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The software implementation and
applications of the
quasi-steady-quasi-homogeneous (QSQH)
theory in near-wall turbulence

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Statement of Originality

I, Yunjiu Yang, hereby declare that this thesis entitled "The software implementation and applications of the quasi-steady-quasi-homogeneous (QSQH) theory in near-wall turbulence" is the result of my own research and efforts. I confirm that: this work has not been submitted, either in whole or in part, for any other degree or qualification at any other university or institution; all information obtained from external sources has been properly cited within the thesis; and all contributions, including suggestions and advice provided by my supervisor as well as other academics both within and outside Imperial College London, are duly acknowledged.

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Abstract

The three-dimensional implementation of the Quasi-Steady-Quasi-Homogeneous (QSQH) theory of scale interaction in near-wall turbulence has been integrated into an online software tool. This tool takes the wall friction distribution of a given flow as input and generates a set of velocity snapshots that exhibit statistics consistent with those predicted by QSQH theory, while simultaneously having the statistical characteristics of the large-scale component of the input wall friction field. Consequently, this tool facilitates straightforward calculation of QSQH predictions for various flow statistics without requiring any derivations. The application of this tool was demonstrated in a number of examples involving plane channel and circular pipe flows in the range of the friction Reynolds numbers from 1000 to 6000. These examples encompass investigations into the relative error associated with Taylor expansions in the amplitude of large-scale motions, the validity of the so-called “QSQH hypotheses per se”, compliance of the large-scale motions with the QSQH theory, variations in Reynolds shear across different Reynolds numbers, and QSQH predictions of two-point correlations at distinct wall-parallel locations. It also included a study of the feasibility of employing the QSQH model for predicting changes in velocity statistics across flows characterized by varying geometry (channel and pipe) and Reynolds numbers. The examined statistics include mean velocity profiles, turbulence intensity profiles, Reynolds shear stress, skewness and flatness measures for velocity fluctuations, and two-point correlation coefficients. A modification of the QSQH theory was proposed to address mass and momentum transport within the mathematical framework of the theory, in particular to ensure the conservation of mass for large-scale velocity components, as well as the conservation of mean longitudinal momentum up to second-order terms in the Taylor expansions of stress terms defined by the QSQH theory.

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Chapter 1

Introduction

1.1 Motivation

To introduce the motivation for this thesis, it is essential to briefly review the historical development of QSQH theory. The foundational work on QSQH theory began over a decade ago ([Chernyshenko et al., 2012](#)). This research was inspired by an empirical prediction model for near-wall turbulence ([Mathis et al., 2009](#)), which was based on observations that large-scale motions in the outer layer exert a superposition and modulation effect on near-wall turbulence. This empirical model successfully reconstructs the near-wall velocity signal using low-pass filtered velocity signals from the outer layer and two empirical coefficients that account for large-scale superposition and modulation effects. A comprehensive review of this model can be found in section 2.1. In ([Chernyshenko et al., 2012](#)), a fundamental theoretical framework for QSQH theory was established, focusing on describing large-scale superposition and modulation while investigating the universality of near-wall turbulence. This is achieved by replacing conventional viscous scaling variables with large-scale variables at the wall during flow non-dimensionalization. The early work demonstrated that superposition and modulation could be approximately described using this alternative scaling method; subsequently, ([Zhang and Chernyshenko, 2016](#)) was so motivated to further develop this theoretical framework and formally introduced QSQH theory as a means to describe both near-wall superposition and modulation, as well as predict near-wall turbulence. While the early version of QSQH theory

specifically addresses superposition and modulation for streamwise velocity, it refines the classical universality hypotheses by positing that near-wall turbulence is in equilibrium with the large-scale component of wall friction rather than mean wall friction. Consequently, QSQH theory assumes that variations in near-wall turbulence are primarily driven by changes in large-scale motions and their associated modulation. The initial version of the QSQH theory was found to be sufficient for predicting one-point statistics up to the second moment of streamwise velocity in turbulent channel flow (Zhang and Chernyshenko, 2016; Zhang, 2019; Chernyshenko et al., 2019). Extensive studies were conducted on the selection of the large-scale filter employed in the theory, aiming to enhance predictive accuracy. However, related research (Agostini and Leschziner, 2018, 2019a) during that period revealed that QSQH theory exhibited quite poor predictions regarding the second moment of spanwise velocity in channel flow. This issue prompted a refinement of the QSQH theory (Chernyshenko, 2021), wherein the direction of the large-scale component of wall friction was incorporated into the framework, with hopes of improving predictions for spanwise velocity statistics. One of the motivations of this thesis is to examine whether the refinement enables the QSQH theory to accurately predict the spanwise velocity statistics, and how the refinement affects the prediction on streamwise velocity statistics.

Although the QSQH theory possesses a comprehensive theoretical framework, much of the prior work necessitates significant effort to derive the Taylor expansions of the second moments of velocities as stipulated by the theory in order to compute their values. For any further investigations into the statistics predicted by QSQH theory, deriving these Taylor expansions will become increasingly cumbersome as the order of moments increases. It is also important to note that previous studies have only derived Taylor expansions up to second order, resulting in higher-order terms being neglected. These higher-order terms could become more significant as the Reynolds number increases; therefore, employing a second-order Taylor expansion risks the predictions by QSQH theory to be inaccurate. Therefore, at this stage, it is essential to develop a model that incorporates QSQH theory and facilitates straightforward and precise calculations of flow statistics.

The development of QSQH theory was initially driven by the potential to "extrapolate" flow statistics from low-Reynolds-number cases to high-Reynolds-number

scenarios. The objective is to predict changes in the flow statistics between two different Reynolds numbers using QSQH theory. If the second moment for the low-Reynolds-number case is known, it becomes possible to determine that for the high-Reynolds-number case. Numerous attempts at extrapolation are documented in (Zhang, 2019), demonstrating that this method is particularly effective for stream-wise velocity when both cases are channel flow. Now an interesting and intuitive question is whether such effective extrapolation still holds, if two cases are of different types of flow of different Reynolds numbers.

Last but not least, the QSQH theory does not necessarily provide the “correct” momentum transport and energy distribution for the predicted flow statistics, as observed in Direct Numerical Simulations (DNS) that are derived from solving the Navier-Stokes equations. The author posits that this limitation of the theory accounts for most of the discrepancies between its predictions and DNS results, particularly concerning the second moments of velocity. This is significant because second moments are predominantly influenced by turbulence energy distribution. To enforce mass and momentum conservation in the QSQH theory, a refinement or modification to the current version of QSQH theory is essential, which is anticipated to substantially improve the accuracy of its predictions.

1.2 The structure of this thesis

There are seven chapters including this current chapter. Their contents are briefly summed up and explained below.

- Chapter 2: this chapter reviews the relevant literature that has supported and informed this study. Given that the QSQH theory aims to describe near-wall modulation, a brief history of universality and scale interactions in near-wall turbulence is presented. One of the objectives of this study is to generate synthetic velocity fields using the QSQH synthetic turbulence model. As a novel approach in synthetic turbulence modelling, attention is also given to providing an overview of other developed synthetic turbulence models. Since the framework for QSQH theory was proposed over a decade ago, an updated introduction is included to facilitate understanding of both QSQH theory and

the mechanisms underlying the QSQH synthetic turbulence model.

- Chapter 3: this chapter offers a comprehensive overview of the novel numerical implementation of the QSQH synthetic turbulence model. The numerical implementation has been developed into a software tool that incorporates both the QSQH synthetic turbulence model and a database of a universal velocity field. Technical issues are explicitly addressed and discussed, particularly in relation to the applications presented in this study.
- Chapter 4: this chapter is composed of two main sections that present the application of the software tool for a plane channel flow exclusively, and in cross-flow scenarios involving both plane channel flow and pipe flow. The section dedicated to channel-exclusive analysis focuses on predicting velocity statistics using the software tool, including, for the first time, the two-point correlation coefficients of velocity fluctuations in wall-parallel directions for plane channel flow at Reynolds numbers of $Re_\tau = 5200$ and 1000. In contrast, the cross-flow section emphasises predictions regarding changes in velocity statistics between pipe flows at different Reynolds numbers, as well as changes from plane channel flow to pipe flow (referred to as extrapolation from one flow regime to another in subsequent discussions).
- Chapter 5: this chapter introduces two novel modifications to the current version of QSQH theory, aimed at addressing the deficiencies in mass and momentum transport inherent within its framework. The original QSQH theory stipulates that the large-scale component of wall-normal velocity is zero, which effectively violates mass conservation principles. In this work, we re-derive the large-scale component of wall-normal velocity and obtain a new expression that ensures the mass conservation. By employing Taylor expansions of parameters within QSQH theory and substituting them into the mean momentum equation in the streamwise direction, we establish a non-trivial constraint for the universal velocity field and the large-scale filter utilized in the QSQH synthetic turbulence model. The objective of these modifications is to enhance the applicability of the QSQH synthetic turbulence model for turbulence prediction.

- Chapter 6 (Discussion) : this chapter contains a discussion examining the limitations of the QSQH theory and the QSQH synthetic turbulence model, along with suggestions for potential approaches to enhance the QSQH theory.
- Chapter 7 (Conclusions): this chapter provides a concise overview of the entire study, while also summarizing the results and issues encountered in this study.

1.3 Database utilised in this research

The QSQH synthetic turbulence model and the software tool developed in the present study are dependent on availability of a quality Direct Numerical Simulation (DNS) database of near-wall turbulence flow. Further study using the synthetic model and the tool also need even more DNS data. Brief data on the DNS databases for plane channel flow and pipe flow used in this study are presented in Table 1.1. This information will be referenced in subsequent chapters in appropriate forms to facilitate comprehension of the content. It is useful to note that the pipe flow database utilized in this study contains only snapshots taken at the radial position closest to the wall.

1.4 Contribution of the author

The QSQH synthetic turbulence model and its software implementation are entirely founded on the mathematical framework of QSQH theory, as derived by Prof. Chernyshenko. Nearly all formulations presented in Section 2.3 have been adapted from previous studies (Chernyshenko et al., 2012; Zhang and Chernyshenko, 2016; Zhang, 2019; Zhang and Chernyshenko, 2016). In addition to the foundational aspects of QSQH theory, my contributions can be delineated as follows: I independently undertook the task of implementing QSQH theory into a functional model; developed the numerical method for this model; and computed the output synthetic velocity field. Furthermore, I conducted all data processing and analysis autonomously. In the rest of the section, the term "my results" will refer specifically to outcomes generated using the QSQH synthetic turbulence model. In highlights,

Type	Re_τ	Domain		Source
		$L_x \times L_y \times L_z$ (L_x/R for pipe)	Snapshots	
Channel	1000	$8\pi h \times 2h \times 3\pi h$	10	(Lee and Moser, 2015)
Channel	5200	$8\pi h \times 2h \times 3\pi h$	3	(Lee and Moser, 2015)
Pipe	1140	15	101	(Pirozzoli et al., 2021)
Pipe	3000	15	31	(Pirozzoli et al., 2021)
Pipe	6000	15	3	(Pirozzoli et al., 2021)

Table 1.1: The DNS database used in this study. For plane channel flow, h is the half channel height and $Re_\tau = \frac{u_\tau h}{\nu}$; for pipe flow, R is the pipe radius and $Re_\tau = \frac{u_\tau R}{\nu}$.

- For the first time, I implemented a three-dimensional version of QSQH theory. I generated the universal velocity field using an available DNS database of near-wall turbulence.;
- I investigated the application of QSQH theory and the QSQH model in pipe flow. I also investigated the feasibility of the QSQH model in extrapolating velocity statistics from channel flow to pipe flow;
- I investigated the prediction of higher-order moments of velocity fluctuations, and two-point correlation in wall-parallel directions, by using the QSQH model.
- I published the software tool of the QSQH model on Github as an open-source project. An up-to-date package is provided including user manual, codes and test samples;
- My results demonstrated that the QSQH synthetic model is a far more appropriate choice for studying and implementing QSQH theory in near-wall turbulence than the utilization of Taylor expansions in early studies ([Chernyshenko](#)

et al., 2012; Zhang and Chernyshenko, 2016; Zhang, 2019; Zhang and Chernyshenko, 2016), especially for high Reynolds numbers ($Re_\tau \gtrsim 5000$);

- My results demonstrated that the QSQH model provides accurate predictions of the changes in velocity statistics including: mean velocity profile; turbulence intensities; Reynolds shear stress; two-point correlations, between the flows of high Reynolds numbers ($Re_\tau \gtrsim 3000$);
- I proposed a new version of QSQH theory, in fulfilment of the partial mass and momentum conservation in the mathematical framework of QSQH theory.

Within the duration of my Ph.D., the QSQH synthetic turbulence model and its accompanying software tool have been presented to the academic community on several occasions: at interdisciplinary-Turbulence-initiative in 2023; during the UKFN SIG session on “Flow Instability, Modelling, and Control” in 2024; and at ICTAM 2024. I am also submitting two papers related to the content of this thesis to “Physical Review Fluids” and “Theoretical and Computational Fluid Dynamics”, respectively.

The content presented in this thesis does not encompass all of my efforts dedicated to this study. At the outset of my PhD study, Prof. Chernyshenko suggested to generate a universal velocity field based on our own produced velocity field rather than relying on existing DNS databases. A work package was devised to create a plane channel flow within the near-wall region that would be independent of Reynolds number, utilizing the available kinematic turbulence model; however, this endeavour proved unsuccessful due to its complexity and resulted in an unreasonable expenditure of time without yielding meaningful outcomes. Following the cessation of this work package, I shifted my focus towards exploring other numerical models capable of generating Reynolds number-independent near-wall turbulence (see Section 3.1 for more details).

Due to time constraints, I am unable to compile materials and present an analysis regarding the application of the QSQH software tool on DNS for plane channel flow concerning minimum flow units provided by Mr Le Yin from École Centrale Lille. In the near-wall turbulence of his DNS, large-scale motions—defined by our Fourier cut-offs of $\lambda_x^+ = 2000\pi$ and $\lambda_z^+ = 250\pi$ —exhibit higher magnitudes compared to

those observed in conventional channel flows as reported by (Lee and Moser, 2015). This particular scenario may be more suitable for applying the QSQH synthetic model, as the influence of large-scale effects prescribed by QSQH theory is likely to be more pronounced.

Chapter 2

Literature Review

2.1 Universality and scale interactions in near-wall turbulence

Classical universality hypothesis of near-wall turbulence states that if the flow parameters are made non-dimensional using the so-called wall units, that is, units based on the mean friction at the wall, density and kinematic viscosity, the near-wall flow statistics are independent of the Reynolds number Re and the large-scale geometry of the flow provided that Re is sufficiently large. This idea can be traced to the seminal work of Ludwig Prandtl (Prandtl, 1925), in which he stated: “These fundamental assumptions can be so expressed ... that the processes in proximity of a wall are not dependent upon the distance of the opposite wall” (cited by the English translation (Prandtl, 1949)). The literature related to this hypothesis and its relation to the topic of this thesis is so large that covering it all is almost impossible. Instead, the reader is referred to the thematic issue (McKeon and Sreenivasan, 2007) and a review (Smits et al., 2011). Here we will only signpost the outline of the topic.

The classical universality hypothesis proved to be extremely productive but also controversial. Even in the simplest case of the mean velocity profile, the statement of its universality, as formulated in many textbooks (see, for example, (Pope, 2000)), continues to be questioned and defended (Luchini, 2017). In practice, the universality hypothesis has been widely used in industrial applications and commercial

CFD codes in the form of the law of the wall. While the foundations of the classical universality hypothesis stemmed largely from dimensional considerations, more recent progress in understanding near-wall turbulence provided other arguments in its favour. For example, the numerical study (Jiménez and Pinelli, 1999) indicated that the existence of the autonomous cycle of near-wall turbulence, which extracts energy from the mean flow and forms a clear peak in the energy spectrum of the flow in the near-wall region, does not require a contribution from the fluctuations of the outer flow. Overall, the validity of the classical universal hypothesis, at least as a good and useful approximation, is beyond any doubt.

On the other hand, there is evidence contradicting the classical universality hypothesis. In high- Re turbulent near-wall flows, two peaks in the turbulent kinetic energy profile are observed (for example, see the review (Smits et al., 2011)). The first peak is located in the buffer layer between 10 and 20 wall units from the wall, while the distance from the wall to the second, outer, peak, if measured in wall units, varies with Re . The outer peak is caused by turbulent motions with the length scales that are significantly larger than those causing the inner peak. It has been long known that the large-scale motions affect the near-wall flow, and more and more confirmations were accumulating. In experiments (Rao et al., 1971) it was observed that the mean period of bursts in the buffer layer scales with outer variables. This led to the conclusion that the turbulence dynamics involve a coupling between outer and near-wall structures. Measurements of velocity correlations across the boundary layer indicated a link between large-scale structures in its outer part and the bursts in the buffer layer (Blackwelder and Kovasznay, 1972). Experimental investigation (Brown and Thomas, 1977) revealed that large-scale motions impose slow variations on the wall shear stress in the turbulent boundary layer. Later, in (Bandyopadhyay and Hussain, 1984), coupling between large and small-scale motions in shear flows was observed in experiments. The experiment in turbulent boundary layer Wark and Nagib (1991) showed that sweeps and ejections of the bursts are associated with large and small-scale structures respectively. Experiments at very high Reynolds number in the atmospheric boundary layer showed that even close to the wall, due to the influence of large-scale motions, the maximum value of the root mean square streamwise velocity, if expressed in wall units, grows as a logarithm of the Reynolds number (Metzger and Klewicki, 2001). Further

numerical and experimental studies, such as (Abe et al., 2004; Hutchins and Marusic, 2007a), also revealed the emergence of very large-scale structures in the outer layer as Re increases, and showed that they impose energetic footprint on the near-wall region. Recent study (Hwang, 2024) reported that as Re_τ increases, the near-wall streamwise turbulence intensity grows due to the spectral contribution of log-layer motions (e.g., self-similar attached eddies), whose energy spans scales from the logarithmic region down to the near-wall region. The near-wall self-sustaining mechanism does play a key role in turbulence production, but the near-wall characteristics are not isolated due to the intrusion of the energy from outer large-scale structures and log-layer structures. In (Hutchins and Marusic, 2007a), in addition to finding the existence of outer large-scale structures at high Re , the large-scale amplitude modulation effect on the near-wall small-scale fluctuations was observed. Frequency modulation was also found (Ganapathisubramani et al., 2012). Analogous effects of large-scale modulation were also observed in experiments for spanwise and wall-normal velocity (Talluru et al., 2014).

These experimental and numerical observations were also expressed in the form of empirical formulae relating the near-wall and large-scale motions. The first and most well-known such formula was based on a decomposition of the flow velocity into small and large scales. It allowed for predicting the near-wall statistics of the turbulent flow on the basis of outer large-scale measurements (Mathis et al., 2009; Marusic et al., 2010; Mathis et al., 2011). Other empirical models include the model (Agostini and Leschziner, 2014), in which the scaling is done by local large-scale-modified friction velocity, and the effect of distortions due to large-scale sweeps and ejections is included in addition to the large-scale superposition and modulation. This modified model was used to successfully reconstruct the near-wall statistics at a single Re_τ of a turbulent channel flow (Agostini and Leschziner, 2016a). Further refinement of the model (Marusic et al., 2010; Mathis et al., 2011) was proposed in (Baars et al., 2016), in which the large-scale motions were defined as the result of the spectral linear stochastic estimation rather than of the application of a Fourier cut-off filter to the flow velocity.

In general, all the models, including the model corresponding to the QSQH theory (Chernyshenko et al., 2012; Zhang and Chernyshenko, 2016; Chernyshenko, 2021) considered in this thesis, share the innovative feature of the model of (Marusic

et al., 2010; Mathis et al., 2011): they have the form of a relationship expressing the turbulent velocity field in terms of the large-scale component of this field and of another, ‘universal’, velocity field, the statistics of which are assumed to be independent of the large-scale motions and often also assumed to be independent of the Reynolds number and the large-scale flow geometry. This allows one to derive, at least in principle, any statistics of the turbulent velocity field if the comprehensive data on the statistics of the large-scale motions and the statistics of the universal velocity field are given. The models differ by the formulae expressing the model, the definitions of the large-scale motions, and the sources of the universal velocity field. The formulae might be purely empirical , or based on some theories . Large-scale motions are often defined using various forms of a Fourier cut-off filter, however, the aforementioned model (Baars et al., 2016) uses stochastic estimation, empirical mode decomposition (Huang et al., 1998) was used in the model of (Agostini and Leschziner, 2014), and a neural network auto-encoder was used in (Agostini and Leschziner, 2022). The universal field can be obtained from a suitable direct numerical simulation results and the main formula, as for example it will be the case in the present work, however, it might also be considered to be the flow field of a flow that is expected not to possess any large scale motions. For example, in (Hwang, 2013; Yin et al., 2018) the minimal turbulent flow unit was generated in the absence of outer motions. Another possible candidate for the universal velocity field is described in (Carney et al., 2020; Carney and Moser, 2023), where the study claims its numerical simulation successfully captures the dynamics of small-scale near-wall turbulence in a small domain.

2.2 Synthetic turbulence model

The QSQH model developed in the study described in this thesis is fundamentally a synthetic turbulence model. The concept of synthetic turbulence modelling dates back to the 1960s. During that time, the Smagorinsky model was introduced in (Smagorinsky, 1963), which brought forth the concept of using subgrid-scale viscosity to represent unresolved turbulence. In 1998, the synthetic eddy method was proposed (Lund et al., 1998). This method aimed to replicate the energy cascade of turbulence by embedding small-scale eddies within a mean flow field. The ap-

plication of the random Fourier modes method has been prominent in kinematic turbulence simulations. For instance, a number of researchers (Fung et al., 1992; Malik and Vassilicos, 1999; Osborne et al., 2006; Clark and Vassilicos, 2011) utilized this approach to generate synthetic turbulence by adding random Fourier modes to the mean flow field. This method offers control over turbulence characteristics, including intensity and length scales. It is worth noting that kinematic turbulence simulations often find use in the study of turbulent particle dispersion. The coherent vortex simulation was introduced in (Farge and Schneider, 2001), which aims to generate coherent vortex structures by superimposing the synthetic streamwise vortex onto a mean flow field. As described below, the QSQH synthetic model we developed uses a large database of ‘universal’ velocity field, which makes it a special case of synthetic turbulence models.

2.3 What is QSQH theory

2.3.1 Preliminaries

The general flow setup pertinent to the QSQH theory, see Fig. 2.1, is that of a wall-bounded turbulent flow, such as, for example flows in a plane channel, pipe, or a boundary layer. Such flows are typically subdivided into an outer layer and a near-wall region consisting of the buffer and viscous layers, with a logarithmic layer being the common, or overlap, part of the outer and near-wall regions.

The overarching presumption of the QSQH theory and other related theories is that in a turbulent flow any field variable can be decomposed into a sum of a large-scale and small-scale parts: $f(t, x, y, z) = f_L(t, x, y, z) + f_S(t, x, y, z)$, where t is time, and x, y, z are streamwise, wall-normal and spanwise coordinates respectively. It is assumed that the large-scale part can be obtained from the total field by applying an operator, usually called a large-scale filter. It will be denoted $\mathcal{L}$, so that $f_L = \mathcal{L} f$. In this thesis the subscripts L and S will always denote the large-scale and small-scale components respectively. Usually, although not always, the mean field $\langle f \rangle$, where the angular bracket represents ensemble average, is assumed to be included into the large-scale part, so that $\langle f_S \rangle = 0$. This is assumed in the present study. Then one can introduce the large-scale component of wall friction $\tau_L^\diamond$ and define the

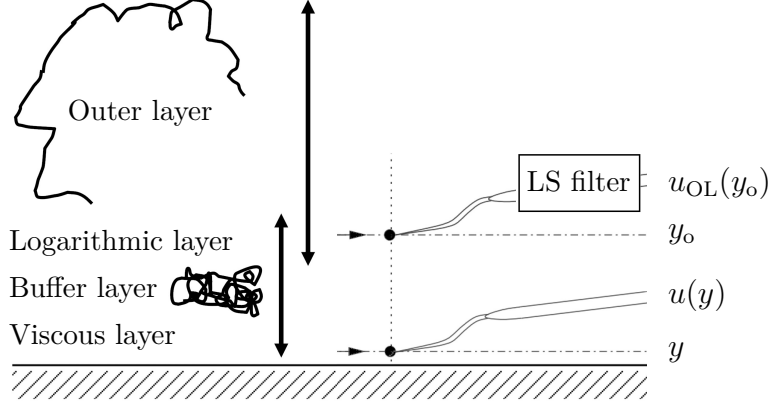


Figure 2.1: Flow setup and the scheme of the experiments of (Mathis et al., 2009; Marusic et al., 2010; Mathis et al., 2011).

large-scale friction velocity as the square root of the magnitude of $\tau_L^\diamond$ divided by the density, $u_{\tau_L}^\diamond = \sqrt{\tau_L^\diamond / \rho^\diamond}$ (presuming $\tau_L^\diamond \geq 0$). Here, the diamond superscript indicates dimensional quantities. Note that $\tau_L^\diamond$ is a wall-parallel vector and that both $\tau_L^\diamond$ and $u_{\tau_L}^\diamond$ vary in time and in wall-parallel directions, so that $u_{\tau_L}^\diamond = u_{\tau_L}^\diamond(t^\diamond, x^\diamond, z^\diamond)$. The flow variables will be non-dimensionalised using the average of $u_{\tau_L}^\diamond$:

$$(u, v, w) = (u^\diamond, v^\diamond, w^\diamond) / \langle u_{\tau_L}^\diamond \rangle, \\ u_{\tau_L} = u_{\tau_L}^\diamond / \langle u_{\tau_L}^\diamond \rangle, \quad (x, y, z) = (x^\diamond, y^\diamond, z^\diamond) \langle u_{\tau_L}^\diamond \rangle / \nu^\diamond, \quad t = t^\diamond \langle u_{\tau_L}^\diamond \rangle^2 / \nu^\diamond. \quad (2.1)$$

Here, $\nu^\diamond$ is the kinematic viscosity of the fluid, and the angular brackets denote averaging. It is noteworthy that there exists an event in near-wall turbulence where the streamwise velocity becomes negative, a phenomenon referred to as backflow in (Lenaers et al., 2012). In channel flow at $Re_\tau = 1000$, the occurrence of backflows is reported to be 0.06%, and these events are observed up to approximately $y^+ \approx 8.5$. Consequently, the assumption of $\tau_L^\diamond \geq 0$ is not entirely accurate. However, backflow occurrences are exceedingly rare for the range of Re_τ investigated in this study. The values of τ_L calculated from snapshots of pipe flow up to $Re_\tau = 6000$ and channel flow up to $Re_\tau = 5200$ remain positive. Therefore, in practical terms, the assumption of $\tau_L^\diamond \geq 0$ holds true within the context of this study.

The well-known dynamic velocity is $u_\tau^\diamond = \sqrt{\langle \tau^\diamond \rangle / \rho^\diamond}$. From the definitions and

properties just stated above, it follows that $u_\tau^\diamond = \sqrt{|\langle \tau_L^\diamond \cos \theta \rangle| / \rho^\diamond} = \sqrt{|\langle u_{\tau_L}^{\diamond 2} \cos \theta \rangle|}$, where θ is the angle between τ_L and the mean flow direction. Therefore, in the non-dimensional units (2.1),

$$u_\tau = \sqrt{|\langle \tau_L \cos \theta \rangle|} = \sqrt{|\langle u_{\tau_L}^2 \cos \theta \rangle|}. \quad (2.2)$$

The variables in the well-known wall units are given by

$$\mathbf{x}^+ = (x^+, y^+, z^+) = (x, y, z)u_\tau, \quad t^+ = tu_\tau^2, \quad (u^+, v^+, w^+) = (u, v, w)/u_\tau. \quad (2.3)$$

If the large-scale component of the wall friction does not fluctuate, that is, if it is equal to the mean wall friction, the non-dimensional units (2.1) coincide with the classical wall units. In units (2.1), the formulae of the QSQH theory are more compact. At moderate values of the Reynolds number, $u'_{\tau_L} = u_{\tau_L} - 1$ and θ are quite small, so that the wall units and the units (2.1) are very close. For this reason, they are often used interchangeably, as well as their one-dimensional version, corresponding to the case of $\theta = 0$, as for example (2.8) in (Chernyshenko, 2021). In general, however, caution is needed when both the wall units and the QSQH units (2.1) are used together (see the appendix A for more detail).

2.3.2 Motivation

In general, the flow velocity expressed in wall units depends not only on time and coordinates but also on the Reynolds number Re and the geometry of the flow:

$$u^+ = u^+(t^+, \mathbf{x}^+, Re).$$

Using wall units, the classical idea of universality (Prandtl, 1925, 1949) can be expressed as a statement that in the logarithmic layer and below it near the wall as the Reynolds number tends to infinity, the Reynolds number and other outer parameters disappear from the set of the arguments, so that one can write

$$u^+ = u^*(t^+, \mathbf{x}^+). \quad (2.4)$$

Whether it is a pipe, channel, or a boundary layer, near the wall the classical idea of universality expects that at sufficiently high Re the statistics of the velocity expressed in wall units are virtually the same.

Obviously, such a statement is not valid in the outer region, where there are large-scale motions with properties that are not universal in wall units. The influence of large-scale motions on the near-wall flow was first quantified in a compact form of the now widely known empirical relation (Marusic et al., 2010; Mathis et al., 2011)

$$u'^+ = \alpha(y^+)u'_{\text{OL}}^+ + (1 + \beta(y^+)u'_{\text{OL}}^+)u^*(t^+, \mathbf{x}^+), \quad (2.5)$$

where ‘OL’ stands for ‘outer large-scale’. As shown in Fig. 2.1, the large scale velocity fluctuation

$$u'_{\text{OL}}^+ = u'_{\text{OL}}^+(t^+, \mathbf{x}^+, Re)$$

is the velocity fluctuation measured in the logarithmic layer at y_o^+ and large-scale filtered. The large-scale filter in the experiment was a Fourier low pass cut-off filter in time domain. The experiments showed that u^* and u'_{OL}^+ were statistically independent provided a suitable choice of the functions $\alpha(y^+)$ and $\beta(y^+)$. This has strong implications. Since the large scale velocity u'_{OL}^+ is affected by the outer layer, statistics of u'_{OL}^+ (again, if measured in wall units) are not universal, that is they can depend on Re and the large-scale geometry of the flow boundaries. Then from (2.5) it follows that the statistics of u^+ cannot be universal, contrary to (2.4).

The magnitude of this deviation from the classical universality depends on the magnitude of fluctuations of the large-scale velocity. For the large-scale filter of Fourier cut-off of fix value in viscous unit, as Re increases, the magnitude increases, too. It is now understood also that it is this increase that is responsible for the increase in the magnitude of total fluctuations. There is very extensive literature on this subject, with (Lee and Moser, 2019; Smits et al., 2021) being recent and relevant. Numerous observations suggest that increase in the magnitude of fluctuations is proportional to the logarithm of Re , which implies that as $Re \rightarrow \infty$, this magnitude increases indefinitely. Whether this is true or not in the limit (Chen and Sreenivasan, 2021; Monkewitz, 2022; Hwang, 2024), this amplitude is not entirely negligible at the values of Re of practical interest. The reason for this is discussed in subsection 2.3.5.

Since the assumption of universality is quite useful, but turns out to be invalid,

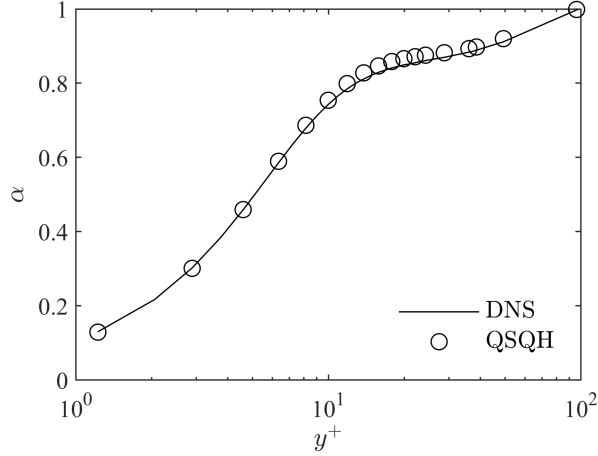


Figure 2.2: Comparison of the superposition coefficient α obtained by direct numerical simulation of a flow in a plane channel at $Re_\tau = 2000$ and the QSQH formula (2.7), after (Chernyshenko et al., 2019).

it is desirable to refine it. The QSQH theory is such a refinement. It describes a simple mechanism by which large-scale motions affect the near-wall turbulence.

An interest in a deeper consideration of the theory can be motivated by the following example of the results this theory provides. By definition as given in (Marusic et al., 2010), the ‘superposition’ coefficient α in (2.5) is

$$\alpha(y) = \frac{\langle u'_L(y)u'_L(y_o) \rangle}{\langle u'^2_L(y_o) \rangle}, \quad (2.6)$$

where prime denotes fluctuations and the angular brackets denote ensemble averaging. Under an additional assumption that the magnitude of the large-scale fluctuations is small, the QSQH theory gives (Chernyshenko et al., 2012; Zhang and Chernyshenko, 2016):

$$\alpha(y) \approx \frac{U(y) + y \frac{dU}{dy}(y)}{U(y_o) + y_o \frac{dU}{dy}(y_o)}, \quad (2.7)$$

where $U(y)$ is the mean velocity profile. Figure 2.2 shows that the QSQH prediction is accurate. The relation between the average of the product of velocity fluctuations and the mean velocity provided by (2.6) and (2.7) is reminiscent of the well-known unsolved turbulence closure problem of expressing Reynolds stresses in terms of

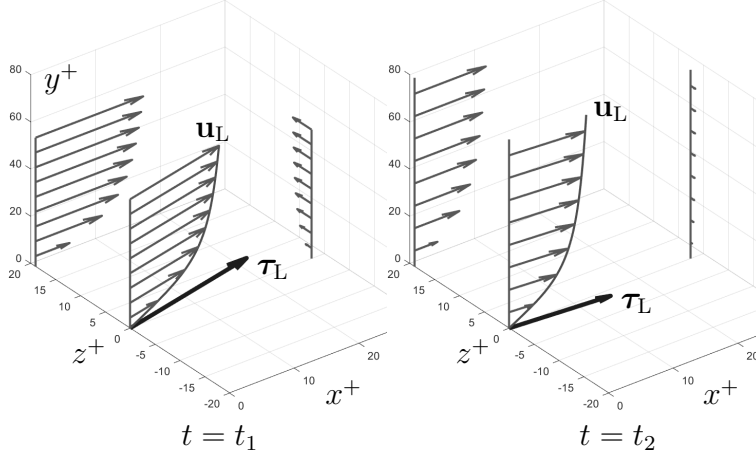


Figure 2.3: Large-scale velocity at two time instances, illustrating the concept of quasi-steady behaviour. The large-scale component of the wall friction τ_L and the large-scale component of the velocity $\mathbf{u}_L$ can depend on time. The scale of y -axis, about 80 wall units, indicates the wall distance where the concept is of primary interest.

the mean velocity. Even approximate results of this nature are rare. Of course (see section 2.3.6), as like any other theory, the QSQH theory has a domain of applicability, the boundaries of which are still being refined.

It should be noted that (2.2) is an estimation under the assumption of small large-scale fluctuation, which is increasingly inaccurate as the Re_τ of the flow increases. The coincidence between the DNS and QSQH result presented in Fig. 2.2 for $Re_\tau = 2000$, effectively confirms the validity of the assumption for such Reynolds number. Later in Section 2.3.6, the deteriorating validity of this assumption will be discussed.

2.3.3 The main idea of QSQH theory

The quasi-steady theory was first introduced in (Chernyshenko et al., 2012) and further developed in (Zhang and Chernyshenko, 2016) for treating only the longitudinal velocity component. In this section, the more general version accounting for all velocity components, introduced in (Chernyshenko, 2021), is briefly described.

To discuss how the large-scale motions affect the small-scale motions, one has first to define what large scale motions are. We will return to this question in subsection 2.3.4. For now we will assume that such definition is available. Figure 2.3

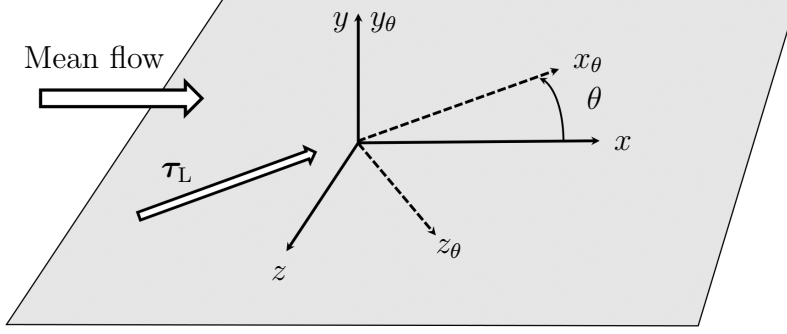


Figure 2.4: The coordinate system: x is in the direction of the mean flow, x_θ is in the direction of the large-scale component of the wall friction, y and y_θ are in the wall-normal direction, z and z_θ are in the wall plane and perpendicular to x and x_θ respectively. The angle of the large-scale wall friction is θ .

shows conceptually the flow at two time instants. The thick line is the large-scale component of the wall friction. Above it, the large scale component of velocity is shown. Both vary with time. Note that both the direction and the magnitude of the wall friction vary. Now suppose that instead there were two flows with different mean wall friction, $\boldsymbol{\tau} = \boldsymbol{\tau}_L(t_1) = \text{const}$ in one flow and $\boldsymbol{\tau} = \boldsymbol{\tau}_L(t_2) = \text{const}$ in the other flow, and that both these flows satisfied the classical universality hypothesis. Then the difference in the mean wall friction direction suggests introducing a new coordinate system, and the difference in the magnitude of the mean wall friction suggests re-scaling the variables, making everything expressed in the standard wall units. After rescaling, from the classic universality hypothesis it would follow that the statistics of these two flows would be the same.

The idea of the QSQH theory is that this is exactly what we should do at every time instant and at every position on the wall, because while the large-scale component of the wall friction varies with time and wall-parallel directions, the rate of this variation is much smaller than the rate of the variation of the small-scale turbulence. Then we replace the mean friction in the classical universality theory with the large-scale component of the wall friction, $\boldsymbol{\tau}_L$. Basically, the QSQH theory amounts to the statement that near the wall turbulence and the large scale component of the wall friction remain in equilibrium.

To express the QHQS theory mathematically, one can introduce variables similar

to variables in wall units:

$$u_\theta(t, x_\theta, y_\theta, z_\theta) = u_{\tau_L} \tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}), \quad (2.8)$$

$$v_\theta(t, x_\theta, y_\theta, z_\theta) = u_{\tau_L} \tilde{v}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}), \quad (2.9)$$

$$w_\theta(t, x_\theta, y_\theta, z_\theta) = u_{\tau_L} \tilde{w}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}), \quad (2.10)$$

where

$$\tilde{t} = tu_{\tau_L}^2, \quad \tilde{x} = x_\theta u_{\tau_L}, \quad \tilde{y} = y_\theta u_{\tau_L}, \quad \text{and} \quad \tilde{z} = z_\theta u_{\tau_L} \quad (2.11)$$

is analogous to the independent variables in wall units in the classical universality assumption. Note that $(u_\theta, v_\theta, w_\theta)$, u_{τ_L} and $(t, x_\theta, y_\theta, z_\theta)$ are non-dimensional parameters which are scaled following the formulas of (2.1). The local coordinate system $x_\theta, y_\theta, z_\theta$ used here is shown in Fig. 2.4, where the coordinate system x, y, z is aligned with the mean flow direction. The flow variables in these coordinate systems are related by the formulae:

$$x_\theta = x \cos \theta - z \sin \theta, \quad y_\theta = y, \quad z_\theta = x \sin \theta + z \cos \theta, \quad (2.12)$$

$$u = u_\theta \cos \theta + w_\theta \sin \theta, \quad v = v_\theta, \quad w = -u_\theta \sin \theta + w_\theta \cos \theta, \quad (2.13)$$

where θ is the angle between the direction of the mean wall friction and the large-scale wall friction.

The QSQH theory assumes that rather than the statistics of the variables in wall units: $u^+(t^+, x^+, y^+, z^+)$, $v^+(t^+, x^+, y^+, z^+)$ and $w^+(t^+, x^+, y^+, z^+)$, it is the statistics of $\tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$, $\tilde{v}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$ and $\tilde{w}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$ that are universal, that is independent of the large-scale motions, outer flow geometry, and the Reynolds number.

From the physical viewpoint, this means that the large-scale structures affect the near-wall turbulence only through the proxy of the large-scale component of the wall friction, τ_L , and in the following way. First, the entire near-wall turbulence structure turns to align with the direction of τ_L . Second, the magnitude of the velocity varies in proportion to the magnitude of τ_L : in Fig. 2.3 the magnitude of $\mathbf{u}_L$ at $t = t_1$ is greater than the magnitude of $\mathbf{u}_L$ at $t = t_2$. In the empirical model (2.5), this amplitude modulation effect is represented by the term $(1 + \beta(y^+) u_{OL}^+) u^*(t^+, x^+, y^+, z^+)$. Third, as it can be seen from (2.11), the time and spatial scales also are modulated, with

time scale modulated in proportion to $1/u_{\tau_L}^2$, and x, y and z modulated in proportion to $1/u_{\tau_L}$. In Fig. 2.3, the modulation of the wall-normal scale can be noticed by the height of the velocity profile at $t = t_1$ being smaller than the height of the velocity profile at $t = t_2$. Note that the frequency (that is time-scale) and x, y, z -scale modulation effects are not present in (2.5). On the other hand, the term $u'^+ = \alpha(y^+)u'_{OL}^+$ in (2.5), usually interpreted as the superposition of large scale motions onto the near-wall turbulence, has no counterpart in the QSQH theory, (2.8-2.13). We will return to this later in subsection 2.3.5, noting now only that, as it turns out, the QSQH theory does predict (2.5) and even provides expression (2.7) for $\alpha(y)$ in terms of the mean velocity profile of the flow, thus giving an alternative interpretation of the physical meaning of the term with α in (2.5).

2.3.4 Large scale motions

The precise definition of large-scale motions can be given by defining a large-scale filter. A large-scale filter is an operator, which we have already denoted $\mathcal{L}$, and which maps any field $f(t, \mathbf{x})$ into its large-scale component $f_L(t, \mathbf{x})$, so that $f_L = \mathcal{L} f$. However, the precise definition of large-scale motions of real flows is elusive. Fortunately, in order to do theoretical derivations within the QSQH theory, the precise definition is not necessary: it is sufficient to postulate the required properties of the large-scale filter (Zhang and Chernyshenko, 2016). These properties are:

- 1) linearity: for any constants a and b and functions $f(t, \mathbf{x})$ and $g(t, \mathbf{x})$

$$\mathcal{L}(af + bg) = a \mathcal{L} f + b \mathcal{L} g, \quad (2.14)$$

- 2) invariance of averages: the averaged variables are large-scale,

$$\mathcal{L} \langle f \rangle = \langle f \rangle, \quad (2.15)$$

- 3) projection property: the large-scale filter does not change an already large-scale-filtered function,

$$\mathcal{L} \mathcal{L} f = \mathcal{L} f, \quad (2.16)$$

4) commuting with averaging,

$$\langle \mathcal{L} f \rangle = \mathcal{L} \langle f \rangle, \quad (2.17)$$

5) scale-separation property: applying the large-scale filter to any function of t , $\mathbf{x}$, and other arguments that are large-scale-filtered variables is equivalent to averaging with the large-scale arguments held constant:

$$\mathcal{L} f(t, \mathbf{x}, Lg_1, \dots, \mathcal{L} g_n) = \langle f(t, \mathbf{x}, \xi_1, \dots, \xi_n) \rangle_{\xi_1=\mathcal{L} g_1, \dots, \xi_n=\mathcal{L} g_n} \quad (2.18)$$

for any functions $f(t, \mathbf{x}, \xi_1, \dots, \xi_n)$ and $g_i(t, \mathbf{x})$, $i = 1, \dots, n$. The fifth property means that the characteristic dimensions and times of the large scales are much larger than that of small scales.

Given (2.8–2.18), a large variety of useful relationships can be derived rigorously. Numerous examples can be found in (Zhang and Chernyshenko, 2016; Chernyshenko, 2021; Chernyshenko et al., 2019; Zhang, 2019). In cases when such a relationship involves a large-scale-filtered quantity, one needs to define the filter itself. If such a filter does not satisfy any of the above properties, the results of the theory have to be considered as approximate. An example of this situation is given by (2.6) and (2.7), where (2.6) involves the large-scale velocity $u'_L(y)$. This is why (2.7) is approximate.

Remarkably, some of the relationships derived by the theory do not involve large-scale quantities. To use them, knowledge of the large-scale filter is not required. However, their validity still depends on the existence of such a filter, and, of course, on the validity of the QSQH universality hypothesis itself. Formulae (3.15) and (3.16) and Fig. 3 in (Chernyshenko, 2021) give an example of such a case.

2.3.5 Taylor expansions in the magnitude of large-scale fluctuations and the superposition of large-scale motions

For the values of the Reynolds number achievable in direct numerical simulations, the amplitude of fluctuation of large-scale motions turns out to be small. While it is observed to grow with the Reynolds number, this growth is not fast. Hence, it makes sense to simplify the expressions obtained with the theory by expanding

them into Taylor series in powers of parameters characterizing the magnitude of the large-scale motions.

The Taylor expansions proved to be useful because the simplified formulae often reveal nontrivial physics. For example, the precise QSQH formula for the mean velocity, derived in (Chernyshenko, 2021), is

$$U(y) = \langle u_{\tau_L} \tilde{U}(y u_{\tau_L}) \cos \theta \rangle, \quad (2.19)$$

and the expression for the large-scale x -component of the velocity is

$$u_L = u_{\tau_L} \tilde{U}(y u_{\tau_L}) \cos \theta, \quad (2.20)$$

where $\tilde{U}(\tilde{y})$ is the mean of $\tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$. The fluctuation of u_{τ_L} is $u'_{\tau_L} = u_{\tau_L} - 1$. Assuming that $u'_{\tau_L} \ll 1$ and $\theta \ll 1$, expanding (2.20) into a power series, and keeping only the first-order term gives the formula for the fluctuation of the large-scale velocity

$$u'_L = u'_{\tau_L} \left(\tilde{U}(y) + y \frac{d\tilde{U}(y)}{dy} \right) + \dots$$

Substituting this into (2.6) gives the remarkable formula (2.7). Moreover, a comparison with (2.5) shows that up to the higher-order terms of the expansion in the magnitude of the fluctuation of the large-scale wall friction the ‘superposition’ term $\alpha(y^+) u'^+_{\text{OL}}$ is equal to $u'_{\tau_L} \left(\tilde{U}(y) + y \frac{d\tilde{U}(y)}{dy} \right)$. Notice now that in this formula $u'_{\tau_L} \tilde{U}(y)$ originates from the amplitude modulation effect described by (2.8) and (2.11), while $u'_{\tau_L} y \frac{d\tilde{U}(y)}{dy}$ originates from the y -scale modulation. Thus, while the term $\alpha(y^+) u'^+_{\text{OL}}$ in (2.5) appears to describe superposition, its true physical meaning is that of amplitude and scale modulation masqueraded by small amplitude of large-scale fluctuations and also by rewriting (2.8) in terms of fluctuations.

As it has already been mentioned, at the values of the Reynolds numbers currently achievable in numerical simulations the magnitude of large-scale fluctuations is not so large. If this is true, why cannot these fluctuations be neglected? In fact, often they can, but not always. It turns out that in some of the formulae the coefficients of the Taylor expansions are large. For example, consider the question of the variation of the root mean square of the longitudinal velocity fluctuation, u^2_{rms} , with

Re . The Taylor expansion of the QSQH theory results gives (Chernyshenko, 2021) that for any two sufficiently large values Re_1 and Re_2 of the Reynolds number

$$u_{\text{rms}}^2(Re_2) - u_{\text{rms}}^2(Re_1) = \left(\langle u_{\tau_L}'^2 \rangle \Big|_{Re=Re_2} - \langle u_{\tau_L}'^2 \rangle \Big|_{Re=Re_1} \right) \left(\frac{dyU}{dy} \right)^2 + \dots$$

In this formula the terms replaced with dots are indeed small and can be neglected at moderate Re apart maybe the close vicinity of the wall. The exact value of $\langle u_{\tau_L}'^2 \rangle \Big|_{Re=Re_2} - \langle u_{\tau_L}'^2 \rangle \Big|_{Re=Re_1}$ depends on the choice of the filter, but the assumptions of the theory imply that this dependence is weak (Chernyshenko et al., 2019). For $Re_{\tau,2} = 5200$ and $Re_{\tau,1} = 2000$ it was estimated that $\langle u_{\tau_L}'^2 \rangle \Big|_{Re=Re_2} - \langle u_{\tau_L}'^2 \rangle \Big|_{Re=Re_1} \approx 0.0026$ (Chernyshenko, 2021). This is, indeed, quite small. However, at $y^+ = 40$, $\left(\frac{dyU}{dy} \right)^2 \approx 290$. This gives $u_{\text{rms}}^2(Re_2) - u_{\text{rms}}^2(Re_1) \approx 0.75$, which is about 12% of the $u_{\text{rms}}^2 \approx 6$ value at this distance from the wall. This is not so small (and is in a reasonable agreement with the value obtained from direct numerical simulations). A similar situation arises in some other formulae.

2.3.6 Accuracy of QSQH theory

The QSQH theory is approximate. First, in the same way as the classical universality hypothesis, it assumes that Re is sufficiently large. Any application will, however, be at a possibly large but finite Reynolds number. Second, it assumes separation of scales: the large-scale motions are expected to have characteristic dimensions that are much larger than the characteristic dimensions of small scales. However, if, for example, a Fourier cut-off filter is used as the large-scale filter, the wavelengths of the Fourier modes just above the threshold, although counted as large-scales, are never much larger than the wavelengths of the modes just below the threshold. Third, at the values of Re currently attainable in numerical simulations and many experiments the magnitude of the large-scale motions is often small. Hence, the effects described by the QSQH theory compete with other physical effects. Fourth, when the Taylor expansion in the amplitude of the large-scale fluctuations is performed, dropping higher-order terms causes error. While this error is not the error of the theory per se, it is sometimes interpreted as such. Fifth, similarly, neglecting the fluctuation of the direction of the large-scale wall friction also causes an error which can be

misinterpreted as the error of the QSQH hypothesis itself.

Comparisons between the predictions of the QSQH theory and direct numerical simulations or experiments often require the knowledge of the large-scale component of the wall friction, and such data are rarely available. The (necessary incomplete) list of papers with comparisons or containing data that allow comparisons is (Chernyshenko et al., 2012; Agostini and Leschziner, 2014, 2018; Zhang and Chernyshenko, 2016; Baars et al., 2016, 2017; Agostini et al., 2017; Howland and Yang, 2018; Chernyshenko et al., 2019; Zhang, 2019; Agostini and Leschziner, 2019b; Lozier et al., 2019; Chernyshenko, 2021; Agostini and Leschziner, 2019a, 2022, 2016b). Overall, as it can be expected, the QSQH theory performs better for higher Re and smaller distances to the wall. The closest agreement between the QSQH theory prediction and the actual data so far observed is probably in the superposition coefficient in Fig. 2.2. So far, the largest, up to an order of magnitude, discrepancy was observed for the Reynolds-number dependence of the Reynolds stress, see Fig. 6 in (Chernyshenko, 2021). The reason for that large discrepancy is that for Re_τ between 2000 and 5200 as in (Chernyshenko, 2021), the effect of the pressure gradient on the Reynolds stress turns out to be much greater than the effect of large-scale motions.

The nature of the discrepancies and the limits of applicability of the QSQH hypothesis are further addressed in section 4.1 on the basis of the new results obtained in the present work, and in the discussion of Chapter 6.

Chapter 3

A tool based on QSQH theory

3.1 The general idea of the QSQH synthetic turbulence model and an attempt to construct an analytic universal field

The QSQH theory employs a representation of the turbulent flow field characterized by a ‘universal’ velocity field, alongside the magnitude (denoted as u_{τ_L}) and direction (represented by θ) of the large-scale component of wall friction (2.8-2.13). It is assumed that the statistical properties of this universal velocity field are consistent across all high-Reynolds-number flows, while those for u_{τ_L} and θ may vary. It is logical to pursue the development of a kinematic model for the universal velocity field, subsequently utilizing it to model actual flows through equations (2.8-2.13). Initial attempts in this direction were conducted by undergraduate students under the supervision of Prof. Chernyshenko (Yue, 2019; Loynes, 2020), with an emphasis on creating a tool capable of verifying results derived from complex QSQH calculations pertaining to a velocity field that fully adheres to QSQH theoretical assumptions. For this purpose, even very simple random velocity fields proved sufficient. In the present study, we aimed to use such a model for making predictions about real flows. This endeavour necessitates that the universal field possesses realistic characteristics.

In the initial phase of this study, we attempted to utilize the previously developed kinematic turbulence model (Clark and Vassilicos, 2011) to characterize the

universal velocity field. This task proved to be both challenging and time-consuming, primarily due to the fact that the properties of this field were not sufficiently aligned with real fluid flows. Additionally, developing a code capable of executing it efficiently presented significant difficulties. Ultimately, our findings indicated that an alternative approach, which is described below, offers a considerably more efficient solution.

3.2 QSQH software tools

In the study described in this thesis two ‘QSQH’ software tools were developed. The first tool, publicly available at (Yang, 2024), is an implementation of the QSQH synthetic turbulence model, described below. It allows the user to easily obtain statistics of the flow as they would be predicted by the QSQH theory, provided that the user has enough data on the wall friction in that flow to contain the statistics of the large-scale component of the wall friction. The second tool is a set of in-house codes that are needed for generating universal velocity field used by the first tool. Users are not required to utilize the second tool; however, it offers the autonomy to explore the universal velocity field as defined by QSQH theory.

The flow for which the QSQH prediction of statistics is desired and for which the necessary data on the shear stress distribution at the wall are available will be referred to as the target flow. A comprehensive set of QSQH-predicted statistics will be represented by the output of the model, which will be a database of snapshots of a synthetic velocity field such that in it the near-wall modulation due to large-scale motions is described by the QSQH theory presented in Section 2.3. Once the database is available, the desired statistics can be calculated by usual methods.

We will describe now the processes and formulae of the QSQH synthetic model.

3.2.1 Outline

The QSQH synthetic model comprises two main components: a database of the universal velocity field and a code for generating the QSQH synthetic velocity field. The current implementation is not time-resolved, even though an extension to a time-resolved case would be straightforward. The universal velocity field database

consists of N instantaneous snapshots of $(\tilde{u}(\tilde{x}, \tilde{y}, \tilde{z}), \tilde{v}(\tilde{x}, \tilde{y}, \tilde{z}), \tilde{w}(\tilde{x}, \tilde{y}, \tilde{z}))$ on a grid in a rectangular box:

$$\begin{aligned} -\tilde{L}_x/2 &\leq \tilde{x} \leq \tilde{L}_x/2, \\ 0 &\leq \tilde{y} \leq \tilde{L}_y, \\ -\tilde{L}_z/2 &\leq \tilde{z} \leq \tilde{L}_z/2. \end{aligned} \tag{3.1}$$

The values of N , $\tilde{L}_x$, $\tilde{L}_y$, $\tilde{L}_z$ and the parameters of the grid are specific for each version of the universal flow database.

In essence, the code takes the skin friction of the target flow in the form of the wall friction components $\tau_x^+(x^+, z^+)$ and $\tau_z^+(x^+, z^+)$ as an input, from which it generates a set of pairs $(u_{\tau_L, i}, \theta_i)$, $i = 1, \dots, I$, where I is the required number of velocity snapshots in the output database. It then allocates to each pair a snapshot of the universal velocity field, $(\tilde{u}_i, \tilde{v}_i, \tilde{w}_i)$, randomly selected from the universal flow database. Then I snapshots of QSQH synthetic velocity field are generated using (2.8 - 2.13) rewritten as:

$$\begin{aligned} u_i^+(x_i^+, y_i^+, z_i^+) &= u_{\tau_L, i}^+ [\tilde{u}_i(\tilde{x}, \tilde{y}, \tilde{z}) \cos \theta_i + \\ &\quad \tilde{w}_i(\tilde{x}, \tilde{y}, \tilde{z}) \sin \theta_i], \\ v_i^+(x_i^+, y_i^+, z_i^+) &= u_{\tau_L, i}^+ \tilde{v}_i(\tilde{x}, \tilde{y}, \tilde{z}), \\ w_i^+(x_i^+, y_i^+, z_i^+) &= u_{\tau_L, i}^+ [-\tilde{u}_i(\tilde{x}, \tilde{y}, \tilde{z}) \sin \theta_i + \\ &\quad \tilde{w}_i(\tilde{x}, \tilde{y}, \tilde{z}) \cos \theta_i], \end{aligned} \tag{3.2}$$

$$\begin{aligned} x_i^+ &= (\tilde{x} \cos \theta_i + \tilde{z} \sin \theta_i) / u_{\tau_L, i}^+, \\ y_i^+ &= \tilde{y} / u_{\tau_L, i}^+, \\ z_i^+ &= (-\tilde{x} \sin \theta_i + \tilde{z} \cos \theta_i) / u_{\tau_L, i}^+. \end{aligned} \tag{3.3}$$

These snapshots are the output of the code. Several technical issues are resolved in the particular ways described below.

3.2.2 Selecting representative values of the large-scale component of wall friction

The large scale filter $\mathcal{L}$ applied to the input field to obtain u_{τ_L} and θ must be the same as the large-scale filter used for generating the universal velocity field. To ensure this, the input required by the code is the unfiltered skin friction (τ_x^+, τ_z^+) rather than its large-scale component, and the filtering is done inside the synthetic model code, giving

$$\begin{aligned}\tau_{L,x}^+(x^+, z^+) &= \mathcal{L} \tau_x^+(x^+, z^+), \\ \tau_{L,z}^+(x^+, z^+) &= \mathcal{L} \tau_z^+(x^+, z^+).\end{aligned}\tag{3.4}$$

Then, u_{τ_L} and θ can be obtained from the formulae

$$\tau_L^+ = \sqrt{\tau_{L,x}^{+2} + \tau_{L,z}^{+2}}, \quad u_{\tau_L}^+ = \sqrt{\tau_L^+}, \quad \theta = -\tan^{-1} \frac{\tau_{L,z}^+}{\tau_{L,x}^+}.\tag{3.5}$$

For this procedure to be fully consistent, the physical domain size represented by each input snapshot must be sufficiently large for the application of the large-scale filter to be meaningful. For example, when $\mathcal{L}$ is a Fourier cut-off filter in the wall-parallel directions (x and z), the domain dimensions in these directions (i.e., the total span of the grid in x^+ and z^+ coordinates) should exceed the filter's cut-off lengths. This ensures that the Fourier transform, applied to a single snapshot over its x-z plane, can resolve scales larger than the cut-off. The input is assumed to consist of S snapshots on a uniform grid with points at x_j^+ and z_k^+ , $j = 1, \dots, J$, $k = 1, \dots, K$, so that the total number of grid points per snapshot is $J \times K$, with J not necessarily equal to K .

To generate a representative set of (u_{τ_L}, θ) pairs, the spatial sampling of these fields must account for their slow variation in the x and z directions. Specifically, sampling locations should be spaced sufficiently far apart in the physical domain to ensure that each pair (u_{τ_L}, θ) captures distinct values, avoiding redundancy from correlated or overly similar measurements. This spacing ensures coverage of the full parameter space spanned by u_{τ_L} and θ , even with their gradual spatial evolution. Also, while this may not be always the case, it might well happen that the input snapshots would be generated in a single numerical calculation and would correspond to consecutive time instants. In this case, the same reasoning applies to the sequence

of the snapshots. Without the use of interpolation, the points on the grid and the snapshots from which u_{τ_L} and θ are taken can be selected to be as close as possible to a uniform distribution in the following way. Given the number of such points in x and z directions to be P and Q respectively, and R to be the number of snapshots to be used from the set of all snapshots of the wall friction, we take

$$\begin{aligned} j_p &= [1.5 + (J - 1)(p - 1)/(P - 1)], \quad p = 1, \dots, P, \\ k_q &= [1.5 + (K - 1)(q - 1)/(Q - 1)], \quad q = 1, \dots, Q, \\ s_r &= [1.5 + (S - 1)(r - 1)/(R - 1)], \quad r = 1, \dots, R, \end{aligned}$$

where $[.]$ denotes an integer part. Then the values of u_{τ_L} and θ at the input grid points $(x_{j_p}^+, z_{k_q}^+)$ from the snapshots with numbers s_r are selected to constitute a set of $(u_{\tau_L, i}, \theta_i)$:

$$\begin{aligned} u_{\tau_L, i} &= u_{\tau_L}(s_r, x_{j_p}^+, z_{k_q}^+), \quad \theta_i = \theta(s_r, x_{j_p}^+, z_{k_q}^+), \\ p &= 1, \dots, P, \quad q = 1, \dots, Q, \quad r = 1, \dots, R, \\ i &= p + P(q - 1) + PQ(r - 1), \end{aligned} \tag{3.6}$$

where $u_{\tau_L}(s, x_j^+, y_k^+)$ and $\theta(s, x_j^+, y_k^+)$ are u_{τ_L} and θ in the snapshot number s . The size of the resulting set of $(u_{\tau_L, i}, \theta_i)$ will be $I = P \times Q \times R$.

3.2.3 Resolving rotation of the universal field samples

The snapshots of the output QSQH velocity field that are generated by the tool fit a rectangular box of fixed dimensions

$$\begin{aligned} -L_x^+/2 &\leq x^+ \leq L_x^+/2, \\ 0 &\leq y^+ \leq L_y^+, \\ -L_z^+/2 &\leq z^+ \leq L_z^+/2, \end{aligned}$$

aligned with the directions of x and z axis, that is aligned to the mean flow. However, the images of the box (3.1) of the snapshots of the universal velocity field are stretched and rotated, see (3.3), when the synthetic field is produced. The resulting situation is illustrated in Fig. 3.1a). The stretching factor $1/u_{\tau_L, i}^+$ and the rotation angle θ_i are different for different snapshots. All the output snapshots share the

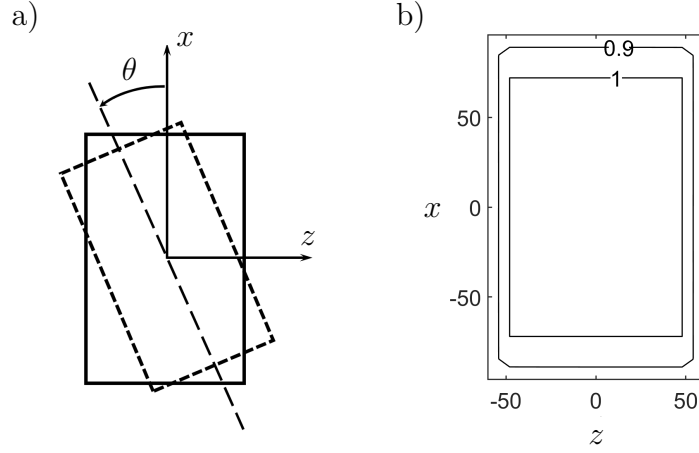


Figure 3.1: a) the output snapshot (solid) and the image of the universal snapshot (dashed); b) two contours of the probability of the data point in the output snapshots to be inside the image of the universal flow snapshot.

same grid, and the values of the velocity at the grid points are obtained by interpolation from the image of the universal flow field snapshot. Unless the length, width, and height of the output snapshots are noticeably smaller than the dimensions of the universal flow snapshot, some of the grid points on the periphery of the output snapshot will occasionally be outside the image of the universal flow snapshot. It would be possible to perform an extrapolation to obtain the missing values. However, extrapolation is not a reliable procedure, and the resulting errors would be hard to control. For this reason, in such cases, no extrapolation is done. Instead, NaN (the standard shorthand for ‘Not a number’) is written to the output snapshot instead of a numerical value. When statistics are calculated by averaging over all snapshots in the output database, the terms containing NaNs have to be removed. The resulting average is then conditional on $|u_{\tau_L}|$ and $|\theta|$ being small enough for the points involved in the calculation of the particular statistics to be inside the image of the universal flow snapshot. Alongside the output database, the output of the software tool includes data on the number of cases NaN was recorded for each point. This allows one to judge how strongly the statistics involving each point were affected by the condition that averaging is done only over snapshots with $|u_{\tau_L}|$ and $|\theta|$ small enough for the point to be inside the image of the universal flow snapshot. One can then use only the points at which there are no NaNs, or estimate, however

approximately, the error caused by dropping out NaN values.

How significant this phenomenon is depends on the particular universal velocity database, the size of the output box, and on the magnitude of the fluctuation of the large-scale component of the wall friction in the model input. As an example, figure 3.1b) shows the contour plot of the probability of a numerical value being obtained at a data point on the x - z plane. In this case, the universal velocity field was generated from the turbulent channel flow at $Re_\tau = 5200$, and the input wall friction was taken from the same flow. The output box dimensions were $L_x^+ = 204$, $L_y^+ = 10$ and $L_z^+ = 120$. Figure 3.1b) refers to the grid points in the layer at $y^+ = 0.711$ from the wall. This value is relatively small compared to $L_y^+ = 10$, and for this reason, in none of the output snapshots this layer was above the upper bound of any image of the universal box.

The two contours shown in Fig. 3.1b) correspond to the probability levels 0.9 and 1. Inside the contour of level of 1 the data had numerical values in all the output snapshots. Moving outside the level of 1, the number of occurrences of NaNs rises up to 10% at the contour level 0.9, which is covering most of the data points of the output snapshots. Figure 3.1 indicates the rarity of NaN in the output snapshots, thereby leading to the conclusion that in this particular case, the stretching and rotation of the grid have a negligible effect on the calculation of the statistics of the output snapshots, due to the rare occurrences of NaN corresponding to the typically small values of $u_{\tau_L}^{\prime 2}$ and θ^2 .

3.2.4 Selecting parameters for generating output snapshots

Before generating the output database, one needs to choose the dimensions of the large-scale subgrid n_L and m_L , and the number of the input snapshots to be used s_L , which then determines the number of the output snapshots N .

Since the amplitude of fluctuation of large scale is small in the range of Re of the currently available and realistic for the near future direct numerical simulations, one can simply take $L_x^+ = \tilde{L}_x$, $L_y^+ = \tilde{L}_y$, and $L_z^+ = \tilde{L}_z$. Smaller dimensions can be used to save computational time and storage space, but larger values would require extrapolation, see section 3.2.3.

Generating the output database will require matching each of I pairs $(u_{\tau_L,i}, \theta_i)$

with a snapshot of the universal velocity. To satisfy the QSQH requirement of the statistical independence of the large-scale motions and the universal velocity, one can select the universal field snapshots randomly with equal probability from the N snapshots of the universal flow database. This might lead to a repeated use of the same universal flow snapshots, in particular if $I > N$. This, however, is acceptable as far as the universal flow database is large enough to provide a comprehensive statistical representation of the universal velocity field.

However, selecting P , Q and R , that is the number of points in x and z directions and the number of input snapshots used (3.6), involves a nontrivial trade-off between the accuracy of the statistics to be eventually calculated using the output database and the computation time and storage space required.

The approach described below was used in the examples given later. It is anticipated that the aspect ratio of the input snapshots closely aligns with that of the large-scale motions. The input snapshot set contains a sufficient number of samples to ensure that the wall friction data is statistically representative. We opted to establish a representative subset of snapshots and grid points, maintaining an equal number of points in both spatial and temporal dimensions, denoted as $P = Q = R$, which serves as the initial condition for our computations. The accuracy of the statistics under consideration can be monitored by incrementally calculating the database over several steps with increasing values of I . At each step, we compute the desired statistics and observe their variations from one step to another. To reduce computational resources, it is desirable to reuse the result obtained in previous steps. This can be achieved by taking

$$P = Q = R = 2^{b-1}, \text{ for } 2^{b-1} < (J, K, S),$$

where b is the step number. As b increases, 2^{b-1} might exceed J , K or S . Then one can take

$$\begin{aligned} P &= J, \text{ for } 2^{b-1} \geq J, \\ Q &= K, \text{ for } 2^{b-1} \geq K, \\ R &= S, \text{ for } 2^{b-1} \geq S. \end{aligned}$$

This procedure was used in the examples given in the present paper.

In a frequent case when the statistics of interest involve only the velocity values

at the same x and z , significant savings of computational time can be made. In this case, the QSQH software we developed generates the 1D synthetic velocity field as

$$\begin{aligned} u_i^+(y_i^+) &= u_{\tau_L,i}^+[\tilde{u}_i(\tilde{y}) \cos \theta_i + \tilde{w}_i(\tilde{y}) \sin \theta_i], \\ v_i^+(y_i^+) &= u_{\tau_L,i}^+ \tilde{v}_i(\tilde{y}), \\ w_i^+(y_i^+) &= u_{\tau_L,i}^+[-\tilde{u}_i(\tilde{y}) \sin \theta_i + \tilde{w}_i(\tilde{y}) \cos \theta_i], \end{aligned} \tag{3.7}$$

where $y_i^+ = \tilde{y}/u_{\tau_L,i}^+$. The universal velocity field used in (3.7) is 1D, meaning that the database of 1D $(\tilde{u}_i, \tilde{v}_i, \tilde{w}_i)$ consists of the instantaneous velocity snapshots in terms of $\tilde{y}$ only. Therefore, the output velocity field is also in terms of y^+ only. The generation of the 1D synthetic velocity field follows exactly the procedures in section 3.2.2 and 3.2.4, without considering the deformation and rotation of the box.

3.3 Generation of the universal velocity field (the second QSQH tool)

This section outlines the process of selecting a large-scale filter and generating the universal velocity field database. The universal velocity field databases are created using the second QSQH tool developed in the present study. For understanding the application of the first QSQH software this subsection is not absolutely necessary. One can skip this subsection and proceed directly to Chapter 4.

In outline, the universal field can be obtained by selecting a suitable database of a high-Reynolds-number fluid flow, referred below as the base flow, and re-engineering the universal field under the assumption that the base flow satisfies the QSQH theory. This process requires a large-scale filter to be defined.

Both the choice of the large-scale filter and the resulting universal velocity field thus obtained depend on the base flow. The description below is general, while the specific examples were obtained using the data for plane channel flows at $Re_\tau = 5200$ and $Re_\tau = 1000$ available at ([University](#)). The corresponding direct numerical calculations are described in ([Lee and Moser, 2015](#)). The case of $Re_\tau = 5200$ was used as the base flow in all sections apart from Section 4.1.2, where two base flows and, correspondingly, two universal fields are considered.

3.3.1 Large-scale filter

A large-scale filter provides a precise definition of large-scale motions. Various filters were used, see for example (Baars et al., 2016; Huang et al., 1998; Agostini and Leschziner, 2014, 2022) and the discussion of large-scale filters and methods of obtaining the universal velocity field in Section 2.1. For users of the QSQH synthetic model software tool the large-scale filter implemented in the model is largely transparent, since neither the input nor the main output of it refer explicitly to the large-scale filter used in the generation of the universal velocity field. However, knowledge of it is useful in interpreting the results.

The specific filter used to generate the universal velocity field, with which the results in Chapter 4 were obtained, is a Fourier cut-off filter. Fourier cut-off filters are commonly used for scale decomposition of wall-bounded turbulence, see for example (Mathis et al., 2009, 2011), which implemented a cut-off filter in time domain transformed into the streamwise cut-off using the Taylor hypothesis (see section 7.1.2 in (Mathis et al., 2009)), or the QSQH study on streamwise velocity component in turbulent channel flow (Zhang and Chernyshenko, 2016), which used cut-offs in wall-parallel directions and time, or (Dogan et al., 2019), in which a Fourier cut-off filter was used to separate the scales in the results of large-eddy simulations (LES) at $Re_\tau = 8200$. In the latter, it was found that the cut-off in span-wise direction is effective in separating the small and large scales, and the slowly-varying structures captured by the cut-off in temporal direction have negligible contribution to the near-wall modulation. This result coincides with the finding of negligible influence of the temporal scale decomposition on the QSQH predictions made in (Zhang, 2019).

Before describing how the cut-off lengths λ_x^+ and λ_z^+ have been selected for the use in the current implementation of the QSQH synthetic model, we will briefly discuss a possible alternative approach. One can attempt simply to choose λ_x^+ and λ_z^+ , that will give the best agreement for the QSQH theory predictions. For example, in (Chernyshenko, 2021), the differences in the second moments of the wall-parallel velocity fluctuations between $Re_\tau = 1000$ and 5200, $\Delta u_{rms}^2(y) = u_{rms}^2|_{Re_\tau=5200} - u_{rms}^2|_{Re_\tau=1000}$ and $\Delta w_{rms}^2(y) = w_{rms}^2|_{Re_\tau=5200} - w_{rms}^2|_{Re_\tau=1000}$, were fitted by the simplified QSQH predictions $\Delta u_{rms}^2(y) \approx \Delta \langle u_{\tau L}^2 \rangle \left(\frac{dy \tilde{U}}{dy} \right)^2$ and $\Delta w_{rms}^2(y) \approx \Delta \langle \theta^2 \rangle \tilde{U}^2$,

Cut-offs/method	$\Delta\langle u_{\tau_L}^{\prime 2} \rangle$	$\Delta\langle \theta^2 \rangle$
Fitting	0.004897	0.002817
$\lambda_x^+ = 1000\pi, \lambda_z^+ = 125\pi$	0.004377	0.002411
$\lambda_x^+ = 2000\pi, \lambda_z^+ = 250\pi$	0.004338	0.001860
$\lambda_x^+ = 2000\pi, \lambda_z^+ = 500\pi$	0.004290	0.001679

Table 3.1: Increment in $\langle u_{\tau_L}^{\prime 2} \rangle$ and $\langle \theta^2 \rangle$ from $Re_\tau = 1000$ to 5200 by fitting and scale decomposition.

with $\Delta\langle u_{\tau_L}^{\prime 2} \rangle = \langle u_{\tau_L}^{\prime 2} \rangle_{Re_\tau=5200} - \langle u_{\tau_L}^{\prime 2} \rangle_{Re_\tau=1000}$ and $\Delta\langle \theta^2 \rangle = \langle \theta^2 \rangle_{Re_\tau=5200} - \langle \theta^2 \rangle_{Re_\tau=1000}$ used as the fitting parameters. The fitted values from (Chernyshenko, 2021) and the values calculated for a few combinations of λ_x^+ and λ_z^+ are presented in Table 3.1. It can be seen that the fitted values are higher than the actual values for all three pairs of Fourier cut-offs. Notably, the choice of the Fourier cut-offs does not significantly affect $\Delta\langle u_{\tau_L}^{\prime 2} \rangle$, which is what is implied by the QSQH theory (see (Chernyshenko et al., 2019) for an extensive discussion of this issue). However, it is different for $\Delta\langle \theta^2 \rangle$. This might be a reflection of the difference in the energy spectra of the streamwise and spanwise velocities of the turbulent channel flow. In any case, adjusting the cut-offs cannot significantly change $\Delta\langle u_{\tau_L}^{\prime 2} \rangle$, which would be needed to fit the direct numerical simulation data. This might be an inherent disadvantage of the Fourier cut-off filter to be used with the QSQH theory, and other filters might turn out to be better for this purpose.

In the present work, the cut-offs will be selected using the same procedure as in (Zhang and Chernyshenko, 2016; Zhang, 2019). A Fourier cut-off filter with any thresholds satisfies the first four properties of the large-scale filter (2.14–2.17) required by the QSQH theory. However, satisfying the property of scale separation (2.18) is challenging because of the continuity of the spectrum of scales of turbulence. The logic of the QSQH theory implies that small scales are much smaller than the large scales and that the theory applies in a near-wall layer which is thin as compared to the wall-parallel dimensions of the large-scale motions. This leads to the understanding that the correlation between the large-scale velocity at a point very close to the wall and the large scale velocity just outside this layer should

be close to 1. This also follows from the QSQH formula (2.20), although it was obtained with additional assumption of the amplitude of the large-scale fluctuations being small. At the same time, a similar correlation between the small scales should be close to zero, as otherwise the small-scale motions would protrude outside the near-wall layer.

Quantitatively, this can be measured by the correlation coefficients

$$\begin{aligned}
R_{u_L,0u_L}(y_o^+) &= \frac{\langle u_L'(+0)u_L'(y_o^+) \rangle}{\sqrt{\langle (u_L'(+0))^2 \rangle \langle (u_L'(y_o^+))^2 \rangle}}, \\
R_{w_L,0w_L}(y_o^+) &= \frac{\langle w_L'(+0)w_L'(y_o^+) \rangle}{\sqrt{\langle (w_L'(+0))^2 \rangle \langle (w_L'(y_o^+))^2 \rangle}}, \\
R_{u_S,0u_S}(y_o^+) &= \frac{\langle u_S'(+0)u_S'(y_o^+) \rangle}{\sqrt{\langle (u_S'(+0))^2 \rangle \langle (u_S'(y_o^+))^2 \rangle}}, \\
R_{w_S,0w_S}(y_o^+) &= \frac{\langle w_S'(+0)w_S'(y_o^+) \rangle}{\sqrt{\langle (w_S'(+0))^2 \rangle \langle (w_S'(y_o^+))^2 \rangle}},
\end{aligned}$$

where y_o^+ is the point just outside the layer of interest. The optimal filter would then be the one maximising $R_{u_L,0u_L}$ and $R_{w_L,0w_L}$ and minimising $R_{u_S,0u_S}$ and $R_{w_S,0w_S}$ over the cut-offs at the same time. Moreover, as we intend to use the synthetic model to study the variation with Re , we would like this to be done with the correlations for at least two values of Re covering the range of interest in the Reynolds number. This amounts to a multi-objective optimisation problem with eight objectives, which is well beyond what is feasible. Note that in (Zhang and Chernyshenko, 2016; Zhang, 2019) the Pareto fronts were in two dimensions only. Instead, we just made a few trials, choosing $y_o^+ = 100$, that is just to include the viscous and buffer layers in the analysis.

Five pairs of the Fourier cut-offs were tested for the turbulent channel flow at $Re_\tau = 1000$. The resulting correlation coefficients are presented in Table 3.2. The pair of lowest thresholds, $\lambda_x^+ = 1000\pi$, $\lambda_z^+ = 125\pi$, coincides with the thresholds used in (Zhang and Chernyshenko, 2016) without the inclusion of a temporal threshold. As the thresholds are increased, the values of large-scale correlation coefficients tend toward unity, because fewer intermediate scales are included in the large scales.

λ_x^+	λ_z^+	$R_{u_L,0u_L}$	$R_{u_S,0u_S}$	$R_{w_L,0w_L}$	$R_{w_S,0w_S}$
1000π	125π	0.780	0.017	0.355	-0.038
1600π	125π	0.855	0.036	0.453	-0.036
1600π	200π	0.885	0.061	0.552	-0.036
2000π	250π	0.913	0.087	0.630	-0.032
2000π	500π	0.927	0.135	0.653	-0.025

Table 3.2: Large and small-scale correlation coefficients of the wall-parallel velocities of the turbulent channel flow at $Re_\tau = 1000$, with $y_o^+ = 100$.

λ_x^+	λ_z^+	$R_{u_L,0u_L}$	$R_{u_S,0u_S}$	$R_{w_L,0w_L}$	$R_{w_S,0w_S}$
2000π	250π	0.951	0.091	0.812	-0.022
2000π	500π	0.965	0.137	0.865	-0.012

Table 3.3: Large and small-scale correlation coefficients of the wall-parallel velocities of the turbulent channel flow (Lee and Moser, 2015) at $Re_\tau = 5200$, with $y_o^+ = 100$.

Conversely, this leads to an opposite effect on the small scales, with more intermediate scales becoming a part of the small scales and consequently resulting in undesirably larger small-scale correlation coefficients. Two pairs of Fourier cut-offs were also tested for the turbulent channel flow at $Re_\tau = 5200$ provided in (Lee and Moser, 2015), and the corresponding correlation coefficients are presented in Table 3.3. By comparing the correlation coefficients in both tables, the Fourier cut-off of $\lambda_x^+ = 2000\pi$, $\lambda_z^+ = 250\pi$ is selected for the large-scale filter to be used in the following. This choice yields $R_{u_L,0u_L} > 0.9$ and $R_{u_S,0u_S} < 0.1$. The large-scale correlation coefficients of both wall-parallel velocities, when filtered with the same Fourier cut-offs, exhibit higher values for flows with higher Reynolds numbers, as demonstrated in Table 3.2 and 3.3. This behaviour is attributed to the shifting of the large-scale energy peak away from the small-scale energy peak in both streamwise and spanwise wave modes within the Fourier space as the Reynolds number increases for turbulent channel flow as shown, for example, in (Mathis et al., 2009). However, it is important to note that the energy spectra of the two velocity components differ from each other. In particular, the large-scale correlation of the spanwise velocity is

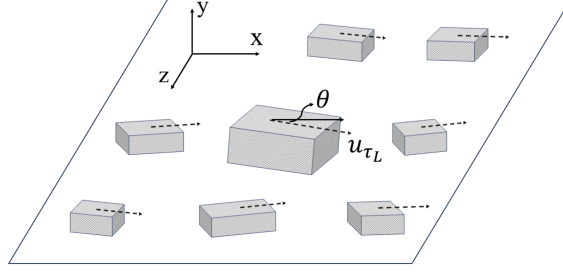


Figure 3.2: The base flow boxes. The magnitude and direction of the large-scale component of the wall friction velocity u_{τ_L} and θ are presented.

smaller than that of the streamwise velocity when the same Fourier cut-off is applied to the same flow. This observation suggests that, in contrast to the streamwise velocity, the large-scale component of spanwise velocity does not exhibit a sufficiently strong correlation between the wall and an outer position, such as at $y_o^+ = 100$. Furthermore, this finding implies a limited accuracy in predicting spanwise velocity statistics using the QSQH tool, which will be discussed in Section 4.1 and 4.2.

3.3.2 Universal velocity field

The outline

The universal velocity field is generated from a base flow. In the base flow the near-wall turbulence should be statistically homogeneous in streamwise and spanwise directions and time.

The universal velocity field will be represented by a set of velocity snapshots $(\tilde{u}_n, \tilde{v}_n, \tilde{w}_n), n = 1, \dots, N$ in a rectangular box:

$$-\tilde{L}_x/2 \leq \tilde{x} \leq \tilde{L}_x/2, \quad 0 \leq \tilde{y} \leq \tilde{L}_y, \quad -\tilde{L}_z/2 \leq \tilde{z} \leq \tilde{L}_z/2.$$

This set of snapshots will constitute the universal database. To obtain the velocity distribution of each of these snapshots we will allocate to each snapshot a rectangular box in the base flow (see Fig. 3.2), with the bottom face centred at the point $(x_{c,n}^+, 0, z_{c,n}^+)$ and a pair $(u_{\tau_L,n}^+, \theta_n)$ equal to the values of u_{τ_L} and θ at the points $(x_{c,n}^+, 0, z_{c,n}^+)$.

The size of these corresponding boxes is $(L_{x,n}^+, L_{y,n}^+, L_{z,n}^+) = (\tilde{L}_x, \tilde{L}_y, \tilde{L}_z)/u_{\tau_L,n}^+$. The boxes are turned by θ_n to align them with the direction of the large-scale component of the wall friction. As the result, the coordinates in the base flow are related to the coordinates in the universal flow snapshot similarly to (3.8) but centered at $(x_{c,n}, z_{c,n})$:

$$\begin{aligned} x^+ &= (\tilde{x} \cos \theta_n + \tilde{z} \sin \theta_n) / u_{\tau_L,n}^+ + x_{c,n}, \\ y^+ &= \tilde{y} / u_{\tau_L,n}^+, \\ z^+ &= (-\tilde{x} \sin \theta_n + \tilde{z} \cos \theta_n) / u_{\tau_L,n}^+ + z_{c,n}. \end{aligned} \quad (3.8)$$

The universal velocity in the snapshot is then given by the QSQH relation (3.3) rearranged as

$$\begin{aligned} \tilde{u}_n(\tilde{x}, \tilde{y}, \tilde{z}) &= (u^+ \cos \theta_n - w^+ \sin \theta_n) / u_{\tau_L,n}^+, \\ \tilde{v}_n(\tilde{x}, \tilde{y}, \tilde{z}) &= v^+ / u_{\tau_L,n}^+, \\ \tilde{w}_n(\tilde{x}, \tilde{y}, \tilde{z}) &= (u^+ \sin \theta_n + w^+ \cos \theta_n) / u_{\tau_L,n}^+, \end{aligned} \quad (3.9)$$

where (u^+, v^+, w^+) is the base flow velocity at the point (x^+, y^+, z^+) given by (3.8). If the base flow grid is uniform in the wall-parallel directions, as is often the case, it is convenient to choose $(x_{c,n}^+, z_{c,n}^+)$ to coincide with certain grid points of the base flow data. In the special case of $u_{\tau_L,n}^+ = 1$ and $\theta_n = 0$ the box dimensions are equal: $(L_{x,n}^+, L_{y,n}^+, L_{z,n}^+) = (\tilde{L}_x, \tilde{L}_y, \tilde{L}_z)$. It is natural to use this to define the grid in the universal flow snapshots boxes by simply copying the grid in the base flow box. Then the same grid is used in all the universal flow snapshots.

The universal flow snapshots are populated with the values of (u_n^+, v_n^+, w_n^+) by determining (x^+, y^+, z^+) for each universal snapshot grid point using (3.8) and then calculating (u_n^+, v_n^+, w_n^+) using (3.9). Due to the deformation and rotation of the base flow boxes when $u_{\tau_L,n}^+ \neq 1$ and/or $\theta_n \neq 0$, the points in the base flow corresponding to the grid points in the universal flow snapshots might not coincide with any grid point of the base flow. Then cubic spline interpolation is used to determine the sampled velocity field (u_n^+, v_n^+, w_n^+) .

To complete the definition of the procedure, the centers of the base flow boxes and the box dimensions need to be selected. This involves a trade-off between several

requirements.

Selection of the centers of the boxes

The number of universal velocity snapshots should be large enough to be statistically representative. For a base flow database of a fixed size, increasing the number of the boxes inevitably reduces the distance between them. However, it is desirable that the elements of these sets are uncorrelated, so that the velocity fields of $(\tilde{u}_n, \tilde{v}_n, \tilde{w}_n)$ were statistically independent. These contradictory requirements are satisfied in the following way.

Let the wall-parallel coordinates of the grid points of the base flow be denoted (x_m^+, z_l^+) , $m = 1, \dots, M$, $l = 1, \dots, L$, with the grid spacing Δx_{base}^+ and Δz_{base}^+ . From these points we select a subset of uniformly spaced points, which we denote $(x_{m_p}^+, z_{l_q}^+)$, $p = 1, \dots, P_u$, $q = 1, \dots, Q_u$. These points will be the centres of the boxes of velocity data (see figure 3.2). The grid spacings $\Delta x_{\text{sample}}^+ = x_{m_{p+1}}^+ - x_{m_p}^+$ and $\Delta z_{\text{sample}}^+ = z_{l_{q+1}}^+ - z_{l_q}^+$ in x and z directions, respectively, are chosen to provide a reasonable compromise between the correlations of the neighbouring velocity snapshots in both x and z directions, which is desired to be small, on the one hand and the number of the resulting snapshots, which is desired to be large, on the other hand. To this end, we calculate the two-point correlation coefficients of the velocity fluctuation u'^+ of the base flow,

$$R_{x,u}(\Delta x^+) = \frac{\langle u'(x^+, y_0^+, z^+) u'(x^+ + \Delta x^+, y_0^+, z^+) \rangle}{\langle u'^2(x^+, y_0^+, z^+) \rangle},$$

$$R_{z,u}(\Delta z^+) = \frac{\langle u'(x^+, y_0^+, z^+) u'(x^+, y_0^+, z^+ + \Delta z^+) \rangle}{\langle u'^2(x^+, y_0^+, z^+) \rangle}.$$

Here, y_0 is the coordinate of the base flow grid layer nearest to the wall, and the base flow is assumed to be statistically homogeneous in the wall-parallel directions. Then $\Delta x_{\text{sample}}^+$ and $\Delta z_{\text{sample}}^+$ are selected so that $R_{x,u}(\Delta x_{\text{sample}}^+)$ and $R_{z,u}(\Delta z_{\text{sample}}^+)$ are reasonably small, while the subgrid dimensions $P_u = [M/(\Delta x_{\text{sample}}^+/\Delta x_{\text{base}}^+)]$ and $Q_u = [L/(\Delta z_{\text{sample}}^+/\Delta z_{\text{base}}^+)]$, where $[.]$ denotes the integer part, are reasonably large.

With $\Delta x_{\text{sample}}^+$, $\Delta z_{\text{sample}}^+$, P_u , and Q_u determined, $(x_{m_p}^+, z_{l_q}^+)$ are defined by taking

$$m_p = 1 + (p - 1) \frac{\Delta x_{\text{sample}}^+}{\Delta x_{\text{base}}^+}, \quad p = 1, \dots, P_u, \quad (3.10)$$

$$l_q = 1 + (q - 1) \frac{\Delta z_{\text{sample}}^+}{\Delta z_{\text{base}}^+}, \quad q = 1, \dots, Q_u. \quad (3.11)$$

Thereby, the grid points where $(u_{\tau_L, n}^+, \theta_n)$ are sampled are $(x_{m_p}^+, z_{l_q}^+)$, $p = 1, \dots, P_u$, $q = 1, \dots, Q_u$ for each snapshot of the base flow. The order in which $(x_{m_p}^+, z_{l_q}^+)$ are allocated to be the centers $(x_{c, n}^+, z_{c, n}^+)$ does not matter. If there is more than one snapshots of the base flow, the described procedure is repeated for each snapshot, thus increasing the number of the snapshots of the universal velocity field $N = P_u \times Q_u \times S$, where S is the number of snapshots of the base flow. (Note the similarity of P_u and Q_u with P and Q introduced in Section 3.2.4.)

Selection of the box size

Equations (3.9) imply that $(u_{\tau_L}^+, \theta)$ is approximately constant and equal to $(u_{\tau_L, n}^+, \theta_n)$ inside each box, which effectively sets a limit to the dimensions of the box, since in reality the large-scale motions are varying, albeit slowly, in x and z directions (Zhang and Chernyshenko, 2016; Zhang, 2019). On the other hand, it is desirable to have the universal flow box dimensions to be large in order to be able to obtain a broader range of two-point statistics from the model.

The degree of typical inhomogeneity of the large-scale motions in the vicinity of the wall can be measured by the two-point correlation coefficients of u'_{τ_L} of the base flow in the wall-parallel directions:

$$R_{u'_{\tau_L}}(\Delta x^+) = \frac{\langle u'_{\tau_L}(x^+) u'_{\tau_L}(x^+ + \Delta x^+) \rangle}{\sqrt{\langle u'^2_{\tau_L}(x^+) \rangle \langle u'^2_{\tau_L}(x^+ + \Delta x^+) \rangle}},$$

$$R_{u'_{\tau_L}}(\Delta z^+) = \frac{\langle u'_{\tau_L}(z^+) u'_{\tau_L}(z^+ + \Delta z^+) \rangle}{\sqrt{\langle u'^2_{\tau_L}(z^+) \rangle \langle u'^2_{\tau_L}(z^+ + \Delta z^+) \rangle}}.$$

Since all the universal velocity snapshots have the same dimensions, it makes sense to assign values to $\tilde{L}_x$ and $\tilde{L}_z$, instead of choosing values for $L_{x, n}^+$ and $L_{z, n}^+$ for each box. Accordingly, we select these dimensions to be as large as possible such that

$R_{u'_{\tau_L}}(\tilde{L}_x/2)$ and $R_{u'_{\tau_L}}(\tilde{L}_z)$ are sufficiently close to 1.

The actual values of Δx_{sample} , Δz_{sample} and $\tilde{L}_x$ and $\tilde{L}_z$ depend on the base flow and the desired or acceptable size of the universal flow database. Specific choices for universal databases used in the following parts of the paper are described in subsection 3.3.1.

One-dimensional universal velocity field

By 1D universal velocity field we understand the very special, yet important case when the universal velocity snapshots contain only one wall-normal grid line. This means setting $\tilde{x} = \tilde{z} = 0$ in (3.8) and (3.9). In this scenario, it is not possible to calculate multi-point correlations unless all the points involved differ only by their wall-normal coordinates. However, this is acceptable often enough to justify giving this case a special treatment. In this specific case, it is convenient and computationally inexpensive to use all local velocity profiles at all grid points in x and z directions of the base flow instead of following the sampling procedures. Effectively, $n = 1, \dots, N$ and N is equal to the total number of grid points in a single wall-parallel grid layer of the base flow.

The currently available universal velocity field databases

The current version of the software implementation of the QSQH model is available at (Yang, 2024). It gives a choice between two universal velocity fields, 1D and 3D. Both fields were generated from the base flow of a single snapshot taken from turbulent channel flow of $Re_\tau = 5200$ (Lee and Moser, 2015). The details of the selection of the distances between the centres of the base flow boxes $\Delta x_{\text{sample}}^+$ and $\Delta z_{\text{sample}}^+$, the universal flow grid, and the universal flow snapshot size $(\tilde{L}_x, \tilde{L}_y, \tilde{L}_z)$ for the 3D case are described below.

The two-point correlation coefficients of u'^+ of the base flow in x and z directions are presented in Figure 3.3. We chose $\Delta x_{\text{sample}}^+ = 1270$ and $\Delta z_{\text{sample}}^+ = 640$, as shown by the dash-dotted lines in the figure. This choice yields $R_{x,u}(\Delta x_{\text{sample}}^+) \approx 0.105$ and $R_{z,u}(\Delta z_{\text{sample}}^+) \approx 0.025$. which are small enough for the sampled velocity snapshots to be approximately independent of each other.

The wall-parallel grid spacing of the universal velocity snapshots was chosen to

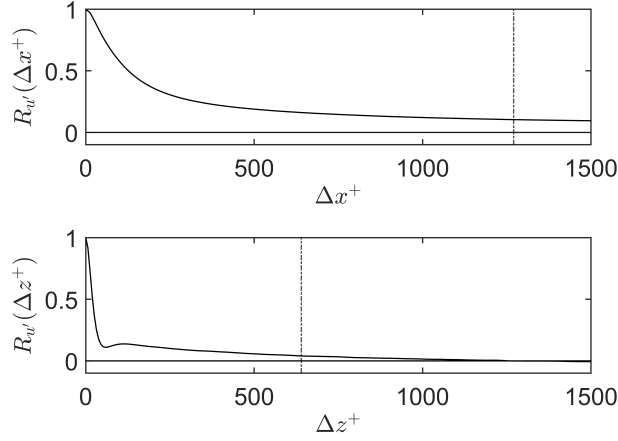


Figure 3.3: The two-points wall-parallel correlation coefficients of u'^+ in the turbulent channel flow at $Re_\tau = 5200$.

be $\Delta\tilde{x} = 12$ and $\Delta\tilde{z} = 6$. This is slightly less than the grid spacing of the base flow $\Delta x_{\text{base}}^+ = 12.7$ and $\Delta z_{\text{base}}^+ = 6.4$, just to compensate for possible shrinking and stretching of the base flow grid when it is mapped to the universal flow grid.

The dimensions of the universal velocity snapshots $\tilde{L}_x$ and $\tilde{L}_z$ were chosen on the basis of the two-point correlation coefficients of $u'_{\tau_L}{}^+$ of the base flow in x and z directions, shown in Figure 3.4. The values $\tilde{L}_x = 204$ and $\tilde{L}_z = 120$ correspond to $\Delta x^+ = 102$ and $\Delta z^+ = 60$ shown with the dashed lines in Figure 3.4. This choice yields $R_{u'_{\tau_L}}(\Delta x^+) \approx 0.999$ and $R_{u'_{\tau_L}}(\Delta z^+) \approx 0.980$, which values are large enough to satisfy the requirement that $u_{\tau_L}^+$ remains close to $u_{\tau_L,n}^+$ within each box. The wall-normal grid of the universal velocity field was reproduced from the wall-normal grid of the base flow, and $\tilde{L}_y = 100$.

The 1D database contains about 3.1 million universal velocity instances, and 3D database contains 7931 universal velocity snapshots. For the 3D version the maximum value of $u_{\tau_L,n}^+$ is 1.255, which yields the maximum possible box size as $L_{x,n} \approx 254$ and $L_{z,n} \approx 150$. Even if this largest box was turned by 90° (in fact for this particular box the maximum turning angle among the boxes is 0.9°), it would still be in the region of high correlations, as can be seen from Figure 3.4.

Figure 3.5 shows the probability density functions for $u_{\tau_L}^+$ and $u_{\tau_L,n}^+$, and θ and θ_n . The sampled $(u_{\tau_L,n}^+, \theta_n)$ are indeed statistically representative of the large-scale component of the wall friction of the base flow. This figure also shows that extreme

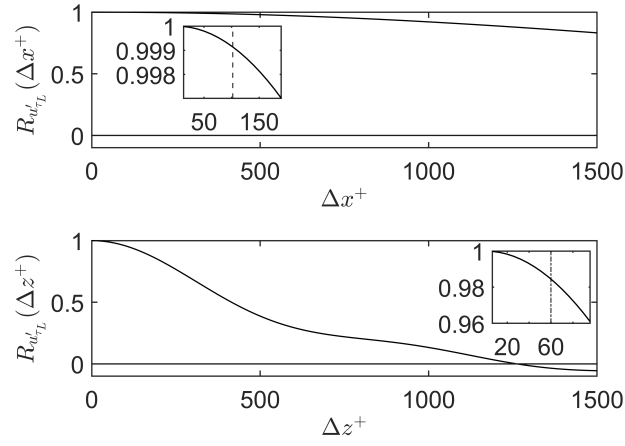


Figure 3.4: The two-points correlation coefficients of u_{τ_L} of the turbulent channel flow of $\text{Re}_\tau = 5200$ in wall-parallel directions.

cases where the box suffers from substantial deformation and rotation are rare.

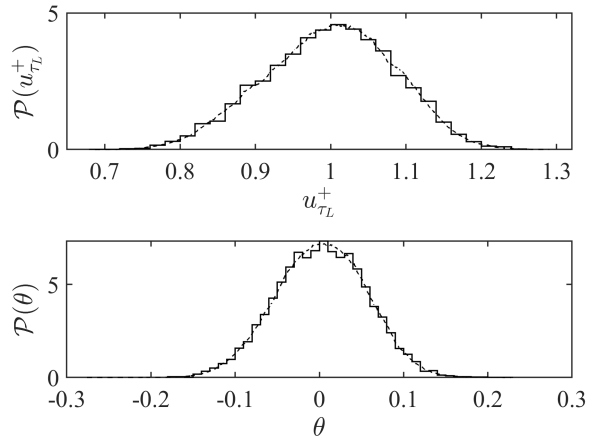


Figure 3.5: Probability density functions of $(u_{\tau_L,n}^+, \theta_n)$ (solid line) and $(u_{\tau_L}^+, \theta)$ (dashed line) of the base flow of turbulent channel flow at $Re_\tau = 5200$.

Chapter 4

Applications of the QSQH software tool

4.1 Plane channel flow

In the examples given in this section the synthetic velocity fields for different target flows are generated using different versions of the universal velocity fields. For reference all these cases are summarised in Table 4.1, where LM5200 and LM1000 denote the turbulent channel flow at $Re_\tau = 5200$ and 1000 calculated by Lee and Moser (Lee and Moser, 2015).

Subsection 4.1.5 investigates the Re -dependence of the universal velocity fields and thus does not use synthetic velocity fields.

4.1.1 Accuracy of Taylor expansion in the amplitude of the fluctuation of the large-scale wall friction

While the formulae obtained using the QSQH theory are often short and neat, as seen in (2.19), they are not easy to use because they often require knowledge of the statistics of the universal velocity field, such as $\tilde{U}(\tilde{y})$ in (2.19). In contrast, using the Taylor expansions in the amplitude of the fluctuation of the large-scale wall friction, as described in Section 2.3.5, meaningful results can sometimes be obtained without any knowledge of the universal field apart from the assumption that a universal field exists.

Dimension	Subsection	Base flow	Target flow
1D	4.1.1	LM5200	LM5200
			LM1000
	4.1.2	LM5200	LM5200
		LM1000	LM1000
	4.1.3	LM5200	LM5200
			LM1000
3D	4.1.4	LM5200	LM5200
			LM1000

Table 4.1: The cases of generating different synthetic velocity fields in the subsections of section 4.1. The large-scale filter is a Fourier cut-off filter with $\lambda_x = 2000\pi$ and $\lambda_z = 250\pi$, which gives $\langle u_{\tau_L}^{\prime 2} \rangle = 0.007348$, $\langle \theta^2 \rangle = 0.002874$ at $Re_\tau = 5200$, and $\langle u_{\tau_L}^{\prime 2} \rangle = 0.003010$, $\langle \theta^2 \rangle = 0.000983$ at $Re_\tau = 1000$.

An example of this is given by the formula (2.7) for the superposition coefficient. Results obtained with Taylor expansions are also useful as they often reveal the underlying physics of various scale interactions, such as explaining why the root mean square of the fluctuations of the streamwise and spanwise velocities is much more sensitive to changes in Re than other Reynolds stress components and mean velocity (Chernyshenko, 2021). However, using Taylor expansions has disadvantages too. When QSQH predictions obtained with Taylor expansions are compared with real observations, attributing the discrepancy is challenging. First, derivations of the Taylor expansions are often cumbersome. Second, there is a truncation error. The QSQH synthetic model resolves these issues. Since the synthetic flow database provided by the model satisfies the QSQH theory exactly, the difference between the statistics of this database and the Taylor expansion prediction is caused only by truncation. Reducing the amplitude of large scales will reduce this difference unless there was a derivation error, thus allowing for the detection of the error. The difference between the statistics of the synthetic model database and the real data is due to the approximate nature of the QSQH theory itself.

We will investigate the truncation errors using the example of the mean velocity profile and second-order moments of velocities. The Taylor expansions of the QSQH

formulae

$$U(y) = \tilde{U}(y) - \frac{1}{2}\langle\theta^2\rangle\tilde{U}(y) + \frac{\langle u_{\tau_L}'^2\rangle}{2}\frac{d}{dy}y^2\frac{d\tilde{U}}{dy} + \dots \quad (4.1)$$

$$u_{rms}^2(y) = \tilde{u}_{rms}^2(y) + \langle\theta^2\rangle(\tilde{w}_{rms}^2 - \tilde{u}_{rms}^2) + \langle u_{\tau_L}'^2\rangle \left[\left(\frac{dy\tilde{U}}{dy} \right)^2 + \frac{1}{2} \frac{d^2y^2\tilde{u}_{rms}^2}{dy^2} \right] + \dots \quad (4.2)$$

$$w_{rms}^2(y) = \tilde{w}_{rms}^2(y) + \langle\theta^2\rangle \left(\tilde{U}^2 + \tilde{u}_{rms}^2 - \tilde{w}_{rms}^2 \right) + \frac{\langle u_{\tau_L}'^2\rangle}{2} \frac{d^2y^2\tilde{w}_{rms}^2}{dy^2} + \dots, \quad (4.3)$$

were extensively used in (Zhang and Chernyshenko, 2016; Zhang, 2019; Chernyshenko, 2021). Here, $\tilde{u}_{rms}$ and $\tilde{w}_{rms}$ are the root mean squares of the fluctuations of the corresponding components of the universal velocity, and u_{τ_L}' is the fluctuation of u_{τ_L} , so that $u_{\tau_L} = 1 + u_{\tau_L}'$, $\langle u_{\tau_L}' \rangle = 0$. Note that the universal flow statistics in these formulas are independent of Re_τ , while the large-scale related parameters $\langle u_{\tau_L}'^2 \rangle$ and $\langle\theta^2\rangle$ increase as Re_τ increases. The expansions were derived assuming $|u_{\tau_L}'| \ll 1$ and $|\theta| \ll 1$. In what follows, we will often use relative errors, which we define for any approximate quantity, for example g_{app} , as

$$\epsilon_g = \frac{g_{app} - g}{|g|},$$

where g is the exact value of this quantity. In this subsection, the values considered as exact are the statistics of the QSQH synthetic field, while the approximate values are given by (4.1-4.3). It is important to note that v_{rms}^2 is not considered in this study, as the current version of QSQH theory predicts an incorrect value for v_{rms}^2 . In (Chernyshenko, 2021), QSQH theory stipulates that the large-scale component of wall-normal velocity is zero (i.e., $v_{L,rms}^2 = 0$), which implies that any prediction of v_{rms}^2 by QSQH theory can be anticipated to be erroneous even prior to calculation. Consequently, results pertaining to v_{rms}^2 will not be addressed in this study. Nevertheless, a proposed approach to address this issue is presented in Sections 5.1 and 5.2.

Figure 4.1 shows the relative errors of (4.1-4.3) as compared to the mean velocity profile and second moments of wall-parallel velocity fluctuations of the synthetic velocity fields for $Re_\tau = 1000$ and 5200. The absolute values of the relative errors are all below 0.01, indicating that at these Reynolds numbers, the accuracy of the Taylor expansions is good compared to the accuracy of the QSQH theory itself

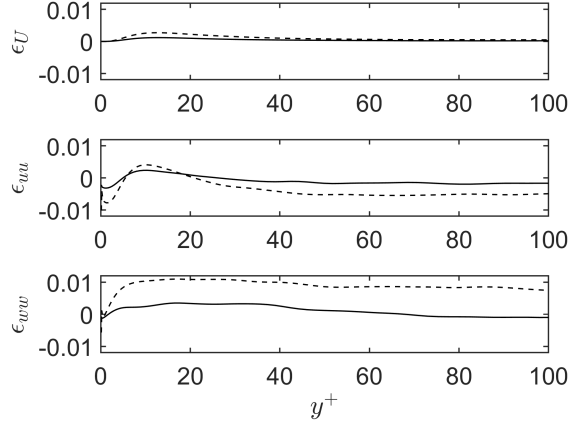


Figure 4.1: Relative errors of the Taylor expansions for mean velocity and second moments of wall-parallel velocity fluctuations of the synthetic velocity field, $Re_\tau = 1000$ (solid) and 5200 (dashed).

(see the following sections). Notably, the absolute values of the relative errors at higher Re_τ are higher. This is to be expected since the amplitude of fluctuations of large-scale motions increases with Re . It can also be noticed that the shape of the error curve at both values of Re_τ is similar. This suggests that for each quantity there might be only one specific higher-order term of the expansion that is largely responsible for the observed truncation error, or that $\langle u_{\tau_L}^2 \rangle$ and $\langle \theta^2 \rangle$ increase with their ratios remaining approximately constant as Re_τ increases.

4.1.2 Statistical independence of large-scale friction velocity and universal velocity field

The QSQH theory postulates statistical independence of the large-scale related parameters (u_{τ_L}, θ) and the velocity field $(\tilde{u}, \tilde{v}, \tilde{w})$ defined by (2.8–2.13). This part of the QSQH universality hypothesis is referred to as the QSQH hypothesis per se, to distinguish it from the second hypothesis: that the statistical properties of $(\tilde{u}, \tilde{v}, \tilde{w})$ defined by (2.8–2.13) are the same for flows at different but sufficiently large Re and different large-scale geometries, such as a plane channel or pipe (Zhang and Chernyshenko, 2016). This is the reason for calling $(\tilde{u}, \tilde{v}, \tilde{w})$ universal. For real flows, the accuracy of the QSQH hypothesis per se depends on the quality of the scale decomposition by the large-scale filter and is never perfect. The synthetic model field,

however, satisfies this hypothesis exactly, because the instances of (u_{τ_L}, θ) from the input wall-friction field and the instances of the universal velocity field $(\tilde{u}, \tilde{v}, \tilde{w})$ are randomly paired in the model. This allows an interesting way of verifying to what extent the base flow, that is, the flow that was used to generate the universal velocity field, satisfies this hypothesis. To do this, the synthetic model can be applied to the base flow itself. If the pairing of the instances of (u_{τ_L}, θ) and $(\tilde{u}, \tilde{v}, \tilde{w})$ happened to be exactly as they were paired in the base flow, the synthetic model database statistics would coincide with the statistics of the base flow. Such occurrence is, however, extremely unlikely. In synthetic model the pairing is mixed randomly, or ‘scrambled’. As a result, the difference in these statistics indicates the deviation of the base flow from satisfying the QSQH hypothesis per se. Once a synthetic model is available, this approach is much easier than comparing the conditional statistics of the base flow conditioned on (u_{τ_L}, θ) , as it was done (without taking θ into account, and in a variety of ways) in many works, exemplified by (Zhang et al., 2022; Agostini and Leschziner, 2019a; Ganapathisubramani et al., 2012; Agostini et al., 2017; Howland and Yang, 2018).

Total velocity field

To investigate this further, the turbulent channel flows at $Re_\tau = 1000$ and 5200 were treated as the base flows, and the corresponding universal fields were generated using the same large-scale filter. Then the QSQH software tool generated the synthetic flows for each case. The relative errors of the mean velocity profile and the second moments of wall-parallel velocity fluctuations of the synthetic field as compared to the DNS data are presented in Fig. 4.2. If the QSQH hypothesis is valid, the flow statistics calculated from the DNS data and the synthetic velocity field should coincide. As the figure shows, for U they coincide. The negligible error of the mean velocity profile for the flows at both values of Re_τ was expected. As the Taylor expansions of the QSQH formula for mean velocity in magnitude show, both with the variation of the direction of large scale wall friction neglected (Zhang and Chernyshenko, 2016) and accounted for (Chernyshenko, 2021), the mean profile is not sensitive to large-scale motions, because apart from $\tilde{U}(\tilde{y})$, all the terms in (4.1) are negligible in the range of Re considered here.

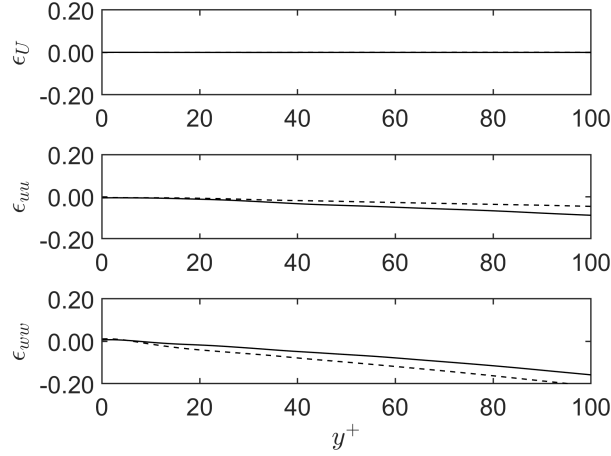


Figure 4.2: Relative errors of mean velocity profile and second moments of wall-parallel velocity fluctuations $\langle u'^2 \rangle, \langle w'^2 \rangle$ of the synthetic velocity field as compared to the DNS velocity field at $Re_\tau = 1000$ (solid line) and 5200 (dashed line).

A similar statement is not true for expansions (4.2,4.3), which contain non-negligible terms $\langle u'^2_{\tau_L} \rangle \left(\frac{dy\tilde{U}}{dy} \right)^2$ and $\langle \theta^2 \rangle \tilde{U}^2$ (Zhang and Chernyshenko, 2016; Chernyshenko, 2021). As Fig. 4.2 shows, the relative errors in the second moments have a constant and low level close to the wall, approximately for $y^+ < 10$, and then increase as one moves away from the wall. This trend indicates that the proposed statistical independence is effectively achieved close to the wall but deteriorates with increasing distance from the wall. The relative errors also demonstrate a dependence on Re . The error of the streamwise component decreases as Re_τ increases. This is expected since the QSQH hypothesis is anticipated to become increasingly accurate for near-wall turbulence with higher Re , attributed to a more prominent outer energy peak centred farther away from the inner peak, and consequently a more reliable definition of large and small scales, and to better applicability of the universality hypothesis at larger Re for the same reasons as for the classical universality Zhang and Chernyshenko (2016); Chernyshenko (2021).

In contrast, the relative error of the spanwise component demonstrates opposite behavior. This error tends to grow with increasing Re , suggesting that further investigation is required. In the following section, it is shown that the spanwise velocity is simultaneously affected by large and small-scale components, and the agreement of these two components with the QSQH theory responds differently

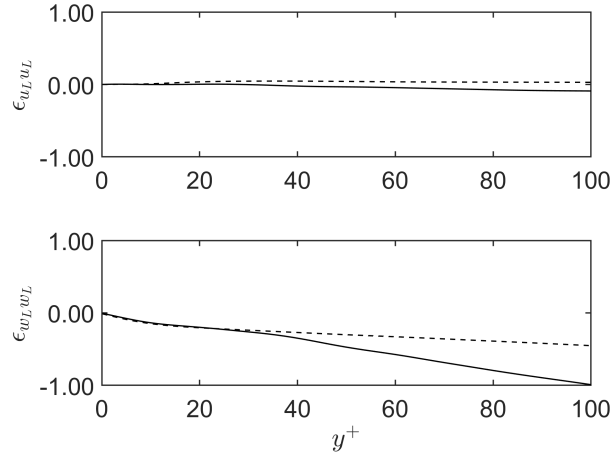


Figure 4.3: Relative errors of second moments of wall-parallel large-scale components of velocity fluctuations $\langle u_L'^2 \rangle, \langle w_L'^2 \rangle$ of the synthetic QSQH velocity field as compared to the DNS velocity field at $Re_\tau = 1000$ (solid line) and 5200 (dashed line).

with respect to Re .

Overall, for this range of Re_τ in the channel flow, the QSQH hypothesis of the proposed statistical independence appears to be accurate near the wall, approximately below $y^+ \approx 20$ for the streamwise component and $y^+ \approx 10$ for the spanwise component. However, the relative error grows with the distance from the wall, reaching approximately 10% at $y^+ \sim 70 - 80$.

Large-scale motions

The QSQH software generates the synthetic velocity field in boxes of dimensions too small for calculating the large-scale component. However, the large-scale component of the synthetic field can be calculated using the formulae given by the QSQH theory itself (Chernyshenko, 2021):

$$\begin{aligned} u_L(y) &= u_{\tau_L} \tilde{U}(y u_{\tau_L}) \cos \theta, \\ w_L(y) &= -u_{\tau_L} \tilde{U}(y u_{\tau_L}) \sin \theta. \end{aligned} \tag{4.4}$$

Figure 4.3 presents the relative errors of the second moments of the wall-parallel large-scale velocity fluctuations. As can be seen, the relative errors of fluctuations of both wall-parallel large-scale velocity components decrease as Re_τ is increases.

This trend suggests that the statistical characteristics of the large-scale motions increasingly align with the QSQH hypothesis as Re_τ increases. This observation is consistent with the behaviour of the large-scale correlation coefficients presented in Tables 3.2 and 3.3, which shows that the large-scale correlations are greater for the flow with higher Re_τ . Note that the large-scale motions referred to in Tables 3.2 and 3.3 and in Figure 4.3 were defined using the same large-scale filter.

Markedly, the behaviour of the relative errors of streamwise and spanwise velocity components is quite different. The relative errors of the streamwise component are nearly zero even at lower Re_τ , whereas the magnitude of the relative errors of the spanwise component is greater and reaches about 50% even at $Re_\tau = 5200$. This discrepancy suggests that the large-scale spanwise motions comply far less with the QSQH hypothesis compared to the streamwise motions at the same Re_τ and defined with the same large-scale filter. This observation also agrees with Tables 3.2 and 3.3, which show that the large-scale correlation of the spanwise velocity is smaller than that of the streamwise velocity.

Small-scale motions

The small-scale component of the synthetic velocity field can be calculated using the formulae

$$\begin{aligned} u_S &= u_{\tau_L}(\tilde{u}'(yu_{\tau_L}) \cos \theta + \tilde{w}'(yu_{\tau_L}) \sin \theta), \\ w_S &= u_{\tau_L}(\tilde{w}'(yu_{\tau_L}) \cos \theta + \tilde{u}'(yu_{\tau_L}) \sin \theta) \end{aligned}$$

obtainable by subtracting the large-scale component given by (4.4) from the total synthetic field given by (2.8-2.13). Here, $\tilde{u}' = \tilde{u} - \tilde{U}$ and $\tilde{w}' = \tilde{w}$. Figure 4.4 shows the relative errors of the second moments of the wall-parallel small-scale velocity fluctuations. The relative errors are close to zero at the wall and increase with distance from the wall. These errors are noticeably smaller than the relative errors of the large-scale component. Interestingly, in contrast to large scales the relative errors of the small-scale statistics for both streamwise and spanwise velocity are greater at the larger Re_τ . This observation matches the behaviour of the small-scale correlation coefficients, which, as shown in Tables 3.2 and 3.3, increase as Re_τ is increased, if the same large-scale filter is used for different Re_τ .

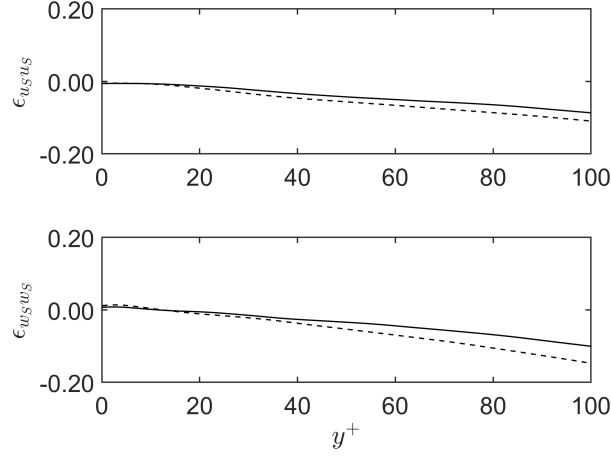


Figure 4.4: Relative errors in small-scale components of second moments of wall-parallel velocity fluctuation $\langle u_S'^2 \rangle, \langle w_S'^2 \rangle$ between DNS field and synthetic QSQH field at $Re_\tau = 1000$ (solid line) and 5200 (dashed line) respectively. Note the difference in ordinate scale with Fig. 4.3.

For the large-scale filter and the DNS field that we used, the ratio of the magnitude of the second moment of the small-scale streamwise velocity component to the magnitude of the second moment of the large-scale streamwise velocity component is about 1.2. For the spanwise velocity, this ratio can reach approximately 14. Consequently, as far as the moments of the total velocity (that is the sum of small- and large-scale components) are concerned, as Re_τ changes, the change in large scales predominantly outweighs the change in small scales of the streamwise velocity. Conversely, for the spanwise velocity component the opposite trend is observed.

4.1.3 Reynolds stress dependence on the Reynolds number

The QSQH theory predicts that the Reynolds stress can be expressed as (see (3.8) in (Chernyshenko, 2021))

$$\langle u'v' \rangle = \tilde{\tau}_R + \frac{\langle u_{\tau_L}^{\prime 2} \rangle}{2} \frac{d^2 y^2 \tilde{\tau}_R}{dy^2} - \frac{\Delta \langle \theta^2 \rangle}{2} \tilde{\tau}_R + \dots, \quad (4.5)$$

where $\tilde{\tau}_R = \langle \tilde{u}'\tilde{v}' \rangle$ is the Reynolds shear stress in the universal velocity field, and the truncated expansion is in the amplitude of the fluctuation of large-scale wall friction. Then the difference between the Reynolds stress at the same y in any two

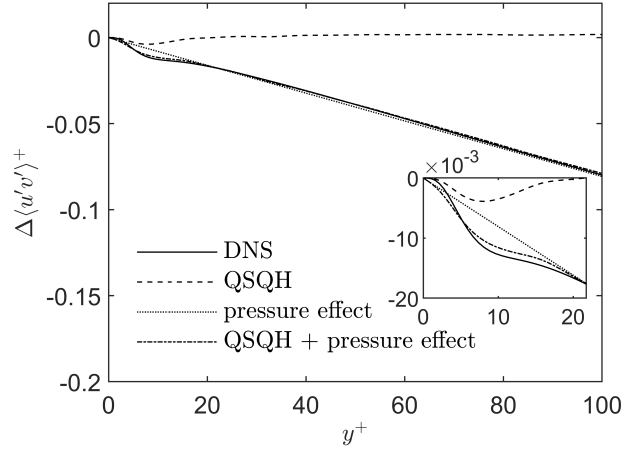


Figure 4.5: The difference of the Reynolds shear stress at $Re_\tau = 1000$ and 5200 . QSQH results were calculated using the synthetic model.

flows is ((3.13) in (Chernyshenko, 2021))

$$\Delta \langle u'v' \rangle|_{y=\text{const}} = \frac{\Delta \langle u_{\tau_L}^{\prime 2} \rangle}{2} \frac{d^2 y^2 \tilde{\tau}_R}{dy^2} - \frac{\Delta \langle \theta^2 \rangle}{2} \tilde{\tau}_R + \dots, \quad (4.6)$$

where $\Delta \langle u_{\tau_L}^{\prime 2} \rangle$ and $\Delta \langle \theta^2 \rangle$ are the differences of the values of $\langle u_{\tau_L}^{\prime 2} \rangle$ and $\langle \theta^2 \rangle$ in these flows. If the comparison is made between the values at the same y^+ then (see Appendix A) the difference is

$$\Delta \langle u'v' \rangle|_{y^+=\text{const}} = \frac{\Delta \langle u_{\tau_L}^{\prime 2} \rangle}{2} \frac{d^2 y^2 \tilde{\tau}_R}{dy^2} - \frac{\Delta \langle \theta^2 \rangle}{2} \tilde{\tau}_R - \left(\langle u_{\tau_L}^{\prime 2} \rangle - \frac{\langle \theta^2 \rangle}{2} \right) \frac{d\tilde{\tau}_R}{dy} + \dots \quad (4.7)$$

In (Chernyshenko, 2021), (4.6) was compared with the results of direct numerical simulations of a plain channel flow at different Reynolds numbers, with the values of the Reynolds stress taken at the same y^+ . However, using (4.7) would lead to the same conclusion. The data in the Fig. 4.5 of the present paper are different in several respects but the most obvious conclusion is the same: the direct numerical simulation result and the QSQH prediction are very different. The primary reason for this is the effect of the pressure gradient, which for the Reynolds shear stress at these conditions is much greater than the effect of large-scale motions (Chernyshenko, 2021).

The pressure effect can be quantified by the contribution of the mean pressure gradient to the total stress, $y^+ d \langle p^+ \rangle / dx^+ = -y^+ / Re_\tau$, which also gives the estimate of the contribution to the difference between the two flows:

$$y^+ \Delta \frac{d \langle p^+ \rangle}{dx^+} = y^+ \Delta \frac{1}{Re_\tau}, \quad (4.8)$$

which is shown as the pressure effect in Fig. 6 in (Chernyshenko, 2021) and Fig. 4.5 here. One of the conclusions in (Chernyshenko, 2021) was that even though the effect of large-scale motions increases with Re_τ while the effect of pressure gradient tends to zero as $Re_\tau \rightarrow \infty$, in the range of the Reynolds numbers considered, the QSQH theory cannot be used for estimating the dependence of the Reynolds stress on the Reynolds number.

It can be observed, however, that near the wall, roughly for $y^+ < 20$, the pressure contribution and the contribution of the large-scale motions as described by the QSQH theory are comparable. Moreover, at these distances from the wall, both effects are small. Then one can expect that these two effects are additive. Verifying this requires overcoming certain difficulties. First, purely technical, is the choice between doing comparisons for fixed values of y or for fixed values of y^+ , that is, using (4.6) or (4.7), with both choices requiring conversions between different units. Second, unlike many other quantities, while the Taylor expansion up to the quadratic terms in the amplitude of the fluctuations of the large-scale component of the wall friction (4.6) does not contain terms with large coefficients (see Section 2.3.5), the higher order terms can contain such terms. This can affect the results but is difficult to explore. The synthetic model developed in the present study resolves this difficulty since the predictions obtained with it do not involve the Taylor expansion and because it gives the predictions of the QSQH theory already in wall units, so no conversion is required for comparisons.

Figure 4.5 presents the changes in the Reynolds shear stress between $Re_\tau = 1000$ and $Re_\tau = 5200$ calculated from the direct numerical calculations dataset (Lee and Moser, 2015), by the QSQH prediction made using the synthetic model, the change due to the pressure gradient as described by (4.8), and the sum of the last two effects. The inset shows more detail and allows us to conclude that the contributions of the QSQH effect and the pressure gradient are indeed approximately additive.

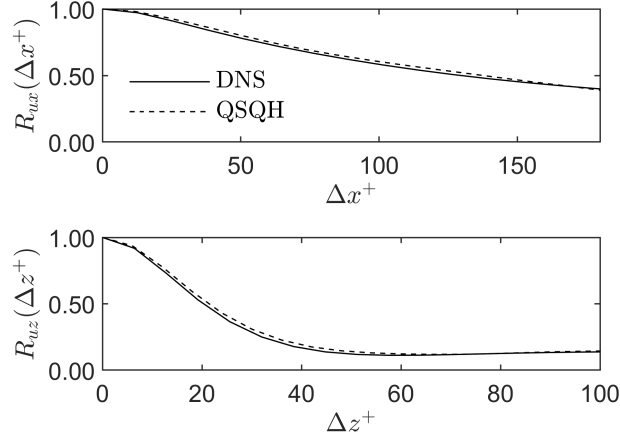


Figure 4.6: Streamwise velocity two-point correlation coefficients at $Re_\tau = 5200$: DNS (solid) and QSQH synthetic model (dashed).

This is important as a similar situation might occur in other cases, and thus opens a route for extending the domain of applicability of the QSQH theory.

4.1.4 Two-point correlations

The QSQH synthetic model software can generate three-dimensional synthetic velocity fields. This section provides a simple example of its use. The three-dimensional synthetic velocity fields considered here were generated using the universal velocity field that was produced from the turbulent channel flow of $Re_\tau = 5200$ and then used to calculate the two-point correlation coefficients

$$\begin{aligned}
 R_{ux}(\Delta x^+) &= \frac{\langle u'(x^+)u'(x^+ + \Delta x^+) \rangle}{\sqrt{\langle u'^2(x^+) \rangle \langle u'^2(x^+ + \Delta x^+) \rangle}}, \\
 R_{wx}(\Delta x^+) &= \frac{\langle w'(x^+)w'(x^+ + \Delta x^+) \rangle}{\sqrt{\langle w'^2(x^+) \rangle \langle w'^2(x^+ + \Delta x^+) \rangle}},
 \end{aligned} \tag{4.9}$$

$$\begin{aligned}
 R_{uz}(\Delta z^+) &= \frac{\langle u'(z^+)u'(z^+ + \Delta z^+) \rangle}{\sqrt{\langle u'^2(z^+) \rangle \langle u'^2(z^+ + \Delta z^+) \rangle}}, \\
 R_{wz}(\Delta z^+) &= \frac{\langle w'(z^+)w'(z^+ + \Delta z^+) \rangle}{\sqrt{\langle w'^2(z^+) \rangle \langle w'^2(z^+ + \Delta z^+) \rangle}}
 \end{aligned} \tag{4.10}$$

at $y^+ = 0.711$, that is very close to the wall.

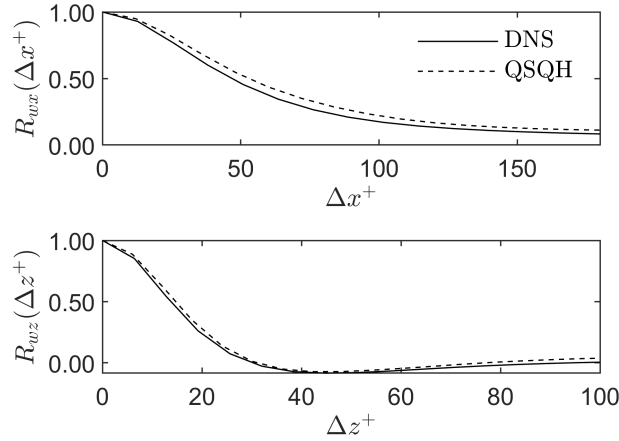


Figure 4.7: Spanwise velocity two-point correlation coefficients at $Re_\tau = 5200$: DNS (solid) and QSQH synthetic model (dashed).

Figs. 4.6 and 4.7 correspond to $Re_\tau = 5200$. In this case the synthetic model is applied to the base flow itself, so that Figs. 4.6 and 4.7 just show to what extent the base flow satisfies the QSQH hypothesis per se (see Section 4.1.2) as far as two-point correlations are concerned.

For $Re_\tau = 5200$, the minimum value of $R_{uz}(\Delta z^+)$ is positive, contrary to the often reported behaviour (see (Ganapathisubramani et al., 2005), (Teng et al., 2018) and (Leoni et al., 2021)) at lower Re_τ . The lowest point of $R_{uz}(\Delta z^+)$ should correspond to the near-wall streak spacing $2\Delta z^+ \approx 100$ but is slightly larger. The positive value at the minimum is due to the existence of strong large-scale motions, which also cause the minimum of $R_{uz}(\Delta z^+)$ to shift to a larger value of Δz^+ for higher Re_τ (Lagraa et al., 2004).

It is interesting to check what the streak spacing defined as twice the value of Δz^+ at the minimum of $R_{uz}(\Delta z^+)$ would be in the case if there were no fluctuations of the large-scale motions. This is the same as the streak spacing in the universal velocity field. The QSQH software tool allows to easily obtain the database of the universal velocity simply by providing a constant wall shear as the input. Figure 4.8 shows the two-point correlations of the universal velocity field. Indeed, the minimum of $R_{uz}(\Delta z^+)$ is negative.

Figures 4.9 and 4.10 are for $Re_\tau = 1000$. These figures compare the true predictions, since the QSQH results are obtained based on limited information on the flow of $Re_\tau = 1000$, that is, the statistics of the large-scale component of the wall

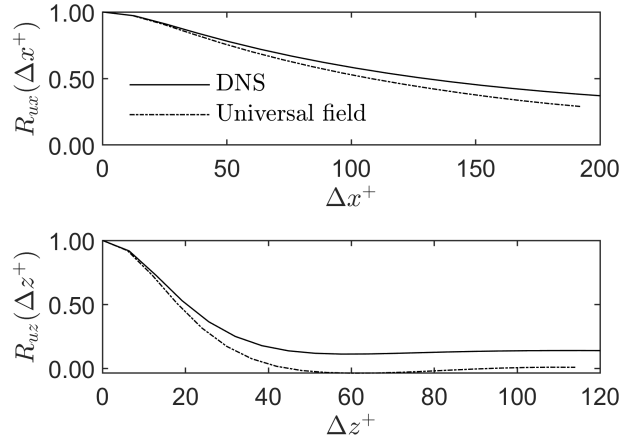


Figure 4.8: Streamwise velocity two-point correlation coefficients: DNS at $Re_\tau = 5200$ (solid) and universal velocity of the QSQH synthetic model (dash-dotted).

friction only. The universal velocity field used in this case is also generated from the channel flow of $Re_\tau = 5200$. Overall, the variation of two-point correlations with Re_τ is not large, but comparing the plots for in Figs. 4.6 and 4.9 and one can still observe that the QSQH prediction of $R_{uz}(\Delta z^+)$ does agree with the explanation of the change given in (Lagraa et al., 2004).

4.1.5 Reynolds number universality

The results in this section do not require the publicly available synthetic model software tool and do not, therefore, give an example of its use. These results were obtained using our in-house generator of the universal field and are included here to be used in the discussion that follows.

As already described in section 4.1.2, the QSQH theory has two hypotheses. The first hypothesis, the QSQH hypothesis per se, was already tested in that section. The second, the universality hypothesis, states that the statistical properties of $(\tilde{u}, \tilde{v}, \tilde{w})$ defined by (2.8–2.13) in near-wall region are the same in flows with different geometries and Reynolds numbers, provided that Re_τ is sufficiently large and the wall is sufficiently close. To inspect this, in Fig. 4.11 we plot the root mean square of the fluctuations of the streamwise and spanwise velocities for $Re_\tau = 1000$ and 5200. The difference between the curves corresponding to different Re_τ characterises the

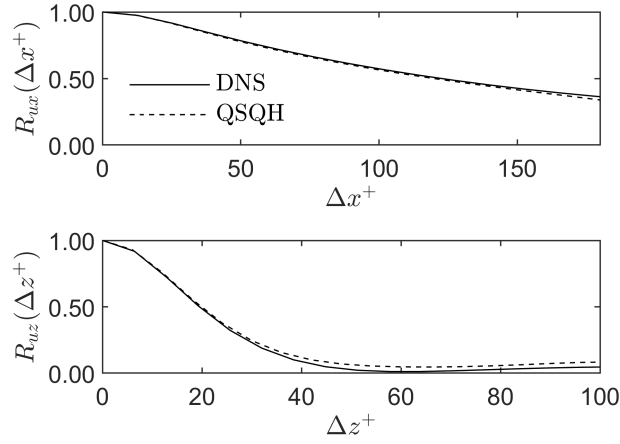


Figure 4.9: Streamwise velocity two-point correlation coefficients at $Re_\tau = 1000$: DNS (solid) and QSQH synthetic model (dashed).

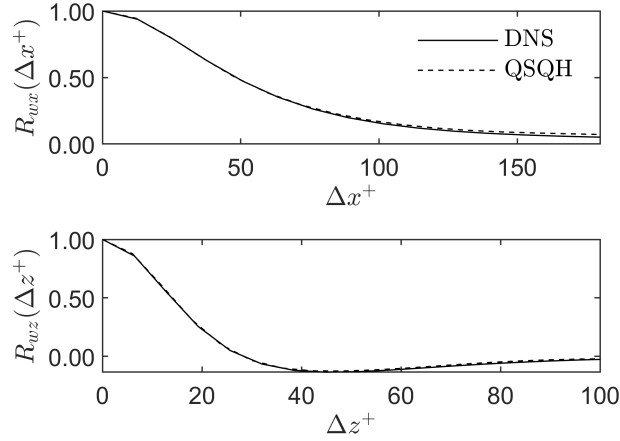


Figure 4.10: Spanwise velocity two-point correlation coefficients at $Re_\tau = 1000$: DNS (solid) and QSQH synthetic model (dashed).

magnitude of the deviation from the QSQH universality assumption. In line with other observations, the deviation grows with the distance to the wall, and is greater for the spanwise velocity than for the longitudinal velocity.

4.2 Pipe flow

We have presented the results of applying the QSQH synthetic turbulence model exclusively to plane channel flow at Reynolds numbers $Re_\tau = 5200$ and 1000 , focusing on the predictions of velocity statistics. However, as we review studies related to QSQH theory in (Zhang and Chernyshenko, 2016; Zhang, 2019; Chernyshenko,

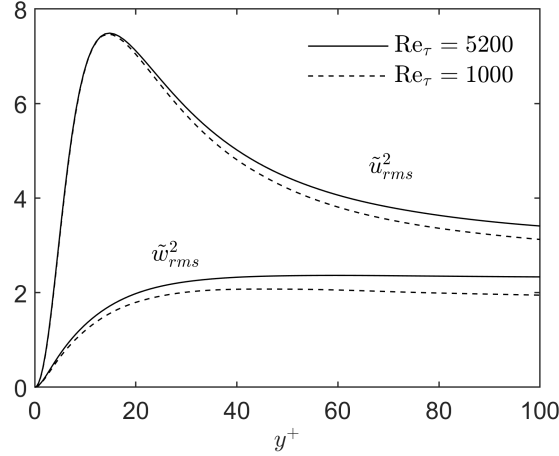


Figure 4.11: Wall-parallel stress profiles of the universal velocity fields generated from $Re_\tau = 1000$ and 5200 .

2021), one of the key questions is whether the QSQH theory can accurately predict changes in velocity statistics between two different Reynolds numbers. With knowledge of the velocity statistics at a lower Reynolds number, an accurate prediction of changes in these statistics naturally leads to precise predictions for higher Reynolds numbers. By comparing QSQH predictions with DNS results regarding changes in velocity statistics, we can elucidate how large-scale effects as prescribed by QSQH theory contribute to change in velocity statistics as the Reynolds number changes. Given that only two Reynolds numbers are available for application in plane channel flow, as discussed in section 4.1, it becomes pointless to predict changes in velocity statistics using the QSQH model. This limitation arises because QSQH predictions exhibit poor accuracy for channel flow at $Re_\tau = 1000$, meanwhile it is impossible to present a clear Reynolds number dependency of changing velocity statistics.

For this reason, this section presents the differences in flow velocity statistics between flows at multiple Reynolds numbers for both plane channel flow and pipe flow. Part of these statistics are for the turbulent pipe flows direct numerical simulations for $Re_\tau = 1140, 3000$ and 6000 conducted by (Pirozzoli et al., 2021). The cross-flow differences between the turbulent pipe and channel flow are also calculated, where the channel flow statistics are obtained from the DNS of the turbulent channel flow at $Re_\tau = 5200$ provided by (Lee and Moser, 2015), available at (University). Note that the DNS pipe and channel flows are presented in cylindrical and Cartesian

coordinate systems respectively. In this study, for simplicity the properties of the pipe flow are interpreted as in the Cartesian coordinate system: the longitudinal, radial and azimuthal directions are denoted x , y and z respectively, the corresponding velocity components are denoted u , v and w , which will be referred to as the stream-wise, wall-normal and span-wise velocity components. Time is denoted t , if used in this context.

The QSQH model generates the synthetic turbulent velocity field corresponding to the given set of wall friction snapshots. These snapshots can be obtained from DNS of a pipe flow or a channel flow, and for different Reynolds numbers. Then the statistics of the flows obtained from these DNS could be compared with the statistics of the fields obtained from the QSQH model. However, within the range of the Reynolds numbers for which DNS data are available, the differences of many such statistics are small. Hence, as we have often done in other sections, we will compare the differences in various statistics between flows obtained by DNS and by QSQH model. This provides much more stringent and more informative tests.

The snapshots of the wall friction $\tau(t, x, z)$ required as an input to the QSQH model were obtained from the DNS pipe and channel flow calculation (Pirozzoli et al., 2021) and (Lee and Moser, 2015), provided by the authors of these works as sets of temporal snapshots. There were enough snapshots for all $\tau(t, x, z)$ sets to be statistically representative of the large-scale component of wall friction of the corresponding flows. The QSQH model itself is publicly available at (Yang, 2024).

The DNS results and the QSQH predictions are compared for different cases as shown in Table 4.3. The turbulent pipe flows at $Re_\tau = 1140, 3000$ and 6000 are tagged P1140, P3000 and P6000 respectively. The turbulent channel flows at $Re_\tau = 1000$ and $Re_\tau = 5200$ are tagged C1000 and C5200. Since all the values of Re_τ considered differ, the statistics can be identified by the values of Re_τ . Then the difference in the velocity statistics, for example, mean velocities U , between two cases is calculated as

$$\Delta U = [U]_{Re_{\tau,1}}^{Re_{\tau,2}} = (U)_{Re_{\tau,2}} - (U)_{Re_{\tau,1}}. \quad (4.11)$$

The comparison cases will be referred to by a four-symbols code, with two first

Flow	Re_τ	$\langle u_{\tau_L}'^2 \rangle$	$\langle \theta^2 \rangle$
Pipe	1140	0.004097	0.001243
	3000	0.006178	0.002232
	6000	0.007694	0.002894
Channel	1000	0.003010	0.000978
	5200	0.007348	0.002874

Table 4.2: The flows for which the DNS statistics and the wall friction are taken, and their corresponding large-scale wall friction statistics $\langle u_{\tau_L}'^2 \rangle$ and $\langle \theta^2 \rangle$, with the Fourier cut-off of $\lambda_x^+ = 2000\pi$ and $\lambda_z^+ = 250\pi$ as the large-scale filter.

symbols corresponding to the first flow and the rest to the second flow. The letters encode ‘channel’ with ‘C’ and ‘pipe’ with ‘P’, and the digits are the first digit of the Reynolds number of the corresponding flow. For example, if in (4.11) $Re_{\tau,1} = 1140$, corresponding therefore to the pipe flow P1140, and $Re_{\tau,2} = 5200$, corresponding therefore to the channel flow C5200, then ΔU would be the value for the case P1C5.

The two parameters $\Delta \langle u_{\tau_L}'^2 \rangle$ and $\Delta \langle \theta^2 \rangle$ presented in Table 4.3 are the measures of the difference in the strength of the large-scale motions between two flows. They are calculated from the $\tau(t, x, z)$ sets by the QSQH model as the “byproducts” of the generation of the synthetic field. The QSQH model applies the same large-scale filter $\mathcal{L}$ to the τ sets: $\tau_L = \mathcal{L}\tau$, and calculates u_{τ_L} and θ as

$$u_{\tau_L}(t, x, z) = \sqrt{\tau_L / \rho}$$

$$\theta(t, x, z) = \tan^{-1} \tau_{L,z} / \tau_{L,x}$$

where ρ is the density of the flow, $\tau_{L,z}$ and $\tau_{L,x}$ are the stream-wise and span-wise components of instantaneous wall friction. The angular bracket stands for averaging over t , x and z directions in this context.

The comparisons in this section aim to address these three key points:

1. The accuracy of the QSQH predictions made for the pipe flow,
2. The accuracy of the QSQH extrapolation of turbulence statistics from channel to pipe,

Case	$Re_{\tau,1}$	$Re_{\tau,2}$	$\Delta\langle u_{\tau_L}^{\prime 2} \rangle$	$\Delta\langle \theta^2 \rangle$	DNS	QSQH
C1C5	C1000	C5200	0.004338	0.001896		N/A
P1P6	P1140	P6000	0.003603	0.001651		N/A
P1P3	P1140	P3000	0.002081	0.000989		-----
P3P6	P3000	P6000	0.001522	0.000662		-----
P1C5	P1140	C5200	0.003251	0.001631		-----
P3C5	P3000	C5200	0.001170	0.000642		-----
P6C5	P6000	C5200	-0.000346	-0.000020		-----

Table 4.3: The cases for the changes in statistics changes pairs of flows. P1140, P3000 and P6000 stands for the pipe flows at $Re_\tau = 1140$, 3000 and 6000 respectively; C1000 and C5200 stand for the channel flow at $Re_\tau = 1000$ and 5200. $\Delta\langle u_{\tau_L}^{\prime 2} \rangle = [\langle u_{\tau_L}^{\prime 2} \rangle]_{Re_{\tau,1}}^{Re_{\tau,2}}$ and $\Delta\langle \theta^2 \rangle = [\langle \theta^2 \rangle]_{Re_{\tau,1}}^{Re_{\tau,2}}$. In Cases C1C5 and P1P6, only DNS statistics are presented. DNS results and QSQH predictions are compared in the rest of the cases where their plots, where applicable, differentiate into solid and dashed lines.

3. The Reynolds-number-dependency of the QSQH predictions and extrapolations.

The QSQH predictions regarding changes in velocity statistics are derived from the synthetic velocity field output generated by the QSQH synthetic model for the corresponding Reynolds numbers, as indicated in Table 4.3. The QSQH synthetic turbulence model utilizes a built-in universal velocity field provided by the QSQH software tool, which is produced from plane channel flow at $Re_\tau = 5200$. The changes in velocity statistics are calculated using the formula defined in (4.11). For instance, the change in mean velocity profile for Case P1P3 is calculated as follows:

$$\Delta U = [U]_{P1000}^{P3000} = (U)_{\text{pipe}, Re_\tau=3000} - (U)_{\text{pipe}, Re_\tau=1140}.$$

This formula is also applied to Cases P1C5, P3C5, and P6C5, where the changes in velocity statistics from channel flow to pipe flow are examined. In these instances, the QSQH predictions are also calculated utilizing both the QSQH model and built-in universal velocity field.

The QSQH prediction regarding the change in velocity statistics from one flow to another can be interpreted as QSQH-enabled extrapolation of the statistics of

the first flow to the second flow. For instance, utilizing known statistics obtained from DNS of a flow characterized by $Re_{\tau,1}$, the accurate QSQH predictions of the difference in these statistics between $Re_{\tau,1}$ and $Re_{\tau,2}$ will yield precise statistical representations for the flow at $Re_{\tau,2}$. In this study, in order to distinguish and remark the cross-flow cases (P1C5, P3C5 and P6C5) from other cases, the QSQH predictions regarding the changes from channel to pipe for these three cases are referred to as the extrapolation from channel to pipe.

4.2.1 Mean velocity profiles

The mean velocity profile of wall-bounded turbulent flow is dependent on the Reynolds number and the geometry of the flow. In this subsection, the differences in the mean velocity profile at two Reynolds numbers are calculated, and comparisons are made between the DNS results and QSQH predictions to reveal the quality of QSQH prediction and the large-scale effects contributing to the change in the mean velocity profile. Note that since the values of ΔU^+ are generally small as compared to the values of U^+ , ΔU^+ is a very sensitive quantity, so that the subtle differences between the channel and pipe flow can be captured and be pronounced in the plots.

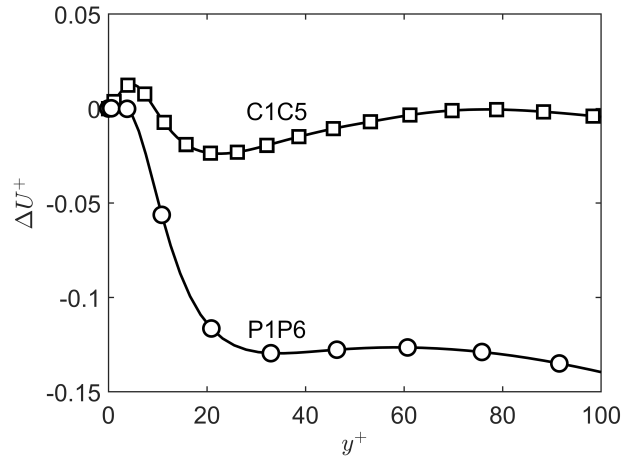


Figure 4.12: The DNS results of the change in the mean velocity profile of the pipe flows between two Reynolds number for Case C1C5 (C5200-C1000, squares) and Case P1P6 (P6000-P1140, circles).

The differences in the mean velocity profile of the DNS of a pipe and chan-

nels flows for similar Re_τ , $\Delta Re_\tau = 5200 - 1000 = 4200$ (channel) and $\Delta Re_\tau = 6000 - 1140 = 4860$ (pipe) are plotted up to $y^+ = 100$ in Fig. 4.12. The plots of $\Delta U^+(y^+)$ of Case C1C5 (channel) and Case P1P6 (pipe) have a similar shape but differ in magnitude even very close to the wall. One effect contributing to $\Delta U^+(y^+)$ is the extension of the logarithmic region in the mean velocity profile as the Reynolds number increases. However, the studied region in this case is of a limited range and very close to the wall. Hence, the Reynolds number effect on the size of the log layer will have negligible contribution to $\Delta U^+(0 \leq y^+ \leq 100)$. Another Reynolds number related effect is the large-scale motions whose strength is increased as the outer-layer large-scale peak is more pronounced in the energy spectra at higher Reynolds number. The outer-layer large-scale motions affect the near-wall turbulence all the way to the wall. One can quantify the strength of the large-scale motions by calculating $\langle u_{\tau_L}^2 \rangle$ and $\langle \theta^2 \rangle$ at the wall, as shown in Table 4.2. As Table 4.3 indicates, the change in the strength of the large-scale motions in Case C1C5 is larger than that in Case P1P6, and they differ by about 17% in $\Delta \langle u_{\tau_L}^2 \rangle$ and 13% in $\Delta \langle \theta^2 \rangle$. Note that the values of $\Delta \langle u_{\tau_L}^2 \rangle$ and $\Delta \langle \theta^2 \rangle$ here are used as indicators of the contribution of the change in the strength of large-scale motions to any velocity statistics, when the full-volume turbulent pipe flow database is unavailable. Now, the plots of $\Delta U^+(y^+)$ of Case C1C5 and Case P1P6 do not match the behaviour of the large-scale indicators in two respects. First, if the change in the large-scale motions was the main cause of ΔU^+ , Case C1C5 was supposed to have a larger ΔU^+ than Case P1P6 since Case C1C5 has larger $\Delta \langle u_{\tau_L}^2 \rangle$ and $\Delta \langle \theta^2 \rangle$, whereas Fig. 4.12 shows that ΔU^+ in Case C1C5 is smaller than that in Case P1P6. Second, ΔU^+ in Case P1P6 is significantly larger in comparison with Case C1C5: they differ by about 14 times at their largest difference, implying the existence of another effect influencing ΔU^+ in these flows. One crucial difference between the channel and pipe flow is their geometry. Pipe flow experiences complex physical effects imposed by the curved wall, whereas such effects are absent in a plane channel flow. Since these effects are not explicitly presented or quantified in the present work, they will be referred to as the “geometry effects of pipe flow” in general. Therefore, the geometry effects of pipe flow contribute to $\Delta U^+(y^+)$ in addition to the relatively weaker large-scale contribution, yielding the large difference between Cases C1C5 and P1P6 seen in Fig. 4.12. Note that these are observations of the actual (DNS) flows, not their

QSQH models.

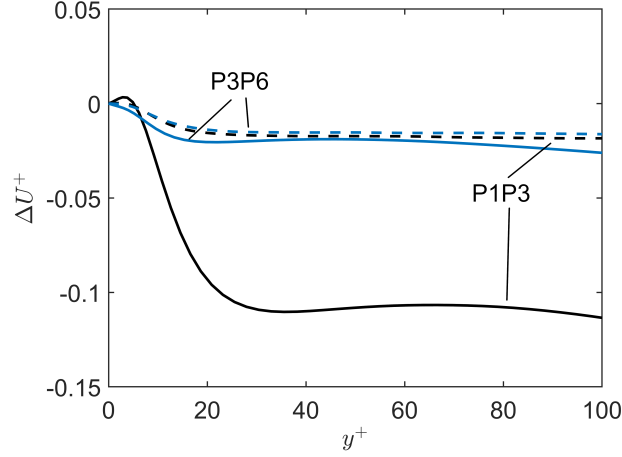


Figure 4.13: The DNS results (solid lines) and QSQH predictions (dashed lines) of the change in the mean velocity profile between the pipe flows of different Reynolds numbers for Case P1P3 (P3000 - P1140, black) and Case P3P6 (P600 - P3000, blue).

The changes in the mean velocity profile ΔU^+ of Cases P1P3 and P3P6 are plotted up to $y^+ = 100$ in Fig. 4.13. As it was seen from Fig. 4.12, the DNS result for ΔU^+ in case P1P6 (that is for $\Delta Re_\tau = P6000 - P1140$) shows mostly the geometry effects of pipe flow on this quantity. A similar observation can be made from the plots of ΔU^+ in Case P1P3 ($\Delta Re_\tau = P3000 - P1140$) shown in Fig. 4.13, where for $y^+ > 20$ the magnitude of ΔU^+ in Case P3P6 can be seen to be about ten times less than that of Case P1P3. The QSQH predictions for Cases P1P3 and P3P6 in Fig. 4.13 show the ΔU^+ between the QSQH synthetic velocity fields for the pipe flows at two Reynolds numbers. These synthetic velocity fields are generated by the QSQH synthetic model, which is based on the QSQH theory. Since this theory describes the quasi-steady effect of modulation of near-wall turbulence by the large-scale motions (Chernyshenko et al., 2012; Zhang and Chernyshenko, 2016; Chernyshenko, 2021), the differences between the two synthetic velocity fields are caused by the difference in the strength of large-scale motions. The truncated Taylor expansions of the QSQH prediction of the mean velocity profile derived from

the QSQH theory (Zhang and Chernyshenko, 2016; Chernyshenko, 2021) gives that

$$\Delta U(y) = \frac{1}{2} \Delta \langle u_{\tau_L}^{\prime 2} \rangle \frac{d}{dy} y^2 \frac{d\tilde{U}}{dy} - \frac{1}{2} \Delta \langle \theta^2 \rangle \tilde{U}(y) + \dots, \quad (4.12)$$

where $\tilde{U}$ is the mean velocity profile of the universal velocity field. Hence, (4.12), that is effectively the QSQH theory, implies that the change in the mean velocity profile is due to the change in the strength of large-scale motion. If the values of the large-scale parameters $\Delta \langle u_{\tau_L}^{\prime 2} \rangle$ and $\Delta \langle \theta^2 \rangle$ of Cases P1P3 and P3P6 given in Table 4.3 are substituted into (4.12), it gives that the QSQH prediction of ΔU^+ of Case P1P3 is higher in magnitude than that in Case P3P6. This is also observed for the small differences between the QSQH predictions of Cases P1P3 and P3P6 in Fig. 4.13.

In Case P1P3, a substantial, about 10 times, deviation of the QSQH prediction from the DNS result is observed, which can be explained by the difference that the DNS result of ΔU^+ mainly attributes to the geometry effect of pipe flow, whereas the QSQH prediction amounts to the large-scale effect prescribed by the QSQH theory. Remarkably, in Case P3P6, the QSQH prediction has a significant improvement that its plot, although not closely coinciding with the plot of DNS result in the figure, has the magnitude of the same order as the DNS result.

Overall, Figs. 4.12-4.13 suggest that in the pipe flow at relatively low Reynolds numbers the geometry effects have greater contribution to ΔU^+ than the large-scale effects described by the QSQH theory: this is shown by Case P1P3. However, for the pipe flow at higher Reynolds number, such as Case P3P6, the large-scale effects become predominant in the contributions to ΔU^+ , which effectively enables the QSQH synthetic model to have improving accurate prediction of ΔU^+ for higher Reynolds numbers. It was theoretically predicted in (Zhang and Chernyshenko, 2016; Zhang, 2019) and observed in Section 4.1 of this thesis that the QSQH theory becomes increasingly accurate as the Reynolds number of plane channel flow increases. It should be noted that the QSQH synthetic model uses a universal velocity field that is generated from a plane channel flow. Then it is used for the QSQH prediction as shown in (4.12). Therefore, the hypothesis that the the universal velocity field is the same for flows with different geometry becomes more accurate as the Reynolds number increases. This is demonstrated in the application of QSQH model to a pipe flow while the universal velocity field was generated from the plane channel

near-wall turbulence.

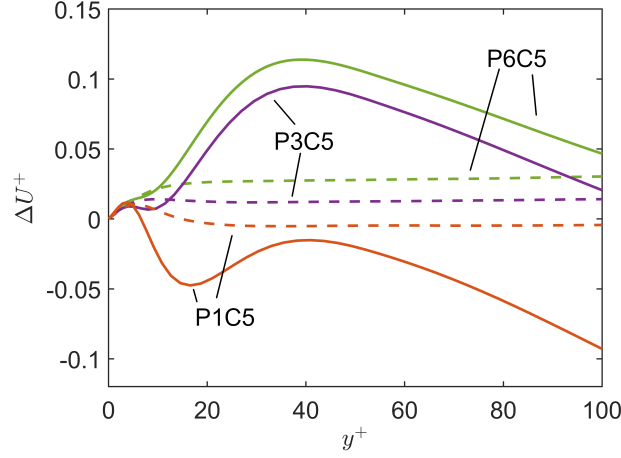


Figure 4.14: The DNS results (solid lines) and QSQH predictions (dashed lines) of the change in the mean velocity profile between three pipe flows and one channel flow of different Reynolds numbers for Case P1C5 (C5200 - P1140, orange), Case P3C5 (C5200 - P3000, purple), and Case P6C5 (C5200 - P6000, green).

The differences in the mean velocity profiles ΔU^+ between the plane channel flow and pipe flow are presented in Fig. 4.14. For all three cases the DNS results exhibit similar behaviour as a function of y , with the curve rising up and down to reach two peaks and a valley. The general shape of the curves is, however, quite different from the plots of ΔU^+ obtained for flows of the same geometry but for different Re_τ , shown in Fig. 4.12. This difference reflects the contribution of geometric variations between channel and pipe flow to $\Delta U^+(y^+)$. For Case P1C5, with $Re_\tau = 1140$ for pipe flow, the plot of ΔU^+ predominantly stays in the negative region. The flow C5200 has higher Reynolds number and expanded logarithmic layer as compared to P1140, and the mean velocity profile of C5200 also expand and has slightly smaller value than that of C1000 for $y^+ < 100$ (Pirozzoli et al., 2021; El Khoury et al., 2013; Ahn et al., 2013). As the value of Re_τ for pipe flow increases from 1140 to 3000 and 6000, we observe an elevation in the plot of ΔU^+ from the negative region to positive region. This is due to the fact that the logarithmic layer of pipe flow also expands, resulting in a decrease in the mean velocity profile under $y^+ = 100$ as the Reynolds number increases (Pirozzoli et al., 2021).

For all three cases, the QSQH predictions of $\Delta U^+(y^+)$ present a relatively simple

behaviour that the plot initially increases, reaching a peak, then gradually decreases or reaches to an approximately constant value. The QSQH predictions generally follow the trend observed in the DNS results but with significant deviations, particularly in the magnitude of ΔU^+ beyond $y^+ = 10$. The dashed lines fail to capture the near-wall peaks and valleys as shown in the DNS results. This is because the QSQH theory oversimplifies the differences between channel and pipe flow, reducing them down to just the modulation effect due to large-scale motions, meaning that it fails to account for the significant contribution of geometry differences to $\Delta U^+(y^+)$ across the flows. As the Reynolds number increases, there appears to be an improvement in accuracy for the QSQH prediction up to $y^+ = 10$, with the plot of the QSQH prediction almost coinciding with the DNS result for Case P6C5. This observation suggests that as Reynolds number increases, large-scale effects become more pronounced for ΔU^+ but have not yet become predominant within our current range of Reynolds numbers considered in this study.

4.2.2 Turbulence intensity profiles

From Fig. 4.12, we have observed that within the same type of flow, for roughly the same Reynolds number difference, $\Delta U^+(y^+)$ is different for the channel and pipe flow, implying the contribution due to the significant geometry difference between channel and pipe. In this section, at first the same investigation is done for the difference in the turbulence intensity profiles Δu_{rms}^{+2} and Δw_{rms}^{+2} between channel and pipe flow, using the DNS data, as shown in Fig. 4.15. The figure indicates that the turbulence intensities, both in the longitudinal and azimuthal directions, increase with Re_τ , which is expected as higher Reynolds numbers typically lead to more vigorous turbulence. Unlike the substantially different $\Delta U^+(y^+)$ for channel and pipe, the changes in the turbulence intensities for these two flows have a relatively decent coincidence, implying that the geometric effect does not have significant impact on Δu_{rms}^{+2} and Δw_{rms}^{+2} . However, it is interesting to note that, despite of the ΔRe_τ for the channel and pipe flow being approximately the same, we have observed that the Δw_{rms}^{+2} for these two cases coincide with each other, while the Δu_{rms}^{+2} for the pipe is higher, especially at $y^+ \approx 15$. This observation implies that when the Reynolds number increases, the pipe flow tends to distribute more energy to the

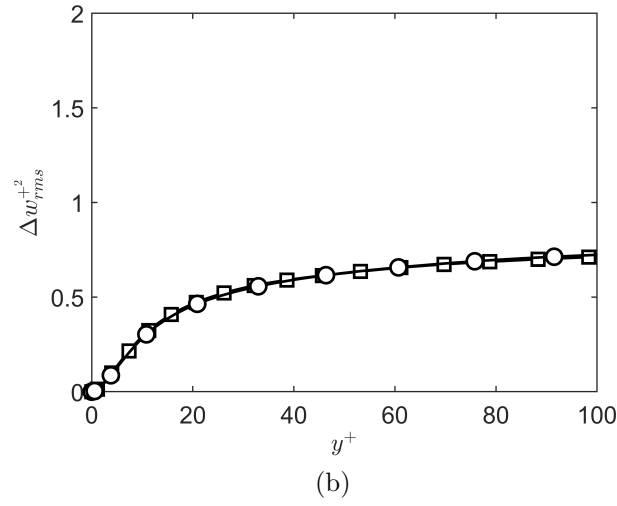
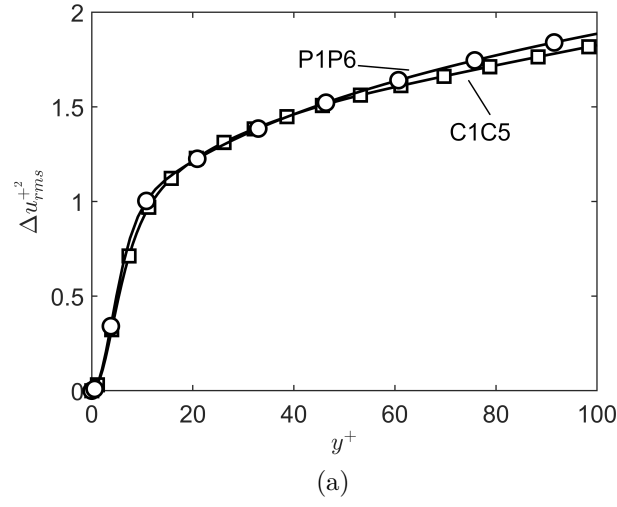


Figure 4.15: The DNS results of the change in the turbulence intensity profiles for Case C1C5 (C5200 - C1000, square) and Case P1P6 (P6000 - P1140, circle), for: (a) streamwise/longitudinal velocity; (b) spanwise/azimuthal velocity.

stream-wise velocity than the channel flow, due to many potential reasons, a major one being the different energy distribution due to different flow geometry (Saito, 2014; del Alamo and Jiménez, 2013).

In Fig. 4.16, the QSQH predictions regarding changes in turbulence intensities of pipe flow between two Reynolds numbers are compared with the corresponding DNS results. The change in longitudinal turbulence intensity Δu_{rms}^{+2} for Case P1P3, where $\Delta Re_\tau = 3000 - 1140$, reveals a substantial deviation between the QSQH pre-

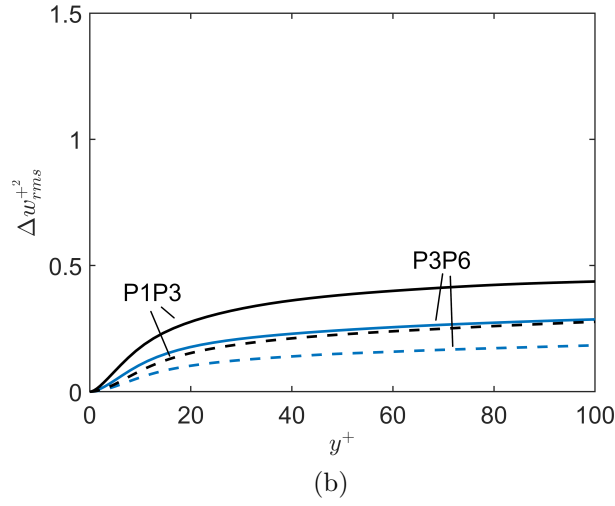
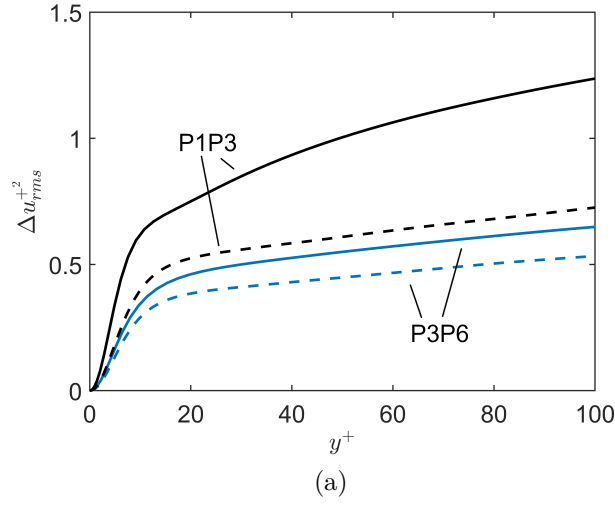


Figure 4.16: The DNS results (solid lines) and QSQH predictions (dashed lines) of the change in the turbulence intensities for Case P1P3 (P3000 - P1140, black) and Case P3P6 (P6000 - P3000, blue), for: a. longitudinal velocity; b. azimuthal velocity.

diction and DNS results, as illustrated in Fig. 4.16a. Notably, it is evident that the QSQH prediction fails to quantitatively replicate the DNS result plot. This indicates that it does not adequately capture the changes in energy distribution, particularly concerning longitudinal velocity, in pipe flow for such a difference in Re_τ . Figure 4.16a also presents Δu_{rms}^{+2} for Case P3P6, where $\Delta Re_\tau = 6000 - 3000$. In this case an improvement is observed in the QSQH prediction. As we increase the Reynolds numbers from Case P1P3, both quantitatively and qualitatively, the plot

representing QSQH predictions approaches that of the DNS results. This observation leads to an intuitive conclusion: the QSQH prediction of $\Delta u_{\text{rms}}^{+2}$ becomes more accurate at higher Reynolds numbers. This enhancement is likely due to improved predictions of energy distribution by the QSQH theory. The observed improvement can be attributed to two main factors: a less pronounced geometric effect and an enhanced large-scale effect. At lower Reynolds numbers, the turbulence intensity of pipe flow is significantly influenced by its geometry; for instance, the curvature of the pipe affects the development of secondary flow, which in turn contributes to turbulence intensity (Liu et al., 2022). Meanwhile, at such low Reynolds numbers in pipe flow, the outer-layer large-scale energy peak has yet to emerge (Samie et al., 2018), resulting in a weak presence of large-scale motions. As the Reynolds number increases, it has been observed that geometric effects diminish (Mollicone, 2017). In contrast, numerous studies have reported an intensification of large-scale motions as indicated by works such as (Gad-el Hak and Bandyopadhyay, 1994; Mathis et al., 2009; Agostini and Leschziner, 2019a; Andreolli et al., 2023) and others over recent years. Notably, very large-scale streamwise elongated motions dominate energy distribution and momentum transport in channel flows at high Reynolds numbers. These large-scale motions also modulate near-wall dynamics and contribute to the dependence of near-wall turbulence intensity on Reynolds number (Lee and Moser, 2019). Therefore, the observed improvement in the accuracy of QSQH predictions for $\Delta u_{\text{rms}}^{+2}$ presented in Fig. 4.16a essentially indicates that large-scale effects prevail over geometric influences regarding momentum transport and energy distribution as the Reynolds number increases within pipe flow.

In Fig. 4.16b the differences in the azimuthal turbulence intensity $\Delta w_{\text{rms}}^{+2}$ for Cases P1P3 and P3P6 are presented. In contrast to the observations made for $\Delta u_{\text{rms}}^{+2}$, there is no distinctive qualitative or quantitative improvement on the QSQH prediction of $\Delta u_{\text{rms}}^{+2}$ as the Reynolds number of the pipe flow increases. First, recall the negligible difference of $\Delta w_{\text{rms}}^{+2}$ between the channel and pipe flow as shown by Fig. 4.15b. It reveals the relatively weakly pronounced geometric effect in the azimuthal turbulence intensity of the pipe flow. Therefore, the deviations of the QSQH predictions from the DNS results as shown in Fig. 4.16b should not be attributed to the geometric effect either. Interestingly, these deviations have the similar magnitude in percentage as compared to the DNS results, implying that these deviations

are due to one or multiple certain physical mechanisms which the QSQH theory is incapable of describing. A major one of the potential missed physics by the QSQH theory is increased turbulence anisotropy of pipe flow. With increasing Reynolds number, the turbulent kinetic energy is more efficiently redistributed across all velocity components, including the azimuthal component. This redistribution process enhances the variance of the azimuthal velocity because more energy is transferred into this direction from the other velocity components, especially in regions away from the pipe centre.

One of the main motivations for this study, as well as for QSQH theory, is to "extrapolate" flow statistics from one flow to another. The attempts of extrapolating the turbulence intensity from channel to pipe flow are presented in Fig. 4.17, as the QSQH predictions of the changes in the turbulence intensities from channel to pipe flow. Figure 4.17 presents a comparison of the variations in streamwise (longitudinal) turbulence intensity $\Delta u_{\text{rms}}^{+2}$ and spanwise (azimuthal) turbulence intensity $\Delta w_{\text{rms}}^{+2}$ between channel and pipe flows at different Reynolds numbers. The DNS results depicted in Fig. 4.17a indicate that, with the exception of the orange curve, the differences in $\Delta u_{\text{rms}}^{+2}$ are most pronounced at lower values of y^+ , diminishing as y^+ increases. The orange curve exhibits the largest difference, indicating that the disparity between channel flow at $Re_\tau = 5200$ and pipe flow at $Re_\tau = 1140$ is more significant than those observed at higher Reynolds numbers; this can be attributed primarily to their substantial difference in Reynolds number. There exists varying degrees of agreement between the DNS results and QSQH predictions across all comparisons; however, discrepancies are particularly evident in the peak region around $y^+ \approx 20$ to 40. This observation implies that while the QSQH model endeavours to capture overall trends and near-wall modulation effects, it has limitations when extrapolating precise turbulence characteristics related to flow geometry within specific regions. However, the accuracy of extrapolation using the QSQH model exhibits both quantitative and qualitative improvements as the Reynolds number of the pipe flow increases. This indicates that at higher Reynolds numbers, the turbulence characteristics of channel and pipe flows become increasingly similar. In other words, the differences between channel and pipe flow significantly contribute to the QSQH effect, while other influences such as geometric effects diminish. In Fig. 4.17b, it is observed that the overall values of $\Delta w_{\text{rms}}^{+2}$ are lower in comparison to stream-

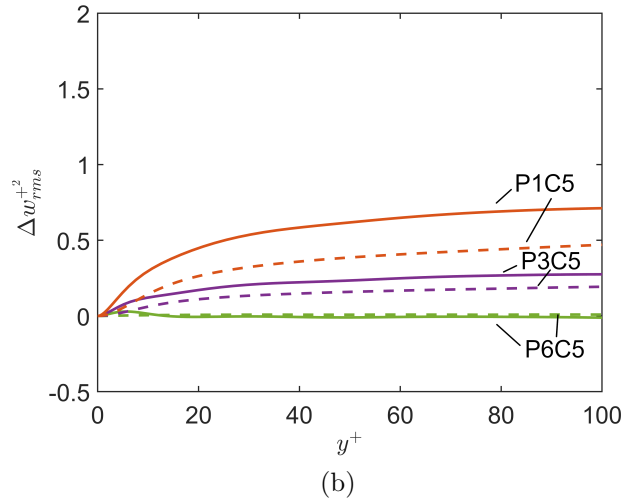
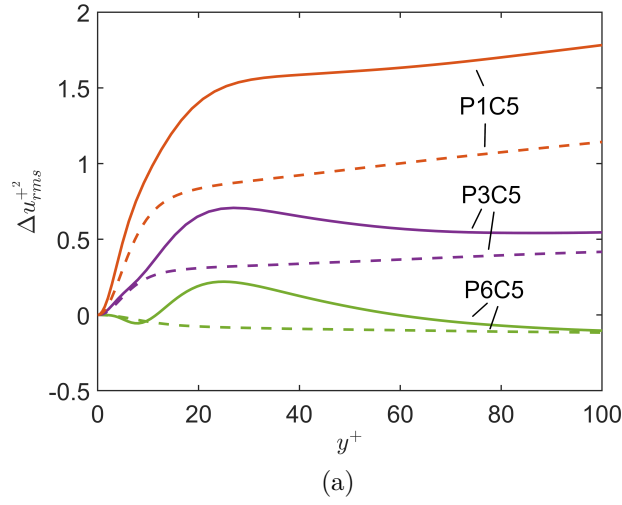


Figure 4.17: The DNS results (solid lines) and QSQH predictions (dashed lines) of the change in the turbulence intensity profiles for Case P1C5 (C5200 - P1140, orange), Case P3C5 (C5200 - P3000, purple) and Case P6C5 (C5200 - P6000, green), for: a. longitudinal velocity; b. azimuthal velocity.

wise intensities. This is typical behaviour since the streamwise component tends to dominate turbulence intensities in wall-bounded flows. Again, the largest deviation between DNS results and QSQH predictions occurs for the orange curve (pipe flow at $Re_\tau = 1140$). The green and purple curves demonstrate progressively smaller deviations as Reynolds number increases. It is particularly noteworthy that for the green curve (pipe flow at $Re_\tau = 6000$), the DNS results exhibit a peak at $y^+ \approx 10$, tapering off to zero towards the outer region of y . In contrast, the QSQH predic-

tions remain approximately zero across the range of y presented here. This serves as further evidence that as the Reynolds number increases, the QSQH effect dominates over other potential influences, except in regions very close to the wall, when extrapolating from channel flows to pipe flows. The studies by (Hoyas and Jiménez, 2008) and (Hutchins and Marusic, 2007b) provide comprehensive discussions on near-wall turbulence and emphasise the significance of capturing large-scale motions in both channel and pipe flows. Their findings corroborate our observations, indicating that large-scale motions are crucial in near-wall turbulence dynamics and that differences between flow geometries diminish as Reynolds number increases. Both figures illustrate that as Reynolds number rises, turbulent characteristics in channel and pipe flows converge more closely. This suggests that at very high Reynolds numbers, specific flow geometry (channel versus pipe) may exert a diminished influence on turbulence characteristics, thereby enhancing the reliability of extrapolation using the QSQH model from high-Reynolds-number channels to pipe flows.

4.2.3 Reynolds shear stress

As we observed and reveal in Fig. 4.5, the change in the Reynolds shear stress of plane channel flow as predicted by the QSQH model and the change in pressure gradient effect are additive. Their sum is equal to the change in the Reynolds shear stress calculated from DNS. This indicates that the magnitude of $\Delta\tau^+$ predicted by the QSQH model is effectively equivalent to $\Delta\frac{dU^+}{dy^+}$. Consequently, in this subsection, we compare the change in viscous stress for pipe flow calculated from DNS with the change in Reynolds shear stress predicted by the QSQH model to validate our earlier findings.

Figure 4.18 illustrates the comparison of variations in the viscous stress term $\frac{dU^+}{dy^+}$ (solid lines), as calculated using Direct Numerical Simulation (DNS), alongside the Reynolds stress (dashed lines) derived from the QSQH model. These variations are presented for pipe flows at different Reynolds numbers. The black lines denote the difference between pipe flows at $Re_\tau = 1140$ and 3000 , while the blue lines indicate the difference between pipe flows at $Re_\tau = 3000$ and 6000 . The viscous stress term, which represents the contribution of viscosity to momentum transfer, is most pronounced near the wall. For the black solid line representing $\Delta\frac{dU^+}{dy^+}$ between

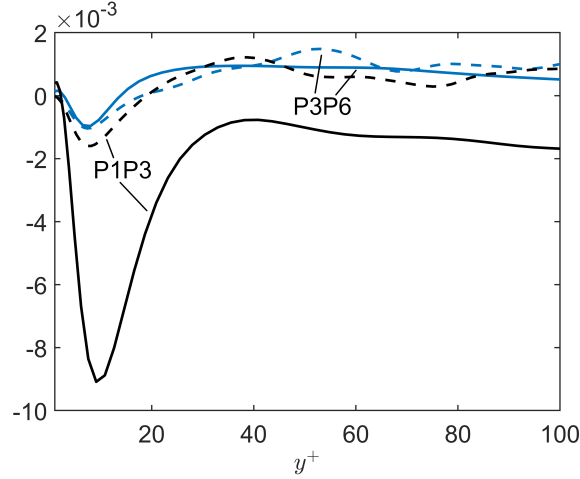


Figure 4.18: The DNS results (solid lines) of the change in the viscous stress, and QSQH predictions (dashed lines) of the change in the Reynolds stress for P3000 - P1140 (black) and P6000 - P3000 (blue).

$Re_\tau = 1140$ and 3000 , there exists a sharp negative peak around $y^+ \approx 10$. This observation indicates that the viscous stress decreases significantly in this region as the Reynolds number increases from 1140 to 3000 . Such behaviour suggests stronger velocity gradients near the wall at lower Reynolds numbers, which dissipate as the Reynolds number rises. Beyond $y^+ \approx 20$, changes in viscous stress become less significant, indicating that viscous effects predominantly influence the near-wall region. In contrast to the black solid line, the blue solid line for $\Delta \frac{dU^+}{dy^+}$ between $Re_\tau = 3000$ and 6000 exhibits much smaller values. This trend signifies that as the Reynolds number increases, viscous effects progressively diminish in importance while turbulent momentum transfer begins to dominate.

The Reynolds stress, represented by the dashed lines, is computed using the QSQH model, which estimates the difference in the turbulent momentum flux resulting from large-scale effects only. The black dashed line illustrating the QSQH prediction of $\Delta \tau_R$ between $Re_\tau = 1140$ and 3000 remains close to zero in the near-wall region. This indicates that the change in Reynolds stress is relatively small between these two Reynolds numbers within this region, as viscous forces remain significant at these values. In contrast, in the outer region ($y^+ > 20$), there is a positive increase in Reynolds stress that gradually rises with increasing Reynolds number. This suggests that turbulent fluctuations become more pronounced as the

Reynolds number increases. The blue dashed line for the QSQH prediction of $\Delta\tau_R$ between $Re_\tau = 3000$ and 6000 exhibits a more marked increase in Reynolds stress within the outer regions of flow ($y^+ > 20$). This implies that as the Reynolds number escalates from 3000 to 6000 , turbulent momentum transfer becomes increasingly significant, consistent with an enhanced dominance of large-scale turbulent structures at higher Reynolds numbers. It should be noted that plots of QSQH predictions exhibit fluctuations beyond $y^+ \approx 20$, highlighting potential averaging errors associated with calculating highly sensitive flow statistics.

The QSQH prediction, represented by the dashed lines, underestimates $\Delta \frac{dU^+}{dy^+}$ in low-Reynolds-number cases where the large-scale effects are anticipated to be weak. As the Reynolds number increases, a significant improvement in the accuracy of the QSQH prediction is observed, particularly in regions close to the wall ($y^+ < 20$). This observation corroborates earlier findings from channel flow depicted in Fig. 4.5, indicating that the QSQH prediction of $\Delta\tau_R$ for one type of flow between two different Reynolds numbers essentially yields the viscous term $\Delta \frac{dU^+}{dy^+}$. Furthermore, it suggests that the QSQH model can effectively capture non-linear behaviour regarding changes in Reynolds stress near the wall ($y^+ < 20$), even within other wall-bounded turbulence scenarios, provided that sufficiently high Reynolds numbers are maintained.

Figure 4.19 illustrates the comparison of variations in the viscous stress term $\frac{dU^+}{dy^+}$ (solid lines), as derived from DNS, with the extrapolated Reynolds stress (dashed lines) obtained through the QSQH model.

The orange solid line representing $\Delta \frac{dU^+}{dy^+}$ for $\Delta Re_\tau = C5200 - P6000$ illustrates the most pronounced variations in viscous stress. The sharp negative peak observed around $y^+ = 10$ signifies significant differences in the near-wall region, reflecting stronger near-wall viscous effects in the lower Reynolds number pipe flow compared to channel flow. As the Reynolds number increases (as indicated by the purple and green solid lines), the magnitude of changes in viscous stress diminishes, suggesting that viscous effects become less prominent with increasing Reynolds numbers, particularly when comparing channel flow to pipe flow at $Re_\tau = 6000$ (green solid line). Beyond $y^+ \approx 20$, discrepancies in viscous stress gradually decrease or smooth out, indicating that these effects are predominantly confined to the near-wall region and lose significance further from the wall.

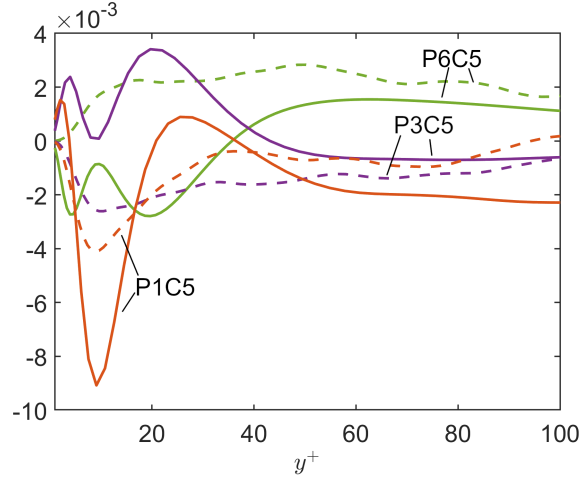


Figure 4.19: The DNS results (solid lines) of the change in the viscous stress, and QSQH predictions (dashed lines) of the change in the Reynolds stress for C5200 - P1140 (orange), C5200 - P3000 (purple) and C5200 - P6000 (green).

In this figure, differences in the viscous term between channel and pipe flows exhibit drastic behaviour close to the wall, contrasting sharply with cases of similar types of flow presented in Fig. 4.18. Generally speaking, while QSQH predictions can provide a trend for $\Delta \frac{dU^+}{dy^+}$, they fail to capture its complex behaviour even qualitatively. Furthermore, as Reynolds numbers increase, there is no improvement in the accuracy of extrapolating QSQH predictions from channel flows to pipe flows.

4.2.4 Skewness and flatness

Figure 4.20a illustrates the variation in the skewness of longitudinal velocity fluctuations, denoted as $\Delta S(u')$, across pipe flows at different Reynolds numbers. The skewness of these velocity fluctuations serves as an indicator of the asymmetry within the velocity distribution, which is closely associated with the occurrence of intermittent turbulent structures in the flow. In this figure, solid lines depict results obtained from DNS, whereas dashed lines represent predictions computed by the QSQH.

In the near-wall region ($y^+ < 10$), the black solid line (DNS for P3000 – P1140) indicates a positive change. This observation suggests that at the higher Reynolds number ($Re_\tau = 3000$), the flow exhibits more asymmetric velocity fluctuations near

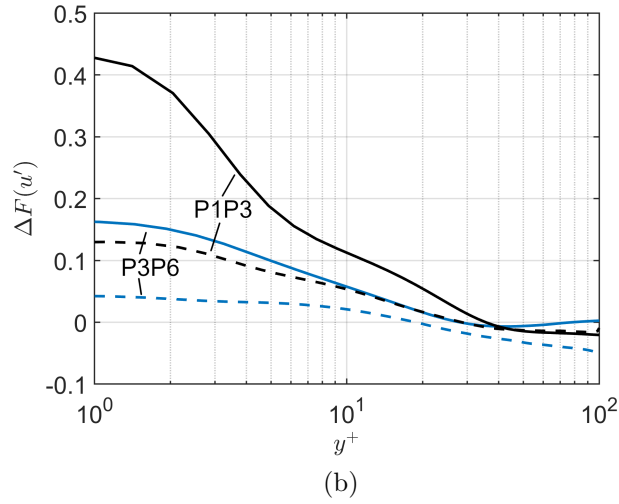
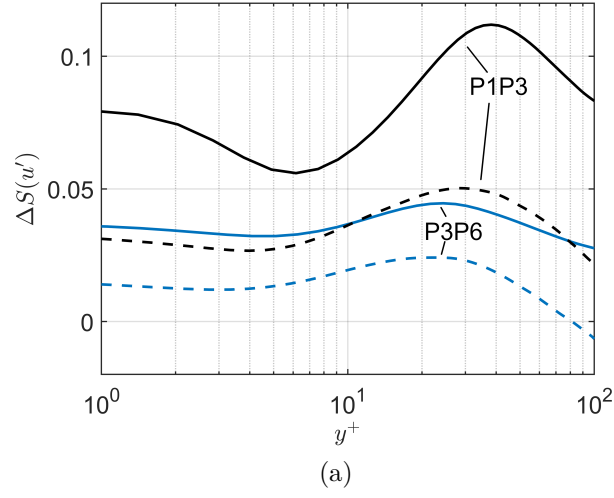


Figure 4.20: The DNS results (solid lines) and QSQH predictions (dashed lines) of the change in the, a. skewness; b. flatness, of the longitudinal velocity fluctuations between the pipe flows of different Reynolds numbers for P3000 - P1140 (black), and P6000 - P3000 (blue).

the wall, likely dominated by intermittent events such as ejections and sweeps. These turbulent structures lead to deviations in the velocity distribution from Gaussian behavior, resulting in increased skewness. Conversely, the blue solid line (DNS for P6000 – P3000) demonstrates a smaller change in skewness within the near-wall region and reveals a pronounced valley similar to that observed for the black line. This indicates that the near-wall behavior of skewness is less sensitive to Reynolds number at elevated values. Consequently, it can be inferred that as Reynolds num-

ber increases, velocity fluctuations become more homogeneous. In the outer region ($y^+ > 10$), the black solid line exhibits a more pronounced peak in skewness at approximately $y^+ \approx 40$. This observation indicates that the variation in skewness remains significant in the outer region between P1140 and P3000, likely attributable to large-scale turbulent structures influencing the velocity distribution. At lower Reynolds numbers, these structures may exhibit reduced symmetry and increased intermittency, resulting in more negative skewness within the outer region, as reported by (Bauer et al., 2017). The blue solid line indicates a smaller yet still discernible increase in skewness within the outer region. In general, the skewness of longitudinal velocity fluctuations in pipe flow tends to increase with rising Reynolds numbers, which aligns with findings from another direct numerical simulation (DNS) study on pipe flow (Bauer et al., 2017). The QSQH predictions for both cases (represented by dashed lines) underestimate the changes in skewness observed in both the near-wall and outer regions, particularly evident for the black dashed line corresponding to lower Reynolds numbers. However, it is noteworthy that the QSQH prediction qualitatively captures the behavior of DNS results, especially for the blue dashed line at higher Reynolds numbers. This observation suggests an enhanced contribution of large-scale effects to the skewness of longitudinal velocity fluctuations in pipe flow as Reynolds numbers increase. Nevertheless, there is no significant quantitative improvement in the accuracy of QSQH predictions for $\Delta S(u')$ as Reynolds number increases.

Figure 4.20b illustrates the variation in flatness of longitudinal velocity fluctuations between two sets of turbulent pipe flows at different Reynolds numbers. Flatness serves as a measure of the "tailedness" of a distribution, indicating the occurrence of extreme velocity fluctuations. The black solid line (DNS for P3000 – P1140) reveals a positive change in flatness near the wall ($y^+ < 10$), suggesting that flatness increases with an elevation in Reynolds number. This observation implies that extreme events become more frequent or pronounced close to the wall as the Reynolds number rises. Conversely, the blue solid line (DNS for P6000 – P3000) indicates a smaller increase in flatness near the wall. This finding suggests that when transitioning from a Reynolds number of 3000 to 6000, flatness is less influenced, implying that extreme velocity fluctuations in the near-wall region exhibit reduced sensitivity to further increases in Reynolds number at higher values. The QSQH predictions

(dashed lines) underestimate changes in flatness near the wall for both cases. This indicates that the model fails to fully capture the increasing influence of intermittent and burst-like events close to the wall as Reynolds number escalates. Within the outer region, the QSQH model performs better by aligning with trends showing decreasing flatness for lower Reynolds numbers. However, it still underestimates magnitude changes compared to DNS for higher Reynolds numbers. This suggests that although QSQH captures large-scale turbulence trends adequately, it falls short of addressing nuances associated with extreme events and their variations with respect to Reynolds number.

The extrapolations of skewness and flatness of longitudinal (streamwise) velocity fluctuations from channel flow to pipe flow, as derived from the QSQH model, are presented in Fig. 4.21. The comparisons between the DNS results and the QSQH predictions generally indicate that the QSQH model is unable to accurately extrapolate the skewness and flatness of longitudinal (streamwise) velocity fluctuations from channel flow to pipe flow. Combining observations from both Figs. 4.20 and 4.21, it becomes evident that the accuracy of QSQH predictions regarding changes in skewness and flatness is inadequate, with no significant improvement in accuracy observed as Reynolds numbers increase. This finding suggests that the skewness and flatness of longitudinal (streamwise) velocity are not predominantly influenced by large-scale effects as proposed by QSQH theory, even at higher Reynolds numbers.

4.2.5 Two-point correlation coefficients

The changes in the two-point correlation coefficients of longitudinal (streamwise) velocity fluctuations in wall-parallel directions are discussed in this section. It is important to note that these correlation coefficients are calculated at radial (wall-normal) positions closest to the wall; however, these positions differ for flows with varying Reynolds numbers. The specified positions are as follows: $y^+ = 0.0711$ for C5200; $y^+ = 0.0263$ for P6000; $y^+ = 0.0255$ for P3000; and $y^+ = 0.0250$ for P1140. Consequently, the changes in the two-point correlation coefficients cannot be rigorously assessed at a fixed radial (wall-normal) position. Therefore, the figures presented in this section emphasize qualitative comparisons rather than quantitative assessments between DNS results and QSQH predictions.

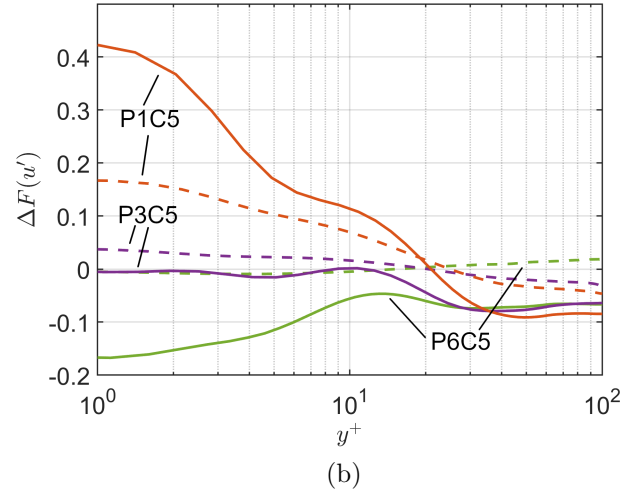
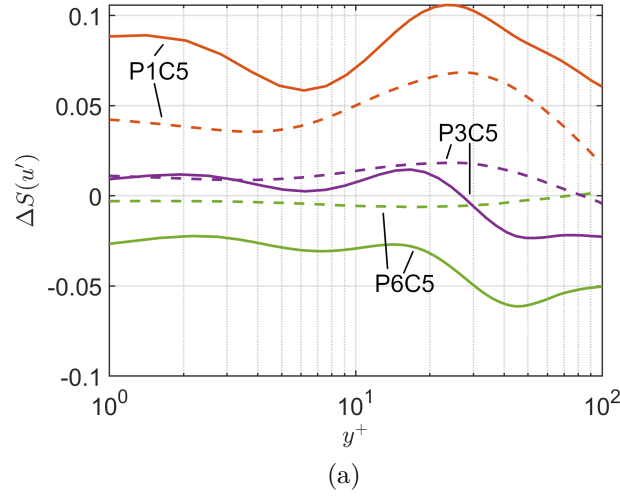


Figure 4.21: The DNS results (solid lines) and QSQH predictions (dashed lines) of the change in the, a. skewness; b. flatness, of the longitudinal (streamwise) velocity fluctuations between three pipe flows and one channel flow of different Reynolds numbers for C5200 - P1140 (orange), C5200 - P3000 (purple), and C5200 - P6000 (green).

Figure 4.22 shows the change in two-point correlation coefficients of the longitudinal velocity fluctuations in the x direction (longitudinal) and the z direction (azimuthal) for turbulent pipe flow, as calculated from DNS (solid lines) and the QSQH synthetic velocity field (dashed lines) at different Reynolds numbers.

In Fig. 4.22a, the black solid line represents the DNS results for the change in two-point correlation in the streamwise direction between the pipe flows at $Re_\tau = 3000$

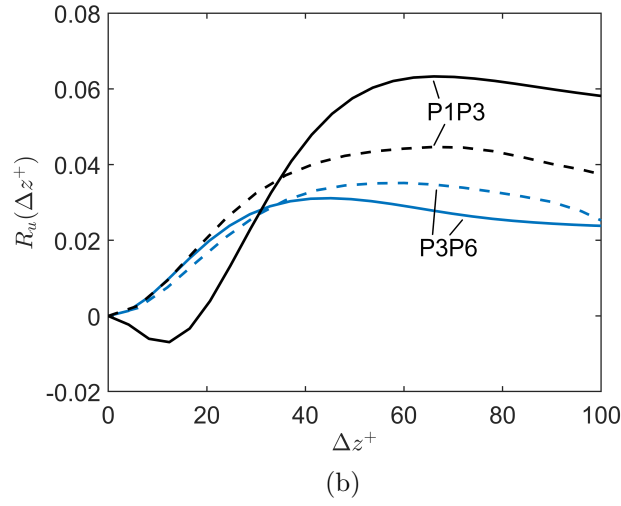
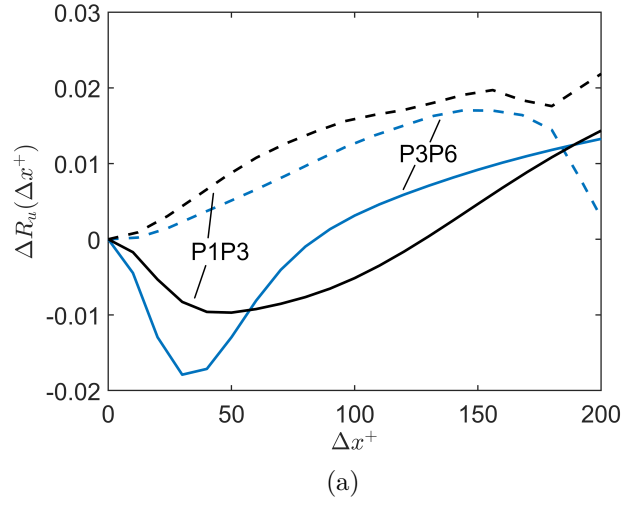


Figure 4.22: The DNS results (solid lines) and QSQH predictions (dashed lines) of the change in the changes in two-point correlation coefficients of the longitudinal velocity fluctuations in (a) x (longitudinal) direction; (b) z (azimuthal) directions, between the pipe flows of different Reynolds numbers: P3000 - P1140 (black), and P6000 - P3000 (blue).

and 1140. It shows an initial negative change in correlation for separations around $\Delta x^+ \approx 50$, followed by a positive change for larger separations ($\Delta x^+ > 100$). This suggests that at smaller scales, the velocity fluctuations are less correlated at the higher Reynolds number, but at larger scales, the correlation increases. The blue solid line shows the change in correlation between $Re_\tau = 6000$ and 3000. The initial negative change at small separations is more pronounced, followed by a significant

positive change in correlation at larger separations ($\Delta x^+ > 100$). This indicates that as the Reynolds number increases, the coherence of large-scale turbulent structures in the streamwise direction becomes more pronounced. The QSQH predictions generally follow the trend of the DNS results but underpredicts the magnitude of the changes in correlation, particularly at larger separations. This suggests that while the QSQH model captures the general effect of large-scale motions on near-wall turbulence, it may not fully represent the long-range coherence of these structures at higher Reynolds numbers. The plots of the QSQH predictions for $\Delta R_u(\Delta x^+)$ presented in Fig. 4.22a exhibit a marked change beyond $\Delta x^+ \approx 170$. This observation highlights a significant averaging error arising from an insufficient number of samples in the longitudinal direction of the QSQH synthetic velocity field, as discussed and evaluated in section 3.2.

In the azimuthal direction, as shown in Fig. 4.22b, the black solid line shows a positive change in correlation for separations around $\Delta x^+ \approx 40$, indicating that the velocity fluctuations in the azimuthal direction become more correlated as the Reynolds number increases from $Re_\tau = 3000$ and 1140. This suggests an increase in coherence of turbulent structures in the azimuthal direction at higher Reynolds numbers. The blue solid line shows a smaller but still positive change in correlation between $Re_\tau = 6000$ and 3000, particularly for separations around $\Delta x^+ \approx 40$. This implies that the increase in coherence of azimuthal structures diminishes slightly at higher Reynolds numbers but still remains positive. In contrast to the performance of the QSQH predictions in longitudinal direction, substantial improvement is observed for the QSQH prediction of $\Delta R_u(\Delta z^+)$ for the case of P6000 - P3000, indicating that the QSQH model is able to accurately predict the $R_u(\Delta z^+)$ for pipe flow of high enough Reynolds number. This essentially suggests that the 3D QSQH model is able to capture the correct streaks spacing, at very near-wall position, for the same type of wall-bounded turbulence as the Reynolds number increases.

Figure 4.23 shows the changes in two-point correlation coefficients of longitudinal (streamwise) velocity fluctuations in both the x (longitudinal/streamwise) direction and the z (azimuthal/spanwise) direction, calculated from DNS and the QSQH synthetic velocity field.

In Fig. 4.23a, the solid lines (DNS) representing $\Delta R_u(\Delta x^+)$ exhibit a consistent behaviour, showing an initial negative change in correlation for separations up to

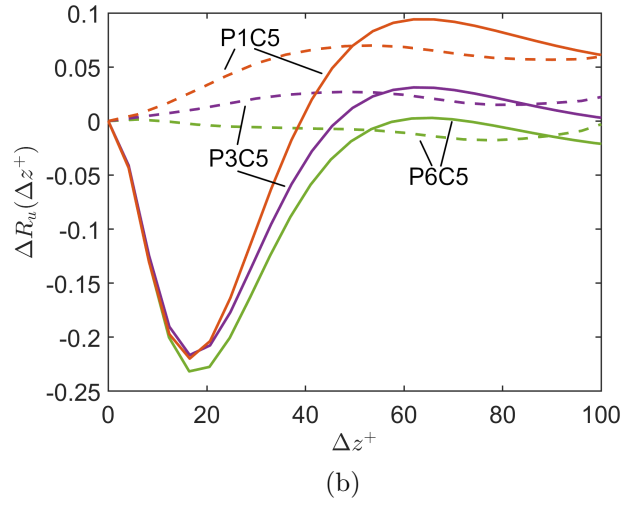
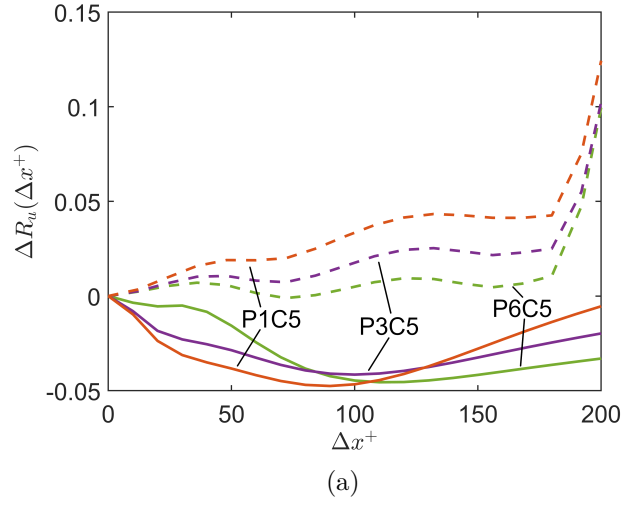


Figure 4.23: The DNS results (solid lines) and QSQH predictions (dashed lines) of the change in the changes in two-point correlation coefficients of the longitudinal velocity fluctuations in, a. x (longitudinal) direction; b. z (azimuthal) directions, between three pipe flows and one channel flow of different Reynolds numbers: C5200 - P1140 (orange), C5200 - P3000 (purple), and C5200 - P6000 (green).

$\Delta x^+ \approx 100$, followed by a rapid increase in correlation beyond this point. As the Reynolds number of the pipe flow increases to $Re_\tau = 6000$, the green solid line demonstrates minimal change in correlation, with almost no variation at small separations and only a slight positive increase at larger separations. This observation suggests that at higher Reynolds numbers, pipe and channel flows become more similar regarding the coherence of streamwise velocity fluctuations. The QSQH predic-

tions indicate a generally increasing trend towards larger separation distances. The plots illustrating the predicted change in correlation from channel to pipe reveals a descending pattern as the Reynolds numbers of both flows becomes similar, suggesting that their large-scale motions are becoming comparable. It is important to note that the rapid increase observed in QSQH predictions beyond $\Delta x^+ \approx 180$ can be attributed to an insufficient number of samples averaged for separations greater than this value. The changes in correlation between channel and pipe due to large-scale effects as prescribed by QSQH theory fail to capture any features present in DNS results. This indicates that the QSQH model is unable to accurately extrapolate $\Delta R_u(\Delta x^+)$, even at high Reynolds numbers.

Similarly, Fig. 4.23b illustrates that the solid lines (DNS) for $\Delta R_u(\Delta z^+)$ exhibit a behaviour of shifting to lower values as the Reynolds number of the pipe flow increases. The solid lines demonstrate a pronounced negative change in correlation for small separations ($\Delta z^+ < 40$), followed by a positive change at larger separations. This observation indicates that azimuthal velocity fluctuations are less correlated in pipe flow compared to channel flow. In line with the QSQH prediction of $\Delta R_u(\Delta x^+)$, the QSQH predictions for $\Delta R_u(\Delta z^+)$, depicted by dashed lines, also show a descending trend with increasing Reynolds numbers from channel to pipe flows. A notable increase in the magnitude of $\Delta R_u(\Delta z^+)$ is observed beyond $\Delta z^+ \approx 90$, attributed to an insufficient number of samples averaged for separations greater than this value. The QSQH prediction fails to capture the valley presented in DNS results at $\Delta z^+ \approx 20$. However, all three cases indicate that QSQH predictions tend to align with the DNS results beyond $\Delta z^+ \approx 50$, following a decreasing trend. This suggests that certain qualitative accuracy in extrapolating $\Delta R_u(\Delta z^+)$ using the QSQH model can be achieved beyond this separation distance if the output synthetic velocity field generated by QSQH model is sufficiently large in the radial (spanwise) direction, thereby mitigating strongly pronounced averaging errors towards the large separation distances, as illustrated in Fig. 4.23b.

4.2.6 Quality of QSQH predictions

The entire section 4.2 highlights the changes in velocity statistics between two distinct flows; however, the magnitudes of certain properties are notably small. This

may lead to skepticism regarding the quality of the QSQH predictions when compared to the actual profiles of these properties. To address this concern, we have included the actual profiles—obtained directly from the channel flow database (Lee and Moser, 2015) and pipe flow database (Pirozzoli et al., 2021)—in the appendix for reference.

Properties such as the mean velocity profile exhibit almost negligible changes with variations in Reynolds number and flow geometry, as illustrated in Fig. C.1. This observation indicates that determining Δu^+ is likely to be less critical for accurately predicting the mean velocity profile of the flows considered in this study. In contrast, other statistics presented in Fig. C.2, C.4, and C.5 demonstrate relatively significant changes with respect to Reynolds number, thereby making predictions of changes in these statistics meaningful.

One noteworthy special case is the QSQH prediction of $\Delta\tau^+$. As previously demonstrated and discussed in section 4.1.3, it is effectively equivalent to the term $\Delta\frac{dU^+}{dy^+}$ of the DNS result. In terms of predicting $\Delta\frac{dU^+}{dy^+}$ through the QSQH prediction of $\Delta\tau^+$, a relatively accurate prediction has been achieved, as illustrated by Case P6P3 in Fig. 4.18. The value of $\Delta\frac{dU^+}{dy^+}$ is of the order of 10^{-3} for the range $0 < y^+ < 100$, which is substantially smaller than the value of Reynolds stress profile as shown in Fig. C.3. This observation may suggest that the QSQH prediction of $\Delta\tau^+$ lacks significance; however, it should be noted that $\Delta\frac{dU^+}{dy^+}$ plays a crucial role in accounting for the notable non-linear behavior observed in actual values of $\Delta\tau^+$ near the wall, as depicted in Fig. 4.5. Also the sum of $\Delta\frac{dU^+}{dy^+}$ and pressure gradient effect $y^+\Delta\frac{dP^+}{dy^+}$ yields the actual $\Delta\tau^+$. Furthermore, as the Reynolds number of the flow increases, the contribution of pressure gradient effect $y^+\Delta\frac{dP^+}{dy^+}$ to $\Delta\tau^+$ gradually diminishes. As a result, $\Delta\tau^+$ become more subtle and increasingly correlated with $\Delta\frac{dU^+}{dy^+}$. For these reasons, the QSQH prediction of $\Delta\tau^+$ remains essential.

Chapter 5

A new version of the QSQH theory

The comparisons for the QSQH theory predictions for the plane channel and pipe flows made in the earlier works and in the previous sections draw a mixed overall picture. A number of the predictions are quite accurate, other are quite poor, and many are accurate only not too far away from the wall. In previous works and in the present study a few specific cases of poor QSQH theory performance were traced to a variety of reasons: not sufficiently high Reynolds numbers, competition with other, stronger, effects, not-quasi-steady behaviour of the large-scale motions even at a moderate distance to the wall, and the geometric differences in the wall shape among them. While each such case is of independent interest, it remains very specific and does not show the bigger picture. It appears that so far no studies on the QSQH theory or in fact any other theory of near-wall scale interaction addressed the question of how well such a theory agrees with the most fundamental laws of classical mechanics, namely, mass and momentum conservation. This section provides a look into this question for the QSQH theory.

Note that the wall-normal velocity statistics predicted by the QSQH model are scarcely addressed in the present study. This is because the QSQH theory gives a zero large-scale component of the wall-normal velocity ([Chernyshenko, 2021](#)), making such attempts unlikely to proceed. Zero large-scale wall-normal component of velocity also inevitably compromises mass conservation within the synthetic velocity field generated by QSQH.

In this chapter, we propose a novel approach aimed at ensuring both mass conservation for large-scale velocity and the mean momentum conservation within the

framework of QSQH theory.

5.1 Mass Conservation

A definitive answer to the question of whether mass conservation holds for the synthetic velocity field generated by the QSQH model is in the negative. The large-scale components of velocities in the x and z directions are defined as (4.4), which we reproduce here for convenience:

$$\begin{aligned} u_L(t, x, y, z) &= u_{\tau_L} \tilde{U}(yu_{\tau_L}) \cos \theta, \\ w_L(t, x, y, z) &= -u_{\tau_L} \tilde{U}(yu_{\tau_L}) \sin \theta, \end{aligned}$$

where $u_{\tau_L} = u_{\tau_L}(t, x, z)$ and $\theta = \theta(t, x, z)$ are functions of time and wall-parallel coordinates x and z . The large-scale component of the wall-normal velocity is determined to be zero through application of a large-scale filter, utilizing properties outlined in equations (2.14) and (2.18):

$$v_L = \mathcal{L}(u_{\tau_L} \tilde{v}(yu_{\tau_L})) = u_{\tau_L} \langle \tilde{v} \rangle = 0.$$

In reality, one cannot expect that the depends of u_{τ_L} and θ on x and z is such that the large-scale components yield $\frac{\partial u_L}{\partial x} + \frac{\partial w_L}{\partial z} = 0$ at any t , x , and z , thus indicating that the original version of the QSQH theory does not adhere to mass conservation.

The large-scale component of the wall-normal velocity as prescribed by the QSQH theory is evidently unrealistic. However, a modification can be introduced to v_L in order to address this flaw by assuming that mass conservation holds for the large-scale components of the velocities. The mass conservation equation in the QSQH units can then be written as follows:

$$\frac{\partial u_L}{\partial x} + \frac{\partial v_L}{\partial y} + \frac{\partial w_L}{\partial z} = 0. \quad (5.1)$$

Equation (5.1) allows to calculate v_L :

$$v_L = - \int_0^y \left(\frac{\partial u_L}{\partial x} + \frac{\partial w_L}{\partial z} \right) dy, \quad (5.2)$$

however, this approach is not straightforward to implement numerically, as it requires the functions $u_L(x, y, z)$ and $w_L(x, y, z)$, which are challenging to generate for the QSQH model. The 1D QSQH model can produce the individual instances of u_L and w_L in terms of y only, while the 3D QSQH model can generate these two fields but with very limited size in the x and z directions. Simplifying numerical computation of (5.2) can be achieved by using formulas for u_L and w_L prescribed by (4.4), which give that

$$\begin{aligned}\frac{\partial u_L}{\partial x} &= [\tilde{U} + y u_{\tau_L} \tilde{U}'(y u_{\tau_L})] \frac{\partial u_{\tau_L}}{\partial x} \cos \theta - \tilde{U}(y u_{\tau_L}) u_{\tau_L} \sin \theta \frac{\partial \theta}{\partial x}, \\ \frac{\partial w_L}{\partial z} &= -[\tilde{U} + y u_{\tau_L} \tilde{U}'(y u_{\tau_L})] \frac{\partial u_{\tau_L}}{\partial z} \sin \theta - \tilde{U}(y u_{\tau_L}) u_{\tau_L} \cos \theta \frac{\partial \theta}{\partial z},\end{aligned}$$

and hence from (5.1)

$$\begin{aligned}\frac{\partial v_L}{\partial y} &= [\tilde{U} + y u_{\tau_L} \tilde{U}'(y u_{\tau_L})] \left(\frac{\partial u_{\tau_L}}{\partial z} \sin \theta - \frac{\partial u_{\tau_L}}{\partial x} \cos \theta \right) + \\ &\quad \tilde{U}(y u_{\tau_L}) u_{\tau_L} \left(\cos \theta \frac{\partial \theta}{\partial z} + \sin \theta \frac{\partial \theta}{\partial x} \right).\end{aligned}$$

Integration by parts gives

$$\int_0^y \tilde{U} + y u_{\tau_L} \tilde{U}'(y u_{\tau_L}) dy = -y \tilde{U}.$$

Using the streamfunction of the universal velocity profile, $\tilde{\Psi}(\tilde{y}) = \int_0^{\tilde{y}} \tilde{U}(\tilde{y}) d(\tilde{y})$, one gets

$$v_L = -y \tilde{U}(y u_{\tau_L}) \left(\frac{\partial u_{\tau_L}}{\partial x} \cos \theta - \frac{\partial u_{\tau_L}}{\partial z} \sin \theta \right) + \tilde{\Psi}(y u_{\tau_L}) \left(\frac{\partial \theta}{\partial x} \sin \theta + \frac{\partial \theta}{\partial z} \cos \theta \right). \quad (5.3)$$

This expression for v_L avoids the use of functions u_L and w_L , instead incorporating y , $\tilde{U}$, $\tilde{\Psi}$, u_{τ_L} , and θ . None of these requires the use of the QSQH model. Therefore, (5.3) is much more computationally efficient than (5.2).

Introducing a modification of (5.3) to the large-scale component of the wall-normal velocity in the original QSQH theory yields:

$$v = u_{\tau_L} \tilde{v} + v_L,$$

which is a new form of the wall-normal velocity prescribed by the modified QSQH theory. The synthetic velocity field based on this modification is expected to satisfy mass conservation of large-scale components of the near-wall turbulence.

5.2 Momentum Conservation

The mean momentum equation in the stream-wise direction for turbulent plane channel flow can be expressed in wall units as:

$$\frac{dU^+}{dy^+} - \tau_R^+ = 1 + y^+ \frac{1}{Re_\tau}, \quad (5.4)$$

where the Reynolds shear stress is defined as $\tau_R^+ = \langle u'v' \rangle^+$. DNS solutions satisfy (5.4) because DNS solves the equations governing the flow. However, the validity of momentum conservation in the QSQH synthetic velocity field is not immediately evident.

As the matter of fact, while the QSQH theory describes near-wall modulation caused by large-scale motions it does not, as shown explicitly by the QSQH formulae, involve the pressure gradient. This is justified by the assumption that the Reynolds number is sufficiently large for the pressure gradient effect to be negligible, in particular as compared to viscous stress and Reynolds stress in (5.4).

To simplify the expansions of the formulae in the subsequent investigation and to maintain consistency with previous QSQH studies, the momentum equation in wall units (5.4) can be rewritten in QSQH units. As follows from (2.2-2.3), in QSQH units The friction velocity is

$$u_\tau = \sqrt{|\langle u_{\tau_L}^2 \cos \theta \rangle|}, \quad (5.5)$$

and one has

$$U^+ = \frac{U}{u_\tau}, \quad \tau^+ = \frac{\tau}{u_\tau^2}, \quad y^+ = y u_\tau. \quad (5.6)$$

Hence in the QSQH units (5.4) is

$$\frac{dU}{dy} - \tau_R = u_\tau^2 - y u_\tau^3 \frac{1}{Re_\tau}. \quad (5.7)$$

Due to the assumption of large Re_τ , the term $yu_\tau^3 \frac{1}{Re_\tau}$ will be neglected in what follows. Further analysis can be made by substituting the QSQH expressions in (5.7) and expanding it into a Taylor series assuming that the amplitude of the fluctuations of large-scale motion is small, that is assuming that $u'_{\tau_L} = u_{\tau_L} - 1 \ll 1$ and $\theta \ll 1$. (It is also possible to find the Taylor expansion (5.4), that is of the momentum conservation written in wall units, which in some cases would be more appropriate, as discussed in the Appendix A, but using QSQH units is more convenient here.)

From (5.5) it follows that

$$u_\tau = \sqrt{\left| \left\langle \left(1 + u'_{\tau_L}\right)^2 \left(1 - \frac{\theta^2}{2} + \dots\right) \right\rangle \right|} = 1 + \frac{\langle u_{\tau_L}^{\prime 2} \rangle}{2} - \frac{\langle \theta^2 \rangle}{4} + \dots . \quad (5.8)$$

$$u_\tau^2 = 1 + \langle u_{\tau_L}^{\prime 2} \rangle - \frac{\langle \theta^2 \rangle}{2} + \frac{\langle u_{\tau_L}^{\prime 2} \rangle^2}{2} - \frac{\langle u_{\tau_L}^{\prime 2} \rangle \langle \theta^2 \rangle}{4} + \frac{\langle \theta^2 \rangle^2}{16} + \dots . \quad (5.9)$$

Derivations of the Taylor expansions of the flow statistics of QSQH synthetic velocity field have been presented in previous works (Zhang and Chernyshenko, 2016; Zhang, 2019; Chernyshenko, 2021), and will not be repeated here. The Taylor expansion of the mean velocity given by the QSQH theory is:

$$U(y) = \tilde{U}(y) + \frac{\langle u_{\tau_L}^{\prime 2} \rangle}{2} \frac{d}{dy} y^2 \frac{d\tilde{U}}{dy} - \frac{\langle \theta^2 \rangle}{2} \tilde{U}(y) + \dots , \quad (5.10)$$

where $\tilde{U}(y)$ represents the mean velocity profile of universal velocity field. The viscous stress, which is the first derivative of (5.10), can be expressed as:

$$\frac{dU}{dy} = \frac{d\tilde{U}}{dy} + \frac{\langle u_{\tau_L}^{\prime 2} \rangle}{2} \frac{d^2}{dy^2} y^2 \frac{d\tilde{U}}{dy} - \frac{\langle \theta^2 \rangle}{2} \frac{d\tilde{U}}{dy} + \dots . \quad (5.11)$$

The Taylor expansion of the Reynolds stress is given by

$$\tau_R(y) = \tilde{\tau}_R(y) + \frac{\langle u_{\tau_L}^{\prime 2} \rangle}{2} \frac{d^2}{dy^2} y^2 \tilde{\tau}_R - \frac{\langle \theta^2 \rangle}{2} \tilde{\tau}_R + \dots , \quad (5.12)$$

where $\tilde{\tau}_R(y)$ represents the Reynolds stress of the universal velocity field. By substituting (5.9), (5.11) and (5.12) into (5.7), the momentum equation can be written as

$$\left(\frac{d\tilde{U}}{dy} - \tilde{\tau}_R\right) + \frac{\langle u_{\tau_L}^{\prime 2} \rangle}{2} \frac{d^2}{dy^2} y^2 \left(\frac{d\tilde{U}}{dy} - \tilde{\tau}_R\right) - \frac{\langle \theta^2 \rangle}{2} \left(\frac{d\tilde{U}}{dy} - \tilde{\tau}_R\right) + \dots = 1 + \langle u_{\tau_L}^{\prime 2} \rangle - \frac{\langle \theta^2 \rangle}{2} + \dots \quad (5.13)$$

The terms with coefficients higher than $\langle u_{\tau_L}^{\prime 2} \rangle$ and $\langle \theta^2 \rangle$ are dropped out here. In order for (5.13) to be valid in all cases, the terms with the of the same order on both sides should be equal. Hence, considering the terms of the leading order gives

$$\frac{d\tilde{U}}{dy} - \tilde{\tau}_R = 1. \quad (5.14)$$

Assuming that this is satisfied then the equation of (5.13) becomes valid. Therefore, (5.14) is the essential condition that is required for the QSQH theory to align with the momentum conservation equation.

Although the analysis of Taylor expansions yields a seemingly straightforward conclusion regarding the condition of a QSQH synthetic velocity field in which the momentum conservation holds, comparing the theoretical condition with actual properties of the QSQH results provides further insight. Figure 5.1 depicts the viscous stress and Reynolds stress of two universal velocity fields generated from the DNS dataset of plane channel flow at $Re_\tau = 1000$ and 5200 respectively. It also examines the condition $\frac{d\tilde{U}}{dy} - \tilde{\tau}_R = 1$ in both universal velocity fields. Both figures in Fig. 5.1 indicate that the condition $\frac{d\tilde{U}}{dy} - \tilde{\tau}_R = 1$ is only approximately valid below $y^+ = 10$ for both universal velocity fields.

For $y > 20$ the curves for $\frac{d\tilde{U}}{dy} - \tilde{\tau}_R$ are almost straight and have a negative slope, similar to what could be expected as an effect of the pressure gradient described by the last, neglected later, term in (5.7). However, the observed discrepancy cannot be attributed to neglecting the pressure gradient. First, it increases as Re_τ increases. Second, the slopes in Fig. 5.1 are roughly -0.0015 for $Re_\tau = 1000$ and -0.002 for $Re_\tau = 5200$, while the slopes of the pressure gradient effect can be estimated as $-1/Re_\tau$ to be approximately -0.001 for $Re_\tau = 1000$ and -0.0002 for $Re_\tau = 5200$. The difference is 1.5 times for $Re_\tau = 1000$ and it increases to about 10 times $Re_\tau = 5200$. It is therefore more likely that the significant deviations of $\frac{d\tilde{U}}{dy} - \tilde{\tau}$ from 1, as shown in Fig. 5.1 is due to the deviation of the large-scale motions from quasi-steady behaviour, already discovered in section 4.1.2. The increase in the discrepancy as

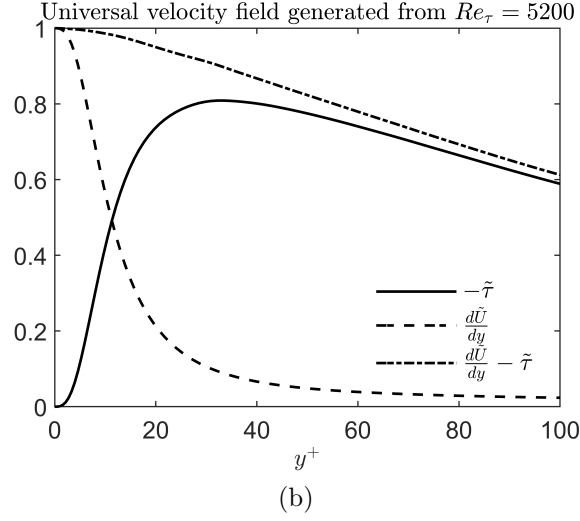
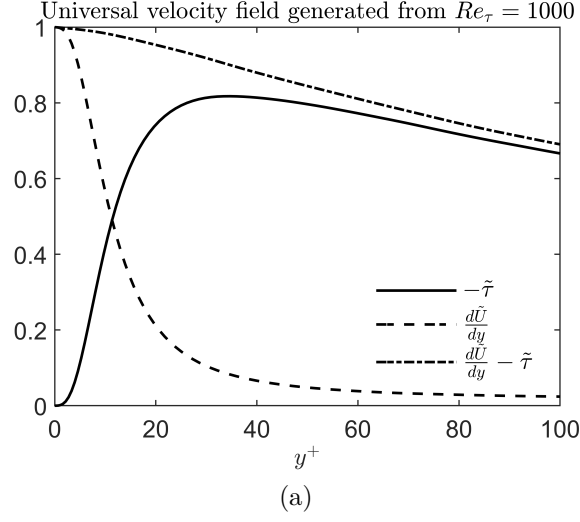


Figure 5.1: The viscous stress $\frac{d\tilde{U}}{dy}$, Reynolds stress $\tilde{\tau}$ and $\frac{d\tilde{U}}{dy} - \tilde{\tau}$ of two universal velocity fields generated from plane channel flow of: (a) $Re_\tau = 1000$; (b) $Re_\tau = 5200$.

Re_τ increases then naturally follows from the increase in the amplitude of the large-scale motions with Re .

The observations made in Fig. 5.1 imply that the universal velocity field inherits certain traces of the large-scale motions from the channel flow from which it is generated.

Furthermore, it is noteworthy to highlight that the condition $\frac{d\tilde{U}}{dy} - \tilde{\tau}_R = 1$ renders the choice of large-scale filter employed in the QSQH model non-trivial. The essence of (5.13) indicates that momentum conservation within the output synthetic velocity

field at sufficiently high Reynolds numbers is solely dependent on the condition $\frac{d\tilde{U}}{dy} - \tilde{\tau}_R = 1$. To satisfy this condition, manipulation to the large-scale filter utilized for generating the universal velocity field are crucial to eliminate the traces of large-scale motions, as illustrated in Fig. 5.1. The same filter is consistently utilized in the QSQH model to process the input data. The current QSQH synthetic turbulence model employs Fourier cut-offs in both the stream-wise and span-wise directions of the flow. Although it is possible to achieve accurate predictions of flow statistics by adjusting these cut-offs, as demonstrated in Table 3.2 and 3.3, along with extensive studies conducted by (Zhang and Chernyshenko, 2016; Chernyshenko et al., 2019; Zhang, 2019), Fourier cut-offs do not necessarily satisfy the condition $\frac{d\tilde{U}}{dy} - \tilde{\tau}_R = 1$. Further research is necessary to explore whether this condition can be met by using more advanced large-scale filters while maintaining the accuracy of predictions made by the QSQH model.

Overall, ensuring the validity of momentum conservation for the QSQH theory proves to be a challenging task. However, further study is needed to investigate whether the proposed modification to the wall-normal velocity as outlined in (5.3) would alter the conditions for momentum conservation for the QSQH theory.

Chapter 6

Discussions

Since the QSQH synthetic model software tool simplifies significantly the use of the QSQH theory, it is appropriate to discuss its application here. As the Reynolds number tends to infinity, the QSQH theory becomes more accurate for several reasons. First, the amplitudes of the competing effects, such as for example the effect of the pressure gradient, tend to zero. Second, the amplitude of the fluctuations of large-scale motions grows. Third, the separation of scales, or at least the ratio of the characteristic dimensions of the bulk of the large-scale motions to the characteristic dimensions of small-scale motions, also increases. The majority of the comparisons of the predictions of the theory made so far were in the range of $Re_\tau \sim 1000\text{--}5000$, and the results are mixed. In practical applications Re_τ can reach the values of the order of 10^5 , that is about 100 times greater. To understand if for such much higher Re_τ the accuracy of the QSQH predictions can improve significantly one can attempt to evaluate the effect of Re_τ on the above three causes of the accuracy improvement.

The effect of the pressure gradient is proportional to y^+/Re_τ (see (4.8)), while the effect of large-scale motions only grows with Re_τ . As we observed in section 4.1.3, for $Re_\tau \sim 1000\text{--}5000$ these two effects might be comparable in the near-wall region up to $y^+ \approx 20$. Hence, increasing Re_τ by a factor of 10, thus bringing it into the middle of the practical application range, will reduce the QSQH error caused by the first reason to an order of 10%, with further reduction to the order of 1% for the largest Re encountered in practice.

The effect of large-scale motions is of the order of $\langle u'_{\tau_L}{}^2 \rangle$ and $\langle \theta^2 \rangle$, for some

quantities of interest multiplied by relatively large but constant coefficients (see, for example, (4.1–4.3) and Section 3.1 in (Chernyshenko, 2021)). The values of $\langle u'_{\tau_L}{}^2 \rangle$ and $\langle \theta^2 \rangle$ depend on the large-scale filter, but according to the QSQH theory and in agreement with observations the increase in the amplitude of the fluctuations in near-wall flows largely results from an increase in the amplitude of the large-scale component. In the range of current observations, the root mean square of the fluctuations of wall friction is given by an empirical formula $\tau_{w,\text{rms}}^+ \approx 0.298 + 0.018 \ln Re_\tau$ (Schlatter and Örlü, 2010). This gives $\tau_{w,\text{rms}} = 0.422, 0.452$ and 0.493 for $Re_\tau = 1000, 5200$, and $5 \cdot 10^4$ respectively. Since at $Re_\tau = 10^3$ the amplitude of the large-scale component is quite small, at the largest side of the practical range at $Re_\tau = 5 \cdot 10^4$ it can be estimated as $0.493 - 0.422 = 0.072$, that about $0.072/0.493 = 15\%$. That is the effects described by the QSQH theory will be noticeable but not dominant.

Separation of scales is the most complicated issue to estimate quantitatively. It is also a question too large to discuss here in general. We will present here just one idea, stemming from the observation in Section 4.1.2 (see Fig. 4.3). It turns out that for the values of Re_τ considered the actual large-scale motions differ noticeably from their QSQH model, and that difference is noticeably greater for the spanwise velocity than for the streamwise velocity. It also diminishes as Re_τ increases. The physical nature of this difference is reasonably easy to see. In the QSQH description the large-scale velocity is just a stretched in wall-normal direction, turned by the angle θ , and multiplied by u_{τ_L} universal mean profile (see (4.4)). It is, therefore, in equilibrium with the local large-scale component of the wall friction. The actual large-scale motions, however, have their own dynamics, which is not in such equilibrium in general. Near the wall, however, it will be, and the deviation can be expected to be of the order of magnitude of the ratio of the distance to the wall to the characteristic vertical dimension of the large-scale motions. The characteristic vertical dimension of the large-scale motions depends on the choice of the filter but also on the location of the outer spectral peak. Let us estimate this dimension as the location of this peak, which is also the middle of the log layer and the position of choice for the large-scale probe of the experiments leading to the famous empirical formula of large-scale modulation of near-wall small scales (see Fig. 2.1 and (Mathis et al., 2009; Marusic et al., 2010; Mathis et al., 2011)): $y_o^+ = 3.9 Re_\tau^{1/2}$. This gives $y_o^+ \approx 123, 281$, and

872 for $Re_\tau = 1000$, 5200 and $5 \cdot 10^4$ respectively. This leads to the corresponding estimates of the relative errors at, for example, $y^+ = 50$ to be $y_o^+/40 \sim 41\%$, 15%, and 6%. The first two of these estimates are in line with Fig. 4.3 for the spanwise velocity case. The difference between the spanwise and streamwise velocity cases might be due to the significant difference of the height of the streamwise velocity and spanwise velocity large-scale structures, as described in (Hwang, 2015).

Overall, all three estimates suggest that at the highest Re_τ encountered in practice the QSQH theory might produce results with accuracy sufficient to represent interest for practical application, but that at the values of Re_τ in the range of what is achievable in direct numerical simulations in the near future the relative error can be of order of 20%, apart from some special cases, when it might be much better or worse. Since the universal velocity field used in the QSQH synthetic model has to be obtained by processing the data obtained by direct numerical simulations, it will inherit the deviation from the QSQH theory at the Re_τ of the simulations.

In view of the above, at the current state of the art the most likely application of the QSQH synthetic model is to use it as a tool for verifying if an effect observed in a particular flow can be attributed to quasi-steady-quasi-homogeneous modulation of the near-wall turbulence by large-scale motions. This can be done simply by constructing the QSQH database of the flow by the QSQH tool and observing whether the effect of interest can be seen in the flow constructed by the tool. The QSQH theory has already revealed physical mechanisms of a number of observed effects, but this required a considerable analytic work, which now will need to be done only when the relevance of the QSQH mechanism has been confirmed.

Importantly, using the QSQH theory for revealing physical mechanisms requires deep understanding of the foundations of this theory. One common and easy to grasp way of describing the basis of the QSQH theory is to state that the theory assumes that “the small-scale near-wall motions adapt to the large-scale fluctuations of the wall shear stress” (Chernyshenko, 2021). This, however, can be misleading, in that it might be seen as stating that the fluctuations of the large-scale wall friction is the cause of the corresponding variations of the properties of the near-wall turbulence, which is not true. In (Andreolli et al., 2023) the Navier-Stokes equations were modified to suppress the fluctuation of the large-scale component of the wall friction, and it was shown by direct numerical simulations that even in this

case the large-scale motions still affect the near-wall flow. The essence of the QSQH assumption is better to be stated along the lines of the statement of the classical universality, which we cited in Prandtl’s formulation in the introduction. Namely, one can state the essence of the QSQH theory is that the processes in proximity of a wall can be so expressed that they will be not dependent upon the fluctuations of the large-scale motions. In the classical universality hypothesis the universality of near-wall turbulence is achieved by re-scaling the variable using the mean wall friction, that is by using wall units. The QSQH universality is achieved by re-scaling the variables using the large-scale wall friction. In neither case causality relationship between the scaling parameter and the variations of the properties of the near-wall turbulence is implied. The existence of such re-scaling is the main point, but the choice of the re-scaling parameter is a matter of convenience. If, for example, one would consider a flow in which the large-scale wall friction is exactly constant as in (Andreolli et al., 2023), one could use for re-scaling the large-scale velocity at a certain distance from the wall, as it was done in (Mathis et al., 2009; Marusic et al., 2010; Mathis et al., 2011).

It remains to discuss the possible ways of improving the QSQH theory. One can try to find a large-scale filter better corresponding to the QSQH requirements. For example, in (Agostini and Leschziner, 2022), separating large and small scales was done using a neural network, which learns the features of the large and small structures in the flow. Thus obtained large-scale filter was then applied within the context of the QSQH theory. While a number of comparisons with this filter were in line with the QSQH theory, a discrepancy was observed specifically in the test involving the length-scale modulation, where the spanwise streak spacing scaled with and conditioned on the large-scale wall friction u_{τ_L} and thus expected to be constant according to the QSQH theory, was found to increase at the smallest values of u_{τ_L} . The likely reason for this, however, is that (Agostini and Leschziner, 2022) did not account for fluctuations of the direction of the large-scale friction. When streaks turn away from the mean flow direction without otherwise changing, their spanwise wavelength increases, which might explain the discrepancy noticeable in Fig. 15b) in (Agostini and Leschziner, 2022).

Another simple suggestion for improving the QSQH theory stems from the intuitive argument that the near-wall flow will react with a certain delay to changes

in the large-scale motions, while the current version of the QSQH theory does not account for this. A time shift of the amplitude modulation is reported in (Guala et al., 2011; Bandyopadhyay and Hussain, 1984; Baars et al., 2015). To take this into account, the values of u_{τ_L} and θ in the QSQH theory can be simply taken with an empirically determined time shift. This would be similar to the streamwise shift of the outer signal in (Mathis et al., 2009; Marusic et al., 2010; Mathis et al., 2011), although there the benefits of the shift are attributed to the mean inclination angle of the large-scale structures.

Yet another possibility to improve the QSQH theory would be to complement it with a model of the large-scale motions. As observed in Sections 4.1.2 and 4.1.2, the relative deviation of the QSQH predictions from the direct numerical simulation results is significantly greater for large-scale motions than for the small-scale motions (compare Figs. 4.3 and 4.4). One can make the QSQH prediction of the velocity field using the existing QSQH theory, subtract from it the large-scale component as it is predicted by the QSQH theory (which is given by the simple formulae (4.4)) and add instead the actual or independently modelled large-scale component, thus obtaining a more accurate result.

Although a modification is proposed in section 5.1 to establish mass conservation for the large-scale components of velocities within the mathematical framework of QSQH theory, the issue of mass conservation for total velocities remains unresolved. The primary challenge in achieving mass conservation for total velocities lies in deriving a new expression for v that does not necessitate the pre-computation of the other two velocities, u and w . This complexity becomes evident when formulating the mass conservation equation for total velocities:

$$\begin{aligned} v &= - \int_0^y \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) dy \\ &= - \int_0^y \left(\frac{\partial u_{\tau_L} (\tilde{u} \cos \theta + \tilde{w} \sin \theta)}{\partial x} + \frac{\partial u_{\tau_L} (-\tilde{u} \sin \theta + \tilde{w} \cos \theta)}{\partial z} \right) dy, \quad (6.1) \end{aligned}$$

where the velocity components of the universal velocity field: $\tilde{u}$ and $\tilde{w}$ are functions of $(\tilde{x}, \tilde{y}, \tilde{z})$. Numerically, the computation of (6.1) is straightforward when utilizing the QSQH synthetic turbulence model; however, this approach necessitates prior knowledge of u and w , which are computed by the model. This requirement can

lead to computational inefficiencies. To date, efforts to derive (6.1) have not achieved the same level of neatness as demonstrated in (5.2).

All the figures of comparisons between the DNS results and the QSQH predictions of changes in velocity statistics are compiled as a montage in Fig. 6.1. They represent of a large variety of flow statistics, which would be hard to produce within a reasonable time without the use of the QSQH synthetic model developed in this study. But done and looked at together, they allow making certain observations.

First, a notable observation can be made regarding the plots associated with pipe flow at $Re_\tau = 1140$, represented by black and orange lines. The comparisons involving P1140 consistently reveal significant quantitative and qualitative discrepancies between the DNS results and QSQH predictions. This suggests that in contrast with the channel flow, a for pipe flows $Re_\tau = 1140$ is too small for the flow to be even in a remote way close to the behaviour described by the QSQH theory.

Second, the spread of the QSQH predictions (dashed curves) is typically smaller than the spread of the DNS results over the same spread of the Reynolds numbers and the flow types (distinguished by colours). This suggests that the large-scale filter used in the present study might underestimate the magnitude of the fluctuations of the large-scale motions.

Third, and related to the second, observation from Fig. 6.1 is that, for about a half or even more velocity statistics, the QSQH predictions qualitatively capture the behaviour of the DNS results.

As demonstrated in Section 4.2, the QSQH prediction regarding changes in velocity statistics is contingent upon variations in the strength of large-scale motions within the output synthetic velocity field. To quantify the changes in large-scale motions, two parameters are employed: $\Delta\langle u_{\tau_L}^{\prime 2} \rangle$ and $\Delta\langle \theta^2 \rangle$. Consequently, to achieve quantitatively accurate QSQH predictions, potential adjustments can be made to the large-scale filter utilized in the QSQH model. This would allow for manipulations to $\Delta\langle u_{\tau_L}^{\prime 2} \rangle$ and $\Delta\langle \theta^2 \rangle$, thereby facilitating a closer alignment of QSQH predictions with DNS results. Such a fitting-like method suggests that there may exist one or multiple suitable large-scale filters for statistical predictions based on QSQH theory. However, several intriguing questions regarding the selection of large-scale filter warrant further investigation. First, the discrepancies between the DNS results and QSQH predictions in Fig. 6.1 are less pronounced at higher Reynolds numbers, sug-

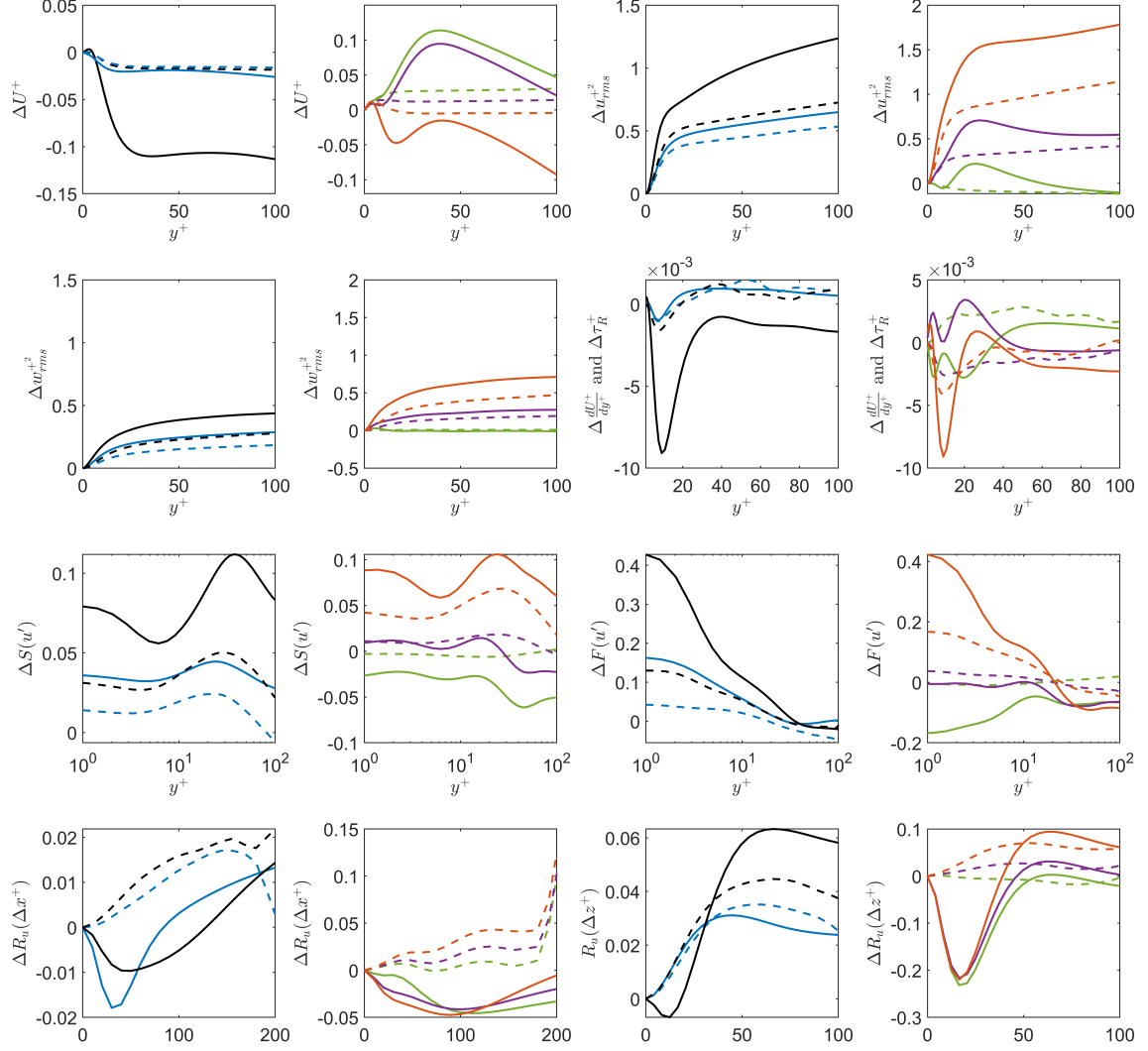


Figure 6.1: A montage of the figures of comparisons between DNS results and QSQH predictions presented in Section 4.2. The DNS results (solid lines) and QSQH predictions (dashed lines) are compared for Case P1P3 (P3000 - P1140, black), Case P3P6 (P6000 - P3000, blue), Case P1C5 (C5200 - P1140, orange), Case P3C5 (C5200 - P3000, purple), and Case P6C5 (C5200 - P6000, green).

gesting that the large-scale filter employed in this study is appropriate for cases with increasing Reynolds numbers. This observation raises an important question: Is the choice of filter specific to Re_τ ? Second, the concept of an energy plateau was discussed in earlier studies on QSQH theory (Zhang and Chernyshenko, 2016; Zhang, 2019), indicating that there exists a certain region within the energy spectra that remains insensitive to changes in cut-off values. If the large-scale filter utilized in the QSQH model falls within this insensitivity region of the input wall friction, then manipulating this large-scale filter will not significantly affect $\Delta\langle u_{\tau_L}'^2 \rangle$ and $\Delta\langle \theta^2 \rangle$. In fact, such observations have already been documented in Table 3.1, where changing the cut-offs of the large-scale yield minimal effects on $\Delta\langle u_{\tau_L}'^2 \rangle$. In addition to exploring manipulations to the large-scale filter, theoretical investigations presented in Section 5.2 reveal a non-trivial selection process for this filter necessary for ensuring momentum conservation within the QSQH model. Collectively, these findings highlight that further investigation into the large-scale filter used in the QSQH model is necessary.

Chapter 7

Conclusions

The main contribution of this study is the QSQH synthetic turbulence model, developed, implemented in a software tool, and made publicly available online at (Yang, 2024). The tool consists of a MATLAB code and a database, which can be downloaded and run on the user's computer. The input to the code is the wall friction distribution of the 'target' flow of interest. The output is a database, which is a set of snapshots of velocity fields such that:

- a) the statistics of the large-scale component of the wall friction are the same as those of the input wall friction field, and
- b) all statistics are such as they would be in the target flow if the target flow would satisfy the assumptions of the QSQH theory.

In effect, the tool allows to calculate easily and without any derivations the QSQH prediction for a variety of flow statistics by calculating the desired statistics of the output database.

The second major contribution is an analysis of a large set of comparisons of the QSQH predictions for flow characteristics, and the more sensitive differences between the same flow characteristics, for flows in plane channel and circular pipe at different Reynolds numbers. These comparisons demonstrate the use of the QSQH synthetic turbulence model and contribute to the development and verification of the QSQH theory. In particular, the following was done in this part of the study.

We found that the relative error of Taylor expansions in the amplitude of the large-scale motion for the QSQH prediction of the second moments of streamwise and spanwise velocities increases with the Reynolds number. This increase is attributed

to the growing amplitude of large-scale fluctuations with Re . It is conjectured that the usefulness of Taylor expansions can deteriorate at higher Re and for flows in which the amplitude of large-scale fluctuations is greater.

The “QSQH hypothesis per se”, which is the hypothesis of statistical independence of the large-scale friction velocity and the universal velocity field, was tested in a novel way, by comparing the actual velocity field with the velocity field generated by the QSQH synthetic model. For $Re_\tau = 1000$ and 5200 , it has been found to be accurate in and below the buffer layer, approximately for $y^+ < 20$ for the streamwise velocity component and $y^+ < 10$ for the spanwise velocity component, and it then becomes less accurate as the distance to the wall increases. The large-scale motions were found to adhere better to the QSQH hypothesis for the higher Re_τ . The small-scale motions exhibited the opposite trend. At both values of Re_τ the spanwise velocity shows less compliance with the QSQH hypothesis, which can now be attributed to the behaviour of the large-scale components.

It was already known that as far as the Reynolds shear stress variation with Re_τ is concerned, the contributions of the QSQH modulation and the pressure gradient are comparable below $y^+ \approx 20$. We have now demonstrated that these effects are approximately additive.

The 3D version of the QSQH synthetic model and tool allowed to obtain for the first time the QSQH predictions of two-point correlations for the points at different wall-parallel locations.

Using the current version of the QSQH synthetic turbulence model, which contains the universal velocity field generated from turbulent channel flow at $Re_\tau = 5200$, for the first time, we are able to examine the accuracy of the prediction of the difference in the turbulence statistics between the pipe flows of different Reynolds numbers, and of the extrapolation of the turbulence statistics from channel to pipe flow.

1. The QSQH predictions regarding the changes in streamwise (longitudinal) velocity statistics of near-wall turbulence, beyond those observed in plane channel flow—such as the pipe flow at varying Reynolds numbers—generally yield results that are qualitatively accurate and quantitatively close to those obtained from Direct Numerical Simulation (DNS). However, potential geometric

effects inherent in different flow configurations, such as pipe flow, may contribute to discrepancies between DNS results and QSQH predictions. Most discrepancies—whether quantitative or qualitative—between the predictions made by the QSQH model and DNS outcomes can be mitigated by increasing the Reynolds number of the target flow.

2. Using the current version of the QSQH synthetic turbulence model to extrapolate velocity statistics from one flow type to another yields limited accuracy. The QSQH theory is primarily constrained in its ability to capture only the differences in large-scale effects between channel and pipe flows. While some qualitative similarities can be observed in the extrapolations of turbulence intensities, Reynolds shear stress, and higher-order moments when compared to DNS results, it is important to note that the accuracy of these extrapolations for both streamwise (longitudinal) and spanwise (azimuthal) velocity statistics has been found to improve quantitatively with increasing Reynolds number.
3. This study, for the first time, investigates the performance of the QSQH theory in predicting and extrapolating skewness and flatness in turbulence velocity. The QSQH predictions and extrapolations regarding changes in the skewness and flatness of streamwise (longitudinal) velocity fluctuations exhibit notably poor accuracy; furthermore, increasing Reynolds numbers does not enhance prediction outcomes. This finding indicates that the large-scale effects described by the QSQH theory do not significantly influence high-order moments such as skewness and flatness of velocity fluctuations, even at elevated Reynolds numbers.

The third distinctive contribution of this study is the analysis of the question of mass and momentum conservation in the QSQH theory, resulting in a proposal of modifying the theory to better fulfil the conservation laws. The main points of this are:

1. The large-scale components of the synthetic turbulence velocity field do not follow mass conservation because the current version of QSQH theory gives that the large-scale component of the wall-normal velocity v_L is zero. This

issue is resolved by introducing the expression derived from the integral of the wall-parallel large-scale velocity components u_L and w_L :

$$v_L = -y\tilde{U}(yu_{\tau_L}) \left(\frac{\partial u_{\tau_L}}{\partial x} \cos \theta - \frac{\partial u_{\tau_L}}{\partial z} \sin \theta \right) + \tilde{\Psi}(yu_{\tau_L}) \left(\frac{\partial \theta}{\partial x} \sin \theta + \frac{\partial \theta}{\partial z} \cos \theta \right),$$

where $\tilde{\Psi}' = \tilde{U}$. Importantly, this does not require pre-computation of u_L and w_L .

2. Conservation of mean longitudinal momentum in the QSQH theory can be achieved, at least approximately, by substituting the parameters prescribed by QSQH into the mean momentum equation along the streamwise (longitudinal) direction. A non-trivial condition is identified that is necessary (but not sufficient) to satisfy this equation: $\frac{d\tilde{U}}{dy} - \tilde{\tau} = 1$. This finding leads to a significant consideration regarding the choice of large-scale filter utilised in the QSQH model, suggesting that the five requirements for large-scale filtering outlined in QSQH theory (Zhang and Chernyshenko, 2016), which have been accepted for over a decade, can be expanded to provide a more rigorous and explicit definition of both large- and small-scale motions within this theoretical framework.

The overarching conclusion is that the current version of the QSQH synthetic turbulence model is suitable for predicting velocity statistics, such as mean velocity profiles, turbulence intensities, and Reynolds shear stress in wall-bounded turbulence. The majority of inaccuracies in the QSQH predictions can be attributed to the insufficiently high Reynolds numbers present in the database utilized for this study. There is considerable optimism that this model will achieve improved accuracy when applied to target flows characterized by higher Reynolds numbers than those examined herein. It is also noteworthy that as the Reynolds number increases, second-order Taylor expansions or truncated versions of the QSQH formulas become increasingly inaccurate. This arises from the enhanced strength of large-scale motions, which renders neglected higher-order terms in these expansions more significant. Consequently, the QSQH synthetic turbulence model emerges as a uniquely accurate choice for applications involving high-Reynolds-number scenarios within QSQH theory. On the other hand, by virtue of the simplicity of the applica-

tion of the QSQH software tool, the QSQH theory becomes best suited for revealing the physical mechanisms of scale interaction in near-wall turbulent flows.

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Appendix A

QSQH Taylor expansion of the differences between cases with wall distance fixed in wall units.

Taylor expansions of the QSQH formulae (see Section 2.3.5 and, for example, (4.1-4.3)) usually have the form

$$f(y) = \tilde{f}(y) + \langle u_{\tau_L}^{\prime 2} \rangle \tilde{g}(y) + \langle \theta^2 \rangle \tilde{h}(y) + \dots, \quad (\text{A.1})$$

where $\tilde{f}(\cdot)$, $\tilde{g}(\cdot)$ and $\tilde{h}(\cdot)$ are certain universal, that is the same for all near-wall flows, functions, while $\langle u_{\tau_L}^{\prime 2} \rangle$ and $\langle \theta^2 \rangle$ depend on the large-scale component of the wall friction, which in turn depends on the Reynolds number and the geometry of the flow. Then the difference in $f(y)$ in two different flows is given by

$$\Delta f(y) = \Delta \langle u_{\tau_L}^{\prime 2} \rangle \tilde{g}(y) + \Delta \langle \theta^2 \rangle \tilde{h}(y) + \dots, \quad (\text{A.2})$$

where $\Delta \langle u_{\tau_L}^{\prime 2} \rangle$ and $\Delta \langle \theta^2 \rangle$ are the differences in $\langle u_{\tau_L}^{\prime 2} \rangle$ and $\langle \theta^2 \rangle$ in these flows. The term with $\tilde{f}(y)$ conveniently disappears, since it is the same in all flows. However, since the results of direct numerical simulations or experiments are often represented in wall units, the comparisons with the QSQH theory is convenient to do using wall units, that is to take the difference between f in the two flows at the same values of y^+ and not at the same values of y . In this case the term with $\tilde{f}$ will not disappear.

Moreover, the function f itself can also be expressed in wall units.

For convenience, we rewrite here parts of (2.2–2.3) as

$$u_\tau = \sqrt{|\langle u_{\tau_L}^2 \cos \theta \rangle|}, \quad y = y^+/u_\tau.$$

We then introduce $u'_\tau = u_\tau - 1$, which together with $\langle u_{\tau_L}^{\prime 2} \rangle \ll 1$ and $\langle \theta^2 \rangle \ll 1$ gives $u'_\tau = \langle u_{\tau_L}^{\prime 2} \rangle / 2 - \langle \theta^2 \rangle / 4 \ll 1$.

$$y = \frac{y^+}{u_\tau} = \frac{y^+}{\sqrt{|\langle u_{\tau_L}^2 \cos \theta \rangle|}} = y^+ (1 - u'_\tau + \dots) = y^+ \left(1 - \langle u_{\tau_L}^{\prime 2} \rangle / 2 + \langle \theta^2 \rangle / 4 \right) + \dots$$

and therefore

$$\tilde{f}(y) = \tilde{f}(y^+) - u'_\tau y^+ \frac{d\tilde{f}}{dy}(y^+) + \dots = \tilde{f}(y^+) - \left(\langle u_{\tau_L}^{\prime 2} \rangle / 2 - \langle \theta^2 \rangle / 4 \right) y^+ \frac{d\tilde{f}}{dy}(y^+) + \dots$$

In many cases $f(y) = u_\tau^\alpha f^+(y^+)$ with a certain value of α , see for example (4.1–4.3) in view of (2.3). Hence (A.1) becomes

$$\begin{aligned} f^+(y) &= (1 + u'_\tau)^{-\alpha} \left(\tilde{f}(y^+) - u'_\tau y^+ \frac{d\tilde{f}}{dy}(y^+) + \langle u_{\tau_L}^{\prime 2} \rangle \tilde{g}(y^+) + \langle \theta^2 \rangle \tilde{h}(y^+) \dots \right) \\ &= \tilde{f}(y^+) - u'_\tau \left(\alpha \tilde{f} + y^+ \frac{d\tilde{f}}{dy}(y^+) \right) + \langle u_{\tau_L}^{\prime 2} \rangle \tilde{g}(y) + \langle \theta^2 \rangle \tilde{h}(y^+) \dots \\ &= \tilde{f}(y^+) + \langle u_{\tau_L}^{\prime 2} \rangle \left(\tilde{g}(y^+) - \frac{\alpha}{2} \tilde{f}(y^+) - \frac{y^+}{2} \frac{d\tilde{f}}{dy}(y^+) \right) + \\ &\quad \langle \theta^2 \rangle \left(\tilde{h}(y^+) + \frac{\alpha}{4} \tilde{f}(y^+) + \frac{y^+}{4} \frac{d\tilde{f}}{dy}(y^+) \right) \dots \quad (\text{A.3}) \end{aligned}$$

Hence, the wall-units counterpart of (A.2) is

$$\begin{aligned} \Delta f(y^+) &= \Delta \langle u_{\tau_L}^{\prime 2} \rangle \left(\tilde{g}(y^+) - \frac{\alpha}{2} \tilde{f}(y^+) - \frac{y^+}{2} \frac{d\tilde{f}}{dy}(y^+) \right) + \\ &\quad \Delta \langle \theta^2 \rangle \left(\tilde{h}(y^+) + \frac{\alpha}{4} \tilde{f}(y^+) + \frac{y^+}{4} \frac{d\tilde{f}}{dy}(y^+) \right) + \dots \quad (\text{A.4}) \end{aligned}$$

The quantitative difference between (A.2) and (A.4) often appears to be small,

probably due to the often happening dominance of a particular term in the expansion (see Section 2.3.5). The simpler form of (A.2) can be more convenient when the Taylor expansion is used to get insight into the physical mechanisms of the differences between the flows. In general, however, a caution is needed when both the wall units and the QSQH units (2.1) are used interchangeably.

Appendix B

QSQH predictions vs DNS results of the change in the wall-normal turbulence intensity profile

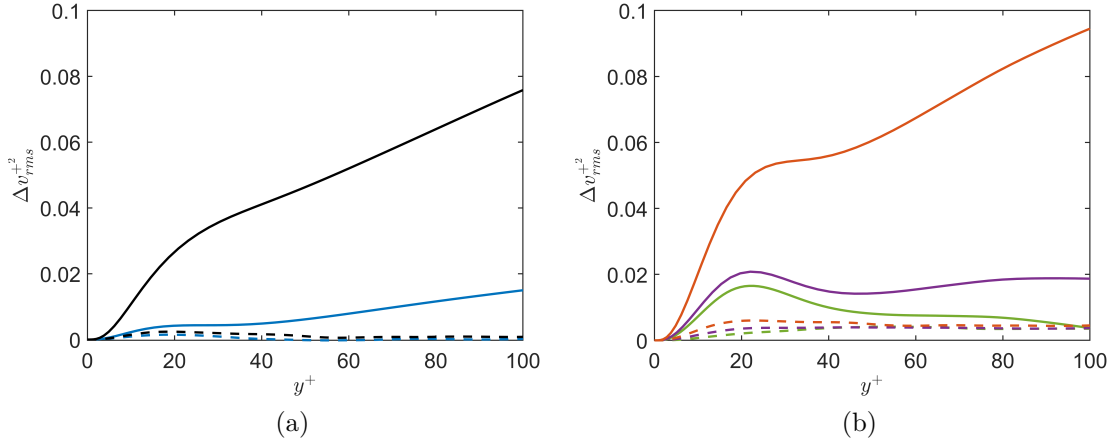


Figure B.1: a. The DNS results (solid lines) and QSQH predictions (dashed lines) of the change in the radial turbulence intensity for Case P1P3 (P3000 - P1140, black) and Case P3P6 (P6000 - P3000, blue); b. The DNS results (solid lines) and QSQH predictions (dashed lines) of the change in the radial turbulence intensity for Case P1C5 (C5200 - P1140, orange), Case P3C5 (C5200 - P3000, purple) and Case P6C5 (C5200 - P6000, green).

Appendix C

Profiles of channel and pipe flow statistics

All the data presented in the following figures are obtained from the simulations conducted by (Lee and Moser, 2015; Pirozzoli et al., 2021).

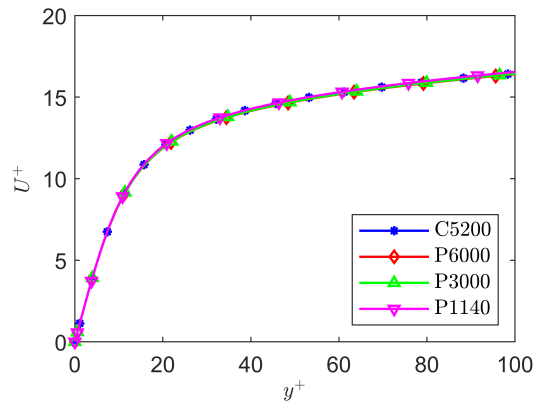


Figure C.1: Mean velocity profiles of C5200 (channel flow at $Re_\tau = 5200$), P6000 (pipe flow at $Re_\tau = 6000$), P3000 (pipe flow at $Re_\tau = 3000$), and P1140 (pipe flow at $Re_\tau = 1140$).

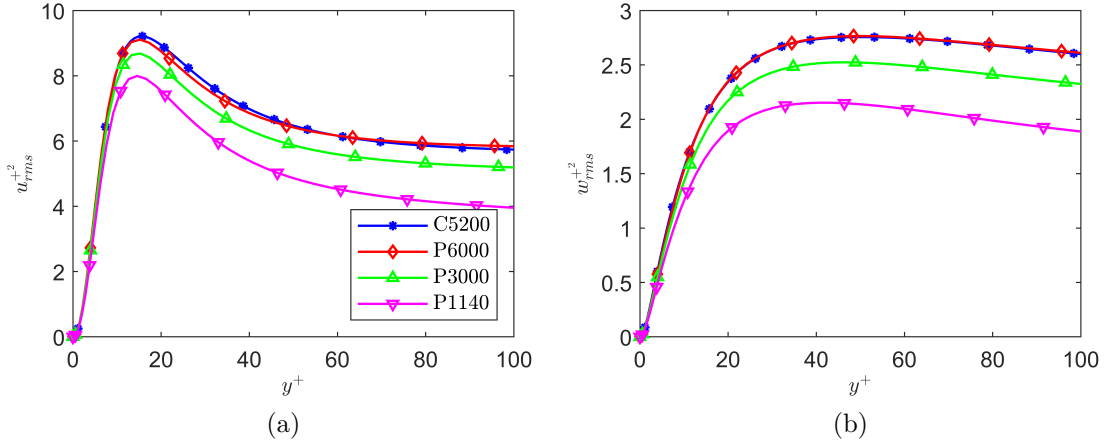


Figure C.2: The intensity profiles of, a.streamwise velocity; b.spanwise velocity, of C5200 (channel flow at $Re_\tau = 5200$), P6000 (pipe flow at $Re_\tau = 6000$), P3000 (pipe flow at $Re_\tau = 3000$), and P1140 (pipe flow at $Re_\tau = 1140$).

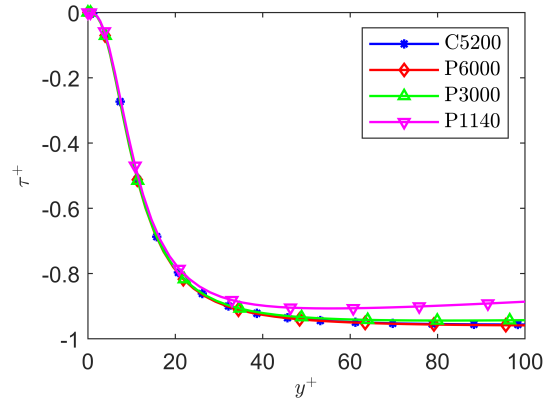


Figure C.3: The profiles of Reynolds stress of C5200 (channel flow at $Re_\tau = 5200$), P6000 (pipe flow at $Re_\tau = 6000$), P3000 (pipe flow at $Re_\tau = 3000$), and P1140 (pipe flow at $Re_\tau = 1140$).

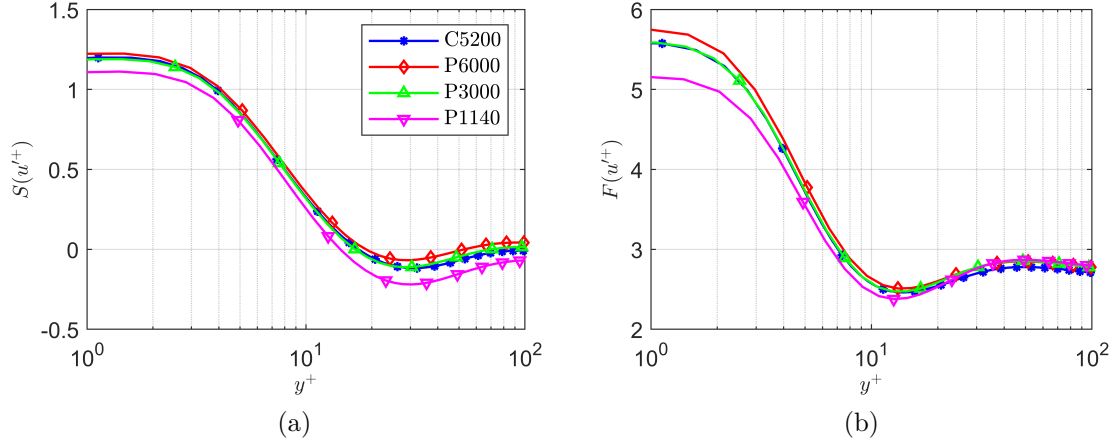


Figure C.4: The, a. skewness; b. flatness of streamwise velocity of C5200 (channel flow at $Re_\tau = 5200$), P6000 (pipe flow at $Re_\tau = 6000$), P3000 (pipe flow at $Re_\tau = 3000$), and P1140 (pipe flow at $Re_\tau = 1140$).

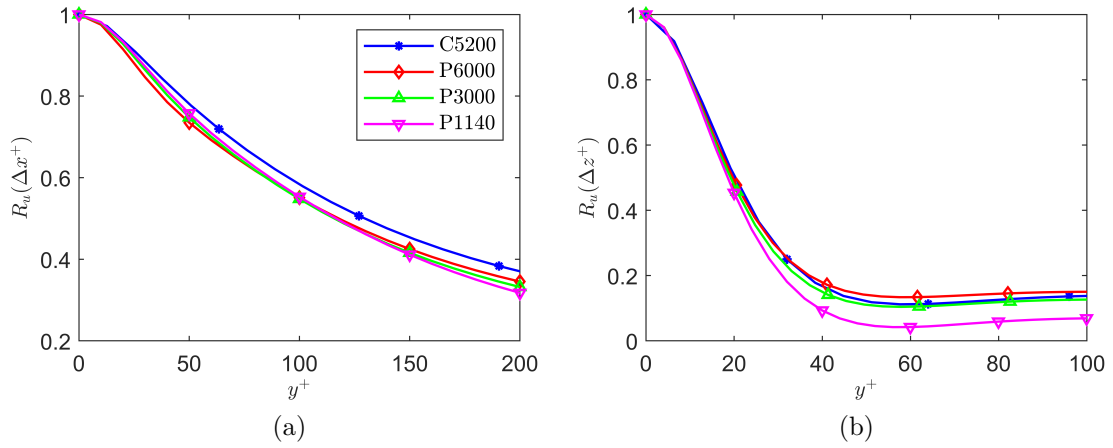


Figure C.5: The two-point correlation coefficients of streamwise velocity in, a. streamwise direction; b. spanwise direction, of C5200 (channel flow at $Re_\tau = 5200$), P6000 (pipe flow at $Re_\tau = 6000$), P3000 (pipe flow at $Re_\tau = 3000$), and P1140 (pipe flow at $Re_\tau = 1140$).