

Imperial College London  
Department of Electrical and Electronic Engineering

# **Beyond Diagonal Reconfigurable Intelligent Surfaces: Mode Analysis, Signal Processing, and Hardware Impairments**

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## **Statement of Originality**

I declare that this thesis is the result of my own work. Information and ideas derived from the work of others have been acknowledged in the text and a list of references is given in the bibliography. The material of this thesis has not been and will not be submitted for another degree at any other university or institution.

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*To all the people I love*



## Abstract

Reconfigurable intelligent surface (RIS) has been envisioned as a promising and cost-effective technique to enhance future wireless communications. Extensive research attention has been drawn to the use of conventional diagonal RIS with diagonal phase shift matrices, where each RIS element is connected to its own load to ground but not connected to other elements. However, the simple architecture of conventional diagonal RIS limits its flexibility of manipulating passive beamforming. To fully exploit the benefits of RIS, a new technical advance, namely, beyond diagonal (BD) RIS, has emerged by introducing inter-element connections. This thesis aims to develop novel architectures and modes of BD-RIS with performance enhancements, study corresponding signal processing solutions, and characterize hardware impairments of BD-RIS.

First, we propose two new branches of BD-RIS, namely BD-RIS with hybrid transmitting and reflecting mode, and BD-RIS with multi-sector mode, which enable enhanced channel gain and enlarged coverage compared to conventional diagonal RIS. Specifically, the modeling, architecture design, discussions, and beamforming design for the proposed two modes are provided.

Second, we study the signal processing techniques for BD-RIS aided multi-antenna systems. Specifically, we start by synergizing BD-RIS with rate-splitting multiple access, which is a non-orthogonal technique providing additional flexibility for the interference management. Under imperfect channel state information conditions at the transmitter, a stochastic optimization problem is formulated and efficiently solved. Then, we propose an efficient pilot sequence and BD-RIS design for channel estimation, which theoretically guarantees to achieve the performance lower-bound.

Finally, we characterize two important hardware impairments at BD-RIS, namely, mutual coupling and frequency dependence in wideband systems. To evaluate the impact of mutual coupling at BD-RIS, we adopt dipole antennas and model the mutual coupling as a function of inter-antenna spacing. To evaluate the impact of frequency dependence at BD-RIS, we adopt lumped circuit models in wideband communication systems.

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*Hongyu Li*

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# Chapter 1

## Introduction

The forthcoming next-generation 6G is expected to address 5G drawbacks, improve existing techniques, and lead new technological trends [1, 2]. Reconfigurable intelligent surface (RIS), which can build controllable radio environments and improve communication quality in a cost-effective manner [3–7], is regarded as one of the promising techniques to drive the 6G architectural evolution [2]. An RIS is a two-dimensional planar surface which consists of numerous nearly passive elements with ultra-low power consumption. Each element has a multi-layer structure composed of rectangle patches and tunable components, e.g., positive intrinsic negative (PIN) diodes, which can be independently switched between “ON” and “OFF” states, thereby generating different responses, i.e., phase shifts and amplitudes, for incident signals [6]. Thus, RISs can be flexibly deployed into various wireless communication systems to modify the propagation environment without much power consumption.

### 1.1 Background and Motivations

#### 1.1.1 Conventional Diagonal RIS

In most existing RIS literature, the simplest architecture, where each element is connected to its own reconfigurable load but is disconnected from other elements, is considered. Therefore, the scattering matrix of this kind of RIS, namely conventional diagonal RIS, is modeled as a simple diagonal phase shift matrix. With this simple architecture, three main research directions, namely, beamforming design, channel estimation, and modeling of hardware impairments, have been well studied and are briefly summarized below.

**Beamforming Design:** With the conventional diagonal RIS, beamforming design for single RIS-aided wireless communication systems has been developed using different metrics, e.g., power minimization, rate maximization, and max-min fairness [8–11]. Considering the

high hardware complexity and cost of realizing RISs with infinite/high-resolution phase shifts, research on practical finite/low-resolution cases has been developed [12, 13]. To further improve the system performance, multiple RISs and coordination problems have also been studied [14–16]. In addition, RIS has also been investigated to provide performance enhancement in various scenarios, e.g., wireless power transfer (WPT) [17], simultaneous wireless information and power transfer (SWIPT) [18], integrated sensing and communication (ISAC) [19], cognitive radio [20], and index modulation [21].

Further, the interplay between RIS and rate-splitting multiple access (RSMA), which is a non-orthogonal transmission strategy enabling more flexible inter-user interference management comparing to other existing multiple access techniques, such as spatial division multiple access (SDMA) [22], has been recently proven to have performance benefits. Specifically, the synergy among the conventional diagonal RIS and 1-layer RSMA in multiuser systems is most commonly considered, whose benefits are shown by focusing on various metrics, such as sum-rate maximization [23], energy efficiency maximization [24], outage probability analysis [25–27], transmit power minimization [28], and max-min fairness [29]. In addition, the interplay between conventional diagonal RIS and RSMA beyond 1-layer architectures, such as 2-layer hierarchical RSMA [30, 31], is also studied. While the abovementioned RIS aided RSMA work [23–31] is restricted to perfect channel state information (CSI) conditions, a more practical imperfect CSI scenario is analyzed in [32].

**Channel Estimation:** It is worth noting that the enhanced performance provided by RIS, as illustrated in [8–21, 23–31], depends heavily on accurate CSI. Therefore, in addition to beamforming at RIS, another important research direction on RIS is channel estimation. In conventional RIS literature on channel estimation, there are in general two strategies to obtain the instantaneous CSI [33]. The first strategy is to *separately* estimate the channels between the base station (BS)/users and the RIS by partially “activating” RIS elements using RF chains. In this strategy, the separate channels can be estimated by leveraging channel reciprocity and time division duplex (TDD) and applying conventional estimation approaches, such as the estimation of signal parameters via rotational invariance technique (ESPRIT) and multiple signal classification (MUSIC) [34], and compressed sensing (CS) [35, 36]. This strategy generally has low training overhead regardless of the number of RIS elements. However, the drawback is that the equipment of RF chains will result in additional cost and power consumption, which violates the original motivation of employing RIS in wireless communication systems. To avoid the equipment of RF chains at RIS, the second strategy has been proposed, whose main idea is to estimate the *cascaded* user-RIS-BS channels relying on purely passive RISs [37]. In this strategy, the BS estimates the cascaded channel through proper design of the pilot sequence and RIS patterns, such as the ON/OFF-based design [38] and the orthogonality-based design [39, 40]. To reduce the training overhead, other estimation schemes, such as a three-phase framework [41] and an anchor-assisted channel estimation [42], have been further proposed making use of the common BS-RIS channels for different users. With the same motivation, a novel cascaded

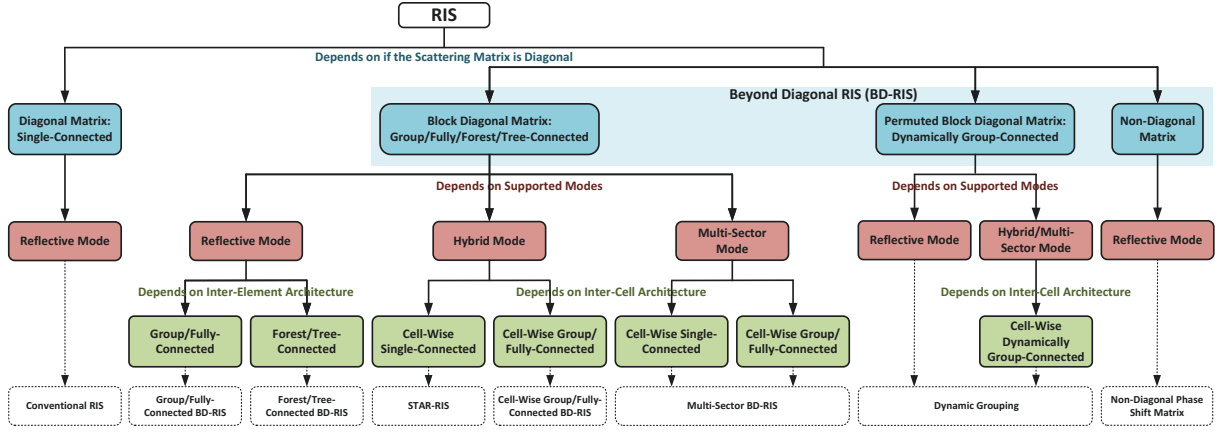


Figure 1.1: RIS classification tree.

channel estimation scheme has been proposed by exploiting the sparsity and correlations of millimeter wave channels [43].

**Hardware Impairments:** The aforementioned research on conventional diagonal RIS assumes an ideal RIS model with no hardware impairments, such as mutual coupling between elements, power loss to realize each element, and dependence on signal frequencies, and focuses on the purely theoretical analysis. However, those hardware impairments are generally not negligible in practice and thus ignoring them might cause performance loss for practical implementation and prototyping. To capture the power loss, practical RIS model analysis based on lumped circuit models has been proposed in narrowband communication systems [44]. Furthermore, the frequency dependence of RIS response in wideband systems has been accurately modeled and analyzed in [45–47], as already experimentally highlighted in [48, 49]. In addition, the impact of mutual coupling at RIS has been analyzed in [50–52], where the strength of mutual coupling has been measured based on the spacing between dipole antennas. To evaluate the performance of RIS in practice, the near- and far-field free-space path loss models for RIS aided wireless systems have also been theoretically derived [53] and experimentally verified [54].

### 1.1.2 Beyond Diagonal RIS

Conventional diagonal RIS can only control the phase of incident signal, which limits the capability for manipulating passive beamforming and thus degrades the performance. To address the limitation of conventional diagonal RIS, beyond diagonal (BD) RIS has been proposed [55] by interconnecting elements to mathematically generate nonzero off-diagonal entries of the RIS scattering matrix. Under the concept of BD-RIS, four main research directions, namely, modeling and architecture design, beamforming design, modeling of hardware impairments, and mode analysis, have been investigated and are briefly summarized below, together with the RIS classification tree illustrated in Fig. 1.1.

**Modeling and Architecture Design:** The modeling of BD-RIS and the concept of group- and fully-connected architectures, named according to the circuit topology of inter-element connections, have been first proposed in [55], followed by the dynamic grouping strategy to decide how to group elements [56]. To find a better performance-complexity trade-off, BD-RIS with tree- and forest-connected architectures have been proposed in [57], which can achieve the performance upper bound with the minimum circuit complexity. In addition, another novel RIS architecture [58] has been proposed, where the signal impinging on one reconfigurable element can be reflected from another one, yielding a non-diagonal phase shift matrix that enables more flexible beam manipulation.

**Beamforming Design:** Based on different architectures of BD-RIS, the corresponding beamforming designs have been studied. Specifically, a closed-form globally optimal solution for group/fully-connected BD-RIS aided single input single output (SISO) systems has been proposed in [59]. Focusing on the same BD-RIS architecture, a low-complexity design for BD-RIS aided multiple input multiple output (MIMO) systems has been proposed in [60] and a unified optimization framework for BD-RIS aided MIMO, adapting to power minimization, energy efficiency and sum-rate maximization problems, has been proposed in [61]. Applications of BD-RIS, such as interplaying with ISAC systems [62, 63], interplaying with RSMA [64], enabling wideband wireless communications [65, 66], enabling the stacked intelligent metasurface [67], and enabling channel reciprocity attacks [68], have also been investigated.

**Hardware Impairments:** While the aforementioned works [55–68] primarily focus on BD-RIS models with perfect hardware, limited works have studied the modeling and impact of hardware impairments, such as the power loss of interconnections and the discrete-value impedance components at BD-RIS. A universal framework has been proposed in [69] which models the mutual coupling and mismatching at antenna arrays. Meanwhile, the practical loss due to the inter-element connections has been investigated based on the transmission line theory [70], which demonstrates the enhanced channel gain provided by BD-RIS over conventional RIS in the presence of losses. In addition, a practical design considering group/fully-connected BD-RIS with discrete values has been proposed in [71]. Results suggest that while four resolution bits are needed in conventional diagonal RIS, only a single resolution bit is sufficient in fully-connected BD-RIS, simplifying significantly the future development of these promising RIS architectures. Beyond this, the resistance loss in the BD-RIS circuit design, which is potentially much higher than that of the diagonal RIS, the mutual coupling between elements, and the frequency variation of antenna gain and the scattering matrix might also have non-negligible impact on BD-RIS performance, while none of existing literature has focused on their modeling and analysis.

**Mode Analysis:** Note that the abovementioned BD-RIS architectures are proposed assuming the BD-RIS works on the reflective mode, that is the incident signals from the transmitter are all reflected from the RIS towards the same side as the transmitter. This, however, physically limits the coverage of BD-RIS. To address this issue, the concept of simultaneously transmitting and reflecting (STAR) RIS or intelligent omni-surface (IOS) [72–76], has been proposed, which

allows the impinged signals to partially be reflected towards one side of the RIS and partially transmit through the RIS to serve users in another side.

### 1.1.3 Motivations

Although BD-RIS has been investigated from multiple perspectives, there still exist many open problems, such as exploring novel architecture designs and mode analysis, beyond the STAR-RIS [72–76] to further enhance the channel gain and enlarge the coverage. More importantly, it remains challenging to analyze the impact of CSI imperfections in BD-RIS aided systems. Due to the unique cascaded channel structures and the new constraints induced by BD-RIS architectures different from conventional diagonal RIS cases, the cascaded channel estimation schemes for conventional diagonal RISs [37–43] do not work for BD-RIS aided scenarios. Therefore, it is also challenging, yet more important, to develop efficient channel estimation methods for BD-RIS aided systems with affordable overhead. In addition, modeling and analyzing other hardware impairments, such as the effect of mutual coupling at BD-RIS and the frequency dependence of BD-RIS in wideband systems, are not straightforward extensions to conventional diagonal RISs since the inter-element connections at BD-RIS introduces correlations between entries of the scattering matrix. This makes it difficult to individually characterize the behavior of each entry of the scattering matrix as in conventional diagonal RISs [45–47].

## 1.2 Contributions

Based on the above literature review, this thesis addresses the mentioned open and challenging problems of BD-RIS by fundamentally modeling its hardware, developing novel signal processing schemes, and providing insights based on theoretical and numerical results. A summary of each chapter/part, highlighting the main contributions, is given as follows.

**Chapter 2: Background Theory** lays the foundations for this thesis by providing an overview of the primary concepts, modeling principle of RIS aided systems, and optimization techniques used throughout the thesis. Specifically, the modeling of general RIS aided wireless communication systems using the multi-port network analysis is first explained. Furthermore, key optimization techniques, including the fractional programming, Karush-Kuhn-Tucker conditions, and Manifold based methods, are briefly reviewed.

**Part I: Mode Analysis of BD-RIS** proposes and analyzes two novel modes of BD-RIS. More specifically:

- **Chapter 3: BD-RIS with Hybrid Transmitting and Reflecting Mode** proposes a novel branch of BD-RIS, namely, BD-RIS with hybrid mode, as illustrated in Fig. 1.1.

This is done by back-to-back placing every two antennas and connecting them to construct a cell, based on which the STAR-RIS [72–76] is one special case and is further generalized to more sophisticated architectures, i.e., BD-RIS with cell-wise group/fully/dynamically group-connected architectures. With the proposed model, the beamforming design problem is formulated and solved using optimization techniques, followed by the initialization strategies, convergence analysis, and complexity analysis.

- **Chapter 4: BD-RIS with Multi-Sector Mode** goes beyond BD-RIS with hybrid mode and proposes a novel branch of BD-RIS, namely, BD-RIS with multi-sector mode as illustrated in Fig. 1.1. This is done by placing multiple antennas at each sector of a polygon and connecting them to construct a cell, based on which BD-RIS with hybrid mode is one special case with number of sectors equal to 2. With the proposed model, the corresponding channel model taking into account the antenna gain is derived. The received power scaling law is derived as a function of the number of sectors, and the beamforming design is proposed to maximize the sum-rate performance for a multiuser system.

**Part II: Signal Processing of BD-RIS** studies the beamforming design of BD-RIS aided wireless systems under imperfect CSI and the channel estimation schemes. More specifically:

- **Chapter 5: Synergizing BD-RIS and Rate-Splitting Multiple Access with Imperfect CSI** studies the performance benefits of synergizing advanced multiple access techniques at the transmitter and BD-RIS. Specifically, the CSI imperfections are taken into account in the beamforming design, which leads to a stochastic problem. An efficient iterative design is proposed applying the relationship between rate maximization and mean square error minimization. The proposed design is also analyzed from the perspectives of initialization, complexity analysis, and convergence.
- **Chapter 6: Channel Estimation and Beamforming for BD-RIS** studies the channel estimation and beamforming design for BD-RIS aided wireless systems. Specifically, the cascaded channel estimation scheme for BD-RIS, which could finely adapt to BD-RIS with different architectures and modes, and to different scenarios, is proposed based on rigorous theoretical derivations and analysis. After obtaining the cascaded channel estimate, the beamforming designs relying on only the cascaded channels are proposed by modifying the algorithms proposed in Chapters 3 and 4. The performance trade-off between the channel estimation and data transmission is also investigated and highlighted.

**Part III: Hardware Impairments of BD-RIS** focuses on the effect of two important hardware impairments at BD-RIS. More specifically:

- **Chapter 7: BD-RIS with Mutual Coupling** focuses on the effect of mutual coupling at BD-RIS. Particularly, the mutual coupling among BD-RIS elements is explicitly

characterized, as a function of inter-element spacing. Based on this, the corresponding beamforming design algorithm capturing the mutual coupling is developed.

- **Chapter 8: Wideband Effect of BD-RIS** focuses on the frequency dependence of BD-RIS in wideband communication systems. Particularly, this effect is mathematically modeled based on specific circuit designs, which can widely adapt to BD-RIS with group/fully/tree/forest-connected architectures. With the proposed wideband model, the beamforming design algorithms for different BD-RIS architectures are also developed.

**Chapter 9: Conclusion** provides the concluding remarks of the thesis and suggests potential future research directions.

The theoretical analysis and algorithms proposed in the six core chapters, i.e., Chapters 3-8, are evaluated through simulations under various parameter settings. For the sake of readability and logical smoothness, lengthy proofs of the main results, written as propositions, lemmas, and theorems, are relegated to the appendices. However, important discussions and insights drawn from such results are mentioned in the chapters.

## 1.3 Publications

This thesis is written as a monograph based on the following one magazine paper, eight journal papers, and two conference papers, that have been accepted and published, or submitted for future publication.

### Magazine and Journal Papers:

- **H. Li**, S. Shen, M. Nerini, and B. Clerckx, “Reconfigurable intelligent surfaces 2.0: Beyond diagonal phase shift matrices,” *IEEE Commun. Mag.*, vol. 62, no. 3, pp. 102-108, 2024. [Chapters 1 and 9]
- **H. Li**, S. Shen, B. Clerckx, “Beyond diagonal reconfigurable intelligent surfaces: From transmitting and reflecting modes to single-, group-, and fully-connected architectures,” *IEEE Trans. Wireless Commun.*, vol. 22, no. 4, pp. 2311-2324, 2023. [Chapter 3]
- **H. Li**, S. Shen, and B. Clerckx, “A dynamic grouping strategy for beyond diagonal reconfigurable intelligent surfaces with hybrid transmitting and reflecting mode,” *IEEE Trans. Veh. Technol.*, vol. 72, no. 12, pp. 16748-16753, 2023. [Chapter 3]
- **H. Li**, S. Shen, B. Clerckx, “Beyond diagonal reconfigurable intelligent surfaces: A multi-sector mode enabling highly directional full-space wireless coverage,” *IEEE J. Sel. Areas Commun.*, vol. 41, no. 8, pp. 2446-2460, 2023. [Chapter 4]

- **H. Li**, S. Shen, and B. Clerckx, “Synergizing beyond diagonal reconfigurable intelligent surface and rate-splitting multiple access,” *IEEE Trans. Wireless Commun.*, vol. 23, no. 8, pp. 8717-8729, 2024. [Chapter 5]
- **H. Li**, Y. Mao, O. Dizdar, and B. Clerckx, “Rate-splitting multiple access for 6G – Part III: Interplay with reconfigurable intelligent surfaces,” *IEEE Commun. Lett.*, vol. 26, no. 10, pp. 2242-2246, 2022. [Chapter 5]
- **H. Li**, S. Shen, Y. Zhang, B. Clerckx, “Channel estimation and beamforming for beyond diagonal reconfigurable intelligent surfaces,” *IEEE Trans. Signal Process.*, vol. 72, pp. 3318-3332, 2024. [Chapter 6]
- **H. Li**, S. Shen, M. Nerini, M. Di Renzo, B. Clerckx, “Beyond diagonal reconfigurable intelligent surfaces with mutual coupling: Modeling and optimization,” *IEEE Commun. Lett.*, vol. 28, no. 4, pp. 937-941, 2024. [Chapter 7]
- **H. Li**, M. Nerini, S. Shen, B. Clerckx, “Beyond diagonal reconfigurable intelligent surfaces in wideband OFDM communications: Circuit-based modeling and optimization,” submitted to *IEEE Trans. Wireless Commun.*, 2024. [Chapter 8]

### Conference Papers:

- **H. Li**, Y. Zhang, and B. Clerckx, “Channel estimation for beyond diagonal reconfigurable intelligent surfaces with group-connected architectures,” in *Proc. IEEE Computational Advances in Multi-Sensor Adaptive Process. (CAMSAP)*, 2023. [Chapter 6]
- **H. Li**, M. Nerini, S. Shen, and B. Clerckx, “Wideband modeling and beamforming for beyond diagonal reconfigurable intelligent surfaces,” in *Proc. IEEE Workshop on Signal Process. Advances in Wireless Commun. (SPAWC)*, 2024. [Chapter 8]

## 1.4 Notations

The following notations are used throughout the thesis.

Boldface lower-case and upper-case letters indicate vectors and matrices, respectively.  $\mathbb{C}$ ,  $\mathbb{R}$ ,  $\mathbb{N}$ , and  $\mathbb{Z}$  denote the set of complex numbers, real numbers, natural numbers, and integers, respectively.  $(\cdot)^T$ ,  $(\cdot)^*$ ,  $(\cdot)^H$ , and  $(\cdot)^{-1}$  denote the transpose, conjugate, conjugate-transpose, and inverse, respectively.  $\otimes$  and  $\odot$  denote the Kronecker product and Hadamard product, respectively.  $\Re\{\cdot\}$  and  $\Im\{\cdot\}$  denote the real and imaginary parts of a complex number, respectively.  $\angle(\cdot)$  returns the angle of complex variables.  $\mathbb{E}\{\cdot\}$  denotes the statistical expectation.  $\text{Tr}(\cdot)$  denotes the summation of diagonal elements of a matrix.  $|\cdot|$  denotes the absolute-value norm

of a scalar or the size of a set.  $\|\cdot\|_2$  and  $\|\cdot\|_F$  denote the Euclidean norm and Frobenius norm, respectively.  $j = \sqrt{-1}$  denotes the imaginary unit.  $\text{diag}(a_1, \dots, a_N)$  and  $\text{blkdiag}(\mathbf{A}_1, \dots, \mathbf{A}_N)$  denote diagonal and block-diagonal matrices, respectively, with diagonal entries  $a_1, \dots, a_N$  and diagonal blocks  $\mathbf{A}_1, \dots, \mathbf{A}_N$ .  $\text{vec}(\cdot)$  and  $\text{unvec}(\cdot)$  denote the vectorization and its reverse operation, respectively.  $\text{circshift}(\mathbf{a}, N)$  rearranges vector  $\mathbf{a}$  by moving the final  $N$  entries to the first  $N$  positions.  $\text{mod}(M, N)$  returns the remainder after  $M$  is divided by  $N$ .  $\nabla(\cdot)$  denotes the first-order Euclidean gradient operation.  $\frac{\partial(\cdot)}{\partial(\cdot)}$  denotes the first-order partial gradient operation.

$\mathbf{I}_N$  denotes the  $N \times N$  identical matrix.  $\mathbf{0}_{M \times N}$  denotes the  $M \times N$  all-zero matrix.  $\mathbf{1}_M$  denotes an  $M \times 1$  all-one vector.  $[\mathbf{A}]_{i,j}$  and  $[\mathbf{a}]_i$ , respectively, denote the  $(i, j)$ -th entry of  $\mathbf{A}$  and the  $i$ -th entry of  $\mathbf{a}$ .  $[\mathbf{A}]_{i:i', j:j'}$  extracts the  $i$ -th to  $i'$ -th rows and the  $j$ -th to  $j'$ -th columns of  $\mathbf{A}$ .  $[\mathbf{a}]_{i:j}$  extracts the  $i$ -th through the  $j$ -th entries of  $\mathbf{a}$ .  $\mathbf{a} \sim \mathcal{CN}(\mathbf{0}_{N \times 1}, \sigma^2 \mathbf{I}_N)$  characterizes a column vector  $\mathbf{a}$  with entries following the circular symmetric complex Gaussian (CSCG) distribution.  $\mathbf{D}_N$  denotes the  $N \times N$  Hadamard matrix.  $\mathbf{F}_N$  and  $\bar{\mathbf{F}}_N$ , respectively, denote the  $N \times N$  discrete Fourier transform (DFT) matrix and its normalized DFT matrix with

$$[\mathbf{F}_N]_{i,j} = \exp\left(-j2\pi \frac{(i-1)(j-1)}{N}\right), \quad [\bar{\mathbf{F}}_N]_{i,j} = \frac{1}{\sqrt{N}} \exp\left(-j2\pi \frac{(i-1)(j-1)}{N}\right). \quad (1.1)$$

A block-circulant matrix  $\mathbf{C} \in \mathbb{C}^{M_1 N \times M_2 N}$  is defined based on a sequence of blocks  $\mathbf{C}_0 \in \mathbb{C}^{M_1 \times M_2}, \dots, \mathbf{C}_{N-1} \in \mathbb{C}^{M_1 \times M_2}$ , such that all columns are composed of the same blocks and each  $M_2$  columns are rotated one block to the bottom relative to the preceding  $M_2$  columns. That is,  $\mathbf{C}$  takes the form

$$\mathbf{C} = \text{blkcirc}(\mathbf{C}_0, \mathbf{C}_1, \dots, \mathbf{C}_{N-1}) = \begin{bmatrix} \mathbf{C}_0 & \mathbf{C}_{N-1} & \cdots & \mathbf{C}_2 & \mathbf{C}_1 \\ \mathbf{C}_1 & \mathbf{C}_0 & \mathbf{C}_{N-1} & & \mathbf{C}_2 \\ \vdots & \mathbf{C}_1 & \mathbf{C}_0 & \ddots & \vdots \\ \mathbf{C}_{N-2} & & \ddots & \ddots & \mathbf{C}_{N-1} \\ \mathbf{C}_{N-1} & \mathbf{C}_{N-2} & \cdots & \mathbf{C}_1 & \mathbf{C}_0 \end{bmatrix}. \quad (1.2)$$

The circulant matrix is a special case of the block-circulant matrix with  $M_1 = M_2 = 1$ .



# Chapter 2

## Background Theory

This chapter lays the foundations for this thesis by explaining the modeling of the RIS aided communication systems using the multi-port network analysis and reviewing the key optimization techniques used in this thesis.

### 2.1 Multi-Port Network Analysis

Multi-port network analysis is a powerful and useful technique for analyzing wireless systems, by regarding each antenna in the wireless system as a port and characterizing the behavior between its terminal voltage, current, incident wave, and reflected wave [77]. This technique has been used in conventional MIMO systems [78–80], where the accurate wireless channel model has been derived capturing the imperfections at the transmitting/receiving antennas, such as the mutual coupling and imperfect matching. In this section, this technique, together with its three important parameters, namely, scattering, impedance, and admittance parameters, has been used to model the RIS aided multi-antenna system. Physically-consistent assumptions at the transmitter, receiver, and RIS have been introduced to derive the RIS aided wireless channel, which will be used throughout Chapters 3-8.

#### 2.1.1 Scattering, Impedance, and Admittance Parameters

Consider an RIS aided multi-antenna system with an  $N$ -antenna transmitter, an  $M$ -element RIS, and a  $K$ -antenna receiver. The whole system is modeled as an  $N_{\text{all}}$ -port network with  $N_{\text{all}} = N + M + K$ , where the incident waves and reflected waves at the ports are respectively given by  $\mathbf{a} \in \mathbb{C}^{N_{\text{all}} \times 1}$  and  $\mathbf{b} \in \mathbb{C}^{N_{\text{all}} \times 1}$ ; the voltages and currents at the ports are respectively given by  $\mathbf{v} \in \mathbb{C}^{N_{\text{all}} \times 1}$  and  $\mathbf{i} \in \mathbb{C}^{N_{\text{all}} \times 1}$ . As such, the  $N_{\text{all}}$ -port network can be characterized by

its scattering matrix  $\mathbf{S} \in \mathbb{C}^{N_{\text{all}} \times N_{\text{all}}}$ , impedance matrix  $\mathbf{Z} \in \mathbb{C}^{N_{\text{all}} \times N_{\text{all}}}$ , and admittance matrix  $\mathbf{Y} \in \mathbb{C}^{N_{\text{all}} \times N_{\text{all}}}$ , each of which relates  $\mathbf{b}$  and  $\mathbf{a}$ , or  $\mathbf{v}$  and  $\mathbf{i}$ , by [77]

$$\mathbf{b} = \mathbf{S}\mathbf{a}, \quad \mathbf{v} = \mathbf{Z}\mathbf{i}, \quad \mathbf{i} = \mathbf{Y}\mathbf{v}, \quad (2.1)$$

where  $\mathbf{v}$  and  $\mathbf{i}$  relate to  $\mathbf{b}$  and  $\mathbf{a}$  by

$$\mathbf{v} = \mathbf{a} + \mathbf{b}, \quad \mathbf{i} = (\mathbf{a} - \mathbf{b})Z_0^{-1} = (\mathbf{a} - \mathbf{b})Y_0, \quad (2.2)$$

with  $Z_0$  the reference impedance and  $Y_0 = Z_0^{-1}$  the reference admittance used to calculate the scattering matrix. Therefore, the three matrices relate to each other by

$$\mathbf{S} = (\mathbf{Z} + Z_0\mathbf{I}_{N_{\text{all}}})^{-1}(\mathbf{Z} - Z_0\mathbf{I}_{N_{\text{all}}}) = (Y_0\mathbf{I}_{N_{\text{all}}} + \mathbf{Y})^{-1}(Y_0\mathbf{I}_{N_{\text{all}}} - \mathbf{Y}), \quad \mathbf{Y} = \mathbf{Z}^{-1}. \quad (2.3)$$

To model the RIS aided communication system, the vectors  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{v}$ , and  $\mathbf{i}$  are formulated in terms of sub-vectors as

$$\mathbf{a} = \begin{bmatrix} \mathbf{a}_T \\ \mathbf{a}_I \\ \mathbf{a}_R \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{b}_T \\ \mathbf{b}_I \\ \mathbf{b}_R \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} \mathbf{v}_T \\ \mathbf{v}_I \\ \mathbf{v}_R \end{bmatrix}, \quad \mathbf{i} = \begin{bmatrix} \mathbf{i}_T \\ \mathbf{i}_I \\ \mathbf{i}_R \end{bmatrix}, \quad (2.4)$$

where sub-vectors with subscripts T, I, and R, respectively, correspond to the incident/reflected waves, voltages, and currents at the antenna ports of the transmitter, the RIS, and the receiver. Accordingly, the matrices  $\mathbf{S}$ ,  $\mathbf{Z}$ , and  $\mathbf{Y}$  can be formulated in terms of sub-matrices as

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{TT} & \mathbf{S}_{TI} & \mathbf{S}_{TR} \\ \mathbf{S}_{IT} & \mathbf{S}_{II} & \mathbf{S}_{IR} \\ \mathbf{S}_{RT} & \mathbf{S}_{RI} & \mathbf{S}_{RR} \end{bmatrix}, \quad \mathbf{Z} = \begin{bmatrix} \mathbf{Z}_{TT} & \mathbf{Z}_{TI} & \mathbf{Z}_{TR} \\ \mathbf{Z}_{IT} & \mathbf{Z}_{II} & \mathbf{Z}_{IR} \\ \mathbf{Z}_{RT} & \mathbf{Z}_{RI} & \mathbf{Z}_{RR} \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} \mathbf{Y}_{TT} & \mathbf{Y}_{TI} & \mathbf{Y}_{TR} \\ \mathbf{Y}_{IT} & \mathbf{Y}_{II} & \mathbf{Y}_{IR} \\ \mathbf{Y}_{RT} & \mathbf{Y}_{RI} & \mathbf{Y}_{RR} \end{bmatrix}, \quad (2.5)$$

where the diagonal sub-matrices refer to the scattering/impedance/admittance matrices that correspond to the antenna arrays at the transmitter, RIS, and the receiver; the off-diagonal sub-matrices refer to the transmission scattering/impedance/admittance matrices between the transmitter, RIS, and the receiver.

**Transmitter:** At the transmitter, each antenna is connected in series to a voltage source and a source impedance (or source admittance), yielding a source impedance matrix  $\mathbf{Z}_T \in \mathbb{C}^{N \times N}$  (or equivalently a source admittance matrix  $\mathbf{Y}_T = \mathbf{Z}_T^{-1} \in \mathbb{C}^{N \times N}$ ). Accordingly, the reflection coefficient matrix at the transmitter is

$$\mathbf{\Gamma}_T = (\mathbf{Z}_T + Z_0\mathbf{I}_N)^{-1}(\mathbf{Z}_T - Z_0\mathbf{I}_N) = (Y_0\mathbf{I}_N + \mathbf{Y}_T)^{-1}(Y_0\mathbf{I}_N - \mathbf{Y}_T). \quad (2.6)$$

The three parameters relate  $\mathbf{b}_T$  and  $\mathbf{a}_T$ , or  $\mathbf{v}_T$  and  $\mathbf{i}_T$ , by [77]

$$\mathbf{a}_T = \mathbf{b}_{s,T} + \mathbf{\Gamma}_T\mathbf{b}_T, \quad \mathbf{v}_T = \mathbf{v}_{s,T} - \mathbf{Z}_T\mathbf{i}_T, \quad \mathbf{i}_T = \mathbf{i}_{s,T} - \mathbf{Y}_T\mathbf{v}_T, \quad (2.7)$$

where  $\mathbf{b}_{s,T} \in \mathbb{C}^{N \times 1}$ ,  $\mathbf{v}_{s,T} \in \mathbb{C}^{N \times 1}$ , and  $\mathbf{i}_{s,T} \in \mathbb{C}^{N \times 1}$ , respectively, denote the source wave vector, the source voltage vector, and the source current vector at the transmitter.

**Receiver:** Each receive antenna is connected to the ground through a load impedance (or admittance), yielding a load impedance matrix  $\mathbf{Z}_R \in \mathbb{C}^{K \times K}$  (or equivalently a load admittance matrix  $\mathbf{Y}_R = \mathbf{Z}_R^{-1} \in \mathbb{C}^{K \times K}$ ). Accordingly, the reflection coefficient matrix at the receiver is

$$\mathbf{\Gamma}_R = (\mathbf{Z}_R + Z_0 \mathbf{I}_K)^{-1}(\mathbf{Z}_R - Z_0 \mathbf{I}_K) = (Y_0 \mathbf{I}_K + \mathbf{Y}_R)^{-1}(Y_0 \mathbf{I}_K - \mathbf{Y}_R). \quad (2.8)$$

The three parameters relate  $\mathbf{b}_R$  and  $\mathbf{a}_R$ , or  $\mathbf{v}_R$  and  $\mathbf{i}_R$ , by [77]

$$\mathbf{a}_R = \mathbf{\Gamma}_R \mathbf{b}_R, \quad \mathbf{v}_R = -\mathbf{Z}_R \mathbf{i}_R, \quad \mathbf{i}_R = -\mathbf{Y}_R \mathbf{v}_R. \quad (2.9)$$

**Reconfigurable Intelligent Surface:** The  $M$  antennas at the RIS are connected to an  $M$ -port reconfigurable impedance/admittance network [55], yielding a reconfigurable impedance matrix  $\mathbf{Z}_I \in \mathbb{C}^{M \times M}$  (or equivalently a reconfigurable admittance matrix  $\mathbf{Y}_I = \mathbf{Z}_I^{-1} \in \mathbb{C}^{M \times M}$ ). Accordingly, the scattering matrix of the reconfigurable impedance/admittance network at the RIS is

$$\mathbf{\Phi} = (\mathbf{Z}_I + Z_0 \mathbf{I}_M)^{-1}(\mathbf{Z}_I - Z_0 \mathbf{I}_M) = (Y_0 \mathbf{I}_M + \mathbf{Y}_I)^{-1}(Y_0 \mathbf{I}_M - \mathbf{Y}_I). \quad (2.10)$$

Specifically when the reconfigurable impedance/admittance network is lossless, the corresponding impedance matrix or admittance matrix are purely imaginary, which makes the scattering matrix unitary, i.e.,  $\mathbf{\Phi}^H \mathbf{\Phi} = \mathbf{I}_M$ . The three parameters relate  $\mathbf{b}_I$  and  $\mathbf{a}_I$ , or  $\mathbf{v}_I$  and  $\mathbf{i}_I$ , by [77]

$$\mathbf{a}_I = \mathbf{\Phi} \mathbf{b}_I, \quad \mathbf{v}_I = -\mathbf{Z}_I \mathbf{i}_I, \quad \mathbf{i}_I = -\mathbf{Y}_I \mathbf{v}_I. \quad (2.11)$$

According to the circuit topology of the reconfigurable impedance/admittance network, the RIS can be generally classified as follows.

- 1) Conventional Diagonal RIS: When each port of the reconfigurable impedance/admittance network is connected to a reconfigurable impedance/admittance component to ground without interconnecting to other ports, the resulting reconfigurable impedance/admittance network has a single-connected architecture. This corresponds to the conventional diagonal RIS [3, 7], which mathematically has diagonal impedance/admittance/scattering matrices, i.e.,  $\mathbf{Z}_I$ ,  $\mathbf{Y}_I$ , and  $\mathbf{\Phi}$  are diagonal matrices.
- 2) BD-RIS: When part of/all the ports are connected to each other via additional reconfigurable impedance/admittance components, the resulting network can have group/fully-connected architectures as introduced in [55], can have forest/tree-connected architectures as introduced in [57], or can support hybrid transmitting and reflecting mode and the multi-sector mode which will be soon explained in Chapters 3 and 4. This corresponds to the BD-RIS,

which mathematically has impedance/admittance/scattering matrices not limited to being diagonal, i.e.,  $\mathbf{Z}_I$ ,  $\mathbf{Y}_I$ , and  $\mathbf{\Phi}$  can have nonzero off-diagonal entries.

### 2.1.2 Modeling the RIS Aided Communication System

The wireless channel from the transmitter and the receiver for the RIS aided communication system, denoted by  $\mathbf{H}_{\text{all}} \in \mathbb{C}^{K \times N}$ , essentially relates the voltage vector  $\mathbf{v}_T$  at the transmitter and the voltage vector  $\mathbf{v}_R$  at the receiver by

$$\mathbf{v}_R = \mathbf{H}_{\text{all}} \mathbf{v}_T, \quad (2.12)$$

such that  $\mathbf{H}_{\text{all}}$  can be equivalently described as a function of scattering, impedance, and admittance parameters, i.e.,

$$\mathbf{H}_{\text{all}} = F_S(\mathbf{S}, \mathbf{\Gamma}_T, \mathbf{\Gamma}_R, \mathbf{\Phi}), \quad \mathbf{H}_{\text{all}} = F_Z(\mathbf{Z}, \mathbf{Z}_T, \mathbf{Z}_R, \mathbf{Z}_I), \quad \mathbf{H}_{\text{all}} = F_Y(\mathbf{Y}, \mathbf{Y}_T, \mathbf{Y}_R, \mathbf{Y}_I). \quad (2.13a)$$

More detailed derivations and comparisons can be found in [69].

In this thesis, the wireless channel models used in Chapters 3-6, and Chapter 8 are derived based on the following three assumptions.

**Assumption 2.1.** *The source impedances at the transmitter and the load impedances at the receiver are equal to the reference impedance  $Z_0$ , i.e.,  $\mathbf{Z}_T = Z_0 \mathbf{I}_N$  and  $\mathbf{Z}_R = Z_0 \mathbf{I}_K$ , such that  $\mathbf{\Gamma}_T = \mathbf{0}_{N \times N}$  and  $\mathbf{\Gamma}_R = \mathbf{0}_{K \times K}$ . This corresponds to the best power matching at the transmitter and receiver, respectively, such that the signals will be transmitted with the maximum power without reflections.*

**Assumption 2.2.** *The distances between the transmitter and receiver, transmitter and BD-RIS, and BD-RIS and receiver are sufficiently large such that the links from the receiving devices to the transmitting devices are negligible, i.e.,  $\mathbf{Z}_{TI} = \mathbf{0}_{N \times M}$ ,  $\mathbf{Z}_{TR} = \mathbf{0}_{N \times K}$ ,  $\mathbf{Z}_{IR} = \mathbf{0}_{M \times K}$ . This corresponds to the unilateral approximation [81], saying that when there are large distances between the transmitter, receiver, and RIS, yielding large signal attenuation between devices, the electrical properties at the transmitter are approximately independent of those at the receiver. The unilateral approximation holds true when the minimum distance between devices, which is a function of the number of antennas, antenna spacing, and radiation pattern, is guaranteed.*

**Assumption 2.3.** *The antennas at the transmitter, RIS, and the receiver are perfectly matched with no mutual coupling, such that  $\mathbf{Z}_{TT} = Z_0 \mathbf{I}_N$ ,  $\mathbf{Z}_{II} = Z_0 \mathbf{I}_M$ , and  $\mathbf{Z}_{RR} = Z_0 \mathbf{I}_K$ . This can be practically realized by individually matching each antenna to the reference impedance  $Z_0$  and keeping the antenna spacing larger than half-wavelength.*

**Result 2.1.** *By Assumptions 2.1-2.3, the scattering, impedance, and admittance parameters*

based expressions of the wireless channel  $\mathbf{H}_{\text{all}}$  are given by [55, 69]

$$\mathbf{H}_{\text{all}} = \mathbf{S}_{\text{RT}} + \mathbf{S}_{\text{RI}}\Phi\mathbf{S}_{\text{IT}}, \quad (2.14a)$$

$$\mathbf{H}_{\text{all}} = \frac{1}{2Z_0}(\mathbf{Z}_{\text{RT}} - \mathbf{Z}_{\text{RI}}(\mathbf{Z}_{\text{I}} + Z_0\mathbf{I}_M)^{-1}\mathbf{Z}_{\text{IT}}), \quad (2.14b)$$

$$\mathbf{H}_{\text{all}} = \frac{1}{2Y_0}(-\mathbf{Y}_{\text{RT}} + \mathbf{Y}_{\text{RI}}(\mathbf{Y}_{\text{I}} + Y_0\mathbf{I}_M)^{-1}\mathbf{Y}_{\text{IT}}), \quad (2.14c)$$

The expressions in (2.14a) and (2.14c) are equivalent to each other with the mappings [69]

$$\mathbf{S}_{\text{RI}} = \frac{\mathbf{Z}_{\text{RI}}}{2Z_0}, \quad \mathbf{S}_{\text{IT}} = \frac{\mathbf{Z}_{\text{IT}}}{2Z_0}, \quad \mathbf{S}_{\text{RT}} = \frac{1}{2Z_0}(\mathbf{Z}_{\text{RT}} - \frac{\mathbf{Z}_{\text{RI}}\mathbf{Z}_{\text{IT}}}{2Z_0}), \quad (2.15a)$$

$$\mathbf{Y}_{\text{RI}} = -2Y_0\mathbf{S}_{\text{RI}}, \quad \mathbf{Y}_{\text{IT}} = -2Y_0\mathbf{S}_{\text{IT}}, \quad \mathbf{Y}_{\text{RT}} = -2Y_0(\mathbf{S}_{\text{RT}} - \mathbf{S}_{\text{RI}}\mathbf{S}_{\text{IT}}). \quad (2.15b)$$

Since the impedance parameters  $\mathbf{Z}_{\text{RT}}$ ,  $\mathbf{Z}_{\text{RI}}$ , and  $\mathbf{Z}_{\text{IT}}$  capture the open-circuit radiation patterns between transmitting and reflecting antennas [77], they essentially characterize the channels between devices in the wireless communication systems. Therefore, we introduce the following fresh notations

$$\mathbf{H}_{\text{d}} = \frac{\mathbf{Z}_{\text{RT}}}{2Z_0}, \quad \mathbf{H} = \frac{\mathbf{Z}_{\text{RI}}}{2Z_0}, \quad \mathbf{G} = \frac{\mathbf{Z}_{\text{IT}}}{2Z_0}, \quad (2.16)$$

where  $\mathbf{H}_{\text{d}} \in \mathbb{C}^{K \times N}$  denotes the direct channel from the transmitter to the receiver,  $\mathbf{H} \in \mathbb{C}^{K \times M}$  denotes the channel from the RIS to receiver, and  $\mathbf{G} \in \mathbb{C}^{M \times N}$  denotes the channel from the transmitter to RIS. Then, (2.14a) can be rewritten as

$$\mathbf{H}_{\text{all}} = \mathbf{H}_{\text{d}} + \mathbf{H}(\Phi - \mathbf{I}_M)\mathbf{G}, \quad (2.17)$$

In existing RIS literature [7, 9, 11, 12, 14, 15], the following channel model has been widely adopted:

$$\mathbf{H}_{\text{all}} = \mathbf{H}_{\text{d}} + \mathbf{H}\Phi\mathbf{G}. \quad (2.18)$$

Comparing (2.17) and (2.18), the physical meaning of the additional term, i.e.,  $-\mathbf{H}\mathbf{G}$ , corresponds to the structural scattering [69], which refers to the “virtual” direct link constructed by RIS when the RIS is turned OFF, i.e.,  $\Phi = \mathbf{0}_{M \times M}$ . The structural scattering exists since the OFF state of RIS is achieved when  $\mathbf{Z}_{\text{I}} = Z_0\mathbf{I}_M$ . This means there is current flowing within the RIS circuit, which scatters electromagnetic waves, even when the RIS is stated OFF.

Based on the analysis in [69], the impact of the structural scattering can be negligible with sufficiently large numbers of RIS elements. Therefore, in this thesis, we apply the expression in (2.18) throughout Chapters 3-6 to ease the modeling and signal processing of BD-RIS with hybrid transmitting and reflecting mode and multi-sector mode, while the expression in (2.14c) will be used in Chapter 8 to facilitate the modeling of hardware impairments at BD-RIS. In addition, in Chapter 7, by relaxing Assumption 2.3, the impedance parameter based channel expression of the wireless channel in (2.14b) will be modified to explicitly capture the impact

of mutual coupling at BD-RIS.

## 2.2 Optimization Techniques

To solve the mathematical problems formulated in this thesis, various optimization techniques are utilized, with the aim to transform the original unconvex problems into a series of convex ones, to find the global optimum of convex problems, and to efficiently deal with the unique constraints induced by BD-RIS architectures. This section briefly describes three key optimization techniques, namely, fractional programming (FP) [82–84], Karush-Kuhn-Tucker (KKT) conditions [85,86], and Manifold based methods [87–89], which are utilized in this thesis.

### 2.2.1 Fractional Programming

FP refers to a family of optimization problems involving ratio terms. The main idea to solve this kind of problems is to transform the ratio terms into more tractable integral expressions capturing the subtraction between the numerator and the denominator of each ratio term. In the following, we review the solutions for three kinds of FP problems, namely, single-ratio FP, multi-ratio FP, and sum-of-function-of-ratio FP.

**Single-Ratio FP:** A simple single-ratio FP problem can be formulated as

$$\max_{\mathbf{x} \in \mathcal{X}} \frac{F_n(\mathbf{x})}{F_d(\mathbf{x})} \quad (2.19)$$

where  $F_n(\mathbf{x}) \in \mathbb{R}$  is a non-negative scalar function,  $F_d(\mathbf{x}) \in \mathbb{R}$  is a positive scalar function, and  $\mathcal{X} \subseteq \mathbb{R}^{X \times 1}$  denotes a non-empty constraint set. This problem has been first solved by the *Dinkelbach's Transform* [82] by reformulating (2.19) as

$$\max_{\mathbf{x} \in \mathcal{X}, \tau \in \mathbb{R}} F_n(\mathbf{x}) - \tau F_d(\mathbf{x}) \quad (2.20)$$

where  $\tau \in \mathbb{R}$  is an auxiliary variable, which is iteratively updated by  $\tau^{o+1} = \frac{F_n(\mathbf{x}^o)}{F_d(\mathbf{x}^o)}$  in the  $o$ -th iteration. Iteratively updating  $\mathbf{x}$  by solving (2.20) and updating  $\tau$  until convergence leads to a locally optimal solution. Specifically, when  $F_n(\mathbf{x})$  is concave and  $F_d(\mathbf{x})$  is convex, the iteration leads to a global optimum.

Dinkelbach's transform works well for single-ratio FP problems, but it might not be feasible for multi-ratio FP problems since the transformed problem in this case and the original problem do not necessarily share the same optimal value. This motivates the *Quadratic Transform* [83] to deal with multi-ratio FP problems, as detailed below.

**Multi-Ratio FP:** A multi-ratio FP problem can be formulated as

$$\max_{\mathbf{x} \in \mathcal{X}} \sum_{j=1}^J \frac{F_{n,j}(\mathbf{x})}{F_{d,j}(\mathbf{x})} \quad (2.21)$$

where  $F_{n,j}(\mathbf{x}) \in \mathbb{R}$ ,  $\forall j \in \mathcal{J} = \{1, \dots, J\}$  are non-negative scalar functions,  $F_{d,j}(\mathbf{x}) \in \mathbb{R}$ ,  $\forall j \in \mathcal{J}$  are positive scalar functions. The quadratic transform says the following [83].

**Theorem 2.1.** *Problem (2.21) can be equivalently transformed to the following problem:*

$$\max_{\mathbf{x} \in \mathcal{X}, \tau_j \in \mathbb{R}, \forall j \in \mathcal{J}} \sum_{j=1}^J (2\tau_j \sqrt{F_{n,j}(\mathbf{x})} - \tau_j^2 F_{d,j}(\mathbf{x})) \quad (2.22)$$

where  $\tau_j \in \mathbb{R}$ ,  $\forall j \in \mathcal{J}$  are auxiliary variables.

*Proof.* Please refer to [83, 84]. □

Theorem 2.1 tells that iteratively updating  $\mathbf{x}$  by solving (2.22) and updating  $\tau_j$ ,  $\forall j \in \mathcal{J}$  by  $\tau_j^o = \frac{\sqrt{F_{n,j}(\mathbf{x}^o)}}{F_{d,j}(\mathbf{x}^o)}$  in the  $o$ -th iteration until convergence leads to a locally optimal solution. Specifically, when  $F_{n,j}(\mathbf{x})$ ,  $\forall j \in \mathcal{J}$  are concave and  $F_{d,j}(\mathbf{x})$ ,  $\forall j \in \mathcal{J}$  are convex, the iteration leads to a global optimum. Then we have the following two important corollaries.

**Corollary 2.1.** *Given a multi-ratio complex FP formulated as*

$$\max_{\mathbf{x} \in \mathcal{X}} \sum_{j=1}^J \frac{|F_{n,j}(\mathbf{x})|^2}{F_{d,j}(\mathbf{x})} \quad (2.23)$$

where  $F_{n,j}(\mathbf{x}) \in \mathbb{C}$ ,  $\forall j \in \mathcal{J}$  are complex scalar functions,  $F_{d,j}(\mathbf{x}) \in \mathbb{R}$ ,  $\forall j \in \mathcal{J}$  are positive scalar functions, and  $\mathcal{X} \subseteq \mathbb{C}^{X \times 1}$  denotes a non-empty constraint set, it is equivalent to

$$\max_{\mathbf{x} \in \mathcal{X}, \tau_j \in \mathbb{C}, \forall j \in \mathcal{J}} \sum_{j=1}^J (2\Re\{\tau_j^* F_{n,j}(\mathbf{x})\} - |\tau_j|^2 F_{d,j}(\mathbf{x})) \quad (2.24)$$

where  $\tau_j \in \mathbb{C}$ ,  $\forall j \in \mathcal{J}$  are auxiliary variables. Iteratively solving problem (2.24) and updating  $\tau_j^o = \frac{F_{n,j}(\mathbf{x}^o)}{F_{d,j}(\mathbf{x}^o)}$ ,  $\forall j \in \mathcal{J}$  in the  $o$ -th iteration leads to a local optimum. Specifically, when  $|F_{n,j}(\mathbf{x})|^2$ ,  $\forall j \in \mathcal{J}$  are concave and  $F_{d,j}(\mathbf{x})$ ,  $\forall j \in \mathcal{J}$  are convex, the iteration leads to a global maximum.

**Corollary 2.2.** *Given a sequence of nondecreasing functions  $F_j(\cdot) \in \mathbb{R}$  and a sequence of ratios  $\frac{|F_{n,j}(\mathbf{x})|^2}{F_{d,j}(\mathbf{x})}$  with  $F_{n,j}(\mathbf{x}) \in \mathbb{C}$  and  $F_{d,j}(\mathbf{x}) > 0$ ,  $\forall j \in \mathcal{J}$ , the sum-of-function-of-ratio problem can be formulated as*

$$\max_{\mathbf{x} \in \mathcal{X}} \sum_{j=1}^J F_j\left(\frac{|F_{n,j}(\mathbf{x})|^2}{F_{d,j}(\mathbf{x})}\right) \quad (2.25)$$

which, by the quadratic transform, is equivalent to

$$\max_{\mathbf{x} \in \mathcal{X}, \tau_j \in \mathbb{C}, \forall j \in \mathcal{J}} \sum_{j=1}^J F_j(2\Re\{\tau_j^* F_{n,j}(\mathbf{x})\} - |\tau_j|^2 F_{d,j}(\mathbf{x})) \quad (2.26)$$

where the optimal  $\tau_j \in \mathbb{C}$  in the  $o$ -th iteration is given by  $\tau_j^o = \frac{F_{n,j}(\mathbf{x}^o)}{F_{d,j}(\mathbf{x}^o)}$ ,  $\forall j \in \mathcal{J}$ .

The expression in (2.25) aligns well with the rate expression for multiuser scenarios in wireless communications, such that the transform from (2.25) to (2.26) is very useful to solve the sum-rate maximization problem, which will be formulated and solved in Chapters 3-6.

### 2.2.2 Karush-Kuhn-Tucker Conditions

Consider a general optimization problem in the following standard form:

$$\min_{\mathbf{x} \in \mathbb{R}^{X \times 1}} F_0(\mathbf{x}) \quad (2.27a)$$

$$\text{s.t. } F_{i,i}(\mathbf{x}) \leq 0, \forall i \in \mathcal{J}_1 = \{1, \dots, J_1\}, \quad (2.27b)$$

$$F_{e,j}(\mathbf{x}) = 0, \forall j \in \mathcal{J}_2 = \{1, \dots, J_2\}, \quad (2.27c)$$

with  $J_1$  inequality constraints and  $J_2$  equality constraints and all the functions in (2.27) differentiable. Let  $p^*$  denote the global minimum value of the primal optimization problem (2.27). Define the Lagrangian of (2.27) as

$$F_L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = F_0(\mathbf{x}) + \sum_{i=1}^{J_1} \lambda_i F_{i,i}(\mathbf{x}) + \sum_{j=1}^{J_2} \mu_j F_{e,j}(\mathbf{x}), \quad (2.28)$$

where  $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_{J_1}]^T \in \mathbb{R}^{J_1 \times 1}$  and  $\boldsymbol{\mu} = [\mu_1, \dots, \mu_{J_2}]^T \in \mathbb{R}^{J_2 \times 1}$  are dual variables (also called Lagrange multipliers). Then the dual function  $F_D(\boldsymbol{\lambda}, \boldsymbol{\mu})$  of (2.27) is given by

$$F_D(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \min_{\mathbf{x}} F_L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) \quad (2.29)$$

which is always concave. Thus, with  $\lambda_i \geq 0$ ,  $\forall i \in \mathcal{J}_1$  and  $F_D(\boldsymbol{\lambda}, \boldsymbol{\mu})$  finite:

- 1) The weak duality says that for any feasible  $\mathbf{x}$  and  $(\boldsymbol{\lambda}, \boldsymbol{\mu})$ , there holds  $F_0(\mathbf{x}) \geq F_D(\boldsymbol{\lambda}, \boldsymbol{\mu})$ , which indicates  $p^* \geq F_D(\boldsymbol{\lambda}, \boldsymbol{\mu})$ . Accordingly, the largest lower bound of  $p^*$  can be found by solving the dual problem

$$\max_{\lambda_i \geq 0, \forall i \in \mathcal{J}_1, \boldsymbol{\mu} \in \mathbb{R}^{J_2 \times 1}} F_D(\boldsymbol{\lambda}, \boldsymbol{\mu}) \quad (2.30)$$

which is always a convex problem leading to the maximum value  $d^* \leq p^*$ .

- 2) The strong duality says  $d^* = p^*$ , which holds true for most convex optimization problems.

The KKT conditions then give the necessary conditions for  $\mathbf{x}^*$  to be a local optimal solution of (2.27), that is, there exists some  $(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$  such that [86]

$$F_{i,i}(\mathbf{x}^*) \leq 0, \forall i \in \mathcal{J}_1, \quad (2.31a)$$

$$F_{e,j}(\mathbf{x}^*) = 0, \forall j \in \mathcal{J}_2, \quad (2.31b)$$

$$\lambda_i^* \geq 0, \forall i \in \mathcal{J}_1, \quad (2.31c)$$

$$\lambda_i^* F_{i,i}(\mathbf{x}^*) = 0, \forall i \in \mathcal{J}_1, \quad (2.31d)$$

$$\nabla_{\mathbf{x}} F_L(\mathbf{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) = 0, \quad (2.31e)$$

where the first two conditions guarantee the feasibility of  $\mathbf{x}^*$ , the third guarantees the feasibility of dual variables, the fourth signifies the complementary slackness of the primary and dual inequality constraints, i.e.,  $F_{i,i}(\mathbf{x}) \leq 0$  and  $\lambda_i \geq 0$ , and the last refers to the first-order condition.

In general, the KKT condition is necessary yet not sufficient for optimality, while it is also sufficient for convex optimization problems. A well-known application of the KKT condition in wireless communications is the water-filling algorithm for power allocation. It will also be used to optimize the transmit precoder in multiuser communication systems in Chapters 3-6.

### 2.2.3 Manifold

Constrained optimization has been well studied, with various solutions working in the Euclidean space, such as convexifying the nonconvex constraints and solving the problem by convex optimization methods, and penalizing the original constraints and decoupling the original problem into several convex sub-problems [85]. Among all constrained optimization problems, a special class of constraints is called geometric constraints, which can be solved on a manifold [87]. The main idea of the manifold based algorithm is to construct all available solutions of a certain constrained problem as a manifold, such that it becomes unconstrained in a constrained search space. In the following, we first review the definition of the manifold [87], and introduce mainly used manifolds in wireless communications.

**Definition 2.1.** *A manifold is a topological space which “locally” resembles the Euclidean space.*

In wireless communications, the following manifolds have been commonly used in existing literature [90–92].

- 1) Complex circle:  $\mathcal{M}_{cc} = \{\mathbf{x} \in \mathbb{C}^{X \times 1} : |[\mathbf{x}]_1| = \cdots = |[\mathbf{x}]_X| = 1\}$ ;
- 2) Complex sphere:  $\mathcal{M}_{cs} = \{\mathbf{X} \in \mathbb{C}^{X_1 \times X_2} : \|\mathbf{X}\|_F = 1\}$ ;
- 3) Complex oblique manifold:  $\mathcal{M}_{co} = \{\mathbf{X} \in \mathbb{C}^{X_1 \times X_2} : \|[\mathbf{X}]_{:,1}\|_2 = \cdots = \|[\mathbf{X}]_{:,X_2}\|_2 = 1\}$ ;

- 4) Complex Stiefel manifold:  $\mathcal{M}_{\text{cst}} = \{\mathbf{X} \in \mathbb{C}^{X_1 \times X_2} : \mathbf{X}^H \mathbf{X} = \mathbf{I}_{X_2}\}^{X_3}$ ;
- 5) Unitary matrices:  $\mathcal{M}_{\text{um}} = \{\mathbf{X} \in \mathbb{C}^{X_1 \times X_1} : \mathbf{X}^H \mathbf{X} = \mathbf{I}_{X_1}\}^{X_2}$ .

Specifically, the complex circle  $\mathcal{M}_{\text{cc}}$  can be mapped to the constraint set of conventional diagonal RIS [92], or that of analog precoder in hybrid beamforming architectures [90], or the constant envelope constraint in symbol-level precoding [91]; the complex sphere  $\mathcal{M}_{\text{cs}}$  can be mapped to the constraint set of the precoder matrix at the transmitter, which has a total power constraint; the complex oblique manifold  $\mathcal{M}_{\text{co}}$  can be mapped to the constraint set of the precoder matrix at the transmitter, which allocates equal power to each stream/user. Moreover, the complex sphere  $\mathcal{M}_{\text{cs}}$ , the complex Stiefel manifold  $\mathcal{M}_{\text{cst}}$ , and the unitary matrices  $\mathcal{M}_{\text{um}}$  can be mapped to the constraint sets of the scattering matrix at BD-RIS, which will be used in Chapters 3, 5, and 6 to solve various optimization problems.

Line-search methods well established on the Euclidean space, e.g., Trust-Region, Conjugate-Gradient, and Broyden-Fletcher-Goldfarb-Shanno methods [88], can be easily extended to the manifold space with some necessary projections. Before explaining the principle of line-search methods on manifold, we give the following definitions.

**Definition 2.2.** *The moving direction of a point on the manifold is referred to as a tangent vector. All these tangent vectors at this point, which include all possible directions this point can move to, form the tangent space.*

**Definition 2.3.** *Each tangent space can be regarded as a Euclidean space which has one tangent vector, namely, Riemannian gradient, pointing to the direction that the objective function decreases fastest.*

Line-search methods are based on the updated formula

$$\mathbf{x}^{o+1} = \mathbf{x}^o + \delta^o \boldsymbol{\eta}^o, \quad (2.32)$$

where  $\boldsymbol{\eta}^o$  refers to the search direction, which, on the manifold, could be the Riemannian gradient at point  $\mathbf{x}^o$ , and  $\delta^o$  denotes the step size at the  $o$ -th iteration. To guarantee that the movement of the search direction always stay on the manifold during iterations of the line-search methods, a retraction is needed to project the updated point back to the manifold. Readers are referred to [87] for various retractions corresponding to different manifolds.

# Part I

## Mode Analysis of BD-RIS



## Chapter 3

# BD-RIS with Hybrid Transmitting and Reflecting Mode

In conventional diagonal RIS, the wave manipulation flexibility is limited and only half-space coverage is supported due to the simple architecture without inter-element connections and the uniform antenna arrangement. To address the first limitation, BD-RIS [55] has been primarily proposed by introducing inter-element connections, which mathematically generates beyond diagonal scattering matrices to provide additional flexibility for wave manipulation. To address the second limitation, STAR-RIS [72–76] has been proposed to achieve full-space coverage. However, it remains unknown if BD-RIS can also enable full-space coverage and go beyond STAR-RIS to further enhance the channel gain. In addition, it remains unknown how exactly the antenna array should be arranged and the circuits should be designed to simultaneously achieve such goal. This chapter thus address this research problem by proposing a new branch of BD-RIS tree in Fig. 1.1, namely, BD-RIS under hybrid mode which provides a full-space coverage, and provides a thorough analysis from modeling, architecture design and mode analysis, to beamforming design. This leads to the following layouts and contributions.

In Section 3.1, a general BD-RIS-aided communication model using microwave network theory has been proposed and analyzed. The proposed model is derived by characterizing an RIS as multiple antennas connected to a group-connected multi-port reconfigurable impedance network. Benefiting from the multi-port network analysis, the hybrid mode of BD-RIS has been proposed by establishing connections among different ports.

In Section 3.2, three modes (reflective, transmissive, and hybrid) and four architectures (cell-wise single-, group-, fully-connected, and dynamically group-connected) have been proposed and summarized, together with implementations and discussions. Specifically, STAR-RIS has been proven to be a special case of BD-RIS with cell-wise single-connected architectures, where every two antennas are back to back placed and connected to each other by a reconfigurable impedance component.

In Section 3.3, with the proposed model, the joint transmit precoder and BD-RIS matrix design to maximize the sum-rate for an RIS-aided multiuser multiple input single output (MU-MISO) system has been illustrated. A general solution suitable for different modes/architectures has been proposed. Then for STAR-RIS case, an efficient solution, which has similar performance, but lower computational complexity compared to the general one has been proposed. In addition, a heuristic solution suitable for cell-wise dynamically group-connected case has been proposed. The initialization and complexity analysis of the proposed designs have been discussed and compared.

In Section 3.4, simulation results have been provided to evaluate the performance of the proposed design. We compare the sum-rate performance when the BD-RIS is under different modes and architectures. Simulation results with specific parameter settings show that under Rayleigh fading conditions, cell-wise fully- and group-connected hybrid BD-RISs can achieve 75% and 37% higher sum-rate than STAR-RIS. In the meanwhile, the sum-rate achieved by cell-wise fully-connected hybrid BD-RISs over that by cell-wise fully-connected reflective/transmissive ones can be around 20% for Rician fading channels.

### 3.1 Hybrid BD-RIS Model

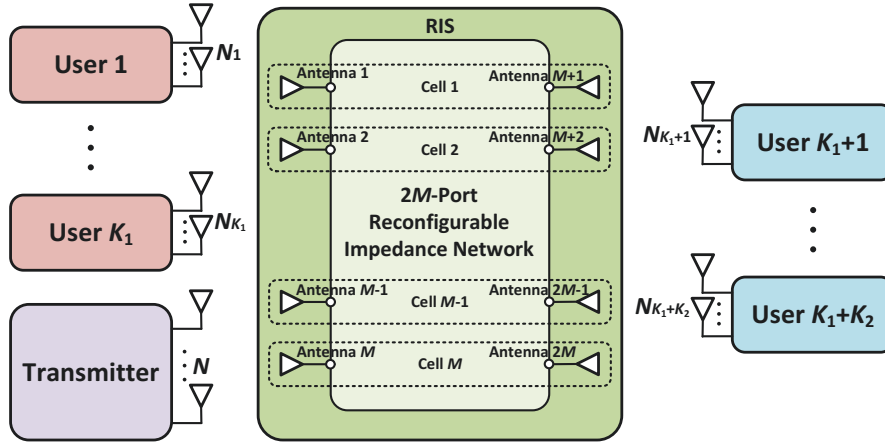


Figure 3.1: Diagram of an  $M$ -cell RIS-aided communication system.

Consider an RIS-aided wireless communication system, where an  $N$ -antenna transmitter indexed by  $\mathcal{N} = \{1, \dots, N\}$  serves  $K$  multi-antenna users with the assistance of an RIS. Each user has  $N_k$  antennas,  $\forall k \in \mathcal{K} = \{1, \dots, K\}$ . In general, we consider an  $M$ -cell<sup>1</sup> RIS indexed by  $\mathcal{M} = \{1, \dots, M\}$  where the  $m$ th cell consists of antenna  $m$  and antenna  $m + M$ . Therefore, in this  $M$ -cell RIS, there are  $2M$  antennas connected to a  $2M$ -port reconfigurable impedance network, as shown in Fig. 3.1. According to [55] and Result 2.1, assuming that the multiple

<sup>1</sup>Here we use “cell” instead of “element” since in existing works each RIS element contains one antenna, while in this work each cell has two antennas. The “cell” is illustrated in Fig. 3.1.

antennas at the transmitter/receiver/RIS are perfectly matched with no mutual coupling, the overall channel matrix between the transmitter and each user,  $\mathbf{H}_{\text{all},k} \in \mathbb{C}^{N_k \times N}$ ,  $k \in \mathcal{K}$ , can be expressed as

$$\mathbf{H}_{\text{all},k} = \mathbf{H}_{\text{d},k} + \bar{\mathbf{H}}_k \Phi \bar{\mathbf{G}}, \forall k \in \mathcal{K}, \quad (3.1)$$

where  $\mathbf{H}_{\text{d},k} \in \mathbb{C}^{N_k \times N}$ ,  $\bar{\mathbf{H}}_k \in \mathbb{C}^{N_k \times 2M}$ ,  $\forall k \in \mathcal{K}$ , and  $\bar{\mathbf{G}} \in \mathbb{C}^{2M \times N}$  denote channel matrices from the transmitter to user  $k$ , from the RIS to user  $k$ , and from the transmitter to the RIS, respectively.  $\Phi \in \mathbb{C}^{2M \times 2M}$  denotes the scattering matrix of the  $2M$ -port reconfigurable impedance network [55]. According to microwave network theory [55, 77],  $\Phi$  should satisfy  $\Phi^H \Phi \preceq \mathbf{I}_{2M}$ . Particularly, when the  $2M$ -port reconfigurable impedance network is lossless, we have  $\Phi^H \Phi = \mathbf{I}_{2M}$ <sup>2</sup>. It is worth noting that the structure of the matrix  $\Phi$  is determined by the circuit topology of the  $2M$ -port reconfigurable impedance network. Therefore, different from traditional RIS model with a diagonal matrix, the matrix  $\Phi$  is not restricted to be diagonal, which is referred to as BD-RIS.

To show that existing RIS-aided communication model [72–74] is a special case of our proposed model, we first make the following assumptions:

**Assumption 3.1.** *Each antenna has a uni-directional radiation pattern*<sup>3</sup>.

**Assumption 3.2.** *In each cell, the two antennas are back to back placed so that each antenna covers half space as shown in Fig. 3.2.*

With these assumptions, we can divide the  $M$ -cell BD-RIS into 2 sectors, where the first sector consists of antennas 1- $M$  and the other sector consists of antennas  $M+1$ - $2M$ , and thus the  $M$ -cell RIS partitions the whole space into two sides, which are respectively covered by the two sectors. Accordingly, we partition the overall channel matrix (3.1) as

$$\begin{aligned} \mathbf{H}_{\text{all},k} &= \mathbf{H}_{\text{d},k} + [\bar{\mathbf{H}}_{k,1} \ \bar{\mathbf{H}}_{k,2}] \begin{bmatrix} \Phi_{1,1} & \Phi_{1,2} \\ \Phi_{2,1} & \Phi_{2,2} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{G}}_1 \\ \bar{\mathbf{G}}_2 \end{bmatrix} \\ &= \mathbf{H}_{\text{d},k} + \bar{\mathbf{H}}_{k,1} \Phi_{1,1} \bar{\mathbf{G}}_1 + \bar{\mathbf{H}}_{k,2} \Phi_{2,1} \bar{\mathbf{G}}_1 + \bar{\mathbf{H}}_{k,1} \Phi_{1,2} \bar{\mathbf{G}}_2 + \bar{\mathbf{H}}_{k,2} \Phi_{2,2} \bar{\mathbf{G}}_2, \forall k \in \mathcal{K}, \end{aligned} \quad (3.2)$$

where  $\bar{\mathbf{H}}_{k,i} = [\bar{\mathbf{H}}_k]_{:, (i-1)M+1:iM} \in \mathbb{C}^{N_k \times M}$ ,  $\forall k \in \mathcal{K}$ , and  $\bar{\mathbf{G}}_i = [\bar{\mathbf{G}}]_{(i-1)M+1:iM, :} \in \mathbb{C}^{M \times N}$ ,  $\forall i \in \{1, 2\}$  are channels between sector  $i$  of the RIS and user  $k$ , and between the transmitter and sector  $i$  of the RIS, respectively.  $\Phi_{i,j} = [\Phi]_{(i-1)M+1:iM, (j-1)M+1:jM} \in \mathbb{C}^{M \times M}$ ,  $\forall i, j \in \{1, 2\}$ .

Without loss of generality, we assume that the transmitter and  $K_1$  users, indexed by  $\mathcal{K}_1 = \{1, \dots, K_1\}$ ,  $0 < K_1 < K$ , are located at one side of the RIS (covered by sector 1), and

<sup>2</sup>Here we should explain that matrix  $\Phi$  is a scattering matrix, which provides an integrated description of a network as seen at its  $2M$  ports. The scattering matrix associates the voltage of incident waves with reflected waves from the ports. When the multi-port network is lossless, no power can be delivered to the network so that the incident and reflected power are equal to each other. In this case, it can be derived that the scattering matrix satisfies a unitary constraint [77].

<sup>3</sup>In practice the uni-directional radiation pattern can be achieved by using microstrip patch antennas or by adding a reflecting ground plane.

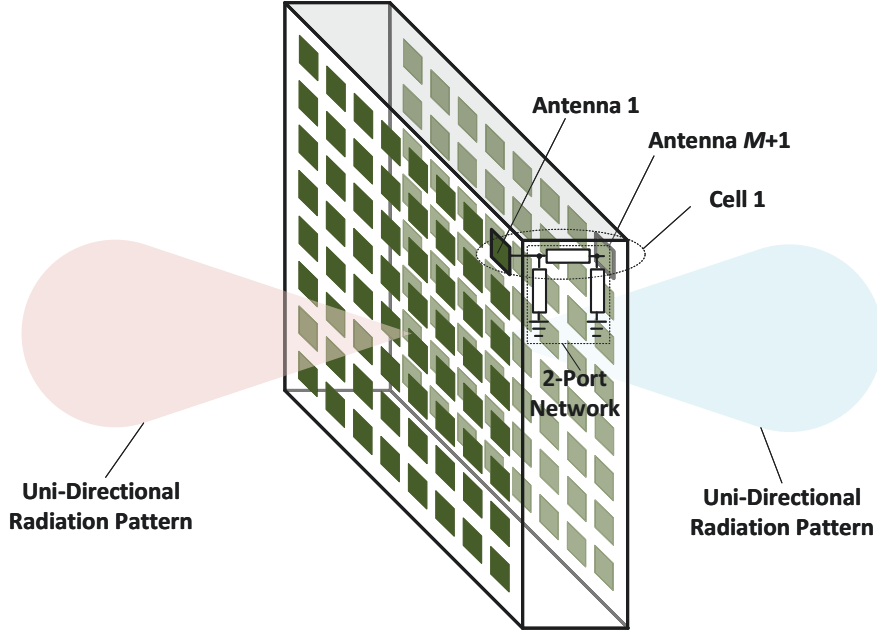


Figure 3.2: An  $M$ -cell BD-RIS with  $2M$  back to back placed uni-directional antennas.

$K_2 = K - K_1$  users, indexed by  $\mathcal{K}_2 = \{K_1 + 1, \dots, K\}$ , are located at the other side of the RIS (covered by sector 2). To facilitate understanding, we illustrate the locations of the transmitter, RIS, and users from a top view as shown in Fig. 3.3. Following Assumptions 3.1 and 3.2, we can deduce the following two results:

**Result 3.1.** *The channel from the transmitter to sector 2 of the BD-RIS is zero, that is,  $\bar{\mathbf{G}}_2 = \mathbf{0}_{M \times N}$ , as the transmitter is not covered by the uni-directional radiation pattern of sector 2 of the BD-RIS.*

**Result 3.2.** *The channel from sector  $i$  of the BD-RIS to the user belonging to  $\mathcal{K}_j$  is zero  $\forall i \neq j$ , that is  $\bar{\mathbf{H}}_{k,i} = \mathbf{0}_{N_k \times M}$ ,  $\forall k \in \mathcal{K}_j$ ,  $\forall i \neq j$ , as the user belonging to  $\mathcal{K}_j$  is not covered by the uni-directional radiation pattern of sector  $i$ .*

Leveraging Results 3.1 and 3.2, we can simplify the channel matrix (3.2) as

$$\mathbf{H}_{\text{all},k} = \begin{cases} \mathbf{H}_{\text{d},k} + \bar{\mathbf{H}}_{k,1} \bar{\mathbf{\Phi}}_{1,1} \bar{\mathbf{G}}_1, & k \in \mathcal{K}_1, \\ \mathbf{H}_{\text{d},k} + \bar{\mathbf{H}}_{k,2} \bar{\mathbf{\Phi}}_{2,1} \bar{\mathbf{G}}_1, & k \in \mathcal{K}_2. \end{cases} \quad (3.3)$$

We denote users within the coverage of sector 1 of the BD-RIS as reflective users, indexed by  $\mathcal{K}_r = \mathcal{K}_1$ ,  $K_r = K_1$ . Similarly, we denote users within the coverage of sector 2 as transmissive users, indexed by  $\mathcal{K}_t = \mathcal{K}_2$ ,  $K_t = K_2 = K - K_r$ . Then, using auxiliary notations,  $\mathbf{H}_k = \bar{\mathbf{H}}_{k,i}$ ,  $\forall k \in \mathcal{K}_i$ ,  $\forall i \in \{1, 2\}$ ,  $\mathbf{\Phi}_r = \bar{\mathbf{\Phi}}_{1,1}$ ,  $\mathbf{\Phi}_t = \bar{\mathbf{\Phi}}_{2,1}$ , and  $\mathbf{G} = \bar{\mathbf{G}}_1$ , we can rewrite (3.3) as

$$\mathbf{H}_{\text{all},k} = \begin{cases} \mathbf{H}_{\text{d},k} + \mathbf{H}_k \mathbf{\Phi}_r \mathbf{G}, & k \in \mathcal{K}_r, \\ \mathbf{H}_{\text{d},k} + \mathbf{H}_k \mathbf{\Phi}_t \mathbf{G}, & k \in \mathcal{K}_t. \end{cases} \quad (3.4)$$

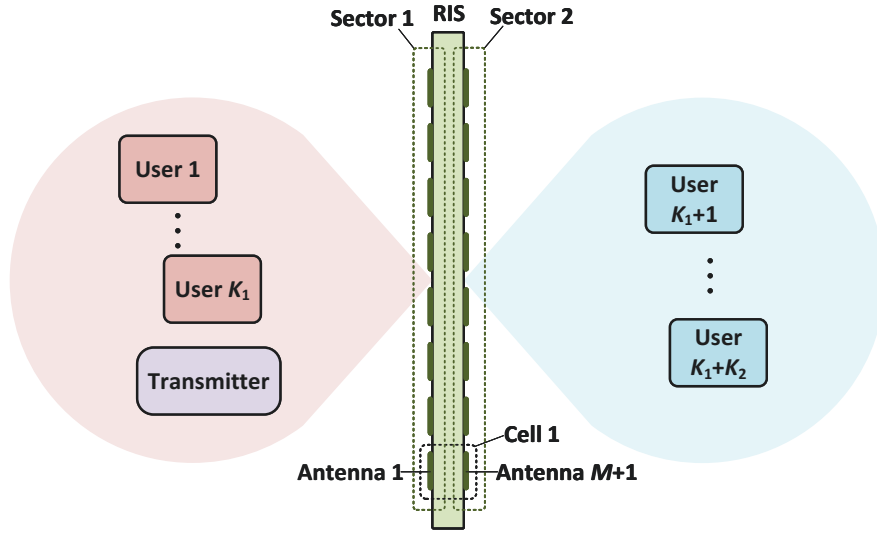


Figure 3.3: Top view for the locations of the  $M$ -cell BD-RIS partitioning the whole space into two sides, the transmitter, and multiple users.

Given that  $\Phi^H \Phi = \mathbf{I}_{2M}$ ,  $\Phi_t$  and  $\Phi_r$  should satisfy the following constraint:

$$\Phi_r^H \Phi_r + \Phi_t^H \Phi_t = \mathbf{I}_M. \quad (3.5)$$

Based on the above analysis, we have verified that the communication model utilized in STAR-RIS [72–74] is a special instance of our models (3.4) and (3.5). In the following section, we will go beyond STAR-RIS and investigate different cases of constraint (3.5).

## 3.2 Architecture/Mode Analysis and Design

### 3.2.1 Architecture Analysis and Design

In [55], various RIS architectures which have different RIS element circuit topologies have been proposed. In this subsection, we generalize this concept by analyzing and designing the  $M$ -cell BD-RIS architecture<sup>4</sup> with different cell circuit topologies as detailed below.

**Cell-Wise Single-Connected (CW-SC) Architecture:** In this architecture, cells of the BD-RIS are not connected to each other. Fig. 3.4(a) provides a simple example of CW-SC BD-RIS with 2 cells. Thus, matrices  $\Phi_r$ ,  $\Phi_t$  are all diagonal, which are given by

$$\Phi_r = \text{diag}(\phi_{r,1}, \dots, \phi_{r,M}), \quad \Phi_t = \text{diag}(\phi_{t,1}, \dots, \phi_{t,M}), \quad (3.6)$$

<sup>4</sup>The following single/fully/group-connected architectures refer to the inter-cell circuit topologies. The 2 antennas within the same cell are connected to a 2-port fully-connected reconfigurable impedance network.

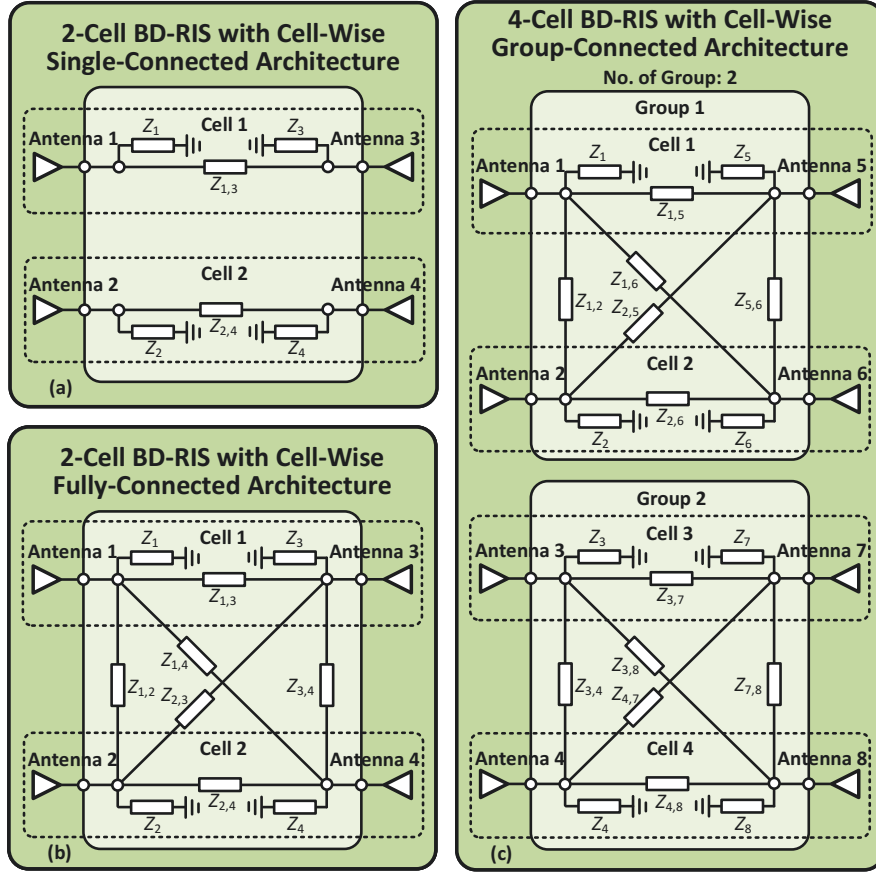


Figure 3.4: Examples of (a) CW-SC BD-RIS, (b) CW-FC BD-RIS, and (c) CW-GC BD-RIS.

where the entries  $\phi_{t/r,m}, \forall m \in \mathcal{M}$ , satisfy the constraints

$$|\phi_{r,m}|^2 + |\phi_{t,m}|^2 = 1, \forall m \in \mathcal{M}. \quad (3.7)$$

The BD-RIS with a CW-SC architecture is essentially the STAR-RIS [72–74].

**Cell-Wise Fully-Connected (CW-FC) Architecture:** We propose and design a generalized RIS architecture referred to as CW-FC architecture where all cells of the BD-RIS are connected to each other through reconfigurable impedance components. In Fig. 3.4(b), we give an example of CW-FC BD-RIS with 2 cells. Thus,  $\Phi_r, \Phi_t$  are all full matrices satisfying the constraint

$$\Phi_r^H \Phi_r + \Phi_t^H \Phi_t = \mathbf{I}_M. \quad (3.8)$$

CW-FC architecture can achieve the best performance due to the most general constraint (3.8).

**Cell-Wise Group-Connected (CW-GC) Architecture:** It is obvious that when the number of cells  $M$  gets larger, the circuit topology of the CW-FC BD-RIS will become extremely complicated. To achieve a good trade-off between the RIS performance and circuit complexity, we propose and design a generalized BD-RIS architecture referred to as CW-GC architecture. In the CW-GC BD-RIS, the  $M$  cells are divided into  $G$  groups indexed by  $\mathcal{G} = \{1, \dots, G\}$  with

each group of cells utilizing the CW-FC architecture. For simplicity, here we assume that all groups have the same size  $\bar{M} = M/G$ , and we define  $\mathcal{G}_g \triangleq \{(g-1)\bar{M} + 1, \dots, g\bar{M}\}$  as the set of cell indexes for group  $g$ . An example of a 4-cell BD-RIS with CW-GC architecture having 2 groups is illustrated in Fig. 3.4(c). Thus,  $\Phi_r$ ,  $\Phi_t$  are block diagonal matrices:

$$\Phi_r = \text{blkdiag}(\Phi_{r,1}, \dots, \Phi_{r,G}), \quad \Phi_t = \text{blkdiag}(\Phi_{t,1}, \dots, \Phi_{t,G}), \quad (3.9)$$

where  $\Phi_{t/r,g} \in \mathbb{C}^{\bar{M} \times \bar{M}}, \forall g \in \mathcal{G}$  satisfies the constraint

$$\Phi_{r,g}^H \Phi_{r,g} + \Phi_{t,g}^H \Phi_{t,g} = \mathbf{I}_{\bar{M}}, \forall g \in \mathcal{G}. \quad (3.10)$$

**Cell-Wise Dynamically Group-Connected (CW-DGC) Architecture:** We also propose a CW-DGC architecture of BD-RIS by adapting the grouping strategy to channel environments. Different from the CW-GC architecture, where  $M$  cells are uniformly divided into  $G$  subsets and every adjacent  $\bar{M}$  cells are connected to each other, CW-DGC is based on a dynamic grouping strategy, which is embodied in two aspects: 1) The number of cells in each group can be different; 2) the positions/indexes of cells in the same group are not restricted to adjacent ones. To model this dynamic grouping strategy, we first present the following definition.

**Definition 3.1.** Define  $G$  subsets  $\mathcal{D}_1, \dots, \mathcal{D}_G$  which store indexes of RIS cells for each group. These subsets satisfy the following constraints:

$$\mathcal{D}_g \neq \emptyset, \forall g \in \mathcal{G}, \quad (3.11a)$$

$$\mathcal{D}_p \cap \mathcal{D}_q = \emptyset, \forall p \neq q, p, q \in \mathcal{G}, \quad (3.11b)$$

$$\bigcup_{g=1}^G \mathcal{D}_g = \mathcal{M}, \quad (3.11c)$$

which indicate that each group contains at least one cell; that each BD-RIS cell can only be mapped into one group; and that there is no overlap among different groups<sup>5</sup>.

Based on Definition 3.1, we have the following result.

**Result 3.3.** The CW-DGC architecture provides additional flexibility by controlling the connections via switches. This mathematically leads to BD-RIS matrices,  $\Phi_r$  and  $\Phi_t$ , satisfying the following constraints

$$[\Phi_t]_{m,n} = 0, \forall m \in \mathcal{D}_p, \forall n \in \mathcal{D}_q, \forall p \neq q, p, q \in \mathcal{G}, \quad (3.12a)$$

$$[\Phi_r]_{m,n} = 0, \forall m \in \mathcal{D}_p, \forall n \in \mathcal{D}_q, \forall p \neq q, p, q \in \mathcal{G}, \quad (3.12b)$$

$$\Phi_{t,\mathcal{D}_g}^H \Phi_{t,\mathcal{D}_g} + \Phi_{r,\mathcal{D}_g}^H \Phi_{r,\mathcal{D}_g} = \mathbf{I}_{|\mathcal{D}_g|}, \forall g \in \mathcal{G}, \quad (3.12c)$$

<sup>5</sup>Herein we provide a simple example to explain Definition 1. For a 4-cell BD-RIS with 2 groups, 2 possible grouping results could be  $\mathcal{D}_1 = \{1, 3\}$ ,  $\mathcal{D}_2 = \{2, 4\}$  or  $\mathcal{D}_1 = \{2\}$ ,  $\mathcal{D}_2 = \{1, 3, 4\}$ .

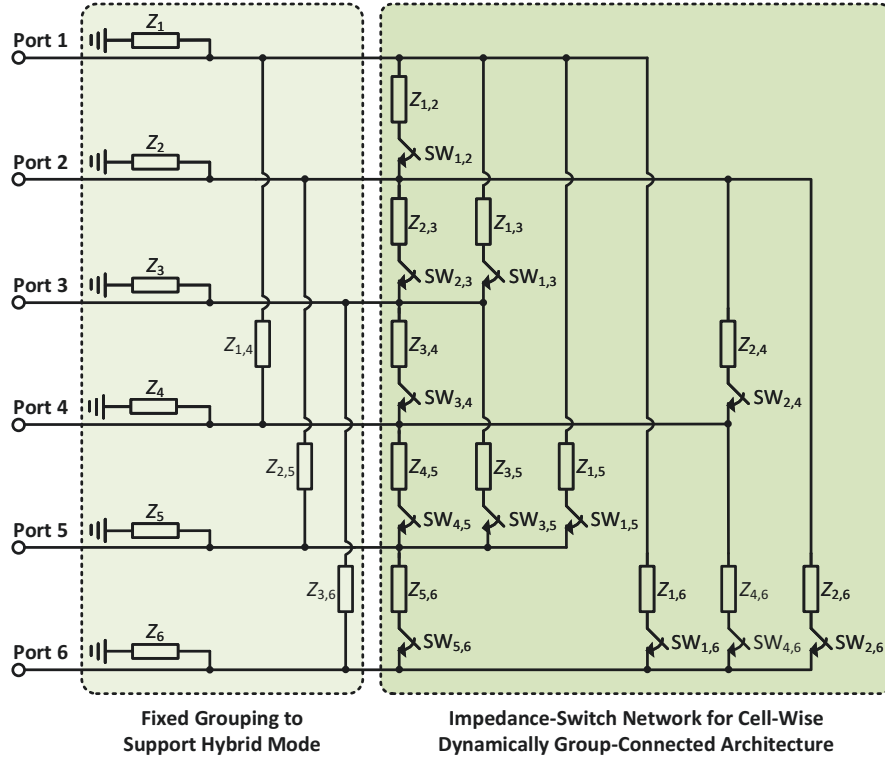


Figure 3.5: Example of a 3-cell BD-RIS with hybrid mode and CW-DGC architecture realized by an impedance-switch network.

where  $\Phi_{t/r, \mathcal{D}_g}$  is a sub-matrix of  $\Phi_{t/r}$  which selects rows and columns of  $\Phi_{t/r}$  according to indexes in the set  $\mathcal{D}_g, \forall g \in \mathcal{G}$ .

Constraint (3.12) indicates that  $\Phi_r$  and  $\Phi_t$  of the CW-DGC BD-RIS are essentially permuted block-diagonal matrices. Specifically, when the BD-RIS has a CW-GC architecture, the  $\bar{M}$  cells within a fixed group  $\mathcal{G}_g, \forall g \in \mathcal{G}$  are connected to each other. In this case, we have  $[\Phi_r]_{m,n} \neq 0, [\Phi_t]_{m,n} \neq 0, \forall m, n \in \mathcal{G}_g$ , and  $\Phi_r$  and  $\Phi_t$  are both block diagonal matrices.

**Implementation and Discussion:** To show the feasibility of the proposed architectures, we employ an impedance-switch network, which enables a joint control of impedance components and the ON/OFF states of switches. Specifically, impedance components can be realized by using inductance and tunable capacitance, such as using varactors. Fig. 3.5 provides an example of the realization of CW-DGC BD-RIS with  $M = 3$ . Specifically, ports  $m$  and  $M + m$  are connected to each other by an impedance to support the hybrid mode (as shown in the left-hand side of Fig. 3.5), while ports  $m$  and  $m', m' \neq M + m$ , are connected by the series of one impedance and one switch to support the dynamic grouping (as shown in the right-hand side of Fig. 3.5). Therefore, to realize an  $M$ -cell BD-RIS with hybrid mode and the CW-DGC architecture,  $M(2M + 1)$  impedance components and  $2M(M - 1)$  switches are required. To realize the dynamic grouping, when cells  $m$  and  $n, m < n$ , are in the same group, the  $(m, n)$ -th,  $(m, M + n)$ -th,  $(n, M + m)$ -th, and  $(M + m, M + n)$ -th switches are turned ON and the corresponding activated impedance components are tuned by the controller based on CSI,

Table 3.1: Twelve Cases of Beyond Diagonal RIS

| Architectures \ Modes | Reflective                                                                                                  | Transmissive                                                                                                | Hybrid                                                                                                                                                        |
|-----------------------|-------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|
| CW-SC                 | $ \phi_{r,m}  = 1, \forall m \in \mathcal{M}$                                                               | $ \phi_{t,m}  = 1, \forall m \in \mathcal{M}$                                                               | $ \phi_{r,m} ^2 +  \phi_{t,m} ^2 = 1, \forall m \in \mathcal{M}$                                                                                              |
| CW-GC                 | $\Phi_{r,g}^H \Phi_{r,g} = \mathbf{I}_{\bar{M}}, \forall g \in \mathcal{G}$                                 | $\Phi_{t,g}^H \Phi_{t,g} = \mathbf{I}_{\bar{M}}, \forall g \in \mathcal{G}$                                 | $\Phi_{r,g}^H \Phi_{r,g} + \Phi_{t,g}^H \Phi_{t,g} = \mathbf{I}_{\bar{M}}, \forall g \in \mathcal{G}$                                                         |
| CW-FC                 | $\Phi_r^H \Phi_r = \mathbf{I}_M$                                                                            | $\Phi_t^H \Phi_t = \mathbf{I}_M$                                                                            | $\Phi_r^H \Phi_r + \Phi_t^H \Phi_t = \mathbf{I}_M$                                                                                                            |
| CW-DGC                | $\Phi_{r,\mathcal{D}_g}^H \Phi_{r,\mathcal{D}_g} = \mathbf{I}_{ \mathcal{D}_g }, \forall g \in \mathcal{G}$ | $\Phi_{t,\mathcal{D}_g}^H \Phi_{t,\mathcal{D}_g} = \mathbf{I}_{ \mathcal{D}_g }, \forall g \in \mathcal{G}$ | $\Phi_{r,\mathcal{D}_g}^H \Phi_{r,\mathcal{D}_g} + \Phi_{t,\mathcal{D}_g}^H \Phi_{t,\mathcal{D}_g} = \mathbf{I}_{ \mathcal{D}_g }, \forall g \in \mathcal{G}$ |

which is obtained based on the channel estimation.

We should highlight that, although the number of connected links, referring to the number of connections between every two ports, of the impedance-switch network as illustrated in Fig. 3.5 scales with  $M^2$ , the activated links, referring to the connections where the switches are turned ON with given CSI and  $G$ , of the CW-DGC architecture could be much smaller than the total number of connected links. Based on the above implementation, with a given number of groups  $G$ , the number of activated links for CW-DGC BD-RIS is  $\sum_{g=1}^G |\mathcal{D}_g|(2|\mathcal{D}_g|+1)$ . Meanwhile, the impedance-switch network is a general framework, which enables CW-FC/GC/SC architectures. Specifically, the CW-FC architecture is realized when all switches are turned ON, while the CW-SC architecture is realized when all switches are turned OFF. This finding indicates that different architectures of BD-RIS could share the same realization, which is preferable for the implementation of BD-RIS in practice.

### 3.2.2 Mode Analysis

According to the proportion of energy split for reflection and transmission, the BD-RIS is able to realize the following three different modes [72]:

**Reflective Mode:** The BD-RIS only reflects signals towards the same side as the transmitter, i.e.,  $\Phi_t = \mathbf{0}_{M \times M}$ ,  $\Phi_r^H \Phi_r = \mathbf{I}_M$ .

**Transmissive Mode:** Incident signals penetrate the BD-RIS, i.e.,  $\Phi_r = \mathbf{0}_{M \times M}$ ,  $\Phi_t^H \Phi_t = \mathbf{I}_M$ .

**Hybrid Reflective and Transmissive Mode:** Incident signals can both reflect from and transmit through the BD-RIS, which leads to a dual function of reflection and transmission<sup>6</sup>, i.e.,  $\Phi_r \neq \mathbf{0}_{M \times M}$ ,  $\Phi_t \neq \mathbf{0}_{M \times M}$ ,  $\Phi_r^H \Phi_r + \Phi_t^H \Phi_t = \mathbf{I}_M$ .

### 3.2.3 Unified Architecture and Mode

Combining four different BD-RIS architectures and three modes, there are in total twelve cases. For clarity, the BD-RIS models for the twelve cases are summarized in Table 3.1. In [55], an

<sup>6</sup>Note that reflective/transmissive mode only is the special case of the hybrid transmissive and reflective mode, and the hybrid mode can efficiently utilize the resources.

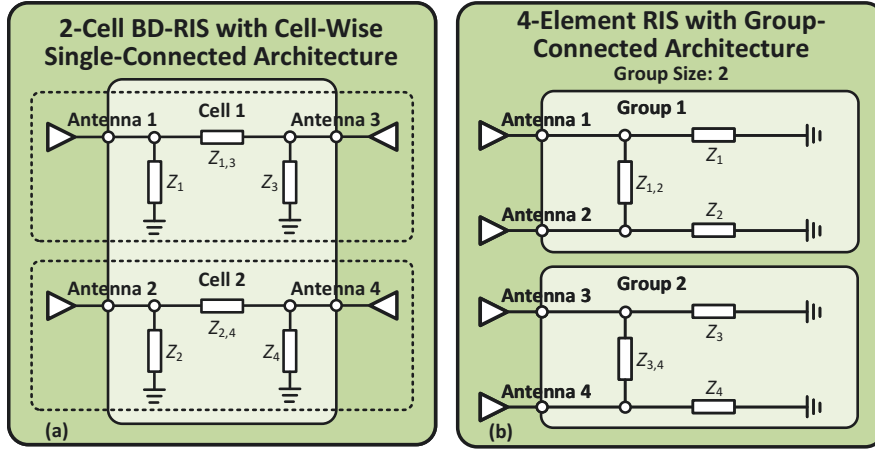


Figure 3.6: (a) 2-cell CW-SC BD-RIS and (b) 4-element group-connected RIS.

$M$ -element RIS is modeled as  $M$  antennas connected to an  $M$ -port reconfigurable impedance network. In this work, we further generalize the RIS model by modeling an  $M$ -cell RIS as  $2M$  antennas connected to a  $2M$ -port network. Namely, each RIS “element” in [55] contains a single antenna, while each RIS “cell” in this work contains two back to back placed unidirectional antennas, which can support reflective and transmissive modes. Particularly, when the RIS works in the reflective mode, our proposed model boils down to the model in [55], as shown in the first row (starting from “Reflective”) of Table 3.1. In this sense, prior works on conventional RIS [8–21] is a subset of group/fully-connected RIS [55] and also a special case of our model as summarized in Table 3.1. Meanwhile, STAR-RIS/IOS, which is essentially the CW-SC hybrid BD-RIS as shown in Table 3.1, is a particular instance of the proposed model when there are no inter-cell connections. More importantly, we go beyond the architecture of STAR-RIS/IOS and propose more general and flexible CW-GC/DGC/FC architectures. The proposed architectures are built on the multi-port reconfigurable impedance network [55], which cannot be derived based on the modeling of STAR-RIS/IOS [72–74] due to the lack of a rigorous analysis using network theory.

Given that the RIS can be generally modeled as multiple antennas connected to a multi-port reconfigurable impedance network, the circuit topology of the reconfigurable impedance network in STAR-RIS is essentially the same as that in the group-connected RIS with group size 2 [55]. To facilitate understanding, we provide a simple example in Fig. 3.6 to show the equivalence in terms of circuit topologies between the CW-SC 2-cell BD-RIS and group-connected 4-elements BD-RIS with group size 2. The difference is that all the antennas of the group-connected BD-RIS with group size 2 are placed in the same direction, while antennas of the CW-SC BD-RIS are divided into two sectors and back to back placed. Therefore, intra-cell connection together with the antenna arrangements supports “modes” while inter-cell connection refers to the “architectures” among different cells.

In the following section, we will focus on the beamforming design for an BD-RIS-aided wireless

communication system to investigate and compare the twelve cases of BD-RIS.

### 3.3 Beamforming Design

#### 3.3.1 Problem Formulation

We consider an RIS-aided MU-MISO system with an  $N$ -antenna base station (BS), an  $M$ -cell BD-RIS, and  $K$  single-antenna users (including  $K_r$  reflective users and  $K_t$  transmissive ones) based on Section 3.1. In this section, we have the following assumptions: 1) We assume direct links between the BS and users are blocked so that there are only BS-RIS-user links<sup>7</sup>. 2) We assume exact and instantaneous channel state information (CSI) is available at the BS, and our proposed design provides an upper bound on the performance of practical systems. Note that here we assume each user has one antenna to simplify the receiver design, while the following model can be readily generalized to scenarios where users have multiple antennas.

Let  $\mathbf{s} \triangleq [s_1, \dots, s_K]^T \in \mathbb{C}^{K \times 1}$  be the transmit symbol vector,  $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_K$ . Transmit symbols are first precoded at the BS by a precoder matrix  $\mathbf{W} \triangleq [\mathbf{w}_1, \dots, \mathbf{w}_K] \in \mathbb{C}^{N \times K}$ , where  $\mathbf{w}_k \in \mathbb{C}^{N \times 1}$  is the precoding vector for user  $k$ ,  $\forall k \in \mathcal{K}$ . Then they are up-converted to the radio frequency (RF) domain via  $N$  RF chains. After propagating through the RIS-aided channels, signals are corrupted by additive Gaussian white noise (AGWN). The received signal for each user is

$$y_k = \mathbf{h}_k \Phi_i \mathbf{G} \mathbf{w}_k s_k + \mathbf{h}_k \Phi_i \mathbf{G} \sum_{p \in \mathcal{K}, p \neq k} \mathbf{w}_p s_p + n_k, \forall i \in \{t, r\}, \forall k \in \mathcal{K}_i, \quad (3.13)$$

where  $\mathbf{h}_k \in \mathbb{C}^{1 \times M}$  denotes the channel vector between the RIS and user  $k$ , and  $n_k \sim \mathcal{CN}(0, \sigma_k^2)$  denotes the AGWN,  $\forall k \in \mathcal{K}$ .

Define  $\tilde{\mathbf{h}}_k \triangleq \mathbf{h}_k \Phi_i \mathbf{G}$ ,  $\forall i \in \{t, r\}$ ,  $\forall k \in \mathcal{K}_i$ . Then the signal-to-interference-plus-noise ratio (SINR) for each user can be calculated as

$$\gamma_k = \frac{|\tilde{\mathbf{h}}_k \mathbf{w}_k|^2}{\sum_{p \in \mathcal{K}, p \neq k} |\tilde{\mathbf{h}}_k \mathbf{w}_p|^2 + \sigma_k^2}, \forall i \in \{t, r\}, \forall k \in \mathcal{K}_i. \quad (3.14)$$

Our goal is to jointly design the transmit precoder and BD-RIS matrix to maximize the sum-rate for the MU-MISO system, subject to power constraint and the BD-RIS constraint shown

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<sup>7</sup>Herein we ignore direct links for simplicity. However, our proposed algorithms still work if the direct link exists.

in Table 3.1. Therefore, the problem is formulated as

$$\max_{\mathbf{W}, \Phi_t, \Phi_r} F_o(\mathbf{W}, \Phi_t, \Phi_r) = \sum_{k \in \mathcal{K}} \log_2(1 + \gamma_k) \quad (3.15a)$$

$$\text{s.t.} \quad \Phi_t \text{ and } \Phi_r \text{ satisfy Table 3.1,} \quad (3.15b)$$

$$\|\mathbf{W}\|_F^2 \leq P, \quad (3.15c)$$

where  $P$  is the total transmit power at the BS. Problem (3.15) is difficult to solve due to the complex form of the objective and the non-convex constraints of the BD-RIS. To tackle this difficulty, in the following subsections, we attempt to first transform problem (3.15) into a more tractable multi-variable/block optimization based on fractional programming theory reviewed in Section 2.2.1 and then iteratively cope with each block.

### 3.3.2 Joint Design Framework

We start by taking  $\gamma_k, \forall k \in \mathcal{K}$  out of the  $\log(\cdot)$  function in  $F_o(\mathbf{W}, \Phi_t, \Phi_r)$ . Based on the *Lagrangian Dual Transform* [83], [84],  $F_o(\mathbf{W}, \Phi_t, \Phi_r)$  can be equivalently transformed into:

$$F_\iota(\mathbf{W}, \Phi_t, \Phi_r, \boldsymbol{\iota}) = \sum_{k \in \mathcal{K}} \left( \log_2(1 + \iota_k) - \iota_k + \frac{(1 + \iota_k)|\tilde{\mathbf{h}}_k \mathbf{w}_k|^2}{\sum_{p \in \mathcal{K}} |\tilde{\mathbf{h}}_k \mathbf{w}_p|^2 + \sigma_k^2} \right), \quad (3.16)$$

where  $\boldsymbol{\iota} \triangleq [\iota_1, \dots, \iota_K]^T \in \mathbb{R}^{K \times 1}$  is an auxiliary vector. It can be seen from (3.16) that we remove the original fractional terms in the  $\log(\cdot)$  function of  $F_o(\mathbf{W}, \Phi_t, \Phi_r)$ , but introduce a summation of new fractional terms. The main difference is that these new fractional terms are independent of the  $\log(\cdot)$  function and thus more tractable. Then we can apply Corollary 2.2 to transform these fractional parts into integral expressions and reformulate the objective function  $F_\iota(\mathbf{W}, \Phi_t, \Phi_r, \boldsymbol{\iota})$  as:

$$\begin{aligned} F_\tau(\mathbf{W}, \Phi_t, \Phi_r, \boldsymbol{\iota}, \boldsymbol{\tau}) = & \sum_{k \in \mathcal{K}} \left( \log_2(1 + \iota_k) - \iota_k + 2\sqrt{1 + \iota_k} \Re\{\tau_k^* \tilde{\mathbf{h}}_k \mathbf{w}_k\} \right. \\ & \left. - |\tau_k|^2 \left( \sum_{p \in \mathcal{K}} |\tilde{\mathbf{h}}_k \mathbf{w}_p|^2 + \sigma_k^2 \right) \right), \end{aligned} \quad (3.17)$$

where  $\boldsymbol{\tau} \triangleq [\tau_1, \dots, \tau_K]^T \in \mathbb{C}^{K \times 1}$  denotes another auxiliary vector.

Now with newly introduced two auxiliary vectors  $\boldsymbol{\iota}$  and  $\boldsymbol{\tau}$  to tackle with the original fractional terms, the original problem (3.15) can be transformed into the following typical multi-

**Algorithm 1** Joint Transmit Precoder and BD-RIS Design**Require:**  $\mathbf{h}_{i,k}$ ,  $\forall i \in \{t, r\}$ ,  $\forall k \in \mathcal{K}_i$ ,  $\mathbf{G}$ ,  $P$ .**Ensure:**  $\Phi_t^*$ ,  $\Phi_r^*$ ,  $\mathbf{W}^*$ .

- 1: Initialize  $\Phi_t$ ,  $\Phi_r$ ,  $\mathbf{W}$ ,  $\mathcal{D}_1, \dots, \mathcal{D}_G$ .
- 2: **while** no convergence of objective (3.15a) **do**
- 3:   Update  $\boldsymbol{\iota}^*$  by (3.19).
- 4:   Update  $\boldsymbol{\tau}^*$  by (3.20).
- 5:   Update  $\mathbf{W}^*$  by (3.22).
- 6:   Update  $\Phi_{t/r}^*$  by solving problem (3.23).
- 7: **end while**
- 8: Return  $\Phi_{t/r}^*$ ,  $\mathbf{W}^*$ .

variable/block problem:

$$\max_{\mathbf{W}, \Phi_t, \Phi_r, \boldsymbol{\iota}, \boldsymbol{\tau}} F_{\boldsymbol{\tau}}(\mathbf{W}, \Phi_t, \Phi_r, \boldsymbol{\iota}, \boldsymbol{\tau}) \quad (3.18a)$$

$$\text{s.t.} \quad (3.15b), (3.15c). \quad (3.18b)$$

A well-known direction to solve problem (3.18) is to apply the block coordinate descent (BCD) iterative algorithms [93]. Given appropriate initial values of  $\mathbf{W}$  and  $\Phi_{t/r}$ , we iteratively update the above blocks until convergence. The proposed joint design framework is summarized in Algorithm 1. We will next decompose problem (3.18) into several sub-problems in an iterative manner and discuss the solution for each block in detail. Specifically, solutions to two auxiliary vectors, i.e., blocks  $\boldsymbol{\iota}$  and  $\boldsymbol{\tau}$ , will be presented in (3.19) and (3.20). Then, the solution to the transmit precoder, i.e., block  $\mathbf{W}$ , will be provided in (3.22). Finally, three solutions to the BD-RIS beamformer, i.e., block  $\{\Phi_t, \Phi_r\}$ , will be proposed.

**Solution to Auxiliary Vectors: Blocks  $\boldsymbol{\iota}$  and  $\boldsymbol{\tau}$ .** When  $\mathbf{W}$ ,  $\Phi_t$ ,  $\Phi_r$ , and  $\boldsymbol{\tau}$  (or  $\boldsymbol{\iota}$ ) are fixed, the sub-problem with respect to  $\boldsymbol{\iota}$  (or  $\boldsymbol{\tau}$ ) is an unconstrained convex optimization, whose solution can be easily derived by setting  $\frac{\partial F_{\boldsymbol{\tau}}(\mathbf{W}, \Phi_t, \Phi_r, \boldsymbol{\iota}, \boldsymbol{\tau})}{\partial \boldsymbol{\iota}} = \mathbf{0}$  (or  $\frac{\partial F_{\boldsymbol{\tau}}(\mathbf{W}, \Phi_t, \Phi_r, \boldsymbol{\iota}, \boldsymbol{\tau})}{\partial \boldsymbol{\tau}} = \mathbf{0}$ ). Then we can obtain the following optimal solutions for each auxiliary variables  $\iota_k$  and  $\tau_k$  as:

$$\iota_k^* = \gamma_k, \forall k \in \mathcal{K}, \quad (3.19)$$

$$\tau_k^* = \frac{\sqrt{1 + \iota_k^*} \tilde{\mathbf{h}}_k \mathbf{w}_k}{\sum_{p \in \mathcal{K}} |\tilde{\mathbf{h}}_k \mathbf{w}_p|^2 + \sigma_k^2}, \forall k \in \mathcal{K}. \quad (3.20)$$

**Solution to the Transmit Precoder: Block  $\mathbf{W}$ .** With given  $\Phi_t$ ,  $\Phi_r$ ,  $\boldsymbol{\tau}$ , and  $\boldsymbol{\iota}$ , the sub-problem with respect to  $\mathbf{W}$  is given by:

$$\max_{\mathbf{W}} \sum_{k \in \mathcal{K}} \left( 2\sqrt{1 + \iota_k} \Re\{\tilde{\mathbf{h}}_k \mathbf{w}_k\} - \mathbf{w}_k^H \left( \sum_{p \in \mathcal{K}} \tilde{\mathbf{h}}_p^H \tilde{\mathbf{h}}_p \right) \mathbf{w}_k \right) \quad (3.21a)$$

$$\text{s.t.} \quad \|\mathbf{W}\|_F^2 \leq P, \quad (3.21b)$$

where  $\ddot{\mathbf{h}}_k \triangleq \tau_k^* \tilde{\mathbf{h}}_k, \forall k \in \mathcal{K}$ . Since both the objective function and the constraint of problem (3.21) are convex, we can check the KKT conditions as illustrated in Section 2.2.2 to find its optimal solution. To do so, we adopt the Lagrange multiplier method and introduce a multiplier  $\lambda \geq 0$  for the power constraint (3.21b). Then by checking the first-order optimum condition, we can obtain the optimal solution of precoder  $\mathbf{W}$ :

$$\mathbf{w}_k^* = \left( \sum_{p \in \mathcal{K}} \ddot{\mathbf{h}}_p^H \ddot{\mathbf{h}}_p + \lambda^* \mathbf{I}_N \right)^{-1} \times \sqrt{1 + \iota_k} \ddot{\mathbf{h}}_k^H, \forall k \in \mathcal{K}, \quad (3.22)$$

where  $\lambda^*$  can be obtained by a simple bisection search.

**Solution to BD-RIS Matrix: Block  $\{\Phi_t, \Phi_r\}$ .** When  $\mathbf{W}$ ,  $\boldsymbol{\tau}$ , and  $\boldsymbol{\iota}$  are determined, the sub-problem with respect to  $\Phi_t$  and  $\Phi_r$  is written as

$$\max_{\Phi_t, \Phi_r} \sum_{i \in \{t, r\}} \sum_{k \in \mathcal{K}_i} \left( 2\Re\{\tilde{\tau}_k^* \mathbf{h}_k \Phi_i \mathbf{g}_k\} - |\tau_k|^2 \sum_{p \in \mathcal{K}} |\mathbf{h}_k \Phi_i \mathbf{g}_p|^2 \right) \quad (3.23a)$$

$$\text{s.t. } \Phi_t \text{ and } \Phi_r \text{ satisfy Table 3.1,} \quad (3.23b)$$

where  $\tilde{\tau}_k = \sqrt{1 + \iota_k} \tau_k$ ,  $\mathbf{g}_k \triangleq \mathbf{G} \mathbf{w}_k, \forall k \in \mathcal{K}$ . It can be easily observed from Table 3.1 that both CW-SC and CW-FC architectures are special cases of the CW-GC architecture. Therefore, we will first provide a general solution for the CW-GC case, which can easily boil down to other two cases, and then propose an efficient algorithm for the CW-SC case to further reduce the complexity. Finally, we will propose a heuristic solution for the CW-DGC case.

1) *A General Solution for CW-GC BD-RIS:* Problem (3.23) can be written as

$$\max_{\Phi_t, \Phi_r} \sum_{i \in \{t, r\}} \left( 2\Re\{\text{Tr}(\Phi_i \mathbf{X}_i)\} - \text{Tr}(\Phi_i \mathbf{Y} \Phi_i^H \mathbf{Z}_i) \right) \quad (3.24a)$$

$$\text{s.t. } \Phi_{t,g}^H \Phi_{t,g} + \Phi_{r,g}^H \Phi_{r,g} = \mathbf{I}_{\bar{M}}, \forall g \in \mathcal{G}, \quad (3.24b)$$

$$\Phi_i = \text{blkdiag}(\Phi_{i,1}, \dots, \Phi_{i,G}), \forall i \in \{t, r\}, \quad (3.24c)$$

where  $\mathbf{X}_i \in \mathbb{C}^{M \times M}$ ,  $\mathbf{Y} \in \mathbb{C}^{M \times M}$ , and  $\mathbf{Z}_i \in \mathbb{C}^{M \times M}$  are respectively defined as  $\mathbf{X}_i \triangleq \sum_{k \in \mathcal{K}_i} \tilde{\tau}_k^* \mathbf{g}_k \mathbf{h}_k$ ,  $\mathbf{Y} \triangleq \sum_{p \in \mathcal{K}} \mathbf{g}_p \mathbf{g}_p^H$ , and  $\mathbf{Z}_i \triangleq \sum_{k \in \mathcal{K}_i} |\tau_k|^2 \mathbf{h}_k^H \mathbf{h}_k, \forall i \in \{t, r\}$ . It is worth noting that there are quadratic terms in problem (3.24), which makes different groups of  $\Phi_{t/r,g}$  mingle with each other. To efficiently solve this problem, we attempt to split out one pair  $\Phi_{t,g}$  and  $\Phi_{r,g}$  for group  $g$  from the objective function (3.24a) with fixed other pairs. This leads to

$$\begin{aligned} & \sum_{i \in \{t, r\}} \left( 2\Re\{\text{Tr}(\Phi_i \mathbf{X}_i)\} - \text{Tr}(\Phi_i \mathbf{Y} \Phi_i^H \mathbf{Z}_i) \right) \\ &= \sum_{i \in \{t, r\}} \left( 2 \sum_{q=1}^G \Re\{\text{Tr}(\Phi_{i,q} \mathbf{X}_{i,q})\} - \text{Tr} \left( \sum_{p=1}^G \Phi_{i,p} \sum_{q=1}^G \mathbf{Y}_{p,q} \Phi_{i,q}^H \mathbf{Z}_{i,q,p} \right) \right), \end{aligned} \quad (3.25)$$

where  $\mathbf{X}_{i,q} = [\mathbf{X}_i]_{(q-1)\bar{M}+1:q\bar{M},(q-1)\bar{M}+1:q\bar{M}} \in \mathbb{C}^{\bar{M} \times \bar{M}}$ ,  $\mathbf{Y}_{p,q} = [\mathbf{Y}]_{(p-1)\bar{M}+1:p\bar{M},(q-1)\bar{M}+1:q\bar{M}} \in \mathbb{C}^{\bar{M} \times \bar{M}}$ , and  $\mathbf{Z}_{i,q,p} = [\mathbf{Z}_i]_{(q-1)\bar{M}+1:q\bar{M},(p-1)\bar{M}+1:p\bar{M}} \in \mathbb{C}^{\bar{M} \times \bar{M}}$ ,  $\forall p, q \in \mathcal{G}$ . Then, when we focus on one pair  $\Phi_{t,g}$  and  $\Phi_{r,g}$  with fixed other pairs, the corresponding sub-objective function is:

$$F_g(\Phi_{t,g}, \Phi_{r,g}) = \sum_{i \in \{t,r\}} \left( \text{Tr}(\Phi_{i,g} \mathbf{Y}_{g,g} \Phi_{i,g}^H \mathbf{Z}_{i,g,g}) - 2\Re \left\{ \text{Tr} \left( \Phi_{i,g} \underbrace{\left( \mathbf{X}_{i,g} - \sum_{p \neq g} \mathbf{Y}_{g,p} \Phi_{i,p}^H \mathbf{Z}_{i,p,g} \right)}_{=\tilde{\mathbf{X}}_{i,g}} \right) \right\} \right). \quad (3.26)$$

Define  $\Phi_g \triangleq [\Phi_{t,g}^H, \Phi_{r,g}^H]^H \in \mathbb{C}^{2\bar{M} \times \bar{M}}$ ,  $\tilde{\mathbf{X}}_g \triangleq [\tilde{\mathbf{X}}_{t,g}, \tilde{\mathbf{X}}_{r,g}] \in \mathbb{C}^{\bar{M} \times 2\bar{M}}$ ,  $\mathbf{Z}_g \triangleq \text{blkdiag}(\mathbf{Z}_{t,g,g}, \mathbf{Z}_{r,g,g}) \in \mathbb{C}^{2\bar{M} \times 2\bar{M}}$ . Then the sub-problem with respect to  $\Phi_g$  is:

$$\min_{\Phi_g} \tilde{F}_g(\Phi_g) = \text{Tr}(\Phi_g \mathbf{Y}_{g,g} \Phi_g^H \mathbf{Z}_g - 2\Re\{\Phi_g \tilde{\mathbf{X}}_g\}) \quad (3.27a)$$

$$\text{s.t. } \Phi_g^H \Phi_g = \mathbf{I}_{\bar{M}}, \forall g \in \mathcal{G}. \quad (3.27b)$$

The main challenge of solving problem (3.27) is the unitary constraint (3.27b). One possible solution is to do some relaxations and solve the relaxed problem. However, it is inevitable that there will be performance loss since the result is not based on the original problem. Moreover, when the value of  $\Phi_g$  is not that “good”, the convergence of the overall BCD procedure might not be guaranteed. In order to get a “good” solution of problem (3.27), we adopt a manifold algorithm [87] as reviewed in Section 2.2.3. In the following we will introduce the procedure of the manifold algorithm by applying the Conjugate-Gradient method.

*Manifold Construction:* Constraint (3.27b) forms a  $2\bar{M}^2$ -dimensional complex Stiefel manifold [88], as explained in Definition 2.1, i.e.,

$$\mathcal{M}_{\text{cst},g} = \{\Phi_g \in \mathbb{C}^{2\bar{M} \times \bar{M}} : \Phi_g^H \Phi_g = \mathbf{I}_{\bar{M}}, \forall g \in \mathcal{G}, \quad (3.28)$$

and then problem (3.27) becomes an unconstrained optimization on the Stiefel manifold, i.e.,

$$\Phi_g^* = \arg \min_{\Phi_g \in \mathcal{M}_{\text{cst},g}} \tilde{F}_g(\Phi_g), \forall g \in \mathcal{G}. \quad (3.29)$$

The tangent space of the manifold (3.28) at point  $\Phi_g$ , as explained in Definition 2.2, is thus given as

$$\mathbf{T}_{\Phi_g} \mathcal{M}_{\text{cst},g} = \{\mathbf{T}_g \in \mathbb{C}^{2\bar{M} \times \bar{M}} : \Re\{\Phi_g^H \mathbf{T}_g\} = \mathbf{0}_{\bar{M} \times \bar{M}}\}, \forall g \in \mathcal{G}. \quad (3.30)$$

*Riemannian Gradient:* In the Conjugate-Gradient method, the Euclidean gradient is required to calculate the Riemannian gradient as explained in Definition 2.3. Therefore, we first calculate the Euclidean gradient of  $\tilde{F}_g(\Phi_g)$  as

$$\nabla \tilde{F}_g(\Phi_g) = 2\mathbf{Z}_g \Phi_g \mathbf{Y}_g - 2\tilde{\mathbf{X}}_g^H, \forall g \in \mathcal{G}. \quad (3.31)$$

Then the Riemannian gradient can be calculated by projecting the Euclidean gradient onto the tangent space [87]:

$$\nabla_{\mathcal{M}_{\text{cst},g}} \tilde{F}_g(\Phi_g) = \text{Pr}_{\Phi_g}(\nabla \tilde{F}_g(\Phi_g)) = \nabla \tilde{F}_g(\Phi_g) - \Phi_g \text{chdiag}(\Phi_g^H \nabla \tilde{F}_g(\Phi_g)), \forall g \in \mathcal{G}, \quad (3.32)$$

where  $\text{Pr}_{\Phi_g}(\cdot)$  denotes the projection function, and  $\text{chdiag}(\cdot)$  chooses all diagonal elements of a matrix to construct a diagonal matrix. Now we can apply the Conjugate-Gradient method. At the  $o$ -th iteration of the Conjugate-Gradient method, we successively do the following steps:

Step 1: Find the descent direction:

$$\Xi_g^o = -\nabla_{\mathcal{M}_{\text{cst},g}} \tilde{F}_g(\Phi_g^o) + \mu_g^o \text{Pr}_{\Phi_g^o}(\Xi_g^{o-1}), \forall g \in \mathcal{G}, \quad (3.33)$$

where  $\mu_g^o$  denotes the Conjugate-Gradient update parameter. There are many choices for updating this parameter, e.g., Fletcher-Reeves formula, Polak-Ribière formula, and Hestenes-Stiefel formula [89]. Here we adopt a Riemannian version of the Polak-Ribière formula, which is given by

$$\mu_g^o = \frac{\Re \left\{ \text{Tr} \left( [\nabla_{\mathcal{M}_{\text{cst},g}} \tilde{F}_g(\Phi_g^o)]^H [\nabla_{\mathcal{M}_{\text{cst},g}} \tilde{F}_g(\Phi_g^o) - \text{Pr}_{\Phi_g^o}(\nabla_{\mathcal{M}_{\text{cst},g}} \tilde{F}_g(\Phi_g^{o-1}))] \right) \right\}}{\text{Tr} \left( [\nabla_{\mathcal{M}_{\text{cst},g}} \tilde{F}_g(\Phi_g^{o-1})]^H \nabla_{\mathcal{M}_{\text{cst},g}} \tilde{F}_g(\Phi_g^{o-1}) \right)}, \forall g \in \mathcal{G}. \quad (3.34)$$

Step 2: Perform a retraction [87]:

$$\Phi_g^{v+1} = \text{Retr}_{\Phi_g^o}(\delta_g^o \Xi_g^o) = (\Phi_g^v + \delta_g^o \Xi_g^o)(\mathbf{I}_{\bar{M}} + (\delta_g^o)^2 (\Xi_g^o)^H \Xi_g^o)^{-1/2}, \forall g \in \mathcal{G}, \quad (3.35)$$

where  $\delta_g^o$  is the step size and can be searched by backtracking algorithms [87]. In addition,  $\text{Retr}_{\Phi_g^o}(\cdot)$  denotes the retraction function which maps a point in the tangent space into the manifold.

Now with proper initial values, the (at least local) optimal  $\Phi_g^*$ ,  $\forall g$  can be obtained by iteratively updating  $\delta_g^{o-1}$ ,  $\Phi_g^o$ ,  $\mu_g^o$ , and  $\Xi_g^o$  until convergence. After solving problem (3.27) for all groups, the optimal reflective and transmissive matrices for each group can be split from  $\Phi_g^*$ , i.e.,

$$\Phi_{t,g}^* = [\Phi_g^*]_{1:\bar{M},:}, \quad \Phi_{r,g}^* = [\Phi_g^*]_{\bar{M}+1:2\bar{M},:}, \forall g \in \mathcal{G}. \quad (3.36)$$

The procedure of the above BD-RIS matrix design is summarized in Algorithm 2. This design is also suitable for BD-RIS with CW-FC and CW-SC architectures.

Case 1: For the CW-FC case ( $G = 1$ ), problem (3.23) can be rewritten as the following form:

$$\min_{\bar{\Phi}} \quad \text{Tr}(\bar{\Phi} \mathbf{Y} \bar{\Phi}^H \mathbf{Z}) - 2\Re\{\text{Tr}(\bar{\Phi} \mathbf{X})\} \quad \text{s.t.} \quad \bar{\Phi}^H \bar{\Phi} = \mathbf{I}_M, \quad (3.37)$$

**Algorithm 2** General Solution for CW-GC BD-RIS**Require:**  $\mathbf{h}_{i,k}, \forall i \in \{t, r\}, \forall k \in \mathcal{K}_i, \mathbf{G}, \boldsymbol{\iota}, \boldsymbol{\tau}, \mathbf{W}, \boldsymbol{\Phi}_t, \boldsymbol{\Phi}_r, G$ .**Ensure:**  $\boldsymbol{\Phi}_t^*, \boldsymbol{\Phi}_r^*$ .

- 1: Calculate  $\mathbf{X}_{t/r}, \mathbf{Y}, \mathbf{Z}_{t/r}$ .
- 2: Initialize  $\boldsymbol{\Phi}^0 = [\boldsymbol{\Phi}_t^H, \boldsymbol{\Phi}_r^H]^H$ .
- 3: Initialize  $\boldsymbol{\Xi}_g^0 = -\nabla_{\mathcal{M}_g} f_g(\boldsymbol{\Phi}_g^0), \forall g \in \mathcal{G}$ .
- 4: **while** no convergence of  $\boldsymbol{\Phi}_t, \boldsymbol{\Phi}_r$  **do**
- 5:   **for**  $g = 1 : G$  **do**
- 6:     Set  $v = 0$ .
- 7:     **while** no convergence of  $\|\nabla_{\mathcal{M}_g} \tilde{f}_g(\boldsymbol{\Phi}_g^v)\|_F$  **do**
- 8:        $v = v + 1$ .
- 9:       Calculate  $\delta_g^{v-1}$  by backtracking algorithms [87].
- 10:       Update  $\boldsymbol{\Phi}_g^v$  by (3.35).
- 11:       Update  $\mu_g^v$  by (3.34).
- 12:       Update  $\boldsymbol{\Xi}_g^v$  by (3.33).
- 13:     **end while**
- 14:     Obtain  $\boldsymbol{\Phi}_g^* = \boldsymbol{\Phi}_g^{v+1}$  and  $\boldsymbol{\Phi}_{t/r,g}^*$  by (3.36).
- 15:   **end for**
- 16: **end while**
- 17: Return  $\boldsymbol{\Phi}_{t/r}^*$  by (3.9).

with  $\bar{\boldsymbol{\Phi}} \triangleq [\boldsymbol{\Phi}_t^H, \boldsymbol{\Phi}_r^H]^H \in \mathbb{C}^{2M \times M}$ ,  $\mathbf{X} \triangleq [\mathbf{X}_t, \mathbf{X}_r] \in \mathbb{C}^{M \times 2M}$ ,  $\mathbf{Z} \triangleq \text{blkdiag}(\mathbf{Z}_t, \mathbf{Z}_r) \in \mathbb{C}^{2M \times 2M}$ , which can be solved by steps 6-15 in Algorithm 2.

Case 2: CW-SC BD-RIS matrix can be directly determined by Algorithm 2 with  $G = M$ . However, CW-SC RIS has a friendly characteristic, i.e.,  $\boldsymbol{\Phi}_{t/r}$  are diagonal matrices, which can facilitate the design and thus motivates us to develop a more efficient algorithm.

2) *An Efficient Solution for CW-SC BD-RIS:* In this case, each non-zero element of  $\boldsymbol{\Phi}_{t/r}$  can be modeled as  $\phi_{t/r,m} = \sqrt{\alpha_{t/r,m}} e^{j\theta_{t/r,m}}$ ,  $\theta_{t/r,m} \in [0, 2\pi)$ ,  $\alpha_{t,m} + \alpha_{r,m} = 1, \forall m \in \mathcal{M}$  [73]. Define  $\boldsymbol{\phi}_i \triangleq [\phi_{i,1}, \dots, \phi_{i,M}]^T$ , and  $\mathbf{v}_{k,p} \triangleq (\mathbf{h}_k \text{diag}(\mathbf{g}_p))^H, \forall p \in \mathcal{K}, \forall k \in \mathcal{K}_i, \forall i \in \{t, r\}$ . Problem (3.23) can be rewritten as

$$\max_{\boldsymbol{\phi}_t, \boldsymbol{\phi}_r} \sum_{i \in \{t, r\}} \left( 2\Re\{\tilde{\mathbf{v}}_i^H \boldsymbol{\phi}_i\} - \boldsymbol{\phi}_i^H \mathbf{V}_i \boldsymbol{\phi}_i \right) \quad \text{s.t.} \quad |\phi_{t,m}|^2 + |\phi_{r,m}|^2 = 1, \forall m \in \mathcal{M}, \quad (3.38)$$

with  $\mathbf{V}_i \triangleq \sum_{k \in \mathcal{K}_i} |\tau_k|^2 \sum_{p \in \mathcal{K}} \mathbf{v}_{k,p} \mathbf{v}_{k,p}^H$ , and  $\tilde{\mathbf{v}}_i \triangleq \sum_{k \in \mathcal{K}_i} \sqrt{1 + \iota_k} \mathbf{v}_{k,i} \tau_k, \forall i \in \{t, r\}$ . We split the objective and focus on the design for one pair, i.e.,  $\phi_{t,m}, \phi_{r,m}$ , while fixing the others. Therefore, the sub-objective for  $\phi_{t,m}$  and  $\phi_{r,m}$  is given by

$$\sum_{i \in \{t, r\}} \left( [\mathbf{V}_i]_{m,m} |\phi_{i,m}|^2 + 2\Re\left\{ \underbrace{\left( \sum_{n \neq m} [\mathbf{V}_i]_{m,n} \phi_{i,n} - [\tilde{\mathbf{v}}_i]_m \right) \phi_{i,m}^*}_{=\chi_{i,m}} \right\} \right). \quad (3.39)$$

Recall  $\phi_{i,m} = \sqrt{\alpha_{i,m}}e^{j\theta_{i,m}}$ ,  $\forall i \in \{t, r\}$ ,  $\forall m \in \mathcal{M}$ , the sub-problem for  $\phi_{t,m}$ , and  $\phi_{r,m}$  is

$$\min_{\substack{\alpha_{t,m}, \alpha_{r,m} \\ \theta_{t,m}, \theta_{r,m}}} \sum_{i \in \{t, r\}} \left( [\mathbf{V}_i]_{m,m} \alpha_{i,m} + 2|\chi_{i,m}| \sqrt{\alpha_{i,m}} \cos(\angle \chi_{i,m} - \theta_{i,m}) \right) \quad (3.40a)$$

$$\text{s.t.} \quad \alpha_{t,m} + \alpha_{r,m} = 1, \quad (3.40b)$$

$$\theta_{t,m} \in [0, 2\pi), \theta_{r,m} \in [0, 2\pi). \quad (3.40c)$$

*Phase Shift:* From problem (3.40) we can observe that the phase shifts  $\theta_{t/r,m}$  and amplitudes  $\alpha_{t/r,m}$  can be separately determined. Therefore, we first directly let the term  $\cos(\angle \chi_{i,m} - \theta_{i,m}) = -1$  to minimize the objective function. Then we can determine the optimal phase shifts:

$$\theta_{i,m}^* = \begin{cases} \angle \chi_{i,m} + \pi, & \angle \chi_{i,m} \in [0, \pi), \\ \angle \chi_{i,m} - \pi, & \angle \chi_{i,m} \in [\pi, 2\pi), \end{cases} \quad \forall i \in \{t, r\}, \forall m \in \mathcal{M}. \quad (3.41)$$

*Amplitude:* With optimal phase shifts, objective (3.40a) becomes a real-value optimization and problem (3.40) can be formulated as a function of  $\alpha_{t,m}$ :

$$\min_{\alpha_{t,m}} v_m \alpha_{t,m} - 2|\chi_{t,m}| \sqrt{\alpha_{t,m}} - 2|\chi_{r,m}| \sqrt{1 - \alpha_{t,m}} \quad \text{s.t.} \quad \alpha_{t,m} \in (0, 1), \quad (3.42)$$

where  $v_m = [\mathbf{V}_t]_{m,m} - [\mathbf{V}_r]_{m,m}$ ,  $\forall m \in \mathcal{M}$ .

**Proposition 3.1.** *The objective function in (3.42) is a convex function within  $\alpha_{t,m} \in (0, 1)$ .*

*Proof.* Please refer to Appendix A. □

Proposition 3.1 indicates that the minimum of (3.42) is achieved at the minimum point, which can be efficiently found by one-dimensional search methods, e.g., golden-section search.

After solving problem (3.42) and getting the optimal amplitude  $\alpha_{t,m}^*$ , we can obtain  $\phi_{t/r,m}^*$  as:

$$\phi_{t,m}^* = \sqrt{\alpha_{t,m}^*} e^{j\theta_{t,m}^*}, \quad \phi_{r,m}^* = \sqrt{1 - \alpha_{t,m}^*} e^{j\theta_{r,m}^*}, \quad \forall m \in \mathcal{M}. \quad (3.43)$$

The details of the single-connected RIS beamforming design are summarized in Algorithm 3.

3) *A Heuristic Solution for CW-DGC BD-RIS:* In this case, the objective function with respect

**Algorithm 3** Efficient Solution for CW-SC BD-RIS**Require:**  $\mathbf{h}_{i,k}$ ,  $\forall i \in \{t, r\}$ ,  $\forall k \in \mathcal{K}_i$ ,  $\mathbf{G}$ ,  $\boldsymbol{\iota}$ ,  $\boldsymbol{\tau}$ ,  $\mathbf{W}$ ,  $\Phi_t$ ,  $\Phi_r$ .**Ensure:**  $\Phi_t^*$ ,  $\Phi_r^*$ .

- 1: Calculate  $\mathbf{V}_{t/r}$  and  $\tilde{\mathbf{v}}_{t/r}$ .
- 2: **while** no convergence of  $\Phi_t$ ,  $\Phi_r$  **do**
- 3:   **for**  $m = 1 : M$  **do**
- 4:     Update  $\theta_{t/r,m}^*$  by (3.41).
- 5:     Update  $\alpha_{t,m}^*$  by golden-section search.
- 6:     Calculate  $\phi_{t/r,m}^*$  by (3.43).
- 7:   **end for**
- 8: **end while**
- 9: Return  $\Phi_{t/r}^* = \text{diag}(\phi_{t/r,M}^*, \dots, \phi_{t/r,1}^*)$ .

to  $\Phi_t$  and  $\Phi_r$  is

$$\sum_{i \in \{t, r\}} \left( \text{Tr}(\Phi_i \mathbf{Y} \Phi_i^H \mathbf{Z}_i) - 2\Re\{\text{Tr}(\Phi_i \mathbf{X}_i)\} \right) \quad (3.44a)$$

$$\stackrel{(a)}{=} \sum_{i \in \{t, r\}} \sum_{g \in \mathcal{G}} \left( \text{Tr}(\Phi_{i, \mathcal{D}_g} \sum_{p \in \mathcal{G}} \mathbf{Y}_{\mathcal{D}_g, \mathcal{D}_p} \Phi_{i, \mathcal{D}_p}^H \mathbf{Z}_{i, \mathcal{D}_p, \mathcal{D}_g}) - 2\Re\{\text{Tr}(\Phi_{i, \mathcal{D}_g} \mathbf{X}_{i, \mathcal{D}_g})\} \right) \quad (3.44b)$$

$$\stackrel{(b)}{\approx} \sum_{g \in \mathcal{G}} \sum_{i \in \{t, r\}} \left( \text{Tr}(\Phi_{i, \mathcal{D}_g} \mathbf{Y}_{\mathcal{D}_g, \mathcal{D}_g} \Phi_{i, \mathcal{D}_g}^H \mathbf{Z}_{i, \mathcal{D}_g, \mathcal{D}_g}) - 2\Re\{\text{Tr}(\Phi_{i, \mathcal{D}_g} \mathbf{X}_{i, \mathcal{D}_g})\} \right) \quad (3.44c)$$

$$= \sum_{g \in \mathcal{G}} \bar{F}_g(\Phi_{t, \mathcal{D}_g}, \Phi_{r, \mathcal{D}_g}), \quad (3.44d)$$

where (a) holds by defining matrices  $\mathbf{X}_{i, \mathcal{D}_g}$ ,  $\mathbf{Y}_{\mathcal{D}_p, \mathcal{D}_g}$  and  $\mathbf{Z}_{i, \mathcal{D}_p, \mathcal{D}_g}$ ,  $\forall g \in \mathcal{G}$ ,  $\forall i \in \{t, r\}$ , which are sub-matrices selecting rows and columns according to  $\mathcal{D}_g$  from  $\mathbf{X}_i$ , and selecting rows according to  $\mathcal{D}_p$  and columns according to  $\mathcal{D}_g$  from  $\mathbf{Y}$  and  $\mathbf{Z}_i$ ,  $\forall i \in \{t, r\}$ , respectively; (b) holds by the finding (based on large amount of simulations) that the value of the term  $\sum_{p \neq g} \mathbf{Y}_{\mathcal{D}_g, \mathcal{D}_p} \Phi_{i, \mathcal{D}_p}^H \mathbf{Z}_{i, \mathcal{D}_p, \mathcal{D}_g}$  is negligible compared to  $\mathbf{X}_{i, \mathcal{D}_g}$  due to the severe double fading of the cascaded BS-RIS-user channels. This approximation generally holds true for Rician fading channels with around 10% difference between the original objective (3.44a) and the approximated objective (3.44d). Based on the above derivations, the sub-problem with respect to  $\Phi_t$  and  $\Phi_r$  is

$$\min_{\substack{\mathcal{D}_g, \forall g \in \mathcal{G} \\ \Phi_{t, \mathcal{D}_g}, \Phi_{r, \mathcal{D}_g}, \forall g \in \mathcal{G}}} \sum_{g \in \mathcal{G}} \bar{F}_g(\Phi_{t, \mathcal{D}_g}, \Phi_{r, \mathcal{D}_g}) \quad \text{s.t. (3.11), (3.12)}. \quad (3.45)$$

Problem (3.45) is still difficult to solve due to the coupling of grouping  $\mathcal{D}_1, \dots, \mathcal{D}_G$  and non-zero parts of the BD-RIS matrices  $\Phi_{t/r, \mathcal{D}_g}$ ,  $\forall g \in \mathcal{G}$ . To further simplify the design, we propose to iteratively<sup>8</sup> update  $\mathcal{D}_g$ ,  $\forall g \in \mathcal{G}$  and  $\Phi_{t/r, \mathcal{D}_g}$ ,  $\forall g \in \mathcal{G}$ , until the convergence is achieved.

<sup>8</sup>Note that the initialization of the dynamic grouping subsets, the BD-RIS matrices and the transmit precoder is required. For simplicity, we initialize dynamic grouping sets as  $\mathcal{D}_g = \mathcal{G}_g$ ,  $\forall g \in \mathcal{G}$ , the BD-RIS matrices as diagonal ones with non-zero entries having constant magnitude  $\frac{1}{\sqrt{2}}$  and random phase, and the transmit precoder as zero-forcing precoder.

Specifically, when BD-RIS matrices are fixed, the grouping problem is

$$\min_{\mathcal{D}_g, \forall g \in \mathcal{G}} \sum_{g \in \mathcal{G}} \bar{F}_g(\mathbf{\Phi}_{t, \mathcal{D}_g}, \mathbf{\Phi}_{r, \mathcal{D}_g}) \quad \text{s.t. (3.11)}. \quad (3.46)$$

We propose a simple yet practical iterative algorithm to solve the grouping problem. Given a proper initialization of  $\mathcal{D}_1, \dots, \mathcal{D}_G$ , we aim to successively determine which group each RIS cell should belong to based on the following steps.

Step 1: For cell  $m$ ,  $\forall m \in \mathcal{M}$ , we first check which group this cell belongs to and tag the exact position as  $g_{\text{tag}}$ . To guarantee each group contains at least one BD-RIS cell, we check the size of  $\mathcal{D}_{g_{\text{tag}}}$ . When  $|\mathcal{D}_{g_{\text{tag}}}| > 1$ , the following Steps 2-4 are executed.

Step 2: We calculate the value of objective (3.46), which is marked as  $\bar{\bar{F}}_{m,g}$ , when cell  $m$  is either in group  $g_{\text{tag}}$ , i.e.,  $g = g_{\text{tag}}$  or moved to other  $G - 1$  groups, i.e.,  $g \neq g_{\text{tag}}$ , according to (3.47):

$$\bar{\bar{F}}_{m,g} = \begin{cases} \sum_{p \in \mathcal{G}} \bar{F}_p(\mathbf{\Phi}_{t, \mathcal{D}_p}, \mathbf{\Phi}_{r, \mathcal{D}_p}), & g = g_{\text{tag}}, \\ \bar{F}_{g_{\text{tag}}}(\mathbf{\Phi}_{t, \tilde{\mathcal{D}}_{g_{\text{tag}}}^m}, \mathbf{\Phi}_{r, \tilde{\mathcal{D}}_{g_{\text{tag}}}^m}) + \bar{F}_g(\mathbf{\Phi}_{t, \hat{\mathcal{D}}_g^m}, \mathbf{\Phi}_{r, \hat{\mathcal{D}}_g^m}) \\ \quad + \sum_{p \in \mathcal{G}, p \neq g} \bar{F}_p(\mathbf{\Phi}_{t, \mathcal{D}_p}, \mathbf{\Phi}_{r, \mathcal{D}_p}), & g \neq g_{\text{tag}}, \end{cases} \quad (3.47)$$

where  $\tilde{\mathcal{D}}_{g_{\text{tag}}}^m = \mathcal{D}_{g_{\text{tag}}} \setminus \{m\}$ ,  $\hat{\mathcal{D}}_g^m = \mathcal{D}_g \cup \{m\}$ .

Step 3: Among all  $\bar{\bar{F}}_{m,g}$ ,  $\forall g \in \mathcal{G}$ , we find the index with the minimum value, i.e.,

$$g_m^* = \arg \min_g \bar{\bar{F}}_{m,g}. \quad (3.48)$$

Step 4: When  $g_m^* \neq g_{\text{tag}}$ , we update the corresponding dynamic subsets as

$$\mathcal{D}_{g_m^*} = \mathcal{D}_{g_m^*} \cup \{m\}, \quad \mathcal{D}_{g_{\text{tag}}} = \mathcal{D}_{g_{\text{tag}}} \setminus \{m\}. \quad (3.49)$$

Successively updating positions of all  $M$  RIS cells gives dynamic grouping subsets  $\mathcal{D}_1, \dots, \mathcal{D}_G$ .

With fixed grouping subsets, the BD-RIS matrix design problem for each group is separated, which is given by

$$\min_{\mathbf{\Phi}_{\mathcal{D}_g}} \text{Tr}(\mathbf{\Phi}_{\mathcal{D}_g} \mathbf{Y}_{\mathcal{D}_g, \mathcal{D}_g} \mathbf{\Phi}_{\mathcal{D}_g}^H \mathbf{Z}_{\mathcal{D}_g}) - 2\Re\{\text{Tr}(\mathbf{\Phi}_{\mathcal{D}_g} \mathbf{X}_{\mathcal{D}_g})\} \quad (3.50a)$$

$$\text{s.t. } \mathbf{\Phi}_{\mathcal{D}_g}^H \mathbf{\Phi}_{\mathcal{D}_g} = \mathbf{I}_{|\mathcal{D}_g|}, \quad (3.50b)$$

where the fresh notations are respectively defined as  $\mathbf{\Phi}_{\mathcal{D}_g} \triangleq [\mathbf{\Phi}_{t, \mathcal{D}_g}^H, \mathbf{\Phi}_{r, \mathcal{D}_g}^H]^H \in \mathbb{C}^{2|\mathcal{D}_g| \times |\mathcal{D}_g|}$ ,  $\mathbf{X}_{\mathcal{D}_g} \triangleq [\mathbf{X}_{t, \mathcal{D}_g}, \mathbf{X}_{r, \mathcal{D}_g}] \in \mathbb{C}^{|\mathcal{D}_g| \times 2|\mathcal{D}_g|}$ , and  $\mathbf{Z}_{\mathcal{D}_g} \triangleq \text{blkdiag}(\mathbf{Z}_{t, \mathcal{D}_g, \mathcal{D}_g}, \mathbf{Z}_{r, \mathcal{D}_g, \mathcal{D}_g}) \in \mathbb{C}^{2|\mathcal{D}_g| \times 2|\mathcal{D}_g|}$ . Problem (3.50) has the same form as (3.37), which can be solved by steps 6-15 in Algorithm 2.

**Algorithm 4** Heuristic Solution for CW-DGC BD-RIS**Require:**  $\mathbf{h}_k, \forall k \in \mathcal{K}, \mathbf{G}, \boldsymbol{\iota}, \boldsymbol{\tau}, \mathbf{W}, \boldsymbol{\Phi}_t, \boldsymbol{\Phi}_r, \mathcal{D}_1, \dots, \mathcal{D}_G$ .**Ensure:**  $\boldsymbol{\Phi}_t^*, \boldsymbol{\Phi}_r^*$ .

```

1: Calculate  $\mathbf{X}_{t/r}, \mathbf{Y}, \mathbf{Z}_{t/r}$ .
2: while no convergence of objective (3.44d) do
3:   for  $m = 1 : M$  do
4:     Find  $m \in \mathcal{D}_g$  and set  $g_{\text{tag}} = g$ .
5:     if  $|\mathcal{D}_g| > 1$  then
6:       Calculate  $\bar{f}_{m,g}, \forall g$  by (3.47).
7:       Find  $g_m^*$  by solving  $g_m^* = \arg \min_g \bar{f}_{m,g}$ .
8:       if  $g_m^* \neq g_{\text{tag}}$  then
9:         Update  $\mathcal{D}_{g_m^*} = \mathcal{D}_{g_m^*} \cup \{m\}$  and  $\mathcal{D}_{g_{\text{tag}}} = \mathcal{D}_{g_{\text{tag}}} \setminus \{m\}$ .
10:      end if
11:    end if
12:  end for
13:  Set  $\boldsymbol{\Phi}_t = \mathbf{0}, \boldsymbol{\Phi}_r = \mathbf{0}$ .
14:  for  $g = 1 : G$  do
15:    Split  $\mathbf{X}_{\mathcal{D}_g}, \mathbf{Y}_{\mathcal{D}_g, \mathcal{D}_g}, \mathbf{Z}_{\mathcal{D}_g}$  and solve problem (3.50) by Algorithm 2.
16:    Obtain  $\boldsymbol{\Phi}_{t, \mathcal{D}_g}^*$  and  $\boldsymbol{\Phi}_{r, \mathcal{D}_g}^*$  by (3.51), and restore  $\boldsymbol{\Phi}_t^*$  and  $\boldsymbol{\Phi}_r^*$  by (3.52).
17:  end for
18: end while
19: Return  $\boldsymbol{\Phi}_t^*, \boldsymbol{\Phi}_r^*$ .

```

After solving (3.50), BD-RIS matrices for each group are given by

$$\boldsymbol{\Phi}_{t, \mathcal{D}_g}^* = [\boldsymbol{\Phi}_{\mathcal{D}_g}^*]_{1:|\mathcal{D}_g^*|, :}, \forall g \in \mathcal{G}, \quad \boldsymbol{\Phi}_{r, \mathcal{D}_g}^* = [\boldsymbol{\Phi}_{\mathcal{D}_g}^*]_{|\mathcal{D}_g^*|+1:2|\mathcal{D}_g^*|, :}, \forall g \in \mathcal{G}. \quad (3.51)$$

With  $\boldsymbol{\Phi}_{t, \mathcal{D}_g}^*$  and  $\boldsymbol{\Phi}_{r, \mathcal{D}_g}^*, \forall g \in \mathcal{G}$ , we need to perform a restoring operation to get the original  $\boldsymbol{\Phi}_t^*$  and  $\boldsymbol{\Phi}_r^*$ . To this end, we introduce  $G$  vectors  $\mathbf{d}_g \in \mathbb{R}^{|\mathcal{D}_g|}, \forall g \in \mathcal{G}$ , which list the indexes in  $\mathcal{D}_g$  in ascending order. Then we have

$$[\boldsymbol{\Phi}_t^*]_{\mathbf{d}_g^T, \mathbf{d}_g^T} = \boldsymbol{\Phi}_{t, \mathcal{D}_g}^*, [\boldsymbol{\Phi}_r^*]_{\mathbf{d}_g^T, \mathbf{d}_g^T} = \boldsymbol{\Phi}_{r, \mathcal{D}_g}^*, \forall g \in \mathcal{G}. \quad (3.52)$$

After iteratively updating grouping subsets and BD-RIS matrices until some convergence thresholds are achieved, we can get  $\mathcal{D}_1^*, \dots, \mathcal{D}_G^*$  and corresponding BD-RIS matrices. For clarity, the CW-DGC BD-RIS design is summarized in Algorithm 4.

### 3.3.3 Analysis

**Initialization:** Choosing an appropriate initialization of BD-RIS coefficients  $\boldsymbol{\Phi}_t, \boldsymbol{\Phi}_r$  and transmit beamformer  $\mathbf{W}$  is important for the proposed algorithm. Unfortunately, it is not that easy to quickly find different and good initial values of  $\boldsymbol{\Phi}_t, \boldsymbol{\Phi}_r$  satisfying different modes/connections. Therefore, we simply adopt CW-SC cases and assume initial  $\boldsymbol{\Phi}_t, \boldsymbol{\Phi}_r$  are all diagonal matrices,

i.e.,  $\Phi_{t/r} = \text{diag}(\phi_{t/r,1}, \dots, \phi_{t/r,M})$ . Each non-zero element has constant amplitude  $\frac{1}{\sqrt{2}}$  and random phase shift within the range  $[0, 2\pi)$ .

Then with the above initial BD-RIS coefficients, we use a typical minimum mean square error (MMSE) precoder to initialize  $\mathbf{W}$ , which is given by

$$\tilde{\mathbf{W}} = (\mathbf{G}^H \Phi^H \mathbf{H}^H \mathbf{H} \Phi \mathbf{G} + \sigma^2 \mathbf{I}_N)^{-1} \mathbf{G}^H \Phi^H \mathbf{H}^H, \quad (3.53)$$

where  $\Phi = [\Phi_t^H, \Phi_r^H]^H$ ,  $\mathbf{H} = \text{blkdiag}(\mathbf{H}_t, \mathbf{H}_r)$  with  $\mathbf{H}_{t/r} = [\mathbf{h}_{t/r,1}; \dots; \mathbf{h}_{t/r,K_{t/r}}]$ ,  $\sigma_{i,k} = \sigma$ ,  $\forall i \in \{t, r\}$ ,  $\forall k \in \mathcal{K}_i$ . Finally, an additional normalization is performed to satisfy the power constraint, i.e.,  $\mathbf{W} = \frac{\sqrt{P}\tilde{\mathbf{W}}}{\|\tilde{\mathbf{W}}\|_F}$ .

**Complexity Analysis:** We provide a broad complexity analysis for algorithms proposed in the previous section. Four blocks are iteratively updated to find a convergent solution. In each iteration, updating blocks  $\boldsymbol{\iota}$  and  $\boldsymbol{\tau}$  require  $\mathcal{O}\{K^2 M^2\}$  operations; updating block  $\mathbf{W}$  has a complexity of approximately  $\mathcal{O}\{K(M^2 + I_{bs} N^3)\}$ , where  $I_{bs}$  is the number of iterations for bisection search. The complexity of the proposed two algorithms for designing block  $\Phi_t$ ,  $\Phi_r$  will be discussed as follows.

1) *The General Solution for CW-GC BD-RIS:* As summarized in Algorithm 2, the optimization of CW-GC BD-RIS is based on an iterative design. In each iteration, the objective is divided into  $G$  sub-problems, each of which is solved by a manifold version of the CG method (steps 7-13 in Algorithm 2) with complexity  $\mathcal{O}\{I_{cg} \bar{M}^3\}$ , where  $I_{cg}$  denotes the number of iterations for the conjugate-gradient method. Therefore, calculating CW-GC BD-RIS requires  $\mathcal{O}\{I_{gc} I_{cg} G \bar{M}^3\}$  operations, where  $I_{gc}$  denotes the number of iterations for Algorithm 2. The overall complexity is  $\mathcal{O}\{I(K^2 M^2 + I_{bs} K N^3 + I_{gc} I_{cg} G \bar{M}^3)\}$ , where  $I$  denotes the number of iterations in Algorithm 1. Since CW-FC and CW-SC BD-RIS are special cases of CW-GC ones, we can easily derive the corresponding complexity as follows:

Case 1: For the CW-FC case, i.e.,  $G = 1$ , there is no need to do iterations for BD-RIS design so that the complexity of updating BD-RIS beamformer is  $\mathcal{O}\{I_{cg} \bar{M}^3\}$ . The overall complexity is thus  $\mathcal{O}\{I(K^2 M^2 + I_{bs} K N^3 + I_{cg} \bar{M}^3)\}$ .

Case 2: For the CW-SC case, i.e.,  $G = M$ , optimizing BD-RIS requires  $\mathcal{O}\{I_{gc} I_{cg} M\}$ . And the overall complexity is  $\mathcal{O}\{I(K^2 M^2 + I_{bs} K N^3 + I_{gc} I_{cg} M)\}$ .

2) *The Efficient Solution for CW-SC BD-RIS:* Updating  $\{\Phi_t, \Phi_r\}$  by Algorithm 3 requires  $\mathcal{O}\{I_{sc} M\}$  operations, where  $I_{sc}$  denotes the number of iterations. Thus, the overall complexity is  $\mathcal{O}\{I(K^2 M^2 + I_{bs} K N^3 + I_{sc} M)\}$ .

3) *The Heuristic Solution for CW-DGC BD-RIS:* The computational complexity of the proposed dynamic grouping optimization mainly comes from the matrix multiplication, which requires around  $\mathcal{O}(\sum_{g=1}^G |\mathcal{D}_g|^{2.37})$ , yielding a worst-case total complexity  $\mathcal{O}(M \sum_{g=1}^G |\mathcal{D}_g|^{2.37})$ .

Table 3.2: The Circuit Topology Complexity and Overall Optimization Complexity for BD-RIS with Different Architectures

| Architecture | No. of Groups | Group Size                                   | No. of Non-Zero Elements           | Circuit Topology Complexity                            | Optimization Complexity                                                                                                          |
|--------------|---------------|----------------------------------------------|------------------------------------|--------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|
| CW-SC        | $M$           | 1                                            | $2M$                               | $3M$                                                   | $\mathcal{O}\{I(K^2M^2 + I_{bs}KN^3 + I_{gc}I_{cg}M)\}$ by Alg. 2<br>$\mathcal{O}\{I(K^2M^2 + I_{bs}KN^3 + I_{sc}M)\}$ by Alg. 3 |
| CW-GC        | $G$           | $\bar{M}$                                    | $2G\bar{M}^2$                      | $M(2\bar{M} + 1)$                                      | $\mathcal{O}\{I(K^2M^2 + I_{bs}KN^3 + I_{gc}I_{cg}G\bar{M}^3)\}$                                                                 |
| CW-FC        | 1             | $M$                                          | $2M^2$                             | $M(2M + 1)$                                            | $\mathcal{O}\{I(K^2M^2 + I_{bs}KN^3 + I_{cg}M^3)\}$                                                                              |
| CW-DGC       | $G$           | $ \mathcal{D}_g , \forall g \in \mathcal{G}$ | $2 \sum_{g=1}^G  \mathcal{D}_g ^2$ | $2 \sum_{g=1}^G  \mathcal{D}_g (2 \mathcal{D}_g  + 1)$ | $\mathcal{O}\{I(K^2M^2 + I_{bs}KN^3 + I_{dgc} \sum_{g=1}^G I_{cg,g} \mathcal{D}_g ^3)\}$                                         |

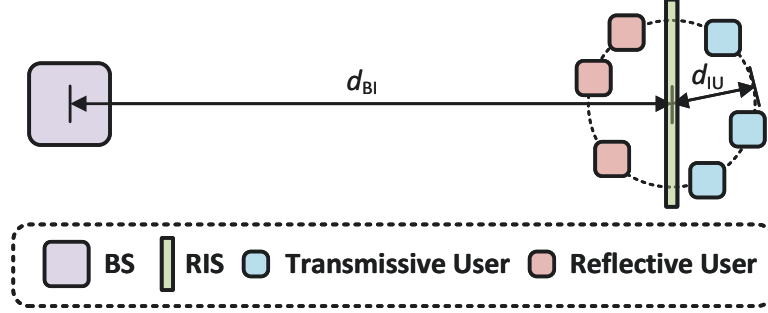


Figure 3.7: An illustration of the relative position among the BS, RIS, and users.

Meanwhile, the BD-RIS matrix optimization has a complexity  $\mathcal{O}(\sum_{g=1}^G I_{cg,g}|\mathcal{D}_g|^3)$  by Algorithm 2. Thus, the overall complexity is  $\mathcal{O}\{I(K^2M^2 + I_{bs}KN^3 + I_{dgc} \sum_{g=1}^G I_{cg,g}|\mathcal{D}_g|^3)\}$ , where  $I_{dgc}$  denotes the number of iterations of Algorithm 4.

To summarize, the group size of the cell-wise group  $\bar{M}$ , the number of cells  $M$ , the number of iterations, and the numbers of transmit antennas and users jointly determine the computational complexity of the proposed algorithms. Specifically, the complexity of designing the CW-SC architecture is dominated by the number of iterations; the complexity of designing the CW-FC architecture is dominated by the number of cells  $M$ ; the complexity of designing group-connected architectures depends on the group size. To provide a clear comparison, we summarize the circuit topology complexity and optimization complexity for different architectures in Table 3.2, showing that the performance improvement (as illustrated in the next section) of cell-wise group/fully-connected BD-RIS is at the expense of higher circuit topology complexity and optimization complexity.

### 3.4 Performance Evaluation

In this section, we present simulation results to demonstrate the performance of the BD-RIS-aided MU-MISO system when the BD-RIS has different modes/architectures. We model the channels from the BS to the RIS and from the RIS to users as a combination of small-scale and large-scale fading in accordance with existing RIS works [9]. Specifically, the distance-dependent pathloss model  $PL_i = \zeta_0(d_i/d_0)^{-\epsilon_i}, \forall i \in \{\text{BI}, \text{IU}\}$  accounts for the large-scale fading

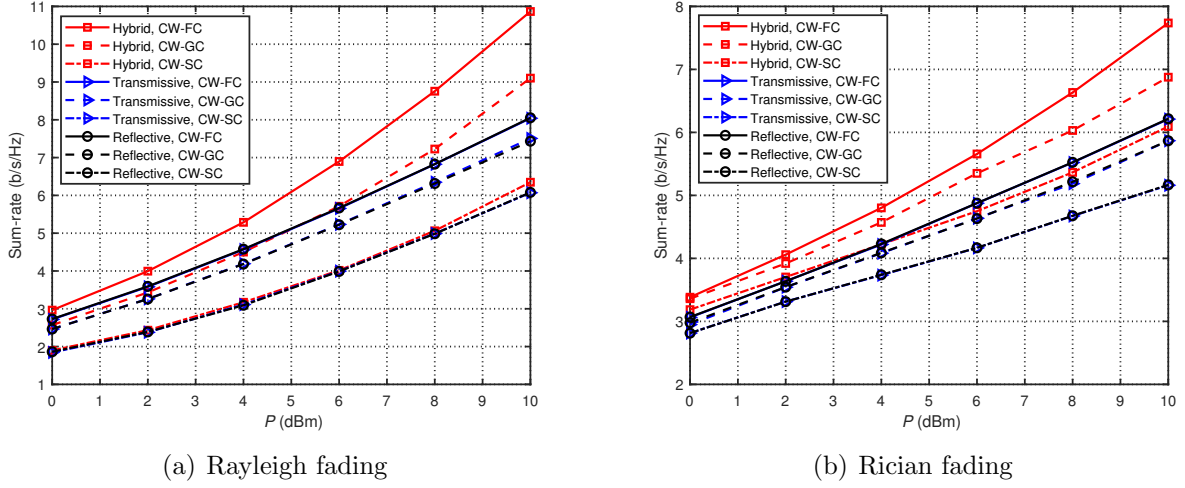


Figure 3.8: Sum-rate versus transmit power  $P$  ( $N = K = 4$ ,  $K_t = K_r = 2$ ,  $M = 32$ ,  $G = 8$ ).

with  $\zeta_0$  referring to the signal attenuation at a reference distance  $d_0$ ,  $d_{\text{BI}}$  and  $d_{\text{IU}}$  referring to the distance between the BS and the RIS, and between the RIS and users, and  $\varepsilon_i$ ,  $\forall i \in \{\text{BI}, \text{IU}\}$  referring to the path loss exponent. Regarding the small-scale fading, we use both Rayleigh fading and Rician fading models with  $\kappa_i$ ,  $\forall i \in \{\text{BI}, \text{IU}\}$  referring to the Rician factor. In the following simulations, we fix the Rician factor for Rician fading channels to  $\kappa_{\text{BI}} = \kappa_{\text{IU}} = 5$  dB for BS-RIS and RIS-user links. The signal attenuation is set as  $\zeta_0 = -30$  dB at a reference distance  $d_0 = 1$  m for all channels. The path loss exponent of BS-RIS and RIS-user channels are all set as  $\varepsilon_{\text{BI}} = \varepsilon_{\text{IU}} = 2.2$ . The noise power at each user is set as  $\sigma_k^2 = -80$  dBm,  $\forall i \in \{\text{t}, \text{r}\}$ ,  $\forall k \in \mathcal{K}_i$ . These parameter settings are typically adopted in sub-6GHz wireless communications. The relative position among the BS, BD-RIS, and users is shown in Fig. 3.7. In Figs. 3.8 and 3.9, the distance between the BS and the RIS is set as  $d_{\text{BI}} = 50$  m, and  $K$  users are randomly located close to the RIS with the same distance  $d_{\text{IU}} = 2.5$  m, while in Figs. 3.10 and 3.11, we set  $d_{\text{BI}} = 100$  m and  $d_{\text{IU}} = 10$  m. Here we assume the users to be located close to the RIS based on the results in [7].

Now we examine the sum-rate performance of the BD-RIS-aided MU-MISO system when BD-RIS has different modes<sup>9</sup> and architectures<sup>10</sup>. Fig. 3.8 shows the sum-rate performance versus the transmit power. From Fig. 3.8 we can obtain the following conclusions: *First*, with the same architecture, the “hybrid” scheme can outperform other modes. For example, for Rician fading channels, the “hybrid, CW-FC” scheme can achieve around 20% higher sum-rate than “Transmissive/Reflective, CW-FC” schemes. This is because the hybrid BD-RIS can serve all the users and flexibly control the signal power split to different users, while part of users are blocked under other modes. Therefore, the “hybrid” scheme can realize a full-dimensional service coverage, which indicates that the scope of application for hybrid RISs can be much

<sup>9</sup>We can easily obtain the RIS beamformer under the reflective (transmissive) mode based on the proposed algorithms by removing transmissive (reflective) users and forcing  $\Phi_t$  ( $\Phi_r$ ) to zero.

<sup>10</sup>In the following simulations, CW-SC BD-RIS beamformer is designed by the efficient solution.

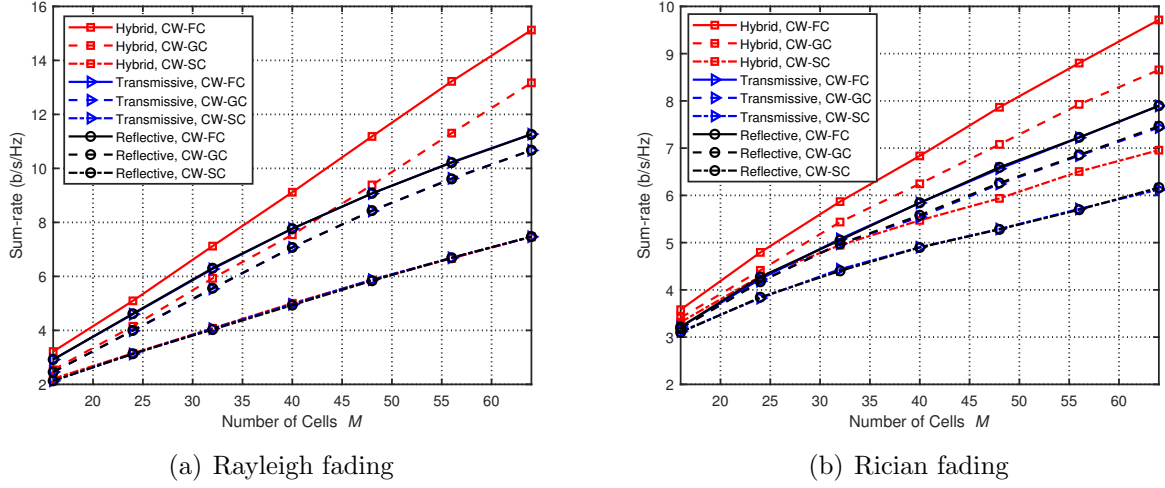


Figure 3.9: Sum-rate versus the number of RIS cells  $M$  ( $N = K = 6$ ,  $K_t = K_r = 3$ ,  $G = 8$ ,  $P = 5$  dBm).

more flexible than “transmissive/reflective” ones. *Second*, under the same mode, the “full” scheme always achieves the best sum-rate performance. For example, for Rayleigh fading channels, the sum-rate of “hybrid, CW-FC” and “hybrid, CW-GC” schemes is around 75% and 37% higher than that of the “hybrid, CW-SC” scheme. *Third*, when the transmit power is relatively small (the sum-rate performance is more related to the BD-RIS matrix than to the BS precoder), with the same inter-cell architecture, the performance gap between the hybrid RIS and transmissive/reflective RIS for Rician fading channels is larger than that for Rayleigh fading channels. It is because in Rician fading channels with strong LoS components, two separate beams of the hybrid BD-RIS can be “sharper” to finely direct to users from different sides. Therefore, hybrid RISs are more suitable in Rician fading environments. *Fourth*, when RISs are under the same mode (in low transmit power case), the performance gap between CW-FC/GC RISs and CW-SC ones for Rayleigh fading channels is larger than that for Rician fading channels. This fact shows that the advantage of CW-FC/GC RIS is more prominent in Rayleigh fading propagations, therefore confirming results in [55].

Then in Fig. 3.9, we plot sum-rate as a function of the number of cells  $M$ . It can be observed from Fig. 3.9 that the sum-rate for all schemes grows with the increase of  $M$ . More importantly, the slope of the sum-rate-versus- $M$  curve for CW-FC/GC BD-RIS is larger than that for CW-SC BD-RIS. This phenomenon can be explained by Table 3.2, which demonstrates that the number of non-zero elements grows in a quadratic manner with increasing  $M$  for CW-FC/GC BD-RIS, but grows linearly for CW-SC case: The more the number of non-zero elements, the higher the beam control flexibility.

Fig. 3.10 shows a simulation result when BD-RIS has 36 cells ( $6 \times 6$  square). Three fixed grouping strategies, namely “Horizontal”, “Vertical”, and “Interlaced”, are used in the simulation as illustrated in 3.10(a), where cells with the same color belong to the same group. For comparison, we also add the performance of CW-FC and CW-SC architectures as upper- and

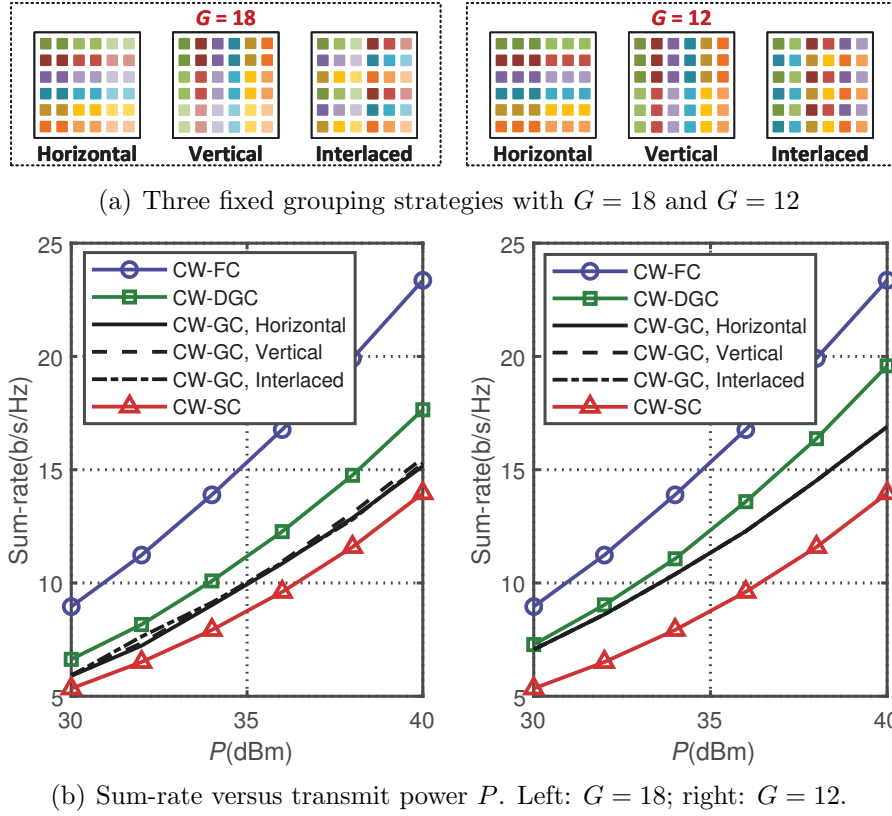


Figure 3.10: Comparison among different grouping strategies ( $M = 36$ ,  $N = K = 6$ ,  $K_t = K_r = 3$ ).

lower-bounds, respectively. We plot sum-rate performance achieved by different architectures of BD-RIS in Fig. 3.10(b). It can be shown that our proposed dynamic grouping strategy outperforms three CW-GC cases. In addition, there are no obvious performance gaps among three CW-GC architectures. These two observations can be explained as follows. 1) With Rayleigh fading channels between the RIS and users and fixed group size, the positional information for users is not a decisive factor. 2) The change of group size, however, boosts the performance by adapting to the strength of different channel coefficients. Therefore, we can deduce from Fig. 3.10 that the performance of CW-GC BD-RIS in this case depends more on the change of group size than on locations of cells.

Finally, in Fig. 3.11<sup>11</sup>, we show the sum-rate performance as a function of the number of groups  $G$ . With increasing number of groups, equivalently the decrease of the number of non-zero elements of the BD-RIS matrix, the performance gap between CW-DGC and CW-GC architectures becomes larger. For example, the performance achieved by a 36-cell CW-DGC architecture improves by 12% when  $G = 12$  while by 15% with  $G = 16$ ; the performance of a 64-cell CW-DGC architecture improves by 13% when  $G = 16$  and by 21% when  $G = 32$ . This fact demonstrates that the benefit of dynamic grouping is much more obvious when the size of each group is relatively small.

<sup>11</sup>Given that three fixed grouping strategies have similar performance, in Fig. 3.11 we only plot the horizontal

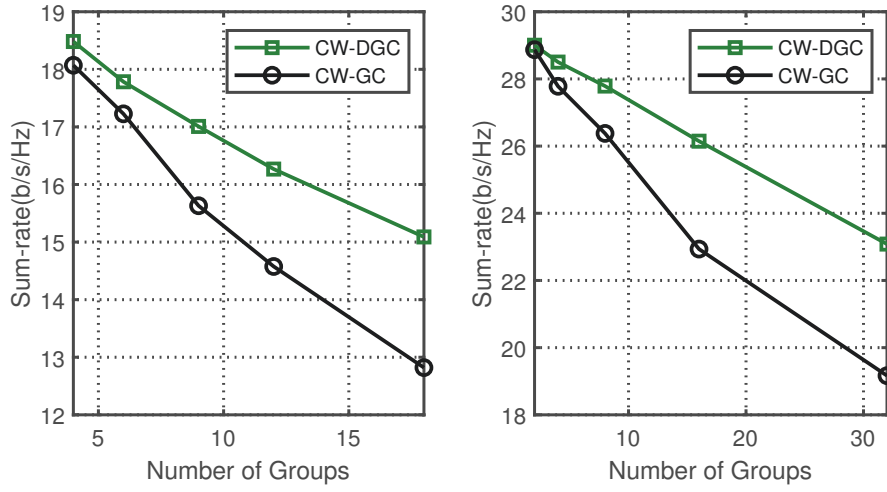


Figure 3.11: Sum-rate versus the number of groups  $G$ . Left:  $M = 36$ ; right:  $M = 64$ . ( $N = K = 6$ ,  $K_t = K_r = 3$ ,  $P = 38$  dBm)

### 3.5 Summary

This chapter analyzes and proposes a general BD-RIS-aided communication model unifying three modes, namely, reflective, transmissive, and hybrid, and three architectures, namely, CW-SC, CW-GC, and CW-FC architectures, totally twelve cases. Particularly, we first prove that the STAR-RIS (hybrid CW-SC RIS) is a special case of our proposed model. Then we go beyond STAR-RIS and propose more flexible CW-GC/FC/DGC architectures.

With the proposed model, we consider the joint transmit precoder and BD-RIS matrix design to maximize the sum-rate for an RIS-aided MU-MISO system. We first transform the original problem into a multi-block optimization based on fractional programming theory. For the design of BD-RIS block, we propose a general algorithm, which is suitable for different architectures, an efficient algorithm specifically for CW-SC BD-RIS, and a heuristic solution suitable for CW-DGC BD-RIS.

Finally, we provide a comprehensive comparison for BD-RIS with different modes/architectures. Simulation results show that under specific parameter settings and channel conditions, CW-FC and CW-GC hybrid BD-RISs can achieve around 75% and 37% higher sum-rate performance than CW-SC hybrid BD-RISs. Meanwhile, using CW-FC hybrid BD-RISs can improve the sum-rate performance by around 20% compared with using CW-FC transmissive/reflective ones. In addition, with Rayleigh fading channels between the RIS and users, the change of group size has a greater impact than that of cell locations on the achievable performance.

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case, which is marked as “CW-GC”.



# Chapter 4

## BD-RIS with Multi-Sector Mode

While the proposed BD-RIS with hybrid mode and different architectures in Chapter 3 achieves significant performance improvement, it is achieved at the expense of heavily increasing circuit complexity. This motivates the exploration of different architectures of BD-RIS with different antenna array arrangements to achieve full-space coverage, enhance performance, and simplify the circuit design. This chapter thus proposes a new branch of BD-RIS tree in Fig. 1.1, namely, BD-RIS under multi-sector mode, and provides a thorough analysis from the modeling, architecture design and mode analysis, to the scaling law and beamforming design. This leads to the following layouts and contributions.

In Section 4.1, a multi-sector BD-RIS has been proposed, which is modeled as a polygon as shown in Fig. 4.1(a). With full-space coverage, highly directional beams, and adaptive structures (based on different numbers of sectors), multi-sector BD-RIS can be flexibly deployed into various situations and is particularly favorable to millimeter wave/Terahertz scenarios and cell-free networks. The constraints of multi-sector BD-RIS have been derived when its reconfigurable elements have either continuous amplitudes/phase shifts or discrete values selected from finite codebooks. The corresponding channel model has also been derived by taking into account the impact of beamwidth and antenna gain.

In Section 4.2, the scaling law of the received signal power as a function of the number of sectors for a multi-sector BD-RIS-assisted single user (SU) SISO system has been derived to show the benefits of using multi-sector BD-RIS.

In Section 4.3, the proposed multi-sector BD-RIS has been applied in MU-MISO systems. Specifically, efficient algorithms have been proposed to maximize the sum-rate performance of the multi-sector BD-RIS-assisted MU-MISO system. The closed-form solution of the multi-sector BD-RIS has been first proposed when its scattering matrix has continuous values. Then, an efficient algorithm has been proposed to obtain the discrete solution.

In Section 4.4, the sum-rate performance of the multi-sector BD-RIS-aided MU-MISO sys-

tem has been evaluated. Numerical results show that with the same sum-rate performance requirement, the number of antennas for the whole multi-sector BD-RIS can be reduced by increasing the number of sectors. In addition, the resolution of the finite codebooks for the multi-sector BD-RIS with discrete values can be reduced with increasing number of sectors, which is beneficial for controlling cost for practical RIS realizations.

## 4.1 Multi-Sector BD-RIS Model

In this section, we propose the multi-sector BD-RIS-assisted communication model based on [55], [94], and specify the channel model based on antenna theory [77].

We consider a general wireless communication system consisting of an  $N$ -antenna transmitter,  $K$  multi-antenna users, each of which has  $N_k$  antennas, and an  $M$ -cell RIS. Denote  $\mathcal{N} = \{1, \dots, N\}$ ,  $\mathcal{K} = \{1, \dots, K\}$ , and  $\mathcal{M} = \{1, \dots, M\}$  as the set of indices of transmitting antennas, users, and cells, respectively. Each cell in this RIS consists of  $L$  antennas,  $L \geq 2$ . Particularly, cell  $m$  contains antennas belonging to the set  $\mathcal{L}_m = \{m, M+m, \dots, (L-1)M+m\}$ ,  $\forall m \in \mathcal{M}$ . Therefore, the  $M$ -cell RIS is modeled as  $LM$  antennas connected to an  $ML$ -port passive reconfigurable impedance network<sup>1</sup> [55]. By Result 2.1, the channel between the transmitter and user  $k$  without mismatching and mutual coupling among antennas is [55]

$$\mathbf{H}_{\text{all},k} = \mathbf{H}_{\text{d},k} + \bar{\mathbf{H}}_k \Phi \bar{\mathbf{G}}, \forall k \in \mathcal{K}, \quad (4.1)$$

where  $\mathbf{H}_{\text{d},k} \in \mathbb{C}^{N_k \times N}$  is the channel matrix for the transmitter-user link,  $\bar{\mathbf{H}}_k \in \mathbb{C}^{N_k \times LM}$ ,  $\forall k \in \mathcal{K}$  and  $\bar{\mathbf{G}} \in \mathbb{C}^{LM \times N}$  are channel matrices for the transmitter-RIS-user link.  $\Phi \in \mathbb{C}^{LM \times LM}$  is the scattering matrix of the  $LM$ -port reconfigurable impedance network satisfying [77]

$$\Phi^H \Phi = \mathbf{I}_{LM}, \quad (4.2)$$

when the  $LM$ -port reconfigurable impedance network is lossless. The scattering matrix  $\Phi$  is solely used to characterize the reconfigurable impedance network, which is independent of the characteristics of the antenna array. More details regarding the impact of mutual coupling from the antenna array can be found in [55], [77].

The channel model (4.1) and the constraint for an  $M$ -cell RIS (4.2) are general expressions regardless of architectures of RIS, spatial arrangements and radiation pattern of RIS antenna elements. In the following, we will propose a novel multi-sector BD-RIS model and derive corresponding multi-sector BD-RIS-assisted channel model by specifying the spatial arrangement of RIS antenna elements and the architecture of the RIS. To this end, we first make the following two assumptions.

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<sup>1</sup>The proposed RIS model in this work is purely passive without power consumption since the reconfigurable impedance network of the RIS consists of passive circuit components.

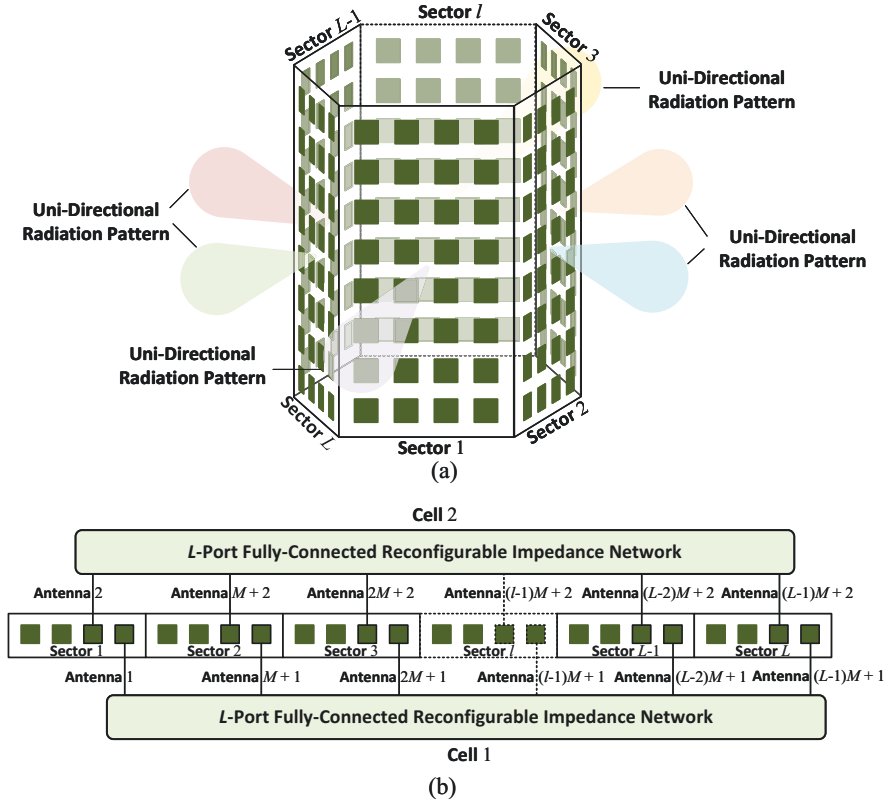


Figure 4.1: (a) A multi-sector BD-RIS with  $LM$  uni-directional antennas modeled as an  $L$ -sector polygon. (b) Illustration of cells 1 and 2 taking the first row of the multi-sector BD-RIS (stretched in a straight line) as an example.

**Assumption 4.1.** Each antenna of the RIS has a uni-directional radiation pattern with the same beamwidth  $\frac{2\pi}{L}$ .

**Assumption 4.2.** The  $L$  antennas in each cell are respectively placed at the side of the polygon such that each antenna covers  $\frac{1}{L}$  space, as illustrated in Fig. 4.1 (a).

Based on Assumptions 4.1 and 4.2, we divide the  $M$ -cell RIS into  $L$  sectors. Specifically, sector  $l$  covers  $1/L$  space containing  $M$  antennas, each of which belongs to  $\mathcal{M}_l = \{(l-1)M+1, \dots, lM\}$ ,  $\forall l \in \mathcal{L} = \{1, \dots, L\}$ . The  $M$ -cell RIS is thus modeled as a polygon with  $L$ -sectors, which is referred to as multi-sector BD-RIS as shown in Fig. 4.1(a). For clarity, in Fig. 4.1(b), we also illustrate the “cell” of the multi-sector BD-RIS, where every  $L$  antennas in the set  $\mathcal{L}_m$  are connected to an  $L$ -port fully-connected reconfigurable impedance network to support the multi-sector mode.

In this work, we focus on a 2D polygon for illustration and tractability. In Fig. 4.2(a), we provide a simple example when the multi-sector BD-RIS is modeled as a 2D polygon with 6 sectors. In this case, we assume the multi-sector BD-RIS has a 2D horizontal coverage at the azimuth plane. However, the concept is also applicable to 3D polygon where antennas are placed on a “sphere”. For ease of understanding, we provide an example when the multi-sector

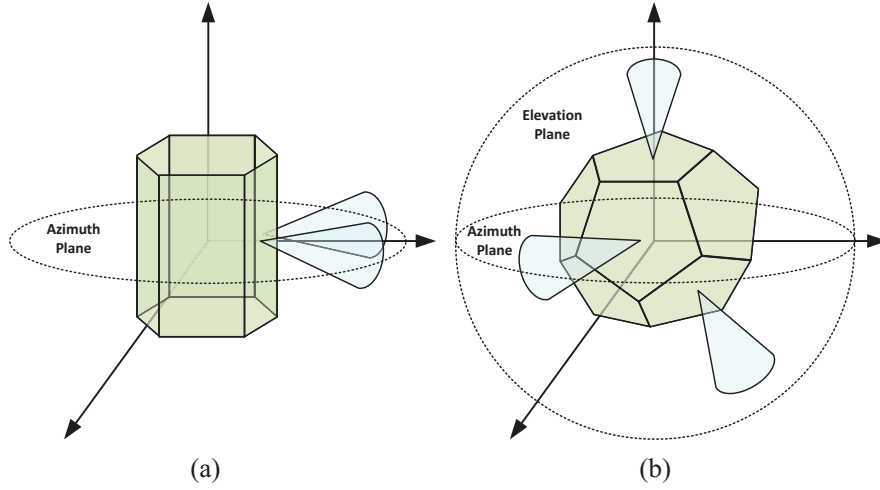


Figure 4.2: Multi-sector BD-RIS modeled as (a) a 2D polygon with 6 sectors and (b) a 3D polygon with 12 sectors.

BD-RIS is modeled as a 3D polygon with 12 sectors in Fig. 4.2(b). In this case, the multi-sector BD-RIS enables a 3D full-space coverage.

With Assumptions 4.1 and 4.2, we can partition channel (4.1) as

$$\begin{aligned}
 \mathbf{H}_{\text{all},k} &= \mathbf{H}_{\text{d},k} + [\bar{\mathbf{H}}_{k,1} \ \bar{\mathbf{H}}_{k,2} \ \cdots \ \bar{\mathbf{H}}_{k,L}] \begin{bmatrix} \Phi_{1,1} & \Phi_{1,2} & \cdots & \Phi_{1,L} \\ \Phi_{2,1} & \Phi_{2,2} & \cdots & \Phi_{2,L} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{L,1} & \Phi_{L,2} & \cdots & \Phi_{L,L} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{G}}_1 \\ \bar{\mathbf{G}}_2 \\ \vdots \\ \bar{\mathbf{G}}_L \end{bmatrix} \\
 &= \mathbf{H}_{\text{d},k} + \sum_{l \in \mathcal{L}} \sum_{l' \in \mathcal{L}} \bar{\mathbf{H}}_{k,l} \Phi_{l,l'} \bar{\mathbf{G}}_{l'}, \forall k \in \mathcal{K},
 \end{aligned} \tag{4.3}$$

where  $\bar{\mathbf{H}}_{k,l} = [\bar{\mathbf{H}}_k]_{:, (l-1)M+1:lM} \in \mathbb{C}^{N_k \times M}$ , and  $\bar{\mathbf{G}}_l = [\bar{\mathbf{G}}]_{(l-1)M+1:lM, :} \in \mathbb{C}^{M \times N}$ ,  $\forall k \in \mathcal{K}$ ,  $\forall l \in \mathcal{L}$  are channels between sector  $l$  of the multi-sector BD-RIS and user  $k$ , and between the transmitter and sector  $l$ , respectively.  $\Phi_{l,l'} = [\Phi]_{(l-1)M+1:lM, (l'-1)M+1:l'M} \in \mathbb{C}^{M \times M}$ ,  $\forall l, l' \in \mathcal{L}$ . To derive the multi-sector BD-RIS-assisted channel model, we further make the following assumption.

**Assumption 4.3.** The transmitter and  $K_1$  users with  $\mathcal{K}_1 = \{1, \dots, K_1\}$ ,  $0 < K_1 < K$  are located within the coverage of sector 1 of the multi-sector BD-RIS;  $K_l$  users with  $\mathcal{K}_l = \{\sum_{i \in \{1, \dots, l-1\}} K_i + 1, \dots, \sum_{i \in \{1, \dots, l\}} K_i\}$ ,  $0 < K_l < K$  are located within the coverage of sector  $l$  of the multi-sector BD-RIS,  $\forall l \in \mathcal{L}, l \neq 1$ , as illustrated in Fig. 4.3.

Based on Assumptions 4.1-4.3, we can deduce two results.

**Result 4.1.** The transmitter is out of the coverage of the uni-directional radiation pattern of sector  $l$  of the multi-sector BD-RIS,  $\forall l \in \mathcal{L}, l \neq 1$ , that is  $\bar{\mathbf{G}}_l = \mathbf{0}_{M \times N}$ ,  $\forall l \in \mathcal{L}, l \neq 1$ .

**Result 4.2.** The users in  $\mathcal{K}'_l$  are out of the coverage of the uni-directional radiation pattern of sector  $l$  of the multi-sector BD-RIS,  $\forall l \neq l'$ , that is  $\bar{\mathbf{H}}_{k,l} = \mathbf{0}_{N_k \times M}$ ,  $\forall k \in \mathcal{K}_{l'}$ ,  $\forall l \neq l', \forall l, l' \in \mathcal{L}$ .

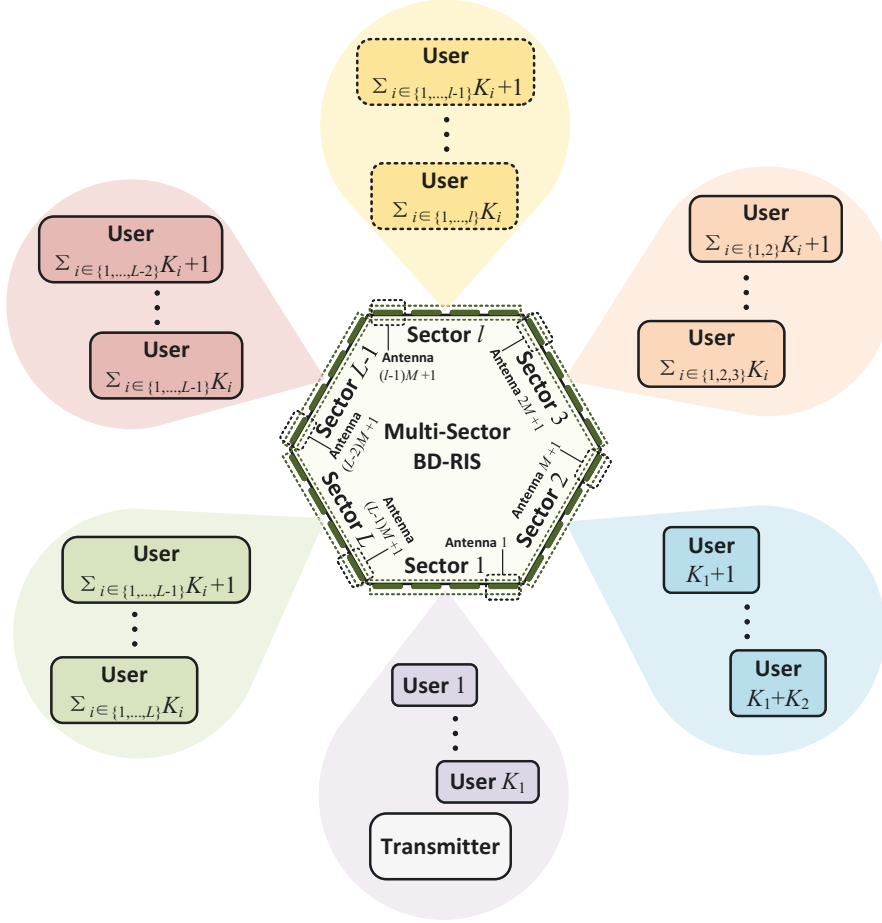


Figure 4.3: Top view for the locations of the  $M$ -cell multi-sector BD-RIS partitioning the whole space into  $L$  sectors, the transmitter, and multiple users.

With Results 4.1 and 4.2, we can simplify (4.3) as

$$\mathbf{H}_{\text{all},k} = \mathbf{H}_{\text{d},k} + \bar{\mathbf{H}}_{k,l} \Phi_{l,1} \bar{\mathbf{G}}_1, \forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}. \quad (4.4)$$

For simplicity, we use auxiliary notations  $\mathbf{H}_k = \bar{\mathbf{H}}_{k,l}$ ,  $\Phi_l = \Phi_{l,1}$ ,  $\mathbf{G} = \bar{\mathbf{G}}_1$ ,  $\forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}$ . We also ignore the channel from the transmitter to the receiver ( $\mathbf{H}_{\text{d},k} = \mathbf{0}_{N_k \times N}$ ,  $\forall k \in \mathcal{K}$ ). Then we can rewrite the simplified channel (4.4) as

$$\mathbf{H}_{\text{all},k} = \mathbf{H}_k \Phi_l \mathbf{G}, \forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}. \quad (4.5)$$

According to the general constraint (4.2), the constraint of matrices  $\Phi_l$ ,  $\forall l \in \mathcal{L}$  of the multi-sector BD-RIS becomes

$$\sum_{\forall l \in \mathcal{L}} \Phi_l^H \Phi_l = \mathbf{I}_M. \quad (4.6)$$

Now we establish the multi-sector BD-RIS-assisted communication model (4.6) and (4.5). In the following subsections, we will show the specific descriptions of matrices  $\Phi_l$ ,  $\forall l \in \mathcal{L}$  when the cell-wise single-connected architecture is applied and channels  $\mathbf{H}_{\text{all},k}$ ,  $\forall k \in \mathcal{K}$  including the

Table 4.1: Three Architectures of Multi-Sector BD-RIS

| (Cell-Wise) Architectures    | Constraint                                                                                                                                                               |
|------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Single-Connected             | $\Phi_l = \text{diag}(\phi_{(l-1)M+1}, \dots, \phi_{lM})$<br>$\sum_{i \in \mathcal{L}_m}  \phi_i ^2 = 1, \forall m \in \mathcal{M}$                                      |
| Group-Connected <sup>†</sup> | $\Phi_l = \text{blkdiag}(\Phi_{l,1}, \dots, \Phi_{l,G})$<br>$\sum_{\forall l \in \mathcal{L}} \Phi_{l,g}^H \Phi_{l,g} = \mathbf{I}_{\bar{M}}, \forall g \in \mathcal{G}$ |
| Fully-Connected              | $\sum_{\forall l \in \mathcal{L}} \Phi_l^H \Phi_l = \mathbf{I}_M$                                                                                                        |

<sup>†</sup>  $G$  is the number of groups.  $\mathcal{G} = \{1, \dots, G\}$ ,  $\bar{M} = M/G$ .

impact of antenna gain.

#### 4.1.1 Cell-Wise Single-Connected Multi-Sector BD-RIS

In this work, we assume the multi-sector BD-RIS has a cell-wise single-connected architecture, that is, different RIS cells are not connected to each other. Similar to the BD-RIS with hybrid mode in Chapter 3, with different inter-cell circuit topologies, the multi-sector BD-RIS can also have different architectures, namely, cell-wise single/group/fully-connected architectures, which are summarized in Table 4.1. The cell-wise group/fully-connected architectures can provide higher performance gains thanks to the more general constraints as shown in Table 4.1, but at the expense of higher circuit topology complexity. Here we focus only on cell-wise single-connected multi-sector BD-RIS and investigating cell-wise group/fully-connected architectures is left for future work. In the cell-wise single-connected case,  $\Phi_l, \forall l \in \mathcal{L}$  can be modeled as diagonal matrices:

$$\Phi_l = \text{diag}(\phi_{(l-1)M+1}, \dots, \phi_{lM}), \forall l \in \mathcal{L}, \quad (4.7)$$

and constraint (4.6) becomes

$$\sum_{i \in \mathcal{L}_m} |\phi_i|^2 = 1, \forall m \in \mathcal{M}. \quad (4.8)$$

We should clarify here that the proposed cell-wise single-connected multi-sector BD-RIS is an application of group-connected reconfigurable impedance network with group size equal to  $L$ . When antennas with different spatial arrangements are connected to the same reconfigurable impedance network, we have RIS models with different modes and architectures. In Fig. 4.4 we provide a simple example to facilitate understanding. We can observe from Fig. 4.4 that the circuit topology of the impedance network for a 2-cell 4-sector BD-RIS with cell-wise single-connected architecture is exactly the same as that for an 8-element group-connected RIS with group size equal to 4 [55]. The difference is that antennas within each cell of the multi-sector BD-RIS are placed at the side of the polygon, while antennas of the RIS in [55] are placed towards one direction. Therefore, the freedom induced by the group-connected reconfigurable impedance network is utilized to support the multi-sector mode of the multi-sector BD-RIS, but to realize more flexible signal reflection for the group-connected RIS [55]. In addition, as per

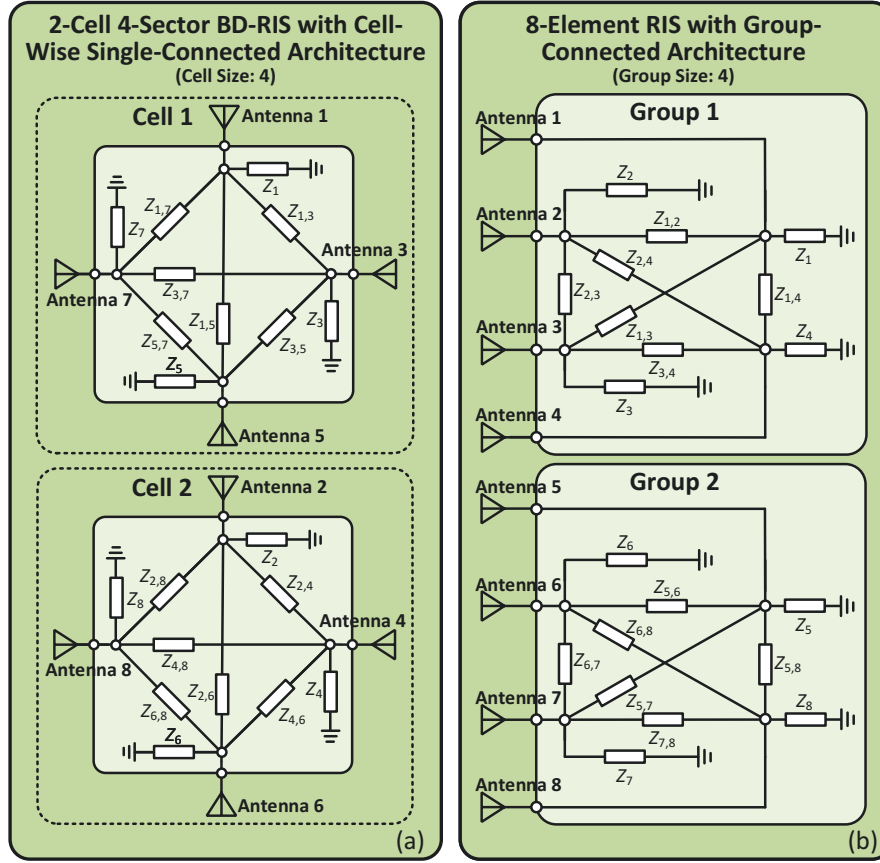


Figure 4.4: Examples of (a) cell-wise single-connected multi-sector BD-RIS and (b) group-connected RIS.

the circuit topology in Fig. 4.4(a), to realize an  $M$ -cell  $L$ -sector BD-RIS with cell-wise single-connected architecture,  $ML$  antennas and  $M$  fully-connected  $L$ -port reconfigurable impedance networks, each of which consists of  $\frac{L(L+1)}{2}$  impedance components, are required. Therefore, the increasing performance with increasing  $L$  is achieved at the expense of higher circuit complexity and cost.

We model  $\phi_i$  as  $\phi_i = \sqrt{\beta_i}e^{j\vartheta_i}$ ,  $\forall i \in \mathcal{L}_m$ ,  $\forall m \in \mathcal{M}$ , where  $0 \leq \sqrt{\beta_i} \leq 1$  and  $0 \leq \vartheta_i \leq 2\pi$  denote the amplitude and phase shift of  $\phi_i$ , respectively. Next, we will consider two cases of the multi-sector BD-RIS: 1) continuous case, that is both the amplitude and the phase shift have continuous values<sup>2</sup>; 2) discrete case, that is both the amplitude and the phase shift are chosen from finite codebooks<sup>3</sup>.

<sup>2</sup>To implement continuously controllable multi-sector BD-RIS, varactors could be used to realize the reconfigurable impedance network.

<sup>3</sup>To implement discrete-value BD-RIS, PIN diodes can be used to realize the reconfigurable impedance network.

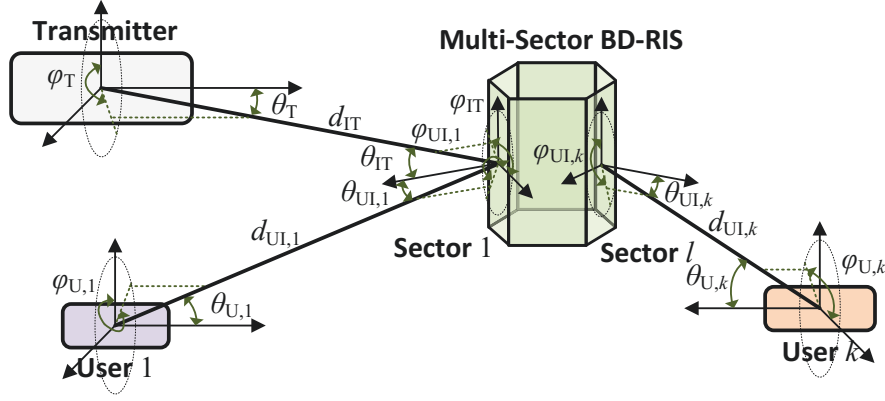


Figure 4.5: Relative locations among the multi-sector BD-RIS, the transmitter, and users.

**Continuous Case:** In this case, constraint (4.8) is transformed into

$$(4.8) \Rightarrow \begin{cases} \phi_i = \sqrt{\beta_i} e^{j\vartheta_i}, \forall i \in \mathcal{L}_m, \forall m \in \mathcal{M}, \\ 0 \leq \beta_i \leq 1, \forall i \in \mathcal{L}_m, \forall m \in \mathcal{M}, \\ \sum_{i \in \mathcal{L}_m} \beta_i = 1, \forall m \in \mathcal{M}, \\ 0 \leq \vartheta_i < 2\pi, \forall i \in \mathcal{L}_m, \forall m \in \mathcal{M}. \end{cases} \quad (4.9)$$

**Discrete Case:** We propose a simple quantization strategy when  $\phi_i$  has discrete values. Specifically, the amplitude square  $\beta_i$  is uniformly spaced within the range  $[0, 1]$ , i.e.,  $\beta_i \in \mathcal{A} = \{\frac{a}{2^A-1} | a = 0, 1, \dots, 2^A - 1\}$  controlled by  $A$  bits. The phase shift  $\vartheta_i$  is uniformly spaced around the unit circle, i.e.,  $\vartheta_i \in \mathcal{B} = \{\frac{2\pi b}{2^B} | b = 0, 1, \dots, 2^B - 1\}$  controlled by  $B$  bits. In this case, constraint (4.8) is transformed into

$$(4.8) \Rightarrow \begin{cases} \phi_i = \sqrt{\beta_i} e^{j\vartheta_i}, \forall i \in \mathcal{L}_m, \forall m \in \mathcal{M}, \\ \beta_i \in \mathcal{A}, \forall i \in \mathcal{L}_m, \forall m \in \mathcal{M}, \\ \sum_{i \in \mathcal{L}_m} \beta_i = 1, \forall m \in \mathcal{M}, \\ \vartheta_i \in \mathcal{B}, \forall i \in \mathcal{L}_m, \forall m \in \mathcal{M}. \end{cases} \quad (4.10)$$

Herein we assume the amplitude and phase shift of each  $\phi_i$  are independent of each other to simplify the design and to provide a theoretical performance upper-bound. In practice, the amplitude and phase shift of each  $\phi_i$  might couple with each other due to specific circuit designs.

#### 4.1.2 Channel Model

We start by analyzing the path loss of the multi-sector BD-RIS-assisted communication system, which will be used to model the multi-sector BD-RIS-assisted channel (4.5).

The relative locations among the transmitter, the multi-sector BD-RIS, and users are illustrated in Fig. 4.5. The power captured by each antenna in sector 1 of the multi-sector BD-RIS from the transmitting antenna is [94]<sup>4</sup>

$$P_{IT} = \bar{P}_T \left( \frac{\lambda}{4\pi} \right)^2 d_{IT}^{-\epsilon_{IT}} G_T(\theta_T, \varphi_T) G_I(\theta_{IT}, \varphi_{IT}), \quad (4.11)$$

which is a generalization of the well-known free-space Friis transmission equation, where the power is allowed to decrease at a rate of  $d_{IT}^{-\epsilon_{IT}}$  [95] with distance  $d_{IT}$  between the transmitter and multi-sector BD-RIS and the path loss exponent  $\epsilon_{IT}$ . Specifically, in free-space propagations, the path loss exponent is 2, while in complicated propagations with obstacles, the path loss component is usually larger than 2. In (4.11),  $\bar{P}_T$  is the power radiated from the transmitting antenna<sup>5</sup>.  $G_T(\theta_T, \varphi_T)$  ( $G_I(\theta_{IT}, \varphi_{IT})$ ) is the gain of each transmitting antenna (each RIS antenna)<sup>6</sup>, where  $\theta_T$  ( $\theta_{IT}$ ) and  $\varphi_T$  ( $\varphi_{IT}$ ) are elevation and azimuth angles from the transmitter to sector 1 of the multi-sector BD-RIS (from sector 1 of the multi-sector BD-RIS to the transmitter), respectively. Then the power scattered from RIS antenna  $m$ ,  $\forall m \in \mathcal{M}_l$  in sector  $l$ ,  $\forall l \in \mathcal{L}$  can be represented as

$$P_{I,m} = P_{IT} |\phi_m|^2 = P_{IT} \beta_m, \forall m \in \mathcal{M}_l, \forall l \in \mathcal{L}. \quad (4.12)$$

Similarly, the power received by user  $k$ ,  $\forall k \in \mathcal{K}_l$  from RIS antenna  $m$ ,  $\forall m \in \mathcal{M}_l$  in sector  $l$ ,  $\forall l \in \mathcal{L}$  is given by

$$P_{U,m,k} = P_{I,m} \left( \frac{\lambda}{4\pi} \right)^2 d_{UI,k}^{-\epsilon_{UI,k}} G_I(\theta_{UI,k}, \varphi_{UI,k}) G_U(\theta_{U,k}, \varphi_{U,k}), \forall m \in \mathcal{M}_l, \forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}, \quad (4.13)$$

where  $\theta_{UI,k}$  ( $\theta_{U,k}$ ) and  $\varphi_{UI,k}$  ( $\varphi_{U,k}$ ) are elevation and azimuth angles from RIS antenna  $m$  to user  $k$  (from user  $k$  to RIS antenna  $m$ ), respectively.  $d_{UI,k}$  is the distance between the multi-sector BD-RIS and user  $k$ , and  $\epsilon_{UI,k}$  is the path loss exponent. Combining equations (4.11)-(4.13), we can obtain

$$P_{U,m,k} = \bar{P}_T \beta_m \left( \frac{\lambda}{4\pi} \right)^4 d_{IT}^{-\epsilon_{IT}} d_{UI,k}^{-\epsilon_{UI,k}} G_T(\theta_T, \varphi_T) G_U(\theta_{U,k}, \varphi_{U,k}) \\ \times G_I(\theta_{IT}, \varphi_{IT}) G_I(\theta_{UI,k}, \varphi_{UI,k}), \forall m \in \mathcal{M}_l, \forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}. \quad (4.14)$$

To separate factors related to the transmitter/users and to the multi-sector BD-RIS, and focus only on the impact from the multi-sector BD-RIS, we assume antennas at the transmitter/users exhibit constant gain, that is  $G_T(\theta_T, \varphi_T) = G_T$  and  $G_U(\theta_{U,k}, \varphi_{U,k}) = G_U$ ,  $\forall k \in \mathcal{K}_l$ ,  $\forall l \in \mathcal{L}$ .

<sup>4</sup>In this paper, we consider a far-field case, that is the distance between the transmitter/users and the multi-sector BD-RIS is much larger than the size of the transmitter/RIS/users. In this case, we ignore the antenna spacing at the transmitter/RIS/users and the distance among different sectors of the multi-sector BD-RIS.

<sup>5</sup>We assume each transmitting antenna radiates the same power.

<sup>6</sup>We assume antennas at the same device (transmitter/RIS/user) have the same normalized power radiation pattern.

Then with equation (4.14), we define the cascaded path loss of the multi-sector BD-RIS-assisted communication system as

$$\zeta_k = \frac{(4\pi)^4}{\lambda^4 G_T G_U} \times \underbrace{\frac{d_{IT}^{\epsilon_{IT}} d_{UI,k}^{\epsilon_{UI,k}}}{G_I(\theta_{IT}, \varphi_{IT}) G_I(\theta_{UI,k}, \varphi_{UI,k})}}_{\text{Vary with RIS antenna gain}}, \forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}. \quad (4.15)$$

It can be observed from equation (4.15) that the path loss is highly associated with the gain of RIS antennas. In the following we will introduce two expressions of the gain of RIS antennas which are suitable for the multi-sector BD-RIS-assisted communication system.

**Idealized Radiation Pattern:** We first consider an idealized radiation pattern function for each antenna of the  $L$ -sector BD-RIS, which is given by

$$F_I^{\text{Idealized}}(\theta_I, \varphi_I) = \begin{cases} 1, & 0 \leq \theta_I \leq \frac{\pi}{L}, 0 \leq \varphi_I \leq 2\pi, \\ 0, & \frac{\pi}{L} < \theta_I \leq \pi, 0 \leq \varphi_I \leq 2\pi. \end{cases} \quad (4.16)$$

According to antenna theory [94], by assuming 100% radiation efficiency, we can calculate the corresponding antenna gain as

$$G_I^{\text{Idealized}}(\theta_I, \varphi_I) = \begin{cases} \frac{2}{1 - \cos \frac{\pi}{L}}, & 0 \leq \theta_I \leq \frac{\pi}{L}, \\ 0, & \frac{\pi}{L} < \theta_I \leq \pi. \end{cases} \quad (4.17)$$

With  $0 \leq \theta_{IT} \leq \frac{\pi}{L}$ , and  $0 \leq \theta_{IU,k} \leq \frac{\pi}{L}$ ,  $\forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}$ , the corresponding path loss is given by

$$\zeta_k^{\text{Idealized}} = \frac{4^3 \pi^4 d_{IT}^{\epsilon_{IT}} d_{UI,k}^{\epsilon_{UI,k}} (1 - \cos \frac{\pi}{L})^2}{\lambda^4 G_T G_U}, \forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}. \quad (4.18)$$

**Practical Radiation Pattern:** We then choose a proper and popular radiation pattern function for each antenna of the  $L$ -sector BD-RIS, which is given by [94]

$$F_I^{\text{Practical}}(\theta_I, \varphi_I) = \begin{cases} \cos^{\alpha_L} \theta_I, & 0 \leq \theta_I \leq \frac{\pi}{2}, 0 \leq \varphi_I \leq 2\pi, \\ 0, & \frac{\pi}{2} < \theta_I \leq \pi, 0 \leq \varphi_I \leq 2\pi, \end{cases} \quad (4.19)$$

where  $\alpha_L > 0$ . Then we can obtain the half-power beamwidth (HPBW) of an  $L$ -sector BD-RIS by setting  $F_I(\theta_I, \varphi_I) = 0.5$ , which yields<sup>7</sup>

$$\text{HPBW}_I^{\text{Practical}} = 2 \arccos \sqrt[{\alpha_L}]{0.5} = \frac{2\pi}{L}, \quad (4.20)$$

<sup>7</sup>Here we set the HPBW of each BD-RIS antenna as  $\frac{2\pi}{L}$  to guarantee the full-space coverage. In this case, adjacent sectors are slightly overlapped with each other.

where  $\alpha_L$  can be determined by solving the equation function  $\sqrt[3]{0.5} = \cos \frac{\pi}{L}$ <sup>8</sup>. The antenna gain with  $\eta = 1$  is

$$G_I^{\text{Practical}}(\theta_I, \varphi_I) = \begin{cases} 2(\alpha_L + 1) \cos^{\alpha_L} \theta_I, & 0 \leq \theta_I \leq \frac{\pi}{2}, \\ 0, & \frac{\pi}{2} < \theta_I \leq \pi. \end{cases} \quad (4.21)$$

With  $0 \leq \theta_{IT} \leq \frac{\pi}{L}$ , and  $0 \leq \theta_{IU,k} \leq \frac{\pi}{L}$ ,  $\forall k \in \mathcal{K}_l$ ,  $\forall l \in \mathcal{L}$ , the path loss is given by

$$\zeta_k^{\text{Practical}} = \frac{4^3 \pi^4 d_{IT}^{\epsilon_{IT}} d_{UI,k}^{\epsilon_{UI,k}}}{\lambda^4 G_T G_U (\alpha_L + 1)^2 \cos^{\alpha_L} \theta_{IT} \cos^{\alpha_L} \theta_{UI,k}}, \forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}. \quad (4.22)$$

With the path loss model (4.18) or (4.22), we can model the multi-sector BD-RIS-assisted channel  $\mathbf{H}_k$  (4.5) as a combination of large-scale and small-scale fading, i.e.,

$$\mathbf{H}_{\text{all},k} = \sqrt{\zeta_k^{-1}} \tilde{\mathbf{H}}_k \Phi_l \tilde{\mathbf{G}}, \forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}, \quad (4.23)$$

where  $\zeta_k$ ,  $\forall k \in \mathcal{K}_l$ ,  $\forall l \in \mathcal{L}$  denote path loss components accounting for the large-scale fading.  $\tilde{\mathbf{H}}_k$  and  $\tilde{\mathbf{G}}$  denote the small-scale fading, such as Rician fading, i.e.,

$$\begin{aligned} \tilde{\mathbf{H}}_k &= \sqrt{\frac{\kappa_k}{\kappa_k + 1}} \tilde{\mathbf{H}}_k^{\text{LoS}} + \sqrt{\frac{1}{\kappa_k + 1}} \tilde{\mathbf{H}}_k^{\text{NLoS}}, \quad \forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}, \\ \tilde{\mathbf{G}} &= \sqrt{\frac{\kappa}{\kappa + 1}} \tilde{\mathbf{G}}^{\text{LoS}} + \sqrt{\frac{1}{\kappa + 1}} \tilde{\mathbf{G}}^{\text{NLoS}}, \end{aligned} \quad (4.24)$$

where  $\kappa_k$  and  $\kappa$  denote the Rician factor, channels with superscript “LoS” and “NLoS” refer to line-of-sight (LoS) component modeled based on the steering vector and non-line-of-sight (NLoS) component modeled as i.i.d. Rayleigh fading, respectively. Specifically, the transmitter and receiver  $k$  are modeled as an  $N$ -antenna uniform linear array (ULA) and an  $N_k$ -antenna ULA with half-wavelength antenna spacing, yielding the steering vectors

$$\begin{aligned} \mathbf{a}_T(\theta_T) &= [1, e^{j\pi \sin \theta_T}, \dots, e^{j\pi(N-1) \sin \theta_T}]^T, \\ \mathbf{a}_{U,k}(\theta_{U,k}) &= [1, e^{j\pi \sin \theta_{U,k}}, \dots, e^{j\pi(N_k-1) \sin \theta_{U,k}}]^T. \end{aligned} \quad (4.25)$$

Each sector of the multi-sector BD-RIS is modeled as an  $M$ -antenna uniform planar array (UPA) with  $M_x$  antennas in the x-direction and  $M_y$  antennas in the y-direction, yielding the

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<sup>8</sup>For positive  $\cos \frac{\pi}{L}$ , the parameter  $\alpha_L$  can also be calculated analytically as  $\alpha_L = \frac{\log(0.5)}{\log(\cos \frac{\pi}{L})}$ .

steering vector

$$\begin{aligned}\mathbf{a}_I(\theta_{IT}, \varphi_{IT}) &= [1, e^{j\pi \sin \theta_{IT} \cos \varphi_{IT}}, \dots, e^{j\pi(M_x-1) \sin \theta_{IT} \cos \varphi_{IT}}]^T \\ &\quad \otimes [1, e^{j\pi \sin \theta_{IT} \sin \varphi_{IT}}, \dots, e^{j\pi(M_y-1) \sin \theta_{IT} \sin \varphi_{IT}}]^T, \\ \mathbf{a}_I(\theta_{UI,k}, \varphi_{UI,k}) &= [1, e^{j\pi \sin \theta_{UI,k} \cos \varphi_{UI,k}}, \dots, e^{j\pi(M_x-1) \sin \theta_{UI,k} \cos \varphi_{UI,k}}]^T \\ &\quad \otimes [1, e^{j\pi \sin \theta_{UI,k} \sin \varphi_{UI,k}}, \dots, e^{j\pi(M_y-1) \sin \theta_{UI,k} \sin \varphi_{UI,k}}]^T,\end{aligned}\tag{4.26}$$

Therefore, the LoS components for the transmitter-RIS channel and for the RIS-user channels are respectively given by

$$\tilde{\mathbf{G}}^{\text{LoS}} = \mathbf{a}_I(\theta_{IT}, \varphi_{IT}) \mathbf{a}_T(\theta_T)^H, \quad \tilde{\mathbf{H}}_k^{\text{LoS}} = \mathbf{a}_{U,k}(\theta_{U,k}) \mathbf{a}_I(\theta_{UI,k}, \varphi_{UI,k})^H, \quad \forall k \in \mathcal{K}.\tag{4.27}$$

Note that there are twofold differences between STAR-RIS/IOS [73,74] and the proposed multi-sector BD-RIS.

- 1) STAR-RIS/IOS is a special instance of our proposed multi-sector BD-RIS with  $L = 2$ .
- 2) Our model includes the effect of antenna beamwidth (and gain). Since the coverage of each sector of the multi-sector BD-RIS decreases with the growth of the number of sectors  $L$ , the antennas utilized to realize the multi-sector BD-RIS have narrower beamwidth, and thus higher gains. The antenna gain has a significant influence on the fading channels and thus on the system performance as illustrated in the following sections.

Now with the channel model (4.23), together with the path loss model (4.18) or (4.22) and the small-scale fading model (4.24)-(4.27), and the cell-wise single-connected multi-sector BD-RIS model with continuous values (4.9) or discrete values (4.10), we finally establish the multi-sector BD-RIS-assisted communication model. In the following, we will first discuss the potential applications of the multi-sector BD-RIS and then use this model to evaluate the performance of multi-sector BD-RIS-assisted communication systems.

## 4.2 Scaling Law

In this section, we analyze how the received power of the multi-sector BD-RIS-assisted system scales as a function of the number of sectors  $L$  to show the benefit of the proposed multi-sector BD-RIS. Different from the scaling law in [13], where the impact of the number of RIS elements to the received power is investigated, here we focus on the impact of the number of sectors  $L$ , which is a novel factor based on the proposed multi-sector BD-RIS model.

We consider a multi-sector BD-RIS-assisted SU-SISO system ( $N = K = N_k = 1$ ) for simplicity. The single-antenna transmitter is allocated within the coverage of sector 1 of the multi-sector

BD-RIS with  $\theta_{IT} = 0$ . The single-antenna user is randomly allocated around the multi-sector BD-RIS, e.g., within the coverage of sector  $l'$ ,  $\forall l' \in \mathcal{L}$ . The transmitting symbol is  $s \in \mathbb{C}$  with  $\mathbb{E}\{|s|^2\} = 1$ . According to equation (4.23), the received signal is expressed as

$$y = \sqrt{P_T} \mathbf{h} \Phi_{l'} \mathbf{g} s + n = \sqrt{P_T \zeta^{-1}} \tilde{\mathbf{h}} \Phi_{l'} \tilde{\mathbf{g}} s + n, \quad (4.28)$$

where  $P_T$  denotes the transmitting power,  $\mathbf{h} \in \mathbb{C}^{1 \times M}$  and  $\mathbf{g} \in \mathbb{C}^{M \times 1}$  denote channels from sector  $l'$  of the multi-sector BD-RIS to the user and from the transmitter to sector 1 of the multi-sector BD-RIS with small-scale fading components  $\tilde{\mathbf{h}}$  and  $\tilde{\mathbf{g}}$ , respectively.  $\zeta$  is the path loss and  $n$  is the noise. The received signal power at the user side is given by

$$P_U = \left| \sqrt{P_T \zeta^{-1}} \tilde{\mathbf{h}} \Phi_{l'} \tilde{\mathbf{g}} \right|^2 = P_T \zeta^{-1} |\tilde{\mathbf{h}} \Phi_{l'} \tilde{\mathbf{g}}|^2. \quad (4.29)$$

To achieve maximum received signal power, we assume that when the user is within the coverage of sector  $l'$ ,  $\forall l' \in \mathcal{L}$ , the corresponding sector is activated. In this case, we have

$$\Phi_l = \begin{cases} \text{diag}(\phi_{(l-1)M+1}, \dots, \phi_{lM}), & l = l', \\ \mathbf{0}, & l \neq l', \end{cases} \quad \forall l \in \mathcal{L}, \quad (4.30)$$

with  $|\phi_j| = 1, \forall j \in \mathcal{M}_{l'}$ . Then, the received power is maximized to

$$P_U^{\max} = P_T \zeta^{-1} \left( \sum_{\forall m \in \mathcal{M}} |\tilde{\mathbf{h}}_m \tilde{\mathbf{g}}_m| \right)^2. \quad (4.31)$$

We assume  $\tilde{\mathbf{h}}$  and  $\tilde{\mathbf{g}}$  are LoS channels for simplicity, i.e.,  $\tilde{\mathbf{h}} = [e^{j\psi_{UI,1}}, \dots, e^{j\psi_{UI,M}}]$  and  $\tilde{\mathbf{g}} = [e^{j\psi_{IT,1}}, \dots, e^{j\psi_{IT,M}}]^T$ , while the following scaling laws can be easily generalized to different small-scale fading conditions. Then with  $\phi_{(l'-1)M+m} = e^{-j(\psi_{UI,m} + \psi_{IT,m})}$ ,  $\forall m \in \mathcal{M}$ , we have

$$P_U^{\max} = P_T \zeta^{-1} M^2. \quad (4.32)$$

It can be observed from equation (4.32) that with fixed  $P_T$ , the maximum received power is determined by the number of cells  $M$  and the path loss, both of which are highly associated with the number of sectors  $L$ . In the following, we will derive the relationship between the maximum received power and  $L$  by considering both idealized and practical radiation patterns.

**Idealized Radiation Pattern:** In this case, the path loss is independent of the location of the user. Therefore, the maximum received signal power is given by

$$P_U^{\text{Idealized}} = \frac{P_T M^2 \lambda^4 G_T G_U}{4^3 \pi^4 d_{IT}^2 d_{UI}^2 (1 - \cos \frac{\pi}{L})^2}, \quad (4.33)$$

where  $d_{UI}$  denotes the distance between the multi-sector BD-RIS and the user and the path loss components are set as  $\epsilon_{IT} = \epsilon_{UI} = 2$  for LoS propagations. From (4.33) we can deduce

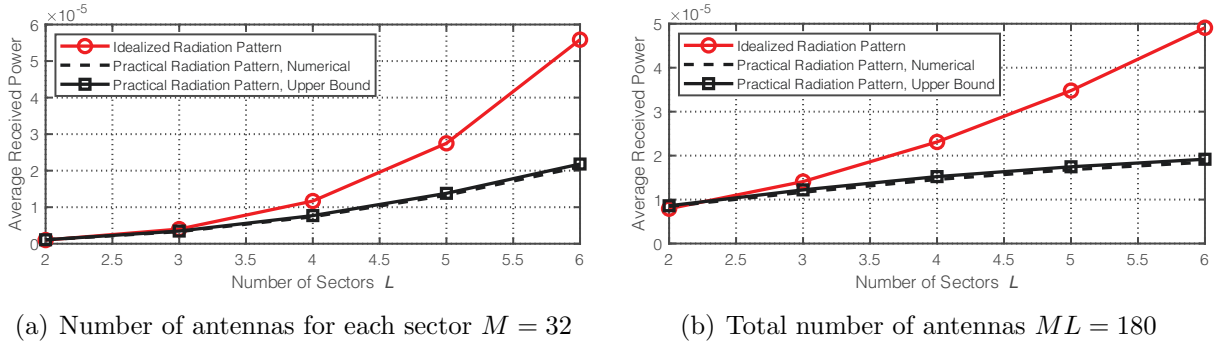


Figure 4.6: Average received power versus the number of sectors  $L$  for the multi-sector BD-RIS. ( $G_T = G_U = 1$ ,  $P_T = 10$  W,  $d_{IT} = 10$  m,  $d_{UI} = 2$  m,  $\lambda = c/f$  with  $f = 2.4$  GHz.)

that the maximum received power is proportional to the number of sectors  $L$  ( $L \geq 2$ ) when other terms in equation (4.33) are fixed. However, when the number of antennas for the whole multi-sector BD-RIS, i.e.,  $ML$ , is fixed, the number of antennas for each sector  $M$  is inversely proportional to the number of sectors  $L$ . There might be a trade-off between  $M$  and  $L$ , which will be discussed in the following simulations.

**Practical Radiation Pattern:** The maximum received signal power with  $\theta_{IT} = 0$  in this case is given by

$$P_U^{\text{Practical}} = \frac{P_T M^2 \lambda^4 G_T G_U}{4^3 \pi^4 d_{IT}^2 d_{UI}^2} (\alpha_L + 1)^2 \cos^{\alpha_L} \theta_{UI}. \quad (4.34)$$

Given that the user is randomly allocated around the multi-sector BD-RIS, we have  $\theta_{UI} \sim \mathcal{U}(0, \frac{\pi}{L})$ , where  $\mathcal{U}(0, \frac{\pi}{L})$  denotes the continuous uniform distribution with boundaries 0 and  $\frac{\pi}{L}$ . Therefore, the average of  $P_U^{\text{Practical}}$  is given by

$$\begin{aligned} \bar{P}_U^{\text{Practical}} &= \mathbb{E}\{P_U^{\text{Practical}}\} \\ &= \frac{P_T M^2 \lambda^4 G_T G_U}{4^3 \pi^4 d_{IT}^2 d_{UI}^2} (\alpha_L + 1)^2 \mathbb{E}\{\cos^{\alpha_L} \theta_{UI}\} \\ &\stackrel{(a)}{\leq} \frac{P_T M^2 \lambda^4 G_T G_U}{4^3 \pi^4 d_{IT}^2 d_{UI}^2} (\alpha_L + 1)^2 \cos^{\alpha_L} \mathbb{E}\{\theta_{UI}\} \\ &= \frac{P_T M^2 \lambda^4 G_T G_U}{4^3 \pi^4 d_{IT}^2 d_{UI}^2} (\alpha_L + 1)^2 \cos^{\alpha_L} \frac{\pi}{2L} = \tilde{P}_U^{\text{Practical}}, \end{aligned} \quad (4.35)$$

where (a) holds following the Jensen's inequality. From the relationship  $\sqrt[4]{0.5} = \cos \frac{\pi}{L}$  we can obtain that the number of sectors  $L$  is proportional to  $\alpha_L$ . In addition, with the growth of  $L$ ,  $\cos^{\alpha_L} \frac{\pi}{2L}$  is always within the range  $0.5 < \cos^{\alpha_L} \frac{\pi}{2L} < 1$ , while  $\alpha_L$  grows significantly, such that  $\tilde{P}_U^{\text{Practical}}$  is proportional to the number of sectors  $L$  (with fixed  $M$ ). Similarly, when we fix the number of antennas  $ML$  for the whole multi-sector BD-RIS, the relationship between the maximum received power and the number of sectors  $L$  might not be that straightforward.

To clarify the relationship between the received power and the number of sectors  $L$ , in Fig. 4.6(a) we first plot the received power  $P_U^{\text{Idealized}}$  using an idealized radiation pattern,  $\bar{P}_U^{\text{Practical}}$

with a practical radiation pattern, and its upper bound  $\tilde{P}_U^{\text{Practical}}$  versus the number of sectors  $L$  when the number of antennas for each sector is fixed to  $M = 32$ . It can be observed from Fig. 4.6 that the received power increase with the growth of  $L$ , and that  $\bar{P}_U^{\text{Practical}}$  can finely approach its upper bound  $\tilde{P}^{\text{Practical}}$ . Then in Fig. 4.6(b), we plot the received power versus the number of sectors when the number of antennas for the whole multi-sector BD-RIS is fixed as  $ML = 180$ . In this case, the dimension of each sector of the multi-sector BD-RIS decreases with the growth of the number of sectors  $L$ . The received power still increases significantly, which indicates the impact of the antenna gain is more dominant than that of the dimension of RIS sectors. Based on the above analysis, the proposed multi-sector BD-RIS-assisted system is expected to achieve better performance when the number of sectors gets larger. To verify this conjecture, in the following section we will focus on the beamforming design to maximize the sum-rate performance for the multi-sector BD-RIS-assisted system.

### 4.3 Beamforming Design

In this section, we apply the multi-sector BD-RIS in MU-MISO communication systems and propose algorithms to design the multi-sector BD-RIS when its scattering matrices have continuous and discrete values.

#### 4.3.1 Problem Formulation and Optimization Framework

We consider a multi-sector BD-RIS-assisted MU-MISO system ( $N_k = 1, \forall k \in \mathcal{K}$ ). The transmitting symbol vector is defined as  $\mathbf{s} \triangleq [s_1, \dots, s_K]^T \in \mathbb{C}^{K \times 1}$  with  $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_K$ . The transmitting symbols are precoded by a precoder matrix  $\mathbf{W} \triangleq [\mathbf{w}_1, \dots, \mathbf{w}_K] \in \mathbb{C}^{N \times K}$  with  $\mathbf{w}_k \in \mathbb{C}^{N \times 1}$  for user  $k, \forall k \in \mathcal{K}$ . With equations (4.5) and (4.7), the received signal  $y_k$  for user  $k$  is thus expressed as follows

$$y_k = \mathbf{h}_k \Phi_l \mathbf{G} \mathbf{W} \mathbf{s} + n_k = \mathbf{v}_{k,k}^H \phi_l s_k + \sum_{\substack{p \in \mathcal{K} \\ p \neq k}} \mathbf{v}_{k,p}^H \phi_l s_p + n_k, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}, \quad (4.36)$$

where  $\mathbf{v}_{k,p} = (\mathbf{h}_k \text{diag}(\mathbf{G} \mathbf{w}_p))^H, \forall p, k \in \mathcal{K}, \phi_l = [\phi_{(l-1)M+1}, \dots, \phi_{lM}]^T, \forall l \in \mathcal{L}, \mathbf{h}_k \in \mathbb{C}^{1 \times M}, \forall k \in \mathcal{K}$  is the channel vector between the multi-sector BD-RIS and user  $k$ , and  $n_k \sim \mathcal{CN}(0, \sigma_k^2)$  is the noise. The sum-rate maximization problem can be formulated as

$$\max_{\substack{\mathbf{W} \\ \{\phi_l\}_{\forall l \in \mathcal{L}}}} \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}_l} \log_2 \left( 1 + \frac{|\mathbf{v}_{k,k}^H \phi_l|^2}{\sum_{\substack{p \in \mathcal{K} \\ p \neq k}} |\mathbf{v}_{k,p}^H \phi_l|^2 + \sigma_k^2} \right) \quad (4.37a)$$

$$\text{s.t.} \quad (4.9) \text{ or } (4.10), \quad (4.37b)$$

$$\|\mathbf{W}\|_F^2 \leq P_T. \quad (4.37c)$$

Problem (4.37) is a typical sum-of-functions-of-ratio optimization, which can be solved by the general algorithm proposed in Chapter 3. However, phase shifts and amplitudes of non-zero multi-sector BD-RIS elements are partitioned based on the illustration in the previous section, which will facilitate the multi-sector BD-RIS design. Therefore, in this work we follow the framework in Chapter 3, whose main idea is to first transform the original problem into a multi-block optimization and then iteratively design each block until convergence, but reconsider the design of the multi-sector BD-RIS block. Specifically, we first apply  $2K$  auxiliary variables,  $\iota_k$  and  $\tau_k$ ,  $\forall k \in \mathcal{K}$ , and transform (4.37) based on the fractional programming theory [83] reviewed in Section 2.2.1 into the following four-block optimization:

$$\begin{aligned} \max_{\substack{\mathbf{W}, \{\iota_k\}_{\forall k \in \mathcal{K}}, \{\tau_k\}_{\forall k \in \mathcal{K}}, \{\phi_l\}_{\forall l \in \mathcal{L}}}} \quad & \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}_l} \left( \log_2(1 + \iota_k) - \iota_k + 2\sqrt{1 + \iota_k} \Re\{\tau_k^* \mathbf{v}_{k,k}^H \phi_l\} - |\tau_k|^2 \left( \sum_{p \in \mathcal{K}} |\mathbf{v}_{k,p}^H \phi_l|^2 + \sigma_k^2 \right) \right) \\ \text{s.t.} \quad & (4.37\text{b}), (4.37\text{c}). \end{aligned} \quad (4.38)$$

Solutions to blocks  $\{\iota_k\}_{\forall k \in \mathcal{K}}$ ,  $\{\tau_k\}_{\forall k \in \mathcal{K}}$ , and  $\mathbf{W}$  are similar to that in Chapter 3 with simple scaling of matrix dimensions, which are thus omitted. In the following subsection, we will propose efficient algorithms to determine block  $\{\phi_l\}_{\forall l \in \mathcal{L}}$  for both continuous and discrete cases.

### 4.3.2 Solution to Multi-Sector BD-RIS

When blocks  $\{\iota_k\}_{\forall k \in \mathcal{K}}$ ,  $\{\tau_k\}_{\forall k \in \mathcal{K}}$ , and  $\mathbf{W}$  are given, we can remove the constant terms in problem (4.38) and formulate the sub-problem with respect to  $\{\phi_l\}_{\forall l \in \mathcal{L}}$  as

$$\max_{\{\phi_l\}_{\forall l \in \mathcal{L}}} \sum_{l \in \mathcal{L}} \left( 2\Re\{\tilde{\mathbf{v}}_l^H \phi_l\} - \phi_l^H \mathbf{V}_l \phi_l \right) \quad \text{s.t.} \quad (4.37\text{b}), \quad (4.39)$$

where  $\tilde{\mathbf{v}}_l = (\sum_{k \in \mathcal{K}_l} \sqrt{1 + \iota_k} \tau_k^* \mathbf{v}_{k,k}^H)^H$ , and  $\mathbf{V}_l = \sum_{k \in \mathcal{K}_l} |\tau_k|^2 \sum_{p \in \mathcal{K}} \mathbf{v}_{k,p} \mathbf{v}_{k,p}^H$ ,  $\forall l \in \mathcal{L}$ . We successively design each cell of the multi-sector BD-RIS with fixed other cells until the convergence is guaranteed. Specifically, the sub-problem for the design of cell  $m$ , i.e.,  $\phi_i$ ,  $\forall i \in \mathcal{L}_m$  while fixing the rest of  $M - 1$  cells is given by

$$\min_{\{\phi_i\}_{\forall i \in \mathcal{L}_m}} \sum_{i \in \mathcal{L}_m} \left( \nu_i |\phi_i|^2 + 2\Re\{\phi_i^* \chi_i\} \right) \quad \text{s.t.} \quad (4.37\text{b}), \quad (4.40\text{a})$$

$$\Leftrightarrow \min_{\substack{\{\beta_i\}_{\forall i \in \mathcal{L}_m} \\ \{\vartheta_i\}_{\forall i \in \mathcal{L}_m}}} \sum_{i \in \mathcal{L}_m} \left( \nu_i \beta_i + 2|\chi_i| \sqrt{\beta_i} \cos(\angle \chi_i - \vartheta_i) \right) \quad \text{s.t.} \quad (4.37\text{b}), \quad (4.40\text{b})$$

where  $\chi_{(l-1)M+m} = \sum_{n \neq m} [\mathbf{V}_l]_{m,n} \phi_{(l-1)M+n} - [\tilde{\mathbf{v}}_l]_m$ , and  $\nu_{(l-1)M+m} = [\mathbf{V}_l]_{m,m}$ ,  $\forall l \in \mathcal{L}$ ,  $\forall m \in \mathcal{M}$ . In the following we will consider the phase shift and amplitude design for both continuous and discrete cases.

**Continuous Case with Constraint (4.9):** In this case, phase shifts  $\vartheta_i$ ,  $\forall i \in \mathcal{L}_m$  can be

easily obtained by setting  $\cos(\angle \chi_i - \vartheta_i) = -1, \forall i \in \mathcal{L}_m$ , which yields

$$\theta_i^* = \angle \chi_i \pm \pi, \forall i \in \mathcal{L}_m. \quad (4.41)$$

Substituting  $\theta_i^*, \forall i \in \mathcal{L}_m$  into objective (4.40b), we can obtain a real-value problem with respect to  $\beta_i, \forall i \in \mathcal{L}_m$  as

$$\min_{\{\beta_i\}_{\forall i \in \mathcal{L}_m}} \sum_{i \in \mathcal{L}_m} \left( \nu_i \beta_i - 2|\chi_i| \sqrt{\beta_i} \right) \quad (4.42a)$$

$$\text{s.t. } 0 \leq \beta_i \leq 1, \forall i \in \mathcal{L}_m, \quad (4.42b)$$

$$\boldsymbol{\beta}_m^T \boldsymbol{\beta}_m = 1, \quad (4.42c)$$

$$\Leftrightarrow \min_{\boldsymbol{\beta}_m} \underbrace{\boldsymbol{\beta}_m^T \boldsymbol{\Psi}_m \boldsymbol{\beta}_m - 2\boldsymbol{\chi}_m^T \boldsymbol{\beta}_m}_{=F(\boldsymbol{\beta}_m)} \quad \text{s.t. (4.42b), (4.42c)}, \quad (4.42d)$$

where  $\boldsymbol{\beta}_m = [\sqrt{\beta_m}, \sqrt{\beta_{M+m}}, \dots, \sqrt{\beta_{(L-1)M+m}}]^T \in \mathbb{R}^{L \times 1}$ ,  $\boldsymbol{\Psi}_m = \text{diag}(\nu_m, \nu_{M+m}, \dots, \nu_{(L-1)M+m}) \in \mathbb{R}^{L \times L}$ , and  $\boldsymbol{\chi}_m = [|\chi_m|, |\chi_{M+m}|, \dots, |\chi_{(L-1)M+m}|]^T \in \mathbb{R}^{L \times 1}$ . Since  $|\chi_i| > 0, \forall i \in \mathcal{L}_m$ , constraint (4.42b) can always be satisfied when problem (4.42) achieve its minimum. This observation motivates us to ignore constraint (4.42b), which yields

$$\boldsymbol{\beta}_m^* = \arg \min_{\boldsymbol{\beta}_m^T \boldsymbol{\beta}_m = 1} F(\boldsymbol{\beta}_m). \quad (4.43)$$

The optimal solution to problem (4.43) can be determined by the following lemma [96].

**Lemma 4.1.**  $\boldsymbol{\beta}_m$  is the optimal solution to problem (4.43) if and only if there exists  $\mu \in \mathbb{R}$  such that  $(\boldsymbol{\Psi}_m + \mu \mathbf{I}_L) \boldsymbol{\beta}_m = \boldsymbol{\chi}_m$ ,  $\boldsymbol{\Psi}_m + \mu \mathbf{I}_L$  positive semi-definite, and  $\boldsymbol{\beta}_m^T \boldsymbol{\beta}_m = 1$ .

*Proof.* Please refer to Appendix B.1. □

According to Lemma 4.1,  $\boldsymbol{\beta}_m$  has a closed-form solution as a function of  $\mu$  constrained by

$$\begin{aligned} F'(\mu) &= \boldsymbol{\beta}_m^{*T} \boldsymbol{\beta}_m^* = \boldsymbol{\chi}_m^T (\boldsymbol{\Psi}_m + \mu \mathbf{I}_L)^{-2} \boldsymbol{\chi}_m \\ &= \text{Tr}\{(\boldsymbol{\Psi}_m + \mu \mathbf{I}_L)^{-2} \boldsymbol{\chi}_m \boldsymbol{\chi}_m^T\} \\ &\stackrel{(a)}{=} \text{Tr}\{(\mathbf{U}_m \boldsymbol{\Sigma}_m \mathbf{U}_m^T + \mu \mathbf{I}_L)^{-2} \boldsymbol{\chi}_m \boldsymbol{\chi}_m^T\} \\ &= \text{Tr}\{(\boldsymbol{\Sigma}_m + \mu \mathbf{I}_L)^{-2} \mathbf{U}_m^T \boldsymbol{\chi}_m \boldsymbol{\chi}_m^T \mathbf{U}_m\} \\ &\stackrel{(b)}{=} \sum_{l \in \mathcal{L}} \frac{\tilde{\chi}_{m,l}^2}{(\varepsilon_{m,l} + \mu)^2} = 1, \end{aligned} \quad (4.44)$$

where (a) holds by performing the eigenvalue decomposition  $\boldsymbol{\Psi}_m = \mathbf{U}_m \boldsymbol{\Sigma}_m \mathbf{U}_m^T$  with  $\boldsymbol{\Sigma}_m = \text{diag}(\varepsilon_{m,1}, \dots, \varepsilon_{m,L})$ ,  $\varepsilon_{m,1} \leq \varepsilon_{m,2} \leq \dots \leq \varepsilon_{m,L}$ , and  $\mathbf{U}_m = [\mathbf{u}_{m,1}, \dots, \mathbf{u}_{m,L}]$ ; (b) holds by defining  $\tilde{\chi}_{m,l} = \boldsymbol{\chi}_m^T \mathbf{u}_{m,l}$ ,  $\forall l \in \mathcal{L}$ . Recall that  $|\chi_i| > 0, \nu_i > 0, \forall i \in \mathcal{L}_m$ . We have  $\tilde{\chi}_{m,l} \neq 0$  and thus  $\varepsilon_{m,l} + \mu \neq 0$ . In addition, according to Lemma 4.1,  $\boldsymbol{\Psi}_m + \mu \mathbf{I}_L$  is positive semi-definite such that

$\varepsilon_{m,l} + \mu \geq 0$ . Therefore, the optimal value of  $\mu$  should be chosen such that  $\varepsilon_{m,l} + \mu > 0, \forall l \in \mathcal{L}$  and  $g(\mu) = 1$ . In the following lemma, we derive the upper bound and lower bound of  $\mu$  [96].

**Lemma 4.2.** Define  $\mathcal{E} = \{l | \varepsilon_{m,l} = \varepsilon_{m,1}, \forall l \in \mathcal{L}\}$ . The optimal solution of  $\mu$  is chosen within the range  $\sqrt{\sum_{\forall l \in \mathcal{E}} \tilde{\chi}_{m,l}^2} - \varepsilon_{m,1} \leq \mu \leq \|\mathbf{\chi}_m\|_2 - \varepsilon_{m,1}$ .

*Proof.* Please refer to Appendix B.2. □

After determining the upper and lower bounds of  $\mu$  according to Lemma 4.2, we can use efficient methods, such as bisection search, to find the optimal  $\mu^*$  such that  $F'(\mu^*) = 1$ . The optimal solution of problem (4.43) is given by

$$\boldsymbol{\beta}_m^* = (\boldsymbol{\Psi}_m + \mu^* \mathbf{I}_L)^{-1} \mathbf{\chi}_m. \quad (4.45)$$

Combining (4.41) and (4.45), we finally obtain  $\phi_i^* = \sqrt{\beta_i^*} e^{j\vartheta_i^*}, \forall i \in \mathcal{L}_m$ .

**Discrete Case with Constraint (4.10):** In this case, we first find the solution of phase shifts  $\vartheta_i^*, \forall i \in \mathcal{L}_m$  for continuous case by (4.41). Then we apply a simple quantization operation to obtain the discrete phase shifts as

$$\vartheta_i^\# = \left\lceil \frac{\vartheta_i^*}{\Delta_B} \right\rceil \times \Delta_B, \forall i \in \mathcal{L}_m, \quad (4.46)$$

where  $\Delta_B = \frac{2\pi}{B}$  denotes the resolution of phase shift and  $\lceil \cdot \rceil$  denotes the rounding operation. Then we substitute (4.46) to problem (4.40), which yields

$$\beta_i^\# = \arg \min_{\substack{\sum_{i \in \mathcal{L}_m} \beta_i = 1 \\ \beta_i \in \mathcal{A}}} \sum_{\forall i \in \mathcal{L}_m} F_i(\beta_i), \quad (4.47)$$

where  $F_i(\beta_i) = \beta_i \nu_i + 2|\chi_i| \sqrt{\beta_i} \cos(\angle \chi_i - \vartheta_i^\#), \forall i \in \mathcal{L}_m$ . Ignoring the discrete constraint and following the procedure for the design of continuous case, we can obtain the continuous solution  $\beta_i^*, \forall i \in \mathcal{L}_m$ , which is further quantized by

$$\beta_i^\# = \left\lceil \frac{\beta_i^*}{\Delta_A} \right\rceil \times \Delta_A, \forall i \in \mathcal{L}_m, \quad (4.48)$$

where  $\Delta_A = \frac{1}{2^A - 1}$  denotes the resolution of amplitude square. It is worth noting that there is no guarantee of  $\sum_{i \in \mathcal{L}_m} \beta_i^\# = 1$ . More specifically, we have  $\sum_{i \in \mathcal{L}_m} \beta_i^\# = 1 + \varsigma \Delta_A, \varsigma \in \mathbb{Z}$  due to the rounding operation. In the following, we will propose a simple yet efficient algorithm to adjust the quantization result (4.48) so that the constraint is satisfied. The proposed algorithm is described in Phases 1-3.

Phase 1: Let  $\varsigma = \frac{1 - \sum_{i \in \mathcal{L}_m} \beta_i^\#}{\Delta_A}$ . If  $\varsigma = 0$ , the procedure stops; if  $\varsigma > 0$ , go to Phase 2; if  $\varsigma < 0$ , go to Phase 3.

Phase 2: Set  $\varsigma' = 1$ . In the  $\varsigma'$ -th loop, do the following procedures until  $\varsigma' > \varsigma$ : For  $\forall i \in \mathcal{L}_m$ , if  $\beta_i^\# < 1$ , calculate

$$\delta_i = F_i(\beta_i^\# + \Delta_A) - F_i(\beta_i^\#). \quad (4.49)$$

Otherwise, set  $\delta_i$  to a sufficient large number, e.g.,  $\infty$ . Then find  $i'$  such that

$$\delta_{i'} = \min_{\forall i \in \mathcal{L}_m} \delta_i, \quad (4.50)$$

and replace  $\beta_{i'}^\#$  with  $\beta_{i'}^\# + \Delta_A$ . Update  $\varsigma' = \varsigma' + 1$ .

Phase 3: Set  $\varsigma' = 1$ . In the  $\varsigma'$ -th loop, do the following procedures until  $\varsigma' > -\varsigma$ : For  $\forall i \in \mathcal{L}_m$ , if  $\beta_i^\# > 0$ , calculate

$$\delta_i = F_i(\beta_i^\# - \Delta_A) - F_i(\beta_i^\#). \quad (4.51)$$

Otherwise, set  $\delta_i$  to  $\infty$ . Then find  $i'$  satisfying

$$\delta_{i'} = \min_{\forall i \in \mathcal{L}_m} \delta_i, \quad (4.52)$$

and replace  $\beta_{i'}^\#$  with  $\beta_{i'}^\# - \Delta_A$ . Update  $\varsigma' = \varsigma' + 1$ .

After processing Phases 1-3, we can obtain the modified  $\beta_i^\#$ ,  $\forall i \in \mathcal{L}_m$  satisfying the constraint  $\sum_{\forall i \in \mathcal{L}_m} \beta_i^\# = 1$ , and the solution of problem (4.40) as  $\phi_i^\# = \sqrt{\beta_i^\#} e^{j\theta_i^\#}$ ,  $\forall i \in \mathcal{L}_m$ .

## 4.4 Performance Evaluation

In this section, we evaluate the performance of the multi-sector BD-RIS-assisted MU-MISO system. Antenna gains for transmitter antennas and user antennas are fixed to  $G_T = G_U = 1$ . The frequency of the transmit signal is set as  $f = 2.4$  GHz. Distances between the transmitter and the multi-sector BD-RIS and between the multi-sector BD-RIS and users are  $d_{\text{TR}} = 100$  m and  $d_{\text{UI},k} = 10$  m,  $\forall k \in \mathcal{K}$ , respectively. The path loss exponents are set as 2.5 when the Rician factors are set as  $\kappa_1 = \dots = \kappa_K = \kappa = 0$  dB, and 2 when the Rician factors are set as  $\kappa_1 = \dots = \kappa_K = \kappa = 10$  dB, given that the richer scattering environment leads to a higher path loss exponent. The noise power is  $\sigma_k^2 = -80$  dBm,  $\forall k \in \mathcal{K}$ . In the following simulations, we assume  $K$  users are uniformly divided into  $L$  groups, each of which contains  $K/L$  users randomly located within the coverage of one sector. We also fix the total number of multi-sector BD-RIS antennas  $ML$  for fair comparison, i.e., the antenna gain of each antenna increases with increasing  $L$ , while the dimension of each sector  $M$  decreases with  $L$ .

Fig. 4.7 shows the sum-rate performance versus the total number of antennas of multi-sector BD-RIS. We consider different numbers of sectors and different Rician factors. We have the following observations from Fig. 4.7. 1) Even with a reduced dimension for each sector, the

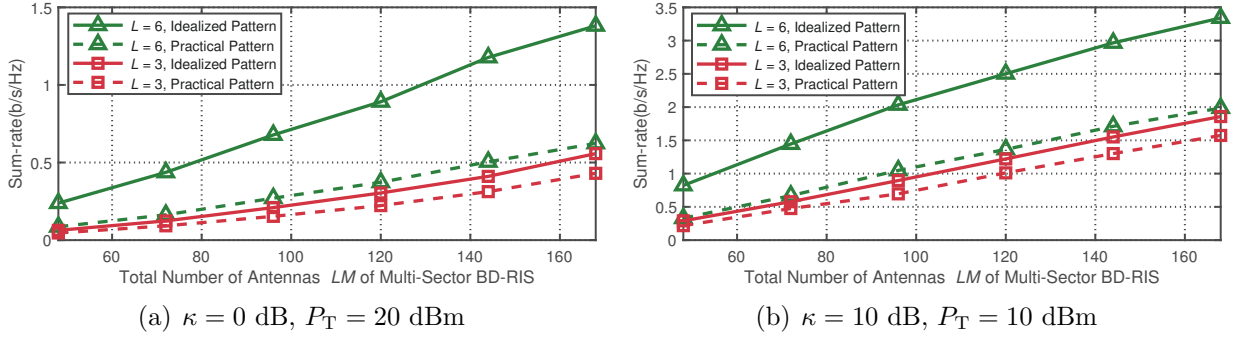


Figure 4.7: Sum-rate versus total number of antennas of multi-sector BD-RIS ( $N = K = 6$ ,  $M_y = 4$ ).

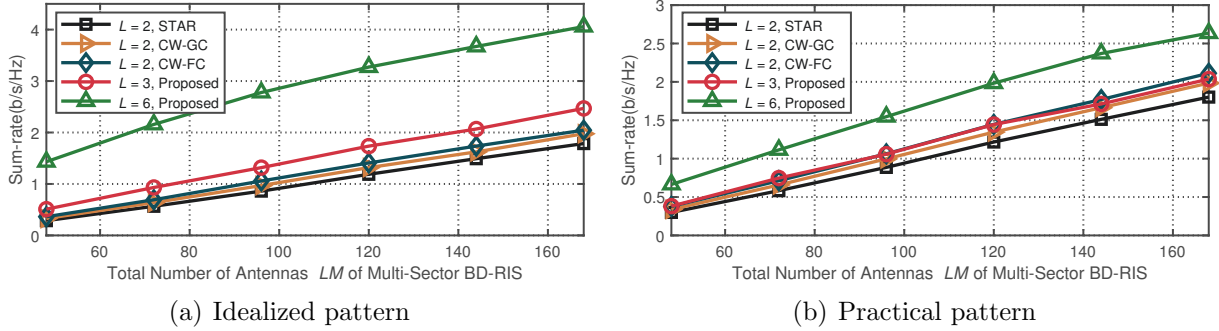


Figure 4.8: Sum-rate versus total number of antennas of multi-sector BD-RIS ( $N = K = 6$ ,  $M_y = 4$ ,  $\kappa = 0$  dB,  $P_T = 30$  dBm).

sum-rate performance achieved by the multi-sector BD-RIS with an increasing number of sectors  $L$  still enhances, which is in accordance with the conclusion in Fig. 4.6(b). This observation indicates that the higher antenna gain due to narrower beamwidth compensates for the reduced dimension of each sector, the reduced power split to each sector, and boosts the performance. 2) Given the same fading conditions, the multi-sector BD-RIS with a smaller number of antennas but a larger number of sectors can achieve similar performance to that with a larger number of antennas but a smaller number of sectors. For example, when the practical radiation pattern is adopted, the sum-rate performance with  $ML = 120$  and  $L = 3$  is similar to that with  $ML = 96$  and  $L = 6$  as shown in Fig. 4.7(a). This phenomenon implies the number of antennas for the whole multi-sector BD-RIS can be reduced by increasing the number of sectors, which is meaningful for practical realizations of the multi-sector BD-RIS. 3) With the fixed number of sectors, the sum-rate performance based on the idealized radiation pattern is always better than that based on the practical radiation pattern, which, again, matches with the result in Fig. 4.6(b).

In Fig. 4.8, we compare the sum-rate performance with existing RIS architectures. Specifically, STAR-RIS is essentially a special case of  $L = 2$  of our proposed model. We also add the performance of BD-RIS with  $L = 2$  and cell-wise group/fully-connected (CW-GC/FC) architectures proposed in Chapter 3, where the group size of CW-GC is fixed as 6, indicating that the CW-

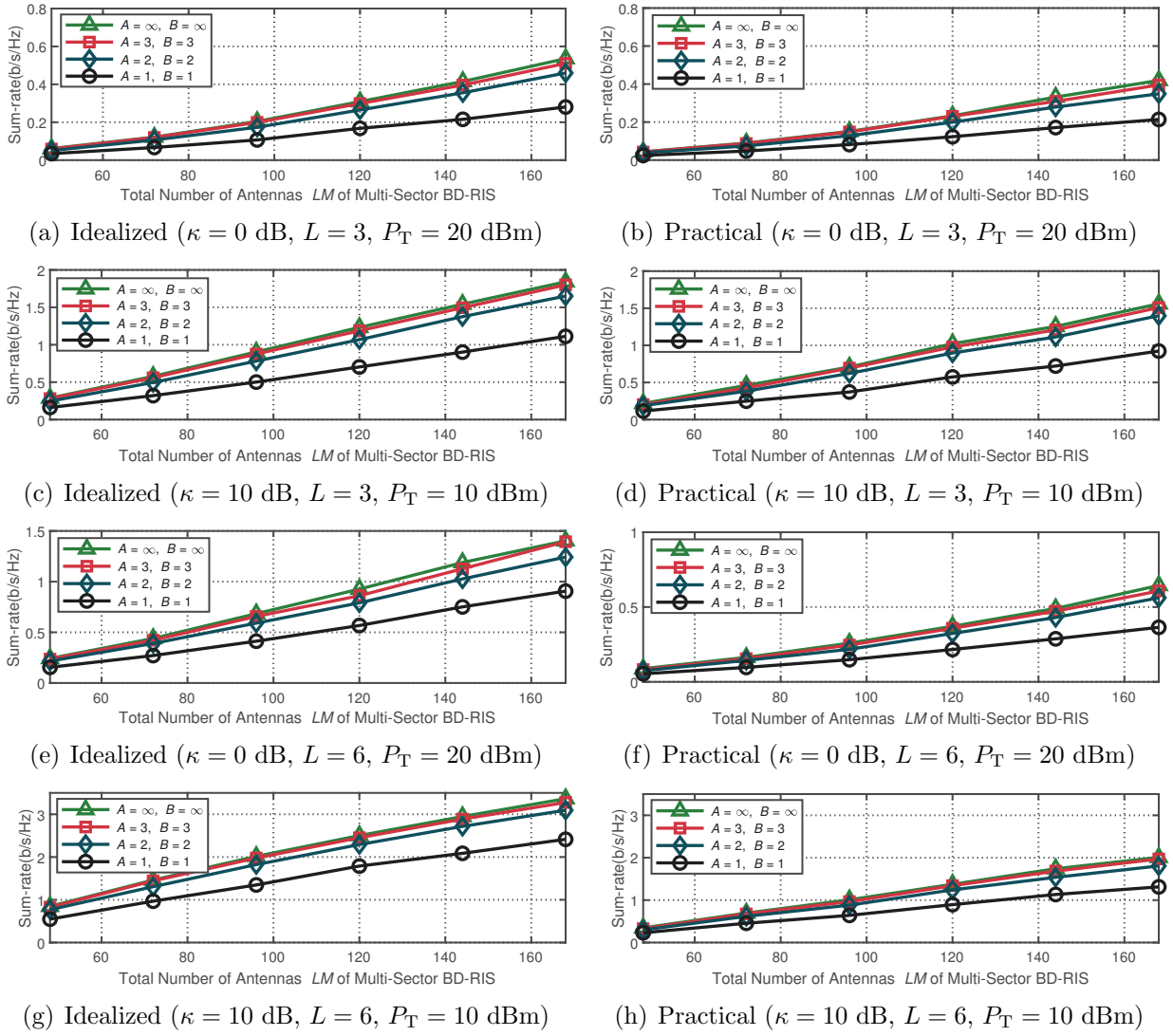


Figure 4.9: Sum-rate versus total number of antennas of multi-sector BD-RIS with different resolutions for amplitudes and phase shifts, different numbers of sectors, and different Rician fading factors ( $N = K = 6$ ,  $M_y = 4$ ).

GC scheme and the proposed model with  $L = 6$  have the same circuit complexity. From Fig. 4.8 we observe that when the idealized radiation pattern is adopted, the proposed multi-sector BD-RIS outperforms both STAR and CW-GC/FC schemes; when the practical radiation pattern is adopted, the proposed multi-sector BD-RIS outperforms STAR and CW-GC schemes, and achieves almost the same performance as the CW-FC scheme. More importantly, with the same circuit complexity of the  $ML$ -port reconfigurable impedance network, the proposed model (with  $L = 6$ ) can achieve much better sum-rate than the CW-GC scheme (with  $L = 2$ ). This observation highlights the benefit of antenna arrangements in boosting performance.

In Fig. 4.9 we plot the sum-rate performance as a function of the total number of antennas of multi-sector BD-RIS when each nonzero element of the multi-sector BD-RIS has discrete amplitudes and phase shifts controlled by different resolutions. Similar conclusions as Fig. 4.7 can be obtained from Fig. 4.9. Moreover, we have the following additional observations. 1)

The sum-rate performance achieved by multi-sector BD-RIS with discrete values increases with the growth of resolutions. Specifically, when the idealized radiation pattern is applied for the modeling of fading channels, the sum-rate with  $A = 3$  and  $B = 3$  is close to that for the continuous case; when the practical radiation pattern is applied for the channel model, the sum-rate with  $A = 2$  and  $B = 2$  is sufficient to achieve satisfactory performance. 2) When the total number of antennas of multi-sector BD-RIS is fixed, the performance gap between multi-sector BD-RIS with different resolutions become smaller with the growth of the number of sectors  $L$ . Recalling that the sum-rate performance increases with  $L$ , we can deploy the multi-sector BD-RIS with a larger number of sectors and lower resolutions to achieve the same performance as that with a smaller number of sectors and higher resolution. For example, the sum-rate performance with  $L = 3$ ,  $A = B = 2$ , and  $M = 40$  in Fig. 4.9(b) is almost the same as that with  $L = 6$ ,  $A = B = 1$ , and  $M = 20$  in Fig. 4.9(f). Such reduction of resolutions is beneficial for reducing the cost of multi-sector BD-RIS realizations.

## 4.5 Summary

This chapter proposes a novel multi-sector BD-RIS model. Particularly, we derive the constraints of the cell-wise single-connected multi-sector BD-RIS for both continuous and discrete cases. We also derive the multi-sector BD-RIS-aided channel model including the impact of antenna gain. The existing STAR-RIS/IOS is a special instance of our proposed multi-sector BD-RIS model.

With the proposed model, we derive the scaling law of the received power as a function of the number of sectors for the multi-sector BD-RIS-aided SU-SISO system. We show that there is a positive correlation between the maximum received power and the number of sectors.

We then apply the multi-sector BD-RIS in the MU-MISO system and propose simple yet efficient algorithms to maximize the sum-rate of the multi-sector BD-RIS-aided MU-MISO system when amplitudes/phase shifts of the RIS have either continuous values or discrete values selected from finite codebooks. Simulation results show that given the same sum-rate requirement, the number of antennas for the whole multi-sector BD-RIS can be reduced by increasing the number of sectors, and that the codebook resolutions can also be reduced by increasing the number of sectors.

## Part II

# Signal Processing of BD-RIS



## Chapter 5

# Synergizing BD-RIS and Rate-Splitting Multiple Access with Imperfect CSI

In the previous chapters, the benefits of BD-RIS in enhancing the channel strength and enlarging the coverage has been theoretically and numerically analyzed, based on perfect channel conditions and simple transmitter designs, i.e., SDMA. An important and unexplored direction is if synergizing advanced transmitter designs and BD-RIS could be a win-win solution in further enhancing the wireless service while easing the requirement of perfect CSI. Due to the potential advantages of RSMA in achieving higher degree of freedom [97], spectral and energy efficiency [98] than SDMA, and being robust to CSI uncertainty [97] and mobility [99], the synergy among RSMA and BD-RIS has recently gained much attention. On one hand, to realize full-space coverage in wireless networks, there are a few results on STAR-RIS aided RSMA considering spatially correlated channels [100], hardware impairments [101], PHY layer security [102], and the coupling of transmission and reflection phases [103]. On the other hand, to enhance the spectrum efficiency of wireless communications, there is only one work [104] considering the employment of BD-RIS with fully-connected architecture and reflective mode in RSMA transmission network, where the sum-rate enhancement compared to SDMA schemes is highlighted. Nevertheless, none of the existing work investigates the integration of BD-RIS and RSMA to simultaneously enlarge the coverage, boost the rate performance [104], and reduce the required number of transmitter/RIS antennas to improve energy efficiency. More importantly, the effect of imperfect CSI at the transmitter has not been captured. This chapter thus focuses on synergizing BD-RIS and RSMA under imperfect CSI conditions to simultaneously achieve the above goals. This leads to the following layouts and contributions.

In Section 5.1, the multi-sector BD-RIS and 1-layer RSMA at the transmitter have been adopted in a multiuser system to provide high performance gains, achieve additional degrees of freedom over conventional multiuser linear precoding and enable better inter-user interference management, and potentially save on the number of both active and passive antennas.

In Section 5.2, a beamforming design algorithm has been proposed to maximize the average sum-rate for multi-sector BD-RIS aided RSMA under imperfect CSI conditions. To solve the stochastic problem, we first approximate it as a deterministic optimization by sample average approximation (SAA) methods [105], and then transform the deterministic problem into a more tractable three-block optimization based on the weighted minimum mean square error (WMMSE)-rate relationship [106]. Initialization, complexity analysis, and convergence analysis of the proposed design are also provided.

In Section 5.3, simulation results have been provided to evaluate the ergodic sum-rate performance of the multi-sector BD-RIS aided RSMA compared to the SDMA scheme. Simulation results show that multi-sector BD-RIS aided RSMA always outperforms multi-sector BD-RIS aided SDMA. More importantly, with the same ergodic sum-rate requirement and quality of service (QoS) thresholds, we show the required numbers of transmitting and BD-RIS antennas can be reduced compared to the SDMA scheme by synergizing BD-RIS with RSMA. Such reduction of antenna numbers leads to a potential energy efficiency gain.

## 5.1 Multi-Sector BD-RIS Aided RSMA

In this section, we describe the multi-sector BD-RIS aided MU-MISO system, explain the architecture of 1-layer RSMA, and illustrate the modeling of CSI uncertainty.

### 5.1.1 Multi-Sector BD-RIS Aided MU-MISO

Consider a MU-MISO system operating in the downlink as illustrated in Chapter 4, where the transmitter equipped with  $N$  antennas communicates to a set of single-antenna users  $\mathcal{K} = \{1, \dots, K\}$  with the aid of an  $M$ -cell multi-sector BD-RIS<sup>1</sup>. We assume the transmitter is within the coverage of sector 1 of the multi-sector BD-RIS and  $K_l$  users in  $\mathcal{K}_l = \{\sum_{i=1}^{l-1} K_i + 1, \sum_{i=1}^{l-1} K_i + 2, \dots, \sum_{i=1}^l K_i\}$  are located within the coverage of sector  $l$ ,  $\forall l \in \mathcal{L}$  of the multi-sector BD-RIS<sup>2</sup>,  $\cup_l \mathcal{K}_l = \mathcal{K}$ , as illustrated in Fig. 4.3. For the multi-sector BD-RIS aided downlink MU-MISO, the signal received by user  $k$ ,  $\forall k \in \mathcal{K}_l$  covered by sector  $l$ ,  $\forall l \in \mathcal{L}$  is

$$y_k = \mathbf{h}_k \Phi_l \mathbf{G} \mathbf{x} + n_k, \quad (5.1)$$

<sup>1</sup>In this work, we ignore the direct links between the transmitter and users for simplicity, while the proposed design in the following section is still feasible when direct links exist.

<sup>2</sup>In this work, we consider the scenario that the relative location between each user and each sector of BD-RIS is fixed for simplicity such that there is no user selection issue. We also assume the sidelobe of each BD-RIS antenna is negligible compared to the mainlobe such that both transmitter and each user are covered by only one sector of the BD-RIS.

where  $\mathbf{h}_k \in \mathbb{C}^{1 \times M}$  and  $\mathbf{G} \in \mathbb{C}^{M \times N}$ , respectively, are the channel between the multi-sector BD-RIS and user  $k$ ,  $\forall k \in \mathcal{K}$ , and the channel between the transmitter and the BD-RIS,  $\mathbf{x} \in \mathbb{C}^{N \times 1}$  is the transmit signal vector, and  $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ ,  $\forall k \in \mathcal{K}$  is the noise. The transmit signal vector  $\mathbf{x}$  is subject to the power constraint  $\mathbb{E}\{\mathbf{x}^H \mathbf{x}\} \leq P$ , where  $P$  is the maximum transmit power. As explained in Chapter 4, the multi-sector BD-RIS is characterized by  $L$  matrices,  $\Phi_l \in \mathbb{C}^{M \times M}$ ,  $\forall l \in \mathcal{L} = \{1, \dots, L\}$ , corresponding to  $L$  sectors, which are sub-matrices of the scattering matrix  $\Phi \in \mathbb{C}^{LM \times LM}$  for the  $LM$ -port reconfigurable impedance network, i.e.,  $\Phi_l = [\Phi]_{(l-1)M+1:lM, 1:M}$ . They satisfy a combined unitary constraint given by

$$\sum_l \Phi_l^H \Phi_l = \mathbf{I}_M. \quad (5.2)$$

As discussed in previous chapters, reconfigurable impedance networks with different circuit topologies have scattering matrices with different mathematical characteristics. In this work, we focus on a cell-wise single-connected architecture of the multi-sector BD-RIS, where the inner-cell antennas are connected to each other by reconfigurable impedance components while the inter-cell antennas are not connected to each other. This architecture of the multi-sector BD-RIS results in diagonal matrices of  $\Phi_l$ ,  $\forall l \in \mathcal{L}$ , each of which can be described as  $\Phi_l = \text{diag}(\phi_{l,1}, \dots, \phi_{l,M})$  with  $\phi_{l,m} \in \mathbb{C}$ ,  $\forall l \in \mathcal{L}$ ,  $\forall m \in \mathcal{M}$ . Therefore, the constraint (5.2) can be re-written as

$$\sum_l |\phi_{l,m}|^2 = 1, \forall m, \quad (5.3)$$

which indicates that the transmit wave impinging on one sector of BD-RIS is flexibly split and reflected by other sectors, where the proportion of energy split to different sectors is determined by constraint (5.3). With  $\Phi_l = \text{diag}(\phi_{l,1}, \dots, \phi_{l,M})$ , we can rewrite equation (5.1) as

$$y_k = \phi_l^T \mathbf{Q}_k^H \mathbf{x} + n_k, \quad (5.4)$$

where  $\mathbf{Q}_k = \mathbf{G}^H \text{diag}(\mathbf{h}_k^H) \in \mathbb{C}^{N \times M}$ ,  $\forall k \in \mathcal{K}$  denotes the cascaded channel between the transmitter and user  $k$ ,  $\phi_l = [\phi_{l,1}, \dots, \phi_{l,M}]^T \in \mathbb{C}^{M \times 1}$ ,  $\forall l \in \mathcal{L}$ .

### 5.1.2 Rate-Splitting

In this work, we adopt an 1-layer RSMA architecture at the transmitter. Fig. 5.1 illustrates the transceiver architectures with 1-layer RSMA, where the splitting, combining, encoding, and precoding of transmit signals at the transmitter and the decoding, splitting, and combining of received signal at the user side will be described in detail.

**Transmitter:** The message  $W_k$  intended to user  $k$ ,  $\forall k \in \mathcal{K}$  is split into a common part  $W_{c,k}$  and a private part  $W_{p,k}$ . As shown in Fig. 5.1(a), the common parts  $W_{c,k}$  from all  $K$  users are merged into a common message  $W_c$  and further encoded into the common stream  $s_c$ , which will be decoded by all users, by a common codebook.  $K$  private parts are encoded

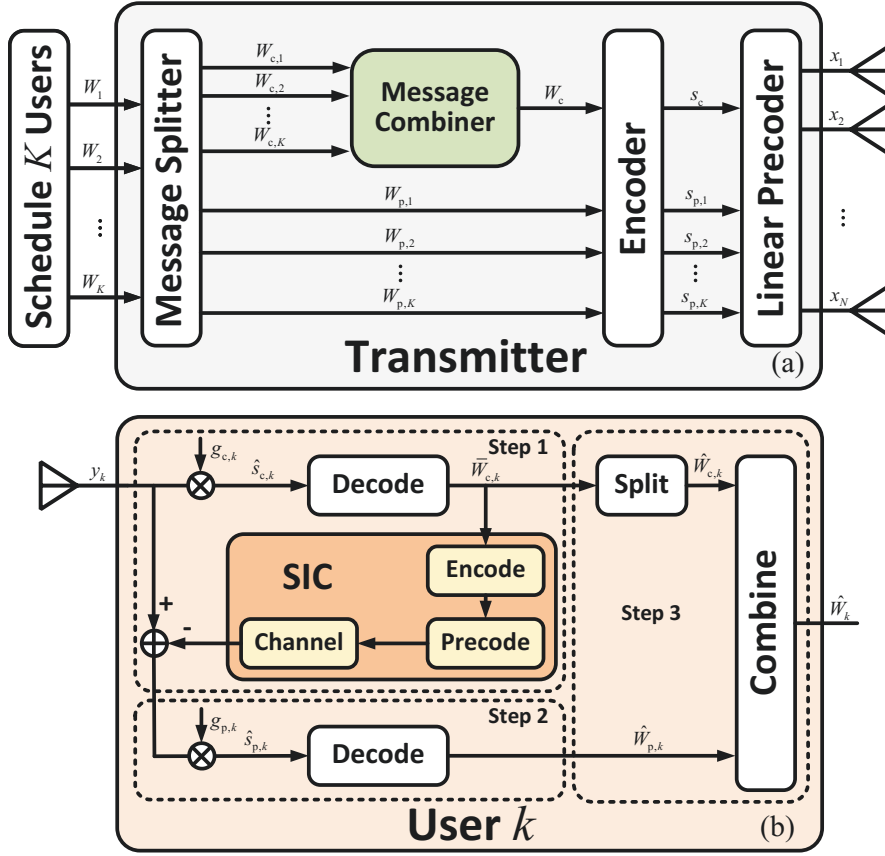


Figure 5.1: (a) Transmitter and (b) receiver architectures with 1-layer RSMA.

into private streams  $s_{p,1}, \dots, s_{p,K}$  and individually decoded by different users. The common stream and  $K$  private streams,  $\mathbf{s} = [s_c, s_{p,1}, \dots, s_{p,K}]^T \in \mathbb{C}^{K+1}$ , are linearly precoded by  $\mathbf{P} = [\mathbf{p}_c, \mathbf{p}_{p,1}, \dots, \mathbf{p}_{p,K}] \in \mathbb{C}^{N \times (K+1)}$ , where  $\mathbf{p}_c \in \mathbb{C}^N$  is the common precoder and  $\mathbf{p}_{p,k} \in \mathbb{C}^N$  is the private precoder for user  $k$ . The resulting transmit signal is

$$\mathbf{x} = \mathbf{P}\mathbf{s} = \mathbf{p}_c s_c + \sum_k \mathbf{p}_{p,k} s_{p,k}. \quad (5.5)$$

Assuming  $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_{K+1}$ , the transmit power constraint becomes  $\mathbb{E}\{\mathbf{x}^H \mathbf{x}\} = \text{Tr}(\mathbf{P}\mathbf{P}^H) = \|\mathbf{P}\|_F^2 \leq P$ .

**Receiver:** Substituting (5.5) into (5.4), we obtain the received signal at user  $k$  covered by sector  $l$ ,  $\forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}$  as

$$y_k = \phi_l^T \mathbf{Q}_k^H \mathbf{p}_c s_c + \phi_l^T \mathbf{Q}_k^H \sum_{k'} \mathbf{p}_{p,k'} s_{p,k'} + n_k. \quad (5.6)$$

Therefore, the average received power of user  $k$  covered by sector  $l$  for given CSI is given by

$$\tau_{c,k} = \mathbb{E}\{|y_k|^2\} = \underbrace{|\phi_l^T \mathbf{Q}_k^H \mathbf{p}_c|^2 + |\phi_l^T \mathbf{Q}_k^H \mathbf{p}_{p,k}|^2}_{=\tau_{p,k}=\tau_{c,k}} + \underbrace{\sum_{k' \neq k} |\phi_l^T \mathbf{Q}_k^H \mathbf{p}_{p,k'}|^2}_{=\tau_{p,k}} + \sigma_k^2. \quad (5.7)$$

At the user side, the signal process, taking user  $k$  covered by sector  $l$ ,  $\forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}$  as an example, is summarized as following three steps and also illustrated in Fig. 5.1(b).

Step 1: User  $k$  covered by sector  $l$  decodes the equalized common stream  $\hat{s}_{c,k} = g_{c,k}y_k$  with equalizer  $g_{c,k} \in \mathbb{C}$  into  $\bar{W}_{c,k}$  by regarding the interference induced by all  $K$  private streams as noise, i.e.,  $\tau_{p,k}$  or  $\iota_{c,k}$  in (5.7). After performing SIC,  $\bar{W}_{c,k}$  is subtracted from the received signal.

Step 2: User  $k$  decodes the equalized private stream  $\hat{s}_{p,k} = g_{p,k}y_k$  with equalizer  $g_{p,k} \in \mathbb{C}$  into  $\widehat{W}_{p,k}$  by regarding the interference induced by remaining  $K-1$  private streams as noise, i.e.,  $\iota_{p,k}$  in (5.7).

Step 3: User  $k$  splits its contribution to the common message,  $\widehat{W}_{c,k}$ , from the decoded  $\bar{W}_{c,k}$ , and combines  $\widehat{W}_{c,k}$  with  $\widehat{W}_{p,k}$  to reconstruct  $\widehat{W}_k$ .

Based on the above signal process, the instantaneous SINRs of the common stream and private streams at user  $k$  covered by sector  $l$ ,  $\forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}$  are given by

$$\gamma_{c,k} = |\phi_l^T \mathbf{Q}_k^H \mathbf{p}_c|^2 \iota_{c,k}^{-1}, \quad \gamma_{p,k} = |\phi_l^T \mathbf{Q}_k^H \mathbf{p}_{p,k}|^2 \iota_{p,k}^{-1}. \quad (5.8)$$

Assuming Gaussian inputs, the instantaneous common rates and private rates for user  $k$  are

$$R_{c,k} = \log_2(1 + \gamma_{c,k}), \quad R_{p,k} = \log_2(1 + \gamma_{p,k}). \quad (5.9)$$

To guarantee the common message  $W_c$  is successfully decoded by all  $K$  users, the common rate should not exceed  $\min_{\forall k} R_{c,k}$ . Given the architecture of RSMA as illustrated above, the common rate  $\min_{\forall k} R_{c,k}$  is split into  $K$  parts, yielding  $\min_{\forall k} R_{c,k} = \sum_k C_k$ . Each part  $C_k \in \mathbb{R}^+$  corresponds to the common part of the achievable rate for user  $k$ . Finally, the total rate for user  $k$  is a summation of common part and private part, i.e.,  $R_k = C_k + R_{p,k}$ ,  $\forall k \in \mathcal{K}$ .

### 5.1.3 Channel State Information

Perfect CSI acquisition in RIS aided systems is challenging due to the passive property of RIS. Therefore, in this work, we assume the transmitter has an imperfect instantaneous CSI  $\widehat{\mathbf{Q}} = [\widehat{\mathbf{Q}}_1, \dots, \widehat{\mathbf{Q}}_K] \in \mathbb{C}^{N \times MK}$ . To model the CSI uncertainty, the perfect cascaded channel matrix  $\mathbf{Q} = [\mathbf{Q}_1, \dots, \mathbf{Q}_K] \in \mathbb{C}^{N \times MK}$  is expressed as a summation of the channel estimate and channel estimation errors, that is  $\mathbf{Q} = \widehat{\mathbf{Q}} + \widetilde{\mathbf{Q}}$ , where the estimation error matrix is defined as  $\widetilde{\mathbf{Q}} = [\widetilde{\mathbf{Q}}_1, \dots, \widetilde{\mathbf{Q}}_K] \in \mathbb{C}^{N \times MK}$ . The channel estimation error could come from the noise and limited uplink training in TDD settings based on the typical MMSE criterion [37]<sup>3</sup>.

<sup>3</sup>The existing popular and effective channel estimation strategy, where the cascaded channel is estimated by pre-defining the RIS pattern during the uplink training process to minimize MSE, can be easily extended to the

In this case, the channel estimation error generally follows the CSCG distribution, that is  $\text{vec}(\tilde{\mathbf{Q}}_k) \sim \mathcal{CN}(\mathbf{0}, \delta_k^2 \mathbf{I}_{MN})$ ,  $\forall k \in \mathcal{K}$ , where  $\delta_k^2$  measures the estimation inaccuracy of the cascaded channel between the transmitter and user  $k$ .

## 5.2 Beamforming with Sample Average Approximation

In this section, we formulate the sum-rate maximization problem for BD-RIS aided RSMA based on imperfect CSI conditions and propose efficient algorithms to jointly design the transmit precoder and BD-RIS.

### 5.2.1 Problem Formulation

Beamforming design for RIS aided systems highly relies on the CSI among the transmitter, RIS, and users. When perfect CSI is available at the transmitter, beamforming design can be developed according to perfect channels such that the instantaneous sum-rate,  $\sum_k R_k$ , is maximized, and thus the long-term ergodic sum-rate given by  $\mathbb{E}_{\mathbf{Q}}\{\sum_k R_k\}$ . However, when the transmitter has only partial instantaneous CSI as illustrated in Section 5.1.3, beamforming design that maximizes instantaneous sum-rate as in perfect CSI case is not possible, which makes it difficult to measure the long-term ergodic sum-rate. One robust approach to tackle this issue is to perform the beamforming design based on the fact that the transmitter has access to the average rate, which captures the statistical property of the channel estimation error with a given estimate. The average rate is defined as follows.

**Definition 5.1.** *The average rate is a measure of the expected performance over a distribution of the channel estimation error, e.g., the distribution of  $\tilde{\mathbf{Q}}$ , with determined estimate  $\hat{\mathbf{Q}}$  [97]. Mathematically, the average common and private rates for user  $k$ ,  $\forall k \in \mathcal{K}$ , depend on the channel estimate  $\hat{\mathbf{Q}}$ , and are, respectively, defined as  $\bar{R}_{c,k} = \mathbb{E}_{\mathbf{Q}|\hat{\mathbf{Q}}}\{R_{c,k}|\hat{\mathbf{Q}}\}$  and  $\bar{R}_{p,k} = \mathbb{E}_{\mathbf{Q}|\hat{\mathbf{Q}}}\{R_{p,k}|\hat{\mathbf{Q}}\}$ , where  $R_{c,k}$  and  $R_{p,k}$  are instantaneous rates determined by the perfect channel  $\mathbf{Q}$  as illustrated in (5.9).*

With Definition 5.1, we can connect the ergodic rates  $\mathbb{E}_{\mathbf{Q}}\{R_{c,k}\}$ ,  $\mathbb{E}_{\mathbf{Q}}\{R_{p,k}\}$  with average rates  $\bar{R}_{c,k}$ ,  $\bar{R}_{p,k}$  as

$$\mathbb{E}_{\mathbf{Q}}\{R_{c,k}\} = \mathbb{E}_{\hat{\mathbf{Q}}}\{\mathbb{E}_{\mathbf{Q}|\hat{\mathbf{Q}}}\{R_{c,k}|\hat{\mathbf{Q}}\}\} = \mathbb{E}_{\hat{\mathbf{Q}}}\{\bar{R}_{c,k}\}, \quad (5.10a)$$

$$\mathbb{E}_{\mathbf{Q}}\{R_{p,k}\} = \mathbb{E}_{\hat{\mathbf{Q}}}\{\mathbb{E}_{\mathbf{Q}|\hat{\mathbf{Q}}}\{R_{p,k}|\hat{\mathbf{Q}}\}\} = \mathbb{E}_{\hat{\mathbf{Q}}}\{\bar{R}_{p,k}\}, \quad (5.10b)$$

---

multi-sector BD-RIS case with cell-wise single-connected architecture. The key and main difference compared with conventional RIS case is the RIS pattern design due to the new constraint of multi-sector BD-RIS. A simple choice is to set identical amplitude for non-zero elements of the BD-RIS matrix, i.e.,  $|\phi_{l,m}| = \frac{1}{\sqrt{L}}$ . Then orthogonal matrices, such as DFT matrix and Hadamard matrix, can be adopted to construct the RIS pattern. The detailed channel estimation scheme will be discussed in Chapter 6.

which indicate that ergodic rates can be characterized by the expectation of average rates over varied channel estimate  $\hat{\mathbf{Q}}$ . Accordingly, maximizing the ergodic sum-rate  $\mathbb{E}_{\mathbf{Q}}\{\sum_k R_k\}$  with partial CSI is transformed into maximizing the average sum-rate for each  $\hat{\mathbf{Q}}$ , given by  $\sum_k \bar{C}_k + \sum_k \bar{R}_{p,k}$ , where  $\bar{C}_k \in \mathbb{R}^+$ ,  $\forall k \in \mathcal{K}$  referring to the common part of the average rate should satisfy  $\sum_k \bar{C}_k = \min_{\forall k} \bar{R}_{c,k}$ . Therefore, the average sum-rate maximization problem can be formulated as

$$\max_{\{\mathbf{P}, \bar{\mathbf{c}}\}, \{\phi_l, \forall l\}} \sum_k \bar{C}_k + \sum_k \bar{R}_{p,k} \quad (5.11a)$$

$$\text{s.t.} \quad \sum_l |\phi_{l,m}|^2 = 1, \forall m, \quad (5.11b)$$

$$\|\mathbf{P}\|_F^2 \leq P, \quad (5.11c)$$

$$\sum_{k'} \bar{C}_{k'} \leq \bar{R}_{c,k}, \forall k, \quad (5.11d)$$

$$\bar{\mathbf{c}} \succeq \mathbf{0}, \quad (5.11e)$$

$$\bar{R}_k \geq R_{\text{th},k}, \forall k, \quad (5.11f)$$

where  $\phi_l = [\phi_{l,1}, \dots, \phi_{l,M}]^T \in \mathbb{C}^M$ ,  $\forall l \in \mathcal{L}$ ,  $\bar{\mathbf{c}} = [\bar{C}_1, \dots, \bar{C}_K]^T \in \mathbb{R}^K$ , and  $R_{\text{th},k} \in \mathbb{R}^+$  is the threshold rate for user  $k$ ,  $\forall k \in \mathcal{K}$ , to guarantee the QoS. Problem (5.11) is a challenging stochastic optimization different from existing optimization problems due to the new constraint (5.11b), the constraint of common rates (5.11d), and the QoS constraint (5.11e), where the BD-RIS matrix and the precoder are always coupled. One reliable solution is to transform problem (5.11) into a deterministic one based on SAA [105], and to deal with the deterministic problem by well-known WMMSE-rate relationship [106]. Details of the two-step transformation will be illustrated in Section 5.2.2.

### 5.2.2 Two-Step Transformation

**Sample Average Approximation:** To facilitate SAA, we give the following definition.

**Definition 5.2.** Given an estimate  $\hat{\mathbf{Q}}_k$  of user  $k$ , a sample of i.i.d. channel realizations is constructed based on  $\mathcal{Q}_k = \{\mathbf{Q}_k^a = \hat{\mathbf{Q}}_k + \tilde{\mathbf{Q}}_k^a | \hat{\mathbf{Q}}_k, \forall a \in \mathcal{A}_s = \{1, \dots, A_s\}\}$ , where  $\tilde{\mathbf{Q}}_k^a$  follows the CSCG distribution, i.e.,  $\text{vec}(\tilde{\mathbf{Q}}_k^a) \sim \mathcal{CN}(\mathbf{0}, \delta_k^2 \mathbf{I}_{MN})$ ,  $\forall a \in \mathcal{A}_s$ . The SAA of average common and private rates are, respectively, defined as  $\hat{R}_{c,k} = \frac{1}{A_s} \sum_a R_{c,k}^a$  and  $\hat{R}_{p,k} = \frac{1}{A_s} \sum_a R_{p,k}^a$ ,  $\forall k \in \mathcal{K}$ .

The SAA of average rate in Definition 5.2 is a tight approximation of the average rate with sufficient large number of samples<sup>5</sup>, i.e.,  $\lim_{A_s \rightarrow \infty} \hat{R}_{c,k} = \bar{R}_{c,k}$ ,  $\lim_{A_s \rightarrow \infty} \hat{R}_{p,k} = \bar{R}_{p,k}$ ,  $\forall k \in \mathcal{K}$ ,

<sup>4</sup>In the following illustrations, notations with superscript “ $(\cdot)^a$ ” have the same expressions as that without “ $(\cdot)^a$ ”, except that the term  $\mathbf{Q}_k$  is replaced by  $\mathbf{Q}_k^a$ ,  $\forall k \in \mathcal{K}_l$ ,  $\forall l \in \mathcal{L}$ ,  $\forall a \in \mathcal{A}_s$ .

<sup>5</sup>In this paper, we fix  $A_s = 50$  to save on the computational time without sacrificing performance based on simulations using the proposed algorithm and the results in [97].

which motivates us to approximate the stochastic problem (5.11) into a deterministic problem

$$\begin{aligned}
& \max_{\{\mathbf{P}, \bar{\mathbf{c}}\}, \{\phi_l, \forall l\}} \sum_k \bar{C}_k + \sum_k \hat{R}_{p,k} \\
& \text{s.t.} \quad (5.11b), (5.11c), (5.11e), \\
& \quad \sum_{k'} \bar{C}_{k'} \leq \hat{R}_{c,k}, \forall k, \\
& \quad \bar{C}_k + \hat{R}_{p,k} \geq R_{\text{th},k}, \forall k.
\end{aligned} \tag{5.12}$$

Problem (5.12) can be further transformed into a more tractable form as detailed below.

**WMMSE-Rate Relationship:** Let  $\hat{s}_{c,k}^a = g_{c,k}^a y_k^a$  be the estimate of  $s_c$  with equalizer  $g_{c,k}^a$  from user  $k$  covered by sector  $l$ ,  $\forall k \in \mathcal{K}_l$ ,  $\forall l \in \mathcal{L}$ ,  $\forall a \in \mathcal{A}_s$ . The estimate of  $s_{p,k}$  with equalizer  $g_{p,k}^a$  after removing the common stream is given by  $\hat{s}_{p,k}^a = g_{p,k}^a (y_k^a - \phi_l^T (\mathbf{Q}_k^a)^H \mathbf{p}_c s_c)$ ,  $\forall k \in \mathcal{K}_l$ ,  $\forall l \in \mathcal{L}$ ,  $\forall a \in \mathcal{A}_s$ . Define MSE functions regarding the common and private streams for user  $k$  covered by sector  $l$  at channel sample  $a$  as  $\epsilon_{c,k}^a = \mathbb{E}\{|\hat{s}_{c,k}^a - s_c|^2\} = |g_{c,k}^a|^2 \tau_{c,k}^a - 2\Re\{g_{c,k}^a \phi_l^T (\mathbf{Q}_k^a)^H \mathbf{p}_c\} + 1$  and  $\epsilon_{p,k}^a = \mathbb{E}\{|\hat{s}_{p,k}^a - s_{p,k}|^2\} = |g_{p,k}^a|^2 \tau_{p,k}^a - 2\Re\{g_{p,k}^a \phi_l^T (\mathbf{Q}_k^a)^H \mathbf{p}_{p,k}\} + 1$ . Then optimal unconstrained equalizers for the common stream and private streams which achieve minimum MSEs are

$$(g_{c,k}^a)^* = \frac{\mathbf{p}_c^H \mathbf{Q}_k^a \phi_l^*}{\tau_{c,k}^a}, \quad (g_{p,k}^a)^* = \frac{\mathbf{p}_{p,k}^H \mathbf{Q}_k^a \phi_l^*}{\tau_{p,k}^a}, \tag{5.13}$$

and the corresponding minimum MSEs are  $(\epsilon_{o,k}^a)^* = \min_{g_{o,k}^a} \epsilon_{o,k}^a = (\tau_{o,k}^a)^{-1} \iota_{o,k}^a = (1 + \gamma_{o,k}^a)^{-1}$ ,  $\forall o \in \{c, p\}$ ,  $\forall k \in \mathcal{K}$ ,  $\forall a \in \mathcal{A}_s$ . Introducing weights  $\lambda_{c,k}^a \in \mathbb{R}^+$  and  $\lambda_{p,k}^a \in \mathbb{R}^+$  associating common and private MSEs for user  $k$ , we can establish the WMMSE-rate relationship  $R_{o,k}^a = \max_{\lambda_{o,k}^a} \log_2 \lambda_{o,k}^a - \lambda_{o,k}^a (\epsilon_{o,k}^a)^* + 1$ ,  $\forall o \in \{c, p\}$ ,  $\forall k \in \mathcal{K}$ ,  $\forall a \in \mathcal{A}_s$ , where the maximum of the right-hand problem is achieved when weights  $\lambda_{c,k}$  and  $\lambda_{p,k}$  satisfy

$$(\lambda_{c,k}^a)^* = 1 + \gamma_{c,k}^a, \quad (\lambda_{p,k}^a)^* = 1 + \gamma_{p,k}^a. \tag{5.14}$$

Based on the above derivations, we reformulate problem (5.12) as a three-block optimization

$$\begin{aligned}
& \max_{\substack{\{\lambda_o^a, \mathbf{g}_o^a, \forall o, \forall a\} \\ \{\mathbf{P}, \bar{\mathbf{c}}\}, \{\phi_l, \forall l\}}} \sum_k \left( \bar{C}_k + \frac{1}{A_s} \sum_a (\log_2 \lambda_{p,k}^a - \lambda_{p,k}^a \epsilon_{p,k}^a) \right) \\
& \text{s.t.} \quad (5.11b), (5.11c), (5.11e), \\
& \quad \frac{1}{A_s} \sum_a (\log_2 \lambda_{c,k}^a - \lambda_{c,k}^a \epsilon_{c,k}^a + 1) \geq \sum_{k'} \bar{C}_{k'}, \forall k,
\end{aligned} \tag{5.15a}$$

$$\bar{C}_k + \frac{1}{A_s} \sum_a (\log_2 \lambda_{p,k}^a - \lambda_{p,k}^a \epsilon_{p,k}^a + 1) \geq R_{\text{th},k}, \forall k, \tag{5.15b}$$

where  $\boldsymbol{\lambda}_o^a = [\lambda_{o,1}^a, \dots, \lambda_{o,K}^a]^T$  and  $\mathbf{g}_o^a = [g_{o,1}^a, \dots, g_{o,K}^a]^T$ ,  $\forall o \in \{c, p\}$ ,  $\forall a \in \mathcal{A}_s$ . Although problem (5.15) is not convex in a joint design manner, the sub-problem regarding one block with fixing other blocks is either convex or more tractable than directly dealing with the original objective. Specifically, the unconstrained solution to block  $\{\boldsymbol{\lambda}_o^a, \mathbf{g}_o^a, \forall o, \forall a\}$  have been given in (5.13) and

(5.14). Solutions to  $\{\mathbf{P}, \bar{\mathbf{c}}\}$  and  $\{\phi_l, \forall l\}$  will be respectively given in Sections 5.2.3 and 5.2.4.

### 5.2.3 Solution to Block $\{\mathbf{P}, \bar{\mathbf{c}}\}$

With fixed equalizers, weights, and BD-RIS matrix, the sub-problem regarding block  $\{\mathbf{P}, \bar{\mathbf{c}}\}$  is

$$\min_{\{\mathbf{P}, \bar{\mathbf{c}}\}} \frac{1}{A_s} \sum_k \sum_a \lambda_{p,k}^a \epsilon_{p,k}^a - \sum_k \bar{C}_k \quad \text{s.t.} \quad (5.11c), (5.11e), (5.15a), (5.15b). \quad (5.16)$$

With the following definitions

$$\psi_{c,k}^a = \lambda_{c,k}^a (g_{c,k}^a)^* \mathbf{Q}_k^a \phi_l^*, \quad \psi_{p,k}^a = \lambda_{p,k}^a (g_{p,k}^a)^* \mathbf{Q}_k^a \phi_l^*, \quad (5.17a)$$

$$\Psi_{c,k}^a = \frac{\psi_{c,k}^a (\psi_{c,k}^a)^H}{\lambda_{c,k}^a}, \quad \Psi_{p,k}^a = \frac{\psi_{p,k}^a (\psi_{p,k}^a)^H}{\lambda_{p,k}^a}, \quad (5.17b)$$

$$\xi_{c,k}^a = \log_2 \lambda_{c,k}^a - \lambda_{c,k}^a |g_{c,k}^a|^2 \sigma_k^2 - \lambda_{c,k}^a + 1, \quad \xi_{p,k}^a = \log_2 \lambda_{p,k}^a - \lambda_{p,k}^a |g_{p,k}^a|^2 \sigma_k^2 - \lambda_{p,k}^a + 1, \quad (5.17c)$$

and their averages over  $A_s$  samples, i.e.,  $\psi_{c,k} = \frac{1}{A_s} \sum_a \psi_{c,k}^a$ ,  $\psi_{p,k} = \frac{1}{A_s} \sum_a \psi_{p,k}^a$ ,  $\Psi_{c,k} = \frac{1}{A_s} \sum_a \Psi_{c,k}^a$ ,  $\Psi_{p,k} = \frac{1}{A_s} \sum_a \Psi_{p,k}^a$ ,  $\xi_{c,k} = \frac{1}{A_s} \sum_a \xi_{c,k}^a$ ,  $\xi_{p,k} = \frac{1}{A_s} \sum_a \xi_{p,k}^a$ ,  $\forall k \in \mathcal{K}$ , problem (5.16) is rewritten as

$$\min_{\dot{\mathbf{c}}, \mathbf{p}_c, \{\mathbf{p}_{p,k}, \forall k\}} \sum_k \left( \dot{C}_k + \sum_{k'} \mathbf{p}_{p,k'}^H \Psi_{p,k} \mathbf{p}_{p,k'} - 2\Re\{\psi_{p,k}^H \mathbf{p}_{p,k}\} \right) \quad (5.18a)$$

$$\text{s.t.} \quad \|\mathbf{p}_c\|_2^2 + \sum_k \|\mathbf{p}_{p,k}\|_2^2 \leq P, \quad (5.18b)$$

$$\mathbf{p}_c^H \Psi_{c,k} \mathbf{p}_c + \sum_{k'} \mathbf{p}_{p,k'}^H \Psi_{c,k} \mathbf{p}_{p,k'} - 2\Re\{\psi_{c,k}^H \mathbf{p}_c\} \leq \sum_{k'} \dot{C}_k + \xi_{c,k}, \forall k, \quad (5.18c)$$

$$\dot{\mathbf{c}} \preceq \mathbf{0}, \quad (5.18d)$$

$$\sum_{k'} \mathbf{p}_{p,k'}^H \Psi_{p,k} \mathbf{p}_{p,k'} - 2\Re\{\psi_{p,k}^H \mathbf{p}_{p,k}\} + \dot{C}_k \leq \xi_{p,k} - R_{\text{th},k}, \forall k, \quad (5.18e)$$

where  $\dot{\mathbf{c}} = [\dot{C}_1, \dots, \dot{C}_K]^T = -\bar{\mathbf{c}}$ . Problem (5.18) is a convex second-order cone program (SOCP) and can be solved by interior-point methods with computational complexity bounded by  $\mathcal{O}((NK + N + K)^{3.5})$  [85].

### 5.2.4 Solution to Block $\{\phi_l, \forall l\}$

Recall that (5.15b) in problem (5.15) is the QoS constraint, which can be guaranteed in the design of block  $\{\mathbf{P}, \bar{\mathbf{c}}\}$ . To simplify the design of block  $\{\phi_l, \forall l\}$ , we temporarily ignore constraint (5.15b). Then, without the QoS constraint (5.15b), we can remove the term  $\bar{\mathbf{c}}$  by putting constraint (5.15a) into the objective. In this case, the original problem (5.15) can be simplified

as the following form

$$\begin{aligned} & \max_{\{\lambda_o^a, \mathbf{g}_o^a, \forall o, \forall a\}} \left\{ \min_{\mathbf{P}, \{\phi_l, \forall l\}} \frac{1}{A_s} \sum_a (\log_2 \lambda_{c,k}^a - \lambda_{c,k}^a \epsilon_{c,k}^a + 1) \right\} + \frac{1}{A_s} \sum_k \sum_a (\log_2 \lambda_{p,k}^a - \lambda_{p,k}^a \epsilon_{p,k}^a) \\ & \text{s.t.} \quad (5.11b), (5.11c), (5.11e). \end{aligned} \quad (5.19)$$

With given equalizers, weights, and the transmit precoder, the sub-problem regarding the multi-sector BD-RIS is

$$\begin{aligned} & \min_{\{\phi_l, \forall l\}} \left\{ \max_{\forall k} \frac{1}{A_s} \sum_a (\lambda_{c,k}^a \epsilon_{c,k}^a - \log_2 \lambda_{c,k}^a - 1) \right\} + \frac{1}{A_s} \sum_a \sum_k \lambda_{p,k}^a \epsilon_{p,k}^a \\ & \text{s.t.} \quad \sum_l |\phi_{l,m}|^2 = 1, \forall m. \end{aligned} \quad (5.20)$$

With the following definitions

$$\mathbf{v}_{c,k}^a = (\mathbf{Q}_k^a)^T \mathbf{p}_c^*, \quad \mathbf{v}_{p,k,k'}^a = (\mathbf{Q}_k^a)^T \mathbf{p}_{p,k'}^*, \quad (5.21a)$$

$$\mathbf{X}_{c,k}^a = \lambda_{c,k}^a |g_{c,k}^a|^2 \mathbf{v}_{c,k}^a (\mathbf{v}_{c,k}^a)^H + \lambda_{c,k}^a |g_{c,k}^a|^2 \sum_{k'} \mathbf{v}_{p,k,k'}^a (\mathbf{v}_{p,k,k'}^a)^H, \quad (5.21b)$$

$$\mathbf{X}_{p,k}^a = \lambda_{p,k}^a |g_{p,k}^a|^2 \sum_{k'} \mathbf{v}_{p,k,k'}^a (\mathbf{v}_{p,k,k'}^a)^H, \quad (5.21c)$$

$$\mathbf{x}_{c,k}^a = \lambda_{c,k}^a (g_{c,k}^a)^* \mathbf{v}_{c,k}^a, \quad \mathbf{x}_{p,k}^a = \lambda_{p,k}^a (g_{p,k}^a)^* \mathbf{v}_{p,k,k}^a, \quad (5.21d)$$

and their averages over  $A_s$  samples, i.e.,  $\mathbf{X}_{c,k} = \frac{1}{A_s} \sum_a \mathbf{X}_{c,k}^a$ ,  $\mathbf{X}_{p,k} = \frac{1}{A_s} \sum_a \mathbf{X}_{p,k}^a$ ,  $\mathbf{x}_{c,k} = \frac{1}{A_s} \sum_a \mathbf{x}_{c,k}^a$ ,  $\mathbf{x}_{p,k} = \frac{1}{A_s} \sum_a \mathbf{x}_{p,k}^a$ ,  $\forall k \in \mathcal{K}$ , problem (5.20) can be rearranged as

$$\begin{aligned} & \min_{\{\phi_l, \forall l\}} \left\{ \max_{\forall k} (\phi_l^H \mathbf{X}_{c,k} \phi_l - 2\Re\{\phi_l^H \mathbf{x}_{c,k}\} - \xi_{c,k}) \right\} + \sum_l (\phi_l^H \bar{\mathbf{X}}_{p,l} \phi_l - 2\Re\{\phi_l^H \bar{\mathbf{x}}_{p,l}\}) \\ & \text{s.t.} \quad \sum_l |\phi_{l,m}|^2 = 1, \forall m, \end{aligned} \quad (5.22)$$

where  $\bar{\mathbf{X}}_{p,l} = \sum_{k' \in \mathcal{K}_l} \mathbf{X}_{p,k'}$  and  $\bar{\mathbf{x}}_{p,l} = \sum_{k' \in \mathcal{K}_l} \mathbf{x}_{p,k'}$ ,  $\forall l \in \mathcal{L}$ . We observe from problem (5.22) that the objective regarding each cell of the multi-sector BD-RIS is coupled with each other, while the constraints for different cells are independent of each other. This fact motivates us to successively design each cell while fixing the rest cells. Therefore, we introduce the following fresh notations

$$\nu_{c,k,m} = [\mathbf{X}_{c,k}]_{m,m}, \quad \nu_{p,l,m} = [\bar{\mathbf{X}}_{p,l}]_{m,m}, \quad (5.23a)$$

$$\chi_{c,k,m} = [\mathbf{x}_{c,k}]_m - \sum_{n \neq m} [\mathbf{X}_{c,k}]_{m,n} \phi_{l,n}, \quad (5.23b)$$

$$\chi_{p,l,m} = [\bar{\mathbf{X}}_{p,l}]_m - \sum_{n \neq m} [\bar{\mathbf{X}}_{p,l}]_{m,n} \phi_{l,n}, \quad (5.23c)$$

$$\xi_{k,m} = \xi_{c,k} - \sum_{n \neq m} \sum_{n' \neq m} [\mathbf{X}_{c,k}]_{n,n'} \phi_{l,n}^* \phi_{l,n'} + 2\Re\left\{ \sum_{n \neq m} \phi_{l,n}^* [\mathbf{x}_{c,k}]_n \right\}, \quad (5.23d)$$

$$\bar{\phi}_m = [\phi_{1,m}, \dots, \phi_{L,m}], \forall m \in \mathcal{M}, \forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}, \quad (5.23e)$$

and formulate the sub-problem regarding cell  $m$ ,  $\forall m \in \mathcal{M}$ , with fixed other cells as

$$\begin{aligned} \min_{\bar{\phi}_m} \quad & \left\{ \max_{\forall k} \underbrace{\nu_{c,k,m} |\phi_{l,m}|^2 - 2\Re\{\phi_{l,m}^* \chi_{c,k,m}\} - \xi_{k,m}}_{=F_{c,k}(\phi_{l,m})} \right\} + \sum_l \underbrace{\nu_{p,l,m} |\phi_{l,m}|^2 - 2\Re\{\phi_{l,m}^* \chi_{p,l,m}\}}_{=F_{p,l}(\phi_{l,m})} \\ \text{s.t.} \quad & \|\bar{\phi}_m\|_2^2 = 1. \end{aligned} \quad (5.24)$$

Problem (5.24) is a non-smooth optimization with a non-convex constraint. To solve this problem, we introduce the well-known LogSumExp (LSE) function to transform the non-smooth part of the objective function into a smooth form, and adopt the manifold theory reviewed in Section 2.2.3 to transform the constrained problem on the Euclidean space into an unconstrained optimization on the manifold space. Each of the two steps is described as follows.

**LogSumExp Approximation:** To facilitate the LSE approximation, we give the following definition.

**Definition 5.3.** Given a set  $\mathcal{F} = \{F_{c,k} \in \mathbb{R} | \forall k \in \mathcal{K}\}$ , an LSE function is defined as the logarithm of the sum of the exponentials of  $F_{c,k}$ ,  $\forall k$ , i.e.,  $\log \sum_k \exp(F_{c,k})$ , which is a smooth approximation of the maximum  $\max_{\forall k} F_{c,k}$  with a lower bound  $\max_{\forall k} F_{c,k}$  and an upper bound  $\max_{\forall k} F_{c,k} + \log |\mathcal{K}|$ , i.e.,  $\max_{\forall k} F_{c,k} \leq \log \sum_k \exp(F_{c,k}) \leq \max_{\forall k} F_{c,k} + \log |\mathcal{K}|$ .

*Proof.* Let  $F_c^{\max} = \max_{\forall k} F_{c,k}$ . Then we have  $\exp(F_c^{\max}) \leq \sum_k \exp(F_{c,k}) \leq |\mathcal{K}| \exp(F_c^{\max})$ . Performing the logarithm to the inequality gives the result  $\max_{\forall k} F_{c,k} \leq \log \sum_k \exp(F_{c,k}) \leq \max_{\forall k} F_{c,k} + \log |\mathcal{K}|$ .  $\square$

With Definition 5.3, we have the following result to obtain an approximation to  $\max_{\forall k} F_{c,k}(\phi_{l,m})$  in problem (5.24).

**Result 5.1.**  $\max_{\forall k} F_{c,k}(\phi_{l,m})$  has the following bounds

$$\max_{\forall k} F_{c,k}(\phi_{l,m}) \stackrel{(a)}{\leq} \underbrace{\varepsilon \log \sum_k \exp\left(\frac{F_{c,k}(\phi_{l,m})}{\varepsilon}\right)}_{=\bar{F}(\bar{\phi}_m)} \stackrel{(b)}{\leq} \max_{\forall k} F_{c,k}(\phi_{l,m}) + \varepsilon \log |\mathcal{K}|, \quad (5.25)$$

with  $\varepsilon > 0$ , where (a) achieves its equality when  $|\mathcal{K}| = 1$  and (b) achieves its equality when  $F_{c,1}(\phi_{l,m}) = \dots = F_{c,K}(\phi_{l,m})$ .

*Proof.* Based on Definition 5.3, by replacing each  $F_{c,k}$  with  $\frac{F_{c,k}}{\varepsilon}$ ,  $\forall k \in \mathcal{K}$ , we have  $\max_{\forall k} \frac{F_{c,k}}{\varepsilon} \leq \log \sum_k \exp\left(\frac{F_{c,k}}{\varepsilon}\right) \leq \max_{\forall k} \frac{F_{c,k}}{\varepsilon} + \log |\mathcal{K}|$ . Multiplying by  $\varepsilon$  gives the inequality (5.25).  $\square$

In Result 5.1,  $\bar{F}(\bar{\phi})$  refers to the LSE function of  $\max_{\forall k} F_{c,k}(\phi_{l,m})$  with  $\varepsilon$  a scalar to make the bounds tighter. In addition, we can easily deduce that the smaller the  $\varepsilon$ , the tighter the LSE

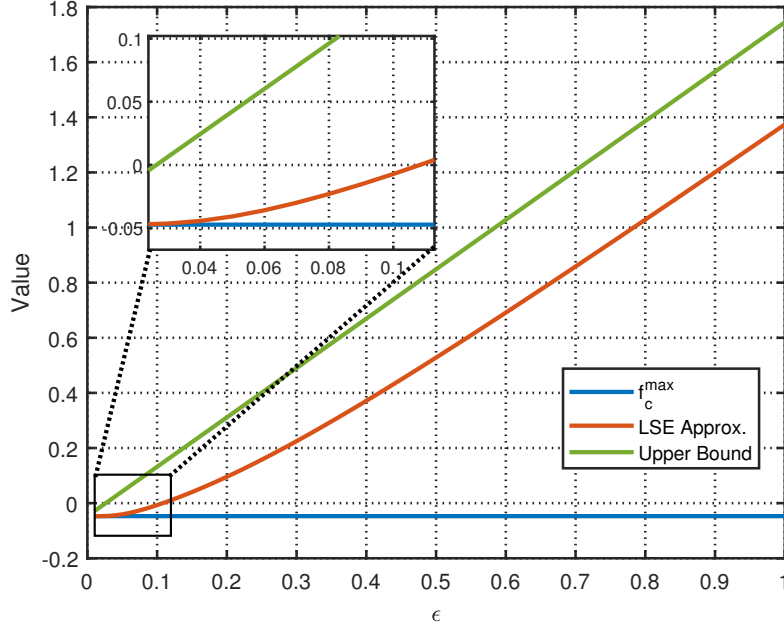


Figure 5.2: The impact of  $\varepsilon$  to the tightness of LSE approximation.

approximation. To get some insights of how the  $\varepsilon$  influences the tightness of the bounds, we compare the values of  $F_c^{\max} = \max_{\forall k} F_{c,k}(\phi_{l,m})$ , its LSE approximation  $\bar{F}(\bar{\phi}_m)$ , and the upper bound  $F_c + \varepsilon \log |\mathcal{K}|$  with different  $\varepsilon$  in Fig. 5.2, which verifies that a smaller  $\varepsilon$  leads to a smaller gap between  $F_c^{\max}$  and the corresponding LSE function.

With Result 5.1, problem (5.24) can be approximated as

$$\bar{\phi}_m^* = \arg \min_{\|\bar{\phi}_m\|_2^2=1} \underbrace{\sum_l F_{p,l}(\phi_{l,m}) + \bar{F}(\bar{\phi}_m)}_{=F(\bar{\phi}_m)}. \quad (5.26)$$

Now problem (5.26) has a smooth objective function  $F(\bar{\phi}_m)$  and only one constraint, which can be solved by the manifold method as explained below.

**Manifold Construction:** We construct the feasible region of problem (5.26), i.e.,  $\|\bar{\phi}_m\|_2^2 = 1$ , as a complex sphere manifold  $\mathcal{M}_{cs}$  as reviewed in Section 2.2.3. More specifically, we say  $\bar{\phi}_m$  is a point on the complex sphere manifold  $\mathcal{M}_{cs}$ , yielding problem (5.26) an unconstrained optimization on  $\mathcal{M}_{cs}$ , which can be solved by many classic algorithms with necessary projections.

The algorithm we use here to solve problem (5.26) is the Riemannian Conjugate-Gradient algorithm similar to Algorithm 2 in Chapter 3, where the key is to determine the descent direction of the objective function. Specifically, the Riemannian gradient, which can be obtained from the Euclidean gradient, is required to calculate the descent direction. Therefore, we first

**Algorithm 5** Riemannian Conjugate Gradient**Require:**  $\nu_{c,k,m}, \nu_{p,l,m}, \chi_{c,k,m}, \chi_{p,l,m}, \xi_{c,k,m}, \xi_{p,l,m}, \forall k, \forall l, \bar{\phi}_m, \varepsilon, m, \zeta, o_{\max}$ .**Ensure:**  $\bar{\phi}_m^*$ .

- 1: Initialize  $\bar{\phi}_m^0 = \bar{\phi}_m, \beta^0 = -\nabla_{\mathcal{M}_{cs}} f(\bar{\phi}_m^0), o = 0$ .
- 2: **repeat**
- 3:   Update  $o = o + 1$ .
- 4:   Calculate stepsize  $\varsigma^{o-1}$  by Armijo backtracking [87].
- 5:   Set  $\bar{\phi}_m^o$  by retraction:  $\bar{\phi}_m^o = \frac{\bar{\phi}_m^{o-1} + \varsigma^{o-1} \beta^{o-1}}{\|\bar{\phi}_m^{o-1} + \varsigma^{o-1} \beta^{o-1}\|_2}$ .
- 6:   Update Polak-Ribière formula  $\mu^o = \frac{\Re\{\text{Tr}([\nabla_{\mathcal{M}_{cs}} F(\bar{\phi}_m^o)]^H [\nabla_{\mathcal{M}_{cs}} F(\bar{\phi}_m^o) - \text{Pr}_{\bar{\phi}_m^o}(\nabla_{\mathcal{M}_{cs}} F(\bar{\phi}_m^{o-1}))])\}}{\text{Tr}([\nabla_{\mathcal{M}_{cs}} F(\bar{\phi}_m^{o-1})]^H \nabla_{\mathcal{M}_{cs}} F(\bar{\phi}_m^{o-1}))}$ .
- 7:   Update search direction  $\beta^o = -\nabla_{\mathcal{M}_{cs}} F(\bar{\phi}_m^o) + \mu^o \text{Pr}_{\bar{\phi}_m^o}(\beta^{o-1})$ .
- 8: **until**  $\|\nabla_{\mathcal{M}_{cs}} F(\bar{\phi}_m^o)\|_2 \leq \zeta$  or  $o \geq o_{\max}$

**Algorithm 6** Multi-Sector BD-RIS: Manifold Based Solution**Require:**  $\mathbf{X}_{c,k}, \bar{\mathbf{X}}_{p,l}, \mathbf{x}_{c,k}, \bar{\mathbf{x}}_{p,l}, \xi_{c,k}, \forall k, \phi_l, \forall l, \varepsilon, \zeta, o_{\max}$ .**Ensure:**  $\phi_l^*$ ,  $\forall l$ .

- 1: Initialize  $\phi_l^0 = \phi_l, \forall l \in \mathcal{L}, o = 0$ .
- 2: **repeat**
- 3:   Update  $o = o + 1$ .
- 4:   **for**  $m \in \mathcal{M}$  **do**
- 5:     Update  $\chi_{c,k,m}^o, \chi_{p,l,m}^o$ , and  $\xi_{k,m}^o, \forall k \in \mathcal{K}$  by (5.23).
- 6:     Update  $\bar{\phi}_m^o$  by Algorithm 5.
- 7:   **end for**
- 8: **until**  $\sum_l \|\phi_l^o - \phi_l^{o-1}\|_2^2 \leq \zeta$  or  $o \geq o_{\max}$

calculate the Euclidean gradient of the objective function  $F(\bar{\phi}_m)$ , which is given by

$$\begin{aligned} \nabla F(\bar{\phi}_m) &= \left[ \frac{\partial F(\bar{\phi}_m)}{\partial \phi_{1,m}}, \dots, \frac{\partial F(\bar{\phi}_m)}{\partial \phi_{L,m}} \right] \\ &= \left[ \nabla F_{p,1}(\phi_{1,m}) + \frac{\partial \bar{F}(\bar{\phi}_m)}{\partial \phi_{1,m}}, \dots, \nabla F_{p,L}(\phi_{L,m}) + \frac{\partial \bar{F}(\bar{\phi}_m)}{\partial \phi_{L,m}} \right], \end{aligned} \quad (5.27a)$$

$$\nabla F_{p,l}(\phi_{l,m}) = 2\nu_{p,l,m}\phi_{l,m} - 2\chi_{p,l,m}, \quad (5.27b)$$

$$\frac{\partial \bar{F}(\bar{\phi}_m)}{\partial \phi_{l,m}} = \frac{\sum_{k \in \mathcal{K}_l} \exp\left(\frac{F_{c,k}(\phi_{l,m})}{\varepsilon}\right) \nabla F_{c,k}(\phi_{l,m})}{\sum_{k \in \mathcal{K}} \exp\left(\frac{F_{c,k}(\phi_{l,m})}{\varepsilon}\right)}, \quad (5.27c)$$

$$\nabla F_{c,k}(\phi_{l,m}) = 2\nu_{c,k,m}\phi_{l,m} - 2\chi_{c,k,m}. \quad (5.27d)$$

Then, we obtain the Riemannian gradient, which projects the Euclidean gradient onto the tangent space, i.e.,  $\nabla_{\mathcal{M}_{cs}} F(\bar{\phi}_m) = \text{Pr}_{\bar{\phi}_m}(\nabla F(\bar{\phi}_m))$ . The Riemannian version of the Conjugate-Gradient method to solve problem (5.26) is summarized in Algorithm 5.

**Summary:** With solutions to each cell of the multi-sector BD-RIS, the procedure of the successive design is straightforward and summarized in Algorithm 6. The complexity of Algorithm 6 is given by  $\mathcal{O}(I_1 L^{1.5})$ , where  $I_1$  is the number of iterations of Algorithm 6.

**Algorithm 7** Joint Design: Block Coordinate Descent**Require:**  $\mathcal{Q}_k, \sigma_k, R_{\text{th},k}, \forall k, \forall l, P, \zeta, v_{\max}$ .**Ensure:**  $\phi_l^*, \forall l, \mathbf{P}^*$ .

- 1: Initialize  $\phi_l^0, \mu_l = \mathbf{0}, \forall l, \mathbf{P}^0, v = 0$ .
- 2: **repeat**
- 3:   Update  $v = v + 1$ .
- 4:   Set weights  $\{\lambda_o^a, \forall o, \forall a\}^{v-1}$  by (5.14).
- 5:   Set equalizers  $\{\mathbf{g}_o^a, \forall o, \forall a\}^{v-1}$  by (5.13).
- 6:   Update BD-RIS matrix  $\{\phi_l^v, \forall l\}$  by Algorithm 6.
- 7:   Update precoder  $\mathbf{P}^v$  by solving SOCP (5.18).
- 8: **until**  $\frac{|\hat{R}^v - \hat{R}^{v-1}|}{\hat{R}^{v-1}} \leq \zeta$  or  $v \geq v_{\max}$

### 5.2.5 Algorithm and Analysis

**Framework of the Algorithm:** The above three blocks are updated in an iterative manner until convergence as summarized in Algorithm 7, where  $\hat{R}^v = \min_{\forall k} \hat{R}_{c,k}^v + \sum_k \hat{R}_{p,k}^v$ .

**Initialization:** In Algorithm 7,  $\phi_l, \forall l \in \mathcal{L}$  are initialized as  $\phi_l^0 = \frac{1}{\sqrt{L}}[e^{j\varphi_{l,1}}, \dots, e^{j\varphi_{l,M}}]^T$ , where  $\varphi_{l,m}$  is randomly selected within the range  $[0, 2\pi]$ ,  $\forall l \in \mathcal{L}, \forall m \in \mathcal{M}$ . The transmit power allocated to the common streams is initialized as  $P_c = 0.7P$ , and thus that allocated to the private streams is  $P_p = 0.3P$ . Accordingly, the common precoder is initialized as  $\mathbf{p}_c = \sum_a \sum_k \mathbf{Q}_k^a \phi_l^*$ , followed by a normalization  $\mathbf{p}_c^0 = \frac{\sqrt{P_c} \mathbf{p}_c}{\|\mathbf{p}_c\|_2}$ . The private precoder is initialized as  $\mathbf{p}_{p,k} = \sum_a \mathbf{Q}_k^a \phi_l^*$ , followed by  $\mathbf{p}_{p,k}^0 = \sqrt{\frac{P_p}{K}} \frac{\mathbf{p}_{p,k}}{\|\mathbf{p}_{p,k}\|_2}, \forall k \in \mathcal{K}$ .

**Complexity Analysis:** The complexity of Algorithm 7 mainly comes from the update of three blocks (steps 4-7). Specifically, updating block  $\{\lambda_o^a, \mathbf{g}_o^a, \forall o, \forall a\}$  requires complexity  $\mathcal{O}(MNKLA)$ . Updating block  $\{\mathbf{P}, \bar{\mathbf{c}}\}$  by solving SOCP has complexity  $\mathcal{O}((NK + N + K)^{3.5})$ , while updating block  $\{\phi_l, \forall l\}$  by Algorithm 6 requires  $\mathcal{O}(I_1 L^{1.5})$ . The overall complexity of Algorithm 7 is thus  $\mathcal{O}(I(NK + N + K)^{3.5})$ , where  $I$  is the number of iterations of Algorithm 7 to achieve the convergence threshold. The overall complexity of the proposed algorithm does not scale with the increasing dimension of BD-RIS, i.e.,  $M$  and  $L$ . Hence, the proposed algorithm has benefits in readily being applied in scenarios where the BD-RIS has numerous elements.

## 5.3 Performance Evaluation

In this section, we evaluate the performance of the proposed robust beamforming design for multi-sector BD-RIS aided RSMA under different CSI conditions. The perfect channels are generated as a combination of small-scale fading and large-scale fading as given in (4.23)-(4.27). Sectors of the multi-sector BD-RIS are modeled as  $M_x \times M_y = M$  uniform planar arrays and the transmitter is modeled as a uniform linear array with half-wavelength inter-antenna spacing.

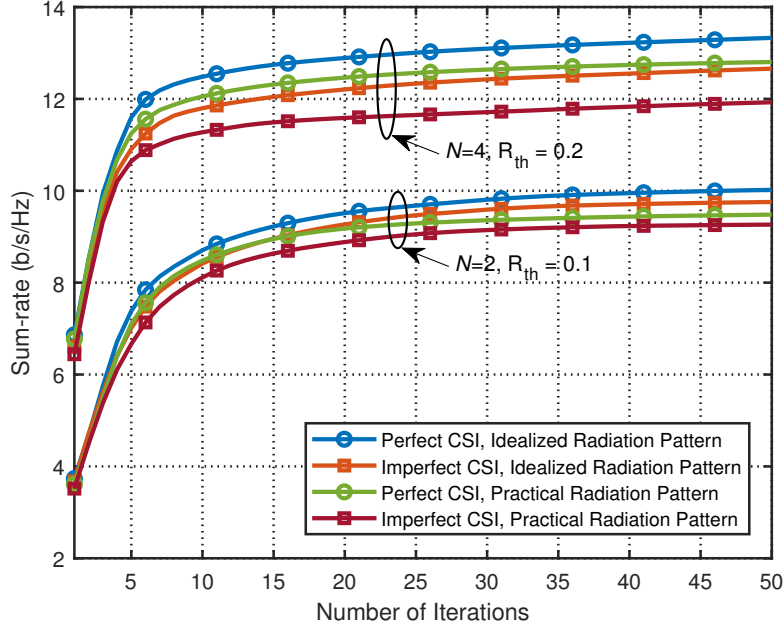
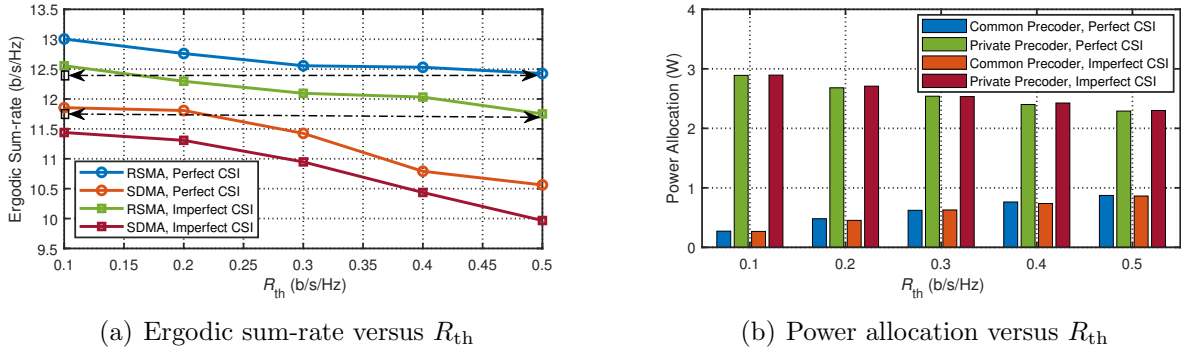
Table 5.1: Simulation Settings

| Parameters                        | Value                                                             |
|-----------------------------------|-------------------------------------------------------------------|
| Rician Factor                     | $\kappa = \kappa_1 = \dots = \kappa_K = 0$ dB                     |
| Transmit/Receive Antenna Gain     | $G_T = G_U = 1$                                                   |
| Transmit Signal Frequency         | $f = 2.4$ GHz                                                     |
| Transmitter-RIS Distance          | $d_{IT} = 100$ m                                                  |
| RIS-User Distance                 | $d_{UI,1} = \dots = d_{UI,K} = 10$ m                              |
| Path Loss Exponent                | $\epsilon_{IT} = \epsilon_{UI} = 2$                               |
| Transmitter-RIS Angle             | $\theta_{IT} \in [0, \frac{\pi}{L}]$                              |
| RIS-User Angle                    | $\theta_{UI,k} \in [0, \frac{\pi}{L}], \forall k \in \mathcal{K}$ |
| Noise Power                       | $\sigma_1^2 = \dots = \sigma_K^2 = -90$ dBm                       |
| Estimation Uncertainty            | $\delta \in \{0, 0.15\}$                                          |
| No. of Samples                    | $A = 50$                                                          |
| No. of Sectors                    | $L = 3$                                                           |
| No. of Users per Sector           | $K_1 = \dots = K_L = 2$                                           |
| No. of BD-RIS antennas per Sector | $M = M_x \times M_y = 5 \times 4 = 20$                            |
| No. of transmit antennas          | $N = 4$                                                           |
| QoS Threshold per User            | $R_{th,1} = \dots = R_{th,K} = R_{th}$                            |

The channel estimation error for each channel realization  $\tilde{\mathbf{Q}}_k$  is generalized such that  $\text{vec}(\tilde{\mathbf{Q}}_k) \sim \mathcal{CN}(\mathbf{0}, \delta_k^2 \mathbf{I}_{MN})$ , where the channel estimation uncertainty  $\delta_k^2$  is set as  $\delta_k^2 = \zeta_k^{-1} \delta^2, \forall k \in \mathcal{K}$  with a scalar  $\delta \in [0, 1)$ . Specifically, the value  $\delta = 0$  refers to the perfect CSI case. For each channel estimate  $\hat{\mathbf{Q}}_k = \mathbf{Q}_k - \tilde{\mathbf{Q}}_k$ , the  $a$ -th sample in the set (available at the transmitter)  $\mathcal{Q}_k$  is given by  $\mathbf{Q}_k^a = \hat{\mathbf{Q}}_k + \tilde{\mathbf{Q}}_k^a$ , where  $\tilde{\mathbf{Q}}_k^a$  follows the same error distribution, that is  $\text{vec}(\tilde{\mathbf{Q}}_k^a) \sim \mathcal{CN}(\mathbf{0}, \delta_k^2 \mathbf{I}_{MN})$ ,  $\forall k \in \mathcal{K}, \forall a \in \mathcal{A}$ . For each channel realization, the precoder and BD-RIS are designed based on sets  $\mathcal{Q}_k, \forall k \in \mathcal{K}$  and the maximization of the average sum-rate as summarized in Algorithm 7, while the sum-rate is calculated based on the perfect instantaneous channel  $\mathbf{Q}$ . The ergodic sum-rate is plotted as an average of sum-rates for 50 perfect channel realizations. The simulation settings, unless otherwise stated, are summarized in Table 5.1.

### 5.3.1 Convergence of the Algorithm

The convergence of Algorithm 7 cannot be theoretically proved since the design of BD-RIS matrix block does not lead to a global optimal solution. Instead, we provide numerical results to show the proposed algorithm can converge within limited iterations. Fig. 5.3 plots the sum-rate versus the number of iterations of Algorithm 7 for different CSI conditions and parameter settings. We can observe from Fig. 5.3 that the proposed algorithm always converges within finite iterations regardless of simulation settings, such as channel conditions (perfect and imperfect CSI), the number of transmit antennas, and QoS requirements.

Figure 5.3: Sum-rate versus the number of iterations ( $P = 35$  dBm).(a) Ergodic sum-rate versus  $R_{th}$ (b) Power allocation versus  $R_{th}$ Figure 5.4: Impact of QoS threshold with idealized radiation pattern and different CSI conditions ( $P = 35$  dBm).

### 5.3.2 Impact of QoS Threshold

Then, in Fig. 5.4, we show the impact of the QoS requirement for each user to the achievable ergodic sum-rate under different CSI conditions when each multi-sector BD-RIS antenna has an idealized radiation pattern. From Fig. 5.4 we have the following observations. 1) The multi-sector BD-RIS aided RSMA always outperforms the SDMA scheme with various QoS thresholds and different CSI conditions. 2) The achievable ergodic sum-rate for both schemes decreases with the growth of the QoS requirement, which can be explained as follows. Since we focus on the sum-rate maximization problem, when the number of transmit antennas is smaller than the number of users ( $4 = N < K = 6$ ), the weakest  $N - K$  users tend to be turned off. In this case, increasing the QoS requirement for all users violates the aim of maximizing sum-rate. 3) The performance gap between the RSMA scheme and the SDMA scheme becomes larger

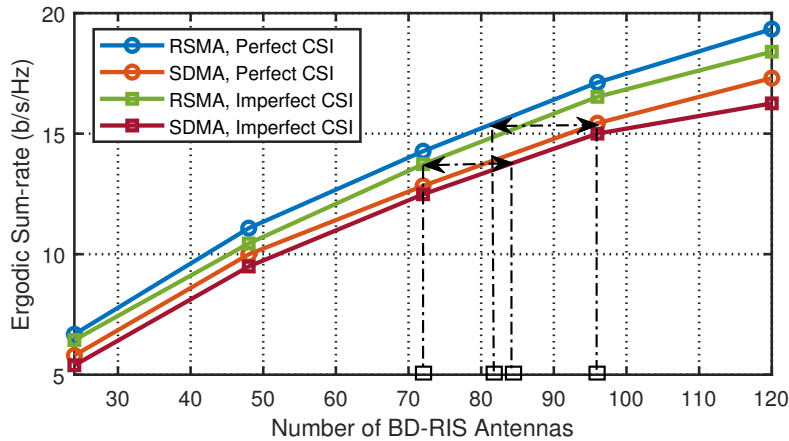


Figure 5.5: Ergodic sum-rate versus the total number of antennas of BD-RIS with idealized radiation pattern ( $P = 35$  dBm,  $R_{th} = 0.2$ ).

with increasing QoS threshold for each user, and thus the RSMA scheme is able to achieve both higher sum-rate and better QoS than the SDMA case, e.g., the comparison between RSMA scheme with  $R_{th} = 0.5$  and SDMA scheme with  $R_{th} = 0.1$ . This is because in RSMA scheme, the power allocated to the common precoders increases with the QoS requirement. This phenomenon demonstrates that the common precoders play an increasingly important role to guarantee the fairness while achieving better sum-rate performance.

### 5.3.3 Impact of Numbers of Transmitter and BD-RIS Antennas

In Fig. 5.5, we plot the ergodic sum-rate versus the total number of antennas of the multi-sector BD-RIS under different CSI conditions. We observe from Fig. 5.5 that the ergodic sum-rate of both RSMA and SDMA schemes grows with increasing number of BD-RIS antennas, and that the RSMA scheme always achieves better performance than the SDMA scheme. More importantly, to achieve the same sum-rate, the required number of BD-RIS antennas can be reduced compared to the SDMA scheme by adopting RSMA at the transmitter. For example, in perfect CSI case, to achieve an ergodic sum-rate of 15 b/s/Hz, the required number of antennas can be reduced by 16%; in imperfect CSI case, to achieve an ergodic sum-rate of 13 b/s/Hz, the required number can be reduced by 14%. This observation highlights the benefit of synergizing BD-RIS and RSMA in saving on BD-RIS antennas, which is friendly to the hardware implementation of multi-sector BD-RIS.

Finally, in Fig. 5.6, we plot the ergodic sum-rate versus the number of transmit antennas under different CSI conditions. Similar conclusions that the RSMA scheme always outperforms the SDMA scheme can be obtained from Fig. 5.6. More importantly, to achieve the same sum-rate, the required number of active antennas at the transmitter can also be reduced compared to the SDMA scheme when RSMA is adopted at the transmitter. For example, an 1-antenna RSMA-

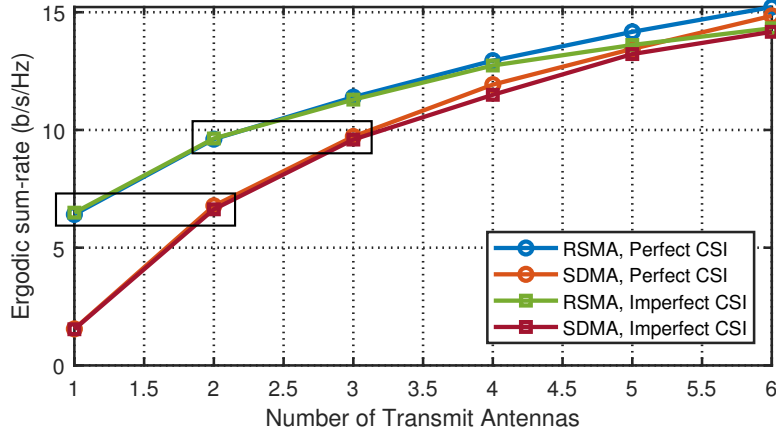


Figure 5.6: Ergodic sum-rate versus the number of transmit antennas when BD-RIS antennas have idealized radiation pattern ( $P = 35$  dBm,  $R_{\text{th}} = 0.15$ ).

enabled transmitter supports the same sum-rate performance as an 2-antenna SDMA-enabled transmitter. Such reduction of the number of transmit antennas demonstrates the benefit of synergizing BD-RIS and RSMA in reducing power consumption and cost, which leads to an energy efficiency gain.

## 5.4 Summary

This chapter considers the integration of multi-sector BD-RIS and RSMA in a MU-MISO communication system. Specifically, we assume a multi-sector BD-RIS and an RSMA-enabled transmitter. Given that the channel estimation for RIS and BD-RIS aided scenarios is challenging, we model the channel estimation error when only partial/imperfect CSI is available at the transmitter.

With the multi-sector BD-RIS aided RSMA model and CSI uncertainty model, we propose a robust design to jointly optimize the transmit precoder and multi-sector BD-RIS for a MU-MISO system. Specifically, SAA and WMMSE-rate relationship are adopted to transform the stochastic problem into a more tractable deterministic multi-block problem. Regarding the design of the BD-RIS matrix block, we adopt the RCG algorithm after transforming the corresponding problem into a smooth optimization.

Finally, we present simulation results, which demonstrate the performance enhancement of multi-sector BD-RIS aided RSMA compared to SDMA schemes with different radiation pattern of BD-RIS antennas and various parameter settings, such as CSI conditions and QoS threshold for users. More importantly, with given sum-rate requirement, the numbers of transmitter and BD-RIS antennas can be effectively reduced by synergizing RSMA with multi-sector BD-RIS. Such reduction of the number of antennas is beneficial for practical implementations.

## Chapter 6

# Channel Estimation and Beamforming for BD-RIS

The enhanced performance achieved by BD-RIS with different modes and architectures highly depends on accurate CSI, while none of the existing works study the channel estimation of BD-RIS. This chapter thus focuses on the channel estimation and beamforming design for BD-RIS aided wireless communication systems. This leads to the following layouts and contributions.

In Section 6.1, the uplink-downlink reciprocity of BD-RIS aided channels have been reviewed, which lays the foundation for the uplink channel estimation and downlink data transmission.

In Section 6.2, a novel channel estimation scheme for a reflective BD-RIS aided MIMO system has been proposed, where the joint design of pilot sequence and BD-RIS matrix during the training process has been investigated. The proposed pilot sequence and BD-RIS design differs from that for conventional RIS cases due to the new structure of the cascaded channel and new constraints originating from the unique BD-RIS architectures. The proposed design is applicable for general time-invariant channel models, such as commonly used i.i.d Rayleigh fading and Rician fading channels. Specifically, a closed-form pilot sequence and BD-RIS design which theoretically guarantees to achieve the MSE of the least square (LS) estimator has been proposed. A comprehensive analysis of the training overhead of the proposed channel estimation scheme has also been provided. In addition, the proposed scheme is readily generalized to multiuser scenarios and to BD-RIS with hybrid transmitting and reflecting mode and multi-sector mode by assuming each sector of the hybrid/multi-sector BD-RIS works in turns.

In Section 6.3, the beamforming designs for a reflective BD-RIS aided point-to-point MIMO system and for a hybrid/multi-sector BD-RIS aided MU-MISO system have been proposed based on the cascaded channel estimates.

In Section 6.4, simulation results have been provided to verify the effectiveness and accuracy of the proposed channel estimation scheme and evaluate the performance of the proposed beam-

forming design algorithms. Simulation results verify that the proposed channel estimation scheme achieves the theoretical minimum MSE. Simulation results also show that with the estimated channel obtained by the proposed channel estimation scheme, the proposed beamforming design algorithms can achieve satisfactory performance close to the perfect CSI cases. More importantly, there exists a practical trade-off between the channel estimation performance and the rate performance, indicating that the improved beamforming flexibility of BD-RIS architectures is achieved at the cost of increasing the training overhead.

## 6.1 Channel Model and Transmission Protocol

In this section, we illustrate the uplink and downlink channel models of general BD-RIS assisted MIMO systems, and introduce the transmission protocol of the whole system.

### 6.1.1 Channel Model

We consider a narrowband system which consists of an  $N$ -antenna BS, an  $M$ -antenna BD-RIS with reflective mode, and a  $K$ -antenna user, as illustrated in Fig. 6.1. The  $M$  antennas of the BD-RIS are connected to an  $M$ -port group-connected reconfigurable impedance network [55], where the  $M$  ports are uniformly divided into  $G_1$  groups with each containing  $\bar{M} = \frac{M}{G_1}$  ports connected to each other<sup>1</sup>. Mathematically, the BD-RIS with group-connected architecture has a block-diagonal scattering matrix<sup>2</sup>  $\bar{\Phi} \in \mathbb{C}^{M \times M}$  with each block  $\bar{\Phi}_g \in \mathbb{C}^{\bar{M} \times \bar{M}}$ ,  $\forall g \in \mathcal{G}_1 = \{1, \dots, G_1\}$ , being unitary [55], i.e.,

$$\bar{\Phi} = \text{blkdiag}(\bar{\Phi}_1, \dots, \bar{\Phi}_{G_1}), \bar{\Phi}_g^H \bar{\Phi}_g = \mathbf{I}_{\bar{M}}, \forall g \in \mathcal{G}_1. \quad (6.1)$$

In practice, the unitary matrix  $\bar{\Phi}_g$  might involve further constraints determined by the circuit topology of the reconfigurable impedance network, such that its entries are jointly controlled by specific numbers of the reconfigurable impedance components. More details about different circuit designs and corresponding mathematical constraints can be found in [55, 57].

In this work, we assume the direct user-BS channel is blocked and focus purely on the estimation of the cascaded user-RIS-BS channel<sup>3</sup>. In the uplink scenario, we denote  $\bar{\mathbf{G}} \in \mathbb{C}^{N \times M}$  and  $\bar{\mathbf{H}} \in \mathbb{C}^{M \times K}$  as the channel from the BD-RIS to BS, and from the user to BD-RIS, respectively.

<sup>1</sup>In practice, the uniform linear or planar array can be utilized for antenna deployment. Besides, varactors or PIN diodes can be used to implement the reconfigurable impedance network.

<sup>2</sup>The group-connected architecture is a general case including both single- and fully-connected architectures as special cases. Specifically,  $G_1 = M$  boils down to single-connected architecture with diagonal scattering matrices;  $G_1 = 1$  represents fully-connected architecture with full scattering matrices.

<sup>3</sup>When the direct BS-user channel exists, the direct channel can be effectively obtained by turning off the BD-RIS and using conventional channel estimation strategies.

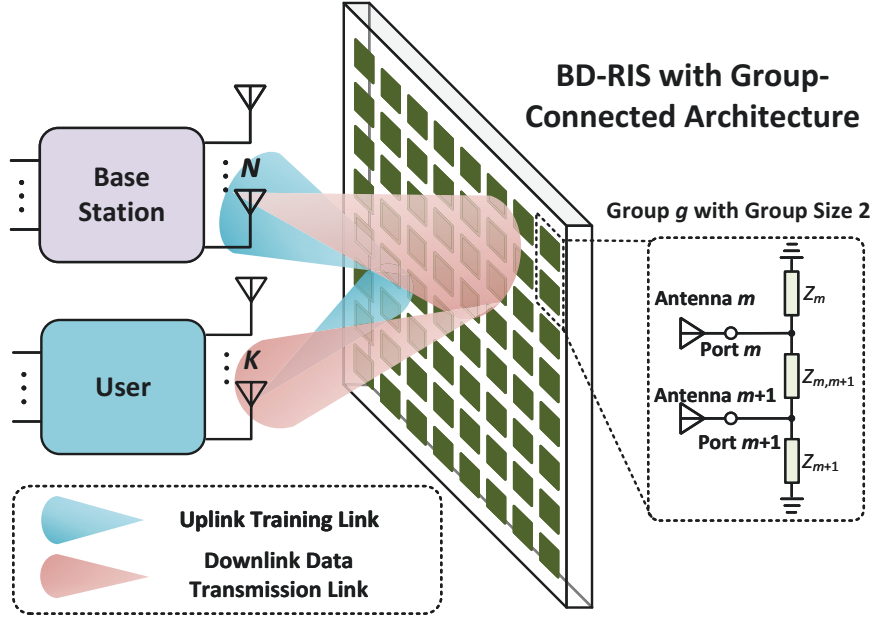


Figure 6.1: A paradigm of a BD-RIS assisted MIMO system.

By Result 2.1, the uplink user-RIS-BS channel  $\mathbf{H}_{\text{up}} \in \mathbb{C}^{N \times K}$  is

$$\mathbf{H}_{\text{up}} = \bar{\mathbf{G}} \bar{\Phi} \bar{\mathbf{H}} = \sum_{g \in \mathcal{G}_1} \bar{\mathbf{G}}_g \bar{\Phi}_g \bar{\mathbf{H}}_g = \text{unvec} \left( \sum_{g \in \mathcal{G}_1} \underbrace{\bar{\mathbf{H}}_g^T \otimes \bar{\mathbf{G}}_g}_{=\bar{\mathbf{Q}}_g \in \mathbb{C}^{NK \times \bar{M}^2}} \text{vec}(\bar{\Phi}_g) \right), \quad (6.2)$$

where  $\bar{\mathbf{G}}_g = [\bar{\mathbf{G}}]_{:, (g-1)\bar{M}+1:g\bar{M}} \in \mathbb{C}^{N \times \bar{M}}$  and  $\bar{\mathbf{H}}_g = [\bar{\mathbf{H}}]_{(g-1)\bar{M}+1:g\bar{M}, :} \in \mathbb{C}^{\bar{M} \times K}$ ,  $\forall g \in \mathcal{G}_1$ . Alternatively, in the downlink scenario, we denote  $\mathbf{G} = [\mathbf{G}_1^T, \dots, \mathbf{G}_{G_1}^T]^T \in \mathbb{C}^{M \times N}$  and  $\mathbf{H} = [\mathbf{H}_1, \dots, \mathbf{H}_{G_1}] \in \mathbb{C}^{K \times M}$ , respectively, as the channels from BS to BD-RIS, and from BD-RIS to the user. The downlink BS-RIS-user channel  $\mathbf{H}_{\text{down}} \in \mathbb{C}^{K \times N}$  is

$$\mathbf{H}_{\text{down}} \stackrel{(a)}{=} \mathbf{H} \bar{\Theta} \mathbf{G} = \sum_{g \in \mathcal{G}_1} \mathbf{H}_g \bar{\Theta}_g \mathbf{G}_g \stackrel{(b)}{=} \text{unvec}^T \left( \sum_{g \in \mathcal{G}_1} (\mathbf{H}_g \otimes \mathbf{G}_g^T) \text{vec}(\bar{\Theta}_g^T) \right), \quad (6.3)$$

where (a) holds by defining the BD-RIS matrix for downlink data transmission as  $\bar{\Theta} = \text{blkdiag}(\bar{\Theta}_1, \dots, \bar{\Theta}_{G_1})$ ; (b) holds by the property of vectorization.

When the communication occurs with TDD and the reciprocity between the uplink and downlink user-RIS and RIS-BS channels exists, i.e.,  $\mathbf{H} = \bar{\mathbf{H}}^T$  and  $\mathbf{G} = \bar{\mathbf{G}}^T$ , we have

$$\mathbf{H}_{\text{down}} = \text{unvec}^T \left( \sum_{g \in \mathcal{G}_1} \bar{\mathbf{Q}}_g \text{vec}(\bar{\Theta}_g^T) \right), \quad (6.4)$$

which indicates that the channel estimate  $\bar{\mathbf{Q}} = [\bar{\mathbf{Q}}_1, \dots, \bar{\mathbf{Q}}_{G_1}] \in \mathbb{C}^{NK \times \bar{M}^2 G_1}$  obtained by the uplink training can be used for downlink data transmission<sup>4</sup>. This motivates us to estimate  $\bar{\mathbf{Q}}$

<sup>4</sup>We should clarify here that  $\bar{\Theta} \neq \bar{\Phi}^T$  since the BD-RIS matrices are designed focusing on different metrics, i.e., minimizing the channel estimation error and maximizing the data transmission rate.

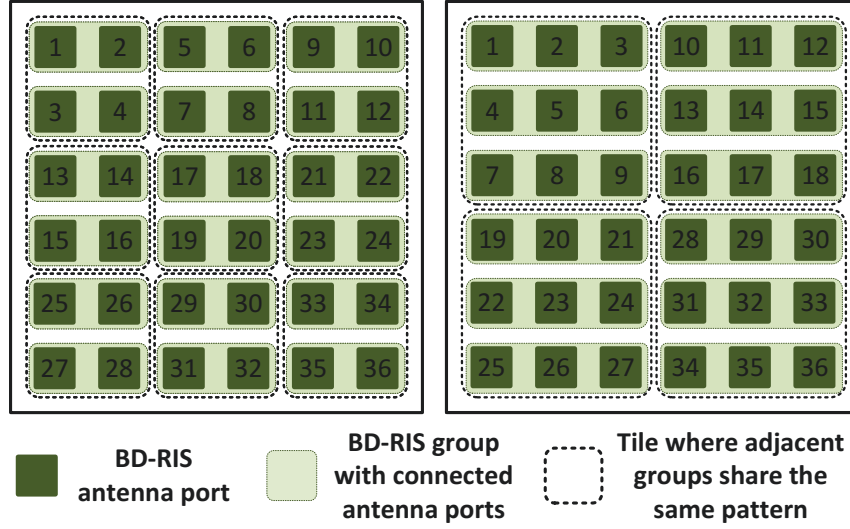


Figure 6.2: Illustration of BD-RIS antenna ports, groups, and tiles. Left: BD-RIS with  $M = 36$ ,  $G_1 = 18$  and  $G_2 = 9$ ; right: BD-RIS with  $M = 36$ ,  $G_1 = 12$  and  $G_2 = 4$ . For example, the left figure says BD-RIS antenna ports with indices  $\{1, 2\}$  and  $\{3, 4\}$  form two different groups, each of which consists of antenna ports connected to each other by reconfigurable impedance components. These two groups further form one tile, within which the groups share the same BD-RIS pattern.

instead of separate channels  $\bar{\mathbf{H}}$  and  $\bar{\mathbf{G}}$ , and design the BD-RIS beamforming with the knowledge of  $\bar{\mathbf{Q}}$  for data transmission.

From (6.2) and (6.3) we observe that the BD-RIS aided cascaded channel for both uplink and downlink, i.e.,  $\bar{\mathbf{Q}}$ , is constructed relying on the non-zero parts of the scattering matrix at BD-RIS, i.e.,  $\bar{\Phi}$  or  $\bar{\Theta}$ , which characterizes inter-element connections in BD-RIS architectures. That is, the channel to be estimated depends on the BD-RIS architectures. In addition, estimating  $\bar{\mathbf{Q}}$  relies on the variation of  $\bar{\Phi}$  during the uplink training, with new constraints (6.1) instead of being diagonal as in conventional RIS. As a result, the passive channel estimation schemes for conventional RIS [38–40] do not work for BD-RIS aided scenarios. Therefore, new passive channel estimation methods adaptive to BD-RIS architectures and  $\bar{\mathbf{Q}}$  are needed.

### 6.1.2 Tile-Based Channel Construction

Estimating  $\bar{\mathbf{Q}}$  requires extremely high complexity and training overhead due to the large number of channel coefficients growing linearly with  $G_1$  and quadratically with  $\bar{M}$ . To effectively reduce the channel estimation complexity and training overhead, we generalize the methods used in conventional RIS channel estimation works [38, 107, 108] by introducing the concept of *tile*, which consists of adjacent *groups*, each having  $\bar{M}$  ports connected to each other. The groups in the same tile share the same BD-RIS pattern during the training. To ease understanding, we provide two examples in Fig. 6.2 to illustrate antenna ports, groups, and tiles. Here, we define

$G_2$  as the number of tiles and  $\bar{G} = \frac{G_1}{G_2}$  as the tile size. We further define the shared pattern  $\Phi_i \in \mathbb{C}^{\bar{M} \times \bar{M}}$  for tile  $i$  with  $\Phi_i^H \Phi_i = \mathbf{I}_{\bar{M}}, \forall i \in \mathcal{G}_2 = \{1, \dots, G_2\}$  and rewrite the BD-RIS matrix  $\bar{\Phi}$  as

$$\bar{\Phi} = \text{blkdiag}(\mathbf{I}_{\bar{G}} \otimes \Phi_1, \dots, \mathbf{I}_{\bar{G}} \otimes \Phi_{G_2}). \quad (6.5)$$

The user-RIS-BS channel (6.2) can thus be re-written as

$$\mathbf{H}_{\text{up}} = \text{unvec} \left( \sum_{i \in \mathcal{G}_2} \underbrace{\sum_{j=1}^{\bar{G}} \bar{\mathbf{Q}}_{(i-1)\bar{G}+j}}_{=\mathbf{Q}_i \in \mathbb{C}^{NK \times \bar{M}^2}} \text{vec}(\Phi_i) \right). \quad (6.6)$$

As such, we estimate the combined cascaded channel of each tile, referred to as the tile-based channel. Specifically, a larger tile size leads to the reduction of training overhead since the channel parameters to be estimated reduces from  $\bar{\mathbf{Q}}$  to  $\mathbf{Q} = [\mathbf{Q}_1, \dots, \mathbf{Q}_{G_2}] \in \mathbb{C}^{NK \times \bar{M}^2 G_2}$ . However, this is achieved at the expense of obtaining reduced CSI, which will limit the beamforming flexibility of BD-RIS and further degrades the performance for data transmission. In other words, although we focus on different objective functions for uplink channel estimation and downlink data transmission, the two designs are tightly related to each other due to the tile-based channel construction. The impact of estimating the tile-based channel instead of the original cascaded channel on the beamforming flexibility of BD-RIS is mathematically reflected in the downlink channel  $\mathbf{H}_{\text{down}}$ , which is re-written as

$$\mathbf{H}_{\text{down}} = \text{unvec}^T \left( \sum_{i \in \mathcal{G}_2} \mathbf{Q}_i \text{vec}(\Theta_i^T) \right), \quad (6.7)$$

with a reformulation of the BD-RIS matrix as

$$\bar{\Theta} = \text{blkdiag}(\mathbf{I}_{\bar{G}} \otimes \Theta_1, \dots, \mathbf{I}_{\bar{G}} \otimes \Theta_{G_2}). \quad (6.8)$$

This indicates that with the knowledge of  $\mathbf{Q}$ , we should design  $\Theta_i, i \in \mathcal{G}_2$ , which provides less beam manipulation flexibility compared with designing  $\bar{\Theta}_g, \forall g \in \mathcal{G}_1$  as in perfect CSI cases, resulting in performance loss for downlink data transmission.

Note that the trade-off between the training overhead for channel estimation and the performance for data transmission comes from twofold perspectives. On the one hand, due to the tile-based channel construction, the larger  $\bar{G}$  reduces the dimension of channels to be estimated, i.e., from  $\bar{\mathbf{Q}}$  to  $\mathbf{Q}$ , which further reduces the overhead. Meanwhile, the larger  $\bar{G}$  reduces the dimension of BD-RIS matrices to be designed, i.e., from  $\bar{\Theta}_g, \forall g \in \mathcal{G}_1$  to  $\Theta_i, \forall i \in \mathcal{G}_2$ . On the other hand, due to the BD-RIS architectures, the larger  $\bar{M}$  increase the dimension of channels to be estimated and further the overhead. Meanwhile, the larger  $\bar{M}$  provides more flexibility to BD-RIS design, which further enhances the performance. This overhead-performance trade-off will be qualitatively and quantitatively studied in the following sections.

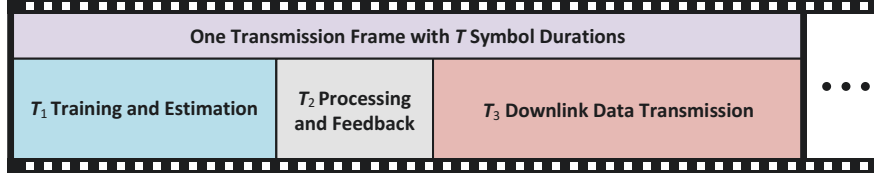


Figure 6.3: One transmission frame consisting of three phases with  $T = T_1 + T_2 + T_3$  symbol durations.

### 6.1.3 Transmission Protocol

With the tile-based channel construction, we instead estimate the reduced dimensional cascaded channel  $\mathbf{Q}$  and design the beamforming with the estimate of  $\mathbf{Q}$  for data transmission. This results in the following protocol [37,38,40] with each transmission frame divided into three phases as illustrated in Fig. 6.3, based on the assumption that user-RIS and RIS-BS channels remain approximately constant within one transmission frame. In this protocol, we propose channel estimation and beamforming designs which are feasible for general time-invariant channel models, while the performance of the proposed designs are evaluated based on commonly-used Rician fading models in Section 6.4.

Phase 1: The BS estimates  $\mathbf{Q}$  by uplink training, where the pilots are consecutively transmitted from the user and reflected by the BD-RIS with varied  $\Phi_i, \forall i \in \mathcal{G}_2$  within  $T_1$  symbol durations. In this phase, the difference compared to conventional RIS comes from the design of the varied BD-RIS pattern due to the different constraint of the scattering matrix, whose details will be given in Section 6.2.

Phase 2: The BS optimizes the beamformers and  $\Theta_i, \forall i \in \mathcal{G}_2$  based on the estimated channel  $\mathbf{Q}$ , and feeds back the optimized  $\Theta_i, \forall i \in \mathcal{G}_2$  to BD-RIS requiring  $T_2$  symbol durations. This phase is also different from conventional RIS cases due to the more general unitary constraint of the BD-RIS, whose details will be given in Section 6.3.

Phase 3: The BS performs the downlink data transmission for  $T_3$  symbol durations based on the optimized beamformers and  $\Theta_i, \forall i \in \mathcal{G}_2$  in Phase 2.

## 6.2 Channel Estimation for BD-RIS

In this section, we describe the proposed channel estimation strategy based on the LS method, formulate the BD-RIS design problem to minimize the MSE of the LS estimator, and provide the closed-form solution.

### 6.2.1 LS Based Channel Estimation

Assuming the user sends pilot symbol vector  $\bar{\mathbf{x}}_t \in \mathbb{C}^{K \times 1}$  with  $|\bar{\mathbf{x}}_t[k]| = 1, \forall k \in \mathcal{K} = \{1, \dots, K\}$ , at time slot  $t, \forall t \in \mathcal{T} = \{1, \dots, T_1\}$ , the signal received at the BS is

$$\begin{aligned} \mathbf{y}_t &= \sqrt{P_{\text{up}}} \mathbf{G} \bar{\bar{\Phi}}_t^H \mathbf{H} \bar{\mathbf{x}}_t + \mathbf{n}_t = \sqrt{P_{\text{up}}} (\bar{\mathbf{x}}_t^T \otimes \mathbf{I}_N) \sum_{i \in \mathcal{G}_2} \mathbf{Q}_i \text{vec}(\Phi_{t,i}) + \mathbf{n}_t \\ &= \sqrt{P_{\text{up}}} (\bar{\mathbf{x}}_t^T \otimes \mathbf{I}_N) \mathbf{Q} \bar{\phi}_t + \mathbf{n}_t = \sqrt{P_{\text{up}}} \underbrace{(\bar{\phi}_t^T \otimes \bar{\mathbf{x}}_t^T \otimes \mathbf{I}_N)}_{=\hat{\Phi}_t \in \mathbb{C}^{N \times NK\bar{M}^2 G_2}} \text{vec}(\mathbf{Q}) + \mathbf{n}_t, \forall t \in \mathcal{T}, \end{aligned} \quad (6.9)$$

where  $P_{\text{up}}$  denotes the transmit power at each user antenna,  $\bar{\bar{\Phi}}_t = \text{blkdiag}(\mathbf{I}_{\bar{G}} \otimes \Phi_{t,1}, \dots, \mathbf{I}_{\bar{G}} \otimes \Phi_{t,G_2})$  with  $\Phi_{t,i} \in \mathbb{C}^{\bar{M} \times \bar{M}}$  denoting the  $i$ -th tile of the BD-RIS matrix at time slot  $t$ , and  $\mathbf{n}_t \in \mathbb{C}^{N \times 1}$  with  $\mathbf{n}_t \sim \mathcal{CN}(\mathbf{0}_{N \times 1}, \sigma^2 \mathbf{I}_N)$ ,  $\forall t \in \mathcal{T}$  denotes the noise. The vector  $\bar{\phi}_t$  is defined as  $\bar{\phi}_t = [\text{vec}^T(\Phi_{t,1}), \dots, \text{vec}^T(\Phi_{t,G_2})]^T \in \mathbb{C}^{\bar{M}^2 G_2 \times 1}, \forall t \in \mathcal{T}$ . To uniquely estimate the cascaded channel  $\mathbf{Q}$  in (6.9),  $T_1 \geq K\bar{M}^2 G_2$  pilot symbols should be transmitted from the user. Combining the data from such pilots together, we have

$$\mathbf{y} = [\mathbf{y}_1^T, \dots, \mathbf{y}_{T_1}^T]^T = \sqrt{P_{\text{up}}} \underbrace{[\hat{\Phi}_1^T, \dots, \hat{\Phi}_{T_1}^T]^T}_{=\hat{\Phi} \in \mathbb{C}^{NT_1 \times NK\bar{M}^2 G_2}} \mathbf{q} + \underbrace{[\mathbf{n}_1^T, \dots, \mathbf{n}_{T_1}^T]^T}_{=\mathbf{n} \in \mathbb{C}^{NT_1 \times 1}}, \quad (6.10)$$

where  $\mathbf{q} = \text{vec}(\mathbf{Q}) \in \mathbb{C}^{NK\bar{M}^2 G_2 \times 1}$ . The simplest way to estimate  $\mathbf{q}$  is to use the LS method, yielding the LS estimator of  $\mathbf{q}$  as the following form

$$\hat{\mathbf{q}} = (\sqrt{P_{\text{up}}})^{-1} \hat{\Phi}^\dagger \mathbf{y} = (\sqrt{P_{\text{up}}})^{-1} (\hat{\Phi}^H \hat{\Phi})^{-1} \hat{\Phi}^H \mathbf{y} = \mathbf{q} + \underbrace{(\sqrt{P_{\text{up}}})^{-1} (\hat{\Phi}^H \hat{\Phi})^{-1} \hat{\Phi}^H \mathbf{n}}_{=\mathbf{q}_e \in \mathbb{C}^{NK\bar{M}^2 G_2 \times 1}}. \quad (6.11)$$

It is worth noting that the LS estimator treats each channel element as an independent unknown parameter, regardless of the strength of each channel coefficients and the correlations between coefficients. Thus, the channel estimation error does not depend on channel models, but only on the transmit power  $P_{\text{up}}$ , the concatenated pilot matrix  $\hat{\Phi}$ , and the noise. The MSE of the LS estimator is

$$\text{e}_{\hat{\mathbf{q}}} = \mathbb{E}\{\|\hat{\mathbf{q}} - \mathbf{q}\|_2^2\} = \mathbb{E}\{\|\mathbf{q}_e\|_2^2\} = \frac{\sigma^2}{P_{\text{up}}} \text{Tr}((\hat{\Phi}^H \hat{\Phi})^{-1}), \quad (6.12)$$

which implies that the MSE of channel estimation depends on the values of  $\bar{\mathbf{x}}_t$  and  $\bar{\phi}_t, \forall t \in \mathcal{T}$ , such that a proper design of them is required. As such, we formulate the following MSE minimization problem

$$\min_{\{\bar{\mathbf{x}}_t, \bar{\phi}_t\}_{\forall t}} \text{Tr}((\hat{\Phi}^H \hat{\Phi})^{-1}) \quad (6.13a)$$

$$\text{s.t.} \quad \Phi_{t,i}^H \Phi_{t,i} = \mathbf{I}_{\bar{M}}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}_2, \quad (6.13b)$$

$$|\bar{\mathbf{x}}_t[k]| = 1, \forall t \in \mathcal{T}, \forall k \in \mathcal{K}, \quad (6.13c)$$

$$\text{rank}(\hat{\Phi}) = NK\bar{M}^2 G_2, \quad (6.13d)$$

| $T_1$ Training and Estimation                                                           |       |                |                                                                                                   |  |                                                                                                           |                |
|-----------------------------------------------------------------------------------------|-------|----------------|---------------------------------------------------------------------------------------------------|--|-----------------------------------------------------------------------------------------------------------|----------------|
| <b>1<sup>st</sup> block with <math>\phi_1</math><br/>repeating <math>K</math> times</b> |       |                | <b>• • •</b>                                                                                      |  | <b><math>T_s^{\text{th}}</math> block with <math>\phi_{T_s}</math><br/>repeating <math>K</math> times</b> |                |
| $\mathbf{x}_1$                                                                          | • • • | $\mathbf{x}_K$ | <b>Sequence <math>\mathbf{x}_1, \dots, \mathbf{x}_K</math><br/>repeats <math>T_s</math> times</b> |  | $\mathbf{x}_1$                                                                                            | $\mathbf{x}_K$ |

Figure 6.4: Training breaking the training time into  $T_s$  blocks of length  $K$ .

where we set  $T_1 = K\bar{M}^2G_2$  to minimize the overhead without introducing estimation ambiguities, which yields  $\hat{\Phi}$  a full-rank square matrix. Problem (6.13) is difficult to solve due to the inverse operation in the objective and the non-convex constraints of group-connected BD-RIS. In the following subsection, we will first simplify the objective function and then propose an efficient approach for optimal  $\bar{\mathbf{x}}_t$  and  $\bar{\phi}_t$ ,  $\forall t \in \mathcal{T}$ .

## 6.2.2 Solution

We start by simplifying the inverse operation in objective (6.13a). To this end, we derive the minimum of the objective (6.13a) based on the following lemma.

**Lemma 6.1.** *The objective function in (6.13a) achieves its minimum, i.e.,  $\min \text{Tr}((\hat{\Phi}^H \hat{\Phi})^{-1}) = N\bar{M}$ , with the condition  $\hat{\Phi}^H \hat{\Phi} = \hat{\Phi} \hat{\Phi}^H = K\bar{M}G_2 \mathbf{I}_{NK\bar{M}^2G_2}$ .*

*Proof.* Please refer to Appendix C.1. □

With Lemma 6.1, we deduce that achieving the minimum MSE in problem (6.13) is equivalent to finding a scaled unitary matrix  $\hat{\Phi}$ , i.e.,  $\hat{\Phi}^H \hat{\Phi} = \hat{\Phi} \hat{\Phi}^H = K\bar{M}G_2 \mathbf{I}_{NK\bar{M}^2G_2}$ . Problem (6.13) is thus transformed into a feasibility-check problem

$$\text{find } \hat{\Phi} \in \mathbb{C}^{NK\bar{M}^2G_2 \times NK\bar{M}^2G_2} \quad (6.14a)$$

$$\text{s.t. } \hat{\Phi}^H \hat{\Phi} = K\bar{M}G_2 \mathbf{I}_{NK\bar{M}^2G_2}, (6.13b), (6.13c). \quad (6.14b)$$

Given the large dimension and complicated structure of matrix  $\hat{\Phi}$ , which is constructed by a coupling of  $\bar{\phi}_t$  and  $\bar{\mathbf{x}}_t$ ,  $\forall t \in \mathcal{T}$ , it is difficult to directly find such a matrix to simultaneously satisfy the three constraints in problem (6.14). To decouple the design of  $\bar{\phi}_t$  and  $\bar{\mathbf{x}}_t$ ,  $\forall t \in \mathcal{T}$ , we apply the most common training approach [109], which breaks the training time into  $T_s = \frac{T_1}{K} = \bar{M}^2G_2$  blocks of length  $K$ , as illustrated in Fig. 6.4. Specifically, the same BD-RIS pattern,  $\phi_s$ , is repeated  $K$  times within each block  $s \in \mathcal{S} = \{1, \dots, T_s\}$ , while it varies from block to block. Meanwhile, the pilots  $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_K]^T$  are chosen as an orthogonal sequence, i.e.,  $\mathbf{X}^H \mathbf{X} = K\mathbf{I}_K$ , and are repeated over the  $T_s$  blocks. In other words, we establish the BD-RIS

pattern set  $\phi_s, \forall s \in \mathcal{S}$ , and the pilot sequence set  $\mathbf{x}_k, \forall k \in \mathcal{K}$ , for periodicity. Specifically, we have the mappings between  $\bar{\phi}_t$  and  $\phi_s$ , and between  $\bar{\mathbf{x}}_t$  and  $\mathbf{x}_k$ , respectively, as

$$\bar{\phi}_{(s-1)K+k} = \phi_s, \bar{\mathbf{x}}_{(s-1)K+k} = \mathbf{x}_k, \forall k \in \mathcal{K}, \forall s \in \mathcal{S}. \quad (6.15)$$

This allows us to rewrite  $\hat{\Phi}^H \hat{\Phi}$  in the following form

$$\begin{aligned} \hat{\Phi}^H \hat{\Phi} &= \sum_{t=1}^{K\bar{M}^2 G_2} (\bar{\phi}_t^* \bar{\phi}_t^T) \otimes (\bar{\mathbf{x}}_t^* \bar{\mathbf{x}}_t^T) \otimes \mathbf{I}_N \\ &\stackrel{(a)}{=} \left( \sum_{s=1}^{T_s} \phi_s^* \phi_s^T \right) \otimes \left( \sum_{k=1}^K \mathbf{x}_k^* \mathbf{x}_k^T \right) \otimes \mathbf{I}_N \stackrel{(b)}{=} K(\ddot{\Phi}^H \ddot{\Phi}) \otimes \mathbf{I}_{NK}, \end{aligned} \quad (6.16)$$

where (a) holds due to the mappings in (6.15); (b) holds with  $\mathbf{X}^H \mathbf{X} = K\mathbf{I}_K$ , and by defining  $\ddot{\Phi} = [\phi_1, \dots, \phi_{T_s}]^T \in \mathbb{C}^{\bar{M}^2 G_2 \times \bar{M}^2 G_2}$ . Based on (6.16), we transform designing  $\bar{\phi}_t$  and  $\bar{\mathbf{x}}_t$  in (6.14) into designing matrices  $\ddot{\Phi}$  and  $\mathbf{X}$ . Specifically, the feasible solution of  $\mathbf{X}$  can be easily obtained, such as using DFT or Hadamard matrices, such that the  $\bar{\mathbf{x}}_t$  is given by (6.15). The design of  $\ddot{\Phi}$ , meanwhile, can be formulated as the following feasibility-check problem

$$\text{find } \ddot{\Phi} \in \mathbb{C}^{\bar{M}^2 G_2 \times \bar{M}^2 G_2} \quad (6.17a)$$

$$\text{s.t. } \ddot{\Phi}^H \ddot{\Phi} = \bar{M} G_2 \mathbf{I}_{\bar{M}^2 G_2}, \quad (6.17b)$$

Problem (6.17) is a generalization of that for conventional RIS when  $\bar{M} = 1$ , in which case the solution can be directly obtained by DFT or Hadamard matrices. However, problem (6.17) is also a new formulation different from conventional RIS case and difficult to solve due to the constraint from BD-RIS architectures, resulting in the complicated construction of  $\ddot{\Phi}$  involving both inter- and intra-row orthogonality and having a large dimension.

To simplify the design of the large-dimensional matrix  $\ddot{\Phi}$ , we propose to decouple problem (6.17) into two reduced-dimensional sub-problems with simpler constraints. This leads to the following lemma.

**Lemma 6.2.** *The feasible matrix  $\ddot{\Phi}$  from problem (6.17) can be constructed as  $\ddot{\Phi} = \mathbf{A} \otimes \check{\Phi}$ , where  $\mathbf{A} \in \mathbb{C}^{G_2 \times G_2}$  is obtained by solving the following problem:*

$$\text{find } \mathbf{A} \in \mathbb{C}^{G_2 \times G_2} \quad (6.18a)$$

$$\text{s.t. } \mathbf{A}^H \mathbf{A} = G_2 \mathbf{I}_{G_2}, \quad (6.18b)$$

$$|[\mathbf{A}]_{i,i'}| = 1, \forall i, i' \in \mathcal{G}_2. \quad (6.18c)$$

$\check{\Phi} \in \mathbb{C}^{\bar{M}^2 \times \bar{M}^2}$  is obtained by solving the following problem:

$$\text{find } \check{\Phi} \in \mathbb{C}^{\bar{M}^2 \times \bar{M}^2} \quad (6.19a)$$

$$\text{s.t. } \check{\Phi}^H \check{\Phi} = \check{\Phi} \check{\Phi}^H = \bar{M} \mathbf{I}_{\bar{M}^2}, \quad (6.19b)$$

$$\check{\Phi}_m^H \check{\Phi}_m = \mathbf{I}_{\bar{M}}, \forall m \in \bar{\mathcal{M}}, \quad (6.19c)$$

where  $\check{\Phi}_m = \text{unvec}([\check{\Phi}^T]_{:,m})$  and  $\bar{\mathcal{M}} = \{1, \dots, \bar{M}^2\}$ .

*Proof.* Please refer to Appendix C.2. □

With Lemma 6.2, we decouple problem (6.17) into two reduced-dimensional problems (6.18) and (6.19). The feasible solution to problem (6.18) can be easily obtained by using a  $G_2 \times G_2$  DFT matrix  $\mathbf{F}_{G_2}$  or a Hadamard matrix  $\mathbf{D}_{G_2}$ , i.e.,  $\mathbf{A}^{\text{DFT}} = \mathbf{F}_{G_2}$  or  $\mathbf{A}^{\text{Hadamard}} = \mathbf{D}_{G_2}$ . However, the solution to problem (6.19) is not a straightforward extension from conventional RIS since we need to find a matrix  $\check{\Phi}$  such that each row constructs a unitary matrix, i.e., constraint (6.19c), and that different rows are orthogonal to each other, i.e., constraint (6.19b). In other words, the matrix  $\check{\Phi}$  should include two-dimensional (intra- and inter-row) orthogonality, which motivates us to construct  $\check{\Phi}$  with two orthogonal bases,  $\mathbf{Z}_1 \in \mathbb{C}^{\bar{M} \times \bar{M}}$  and  $\mathbf{Z}_2 \in \mathbb{C}^{\bar{M} \times \bar{M}}$ . The former guarantees the orthogonality within each row of  $\check{\Phi}$ ;  $\mathbf{Z}_2$  and the circular shift of  $\mathbf{Z}_1$  guarantee that  $\check{\Phi}$  has orthogonal rows. This brings the following theorem.

**Theorem 6.1.** *The matrix  $\check{\Phi}$  satisfying (6.19b) and (6.19c) can be constructed such that each row, i.e.,  $[\check{\Phi}]_{(m-1)\bar{M}+n,:} = \check{\phi}_{m,n}$ ,  $\forall m, n \in \bar{\mathcal{M}} = \{1, \dots, \bar{M}\}$ , has the following structure:*

$$\check{\phi}_{m,n} = \text{circshift}(\text{vec}^T(\mathbf{Z}_1), (n-1)\bar{M}) \odot ([\mathbf{Z}_2]_{m,:} \otimes \mathbf{1}_M^T), \quad (6.20)$$

where  $\mathbf{Z}_1 \in \mathbb{C}^{\bar{M} \times \bar{M}}$  and  $\mathbf{Z}_2 \in \mathbb{C}^{\bar{M} \times \bar{M}}$  are referred to as base matrices satisfying the following constraints:

- 1)  $\mathbf{Z}_1$  is a scaled unitary matrix, i.e.,  $\mathbf{Z}_1^H \mathbf{Z}_1 = \mathbf{Z}_1 \mathbf{Z}_1^H = \alpha_1 \mathbf{I}_{\bar{M}}$  with  $\alpha \in \mathbb{R}$ ,  $\alpha_1 \neq 0$  a scalar;
- 2)  $\mathbf{Z}_2$  is a scaled unitary matrix whose entries have identical modulus, i.e.,  $\mathbf{Z}_2^H \mathbf{Z}_2 = \mathbf{Z}_2 \mathbf{Z}_2^H = \alpha_2 \mathbf{I}_{\bar{M}}$  with  $\alpha_2 \in \mathbb{R}$ ,  $\alpha_2 \neq 0$  a scalar,  $|[\mathbf{Z}_2]_{m,n}| = \sqrt{\frac{\alpha_2}{\bar{M}}}$ ,  $\forall m, n \in \bar{\mathcal{M}}$ ;
- 3) The product of two scalars is a constant, i.e.,  $\alpha_1 \alpha_2 = \bar{M}$ .

*Proof.* Please refer to Appendix C.3. □

Based on Theorem 6.1, we transform problem (6.19) into finding two base matrices  $\mathbf{Z}_1$  and  $\mathbf{Z}_2$  satisfying the above three constraints in Theorem 6.1, which can be easily satisfied using DFT/Hadamard matrices. More specifically, we can simply set  $\mathbf{Z}_1^{\text{DFT}} = \mathbf{F}_{\bar{M}}$  or  $\mathbf{Z}_1^{\text{Hadamard}} = \mathbf{D}_{\bar{M}}$

**Algorithm 8** Channel Estimation for BD-RIS**Require:**  $N, M, \bar{M}, \bar{G}, K$ .**Ensure:**  $\hat{\mathbf{q}}$ .

- 1: Generate pilot sequence  $\mathbf{X}$  such that  $\mathbf{X}^H \mathbf{X} = K \mathbf{I}_K$ .
- 2: Generate  $\mathbf{A}$  by  $\mathbf{A}^{\text{DFT}} = \mathbf{F}_{G_2}$  or  $\mathbf{A}^{\text{Hadamard}} = \mathbf{D}_{G_2}$ .
- 3: Set  $\mathbf{Z}_1$  by  $\mathbf{Z}_1^{\text{DFT}} = \mathbf{F}_{\bar{M}}$  or  $\mathbf{Z}_1^{\text{Hadamard}} = \mathbf{D}_{\bar{M}}$ .
- 4: Set  $\mathbf{Z}_2$  by  $\mathbf{Z}_2^{\text{DFT}} = \frac{1}{\sqrt{\bar{M}}} \mathbf{F}_{\bar{M}}$  or  $\mathbf{Z}_2^{\text{Hadamard}} = \frac{1}{\sqrt{\bar{M}}} \mathbf{D}_{\bar{M}}$ .
- 5: Construct  $\check{\check{\Phi}}$  by (6.20) from Theorem 5.1.
- 6: Construct  $\check{\Phi}$  by  $\check{\Phi} = \mathbf{A} \otimes \check{\check{\Phi}}$  from Lemma 5.2.
- 7: Map  $\mathbf{X}$  and  $\check{\Phi}$  to  $\bar{\mathbf{x}}_t$  and  $\bar{\phi}_t$ ,  $\forall t \in \mathcal{T}$  by (6.15).
- 8: Construct  $\hat{\Phi}$  by (6.9) with  $\bar{\phi}_t$  and  $\bar{\mathbf{x}}_t$ ,  $\forall t \in \mathcal{T}$ .
- 9: Obtain  $\hat{\mathbf{q}}$  by (6.11).

Table 6.1: Training Overhead, Estimation MSE, and Circuit Complexity of BD-RIS with Group-Connected Architectures

| Metrics            | Value                                                                    | Property                                                |
|--------------------|--------------------------------------------------------------------------|---------------------------------------------------------|
| Training Overhead  | $T_1 = KM\bar{M}\bar{G}^{-1}$                                            | $\uparrow$ with $\bar{M}$ ; $\downarrow$ with $\bar{G}$ |
| Estimation MSE     | $e_{\hat{\mathbf{q}}}^{\min} = \frac{\sigma^2}{P_{\text{up}}} N \bar{M}$ | $\uparrow$ with $\bar{M}$                               |
| Circuit Complexity | $C = (\bar{M} + 1) \frac{M}{2}$ [55]                                     | $\uparrow$ with $\bar{M}$                               |

and  $\mathbf{Z}_2^{\text{DFT}} = \frac{1}{\sqrt{\bar{M}}} \mathbf{F}_{\bar{M}}$  or  $\mathbf{Z}_2^{\text{Hadamard}} = \frac{1}{\sqrt{\bar{M}}} \mathbf{D}_{\bar{M}}$  to construct  $\check{\check{\Phi}}$ , and further construct  $\check{\Phi} = \mathbf{A} \otimes \check{\check{\Phi}}$  with  $\mathbf{A}^{\text{DFT}}$  or  $\mathbf{A}^{\text{Hadamard}}$ .

With solutions to construct pilot sequence  $\mathbf{X}$  and matrix  $\check{\check{\Phi}}$ , the procedure of the proposed channel estimation scheme is straightforward, which is summarized in Algorithm 8.

Note that either using DFT matrices or Hadamard matrices to generate  $\check{\check{\Phi}}$ , we achieve the minimum MSE of the LS estimation, i.e.,  $e_{\hat{\mathbf{q}}}^{\min} = N\sigma^2 P_{\text{up}}^{-1} \bar{M}$ , and avoid the time-consuming inversion by  $\hat{\Phi}^\dagger = (K\bar{M}G_2)^{-1} \hat{\Phi}^H$ . Nevertheless, the two ways have pros and cons from different aspects. On one hand, the matrix  $\check{\check{\Phi}}$  constructed by Hadamard bases has the additional advantage of requiring only two states of each entry, which indicates that practical group-connected BD-RIS with discrete values can be effectively tuned to perform accurate channel estimation. However, the values of  $G_2$  and  $\bar{M}$  should be chosen from the set  $\{1, 2, 4n, n \in \mathbb{N}\}$  to guarantee the existence of Hadamard matrix. On the other hand,  $\check{\check{\Phi}}$  constructed by DFT bases requires more states of each entry of group-connected BD-RIS, which complicates the practical implementation of BD-RIS, while there is no such limitation of the values of  $G_2$  and  $\bar{M}$ .

### 6.2.3 Overhead, MSE, and Circuit Complexity

The training overhead  $T_1$  to estimate  $\mathbf{q}$ , the estimation error  $e_{\hat{\mathbf{q}}}^{\min}$ , and the circuit complexity  $C$  of the group-connected BD-RIS indicated by the number of impedances to realize the circuit

topology, are summarized in Table 6.1. We can observe that with fixed number of RIS antennas  $M$  and other parameters, the training overhead, estimation MSE, and circuit complexity all increase with the group size  $\bar{M}$  of the group-connected architecture. Interestingly, the training overhead can be effectively reduced by introducing tiles (increasing tile size  $\bar{G}$ ), although the larger  $\bar{M}$  decreases the number of possible  $\bar{G}$ . More importantly, the larger  $\bar{M}$  provides more beam manipulation flexibility to BD-RIS to enhance the transmission performance, while the larger  $\bar{G}$  leads to reduced CSI and further limits the transmission performance. This phenomenon will be numerically studied in Section 6.4. Therefore, it is important to properly choose the values of  $\bar{M}$  and  $\bar{G}$  to achieve satisfactory channel estimation performance and data transmission performance with affordable training overhead and circuit complexity.

### 6.2.4 Multi-User Scenarios

The proposed channel estimation scheme can be easily generalized to multi-user cases by assuming all users transmit orthogonal pilot sequences such that channels from all users can be simultaneously estimated by the BS. This can be simply done by setting  $K$  as the total number of antennas at all users. Alternatively, cascaded channels from different users can be successively estimated by assuming users transmitting pilots in a TDD manner. In this case, each user can transmit the same pilots and adopt the proposed channel estimation scheme to estimate the cascaded channel.

### 6.2.5 BD-RIS with Hybrid/Multi-Sector Modes

While the proposed channel estimation is based on BD-RIS with reflective mode, that is the user is located at the same side as BS, it is also readily generalized to BD-RIS with hybrid/multi-sector modes. Assume that an  $L$ -sector BD-RIS with  $M$  antennas in each sector,  $L \in \mathcal{L} = \{1, \dots, L\}$ ,  $L \geq 2$ , is adopted in a downlink MU-MISO system with an  $N$ -antenna BS and  $K$  single-antenna users. Each sector covers  $K_l$  users indexed by  $\mathcal{K}_l = \{\sum_{i=1}^{l-1} K_i + 1, \dots, \sum_{i=1}^l K_i\}$ ,  $\sum_{l \in \mathcal{L}} K_l = K$ . In addition, the  $L$ -sector BD-RIS with group-connected architecture is, mathematically, characterized by matrices  $\tilde{\Phi}_l = \text{blkdiag}(\tilde{\Phi}_{l,1}, \dots, \tilde{\Phi}_{l,G_1}) \in \mathbb{C}^{M \times M}$ ,  $\forall l \in \mathcal{L}$  with  $\tilde{\Phi}_{l,g} \in \mathbb{C}^{\bar{M} \times \bar{M}}$ ,  $\forall g \in \mathcal{G}_1$ , satisfying the following constraint

$$\sum_{l \in \mathcal{L}} \tilde{\Phi}_{l,g}^H \tilde{\Phi}_{l,g} = \mathbf{I}_{\bar{M}}, \forall g \in \mathcal{G}_1. \quad (6.21)$$

Let  $\bar{\mathbf{G}} = [\bar{\mathbf{G}}_1, \dots, \bar{\mathbf{G}}_{G_1}] \in \mathbb{C}^{N \times M}$  and  $\bar{\mathbf{h}}_k = [\bar{\mathbf{h}}_{k,1}^T, \dots, \bar{\mathbf{h}}_{k,G_1}^T]^T \in \mathbb{C}^{M \times 1}$  denote the channel from BD-RIS to BS, and from user  $k$ ,  $\forall k \in \mathcal{K}$ , to BD-RIS, respectively. Then the user-RIS-BS uplink

channel can be expressed as

$$\mathbf{h}_{\text{up},k} = \sum_{g \in \mathcal{G}_1} \underbrace{\bar{\mathbf{h}}_{k,g}^T \otimes \bar{\mathbf{G}}_g}_{=\bar{\mathbf{Q}}_{k,g} \in \mathbb{C}^{N \times \bar{M}^2}} \text{vec}(\tilde{\Phi}_{l,g}), \forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}. \quad (6.22)$$

There are two strategies to estimate the above cascaded channels for users covered by different sectors. The first strategy is to successively estimate the channels for users covered by each sector. In this sense, the estimation process for each sector boils down to the case for reflective BD-RIS such that the proposed method can be directly adopted. The second strategy is to simultaneously estimate the channels from users covered by different sectors, while a new BD-RIS pattern design should be considered to adapt to the constraint (6.21). Therefore, in this work, we focus on the first strategy and leave the second one as an interesting future work. Applying the tile-based channel construction<sup>5</sup> proposed in Section 6.1, we instead estimate the channels

$$\mathbf{Q}_{k,i} = \sum_{j=1}^{\bar{G}} \bar{\mathbf{Q}}_{k,(i-1)\bar{G}+j}, \forall i \in \mathcal{G}_2, \forall k \in \mathcal{K}_l, \forall l \in \mathcal{L}. \quad (6.23)$$

This can be done according to the following steps.

Step 1: In every  $T_{1,l} = K_l \bar{M}^2 G_2$  time durations, the  $K_l$  cascaded channels  $\mathbf{Q}_{k,i}, \forall i \in \mathcal{G}_2, \forall k \in \mathcal{K}_l$ , can be simultaneously estimated assuming

- a) that  $\tilde{\Phi}_{l'} = \mathbf{0}_{M \times M}, \forall l' \neq l$ , within the  $T_{1,l}$  durations,
- b) that  $\tilde{\Phi}_l$  and the pilots transmitted by  $K_l$  users are designed using the proposed scheme in Algorithm 8.

This yields the channel estimates  $\hat{\mathbf{Q}}_{k,i}, \forall i \in \mathcal{G}_2, \forall k \in \mathcal{K}_l$ .

Step 2: Repeat Step 1 for all  $L$  sectors.

To facilitate understanding, we provide a simple example for how to estimate the cascaded channels for BD-RIS with hybrid mode ( $L = 2$ ) as illustrated in Fig. 6.5. Based on the above strategy, we can obtain the cascaded channels for BD-RIS with hybrid/multi-sector modes with a total training overhead  $T_1 = \sum_{l \in \mathcal{L}} T_{1,l} = \sum_{l \in \mathcal{L}} K_l \bar{M}^2 G_2 = K \bar{M}^2 G_2 = K M \bar{M} \bar{G}^{-1}$ .

## 6.3 Beamforming Design for BD-RIS

In this section, we formulate two beamforming design problems for BD-RIS with reflective and hybrid/multi-sector modes and different scenarios, and propose efficient beamforming design methods relying on the cascaded channel estimates. The considered problems are different from

<sup>5</sup>For BD-RIS with hybrid/multi-sector mode, the *group* is formed by the antenna ports from the same sector which are connected to each other. The *tile* is formed by groups from the same sector.



notation given by  $\boldsymbol{\theta} = [\text{vec}^T(\boldsymbol{\Theta}_1^T), \dots, \text{vec}^T(\boldsymbol{\Theta}_{G_2}^T)]^T$ . Problem (6.25) is not easy to solve due to the non-convex constraint (6.25b) and the coupling of the BD-RIS constraint, the precoder, and the combiner. To simplify the joint design, the objective function is decoupled into two separate optimizations. Specifically, we first design the BD-RIS to maximize the channel strength. Then, having the effective channel, we determine the precoder and the combiner to further maximize the spectral efficiency.

**Solution to Problem (6.25):** With determined  $\boldsymbol{\theta}$ , it is well-known that the optimal unconstrained precoder and combiner for point-to-point MIMO are obtained from the unitary right and left singular vectors of the effective channel  $\hat{\mathbf{H}}_{\text{down}}$ . Specifically, for the case of  $N = K = N_s$  and equal power allocation for all streams at the BS, we have  $\mathbf{P}\mathbf{P}^H \propto \mathbf{I}_N$  and  $\mathbf{W}\mathbf{W}^H \propto \mathbf{I}_K$ . This motivates us to have the approximations  $\mathbf{W}(\mathbf{W}^H\mathbf{W})^{-1}\mathbf{W}^H \approx \mathbf{I}_K$  and  $\mathbf{P}\mathbf{P}^H \approx \zeta^2\mathbf{I}_N$ , where  $\zeta^2$  denotes the normalization factor. Based on the above discussions, the objective function in (6.25a) has an upper bound as

$$\begin{aligned} \log_2 \left( \left| \mathbf{I}_{N_s} + \sigma'^{-2} \zeta^2 \hat{\mathbf{H}}_{\text{down}} \hat{\mathbf{H}}_{\text{down}}^H \right| \right) &\stackrel{(a)}{=} \log_2 \left( \prod_{n=1}^{N_s} (1 + \lambda_n) \right) \\ &= \sum_{n=1}^{N_s} \log_2(1 + \lambda_n) \stackrel{(b)}{\leq} \frac{1}{\ln 2} \sum_{n=1}^{N_s} \lambda_n = \frac{\zeta^2}{\ln 2 \sigma'^2} \text{Tr}(\hat{\mathbf{H}}_{\text{down}} \hat{\mathbf{H}}_{\text{down}}^H), \end{aligned} \quad (6.26)$$

where (a) holds by defining  $\lambda_1, \dots, \lambda_{N_s}$  as the eigenvalues of the matrix  $\sigma'^{-2} \zeta^2 \hat{\mathbf{H}}_{\text{down}} \hat{\mathbf{H}}_{\text{down}}^H$ , (b) holds since  $\lambda_n \geq 0, \forall n$ . In addition, the objective function in (6.25a) has a lower bound as

$$\begin{aligned} \log_2 \left( \prod_{n=1}^{N_s} (1 + \lambda_n) \right) &= \log_2 \left( 1 + \sum_{n=1}^{N_s} \lambda_n + \sum_{n=1}^{N_s} \sum_{n'=1}^{N_s} \lambda_n \lambda_{n'} + \dots \right) \\ &\geq \log_2 \left( 1 + \sum_{n=1}^{N_s} \lambda_n \right) = \log_2 \left( 1 + \frac{\zeta^2}{\sigma'^2} \text{Tr}(\hat{\mathbf{H}}_{\text{down}} \hat{\mathbf{H}}_{\text{down}}^H) \right). \end{aligned} \quad (6.27)$$

The above derivations indicate that maximizing the sum of eigenvalues helps in increasing both the lower and upper bounds of the rate. This motivates us to develop a two-stage design, where we first design the BD-RIS to maximize the channel strength, i.e., sum of eigenvalues, and then design the precoder and combiner to manage the inter-stream interference once  $\boldsymbol{\theta}$  is determined. As such, we formulate the following BD-RIS design problem

$$\boldsymbol{\theta}^* = \arg \max_{\substack{\boldsymbol{\Theta}_i^H \boldsymbol{\Theta}_i = \mathbf{I}_{\bar{M}}, \forall i}} \text{Tr}(\hat{\mathbf{H}}_{\text{down}} \hat{\mathbf{H}}_{\text{down}}^H) \stackrel{(a)}{=} \arg \max_{\substack{\boldsymbol{\Theta}_i^H \boldsymbol{\Theta}_i = \mathbf{I}_{\bar{M}}, \forall i}} \sum_{i' \in \mathcal{G}_2} \sum_{i \in \mathcal{G}_2} \boldsymbol{\theta}_i^T \mathbf{V}_{i,i'} \boldsymbol{\theta}_{i'}^*, \quad (6.28)$$

where (a) follows with  $\boldsymbol{\theta}_i = \text{vec}(\boldsymbol{\Theta}_i^T)$  and  $\mathbf{V}_{i,i'} = [\hat{\mathbf{Q}}^T \hat{\mathbf{Q}}^*]_{\mathcal{I}_i, \mathcal{I}_{i'}}$ ,  $\mathcal{I}_i = (i-1)\bar{M}^2 + 1 : i\bar{M}^2$ ,  $\forall i, i' \in \mathcal{L}$ . Note that  $\boldsymbol{\theta}_i$  and  $\boldsymbol{\theta}_{i'}$ ,  $\forall i \neq i', i, i' \in \mathcal{G}_2$ , have separate constraints, while they are coupled in the objective function (6.28), which makes it difficult to determine  $\boldsymbol{\theta}_i$ ,  $\forall i \in \mathcal{G}_2$ . To simplify the design, we split the objective with respect to each  $\boldsymbol{\theta}_i$  out from (6.28) with fixed

others. This results in the sub-problem for  $\boldsymbol{\theta}_i$  as

$$\max_{\boldsymbol{\theta}_i} \boldsymbol{\theta}_i^T \mathbf{V}_{i,i} \boldsymbol{\theta}_i^* + 2\Re\{\boldsymbol{\theta}_i^T \boldsymbol{\chi}_i\} \quad (6.29a)$$

$$\text{s.t. } \text{unvec}^*(\boldsymbol{\theta}_i) \text{unvec}^T(\boldsymbol{\theta}_i) = \mathbf{I}_{\bar{M}}, \quad (6.29b)$$

where  $\boldsymbol{\chi}_i = \sum_{i' \neq i} \mathbf{V}_{i,i'} \boldsymbol{\theta}_{i'}^* \in \mathbb{C}^{\bar{M}^2 \times 1}$ ,  $\forall i \in \mathcal{G}_2$ . Constraint (6.29b) constructs a  $\bar{M}^2$ -dimensional complex Stiefel manifold such that problem (6.29) can be formulated as an unconstrained optimization on the Stiefel manifold. Then, the problem can be effectively solved by the conjugate-gradient methods on the manifold space [87].

After the BD-RIS has been designed, we obtain the effective channel as  $\mathbf{H}_{\text{down}}^* = \text{unvec}^T(\hat{\mathbf{Q}}\boldsymbol{\theta}^*)$ . To obtain the precoder and combiner, we adopt the singular value decomposition (SVD) of the effective channel  $\mathbf{H}_{\text{down}}^*$  as  $\mathbf{H}_{\text{down}}^* = \mathbf{U}_1 \boldsymbol{\Sigma} \mathbf{U}_2^H$ , where  $\mathbf{U}_1 \in \mathbb{C}^{K \times K}$  and  $\mathbf{U}_2 \in \mathbb{C}^{N \times N}$  are unitary matrices, and  $\boldsymbol{\Sigma} \in \mathbb{C}^{K \times N}$  is a rectangular diagonal matrix containing eigenvalues of  $\mathbf{H}_{\text{down}}^*$  on the diagonal. Accordingly, we have the precoder and combiner as

$$\mathbf{P}^* = \sqrt{P_{\text{down}}} \frac{[\mathbf{U}_1]_{:,1:N_s}}{\|[\mathbf{U}_1]_{:,1:N_s}\|_F}, \quad \mathbf{W}^* = [\mathbf{U}_2]_{:,1:N_s}. \quad (6.30)$$

### 6.3.2 Hybrid/Multi-Sector BD-RIS Aided MU-MISO

In this subsection, we consider a MU-MISO system aided by a BD-RIS with hybrid/multi-sector mode, as illustrated in Section 6.2. Assume the  $N$ -antenna BS transmits the symbols  $\tilde{\mathbf{s}} = [s_1, \dots, s_K] \in \mathbb{C}^{K \times 1}$  to  $K$  single-antenna users,  $\mathbb{E}\{\tilde{\mathbf{s}}\tilde{\mathbf{s}}^H\} = \mathbf{I}_K$ . Based on (6.22) and (6.23), the received signal at each user covered by sector  $l$ ,  $\forall l \in \mathcal{L}$ , is given by

$$y_k = \sum_{i \in \mathcal{G}_2} \text{vec}^T(\boldsymbol{\Theta}_{l,i}^T) \mathbf{Q}_{k,i}^T \mathbf{p}_k s_k + \sum_{k' \neq k} \sum_{i \in \mathcal{G}_2} \text{vec}^T(\boldsymbol{\Theta}_{l,i}^T) \mathbf{Q}_{k,i}^T \mathbf{p}_{k'} s_{k'} + n_k, \forall k \in \mathcal{K}_l, \quad (6.31)$$

where  $\boldsymbol{\Theta}_{l,i} \in \mathbb{C}^{\bar{M} \times \bar{M}}$ ,  $\forall l \in \mathcal{L}$ , satisfies  $\sum_{l \in \mathcal{L}} \boldsymbol{\Theta}_{l,i}^H \boldsymbol{\Theta}_{l,i} = \mathbf{I}_{\bar{M}}$ ,  $\forall i \in \mathcal{G}_2$ ,  $\mathbf{p}_k \in \mathbb{C}^{N \times 1}$  denotes the precoder for user  $k$ , and  $n_k \in \mathbb{C}$  with  $n_k \sim \mathcal{CN}(0, \sigma^2)$  denotes the noise.

**Problem Formulation:** Based on the channel estimates  $\hat{\mathbf{Q}}_{k,i}$ ,  $\forall k \in \mathcal{K}$ ,  $\forall i \in \mathcal{G}_2$ , the joint transmitter and BD-RIS design problem to maximize the spectral efficiency is

$$\max_{\bar{\boldsymbol{\theta}}_l, \forall l, \mathbf{p}_k, \forall k} \left(1 - \frac{T_1 + T_2}{T}\right) \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}_l} \log_2(1 + \gamma_k) \quad (6.32a)$$

$$\text{s.t. } \sum_{l \in \mathcal{L}} \boldsymbol{\Theta}_{l,i}^H \boldsymbol{\Theta}_{l,i} = \mathbf{I}_{\bar{M}}, \forall i \in \mathcal{G}_2. \quad (6.32b)$$

$$\sum_{k \in \mathcal{K}} \|\mathbf{p}_k\|_2^2 \leq P_{\text{down}}, \quad (6.32c)$$

where  $\gamma_k = \frac{|\tilde{\boldsymbol{\theta}}_l^T \tilde{\mathbf{Q}}_k^T \mathbf{p}_k|^2}{\sum_{k' \neq k} |\tilde{\boldsymbol{\theta}}_l^T \tilde{\mathbf{Q}}_k^T \mathbf{p}_{k'}|^2 + \sigma'^2}$  denotes the signal-to-interference-plus-noise ratio (SINR) with the fresh notations  $\tilde{\boldsymbol{\theta}}_l = [\text{vec}^T(\boldsymbol{\Theta}_{l,1}^T), \dots, \text{vec}^T(\boldsymbol{\Theta}_{l,G_2}^T)]^T$  and  $\tilde{\mathbf{Q}}_k = [\hat{\mathbf{Q}}_{k,1}, \dots, \hat{\mathbf{Q}}_{k,G_2}]$ ,  $\forall k \in \mathcal{K}_l$ ,  $\forall l \in \mathcal{L}$ . Although problem (6.32) is a challenging optimization with complicated ratio terms in the  $\log(\cdot)$  function and non-convex constraints of BD-RIS, we show that the algorithm proposed in Chapter 3 can be easily adapted to solving (6.32) with slight modifications.

**Solution to Problem (6.32):** By introducing auxiliary variables  $\iota_k$  and  $\tau_k$ ,  $\forall k \in \mathcal{K}_l$ ,  $\forall l \in \mathcal{L}$  to each ratio term in (6.32a) based on the fractional programming [83] reviewed in Section 2.2.1, we can transform (6.32a) into a four-block optimization with the objective function  $f(\{\tilde{\boldsymbol{\theta}}_l\}_{\forall l}, \{\mathbf{p}_k\}_{\forall k}, \{\iota_k\}_{\forall k}, \{\tau_k\}_{\forall k}) = \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}_l} (\log_2(1 + \iota_k) - \iota_k + 2\sqrt{1 + \iota_k} \Re\{\tau_k^* \tilde{\boldsymbol{\theta}}_l^T \tilde{\mathbf{Q}}_k^T \mathbf{p}_k\} - |\tau_k|^2 (\sum_{k' \in \mathcal{K}} |\tilde{\boldsymbol{\theta}}_l^T \tilde{\mathbf{Q}}_k^T \mathbf{p}_{k'}|^2 + \sigma'^2))$ . The four blocks are iteratively updated in sequential until convergence. Under the iterative design framework, the solutions to the blocks  $\{\iota_k\}_{\forall k}$  and  $\{\tau_k\}_{\forall k}$  can be easily obtained by checking the first-order conditions of  $f(\{\tilde{\boldsymbol{\theta}}_l\}_{\forall l}, \{\mathbf{p}_k\}_{\forall k}, \{\iota_k\}_{\forall k}, \{\tau_k\}_{\forall k})$ . Meanwhile, the solution to block  $\{\mathbf{p}_k\}_{\forall k}$  can be obtained based on the KKT conditions. The detailed derivations can be done following Section 2.2.2.

When blocks  $\{\iota_k\}_{\forall k}$ ,  $\{\tau_k\}_{\forall k}$ , and  $\{\mathbf{p}_k\}_{\forall k}$  are determined, the sub-problem for designing  $\{\tilde{\boldsymbol{\theta}}_l\}_{\forall l}$  can be formulated as

$$\begin{aligned} \{\tilde{\boldsymbol{\theta}}_l^*\}_{\forall l} &= \arg \max_{\sum_{l \in \mathcal{L}} \boldsymbol{\Theta}_{l,i}^H \boldsymbol{\Theta}_{l,i} = \mathbf{I}_{\bar{M}}, \forall i} \sum_{l \in \mathcal{L}} \left( 2\Re\{\tilde{\boldsymbol{\theta}}_l^T \mathbf{v}_l\} - \tilde{\boldsymbol{\theta}}_l^T \mathbf{V}_l \tilde{\boldsymbol{\theta}}_l^* \right) \\ &= \arg \max_{\sum_{l \in \mathcal{L}} \boldsymbol{\Theta}_{l,i}^H \boldsymbol{\Theta}_{l,i} = \mathbf{I}_{\bar{M}}, \forall i} \sum_{l \in \mathcal{L}} \left( 2 \sum_{i \in \mathcal{G}_2} \Re\{\boldsymbol{\theta}_{l,i}^T \mathbf{v}_{l,i}\} - \sum_{i \in \mathcal{G}_2} \sum_{i' \in \mathcal{G}_2} \boldsymbol{\theta}_{l,i}^T \mathbf{V}_{l,i,i'} \boldsymbol{\theta}_{l,i'}^* \right), \end{aligned} \quad (6.33)$$

where  $\mathbf{v}_l = \sum_{k \in \mathcal{K}_l} \sqrt{1 + \iota_k} \tau_k^* \tilde{\mathbf{Q}}_k^T \mathbf{p}_k$ ,  $\mathbf{V}_l = \sum_{k \in \mathcal{K}_l} |\tau_k|^2 \tilde{\mathbf{Q}}_k^T \sum_{k' \in \mathcal{K}} \mathbf{p}_{k'} \mathbf{p}_{k'}^H \tilde{\mathbf{Q}}_k$ ,  $\forall l \in \mathcal{L}$ ,  $\boldsymbol{\theta}_{l,i} = \text{vec}(\boldsymbol{\Theta}_{l,i}^T)$ ,  $\mathbf{v}_{l,i} = [\mathbf{v}_l]_{\mathcal{I}_i}$ , and  $\mathbf{V}_{l,i,i'} = [\mathbf{V}_l]_{\mathcal{I}_i, \mathcal{I}_{i'}}$ ,  $\forall i, i' \in \mathcal{G}_2$ . Analogous to the derivation in previous subsection, we propose to split the objective with respect to each  $\{\boldsymbol{\theta}_{l,i}\}_{\forall l}$  from the objective in (6.33) with fixed others. Defining  $\ddot{\boldsymbol{\chi}}_i = [\boldsymbol{\chi}_{1,i}^T, \dots, \boldsymbol{\chi}_{L,i}^T]^T$  with  $\boldsymbol{\chi}_{l,i} = \mathbf{v}_{l,i} - \sum_{i' \neq i} \mathbf{V}_{l,i,i'} \boldsymbol{\theta}_{l,i'}^*$ , and  $\ddot{\mathbf{V}}_i = \text{blkdiag}(\mathbf{V}_{1,i,i}, \dots, \mathbf{V}_{L,i,i})$ , we construct the sub-problem for  $\ddot{\boldsymbol{\theta}}_i = \text{vec}([\boldsymbol{\Theta}_{1,i}^T, \dots, \boldsymbol{\Theta}_{L,i}^T])$  as

$$\max_{\ddot{\boldsymbol{\theta}}_i} 2\Re\{\ddot{\boldsymbol{\theta}}_i^T \ddot{\boldsymbol{\chi}}_i\} - \ddot{\boldsymbol{\theta}}_i^T \ddot{\mathbf{V}}_i \ddot{\boldsymbol{\theta}}_i^* \quad (6.34a)$$

$$\text{s.t. } \text{unvec}^*(\ddot{\boldsymbol{\theta}}_i) \text{unvec}^T(\ddot{\boldsymbol{\theta}}_i) = \mathbf{I}_{\bar{M}}, \quad (6.34b)$$

where constraint (6.34b) constructs a  $\bar{M}^2 L$ -dimensional complex Stiefel manifold. Therefore, problem (6.34) can be solved by the manifold based methods [87].

## 6.4 Performance Evaluation

In this section, we perform simulation results to verify the effectiveness of the proposed channel estimation scheme and evaluate the performance of the proposed beamforming design.

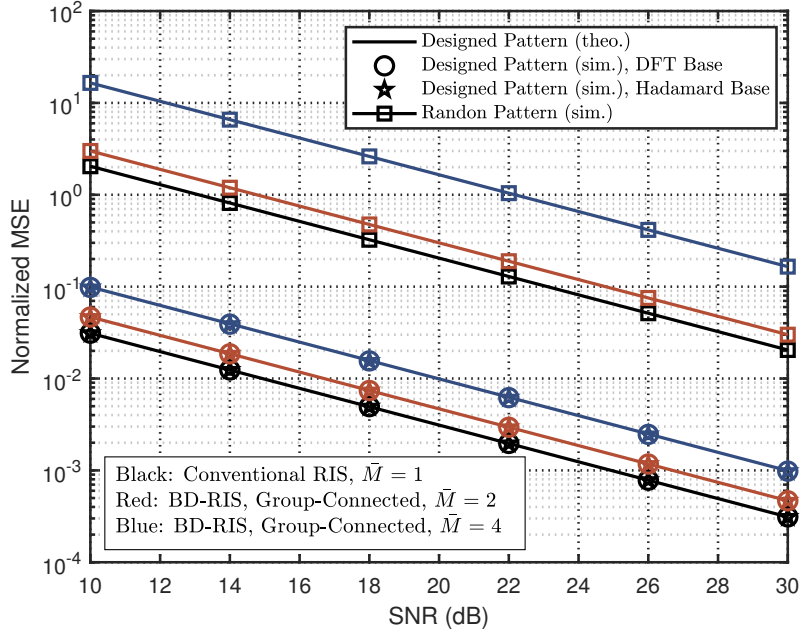


Figure 6.6: Normalized MSE versus SNR ( $M = 32$ ,  $\bar{G} = 4$ ,  $N = K = 2$ ).

The simulation setup is illustrated as follows. Both BS-RIS and RIS-user channels are assumed to have Rician fading with Rician factor  $\kappa = 0$  dB accounting for the small-scale fading and distance-dependent path loss model in Chapter 4 accounting for the large-scale fading. Specifically, we assume each BD-RIS antenna has an idealized radiation pattern, such that the path loss model is expressed as<sup>7</sup>  $\xi = \varrho(1 - \cos \frac{\pi}{L})^2$ , where  $\varrho = 4^3 \pi^4 d_1^{\epsilon_1} d_2^{\epsilon_2} \lambda^{-4} G_t^{-1} G_r^{-1}$  is calculated by the BS-RIS distance  $d_1 = 30$  m and RIS-user distance  $d_2 = 10$  m, path loss exponents  $\epsilon_1 = 2.5$  and  $\epsilon_2 = 2.5$ , wavelength  $\lambda = \frac{c}{f}$  with  $f = 2.4$  GHz, antenna gains at the BS side  $G_t = 1$  and the user side  $G_r = 1$ . The noise powers for uplink channel estimation and downlink transmission are set as  $\sigma^2 = \sigma'^2 = -100$  dBm. In the following simulations, we set the group size  $\bar{M}$  of the group-connected architecture for BD-RIS as  $\bar{M} \in \{1, 2, 4\}$  with an affordable circuit complexity. More comprehensive studies of different BD-RIS architectures and their circuit complexities can be found in [57].

### 6.4.1 Performance for Channel Estimation

We evaluate the channel estimation performance by plotting the normalized MSE versus signal-to-noise ratio (SNR) with different group size of group-connected BD-RIS and different numbers of BS/user antennas<sup>8</sup>. Specifically, the normalized MSE is calculated by  $\bar{e}_{\mathbf{q}} = \mathbb{E}\{\frac{\|\hat{\mathbf{q}} - \mathbf{q}\|_2^2}{\|\mathbf{q}\|_2^2}\}$ . For

<sup>7</sup>For the case of BD-RIS with reflective mode, we calculate the path loss by setting the number of sectors as  $L = 2$ , which indicates that BD-RIS with reflective mode covers half of the space.

<sup>8</sup>Here we only plot the case of BD-RIS with reflective mode since, as explained in Section 6.2, the same scheme can be used in BD-RIS with hybrid/multi-sector modes with the same closed-form MSE performance. The effect of channel estimation error will be studied in the performance evaluation of beamforming design.

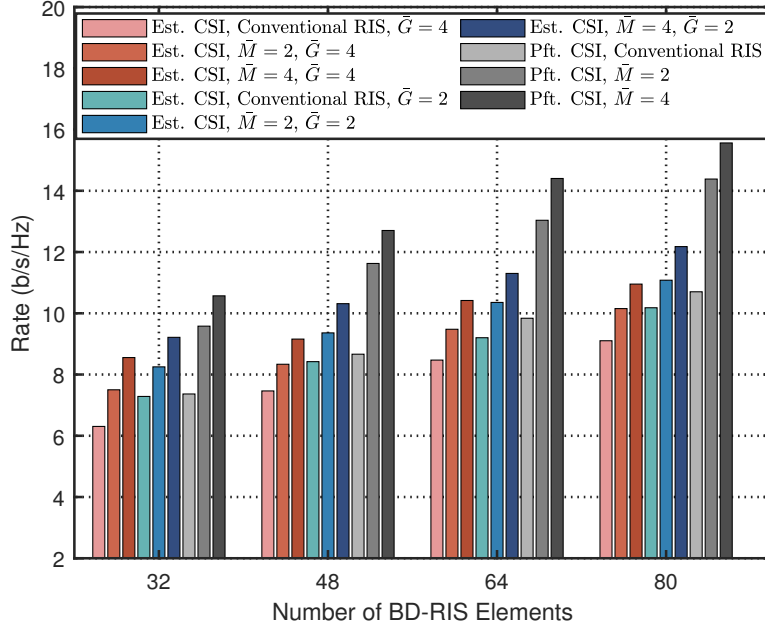


Figure 6.7: Rate versus the number of BD-RIS elements  $M$  ( $N = K = N_s = 2$ ,  $P_{\text{up}} = 250$  mW,  $P_{\text{down}} = K P_{\text{up}}$ ).

fair comparison, the SNR is defined as the ratio between the total power received at each antenna from each transmit antenna through each RIS element for the whole uplink training and the noise power, i.e.,  $\bar{\gamma} = \frac{P_{\text{up}} T_1}{\xi \sigma^2}$ . From Fig. 6.6, we have the following observations<sup>9</sup>. *First*, the matrix  $\hat{\Phi}$  with DFT base or Hadamard base to generate  $\mathbf{Z}_1$ ,  $\mathbf{Z}_2$ , and  $\mathbf{X}$  achieves exactly the same performance, which verifies Theorem 6.1. *Second*, the matrix  $\hat{\Phi}$  with DFT/Hadamard bases achieves exactly the same performance as the theoretical minimum, which verifies Lemma 6.2. *Third*, the normalized MSE performance increases with the group size  $\bar{M}$  of group-connected BD-RIS, which verifies Lemma 6.1. This indicates that, with the same power budget, BD-RIS leads to higher channel estimation error than conventional RIS. The intuition behind is that the same amount of power is split to estimate more channel coefficients, which leads to increasing channel estimation error. *Fourth*, the normalized MSE decreases with SNR, which indicates that larger received SNR leads to smaller channel estimation error. *Fifth*, the matrix  $\hat{\Phi}$  designed by the proposed scheme achieves better MSE performance than that constructed by random unitary matrices, which demonstrates the effectiveness of the proposed design for channel estimation.

<sup>9</sup>In Fig. 6.6, the benchmark scheme, referred to as “Random Pattern”, is generated by replacing Steps 2-6 of the proposed channel estimation method with  $\Phi_{\text{rand}}$ . Specifically,  $\Phi_{\text{rand}}$  is constructed with  $[\Phi_{\text{rand}}]_{i,(l-1)\bar{M}^2+1:l\bar{M}^2} = \text{vec}^T(\mathbf{U}_{i,l})$ , where  $\mathbf{U}_{i,l} \in \mathbb{C}^{\bar{M} \times \bar{M}}$ ,  $\forall i, l$  are randomly generated unitary matrices.

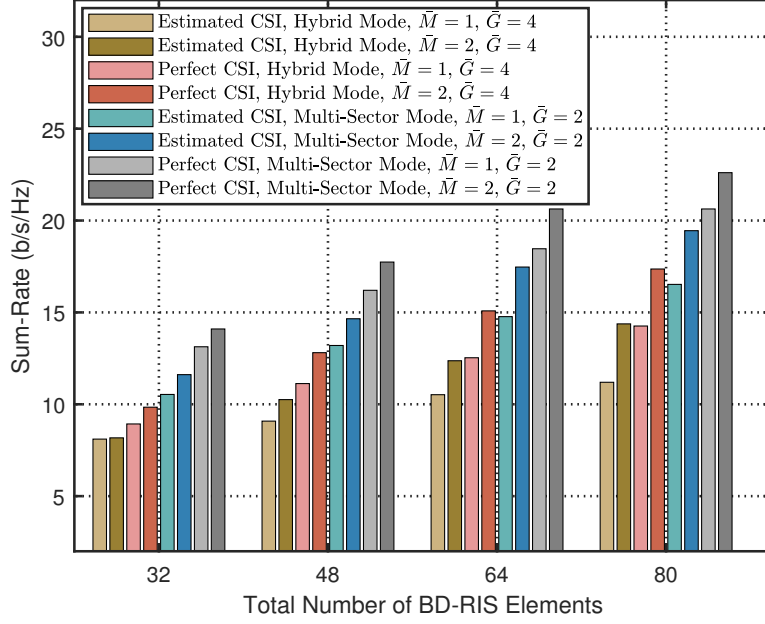


Figure 6.8: Sum-rate versus the total number of BD-RIS elements  $ML$  ( $N = K = 4$ ,  $P_{\text{up}} = 250$  mW,  $P_{\text{down}} = KP_{\text{up}}$ ).

#### 6.4.2 Performance for Beamforming Design

**Reflective BD-RIS Aided Point-to-Point MIMO:** We start by evaluating the performance of the proposed beamforming design algorithm from Section 6.3 in Fig. 6.7. For comparison, we simulate the performance achieved by perfect CSI  $\mathbf{Q}$  as an upper bound. We also ignore the term  $1 - \frac{T_1+T_2}{T}$  in the objective function (6.25a), that is, to evaluate the rate performance for the ease of analyzing the effect of beamforming design. From Fig. 6.7, we have the following observations. *First*, the rate performance achieved based on both the perfect CSI and the estimated CSI increases with increasing group size  $\bar{M}$  of the group-connected architecture, that is, BD-RIS outperforms conventional RIS, which aligns with the results in previous chapters. *Second*, with the fixed group size  $\bar{M}$ , increasing the tile size  $\bar{G}$  during the channel estimation induces the rate performance degradation due to the reduction of CSI, which further limits the BD-RIS design. *Third*, the rate performance gap between the perfect CSI case and the estimated CSI case increases with increasing number of BD-RIS elements. This comes from the fact that more BD-RIS elements provide more flexibility in beamforming design, while the rate performance is more sensitive to CSI conditions.

**Hybrid/Multi-Sector BD-RIS Aided MU-MISO:** We next evaluate the sum-rate performance of the proposed algorithm in Fig. 6.8, where the term  $1 - \frac{T_1+T_2}{T}$  in the objective function (6.32a) is also ignored to obtain the sum-rate. In addition, in the following simulations, we fix the total number of elements  $ML$  for BD-RIS with hybrid mode ( $L = 2$ ) and multi-sector mode ( $L = 4$ ). For BD-RIS with hybrid and multi-sector modes, every  $K/L$  users are located within the coverage of each sector. From Fig. 6.8, we have the following observations. *First*, for both

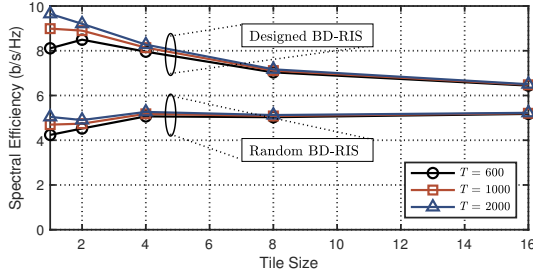
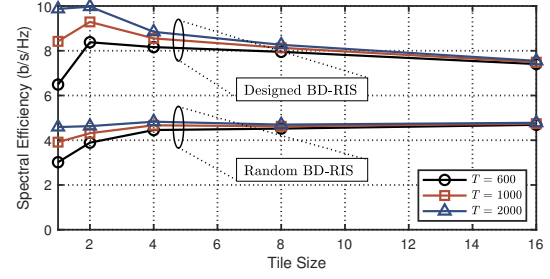
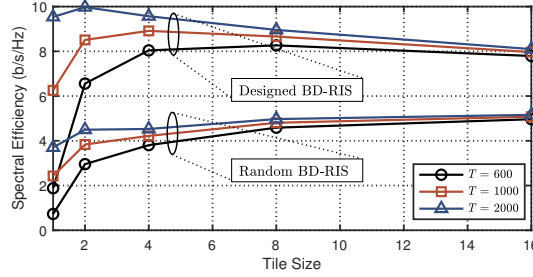
(a) Conventional RIS ( $\bar{M} = 1$ )(b) BD-RIS ( $\bar{M} = 2$ )(c) BD-RIS ( $\bar{M} = 4$ )

Figure 6.9: Spectral efficiency versus tile size  $\bar{G}$  ( $M = 64$ ,  $N = K = N_s = 2$ ,  $P_{\text{up}} = 250$  mW,  $P_{\text{down}} = KP_{\text{up}}$ ).

perfect and estimated CSI cases, BD-RIS with multi-sector mode outperforms BD-RIS with hybrid mode thanks to the use of antennas with narrower beamwidth in the multi-sector mode, which aligns with the results in Chapter 4. *Second*, the sum-rate performance achieved by BD-RIS with hybrid/multi-sector modes increases with  $\bar{M}$  due to the increasing beam manipulation flexibility, which aligns with the results in previous chapters.

### 6.4.3 Overhead and Transmission Performance Trade-Off

To get more insights on the trade-off between the training overhead for channel estimation and the performance for data transmission, we take into account the term  $1 - \frac{T_1+T_2}{T}$  in the following simulations to evaluate the spectral efficiency. For simplicity, we ignore the symbol duration for processing and feedback, i.e.,  $T_2 = 0$ , since the channel estimation overhead  $T_1$  and the symbol duration for data transmission  $T_3$  are generally much larger than  $T_2$ . In addition, we set the symbol duration for one transmission frame  $T$  as  $T \in \{600, 1000, 2000\}$ . In practice, the value of  $T$  depends on multiple factors, such as the carrier frequency of transmit signals, phase differences in the multipath propagation, and mobility [110].

**Reflective BD-RIS Aided Point-to-Point MIMO:** We first plot the spectral efficiency defined in (6.25a) versus the tile size  $\bar{G}$  for BD-RIS with different architectures in Fig. 6.9, from which we have the following observations<sup>10</sup>. *First*, for a relatively small symbol duration,

<sup>10</sup>In Fig. 6.9, the benchmark scheme, referred to as “Random BD-RIS”, is realized by estimating the channel

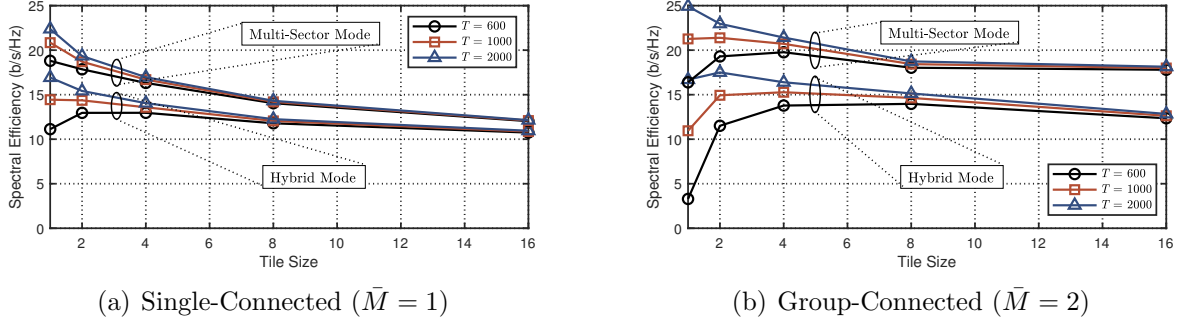


Figure 6.10: Spectral efficiency versus tile size  $\bar{G}$  ( $ML = 128$ ,  $N = K = 4$ ,  $P_{\text{up}} = 250$  mW,  $P_{\text{down}} = KP_{\text{up}}$ ).

e.g.,  $T \in \{600, 1000\}$ , the spectral efficiency achieved by the proposed design in general first increases and then decreases with the increasing tile size  $\bar{G}$ . This is because a larger tile size leads to a smaller training overhead  $T_1$  but a reduced beam manipulation flexibility of BD-RIS. For example, when  $T = 600$ , the best spectral efficiency is achieved at  $\bar{G} = 2$  for group-connected BD-RIS with  $\bar{M} = 2$ , and at  $\bar{G} = 8$  with  $\bar{M} = 4$ . In this case, the rate performance improvement by more flexible BD-RIS design is overwhelmed by the large training overhead. *Second*, for a relatively large symbol duration, e.g.  $T = 2000$ , the spectral efficiency achieved by the proposed design decreases with the increasing tile size. This is because the large  $T$  weakens the impact of the long training overhead at the small tile size during channel estimation, while the rate performance keeps decreasing with increasing tile size. *Third*, with fixed BD-RIS architectures and training overhead, a larger symbol duration leads to better spectral efficiency performance. These observations highlight the importance of adjusting the time slot allocation for uplink and downlink to ensure satisfactory overall performance. *Fourth*, for a relatively small tile size, e.g.  $\bar{G} \in \{1, 2\}$ , the spectral efficiency achieved by BD-RIS with a larger group size performs worse due to the significant training overhead consumption in channel estimation, which overwhelms the rate performance enhancement provided by more flexible BD-RIS architectures. *Fifth*, for a relatively large tile size, e.g.  $\bar{G} \in \{8, 16\}$ , the spectral efficiency achieved by BD-RIS outperforms the conventional RIS thanks to the additional beam manipulation flexibility supported by the group-connected architecture. Therefore, the tile size  $\bar{G}$  and  $\bar{M}$  should be carefully chosen to balance the training overhead for channel estimation, the rate performance, and the circuit complexity for BD-RIS implementation.

**Hybrid/Multi-Sector BD-RIS Aided MU-MISO:** In Fig. 6.10, we plot the spectral efficiency defined in (6.32a) as a function of  $\bar{G}$  for BD-RIS with hybrid/multi-sector modes. From Fig. 6.10 we observe that, in addition to the findings obtained from Fig. 6.9, BD-RIS with multi-sector mode is more robust to the symbol durations  $T$  and the tile size. This is because the significant channel gain enhancement provided by high-gain antennas compensates for the effects of the training overhead and the beamforming flexibility.

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based on the proposed method, randomly generating a block-diagonal unitary matrix  $\Theta$ , and performing the SVD based on the effective channel for downlink transmission.

## 6.5 Summary

This chapter proposes a novel channel estimation scheme for BD-RIS aided multi-antenna systems. Specifically, the BD-RIS has different (single/group/fully-connected) architectures and (reflective/hybrid/multi-sector) modes, which, mathematically, generate unique constraints and complicate the channel estimation. To tackle this difficulty, we propose an efficient pilot sequence and BD-RIS design to estimate the cascaded BD-RIS channel, which achieves the minimum MSE of the LS estimator. The analysis and extensions of the proposed channel estimation scheme are also illustrated.

With the cascaded channel estimates, we consider two BD-RIS scenarios, that is reflective BD-RIS aided point-to-point MIMO, and hybrid/multi-sector BD-RIS aided MU-MISO, and propose efficient algorithms to design the transceiver and BD-RIS with different architectures and modes.

Simulation results demonstrate the advantages of the proposed design. Specifically, the effectiveness of the proposed channel estimation scheme and beamforming algorithms are verified by comparing with the design for perfect CSI cases. In addition, the trade-off between the training overhead for channel estimation and the performance for data transmission is analyzed with different parameter settings.



## **Part III**

# **Hardware Impairments of BD-RIS**



# Chapter 7

## BD-RIS with Mutual Coupling

In Chapters 3-6, modeling and signal processing of BD-RIS with idealized hardware have been studied, while, in practice, the hardware impairments, such as mutual coupling, of BD-RIS cannot be simply ignored and will have non-negligible impact on the system performance. The impact of mutual coupling at conventional diagonal RIS has been analyzed in [50–52], which highlighted the importance of accurately capturing the mutual coupling at RIS for optimization. Its impact at BD-RIS, however, remains unexplored. This chapter thus focuses on the modeling and beamforming of BD-RIS with the mutual coupling effect, and provides a thorough analysis from mathematical modeling to optimization. This leads to the following layouts and contributions.

In Section 7.1, the BD-RIS aided wireless communication model has been derived, which captures the mutual coupling among the RIS elements. This is done by leveraging the equivalence between the scattering and impedance parameters.

In Section 7.2, with the derived model, a general and efficient optimization algorithm has been proposed to maximize the channel gain for a BD-RIS aided SISO system, which can be applied to BD-RIS with single-, group-, and fully-connected architectures.

In Section 7.3, simulation results have been provided to verify the effectiveness of the proposed algorithm and show the performance enhancement of BD-RIS with group/fully-connected architectures when taking into account the mutual coupling at the optimization stage.

### 7.1 Communication Model

In this section, we review the general RIS aided communication model derived in [55] and simplify it while still modeling the mutual coupling among the RIS elements.

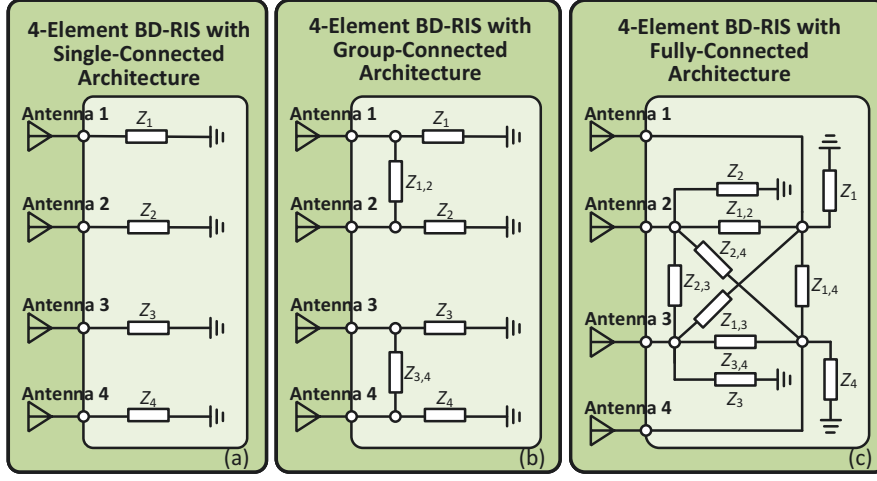


Figure 7.1: Schematic circuit topologies for a 4-element BD-RIS with (a) single-connected, (b) group-connected, and (c) fully-connected architectures.

### 7.1.1 General RIS Aided Communication Model

Consider a BD-RIS aided multi-antenna system with an  $N$ -antenna transmitter, an  $M$ -element BD-RIS, and a  $K$ -antenna receiver. The whole system is modeled as an  $N_{\text{all}} = N + M + K$ -port network characterized by its scattering, impedance, and admittance parameters as illustrated in Section 2.1.

Specifically, the  $M$  scattering elements at the BD-RIS are connected to an  $M$ -port group-connected reconfigurable impedance network [55], where the  $M$  ports are uniformly divided into  $G$  groups, each containing  $\bar{M} = \frac{M}{G}$  ports connected to each other. Mathematically, BD-RIS with a group-connected architecture is characterized by a block-diagonal impedance matrix  $\mathbf{Z}_{\text{I}} \in \mathbb{C}^{M \times M}$ , i.e.,

$$\mathbf{Z}_{\text{I}} = \text{blkdiag}(\mathbf{Z}_{\text{I},1}, \dots, \mathbf{Z}_{\text{I},G}), \quad (7.1)$$

where each block  $\mathbf{Z}_{\text{I},g} \in \mathbb{C}^{\bar{M} \times \bar{M}}$ ,  $\forall g \in \mathcal{G} = \{1, \dots, G\}$ , is symmetric and purely imaginary for reciprocal and lossless reconfigurable impedance networks [55], i.e.,

$$\mathbf{Z}_{\text{I},g} = \mathbf{Z}_{\text{I},g}^T, \quad \Re\{\mathbf{Z}_{\text{I},g}\} = \mathbf{0}_{\bar{M} \times \bar{M}}, \quad \forall g \in \mathcal{G}. \quad (7.2)$$

Specifically, the cases with  $G = 1$  and  $G = M$  refer to the fully- and single-connected architectures of BD-RIS [55], where the corresponding impedance matrices become full matrices and diagonal matrices, respectively. To facilitate understanding, we provide the schematic circuit topologies of BD-RIS with single-, group-, and fully-connected architectures in Fig. 7.1. The impedance matrix in (7.2) results in a symmetric and unitary matrix  $\Phi \in \mathbb{C}^{M \times M}$ .

Using the definitions given in Section 2.1 and the multiport network theory based on the scattering parameters, the general RIS aided channel,  $\mathbf{H}_{\text{all}} \in \mathbb{C}^{K \times N}$ , which relates the voltages

at the receiver ports with those at transmitter ports, is given by [55]

$$\mathbf{H}_{\text{all}} = (\mathbf{\Gamma}_R + \mathbf{I}_K) \mathbf{T}_{\text{RT}} (\mathbf{I}_N + \mathbf{\Gamma}_T \mathbf{T}_{\text{TT}} + \mathbf{T}_{\text{TT}})^{-1}, \quad (7.3)$$

where  $\mathbf{T}_{\text{TT}} \in \mathbb{C}^{N \times N}$  and  $\mathbf{T}_{\text{RT}} \in \mathbb{C}^{K \times N}$  are sub-matrices of  $\mathbf{T} = \mathbf{S}(\mathbf{I}_L - \mathbf{\Gamma}\mathbf{S})^{-1} \in \mathbb{C}^{L \times L}$  with  $\mathbf{\Gamma} = \text{blkdiag}(\mathbf{\Gamma}_T, \mathbf{\Phi}, \mathbf{\Gamma}_R)$ . Specifically,  $\mathbf{T}_{\text{TT}} = [\mathbf{T}]_{1:N, 1:N}$  and  $\mathbf{T}_{\text{RT}} = [\mathbf{T}]_{N+M+1:L, 1:N}$ . The channel model in (7.3) is general enough to include the impact of antenna mismatching and mutual coupling at the transmitter, BD-RIS, and receiver. However, it is too complicated to get insights on the role of BD-RIS, which motivates us to apply further simplifications.

### 7.1.2 Mutual Coupling Aware RIS Aided Communication Model

To simplify the general communication model (7.3), we explore the relationship between the impedance parameter analysis and the scattering parameter analysis. Then, we keep Assumptions 2.1 and 2.2, and relax Assumption 2.3 as follows.

**Assumption 7.1.** *The antennas at the transmitter and receiver are perfectly matched with no mutual coupling, such that  $\mathbf{Z}_{\text{TT}} = Z_0 \mathbf{I}_N$  and  $\mathbf{Z}_{\text{RR}} = Z_0 \mathbf{I}_K$ .*

Based on Assumptions 2.2 and 7.1, and the relationship (2.3), we obtain the following result.

**Result 7.1.** *The transmission scattering matrices from the receiver to the transmitter are zero, i.e.,  $\mathbf{S}_{\text{TI}} = \mathbf{0}_{N \times M}$ ,  $\mathbf{S}_{\text{TR}} = \mathbf{0}_{N \times K}$ ,  $\mathbf{S}_{\text{IR}} = \mathbf{0}_{M \times K}$ . The scattering matrices at the antenna arrays of the transmitter and receiver are zero, i.e.,  $\mathbf{S}_{\text{TT}} = \mathbf{0}_{N \times N}$ ,  $\mathbf{S}_{\text{RR}} = \mathbf{0}_{K \times K}$ .*

Applying Assumption 2.1 and Result 7.1, the expression in (7.3) can be simplified as follows:

$$\mathbf{H}_{\text{all}} = \mathbf{S}_{\text{RT}} + \mathbf{S}_{\text{RI}} (\mathbf{I}_M - \mathbf{\Phi} \mathbf{S}_{\text{IT}})^{-1} \mathbf{\Phi} \mathbf{S}_{\text{IT}}. \quad (7.4)$$

However, (7.4) is still too complex, since the matrix  $\mathbf{\Phi}$  of reflection coefficients appears inside and outside the inverse. To further simplify the expression in (7.4), so as to facilitate the optimization of BD-RIS in the presence of mutual coupling, we leverage Assumptions 2.1, 2.2, and 7.1, and the relationship between  $\mathbf{S}$  and  $\mathbf{Z}$ , and obtain the following result.

**Result 7.2.** *The nonzero sub-matrices of  $\mathbf{S}$  and those of  $\mathbf{Z}$  are related to one another as*

$$\mathbf{S}_{\text{RI}} = \frac{\mathbf{Z}_{\text{RI}}}{2Z_0} (\mathbf{I}_M - (\mathbf{Z}_{\text{II}} + Z_0 \mathbf{I}_M)^{-1} (\mathbf{Z}_{\text{II}} - Z_0 \mathbf{I}_M)), \quad (7.5a)$$

$$\mathbf{S}_{\text{II}} = (\mathbf{Z}_{\text{II}} + Z_0 \mathbf{I}_M)^{-1} (\mathbf{Z}_{\text{II}} - Z_0 \mathbf{I}_M), \quad (7.5b)$$

$$\mathbf{S}_{\text{IT}} = (\mathbf{Z}_{\text{II}} + Z_0 \mathbf{I}_M)^{-1} \mathbf{Z}_{\text{IT}}, \quad (7.5c)$$

$$\mathbf{S}_{\text{RT}} = \frac{\mathbf{Z}_{\text{RT}}}{2Z_0} - \frac{\mathbf{Z}_{\text{RI}}}{2Z_0} (\mathbf{Z}_{\text{II}} + Z_0 \mathbf{I}_M)^{-1} \mathbf{Z}_{\text{IT}}. \quad (7.5d)$$

Plugging (7.5) and (2.10) into (7.4), we obtain a more tractable and convenient expression equivalent to (7.4) as

$$\mathbf{H}_{\text{all}} = \frac{1}{2Z_0}(\mathbf{Z}_{\text{RT}} - \mathbf{Z}_{\text{RI}}(\mathbf{Z}_{\text{II}} + \mathbf{Z}_{\text{I}})^{-1}\mathbf{Z}_{\text{IT}}). \quad (7.6)$$

Equation (7.6) is in accordance with the model in [50], which was derived using the impedance parameter analysis<sup>1</sup>. In (7.6), the role of the beyond-diagonal network of tunable impedances in BD-RIS and the impact of mutual coupling and mismatching at the RIS elements are explicitly visible. The physical meaning of each term in (7.6) is explained as follows.

- 1)  $\mathbf{Z}_{\text{RT}}$ ,  $\mathbf{Z}_{\text{RI}}$ , and  $\mathbf{Z}_{\text{IT}}$  refer to the channels from the transmitter to the receiver, from the BD-RIS to the receiver, and from the transmitter to the BD-RIS, respectively.
- 2)  $\mathbf{Z}_{\text{II}}$  characterizes the mismatching and mutual coupling at the RIS elements. Specifically, the diagonal entries of  $\mathbf{Z}_{\text{II}}$  refer to the self impedance; the off-diagonal entries of  $\mathbf{Z}_{\text{II}}$  account for the mutual coupling, which depends on the inter-element distance. Generally, the larger the inter-element distance, the smaller the mutual coupling.

With the communication model in (7.6) at hand, which explicitly accounts for the mutual coupling at the BD-RIS, in the following section, we aim to design the beyond-diagonal matrix  $\mathbf{Z}_{\text{I}}$  of tunable impedances to maximize the system performance.

We clarify that we start from the general channel model derived in [55], but go beyond that by deriving the new expression in (7.4), which captures explicitly the mutual coupling among the RIS elements. For the first time, more importantly, we show the equivalence between the scattering parameter based channel in (7.4) and the impedance parameter based channel in (7.6) derived in [50]. In addition, only the conventional RIS with single-connected architecture is considered in [50], i.e.,  $\mathbf{Z}_{\text{I}}$  is diagonal, while in this work we consider BD-RIS with  $\mathbf{Z}_{\text{I}}$  not limited to being diagonal.

## 7.2 Mutual Coupling Aware Optimization

For simplicity, we consider a SISO system and focus on the optimization of  $\mathbf{Z}_{\text{I}}$ , which is not limited to being diagonal in BD-RIS. The analysis of the multiple-antenna case is postponed to a future work due to the complexity of considering a joint optimization including the precoder at the transmitter, the beyond-diagonal matrix  $\mathbf{Z}_{\text{I}}$  at the BD-RIS, and the combiner at the receiver.

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<sup>1</sup>The end-to-end model derived in [50] relates the voltage at the receiver ports and the source voltage, which can be easily transformed into (7.6) using the relationship between the source voltage and the voltage at the transmitter.

The corresponding optimization problem to maximize the channel gain can be formulated as<sup>2</sup>

$$\max_{\mathbf{Z}_I} |z_{RT} - \mathbf{z}_{RI}(\mathbf{Z}_{II} + \mathbf{Z}_I)^{-1}\mathbf{z}_{IT}|^2 \quad \text{s.t. (7.1), (7.2),} \quad (7.7)$$

where  $z_{RT} \in \mathbb{C}$ ,  $\mathbf{z}_{RI} \in \mathbb{C}^{1 \times M}$ , and  $\mathbf{z}_{IT} \in \mathbb{C}^{M \times 1}$ . The main difficulties when solving the problem in (7.7) are the following:

- 1) The matrix  $\mathbf{Z}_I$  appears in the inversion  $(\mathbf{Z}_{II} + \mathbf{Z}_I)^{-1}$ .
- 2) The matrix  $\mathbf{Z}_I$  needs to fulfill the constraints (7.1) and (7.2), instead of being diagonal as in conventional RISs.

The first difficulty has been tackled in [51] for diagonal RISs. However, the optimization with mutual coupling among the RIS elements and the unique constraints of BD-RIS have never been investigated. To effectively solve problem (7.7), we apply the idea introduced in [51] and propose to iteratively optimize  $\mathbf{Z}_I$  subject to the constraints (7.1) and (7.2) until convergence. This results in the following optimization framework.

### 7.2.1 Optimization Framework

The main idea of the iterative design in [51] is to slightly modify the value of  $\mathbf{Z}_I$  in each iteration, in order to increase the channel gain. Inspired by this approach, we first introduce an auxiliary variable  $\mathbf{\Omega} \in \mathbb{C}^{M \times M}$  as a small increment to  $\mathbf{Z}_I$  in each iteration. To facilitate the iterative design, we construct  $\mathbf{\Omega}$  based on the following two properties.

Property 1:  $\mathbf{\Omega}$  is chosen in compliance with the mathematical structure of the impedance matrix  $\mathbf{Z}_I$ , such that the updated impedance matrix always satisfies the optimization constraints in each iteration. Specifically, we have

$$\mathbf{\Omega} = \text{blkdiag}(\mathbf{\Omega}_1, \dots, \mathbf{\Omega}_G), \quad \mathbf{\Omega}_g = \mathbf{\Omega}_g^T, \forall g. \quad (7.8)$$

Property 2: Each nonzero entry of  $\mathbf{\Omega}$  is chosen sufficiently small, such that the convergence of the optimization framework is guaranteed [51]. For ease of optimization, we assume that each nonzero entry of  $\mathbf{\Omega}$  has the same amplitude but a different phase, i.e.,

$$|[\mathbf{\Omega}_g]_{m,n}| = \delta, \forall g, \forall m, n \in \bar{\mathcal{M}} = \{1, \dots, \bar{M}\}, \quad (7.9)$$

where  $\delta$  controls the increment of  $\mathbf{Z}_I$  in each iteration, which will be soon explained.

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<sup>2</sup>Here, we assume that channel state information (CSI) is perfectly known at the transmitter to better focus on and highlight the impact of mutual coupling.

Based on these two properties for  $\mathbf{\Omega}$ , the proposed iterative design is summarized by the following steps.

Step 1: At the  $l$ -th iteration, we optimize  $\mathbf{\Omega}^o$ , while keeping  $\mathbf{Z}_I^o$  fixed, by solving the following problem:

$$\max_{\mathbf{\Omega}} |z_{\text{RT}} - \mathbf{z}_{\text{RI}}(\mathbf{Z}_{\text{II}} + \mathbf{Z}_I^o + \mathbf{\Omega})^{-1} \mathbf{z}_{\text{IT}}|^2 \quad \text{s.t. (7.8), (7.9)}. \quad (7.10)$$

Step 2: Given  $\mathbf{\Omega}^o$  as the optimal solution to the problem in (7.10), the impedance matrix at the  $(o+1)$ -th iteration, i.e.,  $\mathbf{Z}_I^{o+1}$ , is updated using the following rule:

$$\mathbf{Z}_I^{o+1} = \mathbf{Z}_I^o + j\Im\{\mathbf{\Omega}^o\}, \quad (7.11)$$

where we only use the imaginary part of  $\mathbf{\Omega}^o$  to guarantee that the impedance matrix is purely imaginary.

Then,  $\mathbf{Z}_I$  is optimized by iteratively solving the problem in (7.10) and updating (7.11) until convergence. The solution of the problem in (7.10) is detailed next.

### 7.2.2 Solution to Problem (7.10)

In (7.10), instead of optimizing  $\mathbf{Z}_I$  as in (7.7), we optimize the small increment  $\mathbf{\Omega}$ , such that  $\mathbf{\Omega}$  can be removed from the matrix inversion in the objective function. Specifically, we apply the Neumann series approximation [51] to the matrix inversion, i.e.,

$$(\mathbf{Z}_{\text{II}} + \mathbf{Z}_I^o + \mathbf{\Omega})^{-1} \approx (\mathbf{Z}_{\text{II}} + \mathbf{Z}_I^o)^{-1} - (\mathbf{Z}_{\text{II}} + \mathbf{Z}_I^o)^{-1} \mathbf{\Omega} (\mathbf{Z}_{\text{II}} + \mathbf{Z}_I^o)^{-1}, \quad (7.12)$$

which achieves a tight approximation provided that the condition  $\delta \ll \frac{1}{M\|(\mathbf{Z}_{\text{II}} + \mathbf{Z}_I^o)^{-1}\|_\infty}$  is fulfilled in each iteration<sup>3</sup>. Accordingly, we have

$$|z_{\text{RT}} - \mathbf{z}_{\text{RI}}(\mathbf{Z}_{\text{II}} + \mathbf{Z}_I^o + \mathbf{\Omega})^{-1} \mathbf{z}_{\text{IT}}|^2 \approx |a^o + \mathbf{b}^o \mathbf{\Omega} \mathbf{c}^o|^2 = |a^o + \sum_{g \in \mathcal{G}} \mathbf{e}_g^o \text{vec}(\mathbf{\Omega}_g)|^2, \quad (7.13)$$

where  $a^o = z_{\text{RT}} - \mathbf{z}_{\text{RI}}(\mathbf{Z}_{\text{II}} + \mathbf{Z}_I^o)^{-1} \mathbf{z}_{\text{IT}}$ ,  $\mathbf{b}^o = \mathbf{z}_{\text{RI}}(\mathbf{Z}_{\text{II}} + \mathbf{Z}_I^o)^{-1}$ ,  $\mathbf{c}^o = (\mathbf{Z}_{\text{II}} + \mathbf{Z}_I^o)^{-1} \mathbf{z}_{\text{IT}}$ ,  $\mathbf{b}_g^o = [\mathbf{b}^o]_{(g-1)\bar{M}+1:g\bar{M}}$ ,  $\mathbf{c}_g^o = [\mathbf{c}^o]_{(g-1)\bar{M}+1:g\bar{M}}$ , and  $\mathbf{e}_g^o = (\mathbf{c}_g^o)^T \otimes \mathbf{b}_g^o$ ,  $\forall g \in \mathcal{G}$ . As a result, the problem in (7.10) becomes

$$\max_{\mathbf{\Omega}_g, \forall g} \left| a^o + \sum_{g \in \mathcal{G}} \mathbf{e}_g^o \text{vec}(\mathbf{\Omega}_g) \right|^2 \quad \text{s.t. } \mathbf{\Omega}_g = \mathbf{\Omega}_g^T, (7.9). \quad (7.14)$$

The solution to the problem in (7.14) is not straightforward due to the symmetric and constant modulus constraints of  $\mathbf{\Omega}_g, \forall g$ . Fortunately, the symmetric constraint of each  $\mathbf{\Omega}_g$  implies that

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<sup>3</sup>For simplicity, in this work, we fix the value of  $\delta$  as  $\delta \ll 1$  to achieve a tight Neumann series approximation.

we do not need to optimize the whole matrices  $\mathbf{\Omega}_g, \forall g$ . In other words, instead of optimizing the symmetric matrices  $\mathbf{\Omega}_g, \forall g$  with  $\bar{M}^2$  variables, we can optimize  $\bar{M}$  diagonal variables and  $\frac{\bar{M}(\bar{M}-1)}{2}$  lower-triangular (or upper-triangular) variables of  $\mathbf{\Omega}_g$ , and reconstruct them to obtain  $\mathbf{\Omega}_g, \forall g$ . Mathematically, this transformation can be done by introducing the following notations:

- 1) A vector  $\boldsymbol{\omega}_g \in \mathbb{C}^{\frac{\bar{M}(\bar{M}+1)}{2} \times 1}$ , which contains the diagonal and lower-triangular entries of  $\mathbf{\Omega}_g$ .
- 2) A binary matrix  $\mathbf{R} \in \{0, 1\}^{\bar{M}^2 \times \frac{\bar{M}(\bar{M}+1)}{2}}$ , which maps  $\boldsymbol{\omega}_g$  into  $\text{vec}(\mathbf{\Omega}_g), \forall g$ .

Specifically, there is only one nonzero entry in each row of the binary matrix  $\mathbf{R}$  defined as

$$[\mathbf{R}]_{\bar{M}(m-1)+n,k} = \begin{cases} 1, & k = \frac{m(m-1)}{2} + n \text{ and } 1 \leq n \leq m, \\ 1, & k = \frac{n(n-1)}{2} + m \text{ and } m < n \leq \bar{M}, \\ 0, & \text{otherwise,} \end{cases} \quad (7.15)$$

where  $\forall m, n \in \bar{\mathcal{M}}$ . Accordingly, we obtain the relationship  $\text{vec}(\mathbf{\Omega}_g) = \mathbf{R}\boldsymbol{\omega}_g, \forall g$ , and the problem in (7.14) becomes

$$\max_{\boldsymbol{\omega}_g, \forall g} \left| a^o + \sum_{g \in \mathcal{G}} \mathbf{e}_g^o \mathbf{R} \boldsymbol{\omega}_g \right|^2 \quad \text{s.t.} \quad |[\boldsymbol{\omega}_g]_i| = \delta, \forall g, \forall i \in \ddot{\mathcal{M}}, \quad (7.16)$$

where  $\ddot{\mathcal{M}} \in \{1, \dots, \frac{\bar{M}(\bar{M}+1)}{2}\}$ . Applying the triangle inequality to the objective function in (7.16), we have

$$|a^o + \sum_{g \in \mathcal{G}} \mathbf{e}_g^o \mathbf{R} \boldsymbol{\omega}_g| \leq |a^o| + \delta \sum_{g \in \mathcal{G}} \sum_{i \in \ddot{\mathcal{M}}} |[\mathbf{e}_g^o \mathbf{R}]_i|, \quad (7.17)$$

where the equality is achieved by rotating each  $[\mathbf{e}_g^o \mathbf{R}]_i$  such that the resulting  $[\mathbf{e}_g^o \mathbf{R}]_i [\boldsymbol{\omega}_g]_i, \forall i$ , are collinear to  $a^o$  on the complex plane. Therefore, the optimal solution of  $\boldsymbol{\omega}_g, \forall g$  in the  $l$ -th iteration can be determined element-by-element as

$$[\boldsymbol{\omega}_g^o]_i = \delta e^{j(\angle a^o - \angle [\mathbf{e}_g^o \mathbf{R}]_i)}, \forall i. \quad (7.18)$$

With the solution (7.18), we can reconstruct  $\mathbf{\Omega}_g$  in the  $l$ -th iteration by  $\mathbf{\Omega}_g^o = \text{unvec}(\mathbf{R}\boldsymbol{\omega}_g^o), \forall g$ . We should highlight that the proposed approach is different from that in [51] since we introduce the auxiliary variable  $\mathbf{\Omega}$  to simplify the design of BD-RIS matrix and the binary matrix  $\mathbf{R}$  to deal with the unique constraint in (7.8) based on BD-RIS architectures, instead of being diagonal as in conventional RISs.

### 7.2.3 Algorithm and Analysis

**Algorithm:** The complete algorithm for solving the problem in (7.7) is summarized in Algorithm 9.

**Algorithm 9** Mutual Coupling Aware BD-RIS Design**Require:**  $\mathbf{z}_{\text{RI}}, \mathbf{Z}_{\text{II}}, \mathbf{z}_{\text{IT}}, \mathbf{R}, M, \bar{M}, o_{\text{max}}, \zeta$ .**Ensure:**  $\mathbf{Z}_{\text{I}}^*$ .

- 1: Initialize  $\mathbf{Z}_{\text{I}}^0$  by solving the problem in (7.7) considering the conventional diagonal RIS with no mutual coupling,  $\mathbf{\Omega}^0 = \mathbf{0}$ , and  $o = 0$ . Calculate  $a^0$ ,  $\mathbf{b}^0$ , and  $\mathbf{c}^0$ .
- 2: **repeat**
- 3:   Update  $o = o + 1$ .
- 4:   Update  $\mathbf{Z}_{\text{I}}^o$  using (7.11),  $a^o$ ,  $\mathbf{b}^o$ , and  $\mathbf{c}^o$ .
- 5:   Update  $\boldsymbol{\omega}_g^o, \forall g$  using (7.18), and  $\mathbf{\Omega}_g^o = \text{unvec}(\mathbf{R}\boldsymbol{\omega}_g^o), \forall g$ .
- 6:   Calculate  $C^o = |a^o + \mathbf{b}^o j \Im\{\mathbf{\Omega}^o\} \mathbf{c}^o|^2$ .
- 7: **until**  $\frac{|C^o - C^{o-1}|}{C^{o-1}} \leq \zeta$  or  $o \geq o_{\text{max}}$

**Complexity:** The complexity of the proposed algorithm mainly comes from the matrix inversion operation for updating  $a^o$ ,  $\mathbf{b}^o$ , and  $\mathbf{c}^o$ , which requires  $\mathcal{O}(M^3)$  complex multiplications. Therefore, the total complexity is  $\mathcal{O}(IM^3)$ , where  $I$  denotes the number of iterations to ensure convergence.

**Convergence:** The convergence of Algorithm 9 is proven in the following proposition.

**Proposition 7.1.** *The convergence of the proposed algorithm is theoretically guaranteed by appropriately setting the value of  $\delta$ .*

*Proof.* Please refer to Appendix D. □

## 7.3 Performance Evaluation

In this section, we perform simulation results to evaluate the performance of the proposed algorithm. The simulation parameters are the same as in [51] and are summarized as follows. Free-space propagation is assumed. The transmitter and receiver are located at (5,-5,3) and (5,5,1), respectively. BD-RIS is located on the  $y$ - $z$  plane and is centered at (0,0,0). All the radiating elements are thin wire dipoles that are parallel to the  $z$ -axis and have radius  $r = \frac{\lambda}{500}$  and length  $\iota = \frac{\lambda}{32}$ , where  $\lambda = \frac{c}{f}$  denotes the wavelength with frequency  $f = 28$  GHz. Thus, the mutual impedance between any two dipoles  $a$  and  $b$  with center-point coordinates  $(a_x, a_y, a_z)$  and  $(b_x, b_y, b_z)$  (and the self impedance at any dipole if  $a = b$ ) is computed as [50]

$$\begin{aligned}
 z_{a,b} = & \int_{a_z - \frac{\iota}{2}}^{a_z + \frac{\iota}{2}} \int_{b_z - \frac{\iota}{2}}^{b_z + \frac{\iota}{2}} \frac{j\eta_0}{4\pi\kappa_0} \left( \frac{(a'_z - b'_z)^2}{d_{a,b}^2} \left( \frac{3}{d_{a,b}^2} + \frac{j3\kappa_0}{d_{a,b}} - \kappa_0^2 \right) - \frac{j\kappa_0 + d_{a,b}^{-1}}{d_{a,b}} + \kappa_0^2 \right) \\
 & \times \frac{\exp(-j\kappa_0 d_{a,b}) \sin(\kappa_0(\frac{\iota}{2} - |b'_z - b_z|)) \sin(\kappa_0(\frac{\iota}{2} - |a'_z - a_z|))}{d_{a,b} \sin^2(\kappa_0 \frac{\iota}{2})} db'_z da'_z,
 \end{aligned} \tag{7.19}$$

where  $\eta_0 = 377 \, \Omega$  denotes the free-space impedance,  $\kappa_0 = \frac{2\pi}{\lambda}$  is the wavenumber, and  $d_{a,b}$  denotes the distance between the dipoles  $a$  and  $b$ , i.e.,  $d_{a,b} = \sqrt{(d_{a,b}^{x,y})^2 + (a'_z - b'_z)^2}$ ,  $d_{a,b}^{x,y} =$

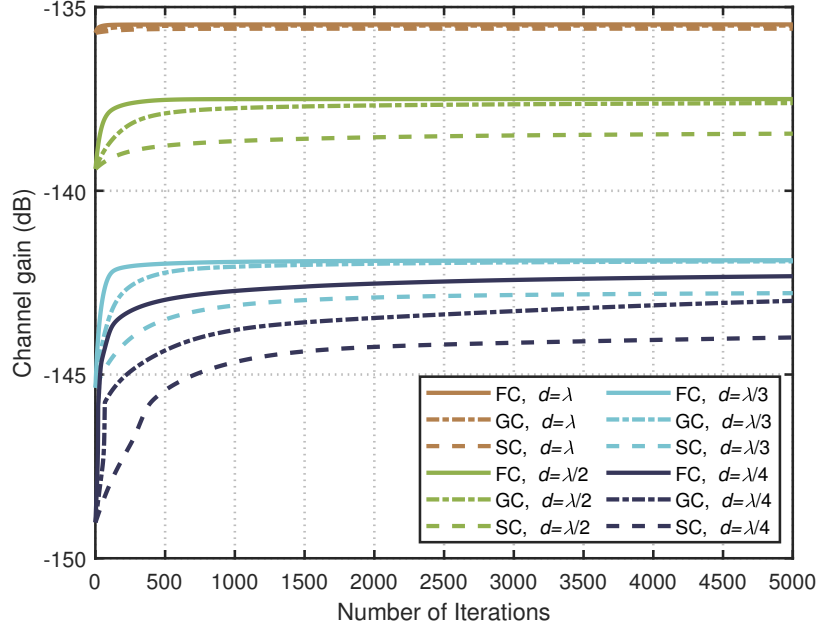


Figure 7.2: Channel gain versus the number of iterations ( $M = 16$ ).

$\sqrt{(a_x - b_x)^2 + (a_y - b_y)^2}$  for  $a \neq b$  and  $d_{a,b}^{x,y} = r$  for  $a = b$ . The elements of  $\mathbf{z}_{\text{RI}}$ ,  $\mathbf{Z}_{\text{II}}$ , and  $\mathbf{z}_{\text{IT}}$  are computed using (7.19), and we set  $z_{\text{RT}} = 0$  to focus on the role of BD-RIS.

Fig. 7.2 evaluates the convergence of the proposed algorithm for different values of the inter-element distance  $d$  and for different BD-RIS architectures. In addition, we observe that, when the number of RIS elements is fixed, the end-to-end channel gain decreases when decreasing the inter-element distance due to the increased mutual coupling. When the aperture (size) of the RIS is kept fixed, on the other hand, a smaller inter-element distance enables the deployment of more RIS elements within the surface, which results in better beamforming gains provided that the mutual coupling is considered in the RIS design [51].

Fig. 7.3 illustrates the channel gain of BD-RIS with different architectures and different inter-element distances. The schemes marked as “FC/GC/SC” refer to the performance achieved by BD-RIS with fully/group/single-connected architectures. We set  $\bar{M} = 4$  for the group-connected case. The value of  $\delta$  is set to  $6 \times 10^{-4}$  to ensure convergence. We observe that the proposed algorithm converges, which verifies our theoretical derivation. In Fig. 7.3, the schemes marked as “w/ MC” are obtained by computing  $\mathbf{Z}_{\text{I}}$  with  $\mathbf{Z}_{\text{II}}$  using (7.19); the schemes marked as “w/o MC” are obtained by setting the off-diagonal entries of  $\mathbf{Z}_{\text{II}}$  to zero. In both cases, the channel gain is  $|\mathbf{z}_{\text{RI}}(\mathbf{Z}_{\text{II}} + \mathbf{Z}_{\text{I}})^{-1}\mathbf{z}_{\text{IT}}|^2$ . We have the following observations. *First*, ignoring the mutual coupling when designing  $\mathbf{Z}_{\text{I}}$ , BD-RIS with single/group/fully-connected architectures achieves exactly the same performance, which is consistent with the conclusion in [55]. *Second*, by taking into account the mutual coupling when designing  $\mathbf{Z}_{\text{I}}$ , BD-RIS with different architectures achieves better performance than the “w/o MC” schemes. *Third*, the performance gap between fully/group-connected BD-RIS and conventional RIS increases when

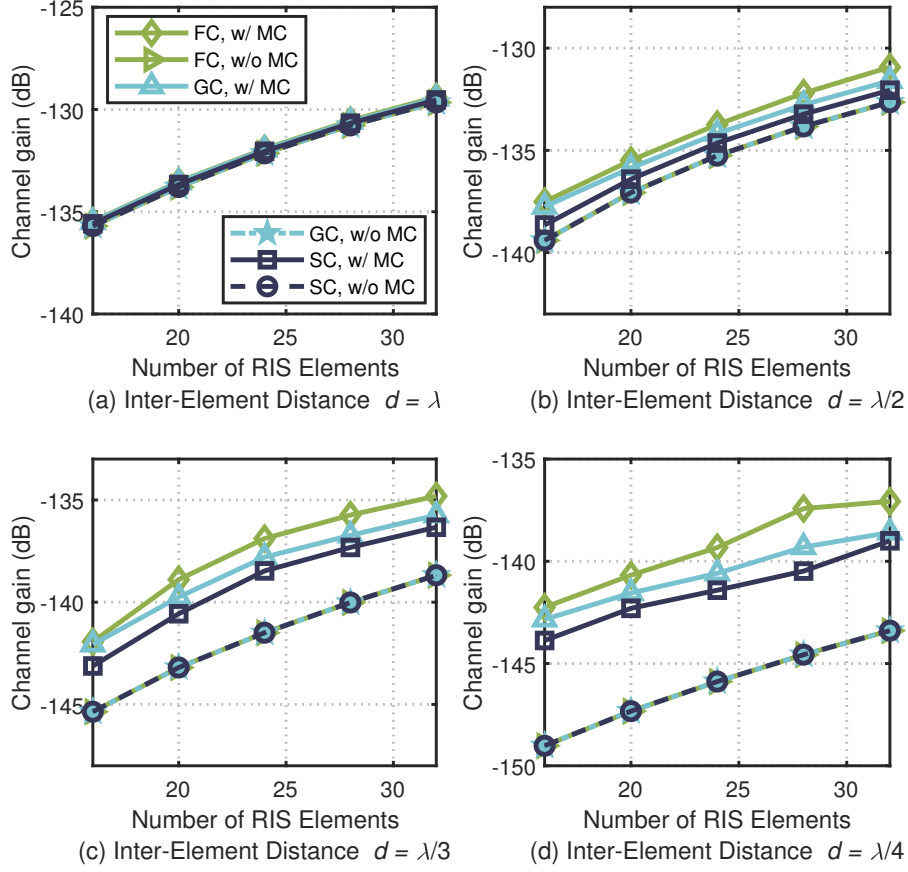


Figure 7.3: Channel gain versus the number of BD-RIS elements.

decreasing the inter-element distance. This can be attributed to the impact of mutual coupling, which becomes more prominent when decreasing the inter-element distance. This, in fact, results in larger values for the off-diagonal entries of  $\mathbf{Z}_{II}$ , which is better exploited by BD-RIS architectures for performance improvement.

## 7.4 Summary

This chapter studies the modeling and optimization of BD-RIS aided wireless communication systems in the presence of mutual coupling among the RIS elements. Specifically, we first derive a mutual coupling aware BD-RIS aided wireless communication model based on the scattering and impedance parameters, proving their equivalence. We then propose a general algorithm to maximize the channel gain of BD-RIS for SISO systems.

We finally illustrate simulation results to analyze the effectiveness of the proposed design and the impact of mutual coupling on BD-RIS architectures. The numerical results show that the larger the mutual coupling the larger the gain offered by BD-RIS.

# Chapter 8

## Wideband Effect of BD-RIS

Since a RIS can be realized using lumped elements, such as inductor, capacitors, and resistors, its scattering matrix is naturally influenced by the signal frequency. This is an intrinsic phenomenon that cannot be simply ignored. The frequency dependence of diagonal RIS in wideband systems has been accurately modeled and analyzed in [45–47]. However, the proposed methods by individually modeling the response of each RIS element [45–47] cannot be generalized to BD-RIS architectures with interconnections and thus coupled response among RIS elements. Therefore, this chapter focuses on the modeling and beamforming of BD-RIS with the wideband effect. This leads to the following layouts and contributions.

In Section 8.1, the signal model for BD-RIS aided SISO orthogonal frequency division multiplexing (OFDM) systems has been first derived. Specifically, we start from the time-domain channels between the transmitter to the receiver, and then bridge them to the frequency-domain forms to gain a comprehensive understanding of the behavior of BD-RIS in both domains.

In Section 8.2, a novel wideband model of BD-RIS based on lumped circuit models has next been proposed. The proposed model has simple linear expressions while capturing accurately the frequency dependence of the BD-RIS admittance matrix within practical frequency ranges. The proposed model is also sufficiently general to adapt to any BD-RIS architecture, such as group- and forest-connected architectures.

In Section 8.3, based on the proposed wideband BD-RIS model, the admittance matrix of BD-RIS with group- and forest-connected architectures has been optimized to maximize the average rate for BD-RIS-aided SISO-OFDM when each tunable admittance has either continuous or discrete values.

In Section 8.4, results show that BD-RIS always outperforms conventional RIS whether the wideband modeling of RIS is taken into account in the optimization process or not. In addition, BD-RIS with the proposed wideband modeling and optimization outperforms that with the frequency-independent model [66]. Furthermore, the impact of accurately modeling BD-RIS

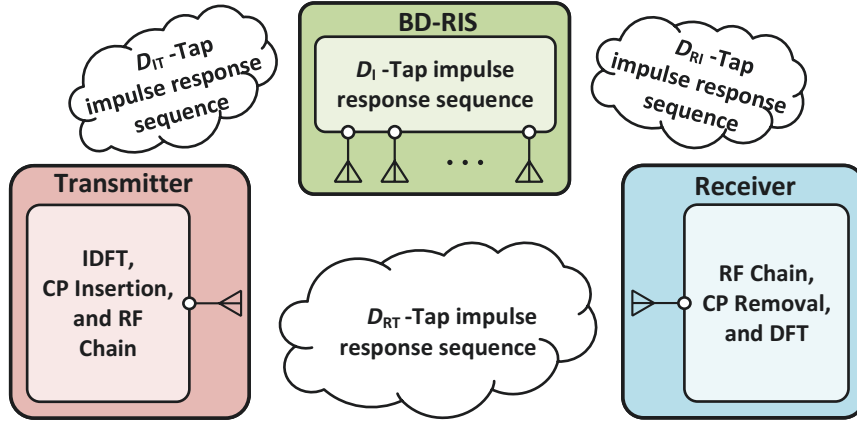


Figure 8.1: Diagram for the BD-RIS-aided SISO-OFDM system.

in wideband scenarios becomes more significant for BD-RIS with more tunable admittance components in the architecture. That is, when using conventional RIS and simple BD-RIS architectures such as group/forest-connected with group size no more than 3, it is reasonable to directly adopt the frequency-independent BD-RIS model; for more advanced architectures such as group-connected with group size 6, proper wideband modeling is advised.

## 8.1 System Model for BD-RIS Aided SISO-OFDM

We consider an  $M$ -element BD-RIS-aided wideband SISO-OFDM system with  $N_c$  subcarriers<sup>1</sup>. The signal model for the BD-RIS-aided SISO-OFDM system is explained below and illustrated in Fig. 8.1.

At the transmitter, we denote  $s_n \in \mathbb{C}$  the transmit symbol associated with subcarrier  $n$ ,  $\mathbb{E}\{|s_n|^2\} = 1$ ,  $\forall n \in \mathcal{N}_c = \{1, \dots, N_c\}$ . The transmitter allocates power  $p_n \geq 0$  to symbol  $s_n$ , and has a total power budget  $P$  such that  $\sum_{n \in \mathcal{N}_c} p_n \leq P$ . This leads to the discrete-time-domain signal  $\bar{\mathbf{s}} \in \mathbb{C}^{N_c \times 1}$  as

$$\bar{\mathbf{s}} = \bar{\mathbf{F}}_{N_c}^H \mathbf{s}, \quad (8.1)$$

where  $\mathbf{s} = [\sqrt{p_1}s_1, \dots, \sqrt{p_{N_c}}s_{N_c}]^T \in \mathbb{C}^{N_c \times 1}$  is the frequency-domain transmit symbol vector. After adding the cyclic prefix (CP), the time-domain signal is up-converted to the RF domain by the RF chain and radiated to the wireless environment through the transmit antenna [111, 112].

In the BD-RIS-aided SISO-OFDM system, we define the channel from the transmit antenna to the receive antenna as a  $D_{RT}$ -tap finite-duration impulse response sequence  $\{\bar{h}_{d,i} \in \mathbb{C} \mid \forall i = 0, \dots, D_{RT} - 1\}$ . In addition, the channel from the transmit antenna to BD-RIS is modeled as a  $D_{IT}$ -tap impulse response sequence  $\{\bar{\mathbf{g}}_i \in \mathbb{C}^{M \times 1} \mid \forall i = 0, \dots, D_{IT} - 1\}$ . The channel

<sup>1</sup>In this work, we focus on BD-RIS-aided SISO-OFDM systems for the ease of illustration, while the signal modeling in this work can be readily extended to multiple input multiple output (MIMO) OFDM systems.

from the BD-RIS to the receive antenna is modeled as a  $D_{\text{RI}}$ -tap impulse response sequence  $\{\bar{\mathbf{h}}_i \in \mathbb{C}^{1 \times M} \mid \forall i = 0, \dots, D_{\text{RI}} - 1\}$ . The response of BD-RIS can be regarded as the channel between the BD-RIS elements and themselves, which is manipulated by the BD-RIS tunable circuit. In contrast to conventional RIS where each element is not connected with each other, in BD-RIS, different elements are connected to each other by additional tunable impedance components [55] such that multiple paths between elements exist. More details about the circuit model of BD-RIS can be found in [55, 57], and will be briefly explained later. The fact that those connections generate multiple paths between BD-RIS elements motivates us to describe BD-RIS as a  $D_{\text{I}}$ -tap impulse response sequence  $\{\bar{\Phi}_i \in \mathbb{C}^{M \times M} \mid \forall i = 0, \dots, D_{\text{I}} - 1\}$  with  $\bar{\Phi}_i$  not being limited to be diagonal thanks to the inter-element connections. As such, the time-domain channel from the transmitter through BD-RIS to the receiver is the series convolution between  $\bar{\mathbf{h}}_i$ ,  $\bar{\Phi}_i$ , and  $\bar{\mathbf{g}}_i$ .

At the receiver, the discrete-time-domain received signal after down-converting to the baseband and removing the CP is the convolution of the time-domain channels and the time-domain symbols [111, 112]. Since the circular convolution of discrete sequences can be expressed as a matrix multiplication by introducing (block) circulant matrices, we write the discrete-time-domain received signal  $\bar{\mathbf{y}} \in \mathbb{C}^{N_c \times 1}$  in the following compact form

$$\bar{\mathbf{y}} = (\bar{\mathbf{H}}_{\text{d}} + \bar{\mathbf{H}}\bar{\Phi}\bar{\mathbf{G}})\bar{\mathbf{s}} + \bar{\mathbf{z}}, \quad (8.2)$$

where  $\bar{\mathbf{z}} \sim \mathcal{CN}(\mathbf{0}_{N_c \times 1}, \sigma^2 \mathbf{I}_{N_c})$  is the noise.  $\bar{\mathbf{H}}_{\text{d}} \in \mathbb{C}^{N_c \times N_c}$ ,  $\bar{\mathbf{H}} \in \mathbb{C}^{N_c \times N_c M}$ ,  $\bar{\Phi} \in \mathbb{C}^{N_c M \times N_c M}$ , and  $\bar{\mathbf{G}} \in \mathbb{C}^{N_c M \times N_c}$  are (block) circulant matrices constructed as in (1.2), based on respectively the following sequence entries/blocks:

$$\begin{aligned} \bar{\mathbf{H}}_{\text{d}} &= \text{blkcirc}(\bar{h}_{\text{d},0}, \dots, \bar{h}_{\text{d},D_{\text{RT}}-1}, \underbrace{0, \dots, 0}_{N_c - D_{\text{RT}} \text{ copies}}), \\ \bar{\mathbf{H}} &= \text{blkcirc}(\bar{\mathbf{h}}_0, \dots, \bar{\mathbf{h}}_{D_{\text{RI}}-1}, \underbrace{\mathbf{0}_{1 \times M}, \dots, \mathbf{0}_{1 \times M}}_{N_c - D_{\text{RI}} \text{ copies}}), \\ \bar{\Phi} &= \text{blkcirc}(\bar{\Phi}_0, \dots, \bar{\Phi}_{D_{\text{I}}-1}, \underbrace{\mathbf{0}_{M \times M}, \dots, \mathbf{0}_{M \times M}}_{N_c - D_{\text{I}} \text{ copies}}), \\ \bar{\mathbf{G}} &= \text{blkcirc}(\bar{\mathbf{g}}_0, \dots, \bar{\mathbf{g}}_{D_{\text{IT}}-1}, \underbrace{\mathbf{0}_{M \times 1}, \dots, \mathbf{0}_{M \times 1}}_{N_c - D_{\text{IT}} \text{ copies}}). \end{aligned} \quad (8.3)$$

After operating DFT, the received signal  $\mathbf{y} \in \mathbb{C}^{N \times 1}$  in the frequency-domain is given by

$$\mathbf{y} = \bar{\mathbf{F}}_{N_c} \bar{\mathbf{y}} = \underbrace{\bar{\mathbf{F}}_{N_c} (\bar{\mathbf{H}}_{\text{d}} + \bar{\mathbf{H}}\bar{\Phi}\bar{\mathbf{G}}) \bar{\mathbf{F}}_{N_c}^H}_{=\mathbf{H}_{\text{all}} \in \mathbb{C}^{N_c \times N_c}} \mathbf{s} + \underbrace{\bar{\mathbf{F}}_{N_c} \bar{\mathbf{z}}}_{=\mathbf{z} \in \mathbb{C}^{N_c \times 1}}, \quad (8.4)$$

with the frequency-domain SISO-OFDM channel  $\mathbf{H}_{\text{all}}$  and the noise  $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}_{N_c \times 1}, \sigma^2 \mathbf{I}_{N_c})$ . In the following proposition, we show that the frequency-domain channel  $\mathbf{H}_{\text{all}}$  can be equivalently described by the frequency-domain channels from the transmitter to the receiver, from the

transmitter to the BD-RIS, and from the BD-RIS to the receiver, as well as the frequency-dependent scattering matrix of BD-RIS.

**Proposition 8.1.** *The frequency-domain BD-RIS-aided SISO-OFDM channel  $\mathbf{H}_{\text{all}}$  is a diagonal matrix, i.e.,  $\mathbf{H}_{\text{all}} = \text{diag}(h_{\text{all},1}, \dots, h_{\text{all},N_c})$ . Specifically, each diagonal entry  $h_{\text{all},n} \in \mathbb{C}$  denotes the BD-RIS-aided channel at subcarrier  $n$ ,  $\forall n \in \mathcal{N}_c$  and is given by*

$$h_{\text{all},n} = h_{\text{d},n} + \mathbf{h}_n \mathbf{\Phi}_n \mathbf{g}_n, \forall n, \quad (8.5)$$

where  $h_{\text{d},n} \in \mathbb{C}$ ,  $\mathbf{h}_n \in \mathbb{C}^{1 \times M}$ , and  $\mathbf{g}_n \in \mathbb{C}^{M \times 1}$ , respectively, are the frequency-domain channels from the transmitter to the receiver, from the BD-RIS to the receiver, and from the transmitter to the BD-RIS, and  $\mathbf{\Phi}_n \in \mathbb{C}^{M \times M}$  is the scattering matrix of BD-RIS at subcarrier  $n$ . These frequency-domain channels are obtained from the time-domain ones based on the mappings:

$$\begin{aligned} h_{\text{d},n} &= [\bar{\mathbf{F}}_{N_c} \bar{\mathbf{H}}_{\text{d}} \bar{\mathbf{F}}_{N_c}^H]_{n,n}, \quad [\mathbf{h}_n]_i = [\bar{\mathbf{F}}_{N_c} \bar{\mathbf{H}}_i \bar{\mathbf{F}}_{N_c}^H]_{n,n}, \forall i \in \mathcal{M}, \forall n, \\ [\mathbf{g}_n]_j &= [\bar{\mathbf{F}}_{N_c} \bar{\mathbf{G}}_j \bar{\mathbf{F}}_{N_c}^H]_{n,n}, \quad [\mathbf{\Phi}_n]_{i,j} = [\bar{\mathbf{F}}_{N_c} \bar{\mathbf{\Phi}}_{i,j} \bar{\mathbf{F}}_{N_c}^H]_{n,n}, \forall i, j \in \mathcal{M}, \forall n, \end{aligned} \quad (8.6)$$

where  $\mathcal{M} = \{1, \dots, M\}$ .  $\bar{\mathbf{H}}_i \in \mathbb{C}^{N \times N}$ ,  $\bar{\mathbf{G}}_j \in \mathbb{C}^{N \times N}$ , and  $\bar{\mathbf{\Phi}}_{i,j} \in \mathbb{C}^{N \times N}$  are circulant matrices extracted respectively from  $\bar{\mathbf{H}}$ ,  $\bar{\mathbf{G}}$ , and  $\bar{\mathbf{\Phi}}$ , with  $[\bar{\mathbf{H}}_i]_{:,p} = [\bar{\mathbf{H}}]_{:,(p-1)M+i}$ ,  $[\bar{\mathbf{G}}_j]_{q,:} = [\bar{\mathbf{G}}]_{(q-1)M+j,:}$ ,  $[\bar{\mathbf{\Phi}}_{i,j}]_{p,q} = [\bar{\mathbf{\Phi}}]_{(p-1)M+i,(q-1)M+j}$ ,  $\forall i, j \in \mathcal{M}, \forall p, q \in \mathcal{N}_c$ .

*Proof.* Please refer to Appendix E. □

From Proposition 8.1, we observe that

- 1) the obtained BD-RIS-aided wideband channel aligns with the intuition in conventional OFDM systems, i.e., the wideband channels in OFDM systems can be described as parallel narrowband channels at different subcarriers;
- 2) the obtained channel has the same expression as the conventional RIS-aided wideband channel [46], except that the BD-RIS response is not limited to being diagonal;
- 3) the scattering matrix of BD-RIS is frequency-dependent, i.e., in general  $\mathbf{\Phi}_n \neq \mathbf{\Phi}_m$  for different subcarriers  $n \neq m$ .

It is worth noting that in conventional RIS with diagonal phase shift matrix  $\mathbf{\Phi}_n$ , it is possible to individually model the phase and amplitude response of each diagonal entry of  $\mathbf{\Phi}_n$  [46]. However, this method does not work for BD-RIS with scattering matrices  $\mathbf{\Phi}_n$ ,  $\forall n \in \mathcal{N}_c$ , not limited to being diagonal, since their entries are highly coupled with each other due to the inter-element connections. To address this challenge, we instead model the frequency-dependent admittance matrix of BD-RIS [69], where each entry of the admittance matrix can be expressed as the linear combination of tunable admittance components with each being independently

controlled by circuit models. As such, the modeling of the scattering matrix with coupled entries boils down to that of each tunable admittance component. To that end, we map the scattering matrix  $\Phi_n$  to the admittance matrix  $\mathbf{Y}_{\text{I},n} \in \mathbb{C}^{M \times M}$ , which directly characterizes the circuit model of BD-RIS, by the relationship in (2.10):

$$\Phi_n = (Y_0 \mathbf{I}_M + \mathbf{Y}_{\text{I},n})^{-1} (Y_0 \mathbf{I}_M - \mathbf{Y}_{\text{I},n}), \forall n, \quad (8.7)$$

where  $Y_0$  refers to the characteristic admittance, e.g.  $Y_0 = \frac{1}{50}$  siemens (S). The modelling of  $\mathbf{Y}_{\text{I},n}$ ,  $\forall n \in \mathcal{N}_c$  will be elaborated based on the circuit model of BD-RIS as follows.

## 8.2 Circuit-Based Wideband Modeling of BD-RIS

In this section, we first introduce the BD-RIS modeling using admittance parameter analysis, and then establish the wideband BD-RIS model by characterizing its frequency dependence based on the circuit model.

### 8.2.1 Admittance Matrix of BD-RIS

Consider an  $M$ -element BD-RIS modeled as  $M$  antennas connected to an  $M$ -port reconfigurable admittance network [55]. There are various architectures of the reconfigurable admittance network determined by the circuit topology design. We focus on the wideband modeling and optimization of two representative architectures, namely group-connected [55] and forest-connected [57], which, respectively, include the fully- and tree-connected architectures as special cases. The mathematical descriptions are explained below.

**Group-Connected Reconfigurable Admittance Network:** For a general group-connected reconfigurable admittance network with  $G$  uniform groups, the  $\bar{M} = \frac{M}{G}$  ports in each group are all connected to each other by tunable admittance components, while ports in different groups are not connected to each other [55]. To facilitate understanding, an illustrative example of a 6-element BD-RIS with the group-connected architecture (of group size 3) is illustrated in Fig. 8.2(a). According to the circuit topology of the group-connected reconfigurable admittance network, the corresponding admittance matrix  $\mathbf{Y}_{\text{I}} \in \mathbb{C}^{M \times M}$  has a block-diagonal structure as

$$\mathbf{Y}_{\text{I}} = \text{blkdiag}(\bar{\mathbf{Y}}_{\text{I},1}, \dots, \bar{\mathbf{Y}}_{\text{I},G}), \quad (8.8)$$

where  $\bar{\mathbf{Y}}_{\text{I},g} \in \mathbb{C}^{\bar{M} \times \bar{M}}$ ,  $\forall g \in \mathcal{G} = \{1, \dots, G\}$  is symmetric and purely imaginary for reciprocal and lossless reconfigurable admittance networks, that is

$$\bar{\mathbf{Y}}_{\text{I},g} = \bar{\mathbf{Y}}_{\text{I},g}^T, \quad \Re\{\bar{\mathbf{Y}}_{\text{I},g}\} = \mathbf{0}_{\bar{M} \times \bar{M}}, \forall g. \quad (8.9)$$

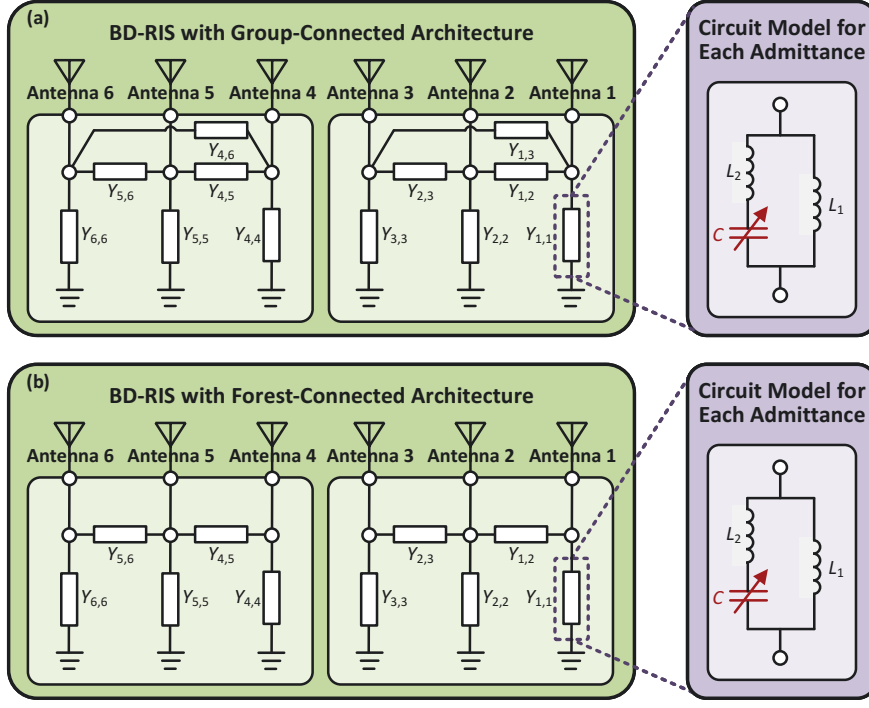


Figure 8.2: Examples of a 6-element BD-RIS with (a) group-connected and (b) forest-connected reconfigurable admittance network of group size 3, and the equivalent circuit for each admittance.

In group  $g$  of the group-connected reconfigurable admittance network, each port  $m_g = (g - 1)\bar{M} + m$  is connected to port  $m'_g = (g - 1)\bar{M} + m'$ , with a tunable admittance<sup>2</sup>  $Y_{m_g, m'_g}$ ,  $\forall m, m' \in \bar{\mathcal{M}} = \{1, \dots, \bar{M}\}$ , which satisfies  $Y_{m_g, m'_g} = Y_{m'_g, m_g}$  for  $m' > m$ . Accordingly,  $\bar{\mathbf{Y}}_{\text{I},g}$ ,  $\forall g \in \mathcal{G}$  writes as [77]

$$[\bar{\mathbf{Y}}_{\text{I},g}]_{m,m'} = \begin{cases} -Y_{m_g, m'_g}, & m_g \neq m'_g \\ \sum_{k=(g-1)\bar{M}+1}^{g\bar{M}} Y_{m_g, k}, & m_g = m'_g \end{cases}. \quad (8.10)$$

**Forest-Connected Reconfigurable Admittance Network:** The circuit complexity of group-connected architecture, that is the number of tunable admittance components, is  $\frac{\bar{M}(\bar{M}+1)}{2}G$ , which grows quadratically with the group size  $\bar{M}$ . This makes it difficult to implement group-connected architecture with larger group size, e.g.,  $\bar{M} > 4$ . To simplify the circuit design and reduce the circuit complexity while maintaining the beam manipulation flexibility, the forest-connected architecture of BD-RIS is proposed in [57]. Different from the group-connected architecture where every two ports in the same group are interconnected via an admittance, the forest-connected architecture is constructed such that in each group, every port is only connected to one other port within the same group, thereby reducing the required number of admittance components. For a forest-connected reconfigurable admittance network with  $G$  uniform groups, every two adjacent ports of the  $\bar{M}$  ports in each group are connected via one tunable admittance component, referred to as tree-connected, while ports from different groups

<sup>2</sup> $Y_{m_g, m_g}$  refers to the admittance which connects port  $m_g$  to ground.

are disconnected [57]. As such, the circuit complexity reduces to  $(2\bar{M} - 1)G$ . To facilitate understanding, an example of a 6-element BD-RIS with the forest-connected architecture is illustrated in Fig. 8.2(b). In this case, the admittance matrix  $\mathbf{Y}_I$  still satisfies (8.8)-(8.10). The difference compared to the group-connected architecture is that in each group  $g$ , ports  $m_g$  and  $m'_g$  are not connected if  $|m_g - m'_g| > 1, \forall m, m' \in \bar{\mathcal{M}}, \forall g \in \mathcal{G}$ . This leads to the following additional constraint:

$$Y_{m_g, m'_g} = 0, \forall m_g, m'_g, |m_g - m'_g| > 1. \quad (8.11)$$

Based on the above illustrations, we observe that group-connected BD-RIS includes conventional RIS (single-connected with  $\bar{M} = 1$ ) and fully-connected BD-RIS ( $\bar{M} = M$ ) as two special cases. Meanwhile, forest-connected BD-RIS includes conventional RIS ( $\bar{M} = 1$ ) and tree-connected BD-RIS ( $\bar{M} = M$ ) as two special cases. Group- and forest-connected BD-RISs with  $\bar{M} = 2$  share the same circuit topology.

In equations (8.8)-(8.11), we have general expressions of the admittance matrix for BD-RIS without specifying its frequency dependence, which will be identified in the following subsection based on the circuit model of each tunable admittance.

### 8.2.2 Wideband Modeling of Tunable Admittance

In this work, we model each tunable admittance component as a lumped circuit consisting of two inductors  $L_1$  and  $L_2$ , and one tunable capacitor  $C$  [113], as illustrated in Fig. 8.2<sup>3</sup>. The admittance of the circuit is thus a function of the value of the capacitor and the frequency  $f$  for the incident signals, which is given by<sup>4</sup>

$$Y_{m_g, m'_g} = \frac{1}{j\omega L_1} + \frac{1}{j\omega L_2 + \frac{1}{j\omega C_{m_g, m'_g}}}, \forall m_g, m'_g, \quad (8.12)$$

where  $\omega = 2\pi f$  denotes the angular frequency. This yields the corresponding susceptance as  $B_{m_g, m'_g} = \Im\{Y_{m_g, m'_g}\}, \forall m, m' \in \bar{\mathcal{M}}, \forall g \in \mathcal{G}$ . From (8.12) we can observe that the tunable admittance values at different frequencies are coupled to each other, that is for a certain value of capacitor  $C$ , the admittance will have different values at different frequencies. More importantly, this is an intrinsic phenomenon coming from the hardware implementation of BD-RIS elements, which cannot be simply ignored in BD-RIS-aided wideband communication systems. This phenomenon is also numerically illustrated in Fig. 8.3(a), where we consider a wideband system with center frequency  $f_c = 2.4$  GHz and adopt a practical varactor diode according to the data sheet for the diode SMV1231-079. Interestingly, the susceptance can be regarded as a linear function with respect to frequency for different values of  $C_{m_g, m'_g}$  within some practical

<sup>3</sup>Here we remove the equivalent resistance in the circuit to make the admittance network lossless.

<sup>4</sup>In this model, we set fixed and same values of inductors  $L_1$  and  $L_2$  for each admittance component, while making the value of the capacitor tunable and different, i.e.,  $C_{m_g, m'_g}$ , for different admittance components.

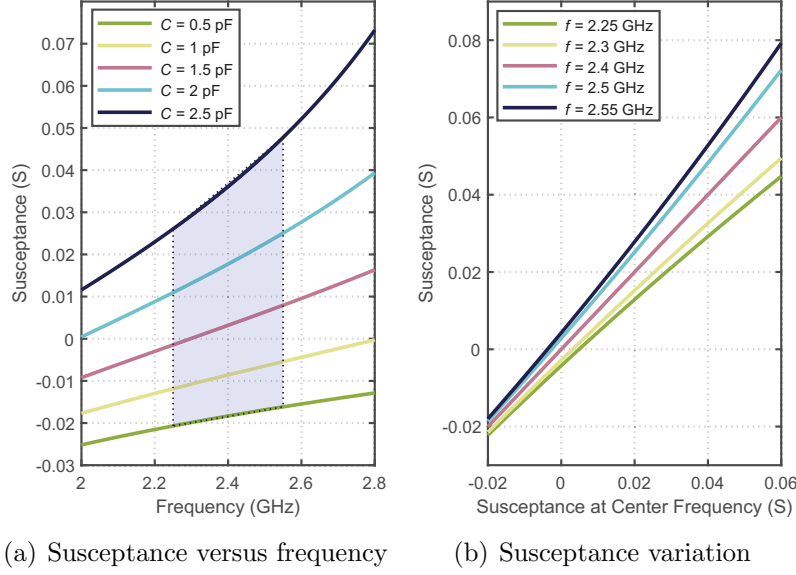


Figure 8.3: The susceptance as a function of (a) frequency and (b) the value of susceptance at the center frequency  $f_c = 2.4$  GHz with  $L_1 = 2.5$  nH,  $L_2 = 0.7$  nH, and  $C \in [0.2, 3]$  pF for a practical varactor diode.

bandwidth for wideband communication systems, such as the range from 2.25 GHz to 2.55 GHz as highlighted in Fig. 8.3(a).

To obtain more insights of how the value of susceptance varies in wideband systems, we define  $B_{c,m_g,m'_g}$  as the susceptance at the center frequency  $f_c$ . According to (8.12), we have the following relationship

$$C_{m_g,m'_g} = \left( \omega_c^2 L_2 + \frac{\omega_c}{B_{c,m_g,m'_g} + \frac{1}{\omega_c L_1}} \right)^{-1}, \quad (8.13)$$

where  $\omega_c = 2\pi f_c$ . Plugging (8.13) into (8.12), we can rewrite  $B_{m_g,m'_g}$ ,  $\forall m, m' \in \bar{\mathcal{M}}, \forall g \in \mathcal{G}$ , as a function of  $B_{c,m_g,m'_g}$  and  $\omega$ , that is

$$B_{m_g,m'_g}(B_{c,m_g,m'_g}, \omega) = \frac{((\omega^2 - \omega_c^2)L_2 + \omega^2 L_1) \left( B_{c,m_g,m'_g} + \frac{1}{\omega_c L_1} \right) - \omega_c}{\omega L_1 \left( \omega_c - (\omega^2 - \omega_c^2)L_2 \left( B_{c,m_g,m'_g} + \frac{1}{\omega_c L_1} \right) \right)}. \quad (8.14)$$

It is difficult to characterize the relationship between  $B_{m_g,m'_g}$  and  $B_{c,m_g,m'_g}$  due to the complicated expression of (8.14). Fortunately, when focusing on the linear range highlighted in Fig. 8.3(a), we observe that  $B_{m_g,m'_g}$  can be regarded as a linear function of  $B_{c,m_g,m'_g}$ , as illustrated in Fig. 8.3(b). Additionally, its slope and intercept can both be regarded as linear functions of  $\omega$ , which are shown in Figs. 8.4(a) and 8.4(b). The above observations motivate us to model the frequency dependence of each admittance component for BD-RIS in wideband communication

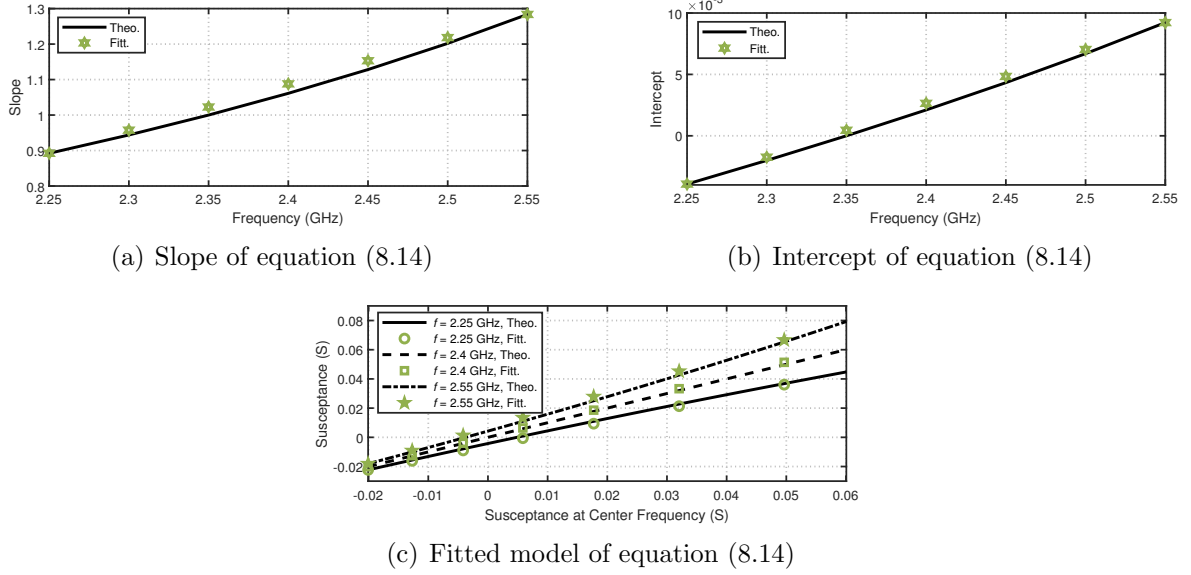


Figure 8.4: (a) The slope and (b) the intercept of equation (8.14) as a function of  $\omega$ ; (c) the fitted result based on equation (8.15). The parameters  $\alpha_j, \beta_j, \forall j \in \{1, 2\}$  in the linear model (8.15) are set as  $\alpha_1 = 2.0046 \times 10^{-10}$ ,  $\alpha_2 = -1.9968$ ,  $\beta_1 = 6.2775 \times 10^{-12}$ ,  $\beta_2 = -0.0942$ .

systems as a simplified linear function, which is given by:

$$\begin{aligned} B_{m_g, m'_g}(B_{c, m_g, m'_g}, \omega) &= F_1(\omega)B_{c, m_g, m'_g} + F_2(\omega), \forall m_g, m'_g, \\ F_1(\omega) &= \alpha_1\omega + \beta_1, \quad F_2(\omega) = \alpha_2\omega + \beta_2, \end{aligned} \quad (8.15)$$

where parameters  $\alpha_j, \beta_j, \forall j \in \{1, 2\}$  are determined by  $L_1, L_2$ , and  $\omega_c$ <sup>5</sup>. For the highlighted curves in Fig. 8.3(a), the theoretical results given by (8.14) and fitted results using the linear model (8.15) are shown in Fig. 8.4. The results in Fig. 8.4(c) guarantee a 0.27% normalized mean square error between the fitted values and theoretical ones, which demonstrate the accuracy of the proposed linear model.

We should highlight that the linear model (8.15) is a generalization of the narrowband model utilized in existing BD-RIS literature [55, 57]. Specifically, the narrowband scenario refers to the case that  $\omega \approx \omega_c$ , yielding  $F_1(\omega) \approx 1$  and  $F_2(\omega) \approx 0$ , such that the susceptance of each tunable admittance component is approximately frequency-independent.

### 8.2.3 BD-RIS with Frequency-Dependent Admittance Matrices

In the BD-RIS-aided SISO-OFDM system, each sub-band  $n$  has a carrier frequency  $f_n$ , within which we assume the admittance matrix of BD-RIS is constant in frequency, and only dependent on the carrier frequency  $f_n$ . Therefore, we characterize BD-RIS in the OFDM system by  $N_c$  frequency-dependent admittance matrices  $\mathbf{Y}_{I,n}, \forall n \in \mathcal{N}_c$ , each of which refers to the admittance

<sup>5</sup>In this work, we obtain the values of  $\alpha_j, \beta_j, \forall j \in \{1, 2\}$  by fitting the slope and intercept of (8.14) as linear functions of  $\omega$  such that each of them can be characterized based on arbitrary two points on the lines.

matrix of BD-RIS at subcarrier  $n$ . In addition, each  $\mathbf{Y}_{I,n}$  is constructed based on equations (8.8)-(8.11) and the linear model (8.15) of each tunable admittance component. In the following, we will summarize the constraint of  $\mathbf{Y}_{I,n}$  for BD-RIS with different architectures.

**Group-Connected BD-RIS:** Given the carrier frequency  $f_n$  for subcarrier  $n$ ,  $\mathbf{Y}_{I,n}$  is

$$\mathbf{Y}_{I,n} = \text{blkdiag}(\mathbf{Y}_{I,1,n}, \dots, \mathbf{Y}_{I,G,n}), \forall n, \quad (8.16a)$$

$$\mathbf{Y}_{I,g,n} = j\mathbf{B}_{g,n}, \quad \mathbf{B}_{g,n} = \mathbf{B}_{g,n}^T, \forall g, n, \quad (8.16b)$$

$$[\mathbf{B}_{g,n}]_{m,m'} = \begin{cases} -B_{m_g,m'_g,n}, & m_g \neq m'_g \\ \sum_{k=(g-1)\bar{M}+1}^{g\bar{M}} B_{m_g,k,n}, & m_g = m'_g \end{cases}, \quad (8.16c)$$

$$B_{m_g,m'_g,n} = F_1(\omega_n)B_{c,m_g,m'_g} + F_2(\omega_n), \forall m_g, m'_g, n, \quad (8.16d)$$

$$B_{c,m_g,m'_g} \in [B_{\min}, B_{\max}], \forall m_g, m'_g, \quad (8.16e)$$

where  $\omega_n = 2\pi f_n$ ,  $B_{\min}$  and  $B_{\max}$  are determined by the practical range of capacitor values. More specifically, we consider the following two cases.

Case 1: When the capacitor has continuously tunable values, each  $B_{c,m_g,m'_g}$  is arbitrarily selected within  $[B_{\min}, B_{\max}]$ ;

Case 2: When the capacitor has finite tunable values, each  $B_{c,m_g,m'_g}$  is selected from the set

$$\mathcal{B}_c = \left\{ B_{\min} + \frac{B_{\max} - B_{\min}}{2^b - 1} x \mid x = 0, 1, \dots, 2^b - 1 \right\}, \quad (8.17)$$

where  $b$  denotes the number of resolution bits.

**Forest-Connected BD-RIS:** Given the carrier frequency  $f_n$  for subcarrier  $n$ ,  $\mathbf{Y}_{I,n}$  is constructed based on (8.16a)-(8.16c) and the following constraints:

$$B_{m_g,m'_g,n} = \begin{cases} 0, & \text{if } |m_g - m'_g| > 1, \\ F_1(\omega_n)B_{c,m_g,m'_g} + F_2(\omega_n), & \text{otherwise,} \end{cases} \quad (8.18a)$$

$$B_{c,m_g,m'_g} \in [B_{\min}, B_{\max}] \text{ or } B_{c,m_g,m'_g} \in \mathcal{B}_c. \quad (8.18b)$$

From (8.16) and (8.18) we observe that  $\mathbf{Y}_{I,n}$  for all carrier frequencies are dependent to each other and jointly tuned by  $B_{c,m_g,m'_g}$ . In [65, 66], however, the frequency-independent BD-RIS model is adopted in wideband systems. That is, the constraint (8.16d) has always  $F_1(\omega_n) = 1$  and  $F_2(\omega_n) = 0$ , such that  $\mathbf{Y}_{I,n} = \mathbf{Y}_I, \forall n \in \mathcal{N}_c$ . This simplified model facilitates the BD-RIS design but will cause performance loss due to the ignorance of frequency dependence of BD-RIS admittance matrices. The performance comparison will be given later.

## 8.3 Optimization for BD-RIS Aided SISO-OFDM

In this section, we apply the proposed wideband BD-RIS model in a SISO-OFDM system to formulate the rate maximization problem and develop corresponding algorithms to optimize BD-RIS admittance matrices.

### 8.3.1 Problem Formulation

According to [69] and Section 2.1, by modeling the transmitter, BD-RIS, and receiver as a multi-port network, the wireless channel can be characterized by its admittance matrix, where the admittance matrix of BD-RIS,  $\mathbf{Y}_{\text{I},n}$ ,  $\forall n \in \mathcal{N}_c$ , is visible in the BD-RIS-aided wireless channel. Specifically, by Result 2.1, the BD-RIS-aided wireless channel  $h_n$  in (8.5) can be re-written as [69]

$$h_n = \frac{1}{2Y_0}(-y_{\text{RT},n} + \mathbf{y}_{\text{RI},n}(\mathbf{Y}_{\text{I},n} + Y_0\mathbf{I}_M)^{-1}\mathbf{y}_{\text{IT},n}), \forall n, \quad (8.19)$$

where  $y_{\text{RT},n} \in \mathbb{C}$ ,  $\mathbf{y}_{\text{RI},n} \in \mathbb{C}^{1 \times M}$ , and  $\mathbf{y}_{\text{IT},n} \in \mathbb{C}^{M \times 1}$ , respectively, refer to the channels from transmitter to receiver, from BD-RIS to receiver, and from transmitter to BD-RIS based on the admittance parameters [69]. According to the discussions in [69], these channels are a characterization of the wireless channels equivalent to the widely used models based on the scattering parameters [7]. The detailed mapping between the channels based on the scattering parameters and those based on the admittance parameters can be found in [69], and is given based on (2.15b) as follows

$$\mathbf{y}_{\text{RI},n} = -2Y_0\mathbf{h}_n, \quad \mathbf{y}_{\text{IT},n} = -2Y_0\mathbf{g}_n, \quad y_{\text{RT},n} = -2Y_0(h_{\text{d},n} - \mathbf{h}_n\mathbf{g}_n), \forall n. \quad (8.20)$$

We aim to jointly design the power allocation at the transmitter and the susceptances  $B_{c,m_g,m'_g}$  at the center frequency to maximize the average rate for the SISO-OFDM system. This yields the following problem:

$$\{B_{c,m_g,m'_g}^*, p_n^*\}_{\forall m_g, m'_g, n} = \arg \max_{(8.16) \text{ or } (8.16a)-(8.16c), (8.18), \sum_n p_n \leq P} \frac{1}{N_c} \sum_{n \in \mathcal{N}_c} \log_2 \left( 1 + \frac{p_n |h_n|^2}{\sigma^2} \right), \quad (8.21)$$

where the constraints in (8.16) are considered for group-connected architectures while the constraints in (8.16a)-(8.16c) and (8.18) for forest-connected architectures. Problem (8.21) is a challenging joint optimization mainly due to the coupling constraints of BD-RIS in wideband systems. To simplify the design, we decouple problem (8.21) into two sub-problems and solve each of them individually.

### 8.3.2 Two-Stage Optimization

Since the wideband modeling of BD-RIS couples the admittances for different subcarriers, which are embedded in the  $\log(\cdot)$  function, it is difficult to directly design the BD-RIS with the original objective function in problem (8.21). In addition, the power allocation and the admittance matrices of BD-RIS are coupled, which further complicates the optimization. To simplify the problem, we decouple the design of BD-RIS and power allocation. This leads to the following two-stage optimization framework:

Stage 1: We first simplify the objective function in (8.21) by using Jensen's inequality, which yields  $\frac{1}{N_c} \sum_{n \in \mathcal{N}_c} \log_2(1 + \frac{p_n |h_n|^2}{\sigma^2}) \leq \log_2(1 + \frac{1}{N_c} \sum_{n \in \mathcal{N}_c} \frac{p_n |h_n|^2}{\sigma^2})$ . In addition, we remove the impact of power allocation when designing BD-RIS by setting  $p_1 = \dots = p_{N_c}$ , while the power allocation will be designed after the admittance matrices of BD-RIS have been determined. Therefore, we solve the following sum channel gain maximization problem:

$$\begin{aligned} \{B_{c,m_g,m'_g}^*\}_{\forall m_g,m'_g} = \arg \max_{\substack{(8.16b)-(8.16e) \text{ or} \\ (8.16b),(8.16c),(8.18)}} \sum_{n \in \mathcal{N}_c} \left| -y_{RT,n} \right. \\ \left. + \sum_{g \in \mathcal{G}} \mathbf{y}_{RI,g,n} (j\mathbf{B}_{g,n} + Y_0 \mathbf{I}_{\bar{M}})^{-1} \mathbf{y}_{IT,g,n} \right|^2, \end{aligned} \quad (8.22)$$

where  $\mathbf{y}_{RI,g,n} = [\mathbf{y}_{RI,n}]_{(g-1)\bar{M}+1:g\bar{M}}$ , and  $\mathbf{y}_{IT,g,n} = [\mathbf{y}_{IT,n}]_{(g-1)\bar{M}+1:g\bar{M}}$ ,  $\forall g \in \mathcal{G}$ ,  $\forall n \in \mathcal{N}_c$ .

Stage 2: When  $B_{c,m_g,m'_g}^*$ ,  $\forall m, m' \in \bar{\mathcal{M}}$ ,  $\forall g \in \mathcal{G}$ , are determined, the BD-RIS-aided wideband channels,  $h_n^*$ ,  $\forall n$ , can be obtained by (8.16), (8.18), and (8.19). Then, problem (8.21) boils down to the following conventional power allocation problem:

$$\{p_n^*\}_{\forall n} = \arg \max_{\sum_n p_n \leq P} \frac{1}{N_c} \sum_{n \in \mathcal{N}_c} \log_2 \left( 1 + \frac{p_n |h_n^*|^2}{\sigma^2} \right), \quad (8.23)$$

which can be solved by the well-known water-filling method.

In the following, we will elaborate on solving problem (8.22) for both group- and forest-connected BD-RISs when each  $B_{c,m_g,m'_g}$  has either continuous values or discrete values controlled by the set  $\mathcal{B}_c$ .

### 8.3.3 Solution to Problem (8.22): Continuous-Value Admittance

We propose to rewrite (8.22) into an unconstrained optimization and solve it using the quasi-Newton method. The detailed mathematical transformations for group- and forest-connected architectures are explained below.

**Group-Connected BD-RIS:** We start by re-describing (8.16c) as a function which maps the matrix  $\bar{\mathbf{B}}_{g,n} \in \mathbb{C}^{\bar{M} \times \bar{M}}$  with entries  $[\bar{\mathbf{B}}_{g,n}]_{m,m'} = B_{m_g,m'_g,n}$  to the matrix  $\mathbf{B}_{g,n}$ , i.e.,  $\mathbf{B}_{g,n} = F_3(\bar{\mathbf{B}}_{g,n})$ , where  $F_3(\cdot)$  is given such that

$$[\mathbf{B}_{g,n}]_{m,m'} = \begin{cases} -[\bar{\mathbf{B}}_{g,n}]_{m,m'}, & m \neq m' \\ \sum_{k \in \bar{\mathcal{M}}} [\bar{\mathbf{B}}_{g,n}]_{m,k}, & m = m' \end{cases}. \quad (8.24)$$

In addition, the symmetric constraint of each  $\mathbf{B}_{g,n}$  in (8.16b) and the linear mapping (8.16c) imply that each  $\bar{\mathbf{B}}_{g,n}$  should be symmetric, i.e.,  $\bar{\mathbf{B}}_{g,n} = \bar{\mathbf{B}}_{g,n}^T$ . In other words, each  $\bar{\mathbf{B}}_{g,n}$  is essentially determined by its  $\bar{M}$  diagonal and  $\frac{\bar{M}(\bar{M}-1)}{2}$  lower-triangular (or upper-triangular) entries. This motivates us to rewrite  $\bar{\mathbf{B}}_{g,n}$  as  $\bar{\mathbf{B}}_{g,n} = \text{unvec}(\mathbf{R}\mathbf{b}_{g,n})$ , where  $\mathbf{b}_{g,n} \in \mathbb{R}^{\frac{\bar{M}(\bar{M}+1)}{2} \times 1}$  contains the diagonal and lower-triangular entries of  $\bar{\mathbf{B}}_{g,n}$ .  $\mathbf{R} \in \{0, 1\}^{\bar{M}^2 \times \frac{\bar{M}(\bar{M}+1)}{2}}$  is the binary matrix mapping  $\mathbf{b}_{g,n}$  into  $\text{vec}(\bar{\mathbf{B}}_{g,n})$ ,  $\forall g \in \mathcal{G}$  defined in 7.15, where  $\forall i, i' \in \bar{\mathcal{M}}$ . According to (8.16d), these vectors  $\mathbf{b}_{g,n}$  for all subcarriers are jointly controlled by  $\mathbf{b}_{c,g} \in \mathbb{R}^{\frac{\bar{M}(\bar{M}+1)}{2} \times 1}$  constructed with the entries  $B_{c,m_g,m'_g}$  for  $m' \geq m$ .

Based on the above illustration, we can reformulate problem (8.22) as the following form:

$$\max_{\mathbf{b}_{c,g}, \forall g} \underbrace{\sum_{n \in \mathcal{N}_c} \left| -y_{\text{RT},n} + \sum_{g \in \mathcal{G}} \mathbf{y}_{\text{RI},g,n} (jF_3(\text{unvec}(\mathbf{R}\mathbf{b}_{g,n})) + Y_0 \mathbf{I}_{\bar{M}})^{-1} \mathbf{y}_{\text{IT},g,n} \right|^2}_{=F(\mathbf{b}_c)} \quad (8.25a)$$

$$\text{s.t. } \mathbf{b}_{g,n} = F_1(\omega_n) \mathbf{b}_{c,g} + F_2(\omega_n), \forall n, g, \quad (8.25b)$$

$$[\mathbf{b}_{c,g}]_l \in [B_{\min}, B_{\max}], \forall g, l \in \bar{\mathcal{M}}, \quad (8.25c)$$

where  $\mathbf{b}_c = [\mathbf{b}_{c,1}^T, \dots, \mathbf{b}_{c,G}^T]^T \in \mathbb{C}^{|\bar{\mathcal{M}}|G \times 1}$ , and  $\bar{\mathcal{M}} = \{1, \dots, \frac{\bar{M}(\bar{M}+1)}{2}\}$ . The linear constraint (8.25c) of each entry of  $\mathbf{b}_{c,g}$  can be further removed by one-to-one mapping variables with arbitrary real values into the range  $[B_{\min}, B_{\max}]$ . This can be done by introducing a tractable mapping function  $[\mathbf{b}_{c,g}]_l = F_4([\mathbf{x}_g]_l)$ ,  $\mathbf{x}_g \in \mathbb{R}^{\frac{\bar{M}(\bar{M}+1)}{2} \times 1}$ ,  $\forall l \in \bar{\mathcal{M}}$ , with the following properties.

Property 1:  $F_4([\mathbf{x}_g]_l)$  is continuous, differentiable, and monotonously increasing in the full range  $[\mathbf{x}_g]_l \in (-\infty, +\infty)$ ;

Property 2:  $\lim_{[\mathbf{x}_g]_l \rightarrow +\infty} F_4([\mathbf{x}_g]_l) = B_{\max}$ ;

Property 3:  $\lim_{[\mathbf{x}_g]_l \rightarrow -\infty} F_4([\mathbf{x}_g]_l) = B_{\min}$ .

An example of such a mapping function is given by:

$$[\mathbf{b}_{c,g}]_l = F_4([\mathbf{x}_g]_l) = \frac{[\mathbf{x}_g]_l}{\sqrt{[\mathbf{x}_g]_l^2 B_-^{-2} + 1}} + B_+, \forall g, l, \quad (8.26)$$

where  $B_- = \frac{B_{\max} - B_{\min}}{2}$  and  $B_+ = \frac{B_{\max} + B_{\min}}{2}$ . By this mapping function, we can transform problem (8.25) into the following unconstrained optimization:

$$\{\mathbf{x}_g^*\}_{\forall g} = \arg \max_{(8.25b), (8.26)} F(\mathbf{b}_c), \quad (8.27)$$

which can be directly solved by the quasi-Newton method.

**Forest-Connected BD-RIS:** The proposed optimization method for group-connected BD-RIS with continuous-value admittance components can be easily adapted to forest-connected BD-RIS. This can be easily done by modifying the mapping  $\bar{\mathbf{B}}_{g,n} = \text{unvec}(\mathbf{P}\mathbf{b}_{g,n})$  for group-connected architecture based on (8.16b), (8.16c), and (8.18a) to align with the forest-connected architecture. More details are explained below.

For forest-connected BD-RIS, (8.16b), (8.16c), and (8.18a) imply that each matrix  $\bar{\mathbf{B}}_{g,n}$  with entries  $[\bar{\mathbf{B}}_{g,n}]_{m,m'} = B_{m_g,m'_g,n}$  is tridiagonal and symmetric. That is,  $\bar{\mathbf{B}}_{g,n}$  is determined by its  $\bar{M}$  diagonal and  $\bar{M} - 1$  non-zero lower/upper diagonal entries. This motivates us to extract these  $2\bar{M} - 1$  entries from each  $\bar{\mathbf{B}}_{g,n}$  to construct the vector  $\check{\mathbf{b}}_{g,n} \in \mathbb{R}^{(2\bar{M}-1) \times 1}$ . For ease of illustration, we define the mapping  $\bar{\mathbf{B}}_{g,n} = F_5(\check{\mathbf{b}}_{g,n})$ , where  $F_5(\cdot)$  is given such that

$$[\bar{\mathbf{B}}_{g,n}]_{m,m'} = \begin{cases} [\check{\mathbf{b}}_{g,n}]_m, & m = m' \\ [\check{\mathbf{b}}_{g,n}]_{\bar{M}+m}, & m' = m + 1 \\ [\check{\mathbf{b}}_{g,n}]_{\bar{M}+m'}, & m = m' + 1 \\ 0, & \text{otherwise} \end{cases}. \quad (8.28)$$

According to (8.18a), these vectors  $\check{\mathbf{b}}_{g,n}$  for all subcarriers are jointly controlled by  $\check{\mathbf{b}}_{c,g} \in \mathbb{R}^{(2\bar{M}-1) \times 1}$  constructed with the entries  $B_{c,m_g,m_g}$  and  $B_{c,m_g,m_g+1_g}$ ,  $\forall m \in \bar{\mathcal{M}}$ . Thus, we can transform problem (8.22) into the following form:

$$\max_{\check{\mathbf{b}}_{c,g}, \forall g} \underbrace{\sum_{n \in \mathcal{N}_c} \left| -y_{RT,n} + \sum_{g \in \mathcal{G}} \mathbf{y}_{RI,g,n} (jF_3(F_5(\check{\mathbf{b}}_{g,n})) + Y_0 \mathbf{I}_{\bar{M}})^{-1} \mathbf{y}_{IT,g,n} \right|^2}_{= \check{F}(\check{\mathbf{b}}_c)} \quad (8.29a)$$

$$\text{s.t. } \check{\mathbf{b}}_{g,n} = F_1(\omega_n) \check{\mathbf{b}}_{c,g} + F_2(\omega_n), \forall n, g, \quad (8.29b)$$

$$[\check{\mathbf{b}}_{c,g}]_l \in [B_{\min}, B_{\max}], \forall g, l \in \bar{\mathcal{L}}, \quad (8.29c)$$

where  $\check{\mathbf{b}}_c = [\check{\mathbf{b}}_{c,1}^T, \dots, \check{\mathbf{b}}_{c,G}^T]^T \in \mathbb{C}^{|\bar{\mathcal{M}}|G \times 1}$ , and  $\bar{\mathcal{M}} = \{1, \dots, 2\bar{M} - 1\}$ . The constraint (8.29c) can be further eliminated by the mapping  $[\check{\mathbf{b}}_{c,g}]_l = F_4([\check{\mathbf{x}}_g]_l)$  with  $\check{\mathbf{x}}_g \in \mathbb{R}^{(2\bar{M}-1) \times 1}$ ,  $\forall l \in \bar{\mathcal{M}}$ , such that problem (8.29) can be transformed into an unconstrained optimization

$$\{\check{\mathbf{x}}_g^*\}_{\forall g} = \arg \max_{(8.29b), [\check{\mathbf{b}}_{c,g}]_l = F_4([\check{\mathbf{x}}_g]_l)} \check{F}(\check{\mathbf{b}}_c), \quad (8.30)$$

and solved by the quasi-Newton method.

### 8.3.4 Solution to Problem (8.22): Discrete-Value Admittance

We propose to iteratively optimize the discrete-value tunable admittance components of BD-RIS using the greedy algorithm. The detailed solutions for group- and forest-connected architectures are explained below.

**Group-Connected BD-RIS:** In this case, the sum channel gain maximization problem (8.25) writes as

$$\mathbf{b}_c^* = \arg \max_{(8.25b), [\mathbf{b}_{c,g}]_l \in \mathcal{B}} F(\mathbf{b}_c). \quad (8.31)$$

To efficiently solve the problem, we propose to uniformly block the variable vector  $\mathbf{b}_c$  into  $U$  sub-vectors, i.e.  $\mathbf{b}_c = [\mathbf{b}_1^T, \dots, \mathbf{b}_U^T]^T$ , where  $\mathbf{b}_u = [\mathbf{b}_c]_{(u-1)\bar{U}+1:u\bar{U}} \in \mathbb{C}^{\bar{U} \times 1}$ ,  $\bar{U} = \frac{|\mathcal{M}|G}{U}$ ,  $\forall u \in \mathcal{U} = \{1, \dots, U\}$ . Accordingly, we construct the codebook for each sub-vector  $\mathbf{b}_u$  based on the original constraint  $[\mathbf{b}_{c,g}]_l \in \mathcal{B}_c$ , as

$$\bar{\mathcal{B}} = \{\mathbf{x} \in \mathbb{C}^{\bar{U} \times 1} \mid [\mathbf{x}]_i \in \mathcal{B}_c, \forall i = 1, \dots, \bar{U}\}, \quad (8.32)$$

with  $|\bar{\mathcal{B}}| = 2^{b\bar{U}}$ .

Then, the greedy algorithm is explained as follows. We first define the initialization  $\mathbf{b}_c^0 = [(\mathbf{b}_1^0)^T, \dots, (\mathbf{b}_U^0)^T]^T$ , where each sub-vector  $\mathbf{b}_u^0$  is randomly chosen from the codebook  $\bar{\mathcal{B}}$ . In the successive optimization of  $\mathbf{b}_u^o$ ,  $\forall u \in \mathcal{U}$ , where the superscript denotes the  $o$ -th iteration, we aim to maximize the objective function  $F([\mathbf{b}_1^o]^T, \dots, (\mathbf{b}_{u-1}^o)^T, \mathbf{b}_u^T, (\mathbf{b}_{u+1}^{o-1})^T, \dots, (\mathbf{b}_U^{o-1})^T]^T)$  with  $\mathbf{b}_u$  being the only variable vector to be optimized and other entries fixed. Then we have the following sub-problem:

$$\mathbf{b}_u^o = \arg \max_{(8.25b), \mathbf{b}_u \in \bar{\mathcal{B}}} F([\mathbf{b}_1^o]^T, \dots, (\mathbf{b}_{u-1}^o)^T, \mathbf{b}_u^T, (\mathbf{b}_{u+1}^{o-1})^T, \dots, (\mathbf{b}_U^{o-1})^T]^T), \quad (8.33)$$

which can be efficiently solved by the exhaustive search over the set  $\bar{\mathcal{B}}$  with proper options of  $\bar{U}$  and  $b$ . By iteratively solving problem (8.33) until the convergence of the objective function is achieved, we finally obtain the solution  $\mathbf{b}_c^*$  to problem (8.31).

**Forest-Connected BD-RIS:** The proposed optimization for group-connected BD-RIS with discrete-value admittance components is also readily extended to forest-connected BD-RIS. In this case, the sum channel gain maximization problem (8.29) writes as

$$\check{\mathbf{b}}_c^* = \arg \max_{(8.29b), [\check{\mathbf{b}}_{c,g}]_l \in \check{\mathcal{B}}} \check{F}(\check{\mathbf{b}}_c), \quad (8.34)$$

where each  $\check{\mathbf{b}}_u = [\check{\mathbf{b}}_c]_{(u-1)\check{U}+1:u\check{U}}$ ,  $\check{U} = \frac{|\check{\mathcal{M}}|G}{U}$ ,  $\forall u \in \mathcal{U}$  is iteratively optimized by the exhaustive search over  $\check{\mathcal{B}}$  given by

$$\check{\mathcal{B}} = \{\mathbf{x} \in \mathbb{C}^{\check{U} \times 1} \mid [\mathbf{x}]_i \in \mathcal{B}_c, \forall i = 1, \dots, \check{U}\}, \quad (8.35)$$

such that the  $\check{\mathbf{b}}_c^*$  is obtained by the greedy algorithm.

## 8.4 Performance Evaluation

In this section, we evaluate the performance of the BD-RIS-aided SISO-OFDM system to show the impact of the wideband modeling for BD-RIS with different architectures.

In the considered SISO-OFDM system, we assume the center frequency as  $f_c = 2.4$  GHz, the bandwidth as  $B = 300$  MHz, and the number of subcarriers as  $N_c = 64$ . Therefore, the carrier frequencies are given by  $f_n = f_c + (n - \frac{N_c+1}{2})\frac{B}{N_c}$ ,  $\forall n \in \mathcal{N}_c$ . The OFDM channels from the transmitter to the receiver, from the transmitter to BD-RIS, and from BD-RIS to the receiver, are modeled with the maximum delay spread of  $D_{\text{RT}} = D_{\text{RI}} = D_{\text{IT}} = 16$  taps using CSCG random variables. The length of the CP is set to be 16. The distance-dependent channel pathloss is modeled as  $\zeta_s = \zeta_0 d_s^{-\varepsilon_s}$ ,  $\forall s \in \{\text{RT}, \text{RI}, \text{IT}\}$ , where  $\zeta_0 = -30$  dB refers to the signal attenuation at reference distance 1 m,  $d_s$  denotes the distance between transmitter/BD-RIS/receiver, and  $\varepsilon_s$  denotes the pathloss exponent. In the following simulations, we set  $d_{\text{RT}} = 33$  m,  $d_{\text{RI}} = 5$  m,  $d_{\text{IT}} = 30$  m,  $\varepsilon_{\text{RT}} = 3.8$ ,  $\varepsilon_{\text{RI}} = 2.2$ , and  $\varepsilon_{\text{IT}} = 2.5$ . The noise power is set as  $\sigma^2 = -80$  dBm.

**Benchmark Scheme 1: Frequency-Independent BD-RIS.** To show how the wideband modeling of BD-RIS impacts the system performance, we include the case where the BD-RIS admittance matrices are designed such that they remain the same for all subcarriers. That is, we assume  $\mathbf{Y}_{\text{I},n} = \mathbf{Y}_{\text{I}}$ ,  $\forall n \in \mathcal{N}_c$  as in [65, 66] where the narrowband BD-RIS model is adopted in wideband scenarios. In other words, we have

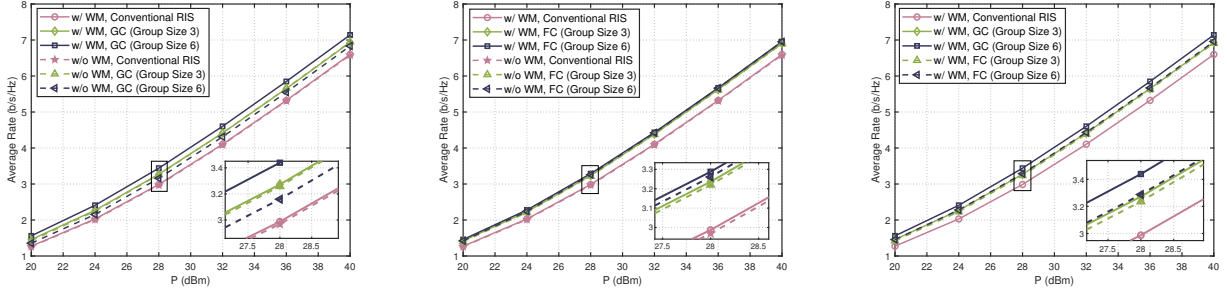
$$\bar{h}_n = \frac{1}{2Y_0}(-y_{\text{RT},n} + \mathbf{y}_{\text{RI},n}(\mathbf{Y}_{\text{I}} + Y_0\mathbf{I}_M)^{-1}\mathbf{y}_{\text{IT},n}), \forall n. \quad (8.36)$$

Then the proposed algorithms for either continuous or discrete-value admittance components can also be adopted to solve the following problem:

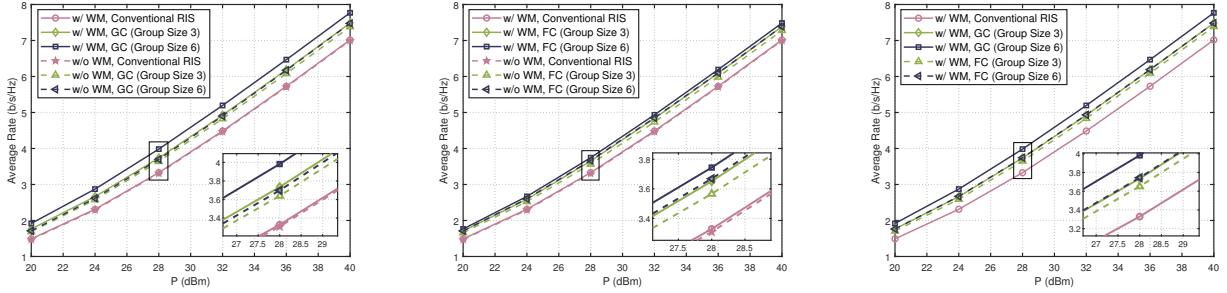
$$\{\{Y_{m_g, m'_g}^\# \}_{\forall m, m', g}, \{p_n^\# \}_{\forall n}\} = \arg \max_{\substack{(8.8)-(8.11), \sum_{n \in \mathcal{N}_c} p_n \leq P \\ \Im\{Y_{m_g, m'_g}\} \in [B_{\min}, B_{\max}]}} \frac{1}{N_c} \sum_{n \in \mathcal{N}_c} \log_2 \left( 1 + \frac{p_n |\bar{h}_n|^2}{\sigma^2} \right), \quad (8.37)$$

where constraints (8.8)-(8.10) refer to the group-connected architecture and (8.8)-(8.11) refer to the forest-connected architecture. The obtained solutions  $Y_{m_g, m'_g}^\#$ ,  $\forall m, m' \in \bar{\mathcal{M}}, \forall g \in \mathcal{G}$  are further plugged into (8.16d) or (8.18a), such that we can obtain

- 1) the admittance matrices  $\mathbf{Y}_{\text{I},n}^\#$ ,  $\forall n \in \mathcal{N}_c$  by (8.8)-(8.11);
- 2) the channels  $h_n^\#$ ,  $\forall n \in \mathcal{N}_c$  by (8.19),



(a) Group-connected BD-RIS with (b) Forest-connected BD-RIS with (c) Group- and forest-connected and without considering wideband and without considering wideband BD-RIS with wideband modeling ( $M = 36$ )



(d) Group-connected BD-RIS with (e) Forest-connected BD-RIS with (f) Group- and forest-connected BD-RIS with wideband modeling ( $M = 48$ )

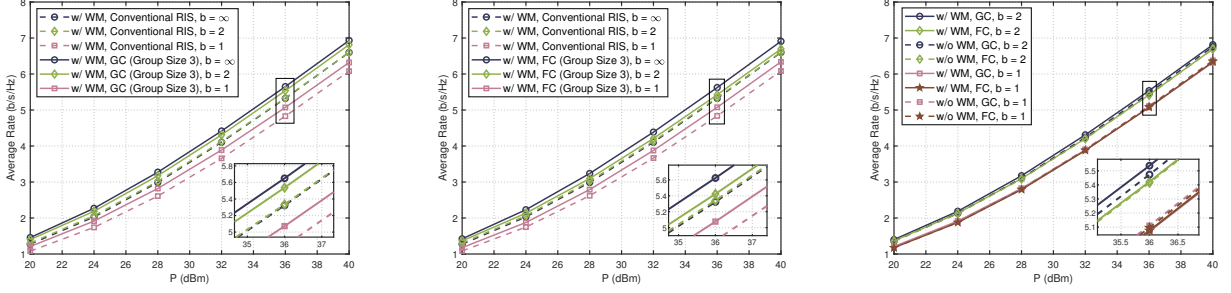
Figure 8.5: Average rate versus transmit power  $P$  with BD-RIS having different architectures ( $M \in \{36, 48\}$ ,  $\bar{M} \in \{1, 3, 6\}$ ). The legend “WM” is short for wideband modeling; “GC” is short for group-connected; “FC” is short for forest-connected. For both “GC” and “FC” architectures, the case of  $\bar{M} = 1$  refers to the conventional RIS.

together with  $p_n^\sharp$ ,  $\forall n \in \mathcal{N}_c$  to be plugged into the original objective function (8.21) for fair performance comparisons.

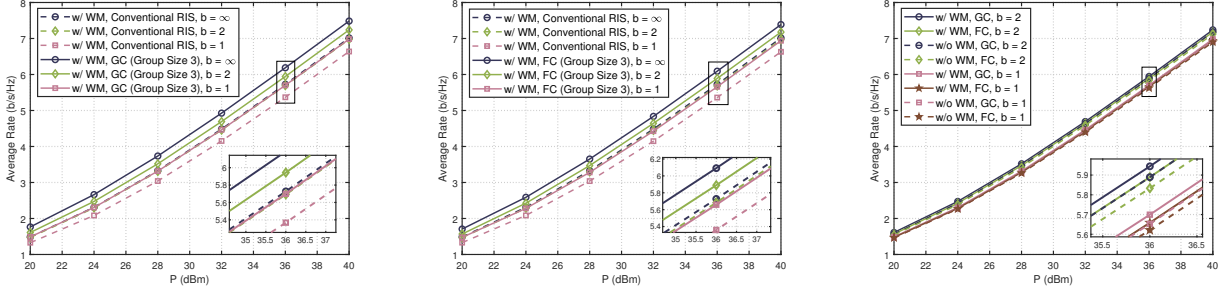
**Benchmark Scheme 2: Conventional RIS.** To show how the performance of BD-RIS compares to that of conventional RIS in the presence or absence of wideband modeling, we include the case where the BD-RIS has diagonal admittance matrices. In other words, the conventional RIS is a special case of the group or forest-connected BD-RIS with  $\bar{M} = 1$  such that the proposed methods can be directly implemented.

### 8.4.1 Continuous-Value Admittance

We evaluate the performance of the proposed wideband modeling and algorithm design by plotting the average rate of the BD-RIS-aided SISO-OFDM system versus the transmit power  $P$  in Fig. 8.5, from which we have the following observations. *First*, when taking into account the wideband modeling, BD-RIS with larger group size of the group-connected architecture achieves better performance within all the considered range of transmit power. This is attributed to



(a) Group-connected BD-RIS with (b) Forest-connected BD-RIS with (c) BD-RIS with and without con-  
discrete-value admittance compo- discrete-value admittance compo- sidering wideband modeling ( $M =$   
nents ( $M = 36$ ) nents ( $M = 36$ )  $36, \bar{M} = 3$ )



(d) Group-connected BD-RIS with (e) Forest-connected BD-RIS with (f) BD-RIS with and without consid-  
discrete-value admittance compo- discrete-value admittance compo- ering wideband modeling ( $M = 48,$   
nents ( $M = 48$ ) nents ( $M = 48$ )  $\bar{M} = 3$ )

Figure 8.6: Average rate versus transmit power  $P$  with BD-RIS having different architectures and discrete-value admittance components ( $M \in \{36, 48\}$ ,  $\bar{M} \in \{1, 3\}$ ,  $b \in \{1, 2\}$ .  $\bar{U} = 4$  for  $b = 1$ ;  $\bar{U} = 3$  for  $b = 2$ ). The legend “WM” is short for wideband modeling; “GC” is short for group-connected; “FC” is short for forest-connected. For both “GC” and “FC” architectures, the case of  $\bar{M} = 1$  refers to the conventional RIS.

the increasing wave manipulation capabilities offered by more complex circuit topologies of BD-RIS. When ignoring the wideband effect for designing group-connected BD-RIS, BD-RIS architectures with larger group size cannot always provide better performance. For example, BD-RIS with group size  $\bar{M} = 3$  outperforms that with  $\bar{M} = 6$  when  $M = 36$ . This is because the variation for BD-RIS matrices between different subcarriers becomes more significant with increasing number of admittances, such that directly designing BD-RIS ignoring this variation will lead to increasing performance loss. *Second*, for the cases of both with or without wideband modeling, BD-RIS with larger group size of the forest-connected architecture achieves better performance within all the considered range of transmit power, while the performance enhancement is not as significant as the group-connected architecture due to the reduced number of tunable admittance components. This also demonstrates that the impact of wideband modeling for BD-RIS to the system performance is proportional to the circuit complexity of BD-RIS architectures: The higher the circuit complexity of BD-RIS architectures, the larger the performance gap between schemes with and without including wideband modeling of BD-RIS. *Third*, when taking into consideration the wideband modeling of BD-RIS, BD-RIS having group-connected architectures outperform that having forest-connected architectures with the

same group size. While group- and forest-connected architectures achieve the same performance in narrowband point-to-point systems [57], wideband OFDM systems benefit from more tunable admittance components in BD-RIS to deal with the frequency selectivity.

### 8.4.2 Discrete-Value Admittance

In Fig. 8.6, we plot the average rate of the BD-RIS-aided SISO-OFDM system versus the transmit power when the BD-RIS has discrete-value admittance components controlled by  $b$  bits. From Fig. 8.6, we have the following observations. *First*, when taking into account the wideband modeling, for both group-connected and forest-connected BD-RIS, increasing the number of resolution bits for each discrete-value admittance leads to better performance. In addition, BD-RIS with 2-bit resolution for each discrete-value admittance can achieve satisfactory performance close to that with continuous-value admittance components. *Second*, similar to the continuous-value admittance cases, in discrete-value cases, there also exists performance gap between including or ignoring the wideband modeling of BD-RIS in the optimization stage. More importantly, with relatively small numbers of resolution bits  $b$  and small group size  $\bar{M}$  of BD-RIS architectures, the performance loss coming from ignoring the wideband modeling is negligible. This observation indicates that it is efficient to directly adopt the narrowband BD-RIS model in wideband scenarios when using simple BD-RIS architectures with limited freedom in tuning admittance components.

## 8.5 Summary

This work studies the wideband modeling and optimization of BD-RIS in a SISO-OFDM system. Particularly, we first derive the system and signal model for BD-RIS-aided OFDM systems, which bridges the time and frequency-domain channels and shows explicitly the frequency dependency of the scattering matrix for BD-RIS. We next characterize this frequency dependency by modeling each tunable admittance of BD-RIS based on lumped circuits. Taking into account the practical frequency bands, we propose a simple linear model with respect to frequency.

With the proposed BD-RIS model and BD-RIS-aided signal model, we consider the optimization of a BD-RIS-aided SISO-OFDM system with the aim of maximizing the average rate. For BD-RIS with continuous-value admittance components, we propose to transform the original problem into an unconstrained optimization problem to be solved by the well-known quasi-Newton method. For BD-RIS with discrete-value admittance components, we adopt the idea of the greedy algorithm to successively design each tunable admittance.

Finally, we provide simulation results to evaluate the effectiveness of the proposed algorithms and the impact of the wideband modeling. Results show that BD-RIS always outperforms

conventional RIS whether the wideband modeling is taken into account during the optimization process or not. In addition, the higher the circuit complexity of BD-RIS, the larger the impact of the wideband modeling on the system performance. This demonstrates the importance of accurately capturing the frequency-dependent response of BD-RIS with complex architectures in wideband communication systems.

# Chapter 9

## Conclusion

This chapter summarizes the main achievements of this thesis, and sheds light on various future directions based on the current results.

### 9.1 Summary of Thesis Achievements

This thesis solved three key open problems of BD-RIS, that is, novel architecture design and mode analysis of BD-RIS to achieve enhanced channel gain and enlarged coverage; signal processing techniques of BD-RIS considering different CSI conditions; and the modeling and analysis of BD-RIS hardware impairments.

Part I focused on the architecture design and mode analysis of BD-RIS by proposing two novel modes, namely hybrid mode, and multi-sector mode. Specifically, Chapter 3 started by presenting a general BD-RIS aided communication model unifying reflective, transmissive, and hybrid modes, and single/group/fully/dynamically group-connected architectures, which include conventional diagonal RIS and STAR-RIS as special cases. To evaluate the performance of the proposed model, the joint precoder and BD-RIS design algorithm has been developed. Simulation results showed the significant sum-rate gains achieved by the proposed BD-RIS with hybrid mode and group/fully/dynamically group-connected architectures compared to conventional diagonal RIS and STAR-RIS. Then, Chapter 4 went beyond BD-RIS with hybrid mode and proposed a novel multi-sector mode by fully exploiting the flexibility of antenna arrangements and circuit topology designs. It has been proven that BD-RIS with hybrid mode was essentially a special case of BD-RIS with multi-sector mode, while the latter generally outperforms the former, benefiting from antennas with higher gains. Theoretical analysis of the power scaling law was provided and beamforming design algorithms for sum-rate maximization were developed. Simulation results verified the theoretical derivations and, more importantly,

highlighted the benefit of multi-sector BD-RIS in enhancing channel gains and reducing the circuit complexity.

Part II focused on the beamforming and channel estimation techniques for BD-RIS. Specifically, Chapter 5 studied the synergy between multi-sector BD-RIS and RSMA under imperfect CSI conditions. A stochastic problem was first formulated, which was transformed to a deterministic problem and solved by iterative solutions. Simulation results showed the benefits of this synergy in saving on the required number of transmitter and BD-RIS antennas. Furthermore, Chapter 6 proposed a novel cascaded channel estimation scheme for BD-RIS and corresponding beamforming design algorithms based on cascaded channel estimates. The proposed channel estimation scheme was developed based on a closed-form solution, which theoretically guaranteed to achieve the MSE lower-bound of the LS estimation. With the cascaded channel estimate, the corresponding beamforming design algorithms were proposed considering different BD-RIS architectures and modes. Simulation results highlighted the performance trade-off between channel estimation and data transmission within one transmission frame.

Part III focused on the modeling and analysis of the hardware impairments of BD-RIS by focusing on the impact of mutual coupling and that of frequency dependence at BD-RIS. Specifically, Chapter 7 modeled the mutual coupling at BD-RIS based on the dipole antennas, followed by the efficient beamforming design. Simulation results demonstrated the importance of taking into account the mutual coupling at BD-RIS in the optimization. Chapter 8 modeled the frequency dependence at BD-RIS based on the lumped circuit model of each tunable admittance. The beamforming design for BD-RIS aided OFDM systems was then developed. Simulation results, again, emphasized the importance of accurately capturing the frequency dependence of BD-RIS architectures.

## 9.2 Future Work

In conclusion of this thesis, some future research directions are proposed.

**Dimensioning of BD-RIS Modes and Architectures:** The size, the number of antenna elements, and the number of sectors of BD-RIS with hybrid/multi-sector modes are jointly determined by the operating frequency of incident waves, the performance requirement, the circuit complexity/cost. Therefore, one meaningful future direction is to carefully determine the dimension of BD-RIS to balance performance in different scenarios, the hardware complexity, and the cost.

**Hardware Implementation of BD-RIS:** The hardware implementation of BD-RIS is a fundamental issue. Currently, only RIS with STAR-RIS/IOS [76] has been implemented, which indicates the physical availability of modeling RIS as antenna arrays connected to reconfigurable impedance network, while the comprehensive hardware implementation of BD-RIS with

different architectures is still on its way. As shown in previous chapters, the reconfigurable impedance network is the key to implement the BD-RIS with different modes and architectures. We can utilize tunable inductance and capacitance, e.g. varactors, to construct the reconfigurable impedance network, so that the continuous value BD-RIS can be implemented. Alternatively, we can use PIN diodes as switches to reconfigure the impedance network to implement discrete value BD-RIS. However, as the group size increases, the circuit complexity and cost also increase, which is unique in BD-RIS due to the inter-element connections that conventional diagonal RIS does not have. Hence, it is challenging but worthwhile to implement and prototype different BD-RIS architectures, taking into account practical hardware impairments, to find the best performance-complexity trade-off.

**BD-RIS Based Beam Training:** Recall that the performance benefits of BD-RIS heavily depend on accurate CSI, which requires appropriate and effective channel estimation strategies. In this thesis, we have developed a novel channel estimation scheme adapting to BD-RIS architectures, which could obtain the cascaded channel for further beamforming design. Instead of acquiring the CSI and designing the beamforming, another direction is to perform BD-RIS based beam training, i.e, to “train” the beam formed by BD-RIS to align with the unknown wireless channel. As such, by letting the transmitter consecutively transmit pilot signals with pre-defined codebooks at BD-RIS, the receiver could obtain the best codeword at BD-RIS which leads to the highest received power without knowing the CSI. Therefore, the requirement of accurate CSI is eased. To facilitate the BD-RIS based beam training, the key open problem is to properly design the codebook for BD-RIS architectures, consisting of codewords which could effectively cover the whole beam space.

**Applications of BD-RIS:** One promising application of BD-RIS is to deploy it in the power grid to relay wireless power. With proper power levels, suitable deployments and locations of BD-RIS, the BD-RIS could effectively aid the wireless power transfer, guaranteeing that the power can be transferred to all receivers. In this case, the transmit waveform and the BD-RIS should be carefully designed. Another interesting application of BD-RIS is to enable flexible and scalable integrated access and backhaul (IAB) [114]. Specifically, the wireless backhauling usually have complicated propagation environments and various obstacles, e.g. trees and high buildings. BD-RIS with full space coverage and high gain performance can be easily incorporated into real environments to bypass the obstacles and assist/enhance the wireless backhaul. Meanwhile, wireless access, especially in millimeter wave or Terahertz wireless frequencies, usually has sparse and highly-directional channels, suffers from high path loss, and is vulnerable to blockages. In this case, BD-RIS is more appealing in providing highly-directional beams to align with low-rank channels, compensate for the severe path loss, and enlarge coverage.



# Appendix A

## Proofs for Chapter 3

This chapter gives the proof for Proposition 3.1. To do so, we first abstract objective in (3.42) as a real-value function

$$\bar{F}(x) = ax^2 + bx + c\sqrt{1-x^2}, \quad (\text{A.1})$$

with one-to-one correlations, i.e.,  $x = \sqrt{\alpha_{t,m}} \in (0, 1)$ ,  $a = v_m \in \mathbb{R}$ ,  $b = -2|\chi_{t,m}| < 0$ ,  $c = -2|\chi_{r,m}| < 0$ . Then the proof is given as follows:

*Proof.* Calculate first- and second-order derivatives of  $\bar{F}(x)$  as

$$\nabla \bar{F}(x) = 2ax + b - \frac{cx}{\sqrt{1-x^2}}, \quad (\text{A.2a})$$

$$\nabla^2 \bar{F}(x) = 2a - \frac{c}{(1-x^2)^{\frac{3}{2}}}. \quad (\text{A.2b})$$

Then we discuss  $\nabla \bar{F}(x)$  under the following two conditions:

Case 1: When  $a \geq 0$ , it is obvious that  $\nabla^2 \bar{F}(x) > 0$  so that  $\nabla \bar{F}(x)$  is a monotonically increasing function with  $x \in (0, 1)$ . Based on the above discussion, we have

$$\lim_{x \rightarrow 0} \nabla \bar{F}(x) = b < 0, \quad \lim_{x \rightarrow 1} \nabla \bar{F}(x) = +\infty > 0, \quad (\text{A.3})$$

which indicate that  $\bar{F}(x)$  is first monotonically decreasing and then monotonically increasing for  $x \in (0, 1)$ .

Case 2: When  $a < 0$ , calculate the third-order derivative of  $\bar{F}(x)$  as

$$\nabla^3 \bar{F}(x) = -3cx(1-x^2)^{-\frac{5}{2}}, \quad (\text{A.4})$$

which is always larger than zero when  $x \in (0, 1)$ . Thus,  $\nabla^2 \bar{F}(x)$  is monotonically

increasing with  $x \in (0, 1)$ . Then we calculate

$$\lim_{x \rightarrow 0} \nabla^2 \bar{F}(x) = 2a - c, \quad (\text{A.5})$$

whose value has the following two cases:

Case 2.1: When  $2a - c \geq 0$ ,  $\nabla^2 \bar{F}(x) > 0$  for  $x \in (0, 1)$ . Thus,  $\nabla \bar{F}(x)$  is monotonically increasing for  $x \in (0, 1)$  and the same conclusion as Case 1 can be obtained.

Case 2.2: When  $2a - c < 0$ , examine

$$\lim_{x \rightarrow 1} \nabla^2 \bar{F}(x) = +\infty > 0, \quad (\text{A.6})$$

which proves that  $\nabla \bar{F}(x)$  is first monotonically decreasing and then increasing for  $x \in (0, 1)$ . Therefore, we can derive that  $\bar{F}(x)$  has the same trend as  $\nabla \bar{F}(x)$  for  $x \in (0, 1)$ .

To summarize,  $\bar{F}(x)$  is a convex function with only one minimum point for  $x \in (0, 1)$ , which completes the proof.  $\square$

# Appendix B

## Proofs for Chapter 4

### B.1 Proof of Lemma 4.1

*Proof.* By applying the Lagrange method, we can easily obtain  $(\Psi_m + \mu \mathbf{I}_L)\beta_m = \chi_m$  with  $\beta_m^T \beta_m = 1$ , where  $\mu \in \mathbb{R}$  is the Lagrange multiplier. Now it remains to show  $\Psi_m + \mu \mathbf{I}_L$  is positive semi-definite. Suppose  $\beta'_m$  is a feasible solution of problem (4.43) with  $\beta_m'^T \beta'_m = 1$ . Then we have  $F(\beta'_m) \geq F(\beta_m^*)$ . Given that  $(\Psi_m + \mu \mathbf{I}_L)\beta_m^* = \chi_m$ , we have

$$\beta_m'^T \Psi_m \beta'_m - 2\beta_m'^T (\Psi_m + \mu \mathbf{I}_L) \beta'_m \geq \beta_m'^T \Psi_m \beta_m^* - 2\beta_m'^T (\Psi_m + \mu \mathbf{I}_L) \beta_m^*. \quad (\text{B.1})$$

Rearranging (B.1), we have

$$(\beta_m'^T - \beta_m^{*T})(\Psi_m + \mu \mathbf{I}_L)(\beta'_m - \beta_m^*) \geq \mu(\beta_m'^T \beta'_m - \beta_m^{*T} \beta_m^*) = 0, \quad (\text{B.2})$$

which completes the proof.  $\square$

### B.2 Proof of Lemma 4.2

*Proof.* With  $\varepsilon_{m,l} + \mu \geq \varepsilon_{m,1} + \mu > 0$ , we have

$$1 = F'(\mu) = \sum_{l \in \mathcal{L}} \frac{\tilde{\chi}_{m,l}^2}{(\varepsilon_{m,l} + \mu)^2} \leq \sum_{l \in \mathcal{L}} \frac{\tilde{\chi}_{m,l}^2}{(\varepsilon_{m,1} + \mu)^2} = \frac{\|\chi_m\|_2^2}{(\varepsilon_{m,1} + \mu)^2}, \quad (\text{B.3})$$

which yields  $\mu \leq \|\chi_m\|_2 - \varepsilon_{m,1}$ . Meanwhile, we have

$$1 = F'(\mu) = \sum_{l \in \mathcal{L}} \frac{\tilde{\chi}_{m,l}^2}{(\varepsilon_{m,l} + \mu)^2} \geq \sum_{l \in \mathcal{E}} \frac{\tilde{\chi}_{m,l}^2}{(\varepsilon_{m,l} + \mu)^2} = \frac{\sum_{l \in \mathcal{E}} \tilde{\chi}_{m,l}^2}{(\varepsilon_{m,1} + \mu)^2}, \quad (\text{B.4})$$

and thus  $\mu \geq \sqrt{\sum_{l \in \mathcal{E}} \tilde{\chi}_{m,l}^2} - \varepsilon_{m,1}$ .  $\square$



# Appendix C

## Proofs for Chapter 6

### C.1 Proof of Lemma 6.1

*Proof.* The objective (6.13a) has the following lower bound

$$\text{Tr}((\hat{\Phi}^H \hat{\Phi})^{-1}) = \sum_{i=1}^{NK\bar{M}^2 G_2} [(\hat{\Phi}^H \hat{\Phi})^{-1}]_{i,i} \quad (\text{C.1a})$$

$$\stackrel{(a)}{\geq} \sum_{i=1}^{NK\bar{M}^2 G_2} [\hat{\Phi}^H \hat{\Phi}]_{i,i}^{-1} \quad (\text{C.1b})$$

$$\stackrel{(b)}{\geq} \frac{N^2 K^2 G_2^2 \bar{M}^4}{\sum_{i=1}^{NK\bar{M}^2 G_2} [\hat{\Phi}^H \hat{\Phi}]_{i,i}} = \frac{N^2 K^2 G_2^2 \bar{M}^4}{\text{Tr}(\hat{\Phi}^H \hat{\Phi})} \quad (\text{C.1c})$$

$$= \frac{N^2 K^2 G_2^2 \bar{M}^4}{\text{Tr}(\sum_{t=1}^{K\bar{M}^2 G_2} (\bar{\phi}_t^* \otimes \bar{\mathbf{x}}_t^* \otimes \mathbf{I}_N)(\bar{\phi}_t^T \otimes \bar{\mathbf{x}}_t^T \otimes \mathbf{I}_N))} \quad (\text{C.1d})$$

$$\stackrel{(c)}{=} \frac{N^2 K^2 G_2^2 \bar{M}^4}{\text{Tr}(\sum_{t=1}^{K\bar{M}^2 G_2} (\bar{\phi}_t^* \bar{\phi}_t^T) \otimes (\bar{\mathbf{x}}_t^* \bar{\mathbf{x}}_t^T) \otimes \mathbf{I}_N)} \quad (\text{C.1e})$$

$$\stackrel{(d)}{=} \frac{NK G_2^2 \bar{M}^4}{\sum_{t=1}^{K\bar{M}^2 G_2} \text{Tr}(\bar{\phi}_t^* \bar{\phi}_t^T)} \stackrel{(e)}{=} N\bar{M}, \quad (\text{C.1f})$$

where the equality of (a) can be attained when  $\hat{\Phi}^H \hat{\Phi}$  is a diagonal matrix [115]; (b) holds due to the relationship between the harmonic mean and the arithmetic mean with equality achieved when  $\hat{\Phi}^H \hat{\Phi}$  has identical diagonal entries; (c) holds by the property of the Kronecker product; (d) holds due to the property of the trace operation and the assumption that  $|\mathbf{x}_t[k]| = 1, \forall k$ , yielding  $\text{Tr}(\mathbf{x}_t^* \mathbf{x}_t^T) = K, \forall t$ ; (e) holds due to the constraint (6.13b) which yields  $\text{Tr}(\bar{\phi}_t^* \bar{\phi}_t^T) = G_2 \bar{M}, \forall t$ . Based on the above derivations, we have that  $\min \text{Tr}((\hat{\Phi}^H \hat{\Phi})^{-1}) = N\bar{M}$  with the condition  $\hat{\Phi}^H \hat{\Phi} = \hat{\Phi} \hat{\Phi}^H = K\bar{M}G_2 \mathbf{I}_{NK\bar{M}^2 G_2}$ , which completes the proof.  $\square$

## C.2 Proof of Lemma 6.2

*Proof.* With  $\mathbf{A}$  satisfying (6.18b), (6.18c) and  $\check{\Phi}$  satisfying (6.19b), (6.19c), we have  $\check{\Phi} = \mathbf{A} \otimes \check{\Phi}$  such that

$$\begin{aligned}\check{\Phi}^H \check{\Phi} &= (\mathbf{A}^H \otimes \check{\Phi}^H)(\mathbf{A} \otimes \check{\Phi}) = (\mathbf{A}^H \mathbf{A}) \otimes (\check{\Phi}^H \check{\Phi}) \\ &= (G_2 \mathbf{I}_{G_2}) \otimes (\bar{M} \mathbf{I}_{\bar{M}^2}) = G_2 \bar{M} \mathbf{I}_{G_2 \bar{M}^2},\end{aligned}\tag{C.2}$$

which aligns with (6.14b).

In addition, we define the  $i$ -th sub-block rows of the  $s$ -th row of  $\check{\Phi}$  by  $\phi_{s,i}$ ,  $\forall s \in \mathcal{S}$ ,  $\forall i \in \mathcal{G}_2$ , that is

$$\phi_{s,i} = [\check{\Phi}]_{s,(i-1)\bar{M}^2:i\bar{M}^2} = [\phi_s^T]_{(i-1)\bar{M}^2:i\bar{M}^2}.\tag{C.3}$$

Meanwhile, from (6.15) we have

$$\phi_{s,i} = [\bar{\phi}_t^T]_{(i-1)\bar{M}^2:i\bar{M}^2} = \text{vec}^T(\Phi_{t,i}),\tag{C.4}$$

where  $t = (s-1)K + k$ ,  $\forall k \in \mathcal{K}$ . Furthermore, from Lemma 2 we have  $\phi_{s,i} = [\mathbf{A}]_{i',i} [\check{\Phi}]_{m,:}$ , where  $s = (i'-1)\bar{M}^2 + m$ ,  $\forall m \in \bar{\mathcal{M}}$ . Hence, (6.13b) is satisfied, i.e.,

$$\Phi_{t,i}^H \Phi_{t,i} = |[\mathbf{A}]_{i',i}|^2 \check{\Phi}_m^H \check{\Phi}_m = \mathbf{I}_{\bar{M}},\tag{C.5}$$

where  $\check{\Phi}_m = \text{unvec}([\check{\Phi}^T]_{:,m})$ . The proof is completed.  $\square$

## C.3 Proof of Theorem 6.1

*Proof.* We start by proving  $\check{\phi}_{m,n}$ ,  $\forall m, n \in \bar{\mathcal{M}}$  satisfies (6.19c). To this end, we define  $\check{\phi}_{m,n,i} = [\check{\phi}_{m,n}]_{(i-1)\bar{M}+1:i\bar{M}}$ ,  $\forall i \in \bar{\mathcal{M}}$ . We have

$$\check{\phi}_{m,n,i} = [\mathbf{Z}_2]_{m,i} [\mathbf{Z}_1^T]_{\text{mod}(i-n,\bar{M})+1,:}.\tag{C.6}$$

Then we calculate

$$\check{\phi}_{m,n,i}^* \check{\phi}_{m,n,i'}^T = \underbrace{[\mathbf{Z}_1^H]_{\text{mod}(i-n,\bar{M})+1,:} [\mathbf{Z}_1]_{:, \text{mod}(i'-n,\bar{M})+1}}_{=z_{1,i,i',n}} \underbrace{[\mathbf{Z}_2]_{m,i}^* [\mathbf{Z}_2]_{m,i'}}_{=z_{2,i,i',m}},\tag{C.7}$$

yielding the following two conditions:

- 1) when  $i = i'$ , we have  $z_{1,i,i,n} = \alpha_1$  and  $z_{2,i,i,m} = \frac{\alpha_2}{\bar{M}}$  such that  $\check{\phi}_{m,n,i}^* \check{\phi}_{m,n,i}^T = 1$ ;
- 2) when  $i \neq i'$ , we have  $z_{1,i,i',n} = 0$  such that  $\check{\phi}_{m,n,i}^* \check{\phi}_{m,n,i'}^T = 0$ .

Therefore, (6.19c) is guaranteed.

We next prove  $\check{\Phi}$  constructed by  $\check{\phi}_{m,n}$  satisfies (6.19b). To this end, we calculate

$$\check{\phi}_{m,n}^* \check{\phi}_{m',n'}^T = \sum_{i \in \mathcal{M}} \check{\phi}_{m,n,i}^* \check{\phi}_{m',n',i}^T = \sum_{i \in \mathcal{M}} \underbrace{[\mathbf{Z}_1^H]_{\text{mod}(i-n, \bar{M})+1, :} [\mathbf{Z}_1]_{:, \text{mod}(i-n', \bar{M})+1}}_{=z_{1',n,n',i}} \underbrace{[\mathbf{Z}_2]_{m,i}^* [\mathbf{Z}_2]_{m',i}}_{=z_{2',m,m',i}}, \quad (\text{C.8})$$

yielding the following three conditions:

- 1) when  $m = m'$  and  $n = n'$ , we have  $z_{1',n,n,i} = \alpha_1$  and  $z_{2',m,m,i} = \frac{\alpha_2}{\bar{M}}$  such that  $\check{\phi}_{m,n}^* \check{\phi}_{m,n}^T = \bar{M}$ ;
- 2) when  $n \neq n'$ , we have  $z_{1',n,n',i} = 0$  such that  $\check{\phi}_{m,n}^* \check{\phi}_{m',n'}^T = 0$ ;
- 3) when  $m \neq m'$  and  $n = n'$ , we have  $z_{1',n,n,i} = \alpha_1$  such that  $\check{\phi}_{m,n}^* \check{\phi}_{m',n'}^T = \alpha_1 \sum_{i \in \mathcal{M}} z_{2',m,m',i} = \alpha_1 [\mathbf{Z}_2^*]_{m,:} [\mathbf{Z}_2^T]_{:,m'} = 0$ .

Therefore, (6.19b) is guaranteed. The proof is completed.  $\square$



# Appendix D

## Proofs for Chapter 7

This chapter gives the proof for Proposition 7.1.

*Proof.* To check the convergence of the proposed algorithm, we have the following relationship of the objective function in (7.16) between iterations:

$$C^o = |a^o + \mathbf{b}^o j \Im\{\mathbf{\Omega}^o\} \mathbf{c}^o|^2 \quad (\text{D.1a})$$

$$= |a^o + j \sum_{g \in \mathcal{G}} \mathbf{e}_g^o \mathbf{R} \Im\{\boldsymbol{\omega}_g\}|^2 \quad (\text{D.1b})$$

$$\stackrel{(a)}{=} |j\delta \sum_g \sum_i |[\mathbf{e}_g^o \mathbf{R}]_i| \sin(\angle a^o - \angle[\mathbf{e}_g^o \mathbf{R}]_i) \cos(\angle a^o - \angle[\mathbf{e}_g^o \mathbf{R}]_i) + |a^o| + \delta \sum_g \sum_i |[\mathbf{e}_g^o \mathbf{R}]_i| \sin^2(\angle a^o - \angle[\mathbf{e}_g^o \mathbf{R}]_i)|^2 \quad (\text{D.1c})$$

$$\geq |a^o|^2 \quad (\text{D.1d})$$

$$= |z_{\text{RT}} - \mathbf{z}_{\text{RI}}(\mathbf{Z}_{\text{II}} + \mathbf{Z}_{\text{I}}^o)^{-1} \mathbf{z}_{\text{IT}}|^2 \quad (\text{D.1e})$$

$$= |z_{\text{RT}} - \mathbf{z}_{\text{RI}}(\mathbf{Z}_{\text{II}} + \mathbf{Z}_{\text{I}}^{o-1} + j \Im\{\mathbf{\Omega}^{o-1}\})^{-1} \mathbf{z}_{\text{IT}}|^2 \quad (\text{D.1f})$$

$$\stackrel{(b)}{\approx} |a^{o-1} + \mathbf{b}^{o-1} j \Im\{\mathbf{\Omega}^{o-1}\} \mathbf{c}^{o-1}|^2 = C^{o-1}, \quad (\text{D.1g})$$

where (a) follows by plugging (7.18) into  $C^o$ ; (b) follows from the Neumann series approximation. We have  $C^o \geq |a^o|^2 \approx C^{o-1}$ , which proves that the objective function  $C^o$  is monotonically non-decreasing after each iteration. Therefore, we conclude that the proposed algorithm converges since the value of the objective function  $C^o$  is bounded from above. The proof is completed.  $\square$



# Appendix E

## Proofs for Chapter 8

This chapter gives the proof for Proposition 8.1.

*Proof.* We first rewrite the frequency-domain channel  $\mathbf{H}_{\text{all}}$  by re-arranging the block-circulant matrices. This yields

$$\begin{aligned}\mathbf{H}_{\text{all}} &= \bar{\mathbf{F}}_{N_c}(\bar{\mathbf{H}}_{\text{d}} + \bar{\mathbf{H}}\bar{\mathbf{\Phi}}\bar{\mathbf{G}})\bar{\mathbf{F}}_{N_c}^H \\ &\stackrel{(a)}{=} \bar{\mathbf{F}}_{N_c}(\bar{\mathbf{H}}_{\text{d}} + \bar{\mathbf{H}}\mathbf{\Gamma}\mathbf{\Gamma}^T\bar{\mathbf{\Phi}}\mathbf{\Gamma}\mathbf{\Gamma}^T\bar{\mathbf{G}})\bar{\mathbf{F}}_{N_c}^H,\end{aligned}\tag{E.1}$$

where (a) holds by introducing the permutation matrix  $\mathbf{\Gamma} \in \{0, 1\}^{N_c M \times N_c M}$  with  $\mathbf{\Gamma}\mathbf{\Gamma}^T = \mathbf{I}_{N_c M}$ , which transforms a block-circulant matrix into a row-wise concatenation of circulant matrices [116]. Specifically, we have

$$[\mathbf{\Gamma}]_{i,j} = \begin{cases} 1, & i = (n-1)M + m, j = (m-1)N_c + n, \\ 0, & \text{otherwise} \end{cases},\tag{E.2}$$

with  $\forall n \in \mathcal{N}_c$  and  $\forall m \in \mathcal{M}$ , such that

$$\bar{\mathbf{H}}\mathbf{\Gamma} = [\bar{\mathbf{H}}_1, \dots, \bar{\mathbf{H}}_M], \quad \mathbf{\Gamma}^T\bar{\mathbf{G}} = [\bar{\mathbf{G}}_1^T, \dots, \bar{\mathbf{G}}_M^T]^T, \quad \mathbf{\Gamma}^T\bar{\mathbf{\Phi}}\mathbf{\Gamma} = \begin{bmatrix} \bar{\mathbf{\Phi}}_{1,1} & \cdots & \bar{\mathbf{\Phi}}_{1,M} \\ \vdots & \ddots & \vdots \\ \bar{\mathbf{\Phi}}_{M,1} & \cdots & \bar{\mathbf{\Phi}}_{M,M} \end{bmatrix},\tag{E.3}$$

where  $\bar{\mathbf{H}}_i \in \mathbb{C}^{N_c \times N_c}$ ,  $\bar{\mathbf{H}}_j \in \mathbb{C}^{N_c \times N_c}$ , and  $\bar{\mathbf{\Phi}}_{i,j} \in \mathbb{C}^{N_c \times N_c}$ ,  $\forall i, j \in \mathcal{M}$ , respectively, are circulant matrices with entries

$$\begin{aligned}[\bar{\mathbf{H}}_i]_{:,p} &= [\bar{\mathbf{H}}]_{:,(p-1)M+i}, \forall i, \forall p \in \mathcal{N}_c, \\ [\bar{\mathbf{G}}_j]_{q,:} &= [\bar{\mathbf{G}}]_{(q-1)M+j,:}, \forall j, \forall q \in \mathcal{N}_c, \\ [\bar{\mathbf{\Phi}}_{i,j}]_{p,q} &= [\bar{\mathbf{\Phi}}]_{(p-1)M+i,(q-1)M+j}, \forall i, j, p, q.\end{aligned}\tag{E.4}$$

Then, we can rewrite  $\mathbf{H}_{\text{all}}$  as

$$\begin{aligned} \mathbf{H}_{\text{all}} &= \bar{\mathbf{F}}_{N_c} \bar{\mathbf{H}}_d \bar{\mathbf{F}}_{N_c}^H + \bar{\mathbf{F}}_{N_c} \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{M}} \bar{\mathbf{H}}_i \bar{\boldsymbol{\Phi}}_{i,j} \bar{\mathbf{G}}_j \bar{\mathbf{F}}_{N_c}^H \\ &\stackrel{(b)}{=} \bar{\mathbf{F}}_{N_c} \bar{\mathbf{H}}_d \bar{\mathbf{F}}_{N_c}^H + \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{M}} \bar{\mathbf{F}}_{N_c} \bar{\mathbf{H}}_i \bar{\mathbf{F}}_{N_c}^H \bar{\mathbf{F}}_{N_c} \bar{\boldsymbol{\Phi}}_{i,j} \bar{\mathbf{F}}_{N_c}^H \bar{\mathbf{F}}_{N_c} \bar{\mathbf{G}}_j \bar{\mathbf{F}}_{N_c}^H, \end{aligned} \quad (\text{E.5})$$

where (b) holds due to the property of the normalized DFT, i.e.,  $\bar{\mathbf{F}}_{N_c}^H \bar{\mathbf{F}}_{N_c} = \mathbf{I}_{N_c}$ . Since a circulant matrix can be diagonalized by DFT [117], we have that

$$\begin{aligned} \bar{\mathbf{F}}_{N_c} \bar{\mathbf{H}}_d \bar{\mathbf{F}}_{N_c}^H &= \text{diag}(h_{d,1}, \dots, h_{d,N_c}), \quad \bar{\mathbf{F}}_{N_c} \bar{\mathbf{H}}_i \bar{\mathbf{F}}_{N_c}^H = \text{diag}(h_{i,1}, \dots, h_{i,N_c}), \\ \bar{\mathbf{F}}_{N_c} \bar{\mathbf{G}}_j \bar{\mathbf{F}}_{N_c}^H &= \text{diag}(g_{j,1}, \dots, g_{j,N_c}), \quad \bar{\mathbf{F}}_{N_c} \bar{\boldsymbol{\Phi}}_{i,j} \bar{\mathbf{F}}_{N_c}^H = \text{diag}(\phi_{i,j,1}, \dots, \phi_{i,j,N_c}), \end{aligned} \quad (\text{E.6})$$

where  $h_{d,n} \in \mathbb{C}$ ,  $h_{i,n} \in \mathbb{C}$ , and  $g_{j,n} \in \mathbb{C}$ , respectively, denote frequency-domain channels from the transmitter to receiver, from the  $i$ -th element of BD-RIS to the receiver, and from the transmitter to the  $j$ -th element of BD-RIS at subcarrier  $n$ ,  $\forall i, j \in \mathcal{M}$ ,  $\forall n \in \mathcal{N}_c$ .  $\phi_{i,j,n} \in \mathbb{C}$  denotes the scattering parameter from the  $j$ -th element to the  $i$ -th element of BD-RIS at subcarrier  $n$ . By introducing the frequency-domain channels  $\mathbf{h}_n \in \mathbb{C}^{1 \times M}$ ,  $\mathbf{g}_n \in \mathbb{C}^{M \times 1}$ , and the scattering matrix of BD-RIS  $\boldsymbol{\Phi}_n \in \mathbb{C}^{M \times M}$  at subcarrier  $n$  with entries

$$[\mathbf{h}_n]_i = h_{i,n}, \quad [\mathbf{g}_n]_j = g_{j,n}, \quad [\boldsymbol{\Phi}_n]_{i,j} = \phi_{i,j,n}, \quad \forall i, j, n, \quad (\text{E.7})$$

we finally have

$$\mathbf{H}_{\text{all}} = \text{diag}(h_{d,1} + \mathbf{h}_1 \boldsymbol{\Phi}_1 \mathbf{g}_1, \dots, h_{d,N_c} + \mathbf{h}_{N_c} \boldsymbol{\Phi}_{N_c} \mathbf{g}_{N_c}). \quad (\text{E.8})$$

The proof is completed.  $\square$

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