

THE COMPUTER CONTROL OF A DISTILLATION COLUMN

by

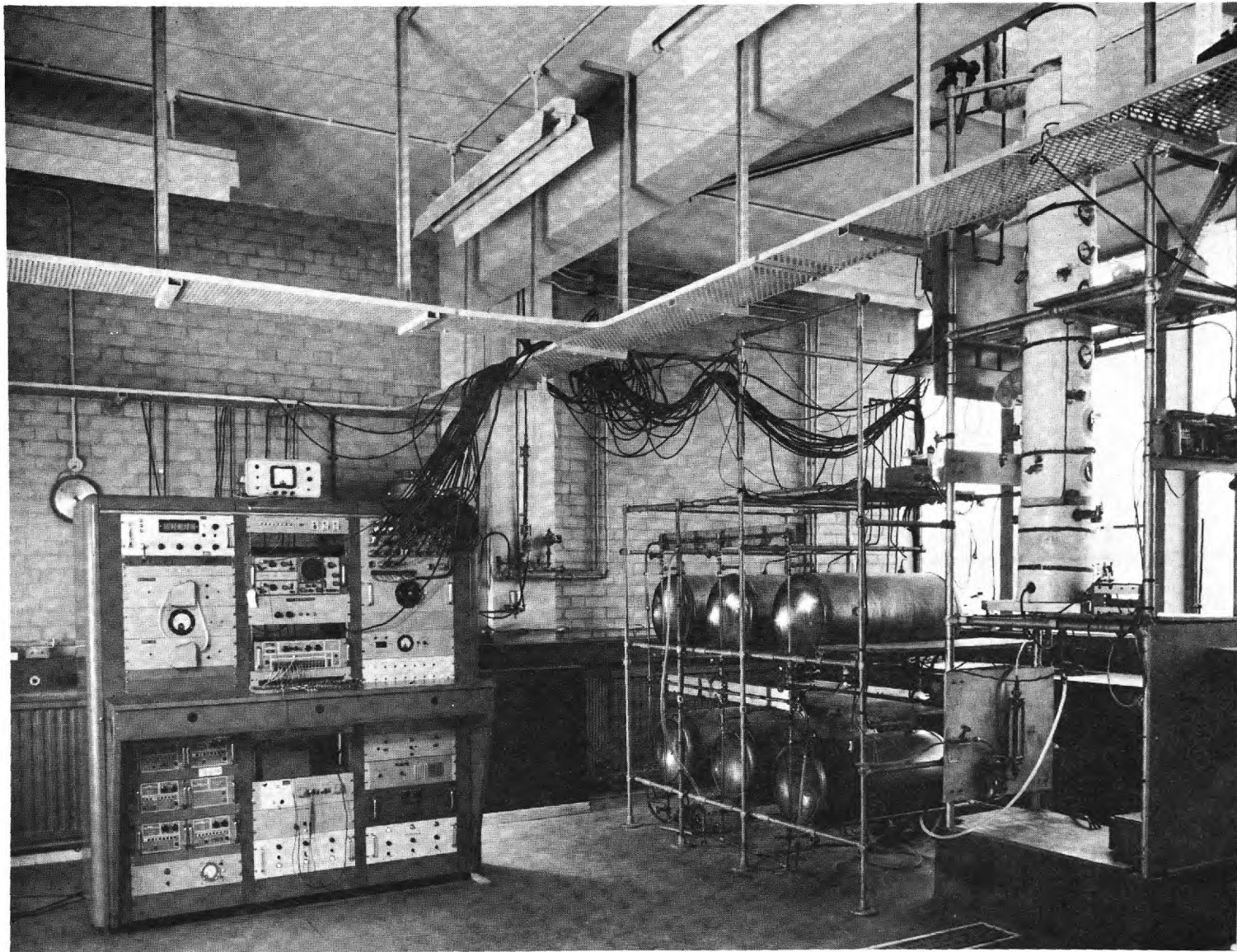
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Ferranti 'Hermes' computer with visual display unit showing logged data.



Pilot scale distillation column and central control panel.

Abstract:

A steady-state model of a binary distillation column is derived from material balance considerations and a digital computer programme is described which calculates the vapour and liquid flow-rates at each plate in the column. Theoretical and experimental results are shown to agree closely, and a dynamic model of the column is derived which, when programmed on a digital computer enables the transient behaviour for large disturbances to be calculated. Agreement with plant data is presented both for step and P.R.B.S. disturbances in the column operating variables. It is further shown how to simplify the complex system of describing equations to a 2nd-order discrete time form, a simplification which is validated by plant measurements. The use of these models in designing closed-loop controllers is discussed and their implementation by a digital computer used to regulate the top-product composition by controlling the Reflux Ratio is described. The same control algorithms are also used to change the set-point of the top-product composition.

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List of Symbols

A, A_1	matrices.
$A(z)$	a polynomial in z .
a_1	coefficient of $A(z)$.
B	binary reflux number or a matrix.
$B(z)$	a polynomial in z .
b_1	coefficient of $B(z)$ or interpolation values for y_r .
$C(z)$	a polynomial in z .
c_1	coefficient of $C(z)$.
D, D_1	vectors.
$D(z)$	a polynomial in z , or controller z transfer function.
d_1	coefficient of $D(z)$.
e	an error term or a plate efficiency.
F	feed molar flow-rate or polynomial in z .
f_1	coefficient of $F(z)$.
$G(z)$	process z transfer function (including zero-order hold).
$G_1(s), G_h(s)$	transfer functions of plant and zero-order hold.
G_T	top-product flow-rate in cc/min.
H	heat supplied to the still.
H_r	hold-up in moles of liquid on plate r .
h_r	hold-up in moles of vapour above plate r .
I	a performance index or the unit matrix.
i, j	indices.
$K(z)$	overall closed-loop z transfer function.
K	open loop plant gain.
K_c	Optimum gain setting for a three term controller.
k	index or fraction of vapour recondensed above each plate.
L_r	molar flow-rate of liquid at plate r .
L_T, L_0	top and bottom product flow-rate in moles/minute.
l	an index.
M	a feedback control vector.
m_1, m_2	elements of M .
m	an index, vector or the feed-plate number.
N	an index or length of PRBS sequence in bits.
n	an index or total number of plates in column, or a noise process.
P, P_1, P_2	matrices or polynomials in z .
p_k	coefficient of $P(z)$.
Q	a heat factor, polynomial in z , or a matrix.
q_k	coefficient of $Q(z)$ or fraction of feed entering column vapourised on entry.
R	Reflux Ratio.

r	index or closed loop system input.
s	Laplace transform variable.
T, T_1	sampling time; pure delay time in plant.
T_R, T_D	integration and differentiation time constants for a 3 term controller.
t, t_0	real time or a temperature.
U	a matrix.
u	a control signal.
V_r	vapour molar flow-rate at plate r .
V_R	voltage controlling reflux ratio.
V_6	top-plate thermistor voltage.
v	noise-free process output.
X	a discrete time state vector.
x, x_1	measured plant output.
x_r	composition of liquid on plate r .
x_s	set-point composition.
x_T, x_0	top and bottom product compositions.
y_r	composition of vapour above plate r .
z	forward shift operator.
z_F	feed composition in % B.M.
α_r, β_r	parameters relating y_r to x_r or coefficients in the controller z transfer function.
γ, δ	zero and pole in a Laplace transfer-function.
Δ	a small increment.
ϵ	a small number or a noise process.
ϕ	a transition matrix.
λ	a scalar parameter.
μ	the mean value.
ζ	a small number or a white noise process.
θ	slope of the process step response at the point of inflexion.
ψ	a general function.
ρ	density in gms/cc.
Σ	the covariance matrix.
σ	standard deviation of a noise process.
τ	dummy time variable.
w_r	coefficient in a noise process.

In addition to the symbols listed above the following abbreviations are also used:

BM	by moles
DCM	dielectric composition meter
DDC	direct digital control
GLS	Generalised Least Squares

LS	Least Squares
ML	maximum likelihood
PRBS	Pseudo-random binary sequence
$\{x\}$,	indicates a sequence of quantities $x(t)$
$E\{ \}$,	the expectation operator
$\text{cov}[e]$,	the covariance of e
$\text{var}[e]$,	the variance of e
$\hat{b}_j$,	the estimated value of a parameter b_j
$\dot{x}$,	the time differential of x
$\text{Prob}\{e\}$,	the probability of e
U^T ,	the transpose of the matrix U
$L[e]$,	the likelihood of e .

Chapter 1. Computer Control of Chemical Processes:

1.1 Introduction:

The Electronic Digital Computer is now an accepted tool in all walks of life particularly in such fields as forecasting, insurance, accounting, medical, police and government. In general, computers are used in any situation where large data handling and storage facilities are required. Use of computers in the process control field is more restricted, but even here it has been recently predicted⁽¹⁾ that by 1970 there will be considerable industrial use of hierarchical control systems. A link between the two aspects of data processing will thus have been effected, since commercial decisions and information will then be used directly to control process operations via the computer hierarchy. Until recently the advancement of process control has been restricted by the theory and measurements available, but the advent of the computer brought with it a new approach and some of these restrictions no longer apply. Recent emphasis in control theory on time-domain system description and control could eventually supersede the frequency domain methods and by using the storage and processing facilities of the computer, indirect methods may make available measurements which have hitherto been unavailable. Computers can be used in a much more complex manner than pneumatic controllers to optimise system performance either statically or dynamically, and it is shown later that in some cases adaptive models may be found by monitoring plant variables on-line. The attempts of theoreticians to advance the state of control theory have been frequently bewailed by those who see their job as applying this theory, and sometimes their criticisms have not been without cause. Many papers have been written without much relation to reality, the authors having simplified the problem to fit the mathematics rather than the reverse. Equally, it may be said that process control engineers are unable to define a general type of control problem, but this is most likely because there is no such thing, each control problem having peculiarities of its own.

1.2 Problems of Computer Control:

There is a considerable difference between normal data processing and the control of a chemical process by digital computer. In the former case the computer is used in an open-loop manner, taking information, storing it, performing calculations and producing a visual output which is used in some fashion by a human. In the latter case the output is sent to a physical process and it is thus necessary to understand the operation of the process before the control programme is written. Because of this fundamental difference in these two aspects of data processing, other problems arise which have resulted in the slow progress in computer control applications. A computer control project may be conveniently split into two parts, the first part being that of defining the plant mathematically and designing a control programme which will satisfy the required performance criteria,

and the second part is the implementation of this control scheme. These objectives are not easily reached and there are several reasons for the slow progress in computer control applications, most of which are due to ignorance. The plant model will usually be unknown and probably non-linear and it may be derived by input-output statistical modelling techniques or from an analysis of the process operation, either way this is not an easy task. Once the model is obtained the control system is to be designed and since control theory is not easily understood and requires some mathematical background, time must be found to absorb a lot of knowledge which may not be of obvious advantage. Ignorance of the economic benefits to be derived from a computer application does not encourage financial investment in this area. Company directors usually require an estimate of the likely return from any investment and since this is usually impossible for a computer control scheme, progress has been slow. If it is decided to proceed with an application, further difficulties to be encountered will probably be shortage of personnel and hardware/software problems. In order to succeed with a computer control project a team effort is required and this team should consist of several experts in such fields as electronics, instrumentation, programming, control theory, etc. led by a control engineer who has a considerable although not necessarily detailed knowledge of each field. Once the software problems have been solved (at present a formidable task in itself), the operational success of the project will depend upon the perfect functioning of many individual components linked together (e.g. computer, scanner, stores, instruments, transducers ...) and as each one of these is often a complex device the probability of a system breakdown is high. Another problem which arises is that of measurement since control strategies may be proposed which involve measuring a variable which is either inaccessible or if the variable is accessible, instrumentation for its measurement may be either unavailable, too expensive or inaccurate. Many people see the lack of adequate instrumentation as the chief impediment to the advancement of computer control at present.

1.3 Feed-Forward Control;

One of the exciting possibilities of using computers in industrial control is the implementation of feed-forward schemes. By using only feed-back methods a disturbance entering a plant may exist for several hours before a correction is applied. Feed-forward concerns itself with correcting the operating conditions as soon as the disturbance is detected and is obviously in principle a superior tactic. In practice feed-back would be used as a trimming device to correct for any drift in the plant and any other indeterminate disturbances. It is most likely that feed-forward will become more widely used in the process industries, particularly on distillation columns which can have settling times of several days.

1.4 Modelling and Control of Distillation Columns:

Distillation columns are one of the most common pieces of equipment used by

the chemical process industry and yet it is remarkable how little substantial work has been published on their modelling and control. In addition, the work that has been published usually considers modelling and control as two separate activities when in fact there should be considerable overlapping. Papers published on modelling normally content themselves with deriving models of varying degrees of complexity without discussing how to use the model to obtain better control. Papers on control of distillation columns usually assume a simple first-order Laplace representation of the plant and proceed from this point to a discussion of various control strategies.

Perhaps the most significant work on modelling of distillation columns is due to Rosenbrock⁽²⁾ who showed how to predict step response behaviour for small perturbations in a binary distillation system. This work was experimentally confirmed by Armstrong and Wilkinson⁽³⁾ and others⁽⁴⁾ have presented experimental results obtained by perturbing a column operating variable and performing frequency response calculations. Implicit in frequency response modelling is the design of a frequency domain controller and this is discussed in detail in the paper by Rijnsdorp et al⁽⁵⁾. It is inefficient however to gather modelling information in real time, derive a frequency domain model and use this to design a controller which, for a digital compensator, has to be transformed back to the original time domain. In the case where it is desired to use a digital compensator it is better to perform all calculations in the time domain.

1.5 Aims of this work:

The work described here exemplifies many of the points discussed above. It is an attempt to apply some of the recent ideas of control theory modified where necessary to fit the practical situation, and it also shows how both modelling and control calculations can be performed entirely in the time domain. The work is specific in that a particular distillation column is investigated rather than a general type, but it is the writer's belief that much progress can be made in the general case by studying specific examples. The advantages of Feed-Forward in combination with Feed-back are also considered. Many difficulties have been encountered in the course of this work, particularly with the hardware and instrumentation and it is pointed out that the experimental results presented here cost much in time and frustration.

1.6 Outline of Chapters:

Chapter 2 discusses the behaviour of the distillation column at steady-state operating conditions and formulates the equations of operation in such a manner that a computer programme can be written which when given the operating conditions will calculate the vapour and liquid flow-rates and compositions at each plate in the column.

Chapter 3 shows how a Dynamic model of the column can be formulated in vector-

matrix form and how the parameters which appear in this model can be found from the steady-state programme. The continuous time vector-matrix model is converted to a discrete time form and is then used to simulate plant behaviour to steps and perturbations in the operating conditions. A method of reducing the full vector-matrix model to a second-order z transfer function model is then proposed and finally the theoretical model is validated by comparison with plant data.

Chapter 4 describes briefly some well-known techniques for estimating discrete-time plant transfer functions from noisy data and the reasons for choosing a particular technique are discussed.

Chapter 5 shows how discrete plant models obtained by using the methods discussed in chapter 4 can be used to design closed loop control systems.

Chapter 6 details some example of the methods of chapter 5 applied to models of the distillation column found from the work discussed in the previous chapters.

1.7 Original Contributions:

To the best of the writer's knowledge this work contains the following original contributions to control theory and practice:

- 1) The derivation of a theoretical model of a binary distillation column in vector-matrix form such that by eliminating the enthalpy equations and taking discrete-time measurements an on-line adaptive model can be derived without the need to perform perturbation experiments.

- 2) The method of simplification of the vector-matrix model to a discrete transfer function.

- 3) A comparison of the models obtained by the application of two and three level sequences to a process.

- 4) The on-line application of plant controllers designed using the method of Dynamic Programming and the criterion of minimum squared error.

- 5) A comparison of a steady-state feed-forward control scheme (combined with feed-back), versus a pure feed-back control scheme.

CHAPTER 2. A Steady-State Model for Binary Distillation

2.1 Introduction

This chapter describes a method of calculating the steady-state behaviour of a binary distillation column, given the operating conditions. Equations describing the steady-state operation are derived from mass-balance considerations only, the enthalpy equations being eliminated by certain assumptions which are later justified. The final result is a general digital computer programme for binary systems which can be used to infer vapour and liquid flow-rates and compositions at each plate in the column. It is then shown in the next chapter how this programme may be used in obtaining the parameters of a linearised Dynamic Model.

2.2 Material Balance Equations

Figure 2.2.1 illustrates the conditions at 3 adjacent plates $r-1$, r , $r+1$ in a binary distillation column:

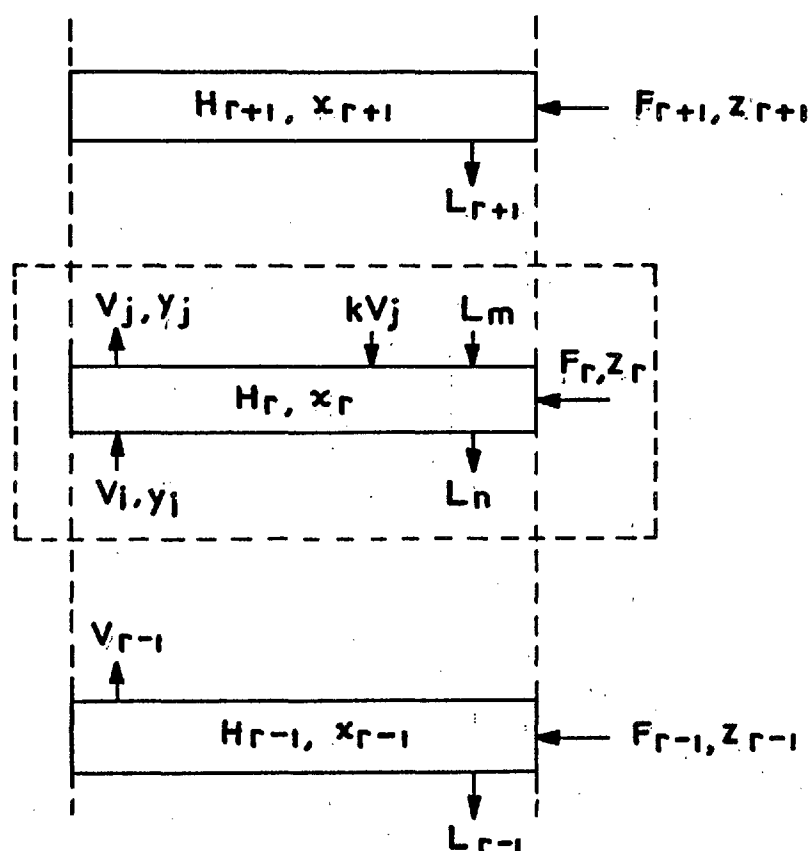


Figure 2.2.1

Conditions at three adjacent plates

At plate r , liquid of composition z_r and at a flow-rate of F_r is fed into the column. Vapour V_j of composition y_j and liquid L_n of composition x_r are leaving the plate while vapour V_1 of composition y_1 is arriving. It is assumed that a fraction q_r of the feed F_r is vapourised on entry to the column. It is also assumed that a fraction k of the vapour which leaves the plate is condensed to liquid above it and that this liquid falls back onto the plate. It can be seen that for plate r a mass-balance of the more volatile component can be written as

$$\frac{d}{dt} (H_r x_r + h_r y_r) = V_1 y_1 + (kV_j) y_j + L_m x_m + F_r z_r (1-q_r) - V_j y_j - L_n x_n$$

and the following relationships are also seen to hold:

$$\begin{aligned} V_1 &= V_{r-1} (1-k) = V_r & x_m &= x_{r+1} \\ V_j &= V_r & x_n &= x_r \\ y_1 &= y_{r-1} & L_m &= L_{r+1} \\ y_j &= y_r & L_n &= L_r \end{aligned}$$

Thus the mass-balance equation can now be written

$$\frac{d}{dt} (H_r x_r + h_r y_r) = V_r y_{r-1} - V_{r+1} y_r + L_{r+1} x_{r+1} - L_r x_r + F_r z_r (1-q_r) \quad \dots (2.2.1)$$

2.3 Flow Relationships

The assumption is now made that at steady-state the heat supplied to the boiler is used to overcome the latent heat of the liquid mixture in the still. If the still mixture contains $x_0\%$ by weight of alcohol, then 1 gram of still mixture contains x_0 gms of alcohol and $(1 - x_0)$ gms

of water and since the latent heats of alcohol and water at their boiling points are 204.26 cal/gm and 539 cal/gm respectively, the latent heat of a mixture $x_0\%$ B.W. will be $[204.26 x_0 + 539 (1-x_0)]$ cal per gram. In addition, 1 gm contains $\left[\frac{x_0}{46} + \frac{1-x_0}{18} \right]$ moles and H K.W. will produce

$$\frac{\frac{x_0}{46} + \frac{1-x_0}{18}}{204.26 x_0 + 539 (1-x_0)} \times \frac{H \times 10^3 \times 60}{4.18} \text{ moles/minute.}$$

$$= 1.48 H \left(1 + \frac{0.011 x_0}{1 - 0.621 x_0} \right)$$

$$\doteq 1.48 H, \text{ since } x_0 \text{ is usually small.}$$

Thus the vapour flow-rate from the boiler may be written as

$$V_0 = 1.48 H \quad \dots (2.3.1)$$

and the vapour flow-rate is virtually independent of still composition.

Under operating conditions the vapour passing up the column will lose a certain amount of heat to the column walls, the vapour being the heat source, the walls acting as a heat sink. The heat which is lost from the vapour is considered to be a latent heat loss and thus some fraction, k , of the vapour flow-rate will be recondensed above each plate. This method of considering the heat loss as a decrease in vapour flow-rate results in a reduced number of equations. (In this work, it is assumed that k is the same for each section.) Thus

$$V_1 = (1-k) V_0$$

⋮

$$\text{and } \therefore V_r = (1-k)^r V_0, \quad 0 < r < m \quad \dots (2.3.2)$$

where r lies below the feed plate m . Above the feed plate this equation becomes

$$V_r = (1-k)^r V_0 + q F (1-k)^{r-m}, m \leq r \leq n \quad \dots (2.3.3)$$

In the type of column under consideration a total condenser is used and this means that all the vapour leaving the top plate is condensed to liquid, i.e.,

$$L_{n+1} + L_T = (1-k) V_n \quad \dots (2.3.4)$$

Where L_T is the top-product flow-rate and L_{n+1} is returned (or refluxed) to the column. The relationship between L_{n+1} and L_T is

$$R = \frac{L_{n+1}}{L_T} \quad \dots (2.3.5)$$

where R is called the Reflux Ratio. Thus the liquid flowing onto plate r may be written down as

$$L_r = L_{n+1} + (1-q) F_m + k \sum_{i=r}^n V_i \quad \dots (2.3.6)$$

$$(0 < r \leq m)$$

and

$$L_r = L_{n+1} + k \sum_{i=r}^n V_i \quad \dots (2.3.7)$$

$$(m < r \leq n)$$

In practice the top product molar flow-rate is found from measuring the composition x_T and the volumetric flow-rate, G_T (cc/min). Then

$$L_T = \frac{G_T \times \rho_T}{18 + 28 \times x_T} \quad \dots (2.3.8)$$

At steady-state a mass-balance for both components gives

$$F = L_T + L_o \quad \dots (2.3.9)$$

and for the more volatile component we have

$$Fz_F = L_T x_T + L_o x_o \quad \dots (2.3.10)$$

2.4 Plate Efficiency

The vapour-liquid equilibrium diagram expresses the relationship between the amounts of one component in the liquid and in the vapour at a constant pressure when the phases are in equilibrium. Figure 2.4.1 shows a series of curves for ethylalcohol-water at various pressures from which the equilibrium diagrams may be constructed. Figure 2.4.2 shows 3 curves for the same pressure obtained by different experimenters, the considerable discrepancies between these curves being due to the difficulty in determining when equilibrium has been attained. In practice, except for the case of small diameter columns, equilibrium is not attained and the extent of departure from equilibrium is given by the plate (or Murphree) efficiency. If the "ideal" curve has the relation

$$y^* = f(x) ,$$

then with reference to Figure 2.4.3 the plate efficiency for the section of the column below the feed is defined as

$$e_r = \frac{BC}{AC} = \frac{y_r - y_{r-1}}{y_r^* - y_{r-1}} \quad \dots (2.4.1)$$

while for the section of the column above the feed plate the efficiency is defined as

$$e_i = \frac{EF}{DF} = \frac{x_{i+1} - x_i}{x_{i+1} - x_i^*} \quad \dots (2.4.2)$$

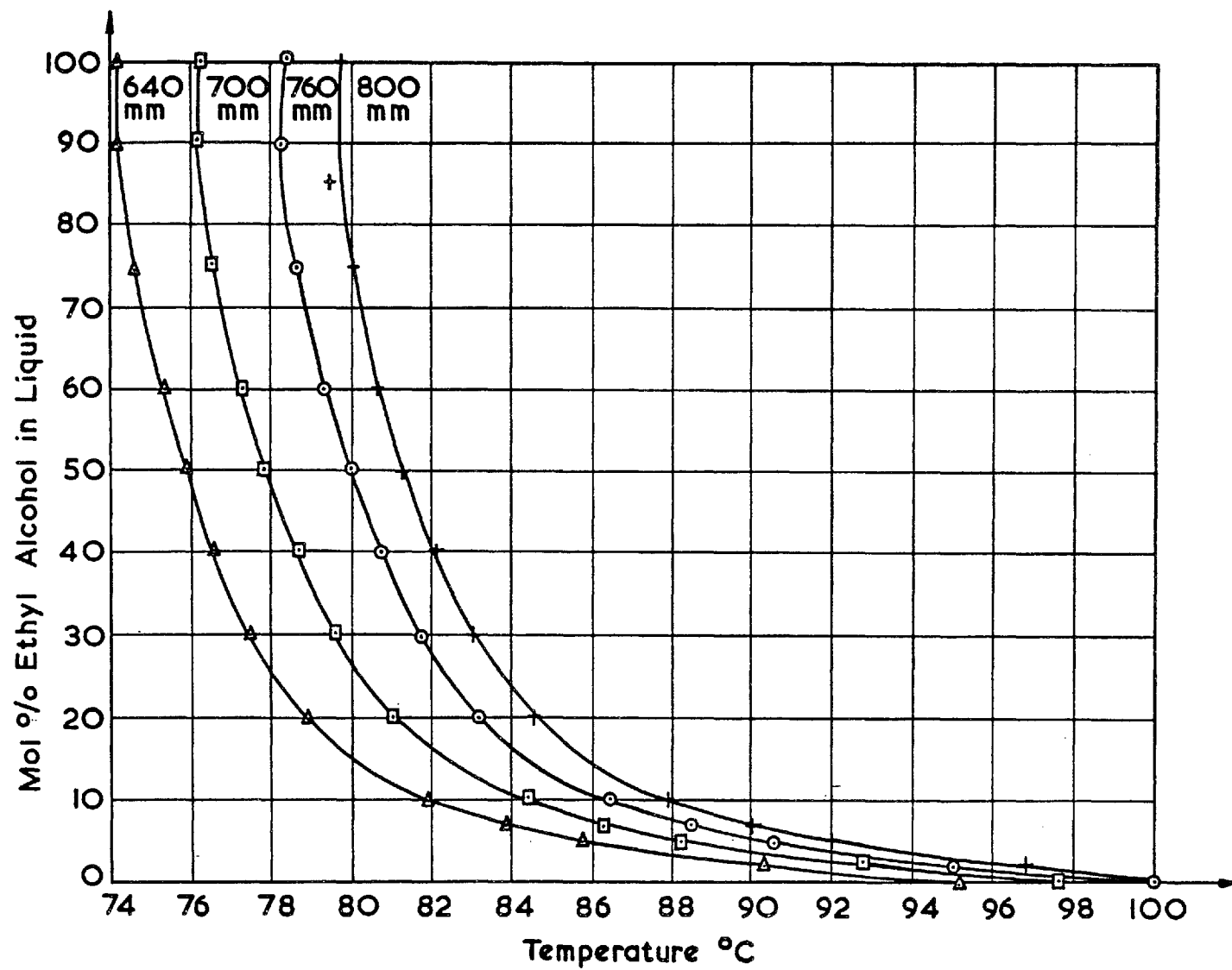


Fig. 2.4.1. MOL % VERSUS TEMPERATURE FOR CONSTANT PRESSURE.

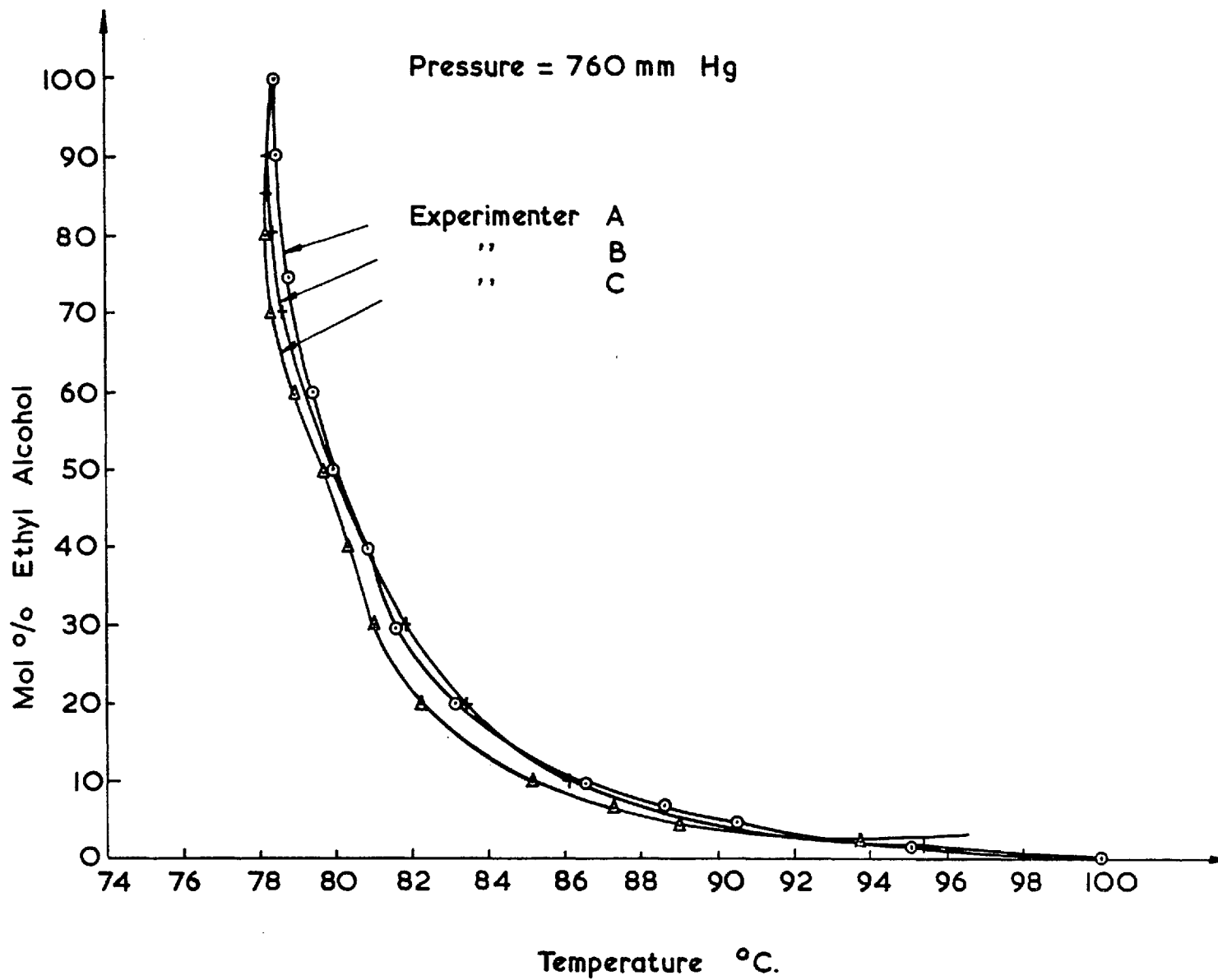


Fig. 2.4.2. BOILING POINT CURVES FOR ETHYL ALCOHOL-WATER.

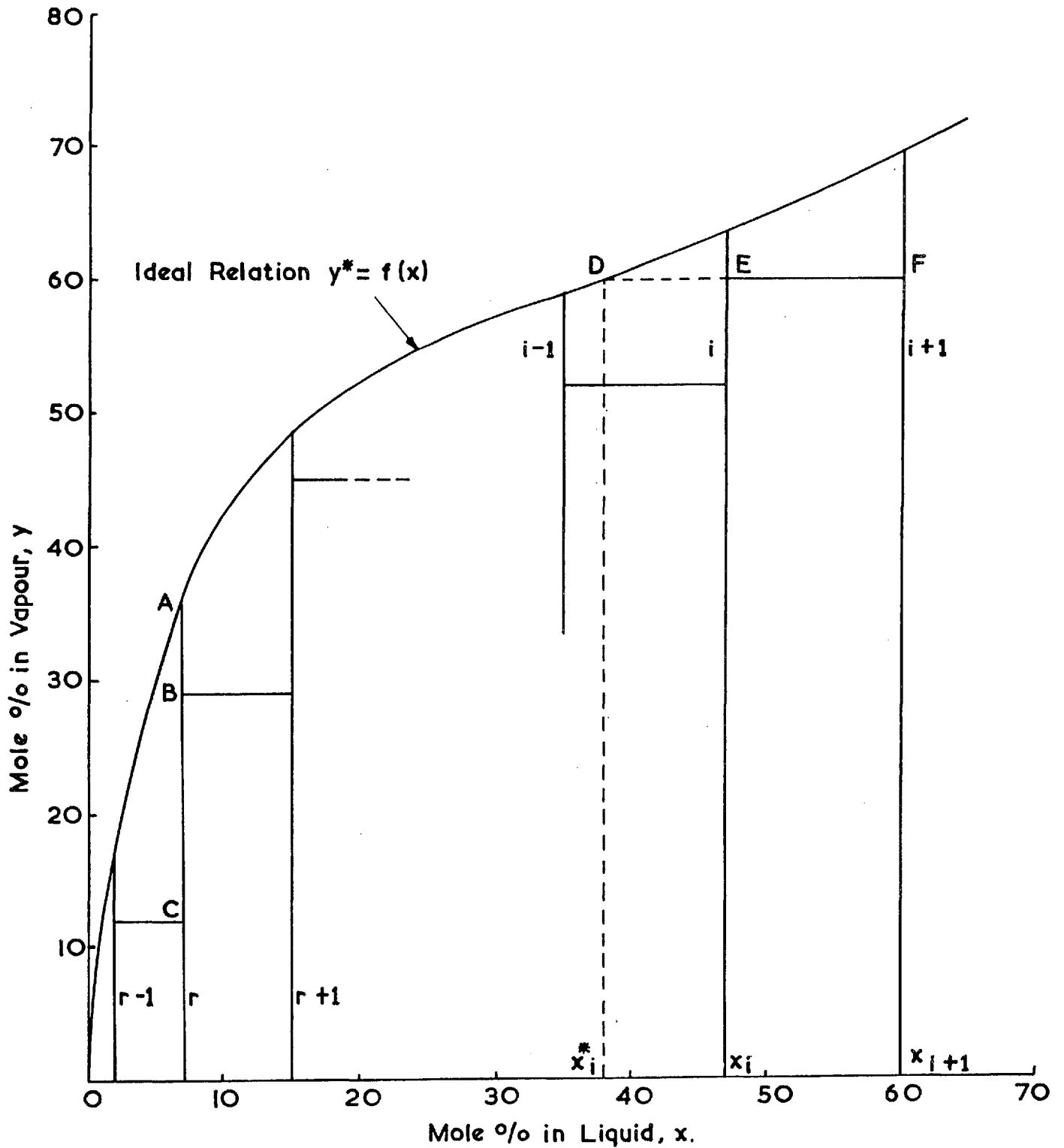


Fig. 2.4.3.

DEFINITION OF PLATE EFFICIENCY

2.5 The Idealised McCabe - Thiele Diagram

If $k = q = 0$ then from 2.3.2

$$V_0 = V_1 = V_2 = \dots V_r = \dots = V_n$$

and 2.3.4 can be written

$$L_{n+1} + L_T = V_n$$

while 2.3.7 becomes

$$L_r = L_{n+1}$$

Thus

$$L_r + L_T = V_n = V_r$$

and taking a balance for the more volatile component

$$L_r x_r + L_T x_T = V_r y_r$$

$$\text{i.e.} \quad y_r = \frac{L_r}{V_r} x_r + \frac{L_T}{V_r} x_T, (m < r \leq n) \quad \dots (2.5.1)$$

If y_r is plotted against x_r there will result a straight line of slope

$$\frac{L_r}{V_r} = \frac{R}{R+1}$$

and intercept

$$\frac{L_T x_T}{V_r} = \frac{x_T}{R+1}$$

Equation 2.5.1 is valid for the upper part (or "rectifying" section) above the feed plate while a similar expression

$$y_j = \frac{L_j}{V_j} x_j - \frac{L_0}{V_j} x_0 \quad \dots (2.5.2)$$

is valid for the lower ("stripping") section below the feed-plate. Equation 2.5.2 is a straight line of slope

$$\frac{L_j}{V_j} = \frac{F}{V_o} + \frac{R}{R+1}$$

and intercept $\frac{L_o}{V_j} x_o$ on the y-axis. The stripping and rectifying lines intersect the line $y = x$ at x_o and x_T respectively. Their point of intersection may be found by equating 2.5.1 and 2.5.2 and setting $x_r = x_j = x$, then

$$\frac{L_r}{V} x + \frac{L_T}{V} x_T = \frac{L_j}{V} x - \frac{L_o}{V} x_o$$

$$(L_j - L_r) x = L_T x_T + L_o x_o = F z_F$$

$$\text{i.e. } x = z_F$$

and the two lines intersect at the feed composition as shown in Figure 2.5.1. It is now possible to determine the plate compositions by starting from x_T and drawing a line parallel to the x-axis to intersect the equilibrium curve. The plate efficiency as defined in equation 2.4.2 is now used to find x_i , since x_{i+1} and x_i^* are known. At x_i a line parallel to the y-axis is drawn to intersect the rectifying line and from this point the procedure is repeated. The same thing occurs from x_o except that 2.4.1 is used to determine y_r . Figure 2.5.1 is termed a McCabe - Thiele diagram and it is much used in the steady-state design of distillation columns. The computer programme described in section 2.10 is essentially a means of constructing such diagrams under non-ideal conditions (i.e. $k \neq q \neq 0$).

2.6 Plate Efficiency determined by Experiment

If the column is operating under steady conditions and $q = F = 0$, then from equation 2.2.1

$$0 = L_{r+1} x_{r+1} - L_r x_r + V_r y_{r-1} - V_{r+1} y_r$$

If the column is under total reflux and the heat losses are neglected then

$$L_T = 0 \text{ (i.e. } V_r = L_{r+1} \text{)}$$

and
$$V_r = V_{r-1}, \quad L_r = L_{r-1} \quad .$$

Therefore

$$0 = L_{r+1} x_{r+1} - L_r x_r + V_{r-1} y_{r-1} - V_r y_r$$

$$\text{i.e.} \quad y_r - y_{r-1} = x_{r+1} - x_r \quad \dots (2.6.1)$$

At the top of the column the vapour is totally condensed and

$$x_{n+1} = y_n \quad \dots (2.6.2)$$

Thus from 2.6.1

$$x_n = y_{n-1}$$

and it can be seen that under these conditions

$$x_r = y_{r-1} \quad .$$

This means that the composition of the liquid on a plate is the same as that of the vapour from the plate below and this relationship enables the plate efficiencies to be found. The plant is placed under total reflux with zero feed and when steady-state has been attained the composition of a sample of the liquid taken from each plate is found. Then

$$e_s = \frac{y_r - y_{r-1}}{y_r^* - y_{r-1}} = \frac{x_{r+1} - x_r}{y_r^* - x_r} \quad (\text{for the stripping section})$$

$$e_r = \frac{x_r - x_{r-1}}{x_r^* - x_r} \quad (\text{for the rectifying section}).$$

Alternatively the plate compositions can be found by determining the temperature and pressure on each plate and using the information contained in Figure 2.4.1. (6)

Unfortunately neither of these methods is successful since in the former there are difficulties in ensuring that the sample represents an average value for the plate, and in the latter, discrepancies in the published data give rise to considerable disagreement particularly at high compositions. It was found however by repeated testing at different steady-state conditions that plate efficiencies of approximately 100% existed throughout the column and for the remainder of this work $e_s = e_r = 1$.

2.7 Feed Conditions

Reference has been made in section 2.2 to the factor q which represents the amount of feed which is vapourised on entry. Perry⁽²⁾ shows how to calculate this factor. The feed, on entry to the feed-plate (F.P.) will absorb a certain amount of heat which is given by

$F \times \text{specific heat} \times (\text{Boiling Point Temperature} - \text{Temperature of Feed})$.

Three sets of conditions are possible according to whether the feed temperature is greater than, equal to, or less than the Boiling Point of the feed. In the ideal case the temperature of the feed, feed-plate liquid and Boiling Point of the feed are equal. If the feed is a liquid below its Boiling Point and below the temperature of the feed plate it will absorb heat from the feed-plate and this heat (which comes from the vapour) means that less vapour will leave the F.P. The converse is true if the feed temperature is greater.

A factor Q is defined such that

$$Q \triangleq \frac{\text{Heat required to bring 1 mole of feed to B.P.T. and vapourise it}}{\text{Molar Latent Heat of Vapourisation of feed (L.H.)}}$$

$$= \frac{1 \times s \times \Delta t + L.H.}{L.H.} = \frac{s \cdot \Delta t}{L.H.} + 1$$

$$= \frac{L_m - L_{m+1}}{F}$$

$$= (1-q)$$

Thus

$$q = - \frac{s \cdot \Delta t}{L.H.} \quad \dots (2.7.1)$$

For a feed at its boiling point $\Delta t = 0$, $\therefore q = 0$. Equation 2.7.1 is valid only if:-

- (1) Molar latent heats of vapourisation of the two components are equal.
- (2) Sensible heat changes are negligible in comparison with latent heats.
- (3) Heats of solution in both phases are absent.

That these 3 conditions are satisfied can be seen from the following:-

- (i) The latent heat of alcohol is 204.3 cal/gm

$$= 204.3 \times 46 = 9400 \text{ cal/mole.}$$

The latent heat of water is 539 cal/gm

$$= 18 \times 539 = 9700 \text{ cal/mole.}$$

- (ii) The sensible heat of an alcohol-water mixture for compositions up to 10% and temperatures up to c 100°C may be reasonably taken as 18.4 cal/mole-°C.

- (iii) The heats of solution are negligible.

For a given case it is easy to calculate the value of q since a temperature measurement is available of the feed and the F.P., and the molar flow-rate is calculable from the volumetric flow and composition and Equation 2.3.8. As Perry shows (page 13-24) the effect of a non-zero q is to shift the intersection of the rectifying and stripping lines, only at $q = 0$ do they intersect at the feed composition.

2.8 Heat Losses

A heat loss at steady-state operation will be due to heat being removed from the vapour and this loss of latent heat will cause some of the

vapour to be recondensed. Consequently the top product flow rate will be reduced by a small amount and this means the top composition will be increased. If it is assumed that a small percentage of the vapour, k is recondensed above each plate then, if $q = 0$, k may be found simply from equations 2.3.3, 2.3.4, 2.3.5

$$\begin{aligned} \text{i.e. } (1-k)^{n+1} &= \frac{\text{Measured Top - product flow-rate}}{\text{Calculated Top - product flow-rate}} \\ &= \frac{G_T \times \rho_T}{18 + 28 \times x_T} \times \frac{(R+1)}{1.48 \times H} \quad \dots (2.8.1) \end{aligned}$$

Where $n = 6$ for the NPL column.

The requirement that $q = 0$ implies adiabatic feed entry and this is discussed in the next section.

If the column is not insulated against heat losses and q has been calculated from 2.7.1, k may be found from

$$V_n = (1-k)^{n+1} V_o + q F (1-k)^{n-m+1} = (R+1) L_T \quad \dots (2.8.2)$$

The solution of this equation is most easily performed on a digital computer.

2.9 Adiabatic Feed Entry

Purely for ease of calculating k from equation 2.8.2, it was decided to set $q = 0$, then 2.8.1 applies and the computer programme described in section 2.10 will find k directly. To maintain $q = 0$ in practice it is necessary to maintain the feed temperature at its Boiling Point and this could easily be done by a servo. The set-point would be the Boiling Point and the feed-back signal would be the measured temperature of the feed. The error signal could then drive a silicon controlled rectifier supplying power to the feed pre-heater. The Boiling Point would be most conveniently found from a series of Boiling Point - composition curves such as Figure 2.4.1 and the pressure could also be measured or a reasonable value of 760 mm could be assumed, since an error of

1 or 2°C has little effect on q which in turn has little effect on the top-product composition. Roughly speaking, negative changes of q of 1% produce positive changes of x_T of 1% and vice versa.

As an experiment, it was decided to plot the feed temperature against the square of the applied (pre-heater) voltage, i.e. against values proportional to the pre-heater power and Figure 2.9.1 shows the result of doing this on a feed-stock of 6½%. It is assumed that the junction of the two lines indicates a change from the liquid phase to liquid + vapour phase and the change appears at about 89.3°C which compares favourably with 89.2°C read from Figure 2.4.1.

2.10 Computer Programme for Calculating the Steady-State Conditions

A programme has been written in Kildsgrove Algol for the KDF9 which will perform the plate-to-plate calculations for steady-state. The flow diagram for this programme is shown in Figure 2.10.1 and the description of the input parameters is shown in Table 2.10.1. An iterative procedure due to Peiser⁽⁸⁾ has been used to perform the plate-to-plate calculations and to start this procedure an initial guess is made for the top-product composition x_T . Equations 2.3.1, 2.3.2, 2.3.3, 2.3.4, 2.3.5, 2.3.6, 2.3.7 are then used to calculate the vapour and liquid flow rates at each plate and these are stored since they depend only on the Heat, Reflux and k and q . Using equations 2.3.9 and 2.3.10 an initial value for x_0 is found from the initial value of x_T and it is also possible to write down 4 further mass balance equations for the stripping and rectifying sections. From Figure 2.10.2 it can be seen that

$$q F + V_m = L_{m+1} + L_T \quad \dots (2.10.1)$$

$$(1-q) F + L_{m+1} = V_m + L_0 \quad \dots (2.10.2)$$

$$q F z_F + V_m y_m = L_{m+1} x_{m+1} + L_T x_T \quad \dots (2.10.3)$$

$$(1-q) F z_F + L_{m+1} x_{m+1} = V_m y_m + L_0 x_0 \quad \dots (2.10.4)$$

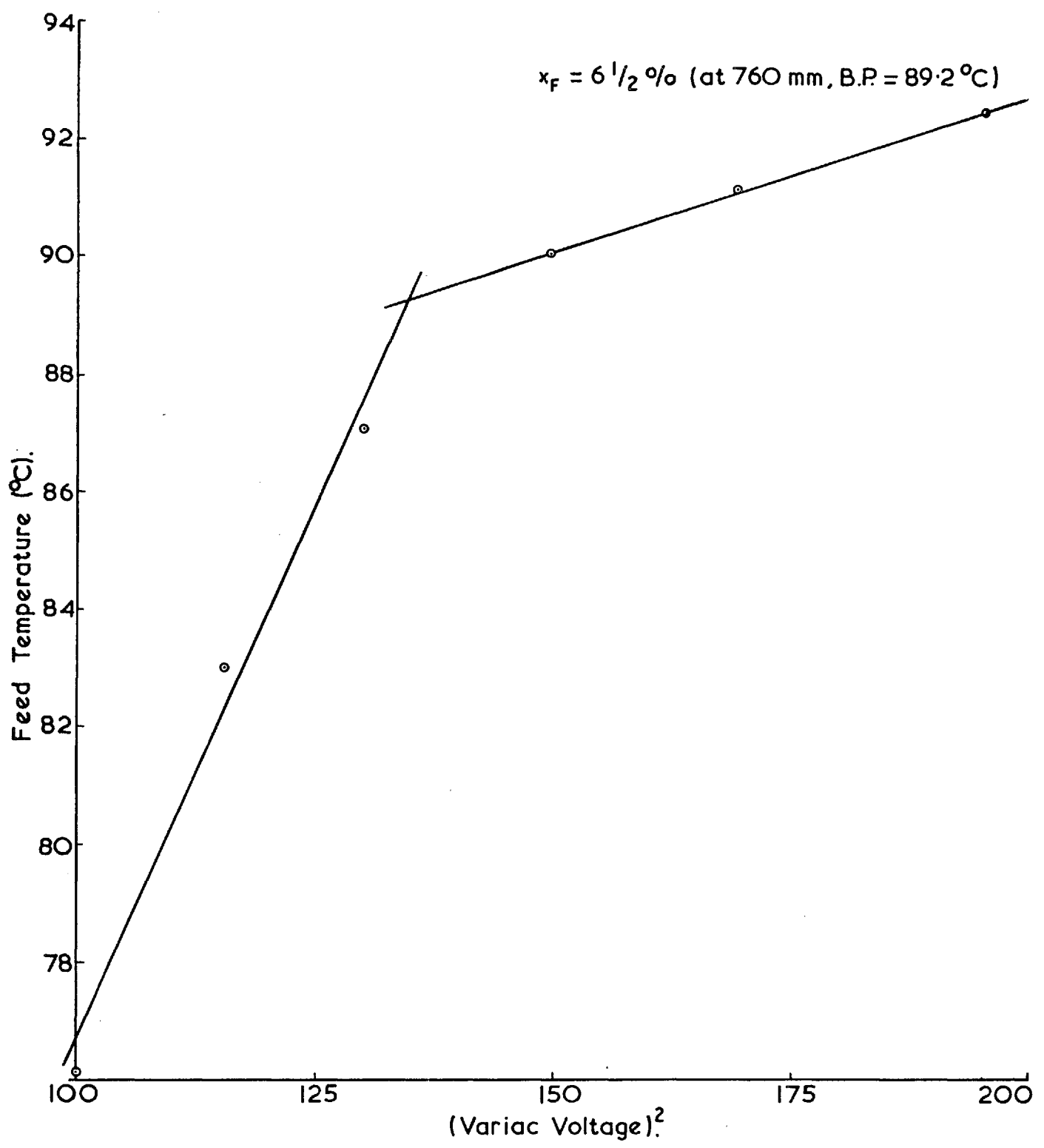


Fig. 2.9.1. EXPERIMENT TO DETERMINE BOILING POINT OF FEED.

TABLE 2.10.1

Input Data For Steady-State Programme

Symbol	Meaning	Typical value
b1	No meaning assigned	0
b2	Heat interval	0
b3	Latent Heat Term	1.48
b4	Heat Loss	0
b5	No. of Plates	6
b6	L_T (measured)	0.521
b7	Feed-Plate	3
b8	q	0
b9	Reflux	5
b10	Closing value for iterations	0.01
b11	Maximum No. of iterations	24
b12	Feed composition (as Molar Fraction)	0.052
b13	Feed-rate (cc/min)	95
b14	Initial guess at x_T	0.82
b15	$[\text{Heat (max)} - \text{Heat (min)}] \div b2$	0
b16	Heat (min)	1.75
b17	No meaning assigned	0

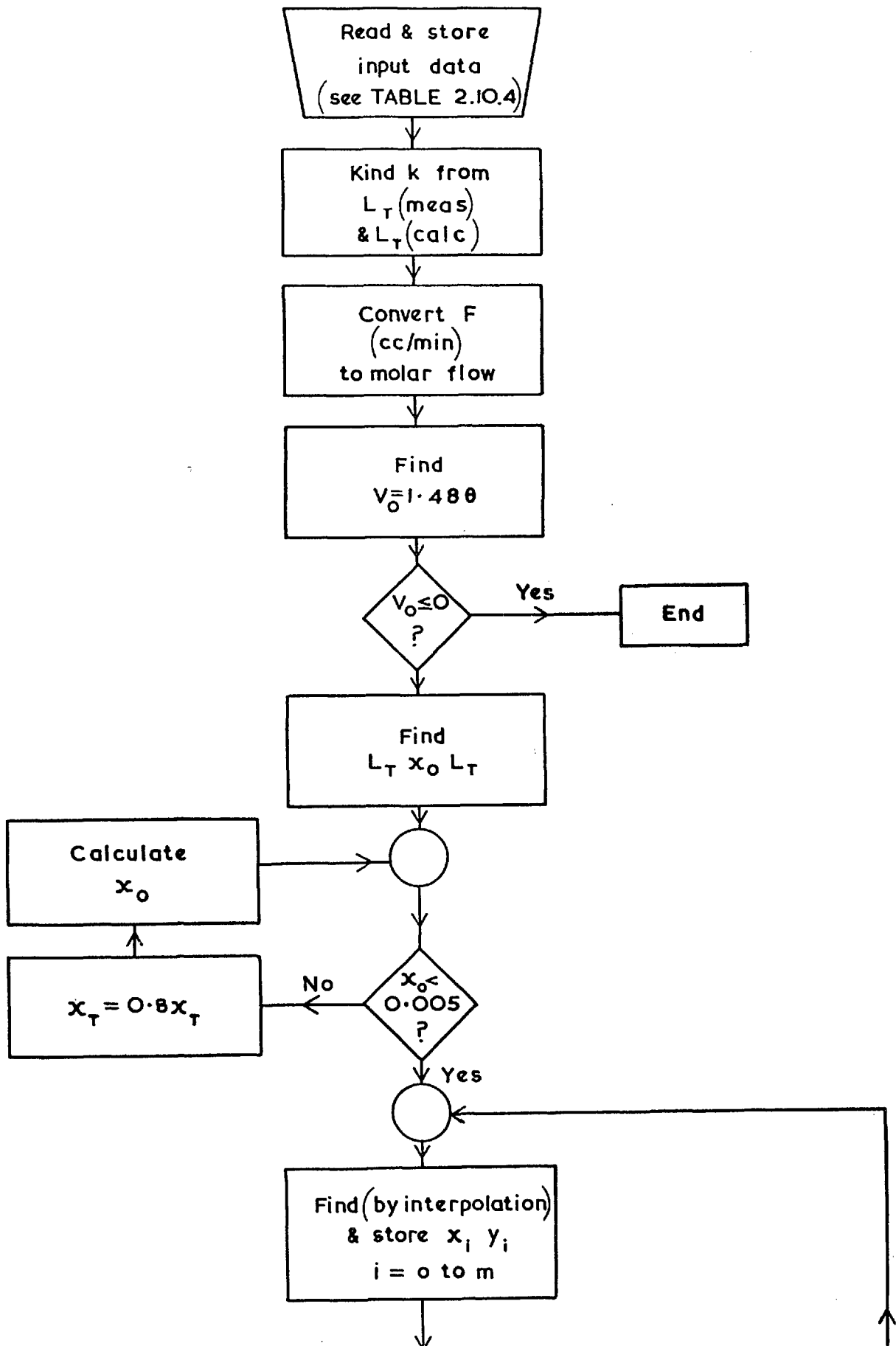


Fig. 2.10.1.

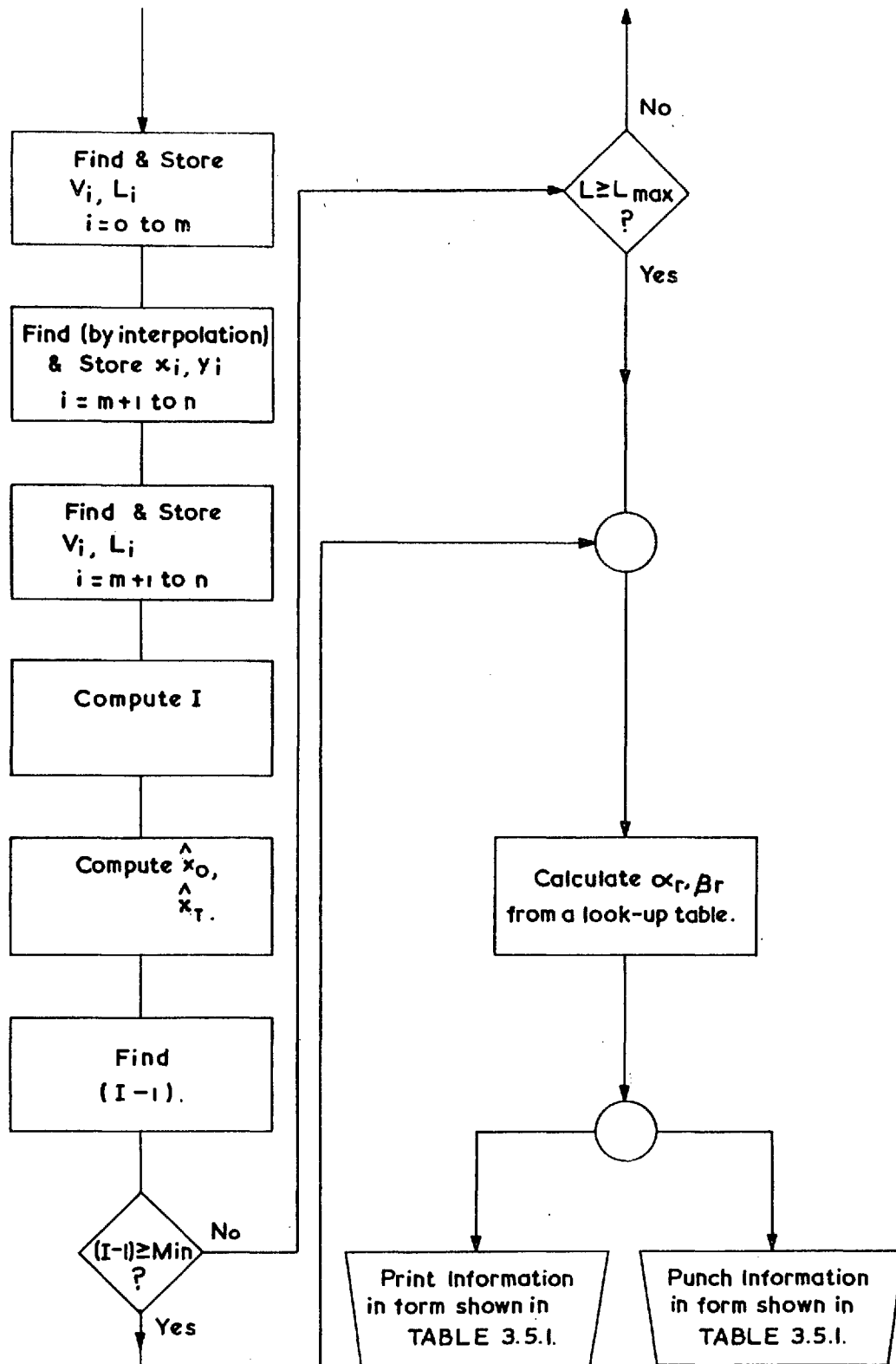


Fig. 2.10.1.
(continued)

FLOW DIAGRAM OF STEADY-STATE PROGRAMME.

Using equation 2.2.1 (with the left-hand side set to zero) and starting from x_0 and x_T , the compositions on each plate are calculated iteratively towards the feed-plate. A criterion function is now found:

$$I^2 = \frac{q F z + V_m y_m}{L_{m+1} x_{m+1} + L_T x_T} \cdot \frac{V_m y_m + L_0 x_0}{(1-q) F z_F + L_{m+1} x_{m+1}} \quad \dots (2.10.5)$$

and this equation has the value unity when the mass balance is attained, as can be seen from equations 2.10.3 and 2.10.4. If $I > 1$, then either x_0 is too rich or x_T is too lean i.e. $\frac{x_T}{x_0}$ is too small and vice versa. Since x_0 has been found from x_T this means that x_T was chosen too lean and hence for the next iteration it should be increased. Peiser proposed that one way of improving the ratio $\frac{x_T}{x_0}$ is to use $\frac{\hat{x}_T}{\hat{x}_0}$ as the new ratio.

Where

$$\frac{\hat{x}_T}{\hat{x}_0} = I \cdot \frac{x_T}{x_0} .$$

Both sets of values for x_0 and x_T are to satisfy the mass-balance equations 2.3.9 and 2.3.10 and hence the absolute values of $\hat{x}_T$ and $\hat{x}_0$ are

$$\hat{x}_0 = \frac{F z_F}{L_0 + L_T I \frac{x_T}{x_0}} \quad \dots (2.10.6)$$

$$\hat{x}_T = I \cdot \frac{x_T}{x_0} \cdot \hat{x}_0 \quad \dots (2.10.7)$$

The iterative procedure is continued until I approaches some value arbitrarily close to unity.

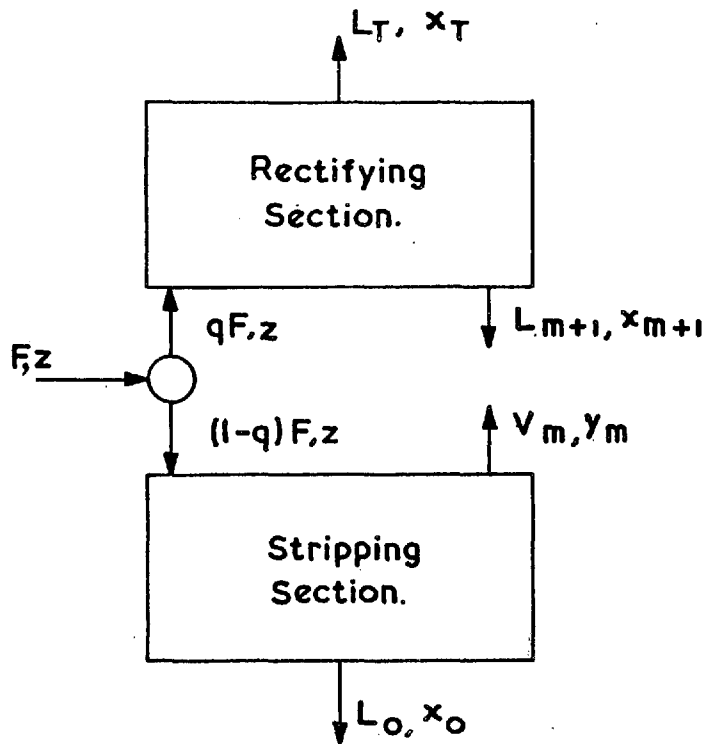


Figure 2.10.2

As an example the following steady-state information was supplied to the programme:

Heat = 1.2 kW, Reflux = 7:1, z_F = 5.85% BM

F = 49 cc/mm, Feed-plate No. 3, L_T = .191 moles/min.

(The measured top and bottom compositions were 70.5% and 0% respectively.)

The programme took 4 iterations to reduce $|I-1|$ to 3.03×10^{-3} and values of x_r , V_r and l_r found were

r	0	1	2	3	4	5	6	7
x_r	1.28×10^{-3}	4.97×10^{-3}	1.7×10^{-2}	5.6×10^{-2}	2.7×10^{-1}	5.3×10^{-1}	6.5×10^{-1}	7.2×10^{-1}
V_r	1.776	1.738	1.701	1.665	1.629	1.595	1.561	1.528
l_r	2.211	3.949	3.912	3.876	1.442	1.407	1.374	1.337

This information is shown on the McCabe-Thiele diagram of Figure 2.10.3 which is similar to that of Figure 2.5.1 except that here the plate efficiencies are assumed to be 100%, $q = 0$ and $k \neq 0$.

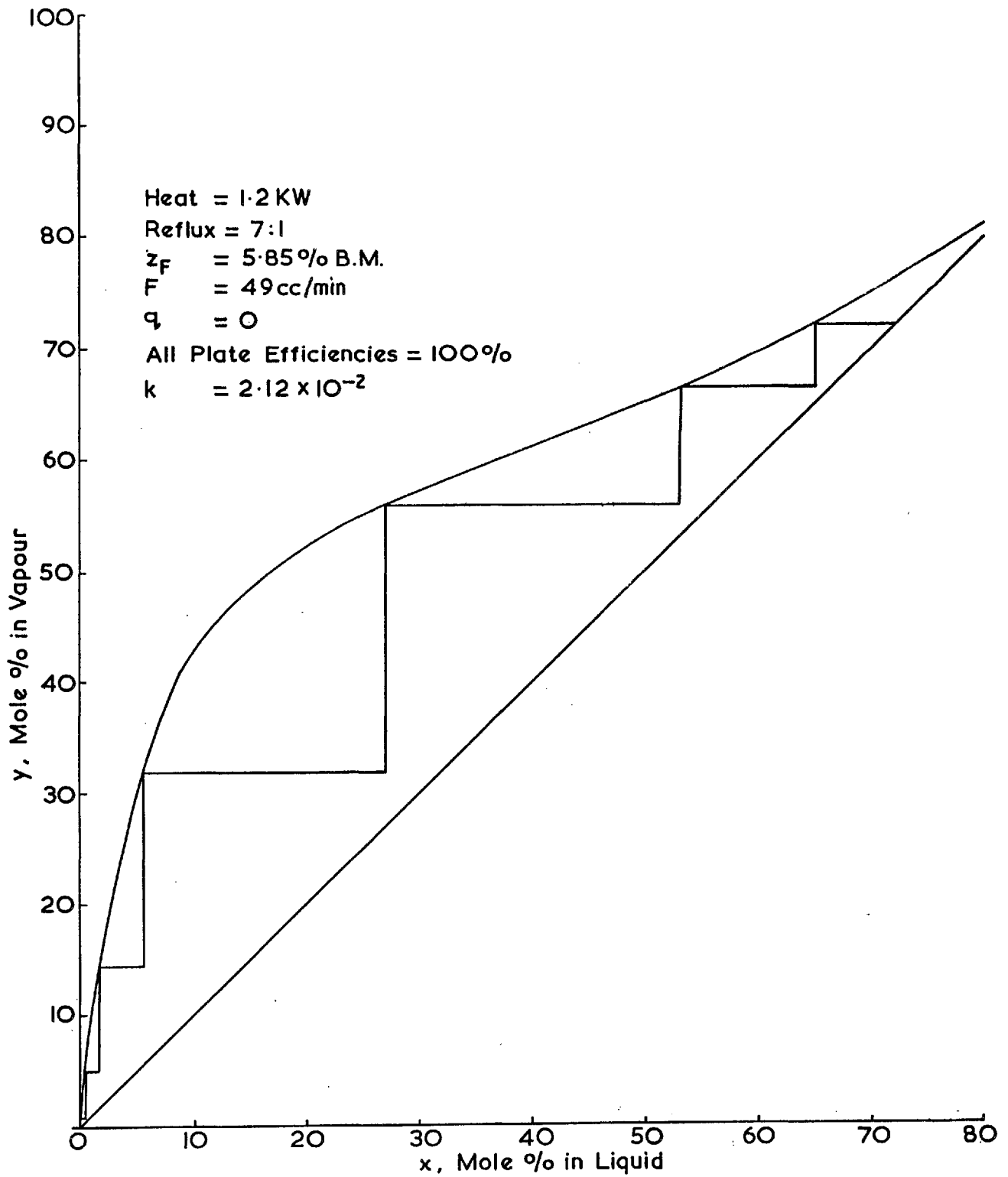


Fig. 2.10.3.

McCABE - THIELE DIAGRAM

2.11 Comparison of Theory with Experiment

The distillation column was run to steady-state under various operating conditions and measurements were taken of top-product flow-rate and composition, bottom composition, and attempts were also made to measure the plate compositions. The latter required the removal of a sample of the liquid from each plate and its weighing in a specific gravity bottle. There is no guarantee however that the sample represented the average composition on the plate, and due to the discrepancy between the calculated values and the measured ones these measurements were ultimately discontinued. TABLE 2.11.1 shows some of the results obtained, the columns marked *m* referring to measured values while *c* refer to calculated values. The point to notice is the close agreement at the extremes of the column rather than the disagreement at intermediate points.

It is considered that the results validate the steady-state model and show how the vapour and liquid flow-rates throughout the column may be easily calculated without recourse to an additional set of equations involving enthalpy balances. In the next chapter it is shown how this programme may be incorporated into a larger scheme for finding dynamic models.

TABLE 2.11.1

ZF (% BM)	F (cc/min)	R	H (K.W.)	m (Feed-Plate)	k	LT (moles/min)	Plate Compositions (% B.M.)															
							0 (STILL)	1		2		3		4		5		6		7		
								c	m	c	m	c	m	c	m	c	m	c	m	c	m	c
4	100	7	1.74	3	.0225	.275	0.16	0	0.52	0.5	1.44	4.0	3.74	6.5	18.8	29.9	48.5	53.5	63.1	63.5	71.2	71.5
4.6	100	5	1.69	3	.0091	.391	0.06	0	0.2	0	0.57	0	1.52	0.95	2.24	0.95	7.9	6.6	33.5	31.5	58.4	57.5
4.87	100	5	2.04	3	.0285	.411	0.04	0	0.16	0	0.49	0.4	1.44	1.25	2.37	1.75	8.45	15.6	34.4	48.5	58.7	62
5.8	100	4	1.575	3	.023	.395	0.18	-	0.59	3.5	1.6	4	4.07	10	16.2	38.5	44.6	57	60.7	64.5	69.9	71.5
6.0	100	4	1.95	3	.0015	.571	0.046	-	.165	-	.519	-	1.56	-	1.79	-	3.76	-	17.5	-	51	54
6.0	100	4	1.51	3	.006	.428	0.13	0	.41	0	1.09	0.6	2.76	2.2	8.16	14.5	31.1	42	55.3	56.5	67.2	67.5
5.0	95	2.95	1.53	3	.038	.438	0.08	0	0.26	0	0.71	0	1.8	1.5	2.4	1.5	5.1	3.5	21.4	34	53.3	56
5.0	95	3.7	1.53	3	.037	.369	0.09	-	0.29	-	0.8	-	2	-	4.3	-	15.9	-	45.3	-	63.1	65.5
4.35	95	5	1.7	3	.034	.330	0.06	-	0.21	-	0.6	-	1.6	-	3.4	-	13.7	-	43.6	-	62.4	65.5
4.35	95	4	1.47	3	.037	.334	0.09	-	0.28	-	0.74	-	1.8	-	3.4	-	12.3	-	40.8	-	61.3	64.5

CHAPTER 3. A Dynamic Model of a Binary Distillation Column

3.1 Introduction

This chapter shows how the equations describing the dynamic behaviour of a Binary Distillation system may be derived. For an n plate column, $n+1$ equations are needed and the resulting description is a vector-matrix one, the state-vector being the composition on each plate. The assumptions involved in deriving the model are discussed and where possible justified. A computer programme designed to simulate the dynamic behaviour of the distillation column is described and simulated step responses are compared with those obtained from plant experiments. A method of simplifying the model from an $(n+1)$ th order vector-matrix form to a 2nd-order z transform type is also discussed and the results of applying this technique to the plant model are compared with PRBS experiments performed on the plant.

3.2 Dynamic Equations

In section 2.2 an equation was derived which described the general mass-balance on a plate of a binary distillation column. This equation is repeated here for convenience;

$$\frac{d}{dt} (H_r x_r + h_r y_r) = V_r y_{r-1} - V_{r+1} y_r + L_{r+1} x_{r+1} - L_r x_r + F_r z_r (1-q_r) \quad \dots (3.2.1)$$

If the vapour hold-up h_r can be ignored we have

$$\frac{d}{dt} (H_r x_r) = V_r y_{r-1} - V_{r+1} y_r + L_{r+1} x_{r+1} - L_r x_r + F_r z_r \times (1-q_r) \quad \dots (3.2.2)$$

Also, if the feed temperature is such that q_r is zero and if variations in H_r can be neglected (3.2.2) becomes

$$H_r \dot{x}_r = V_r y_{r-1} - V_{r+1} y_r + L_{r+1} x_{r+1} - L_r x_r + F_r z_r \quad \dots (3.2.3)$$

This equation describes the general situation at plate r , but it is also applicable to the still of a distillation column (where $r = 0$) if

$y_{r-1} = 0$. For the top-plate of a distillation column, $r = n$ and assuming all the vapour from plate n is condensed to liquid then $x_{n+1} = y_n$ and thus

$$H_n \dot{x}_n = V_n y_{n-1} - V_{n+1} y_n + L_{n+1} y_n - L_n x_n \quad \dots (3.2.4)$$

The complete set of equations for the column can now be written;

$$\dot{x}_r = -\frac{L_r}{H_r} x_r + \frac{L_{r+1}}{H_r} x_{r+1} + \frac{V_r y_{r-1} - V_{r+1} y_r}{H_r} + \frac{F_r z_r}{H_r} \quad \dots (3.2.5)$$

($0 \leq r < n$)

and

$$\dot{x}_n = -\frac{L_n}{H_n} x_n + \frac{V_n y_{n-1} - (V_{n+1} - L_{n+1}) y_n}{H_n} \quad \dots (3.2.6)$$

i.e.,

$$\dot{x} = A_1 x + D_1 + m \quad \dots (3.2.7)$$

Where

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}, \quad A_1 = \begin{bmatrix} -\frac{L_0}{H_0}, & +\frac{L_1}{H_0}, & 0, & \dots \\ 0, & -\frac{L_1}{H_1}, & +\frac{L_2}{H_1}, & 0, \dots \\ 0, & & & \\ \vdots & & & \\ 0 & & & -\frac{L_n}{H_n} \end{bmatrix},$$

$$D_1 = \begin{bmatrix} -\frac{V_1 y_0}{H_0} \\ \frac{V_1 y_0 - V_2 y_1}{H_1} \\ \vdots \\ \frac{V_n y_{n-1} - (V_{n+1} - L_{n+1}) y_n}{H_n} \end{bmatrix}, \quad m = \begin{bmatrix} \frac{F_0 z_0}{H_0} \\ \frac{F_1 z_1}{H_1} \\ \vdots \\ \frac{F_n z_n}{H_n} \end{bmatrix}$$

Although these equations are completely general, in the following, attention will be restricted to a 6 plate column, i.e. $n = 6$.

Attention will also be restricted to the case where $F_r z_r = 0$ except for $r = 3$. Thus

$$m = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{F_3 z_3}{H_3} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

If it is assumed that the relationship between x_r and y_r can be written as

$$y_r = \alpha_r x_r + \beta_r, \quad \dots (3.2.8)$$

Then equations (3.2.5), (3.2.6) can be written

$$\begin{aligned} \dot{x}_r &= \frac{V_r \alpha_{r-1}}{H_r} x_{r-1} + \frac{(-L_r - V_{r+1} \alpha_r)}{H_r} x_r + \frac{L_{r+1} x_{r+1}}{H_r} \\ &+ \frac{V_r \beta_{r-1} - V_{r+1} \beta_r}{H_r} + \frac{F_r z_r}{H_r} \end{aligned} \quad \dots (3.2.9)$$

$$\begin{aligned} \dot{x}_n &= \frac{V_n \alpha_{n-1}}{H_n} x_{n-1} + \frac{(L_{n+1} \alpha_n - L_n - V_{n+1} \alpha_n)}{H_n} x_n \\ &+ \frac{V_n \beta_{n-1} - V_{n+1} \beta_n + L_{n+1} \beta_n}{H_n} + \frac{F_n z_n}{H_n} \end{aligned} \quad \dots (3.2.10)$$

and these equations may be written in the form

$$\dot{\bar{x}} = A\bar{x} + D + m \quad \dots (3.2.11)$$

where x , m are as defined above, but now,

$$A = \begin{bmatrix} -\frac{L_0 - V_1 \alpha_0}{H_0}, \frac{L_1}{H_0}, 0, 0, \dots \\ \frac{V_1 \alpha_0}{H_1}, -\frac{L_1 - V_2 \alpha_1}{H_1}, \frac{L_2}{H_1}, 0, \dots \\ 0, \frac{V_2 \alpha_1}{H_2}, \dots \\ 0 \\ \vdots \\ 0, \dots, 0, \frac{V_6 \alpha_5}{H_6}, \frac{L_7 \alpha_6 - L_6 - V_7 \alpha_6}{H_6} \end{bmatrix}$$

$$D = \begin{bmatrix} -\frac{V_1 \beta_0}{H_0} \\ \frac{V_1 \beta_0 - V_2 \beta_1}{H_1} \\ \vdots \\ \frac{V_6 \beta_5 - V_7 \beta_6 + L_7 \beta_6}{H_6} \end{bmatrix}$$

The use of the alternative sets of equations (3.2.7) and (3.2.11) in describing the process will be discussed later.

3.3 Assumptions

In deriving equation (3.2.3) the following assumptions have been made:

- 1) The plate liquid composition is homogeneous.
- 2) The vapour composition is homogeneous.
- 3) The vapour hold-up is negligible.
- 4) Changes in the molar hold-up are negligible.
- 5) The pressure throughout the column is constant.

In the work which follows it is further assumed that the static and dynamic plate efficiencies are 100% and that the pressure in the column is 760 mm of mercury. When using the relationship (3.2.8) it is assumed that α_r , β_r are functions of x_r only.

None of these assumptions are drastic and they could be modified if thought to introduce serious error. Since the particular column under

consideration has a small diameter then assumption 1) is not unreasonable, and the author has shown ⁽⁹⁾ for the worst possible case that the vapour hold-up is less than 10% of the liquid hold-up, justifying assumption 3). In section 3.4 it is assumed that the volumetric hold-up on each plate is constant and known and thus the molar hold-up can be recomputed at intervals. The assumption of a constant pressure of 760 mm is justified on the grounds of observation: ± 10 mm being the maximum observed extremes.

3.4 Discussion of Dynamics Simulation

Using equations (3.2.7) or (3.2.11), it is possible to write a digital computer programme to simulate dynamic changes in the plant. It is known that the solution to ⁽¹⁰⁾

$$\dot{x}(t) = A(t) x(t) + B(t) u(t) \quad \dots (3.4.1)$$

is

$$x(t) = \phi(t, t_0) x(t_0) + \int_{t_0}^t \phi(t, \tau) B(\tau) u(\tau) d\tau \quad \dots (3.4.2)$$

and the solution to (3.2.11) is thus

$$x(t) = \phi(t, t_0) x(t_0) + \int_{t_0}^t \phi(t, \tau) [D(\tau) + m(\tau)] d\tau \quad \dots (3.4.3)$$

If the process is stationary then A and D are time invariant and (3.4.3) becomes

$$x(t) = \phi(t-t_0) x(t_0) + \int_{t_0}^t \phi(t-\tau) [D + m(\tau)] d\tau \quad \dots (3.4.4)$$

where

$$\begin{aligned} \phi &= e^{AT} \\ &= I + AT + \frac{A^2 T^2}{2!} + \dots + \frac{A^r T^r}{r!} \dots \end{aligned} \quad \dots (3.4.5)$$

T = Sampling Interval.

If the time interval $(t-t_0)$ is the sampling interval T i.e.

$$t_0 = kT$$

$$t = (k+1)T \quad \text{and}$$

if $m(\tau)$ is constant over the time interval t_0 to t and equal to $m(kT)$ then (3.4.4) becomes

$$x(k+1)T = \phi(T) x(kT) + \int_0^T \phi(\tau) d\tau \times [D + m(kT)] \quad \dots (3.4.6)$$

$$= \phi(T) x(kT) + A^{-1} (e^{AT} - I) [D + m(kT)] \quad \dots (3.4.7)$$

In the column under consideration, 4 independent variables were available: the Reflux R , Heat H , feed flow-rate F , and feed composition x_F . Only changes in the latter two affect m , while R , H , F affect the vapour and liquid flow-rates throughout the column, and from equations (3.2.5), (3.2.6), (3.2.9) and (3.2.10) it can be seen that A_1 , A and D_1 , D will be time-varying. If it is assumed however that these matrices are stationary during each sampling interval and they are re-calculated for the next sample interval, then equation (3.4.7) can be used. Using bubble-cap plates (which have an over-flow weir), it is reasonable to assume that the volumetric hold-up on each plate is constant. If the composition is known then the molar hold-up can be found from

$$H_r = \frac{\text{Volume}}{18 + 41 \times x_r} \quad \dots (3.4.8)$$

Several attempts were made to find the volumetric hold-up on each plate, and an average figure of 88 mls per plate was obtained. This makes the total plate hold-up less than 10% of the still hold-up (which was about 5.6 litres) and it was expected that any variations in individual plate hold-ups would have negligible effect on the dynamics. The level of the liquid in the still

was regulated by a photo-electric level controller. The sampling interval used throughout the experiments was 96 seconds as determined from preliminary step response experiments. During 96 seconds the plate compositions change by a very small amount and hence H_r remains constant over this period.

It was found that the response to changes in liquid and vapour flow-rates was very fast and although extensive investigation was not performed, the following experimental evidence was obtained. The values of x_T and L_T were found at steady-state (sample 0 in TABLE 3.4.1) and a step change in Reflux introduced. After 96 secs, and at regular intervals through the ensuing step response, composition and flow measurements were taken enabling the molar flow-rate L_T to be found. Two sets of results are shown in TABLE 3.4.1 where sample 1 represents the values of x_T , L_T after 96 seconds. It can be seen that the top-product flow-rate appears to reach its final value within one sample interval. (Other measurements indicate that it may even be achieved in 32", the timing period of the reflux mechanism).

TABLE 3.4.1

Sample No.	x_T %BM	Time for 25 mls (secs)	L_T (moles/min)	x_T	T (secs)	L_T
0 (t=0)	$62\frac{1}{2}$	-	.268	47	-	.363
1 (t=96")	$62\frac{1}{2}$	1.64	.360	47	3.2	.216
2	$53\frac{1}{2}$	1.8	.358	$59\frac{1}{2}$	2.91	.208
3	51	1.87	.354	$61\frac{1}{2}$	2.8	.213
4	$49\frac{1}{2}$	1.9	.355	$64\frac{1}{2}$	2.8	.210
5	$48\frac{1}{2}$	1.97	.346	66	2.71	.210
6 (t $\rightarrow\infty$)	47	-	.363	70	2.61	.210

Since L_T attains its final value in 96" it seems reasonable to infer that the internal liquid flows do likewise. Responses to vapour flow changes were also found to be very fast, but were more difficult to measure and no qualitative results are presented.

3.5 A Digital Computer Simulation Programme

Using equation (3.4.7) it is possible to write a Digital Computer programme to simulate the dynamic behaviour of the Distillation Column. Starting at $k = 0$, the initial conditions $x(0)$ and the starting matrices $A(0)$, $D(0)$, $m(0)$, $\phi(0)$ are required. These are calculated using the values of V_r , L_r , α_r , β_r etc obtained from the steady-state programme and the equations defining $A(0)$, $D(0)$, $m(0)$ which are (3.2.9), (3.2.10). The transition matrix $\phi(k)$ is found from $A(k)$ using (3.4.5) and summing to 15 terms. The new value of $x(k+1)$ is then found by using (3.4.7) and from this, the new value of H_r is found from (3.4.8). New values of α_r , β_r are calculated by linear interpolation on the vapour-liquid equilibrium curve. With reference to Fig.3.5.1, if x_r is the liquid composition on plate r , then a new value $x_r + \Delta$ is found by incrementing this value by 2½% and the corresponding values y_r and $y_r + \Delta y$ are found from linear interpolation in a look-up table by using the formulae

$$y_r = \frac{x_r}{\Delta} (b_i - b_{i-1}) - i (b_i - b_{i-1}) + b_i$$

$$y_{r+\Delta y} = \frac{x_r + \Delta}{\Delta} (b_{i+1} - b_i) - (i+1) (b_{i+1} - b_i) + b_{i+1}$$

where x_r lies between the i th and the $(i-1)$ th value on the look-up table and b_{i-1} , b_i , b_{i+1} are the values of y in the table.

The new values of α_r , β_r are then found from

$$\alpha_r = ([y_r + \Delta y] - y_r) / \Delta$$

$$\beta_r = y_r - \alpha_r \times x_r.$$

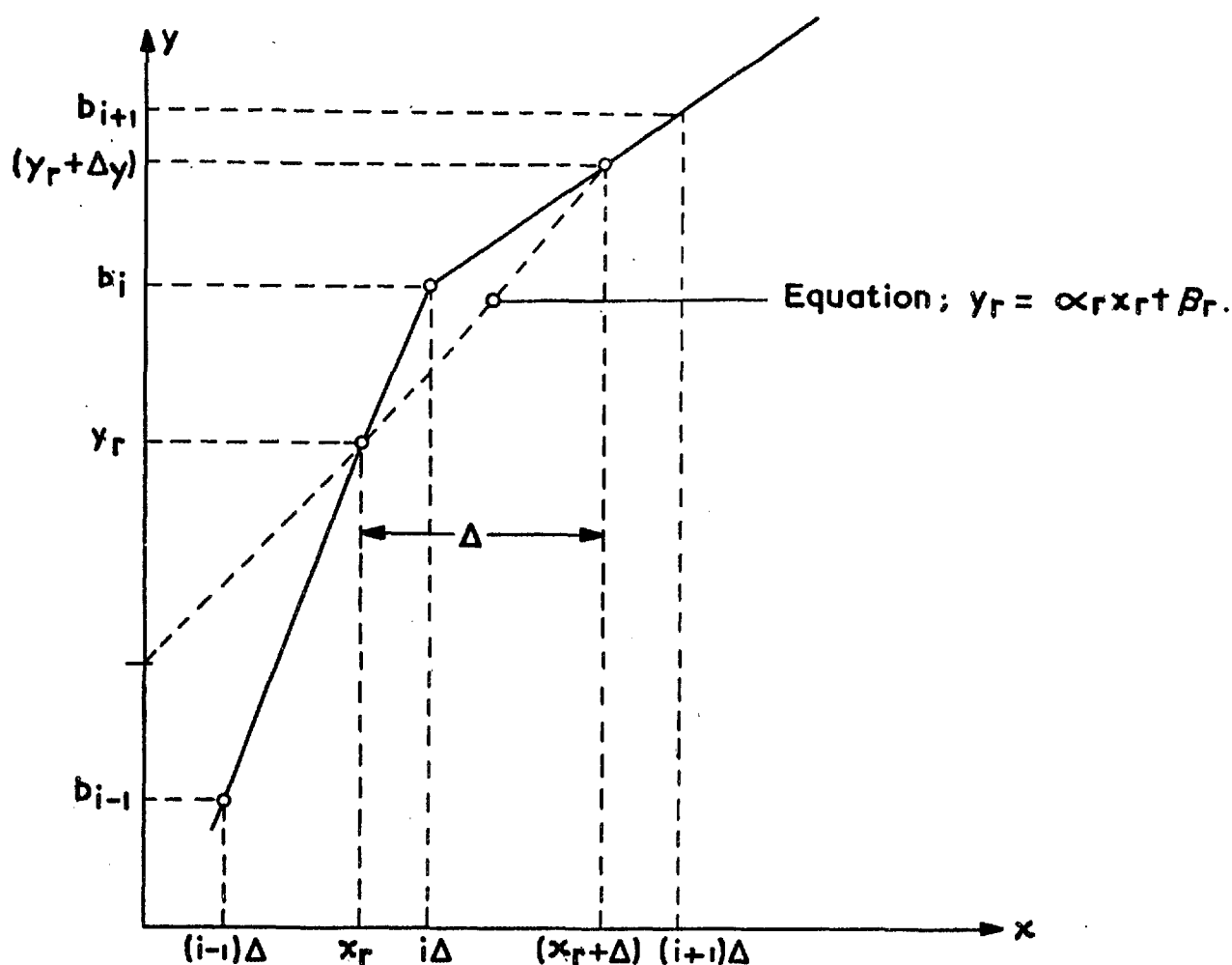


Fig. 3.5.1. INTERPOLATION OF y FROM x

The new values of A , D , m , ϕ are then calculated and the next value of $x(k+1)$ is obtained. This process is outlined in Fig. 3.5.2 which is the computer programme flow diagram used in simulating step and P.R.B.S. responses of the plant.

As it was easy to change the programme to simulate either (3.2.7) or (3.2.11), both sets of equations were tried. It was found however that (3.2.7) gave unstable solutions in some cases, while (3.2.11) was stable, and the reason for this is the assumption of stationarity implicit in (3.4.7). In the stripping section of the column a small change in x produces a large change in y (as can be seen from Fig. 2.5.1), whereas α , β will be almost constant in this section for considerable changes in y . Hence, over a certain (undetermined) period of time, D , in (3.2.7) will be non-stationary

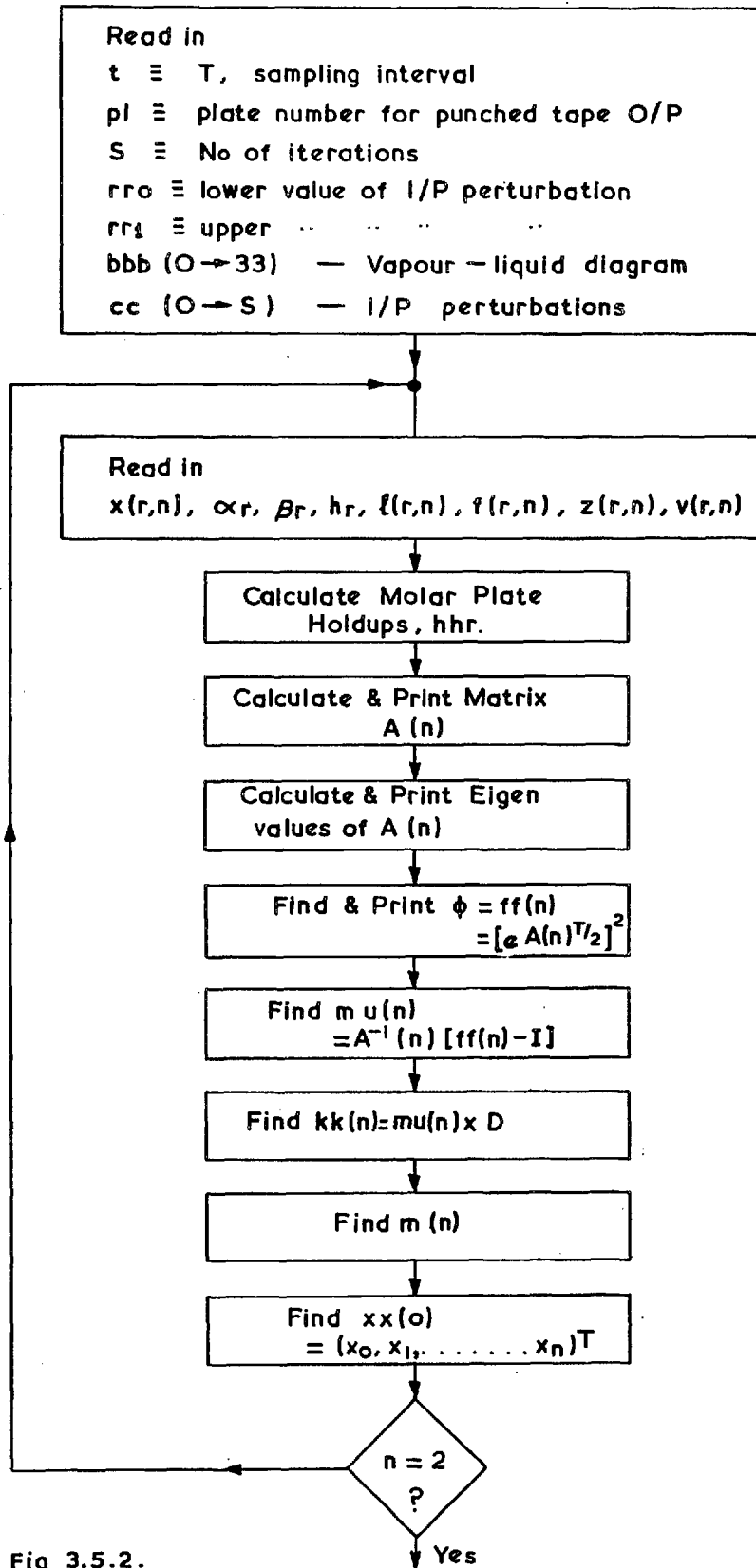


Fig 3.5.2.

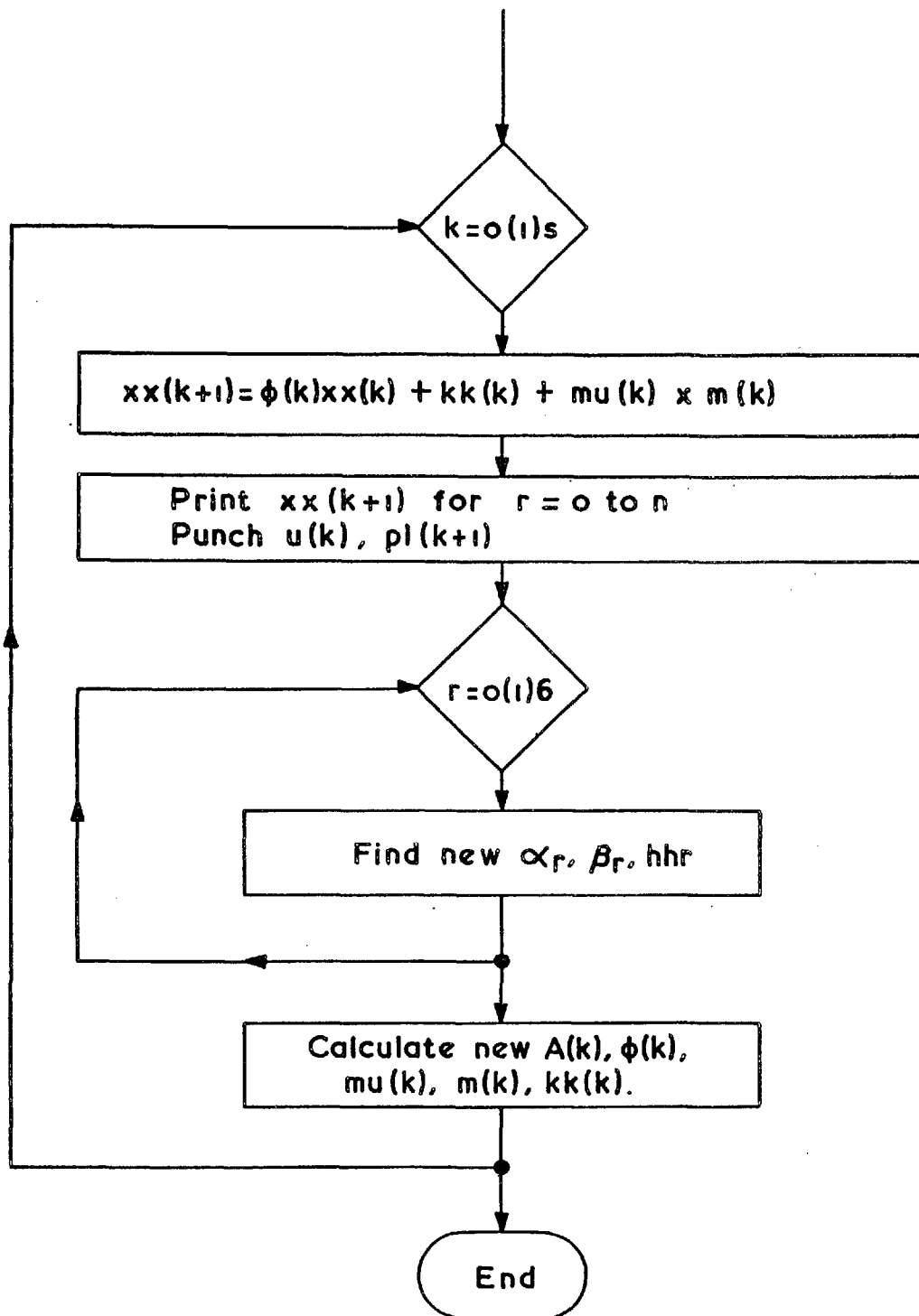


Fig. 3.5.2. FLOW DIAGRAM OF DYNAMIC SIMULATION PROGRAMME
(continued)

whereas D in (3.2.11) will be stationary. The results presented later were obtained using equations (3.2.8) to (3.2.11).

Since the vapour and liquid molar flow-rates change very quickly compared with the composition changes, in perturbing from one flow-rate to another, it is merely necessary to use the two Steady-State flow-rate values concerned and not intermediate values. For example, TABLE 3.5.1 shows two sets of information obtained from the steady-state programme, xr_1 is the initial composition vector and xr_2 is the final composition vector.

Operating Data 1	r	xr_1	αr_1	βr_1	hhr_1	lr_1	fr_1	zr_1	vr_1
Reflux 7:1	0	1.6×10^{-4}	9.7	0	5680	2.11	0	0	2.15
Heat 1.45 KW	1	7.02×10^{-4}	9.7	0	88	4.18	0	0	2.07
F 46.5 cc/min	2	2.7×10^{-3}	9.7	0	88	4.11	0	0	2.0
z_F 5% BM	3	9.98×10^{-3}	9.7	0	88	4.04	2.32	.05	1.94
	4	1.63×10^{-2}	6.3	.03	88	1.67	0	0	1.87
	5	4.84×10^{-2}	3.63	.115	88	1.61	0	0	1.81
	6	2.45×10^{-1}	.5	.42	88	1.55	0	0	1.75
	7	5.48×10^{-1}	.416	.447	0	1.48	0	0	1.68
Operating Data 2	r	xr_2	αr_2	βr_2	hhr_2	lr_2	fr_2	zr_2	vr_2
Reflux 10:1	0	2.69×10^{-4}	9.7	0	5680	2.15	0	0	2.15
Heat 1.45 KW	1	1.14×10^{-3}	9.7	0	88	4.25	0	0	2.10
F 46.5 cc/min	2	4.37×10^{-3}	9.7	0	88	4.21	0	0	2.06
z_F 5% BM	3	1.61×10^{-2}	6.3	0.03	88	4.17	2.32	0.05	2.02
	4	6.88×10^{-2}	2.68	0.18	88	1.82	0	0	1.98
	5	3.14×10^{-1}	.369	0.46	88	1.78	0	0	1.94
	6	5.64×10^{-1}	.460	0.415	88	1.74	0	0	1.91
	7	6.79×10^{-1}	.566	0.354	0	1.70	0	0	1.87

TABLE 3.5.1

It is desired to simulate a step response in Reflux from 7:1 to 10:1 with the other operating conditions as shown in the table. The simulation programme reads in the data (shown marked in heavy lines) and uses αr_1 , βr_1 , lr_2 , hhr_2 , fr_2 , zr_2 , vr_2 to calculate A , ϕ etc since it is assumed that the final values of v, l are attained during one sample interval. The values of αr , βr , hhr are recalculated at each sample interval since these are dependent on x . To simulate a step from 10:1 to 7:1 the reverse procedure is followed. To simulate P.R.B.S. experiments the programme selects either $(v, l, f, z)_1$ or $(v, l, f, z)_2$ depending upon whether the reflux is at one value or

the other. It would be a simple matter to extend the programme to handle 3-level signals or multi-channel perturbations if this was of interest.

3.6 Step Response Results

In order to validate the theoretical model, it is necessary to compare simulated step responses with those measured from the plant under the same conditions. For the present work, a comparison is made of the time responses of the top-product composition (measured in % By Moles). It was found that two sources of error were present in the results, both due to measurement difficulties. The first is that due to drift in the Dielectric Composition Meter (D.C.M.) and the second is an error which appears to be due to inaccurate measurement of feed flow rate. The drift in the D.C.M. was never greater than $\pm 2\%$ in the results quoted here and could be corrected for by taking (manual) composition measurements at the beginning and end of the transient. The second error source is a consistently low (calculated) value for top product composition at steady-state compared with the measured one, and is accounted for by increasing the measured feed flow-rate by about 5%. This correction merely has the effect of shifting the computed step response vertically by about 3%. As an example of this difficulty, the following typical result is presented. At steady-state the top-product molar flow-rate was found to be 3.63 moles/min at 47% B.M. The feed flow-rate was (apparently) 46.5 cc/min at 7.2% B.M. and using

$$F \text{ (moles/min)} = \frac{\text{cc/min} \times \text{density}}{78 + 28 \times \frac{F}{F_F}}$$

the feed molar flow-rate was 2.26 moles/min. Since the measured still composition was 0%, Fz_F should equal $L_T x_T$. It is found however that to obtain the measured value of $L_T x_T$ that F must have been about 2.37 moles/min i.e. there were 48.7 cc/min and not 46.5 cc/min being fed into the column. It is difficult to explain the discrepancy as all variables have been carefully measured, but it was concluded that the feed flow-rate was in error (as distinct from the other possibilities) as it was the most difficult to measure and seemed the most likely source of error.

With these corrections in mind, the results of 8 (corrected) step response tests are shown in Figures 3.6.1 to 3.6.8. Throughout all these tests the time axis scale is in units of 96 secs. There is also a delay of approximately two sample intervals between applying a Reflux step and top composition response. This is due to the delay in the condenser and piping and the D.C.M. and to make a fair comparison between the two curves, the theoretical response has been shifted forward by two time delays. Figures 3.6.1 to 3.6.4 show responses to step changes in reflux with the theoretical and experimental curves plotted on the same graph. The maximum error is found to be about 3% (Figure 3.6.1). Figure 3.6.5 shows the response to a step change in heat and there is an additional delay of 2 units, as predicted by the programme. The maximum error in this case is about $1\frac{1}{2}\%$. Figures 3.6.6 and 3.6.7 show the responses to step changes in feed composition. There is a delay of 4 time units due to the feed piping and valves etc., 2 unit delays for the condenser/DCM and 3 for the change in feed composition to reach the top plate. Finally, Figure 3.6.8 shows the response to a step change in feed flow-rate.

The agreement between theory and experiment is considered quite close, particularly in view of the possible sources of measurement error and the numerous assumptions involved. A further method of checking model validity is to simulate P.R.B.S. perturbations and compare the results with those obtained from the plant. This method was also investigated and the results are presented in section 3.9.

3.7 Model Simplification

The vector-matrix model described by equation (3.2.11) can be used in control schemes in two ways. Either the complete vector-matrix model may be incorporated into an optimal control scheme ⁽¹¹⁾ or, the model can be simplified to a form which is still an accurate representation of the process, but which enables less complex control theory to be applied. Vector-matrix control schemes require the state-vector to be measured or estimated at each control instant and in the case discussed here the elements of the state-vector

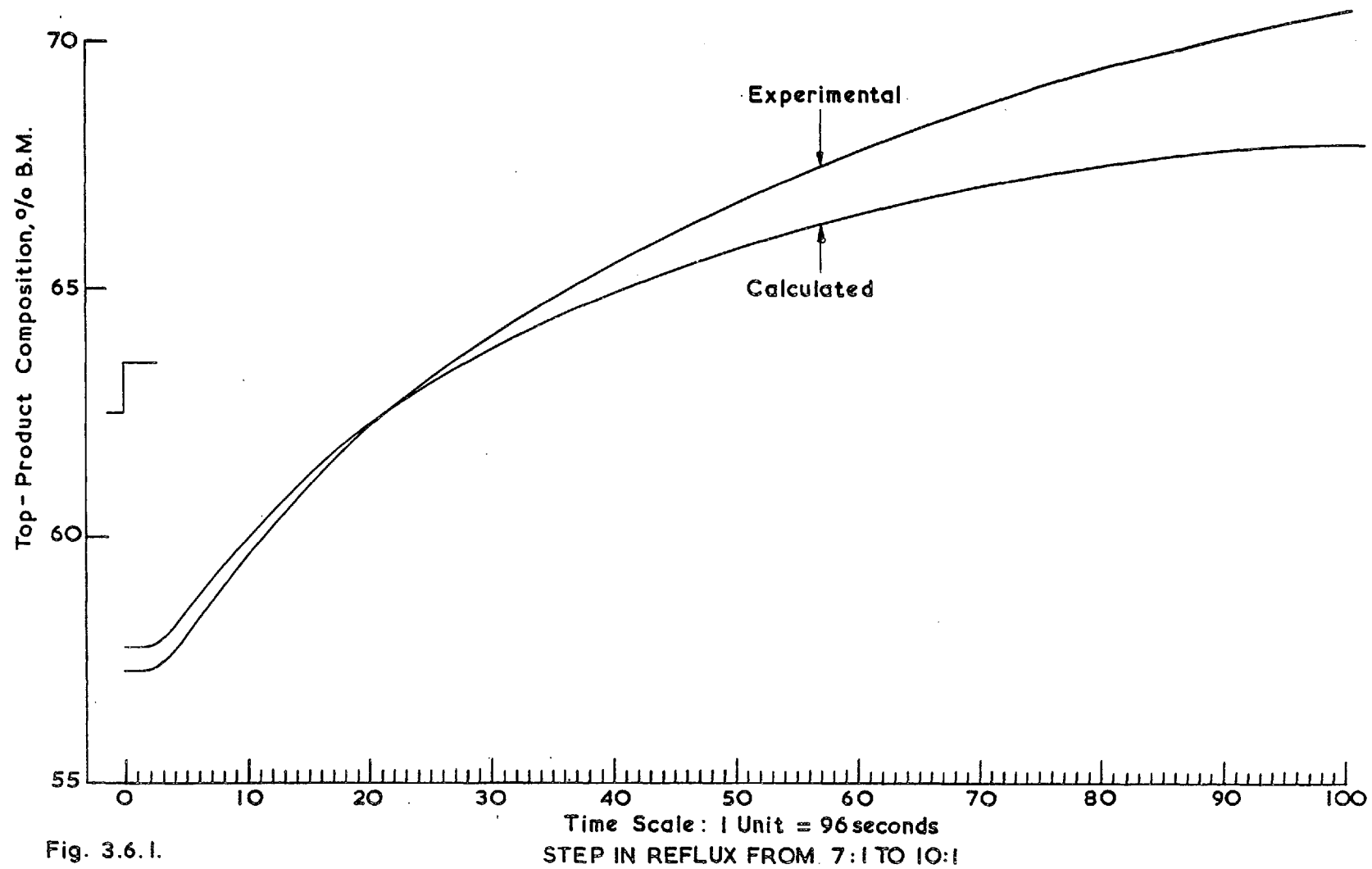
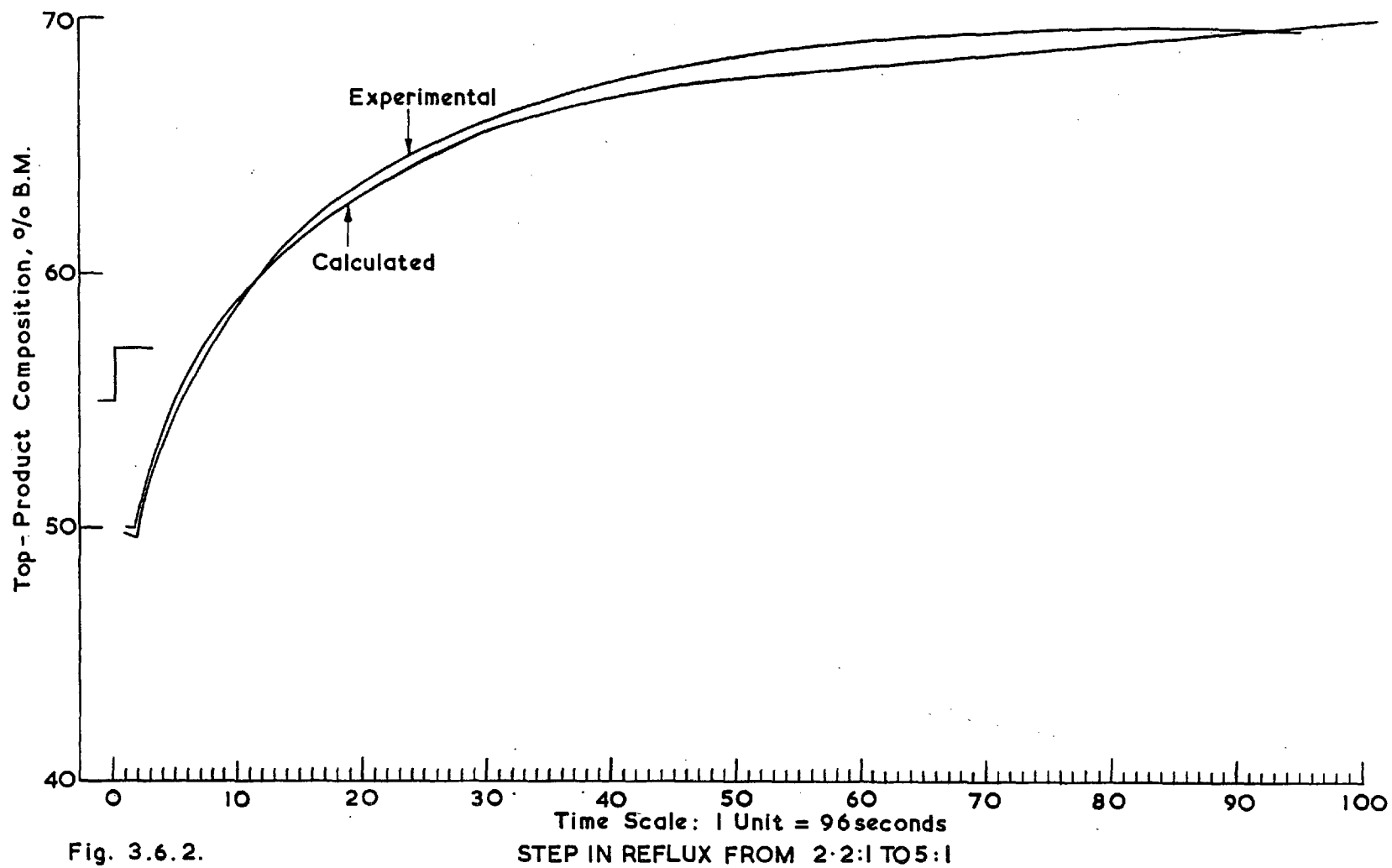


Fig. 3.6.1.



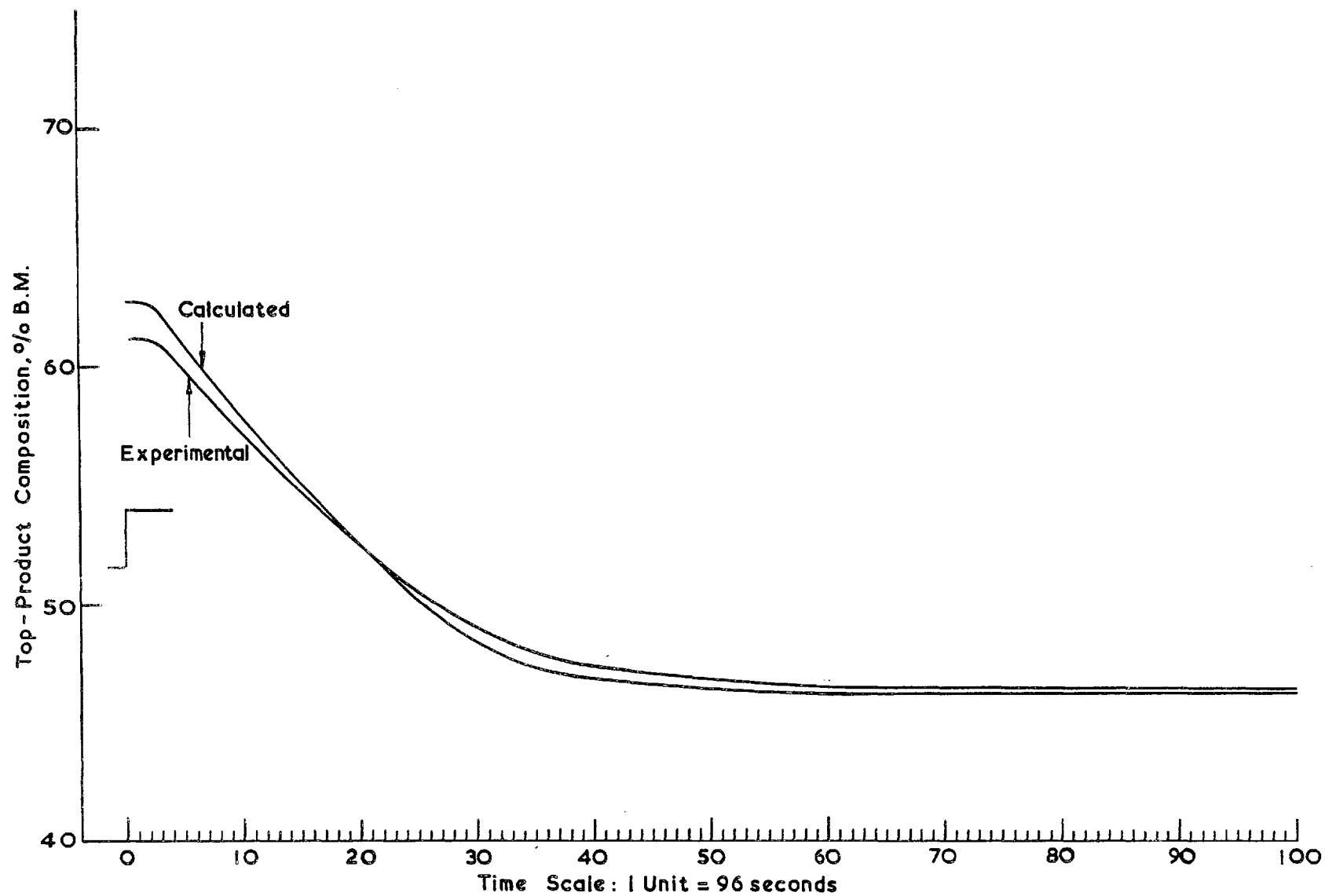


Fig. 3.6. 3.

STEP IN REFLUX FROM 5:1 TO 3:1

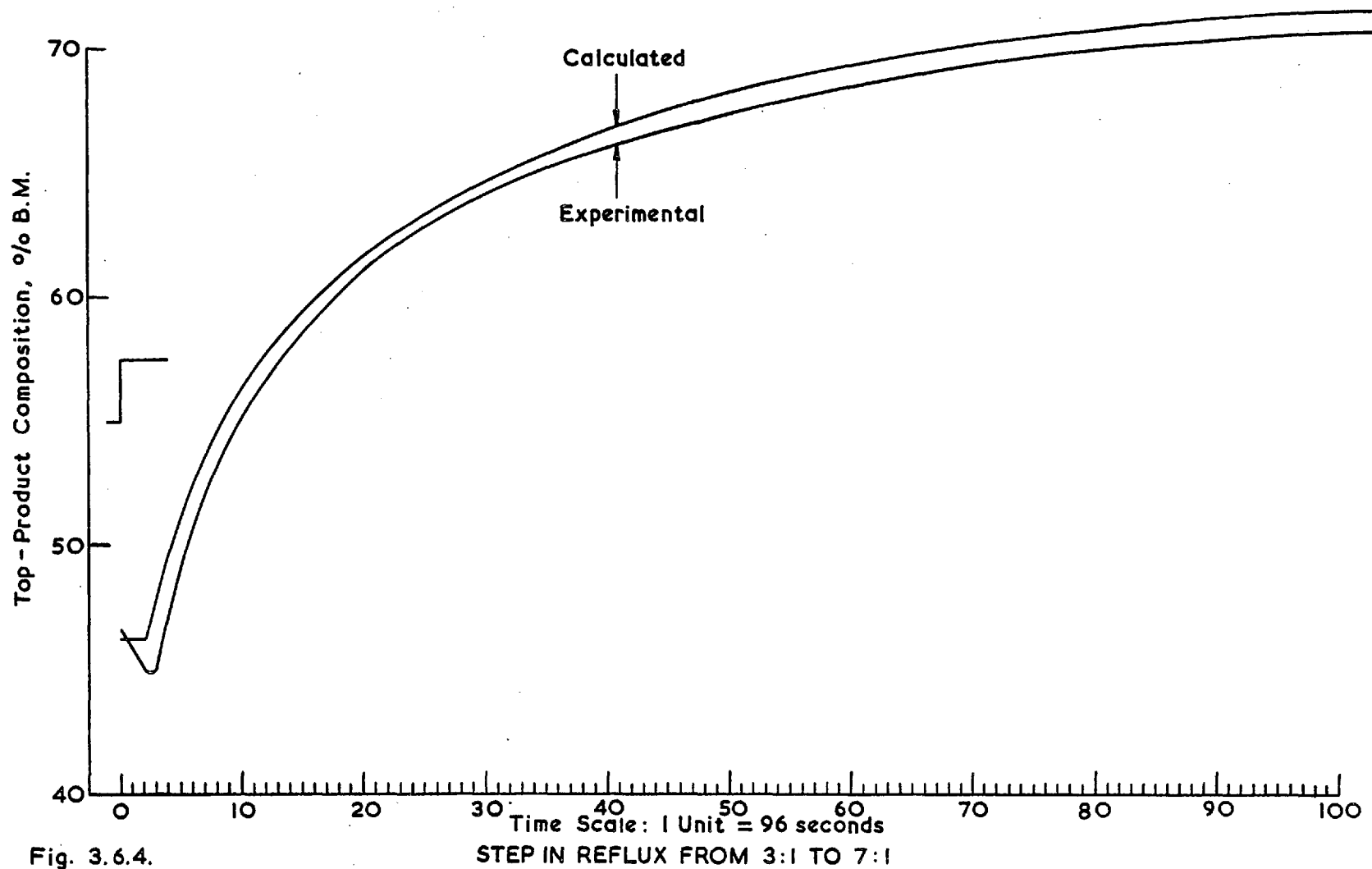


Fig. 3.6.4.

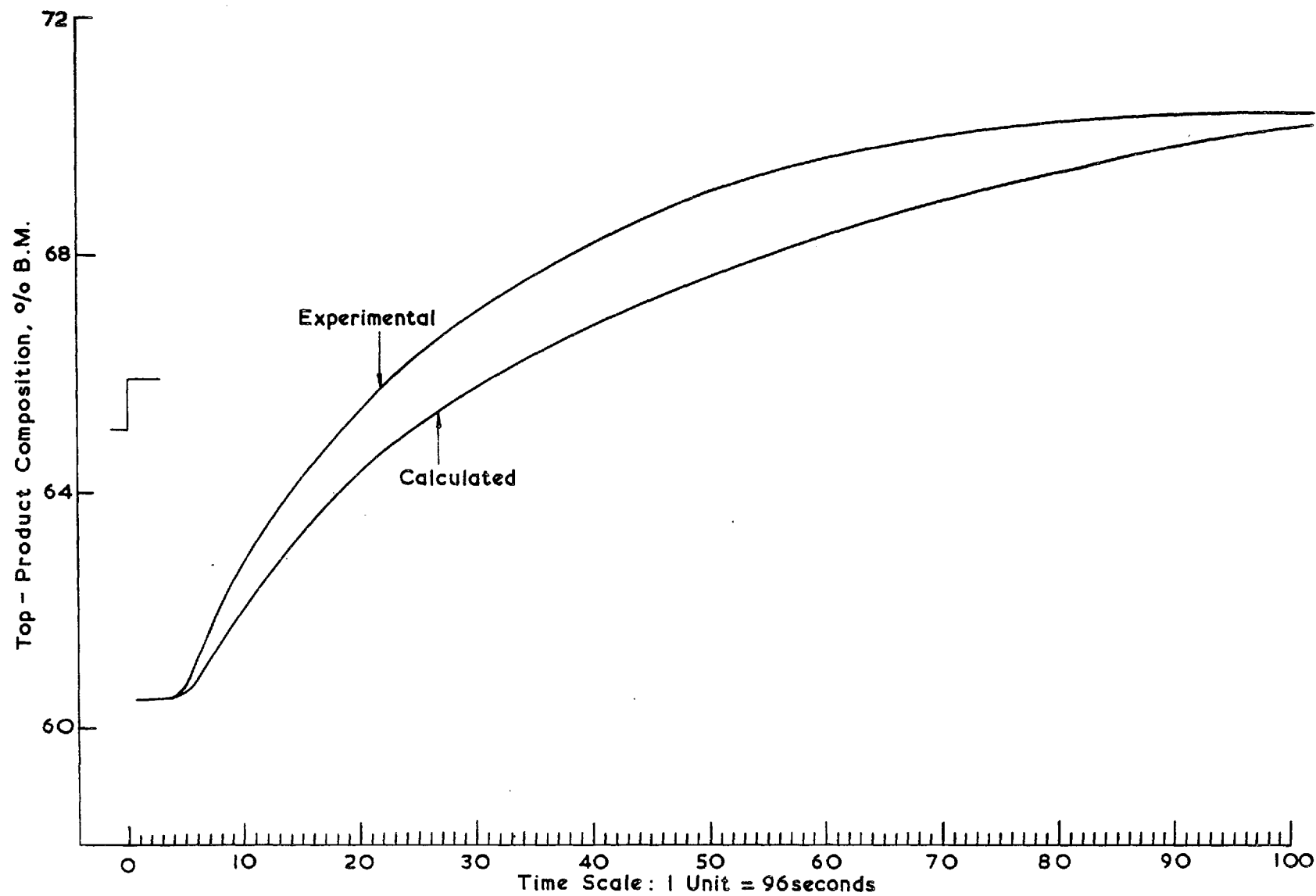


Fig. 3.6.5.

STEP IN HEAT FROM 1.5 TO 1.2 KW.

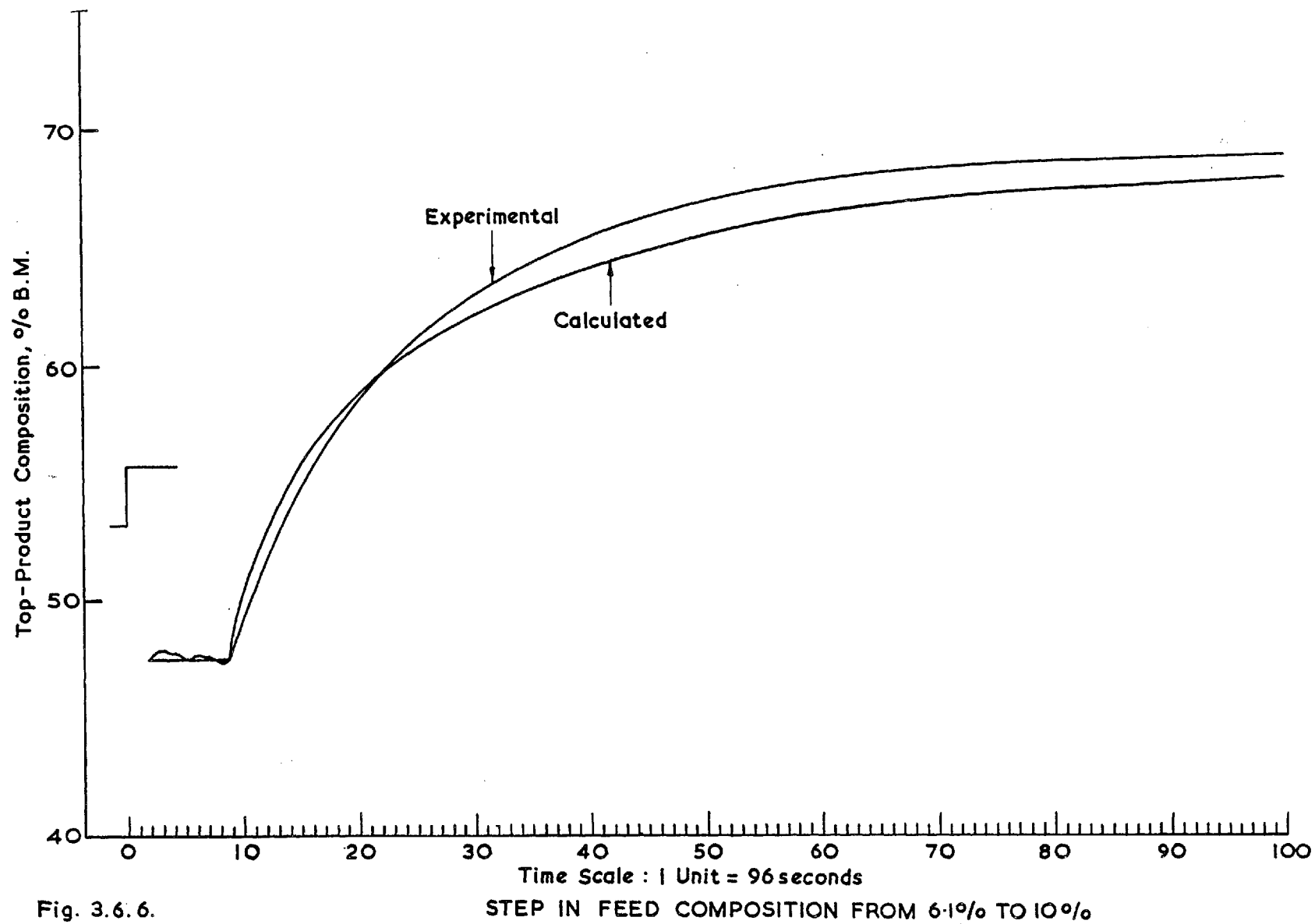


Fig. 3.6.6.

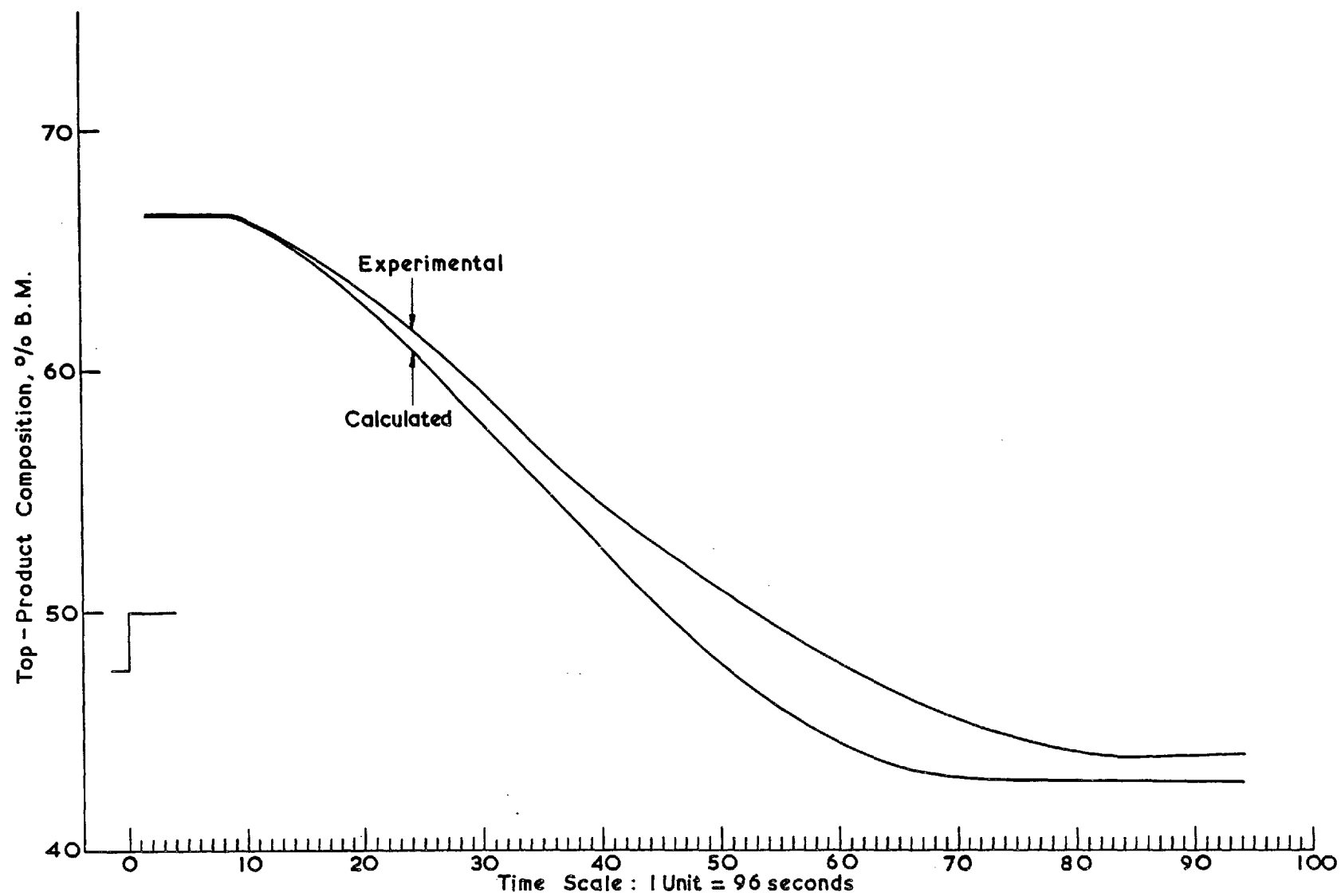


Fig. 3.6. 7.

STEP IN FEED COMPOSITION FROM 7.4% TO 3.9%

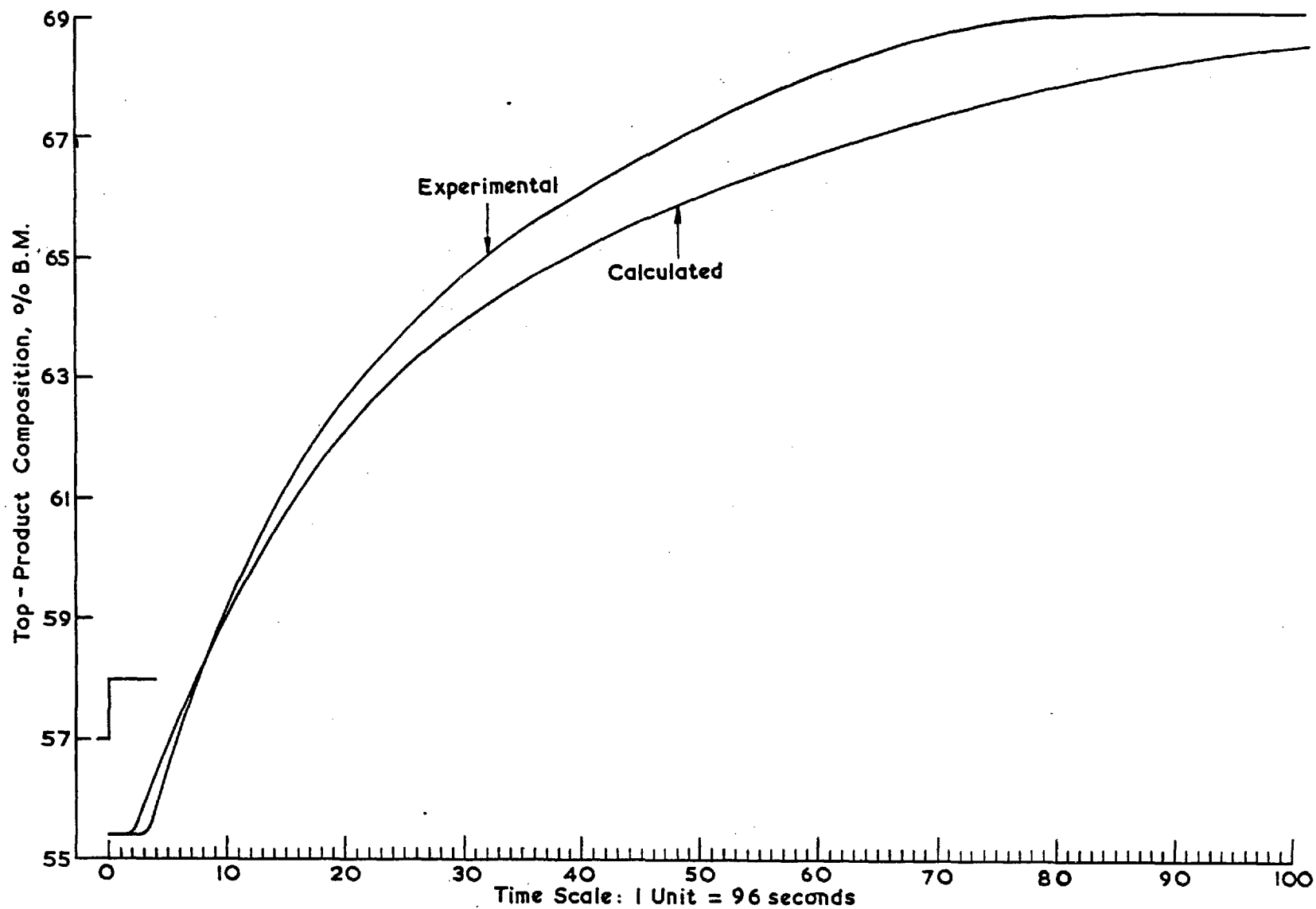


Fig. 3.6. 8.

STEP IN FEED FLOW-RATE FROM 52.4 cc/min. TO 71.4 cc/min.

are the compositions on each plate. The choice is therefore either to use a costly measuring system, or noisy estimates of the state vector, or to use a simpler (but still accurate) process model. The latter course was followed here.

Equation (3.4.7) describes the behaviour of the plant in terms of a transition matrix whose elements are functions of the vapour and liquid flow-rates and the terms α_r, β_r at each plate in the column. If a variable is changed from one level to another then the vapour and liquid molar flow-rates will assume the values which correspond to the steady-state levels of the perturbed variable if the time between perturbations is equal to or greater than 96 seconds. Thus at each sampling instant one or the other set of steady-state vapour and liquid flow-rates are used to calculate the matrices A, ϕ and D depending on the value of the perturbed variable. The values of α_r, β_r required in calculating these matrices are found from the previous values of the plate compositions. To simplify the model it is merely necessary to analyse the stored values of the perturbed variable and the element of the x-vector of interest. In the cases discussed later this element was x_7 corresponding to the top product composition and Generalised least Squares analysis was used. It was found by analysing many cases that a second-order z transfer function of the form

$$\frac{\Delta x_T}{\Delta v_R} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

adequately represented the 7th order vector matrix model.

3.8 A Reflux-Top Product Relationship

By combining equations (2.3.9.), (2.3.10.) and (2.8.1), it can be shown that

$$x_T = \frac{F(z_F - x_0) (R+1)}{1.48 (1-k)^R H} + x_0 \quad \dots (3.8.1)$$

and if x_0 is small

$$x_T = \frac{Fz_F (R+1)}{1.48 (1-k)^7 H} \quad \dots (3.8.2)$$

R is the Reflux Ratio which on the NPL column was altered by varying the cycle time of a tipping bucket mechanism. This time was controlled by an electromagnet which, when energised caused the bucket to swing to a new position and allow the top-product to pass out of the column, and when de-energized the bucket swung to its normal position in which the top-product flowed onto plate 6. The ratio of de-energisation period to energisation period is the Reflux Ratio and the energisation was controlled by a voltage from a buffer store which in turn was supplied with a digital voltage from a tape-reader, keyboard or computer. The buffer store worked over the range 0 to 12.75 volts corresponding to a range of 256 binary digits and the Reflux Ratio was given by

$$R+1 = \frac{256}{B}$$

where B is the binary input and thus total reflux occurs when $B = 0$. It can be seen from (3.8.2) that

$$x_T = \frac{256 Fz_F}{1.48B (1-k)^7 H} \quad \dots (3.8.3)$$

and if F, z_F, k, H are constant

$$x_T = \frac{\text{Const.}}{B}$$

$$\frac{\Delta x_T}{\Delta B} = - \frac{\text{Const.}}{B^2}$$

Experiments were carried out in which the top product composition was measured for different values of B with F , z_F , H held constant. These results are plotted for three values of z_F in Figure 3.8.1 where it is seen that the relationship is not hyperbolic but linear, although over a considerable part of each curve in the lower range of x_T , the product x_TB is constant. The departure from a hyperbolic relationship to a linear one is due to the fact that at high x_T equation (3.8.3) no longer applies since x_0 is no longer zero and it is now necessary to use equation (3.8.1).

The results shown in Figure 3.8.1 were for a feed flow-rate of 46.5 cc/min and a heat of 1.4 KW and if F or H are changed then the results would appear differently. The way in which the gain varies with z_F is shown in Figure 3.8.2.

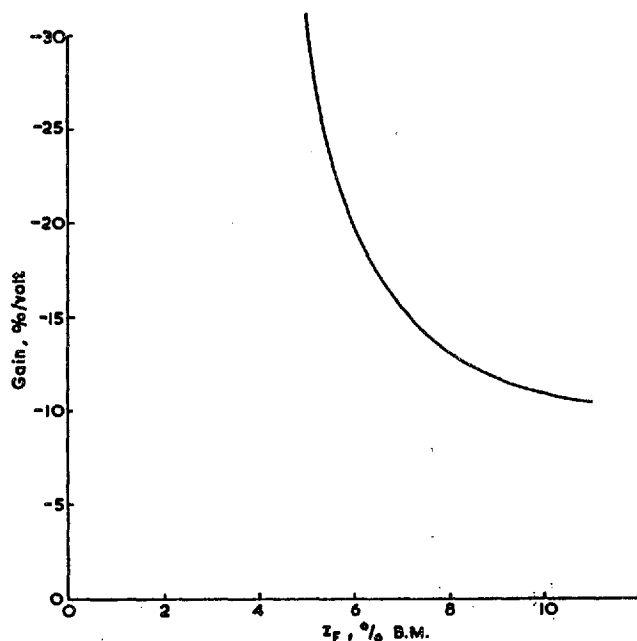


Fig. 3.8.2. VARIATION OF GAIN WITH FEED COMPOSITION

It was also found that the voltage on thermistor 6 (i.e. the top-plate temperature) varied linearly with B as shown in Figure 3.8.3 for $z_F = 5\%$.

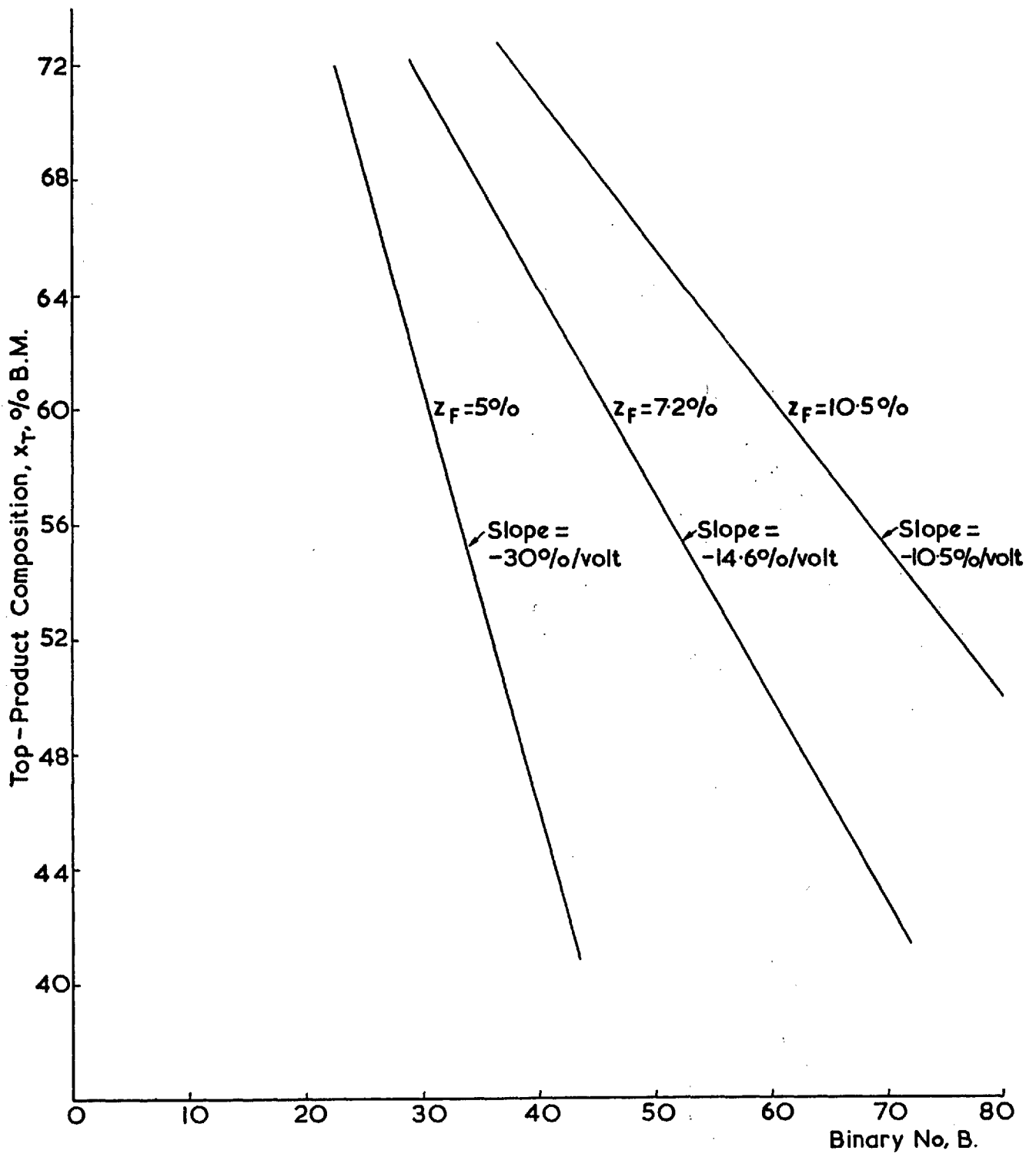


Fig. 3.8.1. VARIATION OF TOP-PRODUCT COMPOSITION WITH B.

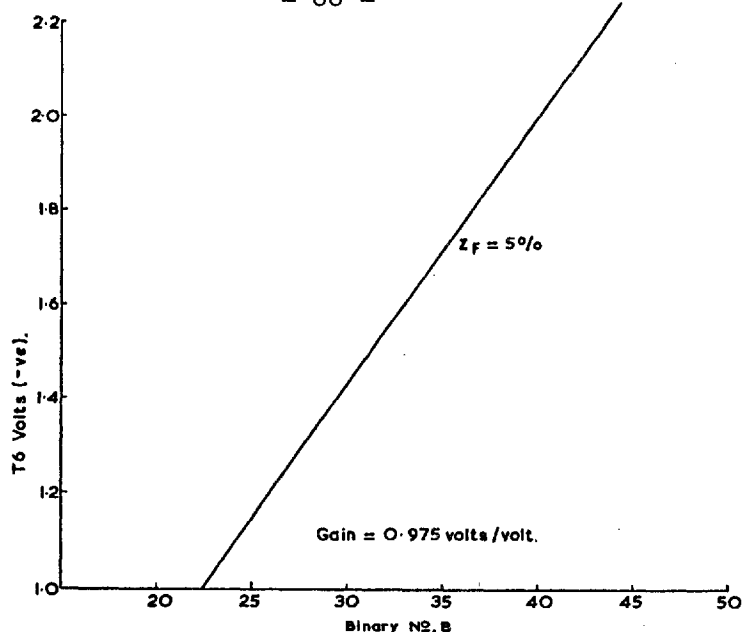


Fig. 3.8.3. VARIATION OF TOP-PLATE TEMPERATURE WITH B.

A linear relationship such as that shown in Figures 3.8.1 and 3.8.3 is quite convenient for control purposes since the system gain is the slope of this line. It also makes the problem of identifying a linear model easier, since a large signal:noise ratio may be obtained by perturbing the Reflux over a large amplitude without violating the assumption of linearity.

3.9 P.R.B.S. Testing: (12)

Extensive plant testing using Pseudo-Random Binary Sequences (PRBS) has been carried out and a range of models for various operating conditions has been built up. Throughout this work emphasis has been placed on models based on Reflux perturbations (although some others were obtained) since Reflux is the control variable used in practice to control the top-product composition. Also, attention was confined to the feed entering the column on plate 3. However the work described could be easily extended to other aspects of distillation column control, for example bottom-product control or side-stream off-take.

The P.R.B.S. experiments were carried out mainly at a 96 seconds basic interval and usually with a sequence length (N) of 127 bits. In order to check the linearity assumption, an 80-bit 3 level m sequence has also been used. For each experiment a z transfer function model between x_{T1} and the

perturbed variable, and the cross-correlation between these two signals were calculated. The cross-correlation was found to be a useful check on the order of the time delay (τ) found by G.I.S. estimation and as a safeguard against non-sensical G.I.S. models. It was found to be useful to calculate the $\frac{\Delta V_e}{\Delta B}$ model as well as the $\frac{\Delta x_T}{\Delta B}$ model since it was frequently found that errors appeared in one which did not appear in the other and this cross-checking enabled the trouble to be rectified.

Figures 3.9.1 , 3.9.2 , and 3.9.3 show some typical results obtained from the PRBS perturbation experiments. The perturbed variable and the feed conditions during the experiment are indicated and two curves are shown, one being the cross-correlation between the top-product composition and the perturbed variable, and the other being the pulse response of a z transform model which has been obtained by analysing the same data using the Generalised Least Squares Technique. This latter technique is discussed in greater detail in chapter 4. The amplitude scale in these figures is not of particular interest and the values shown are arbitrary. Figure 3.9.4 shows a typical plot of the measured plant output (V_e) compared with that predicted by the model which in this case is given by

$$G(z) = \frac{-0.028 z^{-1} - .015 z^{-2}}{1 - .56 z^{-1} - .4 z^{-2}}$$

The output mean of the plant was -1.52 volts with a variance of 1.9×10^{-3} volts and the mean square predicted error was 9.75×10^{-5} volts.

In section 3.8 it was shown that the top-product composition appeared linear with B over a considerable range, and it was pointed out that this would enable linear perturbation models to be readily obtained since large amplitude perturbations could be applied. Table 3.9.1 shows the results of 3 experiments in which 2 level and 3 level sequences were used to perturb the column under 3 different sets of operating conditions. The general agreement between the results indicates no serious non-linearities. It is noticed that the agreement between the steady-state plant gains is not particularly close.

Reflux = 2.95:1 $\rightarrow$ 5:1

Heat = 1.73 KW

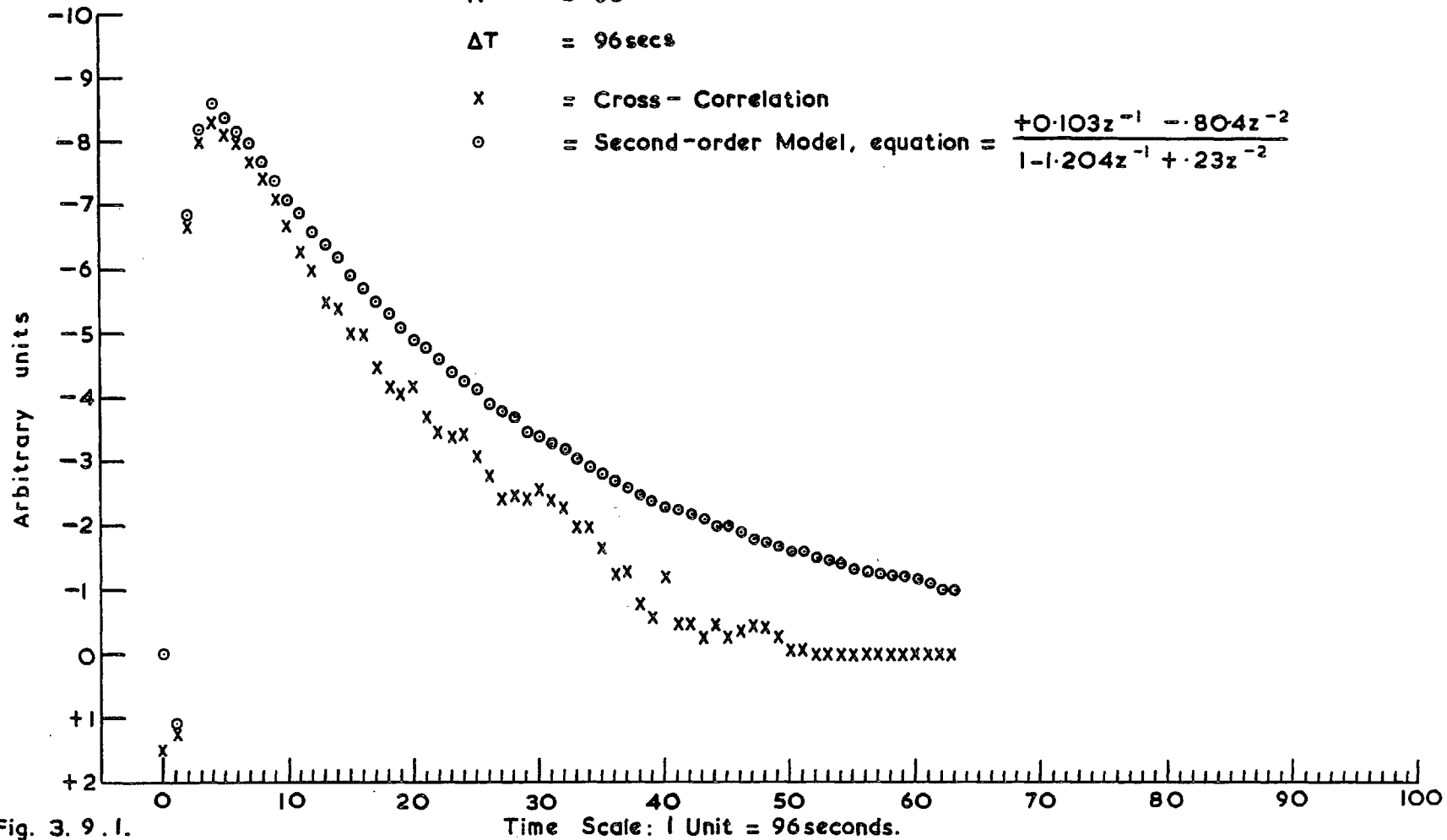
Feed = 90 cc/min at 40°C

N = 63

ΔT = 96 secs

x = Cross - Correlation

o = Second-order Model, equation = $\frac{+0.103z^{-1} - .804z^{-2}}{1 - 1.204z^{-1} + .23z^{-2}}$



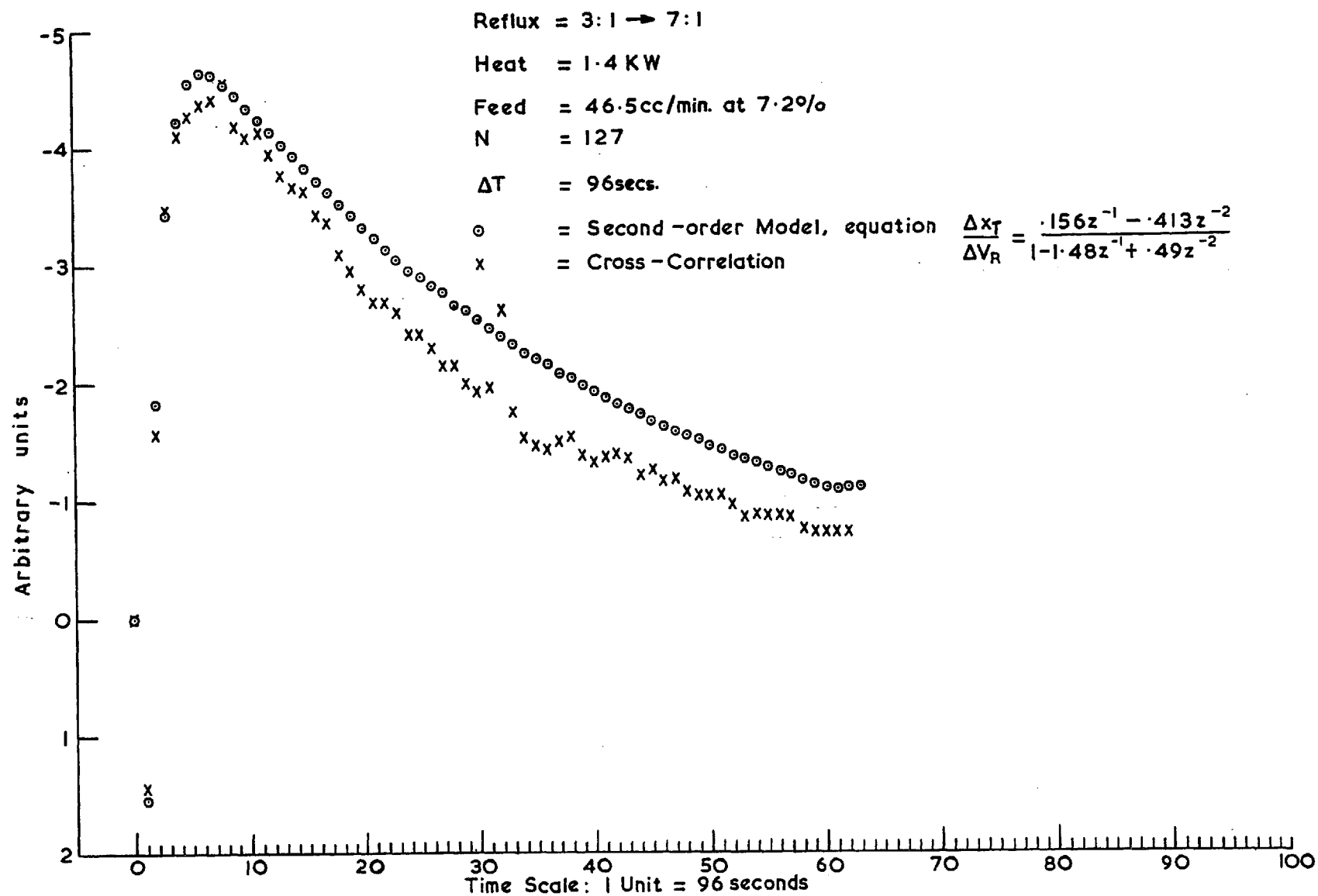


Fig. 3. 9.2.

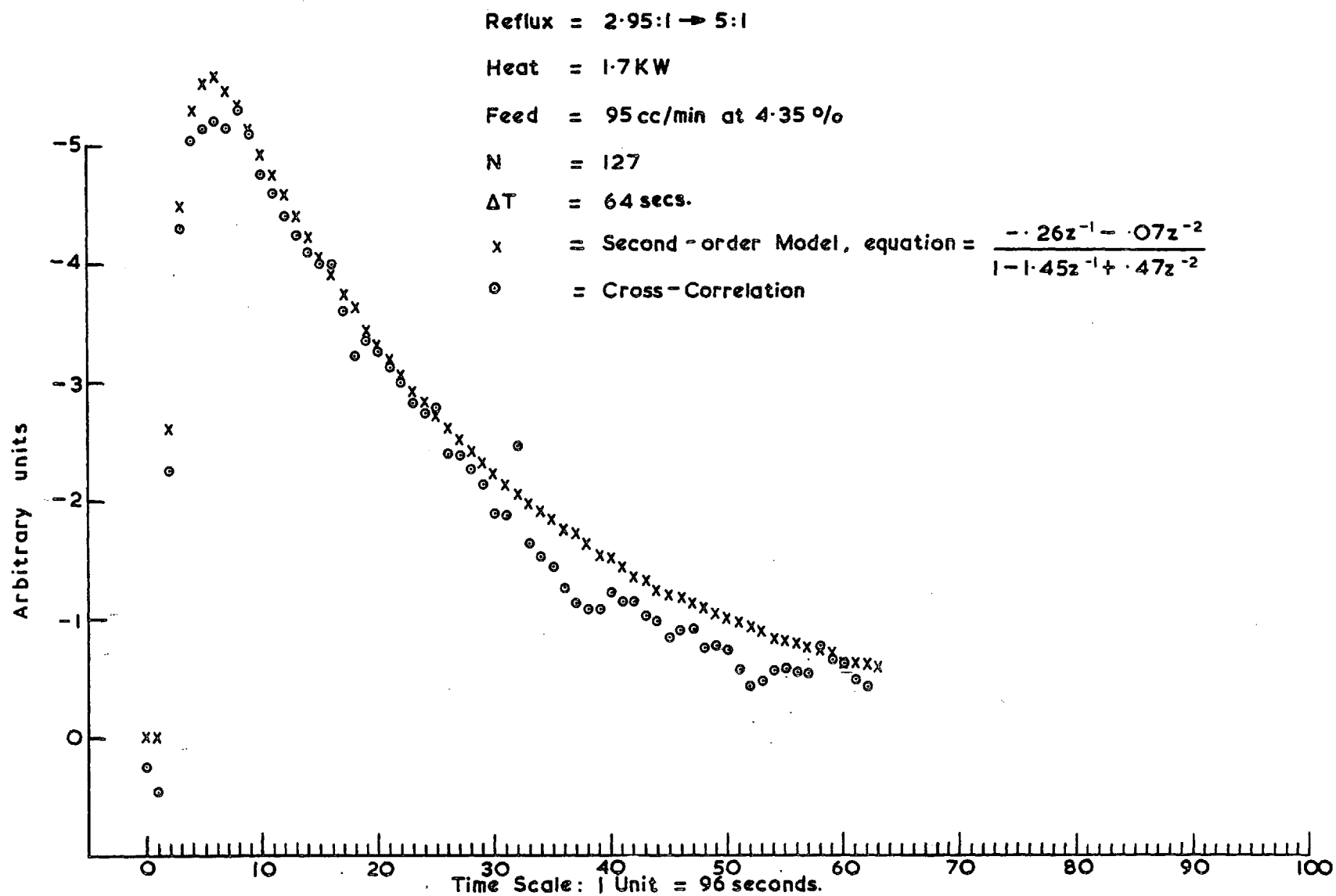


Fig. 3. 9. 3.

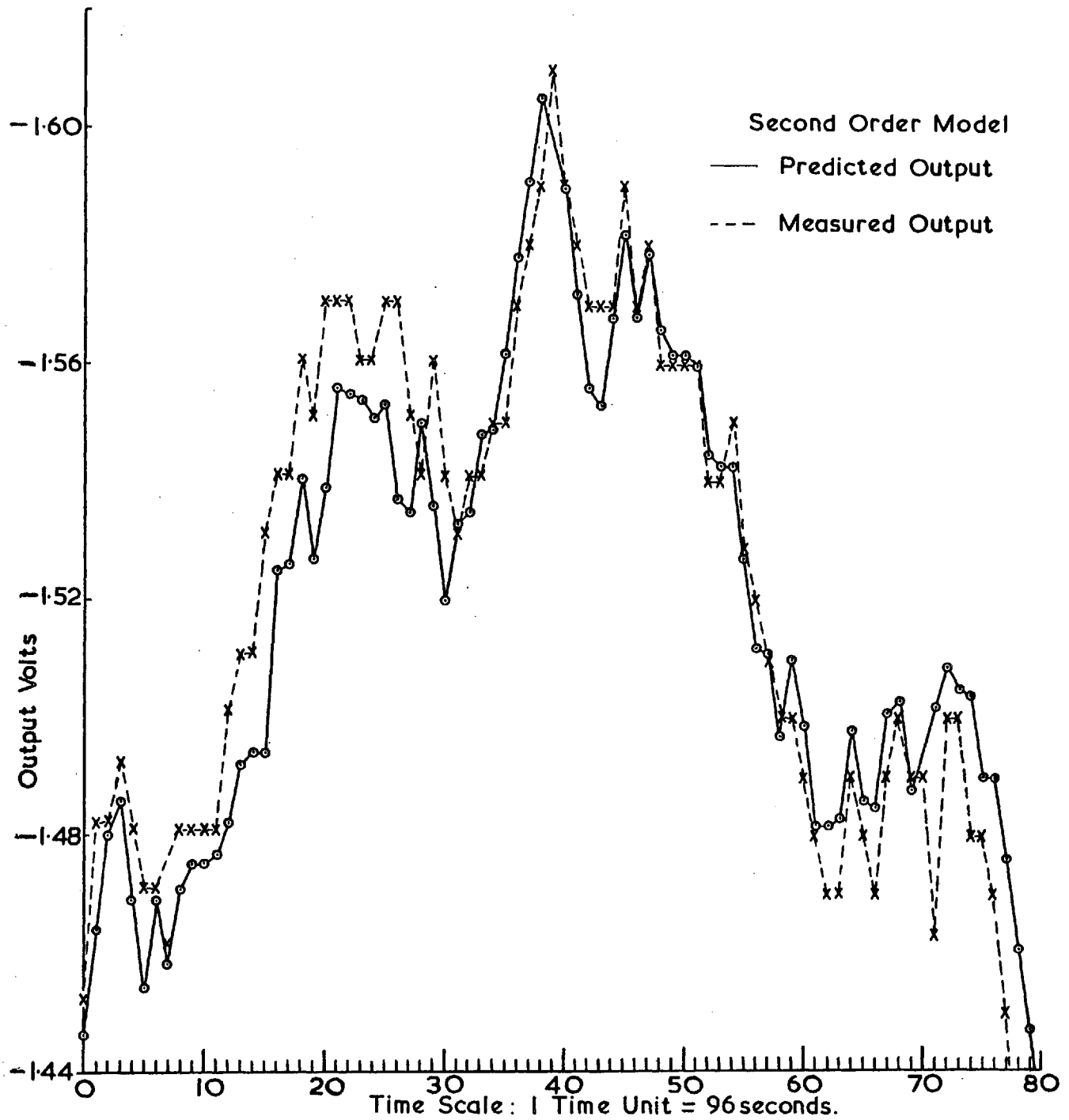


Fig. 3. 9 .4.

PREDICTED OUTPUT & MEASURED OUTPUT
FROM A TYPICAL P.R.B.S. EXPERIMENT.

It is thought that this is due to the method used to obtain the plant gain in which the final value theorem is applied to $G(z)$;

$$\text{i.e. } |G(z)| = \frac{\sum_{i=1}^n b_i}{1 + \sum_{i=1}^n a_i}$$

Since $\sum a_i$ is very close to -1 any small variations in the parameters a_i will result in significant changes in $|G(z)|$. It is considered that an improvement in this method of calculating the plant gain is desirable.

Operating Conditions	Two Level Model 127 Bit PRBS 96 secs	Three Level Model 80 Bit 96 secs
$z_F = 5\%$ $H = 1.56 \text{ KW}$ $F = 47 \text{ cc/min}$ $R = 5:1 \text{ to } 10:1$	$\frac{.239 z^{-1} - .602 z^{-2}}{1 - 1.55 z^{-1} + .567 z^{-2}}$ Gain = -24.6 ± 12.7 Poles = $+.96, +.58$	$\frac{.245 z^{-1} - .604 z^{-2}}{1 - 1.53 z^{-1} + .55 z^{-2}}$ Gain = -17.9 ± 4.8 Poles = $+.95, +.58$
$z_F = 7.2\%$ $H = 1.4 \text{ KW}$ $F = 47 \text{ cc/min}$ $R = 3:1 \text{ to } 7:1$	$\frac{.156 z^{-1} - .413 z^{-2}}{1 - 1.48 z^{-1} + .49 z^{-2}}$ Gain = -19.4 ± 5.4 Poles = $+.98, +.5$	$\frac{.16 z^{-1} - .375 z^{-2}}{1 - 1.46 z^{-1} + .47 z^{-2}}$ Gain = -20.4 ± 8.1 Poles = $+.99, +.47$
$z_F = 10.5\%$ $H = 1.4 \text{ KW}$ $F = 47 \text{ cc/min}$ $R = 2.2:1 \text{ to } 5:1$	$\frac{.176 z^{-1} - .44 z^{-2}}{1 - 1.23 z^{-1} + .243 z^{-2}}$ Gain = -15 ± 4.2 Poles = $+.98, +.25$	$\frac{.188 z^{-1} - .369 z^{-2}}{1 - 1.17 z^{-1} + .185 z^{-2}}$ Gain = -10.9 ± 4.6 Poles = $+.98, +.19$

TABLE 3.9.1

3.10 Comparison of Discrete Models:

In section 3.6 a comparison was made between step responses obtained from the plant and those obtained from theory. As a further attempt to

validate the model, a comparison is now made of the z transfer function models obtained from plant testing, with those obtained from the theoretical model and simplified by the method of section 3.7. TABLE 3.10.1 shows the results of 7 experiments in which the gain and two dominant poles of the plant as computed from theory and experiment are tabulated. Second order models only are shown since higher order models displayed no significant improvement. In general the agreement is good, particularly for the major pole but less so for the minor one. For all models there appears to be a major pole of about 0.97 even though the experimental conditions varied considerably and the plant gain showed an overall variation of 2:1. This suggests that an effective adaptive control scheme could be based on compensating the major time constant with the plant gain calculated from the steady state programme. The minor time constant is ignored since it is negligible by comparison.

Perturbed Variable	Second Order Models	
	Theory	Experiment
Reflux	Poles 0.96, 0.26 Gain -21.6	0.96, 0.25 -24.1
Reflux	Poles .98, .26 Gain -18.2	.96, .58 -24.6
Feed Composition	Poles .97, .50 Gain -23.7	.97, .53 -16.1
Reflux	Poles .96, .23 Gain -13	.97, .26 -15
Heat	Poles .93, .15 Gain -22	.97, .35 -32.9
Reflux	Poles .97, .5 Gain -19.3	.97, .51 -19.4
Reflux	Poles .96, .5 Gain -19.3	.96, .24 -27.1

TABLE 3.10.1

3.11 Conclusions:

The results of this chapter show that an accurate model for binary distillation can be obtained from theoretical considerations assisted by certain measurements. The method depends upon the fact that vapour and liquid flow-rate changes are much faster than composition changes. Given accurate measurements of feed and top-product flow-rates and compositions, the method could easily be extended to on-line use allowing an (adaptive) lower-order model to be found for use in feed back control schemes. The use of such models in feed-forward control schemes could also be implemented.

CHAPTER 4. Parameter Estimation

4.1 Introduction

In the two previous chapters a method of deriving a vector-matrix model of a binary distillation column has been described and the method has been validated by the comparison of theoretical and experimental results. One method of validation was to compare the second-order discrete time model obtained by perturbing the plant and the model and analysing the input/output records with a Generalised Least Squares (G.L.S.) modelling Technique. This chapter discusses some well known modelling techniques, in particular that of G.L.S. when used in deriving discrete plant models. Discrete models rather than continuous ones are preferred for two reasons. First, the modelling techniques will be applied using noisy data by a digital computer and this will require storage of the input/output records in sampled form. Second, assuming a computer is to be included in the closed loop as the controlling device, a discrete time model leads naturally to a discrete-time controller (as shown in chapter 5). The data will be sampled at intervals of time (T) and a pure time delay (T_d) associated with the plant can be approximated by kT time units where k is defined as

$$k = \text{integer part of } \left(\frac{T_d}{T} + 0.5 \right)$$

If it is assumed that even with zero time delay the plant can give no output for any input at $t = 0$, then it is seen that the output $x(n)$ can be expressed as

$$x(n) = \psi \left[x(n-1), x(n-2), u(n-1), u(n-2), \dots \right] \quad \dots 4.1.1$$

Where $u(i)$ are the successive inputs. Amongst the large number of possible types of ψ , a weighted sum is the simplest type:

$$x(n) = -a_1 x(n-1) - a_2 x(n-2) - \dots + b_1 u(n-1) + b_2 u(n-2) + \dots$$

where the a_i, b_i are constant weighting coefficients and using the operator $z (=e^{sT})$ defined as

$$z^{-1} x(n) \triangleq x(n-1)$$

we have

$$x(n) [1 + a_1 z^{-1} + a_2 z^{-2} + \dots] = u(n) [b_1 z^{-1} + b_2 z^{-2} + \dots]$$

$$\text{or} \quad \frac{x(n)}{u(n)} = \frac{b_1 z^{-1} + b_2 z^{-2} + \dots}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots} \quad \dots 4.1.2$$

By using the substitution $z = e^{sT}$ it may be shown that the general Laplace transfer function

$$G(s) = \frac{1 - e^{-sT}}{s} \times \frac{(s+a_0)(s+b_0)}{(s+a)(s+b)(s+c) \dots} \quad \dots 4.1.3$$

reduces to the form of 4.1.2. The reason for choosing the linear models 4.1.2 or 4.1.3 is that extensive work has been done on the stability and control of such models while little is known about these aspects of non-linear models. In this chapter attention is confined to deriving models of the form 4.1.2 by analysing input/output records using the techniques of Least Squares (L.S.), Maximum Likelihood (M.L.) and Generalised Least Squares (G.L.S.). Before describing these procedures however it is necessary to define some statistical properties which will be used to compare the results of the three methods.

(14)

4.2 Estimation Criteria

(i) Unbiasedness

In estimating a parameter b_j of a model from a record of n

samples, the estimated parameter $\hat{b}_j$ is said to be unbiased if

$$E \{ \hat{b}_j \} = b_j$$

ie the expected or average value over all n such samples equals the true value of the parameter.

(ii) Consistency

An estimate $\hat{b}_j$ of a parameter b_j from n samples is said to be consistent if, for any positive ϵ and ζ , however small, there is some N such that

$$\text{Prob. } \{ | \hat{b}_j - b_j | < \epsilon \} > 1 - \zeta \text{ for all } n > N.$$

(iii) Minimum Variance Estimator

$$\text{If } E \{ \hat{b}_j \} = b_j$$

then the variance of $\hat{b}_j$ is given by

$$\text{Var } [\hat{b}_j] = E \{ \hat{b}_j - b_j \}^2$$

and this can be shown to be greater than or equal to a lower value which is called the minimum variance bound or Cramer-Rao lower bound. An estimator which attains this value is called an efficient estimator.

4.3 Least Squares Estimation

In the following the notation $>$ will indicate a column vector and $<$ a row vector. Thus $<.,>$ indicates a scalar quantity and $>.<$ refers to a matrix.

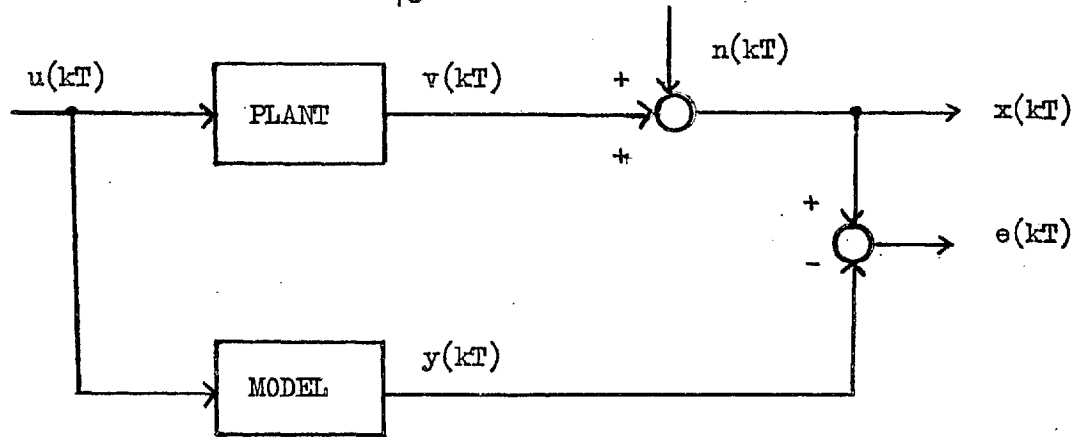


Figure 4.3.1

"Least Squares Plant Model"

Figure 4.3.1 shows a plant with an input signal $u(kT)$ and an output $v(kT)$. This output is corrupted by noise $n(kT)$ and the only available output measurement is $x(kT)$. A model of the plant is assumed to have the weighting sequence form

$$x(kT) = u(\overline{k-1} T)b_1 + u(\overline{k-2} T)b_2 + \dots + \dots + u(\overline{k-m} T)b_m + e(kT) \quad \dots 4.3.1$$

where

$$x(kT) = y(kT) + e(kT)$$

Equation 4.3.1 can be written

$$x(kT) = \langle u(\overline{k-j} T) \cdot b_j \rangle + e(kT) \quad \dots 4.3.2$$

Where $\langle u(\overline{k-j}) = [u(k-1), u(k-2), \dots, u(k-m)]$

$$\langle b_j = [b_1, b_2, \dots, b_m]$$

and it is desired to estimate the parameters b_j , given sequences of input and output measurements $\{u\}$, $\{x\}$, such that

$$I = \langle e(kT) \cdot e(kT) \rangle$$

is a minimum. Equation 4.3.2 can be written (dropping the T)

$$\begin{aligned}
 x(k) &= \langle u(k-j) \ b_j \rangle + e(k) \\
 x(k-1) &= \langle u(k-1-j) \ b_j \rangle + e(k-1) \\
 &\vdots \\
 &\vdots \\
 &\vdots \\
 &\vdots \\
 &\vdots \\
 x(k-n) &= \langle u(k-n-j) \ b_j \rangle + e(k-n)
 \end{aligned}$$

and hence

$$x > = U b_j > + e > \quad \dots 4.3.3$$

Where $x >$ is the n vector of output measurements and U is the matrix

$$\begin{bmatrix}
 u(k-1), u(k-2), u(k-3), \dots, u(k-m) \\
 u(k-2), u(k-3), \dots, u(k-m-1) \\
 \vdots \\
 \vdots \\
 \vdots \\
 u(k-1-n), \dots, u(k-m-n)
 \end{bmatrix}$$

$$\begin{aligned}
 \text{Thus } I &= \langle e.e \rangle = \left[\langle x - \langle b_j U^T \rangle \right] \left[x > - U b_j > \right] \\
 &= \langle x.x \rangle - \langle x U b_j \rangle - \langle b_j U^T x \rangle + \langle b_j U^T U b_j \rangle \\
 \frac{\partial I}{\partial b_j} &= - U^T x > - U^T x > + U^T U b_j > + U^T U b_j >
 \end{aligned}$$

and for $\frac{\partial I}{\partial b_j}$ to be a minimum

$$\frac{\partial I}{\partial b_j} = 0 \quad \text{ie } \hat{b_j} > = \{U^T U\}^{-1} U^T x > \quad \dots 4.3.4$$

and $\frac{\partial^2 I}{\partial b_j^2} = 2 U^T U > 0 \therefore$ the above condition is a minimum.

Substituting 4.3.3 into 4.3.4

$$\begin{aligned} \hat{b}_j &= [U^T U]^{-1} U^T [U b_j + e] \\ &= b_j + [U^T U]^{-1} U^T e \end{aligned} \quad \dots 4.3.5$$

If the elements of the columns of U are uncorrelated with e then

$$E [\hat{b}_j] = E [b_j] \quad \dots 4.3.6$$

and the estimates are unbiased.

Also, the covariance of the estimates $\hat{b}_j$ is defined as

$$\begin{aligned} \text{cov } \hat{b}_j &= E \{ [\hat{b}_j - b_j] [\hat{b}_j - b_j]^T \} \\ &= E \{ [U^T U]^{-1} U^T e e^T U [U^T U]^{-1} \} \end{aligned} \quad \dots 4.3.7$$

and if $E \{ e e^T \} = \sigma^2 I$ i.e. the sequence $\{e\}$ is uncorrelated with common variance σ^2 ("White noise"), then $\text{cov } \hat{b}_j = \sigma^2 [U^T U]^{-1}$.

In practice the sequence $\{e\}$ will not be uncorrelated and hence $E \{ e e^T \}$ is not simply diagonal. Thus L.S. estimates will be biased and therefore unsatisfactory.

(15)

4.4 Maximum Likelihood Estimation

Åström et al considered the process model shown in Figure 4.4.1 and defined by

$$A(z^{-1}) x(t) = B(z^{-1}) u(t) + C(z^{-1}) \varepsilon(t)$$

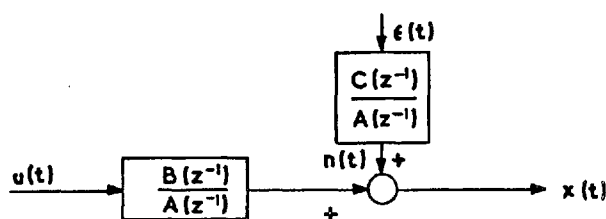


Fig. 4.4.1. PROCESS & NOISE MODEL ASSUMED BY ÅSTRÖM ET AL.

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}$$

$$B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-n}$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2} + \dots + c_n z^{-n}$$

$\{e\}$ is an independently and normally distributed sequence with zero mean and variance σ^2 . Although $\{e\}$ is independent, $\{n\}$ will not necessarily be the same unless $A = C$. Since $\varepsilon(t)$ is normally distributed its distribution function is given by

$$\Delta F = \frac{1}{(2\pi)^{k/2} (\det \Sigma)^{1/2}} \exp \left[-\frac{1}{2} (\varepsilon - \mu) \sum_{j=1}^k (\varepsilon - \mu) > \frac{1}{2} \right] \prod_{j=1}^k \Delta \varepsilon_j$$

where

$$\varepsilon = \begin{bmatrix} \varepsilon(1) \\ \varepsilon(2) \\ \vdots \\ \varepsilon(k) \end{bmatrix}, \quad \mu = E[\varepsilon], \quad \Sigma = E\{(\varepsilon - \mu) > (\varepsilon - \mu)\}$$

since $\mu = 0$

$$\Sigma = E \begin{bmatrix} \varepsilon_1 \varepsilon_1 & \varepsilon_1 \varepsilon_2 & \dots & \varepsilon_1 \varepsilon_k \\ \varepsilon_2 \varepsilon_1 & \varepsilon_2 \varepsilon_2 & \dots & \varepsilon_2 \varepsilon_k \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_k \varepsilon_1 & \varepsilon_k \varepsilon_2 & \dots & \varepsilon_k \varepsilon_k \end{bmatrix}$$

$$= \sigma^2 I \quad \text{since} \quad E[\varepsilon_i \varepsilon_j] = \begin{cases} 0 & i \neq j \\ \sigma^2 & i = j \end{cases}$$

The distribution function is now

$$\Delta F = \frac{1}{(2\pi)^{k/2} \sigma^k} \exp \left[-\frac{1}{2\sigma^2} \langle \varepsilon \varepsilon \rangle \right] \prod_j \Delta \varepsilon_j$$

and the logarithm of the probability density function is

$$\log e [f(\varepsilon)] = -\frac{k}{2} \log (2\pi) - k \log \sigma - \frac{1}{2\sigma^2} \langle \varepsilon \varepsilon \rangle$$

The estimation problem is to find the parameters A, B, C, σ such that $x(t)$ is the most likely when $u(t)$ is given, ie to find the maximum of

$$L [x(t) / u(t), A, B, C, \sigma]$$

and Åstrom shows that this is the same as finding the maximum of

$$L [\varepsilon(t) / u(t), A, B, C, \sigma]$$

which occurs when $\frac{1}{2\sigma^2} \langle \varepsilon \varepsilon \rangle$ is a minimum.

Åstrom also shows that this estimator is asymptotically efficient and this means that in theory an estimator with greater accuracy cannot be found.

4.5 Generalised Least Squares (G.L.S.) Estimation (16)

In equation 4.3.7 if $\text{cov}(\varepsilon) = E \{ \varepsilon \varepsilon^T \}$ is written

$$\text{cov}(\varepsilon) = P = P_1 P_1^T \quad \dots 4.5.1$$

where P_1 is a lower triangular matrix, then multiplying 4.3.3 by P_1^{-1} we have

$$P_1^{-1} x > = P_1^{-1} U b_j > + P_1^{-1} e > \quad \dots 4.5.2$$

Then

$$\begin{aligned} \text{cov } [P_1^{-1} e >] &= E \{P_1^{-1} e > < e (P_1^{-1})^T\} \\ &= P_1^{-1} E \{e > < e\} (P_1^{-1})^T \\ &= P_1^{-1} P (P_1^{-1})^T \\ &= I \end{aligned}$$

ie the covariance of $(P_1^{-1} e >)$ has been found to be a unit matrix and thus the method of L.S. is applicable to the filtered data, ie to $P_1^{-1} x >$ and $P_1^{-1} u >$. The difficulty is that P_1 must be known before the calculations are started and this is tantamount to knowing the answers before the problem is solved! In addition the calculations will be quite involved since now

$$b_j > = [U^T P_1^{-1} U]^{-1} U^T P_1^{-1} x >$$

One way of overcoming these problems was suggested by Clarke who wrote a computer programme using an iterative filtering procedure in which the L.S. residuals (e) are used to derive filters which transform the input/output data pairs so that the new residuals approximate more nearly to white noise. This iterative process is carried out until some constraint is violated. The process and noise model assumed is shown in Figure 4.5.1 and is defined by the following equations:

$$\frac{v(z^{-1})}{u(z^{-1})} = \frac{z^{-k} B(z^{-1})}{A(z^{-1})} \quad \dots 4.5.3$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}$$

and $B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-n}$

The corrupted measurements are given by

$$x(t) = v(t) + \varepsilon(t) \quad \dots 4.5.4$$

and the noise process is defined by

$$C(z^{-1}) \varepsilon(t) = D(z^{-1}) \zeta(t) \quad \dots 4.5.5$$

where

$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2} + \dots + c_m z^{-m}$$

$$D(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2} + \dots + d_m z^{-m}$$

and $\zeta(t)$ is white noise with zero mean and variance σ^2 .

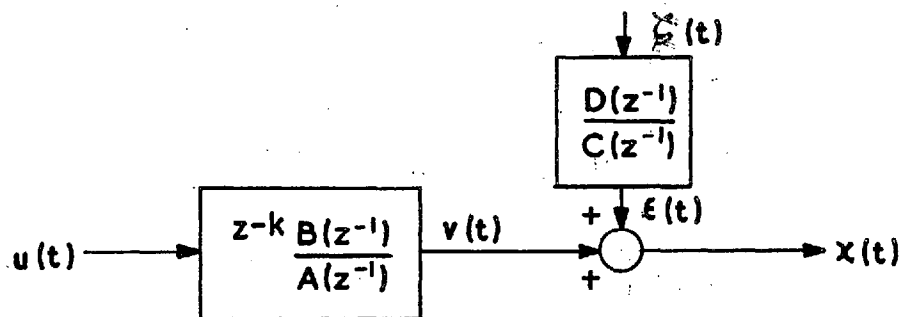


Fig. 4.5.1. PROCESS & NOISE MODEL ASSUMED BY CLARKE.

Combining equations 4.5.3, 4.5.4, and 4.5.5

$$A x(t) = z^{-k} B u(t) + e(t) \quad \dots 4.5.6$$

where
$$e(t) = \frac{A \cdot D}{C} \zeta(t) \quad \dots 4.5.7$$

$$= \langle \omega_x \zeta(t - rT) \rangle \quad \dots 4.5.8$$

where the ω_r are functions of the a_i, d_i, o_i . If $\{x\}, \{u\}, \{e\}$ now stand for sequences of $x(t), u(t), e(t)$ respectively then for several measurements, equation 4.5.6 becomes

$$\{x\} = [-z^{-1}\{x\}, -z^{-2}\{x\}, \dots, -z^{-n}\{x\}, -z^{-k+1}\{u\}, \dots] \begin{bmatrix} -a \\ b \end{bmatrix} + \{e\}$$

... 4.5.9

where

$$[-a \ b] = [-a_1, -a_2, \dots, -a_n, b_1, b_2, \dots, b_n]$$

Equation 4.5.9 can also be written

$$x > = U b j > + e >$$

which is in the form of equation 4.3.3 and if $x >$ was independent of U then L.S. estimation would be directly applicable. However L.S. estimation may be shown to be biased here since U is now a function of x and u , and G.L.S. or M.L. estimation must be used. From 4.5.7 it is seen that $e(t)$ can be written as

$$e(t) \doteq F^{-1} \times \zeta(t) \quad \dots 4.5.10$$

where

$$F = (f_1 z^{-1} + \dots f_r z^{-r})$$

and F is a low order polynomial in z . Equation 4.5.10 is an autoregressive process since $\zeta(t)$ is approximated by a finite weighted sum of previous and present values of $e(t)$. The weighting coefficients f_r can be

estimated by L.S. and then used to filter the input/output sequences $\{u\}$, $\{x\}$ so that the new residuals approximate more closely to uncorrelated noise.

The (computerised) steps in the modelling procedure are

- 1) Find a L.S. model estimate from the data $\{u\}$, $\{x\}$

$$\begin{bmatrix} -a \\ b \end{bmatrix} = \hat{b}_j > = [U^T U]^{-1} U^T x >$$

- 2) Find the errors

$$e > = x > - U \hat{b}_j >$$

- 3) Find a L.S. model for F where

$$e > = F^{-1} \times \zeta >$$

- 4) Filter the data $\{u\}$, $\{x\}$ with F to produce new data $\{u_1\}$, $\{x_1\}$.
- 5) Repeat from 1) again with $\{u_1\}$, $\{x_1\}$ until one of the error criteria are satisfied.

The error criteria are

- i) Divergence of the quantity $\langle e.e \rangle$
- ii) The number of iterations reaches a pre-arranged maximum.
- iii) $\langle e.e \rangle$ becomes less than a pre-arranged value.

From equations 4.5.7 and 4.5.10 it can be seen that $ADC^{-1} = F^{-1}$

$$\text{ie } DC^{-1} = A^{-1} F^{-1}.$$

D , C are not computed explicitly in Clarke's programme but are found as a function of A and F . This compares closely with Åstrom's model where the noise model is of the form $A^{-1} C$.

Because Clarke's method is computer-oriented there is no theoretical proof available of the statistical goodness of the results. It cannot be

shown that the results will be unbiased, consistent or efficient. In practice sufficient information is printed out with the results to ensure their acceptance or rejection on statistical grounds. The method is essentially that of Box and Jenkins except that its assumed model is more useful in Process Control work and the formulation enables automatic selection of the best model by the computer to be more readily facilitated. Since there are fewer parameters to be found in Clarke's model the computing times are faster than those for Åstrom's model and in addition, if the residuals are uncorrelated (or nearly so) then the statistical criteria of section 4.2 are satisfied and the method will be as good as the M.L. method. It has been found in practice that, for most cases investigated, the residuals were very nearly uncorrelated.

4.6 Conclusions

In practice, the residuals between plant and model outputs will be correlated since the noise encountered in process control work will not be white and in addition the structure of the assumed model will be an approximation to reality. The use of L.S. estimation will therefore give inaccurate results. In theory M.L. estimation is the best procedure to adopt but in practice G.L.S. estimation using the programme developed by Clarke is quicker and just as accurate. For these two reasons plus its availability at the NPL, Clarke's programme has been used throughout this work.

CHAPTER 5. The Design of Digital Compensators:

5.1 Introduction:

It has previously been shown that the 7th order vector-matrix model describing the dynamic behaviour of the distillation column may be reduced to a second-order transfer function model which adequately describes the plant. This exemplifies the contention⁽¹⁷⁾ that many industrial plants may be represented by

$$G_1(s) = \frac{x}{u}(s) = \frac{Ke^{-sT_1}(s+\gamma)}{(s+\delta_1)(s+\delta_2)} \quad \dots 5.1.1$$

If γ is negative then the plant will be non-minimum phase. By cascading a zero-order hold whose transfer function is

$$G_h(s) = \frac{1-e^{-sT}}{s},$$

with the model described by 5.1.1, then in the discrete time domain the z transfer function of the two elements becomes

$$G(z) = G_1 G_h(z) = \frac{x}{u}(z) = z^{-r} \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad \dots 5.1.2$$

where T is the sampling interval

and $r = T_1/T$.

It can be seen that only 5 parameters are required to define the plant model whereas N would be required for a weighting sequence model estimated by correlation analysis using an N -bit P.R.B.S. Thus 5.1.2 is an economical description of the system and in addition, it is one which is based on the time domain and should therefore allow time domain controllers to be readily designed.

The problem discussed in this chapter is that of using models of the type defined by equation 5.1.2 in designing control systems, and in the next

chapter on-line applications of the various design procedures to the distillation column are described. In order to justify the effort involved in obtaining models described by equation 5.1.2, the performance of any closed loop system designed with such a model should show an improvement over the conventional approach and for this reason a brief discussion of the Ziegler-Nichols method of designing 3-term controllers is also included in this chapter.

5.2 A Closed-Loop Control System with Unit Feedback: ⁽¹⁸⁾

Figure 5.2.1 shows the block diagram of the closed-loop control system under discussion in this chapter. The plant z transfer function is $G_p(z)$ and when cascaded with the zero-order hold $G_h(z)$ the overall transfer function becomes $G(z)$ which is the model obtained by G.L.S. estimation. The controller, $D(z)$ whose function is performed by a digital computer, is to be designed such that some control criterion is satisfied. The expression $K(z)$ represents the overall closed-loop transfer function between system input and output.

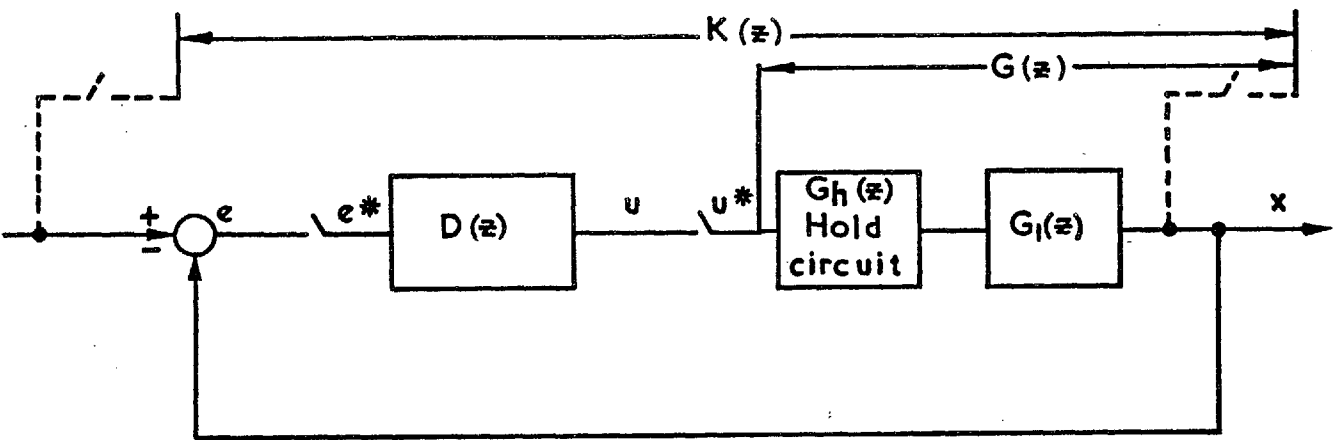


Fig. 5.2.1

"Closed-Loop Control System with Unit Feedback".

In general, two types of control problem confront the process operator, those of regulation and those associated with the change of set-point. Regulation concerns itself with holding some product output at a set level in spite of fluctuations in the process, its feed or operating conditions,

or changes in load. Set point control problems are concerned with altering the level of an output from one value to another such that the new level is attained as quickly as possible and with minimum deviation from this new level once it has been reached. At present it is desirable that $D(z)$ should account for both these functions since available computer storage is usually too small to accommodate separate algorithms. For the same reason $D(z)$ should have as few terms as possible and its output at a sampling instant should be a function only of its previous output values and its present and previous input values. Assuming linear combinations of these terms it can be seen that $D(z)$ may be written in the form

$$D(z) = \frac{u(z)}{e(z)} = \frac{\alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \dots + \alpha_m z^{-m}}{1 + \beta_1 z^{-1} + \beta_2 z^{-2} + \dots + \beta_m z^{-m}}$$

$$= \frac{\sum_{j=0}^m \alpha_j z^{-j}}{\sum_{j=0}^m \beta_j z^{-j}} \quad \dots 5.2.1$$

From Fig. 5.2.1 it can be seen that

$$K(z) = \frac{D(z) G(z)}{1 + D(z) G(z)} \quad \dots 5.2.2$$

Therefore

$$D(z) = \frac{1}{G(z)} \cdot \frac{K(z)}{1 - K(z)} \quad \dots 5.2.3$$

Where $K(z)$ is the overall pulse transfer function relating output to input. Substituting 5.1.2 and 5.2.1 into 5.2.2

$$\begin{aligned}
 K(z) &= \frac{\sum_{j=0}^m \alpha_j z^{-j}}{\sum_{j=0}^m \beta_j z^{-j}} \times z^{-r} \times \frac{\sum_{i=1}^2 b_i z^{-i}}{\sum_{i=0}^2 a_i z^{-i}} \\
 &= 1 + \frac{\sum_{j=0}^m \alpha_j z^{-j}}{\sum_{j=0}^m \beta_j z^{-j}} \times z^{-r} \times \frac{\sum_{i=1}^2 b_i z^{-i}}{\sum_{i=0}^2 a_i z^{-i}} \\
 &= \frac{z^{-r} \left[\sum_{j=0}^m \alpha_j z^{-j} \times \sum_{i=0}^2 b_i z^{-i} \right]}{\sum_{j=0}^m \beta_j z^{-j} \times \sum_{i=0}^2 a_i z^{-i} + z^{-r} \left[\sum_{j=0}^m \alpha_j z^{-j} \times \sum_{i=0}^2 b_i z^{-i} \right]} \\
 &= z^{-r} \times \frac{\sum_{k=1}^{m+2} p_k z^{-k}}{\sum_{k=0}^{(m+r+2)} q_k z^{-k}} \quad \dots 5.2.4
 \end{aligned}$$

Where the p_k, q_k are combinations of the $\alpha_j, \beta_j, a_i, b_i$ above. It can be seen from 5.2.4 that a delay of r time units associated with the plant will produce a delay of r time units on the overall pulse transfer function.

5.3 The Ziegler-Nichols Method of Three-Term Controller Design:

This method is probably the best known one for designing 3 term controllers and was evolved as long ago as 1942⁽¹⁹⁾. The "reaction curve" method is based on the open-loop step response of the plant as shown in Fig. 5.3.1. Two measurements θ and T_d are taken from this Figure

where T_1 is the approximate time delay in the plant and θ is the slope at the point of inflexion. The step response is either taken from an experiment on the plant or from the equation of the model, 5.1.2 if this is known.

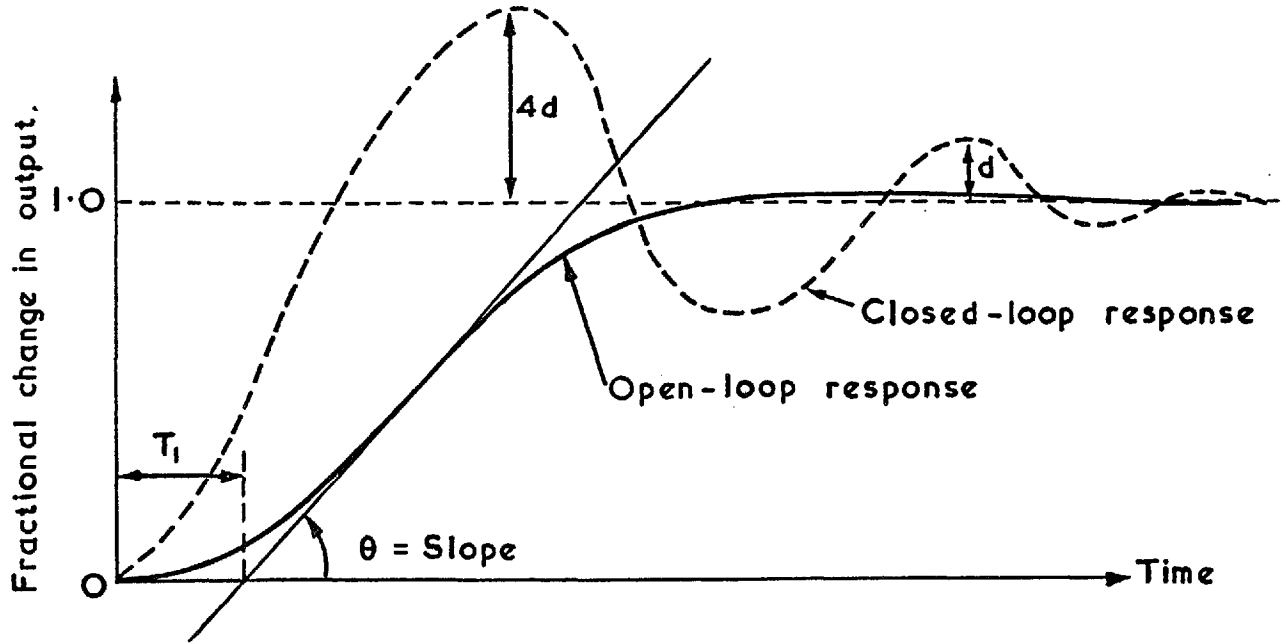


Figure 5.3.1

Three Term Controller design using the Ziegler-Nichols
"Reaction Curve" Method.

The criterion Ziegler and Nichols used for the controller settings was that of a decay ratio of 4:1. This is shown in Fig. 5.3.1 where it can be seen that the second overshoot is $\frac{1}{4}$ of the first. With this criterion the following controller settings were proposed by them:

$$K_c = \frac{1.2}{\theta T_1} \text{ to } \frac{2}{\theta T_1}, \quad T_R = 2T_1, \quad T_D = T_1/2$$

Where the controller is given by

$$D_1(s) = K_c \left[1 + \frac{1}{sT_R} + sT_D \right]$$

In order to approximate this continuous function using a digital computer, it is necessary to find the z transform of

$$D(s) = \frac{1-e^{-sT}}{s} \times K_o \left[1 + \frac{1}{sT_R} + sT_D \right] .$$

With unit sampling time ($T = 1$) and T_R , T_D measured in multiples of T the z transform becomes

$$\begin{aligned} D(z) &= K_o (1-z^{-1}) \left[\frac{1}{1-z^{-1}} + \frac{T/T_R z}{(1-z^{-1})^2} + T_D \right] \\ &= \frac{K_o [(1+T_D) + (1/T_R - 1 - 2T_D) z^{-1} + T_D z^{-2}]}{(1-z^{-1})} \end{aligned} \quad \dots 5.3.1$$

In practice a decay ratio of 4:1 may be unacceptable and in addition, the plant model is usually inaccurately known, so plant operators invariably "tune" the controller in situ. This method is tedious and time-consuming and it is thought that algorithms other than 3 term may lead to superior control since the more flexible structure of the controller (5.2.1) may be better suited to account for process time delays.

5.4 An Optimum Controller via Dynamic Programming:

Methods of designing optimum controllers based on the method of Dynamic Programming have been well discussed elsewhere⁽¹⁰⁾⁽¹¹⁾ and for this reason only a brief outline of the method is discussed here. The process model is written in the form

$$X(k+1) = \phi(k) X(k) + D(k) u(k) \quad \dots 5.4.1$$

and it is required to minimise the Performance Index

$$I_N = \sum_{k=1}^N [X^T(k) Q X(k) + \lambda u^2(k)] \quad \dots 5.4.2$$

$X(k)$ represents an n -th order state vector, $u(k)$ is the single input control signal and Q is a symmetric positive definite matrix. By applying

the Principle of Optimality⁽¹¹⁾ the optimum feedback signal becomes

$$u(j) = M(N-j) X(j) \quad \dots 5.4.3$$

where

$$M(N-j) = - \frac{D^T [Q + P(N-j+1)] \phi}{D^T [Q + P(N-j+1)] D + \lambda} \quad \dots 5.4.4$$

$$P(N-j) = [\phi + DM(N-j)]^T [Q + P(N-j+1)] [\phi + DM(N-j)] + \lambda M^T(N-j) M(N-j) \quad \dots 5.4.5$$

with $P(0) = 0$.

The method requires iterative calculations to be made of $P(N-j)$, $M(N-j)$, $u(j)$ (hence the name of Dynamic Programming) and for this purpose a computer would be needed. To find the new value of $u(j)$, $X(j)$ must be available and as some of the elements of the state vector are usually inaccessible, an estimate of their value must be found. The discrete time state description 5.4.1 is not available directly but must be found from 5.1.2, this is not however a difficult task as shown later.

The problem is simplified if an infinite stage process (i.e. $N \rightarrow \infty$) is considered since in this case the matrix $P(N-j)$ becomes time-invariant, $M(N-j)$ becomes a constant vector and $u(j)$ becomes a weighted sum of the state vectors. A computer programme has been written by D.C. Witt of Oxford University (to whom the author is indebted for calculating the controllers 6.8.5, 6.6.6) which performs the following functions:

1) Given a model 5.1.2 it then converts it to the form 5.1.1, changes this to the continuous-time state model

$$\dot{x} = Ax + Bu$$

and then finds the discrete time form 5.4.1. It is pointed out that only second order models 5.1.2 with an adjoined state variable defined by

$$\dot{x}_3 = x_s - x_1$$

can be handled at present. The adjoined state variable x_3 is used to account for the integral of error between the set-point x_s and the process output x_1 .

2) Assuming an infinite-stage process and given Q, λ , the matrix $P(\infty)$ is found and hence $M(\infty)$ can be calculated.

3) Since the state-vector model is known it is now possible to find by simulation the sequences $\{e\}, \{u\}$ which are the input and output of an equivalent digital controller which would have the same effect as the state variable feed-back controller. By taking the first $2m-1$ terms of $\{e\}, \{u\}$, a matrix inversion procedure is used to derive the parameters of an m -th order digital controller.

An alternative method of obtaining the discrete time model 5.4.1 from the z transfer function model 5.1.2 is described below, this method avoids using the frequency domain and the calculation of the discrete controller $D(z)$ is easily and directly performed. For the general model

$$\frac{x}{u}(z) = \frac{\sum_{i=1}^n b_i z^{-i}}{\sum_{i=0}^n a_i z^{-i}}$$

it is seen that

$$\begin{aligned} x(k+1) = & -a_1 x(k) - a_2 x(k-1) - \dots - a_n x(k-n+1) \\ & + b_1 u(k) + b_2 u(k-1) + \dots + b_n u(k-n+1) \end{aligned}$$

by writing $x(k+1) = x_1(k+1)$ it can be shown that

$$\begin{aligned} x_1(k+1) &= -a_1 x_1(k) + x_2(k) + b_1 u(k) \\ x_2(k+1) &= -a_2 x_1(k) + x_3(k) + b_2 u(k) \\ x_3(k+1) &= -a_3 x_1(k) + x_4(k) + b_3 u(k) \\ &\vdots \\ x_n(k+1) &= -a_n x_1(k) + b_n u(k) . \end{aligned}$$

And the discrete state-vector $X(k+1)$ can be written

$$X(k+1) = \phi X(k) + D u(k)$$

Where

$$X(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_n(k+1) \end{bmatrix}, \quad D = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$\phi = \begin{bmatrix} -a_1 & 1 & 0 & 0 & \dots & 0 \\ -a_2 & 0 & 1 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -a_{n-1} & 0 & 0 & 0 & \dots & 1 \\ -a_n & 0 & 0 & \dots & \dots & 0 \end{bmatrix}$$

With the same performance index (5.4.2) and following the standard Dynamic Programming method as before a control signal $u(k)$ will result where

$$u(k) = -M_1 X(k),$$

i.e. the optimal control signal is a constant linear function at the state variables. In this case however the state vector is accessible since it is a function of the values $a_1, b_1, x(k-1), u(k-1)$ which are all known. For example consider a second order model

$$x(k+1) = -a_1 x(k) - a_2 x(k-1) + b_1 u(k) + b_2 u(k-1)$$

The optimal control signal will be of the form

$$u(k) = (m_1, m_2) \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix}$$

and

$$x_1(k) = -a_1 x_1(k-1) + x_2(k-1) + b_1 u(k-1)$$

$$x_2(k) = -a_2 x_1(k-1) + b_2 u(k-1)$$

Thus

$$\begin{aligned}u(k) &= m_1 x_1(k) + m_2 x_2(k) \\&= m_1 x_1(k) + m_2 [-a_2 x_1(k-1) + b_2 u(k-1)]\end{aligned}$$

which is seen to be in the form of 5.2.1, the general linear discrete controller. The advantage of following this alternative approach is the reduction in computation needed since all calculations are now performed in the time domain. However a computer programme is still required to find the values of $M(\infty)$ and $P(\infty)$ although the equation of the discrete controller is more simply and reliably obtained. Since the computer programme to perform the calculations for the first method was freely made available, results are presented in the next chapter for this method only, although it seems reasonable to suppose that the alternative method would produce exactly the same controller. Both methods would require several trials with various values of λ in order to select the controller with the most acceptable response.

5.5 Plant Compensation by Pole-Zero Cancellation:

Given a model in the form 5.1.2, it would seem possible to obtain improved closed-loop performance by cancelling the unwanted poles and zeros of the plant with the digital compensator. It can be shown⁽²⁰⁾ however that, for poles and zeros which lie on or outside the unit circle, attempts at cancellation can lead to closed-loop instability if the cancellation is imperfect. In practice, models described by equation 5.1.2 would be composed of estimates of the parameters a_i , b_i and these have uncertainties related to the variance of these estimates, so if the parameters lie close to or outside the unit circle attempts at cancellation would lead to instability. Since cancellation by $D(z)$ of process poles and zeros which lie on or outside the unit circle is to be disallowed, then a restriction will also exist on $K(z)$ and this can be determined from the following: suppose the plant transfer function can be written as

$$G(z) = \frac{\prod_{i=1}^I (1-b_i z^{-1})}{\prod_{j=1}^J (1-a_j z^{-1})} \times F(z) \quad \dots 5.5.1$$

where the b_i, a_j are the zeros and poles (respectively) which lie on or outside the unit circle and $F(z)$ represents the remainder of $G(z)$ which lies inside the unit circle. Then from 5.2.3

$$D(z) = \frac{\prod_{j=1}^J (1-a_j z^{-1})}{\prod_{i=1}^I (1-b_i z^{-1}) F(z)} \times \frac{K(z)}{1-K(z)} \quad \dots 5.5.2$$

and if the poles and zeros outside the unit circle are not to appear in $D(z)$ then

$$1-K(z) = \prod_{j=1}^J (1-a_j z^{-1}) \times P(z) \quad \dots 5.5.3$$

$$K(z) = \prod_{i=1}^I (1-b_i z^{-1}) \times Q(z) \quad \dots 5.5.4$$

where $P(z), Q(z)$ are polynomials in z^{-1} . It can be seen therefore that $1-K(z)$ is to contain the poles of $G(z)$ which lie outside the unit circle, while $K(z)$ is to contain the zeros of $G(z)$ which lie outside the unit circle. It will be shown in the next section that to eliminate intersample ripple $K(z)$ should contain all the zeros of $G(z)$.

5.6 Elimination of Intersample Ripple:

The transfer function $K(z)$ relates the closed loop system output to the input at sampling intervals only, and to ascertain system behaviour between these intervals use is made of the modified z transform. It can be seen from Fig. 5.2.1 the overall modified z transfer function, $K(z, m)$ is given by

$$K(z, m) = \frac{x(z, m)}{r(z)} = G(z, m) \times \frac{K(z)}{G(z)} \quad \dots 5.6.1$$

and it may also be shown that the poles of $G(z, m), G(z)$ are the same while the zeros of $G(z, m)$ are functions of m, z , and those of $G(z)$ are

purely functions of z . It is therefore possible to write 5.6.1 in the form

$$K(z, m) = \frac{\text{Zeros of } G(z, m)}{\text{Zeros of } G(z)} \times K(z) \quad \dots 5.6.2$$

from which it can be seen that only if $K(z)$ is a finite polynomial containing all the zeros of $G(z)$ will $K(z, m)$ be a finite polynomial, otherwise it will be an infinite polynomial in which each term is a function of m . In order that the closed-loop output shall be ripple-free after a finite time to a standard input, it can be seen that $K(z, m)$ should be a finite polynomial and that it should become independent of m after the first few terms. Thus $K(z)$ should contain all the zeros of $G(z)$ to achieve ripple-free operation at steady-state, i.e. $K(z)$ should be of the form

$$K(z) = \prod_{i=1}^i (1 - b_i z^{-1}) \times Q(z) \quad \dots 5.6.3$$

where b_i accounts for all the zeros of $G(z)$.

5.7 A Minimum Time Controller:

The objective of this type of controller is to change the plant output from one value to another in the shortest possible time. From Figure 5.2.1 it can be seen that

$$\begin{aligned} e(z) &= r(z) - x(z) \\ &= [1 - K(z)] r(z) \end{aligned} \quad \dots 5.7.1$$

The Final-value theorem⁽¹⁸⁾ states that

$$\begin{aligned} \lim_{t \rightarrow \infty} e(t) &= \lim_{z \rightarrow 1} [(1 - z^{-1}) e(z)] \\ &= \lim_{z \rightarrow 1} [(1 - z^{-1}) \{1 - K(z)\} r(z)] \end{aligned}$$

If $r(z)$ is an impulse, step, ramp, or acceleration input then $r(z)$ can always be written in the form

$$r(z) = \frac{P_1(z)}{(1-z^{-1})^k}$$

where k is an integer and $P_1(z)$ is a polynomial in z^{-1} which has no factors in $(1-z^{-1})$. Hence

$$\begin{aligned} \lim_{t \rightarrow \infty} e(t) &= \lim_{z \rightarrow 1} [(1-z^{-1}) \{1-K(z)\} \frac{P_1(z)}{(1-z^{-1})^k}] \\ &= 0 \quad \text{if} \end{aligned}$$

$$1-K(z) = (1-z^{-1})^k P_2(z) \quad \dots 5.7.2$$

where $P_2(z)$ is a polynomial in z^{-1} .

If

$$G(z) = z^{-r} \times \frac{\sum_{i=1}^n b_i z^{-i}}{\sum_{i=0}^n a_i z^{-i}}$$

then $K(z)$ should be of the form

$$K(z) = z^{-r} Q(z) \quad \dots 5.7.3$$

Equations 5.7.2 and 5.7.3 should be satisfied simultaneously and if the minimum number of terms are used in $P_2(z)$ and $Q(z)$ the resulting controller is termed the minimum prototype. If $k=1$ for example then

$$1-K(z) = (1-z^{-1}) P_2(z) = 1-z^{-r} Q(z)$$

If $Q(z) = 1$

then $K(z) = z^{-r}$

and from 5.2.3

$$D(z) = \frac{\sum_{i=0}^n a_i z^{-i}}{z^{-r} \sum_{i=1}^n b_i z^{-i}} \times \frac{z^{-r}}{1-z^{-r}}$$

$$= \frac{\sum_{i=0}^n a_i z^{-i}}{(1-z^{-r}) \sum_{i=1}^n b_i z^{-i}}$$

This is the minimum prototype controller. Note that $K(z)$ does not contain all the plant zeros and hence intersample ripple will be present. Also, the controller is attempting to cancel the plant poles and if some of these lie near the unit circle a lightly damped control system may result. It is probable that pure minimum time controllers are unsatisfactory in chemical process work since it is desired not only to reach the new set-point but to hold this level without significant variation. This requirement contrasts with that of ballistic control where, once the missile has destroyed its target, system behaviour is of no further interest.

5.8 A Controller to Minimise a Quadratic Performance Index

A similar Performance Index to that used in section 5.4 is proposed, viz

$$I = \sum_{n=0}^{\infty} [e^2(nT) + \lambda u^2(nT)] \quad \dots 5.8.1$$

The objective is to minimise the sum of the error-squared terms with an additional term to account for the control effort. Equation 5.8.1 can be written⁽¹⁸⁾

$$I = \text{Sum of Residues of } [e(z) e(z^{-1}) z^{-1} + \lambda u(z) u(z^{-1}) z^{-1}] \quad \dots 5.8.2$$

Since

$$\begin{aligned} e(z) &= [1-K(z)] r(z) , \\ u(z) &= D(z) e(z) \\ &= \frac{K(z) r(z)}{G(z)} \\ &= \frac{r(z) - e(z)}{G(z)} , \end{aligned}$$

I can be found as a function of $K(z)$ since $G(z)$, $r(z)$ and λ will be known. Equations 5.2.4, 5.5.3 and 5.6.3 specify $K(z)$ and if the minimum number of terms are used in the polynomials P , Q then the minimum response time controller can be found. By including extra terms in these polynomials however it is possible to select them to optimise some criterion, such as the one proposed above. In this case I becomes a function of the unknown terms and can be minimised by using 5.8.2, but to do this considerable algebraic calculations are required and the method is of dubious value.

One of the arguments for weighting the control effort is to prevent it from saturating in the case of a large input signal, since there is then the possibility that instability may occur. There is no guarantee that the stable closed-loop system may not become unstable when the saturation limits are reached. In spite of this argument it is thought that such cases will be rare and can probably be handled as they arise. A simpler set of calculations result if $\lambda = 0$ and this controller is considered in the next section.

5.9 A Mean Squared Error Controller:

If $\lambda = 0$ in equation 5.8.1 then the Performance Index becomes the sum of the errors squared:

$$I = \sum_{n=0}^{\infty} [e^2(nT)]$$

... 5.9.1

As before this may be written

$$I = \text{Sum of Residues of } [e(z) e(z^{-1}) z^{-1}] , \quad \dots 5.9.2$$

and from 5.6.1

$$e(z) = [1-K(z)] r(z) \quad \dots 5.9.3$$

Assuming the plant model is known then 5.2.4, 5.5.3 and 5.6.3 must be satisfied and these equations enable an expression to be found for $K(z)$. The input $r(z)$ is specified and thus $e(z)$ is found, leading to the minimisation of I . The steps in the calculation are enumerated below where it is assumed that the system is to respond to a step input with zero steady-state error and the controller is to be optimised with respect to a pulse input.

1) Given the plant model

$$G(z) = z^{-r} \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} ,$$

it can be seen from sections 5.2 and 5.6 that $K(z)$ should be of the form

$$K(z) = z^{-r} (b_1 z^{-1} + b_2 z^{-2}) (q_0 + q_1 z^{-1} + \dots + q_m z^{-m}) \quad \dots 5.9.4$$

2) To satisfy the requirements of section 5.5

$$1-K(z) = \prod_{j=1}^J (1-a_j z^{-1}) P(z)$$

where $P(z)$ is a polynomial in z^{-1} and a_j are the poles of $G(z)$ which lie on or outside the unit circle. Since the G.I.S. estimation scheme will fail if the plant poles lie on or outside the unit circle it is assumed here that the a_j will always lie within the unit circle. Hence

$$1-K(z) = P(z)$$

3) To settle to steady-state after a step input 5.7.2 must be satisfied and hence

$$\begin{aligned} 1-K(z) &= (1-z^{-1}) P_2(z) \\ &= (1-z^{-1}) (p_0 + p_1 z^{-1} + \dots + p_L z^{-L}) \dots 5.9.5 \end{aligned}$$

and this also agrees with step 2.

4) The coefficients of the highest power in z^{-1} in 5.9.4 and 5.9.5 must be the same hence

$$\begin{aligned} -l-1 &= -m-2-r \\ \text{i.e. } l &= m+r+1 \end{aligned}$$

and these equations can be used to eliminate some of the unknowns p_i, q_i ;

$$\begin{aligned} 1-z^{-r} (b_1 z^{-1} + b_2 z^{-2}) (q_0 + q_1 z^{-1} + \dots + q_m z^{-m}) \\ = (1-z^{-1}) [p_0 + p_1 z^{-1} + \dots + p_{r+m+1} z^{-(r+m+1)}] \end{aligned}$$

it can be seen that

$$p_0 = 1, (p_1 - p_0) = (p_2 - p_1) = \dots = (p_r - p_{r-1}) = 0$$

there remain $m+1$ unknowns in q_i , $m+1$ unknowns in p_i , and $m+1$ equations. Thus the $m+1$ unknowns may be assigned arbitrary values, (as long as they satisfy the relationships between the p_i and q_i), but their values can also be selected by optimising the performance index I .

6) For a unit pulse input

$$e(z) = [1-K(z)] = 1-z^{-r} (b_1 z^{-1} + b_2 z^{-2}) (q_0 + q_1 z^{-1} + \dots + q_m z^{-m}) \quad \dots 5.9.6$$

By substituting 5.9.6 into 5.9.2 and finding $\frac{\partial I}{\partial q_j}$ for the values of q_j not yet determined, it is possible to find all the values of p_1, q_1 which will make I a minimum.

7) The optimum digital controller is then given by

$$\begin{aligned} D(z) &= \frac{1}{G(z)} \times \frac{K(z)}{1-K(z)} \\ &= \frac{1+a_1 z^{-1}+a_2 z^{-2}}{z^{-r} (b_1 z^{-1}+b_2 z^{-2})} \times \frac{z^{-r} (b_1 z^{-1}+b_2 z^{-2}) (q_0+q_1 z^{-1} + \dots + q_m z^{-m})}{(1-z^{-1}) [p_0+p_1 z^{-1} + \dots + p_{r+m+1} z^{-(r+m+1)}]} \\ &= \frac{(1+a_1 z^{-1}+a_2 z^{-2}) (q_0+q_1 z^{-1} + \dots + q_m z^{-m})}{(1-z^{-1}) [p_0+p_1 z^{-1} + \dots + p_{r+m+1} z^{-(r+m+1)}]} \end{aligned}$$

The unknown factor in this method of design is the number of terms to include in $K(z)$. The minimum number of terms which satisfies both 5.9.5 and 5.9.6 will lead to a minimum time response, but as shown in the examples in the next chapter it is only necessary to add one extra term to the resulting $K(z)$ to satisfactorily account for a minimum error-squared criterion. Step 6 appears to be a complicated one mathematically, but because $e(z)$ is of a fortunate type, the calculation of the residue is a very simple task.

5.10 Conclusions:

In this chapter methods of designing Feedback control systems for processes described by a discrete transfer function have been reviewed. Steps involved in the design procedures have been outlined without reference to numerical examples which will be discussed in detail in the next chapter.

CHAPTER 6. On-Line Control of the Distillation Column:

6.1 Introduction:

So far it has been shown how a satisfactory model of the Distillation column can be obtained from theoretical considerations with the aid of certain measurements. Methods of using these models in designing closed-loop control schemes were discussed in chapter 5 and in this chapter the realisation of these schemes is described. The types of controller discussed are three term, minimum mean squared error and those designed by the technique of Dynamic Programming. Before discussing the results obtained a brief description of the on-line control programme used in these experiments will be given.

6.2 The on-line control programme:

The control programme was written to the author's specification by Mr. Keith Wilkinson of the N.P.L. in mnemonic autocode for the Ferranti Hermes computer. The control algorithm is written in the general form

$$D(z) = \frac{\sum_{i=0}^{n-1} \alpha_i z^{-i}}{\sum_{i=0}^{n-1} \beta_i z^{-i}}$$

and $\beta_0 = 1$. The maximum allowable value of n was 12 but normally n is about 3 or 4. The control programme is stored on the Hermes drum and is brought into core-store by a call-tape. Merely by pressing a key on the computer console, a standard format tape would be read into the tape reader and thereby a "new" control algorithm would replace an "old" one. Fig. 6.2.1 shows a block diagram of the computer hesitate and interrupt operations which are initiated by a crystal controlled digital clock situated on the distillation column control panel. Each sampling interval 16 variables are scanned, one per second and they are quantised by a digital voltmeter and sent in parallel form to a binary coded decimal (B.C.D.) encoder from which they are hesitated serially into the core store. There are 6 B.C.D. characters (consisting of 8

bits each) per variable and each bit requires 6 μ s for hesitation.

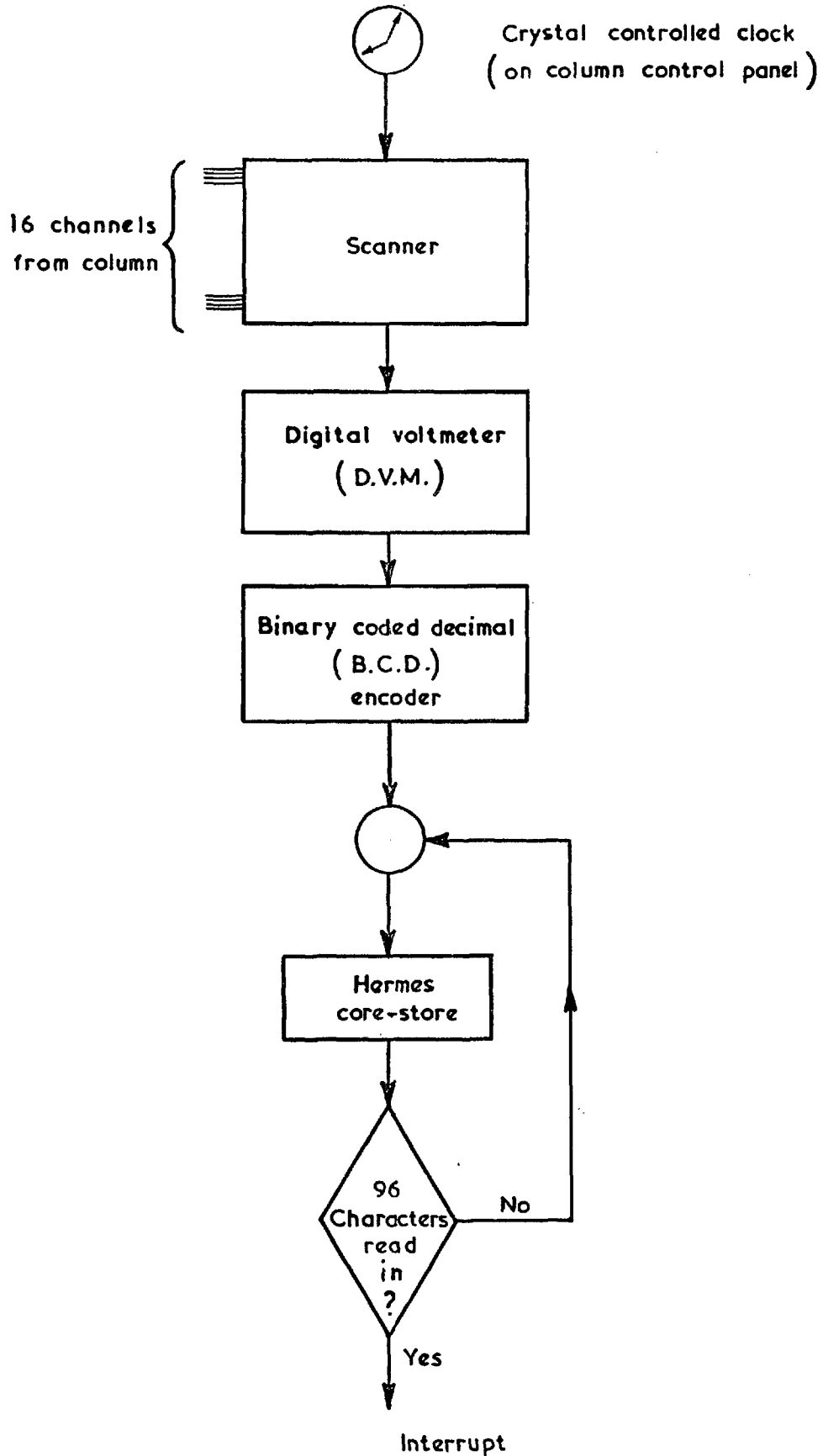


FIG. 6.2.1.

A counter interrupts the main programme when 96 (i.e. 16×6) characters have been read into the core-store and the DDC programme is then initiated. A cathode ray tube display is available of up to 100 past values of selected variables (reflux, top-product composition, temperatures, etc.)

6.3 Control Signal Constraints:

In this work the top-product composition was controlled by the Reflux Ratio which in turn was controlled by a voltage applied from the computer via a keyboard to a buffer store. The maximum and minimum values that can be output by the buffer store is 12.8 volts and 0 volts respectively. Unfortunately, if 0 volts are applied to the tipping bucket Reflux controller, all the condensed vapour re-enters the column and none will leave. This means that the top-product composition meter (which requires a constant flow of liquid through its measuring cell in order to detect changes in composition) will detect no deviation from the last reading, and so for this reason the minimum reflux was changed from 0 volts to 0.8 volts.

6.4 Experimental Conditions:

To compare the performances of the different control schemes accurately it was important to run them on-line under identical conditions. A P.R.B.S. modelling experiment at a feed flow-rate of 4.7 cc/min, feed composition of 7.2%, heat of 1.4 KW and at a 96 second sampling interval produced the following second-order model (see Fig. 3.10.2):

$$G(z) = \frac{\Delta x_T}{\Delta V_R} = \frac{0.156z^{-1} - 0.413z^{-2}}{1 - 1.48z^{-1} + 0.491z^{-2}} = \frac{0.156z^{-1} (1 - 2.63z^{-1})}{(1 - 0.966z^{-1})(1 - 0.512z^{-1})} \quad \dots 6.4.1$$

The steady-state gain estimated from this model was -19.4%/volt and the mean output level was 61.1%. The theoretical first-order model found from the modelling procedure described in chapters 2 and 3 was

$$G(z) = z^{-2} \frac{-0.659z^{-1}}{1-0.966z^{-1}} \quad \dots 6.4.2$$

The steady-state gain was -19.3%/volt and the above conditions and models are used throughout the control experiments described in this chapter. The double time delay which appears in the model 6.4.2 was found to be present in the system from the step response tests described previously. There is evidence that this time delay is an approximation to a non-minimum phase effect associated with the dielectric composition meter which was calibrated under steady-state conditions of flow only. The top-plate temperature measurement (which should correspond closely to the top-product composition), and also manual calculations of the top-product composition before it entered the composition meter did not exhibit the non-minimum phase effect.

Control schemes were ^{also} designed using the model 6.4.2 (which is an approximation to the plant behaviour) because:

1) The modelling scheme usually obtained accurate estimates of the gain and major time constant, but the secondary time constant was not so well defined, probably because of measurement inaccuracies.

2) It is thought that adequate regulation and set-point controllers will result from the efficient use of a well-defined first-order model.

Point 2 is later confirmed by the experimental results.

6.5 Three Term Controller Design:

Following the method of Ziegler and Nichols as outlined in section 5.3 the following values are found: $K_o = -1.096$, $T_D = 1$, $T_R = 4$, $T_1 = 2$, $T = 1$. Substituting these values into 5.3.1 it is found that

$$D(z) = \frac{-2.2 + 3.02z^{-1} - 1.1z^{-2}}{1-z^{-1}} \quad \dots 6.5.1$$

When this controller was applied to the plant the response to set-point changes was highly oscillatory and in common with normal practice an attempt was made to tune the controller on line. In this case the gain was reduced

by a factor of 2. The results are shown in Fig. 6.5.1 where it can be seen that the overshoot and settling time are acceptable by normal industrial standards. Further adjustment of the constants in this controller could possibly result in better behaviour but this was not attempted.

6.6 An Optimum Controller via Dynamic Programming:

Starting with the general form

$$G(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad \dots 6.6.1$$

the Laplace transform of the plant is found to be

$$G(s) = \frac{K(s+\gamma)}{(s+\delta_1)(s+\delta_2)} \quad \dots 6.6.2$$

and by writing

$$\begin{aligned} \dot{x}_1 &= -\delta_1 x_1 + x_2 + Ku \\ \dot{x}_2 &= -\delta_2 x_2 + (\gamma - \delta_2)Ku \\ \dot{x}_3 &= x_1 - x_s \end{aligned}$$

the state variable model with states x_1, x_2, x_3 is derived. The state variable diagram is shown in Figure 6.6.1 and x_s is the set-point of the plant.

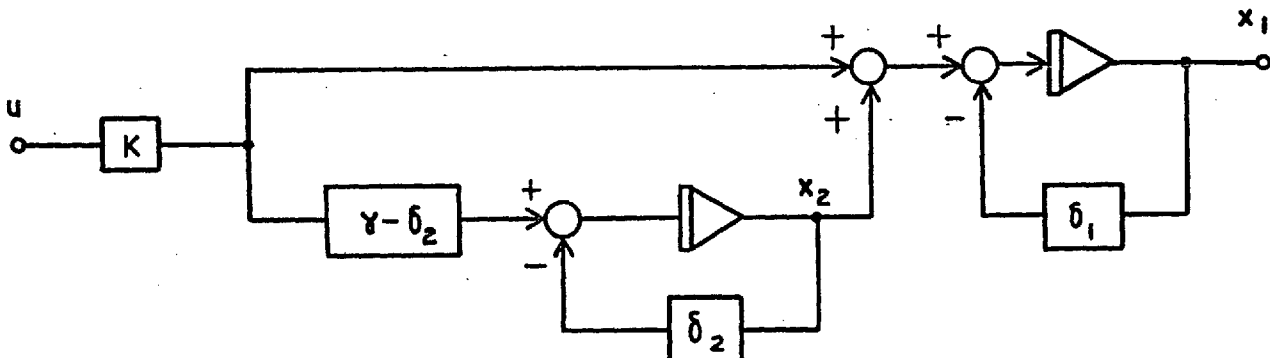


FIGURE 6.6.1. STATE VARIABLE MODEL

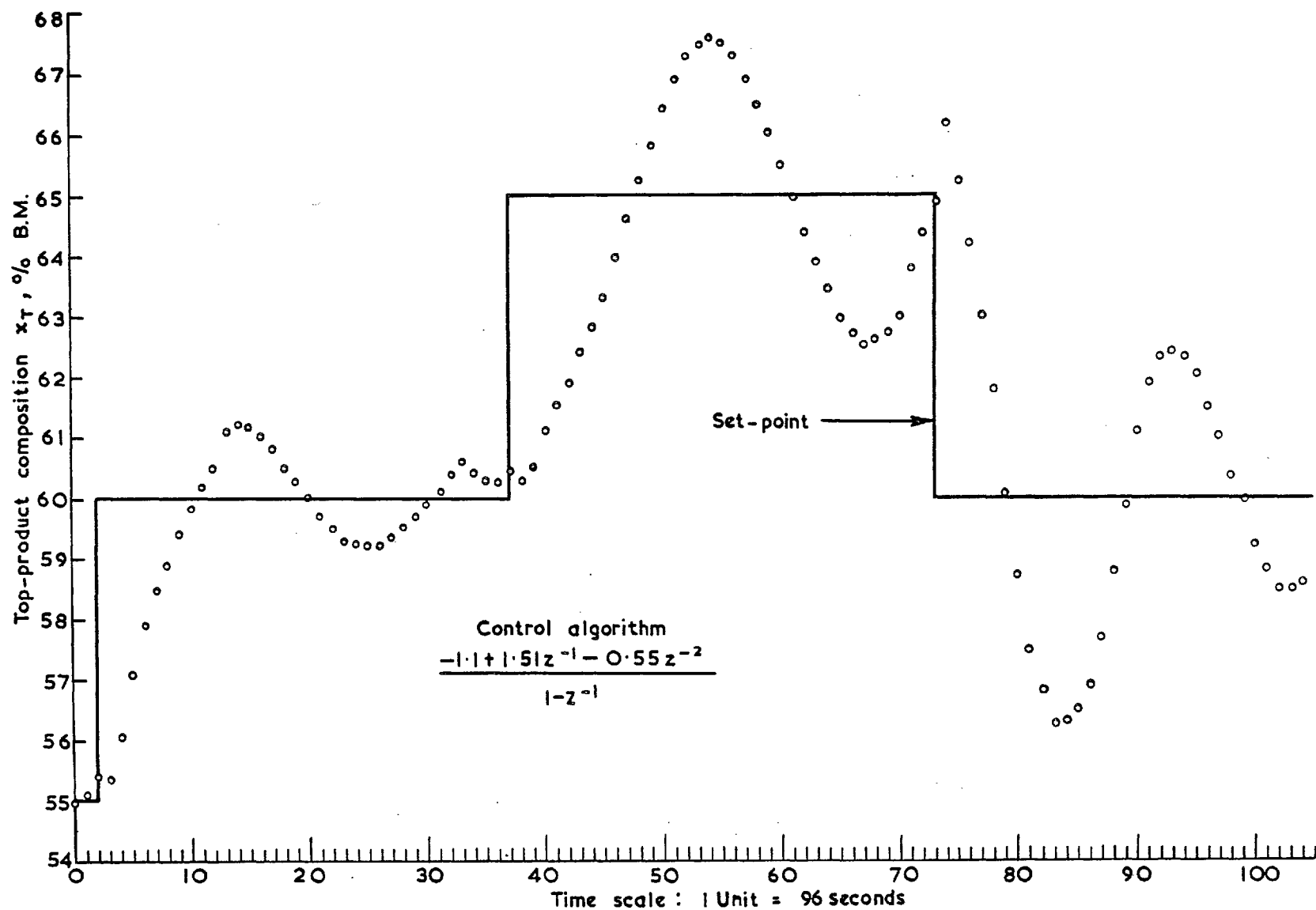


FIG. 6.5.1. ON-LINE CONTROL OF TOP-PRODUCT COMPOSITION USING A THREE-TERM CONTROLLER

The vector, matrix model is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\delta_1 & 1 & 0 \\ 0 & -\delta_2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} k \\ K(\gamma - \delta_2) \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_s \quad \dots 6.6.3$$

and since $b_1 = 0.156$, $b_2 = -0.413$, $a_1 = -1.48$, $a_2 = 0.491$

$$\delta_1 = 0.03, \quad \delta_2 = 0.7, \quad \gamma = -0.85, \quad k = .42$$

and the continuous state-vector form 6.6.3 becomes the discrete state vector form

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0.966 & 0.72 & 0 \\ 0 & 0.512 & 0 \\ 0.983 & 0.4 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0.156 \\ -0.461 \\ 0.112 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} x_s \quad \dots 6.6.4$$

The matrix Q (see section 5.4) is

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and λ can take any arbitrary value. Taking values of $\lambda = 1$ and $\lambda = 4$ the following controllers are found:

$$D(z) \big|_{\lambda = 1} = \frac{-3.163 + 4.18z^{-1} - 1.312z^{-2}}{1 - 0.09z^{-1} - 0.91z^{-2}} \quad \dots 6.6.5$$

$$D(z) \big|_{\lambda = 4} = \frac{2.083 + 2.8z^{-1} - 0.886z^{-2}}{1 - 0.516z^{-1} - 0.484z^{-2}} \quad \dots 6.6.6$$

Plant response to set point changes with the controllers defined by equations 6.6.5, 6.6.6 are shown in Figures 6.6.2, 6.6.3 and 6.6.4. It can be seen that $D(z) \big|_{\lambda = 4}$ gives a better response and this controller was selected for use

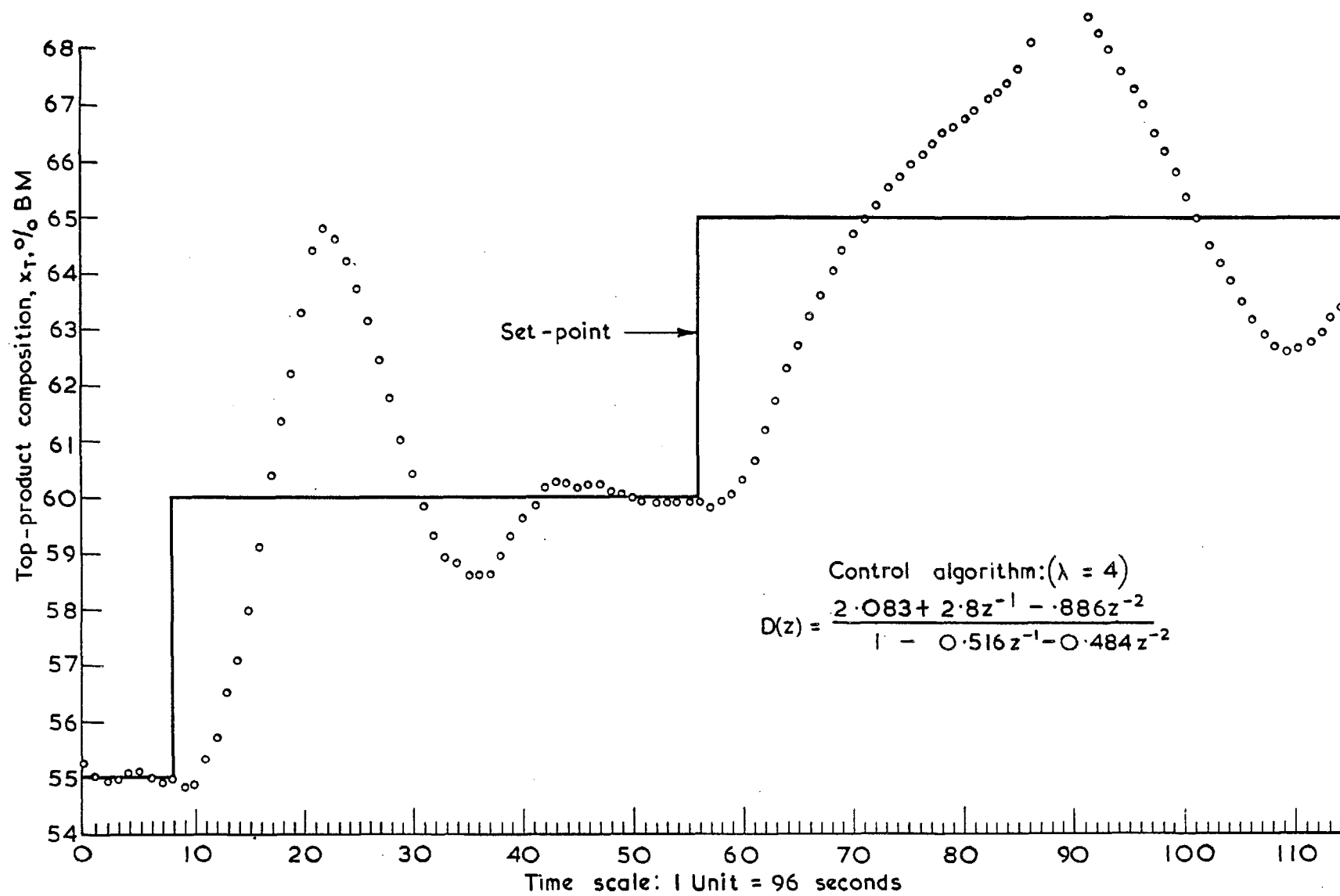


FIG. 6.6.2. ON-LINE CONTROL OF TOP-PRODUCT COMPOSITION

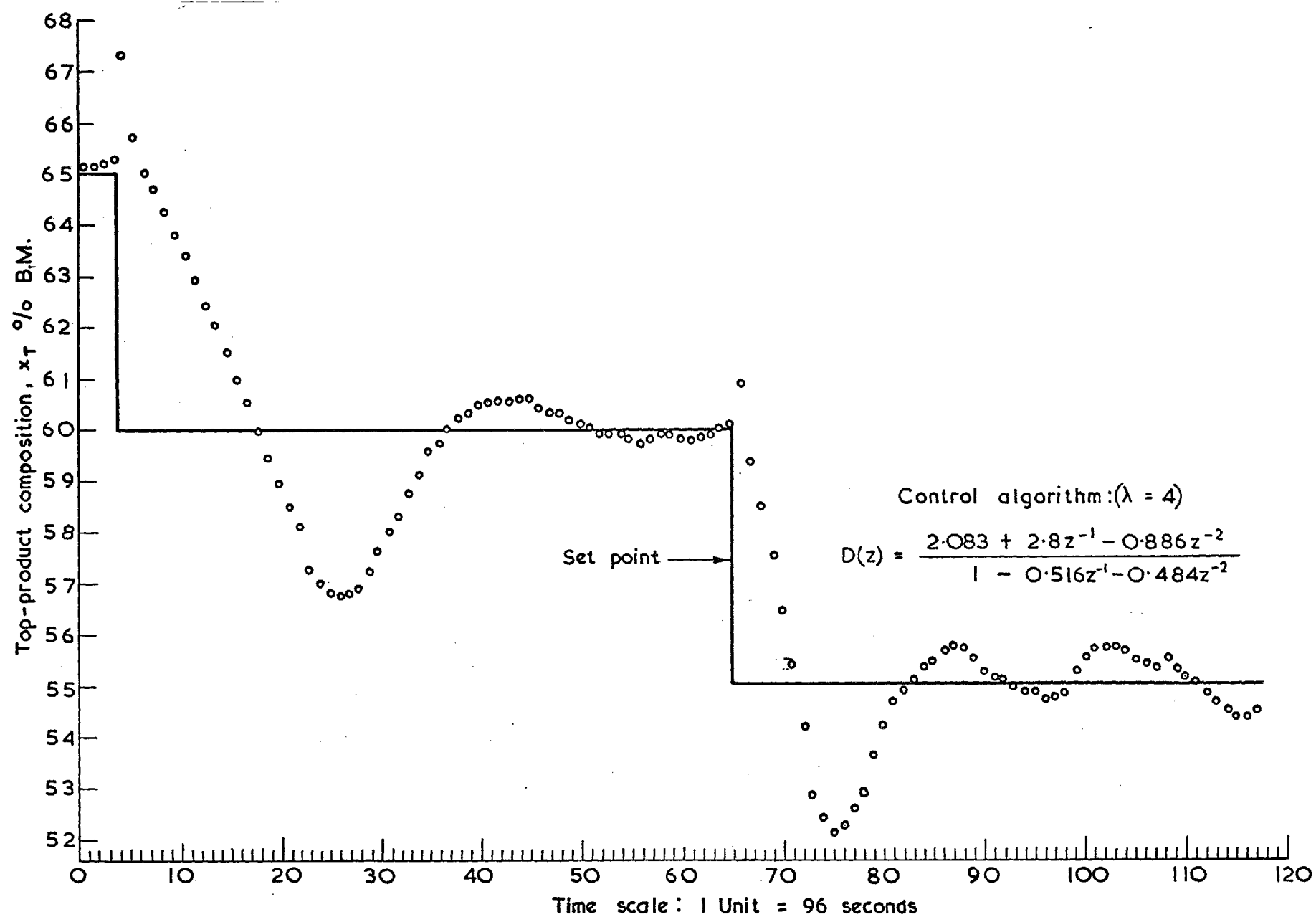


FIG. 6.6.3. ON-LINE CONTROL OF TOP-PRODUCT COMPOSITION

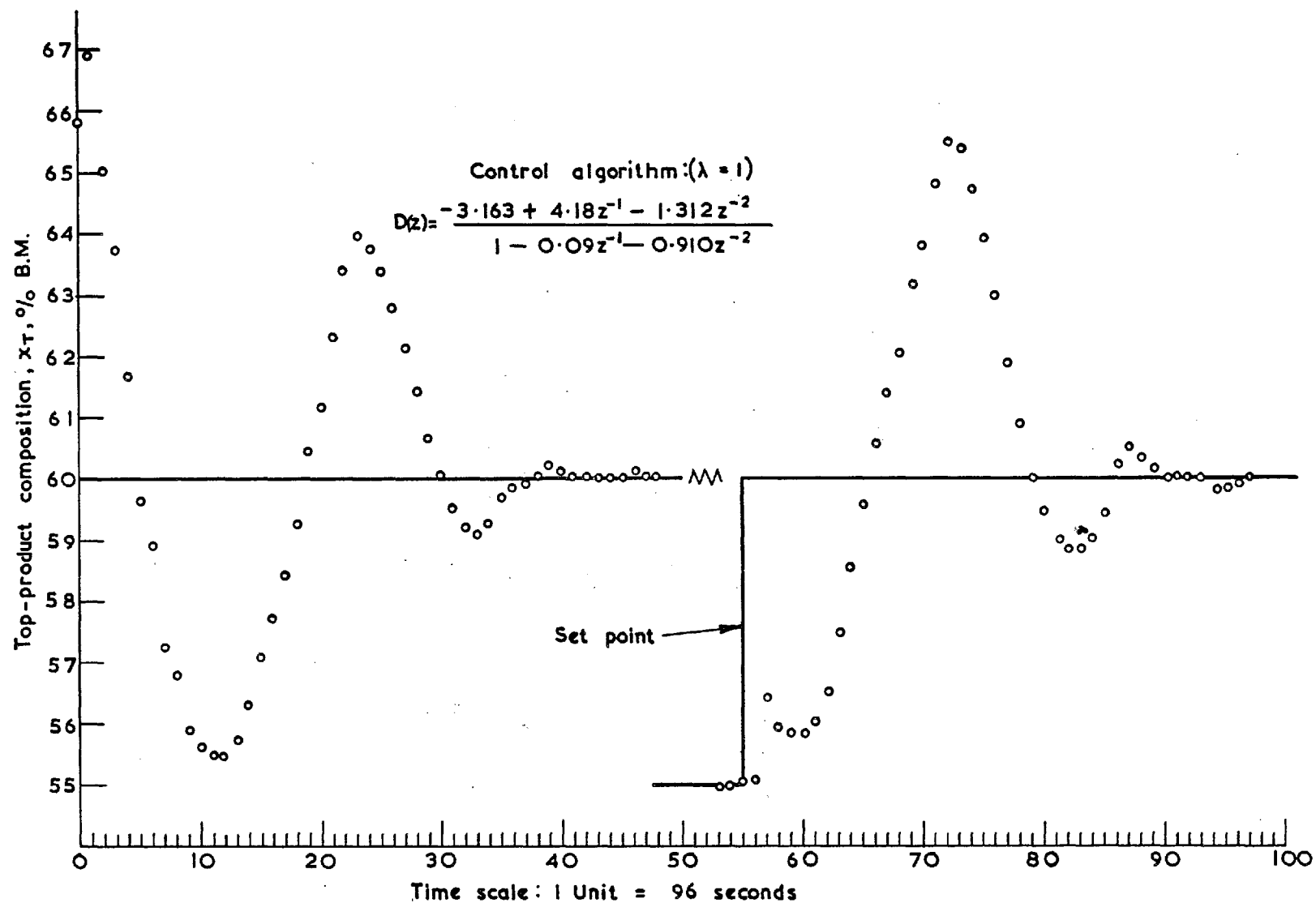


FIG. 6.6.4. ON-LINE CONTROL OF TOP-PRODUCT COMPOSITION.

in the regulation experiments which are discussed in section 6.9.

6.7 A Minimum Time Controller:

Considering the model 6.4.1 it can be seen that $K(z)$ should be of the form

$$K(z) = (1 - 2.63z^{-1}) (q_0 + q_1 z^{-1})$$

and to respond to a step in minimum time

$$1 - K(z) = (1 - z^{-1}) (p_0 + p_1 z^{-1})$$

Hence $p_0 = 1$, $q_0 = 0$, $p_1 = 1.6$, $q_1 = -0.6$
and

$$\begin{aligned} D(z) &= \frac{1}{G(z)} \times \frac{-(1 - 2.63z^{-1}) 0.6z^{-1}}{(1 - z^{-1}) (1 + 1.6z^{-1})} \\ &= \frac{-0.6 (1 - 0.966z^{-1}) (1 - 0.512z^{-1})}{0.156 (1 - z^{-1}) (1 + 1.6z^{-1})} \end{aligned} \quad \dots 6.7.1$$

Alternatively, using the model 6.4.2 it can be shown that

$$D(z) = \frac{(1 - 0.966z^{-1})}{-0.659 (1 - z^{-3})} \quad \dots 6.7.2$$

The result of applying the controller 6.7.2 to the plant is shown in Fig. 6.7.1 where it will be seen by comparison with the later results that the response is the fastest but that instability commenced once the set-point had been attained. The reason for this is the attempt (in either of the above cases) to directly cancel a pole which lies close to the unit circle.

6.8 A Minimum Squared Error Controller:

Starting with the model 6.4.1 and following the method described in section 5.9

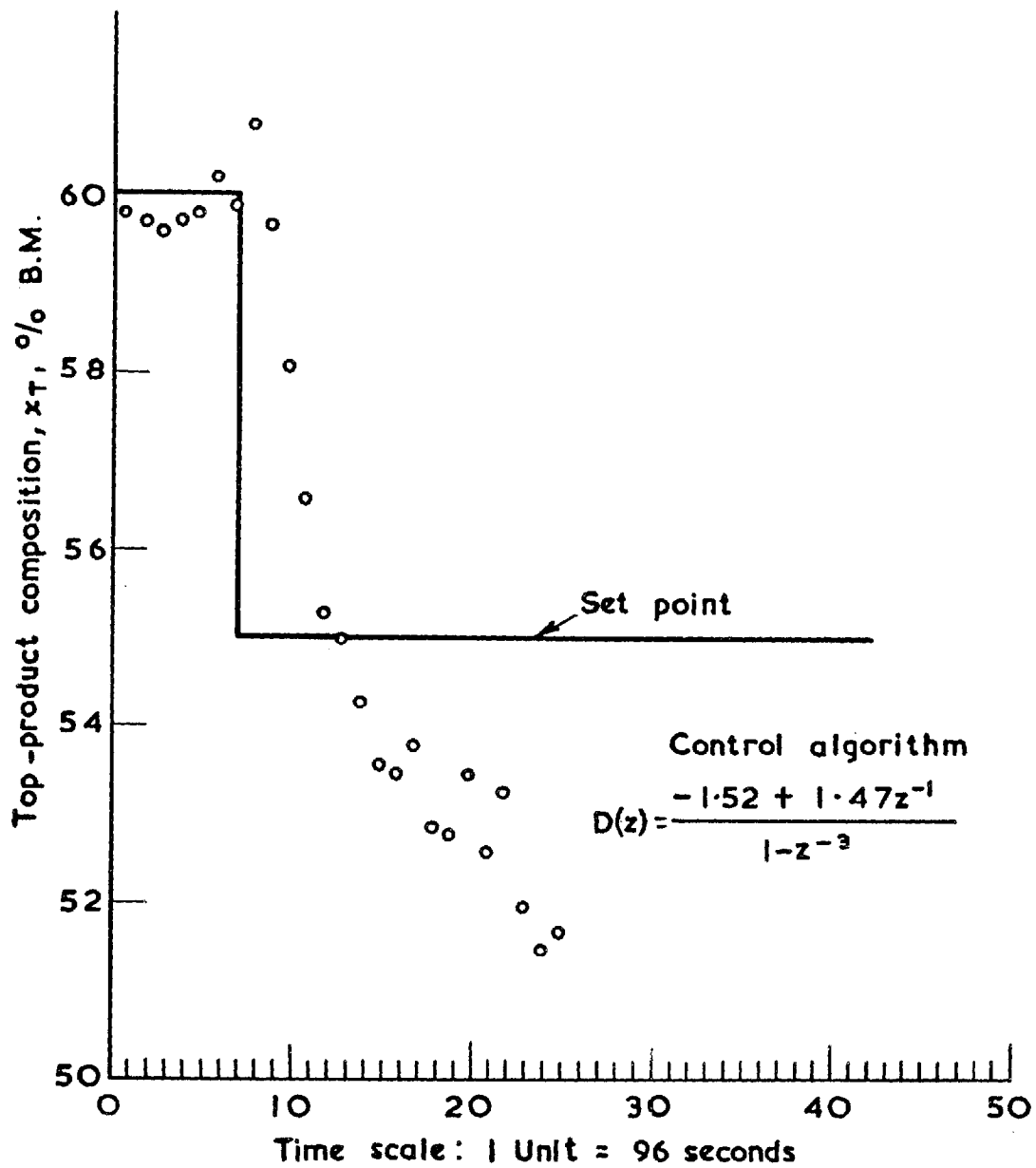


FIG. 6.7.1. MINIMUM RESPONSE-TIME CONTROLLER

$$K(z) = z^{-1}(1-2.63z^{-1})(q_0+q_1z^{-1})$$

$$1-K(z) = 1-q_0z^{-1}-(q_1-2.63q_0)z^{-2}+2.63q_1z^{-3} \quad \dots 6.8.1$$

For the system to settle to steady-state after a step input

$$\begin{aligned} 1-K(z) &= (1-z^{-1})(p_0+p_1z^{-1}+p_2z^{-2}) \\ &= p_0+(p_1-p_0)z^{-1}+(p_2-p_1)z^{-2}-p_2z^{-3} \end{aligned} \quad \dots 6.8.2$$

Thus equating like powers of z^{-1} in 6.8.1, 6.8.2 it can be seen that

$$p_0 = 1, \quad q_0+q_1 = -0.6$$

The error for a pulse input can be written

$$\begin{aligned} e(z) &= [1-K(z)] r(z) \\ &= 1-q_0z^{-1}-(-0.6-q_0-2.63q_0)z^{-2}+2.63(-0.6-q_0)z^{-3} \end{aligned}$$

Thus $I = \text{Sum of Residues of } [e(z) e(z^{-1}) z^{-1}]$

$$\begin{aligned} &= 1+q_0^2+(0.6+q_0+2.63q_0)^2+2.63^2(0.6+q_0)^2 \\ \frac{dI}{dq_0} &= 42.15q_0+12.66 = 0 \quad (\text{for a minimum, since } \frac{d^2I}{dq_0^2} > 0) \\ \text{giving } q_0 &= -0.3 \\ q_1 &= -0.3 \end{aligned}$$

Thus

$$D(z) = \frac{1-1.48z^{-1}+0.491z^{-2}}{0.156z^{-1}(1-2.63z^{-1})} \times \frac{z^{-1}(1-2.63z^{-1}) \times -0.3(1+z^{-1})}{(1-z^{-1})(1+1.3z^{-1}+0.79z^{-2})}$$

$$\text{i.e.} \quad D(z) = \frac{-1.92+0.92z^{-1}+1.9z^{-2}-0.943z^{-3}}{1+0.3z^{-1}-0.51z^{-2}-0.79z^{-3}} \quad \dots 6.8.3$$

Similarly, by using the model 6.4.2 in conjunction with

$$K(z) = z^{-3}(q_0 + q_1 z^{-1})$$

$$\text{and } 1-K(z) = (1-z^{-1})(p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3})$$

it can be shown that

$$p_0 = p_1 = p_2 = 1, \quad q_0 + q_1 = 1$$

and the optimum values of q_0, q_1 are $q_0 = q_1 = \frac{1}{2}$. The optimum controller is given by

$$\begin{aligned} D(z) &= \frac{1-0.966z^{-1}}{-0.659z^{-1}} \times \frac{\frac{1}{2}(1+z^{-1})z^{-3}}{(1-z^{-1})(1+z^{-1}+z^{-2}+\frac{1}{2}z^{-3})} \\ &= \frac{-0.76-0.026z^{-1}+0.73z^{-2}}{1-0.5z^{-3}-0.5z^{-4}} \end{aligned} \quad \dots 6.8.4$$

If the model 6.4.2 is now used in conjunction with

$$\begin{aligned} K(z) &= z^{-3}(q_0 + q_1 z^{-1} + q_2 z^{-2}) \\ 1-K(z) &= (1-z^{-1})(p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3} + p_4 z^{-4}) \end{aligned}$$

and by following the same procedure as above it will be found that

$$q_0 + q_1 + q_2 = 1, \quad p_3 = 1 - q_0$$

and to minimise I it will be found necessary to set $\frac{\partial I}{\partial q_0} = 0, \quad \frac{\partial I}{\partial q_1} = 0$
which will lead to the simultaneous equations

$$4q_0 + 2q_1 = 2$$

$$4q_1 + 2q_0 = 2$$

from which $q_0 = q_1 = \frac{1}{3}$ and the controller is

$$D(z) = \frac{-0.506 - 0.0172z^{-1} - 0.0172z^{-2} + 0.489z^{-3}}{1 - \frac{1}{3}z^{-3} - \frac{1}{3}z^{-4} - \frac{1}{3}z^{-5}} \quad \dots 6.8.5$$

The controllers defined by 6.8.3, 6.8.4, 6.8.5 were applied to the plant in conjunction with set-point changes and the results are shown in Figs. 6.8.1 to 6.8.3. There is little significant difference between the results presented here which confirm that it is only necessary to add one extra term to the minimum-time $K(z)$ to achieve a stable controller, and that a satisfactory control scheme can be designed using a first-order model.

6.9 Regulation Experiments:

It is envisaged that except in extreme cases the same algorithm should be capable of performing both the tasks of changing the set-point and regulation. If an algorithm has been successful in changing the set-point from one level to another, it is likely that it will also be acceptable in regulation. This supposition has been checked by considering 3 algorithms used in regulating the top-product of the distillation column against disturbances in the feed composition. The regulation set-point was 60% B.M and disturbances were introduced into the system by changing the feed composition from the nominal value of 7.2%BM to 4.4%BM so that it alternated for seven periods of 12.8 minutes each between one value and the other. The top product composition was logged on paper tape and later analysed by computer which found the output mean and variance and these results are tabulated in TABLE 6.9.1 from which it may be concluded that these algorithms are quite successful in regulation as well as set-point changes, particularly in view of the large disturbances in the feed.

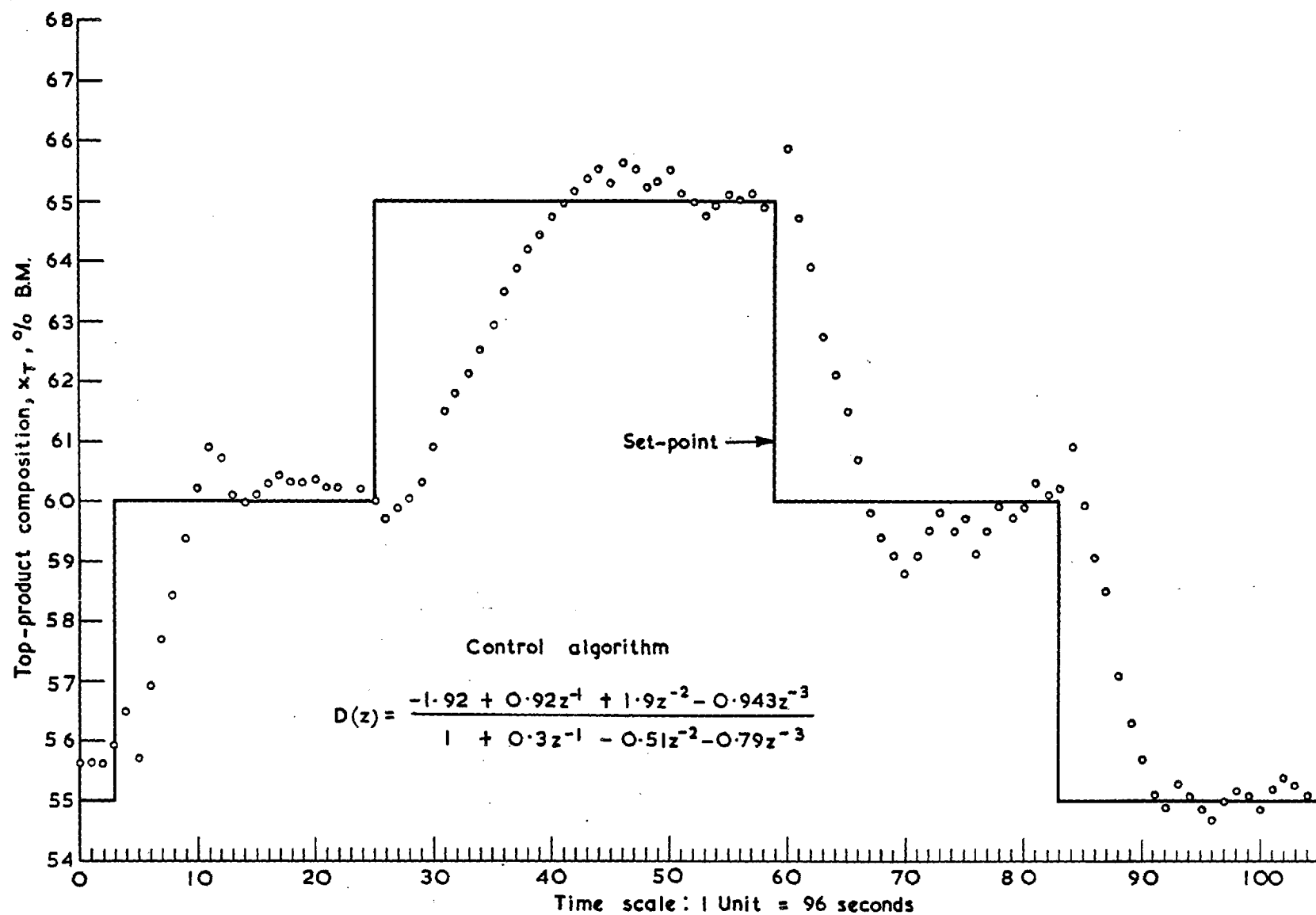


FIG. 6.8.1. ON-LINE CONTROL OF TOP-PRODUCT COMPOSITION

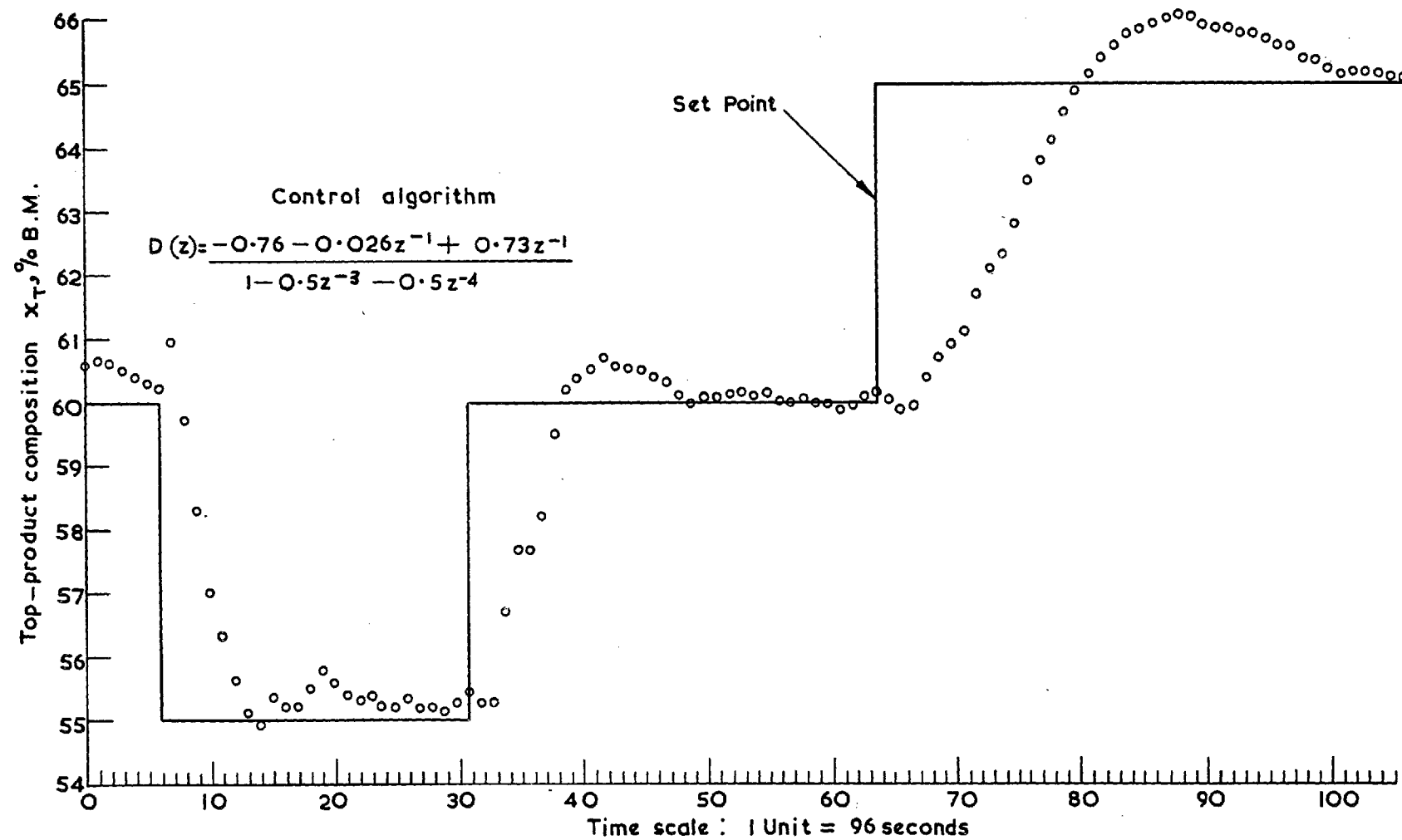


FIG. 6.8.2 ON-LINE CONTROL OF TOP PRODUCT COMPOSITION

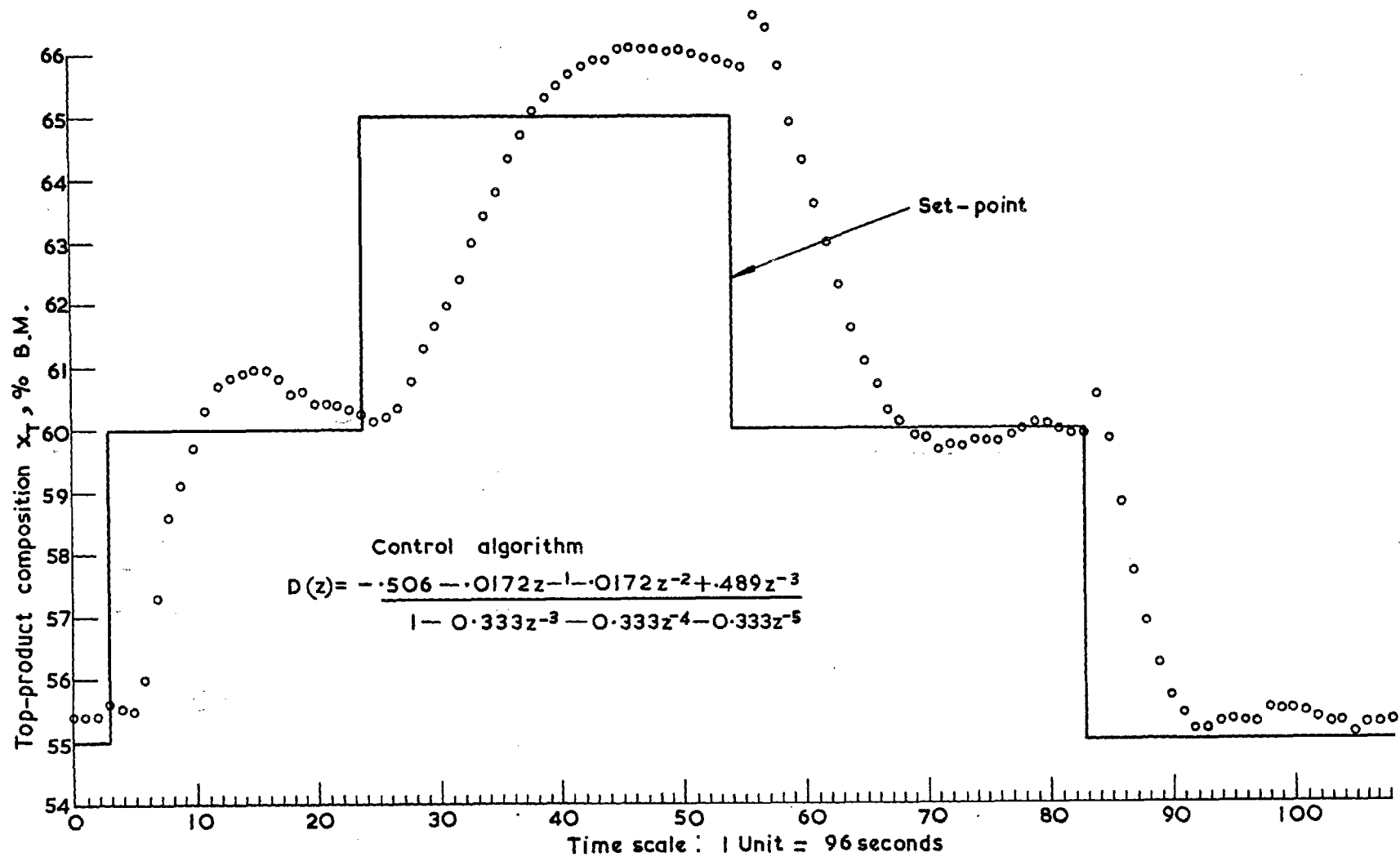


FIG.6.8.3 ON-LINE CONTROL OF TOP PRODUCT COMPOSITION

TABLE 6.9.1

Control Algorithm	Mean Value of Top-product Composition % B.M.	Standard Deviation
$\frac{-0.76 + 0.026z^{-1} + 0.73z^{-2}}{1 - 0.5z^{-3} - 0.5z^{-4}}$	60.47	1.14
$\frac{-1.1 + 1.51z^{-1} - 0.55z^{-2}}{1 - z^{-1}}$	60.2	1.45
$\frac{-2.083 + 2.8z^{-1} - 0.886z^{-2}}{1 - 0.516z^{-1} - 0.484z^{-2}}$	60.35	1.78

The control algorithm 6.8.4 designed using a simple first-order model obtained from theoretical considerations however appears to be the superior of the three algorithms studied in this case.

6.10 Regulation by Feed-Forward:

If the feed composition z_F (considered as an uncontrolled variable) changes, then $\dot{x}$ in equation 3.2.11 will remain equal to zero if A , D , Fz_F are constant. Since (in theory) the vapour flow-rates are dependent only on the applied heat (equation 2.3.1) and the liquid flows in the rectifying section are independent of the feed flow-rate, then D will be constant while only small changes in A will occur due to the influence of the feed flow-rate changes in the stripping section. It can be seen therefore that $\dot{x}$ will remain approximately zero if the product Fz_F is held constant, i.e. if a change in z_F is compensated by a change in F such that their product, (which is the molar flow-rate of the more volatile feed component) remains constant. The time delays between changes in F and z_F and their effects on the top-product composition are also known and these may be incorporated into the control scheme. As an example, the column was regulated using the controller 6.8.4 and the feed composition was changed from 7.2% to 4.4% and when it reached steady-state it was changed back again to the original steady-state values. The results are shown in Fig. 6.10.1. The experiment was repeated but this time feed-forward of the flow-rate was incorporated, the flow-rate being altered from 47 cc/min to 72 cc/min 3 sample intervals after the disturbance in feed had occurred. After steady-state had been attained conditions were changed back to the original values and the results are shown in Fig. 6.10.2. It is obvious that feed-forward has been effective in improving the regulation.

6.11 Discussion of Results:

From the results presented in this chapter the following points are noted:

- 1) The response of the process is distinctly non-linear (see eg. Fig. 6.8.3), the transient behaviour over all the 4 regions considered being completely different.

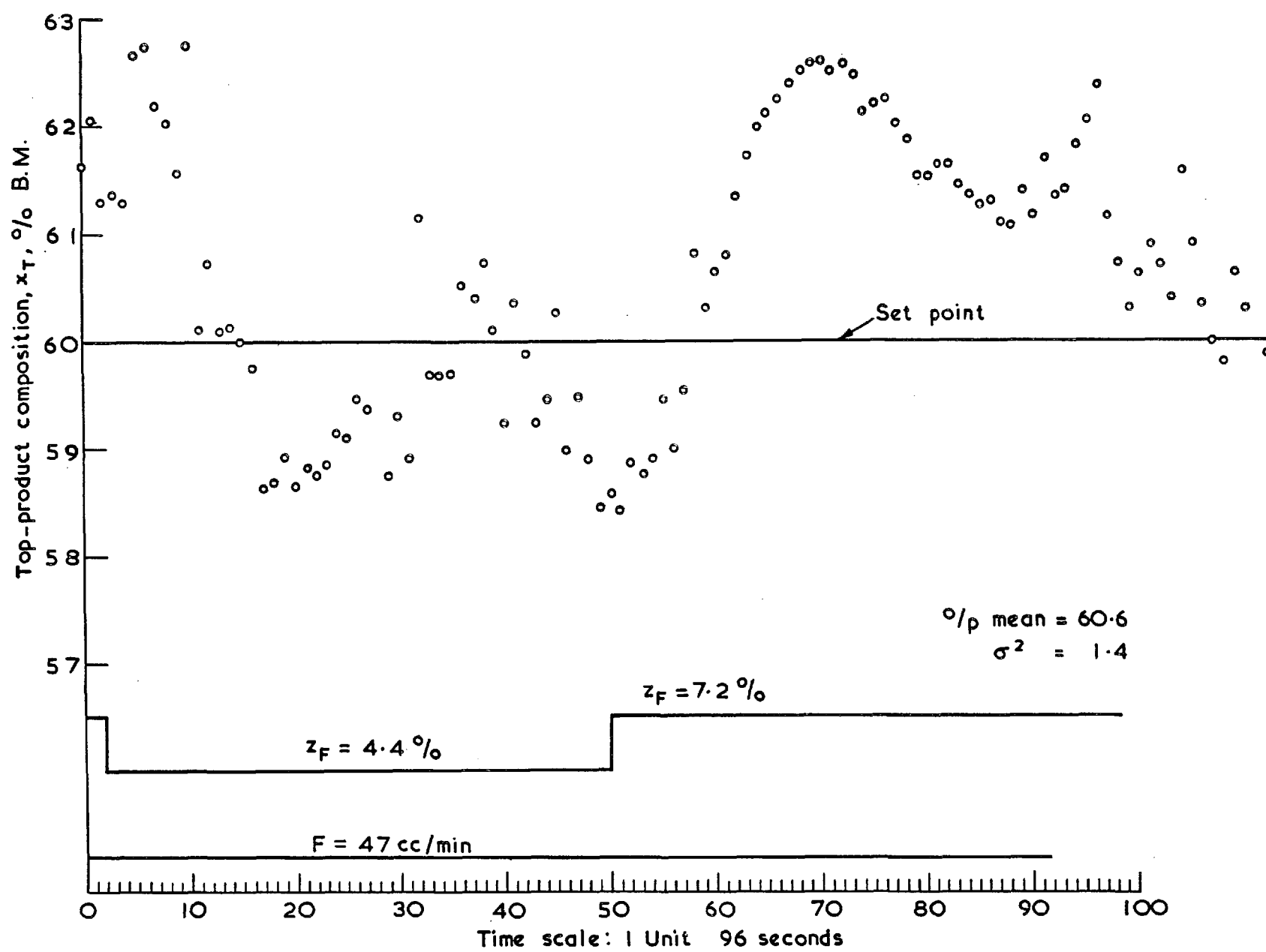


FIG 6.10.1. REGULATION WITHOUT FEED FORWARD

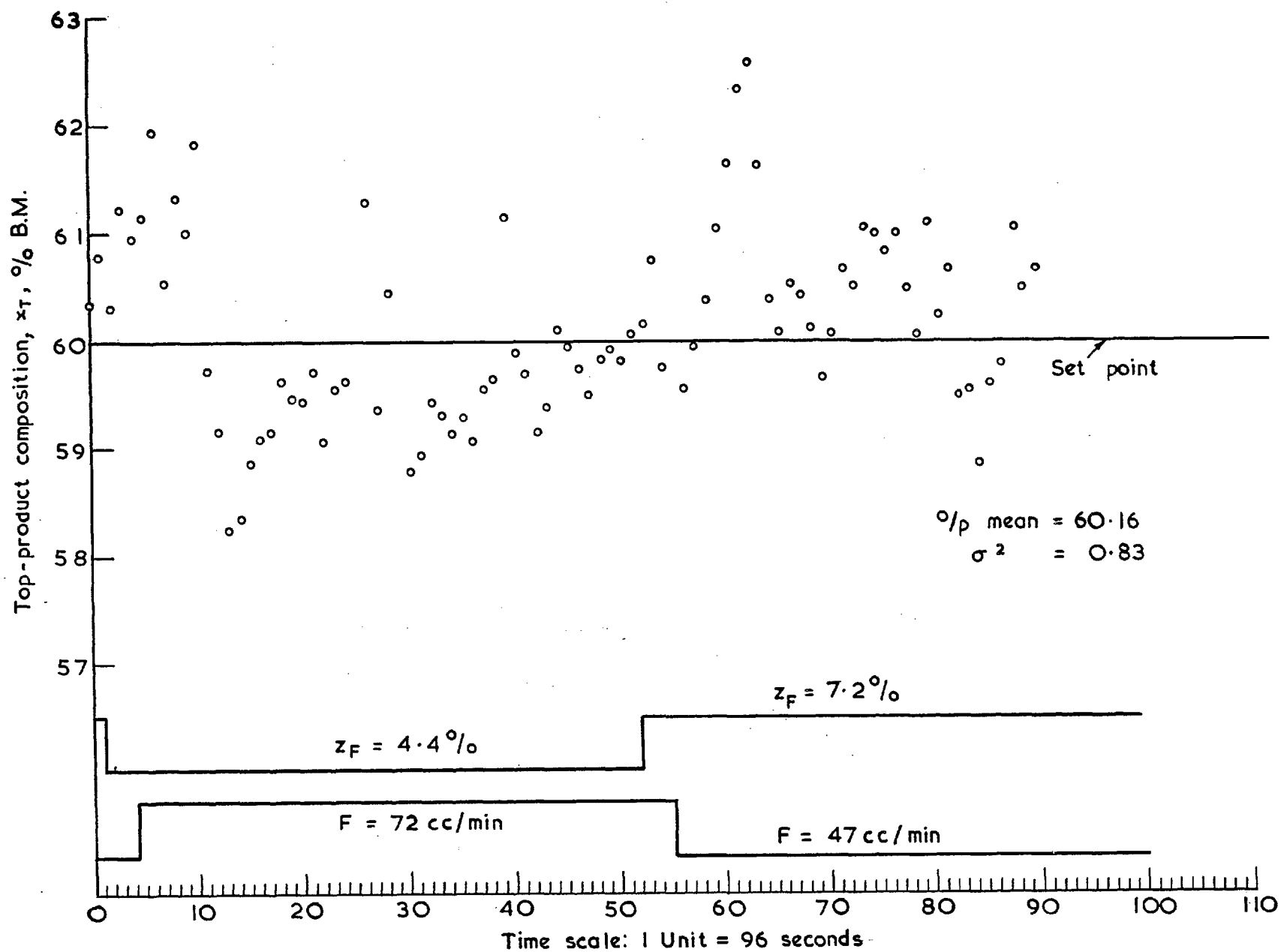


FIG. 6.10.2. REGULATION WITH FEED FORWARD

- ii) Models obtained from perturbations about a set-point will be an "average" for positive or negative perturbations and can therefore be used in designing controllers for changes in these two directions.
- iii) After tuning the three-term controller, the system response to set-point changes and its performance as a regulator was acceptable. However, on both these counts the minimum mean-squared error controller was superior and when it is considered that tuning this controller was unnecessary its superiority becomes more apparent. Attempts have recently been made to find an optimum three-term controller for this system⁽²¹⁾ but so far the results have proved negative.
- iv) The behaviour, for set-point changes, of the closed-loop control systems designed using
 - a) a theoretical first-order model and
 - b) a second-order model obtained from plant experimentswere indistinguishable, a result which confirms the proposition that, for certain cases, a satisfactory controller can be designed using a first order model.
- v) It is only necessary to add one extra term to the minimum prototype $K(z)$ to obtain a satisfactory minimum mean squared error controller. This results in two extra terms in $D(z)$ as can be seen from equation 5.2.3, but the controller is not unduly complex.
- vi) The modelling technique has the following features:
 - a) It avoids perturbation-type experiments
 - b) It can be implemented on-line and can also be adaptive
 - c) It results in controllers which are superior to those in use at present.

6.12 Conclusions:

Based on simple assumptions and using a minimum number of measurements it has been demonstrated that a simple dynamic model of a distillation column can be derived using a digital computer. In spite of difficulties with

measurements, the examples quoted here confirmed the validity of the method, particularly in the case of a first-order model. This discrete-time model is well suited for on-line control applications either of the feed-back or feed-forward variety and a simple method of designing feed-back controllers using an error-squared criterion has also been described. Controllers can be designed using the method of Dynamic Programming but it is doubtful if the additional complexity is justified. It is most likely that the methods described here could be implemented using a hierarchical system, the higher level computer being used to up-date the model and controller while the lower level computer is used to measure the variables and implement the control action.

6.13 Future Work:

It is pointed out that the considerable effort involved in the present work has only resulted in a single input-single output computer control system. Multivariable modelling and control have not been investigated and the effort involved in achieving this aim would be formidable. It is also regretted that time was insufficient to allow a fuller investigation into dynamic feed forward. However it is thought that much future work can be based on the foundations laid here. Provided accurate and reliable hardware is available the author believes that the following can be accomplished: an on-line adaptive modelling procedure can be designed which will monitor the measurements required by the procedure described in chapters 2 and 3 and this will lead to on-line control system design. The next step will be to extend the work to multivariable modelling and control and the implementation of dynamic or steady-state feed-forward. A further, but very difficult step, will be to apply the method to multicomponent distillation columns and finally to control groups of plants.

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