

IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY
UNIVERSITY OF LONDON.

DESIGN AND PERFORMANCE CRITERIA ^{FOR} ~~OF~~ SUPERSONIC
PARALLEL DIFFUSER. - EJECTOR SYSTEMS.

Vol. II

by

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A thesis submitted for the degree of Doctor of
Philosophy, in the Faculty of Engineering,
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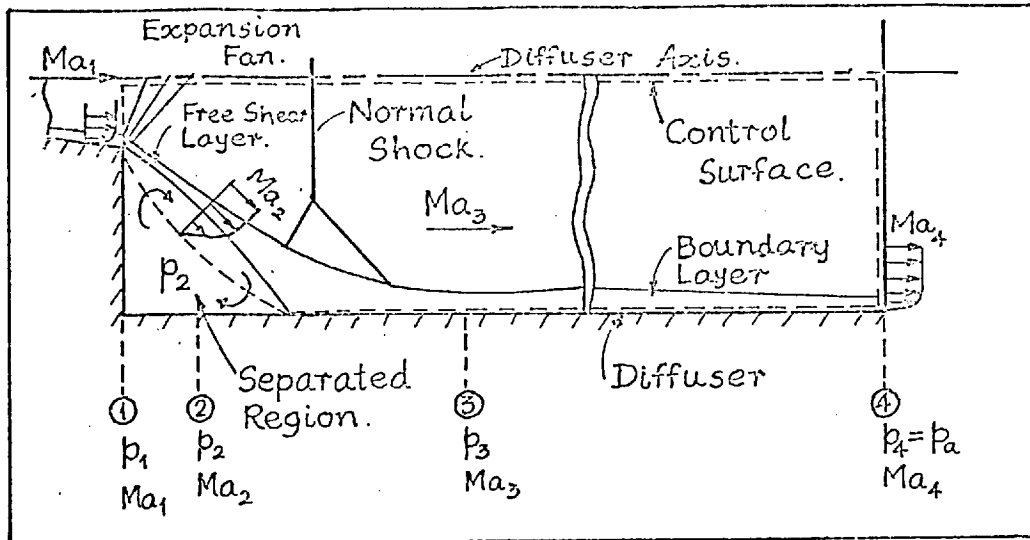


Fig.2.2.1. Flow Model For Unstarted Diffuser.

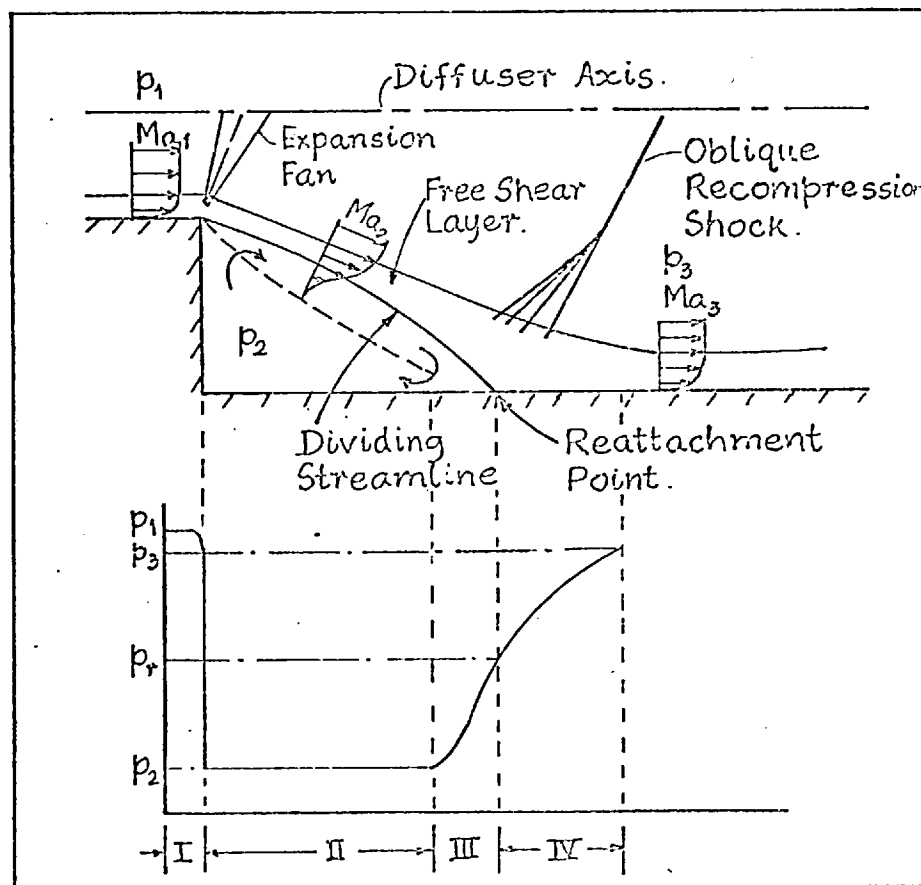


Fig.2.3.1. Flow Model At Started Diffuser Entry. (After Korst-Chapman)

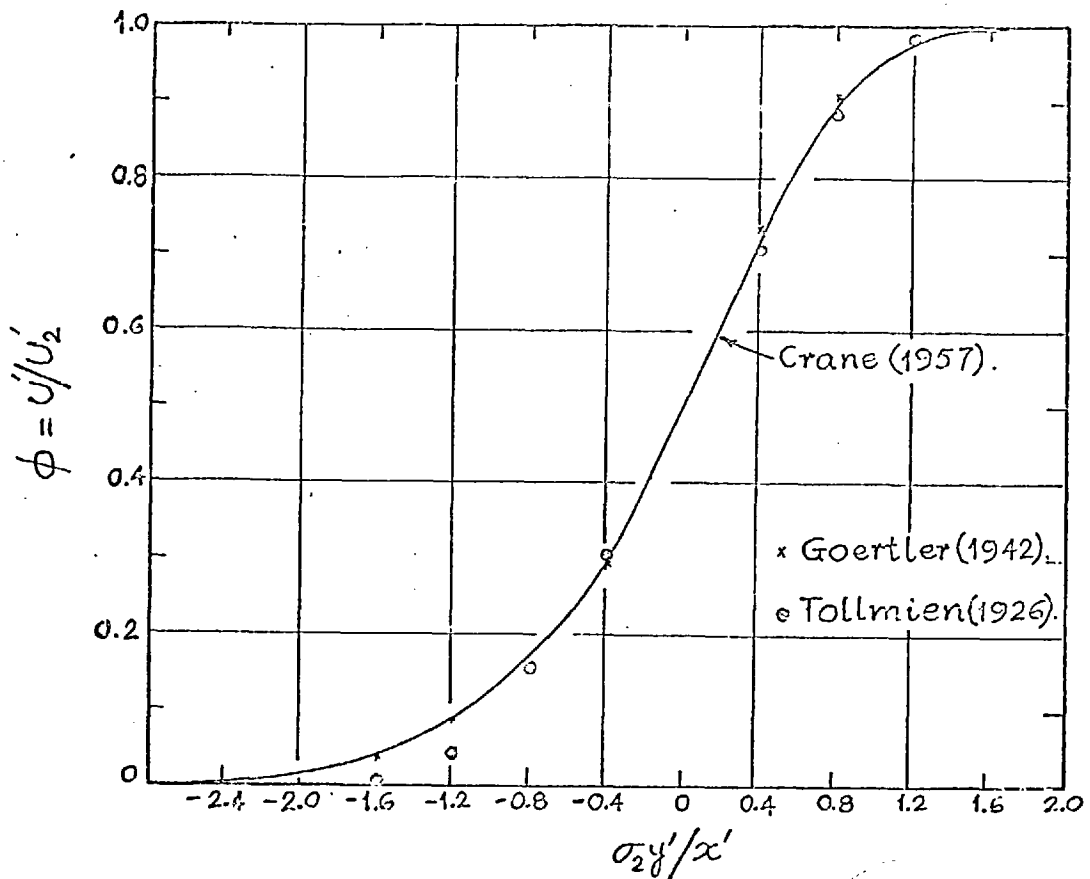


Fig.2.3.2a. Velocity distribution in two-dimensional incompressible free shear layer.

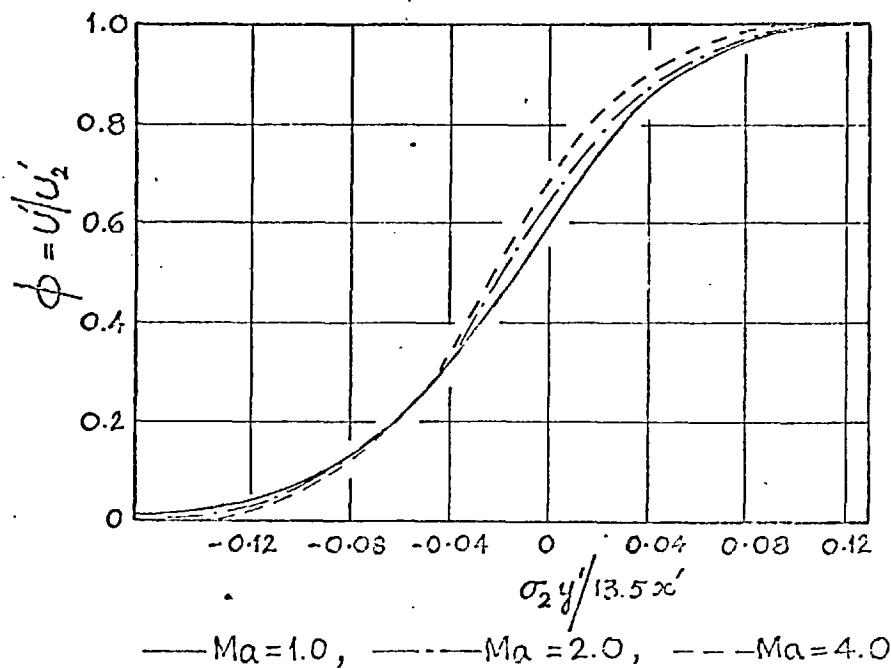


Fig.2.3.2b. Effect of Mach number on two-dimensional shear layer velocity distribution, (Jacques and Gailly, 1966).

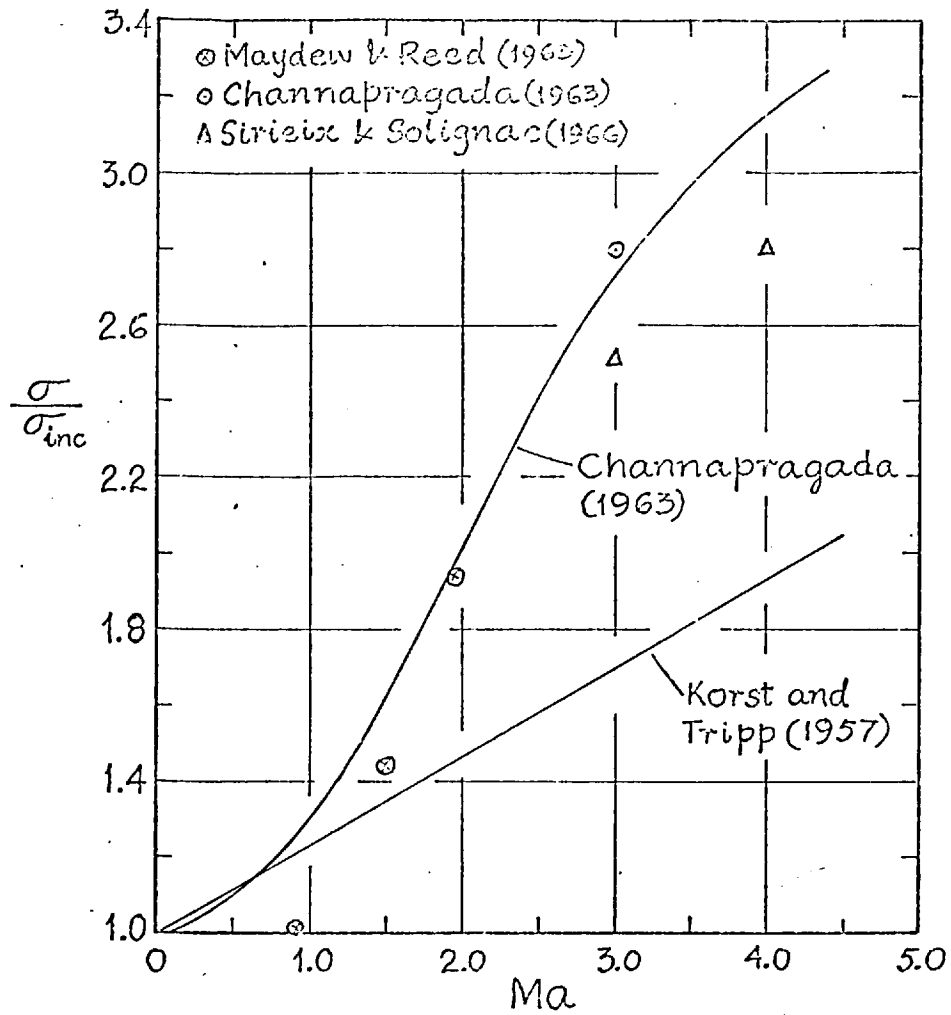


Fig.2.3.3. Variation of compressible jet spread parameter with Mach number.

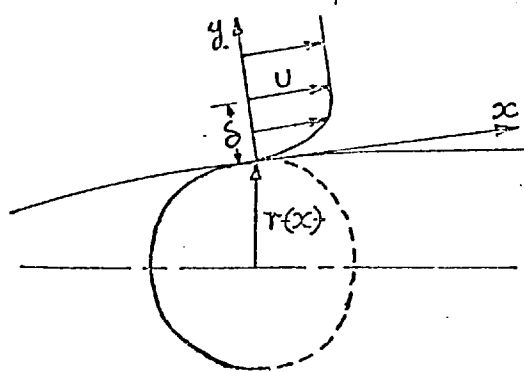


Fig.2.3.4. The co-ordinate system.

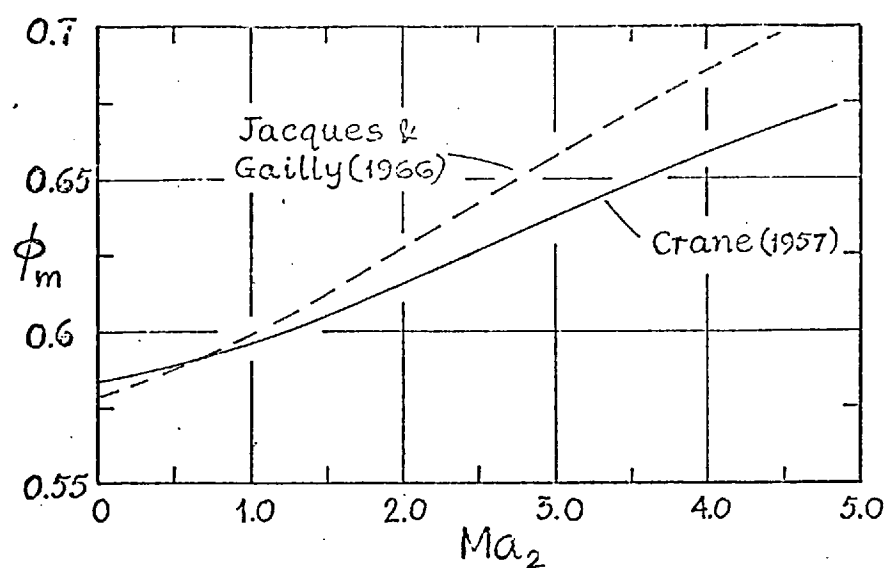


Fig.2.3.5. Variation of median stream-line velocity ratio with Mach number.

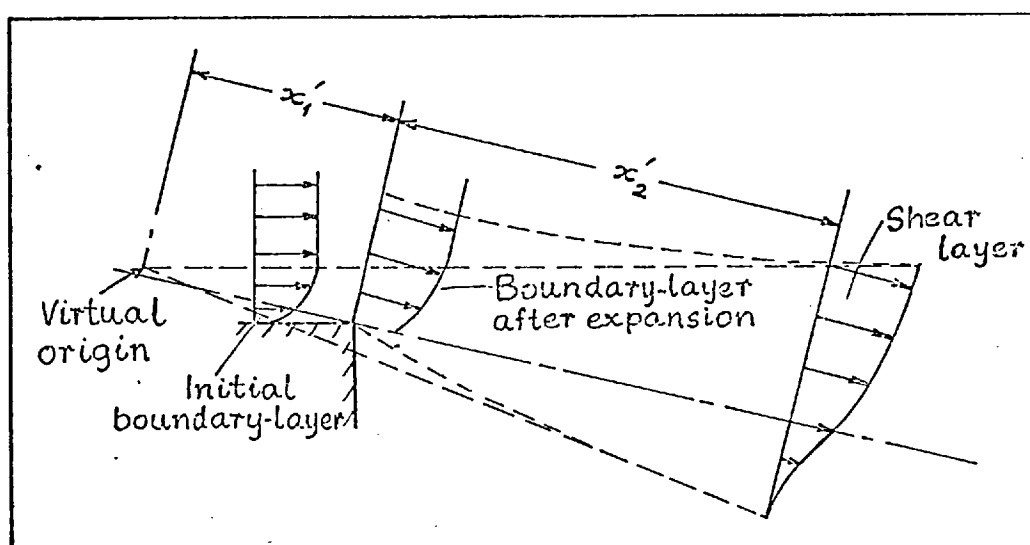


Fig.2.3.6. Effect of initial boundary-layer on shear layer development.

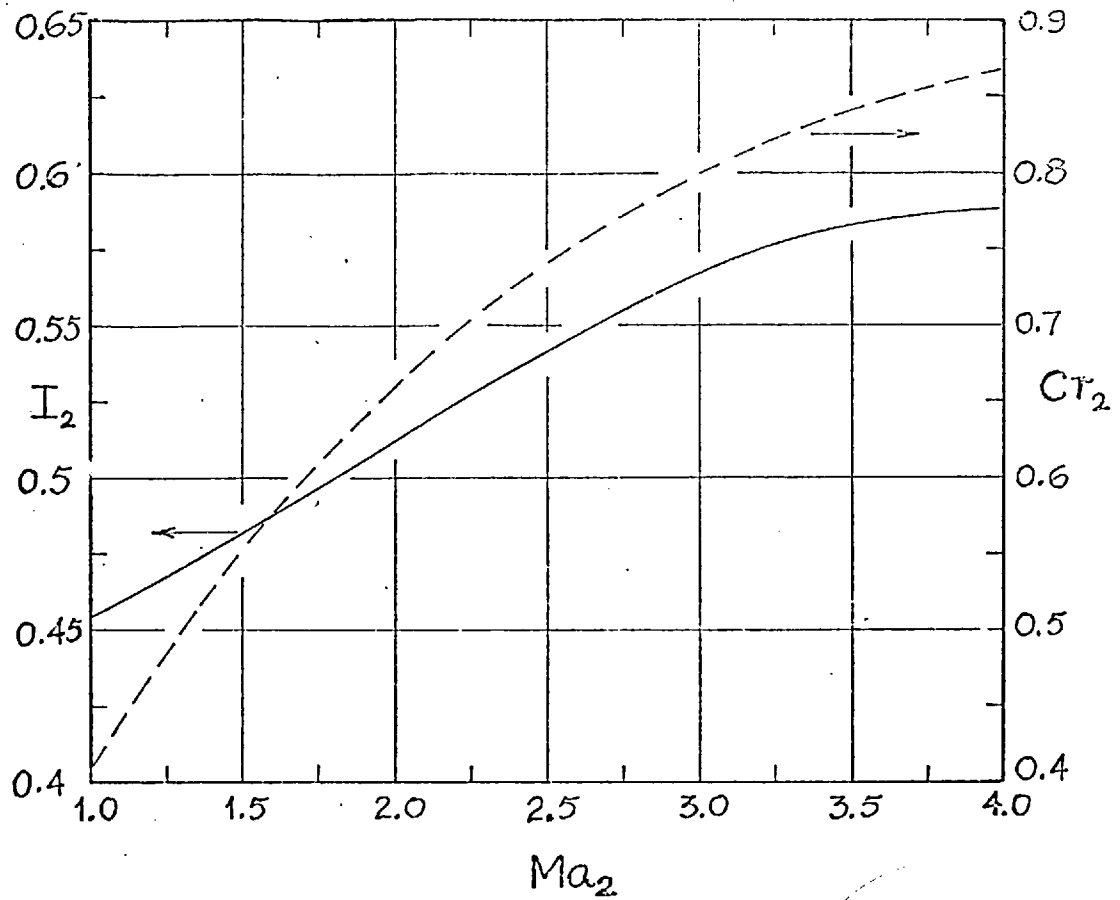


Fig.2.3.7. Variation of I_2 and Crocco number with Mach number.

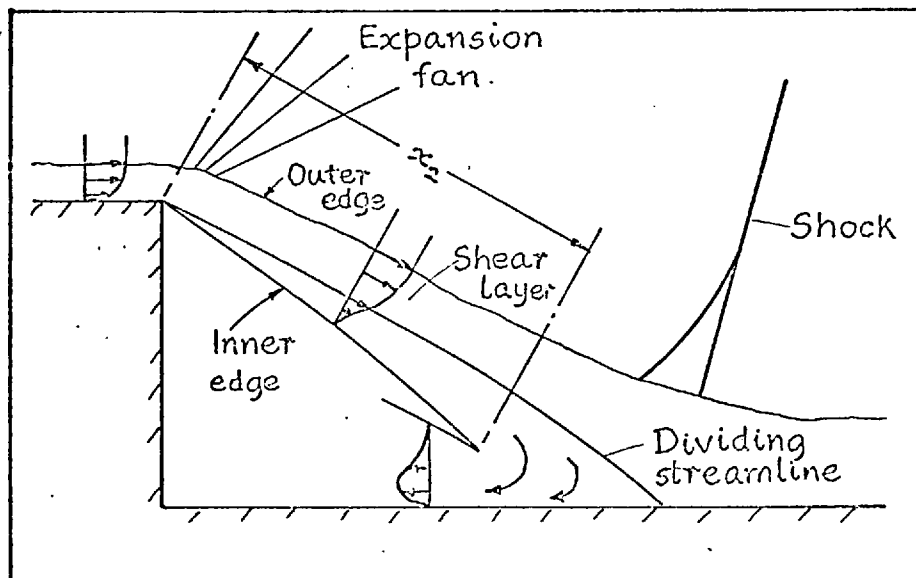


Fig.2.3.8. Shear layer development length x_2 .

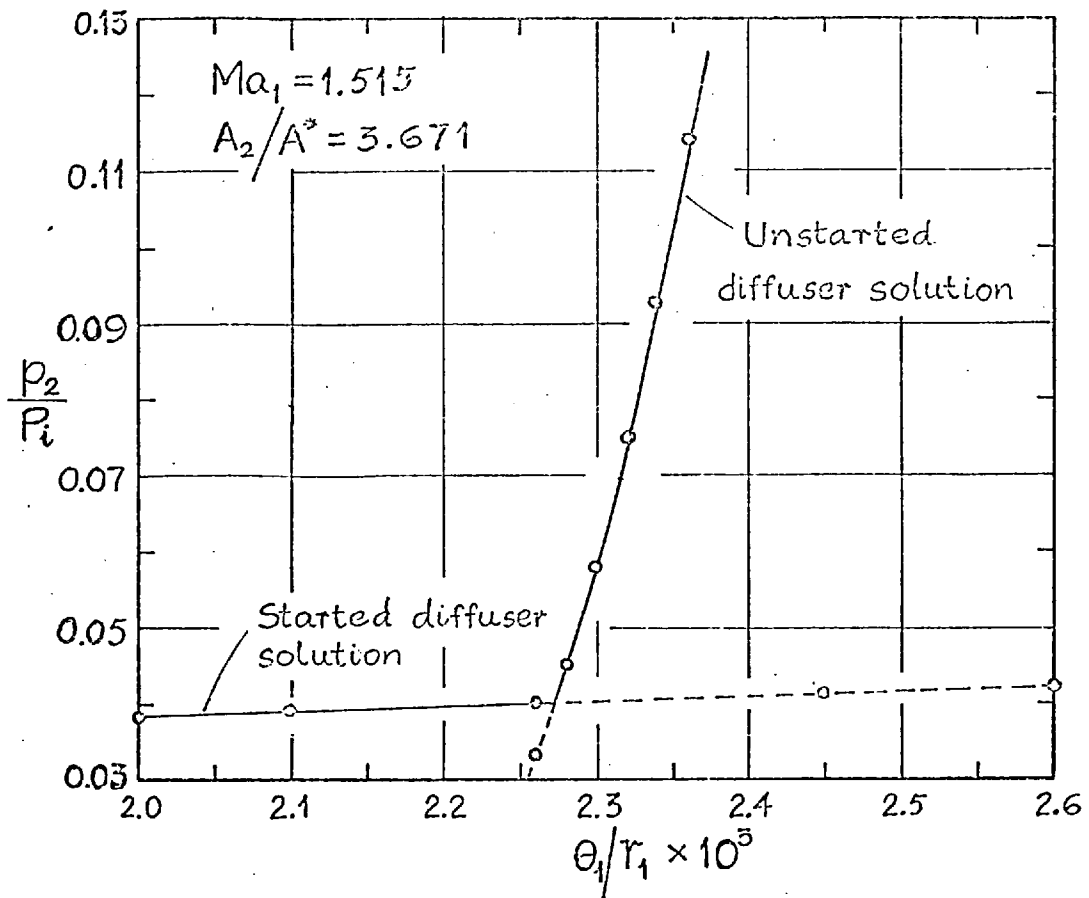


Fig.2.3.9. Variation of nozzle back-pressure ratio with initial boundary-layer momentum thickness.

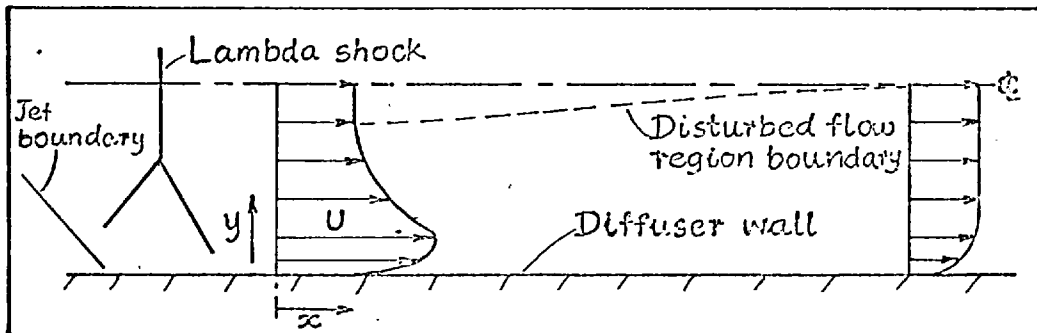


Fig.3.1. Flow model downstream of lambda shock in an unstarted diffuser.

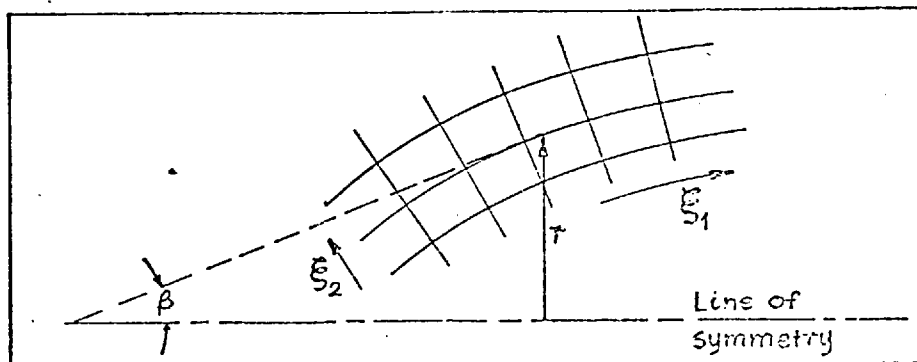


Fig.3.2. Co-ordinate system for axisymmetric flow.

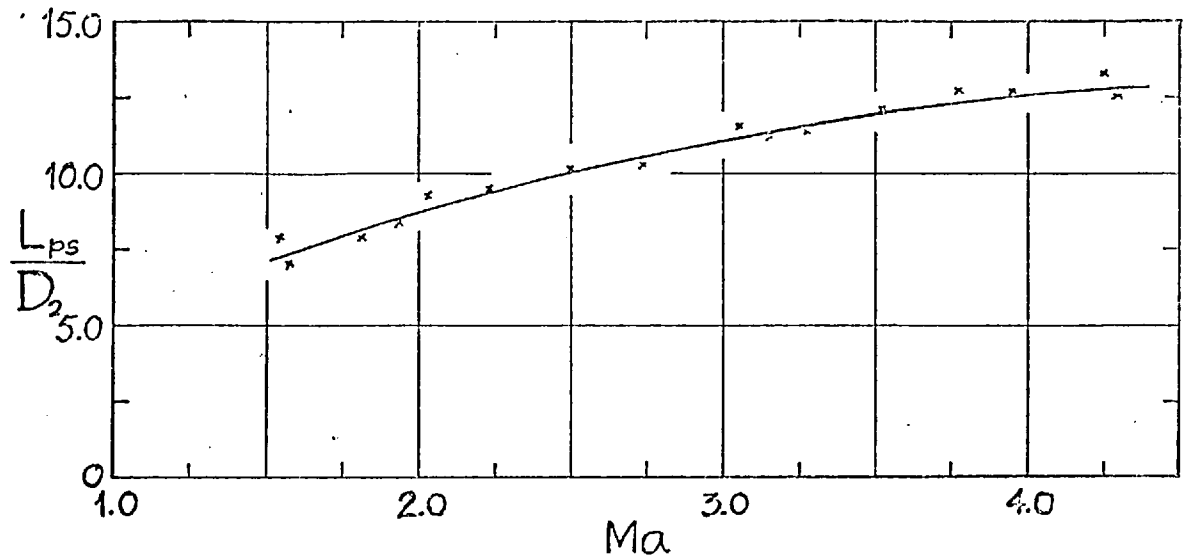


Fig.3.3. Length of pseudo-shock at various Mach numbers (Neumann & Lustwerk, 1949).

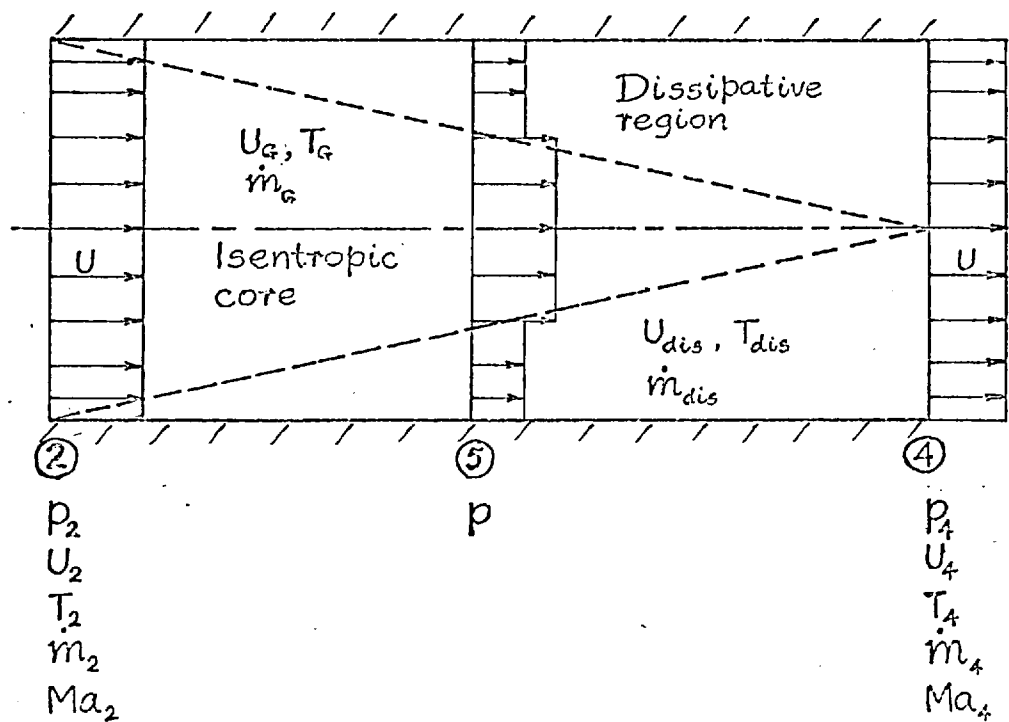


Fig.3.4. Flow model downstream of reattachment in a started diffuser (after Crocco, 1958).

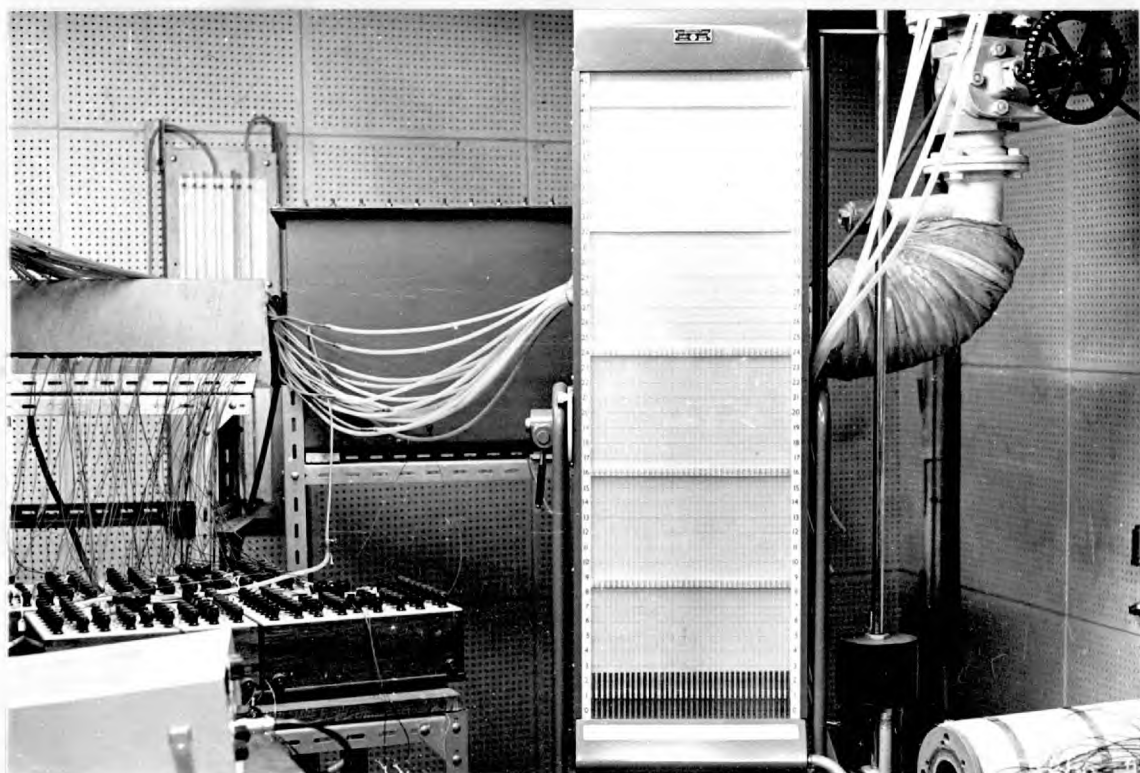


Fig. 4.1. Photograph of the general layout of apparatus.



Fig. 4.2.b Axisymmetric convergent-divergent bell-mouth nozzle - $Ma_1 = 1.49$.

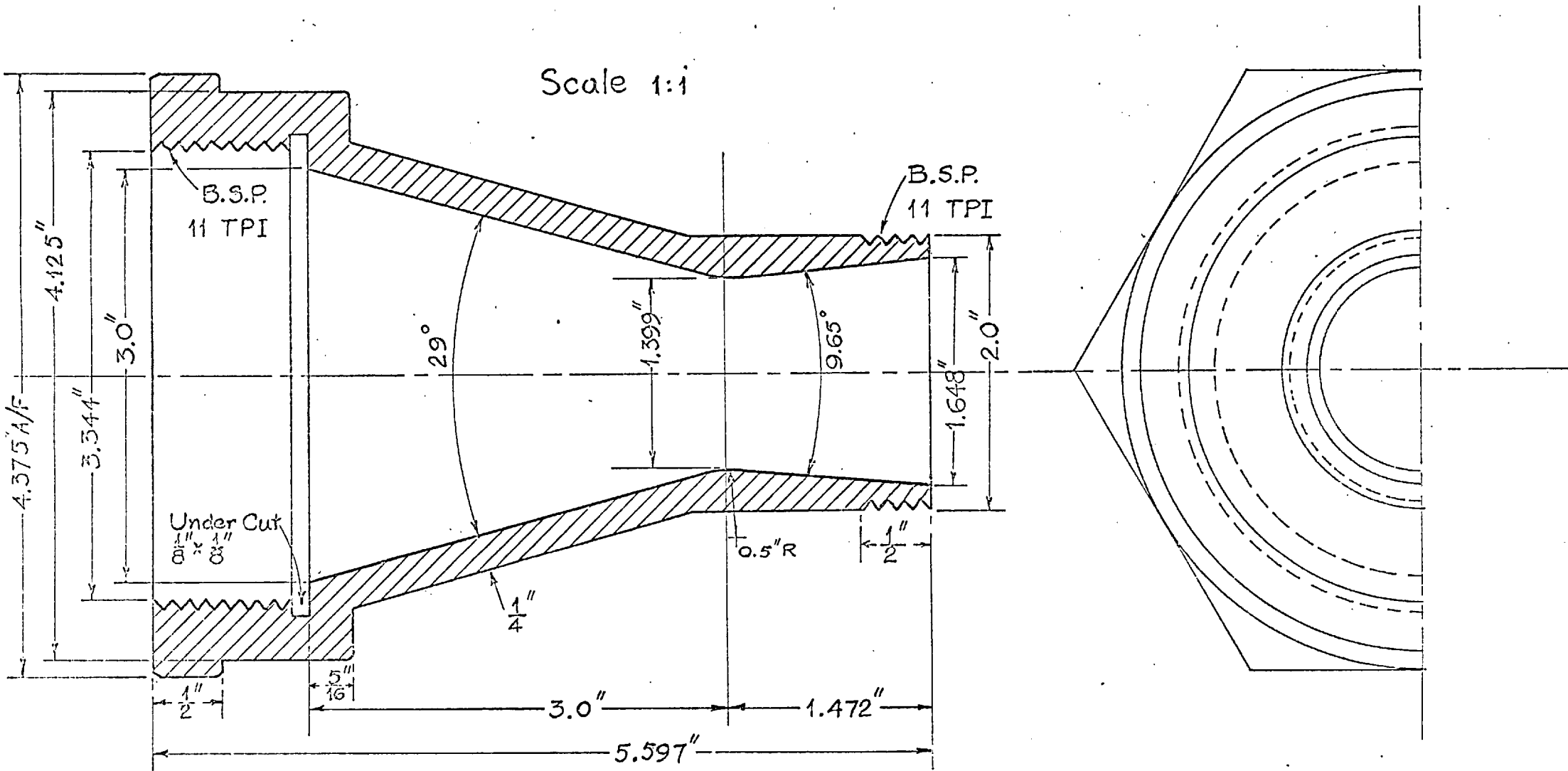


Fig. 4.2.c Axisymmetric convergent-divergent nozzle — $Ma_1 = 1.7$.

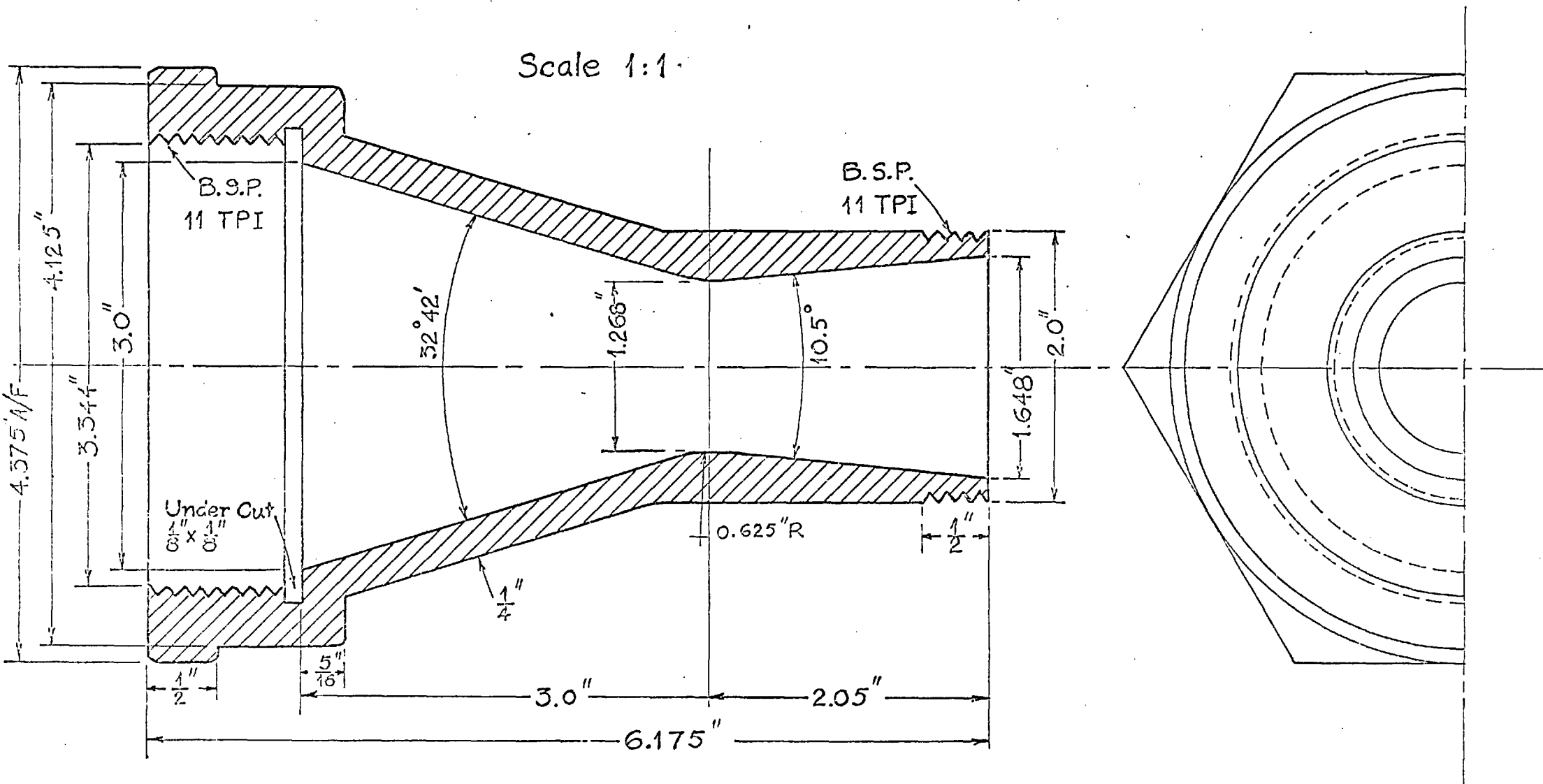


Fig.4.2.d Axisymmetric convergent-divergent nozzle - $Ma_1=1.95$.



Fig. 4.3. Photograph of convergent-divergent nozzles.

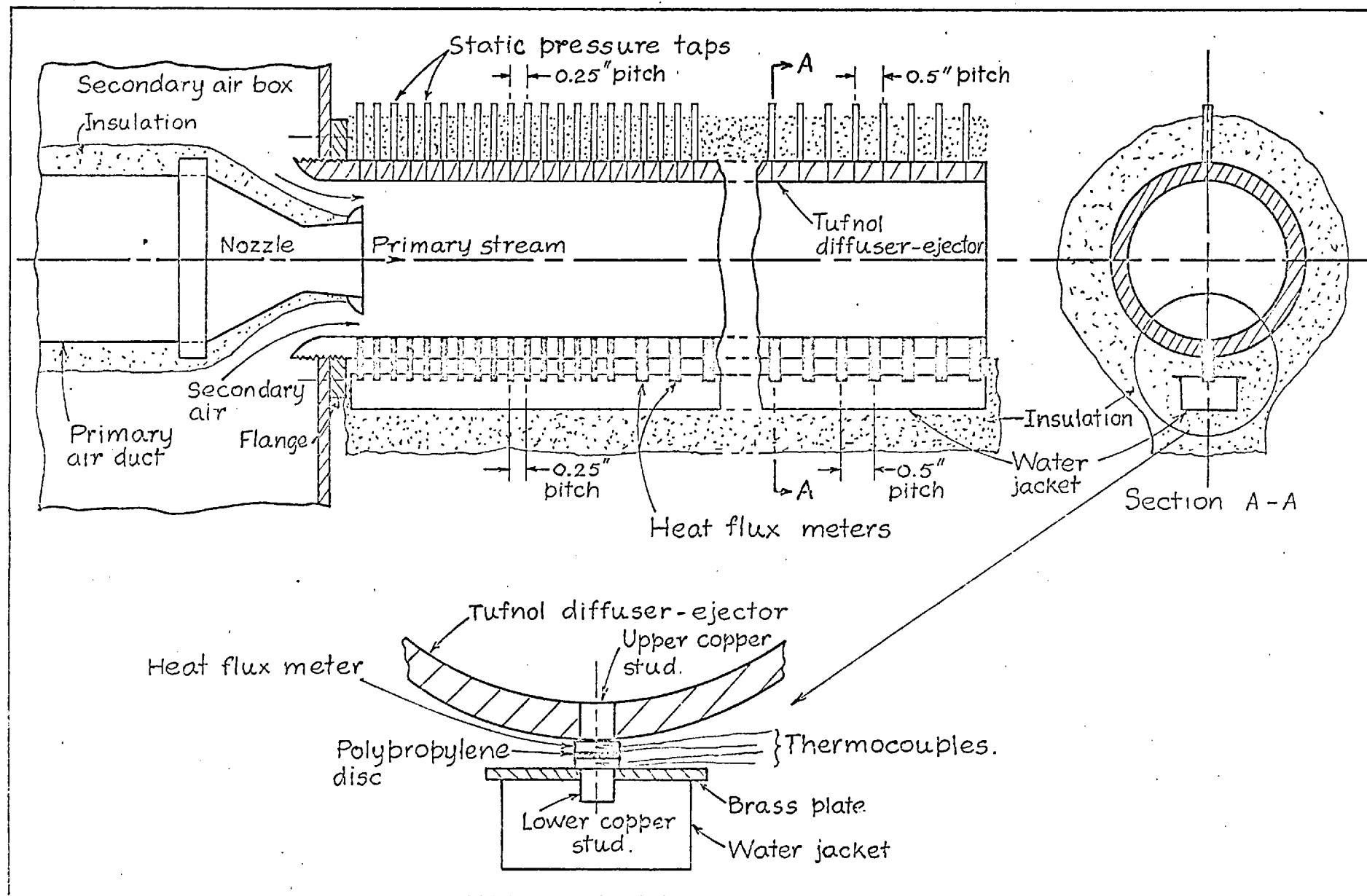


Fig.4.4 Sectional view of axisymmetric diffuser-ejector and heat flux meter. 196



Fig. 4.5. Photograph of Tufnol Diffuser - Ejector with static pressure connections, thermocouples from heat flux meters and the underslung water jacket.



Fig. 4.6. Photograph of Tufnol Diffuser - Ejector, air-exit end.

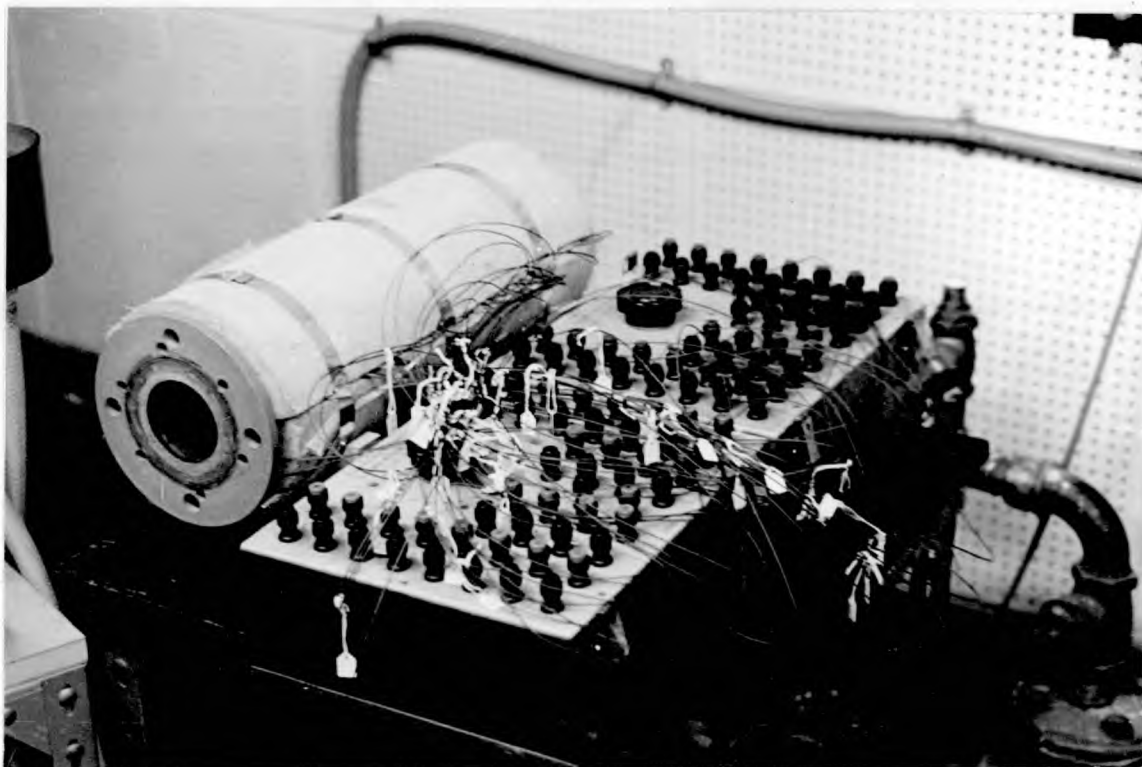


Fig. 4.7. Photograph of adiabatic diffuser-ejector with thermocouples connected to multiple switch boxes.

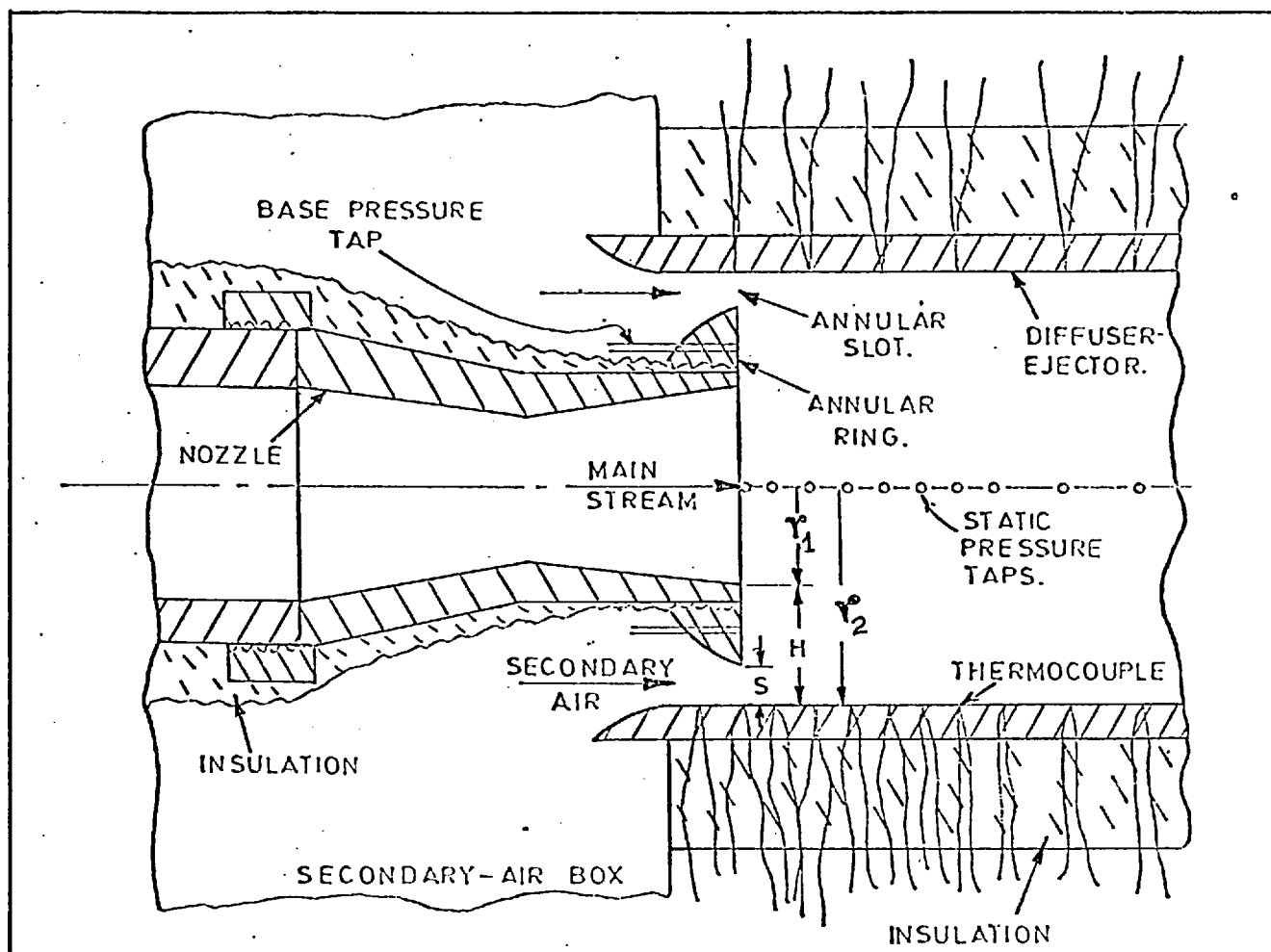


Fig. 4.8 Adiabatic diffuser - ejector entry region.

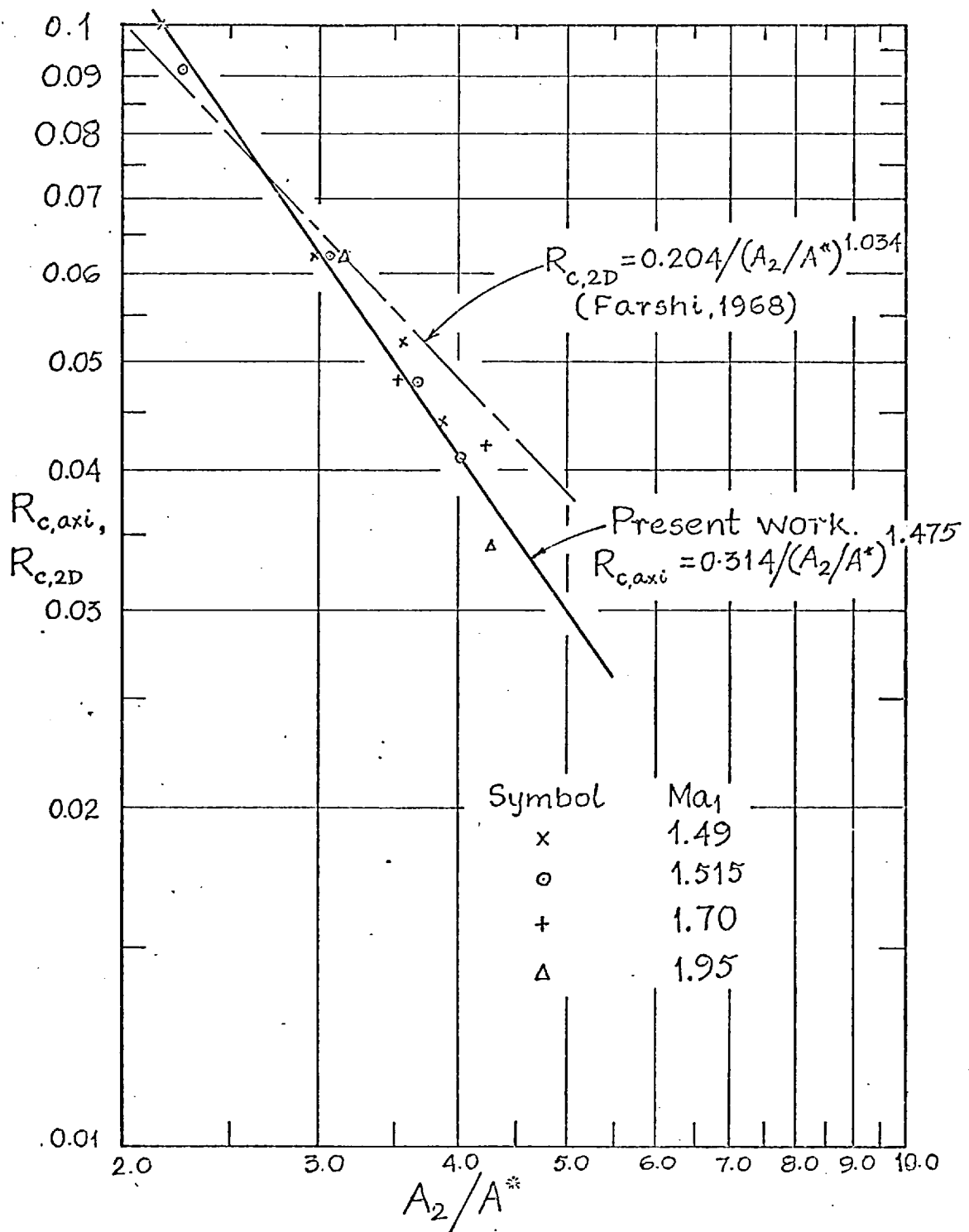


Fig.6.1. Variation of reattachment criterion with area ratio for axisymmetric and two-dimensional confined flow.

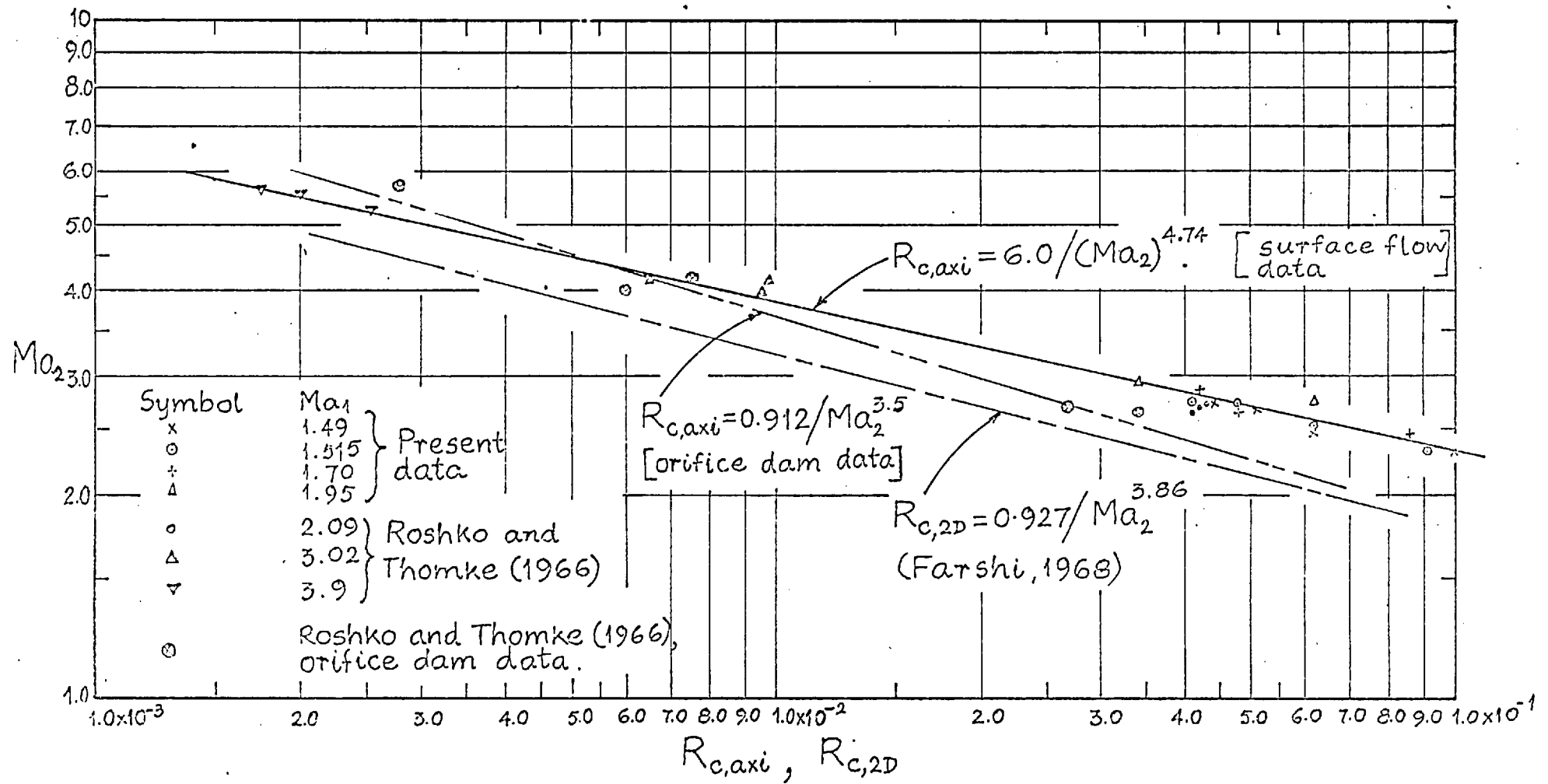


Fig.6.2. Variation of reattachment criterion with Mach number for axisymmetric and two-dimensional confined and unconfined flows.

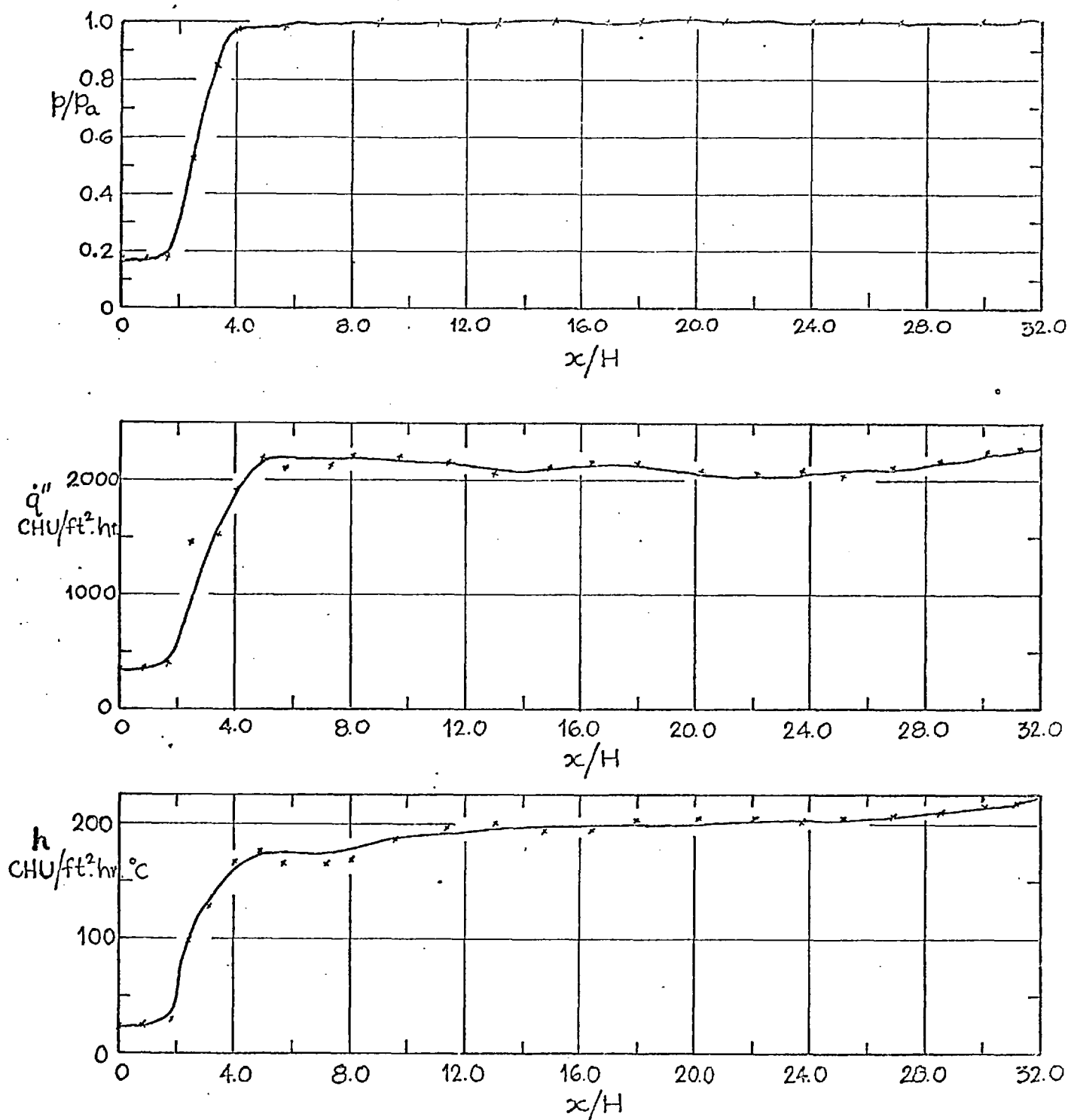


Fig. 7.2.1. Variation of wall static pressure, heat flux and heat transfer coefficient along unstarted diffuser, $Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 3.857$.

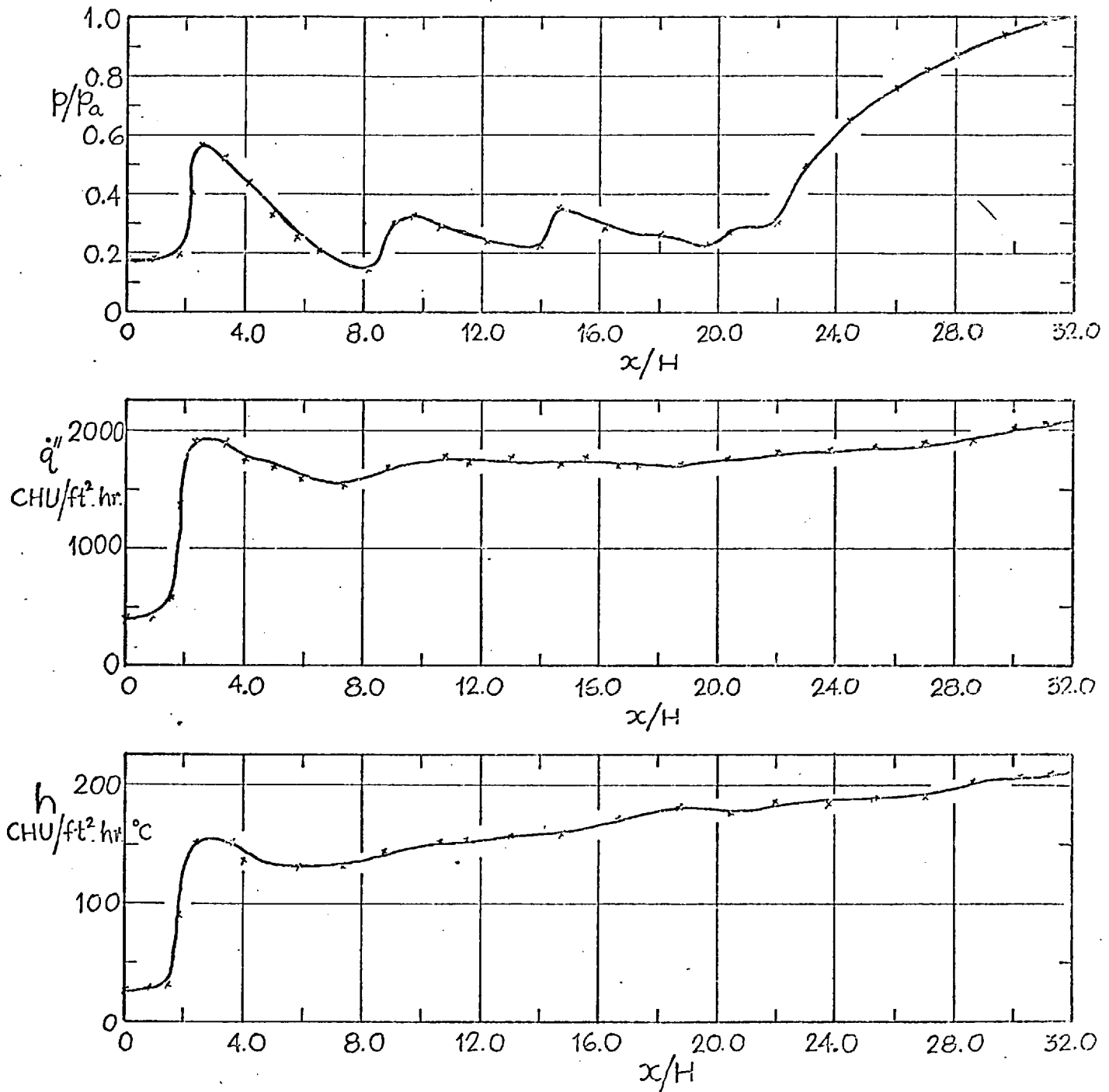
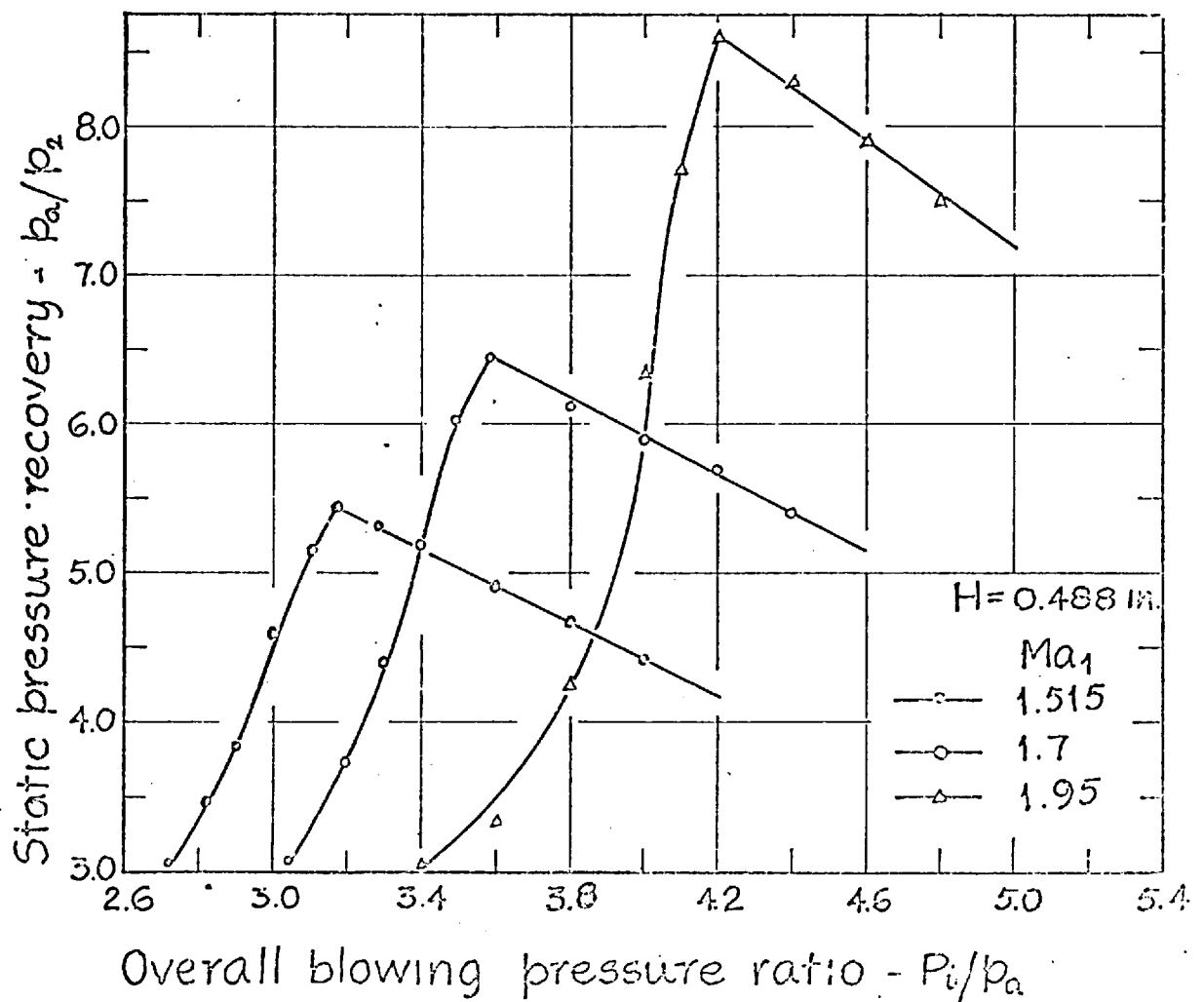
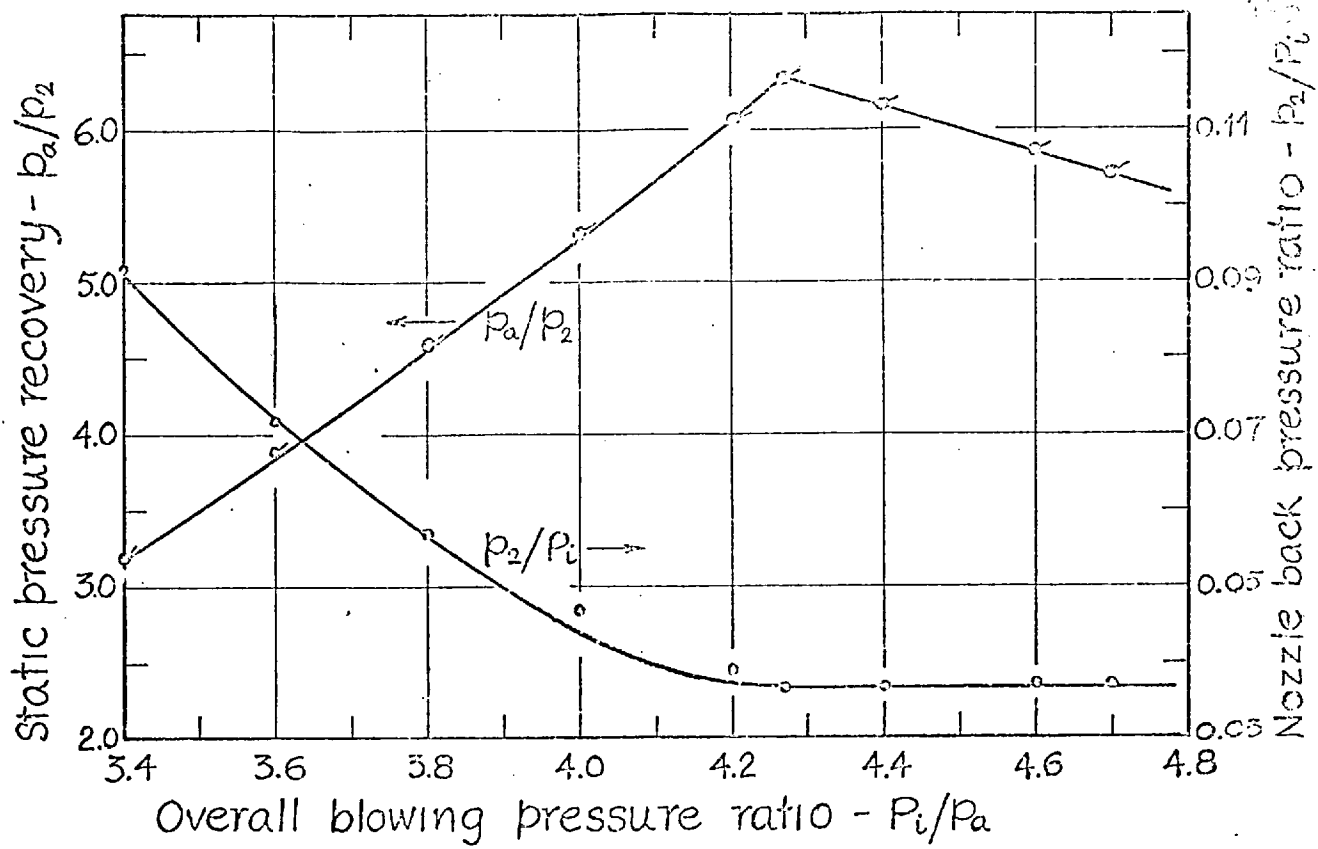
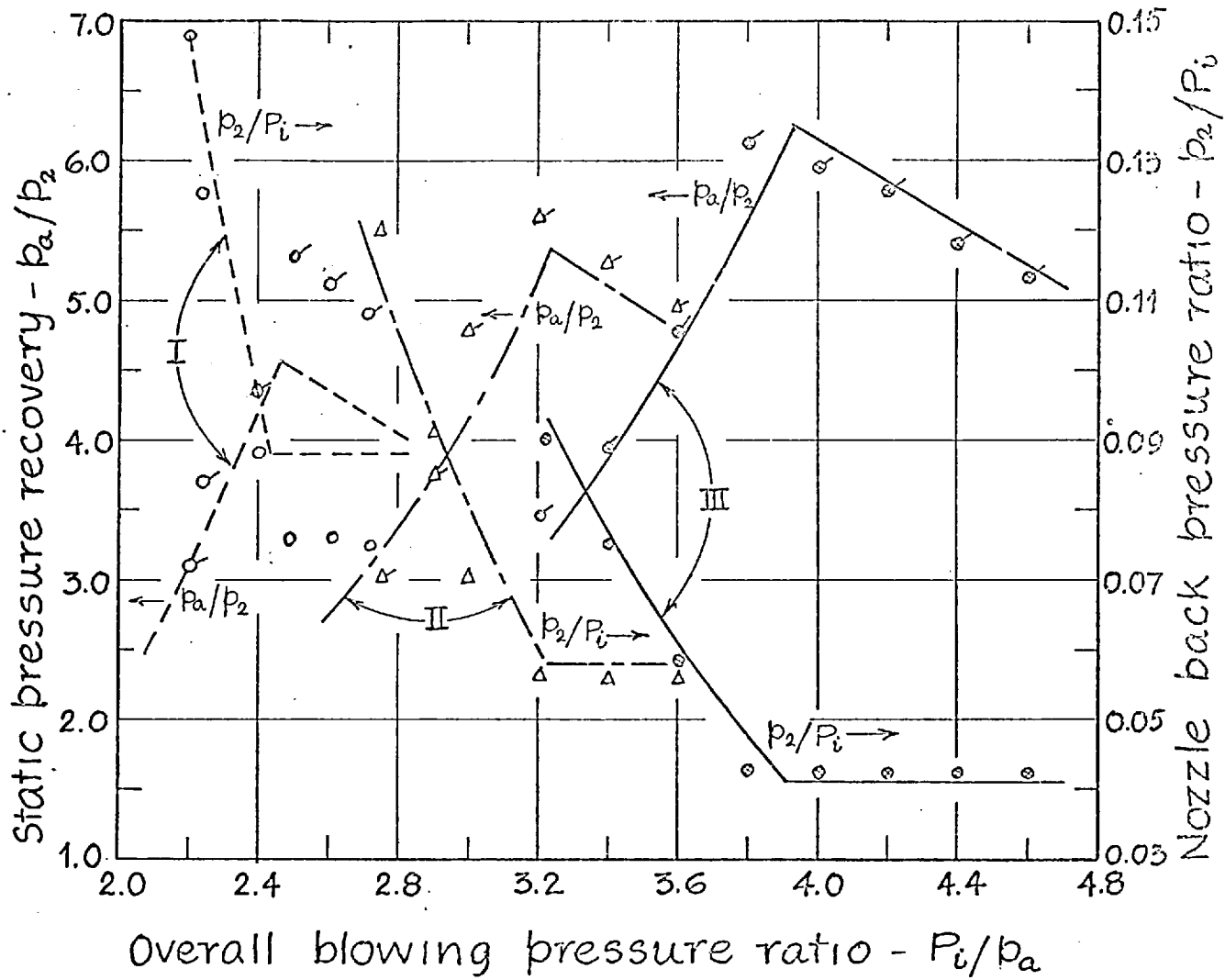


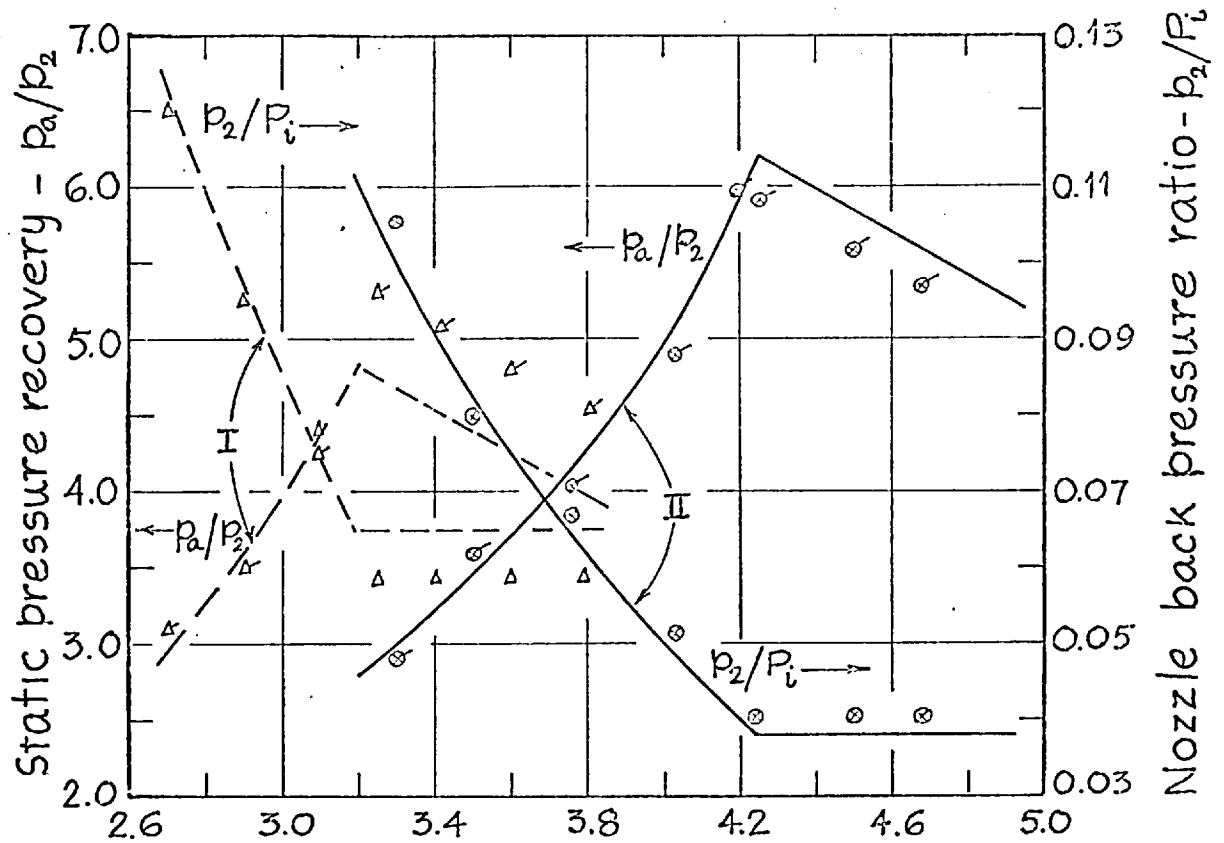
Fig.7.2.2. Variation of wall pressure, heat flux and heat transfer coefficient along started diffuser, $Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 4.129$.





	A_2/A^*	Theory	Experimental data
I	2.25	-----	$\circ$ for p_2/P_i $\checkmark$ for p_a/p_2
II	3.06	- - - - -	Δ for p_2/P_i $\checkmark$ for p_a/p_2
III	3.671	—————	$\circ$ for p_2/P_i $\checkmark$ for p_a/p_2

Fig. 7.2.4. Comparison between predicted and experimental nozzle back pressure ratio and diffuser static pressure recovery for different area ratios when $Ma_1 = 1.515$.

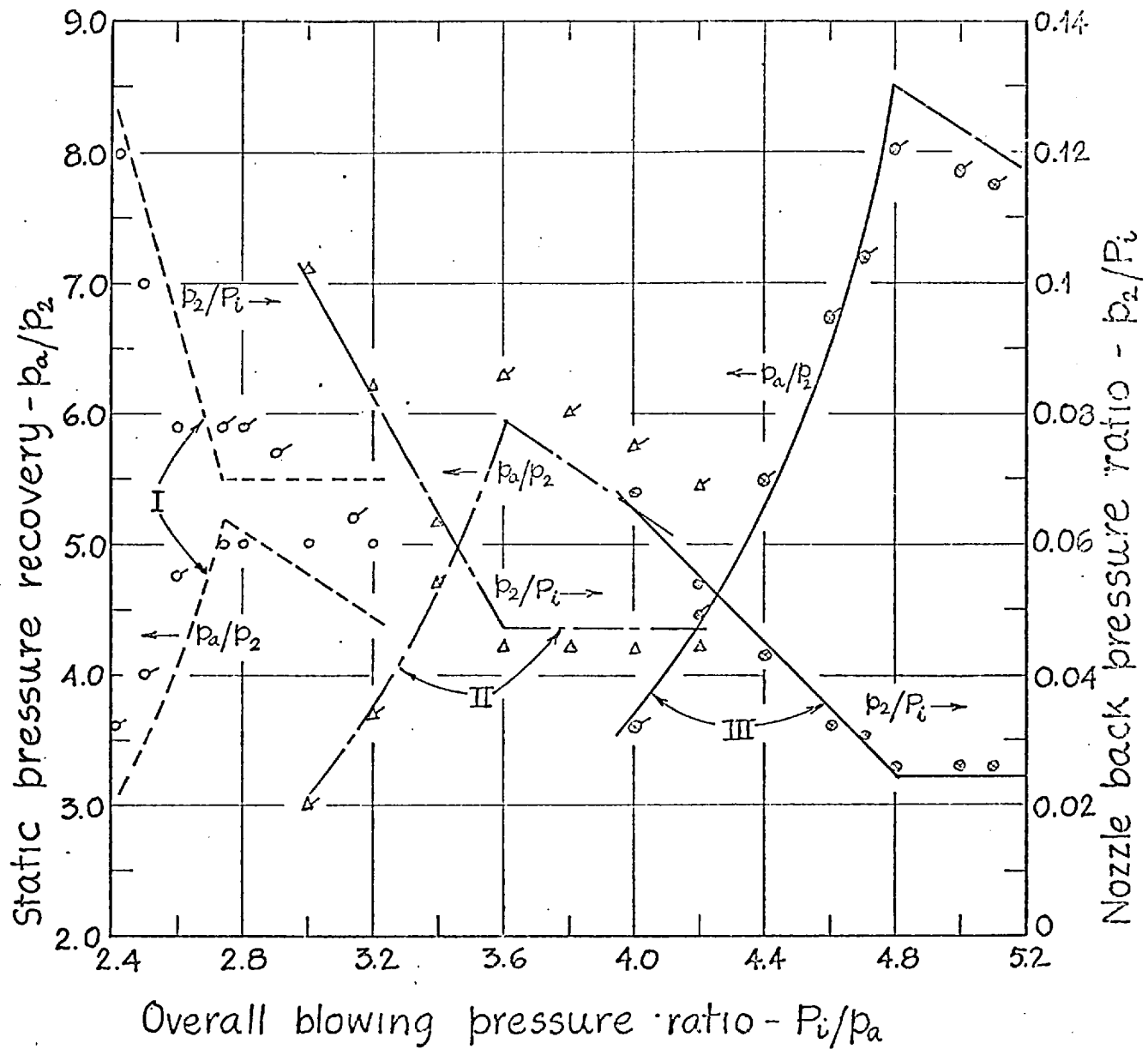


Overall blowing pressure ratio - P_i/p_a

I - $A_2/A^* = 2.97$, ---- Theory, Experimental data
 Δ - for p_2/P_i
 Δ - for p_a/p_2

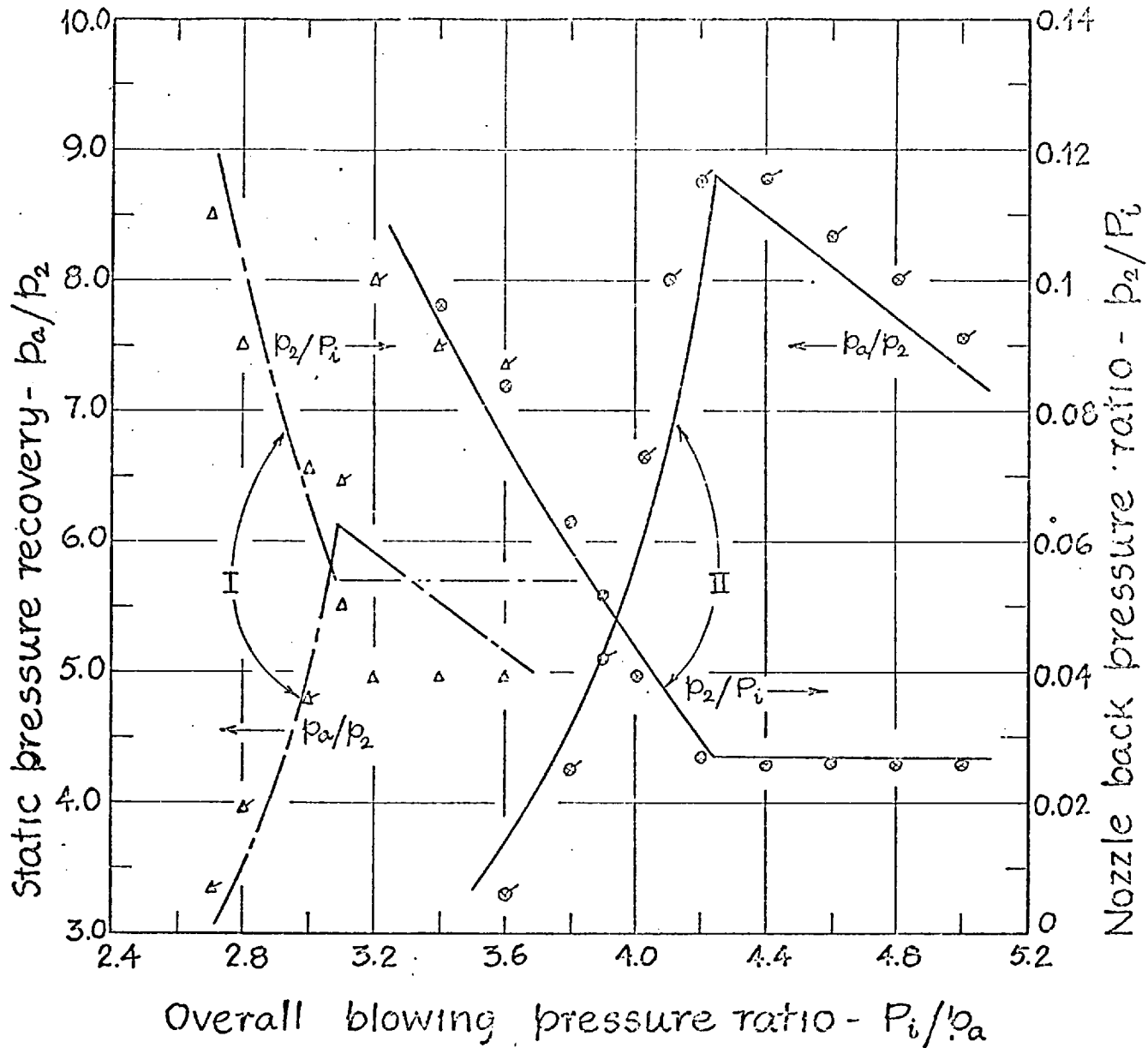
II - $A_2/A^* = 3.88$, —— Theory, Experimental data
 $\odot$ - for p_2/P_i
 $\odot$ - for p_a/p_2

Fig. 7.2.5. Comparison between predicted and experimental nozzle back pressure ratio and diffuser static pressure recovery for different area ratios when $Ma_1 = 1.49$ (Foelsch nozzle).



	A_2/A^*	Theory	Experimental data
I	2.58	-----	$\circ$ for p_2/P_i $\odot$ for p_a/p_2
II	3.52	- - - - -	Δ for p_2/P_i $\triangle$ for p_a/p_2
III	4.6	—————	$\odot$ for p_2/P_i $\oslash$ for p_a/p_2

Fig. 7.2.6. Comparison between predicted and experimental nozzle back pressure ratio and diffuser static pressure recovery for different area ratios when $Ma_1 = 1.7$.



I - $A_2/A^* = 3.147$, --- Theory, Experimental data
 Δ for p_2/P_i
 $\times$ for p_a/p_2

II - $A_2/A^* = 4.285$, — Theory, Experimental data
 $\odot$ for p_2/P_i
 $\otimes$ for p_a/p_2

Fig. 7.2.7. Comparison between predicted and experimental nozzle back pressure ratio and diffuser static pressure recovery for different area ratios when $Ma_1 = 1.95$.

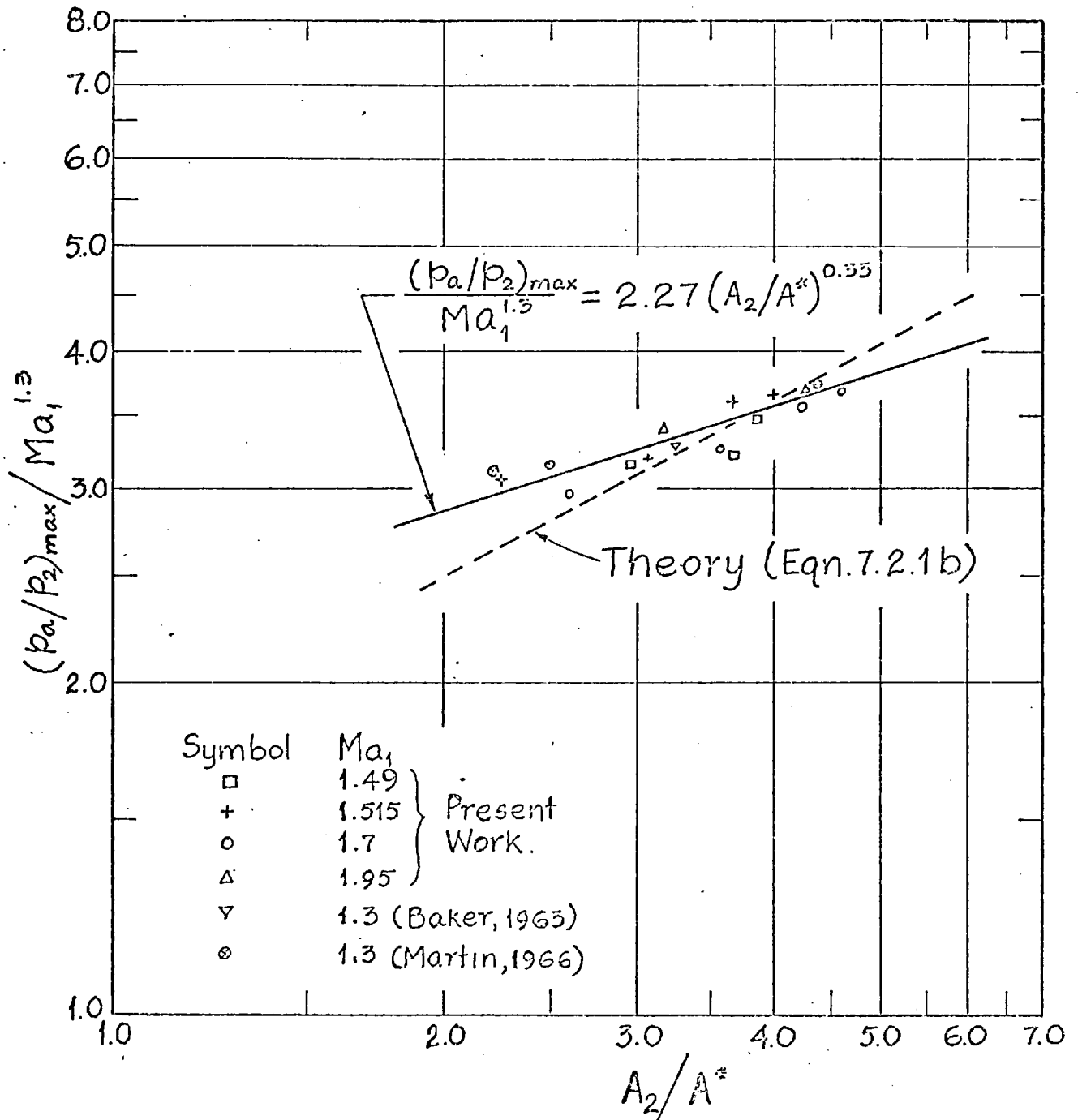


Fig.7.2.8. Effect of varying Mach number and diffuser area ratio on peak pressure recovery.

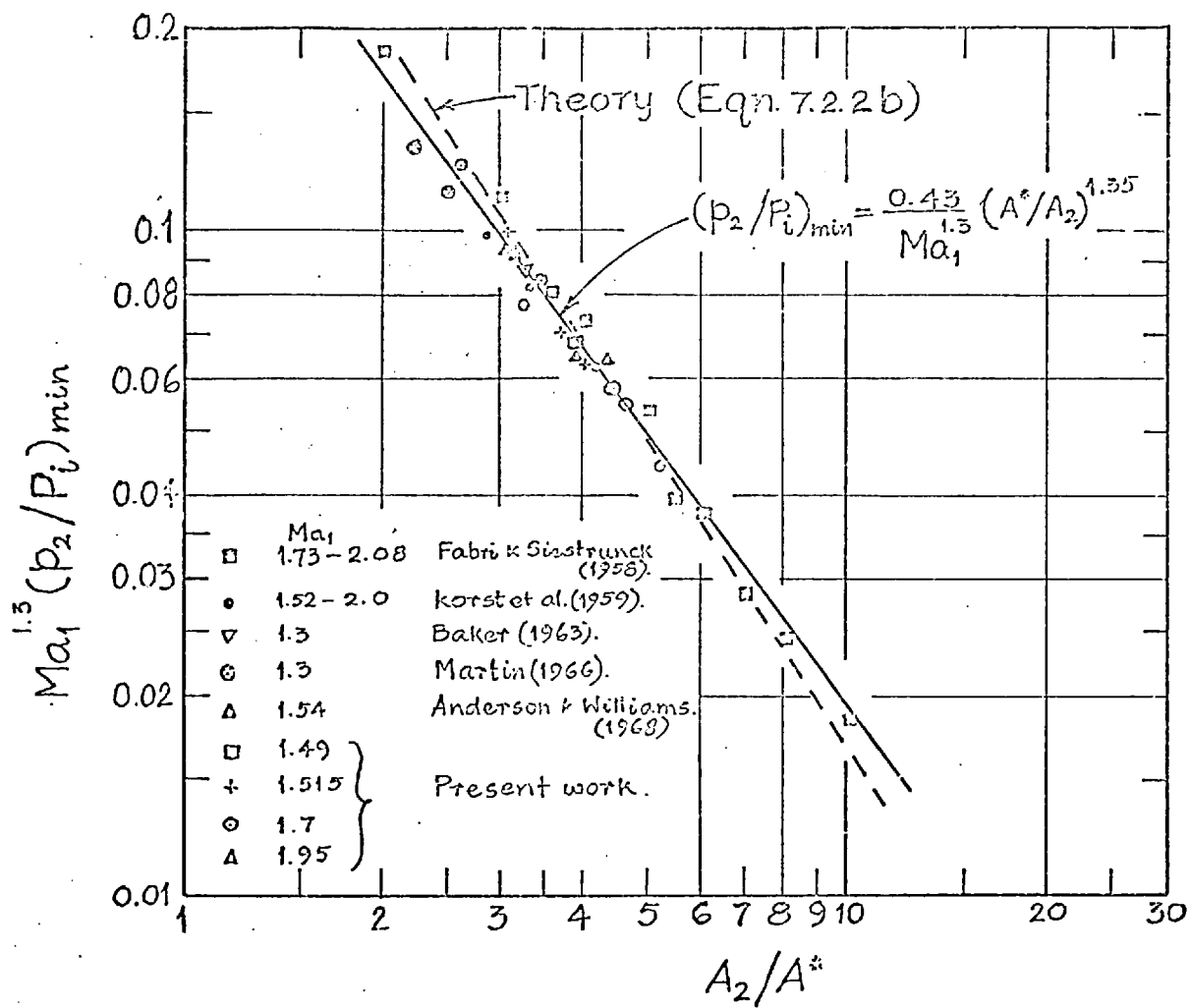


Fig. 7.2.9. Effect of varying Mach number and area ratio on $(p_2/p_i)_{min}$.

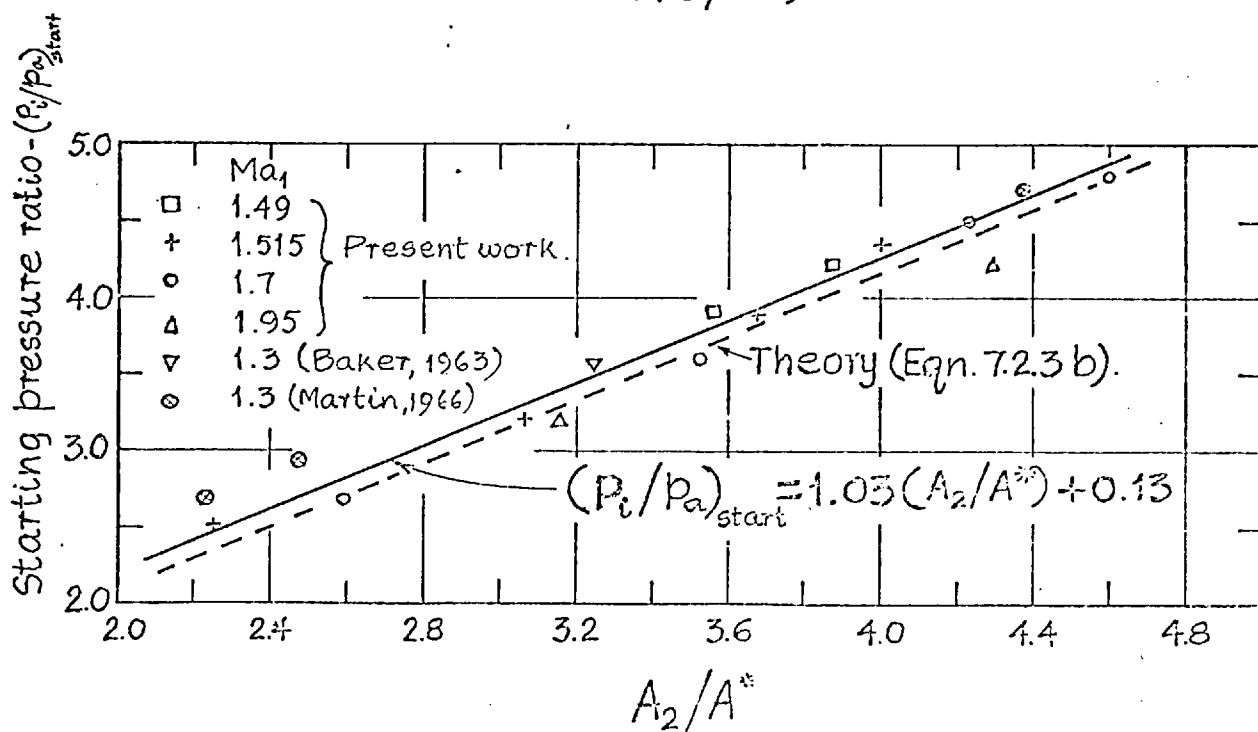


Fig. 7.2.10. Effect of area ratio on diffuser starting pressure ratio.

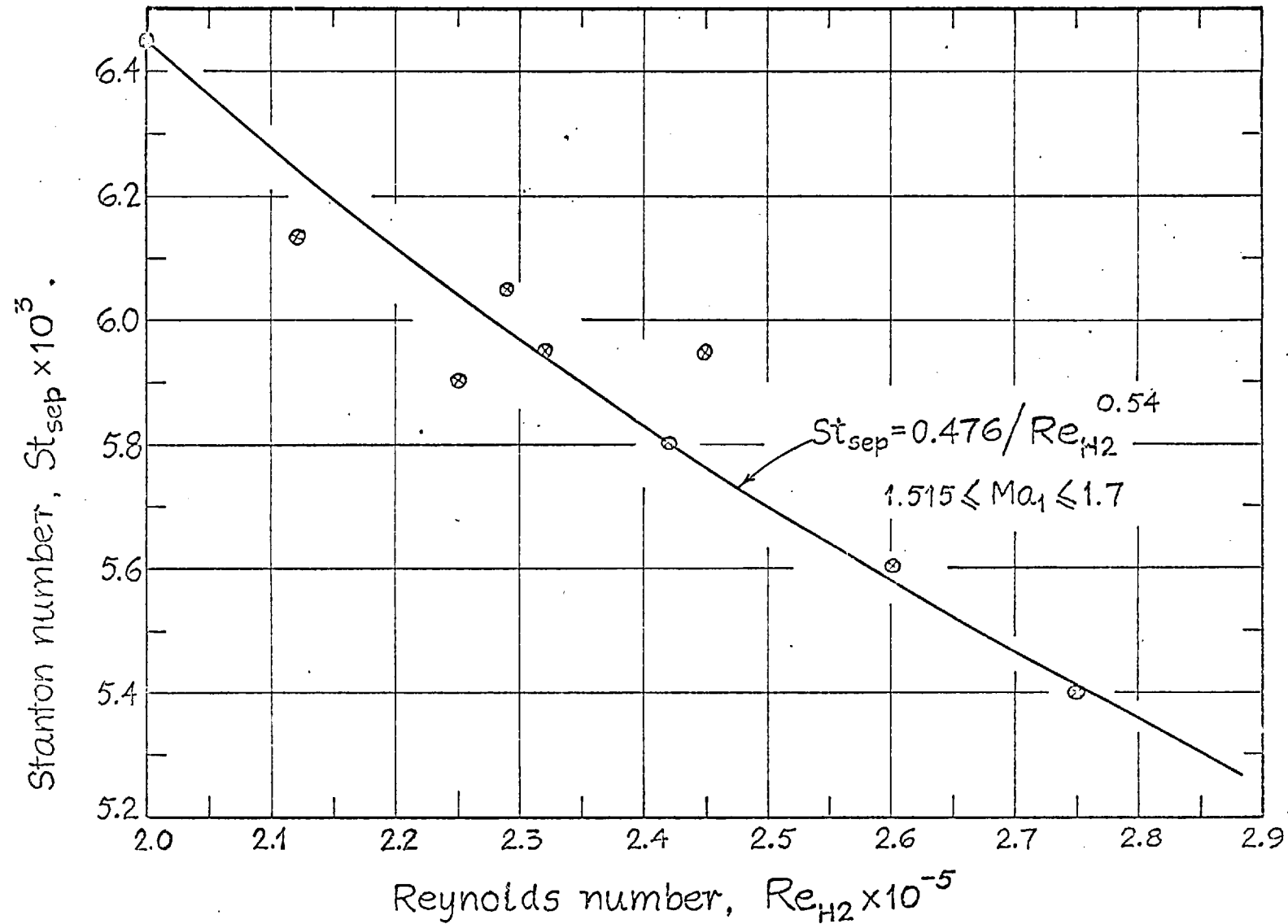


Fig.7.2.11. Variation of Stanton number in the separated flow region of started and unstarted diffusers with Reynolds number.

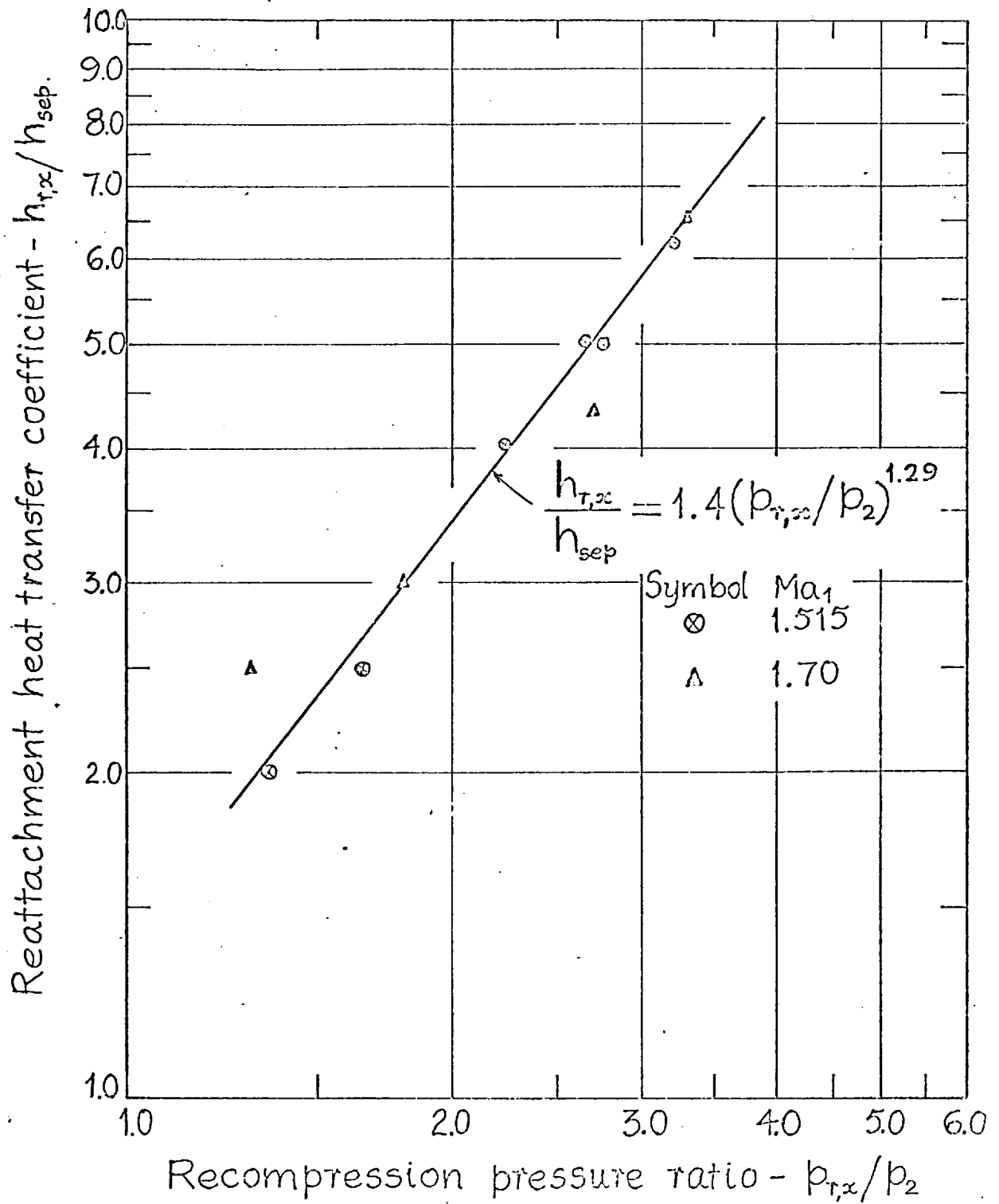
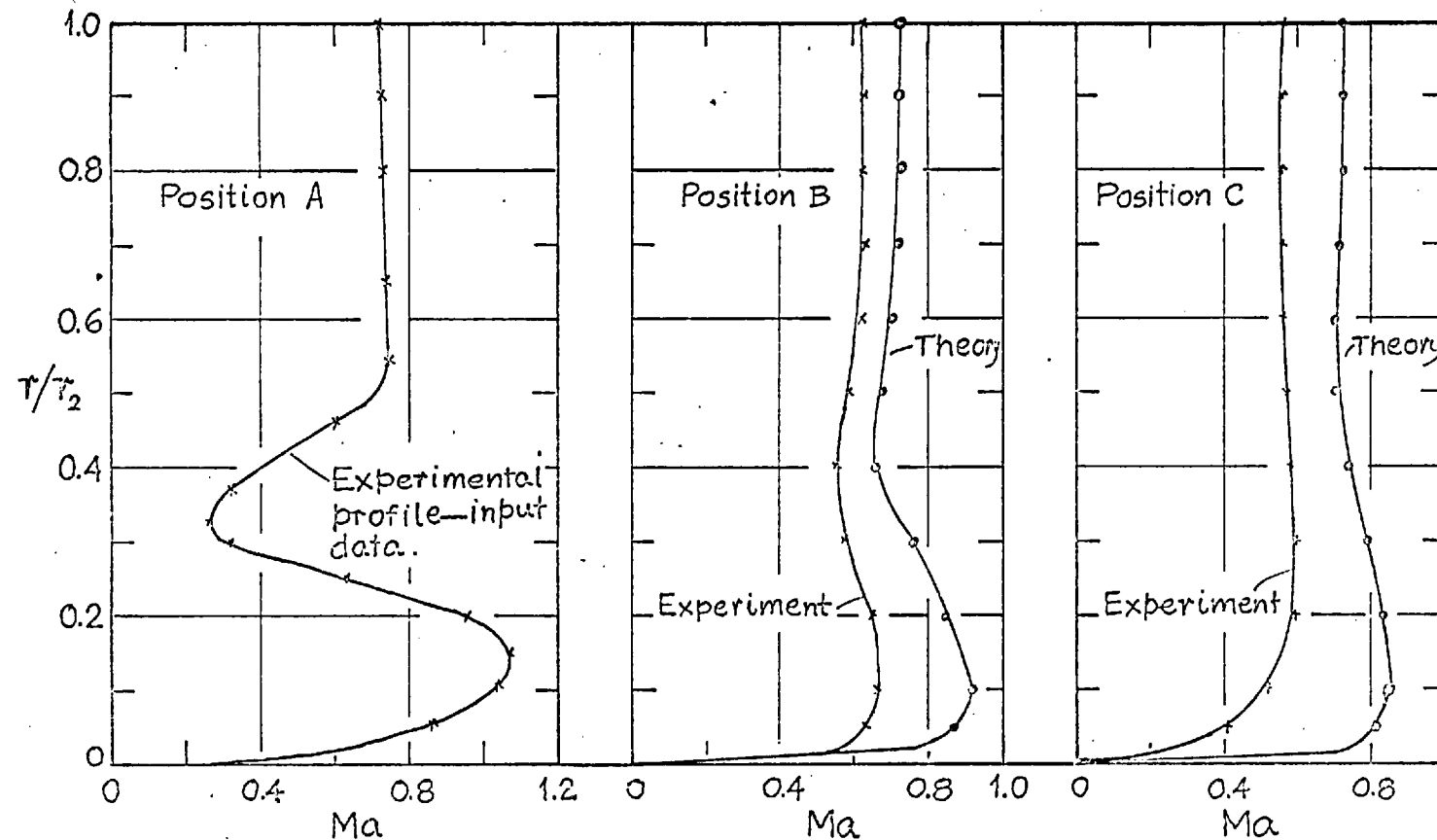


Fig.7.2.12. Effect of recompression pressure rise on reattachment heat transfer in a started diffuser.



Position A - 6.25" from diffuser exit plane.

Position B - 3.25" " " "

Position C - Diffuser exit plane.

Fig.7.2.13. Comparison of theoretical Mach number profiles with experimental data in unstalled diffuser, $Ma_1=1.515$, $A_2/A^*=3.671$, $P_i/p_2=3.84$, $L_2/D_2=2.8$.

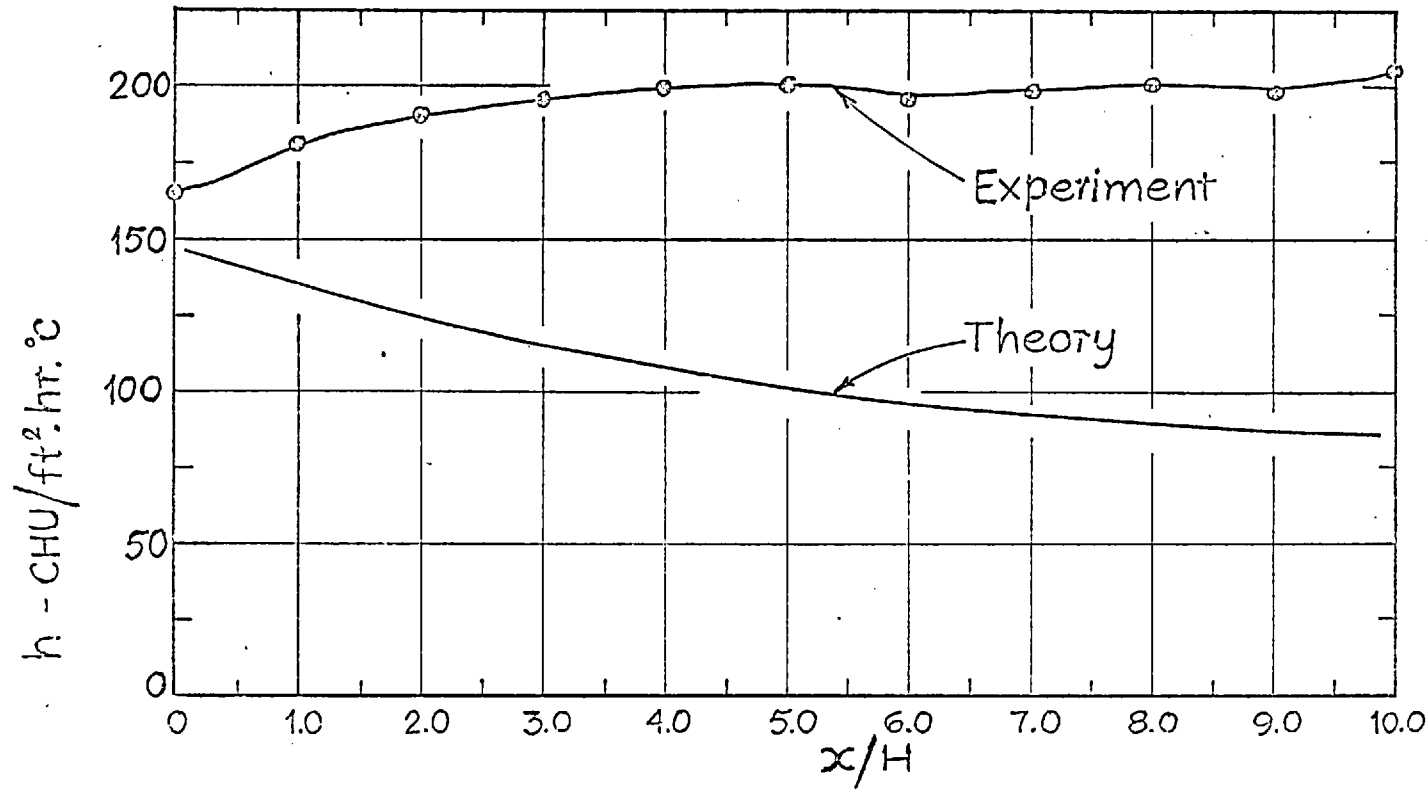
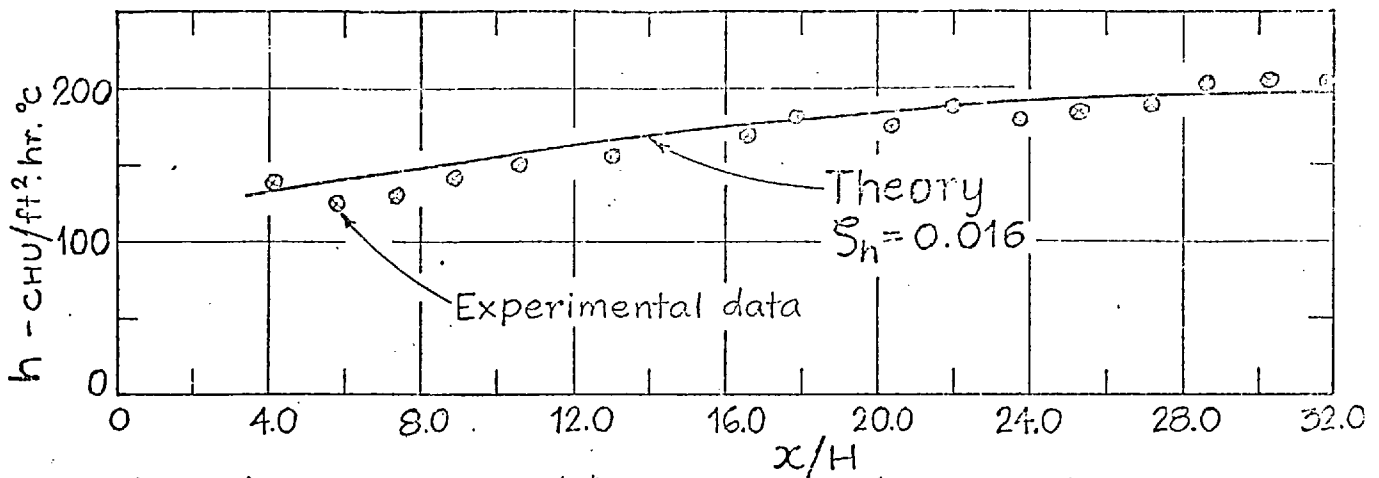
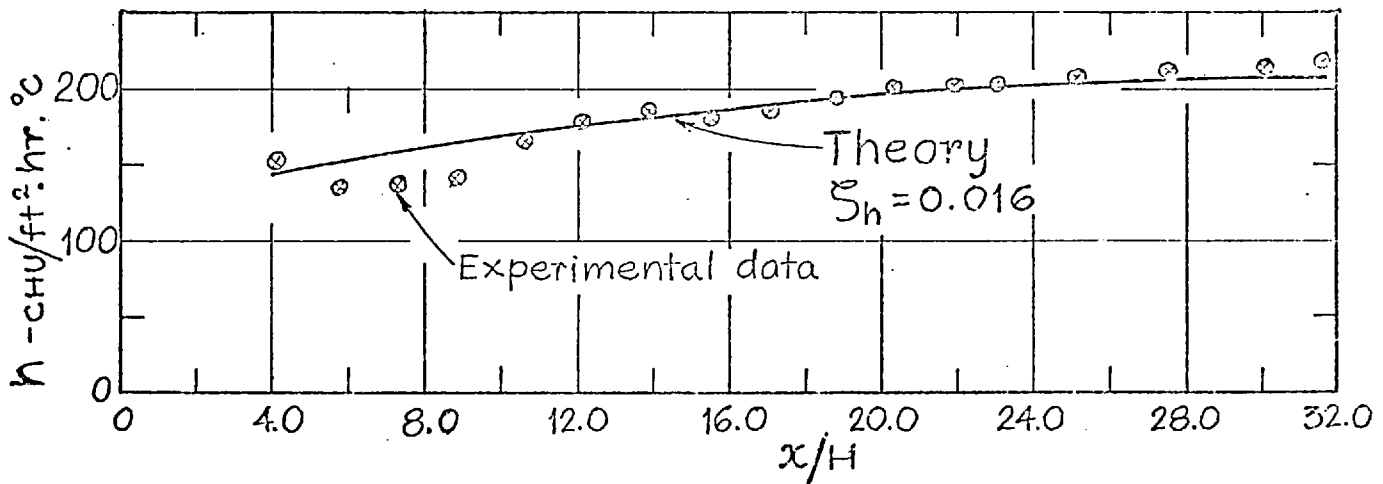


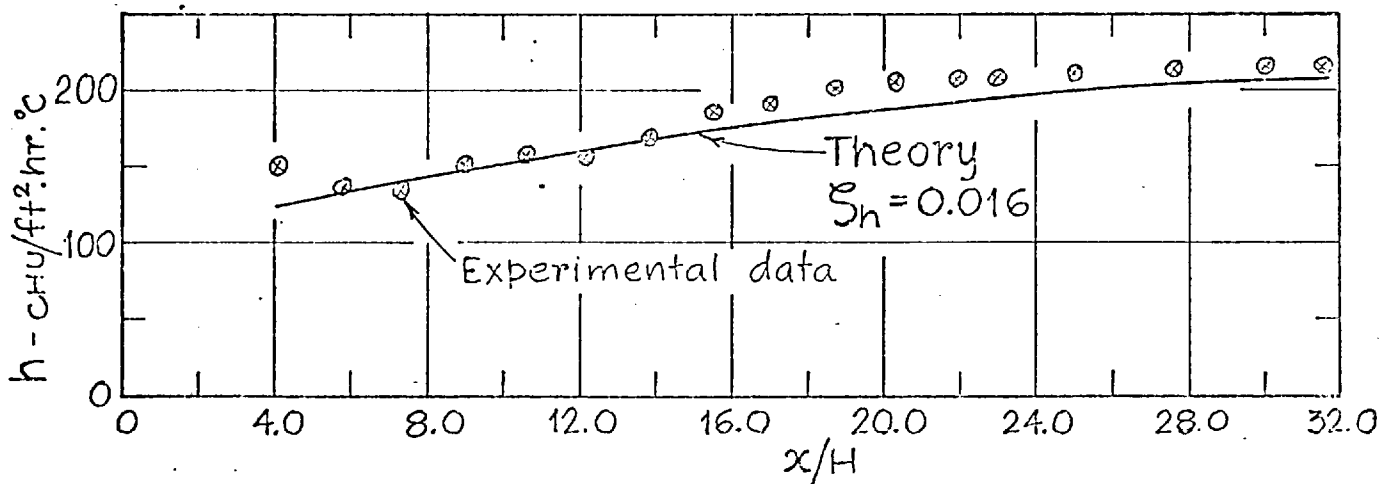
Fig.7.2.14. Comparison of theoretical and experimental variations of heat transfer coefficient in an unstarted diffuser, $Ma_1=1.515$, $A_2/A^*=3.671$ and $P_i/p_a=3.84$. ($L_2/D_2=2.8$).



(a) $Ma_1 = 1.515$, $A_2/A^* = 3.671$, $p_2/P_i = 0.041$, $P_i/p_a = 4.129$

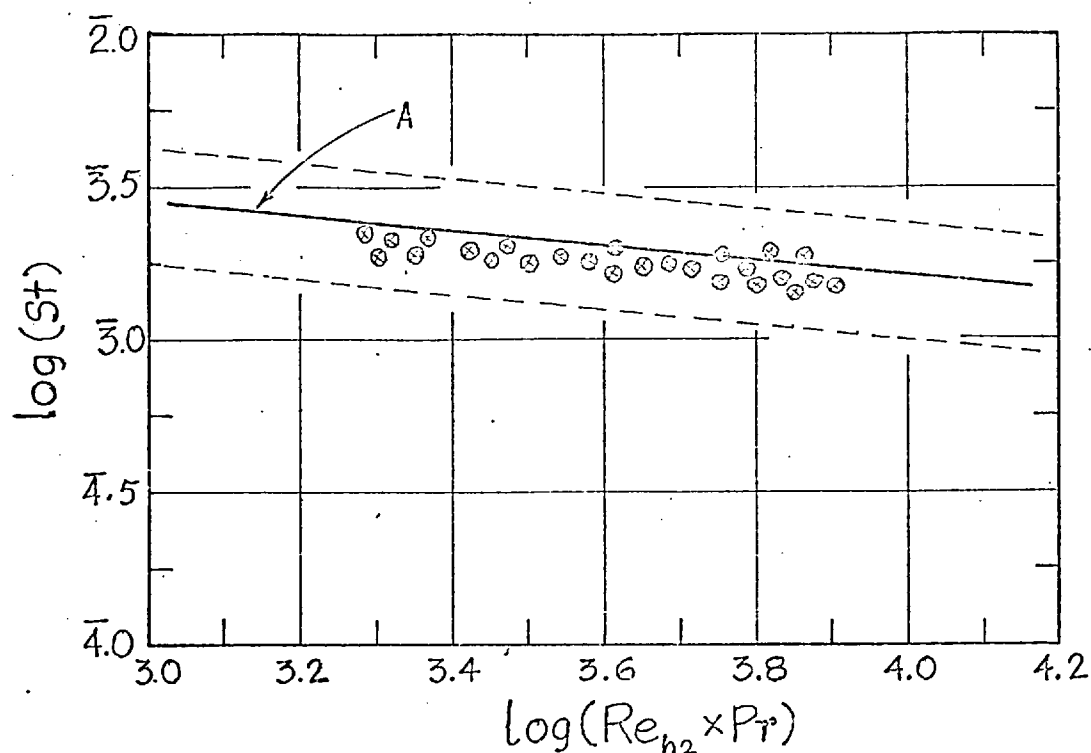


(b) $Ma_1 = 1.515$, $A_2/A^* = 3.671$, $p_2/P_i = 0.041$, $P_i/p_a = 4.401$



(c) $Ma_1 = 1.7$, $A_2/A^* = 4.223$, $p_2/P_i = 0.031$, $P_i/p_a = 4.537$.

Fig. 7.2.15 Comparison between predicted and experimental variations of heat transfer coefficient along started diffuser after reattachment.



A - Correlation of Kutateladze & Leont'ev (1964)

$$St = 0.0143 Re_{h2}^{-0.25} \cdot Pr^{-0.75}$$

--- Limits of scatter in the above reference.

⊗ Present experimental results.

Fig.7.2.16 Comparison of present data with the correlation of Kutateladze and Leont'ev for convective heat transfer in compressible turbulent boundary layer.

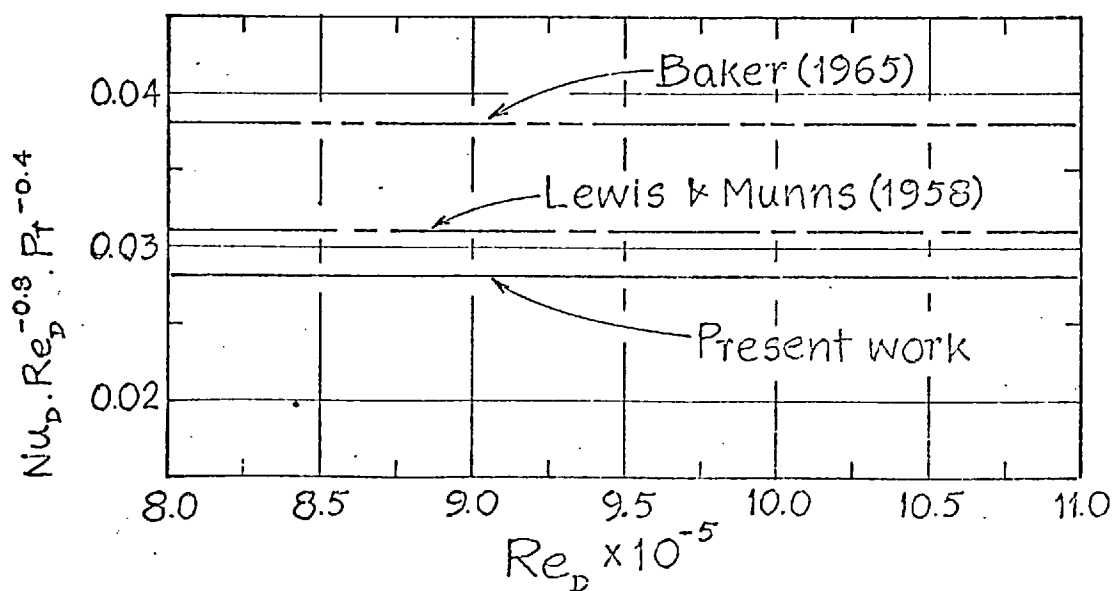


Fig.7.2.17. Comparison of heat transfer correlations for supersonic started diffusers.

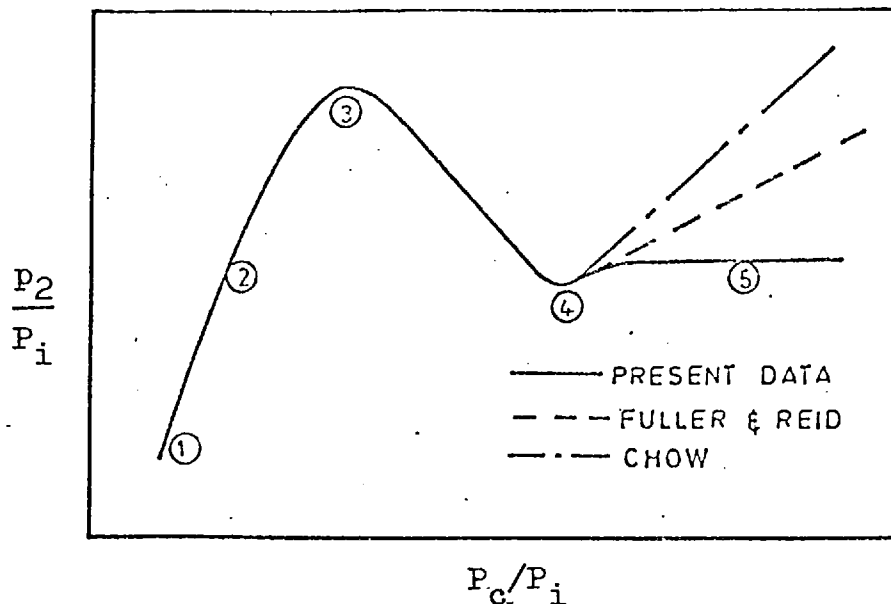
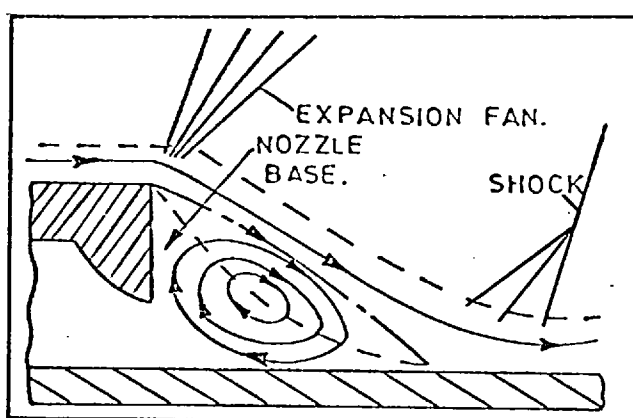
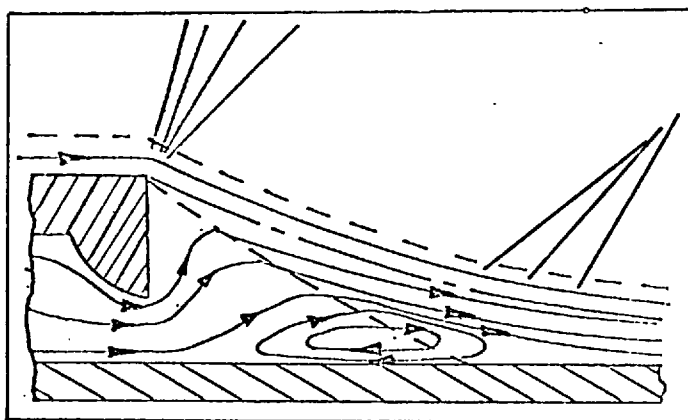


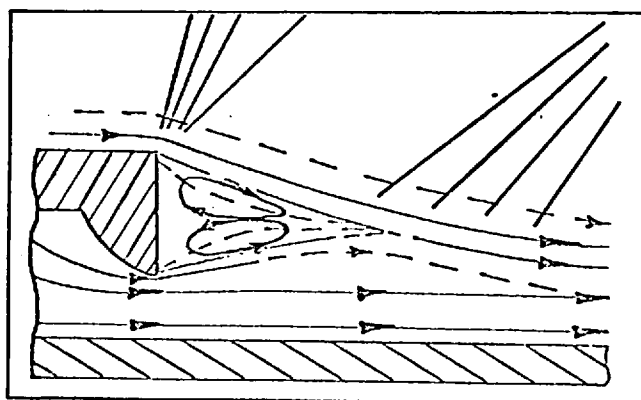
Fig. 7.3.1 Effect of secondary flow on base pressure.



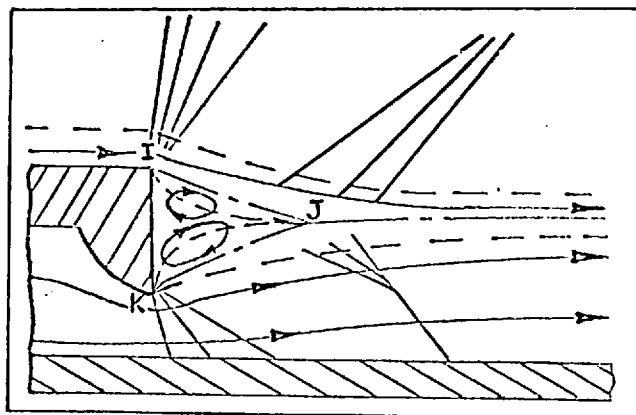
(a) No secondary flow.
(Point 1 in Fig. 7.3.1)



(b) Secondary flow supplies part of entrained air.
(Point 2 in Fig. 7.3.1)



(c) Secondary flow escapes without entering mixing region. (Point 3 in Fig. 7.3.1)



(d) Sonic secondary air flow.
(Points 4 and 5 in Fig. 7.3.1)

----- Dividing streamline, —> Streamline, --- Mixing region boundary.

Fig. 7.3.2 Effect of secondary injection on flow patterns at diffuser - ejector entry.

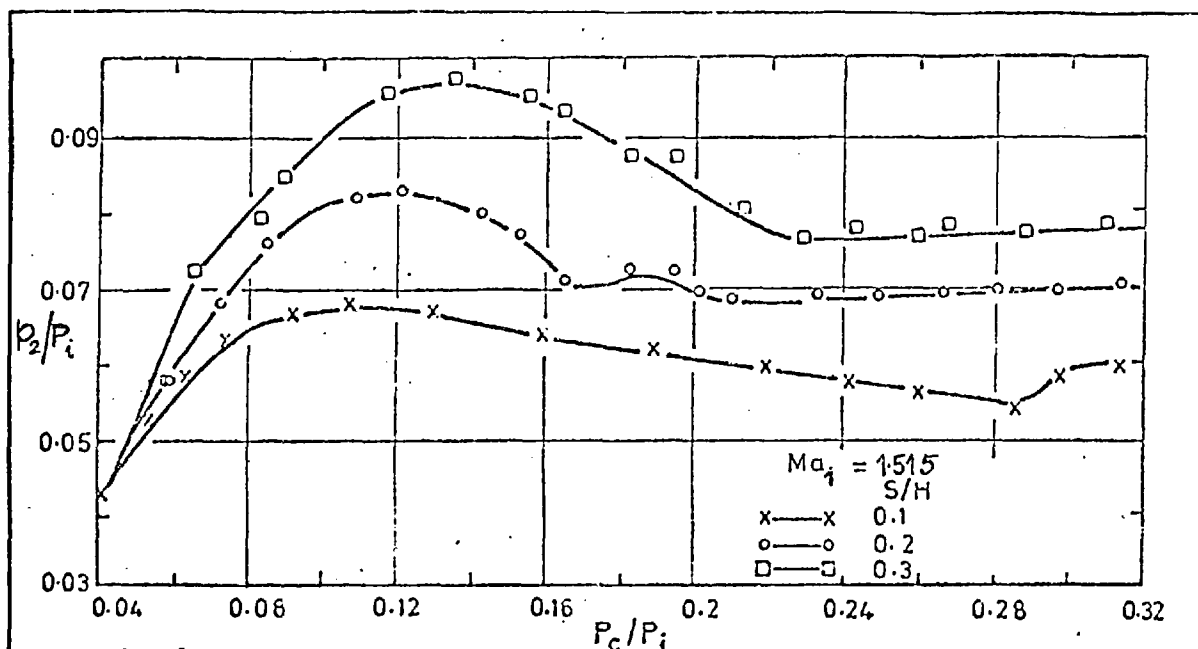


Fig. 7.3.3a Effect of secondary air pressure ratio on base pressure ratio of diffuser - ejector.

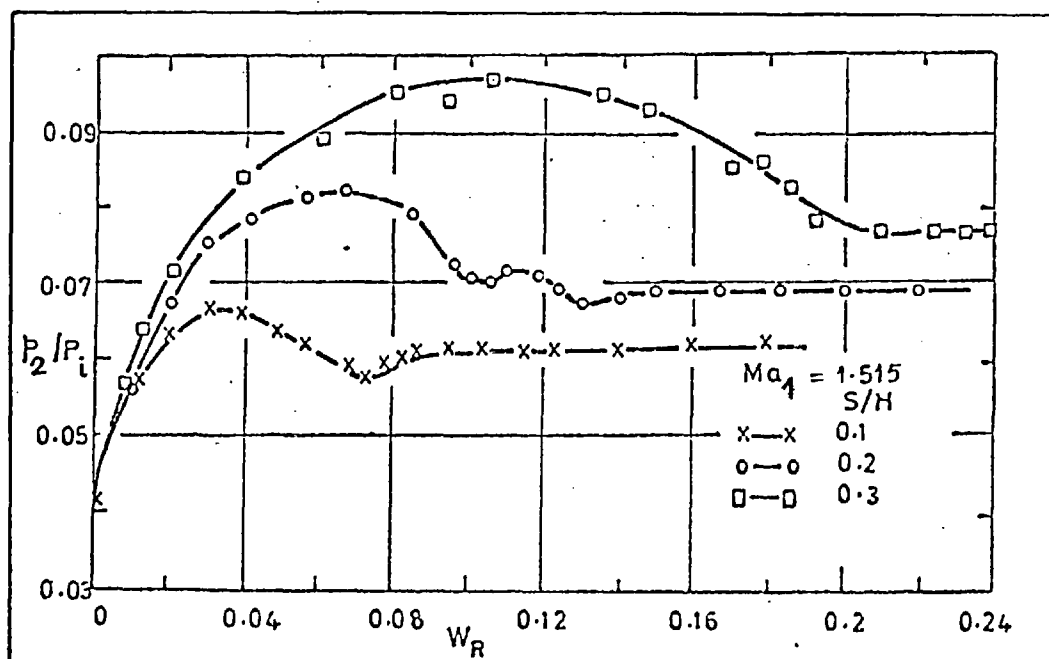
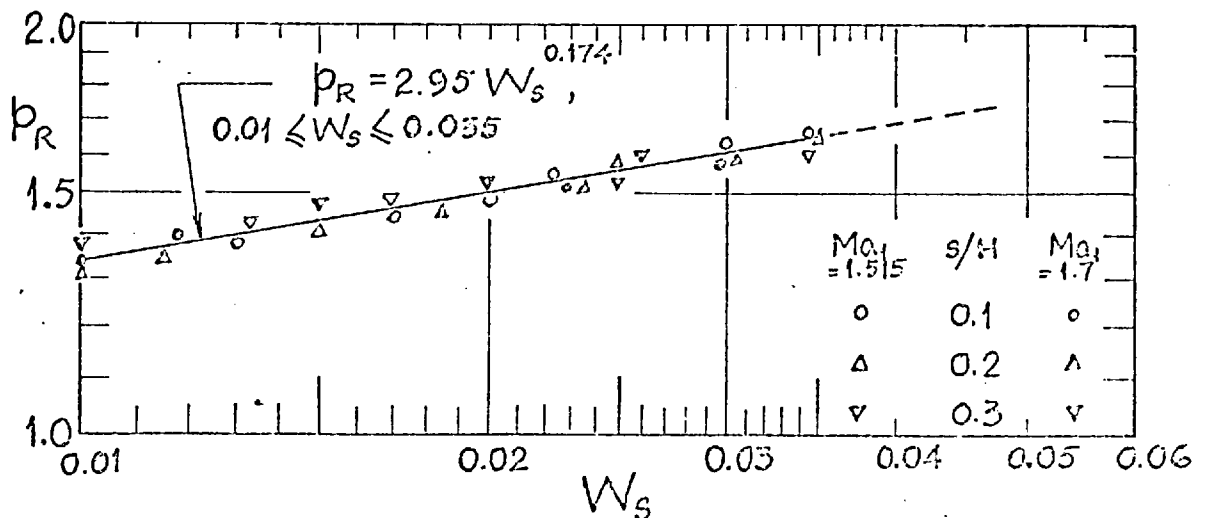
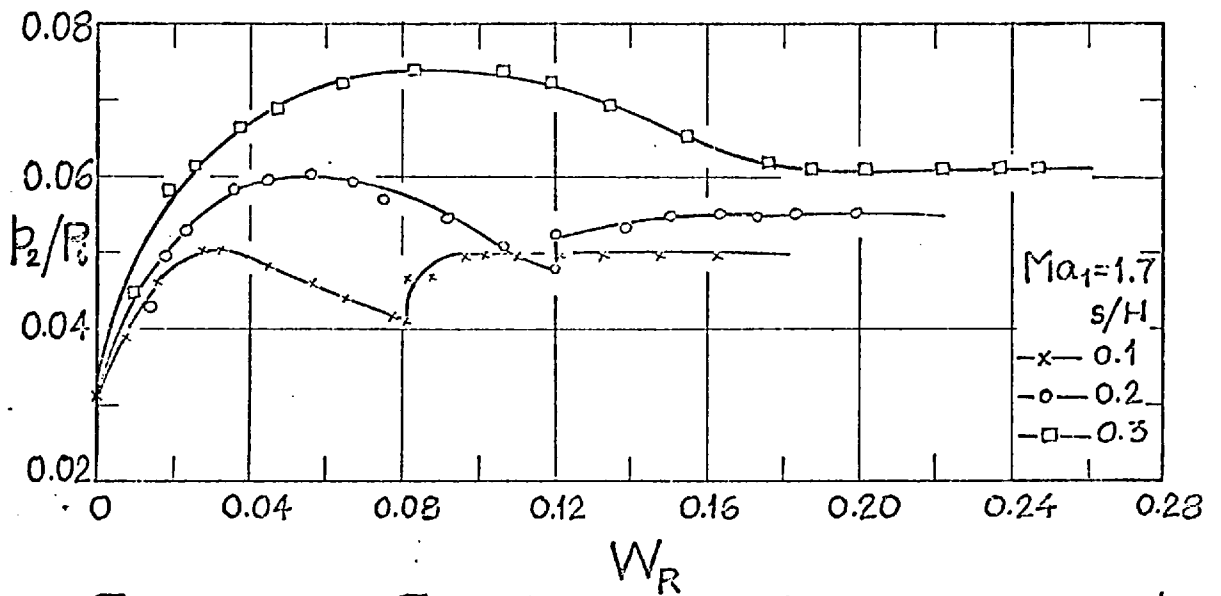
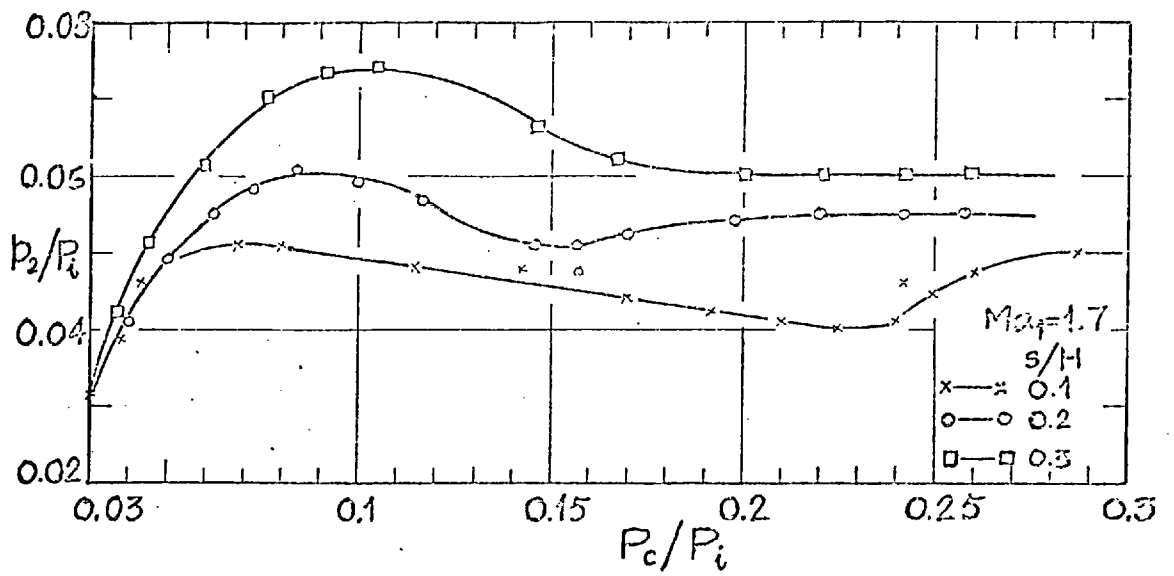


Fig. 7.3.3b Effect of secondary air mass ratio on base pressure ratio of diffuser - ejector.



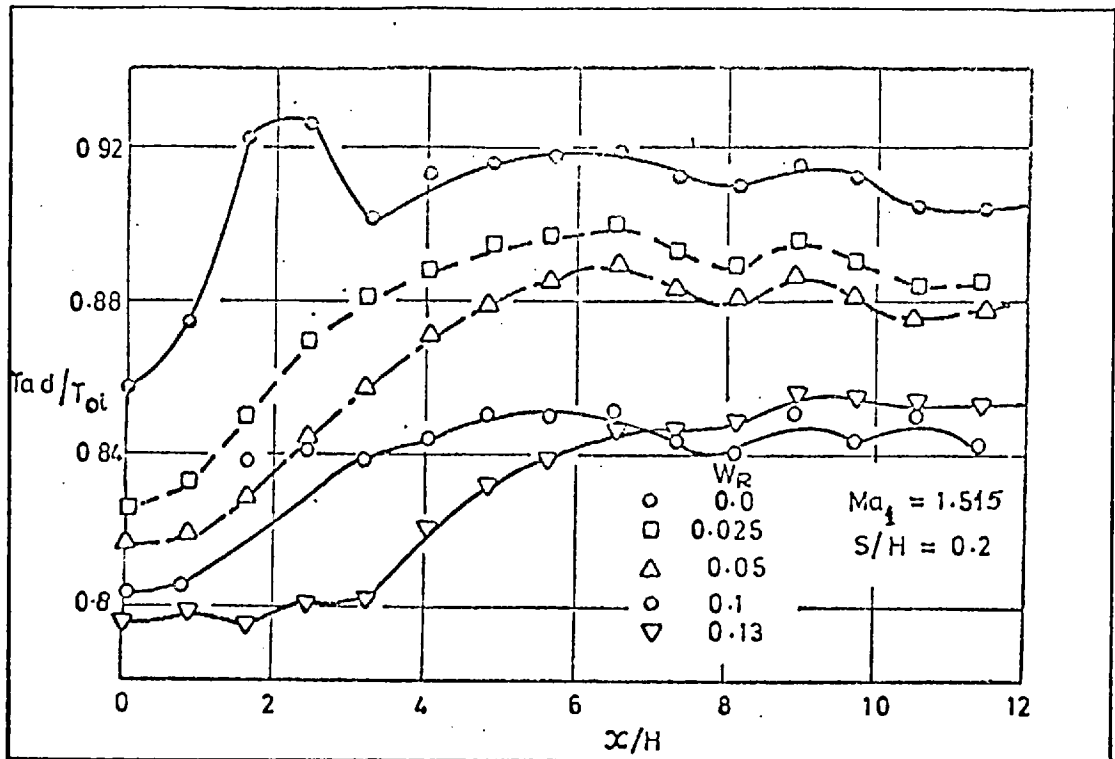


Fig. 7.3.6 Effect of secondary air mass ratio on adiabatic wall temperature distribution.

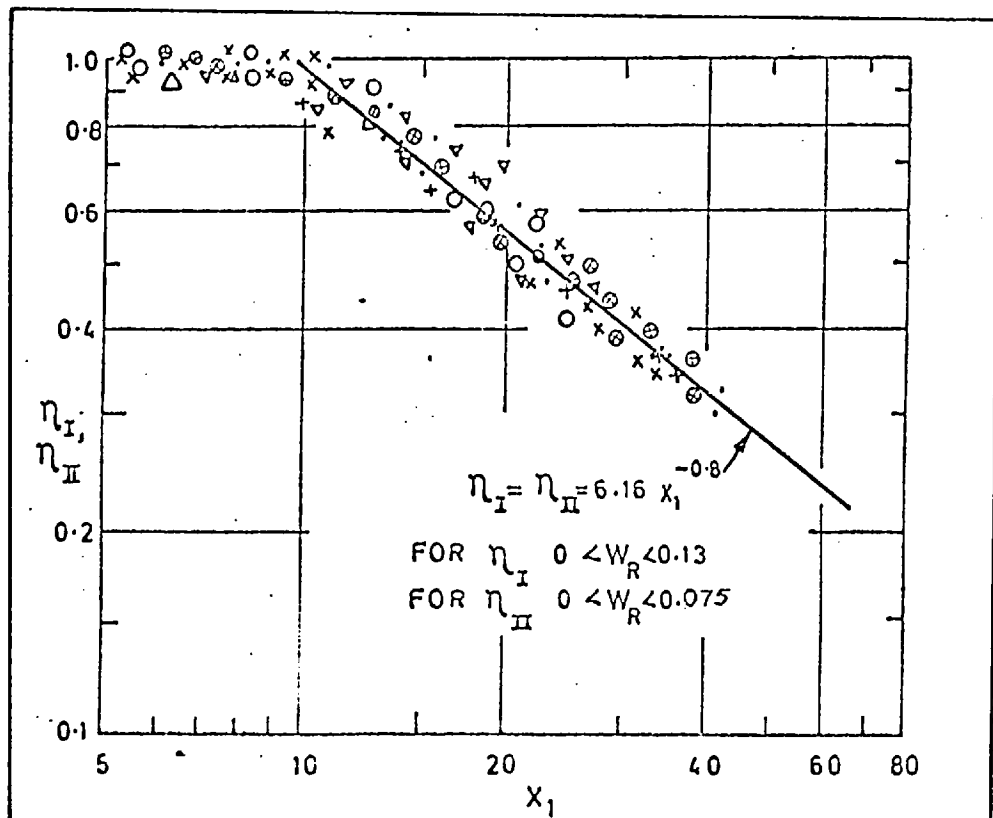


Fig. 7.3.7 Effectiveness correlation at low secondary air mass ratio.

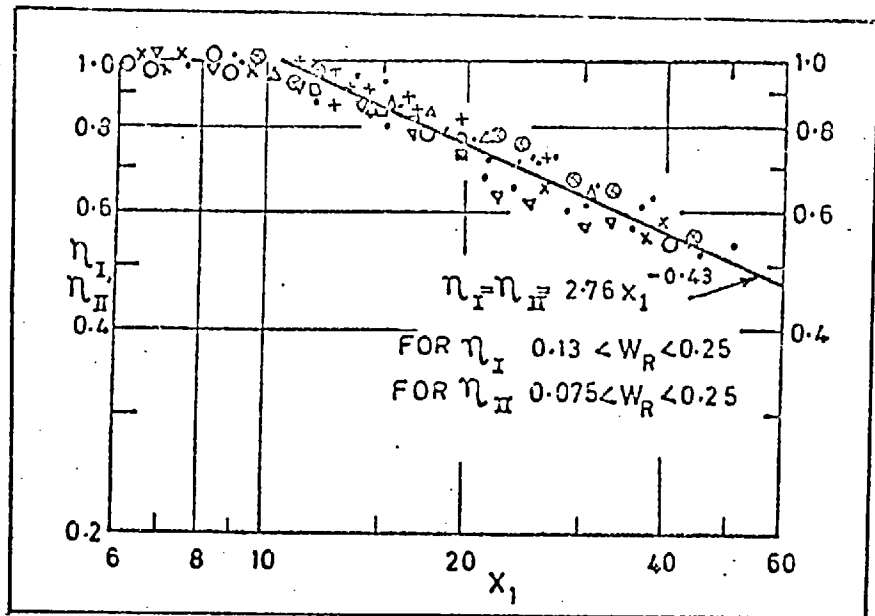


Fig. 7.3.8 Effectiveness Correlation at High Secondary Air Mass Ratios.

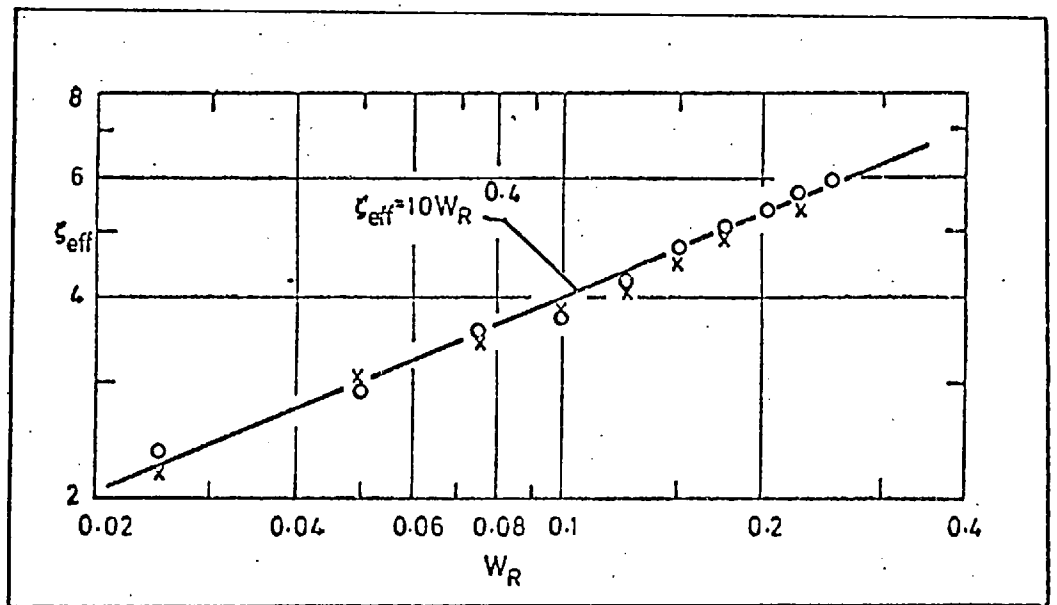


Fig. 7.3.9 Effect of Secondary Air Mass Ratio on Effectively-cooled Length of Diffuser - Ejector.

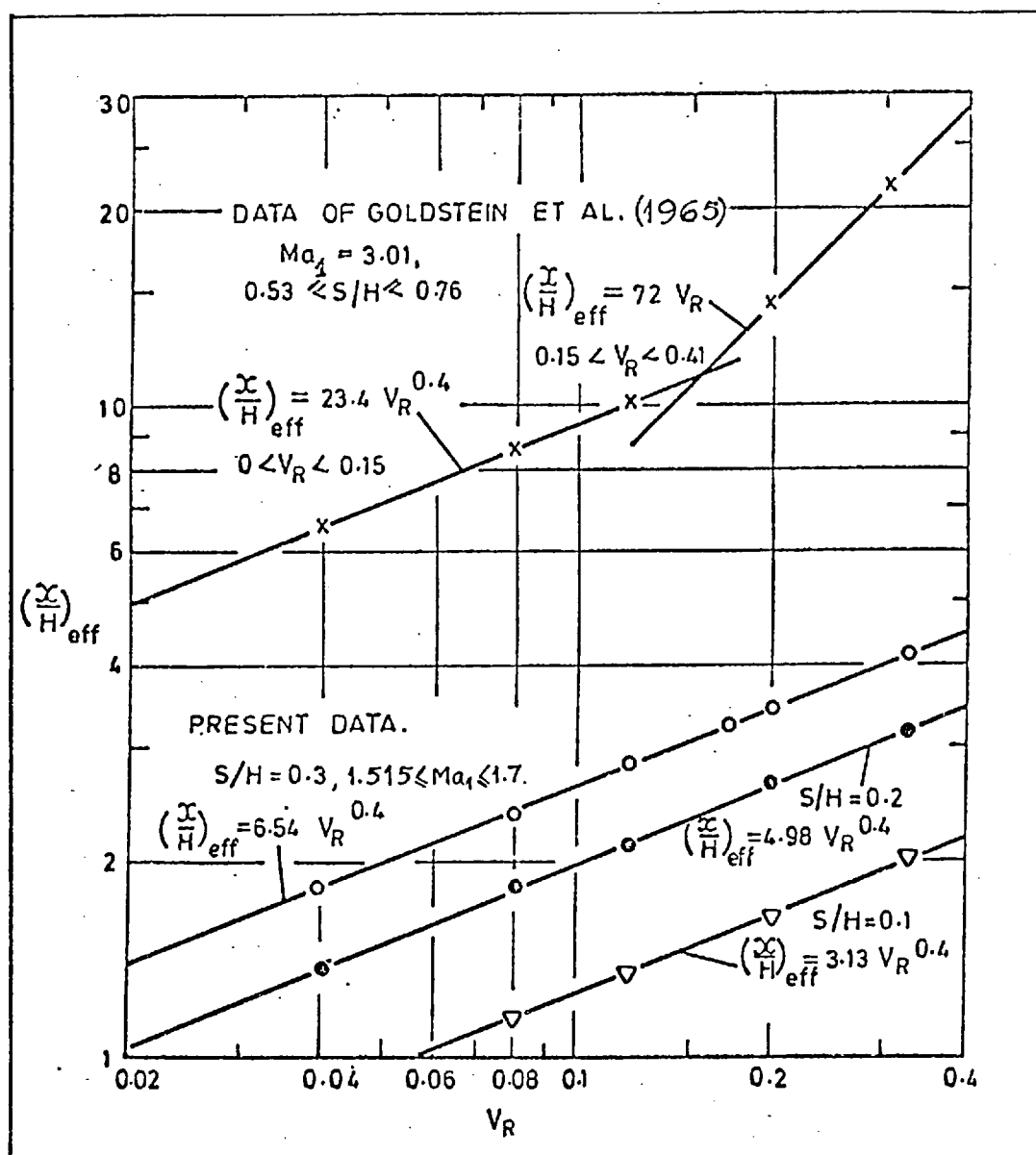


Fig. 7.3.10 Effect of Secondary Air Velocity Ratio on Effectively-cooled Length of Diffuser - Ejector.

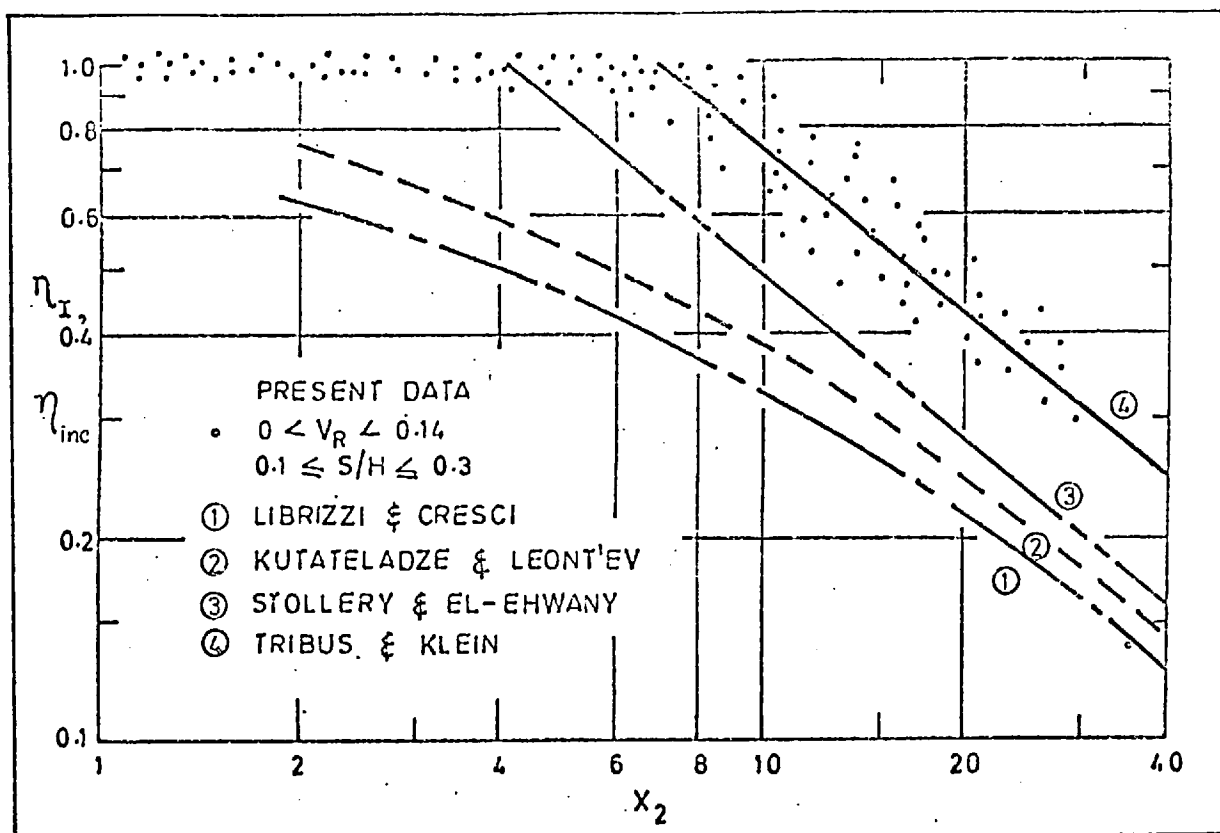


Fig. 7.3.11 Comparison of Present Data with Subsonic Correlations.

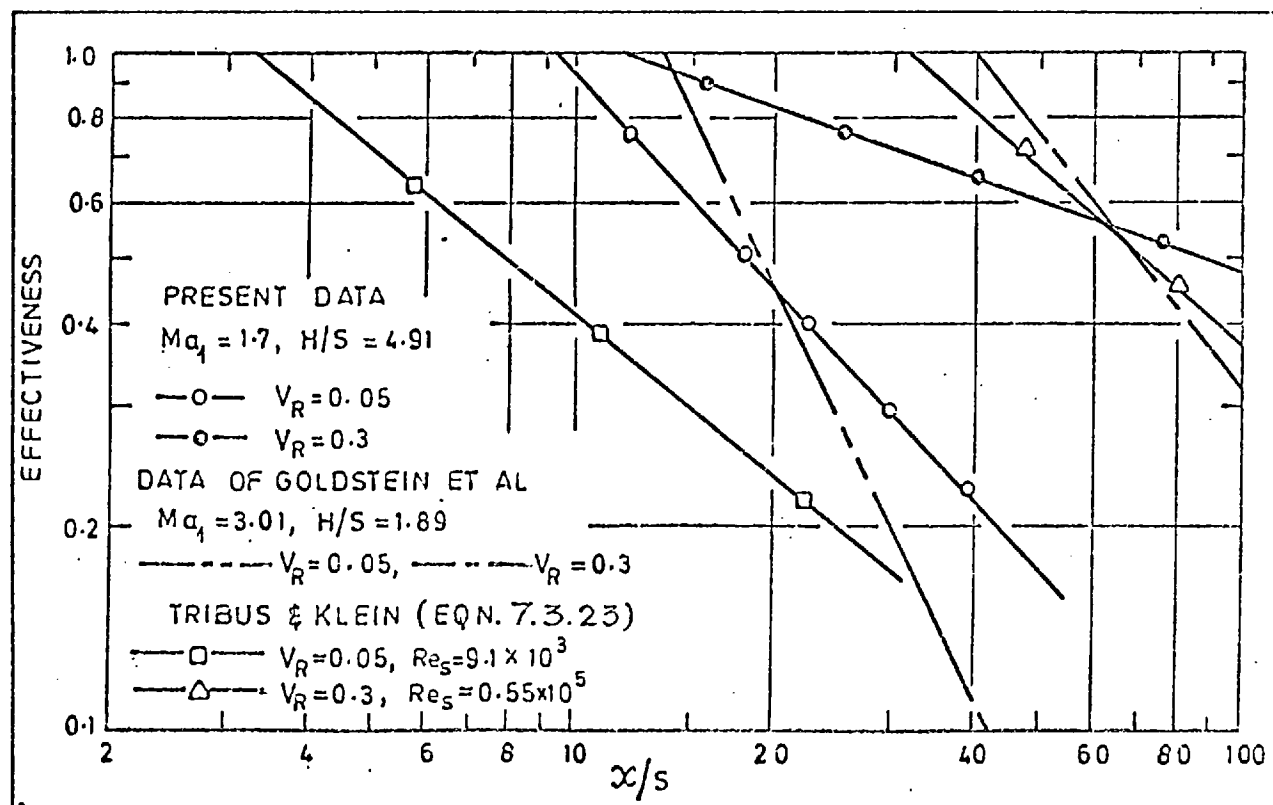


Fig. 7.3.12 Comparison of Present Data with Subsonic and Supersonic Correlations.

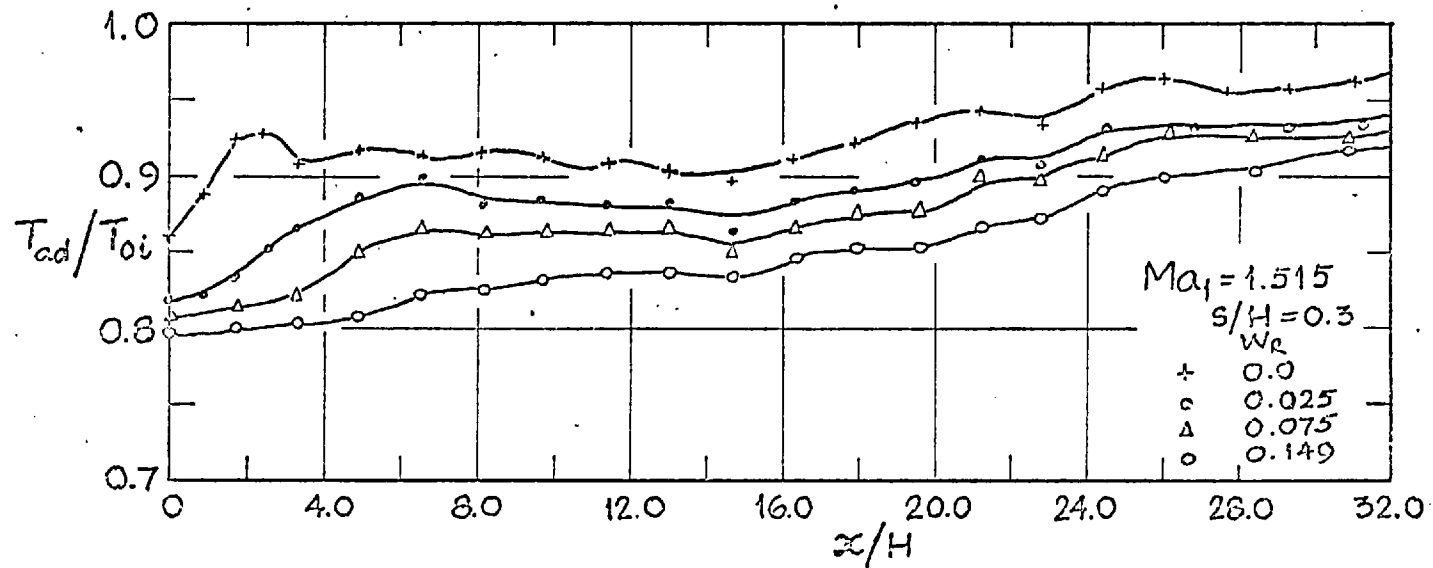


Fig. 7.3.13 Effect of secondary air mass ratio on adiabatic wall temperature along diffuser-ejector.

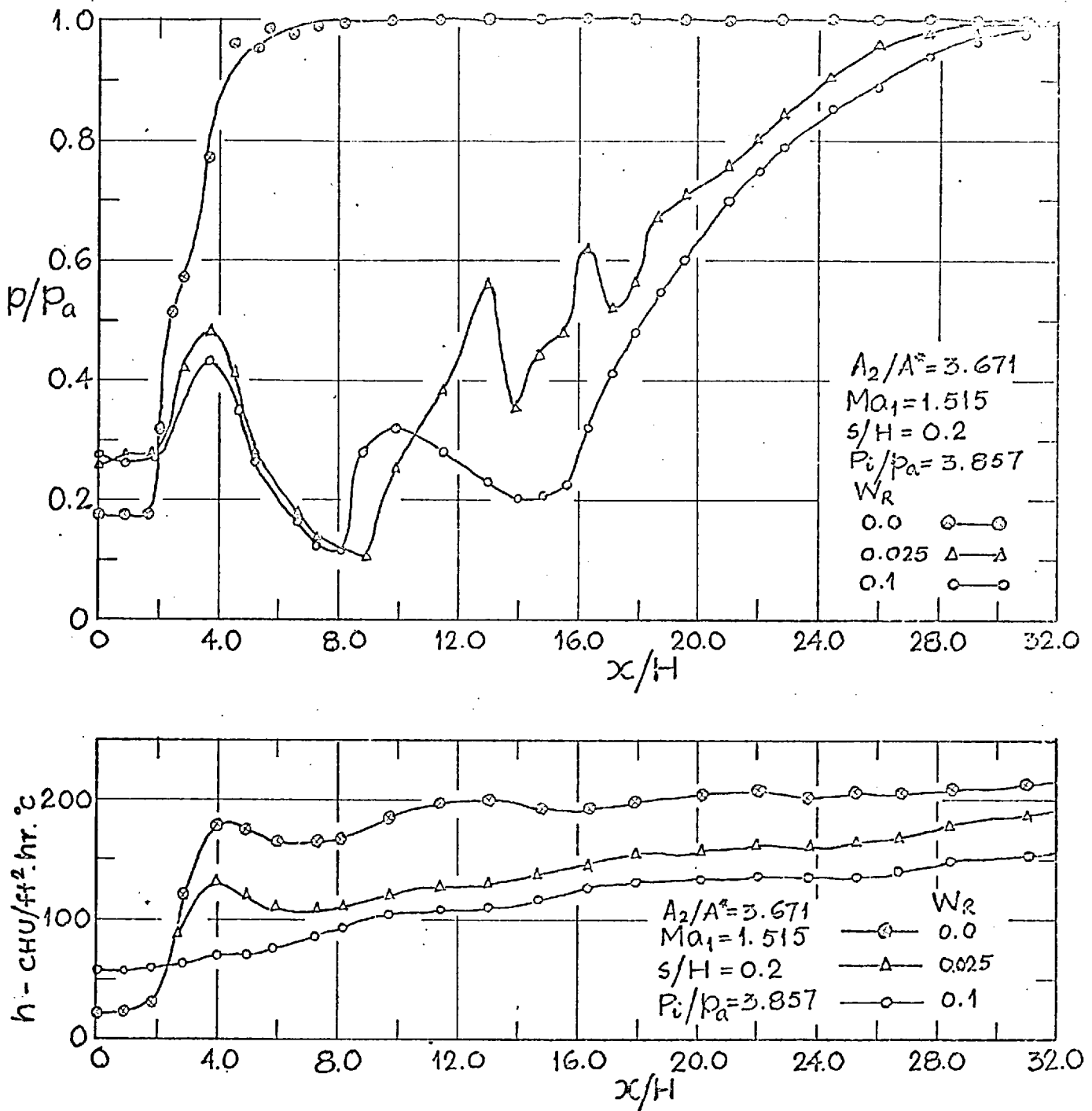


Fig.7.3.14 Effect of secondary air mass ratio on wall pressure and heat transfer coefficient distributions in diffuser-ejector (initially unstirred diffuser).

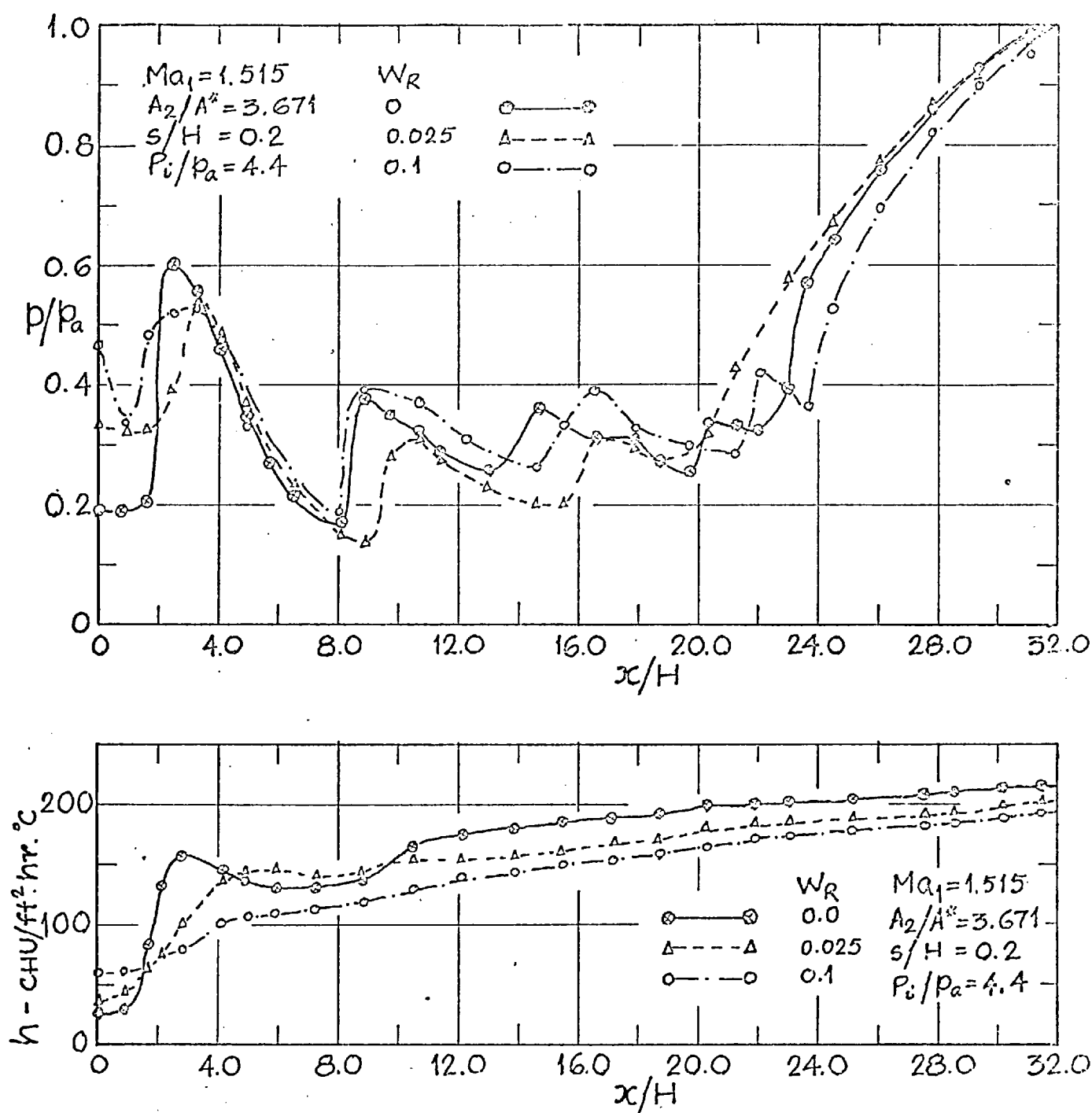


Fig. 7.3.15. Effect of secondary air mass ratio on wall pressure and heat transfer coefficient distributions in diffuser-ejector (initially started diffuser):

N O M E N C L A T U R E.

<u>Symbol</u>	<u>Meaning</u>	<u>Eqn. of first mention.</u>	<u>Units, if any.</u>
A	Flow area.	-	ft. ²
A _c	Area of secondary air slot.	-	ft. ²
A _{dis}	Area of dissipative flow region.	(3.4.9)	ft. ²
A _G	Area of isentropic core.	(3.4.9)	ft. ²
A ₁	Nozzle exit area.		ft. ²
A ₂	Diffuser flow area.	(3.4.9)	ft. ²
A*	Nozzle throat area.	(3.4.30)	ft. ²
a	A ₁ /A ₂ .	(2.2.2)	-
b	Pressure differential across secondary air orifice.	(A4.1)	in. w.g.
C	Integration constant.	(3.4.38)	-
Cr	Crocco number = $U/U_{\max}$.	(2.3.21)	-
Cr ₂	Crocco number of main stream at diffuser entry, $= U_2/U_{\max}$.	(2.3.22)	-
C _{sep}	Constant for separated flow region.	(3.2.2)	-
C _r	Pressure gradient correction factor.	(3.3.1)	-
c _p	Specific heat of fluid at constant pressure.	(3.2.2)	CHU/lb _m ^o C
c _v	Specific heat of fluid at constant volume.	-	CHU/lb _m ^o C
D	Diameter.	-	ft.

D_1	Nozzle exit diameter.	-	ft.
D_2	Diffuser diameter.	-	ft.
D^*	Nozzle throat diameter.	-	ft.
F_h	Form parameter.	(3.4.35)	-
G_1	Mass velocity in direction $1 = \rho U_1$.	(3.4.2)	$\frac{\text{lb}_m}{\text{ft}^2 \text{ sec.}}$
G_2	Mass velocity in direction $2 = \rho U_2$.	(3.4.2)	$\frac{\text{lb}_m}{\text{ft}^2 \text{ sec.}}$
g_0	Gravitational constant.	(3.4.9)	$\frac{\text{lb}_m \text{ ft.}}{\text{lb}_f \text{ sec}^2}$
H	Base height = $\frac{(D_2 - D_1)}{2}$.	-	ft.
$\mathcal{H}_1$	Shape parameter at nozzle exit = δ_1 / θ_1 .	(2.2.11)	-
h	Convective heat transfer coefficient.	-	$\text{CHU}/\text{ft}^2 \text{ h}^\circ \text{C}$
h_{lam}	Coefficient of heat transfer for laminar flow at separating edge.	-	$\text{CHU}/\text{ft}^2 \text{ h}^\circ \text{C}$
h_r	Heat transfer coefficient at shear layer reattachment.	(3.3.3)	$\text{CHU}/\text{ft}^2 \text{ h}^\circ \text{C}$
h_{sep}	Heat transfer coefficient in the separated region.	(3.2.2)	$\text{CHU}/\text{ft}^2 \text{ h}^\circ \text{C}$
h_x	Heat transfer coefficient at x.	(3.4.43)	$\text{CHU}/\text{ft}^2 \text{ h}^\circ \text{C}$
$\tilde{h}$	Stagnation enthalpy.	(3.4.4)	CHU/lb_m
h'	Enthalpy.	(3.4.4)	CHU/lb_m
h'_{dis}	Enthalpy of fluid in dissipative region.	(3.4.10)	CHU/lb_m
h'_G	Enthalpy of fluid in isentropic core.	(3.4.10)	CHU/lb_m

h'_2	Enthalpy of fluid at diffuser entry.	(3.4.10)	CHU/lb _m .
I_1	Integral.	(2.3.24)	-
I_2	Integral.	(2.3.30)	-
J	Mechanical equivalent of heat.	(3.4.10)	ft.lb _f /CHU
k	Mixing length constant.	(3.4.6)	-
k_m	Thermal conductivity at film temperature.	-	CHU/ft.h°C
L	Length of duct.	-	ft.
L_{ps}	Length of pseudo-shock.	(3.4.27)	ft.
L_2	Diffuser length.	-	ft.
l	Flat plate length.	-	ft.
l_m	Mixing length.	(3.4.6)	ft.
l_n	Nozzle divergence length.	-	ft.
l_1	Length scale in direction 1	(3.4.1)	-
l_2	Length scale in direction 2	(3.4.1)	-
Ma	Mach number.	(2.2.1)	-
Ma_a	Jet Mach number at p_a .	-	-
Ma_{dis}	Flow Mach number in dissi- pative region.	(3.4.16)	-
Ma_G	Mach number of the flow in isentropic core.	(3.4.15)	-
Ma_r	Mach number of main stream at reattachment.	(6.1.2)	-
Ma_1	Actual Mach number at nozzle exit.	(2.2.10)	-
Ma_2	Mach number of free stream at diffuser entry.	(2.3.1)	-

Ma_3	Mach number downstream of reattachment.	(6.1.2)	-
Ma_4	Mach number at diffuser exit.	(2.2.12)	-
m_f	Mass fraction in dissipative region = $\dot{m}_{dis}/\dot{m}_2$.	(3.4.13)	-
m_r	Exponent of reattachment pressure ratio.	(3.3.1)	-
m_{sep}	Exponent of Re_{H2} .	(3.2.2)	-
$\dot{m}_b$	Equivalent bleed mass flux.	(2.3.36)	$lb_m/ft^2 \cdot sec.$
$\dot{m}_c$	Secondary air flow rate $= \rho_c \cdot U_c \cdot A_c$.	(A4.2)	$lb_m/sec.$
$\dot{m}_{dis}$	Mass flow rate in dissipative region.	(3.4.8)	$lb_m/sec.$
$\dot{m}_G$	Mass flow rate in isentropic core.	(3.4.8)	$lb_m/sec.$
$\dot{m}_1$	Mass flow rate at nozzle exit = $\rho_1 \cdot U_1 \cdot A_1$.	-	$lb_m/sec.$
$\dot{m}_2$	Mass flow rate at diffuser entry.	(3.4.8)	$lb_m/sec.$
N_{Ma}	Reattachment criterion due to Roberts(1964).	(6.1.2)	-
N_p	Reattachment criterion due to Nash (1962).	(6.1.1)	-
Nu_D	Nusselt number - $h_x D_2 / k_m$.	(7.2.11)	-
n	Turbulent velocity profile power-law parameter defining Ma and P distributions.	-	-

P	Stagnation pressure.	(A3.1)	lb_f/ft^2
P_c	Stagnation pressure in secondary air box.	-	lb_f/ft^2
P_i	Stagnation pressure at nozzle inlet.	(2.2.4)	lb_f/ft^2
Pr	Prandtl number - $c_p \mu_m / k_m$.	(7.2.8)	-
p	Static pressure.	(3.4.9)	lb_f/ft^2
p_a	Ambient atmospheric pressure.	(2.2.14)	lb_f/ft^2
p_J	Static pressure at J	-	lb_f/ft^2
P_R	Pressure ratio.	(7.3.1)	-
p_r	Wall pressure at dividing streamline reattachment.	(6.4.1)	lb_f/ft^2
p_1	Nozzle exit pressure.	(2.2.1)	lb_f/ft^2
p_2	Base or diffuser entry pressure.	(2.2.1)	lb_f/ft^2
p_3	Wall pressure downstream of reattachment.	(6.1.1)	lb_f/ft^2
p_4	Diffuser exit pressure.	(2.2.1)	lb_f/ft^2
$\dot{Q}$	Volumetric air flow rate.	(A4.1)	$\text{ft}^3/\text{sec.}$
$\dot{q}''$	Heat transfer rate.	(5.1)	$\text{CHU}/\text{ft}^2\text{h}$
R	Radial length.	(A1.3)	ft.
$\mathcal{R}$	Gas constant.	(3.4.5)	$\text{lb}_f\text{ft.}/\text{lb}_m^\circ\text{K}$
R_c	Reattachment criterion.	(6.2.2)	-
Re_D	$U_G D_2 / \nu_m$.	(7.2.11)	-
Re_{H1}	$\rho_1 U_1 H / \mu_1$.	(7.3.29)	-
Re_{H2}	$\rho_2 U_2 H / \mu_2$.	(3.2.1)	-
Re_{h2}	$\rho_G U_G \Delta / \mu_o$.	(3.4.33)	-
Re_L	$\rho_G U_G L_{ps} / \mu_o$.	(3.4.33)	-

Re_1	$\rho_1 U_1 l / \mu_1$	-	-
Re_{nl}	$\rho_1 U_1 l_n / \mu_1$	-	-
Re_s	$(\frac{SU}{\nu_c})(\frac{\mu_c}{\mu_1})$	(7.3.23)	-
Re_x	$\rho_G U_G x / \mu_0$	(7.2.10)	-
Re^*	$\rho^* U^* D^* / \mu^*$	-	-
Re_1'	$\rho_1 U_1 / \mu_1$	-	in. ⁻¹
r	Radius.	(2.2.2)	ft.
r^*	Nozzle throat radius.	(2.2.14)	ft.
r_1	Nozzle exit radius.	(2.2.1)	ft.
r_4	Radius at diffuser exit = r_2 .	(2.2.1)	ft.
S	Parameter related to slot width = $(1.0 - \frac{s}{H})^{-1}$	(7.3.7)	-
St	Stanton number = $\frac{h_x}{\rho_G U_G c_p}$	(3.4.33)	-
St_r	$h_r / \rho_2 U_2 c_p$	(3.3.3)	-
St_{sep}	$h_{sep} / \rho_2 U_2 c_p$	(3.2.1)	-
s	Secondary air slot width.	(7.3.2)	ft.
T	Absolute temperature.	-	°K
T_{ad}	Adiabatic wall temperature.	(5.1)	°K
T_{dis}	Temperature in dissipative region.	(3.4.16)	°K
T_G	Isentropic core stream temperature.	(3.4.15)	°K
T_w	Wall temperature.	(5.1)	°K
T_2	Stream temperature at diffuser entry.	(3.4.17)	°K
T_0	Stagnation temperature.	-	°K
T_{odis}	Stagnation temperature in dissipative region.	(3.4.11)	°K

T_{oc}	Secondary air stagnation temperature.	-	$^{\circ}K$
T_{oG}	Stagnation temperature in isentropic core.	(3.4.11)	$^{\circ}K$
T_{oi}	Stagnation temperature at nozzle inlet.	-	$^{\circ}K$
T_{o2}	Stagnation temperature at diffuser entry.	(3.4.11)	$^{\circ}K$
T_{o4}	Stagnation temperature at diffuser exit.	(3.4.11)	$^{\circ}K$
U	Flow velocity along x.	(2.2.5)	ft./sec.
U_c	Velocity at the slot.	-	ft./sec.
U_{dis}	Velocity in dissipative region.	(3.4.9)	ft./sec.
U_G	Velocity in isentropic core.	(3.4.9)	ft./sec.
U_m	Median streamline velocity.	-	ft./sec.
U_{max}	Flow velocity at $T = 0$.	-	ft./sec.
U_1	Nozzle exit velocity.	-	ft./sec.
U_2	Flow velocity at diffuser entry.	(2.3.10)	ft./sec.
U_{∞}	Free stream velocity.	(2.2.5)	ft./sec.
U'	Transformed velocity in two-dimensional plane along x.	(2.3.13)	ft./sec.
U'_d	Transformed dividing-streamline velocity.	-	ft./sec.
U'_m	Transformed median-streamline velocity.	-	ft./sec.
U'_2	Transformed velocity at diffuser entry.	-	ft./sec.

U'_{∞}	Transformed free stream velocity. (2.3.17)	ft./sec.
U^*	Throat velocity of choked nozzle. (3.4.30)	ft./sec.
V	Transverse velocity in y direction. (2.3.16)	ft./sec.
V'	Transformed velocity in y direction. (2.3.16)	ft./sec.
V_R	Secondary air mass velocity ratio = $\rho_c U_c / \rho_1 U_1$. (7.3.21)	-
v_1	Flow velocity in direction 1 -	ft./sec.
v_2	Flow velocity in direction 2 -	ft./sec.
W_R	Secondary air mass ratio = $\dot{m}_c / \dot{m}_1$ (7.3.3)	-
W_S	Mass ratio parameter. (7.3.1)	-
X	Distance x/L_{ps} from the section where $\Delta_2 = 0$. (3.4.33)	-
X_1	Length parameter = $\xi W_R^{-0.4}$ (7.3.14)	-
X_2	Length parameter = $(x/V_R s) \cdot Re_s^{-0.25}$ (7.3.27)	-
x	Distance from diffuser entry plane or the slot; also abscissa. (7.3.15)	ft.
x_{dis}	Length of dissipative region. (3.4.27)	ft.
x'	Transformed abscissa. (2.3.5)	ft.
x'_1	Transformed virtual origin shift. (2.3.26)	ft.
x'_2	Transformed shear layer development length. (2.3.26)	ft.

x'_{tot}	$x'_1 + x'_2$	(2.3.26)	ft.
y	Transverse distance, also co-ordinate.	-	ft.
y_d	Ordinate of dividing streamline.	(2.3.11)	ft.
y_l	Characteristic thickness of layer used to calculate mixing length.	(3.4.6)	ft.
y_m	Ordinate of the median streamline.	(2.3.10)	ft.
y'	Transformed ordinate.	(2.3.6)	ft.
Z	Secondary orifice to pipe flow area ratio.	-	-
α	Flow expansion angle at nozzle exit.	(A1.1)	Radians.
β	Flow turning angle.	(A1.2)	Radians.
β_j	Nozzle half-divergence angle.	(A1.1)	Radians.
γ	c_p/c_v .	(2.2.1)	-
Δ_2	Enthalpy-deficit thick- ness of boundary layer.	(3.4.35)	ft.
Δx	$(x_{n+1} - x_n)$.	(A1.7)	ft.
ΔT	$(T_{ad} - T_w)$.	(3.4.33)	$^{\circ}K$
δ_{sh}	Axisymmetric shear-layer thickness.	(2.3.8)	ft.
δ'_{sh}	Transformed shear-layer ^{thickness} in two-dimensional plane.	(2.3.8)	ft.
δ^*	Velocity boundary layer displacement thickness.	(2.2.5)	ft.
δ_1^*	Boundary layer displacement thickness at nozzle exit.	(2.2.10)	ft.

ϵ	Eddy viscosity.	(2.3.8)	lb _m /ft.h
ξ	Length parameter $= (x/H)(H/s)^{0.263}$	(7.3.14)	-
ξ_h	A constant = St.Re _{h2} ^{-0.25}	(3.4.36)	-
η	$\sigma y/x$.	-	-
η'	Transformed η .	-	-
η'_d	η' corresponding to dividing streamline.	(2.3.23)	-
η'_m	η' corresponding to median streamline.	(2.3.23)	-
η_{inc}	Film-cooling effectiveness for incompressible flow.	(7.3.12)	-
η_I	Film-cooling effectiveness for compressible flow.	(7.3.10)	-
η_{II}	Film-cooling effectiveness for compressible flow.	(7.3.13)	-
η_{III}	Film-cooling effectiveness for compressible flow.	(7.3.11)	-
θ	Velocity boundary layer momentum thickness.	(2.2.6)	ft.
θ_{sh}	Shear layer momentum thickness.	(2.3.27)	ft.
θ_1	Boundary layer momentum thickness at nozzle exit.	(2.2.10)	ft.
θ_2	Boundary layer momentum thickness at nozzle edge after expansion.	(2.3.1)	ft.
θ_4	Boundary layer momentum thickness at diffuser exit.	-	ft.
λ	Impulse parameter.	(2.2.2)	-

λ_1	λ at nozzle exit.	(2.2.3)	-
λ_4	λ at diffuser exit.	(2.2.3)	-
λ_G	A. mixing length constant.	(3.4.6)	-
λ_p	Pressure ratio.	(7.3.7)	-
μ_c	Viscosity of secondary air.	-	$lb_m/ft.h$
μ_m	Viscosity of fluid at film temperature.	-	$lb_m/ft.h$
μ^*	Viscosity of fluid at nozzle throat.	-	$lb_m/ft.h$
μ_o	Viscosity of stream at stagnation temperature.	-	$lb_m/ft.h$
μ_1	Viscosity of free stream at nozzle exit.	(7.3.29)	$lb_m/ft.h$
$\mu_{1,eff}$	Effective viscosity of flow in direction 1.	(3.4.3)	$lb_m/ft.h$
μ_2	Viscosity of free stream at diffuser entry.	(3.2.2)	$lb_m/ft.h$
ν	Prandtl - Meyer angle.	(A1.1)	Radians.
ν_c	Kinematic viscosity of secondary air at slot.	-	$ft.^2/h$
ν_m	Kinematic viscosity of fluid at film temperature.	-	$ft.^2/h$
ξ_1	Co-ordinate length in direction 1.	(3.4.1)	ft.
ξ_2	Co-ordinate length in direction 2.	(3.4.1)	ft.
ρ	Fluid density.	(2.2.5)	$lb_m/ft.^3$
ρ_c	Secondary air density at the slot.	-	$lb_m/ft.^3$

ρ_{dis}	Density of fluid in dissipative region.	(3.4.9)	lb _m /ft. ³
ρ_G	Density of fluid in isentropic core.	(3.4.9)	lb _m /ft. ³
ρ_1	Fluid density at nozzle exit.	(2.3.1)	lb _m /ft. ³
ρ_2	Fluid density at diffuser entry.	(2.3.1)	lb _m /ft. ³
ρ_∞	Free stream density.	(2.2.5)	lb _m /ft. ³
ρ^*	Fluid density at nozzle throat.	(3.4.30)	lb _m /ft. ³
σ	Compressible jet spread parameter.	(2.3.2)	-
σ_{inc}	Incompressible jet spread parameter.	-	-
σ_2	Compressible jet spread parameter at Ma ₂ .	(2.3.23)	-
ϕ	Ratio of local velocity in shear layer to free stream velocity - U/U_∞ , also U'/U'_2 .	(2.3.2)	-
ϕ_d	Dividing streamline velocity ratio = U'_d/U'_2 .	(2.3.39)	-
ϕ_m	Median streamline velocity ratio = U'_m/U'_2 .	(2.3.18)	-
χ	Slope of shear layer velocity profile.	(2.3.39)	-
ψ	Stream function	(2.3.7)	ft. ² /h
ψ'	Transformed ψ in two- dimensional plane.	(2.3.7)	ft. ² /h
ω	Angle.	(A1.3)	Radian.

Subscripts.

<u>Symbol</u>	<u>State or condition referred to.</u>
a	Ambient atmosphere.
dif	Adiabatic diffuser.
e	Adiabatic ejector.
eff	Effective.
iso	Isoenergetic, $T_{oc} = T_{oi}$.
j	Fluid jet.
max	Maximum.
min	Minimum.
n	At point n.
od	Obtained by orifice-dam technique.
opt	Optimum.
pl	Plateau value corresponding to ⑤ in Fig. 7.3.1.
pln	Plane.
r	Reattachment.
sf	Obtained by surface-flow technique.
sp	Spherical.
start	Diffuser starting condition.
x	At x; also co-ordinate direction.
y	At y; also co-ordinate direction.
2D	Two-dimensional geometry.
O	$x = 0$.

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R E F E R E N C E S.

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A P P E N D I X 1Latvala's quasi-theoretical procedure to calculate
supersonic jet boundary.A1.1 Introduction.

When a supersonic nozzle exhausts into a low pressure region, the jet expands until the pressure along the jet boundary is the same as the ambient pressure. To satisfy the condition of constant pressure along the jet boundary, expansion waves originating within the jet are reflected as compression waves. These reflected waves coalesce into a shock near the jet boundary. There is a mixing region along the boundary, whose thickness increases with distance from the nozzle exit. The ambient pressure and jet Mach number also influence the characteristics of the mixing region. However, close to the nozzle exit the mixing region is relatively thin compared to the overall jet diameter and, hence, for practical purposes can be ignored.

Numerous theoretical studies of the spreading of underexpanded supersonic jets have been reported in the literature. Since the jet is supersonic, the method of characteristics has been used in most cases to calculate the jet structure (Love et al.(1959), Wang and Peterson (1958), Vick et al.(1964)). However, the calculations are lengthy, requiring the use of electronic computers, and the results are available for specific cases only.

Attempts have also been made to develop a more convenient and quicker method to estimate jet boundaries from underexpanded supersonic nozzles exhausting into still air. A modified version of the approximate method due to Adamson and Nicholls (1959) has been reported by Latvala (1959) which estimates the jet boundary quite satisfactorily. In the following this method is described.

A1.2 Latvala's approximate analysis.

Consider the flow at the lip of an underexpanded convergent-divergent nozzle (Fig.A1.1) of half-divergence angle β_j . The nozzle exit Mach number is Ma_j and the corresponding Prandtl - Meyer angle is ν_j . After expanding to atmospheric pressure p_a , the Mach number is Ma_a with a corresponding Prandtl - Meyer angle of ν_a . The overall flow expansion angle with respect to the flow axis is given by

$$\alpha_a = \nu_a - \nu_j + \beta_j \quad (A1.1)$$

The flow is turned from this initial expansion angle by the intersection of expansion waves with the boundary, reflected as compression waves, so that the condition of constant pressure on the boundary is satisfied at every point. It is now assumed that the effects of expansion waves intersecting the flow near the boundary can be replaced by the effects of a quasi-one-dimensional area increase. This means that, just as in one-dimensional nozzle flow, the average Mach number and pressure at any axial position in an expanding flow can be found

from the area ratio at the given point, instead of going through a characteristic calculation.

It therefore follows that at any point on the jet boundary, the turning angle, β , with respect to the initial expansion angle α_a , can be expressed as the difference between the Prandtl - Meyer angle associated with the Mach number corresponding to the area ratio A/A^* at that point and the Prandtl - Meyer angle associated with the Mach number at the nozzle lip Ma_a (Fig. A1.2). With β known, the boundary can be calculated in terms of r/r_j and x/r_j from the following:

$$d(x/r_j) = d(r/r_j)/\tan(\alpha_a - \beta) \quad (A1.2)$$

Assuming radial flow to exist in the jet (see Fig. A1.3), the following expressions for the spherical and plane areas at any point 'n' can be written:

$$A_{sp} = \int_0^{\alpha_n} 2\pi R_{sp}^2 \sin\varpi d\varpi = 2\pi R_{sp}^2 (1 - \cos\alpha_n); \quad (A1.3)$$

$$A_{pln} = \pi R_{sp}^2 \sin^2\alpha_n, \quad (A1.4)$$

$$\text{or } A_{pln} = A_{sp} \left(\frac{1 + \cos\alpha_n}{2} \right). \quad (A1.5)$$

Now, if the area ratio which corresponds to the Prandtl - Meyer angle at a particular point is assumed to be related to the spherical area A_{sp} , then the radius ratio, r_{n+1}/r_n , between two adjacent points, n and n+1 (Fig. A1.4) is given by

$$\frac{r_{n+1}}{r_n} = \sqrt{\frac{1 + \cos\alpha_{n+1}}{1 + \cos\alpha_n} \cdot \frac{(A/A^*)_{n+1}}{(A/A^*)_n}}, \quad (A1.6)$$

where $(A/A^*)_n$ and $(A/A^*)_{n+1}$ are the area ratios corresponding to the Prandtl - Meyer angles, $\mathcal{V}_n$ and $\mathcal{V}_{n+1}$ respectively, and α_n and α_{n+1} are the angles between the tangent to the boundary and the jet axis. If small intervals of the boundary turning angle, $\Delta\alpha = \alpha_n - \alpha_{n+1}$, are taken then equation (A1.2) representing jet boundary can be written in the following form:

$$\frac{\Delta x}{r_n} = \left(\frac{r_{n+1}}{r_n} - 1 \right) \cot(\alpha_n - \frac{\Delta\alpha}{2}) \quad (A1.7)$$

A1.3 Calculation procedure.

Given data: Nozzle exit area ratio, $(A/A^*)_j$, and nozzle half-divergence angle, β_j .

- (i) From isentropic flow tables find Ma_j and $\mathcal{V}_j$.
- (ii) For the given value of p_a/P_i determine Ma_a and $\mathcal{V}_a$ from isentropic flow tables.
- (iii) For the first point i.e. $n = 1$, determine $(A/A^*)_1$ from isentropic flow tables corresponding to Ma_a .
- (iv) Determine α_1 for the first point from equation (A1.1).
- (v) Choose $\Delta\alpha$; a value of 2 degrees, or 1 degree for higher accuracy, may be chosen.
- (vi) Find α_2 which is now equal to $(\alpha_1 - \Delta\alpha)$.
- (vii) Find $\mathcal{V}_2$ which is equal to $(\mathcal{V}_1 + \Delta\alpha)$ and hence determine $(A/A^*)_2$ from isentropic flow tables.
- (viii) Calculate $(r/r_j)_1$ and $(x/r_j)_1$ from equations (A1.6) and (A1.7).

Repeat the above procedure for each point along the jet boundary using the previously calculated data for the new starting point.

In Fig. A1.5 a plot of r/r_j and x/r_j obtained by the method of characteristics is shown for $Ma_1 = 1.5$, $\beta_j = 0$ and $\gamma = 1.4$ for a pressure ratio of $p_a/P_i = 0.04$. For comparison the solution obtained by the method due to Latvala is also shown. It can be seen that agreement between the two solutions is improved if a fictitious nozzle divergence angle of $\beta_j = 3.5$ degrees is assumed in Latvala's solution. In the Mach number range of the present investigation, $1.49 \leq Ma_j \leq 1.95$, the jet boundaries were therefore calculated after making this empirical correction of 3.5 degrees to the actual nozzle half-divergence angle.

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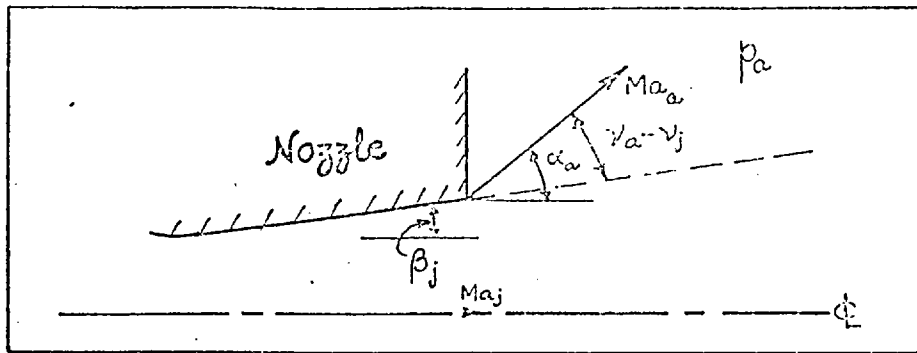


Fig. A1.1. Total Angle Turned Through By The Flow At The Nozzle Lip.

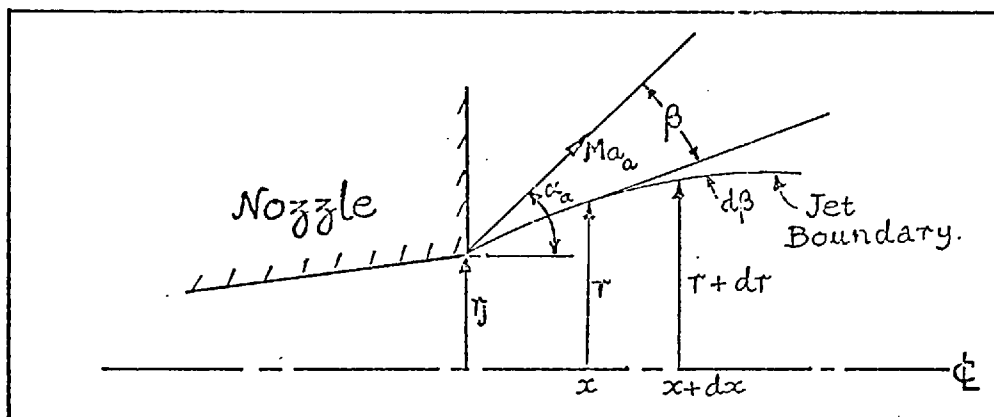


Fig. A.1.2. Incremental Angle And Radius Changes In Jet Boundary Due To An Incremental Increase In Axial Coordinate x .

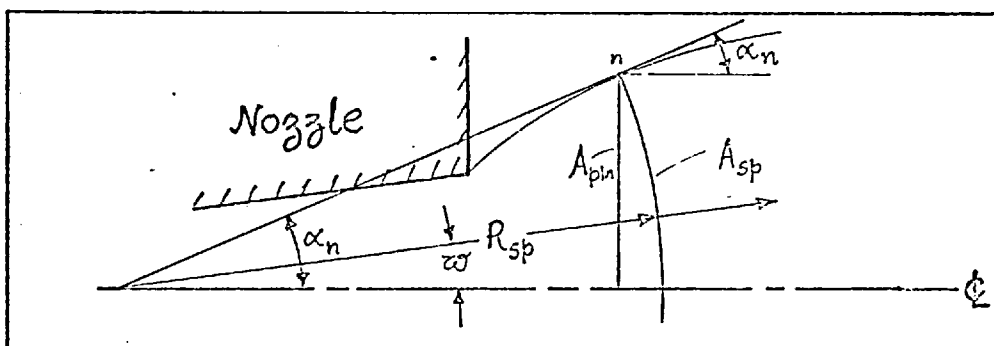


Fig. A.1.3. Radial Flow Of Jet.

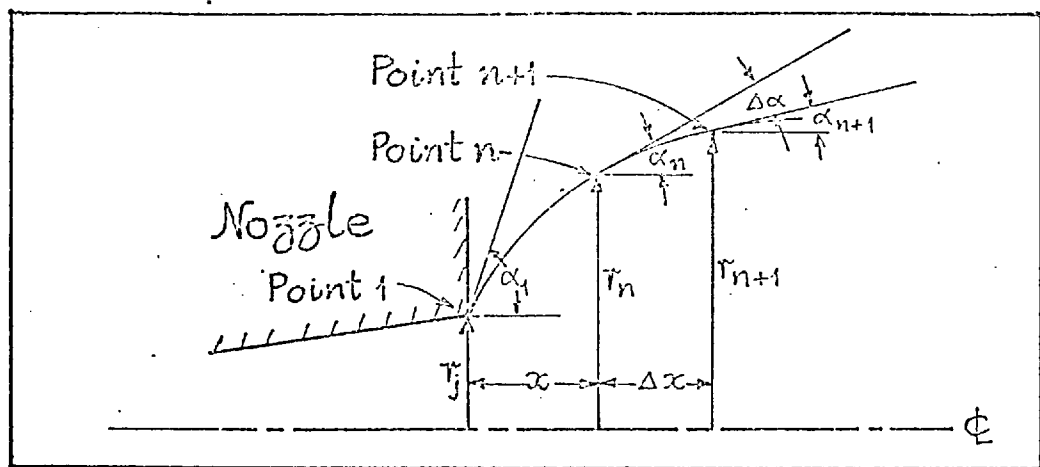


Fig. A.1.4. Jet Boundary Coordinate System.

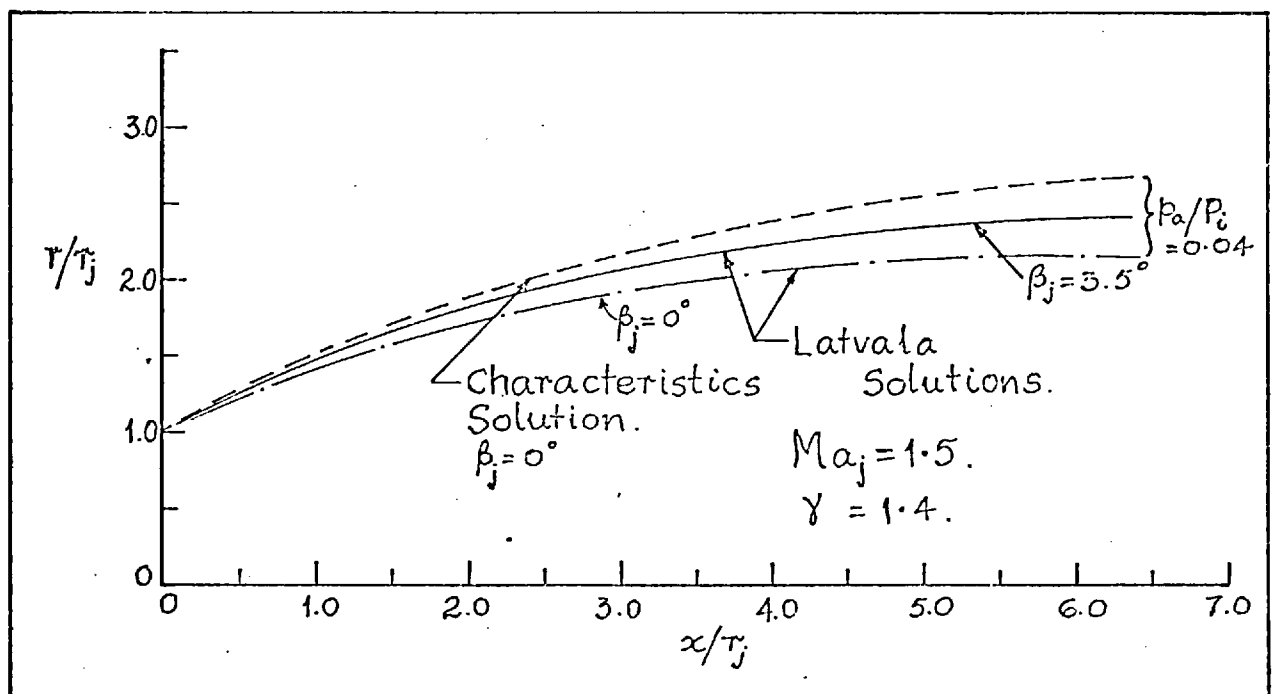


Fig. A.1.5. Comparison Of Jet Boundary Solutions Obtained By The Method Of Characteristics And Latvala's Approximate Method.

A P P E N D I X 2

Free shear layer velocity profiles.

$\frac{\sigma y}{x}$	U/U_2			
	Goertler	Crane	Error Function	Tollmien
$-\infty$	0	0	0	0
-2.6	0.003	-	-	-
-2.4	0.006	-	-	-
-2.2	0.010	-	0.001	-
-2.0	0.017	0.023	0.002	-
-1.8	0.026	0.032	0.005	-
-1.6	0.039	0.045	0.012	0.003
-1.4	0.056	0.064	0.021	0.017
-1.2	0.079	0.089	0.045	0.044
-1.0	0.111	0.123	0.079	0.092
-0.8	0.155	0.169	0.129	0.151
-0.6	0.214	0.228	0.198	0.220
-0.4	0.292	0.304	0.286	0.306
-0.2	0.388	0.395	0.389	0.400
0	0.500	0.500	0.500	0.500
0.2	0.619	0.612	0.611	0.605
0.4	0.733	0.722	0.714	0.707
0.6	0.832	0.819	0.802	0.802
0.8	0.905	0.894	0.871	0.887
1.0	0.953	0.945	0.921	0.949
1.2	0.980	0.975	0.955	0.983
1.4	0.993	0.990	0.976	-
1.6	0.998	0.997	0.988	-
1.8	-	0.999	0.995	-
2.0	-	-	0.998	-
$+\infty$	1.0	1.0	1.0	1.0

A P P E N D I X - 3DETERMINATION OF NOZZLE EXIT MACH NUMBER AND
PRESSURE RATIO.A3.1 Introduction.

In order to calculate the variation of base pressure ratio, p_2/P_i , with overall blowing pressure ratio, P_i/p_a , from equation (2.2.13), nozzle exit pressure ratio, p_1/P_i , and Mach number Ma_1 need to be known experimentally. As indicated in section 4.3 of Chapter 4, each nozzle is provided with a 0.015 in. diameter static pressure tap located 0.02 in. upstream of its exit plane to measure p_1/P_i when the nozzle is fully expanded.

Because of limitations on the available air supply, none of the nozzles was fully expanded or underexpanded when the nozzle back pressure was as high as ^{the} ambient atmospheric pressure, p_a . To achieve the required low back pressures a 2.875 in. internal diameter diffuser was fixed at the nozzle exit.

A3.2 Experimental method and results.

With the diffuser receiving the air jet from an overexpanded nozzle, the back pressure ratio p_2/P_i rapidly dropped when P_i/p_a was increased. P_i/p_a was adjusted such that the back pressure p_2 was equal to the nozzle exit pressure p_1 . The nozzle was now fully expanded. Special care was taken to hold P_i/p_a steady in the fully-expanded nozzle-operating condition, because expansion waves in the underexpanded condition,

and shock waves in the overexpanded condition would otherwise interfere with measurements to yield inaccurate results.

A stainless steel pitot tube of 0.021 in. inner diameter and 0.031 in. outer diameter was then aligned with the nozzle exit plane and traversed across the nozzle to record the total pressure, $P(r)$. Since p_1 is known, the following Rayleigh pitot-tube formula for supersonic flow was used to evaluate the variation of Mach number, $\{Ma(r)\}_1$, across the nozzle exit section:-

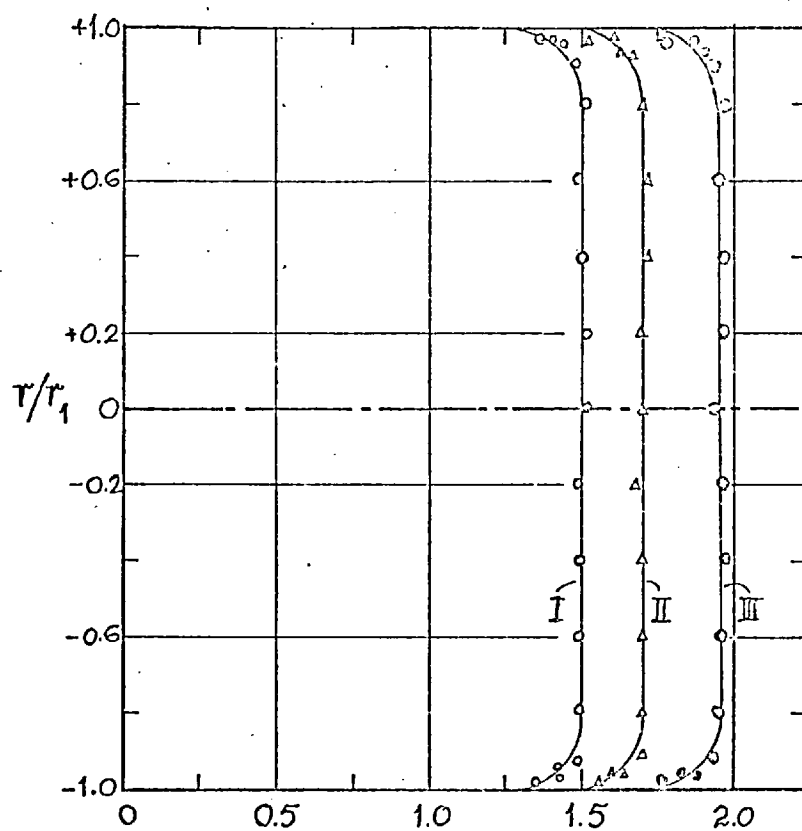
$$\frac{p_1}{\{P(r)\}} = \frac{\left[\frac{2}{\gamma+1} \{Ma(r)\}_1^2 - \frac{\gamma-1}{\gamma+1} \right]^{1/(\gamma-1)}}{\left[\frac{\gamma+1}{2} \{Ma(r)\}_1^2 \right]^{\gamma/(\gamma-1)}} \quad (A3.1)$$

In the above p_1 was assumed constant across the whole section.

In Fig. A3.1 the distributions of nozzle exit Mach number for conical nozzles of design Mach numbers 1.54, 1.75 and 2.0 are shown. The nozzle -axis Mach number may be taken to be the mean "free stream" Mach number. Thus $Ma_1 = 1.515$ for the Mach 1.54 nozzle, $Ma_1 = 1.7$ for the Mach 1.75 nozzle and $Ma_1 = 1.95$ for the Mach 2.0 nozzle. For clarity, the measurements for the Mach 1.54 Foelsch nozzle ($Ma_1 = 1.49$) are not shown. In the legend of the above figure the experimental values of p_1/P_1 for the three nozzles are also shown.

Fig. A3.1 also shows that during the tests the boundary layers at the exit of the fully-expanded nozzles were thin and turbulent when $5.8 \times 10^6 \leq Re_{n1} \leq 7.9 \times 10^6$. Assuming that the boundary layer started from the throat section of the nozzle, following Tucker (1951) the calculated values of boundary-layer thickness at nozzle exit were found to compare satisfactorily with the measurements. This suggests that for estimating boundary-layer thickness at higher Re_{n1} , Tucker's approximate method may be used without appreciable error.

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Mach Number - Ma_1

Curve	Design Ma_1	Exptl. Ma_1	Exptl. P_1/P_i
I	1.54	1.515	0.265
II	1.75	1.70	0.199
III	2.0	1.95	0.136

Fig. A.3.1. Mach Number Distributions
At Nozzle Exit.

A P P E N D I X 4

SECONDARY AIR FLOWMETER DESIGN.

The flowmeter design requirements were as follows:

(a) The meter was to measure accurately air flow rates varying from 0.05 to 0.8 pound per second at approximately atmospheric pressure and ambient temperature of 15°C.

(b) The ratio of meter flow area to secondary air pipe flow area, Z , for a given pipe length should be such that the pressure drop across the system was small.

To satisfy the above requirements an I.S.A. nozzle was chosen of $Z = 0.247$. This allowed not only a low pressure drop across the secondary air flow system, but also a lower limit of differential pressure that could be measured within an accuracy of one per cent with the available manometer. A seamless steel pipe of 3.5 in. inside diameter was chosen; for $Z = 0.247$, the corresponding nozzle throat diameter was then 1.74 in. In Fig. A4.1 the nozzle dimensions are shown in details.

With the above data, according to B.S.1042 (1943), the volumetric air flow rate through the nozzle could be calculated from the following:-

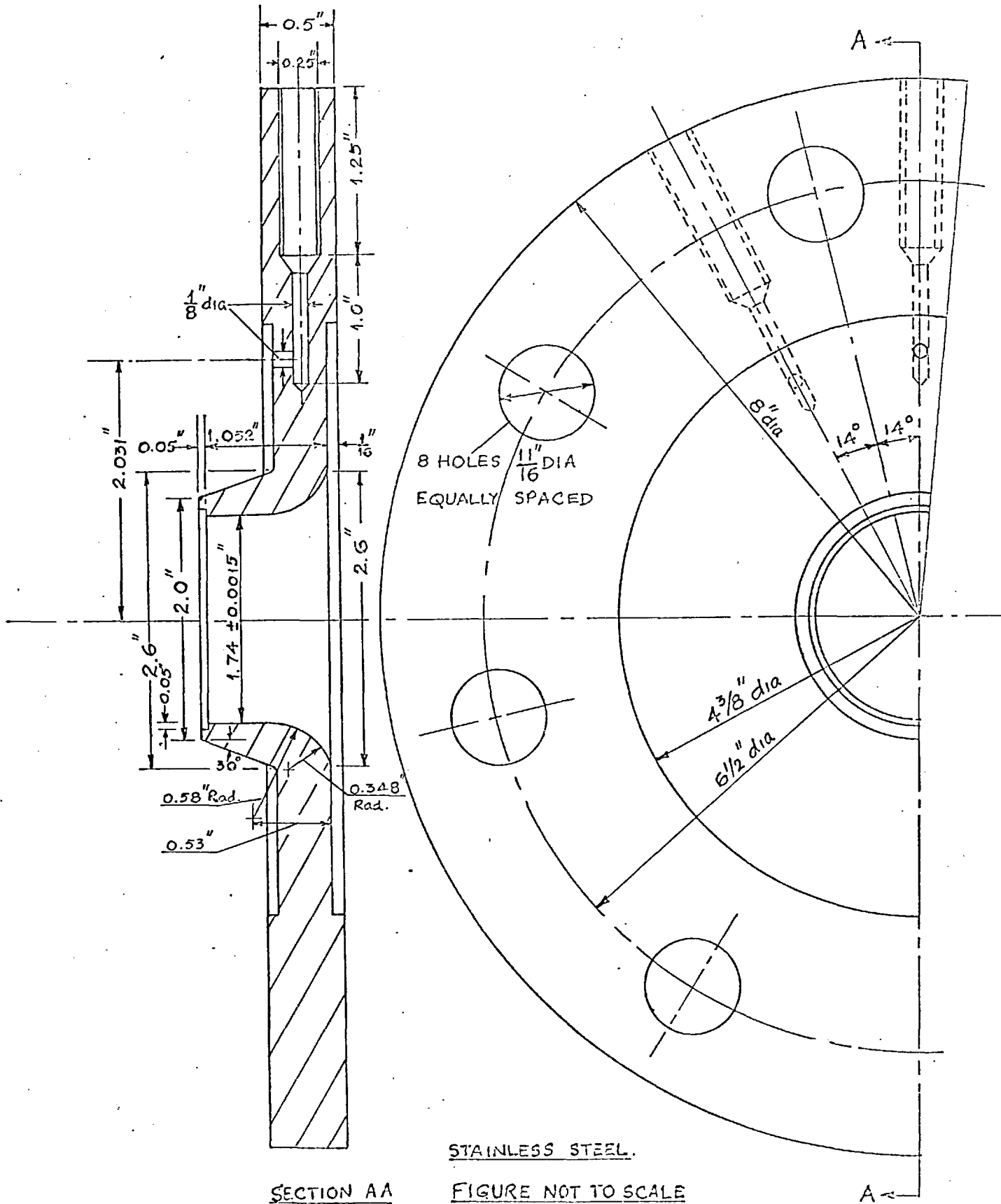
$$\dot{Q} = 1.04 (b)^{0.5}, \quad (A4.1)$$

where $\dot{Q}$ is the air flow rate in cuft/sec., and b is the pressure differential across the nozzle, in inches of water. When air pressure of 14.5 p.s.i.a and ^{air} temperature of 288°K are assumed at the nozzle inlet, the secondary air flow rate $\dot{m}_c$ is given by the following:

$$\dot{m}_c = 0.078 (b)^{0.5}. \quad (A4.2)$$

Unobstructed straight pipe lengths of 11 times and 5 times the pipe inner diameter of 3.5 in. were provided upstream and downstream of the flowmeter respectively.

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A P P E N D I X 5

Summary of experimental reattachment data.

Ma_1	Re_1 (10^8 /in.)	H in.	$1/H$ 10^{-2}	p_2/P_i	Ma_2	R_c	$(x/H)_{sf}$	$(x/H)_{od}$
1.49 ⁺	1.23	0.30	1.03	0.077	2.325	0.1	1.62	—
	1.58	0.48	0.60	0.059	2.495	0.062	1.30	—
	1.88	0.61	0.47	0.044	2.68	0.052	1.56	—
	2.04	0.68	0.42	0.04	2.74	0.044	1.50	—
1.515 ⁺	1.22	0.30	0.65	0.077	2.32	0.091	2.5	—
	1.575	0.48	0.38	0.056	2.525	0.062	1.22	—
	1.88	0.61	0.30	0.042	2.715	0.048	1.52	—
	2.03	0.68	0.27	0.039	2.76	0.041	1.3	—
1.7 ⁺	1.21	0.30	0.64	0.062	2.46	0.086	1.98	—
	1.58	0.48	0.38	0.044	2.68	0.048	1.03	—
	1.89	0.61	0.293	0.033	2.87	0.042	1.42	—
1.95 ⁺	1.1	0.30	0.39	0.039	2.76	0.062	1.50	—
	1.45	0.48	0.23	0.03	2.935	0.034	0.90	—
2.09*	0.0095	0.25	3.89	0.041	2.72	0.044	3.8	—
	0.0095	1.02	0.95	0.043	2.69	0.042	3.34	3.0
	0.0095	1.675	0.58	0.046	2.65	0.041	3.25	3.45
3.02*	0.0136	0.25	4.02	0.0056	4.12	0.0065	2.32	—
	0.0136	1.02	0.99	0.0052	4.17	0.0096	2.65	2.5
	0.0136	1.675	0.06	0.0067	3.99	0.0095	3.1	2.7
3.9*	0.013	0.25	3.59	0.0019	5.23	0.0025	2.3	—
	0.013	1.02	0.88	0.001	5.57	0.002	2.0	—
	0.013	1.675	0.54	0.0009	5.65	0.0017	—	2.4

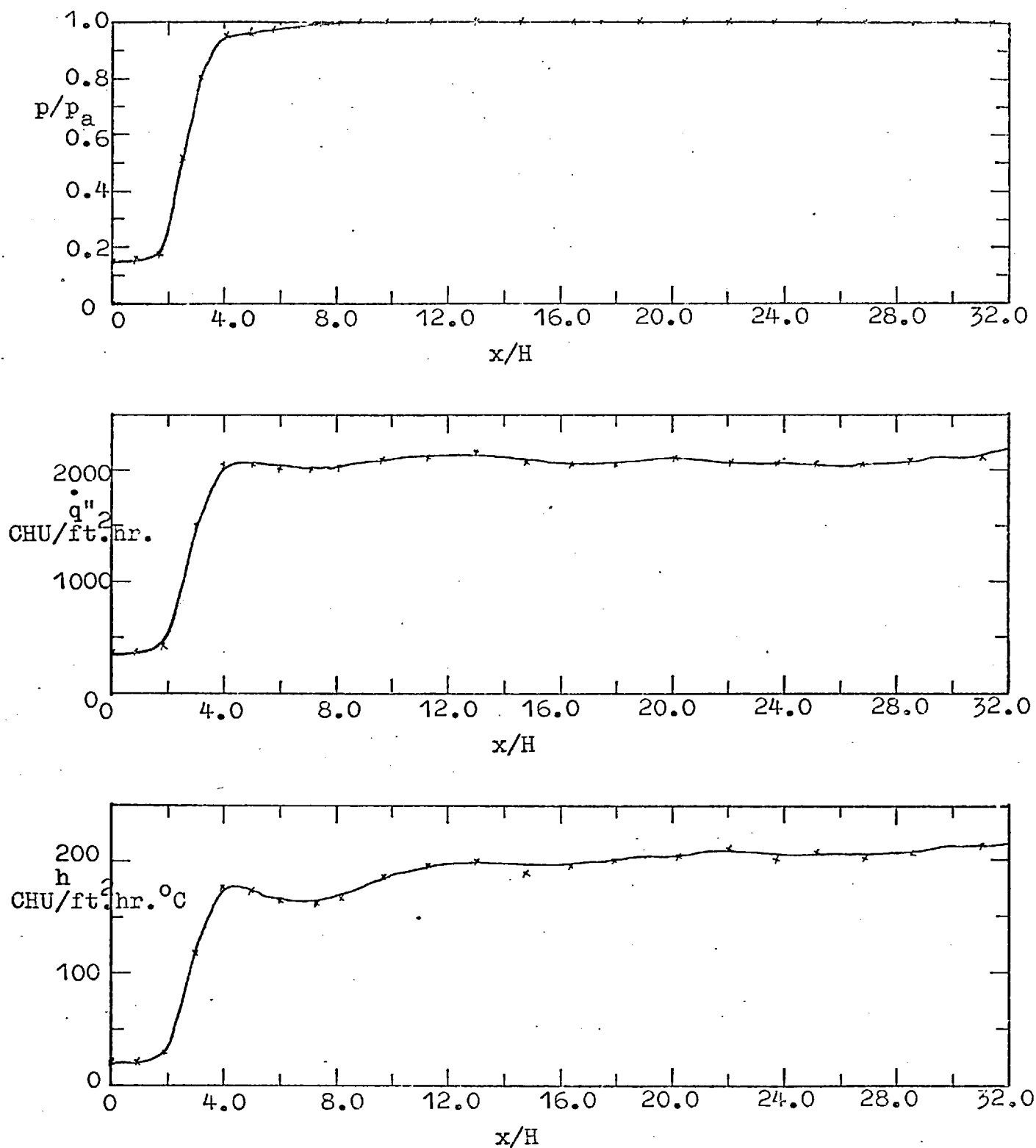
+ - Present data.

* - Roshko and Thomke data (1966).

- - - o o o o o o o - - -

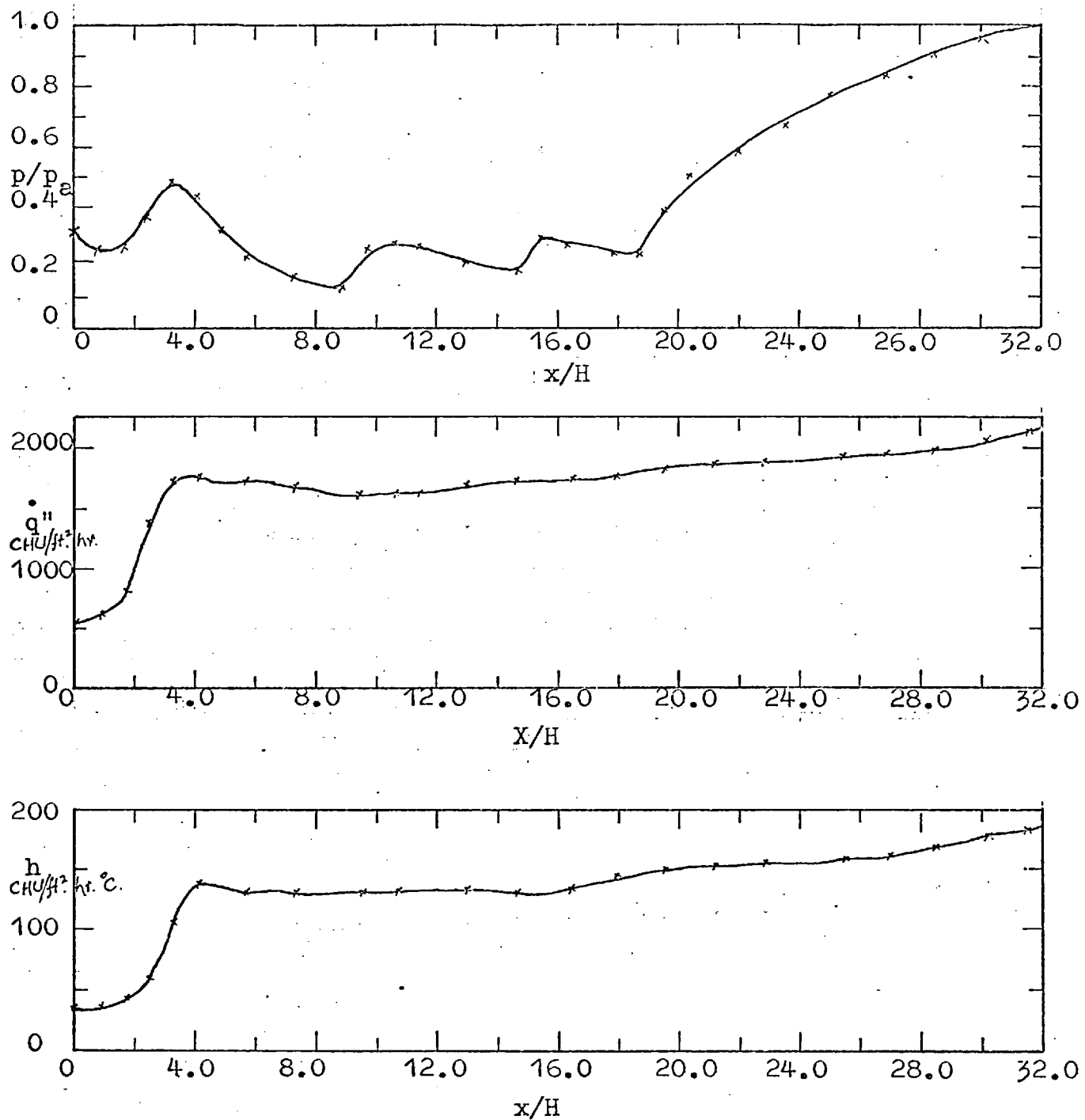
A P P E N D I X 6EXPERIMENTAL GRAPHS.

This appendix gives complete sets of experimental graphs for both diffusers and diffuser-ejectors.



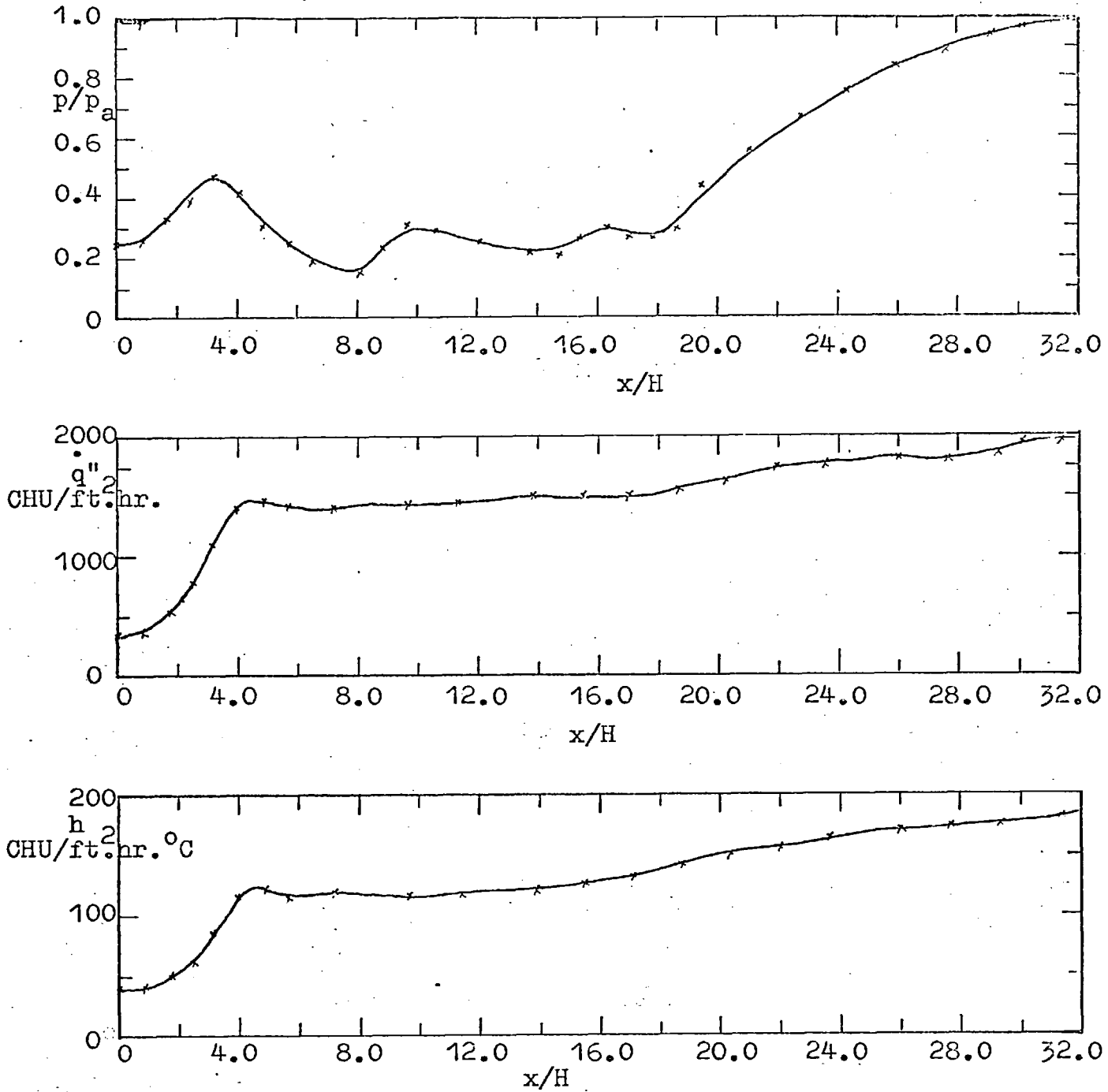
$$\text{Ma}_1 = 1.515, A_2/A^* = 3.671, P_i/p_a = 3.857, w_R = 0.0$$

Fig. A6.1 Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Unstarted Diffuser Length.



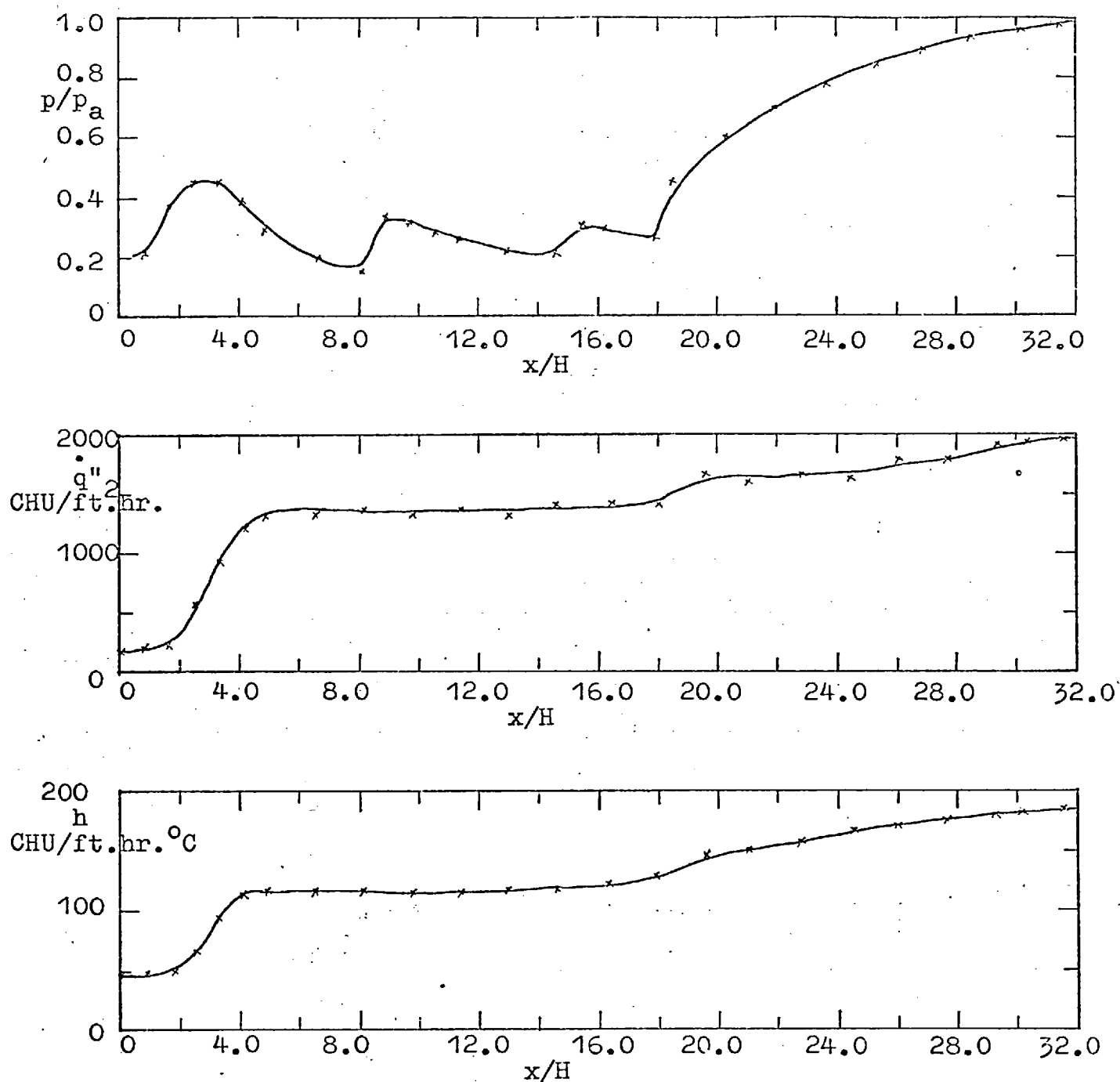
$$\text{Ma}_1 = 1.515, A_2/A^* = 3.671, P_i/p_a = 3.857, W_R = 0.025, s = 1/16''$$

Fig.A6.2a. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser-Ejector Length.



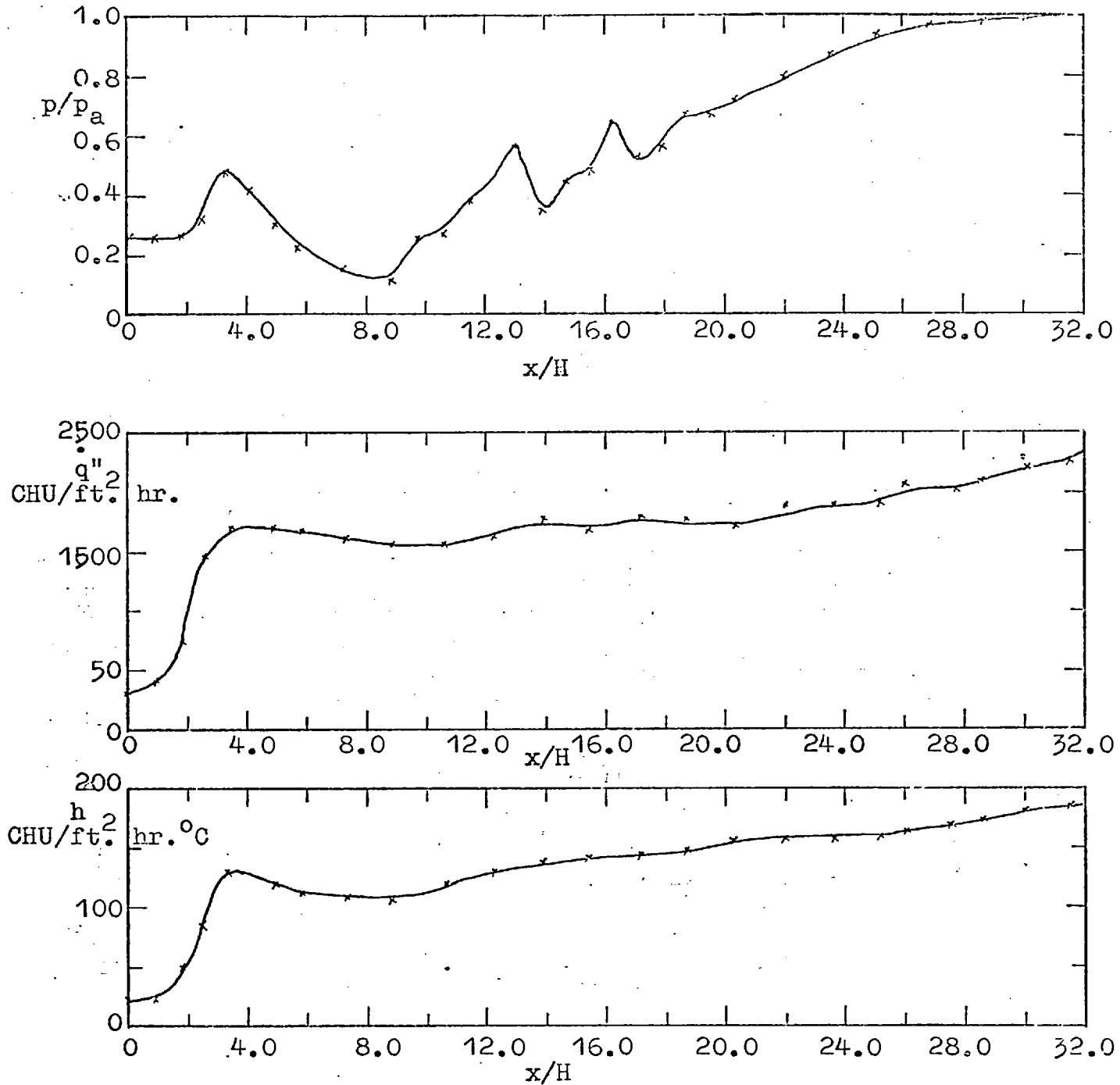
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 3.857$, $W_R = 0.05$, $s = 1/16''$

Fig. A6.2b. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



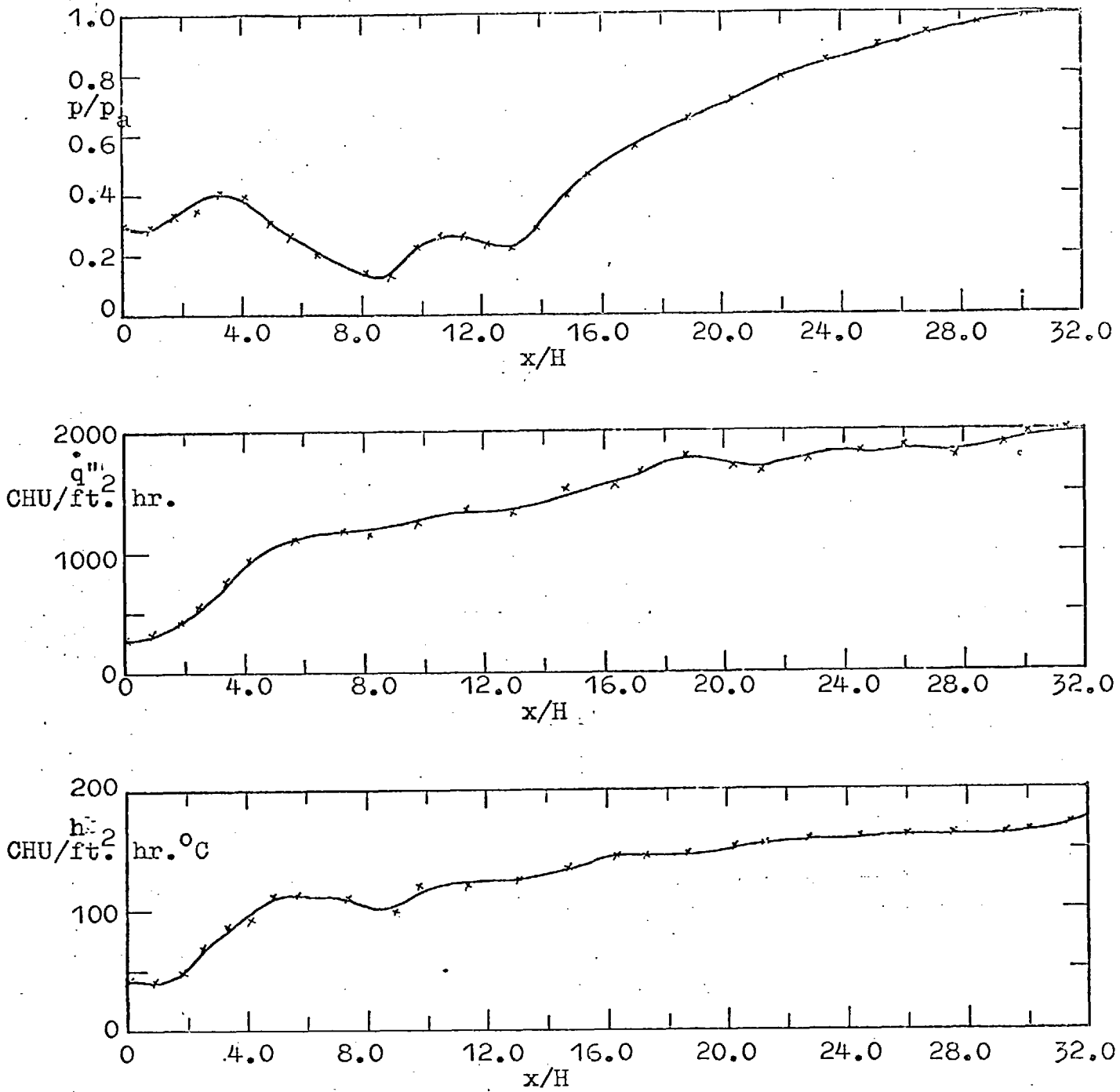
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 3.857$, $W_R = 0.075$, $s = 1/16"$.

Fig.A6.2c. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



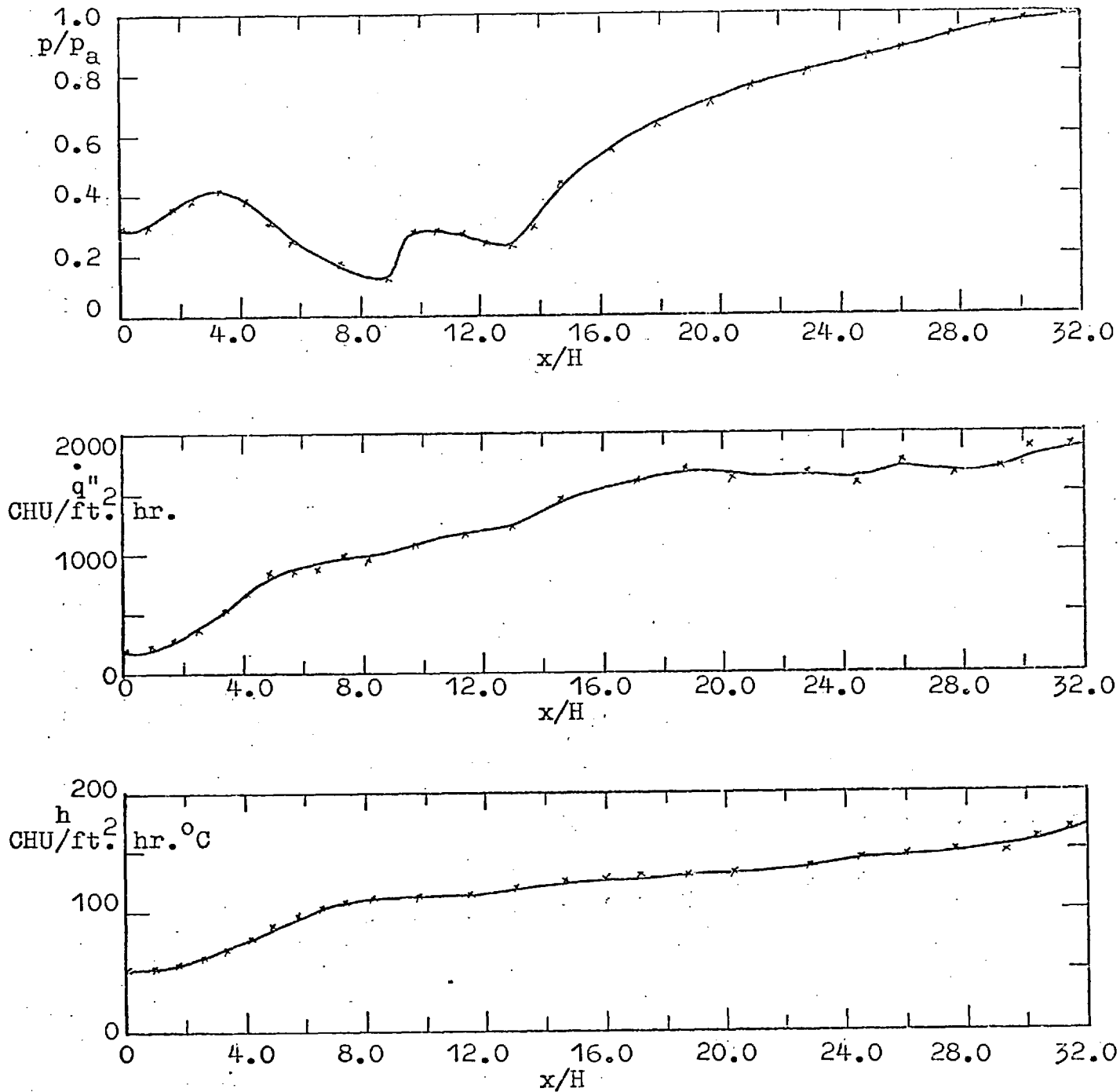
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 3.857$, $W_R = 0.025$, $s = 1/8''$.

Fig.A6.3a. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



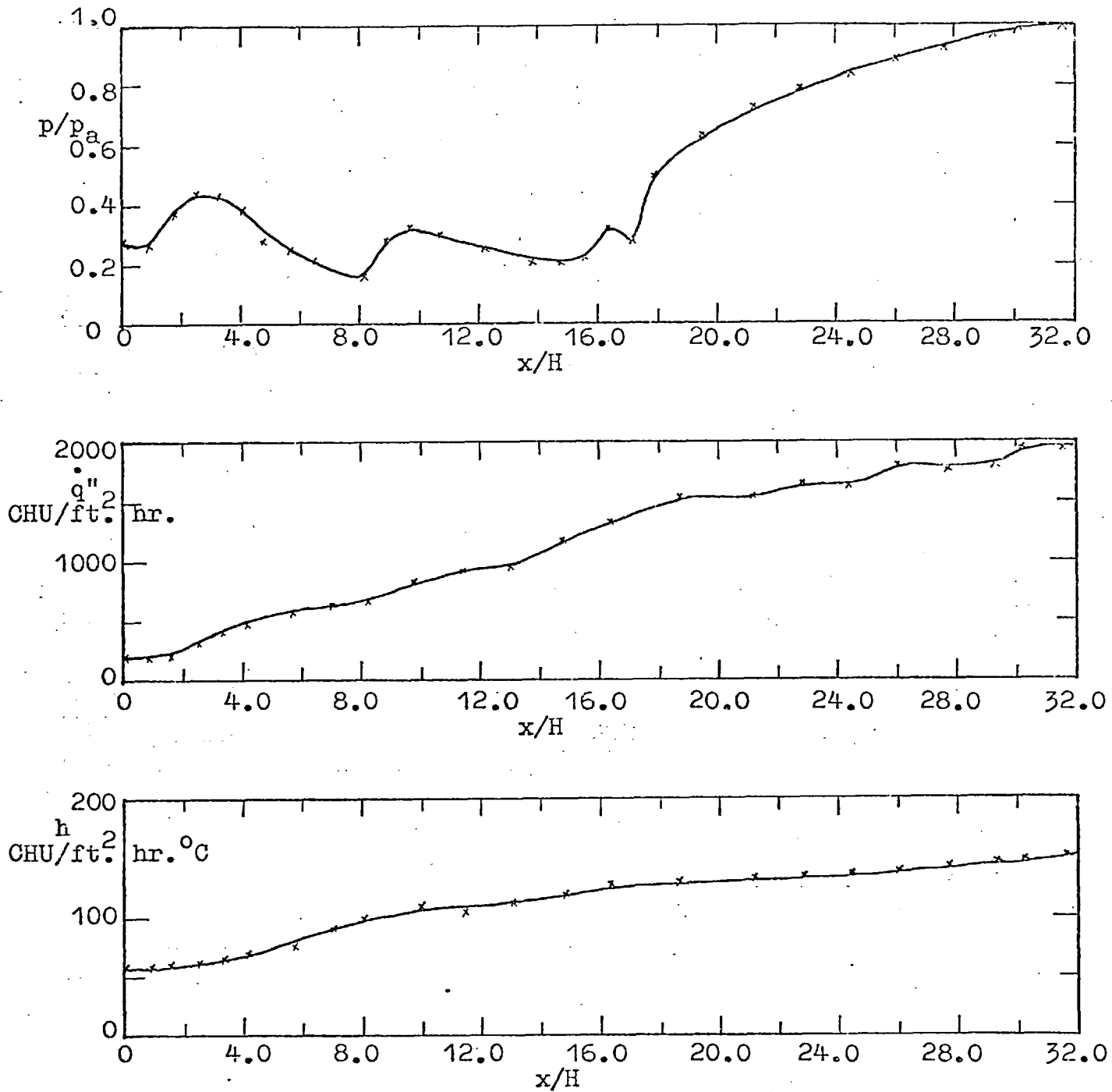
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 3.857$, $w_R = 0.05$, $s = 1/8''$.

Fig. A6.3b. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



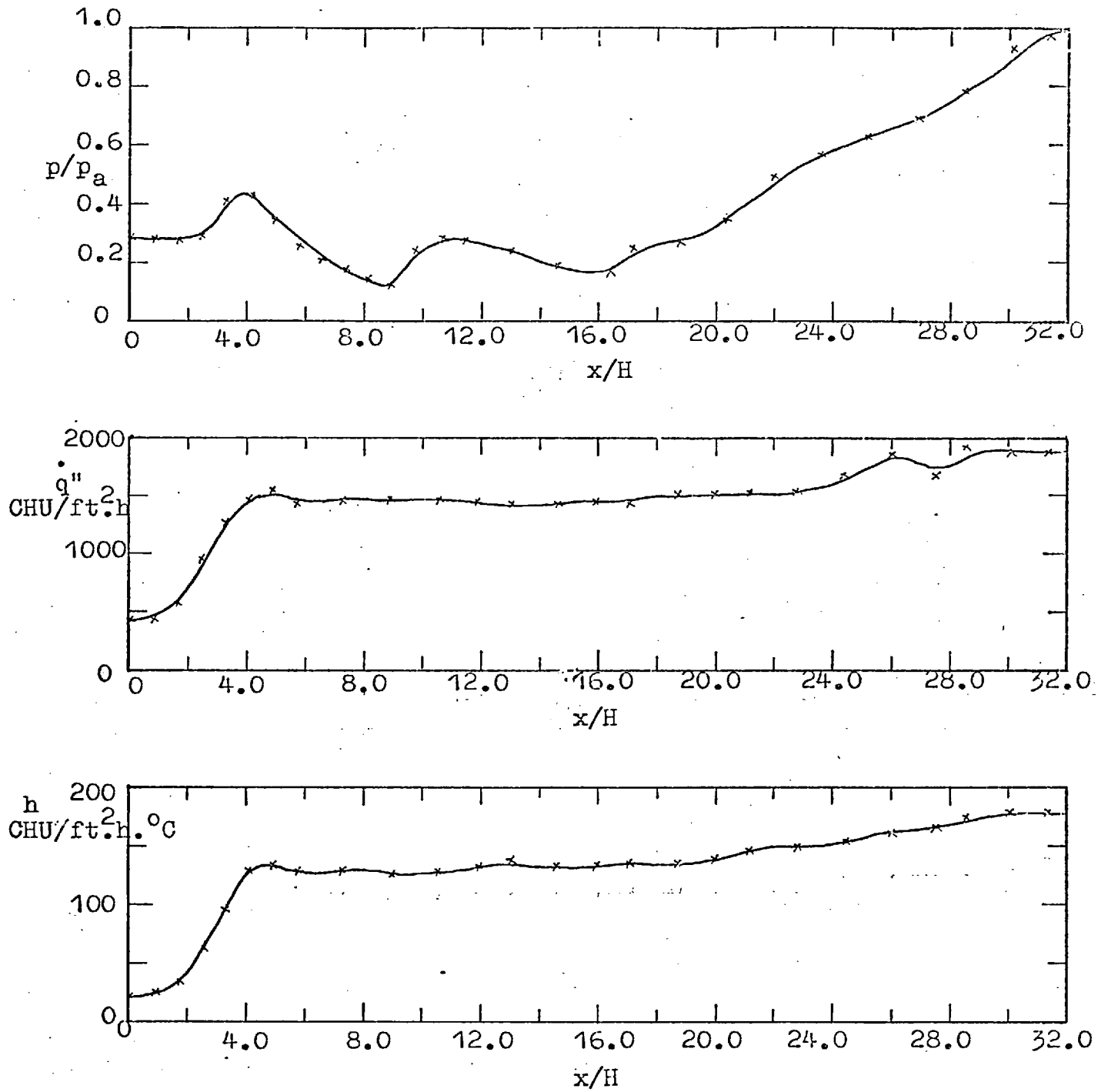
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 3.857$, $W_R = 0.075$, $s = 1/8''$.

Fig. A6.3c. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



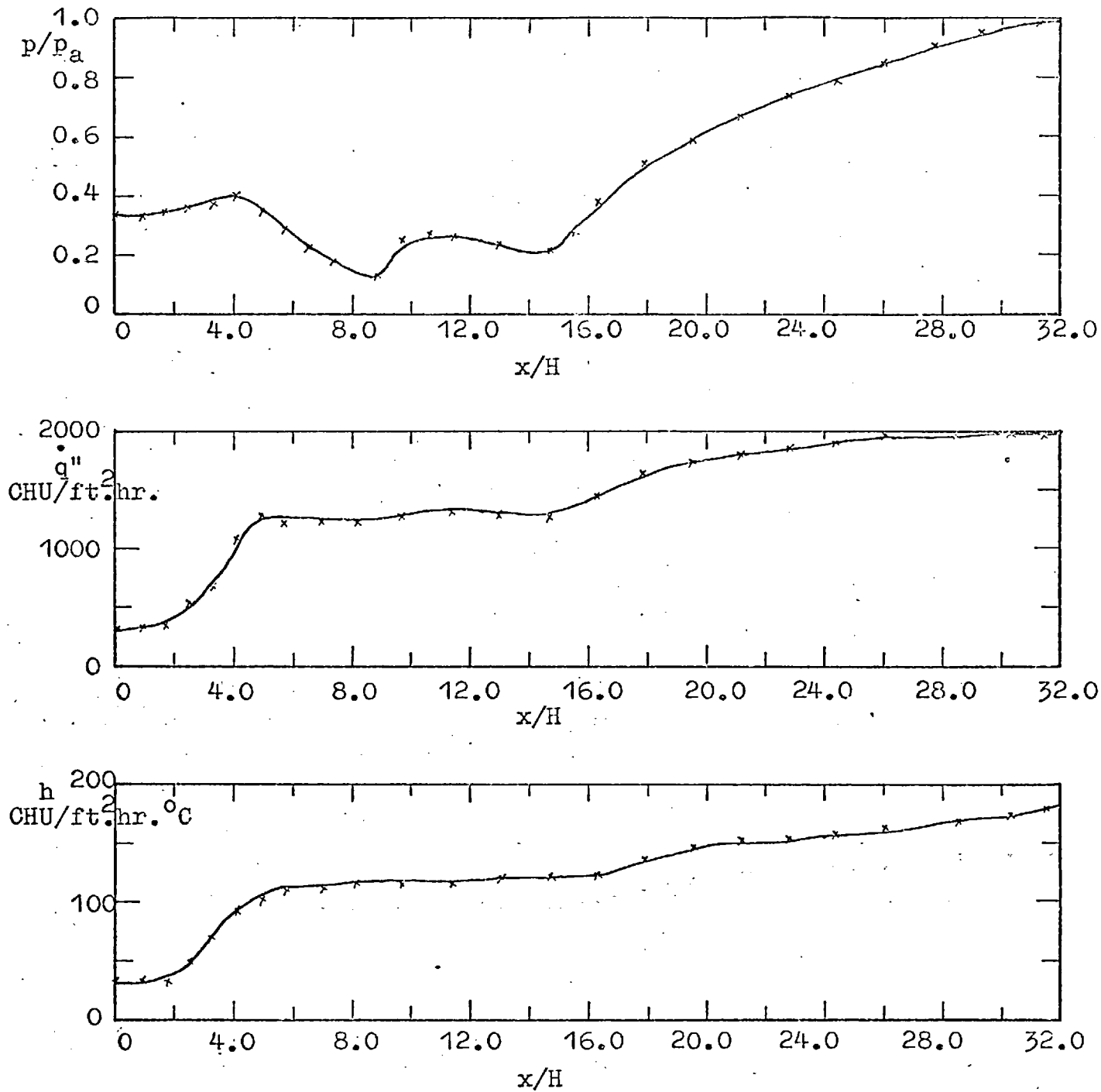
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 3.857$, $w_R = 0.1$, $s = 1/8''$.

Fig.A6.3d Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



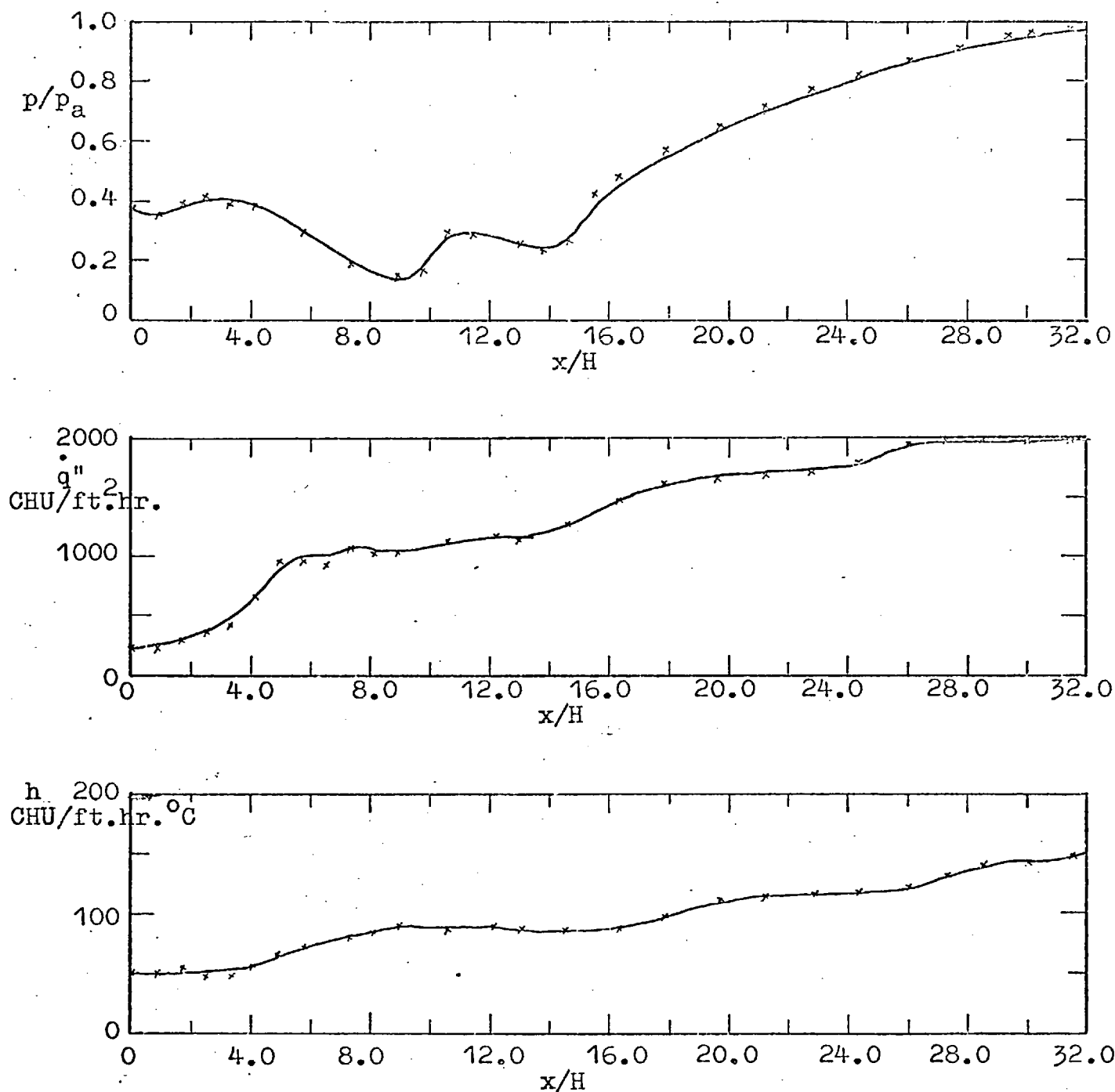
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 3.857$, $W_R = 0.025$, $s = 3/16"$

Fig. A6.4a. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



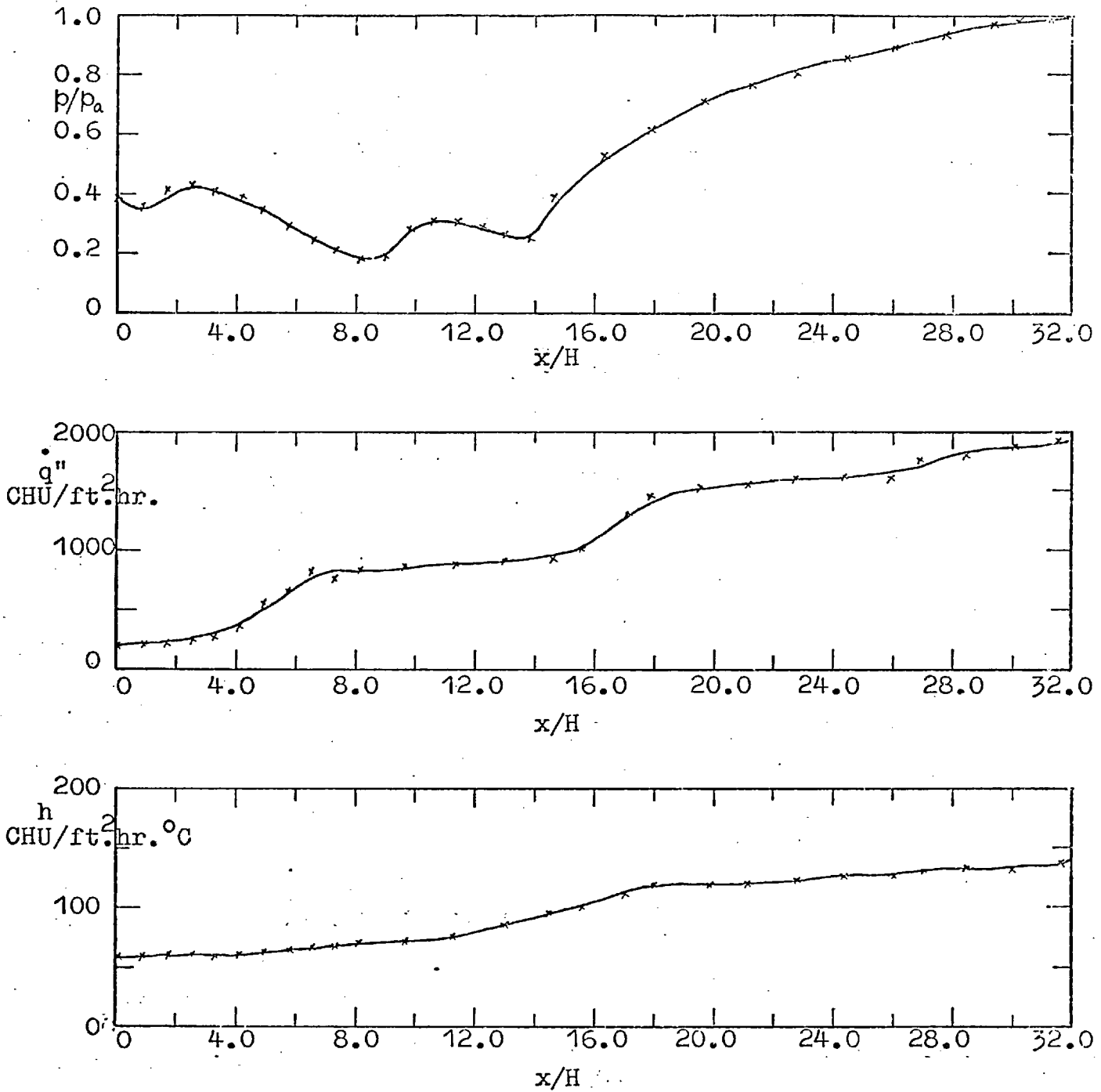
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 3.857$, $W_R = 0.05$, $s = 3/16"$.

Fig.A6.4b. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



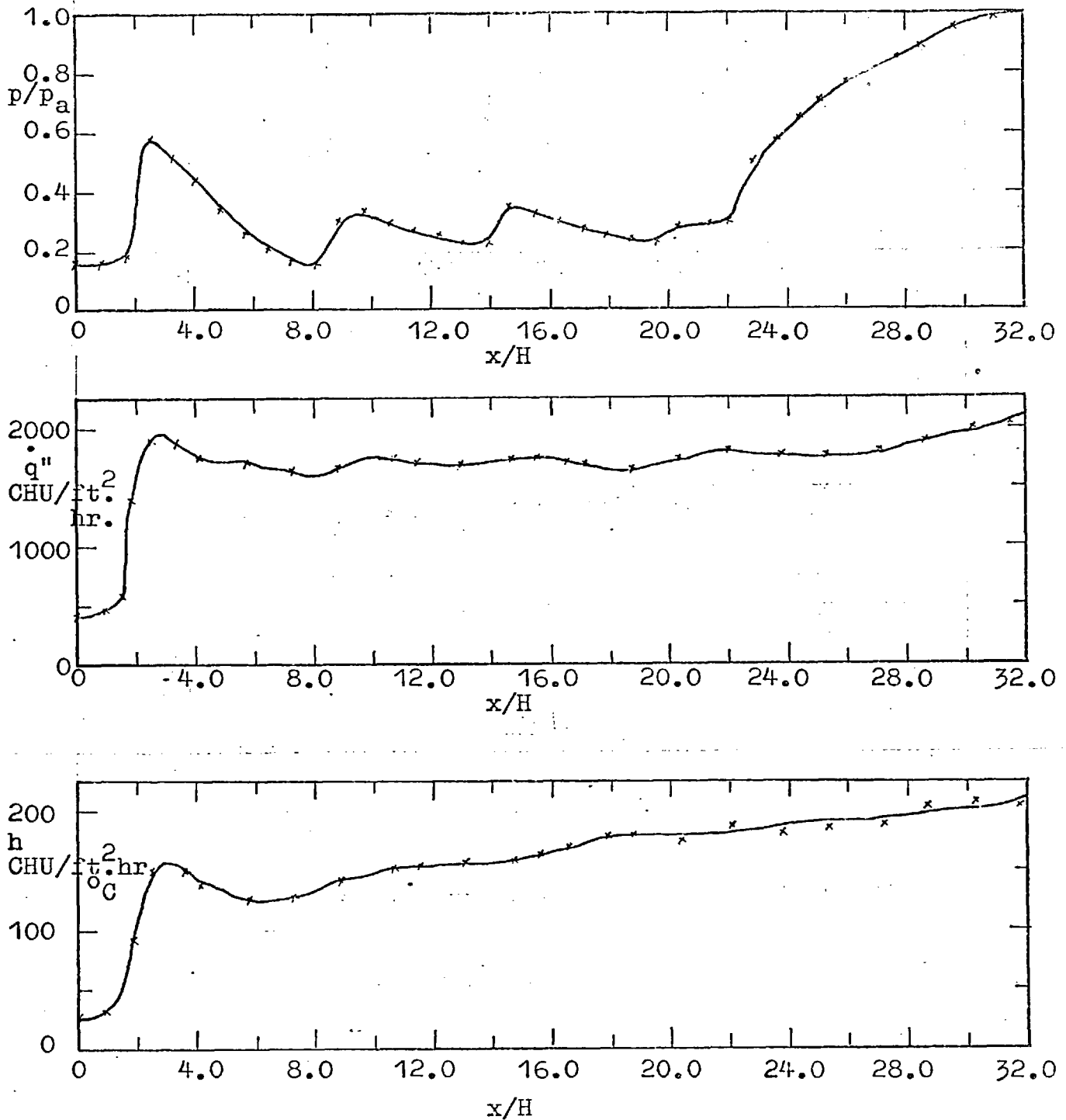
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 3.857$, $w_R = 0.075$, $s = 3/16''$.

Fig.A6.4c. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



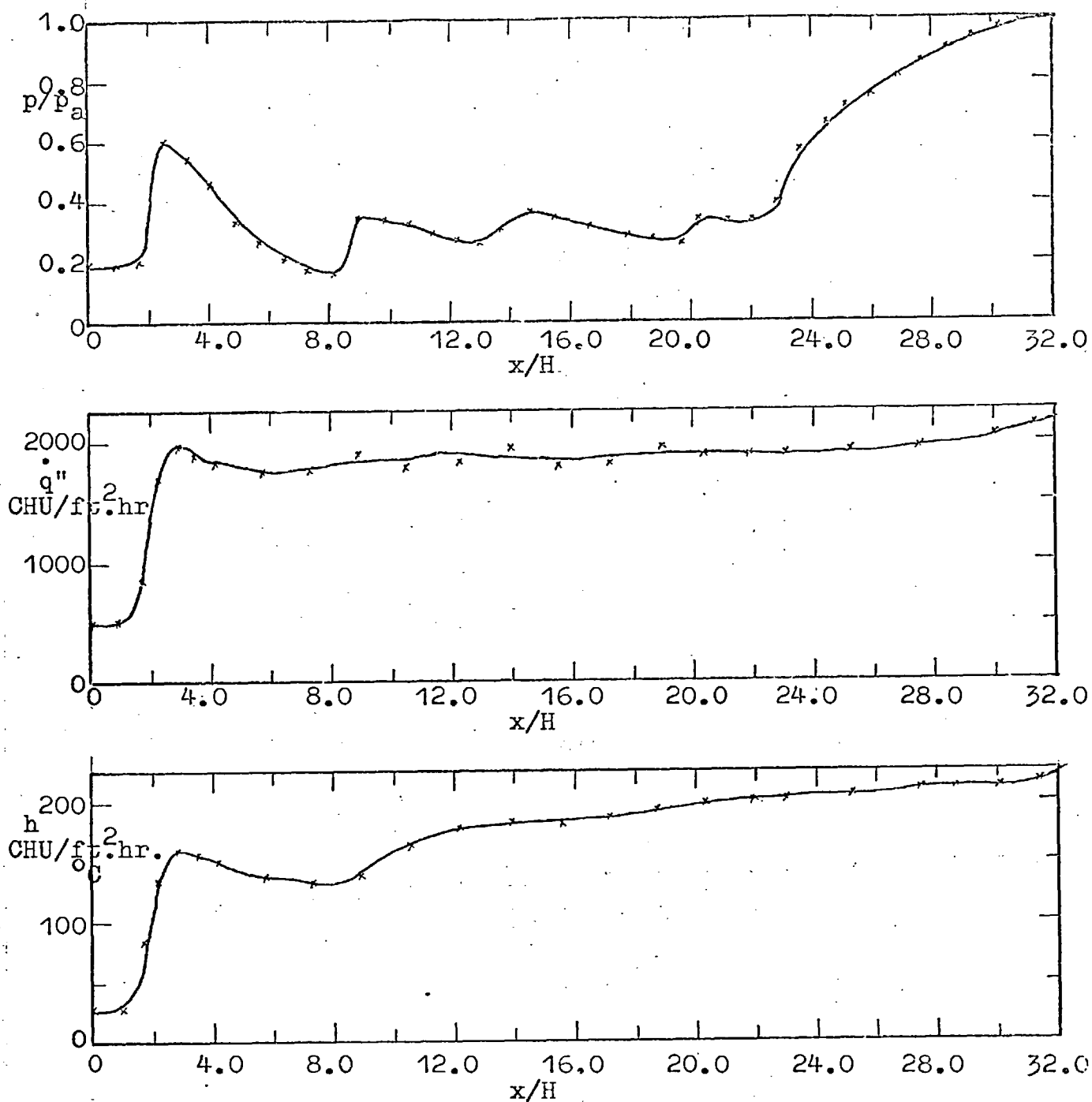
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 3.857$, $W_R = 0.1$, $s = 3/16"$.

Fig. A6.4d. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



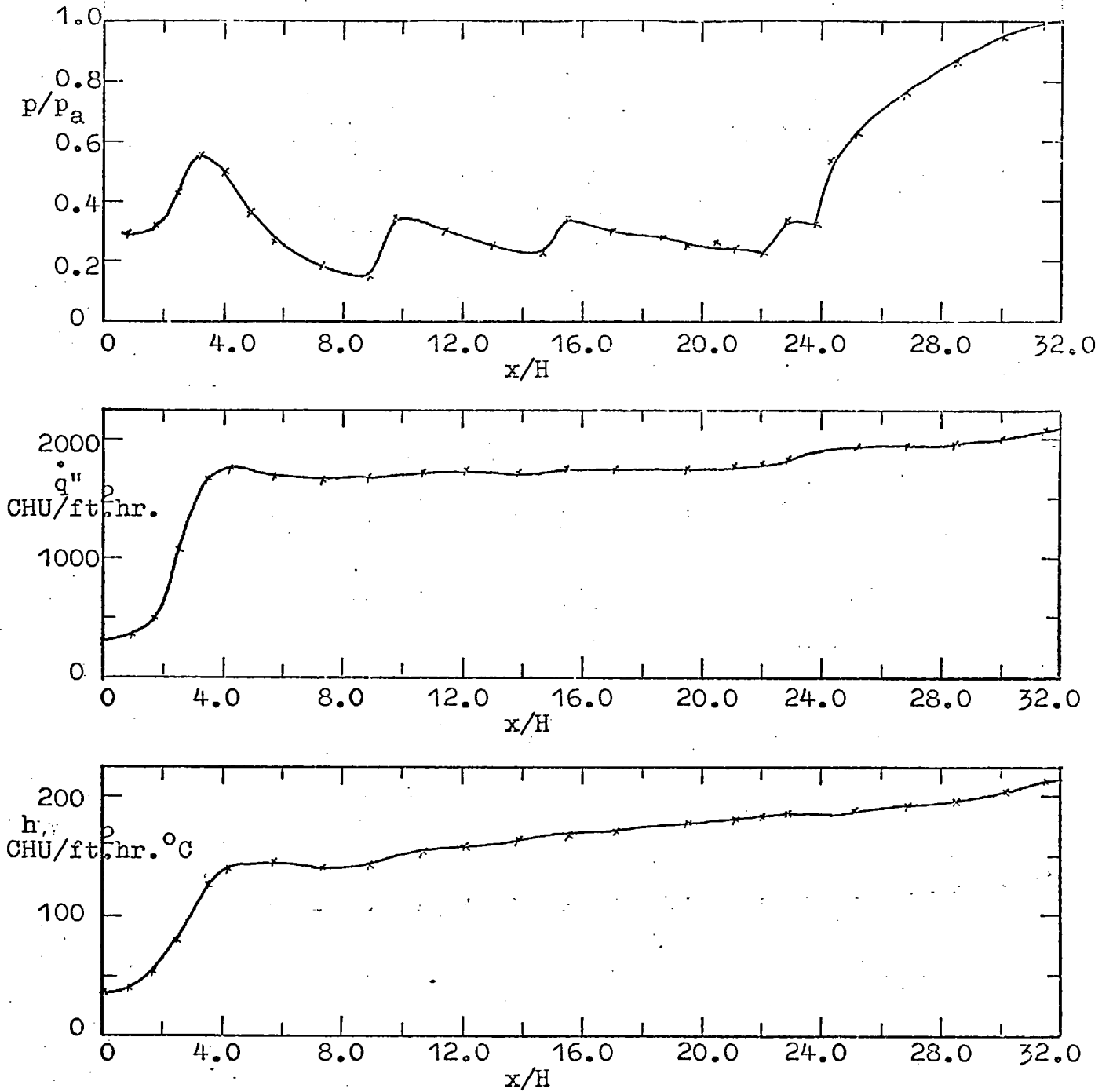
$Ma_1=1.515$, $A_2/A^*=3.671$, $P_i/p_a=4.129$, $p_2/P_i=0.041$, $w_R=0.0$

Fig.A6.5. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Started Diffuser Length.



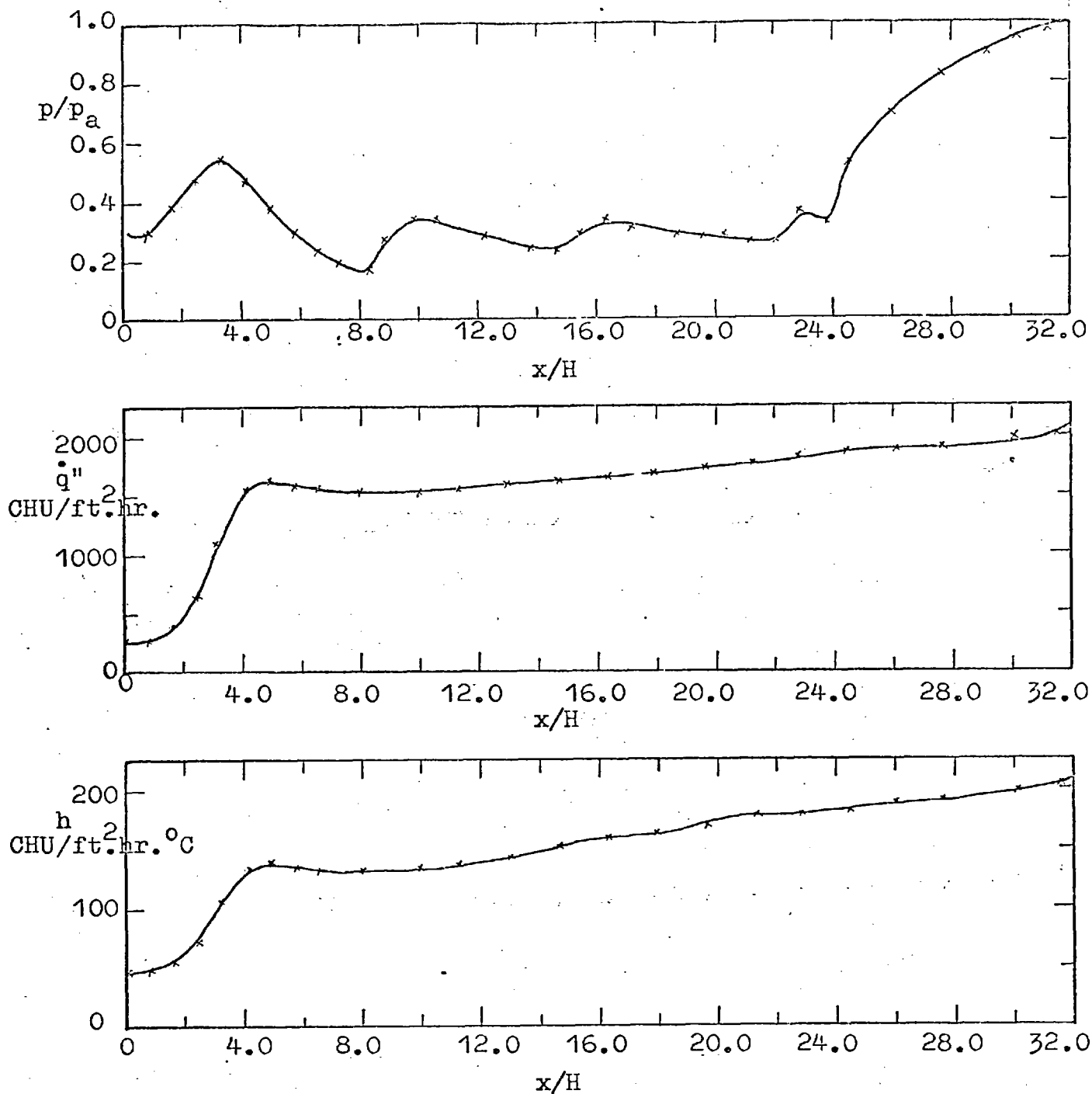
$Ma_1=1.515$, $A_2/A^*=3.671$, $P_i/p_a=4.401$, $W_R=0.0$, $p_2/P_i=0.041$.

Fig. A6.6. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Started Diffuser Length.



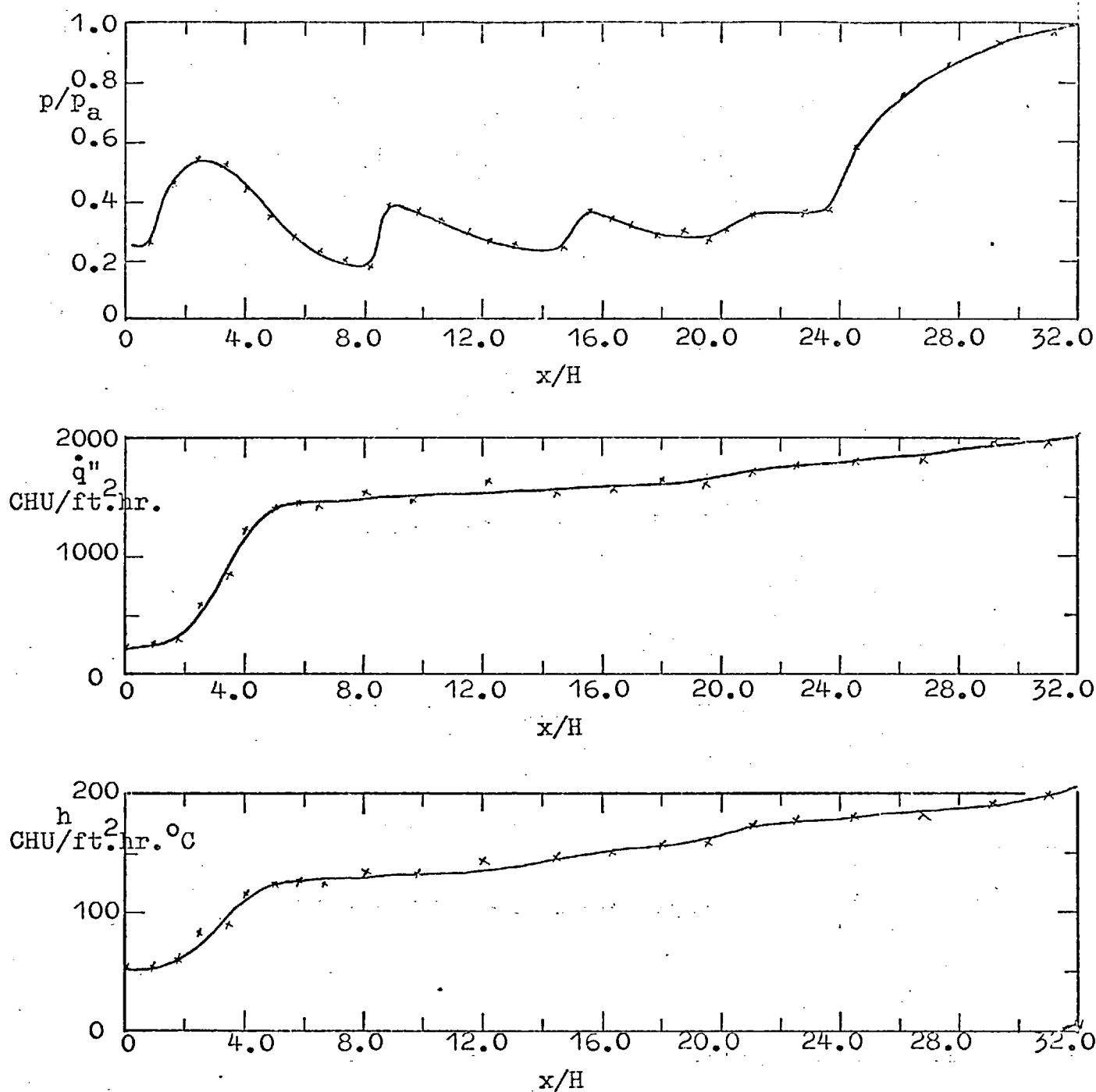
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_1/p_a = 4.401$, $W_R = 0.025$, $s = 1/16''$.

Fig. A6.7. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



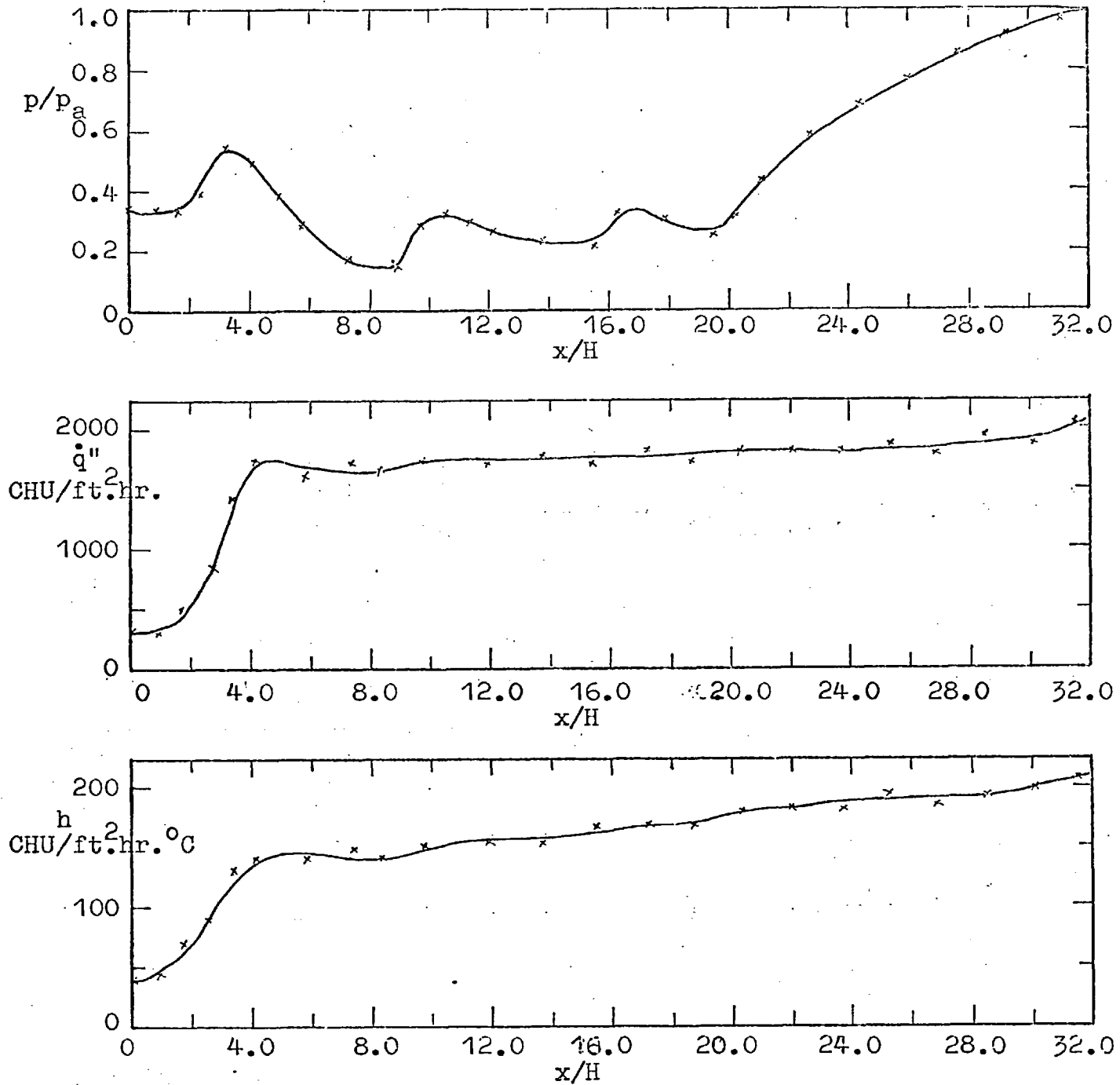
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 4.401$, $W_R = 0.05$, $s = 1/16"$.

Fig. A6.7b. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



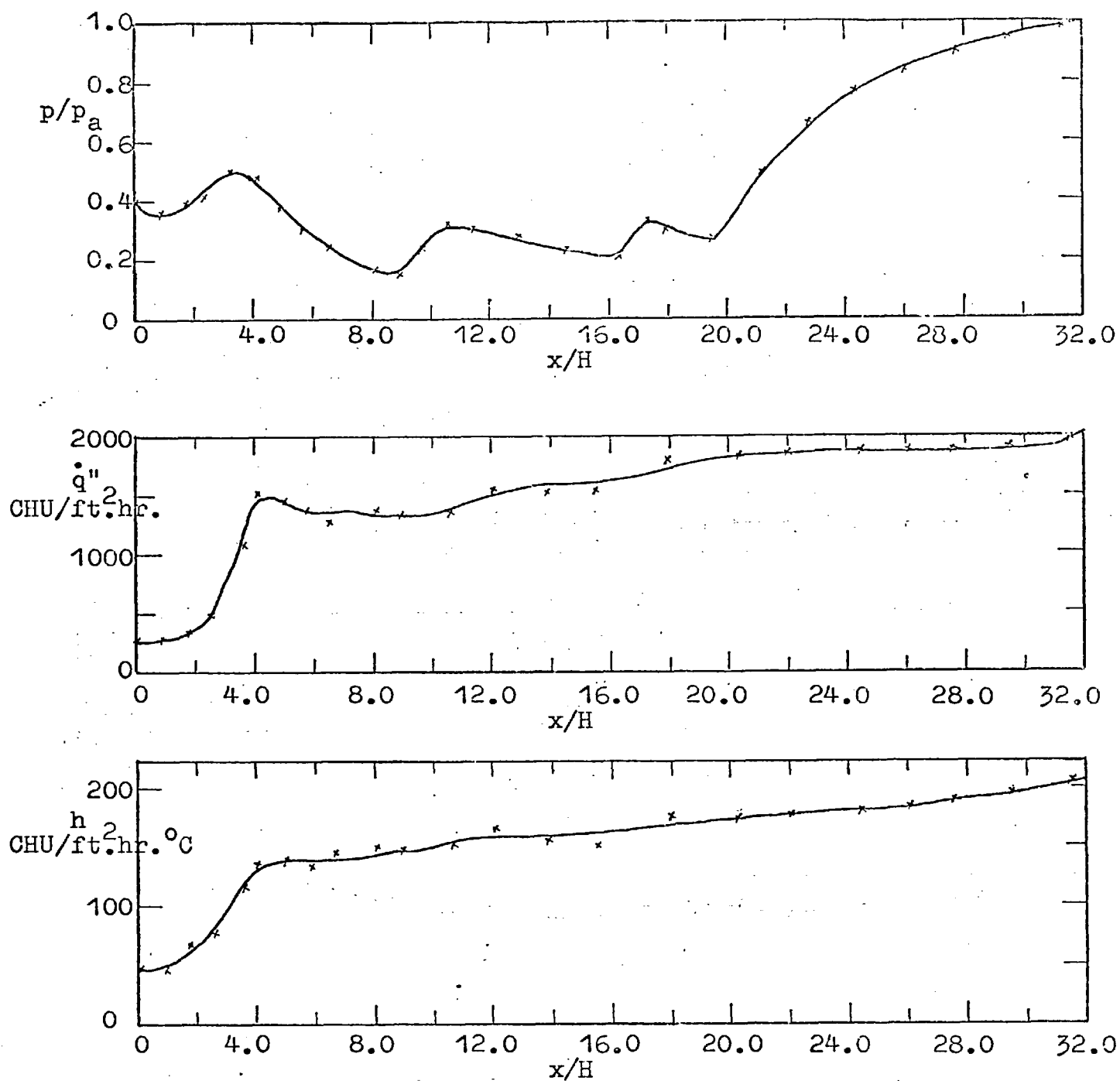
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 4.401$, $W_R = 0.075$, $s = 1/16''$.

Fig. A6.7c. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



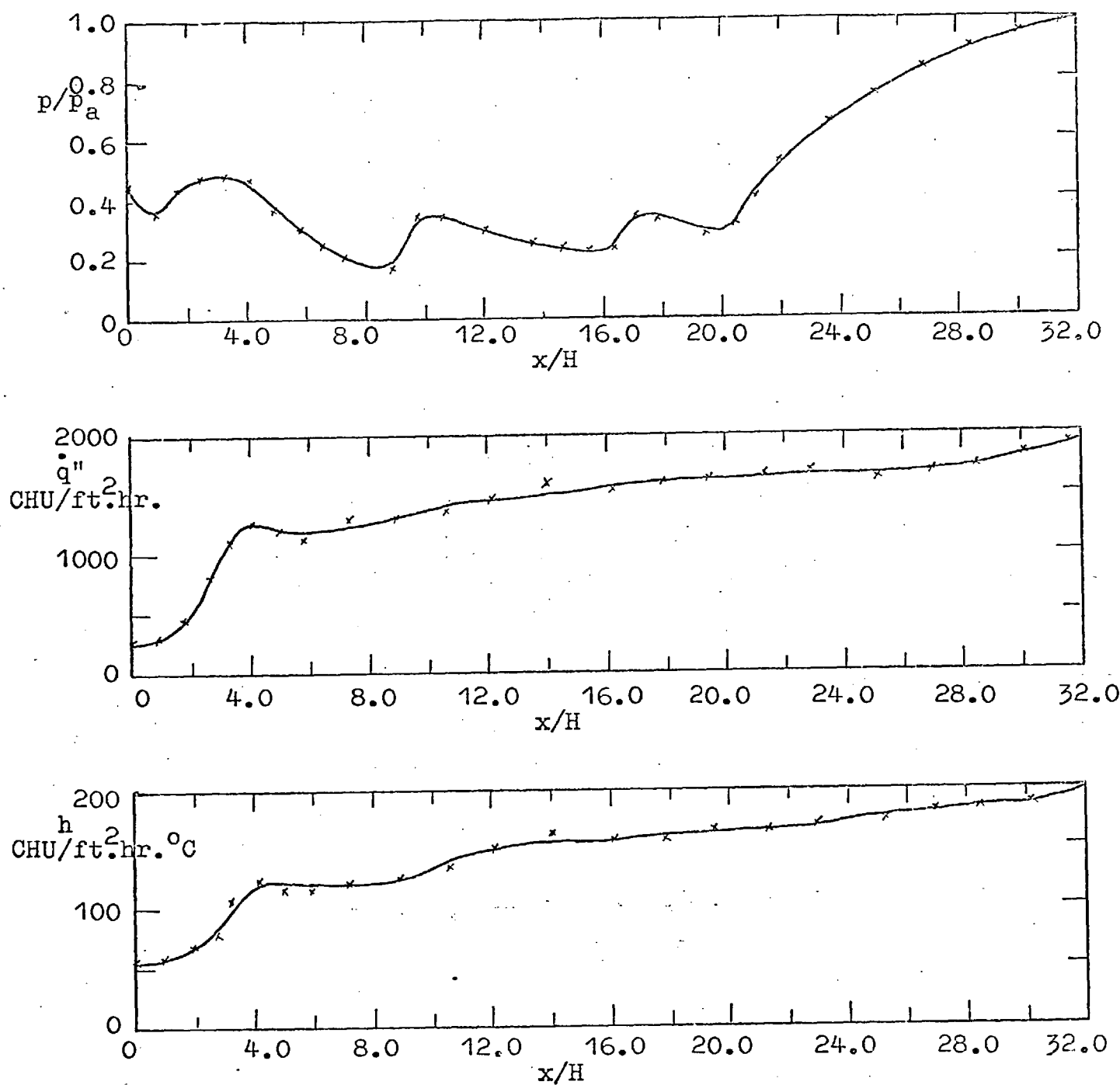
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 4.401$, $W_R = 0.025$, $s = 1/8''$.

Fig.A6.8a. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



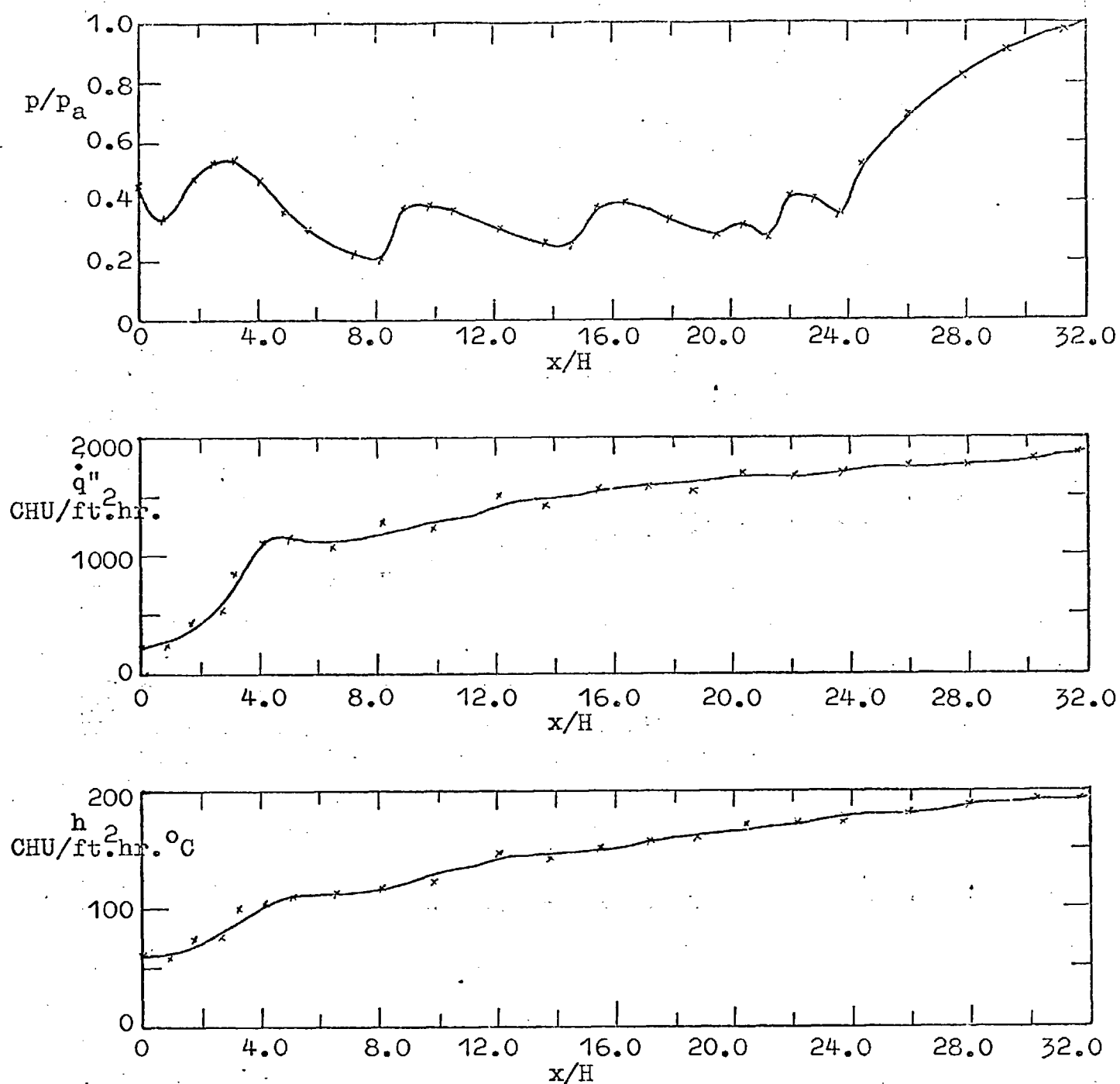
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 4.401$, $W_R = 0.05$, $s = 1/8''$.

Fig.A6.8b. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



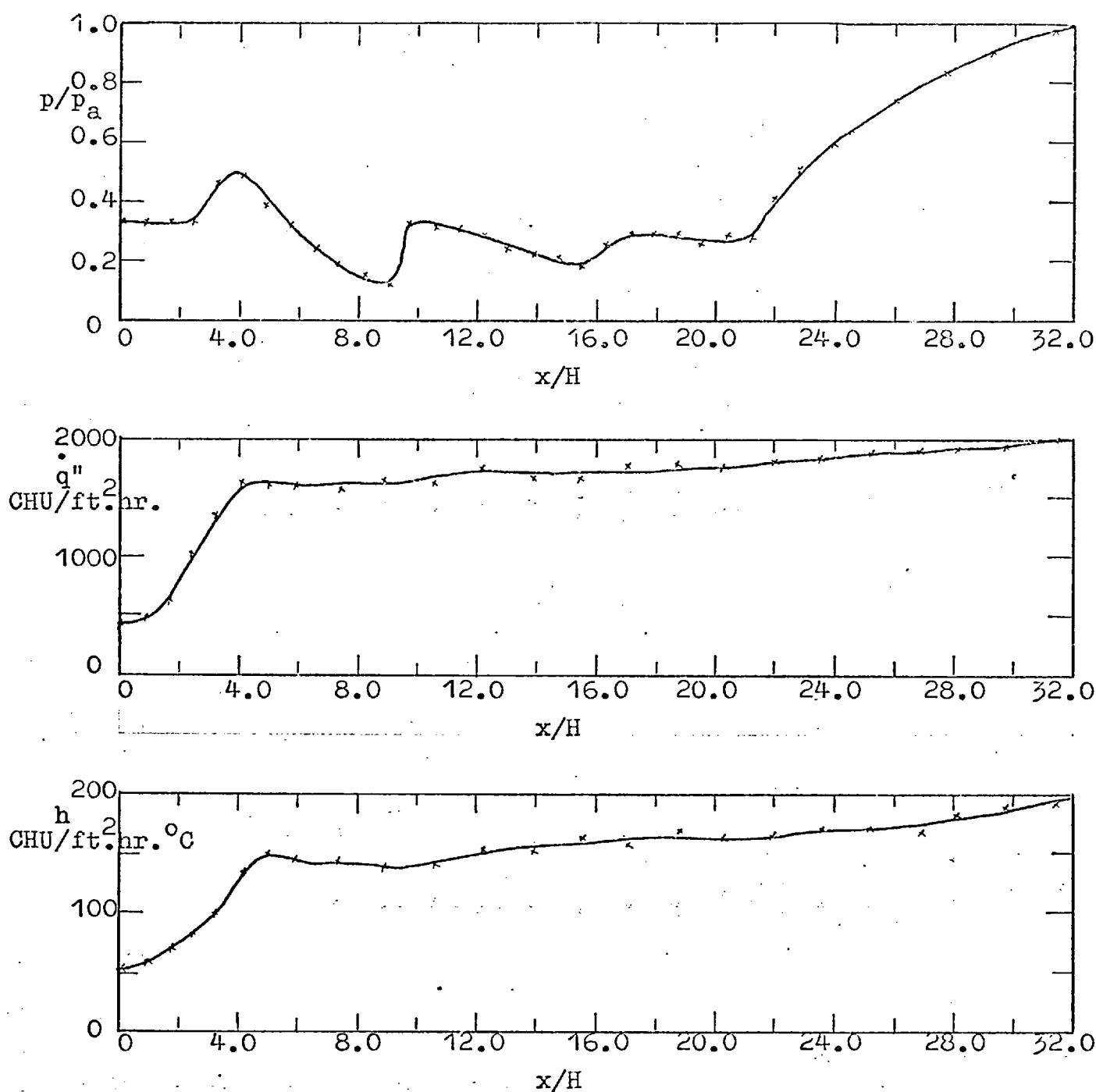
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_1/p_a = 4.401$, $W_R = 0.075$, $s = 1/8''$.

Fig.A6.8c. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



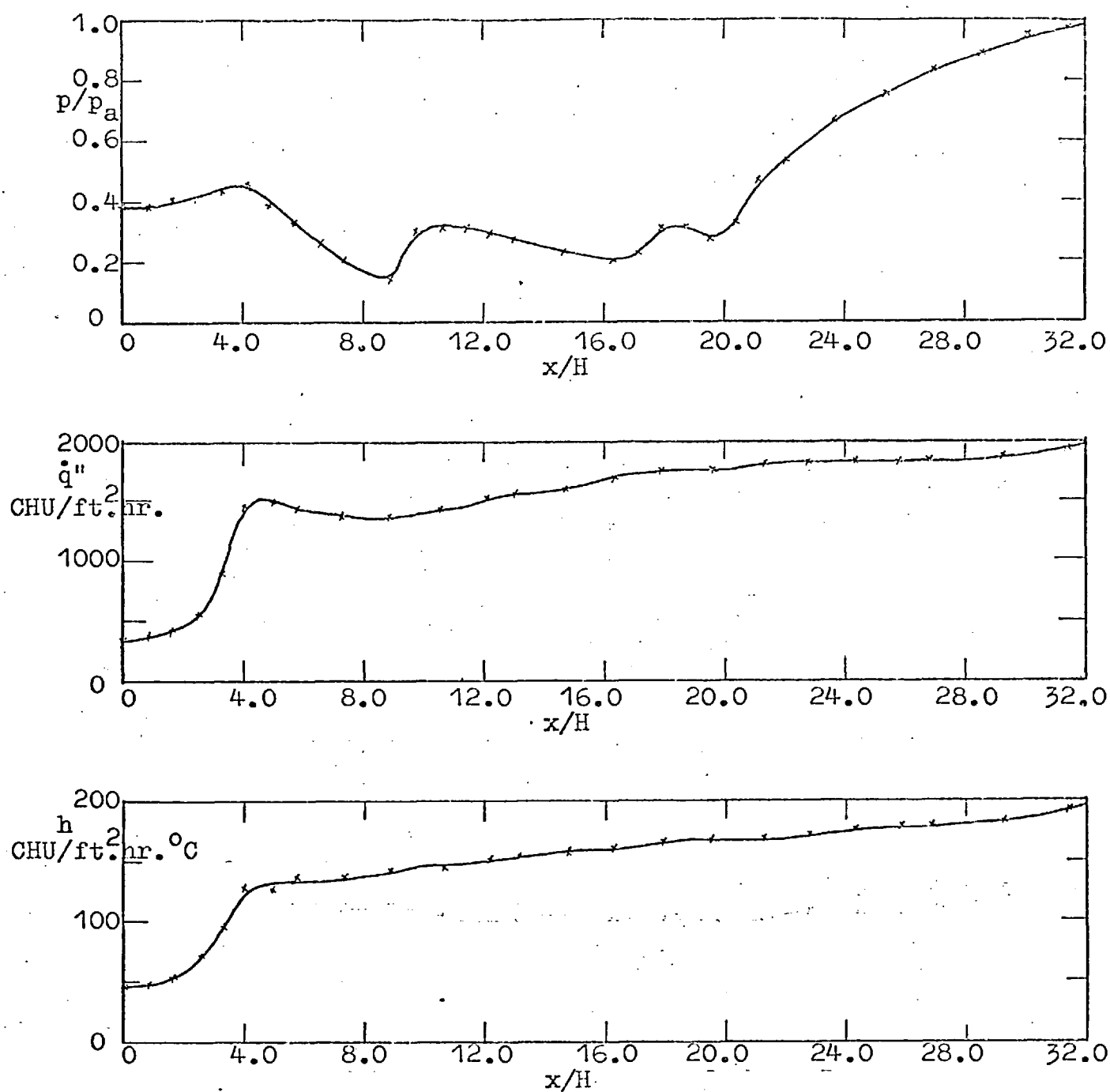
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 4.401$, $w_R = 0.1$, $s = 1/8''$.

Fig. A6.8d. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



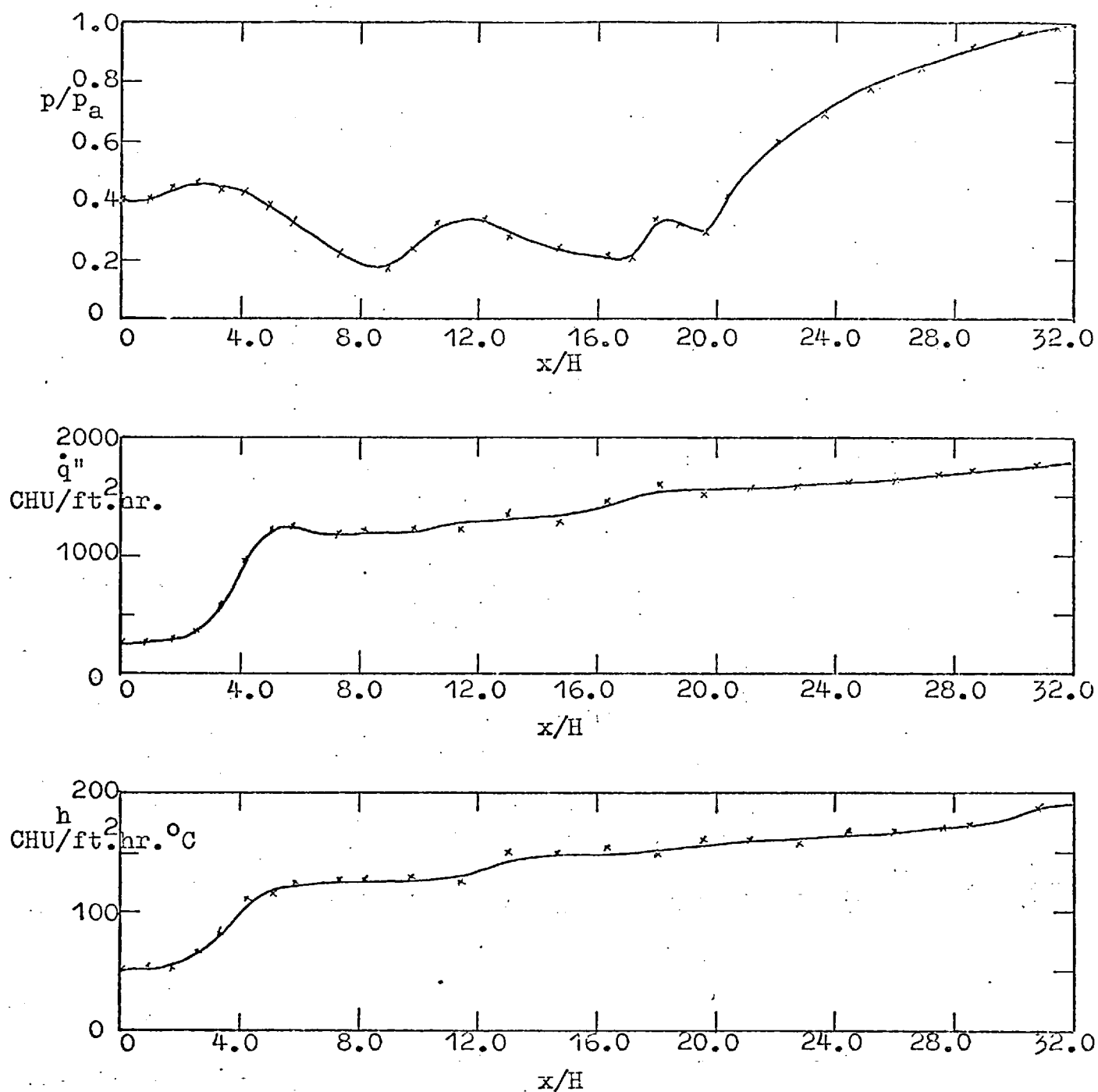
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 4.401$, $W_R = 0.025$, $s = 3/16"$.

Fig. A6.9a. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



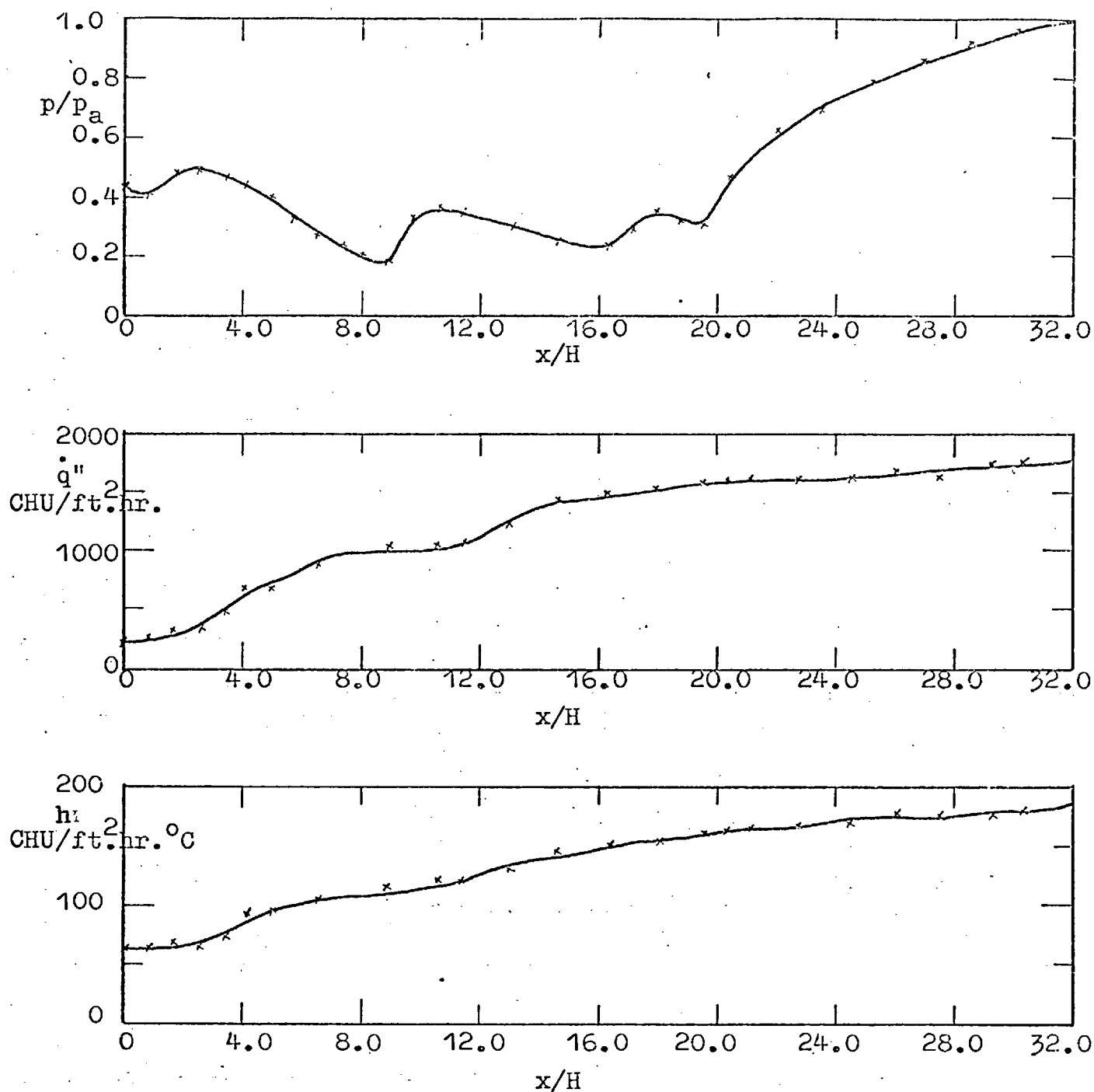
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 4.401$, $W_R = 0.05$, $s = 3/16''$.

Fig.A6.9b. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



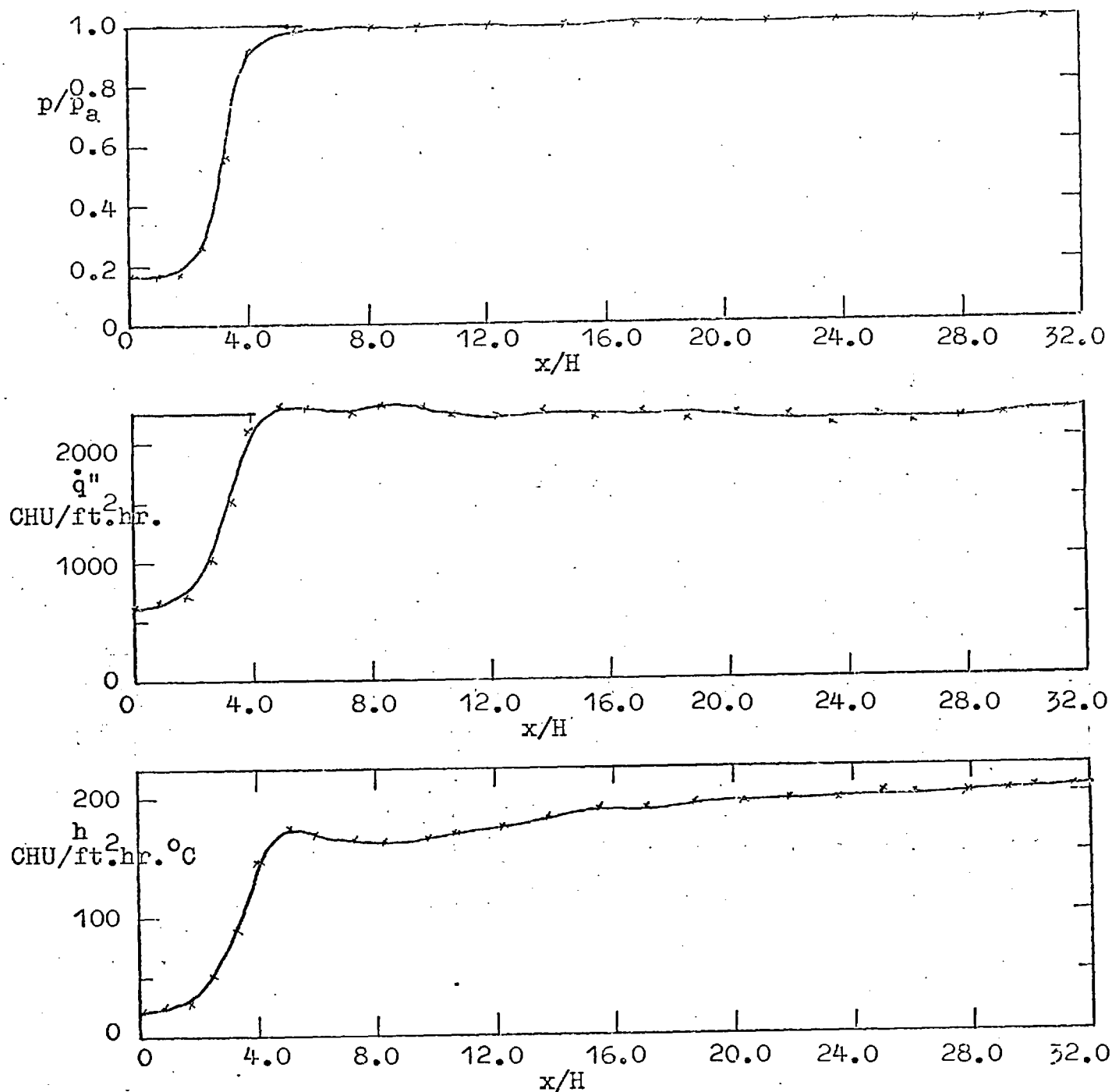
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 4.401$, $W_R = 0.075$, $s = 3/16"$.

Fig.A6.9c. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



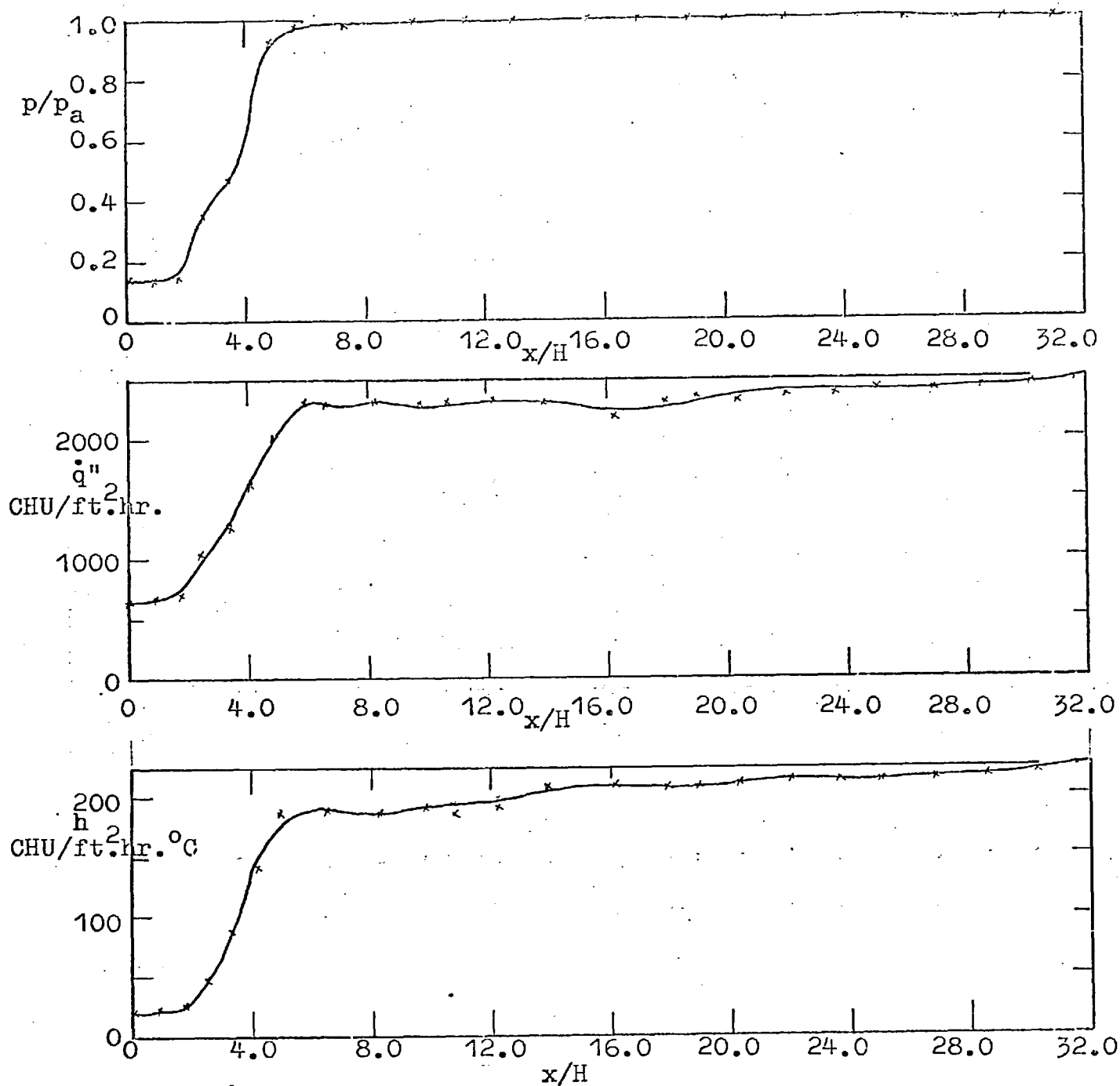
$Ma_1 = 1.515$, $A_2/A^* = 3.671$, $P_i/p_a = 4.401$, $W_R = 0.1$, $s = 3/16"$.

Fig.A6.9d. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



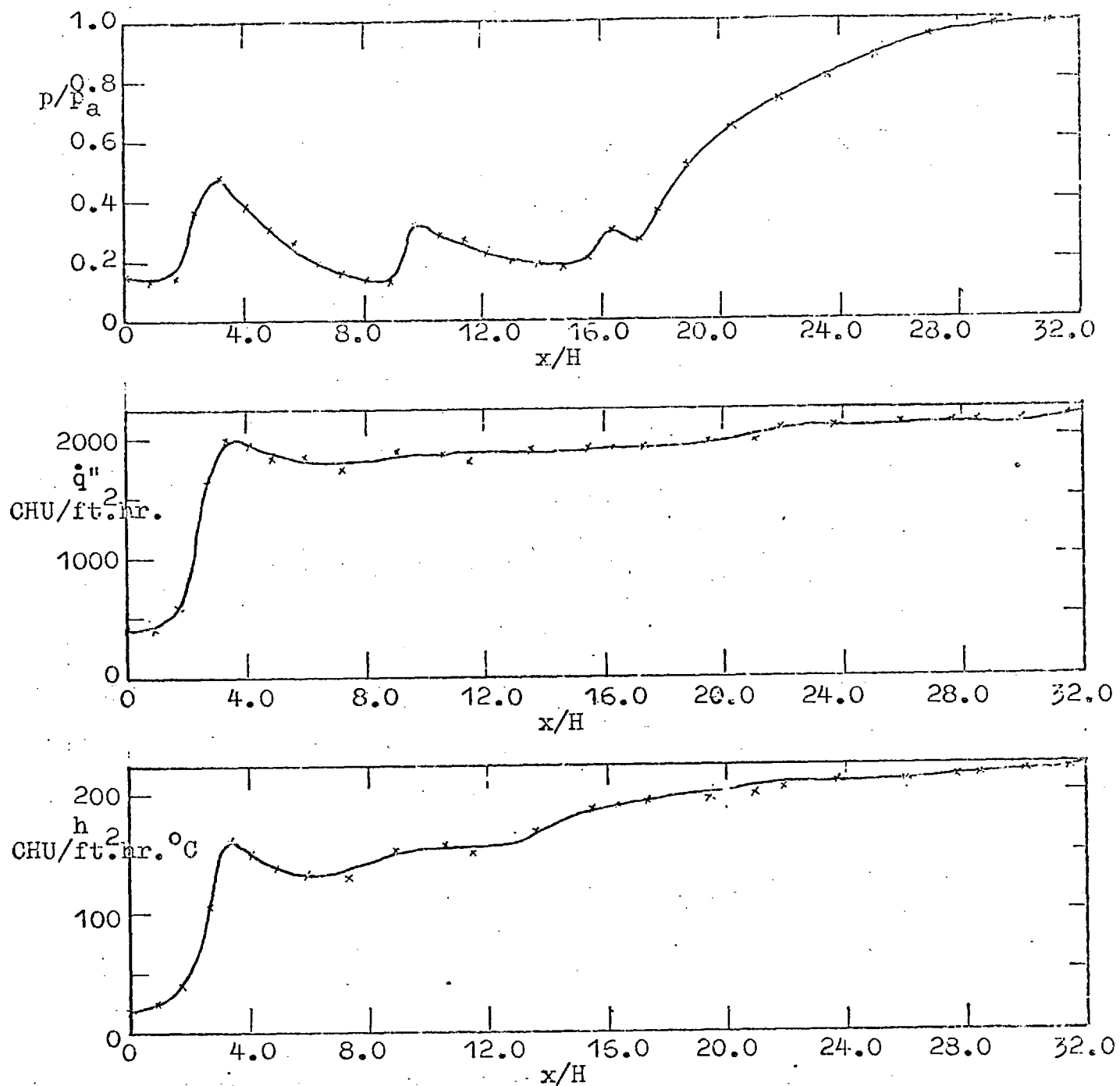
$Ma_1 = 1.7$, $A_2/A^* = 4.223$, $P_i/p_a = 4.129$, $w_R = 0.0$, $p_2/P_i = 0.042$

Fig. A6.10. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Unstarted Diffuser Length.



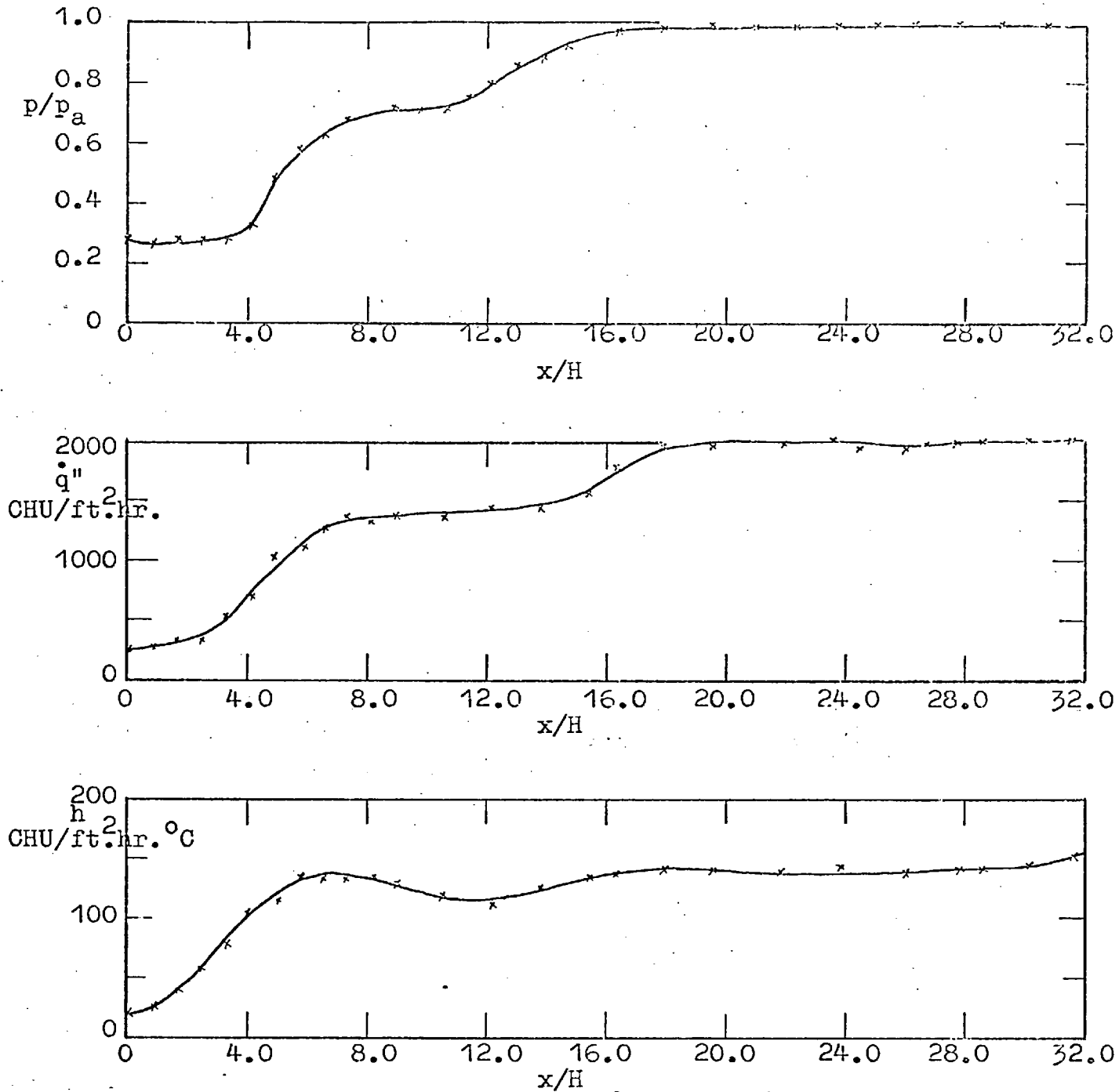
$Ma_1 = 1.7$, $A_2/A^* = 4.223$, $P_i/p_a = 4.401$, $W_R = 0.0$, $p_2/P_i = 0.032$

Fig. A6.11. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Unstarted Diffuser Length.



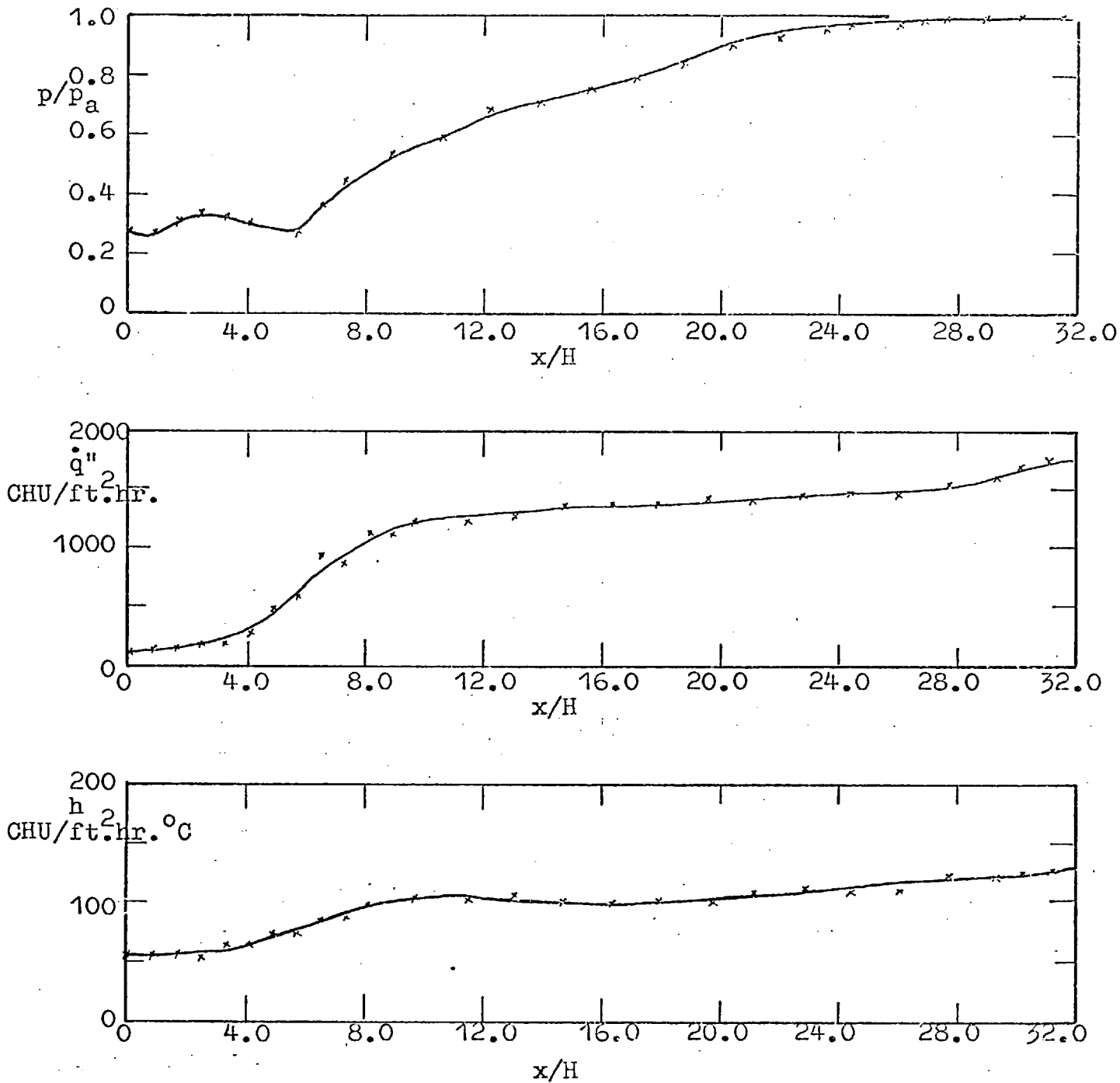
$Ma_1 = 1.7$, $A_2/A^* = 4.223$, $P_i/p_a = 4.537$, $W_R = 0.0$, $p_2/P_i = 0.032$.

Fig. A6.12. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Started Diffuser Length.



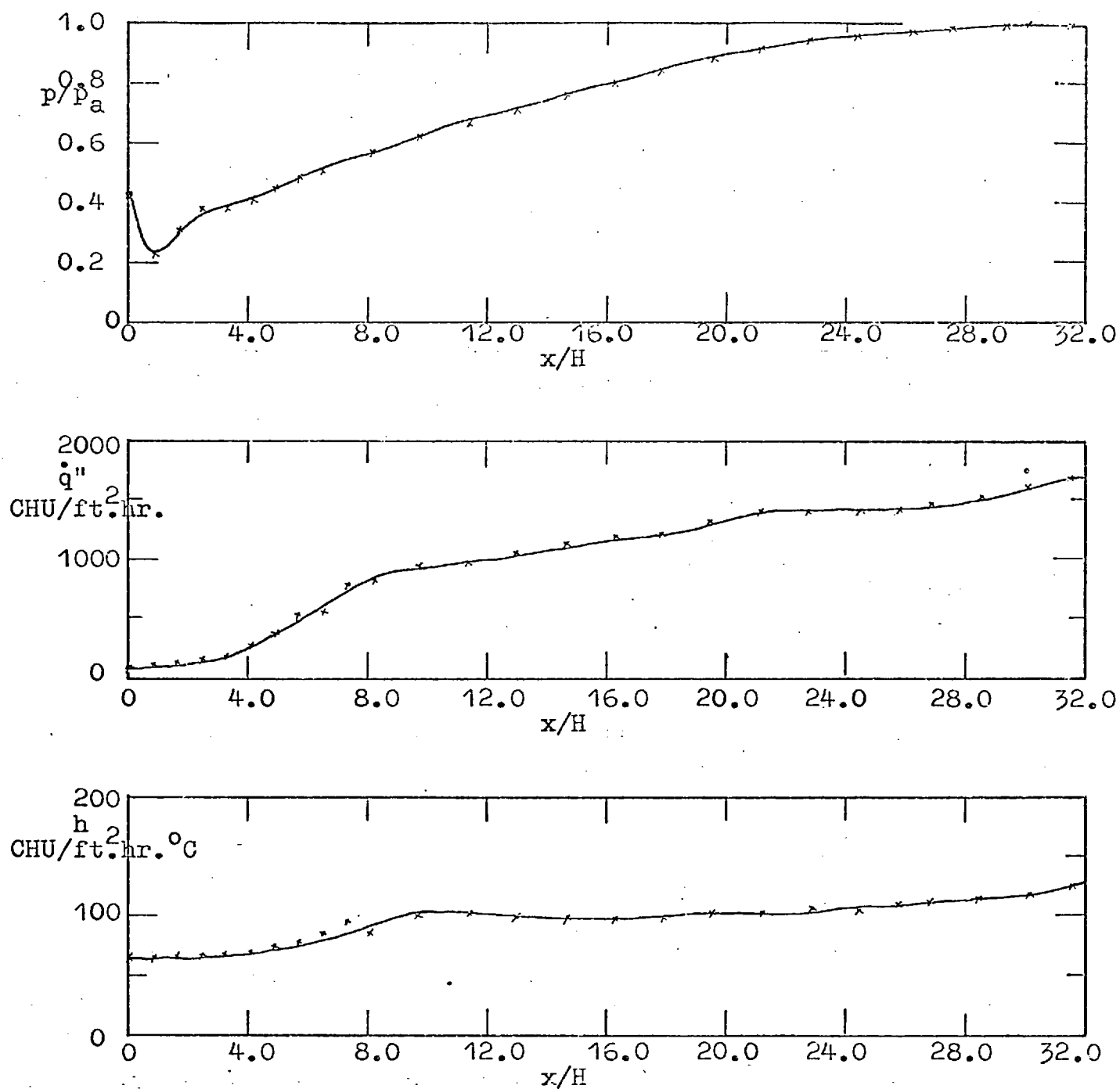
$Ma_1 = 1.7$, $A_2/A^* = 4.223$, $P_i/p_a = 3.857$, $W_R = 0.05$, $s = 3/16"$.

Fig.A6.13a. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



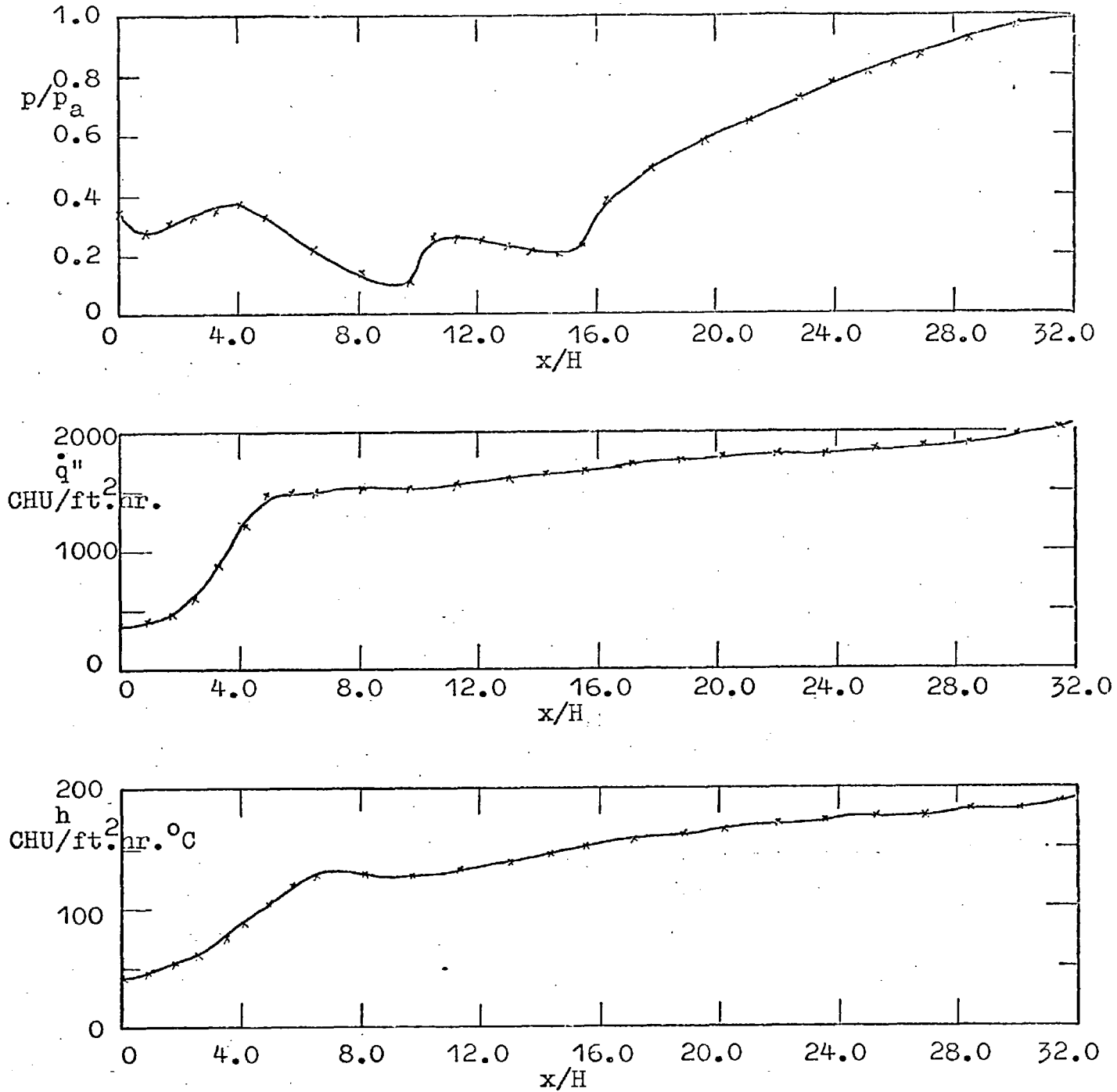
$Ma_1 = 1.7$, $A_2/A^* = 4.223$, $P_i/p_a = 3.857$, $w_R = 0.1$, $s = 3/16''$.

Fig.A6.13b. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser-Ejector Length.



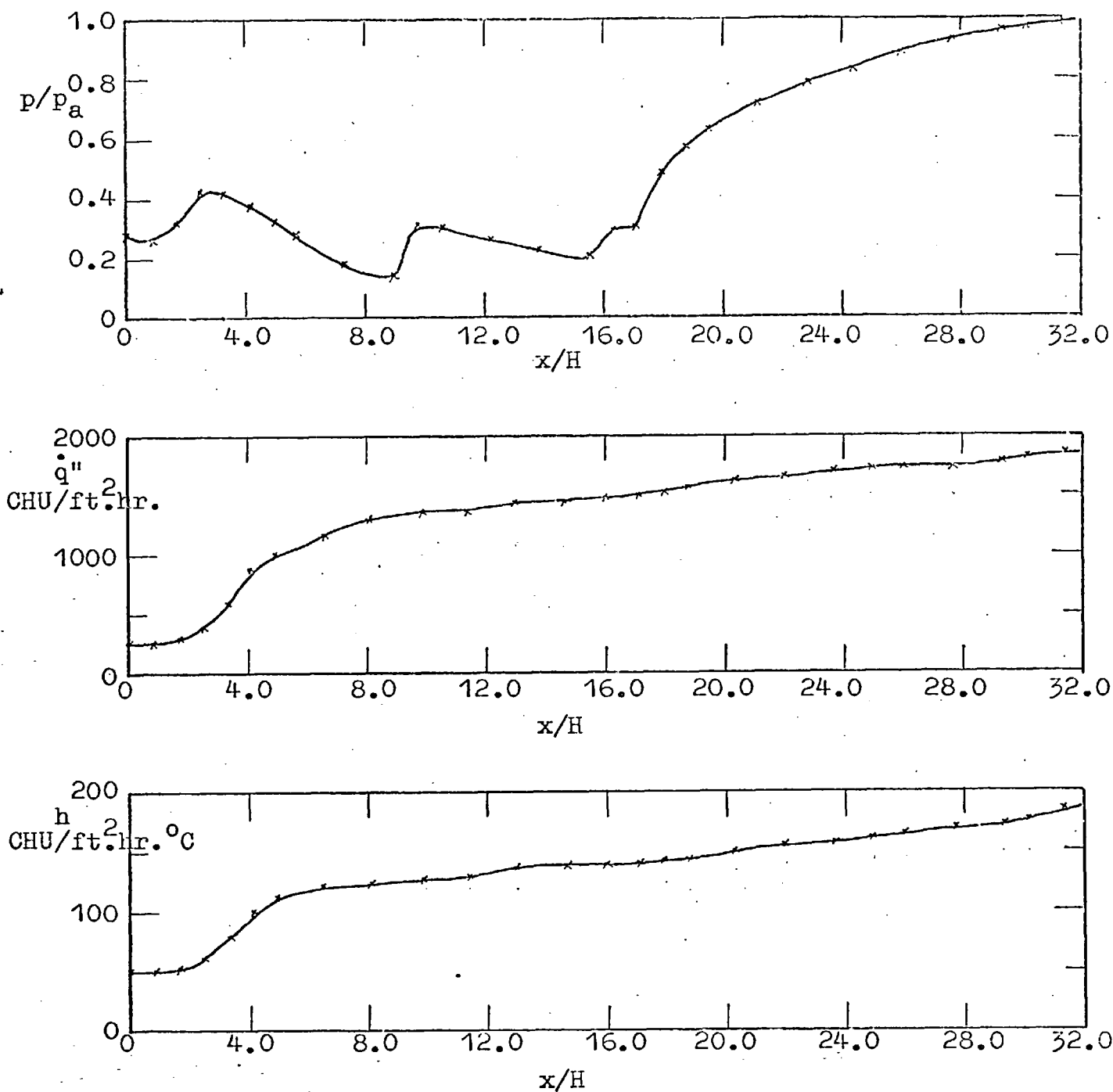
$Ma_1 = 1.7$, $A_2/A^* = 4.223$, $P_i/p_a = 3.857$, $w_R = 0.15$, $s = 3/16"$.

Fig.A6.13c. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



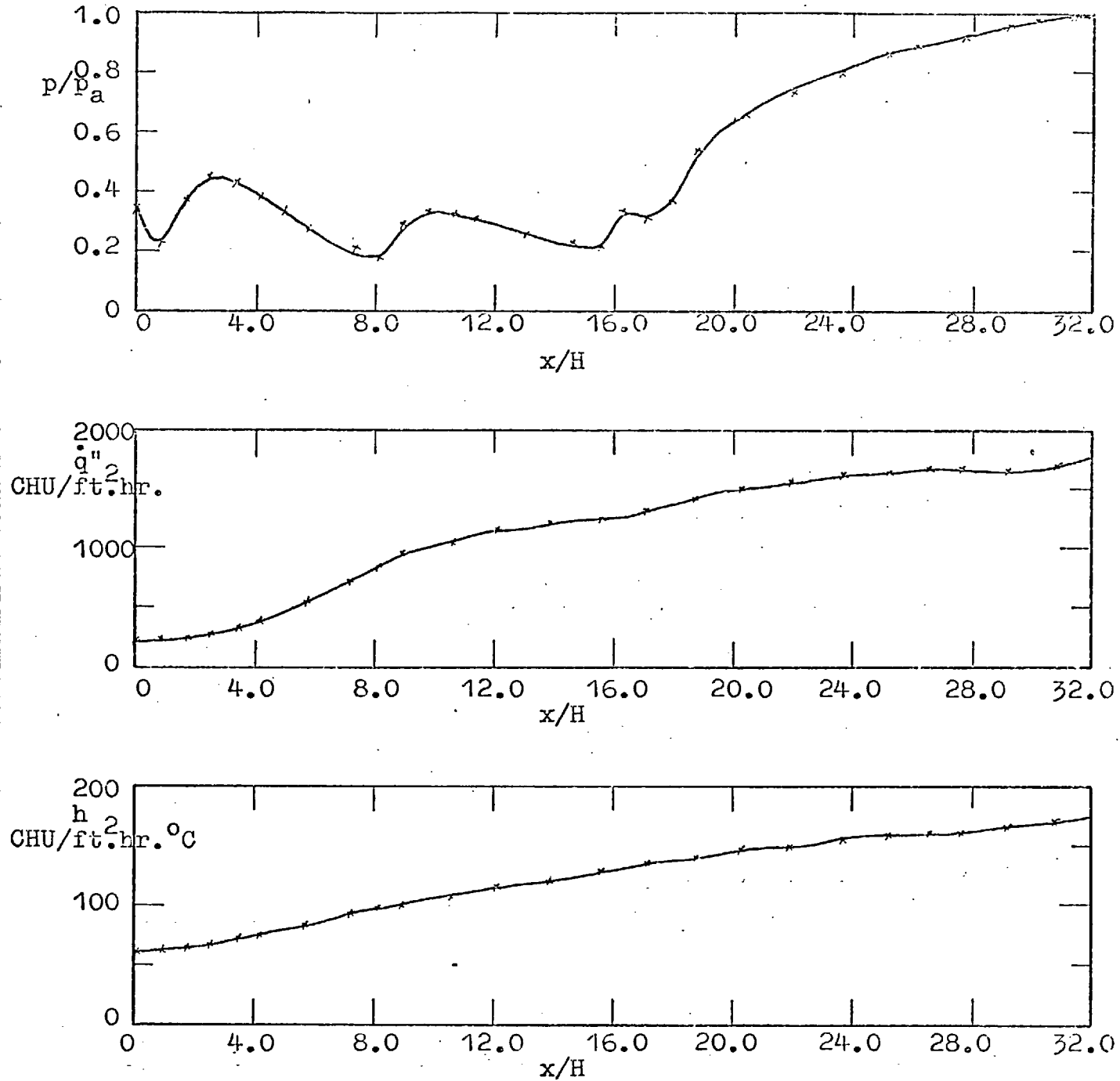
$Ma_1 = 1.7$, $A_2/A^* = 4.223$, $P_i/p_a = 4.401$, $W_R = 0.05$, $s = 1/8''$.

Fig. A6.14a. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



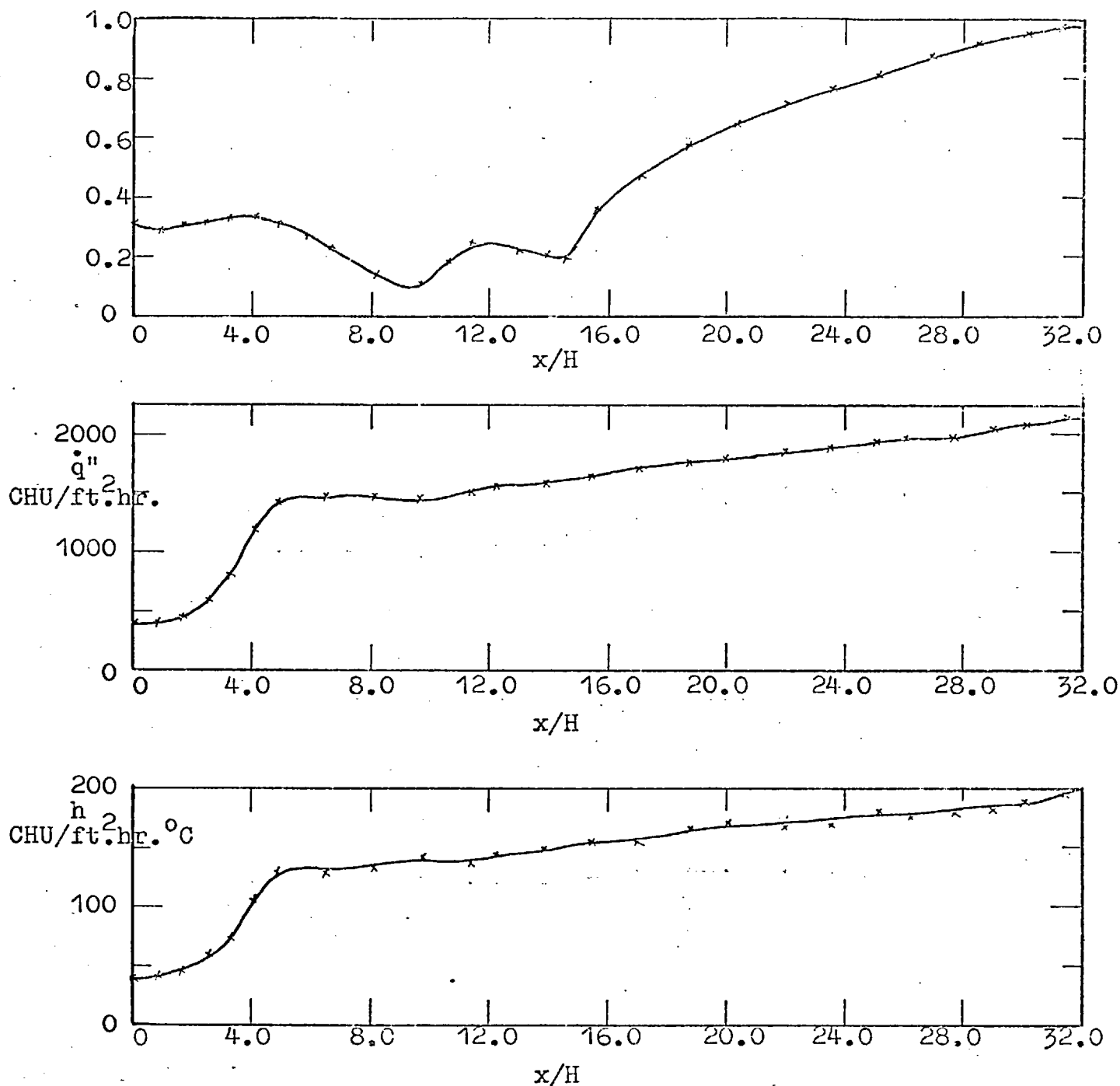
$Ma_1 = 1.7$, $A_2/A^* = 4.223$, $P_1/p_a = 4.401$, $W_R = 0.1$, $s = 1/8''$.

Fig. A6.14b. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



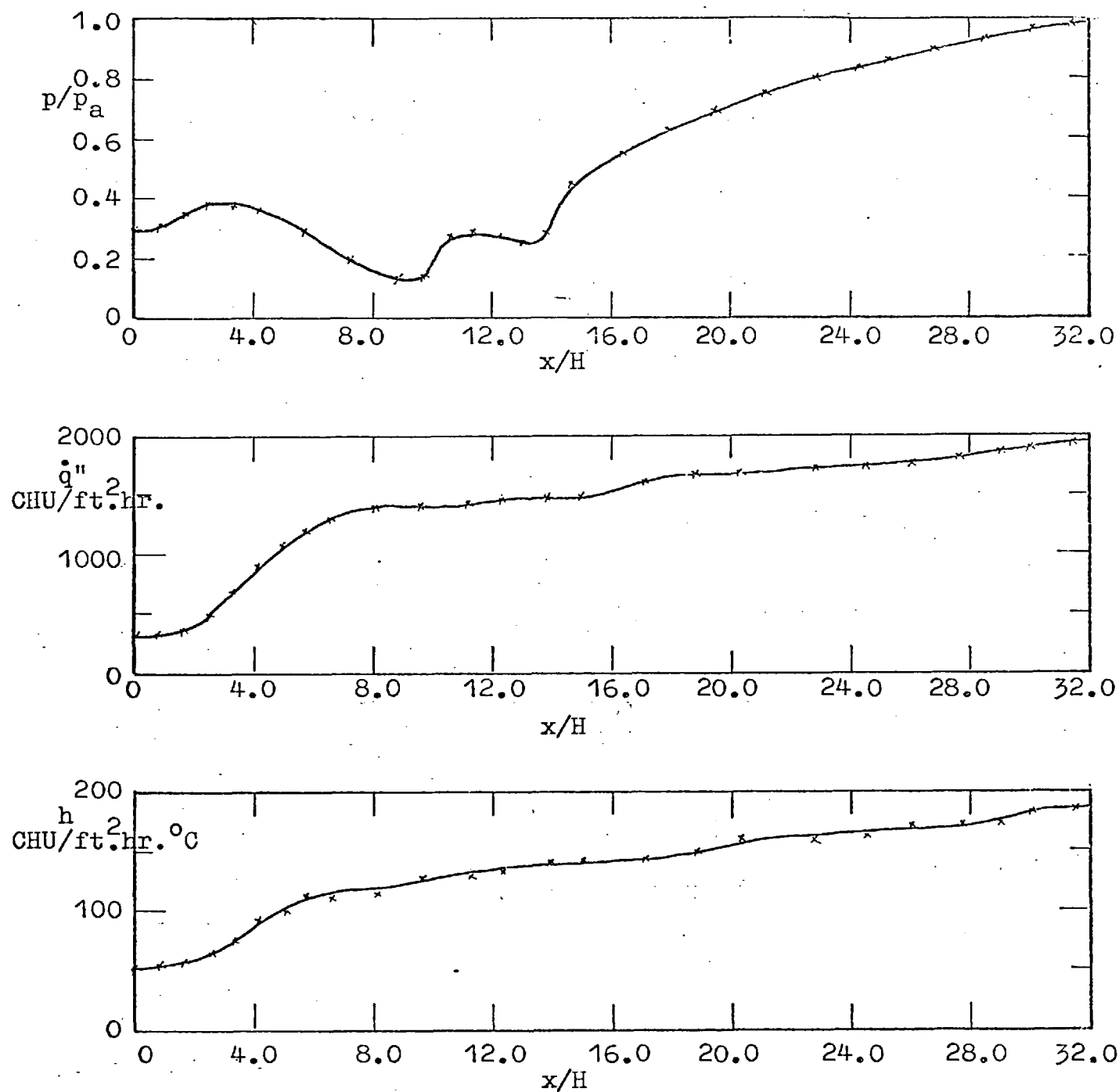
$Ma_1 = 1.7$, $A_2/A^* = 4.223$, $P_i/p_a = 4.401$, $W_R = 0.15$, $s = 1/8"$.

Fig. A6.14c. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



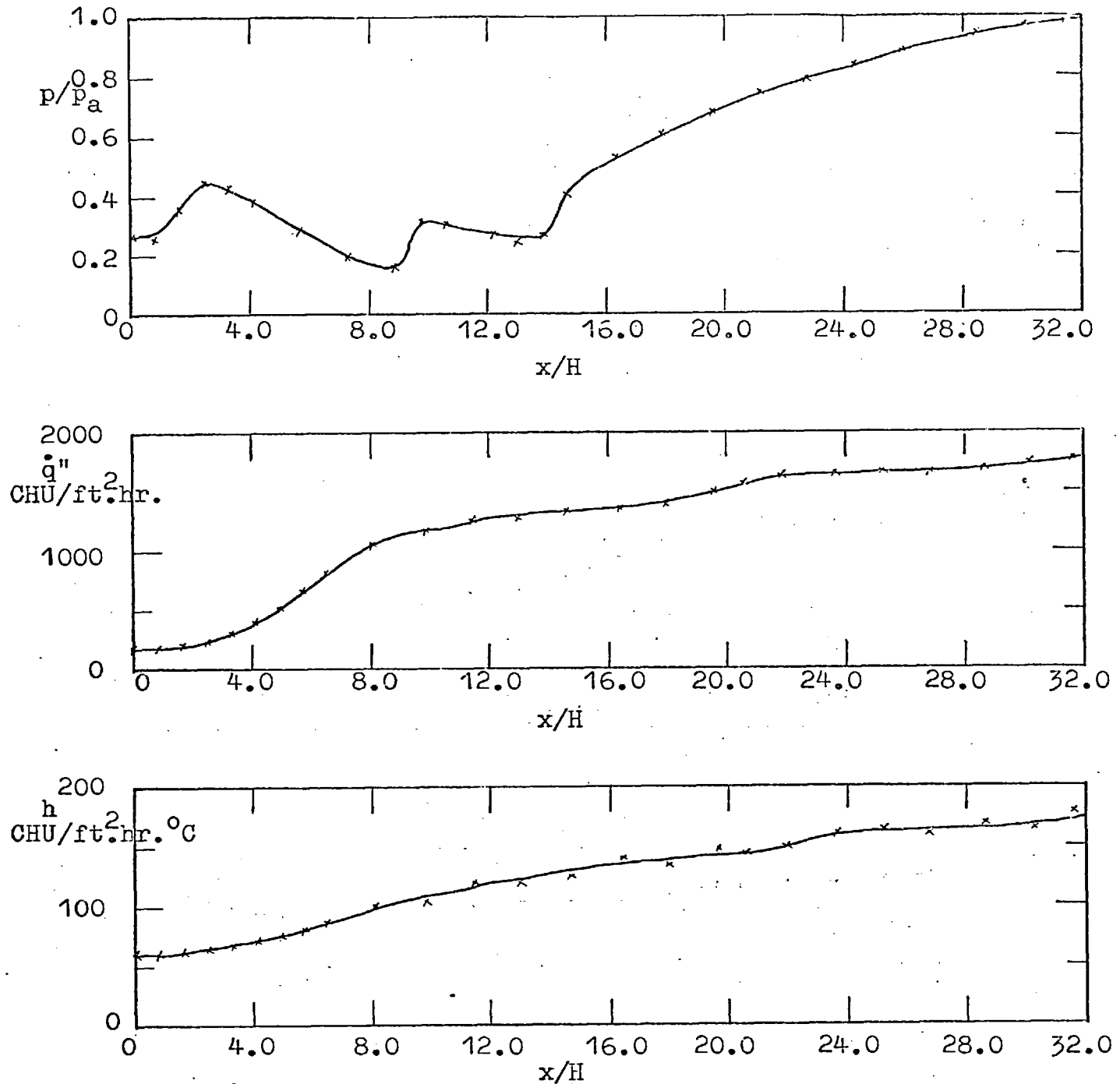
$Ma_1 = 1.7$, $A_2/A^* = 4.223$, $P_i/p_a = 4.401$, $w_R = 0.05$, $s = 3/16''$.

Fig. A6.15a. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



$Ma_1 = 1.7$, $A_2/A^* = 4.223$, $P_i/p_a = 4.401$, $w_R = 0.1$, $s = 3/16"$.

Fig.A6.15b.Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.



$Ma_1 = 1.7$, $A_2/A^* = 4.223$, $P_i/P_a = 4.401$, $W_R = 0.15$, $s = 3/16"$.

Fig.A6.15c. Variation of Wall Static Pressure, Heat Flux and Heat Transfer Coefficient Along Diffuser - Ejector Length.

IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

Exhibition Road, London, S.W.7.

Film cooling by air injection through a backward-facing
annular tangential slot in a supersonic axisymmetric parallel
diffuser *

by

T. Mukerjee and B.W. Martin

March 1968

SF/TN/13

* Also in the Proceedings of the 1968 HEAT
TRANSFER AND FLUID MECHANICS INSTITUTE,
A.F. Emery and C.A. Depew, Editors, Stanford
University Press, 1968.

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FILM COOLING BY AIR INJECTION THROUGH A BACKWARD-FACING ANNULAR TANGENTIAL SLOT INTO A SUPERSONIC AXISYMMETRIC PARALLEL DIFFUSER

by

T. Mukerjee* and B.W. Martin*

ABSTRACT

Measurements are presented of the effect on base pressure and wall temperature of secondary injection into an adiabatic parallel started diffuser through the sudden enlargement at the junction with the nozzle generating the supersonic primary flow. The range of mass ratio covers both subsonic and supersonic flow through the variable-area secondary port. The stagnation temperature of the primary flow is varied, while that of the secondary flow is kept constant.

With increasing mass ratio the base pressure passes first through a maximum and then a minimum to reach a plateau value greater than that for zero secondary flow. Correlations of film-cooling effectiveness with downstream distance from the point of secondary injection show mainly qualitative agreement with subsonic film-cooling predictions and for low mass-velocity ratios suggest greater effective lengths in supersonic flow. The optimisation of diffuser design is considered in the light of the investigation.

INTRODUCTION

Because over wide ranges of gas flow rates and stagnation temperatures the parallel supersonic diffuser is more efficient (and cheaper to fabricate) than its varying-area counterpart, it is used in large-scale engine test facilities to achieve some recompression of the high-velocity exhaust of a jet engine. The resultant simulation of higher altitude conditions than would otherwise be possible in a given test facility involves the discharge of the engine exhaust jet at a low ambient pressure. This is also the pressure at entry to the axisymmetric parallel diffuser located to accept and recompress the exhaust jet by a shock system.

The diffuser is usually of larger diameter than that of the propelling-nozzle outlet and hence there is a sudden annular enlargement in flow area at the diffuser entry. Through this annulus air from the test chamber may be ejected with the high-temperature exhaust gases and discharged to atmosphere at the diffuser outlet. Such secondary flow not only helps to maintain altitude conditions in the test chamber. It also provides film cooling for the diffuser walls which would otherwise be exposed to maximum exhaust jet temperature. However, against these benefits must be set the probable loss in diffuser static pressure recovery which at least initially may be expected to increase with the proportion of secondary fluid.

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When a slightly underexpanded supersonic jet enters a long axisymmetric parallel diffuser via a sudden enlargement, the flow (with no secondary injection) has a multi-celled structure with a terminal shock to compress the fluid to the higher outlet pressure. The jet then reattaches on the diffuser wall far downstream of the entry. As the blowing pressure ratio (P_i/p_a) is increased, the nozzle back pressure ratio (p_b/P_i) falls rapidly, the point of reattachment moves upstream and a strong normal shock appears within the first jet-cell. At sufficiently high P_i/p_a the normal shock breaks down into an oblique shock system extending over an increasing proportion of the diffuser length. With jet reattachment close to the diffuser entry plane, p_b/P_i now reaches a minimum and remains almost unaffected by any further increase in P_i/p_a . The diffuser is then said to have "started", having previously been in the "unstarted" condition.

Based on consideration of the boundary-layer momentum thickness at the nozzle exit, Martin and Mukerjee (1) have recently developed a method of predicting p_b/P_i and diffuser static pressure recovery for a given geometry and initial conditions; calculated values compare favourably with measurements for both the unstarted and started conditions.

With a sudden entry enlargement the flow at the nozzle exit and zone of reattachment is analogous to supersonic separated flow over a backward-facing step. Near reattachment of the separated shear layer very high wall heat transfer has been measured in turbulent or suspected transitional separation by Thomann (2), Gadd et al. (3), Naysmith (4) and Baker and Martin (5); the latter investigation employed a two-dimensional diffuser. In such circumstances the benefits of insulating the diffuser wall from the high temperature exhaust jet by a film of cool secondary air are likely to be considerable.

The need likewise to protect the flame-tube walls of gas-turbine combustion chambers and the components of jet and rocket engines has led to many theoretical and experimental studies of film cooling; unless otherwise stated, those mentioned below are all concerned with low-speed subsonic flow in both primary and secondary streams. The theories of Wieghardt (6), Tribus and Klein (7), Hartnett et al. (8), Seban and Back (9), Kutateladze and Leont'ev (10), Librizzi and Cresci (11) and Stollery and El-Ehwany (12) for calculating the adiabatic wall temperature T_{ad} with tangential secondary fluid injection, when the mass velocity ratio $V_R = \rho_c U_c / \rho_g U_g$ is below unity, are all based on a boundary-layer model and differ only in the intermediate stages of analysis. They necessarily assume mixing of coolant with mainstream and are strictly asymptotic solutions whose agreement with parallel experimental work and also with that of Chin et al. (13, 14), Papell and Trout (15), Seban (16) and Nishiwaki et al. (17) improves with increasing dimensionless distance downstream of the slot x/s .

The theoretical model adopted by Hatch and Papell (18) for $V_R < 1.0$ differs in supposing a discrete coolant film which does not mix with the main stream, and leads to predictions which accord well with their experimental measurements of T_{ad} near the

slot, such that $0 < x/s < 150$. Downstream mixing is taken into account by two empirical modifications to the initial theory.

Dannenberg (19) reported film cooling of a hemisphere in hypersonic flow ($Ma_G = 10$) by helium. Goldstein et al. (20) used both helium and air as coolants for film-cooling experiments in two-dimensional supersonic flow ($Ma_G = 3.01$) over a flat plate. They compared film cooling effectiveness η' (defined by equation 6 below) obtained from measured adiabatic wall temperatures with the incompressible film cooling theories of Kutateladze and Leont'ev (10), Librizzi and Cresci (11) and Tribus and Klein (7), and found satisfactory agreement with the latter (7) for $0.41 > V_R > 0.12$ only. Redeker and Miller (21) report film-cooling experiments on a hemi-cylinder slab in a hypersonic stream ($Ma_G = 16$). They modify the theory of Hatch and Papell (18) to the conditions of their experimental programme to secure agreement with their measurements. In supersonic flow no satisfactory analytical solution of film cooling is yet available.

The already-mentioned adverse effect of secondary injection on p_b/P_i and hence on the diffuser operating range may be inferred from the work of Wimbrow (22), Fuller and Reid (23), Chow (24) and Ginoux (25). Despite the extensive relevant literature, the optimum secondary injection rate which combines sufficient film cooling with acceptable diffuser/ejector performance cannot yet be predicted, especially under the highly non-uniform conditions in the initially-supersonic primary flow. This paper therefore reports measurements with air on an axisymmetric scaled-down model of the diffusers used in engine test facilities.

Secondary air is naturally aspirated from atmosphere into the nozzle base region through an annular slot. Measurements of wall static pressure distribution, nozzle back pressure and adiabatic wall temperature are presented as functions of dimensionless slot width s/H , downstream distance x/H and secondary mass ratio $W_R = \dot{m}_c/\dot{m}_G$ (which is here a convenient alternative to V_R), together with a description of the flow in the base region. Empirical correlations for film cooling effectiveness are also derived and compared with other recommendations and findings for subsonic and supersonic film cooling.

APPARATUS AND EXPERIMENTAL PROCEDURE

The apparatus in Fig. 1 basically consists of an axisymmetric convergent-divergent nozzle discharging air into a $2\frac{7}{8}$ in. inside diameter and 20 in. long parallel diffuser-ejector made of low conductivity 'Tufnol'. From the nozzle the air abruptly expands into the diffuser-ejector where it is compressed by a shock system followed by subsonic diffusion to atmospheric pressure at the diffuser exit.

Actual nozzle outlet Mach numbers of 1.52 and 1.7 are obtained by using nozzles corresponding to values of the area ratio A_D/A^* of 3.67 and 4.22 respectively. A two-stage rotary compressor supplies air continuously at 75 lbf/in² abs. up to a maximum of 4.0 lb/s. to the nozzle through a throttle valve. The air flow approaches the nozzle through a 3.0 in. inside diameter

pipe of 36 in. unobstructed length followed by a 9 in. length containing honeycomb flow straighteners.

The primary stream total temperature may be varied between 65°F and 212°F by partially or wholly by-passing the compressor aftercoolers, the upper limit being determined by the need to avoid distortion of the Tufnol. Measurements were made at these limiting temperatures and also at 140°F.

The secondary air is drawn from ambient atmosphere at 65°F through an orifice plate calibrated according to BS 1042 (1943) and a throttle valve into an air-tight chamber of capacity 9 ft³. It then enters the nozzle base region without preheating through the annular slot shown in Fig. 1. The base height H is maintained constant at 0.61 in., while the slot width s may be altered by varying the size of the ring screwed on the nozzle. Three different rings are used to obtain values of s of 1/16 in., 1/8 in. and 3/16 in.

The 5/16 in. thick diffuser wall is further insulated by 2½ in. thick fibre-glass to minimise heat loss. The adiabatic wall temperature is measured by calibrated copper-constantan thermocouples embedded flush with the diffuser-ejector inner wall. To minimise errors by conduction along the leads the thermocouple wires are led through the isothermal zone formed by a 3 in. long groove situated 1/16 in. below the inner surface of the diffuser-ejector wall.

The primary stream stagnation temperature at the nozzle entry T_i and the secondary air stagnation temperature T_c in the box are also measured by thermocouples. Both the nozzle and the pipe leading to it are well insulated by fibreglass and asbestos rope. All thermocouple outputs are measured to within ± 0.015 mV by Solartron Digital Voltmeter.

The thermocouple and wall static pressure taps are located at 1/4 in. pitch over a 5 in. length of the diffuser-ejector measured from the nozzle exit plane; over the remaining length they are at ½ in. pitch. The nozzle base pressure p_b is measured by two 3/64 in. dia. static pressure taps located in the nozzle ring at opposite ends of a mean diameter, as shown in Fig. 1.

For given s and Ma_G , the initial test was under isoenergetic conditions with $T_G = T_c = 65^\circ\text{F}$. When the diffuser-ejector was started, P_i was controlled at 50 lbf/in². The desired mass ratio W_R was obtained by regulating the secondary air throttle valve; the adiabatic wall temperature T_{ad} , the secondary air stagnation pressure P_c , the base pressure p_b and the wall static pressure distribution p_x were then recorded. Further tests were carried out for higher values of T_G and the entire sequence repeated for other values of s and Ma_G .

Before each run the secondary air pipe joints were checked for leaks to which the pressure measurements were particularly sensitive at small injection rates. Up to 12 min. were allowed to obtain a steady state before recording any observation. The accuracy of temperature and pressure measurements was within

± 1.5 per cent, and for secondary air flow measurements it was estimated to be within ± 3 per cent.

DISCUSSION OF RESULTS

Effect of secondary air on the flow field in the base region.

Without secondary flow, the minimum nozzle back pressure ratio (p_b/P_i) is reached only when the diffuser is started. As shown in Fig. 2a, the supersonic stream then leaving the nozzle with finite boundary-layer thickness undergoes sudden expansion which turns the flow through the appropriate Prandtl-Meyer angle. The boundary layer becomes a free shear layer below which lies the separated region wherein the velocities are considered to be negligibly small and the pressure p_b (marked as point 1 in Fig. 3a) is uniform.

The shear layer initially entrains fluid from the separated region. As the shear layer approaches the diffuser wall to reattach it is compressed and turned by the oblique shock to flow along the wall. Part of the shear layer reverses into the separated region such that the mass of fluid therein is conserved. The remainder of the shear layer negotiates the pressure rise to form the boundary layer downstream of reattachment. As already stated, a detailed theoretical analysis of the above flow field has been reported by the authors elsewhere (1).

Fig. 2b shows the flow pattern observed by Fuller and Reid (22) at low P_c/P_i . Their interpretation is that part of the air entrained by the main flow is supplied by the secondary stream. This reduces the size of the separated bubble to a region adjacent to the diffuser wall and p_b/P_i increases with P_c/P_i through point 2 as shown in Fig. 3a. This trend continues until the secondary air can just escape without entering the mixing region as shown in Fig. 2c. Here p_b/P_i is a maximum, shown as point 3 in Fig. 3a, the secondary stream being still subsonic.

Further increases in P_c/P_i increase the velocity of the secondary stream until it becomes sonic at the slot. Thereafter an expansion fan originates at K (Fig. 2d) and the secondary flow downstream of the base is supersonic. Increased momentum transfer by viscous mixing to the pair of opposed vortices formed in the region bounded by the base and the two streams generates an increased adverse pressure gradient along KJ and IJ, so that p_b decreases relative to p_J .

At J both the flow direction and the pressure p_J , which is the same for both streams, can be calculated for given values of P_c and P_i . For the constant P_i of the present investigation it can be shown that p_J increases with P_c . If the decrease in p_b/p_J is greater than the increase in p_J/P_i , then p_b/P_i decreases with increasing P_c/P_i as shown by the reduction from 3 to 4 in Fig. 3a.

In the region between 4 and 5 it appears that while initially this trend is reversed, any subsequent decrease in p_b/p_J is exactly offset by the increase in p_J/P_i and p_b/P_i becomes constant and independent of P_c/P_i and therefore of p_J/P_i . By con-

trast, Fuller and Reid (23) and Chow (24) both found by experiment that, as indicated qualitatively in Fig. 3a, between 4 and 5 the increase in p_J/P_i always exceeded the decrease in p_b/p_J , and no plateau in p_b/P_i was achieved. However, the divergence between their curves suggests different offsets between p_J/P_i and p_b/p_J .

These discrepancies may be attributed to the different geometries employed and the consequent differences in the complex shock pattern illustrated in Fig. 2d, especially when shock interactions and reflections from the wind-tunnel or diffuser wall are taken into account. These will determine the degree of interference with the flow downstream of the nozzle and its effect on p_J and p_b , a conclusion which is supported by the schlieren observations of Dedman (26).

Figs. 3b and 3c show respectively the measured dependence of p_b/P_i on P_c/P_i and W_R with dimensionless slot width s/H as parameter for $Ma_G = 1.52$. Only for choked secondary flow is P_c/P_i linearly related to W_R i.e. for greater values of these parameters than those corresponding to maximum p_b/P_i . For given s/H the best ejector operating condition will be the value of W_R (or P_c/P_i) which, with adequate film cooling, gives either least p_b/P_i , or more generally, that value at which the plateau in p_b/P_i is first attained. For this condition the measurements in Fig. 3c for $Ma_G = 1.52$ (and also the unrepresented corresponding measurements for $Ma_G = 1.7$) are correlated by the empirical relations

$$W_R \geq 0.69 s/H, \quad (1)$$

$$\lambda = -4.17 + 7.87\sigma - 2.55\sigma^2, \quad (2)$$

for $\sigma > 1$, where

$$\lambda = (p_b/P_i)_5 / (p_b/P_i)_1 \quad (3)$$

$$\text{and } \sigma = (1 - \frac{s}{H})^{-1}. \quad (4)$$

Equation (2) implies that λ has an extrapolated but unconfirmed maximum of 1.894 for $s/H = 0.352$.

The effects of secondary air injection on diffuser-ejector wall static pressure p_x/p_a and adiabatic wall temperature T_{ad}/T_i are shown in Figs. 4 and 5 respectively for $Ma_G = 1.52$ and $s/H = 0.2$ up to $x/H = 12$. The non-dimensional distance x/H is measured downstream from the slot. The pressure p_x/p_a in Fig. 4 is most sensitive to changes in mass ratio W_R in regions of sudden pressure increase caused by compression shocks; elsewhere the influence of mass ratio is small. In Fig. 5 the peak in T_{ad}/T_i with zero secondary flow corresponds to the zone of re-attachment of the free shear layer. Even secondary flows as small as 2.5% of the main flow appear sufficient to prevent re-attachment and thus to eliminate the temperature peak. Further increases in W_R extend the range of x/H over which the value of T_{ad}/T_i at $x = 0$ is maintained.

Film cooling effectiveness and correlations.

To determine the effectiveness of secondary air injection in cooling the diffuser-ejector wall, and also to correlate the data, a film cooling effectiveness is defined as follows:

$$\eta_I = \frac{(T_{ad_x}/T_G) - (T_{ad_x}/T_G)_{iso}}{(T_{ad_0}/T_G) - (T_{ad_0}/T_G)_{iso}}, \quad (5)$$

where T_{ad_x} is the adiabatic wall temperature at distance x from the slot and T_{ad_0} is the adiabatic wall temperature at $x = 0$; T_G is the primary stream stagnation temperature. The subscript 'iso' indicates the isoenergetic condition when T_G is equal to the secondary air stagnation temperature T_c and its absence implies that T_G differs from T_c .

The definition is general in providing not only for the present condition of constant T_c ; for the special case when T_G is constant and only T_c is varied, such that $T_c \geq T_G$, equation (5) reduces to

$$\eta' = \frac{T_{ad_x} - (T_{ad_x})_{iso}}{T_{ad_0} - (T_{ad_0})_{iso}}, \quad (6)$$

which is the definition used by Goldstein et al. (21). With uniform property, incompressible primary and secondary flows $T_{ad} = T_G$ under isoenergetic conditions for all x . When T_c differs from T_G , $T_{ad_0} = T_c$ and equations (5) and (6) reduce to the familiar subsonic definition of effectiveness given by

$$\eta = \frac{T_{ad_x} - T_G}{T_c - T_G}. \quad (7)$$

An equally suitable alternative parameter for film-cooling effectiveness to that defined in equation (5) for compressible flow is

$$\eta_{II} = \frac{(T_{ad_x}/T_i)_d - (T_{ad_x}/T_i)_e}{(T_{ad_x}/T_i)_d - (T_{ad_0}/T_i)_e}, \quad (8)$$

where subscripts d and e refer respectively to the adiabatic diffuser without and with secondary injection. The readily-discernible effect of secondary injection on the diffuser wall temperature with no film cooling may be of advantage to the designer.

When η_I is plotted against x/H , the observations fall on separate curves which imply dependence on secondary air flow rate and slot width. The correlation is improved if instead η_I is plotted against a parameter X_1 defined by

$$X_1 = \zeta W_R^{-0.4}, \quad (9)$$

where ζ is a dimensionless length such that

$$\zeta = \left(\frac{x}{H}\right) \left(\frac{H}{s}\right)^{0.263} . \quad (10)$$

As shown on logarithmic co-ordinates in Figs. 6 and 7 (where both η_I and η_{II} are plotted), even this procedure necessitates separate consideration of small and large mass ratios, though the illustrations indicate similar trends. The effectiveness is close to unity and therefore effectively independent of X_1 in the range up to about $X_1 = 10$; with larger X_1 , the effectiveness diminishes uniformly but at a greater rate for the small than for the large mass ratios. In this range least-squares analysis yields the following power relationships:-

a) For η_I , $0 < W_R < 0.13$ and for η_{II} , $0 < W_R < 0.075$ in Fig. 6,

$$\eta_I = \eta_{II} = 6.16 X_1^{-0.8} , \quad (11)$$

such that the limiting dimensionless distance downstream of the slot ζ_{eff} for a film-cooling effectiveness of unity is given by

$$\zeta_{eff} = 9.6 W_R^{0.4} . \quad (12)$$

b) For η_I , $0.13 < W_R < 0.25$ and for η_{II} , $0.075 < W_R < 0.25$ in Fig. 7

$$\eta_I = \eta_{II} = 2.76 X_1^{-0.43} , \quad (13)$$

such that ζ_{eff} is then given by

$$\zeta_{eff} = 10.48 W_R^{0.4} . \quad (14)$$

The correspondence between (12) and (14) is so close that a satisfactory approximate expression for ζ_{eff} over the range $0 < W_R < 0.25$ is given by the straight line through the experimental points in Fig. 8, such that

$$\zeta_{eff} = (x/H)_{eff} \left(\frac{H}{s}\right)^{0.263} = 10 W_R^{0.4} . \quad (15)$$

For the present geometrical configuration the mass and mass velocity ratios are related by the equation

$$V_R = 0.4 W_R (H/s) . \quad (16)$$

The results in Fig. 8 may thereby be converted to the form

$$(x/H)_{eff} = 14.42 V_R^{0.4} (s/H)^{0.663} , \quad (17)$$

which is convenient for comparison with those of Goldstein et al. (20) as shown in Fig. 9. Here the effectively-cooled length of diffuser-ejector $(x/H)_{eff}$ is logarithmically plotted against V_R

with s/H as parameter.

For $0.3 > s/H > 0.1$, $(x/H)_{\text{eff}}$ increases with W_R , V_R and s/H . This contrasts with the findings of Goldstein et al. (20) that for $0.76 \geq s/H \geq 0.53$, s/H has no measurable effect either on $(x/H)_{\text{eff}}$ or on η_I . These conclusions are not necessarily in conflict, though insensitivity of $(x/H)_{\text{eff}}$ to s/H is hardly likely when s/H tends to zero. Apart from the much higher values of $(x/H)_{\text{eff}}$ obtained by Goldstein et al. (20) at a given V_R , (which lie above those derived from application of equation (17) with $0.76 \geq s/H \geq 0.53$ by a factor of between 1.9 and 2.4), Fig. 9 shows the sudden increase from 0.4 to unity in the exponent relating V_R to $(x/H)_{\text{eff}}$ reported by these workers when V_R exceeds 0.15. This is absent in the present range of s/H . Again these differences may be associated with differences in geometry and hence in Mach numbers, shock systems and flow patterns downstream of the slot.

The subsonic film-cooling predictions of Tribus and Klein (7), Kutateladze and Leont'ev (10), Librizzi and Cresci (11) and Stollery and El-Ehwany (12), for V_R below unity, are all in terms of effectiveness η according to equation (7). This is related to V_R , the dimensionless distance x/s and the secondary stream Reynolds number at the slot exit, $Re_s = (sU_c/\nu_c \cdot \mu_c/\mu_g)$ as follows:-

Tribus and Klein:-

$$\eta = 4.62 \left(\frac{x}{V_R \cdot s} \right)^{-0.8} Re_s^{0.2}, \quad (18)$$

Kutateladze and Leont'ev:-

$$\eta = 3.1 \left\{ 4.16 + \left(\frac{x}{V_R \cdot s} \right) Re_s^{-0.25} \right\}^{-0.8}, \quad (19)$$

Librizzi and Cresci:-

$$\eta = 3.0 \left\{ 3.0 + \left(\frac{x}{V_R \cdot s} \right)^{0.8} Re_s^{-0.2} \right\}^{-1}, \quad (20)$$

Stollery and El-Ehwany:-

$$\eta = 3.09 \left(\frac{x}{V_R \cdot s} \right)^{-0.8} Re_s^{0.2}. \quad (21)$$

Goldstein et al. (20) compared their effectiveness measurements with those predicted from equations (18), (19) and (20) using as abscissa a parameter X_2 defined by

$$X_2 = \left(\frac{x}{V_R \cdot s} \right) Re_s^{-0.25}, \quad (22)$$

such that

$$\frac{X_1}{X_2} = 0.7 V_R^{0.85} Re_H^{0.25} (s/H)^{0.6}, \quad (23)$$

where

$$Re_H = \frac{H \cdot U_G}{v_G} \quad (24)$$

Goldstein et al. (20) found satisfactory agreement only with equation (18) in the range $0.15 \leq V_R \leq 0.41$.

A comparison is presented on logarithmic scales in Fig. 10 between predictions of η based on equations (18), (19), (20) and (21) and current measurements of η_I using X_2 as abscissa. As with Goldstein et al. (20), only the theory of Tribus and Klein (7) gives satisfactory agreement with measurements, but this is in the range of V_R below 0.14 rather than above. The discrepancy becomes large for $V_R > 0.14$ and for clarity measurements of η_I in this range of large mass velocity ratios are omitted. The other subsonic theories predict values of effectiveness at least 35 per cent less than those currently measured in the range $4 \leq X_2 \leq 30$. Ma_G appears to have little influence on effectiveness in the present narrow range of investigation, the upper limit being determined by the air supply available.

The comparison of present measurements with η obtained from equation (18) plotted against x/s in Fig. 11 suggests that with supersonic primary flow, the effectiveness remains near unity over a considerably greater range of x/s (and therefore of diffuser-ejector length) for $V_R < 0.14$ than with subsonic incompressible flow. However, the subsequent decay is also more rapid with supersonic primary flow than with subsonic flow. For $V_R > 0.14$, on the other hand, these effects are reversed. The data of Goldstein et al. (20) are also shown in Fig. 11 for two values of V_R (0.05 and 0.3), $s/H = 0.53$ and $Ma_G = 3.01$. A qualitative comparison with the present data indicates significant effects of geometry, Ma_G and s/H . An increase in Ma_G and s/H appears not only to increase $(x/H)_{eff}$ but also the rate of decline in effectiveness.

In the form commonly used in subsonic film-cooling expressions the present results are best correlated by

$$\eta_I = 8.22 \left\{ \left(\frac{x}{s \cdot V_R} \right)^{0.8} R^{0.48} \left(\frac{H}{s} \right)^{0.21} \right\}^{-1}, \quad 0 < R < 0.05, \quad (25)$$

$$\eta_I = 3.16 \left\{ \left(\frac{x}{s \cdot V_R} \right)^{0.43} R^{0.26} \left(\frac{H}{s} \right)^{0.11} \right\}^{-1}, \quad 0.05 < R < 0.1, \quad (26)$$

$$\text{where } R = Re_s / Re_H \quad (27)$$

Optimum slot width and mass ratio.

An increase in dimensionless slot width s/H at given W_R has been shown to (a) reduce diffuser static pressure recovery by increasing p_b/P_i according to Figs. 3b and 3c and equation (2), (b) improve film cooling by increasing $(x/H)_{eff}$ according to Fig. 8 and equations (15) and (17).

Diffuser-ejector design in connection with engine test facilities is therefore likely to involve some compromise such that the chosen s/H yields a sufficiently large value of $(x/H)_{\text{eff}}$ to eliminate the peak in heat transfer near reattachment of the separated shear layer for $W_R = 0$ as suggested in Fig. 5. This approximates to the distance $(x/H)_r$ from the nozzle base to the peak in diffuser wall static pressure for $W_R = 0$ shown in Fig. 4. This distance may be calculated by construction of the equivalent inviscid jet boundary in conjunction with the ^{used} empirical reattachment criteria of Martin and Mukerjee (1) and together with relation (1) and equation (15) to estimate the maximum value of s/H and the corresponding minimum value of W_R . Larger W_R and smaller s/H imply lower simulated altitude conditions in the engine test cell. Equation (2) then yields the corresponding value of λ and hence the base pressure ratio p_{b5}/p_{b1} , where p_{b1} is again calculable by the procedures of Martin and Mukerjee (1). The diffuser static pressure recovery p_a/p_{b5} may be determined when p_{b5} is known.

If further research reveals that the plateau in p_b/P_i shown in Fig. 3a between 4 and 5 is generally not achieved (or is achieved only over a limited range of conditions) and a further increase with P_c/P_i or W_R may be expected, this will clearly imply a penalty on diffuser static pressure recovery. A corresponding limitation is set on the stagnation pressure ratio across the engine under test if a given range of altitude conditions is to be simulated in the facility.

CONCLUSIONS

This paper reports measurements of base pressure and adiabatic wall temperature when secondary air at uniform temperature is admitted to the enlargement region between an axisymmetric parallel diffuser and an upstream nozzle generating a supersonic primary air flow of variable Mach number and temperature. The area for secondary admission may also be varied, and the range of mass ratio investigated covers both subsonic and supersonic secondary admission. The initial primary stagnation pressure was kept constant throughout the investigation, such that the diffuser was always started.

With increasing mass ratio the base pressure passes through a maximum and a subsequent minimum before becoming insensitive to further increases in mass ratio. The plateau value increases with secondary admission area or slot width from that for zero secondary flow. Over the limited range investigated, the effect of primary Mach number on the base pressure is negligible. The peak in adiabatic wall temperature near reattachment of the primary stream is eliminated even by small proportions of secondary air and further increases in mass ratio extend the length of diffuser over which the adiabatic wall temperature is uniform.

The effectively-cooled length also increases with slot width. The film-cooling effectiveness declines more rapidly from unity at low secondary injection rates than at higher values. Of the many incompressible film-cooling theories in the literature, only the predictions of Tribus and Klein (7) give reasonable agreement with present measurements of film-cooling effect-

iveness. Supersonic flow appears to yield greater effectively-cooled lengths than subsonic flow at low mass-velocity ratios; for higher ratios the effect is reversed.

The conflicting effects of slot width and mass ratio on diffuser static pressure recovery and film cooling of the diffuser walls probably imply a design compromise, such that the effectively-cooled length is sufficient only to offset the peak heat transfer near reattachment of the primary stream in the absence of secondary flow.

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NOMENCLATURE

A	-	Flow area.
H	-	Base height ($r_D - r_G$).
$\dot{m}$	-	Mass flow rate.
Ma	-	Mach number.
P	-	Stagnation pressure.
p	-	Static pressure.
r	-	Radius.
Re _H	-	Main stream Reynolds number at nozzle exit ($U_G \cdot H / \nu_G$)
Re _s	-	Secondary stream Reynolds number at slot ($U_c \cdot s / \nu_c \cdot \mu_c / \mu_G$).
R	-	Ratio of Reynolds numbers (Re_s / Re_H).
s	-	Slot width.
T	-	Total temperature.
T _{ad}	-	Adiabatic wall temperature.
U	-	Stream velocity.
V _R	-	Secondary to main stream mass velocity ratio ($\rho_c U_c / \rho_G U_G$).
W _R	-	Secondary to main stream mass ratio ($\dot{m}_c / \dot{m}_G$).
x	-	Distance along wall from slot exit.
X ₁	-	Non-dimensional parameter defined by equation (9).
X ₂	-	Non-dimensional parameter defined by equation (22).

Greek symbols:

ζ	-	Non-dimensional distance defined by equation (10).
η	-	Effectiveness defined by equation (7).
η'	-	Effectiveness defined by equation (6).
η_I	-	Effectiveness defined by equation (5).
η_{II}	-	Effectiveness defined by equation (8).
λ	-	Base pressure ratio defined by equation (3).
μ	-	Viscosity.
ν	-	Kinematic viscosity (μ / ρ).
ρ	-	Density.
σ	-	Non-dimensional parameter defined by equation (4).

Subscripts

- a - Ambient atmosphere.
- b - Nozzle base region.
- c - Secondary stream at slot exit.
- d - Zero secondary flow.
- D - Diffuser-ejector.
- e - With secondary flow.
- eff - Effectively cooled i.e. effectiveness is unity.
- G - Main stream at nozzle exit.
- i - Nozzle inlet.
- iso - Isoenergetic condition i.e. $T_c = T_G$.
- J - Zone of confluence of main and secondary stream. free shear layers.
- r - Free shear layer reattachment region with zero secondary flow.
- x - At distance x from slot exit.
- 0 - At x = 0.
- 1 - Refers to point 1 in Fig. 3a.
- 5 - Refers to point 5 in Fig. 3a.

Superscript

- * - Nozzle throat.

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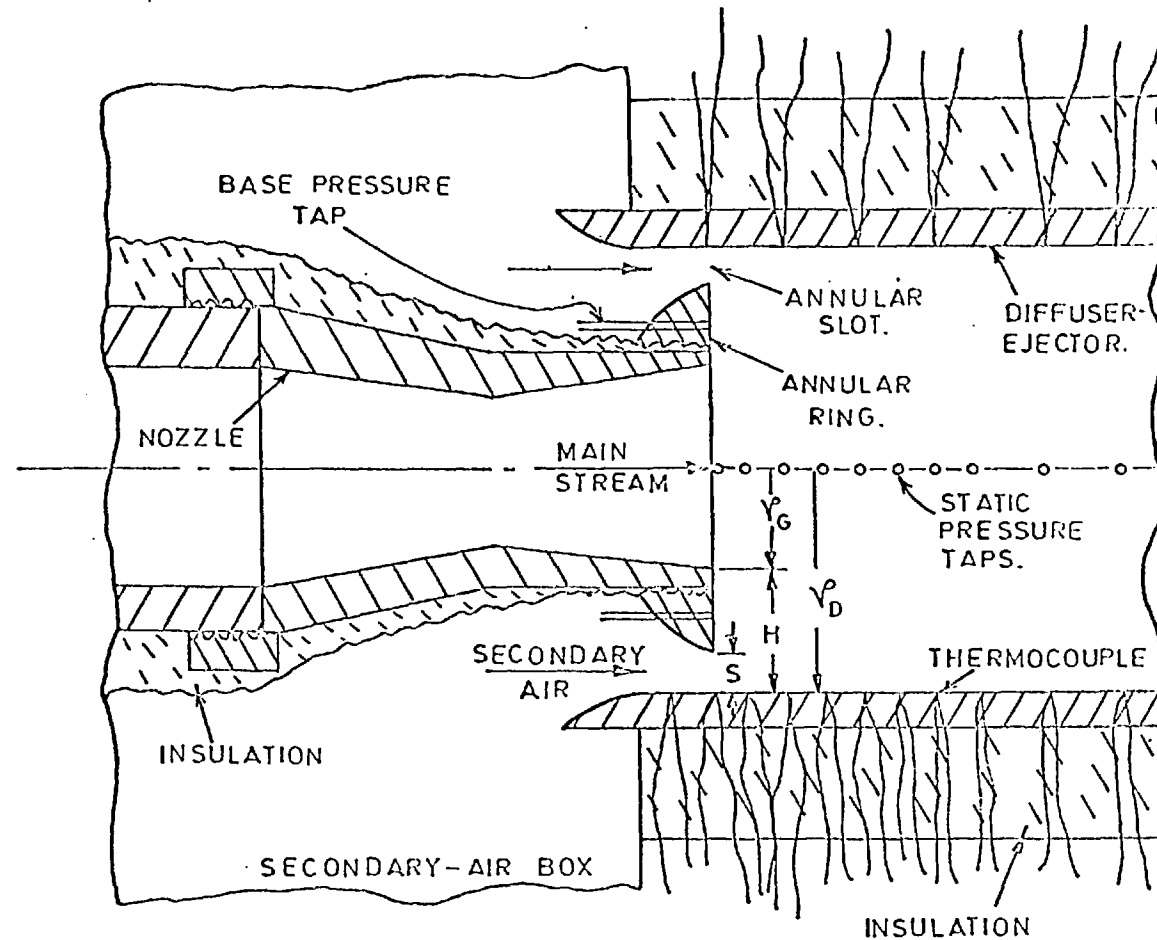
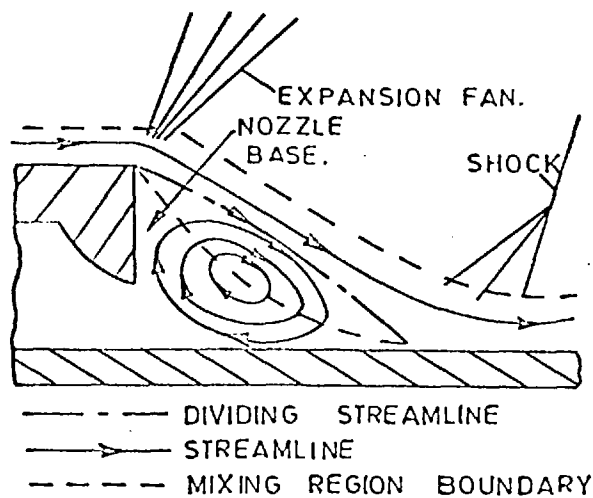
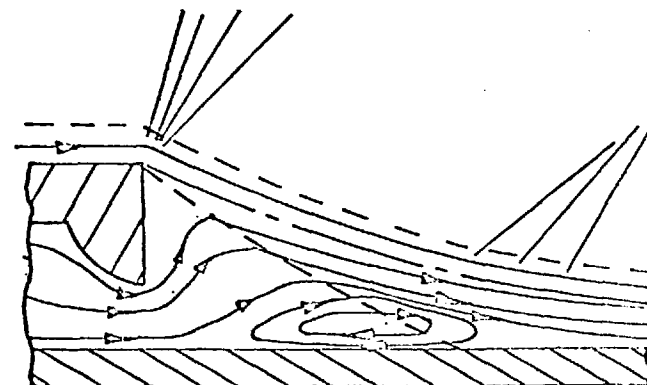


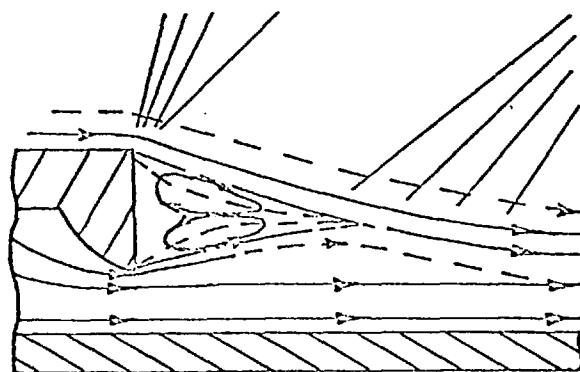
Fig.1. Film-Cooled Diffuser-Ejector.



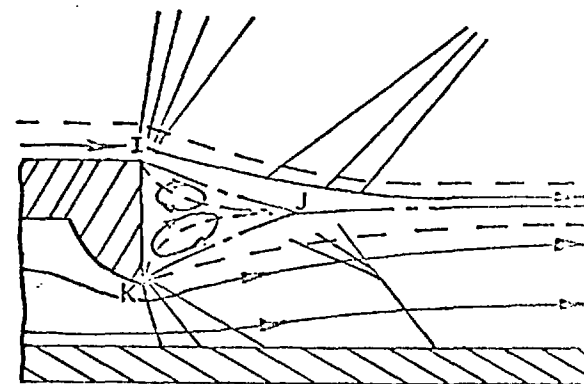
(a) No Secondary Flow (Point 1 in Fig. 3A)



(b) Secondary Flow Supplies Part of Entrained Air (Point 2 in Fig. 3A)



(c) Secondary Flow Escapes Without Entering Mixing Region (Point 3 in Fig. 3A)



(d) Sonic Secondary Air Flow (Points 4 and 5 in Fig. 3A)

Fig. 2. Effect of Secondary Injection on Flow Patterns at Diffuser Entry

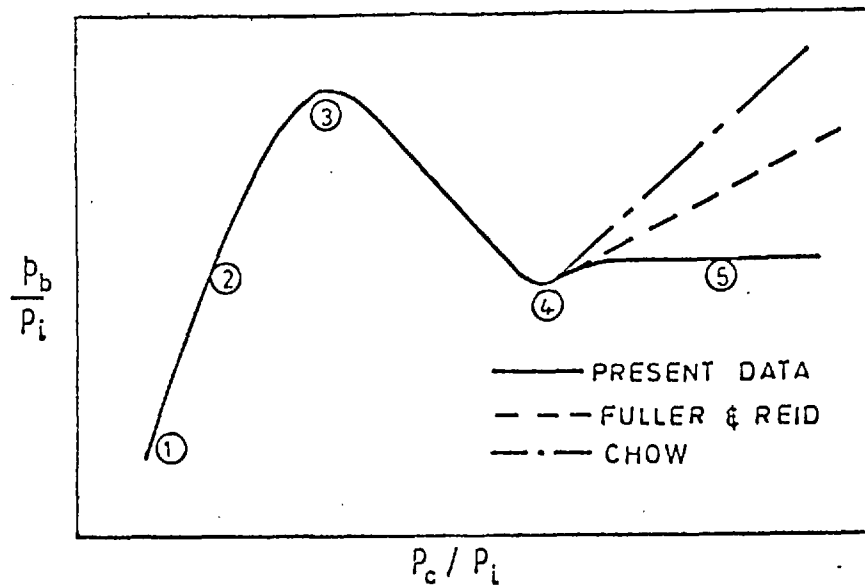
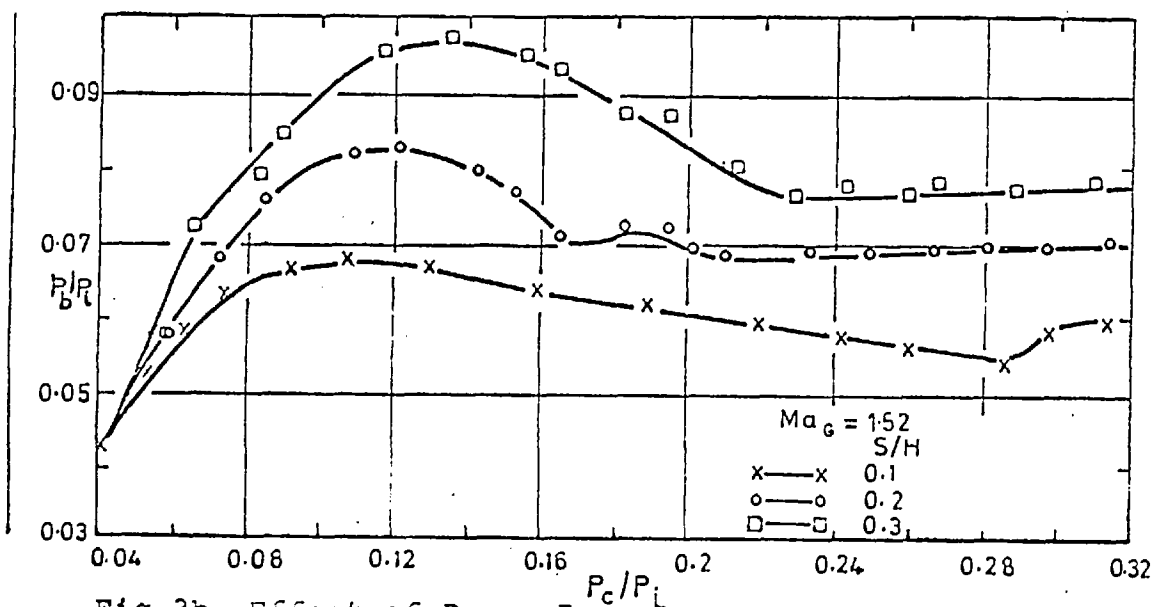


Fig.3a. Effect of Secondary Flow on Base Pressure



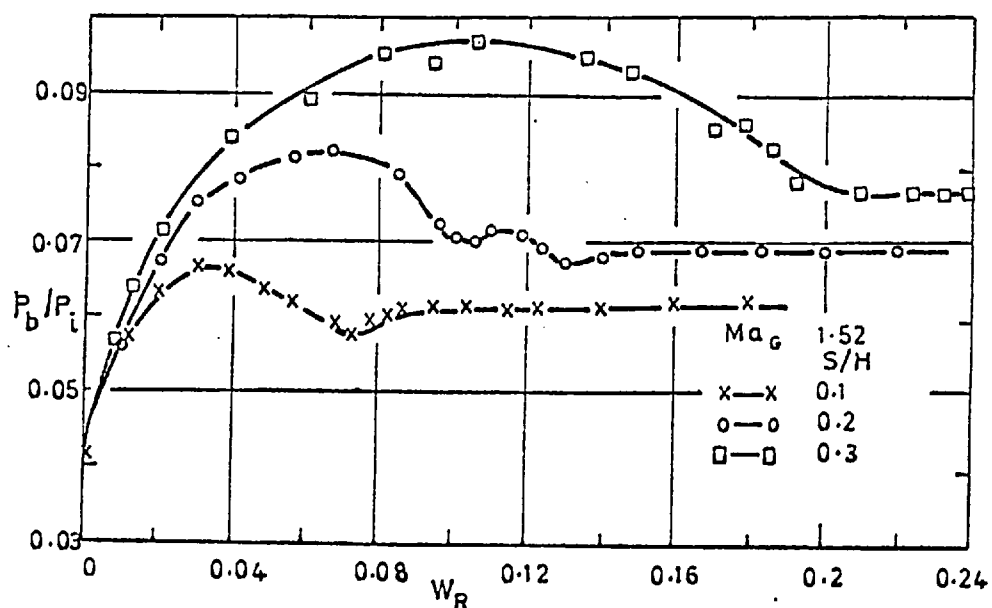


Fig.3c. Effect of W_R on Base Pressure

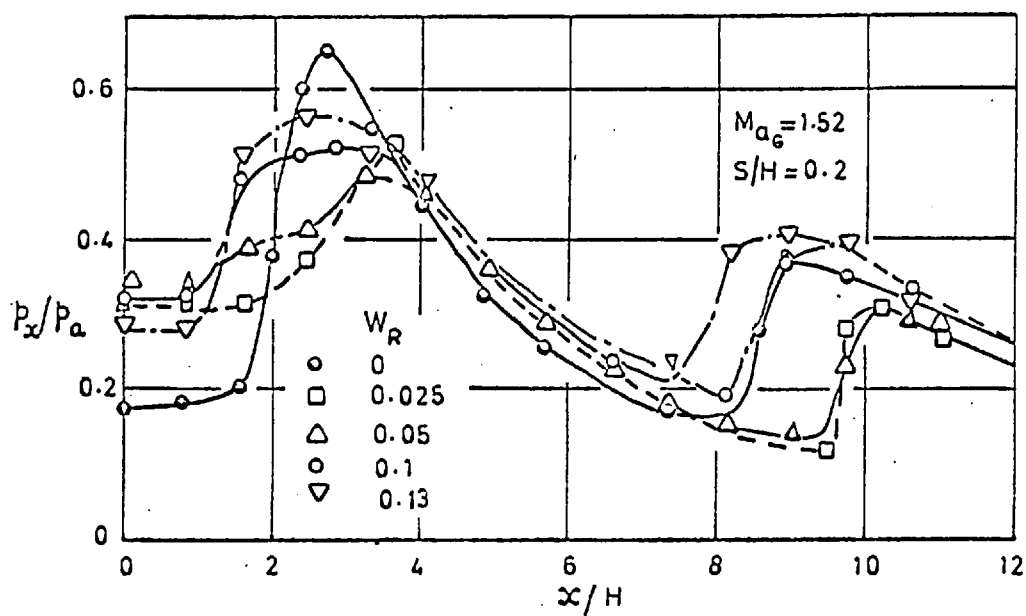


Fig.4. Effect of W_R on Wall Pressure.

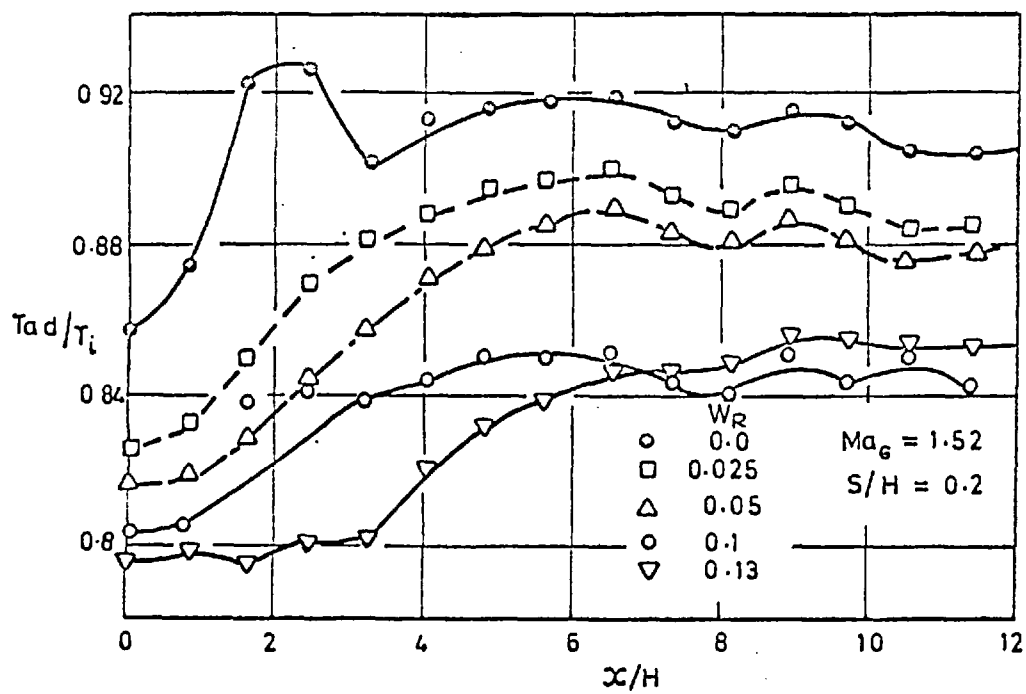


Fig.5. Effect of W_R on Adiabatic Wall Temperatures

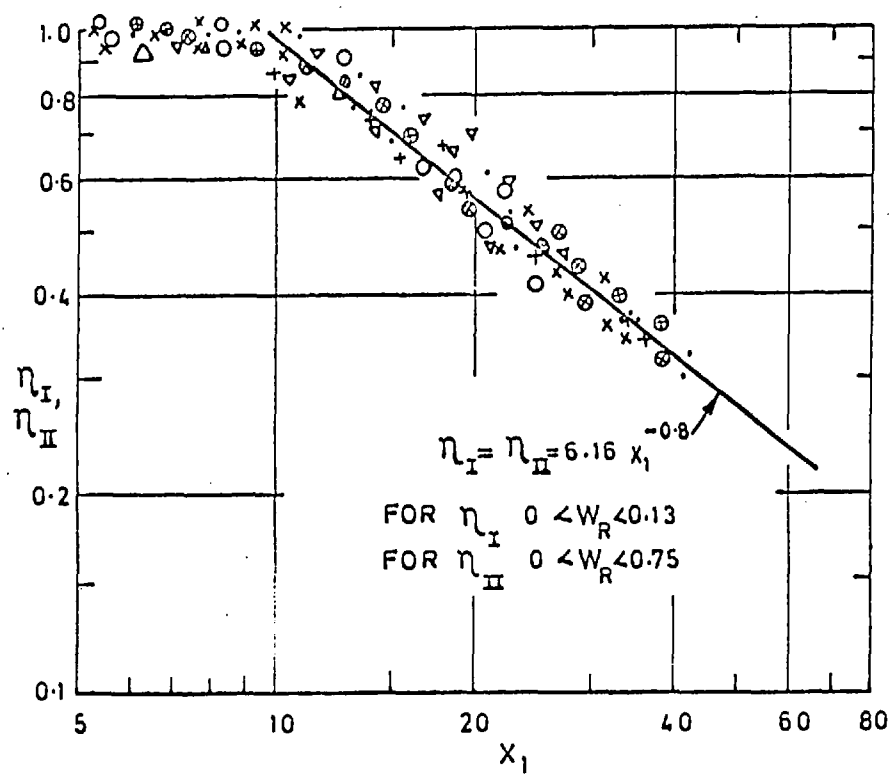


Fig.6. Effectiveness Correlation at Low W_R

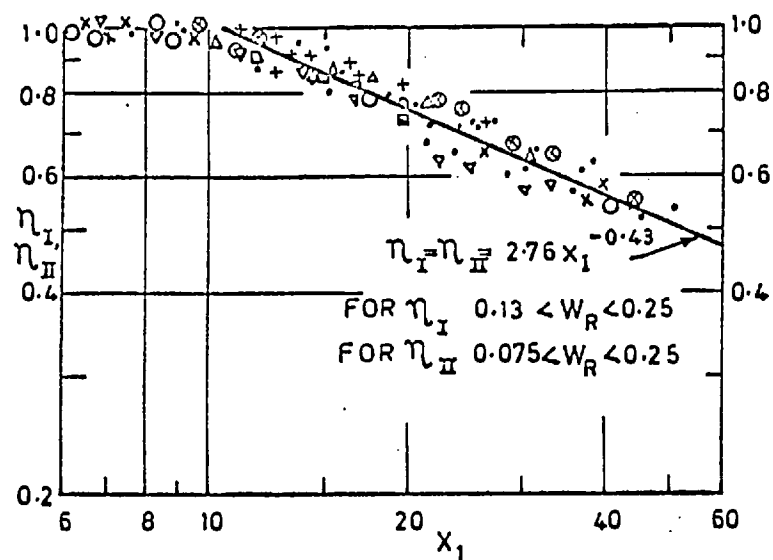


Fig.7. Effectiveness Correlation at High W_R

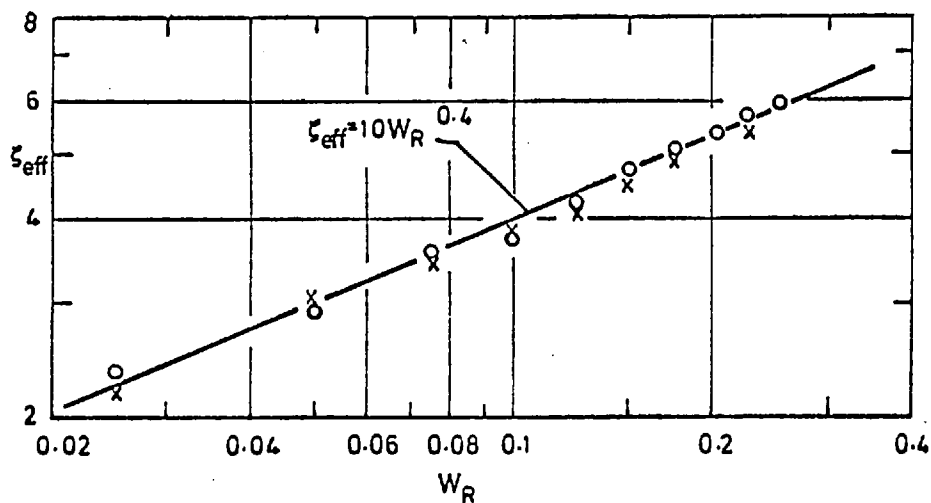


Fig.8. Effect of W_R on ζ_{eff}

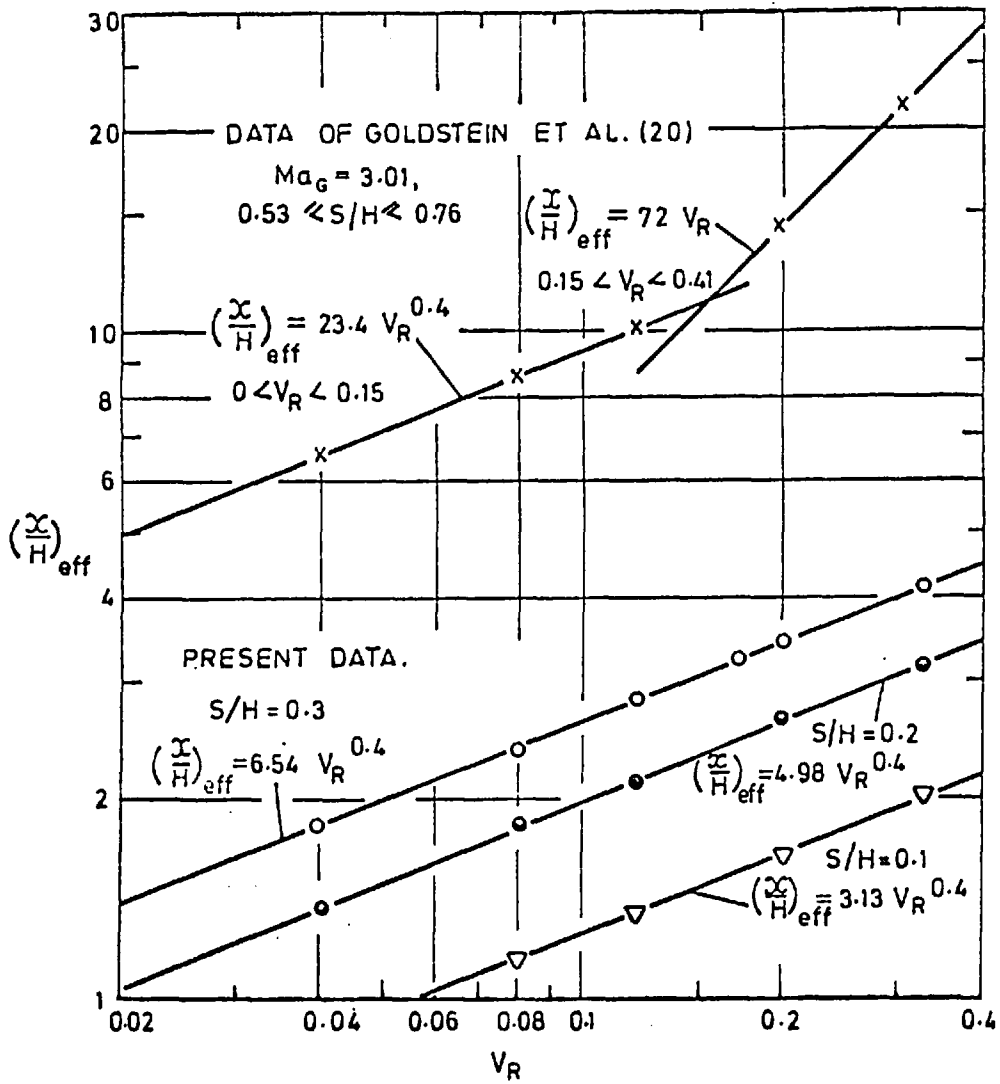


Fig.9. Effect of V_R on $(x/H)_{eff}$

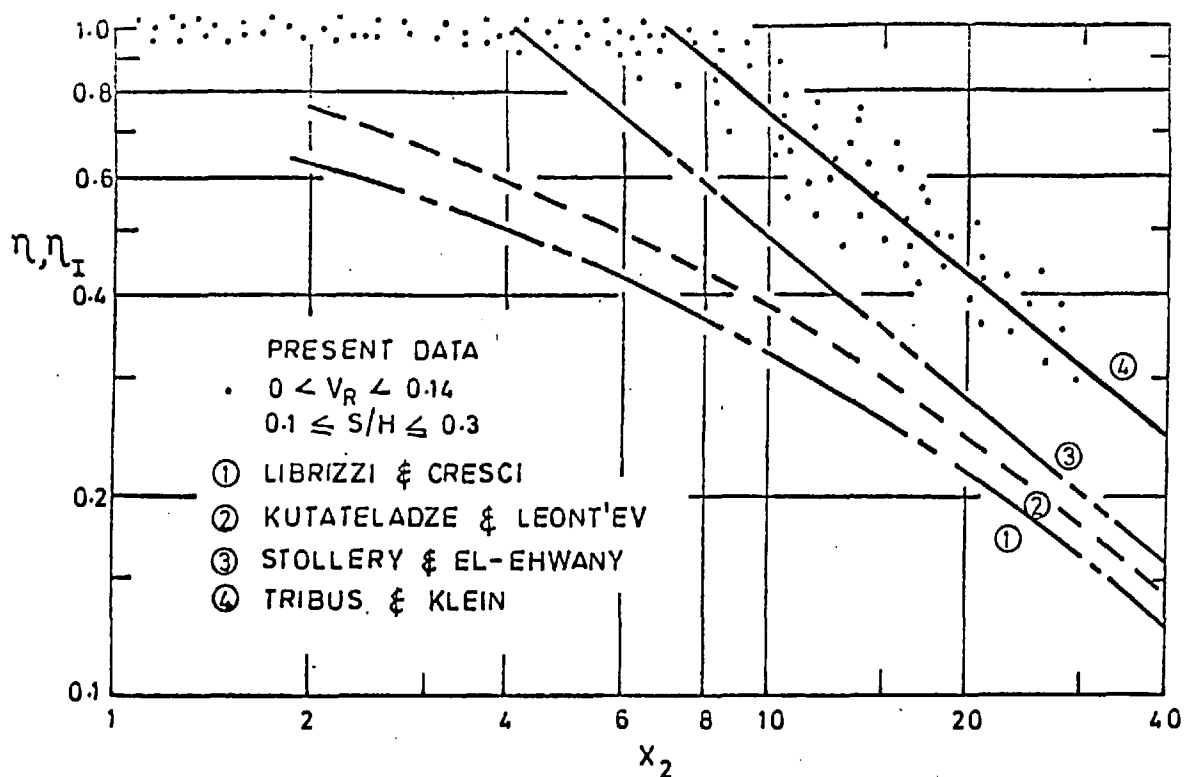


Fig.10. Comparison of Present Date with Subsonic Correlations

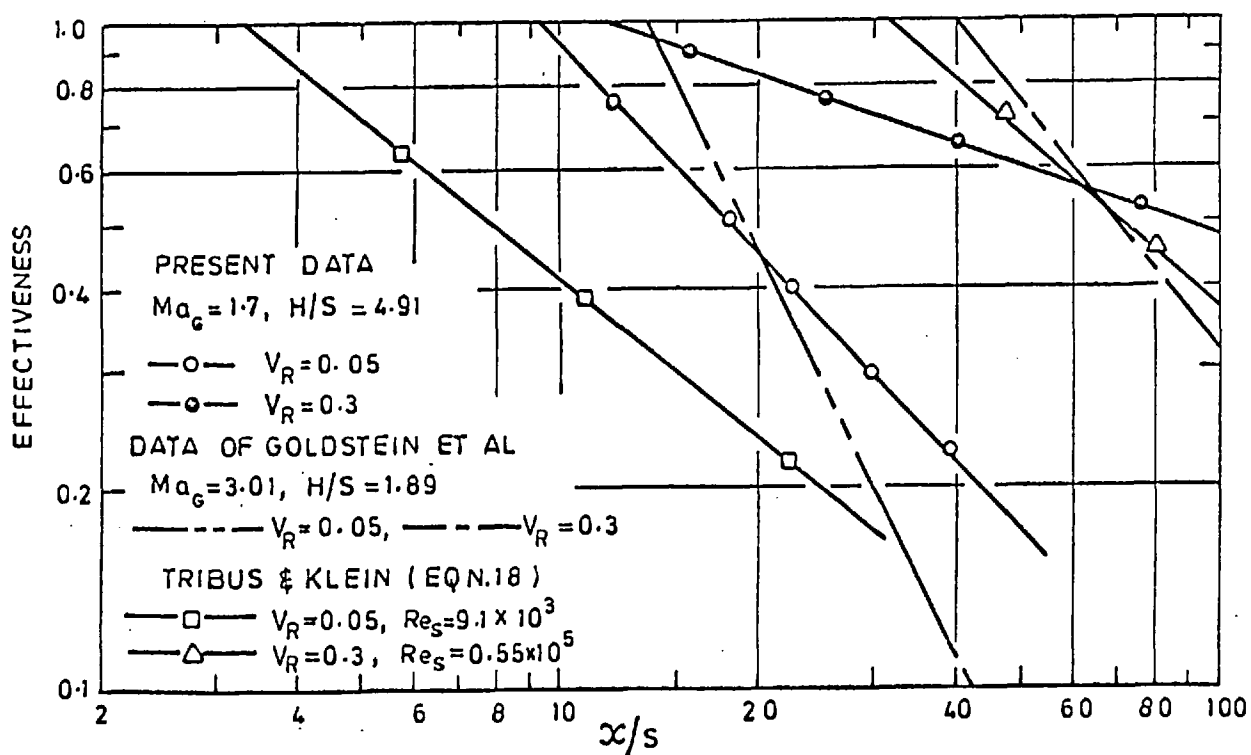


Fig.11. Comparison of Present Data with Subsonic and Supersonic Correlations

THERMODYNAMICS AND FLUID MECHANICS CONVENTION

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PAPER 22

The performance of a supersonic parallel diffuser

B. W. MARTIN and T. MUKERJEE

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THE PERFORMANCE OF A SUPERSONIC PARALLEL DIFFUSER

By B. W. Martin* and T. Mukerjee*

INTRODUCTION

THE EFFICIENT RECOMPRESSION of a confined high-velocity gas stream poses many problems to designers of wind tunnels, turbomachinery, jet propulsion engines, and their test facilities. When the flow entering a duct is supersonic, its deceleration to a low subsonic Mach number ideally requires convergent-divergent geometry. However, in such a diffuser the flow tends to separate from the wall of the divergence and the resultant eddies give rise to excessive and unacceptable losses. If, on the other hand, the walls of the supersonic diffuser are parallel, then pressure recovery comparable with that across normal shock waves is obtainable, particularly for low blowing pressure ratios. At higher p_{01}/p_a , compression is due to a system of multiple oblique shocks and pressure recovery is correspondingly reduced.

The exhaust diffusers used in engine test facilities are required to meet a wide range of gas flow rates and total temperatures. In such circumstances the simple parallel diffuser is used because, over the working range, it gives generally better efficiencies than the fixed geometry convergent-divergent type. Although the behaviour of the parallel diffuser is better understood, as a result of experimental researches by Martin and Baker (1)† (2) and Fortini (3), it is still not possible to anticipate its performance. The purpose of this paper is, therefore, to present a method of predicting static pressure recovery in the axisymmetric supersonic parallel diffuser for specified flow inlet conditions and given geometry.

In test facilities, performance evaluation of jet engines at design altitudes involves the discharge of the engine exhaust jet at a low ambient pressure. This is also the pressure at entry to a parallel diffuser located to accept and recompress the exhaust jet by a shock system.

Usually the diffuser is of larger diameter than that at the nozzle exit. Hence, there is a sudden enlargement in flow area where the supersonic stream from the exhaust nozzle enters the diffuser. This allows for the injection of a secondary fluid when the device serves as an ejector.

When a slightly underexpanded supersonic jet enters a long axisymmetric parallel diffuser the flow has a multicelled structure with a terminal shock to compress the fluid to the diffuser outlet pressure. The jet then reattaches on the diffuser wall far downstream of the diffuser entry, where the flow is very unstable. As p_{01}/p_a is increased, the nozzle back pressure ratio (p_2/p_{01}) falls rapidly and the point of reattachment moves upstream. 'Buzzing' begins accompanied by the appearance of a strong normal shock within the first jet cell.

At higher p_{01}/p_a , the flow becomes steady and 'buzzing' ceases with the jet reattachment zone close to the diffuser entry plane. The nozzle back-pressure ratio continues to fall with further increase in p_{01}/p_a , while the normal shock with forked ends (caused by interaction with the free shear layer) moves very slowly downstream. When the blowing pressure ratio becomes sufficiently high the normal shock breaks down into an oblique shock system extending over an increasing proportion of the diffuser length. The nozzle back-pressure ratio, p_2/p_{01} , now reaches a minimum and remains almost unaffected by any further increase in p_{01}/p_a . The diffuser is then said to have 'started', having previously been in the 'unstarted' condition.

The experimental investigations of supersonic parallel diffusers reported by Martin and Baker (1) (2) for Reynolds numbers (based on diffuser hydraulic diameter) between 4.4×10^5 and 1.1×10^6 have shown that for the unstarted condition the flow at the diffuser outlet is virtually one-dimensional. They have also led Martin (4) to predict an optimum diffuser to nozzle-throat area ratio for maximum possible static pressure recovery. Korst *et al.* (5) reported theoretical and experimental investigations of transonic and supersonic turbulent flow at abrupt

The MS. of this paper was received at the Institution on 30th September 1967. 33

* Imperial College of Science and Technology, London.

† References are given in Appendix 22.3.

increases in cross-section. According to these workers a one-dimensional analysis of the unstarted diffuser predicts the variation of p_2/p_{01} with p_{01}/p_a quite accurately in the absence of a boundary layer at the nozzle outlet. The discrepancy between the predictions of Korst *et al.* and experimental values of p_2/p_{01} increases with nozzle outlet boundary layer thickness.

In a started diffuser the flow at the nozzle exit and zone of reattachment is analogous to that along a backward-facing step in a supersonic stream. Based on a suitable two-dimensional turbulent flow model, Korst (6) predicted the base pressure for a given flow Mach number when the boundary layer thickness at the point of separation was zero. Discrepancies between theoretical and measured values of p_2 were shown by Nash (7) to be due to neglect of the boundary layer thickness at the point of separation, and inaccurate assumptions regarding the pressure rise up to reattachment of the separated shear layer.

The present analysis covers both unstarted and started conditions, the two cases being separately treated. Consideration of the unstarted diffuser rests on a force-momentum balance across a specified control volume, assuming a turbulent boundary layer to grow from the nozzle throat, and at the nozzle outlet to be describable in terms of displacement and momentum thicknesses, δ_1 and θ_1 respectively. Following Liepmann and Roshko (8), wall friction in the diffuser in the presence of an adverse pressure gradient is neglected.

The Korst-Chapman model for the analogous step-down cylinder base flow problem is used to analyse the inlet region of the started diffuser. This leads to equations defining the shear-layer momentum thickness, θ_2 , and relating it to that before separation, θ_1 . Transformation of the axisymmetric equations to their two-dimensional equivalents, and the incorporation of a suitable velocity distribution for the mixing layer, provides a relation between θ_2 and the velocity ratio for the dividing streamline, ϕ_d . This is further related to an experimentally determined reattachment parameter R normalized with respect to upstream conditions. The nozzle back-pressure ratio p_2/p_{01} may then be evaluated by either a graphical or iterative procedure for given initial conditions. Good agreement is obtained between predictions and measurements for both the unstarted and the started diffuser, especially for larger values of $(r_2/r^*)^2$.

Notation

A	Cross-sectional area.
C	Crocco number ($= U/U_{\max}$).
H	Boundary-layer shape parameter.
I_1	Integral defined by equation (22.29).
I_2	Integral defined by equation (22.32).
M	Mach number.
n	Turbulent velocity profile power law parameter ($= \frac{1}{n}$) defining Mach number and total pressure distribution.
p	Pressure.

R	Reattachment parameter defined by equation (22.40).
Re	Reynolds number.
r	Radius or distance from axis of symmetry.
T	Absolute temperature.
U	Velocity in x direction.
V	Velocity in y direction.
x	Distance along inviscid jet boundary.
y	Distance normal to x .
z	Distance along diffuser wall.
α	Diffuser to nozzle-exit area ratio.
γ	Specific heat ratio.
δ	Boundary-layer displacement thickness.
θ	Boundary-layer momentum thickness.
λ	Momentum function defined by equation (22.2).
ψ	Stream function.
ξ	Mean slope defined by equation (22.34).
η'	Transformed length variable defined by equation (22.27).
σ	Jet spreading parameter.
ϕ	Velocity ratio ($= U/U_e$).
ρ	Density.

Subscripts

a	Atmospheric condition at diffuser exit.
d	Dividing streamline.
e	Free stream condition outside viscous layer.
i	Nozzle inlet condition.
m	Median streamline.
r	Reattachment condition.
s	Free shear layer.
max	Maximum value at $T = 0$.
0	Stagnation condition.
1	Nozzle exit plane.
2	Diffuser inlet condition.
3	Diffuser condition downstream of reattachment.
—	Equivalent transformed variable for two-dimensional flow.

Superscript

*	Nozzle throat condition.
---	--------------------------

THE UNSTARTED DIFFUSER

The flow model for the unstarted condition, illustrated in Fig. 22.1, shows a normal shock with wall bifurcations near the diffuser entry. The supersonic stream leaving the nozzle with a finite boundary layer undergoes a sudden expansion which turns the flow through the appropriate Prandtl-Meyer angle. The boundary layer becomes a free shear layer below which lies the separated region wherein the velocities are assumed negligible and the pressure, p_2 , is uniform.

The shear layer initially entrains fluid from the separated region. As the shear layer approaches the diffuser wall to reattach, it is compressed and turned by the shock to flow along the wall. Part of the shear layer reverses into

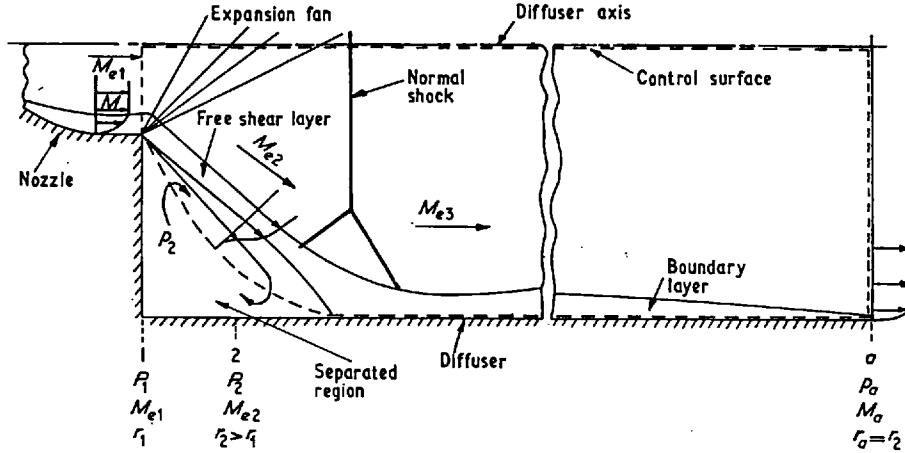


Fig. 22.1. Flow model for unstarted diffuser

the separated region such that the mass of fluid therein is conserved. The remainder of the shear layer negotiates the pressure rise to form the boundary layer downstream of reattachment. This description is equally valid for the started diffuser, when the normal shock is superseded by an oblique shock system, except that in the unstarted diffuser the separated region is not isolated from downstream disturbances, while in the started diffuser communication appears to be severed. This helps to explain the difference in treatment of the two diffuser conditions.

The control volume analysis of Fig. 22.1 is based on the following additional assumptions:

- (a) the static pressure is uniform across sections 1 and a , and also in the base region 2;
- (b) the stagnation pressure and Mach number distributions at section 1 correspond to a conventional turbulent boundary-layer velocity profile;
- (c) the flow is one-dimensional at section a , where the static pressure is equal to the ambient pressure;
- (d) the stagnation temperature is uniform throughout the flow field.

With the exception of the irrelevant assumption (c), these are also applicable to the started diffuser analysed below.

The force-momentum balance requires that, if wall friction be neglected, the sum of the momentum inflows at stations 1 and 2 must equal the momentum outflow at station a , as follows:

$$\frac{p_1 \pi r_1^2}{2} \frac{2}{r_1^2} \int_0^{r_1} (1 + \gamma M^2) r \, dr + \frac{p_2 \pi}{2} (r_a^2 - r_1^2) = \frac{p_a \pi r_a^2}{2} \frac{2}{r_a^2} \int_0^{r_a} (1 + \gamma M^2) r \, dr \quad (22.1)$$

Let

$$\alpha = \frac{r_1^2}{r_a^2} \quad \text{and} \quad \lambda = \frac{2}{r^2} \int_0^r (1 + \gamma M^2) r \, dr \quad (22.2)$$

then equation (22.1) can be written

$$\alpha \lambda_1 + \frac{p_2}{p_1} (1 - \alpha) = \frac{p_a}{p_1} \lambda_a \quad (22.3)$$

With slight rearrangement, this becomes

$$\frac{p_2}{p_{01}} = \frac{\alpha}{(1 - \alpha)} \frac{p_1}{p_{01}} \left[\frac{\lambda_a p_a}{\alpha p_1} - \lambda_1 \right] \quad (22.4)$$

Equation (22.4), which governs the flow, determines the nozzle back pressure ratio p_2/p_{01} . It is, however, more usefully expressed in terms of boundary-layer displacement and momentum thicknesses (δ_1 and θ_1 respectively), which are conventionally defined for compressible flow as follows:

$$\frac{\delta}{r} = \frac{1}{r^2} \int_0^r \left(1 - \frac{\rho U}{\rho_e U_e} \right) r \, dr \quad (22.5)$$

and

$$\frac{\theta}{r} = \frac{1}{r^2} \int_0^r \frac{\rho U}{\rho_e U_e} \left(1 - \frac{U}{U_e} \right) r \, dr \quad (22.6)$$

where subscript e refers to centre-line values. Addition of equations (22.5) and (22.6) gives

$$\frac{\delta + \theta}{r} = \frac{1}{2} \frac{1}{r^2} \int_0^r \frac{\rho U^2}{\rho_e U_e^2} r \, dr \quad (22.7)$$

and since

$$\rho U^2 = \gamma p M^2 \quad (22.8)$$

equation (22.7) can be rewritten

$$\int_0^r \gamma M^2 r \, dr = \gamma M_e^2 r^2 \left[\frac{1}{2} - \frac{(\delta + \theta)}{r} \right] \quad (22.9)$$

Combination of equations (22.2) and (22.9) applied to the nozzle outlet gives

$$\lambda_1 = 1 + \gamma M_{e1}^2 \left[1 - \frac{2(\delta_1 + \theta_1)}{r_1} \right] \quad (22.10)$$

By introducing a shape parameter $H_1 = \delta_1/\theta_1$, equation (22.10) can be rewritten

$$\lambda_1 = 1 + \gamma M_{e1}^2 \left[1 - \frac{2\theta_1(H_1 + 1)}{r_1} \right] \quad (22.11)$$

Since the flow is assumed one-dimensional at the diffuser outlet station a , then $M_{ea} = M_a$ and $\theta_a = 0$. Hence

$$\lambda_a = 1 + \gamma M_a^2 \quad (22.12)$$

Substitution for λ_1 and λ_a from equations (22.11) and (22.12) in (22.4) gives

$$\frac{p_2}{p_{01}} = \frac{\alpha}{(1-\alpha)} \left(\frac{p_a}{p_{01}} \frac{(1+\gamma M_a^2)}{\alpha} \frac{p_1}{p_{01}} \times \left\{ 1 + \gamma M_{e1}^2 \left[1 - \frac{2\theta_1(H_1+1)}{r_1} \right] \right\} \right) \quad (22.13)$$

In equation (22.13), H_1 and θ_1 may be evaluated from the data of Tucker (9) for $n = \frac{1}{\gamma}$ and, given M_{e1} (or p_1/p_{01}) and p_a/p_{01} , M_a can be found by applying continuity between the nozzle throat and diffuser exit. This yields the equation

$$\frac{p_{01}}{p_a} \left[\frac{r^*}{r_a} \right]^2 \left(\frac{2}{\gamma+1} \right)^{(\gamma+1)/(2(\gamma-1))} = M_a \left[1 + \frac{(\gamma-1)}{2} M_a^2 \right]^{1/2} \quad (22.14)$$

Between equations (22.13) and (22.14), p_2/p_{01} may then be determined for a given geometry.

THE STARTED DIFFUSER

The Korst-Chapman flow model is divided into several regions which are treated separately; the results are subsequently recombined in order to calculate p_2/p_{01} . Region I in Fig. 22.2 deals with flow in the nozzle close to the point of separation, where it undergoes Prandtl-Meyer expansion to the uniform base pressure p_2 . In region II the jet entrains and mixes with the fluid from the recirculating region at constant pressure p_2 . In regions III and IV the shear layer is recompressed; the dividing streamline reattaches to the diffuser wall at pressure p_r equal to the local static pressure. As in the unstarted diffuser, fluid below the dividing streamline suffers flow reversal towards the backward-facing step, while the high-velocity fluid above the dividing streamline negotiates the pressure rise to form the boundary layer downstream of reattachment, where, as seen in Fig. 22.2, the local pressure is greater than p_r for reasons which are self-evident. Sirieix, Mirande, and Delery (10) have shown that downstream conditions have virtually no effect on those at reattachment. Attention therefore centres on regions II and III.

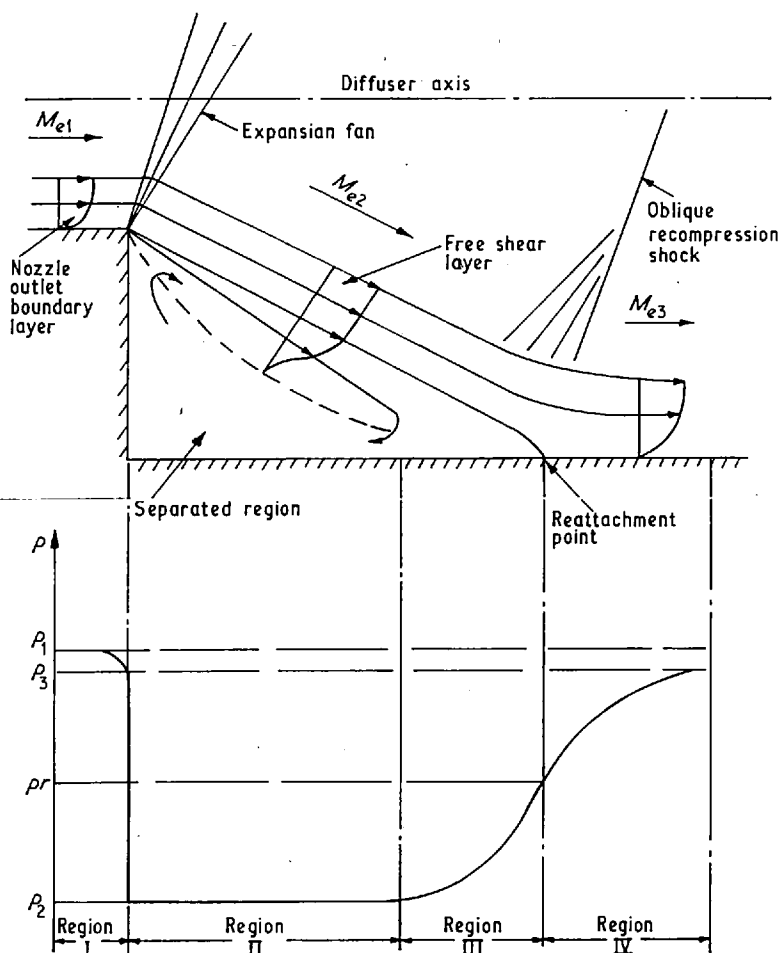


Fig. 22.2. Supersonic internal axisymmetrical flow model in started diffuser (after Korst-Chapman)

The separated shear layer

As in the unstarted diffuser, the present procedure presupposes known or calculable conditions (M_{e1} , θ_1 , p_1/p_{01} , r_2/r^* , r_1/r_2) at the point of separation. In the Korst-Chapman model, the treatment of region II involves two co-ordinate systems. In addition to the intrinsic system to which the velocity profile of the real jet is referred, a reference system is introduced which is based on the boundary of an inviscid free jet under the same geometrical and pressure constraints as the real jet, such that M_{e2} is uniform along the inviscid jet boundary.

This is constructed, using the Prandtl-Meyer relation, by the method of characteristics, starting at the expansion fan centred at the nozzle outlet and terminating where the jet boundary meets the diffuser wall. In the reference system, distance x is measured from the nozzle outlet along the curvilinear constant Mach number boundary, while $r = r(x)$ is measured from, and normal to, the diffuser axis of symmetry.

In general, the two co-ordinate systems do not coincide but, following McDonald (11), because θ_1 is small compared with r_1 , the displacement between the co-ordinate systems is neglected and they are treated as coincident. Furthermore, the r co-ordinate is, when necessary, supplemented by a y co-ordinate (associated with the intrinsic system) measured along a normal from the curvilinear x -axis to the point under consideration. It should be noted that the curvilinear x -axis and the dividing streamline do not, in general, coincide.

In the case of a uniform jet mixing with fluid at rest, a median streamline parallel to the original flow direction can be distinguished, above which the fluid mass in the approaching uniform stream is conserved. Continuity and momentum considerations in the shear layer show that for this reference case, for which $\theta_1 = 0$, the median streamline is defined by

$$\int_{y_m}^{\infty} \frac{r(x)}{r_1} \frac{\rho U}{\rho_{e2} U_{e2}} dy = \int_{-\infty}^{\infty} \frac{r(x)}{r_1} \frac{\rho U^2}{\rho_{e2} U_{e2}^2} dy \quad (22.15)$$

where y_m is the ordinate of the median streamline, which in this reference case coincides with the dividing streamline because $\theta_1 = 0$. Where the flow is inviscid throughout, the median streamline would coincide with the x -axis.

If the flow pattern for the real jet, where θ_1 is not zero, is superimposed on that for $\theta_1 = 0$, continuity and momentum considerations give

$$\int_{y_d}^{\infty} \frac{r(x)}{r_1} \frac{\rho U}{\rho_{e2} U_{e2}} dy + \int_{-\infty}^{\infty} \frac{r(x)}{r_1} \frac{\rho U^2}{\rho_{e2} U_{e2}^2} dy + \theta_2 \quad (22.16)$$

Combination of equations (22.15) and (22.16) yields

$$\theta_2 = \int_{y_d}^{y_m} \frac{r(x)}{r_1} \frac{\rho U}{\rho_{e2} U_{e2}} dy \quad (22.17)$$

which indicates that θ_2 is proportional to the mass flow between the median and dividing streamlines. Nash (7) shows that the momentum thickness after the sudden

expansion at the nozzle exit is also related to upstream conditions by the equation

$$\frac{\theta_2}{\theta_1} = \frac{\rho_{e1} U_{e1} M_{e1}^2}{\rho_{e2} U_{e2} M_{e2}^2} \quad (22.18)$$

provided that (a) θ_1 is not zero and (b) the expansion fan is centred at the nozzle exit.

Equations (22.17) and (22.18), together with a re-attachment criterion defined below, form the basis of the solution by which p_2/p_{01} is determined. Because the shear layer velocity profiles to be used in the integration of equation (22.17) are based on two-dimensional flow, Mangler's transformation (12) is employed to reduce the axisymmetric equations to their two-dimensional equivalents. Again, following McDonald (13), this is permissible if θ_1 is small compared with r_1 and if the eddy viscosities at corresponding points in the real and transformed flows are equal. The transformed variables are given by

$$x' = \int_0^x \left[\frac{r(x)}{r_1} \right]^2 dx \quad (22.19)$$

$$y' = \frac{r(x)}{r_1} y \quad (22.20)$$

and

$$\psi(x', y') = \frac{1}{r_1} \psi(x, y) \quad (22.21)$$

where r_1 is the reference length, with x and y respectively measured parallel and normal to the tangent to the meridional arc of the body. Equations (22.15) and (22.17) then become

$$\int_{y'_m}^{\infty} \frac{\rho U'}{\rho_{e2} U'_{e2}} dy' = \int_{-\infty}^{\infty} \frac{\rho U'^2}{\rho_{e2} U'_{e2}^2} dy' \quad (22.22)$$

$$\theta_2 = \int_{y'_d}^{y'_m} \frac{\rho U'}{\rho_{e2} U'_{e2}} dy' \quad (22.23)$$

where, since at $x = 0$, $r(x) = r_1$, θ_2 is unchanged by the transformation.

Equation (22.22) specifies the median streamline in two-dimensional flow. The transform of the dividing streamline is therefore the dividing streamline in two-dimensional flow. Transformation theory shows that velocities in the two planes at corresponding points are related as follows:

$$U = U' \quad (22.24)$$

$$V = \frac{r(x)}{r_1} V' - \frac{r_1}{[r(x)]^2} \frac{\partial r(x)}{\partial x} U' y' \quad (22.25)$$

$$U_e = U'_e \quad (22.26)$$

When θ_1 is small, no great error is introduced by assuming that the transverse velocity V parallel to the y -axis is negligibly small at $y = 0$, though this is only strictly true on the dividing streamline. All velocities then remain invariant in the transformation, so that the velocity ratio on the dividing streamline ϕ_d may be obtained from the two-dimensional velocity profile growing from the same θ_2 but over a different length scale defined by equation (22.19).

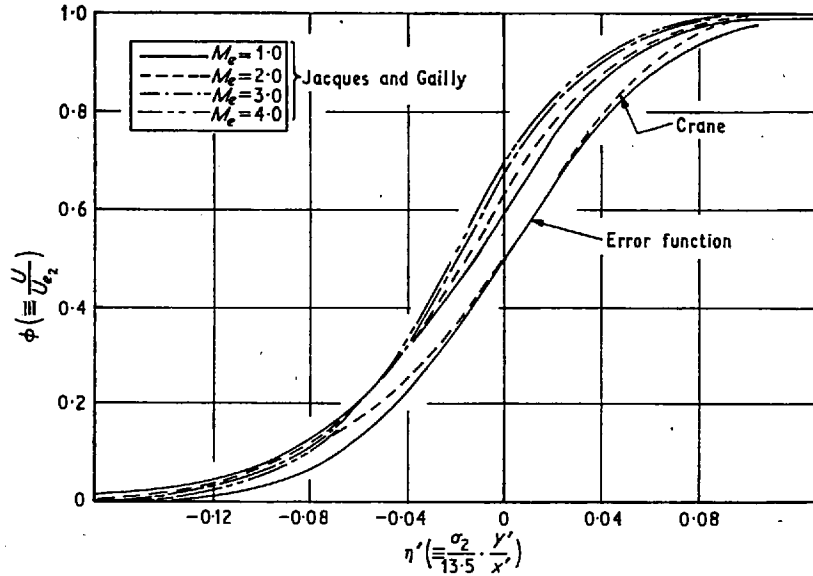


Fig. 22.3. Velocity distributions in the free shear layer (two-dimensional turbulent flow)

It remains to specify a shear layer velocity profile. Following the work of Tollmein (14) (who applied Prandtl's mixing-length theory to the incompressible mixing of a two-dimensional jet with a free boundary), and other workers (15)–(17), Jacques and Gailly (18) used the concept of a virtual kinematic viscosity to obtain an exact solution for two-dimensional turbulent free shear layers in terms of a jet spreading parameter σ . Their predicted velocity profiles, reproduced in Fig. 22.3 and used in the present treatment, show a significant Mach number effect; the profiles accord well with the measurements of Maydew and Reed (19), Gooderum *et al.* (20) and Charwat and Yakura (21).

In Fig. 22.3 Jacques and Gailly (18) use the following reduced variable for iso-energetic flows:

$$\eta' = \frac{\sigma}{13.5} \frac{y'}{x'} \quad (22.27)$$

It is shown in Appendix 22.1 that equation (22.23) can accordingly be rewritten

$$\frac{\theta_2}{x'} = \frac{13.5}{\sigma_2} (1 - C_{e2}^2) I_1 \quad (22.28)$$

where

$$I_1 = \int_{\eta'_a}^{\eta'_m} \frac{\phi \, d\eta'}{(1 - C_{e2}^2 \phi^2)} \quad (22.29)$$

C is the Crocco number and x' is transformed distance from the point of separation to the start of the pressure rise (at the boundary between regions II and III in Fig. 22.2) measured along the inviscid jet boundary. This is valid when $\theta_1 \rightarrow 0$. Otherwise Kirk's treatment (22) is used to account for the effect of the initial boundary layer on shear-layer development. As shown in Fig. 22.4, the real shear-layer development is replaced by an equivalent 'asymptotic' shear layer growing from zero thickness at a

virtual origin a distance x'_1 upstream of the separating edge. This is related to θ_2 to give an equation for x'_1 analogous to (22.28), the result (derived in Appendix 22.2) being

$$\frac{\theta_2}{x'_1} = \frac{13.5(1 - C_{e2}^2)}{\sigma_2} \int_{-\infty}^{\eta'_m} \frac{\phi \, d\eta'}{(1 - C_{e2}^2 \phi^2)} \quad (22.30)$$

It follows from equations (22.19) and (22.30) that the required transformed distance x' is given by

$$x' = x'_1 + x'_2 = \frac{\theta_2 \sigma_2}{13.5(1 - C_{e2}^2)} \frac{1}{I_2} + \int_0^{x_2} \left[\frac{r(x)}{r_1} \right]^2 dx \quad (22.31)$$

where

$$I_2 = \int_{-\infty}^{\eta'_m} \frac{\phi \, d\eta'}{(1 - C_{e2}^2 \phi^2)} \quad (22.32)$$

and x'_2 is the transformed distance from the point of separation to the start of the pressure rise, due to jet re-attachment. The determination of x'_2 is also given in Appendix 22.2. I_2 is a function of Crocco number C_{e2} , and the shape of the shear-layer velocity profile. Using the profiles of Jacques and Gailly (18) in Fig. 22.3 the variation of I_2 with C_{e2} as computed from equation (22.32) is shown in Table 22.1.

Table 22.1

C_{e2}	$I_2 \times 10^2$	C_{e2}	$I_2 \times 10^2$
0.40	1.358	0.70	1.494
0.45	1.365	0.75	1.551
0.50	1.379	0.80	1.625
0.55	1.392	0.85	1.702
0.60	1.413	0.90	1.784
0.65	1.447		

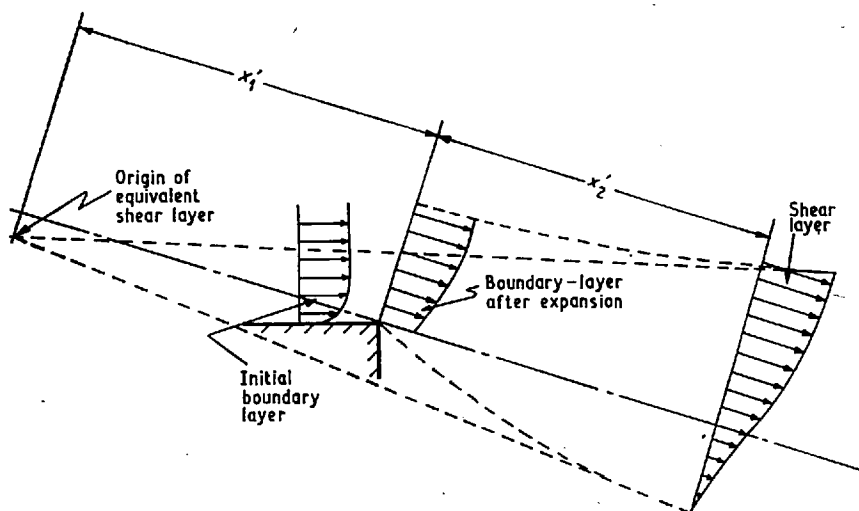


Fig. 22.4. Effect of initial boundary layer on the development of the free shear layer

Elimination of x' between equations (22.28) and (22.31) gives, after some rearrangement,

$$\frac{\theta_2}{r_1} = \frac{13.5(1-C_{e2}^2)}{r_1\sigma_2} x'_2 \left(\frac{1}{I_1} - \frac{1}{I_2} \right)^{-1} \quad (22.33)$$

When θ_1 is small and the mass of fluid in the separated region is conserved, the median and dividing streamlines are sufficiently close together to approximate the slope of the velocity profiles in Fig. 22.3 by a suitable mean value ξ covering the ranges $0.45 < \phi < 0.9$, $2.0 < M_{e2} < 3.5$, as follows:

$$\xi = \frac{d\eta'}{d\phi} = 1.964(\phi_m + \phi_d) - 0.75(\phi_m + \phi_d)^2 - 1.143 \quad (22.34)$$

With equation (22.34) a change of variable from η' to ϕ in equation (22.29) is possible, and I_1 can be evaluated as follows:

$$I_1 = \frac{\xi}{2C_{e2}^2} \ln \left[\frac{1-C_{e2}^2\phi_d^2}{1-C_{e2}^2\phi_m^2} \right] \quad (22.35)$$

Equation (22.33) can now be rewritten as

$$\frac{\theta_2}{r_1} = \frac{13.5(1-C_{e2}^2)}{r_1\sigma_2} x'_2 \left\{ \frac{2C_{e2}^2}{\xi \ln \left[\frac{1-C_{e2}^2\phi_d^2}{1-C_{e2}^2\phi_m^2} \right]} - \frac{1}{I_2} \right\}^{-1} \quad (22.36)$$

In the particular instance when the median and dividing streamlines coincide, $\theta_1 = 0$, $x'_1 = 0$, $\phi_m = \phi_d$ and hence $\theta_2 = 0$.

For the range $1.5 < M_{e2} < 4$, we approximate the variation of median streamline velocity ratio ϕ_m from the predictions of Jacques and Gailly (18) by the following linearized relation:

$$\phi_m^2 = 0.321 + 0.038M_{e2} \quad (22.37)$$

The jet-spreading parameter σ_2 is also Mach-number dependent. Korst and Tripp (23) show that for $M_{e2} < 3$,

$$\sigma_2 = 12 + 2.76M_{e2} \quad (22.38)$$

When considered with equations (22.34), (22.37), and (22.38), equation (22.36) may, therefore, for a given γ , be functionally represented by

$$\theta_2 = f(\phi_d, M_{e2}) \int_0^{x'_2} \left[\frac{r(x)}{r_1} \right]^2 dx \quad (22.39)$$

The reattachment criterion

A search of the literature suggests some difficulty in defining a satisfactory reattachment criterion. As previously stated, Korst's treatment (6) of two-dimensional base flow was shown by Nash (7) to be founded on inaccurate assumptions regarding (a) boundary layer thickness at separation and (b) the pressure rise in the dividing streamline up to reattachment. Nash's own experimentally based criterion of 0.35 for the ratio $(p_i - p_2)/(p_3 - p_2)$ in Fig. 22.2 was in turn shown by McDonald (24) to be unsatisfactory since the pressure ratio as defined depends on both Reynolds and Mach numbers. Instead, McDonald (24) proposed the dimensionless shape parameter H of the redeveloping boundary layer downstream of reattachment as a reattachment criterion. The value of 1.4 assigned to H by McDonald (24) was subsequently shown by Roberts (25) to be unrepresentative, since H is also a function of Mach number. Roberts's own suggestion, based on experimental evidence and assuming oblique shock recompression, was the ratio of free-stream Mach number after reattachment to that at reattachment.

The use of these two-dimensional reattachment criteria in diffuser analyses involves reference to the highly non-uniform flow field downstream of reattachment, where the influence of expansion waves radiating from the point of separation is especially marked. Even if it were possible to select a representative normalizing parameter in this region, the work of Sirieix *et al.* (10) shows that, within limits, perturbations in p_3 have little or no effect on p_r . This implies that reattachment conditions are unaffected

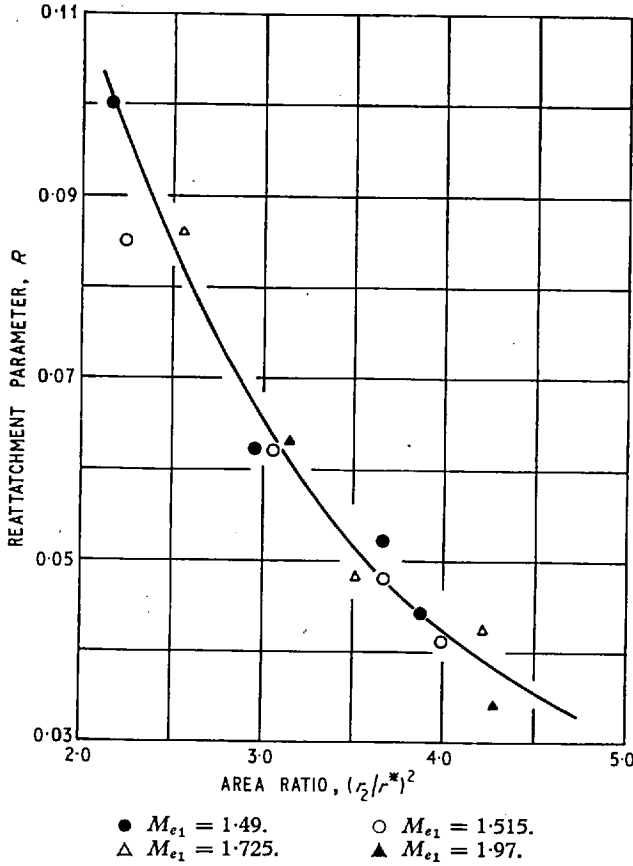


Fig. 22.5. Variation of reattachment parameter with diffuser area ratio

by those downstream. Therefore, the reattachment parameter used by the authors is so defined that it refers to reattachment and upstream pressures only, as follows:

$$R = \frac{p_r/p_2 - 1}{p_{01}/p_2 - 1} \quad (22.40)$$

The authors' measurements of p_{01}/p_2 and p_r/p_2 in started axisymmetric diffusers are presented in this form in Fig. 22.5 as a function of $(r_2/r^*)^2$. The zone of reattachment on the transparent diffuser wall was located by observing the 'line' of flow reversal traced in a film of Ragosine grease mixed with yellow dye. Closely pitched wall static pressure taps yielded the pressure rise to reattachment. The effect of nozzle exit Mach number on R in the range $1.5 < M_{e1} < 2.0$ was negligibly small. An acceptable correlation for the data in Fig. 22.5 appears to be

$$R = 0.2327 - 0.0776 \left(\frac{r_2}{r^*} \right)^2 + 0.0074 \left(\frac{r_2}{r^*} \right)^4 \quad (22.41)$$

for $2.25 < (r_2/r^*)^2 < 4.5$.

Finally, we note that if the dividing streamline is assumed to stagnate isentropically, p_r is related to p_2 by the equations

$$\frac{p_r}{p_2} = (1 - C_{e2}^2 \phi_d^2)^{-\gamma/(\gamma-1)} \quad \text{if } M_d < 1 \quad (22.42)$$

$$\frac{p_r}{p_2} = \frac{\left[\frac{(\gamma+1)}{(\gamma-1)} \frac{C_{e2}^2 \phi_d^2}{(1 - C_{e2}^2 \phi_d^2)} \right]^{\gamma/(\gamma-1)}}{\left[\frac{4\gamma}{(\gamma^2-1)} \frac{C_{e2}^2 \phi_d^2}{(1 - C_{e2}^2 \phi_d^2)} \left(-\frac{\gamma-1}{\gamma+1} \right) \right]^{1/(\gamma-1)}} \quad \text{if } M_d > 1 \quad (22.43)$$

Equation (22.43) is based on the Rayleigh pitot-tube formula for supersonic flow. Elimination of R and p_r/p_2 between equations (22.40), (22.41), and (22.42) yields

$$\phi_d = \frac{1}{C_{e2}} \left[1 - \left\{ 1 + \left(\frac{p_{01}}{p_2} - 1 \right) \times \left(0.2327 - 0.0776 \frac{r_2^2}{r^{*2}} + 0.0074 \frac{r_2^4}{r^{*4}} \right) \right\}^{(1-\gamma)/\gamma} \right]^{1/2} \quad (22.44)$$

a more complex expression for ϕ_d resulting if equation (22.43) is used instead of (22.42), but still of the functional form

$$\phi_d = \phi(C_{e2}, p_{01}/p_2, r_2^2/r^{*2}, \gamma) \quad (22.45)$$

These alternatives are relevant to the above discussion for the following reasons. In the started diffuser R is independent of p_{01}/p_a and M_{e1} ; this implies that p_r/p_{01} and p_2/p_{01} are not affected by changes in p_{01}/p_a . The measured pressure rise to reattachment yields a dividing-streamline Mach number, M_d , such that $1.02 < M_d < 1.18$ for $2.25 < (r_2/r^*)^2 < 5.23$; the stagnation of this streamline must therefore involve a shock. These effects can only isolate the separated region and the point of reattachment from downstream influence, a conclusion which accords with the observations of Sirieix *et al.* (10). Thus, the change from the unstarted to the started condition appears to correspond to the attainment of unit Mach number in the dividing streamline; this is analogous to the establishment of critical pressure ratio in a convergent-divergent nozzle, where the throat pressure is insensitive to downstream pressure changes, but responds linearly to an increase in upstream stagnation pressure. In the started diffuser p_2 also increases linearly with p_{01} .

PREDICTION OF DIFFUSER PERFORMANCE AND COMPARISON WITH MEASUREMENTS

For the unstarted diffuser equations (22.13) and (22.14) are used to calculate p_2/p_{01} as a function of θ_1/r_1 for prescribed values of p_{01}/p_a , r_1/r_2 , r_2/r^* and hence M_{e1} and $\theta_1 = \theta_1(p_{01}/p_a)$. A reduction in θ_1/r_1 implies an increase in nozzle Reynolds number, which can be achieved by increasing p_{01}/p_a . These parameters are related in both the unstarted and started conditions.

A suitable calculation procedure for evaluating p_2/p_{01} as a function of θ_1/r_1 in the started diffuser is as follows:

1. Assume a value of p_2/p_{01} and from the isentropic relationship calculate M_{e2} and hence C_{e2} .

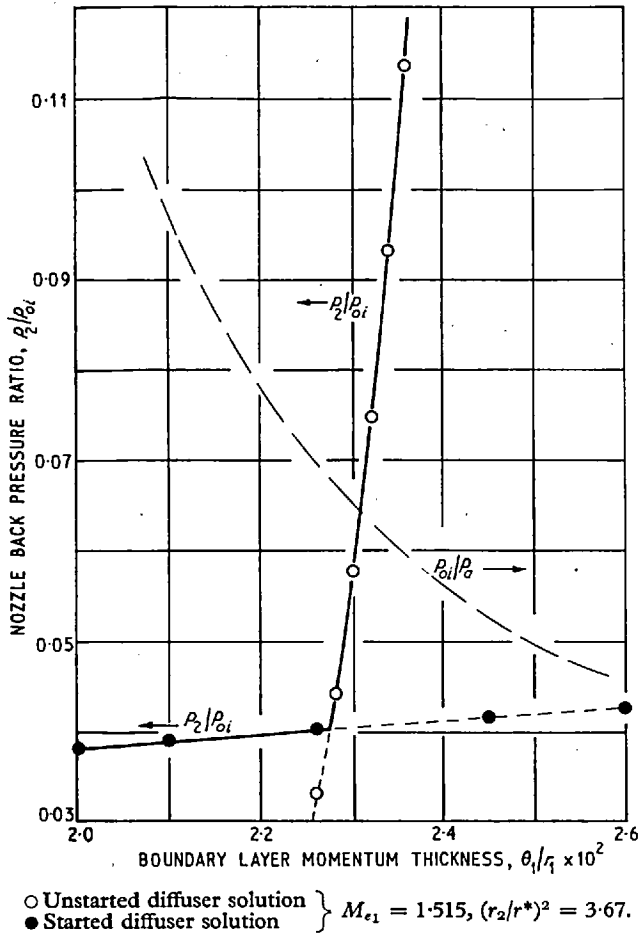


Fig. 22.6. Variation of nozzle back-pressure ratio and overall blowing pressure ratio with boundary layer momentum thickness

2. From the Prandtl-Meyer relation and the method of characteristics construct, knowing M_{e2} , the inviscid jet boundary.

3. Hence determine $x'_2 = \int_0^{x_2} [r(x)/r_1]^2 dx$, where x_2 is length measured along the jet boundary from the nozzle edge to the start of the pressure rise.

4. Knowing C_{e2} , calculate ϕ_d from equation (22.45) and ϕ_m from (22.37).

5. Calculate σ_2 from equation (22.38) and I_2 from Table 22.1.

6. Knowing C_{e2} , x'_2 , ϕ_d , ϕ_m , σ_2 , and I_2 , calculate θ_2/r_1 from (22.36).

7. Determine θ_1/r_1 from equation (22.18) for the calculated values of θ_2/r_1 and M_{e2} .

8. Repeat steps 1 to 7 for different p_2/p_{0i} .

Fig. 22.6 shows p_2/p_{0i} as the above functions of θ_1/r_1 for $M_{e1} = 1.515$ and $(r_2/r^*)^2 = 3.673$. The two curves intersect at the change from the unstarted to the started condition, so that only the full lines correspond to reality. The importance of the nozzle boundary layer in determining

diffuser performance is clearly shown. Also included in Fig. 22.6 is the corresponding variation of p_{0i}/p_a .

Predictions of p_2/p_{0i} and p_a/p_2 as functions of p_{0i}/p_a for $M_{e1} = 1.515$ and area ratios $(r_2/r^*)^2$ of 2.25, 3.06, and 3.67 are displayed in Fig. 22.7. Agreement between predicted and measured performance is best at high $(r_2/r^*)^2$, but even for $(r_2/r^*)^2 = 2.25$ the discrepancies in p_2/p_{0i} and p_a/p_2 do not exceed 10 per cent, and the value of p_{0i}/p_a for optimum performance is satisfactorily predicted for all area ratios.

The discrepancies may reasonably be attributed to delayed transition to turbulence in the presence of the favourable pressure gradient within the nozzle. For $(r_2/r^*)^2 = 2.25$ the diffuser remains unstarted up to $Re^* = 1.39 \times 10^6$. The persistence of a laminar separating boundary layer would modify the relation between θ_1 and p_2/p_{0i} . At higher Re^* , the establishment of a turbulent boundary layer would bring predictions into better accord with measurements, as the agreement in Fig. 22.7 for starting values of $Re^* > 2 \times 10^6$ suggests.

Differentiation of equation (22.41) shows that R reaches a minimum of 0.0293 when $(r_2/r^*)^2 = 5.24$. For this optimum area ratio and $M_{e1} = 1.515$, it may be shown that the diffuser would start at $p_2/p_{0i} = 0.0215$ and would yield the maximum pressure recovery p_a/p_2 of 7.5. Unfortunately, limitations on the air supply available precluded verification of this extrapolated prediction of optimum performance, which exceeds that of $p_a/p_2 = 5.9$ at $(r_2/r^*)^2 = 4.15$ made by Martin (4).

The above treatment rests on a large number of assumptions which require critical scrutiny. In applying equation (22.18) the expansion fan is assumed centred at the nozzle edge, but the experience of the authors (and others) suggests that the fan spreads over a finite distance upstream. This renders reliable measurements of nozzle exit pressure extremely difficult. Neglect of the transverse velocity component along the x -axis in the Mangler transformation, and the assumption of uniform pressure in the separated region, may legitimately be queried. Further work is required to test the empirical reattachment criterion outside the measured ranges of $(r_2/r^*)^2$ and M_{e1} , and if possible to place it on a wholly theoretical basis. Nevertheless, current predictions accord sufficiently well with available measurements for design purposes, though agreement may be due to a fortuitous cancellation of several errors introduced by the above assumptions.

CONCLUSIONS

This paper describes methods of predicting base pressure and static pressure recovery in an axisymmetric parallel diffuser following a sudden enlargement in flow area of an initially supersonic stream. For the unstarted diffuser, with a strong normal shock near the diffuser inlet, the base pressure is calculated from a simple force-momentum balance, while for the started diffuser, the Korst-Chapman model of flow over a backward-facing step is the basis of analysis for the diffuser entry region up to jet

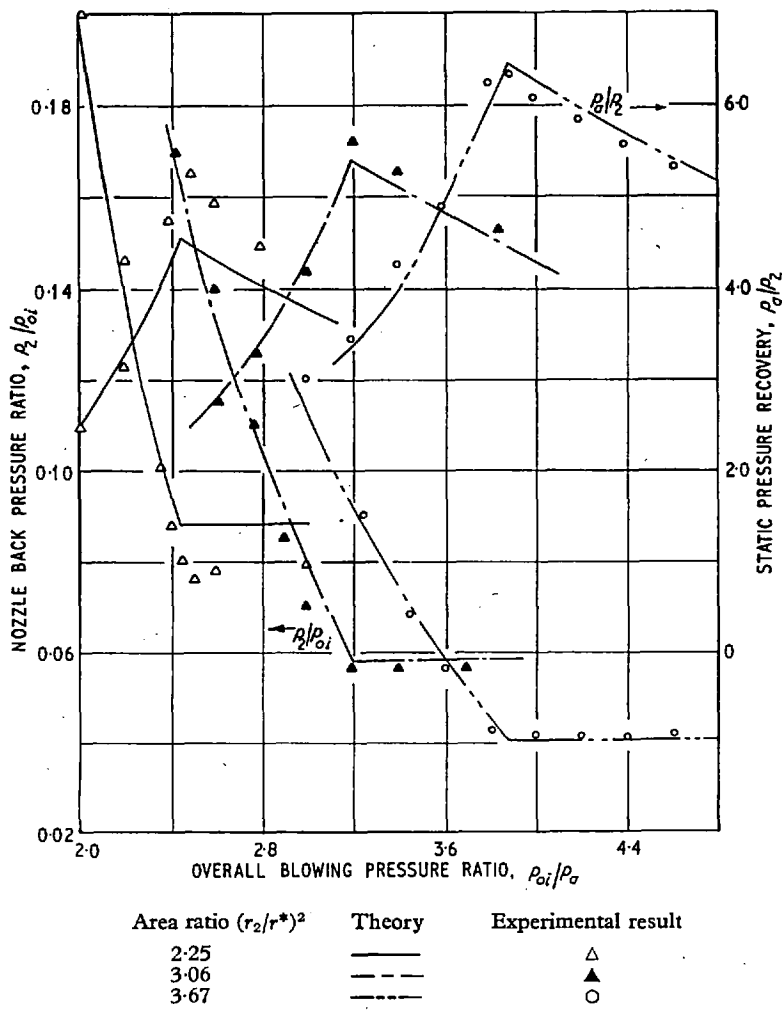


Fig. 22.7. Comparison between measured and predicted nozzle back-pressure ratio and static pressure recovery at $M_{e1} = 1.515$ for different area ratios

reattachment. The treatment is entirely theoretical, apart from the use of an empirical reattachment criterion for which there is at present no theoretical foundation, but which appears superior to those previously employed. Calculated values of base pressure compare favourably with those measured by the authors in axisymmetric diffusers. The treatment indicates clearly the dominant effect of boundary-layer momentum thickness at separation. It also provides a plausible reason for the attainment of maximum pressure recovery in terms of a breakdown of communication between the separated region and conditions downstream when the Mach number along the dividing streamline reaches or exceeds unity. The analysis concludes with a prediction of area ratio for maximum possible static pressure recovery.

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APPENDIX 22.1

DERIVATION OF EQUATION (22.28)

The momentum thickness of the shear layer immediately after expansion to p_2 is given by

$$\theta_2 = \int_{y'_d}^{y'_m} \frac{\rho U'}{\rho_{e2} U'_{e2}} dy' \quad \dots \quad (22.23)$$

With adiabatic flow and uniform static pressure across the shear layer we have, for $\gamma = 1.4$,

$$\frac{\rho}{\rho_{e2}} = \frac{T_{e2}}{T_e} = \frac{5 + M_{e2}^2}{5 + M_e^2} \quad \dots \quad (22.46)$$

The Crocco number is then related to Mach number and flow velocity by

$$C = \left(1 + \frac{5}{M^2}\right)^{-1/2} = \frac{U}{U_{\max}} \quad (22.47)$$

where $U_{\max}$ is the flow velocity at $T = 0$. Elimination of Mach number between these equations gives

$$\frac{\rho}{\rho_{e2}} = \frac{1 - C_{e2}^2}{1 - C_{e2}^2 (U'/U'_{e2})^2} \quad (22.48)$$

Substituting for ρ/ρ_{e2} in (22.23) and writing $\phi = U'/U'_{e2}$, $y' = 13.5x'\eta'/\sigma_2$ according to (22.27), we have

$$\frac{\theta_2}{x'} = \frac{13.5}{\sigma_2} (1 - C_{e2}^2) \int_{\eta'_d}^{\eta'_m} \frac{\phi d\eta'}{(1 - C_{e2}^2 \phi^2)} \quad (22.28)$$

APPENDIX 22.2

ESTIMATION OF DISTANCE x'

The transformed distance from the virtual origin of the shear layer to the start of pressure rise before reattachment is

$$x' = x'_1 + x'_2$$

where, with reference to the nozzle edge, x'_1 is the origin shift upstream and x'_2 is measured downstream along the inviscid jet boundary to the start of the pressure rise. The former is found by equating the boundary-layer momentum thickness after separation θ_2 to that of a free shear layer θ_s that has grown over a distance x'_1 . This is given in general terms by

$$\theta_s = \int_{-\infty}^{\infty} \frac{\rho U}{\rho_{e2} U'_{e2}} \left(1 - \frac{U}{U'_{e2}}\right) dy \quad (22.49)$$

Using (22.27) and (22.48) with $\theta_s = \theta_2$, equation (22.49) can be rewritten in the relevant nomenclature as

$$\frac{\theta_2}{x'_1} = \frac{13.5}{\sigma_2} (1 - C_{e2}^2) \int_{-\infty}^{\infty} \frac{\phi(1-\phi)}{(1 - C_{e2}^2 \phi^2)} d\eta' \quad (22.50)$$

The integral in (22.50) can be expanded as follows:

$$\int_{-\infty}^{\infty} \frac{\phi(1-\phi)}{(1 - C_{e2}^2 \phi^2)} d\eta'_1 = \int_{-\infty}^{\eta'_m} \frac{\phi d\eta'}{(1 - C_{e2}^2 \phi^2)} + \int_{\eta'_m}^{\infty} \frac{\phi d\eta'}{(1 - C_{e2}^2 \phi^2)} - \int_{-\infty}^{\infty} \frac{\phi^2 d\eta'}{(1 - C_{e2}^2 \phi^2)} \quad (22.51)$$

It follows from the definition of median streamline that

$$\int_{\eta'_m}^{\infty} \frac{\phi d\eta'}{(1 - C_{e2}^2 \phi^2)} = \int_{-\infty}^{\infty} \frac{\phi^2 d\eta'}{(1 - C_{e2}^2 \phi^2)} \quad (22.52)$$

whence (22.50) reduces on rewriting to

$$x'_1 = \frac{\sigma_2 \theta_2}{13.5(1 - C_{e2}^2) I_2} \quad (22.53)$$

where

$$I_2 = \int_{-\infty}^{\eta'_m} \frac{\phi d\eta'}{(1 - C_{e2}^2 \phi^2)} \quad (22.32)$$

Calculation of x'_2 is based on measurements of distance z along the diffuser wall over which the pressure rise to reattachment occurs; these measurements are thus associated with the pressure measurements used to determine the empirical reattachment criterion R . Over the range $2.25 < (r_2/r^*)^2 < 4.5$, the data are correlated by

$$\frac{z_2}{z_r} = 1.935 - 0.59 \left(\frac{r_2}{r^*}\right)^2 + 0.059 \left(\frac{r_2}{r^*}\right)^4 \quad (22.54)$$

where z_2 and z_r are measured from the diffuser inlet to the start of the pressure rise and to the point of reattachment respectively. Neglecting the curvature of the axisymmetric inviscid jet boundary, the corresponding distances x_2 and x_r along this boundary from the nozzle edge are by similar triangles in fixed ratio with z_2 and z_r . It follows that

$$\frac{x_2}{x_r} = 1.935 - 0.59 \left(\frac{r_2}{r^*}\right)^2 + 0.059 \left(\frac{r_2}{r^*}\right)^4 \quad (22.55)$$

For given M_{e1} , p_2/p_{01} and r_2/r^* , x_r is obtained by construction

of the inviscid jet boundary; x_2 is subsequently evaluated from (22.55). The transformed distance x'_2 is then determined from

$$x'_2 = \int_0^{x_2} \left(\frac{r(x)}{r_1}\right)^2 dx \quad (22.56)$$

APPENDIX 22.3

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