

DIGITAL PHASE-SPLITTING NETWORK DESIGN FOR
DIGITAL FREQUENCY-DIVISION-MULTIPLEX APPLICATIONS.

by

Ian Colyer

A thesis submitted for the degree of
Doctor of Philosophy in the Faculty
of Engineering, University of London.

Department of Electrical Engineering,
Imperial College of Science and Technology,
University of London.

October 1977.

ABSTRACT

Certain problems of a general nature which occur in the digital processing of telephone speech signals are examined with particular emphasis given to the formation of the Frequency-Division-Multiplexed Group types 'A' or 'B'. Both the digital phase-splitting method and the digital Weaver method of producing single-sideband signals are reviewed and it is shown that the digital phase-splitting approach is advantageous.

The system that corresponds to this approach consists of two signal processing stages - these being (i) a low sample rate (16 kHz.) complex baseband channel filter followed by (ii) a high sample rate (112 kHz.) signal interpolator and high frequency modulation stage. A second stage alternative of a combined signal interpolation and modulation or 'block processing' operation is outlined in addition.

The production of a specified channel filter frequency response using a phase-splitting network is considered in some detail. In an effort to minimize the complexity of the overall system various recursive and non-recursive structures are individually examined to determine their suitability in realizing the phase-splitting network.

A new digital phase-splitting structure, interpreted in terms of polyphase concepts, is introduced. The polyphase network differs from other networks examined in that it is notionally complex (i.e. the complex transmittance of the phase-splitting network is realized directly compared with a synthesis using two real and separate transmittances). An analysis assuming the use of fixed-point arithmetic hardware shows this structure to be fairly insensitive to the effects arising from the use of finite accuracy arithmetic.

Finally using an optimization design method based on the univariate search procedure and making use of the low sensitivity of the polyphase structure to coefficient quantization, a network is designed for use in the 'A' or 'B' channel bank Frequency-Division-Multiplex system. This network consisting of an amplitude correcting filter in cascade with a polyphase (phase-splitting) network is seen to be competitive with other, already proposed, networks after comparisons regarding multiplier complexities and roundoff noise performances are made.

To VIVIEN.

ACKNOWLEDGEMENTS.

I would like to thank my supervisor, Dr. A.G. Constantinides, for his encouragement and guidance throughout the course of this work. I am also indebted to Dr. W. Saraga, whose helpful discussions provoked much further thought.

Thanks are also due to my many colleagues, in particular, Stuart Lawson, Suman Dasgupta, Mun Chen and Bill Hodgkiss, for their support and friendship.

I am also grateful to the Science Research Council and Imperial College for the financial assistance received and to Professor C. Cherry in whose department this research was done.

Finally, I would like to express my appreciation of the contribution made by the typists of Scicon Ltd. in the careful preparation of this manuscript.

Ian Colyer

October 1977.

CONTENTS

	Page No.
Title Page	1
Abstract	2
Dedication	4
Acknowledgement	5
Contents	6
List of Illustrations	8
Definitions	11
Chapter One	
1.0 Introduction	14
1.1 Analogue SSB/FDM theory	17
1.2 SSB filtering at baseband frequencies	19
1.3 Digital SSB/FDM systems	23
1.4 Digital interpolation	28
1.5 Block processing	33
1.6 Summary	37
Chapter Two	
2.0 Digital SSB/FDM theory	46
2.1 The digital Weaver modulator	47
2.2 The digital quadrature or phase-difference modulator	48
2.3 Comparisons of the system hardware requirements	51
2.4 90° phase-splitting networks	55
2.5 Summary	79
Chapter Three	
3.0 The passive RC-polyphase network	96
3.1 The derivation of the basic digital section	100
3.2 Coefficient selection by optimization	108
3.3 Sensitivity considerations	113
3.4 Summary	121
Chapter Four	
4.0 Quantization effects in digital systems	138
4.1 Quantization of the products of multiplications	142
4.2 Roundoff noise analysis	144
4.3 Polyphase network structures and their roundoff noise characteristics	147

4.4	Dynamic range constraints	154
4.5	The effects due to correlated errors	160
4.6	Summary	163
Chapter Five		
5.0	Carrier-telephone system specifications	184
5.1	The channel-filter design procedure	185
5.2	Channel-filter designs meeting stopband requirements	187
5.3	The polyphase network amplitude response correction	197
5.4	The proposed baseband channel-filter	201
5.5	Summary	207
Chapter Six		
6.0	Conclusions and comments	234
6.1	Future research	240
Appendix A		243
References		246

LIST OF ILLUSTRATIONS

Page No.

Fig. 1.1	A representation of the digital system	39
1.2	Formation of the basic group (type B)	40
1.3	The Hartley or phase-splitting modulator	41
1.4	The Weaver modulator	"
1.5a	The baseband spectrum	42
1.5b	Double sideband ($f_c > 2f_2$)	"
1.5c	" " with aliasing	"
1.6	The 'sampled' Weaver modulator	43
1.7	Spectrum evolution	"
1.8	A non-recursive filter (lowpass) amplitude response	44
1.8b	The response after equi-value sample insertion	"
Fig. 2.1	The channel-filter passband (CCITT recommendations)	80
2.2	The digital Weaver modulator (European norms)	81
2.3	Quadrature or phase-difference modulator	82
2.4	Modulation of the baseband	83
2.5	The serial-parallel pipeline type multiplier	84
2.6a	Non-recursive Hilbert Transformer (non-min multiplier)	85
2.6b	" " " (min multiplier)	"
2.7a	" " " (n=55, ideal coeffs) response	86
2.7b	" " " (6-bit coeffs)	"
2.8	The lowpass-highpass 'phase-splitting' network	87
2.9	The 1-st degree allpass network	88
2.10	The allpass interwoven structure	"
2.11	The 'nested' allpass structure	89
2.12	The interwoven structure phase-difference response	90
2.13	1-st degree cascade structure " "	"
2.14	The nested " " "	"
2.15	The one-port ladder network	91
2.16	The digital wave-filter representation	"
2.17	The two-port 'series-L'	92
2.18	" 'series-C'	"
2.19	Digital wave-filter plus input adaptor	93
2.20	" " (types 1,2) phase-difference responses	94

Fig. 3.1	One section of the 4-phase RC-network	122
3.2	Positive and negative sequence phasor relationships	"
3.3	The SSB modulator utilizing the polyphase network	123
3.4	The input signal resolved into symmetric components	"
3.5	The non-symmetrical baseband frequency response	124
3.6	Digital equivalent of the 4-phase RC-network	125
3.7a	The digital i^{th} section showing 4 paths	126
3.7b	The minimum multiplier realization	"
3.8	Phase-difference response for various L_2 approxs.	127
3.9	" " " 4-section networks	128
3.10	" " " 8-section "	129
3.11	Sideband filter & phase-difference relationships	130
3.12	Stopband sensitivity index (8-section network)	131
3.13	" " " (allpass ")	132
3.14	8-section responses - various coefficient accuracies	133
3.15	The sideband filter stopband response	134
3.16	Trivial section network responses	135
3.17a	Section no. vs 0.1° low-frequency extremal	136
3.17b	" " " high-frequency "	"

Fig. 4.1	The general digital network model	165
4.2	The i^{th} to j^{th} node relationship	"
4.3	The digital polyphase network model	166
4.4a	The type 1A structure	167
4.4b	" 1B "	"
4.5a	" i sub-structure	168
4.5b	" ii "	"
4.6a	" 2A structure	169
4.6b	" 2B "	"
4.7a	" iii sub-structure	170
4.7b	" iv "	"
4.8	Predicted noise spectrum - type 1Ai	171
4.9	" " " " 1Aii	"
4.10	Estimated " " " 1Ai	172
4.11	" " " " 1Aii	"
4.12	Predicted " " " 1Bi	173
4.13	" " " " 1Bii	"
4.14	Estimated " " " 1Bi	174
4.15	" " " " 1Bii	"

4.16	Predicted noise spectrum - all-triv. sect.	175
4.17	" " " mod. " "	"
4.18	Estimated " " mod. " "	176
4.19	" " " " "	"
4.20	Predicted " " type 1Ai (mod. 1 st sect)	177
4.21	Estimated " " " "	"
4.22	Predictedd " " type 1Ai scaled	178
4.23	" " " " 1Bi "	"
4.24	Estimated " " " 1Ai "	179
4.25	" " " " 1Bi "	"
4.26	The i sub-structure decay response	180
4.27	Polyphase network decay response	181
4.28	" " transient response	182
Fig. 5.1	British channel-filter specs.	209
5.2	" " modified design	210
5.3	Trivial sect. network order vs bandwidth	211
5.4	Channel-filter attenuation - 7-section network	212
5.5	" " 8-section "	213
5.6	" " " 4-bit coeffs.	214
5.7	" " " 3-bit "	215
5.8	" " (coeffs opt in discrete space)	216
5.9	Univariate search performance $q = 0.125$	217
5.10	" " " $= 0.0625$	218
5.11	" " " $= 0.03125$	219
5.12	9-sect (2-stage) network stopband response	220
5.13	" " " amplitude "	221
5.14	Corrected amplitude response	222
5.15	" " " "	223
5.16	Modified trivial section	224
5.17	Amplitude correcting filter	224
5.18	Corrected passband response (ideal coeffs)	225
5.19	" " " 5,6-bit "	226
5.20	" " " (6-bit,disc space opt)	227
5.21	Proposed phase-splitting network	228
5.22	Effective frequency response	229
5.23	Passband with addition of guardband filter	230
5.24	Stopband of the 10-section network	231
5.25	Simulated lower & upper sideband time response	232

LIST OF SYMBOLS AND ABBREVIATIONS.

A/D	analogue to digital	
D/A	digital to analogue	
C.C.I.T.T.	Consultative Committee on International Telephony and Telegraphy	
F.D.M.	Frequency Division Multiplex	
S.S.B.	single sideband	
F.I.R.	finite impulse response	
T.D.M.	Time Division Multiplex	
z	unit advance	$z = e^{j\omega T}$
z^{-1}	unit delay	$z^{-1} = e^{-j\omega T}$
ω	angular frequency	$\omega = 2\pi f$ (rads.)
Ω_s	angular sampling frequency	(rads.)
ω_c	angular carrier frequency	(rads.)
T	sampling period	$T = 1/f_s$ (secs.)
$x(t)$ ($y(t)$)	continuous time input (output) signal	
$x(n)$ ($y(n)$)	discrete time input (output) signal	
$H(p)$	analogue complex transfer function	$p = \sigma + j\omega$
$H(z)$	digital complex transfer function	

n_o	wordlength leaving A/D convertor
n_a	bits added to reduce internally generated noise
q	quantum step size
$e_j(n)$	rounding error introduced at j^{th} node
$N_y(\omega)$	output roundoff noise spectrum
σ_o^2	white noise variance $= q^2/12$
$\hat{N}_y(\omega)$	normalized output roundoff noise spectrum $= N_y(\omega) / \sigma_o^2$
$ H $	amplitude response
S	sensitivity index
H^*	complex conjugate
$\beta(\omega T), \varphi(\omega T)$	phase response
δ	phase deviation (from 90°)
β_1, β_2	real digital network coefficients

note: symbols used otherwise will be defined specifically.

CHAPTER ONE.

Lend thy serious hearing to what I shall unfold.

Shakespeare.

1.0 Introduction

On the 10th March 1876, a brief conversation between two men demonstrated the then novel idea of 'electric speech'. Alexander Graham Bell had shown that speech communication, other than by acoustic means, was entirely practicable. The foundations had thus been laid for what has become a vast international telecommunication network. Throughout the development of this network, the concept that speech communication must be possible between two parties, regardless of distance or separating terrain, has been clearly defined.

Because of the ever-increasing demand for more communication channels, the telecommunication industry is constantly seeking technological improvements. These improvements, whether via an increased cost-efficiency or perhaps an improved equipment reliability, are currently being sought. One particular area of research attracting a great deal of interest is that of speech-signal processing by 'digital' means. Some of the problems encountered in the 'digital' processing of telephone speech signals are examined in this thesis. In particular, emphasis is given to the use of digital signal processing techniques in the production of a 'Frequency-Division-Multiplexed' group of telephone speech signals.

A signal (defined as a waveform conveying information) is represented mathematically as a function of one or more independent variables. Following convention, a speech signal is represented as a function of time or frequency. Signals(systems) defined in continuous time

are known as continuous-time signals (systems), whilst those defined only at discrete time intervals are known as discrete-time signals (systems). In addition, the dependent variable of the signal (system) may itself be represented in a continuous or discrete form. By definition, digital signals (or signal-processing systems) are those signals (systems) for which all variables are discrete - analogue signals (systems) being represented by continuous variables. The equivalence of signals represented in either form has been shown to exist⁽¹⁾.

Digital systems, especially digital computers designed for use as computational aids, have been in existence for many years⁽²⁾ and, with their advancing sophistication, came the facility of digitally processing signals either in 'non-real' or 'real' time. It was at last possible to process signals which could only be handled with great difficulty in the corresponding analogue systems (for example, the analysis of low frequency signals encountered in seismology).

Furthermore, the introduction of special-purpose algorithms (e.g. the Fast Fourier Transform⁽³⁾) made possible techniques such as spectrum analysis, correlation measurements etc. The technical innovations resulting from the development of the large general-purpose computer were applied in the design and implementation of small special-purpose 'mini'- or 'micro'- computers. Uses for these small computers were soon found in almost all branches of science and technology. This large-scale use thus encouraged further development and increased production of the digital hardware used in the computer construction (e.g. logic devices, storage registers etc) thereby reducing the overall manufacturing costs per item. Better production techniques

and increased demand allow still further cost reductions whilst the design sophistication (e.g. operation speeds, power consumption, packaging etc.) is constantly being improved on. In present-day signal-processor system design, the distinction is generally made between (i) processing performed on a programmed mini- or micro-computer and (ii) the 'hard-wired' signal-processor (i.e. a particular algorithm is realized via software in the computer compared with a hardware realization using digital devices), although the performance of the systems can be made identical. How a particular algorithm is realized is governed basically by economic considerations - a hardware realization would be used in a large-scale production of a particular digital design whereas the programmed computer is more useful for research purposes.

Although digital technology is not static, a defined system terminology⁽⁴⁾ has been agreed upon. The digital system, with its three basic comprising elements (the adder, the delay and the coefficient multiplier) extracted, is as shown in fig. 1.1. Literature introducing digital signal processing techniques and applying the mathematical ideas of linearity, causality and time-invariance is readily available⁽⁵⁻⁹⁾. Many of the mathematical techniques known in control theory (e.g. linear difference equations⁽¹⁰⁾ z-transform theory⁽¹¹⁾ etc.) have been adopted for use in digital system theory. With regard to network theory, design procedures have been proposed for digital structures possessing analogue equivalents⁽¹²⁻¹⁴⁾ and also for structures which may be confined to a digital implementation only^(15,16). It is important to note, however, that in any digital design, the stability of the system must be ensured - that is, it is necessary that⁽⁸⁾ :-

$$\sum_{n=0}^{\infty} |h(n)| < \infty$$

and

$$\lim_{n \rightarrow \infty} h(n) = 0$$

1.0

where:- $h(n)$ is the unit impulse response of the digital system.

The references given only briefly cover some of the many applications to which digital signal processing techniques have been applied. The general topic of this thesis is one such application - that is, the use of digital signal processing in the production of a 'Frequency-Division-Multiplexed' group of single-sideband signals; particularly the multiplexing of telephone speech signals to form the basic channel group⁽¹⁷⁾. The problems encountered in designing an SSB/FDM (digital) system can be seen more clearly if the system operation is examined in detail. However, the problems peculiar to digital SSB/FDM systems will become more apparent if initial consideration is given to the various analogue SSB/FDM designs.

1.1 Analogue SSB/FDM system theory

From about 1920 to the present day, the number of telephones throughout the world is estimated to have increased from approximately 20 million to a figure in excess of 400 million. This level of expansion could not have been achieved without the facility to transmit more than one channel simultaneously along a cable or on a radio carrier. The process used to obtain this multi-channel capability is known as multiplexing. A frequency multiplexing system requires that the baseband channel (the speech-signal frequency spectrum extends from 300 to 3400 Hz.,

although the total channel bandwidth extends from 0 to 4000 Hz.) be shifted to some higher frequency; each channel being allocated a separate 'slot'. Following C.C.I.T.T. recommendations⁽¹⁷⁾, twelve single-sideband suppressed-carrier speech signals are individually modulated to form a basic group: either the group type A (12 to 60 KHz.) or the group type B (60 to 108 KHz.), shown in fig. 1:2. Five of these groups are combined to form a supergroup (312 to 552 KHz) and, in turn, a combination of five supergroups forms a mastergroup (a 1.3 MHz. bandwidth 300-channel system). Technological developments have made possible the formation of still higher order groups: for example, the present European 60 MHz. system⁽¹⁸⁾ (allowing 10,800 speech channels or a combination of speech and video channels) is formed from twelve supermastergroups (1 supermastergroup = 3 master-groups). However, as the overall accuracy of these higher order groups can be related to that of the basic groups and because the multiplexing operation is similar in principle at all stages, only the formation of the basic group type A or type B is considered. This is particularly relevant with regard to the digital FDM system as the operation speed limitations of present-day digital hardware rule out signal processing on all but the basic group.

In forming a multiplexed group from the baseband channels, it is necessary to consider the stage at which the single-sideband is produced (the possibilities being that of bandpass filtering at the carrier frequency or a non-symmetrical bandpass filtering about zero frequency). For systems operating at a high carrier frequency, however, the accuracy requirements placed on the bandpass filter can prove too severe (i.e. for a bandpass filter of 4 KHz.

bandwidth, working in the range of 10^5 Hz. and greater, a satisfactory transition ratio of stopband to passband could only be produced, if at all, by filters of such a high order as to be impracticable). This disadvantage is overcome by utilizing several stages of modulation and filtering - with, of course, the production of sidebands at the baseband frequency proving a useful alternative method. This latter method has proved so important (especially with regard to digital modulation systems) that it is examined in more detail.

1.2 Single-sideband filtering at baseband frequencies

In 1928, a modulation system was proposed by Hartley⁽¹⁹⁾, the outline of which is shown in fig. 1.3. In this 'Hartley' (phase-difference, phase-splitting or quadrature) modulator, the input signal $x(t)$ is filtered by two networks in parallel; each network having unity gain and producing a phase-shift such that the difference between the two outputs is approximately 90° over a finite frequency range f_1 to f_2 . The output signals ($u_a(t)$, $u_b(t)$) from the 'phase-splitting' network, in adjacent paths, are modulated by cosinusoidal and sinusoidal carrier waves respectively. Then, following addition or subtraction, the lower or upper sideband is formed, situated to one side of the carrier. The phase-splitting network is represented by a complex transfer function:-

$$H(p) = H_a(p) + j.H_b(p) \quad 1.1$$

where:- $H_a(p)$, $H_b(p)$ are two real transfer functions representing two real and separate networks.

(Note: these networks are real in that they can be realized using real coefficients.)

Use of this complex transfer function allows us to think in terms of a 'non-real' sideband filter (i.e. the filter is non-symmetrical about the origin). For an input $x(t) = e^{j\omega t}$, we obtain at the phase-splitter output:-

$$\begin{aligned} u_a(t) &= H_a(p) \cdot e^{j\omega t} \\ u_b(t) &= H_b(p) \cdot e^{j\omega t} \end{aligned} \quad 1.2$$

and, after modulation and subtraction:-

$$\begin{aligned} z(t) &= \frac{1}{2} \cdot \left\{ H_a(p) + j \cdot H_b(p) \right\} \cdot e^{j(\omega + \omega_c)t} \\ &+ \frac{1}{2} \cdot \left\{ H_a(p) - j \cdot H_b(p) \right\} \cdot e^{j(\omega - \omega_c)t} \end{aligned} \quad 1.3$$

It is now conventional to represent the carrier by a sinusoid and the input signal by an exponential⁽²⁰⁾. In this way, the repeated use of trigonometric functions is avoided whereas the production of two output frequencies for each modulated input frequency can be clearly seen.

For the ideal phase-splitting network we require:-

$$\begin{aligned} H_a(j\omega) &= e^{-j\beta_1} \\ H_b(j\omega) &= e^{-j\beta_2} \end{aligned}$$

and

$$\beta_1 - \beta_2 = \pi/2 \quad 1.4$$

(Note: $\beta = \beta(\omega T)$ is an odd function of frequency)

By noting:-

$$-j.H_b(p) = j.H_b(-p)$$

and

$$H_b(-j\omega) = e^{j\beta_2} \quad 1.5$$

equation 1.3 can be rewritten as:-

$$\begin{aligned} z(t) = & \frac{1}{2} \cdot \left\{ H_a(p) + j.H_b(p) \right\} \cdot e^{j(\omega + \omega_c)t} \\ & + \frac{1}{2} \cdot \left\{ H_a(-p) + j.H_b(-p) \right\} \cdot e^{j(\omega_c - \omega)t} \end{aligned} \quad 1.6$$

and, on substituting eqns. 1.4 and 1.5 in 1.6, we obtain:-

$$z(t) = e^{j\psi} \cdot e^{j(\omega_c - \omega)t} \quad 1.7$$

Equation 1.7 represents the lower sideband plus phase-shift (ψ). The term $e^{-j\psi} \cdot e^{j(-\omega_c + \omega)t}$ (the negative frequency lower sideband) is not obtained as only the positive frequency input ($e^{j\omega t}$) has been considered.

More detailed theory⁽²¹⁾ and literature giving the analytic signal representation of modulated waveforms^(22,23) has been published although in this research area the major topic of interest has become the design of wideband 90° phase-splitting networks. In solving the phase-splitting network design problem, solutions of an empirical nature were proposed by Dome⁽²⁴⁾, Luck⁽²⁵⁾ and Rowlands⁽²⁶⁾ but, with later publications by Darlington⁽²⁷⁾, Saraga⁽²⁸⁾ and Orchard⁽²⁹⁾, the whole design theory was placed on an analytical basis. (The equivalence

of these latter solutions has in fact been shown⁽³⁰⁾.) With the availability of digital computer analysis, the publication of normalized design tables has followed. Theoretical designs of a high degree of accuracy can now be produced but, because of the limitations of the phase-splitting network's active or passive components⁽³²⁾ (i.e. their degree of stability through temperature changes, the effects of ageing etc.), the theoretically predicted phase-response cannot be maintained in practice. It is for this reason that the phase-splitting modulation method cannot compete with the conventional analogue system utilizing one or more stages of modulation plus bandpass filtering.

Alternative baseband SSB modulation systems have been proposed⁽³⁰⁻³⁵⁾; the most notable being that due to Weaver⁽³⁶⁾. This system, shown schematically in fig 1.4, differs from the quadrature modulator in that it uses two stages of modulation. In addition, the phase-splitting network is replaced by two lowpass filters (one per signal path). Because of the double modulation, four sidebands are produced of which only one is required. Two of these unwanted bands are suppressed by the lowpass filters; the remaining unwanted band being eliminated by a mutual-cancellation. The Weaver system proves advantageous in that any sideband leak occurring (due to an imperfect cancellation) appears 'in-band' as opposed to the 'out-of-band' leak of the quadrature modulator. The problems associated with carrier leak are increased, however, as this occurs in the middle of the signal band and not at the band edge as is the case in the quadrature modulator. The need for the additional modulators must also be borne in mind if comparisons of this and other systems are made.

The problems common to these various analogue modulation systems can be summarised as follows:-

- (i) a theoretically designed response is often unobtainable due to the limited precision of the passive components. (Although a one-off design can be 'tailored' to meet a desired performance characteristic, for large-scale production, the system response should be related in some way to the component tolerancing.)
- (ii) the analogue networks are sensitive to the effects of components ageing or temperature changes thus making it difficult to maintain a constant response.
- (iii) difficulties are encountered in the design and use of the modulators (i.e. the modulator, being an analogue circuit, suffers from the accuracy limitations of (i) and (ii).

In a digital system, however, these problems do not occur and, furthermore, the system accuracy is only a function of the cost (within reasonable operating frequencies). It is therefore understandable that a great deal of interest has been shown in the digital realization of the group type A or type B FDM system. Some digital SSB/FDM designs proposed by various authors are now briefly described.

1.3 Digital SSB/FDM systems

Consideration is now given to single-sideband filtering utilizing digital signal processing techniques. Because of the repeating

spectra associated with sampled signals⁽⁵⁾, it is necessary to assume a baseband signal restricted to a frequency $\leq f_o$, with a sampling frequency $(f_s) > 2 \cdot f_o$. (It is also assumed that the baseband signal has a lower cutoff frequency greater than zero - a necessary condition of SSB modulation⁽²³⁾). We see, by reference to fig 1.5, that the required single-sideband signal is not obtained on using the displacement theorem as this refers to a shift in the complex spectrum.

From the sampling theorem⁽⁵⁾, however, where:-

$$x(t) = \sum_{k=-\infty}^{\infty} x(kT) \cdot \frac{\sin\{\omega_o/2 \cdot (t - kT)\}}{\omega_o/2 \cdot (t - kT)} \quad 1.8$$

and:-

$$T = 1/f_s = \pi/\omega_o \quad 1.9$$

also:-

$$x(t) = 1/\pi \cdot \text{Real} \int_0^{\omega_o} X(j\omega) \cdot e^{j\omega t} d\omega \quad 1.10$$

and:-

$$X(j\omega) = T \cdot \sum_{k=-\infty}^{\infty} x(kT) \cdot e^{-j\omega kT} \quad 1.11$$

it is seen that for the real amplitude spectrum $Y(j\omega)$ (the single-sideband spectrum), it is necessary that:-

$$Y(j\omega) = X(j\omega) \cdot e^{j\omega_c kT} \cdot F(j\omega) \quad 1.12$$

(for a positive frequency shift $e^{j\omega_c kT}$ only)

where:-

$$\begin{aligned} F(j\omega) &= 1 & \omega_c < |\omega| < \omega_c + \omega_o \\ &= 0 & \text{elsewhere} \end{aligned} \quad 1.13$$

For the sideband signal $y(t)$, we have:-

$$\begin{aligned} y(t) &= 1/\pi \cdot \text{Real} \int_0^{\infty} Y(j\omega) \cdot e^{j\omega t} d\omega \\ &= 1/\pi \cdot \text{Real} \int_{\omega_c}^{\omega_c + \omega_o} X(j\omega) \cdot e^{j\omega_c kT} \cdot e^{j\omega t} d\omega \end{aligned} \quad 1.14$$

which, on substituting eqn 1.11, becomes:-

$$y(t) = T/\pi \cdot \sum_{k=-\infty}^{\infty} x(kT) \text{Real} \int_{\omega_c}^{\omega_c + \omega_o} e^{j(t - kT)\omega} \cdot e^{j\omega_c kT} d\omega \quad 1.15$$

Using the substitution $\omega_m = \omega_c + \omega_o/2$, it can be shown that eqn 1.15 becomes:-

$$y(t) = \sum_{k=-\infty}^{\infty} x(kT) \cdot \frac{\sin\{\omega_o/2 \cdot (t - kT)\}}{\omega_o/2 \cdot (t - kT)} \cdot \cos(\omega_m t - k \cdot \pi/2) \quad 1.16$$

A circuit, derived from eqn 1.16, was proposed by Darlington⁽³⁷⁾. A closer examination of this circuit (represented in fig. 1.6) shows it to be a discrete-time version of the analogue Weaver Modulator⁽³⁶⁾. The sampled impulse response of the required filter is given by the sinc term in eqn. 1.16 (i.e. a lowpass filter with a cut-off frequency $\omega_o/2$). A problem peculiar to any discrete-time modulation circuit can be identified whilst examining the operation of this particular circuit. The operation (with the spectra, at various points in the circuit, shown in fig. 1.7) is as follows:- the 4-tap commutator routes samples to the positive or negative (inverting) inputs of the lowpass filters (this being a simple method of implementing the first stage modulators $\sin(\omega_o kT/2)$, $\cos(\omega_o kT/2)$). Then, after lowpass filtering (in which two out of four sidebands are suppressed), the second stage of modulation follows. The third sideband is suppressed by a mutual cancellation leaving the required single-sideband. It is clear from fig. 1.7, however, that, unless $\omega_s \gg 2\omega_o$, no second stage modulation can be performed without producing aliasing. A similar situation now exists as in the analogue modulation method utilizing bandpass filtering at the carrier frequency - that is, the stopband to passband transition bandwidth of the lowpass filter will be made extremely small in satisfying the sampling frequency constraint $\omega_s \gg 2\omega_o$ and, as such, the required filter complexity will be high. The advantage of processing the signals at the baseband frequency at first seems lost; however,

after incorporating a modification suggested by Darlington⁽³⁸⁾ (which involves making the lowpass filtering a two-stage operation) this particular problem can be overcome. The two-stage filtering operation is in fact the required lowpass filter, operating at a low sample-rate (thereby eliminating the requirement for a filter of high complexity), in cascade with an interpolating filter. The interpolation process increases the effective sample-rate (and therefore the interpolator must operate at this high sample-rate) before the high frequency modulation is performed. Several system designs, all based on the digital Weaver modulator and utilizing this two-stage filtering operation, have been proposed⁽³⁹⁻⁴²⁾.

A digital system design based on the quadrature or phase-difference modulator (but once again using the two-stage filtering operation) has been proposed⁽⁴³⁾. For the phase-splitting filter - two allpass networks (each operating at the low sample-rate of 16 KHz) are used. The required high sample-rate is then obtained after a digital interpolation is performed. Also required in this system, however, is some form of pre-filtering (lowpass) to eliminate the redundant spectrum (4KHz. to $\omega_s/2$) as, in the digital Weaver circuit, the lowpass filters following the first-stage modulators satisfy this requirement. This does not prove too disadvantageous and, furthermore, comparisons between designs based on either modulation method have shown the digital phase-difference system to be highly competitive⁽⁴³⁾. This is considered in greater detail (Chapter 2) when the C.C.I.T.T.⁽¹⁷⁾ recommendations on telephony systems are included.

The advantage of processing low sample-rate baseband signals followed by a sample-rate increase (obtained using a digital interpolator) has been emphasised. The design of a suitable interpolating filter is therefore considered.

1.4 Digital Interpolation

The method of obtaining an unknown intermediate sample value between two known sample values is defined as interpolation. For a digital sample-rate increase, the known sample values are used in calculating the unknown values. In the digital interpolator, this can be performed in two separate ways as follows:-

- (i) the sample stream is in a serial form and the interpolated values are obtained serially⁽⁴⁴⁾ (i.e. all operations are performed at the fast sample-rate).
- (ii) the serial sample stream is converted to a parallel form and the interpolated values are obtained by arithmetic operations done in parallel using the original slow sample-rate. The various outputs are then added, with appropriate delays, to give a serial output at the required fast rate^(45,46).

In method (i), before the interpolation can be performed, the sample-rate must be made equal to the working speed of the filter. This is simply achieved by inserting $(L-1)$ equi-value or zero magnitude samples (for a new sample spacing T' where $T' = T/L$, L integer) between the known samples.

For the equi-sample insertion, we have:-

the input samples = $x(nT)$

the required fast sample-rate = $v(mT')$

Now:-

$$\begin{aligned} v(mT') &= x(nT + iT/L) \\ (0 \leq i \leq L-1) \end{aligned} \quad 1.17$$

and therefore:-

$$V(z) = \sum_{i=0}^{L-1} z^{-i} \cdot X_i(z^L) \quad 1.18$$

where:-

$$z = e^{j\omega T/L}$$

As the samples inserted are equi-value, it is clear that:-

$$X(z^L) = X_1(z^L) = X_2(z^L) \dots = X_{L-1}(z^L)$$

and therefore eqn. 1.18 can be rewritten as:-

$$\begin{aligned} V(z) &= X(z^L) \cdot \{ 1 + z^{-1} \dots + z^{-(L-1)} \} \\ \therefore V(e^{j\omega t}) &= X(e^{j\omega t}) \cdot F(e^{j\omega T'}) \end{aligned} \quad 1.19$$

i.e. the sample-rate increase is obtained although the lowpass filter response ($F(e^{j\omega T'})$) is superimposed on the folded signal spectrum (see fig. 1.8). The filter response is dependent on the number of samples inserted and although some lowpass filtering is desirable, the passband deviation it causes is not.

For the zero-magnitude sample insertion, we have:-

$$\begin{aligned} v(nT') &= x(nT/L) \quad n = 0, \pm L, \pm 2L, \dots \\ &= 0 \text{ elsewhere} \end{aligned}$$

and therefore:-

$$\begin{aligned} V(z) &= \sum_{n=-\infty}^{\infty} x(nT/L) \cdot z^{-n} \\ &= \sum_{n=-\infty}^{\infty} x(nT) \cdot z^{-nL} \\ &= X(z^L) \end{aligned} \quad 1.20$$

i.e. $V(e^{j\omega T'})$ is periodic with its effective period equal to $2\pi/T$ (the period of $X(e^{j\omega T})$). The required fast sample-rate can therefore be accurately obtained by inserting zero-magnitude samples and removing the repeat spectra (positioned between $\pi/T \leq \omega \leq \pi/T'$) using an interpolation filter operating at the $2\pi/T'$ sample rate.

It has been shown⁽⁴⁴⁾ that the interpolation filter requirements can be obtained by considering the sampling and reconstruction process.

We assume the input signal for interpolation to be bandlimited to ω_o and, as such, the continuous signal $x(t)$ can be reconstructed from its sampled signal by passing the samples $x(kT)$ through a lowpass filter with a cut-off frequency $\omega_o/2$, that is:-

$$x(t) = \sum_{k=-\infty}^{\infty} x(kT) \cdot \frac{\sin\{\omega_o/2 \cdot (t - kT)\}}{\omega_o/2 \cdot (t - kT)} \quad 1.21$$

Similarly, the same signal $x(t)$ would be reproduced from $x(mT')$

(where $T' = T/L$) as follows:-

$$x(t) = \sum_{m=-\infty}^{\infty} x(mT') \cdot \frac{\sin\{\pi/T' \cdot (t - mT')\}}{\pi/T' \cdot (t - mT')} \quad 1.22$$

Now, on evaluating eqn. 1.21 with $t = mT'$, a sequence $x(mT')$ is

obtained which, if substituted in eqn 1.22, shows that the

reconstruction to give $x(t)$ is still valid, i.e.:-

$$x(t) = \sum_{m=-\infty}^{\infty} \left\{ \sum_{k=-\infty}^{\infty} x(kT) \cdot \frac{\sin \omega_o/2 \cdot (mT' - kT)}{\omega_o/2 \cdot (mT' - kT)} \right\} \cdot \frac{\sin \pi/T' \cdot (t - mT')}{\pi/T' \cdot (t - mT')}$$

and therefore:-

1.23

$$x(mT') = \sum_{k=-\infty}^{\infty} x(kT) \cdot \frac{\sin\{\omega_o/2 \cdot (mT' - kT)\}}{\omega_o/2 \cdot (mT' - kT)}$$

1.24

From eqn. 1.24 (the impulse response of the interpolating filter),

it is immediately obvious that the summation must be truncated before

a practical filter can be realized. Only an approximation to the ideal

interpolator can therefore be obtained although design procedures for

filters which are optimal in some sense have been proposed^(44,47).

If we now consider the required interpolating filter for the group type B F.D.M. system, it is clear that a final sample-rate of at least 108 KHz is required. A sample-rate of 112 KHz is in fact chosen to allow for a guardband of 4KHz. In the first-stage filtering of the various F.D.M. systems, a 16KHz sample-rate is chosen as optimal in the compromise between (i) minimising the sample-rate in order to reduce the complexity of the Weaver modulator lowpass filters (a function of the passband to stopband transition bandwidth) or to reduce the complexity of the phase-difference network (a function of the low-frequency phase constraint - although reducing the sampling frequency increases the effective phase-response bandwidth thereby adding to the network complexity) and (ii) choosing a rate high enough such that the interpolating filters' complexity is not too severe. For the 16KHz. first-stage sample-rate, it is necessary for the interpolating filter to remove, in the half sampling range, three repeat spectra situated at 16, 32 and 48 KHz. (i.e. the baseband rate is then increased by a factor of seven). In the system design of Freeny⁽³⁹⁾, a 21-tap non-recursive interpolating filter was proposed for use in the digital Weaver circuit. For the sample-rates quoted, stopband attenuation (in 4KHz wide bands) at 16, 32 and 48 KHz, with a flat passband extending from zero to 2KHz, is a necessity. In the proposed design, the attenuation requirements are satisfied although the passband requirements are not; however, it was suggested that, rather than increase the complexity of this useful and relatively easily implemented filter, the passband of the first-stage filters should be redesigned to compensate for this distortion. This design has also proved useful in the realization of the digital phase-difference S.S.B. system⁽⁴³⁾.

As an alternative to single channel interpolation plus modulation, a method in which a group of channels are processed to form the required multiplex signal is sometimes employed. This group or 'block' processing method is now briefly introduced.

1.5 Block Processing

Several proposals on block processing are similar in that each uses a digital 'Fourier processor' working on a group of signals so that a multiplex signal is obtained with maximum computational efficiency. In certain of these block-processor schemes^(38, 48, 49) consideration is given to how the individual channels may be optimally combined. An alternative approach⁽⁵⁰⁾ assumes that the Fourier processor is used and the input signals required in producing the multiplex output are subsequently determined.

The original development⁽³⁸⁾ of a block-processing scheme followed on from the idea of making the lowpass filter (in the digital Weaver modulator) a two-stage operation - i.e. a 'slow' recursive filter followed by a 'fast' non-recursive interpolating filter. On considering a bank of these circuits, it is seen that advantages can be gained by transforming the individual interpolating filters plus modulators to become a block processing system.

For the group type B F.D.M. system, we have:-

number of channel slots plus guardbands $N = N_c + N_g$

baseband sample-rate $= 1/T$ (samples/sec)

multiplexed output signal sample-rate ... $1/T' = L/T$ samples/sec

signal input to the i^{th} lowpass filter	$= x_i(mT/L)$
signal input to the i^{th} interpolator	$= u_i(mT/L)$
i^{th} interpolator output signal	$= v_i(nT/L)$
i^{th} product-modulator output	$= p_i(nT/L)$
multiplex output signal	$= y(nT/L)$

where:-

$$0 \leq i \leq (2.N_c - 1)$$

As the baseband sample-rate is $1/T$ samples per second, it is clear that $x_i(mT/L) = 0$ except when $m = L.j$ (j integer). The output from the i^{th} interpolating filter is defined as:-

$$v_i(nT/L) = \sum_{m=n-kL+1}^n g[(n-m)T/L] \cdot u_i(mT/L) \quad 1.25$$

and the multiplex output signal is therefore given by:-

$$y(nT/L) = \sum_{i=0}^{2.N_c-1} p_i(nT/L) \cdot \sum_{m=n-kL+1}^n g[(n-m)T/L] \cdot u_i(mT/L) \quad 1.26$$

As $g(.)$ (the interpolating filter impulse response) is independent of the channel number, eqn. 1.26 can be rewritten as:-

$$y(nT/L) = \sum_{m=n-kL+1}^n g[(n-m)T/L] \cdot \sum_{i=0}^{2.N_c-1} p_i(nT/L) \cdot u_i(mT/L) \quad 1.27$$

As $p_i(nT/L) \cdot u_i(mT/L)$ is zero, except at $m = L.j$, the summation over m (from $n-kL + 1$ to n) need only be calculated k times per computation cycle (or only once per cycle for an interpolation filter of length L). We see therefore that by combining the fast interpolating filter with the modulators (so as to reverse the convolution and modulation),

an increase in the computational efficiency results. Against this, however, is the slight disadvantage that the amount of storage memory required is increased.

Finally, we examine a system^(50, 51) designed around the Fourier processor (i.e. the input signals required in producing the multiplex signal are determined). Once again, we define:-

the input sequence $\quad \quad \quad = x(mT)$

and $Z \{x(mT)\} \Rightarrow X(z^L)$

the output multiplex sequence $\quad \quad \quad = y(nT/L)$

and $Z \{y(nT/L)\} \Rightarrow Y(z)$

where:-

$$z = e^{j\omega T/L} = e^{j\omega T'}$$

Considering the output sequences as L interleaved (but originally separate) sequences i.e.

$$y(nT/L) \equiv y(nT + iT/L) \\ (0 \leq i \leq L-1)$$

gives:-

$$Z \left\{ y(nT + iT/L) \right\} = \sum_{i=0}^{L-1} z^{-i} \cdot Y_i(z^L)$$

1.28

In forming the multiplex sequence, a suitably positioned filter must effectively select the required spectra; the slow input being increased to a fast sample-rate during the interpolation process

(i.e. $x(mT)$ becomes $\hat{x}(nT/L)$). Again the fast sample stream $\hat{x}(nT/L)$ can be considered as L interleaved streams ($\hat{x}(nT + iT/L)$); this fast rate being produced by a filtering and interpolation operation performed by L slow filters working in parallel. The i^{th} sample of the sequence $\hat{x}(nT + iT/L)$ can therefore be related to the input sequence by the transfer function $F_i(z^L)$ (the i^{th} slow filter); the channel filter transfer function being given by:-

$$F(z) = \sum_{i=0}^{L-1} z^{-i} \cdot F_i(z^L) \quad 1.29$$

The relationship between the input and output for the first channel slot must be given by:-

$$X_o^{\wedge}(z) = \sum_{i=0}^{L-1} z^{-i} \cdot F_i(z^L) \cdot X_o(z^L) \quad 1.30$$

For a second signal to occupy the channel 1 slot, the filter $F(z)$ must be shifted along the frequency axis by an amount $1/T$ such that:-

$$F(e^{j(\omega - 2\pi/T)T/L}) = F(z \cdot e^{-j2\pi/L}) \quad 1.31$$

and therefore:-

$$X_1^{\wedge}(z) = \sum_{i=0}^{L-1} z^{-i} \cdot e^{j2\pi i/L} \cdot F_i(z^L) \cdot X_1(z^L) \quad 1.32$$

The multiplex signal is therefore formed by summing the $\hat{x}_k$ thus:-

$$\begin{aligned}
 Y(z) &= \sum_{k=0}^{L-1} \hat{x}_k(z) \\
 &= \sum_{k=0}^{L-1} \sum_{i=0}^{L-1} z^{-i} \cdot e^{j2\pi ik/L} \cdot F_i(z^L) \cdot X_k(z^L)
 \end{aligned}
 \tag{1.33}$$

Reference to eqn. 1.28 shows that eqn. 1.33 can be rewritten (using the substitution $w = e^{-j2\pi/L}$) as:-

$$Y_i(z^L) = \sum_{i=0}^{L-1} w^{-ik} \cdot F_i(z^L) \cdot X_k(z^L)
 \tag{1.34}$$

that is, the output samples are produced at the slow rate ($1/T$ samples per second) throughout; the output multiplex sequence being formed by interleaving the individual sequences.

It is important to note that as the channel passband was shifted by an amount $1/T$, the signal occupying the baseband spectrum ($-1/2T$ to $1/2T$) must be non-symmetrical (i.e. a complex signal) if the single-sideband is to be produced. A digital phase-splitting network (or the low-pass filters plus modulators of the Weaver circuit) must therefore be used for producing the required complex signal sequences.

1.6 Summary

The need for the facility to transmit more than one speech channel simultaneously along a cable or on a radio carrier has been expressed.

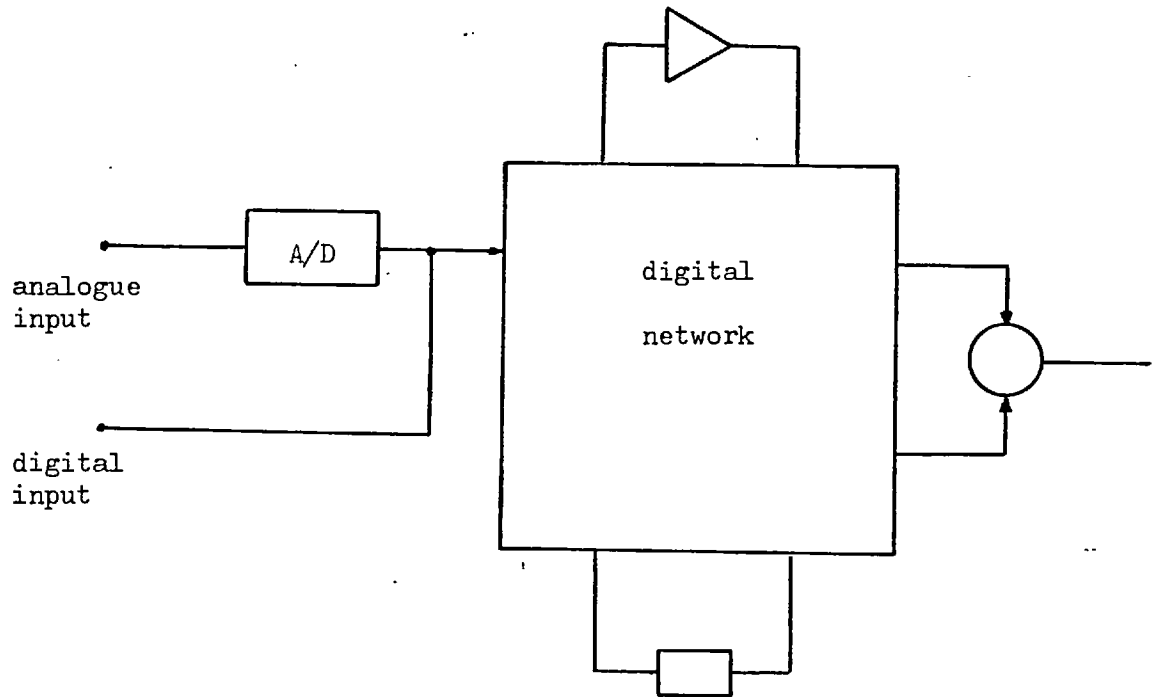
Presently operating F.D.M. systems have been realized using analogue circuitry but, with the increasing usage of digital technology, the trend towards the digital S.S.B./F.D.M. system is foreseen (this will prove especially useful for satisfying the interface requirements of the digital T.D.M. system⁽³⁹⁾). The problems encountered in F.D.M. system design have been outlined but with a greater emphasis placed on the digital F.D.M. system.

It has been shown that the digital S.S.B./F.D.M. system realization can be considered in two stages - that is,

- (i) a complex signal is produced at the baseband frequency. At present the digital realization of the phase-difference modulator or the Weaver modulator is favoured.
- (ii) the complex signal is processed so as to give a single sideband at the carrier frequency. This is achieved after an interpolation and modulation process or by block processing a group of baseband signals..

A more detailed comparison of digital F.D.M. designs based on the phase-difference and Weaver modulator is to be made. Possible improvements are sought, although the C.C.I.T.T. recommendations regarding channel filtering are to be satisfied at all times.

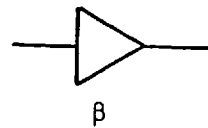
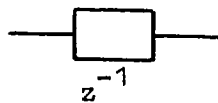
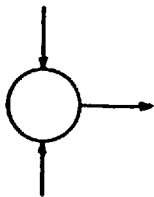
Fig. 1.1 A representation of the digital system.



adder

delay

coefficient multiplier



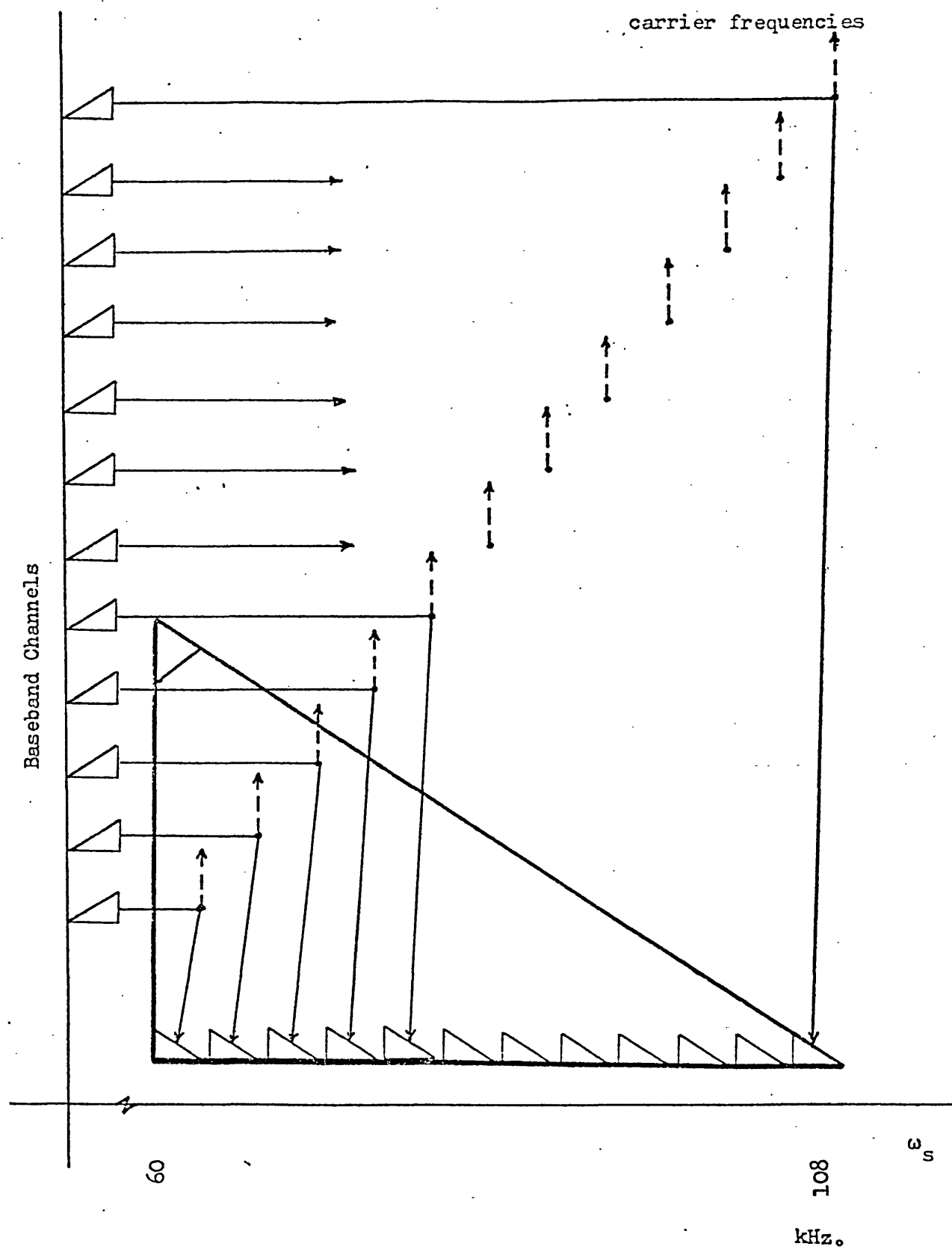


Fig. 1.2 Formation of the basic group (type B)

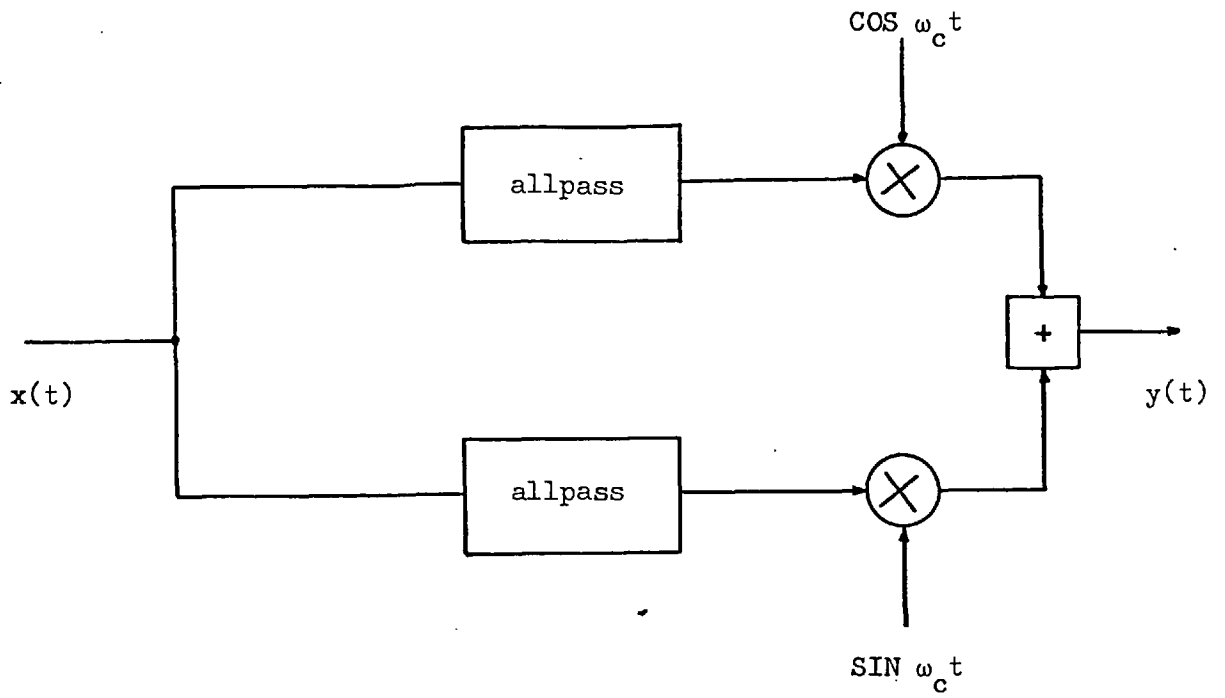


Fig. 1.3 The Hartley or phase-splitting modulator.

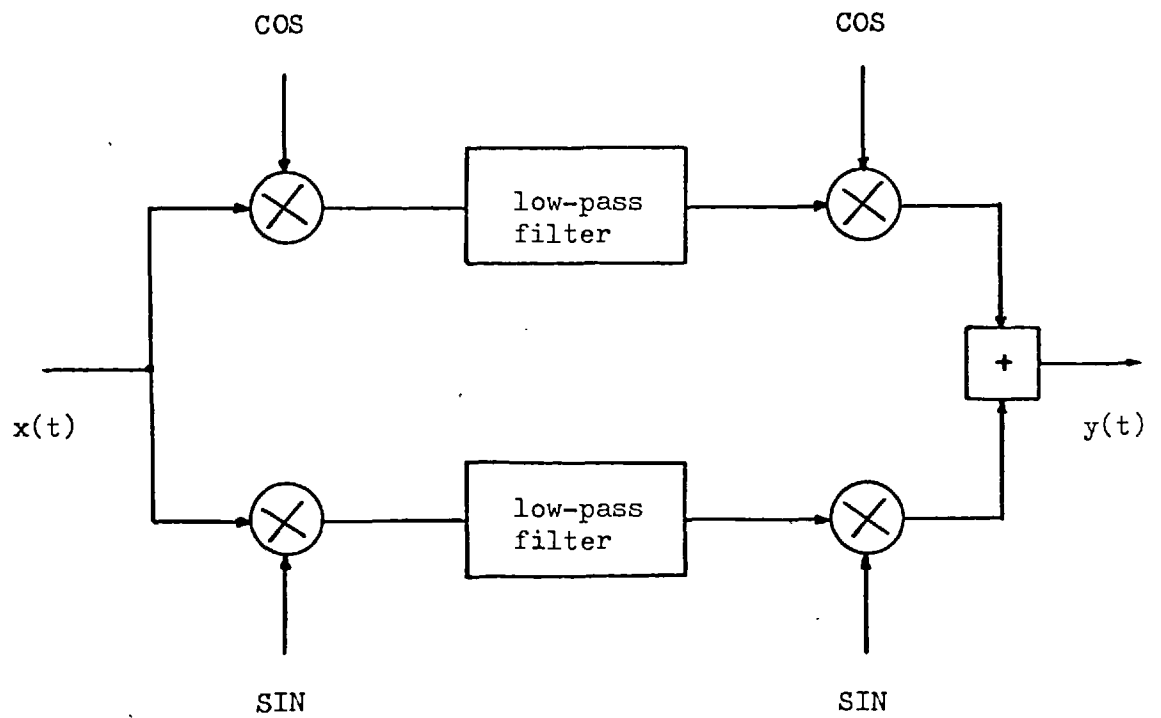


Fig. 1.4 The Weaver modulator.



Fig. 1.5a The baseband spectrum.

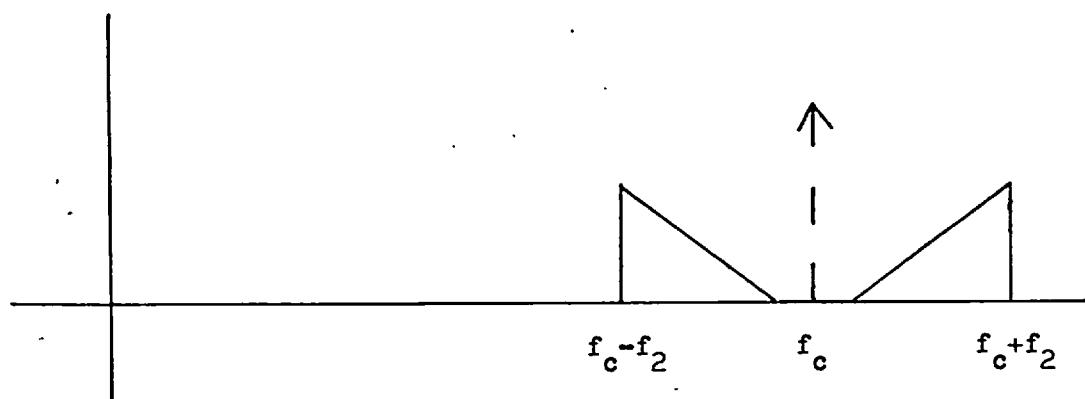


Fig. 1.5b The double sideband ($f_c \geq 2f_2$)

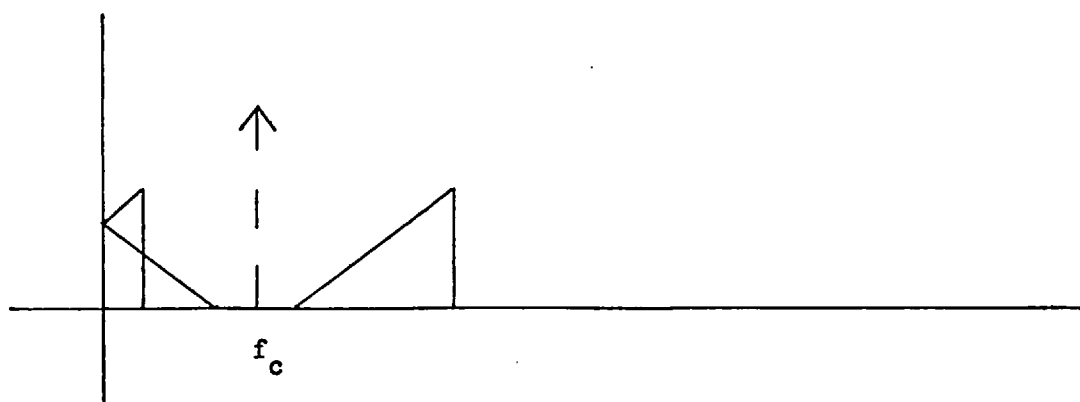


Fig. 1.5c The double sideband with aliasing ($f_c < 2f_2$)

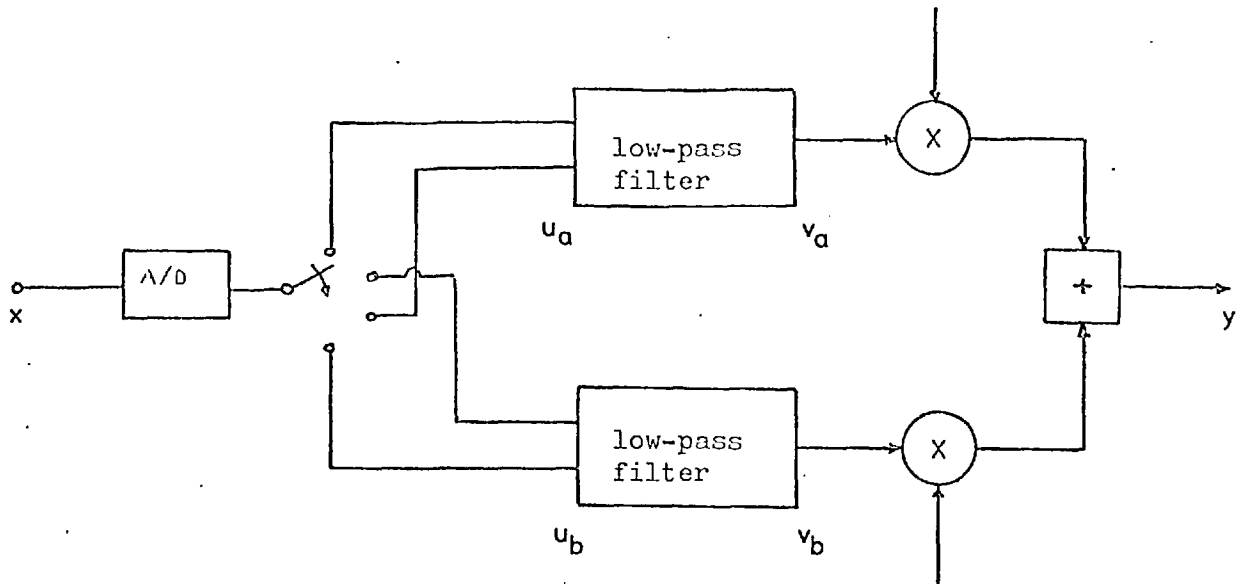


Fig. 1.6 The 'sampled' Weaver modulator.

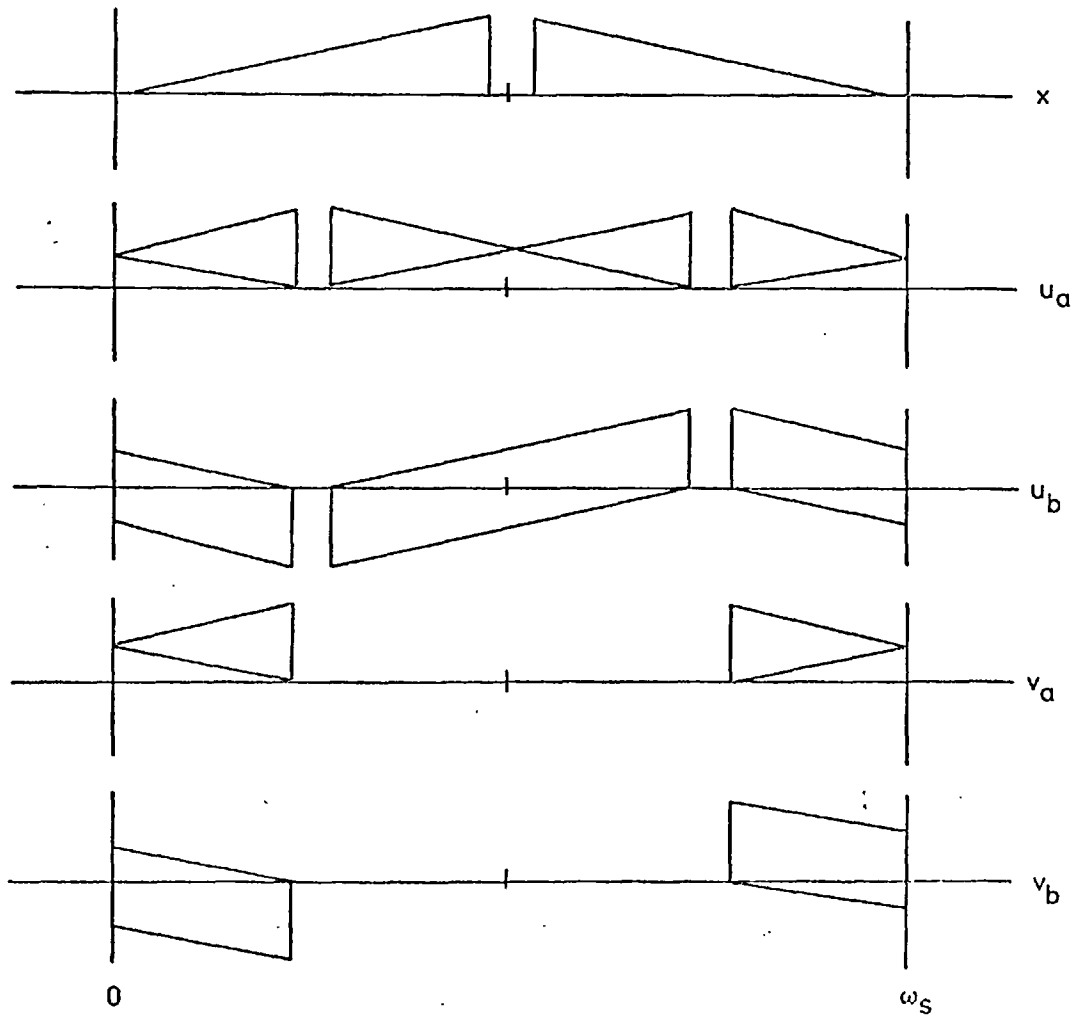


Fig. 1.7 Spectrum evolution in the sampled Weaver modulator.

dBs. Attenuation

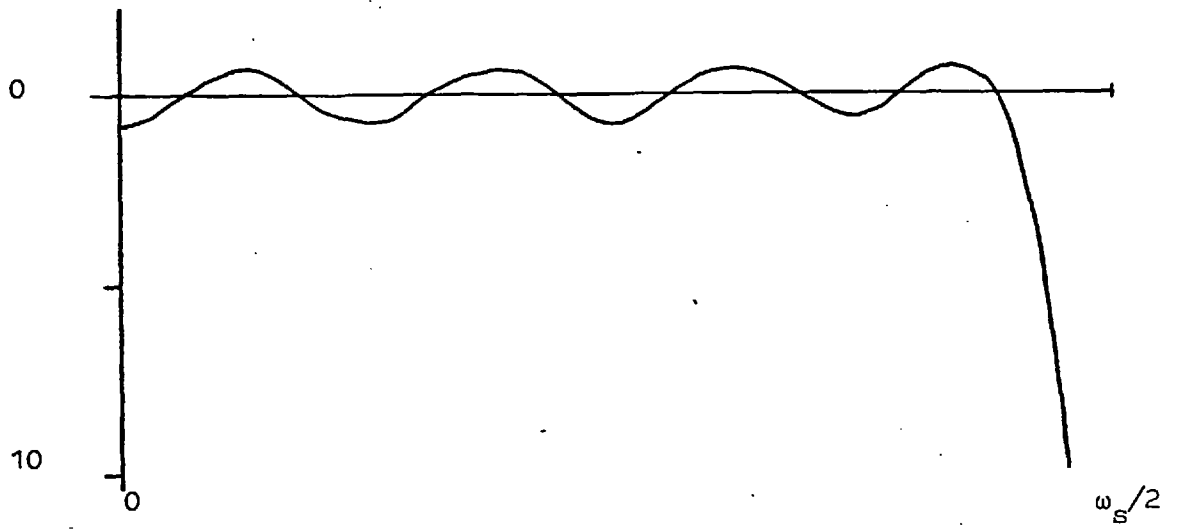


Fig. 1.8a A non-recursive filter (low-pass) amplitude response.

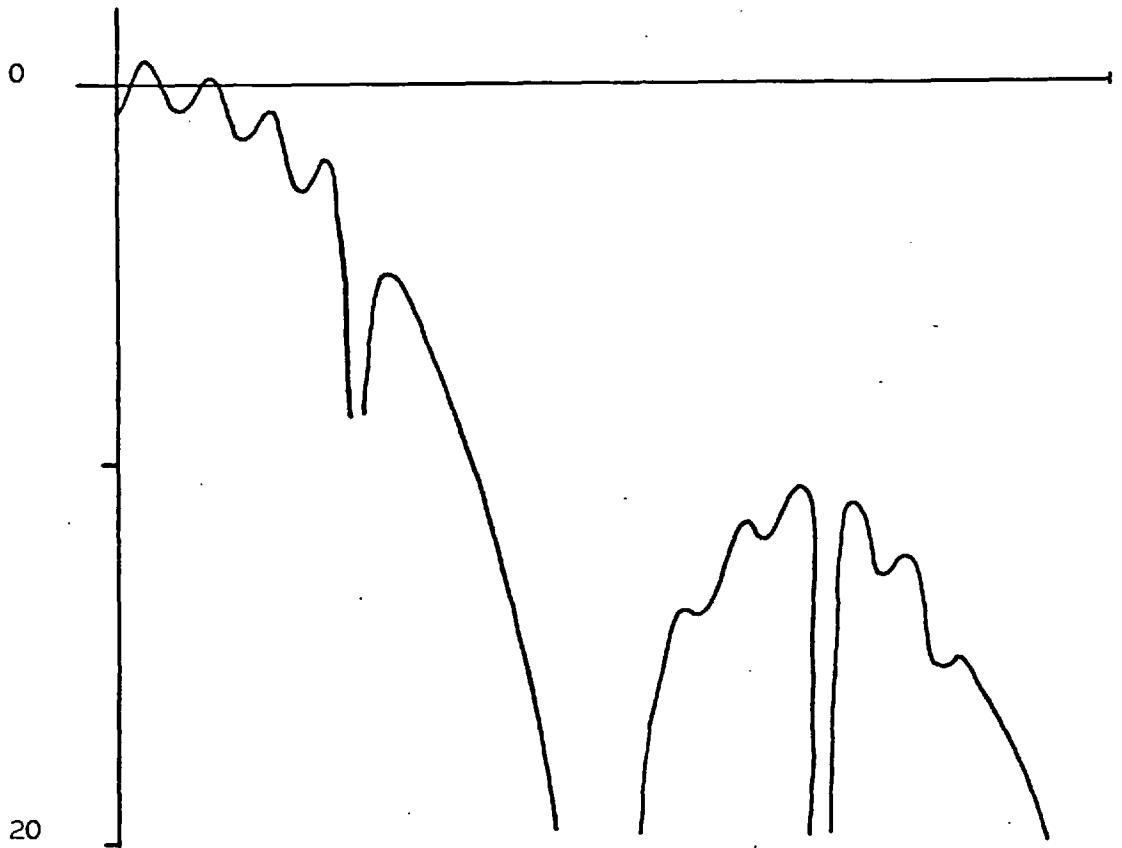


Fig. 1.8b The amplitude response after equisample insertion ($\omega_s^A = \omega_s \cdot 4$)

CHAPTER TWO.

2.0 DIGITAL S.S.B./F.D.M. THEORY

Several methods of realizing an S.S.B./F.D.M. system have been described. Two systems, the digital versions of the Weaver modulator and the quadrature or phase-difference modulator, are now examined in more detail with the hope that an improvement in the system performance (via a reduction in the required hardware complexity, for example) can be gained. In considering the performance of a particular circuit or system, it is necessary to make comparisons with a desired design specification. Throughout the present study, the specifications to be used are the C.C.I.T.T. recommendations on channel filtering ⁽¹⁷⁾ - these are outlined briefly in fig. 2.1.

The schematic representation of the digital Weaver and quadrature modulation circuits, designed ⁽⁴³⁾ to satisfy European norms, are shown in figs. 2.2 and 2.3 respectively. By comparing these circuits with the fairly general representations (figs. 1.3 and 1.4) it can be seen that various modifications have been incorporated. An analysis of the operation of each modulation method shows why these modifications are required. Firstly, however, the factors common to both systems are defined as follows:-

- (i) All input signals are (R.C.-active) pre-filtered before A/D conversion.
- (ii) The non-symmetrical or complex baseband signal is obtained with all signal processing performed at the low sample-rate of 16 KHz.

- (iii) The fast sample-rate of 112 KHz is obtained via an interpolating process. A non-recursive interpolating filter (length 21, 10-bit coefficient wordlength) is used to increase the sample-rate by a factor of seven.

The possibility of simplifying the interpolating filter seems unlikely as its exacting design specifications have already been relaxed by allowing its passband to 'droop'. (This must be compensated for at some stage in the overall system design, however.) For this reason, the fast sample-rate processing stage of each modulator is not examined in any greater detail. In addition, this stage will not be considered when system comparisons are made as the interpolating filters and high-frequency modulators are chosen to be identical in each system design. We therefore examine only the first stage of each modulator and make comparisons of one system with another on this basis.

2.1 The digital Weaver modulator

In the digital Weaver modulator (fig. 2.2), the baseband signal is split into two paths and, following various stages of modulation and filtering, the two signals are added together so that the unwanted sideband is cancelled out. In the earlier digital Weaver modulator designs⁽³⁸⁾, an initial modulation frequency of 2000 Hz. was specified; however, this has since been changed to 1850 Hz.

In this way, the complex signal spectrum, positioned between -1.7 to 1.4 KHz after a 2000 Hz. modulation, is shifted so that it occupies the symmetrical band between ± 1.55 KHz. (see fig. 2.4). Although this is advantageous

with regard to implementing the lowpass filters, the resulting complexity of the high-frequency modulators is thereby increased. A supplementary shift of 150 Hz. is therefore added to simplify the implementation of these high-frequency modulators (i.e. the final modulation frequency once again becomes 2000 Hz. added to the value, 4000 Hz. multiplied by the channel number). One further modification necessary is that of a partial redesign of the lowpass filters. This is required to account for the deviation (or 'droop') occurring in the passband of the interpolating filter. It has been shown ⁽³⁹⁾ that the redesign, apart from modifying the passband response and the numerical values of the multiplier coefficients, leaves all other performance characteristics unchanged.

2.2 The digital quadrature or phase-difference modulator

In the digital phase-difference modulation system (fig. 2.3), the complex signal is obtained at the output of a phase-difference or phase-splitting network. The complex transfer function, of which the real and imaginary parts correspond to the two paths of the phase-splitting network, is produced using two real allpass networks. At first sight, the phase-difference modulation method appears somewhat more complex than the Weaver method because an additional lowpass filter is required. This is used to eliminate the redundant spectrum extending from 4 to 12 KHz. (the lowpass filters in the Weaver modulator satisfy this requirement). However, the non-symmetrical passband

problem does not exist and, by including a supplementary shift of 2KHz., the fairly simple interpolating filter of the Weaver modulator design may be utilized (i.e. the resulting complex spectrum is shifted so as to be perfectly aligned with the passband of the interpolating filter). The problems associated with realizing this system are therefore confined to the design and implementation of the lowpass filter and the phase-splitting network.

The required accuracy of the phase-splitting network response, over a defined region $\omega_1 < |\omega| < \omega_2$, is obtained as follows:-

We assume an input signal $x(n) = e^{j(\omega nT + \phi)}$

and, at the output of the phase-splitting network, we have:-

$$v_a(n) = e^{j(\omega nT + \phi)}$$

$$v_b(n) = e^{j(\omega nT + \phi + \pi/2 + \delta)}$$

(δ is the deviation away from the required phase-difference response).

The two carriers (cosinusoidal and sinusoidal oscillations of frequency f_c Hz.) are modulated by $v_a(n)$ and $v_b(n)$ respectively. (By assuming $1/T \gg 2\omega_c$, we can neglect the increased sample-rate requirements.) After modulation and summing the signals, we obtain:-

$$\begin{aligned}
y(n) &= e^{j(\omega nT + \varphi)} \cdot \frac{e^{j\omega_c nT} + e^{-j\omega_c nT}}{2} \\
&+ e^{j(\omega nT + \varphi + \pi/2 + \delta)} \cdot \frac{e^{j\omega_c nT} - e^{-j\omega_c nT}}{2j} \\
&= \frac{e}{2} e^{j\{(\omega + \omega_c)nT + \varphi\}} \cdot \left\{ 1 + e^{j\delta} \right\} \\
&+ \frac{e}{2} e^{j\{(\omega - \omega_c)nT + \varphi\}} \cdot \left\{ 1 - e^{j\delta} \right\} \\
&= \Omega_u^+ \left\{ \frac{1 + e^{j\delta}}{2} \right\} + \Omega_L^- \left\{ \frac{1 - e^{j\delta}}{2} \right\} \quad 2.1
\end{aligned}$$

Now, for an exact phase-difference response (zero deviation), we obtain:-

$$y(n) = \Omega_u^+$$

i.e. the upper sideband. (The negative frequency upper sideband is not obtained as the input signal was considered to be a positive frequency signal.)

If, however, the deviation away from the 90° phase-difference response is finite, we see (from eqn. 2.1) that a fraction of the lower sideband would also be transmitted. As this is so in any practical design, we require some measure of the ratio of lower to upper sideband so that the relative stopband to passband attenuation may be determined. For an arbitrary 60 dBs attenuation, we have (assuming the negative frequency lower sideband is equivalent to the positive frequency lower sideband):-

$$20 \cdot \log_{10} \frac{-(e^{j\delta} - 1)}{(e^{j\delta} + 1)} = -60$$

$$j \cdot \tan(\delta/2) = 10^{-3}$$

$$\tanh(\delta/2) \approx \delta/2 = 10^{-3} \text{ rads.}$$

$$\text{or:- } \delta = 2 \cdot 10^{-3} \cdot 180/\pi = 0.12^\circ$$

2.2

i.e. to maintain a stopband to passband attenuation ratio of 60 dBs, the maximum deviation of $\pm 0.12^\circ$ must not be exceeded.

2.3 Comparisons of the hardware requirements of each modulation system

The digital Weaver and quadrature modulator first stages are now compared with regard to their respective hardware requirements. Due to the rapid changes in the technology of digital circuitry and because of the varied technologies already available (52), it is unrealistic to consider comparisons of one system with another on all but a simple level. The two systems are therefore compared on the basis of multiplier complexity: at present, the major factor contributing to the overall costs.

To facilitate the comparison, the following assumptions are made:-

- (i) Each network (lowpass filter or phase-splitting network) is specified as operating in the fixed-point numerical representation⁽⁸⁾ - a low-cost and relatively easily implemented mode of operation (should this not satisfy, the system performance can usually be improved, but at a cost, using the floating-point mode⁽⁵⁴⁾).
- (ii) The hardware multiplier chosen is of the serial-parallel 'pipeline' type⁽⁵³⁾ (see fig. 2.5). This is commercially available and is useful in that it allows coefficient multiplexing.

Using the terminology given in (39), it is seen that a measure of the multiplier complexity of a system is obtained from:-

$$M = N_m \cdot N_p \cdot f_s \cdot B_q / f_b$$

2.3

(hardware multipliers)

where:-

N_m = the number of simultaneous multiplications per data sample.

N_p = the number of paths in the multiplex system (two/channel).

f_s = the sampling rate.

B_q = the internal variable wordlength $\equiv (n_o + n_a)$
 $(n_o = \text{wordlength leaving the A/D converter})$
 $(n_a = \text{bits added to reduce internally generated noise})$

f_b = maximum allowable clock-rate making M integer.

The choice of the clock-rate (f_b) is dependent on the 'state-of-the-art' digital circuitry available. However, choosing fewer high clock-rate devices is often less preferable than choosing what will be a larger number of lower operating speed devices. This is because the high speed device generally costs significantly more than several low speed devices. Alternatively, using a small number of high speed devices can be advantageous in that the packing density is minimized although this, coupled with the high power requirements of these devices, magnifies the layout design problem. For these reasons, a conservative estimate of f_b is made by arbitrarily specifying the ratio f_s/f_b to be $1/10^3$ (this assumes the reasonably low clock-rate (f_b) of 16 MHz.). In the comparisons to follow, no attempt is made to make M integer by optimally choosing f_b . Instead, however, M is rounded to become integer which effectively reduces the value of f_b .

In calculating the number of multipliers required, it is necessary to include the hardware used in realizing the modulators. A modulator requires one multiplication per input sample : the effective multiplier coefficients are in fact samples of the carrier wave. Coefficient wordlengths

of twelve bits are used to represent the carrier samples although, for a 2KHz. carrier signal, it is seen that only an 8-bit wordlength is required.

The total pipeline multiplier hardware required is given by:-

$$H_m = M \cdot A \quad (\text{bits})$$

where:-

A is the coefficient accuracy (wordlength) in bits

M is obtained from equation 2.3

For the respective modulator 1st-stages, we have (for the 12-channel F.D.M. system):-

(i) the Weaver modulator

	N_m	N_p	B_q	M
lowpass filters	11	2 x 12	21	5.54
low frequency modulators	1	6 x 12	21	1.51

and therefore:-

$$H_m = (6 \times 7) + (2 \times 12) = 68 \text{ bits}$$

(ii) the phase-difference modulator

	N_m	N_p	B_q	M
lowpass filter	11	1 x 12	20	2.64
phase-splitting network	4	2 x 12	19	1.83
low frequency modulator	1	4 x 12	19	0.91

and therefore:-

$$H_m = (3 \times 6) + (2 \times 11) + (1 \times 8) = 48 \text{ bits}$$

For the commercially available multiplier chips, it is more likely that the coefficient bits would be arranged in groups of four bits (53). Allowing for this, the figures for the number of pipeline multiplier bits required are recalculated to be 72 and 56 respectively.

It is apparent from this simple comparison that the phase-difference or quadrature modulation method is competitive and in fact warrants further attention. Various methods of digitally producing the required 90° phase-difference response are therefore examined.

2.4 90° phase-splitting networks

The production of a 90° phase-difference response using an approximation to the ideal Hilbert Transformer (realised in either an analogue or digital form) is well known (55-60).

Several digital methods of producing the 90° phase-difference response are now examined and the performance characteristics of some of the phase-splitting networks are compared.

(i) Non-recursive networks

For the ideal Hilbert Transformer (digital), we have:-

$$\begin{aligned} H(e^{j\omega}) &= -j & 0 < \omega < \pi \\ &= j & \pi < \omega < 2\pi \end{aligned} \quad 2.5$$

and therefore:-

$$\begin{aligned} h(n) &= \frac{1}{2\pi j} \oint H(z) \cdot z^n \frac{dz}{z} \\ &= \frac{1 - e^{jn\pi}}{\pi n} \equiv \frac{2}{\pi n} \quad (n \text{ odd}) \end{aligned}$$

2.6

This impulse-response can be realized using a non-recursive digital filter as long as a finite constant delay (τ) is introduced (otherwise the impulse-response is non-causal). The impulse-response coefficients $h(n)$ are the coefficients of the Fourier series and are infinite in number. For the realizable network, therefore, the Fourier series must be truncated. A direct truncation leads to the well-known Gibbs phenomenon ⁽⁵⁾ (i.e. an undesirable overshoot of the amplitude response); however, this can be minimized, in some sense, using one of the well-known weighting (or window) functions ⁽⁵⁵⁾.

Alternatively, a Chebychev approximation to the ideal frequency response of the Hilbert Transformer can be obtained by solving a set of non-linear equations. It has been shown ⁽⁵⁶⁾ that these solutions correspond to systems of a lower degree than those obtained by modifying (weighting) the Fourier coefficients. Design data ⁽⁵⁹⁾ on wideband Hilbert Transformers is now available and a computer program (utilizing a Remez optimization procedure) for designing optimum (minimax) linear-phase F.I.R. Hilbert Transformers has been published ⁽⁶²⁾. In ⁽⁵⁹⁾, certain conclusions were drawn such that an optimum linear-phase F.I.R. Hilbert Transformer could be efficiently and easily designed. Those conclusions, particularly relevant to the design of a wideband Hilbert Transformer (for use in the phase-difference modulator), are summarised as follows:-

- (i) The peak approximation error for N odd (the impulse duration) is primarily determined by the minimum of the values of the upper and lower transition ratio widths.
- (ii) The peak approximation error for N even is primarily determined by the lower transition ratio width.
- (iii) The smaller the bandwidth, the faster the decrease of peak error of the approximation with decreasing bandwidth of the Hilbert Transformer.

- (iv) The negligible decrease in peak error obtained using unequal transition bandwidths is more than offset by the undesired effects in the magnitude and phase responses.
- (v) In the case of equal upper and lower transition widths, Hilbert Transformers with N odd can be realized more efficiently (i.e. every other sample value is zero).

In the F.D.M. system design problem, the required phase-splitting network is specified as having a fixed lower transition region and passband, although the upper transition region is undefined. Reference to point (iv) indicates, however, that the upper and lower transition region should be made equal. An efficient realization then follows if the impulse response, length N, is chosen to be an odd number of samples (point v). The required length (N) is obtained from the relationship (9):-

$$N \cdot \Delta F \approx -0.61 \cdot \log_{10}(\delta)$$

where:-

Δf = transition bandwidth ratio

δ = peak approximation error

To meet the design specifications, it is seen that:-

$$N \approx \frac{-0.61 \cdot \log_{10} (0.01)}{0.02125}$$

giving:-

$$N = 55$$

Because the transfer function consists of a mirror-image polynomial (giving the linear phase response) and also every other sample value is zero, the phase-splitting network can be realized using only 14 multipliers - see fig. 2.6.

The exact 90° phase-difference is obtained at all times (regardless of the coefficient precision) whereas the amplitude response, although meeting the specifications in the initial design (fig. 2.7a), deviates away from the required response when coefficients of a reduced accuracy are used (fig. 2.7b. - i.e. the ideal coefficient is replaced by its 6-bit representation). It is clear therefore that, for this network structure (fig. 2.6), the coefficient accuracy must be increased if both the phase and amplitude response specifications are to be satisfied. Alternatively, it has been shown (71, 72) that realizing the non-recursive structure in the form of a ladder structure can give a reduction in the sensitivity to coefficient variation. The ladder structure is derived from the non-recursive network using a continued fraction expansion. The resulting network is a recursive structure and is unusual in that it possesses a finite impulse response.

The useful 90° phase-difference response between the outputs of an even-length non-recursive lowpass filter and its directly derived highpass transform (i.e. the required lowpass-highpass transformation is obtained on replacing z^{-1} by $-z^{-1}$ in the lowpass filter transfer function) has been noted ⁽⁶⁴⁾. Considering the simple, evenlength lowpass filter (length 2), we have:-

$$\begin{aligned}
 F_{LP}(z) &= 1 + z^{-1} \\
 F_{LP}(e^{j\omega T}) &= 1 + \cos(\omega T) - j.\sin(\omega T) \\
 \varphi_{LP}(\omega T) &= \tan^{-1} \frac{\angle F_{LP}}{\phantom{F_{LP}}} = \frac{-\sin(\omega T)}{1 + \cos(\omega T)}
 \end{aligned} \tag{2.8}$$

For the directly derived highpass filter, we therefore have:-

$$\varphi_{HP}(\omega T) = \tan^{-1} \frac{\angle F_{HP}}{\phantom{F_{HP}}} = \frac{\sin(\omega T)}{1 - \cos(\omega T)} \tag{2.9}$$

The phase-difference response is given by:-

$$\varphi = \varphi_{LP} - \varphi_{HP} = \tan^{-1} \frac{\angle F_{LP}}{\phantom{F_{LP}}} - \tan^{-1} \frac{\angle F_{HP}}{\phantom{F_{HP}}}$$

or:-

$$\tan \varphi = \frac{\frac{\angle F_{LP}}{\phantom{F_{LP}}} - \frac{\angle F_{HP}}{\phantom{F_{HP}}}}{1 + \frac{\angle F_{LP}}{\phantom{F_{LP}}} \cdot \frac{\angle F_{HP}}{\phantom{F_{HP}}}}$$

now:-

$$\tan \phi = \frac{-2.\sin(\omega T)}{0} \rightarrow \infty$$

and therefore:-

$$\phi \rightarrow 90^\circ$$

The phase-splitting network is simply implemented and, by using the realization shown in fig. 2.8, the 90° phase-difference response is exactly maintained regardless of the coefficient precision (the mirror-image polynomial is obtained as long as coefficient pairs α_0 and α_{N-1} , α_1 and α_{N-2} etc. are made equal). The constraints affecting the choice of filter length are (i) the highpass filter stopband to passband transition bandwidth (this automatically constrains the lowpass filter transition bandwidth as there must be an exact one-to-one transformation) and (ii) the passband amplitude ripple. As the stopband attenuation is unimportant (the removal of the redundant spectra using an additional filtering stage is assumed), the filter design can be weighted such that a minimum passband ripple is obtained for any particular filter length. The computer program of (62) is particularly suitable for designing the linear phase F.I.R. digital filter in that it permits the necessary passband weighting. The required filter length is obtained from the relationship (63):-

$$N = \frac{-10 \log_{10}(\delta_1 \delta_2) - 15}{14 \cdot \Delta F} + 1$$

2.10

where:-

δ_1, δ_2 are the passband and stopband deviation respectively.

ΔF is the filter stopband to passband transition bandwidth ratio.

For a response similar to that of the non-recursive Hilbert Transformer (fig. 2.7), it can be shown that a filter of length 28 is required (in this design there is no effective stopband as the transition bandwidth is made as wide as possible). This method of realizing the 90° phase-difference response is therefore less efficient than that of the Hilbert Transformer because a larger number of hardware multipliers must be used to produce the equivalent phase and amplitude responses.

(ii) Recursive networks

The complex transfer function

$$H(z^{-1}) = H_a(z^{-1}) + j \cdot H_b(z^{-1})$$

of which the real and imaginary parts correspond to the two paths of the phase-splitting network, can also be realized

using recursive networks. Two real networks, based on the allpass sub-network:-

$$T(z^{-1}) = \frac{z^{-1} - \beta}{1 - \beta \cdot z^{-1}} \quad 2.11$$

are designed such that the required 90° phase-difference response is obtained over a defined frequency range ω_1 to ω_2 . The pole occurring at $z^{-1} = 1/\beta$ is cancelled by the zero at $z^{-1} = \beta$ and, if both the pole and the zero of each sub-network are obtained using a single multiplier (fig. 2.9), the response of each section will remain strictly allpass.

In (43), an allpass phase-splitting network, implemented in the form of an interwoven structure (fig. 2.10), has been proposed. A simple algebraic method of deriving the coefficient values β_i (for the phase-difference specification over an arbitrary frequency range) was given. In addition to this and other realizations of the 1st-order section ⁽⁶⁵⁾, a variety of 2nd-order sections have been shown to exist ⁽⁶⁶⁾. A practical analysis (see figs. 2.12 and 2.13) shows there to be no difference between the phase-difference response of the interwoven structure and that of the cascade of 1st-order sections. This applies also to realizations using other 1st- and 2nd-order sections.

For the following ideal coefficient values (designed ⁽⁴³⁾ with the C.C.I.T.T. recommendations on channel filtering

kept in mind), the cascade and interwoven structure phase-difference responses are shown plotted in figs. 2.12 and 2.13 respectively.

i	$T_a(z^{-1}) \alpha_i$	$T_b(z^{-1}) \alpha_i$
1	0.803 477 80	0.658 857 36
2	0.952 405 96	0.896 309 00
3	0.440 844 83	0.986 351 54
4	-0.456 269 20	0.111 763 07

The effect of reducing the coefficient accuracy of each network is also shown (the ideal coefficient is represented by its nearest 3-decimal place value). This reduced coefficient accuracy check allows us to make a simple comparison (on the basis of network sensitivity) of one network with another.

An interesting alternative to the cascade of 1st-order sections has been suggested ⁽⁶⁷⁾; that is, the nested allpass network (fig. 2.11). The nested structure is developed from a single 1st-order section (fig. 2.9) as follows. In the nth section:-

$$T_n(z^{-1}) = \frac{z^{-1} - \beta_o}{1 - \beta_o \cdot z^{-1}}$$

2.12

z^{-1} is replaced by $z^{-1} \cdot T_{n-1}(z^{-1})$, to give:-

$$T_n(z^{-1}) = \frac{z^{-1} \cdot T_{n-1}(z^{-1}) - \beta_0}{1 - \beta_0 \cdot z^{-1} \cdot T_{n-1}(z^{-1})} \quad 2.13$$

Similarly, $T_{n-1}(z^{-1})$ is modified, i.e.:-

$$T_{n-1}(z^{-1}) = \frac{z^{-1} \cdot T_{n-2}(z^{-1}) - \beta_1}{1 - \beta_1 \cdot z^{-1} \cdot T_{n-2}(z^{-1})} \quad 2.14$$

The problem now remains in finding the multiplier coefficient values in order to satisfy a particular phase-response (using the structure shown, the amplitude response will always be strictly flat). As a satisfactory solution to the phase-difference problem already exists (the cascade of 1st-order sections in (43) from which the direct form is obtained), the problem of finding the multiplier coefficient values can be simply solved by modifying the nested allpass transmittance so that its coefficients can be compared, in some way, with those of the direct form.

For the direct form (order n) we have:-

$$T_n(z^{-1}) = \frac{\alpha_n \cdot z^{-n} + \alpha_{n-1} \cdot z^{-(n-1)} \dots + \alpha_0}{\alpha_0 \cdot z^{-n} + \alpha_1 \cdot z^{-(n-1)} \dots + \alpha_n} \quad 2.15$$

Equation 2.13 is rearranged to give:-

$$T_{n-1}(z^{-1}) = \frac{(\beta_o + T_n(z^{-1}))}{z^{-1} \cdot (1 + \beta_o \cdot T_n(z^{-1}))} \quad 2.16$$

Now, substituting equation 2.15 into 2.16 and rearranging, gives:-

$$T_{n-1}(z^{-1}) = \frac{(\beta_o \alpha_o + \alpha_o) \cdot z^{-n} \dots + (\beta_o \alpha_n + \alpha_o)}{z^{-1} \{ (\beta_o \alpha_n + \alpha_o) \cdot z^{-n} \dots + (\beta_o \alpha_o + \alpha_n) \}} \quad 2.17$$

However, for degree $T_{n-1}(z^{-1})$ less than $T_n(z^{-1})$, it is apparent from equation 2.17 that:-

$$\begin{aligned} \beta_o \cdot \alpha_n + \alpha_o &= 0 \\ \beta_o &= \frac{-\alpha_o}{\alpha_n} \end{aligned} \quad 2.18$$

For the 4-section network, equation 2.17 therefore becomes:-

$$T_3(z^{-1}) = \frac{\alpha'_3 \cdot z^{-3} + \alpha'_2 \cdot z^{-2} \dots + \alpha'_o}{\alpha'_o \cdot z^{-3} + \alpha'_1 \cdot z^{-2} \dots + \alpha'_3} \quad 2.19$$

where:-

$$\begin{aligned} \alpha'_3 &= \frac{(\alpha_4 - \alpha_o \cdot \alpha_o)}{\alpha_4} & \alpha'_2 &= \frac{(\alpha_3 - \alpha_o \cdot \alpha_1)}{\alpha_4} \\ \alpha'_1 &= \frac{(\alpha_2 - \alpha_o \cdot \alpha_2)}{\alpha_4} & \alpha'_o &= \frac{(\alpha_1 - \alpha_o \cdot \alpha_3)}{\alpha_4} \end{aligned}$$

Using:-

$$T_2(z^{-1}) = \frac{\beta_1 + T_3(z^{-1})}{z^{-1} \cdot \{1 + \beta_1 \cdot T_3(z^{-1})\}} \quad 2.20$$

and, substituting eqn. 2.19 in eqn. 2.20, we obtain:-

$$\beta_1 \cdot \alpha_3' + \alpha_0' = 0$$

and therefore:-

$$\begin{aligned} \beta_1 &= - \frac{\alpha_0'}{\alpha_3'} \\ &= - \frac{(\alpha_1 - \frac{\alpha_0}{\alpha_4} \cdot \alpha_3)}{(\alpha_4 - \frac{\alpha_0}{\alpha_4} \cdot \alpha_0)} \end{aligned}$$

2.21

Repeating this process, we obtain for $T_2(z^{-1})$:-

$$T_2(z^{-1}) = \frac{\alpha_2'' \cdot z^{-2} + \alpha_1'' \cdot z^{-1} + \alpha_0''}{\alpha_0'' \cdot z^{-2} + \alpha_1'' \cdot z^{-1} + \alpha_2''}$$

2.22

where:-

$$\alpha_2'' = (\alpha_3' - \frac{\alpha_0'}{\alpha_3'} \cdot \alpha_0')$$

$$\alpha_1'' = (\alpha_2' - \frac{\alpha_0'}{\alpha_3'} \cdot \alpha_2')$$

$$\alpha_0'' = (\alpha_1' - \frac{\alpha_0'}{\alpha_3'} \cdot \alpha_1')$$

giving:-

$$\beta_2 \cdot \alpha_2'' + \alpha_0'' = 0$$

and therefore:-

$$\beta_2 = - \frac{\alpha_0''}{\alpha_2''}$$

2.23

and for $T_1(z^{-1})$:-

$$T_1(z^{-1}) = \frac{\alpha_1'''' \cdot z^{-1} + \alpha_0''''}{\alpha_0'''' \cdot z^{-1} + \alpha_1''''}$$

2.24

where:-

$$\alpha_1''' = (\alpha_2'' - \frac{\alpha_0''}{\alpha_2''} \alpha_0'')$$

$$\alpha_0''' = (\alpha_1'' - \frac{\alpha_0''}{\alpha_2''} \alpha_1'')$$

giving:-

$$\beta_3 \cdot \alpha_1''' + \alpha_0''' = 0$$

and therefore:-

$$\alpha_3 = - \frac{\alpha_0'''}{\alpha_1'''}$$

2.25

From the allpass transfer function (eqn. 2.15) (for $n=4$),

where:-

i	$T_a(z^{-1}) \alpha_i$	$T_b(z^{-1}) \alpha_i$
0	-0.1539 2278	0.0650 9973
1	0.3649 8864	-0.8199 1780
2	0.5370 0973	2.4085 2840
3	-1.7404 5940	-2.6532 2810
4	1.0	1.0

the nested structure coefficients are calculated to be:-

i	$T_a \beta_i$	$T_b \beta_i$
0	0.153 922 78	-0.065 099 73
1	-0.099 448 45	0.649 944 85
2	-0.814 322 67	-0.977 047 84
3	0.995 487 28	0.999 441 79

The resulting phase-difference response, both for ideal and 3-decimal place coefficients, is shown plotted in fig. 2.14.

Although the nested structure is implemented with the same number of adders, delays and multipliers as the previous structures, its high sensitivity to coefficient variation precludes its use.

A digital phase-splitting network that requires the same number of delays and multipliers (but not adders) as the interwoven and nested allpass network is now derived using digital wave-filter techniques (68, 69). Using the allpass network direct form and applying the bilinear transformation (9), we obtain:-

$$T(p) = \frac{a_n p^n - a_{n-1} p^{n-1} \dots + a_0}{a_n p^n + a_{n-1} p^{n-1} \dots + a_0} \quad 2.26$$

(n even)

The transfer function $T(p)$ remains strictly allpass as this property is unaffected in the transformation. Now any allpass function can be written as:-

$$T(p) = \frac{Q(-p)}{Q(p)} \equiv \frac{Q_e(p) - Q_o(p)}{Q_e(p) + Q_o(p)} \quad 2.27$$

where $Q_e(p)$ and $Q_o(p)$ are the even and odd parts of a Hurwitz polynomial (70). For any Hurwitz polynomial, the ratio of even to odd parts represents a reactance function and, for the allpass network (n=4), we therefore have:-

$$T(p) = \frac{1 - Y(p)}{1 + Y(p)} \equiv \frac{G_0 - Y(p)}{G_0 + Y(p)}$$

$$G_0 = 1$$

2.28

with the driving-point admittance given by:-

$$Y(p) = \frac{a_1 p + a_3 p^3}{a_0 + a_2 p^2 + a_4 p^4}$$

2.29

Equation 2.28 can be realized in the form of a lattice network or, as is required in the realization of the digital wave-filter, in the form of a ladder network. We therefore rewrite eqn. 2.29 as:-

$$Y(p) = \frac{1}{b_0 p + \frac{1}{b_2 p + \frac{1}{b_4 p + \dots}}}$$

2.30

from which the one-port ladder network (the first Cauer form), shown in fig. 2.15, is obtained. The digital wave-filter is now simply obtained by replacing each series/parallel inductor/ capacitor with its digital wave-filter equivalent (figs. 2.17 and 2.18); the digital wave-filter is represented as shown in fig. 2.16. (No explanation is offered as to how these individual elements are derived as this is covered in great detail in (69)). Some consideration must be given, however, as to how the wave-filter is to be terminated

(the replaced elements have been considered as two-ports).
 Capacitor C_4 is therefore considered to be a one-port
 termination and is handled as follows:-

Looking into the capacitor (using wave-filter theory (68)),
 we have:-

$$A_4 = V + IR_3$$

$$B_4 = V - IR_3$$

and $V = I/pC_4$

and therefore:-

$$B_4/A_4 = \frac{1/pC_4 - R_3}{1/pC_4 + R_3}$$

2.31

and, after performing a bilinear transformation, we obtain:-

$$B_4/A_4 = \frac{z^{-1} + \alpha_4}{1 + \alpha_4 \cdot z^{-1}}$$

2.32

where:-

$$\alpha_4 = \frac{1 - R_3 C_4}{1 + R_3 C_4}$$

The remaining unknown coefficient values are obtained
 from (69):

$$\alpha_1 = 1/R_1 = G_1$$

$$\alpha_2 = G_1/G_2$$

$$\alpha_3 = R_2/R_3$$

where:-

$$R_1 = R_0 + L_1$$

$$G_2 = C_2 + G_1$$

$$R_3 = L_3 + R_2$$

2.33

To be dimensionally correct it must be noted that

$R_3 = (k) \cdot L_3 + R_2$ etc. as the bilinear transform is strictly

$$p = (k) \cdot \frac{(1 - z^{-1})}{(1 + z^{-1})}$$

although (k) is normalized to unity throughout.

For the coefficients of eqn. 2.26, we obtain:-

i	$T_a(p)$ a_i	$T_b(p)$ a_i
0	0.0076 1620	0.0004 2883
1	0.4047 9505	0.0728 7570
2	4.0024 4390	1.5735 4168
3	8.8265 8720	7.4063 2640
4	2.7585 5770	6.9468 2740

and for eqn. 2.28, where:-

$b_0 = a_4/a_3$	$b_1 = (a_2 - a_4 \cdot a_1/a_3)$
$b_2 = a_3/b_1$	$b_3 = (a_1 - a_0 \cdot b_2)$
$b_4 = b_1/b_3$	$b_5 = a_0$
$b_6 = b_3/a_0$	

i	$Y_a(p)$ b_i	$Y_b(p)$ b_i	
0	0.3125 2823	0.9379 5857	L_1
1	3.8759 3390	1.5051 8730	
2	2.2772 8010	4.9205 3440	C_2
3	0.3752 7521	0.0707 6580	
4	10.3282 4500	21.2699 8100	L_3
5	0.0076 1619	0.0004 2883	
6	49.2732 7800	165.0321 8000	C_4

Using these values in eqn. 2.33, the multiplier coefficient values are calculated to be:-

i	α_{ia}	α_{ib}
1	0.7613 8837	0.5160 0690
2	0.2506 8957	0.0949 1455
3	0.0318 4422	0.0085 7372
4	-0.9962 0237	-0.9994 3523

2.34

A somewhat different realization (with a new set of coefficients) is obtained if the coefficients are derived by working from the output to input. The two-port representations of L_1 , C_2 , L_3 are still as in figs. 2.17 and 2.18 but the termination C_4 is now defined equal to G_3 . We now have:-

$$\begin{aligned} G_3 &= C_4 & R_2 &= R_3 + L_3 \\ G_1 &= G_2 + C_2 & R_0 &= R_1 + L_1 \end{aligned}$$

giving:-

$$\alpha_3 = R_3/R_2 \quad \alpha_2 = G_2/G_1 \quad \alpha_1 = R_1/R_0$$

2.35

From equation 2.32, we now find:-

$$\alpha_4 = \frac{1 - C_4 R_3}{1 + C_4 R_3} = 0$$

giving: $B_4/A_4 = z^{-1}$

i.e. the termination is just the unit delay (z^{-1}).

The apparent loss of the multiplier α_4 is offset by the requirement for an additional multiplier α_0 at the input (i.e. R_0 is no longer unity as required). We have therefore:-

$$\hat{B}_1/\hat{A}_1 = \hat{T} = \frac{1 - Y \cdot \hat{R}_0}{1 + Y \cdot \hat{R}_0} \quad 2.36$$

but require:-

$$B_1/A_1 = T = \frac{1 - Y}{1 + Y} \quad 2.37$$

From equations 2.36 and 2.37, we obtain:-

$$Y = \frac{1 - T}{1 + T} = \frac{1}{R_0} \cdot \frac{1 - \hat{T}}{1 + \hat{T}} \quad 2.38$$

and therefore:-

$$T = \frac{\hat{T} - \alpha_o}{1 - \alpha_o \cdot \hat{T}}$$

where:-

$$\alpha_o = \frac{1 - \hat{R}_o}{1 + \hat{R}_o}$$

2.39

Equation 2.37 is realized therefore by adding the adaptor at the input as shown in fig. 2.19. The modified coefficient set is then found to be:-

i	α_{ia}	α_{ib}
0	0.1539 2280	-0.0650 9974
1	0.5737 5824	0.1766 9897
2	-0.0419 7105	-0.0094 6162
3	0.0019 6114	0.0002 8486

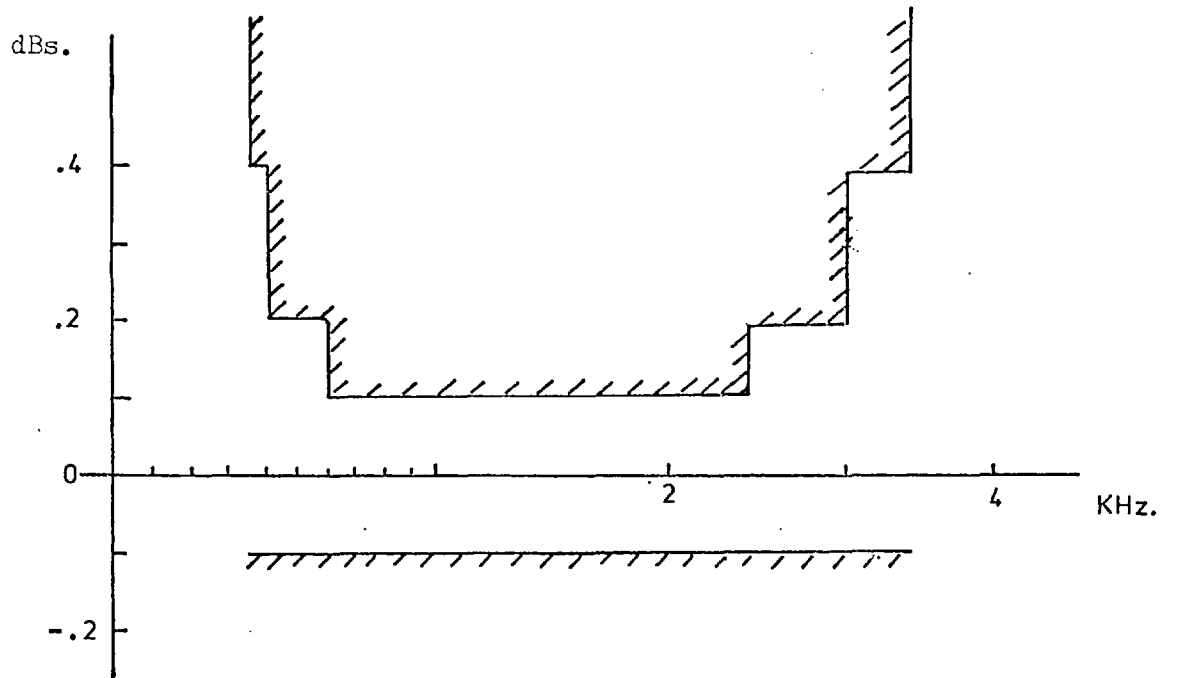
The phase-difference response for the type 1 and type 2 wave-filter realization is shown plotted in fig. 2.20 (for ideal and 3-decimal place coefficients). From the simple estimation of the wave-filter's sensitivity to coefficient variation, it seems safe to conclude that no particular advantage will be gained in using this phase-splitting network in preference to the interwoven or cascaded structures.

2.5 Summary

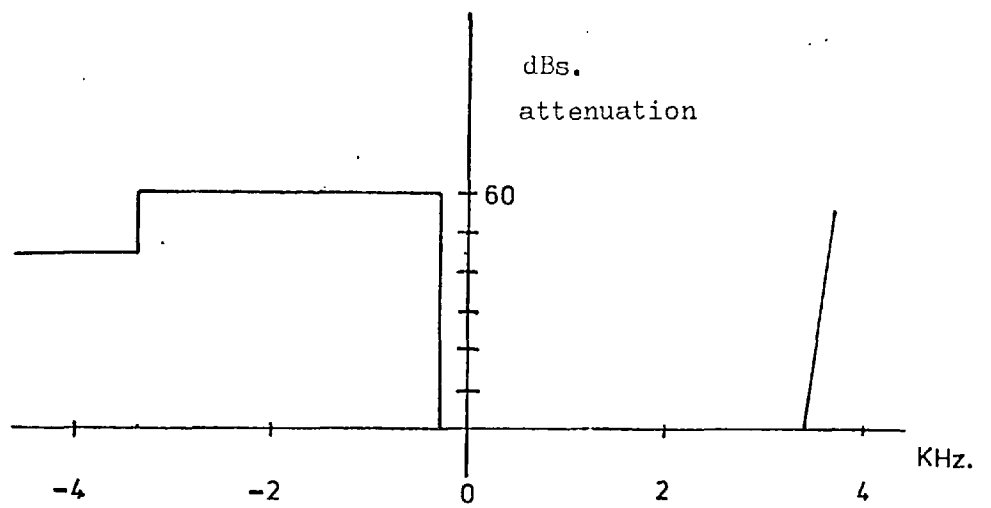
The operation of both the digital Weaver modulator and the digital phase-difference modulator has been described. Multiplier complexity has been used as the basis for comparing these systems as it is the major contributing factor to the overall system costs. It has been shown that, by assuming the commercial availability of a fairly low-cost pipeline type multiplier, the digital phase-difference modulator can be designed to be competitive.

With the hope of improving the phase-difference modulator design, various types of phase-splitting networks (consisting of both recursive and non-recursive structures) have been examined. We see from this brief study that, at present, the non-recursive Hilbert Transformer seems at a disadvantage because, in general, it requires more multipliers than certain recursive structures. The introduction of C.C.D. technology could possibly alter this situation as the long tapped delay-line using simple hardware multipliers (e.g. resistive weightings ⁽⁷³⁾) can be used to good advantage in system designs specifying wideband phase-difference responses.

In the work to follow, the recursive structure is concentrated upon; in particular, a new recursive phase-splitting network interpreted in terms of polyphase concepts is proposed.



(a) passband



(b) stopband

Fig. 2.1 C.C.I.T.T. recommendations for the channel filter.

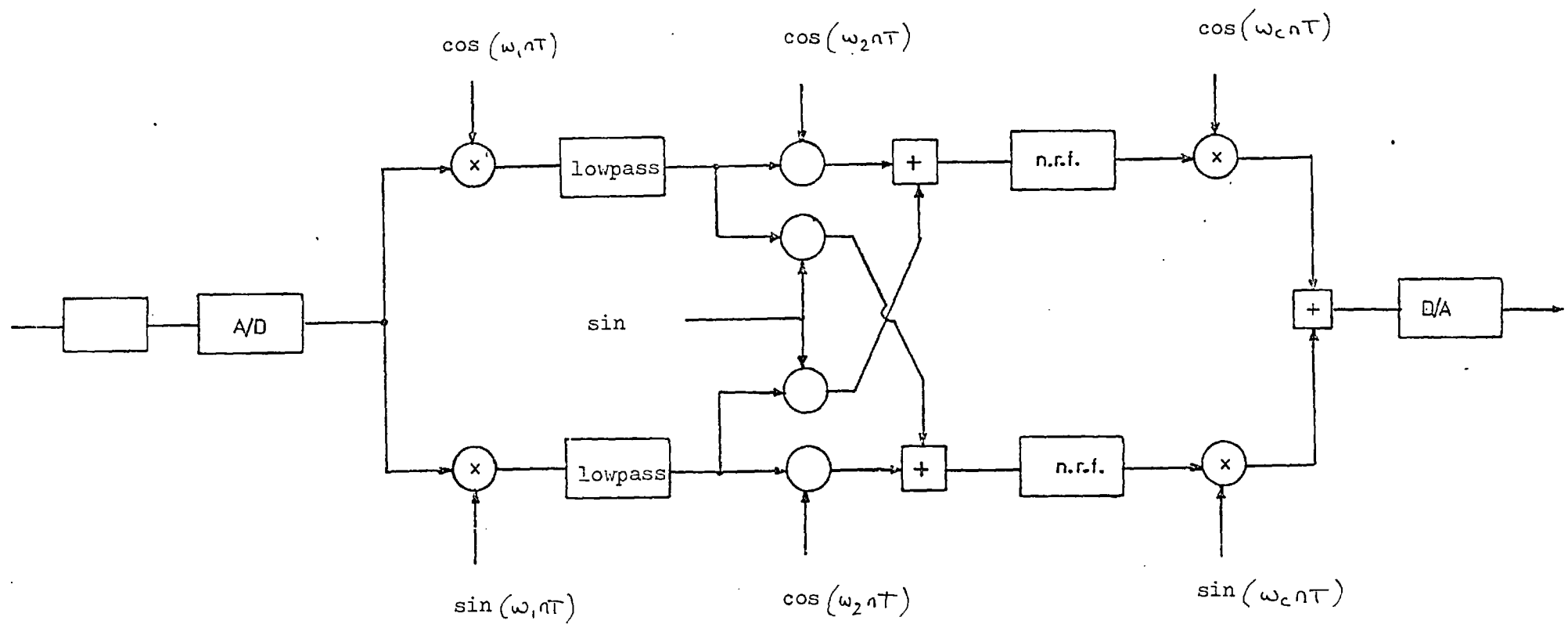


Fig. 2.2 The digital Weaver modulator (for European norms).

$$f_1 = 1850 \text{ Hz.}$$

$$f_2 = 150 \text{ Hz.}$$

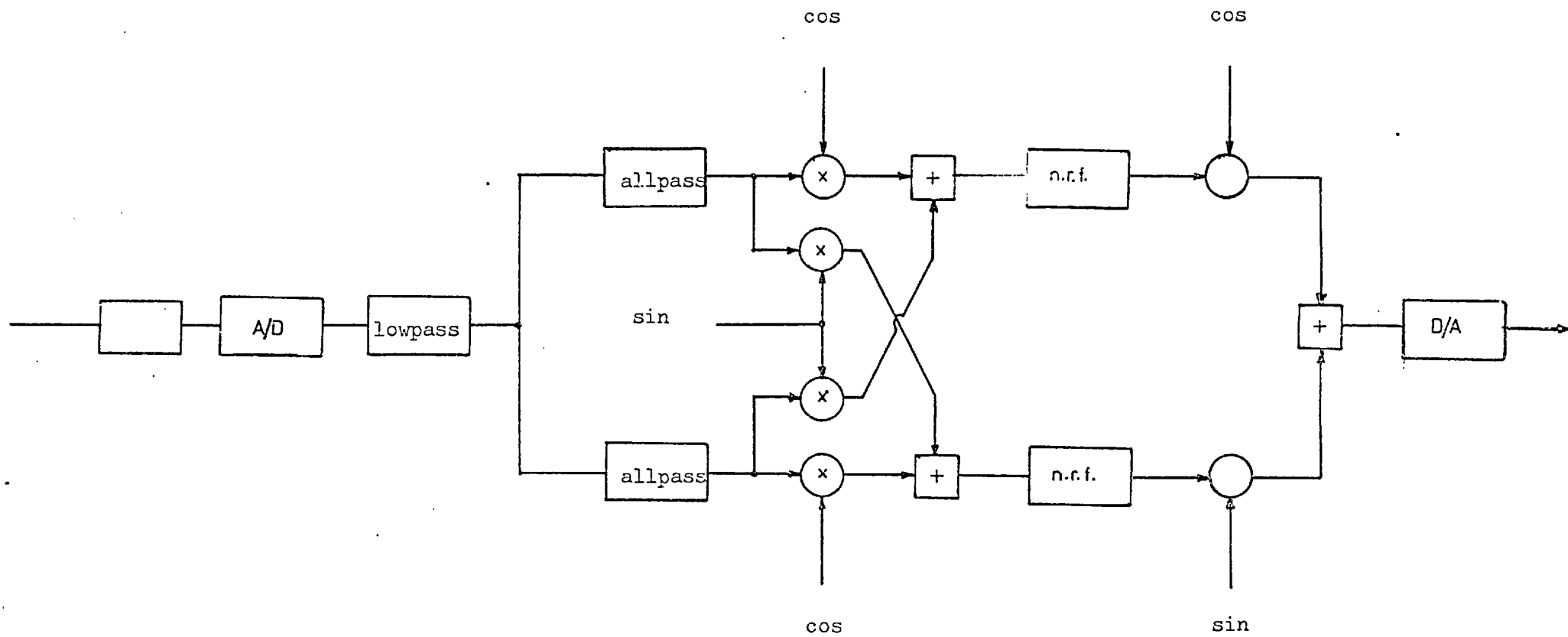


Fig. 2.3 The digital quadrature or phase-difference modulator.

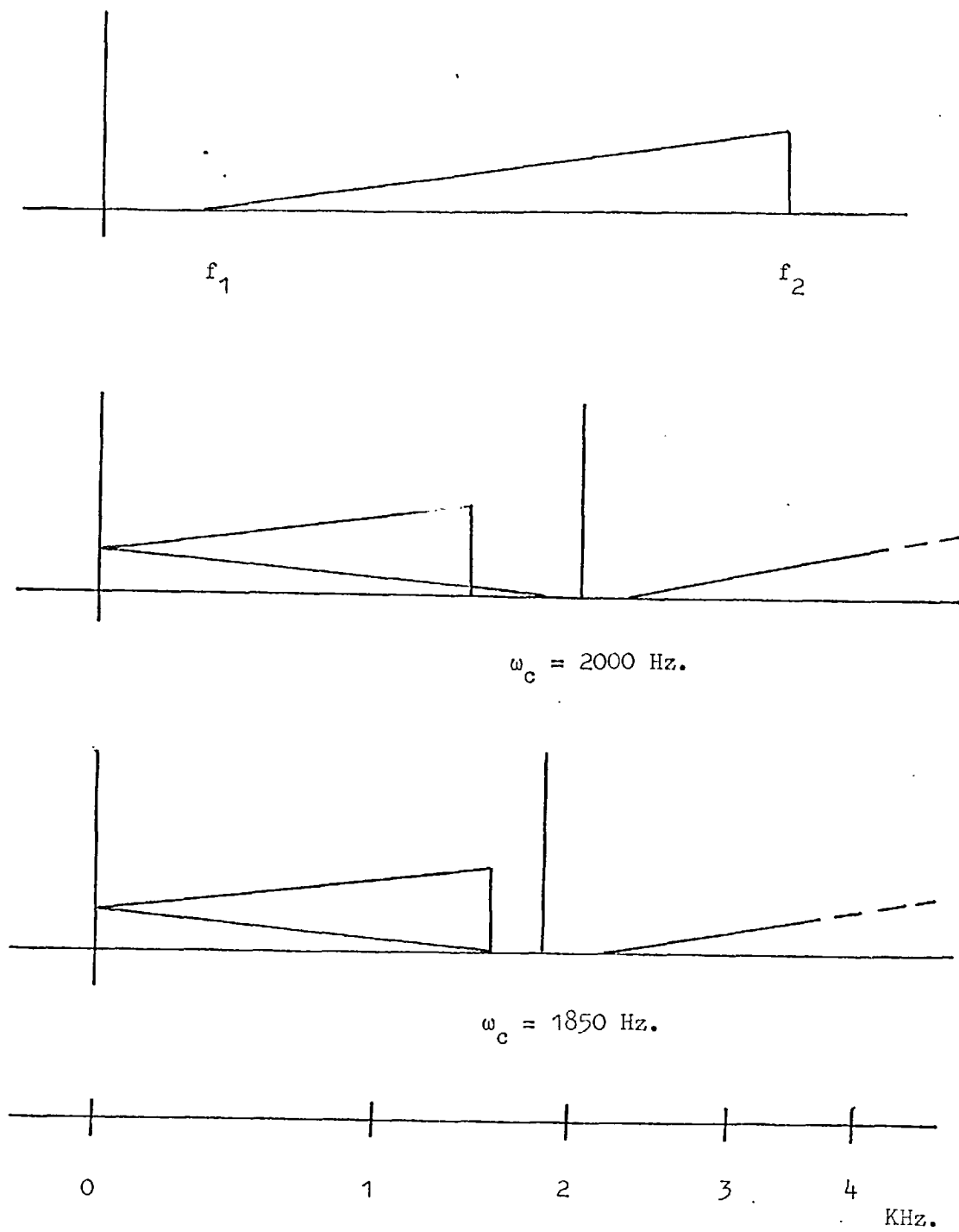


Fig. 2.4 Modulation of the baseband for differing carrier frequencies.

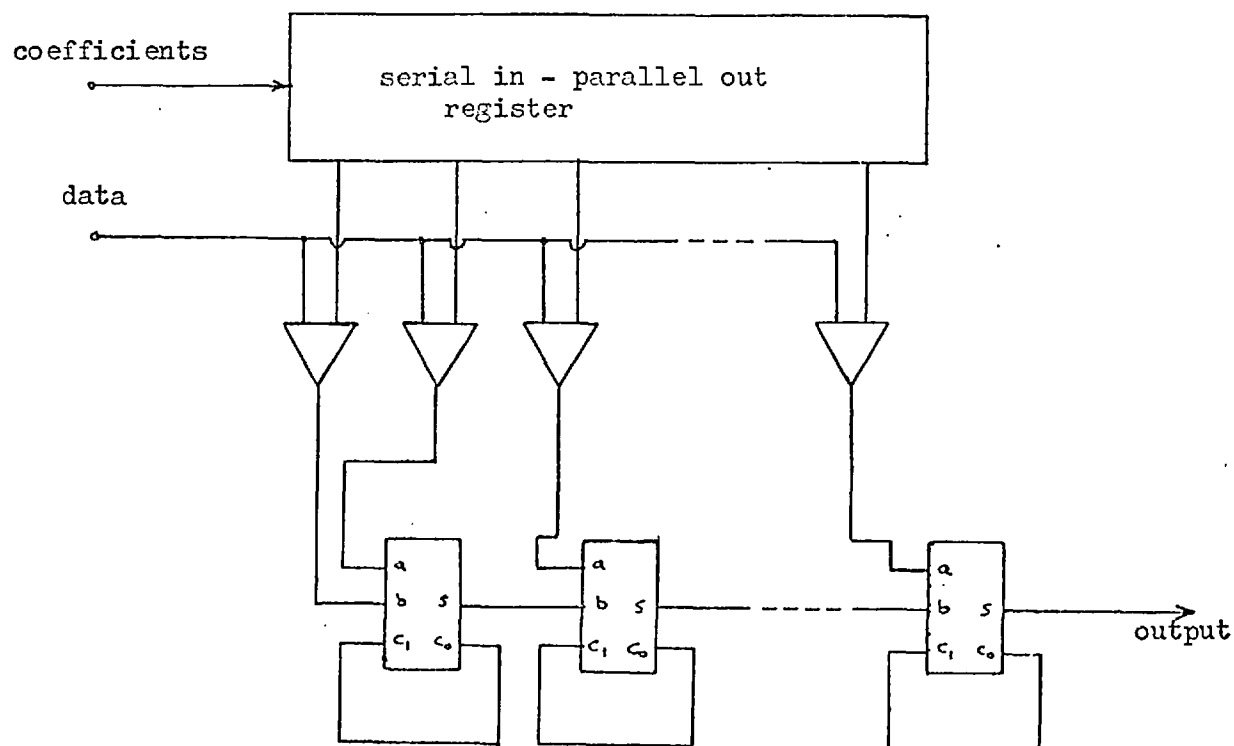


Fig. 2.5 The serial-parallel pipeline type multiplier.

m -bit data (signal)

n -bit coefficient

$m + n$ bit resultant (before rounding or truncation)

total multiplication time = $m + n$ clock cycles

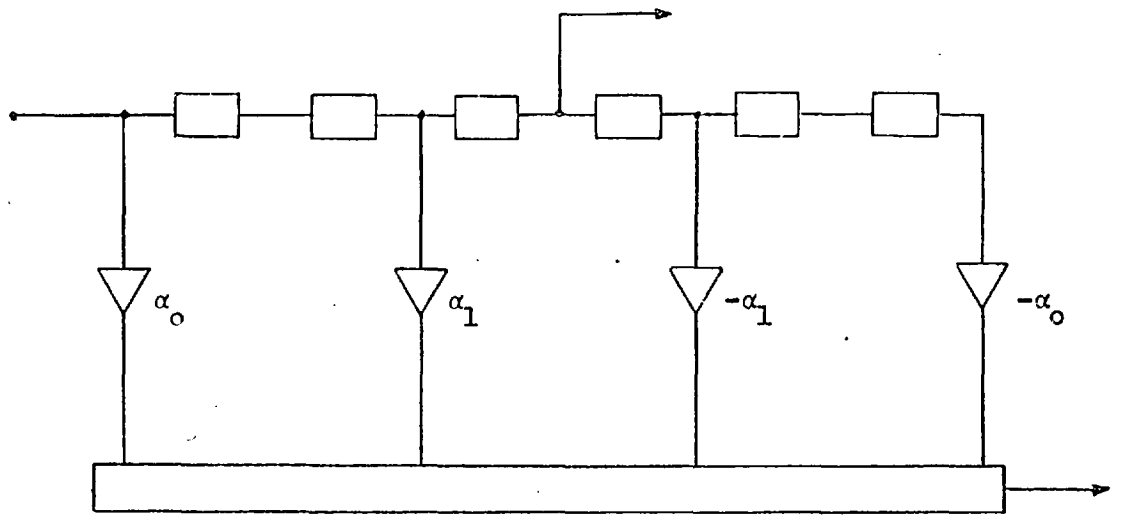


Fig. 2.6a The non-recursive Hilbert Transformer (non-minimum multiplier)

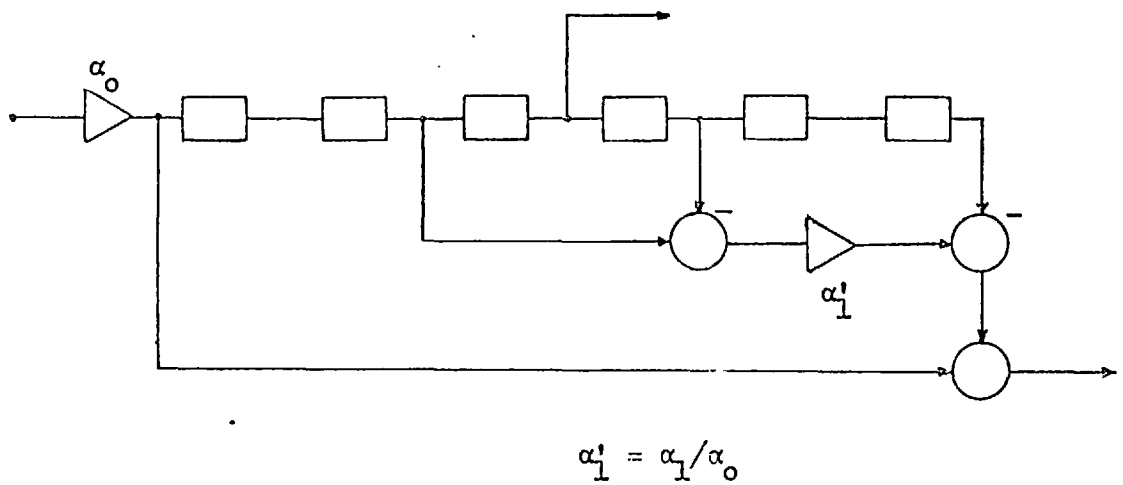


Fig. 2.6b The non-recursive Hilbert Transformer (minimum multiplier)

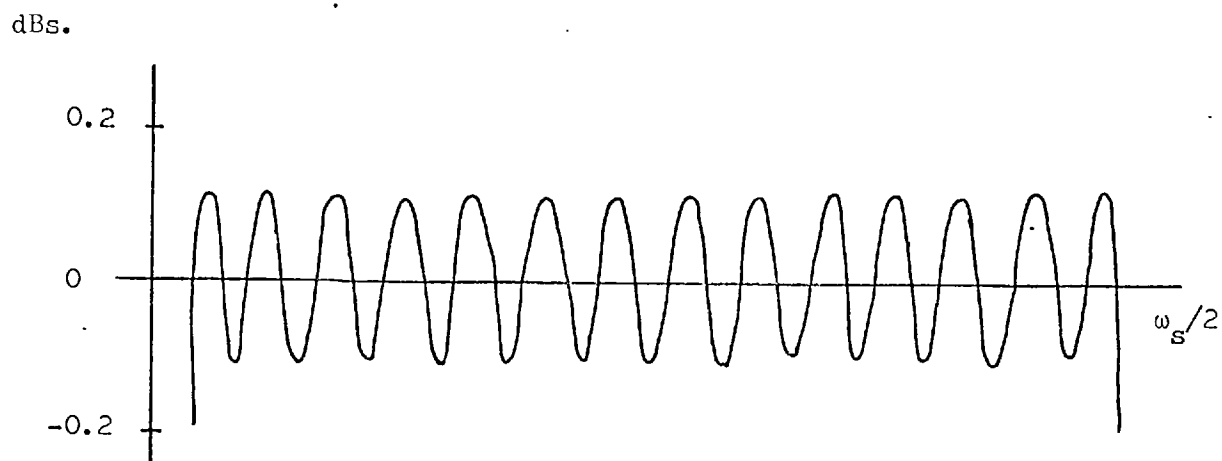


Fig. 2.7a The Hilbert Transformer amplitude response.

(length 55, 14 multipliers - ideal values)

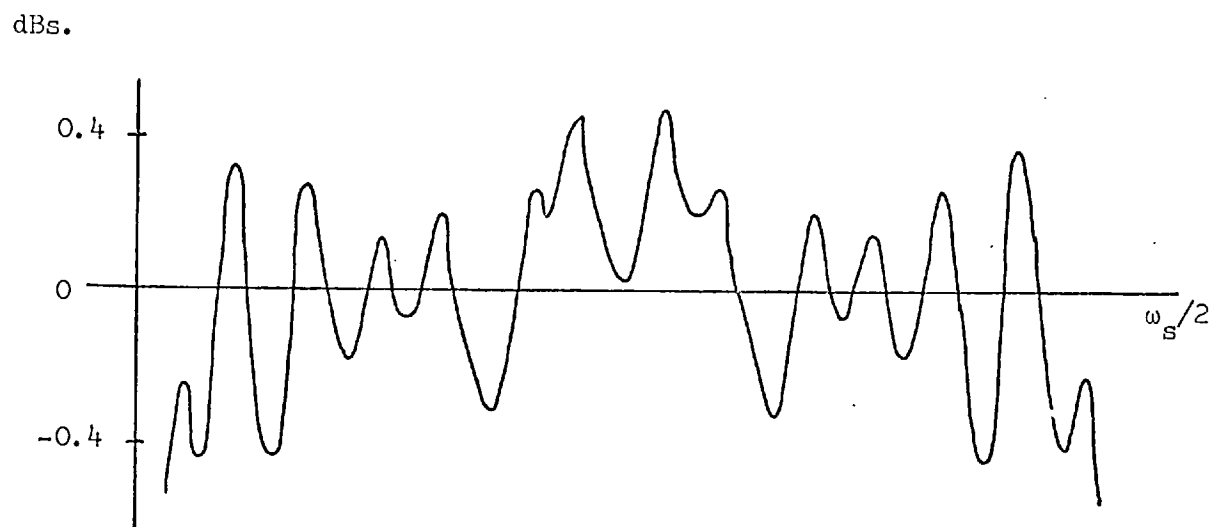


Fig. 2.7b The amplitude response for quantized coefficients.

(6-bit representation)

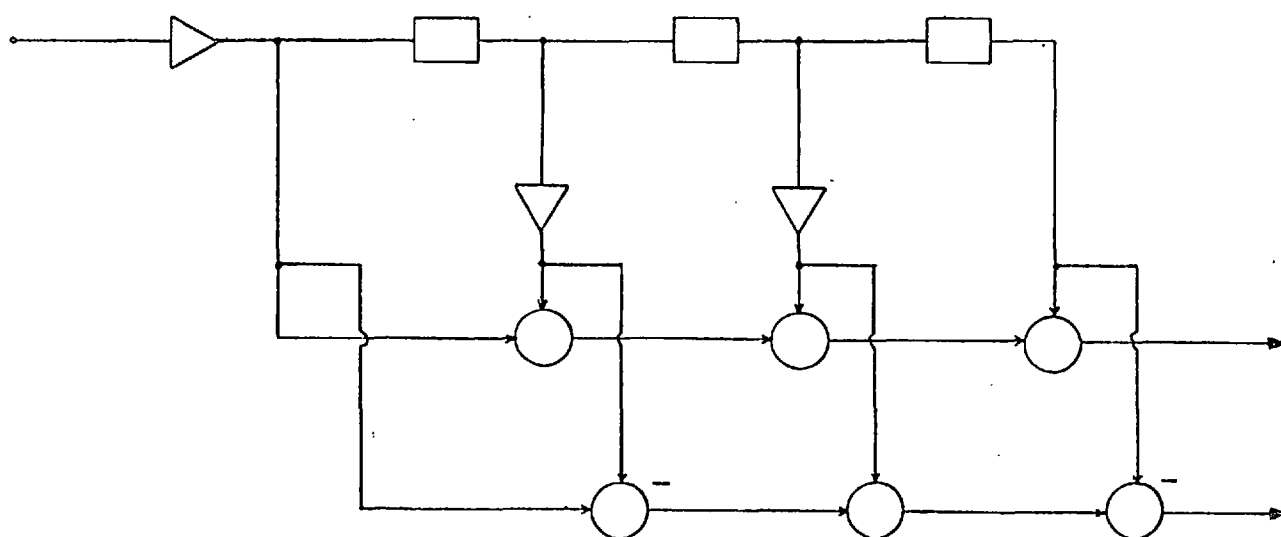


Fig. 2.8 The lowpass-highpass 'phase-splitting' network.

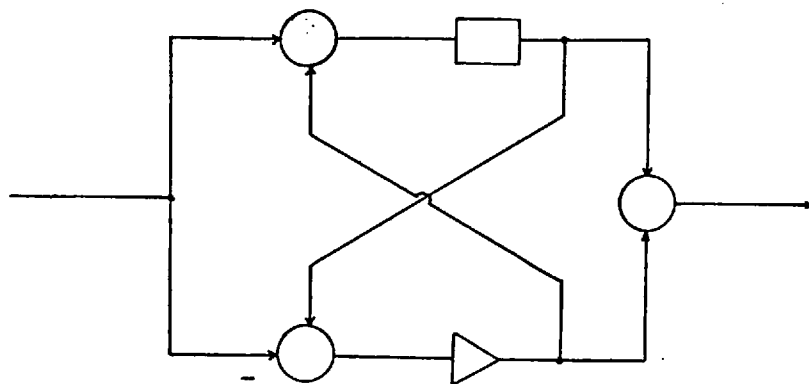


Fig. 2.9 The 1st-degree allpass network.

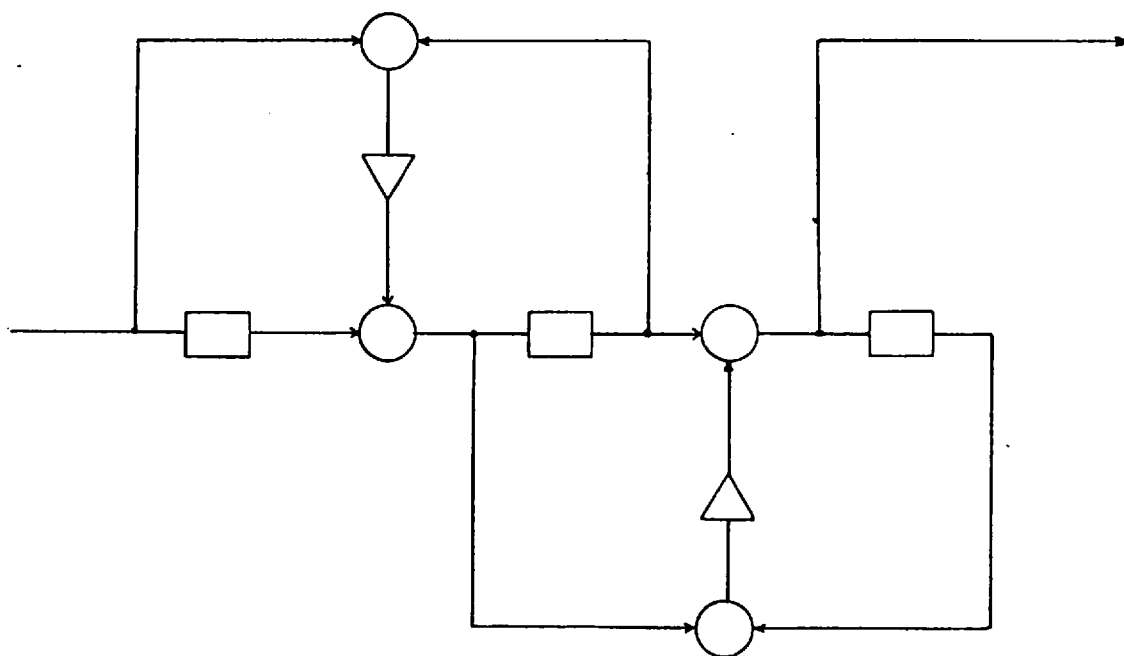


Fig. 2.10 The allpass interwoven structure.

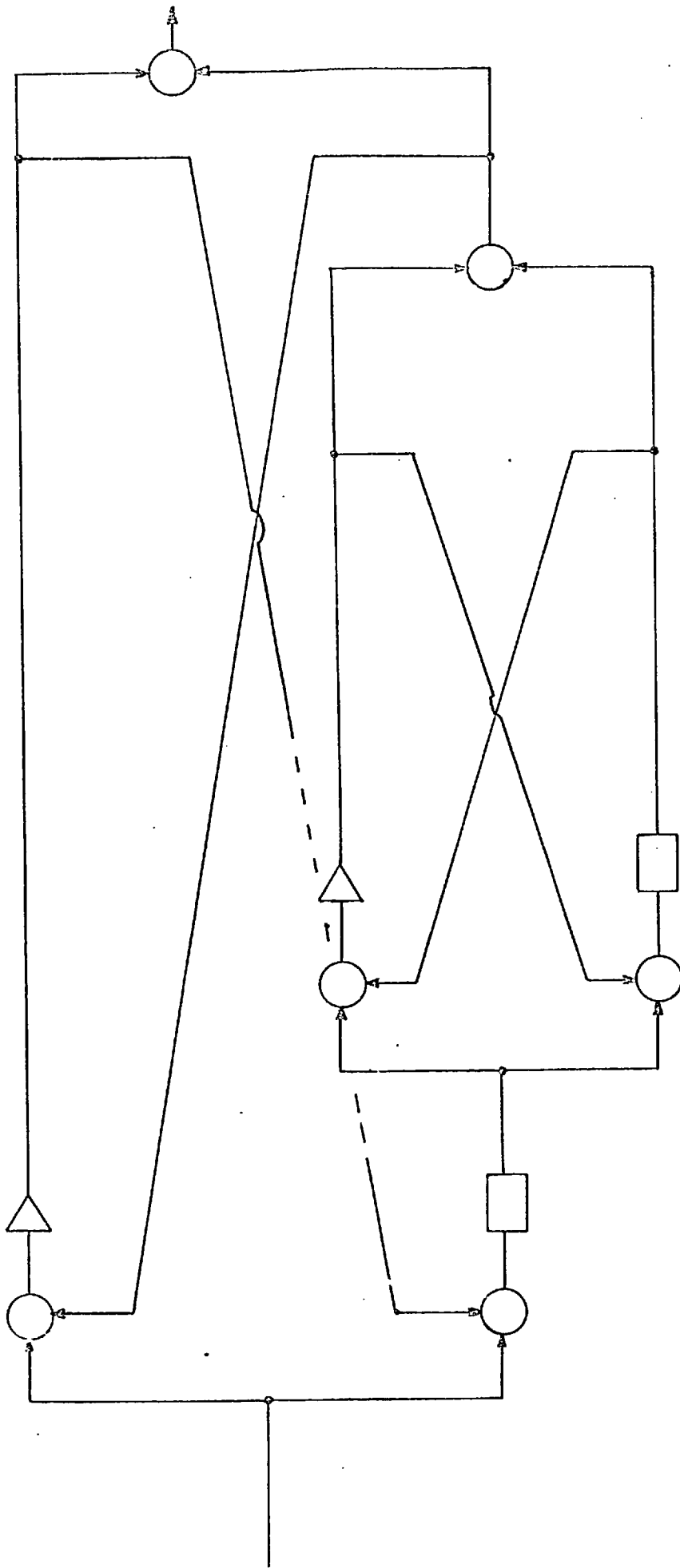


Fig. 2.11 The 'nested' allpass structure.

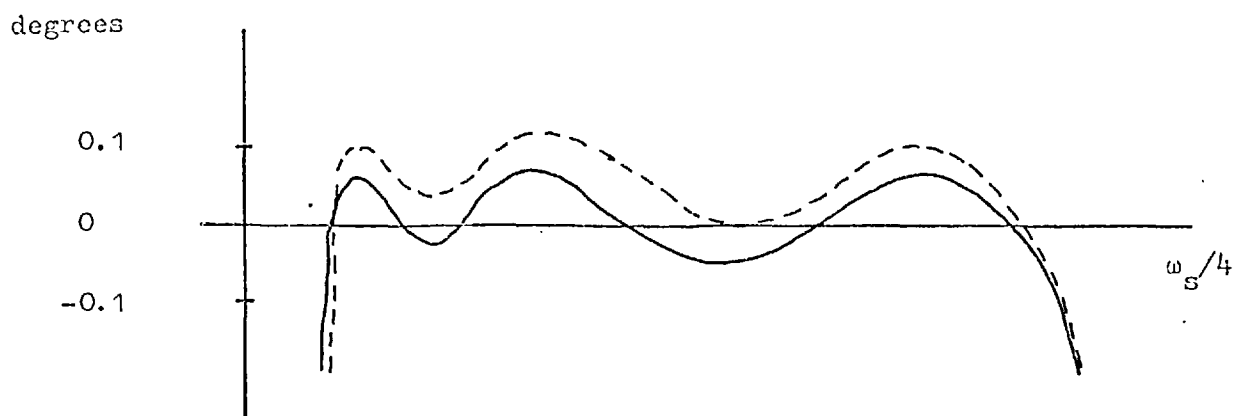


Fig. 2.12 The interwoven structure phase-difference response.

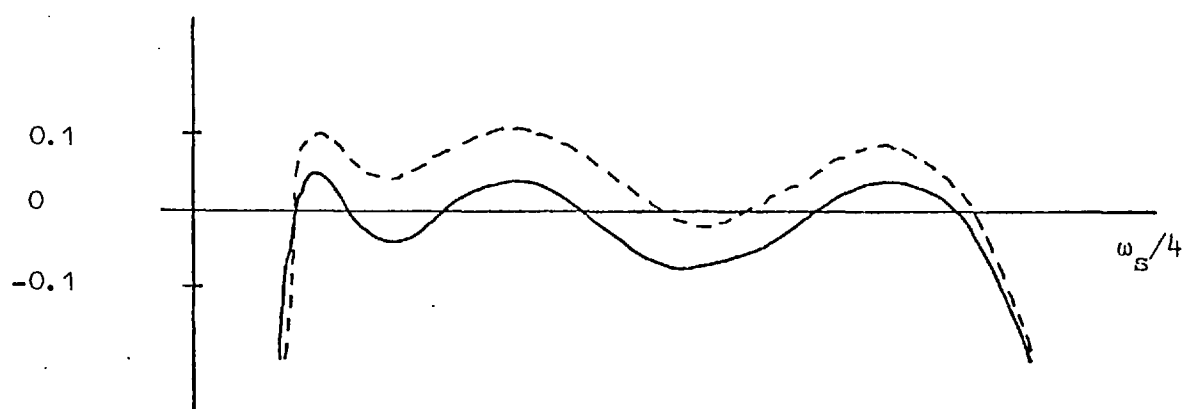


Fig. 2.13 The 1st-degree cascade phase-difference response.

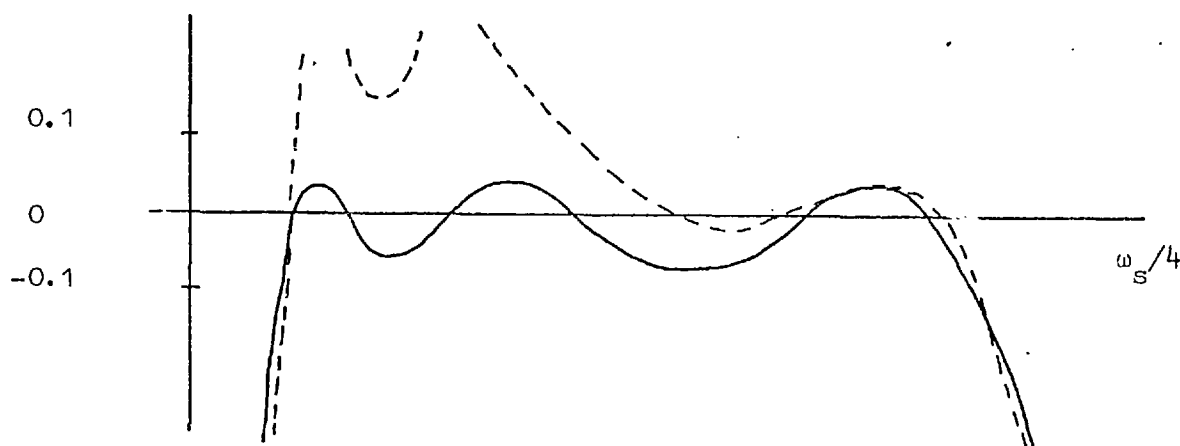


Fig. 2.14 The nested structure phase-difference response.

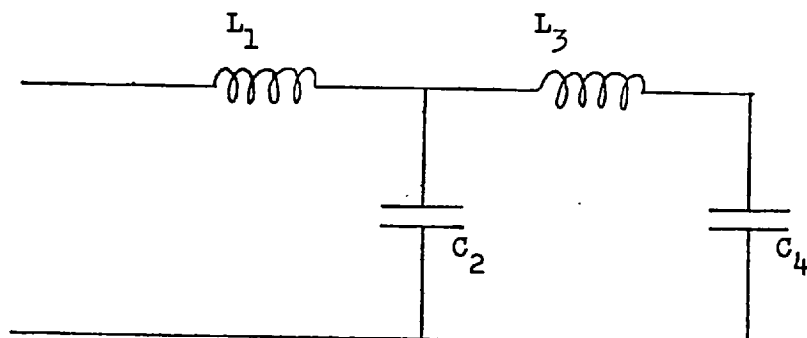


Fig. 2.15 The one-port ladder network.

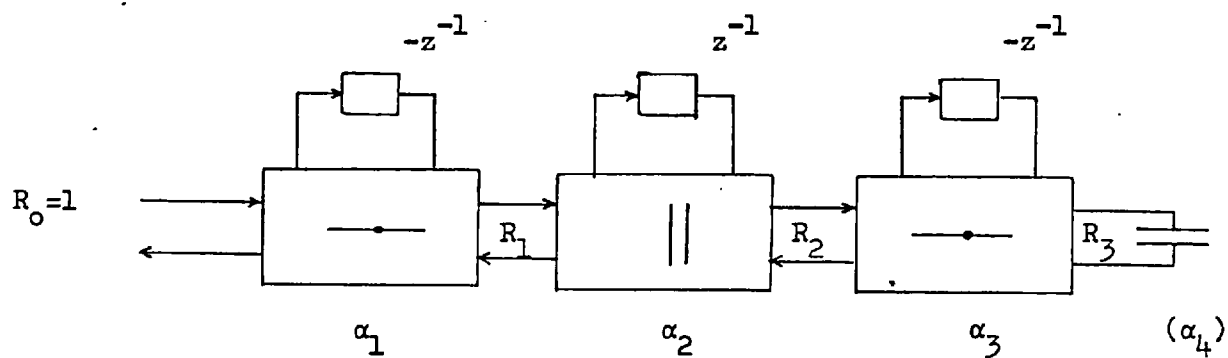


Fig. 2.16 The digital wave-filter representation.

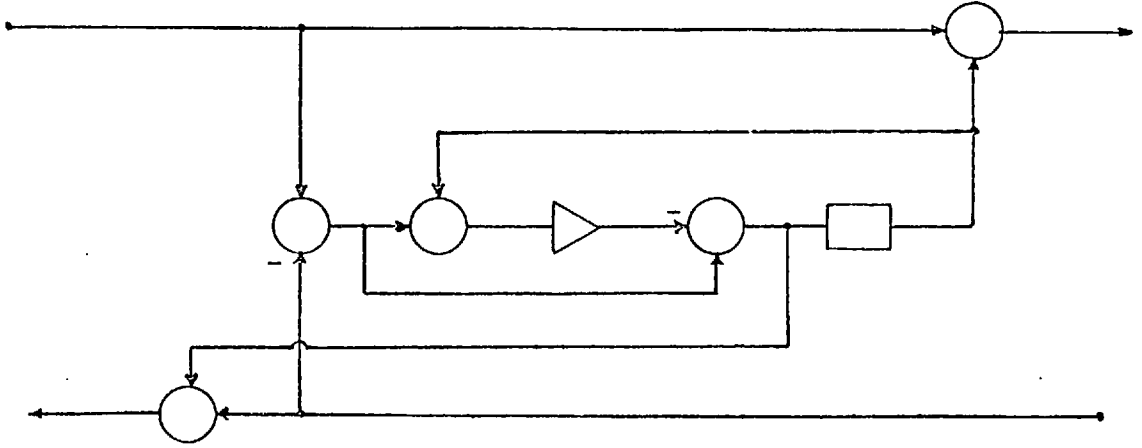


Fig. 2.17 The two-port representation of the 'series-L'

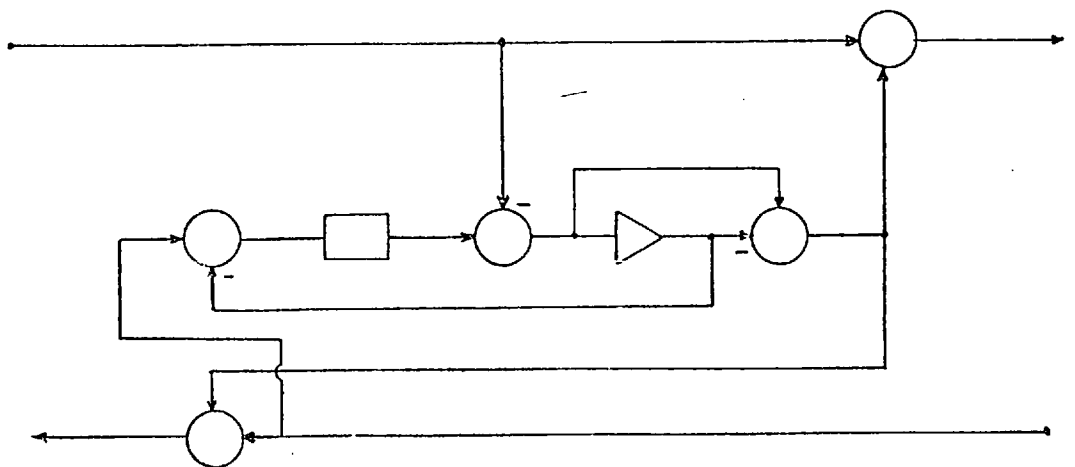


Fig. 2.18 The two-port representation of the 'parallel-C'

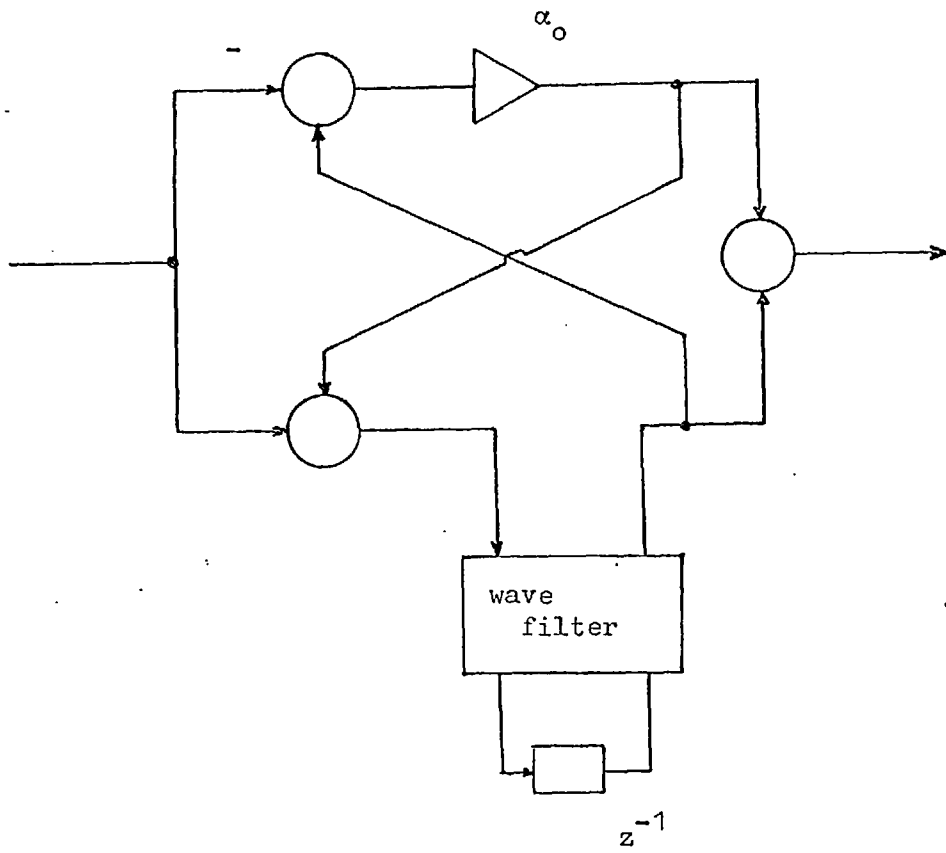


Fig. 2.19 The digital wave-filter plus input adaptor.

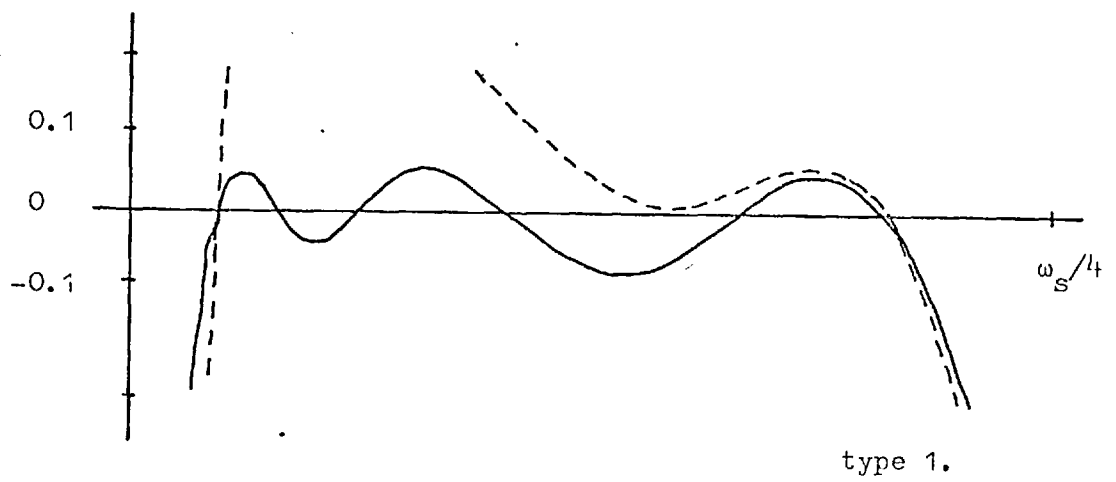
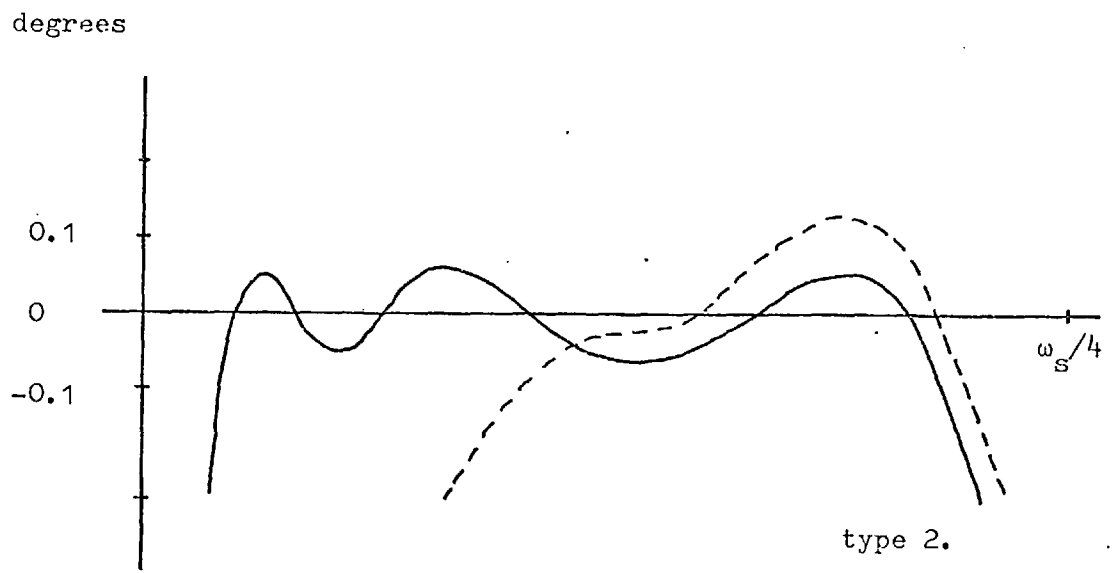


Fig. 2.20 The digital wave-filter (type 1,2) phase-difference response.

CHAPTER THREE.

/

3.0 The passive R.C. - polyphase network

In the phase-splitting networks considered so far, the required 90° phase-difference response has been obtained between the outputs of two paths each containing an appropriate network. Varying the component values (in the analogue networks) or quantizing the coefficients (in the digital networks) will cause the responses of the two separate networks to vary independently. These independent variations give rise to fairly large deviations from the desired phase-difference response. An alternative to the two-network phase-splitter has been proposed⁽³⁵⁾, based on a single passive R.C.-network, and this may be interpreted in terms of polyphase concepts. Furthermore, it has been shown that this network has a lower sensitivity to the variations in component values than that of other passive phase-splitting networks⁽³⁵⁾. The polyphase network concept is adopted in this chapter for use in the digital phase-difference S.S.B. modulator as it is expected that the low sensitivity properties of the analogue polyphase network will exist in its digital counterpart.

In the analogue phase-splitting polyphase network (shown in fig 3.1 for a 1-section 4-phase R.C.-network) four phases (or signal paths) are required in the production of the 90° phase-difference approximation although the desired complex signal is obtained from any two adjacent paths. A considerable insight into the operation of this circuit is gained by considering the polyphase signal in terms of symmetric components⁽³⁴⁾. Figure 3.2 shows the relationship between two phasors; sequence one and two are being defined as positive and negative respectively. Reversing sequence two shows it to be the same as

sequence one but negative in frequency. It is useful, therefore, when handling both positive and negative frequency input signals, to consider these signals as symmetric components of opposite sequence. The input to the polyphase network (fig 3.3) is then represented by (fig. 3.4):-

$$v_1 = v/2. (1 + j) + v/2. (1 - j) \quad 3.1$$

and similarly for v_2 , v_3 and v_4 . For the positive and negative sequence inputs, the polyphase network response can be defined as $H^+(p)$ and $H^-(p)$ or (considering positive and negative frequency) as $H(p)$ and $H(-p)$. Therefore, at the network output, we have:-

$$\begin{aligned} v_1 &= v/2 \{ (1 + j).H(p) + (1 - j).H(-p) \} \\ v_2 &= v/2 \{ (1 - j).H(p) + (1 + j).H(-p) \} \\ v_3 &= v/2 \{ (-1 - j).H(p) + (-1 + j).H(-p) \} \\ v_4 &= v/2 \{ (-1 + j).H(p) + (-1 - j).H(-p) \} \end{aligned} \quad 3.2$$

If each output modulates the carriers ($\cos \omega_c t$, $\sin \omega_c t$, $-\cos \omega_c t$ and $-\sin \omega_c t$) then, following summation, we obtain the output signal V_o given by:-

$$V_o(p) = \frac{e^{-j\phi}}{\sqrt{2}}.H(p).V(p-p_c) + \frac{e^{j\phi}}{\sqrt{2}}.H(-p).V(p+p_c) \quad 3.3a$$

$$\phi \equiv \phi(\omega\tau)$$

The first term in eqn. 3.3a represents the lower sideband (negative frequency) filtered by $H(p)$ and the second term represents the upper sideband (positive frequency) filtered by $H(-p)$. Note: the lower sideband (positive frequency) and the upper sideband (negative frequency) are simply obtained on considering $-p$ replacing p , that is:-

$$V_0(-p) = \frac{e^{j\phi}}{\sqrt{2}} \cdot H(-p) \cdot V(-p - p_c) + \frac{e^{-j\phi}}{\sqrt{2}} \cdot H(p) \cdot V(-p + p_c) \quad 3.3b$$

With regard to equations 3.3a and 3.3b, it can be seen that if a single sideband is to be produced as a result of a filtering operation followed by a high-frequency modulation, then the polyphase network response $H(p)$ is required to be non-symmetrical about the frequency origin - that is, one sideband is effectively removed using the 'non-real' sideband filter. Clearly, this sideband filter has to be represented by a complex transfer function which, for the 4-phase network, we define as:-

$$H(p) = H_a(p) + j \cdot H_b(p) - H_c(p) - j \cdot H_d(p) \quad 3.4$$

However, because of the symmetry of this polyphase network, it is sufficient to verify that the complex transfer function (eqn. 1.1)

$$H(p) = H_a(p) + j \cdot H_b(p)$$

is obtainable - that is, that it is possible to produce the wideband approximation to the 90° phase-difference response between any two adjacent outputs.

We now consider only one phase (or signal path) of the R.C.-network (fig 3.1) as the individual phases of each section are identical (i.e. between the input and output of each path is a resistor and skewed across the paths is a capacitor). For a positive frequency input ($v_{in} \cdot e^{jk\pi/2}$ - where k equals the path number) we derive the chain matrix, for one section, to be:-

$$\begin{bmatrix} V_{in} \\ I_{in} \end{bmatrix} = \begin{bmatrix} \frac{1 + pCR}{1 - jpCR} & \frac{-R}{1 - jpCR} \\ \frac{.2pCR}{1 - jpCR} & \frac{-(1+pCR)}{1 - jpCR} \end{bmatrix} \cdot \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

3.5

giving⁽⁷⁰⁾ :-

$$I_{in} = \frac{(1 + pCR) \cdot V_{in}}{R} + \frac{(1 - jpCR) \cdot V_o}{R}$$

$$I_o = \frac{(1 - jpCR) \cdot V_{in}}{R} + \frac{(1 + pCR) \cdot V_o}{R}$$

$$\underline{I} = [y] \cdot \underline{V} \quad 3.6$$

Now, from the voltage transfer ratio:-

$$T_{12}(p) = \frac{V_o}{V_{in}} = \frac{y_{12}}{y_{22}} = \frac{1 - jpCR}{1 + pCR} \quad 3.7$$

it can be seen that minimum (zero) gain occurs at $\omega = -1/CR$ with the maximum gain occurring at $\omega \approx 1/CR$ (fig 3.5)- that is, we have a non-symmetrical response about the frequency origin. It is noticeable, however, that the resulting passband is not flat. By

cascading further sections and choosing suitable capacitor and resistor values, the required sideband filter may be obtained except for the flat passband response which will necessitate incorporating some form of amplitude correction. A synthesis technique has been used⁽³⁵⁾ as follows; the passive R.C.-network is analysed and the resulting coefficients are equated with some known transfer function to give the unknown component values but for designing high-order networks this synthesis method becomes intractable and the use of a computer optimization procedure is necessary.

An analogue network, based on a cascade of single sections, cannot be designed using the single-section voltage transfer function unless ideal buffering separates each section - that is, the overall network transfer function must be derived. However, in a digital system this is not so in that cascading sections involves no interaction and hence we can employ a basic single digital section which can be cascaded with similar sections as many times as is necessary to produce the required response. The derivation of this digital section is carried out in the following section.

3.1 The derivation of the basic digital section

We shall base our preliminary development of the basic digital section on the relationships that exist in the corresponding analogue sections which are transformed to the digital domain by means of the impulse invariance technique^(5,8). Further refinements will then be carried out on the resulting digital section relationships to produce a useful structure. The impulse invariance transformation is made use of as

it is a relatively simple method of transforming single pole (or single pole plus zero) networks.

Thus for the i^{th} - section of the analogue polyphase network, with input voltages $v_{i1}, v_{i2}, v_{i3}, v_{i4}$, and output voltages $\hat{v}_{i1}, \hat{v}_{i2}, \hat{v}_{i3}, \hat{v}_{i4}$, we have:-

$$\frac{v_{i1} - \hat{v}_{i1}}{R_i} = - (v_{i4} - v_{i1}) \cdot pC_i$$

and therefore:-

$$\hat{v}_{i1} = \frac{v_{i1}}{1 + pC_i R_i} + \frac{v_{i4} \cdot pC_i R_i}{1 + pC_i R_i} \quad 3.8$$

Similarly $\hat{v}_{i2}, \hat{v}_{i3}, \hat{v}_{i4}$, are derived giving:-

$$\begin{vmatrix} \hat{v}_{i1} \\ \hat{v}_{i2} \\ \hat{v}_{i3} \\ \hat{v}_{i4} \end{vmatrix} = \begin{vmatrix} a_i & 0 & 0 & b_i \\ b_i & a_i & 0 & 0 \\ 0 & b_i & a_i & 0 \\ 0 & 0 & b_i & a_i \end{vmatrix} \cdot \begin{vmatrix} v_{i1} \\ v_{i2} \\ v_{i3} \\ v_{i4} \end{vmatrix} \quad 3.9$$

where

$$a_i(p) = \frac{1}{1 + p \cdot C_i R_i} \quad b_i(p) = \frac{p \cdot C_i R_i}{1 + p \cdot C_i R_i}$$

from which it is seen that:-

$$a_i(p) + b_i(p) = 1 \quad 3.10$$

and hence the voltage transfer matrix can be rewritten as:-

$$\begin{vmatrix} a_i & 0 & 0 & 1 - a_i \\ 1 - a_i & a_i & 0 & 0 \\ 0 & 1 - a_i & a_i & 0 \\ 0 & 0 & 1 - a_i & a_i \end{vmatrix}$$

3.11

In the analogue to digital transformation, we require that the imaginary axis of the p-plane maps onto the unit circle in the z-plane. Furthermore, the stable analogue network (its stability is assured as it is a passive network) must be transformed into a stable digital system. A simple transformation satisfying these conditions is obtained using the impulse invariance design method^(5,8) in which the unit impulse-response samples of the digital network are chosen to be equal to evenly spaced samples of the impulse response of the analogue network.

It can be shown⁽⁵⁾ that the z-transform of the digital impulse response ($h(n)$) is related to the Laplace Transform of the samples of the analogue network response ($h_A(nT)$) by:-

$$H(z) \Big|_{z=e^{j\omega T}} = 1/T \cdot \sum_{n=-\infty}^{\infty} H_A(p + jn\omega_s)$$

3.12

It is therefore necessary to have:-

$$\left| H_A(p) \right| = 0 \quad \text{for} \quad \left| \omega \right| > \omega_s/2$$

3.13

in order to avoid added frequency spectra being folded or aliased into the baseband - that is, the analogue network must be bandlimited if the impulse invariance design is to be utilized otherwise the frequency response of the resulting digital network will not be identical to that of the original analogue network. However, in the polyphase network analogue to digital transformation, we are mainly interested in the resulting phase-difference between the two outputs and, although the phase response of each path is modified, the difference between the paths remains unchanged. The impulse invariance design method therefore gives an acceptable analogue to digital transformation of the polyphase network.

For a partial-fraction expansion of $H_A(p)$, we have:-

$$H_A(p) = \sum_{k=1}^N \frac{A_k}{p + p_k} \quad 3.14$$

giving:-

$$h_A(t) = \sum_{k=1}^N A_k \cdot e^{-p_k t} \quad 3.15$$

and therefore:-

$$h_A(nT) = \sum_{k=1}^N A_k \cdot e^{-p_k nT} \quad 3.16$$

Now:-

$$\begin{aligned} H(z) &= \sum_{n=0}^{\infty} h(n) \cdot z^{-n} = \sum_{k=1}^N A_k \cdot \sum_{n=0}^{\infty} e^{-p_k nT} \cdot z^{-n} \\ &= \sum_{k=1}^N \frac{A_k}{1 - e^{p_k T} \cdot z^{-1}} \end{aligned}$$

For high sampling frequencies (i.e. small T), the resulting filter gain may be high and it is therefore convenient to use⁽⁸⁾:-

$$h(n) = T \cdot h_A(nT) \quad 3.18$$

We have for the polyphase subsection:-

$$a_i(p) = \frac{1}{1 + pC_i R_i} \equiv \frac{\alpha_i}{p + \alpha_i} \quad 3.19$$

Now, using eqns. 3.17 and 3.18, $a_i(p)$ is transformed to give $\rho_i(z^{-1})$:-

$$\rho_i(z^{-1}) = \frac{\alpha_i \cdot T}{1 - e^{-\alpha_i T} \cdot z^{-1}}$$

However, to keep the gain of $\rho_i(z^{-1})$ at zero frequency equal to unity, corresponding to eqn. 3.19 with $p = 0$, we modify the above expression to:-

$$\rho_i(z^{-1}) = \frac{1 - \beta_i}{1 - \beta_i \cdot z^{-1}} \quad 3.20$$

which has unity gain at $z^{-1} = 1$ i.e. at zero frequency.

To obtain the complementary part of $b_i(p)$ in the digital domain, we may proceed as above or make use of eqn. 3.10 from which we obtain:-

$$\rho_i(z^{-1}) + \lambda_i(z^{-1}) = 1$$

giving:-

$$\lambda_i(z^{-1}) = \frac{\beta_i \cdot (1 - z^{-1})}{1 - \beta_i \cdot z^{-1}} \quad 3.21$$

from which it is seen that at zero frequency (i.e. at $z^{-1} = 1$) the transmission is zero as expected.

The voltage transfer matrix for the digital i^{th} - section is, therefore, given by:-

$$\begin{bmatrix} I_i \end{bmatrix} = \begin{bmatrix} \rho_i & 0 & 0 & \lambda_i \\ \lambda_i & \rho_i & 0 & 0 \\ 0 & \lambda_i & \rho_i & 0 \\ 0 & 0 & \lambda_i & \rho_i \end{bmatrix} \quad 3.22$$

with its minimum-multiplier structure represented as shown in fig 3.6. It is apparent that the complexity of this structure is high which would in its present form prohibit its use. However, we now obtain a reduction in this complexity where the total number of multipliers, adders and delays is halved without affecting the polyphase phase-splitting properties.

For an N -section network, the transfer ratio is given by:-

$$\begin{bmatrix} \hat{v}_{i1} \\ \hat{v}_{i2} \\ \hat{v}_{i3} \\ \hat{v}_{i4} \end{bmatrix} = \left\{ \prod_{i=1}^N \begin{bmatrix} I_i \end{bmatrix} \right\} \cdot \begin{bmatrix} v_{i1} \\ v_{i2} \\ v_{i3} \\ v_{i4} \end{bmatrix} \quad 3.23$$

If we now consider the first section and note that:-

$$\begin{aligned} v_{11} &= v_{12} = v_{in} \\ v_{13} &= v_{14} = -v_{in} \end{aligned} \quad 3.24$$

we can rewrite equation 3.23 for the first section as:-

$$\begin{vmatrix} v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \end{vmatrix} = \begin{vmatrix} \rho_1 - \lambda_1 & 0 & 0 & 0 \\ 0 & \rho_1 + \lambda_1 & 0 & 0 \\ 0 & 0 & -\rho_1 + \lambda_1 & 0 \\ 0 & 0 & 0 & -\rho_1 - \lambda_1 \end{vmatrix} \begin{vmatrix} v_{in} \\ v_{in} \\ v_{in} \\ v_{in} \end{vmatrix}$$

3.25

and therefore:-

$$\begin{aligned}
 \hat{v}_{13} &= -\hat{v}_{11} \\
 \hat{v}_{14} &= -\hat{v}_{12}
 \end{aligned}$$

3.26

Cascading a second section, we now obtain using eqn 3.23:-

$$\begin{aligned}
 v_{21} &= \{ \rho_2(\rho_1 - \lambda_1) + \lambda_2(-\rho_1 - \lambda_1) \} \cdot v_{in} \\
 v_{22} &= \{ \lambda_2(\rho_1 - \lambda_1) + \rho_2(\rho_1 + \lambda_1) \} \cdot v_{in} \\
 v_{23} &= \{ \lambda_2(\rho_1 + \lambda_1) + \rho_2(-\rho_1 + \lambda_1) \} \cdot v_{in} \\
 v_{24} &= \{ \lambda_2(-\rho_1 + \lambda_1) + \rho_2(-\rho_1 - \lambda_1) \} \cdot v_{in}
 \end{aligned}$$

and therefore:-

$$\begin{aligned}
 \hat{v}_{23} &= -\hat{v}_{21} \\
 \hat{v}_{24} &= -\hat{v}_{22}
 \end{aligned}$$

3.27

From eqns 3.26 and 3.27, it can be seen that, for any number of sections, only two signals need be considered since a simple inversion of $\hat{v}_{i1}$, $\hat{v}_{i2}$ gives $\hat{v}_{i3}$, $\hat{v}_{i4}$. By considering only two outputs ($\hat{v}_{i1}$ and $\hat{v}_{i2}$) and factorizing equation 3.27, we now obtain the modified transfer ratio equation generalized for an N-section network -

i.e:-

$$\begin{bmatrix} \hat{v}_{i1} \\ \hat{v}_{i2} \end{bmatrix} = \left\{ \prod_{i=1}^N \begin{bmatrix} I_i \end{bmatrix} \right\} \cdot \begin{bmatrix} v_{in} \\ v_{in} \end{bmatrix} \quad 3.28$$

where:-

$$\begin{bmatrix} I_i \end{bmatrix} = \begin{bmatrix} \rho_i(z^{-1}) & -\lambda_i(z^{-1}) \\ \lambda_i(z^{-1}) & \rho_i(z^{-1}) \end{bmatrix} \quad 3.29$$

The modified i^{th} -section above of the digital polyphase network is represented in fig 3.7. In reducing the complexity of the initial design we have produced a digital polyphase structure with two input and output paths whereas the analogue network requires four input and output paths. However, a closer examination of the digital polyphase structure shows that the four phase-producing paths exist as in the analogue network. Using the above structure, we can obtain solutions to the phase-difference approximation problem utilizing computer optimization techniques. This will be examined in the following section.

3.2 Coefficient selection by optimization

The required phase-difference response between the two outputs of the digital polyphase network is defined by:-

$$\left| \tan^{-1} \angle H_a(z^{-1}) - \tan^{-1} \angle H_b(z^{-1}) \right| = \pi/2 \pm \epsilon$$

$$\omega_1 T \leq \omega \leq \omega_2 T$$

3.30

and ϵ is a small deviation from $\pi/2$ which may be controlled by the order of the approximation employed.

The transfer functions $H_a(z^{-1})$ and $H_b(z^{-1})$ are given by:-

$$\begin{bmatrix} H_a(z^{-1}) \\ H_b(z^{-1}) \end{bmatrix} = \left\{ \prod_{i=1}^N \begin{bmatrix} \rho_i(z^{-1}) & -\lambda_i(z^{-1}) \\ \lambda_i(z^{-1}) & \rho_i(z^{-1}) \end{bmatrix} \right\} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and

$$\rho_i(z^{-1}) = (1 - \beta_i) / (1 - \beta_i z^{-1})$$

$$\lambda_i(z^{-1}) = \beta_i (1 - z^{-1}) / (1 - \beta_i z^{-1})$$

3.31

Handling this transmittance analytically becomes difficult for high order networks and therefore solutions, yielding approximations to the ideal response, are sought utilizing computer optimization techniques. The problem must first be clearly defined before an appropriate optimization routine can be chosen. The equations for the phase responses resulting from equation 3.31 are non-linear in the coefficients β_i and the use of a non-linear optimization routine is thus envisaged. The occurrence of any phase-discontinuity is undesirable as it cannot be easily handled within the optimization program. We therefore modify equation 3.30 (by taking tangents of both sides):-

$$\frac{\tan \tan^{-1} \angle H_a - \tan \tan^{-1} \angle H_b}{1 + \tan \tan^{-1} \angle H_a \cdot \tan \tan^{-1} \angle H_b} = \tan(\pi/2 \pm \epsilon)$$

giving:-

$$\frac{\angle H_a - \angle H_b}{1 + \angle H_a \cdot \angle H_b} = \frac{1 + \tan(+\epsilon)/\tan(\pi/2)}{1/\tan(\pi/2) - \tan(+\epsilon)} \quad 3.32$$

We now define the error δ , the variable to be minimized, as:-

$$\pm \delta = \tan(+\epsilon) = \frac{1 + \angle H_a \angle H_b}{\angle H_a - \angle H_b} \quad 3.33$$

Using equation 3.33 proves advantageous in that only the arguments of the real transmittances H_a , H_b , need be computed. The problems associated with phase-discontinuities are thus eliminated. In addition, the inaccuracies occurring in the repeated computer calculations of tangents and inverse tangents are obviated. The algorithm used to calculate the unknown coefficients (β_i) , for a minimum δ , can now be chosen.

There are several possible ways in which an approximation to an ideal response can be given⁽⁷⁴⁾; for example (i) the approximation to the ideal curve can be obtained at one point only or (ii) the ideal curve can be crossed at several points by the approximating curve. In many network design problems⁽⁷⁵⁻⁷⁸⁾, however, the use of the Least $-p^{\text{th}}$ error norm has found favour. Only two special cases of the $L-p$ norm need to be considered⁽⁷⁹⁾; these being the L_2 (least squares) error norm and the L_∞ (Chebychev or minimax) error norm. For the ideal response $D(\omega)$, defined for the set ω_j , and the approximating response $H(\beta_i, \omega)$, where the parameters β_i are to be optimized, the required error norms are defined by:-

L_2 error norm

$$\left\| D(\omega) - H(\beta, \omega) \right\|_2 = \left\{ \int_0^\pi \left| D(\omega) - H(\beta, \omega) \right|^2 d\omega \right\}^{\frac{1}{2}} \quad 3.34$$

L_∞ error norm

$$\left\| D(\omega) - H(\beta, \omega) \right\|_\infty = \max_{\omega_j} \left| D(\omega) - H(\beta, \omega) \right| \quad 3.35$$

The least squares error norm is simply implemented and, although satisfactory results can be obtained, it is well known that, in some circumstances, a problem is caused by the response overshooting - that is, Gibbs' phenomena^(5,12). A 'best' (optimal) approximation is, however, to be expected from the Chebychev error norm in which the worst-case deviation is kept as small as possible. This error norm is characterized by the alternation theorem⁽⁷⁴⁾ which states that the best approximation alternates or ripples about the ideal. Both of these error norms are considered in the design of the polyphase phase-splitting network.

Various optimization algorithms are now available for use with non-linear equations; the choice of algorithm being governed by the design problem at hand. In certain of the originally proposed algorithms⁽⁸⁰⁾, the partial differentials $\partial H_1 / \partial \beta_i$, required in gradient calculations, were to be supplied by the user. For simplicity, however, an algorithm is chosen for the polyphase design problem in which this is not the case - it is assumed that partial differentials exist and are continuous. The algorithm⁽⁸¹⁾, after Peckham⁽⁸²⁾, finds the sum of squares of m non-linear functions, each of n variables (β_i , $i = 1, n$) - that is:-

$$S(\beta) = \underline{f}^T \cdot \underline{f} = \sum_{j=1}^m \left[f_j(\beta_1, \beta_2, \dots, \beta_n) \right]^2$$

where $m > n$.

3.36

The Jacobian matrix:-

$$\begin{bmatrix} J \end{bmatrix} = J_{ij} \left(\frac{\partial H_j}{\partial \beta_i} \right) \omega \quad 3.37$$

is used in estimating the minimum of the sum of the squares; the matrix being obtained from an approximation in which the functions are differenced over at least $n + 1$ points. An initial set of values of the variables (β_i) must be supplied by the user. In our case, for a stable system, it is necessary that the initial and subsequent values must satisfy $0 < \beta_i < 1$: also required is the set of m non-linear equations. Via an iterative procedure (computed 'interactively'), new values of β_i are calculated until a satisfactory convergence is obtained or the operation is terminated by the user.

The computer programme 'OPT' (see Appendix A) has been used to design polyphase networks of varying lengths. In this program, it is necessary to specify (i) n - the number of unknown variables (β_i) (equivalent to the number of single sections) (ii) m - the number of frequency sample points (equivalent to the number of non-linear equations) (iii) the extremal frequency points of the constrained region - that is, the region over which the approximation is to be obtained. For each variable β_i , a weight v_i can be applied to give more emphasis to function values near the minimum.

The phase-difference response plots of networks of four and five and six sections, designed for a constrained region extending from 340 to 3400 Hz. (with respect to 16kHz sample-rate), are shown in fig 3.8. In these plots, it is possible that certain ripples will not be evident if they occur between the plotted frequency points because only a finite number of frequency points can be handled. However, as this

'loss' is more likely at low frequencies, 'expanding' this portion of frequency axis, by calculating additional frequency points, minimizes this problem. The effect of varying the extremal frequency points of the constrained region is shown in fig 3.9. By 'squeezing' the response to fit at either extremal frequency point, we cause some additional 'bulging' in the remainder of the curve.

The Chebychev or minimax error norm is obtained by modifying equation 3.36 thus:-

$$S(\beta) = \underline{f^T} \cdot \underline{f} = \sum_{j=1}^m \left[f_j(\beta_1, \beta_2, \dots, \beta_n) \right]^p \quad 3.38$$

and, with $p \rightarrow \infty$, a minimax approximation results. In practice, it is sufficient to use even values of $p \approx 10$ to 20, then with the inclusion of a scaling factor, the worst-case errors are minimized.

Although a 'best' fit is to be expected from the Chebychev approximation, it is seen from the practical result of fig 3.10 (the phase-difference responses of an 8-section polyphase network designed using both L_2 and L_∞ approximations) that, with regard to the desired channel filter specifications, the least-squares error approximation gives a more satisfactory result. This is simply explained by the fact that the Chebychev error norm gives more 'weight' to the extremal frequency points than is required by our design specification. (Note: this is only true because the error maxima occur at the band edges). The resulting response for the L_∞ approximation is 'best' in the sense that the total error is minimum; however, because of the permitted relaxation of the phase-difference response at the band edges the results from the L_2 error norm approximation are preferred.

3.3 Sensitivity Considerations

We now require a measure of the sensitivity of the non-real sideband filter's amplitude response to variations in the coefficient values of the phase-splitting network. In order to obtain this sensitivity measure, the sideband-filter amplitude response must be related, in some way, to the phase-difference response of the polyphase network.

For the non-real sideband-filter transmittance, we have:-

$$H(z^{-1}) \triangleq \frac{1}{2} \left\{ H_a(z^{-1}) + j \cdot H_b(z^{-1}) \right\} \quad 3.39$$

where:-

$$\begin{aligned} H_a(e^{-j\omega t}) &= \left| H_a(e^{-j\omega t}) \right| \cdot e^{j\phi_a(\omega T)} \\ H_b(e^{-j\omega t}) &= \left| H_b(e^{-j\omega t}) \right| \cdot e^{j\phi_b(\omega T)} \\ \phi(\omega T) &= \phi_a(\omega T) - \phi_b(\omega T) \\ \phi(-\omega T) &= -\phi(\omega T) \end{aligned} \quad 3.40$$

From equation 3.7, we know that:-

$$\left| H_a(e^{-j\omega t}) \right| = \left| H_b(e^{-j\omega t}) \right| = \left| H_o(e^{-j\omega t}) \right| \quad 3.41$$

that is, the amplitude response of each path is the same although it is not necessarily flat. Using equations 3.39 to 3.41, we derive:-

$$\begin{aligned} \left| H(e^{-j\omega t}) \right|^2 &= H(e^{-j\omega t}) \cdot H^*(e^{-j\omega t}) \\ &= \left| H_0(e^{-j\omega t}) \right|^2 \cdot \left\{ e^{j\phi_a} + je^{j\phi_b} \right\} \cdot \left\{ e^{-j\phi_a} - je^{-j\phi_b} \right\} \end{aligned}$$

$$\phi \cong \phi(\omega\tau)$$

and therefore:-

$$\left| H(e^{-j\omega t}) \right| = \left| H_0(e^{-j\omega t}) \right| \cdot \left\{ \frac{1}{2} (1 + \sin \phi(\omega\tau)) \right\}^{\frac{1}{2}} \quad 3.42$$

giving:-

$$\begin{aligned} \left| H(e^{-j\omega t}) \right| &= \left| H_0(e^{-j\omega t}) \right| & \phi(\omega\tau) &= 90^\circ \\ &= 0 & \phi(\omega\tau) &= -90^\circ \end{aligned} \quad 3.43$$

From equations 3.42 and 3.43, it can be seen that the sideband filter is dependent on the phase-difference response only for its stopband shape whereas its passband is defined by both the phase-difference response and the amplitude response $H_0(e^{-j\omega t})$ of the phase-splitting network. It is apparent therefore that, if the passband of the sideband-filter is to be flat, some form of pre- or post-distortion of the polyphase phase-splitting network's amplitude response will be required. Until a final phase-splitting network design is arrived at, we assume a flat polyphase network amplitude response and neglect it in our sensitivity considerations. This is valid as (i) the polyphase network's symmetry maintains that $H_a(e^{-j\omega\tau}) = H_b(e^{-j\omega\tau})$ - that is, the coefficients in each section are identical and therefore

so are any changes in each path and (ii) we now show that the sideband-filter's specifications need only be satisfied in the stopband for the passband specifications to be automatically satisfied. Condition (ii) requires that the amplitude response sensitivity of the polyphase network should not be significantly higher than its phase-difference sensitivity (we verify this very simply as one calculation yields both the amplitude and phase-difference sensitivity to coefficient changes of the polyphase network).

In the sideband-filter design, we allow the maximum errors τ_1 and τ_2 in the passband & stopband respectively (fig 3.11). The relationship between τ_1, τ_2 and τ (the deviation away from the $\pi/2$ phase-difference response) is (using equation 3.42) as follows:-

sideband-filter passband:

$$\begin{aligned}
 1 - \tau_1 &= \frac{1}{2} \left\{ (1 + \sin(\pi/2 \pm \tau)) \right\}^{\frac{1}{2}} \\
 &\simeq 1 - \tau^2/8 \\
 \tau_1 &\simeq \tau^2/8
 \end{aligned}
 \tag{3.44}$$

sideband-filter stopband:

$$\begin{aligned}
 \tau_2 &= \frac{1}{2} \left\{ (1 + \sin(-\pi/2 \pm \tau)) \right\}^{\frac{1}{2}} \\
 &\simeq (\tau^2/4)^{\frac{1}{2}} \\
 \tau_2 &\simeq \tau/2
 \end{aligned}
 \tag{3.45}$$

This gives the relationship between the sideband-filter's passband and stopband as:-

$$\tau_1 \simeq \tau_2^2 / 2
 \tag{3.46}$$

from which the correspondence between the stopband and passband

attenuation has been shown⁽⁴³⁾ as:-

Stopband	Passband
> 40 dBs	< 4×10^{-4} dBs
60 dBs	4×10^{-6} dBs
80 dBs	4×10^{-8} dBs

3.47

The sensitivity index of the non-real filter's amplitude response is defined by:

$$S_{\beta}^{|H|}(\varphi) \triangleq \left\{ \sum_{i=1}^N \left[S_{\beta_i}^{|H|}(\varphi) \right]^2 \right\}^{\frac{1}{2}} \quad 3.48$$

where:-

$$S_{\beta_i}^{|H|} \triangleq \frac{\beta_i}{|H|} \cdot \frac{\partial |H|}{\partial \beta_i} \quad 3.49$$

This cannot be obtained directly and we therefore make use of the following equation:-

$$\frac{\partial |H|}{\partial \beta_i} = \frac{\partial |H|}{\partial \varphi(\omega T)} \cdot \frac{\partial \varphi(\omega T)}{\partial \beta_i} \quad 3.50$$

Using equation 3.42, we obtain:-

$$\frac{\partial |H|}{\partial \varphi(\omega T)} = \frac{\cos \varphi(\omega T)}{4} \cdot \left\{ \frac{1}{2}(1 + \sin \varphi(\omega T)) \right\}^{-\frac{1}{2}} \quad 3.51$$

or, after normalizing:-

$$\frac{1}{|H|} \cdot \frac{\partial |H|}{\partial \varphi(\omega T)} = \frac{\cos \varphi(\omega T)}{2(1 + \sin \varphi(\omega T))} \quad 3.52$$

We now require $\beta_i \cdot \frac{\partial \varphi(\omega T)}{\partial \beta_i}$ which is derived as follows: differentiating $\varphi(\omega T)$ with respect to β_i and normalizing gives:-

$$\beta_i \cdot \frac{\partial \varphi(\omega T)}{\partial \beta_i} = \beta_i \cdot \frac{\partial \varphi_a(\omega T)}{\partial \beta_i} - \beta_i \cdot \frac{\partial \varphi_b(\omega T)}{\partial \beta_i}$$

3.53

For the sub-transmittance:

$$H_a(z^{-1}) = \left| H_a(e^{-j\omega T}) \right| \cdot e^{j\varphi_a(\omega T)}$$

after taking logarithms of both sides, we obtain:-

$$\ln(H_a) = \ln(|H_a|) + j \cdot \varphi_a \quad 3.54$$

and differentiating, with respect to β_i , gives:-

$$\frac{1}{H_a} \cdot \frac{\partial H_a}{\partial \beta_i} = \frac{1}{|H_a|} \cdot \frac{\partial |H_a|}{\partial \beta_i} + j \cdot \frac{\partial \varphi_a}{\partial \beta_i}$$

or:-

$$\frac{\beta_i}{H_a} \cdot \frac{\partial H_a}{\partial \beta_i} = \frac{\beta_i}{|H_a|} \cdot \frac{\partial |H_a|}{\partial \beta_i} + j \cdot \frac{\beta_i \cdot \partial \varphi_a}{\partial \beta_i}$$

and therefore:-

3.55

$$S_{\beta_i}^{H_a} = S_{\beta_i}^{|H_a|} + j \cdot \varphi_a \cdot S_{\beta_i}^{\varphi_a}$$

3.56

From eqns. 3.55 and 3.56, we have:-

$$S_{\beta_i}^{|H_a|} = \frac{\beta_i}{H_a} \cdot \frac{\partial |H_a|}{\partial \beta_i} = \text{Real} \left[S_{\beta_i}^{H_a} \right]$$

also:-

$$S_{\beta_i}^{\varphi_a} = \frac{\beta_i}{\varphi_a} \cdot \frac{\partial \varphi_a}{\partial \beta_i} = \text{Imag} \left[\frac{1}{\varphi_a} \cdot S_{\beta_i}^{H_a} \right]$$

and therefore:-

$$\beta_i \cdot \frac{\partial \varphi_a}{\partial \beta_i} = \text{Imag} \left[S_{\beta_i}^{H_a} \right]$$

3.57

Similarly:-

$$\beta_i \cdot \frac{\partial \varphi_0}{\partial \beta_i} = \text{Imag} \left[S_{\beta_i}^{H_b} \right] \quad 3.58$$

Equation 3.48 can therefore be rewritten as:-

$$S_{\varphi}^{H(\varphi)} = \left| \frac{\cos \varphi(\omega T)}{2(1 + \sin \varphi(\omega T))} \right| \cdot \left\{ \sum_{i=1}^N \text{Imag} \left(S_{\beta_i}^H \right)^2 \right\}^{\frac{1}{2}}$$

where:-

$$\text{Imag} \left[S_{\beta_i}^H \right] = \text{Imag} \left[S_{\beta_i}^{H_a} \right] - \text{Imag} \left[S_{\beta_i}^{H_b} \right] \quad 3.59$$

Equation 3.59 shows clearly that the sideband-filter amplitude sensitivity is a minimum in the passband with the maximum occurring in the stopband (at $\varphi = \pi/2$). To examine the amplitude sensitivity in the stopband, the partial differentials $\partial \varphi_a / \partial \beta_i$, $\partial \varphi_b / \partial \beta_i$ must be computed. Seviaora and Sablatash⁽⁸³⁾ have derived a simple expression for computing these 1st-order sensitivities; their approach is based on an extension of Tellegen's theorem. For a general transmittance $T(z^{-1})$, it can be shown that, for a small change in branch B_{ij} ,:-

$$\frac{\partial T(z^{-1})}{\partial B_{ij}} = T_{i1}(z^{-1}) \cdot T_{2j}(z^{-1}) \quad 3.60$$

Similar results have also been obtained by Constantinides⁽¹⁰⁰⁾, Fettweis⁽⁸⁴⁾ and Collona & Julio⁽⁸⁵⁾. The stopband sensitivity index, computed using eqns. 3.59 and 3.60, for the 8-section network (designed using the L_2 error norm) is shown in fig. 3.12. For comparisons, the stopband sensitivity index of the allpass phase-splitting network is shown plotted in fig 3.13. Both networks were designed to meet the same phase-difference response constraint. It is interesting to note that, although the polyphase network is realized using twice as many multipliers as in the allpass network, the two sensitivity plots are

somewhat similar thus indicating that the sensitivity contribution per multiplier is higher in the allpass phase-splitting network. This suggests that, for the two networks' phase-difference responses to be equivalent (in network designs using quantized coefficients), the polyphase network should be realized using a larger number of coefficients (but of short wordlengths) than is required in realizing the allpass network (it has been shown⁽⁴³⁾ that eight 11-bit coefficients are required by the allpass network). The effects of reducing the coefficient accuracy, throughout the polyphase network, can be observed by introducing a finite change ΔB_{ij} in the branch B_{ij} . The resulting change $\Delta T(z^{-1})$, in the general transmittance $T(z^{-1})$ is obtained from⁽⁸⁶⁾ :-

$$\Delta T(z^{-1}) = \frac{T_{i1}(z^{-1}) \cdot T_{2j}(z^{-1}) \cdot \Delta B_{ij}}{1 - T_{ij}(z^{-1}) \cdot \Delta B_{ij}}$$

3.61

Alternatively, we can evaluate the network's phase-difference response characteristics for successively reduced coefficient accuracy; this being equivalent to a finite change in all multiplier-containing branches. The coefficient accuracy can be reduced by rounding (or truncating) the ideal coefficient's decimal value or, of greater practical interest, by representing the ideal value with its nearest binary equivalent. Using this latter method, the effects of finite changes in the coefficient values of the polyphase network were investigated. The plots of fig 3.14 show the phase-difference response of the 8-section network realized using ideal 5,4 and 3-bit coefficients and, for ideal and 4-bit coefficients, the resulting stopband of the sideband filter is shown plotted in fig 3.15.

From this initial investigation of the properties of the polyphase network phase-difference response, we deduce by comparing the improved phase-difference response, gained by increasing the network order, with that obtained due to increased coefficient accuracy, that for high order networks the coefficient accuracy becomes of secondary importance. The phase-difference plots of fig 3.16, and the plot of fig 3.17 showing the 0.1° upper and lower frequency extremes for varying network lengths, emphasise this point in that all sections are chosen with $\beta_i = \pm 0.5$ (defined throughout this thesis as trivial coefficients due to their ease of implementation - that is, multiplication by 0.5 involves a shifting operation only⁽⁵²⁾). An advantage can be gained therefore by selecting coefficient values with short wordlengths and using high enough order networks so that particular phase requirements are satisfied. There is, however, one disadvantage which does not occur to such a great extent when low order networks are used; that is, the deviation away from the required flat amplitude response is increased with every section added. This and the problem of selecting an optimal network length will be considered further. Firstly, however, we examine the effects of using finite-precision arithmetic with regard to the noise characteristics of the network.

3.4 Summary

In the analogue domain, R.C.-networks have been proposed for phase-splitting and recently such networks have been interpreted in terms of polyphase concepts. In this chapter, a digital approach for the realization of a sequence asymmetric polyphase network has been proposed. Using the impulse-invariance design method, one section of a passive R.C.-polyphase network has been transformed to yield the digital equivalent structure. Furthermore, it has been shown that the unwanted inputs and outputs of the analogue circuit can be eliminated thus giving a reduction in the overall hardware complexity. Solutions to the phase-difference approximation problem have been sought by utilizing a computer optimization procedure. An 8-section network, satisfying the required phase-difference constraints, has been examined with regard to its sensitivity to coefficient variation. The results of sensitivity studies on this, and other networks of varying lengths, indicate that the coefficient accuracy is of secondary importance to the network length in the production of wideband 90° phase-difference responses.

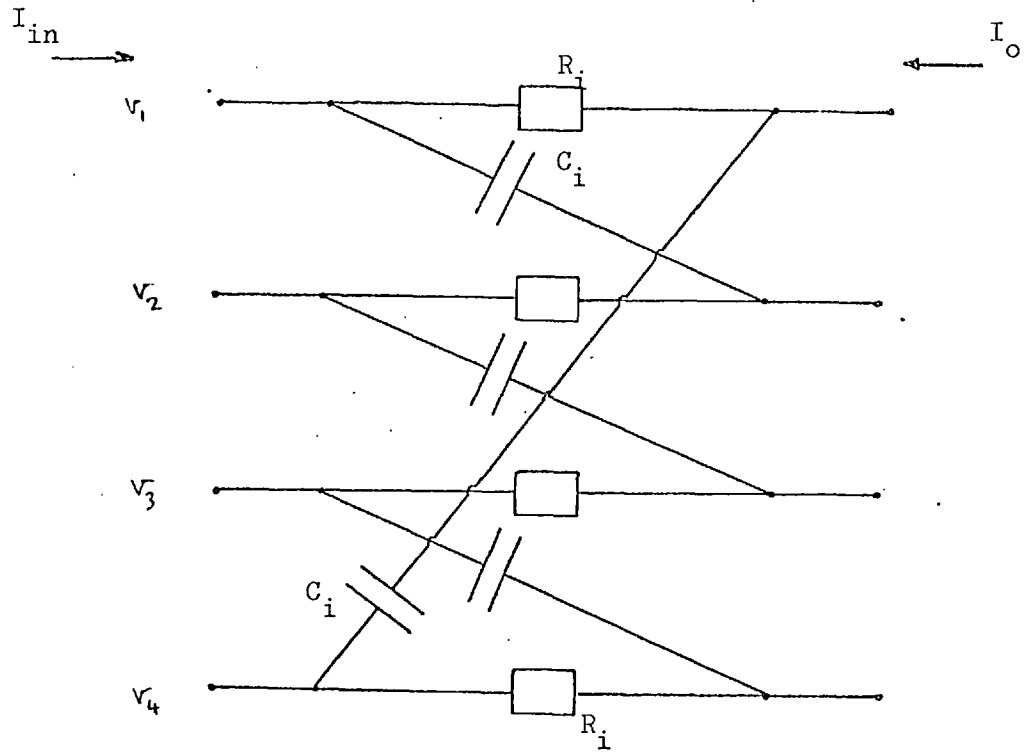


Fig. 3.1 One section of the 4-phase RC-network

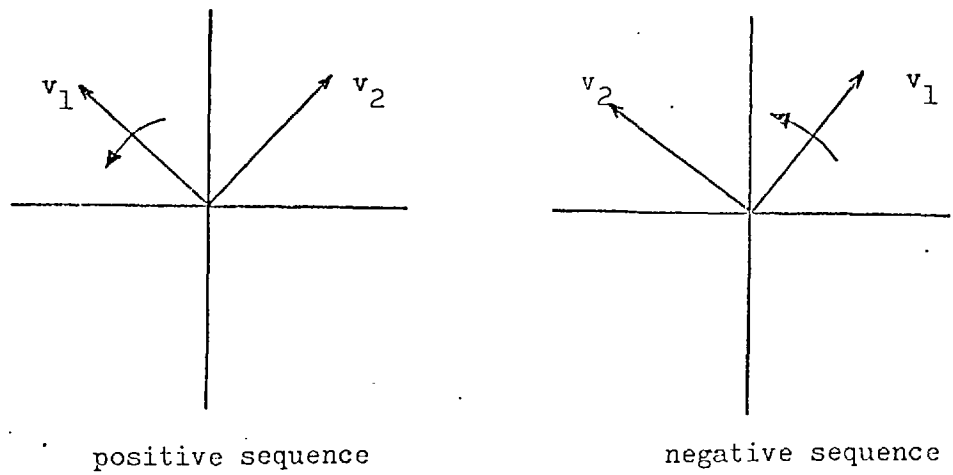


Fig. 3.2 Positive and negative sequence phasor relationships.

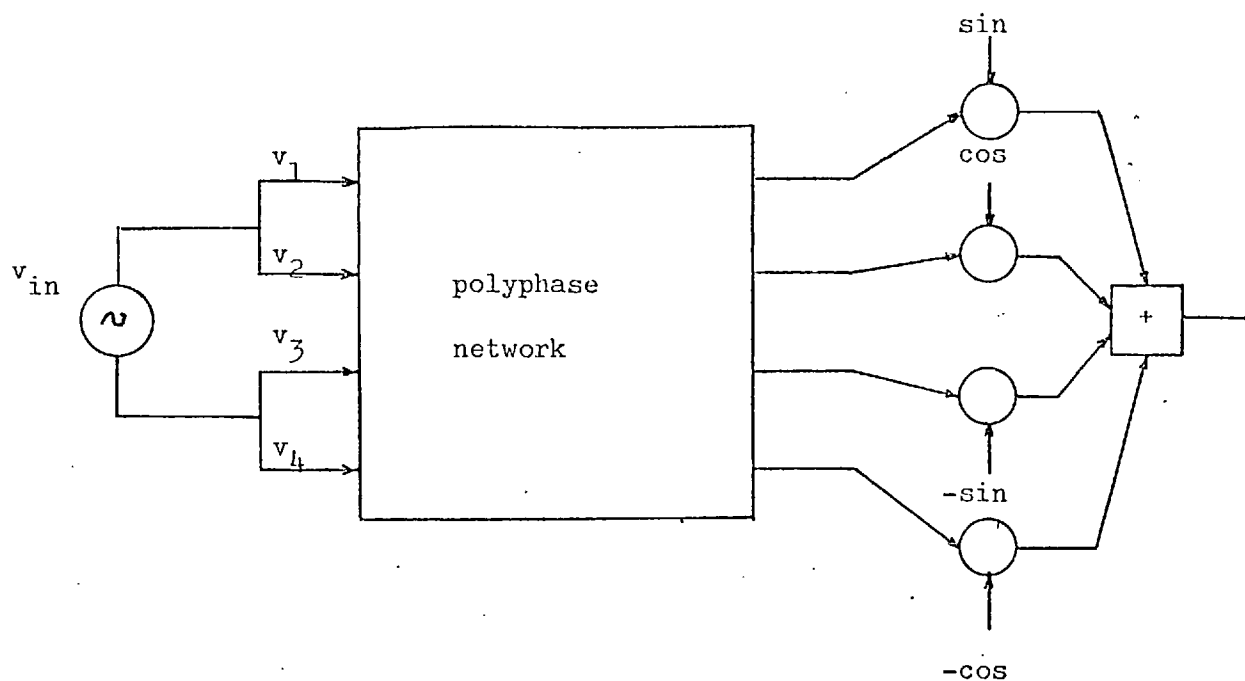


Fig. 3.3 The S.S.B. modulator utilizing the polyphase network.

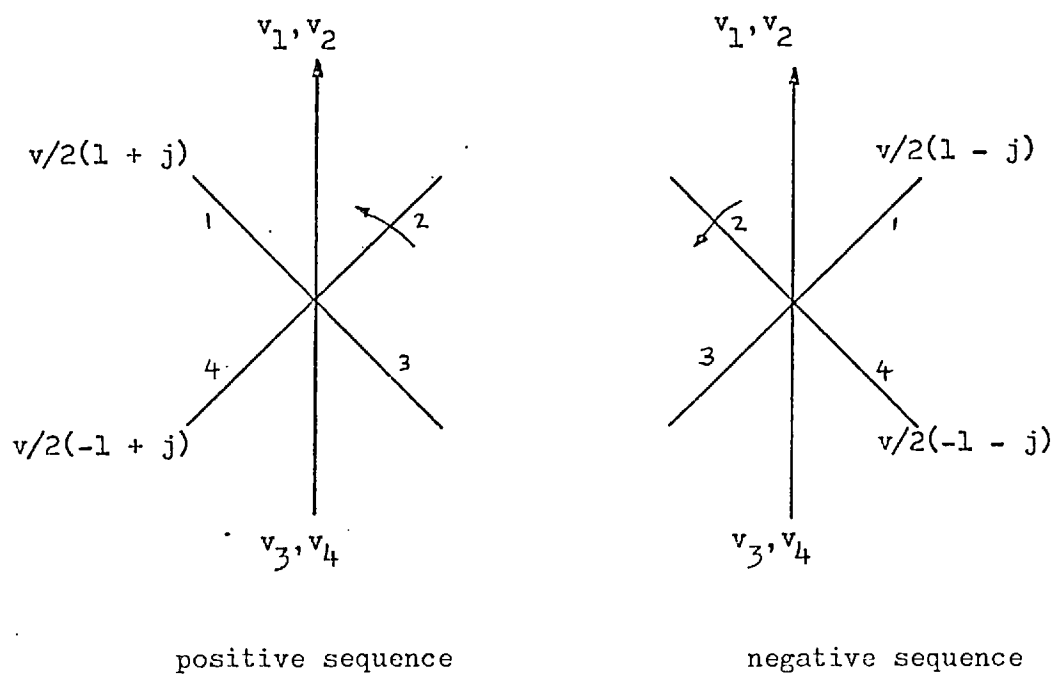


Fig. 3.4 The input signal resolved into symmetric components.

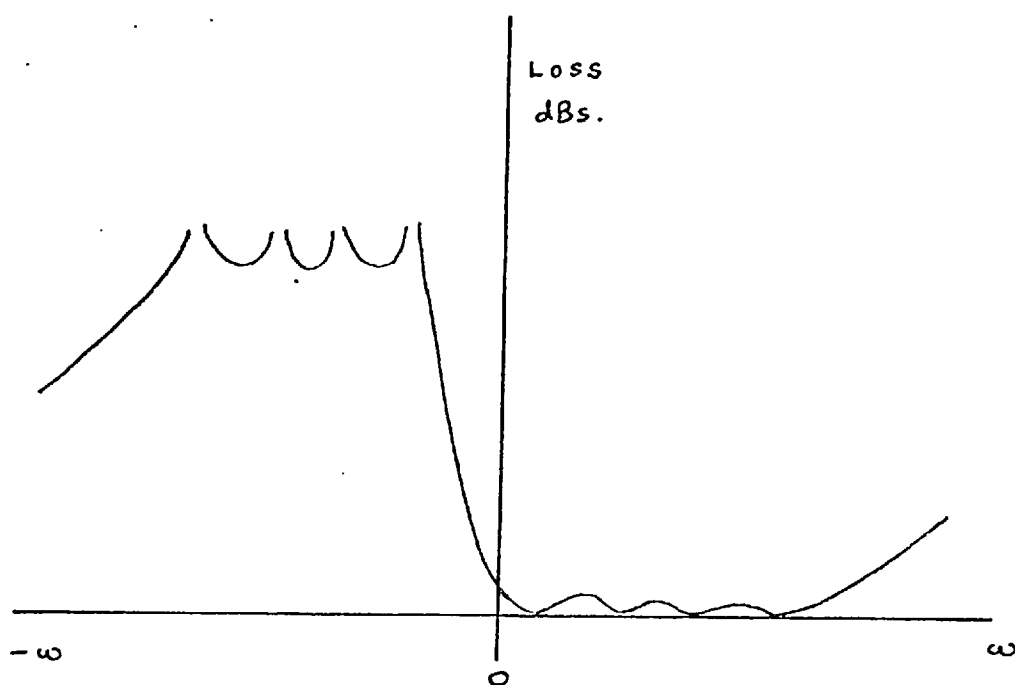


FIG. 3.5 A non-symmetrical baseband frequency response.

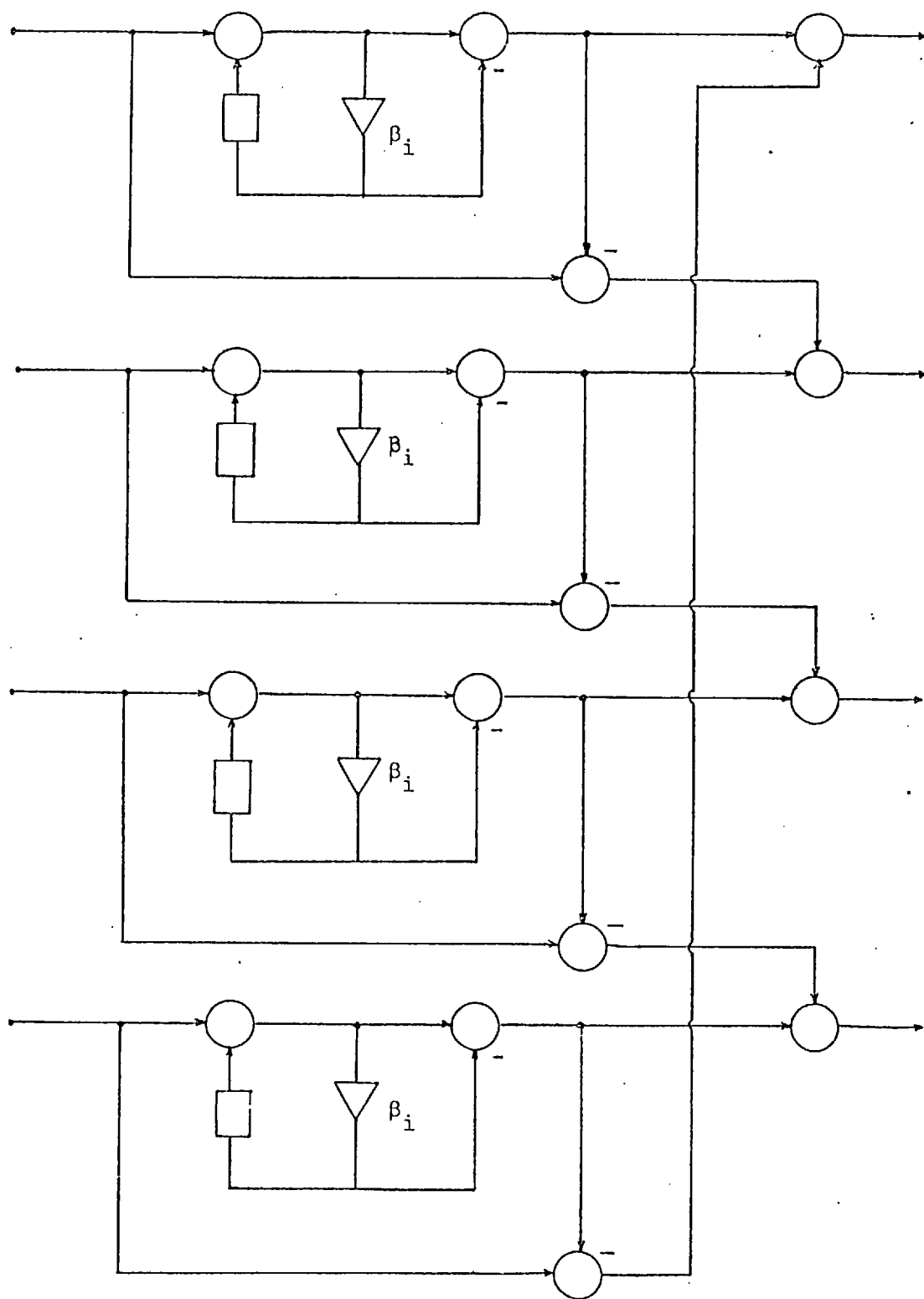
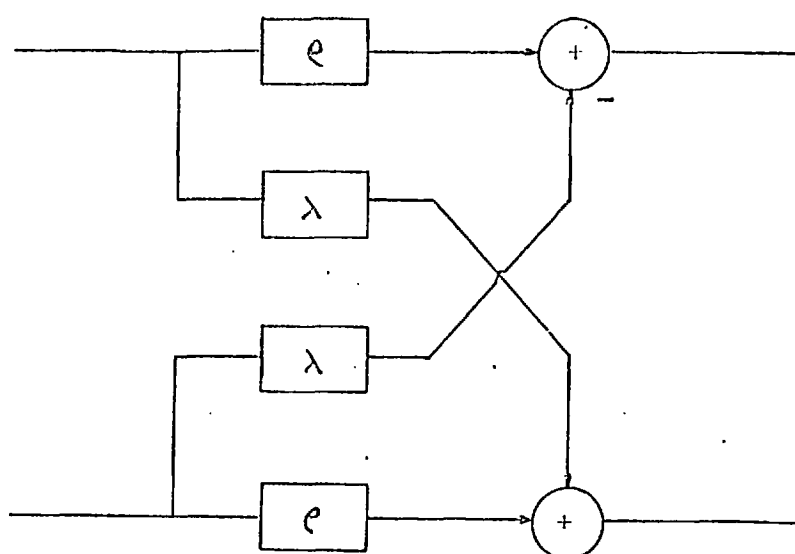


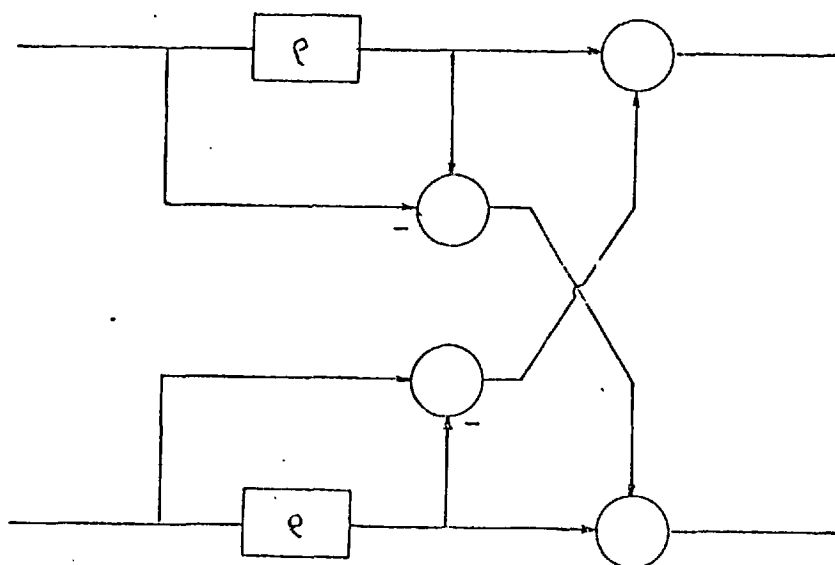
Fig. 3.6 The digital equivalent of the 4-phase RC-network.



(a)

Fig. 3.7 (a) digital i^{th} -section (showing 4 paths).

(b) minimum multiplier realization



(b)

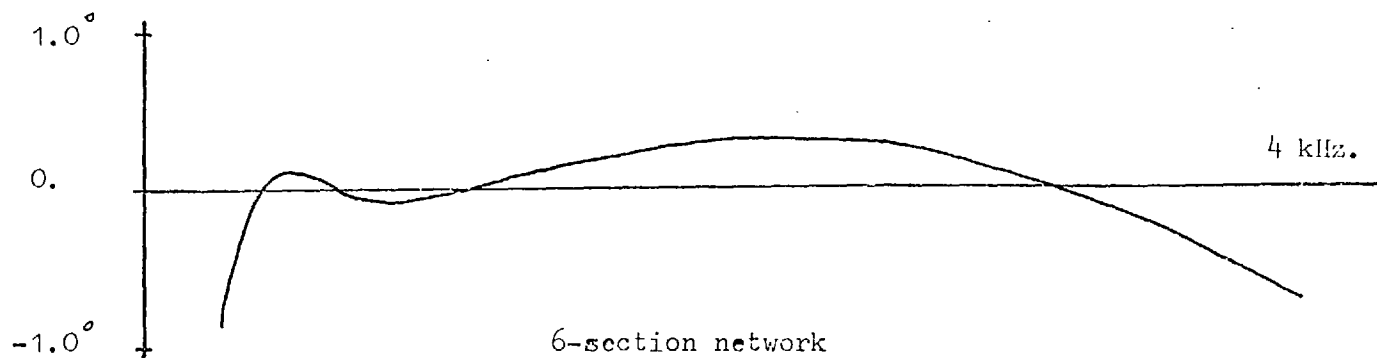
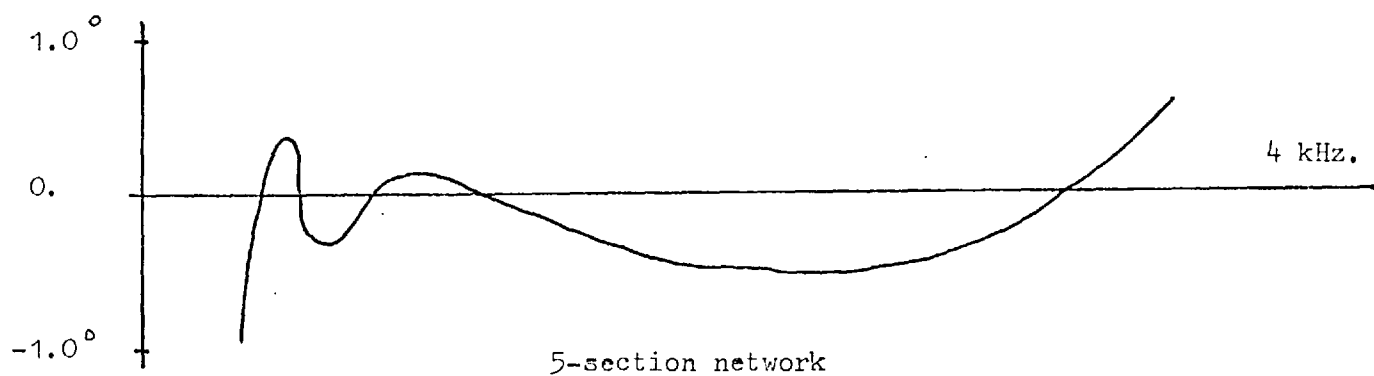
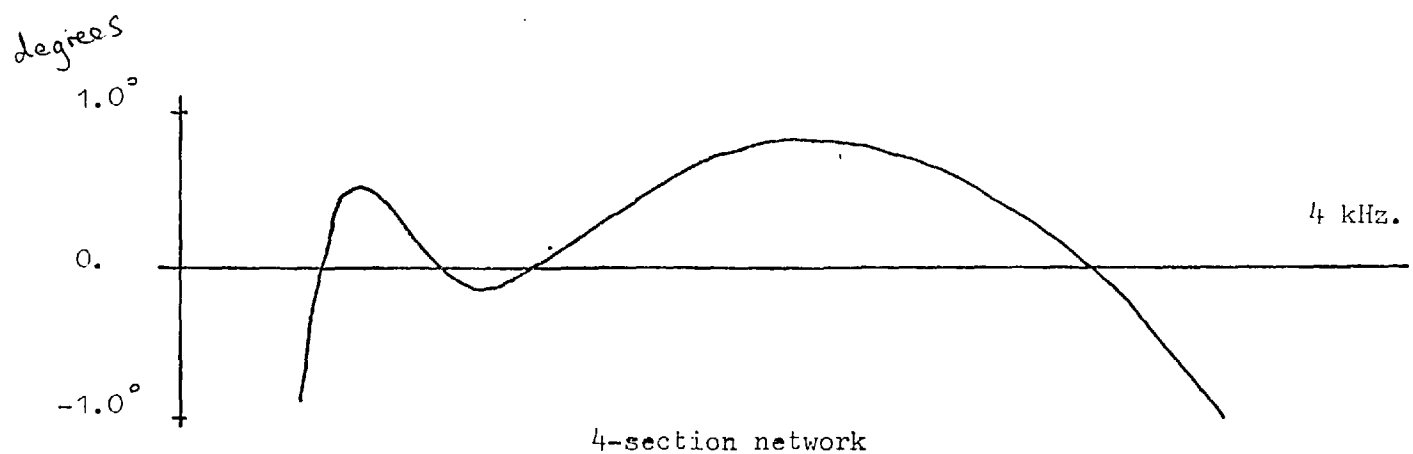
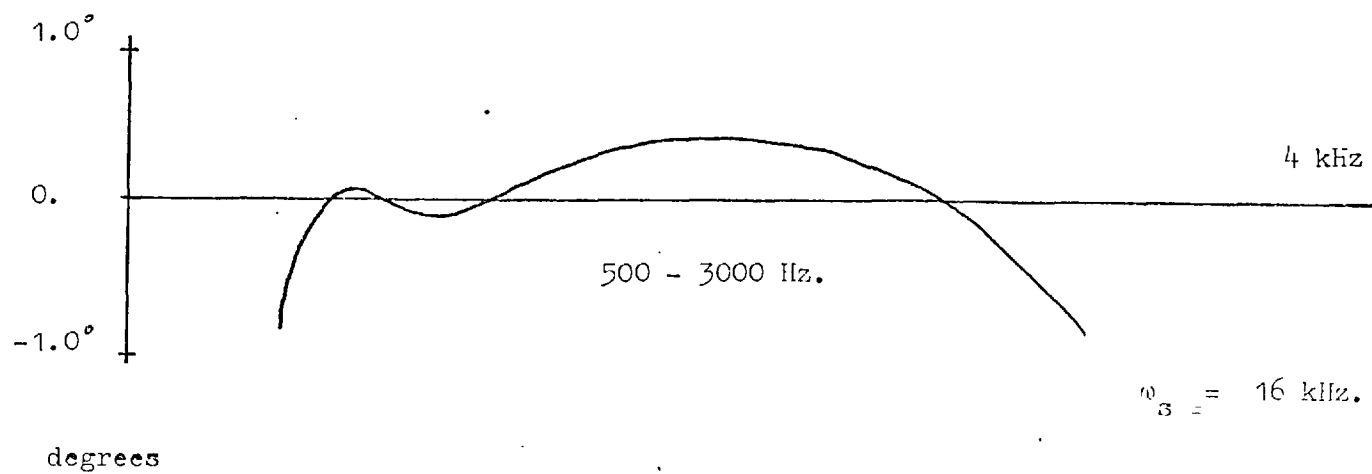
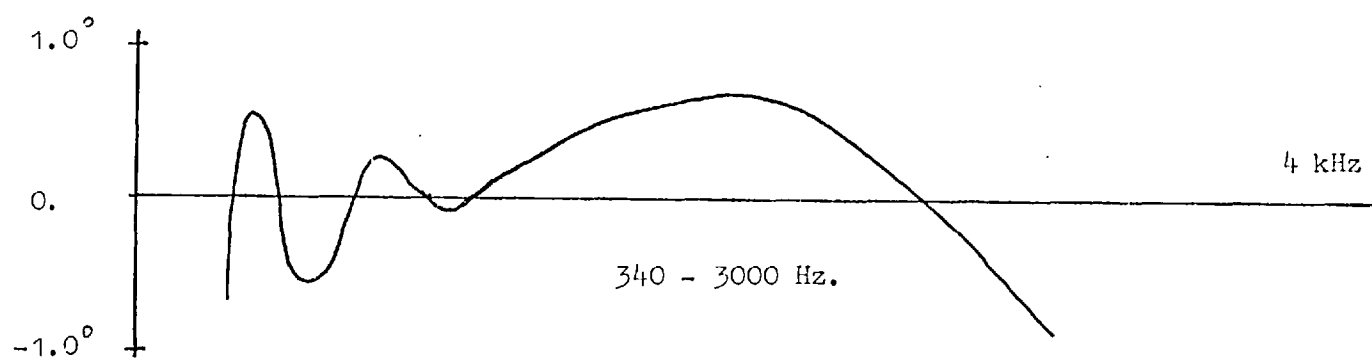
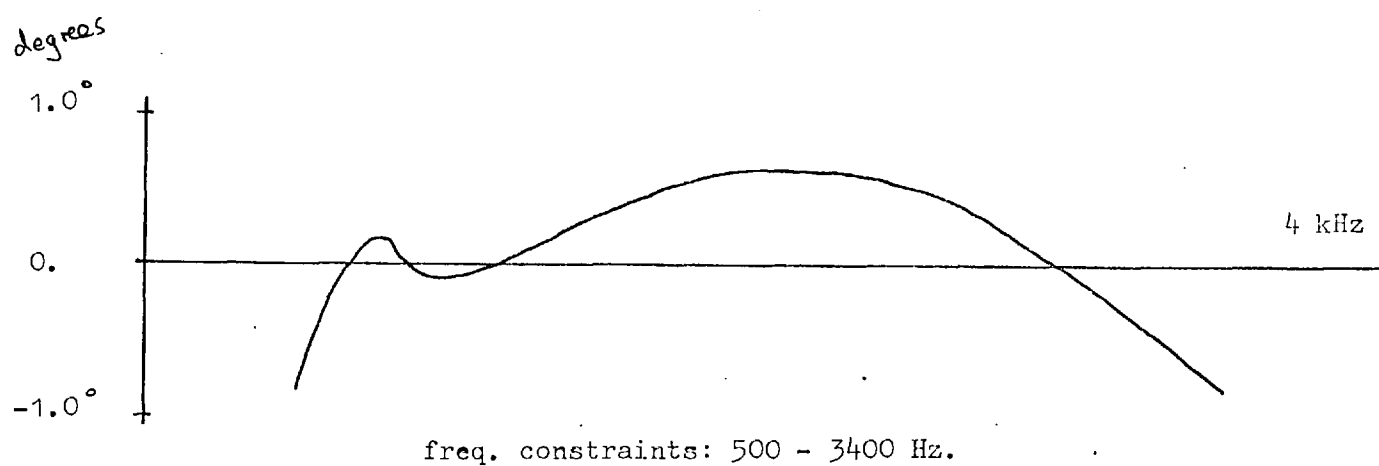


Fig. 3.8 Phase-difference response for various L_2 -approximations.

(frequency constraints: 340-3400 Hz.)

(sample rate: 16 kHz.)

Fig. 3.9 4-section network phase-difference responses



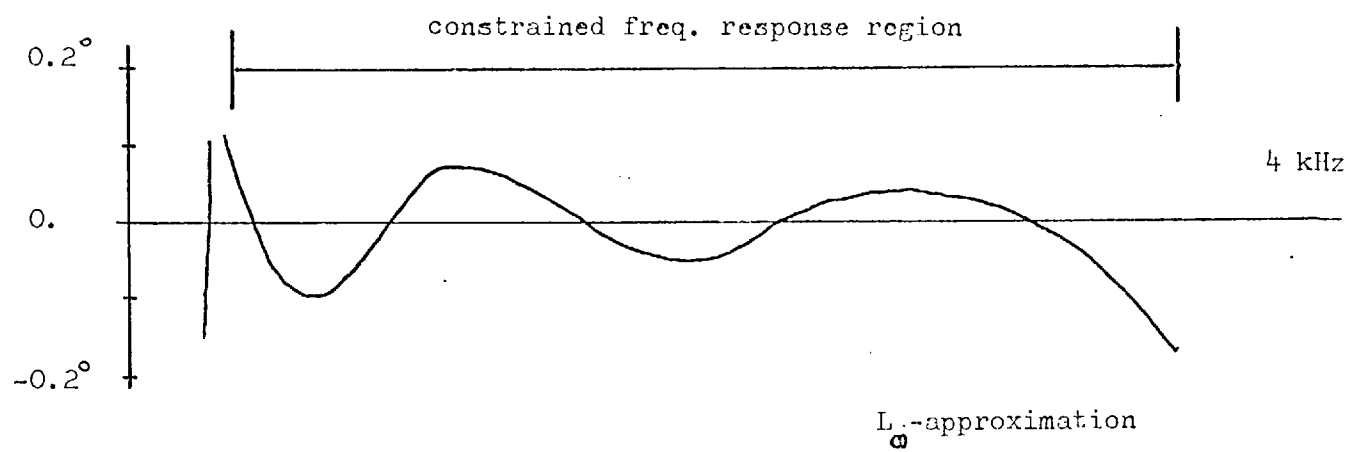
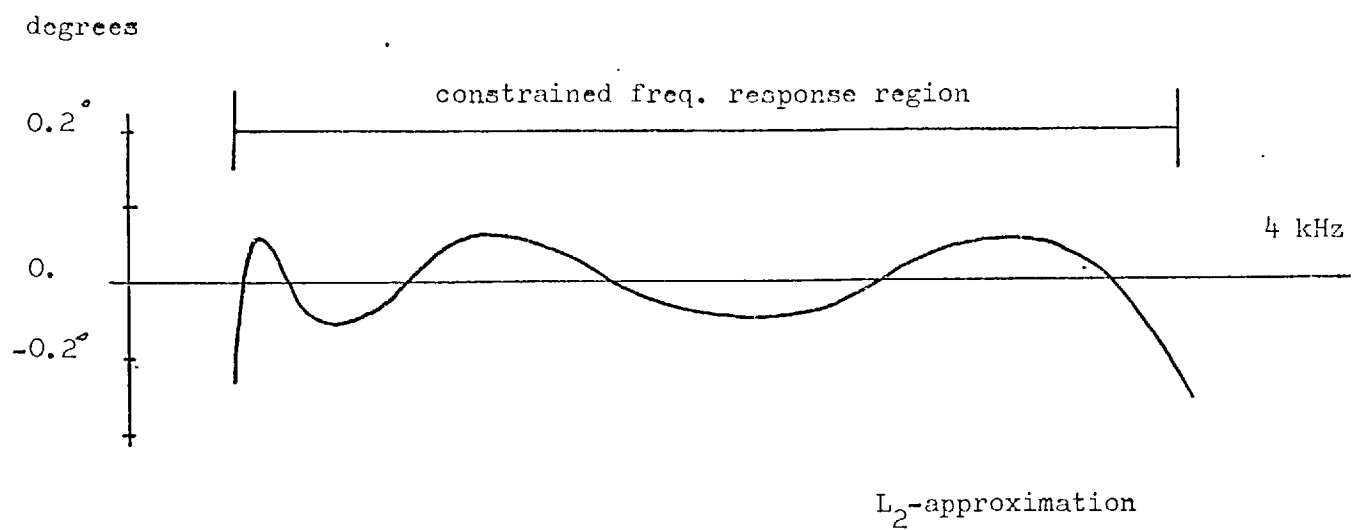


Fig. 3.10 8-section network phase-difference responses for

L_2 and L_∞ error norms.

$$\omega_s = 16 \text{ kHz.}$$

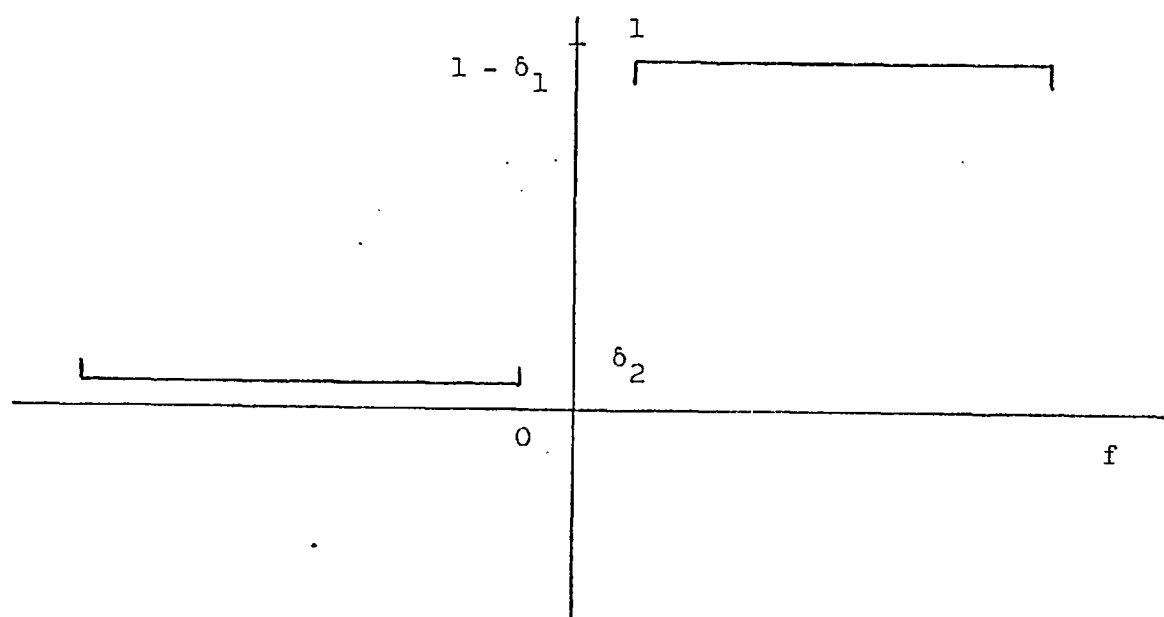
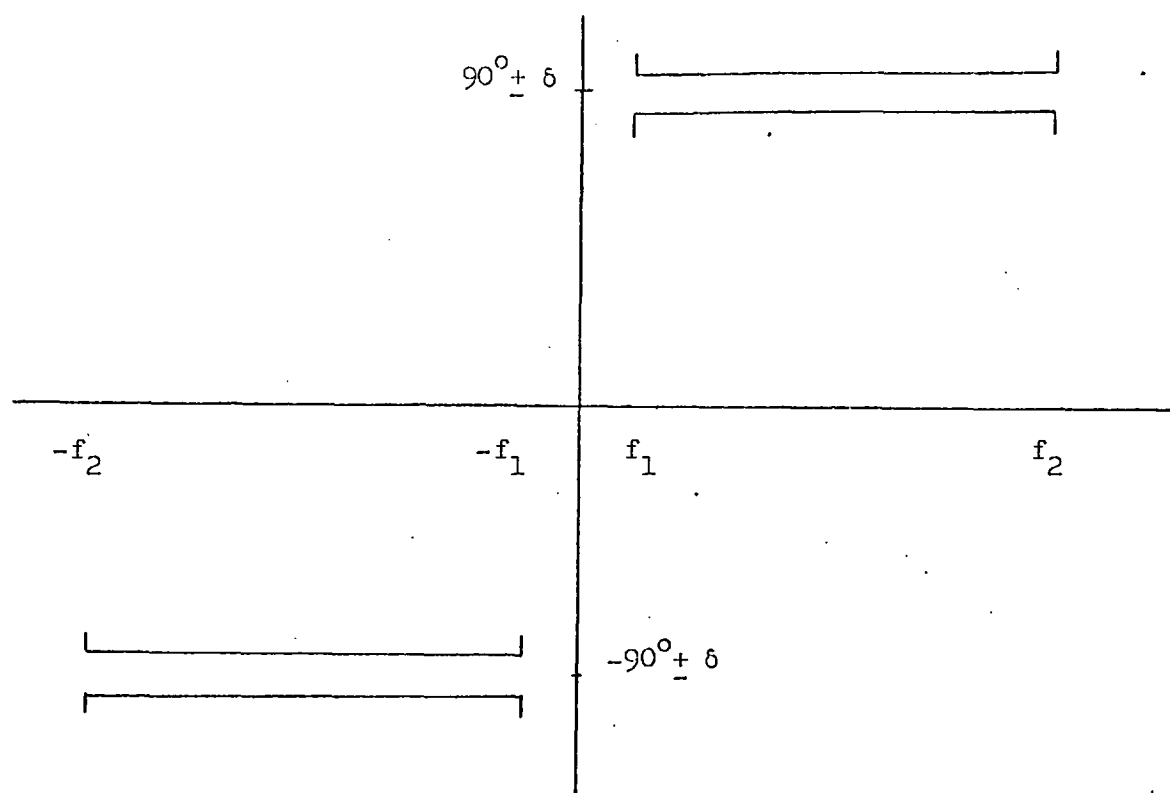


Fig. 3.11 Sideband filter and phase-difference response relationship.

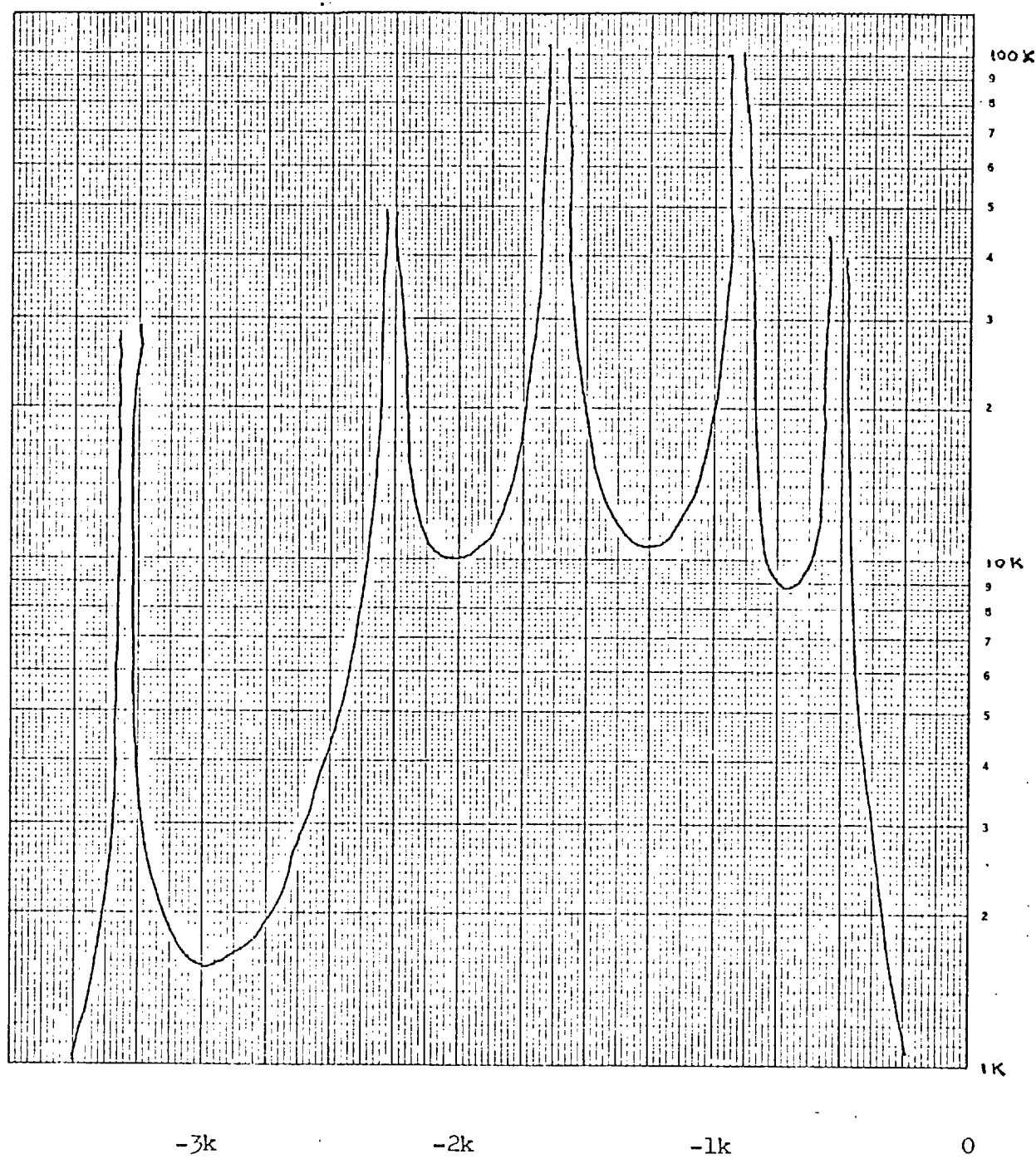


Fig. 3.12 Stopband sensitivity index (8-section polyphase network)

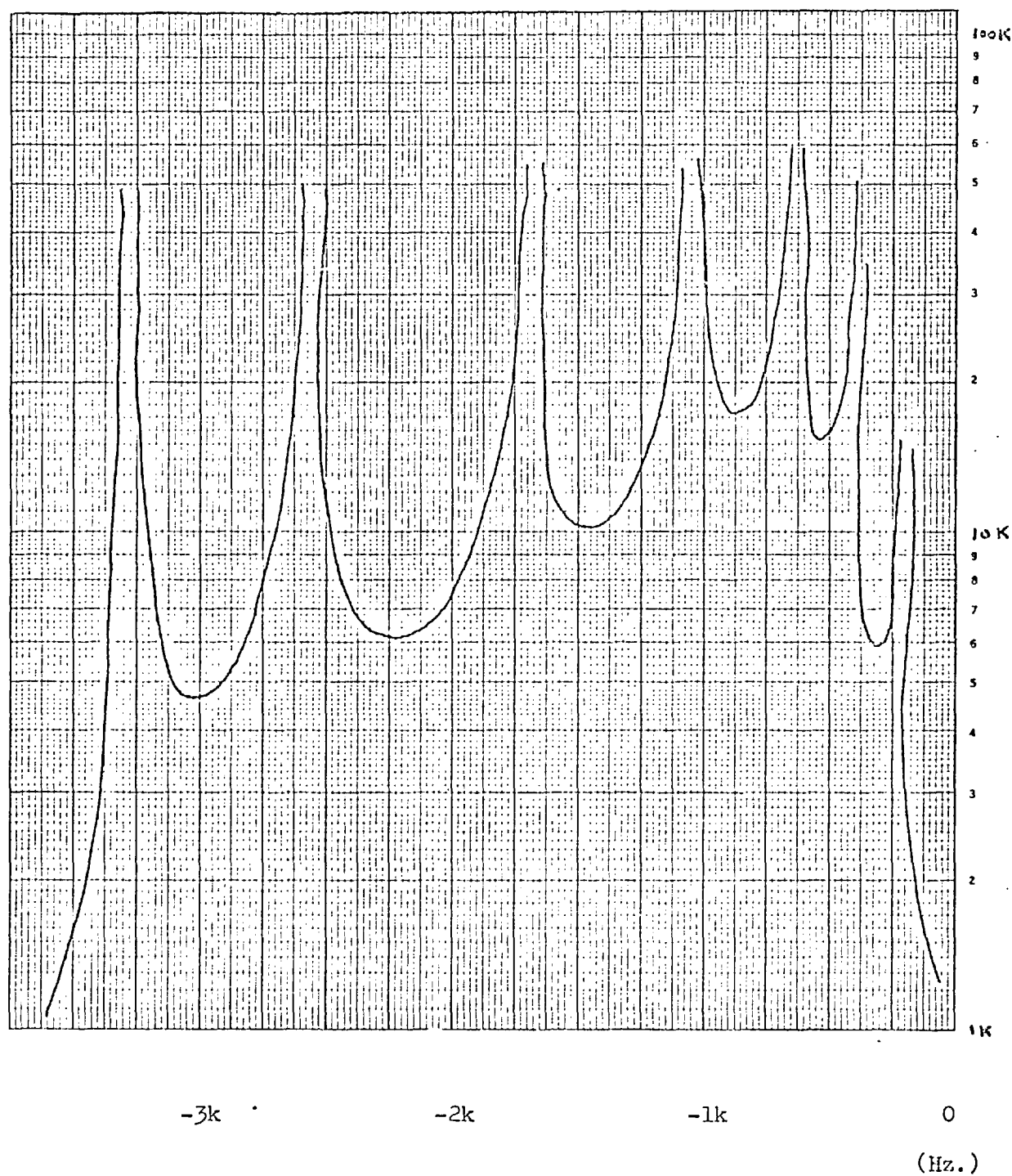
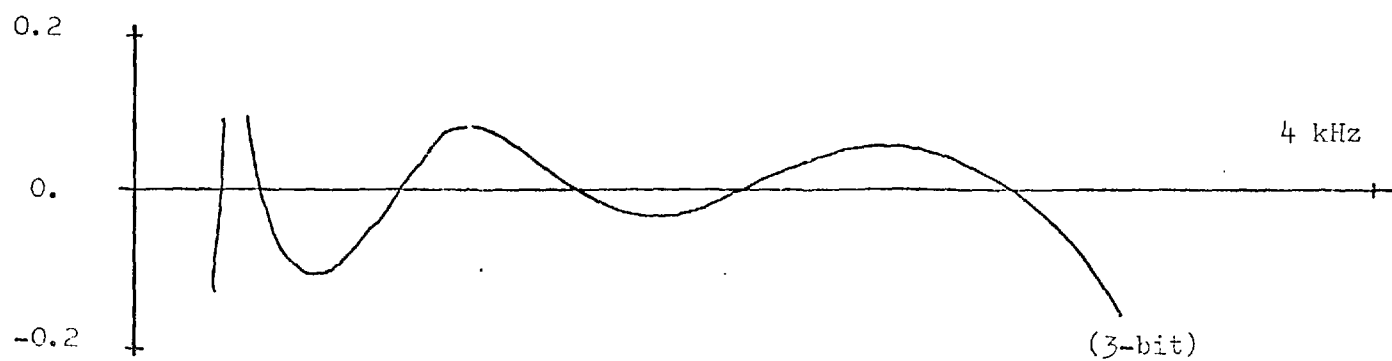
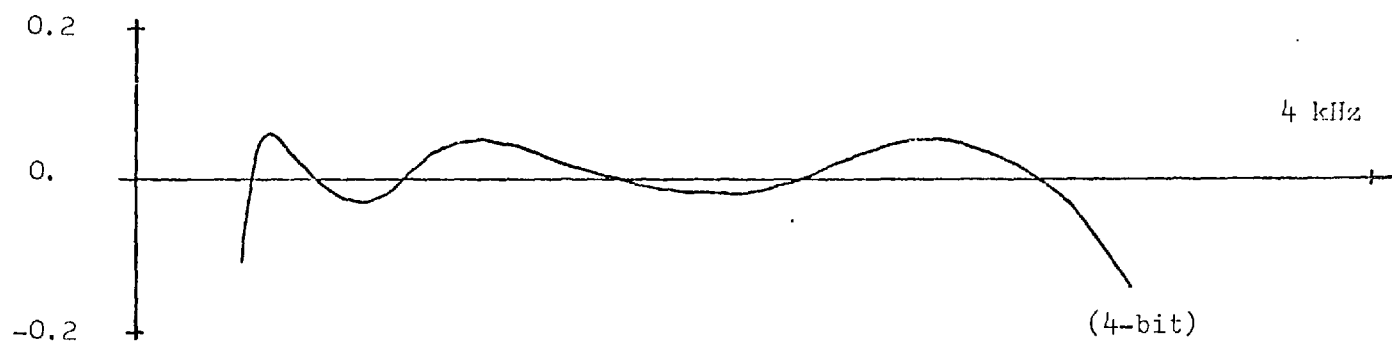
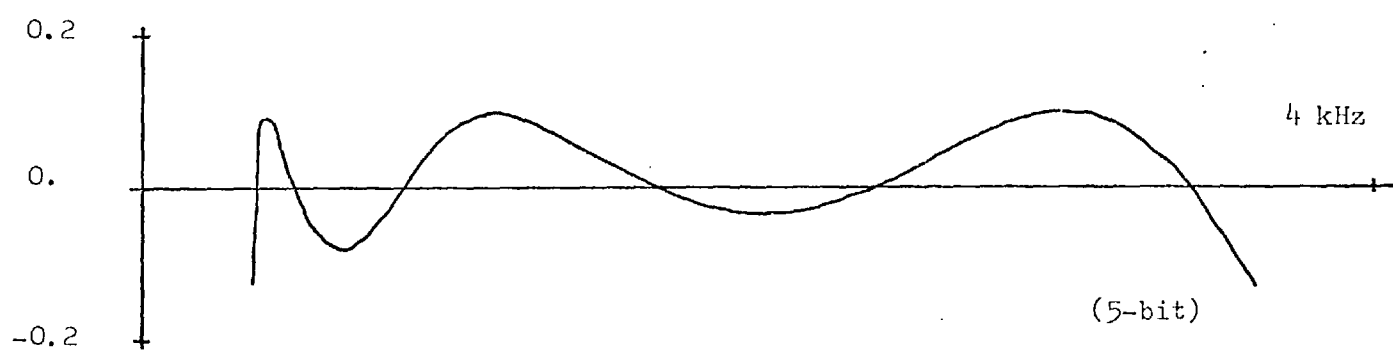
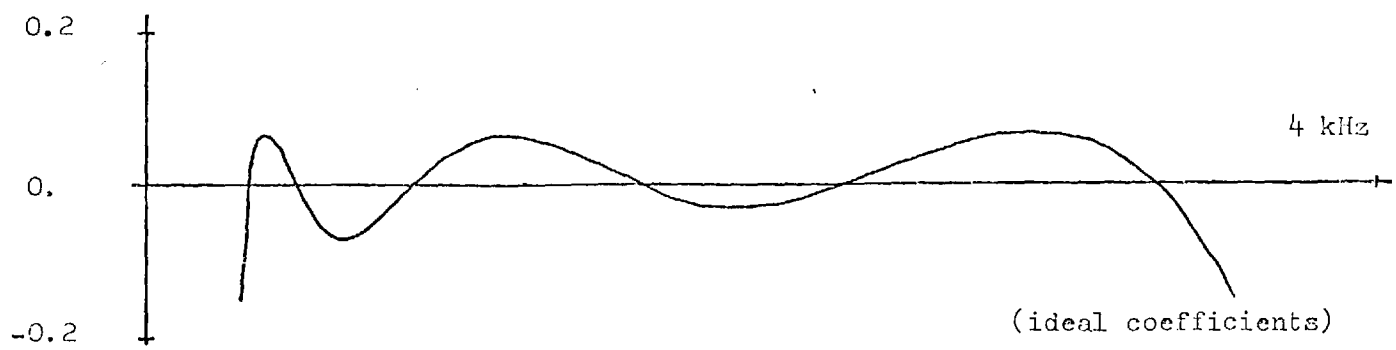


Fig. 3.13 Stopband sensitivity index (allpass phase-splitting network).

Fig. 3.14 8-section network responses - various coefficient accuracies.

$$\omega_s = 16k$$



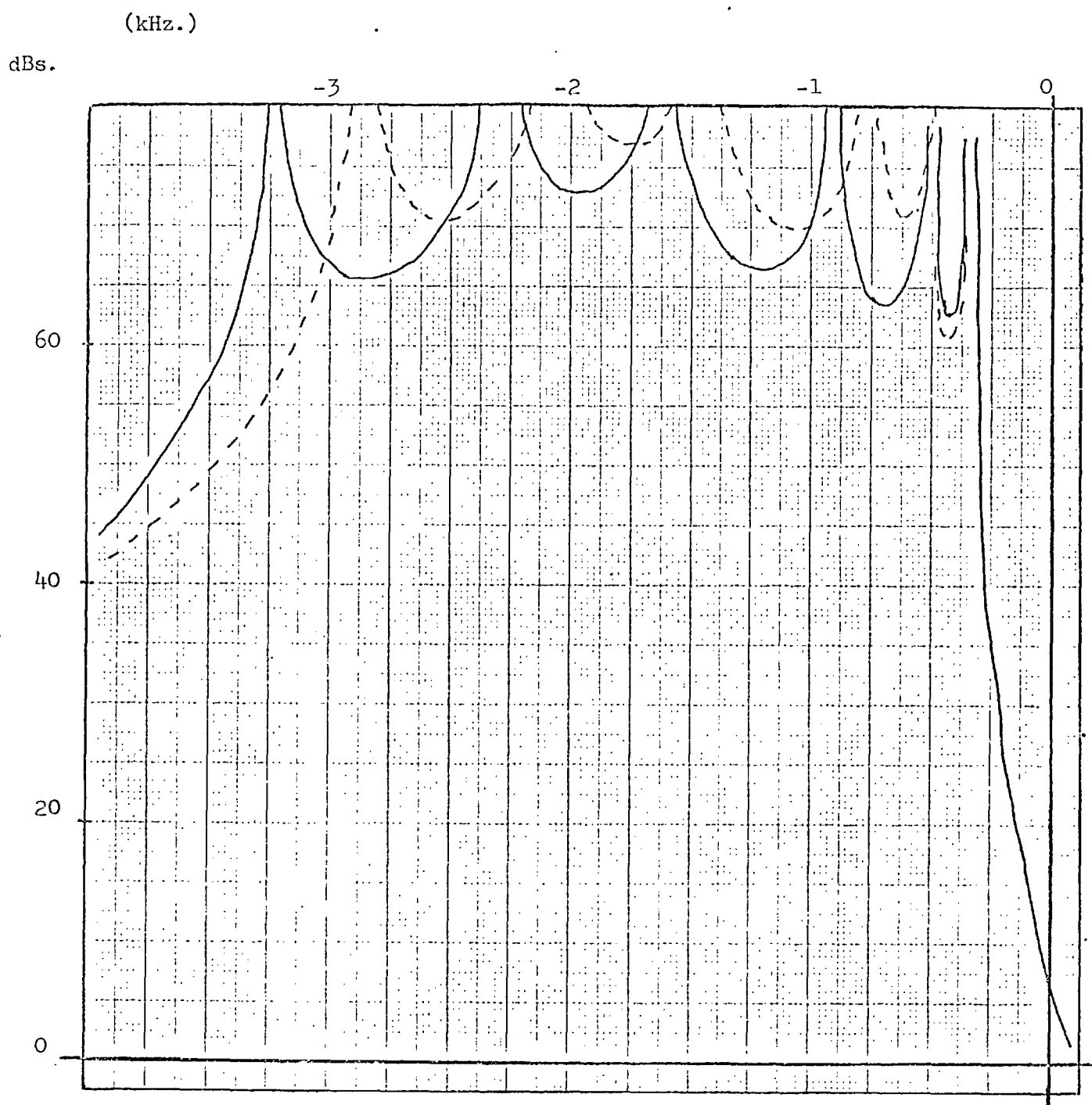


Fig. 3.15 The sideband filter stopband response.

ideal coefficients
4-bit coefficients

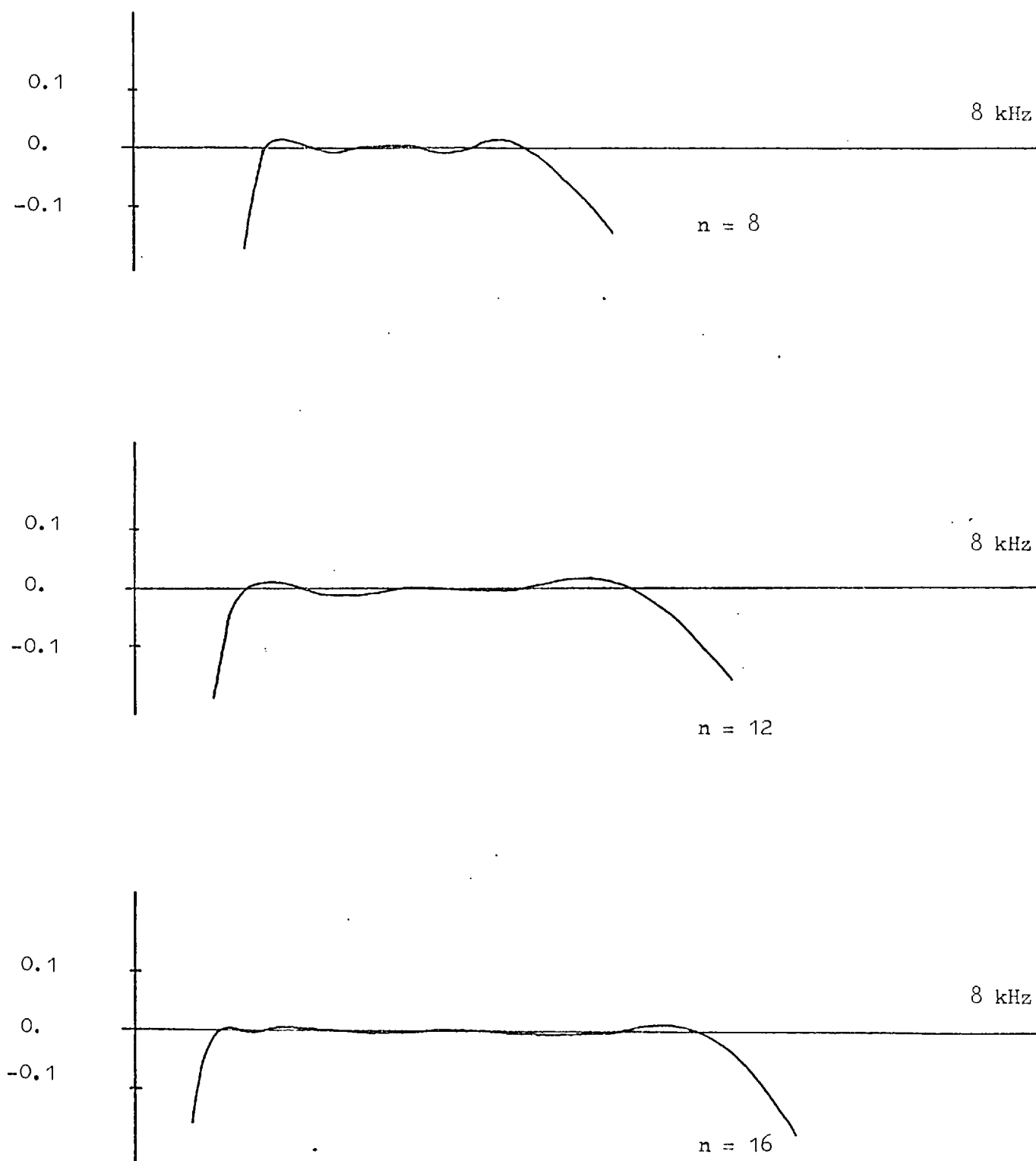


Fig. 3.16 n -section network phase-difference responses

all sections using $\beta_i = 0.5$

(kHz.)

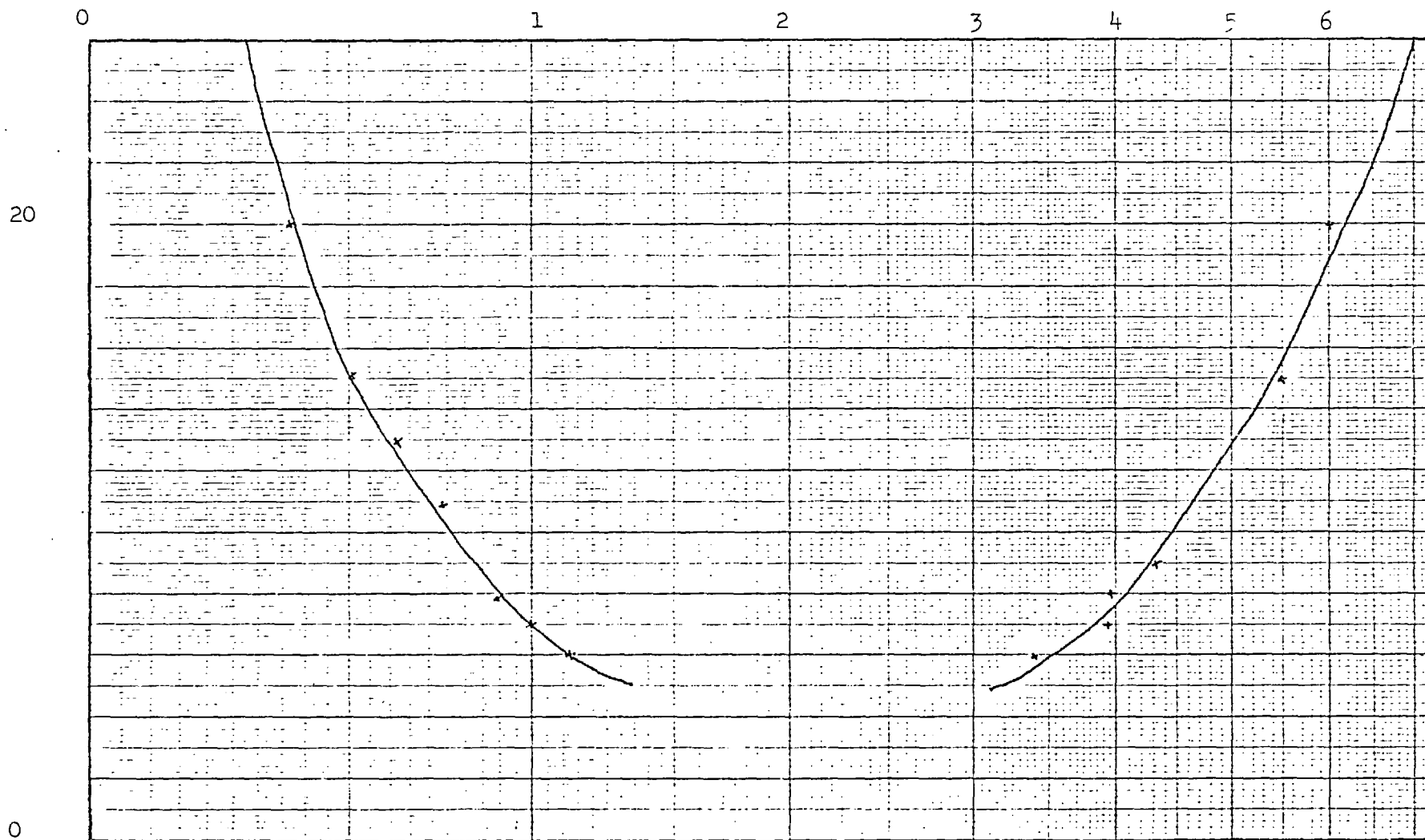


Fig. 3.17 (a) section no. against 0.1° low-frequency extreme.

(b) " " 0.1° high-frequency extreme.

(all sections $\beta_i = 0.5$)

$\omega_s = 16\text{kHz}$

CHAPTER FOUR.

4.0 Quantization effects in digital systems

In any digital system the theoretically ideal signal data and numerical values must be represented at some stage by binary numbers chosen from a finite set of allowable values - that is, the quantized datum is represented by a binary word of finite length. Theoretically the numerical accuracy could be maintained arbitrarily high but in implementing a practical system a tradeoff must be made between accuracy (and therefore performance, noise levels etc.) and overall system costs. Furthermore, the mode of arithmetic (fixed-point, floating - point⁽⁸⁾ or the more recently proposed block-floating-point⁽⁸⁷⁾) employed to represent this quantized data has a significant effect on the overall performance in the system, its complexity and therefore, most important, its cost. Throughout this study, we assume for the following reasons that the fixed-point arithmetic representation is used i.e.:-

- i) The output errors associated with the fixed-point implementations are usually 'worst-case' and therefore the option of improving the system's performance remains open to us.
- ii) Complexity considerations (and therefore costs) make fixed point arithmetic operations most attractive (i.e. fixed-point arithmetic hardware is most easily realized).

- iii) The widespread use of this arithmetic mode facilitates the comparison of one system with another.
- iv) the availability of 'off-the-shelf' fixed-point hardware reduces costs for both 'one-off' designs and also 'mass-produced' systems.

Regardless of the arithmetic chosen the quantization of any numerical data reduces their possible accuracy and consequently some form of error is introduced in the process. The errors likely to occur are well-known (88,89) and are summarised as follows:-

- 1) Quantization noise is introduced by the signal analogue -to-digital conversion⁽⁹⁰⁾. This noise level is reduced with increased A/D accuracy and usually it is arranged by choosing a suitable wordlength so that this noise contribution is of little significance. The effect of the error on the performance of the system can be determined, however, if the statistical properties of the noise are known. The usual assumptions made are that the added noise is white with a zero mean^{*} (for rounding) and a variance σ_o^2 equal to $q^2/12$ (where q is the quantization step size) - that is, the noise source added to the ideal signal is a stationary random signal (discrete) having a uniform power density spectrum.

For this input noise sequence, the resulting output error sequence can be easily determined using linear system noise theory⁽⁸⁹⁾.

^{*} and a uniform probability density function

In addition to the straightforward A/D quantization noise, a problem occurs if the input signal is allowed to exceed the dynamic range of the A/D convertor - that is, amplitude clipping results. However, by simply reducing the input signal amplitude to reduce the clipping distortion, we also reduce the output signal-to-noise ratio. It is therefore necessary for us to consider the amplitude properties of speech signals as, in this study, the A/D conversion forms the first stage of the digital F.D.M. telephony system. It has been shown⁽⁹¹⁾, that a speech signal (conveniently represented as a random waveform) is characterized by a probability distribution peaked around zero (amplitude) falling off rapidly with increasing amplitude; also, the probability of a sample value exceeding four times the r.m.s value of the signal is low. It is apparent, therefore, that if sample values $\sigma^2/4 \cdot x(n)$ are quantized instead of $x(n)$ then, with high probability, clipping distortion will be eliminated. The signal-to-noise ratio is given by⁽⁸⁾:-

$$S/N \text{ ratio} = 6 \cdot n_o - 1.24 \quad (\text{dBs}) \quad 4.1$$

Since in the practical F.D.M. system the use of 14- or 15-bit signal word lengths are envisaged, the effects due to A/D quantization noise can be neglected.

- 2) The multiplier coefficient values must be represented by binary words of a finite accuracy. The error introduced by this quantization is deterministic and as such is easily analysed⁽⁹²⁾ (i.e. we compute the system response using ideal and quantized coefficient values. In this way the deviation away from the ideal response caused by the small changes in the coefficient values is computed). The effects of coefficient quantization are not considered in any greater detail at present since, in this thesis, each system-response is given both for ideal and quantized (or reduced accuracy) coefficient values.

- 3) Quantization of the results of arithmetic operations or, more specifically, the products of multiplications (of signal data by coefficients) being rounded or truncated, introduces noise known generically as 'roundoff' noise. The level of output noise is highly dependent on the network implementation due to the fact that the noise is introduced at various nodes within the network. For this reason, the level of the output noise is potentially much larger than that due to the A/D quantization and as such warrants a more detailed investigation.

The method most used in the analysis of roundoff noise results in a stochastic approach - that is, the assumption is made that the error sequences from different multipliers in the digital network are statistically independent and uncorrelated with each other and with the processed signal. The results obtained using this simple noise model have been shown to be in close agreement with those obtained experimentally⁽⁹³⁾. In some circumstances (e.g. for low level or zero level input signals) the errors become correlated and therefore the previous stochastic approach is unsuitable - an analysis based on a deterministic approach is then required. In analysing the roundoff noise properties of the polyphase network we consider the effects due to both correlated and uncorrelated error sequences.

4.1 Quantization of the products of multiplications

We now consider the quantization of the product of a multiplication utilizing the uncorrelated - error noise model. For investigating the effects caused by rounding the product of a multiplication it is usual to assume⁽⁸⁹⁾ that:

- i) the errors introduced are uncorrelated from sample to sample and that
- ii) they are also uncorrelated from one rounding point to another.

These assumptions are useful in that i) is a necessary condition if the noise injected at each rounding point is to be considered 'white' (i.e. its statistical properties are well-known) and ii) the superposition is allowed at the output of the processed (filtered) noise spectra due to the separate error sources. The assumption of uncorrelated errors applies both for truncation and rounding but for input signals tending towards a non-random nature, the uncorrelated error assumption is less valid for the truncating operation than for the rounding operation.

As in the A/D quantization process, the dynamic range constraints must be met i.e. the data must be suitably scaled to prevent overflow in any multiplying operation. At the same time, however, the maximum signal-to-noise ratio must be maintained.

The inclusion of the dynamic range constraints complicates the analysis problem and, for this reason, the analysis is performed in two stages -- that is,

- i) we compute the roundoff noise for the different realizations of the polyphase network. For ease, we choose arbitrarily a network made up of three single-sections.

- ii) using the dynamic range constraints, we calculate the scaling factors required (if at all) by each section. With the inclusion of the scaling-factors, we then recompute the roundoff noise for the modified (scaled) network.

To facilitate the roundoff noise analysis, the general digital network model (after Jackson⁽⁹³⁾) is modified to represent the digital polyphase network.

4.2 Roundoff noise analysis

A representation of a general digital network (after Jackson⁽⁹³⁾) is shown in fig. 4.1, where the input and output at a time $t = nT$ are given by $u(n)$ and $y(n)$ respectively. The output from the i^{th} branch-node is denoted by $v_i(n)$ whilst the roundoff-error added at the j^{th} summation node is denoted by $e_j(n)$. The relationship between the i^{th} and j^{th} nodes is as shown in fig. 4.2 - that is, the data flow from the i^{th} branch node through the constant fixed-point multiplier to the j^{th} summation node. Using this model and assuming uncorrelated rounding errors, it has been shown⁽⁸⁹⁾ that the spectrum of the output roundoff noise of a digital network is given by:-

$$N_Y(\omega) = \sigma_o^2 \cdot \sum_j k_j \cdot |G_j(\omega)|^2 \quad 4.2$$

where:-

$G_j(\omega)$ are the transfer responses from certain summation nodes to the output.

k_j is an integer value giving the number of error inputs to the j^{th} summation node.

The total average power (or variance) is obtained from:-

$$\sigma_Y^2 = 1/\omega_s \cdot \int_0^{\omega_s} N(\omega) d\omega \quad 4.3$$

or (using the trapezoidal rule):-

$$\sigma_Y^2 \approx 1/2K \cdot \left\{ N_Y(0) + N_Y(\omega_s/2) \right\} + 1/K \cdot \left\{ \sum_{k=1}^{K-1} N_Y(\omega_k) \right\} \quad 4.4$$

where:-

K equals half the number of frequency samples in the interval 0 to ω_s .

To facilitate direct comparisons between the predicted noise response obtained from eqns. 4.2 and 4.4 and an estimated noise response computed by injecting white noise at each rounding point and calculating, from the resulting output sequence, the smoothed power density spectrum and the variance, it is more convenient to use the normalized noise power density spectrum and variance equations, i.e:-

$$\hat{N}_Y(\omega) = N_Y(\omega) / \sigma_o^2 = \sum_j k_j |G_j(\omega)|^2 \quad 4.5$$

and:-

$$\hat{\sigma}_Y^2 = \sigma_Y^2 / \sigma_o^2 \quad 4.6$$

Before these ideas can be applied to the polyphase network analysis, the general digital network model (fig. 4.1) must be slightly modified. The modification is necessary as the transfer response between the input and each output must be represented. The polyphase network model becomes rather complicated and is hence shown in

fig. 4.3 with only a pair of branch and summation nodes extracted: there are N pairs in an N -section network. The outputs from the i^{th} and i_b^{th} branch nodes are denoted by $v_{ia}(n)$ and $v_{ib}(n)$ respectively, with the transfer responses from the input to the i^{th} branch nodes given by F_{iaa} , F_{iab} , F_{iba} and F_{ibb} . The roundoff errors added at the j^{th} summation nodes are denoted by $e_{ja}(n)$ and $e_{jb}(n)$ with the transfer responses from the j^{th} summation nodes to the outputs $y_a(n)$ and $y_b(n)$ given by G_{jaa} , G_{jba} , G_{jab} and G_{jbb} respectively. It is important to note that the coefficient multipliers occur between $v_{ia}(n)$ and $e_{ja}(n)$ or $v_{ib}(n)$ and $e_{jb}(n)$ ($1 \leq i, j \leq N$) only. Now, assuming the uncorrelated-error noise model and referring to fig. 4.3, it can be seen that the roundoff noise spectrum for each output is obtained from:-

$$N_{ya}^{\wedge}(\omega) = \sum_j k_j |G_{jaa}(\omega)|^2 + l_j |G_{jba}(\omega)|^2 \quad 4.7$$

$$N_{yb}^{\wedge}(\omega) = \sum_j l_j |G_{jab}(\omega)|^2 + k_j |G_{jbb}(\omega)|^2 \quad 4.8$$

Reference to figs. 4.3 to 4.7 indicates that equations 4.7 and 4.8 can be simplified as $l_j = k_j = 1$ for all j .

We now examine the noise characteristics of different structures realizing the polyphase network. Firstly, however, such possible realizations of the polyphase network single-section must be derived.

4.3 Polyphase network structures and their roundoff noise characteristics

From the transmittance parameters for the i^{th} single - section:-

$$\begin{bmatrix} y_{ia}(n) \\ y_{ib}(n) \end{bmatrix} = \begin{bmatrix} \rho_i(z^{-1}) & -\lambda_i(z^{-1}) \\ \lambda_i(z^{-1}) & \rho_i(z^{-1}) \end{bmatrix} \cdot \begin{bmatrix} x_{ia}(n) \\ x_{ib}(n) \end{bmatrix}$$

and since:-

$$\rho_i(z^{-1}) + \lambda_i(z^{-1}) = 1 \quad 4.9$$

we have:-

$$\begin{bmatrix} \rho_i(z^{-1}) & -(1 - \rho_i(z^{-1})) \\ (1 - \rho_i(z^{-1})) & \rho_i(z^{-1}) \end{bmatrix} \quad 4.10$$

From eqn. 4.10 the realization of fig. 4.4(a) is obtained which we define as the type 1A structure. By transposing (84,86) this structure, we obtain the realization of fig. 4.4(b) which we define as the type 1B structure. Considering the sub-transmittance $\rho_i(z^{-1})$ as $[1/1 - B_i \cdot z^{-1}] \times (1 - \beta_i)$, we obtain the minimum multiplier realization type i represented in fig. 4.5(a). For the sub-transmittance $(1 - \beta_i) \times 1/(1 - \beta_i \cdot z^{-1})$ we obtain an alternative form - the type ii realization shown in fig. 4.5(b). From the above consideration it is evident that we need to consider four structures for realizing eqn. 4.10:

these are the type 1Ai, 1Aii, 1Bi and 1Bii realizations.

Additionally, if we consider:-

$$\begin{array}{cc} 1 - \lambda_i(z^{-1}) & -\lambda_i(z^{-1}) \\ \lambda_i(z^{-1}) & 1 - \lambda_i(z^{-1}) \end{array} \quad 4.11$$

we now obtain the structure shown in fig. 4.6(a) (type 2A) with its transpose as in fig. 4.6(b) (type 2B). The sub-transmittance $\lambda_i(z^{-1})$ is then realized either as in fig. 4.7(a) (type iii) or in fig. 4.7(b) (type iv). Once again, this gives us four possible realizations; these being the type 2Aiii, 2Aiv, 2Biii and 2Biv structures.

Our initial roundoff noise analysis is now performed by considering the predicted noise contribution from the n^{th} section for the above structures. In this way, any structures possessing similar noise performance characteristics can be identified and the total number of structures requiring noise computations can thus be reduced.

In equations 4.12 and 4.13, we define a sub-transmittance T_{jk} as follows:-

$$T_{jk}^{(i)}{}_n = \frac{-1 + z^{-1}}{1 - \beta_n \cdot z^{-1}} \quad 4.12$$

$$T_{jk}^{(ii)}{}_n = \frac{1}{1 - \beta_n \cdot z^{-1}} \quad 4.13$$

i.e. T is the transfer function between the j^{th} error node and the k^{th} output node of the n^{th} single section (figs. 4.5(a) and (b)). With reference to figs. 4.7(a) and (b), it is observed that:-

$$\begin{aligned} T_{jk}^{(iii)}{}_n &= T_{jk}^{(i)}{}_n \\ T_{jk}^{(iv)}{}_n &= T_{jk}^{(ii)}{}_n \end{aligned} \quad 4.14$$

As the k^{th} output node of the n^{th} section forms the input to the $n+1^{\text{th}}$ section, there being two k^{th} outputs i.e. two noise contributions per section, we require the transfer function for the remaining $n+1$ sections. This is most easily handled as a partial transfer function from the two k^{th} nodes to the two network outputs. These partial transfer functions (we define as H_{aa}^{n+1} , H_{ba}^{n+1} , H_{ab}^{n+1} and H_{bb}^{n+1}) are obtained from:-

$$\begin{vmatrix} H_{aa}^{n+1} & H_{ba}^{n+1} \\ H_{ab}^{n+1} & H_{bb}^{n+1} \end{vmatrix} = \prod_{i=n+1}^N \begin{vmatrix} \rho_i(z^{-1}) & -\lambda_i(z^{-1}) \\ \lambda_i(z^{-1}) & \rho_i(z^{-1}) \end{vmatrix} \quad 4.15$$

The predicted roundoff noise spectrum for each structure is therefore obtained, for the n^{th} -section contribution, as:-

Type 1Ai

$$\begin{aligned} N_{ya}^{\wedge}(\omega) &= \left| T_{jk}^{(i)}{}_n \cdot H_{aa}^{n+1} \right|^2 + \left| T_{jk}^{(i)}{}_n \cdot H_{ba}^{n+1} \right|^2 \\ N_{yb}^{\wedge}(\omega) &= \left| T_{jk}^{(i)}{}_n \cdot H_{ab}^{n+1} \right|^2 + \left| T_{jk}^{(i)}{}_n \cdot H_{bb}^{n+1} \right|^2 \end{aligned} \quad 4.16$$

Type 1Aii

$$N_{ya}^{\wedge}(\omega) = \left| T_{jk}(ii)_n \cdot H_{aa}^{n+1} \right|^2 + \left| T_{jk}(ii)_n \cdot H_{ba}^{n+1} \right|^2$$

$$N_{yb}^{\wedge}(\omega) = \left| T_{jk}(ii)_n \cdot H_{ab}^{n+1} \right|^2 + \left| T_{jk}(ii)_n \cdot H_{bb}^{n+1} \right|^2$$

4.17

Type 1Bi

$$N_{ya}^{\wedge}(\omega) = 2 \cdot \left| T_{jk}(i)_n \cdot H_{aa}^{n+1} \right|^2 + 2 \cdot \left| T_{jk}(i)_n \cdot H_{ba}^{n+1} \right|^2$$

$$N_{yb}^{\wedge}(\omega) = 2 \cdot \left| T_{jk}(i)_n \cdot H_{ab}^{n+1} \right|^2 + 2 \cdot \left| T_{jk}(i)_n \cdot H_{bb}^{n+1} \right|^2$$

4.18

Type 1Bii

$$N_{ya}^{\wedge}(\omega) = 2 \cdot \left| T_{jk}(ii)_n \cdot H_{aa}^{n+1} \right|^2 + 2 \cdot \left| T_{jk}(ii)_n \cdot H_{ba}^{n+1} \right|^2$$

$$N_{yb}^{\wedge}(\omega) = 2 \cdot \left| T_{jk}(ii)_n \cdot H_{ab}^{n+1} \right|^2 + 2 \cdot \left| T_{jk}(ii)_n \cdot H_{bb}^{n+1} \right|^2$$

4.19

By continuing this analysis for the type 2 structures, and by making use of equation 4.14, it becomes apparent that we can have the following equivalences:-

Structure	type 1Ai	=	type 2Ai
	type 1Aii	=	type 2Aii
	type 1Bi	=	type 2Bi
	type 1Bii	=	type 2Bii

Therefore we need only consider the type 1 structures in our roundoff noise analysis.

By comparing equations 4.16 and 4.18, 4.17 and 4.19 it can be seen that:-

$$\hat{N}_{ya,b}^{(\omega)} [1Ai] = \hat{N}_{ya,b}^{(\omega)} [1Bi] - 3 \text{ dBs.}$$

$$\hat{N}_{ya,b}^{(\omega)} [1Aii] = \hat{N}_{ya,b}^{(\omega)} [1Bii] - 3 \text{ dBs.}$$

$$[\text{unit type}]^{4.20}$$

and also that for all structures:-

$$\hat{N}_{ya}^{(\omega)} = \hat{N}_{yb}^{(\omega)}$$

This is verified by computing the predicted roundoff noise spectra for the type 1Ai, 1Aii, 1Bi and 1Bii structures. An estimation of the noise response is computed by inserting white noise sources at the relevant summation nodes and, from the output sequence, we calculate the noise power spectral density and total average power. By comparing figs 4.8, 4.10; 4.9, 4.11; 4.12, 4.14; 4.13, 4.15 - we see that the predicted and estimated results for the type 1Ai, 1Aii, 1Bi, 1Bii respectively are in close agreement. Equation 4.20 can be verified by comparing figs 4.8 and 4.12; 4.9 and 4.13. A three-section network (with coefficients $\beta_1=0.875$, $\beta_2 = 0.875$ and $\beta_3= 0.75$) is used in all calculations.

At this stage, it is worthwhile considering the noise contributions from sections containing trivial multipliers (i.e. $\beta_1 = 0.5$). For simply halving the signal level, the signal-to-noise ratio would be reduced by 6 dBs. However, as the noise is introduced at various nodes within the network, we must once again consider the effect of the different structures on the resulting output noise spectrum. As in the previous analysis, a three-section network is chosen (with $\beta_i = 0.5$, all i).

We now have the choice of realizing the sub-transmittance

$\rho(z^{-1})$ as follows:-

$$\rho_i(z^{-1}) = \frac{1 - 0.5}{1 - 0.5 z^{-1}}$$

giving:-

$$T_{jk} = \frac{-1 + z^{-1}}{1 - 0.5 z^{-1}}$$

i.e. the type (i) j to k transfer response

or:-

$$\rho_i(z^{-1}) = \frac{0.5}{1 - 0.5 z^{-1}}$$

giving:-

$$T_{jk} = \frac{1}{1 - 0.5 z^{-1}}$$

i.e. the type (ii) j to k transfer response.

Hence, at the cost of one extra adder (allowing the $1-0.5$ numerator instead of 0.5), the highpass filtering effect (due to the term $(-1+z^{-1})$) is obtained. This is evident in the predicted and estimated noise spectra shown plotted in figs. 4.16 to 4.19. Whether this 'trivial' type (i) section proves useful or not will depend on the final design of the sideband filter and, in particular, on the ordering of the trivial and non-trivial sections used.

From this analysis, it has become evident that we can eliminate the type 2 structure from further investigations since it is equivalent to the type 1 structure. Furthermore, as the networks using the type (ii) sub-structure produce a larger total average power than those using the type (i) sub-structure, we exclude the type (ii) unit from any further investigations. In this case, these simple comparisons of the noise contributions can be made because any necessary scaling factors precede and follow the sub-structure thereby not affecting the various noise levels. This cannot be said of the type A and B structures, however, as any included scaling factors will affect the output noise levels. We therefore include the dynamic range constraints and recompute the resulting output noise spectra. From these results, a desirable structure, with regard to roundoff noise performance, can then be chosen.

4.4 Dynamic range constraints

It has been shown⁽⁹⁴⁾ that for the dynamic range constraints to be satisfied in any digital network, it is necessary that:-

$$\left\| F_i' \right\|_p \leq 1 \quad 4.22$$

where:-

$F_i'(\omega)$ is the scaled transfer response from the network input to the i^{th} branch node at which the dynamic range constraint is required (i.e. that input of the fixed-point multiplier). The value given to p (equation 4.22) is determined by the assumed input signal spectra and has the effect of bounding the maximum signal amplitude for deterministic signals or the maximum average power for random input signals at the scaled branch node.

Equation 4.22 represents the L-p norm of the response $F_i'(\omega)$, defined for an arbitrary periodic function $A(\omega)$, (with period ω_s) by:-

$$\left\| A \right\|_p \triangleq \left[\frac{1}{\omega_s} \cdot \int_0^{\omega_s} |A(\omega)|^p d\omega \right]^{1/p} \quad 4.23$$

It is also well-known that⁽⁹³⁾:-

$$\left\| F_i' \right\|_p \leq \left\| F_i' \right\|_\infty \quad \forall p \geq 1 \quad 4.24$$

For input signals of a random nature (the case most applicable in this study), it is sufficient to verify the less stringent condition, i.e.:-

$$\left\| F_i' \right\|_2 \leq 1 \quad 4.25$$

For a practical analysis, it has been shown that:-

$$\begin{aligned} \left\| F_i' \right\|_2^2 &\approx 1/2K \cdot \left\{ \left| F_i'(0) \right|^2 + \left| F_i'(\omega_s/2) \right|^2 \right\} \\ &+ 1/K \cdot \sum_{k=1}^{K-1} \left| F_i'(\omega_k) \right|^2 \end{aligned} \quad 4.26$$

where: K equals one half of the number of samples in the interval 0 to ω_s . For non-random or deterministic signals, this is not sufficient and it is necessary to verify:-

$$\left\| F_i' \right\|_{\infty} \leq 1 \quad 4.27$$

where:-

$$\left\| F_i' \right\|_{\infty} = \max_k \left| F_i'(\omega) \right| \quad 4.28$$

For the scaled network, we now obtain the total normalised average power from:-

$$\begin{aligned} \hat{\sigma}_y^2 &= \frac{1}{2K} \cdot \left\{ \hat{N}_y(0) + \hat{N}_y(\omega_s/2) \right\} \\ &+ \frac{1}{K} \cdot \sum_{k=1}^{K-1} \hat{N}_y(\omega_k) \end{aligned} \quad 4.29$$

or, for the peak noise power density, from:-

$$\left\| \hat{N}_y \right\| \simeq \max_k \left| \hat{N}_y(\omega) \right| \quad 4.30$$

where:-

$$\hat{N}_y(\omega) = \sum_j k_j \cdot \left| G_j'(\omega) \right|^2 \quad 4.31$$

i.e. $G_j'(\omega)$ represents the scaled transfer response from the j^{th} rounding point to the system output.

We now consider the scaling constraints as applied to the polyphase network. By referring to fig. 4.3, it can be seen that, although a common input node exists, transfer responses F_{iaa} and F_{iba} , F_{iab} and F_{ibb} must be considered for the i_a^{th} and i_b^{th} branch-nodes respectively. Following the reasoning in section 4.3, we consider the dynamic range constraints for the type 1Ai and type 1Bi structures only. For the type 1Ai section, it becomes evident that on arranging a common input to the first section, part of one path of that section is eliminated (i.e. reference to fig. 4.4(a) shows that, for a common input, the cancellation resulting from the addition and subtraction at the first summing node makes the remainder of that path redundant).

The first section of a type 1Ai structure is therefore realized with only one multiplier, compared with the two multipliers required by any other section. Furthermore, we see that, by connecting a common input to the first section, the roundoff noise at each output is modified - i.e. - compare figs. 4.8 and 4.10 (the predicted and estimated roundoff noise spectra for a separate input 3-section network) with figs. 4.20 and 4.21

(the roundoff noise spectra for the equivalent 3-section network but connected with a common input). The better roundoff noise performance of the type 1Ai structure, coupled with its multiplier reduction, would, at first sight, seem to indicate that this is the most desirable structure. We now show, however, that, after dynamic range constraints have been satisfied, this is not so.

To maximize the signal-to-noise ratio, we require that:-

$$\left\| F_i' \right\|_p = 1 \quad 4.32$$

and this condition is met if the scaling factors s_i , defined by:-

$$F_i'(\omega) = s_i \cdot F_i(\omega) \quad 4.33$$

are given by⁽⁹⁴⁾:-

$$s_i = 1 / \left\| F_i \right\|_p \quad 4.34$$

To keep the transfer response $H(z)$ constant, the signal level is rescaled by $1/s_i$ after the multiplication although for the polyphase single section, this can be done at both outputs. As in many networks, the required scaling factor s_i is less than unity. The rescaling operation therefore increases the noise level and thus reduces the overall signal-to-noise ratio.

The scaling factors computed for a 3-section network ($\beta_i = 0.875; 0.875; 0.75$), using both type 1Ai and type 1Bi structures are now listed:-

Type 1Ai structure.

section i	$\ F_i\ _2$	a	b	s_i (binary value)
1		4.13	-	0.25
2		2.73	2.89	0.25
3		1.91	2.76	0.25

Type 1Bi structure.

section i	$\ F_i\ _2$	a	b	s_i (binary value)
1		2.05	2.05	0.5
2		2.04	1.9	0.5
3		1.73	1.61	0.5

The scaling factor (s_i) is represented by an appropriate binary value for reasons of simplicity. We see from the computed results that a common scaling factor is required by each structure - i.e. type 1Ai structures require a $\frac{1}{4}$ -scaling whereas the type 1Bi structures require a $\frac{1}{2}$ -scaling. By referring to figs 4.4(a) and (b), it can be seen that, as the scaling and rescaling occurs at the input & output of each section respectively, it is sufficient, in practice, to scale at the input to the first section only with the rescaling being performed at the network output. For the rescaling at the network output, it is obvious that the reduction in the signal-to-noise ratio for the type

1Ai and type 1Bi structures will be by 12 dBs and 6dBs respectively i.e. compare figs. 4.20,21 with figs. 4.22,24 (type 1Ai unscaled and scaled) and also figs 4.12,14 with figs. 4.23, 25 (type 1Bi).

From the following summarised results which show the total average power for each scaled structure it becomes clear that the type 1Bi structure has the lowest roundoff noise performance.

Structure (scaled)	Total average power. (dBs.)
1Ai	16.3
1Aii	22.4
1Bi	13.5
1Bii	19.7

(Note: the values given for the type 1Ai and 1Aii structures are 'worst-case' - that is, the two paths do not have identical responses due to the modification in the first section).

4.5

The effects due to correlated roundoff errors

In the previous roundoff noise analysis, we have assumed that the errors introduced into the digital system after multiplication are uncorrelated from sample to sample and also from one rounding point to another. However, for low-level input signals, the roundoff errors become correlated and, in the limit of zero input to the system, the occurrence of periodic signals or limit-cycle oscillations⁽⁹⁵⁾ can be observed at the output of certain recursive structures.

To investigate the likelihood of low-level limit-cycles occurring in the polyphase network, we need only consider the recursive sub-section $\rho(z^{-1}) = (1-\beta) / (1-\beta.z^{-1})$ (realized as shown in fig. 4.6(a)). We obtain for its difference equation:-

$$y(n) = x(n) + d(n) - \left[d(n) * \beta \right]_q$$

where:-

$d(n)$ represents the n^{th} sample from the unit delay element.

$[.]_q$ represents the rounding performed at the output of the multiplier.

Now:-

$$d(n) = \left[(d(n-1) + x(n)) * \beta \right]_q$$

and therefore:-

$$y(n) = x(n) + \left[(d(n-1) + x(n)) * \beta \right]_q - \left[\left[(d(n-1) + x(n)) * \beta \right]_q \right]_q \quad 4.35$$

To investigate the low-level limit-cycle effect, we assume:-

$$x(n) = 0 \quad n > 0$$

and specify that $[.]_q$ gives a rounding to a particular decimal place or, for this example, by including scaling and rescaling to an integer.

$$\begin{aligned} \text{e.g.} \quad & \left[0.888 \right]_q \Rightarrow 0.9 \\ \text{or} \quad & \left[0.888 \times 10 \right]_q^s \Rightarrow 9 \div 10 = 0.9 \\ & \text{(to one decimal place)} \end{aligned}$$

Equation 4.35 can therefore be rewritten as:-

$$\begin{aligned} y(n) &= \left[d(n-1) * \beta \right]_q^s - \left[\left[d(n-1) * \beta \right]_q^s \right]_q^s \\ &\Rightarrow \text{integer}' - \left[\text{integer}' * \beta \right]_q^s \\ &= \text{integer}' - \text{integer}'' \end{aligned}$$

Now, for n increasing, integer" $\rightarrow$ integer' and therefore $y \rightarrow 0$, i.e. no limit-cycle oscillation can occur as we have a forced decay. The sub-structure's output decay, for input $x(0)=1$, is shown in fig. 4.26 ($\beta_1 = 0.7, 0.5$ and 0.3).

We also show (fig. 4.27) the effect on the polyphase network (3 sections - $\beta_1 = 0.875, 0.875$ and 0.75) output response (for $x(0)=1; x(n)=0, n>0$) for different quantization accuracies (i.e. rounding to a particular decimal point is a useful analogy of using finite length internal variables). It is clear from fig. 4.27 that reducing the internal variable wordlength also reduces the number of impulse response samples. From this we can deduce that reducing the internal wordlength will also reduce the transient response of the network (this is confirmed by the time response sequences shown in fig. 4.28, i.e. $y(n)$ for $x(n) = 1.0$ for all n).

4.6 Summary

In this chapter, we have investigated some of the effects, with regard to the polyphase phase - splitting network performance, caused by theoretically ideal signal data and numerical values being represented by binary values of limited accuracy (i.e. binary words of finite length). For the reasons given earlier, the use of the fixed-point arithmetic mode has been assumed although the binary number representation (i.e. 2's complement, sign-magnitude) has not been considered. Of the errors resulting from using a finite number representation - i.e. errors due to:

- i) quantization of the multiplier coefficients
- ii) quantization of the input signal
- iii) quantization of the results of arithmetic operations

- only the effects caused by the rounding of the products of multiplications have been investigated in any detail. This is because (a) the A/D noise contribution is neglected as the use of a 15-bit A/D wordlength gives us approximately a 90dB signal-to-noise ratio and (b) the effects due to reduced coefficient accuracies are considered at the system design stage. A direct and indirect connection between the generation of roundoff noise due to multiplier rounding operations and that due to the limitation on the number of bits available for realizing the multiplier coefficients has been shown to exist⁽⁹⁷⁾. The indirect connection, due to the fact that the multiplier complexity depends on the wordlength of both coefficients and signals, is not of such

great importance in the case of the polyphase network (i.e. we have seen that fairly short coefficient wordlengths can be used in satisfying particular phase responses and therefore the number of bits used to represent the signal can be increased thereby decreasing the roundoff noise without increasing the multiplier complexity). We have, however, considered in detail the generation of roundoff noise for different realizations of the polyphase network, both scaled and unscaled. From this analysis, it has been possible to select an optimal structure (optimal with regard to its minimal noise contribution). Further examination of this structure (by considering the effects due to correlated errors) has shown it to possess a particularly useful property - that is, the absence of limit-cycle oscillation for low-level input signals. Furthermore, the reduction of transients (decay and rise time) for reduced internal variable wordlengths show the network to be consistently stable.

Note: 2's complement arithmetic is generally found to be most useful in digital systems using serial arithmetic because additions can be performed, starting with the least significant bit, before the signs or relative magnitudes of the numbers being added are known⁽⁹⁶⁾.

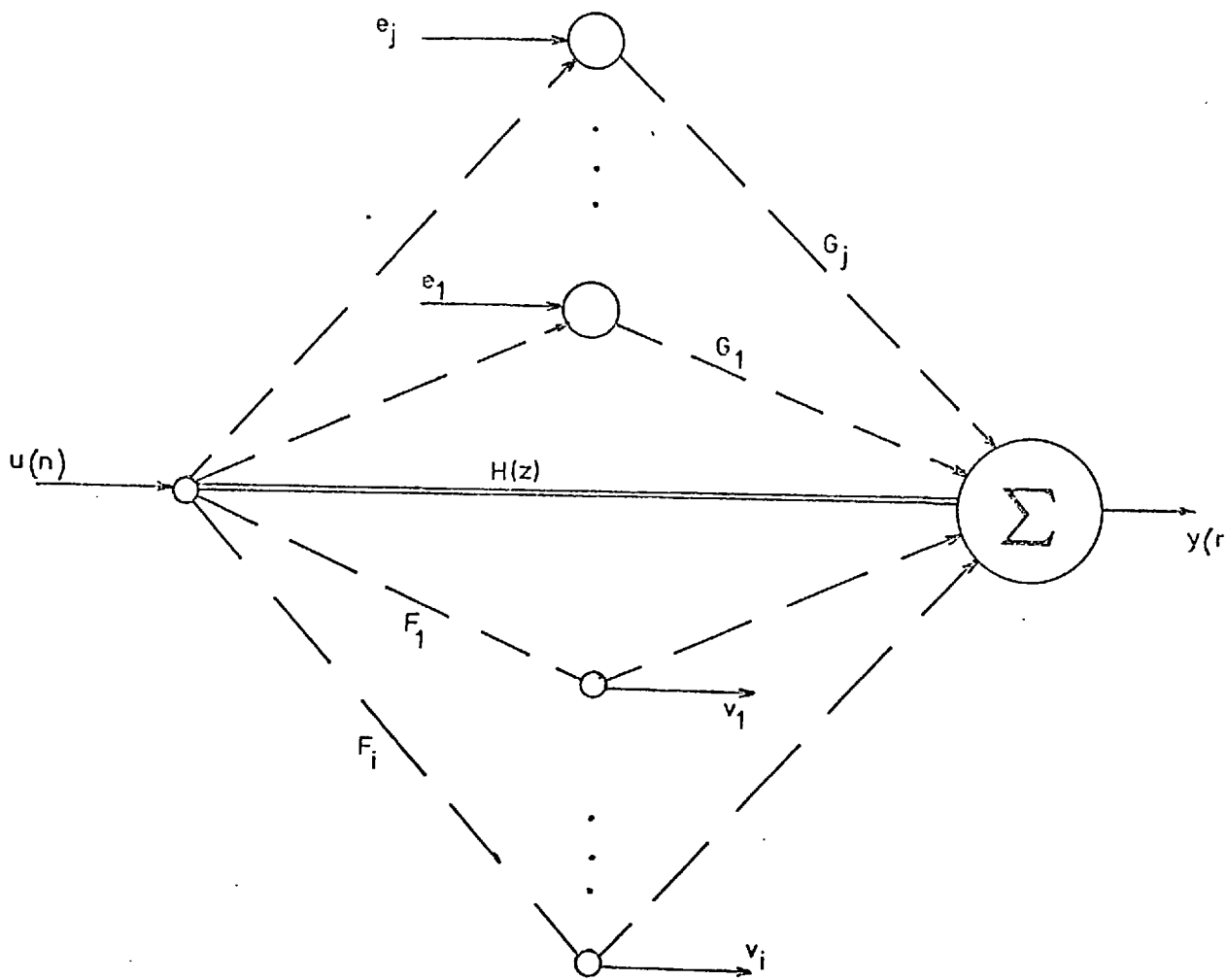


Fig. 4.1 The general digital network model.

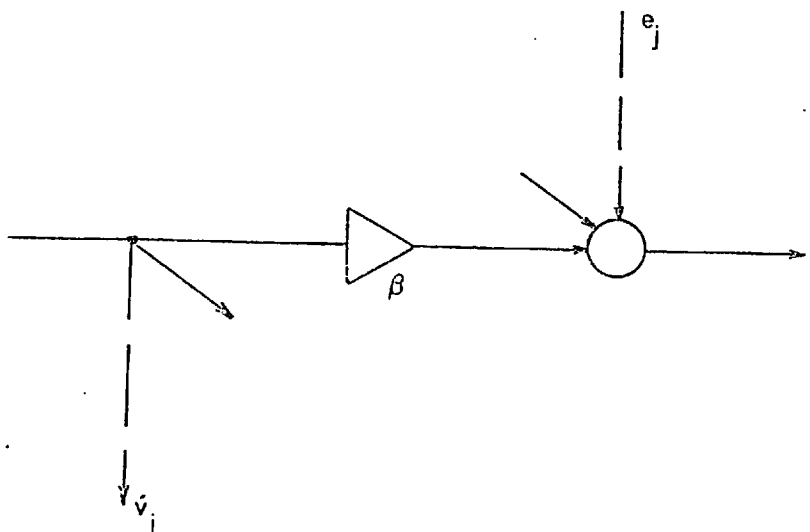


Fig. 4.2 The i^{th} to j^{th} node relationship.

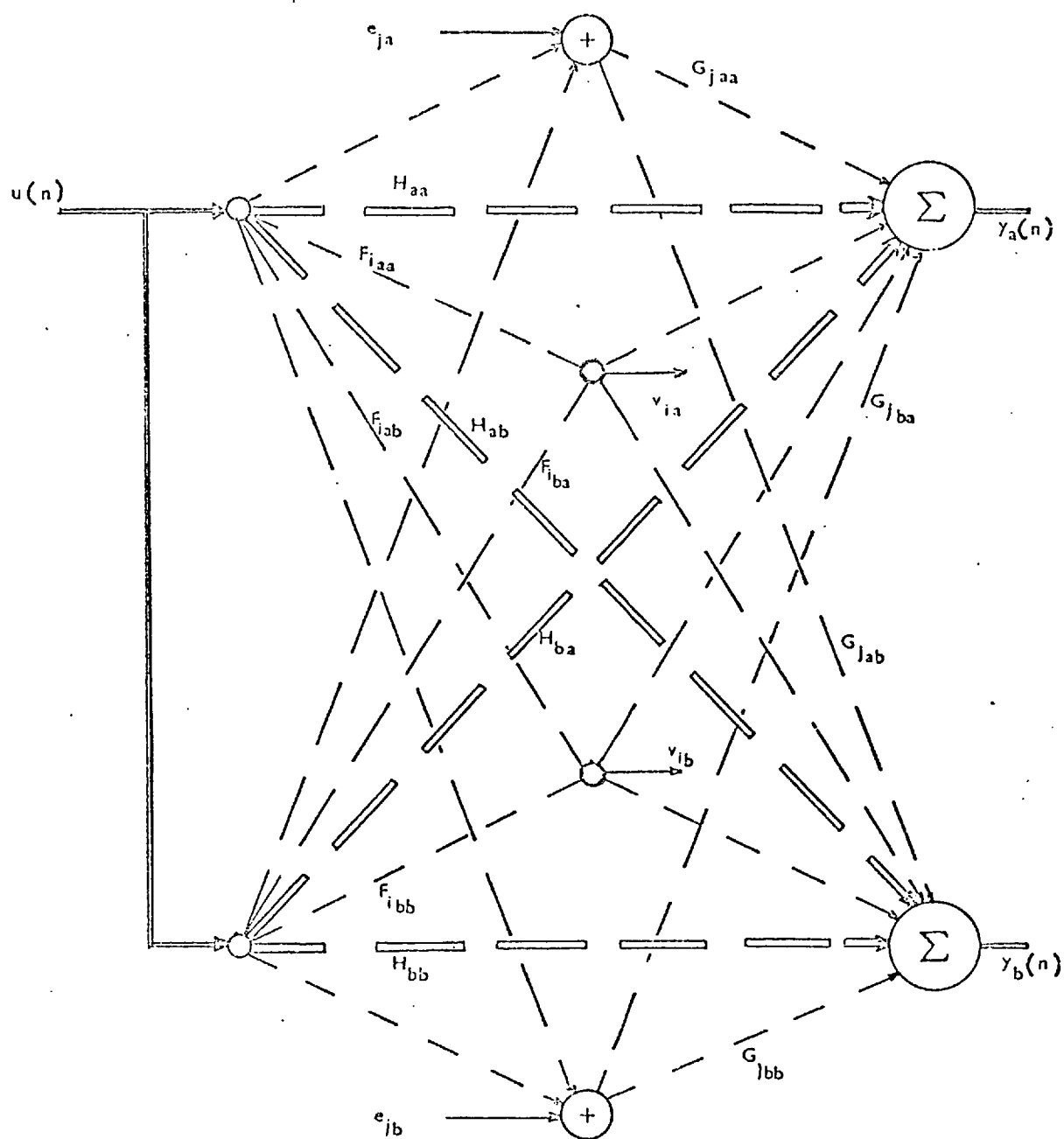


Fig. 4.3 The digital polyphase network model.

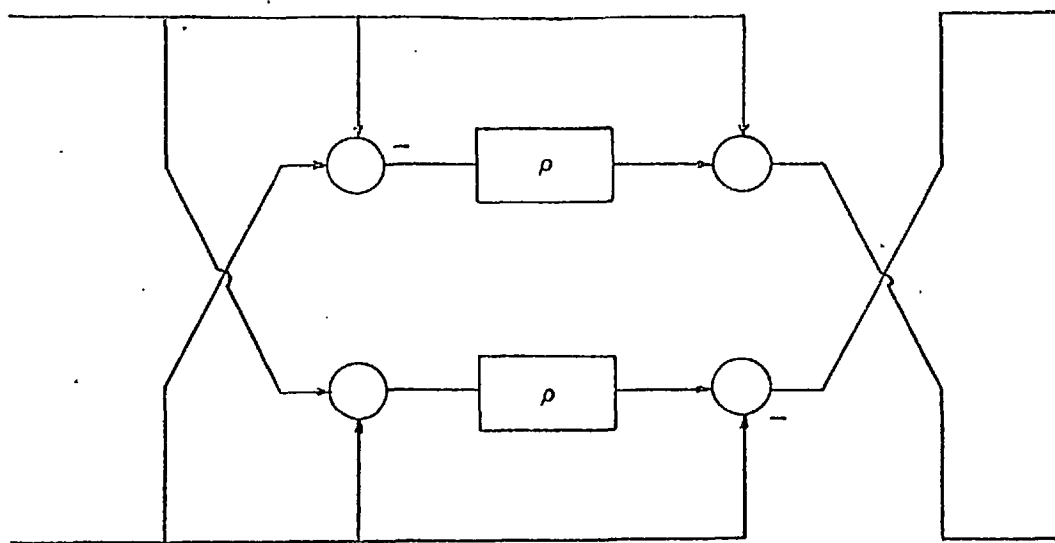


Fig. 4.4(a) The type 1A structure.

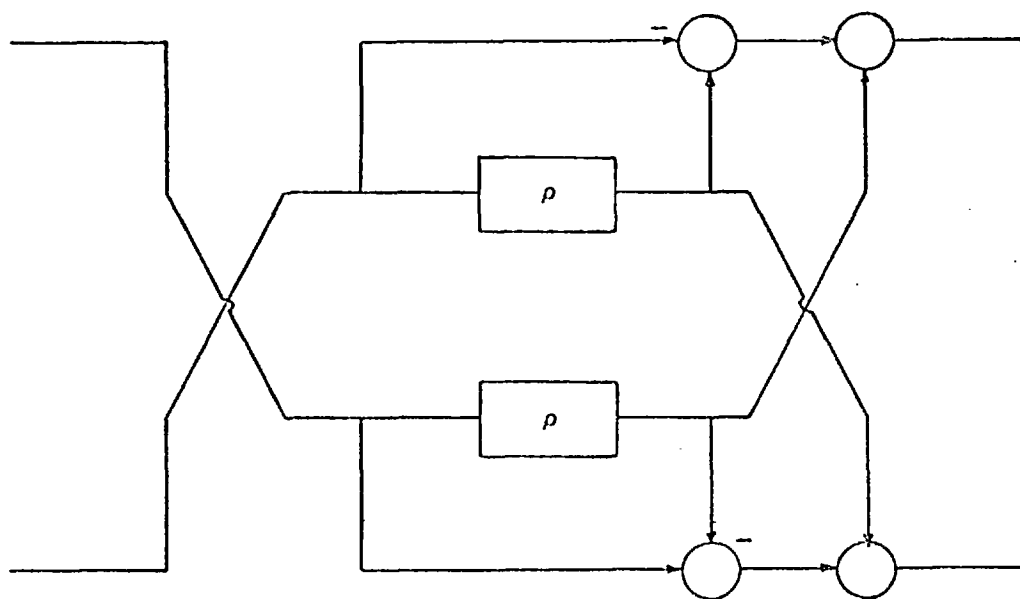


Fig. 4.4(b) The type 1B structure.

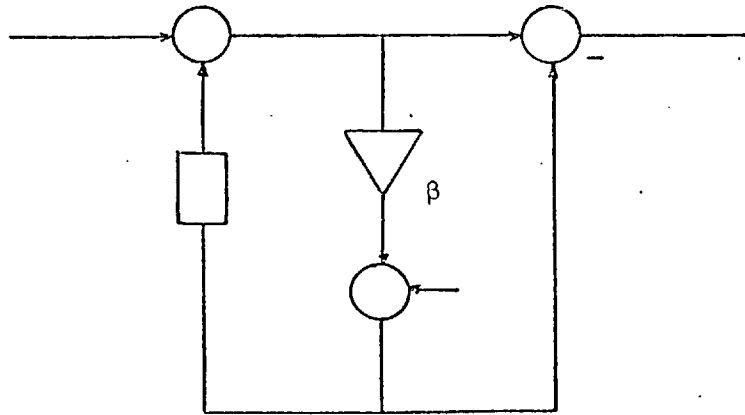


Fig. 4.5(a) The type i sub-structure.

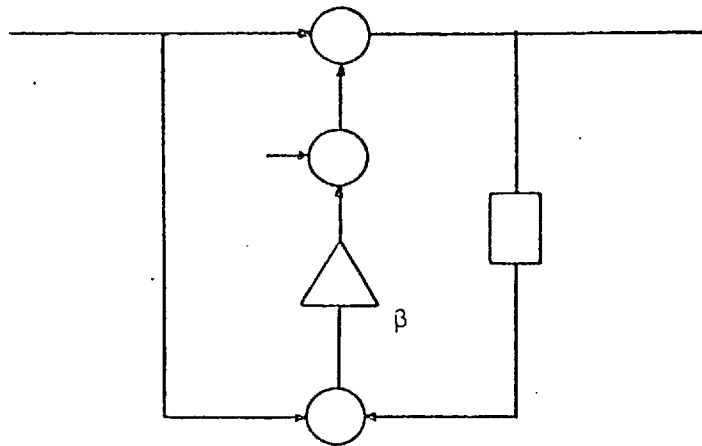


Fig. 4.5(b) The type ii sub-structure.

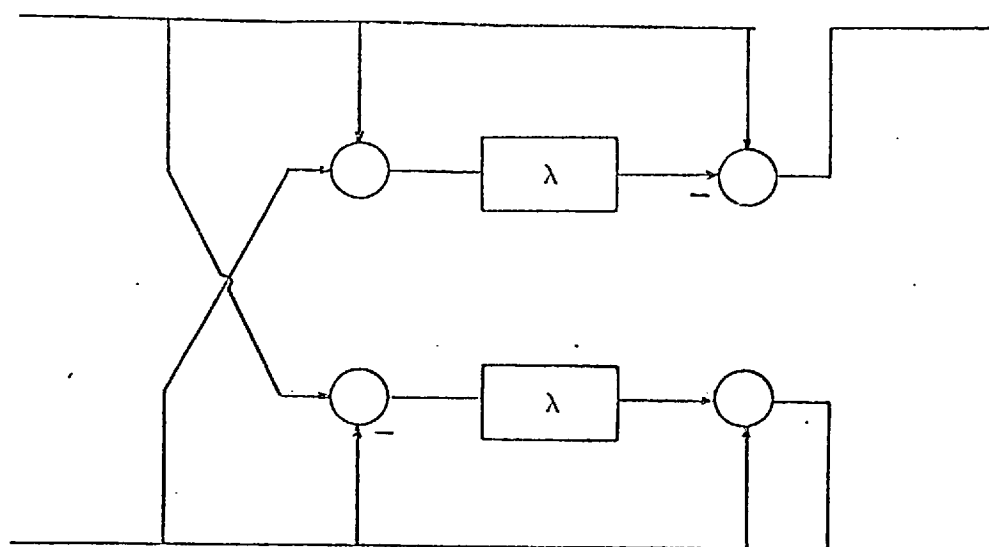


Fig. 4.6(a) The type 2A structure.

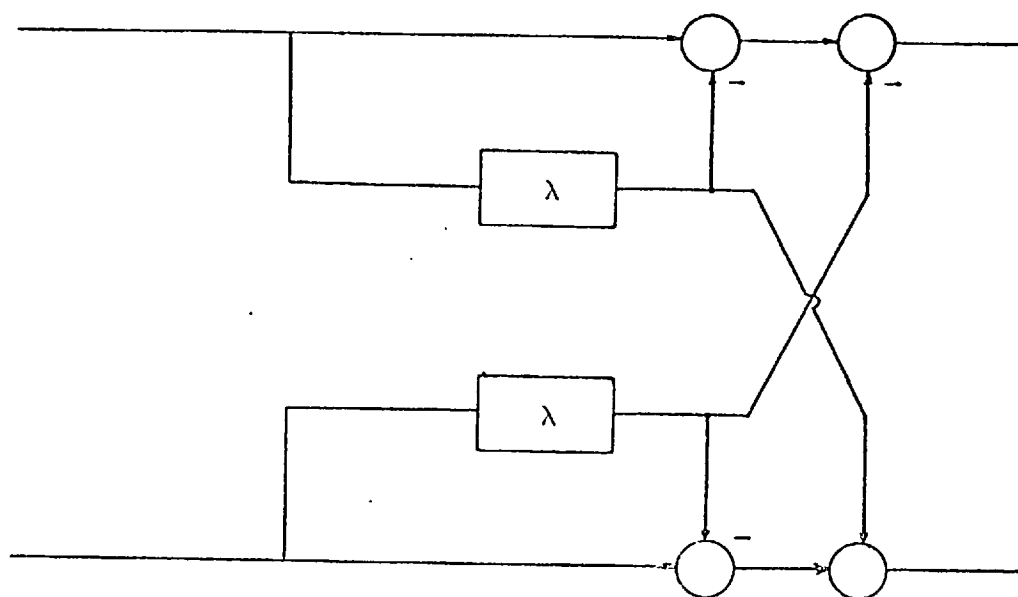


Fig. 4.6(b) The type 2B structure.

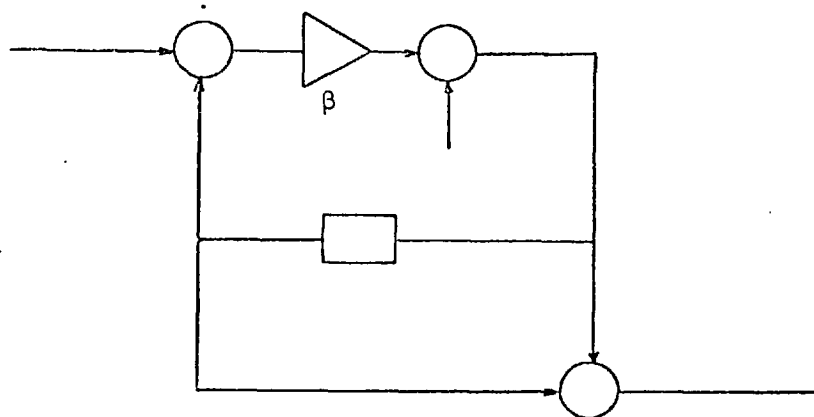


Fig. 4.7(a) The type iii sub-structure.

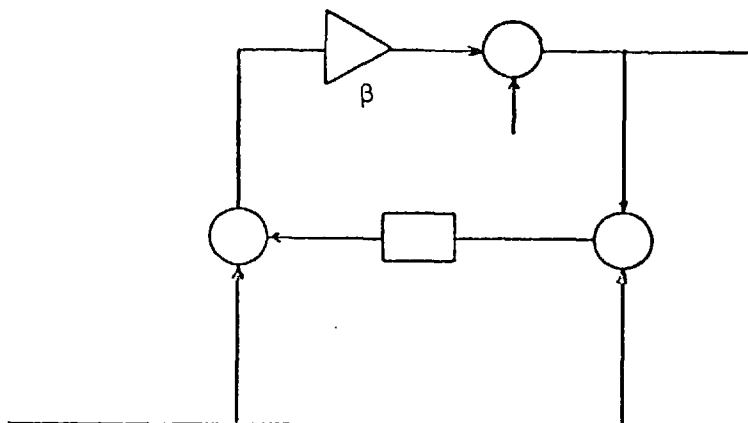


Fig. 4.7(b) The type iv sub-structure.

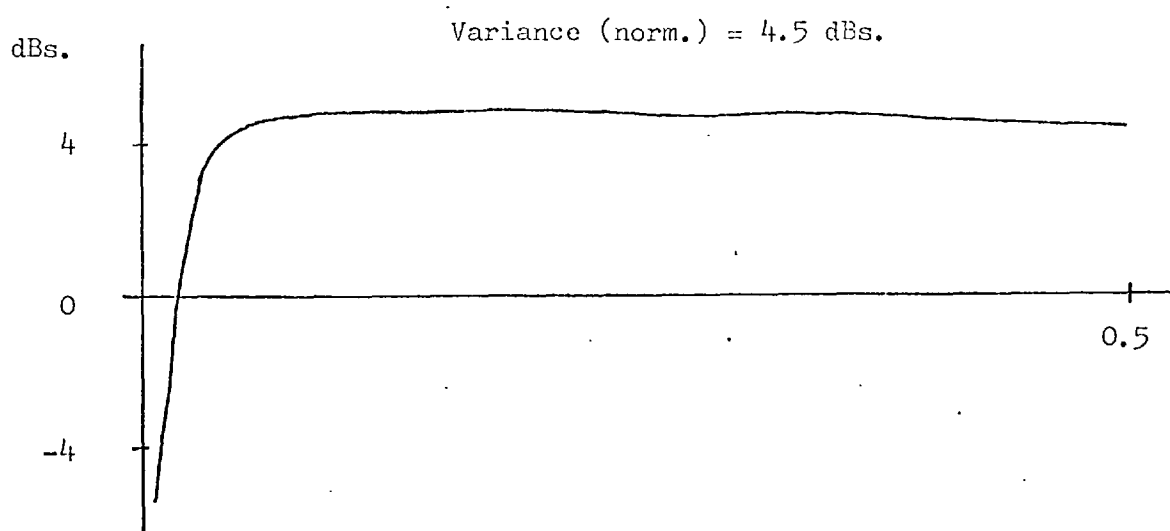


Fig. 4.8 The predicted noise spectrum : type 1Ai.

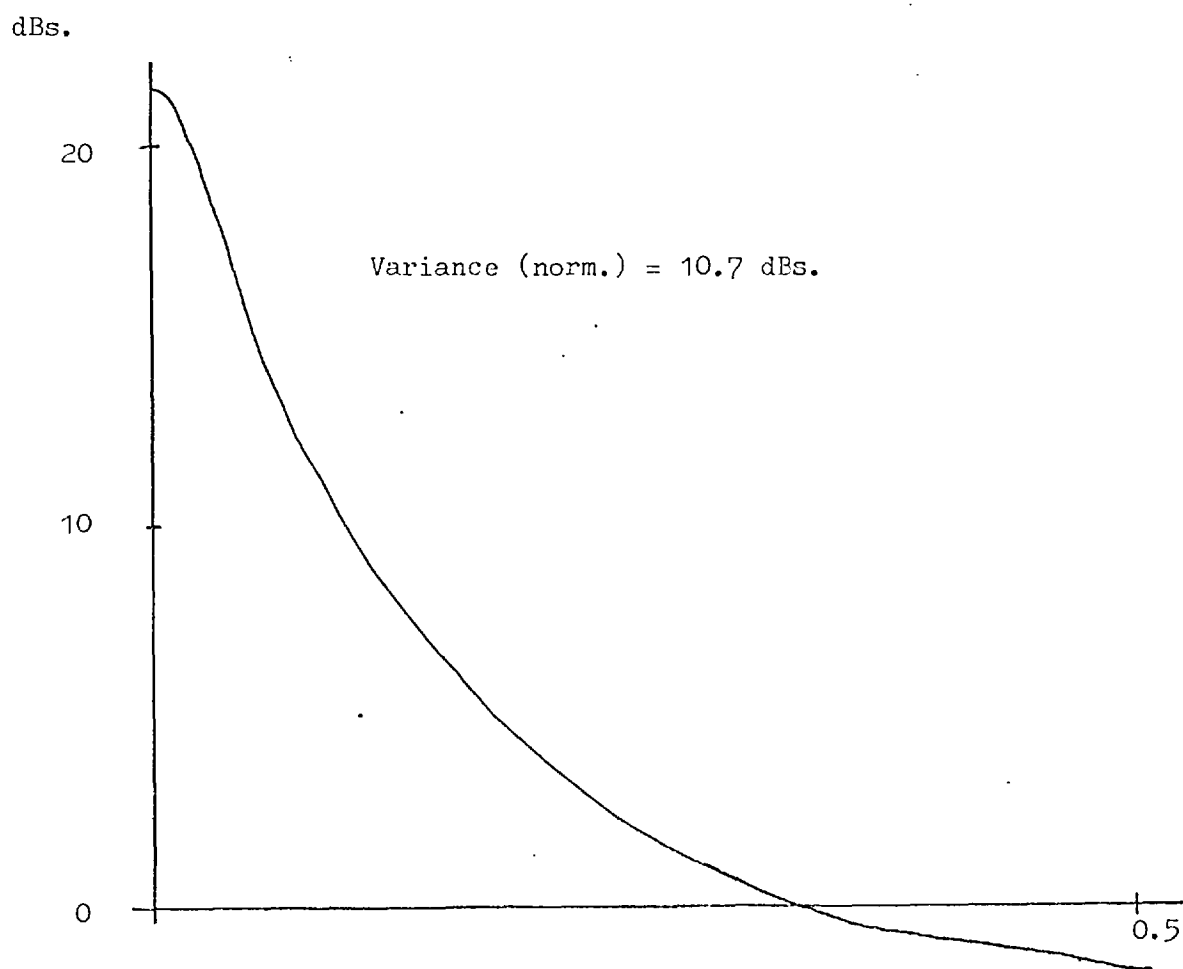


Fig. 4.9 The predicted noise spectrum : type 1Aii.

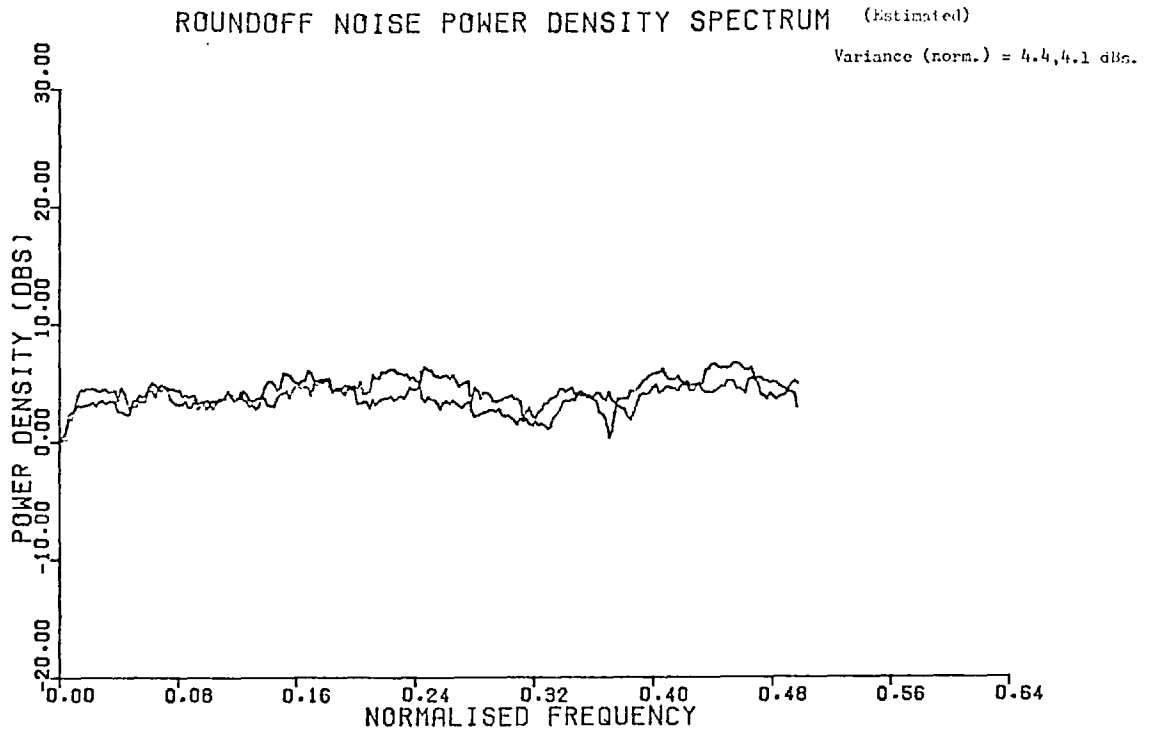


Fig. 4.10 Estimated noise spectrum : type 1Ai.

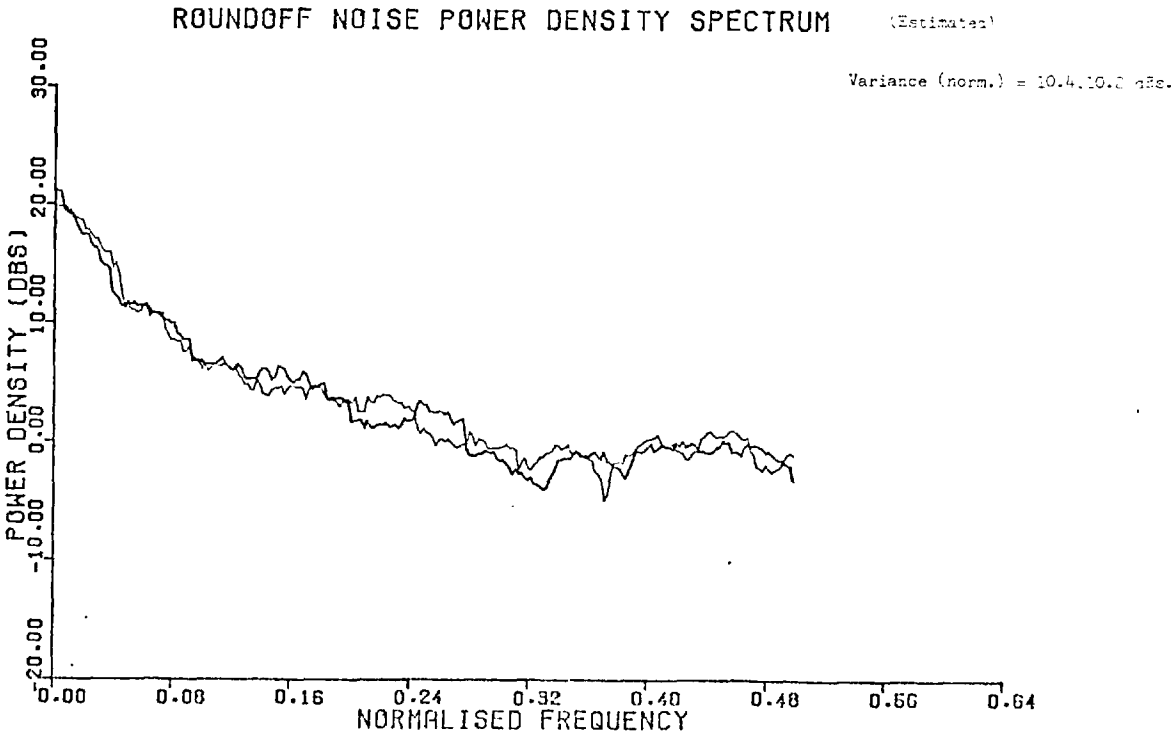


Fig. 4.11 Estimated noise spectrum : type 1Aii.

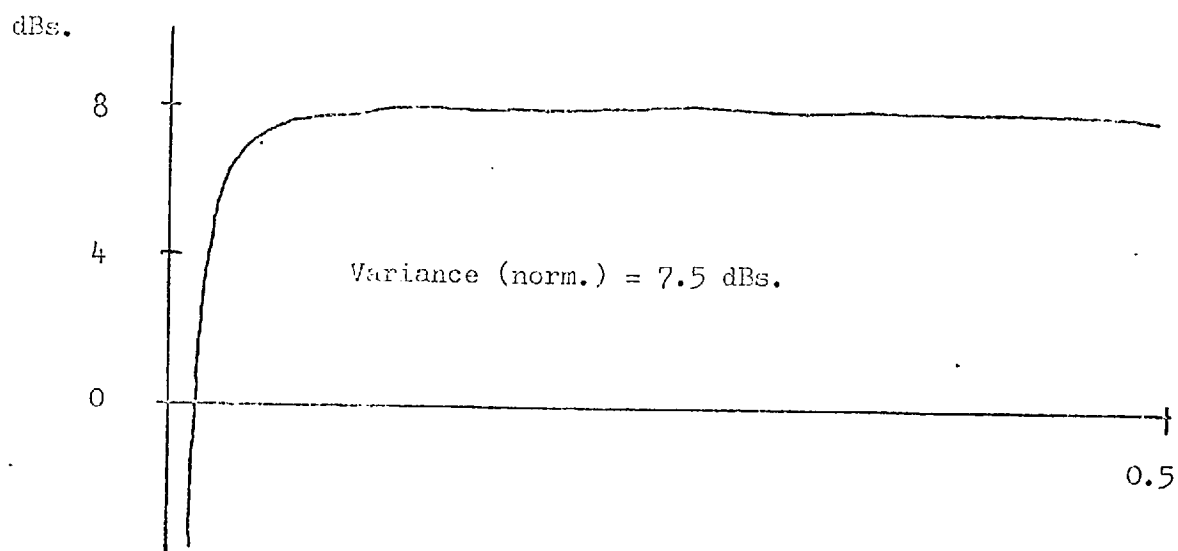


Fig. 4.12 The predicted noise spectrum : type 1Bi.

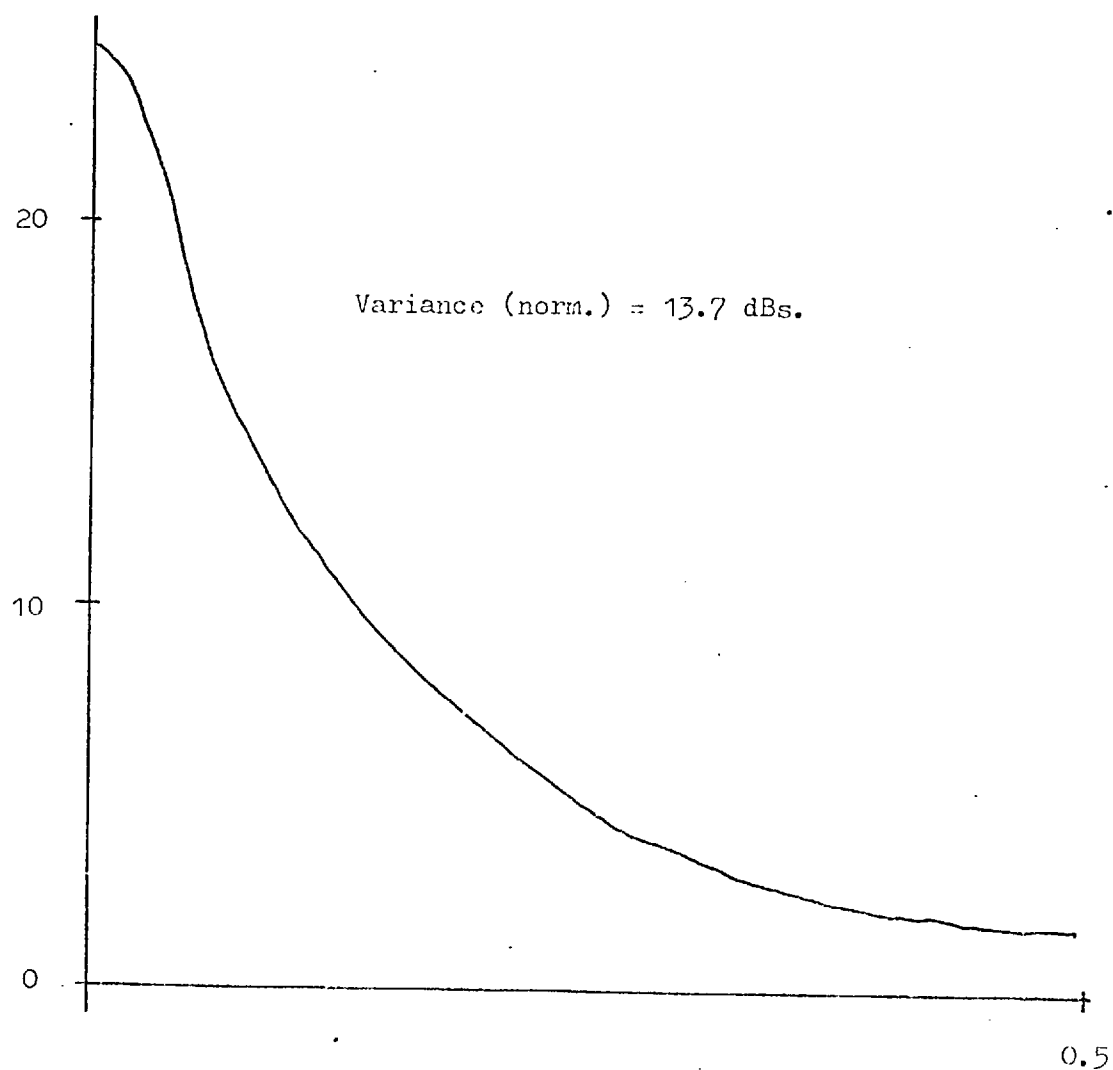


Fig. 4.13 The predicted noise spectrum : type 1Bii.

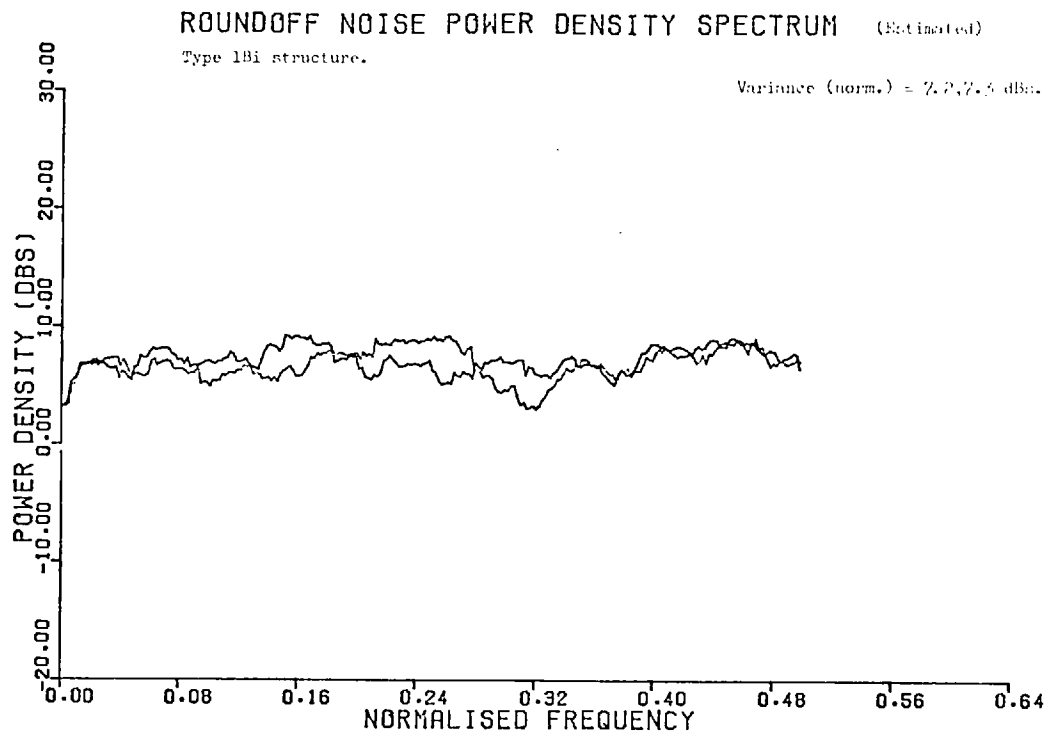


Fig. 4.14 Estimated noise spectrum : type 1Bi.

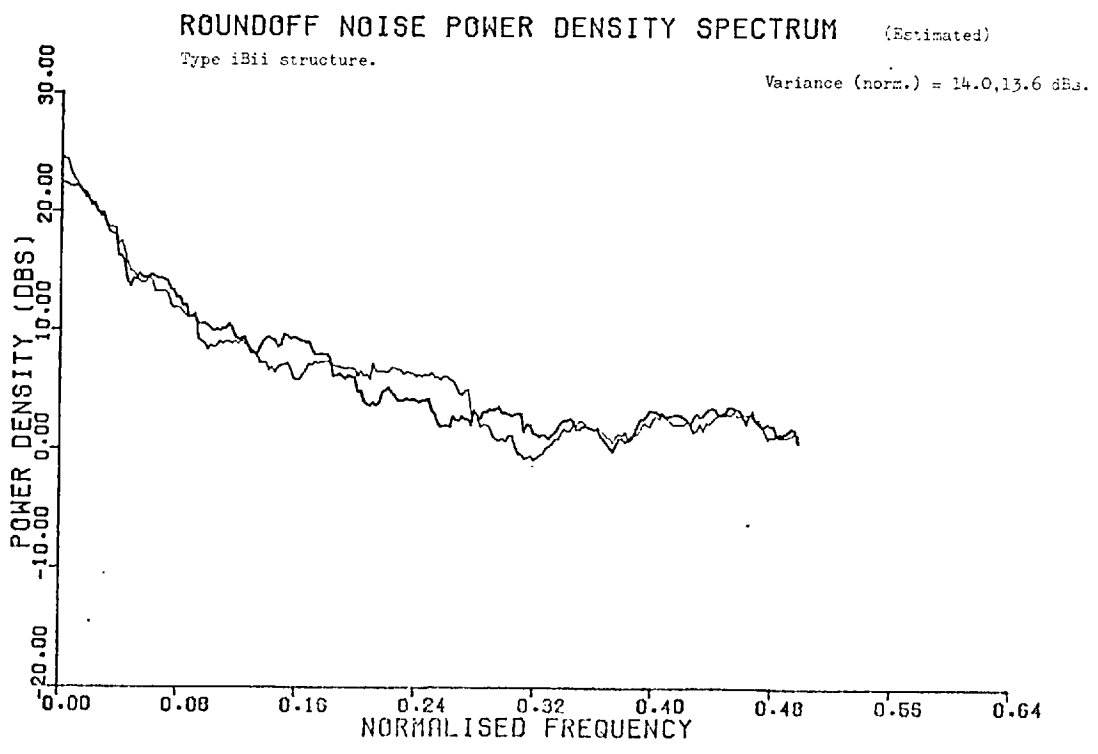


Fig. 4.15 Estimated noise spectrum : type 1Bii.

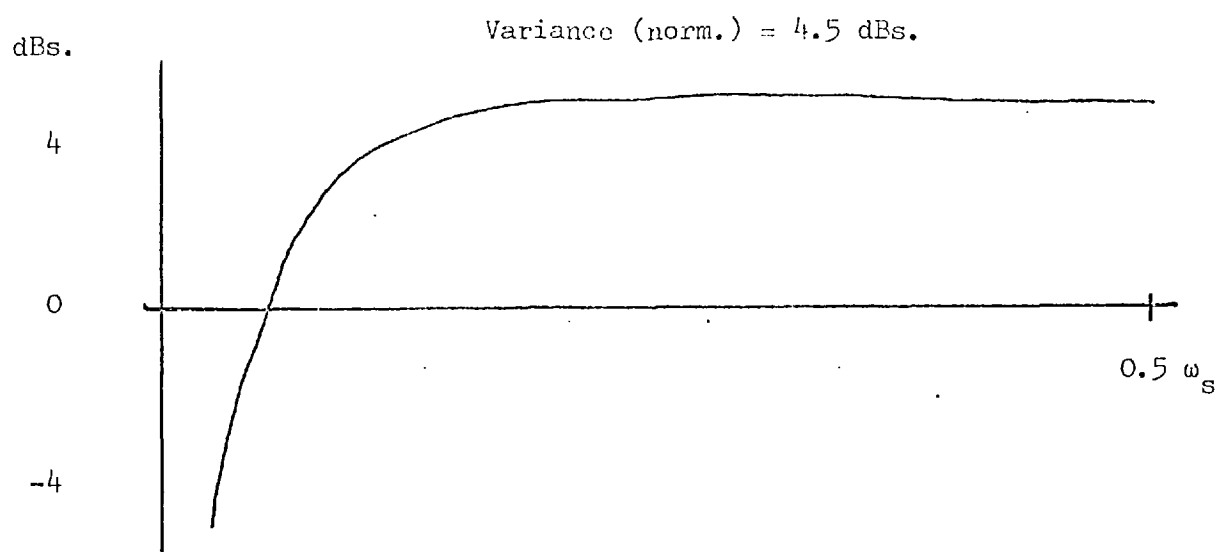


Fig. 4.16 The predicted noise spectrum : all-trivial section.

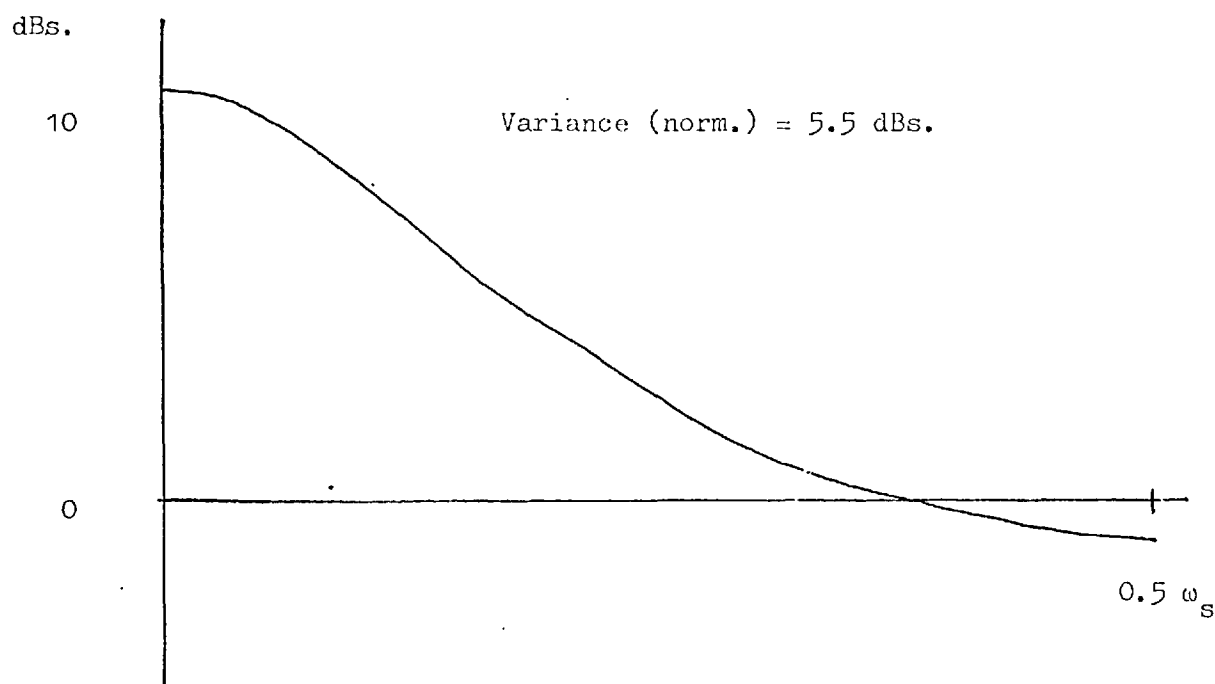


Fig. 4.17 The predicted noise spectrum : modified all-trivial section

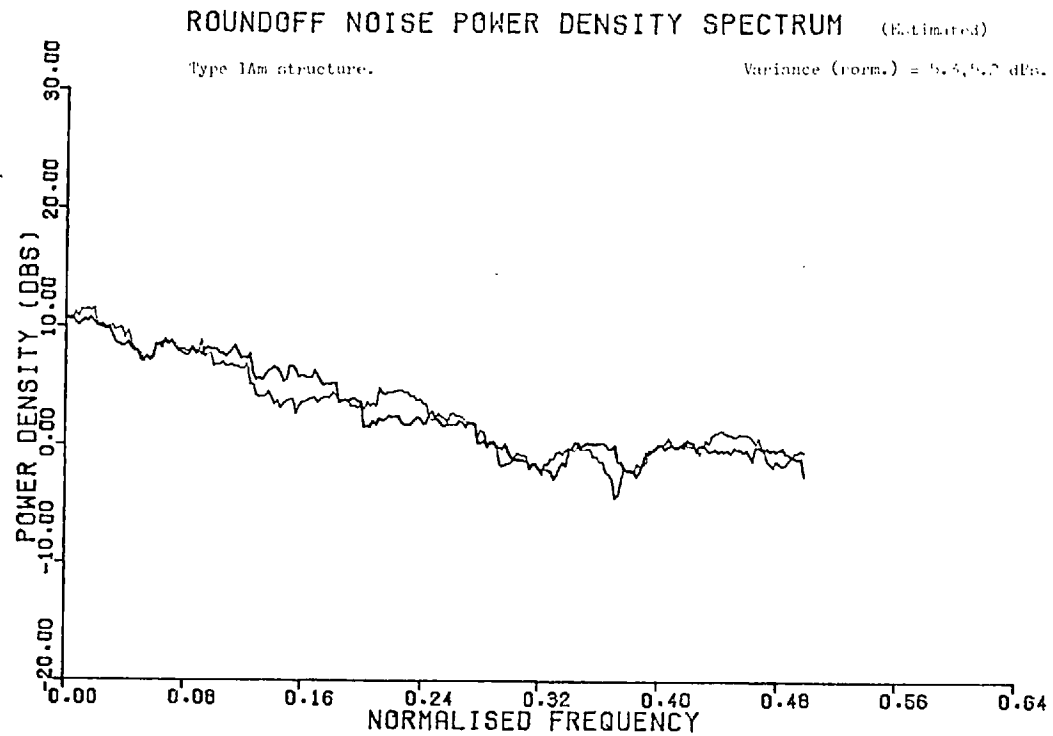


Fig. 4.18 Estimated noise spectrum : all-triv. section (modified).

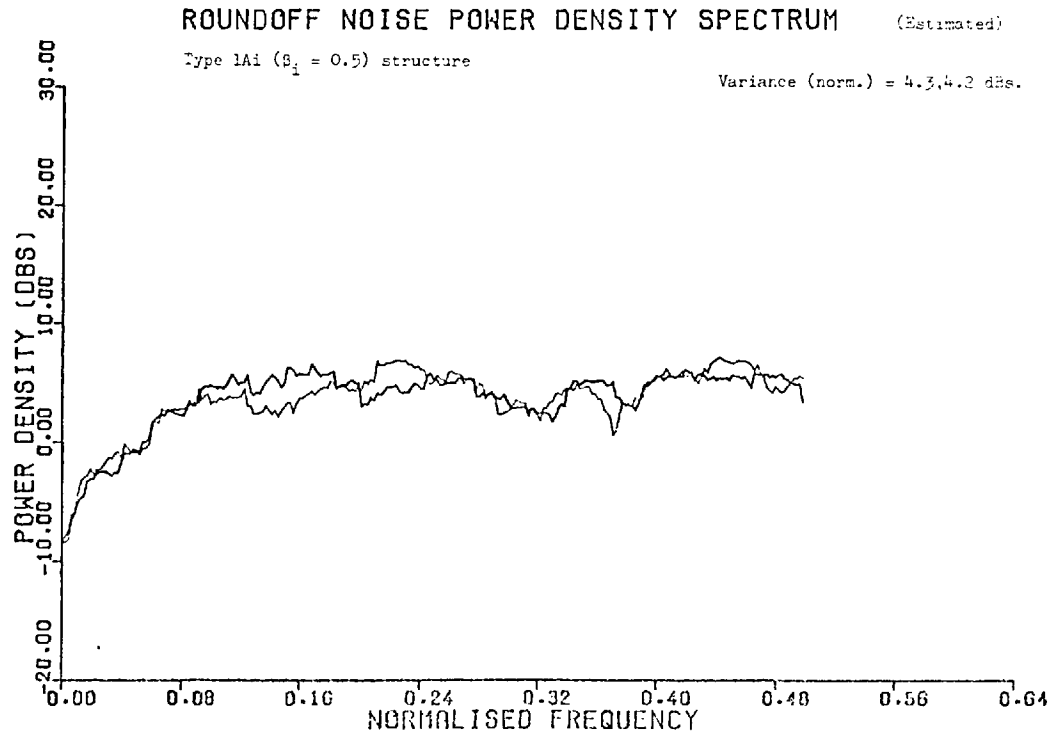


Fig. 4.19 Estimated noise spectrum : all-trivial section.

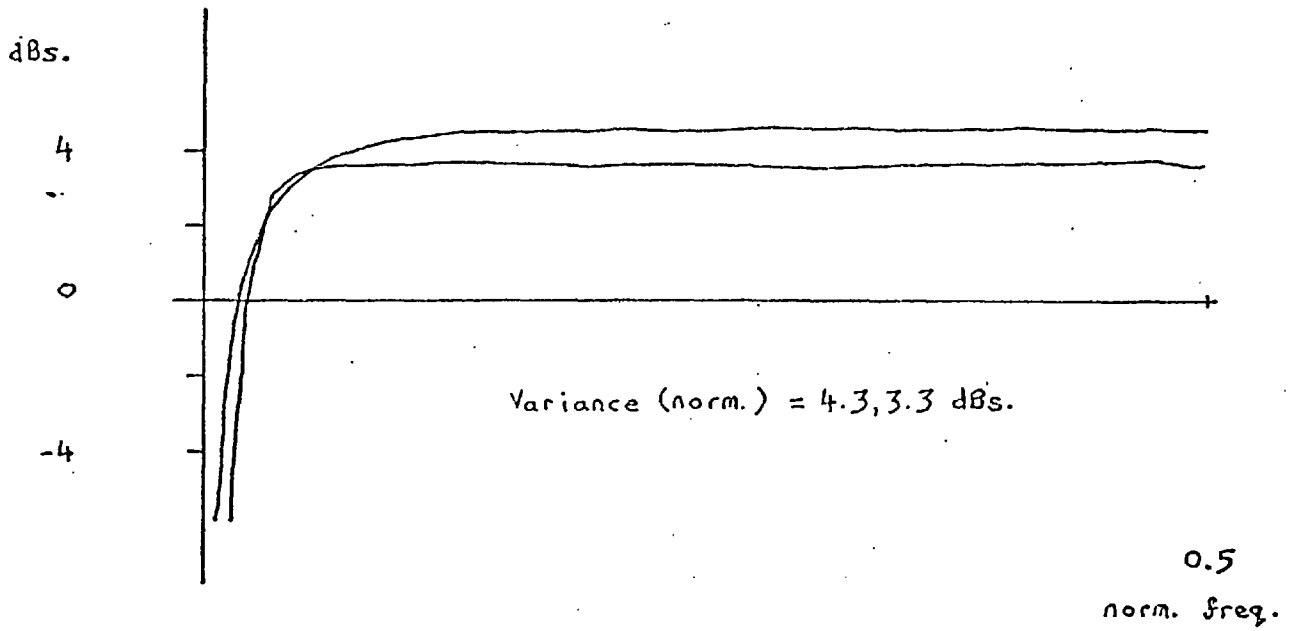


Fig. 4.20 Predicted noise spectrum : type 1Ai (modified 1st-sect.).

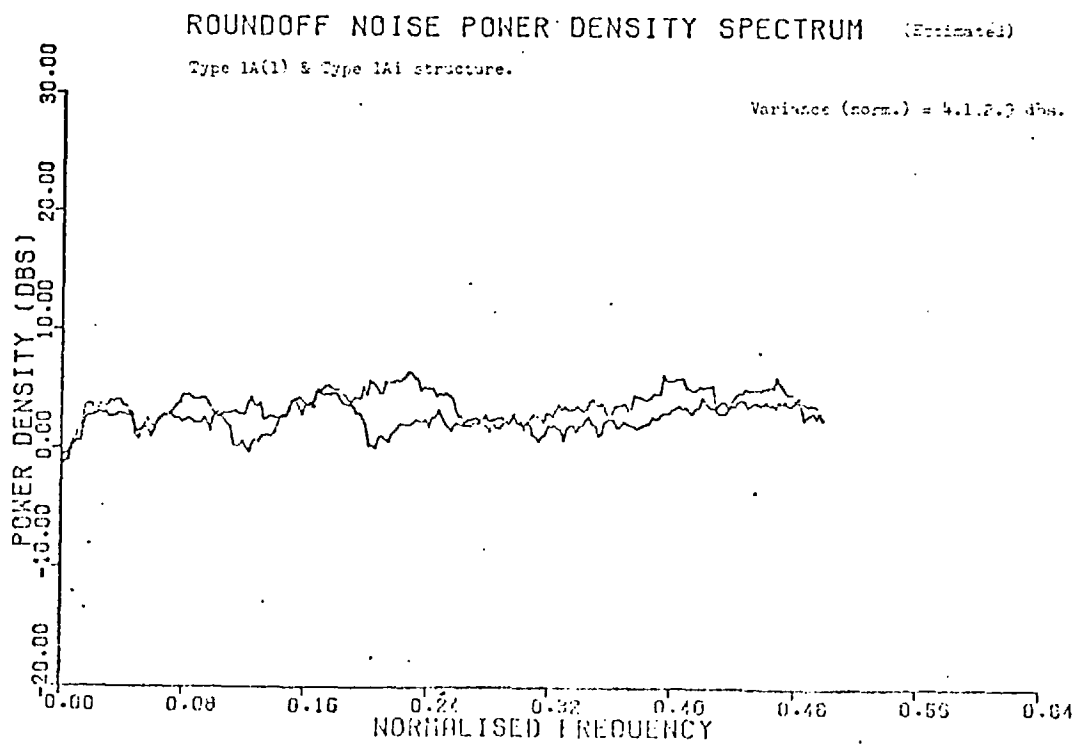


Fig. 4.21 Estimated noise spectrum : type 1Ai (modified 1st-sect.).

dBs.

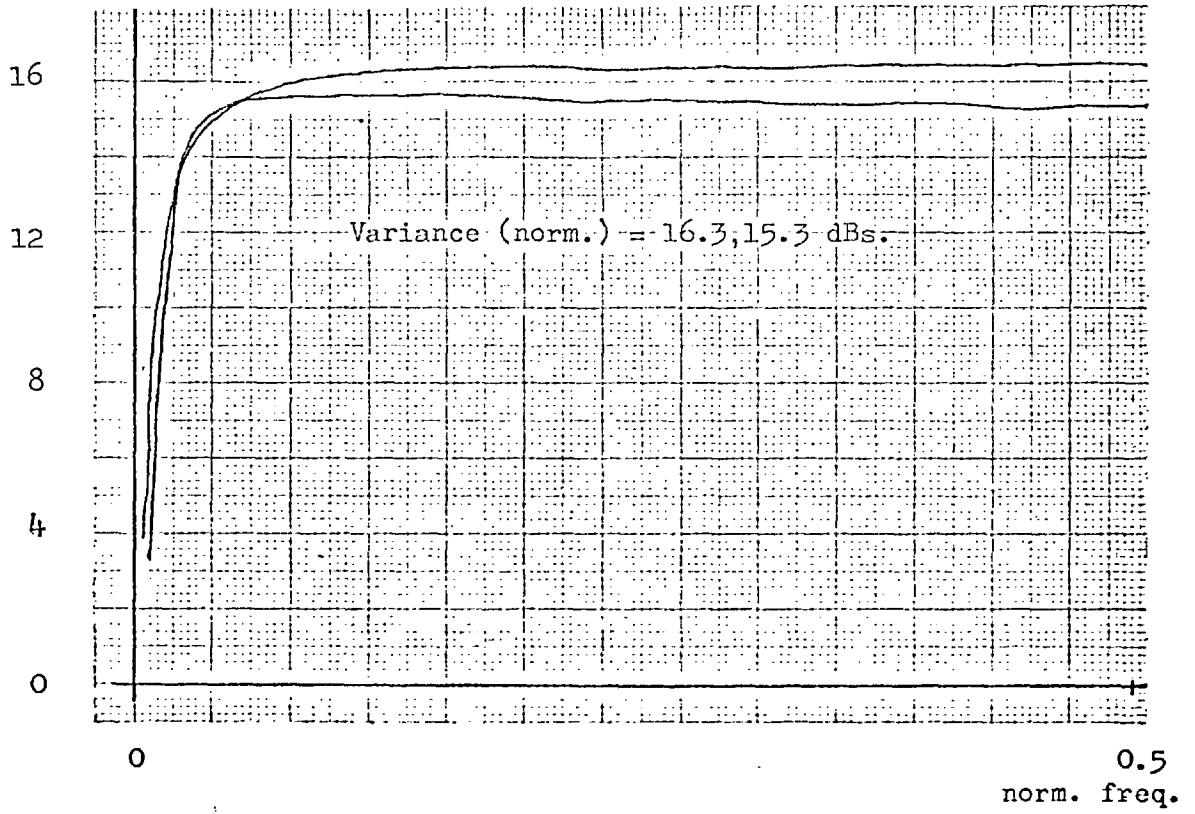


Fig. 4.22 Predicted noise spectrum : type 1Ai SCALED.

dBs.

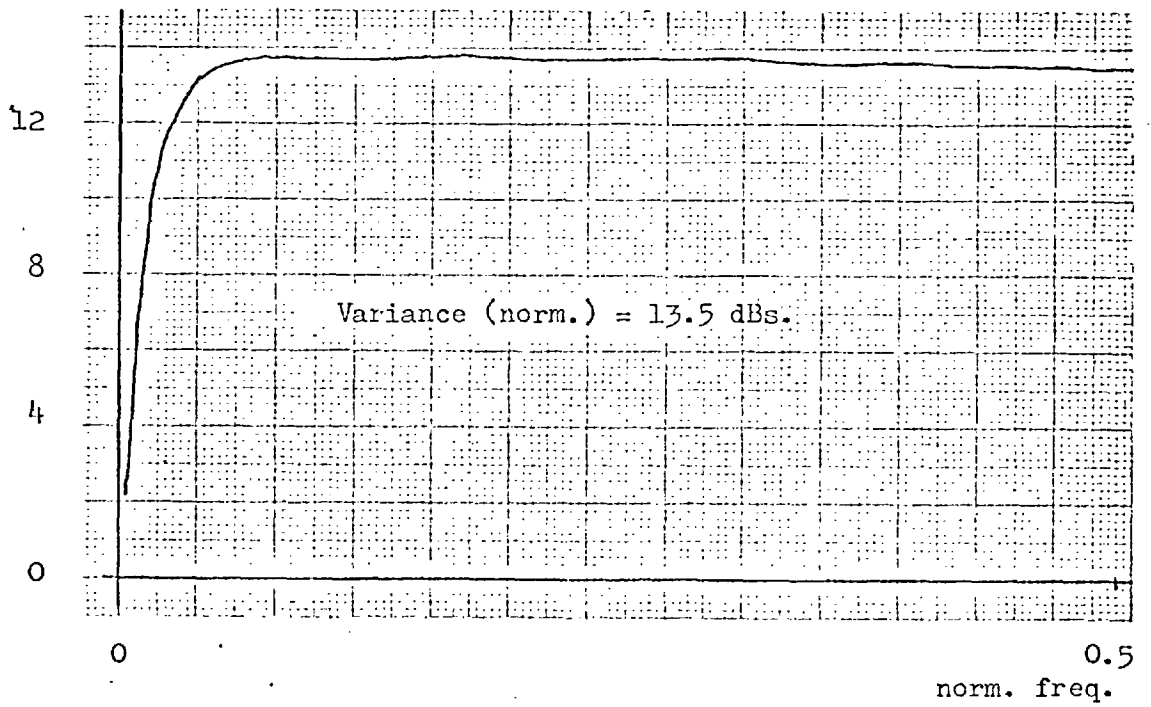


Fig. 4.23 Predicted noise spectrum : type 1Bi SCALED.

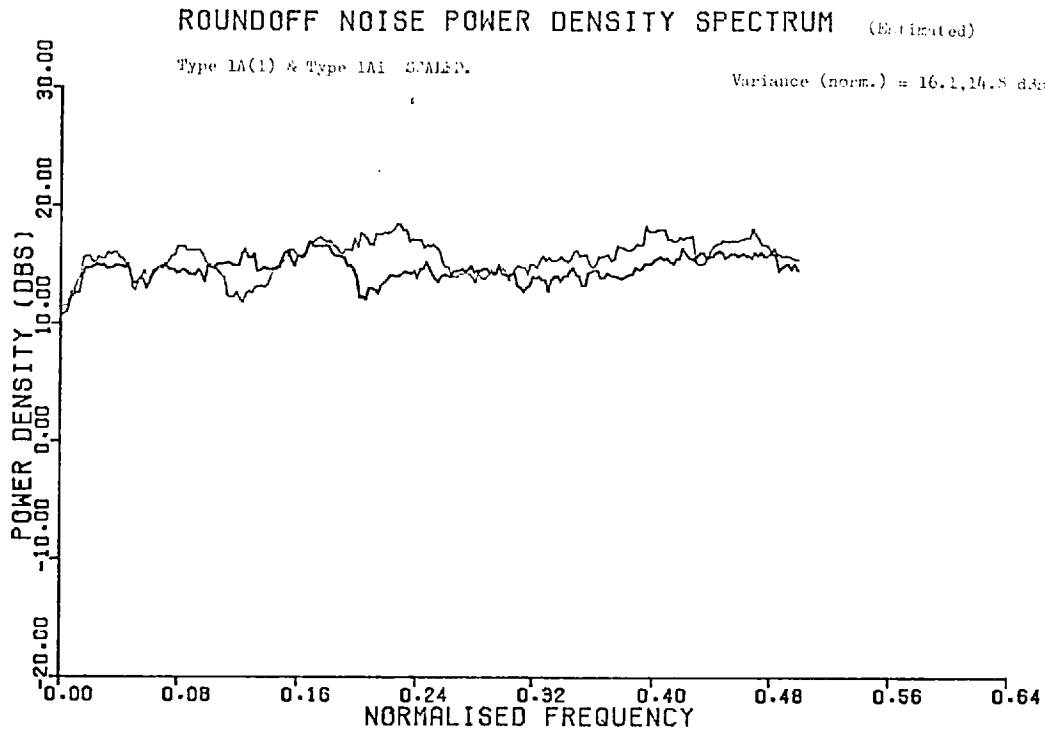


Fig. 4.24 Estimated noise spectrum : type 1Ai SCALED.

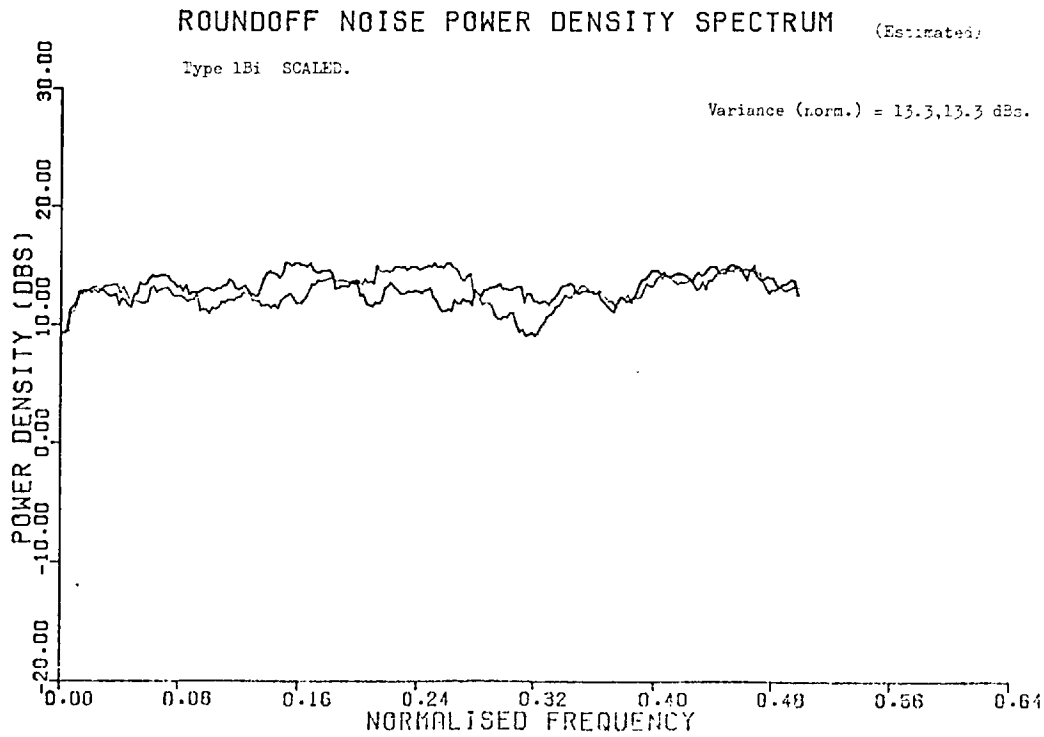


Fig. 4.25 Estimated noise spectrum : type 1Bi SCALED.

Fig. 4.26 The type I sub-structure decay response.

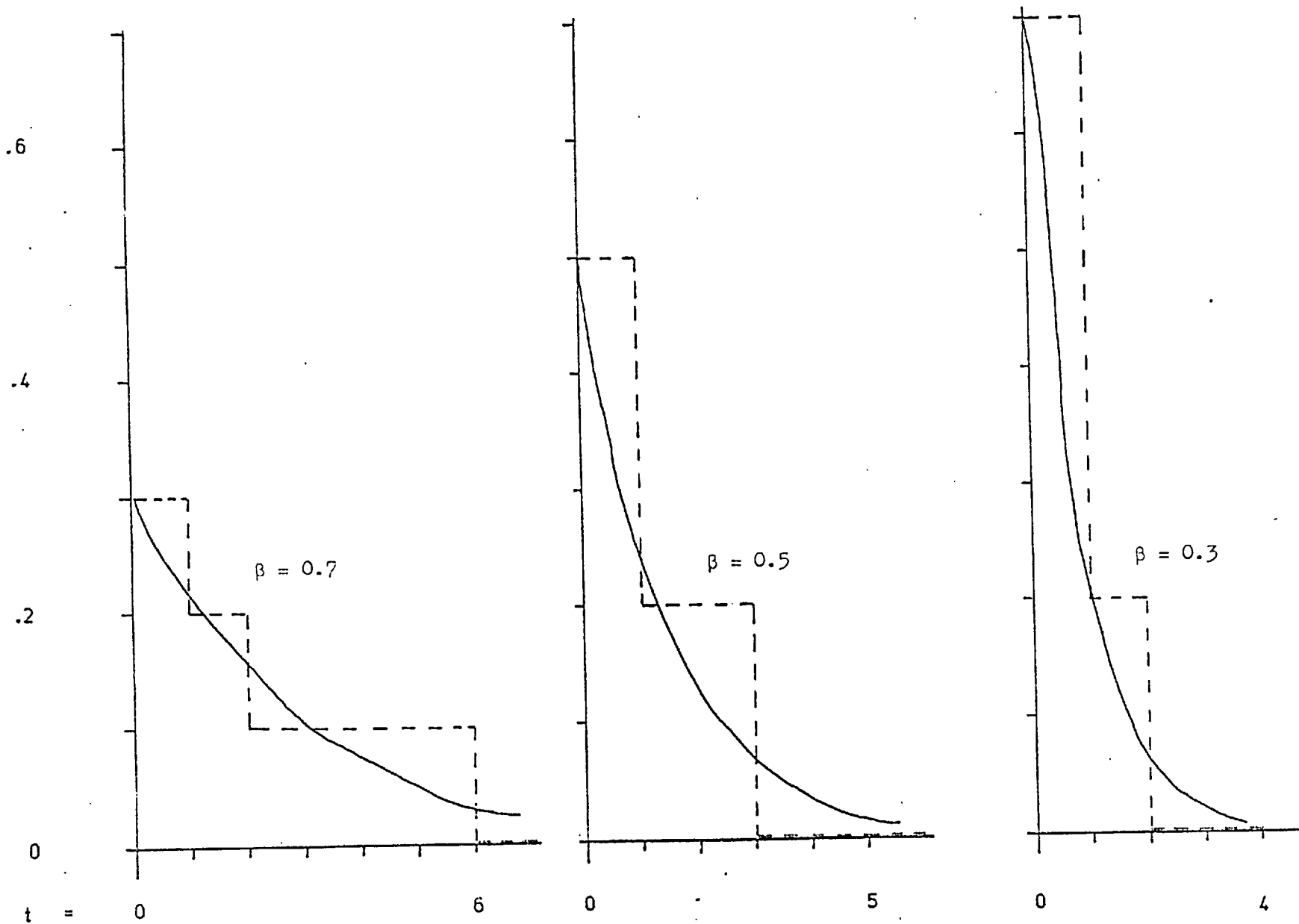


Fig. 4.27 Polyphase network decay response
(different internal variable accuracies)

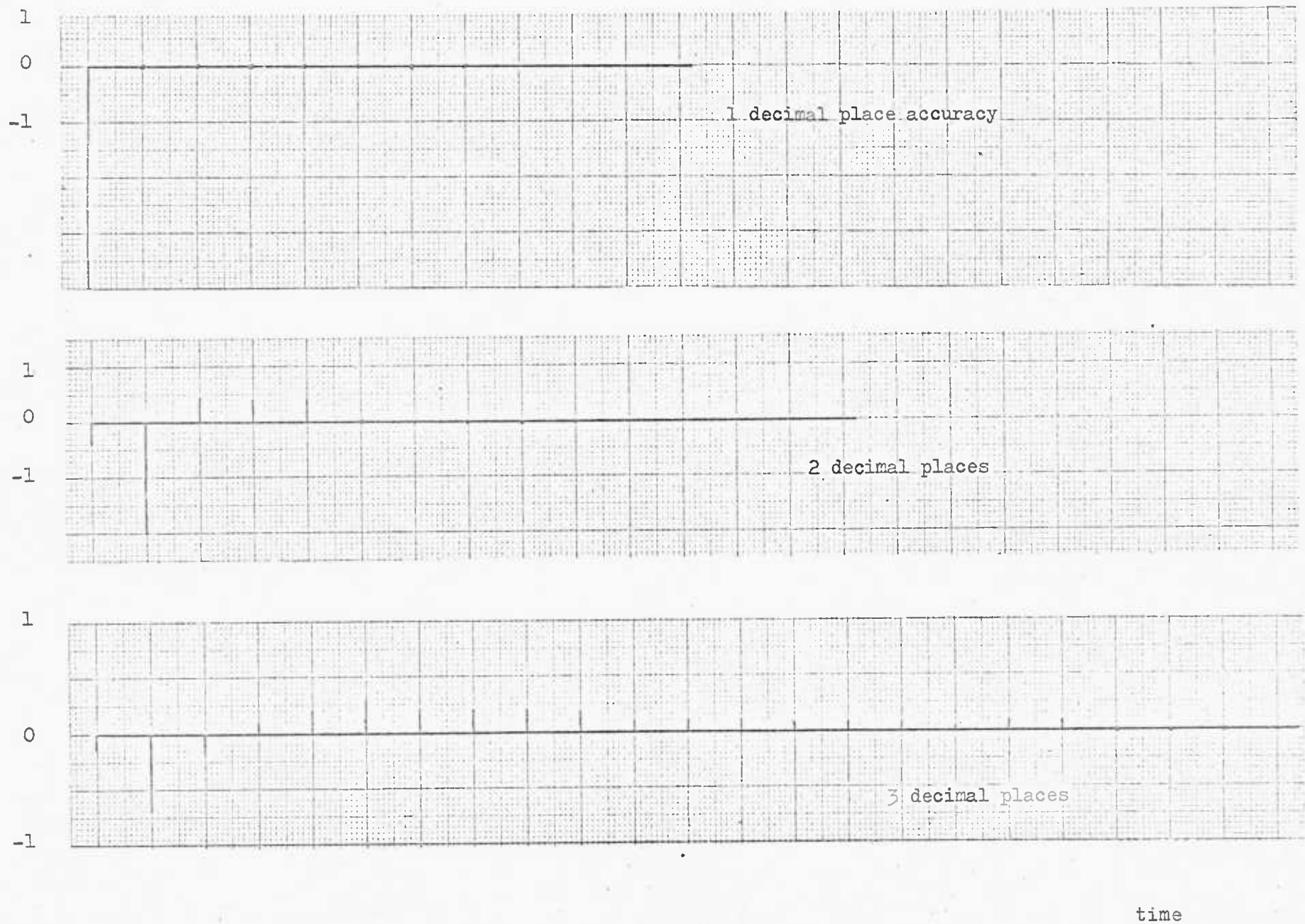
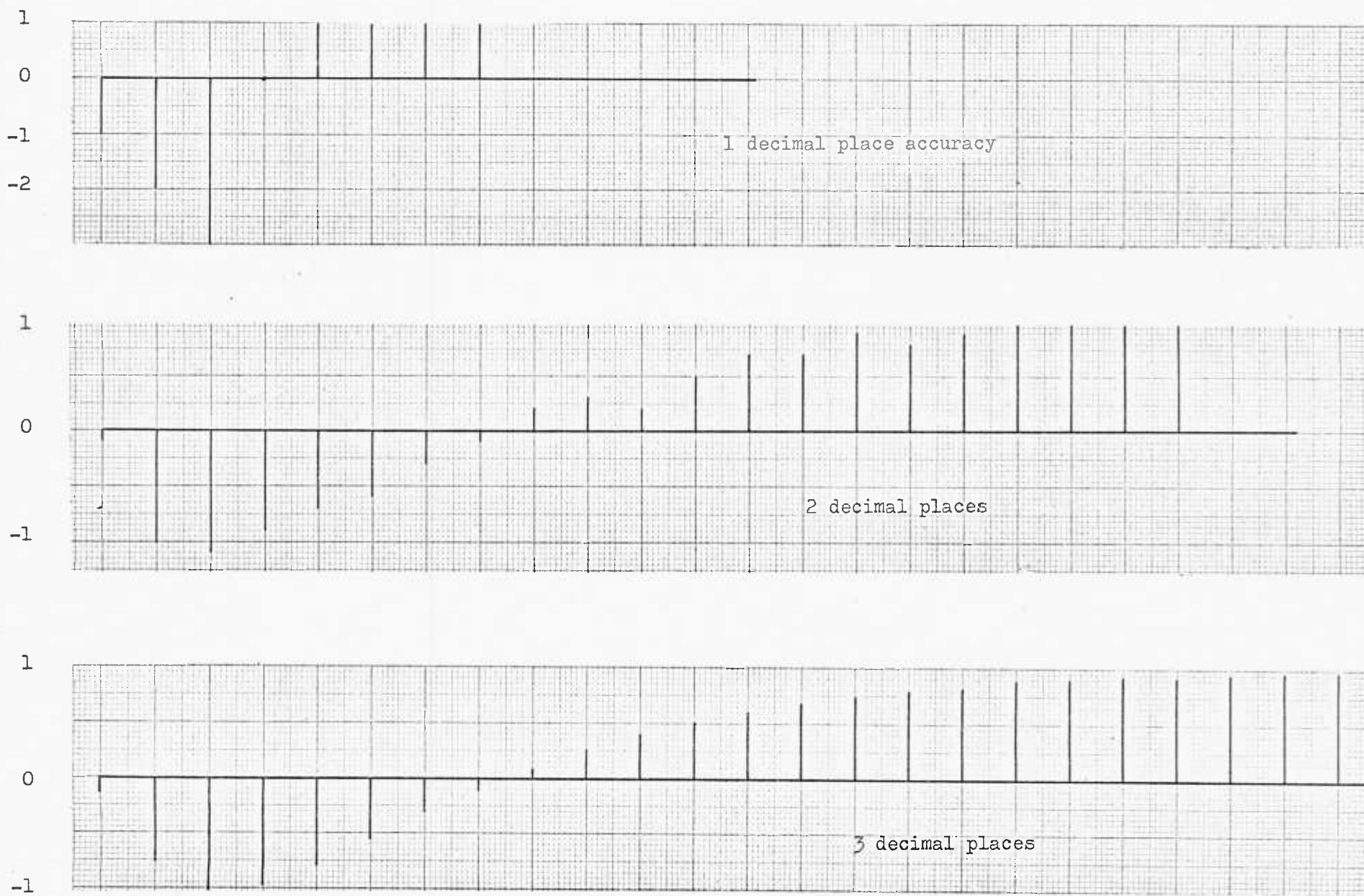


Fig. 4.28 Transient response (different internal variable accuracies).



CHAPTER FIVE.

5.0 Carrier telephone system specifications

In our initial consideration of the various digital S.S.B./F.D.M. telephony systems, the performance characteristics to be satisfied were obtained from the C.C.I.T.T. recommendations ⁽¹⁷⁾ regarding international carrier telephone systems. Within this framework, we also aim to meet the more detailed requirements of the typical British channel-filter specification ⁽¹⁰¹⁾. However, in this thesis, we attach no intrinsic significance to these specifications as they are constantly being updated. For this reason it is the method of designing the channel filter so that it satisfies a particular performance characteristic which is of prime significance.

The C.C.I.T.T. recommendations G232 ⁽¹⁷⁾, regarding the formation of the basic 12-channel group (types A or B), require that a frequency band 300 Hz. to 3400 Hz. be transmitted. The C.C.I.T.T. limits for the attenuation distortion of this passband are as previously shown in fig. 2.1. For the stopband response, an unintelligible crosstalk suppression ratio of 60 dBs, or greater is recommended.

(Unintelligible crosstalk is produced by an unwanted sideband in an adjacent channel and is due to the imperfect suppression of the channel filter).

The desired performance characteristics for the British system⁽¹⁰¹⁾ are as shown in fig. 5.1 - the channel-filter passband requirements are, again, as shown in fig. 2.1.

We now consider the design of a channel filter, satisfying both the British specifications and those recommended by the C.C.I.T.T., in which the digital polyphase (phase-splitting) network is utilized.

5.1 The channel-filter design procedure

In the design of the channel filter (using a cascade of polyphase single-sections), initially, we merely require that the stopband specifications should be satisfied. Our design procedure is therefore arranged accordingly. We employ various design methods in the search for a low complexity network possessing the desired performance characteristics and the use of the non-linear optimization routine, for obtaining approximations to the required response, is again heavily relied upon.

In addition to this 'continuous' optimization; solutions to the approximation problem are obtained using the method of optimizing in the discrete space of coefficients⁽⁹⁸⁻¹⁰⁰⁾.

We also propose an optimization procedure⁽¹⁰²⁾ based on the univariate search method. A network utilizing trivial coefficients is chosen such that the required bandwidth is satisfactory although its positioning may not be. Coefficients of firstly one section, then two sections etc., are individually optimized in the continuous or discrete space of coefficients until the phase-difference response is shifted to the desired position.

We then turn our attention to the channel-filter passband specifications (it has been shown that the polyphase network amplitude response may not be flat and, furthermore, the deviation away from the flat response is increased with every section added). It is apparent therefore that some form of amplitude correction will be required and, for this reason, the design of a suitable amplitude correcting (equalizing) filter is undertaken.

Finally, with the addition of a low-pass guard filter (required to eliminate the redundant out-of-band frequency spectrum: -8 to -4 kHz. and 4 to 8 kHz), the baseband modulation scheme (imaginary channel filter) is completed. The guardband filter design proposed in reference 43 eminently satisfies this requirement and is therefore included in our overall system design.

(i) The use of coefficients from the continuous space

In designing our network so that it possesses the specified channel-filter stopband performance characteristics, the relationship between the phase tolerance, bandwidth and complexity of the polyphase network must be considered. Analogue phase-splitting network design tables do exist⁽³¹⁾ which, for a particular network order, give the phase-difference response for various bandwidth ratios. However, in our particular design problem, it is not immediately clear how the order of a polyphase network could be defined. Furthermore, the simple relationship of phase tolerance versus bandwidth ratio for a particular network order is of limited use because the permitted phase deviation of our design specification is not constant throughout the constrained bandwidth. A useful indication as to the network order required is obtained, however, if:- (a) we make use of the empirically obtained relationship between the polyphase network order, the phase tolerance and the bandwidth ratio. It has been found that, by equating the polyphase network section number with the analogue network order, the design table data relationships between phase tolerance and bandwidth ratio still hold as long as the polyphase network bandwidth ratio is increased by a factor of four. It is also necessary to transform the discrete frequency to its analogue equivalent before using the design tables.

(b) we design the phase-difference response so that it exceeds the required accuracy over the major part of the bandwidth - hence the modified performance specifications shown in fig. 5.2 (over the bandwidth 300 to 3400 Hz., the phase deviation from 90° must not exceed $\pm 0.1^\circ$).

In our resulting network design, it is necessary to verify that the phase-difference response requirements, for the remainder of the constrained region, are also satisfied.

Choice of the network order using this procedure may be appropriate because, by initially exceeding the phase-difference response specifications, we can consider reducing the eventual overall design complexity if some relaxation of the resulting phase response is permitted. Before we can choose the network order, using the analogue design tables, the non-linear warping of the frequency axis⁽⁵⁾ (occurring with the mapping from the z to the p plane) must be taken into account - that is, the discrete frequency variable is transformed using:-

$$\omega_a = 2/T \cdot \tan(\omega_d \cdot T/2)$$

5.1

where:-

ω_d = the discrete frequency variable

and:-

$\omega_{d1} = 300 \text{ Hz.}$

$\omega_{d2} = 3400 \text{ Hz.}$

Using equation 5.1, we obtain the modified analogue bandwidth ratio to be $(13 \times 4 = 52)$ for which the design tables indicate that a network order of $n \approx 8$ is required.

Alternatively, the required network order (number of single sections) can be obtained from the plot of fig. 5.3, in which we show the phase-difference response bandwidth $\phi_{1,2}$ (for the condition $\phi_1 < \phi = 90^\circ \pm 0.1^\circ < \phi_2$) plotted against the network order n - all sections containing trivial coefficients ($\beta_i = 0.5$, all i). From this plot and previous results, we deduce that the network order has the most significant effect on the phase-difference response bandwidth whereas the coefficient values effectively 'position' this response.

Later, it will be seen that the phase-difference response (obtained using an all-trivial section network) may be shifted without drastically effecting its resulting bandwidth. This has been found to be true so long as the 'shift' of response is not too extreme. This is explained by the fact that shifting the phase-difference response effectively increases its bandwidth ratio (assuming a move towards zero frequency) and, therefore, if a significant shift is introduced, an increase in the network order would be required in order to preserve the same phase tolerance.

If we assume that the starting point of our design is an all-trivial section network, fig. 5.3 used in conjunction with fig. 3.16 (the plot of the lower and upper frequency extremals versus the trivial-section network order) provides a useful indication of (a) the approximate phase-difference response for various network orders and (b) the severity of the bandwidth shift. All this has particular relevance since, in the optimization work, a 'guess' is made that each multiplier coefficient has the value 0.5 at the beginning of an optimization run. From fig. 5.3, we see that an 8-section network possesses the required phase-difference response whereas the effect of shifting the response can be observed in a design example.

To test our choice of network order, both 7 and 8-section networks are designed using the non-linear optimization method outlined in chapter three. Equal weighting is given to each frequency point from 300 to 3400 Hz.

The resulting channel-filter stopband attenuation curves (derived from the phase-difference responses for the 7- and 8-section networks) are as shown in fig. 5.4 and fig. 5.5. We see that an 8-section network is required in order to satisfy the fairly tight specifications of fig. 5.2. The effect of reducing the network complexity (via a reduction in the coefficient accuracy) can be seen in figs. 5.6 and 5.7 (the ideal coefficient values have been replaced by their 4- and 3-bit values respectively). It is clear from fig. 5.7 that the 3-bit coefficient network

fails to meet the specifications at both the upper and lower frequency extremal points whereas the 4-bit coefficient network only fails to satisfy at the low frequency extremal (fig. 5.6). However, as the 4-bit coefficient network has exceeded the high frequency extremal, with bandwidth remaining, it is apparent that a better choice of 4-bit coefficients could possibly produce the required fit (i.e. we may obtain the required response after optimizing the coefficients in the discrete space of coefficients⁽⁹⁸⁻¹⁰⁰⁾). This 'discrete' optimization is now considered in more detail.

(ii) Optimization in the discrete space of coefficients

The number of decimal values that can be represented by an N_b -bit coefficient is given by $(2^{N_b} - 1)$. For an M -section polyphase network, this gives us a total number of $(2^{N_b} - 1)^M$ different coefficient sets. Our problem therefore consists of choosing the coefficient set that yields the required phase-difference response. It is obvious that the responses obtained for these coefficient sets cannot be individually examined because of the extremely high amount of computation time this would require. We are therefore in need of some systematic search procedure that leads us to the optimal coefficient set.

Various discrete optimization procedures have been proposed (92,98) in which some knowledge of the suitable coefficient wordlength is required. A statistical estimation of this wordlength results from a comparison of the standard deviation of some response curve (e.g. attenuation $\Rightarrow \sigma^2_{\Delta|A|}$) with the margin of error $\epsilon(\omega T)$ between the ideal and the specified response. The deviations away from the ideal coefficient values are handled as random variables distributed between the values $-q/2$ and $q/2$ ($q = 2^{-N_b}$: the quantum step size).

For an arbitrary set of coefficients, we therefore obtain⁽⁸⁾ :-

$$\begin{aligned}\sigma^2_{\Delta\beta_k} &= \int_{-\infty}^{\infty} \Delta\beta_k^2 \cdot p_{\Delta\beta_k}(\Delta\beta_k) \cdot d\Delta\beta_k \\ &= q^2/12 \quad \equiv \quad (2^{-N_b})^2 / 12\end{aligned}$$

5.2

As the errors introduced $(\Delta\beta_k)$ are independent, this leads to⁽⁹⁹⁾ :-

$$\sigma^2_{\Delta|A|} = \sum_k (S_{\beta_k}^{|A|})^2 \cdot \sigma^2_{\Delta\beta_k}$$

and therefore:-

$$\sigma_{\Delta|A|} = \sum_k S_{\beta_k}^{|A|} \cdot (2^{-N_b}) / \sqrt{12}$$

5.3

The estimated value of N_b is then obtained by satisfying⁽⁹⁹⁾:-

$$(2^{-N_b}) / \sqrt{12} \cdot \sum_k S_{\beta_k}^{|A|} \leq \mathcal{E}(\omega T)$$

or

$$N_b \geq \log_2 \left(\sum_k S_{\beta_k}^{|A|} \right) + \log_2 \left(\mathcal{E}(\omega T) \right)$$

5.4

In the polyphase network design problem, we need only supply $\mathcal{E}(\omega T)$ and replace $S_{\beta_k}^{|A|}$ by $S_{\beta_k}^{\emptyset}$ to obtain the estimated value of N_b .

The value of N_b has not been calculated as we know that the accuracy of the coefficient values is of secondary importance and therefore the coefficient wordlengths are to be kept short. (This might not be so in a typical filter design problem where wordlengths of up to 18-bits could be encountered). We therefore need

only observe the phase-difference responses for 3, 4 and 5-bit representations of the ideal coefficient set, for example, before deciding on the suitable coefficient wordlengths and whether or not we can gain by employing an optimization in the discrete space of coefficients.

It was seen in fig. 5.6 that the response obtained using the 4-bit coefficients just failed to meet the specifications and therefore, using the optimization procedure proposed in⁽¹⁰⁰⁾, we search for a response with a more satisfactory fit. This optimization procedure consists of rounding off a particular coefficient value (β_k , $k = 1, K$), followed by an optimization in the continuous space of the remaining $K-1$ coefficients. Next, coefficient β_{K-1} is rounded and, with β_K and β_{K-1} fixed, the remaining $K-2$ coefficients are optimized in the continuous space. This process is continued until all coefficients have been rounded - the operation is considered successful when an improvement is gained in the fit of the response curve. The suggestion was made in reference⁽¹⁰⁰⁾ that the coefficients should be operated upon in order of decreasing selectivity.

The improvement gained using this optimization procedure can be seen by comparing the attenuation curves of fig. 5.6 and 5.8 (the response for the rounded 4-bit coefficient set compared with the improved response after optimizing in the discrete space of coefficients).

(iii) An optimization method based on the univariate search procedure

In our final approach to designing a low complexity network possessing the required performance characteristics we use an optimization procedure based on the univariate search method⁽¹⁰⁴⁾. Our resulting design is a two-stage network in which the separate stages consist of trivial and non-trivial sections respectively. This design is simply arrived at as follows:-

- (a) a network utilizing only trivial sections is chosen so that the required phase-difference response bandwidth is satisfactory although its positioning may not be.
- (b) coefficients of firstly one section, then two sections are individually optimized in the continuous or discrete space of coefficients until the phase-difference response is shifted to the desired position.

As only one stage of our network employs non-trivial coefficients and since we already know that these coefficients will be represented by short wordlengths, it is probable that the use of an optimization in the discrete space of coefficients will prove advantageous. With regard to the number of sections required and the

necessary wordlengths of the non-trivial coefficients, the plots of figs. 5.9 to 5.11 are informative. In each plot, we show the sum of the squares of the phase-difference response errors (for the constrained bandwidth 300 to 3400 Hz.) plotted against the number of sections containing optimized coefficient values - for different order networks. These plots result from discrete optimizations using the quantization step sizes $q = 0.125, 0.0625$ and 0.03125 respectively. From figs. 5.9 to 5.11, we see that the problem we had in producing the required phase-difference response using the 4-bit coefficient 8-section network probably will be eliminated if we add one further section (i.e. for networks containing three or more non-trivial sections we obtain a reduction by a factor of ten in the sum of the squares of the response errors on increasing the network order from eight to nine. Furthermore, it is apparent that there is little to be gained in using more than three non-trivial sections and also a coefficient accuracy of greater than three bits. Then as we see from fig 5.12 using the proposed two-stage network (comprised of 6 trivial sections and 3 non-trivial 3-bit coefficient sections), the required performance characteristics are obtained with what is evidently a low complexity network.

It now remains for us to satisfy the channel-filter passband specifications (fig. 2.1) which entails modifying the real (symmetrical) amplitude response of the 9- section polyphase network.

5.3 The polyphase network amplitude response correction

In the design procedure so far, the channel-filter passband specifications have been ignored but it has been mentioned that the polyphase network amplitude response may not in general meet these specifications without some form of correction. It is therefore clear that the desired amplitude correction can only be obtained by pre-filtering the input signal to the polyphase network or post-filtering at the output. We choose to pre-filter the input signal as this is achieved using only one filter compared with the two separate filters that would be required to post-filter the output signals. In designing a suitable amplitude correcting (equalizing) filter, the choice must be made as to whether a recursive or non-recursive structure is to be used. Firstly, however, the amplitude response of the 9-section polyphase network is examined to obtain some indication as to the severity of the required correction.

In fig. 5.13, we show the 9-section polyphase network amplitude response over the frequency range 0 to 4 kHz. (note that the passband specifications (fig. 2.1) are only defined from 300 to 3400 Hz). An equalizing filter could be designed so that it produces the exact inverse of this response i.e:-

$$\left| E_q(e^{j\omega T}) \right| = 1 / \left| H_a(e^{j\omega T}) \right| = 1 / \left| H_b(e^{j\omega T}) \right|$$

although this would prove disadvantageous in that:-

- (i) the filter required would be so complicated (ninth order) that the advantage of using the low-complexity polyphase network would be lost.
- (ii) we require that the frequency spectrum extending from 4 to 8 kHz. receives no gain - that is, the addition of the equalizing filter must not significantly reduce the existing out-of-band attenuation level. However, as it can be seen from fig. 5.13, the amplitude response beyond 3400 Hz is asymptotically reducing and, therefore, using an equalizing filter (which produces the inverse of this response) would in fact reduce the out-of-band attenuation level. It is therefore preferable to design the filter so that it produces the inverse of the amplitude response over the frequency range 300 to 3400 Hz. only. We can then introduce the additional condition in our design procedure that the existing out-of-band attenuation level must not be significantly reduced by the addition of the equalizing filter.

Prior to designing the required amplitude correcting filter, a brief investigation is made to see whether or not a simple alteration of the proposed polyphase structure leads to an improvement in the amplitude response of the network. It is fairly evident that the sub-transmittances $H_a(z)$ and $H_b(z)$ must

not be changed with respect to each other if the phase-difference response is to remain the same. We therefore begin this operation by removing all recursive paths, this being equivalent to multiplying both sub-transmittances by the factor:

$$\prod_{i=1}^9 (1 - \beta_i \cdot z^{-1})$$

As seen from fig. 5.13 (curve b), no advantage is gained since the alteration in the amplitude response is too severe. This procedure is repeated, therefore, only for (a) the removal of the recursive paths for various non-trivial sections (fig. 5.14) and for (b) the removal of the recursive paths for different numbers of trivial sections (fig. 5.15). It is apparent from these figures that since the amplitude response curves are 'smooth' it is the ratio of the maximum to the minimum values of the amplitude response that determines the severity of the required correction. For the various multiplying correcting factors, the amplitude maximum to minimum ratios obtained are as given in table 5.1.

Table 5.1

<u>multiplying factor</u>	<u>amplitude max. to min. ratio</u>
1	4.3
$(1 - 0.875 \cdot z^{-1})$	3.8
$(1 - 0.875 \cdot z^{-1})^2$	17.7
$(1 - 0.875 \cdot z^{-1})^2(1 - 0.75 \cdot z^{-1})$	71.5
$(1 - 0.5 \cdot z^{-1})$	2.4
$(1 - 0.5 \cdot z^{-1})^2$	1.9
$(1 - 0.5 \cdot z^{-1})^3$	2.7

We see from table 5.1 that the amplitude response is modified, to good advantage, if the sub-transmittances (H_a, H_b) are multiplied by $(1-0.5z^{-1})^2$. This multiplication can be thought of as the first stage in the design of the amplitude equalizing filter. However, by using the multiplying factor to cancel out the recursive paths of two of the trivial sections, we eliminate the need for any additional hardware and also reduce the total number of adders required by two per modified section. The polyphase network transfer response is then represented by:-

$$\begin{bmatrix} H_a(z^{-1}) \\ H_b(z^{-1}) \end{bmatrix} = \prod_{i=1}^7 \begin{bmatrix} \rho_i(z^{-1}) & -\lambda_i(z^{-1}) \\ \lambda_i(z^{-1}) & \rho_i(z^{-1}) \end{bmatrix} \prod_{i=8}^9 \begin{bmatrix} 1-\beta_i & -\beta_i(1-z^{-1}) \\ \beta_i(1-z^{-1}) & 1-\beta_i \end{bmatrix}$$

5.5

with the modified structure (type 1Bm) as shown in fig. 5.16.

In practice the inclusion of this simple modification has more than halved the amplitude maximum to minimum ratio thereby reducing the complexity of the required equalizing filter. We now find that, for producing the ideal equalizing filter amplitude response (this is obtained by taking the inverse of the curve of fig. 5.15(b) over the frequency range 300 to 3400 Hz and, outside these limits, constraining the amplitude response so that its maximum value does not exceed the maximum of the inverted response curve), all that is required is a single 2nd-order recursive section (fig. 5.17). The unknown coefficient values are simply computed

using a non-linear optimisation routine. For the ideal coefficient values, we obtain the response shown in fig. 5.18 (the corrected polyphase network amplitude response). By simply rounding the coefficients to their nearest 6-bit values, we see from fig. 5.19 that the response fails to meet the passband specifications. If, however, we optimize these coefficients in the discrete space of coefficients⁽¹⁰⁰⁾, it can be seen from fig. 5.20 that the amplitude correcting filter using the optimized set of 6-bit coefficients produces the necessary correction - that is, the polyphase network amplitude response lies within the C.C.I.T.T. passband specifications.

5.4 The proposed baseband channel-filter

As the baseband channel-filter design procedure involved satisfying the stopband and passband specifications separately (using the amplitude correcting filter in cascade with the polyphase (phase-splitting) network), the complete system is to be evaluated with regard to its performance characteristics and overall complexity. Firstly, however, the optimal polyphase network section ordering is chosen by examining the roundoff noise produced by the different realizations. At the same time, we obtain (from the output noise spectrum of the chosen realization) the necessary internal variable wordlength for this noise contribution to be made insignificant.

In the preliminary roundoff noise analysis, it was seen that for realizing a non-trivial section, the type 1Bi structure proved optimal, whereas for realizing the trivial version of this section, the choice between using $\rho_i = (1-0.5)/(1-0.5 z^{-1})$ (giving the types 1Bi structure) or $\rho_i = (0.5)/(1-0.5 z^{-1})$ (defined as the type 1Bt structure) could only be decided by comparing noise performances of particular network configurations. We therefore compute the total average power for all possible section orderings (including the two modified type 1Bm sections) of the 9-section network. From these results (listed following), the optimal section ordering is then selected.

<u>Section Ordering</u>	<u>Total average noise power (dBa)</u>
1Bi , 1Bm	12.4
1Bm , 1Bi	13.3
1Bi , 1Bt , 1Bm	11.5
1Bi , 1Bm , 1Bt	13.1
1Bt , 1Bi , 1Bm	12.5
1Bt , 1Bm , 1Bi	13.0
1Bm , 1Bi , 1Bt	13.4
1Bm , 1Bt , 1Bi	16.2

The network consisting of 3 type 1Bi sections, 4 type 1Bt sections and 2 type 1Bm sections, in that order, is therefore selected as desirable. For realizing the amplitude equalizing filter, we

compare the noise performances of the two most commonly used 2nd-order structures; these being the pole-zero structure of fig. 5.17 and its transpose. The total average power at the system output, for each structure, is found to be:-

$$\begin{aligned}\text{Pole-zero structure} &= 9.8 \text{ dBs} \\ \text{Zero-pole structure} &= 11.6 \text{ dBs}\end{aligned}$$

We therefore propose the system outlined in fig. 5.21 as optimal with regard to the roundoff noise performance. By examining at the system output the roundoff noise produced by each stage, it is seen that the internal variable wordlength must be two bits longer than the wordlength leaving the A/D converter for the noise generated by the multiplier rounding operations to become negligible. (Note that the roundoff noise spectrum is reduced by 6dBs for every bit added to the internal variable wordlength⁽³⁹⁾).

We now turn our attention to the non-real channel-filter amplitude response. It was shown previously (eqn. 3.42) that this imaginary response is dependent on both the phase-difference and amplitude response of the polyphase (phase-splitting) network - that is:-

$$\left| H(e^{j\omega T}) \right| = \left| A(e^{j\omega T}) \right| \cdot \frac{1}{2} (1 + \sin \varphi(\omega T))^{1/2}$$

where:-

$$\left| A(e^{j\omega T}) \right| = \left| H_a(e^{j\omega T}) \right| = \left| H_b(e^{j\omega T}) \right|$$

$$\varphi(\omega T) = \varphi_a(\omega T) - \varphi_b(\omega T)$$

Working in power ratios (i.e. taking $20 \log_{10}(\cdot)$ of both sides to give dBs), we see that the overall channel-filter response is equal to the effective amplitude response obtained from the phase-difference plus the amplitude response of the polyphase network. Over the frequency region 300 to 3400 Hz., we have an effective stopband of 60dBs or greater and therefore a passband response of 4×10^{-6} dBs or less. This passband response is added to the polyphase network amplitude response (which has been corrected to give $20 \log_{10} |A| \approx 0$ dBs). Outside the passband region, the out-of-band gain constraint imposed on the equalizing filter design ensures that the attenuation level, due to the phase-difference response, is not reduced. For example, at zero frequency, the phase-difference response is zero and therefore:-

$$\left| H(e^{j\omega T}) \right| = \left| A(e^{j\omega T}) \right| \cdot \sqrt{0.5}$$

In practice, we obtain an increase in the attenuation level from 3 to 8.5 dBs on adding the amplitude correcting filter.

The full baseband amplitude response ($-\omega_s/2$ to $\omega_s/2$), for the proposed system, is as shown in fig. 5.22. It is clear that with the addition of the out-of-band guard filter given in reference 43, the specifications of both fig. 5.1 and 5.2 are completely satisfied (see fig. 5.23).

In order to obtain a measure of the multiplier complexity of our proposed system and compare this with the allpass network design⁽⁴³⁾, we again make use of equations 2.2 and 2.3. For the amplitude correcting filter using five 6-bit coefficients and the three non-trivial sections using six 3-bit coefficients, we obtain:-

$$H_m = \left[N_m \cdot N_p \cdot B_q \cdot f_s / f_b \right]_{\text{Int.}} \times A$$

$$\Rightarrow \left[5 \times (1 \times 12) \times 17 \times f_s / f_b \right]_I \times 6$$

$$+ \left[3 \times (2 \times 12) \times 17 \times f_s / f_b \right]_I \times 3$$

and for f_s / f_b chosen to be $1/10^3$:-

$$H_m = \left[1.224 \right]_I \times 3 + \left[1.02 \right]_I \times 6$$

$\Rightarrow$ 12 multiplier bits.

We therefore require 12 multiplier bits compared with the 22 bits of the allpass network of ref. 43 or, assuming commercially available multiplier chips in groups of four bits are used^(52,53), the figures become 16 and 22 respectively.

The validity of this comparison can be questioned for two reasons - these are:-

- (i) the proposed system requires a larger amount of basic hardware (e.g. adders and one-bit multiplications) than is required in realizing the allpass network. However, it is felt that, in the proposed system, the multiplier

hardware saving does outweigh the additional basic hardware requirements although it is difficult to measure, objectively, the relative costs.

- (ii) the design specification of fig. 5.2 is not identical to that in ref. (43) (in the stopband, from 300 to 3400 Hz, the specified attenuation curve is 'tighter' although it is less 'exacting' from 100 to 300 Hz). Modifying our design to meet the specifications of (43) allows a more real comparison of one network with another. It is found that, on increasing the constrained phase-difference response bandwidth, we obtain $\omega_{a2}/\omega_{a1} \approx (4000/100) \times 4 = 160$, for which a value of $n = 10$ is indicated. Using the univariate search procedure, we find that a network of four non-trivial sections (using 4-bit coefficients) plus six trivial sections produces the required performance characteristics (fig. 5.24). The hardware requirements (Hm) are therefore unchanged, except for the addition of one non-trivial section.

Finally, to simulate the baseband channel-filter performance, we apply a sampled 1kHz. sinusoidal time sequence and process this with the guardband filter, the amplitude correcting filter and the polyphase network. Either the upper or lower sideband is then derived from the output sequences after the signals (the complex pair) have been modulated by a cosinusoidal and sinusoidal carrier respectively. In fig. 5.25, we show both upper and lower sidebands

produced using a 4 kHz sampled carrier. (Note: the single-sideband is only a sinusoidal frequency of either $\omega_c - \omega_m$ (lower) or $\omega_c + \omega_m$ (upper) whereas, for a double sideband, we would have an envelope waveform).

5.5 Summary

The aim of this chapter has been to design a low complexity phase-splitting network, using the polyphase structure, so that a particular channel-filter design specification is satisfied. It has been emphasised that the design specifications have no intrinsic significance and it is the method of designing the channel filter (and the resulting network complexity this produces) that is important. For this reason, fairly general design specifications (meeting British and C.C.I.T.T. recommendations) have been used.

The channel-filter design procedure involves satisfying the stopband and passband specifications separately. Initially, a network consisting of a cascade of polyphase single-sections is designed to produce the required stopband attenuation. Several design methods are used in the search for a suitable low-complexity network. In all these methods, computer optimization techniques are widely used. An investigation of the resulting network's amplitude response shows that, to satisfy the passband specifications, some form of amplitude correction is required. The design of a suitable amplitude correcting filter is therefore undertaken.

The proposed phase-splitting network design, consisting of a 2nd-order recursive amplitude correcting filter in cascade with a 9-section polyphase network, is shown to produce the required performance characteristics. A roundoff noise analysis is performed, on this network, to ascertain the optimal section ordering and also the necessary internal variable wordlength.

Finally, by obtaining a measure of the multiplier complexity, this proposed phase-splitting network is shown to be competitive with other, already proposed, networks after comparisons regarding hardware complexity (and therefore costs) are made.

Fig. 5.1 British channel-filter specifications.

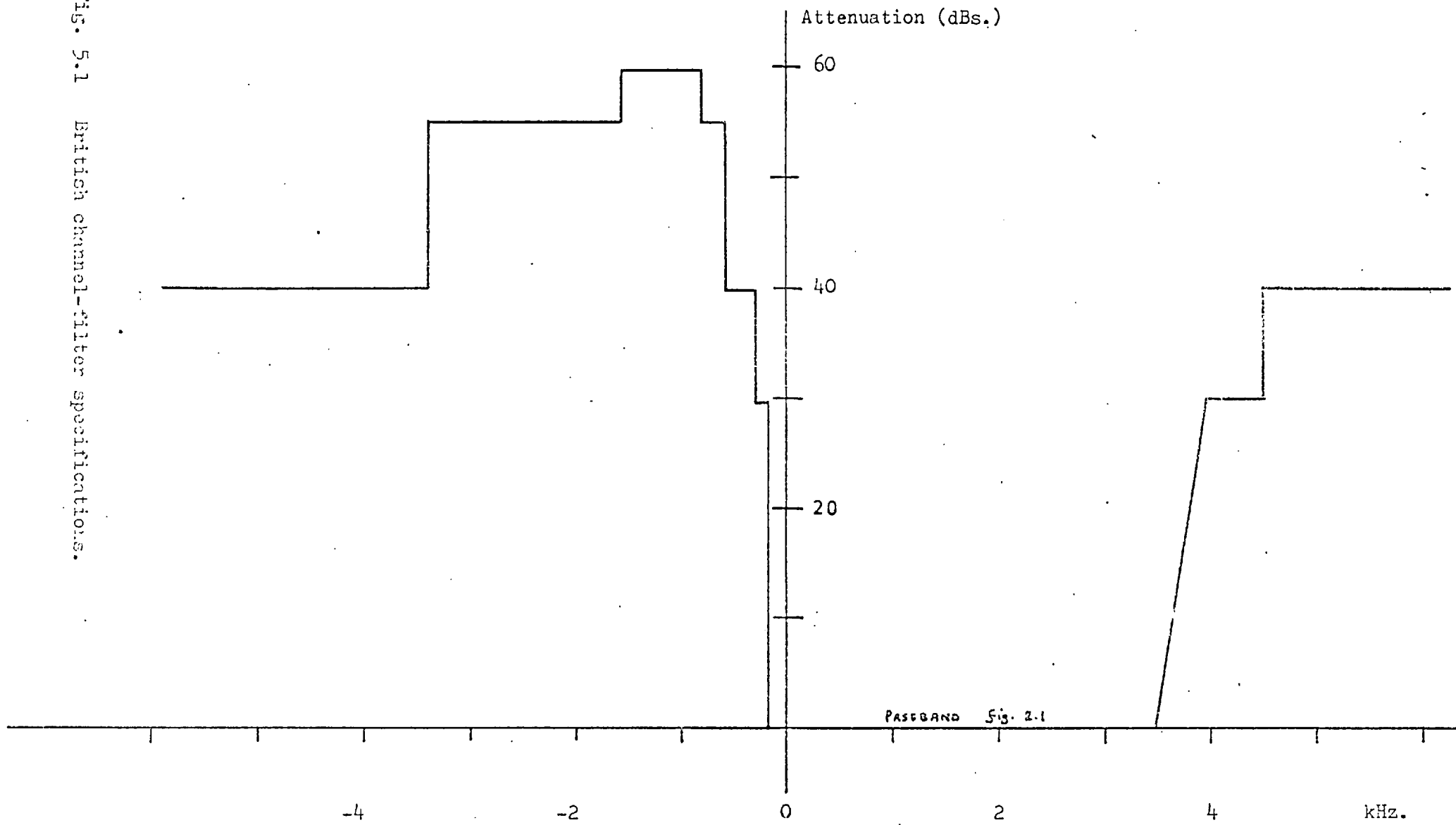
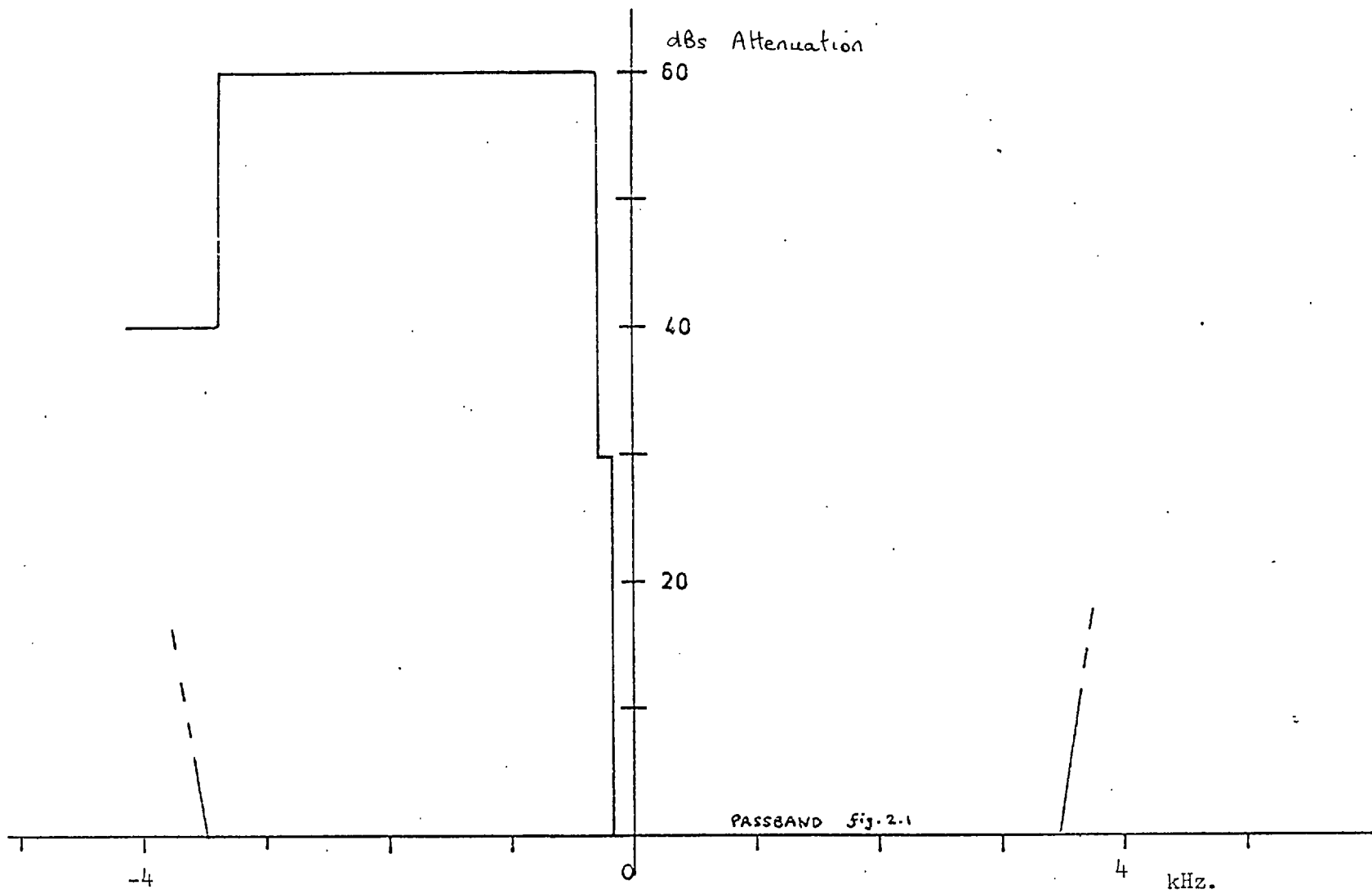


Fig. 5.2 The modified design specifications.



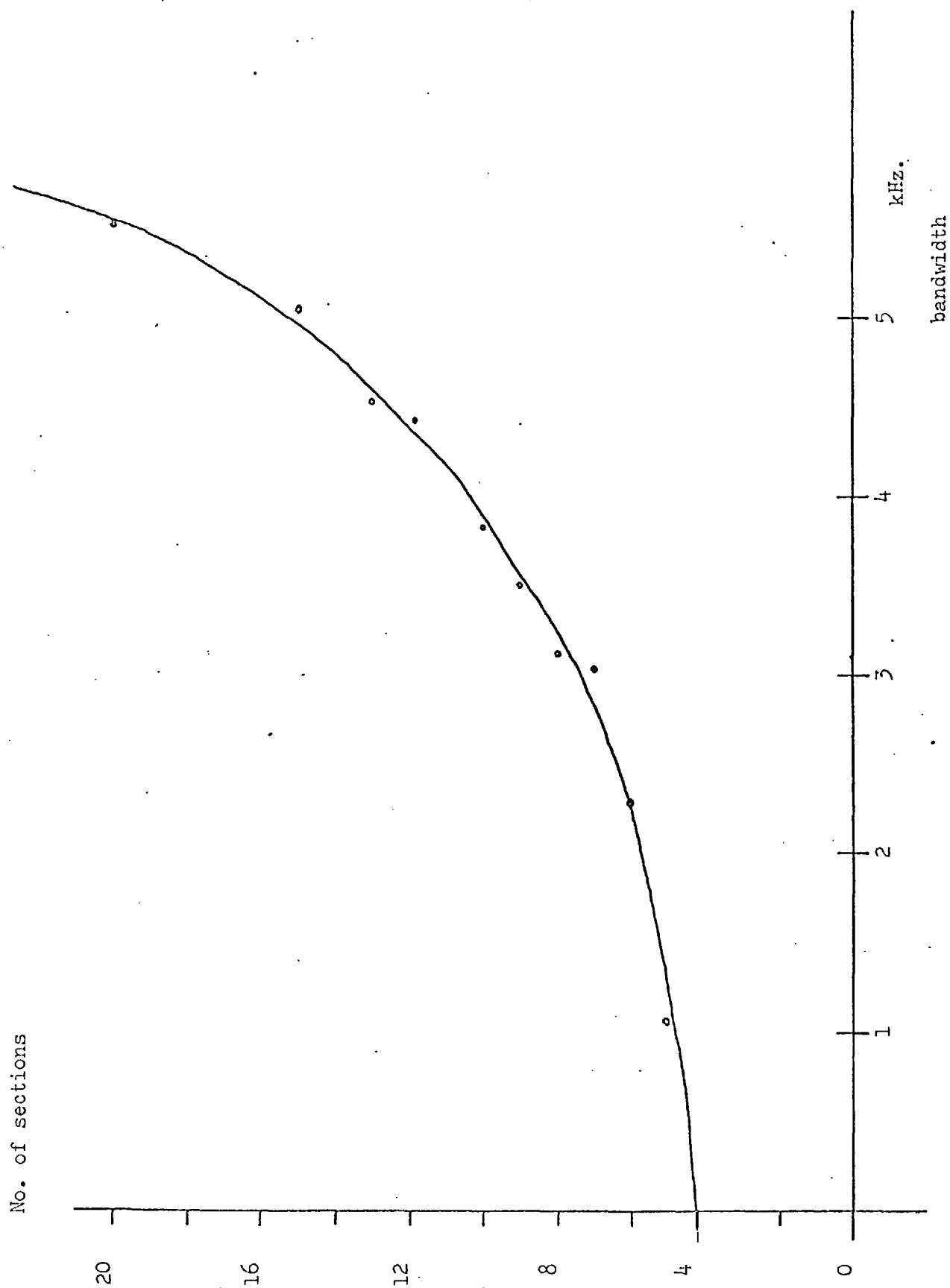


Fig. 5.3 Trivial section network order versus bandwidth.

($\pm 0.1^\circ$ phase-dev. constraint)

Fig. 5.4 Channel-filter attenuation : 7-section network.

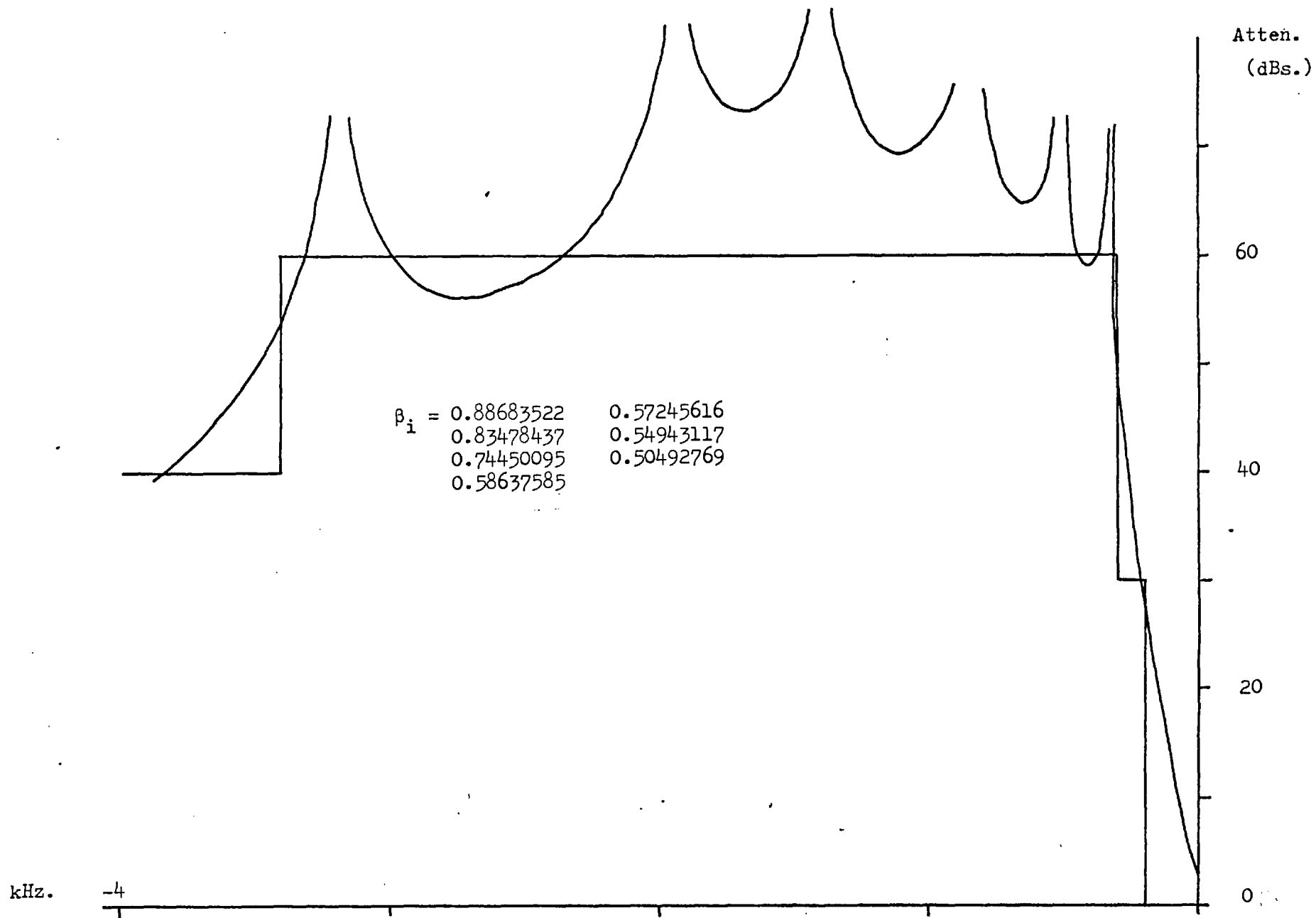


Fig. 5.5 Channel-filter attenuation : 8-section network.

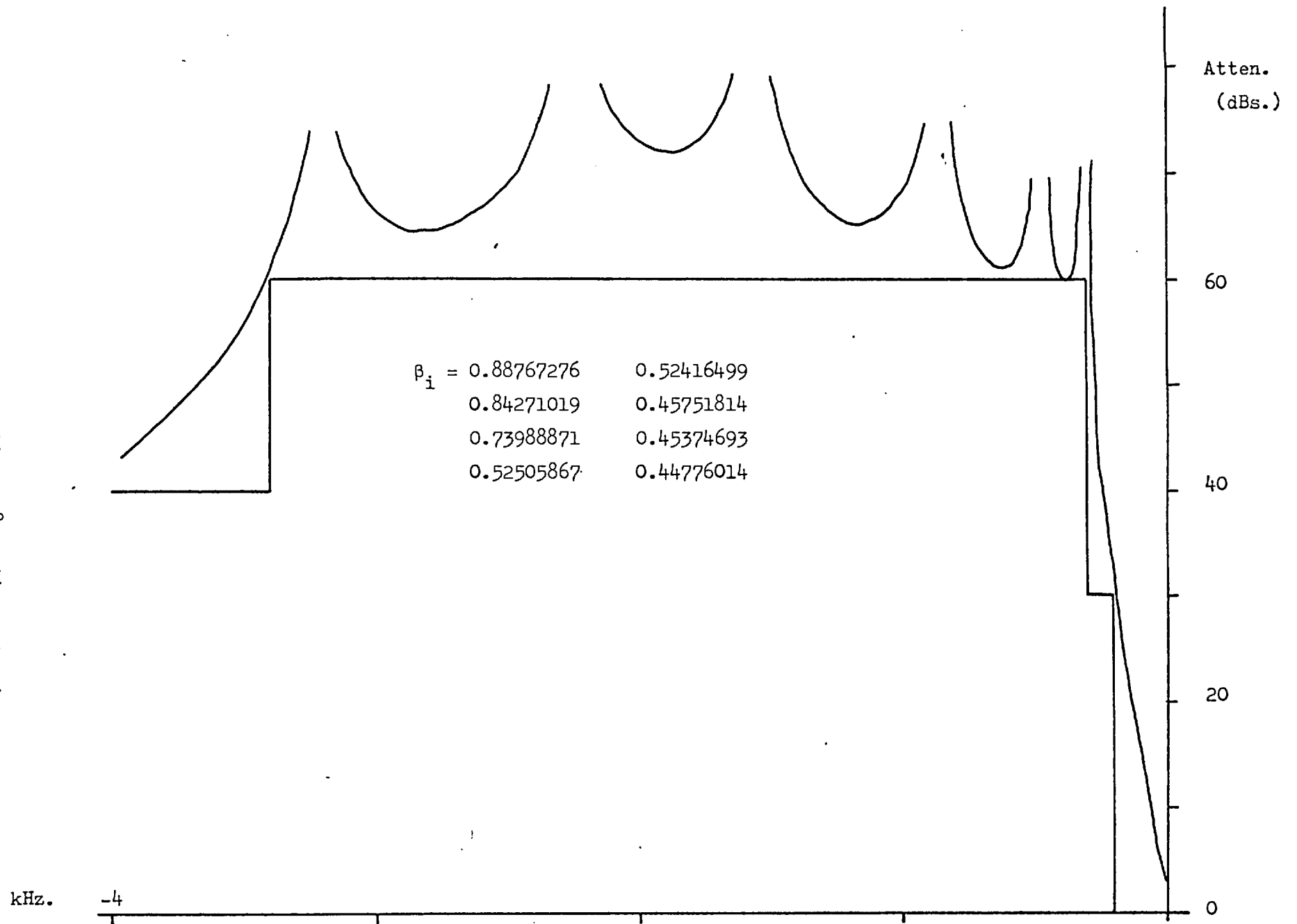


Fig. 5.6 8-section network response : 4-bit coefficients.

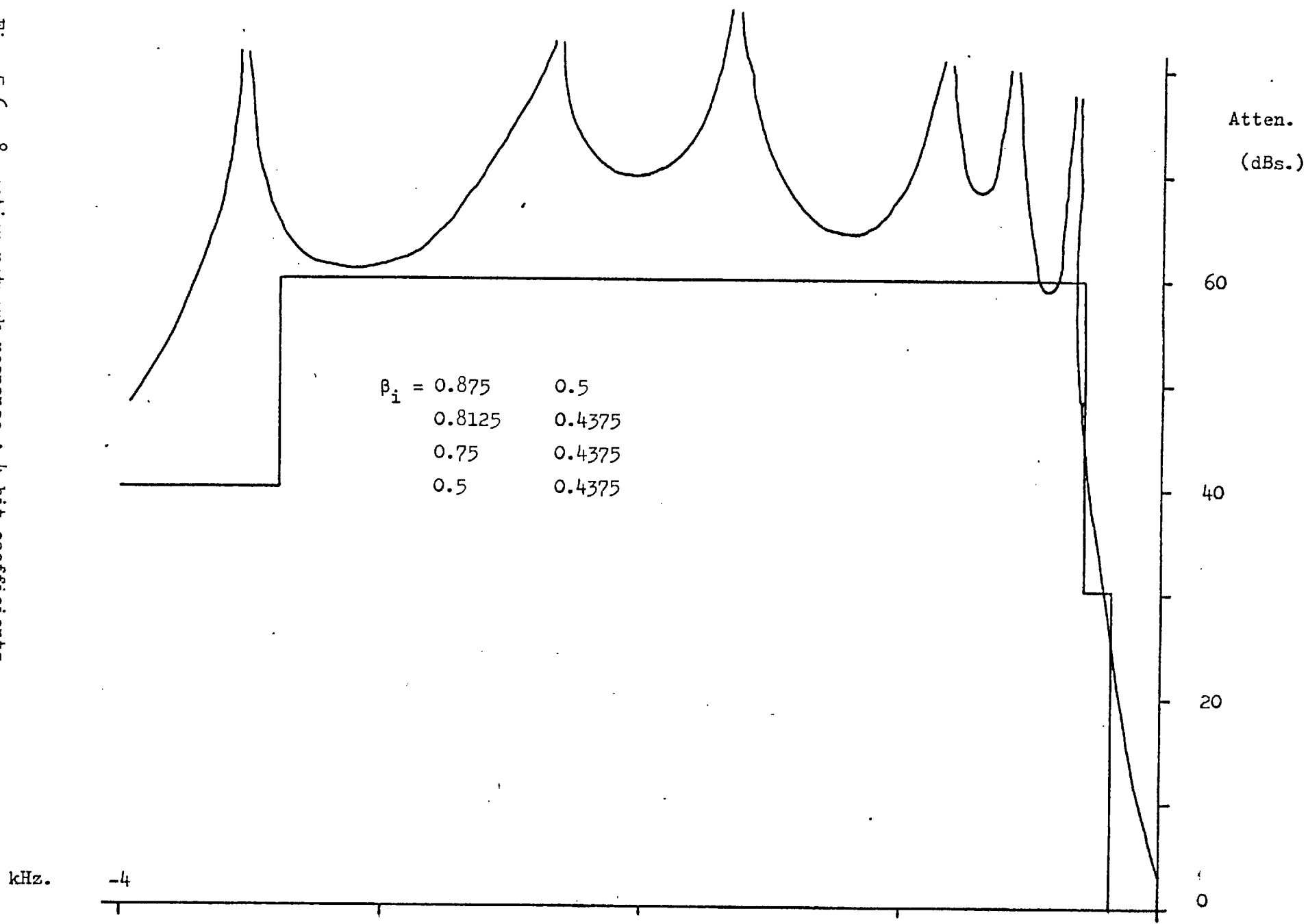


Fig. 5.7 8-section network response : 3-bit coefficients.

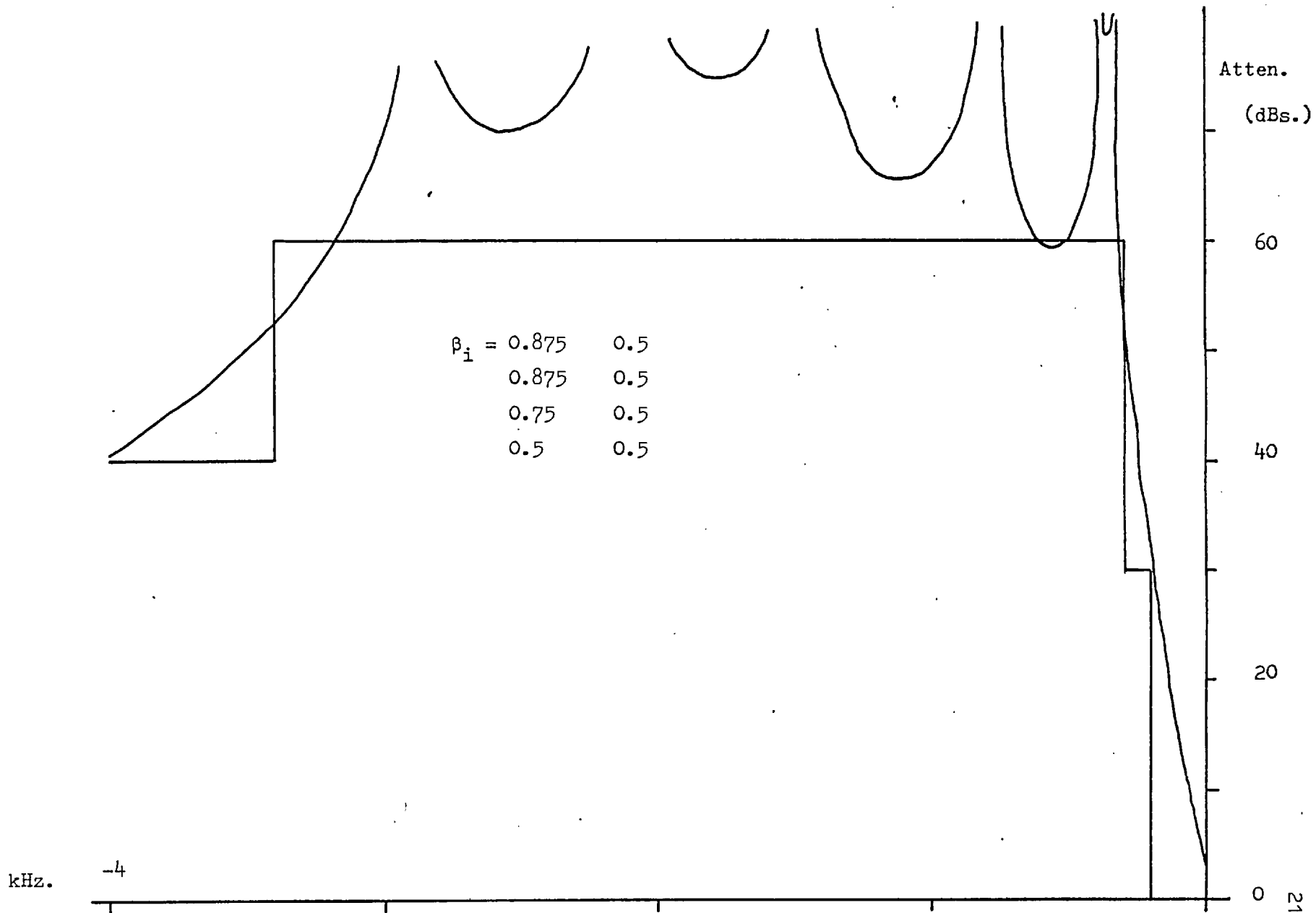
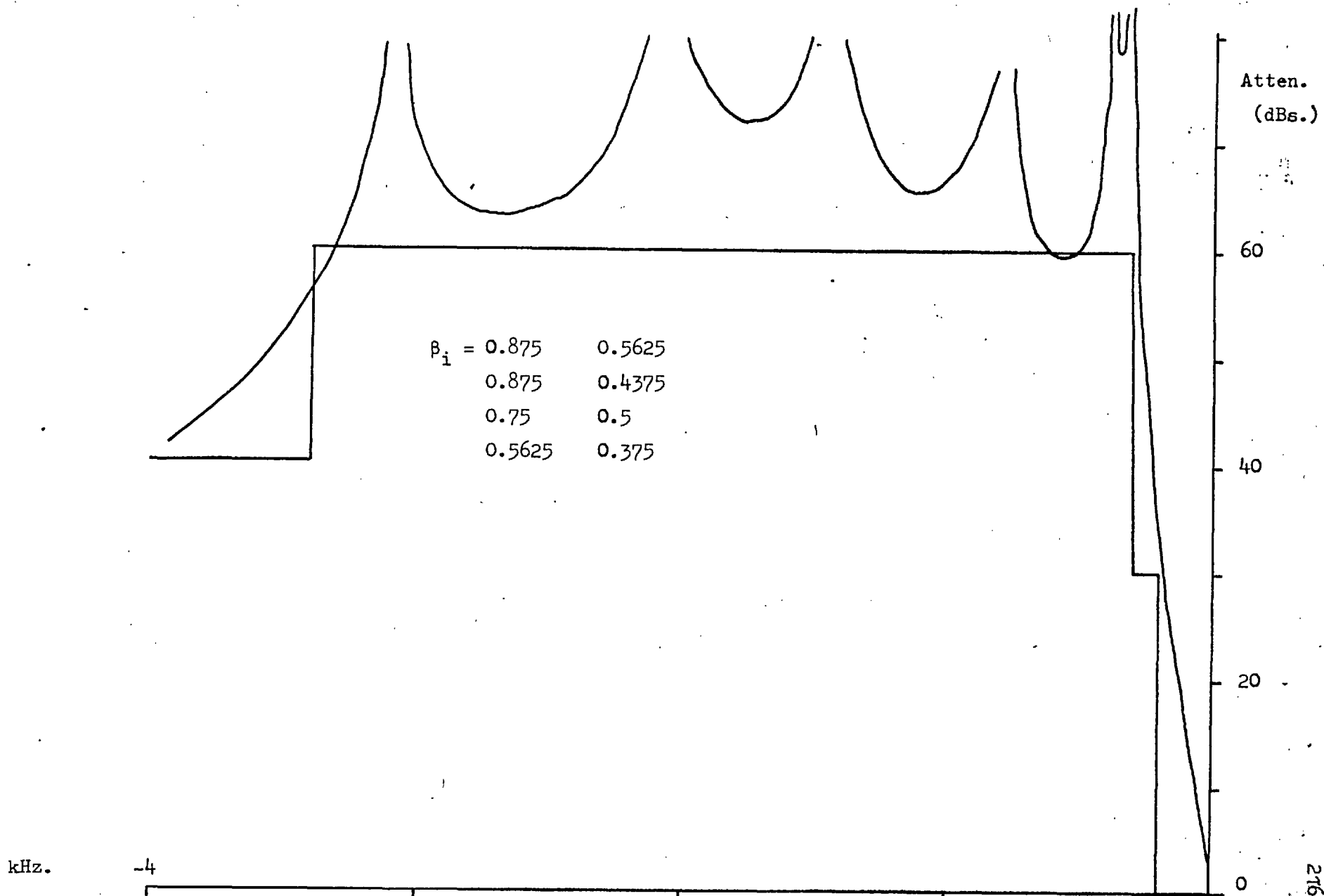


Fig. 5.8 8-section network response : 4-bit coeffs. (opt. in discrete space).



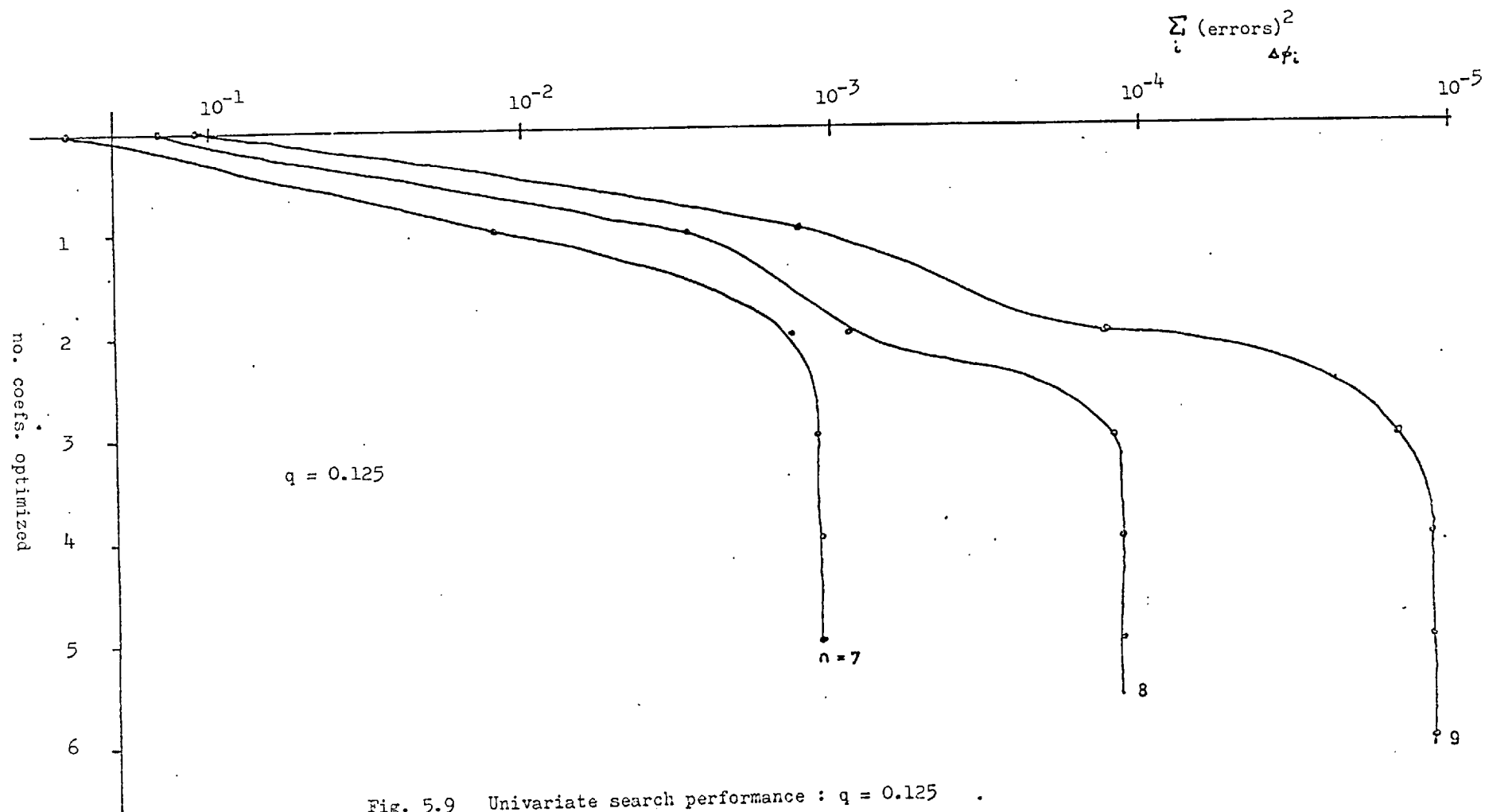


Fig. 5.9 Univariate search performance : $q = 0.125$.

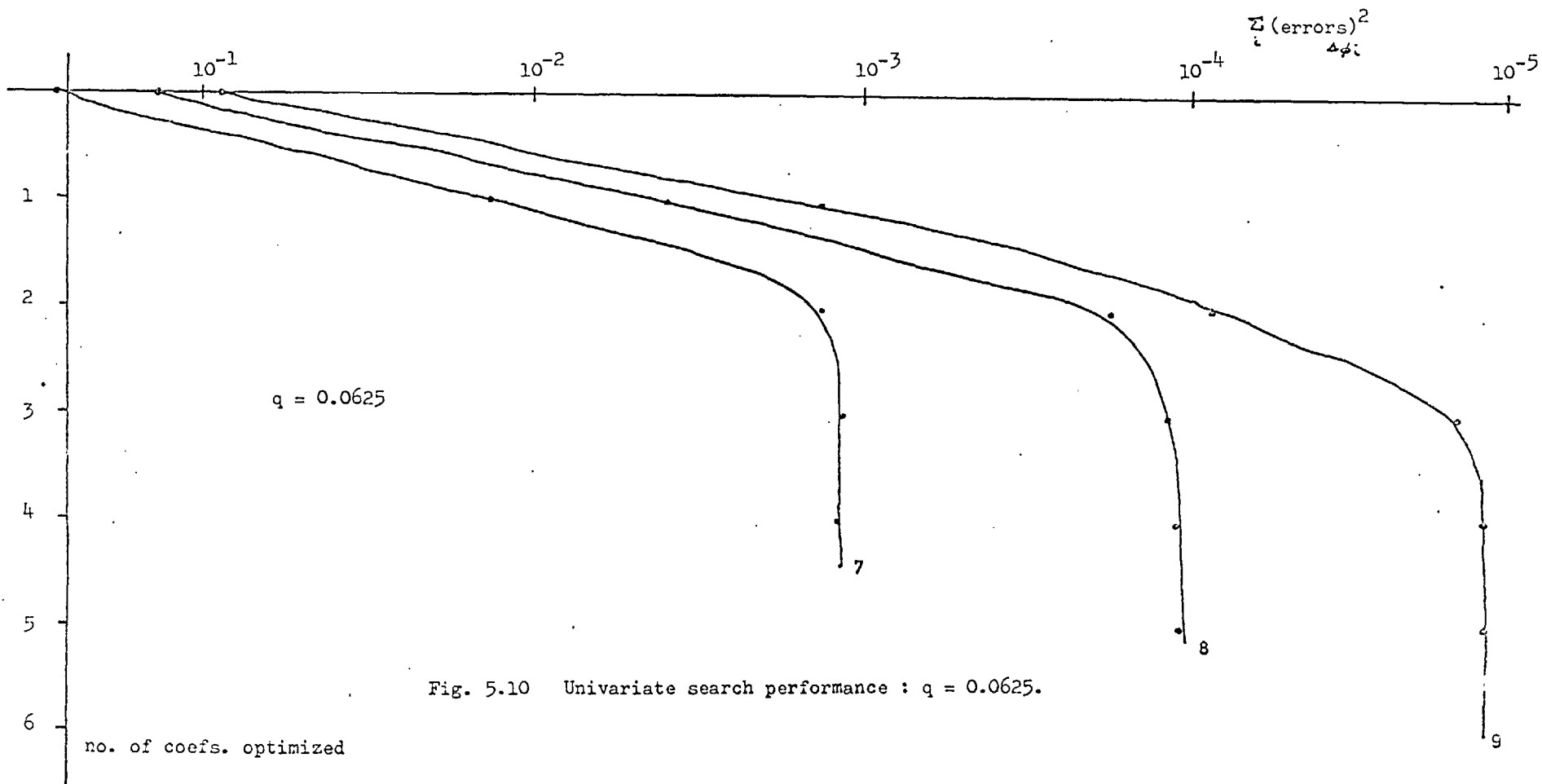


Fig. 5.10 Univariate search performance : $q = 0.0625$.

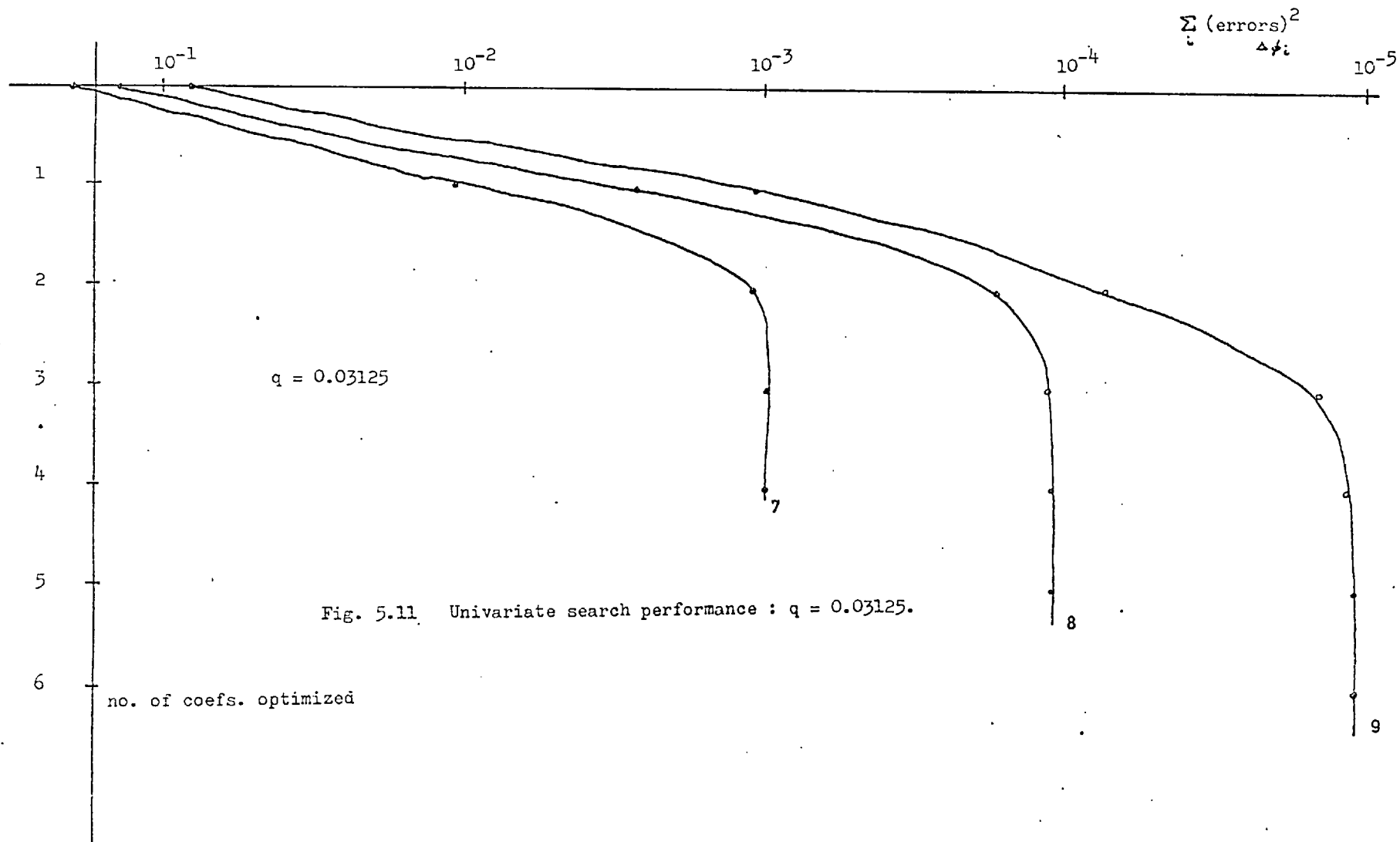
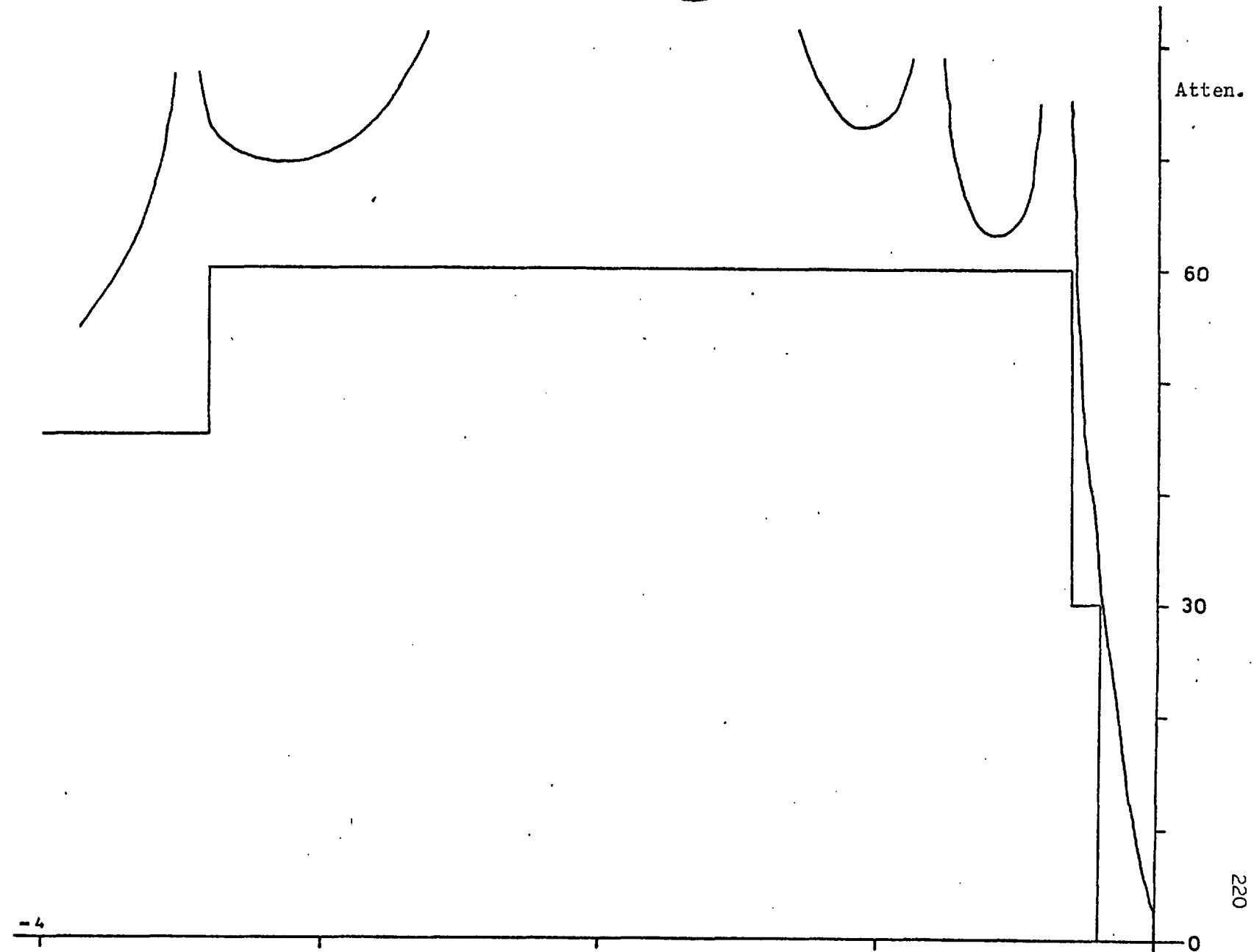


Fig. 5.11 Univariate search performance : $q = 0.03125$.

Fig. 5.12 9-section network (two stage) stopband response.

kHz.

-4



220

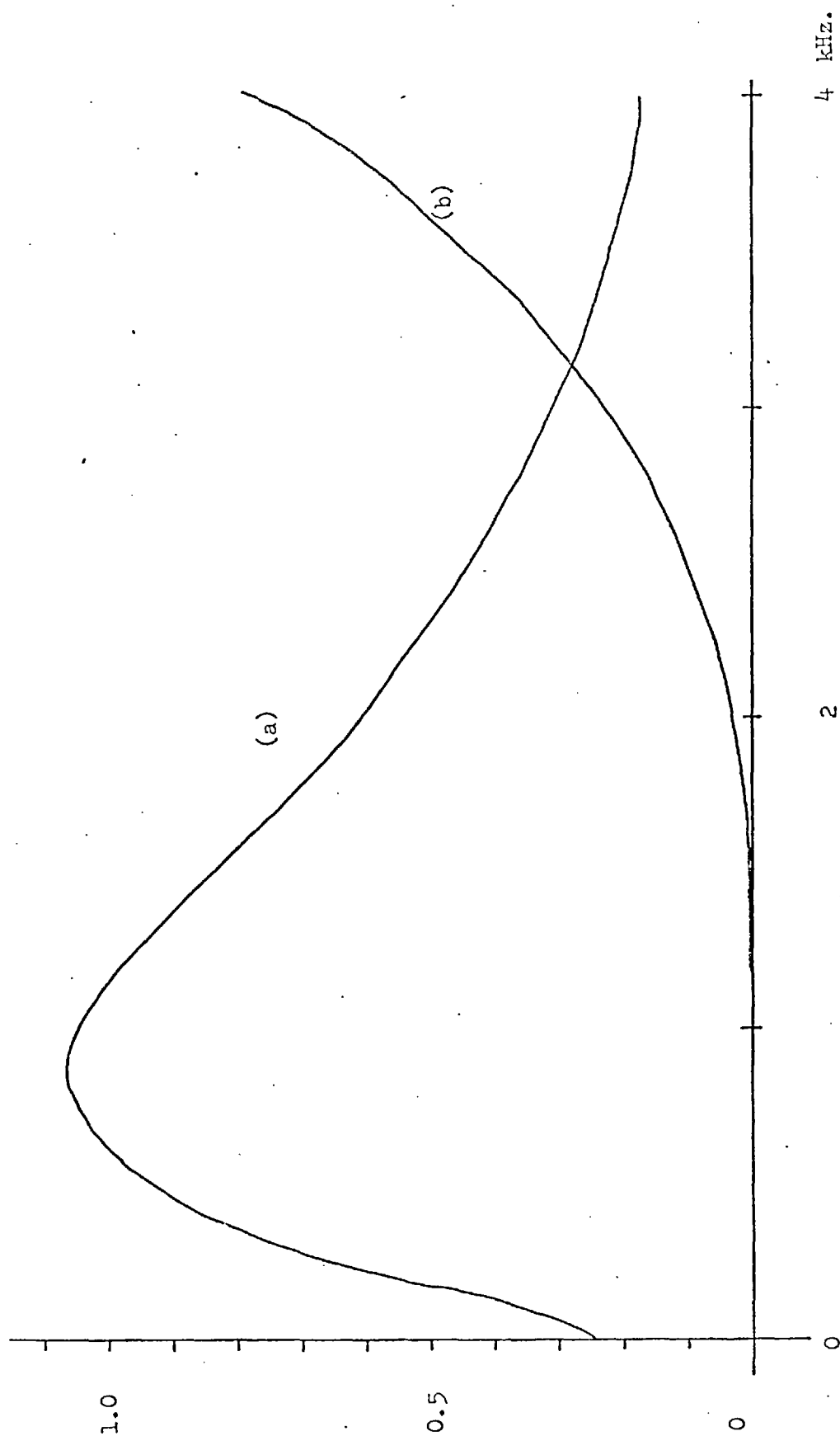


Fig. 5.13(a) 9-section network amplitude response.

(b) the response with the recursive paths removed.

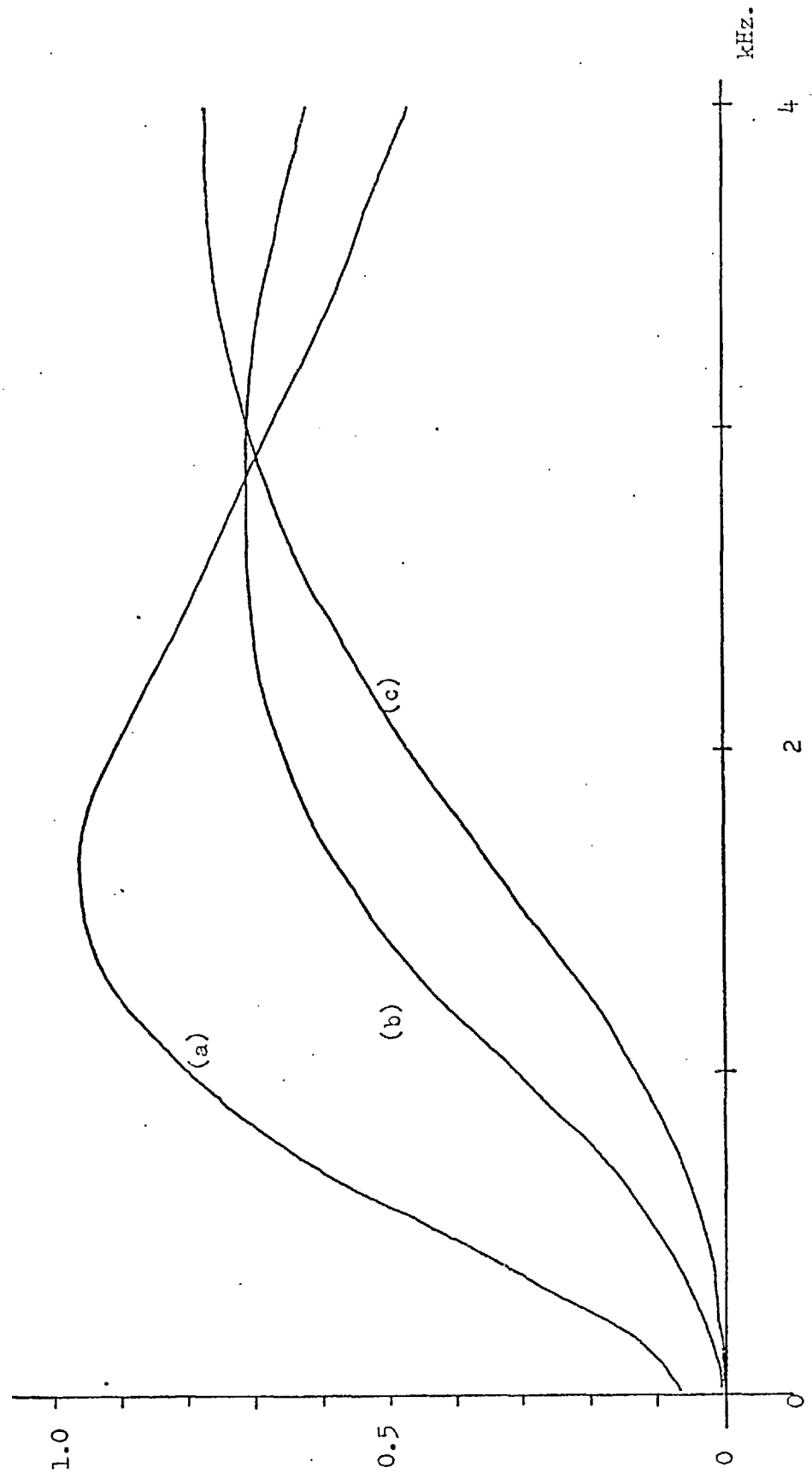


Fig. 5.14 Amplitude response with the following input corrections.

(a) $1 - 0.875 \cdot z^{-1}$ (b) $(1 - 0.875 \cdot z^{-1})^2$ (c) $(1 - 0.875 \cdot z^{-1})^2 \cdot (1 - 0.75 \cdot z^{-1})$

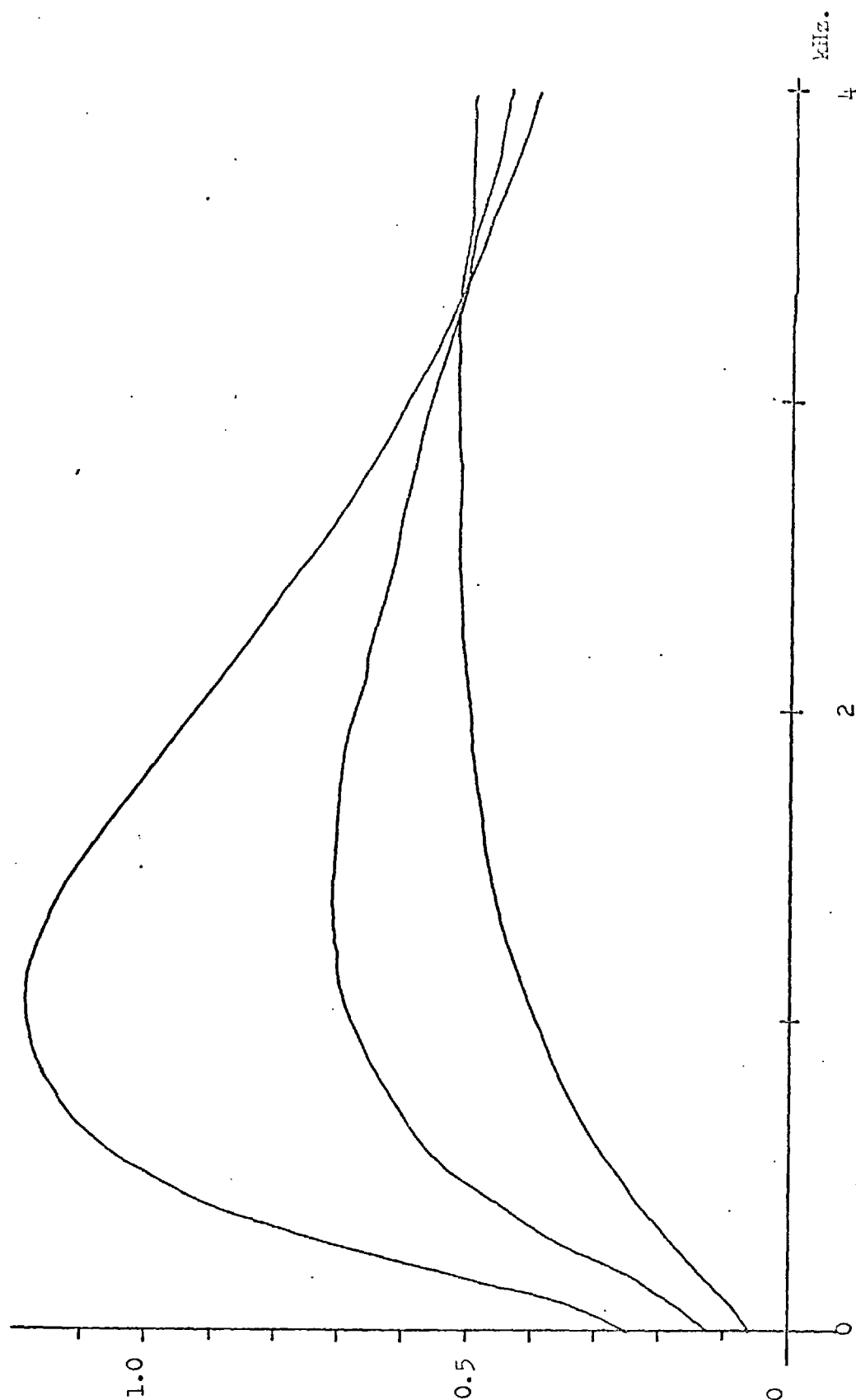


Fig. 5.15. Amplitude response with the following input corrections.

(a) $(1-0.5.z^{-1})$ (b) $(1-0.5.z^{-1})^2$ (c) $(1-0.5.z^{-1})^3$

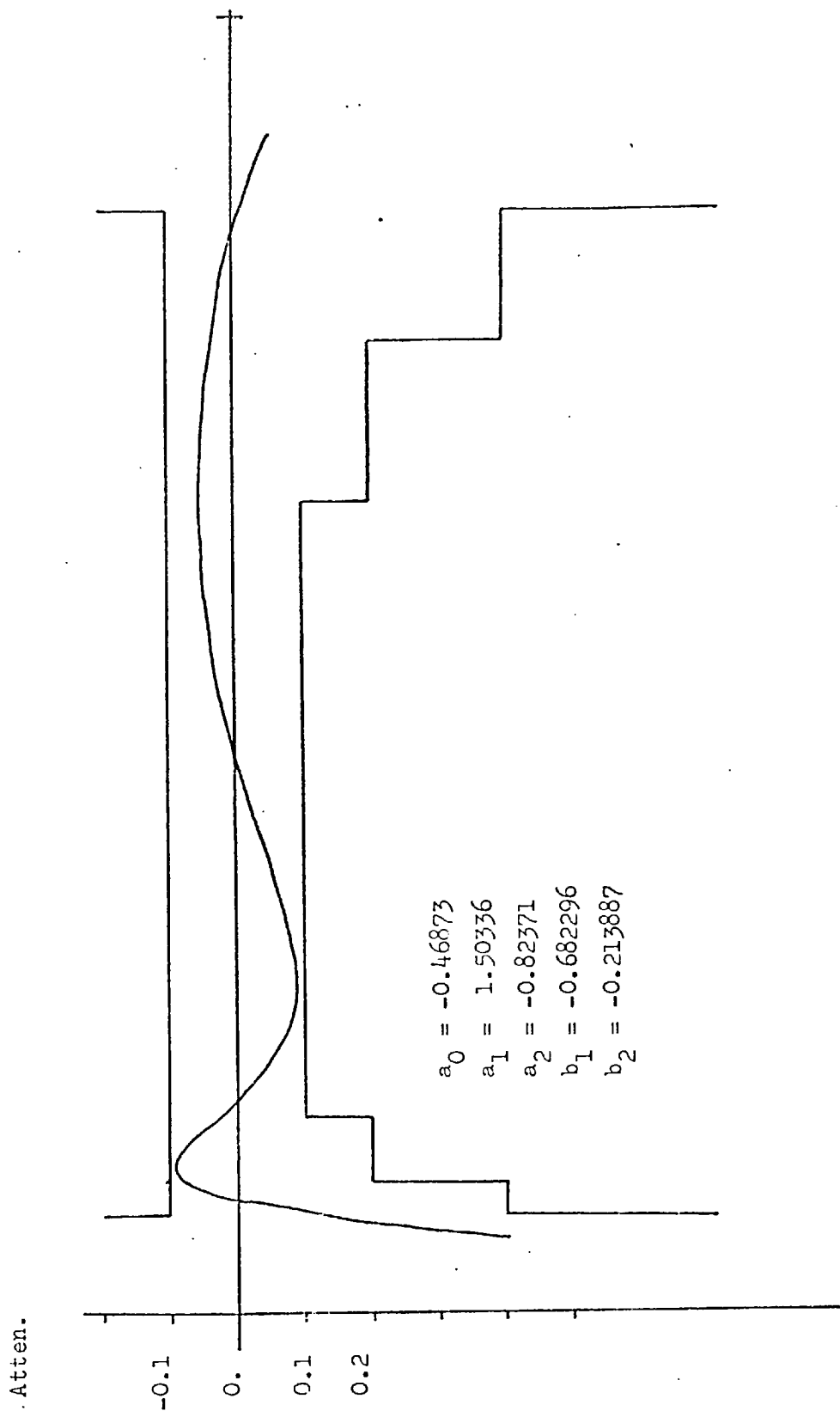
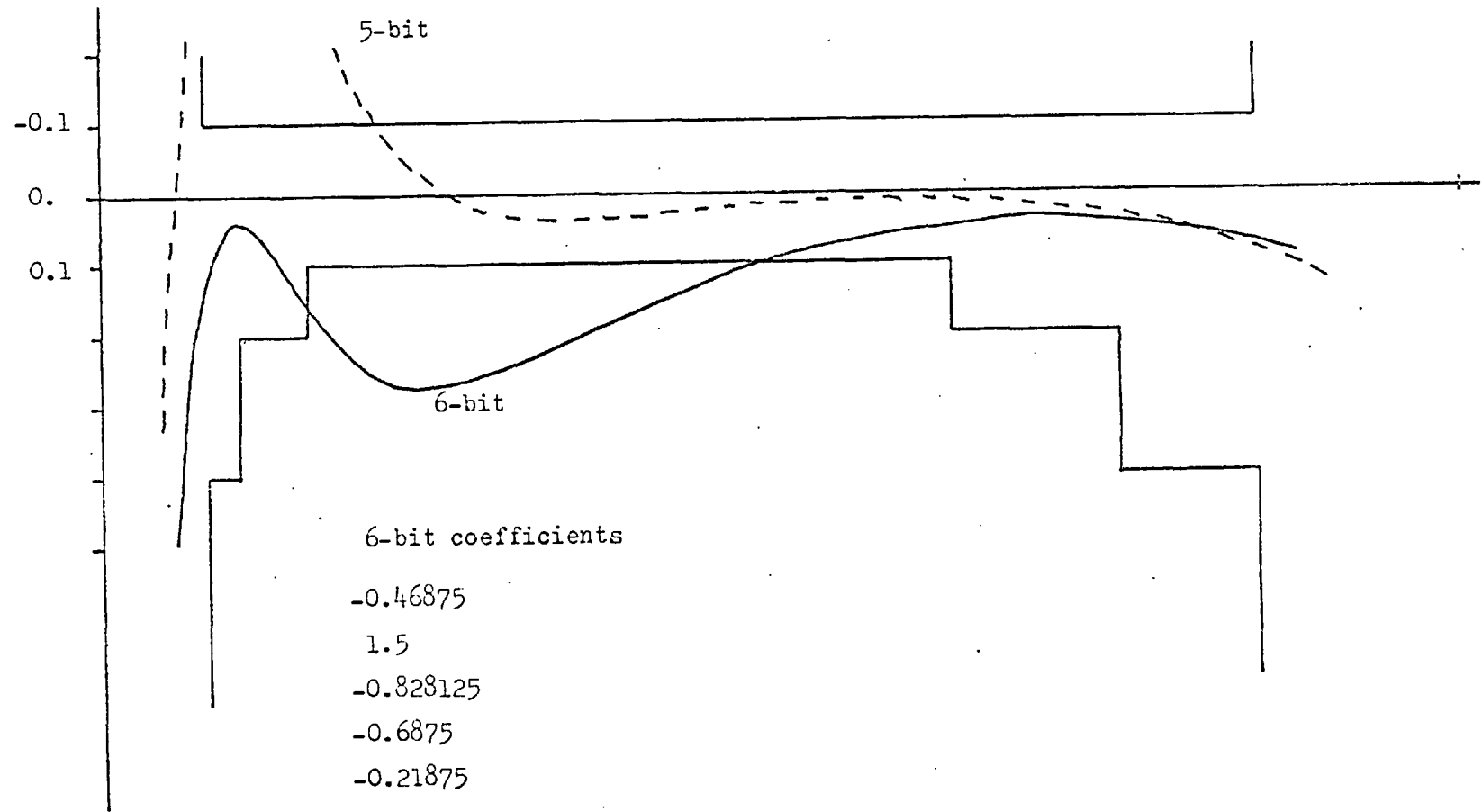


Fig. 5.18 The corrected passband response (using ideal coefficients).

Atten.

Fig. 5.19 Corrected amplitude response (5-bit, 6-bit coefficients).



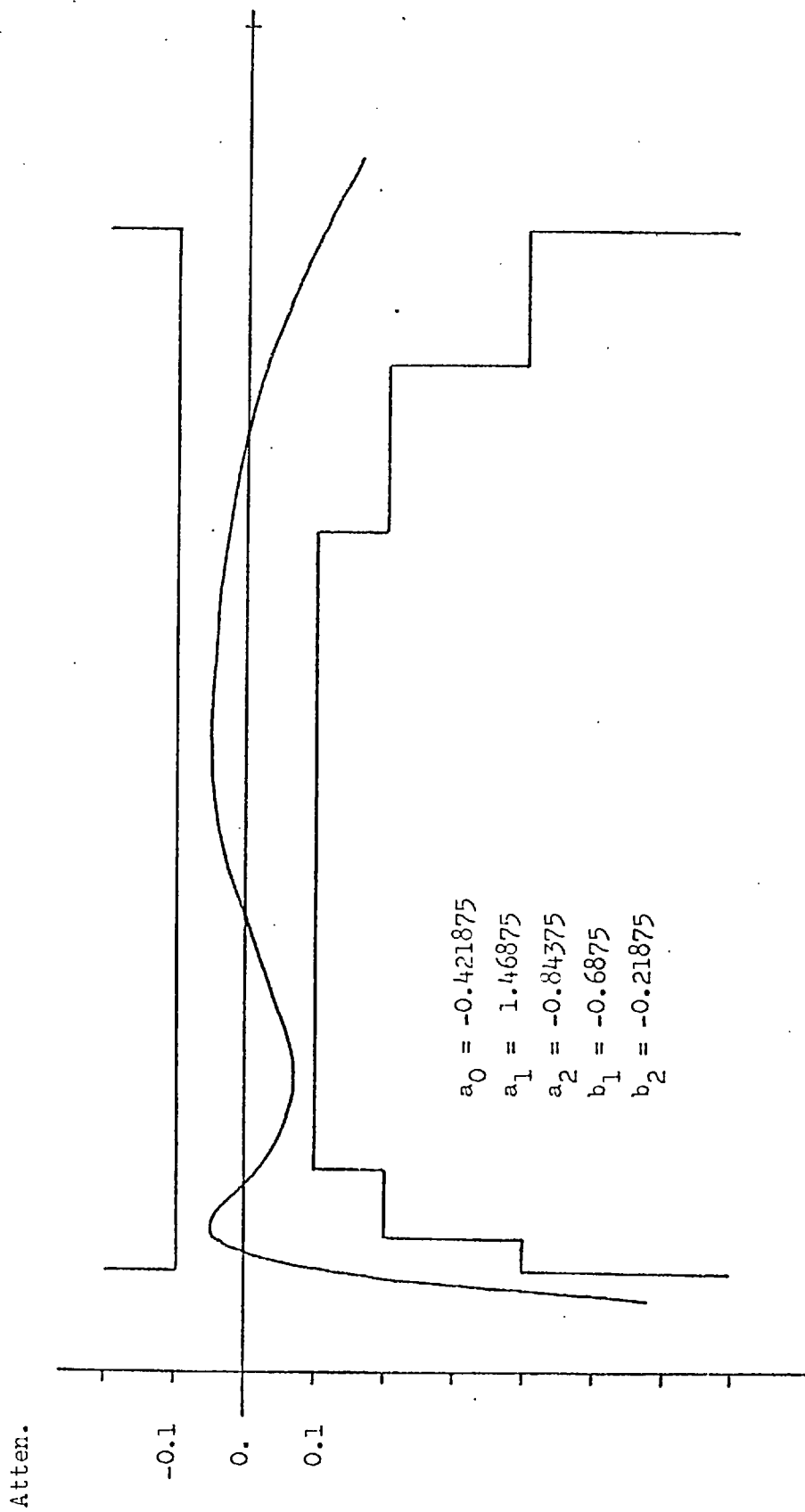
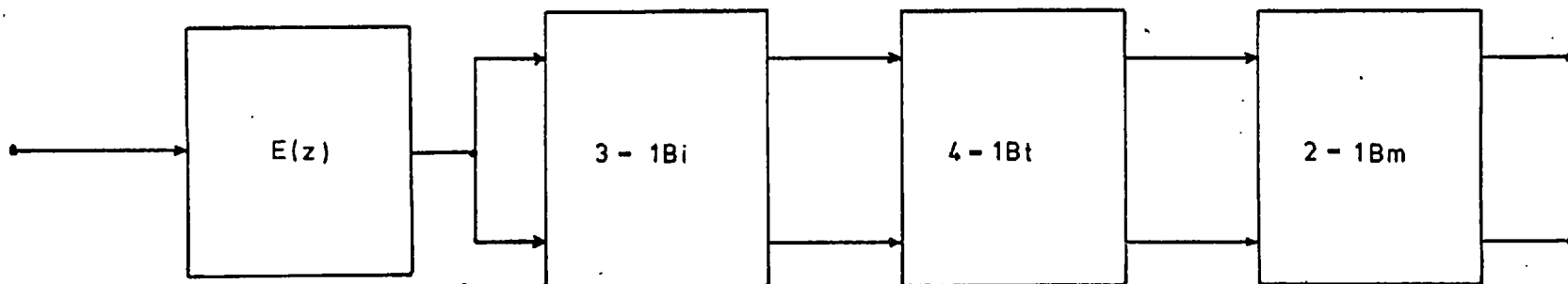


Fig. 5.20 Corrected passband response (6-bit coefficients).

(coefficient set optimized in the discrete space).

Fig. 5.21. The proposed 'phase-splitting' network.



Amplitude
Correcting
Filter. (fig.5.17)

Polyphase network.

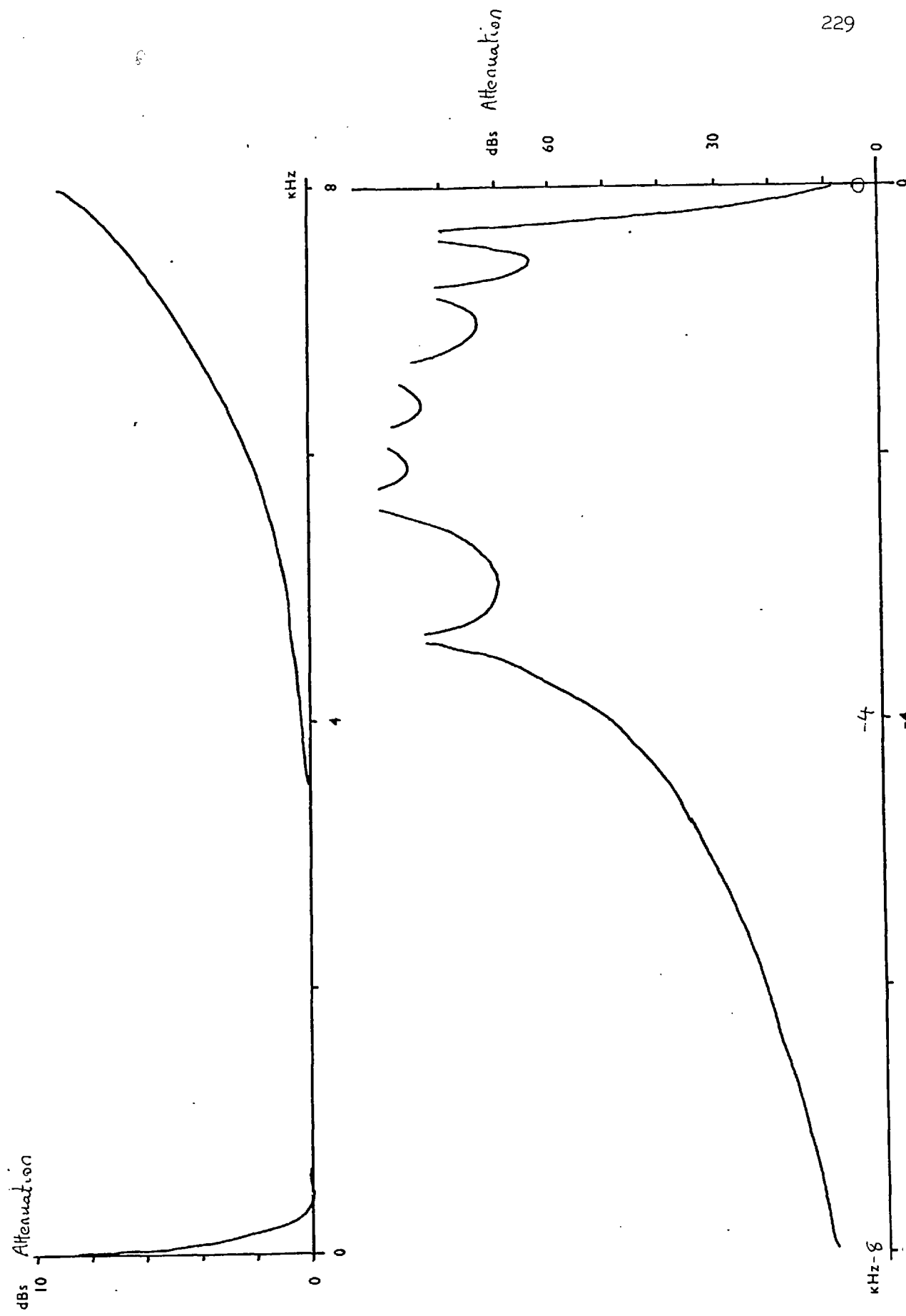


Fig. 5.22. The effective frequency response of the proposed phase-splitting network.

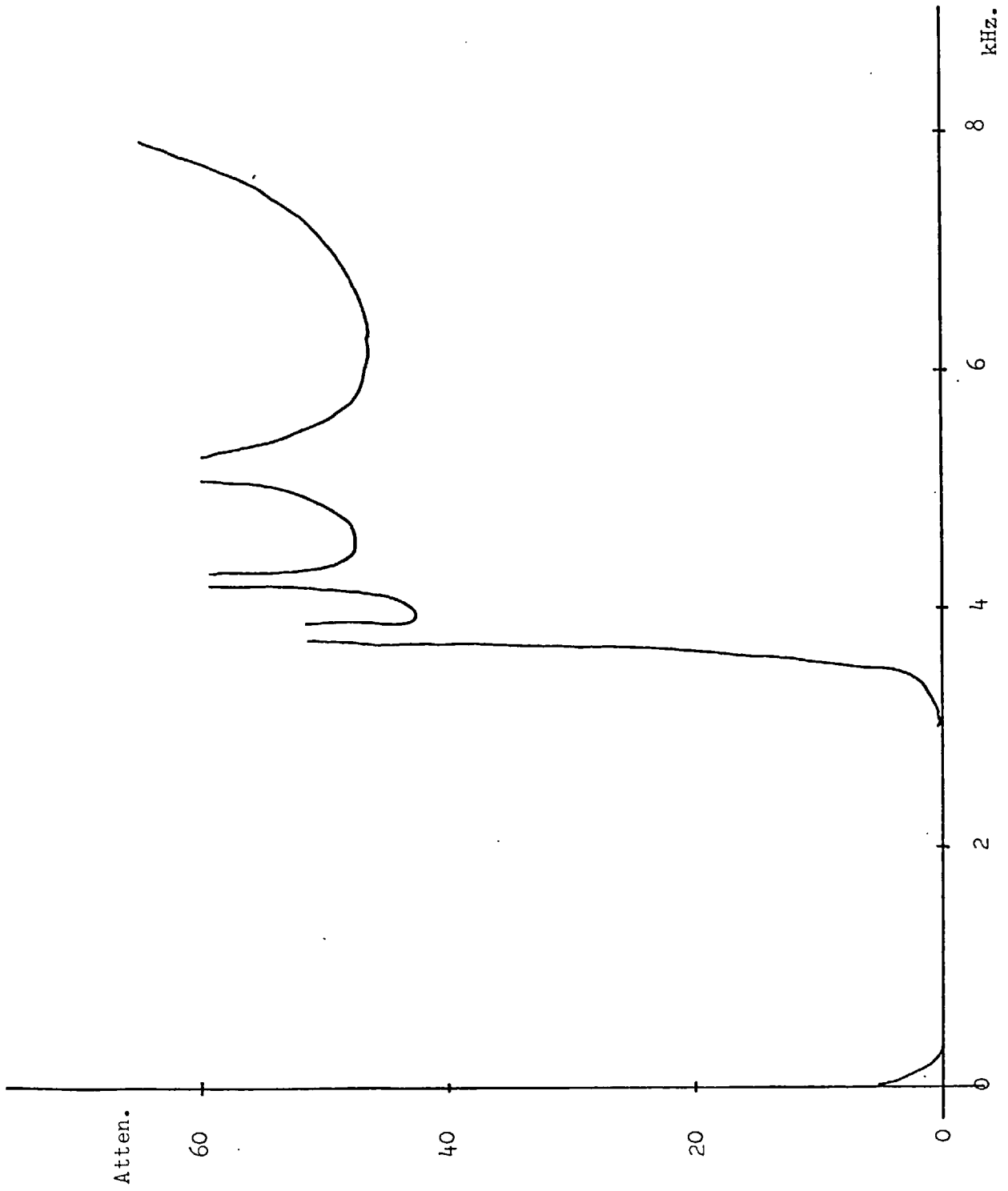


Fig. 5.23 The passband response after adding the guardband filter.

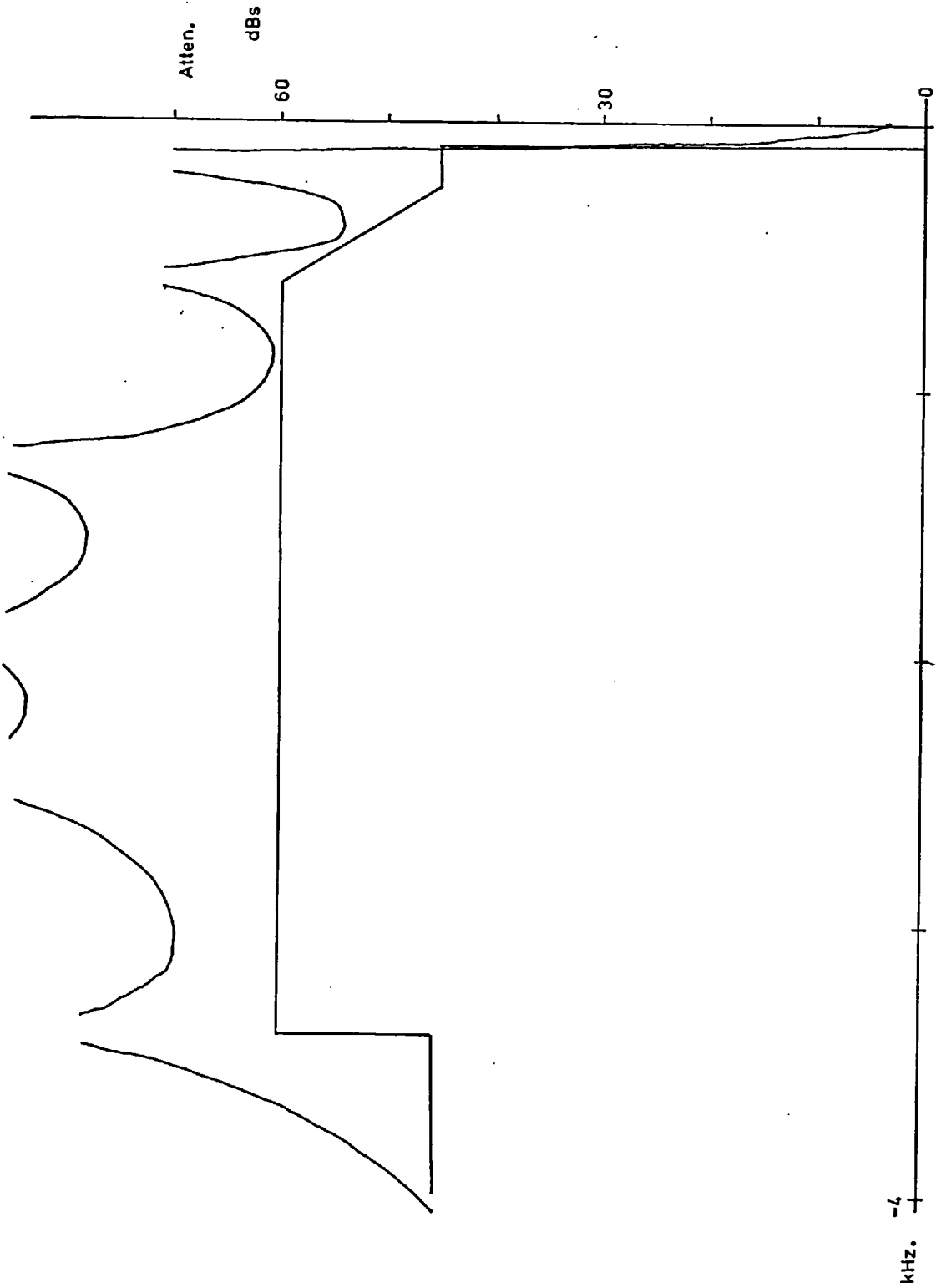


Fig. 5.24 The stopband attenuation for the ten-section network.

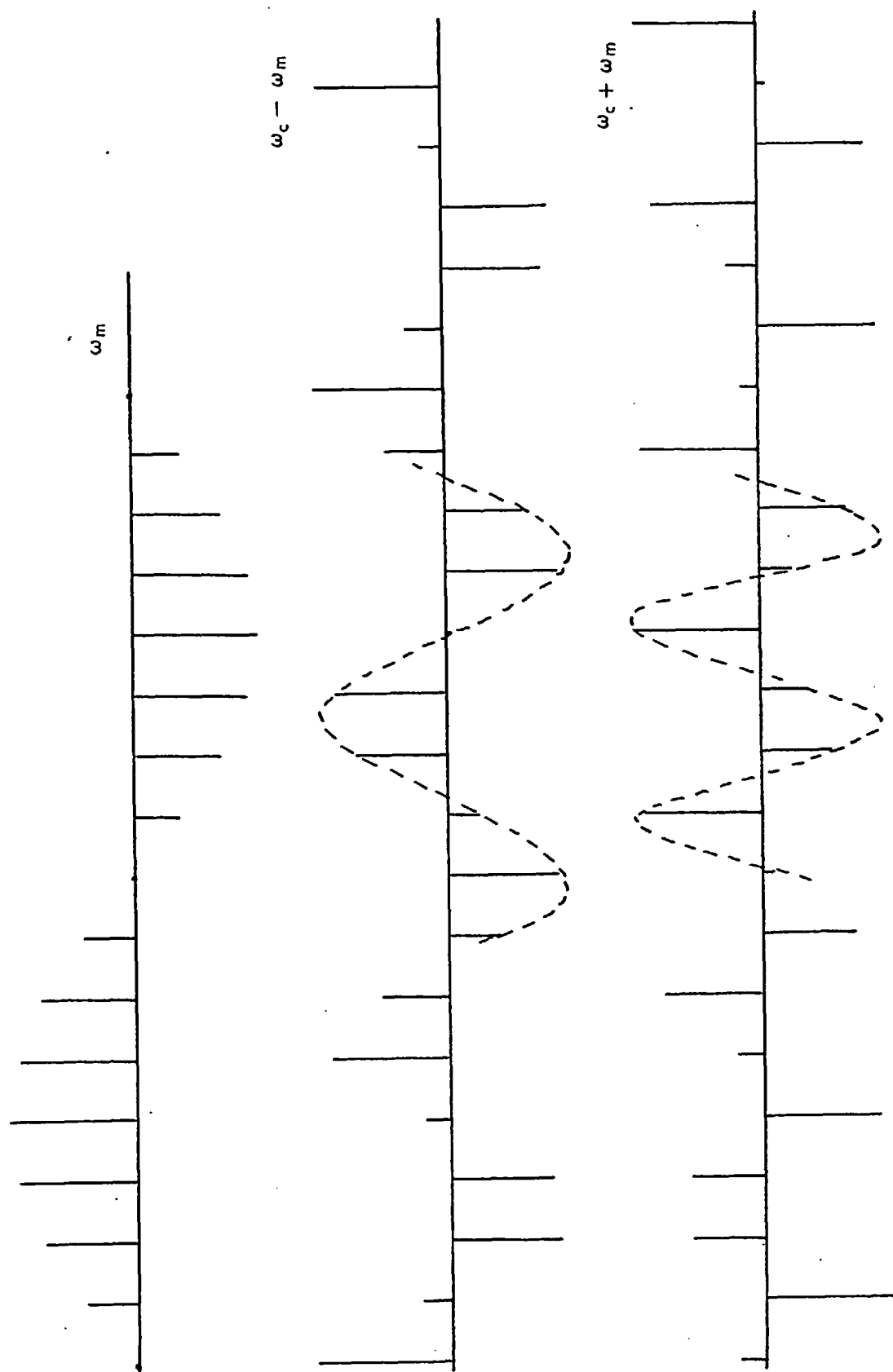


Fig. 5.25 The simulated lower & upper sideband time response.
(steady-state)

CHAPTER SIX.

My pen is at the bottom of a page,
Which being finished, here the story ends.

Byron.

6.0 Conclusions and Comments

The world's telecommunication system forms the biggest electronic complex yet created. At an early stage in the development of this vast communication network, it became advantageous to increase the efficiency of use of the communication channels (rather than add extra channels) to cope with the growing telecommunication traffic demands. This was made possible with the introduction of the F.D.M. system and, more recently using digital signal processing techniques, the T.D.M. system. As these two systems will be required to coexist and be interconnected it is of great practical interest to consider realizing the F.D.M. system digitally.

In this thesis the production of single-sideband signals required in forming the F.D.M. group, using both the digital phase-splitting method and the digital Weaver method, have been examined. The use of a proposed digital phase-splitting modulation system⁽⁴³⁾ is favoured as this system is shown to have several advantages. The chosen system is considered in two stages: (i) a low sample-rate (16 kHz.) complex baseband channel filter followed by (ii) a high sample-rate (112 kHz) signal interpolation and high frequency modulation stage. A second stage alternative of a block processing^(38,48-51) operation is outlined. At present, the use of the interpolating filter plus high frequency modulators seems most easily implemented - however, the large-scale manufacture of 'purpose-built' Fast Fourier Processors (to perform the block processing) could alter this situation.

The production of the complex channel filter frequency response (i.e. an amplitude response of odd symmetry about the origin), using a phase-splitting network, is considered in some detail. In an effort to improve the chosen system via a reduction in the hardware complexity, various recursive and non-recursive structures are individually examined to determine which should be used in realizing the phase-splitting network. Some justification is given as to why this part of the system receives attention regarding the hardware complexity reduction.

Using non-recursive structures, approximations to the ideal Hilbert Transformer are sought. Exact phase-difference response solutions are obtained regardless of the network order (length) but the poor amplitude response properties of low order networks limit their usefulness. The order of network required is such that a digital non-recursive structure cannot compete with a recursive structure, producing an equivalent amplitude and phase-difference response, on the comparisons of multiplier complexity. With the introduction of the C.C.D. 'shift register' and more widely available 'Integer Programming' techniques, a non-recursive Hilbert Transformer, realized in the form of a binary transversal filter⁽⁷³⁾, could perhaps reverse this situation.

Using recursive structures, two real networks are designed such that the required phase-difference response between their respective outputs is obtained over a frequency range ω_1 to ω_2 . The real and imaginary parts of the required complex transmittance $H(z)$ correspond to the two paths of the phase-splitting network. It

is arranged so that each real network possesses an allpass response by ensuring that a pole at $z = \beta$ is cancelled by a zero at $z = 1/\beta$. Although the resulting phase-splitting networks are computationally efficient, it is found that their related stopband attenuation is extremely sensitive to coefficient inaccuracies.

For the various reasons given, the polyphase (phase-splitting) network is specified as using fixed-point arithmetic hardware. A detailed analysis (using a network of arbitrary design) shows the polyphase structure to be relatively insensitive to the effects resulting from the use of finite accuracy arithmetic. The coefficient sensitivity is found to be particularly low and, furthermore, it is shown that the accuracy of the coefficient values is only of secondary importance (i.e. the network order is responsible for the phase-difference response bandwidth, with the coefficient values positioning this response). This knowledge is put to good use in deriving a phase-splitting network design procedure.

In the design of the required phase-splitting network, handling the polyphase network transmittance proves intractable and therefore solutions, yielding approximations to the ideal response, are sought using computer optimization techniques. A non-linear optimization routine, based on the least-squares error norm, is used. We show that, although the L_∞ error norm gives the most accurate approximation, the L_2 error norm gives the most satisfactory approximation of the desired phase-difference response. The frequency bandwidth, over which the approximation is to be

obtained, is specified (a weighting can be applied to any part of this region) along with the number of unknowns (β_1) and the number of non-linear equations (one per frequency sample). Although the results obtained are theoretically accurate, it is found that, for practical applications, using an optimization in the discrete space of coefficients produces better results (i.e. for coefficients of any particular wordlength, we then have the optimal response).

On investigating the effects due to coefficient quantization in the structures under examination, it is found that the coefficient value accuracy is of secondary importance to the network order (number of sections). Furthermore, we show that wideband approximations, to the 90° phase-difference response, result from using a cascade of 'trivial' sections (i.e. sections using the 1-bit multiplication ($\beta = 0.5$) are defined as 'trivial'). A design procedure is proposed which makes use of this trivial-section property - that is, we choose an all-trivial section network that produces the required phase-difference response although the resulting positioning of this response fails to satisfy our design specification. An optimization procedure, based on the univariate search method, is then used (i.e. coefficients of firstly one section, then two sections, are individually optimized - preferably in the discrete space of coefficients - until the phase-difference response is shifted to the desired position). It is observed in this design procedure that any significant shift of the phase-difference response, towards the origin, tends to reduce its effective bandwidth. However the use of an empirically

obtained relationship between the phase-difference response bandwidth and the polyphase network order is used in defining the necessary network order (i.e. from the analogue design tables⁽³¹⁾, and using the transformed frequency variable, it is found that the bandwidth ratio multiplied by a factor of four defines the approximate number of sections required for a particular phase tolerance). Using this complete design procedure, we obtain a low complexity polyphase network (consisting of two stages, one containing trivial sections, the other non-trivial sections) producing the specified phase-difference response.

Although the proposed low-complexity polyphase network produces the specified phase-difference response (and therefore the effective stopband attenuation), we see that this network does not have the flat amplitude response necessary in satisfying the passband specifications. We therefore design a filter to flatten (or equalize) the polyphase network amplitude response. A modification in the polyphase network structure (in fact, a slight hardware reduction) simplifies the amount of correction required to produce the passband response. For this modified network (i.e. the recursive paths of two trivial sections are eliminated), it is found that a single 2nd-order recursive section, using five 6-bit coefficients, provides the desired amplitude correction.

The complete phase-splitting network (the amplitude correcting filter in cascade with the polyphase network) is evaluated on the basis of multiplier complexity: the use of the fixed-point pipeline multiplier is assumed throughout. Comparisons with other, already proposed, phase-splitting networks show it to be competitive even after allowing for the additional basic hardware (storage, addition and 1-bit multipliers) requirements.

To conclude we now summarise this research work as a whole and suggest some areas for future research.

Certain problems of a general nature, occurring in the digital processing of telephone speech signals, are examined. In particular, the formation of the F.D.M. Group (types A or B) is considered in some detail. The use of the digital phase-splitting modulation approach for producing single-sideband signals is shown to be advantageous. A new digital structure interpreted in terms of polyphase concepts and possessing phase-splitting properties is proposed. We show that this structure can be used, to good effect, in the system based upon the digital phase-splitting modulation approach.

We have, therefore, added to the already attractive attributes of the digital phase-splitting modulation system.

6.1 Future Research.

At various stages throughout the course of this research work, some possible areas for further research were identified. These areas are:-

(a) the practical implementation of general digital signal processing systems (b) further properties of polyphase networks and (c) digital computer applications. Various topics within these categories are now outlined.

(a) the practical implementation of general digital signal processing systems.

:- choice of technologies and circuitry techniques.

Although the choice of technology/circuitry technique is usually limited to bipolar/ I^2L and MOS/ μ -phase dynamic (105), it would be helpful to the design engineer if some detailed and up-to-date survey of the 'state-of-the-art' was undertaken.

:- design rules.

In an operational digital system, problems caused by delays in signal throughput, low/high signal level fanout, timing skew etc. are envisaged. It would therefore be advantageous if design rules were defined aimed at minimizing such problems.

:- use of mixed arithmetic.

At present, for ease of realization, mixed arithmetic is often used in practical digital circuits i.e. sign-magnitude arithmetic for multiplication and 2's complement arithmetic for addition/subtraction. Although the required code conversion is easily

performed, it is felt that hardware multiplication techniques leave room for improvement.

(note: see also digital computer applications)

(b) further properties of polyphase networks.

The digital structure used throughout this thesis for designing phase-splitting networks was obtained by transforming an analogue polyphase network to its digital equivalent. Recently a publication ⁽¹⁰⁶⁾ introduced a new quadrature phase-shift or phase-splitting network based on a polyphase active-ladder network. It would therefore be of great interest if comparisons were made of the digital counterparts of both the passive and active polyphase networks.

If the polyphase network is to be assessed with regard to its complexity, it would be useful if comparisons are also made with complex digital structures (i.e. networks realized with complex coefficients) designed under similar constraints.

(c) digital computer applications.

:- digital signal processing by micro-computers making use of stored program control.

Already, there are various systems in which digital signal processing is performed by small and relatively inexpensive micro-computers making use of stored program control. It is now becoming apparent, however, that the computer capabilities are not being used to their full extent. In ref. (107), for example, a micro-processor-based system handles computer data prior to it being

transmitted over an H.F. radio link. A coding scheme is used in an attempt to protect the data from the effects of extraneous noise - that is, for each 'packet' of 12-bit character words transmitted, an error of 1-bit in any word is corrected or a 2-bit error in any word causes a retransmission of that packet. The transmitting baud-rate is adapted to suit the channel condition although the presently available modems require that active band-pass filters be optimized for one baud-rate. Future requirements therefore indicate the need for an adaptive band-pass filter i.e. the filter bandwidth is altered to match the particular baud-rate in use.

:- table look-up multiplication.

The declining price of read-only memory (ROM) makes the use of table look-up techniques feasible. However, for multiplication, the total number of ROM bits required is prohibitively large. Recently, a compromise technique ⁽¹⁰⁸⁾ combining ROM tables of convenient sizes with adder blocks was introduced. This encourages further investigation as to how effective the use of table look-up multiplication can be.

```

C      THIS OPTIMIZATION PROGRAM REQUIRES AS VARIABLES
C      (AND THEREFORE MUST BE SUPPLIED WITH) THE FOLLOWING
C
C  NZ      NO. OF SECTIONS WITH FIXED VALUE COEFFICIENTS
C  NS1     LOW FREQ. SAMPLE EXTREMAL
C  NS2     HIGH FREQ. SAMPLE EXTREMAL
C  N       NO. OF UNKNOWNNS
C
C      THERE ARE 400 POINTS IN THE HALF SAMPLING RANGE
C      GIVING 20 HZ. PER SAMPLE
C
C      ACCESS IS REQUIRED TO THE NAG4F LIBRARY
C
C      PROGRAM OPT(INPUT,OUTPUT,TAPES=INPUT,TAPE6=OUTPUT)
C      DIMENSION W(5000),PH(180),BB(12),V(12),RE(180)
C      COMPLEX Z(180)
C      COMMON PI2,NS1,NS2,DN,Z,NZ
C      EXTERNAL SUMSQ,MONIT
C
C      PI=3.1415926536
C      PI2=2.*PI
C
C      MAX=25
C      ALF=1.E-2
C      EPS=1.E-8
C      IFAIL=1
C      NS=900
C      DN=FLOAT(NS)
C
C      WRITE(6,100)
100    FORMAT(*GIVE NZ NS1 NS2 N ... FORMAT = 4I3 *)
101    READ(5,101)NZ,NS1,NS2,N
C      FORMAT(4I3)
C
C      M=NS2-NS1+1
C      IP=N+3+N/3
C      IW=2*M+4*N+M*N+(N*N+N)/2+IP*(M+2+2*N)
C
C      DO 10 I=NS1,NS2
10     Z(I)=CMPLX(COS(PI2*FLOAT(I-1)/DN),-SIN(PI2*FLOAT(I-1)/DN))
C
C      ESTIMATES OF COEFFS & WEIGHTING
C      DO 12 I=1,N
12     BB(I)=0.5
C      V(I)=0.2
C
C      NAG4F LIBRARY NON-LINEAR OPTIMIZATION ROUTINE
C      CALL E04FDF(M,N,BB,RE,SSQ,EPS,ALF,V,W,IW,SUMSQ,MONIT,2,MAX,IFAIL)
C
C      WRITE(6,102)IFAIL,SSQ
102    FORMAT(*      IFAIL = *,I2,*      SSQ = *,F17.10,/)
103    WRITE(6,103)(BB(I),I=1,N+NZ)
103    FORMAT(12X,F16.10,/)
104    WRITE(6,104)
104    FORMAT(*ERROR SQUARED      PHASE DEVIATION*)
103    FORMAT(2(2X,F14.8))
C
C      DO 111 J=1,M+2
111    RE(J)=RE(J)**0.5
C      PH(J)=(180./PI)*ATAN(RE(J))
111    WRITE(6,103)RE(J),PH(J)
C
C      STOP
C      END

```

```

C      THIS SUBROUTINE COMPUTES THE SQUARE OF THE PHASE
C      ERROR BETWEEN NS1 AND NS2 FOR EACH SAMPLE POINT
C
C      SUBROUTINE SUNSQ(M,N,BB,RE)
C      DIMENSION HA(180),HB(180),BB(12),V(12),Q(180),RE(180)
C      COMPLEX Z(180),R(180),Y(180),R1(180),Y1(180),A1(180),A2(180)
C      COMMON F12,NS1,NS2,DN,NZ,Z
C
C      DO 22 I=1,NS2
22      RE(I)=0.
C
C      DO 10 I=NS1,NS2
C
C      R1(I)=(1,-BB(1))/(1,-BB(1)*Z(I))
C      Y1(I)=1,-R1(I)
C
C      DO 12 L=2,NZ+N
C
C      R(I)=(1,-BB(L))/(1,-BB(L)*Z(I))
C      Y(I)=1,-R(I)
C
C      A1(I)=(R(I)*R1(I)-Y(I)*Y1(I))
C      A2(I)=-(Y(I)*R1(I)+R(I)*Y1(I))
C
C      R1(I)= A1(I)
C      Y1(I)=-A2(I)
C
C      12      CONTINUE
C
C      A1(I)=R1(I)-Y1(I)
C      A2(I)=R1(I)+Y1(I)
C
C      HA(I)=AIMAG(A1(I))/REAL(A1(I))
C      HB(I)=AIMAG(A2(I))/REAL(A2(I))
C
C      RE(I)=((1,+HA(I)*HB(I))/(HA(I)-HB(I)))*2
C
C      10      CONTINUE
C
C      JJ=0
C      DO 24 J=NS1,NS2
C      JJ=JJ+1
C      Q(JJ)=RE(J)
C      RE(JJ)=Q(J)
24      CONTINUE
C
C      RETURN
C      END
*

```

REFERENCES.

1. STEIGLITZ, K.: The equiv. of digital and analogue signal processing.
Inform. & Contr., Vol. 8, pp 455-67, 1967.
2. ROSEN, S.: Electronic Computers - 'An historical survey'.
Comp. Surveys, Vol. 1, No. 1, 1969.
3. COOLEY, J.W. & TUKEY, J.W.: An algorithm for the machine calculation
of the complex Fourier series.
Digital Signal Proc., I.E.E.E. Press, pp 223-28, 1972.
4. COOLEY, J.W. et al.: Terminology in digital signal processing.
I.E.E.E. Trans. A.&E.A., Vol. AU20, pp 332-40, Dec. 1972.
5. GOLD, B. & RADER, C.M.: Digital Processing of Signals.
publ. McGraw-Hill, New York, 1969.
6. BOGNER, B. & CONSTANTINIDES, A.G.: Introduction to Digital Filtering.
publ. Wiley & Sons, London, 1975.
7. STEIGLITZ, K.: Introduction to Discrete Systems.
publ. Wiley & Sons, New York, 1975.
8. OPPENHEIM, A.V. & SCHAFER, R.W.: Digital Signal Processing.
publ. Prentice Hall, 1975.
9. KUO, F. & KAISER, J.F.: System analysis by Digital Computer.
publ. Wiley & Sons, New York, 1966.
10. REGAZZINI, J.R. & FRANKLIN, F.F.: Sampled Data Control Systems.
publ. McGraw-Hill, New York, 1966.
11. JURY, E.I.: Theory & Application of the Z-Transform Method.
publ. Wiley & Sons, New York, 1964.
12. KAISER, J.F.: Design methods for sampled-data filters.
Digital Sig. Proc., I.E.E.E. Press, 1972.

13. CONSTANTINIDES, A.G.: Synthesis of Chebychev digital filters.
Elec. Letters, Vol. 3, No. 3, March 1967.
14. CONSTANTINIDES, A.G.: Synthesis of Elliptic digital filters.
Elec. Letters, Vol. 3, No. 6, June 1967.
15. HERRMANN, O.: On the design of non-recursive filters with linear phase.
Elec. Letters, Vol. 6, No. 11, pp 328-9, 1970.
16. HERRMANN, O. & SCHUSSLER, W.: The design of non-recursive filters
with minimum phase.
Elec. Letters, Vol. 6, No. 11, Nov. 1970.
17. C.C.I.T.T. Line Transmission Recommendations (G232).
C.C.I.T.T. White Book, Mar del Plata, 1968.
18. KOCHER, K. et al.: F.D.M. coaxial cable system up to 60 MHz..
Elec. Comm., Vol. 50, pp 64-69, 1975.
19. HARTLEY, R.V.L.: Patent No. 1 666 206.
U.S.A. Patents, 1928.
20. SARAGA, W.: Some aspects of N-path & quadrature modulation S.S.B.
Conference Proc., Knokke, 1968.
21. NORGAARD, D.: Phase-shift method of S.S.B. generation.
Proc. I.R.E., Vol. 44, pp 1718-35, Dec. 1956.
22. BEDROSIAN, E.: The analytic signal representation of mod. waveforms.
Proc. I.R.E., Vol. 50, pp 2071-76, 1962.
23. SCHWARTZ, M. et al.: Communication Systems & Techniques.
publ. McGraw-Hill, New York, 1966.
24. DOME, R.B.: Wideband phase-shift networks.
Electronics, Vol. 19, pp 115-22, 1946.
25. LUCK, D.G.C.: Properties of some wideband phase-splitting networks.
Proc. I.R.E., Vol. 37, pp 147-51, Feb. 1949.

26. ROWLANDS, R.O.: Constant phase-shift networks.
Wireless Engr., Vol. 26, pp 72-81, Sept. 1949.
27. DARLINGTON, S.: Realization of a constant phase-difference.
B.S.T.J., Vol. 29, pp 94-104, Jan. 1950.
28. SARAGA, W.: The design of wideband phase-splitting networks.
Proc. I.R.E., Vol. 38, pp 754-70, July 1950.
29. ORCHARD, H.J.: Synthesis of wideband two-phase networks.
Wireless Engr., Vol. 27, pp 72-81, Jan. 1950.
30. SARAGA, W.: Wideband two-phase networks.
Wireless Engr., Vol. 28, pp 30-31, Jan. 1951.
31. BEDROSIAN, S.D.: Normalized design of 90° phase-difference networks.
I.R.E. Trans. Cct. Theory, Vol. CT7, pp 128-36, June 1960.
32. GALPIN, R. et al.: A practical tantalum thin-film S.S.B. demodulator using R.C. time-varying and active networks.
Proc. 2nd Int. Symp. on Microelectronics, Munich 1966.
33. SARAGA, W.: S.S.B. generation - a new principle.
Elec. Tech., Vol. 39, pp 168-71, May 1962.
34. BARBER, D.R. and GINGELL, M.J.: A polyphase modem for F.D.M..
Elec. Comm., Vol. 44, pp 108-14, 1969.
35. GINGELL, M.J.: S.S.B. modulation using sequence asymmetric polyphase networks.
Elec. Comm., Vol. 48, pp 21-25, 1972.
36. WEAVER, D.K.: A 3rd method of generation and detection of SSB signals.
Proc. I.R.E., Vol. 44, pp 1703-05, Dec. 1956.
37. DARLINGTON, S.: Some circuits for S.S.B. modulation.
Proc. 3rd Ann. Princeton Conf., 1969.
38. DARLINGTON, S.: On digital S.S.B. modulators.
I.E.E.E. Trans. Cct. Th., Vol. CT17, pp 409-14, Aug. 1970.

39. FREENY, S.L. et al.: Design of digital filters for an all-digital FDM-TDM translator.
I.E.E.E. Trans. Cct. Th., Vol. CT18, pp 702, Nov. 1971.
40. FREENY et al.: Analysis of an all-digital FDM-TDM translator.
I.E.E.E. Trans. Comm. Tech., Vol. COMM19, pp 1050-59, 1971.
41. GUPTA, S et al.: Digital S.S.B. modulation.
I.E.E.E. Trans. Commun., Vol. COM21, pp 255-62, March 1973.
42. KURTH, C.F.: SSB-FDM utilizing TDM digital filters.
I.E.E.E. Trans. Comm. Tech., Vol. COMM19, pp 63-71, 1971.
43. BOITE, R. and LEICH, H.: On the use of the numerical phase-difference filter for the digital channel bank.
Proc. I.E.E., Vol. 122, pp 340-44, April 1975.
44. SCHAFER and RABINER: A digital sig. proc. approach to interpolation.
Proc. I.E.E.E., Vol. 61, pp 692-702, June 1973.
45. WHITE, S.A.: A slow approach to mechanizing a fast interpolation filter.
I.E.E.E. Trans. A. and E.A., Vol. AU20, pp 90-93, March 1972.
46. URKOWITZ, H.: Parallel realizations of digital interpolation filters.
Int. Report, RCA Missile and Surface Radar Division, 1972.
47. OETKEN and SCHUSSLER: On the design of dig. filters for interpolation.
A.E.U., Band 27, Heft 11, pp 471-76, 1973.
48. TERRELL, P.M. and RAYNER, P.J.W.: Digital S.S.B. modulators.
Elec. Letters, Vol. 10, No. 13, pp 257-58, June 1974.
49. TERRELL and RAYNER: A digital block processor for SSB-FDM modulation.
I.E.E.E. Trans. Commun., Vol. COM23, pp 282-86, Feb. 1975.
50. BELLANGER, M.G. and DAGUET, J.L.: TDM-FDM transmultiplexer utilizing digital polyphase and F.F.T. processing.
I.E.E.E. Trans. Commun., Vol. COM22, pp 1199-1204, 1974.

51. BELLANGER, M.G. et al.: Digital filtering by polyphase networks -
Applications to sample-rate alteration and filter banks.
I.E.E.E. Trans. ASSP, Vol. ASSP24, pp 109-14, April 1976.
52. FREENY, S.L.: Special purpose hardware for digital filtering.
Proc. I.E.E.E., Vol. 63, pp 633-48, April 1975.
53. FREENY, S.L.: A survey of special purpose hardware for dig. filters.
Proc. of Florence Conf. on Dig. Signal Proc., Sept. 1975.
54. WEINSTEIN, C.J. and OPPENHEIM, A.V.: A comparison of roundoff noise in
floating-pt. and fixed-pt. dig. filter realizations.
Proc. I.E.E.E., Vol. 57, pp 1181-83, June 1969.
55. GOLD, B. et al.: Theory and implementation of discrete Hilbert Transform.
in Dig. Signal Proc., I.E.E.E. Press, pp 94-109, 1972.
56. HERRMANN, O.: Transversal filter zur Hilbert Transformation.
A.E.U., Band 23, Heft 12, pp 581-87, Dec. 1969. (German)
57. CIZEK, V.: The discrete Hilbert Transform.
I.E.E.E. Trans. A. and E.A., Vol. AU18, pp 340-43, Dec. 1970.
58. BONZANIGO, F.: A note on the discrete Hilbert Transform.
I.E.E.E. Trans. A. and E.A., Vol. AU20, Dec. 1972.
59. RABINER, L.R. and SCHAFER, R.W.: On the behaviour of the minimax
F.I.R. digital Hilbert Transformer.
B.S.T.J., Vol. 53, No. 2, pp 363-90, Feb. 1974.
60. HAYKIN, S.S.: Different forms of the Hilbert Transformer as applied
to digital filters and sequences.
Proc. I.E.E.E., Vol. 121, No. 7, July 1974.
61. RABINER, L.R.: Techniques for designing F.I.R. digital filters.
I.E.E.E. Trans. Comm Tech., Vol. COM19, pp 188-95, April 1971.
62. McCLELLLEN, J.H. et al.: A computer programme for designing linear-
phase F.I.R. digital filters.
I.E.E.E. Trans. A. and E.A., Vol. AU21, pp 506-26, Dec. 1973.

63. RABINER, L.R.: Some comparisons between F.I.R. and I.I.R. dig. filters.
B.S.T.J., Vol. 53, No. 2, Feb. 1974.
64. CONSTANTINIDES, A.G.: Private Communication.
65. LEICH, H.: Filtres numerique a difference de phase, sensibilité
et structure.
Internal Report, Faculté Polytechnique de Mons, 1973. (French)
66. MITRA, S.K. and HIRANO, K.: Digital all-pass networks.
I.E.E.E. Trans. Cct. and Syst., Vol. CAS21, pp 688-700, 1974.
67. CONSTANTINIDES, A.G.: The nested all-pass network (to be published).
68. FETTWEIS, A.: Digital filter structures related to classical structures.
A.E.U., Vol. 25, pp 79-89, Feb. 1971.
69. LAWSON, S.S.: Dig. filt. structs. derived from classical analogue structs.
Ph.D. Thesis, Univ. of London, 1975.
70. BALABANIAN, N.: Network Synthesis.
publ. Prentice Hall, New Jersey, 1958.
71. CONSTANTINIDES, A.G.: Realization structures for non-rec. digital filters.
Proc. Florence Sem. on Digital Filters, Sept. 1972.
72. DASGUPTA, S.: Realization structures for non-recursive digital filters.
Ph.D. Thesis, Univ. of London, 1976.
73. LOCKHART, G.B.: Binary transversal filters with quantized coefficients.
Elec. Letters, Vol. 7, No. 11, pp 305-07, June 1971.
74. VLACH, J.: Computerized approximation and synthesis of linear networks.
publ. Wiley and Sons, New York, 1969.
75. STEIGLITZ, K.: Computer-aided design of non-rec. digital filters.
I.E.E.E. Trans. A. and E.A., Vol. AU18, pp 123-29, 1970.
76. DECZKY, A.G.: Synthesis of recursive digital filters using the
minimum p-error criterion.
I.E.E.E. Trans. A. and E.A., Vol. AU20, pp 257-63, 1972.

77. BANDLER, J.W. and BARDAKJIAN, B.L.: Least- p^{th} optimization of recursive digital filters.
I.E.E.E. Trans. A. and E.A., Vol. AU21, pp 460-70, 1973.
78. BROPHY, F.J. and SALAZAR, A.: Two design techniques for digital phase networks.
B.S.T.J., Vol. 54, pp 767-81, April 1975.
79. MCCLELLEN, J.H.: Computer opt. algorithms for dig. filter design.
I.E.E.E. Proc. ISCAS/76, pp 779-82, May 1976.
80. FLETCHER and POWELL: A rapidly convergent descent method for minimiz..
Brit. Computer J., Vol. 6, pp 163-68, 1963.
81. - Non-linear optimization routines (EO4FAF).
N.A.G. Library Manual, Document 328, May 1972.
82. PECKHAM, G.: A new method for minimizing the sum of squares without calculating the gradient.
Brit. Computer J., Vol. 13, pp 418-20, 1970.
83. SEVIOIRA and SABLATASH: A Tellegen's theorem for digital filters.
I.E.E.E. Trans Cct Th., Vol. CT18, pp 201-03, Jan. 1971.
84. FETTWEIS, A.: General theorem for signal-flow networks, with applications.
A.E.U., Vol. 25, pp 557-61, 1971.
85. COLLONA, S. et al.: Determination of sensitivity functions of dig. filts..
I.E.E.E. Trans. Cct. Th., Vol. CT19, pp 538-39, Sept. 1972.
86. CROCHIERE, R.E.: Some network properties of digital filters.
Proc. I.E.E.E. Int. Symp. on Cct Theory, pp 144-48, 1973.
87. OPPENHEIM, A.V.: Realization of digital filters using block-floating-point arithmetic.
I.E.E.E. Trans. A. and E.A., Vol. AU18, pp 130-36, 1970.
88. LIU, B.: Effect of finite wordlength on the accuracy of dig. filters.
I.E.E.E. Trans. Cct Th., Vol. CT18, pp 670-78, Nov. 1971.

89. KNOWLES, J.B. and EDWARDS, R.: The effect of a finite wordlength computer in sampled-data feedback system.
Proc. I.E.E., Vol. 112, pp 1197-1207, June 1965.
90. BENNET, W.R.: Spectra of quantized signals.
B.S.T.J., Vol. 47, pp 446-72, July 1948.
91. ROTHHAUSER, R.H.: Speech in digital communication systems.
I.E.E.E. Trans. A. and E.A., Vol. AU 21, pp 21-26, 1973.
92. KNOWLES and OLCAYTO: Coefficient accuracy and dig. filter response.
I.E.E.E. Trans. Cct. Th., Vol. CT15, pp 31-41, 1968.
93. JACKSON, L.B.: Analysis of roundoff noise in digital filters.
D.Sc. Dissertation, Stevens Inst. of Technology, 1969.
94. JACKSON, L.B.: On the interaction of roundoff noise and dynamic range in digital filters.
B.S.T.J., Vol. 49, pp 159-84, Feb. 1970.
95. PARKER and HESS: Limit-cycle oscillations in digital filters.
I.E.E.E. Trans. Cct. Th., Vol. CT18, No. 6, Nov. 1971.
96. JACKSON, L.B. et al.: An approach to the implementation of dig. filters.
I.E.E.E. Trans. A. and E.A., Vol. AU16, pp 413-21, 1968.
97. FEITWEIS, S.: On the connection between multiplier wordlength limitation and roundoff noise in digital filters.
I.E.E.E. Trans. Cct. Th., Vol. CT19, pp 486-91, 1972.
98. AVENHAUS, E.: On the design of digital filters with coefficients of limited wordlength.
I.E.E.E. Trans. A. and E.A., Vol. AU20, pp 206-12, 1970.
99. DUBOIS, H.: Optimisation d'une transmittance de filtre numerique recursif dans l'espace discret des coefficients.
D.Sc.A. Thesis, Faculté Polytechnique de Mons, 1975.

100. CONSTANTINIDES, A.G.: Some sensitivity aspects concerning dig. filts.
I.E.E. Colloquium Digest No. 1973/3 Jan. 1973.
101. BOITE, R. et al.: Optimization of digital filter coefficients in the
discrete space of coefficients.
Elec. Letters, Vol. 10, pp 179-80, 1974.
102. MUIR, A.J.L. and LAWSON, S.S.: Private Communication.
Plesseys Ltd.
103. CONSTANTINIDES, A.G. and COLYER, I.: Digital phase-splitting network
design for digital F.D.M. applications.
Proc. I.E.E., Vol. 123, No. 12, Dec. 1976.
104. BOX, M.; DAVIES, D.; SWANN, W.: Non-linear optimization techniques.
Math. and Stat. Tech., I.C.I. Monograph No. 5, 1969.
105. EDWARDS, G.P. and ASH, C.P.: Recent advances in digital filtering.
PYE TMC Ltd., 1976.
106. HAINE, J.: New active quadrature phase-shift network.
Elec. Letters, Vol. 13, No. 7, pp 216-18, 1977.
107. : 'SPARQ' Functional Specification.
Project Doc. No. 01-1261/1/3, Scicon Consultancy Ltd., 1977.
108. PARASURAMAN, B.: Hardware multiplication techniques.
Computer Design, pp 75-82, April 1977.