

STRESSES MEASURED BY PHOTOELASTICITY
IN LUBRICATED LINE CONTACT

by
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ABSTRACT

Photoelasticity has been used to determine the contact stresses in a glass disc, which is in rolling contact with a steel disc. By postulating various polynomials for the pressure curve in different parts of the contact it is possible, by knowing the position of the principal stress difference, to find the shape of the complete pressure curve. An alternative method has also been developed, which calculates the pressure curve direct from the isochromatics. At the moment this method is restricted by the quality of the experimental stress field. A method is foreseen however, of overcoming this difficulty.

Elastic deformations due to the pressure curves have been produced, these have been compared with the film shapes obtained from the inverse solution of Reynolds equation. The film thickness was obtained by graphical differentiation.

Oil entrapment effects were demonstrated. The stress patterns under these conditions indicated that pressures higher than the Hertz maximum were being created. Oil starvation effects were also demonstrated. The shape of the stress patterns under these conditions gradually changed to the shape normally associated with dry rolling.

Finally, a series of tests were performed to investigate the effect of surface roughness on lubricated contacts. It was observed that as the roughness increased (all other conditions being equal) the shape of the stress patterns changed to the shape normally associated with dry rolling. These conditions of partial surface contact with a lubricant, are probable causes of failure by pitting.

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SYMBOLS

Symbols which only arise once, and are defined on the spot, are not listed.

2B	width of Hertzian contact zone
D	Dawson number, defined on page 89
E_s, E_g	elastic moduli for steel and glass discs
E'	effective elastic moduli, defined on page 110
G	$\alpha E'$
h	lubricant film thickness
h_m	film thickness at point of maximum pressure
h_o	film thickness on line of centres
$\bar{h}$	h/h_m
p	pressure intensity
p_1-p_2	principal stress difference
P	normal line load per unit width of disc
PM	maximum Hertz pressure
q	reduced pressure, defined on page 45
R_s, R_g	radii of steel and glass discs
R	effective radius of disc pair, defined on page 110
s	additional co-ordinate in x-direction
u_1, u_2	surface velocity of solids in x-direction
u	$\frac{1}{2}(u_1+u_2)$
U	$\gamma_o u/E'R$
v	displacement in y-direction
W'	total load per unit width of disc
W	$W'/E'R$
x,y,z	co-ordinates
α	pressure exponent of viscosity
γ	viscosity
γ_o	viscosity at conditions of entry to contact
μ_s, μ_g	Poisson's ratio for steel and glass.

CHAPTER 1INTRODUCTION1.01 Review of previous work.

The aim of this work is to show how photoelasticity can be used to determine the pressure distribution and the film shape, in lubricated line contact. This form of lubrication, which involves elastic deformation of the rolling elements, is generally known as Elastohydrodynamic Lubrication. The development of this subject has progressed considerably within the last twenty years, both theoretically and experimentally. One of the most recent reviews on this topic was made by Dowson⁽¹¹⁾ in 1967. This paper, together with books by Dowson and Higginson⁽¹³⁾ and Cameron⁽⁴⁾ form a comprehensive survey of work covered up to mid 1967. Reference to more recent work will appear later, where appropriate. The purpose of this survey is to show only how photoelasticity has been used to study the stresses in elastic line contact.

In general, photoelasticity has been used as an off-shoot of their main research projects. A typical example of this was Way⁽⁴⁶⁾. This paper, published in 1935, is often quoted for its valuable work on pitting, but a very small section of the report is devoted to a photoelastic experiment. This experiment compares the contours of the principal stress difference, calculated from a Hertzian pressure distribution, with the photoelastic fringes in two bakelite cylinders, which were pressed together. Good agreement between theoretical and experimental results was obtained.

Similarly, Liu⁽³¹⁾ in 1950 produced some theoretical work on internal stresses due to tangential and normal loads on an elastic medium. Also included in the report were some pictures of photoelastic stresses caused by two cylinders in line contact subject to traction. Very good fringe patterns were obtained showing a distinct tilt of the inner fringes. However, no information was given regarding the size of the cylinders or of their material. Nor was any attempt made to relate the theoretical and experimental work.

Later in 1959 Ollerton⁽³⁷⁾ examined the contact stresses, photoelastically, between curved surfaces under radial loads. The surfaces remained stationary and very good stress patterns were obtained. This was probaly due to the fact that a frozen stress technique was used, this enabled thin slices to be examined eliminating possible end effects.

In 1961, Sternlicht, Lewis and Flynn⁽⁴²⁾ investigated the photoelastic stresses in the contact region of two discs, for dry rolling contact and for lubricated rolling and sliding contact. One disc was made of steel 6 inches in diameter and $\frac{3}{8}$ inch thick. the other was made of a photoelastic material (not specified) also 6 inches in diameter but $\frac{1}{4}$ inch thick. Only the steel disc was driven, which would account for slipping between the discs when lubricated. The authors reported that the stress pattern obtained under lubricated conditions was asymmetrical or 'tipped'. They conluded that this tipping was caused by the tangential forces acting between the sliding discs. The authors did not indicate how close to the contact region they made the comparision of the lubricated and dry rolling

stress patterns. Hardly any difference in the stress patterns could be detected from the prints of the photographs available. No attempt had been made to produce a contact pressure curve.

In 1964, Munnich and Milton⁽³⁵⁾ examined photoelastically, the distribution of load through a roller bearing. They were able to show that the height of the chock greatly influenced the distribution of load. The paper is of interest since the photographs of the stress distribution in the rollers, showed severe blurring of the fringes. There are a number of factors which may account for this, these will be commented on later.

Remarkable photographs were obtained by Hamilton⁽²⁵⁾ in 1964 when he investigated the rolling contact stresses between a glass disc and a flat aluminium plate. Yielding occurred in the aluminium plate. The photoelastic stress pattern in the glass disc, indicated a distinct tilt of the inner fringe in the rolling direction. Dr. Hamilton also calculated the contact pressure distribution from the stress pattern by a method similar to that adopted later in this report. Dr. Ollerton, in his communication on Hamiltons' paper, had also noted a tilt of fringes in an Araldite disc subjected to traction on a steel plate. Experimental conditions were, however, completely different.

Also in 1964, Jones and Reason⁽²⁸⁾ investigated, by a photostress technique, the stress pattern at the end faces of two steel discs in line contact. A stress pattern different from the one normally associated with line contact

was observed, due mainly to the bulging effects of the ends. Rolling contact stresses were to be investigated at a later date, but a private communication with Mr. Reason revealed that this experiment was abandoned due to failure of a disc shaft. However, tilting of fringes had been observed before failure.

In 1965, Jacek Stupnicki⁽⁴³⁾ published a comprehensive report on the photoelastic stresses in dry and lubricated line contact. In high load tests (maximum Hertz pressure 3000 kg/cm^2 , $42000 \text{ lb}_f/\text{in}^2$ or 290 MN/m^2) he used flint glass discs 120 mm in diameter, 30 mm thick. In low load tests (400 kg/cm^2 , $5800 \text{ lb}_f/\text{in}^2$ or 40 MN/m^2) he used composite discs also 120 mm in diameter, 30 mm thick. Composite discs were used so that the stress patterns in the central region of the contact could be observed. The inner region of the composite disc was made of Columbia Resin 39 (CR-39), while the outer parts were of organic glass, made especially for the purpose. The author claimed that the organic glass (a plastic) had an optical sensitivity over 100 times smaller than CR-39, while the elastic moduli for the two materials were similar. The report stated that a tilt of the fringes was observed in the lubricated case and no tilt when rolling dry. It was however, very difficult to determine any tilt of the fringes from the photographic prints available. From the stress patterns, Stupnicki determined the pressure distribution in the contact region, by the method of Characteristics. The following conclusions were drawn by Stupnicki:

(1) The presence of an oil film between two rotating discs reduces the maximum pressure by an average amount of 20% relative to the maximum Hertz value.

(2) The degree of reduction of pressure is effected by the initial viscosity of the oil.

(3) A pressure spike could not be detected.

Cope⁽⁶⁾ 1968, developed an optical method of determining not only the principal stress difference, but also the principal stress sum, in the contact region of two discs in rolling line contact. The contact pressure distribution was obtained directly from the principal stress sum at the contact edge. His experiments were conducted in the elastohydrodynamic region with high surface speeds. The theoretical pressure curve as shown in Figure 1, predicted by Dowson, Higginson and Whitaker⁽¹⁶⁾ was more or less confirmed by Cope. He produced experimental points from the inlet region to the spike which followed the theoretical curve. Points beyond this region were however, unreliable. Fortunately, the area under his inlet sweep of the curve was slightly less than the applied load. Cope therefore reasonably assumed that there would be a sudden drop in pressure, so that the total area under the pressure curve would equal the applied load.

Finally Khayyat and Lancaster³⁰⁽³⁰⁾ in 1969, examined photoelastically the effect of rolling a lead strip. The lead was rolled between two discs made from Columbia Resin 39. The load was large enough to cause the lead to yield when passing between the discs. The contact

length was about 10 mm., consequently very clear stress patterns were obtained. The normal and shear stress distributions at the surface were obtained by a Shear Difference method.

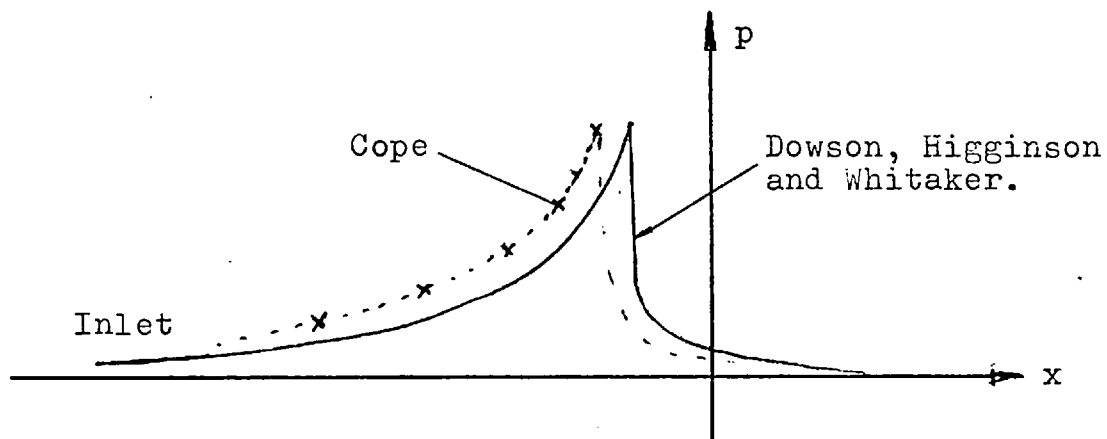


Fig. 1. Comparison of experimental and theoretical pressure curves.

1.02 General notes on photoelasticity.

Photoelasticity, is the study of the internal stresses in an elastic transparent material by observing the effects of the passage of polarised light through it. The theory of photoelasticity has been described many times. Textbooks by Frocht⁽²⁴⁾ and also Jessop and Harris⁽²⁷⁾ are two examples. Only a brief summary therefore, will be given here.

Fringe patterns

When plane polarised light passes through a piece of stressed glass, the incident ray is split into two components in the directions of the two principal stresses. One component is retarded relative the other by an amount proportional to the thickness of the glass and to the difference of the principal stresses, at that point. The two components, on leaving the glass, are resolved at the analyser. A resultant ray is formed from which, the stresses at a point in the glass can be determined.

Interference

Suppose the light wave emerging from the polariser can be considered as a vibration of the form:

$$V = a \sin \frac{2\pi}{\lambda} (ct-z), \quad \text{occurring in the plane OP} \\ \text{as shown in Figure 2.}$$

Where V = the displacement perpendicular to the axis of the polariscope measured in the direction of polarisation at a time t .

a = amplitude of vibration

c = velocity of lightwave

λ = the wavelength of the light

z = constant distance along the axis of the polariscope.

As this polarised wave passes into the stressed material it is resolved into the planes of principal stress Ox and Oy with components $x = V \cos\phi$ and $y = V \sin\phi$.

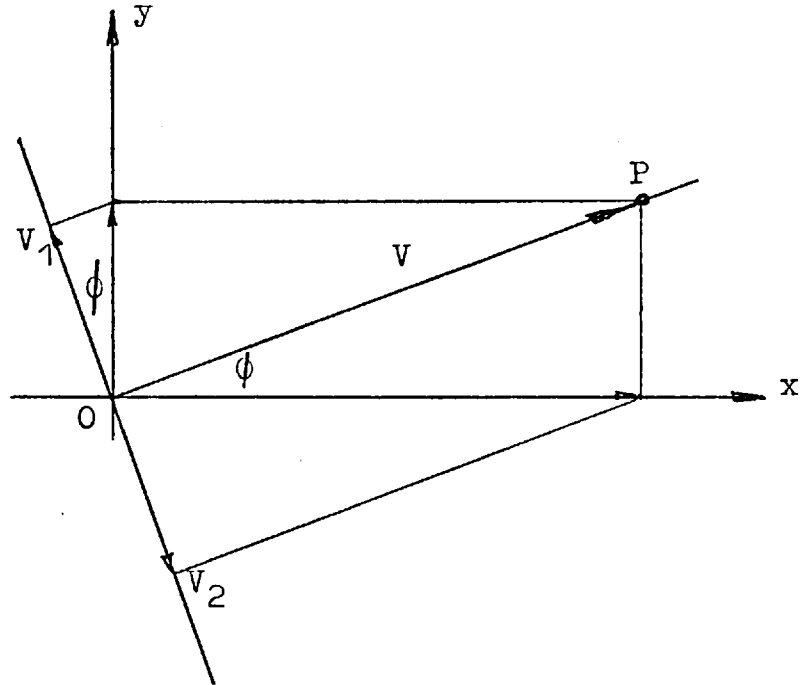


Fig. 2. Polarised wave components.

During the passage through the stressed glass the waves will be retarded relative to one another by an amount r , given by

$$r = C (p_1 - p_2) d$$

Where $p_1 - p_2$ is the difference in principal stresses,

d is the thickness of the glass and

C is the constant of proportionality, called the Stress Optical Coefficient.

The two waves emerging from the stressed plate will be expressed as

$$x_1 = a \cos \phi \sin \frac{2\pi}{\lambda} (ct - z - r) \quad \text{and}$$

$$y_1 = a \sin \phi \sin \frac{2\pi}{\lambda} (ct - z).$$

In order to observe the interference caused by

the phase difference, the two waves are now resolved perpendicular to OP by the analyser, giving components

$$V_2 = y_1 \cos \phi \quad \text{and} \quad V_1 = x_1 \sin \phi.$$

Therefore,

$$\begin{aligned} V_2 - V_1 &= a \sin \phi \cos \phi \sin \frac{2\pi}{\lambda} (ct - z) \\ &\quad - a \cos \phi \sin \phi \sin \frac{2\pi}{\lambda} (ct - z - r) \\ &= a \sin 2\phi \sin \frac{\pi r}{\lambda} \cos \frac{2\pi}{\lambda} (ct - z - \frac{r}{2}). \end{aligned}$$

This is a vibration whose amplitude is

$$a \sin 2\phi \sin \frac{\pi r}{\lambda}.$$

The intensity of light is proportional to the square of the amplitude. Therefore, the observed intensity will be zero, at the point considered,

$$\begin{aligned} \text{when either } \frac{\pi r}{\lambda} &= n\pi, \\ \text{or } 2\phi &= n\pi. \end{aligned}$$

Isochromatics

The first condition $\frac{\pi r}{\lambda} = n\pi$, can be rewritten as; $r = C (p_1 - p_2) d = n\lambda$.

At all points where the relative retardation is zero dark spots will occur, forming a dark line. Similar dark lines (if monochromatic light is used) will appear forming the locus of points where the relative retardation is one wavelength, and similarly for n wavelengths. These dark lines are called Isochromatics (lines of constant colour), or simply, fringes. In the case of monochromatic light, complete extinction will occur producing black lines.

White light, however, has a continuous spectrum with a range of wavelengths from about 3900 Å (Violet), to 7200 Å (Red). In this case separate colours will be extinguished at slightly differing stresses. Complete extinction will not occur. Furthermore, the higher the order of fringe, the less clearly defined the stress becomes.

Isoclinics

The second condition for zero intensity was $\phi = \frac{n\pi}{2}$. This is the condition that the directions of the principal stresses should be along the axes of the polariser and analyser. At every point in the glass where the principal stresses are parallel to the polariser and analyser axes, a dark spot will appear. These spots will, in general, form a dark line across the model, called an isoclinic line.

If the glass disc is viewed in plane polarised light, isoclinic lines will appear passing through the contact region. This obscures the isochromatics in that region. The isoclinics can be eliminated, without affecting the isochromatics, by using quarter-wave ($\lambda/4$) plates, to give circular polarised light.

Circular polarised light

Consider the case of two plane polarised beams from the same source, of equal amplitude, emerging from a birefringent material out of phase by ϵ .

That is:

$$x = a \sin \omega t \quad \text{and} \quad y = a \sin (\omega t - \epsilon)$$

Eliminating t , gives

$$y = a (\sin \omega t \cos \epsilon - \cos \omega t \sin \epsilon)$$

$$= x \cos \epsilon - \sqrt{a^2 - x^2} \sin \epsilon$$

$$(a^2 - x^2) \sin^2 \epsilon = y^2 - 2xy \cos \epsilon + x^2 \cos^2 \epsilon$$

$$\text{i.e.} \quad a^2 \sin^2 \epsilon = x^2 - 2xy \cos \epsilon + y^2$$

which is an equation of an ellipse.

Suppose $\epsilon = \frac{\pi}{2}$ the equation then becomes $x^2 + y^2 = a^2$ which is that of a circle.

As t increases from 0 to $\pi/2\omega$, x increases from 0 to a , while y increases from $-a$ to zero.

The light produced by the two waves out of phase by a quarter wavelength, is represented by a point rotating in a circular path in an anti-clockwise direction. A plate which introduces this amount of retardation in order to produce circular polarised light, is called a quarter-wave plate. It should be noted that a given quarter-wave plate matches only one particular wavelength and that it is not possible to produce a quarter-wave plate which suits white light.

There are many kinds of polariscopes in existence. The type chosen was a Mixed (light field) Circular Polariscopes. A schematic layout of the polariscopes is shown in Figure 3.

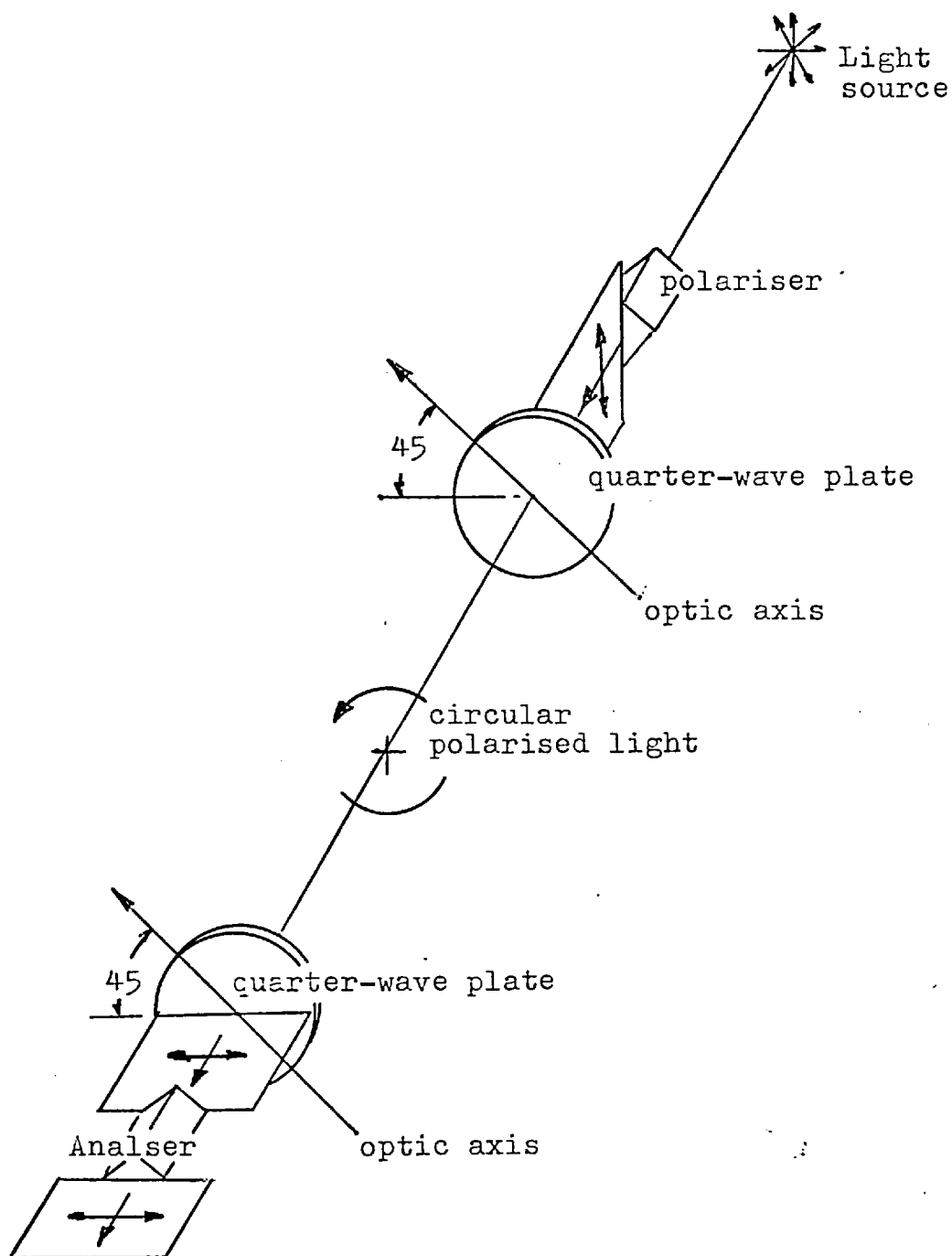


Fig. 3. Schematic diagram - arrangement of elements in a Mixed Circular Polariscope. Polariser and analyser are crossed, quarter-wave plates are parallel. Light passes through the analyser giving a light background.

CHAPTER 2APPARATUS

The apparatus consisted of a modified David Brown Two-Disc Machine. A general view of the apparatus with sketches is shown in Figures 7 to 10.

2.01 Discs

One disc was made from plate glass. Glass was chosen because it is photoelastic, and because it is capable of withstanding the very high stresses encountered in elastohydrodynamic lubrication without yielding. The glass disc was 6 inches in diameter and 1 inch thick. The other disc was made from steel also 6 inches in diameter but only $7/8$ of an inch thick. The glass disc was made wider than the steel disc for two reasons:-

(1) The Hertz pressure distribution increases at the ends of the contact region, and it was thought that this increase in pressure might cause failure at the glass edge.

(Hamilton⁽²⁵⁾ reported small chips breaking away from his glass disc.) By making the glass disc wider than the steel, the high concentration of stress at the edge would be avoided.

(2) Some of the oil passing through the contact region is squeezed out axially. The glass disc being wider than the steel would prevent the oil from overlapping the glass edge and blurring the fringe pattern. The oil could be prevented from obscuring the fringe pattern by other methods, for example by the use of air jets blowing the oil away from the glass edge, as shown in Figure 4(a); or by the use of thin stationary glass plates at either side of

the two discs, so that the whole enclosure would be filled with oil, as shown in Figure 4(b). The oil having the same refractive index as the glass would not cause any distortion of the fringe pattern.

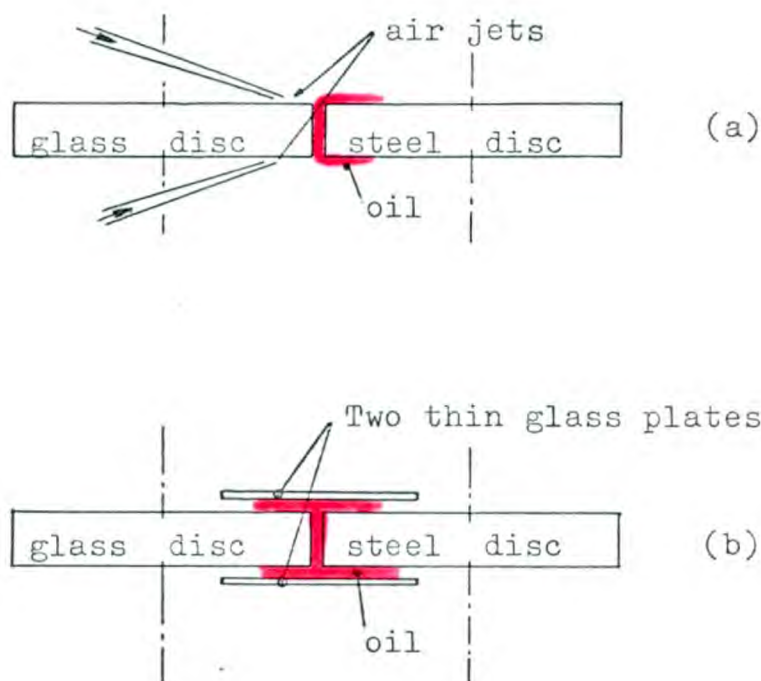


Fig. 4. Methods of preventing oil blurring of fringe pattern.

There would however, still be the tendency for the glass edge to chip, so the method of using the discs of unequal widths was chosen. Nevertheless, it was realised that the effect of having different widths would introduce additional 'end effects' which would tend to distort the fringes. However as thick discs were used the end effects were likely to be small. Unfortunately, in the investigation of a pressure spike, the fringe

pattern within 0.001 inches from the contact surface would have to be analysed. This means that end effects would have to be completely eliminated. A method of achieving this is proposed on the section on Future Work, page 104.

The glass and steel discs were specially manufactured by Optical Works, Ealing Broadway, London. The steel disc had a surface finish of approximately 2.5 micro-inches C.L.A. (Centre Line Average). Page 90. While the glass disc had a finish so smooth that it was not measurable on the Talysurf.

An additional steel disc was machined in the College Workshop in order to investigate the effect of roughness on the fringe pattern. The dimensions of this disc were similar to the other steel disc apart from the surface roughness. The disc surface was initially ground giving a C.L.A. value of approximately twenty. By using grades '2.5' and '5' emery cloth, a range of surface roughnesses were obtained. The Tally Surf traces are shown in Figure 33.

2.02 Load

The steel disc was mounted on fixed roller bearings. The glass disc, which was mounted on self-aligning bearings, was free to swing on two rocker arms as shown in Figure 9. The radial load to the two discs was achieved by independent lever arms on either side of the glass disc shaft. The calibration chart for the loads is shown in Figure 57.

2.03 Lubricant

The lubricating oil used was BP 1065. This oil was used because of its high pressure exponent of viscosity. ($\alpha = 2.1 \times 10^{-9} \text{ cm}^2/\text{dyne}$; $2.1 \times 10^{-8} \text{ m}^2/\text{N}$). This was necessary in order to make up for the low value, due to the glass, of the equivalent elastic modulus (E'). Dowson and Higginson⁽¹⁵⁾ have produced theoretical results for values of $\alpha E'$ equal to 2500 and 5000. Using BP 1065 as a lubricant gives an $\alpha E'$ value of 2300. A Kinematic viscosity-temperature chart is shown in Figure 58.

2.04 Drive

The steel disc was driven, via a Vee belt, by a 1/4 horse power D.C. shunt wound NECO geared motor. The motor speed was adjusted by means of a 'Servo-Tek' NECO speed controller. A 'Leindrive' flexible cable was connected from the shaft of the steel disc to the shaft of the steel disc so that the two discs would rotate at the same speed in opposite directions. Experiments were carried out within a speed range of 5 to 80 revolutions per minute.

2.05 Optical Bench

The layout of the optical bench is shown in Figure 10.

The monochromatic light unit is shown in Fig. 5. An Osram, 250 W, compact source mercury vapour lamp was used to produce monochromatic light.



Fig. 5 Monochromatic light unit.

This lamp produces two visible radiation lines: a green line of 5461 Å and a violet line of 4358 Å. By passing the light through a No 77 Wratten filter, the violet ray can be absorbed and a strong green line obtained. The electrical circuit requires a choke to control the current to the lamp.

Figure 6 shows the white light unit. White light was obtained by using an Atlas tungsten-halogen projector lamp 100 W, 12 V. A 20:1 transformer, variac and voltmeter were included in the circuit. The voltmeter was later discarded as it was not considered reliable. Instead, a lamp voltage-variatic dial reading chart was constructed by means of an AVO-meter. The calibration chart is shown in Figure 59.

Tungsten-halogen lamp



Fig. 6. White light unit.

The polariser and analyser was made from polioriod sheeting Ref. HN32. This gives a total luminous transmittance of 32% and an extinction of about 0.005%. It represents the best balance between transmission and extinction for general use and is recommended for photoelastic stress analysis. The quarter-wave plates were selected to suit the wavelength of the green line, transmitted by the mercury vapour lamp. To prevent their damage, the quarter-wave plates were mounted between 1 inch diameter glass discs.

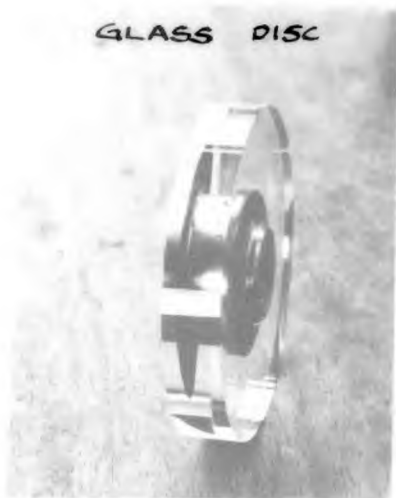
By a trial and error method, it was found that the simplest optical arrangement gave the best results. Theoretically a parallel beam of polarised light should be used, but in practice, it was found that a slightly converging beam gave better results. The converging beam

was obtained by placing a collecting lens at a distance from the light source, slightly greater than the focal length of the lens. A Beck microscope, with a telescope objective, was used to observe the fringes through another lens, placed at its focal length from the model.

Photographs were taken with a 35 mm. Pentax camera. For black and white fringes, HP4 film was used. The developer was Microphen used for 5.5 minutes. The fixer was Amphix, used for 3 minutes. For coloured fringes, Anscochrome 500 was used, the films were developed by the makers.



GLASS DISC



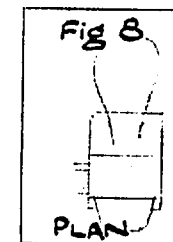
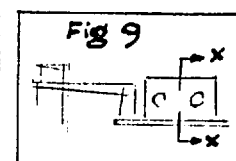
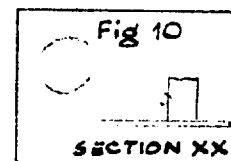
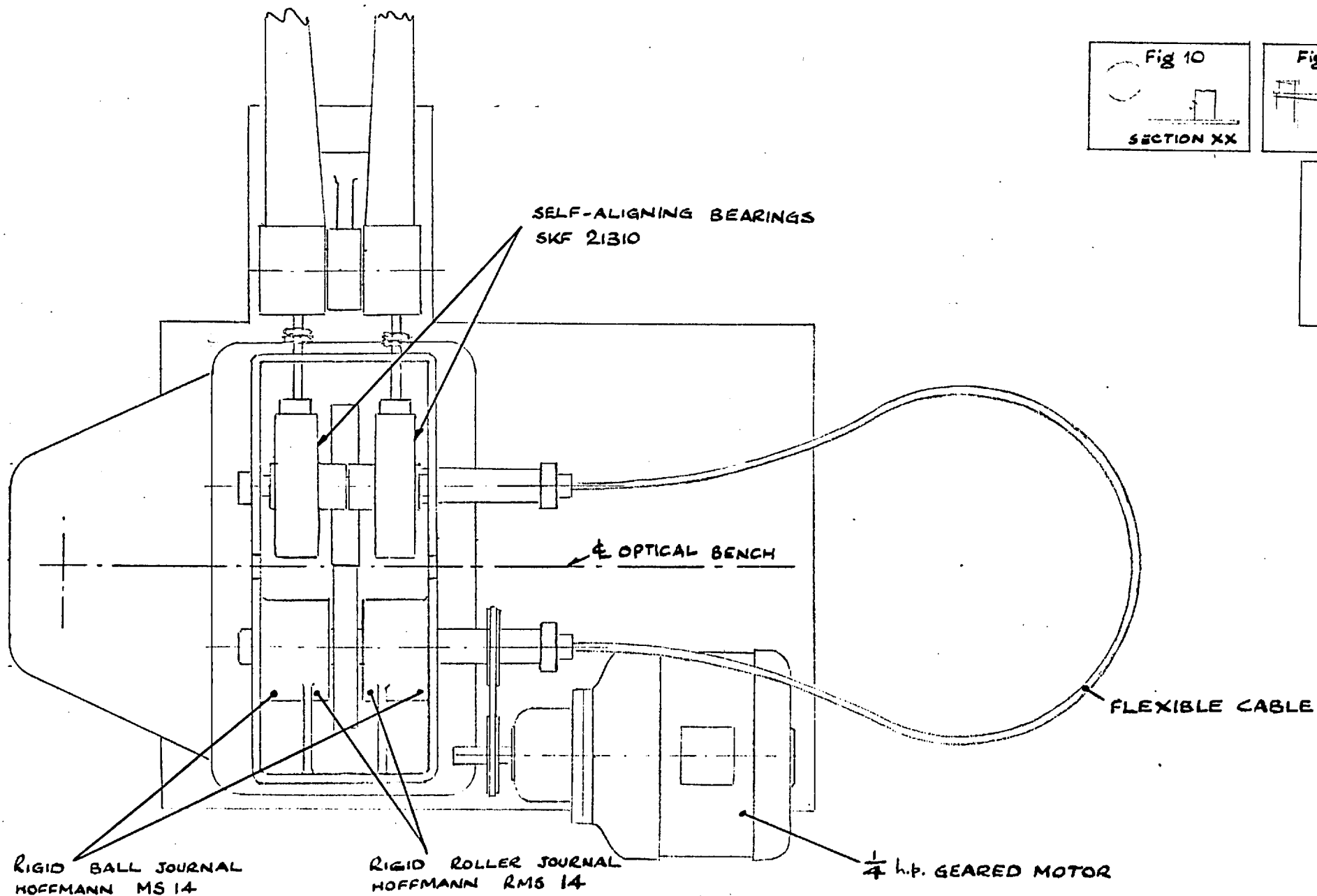
STEEL DISC



FIG. 7. GENERAL VIEW OF APPARATUS.



Fig. 8. PLAN VIEW OF APPARATUS



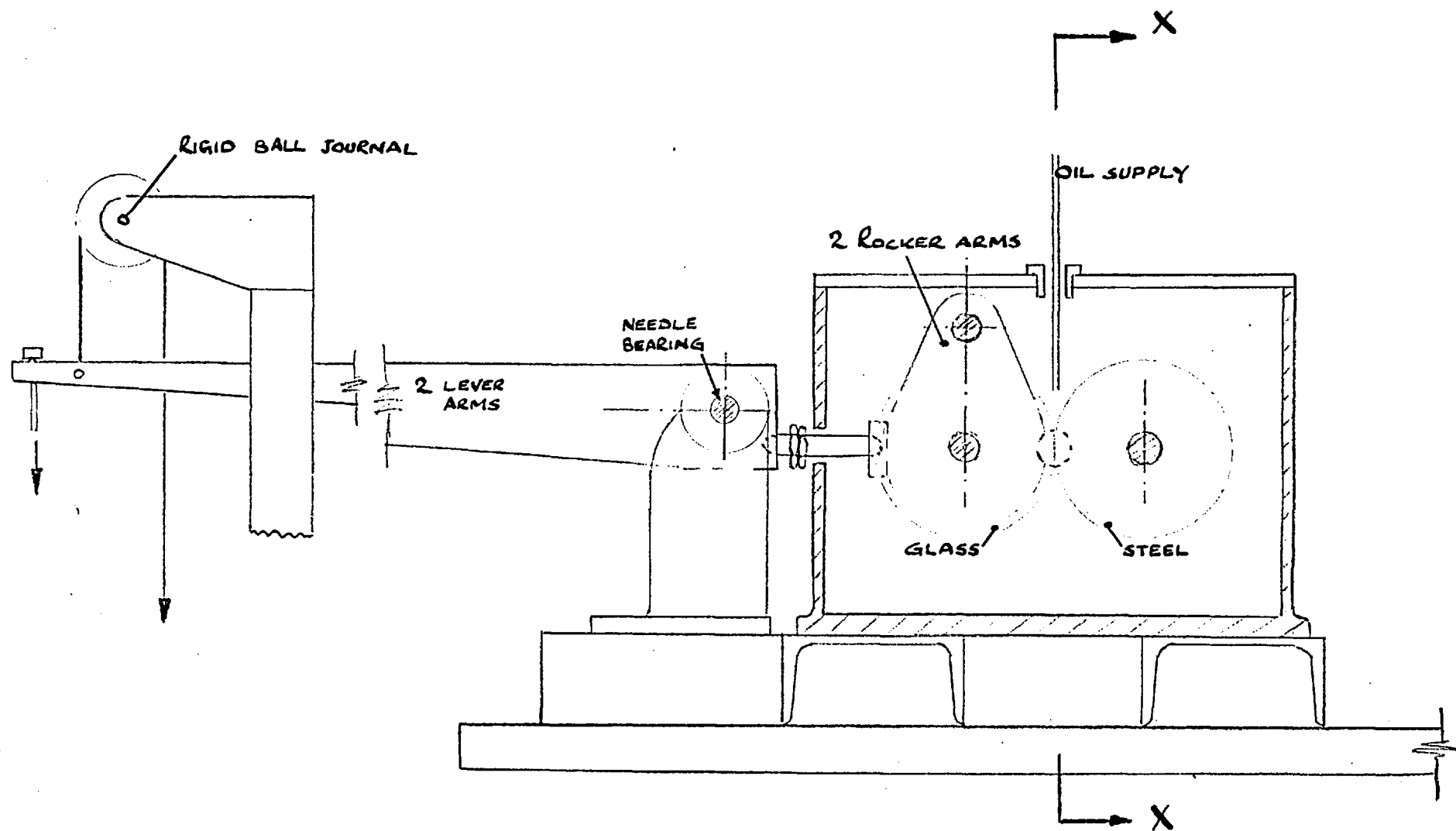


Fig. 9. FRONT ELEVATION of APPARATUS.

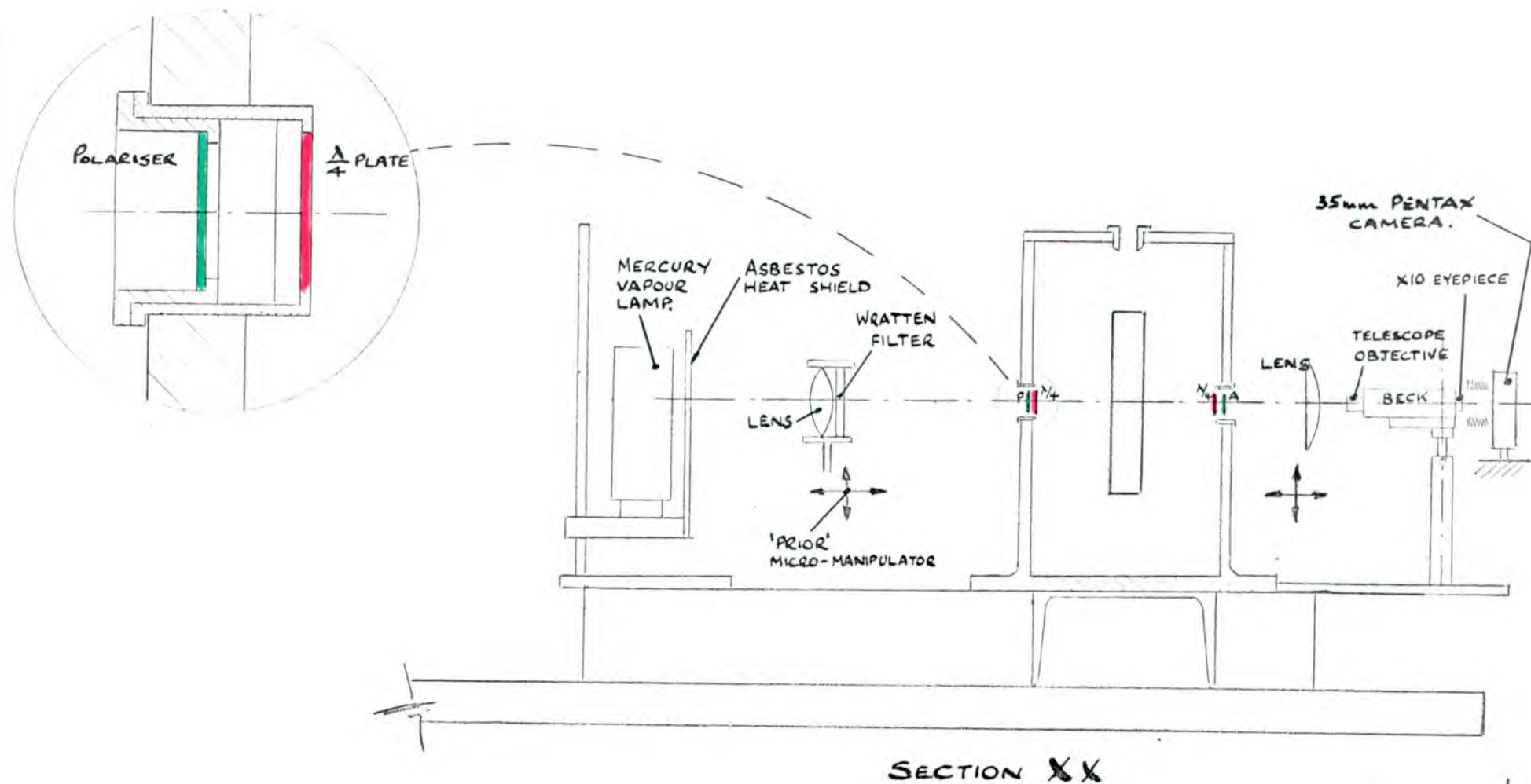


Fig. 10. OPTICAL BENCH

CHAPTER 3ANALYSIS3.01 Calibration of Isochromatics

The isochromatic fringes, represent lines of constant principal stress difference. As the contact pressure distribution changes, so does the shape of the fringes. Their values however, remain constant. In order to obtain a pressure curve from a known fringe pattern, the value for each fringe must first be ascertained. This was achieved by assuming a Hertz pressure distribution existed between the two discs in static contact. Comparing the computed values of the principal stress difference with the experimental fringes, enabled the calibration to be made.

It has been shown by Ford⁽²²⁾ that the radial stress at the point F, as shown in Figure 11, due to a normal line load P can be expressed in polar co-ordinates as:

$$p_r = -\frac{2P}{\pi} \frac{\cos\theta}{r}, \quad p_\theta = 0 \quad \text{and} \quad p_{r\theta} = 0.$$

It is more convenient in the present problem to use cartesian co-ordinates. That is;

$$\sigma_x = \frac{-2P x^2 y}{\pi(x^2 + y^2)^2}$$

$$\sigma_y = \frac{-2P y^3}{\pi(x^2 + y^2)^2}$$

$$\tau_{xy} = \frac{2P x y^2}{\pi(x^2 + y^2)^2}$$

where σ_x and σ_y are the normal stress components in the x and y directions and τ_{xy} the shear stress in the plane xy.

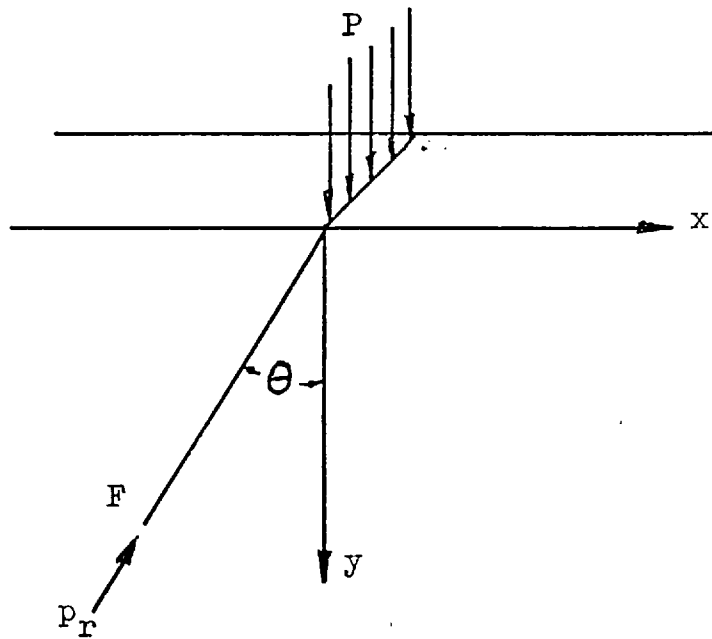


Fig. 11. Co-ordinate system for stresses.

Radzimovsky⁽⁴⁰⁾ showed that the stresses at a point, due to a pressure distribution, can be obtained by assuming the pressure to consist of a number of concentrated loads. The summation of the stresses due to each of the concentrated loads would represent the stress for the pressure curve. Hence, the principal stress difference at a point, due to a distributed load, is:

$$p_1 - p_2 = \sqrt{\left(\sum \sigma_x - \sum \sigma_y\right)^2 + 4\left(\sum \tau_{xy}\right)^2}$$

or

$$p_1 - p_2 = \sqrt{\left(\sum (\sigma_x - \sigma_y)\right)^2 + 4\left(\sum \tau_{xy}\right)^2}$$

For programming details of Calibration Chart see Appendix I.

3.02 Contact pressure distribution

One way of obtaining a pressure distribution from a pattern of isochromatics, would be by a trial and error method. That is, to assume a pressure curve equivalent to the applied load, and to obtain from this curve a map of contours representing the lines of principal stress difference. Comparision of these contours with the experimental isochromatics will enable a modification to the assumed pressure to be made. Eventually, after several iterations, a pressure curve can be obtained which conforms to the experimental stress pattern.

3.03 Pressure distribution consisting of concentrated loads

One method of calculation was to assume the pressure distribution to be formed out of several concentrated loads. (Usually about two to three hundred.)

By constructing a fine grid (80 by 40) in the contact region, the principal stress difference at the mesh points can be computed. Using a contour sub-routine developed by Monroe⁽³⁴⁾ enabled a plot of stress contours to be interpolated from the mesh points.

For programming details see Appendix II.

3.04 Pressure distribution consisting of parabolas

There are several disadvantages in this method of contour plotting based on concentrated loads, they are:

- (i) In order to obtain a smooth pressure curve at least two hundred concentrated

loads are required, in some cases nearly three hundred were used. This meant a considerable amount of time being used in the preparation of data cards.

(ii) The principal stress difference very near the contact region would be influenced by the concentrated load near that point; the resulting value would not conform to the true value based on a 'smooth' pressure curve.

(iii) Computation time was from about two to three minutes, which was considered excessive when one takes into account that up to ten attempts may be required before obtaining the correct pressure curve.

These objections were overcome by adopting the method suggested by Dowson, Higginson and Whitaker.⁽¹⁷⁾ They showed how the stresses at a point could be calculated by formal integration, when the pressure consisted of parabolic blocks. Although the expressions are cumbersome they can be evaluated on a digital computer.

This method was tried and it was found that all the disadvantages of the previous method were overcome. That is;

(i) Only about six parabolas were required to represent a pressure curve. The number of data points being reduced by a factor greater than twenty.

(ii) The principal stress difference could be computed up to the contact edge without distortion.

(iii) Computation time had been reduced to about one minute. That is, over one hundred per cent saving in computer time.

For programming details see Appendix III.

3.05 Pressure distribution from isochromatics

The trial and error method of obtaining the pressure distribution, requires up to about ten attempts before a satisfactory solution is obtained. The number of tries, depends to a certain extent, upon the skill and experience of the investigator. It was decided, therefore, to investigate the possibility of obtaining the pressure curve, direct from the isochromatics.

It has already been shown that the isochromatics, which represent lines of constant principal stress difference (PDIFF), can be expressed in the form:

$$PDIFF = \sqrt{\left[\sum (\sigma_x - \sigma_y) \right]^2 + 4\tau_{xy}^2}$$

$$\text{i.e.} \quad \left[\sum (\sigma_x - \sigma_y) \right]^2 + 4\tau_{xy}^2 = PDIFF^2 \quad \text{---(3.1)}$$

It has also been shown that σ_x , σ_y and τ_{xy} are functions of P (the load/unit length) so that equation (3.1) becomes:

$$\left[\sum a_{ij} P_j \right]^2 + \sum b_{ij} P_j^2 = c_i \quad \text{---(3.2)}$$

This expression represents a set of SIMULTANEOUS QUADRATIC EQUATIONS.

Where P_j are the loads (to be found) representing a pressure distribution. The ultimate aim was one hundred loads.

a_{ij} and b_{ij} are coefficients, which can be calculated, and c_i are the experimental values, obtained from the isochromatics.

A digital computer was used to solve equation (3.2) using a Newton-Raphson method. The method depended on a first approximation to the values of P_j being supplied. The programme was checked using 'experimental' points based on a Hertzian pressure curve. It was decided, as a first approximation, to assume a uniform pressure curve of value about half the maximum Hertz pressure. Such a wild guess was thought to be a severe but just check on the programme. It was found that the uniform pressure rapidly converged to the Hertzian curve in about three iterations, as shown in Figure 52.

For programming details see Appendix IV.

The programme solved Twelve equations and there was no reason why it could not be extended to solve one hundred equations. Unfortunately a serious setback was encountered in the development of the programme. As the number of equations increased, so the experimental values had to be taken nearer the contact region in order for

convergence to occur. For example; for seven equations, the experimental points must be within $0.25B$ of the contact surface. For twelve equations, this distance had been reduced to $0.05B$. Clearly, for the solution of one hundred equations the experimental points would have to be so close to the contact region as to make their accurate reading a practical impossibility.

It is possible, however, that just as the trial and error method had been modified from concentrated loads to parabolas; so the quadratic equations, which were based on concentrated loads, could also be reformed on the basis of parabolas. If this is possible then the number of equations required for a smooth pressure curve would be drastically reduced.

3.06 Accuracy of pressure curves

It is now possible to obtain pressure curves from the isochromatics. It was found, however, that the accuracy of these curves depends to a large extent, on the ability to obtain clear fringes very close to the contact. This is in accordance with de Saint Venant's principle, which states as follows: "A change in the stress distribution, caused by a change in the pressure distribution at the point of contact, is restricted to an area with dimensions of the same order as the width of the field of contact."

It was thought that a method of checking the accuracy of the pressure curves, apart from comparing with

other research workers, would be to compare the elastic deflexion with the film shape, both being calculated from the same pressure curve.

3.07 Elastic deflexion

Dowson and Higginson,⁽¹²⁾ expressed the elastic equation in the form:

$$v = \frac{-2}{\pi E'} \int_{s_1}^{s_2} p(s) \log(x-s) ds + \text{Constant}$$

for the configuration shown in Figure 12.

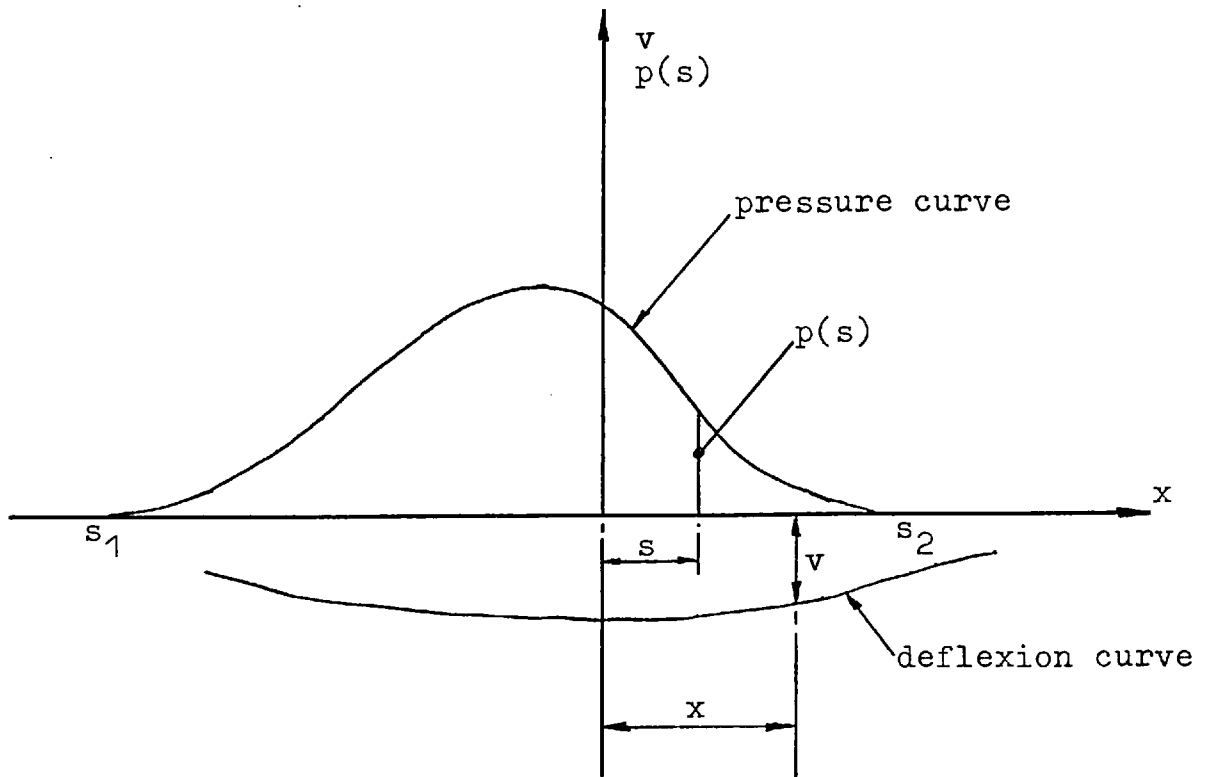


Fig. 12. Co-ordinate system for deflexion integrals.

Cameron⁽⁴⁾ noted that since the constant could be ignored as only the differences in deflexion is required, the equation reduces to:

$$v = \frac{-2}{\pi E'} \int_{s_1}^{s_2} p(s) \log(x-s) ds \quad \text{-----} (3.3)$$

Difficulty is encountered in the numerical integration of this equation, at the point $x=s$. Archard, Gair and Hirst⁽²⁾ overcame this difficulty by considering the pressure distribution to be uniform for a small region, abounding the point at $x=s$. The integral at that point was then solved formally, by Parts. Weber,⁽⁴⁷⁾ avoided this problem by integrating his pressure curve formally. Dowson and Higginson⁽¹²⁾ also solved the integral formally by using their established technique of dividing the pressure curve into parabolic blocks and evaluating the integrals. The writer has also adopted this method, in a slightly modified form.

For programming details see Appendix V.

3.08 Film shape - Inverse solution of Reynolds equation

For the configuration shown in Figure 13, Reynolds equation for one dimensional flow can be expressed as

$$\frac{d}{dx} \left(\frac{h^3}{2} \frac{dp}{dx} \right) = 12u \frac{dh}{dx} \quad \text{-----} (3.4)$$

where $u = \frac{1}{2}(u_1 + u_2)$

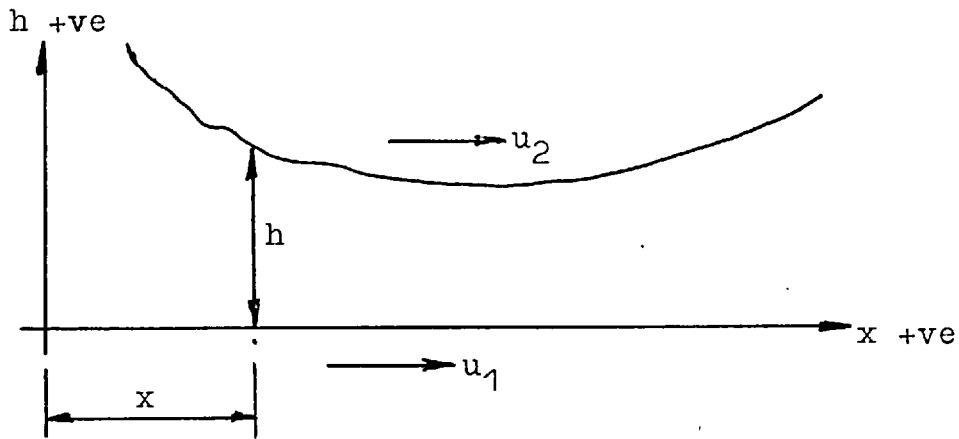


Fig. 13. Co-ordinate system.

Integrating Reynolds equation w.r.t. x and substituting the boundary conditions $\frac{dp}{dx} = 0$, when $h=h_m$ gives

$$\frac{dp}{dx} = 12u\eta \frac{h-h_m}{h^3} \quad \text{-----} (3.5)$$

Rearranging gives,

$$K \bar{h}^3 - \bar{h} + 1 = 0 \quad \text{-----} (3.6)$$

where $K = \frac{dp}{dx} \frac{h_m^2}{12u\eta}$ and $\bar{h} = h/h_m$

This equation can be solved for a given pressure curve providing h_m is known. Dowson and Higginson⁽¹²⁾ have shown how h_m could be estimated for a constant viscosity condition. The method described here extends their reasoning to the case where $\eta = \eta_0 e^{\alpha p}$

Differentiating the L.H.S. of equation (3.4) gives,

$$\frac{h^3}{\gamma} \frac{dp^2}{dx^2} + \frac{3h^2}{\gamma} \frac{dh}{dx} \frac{dp}{dx} - \frac{h^3}{\gamma^2} \frac{d\gamma}{dx} \frac{dp}{dx} = 12u \frac{dh}{dx}$$

$$\text{i.e.} \quad \frac{dh}{dx} \left[\frac{3h^2}{\gamma} \frac{dp}{dx} - 12u \right] + \frac{h^3}{\gamma} \left[\frac{d^2p}{dx^2} - \frac{1}{\gamma} \frac{d\gamma}{dx} \frac{dp}{dx} \right] = 0$$

$$\text{but } \gamma = \gamma_0 e^{\alpha p} \quad \text{so that} \quad \frac{d\gamma}{dx} = \alpha \gamma_0 e^{\alpha p} \frac{dp}{dx} = \alpha \gamma \frac{dp}{dx}$$

$$\frac{1}{\gamma} \frac{d\gamma}{dx} = \alpha \frac{dp}{dx} \quad \text{then,}$$

$$\frac{dh}{dx} \left[\frac{3h^2}{\gamma} \frac{dp}{dx} - 12u \right] + \frac{h^3}{\gamma} \left[\frac{d^2p}{dx^2} - \alpha \left(\frac{dp}{dx} \right)^2 \right] = 0$$

A possible solution to this condition would be when

$$\frac{d^2p}{dx^2} - \alpha \left(\frac{dp}{dx} \right)^2 = 0 \quad \text{----- (3.7)}$$

When this is satisfied

$$\frac{dh}{dx} \left[\frac{3h^2}{\gamma} \frac{dp}{dx} - 12u \right] = 0$$

then, either $\frac{dh}{dx} = 0$ (Minimum film thickness)

$$\text{or} \quad \frac{3h^2}{\gamma} \frac{dp}{dx} = 12u$$

$$\text{i.e.} \quad h = \sqrt{\frac{4u\gamma}{dp/dx}} \quad \text{----- (3.8)}$$

Clearly dp/dx must be positive, which implies inlet sweep of pressure curve for a solution.

Equation (3.8) gives the film thickness at the point where

$$\frac{d^2p}{dx^2} - \alpha \left(\frac{dp}{dx}\right)^2 = 0.$$

The film thickness h_m can be obtained by combining equations (3.8) and (3.5).

$$\text{i.e. } \frac{dp}{dx} = \frac{4u\eta}{h^2} = 12u\eta \frac{h-h_m}{h^3}$$

$$\frac{h}{3} = h-h_m \quad \text{hence } h_m = \frac{2}{3} h.$$

Procedure for finding h_m is as follows. From the pressure curve obtain the point where equation (3.7) is satisfied.

$$\text{At this point } h = \sqrt{\frac{4u\eta}{dp/dx}}$$

$$\text{and then } h_m = \frac{2}{3} h.$$

Dowson and Higginson in their textbook on Elastohydrodynamic Lubrication, noted that the condition

$\frac{d}{dx} \left(\frac{1}{\eta} \frac{dp}{dx} \right) = 0$ is satisfied at the points of inflexion of the reduced pressure curve. (This is defined as the pressure curve which would be generated in an isoviscous lubricant of viscosity η_0).

Let q equal the reduced pressure, then

$$\frac{dq}{dx} = 12 u_0 \frac{h-h_m}{h^3} = e^{-\alpha p} \frac{dp}{dx}$$

Points of inflexion when $\frac{d}{dx} \left(\frac{dq}{dx} \right) = 0$ ————— (3.9)

i.e. $e^{-\alpha p} \left[\frac{d^2 p}{dx^2} - \alpha \left(\frac{dp}{dx} \right)^2 \right] = 0$ which, of course, agrees with equation (3.7).

A graphical method of differentiation⁽¹⁾ was used to solve equations (3.7) and (3.9). A complete solution is given in Appendix VI.

Solution of cubic equation ————— $K \bar{h}^3 - \bar{h} + 1 = 0$

It is now possible to estimate K for any point in the contact region. The solution of the equation yields, in general, three values of $\bar{h}$ for each value of K. A practical solution suggests that $\bar{h}$ must be real and positive. The choice between two possible positive roots can be determined by considering how the graphical solution of the cubic equation, is effected by changes in the value of K.

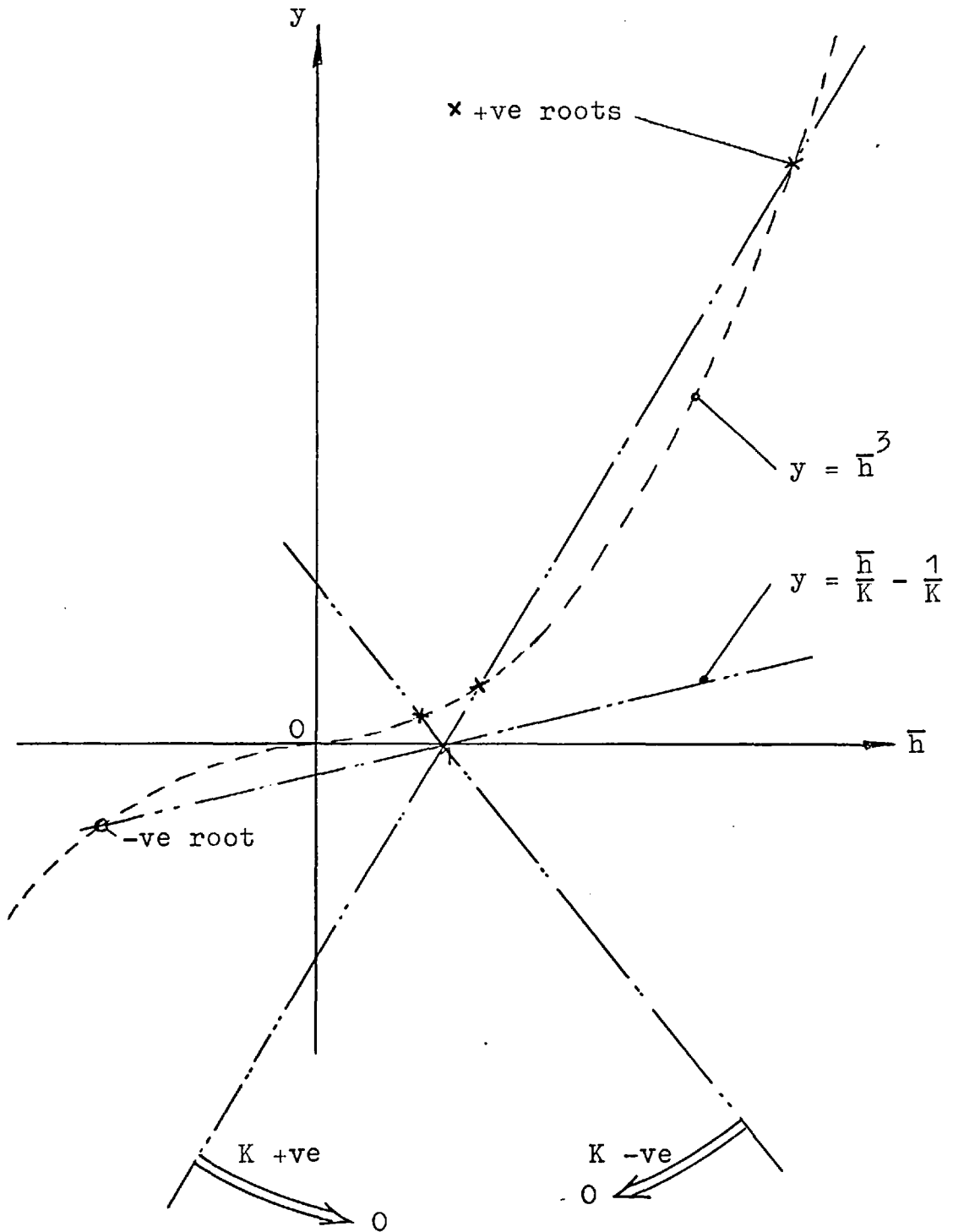
Rearranging equation (3.6) gives $\bar{h}^3 = \frac{\bar{h}}{K} - \frac{1}{K}$

The solution of $\bar{h}$ would be the graphical intersection of two curves:

$$y = \bar{h}^3 \quad \text{and} \quad y = \frac{\bar{h}}{K} - \frac{1}{K}$$

as shown in Figure 14.

Note $y = \frac{\bar{h}}{K} - \frac{1}{K}$ represents a family of straight lines passing through the point $\bar{h} = 1, y = 0$.



Two positive real roots if K is positive (and small).
 One positive real root if K is negative.

Fig. 14. Graphical solution of the Inverse Reynolds equation.

Selection of positive root.

It has been shown in Figure 14 that when K is positive, two positive real roots may occur. The roots converge as K increases. Clearly, in the inlet region the film thickness must decrease. Hence choice of root is obvious. By continuing the analysis, a complete picture of the film shape can be obtained, as shown in Figure 15.

It is possible to show by this method, that for a pressure curve with a spike, the film thickness increases just before the final restriction. This has been shown experimentally, using optical interferometry with point contact, by Westlake.⁽⁴⁹⁾

For the programming details of the Inverse solution of Reynolds equation see Appendix VII.

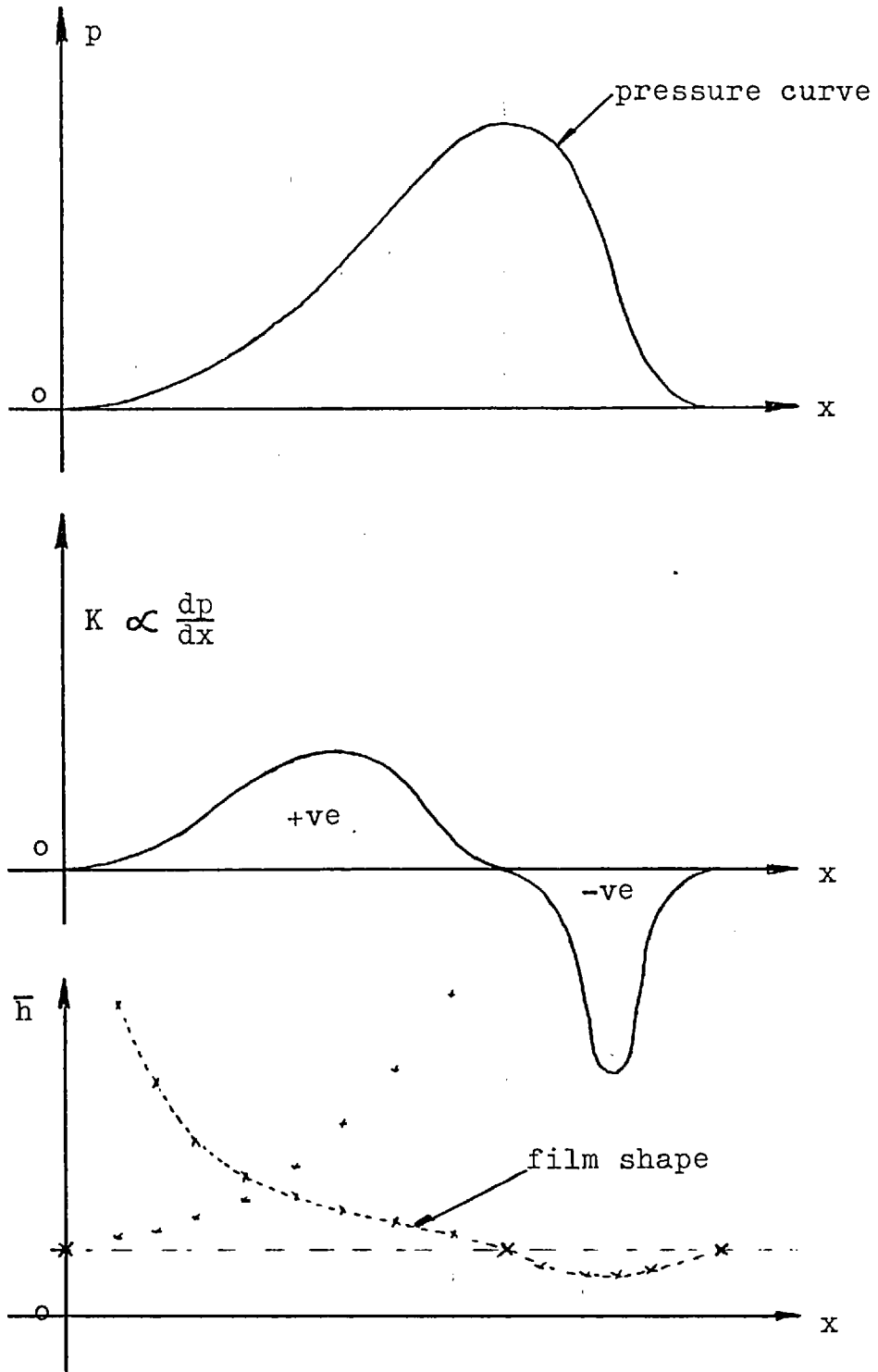


Fig. 15. Graphical determination of positive root.
and film shape.

3.09 Magnification of fringes

The following procedure was adopted to obtain the magnification of the fringes.

(1) A radial load was applied to the stationary discs, and the camera was adjusted so that the fringes were in sharp focus. A photograph of the fringes was taken.

(2) The load was removed and the two discs were separated to leave a small gap.

(3) A piece of fine wire of known diameter was inserted into the gap. The wire was positioned so that it came into sharp focus with the camera. A further picture was taken.

(4) The tests were continued without further movement of the camera.

A photograph showing the wire in focus between the two discs and the calculation of magnification is shown in Figure 16.



$$\text{Magnification} = \frac{0.28}{0.0192} = 14.6$$

Fig. 16. Magnification of fringes.

3.10 Position of 'hairline'.

Photographs of the fringes were taken without a 'hairline' in the eyepiece. The hairline (line of centres) was afterwards superimposed on the prints. It was possible to do this because the position of the line of centres relative to the exposed film is constant for each frame. This was because the axes of the discs, the optical bench and the camera remained rigid throughout the experiment. The line of centres is easily determined from the static stress pattern, Hence it's relative position for other stress patterns is known.

CHAPTER 4

'SMOOTH DISCS' - EXPERIMENTAL RESULTS
AND COMPUTATIONS.

4.01 Complete analysis of the stress patterns to obtain the contact pressure distribution and film shape.

Conditions of experiment.

Monochromatic light source.

Steel disc roughness approximately $2.5 \mu\text{in. C.L.A.}$

Oil temperature 24.2°C

Radial load $265 \text{ lb}_f/\text{in.}$ (46.5 KN/m)

Maximum Hertz pressure $21,200 \text{ lb}_f/\text{in}^2$ (146 MN/m^2)

Photographs of the fringes were taken for the following discs speeds:

0.0, 5.82, 17.5, 36.0 and 58.2 rev/min.

Analysis of results.

The fringes were calibrated from a photograph of the static stress pattern using the Calibration Chart, Figure 48. This calibration enabled the pressure curves to be computed from the remaining pictures. The comparison of experimental with theoretical contours, together with the computed pressure distributions, are shown in Figures 17 to 21.

Figure 17 compares the theoretical contours computed from a Hertz pressure distribution, with the isochromatics. Agreement is poor near the contact edge at ± 1.5 . This is probably due to the actual pressure curve being distributed over a wider range (due to surface roughness) than the Hertzian curve.

Figure 18 compares the theoretical contours computed from a guessed pressure curve (fourth attempt), with the isochromatics obtained when the discs were rotating at 5.82 revolutions per minute. Fairly good agreement has been achieved so that the 'guessed' pressure curve represents the actual pressure distribution between the discs. For this particular test only, use was made of Programme 4a to compute the pressure distribution direct from the isochromatics. Seven simultaneous quadratic equations were solved. The experimental points were interpolated from a photograph of the fringes at a distance of 0.2B from the contact edge and 0.4B apart. The computed values of $P(J)$ represent the concentrated loads.

$$\text{The equivalent pressure ratio} = \frac{P(J)}{\text{area} \times PM}$$

$$\text{reduces to} \quad \frac{p}{PM} = \frac{P(J)}{68}$$

The pressure curve, obtained from only seven experimental points is shown in Figure 18(b).

Figure 19 compares the theoretical contours computed from a guessed pressure curve (eighth attempt), with the isochromatics obtained when the discs were rotating at 17.5 revolutions per minute. Good agreement has been achieved except in the region (1.5, 0.5). This could be corrected by extending the pressure curve very slightly in the outlet region.

Figures 20 and 21 compare the theoretical contours computed from guessed pressure curves (third and sixth attempt), with the isochromatics obtained when discs were rotating at 36.0 and 58.2 revolutions per minute respectively. Very good agreement was achieved.

It can be seen that as the speed increases, so the pressure distribution spreads over a wider contact region, with a consequent reduction in maximum pressure. This has been illustrated in Figure 22.

It was suggested in Chapter 3.06 that one method of checking the accuracy of a pressure curve would be to compare the elastic deflexion with the film shape; both being calculated from the same pressure curve. Figure 23 makes this comparison for a pressure curve obtained when the discs were rotating at 5.82 revolutions per minute. Agreement is poor at the inlet and outlet regions. This is to be expected since the solution of Reynolds equation suggests that when dp/dx is zero, the film thickness is either infinite or equal to h_m . This has been illustrated in Figure 15.

Figure 24 shows how the film thickness and shape changes with the speed for a constant load. The centre line and minimum film thickness values have been compared with the theoretical film thickness based on formulae produced by Martin⁽³³⁾ and also by Dowson and Higginson.⁽¹⁴⁾

The Martin formula gives
$$\frac{h_o}{R} = \frac{4.9 \gamma_o u}{W'}$$

for $\gamma_o = 35$ poise (3.5 kg/m.s., 0.073 Slug/ft.s.)

$W' = 265$ lb_f/in. (46.5 kN/m.)

then
$$\frac{h_o}{R} = \frac{4.9 \times 0.073}{265 \times 12} \times \frac{u}{12} \quad \text{Note } u \text{ in inches/sec.}$$

i.e.
$$\frac{h_o}{R} = 9.4u \times 10^{-6}$$

The Dowson and Higginson formula for minimum film thickness is

$$\frac{h}{R} = \frac{1.6 \alpha^{.6} (\gamma_o u)^{.7} (E')^{.03}}{(W')^{.13} R^{.57}}$$

for $E' = 15.9 \times 10^6$ lb_f/in² and $R = 1.5$ in.

then
$$\frac{h}{R} = 27.2 u^{.7} \times 10^{-6}$$

The relationship between these theoretical curves and the experimental points is shown in Figure 25.

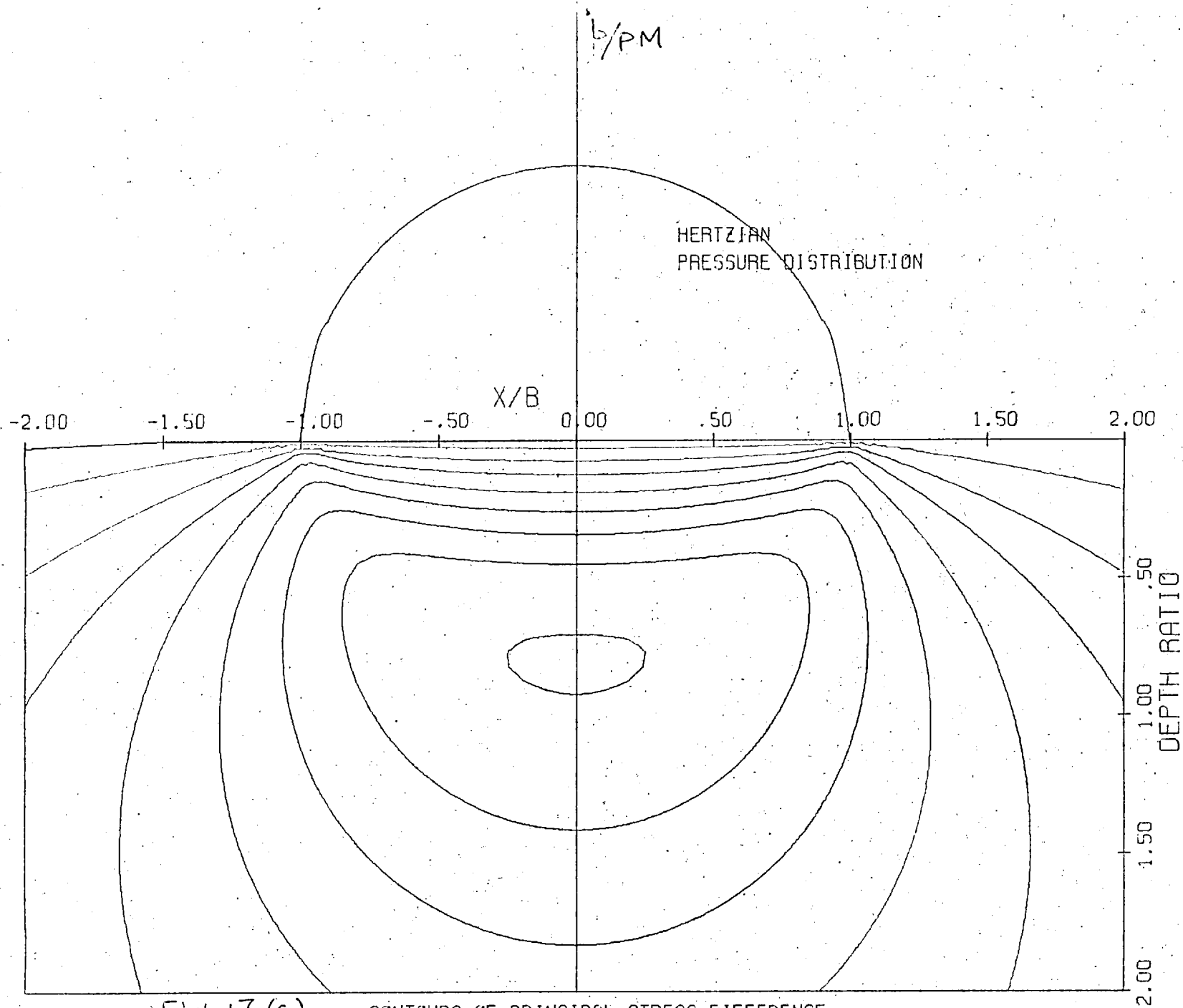


Fig. 17 (a)

CONTOURS OF PRINCIPAL STRESS DIFFERENCE

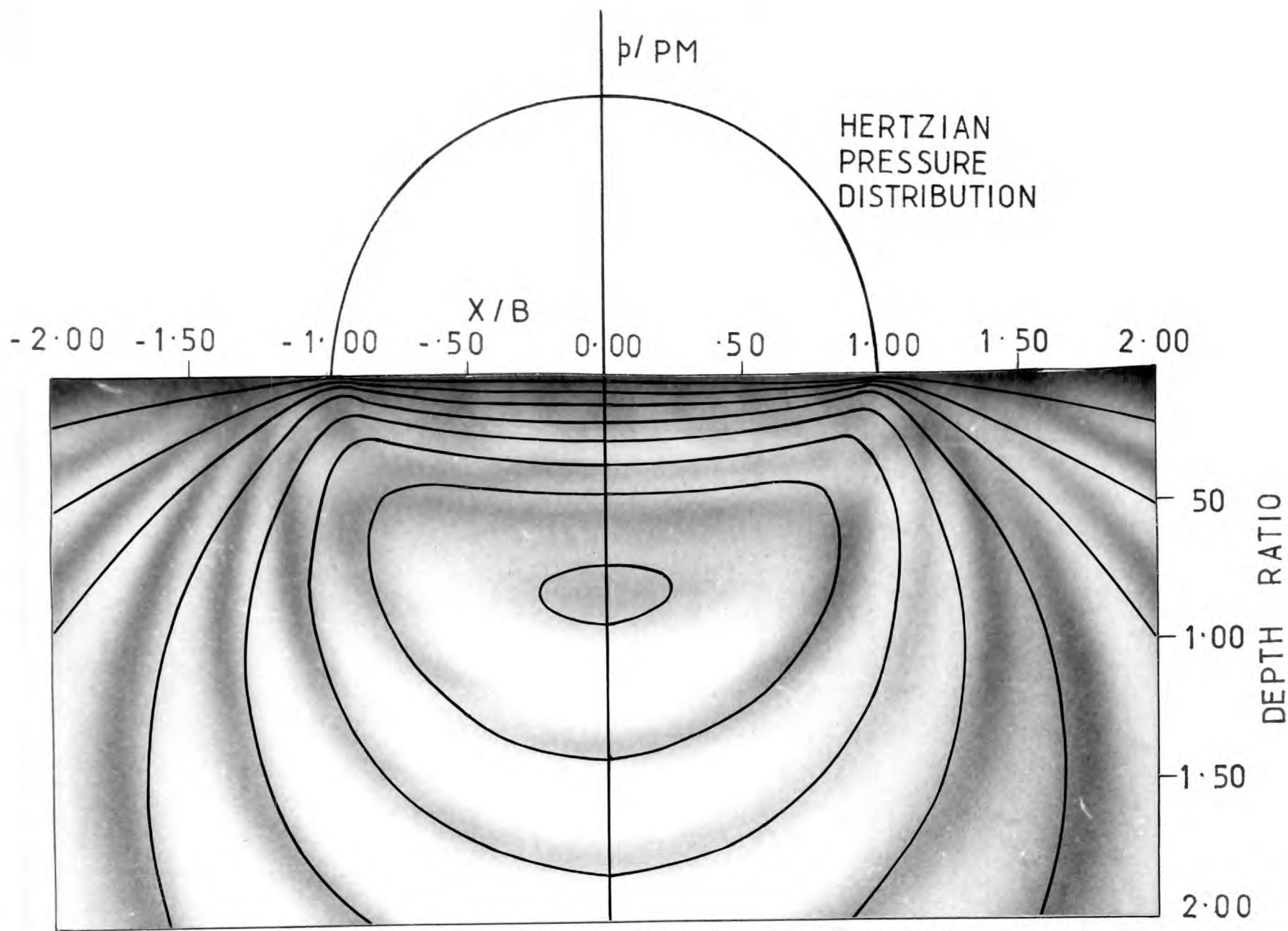


FIG. 17(a) CONTOURS OF PRINCIPLE STRESS DIFFERENCE

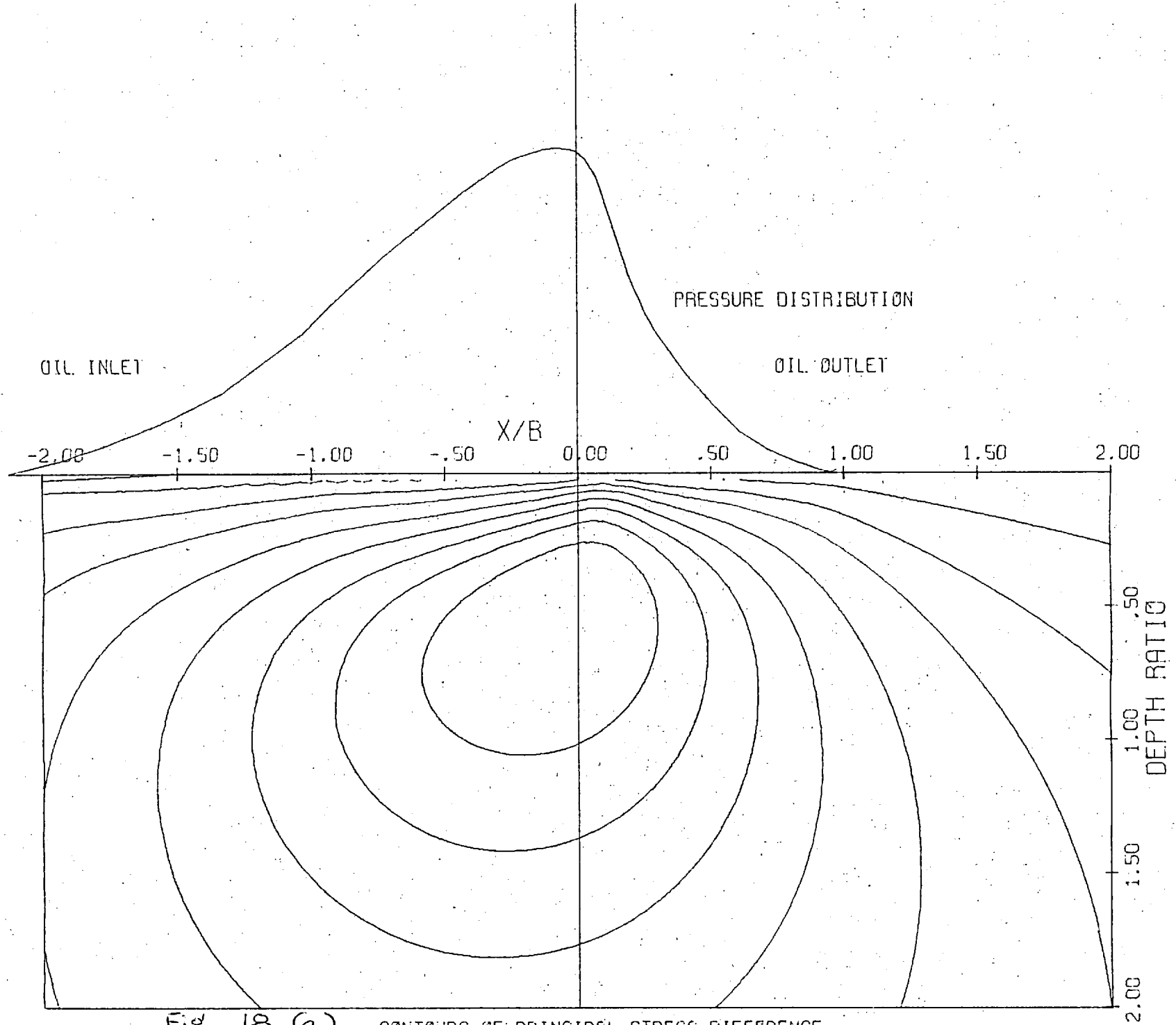


Fig. 18 (a) CONTOURS OF PRINCIPAL STRESS DIFFERENCE

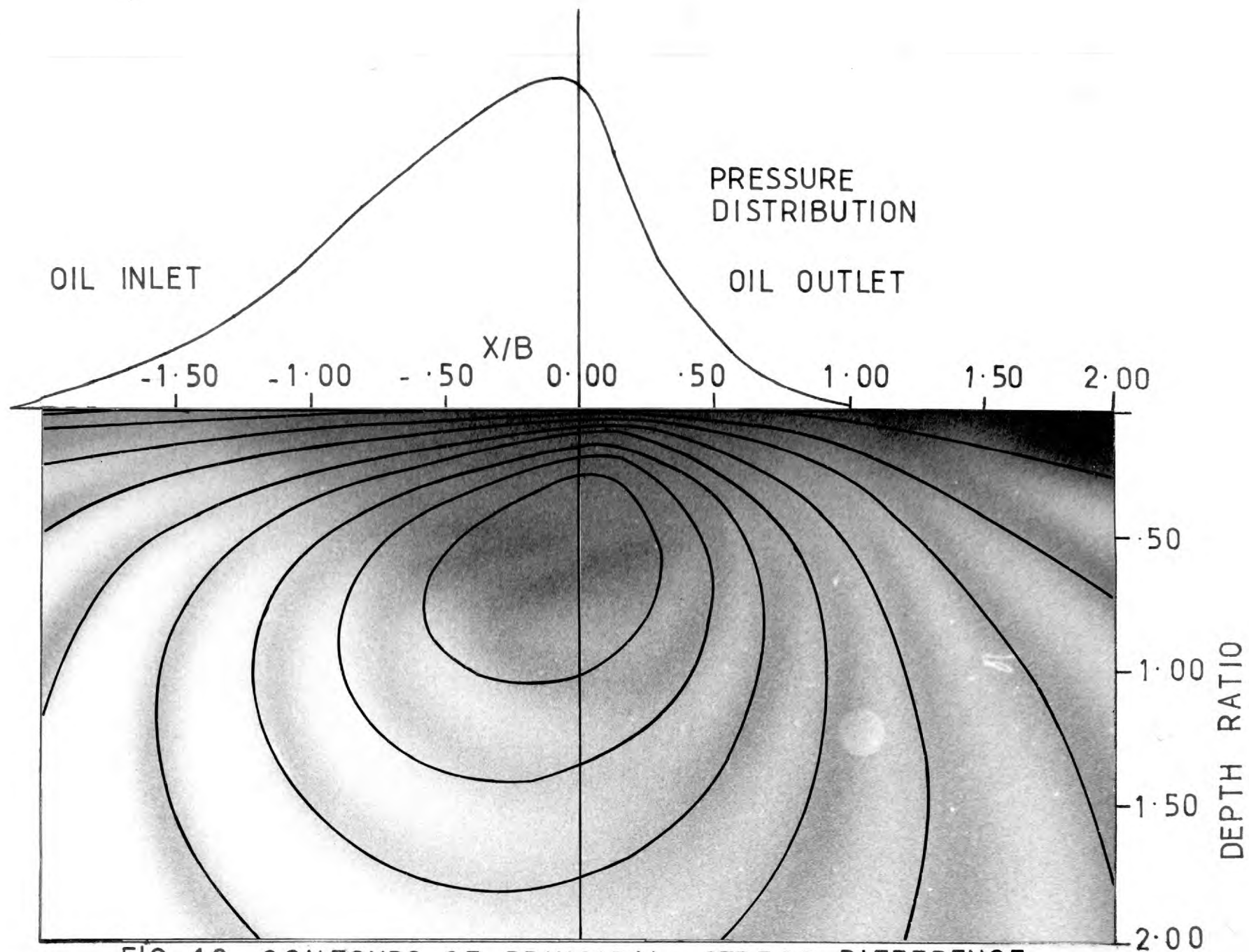


FIG 18. CONTOURS OF PRINCIPAL STRESS DIFFERENCE

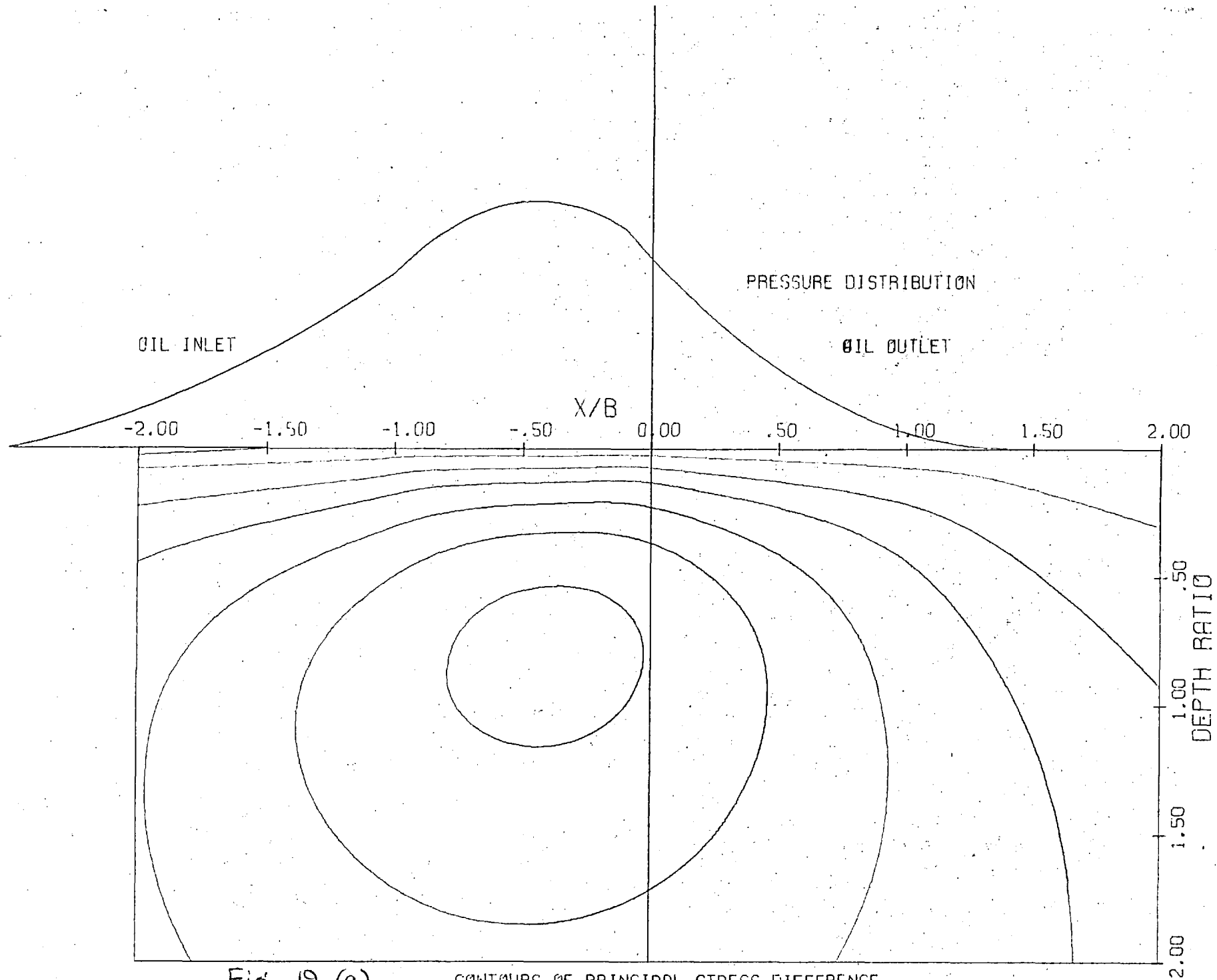


Fig. 19 (a)

CONTOURS OF PRINCIPAL STRESS DIFFERENCE

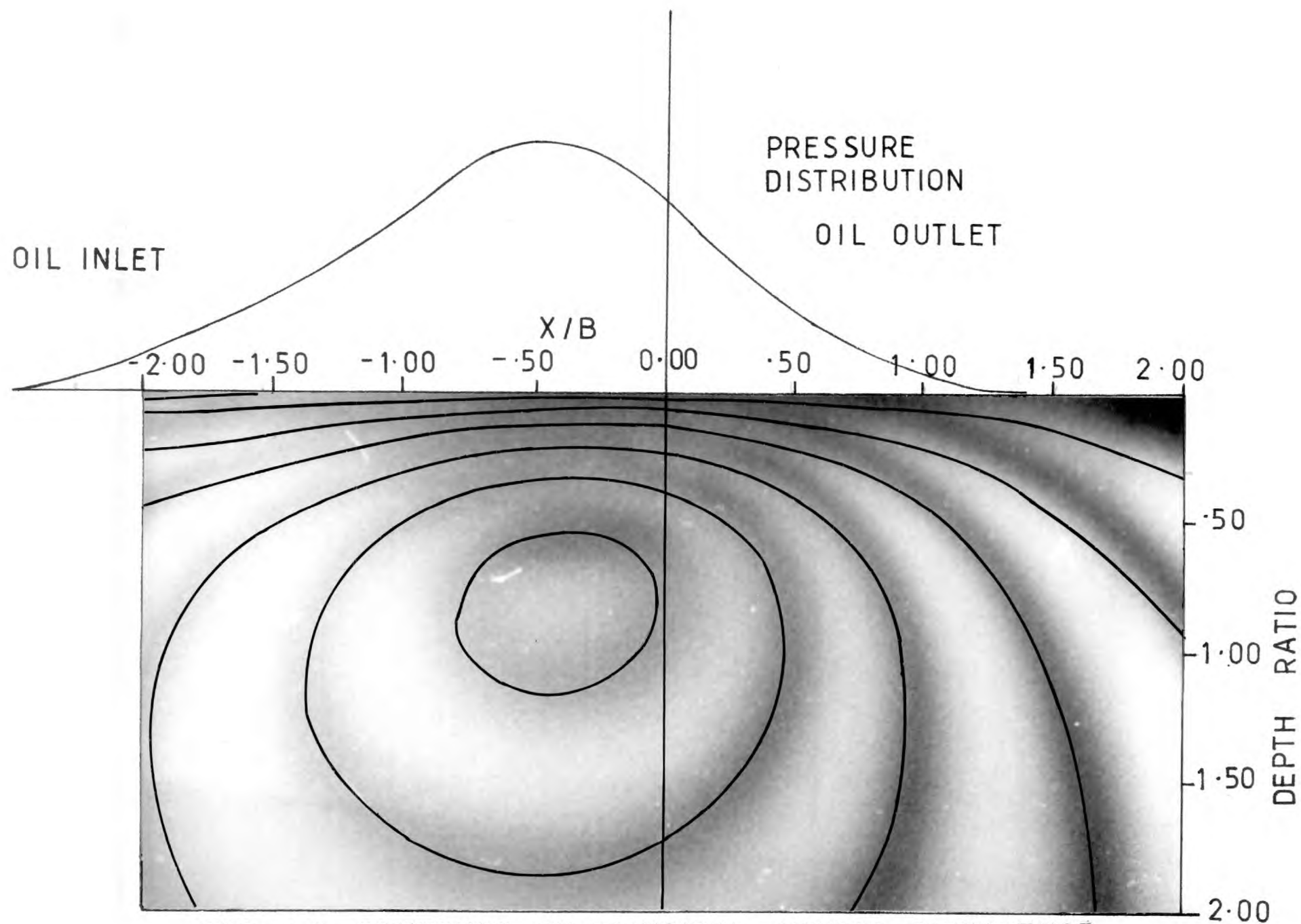


FIG. 19. CONTOURS OF PRINCIPAL STRESS DIFFERENCE

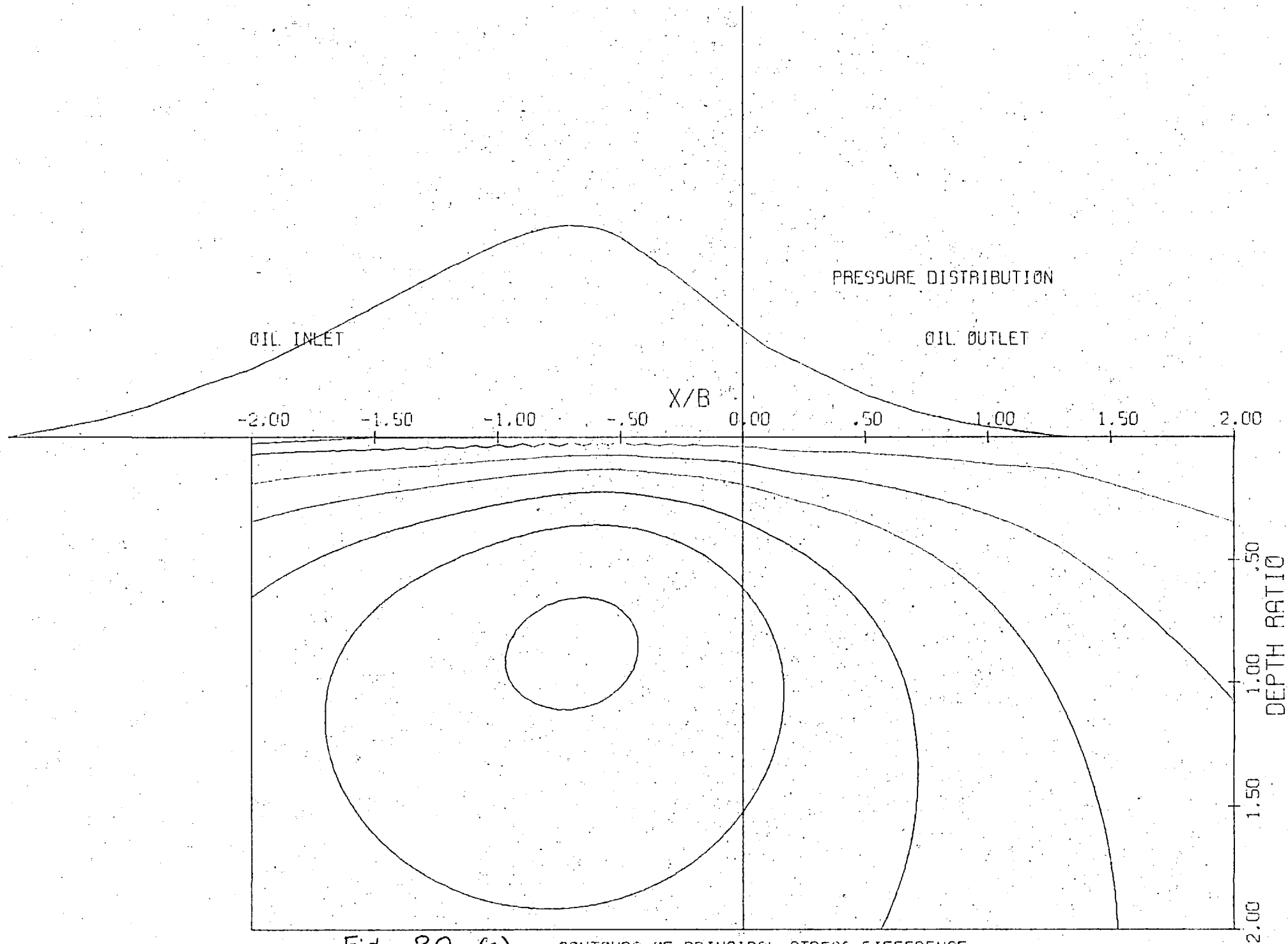
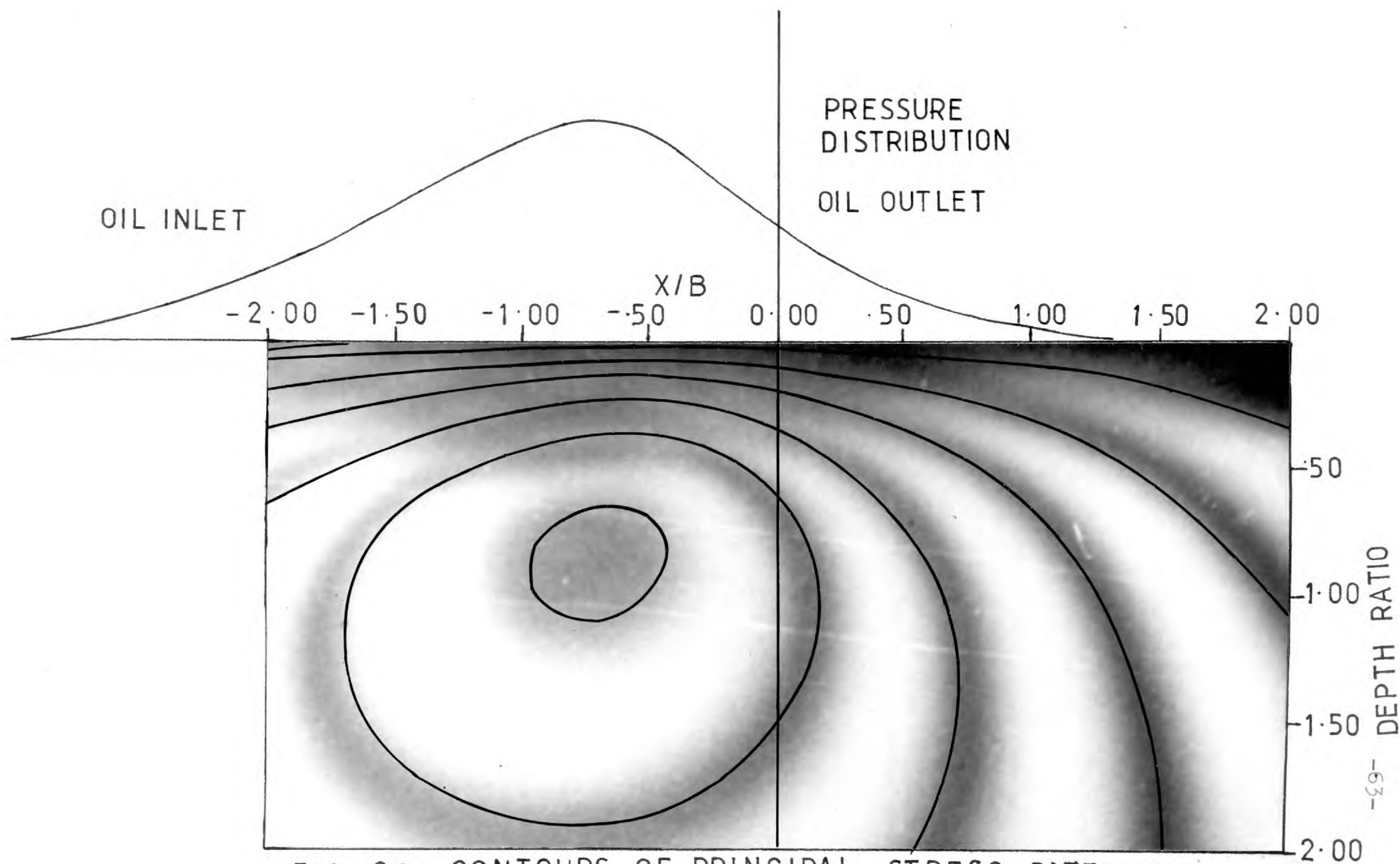


Fig. 20 (a) CONTOURS OF PRINCIPAL STRESS DIFFERENCE



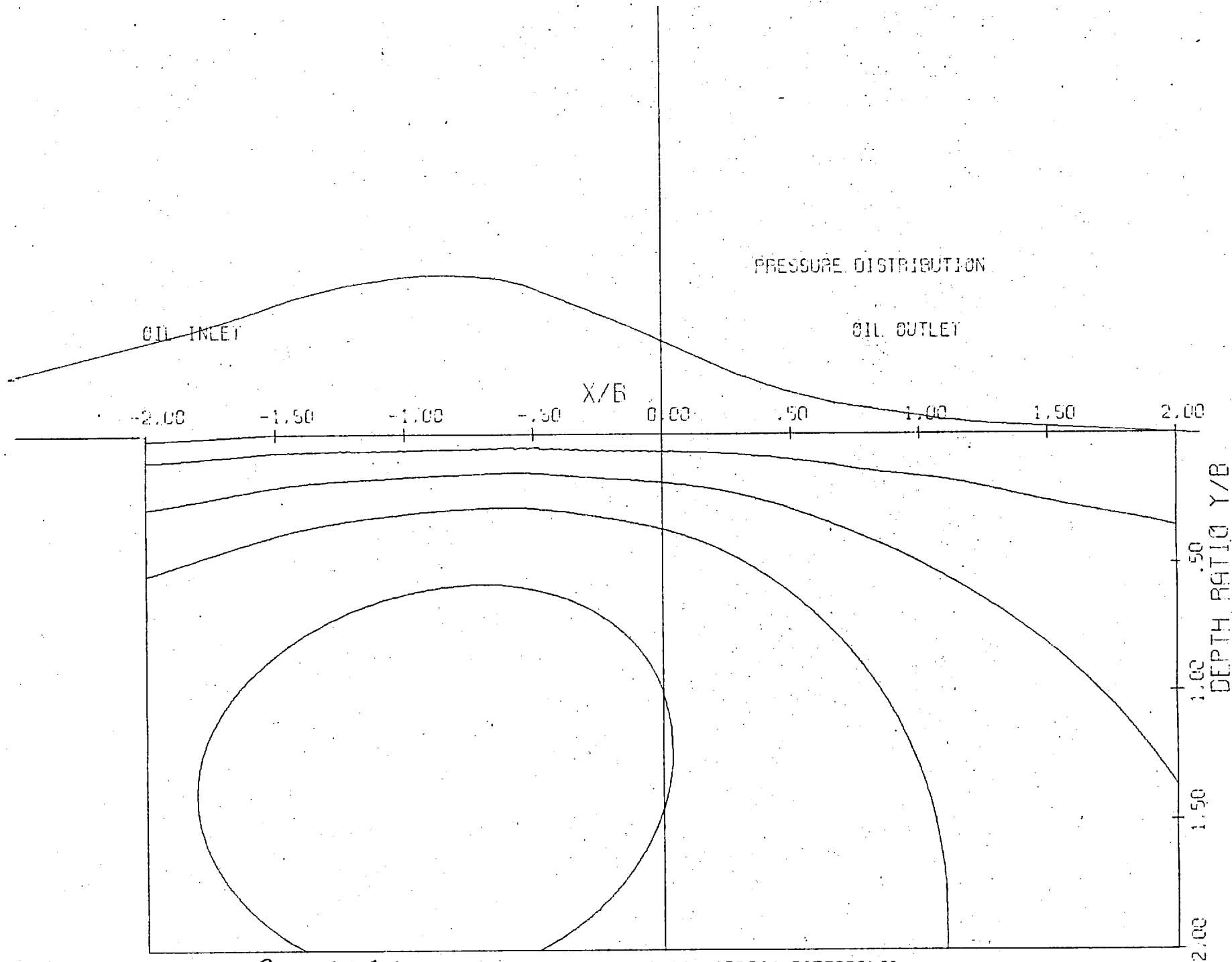


Fig. 21(a)

CONTOURS OF PRINCIPAL STRESS DIFFERENCE

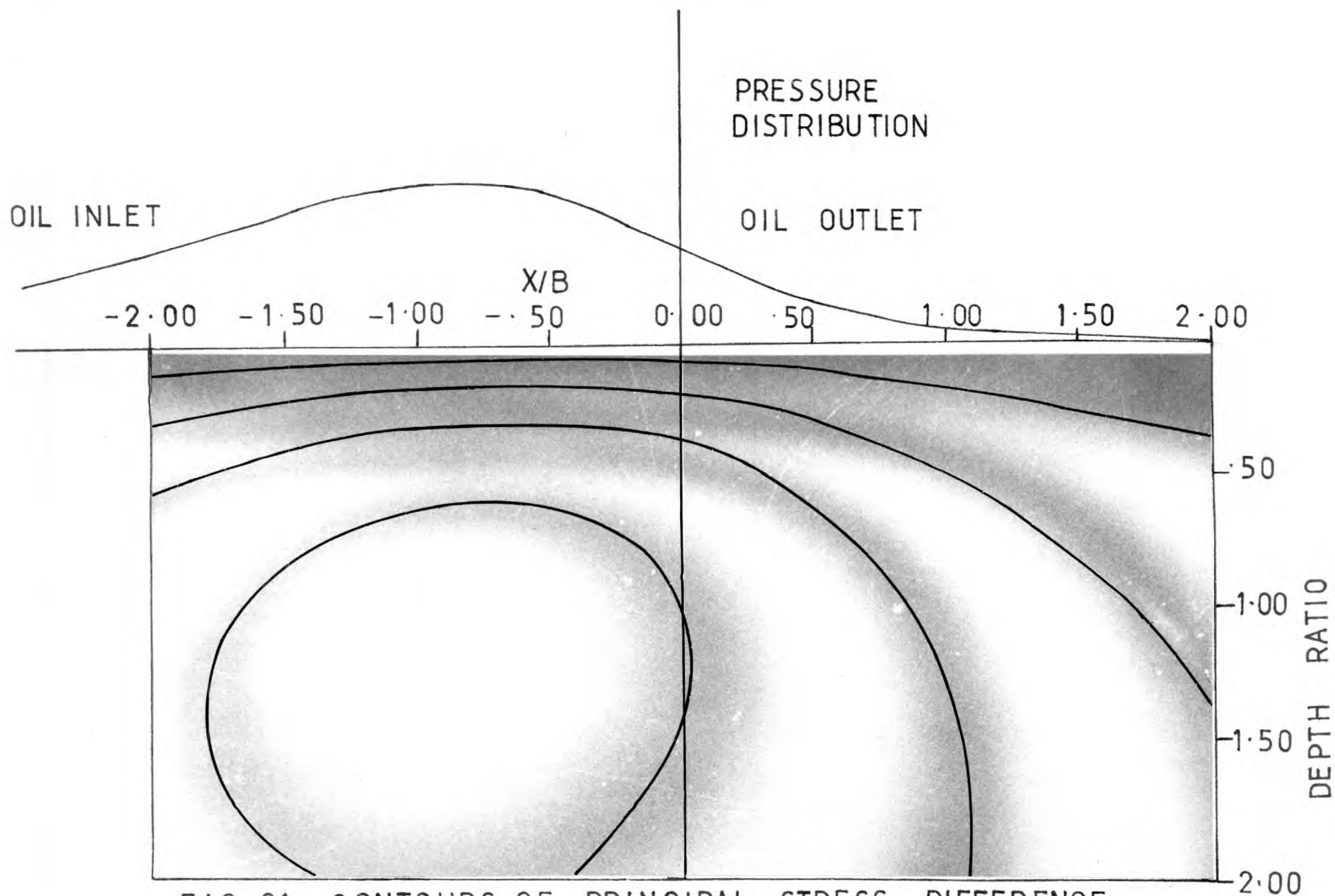


FIG. 21. CONTOURS OF PRINCIPAL STRESS DIFFERENCE

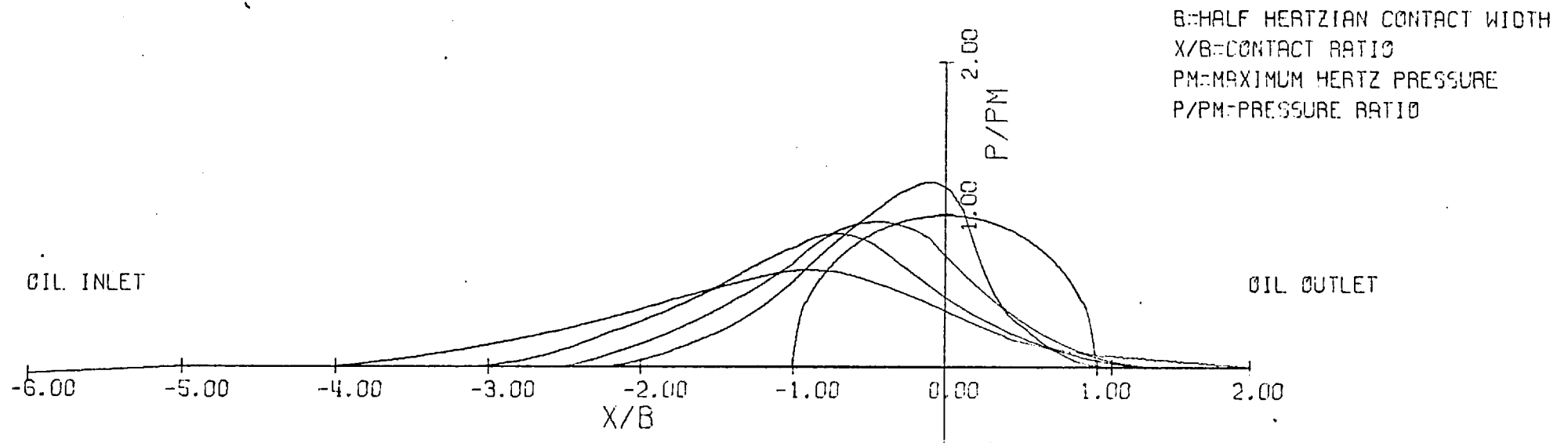


Fig.22. EFFECT OF SPEED ON PRESSURE DISTRIBUTION

PRESSURE AND ELASTIC DEFLEXION CURVES

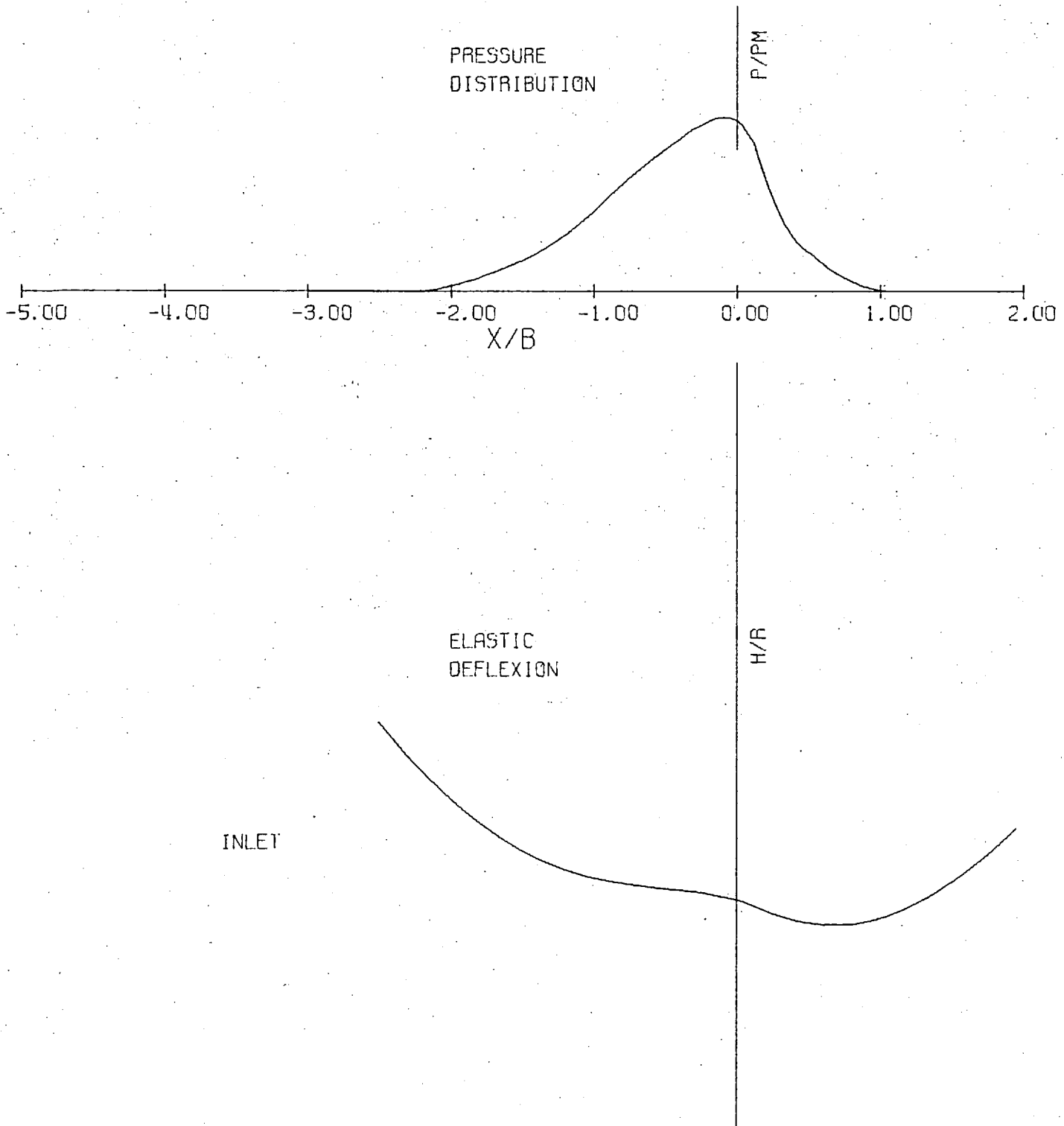
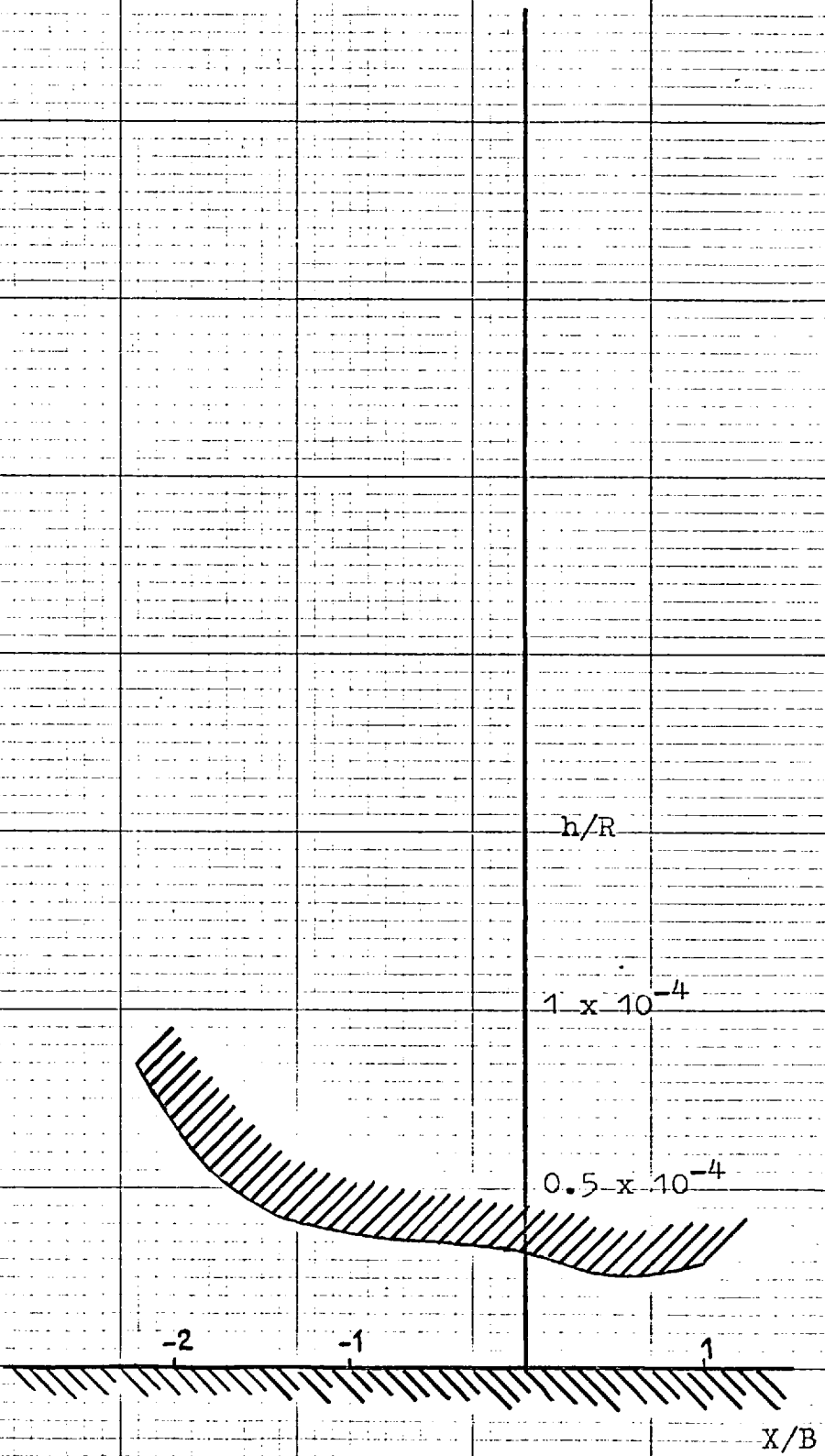


Fig. 23. Comparison of deflexion curves.
(a) Elastic.

Film Shape-Inverse solution of Reynolds equation.



(b) Reynolds.

Maximum Hertz Pressure = $21,200 \text{ lb}_f/\text{in}^2$

1. Static load only
2. Speed = 5.82 rev/min. $u = 1.83 \text{ in/s}$.
3. 17.5 rev/min 5.5 in/s
4. 36.0 rev/min 11.32 in/s
5. 58.2 rev/min 18.3 in/s

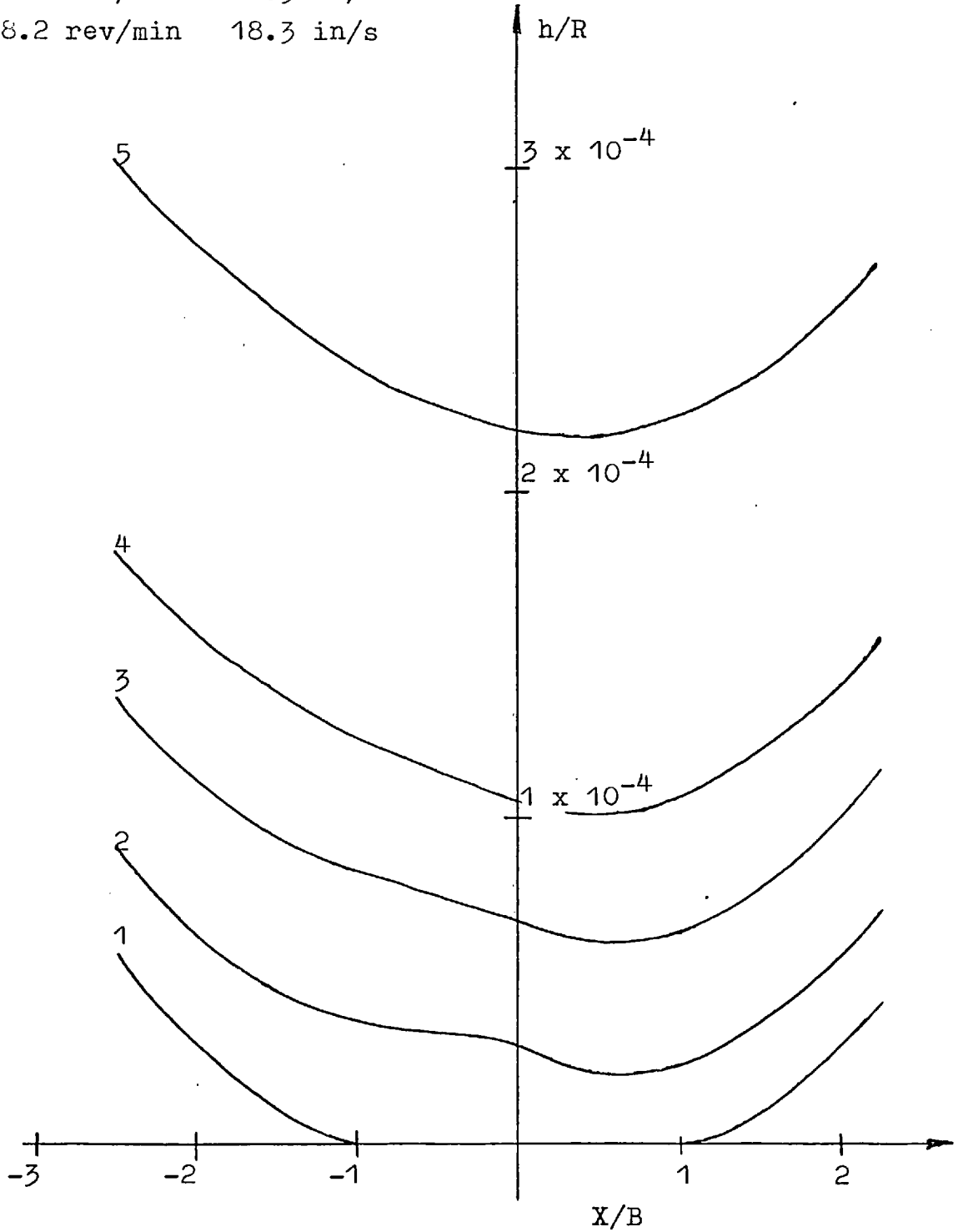


Fig. 24. Effect of speed on film shape.

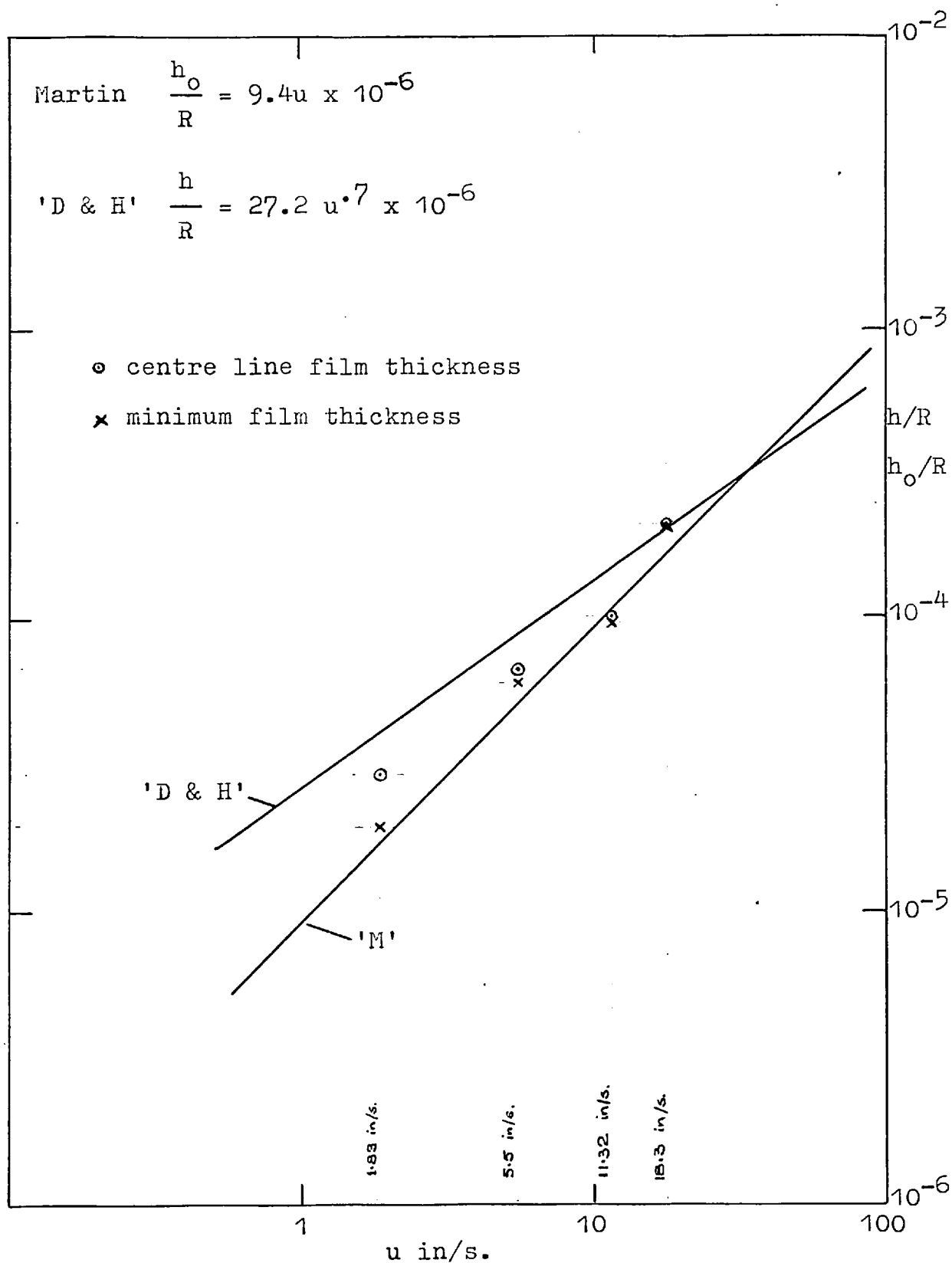


Fig. 25. Theoretical and experimental relationship between film thickness and surface speed.

4.02 Pressure spike

Conditions of experiment.

White light source.

Steel disc roughness approximately 2.5 μ in. C.L.A.

Oil temperature 25°C.

Radial load 388 lb_f/in. (68.0 KN/m.)

Maximum Hertz pressure 25,800 lb_f/in². (177 MN/m²)

Photographs, shown in Figure 26, were taken when the discs were (a) stationary

(b) rotating at 5.1 revolutions per minute.

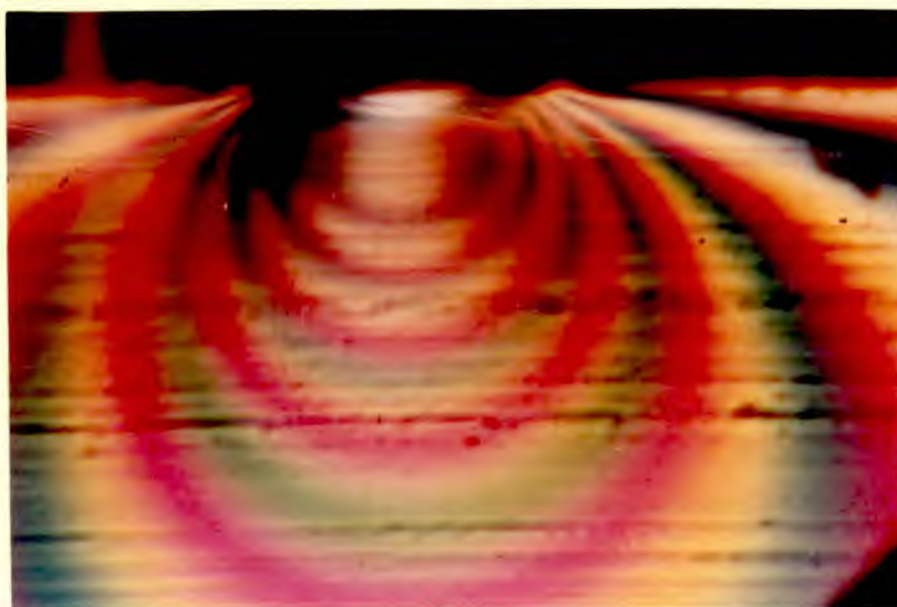
With these experimental conditions; the values of the Load, Speed and Material parameters become:

$$W = \frac{W'}{E'R} = 1.63 \times 10^{-5}$$

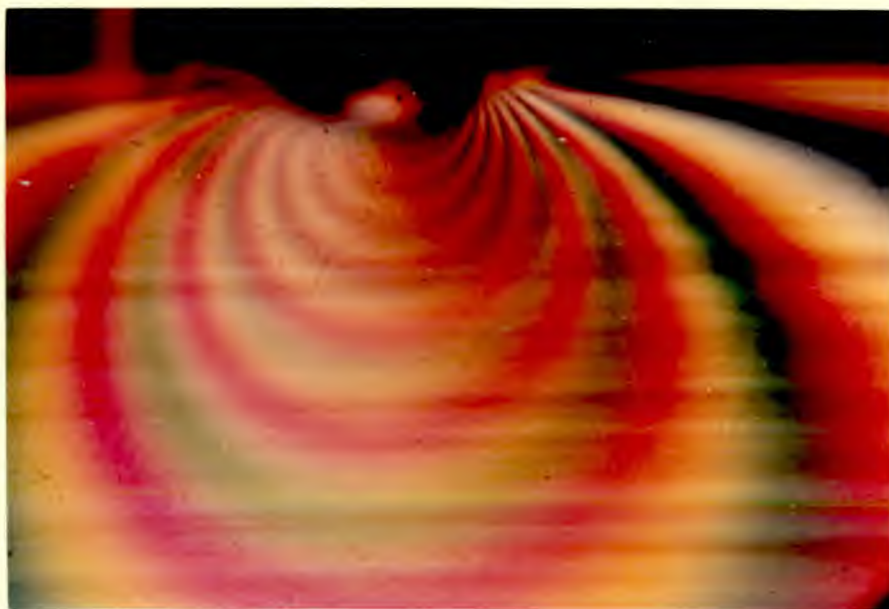
$$U = \frac{\gamma_0^u}{E'R} = 3.4 \times 10^{-11}$$

$$G = \alpha E' = 2300$$

A chart has been devised by Dowson and Whitaker⁽²¹⁾ which enables a particular problem to be located in the 'rigid', 'intermediate' or 'elastic' zones. Use of this chart indicated that elastic conditions probably existed for this experiment.



(a) Stationary



(b) Rotating at 5.1 rev/min.

Fig. 26. Coloured fringes.

Three pressure curves (similar in shape except for the outlet region) together with their corresponding stress contours are compared with the experimental stress pattern, in Figures 27 (a) to (d).

It would appear that the second pressure curve (Fig. 27(b)) gives the best fit to the experimental pattern. The choice of curve, however, is dependent on the shape of part of the inner stress line. Although aware that the experimental evidence is not absolutely conclusive, it is nevertheless gratifying to record that other experimenters, such as Orcutt⁽³⁸⁾ and Kannel,⁽²⁹⁾ have obtained experimental pressure curves with a slight pressure rise in the exit region.

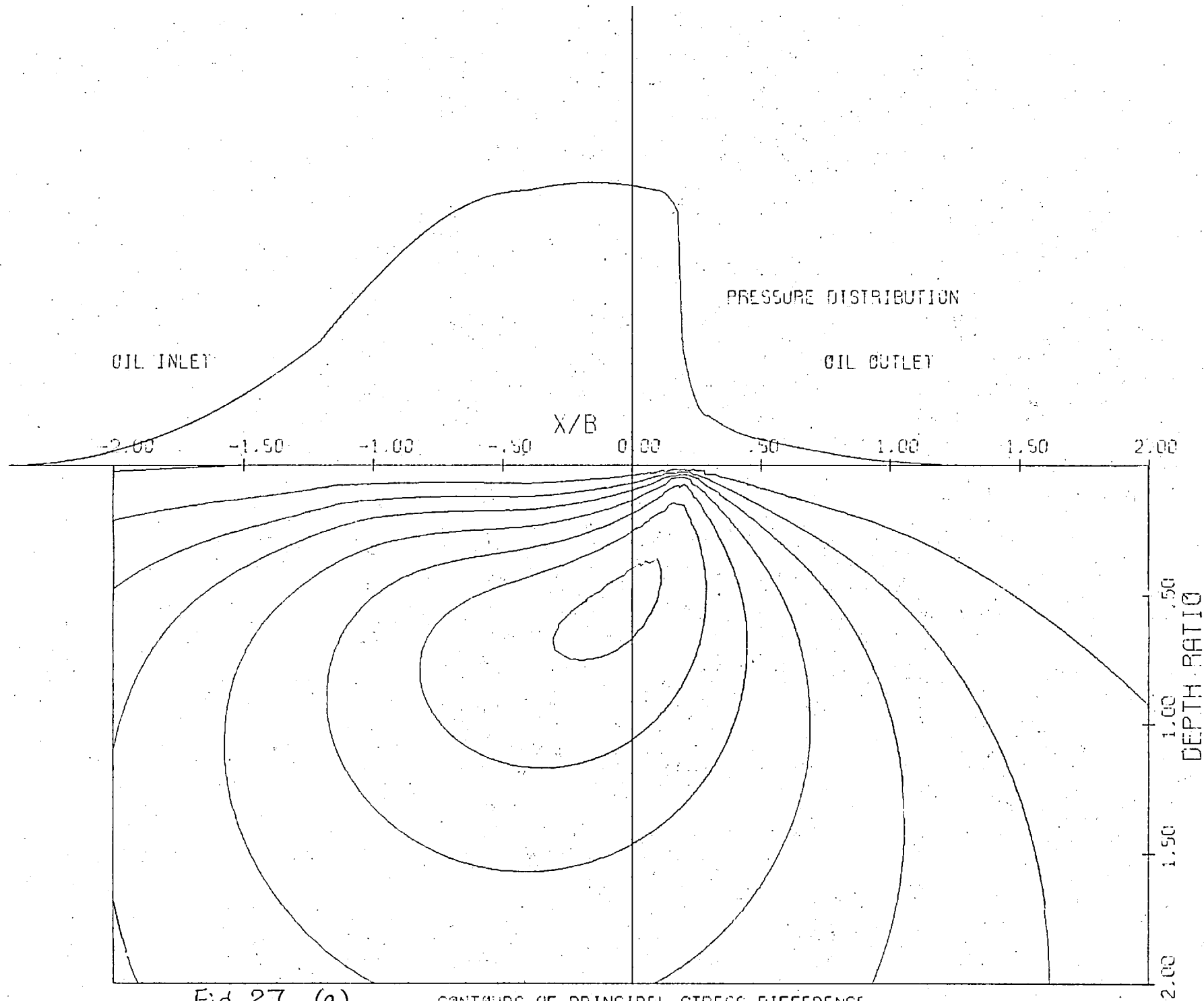
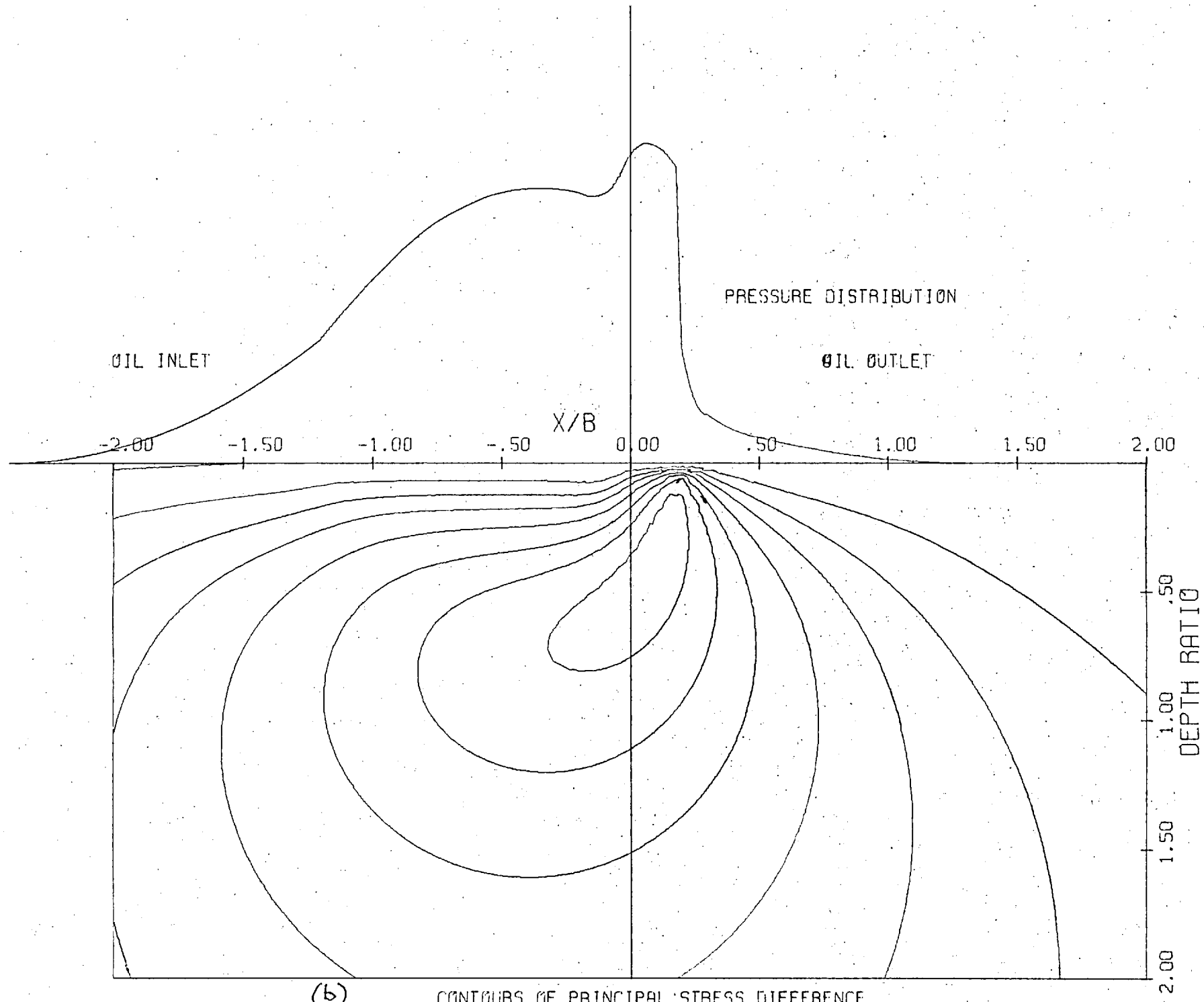
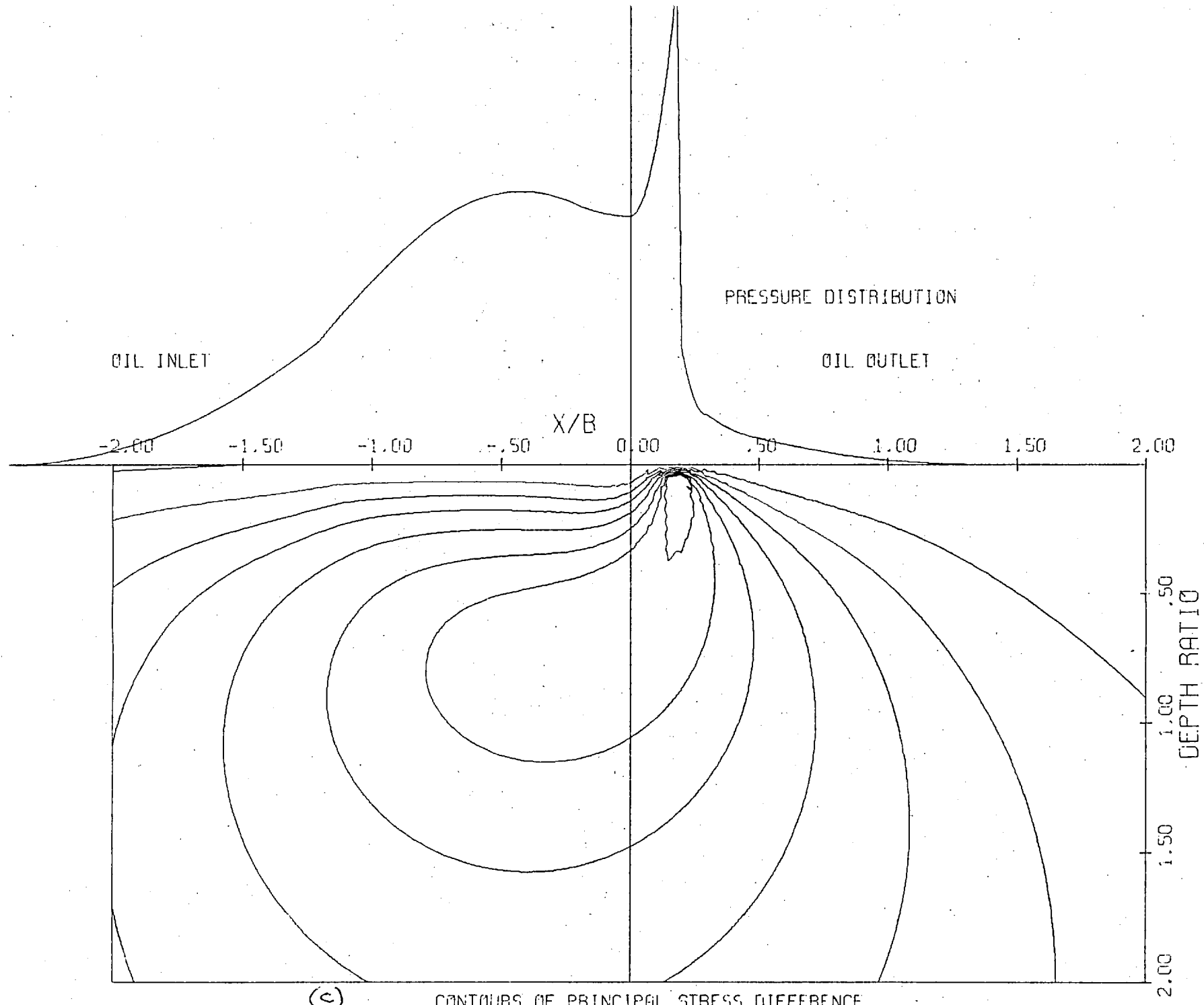
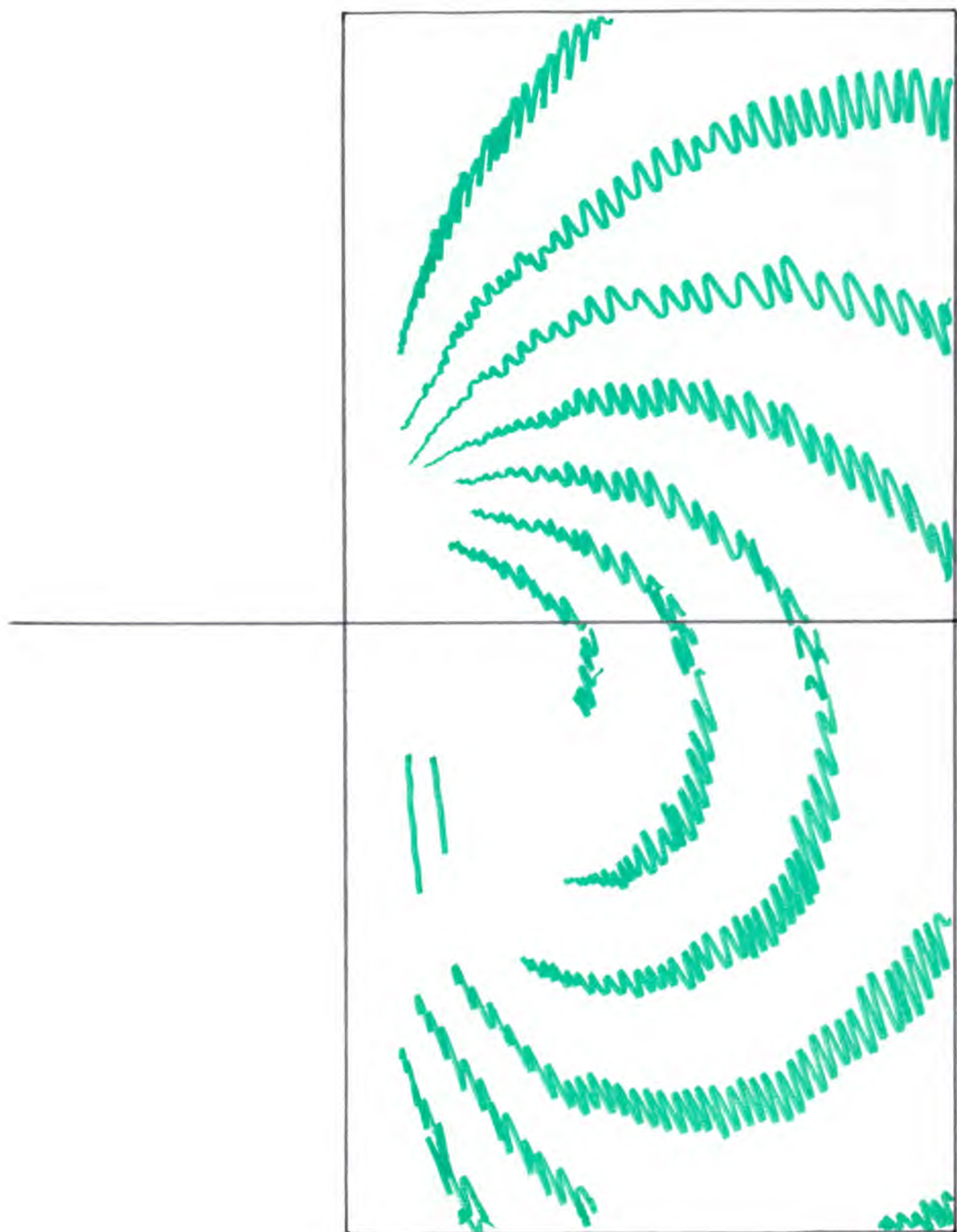


Fig. 27 (a)

CONTOURS OF PRINCIPAL STRESS DIFFERENCE

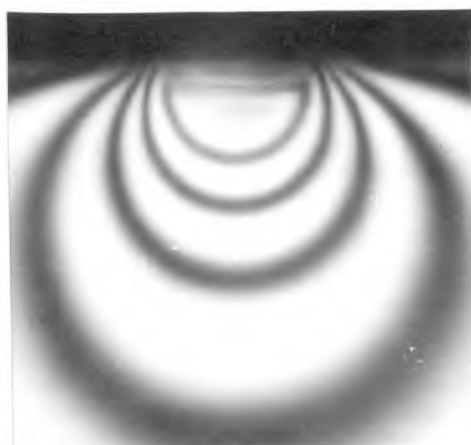




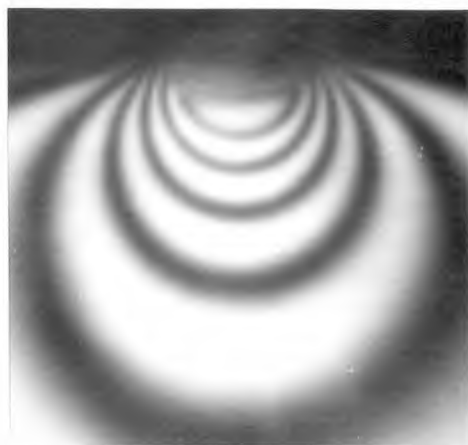


4.03 Dry and Lubricated Rolling Contact

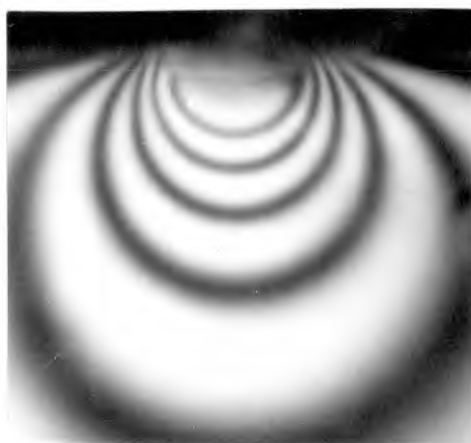
The series of pictures shown in Figure 28 shows the effect on the stress pattern of dry and lubricated rolling contact. Figure 28(a) shows the stress pattern obtained when the two discs are in static contact. The pattern is typical of the shape associated with a Hertzian contact pressure distribution. The sixth (inner) fringe is just appearing as a dark patch. Picture (b) shows the effect of rotating the discs without a lubricant. The shape of the fringes remains basically unchanged although it is noted that the sixth fringe has now become clearly defined. This could be due to the surface asperities becoming less pronounced due to wear. The effect of this would be to reduce the contact area and so increase the pressure intensity. The increase in pressure intensity would tend to increase the number of fringes. Pictures (c), (d) and (e) show the effect of stopping, rotating and stopping again, the discs without a lubricant. The shape remains basically unchanged. The final picture (f) shows the dramatic change in the stress pattern when the discs rotate with a lubricant. The number of fringes has been reduced to three, indicating that the pressure distribution has been spread over a wider contact area.



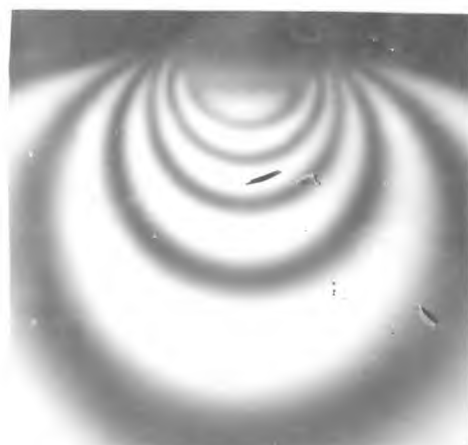
(a) Static (dry)



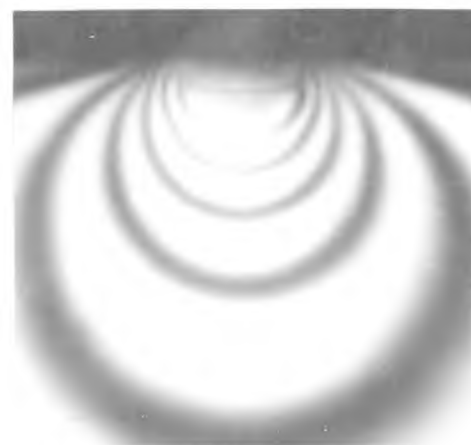
(b) Rotating (dry)



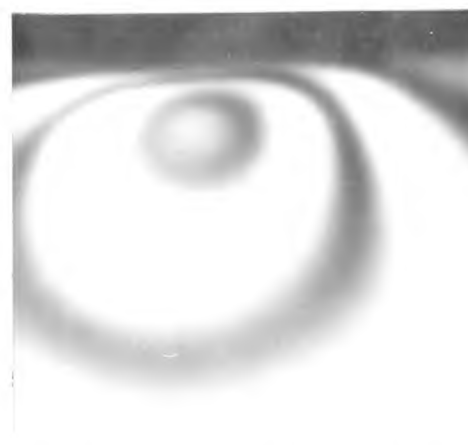
(c) Stopped (dry)



(d) Rotating again
(dry)



(e) Stopped again
(dry)



(f) Rotating (lubricated)

Fig. 28. Dry and Lubricated Rolling Contact.

4.04 Oil Entrapment

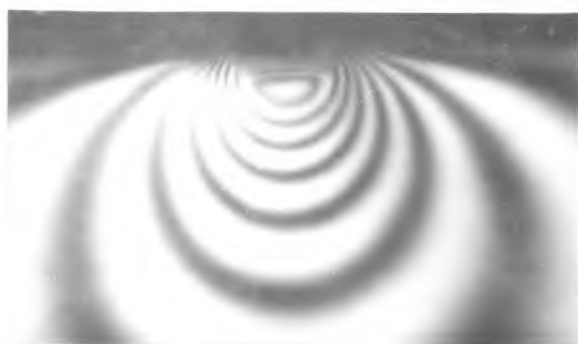
Oil entrapment can occur when two lubricated surfaces are impacted together. It is thought by the writer, that one of the first persons to comment on this phenomenon was Rabinowicz⁽³⁹⁾ in 1952. Rabinowicz observed that the transfer of metal occurred when a sphere was impacted against a steel plate, both for dry and lubricated conditions. However, in the lubricated case a small region in the centre of the indentation was free of metal transfer. Rabinowicz concluded from this, that a small amount of oil must have been trapped between the two impacting surfaces.

Ten years later, Christensen⁽⁵⁾ analysed the effect of two elastic cylinders approaching each other in lubricated line contact. He showed that oil could be trapped between the two cylinders at a pressure higher than the maximum Hertz pressure. Since then, several experimenters have shown by optical interferometry the existence of oil trapped between two impacting surfaces. Dowson and Jones⁽¹⁸⁾ found it when dropping a steel ball on to a glass plate. Westlake and Cameron⁽⁵⁰⁾ also found it by dropping a steel ball on to a glass plate, and by using high speed photography and multiple beam interferometry the formation and dispersion of trapped oil was observed. Finally Roberts and Tabor⁽⁴¹⁾ using a rubber hemisphere and hemicylinder against glass, also observed the entrapment effects.

It has been shown that there is no change in the stress pattern when stopping and starting dry discs (Fig.28)

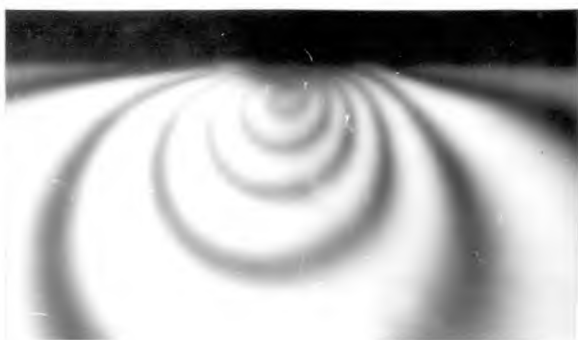
However, if the discs are lubricated the number of fringes increases considerably when the rotating discs are suddenly stopped. Several minutes then elapse before the fringes gradually revert to the form normally associated with static contact. This phenomenon may be due to the oil being trapped between the two surfaces. A series of pictures showing the possible entrapment and dispersion of oil is shown in Figure 29. Pictures (a) and (b) show the stress patterns for lubricated static and rolling contact respectively. Eight fringes are visible for the static contact, reducing to six fringes for the rolling contact. The stress pattern, three seconds after the discs were stopped, is shown in picture (c). It can be seen that the number of fringes has increased over the static case by about fifty per cent. This would suggest that the maximum pressure is greater than the Hertz maximum. Unfortunately it is not possible to calculate the pressure distribution due to the entrapment, because of the dark patch which obscures the fringe pattern in the contact region. As the trapped oil gradually leaks away, so the pressure is reduced and the stress pattern changes, as shown in pictures (d) and (e), until the normal Hertz contact is obtained. This can be seen by comparing pictures (e) and (a). It was also observed that the dark patch gradually disappeared. The cause of this dark patch may be the uneven axial film shape and pressure caused by the entrapment of the oil. A similar occurrence has been commented on in the section on Oil Starvation.

The design of rolling elements subjected to starting and stopping conditions, should allow for the fact that pressures greater than the Hertz maximum can occur if the rolling surfaces are lubricated. In some cases, perhaps dry operating conditions may be appropriate.



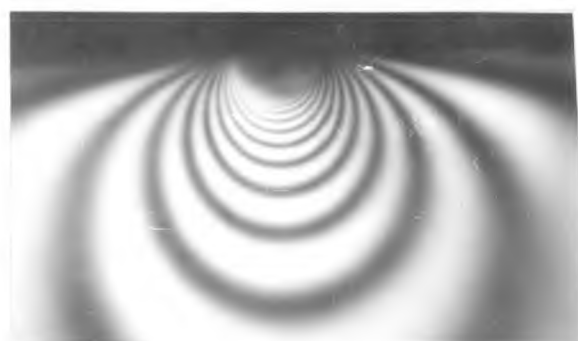
(a)

Static
8 fringes



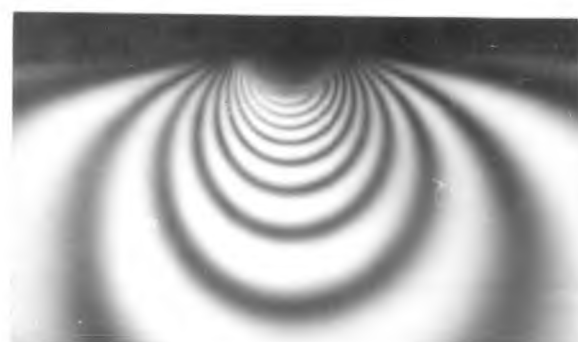
(b)

Rotating 17 rev/min.
6 fringes



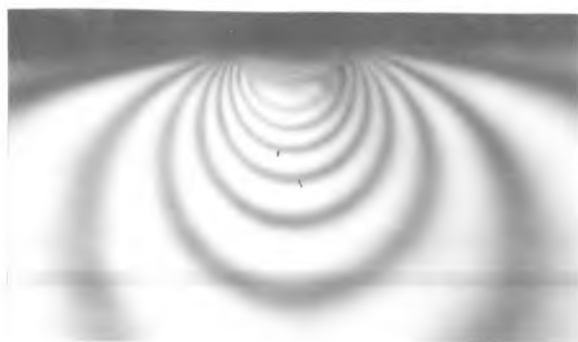
(c)

Time t 3s. from
stopping discs
at least 12 fringes



(d)

t 10s.
at least 10 fringes



(e)

t 3min.
8 fringes

X 30

Fig. 29. Oil Entrapment.

4.05 Oil Starvation

The series of pictures shown in Figure 32 shows the effect on the stress pattern of cutting off the oil supply. Pictures (a) and (b) show the stress patterns for dry static contact and lubricated rolling contact respectively. With the removal of the lubricant, the stress patterns gradually changed to the shape normally associated with dry rolling contact, as shown in pictures (c) and (d). After running for about 45 minutes without additional oil being added, the discs were stopped. Continuation of the test seemed unnecessary since it was noted that after a time the shape of the stress pattern remained more or less constant. The noise caused by the experiment was also a governing factor which decided the stopping of the test. The sound was similar to that experienced when unrolling rapidly, a strip of self-adhesive tape. It was feared that permanent damage to the discs would result if the test continued for too long a time.

It might appear from the pictures that a rough estimation of the amount of oil on the discs could be obtained by direct measurement. This however, was not possible as the glass was wider than the steel disc. This meant that the oil thickness observed on the photographs was really the thickness of the oil on the glass edge and not on the contact surface. This is illustrated in Figure 30.

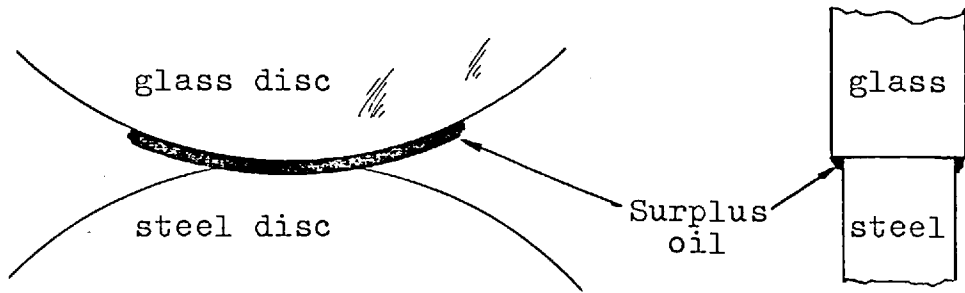


Fig. 30. Surplus oil film.

The surplus film of oil gradually disappears so that either it spreads over the sides of the steel disc, or it acts as a reservoir of oil which partially helps to lubricate the contact zone.

Another interesting feature that can be observed from these pictures is that the stress pattern is even more blurred than usual in the contact region. This may be due to the oil meniscus being curved at entry so causing a variable axial pressure distribution, as shown in Figure 31.

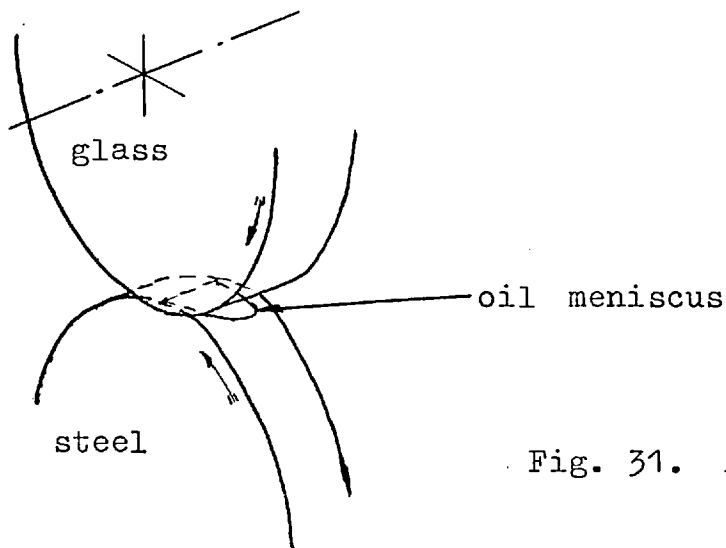


Fig. 31. Axial oil meniscus.

In 1958, Crook⁽⁷⁾ carried out an experiment on a Two-Disc machine in which he measured the film thickness and the oil inlet temperature, during the period when the oil supply was cut off. This period lasted about 25 minutes, and he found that while the oil film thickness decreased, the oil temperature increased. This increase in temperature with the resulting decrease in viscosity was sufficient to account for the reduced film thickness.

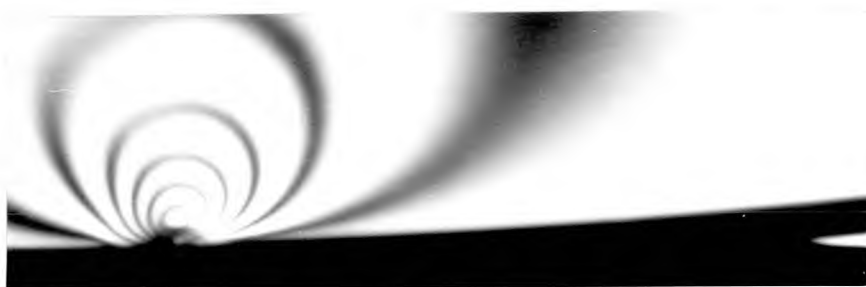
Wedeven⁽⁴⁸⁾ has recently investigated, for rolling point contact, the effect of oil starvation on film thickness. Optical interferometry was used to observe, not only the film shape, but also the amount of oil in the outlet region. It was noted that the starvation of oil did not effect the film thickness, until the amount of oil in the inlet region fell below a certain value, after which the film thickness decreased. It was not thought possible, as in Crooks' experiment, for the reduced film thickness to be accounted for by the increase in the oil inlet temperature, since the rolling speed was about one tenth of the value used by Crook. However, the point to remember is that any reduction in film thickness, whether due to the oil starvation or the temperature rise, may result in asperity contact with the subsequent risk of wear.

Maximum Hertz pressure $21,200 \text{ lb}_f/\text{in}^2$ speed 17.5 rev/min.



8 fringes

(a) Dry static contact



6 fringes

(b) Lubricated rolling contact



8 fringes

(c) After 10 mins. without additional oil



8 fringes

(d) After 45 mins.

X 16.5

Fig. 32. Oil Starvation.

CHAPTER 5 EXPERIMENTAL RESULTS - EFFECT of ROUGHNESS.

5.01 Object and reasons for tests

The object of these tests, was to observe the change in the contact stress patterns with the change in the roughness of the discs. It was thought that some evidence might be forthcoming on the effect of film thickness and asperity heights on pitting.

Most of the initial work on pitting was carried out by Way,⁽⁴⁶⁾ who demonstrated that pitting was a surface fatigue phenomenon that could only occur if a lubricant was used. He also showed that rough discs were more likely to fail due to pitting than smooth discs. In a later paper entitled 'How to Reduce Surface Failure', Way⁽⁴⁵⁾ suggested that a very high alternating shearing stress below the contact surface, is not necessarily dangerous as far as fatigue is concerned. This was verified by Dawson⁽⁹⁾⁽¹⁰⁾ who produced evidence that even for a high load, (maximum Hertz pressure $175 \times 10^3 \text{ lb}_f/\text{in}^2$) in the complete absence of metallic contact pitting does not occur. It would therefore appear that surface contact must exist if pitting is to start. It was suggested by Cameron in a discussion of the paper by Beeching and Nicholls,⁽³⁾ that the higher local stresses at asperity contact may account for the increased pitting with roughness. These higher local stresses have been shown, theoretically to exist, by Fowles.⁽²³⁾

Dawson⁽⁹⁾ noted that increasing the oil film

thickness has the same effect as reducing the surface roughness. A dimensionless ratio was defined as:

$$D = \frac{\text{The total initial surface roughness of the two mating discs. (Peak to Valley)}}{\text{Elastohydrodynamic oil-film thickness between the discs.}}$$

The Dawson experiments indicated that pitting did not occur when $D < 0.1$ so that a range of tests were conducted for $0.025 < D < 7.0$

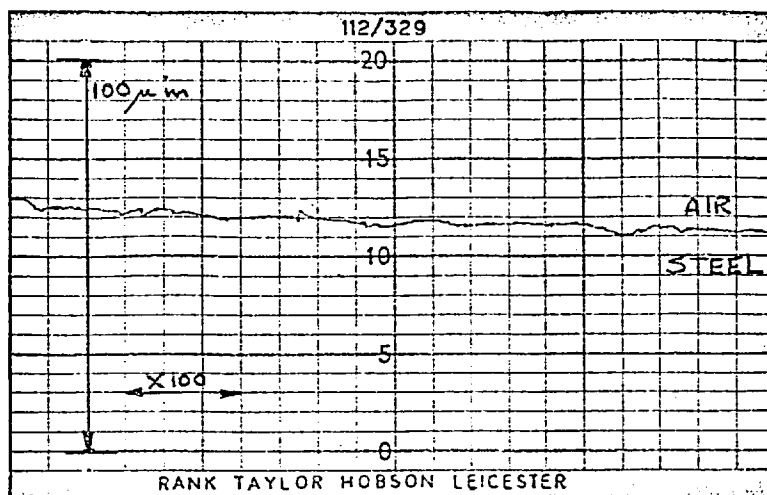
5.02 Details of surface finish

The Talysurf trace for each disc is shown in Figure 33.

Surface finish	C.L.A.	P to V
lapped	2.5	5.0
ground	20.0	60.0
grade '2½' emery paper	90.0	250.0
grade '5' emery paper	125.0	350.0

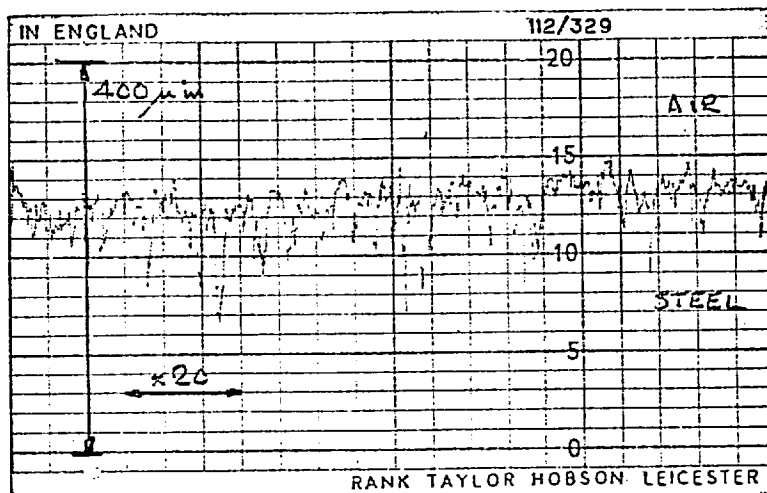
5.03 'D' Values

		'D' Values			
surface speed in/s	film thickness $\times 10^{-6}$	lapped	ground	'2½'	'5'
1.83	54.3	.093	1.128	4.6	6.46
11.32	194.0	.026	.308	1.286	1.8



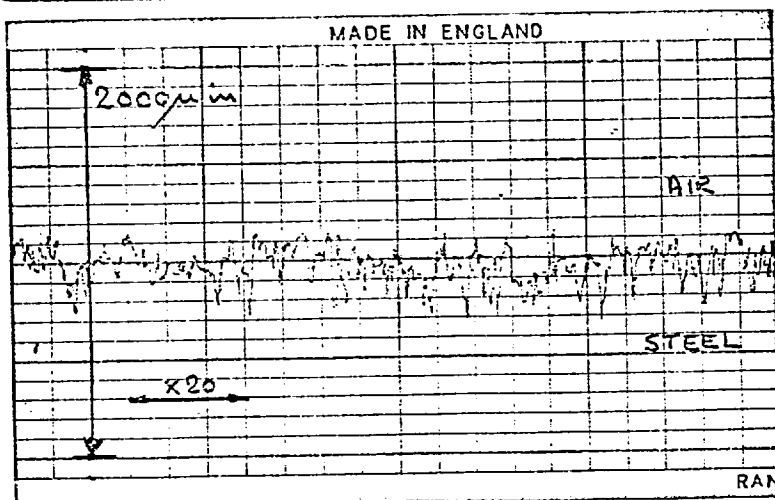
'Lapped'

2.5 in. C.L.A.



'Ground'

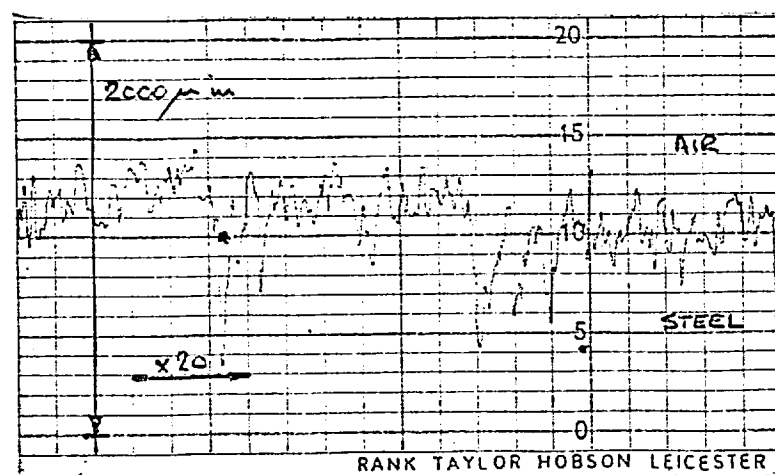
20 in. C.L.A.



'21' grade

emery paper

90 in. C.L.A.



'5' grade

emery paper

125 in. C.L.A.

Fig. 33. Talysurf traces for steel discs.
(across track)

5.04 Results

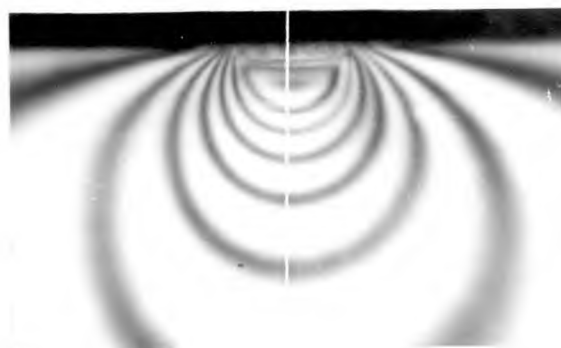
Figure 34 shows the change in the static stress pattern as the surface roughness increases, as in pictures (a) to (d). As the roughness increases, the apparent contact width increases and the number of fringes decreases. This is to be expected since the rough surface would distribute the applied load over a wider area, reducing the maximum pressure and hence the number of fringes.

The corresponding stress patterns are shown in Figure 35 for the same load and degrees of roughness, as in the previous case, but with the discs rotating with a surface speed of 1.83 in/s. These pictures indicate that as the roughness increases, so the stress pattern reverts to the shape normally associated with its equivalent static shape. This is clearly seen by comparing Figure 35(c) with 34(c) and also Figure 35(d) with 34(d). The similarity between the static and dynamic stress patterns is to be expected, since increasing the roughness increases the possibility of surface contact. It has been shown in Chapter 4.03 that the stress patterns remain unchanged from static to dry rolling contact. Thus two conditions ideal for pitting are present, namely, a lubricant and surface contact.

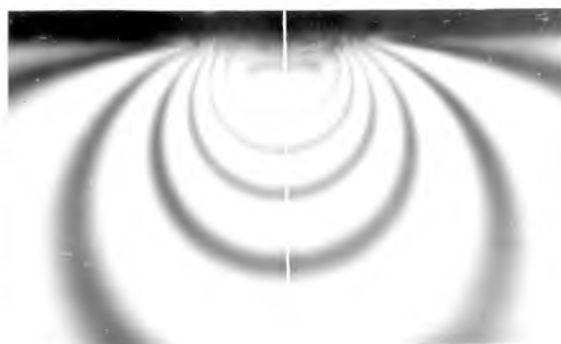
Increasing the speed of the discs increases the film thickness, which delays the tendency for surface contact to be made. This has been illustrated in pictures (a) to (d) of Figure 36 where the speed of the discs has

been increased to 11.32 in/s. Picture (a) shows the stress pattern when a fully developed oil film exists. Pictures (b),(c) and(d) show how the stress pattern gradually changes to the shape normally associated with dry rolling. The change, of course, is not so noticable as in Figure 35 due to the higher disc speed.

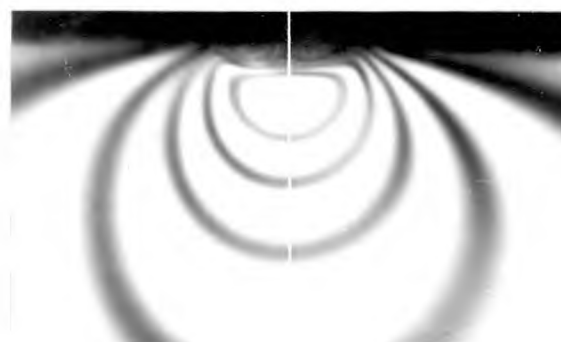
The results suggest that almost complete surface contact is obtained for 'D' values greater than two.



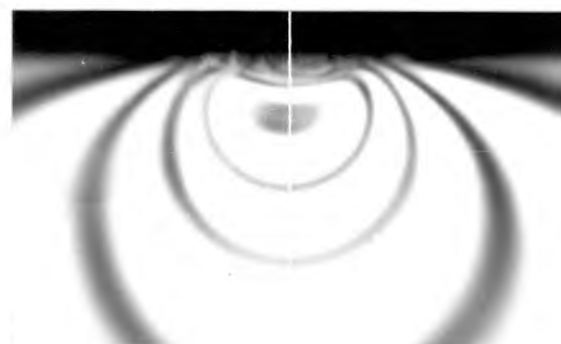
(a)
 $2.5 \mu\text{in. C.L.A.}$



(b)
 $20.0 \mu\text{in. C.L.A.}$



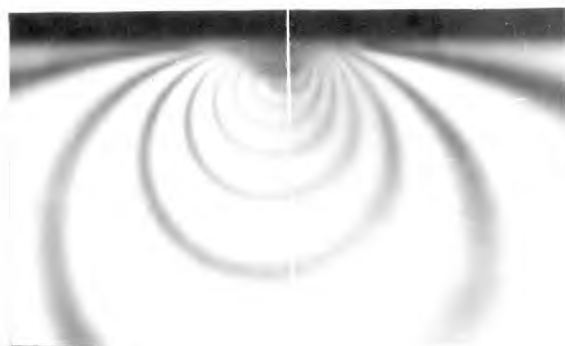
(c)
 $90.0 \mu\text{in. C.L.A.}$



(d)
 $125 \mu\text{in. C.L.A.}$

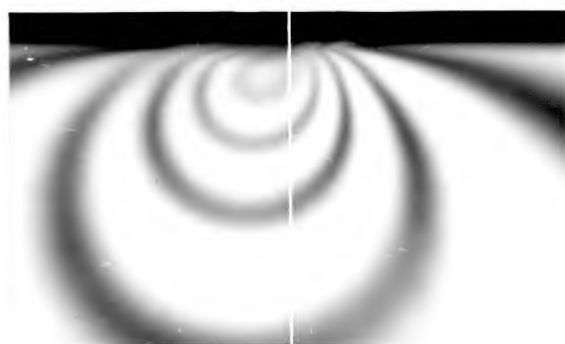
X 28

Fig. 34. Effect of roughness on static fringe pattern.



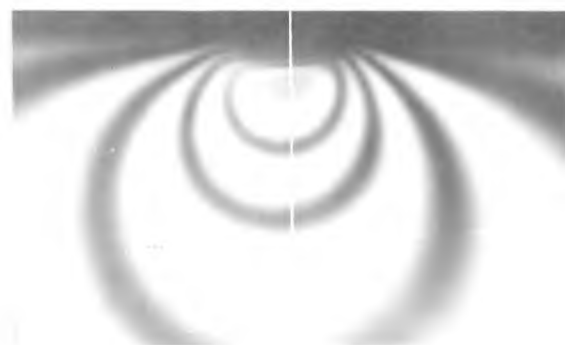
(a)

$D = .093$



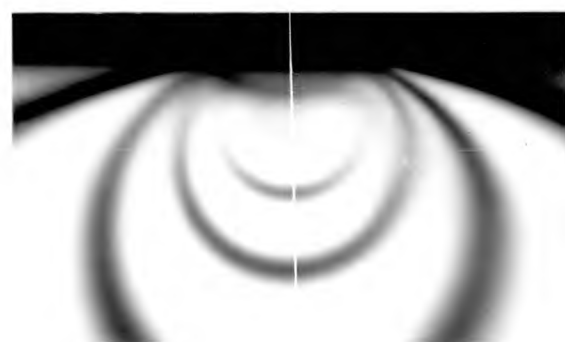
(b)

$D = 1.13$



(c)

$D = 4.6$

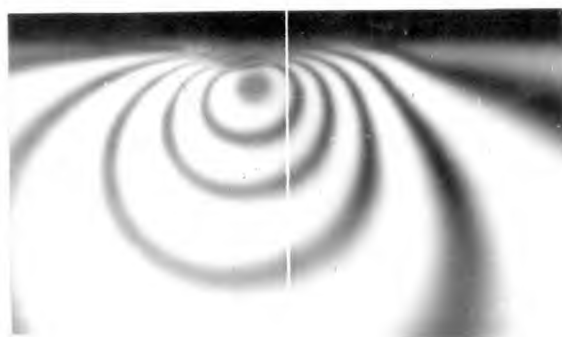


(d)

$D = 6.46$

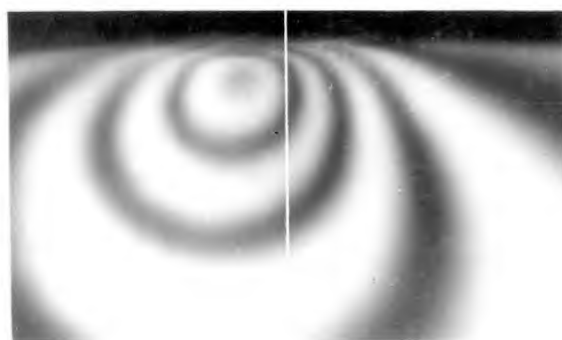
X 28

Fig. 35. Effect of roughness
- discs rotating.



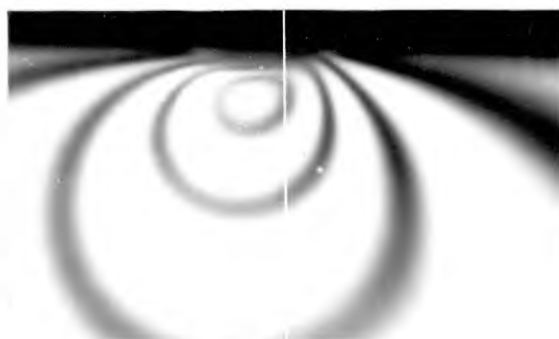
(a)

$D = .026$



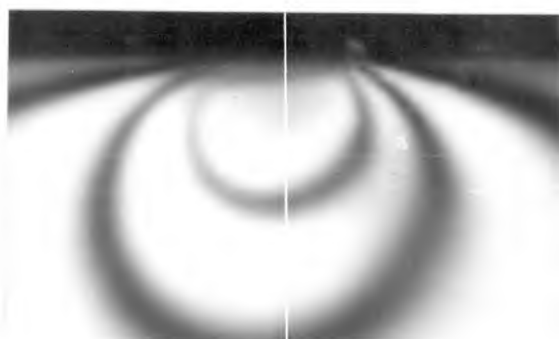
(b)

$D = .31$



(c)

$D = 1.29$



(d)

$D = 1.8$

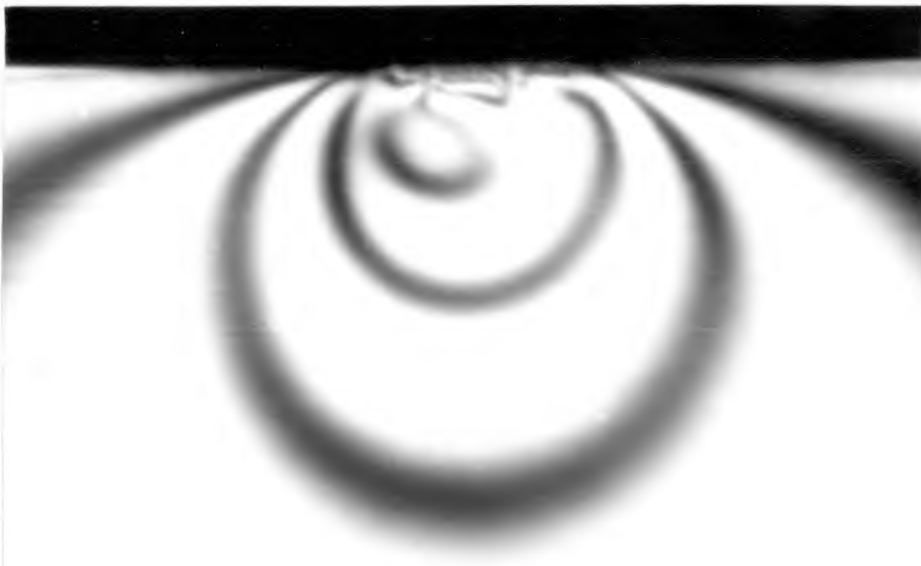
X 28

Fig. 36. Effect of roughness
- increased speed.

5.05 Effect of large asperity contact on the static stress pattern

It was noted when investigating the effect of surface roughness, that the inner fringe would tend to 'float about' for different contact positions. It was thought that this may have been due to an unusually large surface protrusion.

A typical stress field is shown in Figure 37 where the inner fringe is distorted. A slight movement of the discs caused this fringe to change its shape completely. An analysis of this picture would clearly be inappropriate, as the axial pressure distribution would not be uniform. However, a contour plot has been produced in Figure 38 which shows the effect on the stress field of an undulating pressure curve. The curve was spread over a wide contact length to show the corresponding reduction in contours.



Applied
load 265 lb/in

Roughness
125 CLA μ in.

Four fringes
instead of
the usual
eight for
'smooth'
contact.

Fig. 37. Isochromatics - very rough surface contact.

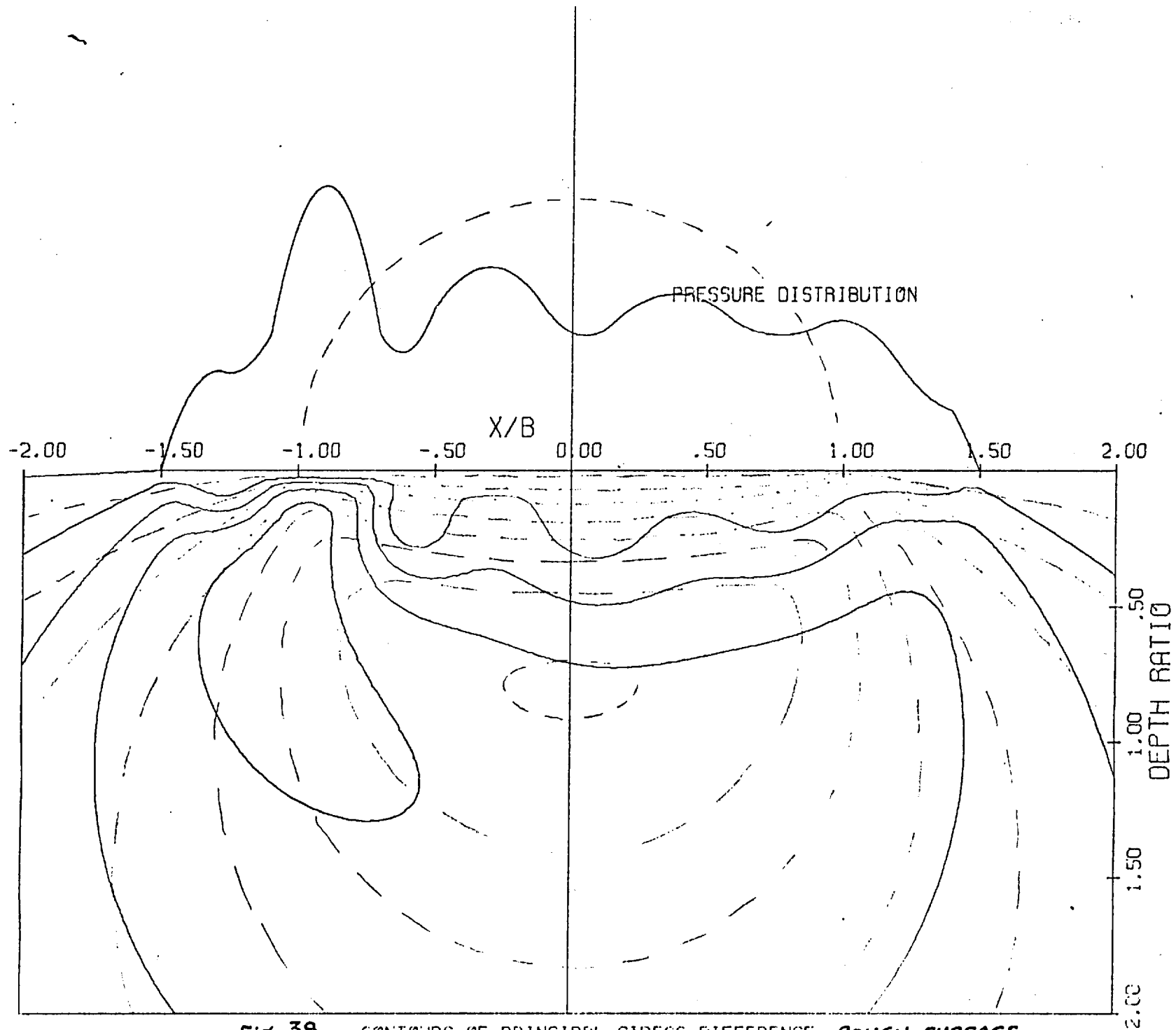


Fig. 38. CONTOURS OF PRINCIPAL STRESS DIFFERENCE - ROUGH SURFACE

CHAPTER 6SUMMARY6.01 Errors

The errors arising from the project can be classified into two groups; namely, errors in the physical measurements and errors in the theoretical computations.

Errors in the physical measurementsLoad

A simple lever using standard weights, was used to apply the load to the discs. The error in the standard weights can be considered negligible. The length of the shortest arm of the lever was 1.5 inches. If the maximum error in measuring this length was ± 0.015 inches. Then the possible error in measuring the load was $\pm 1.0\%$

Speed

The range of speed covered in the tests was from about 0.1 to 1.0 rev/s. For this range, it was considered that the most economical method of measuring the speed (without sacrifice of accuracy) was by simply noting the time taken to count a set number of revolutions. This method produces errors less than 1.0%. Unfortunately a certain amount of slip occurred at the fastest speed, due probably to the flexible cable. A more realistic estimation of the error would be $\pm 3.0\%$.

Temperature

The temperature of the oil entering the contact region was recorded. This value would be slightly less

than the disc surface temperature required for estimating the oil inlet viscosity. Since however, very low speeds and only moderate loads were used, the difference in temperature was thought to be less than one degree. The effect of this error on the oil viscosity reading was estimated to be $\pm 4.0\%$.

Surface roughness

The specification of the surface roughness was not a straightforward matter. Several Talysurf readings were taken in order to obtain a representative value. Readings deviated by as much as 12.0%.

Film thickness and 'D' values

The effect of these errors on the calculation of the film thickness and 'D' values was estimated to be $\pm 5.0\%$ and $\pm 17.0\%$ respectively.

Errors in the theoretical computations

Calibration Chart

The validity of this chart is based on the fact that the magnitude and position of the maximum shearing stress, corresponds with the values obtained by other writers.⁽⁴⁴⁾⁽³⁾

Contour plot - parabolas

Stress values on the centre line of the contour plot, based on a Hertz pressure curve, correspond to the values on the Calibration Chart. On this basis, the

programme was considered reliable for other pressure curves. Extreme care must be taken however when splitting the pressure curve into parabolic blocks. It is possible that the shape of the computed parabola, based on three points, would be different from the shape required. This has been illustrated in Figure 39. The possibility of this happening undetected was overcome by instructing the computer to plot the pressure curve so that a visible check on its shape could be made.

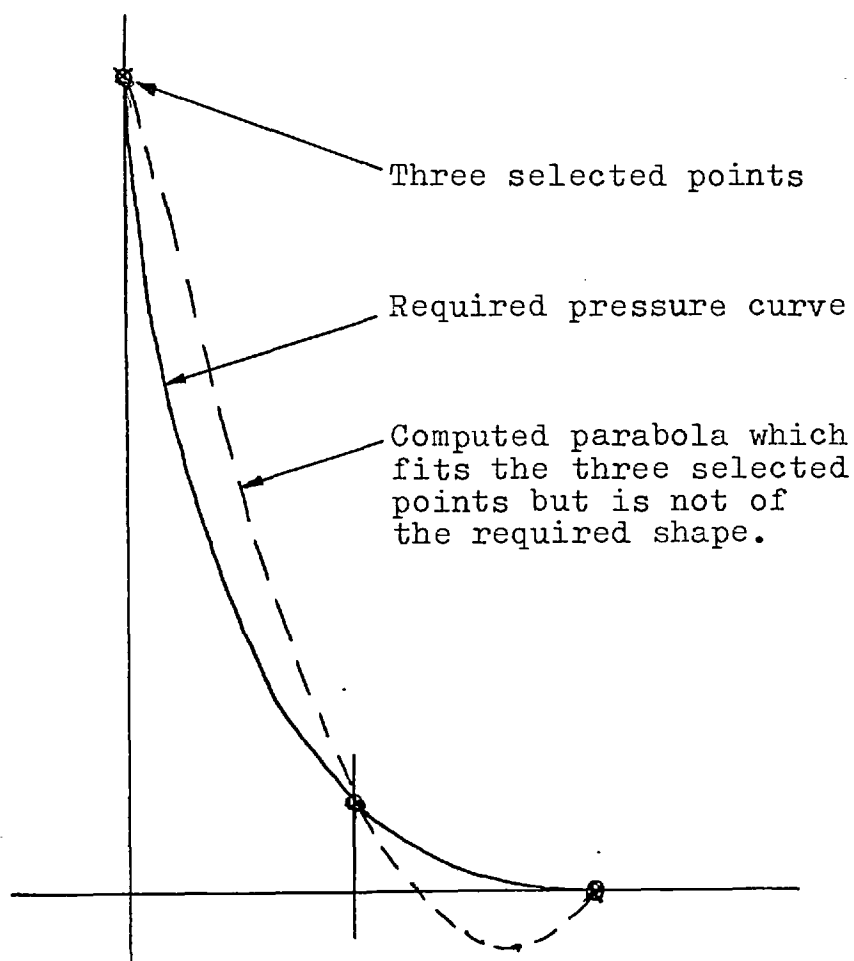


Fig. 39. Possible source of error - required and computed pressure curves.

Elastic deflexion

The elastic deflexion programme was used to compute and plot the deformed surface, caused by a Hertz pressure distribution effect. The result, shown in Figure 54 shows the flat deformed surface justifying the use of the programme. The programme would have to be modified however, if extremely narrow parabolic sections were to be used. This is because small differences would then be involved in the calculations of the deflexions. The method of modifying the programme under these circumstances has been admirably explained by Dowson and Higginson.⁽¹³⁾

Inverse solution of Reynolds equation

The accuracy of this programme depends mainly on the accuracy of the computed pressure gradients. The pressure profiles is made up of several parabolic curves so that discontinuities in the pressure gradients are inevitable. The differences in the Elastic deflexion curve and Reynolds curve shown in Figure 23(a) and (b) are thought to be due to errors in calculating the pressure gradient.

6.02 Discussion

Some of the experimental pressure curves and film shapes found by Dowson and Longfield,⁽¹⁹⁾⁽²⁰⁾ Longfield⁽³²⁾ and Niemann and Gartner,⁽³⁶⁾ show a marked similarity with those obtained in this report. Absolute agreement is, of course impossible, because of the difficulty of ensuring the similarity of the various parameters.

The slight pressure rise in the outlet zone of the pressure curve (shown in Figure 27(b)) is of interest, since it is an experimental indication of a pressure spike. However, too much reliance should not be placed on this section, because the prediction of the pressure rise depended on the observation of part of an isochromatic fringe. One of the experimental difficulties of obtaining clear isochromatic fringes, is that their observations are required over a very small area and through a very long line of contact. The line of contact could be reduced considerably to give sharp fringes, if the method of reducing end effects suggested in Chapter 7.01 is successful.

The effect of the tangential surface stresses on the contours was considered negligible, since pure rolling was assumed to exist. Even if a small amount of sliding was present, the tangential stresses would be small in comparison with the direct stresses. Calculations of the tangential stresses for pure rolling and sliding contacts have been given by Dowson and Higginson.⁽¹²⁾

6.03 Conclusions

The photoelastic technique has been shown to be suitable for investigating the pressure distribution and film shape in rolling line contact.

The Trial and Error method of determining the pressure distribution is simple in concept but may be lengthy in application, depending on the 'turn around' time of the computer. The other method of computing the pressure curve direct from the Isochromatics requires very careful measurement of the experimental stresses. This method, which has yet to be developed to its full potential, is capable of producing a result in a fraction of the time taken by the Trial and Error method.

CHAPTER 7FUTURE WORK

It should be pointed out that the suggestions listed below for future work are not entirely those of the writer. They are the result of general discussions with staff and students of the Lubrication Laboratory and visiting research workers.

7.01 Elimination of end effects

A sketch of a stress pattern corresponding to a pressure with a spike is shown in Figure 40 reproduced from a paper from Dowson and Higginson.⁽¹⁵⁾

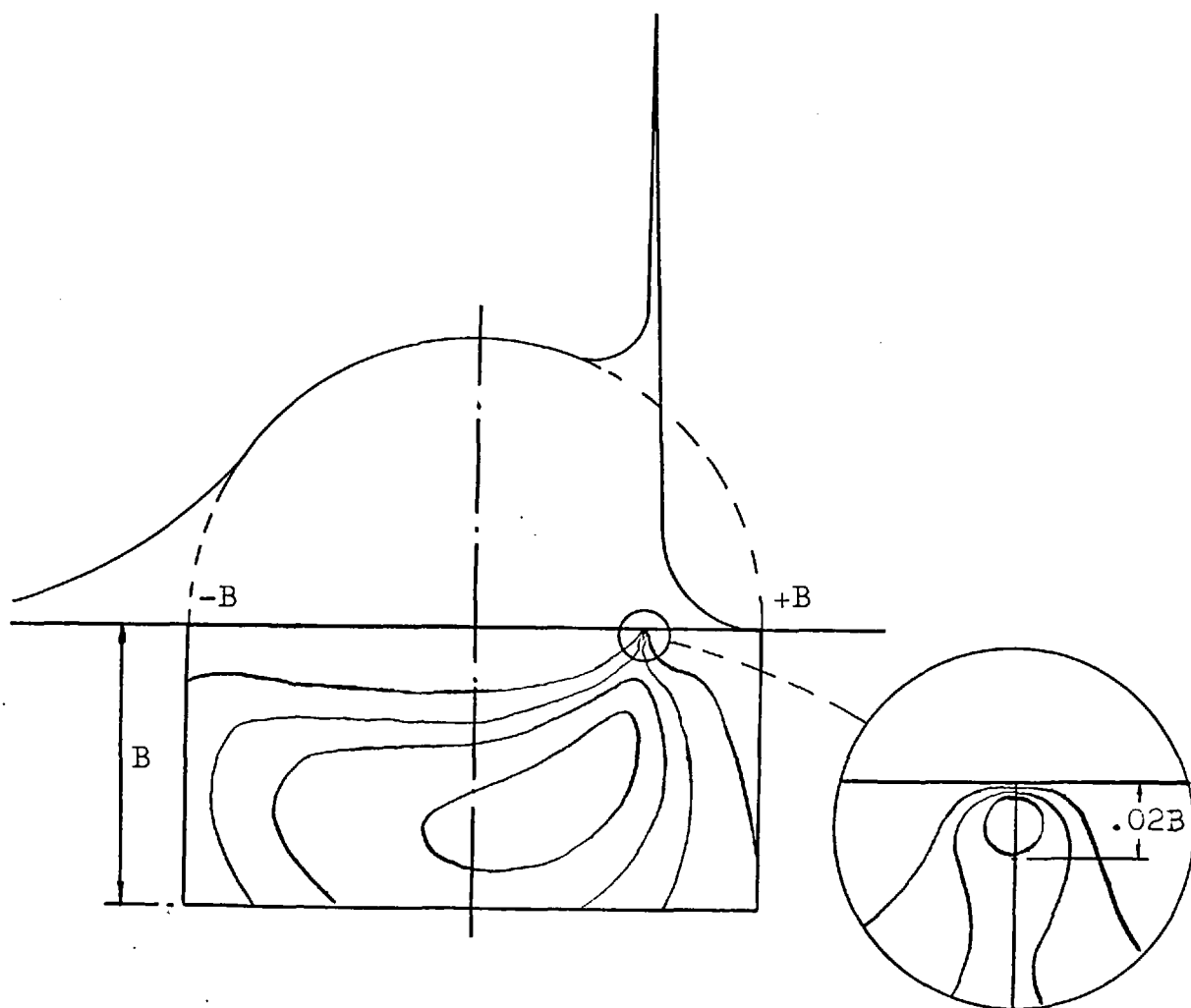
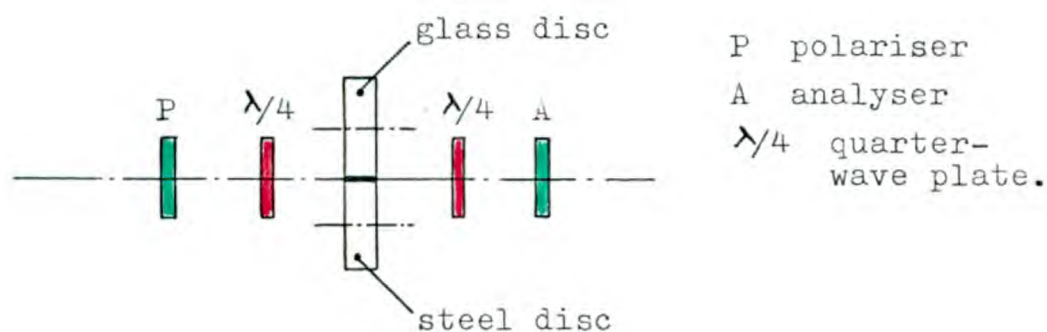
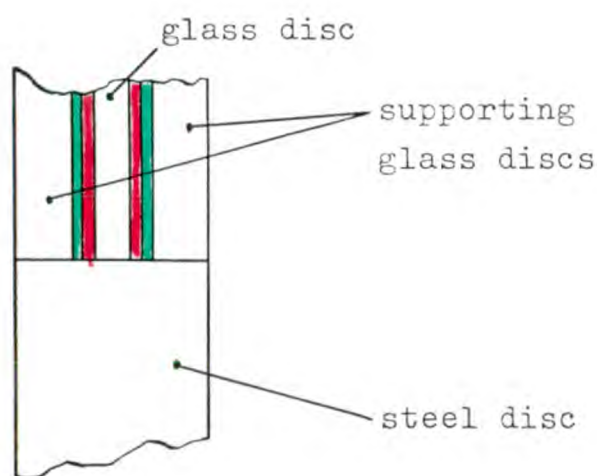


Fig. 40. Effect of pressure spike on stress contours.

If this stress pattern is to be observed experimentally then 'end effects' must be eliminated. One way of achieving this would be to combine the polariser, analyser and quarter-wave plates with the glass disc; together with supporting glass discs on each side, as shown in Figure 41. When stressed, only the centre glass disc would show photoelastic fringes, which should be free from 'end effects'.



(a) Existing arrangement



(b) Proposed arrangement

Fig. 41. Optical layout.

There is however, a possibility that local distortion may be suffered by the polariser, analyser and quarter-wave plates when in contact with the steel disc. Or indeed, that the polarising properties of the plates may be affected when stressed. These factors could cause distortion of the fringe pattern. To check this possibility it would be first advisable to make a compound glass block, and to load it against a steel roller, as shown in Fig. 42.

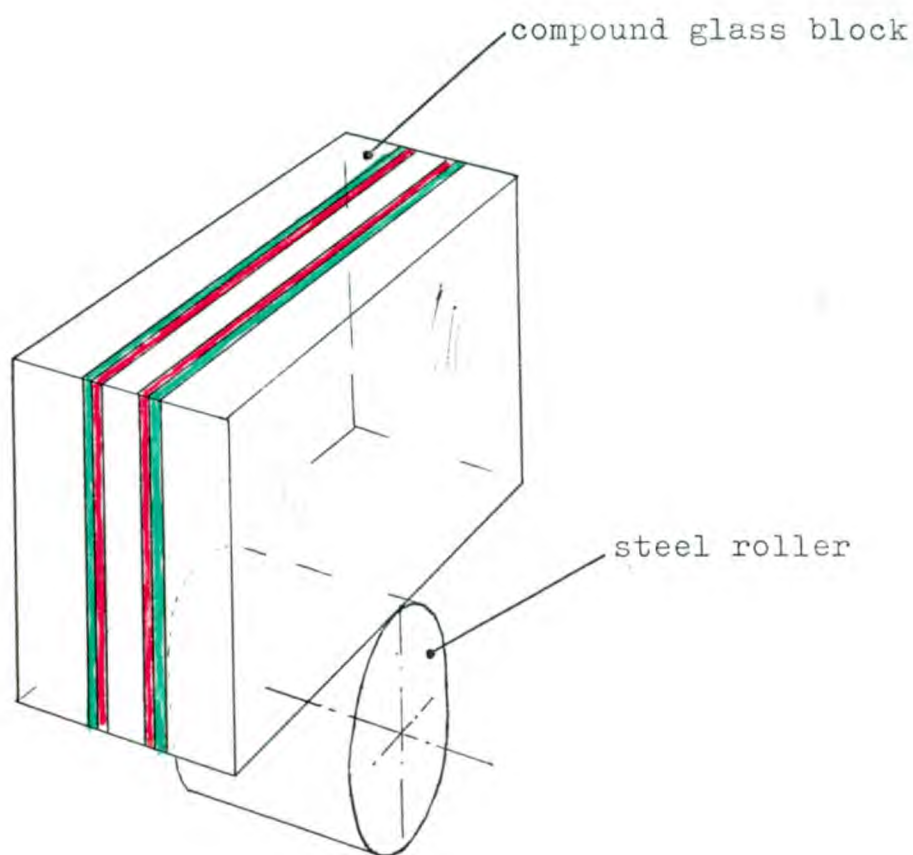


Fig. 42. Experiment to check clarity of photoelastic contact stresses.

Another way in which 'end effects' may be eliminated would be by blending the ends of the steel disc, shown in Figure 43. The object of blending the steel disc is to ensure that when it is loaded against the glass the axial pressure distribution between them remains uniform.

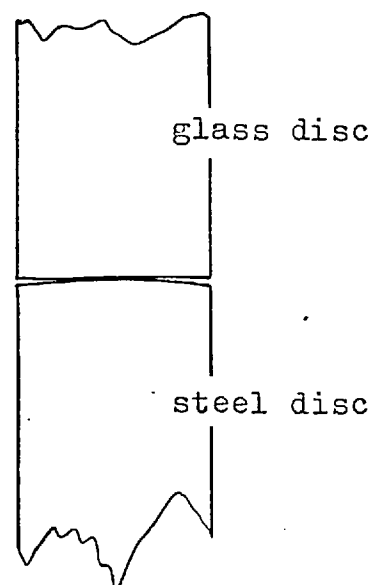


Fig. 43. Blended steel disc.

7.02 Combination of optical methods

The optical interferometry method of determining the film shape in point and line contact has been developed to a fine art; particularly in the Lubrication Laboratory at Imperial College. An experiment which combines the interference film shape with its photoelastic stress pattern would be worth developing. The suggested apparatus shown in Figure 44 would be basically a four-disc machine the inner disc being made of glass.

Hamilton and Moore⁽²⁶⁾ combined Crook's⁽⁸⁾ method for obtaining the film shape, with Kannel's⁽²⁹⁾ method for obtaining the pressure distribution in line contact, into one gauge. The gauge consisted of a thin manganin strip vapour-deposited on to the rolling surface of a glass disc.

If a similar metallic strip was vapour-deposited on to the glass at A, as shown in Figure 44, then the effect of its presence as it passed through the contact region, could be observed optically.

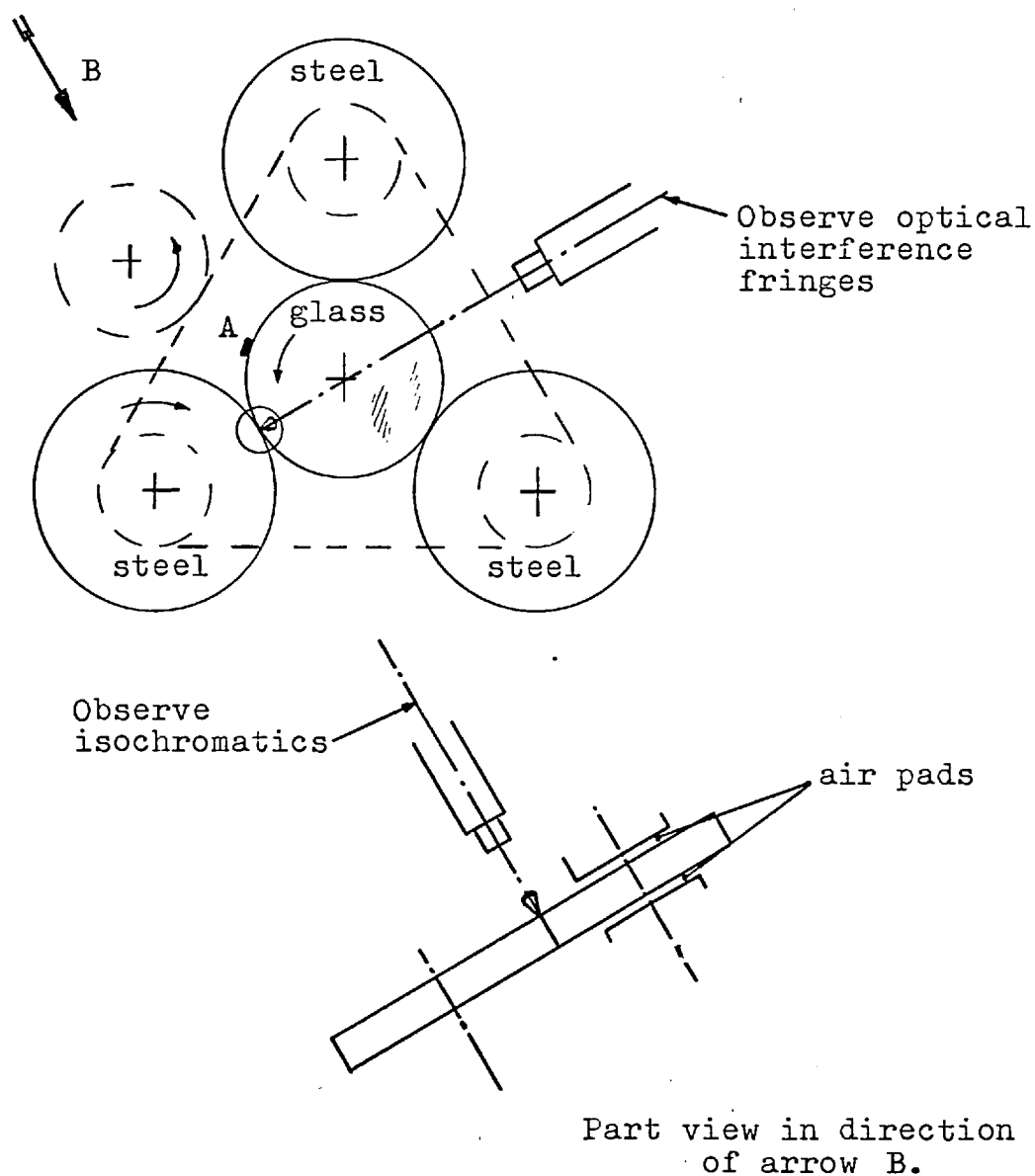


Fig. 44. Four disc machine, combination of optical methods.

7.03 Sapphire insert

Another way in which the two optical methods could be combined is by using a sapphire insert as shown in Figure 45. Sapphire has the advantage of having a much higher elastic modulus than glass. It would therefore be able to withstand a much higher contact stress.

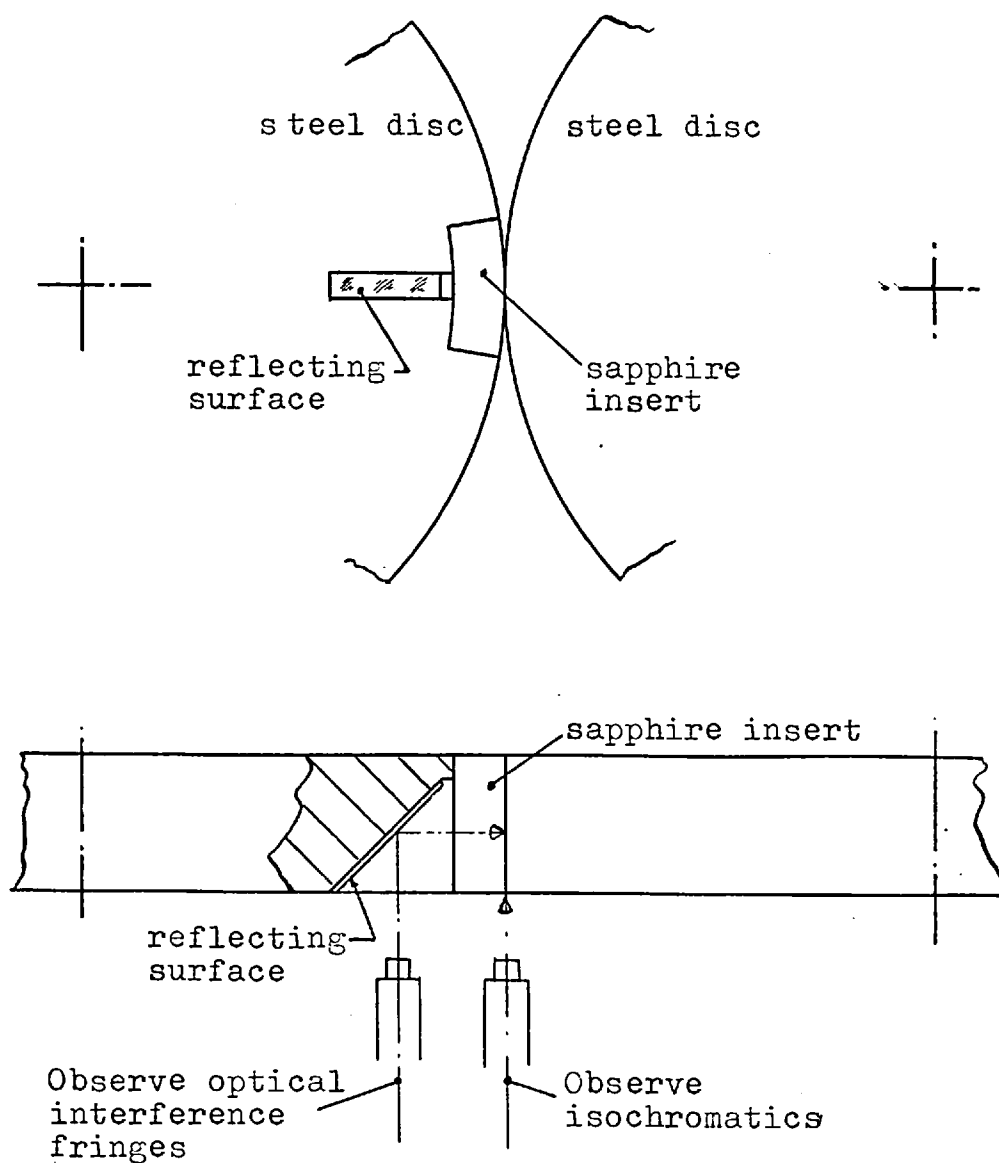


Fig. 45. Sapphire insert.

APPENDIX I

Calibration Chart

This chart calibrates the Isochromatics due to a Hertz pressure distribution. The chart has been devised in nondimensional form to enable one curve to cover the whole range of experimental conditions.

The programme computes and then plots the ratio of Principal stress difference to maximum Hertz pressure; against the ratio, radial distance from contact, to half the Hertzian contact width.

According to Hertz,⁽⁴⁴⁾ the pressure distribution on the surface of two compressed elastic bodies, as in Figure 46, is defined by the expression:

$$p = PM \sqrt{1 - \frac{x^2}{B^2}} \quad \text{and} \quad B = 2 \sqrt{\frac{W'R}{\pi E'}}$$

where $\frac{1}{E'} = \left[\frac{1 - \mu_g^2}{E_g} + \frac{1 - \mu_s^2}{E_s} \right] \cdot \frac{1}{2}$

and $\frac{1}{R} = \frac{1}{R_g} + \frac{1}{R_s}$

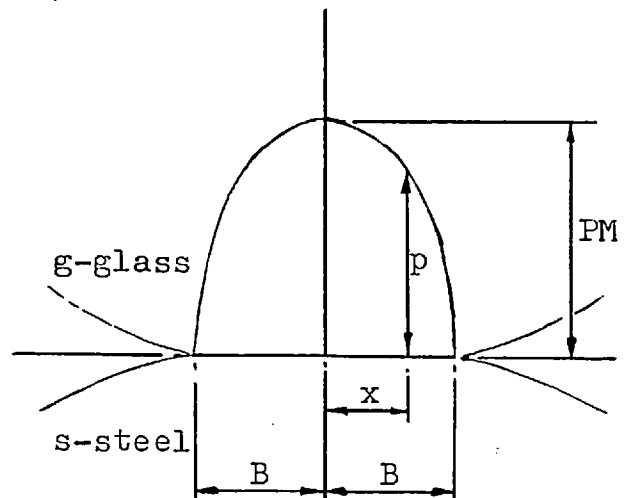


Fig. 46. Hertz pressure curve.

$$W' = \int_{-B}^{+B} p \, dx = PM \int_{-B}^{+B} \left[1 - \frac{x^2}{B^2} \right] dx$$

$$\text{i.e. } W' = \frac{\pi B PM}{2}$$

$$\therefore PM = \frac{2W'}{\pi B}$$

$$\text{Given that } R_g = R_s = 3.0 \text{ in.}$$

$$\mu_g = 0.35$$

$$\mu_s = 0.25$$

$$E_g = 9.5 \times 10^6 \text{ lb}_f/\text{in}^2$$

$$E_s = 29.0 \times 10^6 \text{ lb}_f/\text{in}^2$$

$$\text{then } B = 0.000485 \sqrt{W'} \text{ in.}$$

$$\text{and } PM = 1310 \sqrt{W'} \text{ lb}_f/\text{in}^2$$

when W' is measured in $\text{lb}_f/\text{in.}$

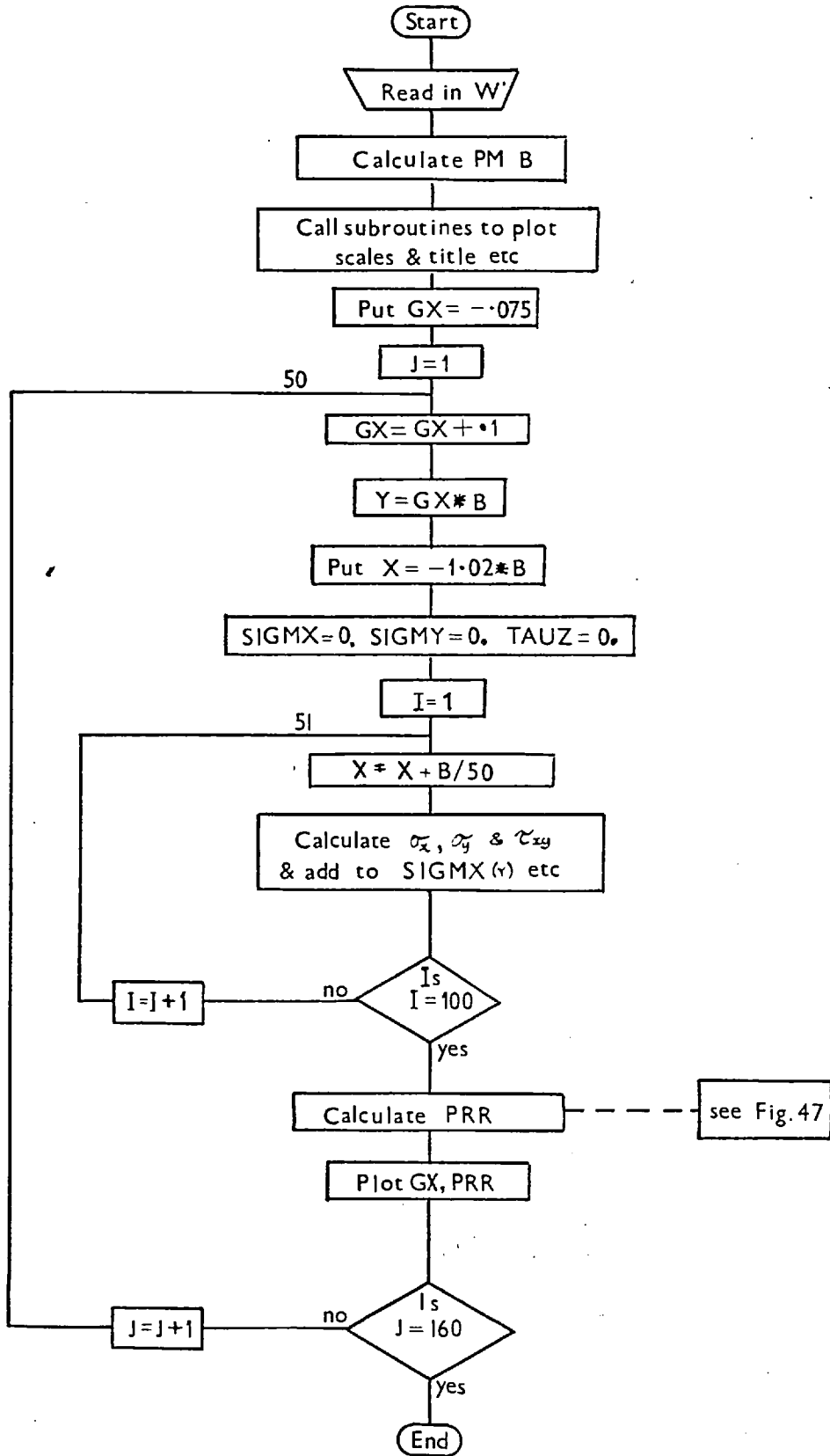
Notation and method of computation

The distribution of concentrated loads within the Hertz pressure curve, together with the positions in which the values of the principal stress difference have to be calculated, are shown in Figure 47.

From this Figure it can be seen that

$$P = p \, d(X/B) = PM \sqrt{1 - (X/B)^2} \cdot \frac{B}{50}$$

Flow chart 1



Programme 1

-114-

// JOB WH001

// FOR

*IOCS(PLOTTER,TYPEWRITER)

1130 IBM

*LIST SOURCE PROGRAM

West Ham College

** B KLEMZ. MECH DEPT.

C CALIBRATION CHART

C INSTRUCT OPERATOR TO SET PLOTTER AT CENTRE

WRITE(1,99)

PAUSE 3

99 FORMAT(1H,'PLEASE LOCATE PLOTTER IN CENTRAL POSITION')

PI =3.14159

TH=PI/2.0

W=388.

B=.00049*SQRT(W)

PM=1310*SQRT(W)

C----TO FIX SCALES AND ORIGIN

CALL SCALF(1.0,10.0,-1.0,0.4)

C----DRAW GRID

C----PLOT +X AXIS

CALL FGRID (0,0.0,0.0,1.0,16)

C----PLOT +Y AXIS

CALL FGRID(1,0.0,0.0,0.1,6)

C----TITLE

CALL FCHAR(6.0,0.7,0.2,0.2,0.0)

WRITE(7,200)

200 FORMAT(17H CALIBRATION CHART)

CALL FCHAR(6.8,0.67,0.1,0.1,0.0)

WRITE(7,201)

201 FORMAT(17H(NON-DIMENSIONAL))

C--TO TABULATE X SCALE

XN=-0.1

IX=0

DO 20 I=1,17

CALL FCHAR(XN,-0.03,0.2,0.2,0.0)

WRITE(7,202)IX

202 FORMAT(I2)

XN =XN+1.0

IX=IX+1

20 CONTINUE

CALL FCHAR(5.0,-0.09,0.2,0.2,0.0)

WRITE(7,203)

203 FORMAT(16H DEPTH RATIO Y/B)

CALL FCHAR(10.0,-0.09,0.15,0.15,0.0)

WRITE(7,204)

204 FORMAT(29HB=HALF HERTZIAN CONTACT WIDTH)

C--TO TABULATE Y SCALE

YN=0.0

YA=0.0

DO 21 I=1,7

CALL FCHAR(-0.4,YN,0.1,0.1,0.0)

WRITE(7,205)YAU

205 FORMAT(F3.1)

YN=YN+0.1

YA=YA+0.1

YAU=YAU+0.01

21 CONTINUE

CALL FCHAR(-0.7,0.1,0.1,0.1,TH)

WRITE(7,206)

206 FORMAT(50H PRINCIPAL STRESS DIFFERENCE/MAXIMUM HERTZ PRESSURE)

GX=-0.075

DO 50 J=1,160

```
GX=GX+0.1
Y=GX*B
X=-1.02*B
SIGMX=0.
SIGMY=0.
TAUZ=0.
DO 51 I=1,100
X=X+(B/50.0)
P=26.2*SQR((.00049*W)**2-X*X*W)
C=(-2.*Y*P)/(PI*((X**2+Y**2)**2))
SIGMX=SIGMX+C*(Y*Y)
SIGMY=SIGMY+C*(X*X)
TAUZ=TAUZ+C*(X*Y)
51 CONTINUE
PDIFF=SQR((SIGMX-SIGMY)**2+4.*TAUZ**2)
PRR=PDIFF/PM
CALL FPL0T(-2,GX,PRR)
50 CONTINUE
CALL EXIT
END
```

FEATURES SUPPORTED
IOCS

CORE REQUIREMENTS FOR
COMMON 0 VARIABLES 50 PROGRAM 616

END OF COMPILATION

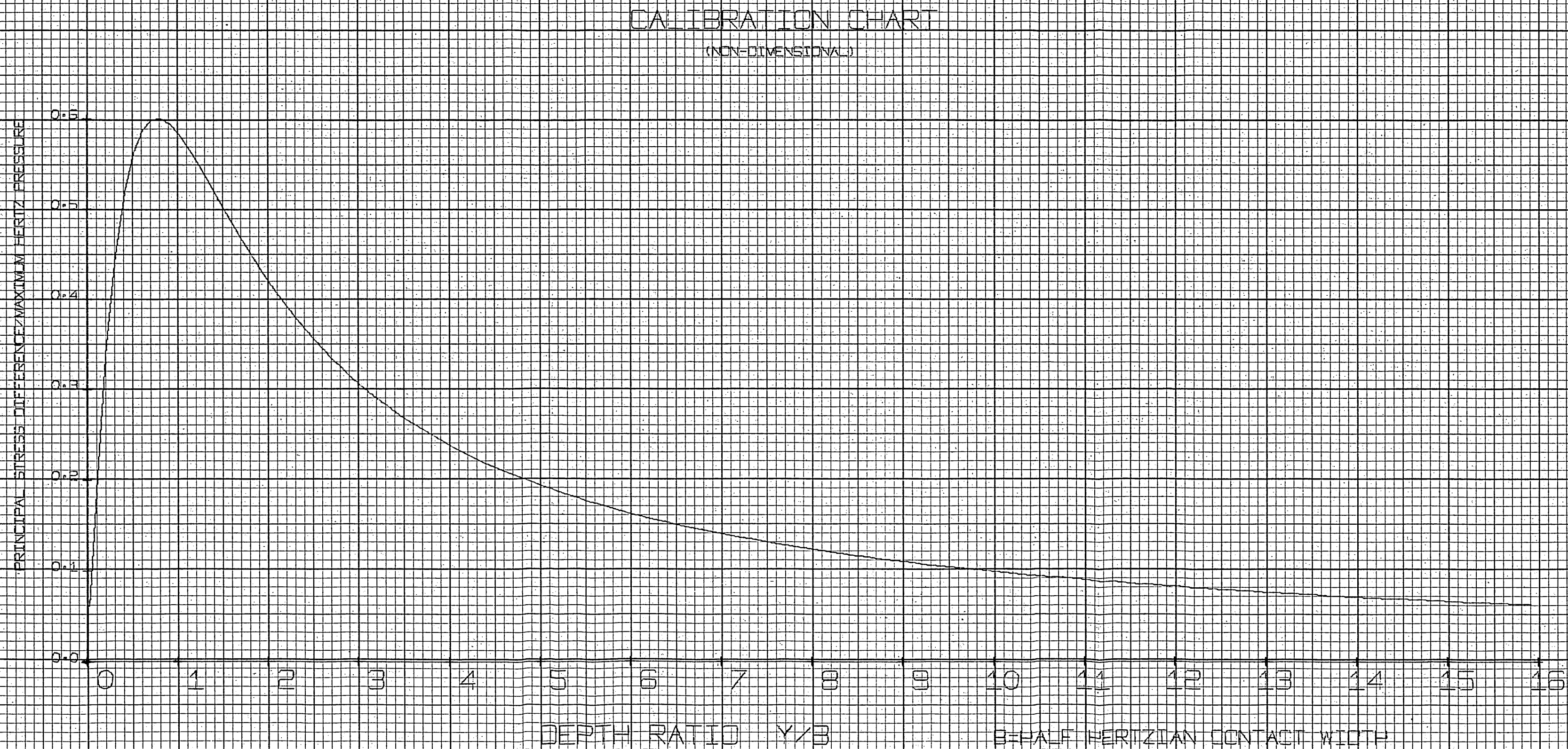


Fig. 48,

APPENDIX II

Contours - pressure curve based on concentrated loads

Data fed into the computer was magnitude and position of each load.

The programme plots the pressure curve and from this curve computes and then plots, the contours of principal stress difference/maximum Hertz pressure.

Provision was made in the programme to:

- (i) move the pressure curve either to the left or right of the line of centres,
- (ii) check and modify the area under the pressure curve to agree with the applied load.

Notation and method of computation

The distribution of concentrated loads within the pressure curve, together with the mesh points in which the values of the principal stress difference have to be calculated, are shown in Figure 49.

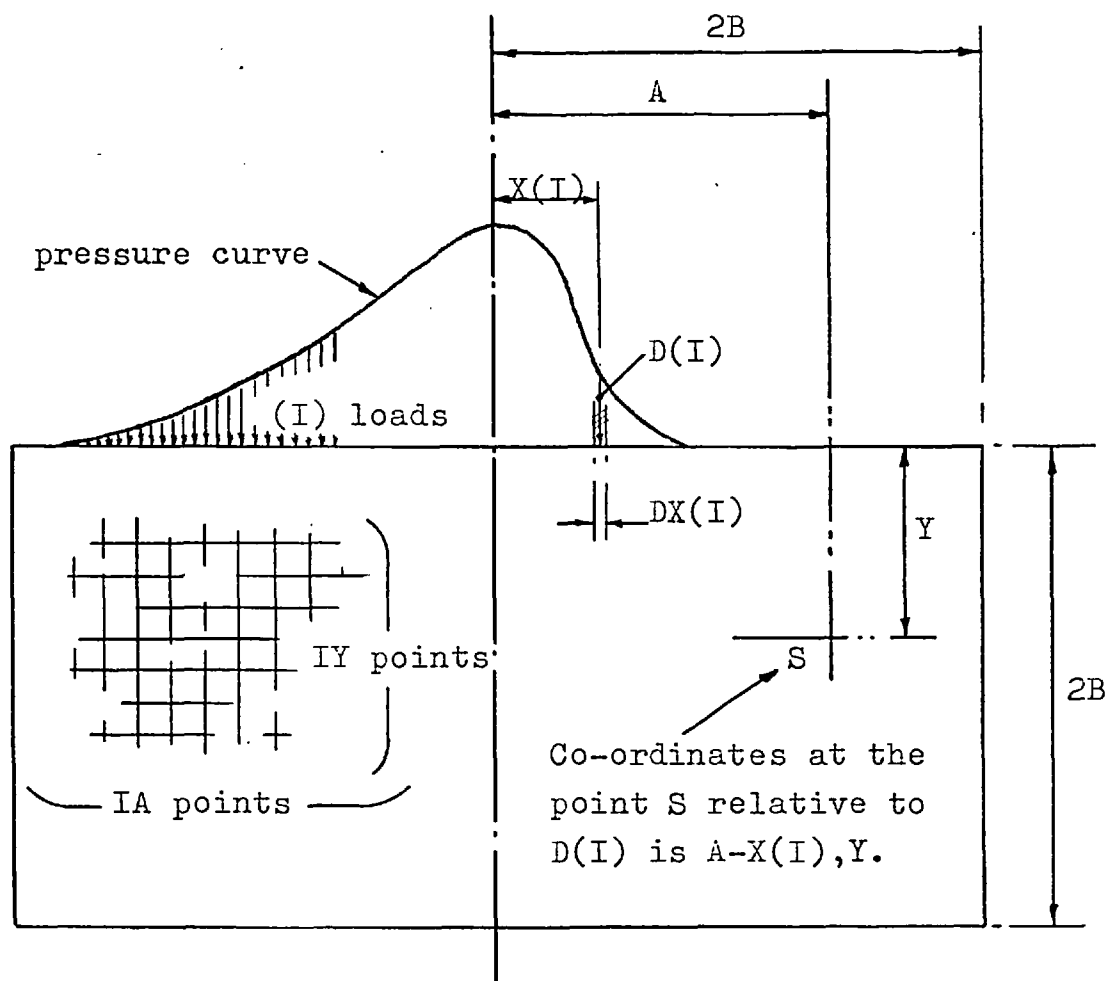


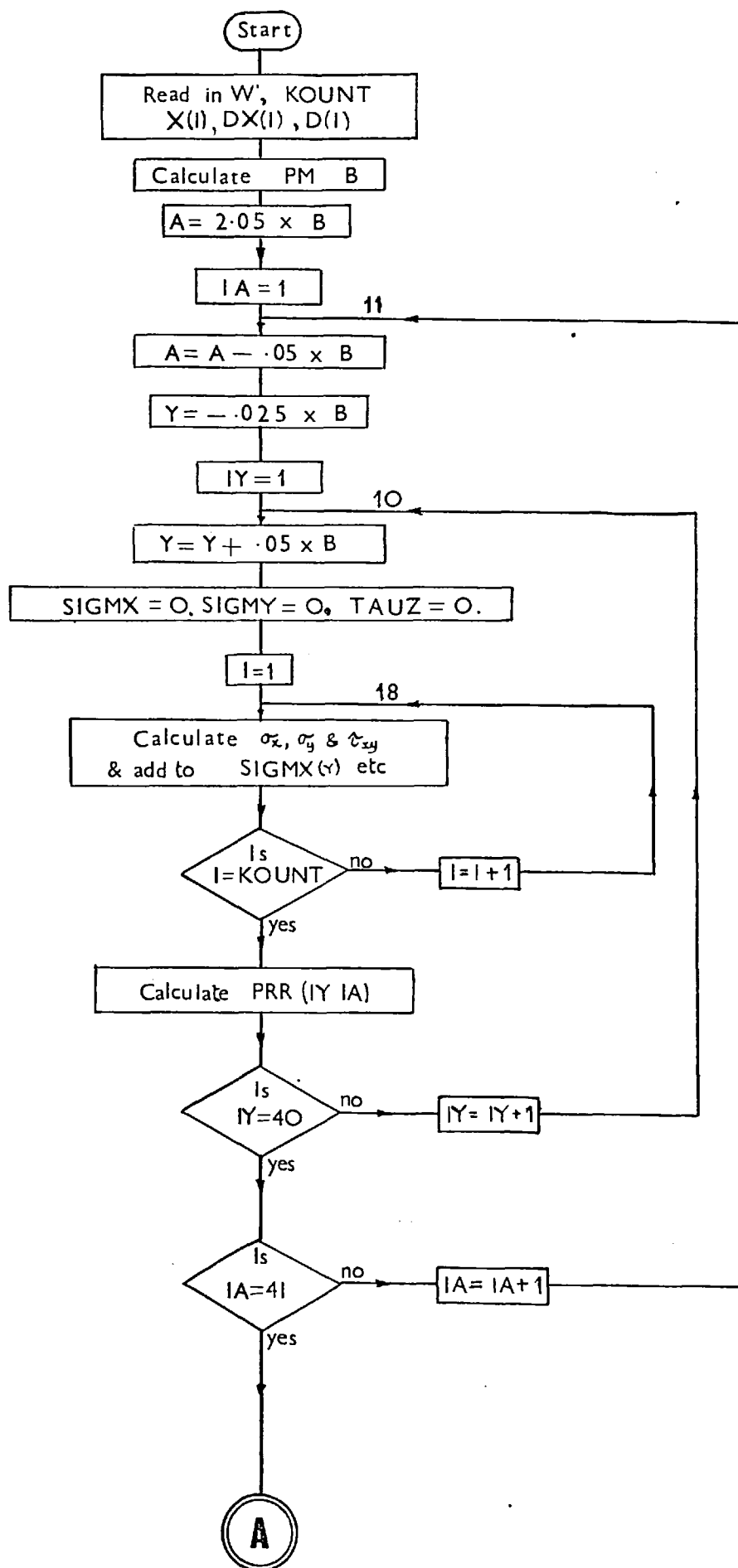
Fig. 49. Co-ordinate system for contours.

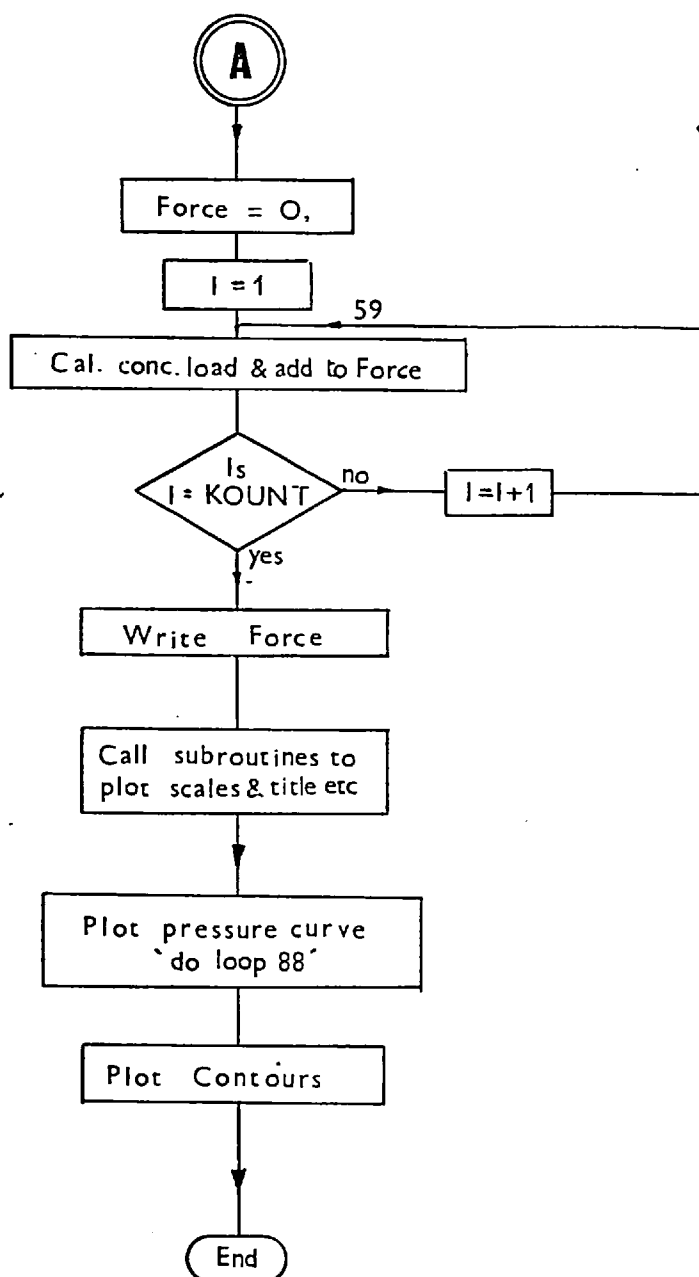
Mesh details.

Number of mesh points $40 \times 80 = 3,200$

Mesh constructed from 0.1 inch squares to give a contour plot 4 inches by 8 inches.

Flow chart 2





Programme 2

7094 IBM Imperial College

\$IBFTC BLK

BLK - EFN SOURCE STATEMENT - IFN(S) -

```

0DIMENSION X(500),DX(500),D(500),PRR(40,81),ISCAN(40),
1IXST(400),IYST(400)
KOUNT=306
FW=265.0
WW=265.0
AAAA=0.025
READ (5,28)(X(I),DX(I),D(I),I=1,KOUNT)
28 FORMAT(3(3F7.4,3X,))
PI=3.14159
W=265.0
B=.00049*SQRT(W)
PM=1310.*SQRT(W)
A=2.050*B
DO 11 IA=1,81
A=A-0.05*B
Y=-0.025*B
DO 10 IY=1,40
Y=Y+.05*B
SIGMX=0.
SIGMY=0.
TAUZ=0.
DO 18 I=1,KOUNT
P=D(I)*DX(I)*PM*B*FW/WW
C=(-2.*Y*P)/(PI*(((X(I)-AAAA)*B-A)**2+Y*Y)**2)
SIGMX=SIGMX+C*Y*Y
SIGMY=SIGMY+C*((X(I)-AAAA)*B-A)**2
TAUZ=TAUZ+C*((X(I)-AAAA)*B-A)*Y
18 CONTINUE
PDIFF=SQRT((SIGMX-SIGMY)**2+4.*TAUZ**2)
PRR(IY,IA)=PDIFF/PM
10 CONTINUE
11 CONTINUE
FORCE=0.0
DO 59 I=1,KOUNT
FORCE=FORCE+D(I)*PM*DX(I)*B*FW/WW
59 CONTINUE
WRITE(6,58)FORCE
58 FORMAT(10X,38HRAIAL FORCE BETWEEN DISCS,IN POUNDS =F6.1)
CALL START
DO 55 JJ=1,2
XD=5.0
YD=7.0
3 CALL PLOT(XD,YD,-3)
CALL AXIS(0.0,-4.0,3HX/B,3,8.0,90.0,-2.0,0.5)
CALL PLOT(0.0,-4.0,3)
CALL PLOT(4.0,-4.0,2)
CALL PLOT(4.0,4.0,2)
CALL AXIS(4.0,4.0,15HDEPTH RATIO Y/B,-15,3.0,180.0,2.0,-0.5)

```

B02202 10036170

DATE

ELK - EEN SOURCE STATEMENT - IFN(S) -

CALL PLOT(1.0,4.0,3)
 CALL PLOT(0.0,4.0,2)
 CALL PLOT(4.0,0.0,3)
 CALL PLOT(-4.75,0.0,2)

C--TO WRITE TITLES

CALL SYMBOL(-.75,1.5,0.1,10HCIL OUTLET,+90.0,10)
 CALL SYMBOL(-.75,-4.0,0.1,PHOTIL INLET,+90.0,9)
 CALL SYMBOL(-1.25,0.75,0.1,21HPRESSURE DISTRIBUTION,90.0,21)
 CALL SYMBOL(4.2,-1.5,0.1,32HC CONTOURS OF PRINCIPAL STRESS DIFFERENC
 1E,90.0,39)

C--PLOT PRESSURE CURVE

CALL PLOT(0.0,-4.0,3)
 CALL PLOT(0.0,-5.0,2)
 CALL PLOT(-0.45,-5.0,3)
 DO 88 I=75,KOUNT
 XA=-2.0*D(I)*FW/kk
 YA=(X(I)-AAAA)*2.0
 CALL PLOT(XA,YA,2)

88 CONTINUE

C--MOVE PLOT ORIGIN TO LOWER L H CORNER

CALL PLOT(0.05,-4.0,-3)
 XTEST=0.596
 CALL CONTRS(PRR,40,81,XTEST,3.95,8.,ISCAN,IXST,IYST,400,1)
 XTEST=0.525
 CALL CONTRS(PRR,40,81,XTEST,3.95,8.,ISCAN,IXST,IYST,400,1)
 XTEST=0.455
 CALL CONTRS(PRR,40,81,XTEST,3.95,8.,ISCAN,IXST,IYST,400,1)
 XTEST=0.381
 CALL CONTRS(PRR,40,81,XTEST,3.95,8.,ISCAN,IXST,IYST,400,1)
 XTEST=0.3
 CALL CONTRS(PRR,40,81,XTEST,3.95,8.,ISCAN,IXST,IYST,400,1)
 XTEST=0.210
 CALL CONTRS(PRR,40,81,XTEST,3.95,8.,ISCAN,IXST,IYST,400,1)
 XTEST=0.133
 CALL CONTRS(PRR,40,81,XTEST,3.95,8.,ISCAN,IXST,IYST,400,1)
 XTEST=0.054
 CALL CONTRS(PRR,40,81,XTEST,3.95,8.,ISCAN,IXST,IYST,400,1)
 CALL PLOT(-0.05,4.0,-3)
 IF(Y0-7.0)1,2,4

2 Y0=14.0

X0=0.0

GO TO 3

4 CALL PLOT(6.0,-20.5,-3)

55 CONTINUE

1 CALL ENPLOT(8.0)

STOP

END

APPENDIX III

Contours - pressure curve based on parabolic sections

Data fed into computer was position and magnitude of three ordinates for each parabola. The pressure curve is divided into sections of differing size, according to the shape of the curve. Each section is represented by a parabola with respect to an origin at the centre of the base of the section, as shown in Figure 50.

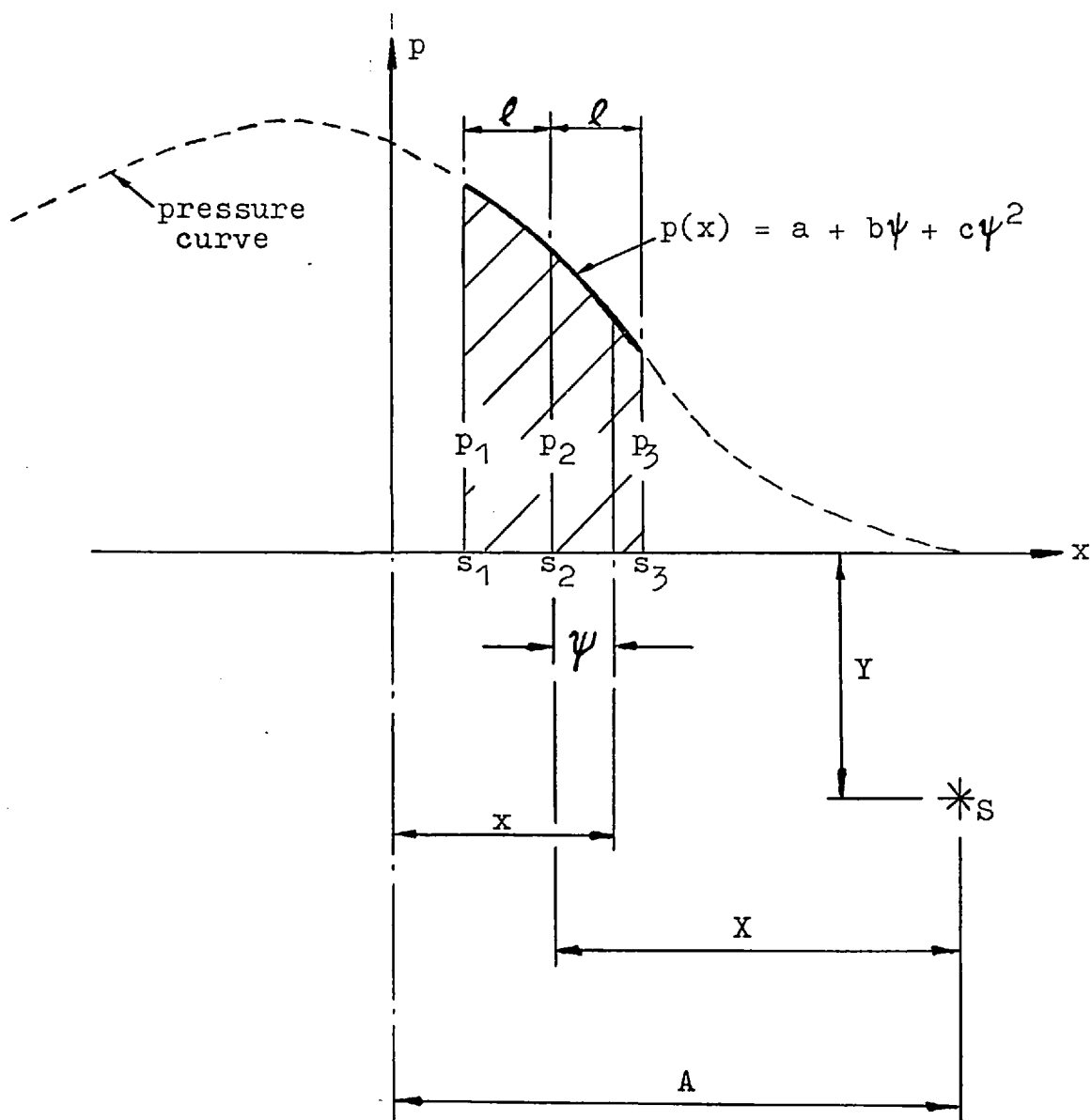


Fig. 50. Co-ordinate system for parabolas.

By modifying the equations shown on page 34 it is possible to show that the stresses at the point S (Figure 50) due to a pressure curve, is given by the following integrals.

$$\sigma_x = -\frac{2}{\pi} \int_{s_1}^{s_3} \frac{(A-x)^2 Y p(x)}{[(A-x)^2 + Y^2]^2} dx \quad \text{---(A3.1)}$$

$$\sigma_y = -\frac{2}{\pi} \int_{s_1}^{s_3} \frac{Y^3 p(x)}{[(A-x)^2 + Y^2]^2} dx \quad \text{---(A3.2)}$$

$$\tau_{xy} = -\frac{2}{\pi} \int_{s_1}^{s_3} \frac{(A-x) Y^2 p(x)}{[(A-x)^2 + Y^2]^2} dx \quad \text{---(A3.3)}$$

The general equation of the parabola is

$$p(x) = a + b\psi + c\psi^2$$

the boundary conditions required to calculate the coefficients a, b and c are:

$$p(x) = p_1 \quad \text{at} \quad \psi = -\ell$$

$$p(x) = p_2 \quad \text{at} \quad \psi = 0$$

and

$$p(x) = p_3 \quad \text{at} \quad \psi = +\ell$$

∴ $a = p_2$ in Fortran IV this becomes

$$SA = P(I+1)$$

$$b = \frac{p_3 - p_1}{2\ell} \quad \text{this becomes}$$

$$SB = (P(I+2) - P(I)) / (2 * SL(I))$$

and $c = \frac{p_3 + p_1 - 2p_2}{2\ell^2}$

i.e. $SC = (P(I+2) + P(I) - 2 * P(I+1)) / (2 * SL(I) ** 2)$

From Figure 49 it can be seen that $\psi = x - s_2$
and $d\psi = dx$ therefore equation (A3.1) becomes:

$$x = -\frac{2}{\pi} \int_{-\ell}^{+\ell} \frac{(A - s_2 - \psi)^2 (a + b\psi + c\psi^2)}{[(A - s_2 - \psi)^2 + Y^2]^2} d\psi \quad \text{--- (A3.4)}$$

but $X = A - s_2$

therefore, if $a + b\psi + c\psi^2 = \alpha + \beta(X - \psi) + \gamma(X - \psi)^2$

then $\alpha = a + bX + cX^2$ in Fortran IV this becomes

$$ALPHA = SA + SB * BX + SC * BX * BX$$

$$\beta = - (2cX + b) \quad \text{this becomes}$$

$$BETA = - (2.0 * SC * BX + SB)$$

and $\gamma = c$ i.e. $GAMA = SC$

∴ equation (A3.4) becomes

$$\sigma_x = -\frac{2}{\pi} \int_{-\ell}^{+\ell} \frac{[\alpha + \beta(X-\psi) + \gamma(X-\psi)^2] (X-\psi)^2 Y}{[(X-\psi)^2 + Y^2]^2} d\psi$$

The solution of this integral has been noted by Dowson, Higginson and Whitaker⁽¹⁷⁾ to be:

$$\sigma_x = \frac{1}{\pi} (\alpha I_2 + \beta Y I_3 + c Y^2 I_4)$$

$$\sigma_y = \frac{1}{\pi} (\alpha I_0 + \beta Y I_1 + c Y^2 I_2)$$

$$\tau_{xy} = \frac{1}{\pi} (\alpha I_1 + \beta Y I_2 + c Y^2 I_3)$$

$$\text{where } I_0 = \left[\tan^{-1} \frac{X-\psi}{Y} + \frac{(X-\psi) Y}{(X-\psi)^2 + Y^2} \right]_{-\ell}^{+\ell} \quad \dots \quad \text{Fortran BIO}$$

$$I_1 = \left[\frac{-Y^2}{(X-\psi)^2 + Y^2} \right]_{-\ell}^{+\ell} \quad \dots \quad \text{BI1}$$

$$I_2 = \left[\tan^{-1} \frac{X-\psi}{Y} - \frac{(X-\psi) Y}{(X-\psi)^2 + Y^2} \right]_{-\ell}^{+\ell} \quad \dots \quad \text{BI2}$$

$$I_3 = \left[\log((X-\psi)^2 + Y^2) + \frac{Y^2}{(X-\psi)^2 + Y^2} \right]_{-\ell}^{+\ell} \quad \dots \quad \text{BI3}$$

$$I_4 = -\frac{4\ell}{Y} + \left[\frac{(X-\psi) Y}{(X-\psi)^2 + Y^2} - 3 \tan^{-1} \frac{X-\psi}{Y} \right]_{-\ell}^{+\ell} \quad \dots \quad \text{BI4}$$

$$\text{let } C1 = \left[\tan^{-1} \frac{X-\psi}{Y} \right]_{-\ell}^{+\ell}$$

$$\text{i.e. } C1 = \text{ATAN}((BX-SL)/Y) - \text{ATAN}((BX+SL)/Y)$$

$$\text{let } C2 = \left[\frac{(X-\psi)}{(X-\psi)^2 + Y^2} \right]_{-\ell}^{+\ell}$$

$$\text{i.e. } C2 = (BX-SL)/((BX-SL)^2 + Y^2) - (BX+SL)/((BX+SL)^2 + Y^2)$$

$$\text{let } C3 = \left[\frac{Y^2}{(X-\psi)^2 + Y^2} \right]_{-\ell}^{+\ell}$$

$$\text{i.e. } C3 = Y^2/((BX-SL)^2 + Y^2) - Y^2/((BX+SL)^2 + Y^2)$$

$$\text{let } C4 = \left[\log((X-\psi)^2 + Y^2) \right]_{-\ell}^{+\ell}$$

$$\text{i.e. } C4 = \text{ALOG}(((BX-SL)^2 + Y^2)/((BX+SL)^2 + Y^2))$$

Therefore

$$BIO = C1 + C2 * Y$$

$$BI1 = -C3$$

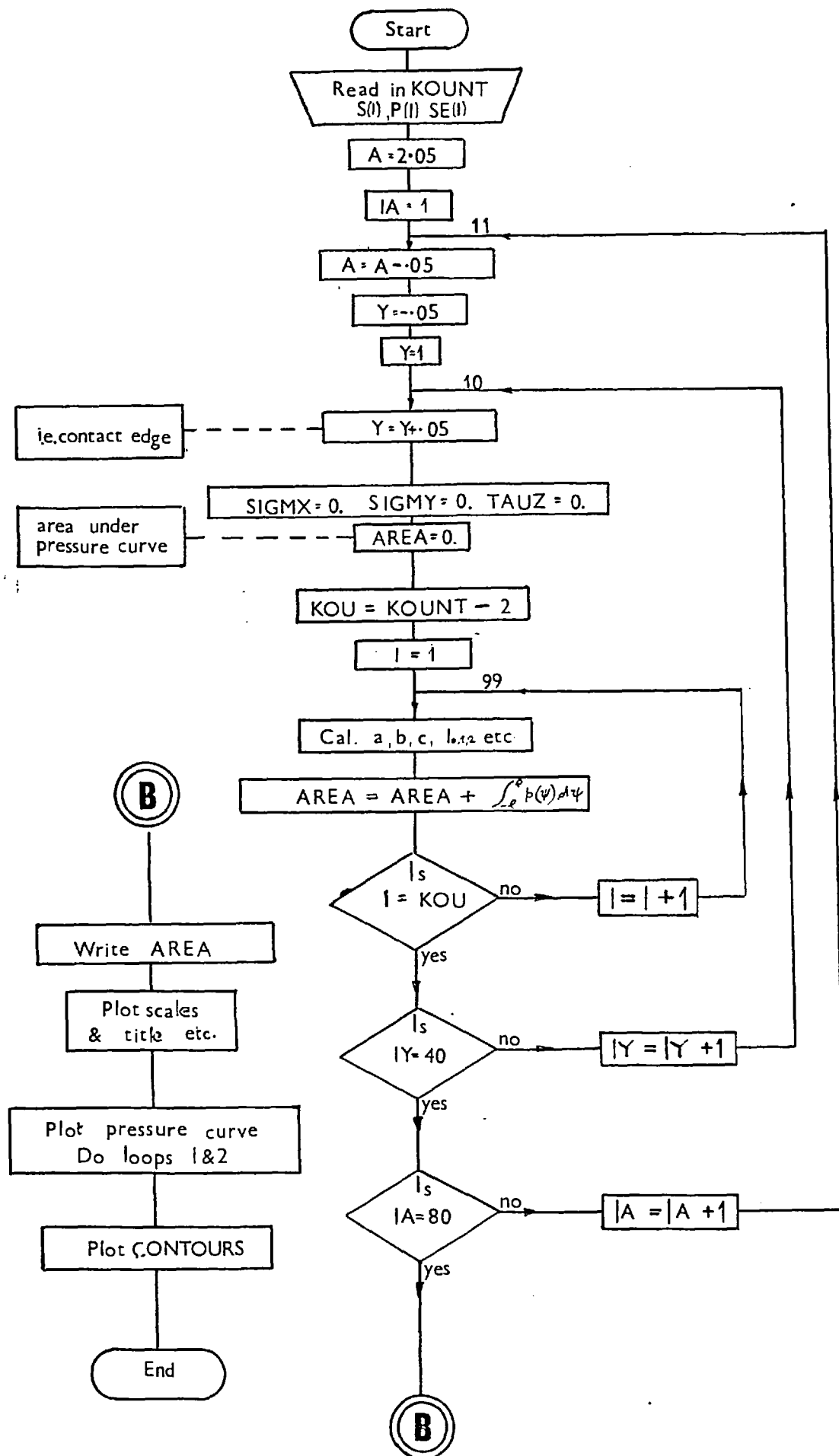
$$BI2 = C1 - C2 * Y$$

$$BI3 = C4 + C3$$

$$BI4 = (-4.0 * SL/Y) + C2 * Y - 3.0 * C1$$

It is therefore possible to write a programme which will compute σ_x , σ_y , and τ_{xy} at a point in a elastic medium due to a parabolic load distribution. By repeating the procedure for other parabolas a summation of σ_x , σ_y and τ_{xy} can be obtained. Hence the principal stress difference.

Flow chart 3



Programme 3

7094 IBM Imperial College

\$IBFTC BLK

BLK - EFN SOURCE STATEMENT - IFN(S) -

```

CDIMENSION S(25),P(25),SE(25),PRR(40,81),ISCAN(40),
IIXST(400),IYST(400)

```

C--TEST C18

KCUNT=23

LINES=1

READ(5,20) (S(I),P(I),SE(I),I=1,KCUNT)

WRITE(6,21) (S(I),P(I),SE(I),I=1,KCUNT)

20 FORMAT(3(3F6.3,2X))

21 FORMAT(4(10X,3F7.3))

PI=3.14159

A=2.05

DO 11 IA=1,81

A=A-0.05

Y=-0.05

DO 10 IY=1,40

Y=Y+0.05

SIGMX=0.0

SIGMY=0.0

TAUXY=0.0

AREA=0.0

KCU=KCUNT-2

DO 99 I=1,KCU,2

SL=SE(I)

SA=P(I+1)

SB=(P(I+2)-P(I))/(2.0*SL)

SC=(P(I+2)+P(I)-2.0*P(I+1))/(2.0*SL*SL)

BX=A-S(I+1)

ALPA=SA+SB*BX+SC*BX*BX

BETA=-(2.0*SC*BX+SB)

GAMA=SC

C1=ATAN((BX-SL)/Y)-ATAN((BX+SL)/Y)

C2=(BX-SL)/((BX-SL)**2+Y*Y)-(BX+SL)/((BX+SL)**2+Y*Y)

C3=Y*Y/((BX-SL)**2+Y*Y)-Y*Y/((BX+SL)**2+Y*Y)

C4=ALOG(((BX-SL)**2+Y*Y)/((BX+SL)**2+Y*Y))

BI0=C1+C2*Y

BI1=-C3

EI2=C1-C2*Y

EI3=C4+C3

BI4=(-4.0*SL/Y)+C2*Y-3.0*C1

SIGMX=SIGMX+(ALPA*BI2+BETA*Y*BI3+SC*Y*Y*BI4)/PI

SIGMY=SIGMY+(ALPA*BI0+BETA*Y*BI1+SC*Y*Y*BI2)/PI

TAUXY=TAUXY+(ALPA*BI1+BETA*Y*BI2+SC*Y*Y*BI3)/PI

AREA=AREA+2.0*SA*SL+(SC*SL**3)*2.0/3.0

99 CONTINUE

PRR(IY,IA)=SQRT((SIGMX-SIGMY)**2+4.0*TAUXY**2)

10 CONTINUE

11 CONTINUE

WRITE(6,22)AREA

22 FORMAT(1CX,5HAREA=E16.8)

CALL START

DO 55 JJ=1,LINES

XO=5.0

YO=7.0

3 CALL PLOT(XO,YO,-3)

CALL AXIS(0.0,-4.0,3HX/B,3,8.0,90.0,-2.0,0.5)

CALL PLOT(0.0,-4.0,3)

CALL PLCT(4.0,-4.0,2)

CALL PLCT(4.0,4.0,2)

CALL AXIS(4.0,4.0,11HDEPTH RATIO,-11,3.0,180.0,2.0,-0.5)

CALL PLOT(1.0,4.0,3)

CALL PLCT(0.0,4.0,2)

CALL PLOT(4.0,0.0,3)

CALL PLCT(-4.75,0.0,2)

C--TO WRITE TITLES

CALL SYMBOL(-.75,1.5,0.1,10HCIL CUTLET,+90.0,10)

CALL SYMBOL(-.75,-4.0,0.1,9HCIL INLET,+90.0,9)

CALL SYMBOL(-1.25,0.75,0.1,21HPRESSURE DISTRIBUTION,90.0,21)

 CCALL SYMBOL(4.2,-1.5,0.1,39HCCNTOURS OF PRINCIPAL STRESS DIFFERENC
 IE,90.0,39)

C--TO PLOT PRESSURE CURVE

CALL PLOT(0.0,-3.0,3)

DO 1 I=1,KOL,2

SL=SE(I)

SA=P(I+1)

SB=(P(I+2)-P(I))/(2.0*SL)

SC=(P(I+2)+P(I)-2.0*P(I+1))/(2.0*SL*SL)

X=S(I)

DO 2 K=1,40

TYA=X*2.0

TXA=-2.0*(SA+SB*(X-S(I+1))+SC*(X-S(I+1))**2)

CALL PLCT(TXA,TYA,2)

X=X+SL/10.0

IF(X.GT.S(I+2)) GO TO 1

2 CONTINUE

1 CONTINUE

C--MOVE PLOT ORIGIN TO LOWER L H CORNER

CALL PLCT(0.00,-4.0,-3)

XTEST=0.640

CALL CNTRIS(PRR,40,81,XTEST,4.00,8.,ISCAN,IXST,IYST,400,1)

XTEST=0.574

CALL CNTRIS(PRR,40,81,XTEST,4.00,8.,ISCAN,IXST,IYST,400,1)

XTEST=0.499

CALL CNTRIS(PRR,40,81,XTEST,4.00,8.,ISCAN,IXST,IYST,400,1)

XTEST=0.41

CALL CNTRIS(PRR,40,81,XTEST,4.00,8.,ISCAN,IXST,IYST,400,1)

XTEST=0.32

CALL CNTRIS(PRR,40,81,XTEST,4.00,8.,ISCAN,IXST,IYST,400,1)

XTEST=0.237

CALL CNTRIS(PRR,40,81,XTEST,4.00,8.,ISCAN,IXST,IYST,400,1)

XTEST=0.14

CALL CNTRIS(PRR,40,81,XTEST,4.00,8.,ISCAN,IXST,IYST,400,1)

CALL PLOT(0.0,4.0,-3)

IF(YO-7.0)4,5,6

5 YO=14.0

XO=0.0

GO TO 3

6 CALL PLOT(6.0,-21.0,-3)

55 CONTINUE

4 CALL ENPLCT(8.0)

STOP

END

APPENDIX IV

Solution of Simultaneous Quadratic Equations

It was shown in Chapter 3.05 that a pressure curve could be obtained from the isochromatics by solving a set of simultaneous quadratic equations. The set of equations to be solved are:

$$\left[\sum_{j=1}^n a_{ij} P_j \right]^2 + \sum_{j=1}^n b_{ij} P_j^2 = c_i$$

$$i = 1 \text{ to } n$$

where n is the number of equations

Method of solution.

$$\text{Let } f(P) = \left[\sum_{j=1}^n a_{ij} P_j \right]^2 + \sum_{j=1}^n b_{ij} P_j^2 - c_i = 0$$

Expanding to the linear term of Taylor's series gives

$$f(P) = f(P') + \left[\frac{\partial (f(P))}{\partial P_j} \right]_{P=P'} \epsilon_j = 0$$

where P' is the guessed value of P .

$$\therefore \left[\frac{\partial (f(P))}{\partial P_j} \right]_{P=P'} \epsilon_j = -f(P')$$

$$\begin{aligned} \text{i.e. } \left[2a_{ij} \sum a_{ij} P'_j + 2b_{ij} P'_j \right] \epsilon_j \\ = c_i - \left[\left[\sum a_{ij} P'_j \right]^2 + \sum b_{ij} P_j'^2 \right] \end{aligned}$$

$$\text{Let } A_{ij} = 2a_{ij} \sum a_{ij} P'_j + 2b_{ij} P'_j$$

$$\text{and } B_i = c_i - \left[\left[\sum a_{ij} P'_j \right]^2 + \sum b_{ij} P_j'^2 \right]$$

$$\text{i.e. } A_{ij} \epsilon_j = B_i$$

This represents a set of simultaneous linear equations. Solution of ϵ_j is obtained by the Gauss-Jordan method. The new value of P_j becomes $P_j'' = P'_j + \epsilon_j$.

Procedure is repeated until ϵ_j is considered sufficiently small.

Fortran symbols:

AA(IJ)	=	A_{ij}
BB(I)	=	B_i
BB(J)	=	ϵ_j
P(J)	=	P_j
DX	=	distance between P(J) loads
DXX	=	distance between experimental stress points
YY	=	distance of stress points from contact surface

L = n (the number of equations)
K = number of iterations
PD(I) = experimental stress values

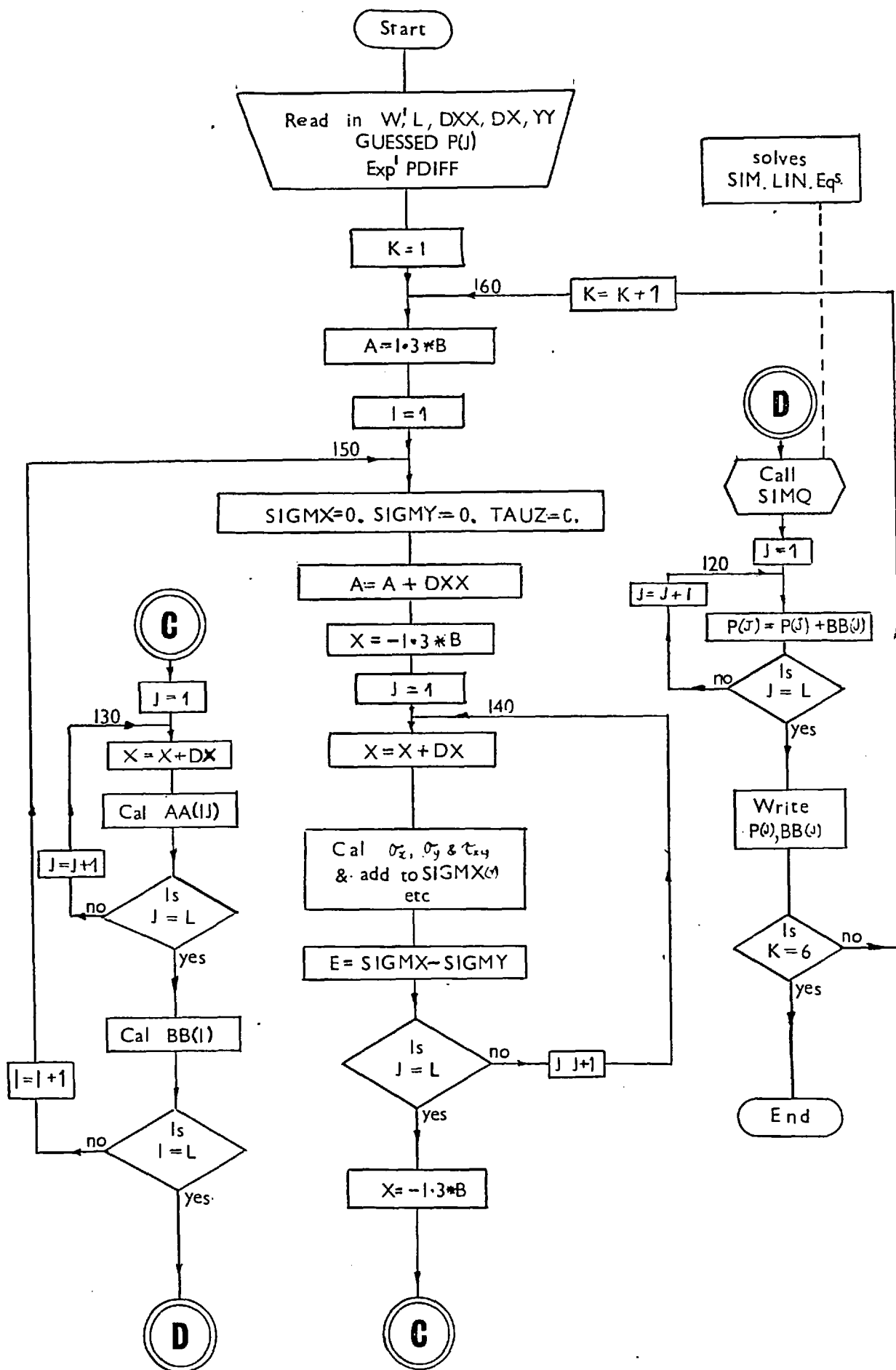
See Flow Chart 4a and Programme 4a.

Programme 4a check.

A set of experimental values of the principal stress difference were required in order to test Programme 4a. However, the use of experimental values was not suitable at this stage, since an accurate knowledge of the pressure curve producing the values would be unknown. It was therefore decided to compute 'experimental values' from a known pressure curve.

Hence Programme 4b

In this programme twelve 'experimental values' were calculated from twelve concentrated loads. The loads were distributed in order to approximate to a Hertzian curve as shown in Figure 51.



```

C---SOLUTION OF 12 SIMULTANEOUS QUADRATIC EQUATIONS
      DIMENSION AA(12,12), BB(12), P(12), PD(12), AAA(12,12)
      WRITE(6,22)
220 FORMAT(10X,52H'EXPERIMENTAL' VALUES OF PRINCIPAL STRESS DIFFE
1)                                     -ENCE
      W=400.
      B=.00049*SQRT(W)
      DXX=.2*B
      DX=.2*B
      YY=.05*B
      L=12
      PI=3.14159
      READ (5,27)(P(J),J=1,L)
27 FORMAT(12F6.2)
      READ (5,25)(PD(I),I=1,L)
25 FORMAT(6F8.2)
      WRITE (6,26)(PD(I),I=1,L)
26 FORMAT(2X,6F10.2,/20X,6F10.2)
      WRITE(6,21)
21 FORMAT(10X,30HINITIAL GUESSED VALUES OF P(J))
      WRITE(6,24)(P(J),J=1,L)
      DO 160 K=1,6
      A=-1.3*B
      DO 150 I=1,L
      SIGMX=0.
      SIGMY=0.
      TAUZ=0.
      A=A+DXX
      X=-1.3*B
      DO 140 J=1,L
C   THIS CALS. E FOR P J
      X=X+DX
      C=(-2.*YY*P(J))/(PI*((A-X)**2+YY*YY)**2)
      SIGMX=SIGMX+C*(A-X)*(A-X)
      SIGMY=SIGMY+C*YY*YY
      TAUZ=TAUZ+C*(A-X)*YY
      E=SIGMX-SIGMY
140 CONTINUE
      X=-1.3*B
      DO 130 J=1,L
C   TO CAL THE PARIAL DIFF TERMS
      X=X+DX
      CD=-2./(PI*((A-X)*(A-X)+YY*YY)**2)
      GAA(I,J)=2.*CD*((A-X)*(A-X)*YY-YY**3)*E+8.*P(J)
      1*((CD*(A-X))**2)*YY**4
130 CONTINUE
C   CAL OF BB SPD-SEPD
      BB(I)=(PD(I)**2)-(E**2)-(4.*(TAUZ**2))
150 CONTINUE
      DO 170 I=1,L
      DO 170 J=1,L
170 AAA(J,I)=AA(I,J)
      WRITE (6,30)
30 FORMAT(10X,20HSOLUTION OF SIMGE IS,)
      CALL SIMQ(AAA,BB,L,JU)
      IF(JU)1,2,1
1   WRITE (6,31)
2   WRITE (6,32)(BB(J),J=1,L)
32 FORMAT(6(2X,E16.8,))

```

```

31 FORMAT(10X,11HNO SOLUTION,)
   DO 120 J=1,L
   P(J)=P(J)+BB(J)
   IF(P(J))12,12,120
12  P(J)=0.0
120 CONTINUE
   WRITE(6,20)
20  FORMAT(10X,18HNEW VALUES OF P(J))
   WRITE (6,24)(P(J),J=1,L)
24  FORMAT(6(2X,E16.8),/10X,6(2X,E16.8),/)
160 CONTINUE
   STOP
   END

```

```

SUBROUTINE SIMQ(A,B,N,KS)
DIMENSION A(144),B(12)

```

C
C
C

```

FORWARD SOLUTION

```

```

TOL=0.0
KS=0
JJ=-N
DO 65 J=1,N
JY=J+1
JJ=JJ+N+1
BIGA=0
IT=JJ-J
DO 30 I=J,N

```

C
C
C

```

SEARCH FOR MAXIMUM COEFFICIENT IN COLUMN

```

```

IJ=IT+1
IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
20 BIGA=A(IJ)
IMAX=I
30 CONTINUE

```

C
C
C

```

TEST FOR PIVOT LESS THAN TOLERANCE (SINGULAR MATRIX)

```

```

IF(ABS(BIGA)-TOL) 35,35,40
35 KS=1
RETURN

```

C
C
C

```

INTERCHANGE ROWS IF NECESSARY

```

```

40 II=J+N*(J-2)
IT=IMAX-J
DO 50 K=J,N
I1=II+N
I2=I1+IT
SAVE=A(I1)
A(I1)=A(I2)
A(I2)=SAVE

```

C
C
C

```

DIVIDE EQUATION BY LEADING COEFFICIENT

```

```

50 A(I1)=A(I1)/BIGA
SAVE=B(IMAX)

```

B(IMAX)=B(J)
B(J)=SAVE/BIGA

C
C
C

ELIMINATE NEXT VARIABLE

IF(J-N) 55,70,55
55 IQS=N*(J-1)
DO 65 IX=JY,N
IXJ=IQS+IX
IT=J-IX
DO 60 JX=JY,N
IXJX=N*(JX-1)+IX
JJX=IXJX+IT
60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
65 B(IX)=B(IX)-(B(J)*A(IXJ))

C
C
C

BACK SOLUTION

70 NY=N-1
IT=N*N
DO 80 J=1,NY
IA=IT-J
IB=N-J
IC=N
DO 80 K=1,J
B(1B)=B(IB)-A(IA)*B(IC)
IA=IA-N
80 IC=IC-1
RETURN
END

0

EXECUTION

'EXPERIMENTAL' VALUES OF PRINCIPAL STRESS DIFFERENCE

3286.35 21769.00 37802.14 50520.38 54466.60 57894.61
 57894.61 54466.60 50520.38 37802.14 21769.00 3286.35

INITIAL GUESSED VALUES OF P(J)

0.30000000E 02 0.30000000E 02 0.30000000E 02 0.30000000E 02 0.30000000E 02 0.30000000E 02
 0.30000000E 02 0.30000000E 02 0.30000000E 02 0.30000000E 02 0.30000000E 02 -0.

SOLUTION OF SIMQ IS

-0.13529630E 02 -0.73082966E 01 0.50487256E 01 0.19368161E 02 0.25428048E 02 0.30406721E 02
 0.30363808E 02 0.25296684E 02 0.19117817E 02 0.46962469E 01 -0.77426587E 01 -0.96065071E-01

NEW VALUES OF P(J)

0.16470369E 02 0.22691703E 02 0.35048725E 02 0.49368162E 02 0.55428047E 02 0.60406721E 02
 0.60363808E 02 0.55296684E 02 0.49117816E 02 0.34696246E 02 0.22257341E 02 0.

SOLUTION OF SIMQ IS

-0.72149490E 01 -0.22928070E 01 -0.97947764E 00 -0.41047435E 01 -0.59866317E 01 -0.76981138E 01
 -0.76738934E 01 -0.59138932E 01 -0.39724196E 01 -0.85110035E 00 -0.24344634E 01 0.30047763E 01

NEW VALUES OF P(J)

0.92554203E 01 0.20398896E 02 0.34069248E 02 0.45263417E 02 0.49441416E 02 0.52708607E 02
 0.52689914E 02 0.49382790E 02 0.45145397E 02 0.33845145E 02 0.19822878E 02 0.30047763E 01

SOLUTION OF SIMQ IS

-0.30163372E 01 0.35407340E-01 0.78707150E-01 -0.18822291E 00 -0.39243029E 00 -0.66594630E 00
 -0.65384145E 00 -0.35558598E 00 -0.11725955E 00 0.20333540E 00 0.34937436E 00 0.47322825E 01

NEW VALUES OF P(J)

0.62390830E 01 0.20434303E 02 0.34147954E 02 0.45075194E 02 0.49048986E 02 0.52042660E 02
 0.52036073E 02 0.49027205E 02 0.45028137E 02 0.34048481E 02 0.20172252E 02 0.77370588E 01

SOLUTION OF SIMQ IS

-0.11723278E 01 -0.99639448E-01 -0.25719715E-01 -0.95030410E-02 -0.38404429E-02 -0.45013002E-02
 0.28157945E-02 0.20545859E-01 0.43042114E-01 0.89087614E-01 0.23011906E 00 -0.20713965E 01

NEW VALUES OF P(J)

0.50667552E 01 0.20334663E 02 0.34122235E 02 0.45065691E 02 0.49045146E 02 0.52038159E 02
 0.52038869E 02 0.49047749E 02 0.45071179E 02 0.34137568E 02 0.20402370E 02 0.56656623E 01

SOLUTION OF SIMQ IS

-0.25352053E 00 -0.54967627E-01 -0.22053504E-01 -0.12125096E-01 -0.84719004E-02 -0.72157821E-02
 -0.73293549E-02 -0.89890161E-02 -0.13007886E-01 -0.27122814E-01 -0.95382296E-01 -0.75918503E 00

NEW VALUES OF P(J)

0.48132347E 01 0.20279696E 02 0.34100181E 02 0.45053566E 02 0.49036673E 02 0.52030943E 02
 0.52031559E 02 0.49033760E 02 0.45059171E 02 0.34110446E 02 0.20306988E 02 0.49064772E 01

SOLUTION OF SIMQ IS

0.46240101E-02 -0.11717720E-01 -0.57927079E-02 -0.37073638E-02 -0.28763742E-02 -0.28420763E-02
 -0.33255361E-02 -0.45245551E-02 -0.73394195E-02 -0.14111336E-01 -0.34990183E-01 -0.95585232E-01

NEW VALUES OF P(J)

0.48178586E 01 0.20267978E 02 0.34094388E 02 0.45049858E 02 0.49033797E 02 0.52028101E 02
 0.52028233E 02 0.49034236E 02 0.45050831E 02 0.34096333E 02 0.20271998E 02 0.48108919E 01

See Fig. 52.

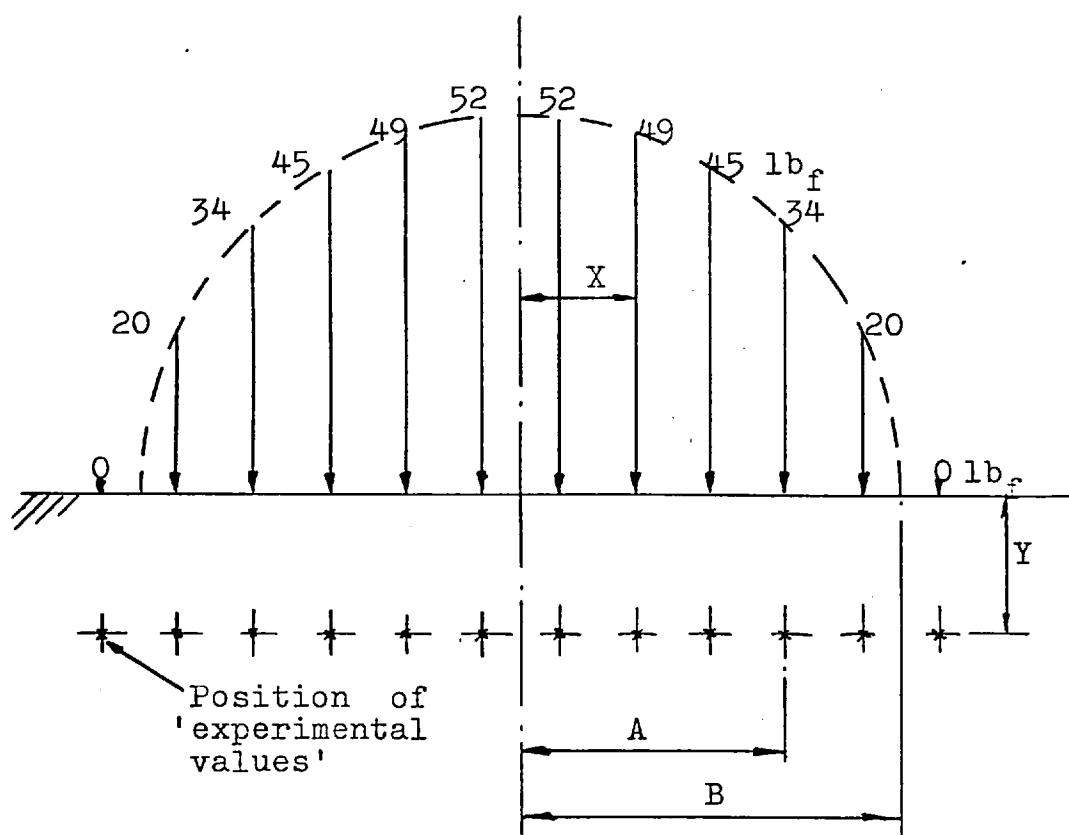
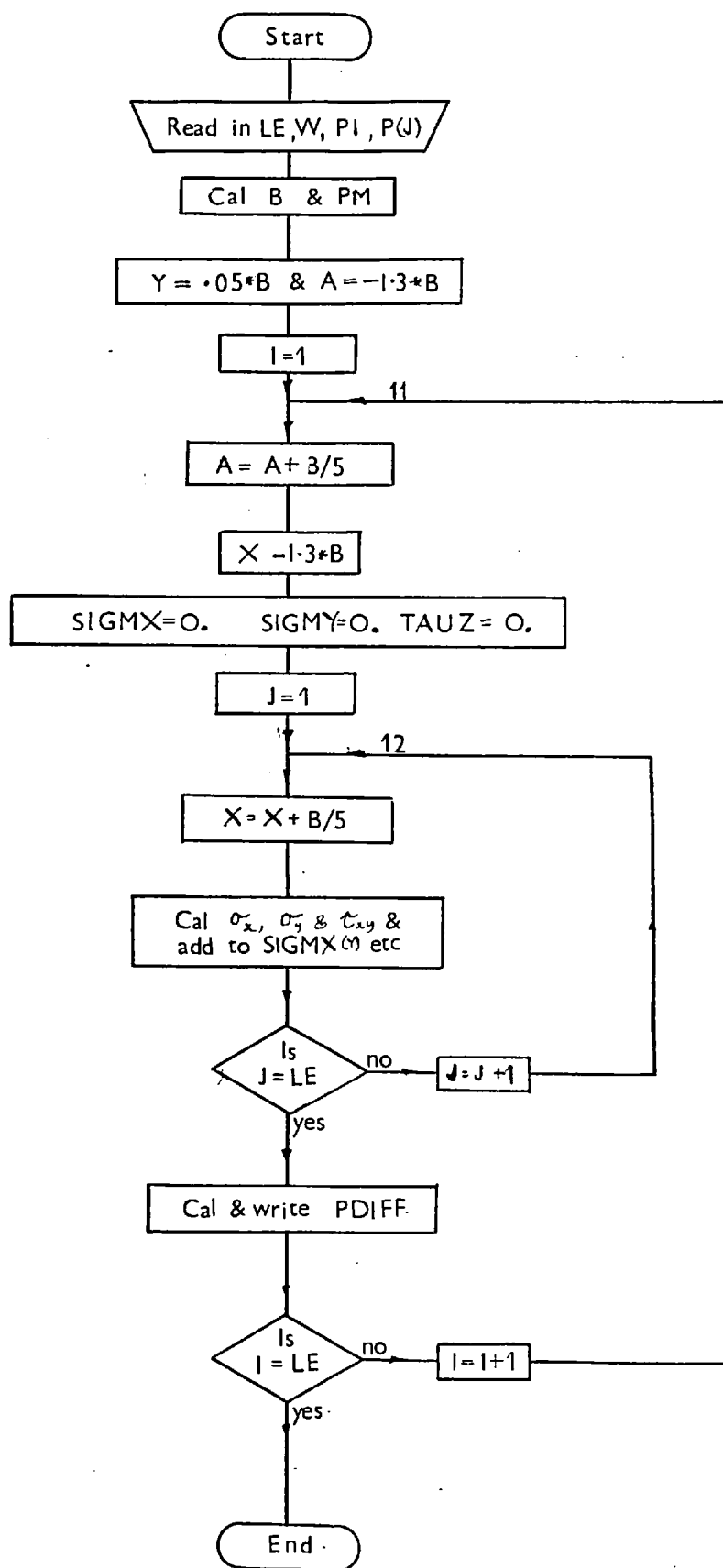


Fig. 51. Co-ordinate system for concentrated loads.

The values of the principal stress difference (page 142) computed from Programme 4b was used to check Programme 4a. The guessed pressure curve was assumed to be uniform and approximately half its known maximum value. This curve converged to the correct shape within three iterations as shown in Figure 52.

Flow chart 4b



C---CALCULATION OF PDIFF FOR 12 CONCENTRATED LOADS.

```

      DIMENSION P(12)
      WRITE(6,102)
      LE=12
      W=400.
      PI =3.14159
      B=.00049*SQRT(W)

      PM=1310.*SQRT(W)
      READ (5,100)(P(J),J=1,LE)
      WRITE (6,101)(P(J),J=1,LE)
      WRITE(6,105)
      WRITE(6,106)
      WRITE(6,107)
      Y=C.05*B
      A=-1.3*B
      WRITE(6,103)
      DO 11 I=1,12
      A=A+B/5.
      X=-1.3*B
      SIGMX=0.
      SIGMY=0.
      TAUZ=0.
      DO 12 J=1,LE
      X=X+.2*B
      C=(-2.*Y*P(J))/(PI*((X-A)**2+Y*Y)**2)
      SIGMX=SIGMX+C*Y*Y
      SIGMY=SIGMY+C*(X-A)*(X-A)
      TAUZ=TAUZ+C*(X-A)*Y
12    CONTINUE
      PDIFF=SQRT((SIGMX-SIGMY)**2+4.*TAUZ**2)
      E=A/B
      WRITE(6,104)E,PDIFF
11    CONTINUE
100  FORMAT(12F6.2)
101  FORMAT(10X,6(2X,F6.2),/)
102  FORMAT(10X,29HDATA--CONCENTRATED LOADS P(J))
103  FORMAT(23X,3H4/B,21X,5HPDIFF)
104  FORMAT(17X,F10.3,15X,F13.2)
105  FORMAT(10X,11HCALCULATION)
106  FORMAT(20X,30H4/B- SPAN RATIO ACROSS CONTACT)
107  FORMAT(20X,34HPDIFF- PRINCIPAL STRESS DIFFERENCE,/)
      STOP
      END

```

0

EXECUTION

DATA--CONCENTRATED LOADS P(J)

0. 20.00 34.00 45.00 49.00 52.00

52.00 49.00 45.00 34.00 20.00 0.

CALCULATION

A/B- SPAN RATIO ACROSS CONTACT

PDIFF- PRINCIPAL STRESS DIFFERENCE

A/B	PDIFF
-1.100	3286.35
-0.900	21769.00
-0.700	37802.14
-0.500	50520.38
-0.300	54466.60
-0.100	57894.61
0.100	57894.61
0.300	54466.60
0.500	50520.38
0.700	37802.14
0.900	21769.00
1.100	3286.35

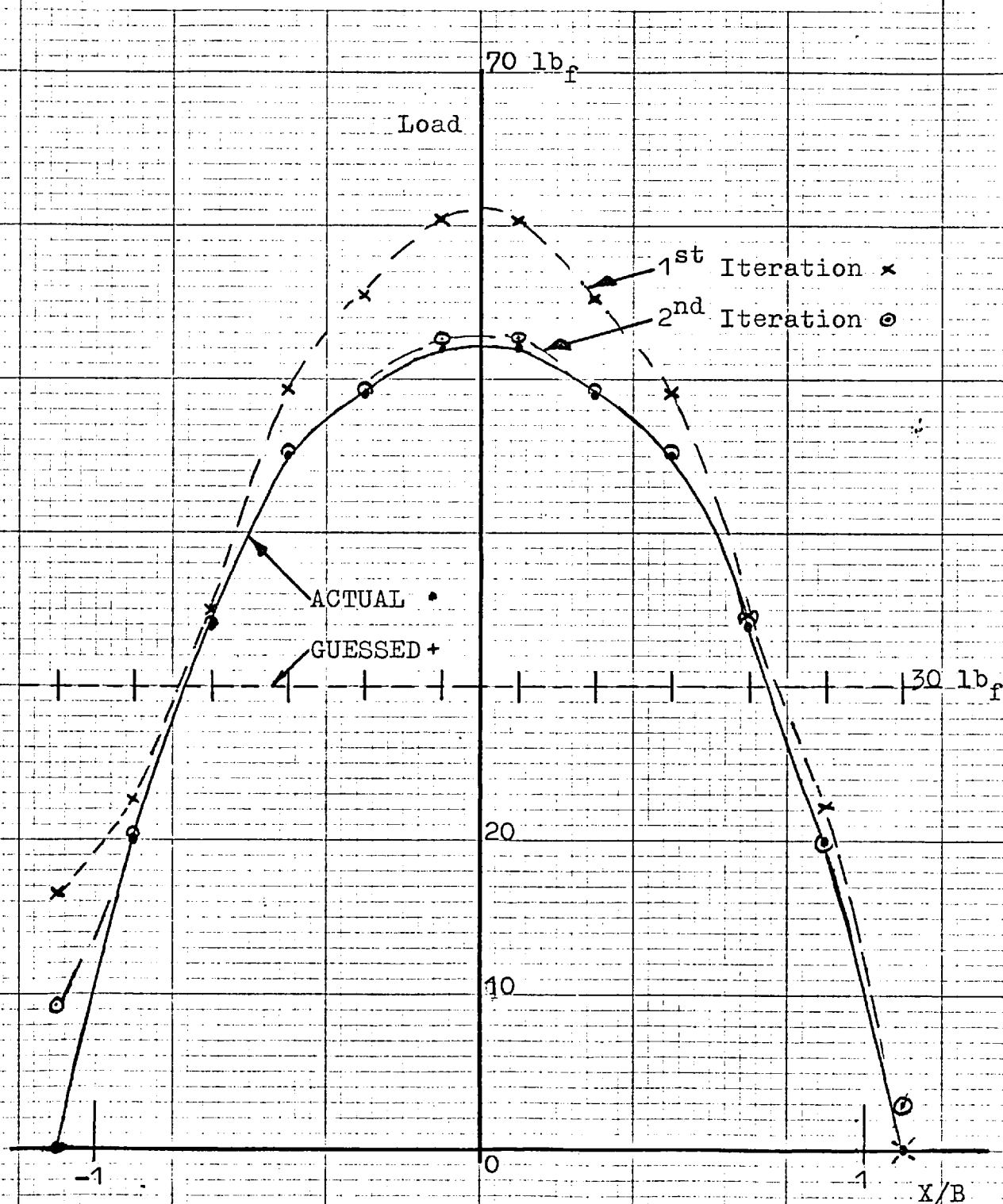
0

EXECUTION TERMINATED BY STOP --- AT OCTAL 60537

Solution of Twelve Simultaneous Equations

$$\left[\sum_{j=1}^{12} a_{ij} P_j \right]^2 + \sum_{j=1}^{12} b_{ij} P_j^2 = c_i$$

$$i = 1 \text{ to } 12$$



For computations see page 138.

Fig. 52. Iteration of Loads.

0

EXECUTION

'EXPERIMENTAL' VALUES OF PRINCIPAL STRESS DIFFERENCE

5100.00	10600.00	18550.00	20100.00	8650.00	3	300.00	1850.00
INITIAL GUESSED VALUES OF P(J)							
30.00	30.00	30.00	30.00	30.00		30.00	30.00
SOLUTION OF SIMQE IS							
-0.42159767E 01	0.14790261E 02	0.55583596E 02		0.66257018E 02		0.10732574E 02	-0.37905351E 01
NEW VALUES OF P(J)							
25.78	44.79	85.58	96.26	40.73		26.21	20.34
SOLUTION OF SIMQE IS							
-0.15615270E 02	-0.10961717E 02	-0.19106717E 02		-0.23564737E 02		-0.16521679E 02	-0.19855244E 02
NEW VALUES OF P(J)							
10.17	33.83	66.48	72.69	24.21		6.35	4.67
SOLUTION OF SIMQE IS							
0.48052413E 02	-0.74773586E 00	-0.63881426E 01		-0.93190240E 01		0.16140045E 01	0.93672755E 01
NEW VALUES OF P(J)							
58.22	33.08	60.69	63.37	25.82		15.72	31.44
SOLUTION OF SIMQE IS							
-0.23145463E 02	0.98904122E 01	0.19851391E 01		0.11970701E 01		0.12940650E 02	0.10105762E 01
NEW VALUES OF P(J)							
35.08	42.97	62.07	64.57	38.77		16.73	19.13
SOLUTION OF SIMQE IS							
-0.10449027E 02	-0.11528311E 01	0.15050117E 01		0.20632212E 01		-0.31195319E 01	0.23595603E 00
NEW VALUES OF P(J)							
24.63	41.32	63.58	66.63	35.65		16.97	12.99
SOLUTION OF SIMQE IS							
-0.46254290E 01	-0.16268538E 01	-0.76850258E 00		-0.78339265E 00		-0.12789179E 01	-0.12423278E 01
NEW VALUES OF P(J)							
20.00	40.19	62.81	65.85	34.37		15.73	9.35
SOLUTION OF SIMQE IS							
-0.80011791E 00	-0.82806874E 00	-0.56580105E 00		-0.52455871E 00		-0.48166084E 00	-0.31058819E 00
NEW VALUES OF P(J)							
19.20	39.37	62.24	65.33	33.89		15.42	7.78
SOLUTION OF SIMQE IS							
0.33155195E 00	-0.12188478E 00	-0.19046506E 00		-0.18400221E 00		-0.64867946E -01	-0.38824067E -01
NEW VALUES OF P(J)							
19.53	39.25	62.05	65.14	33.82		15.38	8.19
SOLUTION OF SIMQE IS							
0.13728914E 00	0.10144221E 00	0.11310426E -01		0.60092150E -02		0.82376761E -01	0.38550158E 00
NEW VALUES OF P(J)							
19.67	39.35	62.07	65.15	33.90		15.76	8.63
SOLUTION OF SIMQE IS							
0.30537879E -02	0.46541028E -01	0.43864941E -01		0.53785184E -01		0.12754691E 00	0.19622726E 00
NEW VALUES OF P(J)							
19.67	39.40	62.11	65.20	34.03		15.96	8.51
SOLUTION OF SIMQE IS							
-0.26139408E -01	-0.14579365E -02	0.14023130E -01		0.25531714E -01		0.26785488E -01	-0.11105823E 00
NEW VALUES OF P(J)							
19.65	39.40	62.12	65.23	34.06		15.85	8.28
SOLUTION OF SIMQE IS							
-0.17859418E -01	-0.14981547E -01	-0.90712892E -02		-0.10801777E -01		-0.48117929E -01	-0.13856658E 00
NEW VALUES OF P(J)							
19.63	39.38	62.11	65.22	34.01		15.71	8.20
SOLUTION OF SIMQE IS							
-0.51993845E -03	-0.81671686E -02	-0.10764916E -01		-0.17568215E -01		-0.37211124E -01	-0.31085890E -01
NEW VALUES OF P(J)							
19.63	39.37	62.10	65.20	33.97		15.68	8.28
SOLUTION OF SIMQE IS							
0.80233523E -02	0.20497935E -02	-0.16750457E -02		-0.42881720E -02		0.15993453E -02	0.57134849E -01
NEW VALUES OF P(J)							
19.64	39.38	62.10	65.19	33.97		15.73	8.35
SOLUTION OF SIMQE IS							
0.48188309E -02	0.49920914E -02	0.41768980E -02		0.60542026E -02		0.19852244E -01	0.41608355E -01
NEW VALUES OF P(J)							
19.64	39.38	62.11	65.20	33.99		15.78	8.35

See Fig. 18(b).

APPENDIX V

Elastic Deflexion

The deflexion diagram shown in Figure 12 has been modified to incorporate the use of the parabolas, as shown in Figure 53.

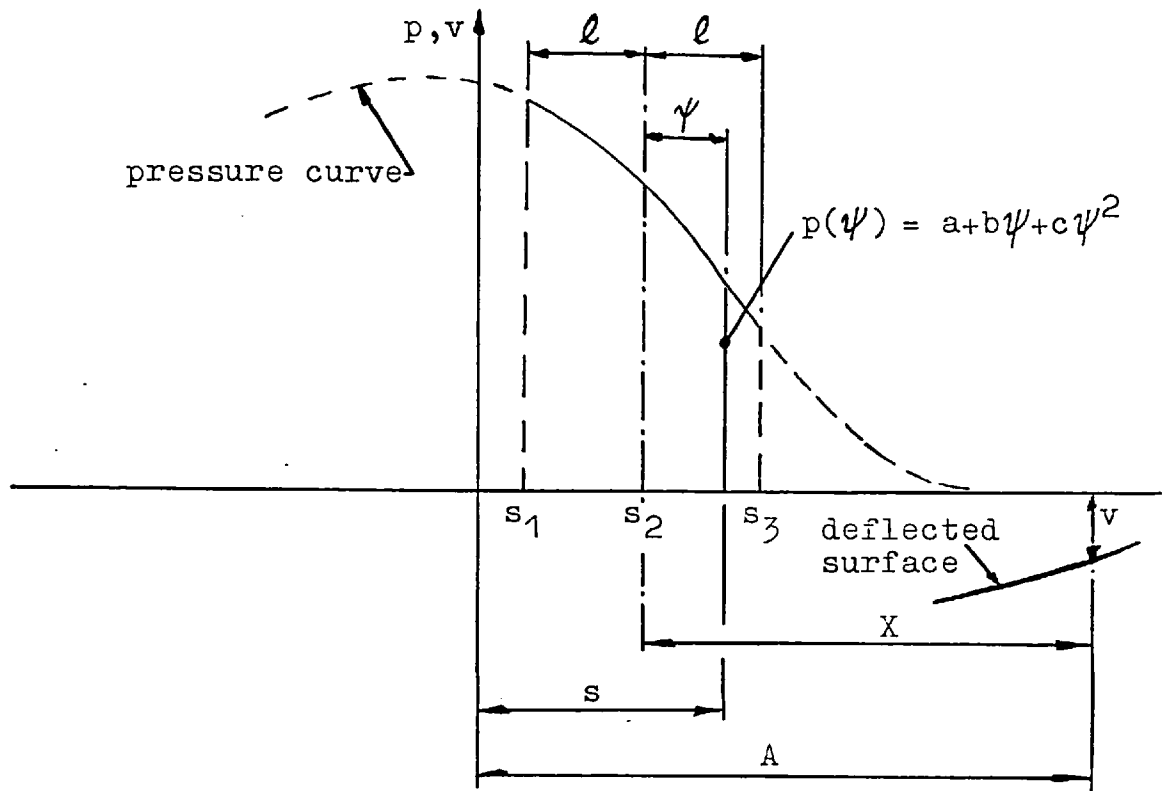


Fig. 53. Modified deflexion diagram.

Equation (3.3) then becomes:

$$v = - \frac{2}{\pi E'} \int_{-l}^{+l} (a + b\psi + c\psi^2) \log(X - \psi) d\psi$$

$$\text{i.e.} \quad v = - \frac{2}{\pi E'} \left[I_1 + I_2 + I_3 \right] \quad \text{-----} \quad (A5.1)$$

$$\text{Where } I_1 = \int_{-\ell}^{+\ell} a \log(X-\psi) d\psi$$

$$I_2 = \int_{-\ell}^{+\ell} b \psi \log(X-\psi) d\psi$$

$$I_3 = \int_{-\ell}^{+\ell} c \psi^2 \log(X-\psi) d\psi$$

These integrals can be solved by Parts, giving:

$$I_1 = a \left[(\ell+X) \log|X+\ell| - (X-\ell) \log|X-\ell| - 2\ell \right]$$

in Fortran IV, this becomes BI1

$$I_2 = \frac{b}{2} \left[(X^2-\ell^2) \log \left| \frac{X+\ell}{X-\ell} \right| - 2X\ell \right] \quad \text{i.e. BI2}$$

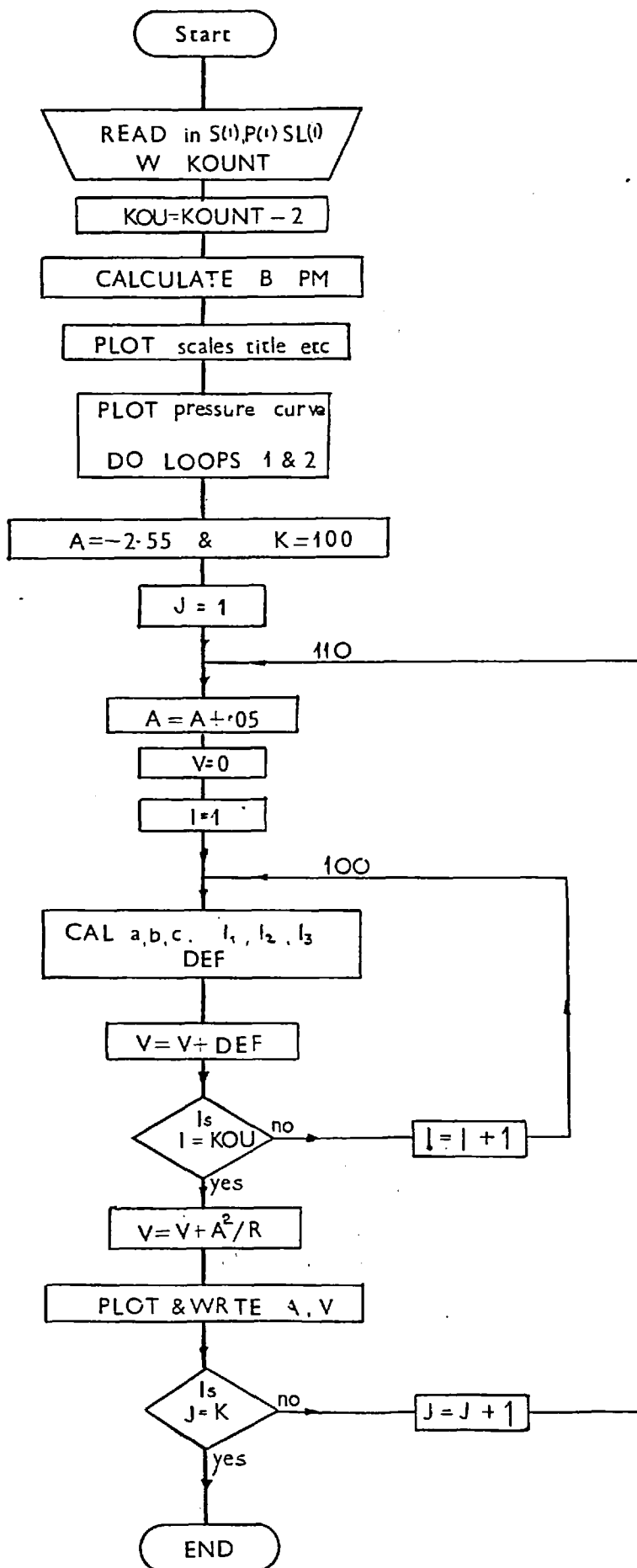
$$I_3 = \frac{c}{3} \left[(X^3+\ell^3) \log|X+\ell| - (X^3-\ell^3) \log|X-\ell| - \frac{2}{3}\ell^3 - 2\ell X^2 \right]$$

which becomes, in Fortran BI3

Substituting into equation (A5.1) gives the elastic deflexion from a flat surface. If it is assumed that similar deflexions would occur on a curved surface, then the total deflexion would equal $v+(A^2/R)$ where R equals the effective radius of the two discs.

The programme was tested by fitting eleven parabolas to a Hertz pressure curve. The computed deflexion curve became flat in the contact region, so justifying the use of the programme for other pressure curves. A print of this computer plot is shown in Figure 54.

Flow chart 5



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Programme 5

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IBFTC BLK

BLK - EFN SOURCE STATEMENT - IFN(S) -

C---PRESSURE P AT DISTANCE S FROM ORIGIN

C---SL IS THE DISTANCE BETWEEN PRESSURE POINTS.

DIMENSION S(25),P(25),SL(25)

WRITE(6,19)

19 FORMAT(10X,31HDATA--DETAILS OF PRESSURE CURVE)

WRITE(6,18)

18 FORMAT(4(14X,1HS,6X,1HP,5X,2HSL,2X))

C--HERTZIAN PRESSURE

KOUNT = 23

KOU=KOUNT-2

READ(5,20) (S(I),P(I),SL(I),I=1,KOUNT)

WRITE(6,21)(S(I),P(I),SL(I),I=1,KOUNT)

20 FORMAT(3(3F6.3,2X))

21 FORMAT(4(10X,3F7.3))

WRITE(6,17)

17 FORMAT(1H0,11X,1HA,16X,1HV,18X,3HGIN,16X,9HDEFLEXION)

W=265.0

B=0.00049*SQRT(W)

PM=1310.0*SQRT(W)

PI=3.14159

POIS=0.25

CC=-2.0*PM*(1.0-POIS**2)/(PI*7500000.0)

CALL START

DO 55 JJ=1,2

XJ=7.0

YJ=3.5

6 CALL PLOT(XJ,YJ,-3)

CALL PLOT(0.0,-6.0,3)

CALL PLOT(0.0,-0.5,2)

CALL PLOT(0.0,1.0,3)

CALL PLOT(0.0,2.0,2)

OCALL SYMBOL(-3.0,3.8,0.15,37HPRESSURE AND ELASTIC DEFLEXION CURV
1,0.0,37)

CALL PLOT(-3.0,3.7,3)

CALL PLOT(1.7,3.7,2)

CALL SYMBOL(-2.0,1.6,0.1,17HHERTZIAN PRESSURE,0.0,17)

CALL SYMBOL(-2.0,1.4,0.1,12HDISTRIBUTION,0.0,12)

CALL SYMBOL(0.2,1.5,0.1,4HP/PM,90.0,4)

CALL AXIS(-3.0,0.0,3HX/B,-3,5.0,0.0,-3.0,1.0)

C---TO PLOT PRESSURE CURVE

CALL PLOT(-3.0,0.0,3)

DO 1 I=1,KOU,2

SE=SL(I)

SA=P(I+1)

SB=(P(I+2)-P(I))/(2.0*SE)

SC=(P(I+2)+P(I)-2.0*P(I+1))/(2.0*SE*SE)

X=S(I)

DO 2 KL=1,40

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- EFN SOURCE STATEMENT - IFN(S) -

```

PY=SA+SB*(X-S(I+1))+SC*(X-S(I+1))**2
CALL PLOT(X,PY,2)
X=X+SE/10.0
IF(X.GT.S(I+2))GO TO 1
2 CONTINUE
1 CONTINUE
K=100
A=-2.55
CALL PLOT(0.0,-5.0,-3)
CALL SYMBOL(-2.0,3.5,0.1,7HELASTIC,0.0,7)
CALL SYMBOL(-2.0,3.3,0.1,9HDEFLEXION,0.0,9)
CALL SYMBOL(0.2,2.4,0.1,3HH/R,90.0,3)
DO 110 J=1,K
C---CALCULATION OF ELASTIC DEFORMATION
A=A+0.05
V=0.0
DO 100 I=1,K00,2
SE=SL(I)*B
SA=P(I+1)
SB=(P(I+2)-P(I))/(2.0*SE)
SC=(P(I+2)+P(I)-2.0*P(I+1))/(2.0*SE*SE)
BX=(A-S(I+1))*B
C1=ALOG(ABS(BX+SE))
C2=ALOG(ABS(BX-SE))
C3=SE+BX
C4=BX-SE
C5=C3*C4
C6=BX**3+SE**3
C7=BX**3-SE**3
BI1=(C3*C1-C4*C2-2.0*SE)*SA
BI2=(C5*C1-C6*C2-2.0*BX*SE)*SB/2.0
BI3=(C6*C1-C7*C2-(2.0/3.0)*SE**3-2.0*SE*BX**2)*SC/3.0
DEF=CC*(BI1+BI2+BI3)
V=V+DEF
100 CONTINUE
GIN=(A*B)**2/3.0
DEFN=GIN+V
YED=(DEFN/1.5)*20000.0
C---TO PLOT ELASTIC DEFLEXION
IF(A+2.5)71,72,71
72 CALL PLOT(A,YED,3)
71 CALL PLOT(A,YED,2)
C---GIN=GEOMETRICAL INCREMENT, V=DEFLEXION FROM FLAT SURFACE
C---DEFN=TOTAL DEFLEXION AT POINT 'A' FROM ORIGIN
WRITE(6,10)A,V,GIN,DEFN,YED
10 FORMAT(10X,F6.3,4(5X,E16.8))
110 CONTINUE
IF(YC-8.5)3,4,5
4 YC=20.0
XC=0.0
GO TO 6
5 CALL PLOT(4.0,-16.5,-3)
55 CONTINUE
3 CALL ENPLOT(5.0)
STOP
END

```

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DATE 04/16/69

TIME 3.0 MINS

PAGE 3

EXECUTION

DATA--DETAILS OF PRESSURE CURVE (HERTZ)

S	P	SL	S	P	SL
-1.000	0.000	0.050	-0.950	0.312	0.050
-0.800	0.600	0.100	-0.700	0.714	0.100
-0.400	0.916	0.100	-0.300	0.954	0.100
0.200	0.980	0.100	0.300	0.954	0.100
0.600	0.800	0.100	0.700	0.714	0.100
0.900	0.436	0.050	0.950	0.312	0.050

S	P	SL	S	P	SL
-0.900	0.436	0.050	-0.850	0.527	0.050
-0.600	0.800	0.100	-0.500	0.866	0.100
-0.200	0.980	0.200	0.000	1.000	0.200
0.400	0.916	0.100	0.500	0.866	0.100
0.800	0.600	0.050	0.850	0.527	0.050
1.000	0.000	0.000			

A	V	GIN
-2.500	0.83542215E-04	0.13255520E-03
-2.450	0.83790127E-04	0.12730602E-03
-2.400	0.84448286E-04	0.12216287E-03
-2.350	0.84916708E-04	0.11712577E-03
-2.300	0.85376290E-04	0.11219472E-03
-2.250	0.85837601E-04	0.10736971E-03
-2.200	0.86391553E-04	0.10265074E-03
-2.150	0.86903386E-04	0.98037818E-04
-2.100	0.87438962E-04	0.93530941E-04
-2.050	0.87984065E-04	0.89130107E-04
-2.000	0.88544491E-04	0.84835317E-04
-1.950	0.89121439E-04	0.80646574E-04
-1.900	0.89715942E-04	0.76563374E-04
-1.850	0.90328922E-04	0.72587217E-04
-1.800	0.90961930E-04	0.68716604E-04
-1.750	0.91616451E-04	0.64952035E-04
-1.700	0.92294111E-04	0.61293511E-04
-1.650	0.92997015E-04	0.57741032E-04
-1.600	0.93727065E-04	0.54294595E-04
-1.550	0.94436868E-04	0.50954205E-04
-1.500	0.95279513E-04	0.47719858E-04
-1.450	0.96108199E-04	0.44591554E-04
-1.400	0.96977311E-04	0.41569296E-04
-1.350	0.97891721E-04	0.38653081E-04
-1.300	0.98858119E-04	0.35842911E-04
-1.250	0.99883961E-04	0.33133785E-04
-1.200	0.10098040E-03	0.30540702E-04
-1.150	0.10216244E-03	0.28048665E-04
-1.100	0.10345279E-03	0.25662672E-04
-1.050	0.10489136E-03	0.23382722E-04
-1.000	0.10653214E-03	0.21208818E-04
-0.950	0.10868366E-03	0.19140957E-04
-0.900	0.11067605E-03	0.17179141E-04
-0.850	0.11253830E-03	0.15323368E-04
-0.800	0.11429600E-03	0.13573640E-04
-0.750	0.11594573E-03	0.11929956E-04
-0.700	0.11749090E-03	0.10392317E-04
-0.650	0.11892946E-03	0.89607213E-05
-0.600	0.12025970E-03	0.76351698E-05
-0.550	0.12148332E-03	0.64156626E-05
-0.500	0.12260085E-03	0.53021998E-05
-0.450	0.12361177E-03	0.42947811E-05
-0.400	0.12451585E-03	0.33934065E-05
-0.350	0.12531439E-03	0.25980761E-05

DEFLECTION	
0.21609742E-03	0.28812988E 01
0.21129614E-03	0.28172818E 01
0.20661116E-03	0.27548154E 01
0.20204248E-03	0.26938997E 01
0.19759101E-03	0.26345468E 01
0.19325731E-03	0.25767641E 01
0.18904229E-03	0.25205639E 01
0.18494620E-03	0.24659494E 01
0.18096990E-03	0.24129320E 01
0.17711417E-03	0.23615223E 01
0.17337981E-03	0.23117307E 01
0.16976801E-03	0.22635735E 01
0.16627981E-03	0.22170642E 01
0.16291614E-03	0.21722152E 01
0.15967853E-03	0.21290471E 01
0.15556849E-03	0.20875798E 01
0.15358762E-03	0.20478349E 01
0.15073805E-03	0.20098406E 01
0.14802166E-03	0.19736221E 01
0.14544107E-03	0.19392143E 01
0.14299937E-03	0.19066582E 01
0.14069975E-03	0.18759967E 01
0.13854661E-03	0.18472881E 01
0.13654480E-03	0.18205974E 01
0.13470103E-03	0.17960137E 01
0.13302275E-03	0.17736366E 01
0.13152110E-03	0.17536146E 01
0.12911547E-03	0.17215395E 01
0.12827409E-03	0.17103211E 01
0.12779096E-03	0.17038794E 01
0.12782462E-03	0.17043283E 01
0.12785519E-03	0.17047358E 01
0.12786217E-03	0.17048269E 01
0.12786963E-03	0.17049284E 01
0.12787568E-03	0.17050091E 01
0.12788321E-03	0.17051095E 01
0.12789019E-03	0.17052025E 01
0.12789487E-03	0.17052649E 01
0.12789898E-03	0.17053198E 01
0.12790305E-03	0.17053740E 01
0.12790655E-03	0.17054206E 01
0.12790925E-03	0.17054567E 01
0.12791247E-03	0.17054995E 01

-0.300 0.12600686E-03
 -0.250 0.12659273E-03
 -0.200 0.12707170E-03
 -0.150 0.12744394E-03
 -0.100 0.12777983E-03
 -0.050 0.12786937E-03
 0.000 0.12792255E-03
 0.050 0.12786937E-03
 0.100 0.12777982E-03
 0.150 0.12744393E-03
 0.200 0.12707168E-03
 0.250 0.12659272E-03
 0.300 0.12600686E-03
 0.350 0.12531438E-03
 0.400 0.12451585E-03
 0.450 0.12361176E-03
 0.500 0.12260084E-03
 0.550 0.12148328E-03
 0.600 0.12025972E-03
 0.650 0.11892940E-03
 0.700 0.11749091E-03
 0.750 0.11594574E-03
 0.800 0.11429597E-03
 0.850 0.11253873E-03
 0.900 0.11067612E-03
 0.950 0.10868359E-03
 1.000 0.10658207E-03
 1.050 0.10437122E-03
 1.100 0.10345273E-03
 1.150 0.10216244E-03
 1.200 0.10093041E-03
 1.250 0.99683981E-04
 1.300 0.98385799E-04
 1.350 0.97891721E-04
 1.400 0.96977261E-04
 1.450 0.96108135E-04
 1.500 0.95273455E-04
 1.550 0.94486822E-04
 1.600 0.93726847E-04
 1.650 0.92996970E-04
 1.700 0.92294201E-04
 1.750 0.91616533E-04
 1.800 0.90962030E-04
 1.850 0.90328338E-04
 1.900 0.89715892E-04
 1.950 0.89121409E-04
 2.000 0.88544594E-04
 2.050 0.87984171E-04
 2.100 0.87438922E-04
 2.150 0.86908389E-04
 2.200 0.86391529E-04
 2.250 0.85887714E-04
 2.300 0.85395282E-04
 2.350 0.84916635E-04
 2.400 0.84447942E-04
 2.450 0.83989830E-04

0.19087899E-05 0.12791565E-03 0.17055420E 01
 0.13255478E-05 0.12791828E-03 0.17055770E 01
 0.84834993E-06 0.12792005E-03 0.17056006E 01
 0.47719620E-06 0.12792114E-03 0.17056151E 01
 0.21208664E-06 0.12792191E-03 0.17056255E 01
 0.53021241E-07 0.12792239E-03 0.17056319E 01
 0.33227287E-17 0.12792255E-03 0.17056340E 01
 0.53022920E-07 0.12792239E-03 0.17056319E 01
 0.21209000E-06 0.12792191E-03 0.17056254E 01
 0.47720124E-06 0.12792113E-03 0.17056151E 01
 0.84835663E-06 0.12792004E-03 0.17056005E 01
 0.13255562E-05 0.12791828E-03 0.17055770E 01
 0.19087998E-05 0.12791566E-03 0.17055422E 01
 0.25980876E-05 0.12791247E-03 0.17054996E 01
 0.33934197E-05 0.12790927E-03 0.17054569E 01
 0.42947958E-05 0.12790655E-03 0.17054206E 01
 0.53022160E-05 0.12790305E-03 0.17053740E 01
 0.64156803E-05 0.12789896E-03 0.17053195E 01
 0.76351889E-05 0.12789490E-03 0.17052654E 01
 0.89607418E-05 0.12789014E-03 0.17052019E 01
 0.10392338E-04 0.12788324E-03 0.17051099E 01
 0.11929980E-04 0.12787572E-03 0.17050096E 01
 0.13573665E-04 0.12786963E-03 0.17049284E 01
 0.15323394E-04 0.12786213E-03 0.17048283E 01
 0.17179168E-04 0.12785529E-03 0.17047372E 01
 0.19140985E-04 0.12782457E-03 0.17043276E 01
 0.21203847E-04 0.12779092E-03 0.17038789E 01
 0.23382753E-04 0.12827397E-03 0.17103196E 01
 0.25662703E-04 0.12911544E-03 0.17215392E 01
 0.28043697E-04 0.13021113E-03 0.17361484E 01
 0.30540735E-04 0.13152115E-03 0.17536153E 01
 0.33138817E-04 0.13302280E-03 0.17736373E 01
 0.35842944E-04 0.13470093E-03 0.17960124E 01
 0.38653116E-04 0.13654484E-03 0.18205978E 01
 0.41569330E-04 0.13854659E-03 0.18472879E 01
 0.44591589E-04 0.14069972E-03 0.18759963E 01
 0.47719892E-04 0.14299935E-03 0.19066579E 01
 0.50954240E-04 0.14544106E-03 0.19392141E 01
 0.54294631E-04 0.14802148E-03 0.19736197E 01
 0.57741067E-04 0.15073804E-03 0.20098405E 01
 0.61293547E-04 0.15358775E-03 0.20478366E 01
 0.64952072E-04 0.15656860E-03 0.20875814E 01
 0.68716639E-04 0.15967867E-03 0.21290489E 01
 0.72587251E-04 0.16291614E-03 0.21722152E 01
 0.76563908E-04 0.16627980E-03 0.22170640E 01
 0.80646609E-04 0.16976802E-03 0.22635735E 01
 0.84835352E-04 0.17337995E-03 0.23117326E 01
 0.89130140E-04 0.17711431E-03 0.23615241E 01
 0.93530973E-04 0.18096989E-03 0.24129319E 01
 0.98037845E-04 0.18494623E-03 0.24659497E 01
 0.10265077E-03 0.18904229E-03 0.25205639E 01
 0.10736973E-03 0.19325744E-03 0.25767659E 01
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 0.12730603E-03 0.21129586E-03 0.28172781E 01

PRESSURE AND ELASTIC DEFLEXION CURVES

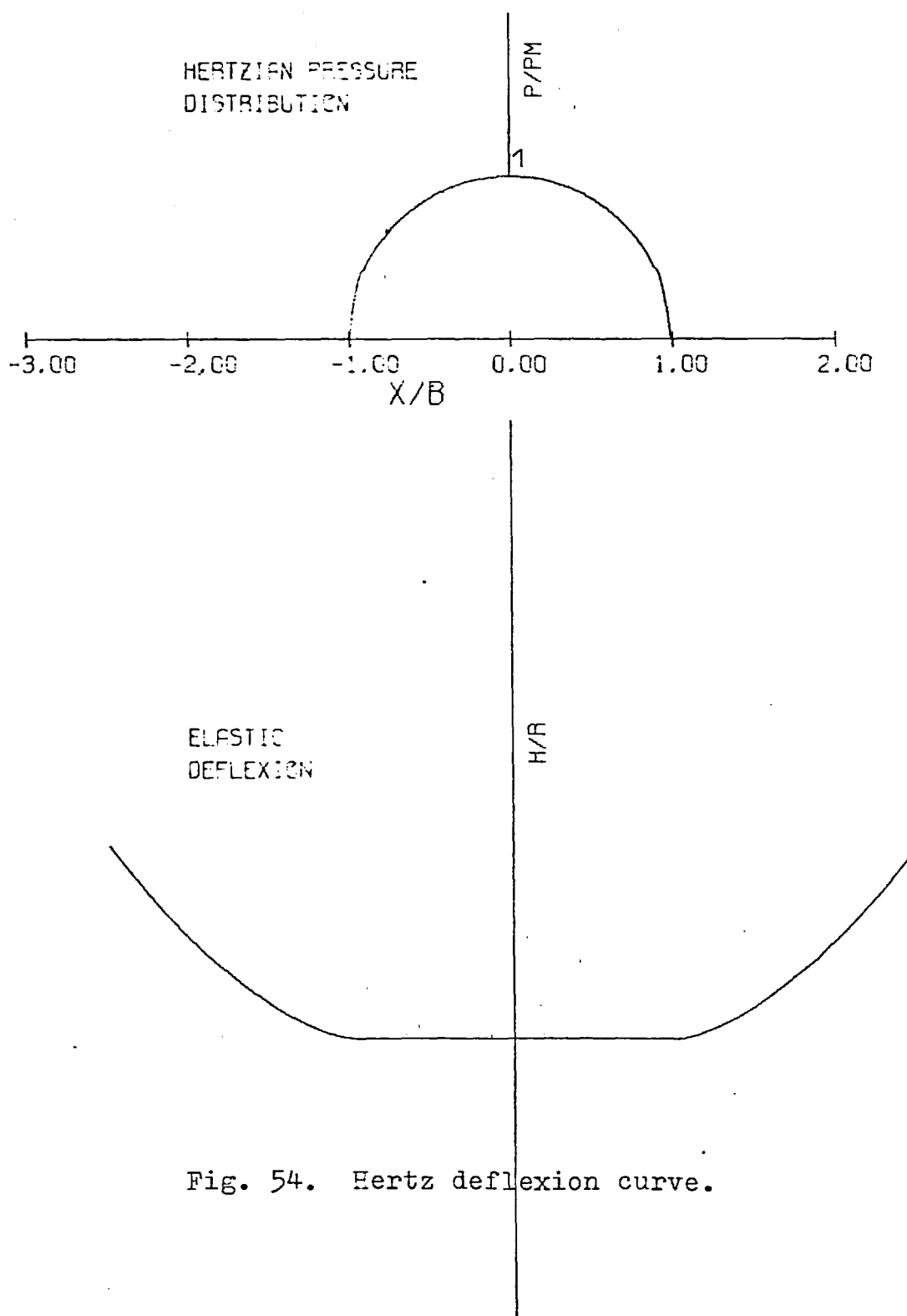


Fig. 54. Hertz deflexion curve.

EXECUTION

DATA--DETAILS OF PRESSURE CURVE (N=5.82 rev/min.)

S	P	SL	S	P	SL	S	P	SL	S	P	SL
-2.200	0.000	0.300	-1.900	0.068	0.300	-1.600	0.180	0.300	-1.300	0.330	0.300
-1.000	0.560	0.300	-0.700	0.840	0.300	-0.400	1.072	0.100	-0.300	1.145	0.100
-0.200	1.192	0.120	-0.080	1.220	0.120	0.040	1.162	0.040	0.080	1.095	0.040
0.120	1.042	0.200	0.320	0.500	0.200	0.520	0.260	0.280	0.800	0.065	0.280
1.080	0.000	-0.000									

A	V	GIN	DEFLECTION
-3.200	0.84147810E-04	0.21717844E-03	0.30132625E-03
-3.150	0.84565346E-04	0.21044464E-03	0.29500998E-03
-3.100	0.84991489E-04	0.20381688E-03	0.28880836E-03
-3.050	0.85426756E-04	0.19729516E-03	0.28272191E-03
-3.000	0.85871385E-04	0.19087948E-03	0.27675087E-03
-2.950	0.86325909E-04	0.18456985E-03	0.27089576E-03
-2.900	0.86790917E-04	0.17836627E-03	0.26515718E-03
-2.850	0.87266790E-04	0.17226873E-03	0.25953551E-03
-2.800	0.87754291E-04	0.16627723E-03	0.25403152E-03
-2.750	0.88253994E-04	0.16039178E-03	0.24864577E-03
-2.700	0.88766586E-04	0.15461237E-03	0.24337896E-03
-2.650	0.89292960E-04	0.14893901E-03	0.23823197E-03
-2.600	0.89833926E-04	0.14337169E-03	0.23320561E-03
-2.550	0.90390562E-04	0.13791041E-03	0.22830098E-03
-2.500	0.90964009E-04	0.13255518E-03	0.22351919E-03
-2.450	0.91555635E-04	0.12730600E-03	0.21886163E-03
-2.400	0.92166951E-04	0.12216285E-03	0.21432980E-03
-2.350	0.92799984E-04	0.11712576E-03	0.20992574E-03
-2.300	0.93457063E-04	0.11219470E-03	0.20565176E-03
-2.250	0.94141543E-04	0.10736969E-03	0.20151123E-03
-2.200	0.94859082E-04	0.10265072E-03	0.19750098E-03
-2.150	0.95619394E-04	0.98037803E-04	0.19365720E-03
-2.100	0.96417748E-04	0.93530925E-04	0.18994867E-03
-2.050	0.97251439E-04	0.89130092E-04	0.18638153E-03
-2.000	0.98119195E-04	0.84835303E-04	0.18295450E-03
-1.950	0.99019999E-04	0.80646560E-04	0.17966656E-03
-1.900	0.99952850E-04	0.76563858E-04	0.17651671E-03
-1.850	0.10091677E-03	0.72587201E-04	0.17350397E-03
-1.800	0.10191071E-03	0.68716590E-04	0.17062730E-03
-1.750	0.10293342E-03	0.64952023E-04	0.16788544E-03
-1.700	0.10398357E-03	0.61293498E-04	0.16527707E-03
-1.650	0.10505945E-03	0.57741019E-04	0.16280047E-03
-1.600	0.10615859E-03	0.54294584E-04	0.16045318E-03
-1.550	0.10727748E-03	0.50954192E-04	0.15823167E-03
-1.500	0.10841992E-03	0.47719845E-04	0.15613976E-03
-1.450	0.10958694E-03	0.44591544E-04	0.15417849E-03
-1.400	0.11077803E-03	0.41569285E-04	0.15234731E-03
-1.350	0.11199171E-03	0.38653071E-04	0.15064478E-03
-1.300	0.11322592E-03	0.35842901E-04	0.14906882E-03
-1.250	0.11447793E-03	0.33138775E-04	0.14761670E-03
-1.200	0.11574451E-03	0.30540693E-04	0.14628521E-03
-1.150	0.11702185E-03	0.28048656E-04	0.14507050E-03
-1.100	0.11830550E-03	0.25662663E-04	0.14396816E-03
-1.050	0.11959053E-03	0.23382714E-04	0.14297324E-03
-1.000	0.12087180E-03	0.21208809E-04	0.14208061E-03

A
-0.950 0.1221433CE-C3
-0.900 0.12338816E-C3
-0.850 0.12459459E-C3
-0.800 0.12575328E-C3
-0.750 0.12685600E-C3
-0.700 0.12789508E-C3
-0.650 0.12886316E-C3
-0.600 0.12975302E-C3
-0.550 0.13055742E-C3
-0.500 0.13126897E-C3
-0.450 0.13188010E-C3
-0.400 0.13238358E-C3
-0.350 0.13277137E-C3
-0.300 0.13301987E-C3
-0.250 0.13311325E-C3
-0.200 0.13304063E-C3
-0.150 0.13279191E-C3
-0.100 0.13233530E-C3
-0.050 0.13165239E-C3
0.000 0.13073042E-C3
0.050 0.12954894E-C3
0.100 0.12810560E-C3
0.150 0.12640133E-C3
0.200 0.12451519E-C3
0.250 0.12257679E-C3
0.300 0.12064868E-C3
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1.200 0.97199048E-C4
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1.300 0.95688739E-C4
1.350 0.94976698E-C4
1.400 0.94290086E-C4
1.450 0.93626880E-C4
1.500 0.92985315E-C4
1.550 0.92363870E-C4
1.600 0.91761162E-C4
1.650 0.91176004E-C4
1.700 0.90607307E-C4
1.750 0.90054085E-C4
1.800 0.89515454E-C4
1.850 0.88990619E-C4
1.900 0.88478836E-C4
1.950 0.87979417E-C4
2.000 0.87491752E-C4

DEFLECTION
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0.76351652E-C5
0.64156585E-C5
0.53021959E-C5
0.42947777E-C5
0.33934034E-C5
0.25980734E-C5
0.19087876E-C5
0.13255459E-C5
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0.47719506E-C6
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0.84835367E-C4

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0.20667489E C1
0.21097614E C1
0.21543718E C1
0.22005701E C1
0.22483472E C1
0.22976949E C1

EXECUTION

DATA--DETAILS OF PRESSURE CURVE (N=17.5 rev/min.)

S	P	SL	S	P	SL
-2.500	0.000	0.750	-1.750	0.250	0.750
-0.100	0.850	0.750	0.650	0.212	0.750

S	P	SL
-1.000	0.680	0.450
1.400	0.000	-0.000

S	P	SL
-0.550	0.950	0.450

A	V	GIN	DEFLEXION
-2.700	0.91973300E-04	0.15461238E-03	0.24658568E-03
-2.650	0.92588670E-04	0.14893902E-03	0.24152769E-03
-2.600	0.93227885E-04	0.14337170E-03	0.23659959E-03
-2.550	0.93894591E-04	0.13791043E-03	0.23180502E-03
-2.500	0.94595351E-04	0.13255520E-03	0.22715055E-03
-2.450	0.95341687E-04	0.12730601E-03	0.22264769E-03
-2.400	0.96125095E-04	0.12216287E-03	0.21828796E-03
-2.350	0.96940666E-04	0.11712577E-03	0.21406643E-03
-2.300	0.97785808E-04	0.11219471E-03	0.20998052E-03
-2.250	0.98658586E-04	0.10736970E-03	0.20602829E-03
-2.200	0.99557377E-04	0.10265074E-03	0.20220811E-03
-2.150	0.10048069E-03	0.98037814E-04	0.19851850E-03
-2.100	0.10142709E-03	0.93530936E-04	0.19495803E-03
-2.050	0.10239515E-03	0.89130102E-04	0.19152525E-03
-2.000	0.10338345E-03	0.84835313E-04	0.18821876E-03
-1.950	0.10439051E-03	0.80646569E-04	0.18503708E-03
-1.900	0.10541480E-03	0.76563869E-04	0.18197867E-03
-1.850	0.10645474E-03	0.72587212E-04	0.17904195E-03
-1.800	0.10750866E-03	0.68716599E-04	0.17622526E-03
-1.750	0.10857482E-03	0.64952032E-04	0.17352685E-03
-1.700	0.10965138E-03	0.61293507E-04	0.17094489E-03
-1.650	0.11073640E-03	0.57741027E-04	0.16847742E-03
-1.600	0.11182782E-03	0.54294592E-04	0.16612241E-03
-1.550	0.11292347E-03	0.50954201E-04	0.16387767E-03
-1.500	0.11402105E-03	0.47719854E-04	0.16174090E-03
-1.450	0.11511812E-03	0.44591551E-04	0.15970967E-03
-1.400	0.11621207E-03	0.41569292E-04	0.15778136E-03
-1.350	0.11730015E-03	0.38653078E-04	0.15595322E-03
-1.300	0.11837942E-03	0.35842907E-04	0.15422232E-03
-1.250	0.11944676E-03	0.33138781E-04	0.15258554E-03
-1.200	0.12049887E-03	0.30540699E-04	0.15103957E-03
-1.150	0.12153227E-03	0.28048662E-04	0.14958093E-03
-1.100	0.12254343E-03	0.25662669E-04	0.14820609E-03
-1.050	0.12352900E-03	0.23382720E-04	0.14691172E-03
-1.000	0.12448775E-03	0.21208815E-04	0.14569656E-03
-0.950	0.12542049E-03	0.19140955E-04	0.14456144E-03
-0.900	0.12629838E-03	0.17179138E-04	0.14347752E-03
-0.850	0.12710375E-03	0.15323366E-04	0.14242712E-03
-0.800	0.12782506E-03	0.13573638E-04	0.14139870E-03
-0.750	0.12845357E-03	0.11929954E-04	0.14038352E-03
-0.700	0.12898236E-03	0.10392315E-04	0.13937467E-03
-0.650	0.12940591E-03	0.89607195E-05	0.13836663E-03
-0.600	0.12971983E-03	0.76351683E-05	0.13735499E-03
			0.32878001E 01
			0.32203692E 01
			0.31546611E 01
			0.30907336E 01
			0.30286739E 01
			0.29686359E 01
			0.29105061E 01
			0.28542191E 01
			0.27997402E 01
			0.27470438E 01
			0.26961081E 01
			0.26469133E 01
			0.25994403E 01
			0.25536700E 01
			0.25095835E 01
			0.24671610E 01
			0.24263822E 01
			0.23872260E 01
			0.23496701E 01
			0.23136913E 01
			0.22792651E 01
			0.22463656E 01
			0.22149654E 01
			0.21850355E 01
			0.21565453E 01
			0.21294622E 01
			0.21037515E 01
			0.20793763E 01
			0.20562976E 01
			0.20344739E 01
			0.20138609E 01
			0.19944124E 01
			0.19760813E 01
			0.19588230E 01
			0.19426208E 01
			0.19274859E 01
			0.19130336E 01
			0.18990282E 01
			0.18853160E 01
			0.18717802E 01
			0.18583290E 01
			0.18448884E 01
			0.18313999E 01

A

DEFLEXION

-0.550	0.12992066E-03	0.64156612E-05	0.13633632E-03	0.18178176E 01
-0.500	0.13000582E-03	0.53021985E-05	0.13530802E-03	0.18041070E 01
-0.450	0.12997352E-03	0.42947800E-05	0.13426830E-03	0.17902439E 01
-0.400	0.12982263E-03	0.33934055E-05	0.13321603E-03	0.17762138E 01
-0.350	0.12955272E-03	0.25980752E-05	0.13215079E-03	0.17620105E 01
-0.300	0.12916392E-03	0.19087891E-05	0.13107270E-03	0.17476360E 01
-0.250	0.12865684E-03	0.13255472E-05	0.12998239E-03	0.17330985E 01
-0.200	0.12803236E-03	0.84834944E-06	0.12888071E-03	0.17184095E 01
-0.150	0.12729096E-03	0.47719583E-06	0.12776816E-03	0.17035754E 01
-0.100	0.12642957E-03	0.21208639E-06	0.12664165E-03	0.16885554E 01
-0.050	0.12544055E-03	0.53021115E-07	0.12549357E-03	0.16732476E 01
0.000	0.12436352E-03	0.43988053E-17	0.12436352E-03	0.16581803E 01
0.050	0.12322257E-03	0.53023046E-07	0.12327559E-03	0.16436745E 01
0.100	0.12203331E-03	0.21209025E-06	0.12224540E-03	0.16299387E 01
0.150	0.12080783E-03	0.47720162E-06	0.12128503E-03	0.16171337E 01
0.200	0.11955609E-03	0.84835715E-06	0.12040445E-03	0.16053926E 01
0.250	0.11828659E-03	0.13255568E-05	0.11961215E-03	0.15948286E 01
0.300	0.11700671E-03	0.19088006E-05	0.11891551E-03	0.15855401E 01
0.350	0.11572293E-03	0.25980885E-05	0.11832102E-03	0.15776136E 01
0.400	0.11444105E-03	0.33934207E-05	0.11783447E-03	0.15711262E 01
0.450	0.11316619E-03	0.42947970E-05	0.11746099E-03	0.15661464E 01
0.500	0.11190297E-03	0.53022172E-05	0.11720519E-03	0.15627359E 01
0.550	0.11065554E-03	0.64156817E-05	0.11707122E-03	0.15609496E 01
0.600	0.10942759E-03	0.76351904E-05	0.11706278E-03	0.15608370E 01
0.650	0.10822243E-03	0.89607433E-05	0.11718317E-03	0.15624423E 01
0.700	0.10704302E-03	0.10392340E-04	0.11743536E-03	0.15658048E 01
0.750	0.10589197E-03	0.11929981E-04	0.11782195E-03	0.15709593E 01
0.800	0.10477156E-03	0.13573667E-04	0.11834522E-03	0.15779363E 01
0.850	0.10368377E-03	0.15323396E-04	0.11900717E-03	0.15867622E 01
0.900	0.10263028E-03	0.17179170E-04	0.11980945E-03	0.15974594E 01
0.950	0.10161247E-03	0.19140988E-04	0.12075346E-03	0.16100461E 01
1.000	0.10063144E-03	0.21208849E-04	0.12184029E-03	0.16245372E 01
1.050	0.99687961E-04	0.23382755E-04	0.12307071E-03	0.16409429E 01
1.100	0.98782540E-04	0.25662706E-04	0.12444525E-03	0.16592699E 01
1.150	0.97915332E-04	0.28048700E-04	0.12596403E-03	0.16795204E 01
1.200	0.97086173E-04	0.30540738E-04	0.12762691E-03	0.17016921E 01
1.250	0.96294501E-04	0.33138821E-04	0.12943332E-03	0.17257776E 01
1.300	0.95539329E-04	0.35842948E-04	0.13138228E-03	0.17517637E 01
1.350	0.94819103E-04	0.38653119E-04	0.13347222E-03	0.17796296E 01
1.400	0.94131444E-04	0.41569334E-04	0.13570078E-03	0.18093437E 01
1.450	0.93472251E-04	0.44591593E-04	0.13806384E-03	0.18408512E 01
1.500	0.92837455E-04	0.47719896E-04	0.14055735E-03	0.18740980E 01
1.550	0.92224479E-04	0.50954243E-04	0.14317872E-03	0.19090496E 01
1.600	0.91631353E-04	0.54294635E-04	0.14592599E-03	0.19456798E 01
1.650	0.91056477E-04	0.57741071E-04	0.14879755E-03	0.19839673E 01
1.700	0.90498497E-04	0.61293551E-04	0.15179205E-03	0.20238940E 01
1.750	0.89956243E-04	0.64952075E-04	0.15490832E-03	0.20654442E 01

EXECUTION

DATA--DETAILS OF PRESSURE CURVE (N=36.0 rev/min.)

S	P	SL	S	P	SL
-3.780	0.000	0.440	-2.640	0.070	0.440
-1.400	0.580	0.220	-1.180	0.705	0.220
-0.400	0.765	0.200	-0.200	0.610	0.200
0.860	0.090	0.260	1.060	0.030	0.260

S	P	SL	S	P	SL
-2.200	0.225	0.400	-1.800	0.380	0.400
-0.960	0.805	0.280	-0.680	0.880	0.280
0.000	0.460	0.400	0.400	0.235	0.400
1.320	0.000	0.000			

A	V	GIN
-3.710	0.84304203E-04	0.29034891E-03
-3.650	0.84721681E-04	0.28255467E-03
-3.600	0.85168759E-04	0.27486546E-03
-3.550	0.85616002E-04	0.26728430E-03
-3.500	0.86073953E-04	0.25980818E-03
-3.450	0.86543313E-04	0.25243811E-03
-3.400	0.87024773E-04	0.24517409E-03
-3.350	0.87519133E-04	0.23801611E-03
-3.300	0.88027509E-04	0.23096417E-03
-3.250	0.88550390E-04	0.22401828E-03
-3.200	0.89090690E-04	0.21717843E-03
-3.150	0.89648674E-04	0.21044462E-03
-3.100	0.90227350E-04	0.20381686E-03
-3.050	0.90831594E-04	0.19729514E-03
-3.000	0.91463239E-04	0.19087947E-03
-2.950	0.92121469E-04	0.18456984E-03
-2.900	0.92806024E-04	0.17836625E-03
-2.850	0.93516860E-04	0.17226871E-03
-2.800	0.94253777E-04	0.16627722E-03
-2.750	0.95016422E-04	0.16039177E-03
-2.700	0.95804313E-04	0.15461236E-03
-2.650	0.96616805E-04	0.14893899E-03
-2.600	0.97453051E-04	0.14337168E-03
-2.550	0.98312071E-04	0.13791040E-03
-2.500	0.99192652E-04	0.13255517E-03
-2.450	0.10000339E-03	0.12730598E-03
-2.400	0.10101261E-03	0.12216284E-03
-2.350	0.10194832E-03	0.11712574E-03
-2.300	0.10289310E-03	0.11219469E-03
-2.250	0.10385884E-03	0.10736968E-03
-2.200	0.10483256E-03	0.10265071E-03
-2.150	0.10579101E-03	0.98037793E-04
-2.100	0.10675806E-03	0.93530914E-04
-2.050	0.10772795E-03	0.89130082E-04
-2.000	0.10870033E-03	0.84825292E-04
-1.950	0.10967437E-03	0.80646550E-04
-1.900	0.11064836E-03	0.76563349E-04
-1.850	0.11162238E-03	0.72587192E-04
-1.800	0.11259330E-03	0.68716581E-04
-1.750	0.11355976E-03	0.64952014E-04
-1.700	0.11451974E-03	0.61293491E-04
-1.650	0.11547099E-03	0.57741011E-04
-1.600	0.11641106E-03	0.54294576E-04
-1.550	0.11733726E-03	0.50954185E-04
-1.500	0.11824668E-03	0.47719338E-04
-1.450	0.11913615E-03	0.44591536E-04

DEFLEXION
0.37465312E-03
0.36728635E-03
0.36003523E-03
0.35290030E-03
0.34588214E-03
0.33898142E-03
0.33219886E-03
0.32553529E-03
0.31899168E-03
0.31256917E-03
0.30626911E-03
0.30009330E-03
0.29404421E-03
0.28812674E-03
0.28234276E-03
0.27669131E-03
0.27117228E-03
0.26578557E-03
0.26053099E-03
0.25540819E-03
0.25041667E-03
0.24555580E-03
0.24082473E-03
0.23622247E-03
0.23174732E-03
0.22739937E-03
0.22317545E-03
0.21907407E-03
0.21509270E-03
0.21122851E-03
0.20747641E-03
0.20382880E-03
0.20028898E-03
0.19685803E-03
0.19353562E-03
0.19032092E-03
0.18721270E-03
0.18420957E-03
0.18130988E-03
0.17851178E-03
0.17581323E-03
0.17321200E-03
0.17070563E-03
0.16829145E-03
0.16596651E-03
0.16372769E-03

0.49953749E 01
0.48971512E 01
0.48004698E 01
0.47053373E 01
0.46117619E 01
0.45197523E 01
0.44293181E 01
0.43404704E 01
0.42532223E 01
0.41675888E 01
0.40835881E 01
0.40012439E 01
0.39205894E 01
0.38416898E 01
0.37645701E 01
0.36892174E 01
0.36156303E 01
0.35438076E 01
0.34737465E 01
0.34054424E 01
0.33388889E 01
0.32740773E 01
0.32109963E 01
0.31496329E 01
0.30899709E 01
0.30319816E 01
0.29756727E 01
0.29209875E 01
0.28679039E 01
0.28163802E 01
0.27663521E 01
0.27177173E 01
0.26705196E 01
0.26247737E 01
0.25804750E 01
0.25376122E 01
0.24961694E 01
0.24561276E 01
0.24174651E 01
0.23801570E 01
0.23441764E 01
0.23094933E 01
0.22760751E 01
0.22438859E 01
0.22128868E 01
0.21830358E 01

A

DEFLEXION

-15

-1.400	C.12000253E-03	0.41569278E-04	0.16157136E-03	0.21542914E 01
-1.350	C.12184250E-03	0.38653065E-04	0.15949556E-03	0.21266074E 01
-1.300	C.12164592E-03	0.35842894E-04	0.15748882E-03	0.20998509E 01
-1.250	C.12240583E-03	0.33138769E-04	0.15554465E-03	0.20739286E 01
-1.200	C.12311690E-03	0.30540687E-04	0.15365759E-03	0.20487679E 01
-1.150	C.12377419E-03	0.28048650E-04	0.15182284E-03	0.20243046E 01
-1.100	C.12437336E-03	0.25662657E-04	0.15003601E-03	0.20004802E 01
-1.050	C.12491030E-03	0.23382709E-04	0.14829301E-03	0.19772402E 01
-1.000	C.12538139E-03	0.21208805E-04	0.14659020E-03	0.19545359E 01
-0.950	C.12578528E-03	0.19140945E-04	0.14492622E-03	0.19323496E 01
-0.900	C.12611656E-03	0.17179129E-04	0.14329569E-03	0.19106092E 01
-0.850	C.12635490E-03	0.15323358E-04	0.14167826E-03	0.18890435E 01
-0.800	C.12648999E-03	0.13573630E-04	0.14006362E-03	0.18675150E 01
-0.750	C.12651563E-03	0.11929947E-04	0.13844563E-03	0.18459417E 01
-0.700	C.12642837E-03	0.10392308E-04	0.13682061E-03	0.18242748E 01
-0.650	C.12622613E-03	0.89607130E-05	0.13518684E-03	0.18024912E 01
-0.600	C.12590916E-03	0.76351627E-05	0.13354432E-03	0.17805909E 01
-0.550	C.12547907E-03	0.64156557E-05	0.13189473E-03	0.17585964E 01
-0.500	C.12493926E-03	0.53021934E-05	0.13024146E-03	0.17365527E 01
-0.450	C.12429512E-03	0.42947754E-05	0.12858990E-03	0.17145319E 01
-0.400	C.12355454E-03	0.33934015E-05	0.12694794E-03	0.16926392E 01
-0.350	C.12273056E-03	0.25980717E-05	0.12532864E-03	0.16710485E 01
-0.300	C.12184025E-03	0.19087861E-05	0.12374904E-03	0.16499872E 01
-0.250	C.12089502E-03	0.13255447E-05	0.12222056E-03	0.16296075E 01
-0.200	C.11990391E-03	0.84834741E-06	0.12075226E-03	0.16100301E 01
-0.150	C.11887483E-03	0.47719431E-06	0.11935203E-03	0.15913604E 01
-0.100	C.11781511E-03	0.21208538E-06	0.11802719E-03	0.15736959E 01
-0.050	C.11673192E-03	0.53020608E-07	0.11678494E-03	0.15571325E 01
0.000	C.11563319E-03	0.10210090E-16	0.11563318E-03	0.15417757E 01
0.050	C.11452872E-03	0.53023552E-07	0.11458174E-03	0.15277565E 01
0.100	C.11342093E-03	0.21209126E-06	0.11363302E-03	0.15151069E 01
0.150	C.11231265E-03	0.47720314E-06	0.11278986E-03	0.15038648E 01
0.200	C.11120702E-03	0.84835918E-06	0.11205538E-03	0.14940717E 01
0.250	C.11010700E-03	0.13255593E-05	0.11143256E-03	0.14857675E 01
0.300	C.10901539E-03	0.19088037E-05	0.11092419E-03	0.14789891E 01
0.350	C.10793469E-03	0.25980921E-05	0.11053279E-03	0.14737705E 01
0.400	C.10686726E-03	0.33934247E-05	0.11026068E-03	0.14701424E 01
0.450	C.10581516E-03	0.42948015E-05	0.11010996E-03	0.14681328E 01
0.500	C.10478029E-03	0.53022222E-05	0.11008251E-03	0.14677668E 01
0.550	C.10376432E-03	0.64156873E-05	0.11018001E-03	0.14690668E 01
0.600	C.10276971E-03	0.76351965E-05	0.11040390E-03	0.14720520E 01
0.650	C.10179469E-03	0.89607497E-05	0.11075544E-03	0.14767392E 01
0.700	C.10034326E-03	0.10392347E-04	0.11123567E-03	0.14831414E 01
0.750	C.99915125E-04	0.11929939E-04	0.11184511E-03	0.14912682E 01
0.800	C.99010475E-04	0.13573675E-04	0.11258415E-03	0.15011220E 01
0.850	C.98128684E-04	0.15323405E-04	0.11345209E-03	0.15126945E 01
0.900	C.97271445E-04	0.17179179E-04	0.11445062E-03	0.15260083E 01
0.950	C.96439879E-04	0.19140997E-04	0.11558088E-03	0.15410783E 01
1.000	C.95634712E-04	0.21208359E-04	0.11684357E-03	0.15579143E 01
1.050	C.94856448E-04	0.23382766E-04	0.11823921E-03	0.15765228E 01
1.100	C.94105407E-04	0.25662717E-04	0.11976812E-03	0.15969083E 01
1.150	C.93381769E-04	0.28048712E-04	0.12143048E-03	0.16190731E 01
1.200	C.92685569E-04	0.30540751E-04	0.12322632E-03	0.16430176E 01
1.250	C.92016721E-04	0.33138834E-04	0.12515555E-03	0.16687407E 01
1.300	C.91375195E-04	0.35842961E-04	0.12721815E-03	0.16962420E 01
1.350	C.90761801E-04	0.38653132E-04	0.12941493E-03	0.17255324E 01
1.400	C.90173215E-04	0.41569348E-04	0.13174256E-03	0.17565675E 01
1.450	C.89605231E-04	0.44591608E-04	0.13419664E-03	0.17892912E 01
1.500	C.89055569E-04	0.47719912E-04	0.13677548E-03	0.18236731E 01
1.550	C.88522553E-04	0.50954260E-04	0.13947681E-03	0.18596908E 01

EXECUTION

DATA--DETAILS OF PRESSURE CURVE (N=58.2 rev/min.)

S	P	SL	S	P	SL
-4.000	0.000	0.400	-3.600	0.045	0.400
-2.400	0.270	0.300	-2.100	0.350	0.300
-1.200	0.600	0.300	-0.900	0.640	0.300
0.000	0.370	0.200	0.200	0.280	0.200
1.000	0.075	0.500	1.500	0.040	0.500

A	V	GIN
-4.000	0.84146424E-04	C.33934132E-03
-3.950	0.84631494E-04	C.32091080E-03
-3.900	0.85133949E-04	C.32258634E-03
-3.850	0.85651586E-04	C.31436791E-03
-3.800	0.86183275E-04	C.30625553E-03
-3.750	0.86728232E-04	C.29824920E-03
-3.700	0.87285815E-04	C.29034890E-03
-3.650	0.87855498E-04	C.28255465E-03
-3.600	0.88436785E-04	C.27486645E-03
-3.550	0.89029213E-04	C.26728429E-03
-3.500	0.89632365E-04	C.25980818E-03
-3.450	0.90245815E-04	C.25243811E-03
-3.400	0.90869136E-04	C.24517408E-03
-3.350	0.91501924E-04	C.23801610E-03
-3.300	0.92143742E-04	C.23096416E-03
-3.250	0.92794173E-04	C.22401827E-03
-3.200	0.93452782E-04	C.21717842E-03
-3.150	0.94119122E-04	C.21044461E-03
-3.100	0.94792727E-04	C.20381685E-03
-3.050	0.95473140E-04	C.19729513E-03
-3.000	0.96159866E-04	C.19087946E-03
-2.950	0.96852409E-04	C.18456983E-03
-2.900	0.97550250E-04	C.17836625E-03
-2.850	0.98252852E-04	C.17226871E-03
-2.800	0.98959667E-04	C.16627721E-03
-2.750	0.99670119E-04	C.16039176E-03
-2.700	0.10038361E-03	C.15461235E-03
-2.650	0.10109954E-03	C.14893859E-03
-2.600	0.10181726E-03	C.14337167E-03
-2.550	0.10253613E-03	C.13791039E-03
-2.500	0.10325548E-03	C.13255516E-03
-2.450	0.10397464E-03	C.12730598E-03
-2.400	0.10469300E-03	C.12216284E-03
-2.350	0.10541008E-03	C.11712574E-03
-2.300	0.10612473E-03	C.11219468E-03
-2.250	0.10683586E-03	C.10736967E-03
-2.200	0.10754238E-03	C.10265071E-03
-2.150	0.10824315E-03	C.98037786E-04
-2.100	0.10893693E-03	C.93530909E-04
-2.050	0.10962244E-03	C.89130077E-04
-2.000	0.11029825E-03	C.84835287E-04
-1.950	0.11096281E-03	C.80646545E-04
-1.900	0.11161439E-03	C.76563844E-04
-1.850	0.11225095E-03	C.72587189E-04
-1.800	0.11286981E-03	C.68716577E-04

S	P	SL	S	P	SL
-3.200	0.105	0.400	-2.800	0.180	0.400
-1.800	0.440	0.300	-1.500	0.522	0.300
-0.600	0.605	0.300	-0.300	0.500	0.300
0.400	0.200	0.333	0.700	0.120	0.300
2.000	0.000	0.000			

DEFLEXION	
C.42348775E-03	0.56465032E C1
C.41554230E-03	0.55405639E C1
C.40772029E-03	0.54362704E C1
C.40001950E-03	0.53335932E C1
C.39243881E-03	0.52325174E C1
C.38497743E-03	0.51320322E C1
C.37763477E-03	0.50351295E C1
C.37041015E-03	0.49388020E C1
C.36330324E-03	0.48440421E C1
C.35631350E-03	0.47508467E C1
C.34944054E-03	0.46592072E C1
C.34268392E-03	0.45691189E C1
C.33604322E-03	0.44805762E C1
C.32951802E-03	0.43925735E C1
C.32310790E-03	0.43081053E C1
C.31681244E-03	0.42241658E C1
C.31063120E-03	0.41417493E C1
C.30456373E-03	0.40608497E C1
C.29860958E-03	0.39814610E C1
C.29276827E-03	0.39025769E C1
C.28703932E-03	0.38271910E C1
C.28142224E-03	0.37522965E C1
C.27591650E-03	0.36788866E C1
C.27052156E-03	0.36069541E C1
C.26523688E-03	0.35364917E C1
C.26006188E-03	0.34674917E C1
C.25499596E-03	0.33999461E C1
C.25003852E-03	0.33328469E C1
C.24518892E-03	0.32691856E C1
C.24044652E-03	0.32059526E C1
C.23581064E-03	0.31441418E C1
C.23128062E-03	0.30837416E C1
C.22685584E-03	0.30247445E C1
C.22253582E-03	0.29671442E C1
C.21831942E-03	0.29109255E C1
C.21420554E-03	0.28560738E C1
C.21019309E-03	0.28025745E C1
C.20628093E-03	0.27504124E C1
C.20246784E-03	0.26995712E C1
C.19875252E-03	0.26500335E C1
C.19513354E-03	0.26017804E C1
C.19160936E-03	0.25547914E C1
C.18817823E-03	0.25090431E C1
C.18483814E-03	0.24645085E C1
C.18158639E-03	0.24211518E C1

A

-1.750 0.11346719E-03
 -1.700 0.11404264E-03
 -1.650 0.11459528E-03
 -1.600 0.11512382E-03
 -1.550 0.11562681E-03
 -1.500 0.11610269E-03
 -1.450 0.11654978E-03
 -1.400 0.11696626E-03
 -1.350 0.11735011E-03
 -1.300 0.11769910E-03
 -1.250 0.11801061E-03
 -1.200 0.11828147E-03
 -1.150 0.11850704E-03
 -1.100 0.11868229E-03
 -1.050 0.11880411E-03
 -1.000 0.11887028E-03
 -0.950 0.11887920E-03
 -0.900 0.11882977E-03
 -0.850 0.11872129E-03
 -0.800 0.11855343E-03
 -0.750 0.11832624E-03
 -0.700 0.11804005E-03
 -0.650 0.11769549E-03
 -0.600 0.11729307E-03
 -0.550 0.11683336E-03
 -0.500 0.11632317E-03
 -0.450 0.11576731E-03
 -0.400 0.11516920E-03
 -0.350 0.11453152E-03
 -0.300 0.11385835E-03
 -0.250 0.11315132E-03
 -0.200 0.11241369E-03
 -0.150 0.11164842E-03
 -0.100 0.11085863E-03
 -0.050 0.11004774E-03
 0.000 0.10921971E-03
 0.050 0.10837965E-03
 0.100 0.10753283E-03
 0.150 0.10668369E-03
 0.200 0.10583672E-03
 0.250 0.10499746E-03
 0.300 0.10417513E-03
 0.350 0.10340023E-03
 0.400 0.10262995E-03
 0.450 0.10184952E-03
 0.500 0.10107920E-03
 0.550 0.99972105E-04
 0.600 0.99178616E-04
 0.650 0.98407723E-04
 0.700 0.97657719E-04
 0.750 0.96927811E-04
 0.800 0.96217683E-04
 0.850 0.95527438E-04
 0.900 0.94857870E-04
 0.950 0.94212083E-04
 1.000 0.93619463E-04
 1.050 0.92952105E-04
 1.100 0.92302462E-04
 1.150 0.91682039E-04
 1.200 0.91078349E-04

DEFLEXION

0.64952009E-04 0.17841920E-03 0.23789226E-01
 0.61293486E-04 0.17533613E-03 0.23378150E-01
 0.57741007E-04 0.17233629E-03 0.22978172E-01
 0.54294572E-04 0.16941839E-03 0.22589118E-01
 0.50954181E-04 0.16658099E-03 0.22210798E-01
 0.47719835E-04 0.16382252E-03 0.21843002E-01
 0.44591532E-04 0.16114131E-03 0.21485508E-01
 0.41569274E-04 0.15853553E-03 0.21138071E-01
 0.38653061E-04 0.15600317E-03 0.20800423E-01
 0.35842892E-04 0.15354199E-03 0.20472265E-01
 0.33138766E-04 0.15114938E-03 0.20153250E-01
 0.30540685E-04 0.14882216E-03 0.19842954E-01
 0.28048647E-04 0.14655569E-03 0.19540758E-01
 0.25662655E-04 0.14434495E-03 0.19245993E-01
 0.23382706E-04 0.14219682E-03 0.18958242E-01
 0.21208802E-04 0.14007908E-03 0.18677211E-01
 0.19140943E-04 0.13802015E-03 0.18402686E-01
 0.17179127E-04 0.13600890E-03 0.18134520E-01
 0.15323355E-04 0.13404464E-03 0.17872619E-01
 0.13573628E-04 0.13212706E-03 0.17616541E-01
 0.11929945E-04 0.13025618E-03 0.17367491E-01
 0.10392306E-04 0.12843236E-03 0.17124314E-01
 0.89607113E-05 0.12665620E-03 0.16887494E-01
 0.76351607E-05 0.12492823E-03 0.16657097E-01
 0.64156543E-05 0.12324896E-03 0.16433194E-01
 0.53021922E-05 0.12162536E-03 0.16216715E-01
 0.42947743E-05 0.12006208E-03 0.16008278E-01
 0.33934004E-05 0.11856260E-03 0.15808347E-01
 0.25980708E-05 0.11712599E-03 0.15617332E-01
 0.19087854E-05 0.11576713E-03 0.15435618E-01
 0.13255440E-05 0.11447687E-03 0.15263582E-01
 0.84834690E-06 0.11326204E-03 0.15101605E-01
 0.47719392E-06 0.11212561E-03 0.14950081E-01
 0.21208513E-06 0.11107071E-03 0.14809428E-01
 0.53020483E-07 0.11010076E-03 0.14680101E-01
 0.12039656E-06 0.10921971E-03 0.14562627E-01
 0.53023679E-07 0.10843267E-03 0.14457685E-01
 0.21209152E-06 0.10774492E-03 0.14365989E-01
 0.47720351E-06 0.10716089E-03 0.14288118E-01
 0.84835967E-06 0.10668508E-03 0.14224677E-01
 0.13255600E-05 0.10632302E-03 0.14176402E-01
 0.19088044E-05 0.10608393E-03 0.14144524E-01
 0.25980929E-05 0.10589833E-03 0.14123110E-01
 0.33934257E-05 0.10602338E-03 0.14136450E-01
 0.42948026E-05 0.10594432E-03 0.14125910E-01
 0.53022236E-05 0.10609427E-03 0.14145603E-01
 0.64156887E-05 0.10638779E-03 0.14185039E-01
 0.76351980E-05 0.10681381E-03 0.14241842E-01
 0.89607515E-05 0.10736847E-03 0.14315796E-01
 0.10392349E-04 0.10805007E-03 0.14406676E-01
 0.11929991E-04 0.10885780E-03 0.14514273E-01
 0.13573677E-04 0.10979136E-03 0.14638848E-01
 0.15323407E-04 0.11085084E-03 0.14780113E-01
 0.17179181E-04 0.11203705E-03 0.14938273E-01
 0.19140999E-04 0.11335308E-03 0.15113744E-01
 0.21208862E-04 0.11482832E-03 0.15310442E-01
 0.23382768E-04 0.11633487E-03 0.15511316E-01
 0.25662719E-04 0.11796518E-03 0.15728691E-01
 0.28048714E-04 0.11973075E-03 0.15964100E-01
 0.30540753E-04 0.12161910E-03 0.16215880E-01

APPENDIX VI

Graphical determination of film thickness at maximum pressure

It was shown in Chapter 3.08 that the film thickness could be obtained at the section which was satisfied by equation (3.7).

$$\text{i.e.} \quad \frac{d^2 p}{dx^2} - \alpha \left(\frac{dp}{dx} \right)^2 = 0$$

or by equation (3.9)

$$\text{i.e.} \quad \frac{d}{dx} \left(\frac{dq}{dx} \right) = 0$$

The pressure curves are available in non-dimensional form, so that equation (3.7) could be expressed as

$$\frac{PM}{B^2} \frac{d\left(\frac{p}{PM}\right)}{d\left(\frac{x}{B}\right)^2} - \frac{PM^2}{B^2} \alpha \left[\frac{d\left(\frac{p}{PM}\right)}{d\left(\frac{x}{B}\right)} \right]^2 = 0$$

$$\text{i.e.} \quad \frac{d^2 p_r}{dx_r^2} - \alpha' \left(\frac{dp_r}{dx_r} \right)^2 = 0 \quad \text{----- (A6.1)}$$

where

$$\begin{aligned} p_r &= \frac{p}{PM} \\ x_r &= \frac{x}{B} \\ \alpha' &= \alpha PM \end{aligned}$$

The graphical method of differentiation⁽¹⁾ is shown in Figure 55. The application of this method in

solving equations (3.7) and (3.9) is shown in Figure 56. The solution of both equations are of course, identical. That this is borne out in practice indicates, to a certain extent, the degree of accuracy of the graphical method.

The intersection of the curves at point E satisfies equation (A6.1). Similarly, equation (3.9) is satisfied at the point F.

Scales (with reference to Figure 55)

It can be shown that $n(c) = \frac{m(B)}{N_1 O_1 h(A)}$

and $o(D) = \frac{n(c)}{N_2 O_2 h(A)}$

Calculation of h_m

Figure 56 shows the actual application of graphical differentiation to the pressure curve shown in Figure 18(a).

Points E and F occur when $p_r = 0.19$

and $\frac{dp_r}{dx_r} = 0.45$

Relevant information:

load $W' = 265.0 \text{ lb}_f/\text{in.}$

speed $N = 5.82 \text{ rev/min.}$

disc radius $r = 3.0 \text{ in.}$

$B = .000485 \sqrt{W'} = 0.0079 \text{ in.}$

$PM = 1310 \sqrt{W'} = 21,300 \text{ lb}_f/\text{in}^2$

$\alpha = 2.1 \times 10^{-9} \text{ cm}^2/\text{dyne}$

and also

$$\eta_0 = 3550 \text{ centi-poise}$$

$$\left[\text{Note: } 1 \text{ lb}_f/\text{in}^2 = 6.88 \times 10^4 \text{ dynes/cm}^2 \right]$$

$$u = \frac{2\pi rN}{60} = \frac{2\pi \times 3 \times 5.82}{60}$$

$$\text{i.e. } u = 1.83 \text{ in/s.}$$

$$\gamma = \eta_0 e^{\alpha p} = \frac{35 \times 10^{-4}}{6.88} e^{(2.1 \times 10^{-9} \times 21300 \times .19 \times 6.88 \times 10^4)}$$

$$\text{i.e. } \gamma = 0.00091 \text{ lb}_f \text{ s in}^{-2}$$

$$\frac{dp}{dx} = \frac{PM}{B} \frac{dp_r}{dx_r} = \frac{21300}{.0079} 0.45$$

$$\text{i.e. } \frac{dp}{dx} = 1,215,000 \text{ lb}_f/\text{in}^3$$

Substituting into equation (3.8)

$$\text{i.e. } h = \sqrt{\frac{4u}{dp/dx}}$$

$$\text{gives } h = \sqrt{\frac{4 \times 1.83 \times 0.00091}{1215000}} \text{ in.}$$

$$\text{i.e. } h = 74.5 \times 10^{-6} \text{ in.}$$

$$\therefore \underline{h_m = \frac{2}{3} h = 49.6 \times 10^{-6} \text{ inches.}}$$

It is now possible to solve the cubic equation

$$K\bar{h}^3 - \bar{h} + 1 = 0$$

at any section within the pressure curve, since K at any point can now be computed.

Subroutine CALL POLRT was used to solve the cubic equation in the form

$$A + Bx + Cx^2 + Dx^3 = 0.$$

Where	A = 1	in Fortran	XCOF(1)=1.0
	B = -1/h _m		XCOF(2)=-1.0/HM
	C = 0		XCOF(3)=0.0
	D = K/h _m ³		XCOF(4)=C/HM**3

NOT TO SCALE

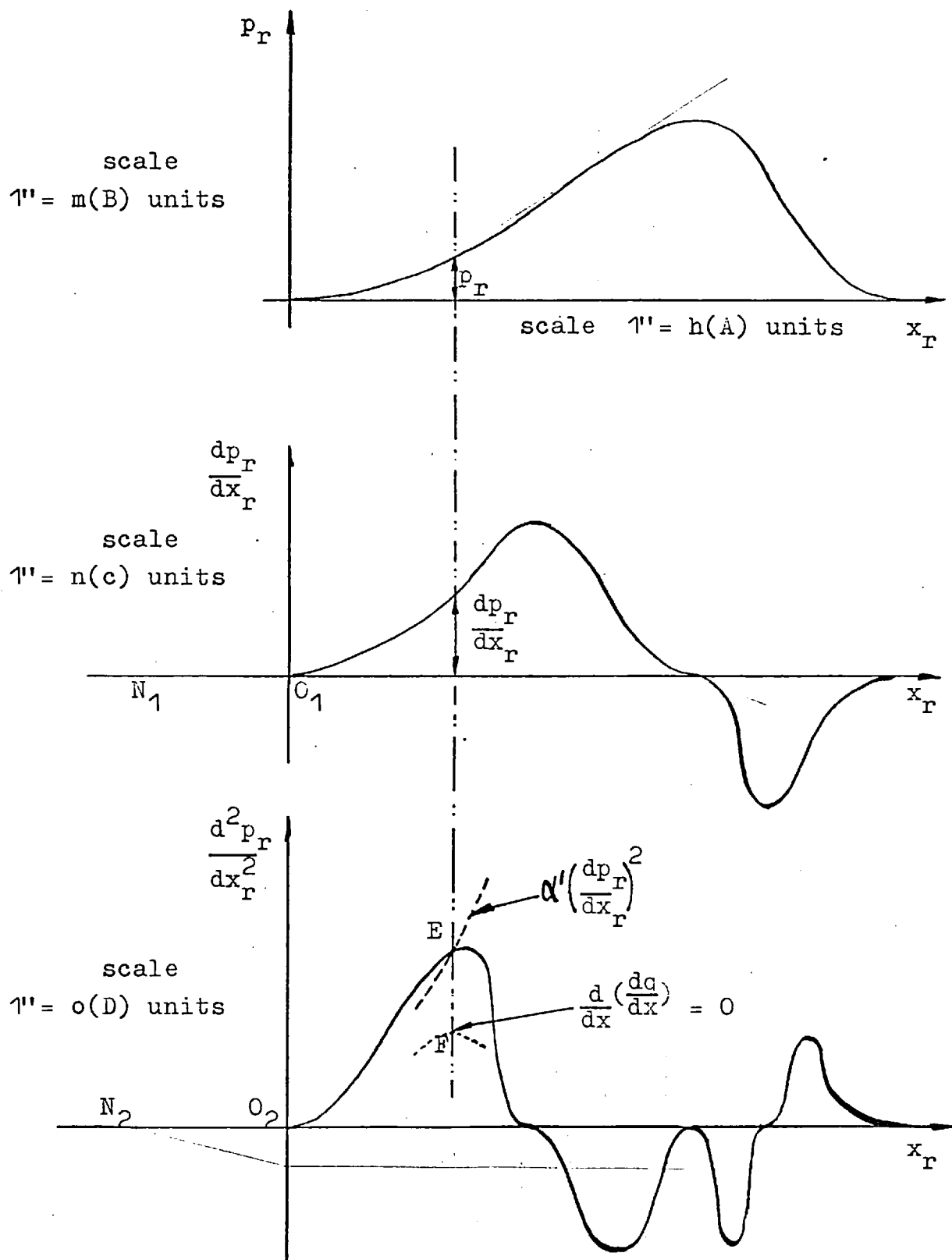


Fig. 55. Graphical Differentiation.

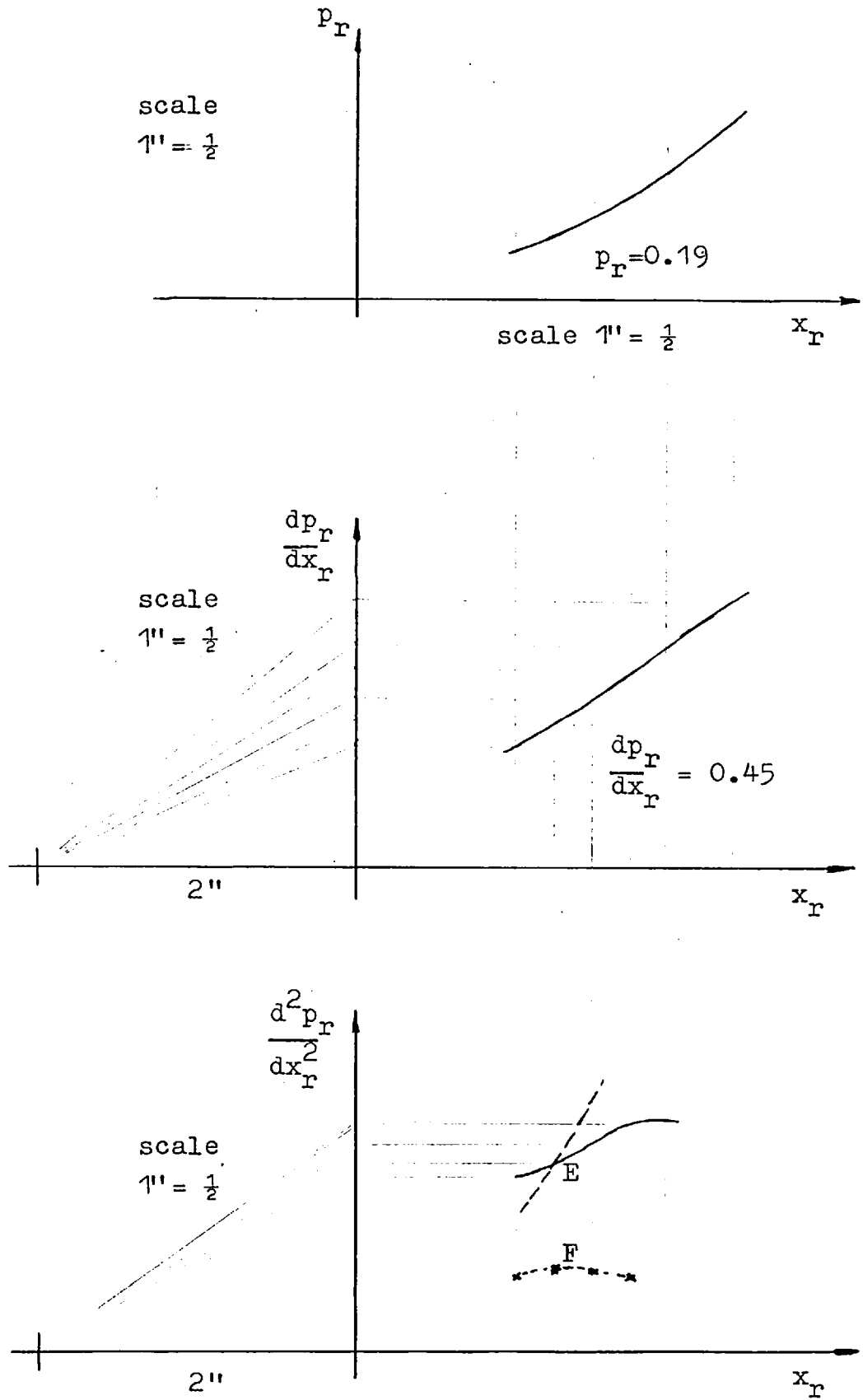
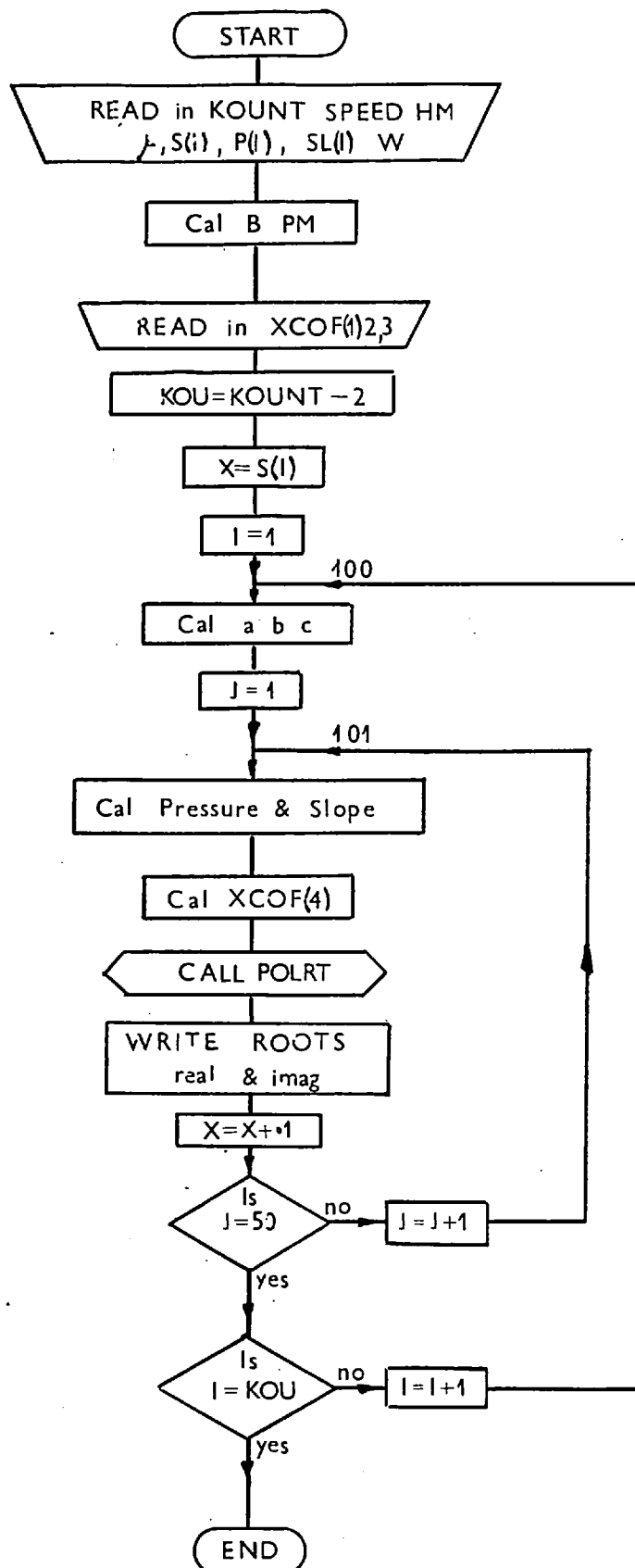


Fig. 56. Graphical Differentiation - application to pressure curve.

Flow chart 6



```

Programme 6      1130 IBM West Ham College
// JOB      WH101
// FOR
*IOCS(CARD,1132 PRINTER,TYPEWRITER)
*LIST SOURCE PROGRAM
**      B.KLEMZ.      MECH. DEPT.
C--FILM THICKNESS FROM REYNOLDS EQUATION.
      DIMENSION S(25),P(25),SL(25),XCOF(4),COF(4),ROOTR(3),ROOTI(3)
C--TEST 4A
      KOUNT=17
      READ (2,20)(S(I),P(I),SL(I),I=1,KOUNT)
      WRITE(3,19)
19  FORMAT(10X,19HPRESSURE CURVE DATA,/)
      WRITE(3,21)
21  FORMAT(2(12X,'S',5X,'P',5X,'SL',2X))
      WRITE(3,99)(S(I),P(I),SL(I),I=1,KOUNT)
20  FORMAT(3(3F6.3,2X))
99  FORMAT(2(10X,3F6.3))
      WRITE(3,22)
22  FORMAT(/,10X,29HCALCULATION OF FILM THICKNESS,/)
      WRITE(3,53)
53  FORMAT(22X,'REAL PART',12X,'IMAGINARY PART')
      W=265.0
      B=0.00049*SQRT(W)
      PM=1310.0*SQRT(W)
      PI=3.14159
      AL=3.06
      HM=0.0000496
      XCOF(1)=1.0
      XCOF(2)=-1.0/HM
      XCOF(3)=0.0
      ZETA=0.095*35.5/45.0
      SPEED=5.82
      U=PI/120.0*SPEED
      CONST=(HM**2)*PM/(B*U*ZETA)
C--CALCULATION OF PARABOLA
      KOU=KOUNT-2
      X=-2.2
      DO 100 I=1,KOU,2
      SA=P(I+1)
      SB=(P(I+2)-P(I))/(2.0*SL(I))
      SC=(P(I+2)+P(I)-2.0*P(I+1))/(2.0*SL(I)**2)
      DO 101 J=1,50
C--CALCULATION OF PRESSURE, SLOPE AND C
      R=SA+SB*(X-S(I+1))+SC*(X-S(I+1))**2
      SLOPE=SB+2.0*SC*(X-S(I+1))
      C=CONST*SLOPE*EXP(-AL*R)
      XCOF(4)=C/HM**3
      CALL POLRT(XCOF,COF,3,ROOTR,ROOTI,IER)
      IF(IER)2,1,2
2  WRITE(3,51)IER
51  FORMAT(10X,I3)
1  WRITE(3,49)X
49  FORMAT(10X,4HX/B=F7.3)
52  FORMAT(20X,E16.8,5X,E16.8)
      WRITE(3,52)(ROOTR(J),ROOTI(J),J=1,3)
      X=X+0.1
      IF(X-S(I+2))101,100,100
101 CONTINUE
100 CONTINUE
      CALL EXIT
      END

```

// XEQ

PRESSURE CURVE DATA

S	P	SL	S	P	SL
-2.200	0.000	0.300	-1.900	0.068	0.300
-1.600	0.180	0.300	-1.300	0.330	0.300
-1.000	0.560	0.300	-0.700	0.840	0.300
-0.400	1.072	0.100	-0.300	1.145	0.100
-0.200	1.192	0.120	-0.080	1.220	0.120
0.040	1.162	0.040	0.080	1.095	0.040
0.120	1.042	0.200	0.320	0.500	0.200
0.520	0.260	0.280	0.800	0.065	0.280
1.080	0.000	0.000			

CALCULATION OF FILM THICKNESS

	REAL PART	IMAGINARY PART
X/B= -2.200	-0.18766563E-03	0.00000000E 00
	0.55855234E-04	0.00000000E 00
	0.13180973E-03	0.00000000E 00
X/B= -2.099	-0.15976057E-03	0.00000000E 00
	0.58642348E-04	0.00000000E 00
	0.11111820E-03	0.00000000E 00
X/B= -1.999	-0.15892827E-03	0.00000000E 00
	0.61951752E-04	0.00000000E 00
	0.96976742E-04	0.00000000E 00
X/B= -1.899	-0.15243151E-03	0.00000000E 00
	0.66076216E-04	-0.38753803E-07
	0.66076216E-04	0.38753803E-07
X/B= -1.799	-0.14896612E-03	0.00000000E 00
	0.71055197E-04	-0.12934970E-05
	0.71055197E-04	0.12934970E-05
X/B= -1.699	-0.14785170E-03	0.00000000E 00
	0.73241171E-04	-0.30861042E-05
	0.73241171E-04	0.30861042E-05
X/B= -1.599	-0.16241180E-03	0.00000000E 00
	0.60629281E-04	0.00000000E 00
	0.10178203E-03	0.00000000E 00
X/B= -1.499	-0.15592121E-03	0.00000000E 00
	0.63458836E-04	0.77509430E-08
	0.63458836E-04	-0.77509430E-08
X/B= -1.399	-0.15417562E-03	0.00000000E 00
	0.64598643E-04	0.00000000E 00
	0.89607740E-04	0.00000000E 00
X/B= -1.299	-0.15618040E-03	0.00000000E 00
	0.63309722E-04	0.74315957E-08
	0.63309722E-04	-0.74315957E-08
X/B= -1.199	-0.16157981E-03	0.00000000E 00

	0.60915117E-04	0.00000000E 00
	0.10066415E-03	0.00000000E 00
X/B= -1.099		
	-0.17039975E-03	0.00000000E 00
	0.58503486E-04	0.00000000E 00
	0.11189626E-03	0.00000000E 00
X/B= -0.999		
	-0.17343059E-03	0.00000000E 00
	0.57898156E-04	0.00000000E 00
	0.11553245E-03	0.00000000E 00
X/B= -0.899		
	-0.20375943E-03	0.00000000E 00
	0.54429037E-04	0.00000000E 00
	0.14933012E-03	0.00000000E 00
X/B= -0.799		
	-0.23838720E-03	0.00000000E 00
	0.52713883E-04	0.00000000E 00
	0.18567292E-03	0.00000000E 00
X/B= -0.699		
	-0.27769064E-03	0.00000000E 00
	0.51713839E-04	0.00000000E 00
	0.22597672E-03	0.00000000E 00
X/B= -0.599		
	-0.32205070E-03	0.00000000E 00
	0.51083166E-04	0.00000000E 00
	0.27096748E-03	0.00000000E 00
X/B= -0.499		
	-0.37185603E-03	0.00000000E 00
	0.50666065E-04	0.00000000E 00
	0.32118993E-03	0.00000000E 00
X/B= -0.399		
	-0.38600387E-03	0.00000000E 00
	0.50580070E-04	0.00000000E 00
	0.33542380E-03	0.00000000E 00
X/B= -0.299		
	-0.50955824E-03	0.00000000E 00
	0.50132497E-04	0.00000000E 00
	0.45942567E-03	0.00000000E 00
X/B= -0.199		
	-0.54961117E-03	0.00000000E 00
	0.50052585E-04	0.00000000E 00
	0.49955863E-03	0.00000000E 00
X/B= -0.099		
	0.49596215E-04	0.00000000E 00
	-0.24798118E-04	-0.56747067E-02
	-0.24798118E-04	0.56747067E-02
X/B= 0.000		
	0.49161608E-04	0.00000000E 00
	-0.24580807E-04	-0.52233762E-03
	-0.24580807E-04	0.52233762E-03
X/B= 0.100		
	0.48266629E-04	0.00000000E 00
	-0.24133318E-04	-0.29339140E-03
	-0.24133318E-04	0.29339140E-03
X/B= 0.200		
	-0.22183874E-04	-0.13479078E-03
	-0.22183874E-04	0.13479078E-03
	0.44367749E-04	0.00000000E 00
X/B= 0.300		
	-0.21204243E-04	-0.10933657E-03
	-0.21204243E-04	0.10933657E-03

	0.42408435E-04	0.00000000E 00
X/B= 0.400	-0.20998661E-04	-0.10518764E-03
	-0.20998661E-04	0.10518764E-03
	0.41996587E-04	0.00000000E 00
X/B= 0.500	-0.22148844E-04	-0.13366167E-03
	-0.22148844E-04	0.13366167E-03
	0.44297688E-04	0.00000000E 00
X/B= 0.600	-0.20969644E-04	-0.10462934E-03
	-0.20969644E-04	0.10462934E-03
	0.41938692E-04	0.00000000E 00
X/B= 0.700	-0.21007792E-04	-0.10536510E-03
	-0.21007792E-04	0.10536510E-03
	0.42014864E-04	0.00000000E 00
X/B= 0.800	-0.21337702E-04	-0.11220497E-03
	-0.21337702E-04	0.11220497E-03
	0.42675565E-04	0.00000000E 00
X/B= 0.900	-0.22041327E-04	-0.13036371E-03
	-0.22041327E-04	0.13036371E-03
	0.44085289E-04	0.00000000E 00
X/B= 1.000	-0.23261593E-04	-0.18533665E-03
	-0.23261593E-04	0.18533665E-03
	0.46523135E-04	0.00000000E 00

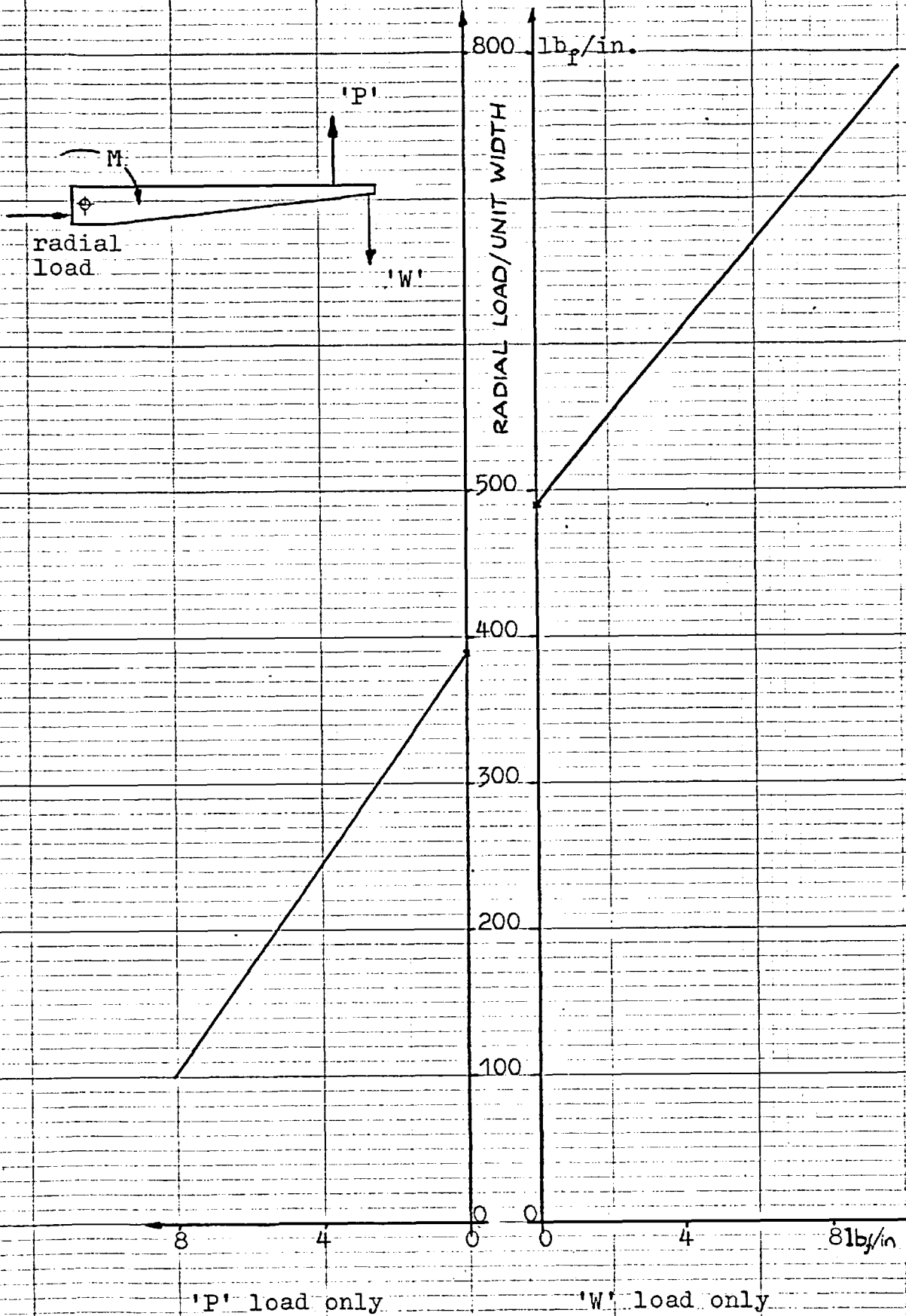


Fig. 57. Calibration Chart - lever arm.

A.S.T.M. STANDARD VISCOSITY-TEMPERATURE CHARTS
FOR LIQUID PETROLEUM PRODUCTS (D 341-43)
CHART C: KINEMATIC VISCOSITY, HIGH RANGE

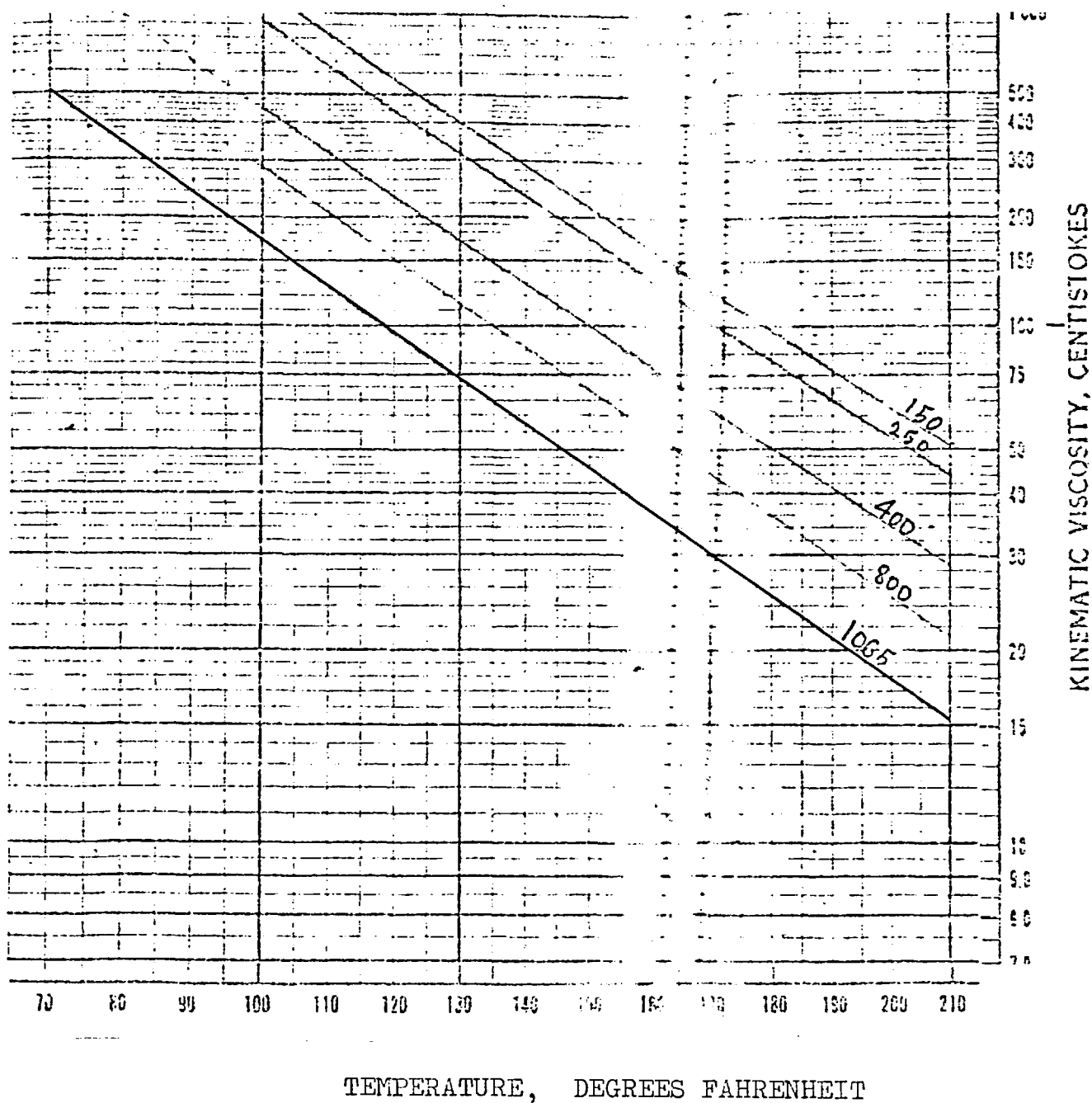


Fig. 58. Viscosity-Temperature Chart. BP 1065.

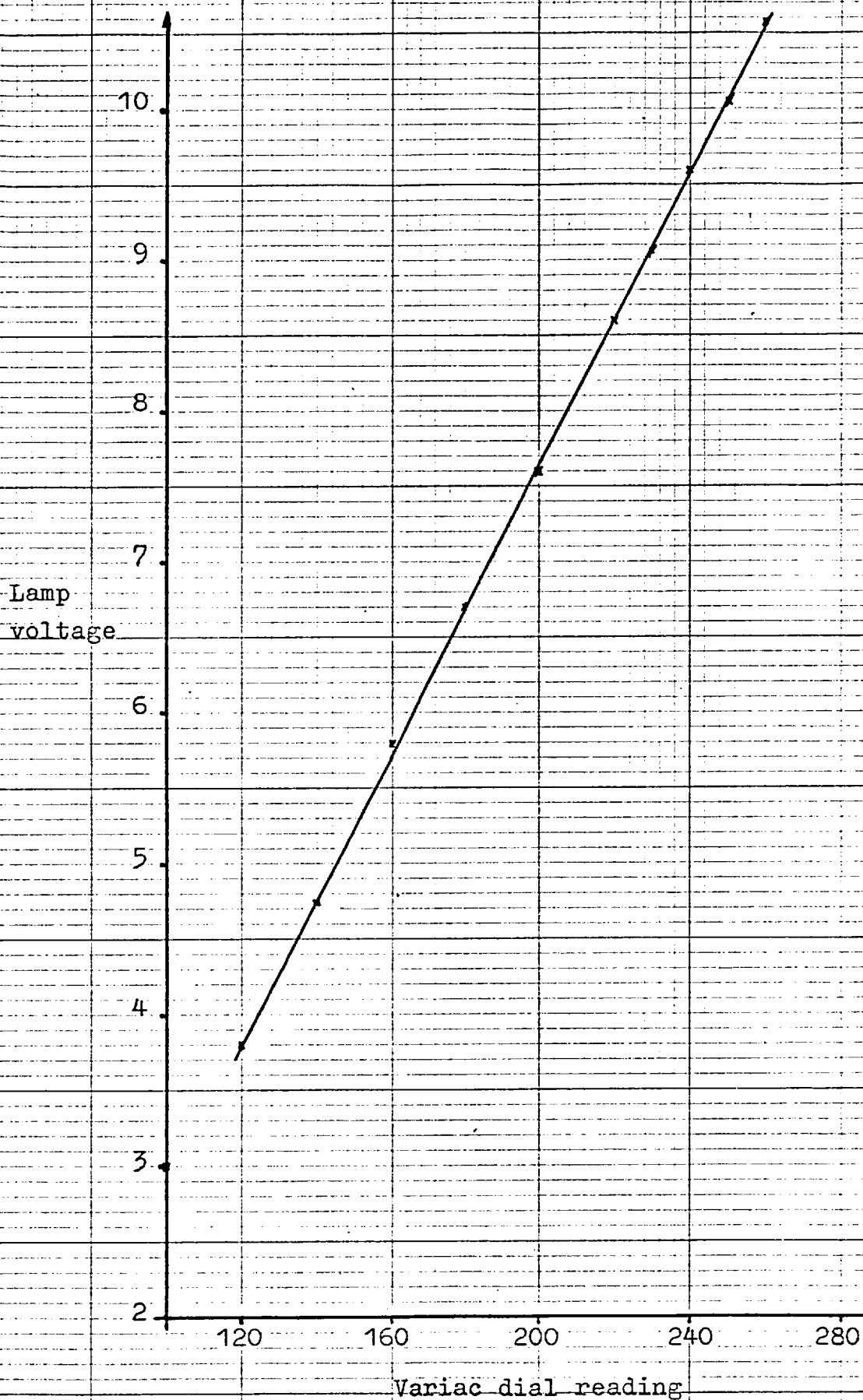


Fig. 59. Calibration Chart-lamp voltage.

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