Understanding the Aggregate Flexibility of Thermostatically Controlled Loads

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***Abstract*—Thermostatically controlled loads (TCLs) are an attractive source of responsive demand. This paper aims to provides a better understanding of the relation between thermal properties of TCLs and their suitability to provide energy arbitrage and frequency services. An approximate analysis on the basis of dimensionless parameters is used to visualise the relative abilities of eight classes of TCLs. The results are compared to those obtained from a formal optimisation approach, in the context of a GB case study. Additional studies are performed to investigate the impact of increasingly flexible frequency services and physical variations of TCL thermal models (thermal conductance and temperature deadband).**

***Index Terms*-- demand side response, thermostatically controlled load, energy storage, frequency control, energy arbitrage.**

I. Introduction

The increasing penetration of different types of Renewable Energy Sources (RES) within the generation portfolio requires a significant growth in the network flexibility. This is mainly due to the intrinsic variability and uncertainty characterising RES output, together with their limited ability to participate in frequency control [1]. Previous research has demonstrated the economic and environmental value for a collection of Thermostatically Controlled Loads (TCLs) to unlock their inherent flexibility, by providing various forms of ancillary services [2, 3]. The main challenges to achieve such a demand-side participation encompass the need to combine compact models, accurately describing the aggregate temperature-power dynamics of several millions of heterogeneous TCLs, and a reliable device-level control strategy and its underlying control infrastructure. In fact, the *on/off* nature of individual TCLs, their temperature constraints and the large number of devices make it difficult to exert precise control over their power consumption [4].

In a recent work [5], we demonstrated how a large heterogeneous population of TCLs can be precisely modelled as a single *leaky energy storage* unit. The accessible energy and power consumption levels of this storage unit are physically limited by the properties of appliances' thermal models and their temperature constraints. Moreover, further constraints are introduced by the particular control strategy adopted [6] together with the inevitable heterogeneity of the appliances within the entire cluster. To account for these issues, a set of six envelope parameters was derived, as a function of 'thermal' parameters, to establish a lowest-common-denominator model of aggregate flexibility of controlled heterogeneous TCLs.

This paper extends the findings in [5]. The first contribution is an intuitive framework that aids understanding of aggregate TCL flexibility. The aggregate ability of appliances to provide ancillary services (short and medium term frequency response) and energy arbitrage is analysed. Simplified expressions are derived in terms of dimensionless parameters, and they are used to visualise relative abilities of appliances. The results from this procedure are compared with results from formal optimisation. The second contribution is a study of the potential benefits of a regulatory framework enabling a flexible allocation ancillary services (on a half-hour base), compared to the traditional settings, which require fixed amounts to be maintained over longer time windows (more than a day) [7]. Finally, the sensitivity of solutions to changes in thermal conductance and temperature deadband parameters is investigated.

II. Preliminaries: Aggregate Control of TCLs

This section summarises key results from [5] that underlie the analysis in following sections. For detailed derivations, we refer the reader to [5]. We consider TCLs that are described by a first order thermal model. Due to the linearity of the thermal model of a single TCL, the average temperature $\bar{T}(t)$ of a homogeneous population of $N \gg 1$ such devices evolves as the temperature of a single large device with a variable cooling/heating rate $\pi(t)$. In particular, $\pi(t)$ is the fraction of units in the *on* state. Moreover, the aggregate temperature distribution also represents the probability distribution for the state of a single TCL with random initial conditions [6], [8]. This implies that $\pi(t)$ is also proportional to the expectation of the appliance power consumption $P^a(t)$:

$$\pi(t) = E[P^a(t)]/P^a_{on}, \quad (1)$$

where a indicates a particular TCL and P^a_{on} is its maximum absorbed power (constant) when the appliance is on.

Furthermore, as demonstrated in [6], (1) effectively relates the expected power consumption $E[P^a(t)]$ and temperature $E[T^a(t)]$ of an appliance, and any controller that modulates the expected power consumption thus changes the expected temperature.

A compact dimensionless expression of aggregate TCL power-temperature (energy) dynamics is then derived [5]:

$$\frac{d\sigma(t)}{dt} = -\frac{1}{\tau}[\sigma(t) - \Pi(t)], \tag{2}$$

It is obtained through the following coordinate transformation

$$\Pi(t) = \frac{\pi(t)}{\pi_0} \qquad \sigma(t) = \frac{T_{off} - E[T^a(t)]}{T_{off} - T_o} \tag{3}$$

where π_o is the steady-state probability of being in the *on* state, in other words the *duty cycle* of the appliance, and T_0 the steady-state expected temperature. For a hysteresis controller, these are given by

$$\pi_o = \frac{T_{off} - T_0}{T_{off} - T_{on}} \tag{4}$$

$$T_0 = \frac{\log\left(\frac{T_{max} - T_{on}}{T_{min} - T_{on}}\right)}{ln\left(\frac{(T_{max} - T_{on})(T_{min} - T_{off})}{(T_{min} - T_{on})(T_{max} - T_{off})}\right)} \tag{5}$$

Here, T_{max}, T_{min} are the upper/lower operating temperature deadband limits, respectively. T_{off} is the ambient temperature, while T_{on} is the asymptotic cooling/heating temperature, which follows from the appliance model and cooling rate. $\Pi(t)$ represents the relative power level and $\sigma(t)$ is a dimensionless measure of stored energy. In the steady state, $\sigma_0 = \Pi_0 = 1$.

The stochastic controller in [6] is used to manage the operation of single TCLs. It enables the power consumption of a single TCL to follow, *in expectation*, a reference signal $\Pi(t)$, in accordance with (1). By achieving this result at device-level, we automatically ensure that the aggregate TCL power is $P(t) \approx N\pi_0 P_{on}\Pi(t)$. The range of feasible reference power levels is constrained by (i) the particular TCL implementation in [6] (linear controller), (ii) requirements on the quality of the primary cooling/heating function of the devices, (iii) appliance heterogeneity. To account for these issues, a set of largely dimensionless envelope parameters was identified in [5]. The envelope parameters define a lowest-common-denominator model of aggregate controlled heterogeneous TCLs. The total power consumption is given by $P(t) = \hat{P}_0 \cdot \Pi(t)$ with $\hat{P}_0 = \sum_a P_0^a = \pi_0^a \cdot P_{on}^a$. The controller introduces constant upper/lower limits on the instantaneous signal $\Pi(t)$, which are depend on TCL thermal parameters. Feasible envelope constraints are given by:

$$\hat{\Pi}_{min} = \max_a \Pi_{min}^a \leq \Pi(t) \leq \hat{\Pi}_{max} = \min_a \Pi_{max}^a. \tag{6}$$

Furthermore, the quality of the service of the primary function of TCL is fairly maintained if the average temperature $\bar{T}(t)$ does not exceed upper/lower temperature limits, typical of thermostatic loads. Expressed in dimensionless units, we obtain

$$\hat{\sigma}_{min} = \max_a \sigma_{min}^a \leq \sigma(t) \leq \hat{\sigma}_{max} = \min_a \sigma_{max}^a \tag{7}$$

Finally, the definition of an envelope time constant is related to the TCL's ability to dynamically follow a power signal $\Pi(t)$, subject to the (6-7). This ability does depend on τ^a, σ_{min}^a and σ_{max}^a, whereas σ_{min}^a and σ_{max}^a are already bounded by (7). A suitable definition of $\hat{\tau}$ is

$$\hat{\tau} = \min_a\left(min\left(\tau_{up}^{a*}, \tau_{down}^{a*}\right)\right) \tag{8}$$

$$\tau_{down}^{a*} = \tau^a\left[ln\left(\frac{\hat{\sigma}_{max} - \hat{\Pi}_{min}}{\sigma_{min}^a - \hat{\Pi}_{min}}\right) - ln\left(\frac{\hat{\sigma}_{max} - \hat{\Pi}_{min}}{\hat{\sigma}_{min} - \hat{\Pi}_{min}}\right)\right] \tag{8a}$$

$$\tau_{up}^{a*} = \tau^a\left[ln\left(\frac{\hat{\Pi}_{max} - \hat{\sigma}_{min}}{\hat{\Pi}_{max} - \sigma_{max}^a}\right) - ln\left(\frac{\hat{\Pi}_{max} - \hat{\sigma}_{min}}{\hat{\Pi}_{max} - \hat{\sigma}_{max}}\right)\right] \tag{8b}$$

Further details on the derivation of (8) are in [5]. With the reference parameters used in [5], it results that $\hat{\tau} \geq \min_a \tau^a$, for each class of cold appliances. Hence, we state the analogue of equation (2) for the dynamics of heterogeneous TCLs using the envelope time-constant $\hat{\tau}$.

$$\frac{d\sigma(t)}{dt} = -\frac{1}{\hat{\tau}}[\sigma(t) - \Pi(t)], \tag{9}$$

Equation (9), coupled with constraints (6-8), forms a compact representation of aggregate flexibility which is easily embedded in optimisation tools. Moreover, any generic power signal $\Pi(t)$ that is compatible with the proposed model is guaranteed to be realizable by all TCLs (see [5], [6] for validation). In addition, the underlying control strategy is non-disruptive i.e. no single TCL ever exceeds admissible temperature limits. It can also be implemented in a semi-autonomous fashion, without the need for a real-time communication infrastructure.

The above approach constructs a worst-case envelope, which can be quite conservative if appliances have very different characteristics. The overall flexibility can be enhanced by first clustering appliances by the similarity of their parameters. In [5] such partitioning was performed by means of *k-means clustering* algorithm, where the appliances were grouped based on their distance in parameter space. Envelope parameters (6-8) can be separately computed for each of the clusters.

III. Intuitive Understanding of TCL Flexibility for Network Services

This section analyses the flexibility model given by equations (6-9) to gain an intuitive understanding of appliance flexibility, and how it depends on the envelope parameters $P_0, \hat{\Pi}_{min}, \hat{\Pi}_{max}, \hat{\sigma}_{min}, \hat{\sigma}_{max}, \hat{\tau}$. We consider eight distinct types of refrigeration units across the domestic (refrigerators, freezers and fridge-freezers) and commercial sectors (bottle coolers, upright refrigerators, upright freezers, and two different multideck refrigerators). Reference parameters for first order dynamic thermal models of each class of appliance are taken from [5]. These parameters have been independently varied by +/-15% to generate a heterogeneous set of appliances for that class. Aggregate envelope parameters are computed using (6-8). We consider the suitability of each appliance class for energy arbitrage and frequency support services. Three different frequency services are modelled in this paper, according to the National Grid (NG) practice: primary response (PR), high frequency response (HFR) and secondary response (SR) [7]. The PR and HFR services are *short-term* services, requiring a constant decrease or increase in power

consumption, respectively, for up to 30 seconds. The SR service requires a *medium term* (30 min) reduction in power.

A. Short Term Services

The ability to provide short term services is directly linked to the constraints on $\Pi(t)$ (see (6)). Energy constraints play a marginal role due the short commitment time of these services. Fig. 1a (white bars) shows the accessible power ranges for each TCL class, based on (6), assuming that appliances within a class are controlled identically. The blue crosses indicate the steady state relative power level, which is equal to one by definition. The green and red areas illustrate the additional range of power excursion that was available to the most flexible cluster after 50000 random devices for each TCL class were partitioned into 25 clusters following the methodology in [5]. Note that this is a stochastic algorithm, so that cluster assignments are likely to slightly differ between repeated cluster attempts. As expected, the single-cluster population shows less flexibility than the most flexible cluster. Interpreting the results, we note that the domestic units have low steady state duty cycles and an ability to significantly increase their short-term power consumption. In contrast, bottle coolers and multidecks are only able to provide limited HFR due to their high nominal duty cycles.

B. Medium Term Services

In order to provide secondary response (SR), a population of TCLs must decrease its overall power consumption by a contracted amount $\Delta\Pi$ and sustain this for 30 minutes [7]. The solution of (9) for a generic and constant power level Π is

$$\sigma(t;\sigma(0),\Pi,\hat{\tau}) = (\sigma(0)-\Pi)e^{-t/\hat{\tau}} + \Pi. \qquad (10)$$

Rearranging (10) for an initial steady state condition $\sigma(0) = 1$ and a constant power reduction $\Pi(t) = 1 - \Delta\Pi$, leads to the following time evolution of the energy level,

$$\sigma(t) = 1 - \Delta\Pi \cdot \left(1 - e^{-t/\hat{\tau}}\right), \qquad (11)$$

with the asymptotic limit $\sigma(t \to \infty) = 1 - \Delta\Pi$. Note that $\Delta\Pi = 1$ represents a 100% reduction of power consumption. When $1 - \Delta\Pi$ is less than $\hat{\sigma}_{min}$ such a demand reduction response can be sustained only for a limited time until $\sigma(t^-) = \hat{\sigma}_{min}$. This maximum response duration is then given by

$$t^- = \hat{\tau} \cdot log\left(\frac{1}{1-(1-\hat{\sigma}_{min})/\Delta\Pi}\right) \qquad (12)$$

The aptitude of different classes of TCLs for providing SR services is obtained by solving (12) for constant $\Delta\Pi$, with $t^- =$ 30 minutes. Fig. 1b shows the resulting iso-power-reduction curves for $\Delta\Pi = 10\%, 20\%, \ldots, 100\%$. The position of a TCL class on this chart thus illustrates the maximum power reduction that could be provided for SR, assuming initialisation in the steady state, an ideal controller and no onerous constraints due to temperature distributions. Each TCL class (single-cluster, 50000 heterogeneous devices) is represented by a coloured outer perimeter that encompasses the appliances' effective energy reduction capacity, $1 - \sigma_{min}$ (x-axis) and thermal relaxation time τ (y-axis, logarithmic scale). Smaller perimeters are also shown for five clusters within each class, highlighting the differences in ability between clusters.

The ability for bottle coolers and upright fridges and freezers to provide long term services is significant as these appliances are above the upper envelope $\Delta\Pi = 1$. Hence, they could (in principle) remain switched off for more than 30 minutes. In contrast, the maximum contribution for the domestic refrigeration is limited to only a fraction of their aggregate power. Ultimately, the multidecks are not suitable for long term services due to their small thermal time constant $\hat{\tau}$, despite a substantial thermal headroom $1 - \hat{\sigma}_{min}$.

C. Energy Arbitrage

We now turn to the ability of TCLs to exploit their intrinsic energy reservoir to profit from energy arbitrage. In order to estimate the TCL performance, we consider how much energy can be provided or absorbed without violating the devices' energy limits; in other words, how much energy can effectively be 'transferred' between time windows. For this generic analysis we ignore the power constraints imposed by the controller so that $\Pi(t) \in [0, 1/\pi_0]$. First (11) is used to determine the time t_{empty} to fully deplete the storage capacity from full at $\Pi = 0$, defined by $\sigma(t_{empty}; \hat{\sigma}_{max}, 0, \hat{\tau}) = \hat{\sigma}_{min}$.

$$t_{empty} = \hat{\tau} \cdot [log(\hat{\sigma}_{max}) - log(\hat{\sigma}_{min})] \qquad (13)$$

The effective energy provided ΔE is the time t_{empty} it takes to deplete the storage capacity multiplied by the reduction in power consumption relative to the steady state:

$$\Delta E = t_{empty} \cdot \hat{P}_0 = \hat{\tau} \cdot \hat{P}_0 \cdot \Delta\sigma + O(\Delta\sigma^2): \qquad (14)$$

where $\Delta\sigma = \hat{\sigma}_{max} - \hat{\sigma}_{min}$ represents the dimensionless accessible storage capacity. For the opposite case of energy absorption, we consider the filling time t_{fill} such that $\sigma(t_{fill}; \hat{\sigma}_{min}, 1/\pi_0, \hat{\tau}) = \hat{\sigma}_{max}$ using $\Pi = 1/\pi_0$, the maximum attainable value when all devices are forced on. This results in

$$t_{fill} = \hat{\tau}\left\{log\left[\frac{1-\pi_0\hat{\sigma}_{min}}{1-\pi_0}\right] - log\left[\frac{(1-\pi_0\hat{\sigma}_{max})}{1-\pi_0}\right]\right\} \qquad (15)$$

The energy stored is the time t_{fill} to full capacity times the increase in power consumption relative to the steady state ($\hat{P}_0(1-\pi_0)/\pi_0$). An expansion in powers of $\Delta\sigma$ again results in $\Delta E = \hat{\tau} \cdot \hat{P}_0 \cdot \Delta\sigma + O(\Delta\sigma^2)$, (see (14)). The approximate energy transfer capacity per installed capacity (in units of P_0) is

$$\Delta e = {}^{\Delta E}/_{\hat{P}_0} \cong \hat{\tau} \cdot \Delta\sigma \quad [MWh/MW] \qquad (16)$$

Fig. 1c shows the appliance types on the $(\Delta\sigma, \tau)$-plane (τ-axis in logarithmic scale). The dashed curves represent iso-lines of the approximate energy arbitrage capacities Δe. As before, coloured outlines indicate the capabilities of 50000 randomised appliances of each class, and the outlines of 5 clusters within each class. The highest potential for energy arbitrage is realised in the upper-right side of Fig. 1c, where a large usable storage capacity is used in conjunction with a large thermal relaxation time. Mirroring the suitability for medium-term services, the commercial upright fridges and freezers and the bottle coolers are the most suitable units for energy arbitrage.

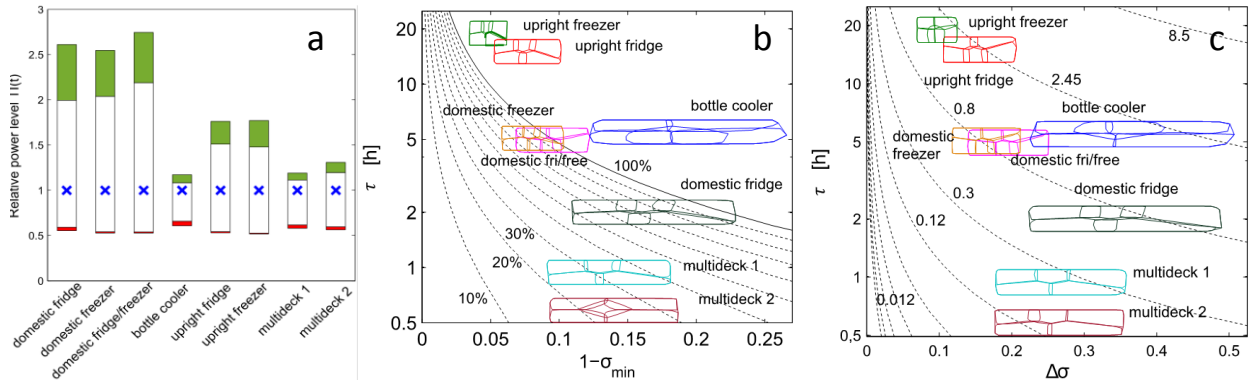


Fig. 1 Ability to reduce/increase the power consumption at short notice (a). Ability to provide 30 min demand reduction for medium term services (b). Approximate energy arbitrage capacity Δe [hours](c).

The results in this section, illustrated in Fig. 1, provide intuitive insights into the ability for a group of TCLs to deliver a particular service (arbitrage or frequency response services) in terms of the parameters $\hat{\Pi}_{min}, \hat{\Pi}_{max}, \hat{\sigma}_{min}, \hat{\sigma}_{max}$ and $\hat{\tau}$. Note that $\hat{P}_0$ does not feature, as it is only a measure of the aggregate power of the TCL population and therefore does not impact relative TCL flexibility. This analysis shows that different classes of TCLs could be clustered together with regard to one or more particular services; for example, domestic fridge-freezers overlap with bottle coolers in their ability to provide energy-intensive services (Figs.1b-c). This similarity does not hold anymore in case of provision of high frequency response (Fig. 1a). In addition, Figs. 1b-c illustrate that clustering of similar appliances within the same class allows to better exploit the intrinsic flexibility of the devices; the units within the top-right area of each class are more suitable for particular services compared to those that lie in the bottom-left. The envelope equations could also be used to increase the robustness of solutions to unknown variations in model parameters.

IV. Optimal Multi-Service Allocation

Whereas the previous section focused on an understanding of aggregate flexibility, this section considers the optimal allocation of multiple network services simultaneously. This is done in the context of a case study using three frequency services (PR, SR and HFR) and energy arbitrage with known prices. The model allows TCLs to deviate from their steady-state consumption to benefit from energy price differentials across the optimisation horizon (w=24 h, with periodic boundary conditions). In addition, the model allocates energy/power buffers to be held to ensure the deliverability of frequency response services. Constant response across the 24h is imposed in line with current practice in the GB regulatory framework. The optimal service allocations are compared with the analysis of envelope parameters in Section III. Then, sensitivities are investigated with respect to service flexibility (constant vs. variable) and domestic fridge parameters.

A. Optimal allocation model

The optimal allocation model is based on [5] with a reduced set of services. It is instructive, for system-oriented applications, to express the envelope model in Sec. II in units of MW and MWh. This is achieved by defining

$$S(t) \equiv \hat{P}_0 \cdot \hat{\tau} \cdot \sigma(t), \tag{17}$$

which leads to the 'leaky storage' equation:

$$\frac{dS(t)}{dt} = -\frac{1}{\hat{\tau}} S(t) + P(t) \tag{18}$$

Note that $P(t) = \hat{P}_0 \cdot \Pi(t)$ as previously defined.

The optimization horizon is divided into m=48 periods i of Δt=30 minutes. Solving (17) on the interval $[0, t]$ and imposing constant electrical power consumption P_i within each interval i results in the discrete energy evolution equation:

$$S_{i+1} = S_i \cdot e^{-\frac{\Delta t}{\hat{\tau}}} + \hat{\tau} P_i \cdot \left(1 - e^{-\frac{\Delta t}{\hat{\tau}}}\right) \tag{19}$$

S_i and S_{i+1} are the energy levels [MWh] at the extremities of interval i. The set of discrete energy levels $\mathcal{S} = \{S_i\}_{i=1}^{m}$ are decision variables, together with the committed frequency services P^p (PR), P^h (HFR), P^s (SR). Note that m = 48 periods i of Δt = 30 min. The cluster of devices is modelled as a price-taking leaky energy storage unit. The optimization therefore takes the form of payment minimisation problem. The revenues due to the commitment of frequency services are subtracted from the cost of energy consumption. Electricity prices ρ_i representative of the electricity GB market for a generic winter day, are as in [5]. By night (22:00 pm-06:00 am) prices are around 30£/MWh. By day achieve 100£/MWh, before the typical spike (peak of 249£/MWh) between 17:30 pm – 21:00 pm. The availability fees for each frequency service are h^p=6 £/MWh, h^h=7 £/MWh and h^s=5 £/MWh [7]. Electricity market prices and frequency service fees are assumed to be known in advance.

$$\min_{\mathcal{S}, P^p, P^h, P^s} \left\{ \sum_{i=1}^{m} [\rho_i P_i(\mathcal{S}) \Delta t] - w \cdot (h^p P^p + h^h P^h + h^s P^s) \right\} \tag{20}$$

subject to (for all i, where applicable)

$$\hat{P}_{min} \le P_i(\mathcal{S}) \le \hat{P}_{max} \tag{21}$$

$$\hat{S}^r_{min} \le S_i \le \hat{S}^r_{max} \tag{22}$$

$$\frac{1}{m} \sum_{j=1}^{m} S_j = S_0 \tag{23}$$

$$0 \le P^p \le P_i(\mathcal{S}) - \hat{P}_{min} \tag{24}$$

$$0 \le P^h \le \hat{P}_{max} - P_i(\mathcal{S}) \tag{25}$$

$$0 \le P^s \le P_i(\mathcal{S}) - \hat{P}_{min} \tag{26}$$

$$S_{i+1} - \hat{\tau}\left(1 - e^{-\frac{\Delta t}{\hat{\tau}}}\right) \cdot P^s \ge S_{min} \tag{27}$$

The optimal solution is bounded by power (21) and energy (22) constraints. The energy buffer for arbitrage $\left(\hat{S}^r_{max} - \hat{S}^r_{min}\right)$ is assumed to be 25% smaller (with respect to S_0) compared to the nominal one (see (7) and (17)). The latter is made available only for SR, as this service is rarely triggered. In addition, (23) forces the average energy (temperature) to equal the steady state energy (temperature). Constraints (24-26) impose sufficient power margins for PR, HFR and SR. Constraint (27) maintains the energy level, after the provision of SR, above $\hat{S}_{min}$. The energy deployment associated with short-term services is neglected as in [3].

B. Service allocation results

Figure 2 illustrates the resulting revenues from the different services for the eight appliance classes, per installed MW of nominal power consumption. The profit from energy arbitrage is defined as the difference in cost between constant power consumption at the nominal level and consumption that is optimally allocated. The differences in service provision between TCL types largely mirrors the ordering shown in Figs. 1, adjusted by the price-driven competition between services. Domestic appliances make most of their profit from contracting HFR (green) and then making energy arbitrage (grey), whereas other services contribute only marginally. In fact, these appliances exploit the large upward power margins (Fig. 1a) and the relatively high availability fee for HFR.

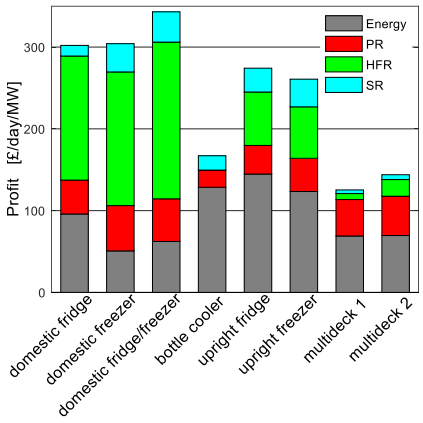


Fig. 2 Daily profit per class of TCLs (single-cluster) realised by energy arbitrage and by selling simultaneous availability for PR, HFR and SR.

In contrast, upright fridges and freezers can profit more from energy arbitrage rather than from HFR, reflecting their small upward power headroom (Fig. 1a) and the position on a high-value iso-energy line in Fig. 1c. Similarly to domestic appliances, commercial fridges and freezers benefit from ample downward power margin (for PR and SR) and a valuable position in Fig. 1b (SR). The bottle coolers and multidecks are limited to smaller potential profits, resulting primarily from their very limited power headroom (Fig. 1a) and – for multidecks – their weak thermal insulation (Figs. 1b-c). It is worth pointing out that bottle coolers do not commit any HFR.

C. Flexible allocation of frequency response services

In the allocation problem (20-27), the provision of contracted services is constant throughout the day. In this section, the potential value increase of relaxing this constraint is investigated (assuming equal prices). The objective function (20) is replaced by the following objective function with 48 decision variables for each of the frequency services.

$$\min_{\mathcal{S},\wp^P,\wp^h,\wp^S} \sum_{i=1}^{m} \left[\rho_i P_i(\mathcal{S})\Delta t - h^p P_i^P - h^h P_i^h - h^s P_i^s\right] \qquad (28)$$

The other constraints still hold for all i. In addition, in the flexible service scenario it is necessary to consider that deployment of frequency services may be triggered at any time within an interval, implying a response from that moment onward. This impacts SR in particular, ensuring that the contracted response can be delivered into the next 30-min period. Equations (29) are therefore included in the 'flexible-formulation' to guarantee that calling upon SR does not conflict with lower power and energy limits during the subsequent interval $i+1$. Similarly, constraints on power margin (like (29a)) are applied for PR and HFR.

$$0 \le P_i^S \le P_{i+1}(\mathcal{S}) - \widehat{\mathrm{P}}_{min} \qquad (29a)$$

$$S_{i+2} - \hat{\tau}\left(1 - e^{-\frac{\Delta t}{\hat{\tau}}}\right) \cdot P_i^S \ge S_{min} \qquad (29b)$$

The relaxation of constraints results in increased profits. Table I quantifies the additional profit realised by the different classes of TCLs under a flexible scheme for the allocation of frequency services. Results are expressed as percent variation compared to the base case (inflexible allocation). Notably, in addition to an increase in overall profits (last column), this is also the case for each source of revenues, showing that, in this case, augmented flexibility does not introduce any conflict among each revenue streams. As a result, the largest percent profit variations occur for those services that were less remunerative in the base case due to TCL physical characteristics and price competition. For example, multidecks improve SR and HFR allocations which were penalised by low downward power margins (Fig. 1a) and low $\hat{\tau}$ (Figs 1b-c). On the contrary, domestic appliances improve HFR profits only marginally since HFR was already largely driving the solution of problem (20-27), due to its high fees and large upward power margins.

TABLE I
FLEXIBLE ALLOCATION OF FREQUENCY SERVICES (PROFIT VARIATION [%])

Class of TCLs	Energy	PR	HFR	SR	Total
Domestic fridge	+16%	+35%	+8%	+165%	+21%
Domestic freezer	+91%	+15%	+4%	+19%	+22%
Domestic fri-freezer	+82%	+20%	+4%	+13%	+21%
Bottle cooler	+16%	+123%	n.a.	+123%	+48%
Upright fridge	+39%	+75%	+23%	+71%	+43%
Upright freezer	+39%	+59%	+20%	+49%	+39%
Multideck 1	+1%	+22%	+135%	+288%	+26%
Multideck 2	+3%	+19%	+49%	+286%	+27%

The largest overall profit increase is for bottle coolers (48%). As in Fig. 1b, this result is driven by the intrinsic high capability to sustain energy-intensive SR. However, in an inflexible environment, this potential value was limited by the lowest power consumption due to energy arbitrage. Similarly, the value for PR has also grown by the same quantity. Bottle coolers now contract HFR, obtaining 11 £/day/MW. Since this service was not committed in the base case, a relative increase cannot be shown.

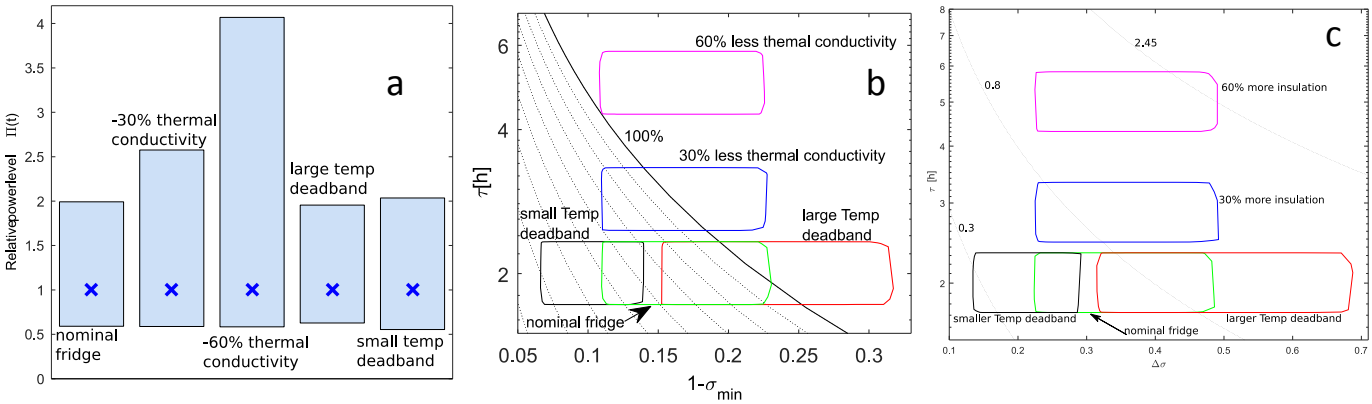


Fig. 3. Varying the domestic refrigerator thermal model (single-cluster). Impact on short-term services (a), secondary response (b) and energy arbitrage (c).

D. Sensitivity to parameter changes

We now assess the impact of parameter variations in TCL parameters on the optimal multi-service allocation. In general, the effect of these variations may not be easily understood without resorting to a detailed quantitative analysis. The use of the charts in Fig. 1 enables to qualitatively predict the impact of models' variations. In this case study, we decreased the thermal conductance of domestic refrigerators by 30% and 60% compared to the nominal value. This has the primary effect of increasing the thermal time constant $\hat{\tau}$, but it also decreases T_{on}, the asymptotic cooling temperature, affecting the other parameters. We also varied the TCL temperature thresholds ($[T_{min} \pm 1\,, T_{max} \mp 1\,]$ °C) so that we increased/decreased the temperature deadband width. For brevity, only a single cluster of domestic refrigerators was considered.

How these variations ultimately affect power/energy scheduling in (20-27) is not straightforward. As a first step, qualitative results are shown in Figs. 3. Increasing $\hat{\tau}$ has the inferable effect of improving the fridge performance in terms of SR and energy arbitrage (Figs 3b-c). It also positively impacts on the power margin $\widehat{\Pi}_{max}$ (Fig. 3a), while the $\widehat{\Pi}_{min}$ is almost constant, in both cases. A comparison the optimal profits with the base case (Table II) shows large improvements in profitability of SR and HFR at the cost of slightly decreasing the profit from PR.

TABLE II
IMPACT OF VARIATIONS, COMPARED TO BASE CASE (DOMESTIC FRIDGE)

Variation	Energy	PR	HFR	SR	Total
-30% thermal conductivity	1.6%	-2.5%	66.0%	87.4%	37.1%
-60% thermal conductivity	35.3%	-14.8%	225.9%	126.5%	128.1%
Large Temp deadband	39.9%	-28.7%	-8.2%	39.2%	6.3%
Small Temp deadband	-39.8%	29.0%	8.6%	-39.5%	-6.0%

Increasing the temperature deadband improves the profitability of energy arbitrage and SR as anticipated in Figs. 3b-c, while it makes short-term services less cost effective (negative percent variation). This result arises from the reduction of the available upward/downward relative power margin (Fig. 3a). These changes are mirrored in the case of a smaller temperature deadband.

V. CONCLUSIONS

This paper proposed an intuitive framework to understand the intrinsic ability of TCLs to provide energy arbitrage and frequency services. The insights gained by means of the proposed qualitative analysis were backed up by a quantitative price-driven optimisation model. The proposed high-level analysis is an effective tool for demand-side response aggregators to understand and cluster TCL appliance types (including extensions to the heating sector), depending on services offered with the System Operator.

Such analysis may also inform reviews/definition of grid codes associated to existing/new ancillary services designed for demand-side actors, leading to a better match between the definition of services' requirements and the intrinsic flexibility of demand-side technologies.

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