$ (5000 REV./MIN.), $\gamma_M = 2.01$ (48°), $\gamma_E = 0.501$ (12°), $\gamma_C = 0.431$ (10.4°), $T_{H0} = 308^\circ K$, $P_{L0} \approx 1 \text{ ATM.}$

—————	WIDTH OF HP PORT	= 39°	, $\gamma_H = 1.63$	, $\gamma_L = 2.13$.
-----	"	"	= 43°	, $\gamma_H = 1.80$, $\gamma_L = 1.96$.
— . — . — .	"	"	= 47°	, $\gamma_H = 1.96$, $\gamma_L = 1.80$.
- - - - -	"	"	= 51°	, $\gamma_H = 2.13$, $\gamma_L = 1.63$.
- - - - -	"	"	= 54°	, $\gamma_H = 2.26$, $\gamma_L = 1.50$.



VARIATION OF EXPERIMENTALLY OBTAINED EQUALISER PERFORMANCE WITH PORT WIDTH AND PHASING. $P_{H0}/P_{L0}=1.6$, $M_{R0}=0.1225$ (5000 REV./MIN.), $\gamma_M=2.01$ (48°), $\gamma_E=0.501$ (12°), $\gamma_C=0.431$ (10.4°), $T_{H0}=308^\circ K$, $P_{L0}\approx 1 \text{ ATM}$.

—————	WIDTH OF HP PORT = 39°	$\tau_H = 1.63$	$\tau_L = 2.13$
-----	" " " " = 43°	$\tau_H = 1.80$	$\tau_L = 1.96$
—————	" " " " = 47°	$\tau_H = 1.96$	$\tau_L = 1.80$
-----	" " " " = 51°	$\tau_H = 2.13$	$\tau_L = 1.63$
-----	" " " " = 54°	$\tau_H = 2.26$	$\tau_L = 1.50$

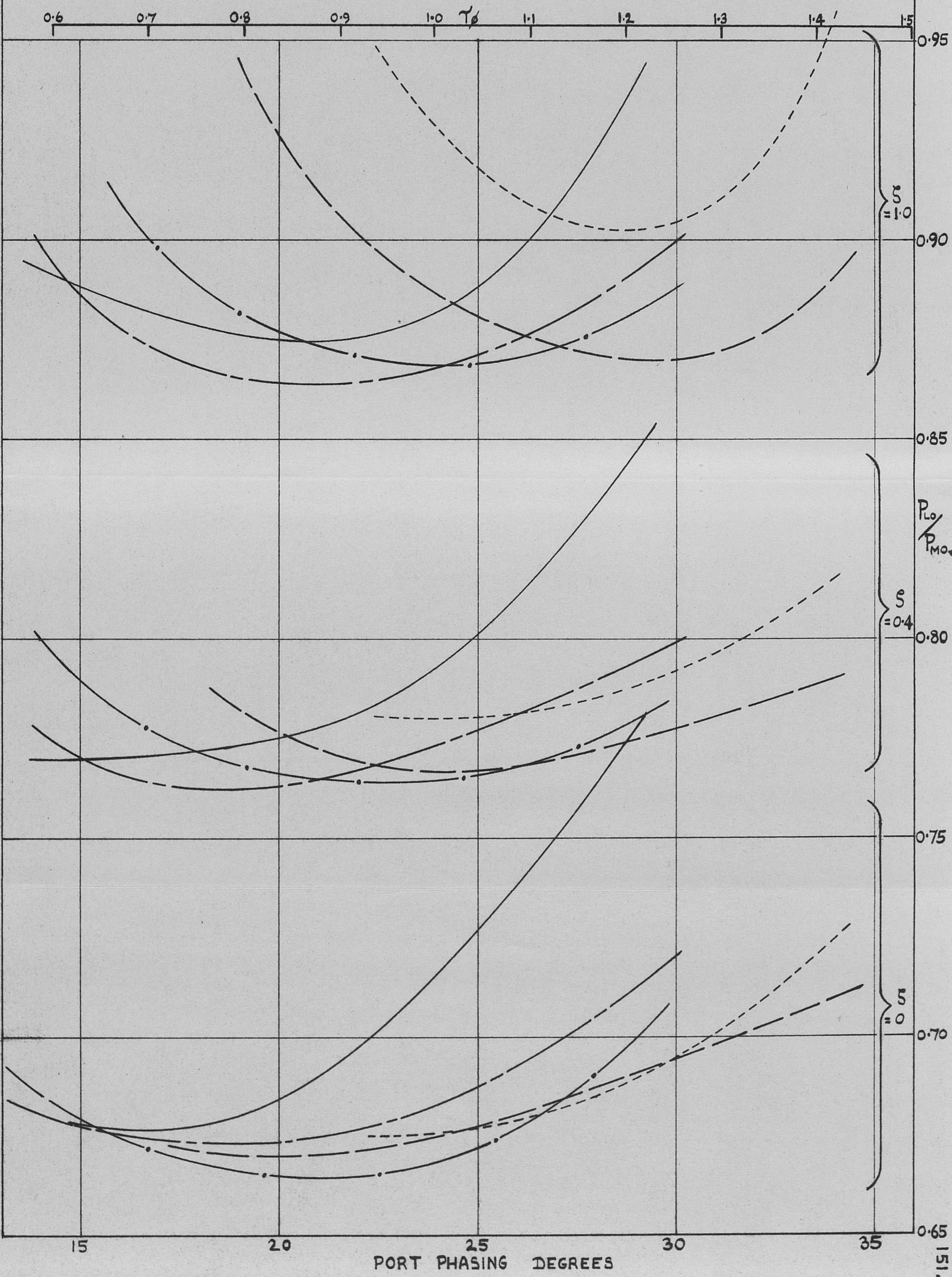


FIG. 12.2-26

VARIATION OF EXPERIMENTALLY OBTAINED EQUALISER PERFORMANCE WITH ROTOR SPEED

$P_{H0}/P_{L0} = 1.3$, HP PORT WIDTH = 47° , LP PORT WIDTH = 43° , MP PORT WIDTH = 48° , PHASING = 2.4° , LAND WIDTH = 12° , CELL WIDTH = 10.4° ,
 $T_{H0} = 308^\circ K$, $P_{L0} = 1 \text{ ATM.}$

(NB. THE MARKED POINTS WERE OBTAINED FROM THE PRESENTATION, ON THE
 P_{H0}/P_{M0} VERSUS P_{H0}/P_{M0} PLANE, OF THE APPROPRIATE INDEPENDENT TEST RESULTS.)

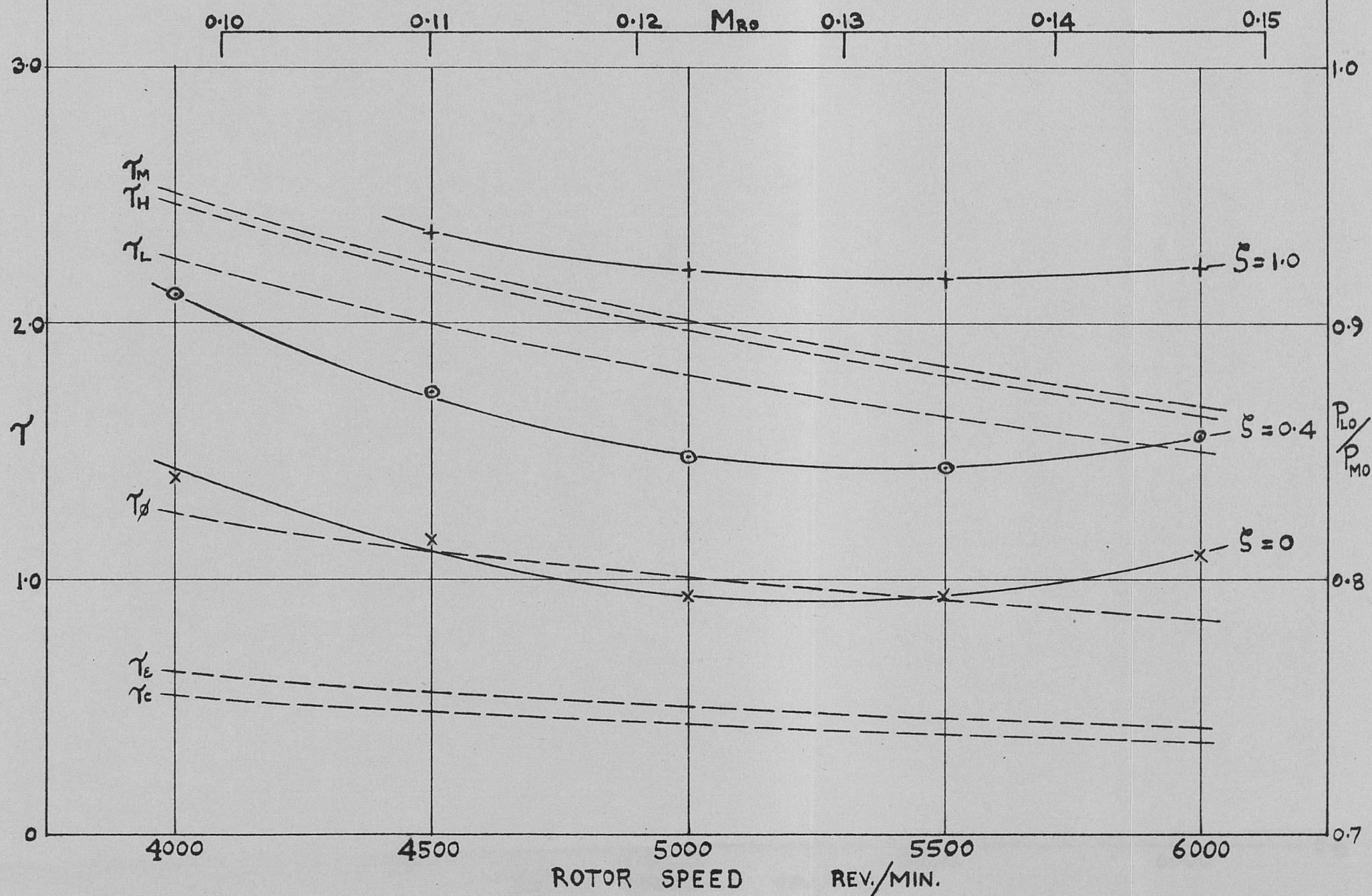
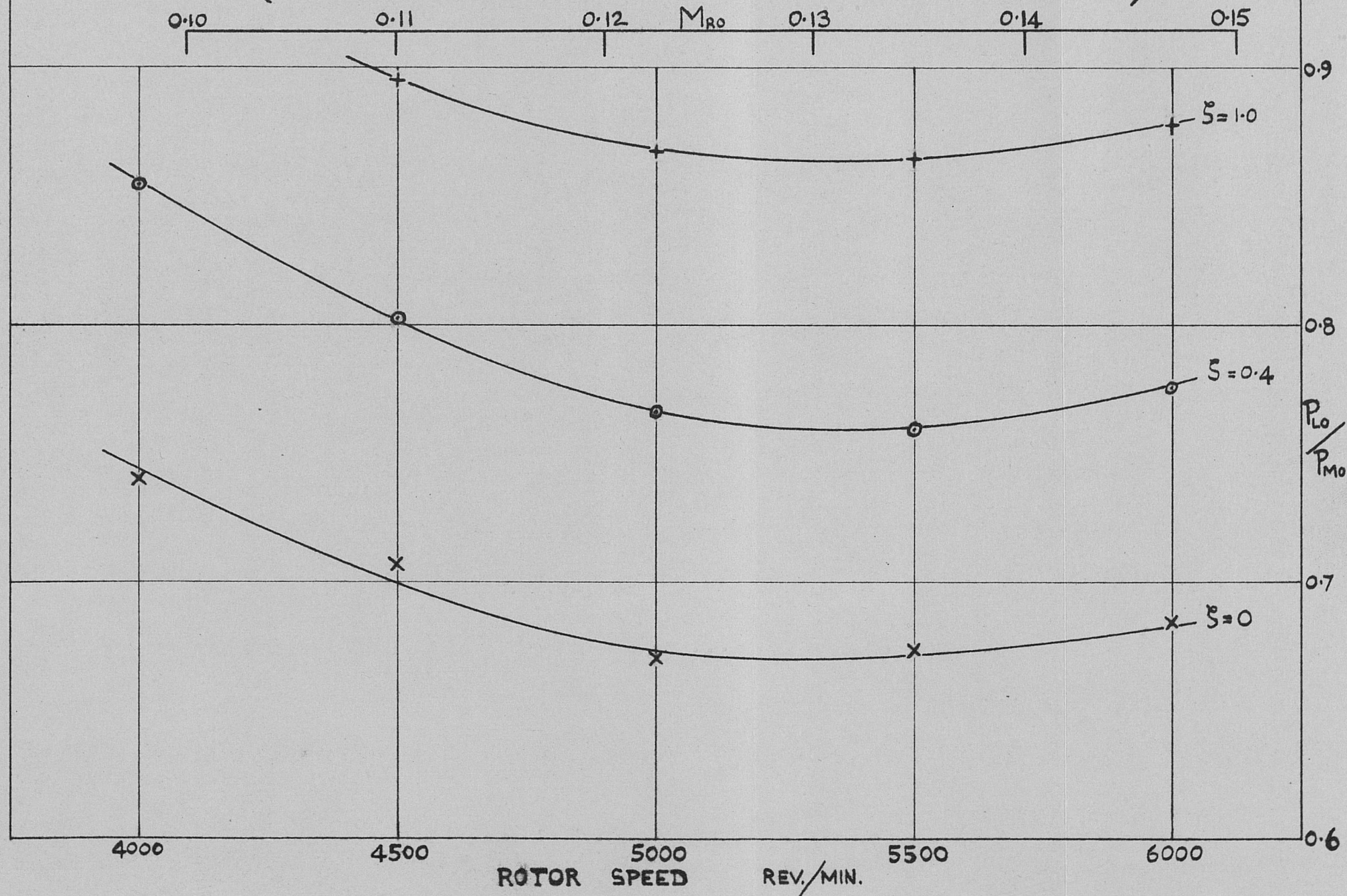


FIG. 12.2-27

VARIATION OF EXPERIMENTALLY OBTAINED EQUALISER PERFORMANCE WITH ROTOR SPEED

$P_{H0}/P_{L0} = 1.6$, HP PORT WIDTH = 47° , LP PORT WIDTH = 43° , MP PORT WIDTH = 48° , PHASING = 2.4° , LAND WIDTH = 12° , CELL WIDTH = 10.4° ,
 $T_{H0} = 308^\circ K$, $P_{L0} \approx 1 \text{ ATM.}$

(NB. THE MARKED POINTS WERE OBTAINED FROM THE PRESENTATION, ON THE P_{L0}/P_{M0} VERSUS P_{H0}/P_{M0} PLANE, OF THE APPROPRIATE INDEPENDENT TEST RESULTS.)



13.

DIAGRAMS AND ILLUSTRATIONS

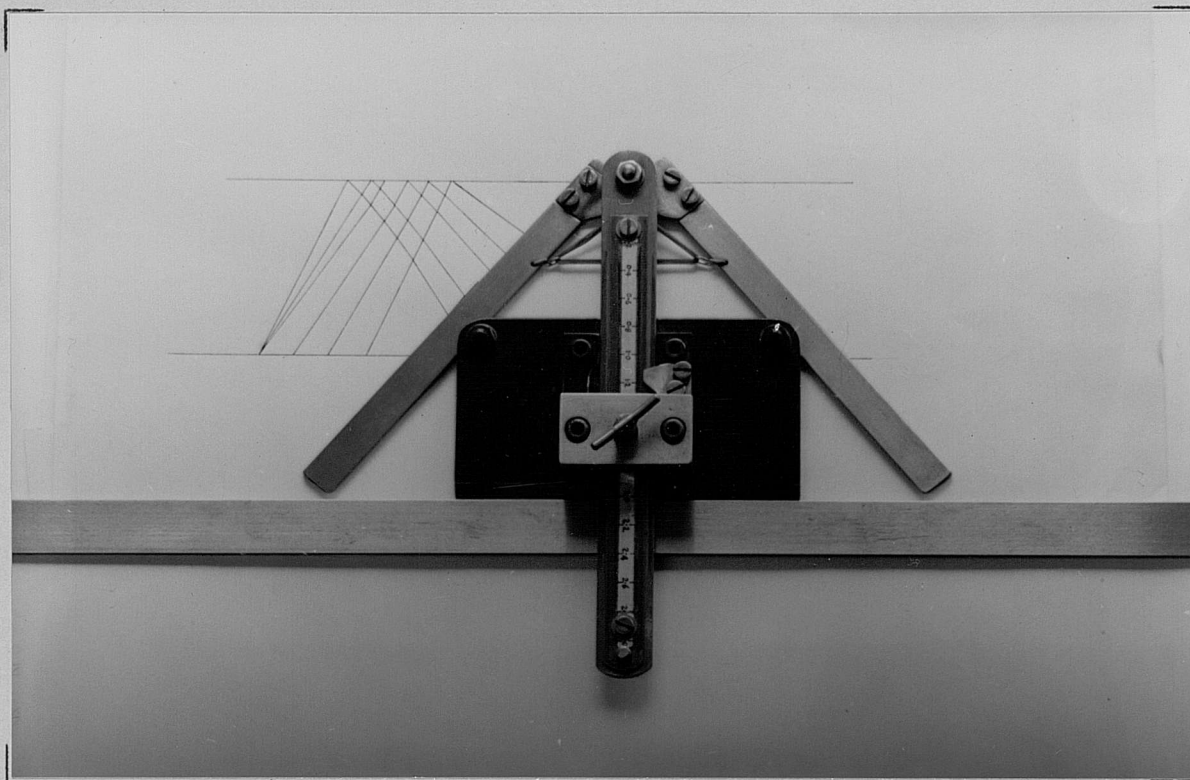


Fig. 13-1 APPARATUS FOR THE CONSTRUCTION OF WAVE DIAGRAMS

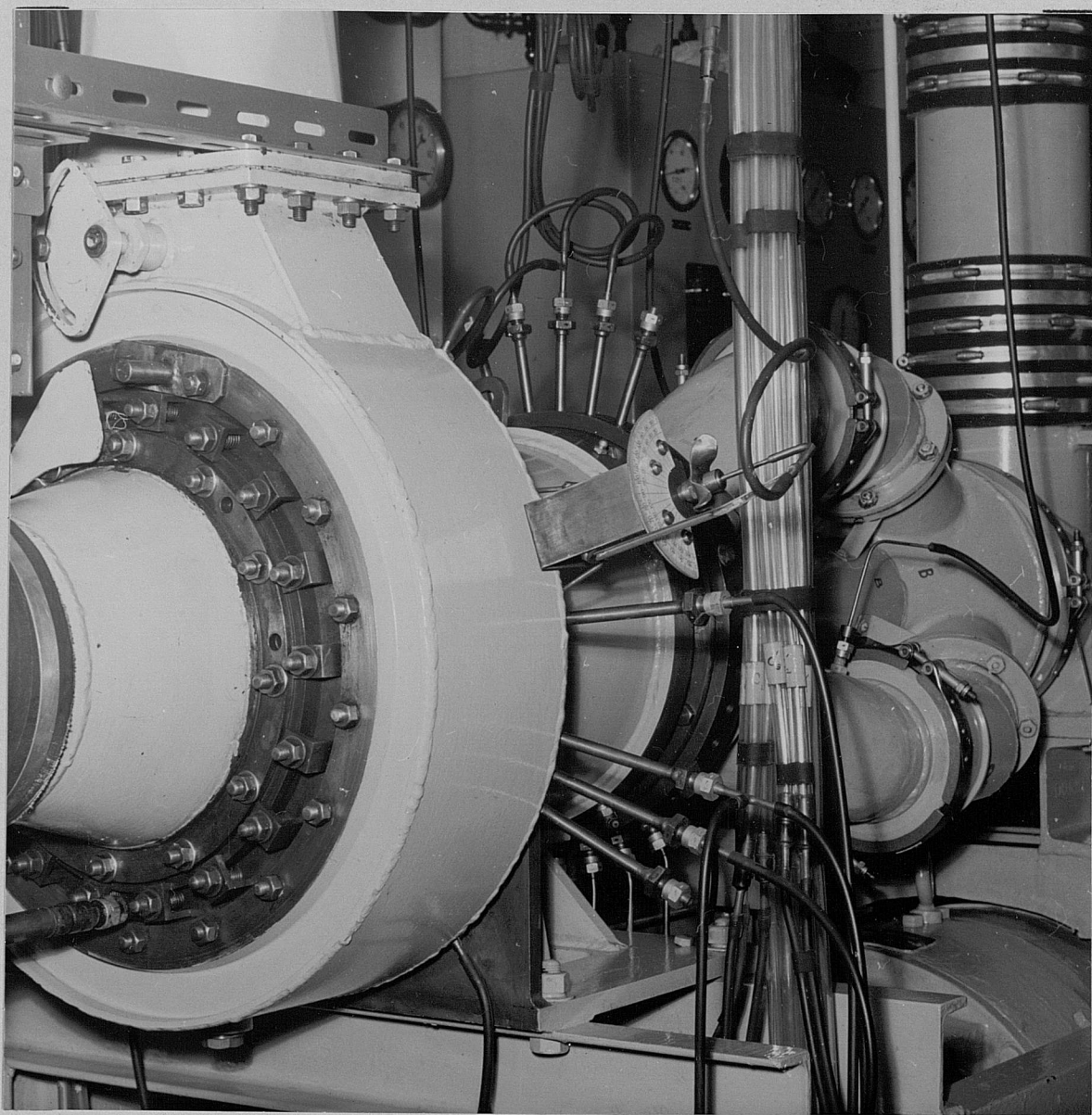


Fig. 13-2

CLOSE-UP PHOTOGRAPH OF THE PRESSURE-EXCHANGER.

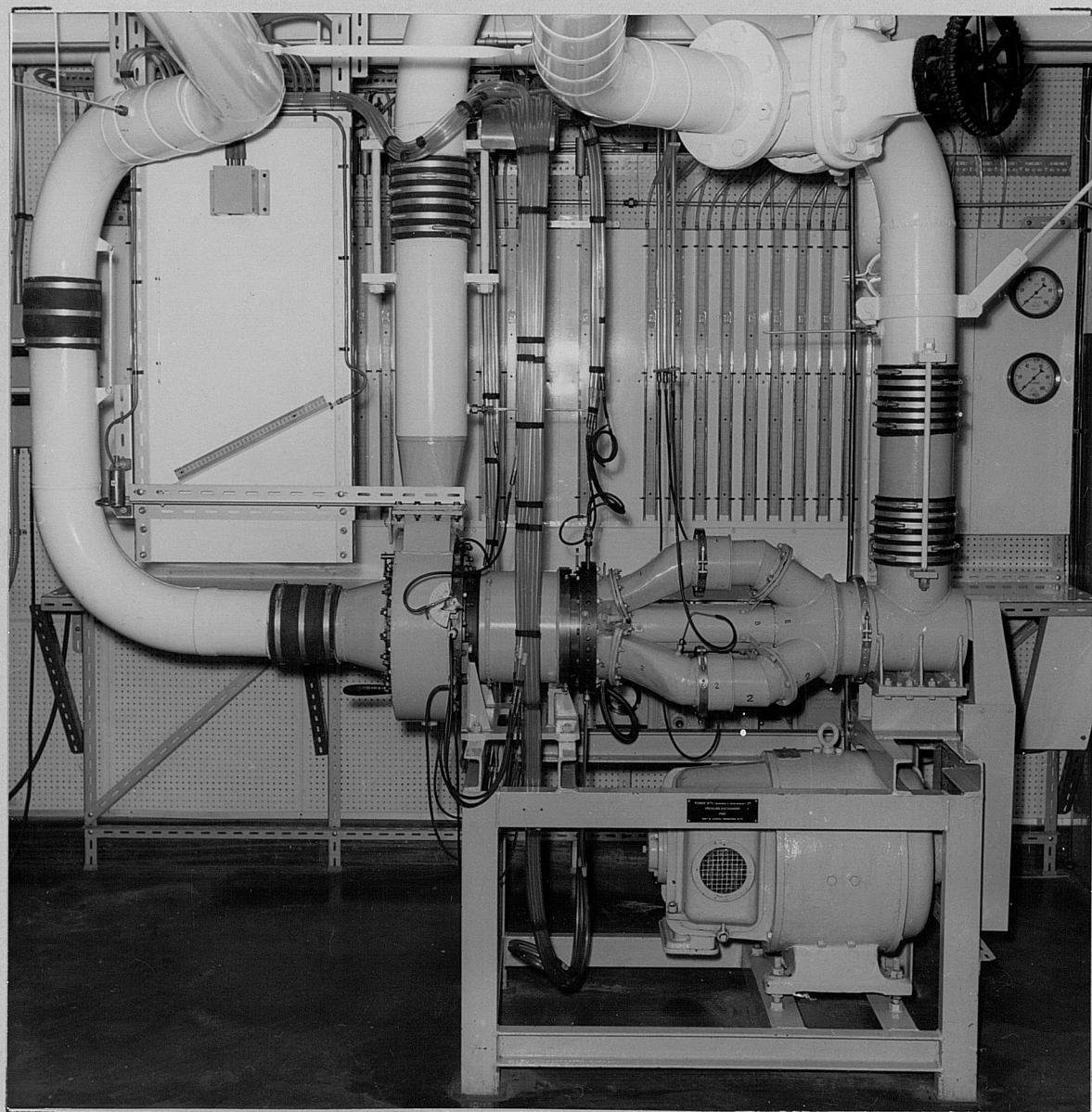


Fig. 13-3 GENERAL VIEW OF THE PRESSURE-EXCHANGER RIG ASSEMBLED
FOR DIVIDER OPERATION.

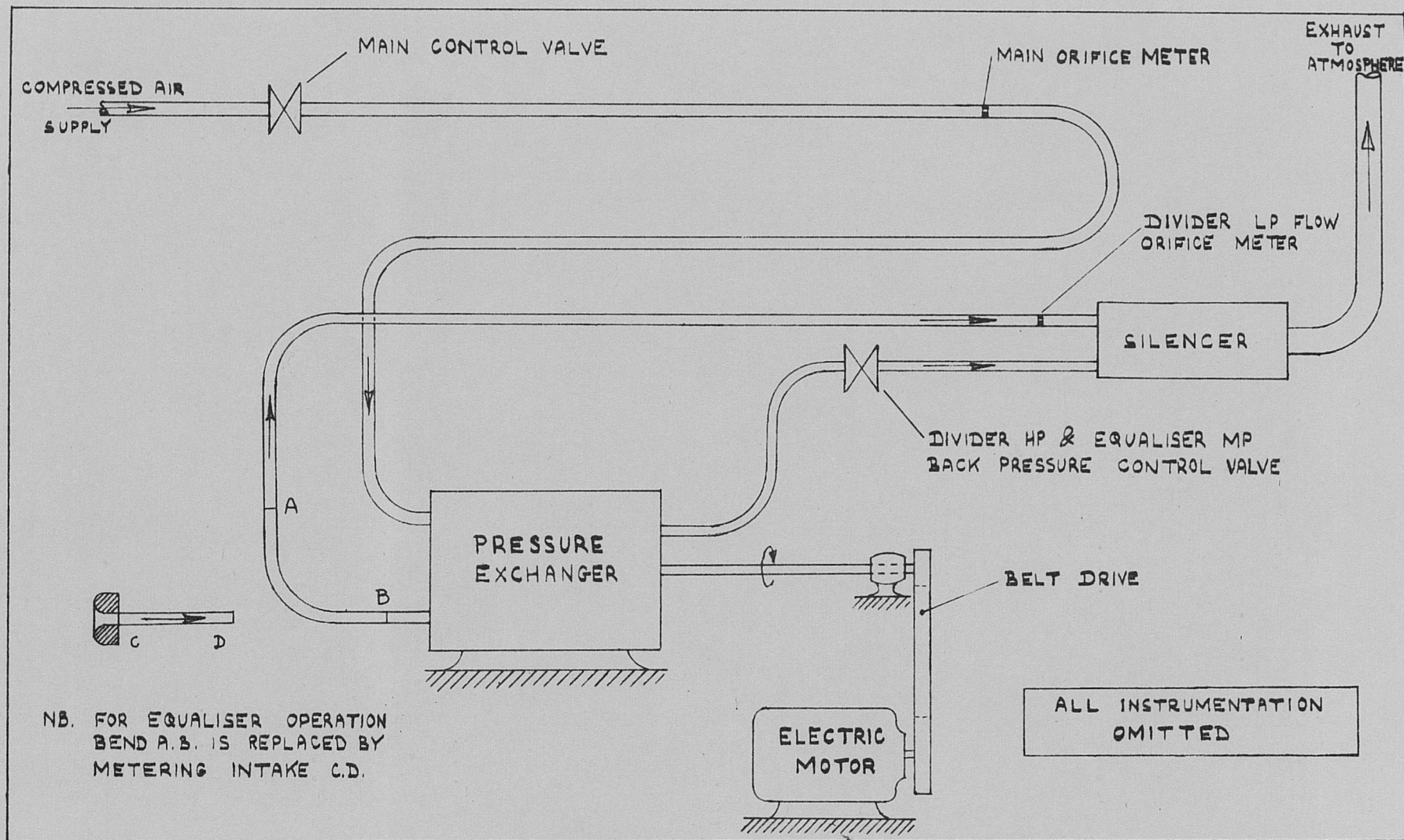


Fig. 13-4 CIRCUIT DIAGRAM OF THE PRESSURE-EXCHANGER DIVIDER & EQUALISER APPARATUS.

14. REFERENCES

1. ALLEN, D.N.DE G.
"Relaxation Methods in Engineering & Science."
McGraw-Hill Book Co.1954 Ch.5.
2. BARNHAM, P.
"The Single Cell Pressure-Exchanger"
Dissertation submitted for Diploma of Imperial College 1961-2
3. BERCHTOLD, M. GULL, H.P.
"Road Performance of a Complex Supercharged Diesel Truck".
Society of Automotive Engineers, Paper 118U, National Diesel
Engine Meeting, Chicago, Oct.27-29, 1959.
4. BRITISH STANDARDS INSTITUTION.
"Flow Measurement."
British Standard Code BS.1042: 1943.
5. EICHACKER, S.S HOGE, H.J.
"Jet Compression Efficiencies as Influenced by the Nature of
the Driving & Driven Gases."
Journal of the Aerospace Sciences, Vol.27, No.8, August, 1960.
6. HILSCH, R.
"The Use of the Expansion of Gases in a Centrifugal Field as
a Cooling Process."
The Engineer's Digest, Vol.IX, March, 1948, P.82.
7. HOGE, H.J. EICHACKER, S.S. FISKE, D.L.
"Studies of Jet Compression I, Apparatus & Methods, Results
with Air at Room Temperature."
Transactions A.S.M.E., Journal of Basic Engineering, Series D.
Sept.1959, P.426.
8. JENDRASSIK, G.
United States Patent Specification 2,946,184.
9. JENNY, E.
"Unidimensional Transient Flow with Consideration of Friction,
Heat Transfer and Change of Section".
The Brown Beveri Review, Vol.37, No.11, Nov.1950 P.447.
10. KEARTON, W.J. KEH, T.H.
"Leakage of Air Through Labyrinth Glands of Staggered Type."
Procds. I.Mech.E. Vol.166, 1952, P.180.
11. RUDINGER, G.
"Wave Diagrams for Non-Steady Flow in Ducts."
D.Van Nostrand & Co. Inc.1955. Ch.V. C.2.2

12. RUDINGER, G.
"Effect of Boundary Layer Growth in a Shock Tube on Shock Reflection from a Closed End."
Project Squid Technical Report CAL-83-P, June 1961.
13. SCHELLER, W.A. BROWN, G.M.
"The Ranque-Hilsch Vortex Tube."
Industrial and Engineering Chemistry, Vol.49, No.6.
June 1957, P.1013.
14. SPALDING, D.B.
British Patent Specification 799,143.
15. SPALDING, D.B.
"A Note on Pressure Equalisers & Dividers."
Power Jets Report No.2251/Px3, May, 1958.
16. SPALDING, D.B.
"Filling, Emptying & Transfer Processes in a Pressure-Exchanger of Finite Cell Width."
Power Jets Report No.2222/X46, August, 1956.
17. SPALDING, D.B.
"Wave Effects in Pressure-Exchangers, Part II."
Power Jets Report No.2199/X40, Feb.1955.
18. SPALDING, D.B.
"Theoretical Considerations On Leakage Effects in Pressure-Exchangers."
Power Jets Report No.2224/X47, Oct.1956.
19. SPIERS, H.M.
"Technical Data on Fuel."
British National Committee World Power Conference, London 1955
Table 81, p.87.
20. SPIERS, H.M.
"Technical Data on Fuel."
British National Committee World Power Conference, London 1955
Table 82, facing P.94.
21. TOPPING, J.
"Errors of Observation and Their Treatment."
The Institute of Physics, Monographs for Students.
22. ZABRISKIE, W. STERNLICHT, B.
"Labyrinth-Seal Leakage Analysis."
Transactions A.S.M.E. Journal of Basic Engineering, Series D,
Sept. 1959, P.332.

15.1 APPENDIX INOTATION

Generally no dimensions are specified for the quantities corresponding to the symbols in the following lists. Usually the symbols are employed in such a way that equations involving them are dimensionless. Where dimensions are required they are given in the text.

ENGLISH-LETTER SYMBOLS

a	Local acoustic velocity
a_{REF}	Reference acoustic velocity. $\left[\equiv a_{M0} \text{ \& } a_{H0} \text{ for dividers \& equalisers respectively.} \right]$
A	Cross-sectional area.
A_h	Cross-sectional area of an averaging manometer pressure pick-up limb.
A_R	Cross-sectional area of an averaging manometer recording limb.
b	Breadth of a cell at mean radial depth.
B	Constant in averaging manometer equations.
c	Clearance between rotor and stator.
C	Leakage coefficient for "simple" leaks.
C_L	Mean value of leakage coefficient for labyrinth leaks.
C_p	Specific heat at constant pressure.
C_v	Specific Heat at constant volume.
d	Radial depth of cells.
D_M	Hydraulic mean diameter of a cell.
f	(without subscript) cross sectional area (Jenny's notation).
f	(with a subscript 1, 2, 3 or 4) leakage functions defined in Appendix V part III.
F	Function of $\frac{P}{P_n}$ (Kearton & Keh's notation).

g	Gravitational acceleration.
G_{SEC}	Mass flow (Jenny's notation).
h	Displacement of fluid from the datum in an averaging manometer pressure pick-up limb.
H	Displacement of fluid from the datum in an averaging manometer recording limb.
K	Ratio; $\frac{\text{Free flow area through a cell}}{\text{cross sectional area of cell plus partition.}}$
l	Length of cells.
m	Mass.
$\dot{m}$	Mass flow.
$\dot{m}_{DUCT}$	Mass flow in an unspecified duct.
$\dot{m}_L$	Mass flow through a leak.
M	Axial component of Mach number.
M_{OZ}	Local Mach number in a cell end, in communication with an inlet port, referred to isentropic stagnation conditions in the inlet.
$M_{\frac{1}{2}}$	Local Mach number in a cell end referred to isentropic static conditions in the outlet port communicating with the cell.
M_t	Total Mach number, i.e. vector sum of whirl and axial components of Mach number, hence; $M_t = \sqrt{M^2 + M_w^2}$
M_w	Whirl component of Mach number at mean radial depth of cells.
n	Number of pressure pick-up limbs on an averaging manometer. Also number of stages in a labyrinth seal (Kearton & Keh's notation).
N	Rotational speed, revs./min.
$\dot{N}$	Number of cells passing a fixed point in unit time.
P	Static pressure.
P_{BACK}	Back pressure on recording limb of an averaging manometer.
P_L	Effective stagnation pressure at entry to a leakage path.

- P_n Static pressure downstream of last (n th) stage of a labyrinth seal.
- P_o Stagnation pressure; subscript indicating the fluid stream is interposed between the "P" and "o".
- P_r Static pressure at radius r
- P_s Static pressure in an unspecified outlet port prior to making allowances for cell partition thickness, viscous, or leakage effects; subscript S replaced by H, L, or M when the port is specified.
- P_x Static pressure on the discharge side of a simple leak.
- $\dot{q}$ Rate of heat transfer.
- $\dot{Q}$ Dimensionless form of $\dot{q}$. $\left[\equiv \frac{\dot{q}}{C_v \dot{m}_1 T_1} \right]$
- r Radius.
- R Gas constant.
- s Pitch of star points in relaxation procedure.
- S Number of velocity heads lost.
- t Time or time interval.
- T Static temperature.
- T_o Stagnation temperature; subscript indicating the fluid stream is interposed between the "T" and "o".
- u Component of local gas velocity parallel to axis of rotor.
- U Dimensionless form of u. $\left[\equiv \frac{u}{U} \right]$
- v Whirl component of local gas velocity at mean radial depth of cells.
- v_w Circumferential velocity of rotor at mean radial depth of cells.
- V Dimensionless form of v. $\left[\equiv \frac{v}{U} \right]$
- V_w Dimensionless form of v_w . $\left[\equiv \frac{v_w}{U} \right]$
- w Circumferential width of port at mean radial depth of cells.
- x Co-ordinate in a direction parallel to axis of rotor.
- X Dimensionless form of x, $\left[\equiv \frac{x}{W} \right]$ (applicable in ducts only).

- y Circumferential co-ordinate.
- Y Dimensionless form of y , $\left[\equiv \frac{y}{\omega} \right]$ (applicable in ducts only).

GREEK-LETTER SYMBOLS

- α Dimensionless form of sonic velocity; $\equiv a/a_{REF}$
- β Divider cycle mass ratio; $\equiv \dot{m}_H/\dot{m}_M$
- β^+ Divider mass ratio from wave diagram calculations allowing for leakage effects in cells (does not take account of leakage effects external to the cells).
- γ Ratio of specific heats; $\equiv C_p/C_v$
- δ Small finite difference.
- Δ Finite difference.
- ζ Equaliser stagnation enthalpy ratio; $\equiv \dot{m}_L T_{Lo}/\dot{m}_H T_{Ho}$
- ζ^+ Equaliser stagnation enthalpy ratio from wave diagram calculations allowing for leakage effects in cells, (does not take account of leakage effects external to the cells.)
- z Dimensionless velocity for wave diagrams; $\equiv u/a_{REF}$
- η_{cis} Isentropic compression efficiency.
- η_{eis} Isentropic expansion efficiency.
- η_{pis} Isentropic product efficiency; $\equiv \eta_{cis} \eta_{eis}$
- Θ Angle of inclination of the walls of a pressure-exchanger inlet duct.
- Θ Criterion of divider cooling cycle performance; $\equiv (T_{Mo} - T_{Lo})/T_{Mo}$
- z Dimensionless form of rotor to stator clearance; $\equiv c/l$
- h " " " breadth of cell; $\equiv b/l$
- λ " " " radial depth of cells; $\equiv d/l$
- ξ " " " displacement along axis of cells; $\equiv x/l$
- $\xi(\psi)$ Vorticity in a pressure-exchanger duct.
- ρ Density.
- σ Static pressure ratio (employed in universal pressure ratio temperature ratio relationships, Appendix IV).

- τ Dimensionless time or time interval; $\equiv t_{a_{REF}}/l$
- τ Static temperature ratio (employed in universal pressure ratio, temperature ratio relationships, Appendix IV).
- ϕ Fraction of the cross-sectional area of a cell communicating with a port during opening or closing.
- Φ Ratio of mass ratios; $\equiv \beta/\beta^*$
- Φ_{lx} Leakage function; $\equiv \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{P_x}{P_L} \right)^{\frac{2}{\gamma}} - \left(\frac{P_x}{P_L} \right)^{\frac{\gamma+1}{\gamma}} \right]}$
- χ Relaxation procedure residual quantity.
- ψ Stream function.
- Ψ Criterion of divider cooling cycle performance; $\equiv \dot{m}_L(T_{M0} - T_{L0})/\dot{m}_M T_{M0}$
- ω Angular velocity.
- Ω Fraction of length of a grid arm S inboard of a boundary (relaxation procedure).

SUBSCRIPTS

- AV Average value; the particular meanings implied, arithmetic average, etc, are specified in the text.
- C Refers to cells.
- H Refers to the high pressure stream or HP port.
- L " " " low " " " LP "
- M " " " medium " " " MP "
- OS The static conditions obtained from isentropic expansion (or compression) from cycle inlet conditions to outlet port static pressure P_s .
- P Refers to a port of unspecified type.
- T Refers to the total time a cell communicates with a port.
- Z Refers to stagnation conditions in an inlet port prior to making allowances for cell partition thickness, viscous, or leakage effects.
- E Refers to the land between divider MP inlet and LP outlet, or equaliser HP and LP inlets.

- ϕ Refers to the phasing of the leading edges of the divider or equaliser HP and HP ports.

Other subscripts are defined on diagrams appearing in the text.

SUPERSCRIPTS

- / Refers to fluid trapped in the cells by cut-off of the last port in the cycle.
- *
- Refers to the end conditions, or the results, of an isentropic process.
- Refers to the mean conditions in a duct.

LIMITS OF INTEGRATION ETC.

- 1 & 2 These are generally used to represent the lower and upper limits of integrations respectively as in; $\int_1^2 \text{function}(z) dz$ which is an abbreviation for; $\int_{z_1}^{z_2} \text{function}(z) dz$.
- r Represents the general (rth) term in a series or summation.
- n Represents the upper (unspecified) limit of a summation of a finite number of terms.

DIAGRAM LABELS ETC.

- (a), (B) Where possible appropriate physical quantities have been written on diagrams. Elsewhere codes of encircled numerals
- (1), (II) Roman and Arabic and encircled letters, capital and small, are used as symbols to link t-x with a-u diagrams and
- etc. γ-○ charts, etc. Where appropriate, symbols have, after removal of the surrounding circles, been used as subscripts for physical quantities, etc. in calculations relevant to the diagrams.

H High pressure port or stream.

L Low " " " "

M Medium " " " "

»» Direction of travel of cells.

→ Direction of fluid flow.

ABBREVIATIONS IN THE TEXT.

HP High pressure.

LP Low "

MP Medium "

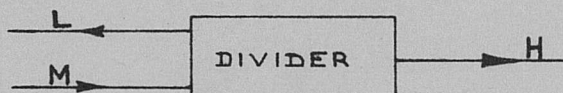
p.e. Pressure-exchanger.

15.2 APPENDIX IISIGNIFICANCE OF PRODUCT EFFICIENCY& CRITERIA OF PERFORMANCE

This Appendix is sub-divided into three sections (a), (b) and (c). Section (a) deals with the significance of mass ratio and product efficiency as exponents of divider performance, section (b) justifies the criteria of divider cooling cycles quoted in 3.2.2. Section (c) covers the derivation of expressions for equaliser performance in terms of enthalpy ratio and product efficiency.

(a) Significance of η_{PIS} and β as Criteria of Divider Performance

The sketch shows a block diagram of an adiabatic divider of unspecified kind;



The energy equation for working fluid having constant specific heat:

$$\dot{m}_H (T_{H0} - T_{M0}) = \dot{m}_L (T_{M0} - T_{L0}) \quad 15.2-1$$

by definition of isentropic efficiency, where the symbols with asterisks refer to states after an isentropic process:

$$\eta_{CIS} \equiv \frac{T_{H0}^* - T_{M0}}{T_{H0} - T_{M0}} \quad 15.2-2$$

$$\eta_{EIS} \equiv \frac{T_{M0} - T_{L0}^*}{T_{M0} - T_{L0}} \quad 15.2-3$$

Substituting from 15.2.2 and 15.2.3 for the temperature differences

appearing in 15.2-1, dividing by T_{M0} , and rearranging;

$$\frac{\dot{m}_H}{\dot{m}_L} \left[\frac{T_{H0}^*}{T_{M0}} - 1 \right] = \eta_{CIS} \eta_{EIS} \left[1 - \frac{T_{L0}^*}{T_{M0}} \right] \quad 15.2-4$$

From isentropic relations:

$$\frac{T_{H0}^*}{T_{M0}} = \left(\frac{P_{H0}}{P_{M0}} \right)^{\frac{\gamma-1}{\gamma}} \quad 15.2-5 \quad \frac{T_{L0}^*}{T_{M0}} = \left(\frac{P_{L0}}{P_{M0}} \right)^{\frac{\gamma-1}{\gamma}} \quad 15.2-6$$

by definition:

$$\frac{\dot{m}_H}{\dot{m}_M} \equiv \beta \quad \therefore \quad \frac{\dot{m}_H}{\dot{m}_L} = \frac{\beta}{1-\beta} \quad 15.2-7$$

by substituting 15.2-5, 6, 7 in 15.2-4 and rearranging, the following result is obtained after invoking the definition of η_{PIS} :

$$\eta_{PIS} \equiv \eta_{CIS} \eta_{EIS} = \left(\frac{\beta}{1-\beta} \right) \frac{\left[\left(\frac{P_{H0}}{P_{M0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{\left[1 - \left(\frac{P_{L0}}{P_{M0}} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad 15.2-8$$

Defining β^* as the mass ratio $\frac{\dot{m}_H}{\dot{m}_M}$ of an isentropic machine, it follows from 15.2-8 that for corresponding pressure ratios:

$$\left(\frac{\beta}{1-\beta} \right) / \left(\frac{\beta^*}{1-\beta^*} \right) = \eta_{PIS} \quad 15.2-9$$

Equation 15.2-8 shows that the product efficiency of an adiabatic divider is a function of the mass and pressure ratios. Alternatively 15.2-9 shows that the product efficiency is a unique function of actual and corresponding isentropic mass ratios, i.e. $\eta_{PIS} = \text{function}(\beta, \beta^*)$. This function is shown in graphical form in Fig.15.2-1.

(b) Criteria of Divider Cooling Cycle Performance

For some applications the most important criterion of divider cooling performance is the magnitude of the obtainable temperature drop. If this is expressed as a fraction of the corresponding temperature drop of an isentropic machine, then the ratio is, by definition, the isentropic expansion efficiency of the divider; see equation 15.2-3.

A more meaningful criterion of temperature performance is the ratio of the temperature drop divided by the input temperature T_{M0} .

Expressing the temperature drop in this way and involving equations 15.2-3

and 15.2-6, the following expression is obtained;

$$\Theta \equiv \frac{T_{M0} - T_{L0}}{T_{M0}} = \eta_{EIS} \left[1 - \left(\frac{P_{L0}}{P_{M0}} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad 15.2-10$$

For some applications the potential cooling effect is more important than the maximum temperature drop. To form a criterion of this, equation 15.2.10 may be converted into an enthalpy ratio, defining this as Ψ , and from the definition of β , we have for the case of constant specific heat:

$$\Psi \equiv \frac{C_p \dot{m}_L (T_{M0} - T_{L0})}{C_p \dot{m}_M T_{M0}} = (1-\beta) \eta_{EIS} \left[1 - \left(\frac{P_{L0}}{P_{M0}} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad 15.2-11.$$

Assigning the symbol Ψ^* to the performance of an isentropic machine, also invoking equation 15.2-9

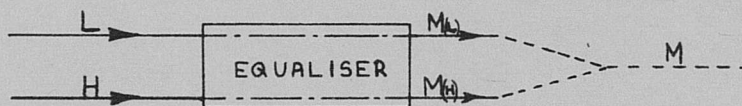
$$\frac{\Psi}{\Psi^*} = \eta_{EIS} \left[\frac{1-\beta}{1-\beta^*} \right] = \frac{\beta}{\beta^* \eta_{CIS}} \quad 15.2-12$$

From equation 15.2-8 it can be seen that if $\eta_{CIS} \rightarrow 0$, then $\beta \rightarrow 0$ hence from 15.2-12, $\frac{\Psi}{\Psi^*}$ can conceivably exceed unity, particularly if $\eta_{EIS} \rightarrow 1$.

The orthodox coefficient of performance criterion for refrigerators is of little use in assessing divider cooling effectiveness because the appropriate expression involves some knowledge of the process or device supplying compressed fluid to the divider.

(c) Significance of η_{PIS} and $\bar{S}$ as Equaliser Performance Criteria:

An equaliser of unspecified type is shown in the block diagram:



The energy equation for a machine discharging two separate medium pressure streams is;

$$C_{PH} \dot{m}_H (T_{H0} - T_{M0H}) = C_{PL} \dot{m}_L (T_{M0L} - T_{L0}) \quad 15.2-13$$

Where the symbols with asterisks refer to states after isentropic processes, the isentropic compression and expansion efficiencies are defined;

$$\eta_{cis} \equiv \frac{T_{MOL}^* - T_{LO}}{T_{MOL} - T_{LO}} \quad 15.2-14$$

$$\eta_{eis} \equiv \frac{T_{HO} - T_{MOH}}{T_{HO} - T_{MOH}^*} \quad 15.2-15$$

Substituting from 15.2-14 and 15.2-15 for the temperature differences appearing in 15.2-13:

$$C_{PH} \dot{m}_H T_{HO} \eta_{eis} \left[1 - \frac{T_{MOH}^*}{T_{HO}} \right] = C_{PL} \dot{m}_L \frac{T_{LO}}{\eta_{cis}} \left[\frac{T_{MOL}^*}{T_{LO}} - 1 \right] \quad 15.2-16$$

From isentropic relations: $\frac{T_{MOH}^*}{T_{HO}} = \left(\frac{P_{MOH}}{P_{HO}} \right)^{\frac{\gamma-1}{\gamma}} \quad 15.2-17.$ $\frac{T_{MOL}^*}{T_{LO}} = \left(\frac{P_{MOL}}{P_{LO}} \right)^{\frac{\gamma-1}{\gamma}} \quad 15.2-18$

Defining ζ such that:

$$\zeta \equiv \frac{C_{PL} \dot{m}_L T_{LO}}{C_{PH} \dot{m}_H T_{HO}} \quad 15.2-19$$

rearranging 15.2-16 and substituting 15.2-17, 18, 19 and invoking the definition of η_{pis} :

$$\eta_{pis} \equiv \eta_{cis} \eta_{eis} = \zeta \frac{\left[\left(\frac{P_{MOL}}{P_{LO}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{\left[1 - \left(\frac{P_{MOH}}{P_{HO}} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad 15.2-20$$

defining ζ^* as the enthalpy ratio of an isentropic machine, then for corresponding pressure ratios:

$$\frac{\zeta}{\zeta^*} = \eta_{pis} \quad 15.2-21$$

In most pressure-exchanger equalisers both the medium pressure streams share the same medium pressure outlet ports, the flows then pass to a common reservoir. This arrangement is shown in diagrammatic form by dotted lines added to the block diagram of the equaliser.

In an idealised machine mixing losses can be avoided by making $P_{MOH} = P_{MOL} = P_{MO}$

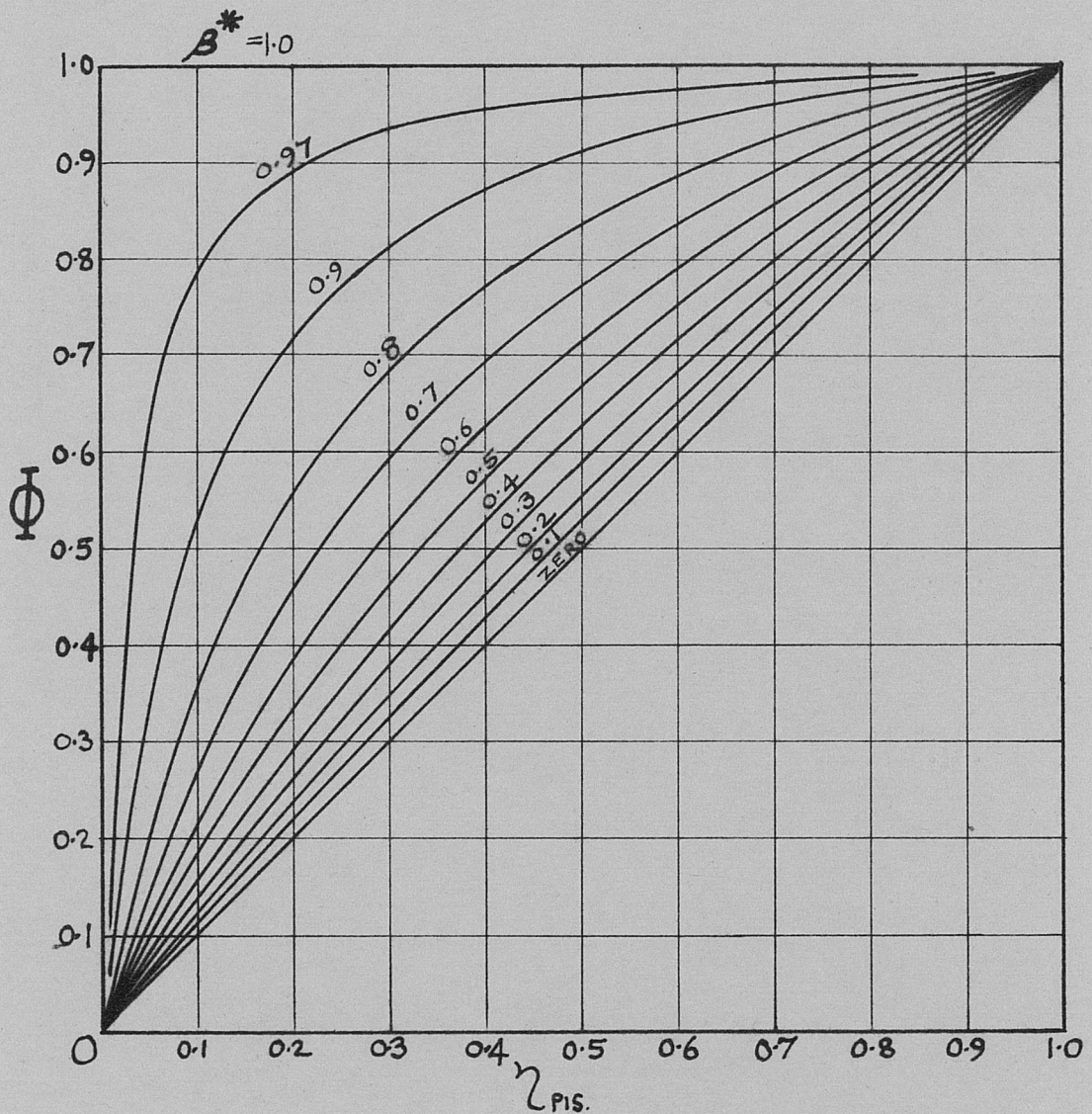
and diffusing each of the separate medium pressure streams to essentially zero velocity prior to mixing. Hence there is no need to allow for mixing losses in the idealised analysis; particularising equation 15.2-20 for the case $P_{MOH} = P_{MOL} = P_{MO}$:

$$\eta_{PIS} \equiv \eta_{CIS} \eta_{EIS} = \sum \frac{\left[\left(\frac{P_{MO}}{P_{LO}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{\left[1 - \left(\frac{P_{MO}}{P_{HO}} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad 15.2-22$$

Equations 15.2-21 and 15.2-22 are those of particular relevance to pressure-exchanger equalisers.

FIG. 15-2-1
RELATIONSHIP BETWEEN DIVIDER MASS RATIO
AND PRODUCT EFFICIENCY

$$\Phi \equiv \frac{\text{MASS RATIO } \beta \text{ OF A DIVIDER OF EFFICIENCY } \eta_{\text{PIS}}}{\text{MASS RATIO } \beta^* \text{ OF AN ISENTROPIC DIVIDER}}$$



15.3 APPENDIX III

PREDICTION OF THE PERFORMANCE
OF ISENTROPIC PRESSURE EXCHANGER
DIVIDERS & EQUALISERS

This Appendix is divided into two parts (a) and (b). Part (a) deals with the derivation of expressions needed for evaluating the performance and performance limits of a hypothetical isentropic pressure-exchanger divider. Section (b) serves a similar function for a hypothetical isentropic pressure-exchanger equaliser.

(a) Pressure Exchanger Divider:

In order to present the performance of an isentropic pressure-exchanger divider on the $P_{H0}/P_{M0} - P_{L0}/P_{M0}$ plane it was necessary to determine:

- (1) the mass ratio β^* in terms of P_{H0}/P_{M0} and P_{L0}/P_{M0} .
- (11) The Mach numbers M_H , M_M , & M_L in the high, medium and low pressure ports respectively, all in terms of P_{H0}/P_{M0} and P_{L0}/P_{M0} .
- (111) the performance limitations, if any, determining the boundaries of the performance regime of the $P_{H0}/P_{M0} - P_{L0}/P_{M0}$ plane.

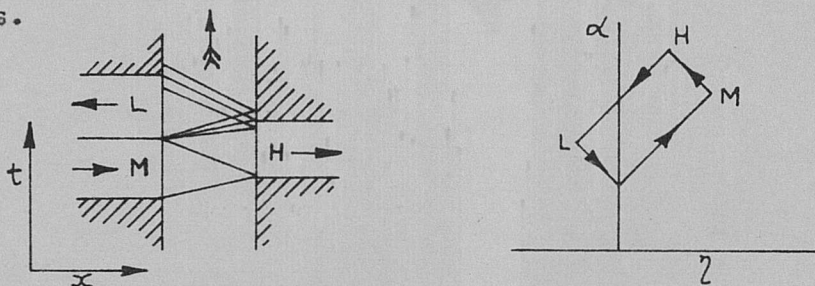
Dealing with these in order:

- (1) The mass ratio of an isentropic divider (pressure-exchanger or otherwise) can be expressed as a unique function of P_{H0}/P_{M0} and P_{L0}/P_{M0} by suitably modifying equation 15.2-8 of Appendix II to suit the isentropic case:

$$\left(\frac{\beta^*}{1 - \beta^*} \right) = \frac{\left[\left(\frac{P_{H0}}{P_{M0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{\left[1 - \left(\frac{P_{L0}}{P_{M0}} \right)^{\frac{\gamma-1}{\gamma}} \right]}$$

Evaluation of 15.3-1 allowed curves of constant β^* to be drawn on the $P_{H0}/P_{M0} - P_{L0}/P_{M0}$ plane.

(11) If the wave processes in a pressure-exchanger are wholly isentropic it is in principle possible (15) to mark the state points on the corners of rectangles on $\alpha - \eta$, or dimensionless $\alpha - \eta$ diagrams. This procedure is well known and will not be further elaborated; restrictions are discussed in 4.2.1. Sketching the $t-x$ and $\alpha - \eta$ diagrams.



From the geometry of the $\alpha - \eta$ diagram;

$$\alpha_H - \alpha_M = \left(\frac{\gamma-1}{2}\right) [\eta_M - \eta_H] \quad 15.3-2$$

$$\alpha_H - \alpha_L = \left(\frac{\gamma-1}{2}\right) [\eta_H - \eta_L] \quad 15.3-3$$

$$\eta_L = \eta_H - \eta_M \quad 15.3-4$$

It was found convenient to expedite the calculations by specifying the sonic velocity at static conditions in the low pressure port as the reference state, hence:

$$\alpha_L = 1 \quad 15.3-5$$

$$\eta_L = M_L \quad 15.3-6$$

$$\alpha_H^{(2\gamma/\gamma-1)} = \frac{P_H}{P_L} \quad 15.3-7$$

$$\alpha_M^{(2\gamma/\gamma-1)} = \frac{P_M}{P_L} \quad 15.3-8$$

The following equations were obtained by invoking the well-known relationship between static and stagnation pressure and Mach number for steady isentropic flow:

$$\frac{P_{H0}}{P_{M0}} = \frac{P_H}{P_M} \left[\frac{1 + \frac{\gamma-1}{2} M_H^2}{1 + \frac{\gamma-1}{2} M_M^2} \right]^{\frac{\gamma}{\gamma-1}} \quad 15.3-9$$

$$\frac{P_{L0}}{P_{M0}} = \frac{P_L}{P_M} \left[\frac{1 + \frac{\gamma-1}{2} M_L^2}{1 + \frac{\gamma-1}{2} M_M^2} \right]^{\frac{\gamma}{\gamma-1}} \quad 15.3-10$$

Appropriate use of equations 15.3-2 to 15.3-10 inclusive allowed curves of constant Mach number to be drawn on the $\frac{P_{H0}}{P_{M0}} - \frac{P_{L0}}{P_{M0}}$ plane.

(111) The flow Mach numbers in the outlet ports cannot exceed unity because the incremental expansion waves which would make the flow supersonic cannot advance into the cells of the pressure-exchanger. To test if there was a medium pressure port Mach number corresponding to a maximum attainable $\frac{P_{H0}}{P_{M0}}$, equation 15.3-9 was differentiated w.r.t. M_M and equated to zero. An expression involving M_M and M_H was first substituted for $\frac{P_H}{P_M}$ in equation 15.3-9. The substituted expression was derived, after some algebra, from equations 15.3-2 to 15.3-8 inclusive. The modified form of 15.3-9 became:

$$\frac{P_{H0}}{P_{M0}} = \left[\frac{M_M + \frac{2}{\gamma-1}}{M_H + \frac{2}{\gamma-1}} \right]^{\frac{2\gamma}{\gamma-1}} \left[\frac{1 + \frac{\gamma-1}{2} M_H^2}{1 + \frac{\gamma-1}{2} M_M^2} \right]^{\frac{\gamma}{\gamma-1}} \quad 15.3-11$$

differentiating 15.3-11 logarithmically and equating to zero:

$$\frac{P_{M0}}{P_{H0}} \frac{d\left(\frac{P_{H0}}{P_{M0}}\right)}{dM_M} = \frac{2\gamma}{(\gamma-1)\left[M_M + \frac{2}{\gamma-1}\right]} - \frac{\gamma M_M}{\left[1 + \left(\frac{\gamma-1}{2}\right) M_M^2\right]} = 0 \quad 15.3-12$$

hence from 15.3-12:

$$M_M = 1 \quad 15.3-13$$

Clearly this must be a maximum, hence no greater value of $\frac{P_{H0}}{P_{M0}}$ can be obtained for a specified M_H than that corresponding to $M_M = 1$

The pressure-exchanger divider isentropic performance characteristics are presented on Fig.12.1-1.

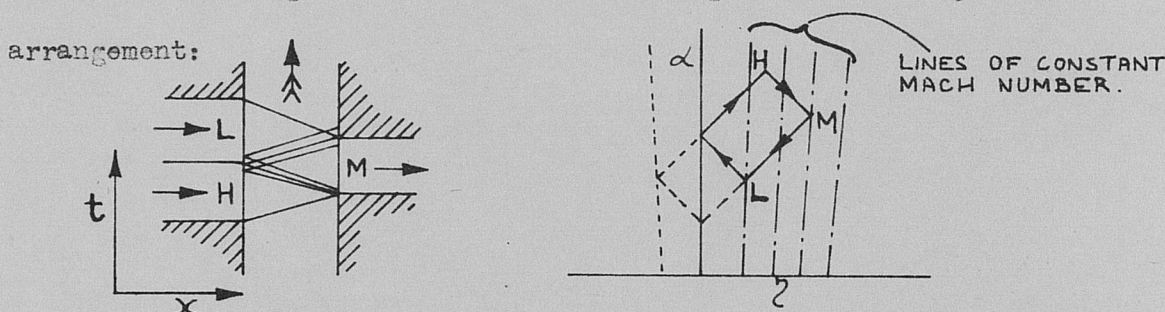
(b) Pressure-Exchanger Equaliser

Following a generally similar procedure to that used in section (a), and sub-dividing the study into three parts.

(1) The enthalpy ratio of an isentropic machine is given by rearranging equation 15.2-22 of Appendix II to suit the isentropic case, namely:

$$\zeta^* = \frac{\left[1 - \left(\frac{P_{Lo}}{P_{Ho}} \right)^{\frac{\gamma-1}{\gamma}} \right]}{\left(\left(\frac{P_{Mo}}{P_{Lo}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right)} \quad 15.3-14$$

(11) Constructing the $t-\chi$ and $\alpha-\eta$ diagrams for the equaliser arrangement:



From the geometry of the dotted extensions of the $\alpha-\eta$ diagram it can be seen that, apart from the sign of the LP Mach number, the Mach number-stagnation pressure ratio relationships are the same for equalisers and dividers. Separate calculations were however carried out for the equaliser case the results being used to check the divider cycle calculations previously described. From the geometry of the equaliser $\alpha-\eta$ diagram:

$$\alpha_M - \alpha_L = \left(\frac{\gamma-1}{2} \right) [\eta_M - \eta_L] \quad 15.3-15$$

$$\alpha_H - \alpha_M = \left(\frac{\gamma-1}{2} \right) [\eta_M - \eta_H] \quad 15.3-16$$

$$\eta_H = \eta_M - \eta_L \quad 15.3-17$$

It was found convenient to specify the velocity of sound at static conditions in the medium pressure port as the reference state, hence:

$$\alpha_M = 1 \quad 15.3-18$$

$$\gamma_M = M_M \quad 15.3-19$$

$$\alpha_L^{\frac{\gamma}{\gamma-1}} = \frac{P_L}{P_M} \quad 15.3-20$$

$$\alpha_H^{\frac{\gamma}{\gamma-1}} = \frac{P_H}{P_M} \quad 15.3-21$$

Using equations 15.3-15 to 15.3-20 inclusive and also equations 15.3-9 and 15.3.10, curves of constant Mach number were constructed on the $P_{L0}/P_{M0} - P_{H0}/P_{M0}$ plane.

(111) The isentropic performance of an equaliser is essentially limited by the restriction to sonic or lower gas velocities in the medium pressure outlet ports. The reasons for this are the same as those given in (a) (111) of this Appendix. With M_M limited to a maximum value of unity, M_L and M_H are automatically restricted to the same upper limit by the geometry of the $\alpha - \gamma$ diagram.

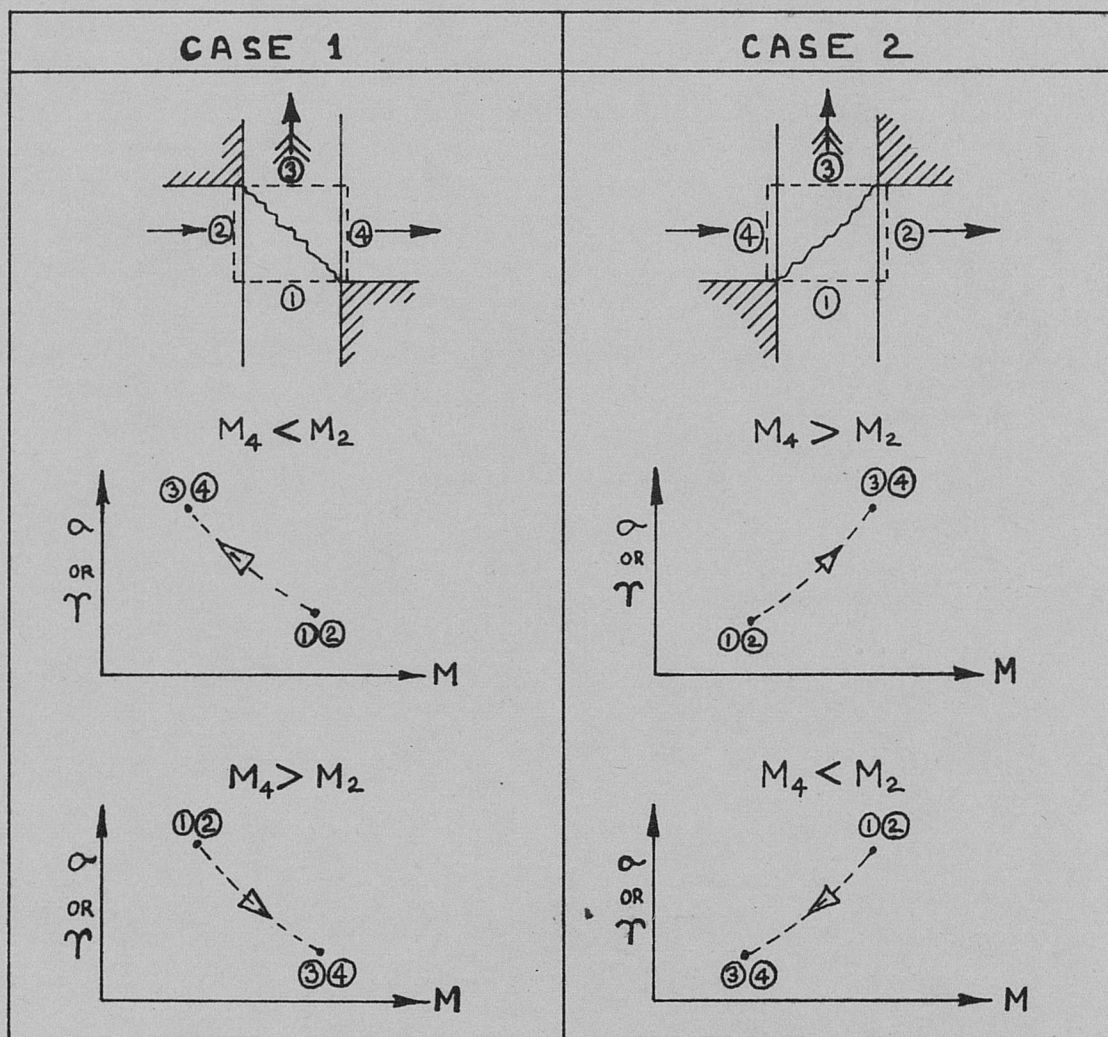
The performance characteristics of an isentropic pressure-exchanger equaliser are shown on Fig.12.1-10.

15.4 APPENDIX IVSIMPLIFIED THEORETICAL TREATMENT

The purpose of this Appendix is to explain the derivations of equations 4.3.1 to 4.3-5 inclusive.

The Generalised Energy Equation (Equation 4.3-1)

Following the pattern set in section 4.3, the energy equation will be deduced for the regions lying within the boundaries shown in the upper sketches. The lower diagrams illustrate the forms of pressure ratio and temperature ratio changes across possible waves:



The derivation offered here is in a form directly applicable to Case 1.

Equation 15.4-1 may be obtained from application of the 1st law of Thermodynamics to individual cells passing through the bounded region shown on the sketch, and multiplying both sides of the resultant expression by the number of cells passing a fixed point in unit time. Alternatively, the same result is obtained by writing an energy equation for the system, bearing in mind that fluid at states (1) and (3) is carried across the boundaries in a rigid assembly of cells of constant volume, hence, cancelling the whirl component of kinetic energy throughout:

$$\dot{q} + C_v \dot{m}_1 T_1 + \dot{m}_1 \frac{u_1^2}{2} + C_p \dot{m}_2 T_2 + \dot{m}_2 \frac{u_2^2}{2} = C_p \dot{m}_4 u_4 + \dot{m}_4 \frac{u_4^2}{2} + C_v \dot{m}_3 T_3 + \dot{m}_3 \frac{u_3^2}{2} \quad 15.4-1$$

Where $\dot{q}$ is the rate of heat transfer from the pressure-exchanger structure into the region enclosed by the boundary.

From the assumption given in section 4.3

$$\left. \begin{array}{l} P_1 = P_2, \quad u_1 = u_2, \quad T_1 = T_2 \\ P_3 = P_4, \quad u_3 = u_4, \quad T_3 = T_4 \end{array} \right\} \quad 15.4-2$$

$$\text{Continuity:} \quad \dot{m}_1 + \dot{m}_2 = \dot{m}_3 + \dot{m}_4 \quad 15.4-3$$

Dividing 15.4-1 by $C_v \dot{m}_1 T_1$, invoking the continuity equation 15.4-3, and making use of the assumptions 15.4-2, the following result is obtained after some algebra:

$$\frac{(\gamma-1) \left[1 - \frac{\dot{m}_3 T_4}{\dot{m}_1 T_2} \right] - \dot{Q}}{\gamma \left[1 + \frac{u_2^2}{2\gamma C_v T_2} - \frac{T_4}{T_2} - \frac{u_4^2}{2\gamma C_v T_2} \right]} = \left[1 + \frac{\dot{m}_2}{\dot{m}_1} \right] \quad 15.4-4$$

$$\text{Where:} \quad \dot{Q} \equiv \frac{\dot{q}}{C_v \dot{m}_1 T_1}$$

Noting that, to be consistent with assumption (e) of section 4.3, the pressure wave has been considered to be infinitely thin and hence; $A_2 = A_4$.

Also invoking the well-known steady flow expression for mass flow allows the following equations to be written:

$$\frac{\dot{m}_4}{\dot{m}_2} = \frac{\rho_4 u_4 A_4}{\rho_2 u_2 A_2} = \frac{P_4 T_2 u_4}{P_2 T_4 u_2}$$

$$\therefore \frac{\dot{m}_4}{\dot{m}_1} = \frac{\dot{m}_2}{\dot{m}_1} \left[\frac{P_4 T_2 u_4}{P_2 T_4 u_2} \right] \quad 15.4-5$$

The following equation is obtained after substituting 15.4-5 in 15.4-3, dividing by $\dot{m}_1$, and rearranging:

$$1 + \frac{\dot{m}_2}{\dot{m}_1} = \frac{\left[\frac{\dot{m}_2}{\dot{m}_1} - 1 \right]}{\left[1 - \frac{P_4 T_2 u_4}{P_2 T_4 u_2} \right]} + 1 \quad 15.4-6$$

Because the volume of each cell remains constant.

$$\frac{\dot{m}_3}{\dot{m}_1} = \frac{P_3 T_1}{P_1 T_3} \quad 15.4-7$$

It may also be shown that:

$$\frac{u_2^2}{2 C_v T_2} = \gamma \left(\frac{\gamma-1}{2} \right) M_2^2 \quad 15.4-8$$

$$\frac{u_4^2}{2 C_v T_2} = \frac{T_4}{T_2} \gamma \left(\frac{\gamma-1}{2} \right) M_4^2 \quad 15.4-9$$

Making substitutions from 15.4-6 to 15.4-9 inclusive in equation 15.4-4, the following result is obtained:

$$\frac{(\gamma-1) \left[1 - \frac{P_4}{P_2} \right] - \dot{Q}}{\gamma \left[\left(1 + \frac{\gamma-1}{2} M_2^2 \right) - \left(1 + \frac{\gamma-1}{2} M_4^2 \right) \frac{T_4}{T_2} \right]} = \frac{\left[\frac{P_4 T_2}{P_2 T_4} - 1 \right]}{\left[1 - \frac{P_4}{P_2} \sqrt{\frac{T_2}{T_4}} \frac{M_4}{M_2} \right]} + 1$$

This is equation 4.3-1; equations 4.3-2 and 4.3-3 follow (after some algebra) when M_2 and M_4 are respectively made equal to zero.

When $M_2=0$ equation 4.3-2 is obtained:

$$1 = \gamma \left[1 + \frac{\gamma-1}{2} M_4^2 \right] \frac{T_4}{T_2} - (\gamma-1) \frac{P_4}{P_2} - \dot{Q}$$

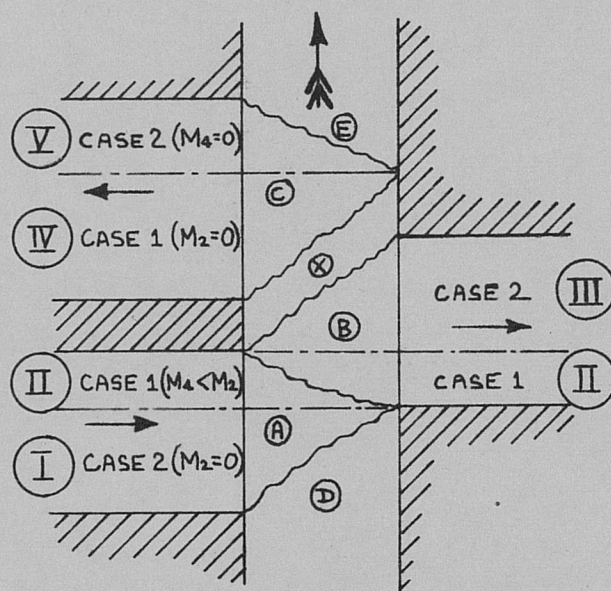
When $M_4=0$ equation 4.3-3 results:

$$\frac{P_4 T_2}{P_2 T_4} \left[\gamma \left(\frac{\gamma-1}{2} \right) M_2^2 + \gamma - \frac{T_4}{T_2} \right] = (\gamma-1) - \dot{Q}$$

Equations 4.3-1 to 4.3-3 can also be obtained from a similar analysis applied to Case 2. The input information for evaluation of equations 4.3-1 to 4.3-3 is shown on Fig.15.4-1 to Fig.15.4-5 inclusive.

Derivation of Equation 4.3-4

Equation 4.3-4 used for evaluating β was obtained by making use of expressions established in the course of deriving equations 4.3-1 for Case 1 or Case 2. The divider cycle was accordingly split into five component parts involving only single pressure waves. These have been labelled I to V in the accompanying sketch:



As explained in the main text, an expression for β was required of the form:

$$\beta = 1 - \frac{\dot{m}_C}{\dot{m}_A}$$

or, since $\dot{m}_A = \dot{m}_I + \dot{m}_{II(IN)}$

and $\dot{m}_C = \dot{m}_{IV} + \dot{m}_V$

$$\therefore \beta = 1 - \left[\frac{\dot{m}_{IV} + \dot{m}_V}{\dot{m}_I + \dot{m}_{II(IN)}} \right]$$

Substitution in 15.4-10 of suitable expressions in terms of pressure ratio, temperature ratio and Mach number allows equation 4.3-4 to be obtained. In the interests of brevity only an outline of the main points of the procedure will be given.

Reference was made to various portions of the derivations of equation 4.3-1, based on Case 1 and Case 2, to abstract various expressions involving mass ratio. These expressions were rewritten, often after rearrangement, in the notation of the divider cycle diagram given here. This process led to the following results:

For region I: from Case 2, ($M_2 = 0$)

$$\frac{\dot{m}_I}{\dot{m}_D} = \left[\frac{P_A T_D}{P_D T_A} - 1 \right] \quad 15.4-11$$

For region II: from Case 1 ($M_4 < M_2$)

$$\frac{\dot{m}_{II(IN)}}{\dot{m}_D} = \frac{\frac{P_A T_D}{P_D T_A} \left[\frac{P_B T_A}{P_A T_B} - 1 \right]}{\left[1 - \frac{P_B}{P_A} \sqrt{\frac{T_A}{T_B}} \frac{M_B}{M_A} \right]} \quad 15.4-12$$

For region V: from Case 2 ($M_4 = 0$)

$$\frac{\dot{m}_V}{\dot{m}_E} = \left[\frac{P_C T_E}{P_E T_C} - 1 \right] \quad 15.4-13$$

For region IV: from Case 1 ($M_2 = 0$)

$$\frac{\dot{m}_{IV}}{\dot{m}_E} = \frac{P_C T_E}{P_E T_C} \left[\frac{P_X T_C}{P_C T_X} - 1 \right] \quad 15.4-14$$

Because $P_D = P_E$, and the volume of each cell remains constant

$$\frac{\dot{m}_E}{\dot{m}_D} = \frac{T_D}{T_E} \quad 15.4-15$$

Hence from substituting equations 15.4-11 to 15.4-16 inclusive in 15.4-10, the following result is obtained:

$$\beta = 1 - \frac{T_D}{T_E} \left[\frac{\frac{P_X T_E}{P_E T_X} - 1}{\frac{P_B T_D}{P_D T_B} \left\{ \frac{1 - \sqrt{\frac{T_B}{T_A}} \frac{M_B}{M_A}}{1 - \frac{P_B}{P_A} \sqrt{\frac{T_A}{T_B}} \frac{M_B}{M_A}} \right\} - 1} \right]$$

This is labelled equation 4.3-4 in the main text.

Derivation of Equation 4.3-5

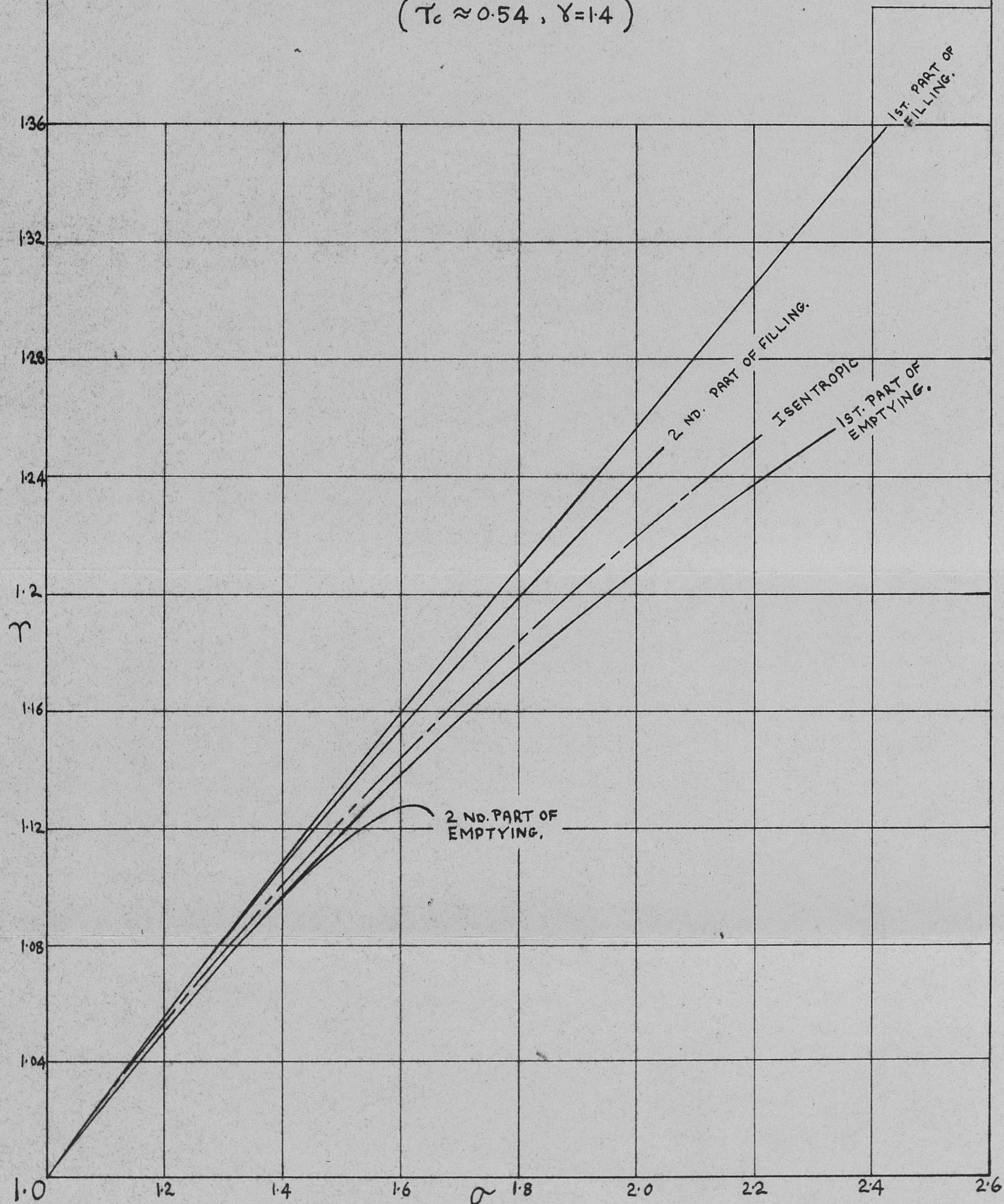
The expression for ζ was obtained by a similar procedure to that given in the outline of the derivation of 4.3-4. Using the notation shown on the diagram in section 4.3.2, the following result was obtained:

$$\zeta = \frac{\dot{m}_L T_{L0}}{\dot{m}_H T_{H0}} = \frac{T_{L0} \cdot T_D}{T_{A0} \cdot T_E} \left[\frac{1 - \frac{P_X T_E}{P_E T_X}}{\frac{P_B T_D}{P_D T_B} \left\{ \frac{1 - \sqrt{\frac{T_B}{T_A}} \frac{M_B}{M_A}}{1 - \frac{P_B}{P_A} \sqrt{\frac{T_A}{T_B}} \frac{M_B}{M_A}} \right\} - 1} \right]$$

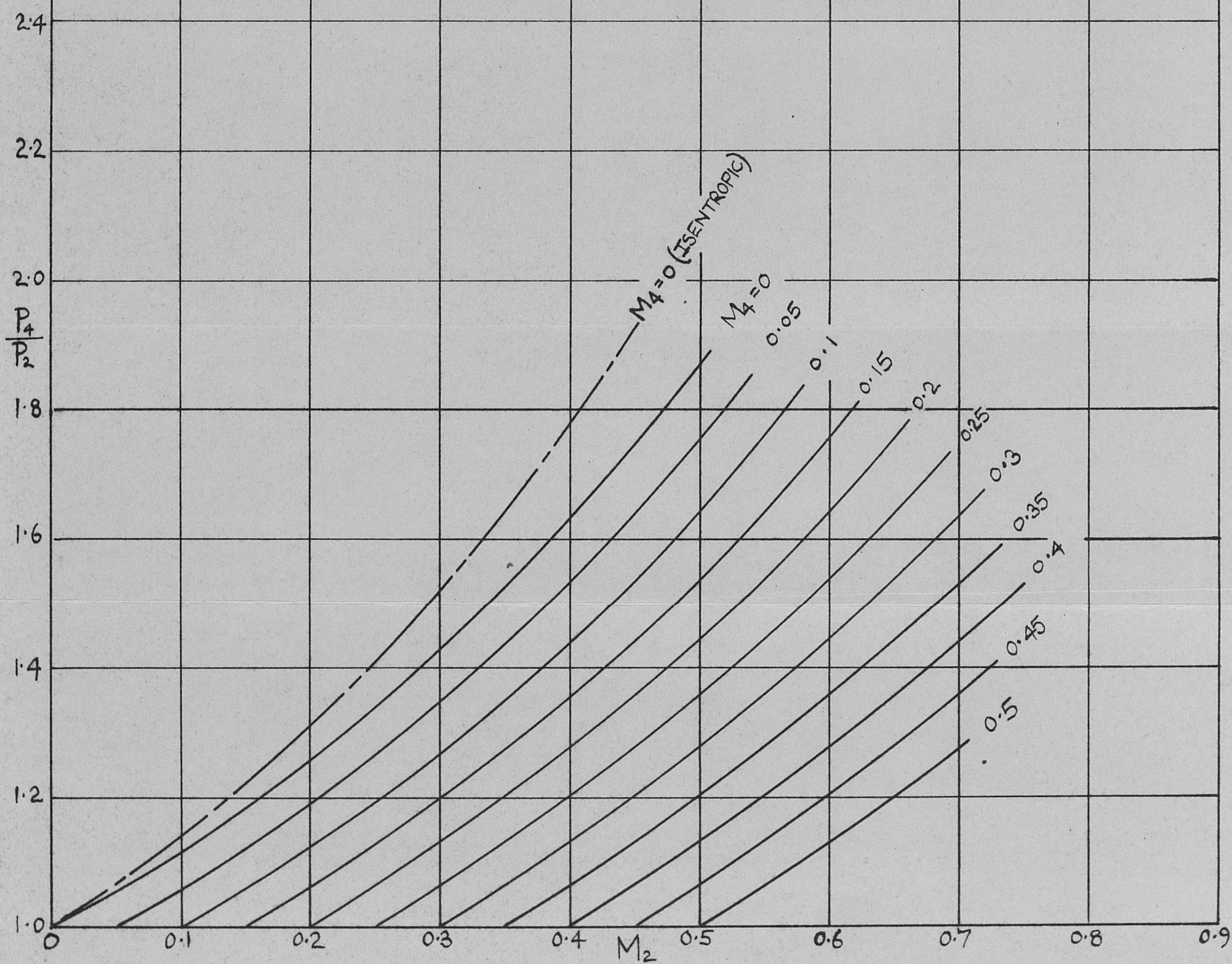
FIG. 15-4-1

EXPERIMENTALLY OBTAINED STATIC TEMPERATURE RATIO (τ)
VERSUS STATIC PRESSURE RATIO (σ) RELATIONSHIP

$$(\gamma_c \approx 0.54, \gamma = 1.4)$$

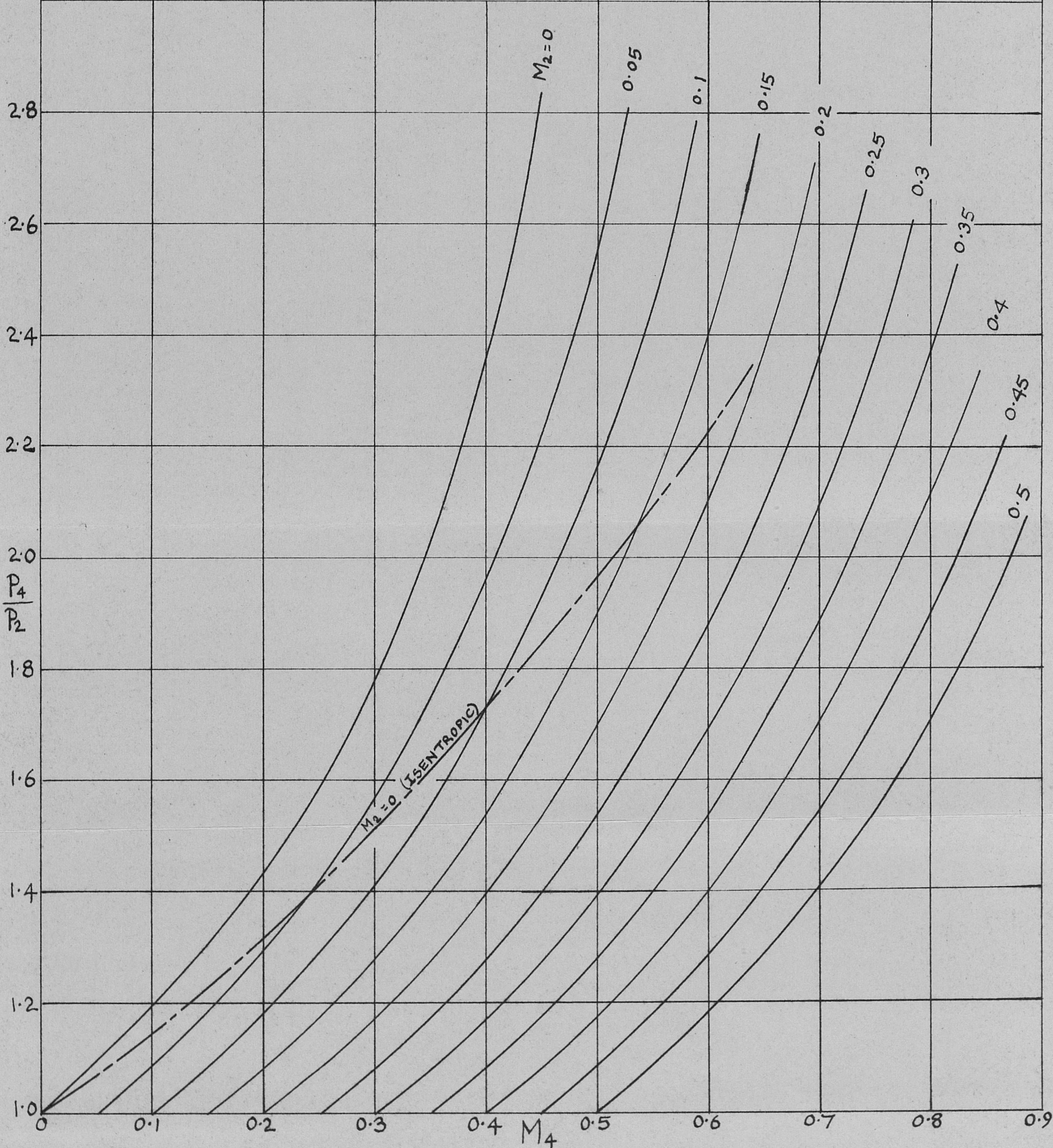


SIMPLIFIED THEORETICAL TREATMENT
 P_4/P_2 VERSUS M_2 FOR CASE 1, 2ND PART OF FILLING,
 $\gamma_c \approx 0.54$, $\gamma = 1.4$.



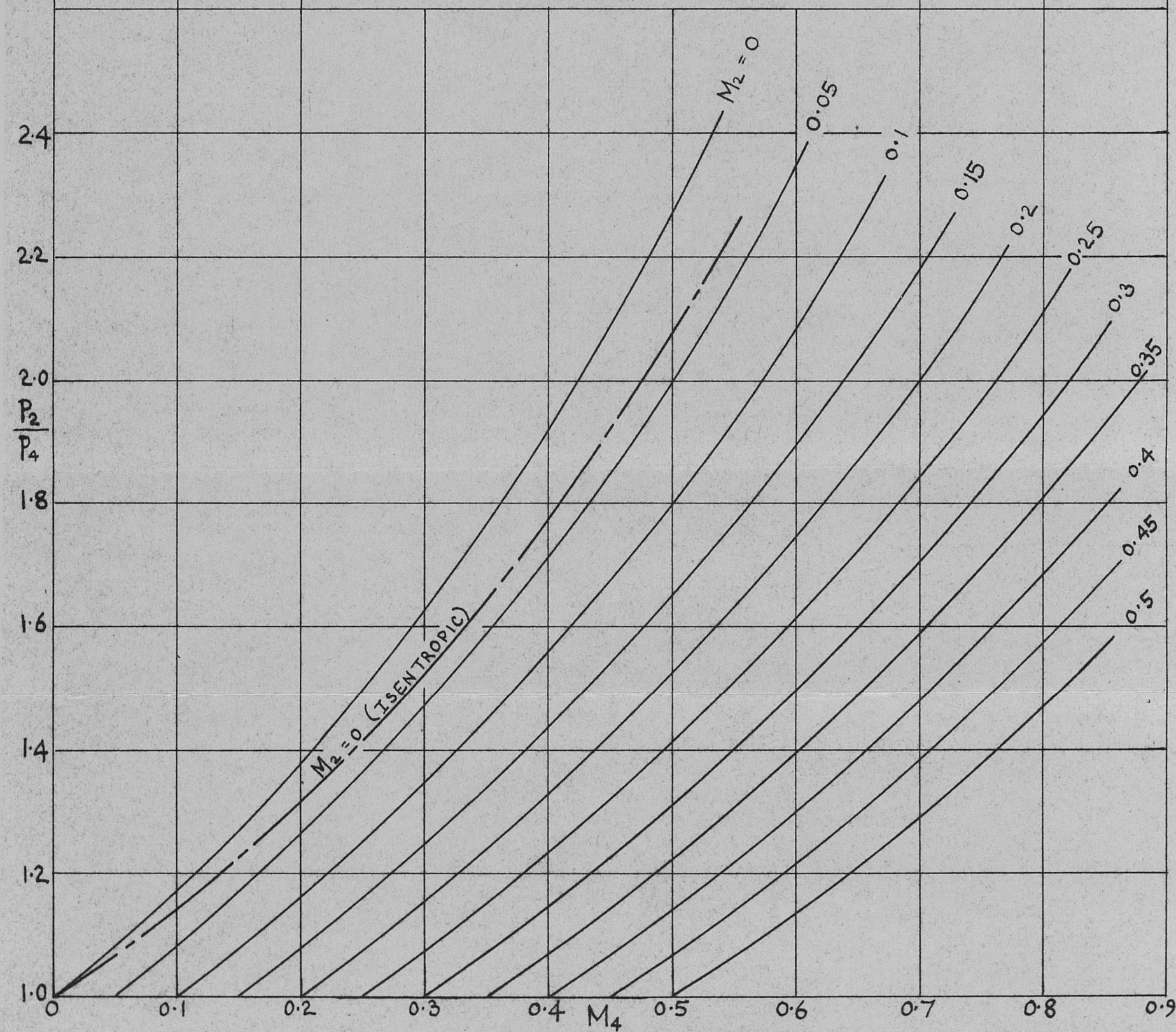
SIMPLIFIED THEORETICAL TREATMENT

P_4/P_2 VERSUS M_4 FOR CASE 2, 1ST PART OF FILLING,
 $\gamma_c \approx 0.54$, $\gamma = 1.4$.



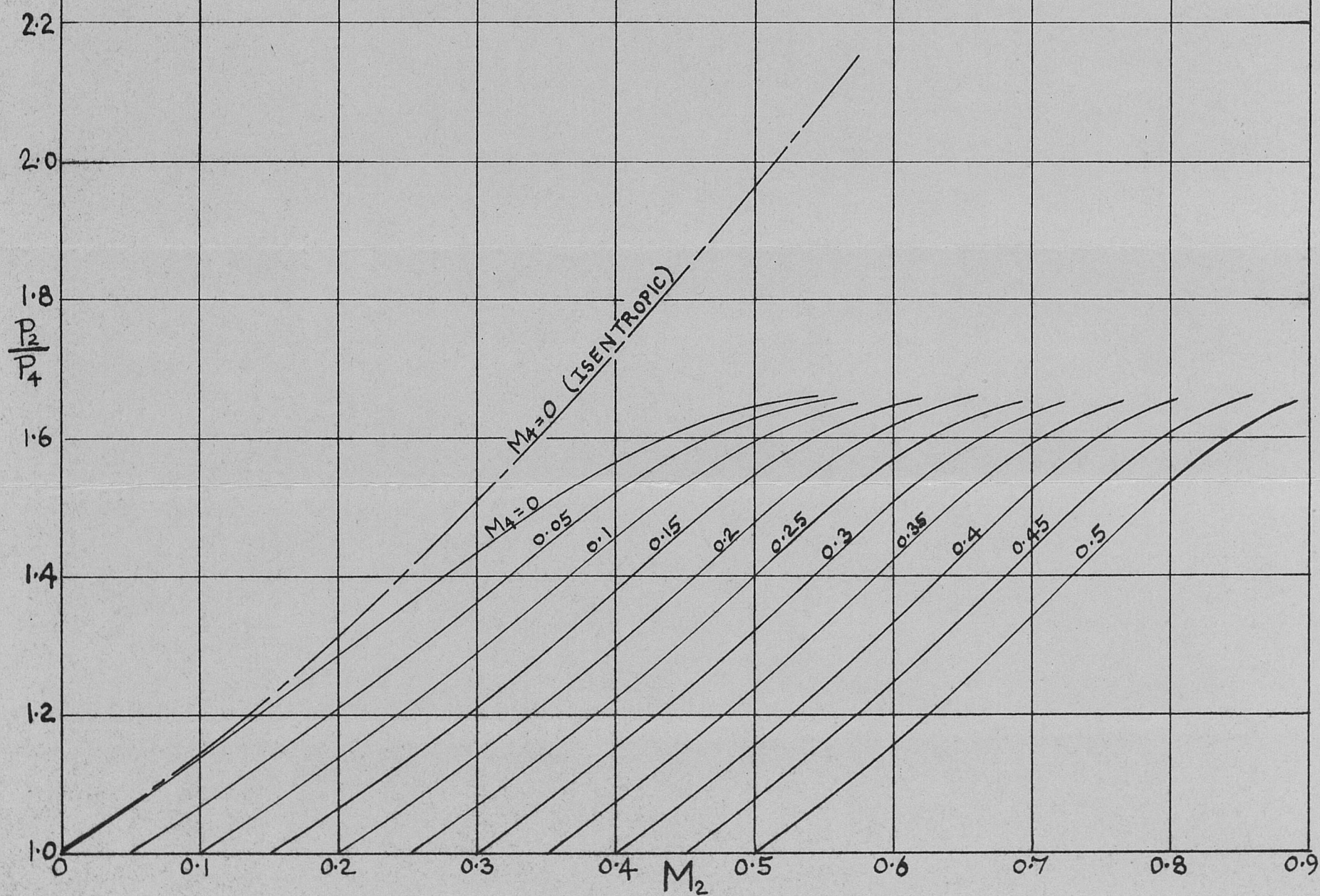
SIMPLIFIED THEORETICAL TREATMENT

P_2/P_4 VERSUS M_4 FOR CASE 1, 1ST PART OF EMPTYING,
 $\gamma_c \approx 0.54$, $\gamma = 1.4$.



SIMPLIFIED THEORETICAL TREATMENT

P_2/P_4 VERSUS M_2 FOR CASE 2, 2ND PART OF EMPTYING,
 $\gamma_c \approx 0.54$, $\gamma = 1.4$.



15.5 APPENDIX V

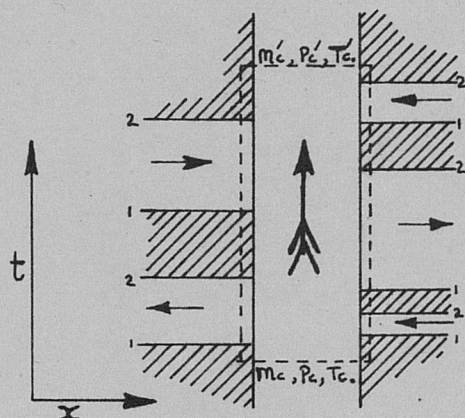
THE ADAPTATION OF THE METHOD-OF-CHARACTERISTICS
TO THE STUDY OF DIVIDER & EQUALISER CYCLES

The purpose of this Appendix is to provide derivations of the equations stated without proof in section 4.4. The Appendix is divided into four parts, each dealing with a specific aspect of the adaptation and extension of the method-of-characteristics.

I. Characteristic Calculations:

Object: to provide derivations of equations 4.4-1 to 4.4-4 inclusive, and to list Jenny's boundary equations.

The diagram shows an array of ports in the endplates of a pressure-exchanger. The port arrangement illustrated is arbitrary:



Subscripts (1) and (2) refer to the times an individual cell communicates with and is cut-off from a particular port. Hence application of the 1st Law to a cell passing through the bounded region, in the direction shown, leads to the following result:

$$\left[\sum_1^2 \dot{m} C_p T_c dt \right]_{\text{INLETS}} - \left[\sum_1^2 \left(\dot{m} C_p T + \frac{\dot{m} u^2}{2} \right) dt \right]_{\text{OUTLETS}} = m'_c C_v T'_c - m_c C_v T_c$$

where $\dot{m}$ refers specifically to the mass flow passing into or out of an individual cell, and subscript z refers to the stagnation conditions in an inlet port.

Assuming C_p is the same in all ports, $C_v = C'_v$, $R = R'$, and noting that for cells of constant volume; $\frac{P'_c}{P_c} = \frac{m'_c T'_c}{m_c T_c}$ then division of 15.5-1 by the original cell internal energy, $m_c C_v T_c$, gives:

$$\left[\sum_{\text{INLETS}} \int \frac{\dot{m} T_z}{m_c T_c} dt \right] - \left[\sum_{\text{OUTLETS}} \int \left(\frac{\dot{m} T}{m_c T_c} + \frac{\dot{m} u^2}{C_p 2 m_c T_c} \right) dt \right] = \frac{1}{\gamma} \left[\frac{P'_c}{P_c} - 1 \right] \quad 15.5-2$$

The notation of equation 15.5-2 may be changed to that of the state, wave, and boundary diagrams by substitution from the following definitions, etc.

For filling and emptying processes:

$$\alpha \equiv \frac{a}{a_{\text{REF}}} \quad 15.5-3, \quad \gamma \equiv \frac{t a_{\text{REF}}}{l} \quad 15.5-4, \quad m_c = \frac{P_c A_c l}{R T_c} \quad 15.5-5$$

For filling processes only:

$$M_{oz} \equiv \frac{\dot{m}}{a_z / \beta_z A_c} \quad 15.5-6 \quad \left[= \frac{G_{\text{SEC}}}{a_z / \beta_z g f} \quad \begin{array}{l} \text{in Jenny's} \\ \text{notation} \end{array} \right]$$

For emptying processes only:

$$M_s^+ \equiv \frac{\dot{m}}{a_{os} / \beta_{os} A_c} \quad 15.5-7 \quad \left[= \frac{G_{\text{SEC}}}{a_{os} / \beta_{os} g f} \quad \begin{array}{l} \text{in Jenny's} \\ \text{notation} \end{array} \right]$$

Bearing in mind that, provided choking does not occur, the actual flow Mach No. of fluid leaving a cell during opening or closing is M_s^+ / ϕ , substitution of equations 15.5-3 to 15.5-7 in 15.5-2 leads to the following result:

$$\left[\sum_{\text{INLETS}} \int M_{oz}^2 \alpha_z \left(\frac{P_z}{P_c} \right) d\gamma \right] - \left[\sum_{\text{OUTLETS}} \int M_s^2 \alpha \left(\frac{P_s}{P_c} \right) \left[1 + \frac{\gamma-1}{2} \left(\frac{M_s^+}{\phi} \right)^2 \right] d\gamma \right] = \frac{1}{\gamma} \left[\frac{P'_c}{P_c} - 1 \right] \quad 15.5-8$$

Setting $\frac{P_c'}{P_c} = 1$, and restating 15.5-8 in the specific

terms of a divider cycle gives:

$$\int_1^2 M_{OZM} \alpha_{ZM} \left(\frac{P_{ZM}}{P_c} \right) d\tau - \int_1^2 M_{SL}^+ \alpha_L \left(\frac{P_L}{P_c} \right) \left[1 + \frac{\gamma-1}{2} \left(\frac{M_{SL}^+}{\phi} \right)^2 \right] d\tau - \int_1^2 M_{SH}^+ \alpha_H \left(\frac{P_H}{P_c} \right) \left[1 + \frac{\gamma-1}{2} \left(\frac{M_{SH}^+}{\phi} \right)^2 \right] d\tau = 0$$

15.5-9

For equalisers:

$$\int_1^2 M_{OZH} \alpha_{ZH} \left(\frac{P_{ZH}}{P_c} \right) d\tau + \int_1^2 M_{OZL} \alpha_{ZL} \left(\frac{P_{ZL}}{P_c} \right) d\tau - \int_1^2 M_{SM}^+ \alpha_M \left(\frac{P_M}{P_c} \right) \left[1 + \frac{\gamma-1}{2} \left(\frac{M_{SM}^+}{\phi} \right)^2 \right] d\tau = 0$$

15.5-10

The results 15.5-9 and 15.5-10 are those labelled in section 4.4.3 as equations 4.4-1 and 4.4-2 respectively.

The mass ratio β for the divider cycle was evaluated, for reasons given in the main text, from the relation:

$$\beta = \frac{\int_1^2 \dot{m}_M dt - \int_1^2 \dot{m}_L dt}{\int_1^2 \dot{m}_M dt} \quad \text{dividing}$$

numerator and denominator by $\dot{m}_c$ gives:

$$\beta = 1 - \frac{\int_1^2 \frac{\dot{m}_L}{\dot{m}_c} dt}{\int_1^2 \frac{\dot{m}_M}{\dot{m}_c} dt}$$

15.5-11

By use of equations 15.5-3 and 15.5-7 and the isentropic relations, equation 15.5-11 may be transformed into the following result:

$$\beta = 1 - \frac{\int_1^2 M_{SL}^+ \alpha_L \left(\frac{P_L}{P_c} \right)^{1/8} d\tau}{\int_1^2 M_{OZ} \alpha_{ZM} \left(\frac{P_{ZM}}{P_c} \right)^{1/8} d\tau}$$

15.5-12

Equation 15.5-12 is quoted in section 4.4 as equation 4.4-3.

The enthalpy ratio for equalisers is defined in integral form by:

$$\xi = \frac{\int_1^2 \dot{m}_L C_{PL} T_{ZL} dt}{\int_1^2 \dot{m}_H C_{PH} T_{ZH} dt}$$

15.5-13

On the assumption that $C_{PL} = C_{PH}$, the same substitutions as for the divider case lead to the result:

$$\zeta = \frac{\int_1^2 M_{0ZL} \alpha_{ZL} \left(\frac{P_{ZL}}{P_c} \right) d\gamma}{\int_1^2 M_{0ZH} \alpha_{ZH} \left(\frac{P_{ZH}}{P_c} \right) d\gamma} \quad 15.5-14$$

This result appears as equation 4.4-4 in the main text.

Simplified forms of equations 15.5-9, 15.5-10, 15.5-12 and 15.5-14 were devised for calculations involving infinitely narrow cells. The expressions were derived in a similar way to those for finite width cells, except that equations 15.5-6 and 15.5-7 were replaced by the definition $\gamma \equiv \frac{u}{a_{REF}}$, and use was made of isentropic relations.

The thin cell results for dividers are:

$$\int_1^2 \gamma_M^2 \frac{P_M}{P_c} \left[1 + \frac{\gamma-1}{2} \left(\frac{\gamma_M}{\alpha_M} \right)^2 \right] d\gamma - \int_1^2 \gamma_L^2 \frac{P_L}{P_c} \left[1 + \frac{\gamma-1}{2} \left(\frac{\gamma_L}{\alpha_L} \right)^2 \right] d\gamma - \int_1^2 \gamma_H^2 \frac{P_H}{P_c} \left[1 + \frac{\gamma-1}{2} \left(\frac{\gamma_H}{\alpha_H} \right)^2 \right] d\gamma = 0 \quad 15.5-15$$

$$\& \quad \beta = 1 - \frac{\int_1^2 \gamma_L^2 \left(\frac{P_L}{P_c} \right)^{1/2} d\gamma}{\int_1^2 \gamma_M^2 \left(\frac{P_M}{P_c} \right)^{1/2} d\gamma} \quad 15.5-16$$

For equalisers:

$$\int_1^2 \gamma_H^2 \frac{P_H}{P_c} \left[1 + \frac{\gamma-1}{2} \left(\frac{\gamma_H}{\alpha_H} \right)^2 \right] d\gamma + \int_1^2 \gamma_L^2 \frac{P_L}{P_c} \left[1 + \frac{\gamma-1}{2} \left(\frac{\gamma_L}{\alpha_L} \right)^2 \right] d\gamma - \int_1^2 \gamma_M^2 \frac{P_M}{P_c} \left[1 + \frac{\gamma-1}{2} \left(\frac{\gamma_M}{\alpha_M} \right)^2 \right] d\gamma = 0 \quad 15.5-17$$

$$\& \quad \zeta = \frac{\int_1^2 \gamma_L^2 \frac{P_L}{P_c} \left[1 + \frac{\gamma-1}{2} \left(\frac{\gamma_L}{\alpha_L} \right)^2 \right] d\gamma}{\int_1^2 \gamma_H^2 \frac{P_H}{P_c} \left[1 + \frac{\gamma-1}{2} \left(\frac{\gamma_H}{\alpha_H} \right)^2 \right] d\gamma} \quad 15.5-18$$

Equations 15.5-15 to 15.5-18 replace the finite cell width equations 15.5-9, 15.5-12, 15.5-10 and 15.5-14 respectively.

Jenny's boundary curve equations are, for the sake of completeness, listed here. Differences between the notation used in this work and Jenny's notation are signified.

(NB. SYMBOLS α , u & P WITHOUT SUBSCRIPTS REFER TO CONDITIONS IN THE CELLS, CLOSE TO THE PLANE OF THE PORT.)

Cell filling, constant ϕ curves:

Sub-critical pressure ratios:

$$\frac{u/a_z}{1 - \frac{\gamma-1}{2} (u/a_z)^2} = \phi \left(\frac{P}{P_z} \right)^{-\frac{(\gamma-1)}{2\gamma}} \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{P}{P_z} \right)^{-\frac{(\gamma-1)}{\gamma}} - 1 \right]}$$
15.5-19

Super-critical pressure ratios:

$$\frac{P}{P_z} = \phi \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{a_z}{u} \left[1 - \frac{\gamma-1}{2} \left(\frac{u}{a_z} \right)^2 \right]$$
15.5-20

Cell filling, mass flow characteristics:

Sub-critical pressure ratios:

$$M_{0z} = \left(\frac{P}{P_z} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \phi \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{P}{P_z} \right)^{-\frac{(\gamma-1)}{\gamma}} - 1 \right]}$$
15.5-21

Super-critical pressure ratios:

$$M_{0z} = \phi \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$
15.5-22

M_{0z} replaces
Jenny's;
 $\frac{G_{SEC}}{a_z/\rho_z g f}$

Cell emptying, constant ϕ curves: (NB. Jenny's symbol P_0 is replaced by P_s .)

Sub-critical pressure ratios:

$$\left(\frac{u}{a_{os}} \right)^2 = \frac{2}{\gamma-1} \left[\left(\frac{P}{P_s} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] / \left[\frac{1}{\phi^2} \left(\frac{P}{P_s} \right)^{\frac{2}{\gamma}} - 1 \right]$$
15.5-23

Super-critical pressure ratios:

$$\frac{u}{a} = \frac{u_{CHOKING}}{a_{CHOKING}} = \text{CONSTANT}$$
15.5-24

Cell emptying, mass flow characteristics:

Sub-critical pressure ratios:

$$M_S^+ = \phi \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{a}{a_{os}} \right)^2 - 1 \right] + \left(\frac{u}{a_{os}} \right)^2}$$
15.5-25

Super-critical pressure ratios:

$$M_S^+ = \phi \left[\frac{a}{a_{os}} / \left(\frac{a}{a_{os}} \right)_{CHOKING} \right]^{\frac{\gamma+1}{\gamma-1}}$$
15.5-26

M_S^+ replaces
Jenny's;
 $\frac{G_{SEC}}{a_{os}/\rho_{os} g f}$

Equations 15.5-19 to 15.5-26 were those used to establish Jenny's

Fig.23. Some of the expressions are quoted in reference (9); in that

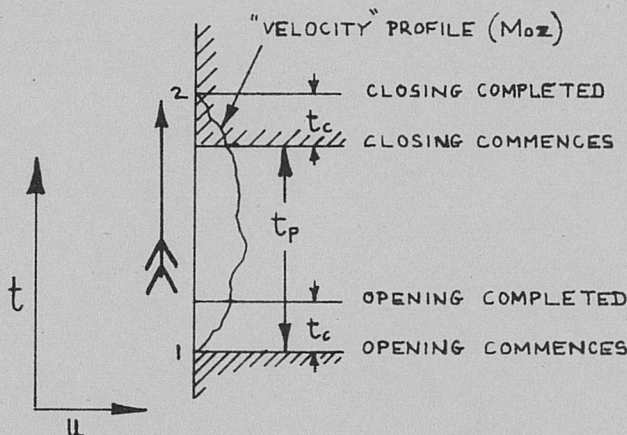
reference Fig.23 of Jenny's thesis appears as Fig.3.

II. Correlation Between Cell & Duct Flows

Object: to form isentropic relations between flow conditions in the cell ends and the assumed steady flow in the ducts of the machine. Two analyses are given, (a) for filling, the other (b) for emptying processes:

(a) Filling Processes:

The diagram shows an inlet port serving cells of finite width. The times at which a cell first communicates with, and is cut-off from, the port are given the subscripts (1) and (2) respectively.



From the definition of $M_{0z} \left[\equiv \frac{\dot{m}}{a_z / z A_c} \right]$, where $\dot{m}$ is the mass flow into a cell], the total mass entering a cell during a filling process is given by:

$$m = \int_1^2 \dot{m} dt = a_z / z A_c \int_1^2 M_{0z} dt \quad 15.5-27$$

If $\dot{N} \left[= \frac{1}{t_c} \right]$ cells pass a fixed point in unit time, then the duct mass flow $\dot{m}_{\text{DUCT}}$ is, after invoking 15.5-27, given by:

$$\dot{m}_{\text{DUCT}} = \bar{\rho} \bar{u} A_p = \frac{a_z / z A_c}{t_c} \int_1^2 M_{0z} dt \quad 15.5-28$$

Taking the thickness of the cell partitions into account:

$$K A_p = A_c \frac{t_p}{t_c} \quad 15.5-29$$

From the energy equation:

$$\frac{T_z}{T} = \left\{ 1 + \left(\frac{\gamma-1}{2} \right) \bar{M}^2 \right\} \quad 15.5-30$$

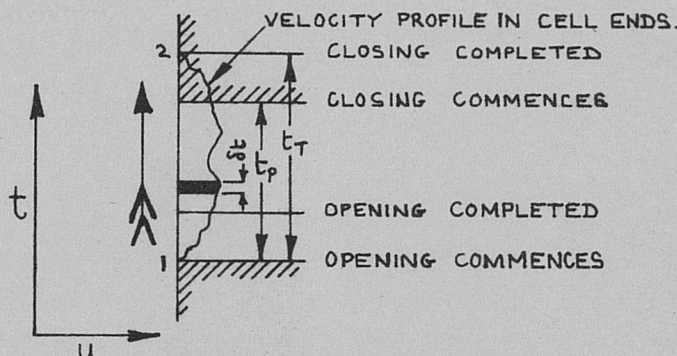
Dividing 15.5-28 by $\bar{A}$, rearranging, substituting from 15.5-29 and 15.5-30 and expressing time in the usual dimensionless units, allows the following result to be obtained:

$$\bar{M} = K \left\{ 1 + \left(\frac{\gamma-1}{2} \right) \bar{M}^2 \right\}^{\frac{\gamma+1}{2(\gamma-1)}} \frac{\int_1^2 \dot{M}_{oz} d\tau}{\tau_p} \quad 15.5-31$$

15.5-31 is the result given as equation 4.4-11 in the main text.

(b) Emptying Processes:

The illustration shows an outlet port; as for the previous case, the time limits are marked (1) and (2).



Mass removed from one cell traversing the port is:

$$m = \int_1^2 \rho u A_c dt \quad 15.5-32$$

Multiplication of 15.5-32 by the number of cells passing a fixed point in unit time gives the duct mass flow $\dot{m}_{\text{DUCT}}$, i.e.

$$\dot{m}_{\text{DUCT}} = \bar{\rho} \bar{u} A_p = \frac{A_c}{t_c} \int_1^2 \rho u dt = \frac{A_c}{t_c} \rho_{AV} u_{AV} t_T \quad 15.5-33$$

From the 2nd and 4th expressions of equation 15.5-33, and substituting for A_p from 15.5-29, the following result is obtained:

$$\bar{u} = K \left(\frac{\rho_{AV}}{\bar{\rho}} \right) u_{AV} \frac{t_T}{t_p} \quad 15.5-34$$

The energy equation, $C_p \bar{T} + \frac{\bar{u}^2}{2} \equiv C_p T_{AV} + \frac{u_{AV}^2}{2}$, after invoking the isentropic laws and rearranging, leads to the result:

$$\frac{\rho_{AV}}{\rho} = \left(\frac{T_{AV}}{T} \right)^{\frac{1}{\gamma-1}} = \left\{ \frac{1 + \left(\frac{\gamma-1}{2} \right) \bar{M}^2}{1 + \left(\frac{\gamma-1}{2} \right) M_{AV}^2} \right\}^{\frac{1}{\gamma-1}} \quad 15.5-35$$

Substituting in 15.5-34 for $\frac{\rho_{AV}}{\rho}$ from 15.5-35 and dividing the resultant expression by $\bar{a}$ gives:

$$\bar{M} = \frac{\bar{u}}{\bar{a}} = K \left\{ \frac{1 + \left(\frac{\gamma-1}{2} \right) \bar{M}^2}{1 + \left(\frac{\gamma-1}{2} \right) M_{AV}^2} \right\}^{\frac{1}{\gamma-1}} \frac{u_{AV}}{a_{AV}} \cdot \frac{a_{AV}}{\bar{a}} \cdot \frac{t_T}{t_P} \quad 15.5-36$$

Replacing t_T and t_P by the dimensionless counterparts leads finally to the result:

$$\bar{M} = K \left\{ \frac{1 + \left(\frac{\gamma-1}{2} \right) \bar{M}^2}{1 + \left(\frac{\gamma-1}{2} \right) M_{AV}^2} \right\}^{\frac{\gamma+1}{2(\gamma-1)}} M_{AV} \left(\frac{\gamma_T}{\gamma_P} \right) \quad 15.5-37$$

Equation 15.5-37 is the result appearing in section 4.4 as equation 4.4-12. Due to the nature of the definition of M_S^+ , 4.4-12 is almost exact if M_{AV} is obtained from integration of the M_S^+ profile, i.e:

$$M_{AV} \gamma_T = \int_1^2 M_S^+ d\gamma \quad 15.5-38$$

15.5-38 implies that ρ_{AV} is in fact taken to be equal to the density in the cell ends when these are fully exposed to the port, i.e. $\rho_{AV} = \rho_{02}$. Equation 4.4-12 is applicable up to the onset of choking.

III. Leakage Effects:

Object: to derive equations 4.4-13 and 4.4-14 quoted in section 4.4.3 and to explain the application of the generalised leakage theory to equaliser and divider cycles. The work is divided into four sub-sections:

- (a) The prediction of $\frac{P_c'}{P_c}$ when leakage effects are present

- (b) The effect of leakage on the mass flow in pressure-exchanger ducts
 (c) Application of the leakage theory to dividers
 (d) " " " " " equalisers.

(a) The Prediction Of $\frac{P_c'}{P_c}$ When Leakage Effects Are Present

Applying the 1st law of thermodynamics to cells passing through the region in which leakage is presumed to be concentrated (see diagrams of the leakage models for dividers and equalisers) leads to the following expression:

$$\dot{N} (m_c T_c - m'_c T'_c) C_v A_c l = C_p \sum \dot{m}_l T_l \quad 15.5-39$$

Where: $\dot{m}_l$ is leakage mass flow (+ve. into cells),

T_l is stagnation temperature of leakage flow.

Substituting in 15.5-39:

$$m_c = \frac{P_c A_c l}{R T_c}, \quad m'_c = \frac{P'_c A_c l}{R T'_c}, \quad \dot{N} = \frac{1}{t} = \frac{Q_{REF}}{T_c l}$$

and dividing the result by $P_c A_c l C_{p/R}$ gives:

$$\left[1 - \frac{P'_c}{P_c} \right] = \frac{R T_c \gamma}{P_c A_c Q_{REF}} \sum \dot{m}_l T_l \quad 15.5-40$$

Kearton & Keh (10) showed that for a "simple" leak up to and including choking:

$$\dot{m}_l = A_l C \sqrt{\frac{2 \gamma \gamma}{(\gamma-1) R T_l} \left[\left(\frac{P_x}{P_l} \right)^{\frac{2}{\gamma}} - \left(\frac{P_x}{P_l} \right)^{\frac{\gamma+1}{\gamma}} \right]} \quad 15.5-41$$

and for leakage past a labyrinth:

$$\dot{m}_l = A_l C_L \sqrt{\frac{F \gamma (P_l^2 - P_n^2)}{n R T_l}} \quad 15.5-42$$

For equations 15.5-41 and 15.5-42, which are written in Kearton & Keh's notation:

P_x = static pressure at the discharge of a simple leak

P_l = (effective) stagnation pressure at entry to leak

P_n = static pressure downstream of last (nth) labyrinth in a labyrinth seal.

F = function of P_l/P_n ; this function is displayed, for staggered labyrinth seals, in Fig.14 of Kearton & Keh's paper (10)

n = number of stages in a labyrinth seal.

A_l = cross-sectional area of leakage path.

C = discharge coefficient of a "simple" seal.

C_L = mean discharge coefficient of a labyrinth seal.

Substituting 15.5-41 and 15.5-42 in equation 15.5-40, and rearranging:

$$\left[1 - \frac{P_c}{P_e}\right] = \gamma \gamma_c \left\{ \left[\frac{A_l P_l}{A_c P_c} \alpha_1 C \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{P_l}{P_c} \right)^{\frac{2}{\gamma}} - \left(\frac{P_l}{P_l} \right)^{\frac{\gamma+1}{\gamma}} \right]} \right] + \left[\frac{A_l}{A_c} \alpha_1 C_L \sqrt{\frac{F \left[\left(\frac{P_l}{P_c} \right)^2 - \left(\frac{P_n}{P_c} \right)^2 \right]}{n \gamma}} \right] \right\}$$

15.5-43

The first expression on the R.H.S. of 15.5-43 represents the contributions due to simple leakage, the latter part of the RHS sums the effects of leakage past labyrinth seals.

The usefulness of 15.5-43 can be enhanced if the factors A_l/A_c are expressed in terms of the dimensionless geometry of the pressure-exchanger, defining:

γ = distance, in dimensionless time units, between adjacent ports.

C = (axial) clearance between the end of the rotor and adjacent port face.

d = (radial) height of the cells.

b = mean (circumferential) breadth of the cells.

l = (axial) length of the cells

Also defining: $\lambda \equiv \frac{C}{l}$, $\lambda \equiv \frac{d}{l}$, $\kappa \equiv \frac{b}{l}$

15.5-44

For radial leakage:

$$\frac{A_l}{A_c} = \frac{\gamma b c}{\gamma_c b d}$$

hence, invoking the definitions 15.5-44

$$\gamma_c \frac{A_l}{A_c} = \frac{\gamma \lambda}{\lambda}$$

15.5-45

For circumferential leakage:

$$\frac{A_l}{A_c} = \frac{c d}{b d} \quad \text{hence from 15.5-44,}$$

$$\frac{A_l}{A_c} = \frac{b}{K}$$

15.5-46

Substituting 15.5-45 and 15.5-46 in 15.5-43 gives:

$$\left[1 - \frac{P'_c}{P_c}\right] = \delta \left\{ \tau_c \sum \frac{b}{K} \frac{P_l}{P_c} C \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{P_l}{P_c} \right)^{\frac{\gamma}{\gamma-1}} - \left(\frac{P_l}{P_c} \right)^{\frac{\gamma+1}{\gamma}} \right]} + \sum \frac{\gamma b}{\lambda} \alpha_l C_L \sqrt{\frac{F}{n\gamma} \left[\left(\frac{P_l}{P_c} \right)^2 - \left(\frac{P_l}{P_c} \right)^2 \right]} \right\}$$

15.5-47

This is the general result 4.4-13 quoted in section 4.4.3. The equation is so arranged that leakage into the cells is positive.

Wave diagrams for complete wave cycles, which have been constructed for the leak-free boundary condition $P'_c/P_c = 1$ can be modified, by re-interpolation, to suit the value of P'_c/P_c obtained from 15.5-47.

Alternatively finite difference equations may be used; no further reference will be made to these as their use appears to offer no advantages over the re-interpolation procedure, despite involving more labour.

(b) The Effect of Leakage On the Mass-Flow In Pressure-Exchanger

Changes of mean stagnation temperature in the ducts into which leakage flowed were ignored. For cycles involving only small temperature differentials the effect of this omission is only of second order importance. The simplification was made to avoid complications arising from the application of isentropic wave theory to the study of wave cycle performance.

Therefore, for the purposes of the theory, leakage into or from a duct is merely assumed to increase or decrease the mass flow therein. A means is thus required for evaluating the leakage mass flow.

Since;
$$\frac{\sum \dot{m}_l}{\dot{m}_c} = \frac{\sum \dot{m}_l}{\dot{m}_c} = \frac{\sum \dot{m}_l}{\dot{N} \dot{m}_c}$$

$$\therefore \frac{\sum \dot{m}_l}{\dot{m}_c} = \frac{\sum \dot{m}_l}{\dot{N} \left(\frac{P_c A_{cl}}{R T_c} \right)} \quad 15.5-48$$

Invoking equations 15.5-41 and 15.5-42 and making similar substitutions in 15.5-48 to those made in the previous case:

$$\frac{\sum \dot{m}_l}{\dot{m}_c} = \tau_c \left\{ \frac{A_l P_c \alpha_c^2}{A_c P_c \alpha_l} C \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{P_x}{P_l} \right)^{\frac{2}{\gamma}} - \left(\frac{P_x}{P_l} \right)^{\frac{\gamma+1}{\gamma}} \right]} + \frac{A_l \alpha_c^2}{A_c \alpha_l} C \sqrt{\frac{F}{n\gamma} \left[\left(\frac{P_l}{P_c} \right)^2 - \left(\frac{P_h}{P_c} \right)^2 \right]} \right\}$$

or, after substituting 15.5-45 and 15.5-46 in 15.5-49:

$$\frac{\sum \dot{m}_l}{\dot{m}_c} = \left\{ \tau_c \frac{L P_l \alpha_c^2}{K P_c \alpha_l} C \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{P_x}{P_l} \right)^{\frac{2}{\gamma}} - \left(\frac{P_x}{P_l} \right)^{\frac{\gamma+1}{\gamma}} \right]} + \frac{\gamma L \alpha_c^2}{\lambda \alpha_l} C \sqrt{\frac{F}{n\gamma} \left[\left(\frac{P_l}{P_c} \right)^2 - \left(\frac{P_h}{P_c} \right)^2 \right]} \right\}$$

15.5-50

Equation 15.5-50 is the result labelled 4.4-14 in the main text.

Specific leakage calculations are described in sub-sections (c) and (d).

(c) Application Of The Leakage Theory To Dividers

The purpose of this sub-section is to describe the application of equations 15.5-47 and 15.5-50 to the leakage model illustrated in section 4.4., and to feed into the equations geometry data, etc. applicable to the Power Jets machine.

To simplify the computational procedure all leaks in the Power Jets machine were regarded as "simple" or non-labyrinth leaks. It was also assumed that the pressure in the leakage sink was equal to the static pressure in the low pressure port; this approximation roughly corresponded to the test conditions.

In order to reduce the number of repetitive calculations the function

$\Phi_{l,x}$ was evaluated over a range of P_l/P_x . $\Phi_{l,x}$ is defined thus:

$$\Phi_{l,x} = \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{P_x}{P_l} \right)^{\frac{2}{\gamma}} - \left(\frac{P_x}{P_l} \right)^{\frac{\gamma+1}{\gamma}} \right]}$$

Fig.15.5-1 displays a curve of $\Phi_{l,x}$ versus P_l/P_x up to the onset of choking.

For the Power Jets machine;

$$\kappa \approx 0.06$$

$$\lambda \approx 0.20$$

$$b \approx 0.0006 \text{ (i.e. } 0.007'' \text{ end clearance)}$$

Assuming the flow coefficients for all leakage paths are identical, then from Snow's contribution to the discussion of Kearton & Keh's paper (10)

$$C \approx 0.8$$

[N.B. Snow showed that C was to some extent a function of pressure ratio.]

Taking τ_c to be the same as for the finite cell width wave diagram calculations::

$$\tau_c = 0.4$$

Thus feeding the appropriate numerical data into equation 15.5-47 also noting that radial leakage occurs at both ends as well as at the top and bottom of each cell, then for closely pitched sets of ports ($\tau_b = \tau_c$), equation 15.5-47 becomes:

$$\left[1 - \frac{P'_c}{P_c} \right] = \left[0.0045 \left\{ 2\alpha_H \frac{P_H}{P_c} \Phi_{H,c} + \alpha_M \frac{P_M}{P_c} \Phi_{M,c} + \alpha_L \frac{P_L}{P_c} \Phi_{L,c} \right\} + 0.0067(\tau_c + 0.4) \alpha_L \frac{P_L}{P_c} \Phi_{L,c} \right]$$

15.5-51

Equation 15.5-51 is therefore the particularised form of equation 4.4-13 applicable to divider leakage calculations. Referring to the diagrams of either the real situation or the simplified leakage model; the first term in the .R.H.S. of 15.5-51 accounts for leaks a & b the second term covers leak c , and the third term accounts for leak f .

The last term of the expression embraces the combined effects of leaks (2) and (6) occurring at the tops and bottoms of the cells.

Particularising equation 15.5-50 to establish the mass flows leaking from or into the low, medium and high pressure ducts produced, using the same numerical data as in the previous case, the following results:

Leakage into the LP duct;

$$\left(\frac{\dot{m}_l}{\dot{m}_c}\right)_{LP} = 0.0032 \alpha_c \left\{ \frac{\alpha_c}{\alpha_M} \frac{P_M}{P_c} \Phi_{M,L} - \frac{\alpha_c}{\alpha_L} \frac{P_L}{P_c} \Phi_{L,c} \right\} \quad 15.5-52$$

Leakage from the MP and HP ducts respectively are given by:

$$\left(\frac{\dot{m}_l}{\dot{m}_c}\right)_{MP} = \frac{\alpha_c^2}{\alpha_M} \frac{P_M}{P_c} \left\{ 0.0032 (\Phi_{M,L} + \Phi_{M,c}) + 0.0048 \tau_M \Phi_{M,L} \right\} \quad 15.5-53$$

$$\left(\frac{\dot{m}_l}{\dot{m}_c}\right)_{HP} = \frac{\alpha_c^2}{\alpha_H} \frac{P_H}{P_c} \left\{ 0.0064 \Phi_{H,c} + 0.0048 \tau_H \Phi_{H,L} \right\} \quad 15.5-54$$

These equations are the required special forms of 4.4-14.

It should be noted that 15.5-52 is not based on an assumption that all the nett leakage from the pressure-exchanger is ducted into the LP tract. As mentioned earlier, it has been assumed however that the leakage sink was maintained at the static pressure in the LP port, and it is this fact which accounts for the omission of a contribution due to leak (1) (see leakage model) from equation 15.5-52.

Mass ratio β undergoes a two-component change as a result of leakage. One portion of the change in β arises from modifications to the wave cycle because, in the general case with leakage present, $P'_c \neq P_c$. The other contribution to the departure from the value of the leak-free mass ratio is the result of leakage into or from the ducts of the machine. The combined effects of leakage on β were taken into account in the following way:

Defining; $f_1 \equiv \frac{(\dot{m}_l)_{HP}}{\dot{m}_H}$, $f_2 \equiv \frac{(\dot{m}_l)_{MP}}{\dot{m}_M}$,

then it may be shown by suitable substitutions in the right hand sides of the definitions of f_1 and f_2 that:

$$f_1 = \frac{\left(\frac{\dot{m}_L}{\dot{m}_C}\right)_{HP}}{\int_1^2 M_{SH}^+ \alpha_H \left(\frac{P_H}{P_C}\right)^{\gamma/2} d\tau} \quad 15.5-55$$

$$f_2 = \frac{\left(\frac{\dot{m}_L}{\dot{m}_C}\right)_{MP}}{\int_1^2 M_{OZM} \alpha_{ZM} \left(\frac{P_{ZM}}{P_C}\right)^{\gamma/2} d\tau} \quad 15.5-56$$

Remembering that leakage has the effect on the mass flows deduced from wave diagram calculations, of decreasing the HP flow, and, due to the supply of extra fluid to maintain the leaks, increasing the MP flow:

$$\beta \text{ (with leakage)} = \beta^+ \frac{(1-f_1)}{(1+f_2)} \quad 15.5-57$$

where β^+ is the massratio predicted from equation 15.5-12, using data from a wave diagram modified to take account of leakage.

Evaluation of equations 15.5-51 to 15.5-57 inclusive allowed the effects of leakage on divider cycle performance to be determined. Such calculations account for the differences between the performances shown on Fig.12.1-5 and 12.1-6. Some of the equations were specially simplified for leakage predictions in the region close to $\frac{P_{H0}}{P_{M0}} = \frac{P_{L0}}{P_{M0}} = 1$

(d) Application Of The Leakage Theory To Equalisers

Equations 4.4-13 and 4.4-14 were particularised by similar procedures to those used in the previous case. The special form of equation 4.4-13 for the equaliser case is:

$$\left[1 - \frac{P'_C}{P_C}\right] = \left[0.0045 \left\{ 2 \alpha_M \frac{P_M}{P_C} \Phi_{M,C} + \alpha_H \frac{P_H}{P_C} \Phi_{H,C} - \alpha_C \Phi_{C,L} \right\} - 0.0067 (\gamma_2 + 0.4) \alpha_C \Phi_{C,L} \right] \quad 15.5-58$$

The term $2 \alpha_M \frac{P_M}{P_C} \Phi_{M,C}$ in equation 15.5-58 should be replaced by $-2 \alpha_C \Phi_{C,M}$ when $\frac{P_M}{P_C} < 1$; this situation corresponds to a reversal of the direction of leakage when P_C exceeds P_M .

On the assumption that all the nett leakage flow was conducted into the low pressure intake, and bearing in mind that the leakage sink was assumed to be maintained at the static pressure in the LP inlet, the specific forms of 4.4-14 are:

Leakage into the LP intake:

$$\left(\frac{\dot{m}_l}{\dot{m}_c}\right)_{LP} = 0.0032 \alpha_c \left\{ \frac{\alpha_c}{\alpha_H} \frac{P_H}{P_c} \Phi_{H,L} + \Phi_{C,L} \right\} + 0.0048 \alpha_c \left\{ \frac{\gamma_M \alpha_c P_H}{\alpha_M P_c} \Phi_{M,L} + \frac{\gamma_H \alpha_c P_H}{\alpha_H P_c} \Phi_{H,L} + (\gamma_2 + 0.4) \Phi_{C,L} \right\}$$

Leakage into the MP duct:

$$\left(\frac{\dot{m}_l}{\dot{m}_c}\right)_{MP} = \alpha_c \left\{ 0.0064 \Phi_{C,M} - \frac{\alpha_c}{\alpha_M} \frac{P_M}{P_c} \gamma_M \Phi_{M,L} \right\} \quad 15.5-60$$

$$\left[\begin{array}{l} \text{NB. if } P_M > P_c, \text{ replace } \Phi_{C,M} \text{ in the first} \\ \text{term of the RHS by } -\frac{\alpha_c}{\alpha_M} \frac{P_M}{P_c} \Phi_{M,C} \end{array} \right]$$

Leakage from the HP duct:

$$\left(\frac{\dot{m}_l}{\dot{m}_c}\right)_{HP} = \frac{\alpha_c^2}{\alpha_H} \frac{P_H}{P_c} \left\{ 0.0032 (\Phi_{H,C} + \Phi_{H,L}) + 0.0048 \gamma_H \Phi_{H,L} \right\} \quad 15.5-61$$

The effect of leakage on enthalpy ratio $\bar{S}$ was determined by a similar method to that used for finding the value of β with leakage present.

Defining: $f_3 \equiv \frac{(\dot{m}_l)_{LP}}{\dot{m}_L}$, $f_4 \equiv \frac{(\dot{m}_l)_{HP}}{\dot{m}_H}$, then

by suitable substitutions:

$$f_3 = \left(\frac{\alpha_{ZL}}{\alpha_c} \right)^2 \left(\frac{\dot{m}_l}{\dot{m}_c} \right)_{LP} \int_1^2 M_{OZL} \alpha_{ZL} \left(\frac{P_{ZL}}{P_c} \right) d\tau \quad 15.5-62$$

$$f_4 = \left(\frac{\alpha_{ZH}}{\alpha_c} \right)^2 \left(\frac{\dot{m}_l}{\dot{m}_c} \right)_{HP} \int_1^2 M_{OZH} \alpha_{ZH} \left(\frac{P_{ZH}}{P_c} \right) d\tau \quad 15.5-63$$

Thus where $\bar{S}^+$ is the value of $\bar{S}$ deduced from equation 15.5-14

using data from a wave diagram modified to take account of leakage then:

$$\bar{S} \text{ (with leakage)} = \bar{S}^+ \frac{(1-f_3)}{(1+f_4)} \quad 15.5-64$$

Evaluation of equations 15.5-58 to 15.5-64 inclusive allowed the performance shown on Fig.12.1-14 to be derived from that on Fig.12.1-13. Specially simplified forms of the equations appearing in this subsection were used to extend the prediction of leakage effects to values of P_{H0}/P_{M0} and P_{L0}/P_{M0} close to unity.

IV. Viscous Effects

Object: to provide quantitative expressions needed for executing the four-part procedure described under the heading "Step (5)" in section 4.4.3; the system of numbering accompanying the description given in the main text will be adhered to.

(I) Steady flow pressure loss in one cell length:

For the Power Jets machine:

$$\frac{l}{D_M} = 12.5$$

where; l = cell length

$$D_M = \text{hydraulic mean diameter of cells} = \frac{4 \times \text{C.S.A.}}{\text{Wetted perimeter.}}$$

From "Technical Data On Fuel" (19) one velocity head is lost in approximately 62 diameters when the pipe bore is fairly smooth ($\frac{R}{D_M} = 0.002$, Relative roughness = 0.0004). Hence number of velocity heads lost in one cell length is:

$$S_{(CELL)} \approx \frac{12.5}{62} = 0.202 \text{ cell velocity heads.}$$

(II) Steady flow pressure losses at cell entry and exit:

Because the edges of the cell partitions of the Power Jets machine are unstreamlined, the appropriate data is that applying to sudden contractions and expansions. From reference (20):

$$\begin{array}{ll} \text{For sudden contractions :} & S = 0.5(1-K) \\ \text{" " expansions:} & S = (1-K)^2 \end{array}$$

Where, to correspond to the notation given in the reference, K should

be read as $\frac{a}{A}$ where:

a = C.S.A. of smaller section

A = " " larger "

Hence, evaluating the entry and exit losses for the appropriate value of K ($= 0.862$) applicable to the Power Jets machine:

$$S(\text{ENTRY}) = 0.069 \text{ cell velocity heads}$$

$$S(\text{EXIT}) = 0.019 \text{ " " "}$$

(III) Total frictional losses for fluid entering or leaving cells:

Total No. of velocity heads lost in filling $\equiv S(\text{IN})$ where:

$$S(\text{IN}) \approx S(\text{ENTRY}) + \frac{1}{2} S(\text{CELL}) = 0.17$$

Total No. of velocity heads lost in emptying $\equiv S(\text{OUT})$ where;

$$S(\text{OUT}) \approx S(\text{EXIT}) + \frac{1}{2} S(\text{CELL}) = 0.12$$

(IV) Modification to the duct total head pressures to allow for frictional effects:

Where P_o^* = Total head pressure without allowing for viscosity.

" P_o = " " " with viscosity effects included.

" P = static pressure in the cells.

$$\text{For inflow: } \frac{P_o}{P} = 1.17 \left[\frac{P_o^*}{P} - 1 \right] + 1 \quad 15.5-65$$

$$\text{For outflow: } \frac{P_o}{P} = 0.88 \left[\frac{P_o^*}{P} - 1 \right] + 1 \quad 15.5-66$$

These results allowed the total heads in the ducts to be computed when friction was taken into account. It is emphasized that the static pressures appearing in equations 15.5-65 and 15.5-66 are those occurring in the ends of the cells, not in the ducts; they are therefore the pressures corresponding to the average flow conditions in the cells traversing a port. The appropriate cell Mach numbers were computed from use of equations 15.5-31 and 15.5-37 with $K = 1$; the values of

P_0^*/P corresponding to these Mach numbers were obtained from tables of the well-known steady flow isentropic relationship between Mach number and pressure ratio.

FIG. 15.5-1

LEAKAGE FUNCTION $\Phi_{l,x}$ VERSUS LEAKAGE PRESSURE RATIO P_l/P_x $\Phi_{l,x}$ VERSUS P_l/P_x FOR $\gamma = 1.4$

WHERE:
$$\Phi_{l,x} = \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{P_x}{P_l} \right)^{2/\gamma} - \left(\frac{P_x}{P_l} \right)^{(\gamma+1)/\gamma} \right]}$$

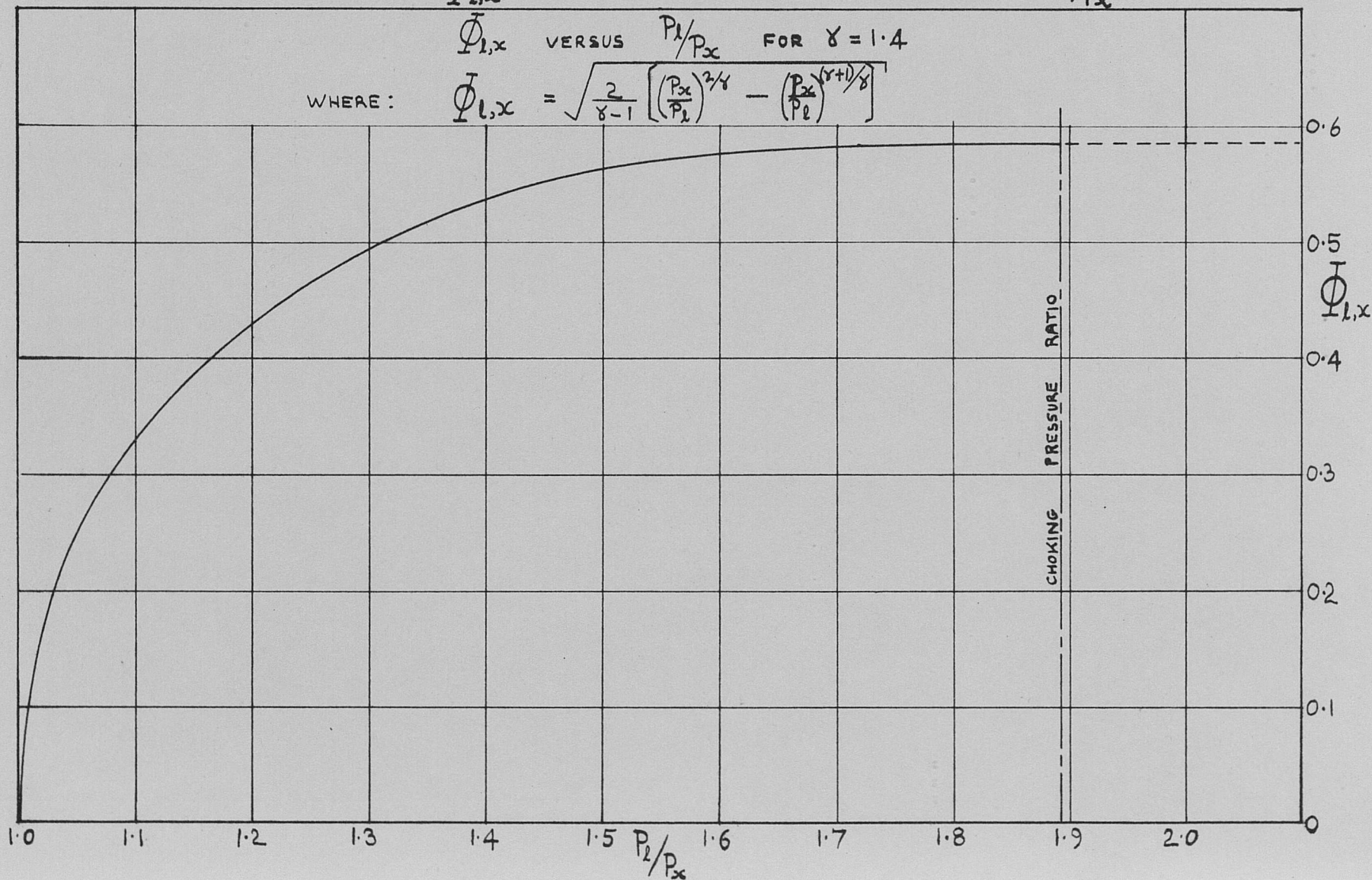
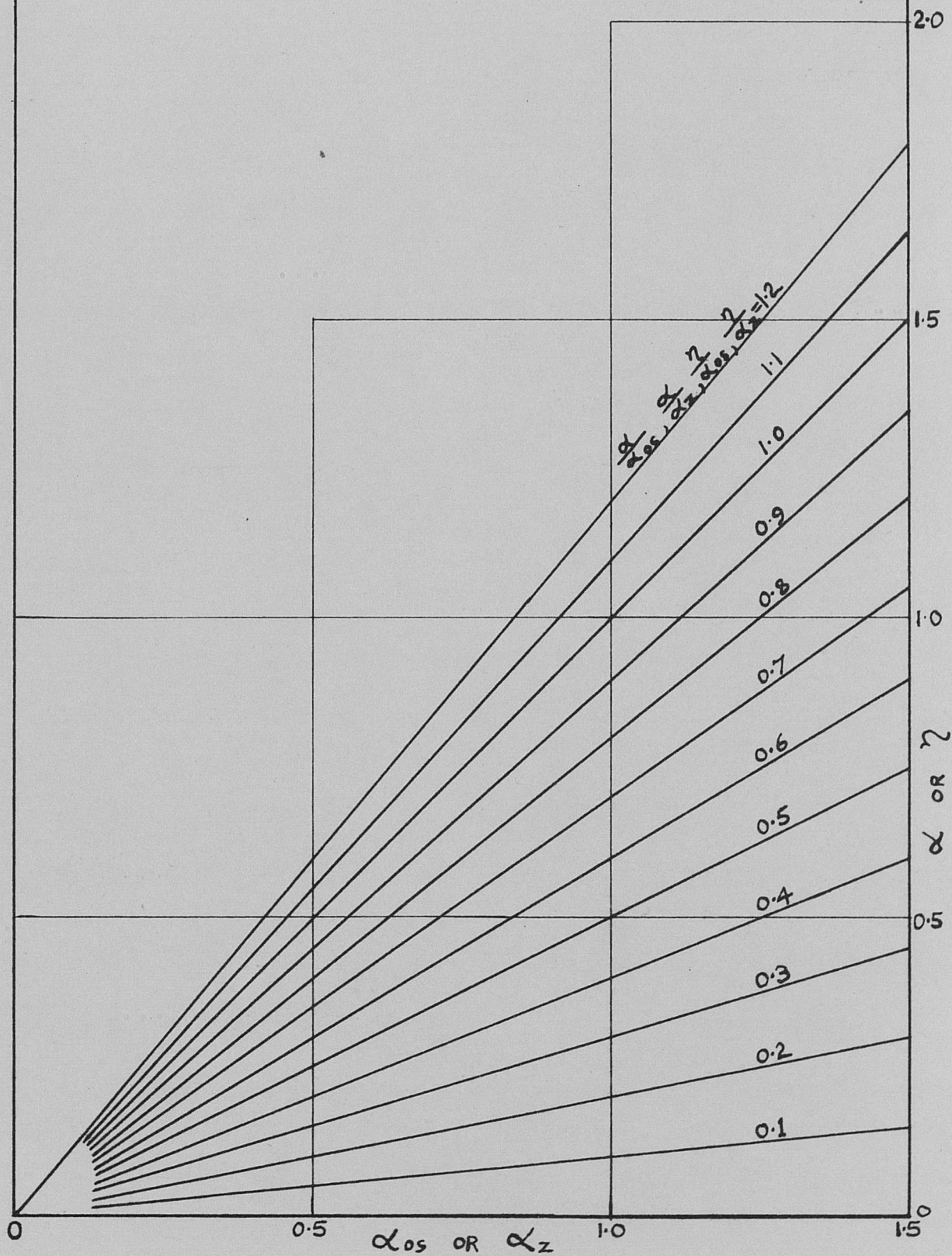


FIG. 15.5-2
 SCALING DIAGRAM FOR LINKING BOUNDARY CURVES
 WITH PORT CONDITIONS



15.6 APPENDIX VIA STUDY OF FLOW IN THE PORTS OF
PRESSURE-EXCHANGERS

Elsewhere references were made to the presence of static pressure gradients in the ports of the pressure-exchanger, typical examples of these are shown on Fig.15.6-1. A special subsidiary study was undertaken to investigate the reasons for these phenomena; an outline of the methods used, and a statement of the main conclusions form the contents of this Appendix.

To establish the basic flow pattern within the ducts of the pressure-exchanger it was assumed that the flow was essentially steady, two-dimensional, and incompressible. The assumption of steady flow is usually made when considering flow in pressure-exchanger ducts. The flow could justifiably be regarded as two-dimensional if the radial pressure gradients were small; these are predicted in (a). The assumption of incompressible flow seemed warrentable from experimental evidence (see Fig.15.6-1).

The method of predicting the flow pattern within the ducts involved determining, by a relaxation procedure, solutions for the Poisson equation:

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = \zeta(\psi) \quad 15.6-1$$

The solution for each problem was conveniently drawn as contours of constant ψ (streamlines) on a plan of the duct. Pressure distributions were established by application of incompressible Bernoulli along the streamlines. The application of relaxation methods to outlet

ducts including the fitment of boundary conditions is explained in (b)

Section (c) covers the prediction of a pressure gradient in an outlet port for comparison with an experimental result; the comparison is shown in Fig.15.6-3; Section (d) described the adaptation of the method to the study of flow in an inlet port, a comparison between a theoretical and an actual result is shown in Fig.15.6-4. Fig.15.6-2 illustrates the outcome of a simple experiment designed to confirm that the static pressure gradient in a port could be influenced by changes in duct geometry. As can be seen the sign of the change was that which would be expected from theoretical considerations.

It was concluded from the studies that the observed static pressure gradients were, in large measure, a consequence of port and duct geometry this being particularly true for outlets. The analysis did not attempt any close marriage of wave diagram and steady flow two-dimensional calculations at the plane of the ports. Such a procedure would have been tedious, but would have allowed the theory to include factors omitted from the analysis reported here.

The static pressure gradients appeared in general to have an adverse effect on the machine's performance, particularly by restricting flow near the trailing edge of outlet ports. The quantitative effects are probably best determined by experiment.

The methods used in (b) and (d) allow the likely two-dimensional steady flow performance of ducts to be studied during the early stages of design.

(a) Radial Pressure Gradients:

By integrating the equation of forces acting on a particle in a

forced vortex field in an incompressible fluid:

$$P_r - \frac{\rho \omega^2 r^2}{2} = \text{constant} \quad 15.6-2$$

Defining r_1 and r_2 as the minimum and maximum radii of a cell respectively, also invoking 15.6-2 and the well-known relationship $\rho = \frac{P}{RT}$, the following equation may be written down for forced vortex motion:

$$\frac{\Delta P_r}{P_1} = \frac{P_r - P_1}{P_1} = \frac{\omega^2 r_1^2}{2RT} \left[\left(\frac{r}{r_1} \right)^2 - 1 \right] \quad 15.6-3$$

The magnitude of the radial pressure gradients existing in the pressure-exchanger can be found by evaluating 15.6-3 over the range $1 \leq r/r_1 \leq 2$, with dimensions, etc. representative of the experiments;

Data used in equation 15.6-3 for evaluation of the table on the right:

$\omega = 200 \pi$ rad/sec (6000 RPM)

$T = 300^\circ \text{K}$ (Typical temperature)

$r_1 = 0.18 \text{ ft.}$ (2.16")

$r_2 = 0.36 \text{ ft.}$ (4.32")

r/r_1	$\Delta P_r / P_1 \%$
1.2	0.303
1.4	0.661
1.6	1.070
1.8	1.545
2.0	2.080

i.e. the difference in static pressure between the top and bottom of a port was only about 2% of the mean static pressure.

(b) Flow in Outlet Ducts Studied by Relaxation Methods

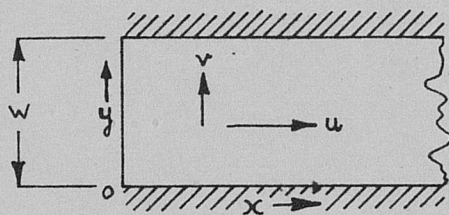
An equation involving vorticity $\zeta(\psi)$ and the stream function ψ for a two-dimensional, incompressible steady flow field is:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \zeta(\psi) \quad 15.6-1$$

$$\text{where } U = \frac{\partial \psi}{\partial Y} \quad 15.6-4 \quad \text{and} \quad V = - \frac{\partial \psi}{\partial X} \quad 15.6-5$$

and where U, V, X and Y are defined, with reference to the sketch, in the following way:

$$U \equiv \frac{u}{w}, \quad V \equiv \frac{v}{w}, \quad X \equiv \frac{x}{w}, \quad Y \equiv \frac{y}{w}.$$



w is the breadth of the port, $\bar{u}$ is the mean velocity of flow normal to the plane of the port of area A ; i.e. $\bar{u} \equiv \frac{\int_0^A u \, dA}{A}$.

15.6-1 is readily amenable to solution by relaxation methods if $\bar{S}(\psi)$ is a constant. Constant $\bar{S}(\psi)$ implies that the velocity profile is linear in the region where $V=0$ well downstream of the plane of the port;

i.e. if $V=0$ then from 15.6-5; $\frac{\partial^2 \psi}{\partial X^2} = 0$,

and if; $U = \text{Constant} + \bar{S}(\psi) Y$ 15.6-6

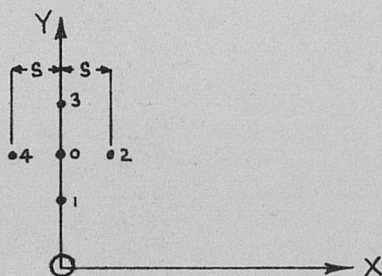
then differentiating 15.6-6 where $\bar{S}(\psi)$ is a constant, and invoking 15.6-4:

$$\frac{\partial U}{\partial Y} = \bar{S}(\psi) = \frac{\partial^2 \psi}{\partial Y^2}$$

Equation 15.6-6 clearly implies a restriction in the complexity of the velocity profiles acceptable for easy theoretical solutions. The particular case of $\bar{S}(\psi)=0$ is that applicable to potential flow.

The general relaxation method is described in detail by Allen (1) however three procedures for particular cases relevant to this problem will be dealt with:

(1) Application of Boundary Conditions at Plane of Port:



Let grid 1~4 be centred on the Y axis as shown, point 4 is a fictitious point lying in the region of the cells, by approximation:

$$\frac{\psi_2 - \psi_4}{2s} = \left(\frac{\partial \psi}{\partial x} \right)_{x=0} \quad 15.6-7$$

but on the Y axis, at an outlet port, $V = V_{\omega}$ and is a specified constant corresponding to the rotor speed, hence:

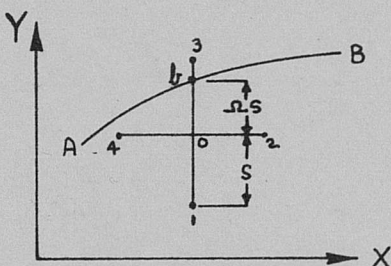
$$-V_{\omega} = \left(\frac{\partial \psi}{\partial x} \right)_{x=0} \quad 15.6-8$$

using 15.6-7 and 15.6-8 to substitute for ψ_4 in the normal relaxation operator, the residual χ_o at o is given by

$$\psi_1 + \psi_3 + 2(\psi_2 + sV_{\omega}) - 4\psi_o - s^2 \nabla^2 \psi = \chi_o \quad 15.6-9$$

Relaxation may then proceed normally using equation 15.6-9 on the plane of the port.

11. Boundaries Intersecting Grid Arms:



Let AB be a boundary cutting the grid arm $o,3$ at b , by approximation;

$$\frac{\psi_3 - \psi_o}{s} = \frac{\psi_b - \psi_o}{\Omega s} \quad 15.6-10$$

substituting for ψ_3 in the normal operator 15.6-11;

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \psi = s^2 \xi_{(\psi)} + \chi_0 \quad 15.6-11$$

the following equation is obtained:

$$\psi_1 + \psi_2 + \psi_4 + \left(\frac{\psi_1 - \psi_0}{2} \right) - 3\psi_0 - s^2 \xi_{(\psi)} = \chi_0 \quad 15.6-12$$

Relaxation may be carried out using the known boundary value ψ_b . Equation 15.6-11 copes with ordinary grid points away from the boundaries.

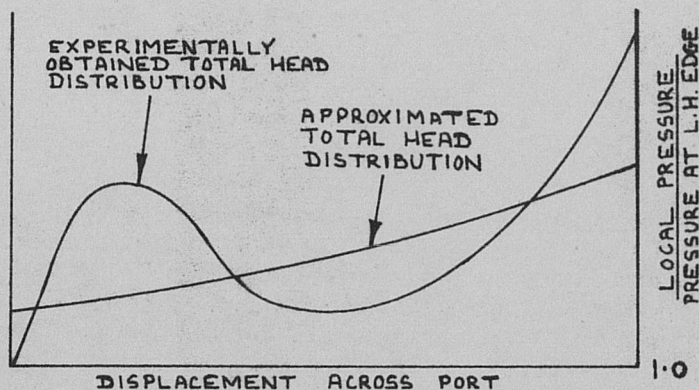
111. Detached Flow:

The relaxation procedure cannot itself determine whether flow will become detached or not. It may often be decided from the geometry, or incompatibility of geometry and boundary conditions, that separation will occur.

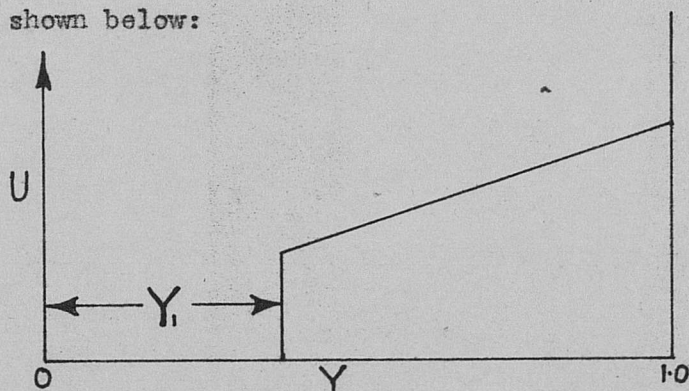
Relaxation may be undertaken for separated flow, albeit with more trouble than for solid boundaries. By taking the pressure in the cut-off stagnant region to be uniform, it follows from application of Bernoulli that the velocity is constant along the streamline separating the stagnant and moving fluids. Hence the demarcation, or free, streamline may be located by the relaxation process, any errors in the initial location being automatically corrected.

(c) Application of Theory to an Outlet Duct

The theory outlined in (b) was applied to a particular case of flow through an unfaired outlet duct lying parallel to the axis of the pressure-exchanger rotor. Experimentally obtained total head and static pressure distributions were available. The total head profile provided input data for commencing the theoretical treatment by allowing a guessed, approximately parabolic, total head curve to be drawn.



Flow presumably becomes detached at the leading edge of the port. This therefore implies that the static pressure at the leading edge of the port is that prevailing in the stagnant fluid cut-off by the free streamline, and hence is also the static pressure all across the duct in the region of parallel flow well downstream of the port. These considerations, coupled with constancy of total head along streamlines, allowed the velocity, based on the approximate total head curve, to be calculated in the parallel flow region. A sketch of the linear velocity profile is shown below:



From the sketch and allied data the value of $\bar{S}(\psi)$ was deduced; equation 15.6-1 was then solved for this value of $\bar{S}(\psi)$ the total head curve so obtained at the plane of the port was compared with the initial guessed value. The guessed total head profile was then modified and the cycle of calculations repeated until the guessed and deduced total head profiles nearly coincided, both being good approximations of the experimental result. When this occurred it was found that, for the

prescribed condition of $\overline{U} = 1.2 V_w$, $S_{(4)} = 2.4$.

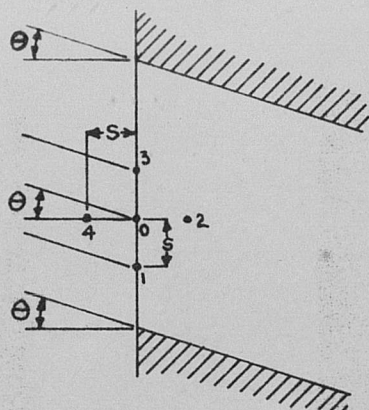
The final result is shown in Fig.15.6-3; from the pressure distributions at the plane of the port, shown at the foot of the flow pattern, it can be seen that the experimental and theoretical static pressure profiles were very similar. The value of Y_1 was found to be approximately 0.4 for the results giving best agreement.

The theoretical treatment indicates that outflow ceases at the point near the right hand wall at which the static pressure in the port equals the total head pressure in the cells; between this point and the wall inflow occurs. The stagnation pressure in the duct is greater, by an amount proportional to the whirl component of velocity, than the total head pressure in the cells recordable with a probe attached to the rotor.

(d) Flow In Inlet Ducts

The flow in inlet ducts may also be investigated by relaxation methods. It is not always possible to justify the same boundary conditions in the port which were used for the studies of the flow in discharge ducts.

It appears to be reasonable to assume that the streamlines are parallel to the walls of the duct at the plane of the port. This assumption is presumably completely justified if the port is adequately faired. It is necessary to make an assumption such as the foregoing in order to specify the boundary conditions across the port opening. The method used for setting-up this boundary condition in a suitable form for the relaxation procedure is shown below:



For stars centred on the plane of the port, geometry and linear interpolation give;

$$\psi_4 = (\tan \theta)(\psi_1 - \psi_0) + \psi_0 \quad 15.6-13$$

The appropriate residual equation is then given by substituting for ψ_4 from 15.6-13 in the normal relaxation operator:

$$\psi_1 + \psi_2 + \psi_3 + (\psi_1 - \psi_0) \tan \theta - 3\psi_0 - s^2 \xi_{(\psi)} = \chi_0 \quad 15.6-14$$

An example of the study of irrotational flow ($\xi_{(\psi)} = 0$) in a divider medium pressure inlet duct is shown in Fig. 15.6-4. The derived static pressure gradient was fairly similar to a corresponding profile obtained by experiment.

FIG. 15-6-1

TYPICAL STATIC PRESSURE DISTRIBUTIONS IN THE PORTS OF THE PRESSURE-EXCHANGER DIVIDER

$\left[P_{Ho}/P_{Mo} \approx 1.27, P_{Lo}/P_{Mo} \approx 0.82, M_{w} \approx M_{Ro} = 0.1470 (6000 \text{ REV/MIN}) \right]$

- HP OUTLET PORTS
- x— MP INLET PORTS
- LP OUTLET PORTS

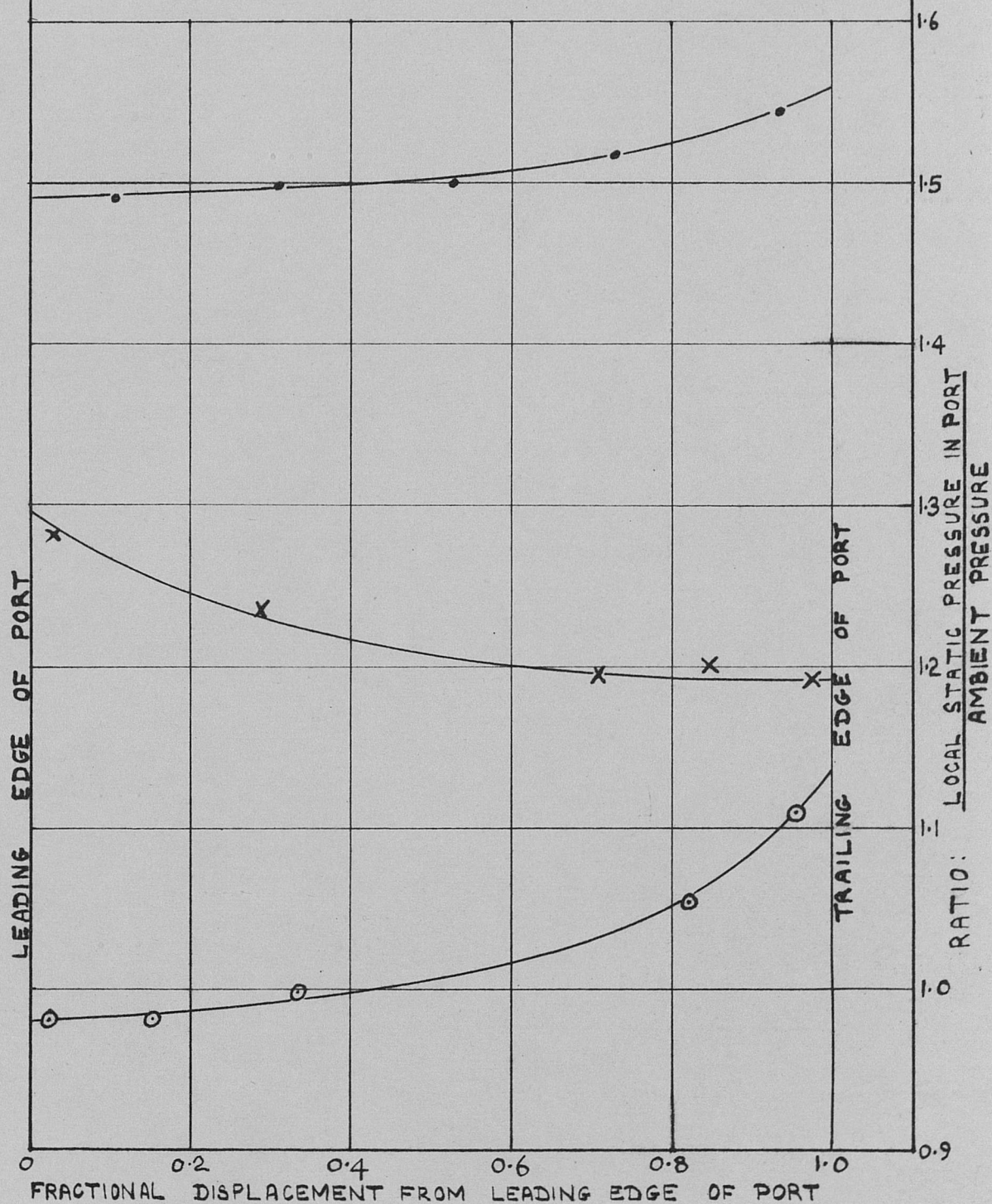
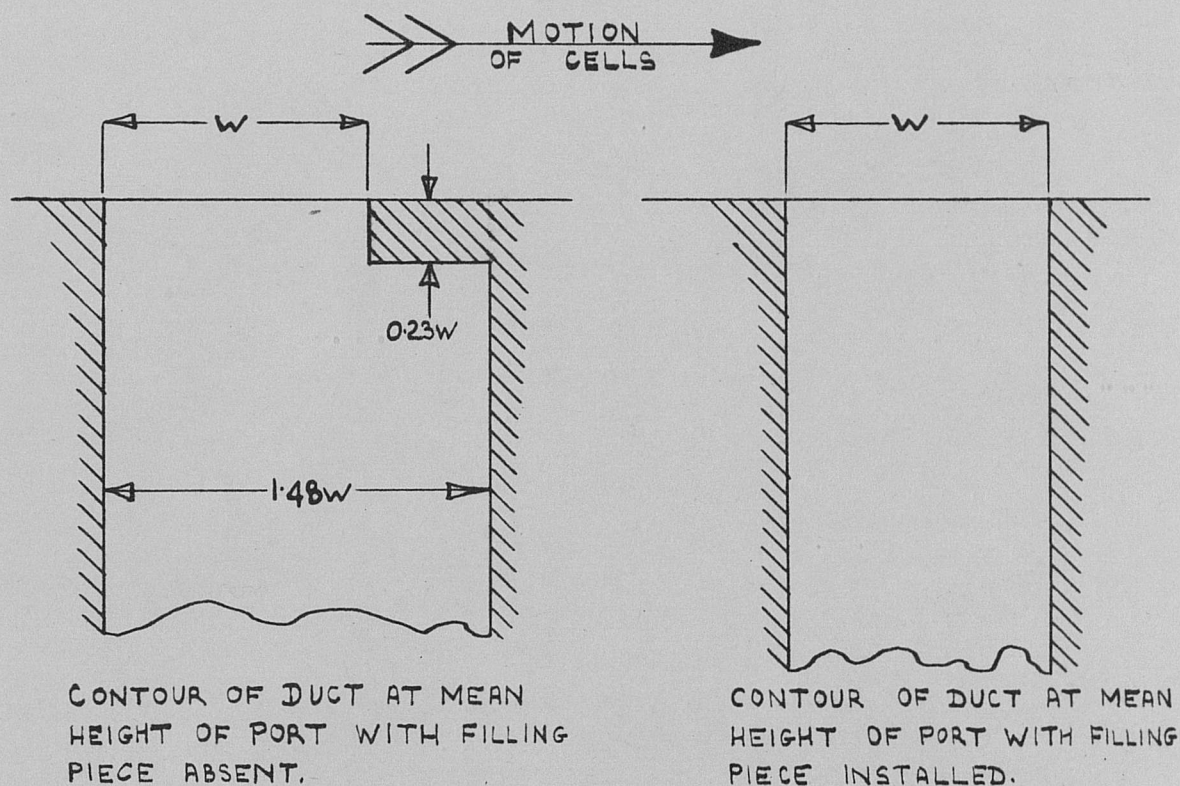
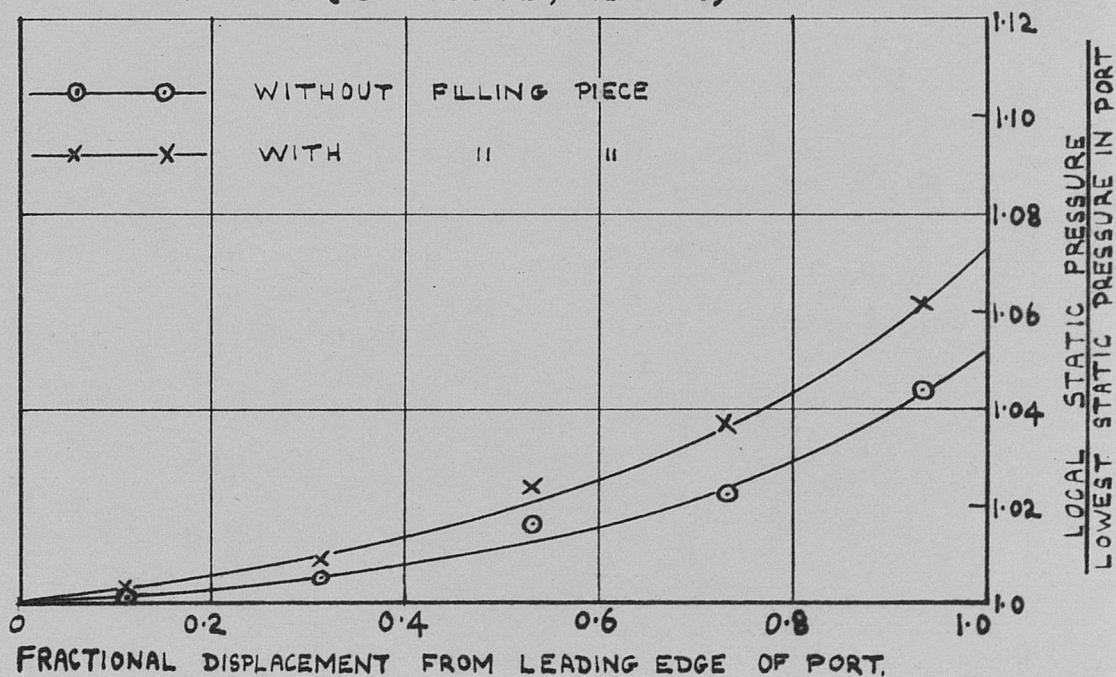


FIG. 15.6-2

EFFECT OF A FILLING PIECE ON THE STATIC PRESSURE DISTRIBUTION IN AN OUTLET PORT

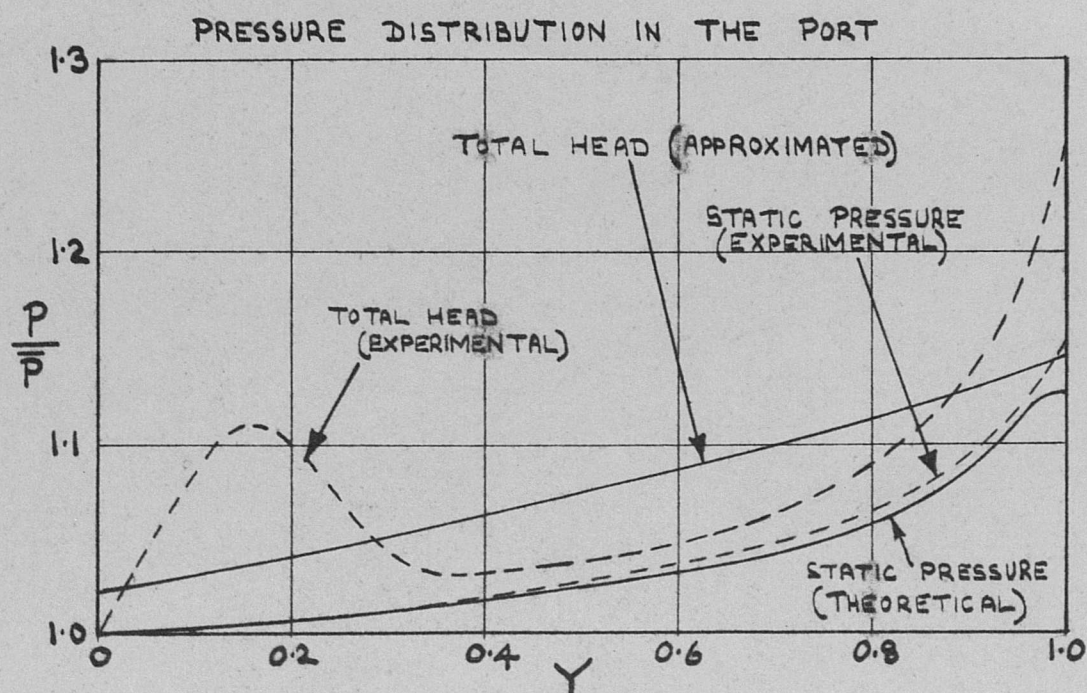
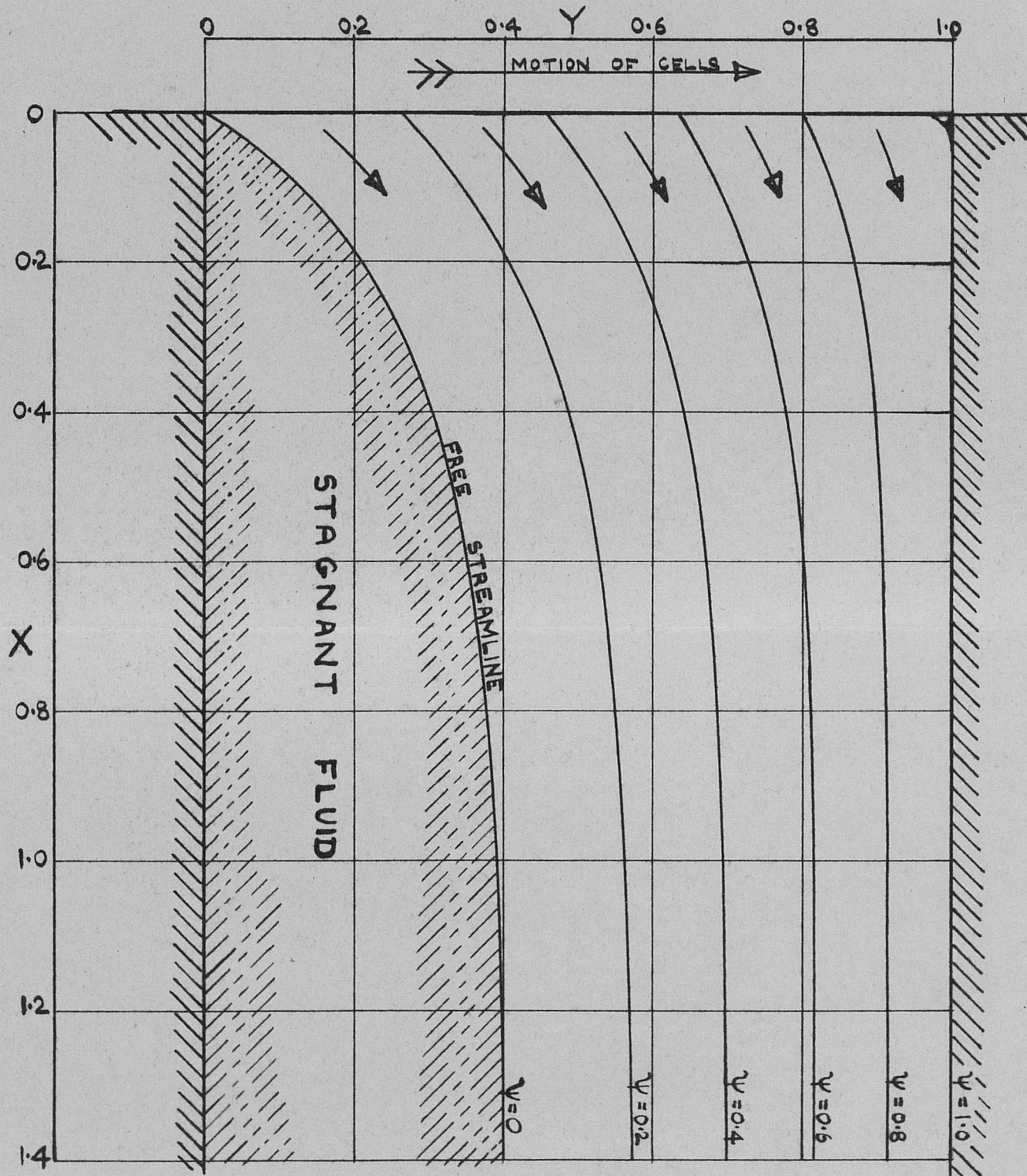


STATIC PRESSURE DISTRIBUTION IN PORT ($\bar{U} = 0.58 V_w$, $M_w \approx 0.14$)



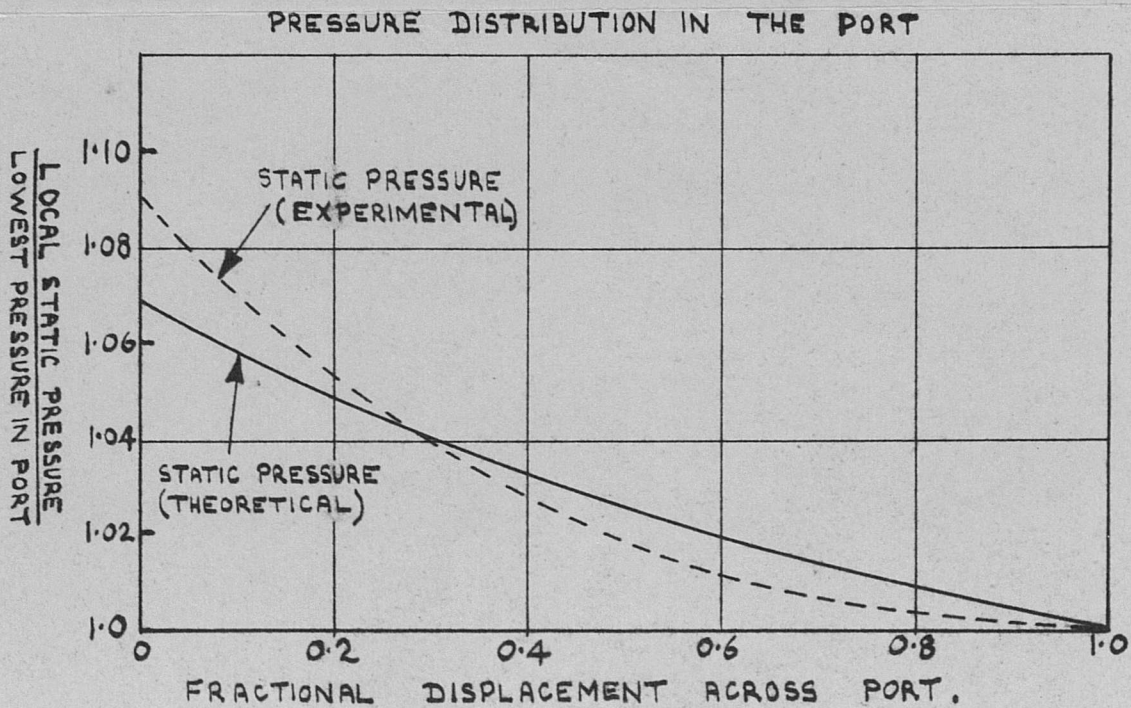
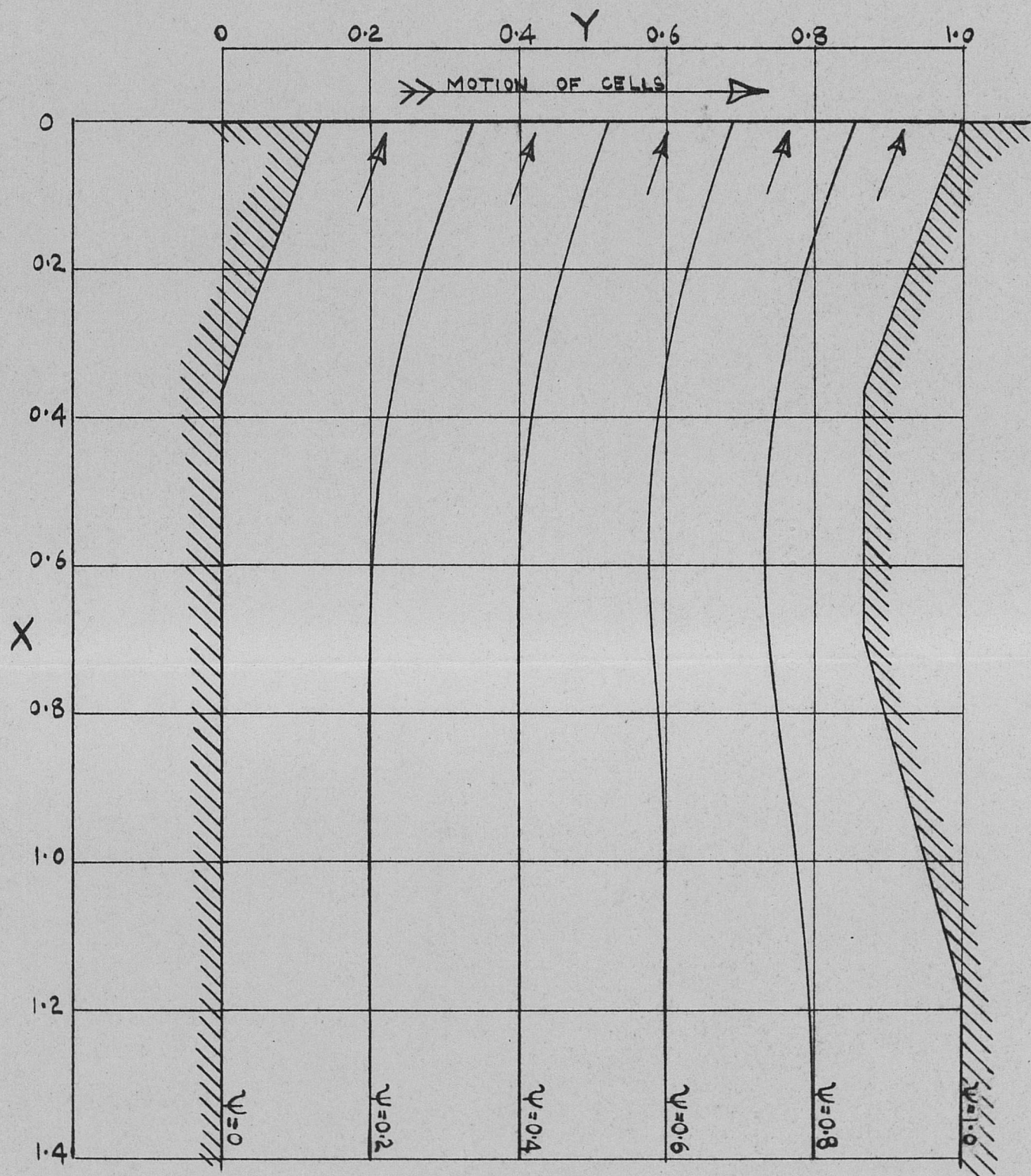
THEORETICAL FLOW PATTERN IN AN UNFAIRED OUTLET DUCT

[NB. FOR THE THEORETICAL CALCULATIONS $\bar{U} = 1.2V\omega$, $\xi(\omega) = +2.4$.
THE DUCT PROFILE SHOWN REPRESENTS THE PLAN-FORM
OF THE DIVIDER LP OUTLET TRACTS.]



THEORETICAL FLOW PATTERN FOR THE DIVIDER MP AND
EQUALISER HP INLET TRACTS

[NB. FOR THE THEORETICAL CALCULATIONS IT WAS ASSUMED THAT $S(\psi) = 0$.
THE DUCT PROFILE ILLUSTRATED CORRESPONDS TO THAT AT THE
MEAN RADIAL DEPTH OF THE DUCT WHEN THE PORT WIDTH = 45°.]



hence:

$$\sum_{r=1}^n (P_r - P_{BACK}) = B \sum_{r=1}^n (H + h_r)$$

15.7-2

dividing both sides of 15.7-2 by the number of stand pipes n , and expanding the summation signs:

$$\frac{\sum_{r=1}^n P_r}{n} - P_{BACK} = BH + B \frac{\sum_{r=1}^n h_r}{n}$$

15.7-3

But

$$\frac{\sum_{r=1}^n P_r}{n} = P_{AV}$$

15.7-4

where P_{AV} is defined as the arithmetic average of the pressures $P_1, P_2, P_3, \dots, P_{(n-1)}, P_n$.

From equation 15.7-1, for the case where stand pipes 1---- n have the same area A_h :

$$\frac{A_R H}{A_h} = \sum_{r=1}^n h_r$$

15.7-5

Substituting equation 15.7-4 and 15.7-5 in 15.7-3:

$$P_{AV} - P_{BACK} = BH \left(1 + \frac{A_R}{n A_h} \right)$$

15.7-6

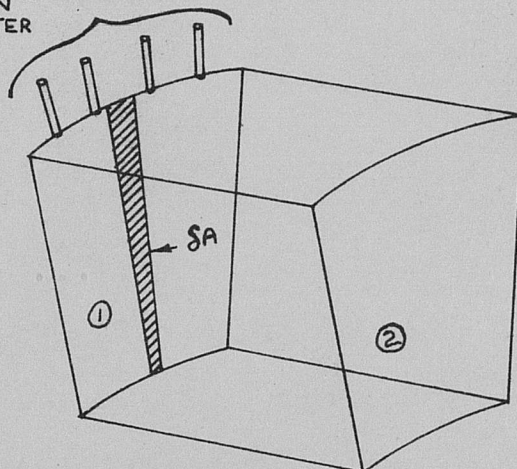
Equation 15.7-6 shows that for a given instrument, H is proportional to the arithmetic average of the pressures applied to the stand pipes 1----- n .

If the individual stand pipes are not all of the same cross-sectional area, H will be proportional to a weighted mean of the applied pressures.

15.8 APPENDIX VIII

JUSTIFICATION FOR THE USE OF AVERAGESTATIC PRESSURES

CONNECTIONS TO PRESSURE
PICK-UP LIMBS OF AN
AVERAGING MANOMETER



The sketch shows two planes spaced apart in a parallel duct. Plane ① is intended to represent a pressure-exchanger port with circumferentially non-uniform distributions of pressure and velocity; connections for an averaging manometer are also shown. Plane ② is a plane on which prevail the hypothetical conditions (subscript av) assumed to exist in the port. For the hypothetical conditions to adequately represent those at plane ①, there should be continuity of mass, energy and momentum between the two planes.

As proved in Appendix VII, as a consequence of the use of an averaging manometer, P_{av} is the arithmetic average of the static pressure at station ① :

$$P_{av} = \frac{\int_0^A P_i dA}{A} \quad 15.8-1$$

From continuity the mass flows at both stations are equal, hence, on the assumption that variations in static temperature are small enough to be ignored:

$$\int_0^A P_i u_i dA = P_{av} u_{av} A \quad 15.8-2$$

$$\begin{aligned} \text{Momentum at station } (1) &= \int_0^A u_1^2 dA \\ \text{" " " } (2) &= P_{AV} u_{AV}^2 A \end{aligned}$$

If the momenta at (1) and (2) are equal and ignoring temperature differences

$$\int_0^A P_1 u_1^2 dA = P_{AV} u_{AV}^2 A \quad 15.8-3$$

$$\text{and from continuity } \int_0^A P_1 u_1 dA = P_{AV} u_{AV} A \quad 15.8-2$$

$$\text{where } \int_0^A P_1 dA = P_{AV} A \quad 15.8-1$$

For the conditions on plane (1) to completely represent these on (2) it is necessary for equations 15.8-1 to 15.8-3 to be satisfied simultaneously. Equations 15.8-1 and 15.8-2 will always be satisfied; 15.8-3 will also be satisfied if u_1 is a constant $= u_{AV}$. The use of an averaging manometer will therefore involve discrepancies in correctly estimating the momentum term, and consequently the total head pressure, when $u_1 \neq u_{AV}$.

Predicting the likely difference in the total head pressures, obtained from the integrated expression $\int_0^A P_1 u_1^2 dA$ and non-integral form $P_{AV} u_{AV}^2 A$ by means of two examples (a) and (b).

(a) Assuming P_1 is constant $= P_{AV}$. Also assuming $u_1 = (1 \pm 0.2) u_{AV}$, then 15.8-2 is satisfied when fluid passing through 50% of the port area has a velocity of $0.8 u_{AV}$, and $1.2 u_{AV}$ for flow through the remainder of the area. This leads to the result:

$$P_{AV} \int_0^A u_1 dA = 1.04 (P_{AV} u_{AV}^2 A)$$

For the circumstances given here this corresponds to the average conditions (subscript AV) under-estimating the total head established

by integration by only 0.2% when the flow Mach number is 0.3, a typical value observed in the experiments.

- (b) A similar example to the foregoing, for the case $u_1 = (1 \pm 0.4) u_{AV}$ gives the result;

$$P_{AV} \int_0^A u_1^2 dA = 1.16 (P_{AV} u_{AV}^2 A)$$

This implies that the estimate of total head pressure based on knowledge of the average conditions under-estimates the total head established by integration over the port area by about 1.1% for a flow Mach number of 0.3

From the foregoing examples it can be seen that the use of an averaging manometer should lead to improved experimental accuracy, compared with that likely from a single point reading, without slowing down the test procedure in any way.