# AN INVESTIGATION OF STRETCH FORMING

# IN RELATION TO

# DEEP-DRAWING AND TESTING SHEET METAL

by

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VOLUME - II

Imperial College,

London, S.W.7.

January, 1966

## APPENDIX -A. FORMULAE USED IN NUMERICAL ANALYSIS

The formulae used in various numerical techniques in the body of the thesis and in the other sections of the appendix are derived below, under the appropriate section headings.

### A.1. Numerical Integration

There are two types of numerical integration: Case (i) corresponds to an indefinite integral in analytical integration, and is used in integrating differential equation(s) with the given boundary conditions. The Runga-Kutta integration is of this type and the reasons for its suitability for sheet metal deformation problems are given in Chapter 5. Case (ii), which is more generally known as "numerical quadrature", corresponds to a definite integral between limits in analytical integration. In connection with the present work, for the same reasons as in Case (i), a "Lagrangian" method of numerical quadrature was found to be satisfactory.

#### A.1.1. Runga-Kutta Method

This method in general can be used in the solution of a set of first order differential equations, and also for those equations of higher orders which can be reduced to a set of first order equations (66).

If the solution has reached the point $\bar{x}$ (a value of

the independent variable), then $y_r(\bar{x}) = \bar{y}_1, \bar{y}_2, .., \bar{y}_q$ (dependent variables). This can be an interior point or a point on a boundary.

$$\text{Since } \frac{dy_r}{dx} = \xi_r(x, y_1, y_2, \ldots, y_q) \qquad (A.1)$$

which can be computed from the given differential equations, in a fourth-order Runga-Kutta process the following intermediate quantities are calculated:

$$k_{ro} = \ell\, \xi_r(\bar{x}, \bar{y}_1, \bar{y}_2, \ldots, \bar{y}_q)$$

$$k_{r1} = \ell\, \xi_r(\bar{x}+0.5\ell, \bar{y}_1+0.5k_{10}, \bar{y}_2+0.5k_{20}, \ldots, \bar{y}_q+0.5k_{qo})$$

$$k_{r2} = \ell\, \xi_r(\bar{x}+0.5\ell, \bar{y}_1+0.5k_{11}, \bar{y}_2+0.5k_{21}, \ldots, \bar{y}_q+0.5k_{q1})$$

$$k_{r3} = \ell\, \xi_r(\bar{x}+\ell, \bar{y}_1+k_{12}, \bar{y}_2+k_{22}, \ldots, \bar{y}_q+k_{q2}) \qquad (A.2)$$

then the next nodal point is:

$$y_r(\bar{x}+\ell) = y_r(\bar{x}) + \frac{1}{6}(k_{ro}+2k_{r1}+2k_{r2}+k_{r3}) \qquad (A.3)$$

This process is repeated until the final boundary of the particular region under investigation is reached. The error introduced by this numerical method is proportional to $\ell^5$ and is also influenced by the higher derivatives of the functions involved. It is therefore necessary to investigate the mathematical stability of the solution by repeating the calculation using different values of $\ell$ and reaching a compromise at which the solution

is sufficiently accurate, but still does not require a formidable amount of computer time. This method is well suited to automatic digital computers because the evaluations of eqns. (A.1), and hence (A.2) can be very laborious.

In the present work this method was applied to a single differential equation at a time (suffix r = 1), and the other details are discussed in Chapter 5.

A.1.2. Lagrangian Method

Supposing that the functional relationship between x, and y is not analytical, but is given at unequal intervals of either the independent or of the dependent variable. It is then possible to approximate this relationship which is given at (s+1) discrete points, with a unique polynomial of degree not greater than s. The form of such a polynomial usually associated with Lagrange is (33, 16):

$$y = \left( \frac{x-x_1}{x_o-x_1} \; \frac{x-x_2}{x_o-x_2} \cdots \frac{x-x_s}{x_o-x_s} \right) y_o + \left( \frac{x-x_o}{x_1-x_o} \right.$$

$$\left. \frac{x-x_2}{x_1-x_2} \cdots \frac{x-x_s}{x_1-x_s} \right) y_1 + \ldots . ; \qquad (A.4)$$

This polynomial should be used with care since it may be unstable and may not represent the function correctly if the function is ill-conditioned, and the range is too

large. The lower degree polynomials are more stable but may lead to more approximate results for the same intervals. Therefore a compromise must be reached between the degree of the polynomial and its range, which can only be done by special investigations. For the present work a polynomial of second degree was found to be adequate since usually three consecutive points at unequal intervals were available in a short range. This can be written as:

$$y = \frac{(x - x_1)(x - x_2)}{(x_o - x_1)(x_o - x_2)} y_o + \frac{(x - x_o)(x - x_2)}{(x_1 - x_o)(x_1 - x_2)} y_1 + \frac{(x - x_o)(x - x_1)}{(x_2 - x_o)(x_2 - x_1)} y_2 \qquad (A.5)$$

To find the integral of y, eqn. (A.5) is integrated to give:

$$\int_{x_1}^{x_2} y\,dx = \frac{y_o}{(x_o - x_1)(x_o - x_2)}\left[\frac{x_2^3 - x_1^3}{3} - \tfrac{1}{2}(x_1 + x_2)(x_2^2 - x_1^2) + x_1 x_2 (x_2 - x_1)\right]$$

$$+ \frac{y_1}{(x_1 - x_o)(x_1 - x_2)}\left[\frac{x_2^3 - x_1^3}{3} - \tfrac{1}{2}(x_o + x_2)(x_2^2 - x_1^2) + x_o x_2 (x_2 - x_1)\right]$$

$$+ \frac{y_2}{(x_2 - x_o)(x_2 - x_1)}\left[\frac{x_2^3 - x_1^3}{3} - \tfrac{1}{2}(x_o + x_1)(x_2^2 - x_1^2) + x_o x_1 (x_2 - x_1)\right] \qquad (A.6)$$

This equation finds the value of the integral between the limits of $x_2$ and $x_1$ using the three points $x_o$, $x_1$ and $x_2$. This can be extended for a tabular data of (w+1) points as follows:

if $x_o, x_1, \ldots, x_w$

and $y_o, y_1, \ldots, y_w$ then

$$\int_{x_1}^{x_w} ydx = \sum_{m=1}^{m=w-1} \frac{y_{(m-1)}}{(x_{(m-1)} - x_m)(x_{(m-1)} - x_{(m+1)})}$$

$$\left[ \frac{x_{(m+1)}^3 - x_m^3}{3} - \frac{1}{2}(x_m + x_{(m+1)})(x_{(m+1)}^2 - x_m^2) + x_m x_{(m+1)} \right.$$

$$\left. (x_{(m+1)} - x_m) \right] + \frac{y_m}{(x_m - x_{(m-1)})(x_m - x_{(m+1)})}$$

$$\left[ \frac{x_{(m+1)}^3 - x_m^3}{3} - \frac{1}{2}(x_{(m-1)} + x_{(m+1)})(x_{(m+1)}^2 - x_m^2) \right.$$

$$\left. + x_{(m-1)} x_{(m+1)} (x_{(m+1)} - x_m) \right] + \frac{y_{(m+1)}}{(x_{(m+1)} - x_{(m-1)})(x_{(m+1)} - x_m)}$$

$$\left[ \frac{x_{(m+1)}^3 - x_m^3}{3} - \frac{1}{2}(x_{(m-1)} + x_m)(x_{(m+1)}^2 - x_m^2) + \right.$$

$$\left. x_{(m-1)}\, x_m\, (x_{(m+1)} - x_m) \right] \qquad (A.7)$$

Hence the quadrature, or the definite integral of a

tabular function can be obtained.

## A.2. Numerical Differentiation

For tabular functions at unequal intervals, the differentiation of the Lagrange polynomial provides approximate derivatives of the function represented either at tabular or intermediate points. The accuracy of the derivatives will depend on how close the polynomial represents the given function, and hence this calls for an investigation of the degree and the range of the polynomial. In the present work, and for the intervals used, a second degree polynomial was found to be adequate.

(i) If $x_o$, $x_1$, $x_2$

$y_o$, $y_1$, $y_2$ are given,

to find $\left(\frac{dy}{dx}\right)_{x_2}$ :

Eqn. (A.5) is differentiated with respect to x and $x = x_2$ is substituted giving:

$$\left(\frac{dy}{dx}\right)_{x_2} = \frac{x_2 - x_1}{(x_o - x_1)(x_o - x_2)}\, y_o + \frac{x_2 - x_o}{(x_1 - x_o)(x_1 - x_2)}\, y_1 + \frac{2x_2 - x_1 - x_o}{(x_2 - x_o)(x_2 - x_1)}\, y_2 \qquad (A.8)$$

alternatively if $\ell_1 = x_1 - x_o$

and $\ell_2 = x_2 - x_1$

$$\left(\frac{dy}{dx}\right)_{x_2} = \frac{\ell_2}{\ell_1(\ell_1+\ell_2)}\,y_o - \frac{\ell_1+\ell_2}{\ell_1\ell_2}\,y_1 + \frac{2\ell_2+\ell_1}{(\ell_1+\ell_2)\ell_2}\,y_2 \qquad (A.9)$$

(ii) If $x_o$, $x_1$, $x_2$

$y_o$, $y_1$, $y_2$ are given,

to find $\left(\frac{dy}{dx}\right)_{x_1}$ :

Eqn. (A.5) is differentiated with respect to x, and $x = x_1$ is substituted giving:

$$\left(\frac{dy}{dx}\right)_{x_1} = \frac{x_1-x_2}{(x_o-x_1)(x_o-x_2)}\,y_o + \frac{2x_1-x_2-x_o}{(x_1-x_o)(x_1-x_2)}\,y_1 + \frac{x_1-x_o}{(x_2-x_o)(x_2-x_1)}\,y_2 \qquad (A.10)$$

or

$$\left(\frac{dy}{dx}\right)_{x_1} = \frac{-\ell_2}{\ell_1(\ell_1+\ell_2)}\,y_o + \frac{\ell_2-\ell_1}{\ell_1\ell_2}\,y_1 + \frac{\ell_1}{(\ell_1+\ell_2)\ell_2} \qquad (A.11)$$

(iii) If $x_1$ $x_2$

$y_1$ $y_2$ and $\left(\frac{dy}{dx}\right)_{x_1}$ are given,

to find $\left(\frac{dy}{dx}\right)_{x_2}$ :

The Lagrange polynomial is rewritten for points $x_1$, $x_2$ and another fictitious point at $(x_1+\delta x)$ having a function value $(y_1+\left(\frac{dy}{dx}\right)_{x_1}\delta x)$ similar to eqn. (A.5), as follows:

$$y = \frac{(x-x_1-\delta x)(x-x_2)}{(x_1-x_1-\delta x)(x_1-x_2)}\, y_1 + \frac{(x-x_1)(x-x_2)}{(x_1+\delta x-x_1)(x_1+\delta x-x_2)}$$

$$(y_1+\left(\frac{dy}{dx}\right)_{x_1}\delta x) + \frac{(x-x_1)(x-x_1-\delta x)}{(x_2-x_1)(x_2-x_1-\delta x)}\, y_2 \qquad (A.12)$$

After simplifying, rearranging, and taking the limit when $\delta x \to 0$ :

$$y = \frac{(2x_1-x-x_2)(x-x_2)}{(x_1-x_2)^2}\, y_1 + \frac{(x-x_1)(x-x_2)}{(x_1-x_2)}\left(\frac{dy}{dx}\right)_{x_1}$$

$$+ \frac{(x-x_1)^2}{(x_2-x_1)^2}\, y_2 \qquad (A.13)$$

Differentiating eqn. (A.13) with respect to x and substituting $x = x_2$ gives:

$$\left(\frac{dy}{dx}\right)_{x_2} = \frac{2(y_1-y_2)}{x_1-x_2} - \left(\frac{dy}{dx}\right)_{x_1} \qquad (A.14)$$

Polynomials involving derivatives such as eqns. (A.13) and (A.14) are generally known as Hermite polynomials.

## A.3. Interpolation

When a function is known at a set of discrete points of the argument or of the function itself, provided that it is continuous, the interior points can be found by interpolation, and the exterior points by extrapolation. Finding the function value for a given interior intermediate value of the argument is known as direct, and the calculation of the argument for a given value of the function is known as inverse interpolation.

For the present work a polynomial of degree two represented the functions sufficiently accurately for the ranges chosen. The following formulae applies for direct and inverse interpolation and extrapolation, at equal or unequal intervals of the function or the argument.

(i) If $x_0$, $x_1$, $x_2$

$y_0$, $y_1$, $y_2$ are given,

y at a given x can be calculated from the Lagrange polynomial (A.5). In the present work the algorithm due to Aitken (1) has been used since this was more adaptable to computer programming,* but it gives identical results with eqn. (A.5 ). The process consists of a sequential combination of"linear cross-means" as shown below which

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* The author expresses his thanks to Mr.D.J.McConalogue, Department of Computer Science, Imperial College, for his valuable advice on numerical interpolation, and his help in writing the interpolation routine (Routine 2) in the computer program for the analysis of stretch forming.

is equivalent to a second degree polynomial interpolation:

$$y_{2,1} = \frac{(x_2 - x)y_1 - (x_1 - x)y_2}{x_2 - x_1}$$

$$y_{1,0} = \frac{(x_1 - x)y_0 - (x_0 - x)y_1}{x_1 - x_0}$$

$$y = y_{2,2} = \frac{(x_2 - x)y_{1,0} - (x_0 - x)\, y_{2,1}}{x_2 - x_0} \qquad (A.15)$$

(ii) If $x_1$, $x_2$

$y_1$, $y_2$ and $\left(\frac{dy}{dx}\right)_{x_1}$ are given,

to find y at a given x, eqn. (A.13) can be used, which is a Hermitean polynomial.

The inverse interpolation or extrapolation can be effected by interchanging the given x's with the corresponding y's, and carrying out the calculation as for direct interpolation. In either case the x's or y's need not be in a numerical sequence.

## A.4 Maximum and Minimum

It is sometimes necessary to find the maximum or a minimum of a function and the value of the argument at which this occurs. In the present work the following method was used in connection with the prediction of the SP instability.

If $y = f(x)$ and the values

$x_o$, $x_1$, $x_2$

$y_o$, $y_1$, $y_2$

are generated in a numerical solution, the point when the signs of

$$\frac{y_2 - y_1}{x_2 - x_1} \quad \text{and} \quad \frac{y_1 - y_o}{x_1 - x_o} \quad \text{become}$$

different must be determined. This would mean that the required maximum or the minimum is between $x_2$ and $x_o$, and the corresponding signs are - and + for a minimum, and + and - for a maximum respectively. In case of a maximum if all the values of y are negated then it will be a problem of finding the minimum, which is a matter of convenience in a computer program. The results can be renegated when obtained in such a case.

Assuming that such a point is reached and the minimum is bound by $x_2$ and $x_o$ as shown in Figs. A.1.a and A.1.b. approaching in an increasing, and a decreasing sequence respectively. Such a function can be approximately represented by a Lagrange polynomial, for the present purposes a second degree one, as given by eqn. (A.5). Differentiating this equation, and equating it to zero:

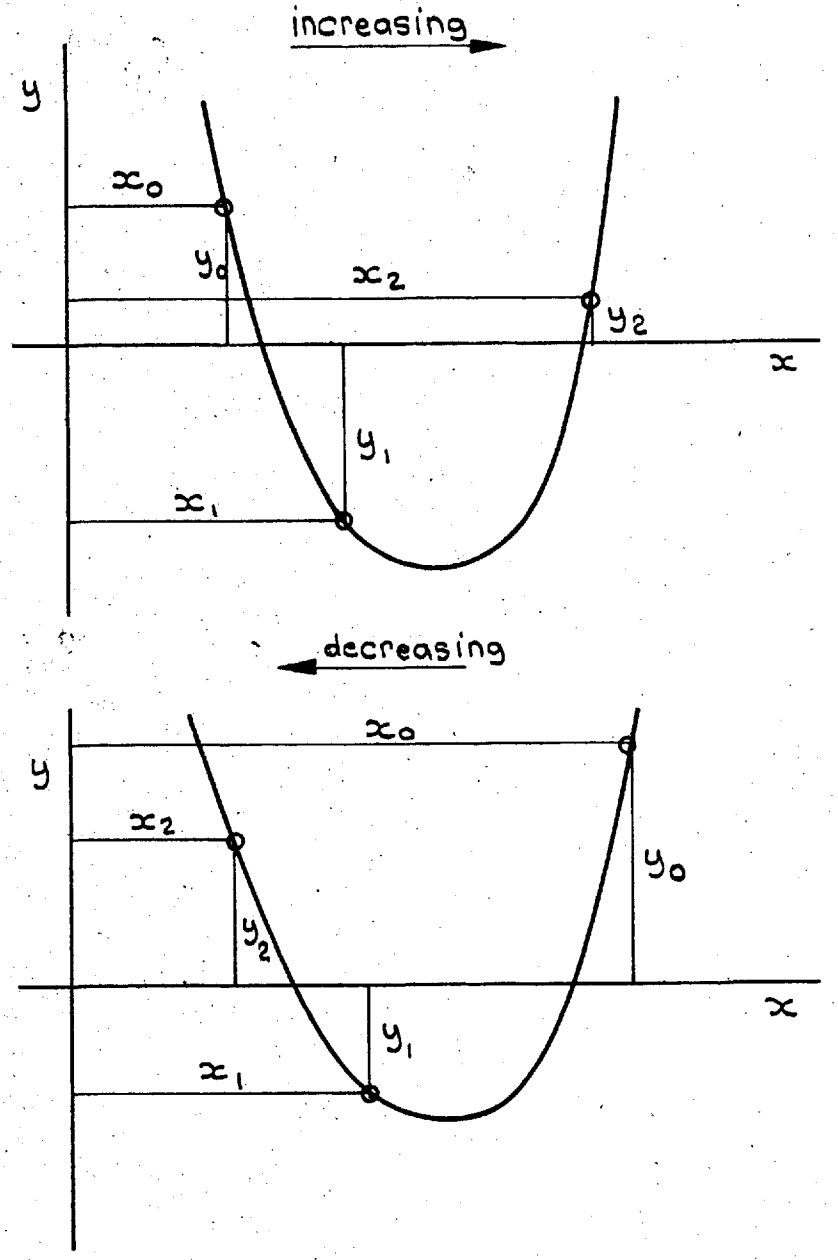



FIG. A.I. - SEQUENCE OF POINTS IN MINIMIZATION.

$$\frac{dy}{dx} = \frac{2x - x_2 - x_1}{(x_o - x_1)(x_o - x_2)} y_o + \frac{2x - x_2 - x_o}{(x_1 - x_o)(x_1 - x_2)} y_1$$

$$+ \frac{2x - x_1 - x_o}{(x_2 - x_1)(x_2 - x_1)} y_2 = 0$$

Solving for the unknown x after some algebraic manipulation gives:

$$x = \frac{(x_1^2 - x_2^2)y_o + (x_2^2 - x_o^2)y_1 + (x_o^2 - x_1^2)y_2}{2\left[(x_1 - x_2)y_o + (x_2 - x_o)y_1 + (x_o - x_1)y_2\right]} \qquad (A.16)$$

Hence a first approximation to the minimum can be obtained by using eqn. (A.16). Since the function minimised in general is not a polynomial, using this x a fuction value y is computed according to the numerical procedure of generating this function. For use in eqn. (A.16), the x-values need not be in a numerical sequence as shown in Fig. A.1, but this is necessary for the following iterative technique.

To improve on the minimum found, the points with suffices 0-2 and the first approximation for x and y would be available. From these available four points the best three are chosen by substituting the new x, y in place of the co-ordinates of one of the points in the initial sequence. This is effected as follows:

If $x_2 > x_o$ (Fig. A.1.a.):

if $y > y_1$ and

- $x > x_1$ : $x_2 = x$ and $y_2 = y$
- $x_1 > x$ : $x_o = x$ and $y_o = y$

if $y_1 > y$ and

- $x > x_1$ : $x_o = x_1$, $y_o = y_1$ and $x_1 = x$, $y_1 = y$
- $x_1 > x$ : $x_2 = x_1$, $y_2 = y_1$ and $x_1 = x$, $y_1 = y$

if $x_o > x_2$ (Fig. A.1.b.):

if $y > y_1$ and

- $x > x_1$ : $x_o = x$ and $y_o = y$
- $x_1 > x$ : $x_2 = x$ and $y_2 = y$

if $y_1 > y$ and

- $x > x_1$ : $x_2 = x_1$, $y_2 = y_1$ and $x_1 = x$, $y_1 = y$
- $x_1 > x$ : $x_o = x_1$, $y_o = y_1$ and $x_1 = x$, $y_1 = y$

When these transfers are made the new

$x_o$, $x_1$, $x_2$

$y_o$, $y_1$, $y_2$ are to be used in equation (A.16) to obtain a better approximation of the minimum of the function with that of the polynomial. The process is carried on until the values of either x or y do not differ more than a specified amount in two consecutive approximations. This technique is rapidly convergent and is very accurate.

## APPENDIX-B. GEOMETRICAL RELATIONSHIPS OF THE AXISYMMETRICAL SHELLS USED

### B.1. Principal Radii of Curvature

In an axisymmetrical shell the two principal radii of curvature ($\rho_1$, and $\rho_2$) are in two mutually perpendicular planes in meridional and circumferential directions as shown in FIG. B.1.

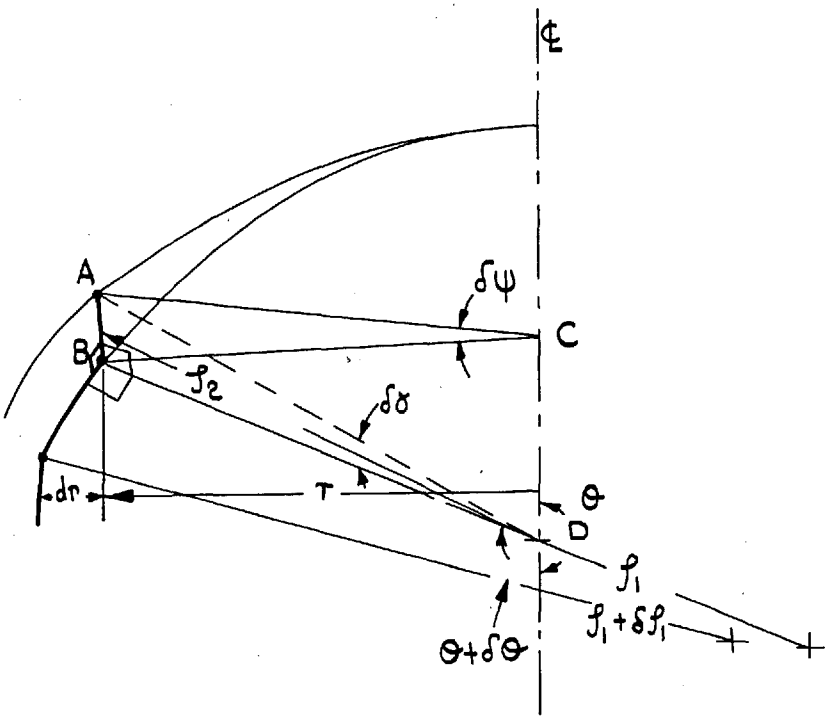



FIG. B.1. PRINCIPAL RADII OF CURVATURE OF AN AXISYMMETRICAL SHELL

To find the relationship between r, θ, and $\rho_1$:

$$(\rho_1 + \frac{\delta\rho_1}{2})(\delta\theta)\left[\cos\left(\theta + \frac{\delta\theta}{2}\right)\right] = \delta r$$

After expanding, simplifying and taking the limit ( $\delta\theta \rightarrow 0$):

$$\rho_1 \cos\theta = \frac{dr}{d\theta} \qquad (B.1)$$

Also

$$r\,\delta\phi = \rho_2\,\delta\gamma \qquad (B.2)$$

and from the right angle triangle BCD

$$d\gamma = \delta\phi \sin\theta \qquad (B.3)$$

Substituting eqn. (B.3) in eqn. (B.2) and rearranging:

$$r = \rho_2 \sin\theta \qquad (B.4)$$

Differentiating eqn. (B.4) totally and also using eqn. (B.1), it can be shown that the two principal radii of curvature ($\rho_1$, and $\rho_2$) are not independent and they are related by the following equation:

$$\rho_1 = \frac{\rho_2}{1 - \frac{r}{\rho_2}\frac{d\rho_2}{dr}} \qquad (B.5)$$

## B.2. Ellipsoidal Shell

Equation of an ellipse in Cartesian co-ordinates is (FIG.B.2):

$$\frac{r^2}{a^2} + \frac{z^2}{b^2} = 1 \qquad (B.6)$$

From FIG. B.2,

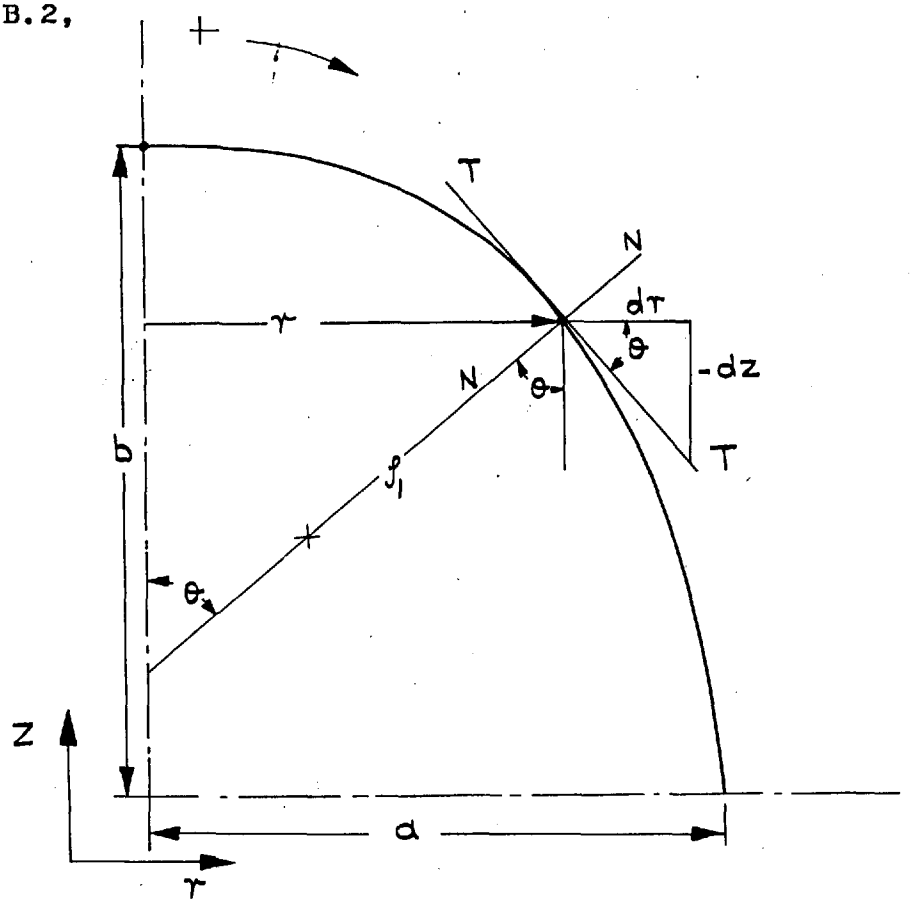



FIG.B.2. GEOMETRICAL PROPERTIES OF ELLIPSE

$$\frac{dz}{dr} = - \tan \theta \qquad (B.7)$$

To find r in terms of a, b, and θ, eqn. (B.6) is differentiated totally, and using eqn. (B.7), z is found in terms of a, b, and θ which is in turn substituted back to eqn.(B.6) to give:

$$r = a\left(1 + \frac{b^2}{a^2} \cot^2 \theta\right)^{-\frac{1}{2}} \qquad (B.8)$$

The instantaneous radius of curvature is given by:

$$\rho_1 = \frac{\left[1 + \left(\frac{dz}{dr}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2z}{dr^2}\right|} \qquad (B.9)$$

Finding $\frac{dz}{dr}$ and $\frac{d^2z}{dr^2}$ from eqns. (B.6) and (B.7) and substituting in (B.9) gives:

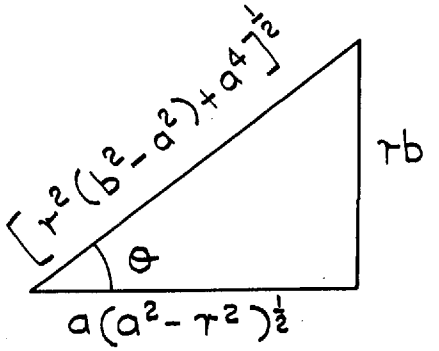
$$\rho_1 = b^2a^2\left[b^2\cos^2\theta + a^2\sin^2\theta\right]^{-\frac{3}{2}} = \frac{b^2r^3}{a^4\sin^3\theta} \qquad (B.10)$$

It is more convenient to find the geometrical relationships in terms of r, a, and b for numerical computation, which can now be obtained as follows:

After rearranging eqn. (B.8),

$$\tan\theta = rb\,(a^2 - r^2)^{-\frac{1}{2}}/a \qquad (B.11)$$

hence from the right angle triangle constructed from eqn. (B.11)




$$\cos\theta = a\,(a^2 - r^2)^{\frac{1}{2}}\left[r^2\,(b^2 - a^2) + a^4\right]^{-\frac{1}{2}} \qquad (B.12)$$

$$\sin\theta = rb\left[r^2\,(b^2 - a^2) + a^4\right]^{-\frac{1}{2}} \qquad (B.13)$$

From eqns. (B.10) and (B.13):

$$\rho_1 = \left[r^2 (b^2 - a^2) + a^4\right]^{\frac{3}{2}} / a^4 b \qquad (B.14)$$

and from differentiating the arctan of eqn. (B.11) with respect to r:

$$\frac{d\theta}{dr} = a^3 b (a^2 - r^2)^{-\frac{1}{2}} \left[r^2 (b^2 - a^2) + a^4\right]^{-1} \qquad (B.15)$$

## B.3. Hemispherical Shell

The geometrical relationships for the hemispherical shell can be obtained by substituting $a = b = \rho_1$ in the equations for the ellipsoidal shell, and the following equations are obtained.

$$\sin \theta = r/\rho_1 \qquad (B.16)$$

$$\cos \theta = (\rho_1^2 - r^2)^{\frac{1}{2}} / \rho_1 \qquad (B.17)$$

$$\tan \theta = r (\rho_1^2 - r^2)^{-\frac{1}{2}} \qquad (B.18)$$

$$\frac{d\theta}{dr} = (\rho_1^2 - r^2)^{-\frac{1}{2}} \qquad (B.19)$$

where $\rho_1$ is a constant.

## B.4. Equations for Bulge Depth

In stretch forming with a rigid punch, and also in hydraulic bulging, the depth of bulge cannot be computed analytically, unless some simplifying assumptions are made about the shape of bulge at places where the sheet metal is not in contact with the rigid tools. The two

methods used in the present investigation are described below:

(i) Exact Method: From the general solution of the problems of stretch forming and hydraulic bulging (see Chapters 3, 5, and 6) the distributions of $\theta$ and r along the shell profile at any stage are available. Bearing in mind that, as shown in FIG. B.3, the bulge depth h as measured in experiments is the distance between the internal face of the pole, and the blank-holder or lower die which rests on the thickest section of the undeformed blank:

$$h = \int_0^{D/2} \tan\theta \, dr - \frac{t_p}{2} - \frac{t_b}{2} \cos\varphi + t_o \qquad (B.20)$$

In eqn. (B.20), the relationship between $\tan\theta$ and r is tabular and not analytical since it is obtained from the general numerical solution. Therefore the integral can be evaluated numerically by the Lagrangian quadrature formula (A.7). Hence it is possible to find the bulge depth at any stage of the deformation in stretch forming, hydraulic bulging and deep-drawing after the numerical solution is completed.

(ii) Approximate Method: In some cases as in Chapter 7 in the calculation of the coefficient of friction, it is more convenient to use an approximate analytical

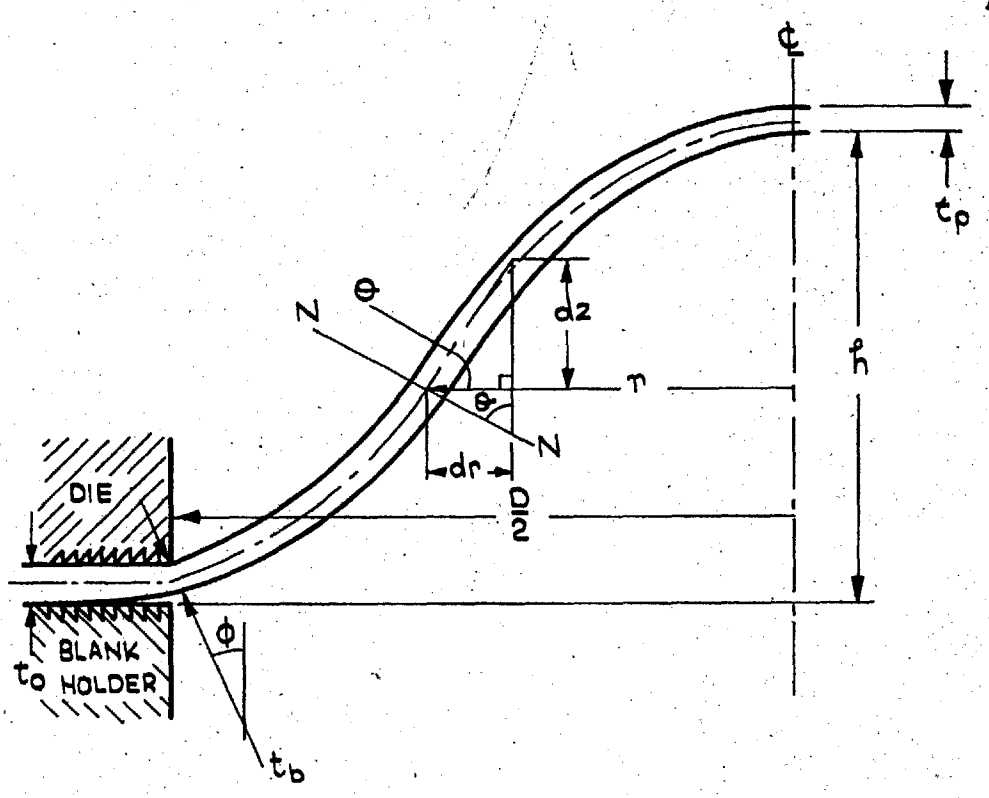



FIG. B.3. BULGE PROFILE IN STRETCH FORMING.

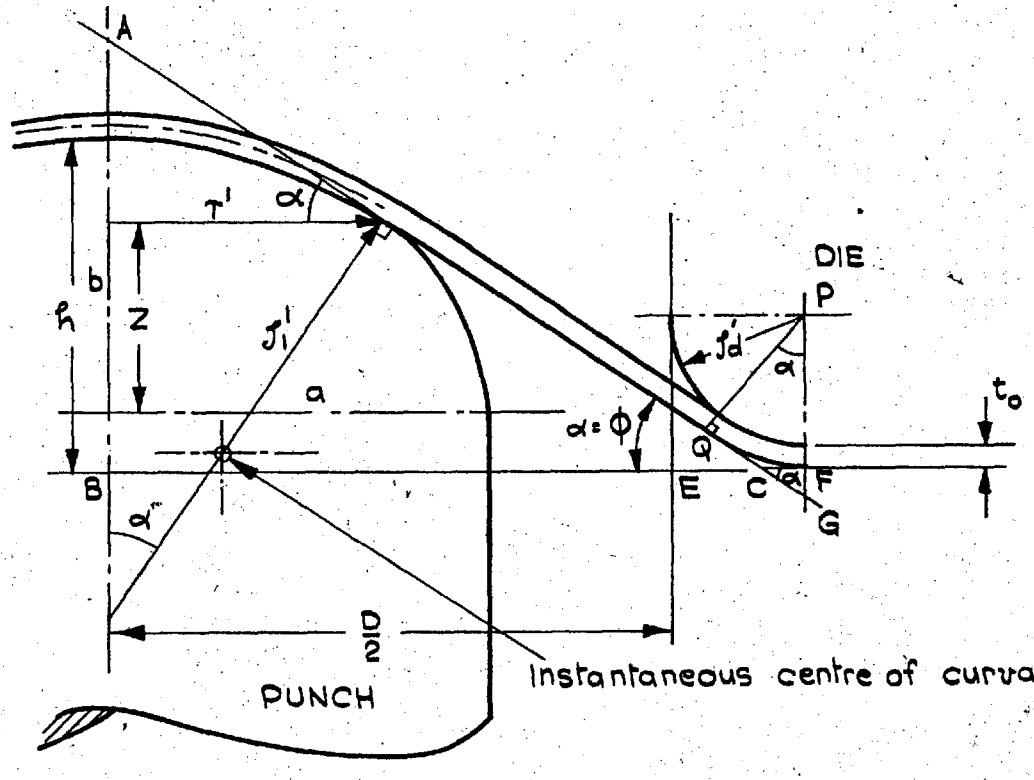



FIG. B.4. IDEALISED BULGE PROFILE FOR ELLIPSOIDAL PUNCH.

formula to calculate the bulge depth. To derive such a formula the following assumptions are made:

a) Thickness remains constant throughout the bulge and it is equal to the original thickness.

b) Angle of embrace on die is equal to that of the punch, i.e. region between die and punch is a truncated cone ($\alpha = \varphi$).

Refering to FIG.B.4, for an ellipsoidal punch:

$$\tan \alpha = \frac{AB}{BC} \tag{B.21}$$

$$AB = h + r' \tan \alpha - (b - z)$$

From eqns. (B.6) and (B.8):

$$AB = h + a\left(1 + \frac{b^2}{a^2} \cot^2 \alpha\right)^{-\frac{1}{2}} \tan \alpha - \left[b - \frac{b^2}{a^2} \left(\tan^2 \alpha + \frac{b^2}{a^2}\right)^{-\frac{1}{2}}\right] \tag{B.22}$$

$$BC = BE + EF - FC \tag{B.23}$$

where $BE = D/2$ and $EF = \rho_d'$, (B.24)

$$\frac{FG}{FC} = \tan \alpha \quad \text{hence} \quad FC = FG \cot \alpha$$

and $$FG = PG - PF = (\rho_d' + t_o)(\sec \alpha - 1)$$

therefore $$FC = (\rho_d' + t_o)(\sec \alpha - 1) \cot \alpha \tag{B.25}$$

substituting eqns. (B.24) and (B.25) in (B.23):

$$BC = D/2 + \rho_d' - (\rho_d' + t_o)(\sec \alpha - 1) \cot \alpha \tag{B.26}$$

substituting eqns. (B.26) and (B.22) in (B.21) and

simplifying:

$$h = \left[ (D/2 + \rho_d') - a\left(1 + \frac{b^2}{a^2}\cot^2\alpha\right)^{-\frac{1}{2}} \right]\tan\alpha$$

$$- (\rho_d' + t_o)(\sec\alpha - 1) + b\left[ 1 - \frac{b}{a}\left(\tan^2\alpha + \frac{b^2}{a^2}\right)^{-\frac{1}{2}} \right] \quad (B.27)$$

The following equation can be similarly derived for a flat punch:

$$h = (D/2 + \rho_d' - r_e)\tan\alpha - (\rho_1' + \rho_d' + t_o)(\sec\alpha - 1) \quad (B.28)$$

The corresponding equation for a hemispherical punch can be obtained either from eqn. (B.27) or from eqn. (B.28):

$$h = (D/2 + \rho_d')\tan\alpha - (\rho_1' + \rho_d' + t_o)(\sec\alpha - 1) \quad (B.29)$$

The accuracy of the above equations will increase with decrease of $t_o/\rho_1$ and $t_o/\rho_d'$ ratios.

APPENDIX-C.COMPUTER PROGRAM FOR THE SOLUTION
OF THE STRETCH FORMING PROBLEM
WHEN USING A HEMISPHERICAL PUNCH

```
>>AUXILIARY PROGRAM
>>JOB DESCRIPTION

JOB
LSC44EC1, KAFTANOGLU MAG.TAPE.INITILIZATION RUN1964
COMPUTING 2000 INSTRUCTIONS
TAPE
1 L0100
OUTPUT
0 LINE PRINTER 200 LINES
STORE 15 BLOCKS
COMPILER EMA

MAIN→600
AUXILIARY(0,0)
DEPTH 0
DUMPS 0
```

```
CHAPTERo

A→515

>>SET RADII=-10
I=0(1)513
AI=-10
REPEAT

>>TRANSFER RADII TO MAGNETIC TAPE
WIND TAPE(1,10)
WRITE(1,Ao)

END
CLOSE

***Z
```

```
 >>MAIN PROGRAM
 >>JOB DESCRIPTION
 JOB
 LSC44EC1, KAFTANOGLU HEMISPHERICAL RUN 37 1965
COMPUTING 240000 INSTRUCTIONS
 TAPE
 1 Lo100
 OUTPUT
 0 LINE PRINTER 2000 LINES
 STORE 32 BLOCKS
 COMPILER EMA

 MAIN→2000
 AUXILIARY(0,0)
 DEPTH 0
 DUMPS 0


 >>LAGRANGIAN INTERPOLATION
 ROUTINE2

 Kn=21(1)22
 Nn=22(-1)Kn
 L=Nn-Kn+20
 ZNn=XDIVIDE(£NnZ(Nn-1)-£LZNn,£Nn-£L)
REPEAT
 REPEAT

 RETURN

 **



 >>HERMITEAN DIFFERENTIATION
 ROUTINE4

 £20=£21-£20
 £21=£22-£21
 £22=£20+£21

 Z20=XDIVIDE(£21Z20,£20£22)
 Z21=XDIVIDE(£21Z21-£20Z21,£20£21)
Z22=XDIVIDE(£20Z22,£21£22)
 Z22=-Z20+Z21+Z22

 RETURN
```

```
**


>>HERMITEAN INTERPOLATION
ROUTINE6

En=£20-£21
Fn=Z22-£20
Gn=Z22-£21
Hn=2£20-Z22-£21
Hn=XDIVIDE(GnHnZ20,EnEn)
£22=FnGn£22/En
Z21=XDIVIDE(FnFnZ21,EnEn)
 Z22=Hn+£22+Z21

RETURN

**


 TITLE
 B.KAFTANOGLU ICST MECH.ENG.JOB.NO.LSC44EC1,1964
TITLE
 THEORETICAL ANALYSIS OF STRETCH-FORMING FOR
TITLE
  ANISOTROPIC MATERIALS - HEMISPHERICAL PUNCH-
```

```
CHAPTER1


>>ALLOCATION OF VARIABLES
A→122
B→122
C→122
D→122
E→122
F→1
G→1
H→1
U→122
V→122
W→62
X→90
Y→1
Z→100
£→650

>>COMPUTATION OF STARTING VALUES AT THE POLE
>>S.1,2 IN 5.1.1
42)£79=0.035
X6=-0.024U+0.03
£92=0.5X6

1)Q=0
JUMP13,Sn>-1
JUMP12,2>P
X5=B3-B1
X10=U3-U1
JUMP14
13)X5=B2-Bo
X10=U2-Uo
JUMP14
12)X5=0.3Zo
X10=-0.5Zo

>>COMPUTATION OF POLAR BOUNDARY CONDITIONS
>>S.1 IN 5-1-1
14)Ao=0
Uo=1
Vo=0
Q=2
JUMP71,Z71>X6
L=20(1)22
£L=Z(L+51)-X6
ZL=Z(L+54)
REPEAT
JUMPDOWN(R2)
```

```
U=Z22
JUMP71,U>0
U=0
71)JUMP82,Z73>X6
U=Z76
82)JUMPDOWN43
ACROSS1/2

>>SP INSTABILITY SEARCH FOR REGION 1
2)JUMP5,0>£16
JUMP83,£23=0
En=XMOD(1-X10/£23)
JUMP10,0.0001>En
83)En=XMOD(X10)
Fn=XMOD(£23)
JUMP84,En>0.000001
JUMP84,Fn>0.000001
JUMP10
84)JUMP40,X36>X34
O=1
JUMP41
40)O=-1
41)JUMPDOWN38

3)L=20(1)22
£L=X(L+14)
ZL=X(L+11)
REPEAT
JUMPDOWN39
£23=X10
X10=Z22
Qn=Qn+1
JUMP90,5>£400
ACROSS53/3
90)ACROSS16/2

5)£18=XMOD(0.3X10)
Qn=0
W=0
X33=£16
X36=X10
JUMP91,5>£400
ACROSS52/3
91)ACROSS15/2

>>SP INSTABILITY SEARCH FOR REGION 2
7)JUMP8,6>£16
JUMP85,£23=0
En=XMOD(1-X10/£23)
JUMP9,0.0001>En
85)En=XMOD(X10)
```

```
Fn=XMOD(£23)
JUMP86,En>0.0000001
JUMP86,Fn>0.0000001
JUMP9
86)JUMPDOWN38
6)L=20(1)22
£L=X(L+14)
ZL=X(L+11)
REPEAT
JUMPDOWN39
£23=X10
X10=Z22
Qn=Qn+1
ACROSS6/4

8)£18=XMOD(0.025X10)
£18=-0£18
Qn=0
X33=£16
X36=X10

ACROSS15/4

9)JUMP10,R≥1
JUMP11,R=0
JUMP80,Mn>0
JUMP81,Mn=0
X61=X32
JUMP80
81)JUMP80,X61>X32
JUMP10,0>Z60
Mn=-1
Z60=-1
ACROSS61/5

80)X5=X5+0.0037Z60
En=XEXP(B(Q-2)+X5)
En=Z5En-A(Q-2)
JUMP10,0.00001>En
Mn=Mn+1
Zn=1
ACROSS9/5
11)X5=X5-X15
X15=0.5X15
En=XMOD(X15)
JUMP10,0.000003>En
ACROSS8/5

 >>RESTART FROM POLE WHEN SP INSTABILITY IS OBTAINED
>>IN REGIONS 1 AND 2
10)JUMP49,£7>0
```

```
£54=Bo-B1
49)£7=10
Nn=Q-2
L=o(2)Nn
Kn=XINTPT(o.5L)
WKn=DL
REPEAT
Nn=XINTPT(o.5Q)
Kn=XINTPT(o.5J)
L=Nn(1)Kn
WL=o
REPEAT

£5=o.5QZo

£54=XMOD(o.5£54)
X6=Bo-£54
Sn=-1
Z41=-10
Z42=-10
£31=-1
£32=o
£88=-10
JUMP1

>>DIAGNOSTIC OUTPUT WHEN NO CONVERGENCE IS OBTAINED
15)JUMP16,o>£24
JUMP29
16)JUMP23,£24>-7
£24=-5
JUMP24
23)NEWLINE
CAPTION
REGION1,NO CONVERGENCE OF DE2,INTERMEDIATE RESULTS-
£24=10
24)ACROSS21/2

17)JUMP18,o>£25
JUMP29
18)JUMP25,£25>-7
£25=-5
JUMP24
25)NEWLINE
CAPTION
REGION1,NO CONVERGENCE OF DX,INTERMEDIATE RESULTS-
£25=10
JUMP24

19)JUMP20,o>£27
JUMP29
20)JUMP26,£27>-7
```

```
 £27=-5
 JUMP27
 26)NEWLINE
 CAPTION
 REGION2,NO CONVERGENCE OF DO,INTERMEDIATE RESULTS-
£27=10
 27)ACROSS4/5

 21)JUMP22,0>£26
 JUMP29
 22)JUMP28,£26>-7
 £26=-5
 JUMP27
 28)NEWLINE
 CAPTION
 REGION2,NO CONVERGENCE OF DE2,INTERMEDIATE RESULTS-
£26=10
 JUMP27

 29)ACROSS3/0

 >>SELECTION ROUTINE FOR THE MINIMIZING ROUTINE
38)JUMP30,X32>£16
 JUMP31,X35>X10
 L=-0
 X(L+32)=£16
 X(L+35)=X10
 JUMP48
 31)X(0+32)=£16
 X(0+35)=X10
 JUMP48
 30)JUMP32,X35>X10
 X(0+32)=X32
 X(0+35)=X35
 JUMP33
 32)L=-0
 X(L+32)=X32
 X(L+35)=X35
 33)X32=£16
 X35=X10

 48)RETURN

 >>MINIMIZING ROUTINE FOR SP INSTABILITY
39)Z1=£20£20
 Z2=£21£21
 Z3=£22£22
 Z4=Z2-Z3
 Z3=Z3-Z1
 Z2=Z1-Z2
```

```
Z1=£21-£22
£22=£22-£20
£21=£20-£21

Z4=Z4Z20+Z3Z21+Z2Z22
£20=Z1Z20+£22Z21+£21Z22
JUMP87,£20=0
Z22=0.5Z4/£20
JUMP88
87)Z22=0.9X35+0.1X36
88)RETURN

>>COMPUTATION OF POLAR BOUNDARY CONDITIONS.S.1 IN 5-1-1
43)Bo=X6
Co=-2Bo
Do=XMOD(Co/£62)
Eo=XLOG(B+£64Do)
Eo=XEXP(Dn+CEo)
Eo=£64Eo
Fn=XEXP(Co)
Fn=FnG
£95=0.5Fn+D
Gn=£95Fn
Hn=DD
Hn=Gn/Hn+1
Eo=£65Eo/Hn
£96=-1.3333333U/£95
£97=0.6666667UEoFn/£95
RETURN

CLOSE
```

CHAPTER2

VARIABLES1

```
>>SETTING UP LOOP PARAMETERS FOR REGION 1-S.3 IN 5-1-1
1)£24=-10
£25=-10

21)Z5=0.5QZ0
S=-1

Rn=1
X75=-1

X2=XEXP(C(Q-2))
X2=X2G/D
X3=1+0.5X2

77)£18=XMOD(0.6X10)
En=£18/Z0
JUMP23,En>0.1
£18=0.2Z0
23)£18=-£18
X10=X10-£18
Ln=-1
Qn=0
W=0
>>BEGINNING OF OUTER LOOP IN REGION 1.S.4 IN 5-1-1
15)X10=X10+£18
16)£3=U(Q-2)+X10
JUMP13,K>Q
Z8=£3
JUMP14
13)Z8=0.5U(Q+1)+0.5£3
14)R=-1
Vn=0
X4=1+Z8
X4=-0.5X2X4X3
En=Z8-X4
Fn=Z8-1
Z54=En+£61Fn
En=RnZ54
JUMP75,0.0000001>En
Z54=XDIVIDE(2X4-Z8-1,Z54)
X15=XMOD(0.6X5)
JUMP2,X15>0.001
X15=0.002
```

```
2)X5=X5-X15
>>BEGINNING OF INNER LOOP IN REGION 1.S.5 IN 5-1-1
8)X5=X5+X15

9)Z6=XEXP(B(Q-2)+X5)
Z6=Z6Z5-A(Q-2)
JUMP10,Z6>0.000001
X5=X5+0.000013
JUMP9

>>S.6 IN 5-1-1
10)X7=B(Q-2)-B(Q+1)+X5
En=RnX7
JUMP11,En>0.0000008
JUMP70,R>0
JUMP71,R>-1
X5=B(Q+1)-B(Q-2)+0.0001Rn
JUMP9
71)X5=X30+0.0001Rn
JUMP9
70)JUMP92,Sn=-1
89)JUMP75,0>X75
JUMP81,X10>0.1
£200=£202
£202=X7
JUMP83,-9990>£200
L=-Rn
JUMP83,£(Rn+201)>£(L+201)
En=XMOD(£18)
JUMP93,En>0.0000001
JUMP81,Rn>0
JUMP80
93)X10=X10-£18
£18=0.5£18
£202=£200
JUMP83

92)JUMP89,£74>4
Q=Q-2
T=Q
ACROSS3/6

11)£4=A(Q-2)+Z6
>>S.7 IN 5-1-1
X9=Z54X7
JUMP91,X9>-8
JUMP75,0>X75
X10=X10+0.000001Rn
JUMP96
91)X8=C(Q+1)+X9
```

```
 JUMPDOWN85
 JUMP61,0>Fn

 >>INTEGRATION OF COMPATIBILITY EQUATION.S.8 IN 5-1-1
INTSTEP(5)

 >>CALCULATION OF DELTA FOR INNER LOOP IN
>> REGION 1-(5-1-1)
 X12=Y1
 X13=Y1-B(Q-2)
 X14=X5-X13

 JUMP4,0>£24
 NEWLINE
 PRINT(X5)0,5
 PRINT(X14)0,5

 4)JUMP24,X5=0
 X17=XMOD(1-X13/X5)
 JUMP36,0.00005>X17
 24)En=XMOD(X5)
 Fn=XMOD(X13)
 JUMP17,En>0.000005
 JUMP17,Fn>0.000005
 JUMP36

 >>CONSTRUCTION OF CHARACTERISTIC OF CONVERGENCE
>> IN REGION 1  -SEE S.9 IN 5-1-1
 17)JUMP35,R>200
 JUMP38,Vn>5
 X25=X26
 X26=X27
 X28=X29
 X29=X30
 X27=X14
 X30=X5
 JUMP86,0>X75
 JUMP18,R≠0
 En=0.0001Rn
 90)X5=X30+En
 Fn=B(Q-2)-B(Q+1)+X5
 Fn=Z54Fn
 JUMP9,Fn>-5
 En=0.5En
 JUMP90
 86)JUMP18,R≠0
 JUMP19,0>X14
 X15=-X15
 19)X15=1.1X15
 JUMP8
```

```
 >>INVERSE LINEAR INTERPOLATION.S.9 IN 5-1-1
18)JUMP37,R>1
 JUMP41,X26≠X27
 R=0
 X5=X29+0.000003Rn
 JUMP9
 41)X5=XDIVIDE(X29-X30,X26-X27)
 X5=X29-X5X26
 JUMP9

 >>SELECTING PREVIOUS TRIALS WITH MINIMUM ERRORS-S-9
>>IN 5-1-1
 38)L=17(1)19
 ZL=XMOD(X(L+8))
 REPEAT
 L=XMAX(Z0,17,19)
 X(L+8)=X14
 X(L+11)=X5

 >>INVERSE LAGRANGIAN INTERPOLATION S.9 IN 5-1-1
37)L=20(1)22
 £L=X(L+5)
 ZL=X(L+8)
 REPEAT
 Vn=10

 JUMPDOWN(R2)
 X5=Z22
 JUMP9

 >>END OF INNER LOOP IN REGION 1 S.9 IN 5-1-1
35)ACROSS15/1

 36)ACROSS3/3

 >>IF Θ>£/2 ADJUST LOCATION OF MOVING BOUNDARY
61)JUMP62,Sn>-1
 T=Q-2
 Q=T
 ACROSS3/6
 62)T=Q
 £33=V(Q-6)
 £34=V(Q-4)
 £35=V(Q-2)
 Z51=1.5707963
 R=T
 S=Q-2
 ACROSS90/6

 >>EVALUATING STRAIN COMPATIBILITY EQUATION.S.8
>>IN 5-1-1
```

```
 5)Jn=Jn+1
 JUMP6,Jn=3
 L=L+1
 6)JUMP3,X>0.000001
 F1=0
 JUMP7
 3)£9=XEXP(2Y1+XL)
 F1=1-£9/X(L+3)
 F1=F1/X
 7)END  AUX  SEQ

 >>CALCULATION OF VARIABLES FOR INSTANTANEOUS
>> PLANE STRAIN  -SEE 5-1-1
 75)JUMP76,K>Q
 Fn=U(Q-2)+1
 En=£70U(Q-2)-£61+0.5X2X3Fn
 Fn=-X2X3Fn-U(Q-2)-1
 Gn=£70+0.5X2X3
 X10=X2X3+1
 X76=XDIVIDE(-13 5000En-RnFn,13 5000Gn-RnX10)
JUMP78
 76)En=0.5U(Q+1)+0.5U(Q-2)
 Fn=En+1
 Gn=-X2X3Fn-En-1
 X10=£70En-£61+0.5X2X3Fn
 X72=X2X3+1
 X73=0.5£70+0.25X2X3
 X76=XDIVIDE(-13 5000X10-RnGn,13 5000X73-0.5RnX72)
78)X10=X76
 X74=B(Q+1)-B(Q-2)+0.000001Rn
  £202=-10000

 £18=XMOD(0.005X76)
 JUMP82,£18>0.0001
 £18=0.0001
 82)£18=Rn£18
 X10=X10-£18
 95)S=-1
 Ln=-1
 Qn=0
 W=0
 83)X10=X10+£18
 96)X5=X74
 X75=1
 JUMP16
 81)Rn=-1
 JUMP75
 80)Rn=1
 S=-1
 X75=-1
 Ln=-1
```

```
Qn=0
W=0
Z8=xDIVIDE(£61-0.5X2X3,1+0.5X2X3+£61)
JUMP72,K>Q
£3=Z8
JUMP84
72)£3=2Z8-U(Q+1)
84)Z6=A(Q+1)-A(Q-2)
X7=0
£4=A(Q+1)
X8=C(Q-2)
JUMPDOWN85
JUMP61,0>Fn
X12=B(Q+1)
X10=-0.01
£18=-0.005
£400=10
ACROSS50/3

>>EVALUATING STRAIN COMPATIBILITY EQUATION.S.8 IN 5-1-1
85)R=R+1
N=1
H=Z6
X=A(Q-2)
Y1=B(Q-2)

Gn=X+0.5H
JUMP48,Q>3
En=xDIVIDE(X8-C(Q-2),Z6Z6)
X48=EnGnGn+C(Q-2)
JUMP49

48)£20=A(Q-4)-Gn
£21=A(Q-2)-Gn
£22=£4-Gn
Z20=C(Q-4)
Z21=C(Q-2)
Z22=X8
JUMPDOWN(R2)
X48=Z22
49)X47=C(Q-2)
X49=X8
Hn=X-0.5H

Jn=47(1)49
Hn=Hn+0.5H
Fn=xEXP(XJn)
Fn=0.5FnG
Gn=xDIVIDE(FnHn,D+Fn)
Gn=Hn-Gn
Fn=DD-GnGn
```

```
JUMP60,Fn>0
JUMP87
60)Fn=XSQRT(Fn)
X(Jn+3)=Fn/D
X(Jn+6)=Gn
REPEAT
Jn=0
L=46

Nn=1

87)RETURN

CLOSE
```

CHAPTER3

VARIABLES1

```
>>SETTING RETURN LABEL FOR INSTANTANEOUS PLANE STRAIN
50)In)=52)
52)X10=X10+£18
53)X9=X10
X8=C(Q+1)+X9
JUMP51


>>S.10-13 IN 5-1-1
3)In)=40)
51)X19=X7X7+X7X9
X19=XSQRT(£67X9X9+£68X19)
X19=£66X19
X20=D(Q+1)+X19
X21=XLOG(B+£64X20)
X21=XEXP(Dn+CX21)
X21=£64X21
En=£3+1
X4=-0.5X2X3En
X22=X4-£3-1
£13=£3£3-2£3+1
X22=£3£3+1+2X4X22+£61£13
X22=XSQRT(£69X22)
X22=X21/X22
£13=XEXP(X8)
£13=G£13
X23=X22£13
X24=XEXP(C(Q-2))
>>EVALUATION OF EQN.OF EQUILIBRIUM.S.14 IN 5-1-1
Y1=X24GE(Q-2)
X=A(Q-2)

Gn=X+0.5H
JUMP48,Q>3
X57=0.25£3+0.25Z6£96+0.75
JUMP49
48)£20=A(Q-4)-Gn
£21=A(Q-2)-Gn
£22=£4-Gn
Z20=U(Q-4)
Z21=U(Q-2)
Z22=£3
JUMPDOWN(R2)
```

```
X57=Z22
49)X56=U(Q-2)
X58=£3
Jn=0
L=46

S=S+1
Pn=On

Nn=1

>>INTEGRATION OF EQN.OF EQUILIBRIUM
INTSTEP(10)

>>CALCULATION OF DELTA IN OUTER LOOP
>>IN REGION ONE.S.15 IN 5-1-1
£14=Y1
£16=Y1-X23
£15=XMOD(1-X23/£14)
JUMP8,0>£25
NEWLINE
PRINT(X10)6,5
PRINT(£16)6,5
8)JUMP7,0>Ln
JUMP36,0.00001>£15
7)JUMP35,S>200
JUMP30,£16>0
On=-1
JUMP32
30)On=1
32)JUMP13,1>Qn
ACROSS2/1
13)JUMP38,Ln>5

>>SELECTION OF THE CORRECT BRANCH OF THE
>>CHARACTERISTIC S.15 IN5-1-1
  X31=X32
X32=X33
X34=X35
X35=X36
X33=£16
X36=X10
JUMP1,Ln=1

JUMP20,0>X75
JUMP5,S>1
JUMP24,S=0
JUMP21,X33>X32
23)£18=2£18
24)£202=-10000
ACROSS83/2
```

```
21)JUMP33,On≠Pn
JUMP22,X32>0
JUMP23
22)X10=X76
£18=0.1£18
ACROSS95/2
20)JUMP5,S≠0
£18=On£18
JUMP(In)
5)Yn=£16-X32
Yn=RnYn£18
JUMP6,Yn>0
JUMP9,3>W
JUMP2
9)£18=XMOD(1.5£18)
26)£18=Rn£18
JUMP(In)

6)JUMP33,On≠Pn
W=5
JUMP34,£16>X32
£18=XMOD(1.2£18)
£18=-Rn£18
JUMP(In)
34)JUMP2,X32>0
£18=XMOD(1.3£18)
JUMP26
>>POSSIBILITY OF SP INSTABILITY
2)ACROSS3/1

>>SELECTING PREVIOUS TRIALS WITH MINIMUM ERRORS
>>AND BINARY CHOPPING.S.15 IN 5-1-1
33)£18=-0.5£18
Ln=1
JUMP57,X33>X32
X37=X33
X38=X36
X39=X32
X40=X35
JUMP(In)
57)X37=X32
X38=X35
X39=X33
X40=X36
JUMP(In)

1)JUMP14,£16>0
X37=£16
X38=X10
JUMP37
14)X39=£16
```

```
X40=X10
JUMP37
38)JUMP80,£16>0
L=XMIN(X0,31,33)
X37=£16
X38=X10
JUMP81
80)L=XMAX(X0,31,33)
X39=£16
X40=X10
81)XL=£16
X(L+3)=X10

37)L=20(1)22
£L=X(L+11)
ZL=X(L+14)
REPEAT
Ln=10
Nn=22
L=20(1)22
Kn=L
JUMP91,22>L
Nn=21
Kn=20
91)En=£Nn-£Kn
JUMP62,En=0
REPEAT
>>INVERSE LAGRANGIAN INTERPOLATION TO
>>FIND A BETTER DX.S.15 IN 5-1-1
JUMPDOWN(R2)
L=-Rn
JUMP61,Z22>X(L+39)
JUMP62
61)JUMP63,X(Rn+39)>Z22
62)Z22=0.5X38+0.5X40
63)X10=Z22
JUMP53,£400>5
42)JUMP25,X75>0
ACROSS16/2
25)£202=-10000
ACROSS96/2
40)JUMP24,X75>0
ACROSS15/2
35)JUMP36,0.001>£15
ACROSS17/1
>>END OF OUTER LOOP.S.15 IN 5-1-1
36)ACROSS1/6

>>EVALUATION OF EQN.OF EQUILIBRIUM.S.14 IN 5-1-1
10)Jn=Jn+1
JUMP11,Jn=3
```

```
L=L+1
11)JUMP4,X>0.00001
F1=£97
JUMP12
4)Fn=XEXP(XL)
Fn=0.5GFn
Fn=D+Fn
Gn=FnX(L+3)
En=XDIVIDE(X(L+9)X(L+6),DX(L+3))
F1=1/Gn+En/X
Hn=X(L+9)-1
F1=UF1+Hn/X
F1=Y1F1
12)END  AUX  SEQ

CLOSE
```

CHAPTER4

VARIABLES1

```
 >>SELECTION OF THE CORRECT BRANCH OF THE CHARACTERISTIC
>>S.6 IN 5-1-2
 48)JUMP49,X7>0
 O=-1
 JUMP51
 49)O=1
 51)En=XMOD(Z)
 JUMP72,En>0.0000001
 X10=-0.00000010
 JUMP73
 72)X10=XMOD(0.001Z)
 X10=-OX10+Z
 73)£18=XMOD(X10)
 £18=-0.0150£18
 12)Xn=-10

 X1=-1

 S=-1
 X10=X10-£18
 Ln=0
 >>BEGINNING OF INNER LOOP IN REGION 2 S.5-14 IN 5-1-2
15)X10=X10+£18
 6)£3=V(Q-2)+X10
 JUMP21,£3>0.0001
 X10=0.9X10
 JUMP6
 21)£8=XTAN(£3)
 £8=1/£8
 JUMP2,Q>Rn
 Z22=2X10/Z6-Bn
 JUMP3

 2)£20=A(Q-2)-A(Q-4)
 £21=Z6
 £22=£20+£21
 Z20=XDIVIDE(£21V(Q-4),£20£22)
 Z21=XDIVIDE(£22V(Q-2),£20£21)
  Z22=XDIVIDE(2£21+£20,£21£22)
  Z22=Z20-Z21+Z22V(Q-2)+Z22X10
```

```
3)Z9=-£4£8Z22
JUMP23,£74>-0.5
Z8=Z9
JUMP24
23)Z8=0.5Z9+0.5U(Q+1)
24)Un=£70Z8-£61
JUMP22,Jn>2
£16=Un
JUMP4
22)En=OUn
JUMP15,0≥En
X9=XDIVIDE(-1-Z8,Un)
X9=X7X9
JUMP15,-10>X9
X19=X7X7+X7X9
X19=XSQRT(£67X9X9+£68X19)
X19=£66X19
X20=D(Q+1)+X19
X21=XLOG(B+£64X20)
X21=XEXP(Dn+CX21)
X21=£64X21
X8=C(Q+1)+X9
£13=XEXP(X8)
£13=G£13
X22=Z9Z9-2Z9+1
X22=Z9Z9+£61X22+1
X22=XSQRT(£69X22)

X22=X21/X22
X23=X22£13
X24=XSIN(£3)
X24=Wn/X24
>>COMPUTATION OF DELTA FOR INNER
>>LOOP IN REGION TWO.S14 IN 5-1-2
£16=X24-X23
£15=XMOD(1-X23/X24)

JUMP5,0>£27
NEWLINE
PRINT(X10)0,5
PRINT(£16)0,5
PRINT(Z9)0,5
PRINT(X9)0,5

5)JUMP4,0>Xn
JUMP36,0.00001>£15
4)S=S+1
Pn=On
JUMP35,S>250
JUMP41,Jn>2
£17=XMOD(£16)
```

```
JUMP41,£17>0.00000001
93)Jn=10
Z=X10
JUMP48
41)JUMP38,Ln>5

JUMP30,£16>0
On=-1
JUMP32
30)On=1
32)JUMP8,1>Qn
ACROSS7/1
8)JUMP9,-1>In
>>CONSTRUCTION OF THE CHARACTERISTIC
>>OF CONVERGENCE FOR REGION 2 S.14 IN 5-1-2
X31=X32
X32=X33
X34=X35
X35=X36
X33=£16
X36=X10

JUMP1,Ln=1
JUMP55,S≠0
JUMP50,Jn>2
£18=On£18
50)JUMP15
55)JUMP52,2>Jn
JUMP52,Xn>0
Yn=£16-X32
Yn=-0£18Yn

JUMP95,X1>0
JUMP95,Yn>0
X1=1
95)JUMP7,Yn>0
96)£18=2£18
JUMP15
7)JUMP56,0>X32
JUMP16,2>S
JUMP96,0>X1

>>POSSIBILITY OF SP INSTABILITY.S.14 IN 5-1-2
ACROSS6/1
16)JUMP96,0.5>£89
X10=Z
£18=XMOD(X10)
JUMP71,£18>0.0000001
£18=0.0000001
71)£18=-0.00001O£18
£89=0.0003
```

```
JUMP12

56)Xn=10
JUMP33,On≠Pn
£18=XMOD(X10)
JUMP74,£18>0.0000001
£18=0.000001
74)£18=-0.080£89£18
JUMP15
52)JUMP33,On≠Pn
£18=1.6£18
JUMP15

>>BINARY CHOPPING OF THE CHARACTERISTIC IN
>>REGION 2,INNER LOOP
33)JUMP57,X33>X32
X37=X33
X38=X36
X39=X32
X40=X35
JUMP58
57)X37=X32
X38=X35
X39=X33
X40=X36
58)JUMP10,2>Jn
In=-6
£18=-0.5£18
JUMP15

10)JUMP37,S>1
X10=XDIVIDE(X35-X36,X32-X33)
X10=X35-X10X32
Ln=1
JUMP6
9)In=In+1
JUMP11,On≠Pn
£18=0.5£18
JUMP13
11)£18=-0.5£18
JUMP13,Jn>12
JUMP17,In=-5
Jn=13
JUMP18
17)Jn=14
18)X32=£16
X35=X10
X37=£16
X38=X10
JUMP14
13)JUMP25,Jn=14
```

```
 JUMP38,In>-5
 25)JUMP19,13>Jn
 Jn=13

 >>SELECTION OF TRIALS WITH MINIMUM ERROR
>>IN REGION 2,INNER LOOP.S.14 IN 5-1-2
19)X31=£16
 X34=X10
 JUMP1
 38)JUMP80,£16>0
 L=XMIN(X0,31,33)
 JUMP81
 80)L=XMAX(X0,31,33)
 81)XL=£16
 X(L+3)=X10

 1)JUMP59,£16>0
 X37=£16
 X38=X10
 JUMP14
 59)X39=£16
 X40=X10
 14)JUMP15,-2>In
 >>INVERSE LAGRANGIAN INTERPOLATION IN REGION 2,
>>INNER LOOP.S.14 IN 5-1-2
 37)L=20(1)22
 £L=X(L+11)
 ZL=X(L+14)
 REPEAT
 Ln=10
 Nn=22
 L=20(1)22
 Kn=L
 JUMP91,22>L
 Nn=21
 Kn=20
 91)En=£Nn-£Kn
 JUMP62,En=0
 REPEAT

 JUMPDOWN(R2)
 JUMP61,Z22>X(0+39)
 JUMP62
 61)L=-0
 JUMP63,X(L+39)>Z22
 62)Z22=0.5X38+0.5X40

 63)X10=Z22
 JUMP6

 35)JUMP94,Jn>2
```

```
JUMP93
94)JUMP36,0.001>£15
In=0
Qn=0
JUMP16,£89>0.5
ACROSS19/1

>>END OF INNER LOOP IN REGION 2
36)Mn=-1
ACROSS2/5

CLOSE
```

CHAPTER 5

VARIABLES 1

```
 >>SETTING PARAMETERS AND STARTING VALUES FOR
>> OUTER AND INNER LOOPS IN REGION 2.S.2 IN 5-1-2
1)£26=-10
 £27=-10

 4)Z5=0.5QZ0

 Zn=-1
 Mn=-1

 Z60=1
 X15=0.005
 61)X5=B(Q+1)-B(Q-2)+0.0000011Z60
 R=-1
 Vn=0
 X5=X5-X15
 8)X5=X5+X15

 9)Z6=XEXP(B(Q-2)+X5)
 Z6=Z5Z6-A(Q-2)
 X7=B(Q-2)-B(Q+1)+X5
 En=XMOD(X7)
 JUMP91,En>0.0000008
 En=XSIGN(X7)
 X5=X5+0.000000923456 7En
 JUMP9
 91)JUMP20,Z6>0.000001
 X5=X5+0.000013
 JUMP9

 20)S=-1
 In=0
 Qn=0
 £89=1
 O=1
 Jn=0
 Ln=0
 £4=A(Q-2)+Z6
 Wn=XDIVIDE(An,6.283185 4£4)
 £18=XMOD(0.18X10)
```

```
JUMP71,£18>0.000001
£18=0.00001
71)X10=X10-£18
ACROSS15/4

>>EVALUATION OF STRAIN COMPATIBILITY EQN. IN REGION 2,
>>OUTER LOOP S.15 IN 5-1-2
2)R=R+1
N=1
H=Z6
X=A(Q-2)
Y1=B(Q-2)

W=X+0.5H
JUMP18,Q>Rn

X43=B(Q-2)+X5
X43=XEXP(2X43+X8)
En=XCOS(£3)
X43=1-X43/En
X43=X43/£4
£20=£50
£21=£51
£22=£4
Z20=X41
Z21=X42
Z22=X43
JUMPDOWN(R4)
Cn=£51Z22+X42
Cn=X45-X46Cn
Cn=X44Cn-2X42

£20=A(Q-2)
£21=£4
£22=Bn
Z20=V(Q-2)
Z21=£3
Z22=W
JUMPDOWN(R6)
X51=Z22
£22=Cn
Z20=C(Q-2)
Z21=X8
Z22=W
JUMPDOWN(R6)
X48=Z22
JUMP19
18)£20=A(Q-4)-W
£21=A(Q-2)-W
£22=£4-W
Z20=V(Q-4)
```

```
Z21=V(Q-2)
Z22=£3
JUMPDOWN(R2)
X51=Z22
Z20=C(Q-4)
Z21=C(Q-2)
Z22=X8
JUMPDOWN(R2)
X48=Z22
19)X47=C(Q-2)
X49=X8
X50=V(Q-2)
X52=£3
L=50(1)52
XL=XCOS(XL)
REPEAT

Nn=1

Kn=0
L=46

 >>INTEGRATION OF STRAIN COMPATIBILITY EQN.IN REGION 2,
>>OUTER LOOP S.15 IN5-1-2
 INTSTEP(5)
 X12=Y1
 X13=Y1-B(Q-2)
 >>CALCULATION OF DELTA IN REGION 2,OUTER LOOP
X14=X5-X13
 JUMP6,0>£26
 NEWLINE
 PRINT(X5)0,5
 PRINT(X14)0,5

 6)JUMP24,X5=0
 X17=XMOD(1-X13/X5)
 JUMP36,0.00005>X17
 24)En=XMOD(X5)
 Fn=XMOD(X13)
 JUMP7,En>0.0000005
 JUMP7,Fn>0.0000005
 JUMP36

 7)JUMP35,R>100
 JUMP38,Vn>5
 >>CONSTRUCTION OF CHARACTERISTIC OF
>>CONVERGENCE IN REGION2,OUTER LOOP
X25=X26
 X26=X27
 X28=X29
 X29=X30
```

```
X27=X14
X30=X5

JUMP10,R≠0
JUMP92,0>Zn
JUMP93,Z60>0
JUMP95,X27>0
En=XMOD(B(Q-2)-B(Q+1)+X30)
JUMP96,En>0.00001
JUMP36
96)X15=0.000008119
JUMP8
95)X15=-0.0001789 45
JUMP8
93)X15=0.0001234 56
JUMP8
92)JUMP8,Z60>0
JUMP36,0≥X14
X15=-X15
JUMP8
>>INVERSE LINEAR INTERPOLATION IN REGION 2,
>>OUTER LOOP S.16
>>IN 5-1-2
10)JUMP40,R=2
X5=XDIVIDE(X29-X30,X26-X27)
 X5=X29-X5X26

97)En=XSIGN(B(Q-2)-B(Q+1)+X5)
 JUMP98,Z60>0
JUMP9,0>En
JUMP36
98)JUMP9,En>0
Z60=-1
X15=0.0005
JUMP61

40)En=XMOD(B(Q-2)-B(Q+1)+X5)
JUMP37,En>0.00001
JUMP36

>>SELECTION OF PREVIOUS TRIAL WITH MINIMUM ERROR
>>IN REGION 2,OUTER LOOP S.16,5-1-2
38)L=17(1)19
ZL=XMOD(X(L+8))
REPEAT

L=XMAX(Z0,17,19)
X(L+8)=X14
X(L+11)=X5
>>INVERSE LAGRANGIAN INTERPOLATION REGION.2,
>>O.L.STEP 16 IN 5-1-2
```

```
37)L=20(1)22
£L=X(L+5)
ZL=X(L+8)
REPEAT
Vn=10
JUMPDOWN(R2)
X5=Z22
JUMP97

35)JUMP36,0.02>X17
ACROSS21/1
36)ACROSS4/6

>>EVALUATION OF STRAIN COMPATIBILITY EQN.IN REGION 2,
>>OUTER LOOP. S.15 IN 5-1-2
5)Kn=Kn+1
JUMP3,Kn=3
L=L+1
3)£9=XEXP(2Y1+XL)
F1=1-£9/X(L+3)
F1=F1/X
END  AUX  SEQ

CLOSE
```

CHAPTER6

VARIABLES1

```
 >>TRANSFER CONVERGED VALUES TO MAIN VARIABLES
>>IN REGION 1
 1)AQ=£4
 BQ=X12
 CQ=X8
 DQ=X20
 EQ=X22
 UQ=£3
 VQ=X55/D
 VQ=XARCTAN(X52,VQ)

 £400=0

 >>CHECK WHETHER A POSSIBILITY FOR SP INSTABILITY
>>EXISTED IN REGION 1
 JUMP73,£88>0
 JUMP73,0>£7
 JUMP73,DQ>D(Q-2)
 L=Q-4
 JUMP72,L>0
 L=0
 72)Kn=XINTPT(0.5L)
 JUMP74,WKn>0.00001
 75)£54=XMOD(0.5£54)
 X6=B0+£54
 Sn=-1
 Z41=-10
 Z42=-10
 £31=-1
 £32=0
 ACROSS1/1
 74)En=XMOD(WKn-DL)
 JUMP75,En>0.01
 £88=10
 >>CHECK WHETHER MOVING BOUNDARY IS REACHED
73)JUMP3,Q≥T
 >>ADVANCE SUFFIX TO COMPUTE MORE POINTS IN REGION 1
```

```
 Q=Q+2
 ACROSS1/2

 >>MOVING BOUNDARY REACHED
 3)JUMP55,0>Z41
 >>INTERPOLATION OF VARIABLES WHEN
>>MOVING BOUNDARY IS BETWEEN TWO NODAL POINTS
En=Z43-Z23
 Fn=Z43-Z40
 Fn=Fn/En
 JUMP40,Fn>0.1
 Z50=-10
 R=Q
 JUMP41
 40)R=Q-2
 Z50=10
 41)S=T
 Z51=Z40
 £33=A(S-4)
 £34=A(S-2)
 £35=AS
 JUMPDOWN42
 AR=Z40
 Q=R

 JUMP5

 >>CALCULATION OF BOUNDARY CONDITIONS AT MOVING BOUNDARY
>>S.WN 5-1-2
 55)Z40=AQ
 5)Rn=Q+3
 Gn=XSIN(VQ)
 £10=BQ
 £12=VQ
 Hn=XEXP(CQ)
 Hn=GHn
 An=6.283185 4AQEQHnGn
 Bn=XTAN(VQ)
 X2=Hn/D
 X3=1+0.5X2
 En=1+UQ
 X4=-0.5X2EnX3
 £71=X4-UQ-1
 £72=UQUQ-2UQ+1
 £71=£61-UQUQ-2X4£71-£61£72
 £71=XSQRT(£73-£70£71)
 £52=£61+£71
 £52=£52/£70
 Bn=-Bn£52/AQ

 Kn=0
```

```
 Nn=Q-2
 L=Nn(2)Q
 Kn=Kn+1
 JUMP9,AL>0.00001
 X(Kn+40)=0
 JUMP10
 9)En=XEXP(2BL+CL)
 Fn=XCOS(VL)
 En=1-En/Fn
 X(Kn+40)=En/AL
 £(Kn+49)=AL
 10)REPEAT
 X44=XEXP(-2BQ-CQ)
 X45=AQX42-1
 X45=X45GnBn
 X46=Fn

 Z6=AQ-A(Q-2)
 X10=BnZ6
 X5=X42Z6
 Q=Q+2
 ACROSS1/5

 >>TRANSFER CONVERGED VALUES TO MAIN VARIABLES
>>IN REGION 2
 4)AQ=£4
 BQ=X12
 CQ=X8
 DQ=X20
 EQ=X22
 UQ=Z9
 VQ=£3

 >>CHECK WHETHER A POSSIBILITY FOR SP INSTABILITY
>>EXISTED IN REGION 2
 JUMP76,£88>0
 JUMP76,0>£7
 JUMP76,DQ>D(Q-2)
 L=Q-4
 Kn=XINTPT(0.5L)
 JUMP78,WKn>0.00001
 77)£54=XMOD(0.5£54)
 X6=B0+£54
 Sn=-1
 Z41=-10
 Z42=-10
 £31=-1
 £32=0
 ACROSS1/1
 78)En=XMOD(WKn-DL)
 JUMP77,En>0.01
```

```
£88=10

>>RESTORE OVERWRITTEN VALUES AT MOVING BOUNDARY
76)JUMP43,0>Z41
L=T+4
JUMP43,Q≠L
JUMP45,0>Z50
R=T-2
AR=Z23
BR=Z24
CR=Z25
DR=Z26
ER=Z27
UR=Z28
VR=Z29
JUMP43

45)S=Q
R=T
Z51=0.5TZ0
UT=£52
Nn=32
L=T(2)Q
Nn=Nn+1
£Nn=XEXP(-BL)
£Nn=AL£Nn
REPEAT
JUMPDOWN42
En=XEXP(BT)
AT=EnZ51

>>CHECK WHETHER REGION 2 IS FULLY PLASTICALLY DEFORMED
43)JUMP11,£74>-0.5
JUMP11,0>Z41
£71=B(Q-2)-B(Q-1)
£72=BQ-B(Q+1)
JUMP11,£72>0
JUMP11,0>£71
£71=XMOD(U(Q-2)-UQ)
£72=XMOD(U(Q-4)-U(Q-2))
£71=£71/£72
JUMP11,2>£71
NEWLINE
CAPTION
REGION 2 IS NOT FULLY PLASTICALLY DEFORMED AT STAGE
PRINT(P)2,0
NEWLINE
NEWLINE
N=T-2
£71=N
£72=M
```

```
 Ln=XINTPT(£71/£72)
 Ln=LnM
 ACROSS7/0

 >>MODIFY MOVING BOUNDARY LOCATION IF TOO MUCH
>> OVERESTIMATED
 11)JUMP44,Q≥J
 JUMP46,UQ>-0.8
 JUMP47,Sn>-1
 JUMPDOWN58
 T=T-2
 Q=T
 £32=5
 JUMP3
 47)JUMP60,Z42>0
 Z42=10
 JUMPDOWN58
 60)£32=5
 Z41=10
 X11=XMOD(0.5X11)
 Z40=Z40-X11
 Q=T
 JUMPDOWN59
 JUMP3

 >>ADVANCE CALCULATION TO NEXT POINT IN REGION 2
46)Q=Q+2
 ACROSS1/5

 >>AT CLAMPED EDGE CHECK IF BOUNDARY CONDITION
>> IS SATISFIED
 44)Sn=Sn+1
 £30=XMOD(BJ)
 JUMP82,30>Sn
 £91=1
 JUMP83,0.0001>£30
 JUMP50
 82)JUMP50,£30>0.000008
 £74=£74+1
 83)ACROSS4/0

 50)£28=£29
 JUMP48,BJ>0
 Pn=0
 £29=-1
 JUMP49
 48)£29=1
 Pn=2
 49)JUMP51,£31>5
 >>CONSTRUCTION OF CHARACTERISTIC FOR BOUNDARY
>>MATCHING.S.1-2 IN 5-1-3
```

```
Z30=Z31
Z31=Z32
Z33=Z34
Z34=Z35
Z32=BJ
Z35=Z40
JUMP52,£31>0
JUMP53,Sn=0
JUMP57,£28≠£29
53)JUMP54,BJ>0
JUMPDOWN58
T=T-2
Q=T
£32=0
JUMP3
54)JUMP56,£32>0
JUMP56,Z41>0
T=T+2
Q=T
X10=U(Q-2)-U(Q-4)
X5=B(Q-2)-B(Q-4)
ACROSS1/2

56)X11=XMOD(0.5X11)
Z40=Z40+X11
JUMP63,Z42>0
T=T+2
63)£32=0
Z41=10
Z42=10
Q=T
JUMPDOWN59
JUMP3
57)JUMP61,Z42>0
T=T+Pn
Z42=10
JUMP61,BJ>0
JUMPDOWN58
>>BINARY CHOPPING OF CHARACTERISTIC FOR BOUNDARY
>>MATCHING.S.3 IN 5-1-3
61)Z40=0.5Z40+0.5Z34
£31=1
JUMP62,Z32>Z31
Z36=Z32
Z37=Z35
Z38=Z31
Z39=Z34
JUMP63
62)Z38=Z32
Z39=Z35
Z36=Z31
```

```
 Z37=Z34
 JUMP63

 >>SELECTION OF PREVIOUS TRIALS WITH MINIMUM
>>ERROR.S.4 IN 5-1-3
 52)JUMP64,BJ>0
 Z36=BJ
 Z37=Z40
 JUMP65
 64)Z38=BJ
 Z39=Z40
 JUMP65
 51)JUMP86,BJ>0
 L=xMIN(Z0,30,32)
 Z36=BJ
 Z37=Z40
 JUMP66
 86)L=xMAX(Z0,30,32)
 Z38=BJ
 Z39=Z40
 66)ZL=BJ
 Z(L+3)=Z40
 65)L=20(1)22
 £L=Z(L+10)
 ZL=Z(L+13)
 REPEAT
 £31=10

 >>INVERSE LAGRANGIAN INTERPOLATION FOR BOUNDARY
>>MATCHING.S.4 IN 5-1-3
 Nn=22
 L=20(1)22
 Kn=L
 JUMP98,22>L
 Nn=21
 Kn=20
 98)En=£Nn-£Kn
 JUMP68,En=0
 REPEAT
 JUMPDOWN(R2)
 JUMP67,Z22>Z39
 JUMP68
 67)JUMP69,Z37>Z22
 68)Z22=xDIVIDE(Z39-Z37,Z38-Z36)
Z22=Z39-Z22Z38
 69)JUMP88,0>£91
 Z22=0.5Z37+0.5Z39
 88)Z40=Z22
 JUMP63

 >>INTERPOLATION OF VALUES AT MOVING BOUNDARY
```

```
>>AT 90 DEGREE.
90)JUMPDOWN42
Ln=S-6
L=20(I)22
Ln=Ln+2
ZL=ALn
REPEAT
JUMPDOWN(R2)
AR=Z22
JUMPDOWN58
Z42=10
Z41=10
X11=XMOD(0.5X11)
Z40=A(T-2)+X11
JUMP3

>>PRESERVES TEMPORARY VALUES AT MOVING BOUNDARY
58)L=-20
Kn=T-2
Nn=Kn(2)T
L=L+20
Z(L+23)=ANn
Z(L+24)=BNn
Z(L+25)=CNn
Z(L+26)=DNn
Z(L+27)=ENn
Z(L+28)=UNn
Z(L+29)=VNn
REPEAT
X11=AT-A(T-2)
RETURN

>>RESTORES TEMPORARY VALUES AT MOVING BOUNDARY
59)L=-20
Kn=T-2
Nn=Kn(2)T
L=L+20
ANn=Z(L+23)
BNn=Z(L+24)
CNn=Z(L+25)
DNn=Z(L+26)
ENn=Z(L+27)
UNn=Z(L+28)
VNn=Z(L+29)
REPEAT
RETURN

>>INTERPOLATION ROUTINE FOR MOVING BOUNDARY
42)Ln=S-6
L=20(I)22
Ln=Ln+2
```

```
FL=£(L+13)-Z51
ZL=BLn
REPEAT
JUMPDOWN(R2)
BR=Z22
Ln=S-6
L=20(1)22
Ln=Ln+2
ZL=CLn
REPEAT
JUMPDOWN(R2)
CR=Z22
Ln=S-6
L=20(1)22
Ln=Ln+2
ZL=DLn
REPEAT
JUMPDOWN(R2)
DR=Z22
Ln=S-6
L=20(1)22
Ln=Ln+2
ZL=ELn
REPEAT
JUMPDOWN(R2)
ER=Z22
Ln=S-6
L=20(1)22
Ln=Ln+2
ZL=ULn
REPEAT
JUMPDOWN(R2)
UR=Z22
Ln=S-6
L=20(1)22
Ln=Ln+2
ZL=VLn
REPEAT
JUMPDOWN(R2)
VR=Z22
RETURN

CLOSE
```

CHAPTER0

VARIABLES1

```
 >>START EXECUTION OF PROGRAM
>>TRANSFER RADII ON MAGNETIC TAPE TO
>>MAIN VARIABLES
 WIND TAPE(1,10)

 READ FWD(1,£101)
 L=0(1)122
 AL=£(L+101)
 REPEAT
 >>TEST FOR SIGN OF RADII
 JUMP75,A10>0
 JUMP70
 >>READ ALL VARIABLES FROM MAGNETIC TAPE
75)READ FWD(1,£101)
 L=0(1)122
 BL=£(L+101)
 REPEAT
 READ FWD(1,£102)
 L=0(1)122
 CL=£(L+101)
 REPEAT
 READ FWD(1,£101)
 L=0(1)122
 DL=£(L+101)
 REPEAT
 READ FWD(1,£101)
 L=0(1)122
 EL=£(L+101)
 REPEAT
 READ FWD(1,£101)
 L=0(1)122
 UL=£(L+101)
 REPEAT

 READ FWD(1,£102)
 L=0(1)122
```

```
VL=£(L+101)
REPEAT
READ FWD(1,£101)
L=0(1)62
WL=£(L+101)
REPEAT
READ FWD(1,£101)
L=0(1)99
XL=£(L+101)
REPEAT
READ FWD(1,£101)
L=0(1)100
ZL=£(L+101)
REPEAT
READ FWD(1,£0)
READ FWD(1,£101)

A=£101
B=£102
C=£103
D=£104
E=£105
F=£106
G=£107
U=£108
Dn=£109
I=XINTPT(£110)
J=XINTPT(£111)
K=XINTPT(£112)
M=XINTPT(£113)
P=XINTPT(£114)
T=XINTPT(£115)

JUMP 71,P>1
>>READ DATA TAPE
70)READ(I)
1)I=I-1

READ(A)
READ(B)
READ(C)
READ(D)
READ(F)
READ(G)
READ(M)
READ(J)
READ(£61)
L=71(1)73
READ(ZL)
READ(Z(L+3))
REPEAT
```

```
 U=Z74
 T=XINTPT(0.2J)
 T=2T

 X80=0

 >>PRINT INPUT VARIABLES WITH CAPTIONS
L=0(1)5
 NEWLINE
 REPEAT
 CAPTION
 PARAMETERS USED IN THE THEORETICAL COMPUTATION-
NEWLINE
 NEWLINE
 CAPTION
 A=
 PRINT(A)6,0
 CAPTION
 B=
 PRINT(B)1,4
 CAPTION
 N=
 PRINT(C)1,3
 NEWLINE
 CAPTION
 PUNCH PROFILE RADIUS=
 PRINT(D)1,3
 NEWLINE
 CAPTION
 DIAMETER OF THE CLAMPED EDGE=
PRINT(F)1,3
 NEWLINE
 CAPTION
 INITIAL THICKNESS=
 PRINT(G)1,4
 NEWLINE
 CAPTION
 NUMBER OF INTEGRATION STEPS=
PRINT(J)3,0
 NEWLINE
 CAPTION
 STRAIN RATIO IN TENSION=
 PRINT(£61)2,2
 NEWLINE
 CAPTION
 GIVEN VARIATION OF COEFFICIENT OF FRICTION WITH POLAR-
CAPTION
  STRAIN-
 NEWLINE
 CAPTION
 POLAR CIRCUMFERENTIAL STRAIN   COEF. OF FRICTION
```

```
NEWLINE
L=71(1)73
Kn=0(1)10
SPACE
REPEAT
PRINT(ZL)1,3
Kn=0(1)19
SPACE
REPEAT
PRINT(Z(L+3))1,3
NEWLINE
REPEAT

L=0(1)7
NEWLINE
REPEAT
£91=-1

X60=0
£74=-1

£90=0

 >>COMPUTE CONSTANTS INVOLVING R
£62=4£61£61£61+12£61£61+9£61+2
£62=XSQRT(0.33333333£62)
 £62=XDIVIDE(2£61+1,£62)
 £63=XDIVIDE(2£61+4,3£61+3)
£63=XSQRT(£63)
 £64=1/£63
 £65=XSQRT(0.66666667+0.33333333£61)
£66=£61+2
 £66=XSQRT(0.66666667£66)
 £66=XDIVIDE(£66,2£61+1)
 £67=2£61£61+3£61+1
 £68=4£61+2
 £69=XDIVIDE(1.5,£61+2)
 £70=£61+1
 £73=£61£61

 >>SET INITIAL CONDITIONS ON BLANK

 Z0=0.5F/J
 J=2J
 Kn=J+1
 L=1(2)Kn
 Ln=L-1
 AL=0.5LnZ0
 BL=0
 CL=0
 DL=0
```

```
EL=0
UL=0
VL=0
REPEAT

>>SET INITIAL PAPAMETERS FOR LOOPS AND COMPUTATION
£31=-1
£32=0
Z41=-10
Z42=-10
£7=-10
£88=-10
K=0
M=2M
£46=0
£49=0
Dn=xLOG(A)
P=1
Sn=-1
£82=0
ACROSS42/1

>>OUTPUT OF RESULTS PER STAGE
4)JUMP6,0>£88
En=0.5TZ0
JUMP98,En>£5
JUMP98,X89>2
Sn=-1
Z41=-10
Z42=-10
£31=-1
£32=0
X6=B0
£91=-1
£88=-10
£7=-10
X89=5
ACROSS1/1
98)NEWLINE
NEWLINE
CAPTION
PLASTIC INSTABILITY AT-
PRINT(£5)1,4
NEWLINE

>>CALCULATION OF BULGE DEPTH
6)£27=0
£40=0
£26=-A2
£29=xTAN(V2)
£6=0
```

```
L=2(2)J
£38=AL
£41=XTAN(VL)
En=£38£38
Fn=£37£37
Gn=En£38-Fn£37
Fn=En-Fn
En=£38-£37

Un=0
Kn=36(1)38
Vn=0
Wn=1
Yn=1
Nn=36(1)38
JUMP 5,Nn=Kn
Vn=£Nn+Vn
Wn=Wn£Nn
Xn=£Kn-£Nn
Yn=YnXn
5)REPEAT
Hn=0.333333333333Gn-0.5VnFn+WnEn
Hn=Hn£(Kn+3)/Yn
Un=Un+Hn
REPEAT
£36=£36+Un
£36=£37
£37=£38
£39=£40
£40=£41
REPEAT

Kn=41
L=0(J)J
Kn=Kn+1
£Kn=XEXP(CL)
£Kn=0.5G£Kn
REPEAT
En=XCOS(VJ)
£94=£6-En£43-£42+G
£94=25.4£94

>>CONTINUATION OF OUTPUT
An=0.000453592743An
NEWLINE
CAPTION
STAGE-
PRINT(P)2.0
CAPTION
PUNCH LOAD-
```

```
 PRINT(An)2,4
 CAPTION
 BULGE DEPTH-
 PRINT(£94)3,3
 CAPTION
 MOVING BOUNDARY IS AT-
 PRINT(Z40)2,4
 PRINT(57.2957795131)2,2
 En=xEXP(-£10)
 En=Z40En
 CAPTION
 INITIAL RADIUS,
 PRINT(En)1,4
 NEWLINE
 NEWLINE
 CAPTION
 CURRENT EFFECTIVE COEFFICIENT OF FRICTION-
PRINT(U)1,3
 NEWLINE
 NEWLINE
 N=J

 Ln=J

 7)Nn=T-2
 CAPTION
 REGION 1
 NEWLINE
 NEWLINE
 Pn=-10

 L=0(M)Ln
 JUMP19,Nn≥L
 JUMP10,Pn>0
 NEWLINE
 CAPTION
 REGION 2
 NEWLINE
 NEWLINE
 Pn=10
 19)Kn=xINTPT(0.5L)
 En=KnZ0
 PRINT(En)1,3
 PRINT(AL)1,3
 Gn=xEXP(CL)
 Gn=GGn

 JUMP12,AL>0.000001
 R=0(1)15
 SPACE
 REPEAT
```

```
 JUMP 13
 12)Fn=xSIN(VL)
 Hn=0.5GnFn
 Un=AL+Hn
 Un=xLOG(Un/En)
 PRINT(Un)1,3
 Vn=AL-Hn
 Un=xLOG(Vn/En)
 PRINT(Un)1,3
 13)PRINT(BL)1,3
 PRINT(CL)1,3
 PRINT(-BL-CL)1,3
 PRINT(DL)1,3
 Un=xLOG(B+£64DL)
 Un=xEXP(Dn+CUn)
 Un=£64Un
 PRINT(Un)5,0
 PRINT(EL)5,0
 Un=ELUL
 PRINT(Un)5,0
 PRINT(UL)1,3
 PRINT(57.29578VL)2,1
 JUMP 18,L>Nn
 Wn=D+0.5Gn
 JUMP 11,AL>0.00001
 Wn=xDIVIDE(2ELGnWn,DD)
 JUMP 10
 11)Wn=ELAL+UnWnFn
 Wn=xDIVIDE(WnGn,DVn)
 10)PRINT(Wn)5,0
 Wn=xDIVIDE(B+£64DL,C£64)
 PRINT(Wn)1,3
 JUMP 0
 16)Wn=AL/Fn
 PRINT(-Wn/UL)2,2
 PRINT(Wn)2,2
 0)Wn=xEXP(C(L+1))
 Wn=A(L+1)E(L+1)GWn
 Un=ALELGn
 JUMP 51,Un≥Wn
 CAPTION
 D
 51)NEWLINE

 REPEAT

 >>OVERWRITE RESULTS OF PREVIOUS STAGE
>>BY THOSE OF CURRENT STAGE
JUMP 2,£88>0
 L=0(2)N
 A(L+1)=AL
```

```
B(L+I)=BL
C(L+I)=CL
D(L+I)=DL
E(L+I)=EL
U(L+I)=UL
V(L+I)=VL
REPEAT

>>TRANSFER ALL NECESSARY VARIABLES FOR RESTART
>>TO MAGNETIC TAPE
WIND TAPE(I,IO)
L=O(I)122
F(L+IOI)=AL
REPEAT
WRITE(I,FIOI)
L=O(I)122
F(L+IOI)=BL
REPEAT
WRITE(I,FIOI)
L=O(I)122
F(L+IOI)=CL
REPEAT
WRITE(I,FIOI)
L=O(I)122
F(L+IOI)=DL
REPEAT
WRITE(I,FIOI)
L=O(I)122
F(L+IOI)=EL
REPEAT
WRITE(I,FIOI)
L=O(I)122
F(L+IOI)=UL
REPEAT
WRITE(I,FIOI)
L=O(I)122
F(L+IOI)=VL
REPEAT
WRITE(I,FIOI)
L=O(I)62
F(L+IOI)=WL
REPEAT
WRITE(I,FIOI)
L=O(I)90
F(L+IOI)=XL
REPEAT
WRITE(I,FIOI)
L=O(I)100
F(L+IOI)=ZL
REPEAT
WRITE(I,FIOI)
```

```
 WRITE(1,£0)
 £101=A
 £102=B
 £103=C
 £104=D
 £105=E
 £106=F
 £107=G
 £108=U
 £109=Dn
 £110=I
 £111=J
 £112=K
 £113=M
 £114=P
 £115=T
 WRITE(1,£101)

 >>BREAK OUTPUT AT EVERY 12 STAGES
71)£90=£90+1
 JUMP30,12>£90
 NEWLINE
 1071,0,0,0
 NEWLINE
 CAPTION
 CONTINUATION OF RESULTS FROM STAGE
PRINT(P)2,0
 NEWLINE
 £90=0
 30)JUMP8,-0.5>£74

 >>RESET PARAMETERS FOR NEXT STAGE
£44=£45
 £45=£46
 £47=£48
 £48=£49
 En=xEXP(-£10)
 £46=EnZ40
 £49=B0
 £80=£81
 £81=£82
 Jn=xMAX(D0.0,J)
 £82=DJn
 JUMP14,£74>0.5
 8)Z22=B0+£92
 JUMP15

 14)En=£46+2Z0
 L=20(1)22
 £L=£(L+24)-En
 ZL=£(L+27)
```

```
REPEAT
JUMPDOWN(R2)
X6=Z22
En=£82+£79
L=20(1)22
£L=£(L+60)-En
ZL=£(L+27)
REPEAT
JUMPDOWN(R2)
JUMP15,X6>Z22
Z22=X6
15)X6=Z22
En=X6-B0
JUMP62,En>0.0025
X6=B0+0.0025
62)Sn=-1
P=P+1
In=T
Z41=-10
Z42=-10
£31=-1
£32=0
JUMP60,-0.5>£74
L=Jn+8
JUMP61,L≥T
T=2T-K
JUMP61
60)T=T+4
61)£91=-1
K=In
ACROSS1/1

>>DIAGNOSTIC OUTPUT WHEN NO CONVERGENCE IS OBTAINED
3)NEWLINE
Kn=Q-2
Nn=Q+1
L=Kn(3)Nn
PRINT(AL)0,5
PRINT(BL)0,5
PRINT(CL)0,5
PRINT(DL)0,5
PRINT(EL)0,5
PRINT(UL)0,5
PRINT(VL)0,5
NEWLINE
REPEAT

NEWLINE
PRINT(P)2,0
PRINT(T)2,0
PRINT(Sn)2,0
```

```
PRINT(Q)2,0

2)L=0(1)15
NEWLINE
REPEAT
JUMP1,I>0

>>END CALCULATION
END


CLOSE



***T
```

APPENDIX-D. COMPUTER PROGRAM FOR CURVE FITTING TO STRAIN HARDENING LAW BY LEAST SQUARES

```
>>MINIMIZATION ROUTINE
ROUTINE 970

Vn vNN+Un
Q=N-1
R=0(1)Q
£R=0
REPEAT
R=0(1)Q
£R=1
X7(RN+Un)£0,N
£R=0
REPEAT
R=0(1)Q
£R=F+F
REPEAT
X7(Vn)£0,N

JUMPDOWN(T)
Fn=F

1)  R=1(1)N
X6(RN-N+Un)£0,N
X6(R-1+Vn)Gn,1
Hn=0
Gn=.5Gn

2) JUMPDOWN 99
JUMPDOWN(T)
JUMP3,R≠0
6) RETURN
3) JUMP4,Fn≥F

Gn=-Gn
JUMPDOWN99
Gn=.5Gn
JUMP2

4) En=F
Gn=2Gn
JUMPDOWN99
JUMPDOWN(T)
JUMP6,R=0
JUMP5,F>En
Fn=En
Hn=.5Gn+Hn
```

```
JUMP 4

5) Fnv Fn-En
Ënv F-En
Fnv x DIVIDE(8Fn+En, 8Fn+1En)
Hn=GnFn+Hn
Gn=GnFn-1.5Gn
JUMPDOWN09
JUMPDOWN(T)
JUMP6, R=0
Fn=F

X7(R-1+Vn)Hn, 1
REPEAT

JUMPDOWN(S)
JUMP6, R=0

X7(N+Vn)Y0, N

R=1(1)N

I=1(1)N
Y(I-1)=0
REPEAT

I=R(1)N
X6(IN-N+Un)£0, N
X6(I-1+Vn)Gn, 1
J=1(1)N
Y(J-1)=Gn£(J-1)+Y(J-1)
REPEAT
REPEAT

JUMP7, R=1

I=R-1
I=I(-1)1
X6(IN-N+Un)£0, N
F=0
J=1(1)N
F=Y(J-1)£(J-1)+F
REPEAT
J=1(1)N
Y(J-1)=-F£(J-1)+Y(J-1)
REPEAT
REPEAT

7)   F=0
J=1(1)N
F=Y(J-1)Y(J-1)+F
```

```
REPEAT
F=XSQRT(1/F)
J=1(1)N
Y(J-1)=FY(J-1)
REPEAT

X7(RN-N+Un)Yo,N

REPEAT

X6(N+Vn)Yo,N

JUMP1

99) I=1(1)N
Y(I-1)=Gnf(I-1)+Y(I-1)
REPEAT
RETURN

**

→
```

```
TITLE
B.KAFTANOGLU,MECH.ENG. ICST JOB NO U-866/2, 1963
TITLE
CURVE FITTING TO STRAIN HARDENING LAW BY LEAST SQUARES
```

```
CHAPTER 1


>>VARIABLE ALLOCATIONS
A→5
B→5
C→200
D→200
E→5
G→5
H→5
Y→5
£→5


>>SCALING AND SETTING OF INITIAL VALUES
8)Y0=0.000001X
Y1=C
Y2=V

>>SETTING OF MINIMIZATION ROUTINE PARAMETERS
F=0.002
N=3
Unv0
S)=10)
T)=20)
Ln=0


G0=0.000001X
G1=C
G2=V

>>ENTER MINIMIZATION ROUTINE
JUMPDOWN(R070)

JUMP32,Y1>0
>>IF 0>B CURVE FIT TO LUDWIK POWER LAW.
ACROSS40/2

>>GO TO PRINTING SECTION
32)ACROSS5/0

>>GENERATE THE ERROR FUNCTION
20)Dn=xLOG(Y0)
F=0
Kn=1(1)P
Y=Y1+CKn
          JUMP30,Y>0
Y=0
```

```
JUMP2,1

20)Y=xLOG(Y)
Y=xEXP(Y2Y+Dn)
21)Y=DKn-1000000Y
F=YY+F
REPEAT
RETURN

>>ACCURACY TESTS FOR CONVERGENCE
10)Mn=0(1)2
HMn=xMOD(YMn/GMn-1)
REPEAT
JUMP9,H0>An
JUMP9,H1>An
JUMP9,H2>An
Ln=Ln+1
JUMP9,3>Ln

R=0


PRINT(1000000G0)0,5
PRINT(G1)0,5
PRINT(G2)0,5
PRINT(Xn)0,7


9)Nn=0(1)2
GNn=YNn
REPEAT
Xn=F

RETURN

xEXP
xLOG
xSQRT

CLOSE
```

```
CHAPTER2

VARIABLES1

>>IF 0>B LUDWIK POWER LAW IS USED FOR CURVE FITTING
40)NEWLINE
CAPTION
LUDWIK POWER LAW IS USED FOR MORE ACCURACY.
NEWLINE

>>SCALING AND SETTING OF INITIAL VALUES
Y0=0.000001X
Y1=V

F=0.002
N=2
Unv0
S)=45)
T)=48)
Ln=0

G0=0.000001X
G1=V

>>ENTER MINIMIZATION ROUTINE
JUMPDOWN(R970)

>>GO TO PRINTING SECTION
ACROSS28/0

>>GENERATE THE ERROR FUNCTION
48)Dn=XLOG(Y0)
F=0
Kn=1(1)P
JUMP41,0≥CKn

Y=XLOG(CKn)
Y=XEXP(Y1Y+Dn)
Y=DKn-1000000Y
F=YY+F
41)REPEAT
RETURN

>>ACCURACY TESTS FOR CONVERGENCE
45)H0=XMOD(Y0/G0-1)
H1=XMOD(Y1/G1-1)

JUMP42,H0>An
JUMP42,H1>An

Ln=Ln+1
```

```
JUMP 42,3>Ln

R=0

PRINT(1000000Go)0,5
PRINT(G1)0,5
PRINT(Xn)0,7

42)Go=Yo
G1=Y1
Xn=F

RETURN

xEXP
xLOG
xSQRT

CLOSE
```

```
CHAPTER0


VARIABLES1

>>READING IN OF DATA
READ(An)

READ(K)
4)K=K-1

0=0(1)2
READ(A0)
READ(B0)
REPEAT


READ(P)
In=1(1)P
READ(CIn)
READ(DIn)
REPEAT

>>CALCULATION OF INITIAL VALUES OF A,B AND N
A=xLOG(B2/B0)
B=xLOG(B2/B1)
B=A/B

M=0
U=0.01
C=0.06

2)C=C+U
M=M+1
Qn=L
Wn=H
JUMP15,M>300
JUMP16
15)M=10
16)D=C+A2
E=C+A1
Z=C+A0
Z=xLOG(D/Z)
E=xLOG(D/E)
E=Z/E

G=E-B
H=xMOD(G)
JUMP1,G>0
JUMP3
1)L=1
JUMP17
```

```
3)L=-1

17)JUMP21,M=1
JUMP22,L≠Qn
JUMP23,H>Wn
JUMP21

23)U=-U
JUMP21
22)U=-0.5U
21)JUMP2,H>0.0001
JUMP24,C>0


C=0.01
D=C+A2
Z=C+A0
Z=XLOG(D/Z)
24)V=A/Z
W=XLOG(C+A1)
X=XLOG(B1)
X=X-VW
X=XEXP(X)

NEWLINE
NEWLINE
PRINT(X)0,5
PRINT(C)0,5
PRINT(V)0,5
NEWLINE
NEWLINE

>>START OF MINIMIZATION
ACROSS8/1

>>PRINT A,B,N,AND ERROR
5)NEWLINE
PRINT(1000000Y0)0,5
PRINT(Y1)0,5
PRINT(Y2)0,5
PRINT(F)0,7
NEWLINE
NEWLINE

>>TABULATION OF INITIAL DATA AND FITTED CURVE
Bn=XLOG(1000000Y0)
Jn=1(1)P
PRINT(CJn)1,3

PRINT(DJn)5,0
Cn=XLOG(Y1+CJn)
Cn=XEXP(Y2Cn+Bn)
```

```
 PRINT(Cn)5,0
 PRINT(DJn-Cn)3,0
 JUMP40≥DJn
 PRINT(100-100Cn/DJn)2,1
 40)NEWLINE
 REPEAT
 JUMP29

 >>PRINT A,B AND ERROR FOR LUDWIK POWER LAW
28)NEWLINE
 NEWLINE
 PRINT(1000000Yo)0,5
 PRINT(Y1)0,5
 PRINT(F)0,7
 NEWLINE
 >>TABULATION OF INITIAL DATA AND FITTED CURVE
>>FOR LUDWIK POWER LAW
 Bn=xLOG(1000000Yo)
 Jn=1(1)P
 PRINT(CJn)1,3
 PRINT(DJn)5,0
 JUMP35,CJn>0
 Cn=0
 JUMP36
 35)Cn=xLOG(CJn)
 Cn=xEXP(Y1Cn+Bn)
 36)PRINT(Cn)5,0
 PRINT(DJn-Cn)3,0
 JUMP41,0≥DJn
 PRINT(100-100Cn/DJn)2,1

 41)NEWLINE
 REPEAT

 29)Pn=0(1)6
 NEWLINE
 REPEAT

 >>CHECK IF THERE IS MORE DATA TO BE FITTED
JUMP4,K>0

 END

 xEXP
 xLOG

 CLOSE

 →
```

APPENDIX-E. COMPUTER PROGRAM FOR THE ANALYSIS OF EXPERIMENTAL RESULTS

```
 JOB
 LSC44EC1, KAFTANOGLU RUN 3-3 1964
COMPUTING 70000 INSTRUCTIONS
OUTPUT
 0 LINE PRINTER 3000 LINES
 STORE 17 BLOCKS
 COMPILER EMA

 MAIN→479
 AUXILIARY(0,0)
 DEPTH 0
 DUMPS 0

 TITLE
 B.KAFTANOGLU,MECH.ENG. ICST JOB NO LSC44EC1, 1964
TITLE
 DATA PROCESSING FOR STRETCH FORMING AND DEEP-DRAWING
TITLE
  ANISOTROPIC
```

```
CHAPTER 1

A→50
B→50
C→50
D→50
E→50
F→50
G→50
H→30
>>CALCULATION OF OUTER AND INNER CIRCUMFERENTIAL STRAINS
50)O=1(2)N
PRINT(AO)1,4
JUMP0,BO>0
BO=DO+EODO/D
0)W=XLOG(BO/AO)
PRINT(W)1,4
PRINT(CO)1,4
JUMP10,DO>0
DO=D+EO
DO=BO-EOBO/DO
10)X=XLOG(DO/CO)
PRINT(X)1,4
>>CALCULATION OF MEAN CIRCUMFERENTIAL        , THICKNESS,
>>MERIDIONAL AND EQUIVALENT STRAINS
Y=0.5AO+0.5CO
PRINT(Y)1,4
Z=0.5BO+0.5DO
An=XLOG(Z/Y)
PRINT(An)1,4
B=XLOG(EO/F)
PRINT(Bn)1,4
Cn=-An-Bn
PRINT(Cn)1,4
Dn=CnCn+AnAn
Dn=XSQRT(H14Dn+H15BnBn)
Dn=H11Dn
PRINT(Dn)1,4
>>CALCULATION OF THE  STRESS RATIOS

JUMP11,DO>E
JUMP12,0>J
JUMP11,DO>D
En=DO/D
Fn=D
H2=XSQRT(DD-DODO)
H2=H2/D
JUMP13
12)JUMP11,DO>G
En=HH-GG
Gn=XSQRT(DODOEn+GGGG)
H3=XSQRT(GG-DODO)
```

```
 H3=GH3/Gn
 En=DOH/Gn
 Fn=xDIVIDE(GnGnGn,GGGGH)

 13)Gn=Fn+0.5EO
 Wn=GnEn
 Xn=xDIVIDE(-0.5EO,FnDO)
 14)Hn=2V-1
 Gn=V+H10
 Un=xDIVIDE(HnAn+BnGn,AnH16Bn)
Vn=V
 V=Z+UnWn
 V=XnV
 Yn=xMOD(1-Vn/V)
 JUMP14,Yn>0.0001
 Zn=10
 JUMP15

 11)Zn=-10
 V=0
 Un=xDIVIDE(-An+BnH10,An+H16Bn)
>>CALCULATION OF EQUIVALENT, MERIDIONAL, AND
>>CIRCUMFERENTIAL STRESSES
 15)PRINT(Un)1,3
 Ho=xLOG(B+H12Dn)
 Ho=xEXP(U+CHo)
 Ho=12Ho
 PRINT(Ho)5,0
 H17=V-1-Un
 H18=UnUn-2Un+1
 H1=UnUn+1+2VH17+H10H18
 H1=xSQRT(1.5H12H1)
 H1=Ho/H1
 PRINT(H1)5,0
 H2=UnH1
 PRINT(H2)5,0
 NEWLINE
 PRINT(BO)1,4
 PRINT(DO)1,4
 PRINT(Z)1,4
 >>CALCULATION OF INTERFACIAL PRESSURE
>>COEFFICIENT OF FRICTION
 JUMP16,0>Zn
 H4=xARCTAN(H3,En)
 SPACE
 SPACE
 PRINT(57.29578H4)2,2
 H5=1-Un
 H6=1+Un
 H7=H4-EnH3
 H8=xDIVIDE(H5EnEn,H6H7)
 PRINT(-2H1V)4,0
 PRINT(H8)1,4
```

```
T6)NEWLINE
NEWLINE
REPEAT
ACROSS60/0

XLOG
XEXP
XSQRT
XARCTAN

CLOSE
```

```
CHAPTER0


VARIABLES1

 >>READING IN OF DATA TAPE. SEE FORMAT IN 11.2
READ(I)
 1)I=I-1
 READ(H10)
 READ(J)
 READ(A)
 READ(B)
 READ(C)
 JUMP0,0>J
 READ(D)
 JUMP2
 0)READ(G)
 READ(H)
 2)READ(E)
 E=1.500-E
 READ(K)
 READ(L)
 READ(F)
 N=2L-1


 >>READING OF GRID MEASUREMENTS FOR OUTER SURFACE,
>>AND THICKNESS
 M=1(2)N
 READ(AM)
 READ(A(M+1))
 READ(BM)
 READ(B(M+1))

 JUMP3,2≥K
 READ(CM)
 READ(C(M+1))

 JUMP3,3≥K
 READ(DM)
 READ(D(M+1))

 3)READ(EM)
 READ(E(M+1))
 JUMP4,2≥K
 READ(FM)
 READ(F(M+1)
 JUMP4,3>K
 READ(GM)
 READ(G(M+1))
 4)REPEAT
```

```
 >>CALCULATION OF RADII BY SUBTRACTING MICROSCOPE READINGS
>> FROM 1.5IN
 O=1(2)N
 AO=1.50000-AO
 A(O+1)=1.50000-A(O+1)
 BO=1.50000-BO
 B(O+1)=1.50000-B(O+1)

 JUMP5,2≥K
 CO=1.500-CO
 C(O+1)=1.50000-C(O+1)
 JUMP5,3≥K
 DO=1.50000-DO
 D(O+1)=1.50000-D(O+1)
 5)REPEAT

 >>AVERAGING RADII AND THICKNESSES
O=1(2)N
 AO=AO+A(O+1)
 AO=0.5AO
 BO=BO+B(O+1)
 BO=0.5BO
 B(O+1)=BO
 EO=EO+E(O+1)
 EO=0.5EO
 E(O+1)=EO

 JUMP6,2≥K
 CO=CO+C(O+1)
 CO=0.5CO
 BO=BO+CO
 BO=0.5BO
 FO=FO+F(O+1)
 FO=0.5FO
 EO=EO+FO
 EO=0.5EO

 JUMP6,3≥K
 DO=DO+D(O+1)
 DO=0.5DO
 BO=B(O+1)+CO+DO
 BO=0.33333BO
 GO=GO+G(O+1)
 GO=0.5GO
 EO=E(O+1)+FO+GO
 EO=0.333333EO
 6)REPEAT

 >>READING OF GRID MEASUREMENTS FOR INNER SURFACE
O=1(2)N
 READ(CO)
 READ(C(O+1))
 READ(DO)
```

```
READ(D(O+I))

JUMP 7,2≥K
READ(FO)
READ(F(O+I))
JUMP 7,3≥K
READ(GO)
READ(G(O+I))
7)REPEAT

>>CALCULATION OF RADII BY SUBTRACTING MICROSCOPE READINGS
>> FROM 1.5IN,AND AVERAGING
O=I(2)N
CO=1.50000-CO
C(O+I)=1.50000-C(O+I)
CO=CO+C(O+I)
CO=0.5CO
DO=1.50000-DO
D(O+I)=1.50000-D(O+I)
DO=DO+D(O+I)
DO=0.5DO
D(O+I)=DO
JUMP8,2≥K
FO=1.50000-FO
F(O+I)=1.50000-F(O+I)
FO=FO+F(O+I)
FO=0.5FO
DO=DO+FO
DO=0.5DO

JUMP8,3≥K
GO=1.50000-GO
G(O+I)=1.50000-G(O+I)
GO=GO+G(O+I)
GO=0.5GO
DO=D(O+I)+FO+GO
DO=0.333333DO
8)REPEAT

>>CALCULATION OF CONSTANTS IN EQUATIONS
U=XLOG(A)
JUMP10,0>J
V=D
JUMP11
10)V=GG/H
11)Zn=6+0.5EI
V=xDIVIDE(-EIZn,VV)

NEWLINE
CAPTION
DATA NO-
PRINT(33-I)2,6
NEWLINE
```

```
 >>CALCULATION OF CONSTANTS INVOLVING R-VALUE
H13=H10+2
 H11=XSQRT(0.6666667H13)
 H11=XDIVIDE(H11,2H10+1)
 H12=XDIVIDE(H10+1,H10+2)
 H12=XSQRT(1.5H12)
 H14=2H10+1
 H15=2H10H10+H10
 H16=H10+1
 H19=1/H13

 ACROSS50/1

 60)In=0(1)10
 NEWLINE
 REPEAT

 JUMP1,I>0

 >>END CALCULATION
 END
 CLOSE
```

APPENDIX - F. COMPUTER PROGRAM FOR ESTIMATING THE LIMITING DRAWING RATIO

```
TITLE
EXPRED  LDR  KAFTANOGLU  ICST  JOBNO.T370
```

```
CHAPTER0
A→3
B→3
C→2
>>READING OF DATA
1)READ(K)
K=K-1
I=0(1)2
READ(AI)
READ(BI)
REPEAT
>>CALCULATION OF INITIAL CONSTANTS
A=A0-A1
B=A0-A2
B=A/B
J=-1
2)J=J+1
C=0.3J+0.20
D=xEXP(-CB0)
E=xEXP(-CB1)
F=xEXP(-CB2)
G=D-E
H=D-F
H=G/H
GJ=H-B
JUMP2,J≠1
U=C1-C0
U=0.3/U
W=0
V=0.20-UC0
>>SUCCESSIVE APPROXIMATION LOOP
3)V=V-UW
D=xEXP(-VB0)
E=xEXP(-VB1)
F=xEXP(-VB2)
G=D-E
H=D-F
H=G/H
W=H-B
Z=xMOD(W)
JUMP3,Z>0.0001
X=A/G
Y=A1-XE
NEWLINE
>>PRINTING OUT OF RESULTS
PRINT(V)0,4
PRINT(X)5,2
PRINT(Y)1,4
PRINT(25.4Y)2,2
PRINT(Y/1.3000)1,3
NEWLINE
```

```
PRINT(Y+0.0325)1,3
PRINT(Y-0.0325)1,3
PRINT(25.4Y+0.8255)3,3
PRINT(25.4Y-0.8255)3,3
PRINT(W)0.4
NEWLINE
NEWLINE
JUMP1,K≠0
END
XEXP
CLOSE
→
```

## APPENDIX-G. THEORY OF PLASTIC BENDING UNDER BIAXIAL TENSION FOR ANISOTROPIC LINEAR STRAIN - HARDENING MATERIALS

An approximate theory to estimate the thinning under plastic bending under biaxial tension for anisotropic materials with linear strain-hardening properties is presented. It is developed along the same lines as that given by Chung and Swift (18) for isotropic materials under conditions of plane strain, and it shows the limitations of their theory. This method can be applied in cases when:

(i) The tensile force under which the plastic bending is taking place is statically determinate, e.g. in bending of a metal strip around a pulley,

(ii) The plastic bending is superimposed on another stress system with a mean tensile stress acting in the plane of bending, e.g. plastic bending under tension at the start of the die curvature in deep-drawing.

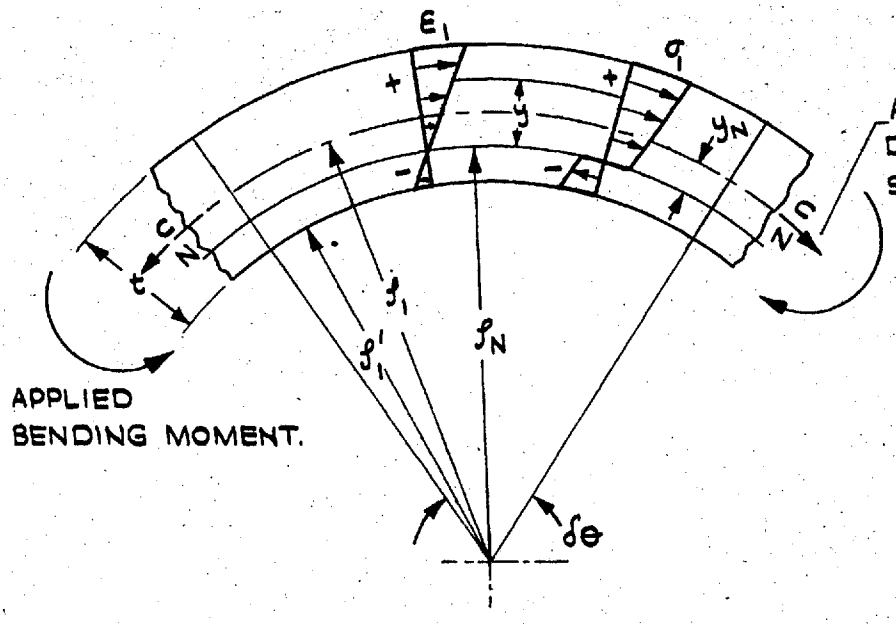
It will be assumed that the material obeys a linear strain-hardening law of the type:

$$\bar{\sigma} = \bar{\sigma}_o + \beta\bar{\epsilon} \qquad (G.1)$$

where $\bar{\sigma}_o$ is the equivalent stress of the material prior to plastic bending, and $\beta$ is a coefficient expressing the rate of linear strain-hardening. It is further assumed that the elastic strains are negligible in comparison with the plastic ones, and plane sections remain plane during

bending. Also the effect of the interfacial pressure between tool and sheet metal will be neglected, and bending will be assumed to take place under a constant stress ratio (ratio of stresses in the plane of the sheet) which may be equal to that, prior to the commencement of bending.

As shown in Fig. G.1, bending takes place under the applied tension resulting from the mean stress $\sigma_{1b}$. The neutral surface of bending NN is displaced from the central surface CC by an amount $y_N$, which can be determined by equating the applied tension to the difference between the bending tension and compression. However, as shown in Fig. G.1, two distinct cases exist in the process. In Case I, the neutral surface remains within the thickness of the sheet, whereas in Case II, it is outside of the thickness, because of the higher applied tension and hence higher mean stress, which completely compensates the compressive stresses due to bending. In the second case all fibers of the sheet suffer tensile strains in contrast to those in Case I. This possibility was not mentioned by Swift (78), and Chung and Swift (18), and their analyses can only apply for the conditions of Case I. Since they have not provided a criterion to find the limiting mean stress for the change over from Case I to Case II, results obtained using their method need to be checked back to see that the



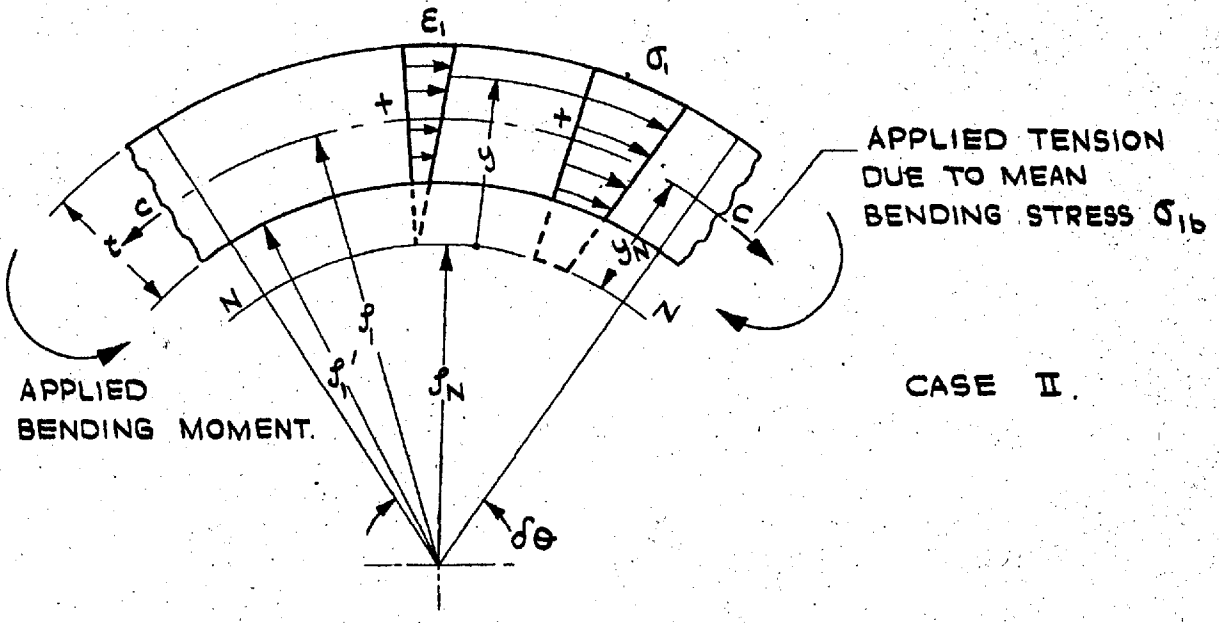




FIG. G.1. CONDITIONS DURING PLASTIC BENDING UNDER TENSION.

neutral surface remains within the thickness of the sheet, otherwise the results may be incorrect.

The present analysis is as follows:

From Fig.G.1:

$$\rho_N = \rho_1' + \frac{t}{2} - y_N \qquad (G.2)$$

The meridional strain at any fibre is given by:

$$\varepsilon_1 = \ln\left(1 + \frac{y}{\rho_N}\right) \qquad (G.3)$$

The ratio of the circumferential stress to the meridional one is: $\frac{\sigma_2}{\sigma_1} = x$, and it is assumed to remain constant during bending. $\sigma_3 = 0$, as mentioned earlier.

The yield criterion for anisotropic materials under these conditions become:

$$\bar{\sigma} = \left\{\frac{3\left[x^2 + R(x^2 - 2x + 1) + 1\right]}{2(2 + R)}\right\}^{\frac{1}{2}} \sigma_1$$

or

$$\bar{\sigma} = V_1 \sigma_1 \qquad (G.4)$$

where $V_1$ is a constant.

By dividing two of the eqns. (3.19) side by side:

$$\frac{d\varepsilon_2^*}{d\varepsilon_1^*} = \frac{R(x - 1) + x}{R(1 - x) + 1} = V_2 \text{ - a constant} \qquad (G.5)$$

Then substituting $V_2 d\varepsilon_1^*$ in eqn. (3.23) for $d\varepsilon_2^*$, and simplifying:

$$d\bar{\varepsilon}^{pl} = \frac{\left[\frac{2}{3}(2+R)\right]^{\frac{1}{2}}}{1+2R}\left[(2R^2+3R+1)(1+V_2^2)+2V_2(R+2R^2)\right]^{\frac{1}{2}} d\varepsilon_1$$

which leads to:

$$\bar{\varepsilon} = V_3\varepsilon_1 \qquad (G.6)$$

where $V_3$ is another constant.

To be consistent with the sign convention adopted for y in eqn. (G.3), the following conditions must be satisfied by eqns. (G.4) and (G.6):

$$\begin{aligned} &\text{when} \quad \sigma_1 \gtrless 0 \quad \text{than} \quad V_1 \gtrless 0 \\ &\text{and} \\ &\qquad \varepsilon_1 \gtrless 0 \quad \text{then} \quad V_3 \gtrless 0 \end{aligned} \qquad (G.7)$$

so that $\bar{\sigma}$ and $\bar{\varepsilon}$ will always remain positive. These can be ensured by taking the appropriate signs of the radicals in eqns. (G.4) and (G.6) to satisfy the above conditions. This distinction was not made by Chung and Swift (18).

From eqns. (G.4), and (G.1):

$$\sigma_1 = \frac{\bar{\sigma}}{V_1} = \frac{\bar{\sigma}_o}{V_1} + \frac{\beta}{V_1}\bar{\varepsilon} \qquad (G.8)$$

Substituting for $\bar{\varepsilon}$ from eqns. (G.6) and (G.3) also with the approximation:

$$\ln\left(1+\frac{y}{\rho_N}\right) \approx \frac{y}{\rho_N}$$

with about 1% error in the relevant range, into eqn. (G.8):

$$\sigma_1 = \frac{\bar{\sigma}_o}{V_1} + \frac{\beta}{V_1} V_3 \frac{y}{\rho_N} \qquad (G.9)$$

This equation gives the stress distribution through the thickness.

For Case I:

Keeping the conditions of inequalities (G.7) in mind, the applied tension can be equated to the difference of bending tension and compression, which is given by the integral of eqn. (G.9). However when $y > 0$ then $V_1$, and $V_3 > 0$, and when $0 > y$ then $0 > V_1$, and $V_3$. This produces a change of sign only in the first term of eqn. (G.9). Hence the equilibrium of the forces results in:

$$\sigma_{1b} t = \int_0^{\frac{t}{2}+y_N} \left[ \frac{\bar{\sigma}_o}{V_1} + \frac{\beta}{V_1} V_3 \frac{y}{\rho_N} \right] dy + \int_{y_N - \frac{t}{2}}^{0} \left[ - \frac{\bar{\sigma}_o}{V_1} + \frac{\beta}{V_1} V_3 \frac{y}{\rho_N} \right] dy \qquad (G.10)$$

Integrating and solving for $y_N$:

$$y_N = \frac{\sigma_{1b} t}{\frac{2\bar{\sigma}_o}{V_1} + \frac{\beta V_3 t}{V_1 \rho_N}} \qquad (G.11)$$

For Case II:

There is no stress discontinuity through the thickness, and the equilibrium between the applied tension and bending stresses is expressed as:

$$\sigma_{1b} t = \int_{y_N - \frac{t}{2}}^{\frac{t}{2} + y_N} \left[ \frac{\sigma_o}{V_1} + \frac{\beta}{V_1} V_3 \frac{y}{\rho_N} \right] dy \qquad (G.12)$$

Integrating and solving for $y_N$:

$$y_N = \frac{V_1 \rho_N}{\beta V_3} \left( \sigma_{1b} - \frac{\bar{\sigma}_o}{V_1} \right) \qquad (G.13)$$

It is seen that eqn. (G.13) for Case I is quite different from eqn. (G.11) for Case II, and the correct one should be used under a given set of conditions.

Criterion to Determine Which Case to Apply:

At the limiting state between the two cases, referring to Fig. G.1:

$$y_N = \frac{t}{2} \quad \text{and} \quad \rho_N = \rho_1' \qquad (G.14)$$

Substituting these conditions in eqn. (G.11), and simplifying:

For Case I: $$\sigma_{1b} \lessgtr \frac{\bar{\sigma}_o}{V_1} + \frac{\beta V_3 t}{2 V_1 \rho_1'} \qquad (G.15)$$

For Case II:

In the estimation of thinning under plastic bending under tension the following sequence should be followed:

(i) From the given conditions, a check should be made by using the inequality (G.15) to see whether the applied stress is greater or less than the limiting stress, to determine which case to apply.

(ii) If Case I: since the value of $\rho_N$ is not known beforehand, a starting value of $\rho_N \approx \rho_1' + \frac{t}{4}$ can be used with the value of the initial thickness in eqn. (G.11), to obtain a value of $y_N$.

If Case II: with the value of the applied mean stress $\sigma_{1b}$, $\frac{y_N}{\rho_N}$ can be directly found from eqn. (G.13).

(iii) The extension of the element is given by the mean strain at the central fibre which is:

$$\varepsilon_{1b} = \ln \left(1 + \frac{y_N}{\rho_N}\right) \qquad (G.16)$$

and for the type of anisotropy considered, the circumferential strain, if bending does not take under plane strain conditions, from eqn. (G.5) is:

$$\varepsilon_{2b} = V_2 \, \varepsilon_{1b} \qquad (G.17)$$

The thickness strain can be found from the constancy of volume using eqns. (G.16), and (G.17):

$$\varepsilon_{3b} = -(1+V_2) \ln \left(1 + \frac{y_N}{\rho_N}\right) \qquad (G.18)$$

(iv) The additional applied stress necessary to supply the work required to impose this bending can be calculated by equating the work done in bending to the work done by the applied stress. In cases when this bending takes place under conditions other than plane strain, work done in the circumferential direction should also be considered in addition to that of in the meridional direction. For this, a similar method followed by Chung and Swift (18) for the plane strain bending of isotropic materials can be used.

However in most cases since the bending strains are small this addition to the applied stress may be negligible.

The first approximations for the values of $\rho_N$, t, and possibly $\sigma_{1b}$ can be improved and the procedure can be repeated including steps (i) to (iv) until no significant changes in the results are obtained. It is important to repeat step (i) in case there is a change over from Case I to Case II or vice versa due to the errors in the first approximations.

It should be mentioned that eqn. (G.11) will reduce to that given by Chung and Swift (18) when $x = 0.5$ and $R = 1$, i.e. plane strain bending under tension for isotropic materials.

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Table 8.1 - Materials tested.

| Material | Composition or Specification | Hardness, VPN | Thickness, x $10^{-3}$ in. (tolerances as measured) |
|---|---|---|---|
| Mild Steel (E.D.D.S.) | Not known (designated E.D.D.S.*) | 107 (5 kg load)<br>102 "<br>109 " | S.W.G. 14 = 80.5 ± 0.5<br>" 16 = 61 ± 0.3<br>" 20 = 34.5 ± 0.3 |
| Soft Copper | Bright-annealed, commercially pure | 50 (1 kg load)<br>54 "<br>54 " | " 16 = 65 ± 0.5<br>" 18 = 48.5 ± 0.5<br>" 20 = 37.5 ± 0.5 |
| Soft Brass | 70 : 30 Bright-annealed | 83 (2.5 kg load)<br>83 "<br>75.5 " | " 16 = 64.7 ± 1<br>" 18 = 47.5 ± 0.5<br>" 20 = 36.2 ± 0.8 |
| Soft Aluminium | Bright-annealed, commercially pure | 26.2(1 kg load)<br>27.2 "<br>27.4 " | " 16 = 61.2 ± 0.5<br>" 18 = 46.9 ± 0.3<br>" 20 = 35.8 ± 0.4 |

* Extra Deep-Drawing Quality Steel

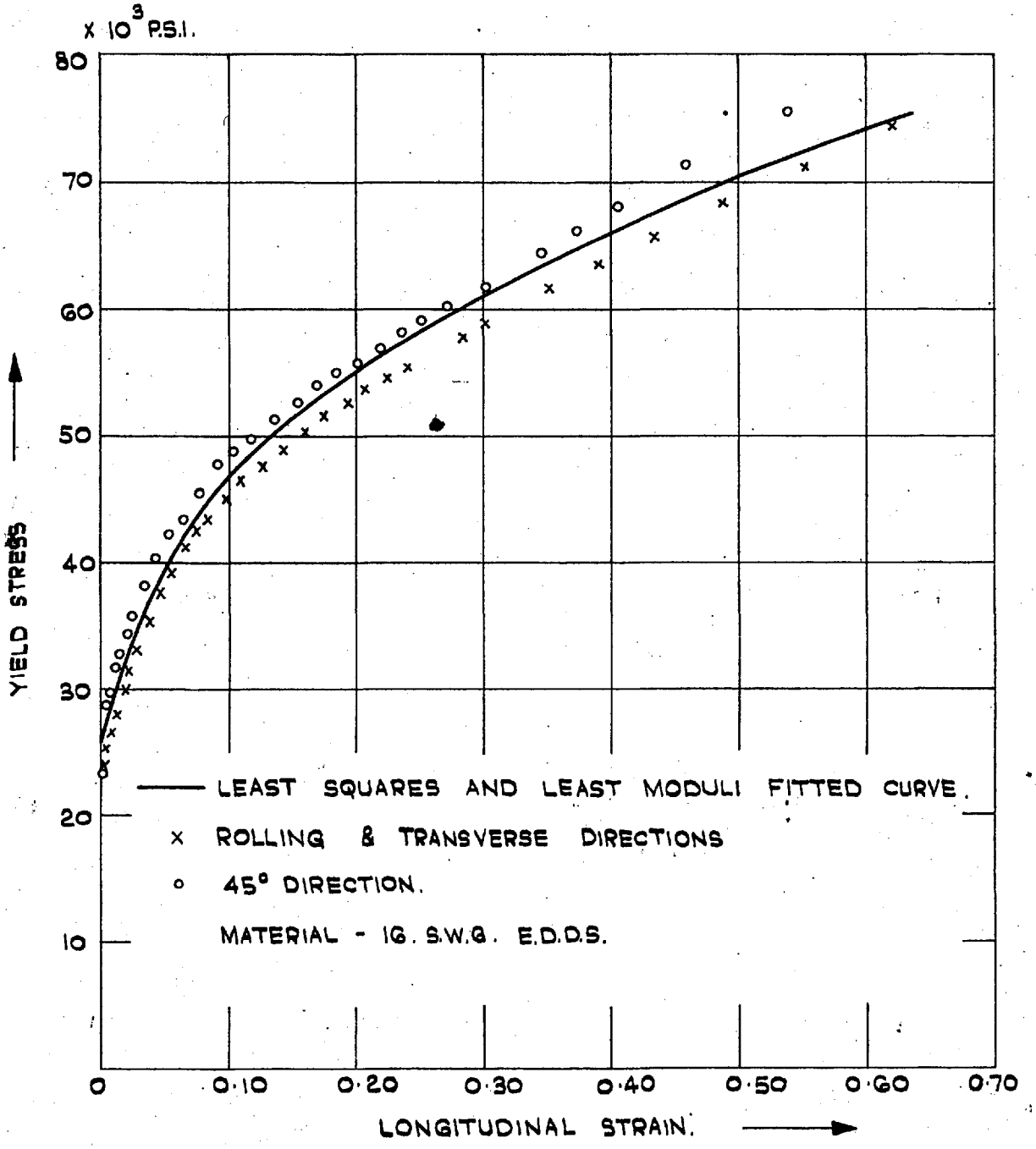



FIG. 21. YIELD STRESS CURVES IN UNIAXIAL TENSION.

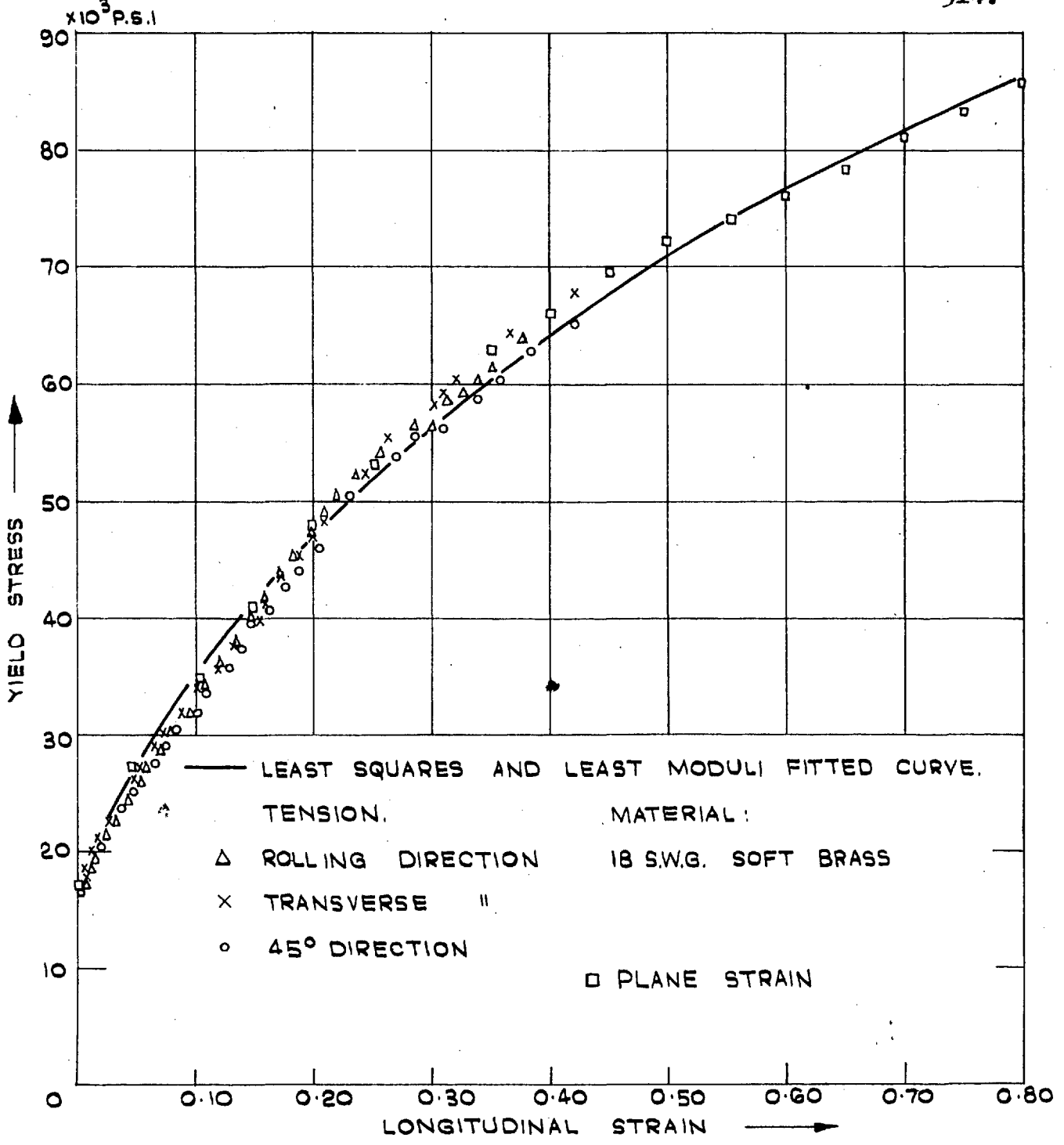



FIG. 22. YIELD STRESS CURVES IN UNIAXIAL TENSION AND PLANE STRAIN.

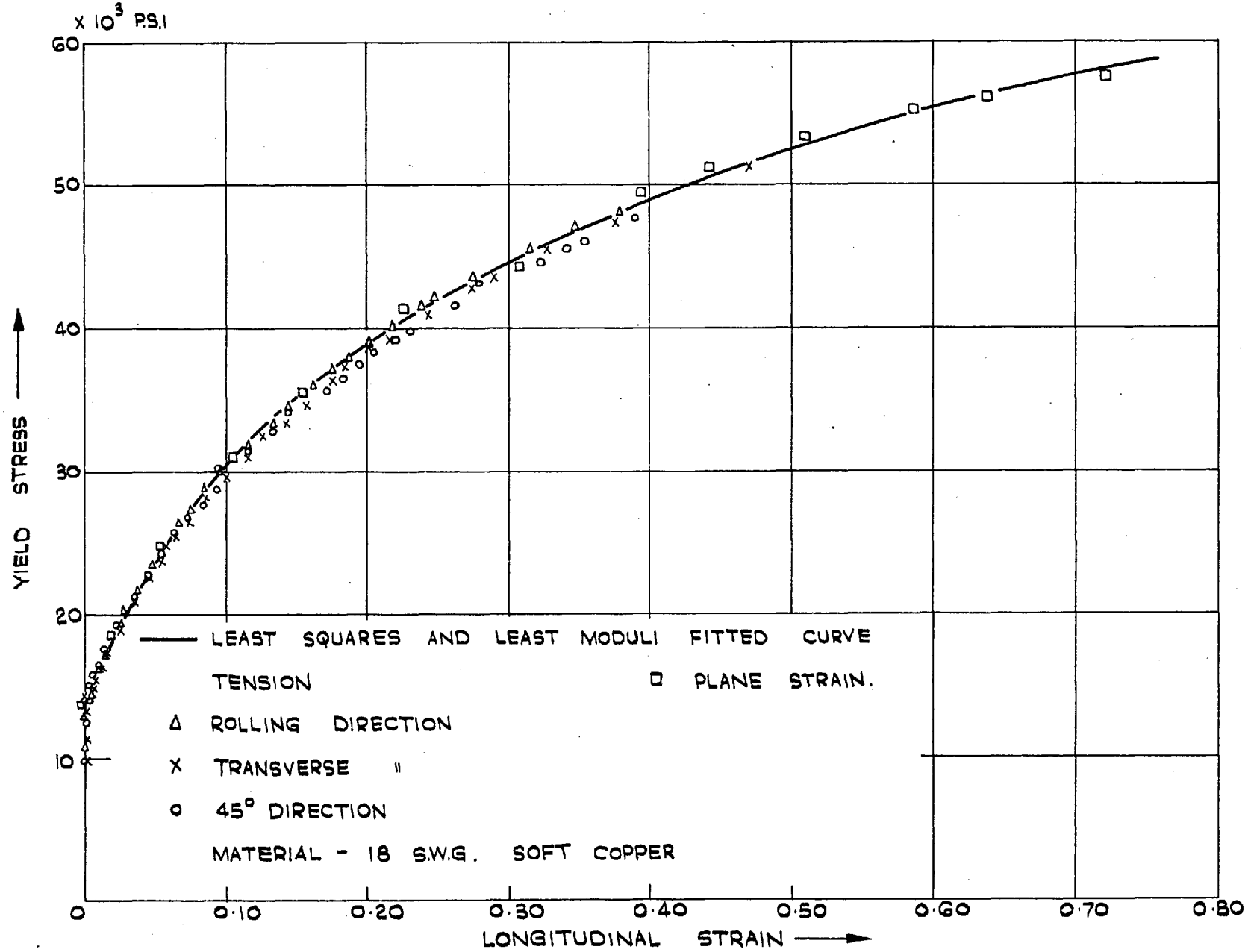



FIG. 23. YIELD STRESS CURVES IN UNIAXIAL TENSION AND PLANE STRAIN.

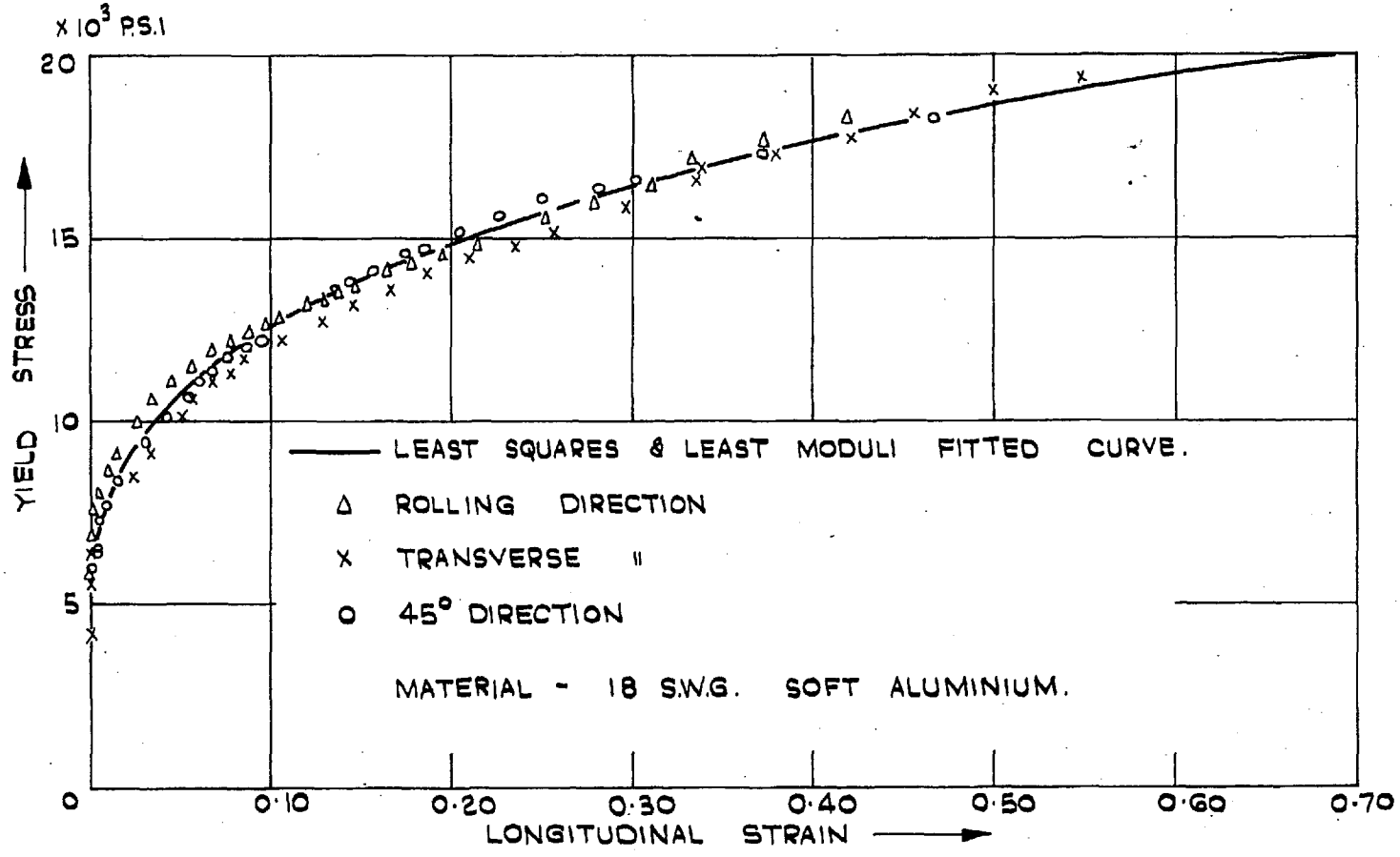



FIG. 24 YIELD STRESS CURVES IN UNIAXIAL TENSION

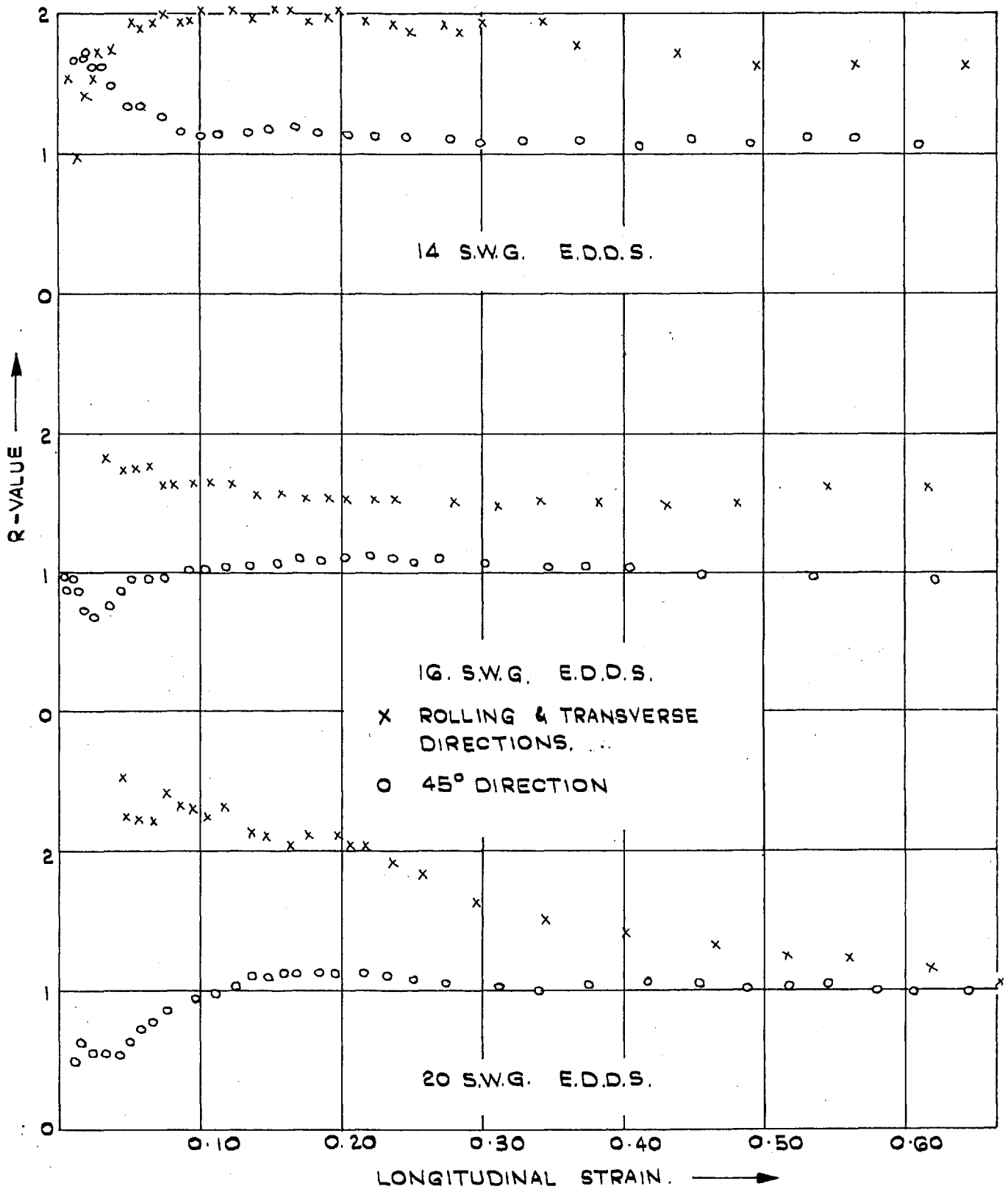



FIG. 25. VARIATION OF THE R-VALUE WITH LONGITUDINAL STRAIN IN THE UNIAXIAL TENSILE TEST.

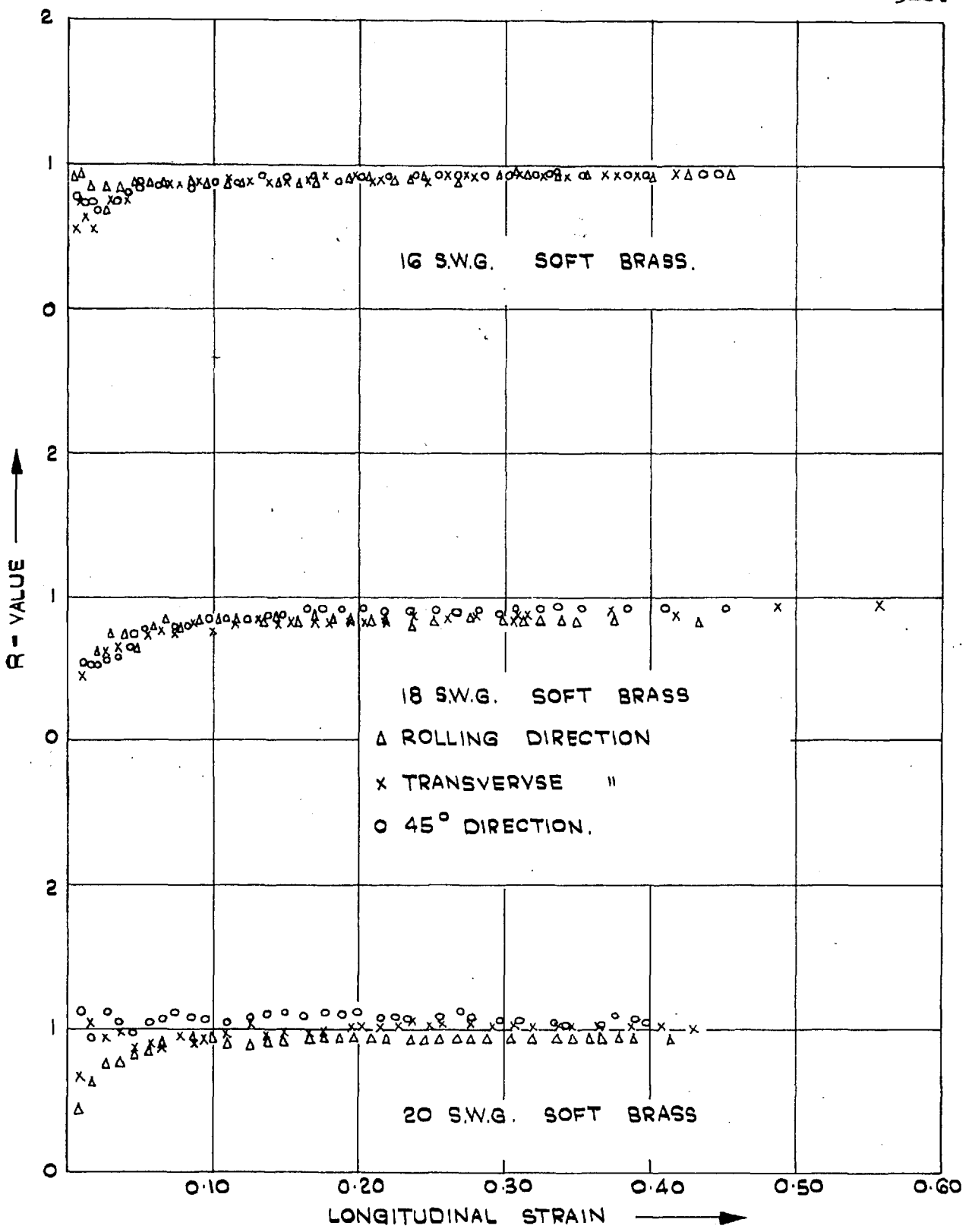



FIG. 26. VARIATION OF THE R-VALUE WITH LONGITUDINAL STRAIN IN THE UNIAXIAL TENSILE TEST.

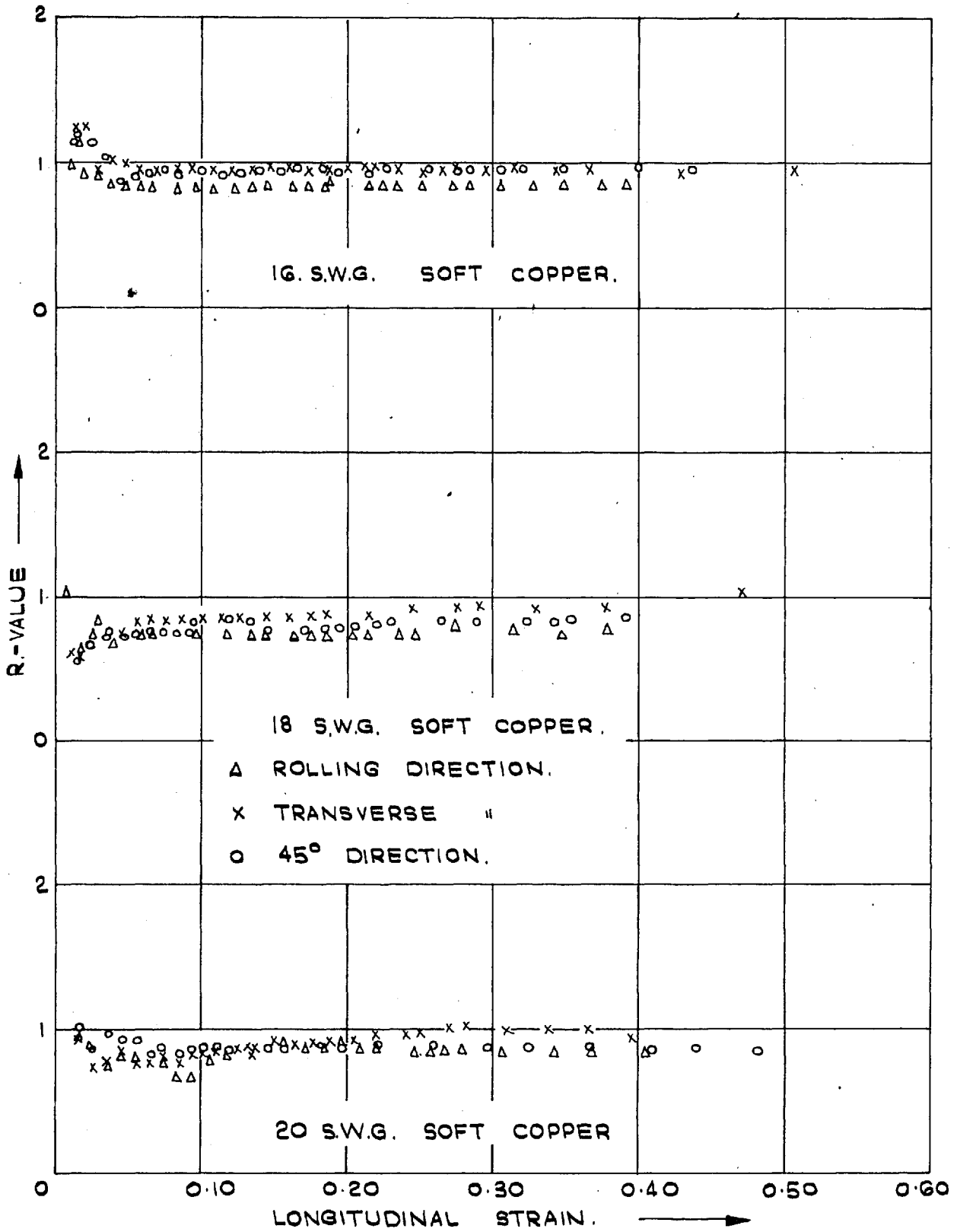



FIG. 27. VARIATION OF THE R-VALUE WITH LONGITUDINAL STRAIN IN THE UNIAXIAL TENSILE TEST.

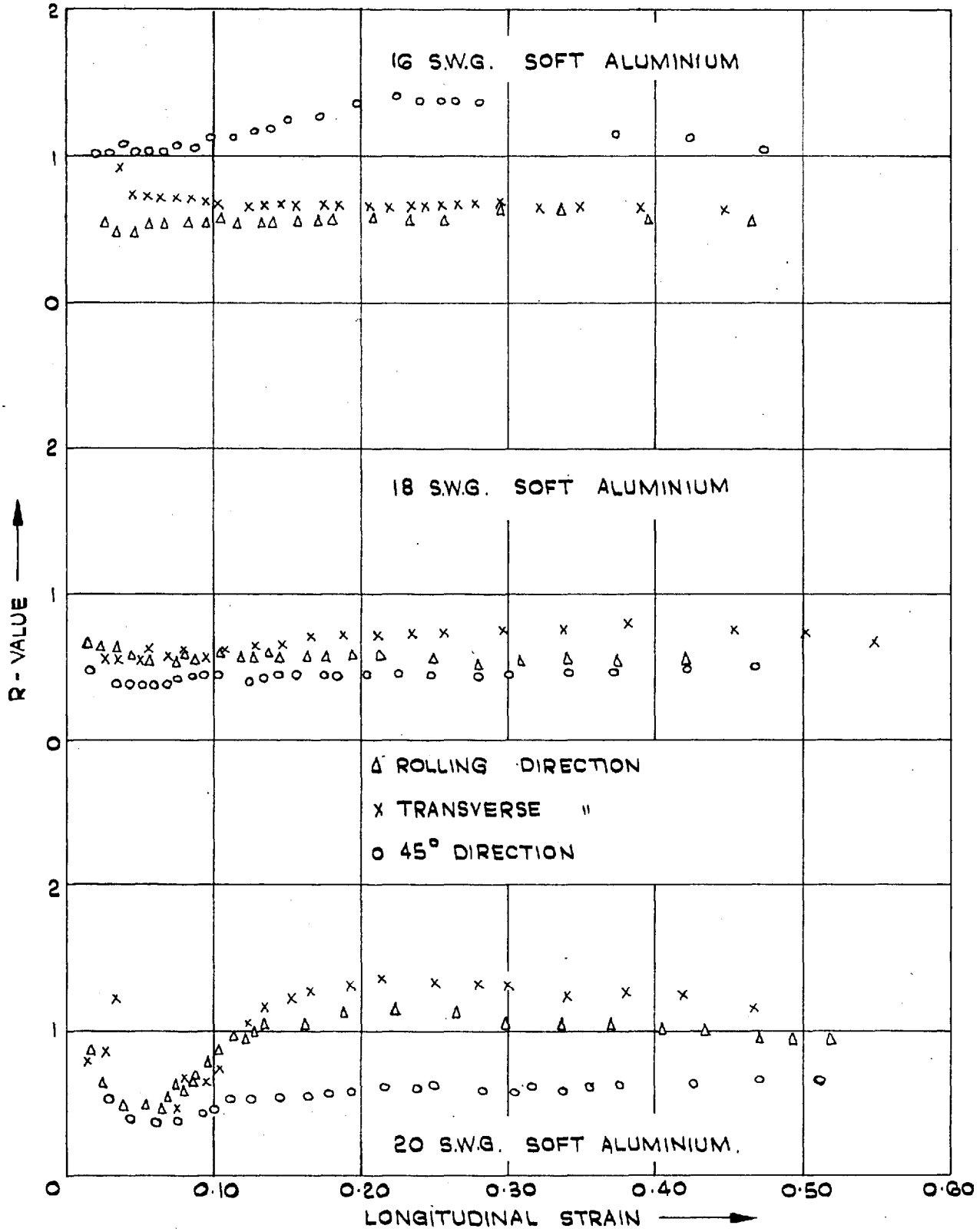



FIG. 28. VARIATION OF THE R-VALUE WITH LONGITUDINAL STRAIN IN THE UNIAXIAL TENSILE TEST.

Table 8.2 - Values of the constants A, B, and n in $X = A(B + \varepsilon_\ell)^n$ obtained by the computer program using the least squares method.

| Material | Direction | A | B | n | Average Approximation with real curve % |
|---|---|---|---|---|---|
| 16 SWG Steel E.D.D.S. | Rolling & Transverse | 81758 | 0.0099 | 0.2687 | <1 |
| 16 SWG E.D.D.S. | 45° | 85063 | 0.0082 | 0.2571 | ~1.5 |
| 16 SWG E.D.D.S. | Mean of T & 45° by Least Squares | 83452 | 0.0092 | 0.2632 | ~3 |
| 16 SWG E.D.D.S. | Arithmetic Mean of T & 45° | 83411 | 0.0090 | 0.2629 | |
| 18 SWG Soft Brass | Mean of R,T & 45° by Least Squares | 93685 | 0.0165 | 0.4373 | ~1.5 |
| 18 SWG Soft Copper | Mean of R,T & 45° by Least Squares | 68394 | 0.0122 | 0.3789 | ~1.5 |
| 18 SWG Aluminium | Rolling | 19905 | 0.0041 | 0.1945 | <1 |
| 18 SWG Aluminium | Transverse | 22701 | 0.0133 | 0.2939 | <1 |
| 18 SWG Aluminium | 45° | 23404 | 0.0082 | 0.2771 | <1 |
| 18 SWG Aluminium | Mean of R,T & 45° by Least Squares | 21952 | 0.0087 | 0.2532 | ~3 |
| 18 SWG Aluminium | Arithmetic Mean of R,T & 45° | 22004 | 0.0085 | 0.2552 | |

Table 8.2 - (Cont..)

| Material | Direction | A | B | n | Average Approximation with real curve % |
|---|---|---|---|---|---|
| 14 SWG E.D.D.S. | Rolling & Transverse | 83748 | 0.0217 | 0.3055 | ~ 1.5 |
| 14 SWG E.D.D.S. | 45° | 84447 | 0.0142 | 0.2792 | ~ 1.5 |
| 14 SWG E.D.D.S. | Mean of T & 45° by Least Squares | 84070 | 0.0186 | 0.2941 | ~ 2.5 |
| 14 SWG E.D.D.S. | Arithmetic Mean of T & 45° | 84097 | 0.0180 | 0.2923 | |
| 20 SWG E.D.D.S. | Rolling & Transverse | 89331 | 0.0968 | 0.3829 | ~ 1.5 |
| 20 SWG E.D.D.S. | 45° | 90064 | 0.0638 | 0.3376 | ~ 1.5 |
| 20 SWG E.D.D.S. | Mean of T & 45° by Least Squares | 89794 | 0.0801 | 0.3607 | ~ 2 |
| 20 SWG E.D.D.S. | Arithmetic Meanof T & 45° | 89697 | 0.0803 | 0.3602 | |
| 16 SWG Soft Brass | Mean of R,T & 45° by Least Squares | 96642 | 0.0112 | 0.4235 | ~ 1 |
| 20 SWG Soft Brass | Mean of R,T & 45° by Least Squares | 92547 | 0.0149 | 0.4403 | ~ 1 |
| 16 SWG Soft Copper | Mean of R,T & 45° by Least Squares | 65485 | 0.01297 | 0.3849 | ~ 2 |
| 20 SWG Soft Copper | Mean of R,T & 45° by Least Squares | 65007 | 0.0170 | 0.3643 | ~ 1.5 |

Table 8.2 - (Cont..)

| Material | Direction | A | B | n | Average Approximation with real curve % |
|---|---|---|---|---|---|
| 16 SWG Soft Aluminium | Rolling | 24778 | 0.0333 | 0.2966 | ~ 1 |
| 16 SWG Soft Aluminium | Transverse | 23242 | 0.02775 | 0.2727 | < 1 |
| 16 SWG Soft Aluminium | 45° | 21536 | 0.0113 | 0.2144 | < 1 |
| 16 SWG Soft Aluminium | Mean of R, T & 45° by Least Squares | 23574 | 0.0274 | 0.2738 | ~ 2 |
| 16 SWG Soft Aluminium | Arithmetic Mean of R, T & 45° | 23185 | 0.0241 | 0.2612 | |
| 20 SWG Soft Aluminium | Rolling | 26577 | 0.0115 | 0.2827 | ~ 1.5 |
| 20 SWG Soft Aluminium | Transverse | 23858 | 0.0126 | 0.2537 | ~ 1.5 |
| 20 SWG Soft Aluminium | 45° | 26006 | 0.0346 | 0.3611 | ~ 1.5 |
| 20 SWG Soft Aluminium | Mean of R, T & 45° by Least Squares | 23863 | 0.0127 | 0.2587 | ~ 4 |
| 20 SWG Soft Aluminium | Arithmetic Mean of R, T & 45° | 25480 | 0.0196 | 0.2992 | |

Table 8.3 - Values of the constants A, B, and n in $X = A(B + \varepsilon_\ell)^n$ obtained by the computer program using the least moduli method.

| Material | Direction | A | B | n | Average Approximation with real curve % |
|---|---|---|---|---|---|
| 16 SWG Steel E.D.D.S. | Rolling & | 80815 | 0.0084 | 0.2606 | $<1$ |
| 16 SWG E.D.D.S. | 45° | 82234 | 0.0063 | 0.2395 | $\sim 1$ |
| 16 SWG E.D.D.S. | Mean of T & 45° by Least Moduli | 81494 | 0.0053 | 0.2403 | $\sim 3$ |
| 16 SWG E.D.D.S. | Arithmetic Mean of T & 45° | 81524 | 0.0073 | 0.25 | |
| 18 SWG Soft Brass | Mean of R,T & 45° by Least Moduli | 92542 | 0.0203 | 0.4427 | $\sim 2.5$ |
| 18 SWG Soft Copper | Mean of R,T & 45° by Least Moduli | 68846 | 0.0134 | 0.3859 | $\sim 1.5$ |
| 18 SWG Aluminium | Rolling | 19899 | 0.0049 | 0.1953 | $<1$ |
| 18 SWG Aluminium | Transverse | 22625 | 0.0141 | 0.2948 | $<1$ |
| 18 SWG Aluminium | 45° | 23275 | 0.0082 | 0.2748 | $\sim 1$ |
| 18 SWG Aluminium | Mean of R,T & 45° by Least Moduli | 20869 | 0.0026 | 0.2188 | $\sim 3.5$ |

Table 8.3 - (Cont..)

| Material | Direction | A | B | n | Average Approximation with real curve % |
|---|---|---|---|---|---|
| 18 SWG Aluminium | Arithmetic Mean of R,T & 45° | 21933 | 0.0096 | 0.2549 | |
| 14 SWG E.D.D.S. | Rolling & Transverse | 81979 | 0.0167 | 0.2864 | ~1.5 |
| 14 SWG E.D.D.S. | 45° | 82436 | 0.0114 | 0.2617 | ~1.5 |
| 14 SWG E.D.D.S. | Mean of T & 45° by Least Moduli | 85022 | 0.0206 | 0.3047 | ~3.0 |
| 14 SWG E.D.D.S. | Arithmetic Mean of R, T & 45° | 82270 | 0.0141 | 0.2741 | |
| 20 SWG E.D.D.S. | Rolling & Transverse | 88131 | 0.1136 | 0.3924 | ~1.5 |
| 20 SWG E.D.D.S. | 45° | 90083 | 0.0669 | 0.3395 | ~1.5 |
| 20 SWG E.D.D.S. | Mean of T & 45° by Least Moduli | 90244 | 0.0913 | 0.3757 | ~2.0 |
| 20 SWG E.D.D.S. | Arithmetic Mean of T & 45° | 89107 | 0.0902 | 0.3659 | |
| 16 SWG Soft Brass | Mean of R, T & 45° by Least Moduli | 98385 | 0.0215 | 0.4285 | ~1.5 |
| 20 SWG Soft Brass | Mean of R, T & 45° by Least Moduli | 93128 | 0.0205 | 0.438 | ~1 |

Table 8.3 - (Cont..)

| Material | Direction | A | B | n | Average Approximation with real curve % |
|---|---|---|---|---|---|
| 16 SWG Soft Copper | Mean of R,T & 45° by Least Moduli | 65753 | 0.013 | 0.3883 | ~1.5 |
| 20 SWG Soft Copper | Mean of R, T & 45° by Least Moduli | 65098 | 0.0160 | 0.3634 | ~1.5 |
| 16 SWG Soft Aluminium | Rolling | 24519 | 0.0318 | 0.2887 | ~1 |
| 16 SWG Soft Aluminium | Transverse | 23260 | 0.0273 | 0.2726 | ~1 |
| 16 SWG Soft Aluminium | 45° | 22308 | 0.0177 | 0.2400 | ~1 |
| 16 SWG Soft Aluminium | Mean of R,T & 45° by Least Moduli | 22631 | 0.0199 | 0.2457 | ~2 |
| 16 SWG Soft Aluminium | Arithmetic Mean of R, T & 45° | 23362 | 0.0256 | 0.2671 | |
| 20 SWG Soft Aluminium | Rolling | 28623 | 0.0214 | 0.3304 | ~1.5 |
| 20 SWG Soft Aluminium | Transverse | 30636 | 0.0579 | 0.4455 | ~1.5 |
| 20 SWG Soft Aluminium | 45° | 25941 | 0.0338 | 0.3579 | ~1.5 |
| 20 SWG Soft Aluminium | Mean of R,T & 45° by Least Moduli | 26060 | 0.0700 | 0.3954 | ~8 |
| 20 SWG Soft Aluminium | Arithmetic Mean of R, T & 45° | 28400 | 0.0377 | 0.3779 | |

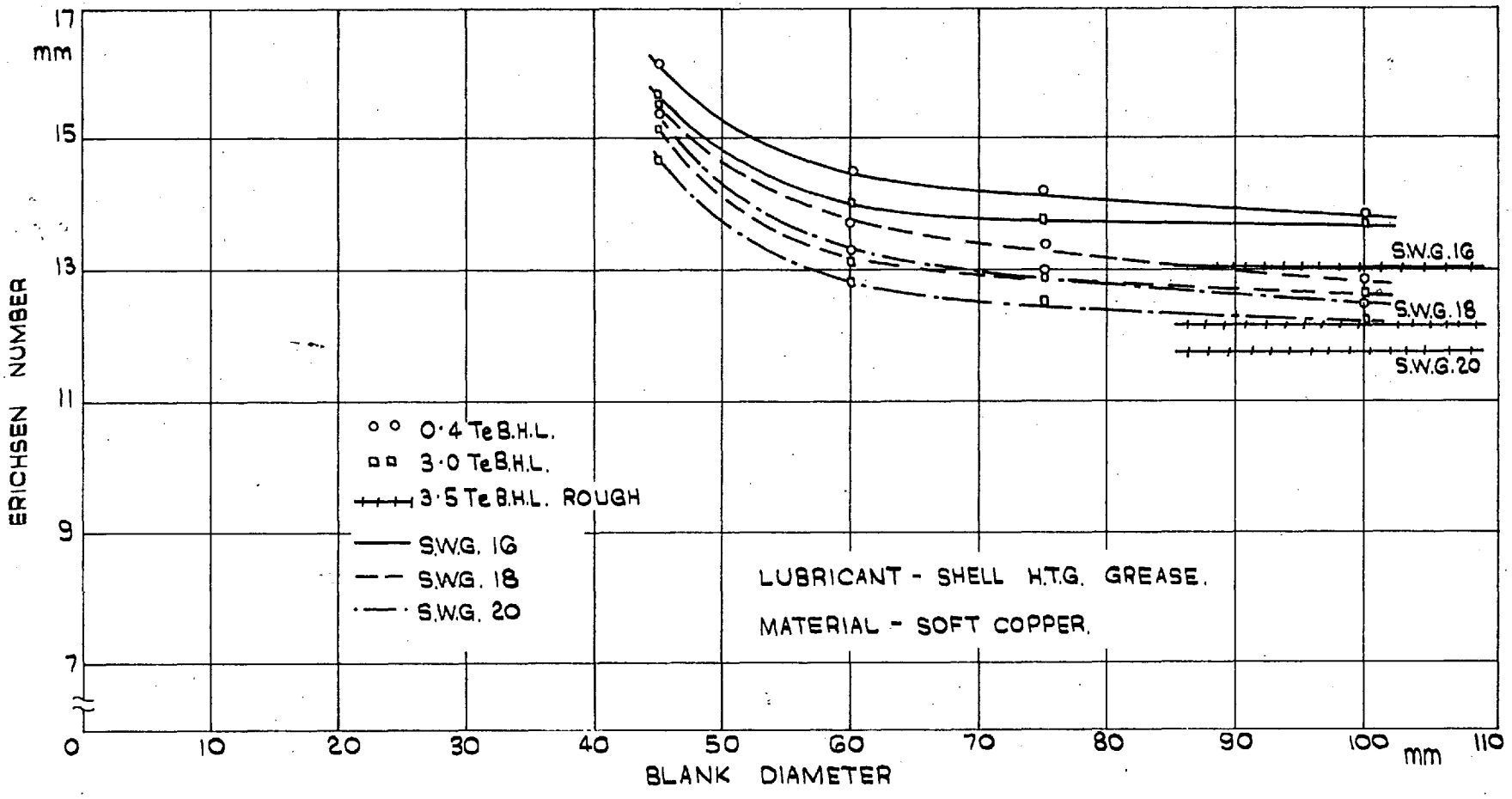



FIG. 40. VARIATIONS IN THE ERICHSEN NUMBER.

ROUND SPECIMENS.

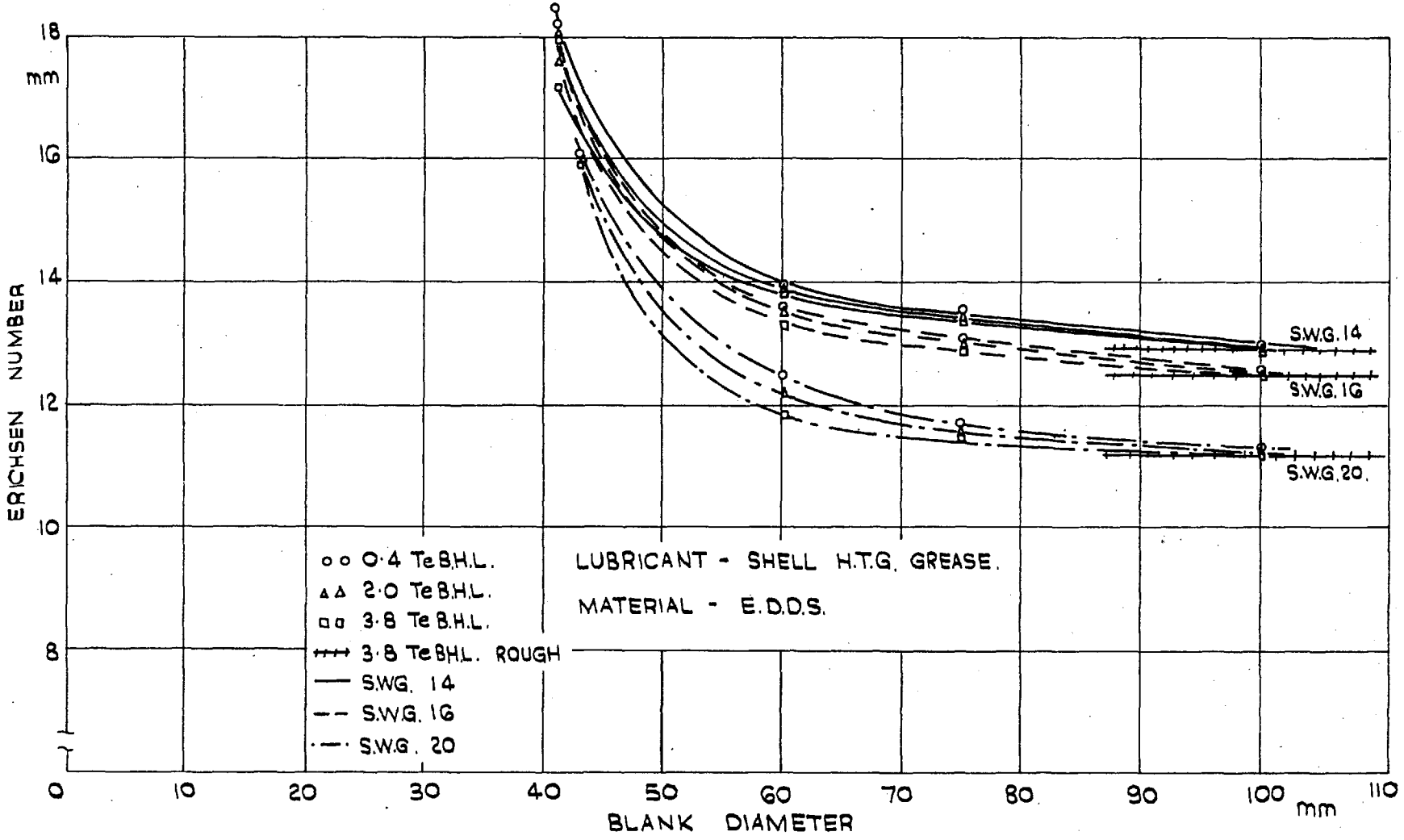



FIG. 41. VARIATIONS IN THE ERICHSEN NUMBER.

ROUND SPECIMENS.

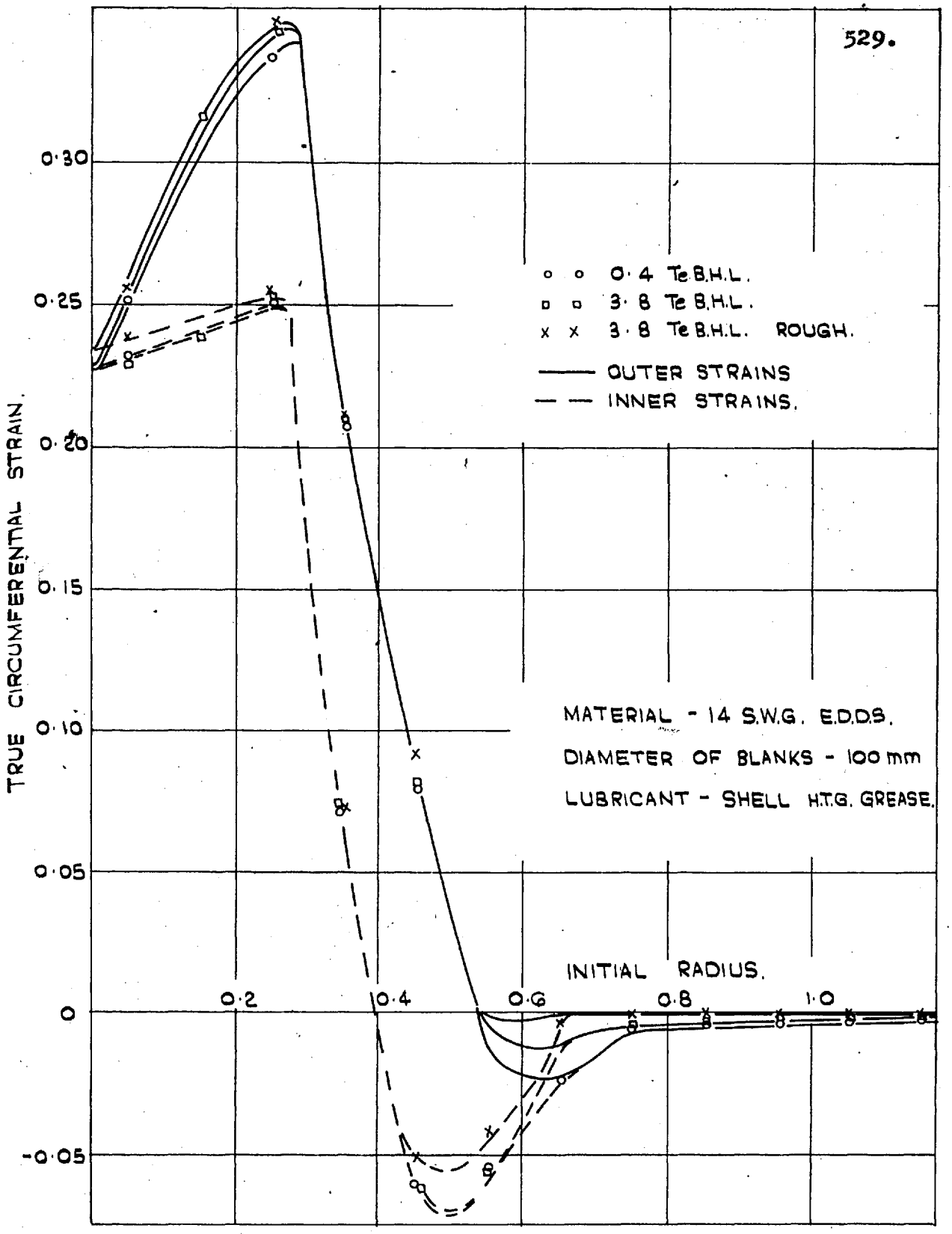



FIG. 42. TRUE CIRCUMFERENTIAL STRAIN IN THE ERICHSEN TEST.

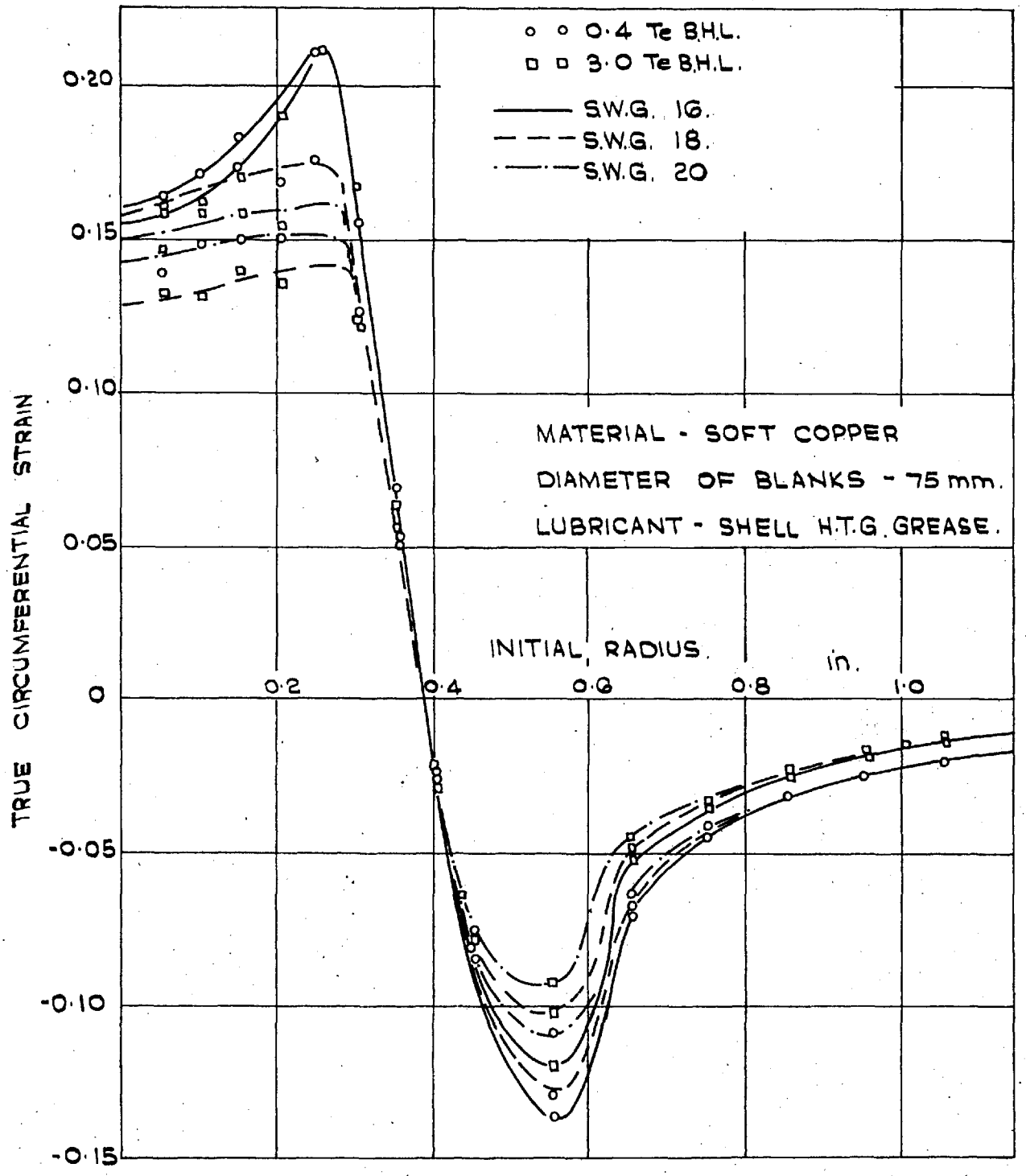



FIG. 43. TRUE INNER CIRCUMFERENTIAL STRAIN IN THE ERICHSEN TEST.

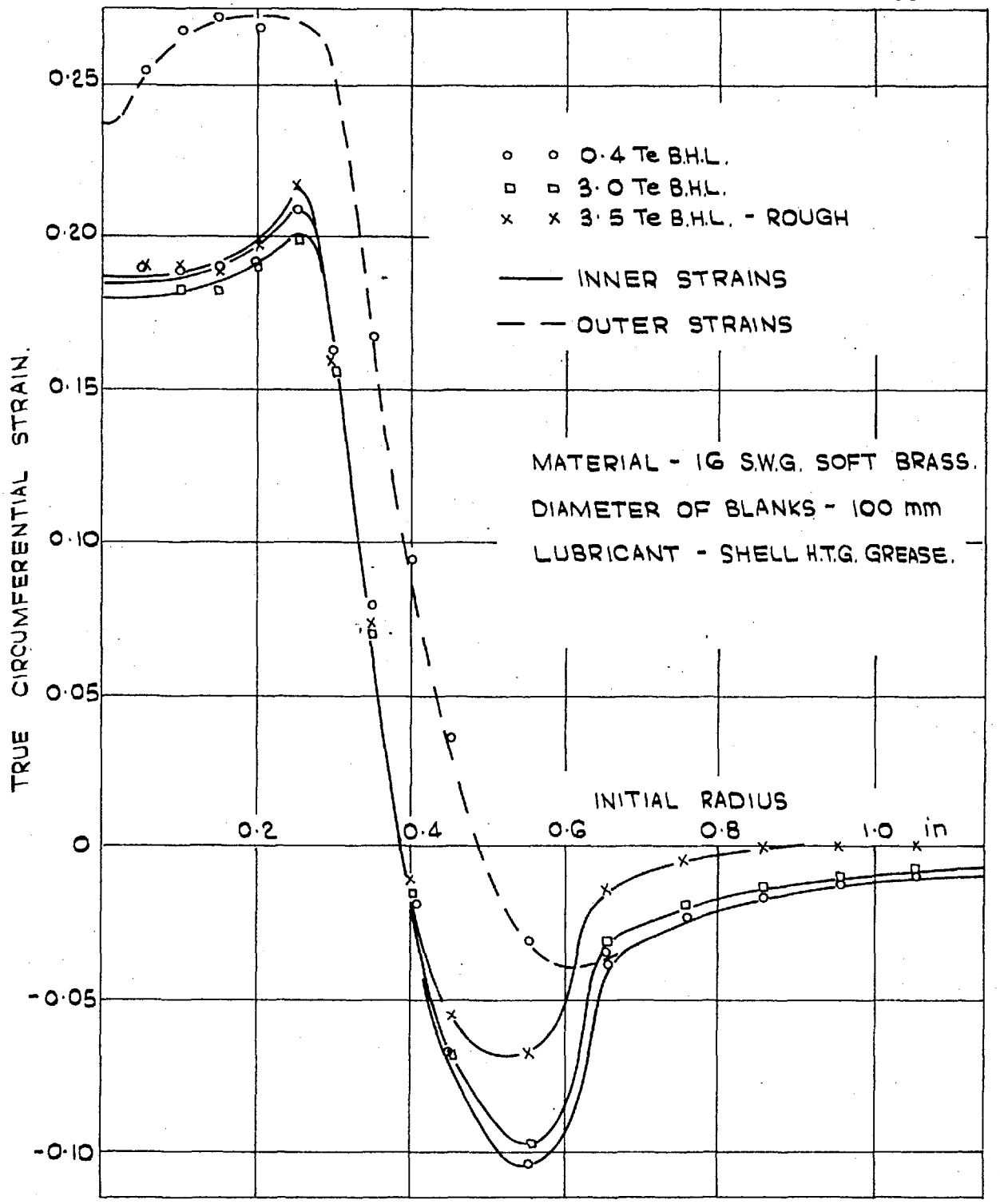



FIG. 44 TRUE CIRCUMFERENTIAL STRAIN IN THE ERICHSEN TEST.

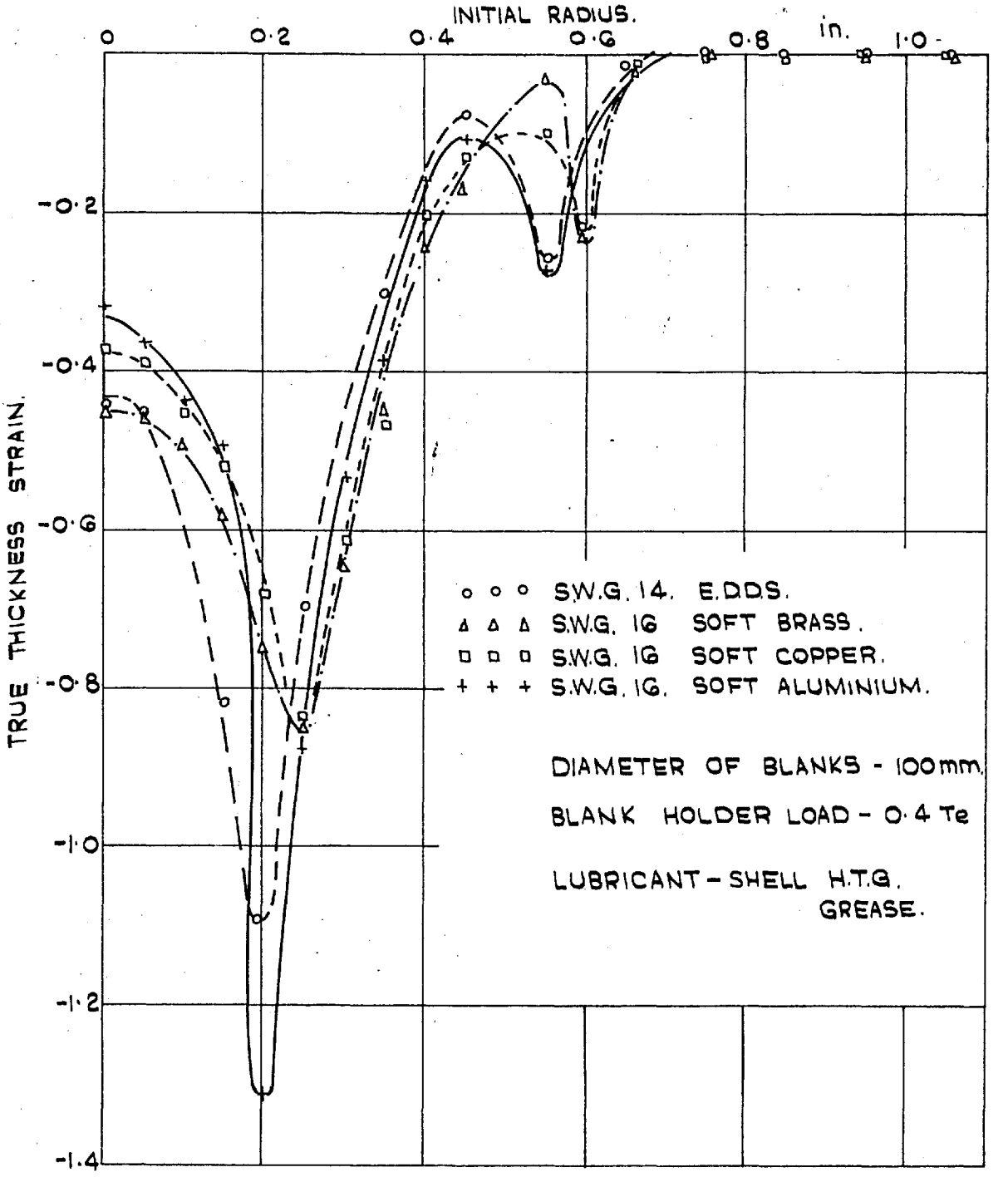



FIG. 45. TRUE THICKNESS STRAIN IN THE ERICHSEN TEST.

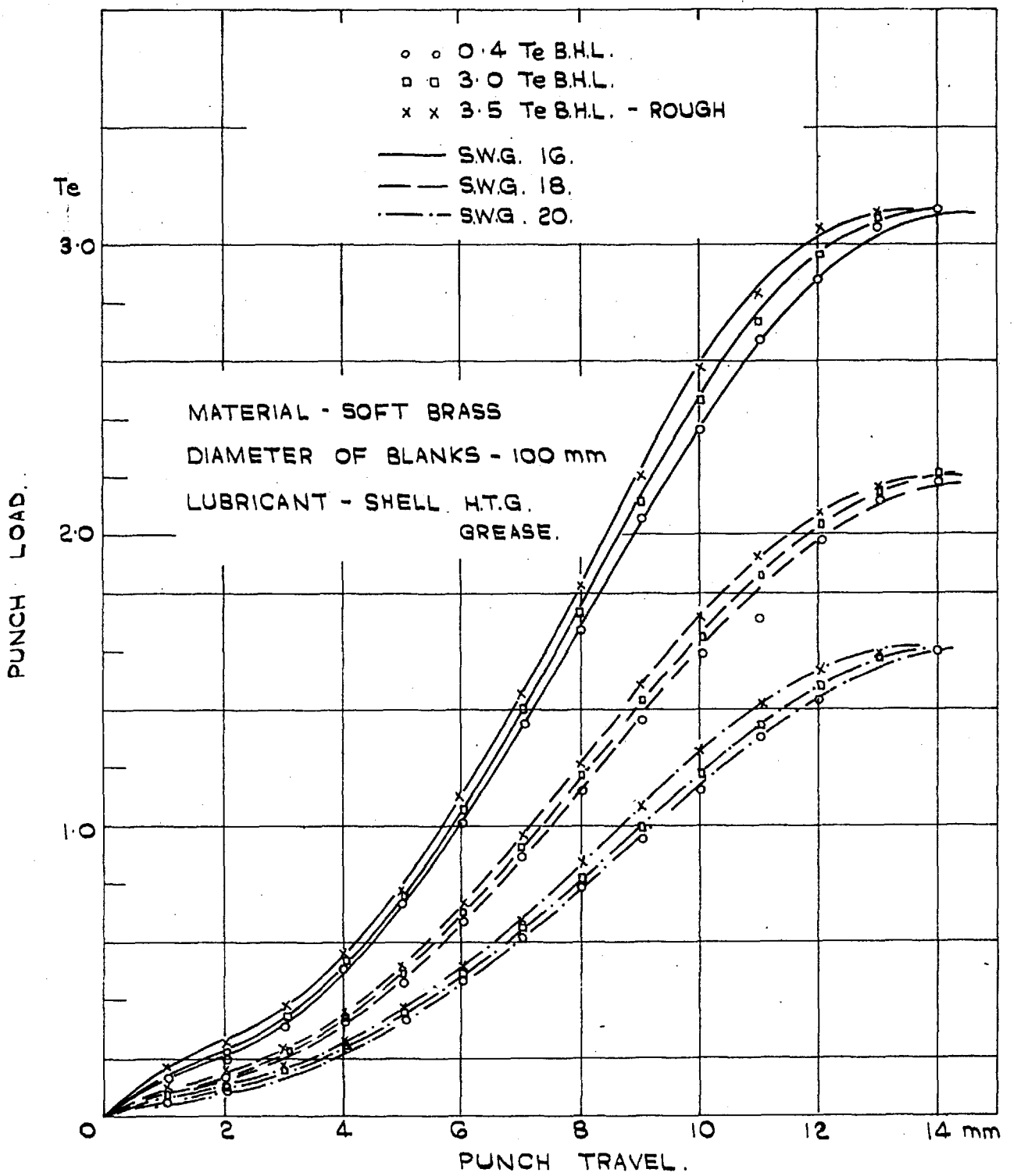
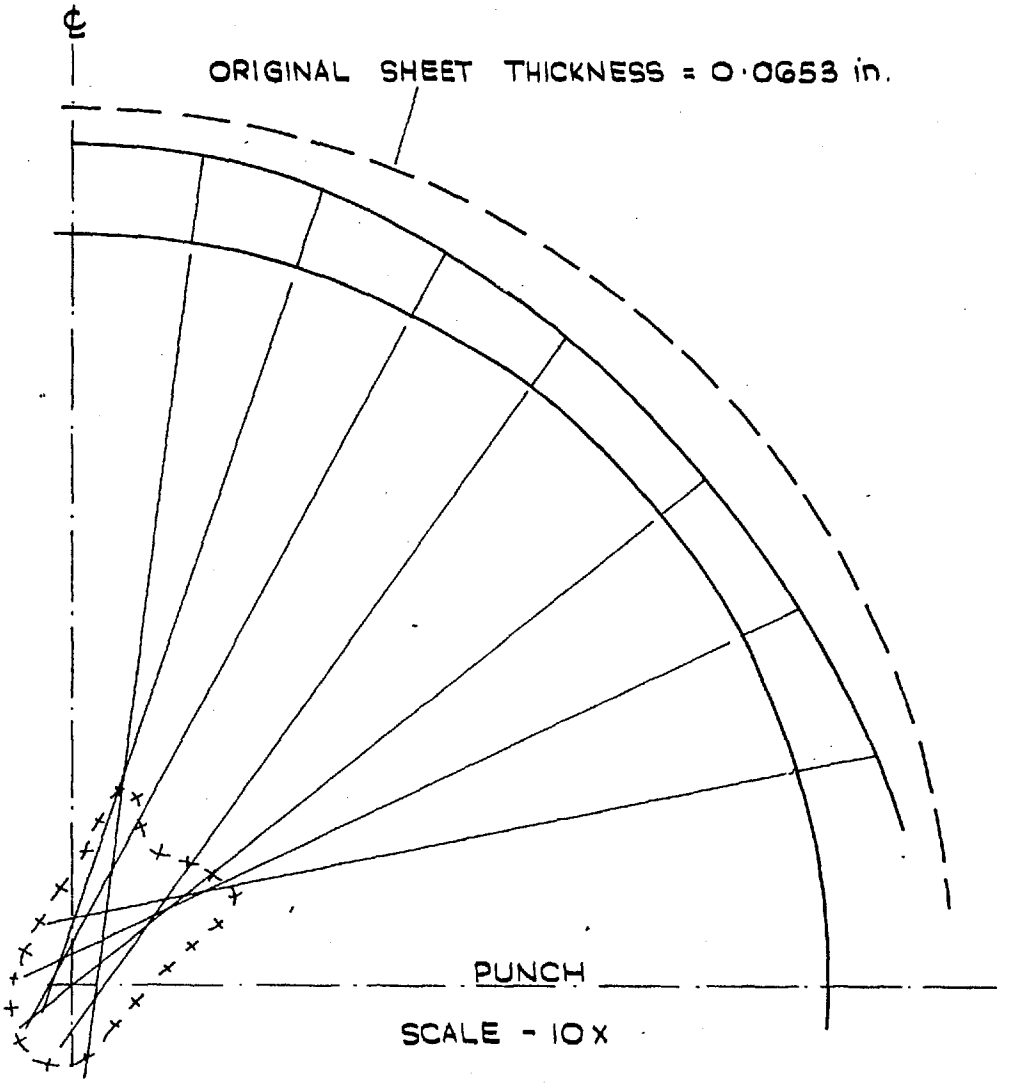



FIG. 46. PUNCH LOAD DIAGRAM.




+ + + Envelope of the individual centres of bending of the elements of the bulge.

LUBRICANT - SHELL H.T.G. GREASE.
MATERIAL - SOFT COPPER.

FIG. 47. ACTUAL GEOMETRY OF THE BULGE USING THE ERICHSEN PUNCH.

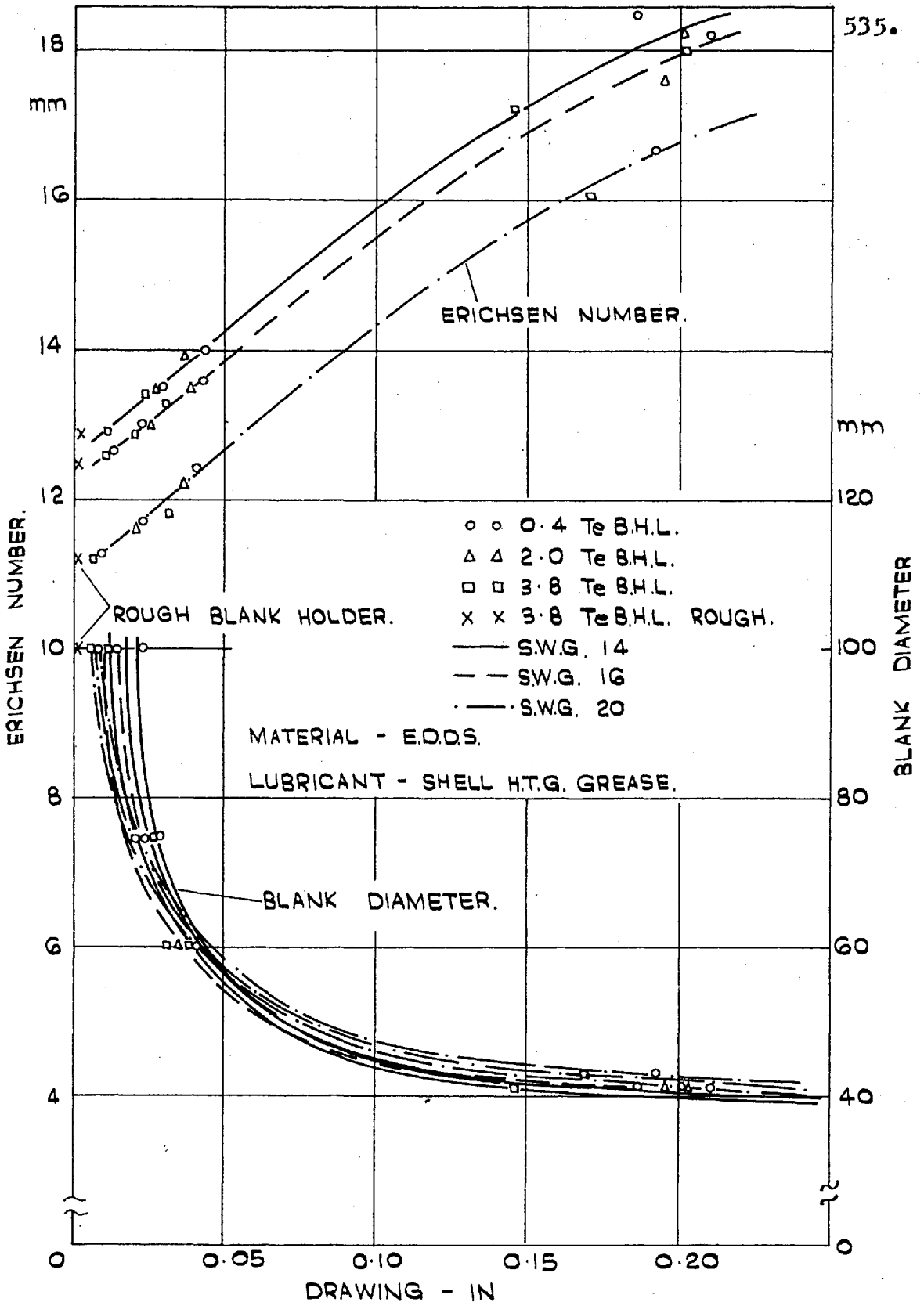



FIG. 48. VARIATION OF THE ERICHSEN Nº & BLANK DIAMETER WITH DRAWING - IN.

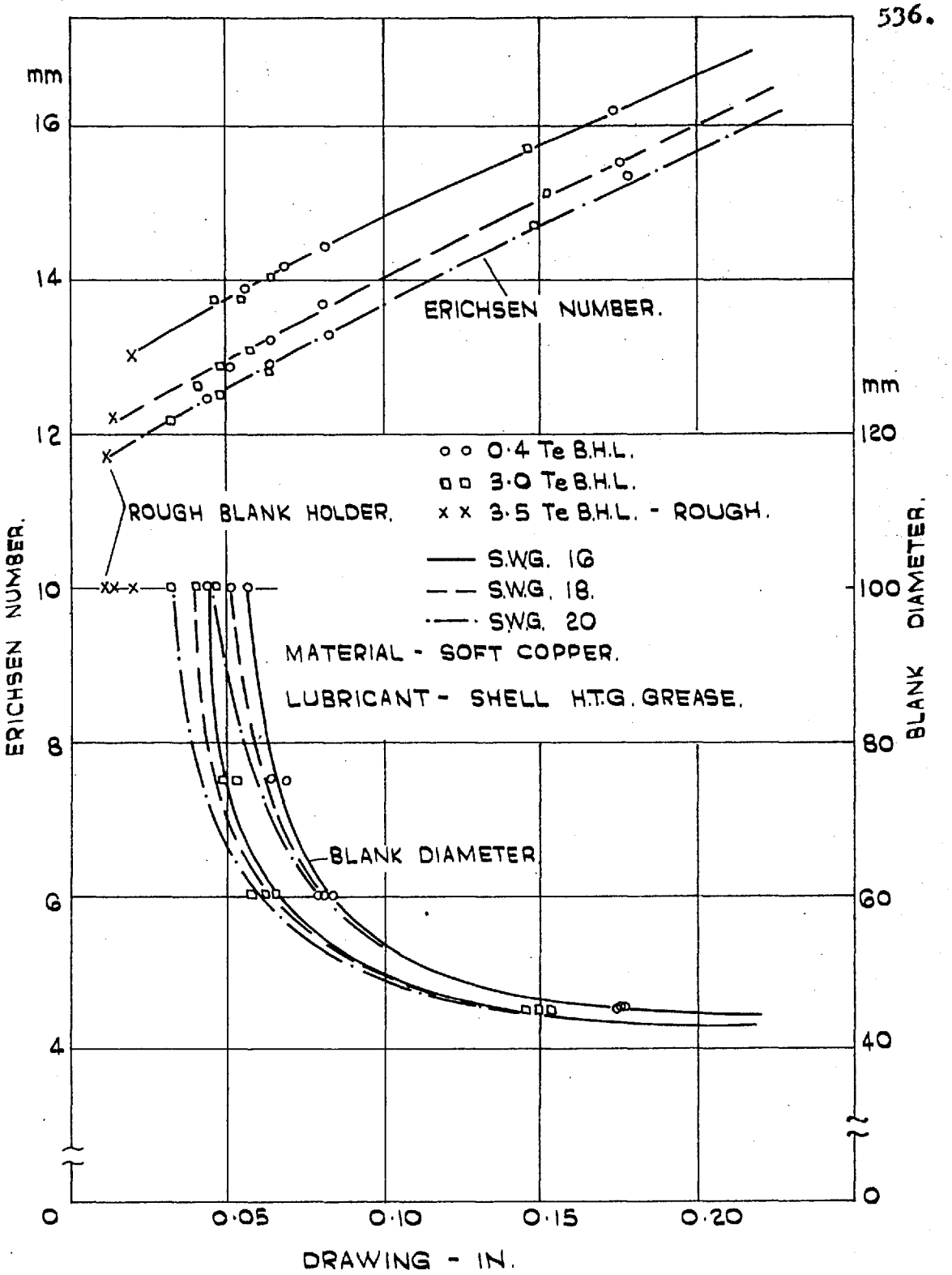



FIG. 49. VARIATION OF THE ERICHSEN Nº & BLANK DIAMETER WITH DRAWING - IN.

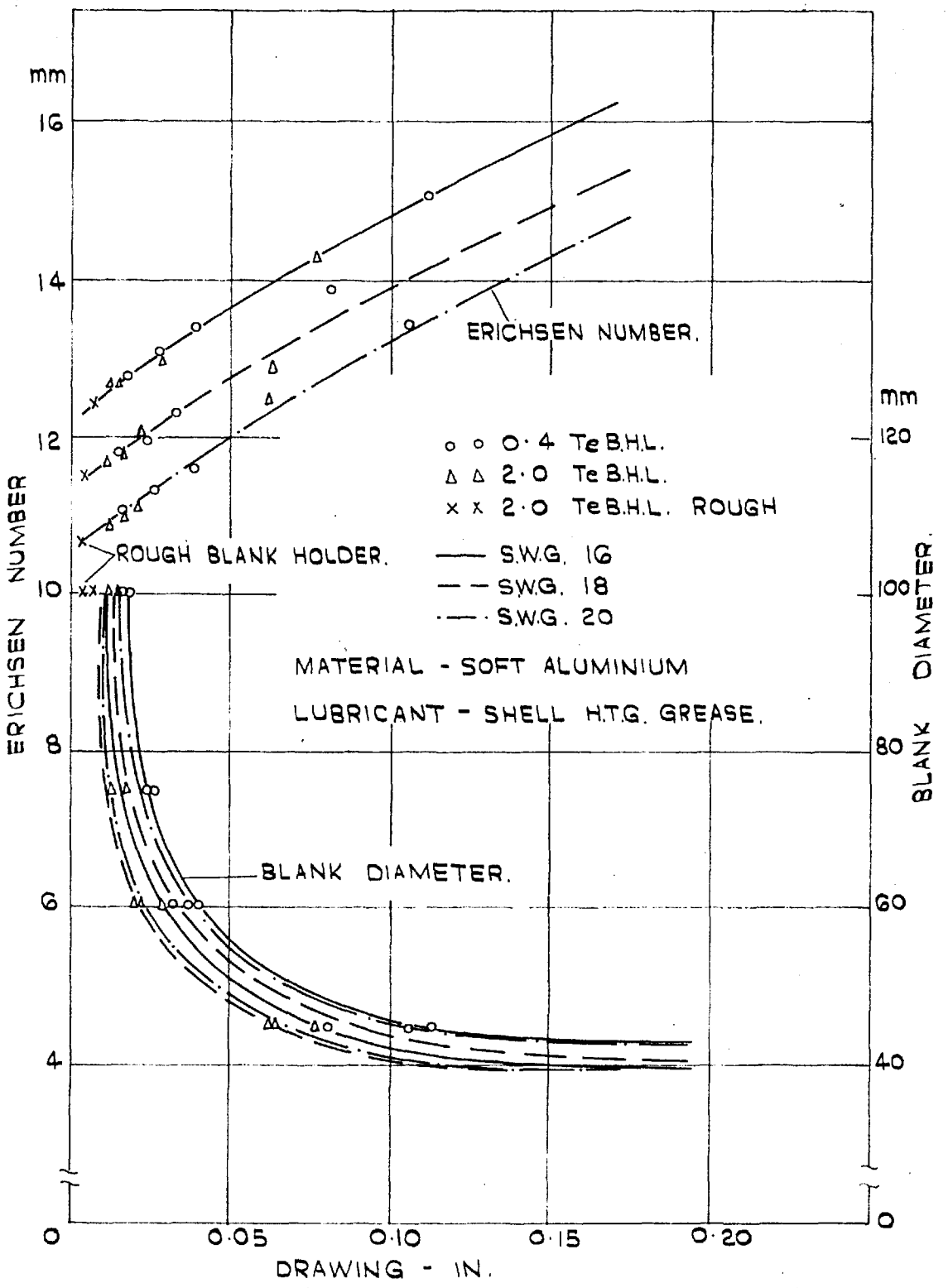



FIG. 51. VARIATION OF THE ERICHSEN Nº & BLANK DIAMETER WITH DRAWING - IN.

Table 11.1 - Necking and fracture in Erichsen test.

| Material | Thickness, SWG | Form of Necking | Form of Fracture | Other Remarks |
|---|---|---|---|---|
| | | | R (=Rolling direction) | R |
| Steel | 14 | All round, severe |  |  for 43 mm strips |
| | 16 | " |  " |  for 45 mm strips |
| | 20 | " |  " |  for 45 mm strips |
| Brass | 16 | Slightly visible |  Mostly one arc present |  Very rarely |
| | 18 | " |  " |  Very rarely |
| | 20 | " |  " |  Very rarely |
| Copper | 16 | All round, severe |  |  Rarely |
| | 18 | " |  | |
| | 20 | " |  |  Rarely |
| Aluminium | 16 | Slightly elliptical, very severe |  |  Some |
| | 18 | " |  |  Some |
| | 20 | " |  R |  Some R |

Note: For Aluminium - Elliptical necking was measured as follows:

S.W.G. 16 $\frac{D_T}{D_R} = \frac{0.559}{0.538} = 1.04$; S.W.G. 18 $\frac{D_T}{D_R} = \frac{0.560}{0.533} = 1.05$; S.W.G.20 $\frac{D_T}{D_R} = \frac{0.549}{0.541} = 1.065$

$D_T$ = dia. of neck in transverse direction $D_R$ = dia. of neck in rolling direction

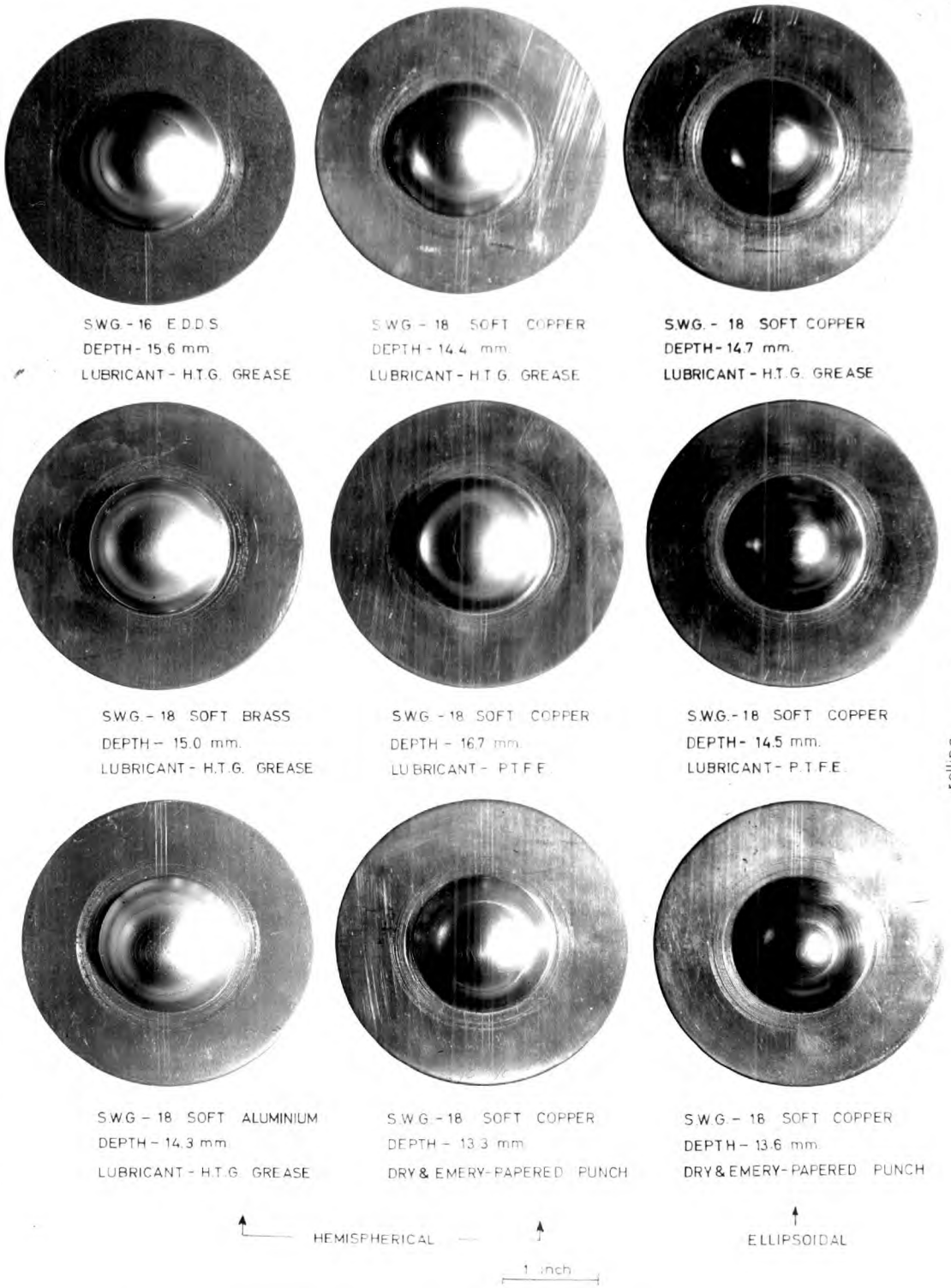
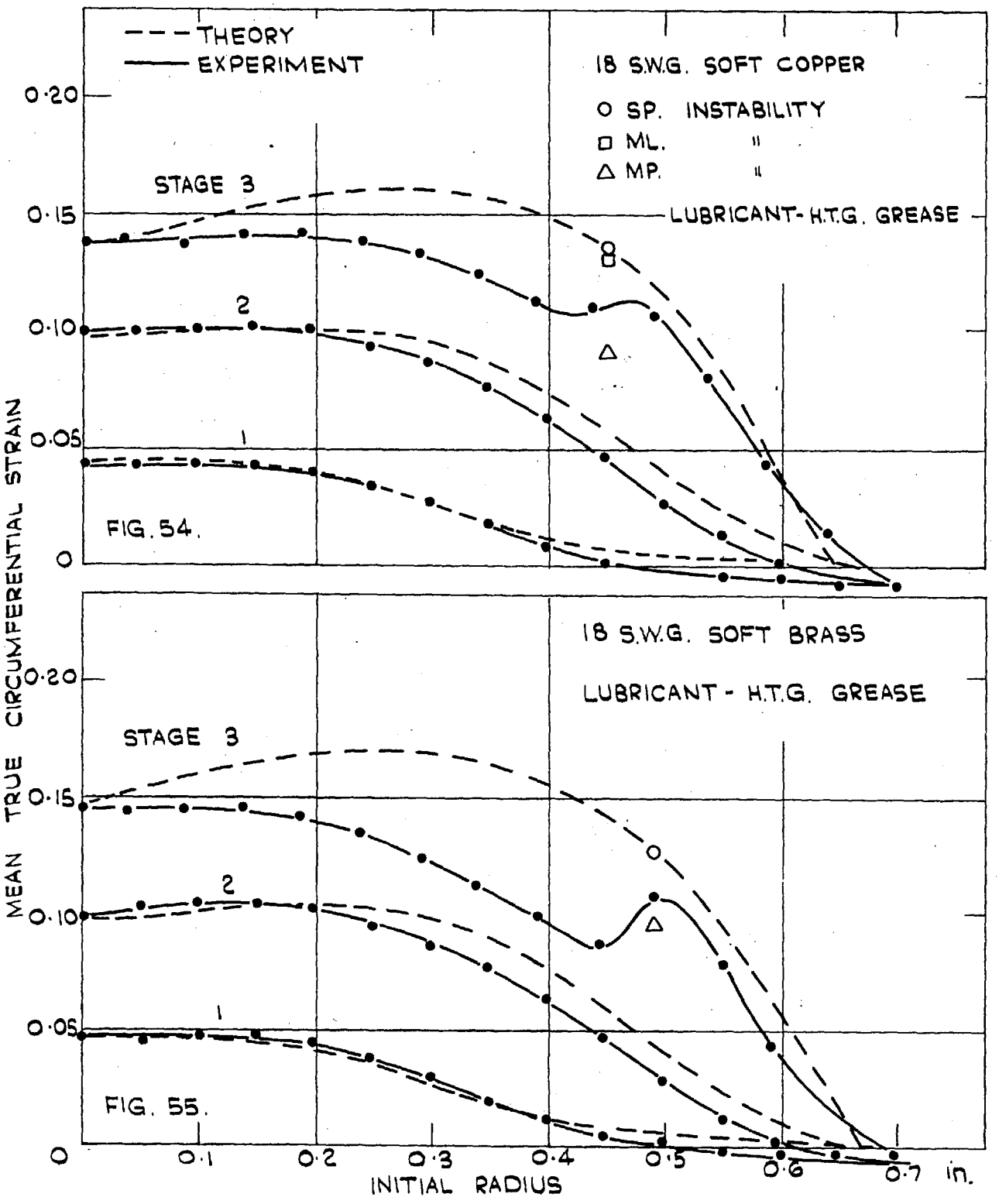
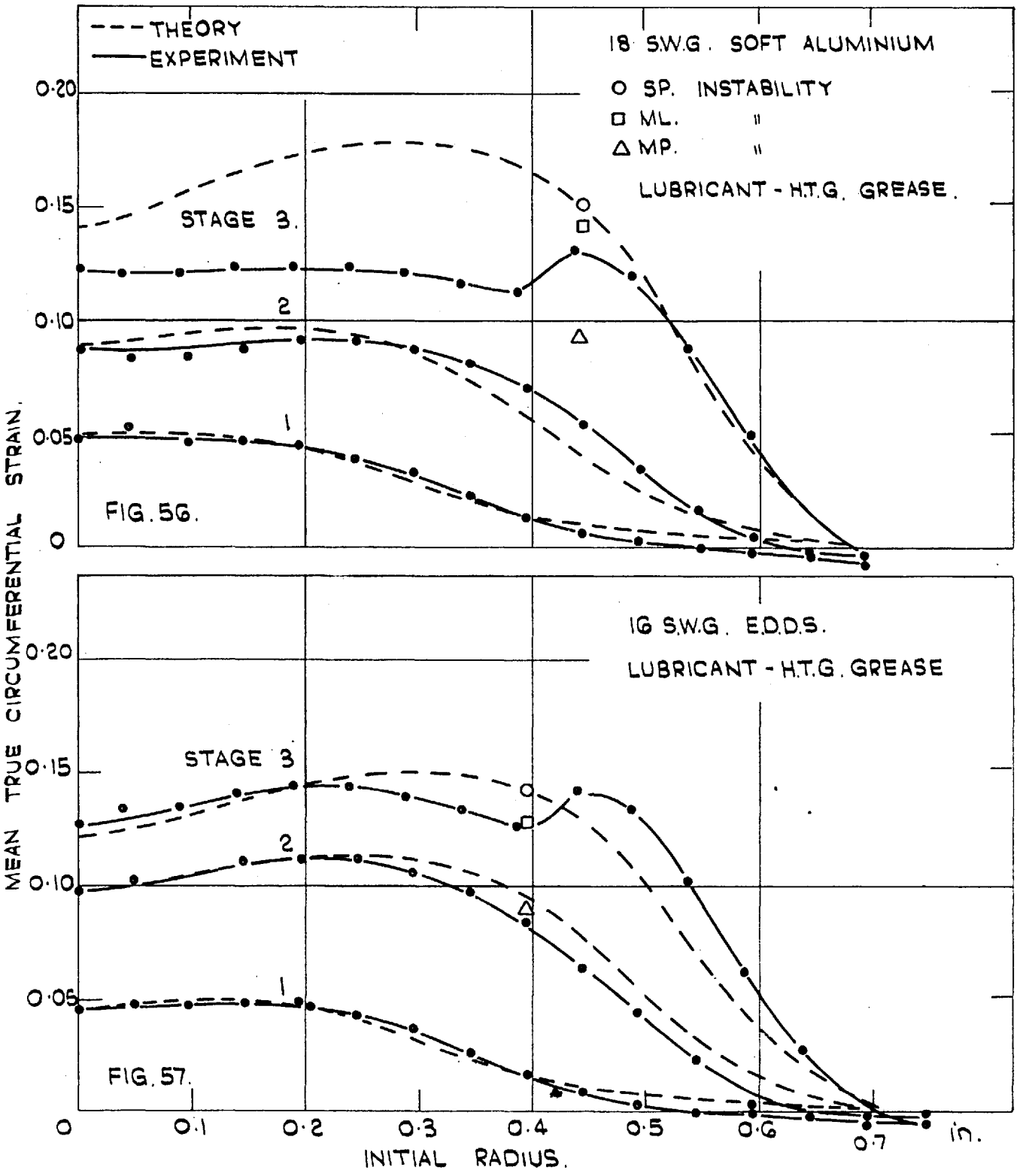


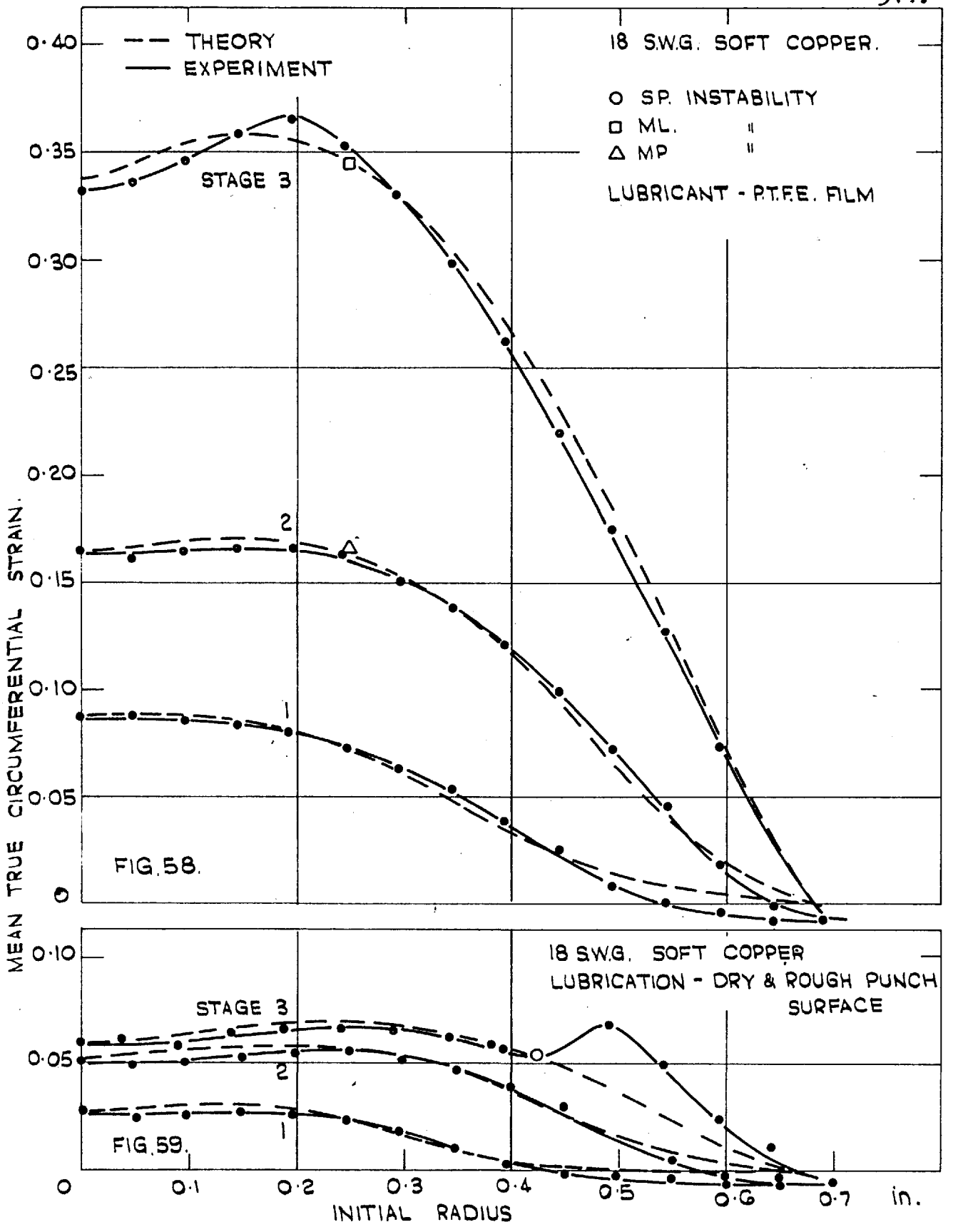

FIG. 53 - STRETCH FORMED SPECIMENS



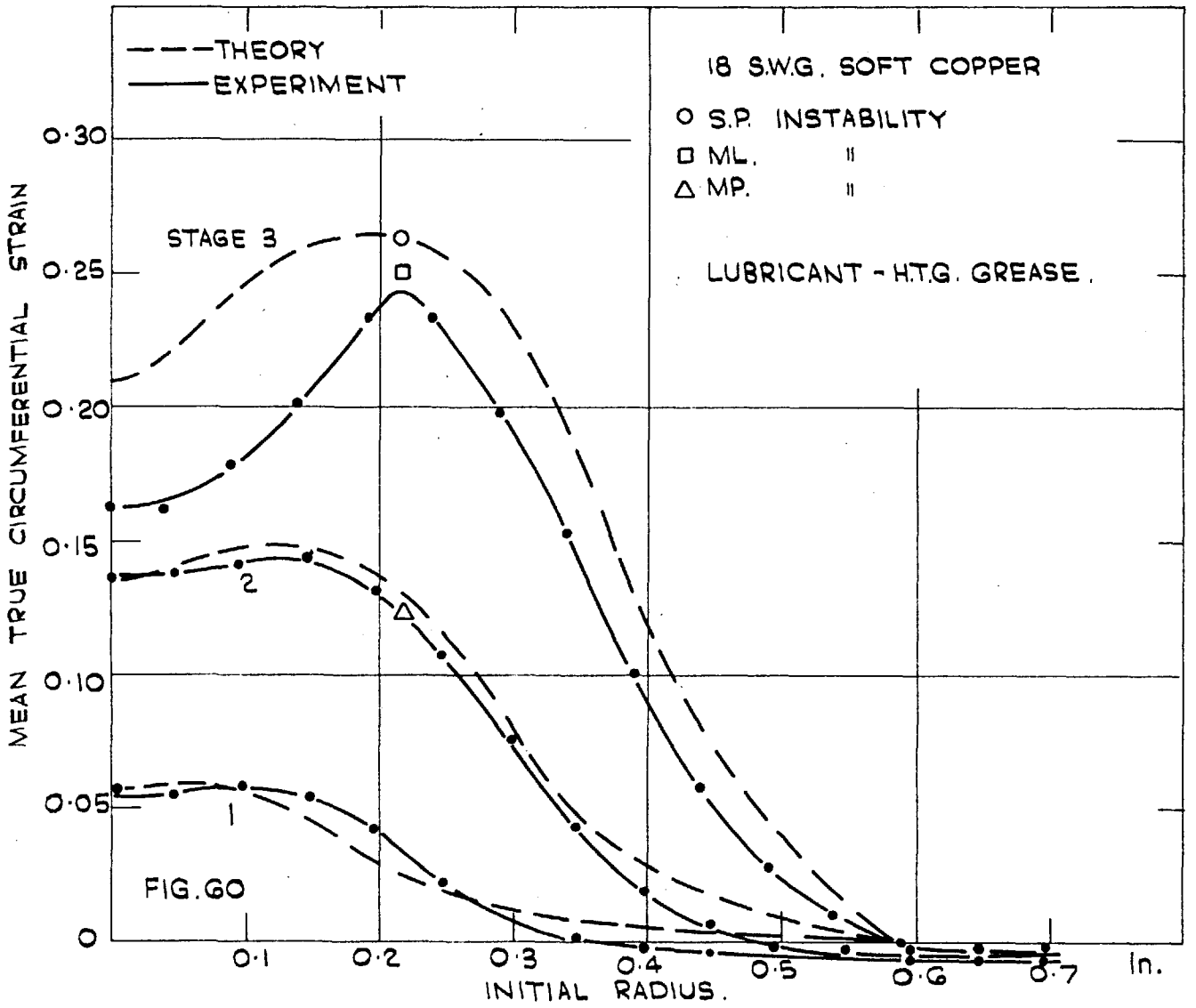

CIRCUMFERENTIAL STRAIN IN STRETCH FORMING BY HEMISPHERICAL PUNCH



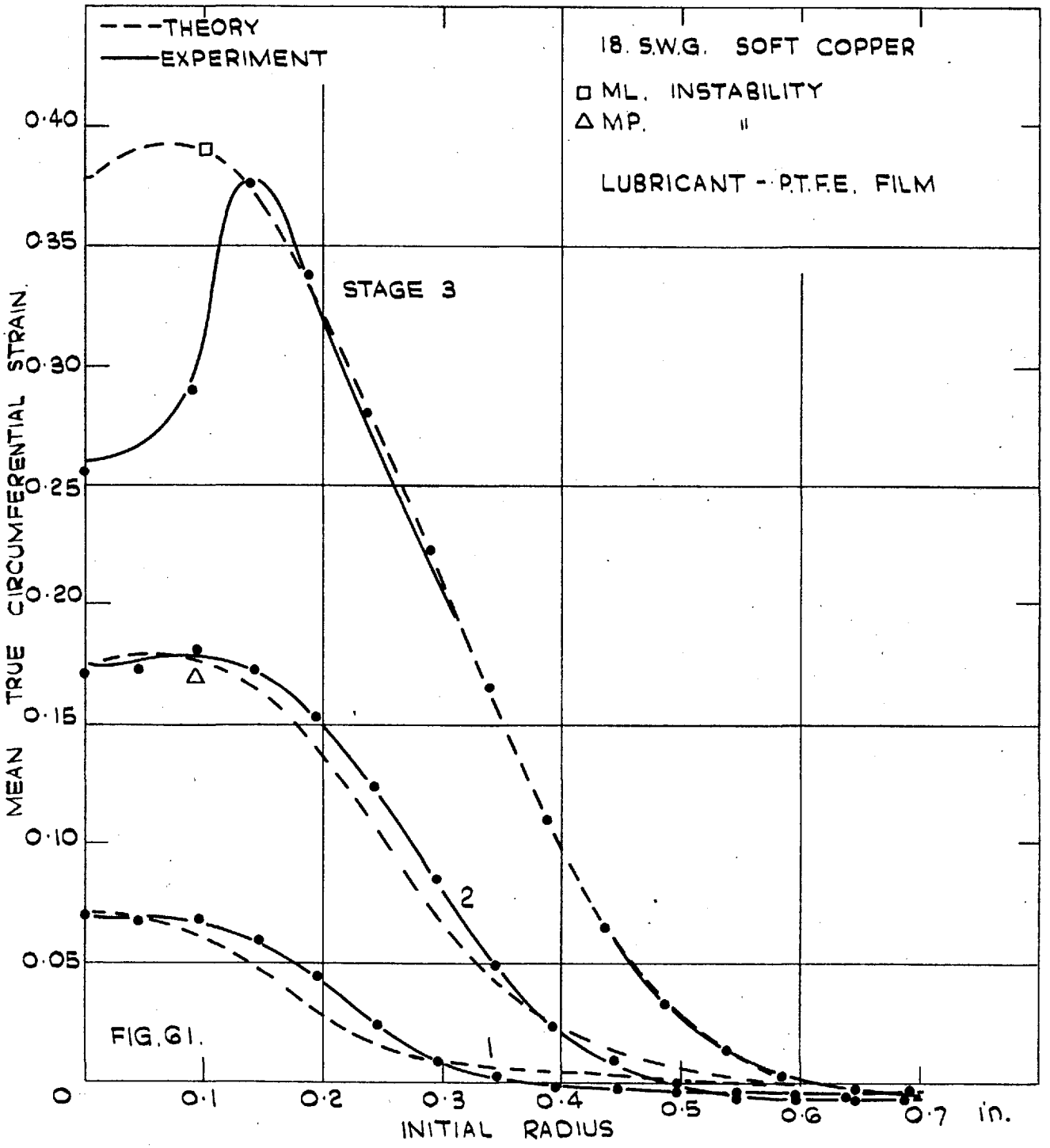

CIRCUMFERENTIAL STRAIN IN STRETCH FORMING BY HEMISPHERICAL PUNCH.




CIRCUMFERENTIAL STRAIN IN STRETCH FORMING BY HEMISPHERICAL PUNCH.




CIRCUMFERENTIAL STRAIN IN STRETCH FORMING BY ELLIPSOIDAL PUNCH.




CIRCUMFERENTIAL STRAIN IN STRETCH FORMING BY ELLIPSOIDAL PUNCH.

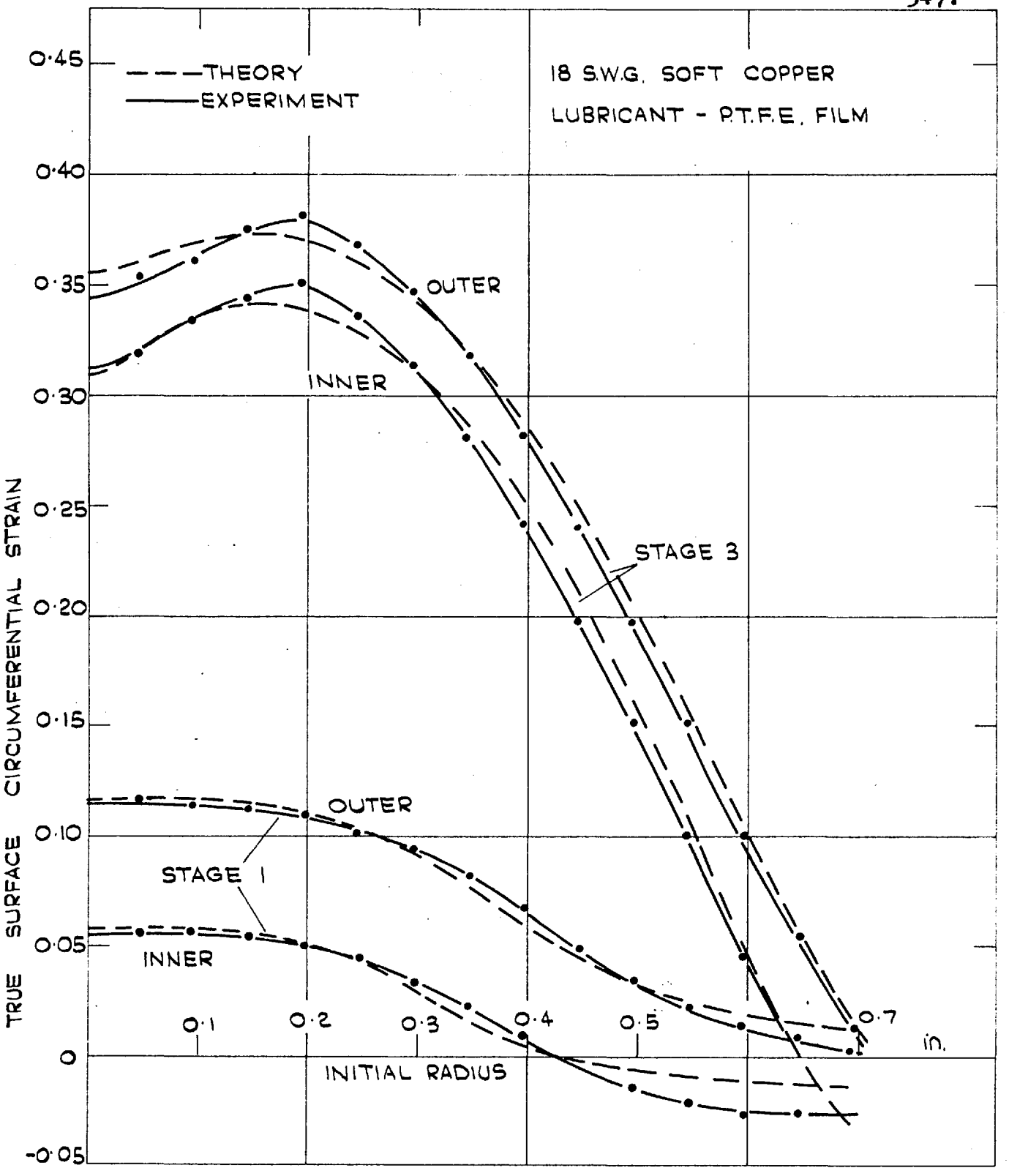
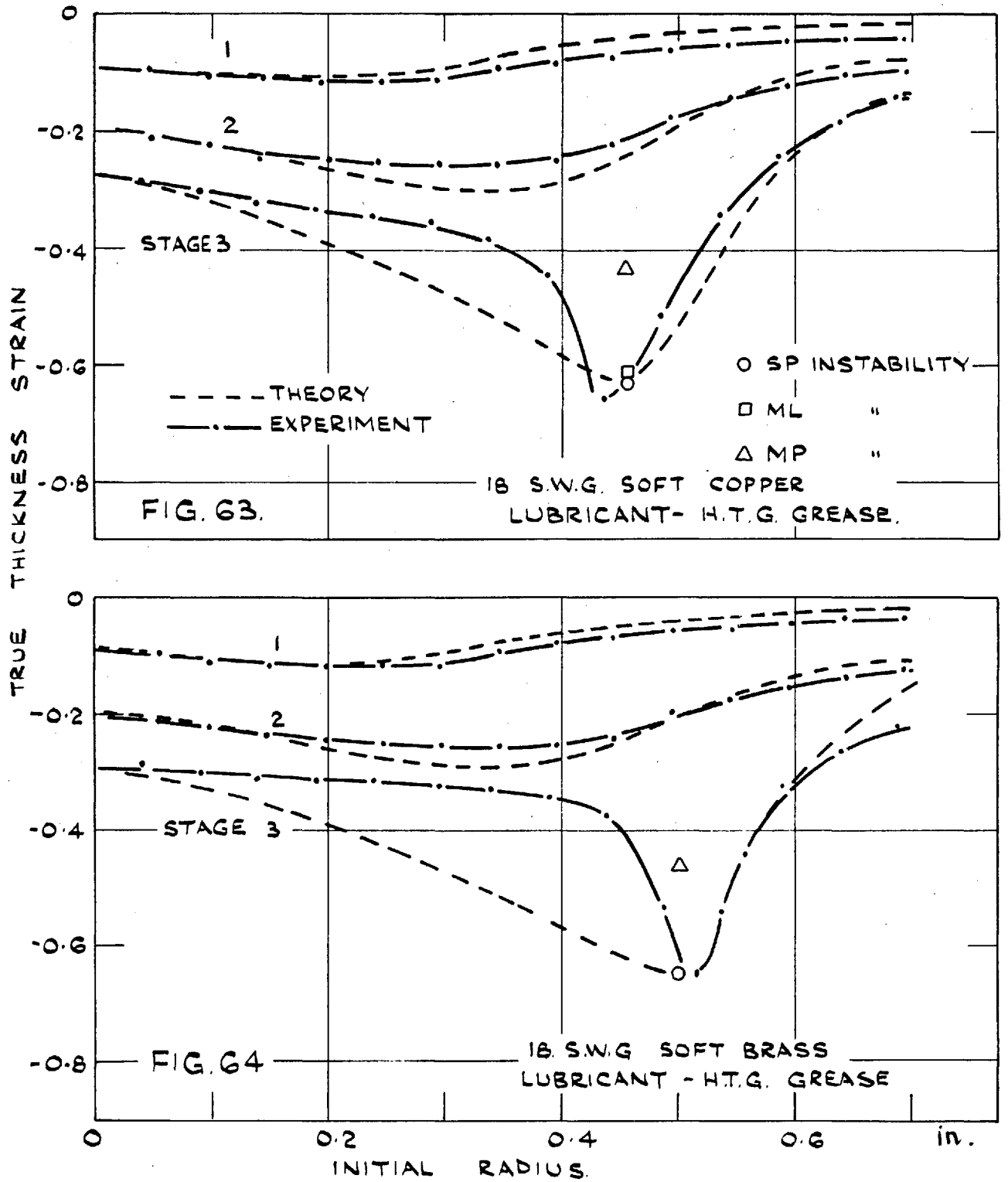
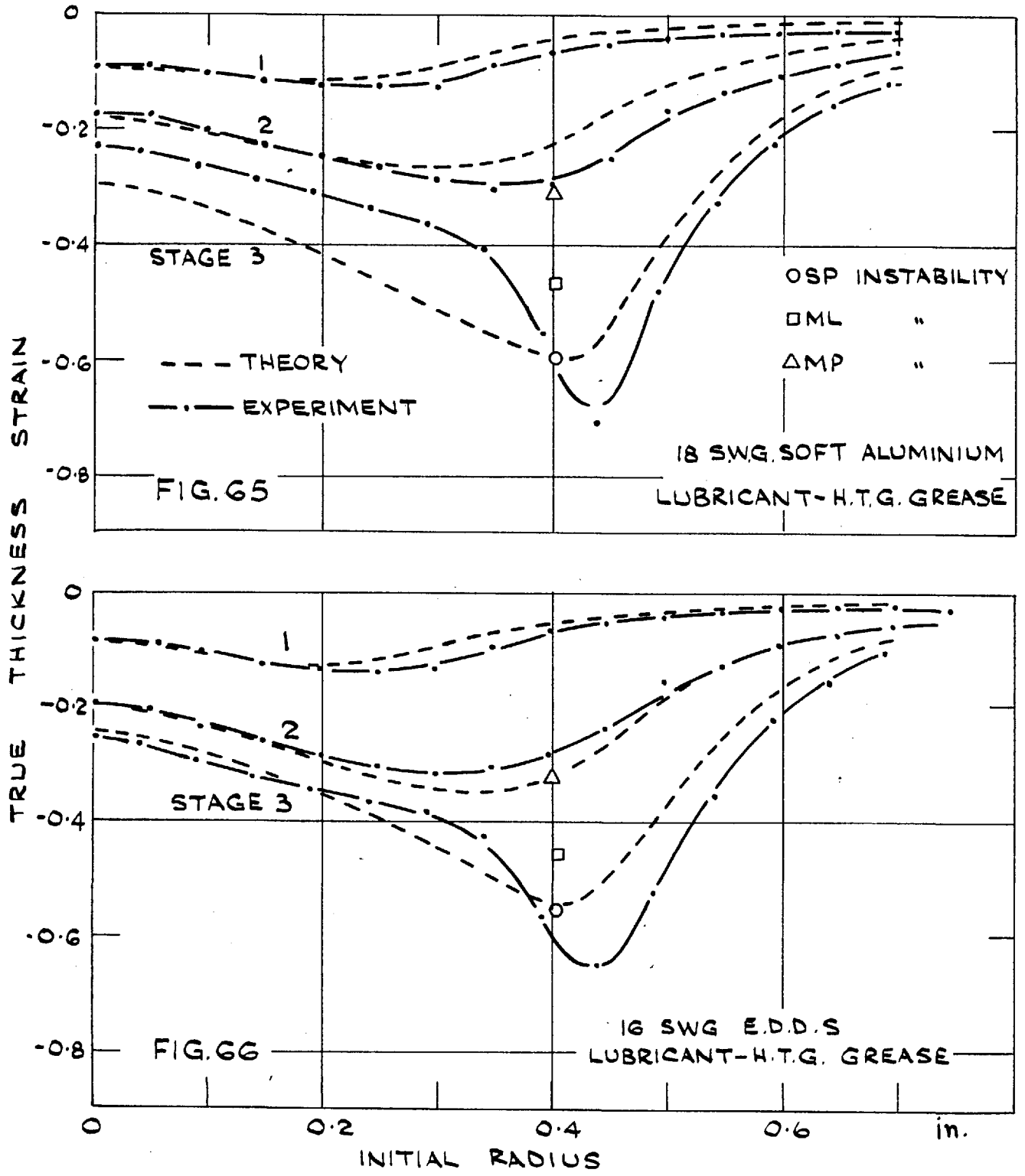



FIG. 62. SURFACE CIRCUMFERENTIAL STRAINS IN STRETCH FORMING BY HEMISPHERICAL PUNCH.




THICKNESS STRAIN IN STRETCH FORMING BY HEMISPHERICAL PUNCH.




THICKNESS STRAIN IN STRETCH FORMING BY HEMISPHERICAL PUNCH.

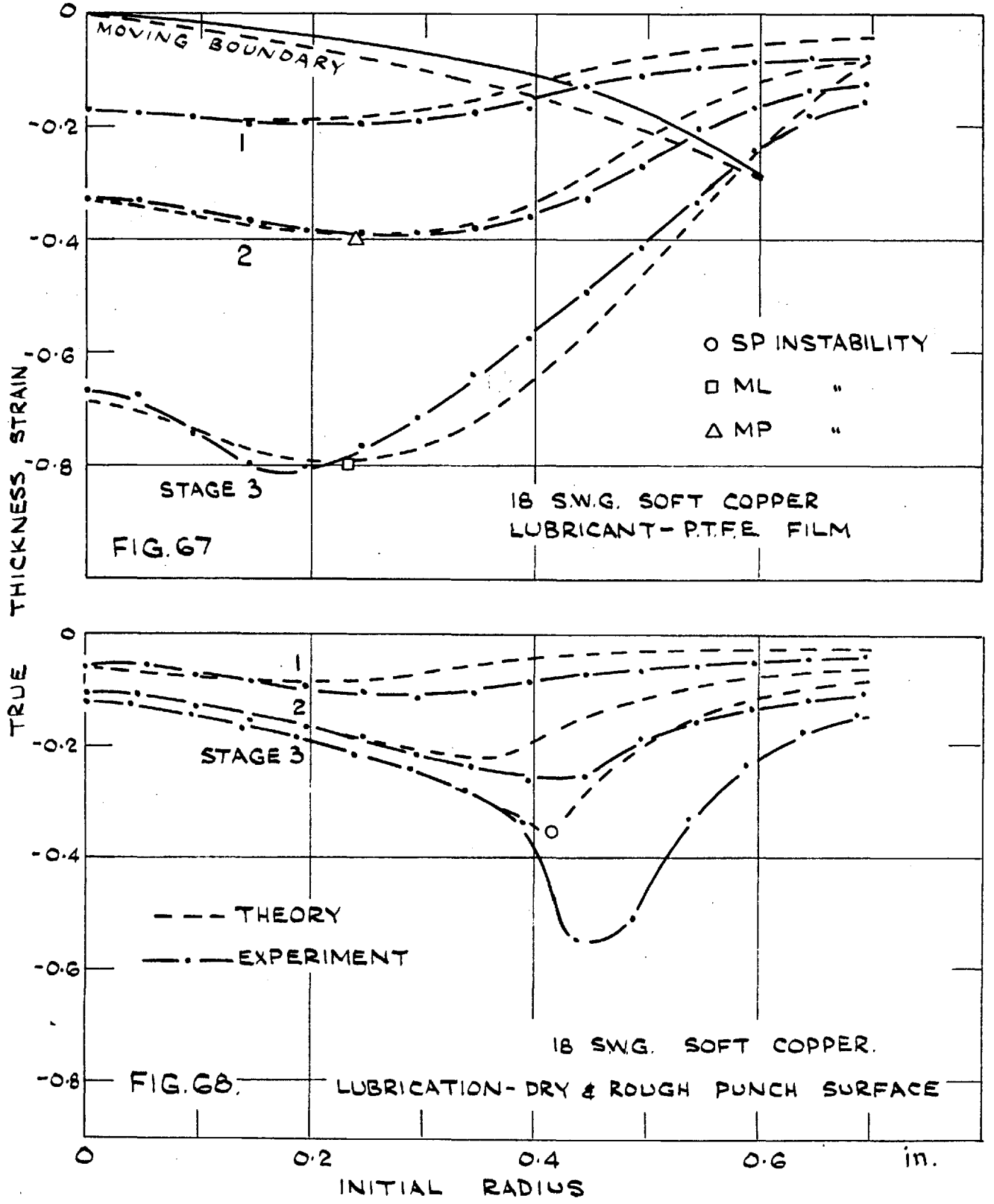



FIG. 67




FIG. 68.

THICKNESS STRAIN IN STRETCH FORMING BY HEMISPHERICAL PUNCH.

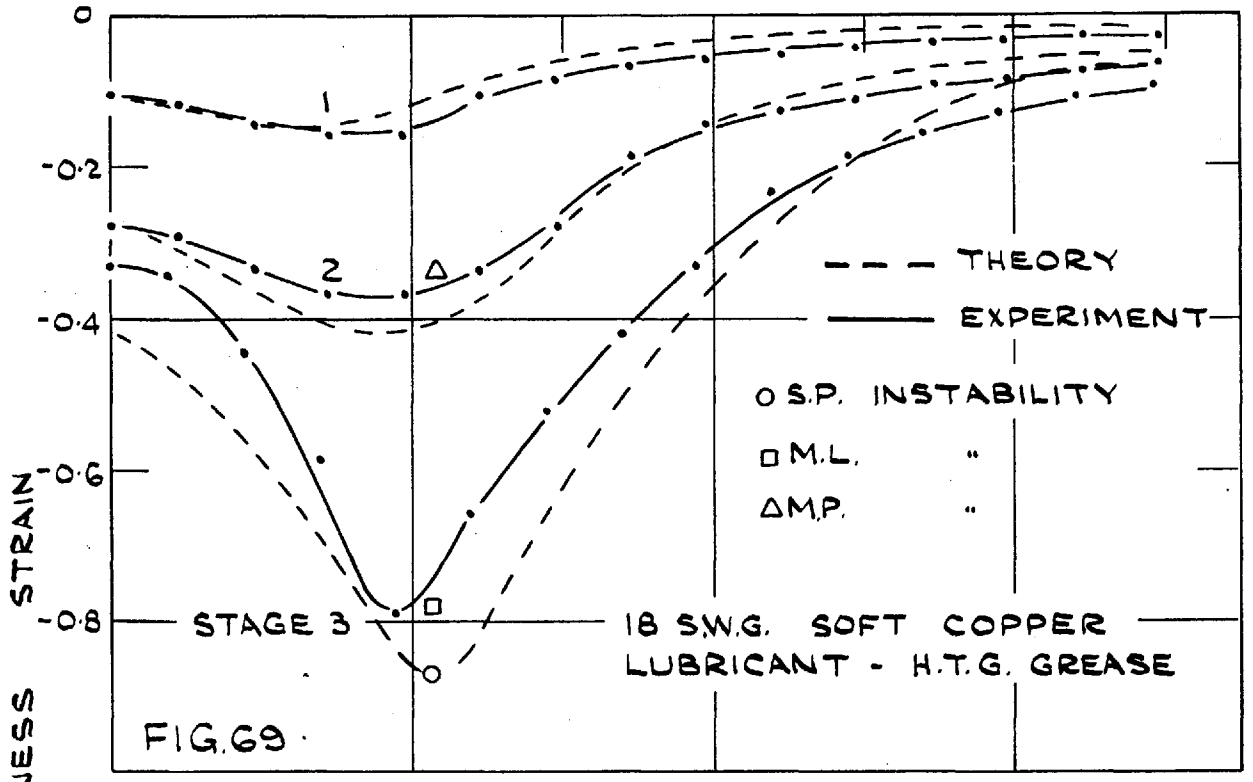



FIG.69.

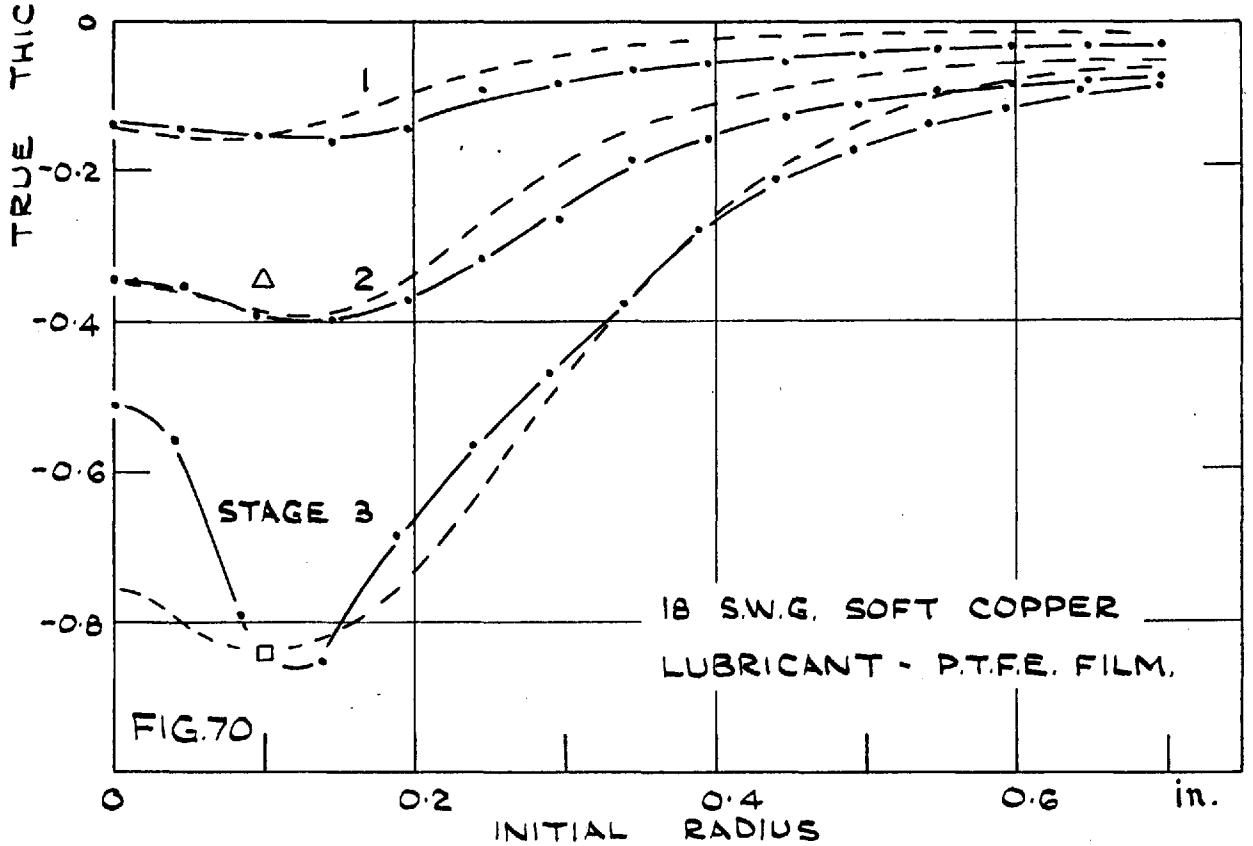
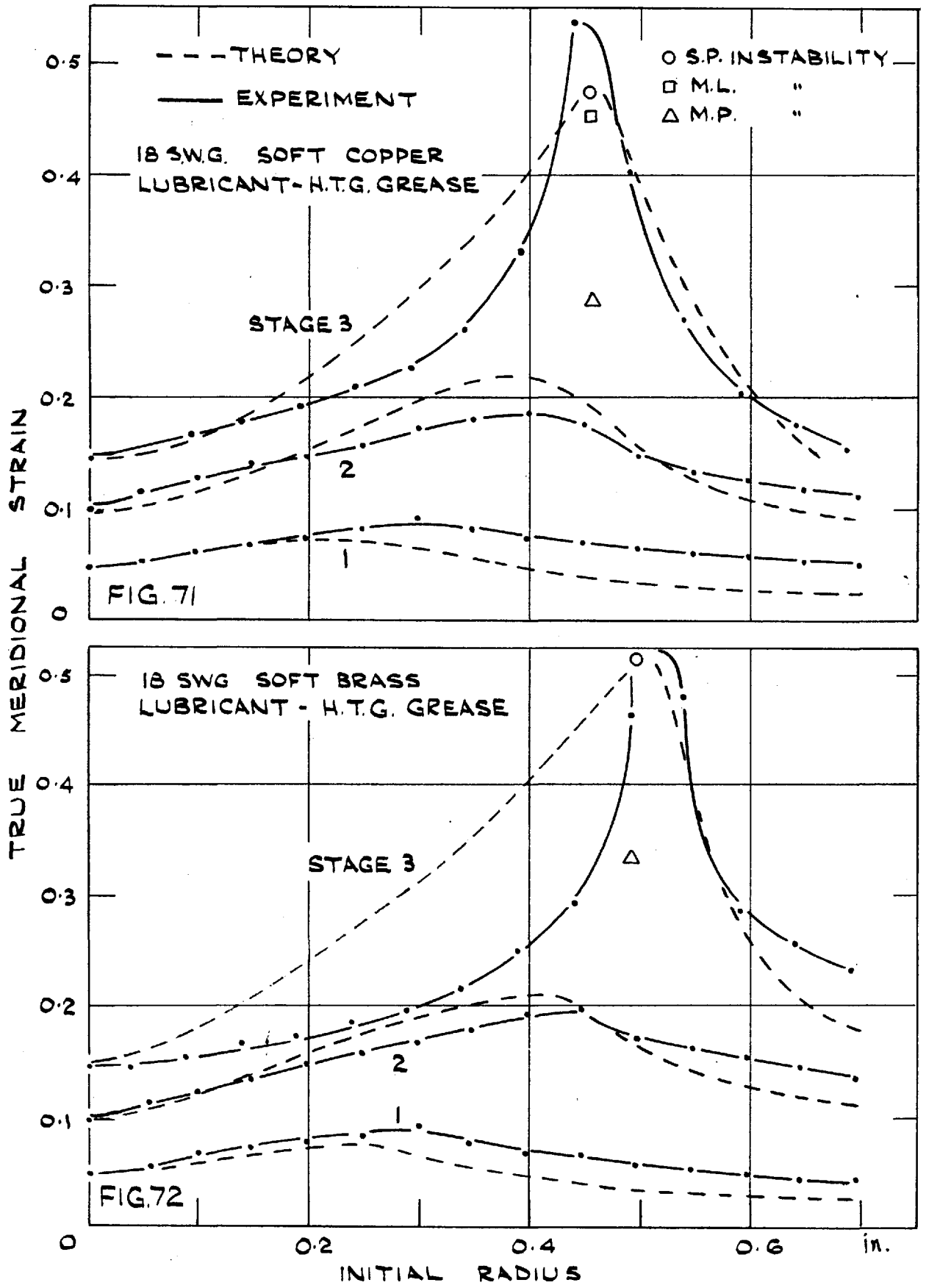
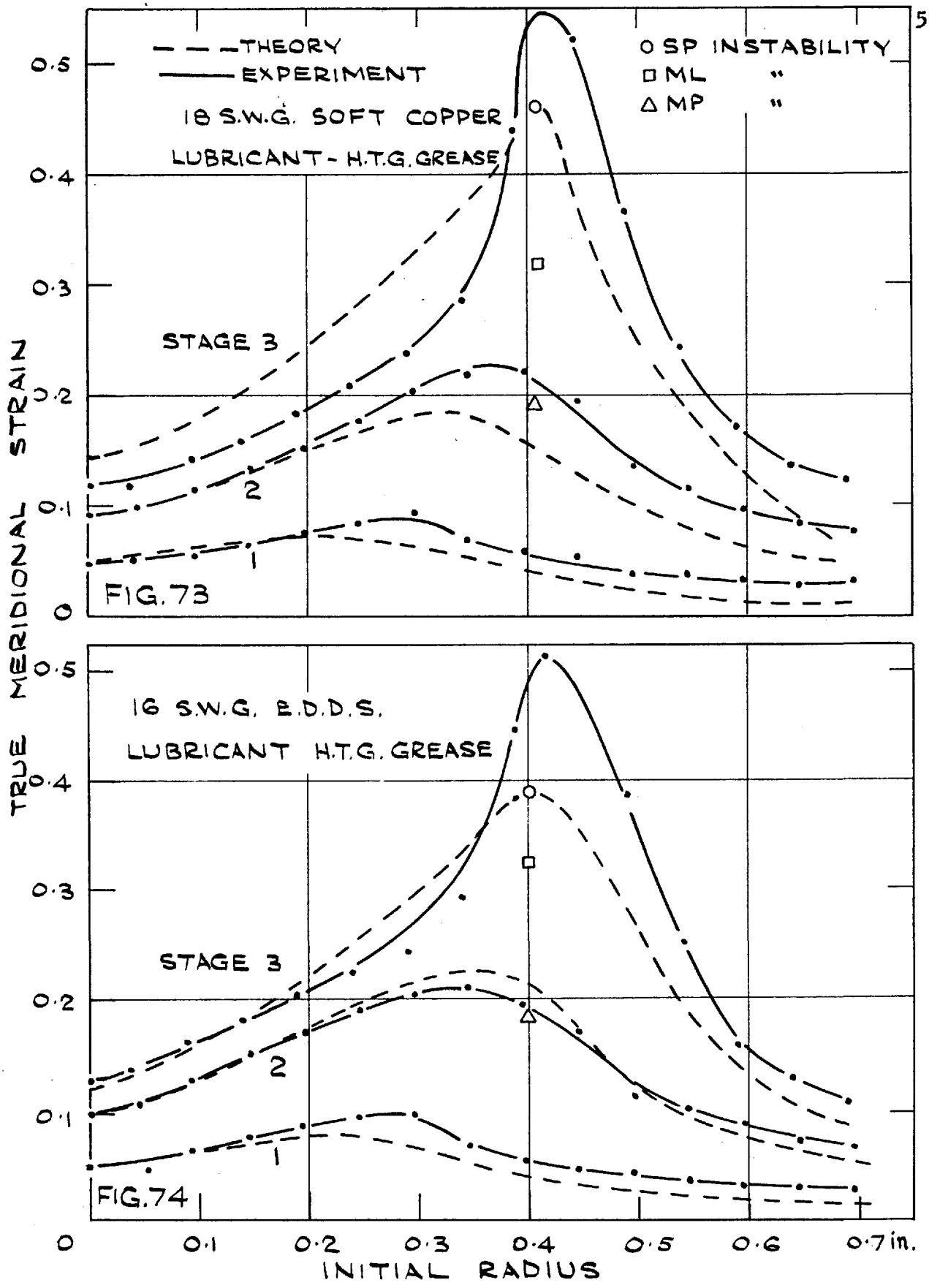



FIG.70.

THICKNESS STRAIN IN STRETCH FORMING BY ELLIPSOIDAL PUNCH.




MERIDIONAL STRAIN IN STRETCH FORMING BY HEMISPHERICAL PUNCH.




MERIDIONAL STRAIN IN STRETCH FORMING BY HEMISPHERICAL PUNCH

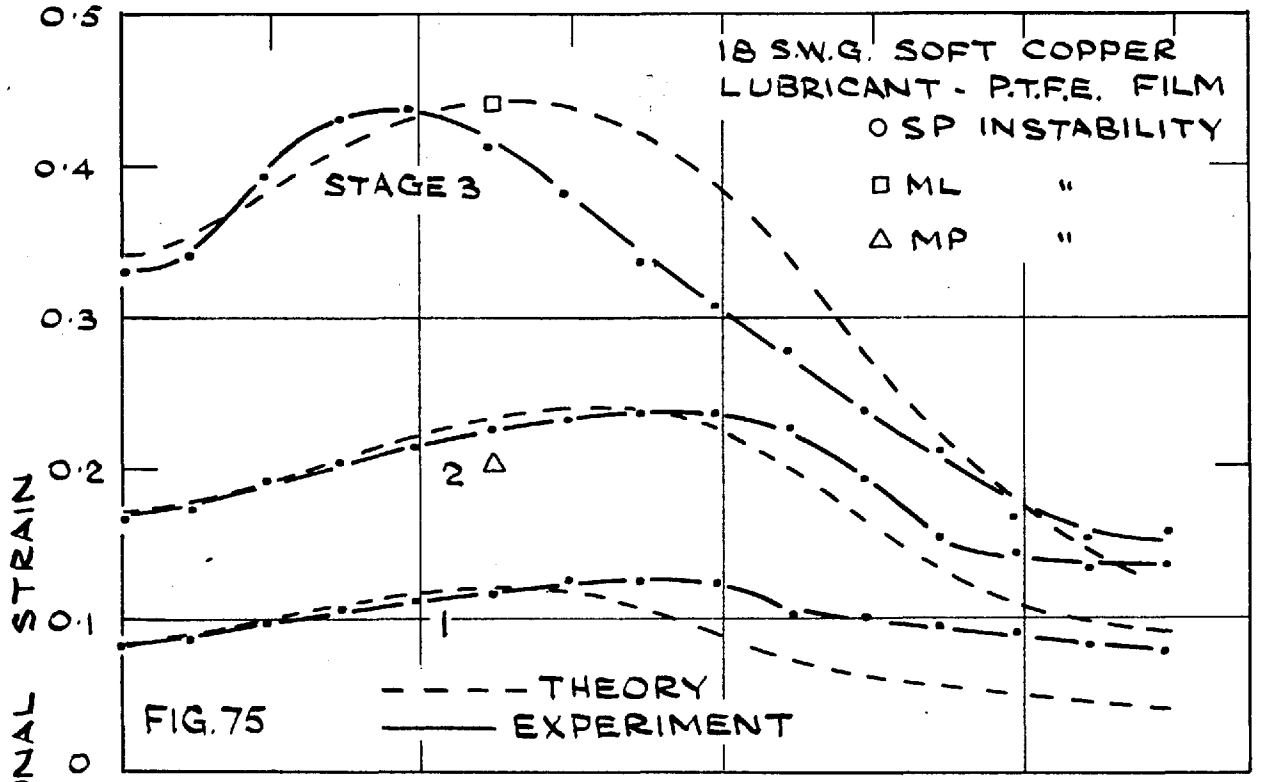



FIG. 75

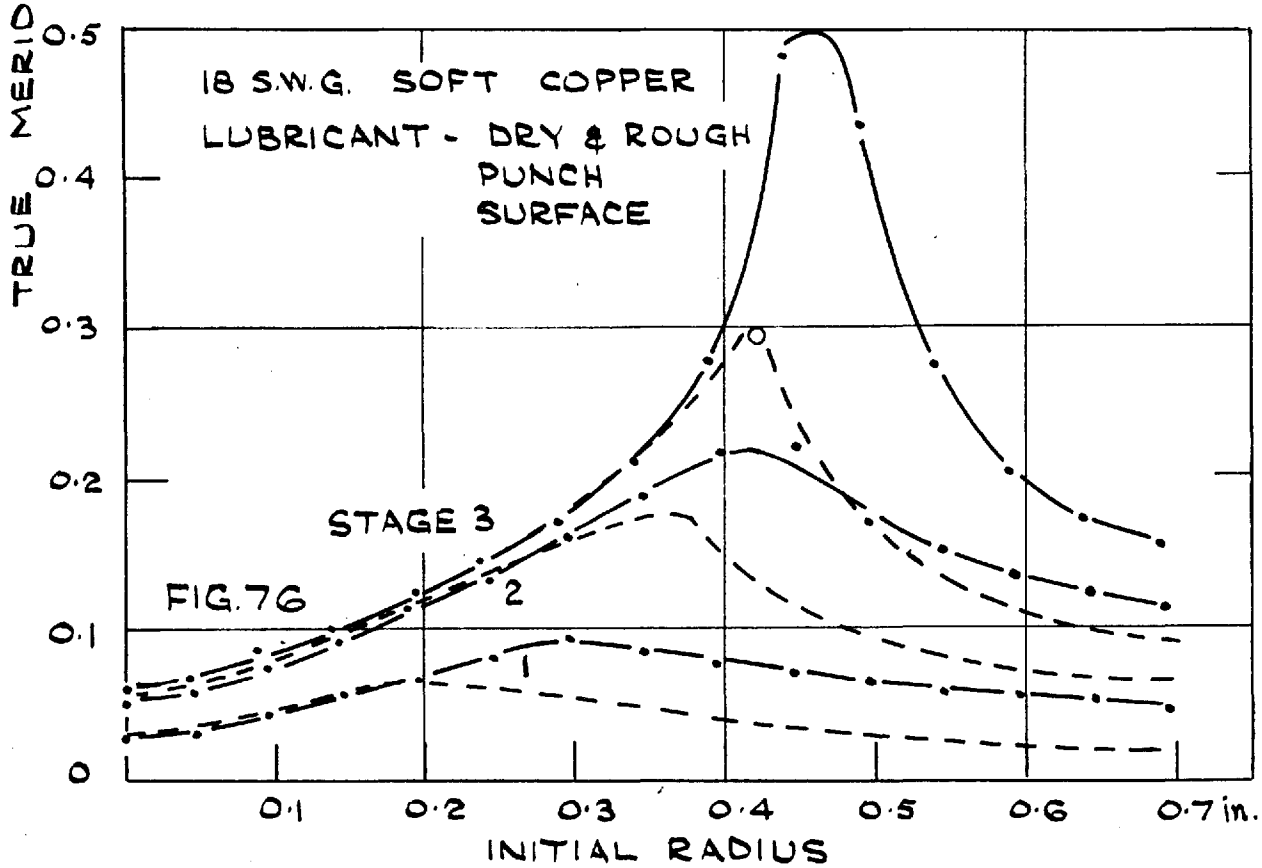



FIG. 76

MERIDIONAL STRAIN IN STRETCH FORMING BY HEMISPHERICAL PUNCH.

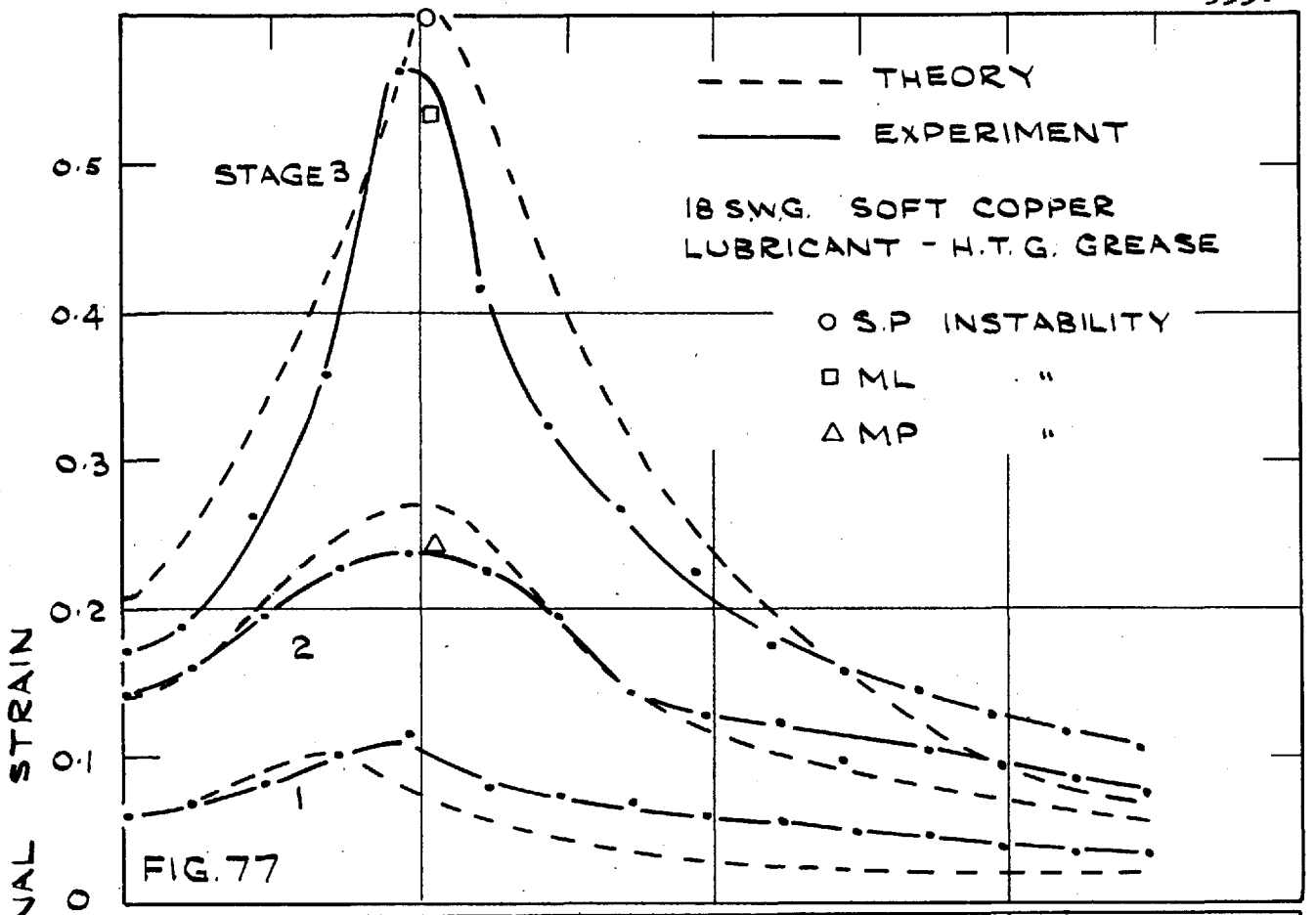



FIG. 77

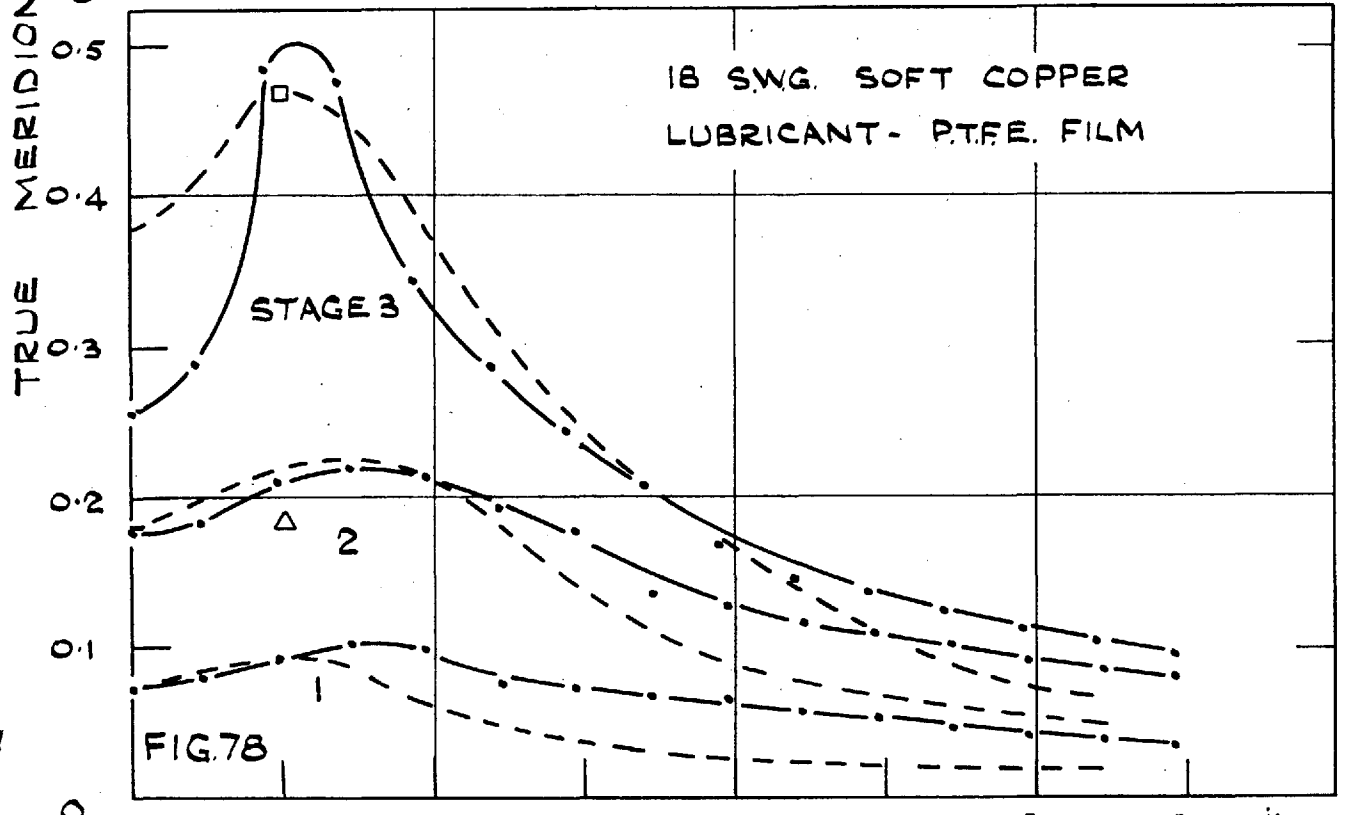



FIG. 78

MERIDIONAL STRAIN IN STRETCH FORMING BY ELLIPSOIDAL PUNCH.

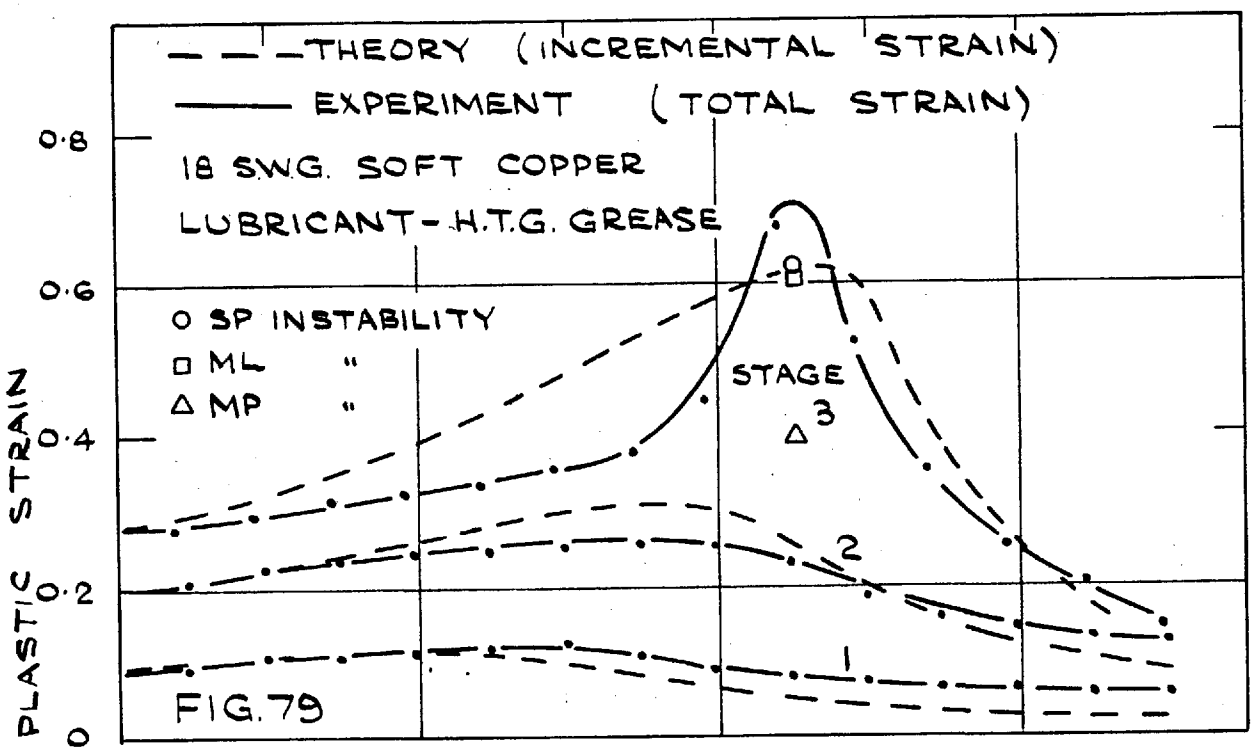



FIG. 79

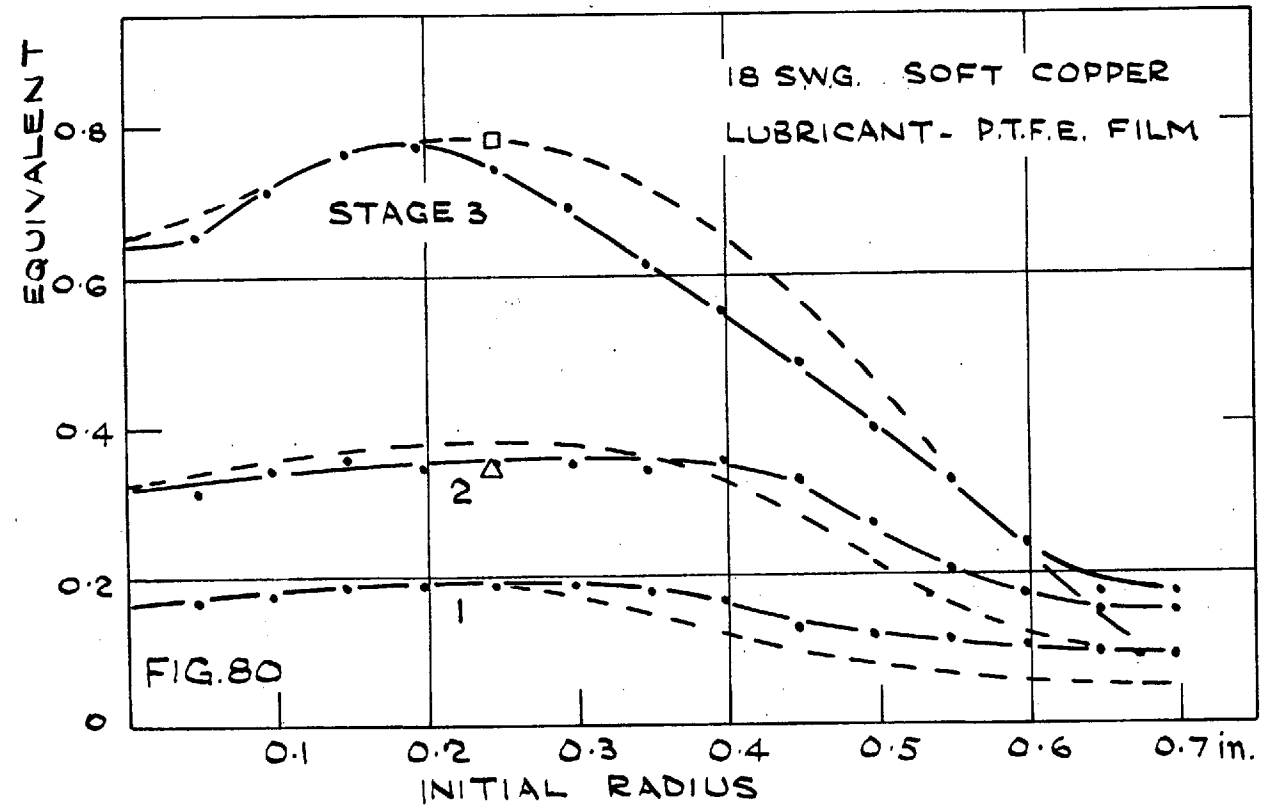



FIG. 80

EQUIVALENT STRAIN IN STRETCH FORMING BY HEMISPHERICAL PUNCH.

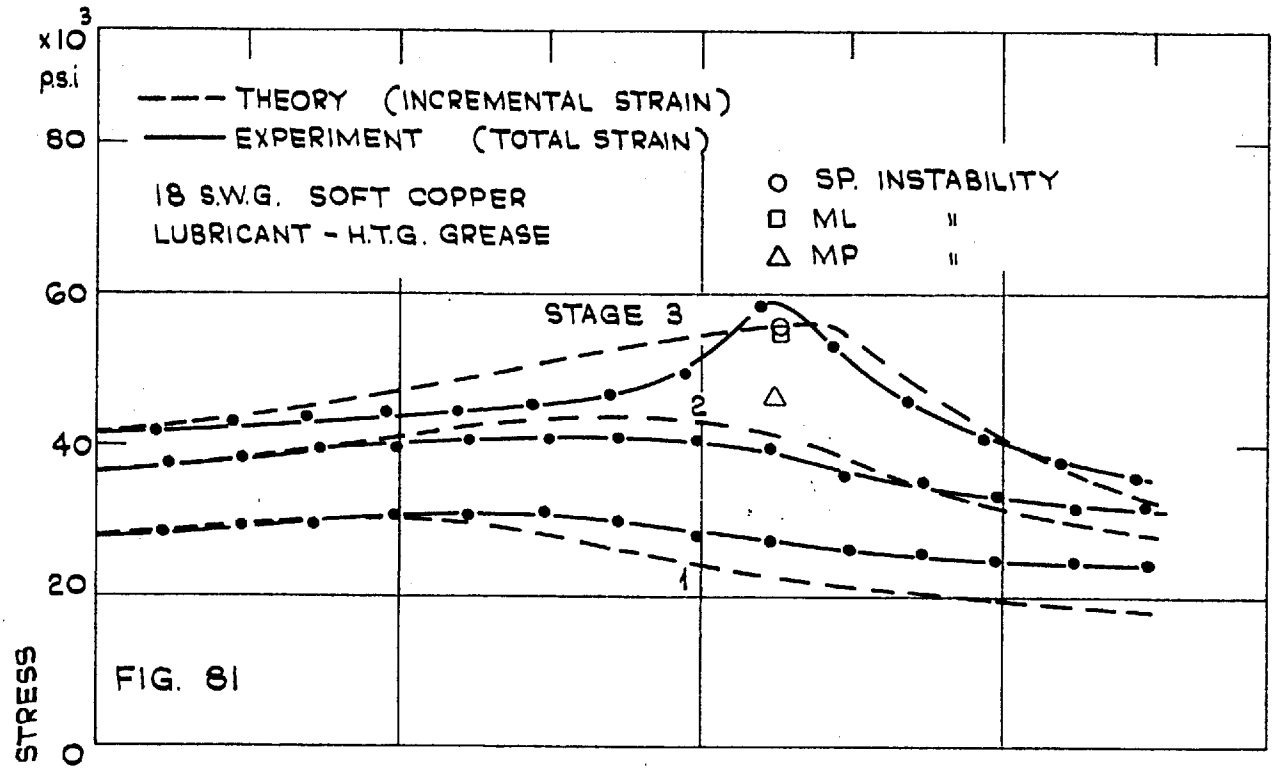



FIG. 81

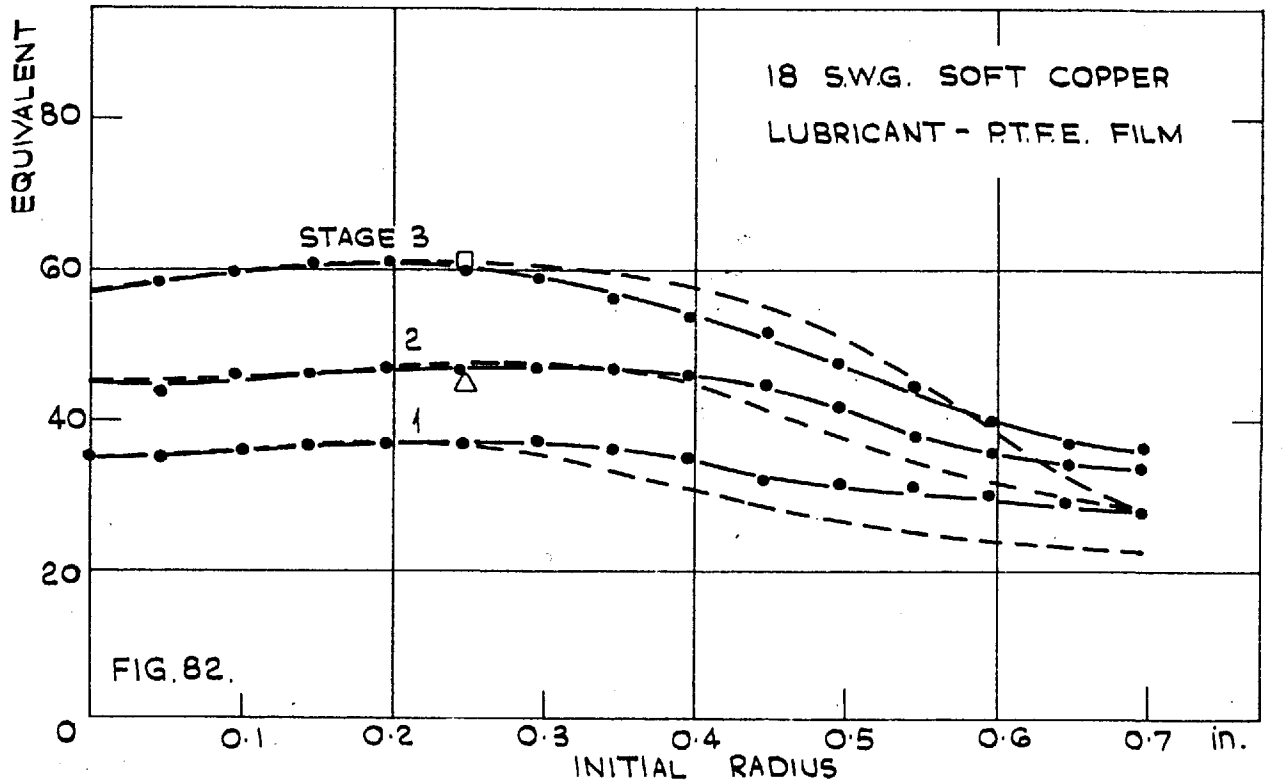
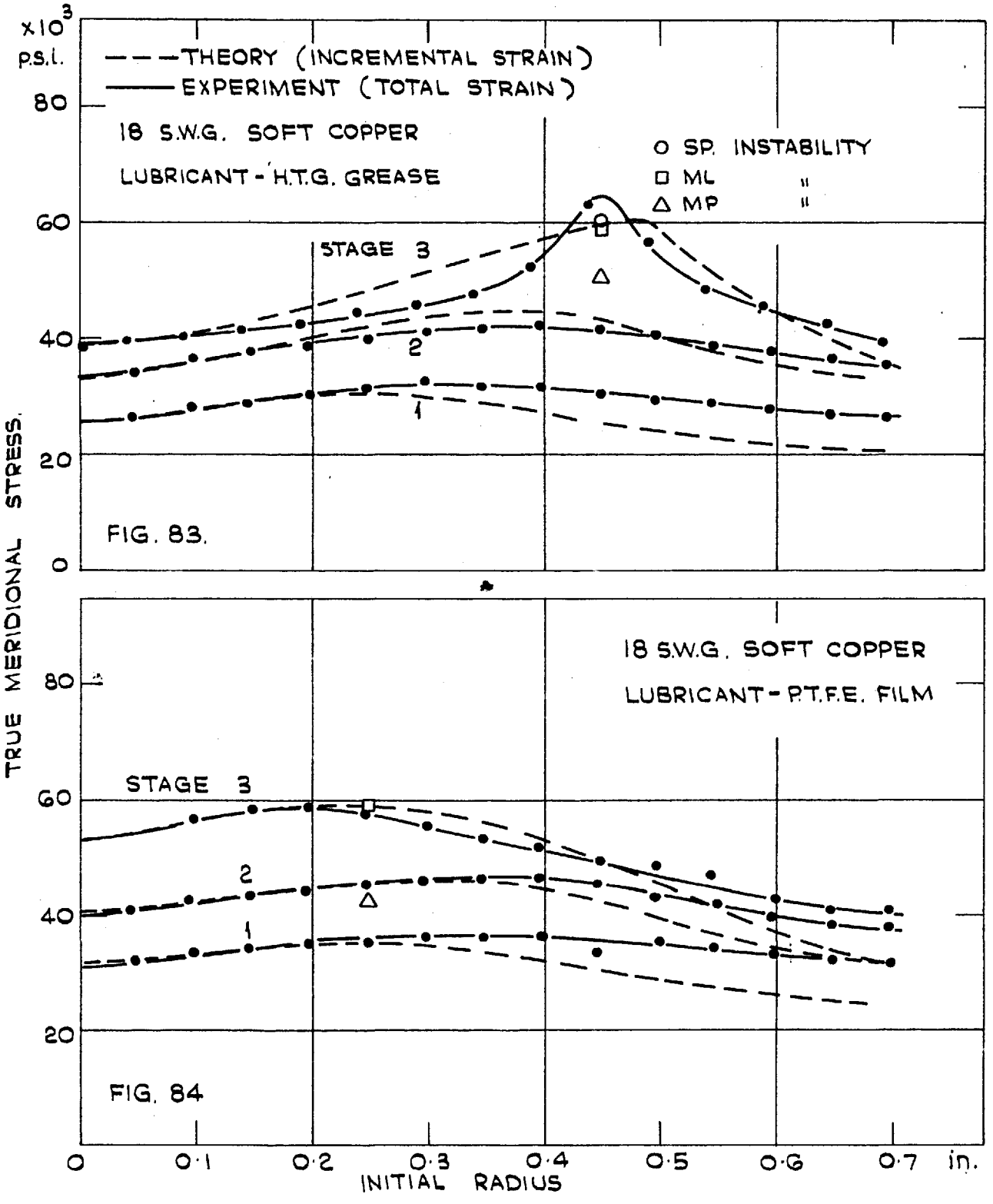
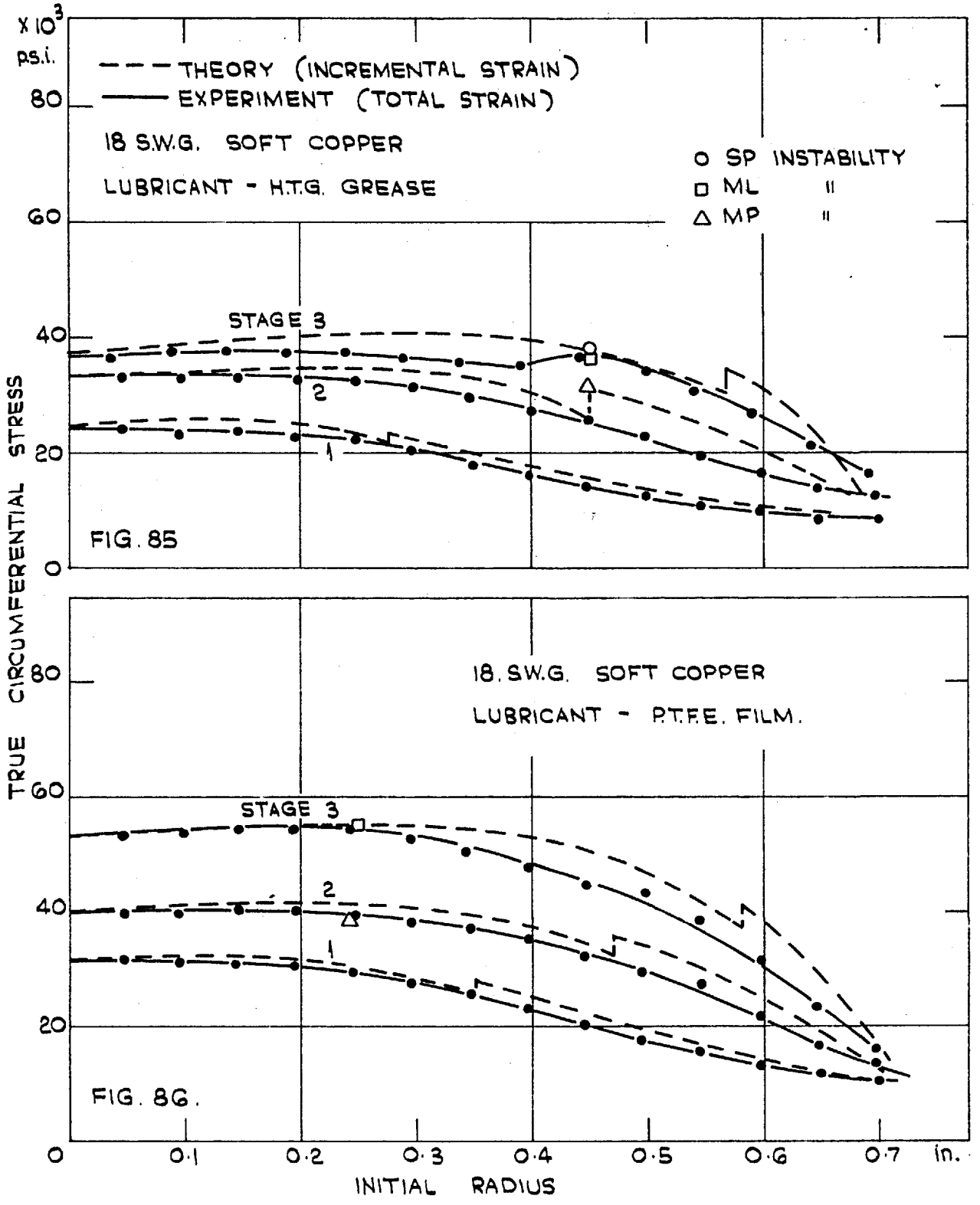



FIG. 82.

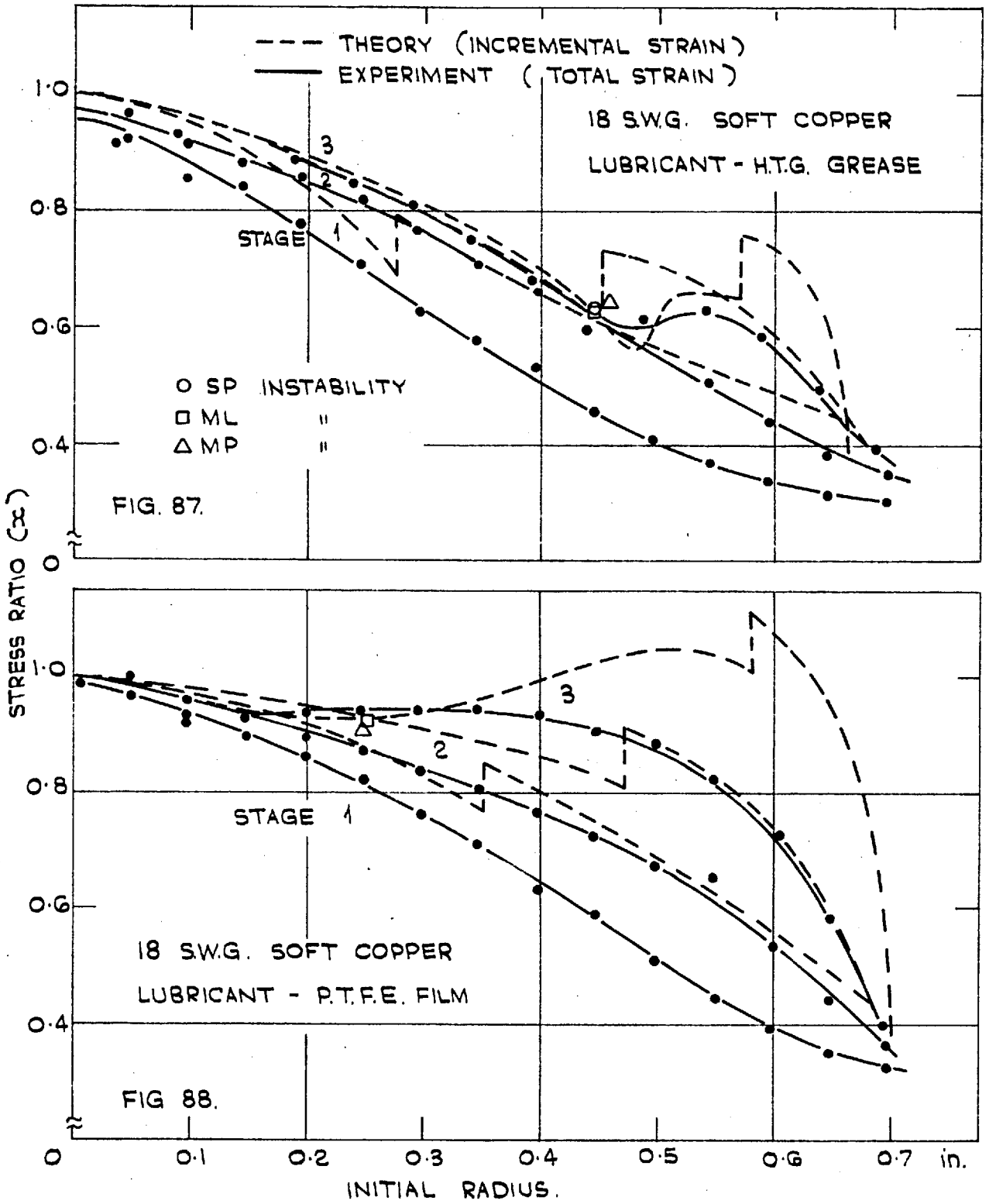
EQUIVALENT STRESS IN STRETCH FORMING BY HEMISPHERICAL PUNCH.




MERIDIONAL STRESS IN STRETCH FORMING BY HEMISPHERICAL PUNCH.




CIRCUMFERENTIAL STRESS IN STRETCH FORMING BY HEMISPHERICAL PUNCH




STRESS RATIO (α) IN STRETCH FORMING BY HEMISPHERICAL PUNCH

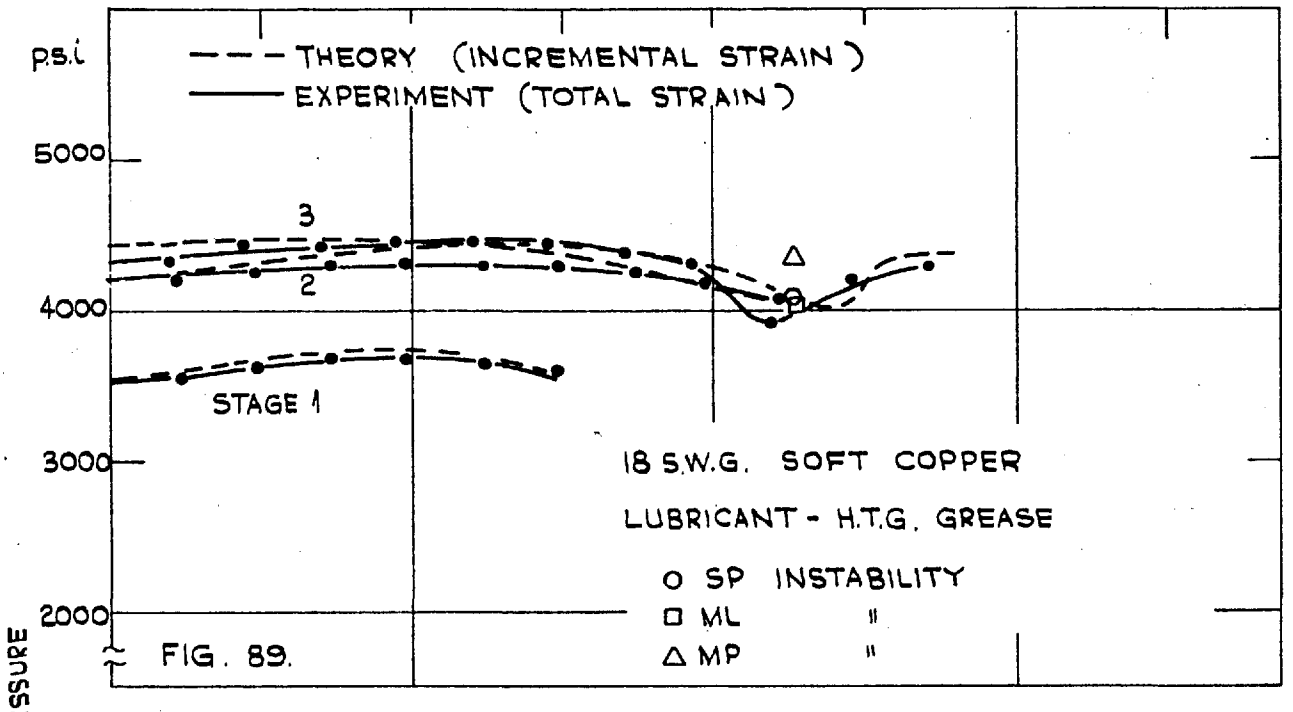



FIG. 89.

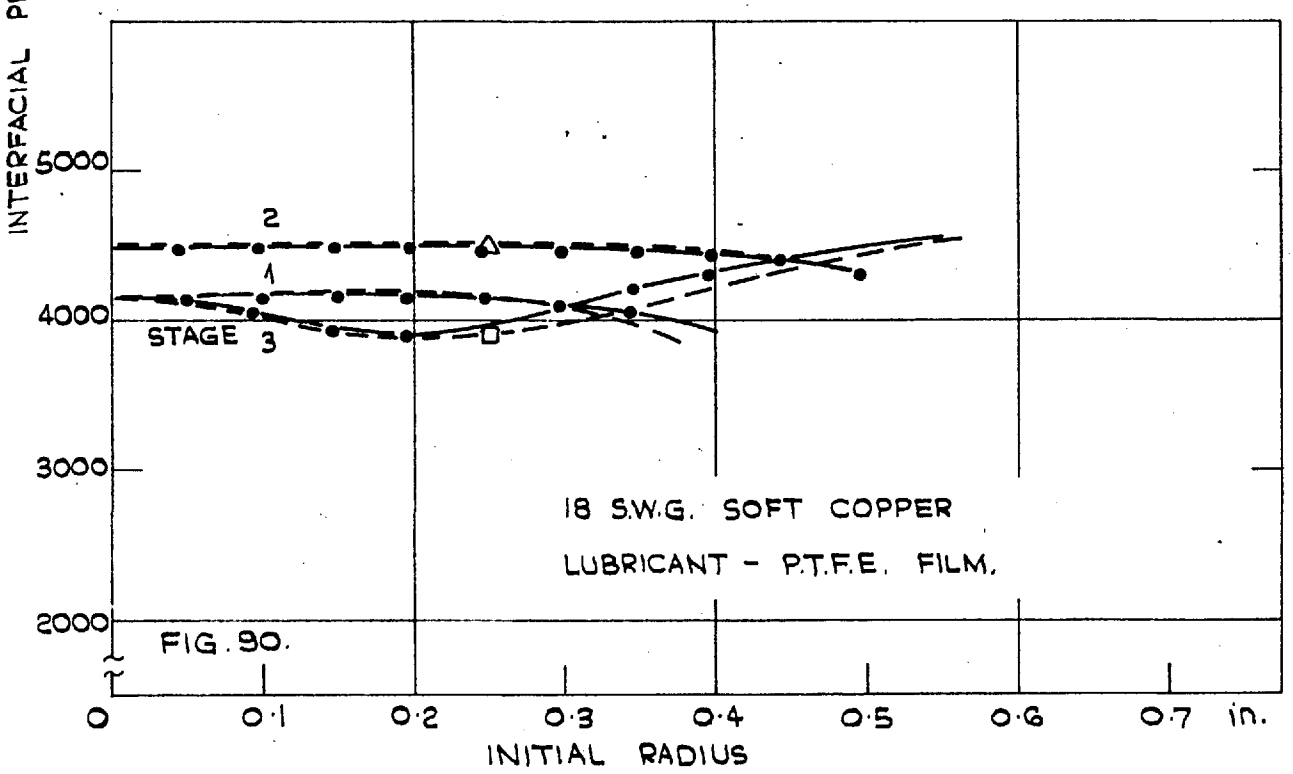



FIG. 90.

INTERFACIAL PRESSURE BETWEEN PUNCH AND SHEET METAL IN STRETCH FORMING BY HEMISPHERICAL PUNCH

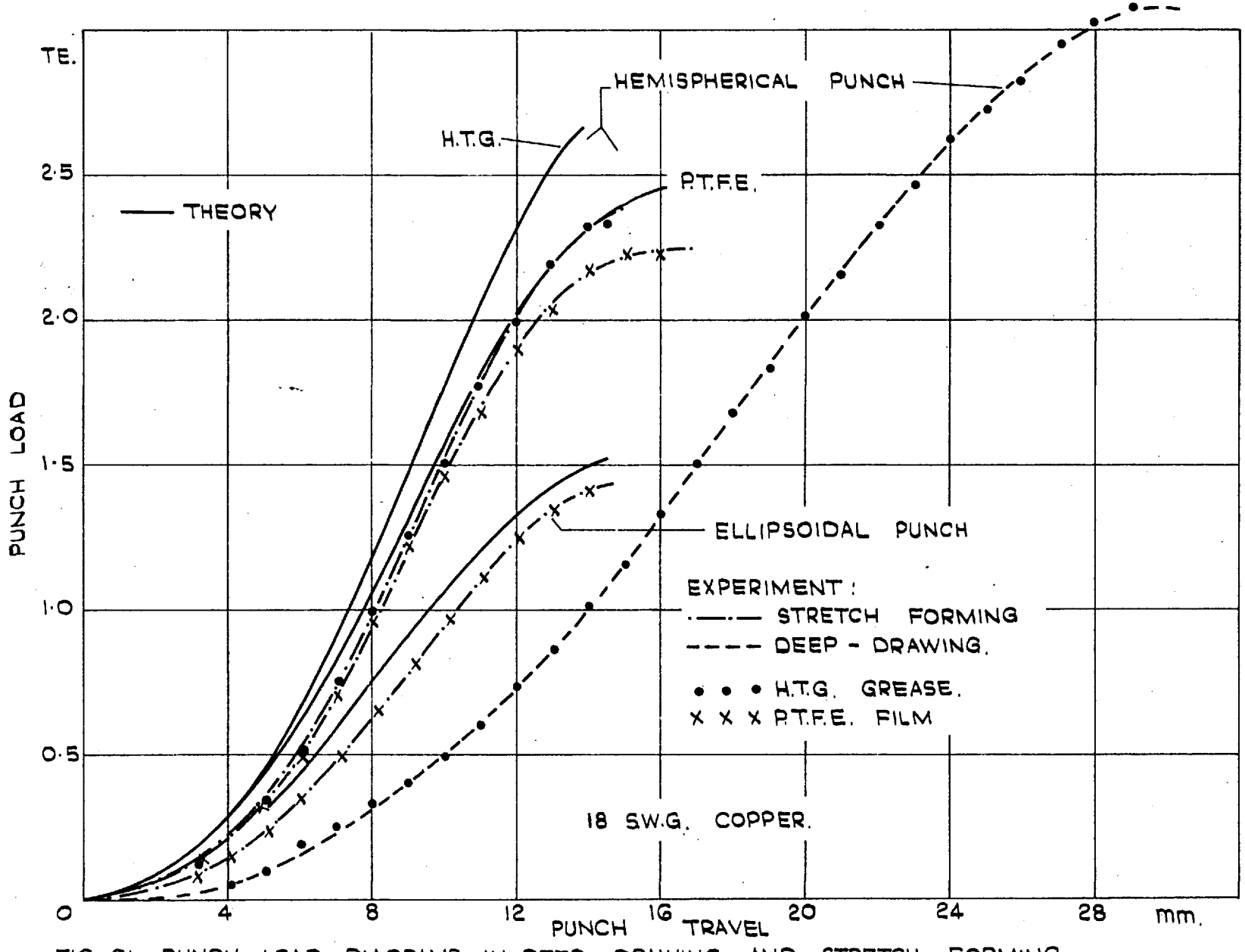



FIG. 91. PUNCH LOAD DIAGRAMS IN DEEP - DRAWING AND STRETCH FORMING.

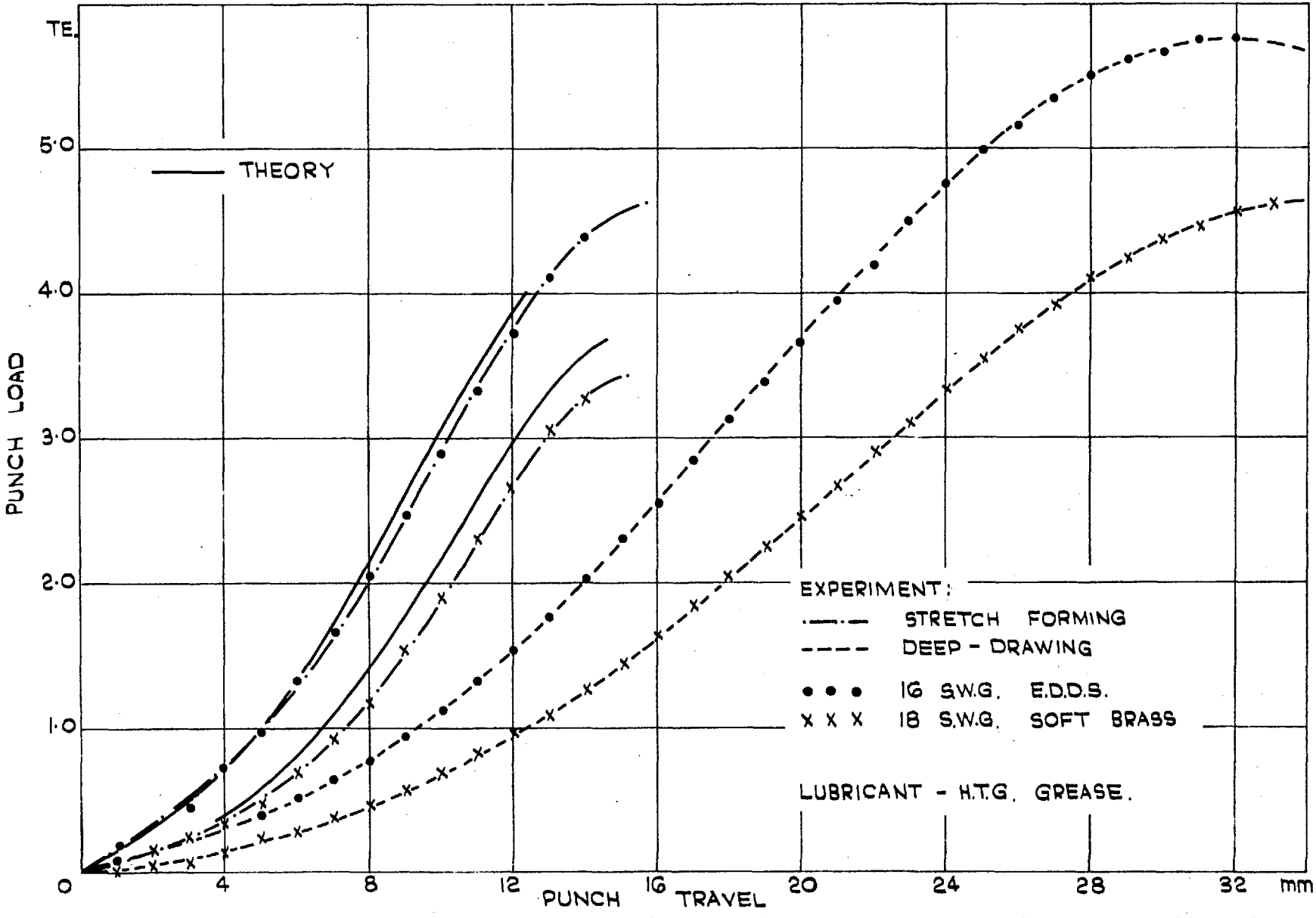



FIG. 92. PUNCH LOAD DIAGRAMS IN DEEP- DRAWING & STRETCH FORMING BY HEMISPHERICAL PUNCH.

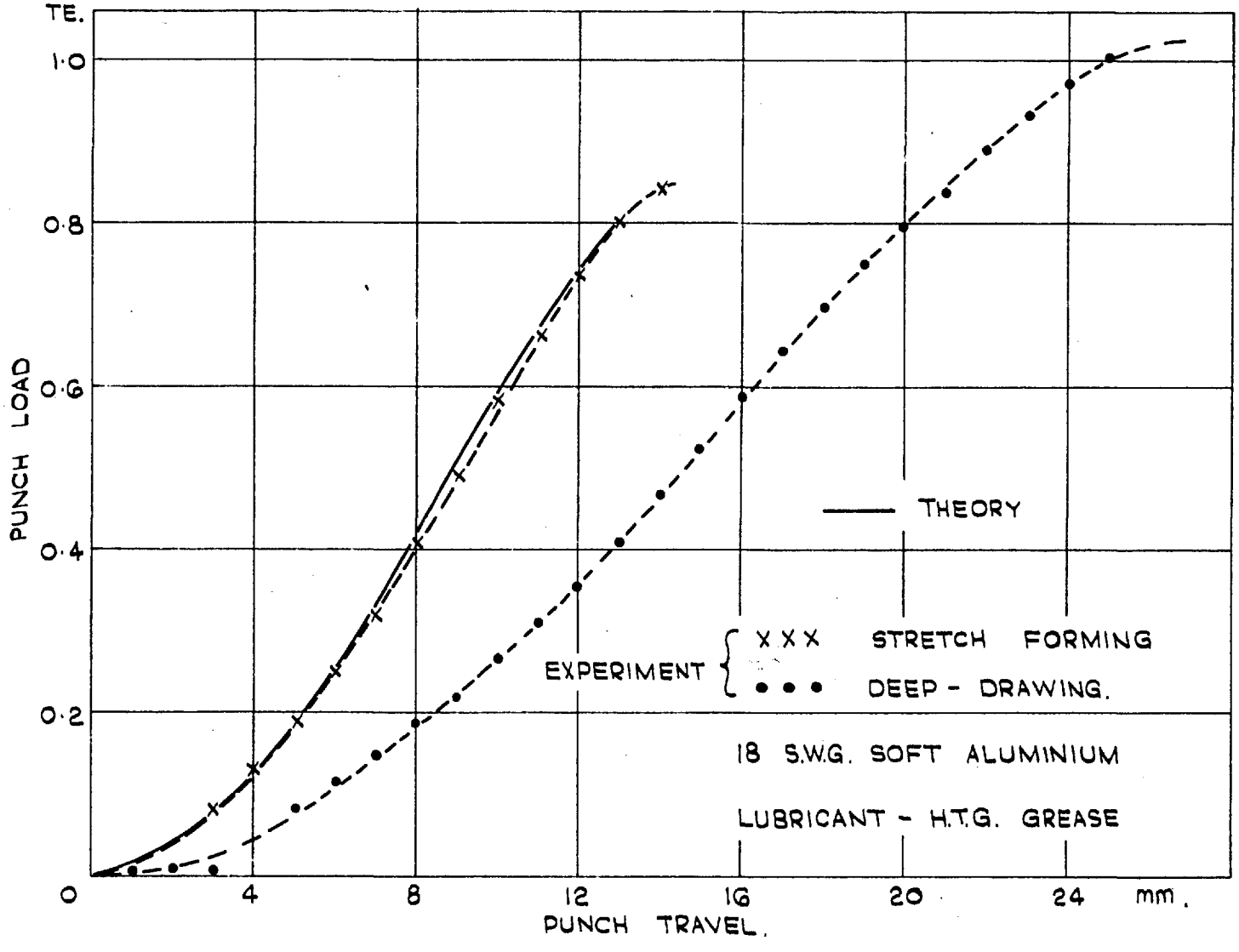



FIG. 93. PUNCH LOAD DIAGRAMS IN DEEP-DRAWING & STRETCH FORMING BY HEMISPHERICAL PUNCH

Table 11.2 - Limiting drawing ratio in deep-drawing.

| Material S.W.G. | | Limiting Blank Diameter in. | Within an increment of 0.025 on L.D.R. | | L.D.R. with respect to | | Constants for the Empirical Eqn. $D_b = Me^{-Nh} + D_{cr}$ | | | Sizes of Blanks used for Prediction of Critical Diameter in. | $\frac{D_{b,1}}{D_{cr}}$ | Comments where L.D.R. was not as predicted within an increment |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Oversize Blank in. | Undersize Blank in. | Punch Diameter | Mean Wall Diameter | N | M | $D_{cr}$ | $D_{b,1} > D_{b,2} > D_{b,3}$ | | |
| Copper | 16 | 3.0270 | 3.059 | 2.994 | 2.328 | 2.219 | 0.45185 | 138391 | 3.0270 | 3.940<br>3.546<br>3.152 | 1.30 | Successful |
| | 18 | 3.067 | 3.100 | 3.035 | 2.359 | 2.275 | 0.5285 | 353053 | 3.067 | 3.938<br>3.545<br>3.151 | 1.28 | Successful |
| | 20 | 3.032 | 3.065 | 3.000 | 2.333 | 2.270 | 0.45555 | 21334 | 3.032 | 3.938<br>3.545<br>3.152 | 1.30 | Successful |
| Aluminium | 16 | 2.902 | 2.935 | 2.870 | 2.333 | 2.128 | 0.44662 | 32711 | 2.902 | 3.345<br>3.150<br>2.953 | 1.155 | Successful Range $(\frac{D_{b,1}}{D_{cr}})$ had to be reduced for Aluminium to obtain accurate results |
| | 18 | 2.837 | 2.869 | 2.804 | 2.182 | 2.105 | 0.47613 | 18628 | 2.837 | 3.340<br>3.150<br>2.953 | 1.175 | Successful " |
| | 20 | 2.839 | 2.872 | 2.807 | 2.185 | 2.126 | 0.57621 | 57443 | 2.839 | 3.346<br>3.150<br>2.953 | 1.185 | Successful " |

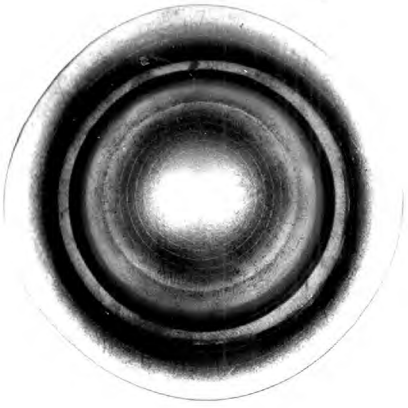
TABLE 11.2 p. 565

Table 11.2. - (Cont..)

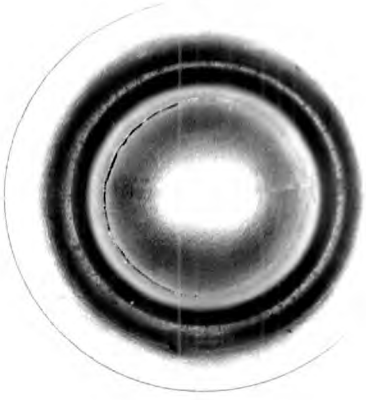
| Material S.W.G. | | Limiting Blank Diameter in. | Within an increment of 0.025 on L.D.R. | | L.D.R. with respect to | | Constants for the Empirical Eqn. $D_b = Me^{-Nh} + D_{cr}$ | | | Sizes of Blanks used for Prediction of Critical Diameter in. | $\frac{D_{b,1}}{D_{cr}}$ | Comments where L.D.R. was not as predicted within an increment |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Oversize Blank in. | Undersize Blank in. | Punch Diameter | Mean Wall Diameter | N | M | $D_{cr}$ | | | |
| | | | | | | | | | | $D_{b,1}$ $D_{b,2}$ $D_{b,3}$ | | |
| Steel | 14 | 2.989 | 3.022 | 2.957 | 2.299 | 2.166 | 0.033465 | 4817 | 2.989 | 3.937<br>3.544<br>3.150 | 1.315 | Successful |
| | 16 | 3.026 | 3.059 | 2.944 | 2.328 | 2.218 | 0.37646 | 6331 | 3.026 | 3.937<br>3.544<br>3.150 | 1.30 | Successful |
| | 20 | 3.095 | 3.128 | 3.063 | 2.381 | 2.317 | 0.44984 | 3628 | 3.095 | 3.939<br>3.545<br>3.150 | 1.27 | Successful |
| Brass | 16 | 3.094 | 3.127 | 3.062 | 2.380 | 2.268 | 0.34822 | 13503 | 3.094 | 3.938<br>3.545<br>3.152 | 1.27 | Successful |
| | 18 | 3.100 | 3.132 | 3.067 | 2.385 | 2.300 | | | | | | Missed by one increment |
| | 20 | 3.115 | 3.148 | 3.083 | 2.396 | 2.332 | 0.32582 | 5100 | 3.115 | 3.938<br>3.544<br>3.150 | 1.265 | It was necessary to choose 3 points from the curve h versus $D_b$ constructed from 6 points. |

TABLE 11.2 p. 566

(cont.)



S.W.G. - 16 E.D.D.S.
DEPTH - 33.8 mm.



S.W.G. - 18 SOFT BRASS
DEPTH - 35.8 mm.



S.W.G. - 18 SOFT COPPER
DEPTH - 30.2 mm.

rolling direction



S.W.G. - 18 SOFT ALUMINIUM
DEPTH - 26.7 mm.

LUBRICANT - H.T.G. GREASE

PUNCH - HEMISPHERICAL

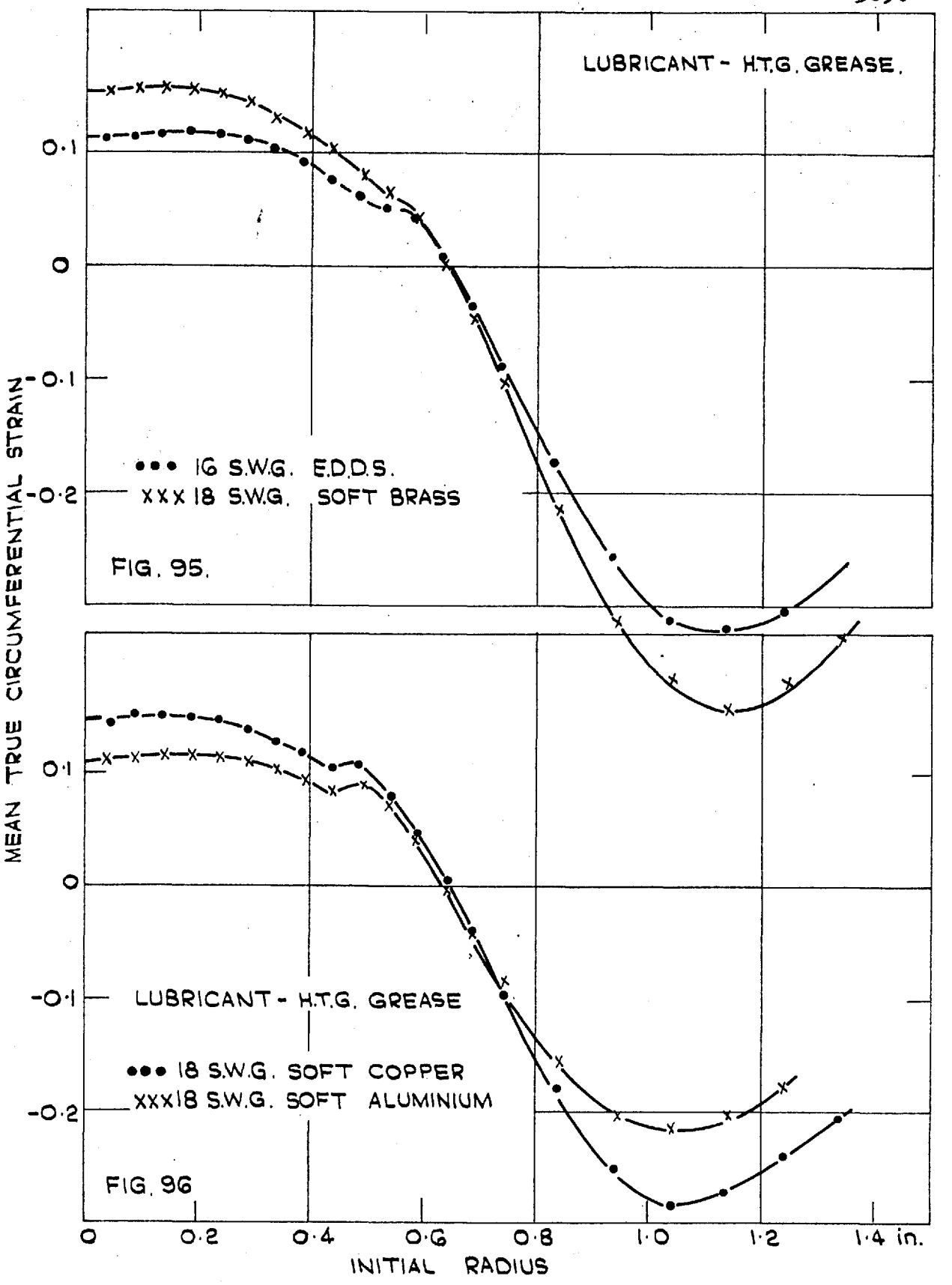
1 inch

FIG. 94 - DEEP-DRAWN SPECIMENS

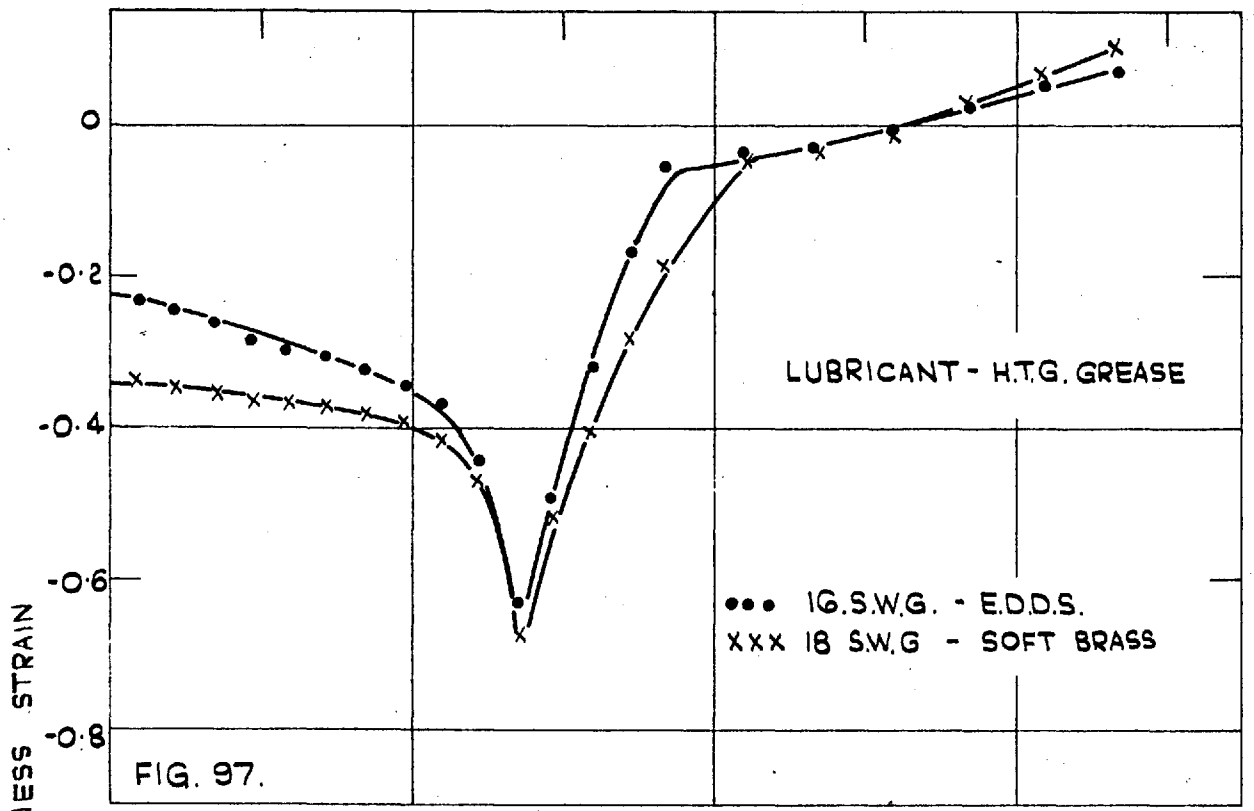
Table 11.3 - Calculated earing units for hemispherical cups

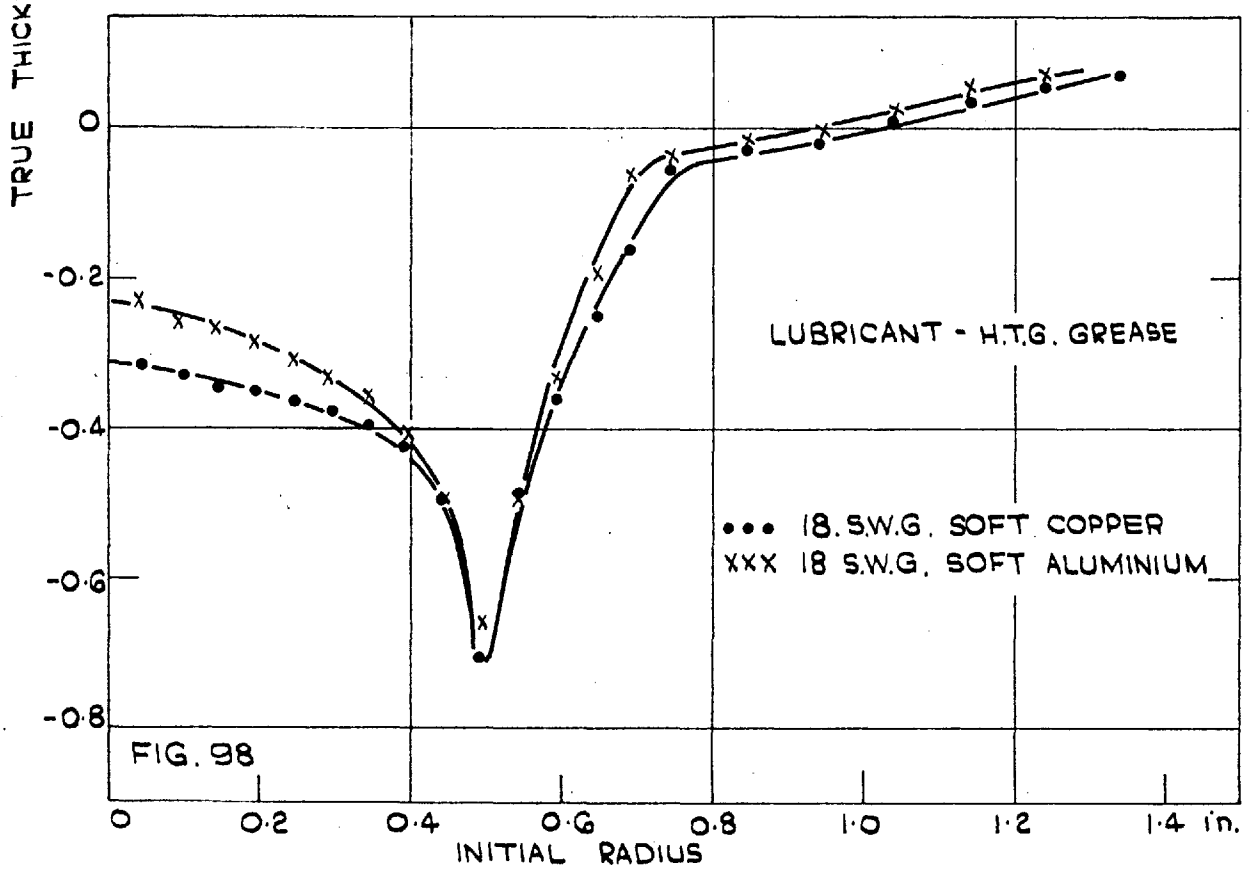
| Material | Thickness, SWG | Mean Height of Ears ($h_e$), in.* | Mean Height of Troughs ($h_t$), in.* | Earing Units $\frac{h_e - h_t}{h_t} \times 100$ | Angle of Ears | No. of Ears |
|---|---|---|---|---|---|---|
| Steel | 14 | 1.618 | 1.480 | 9.33 | 90° | 4 |
| | 16 | 1.612 | 1.502 | 7.3 | 90° | 4 |
| | 20 | 1.664 | 1.555 | 7.02 | 90° | 4 |
| Brass | 16 | 1.585 | 1.556 | 1.86 | 45° | 4 |
| | 18 | | | Negligible | | |
| | 20 | | | Negligible | | |
| Copper | 16 | 1.587 | 1.562 | 1.6 | 45° | 4 |
| | 18 | 1.608 | 1.574 | 2.16 | 90° | 4 |
| | 20 | 1.644 | 1.588 | 3.5 | 90° | 4 |
| Aluminium | 16 | 1.400 | 1.333 | 5.15 | 45° | 4 |
| | 18 | 1.300 | 1.178 | 10.4 | 90° | 4 |
| | 20 | | | Negligible | | |

* $h_e$ and $h_t$ measured from the pole of the hemispherical end.

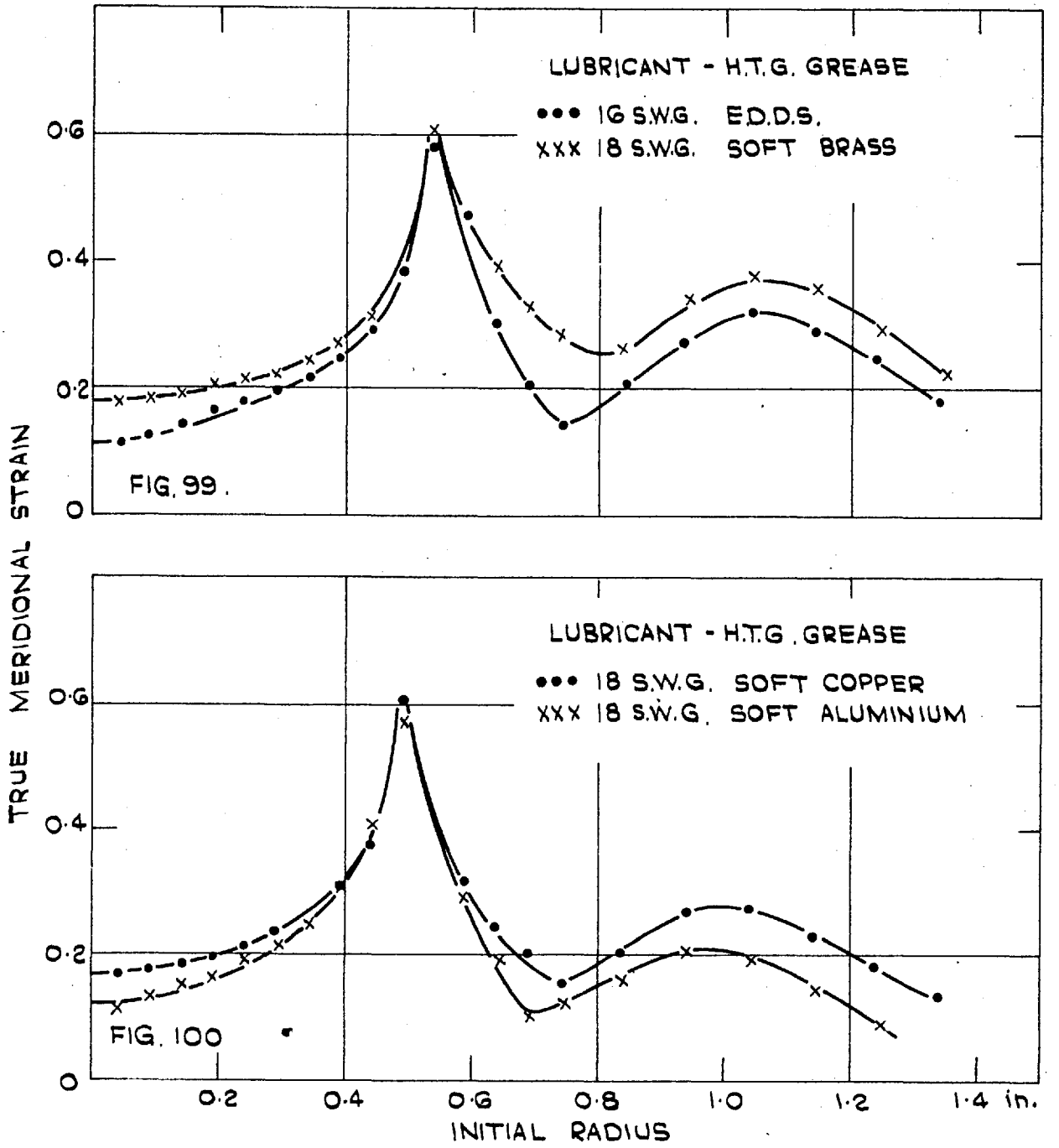



CIRCUMFERENTIAL STRAIN IN DEEP - DRAWING.

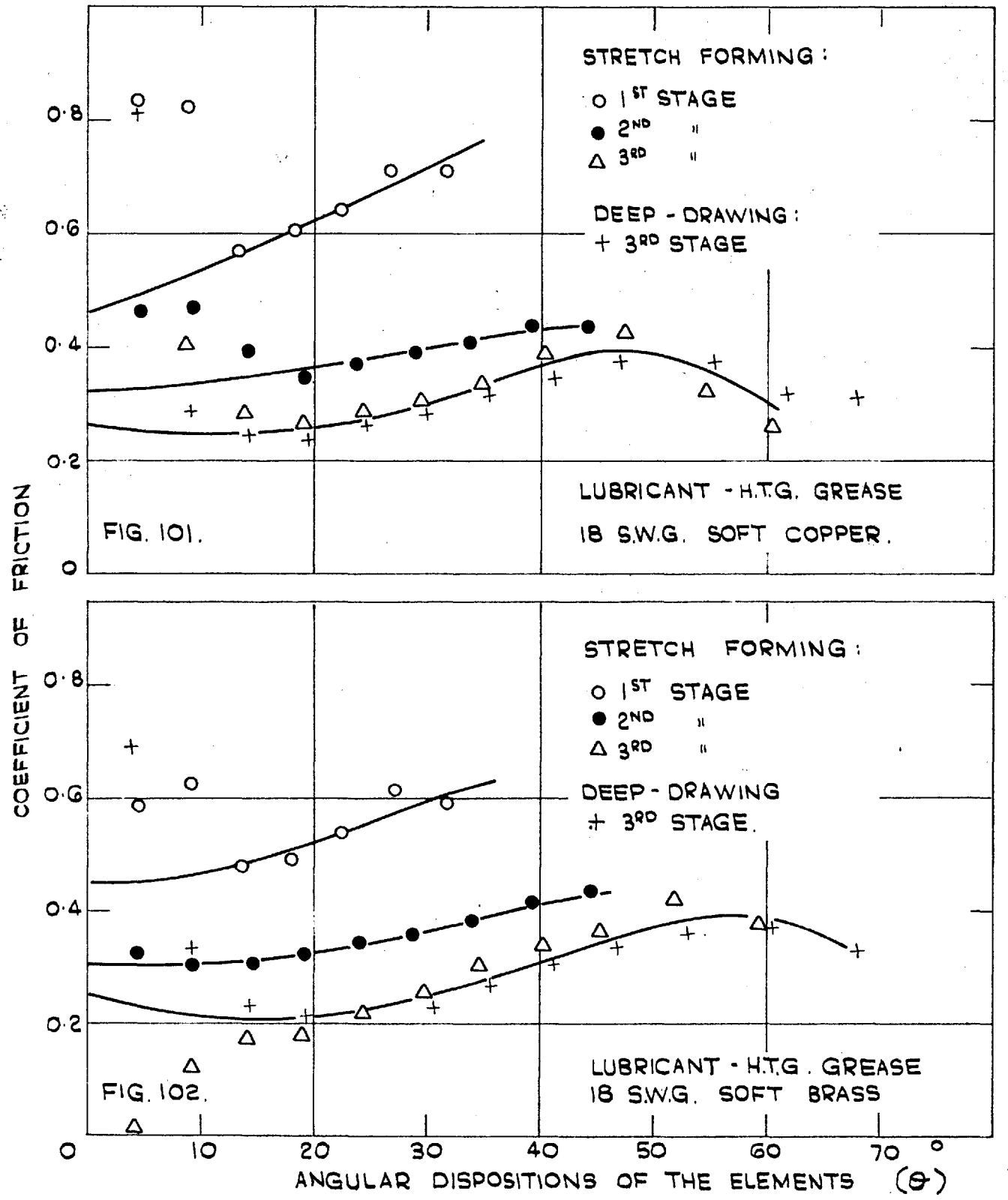


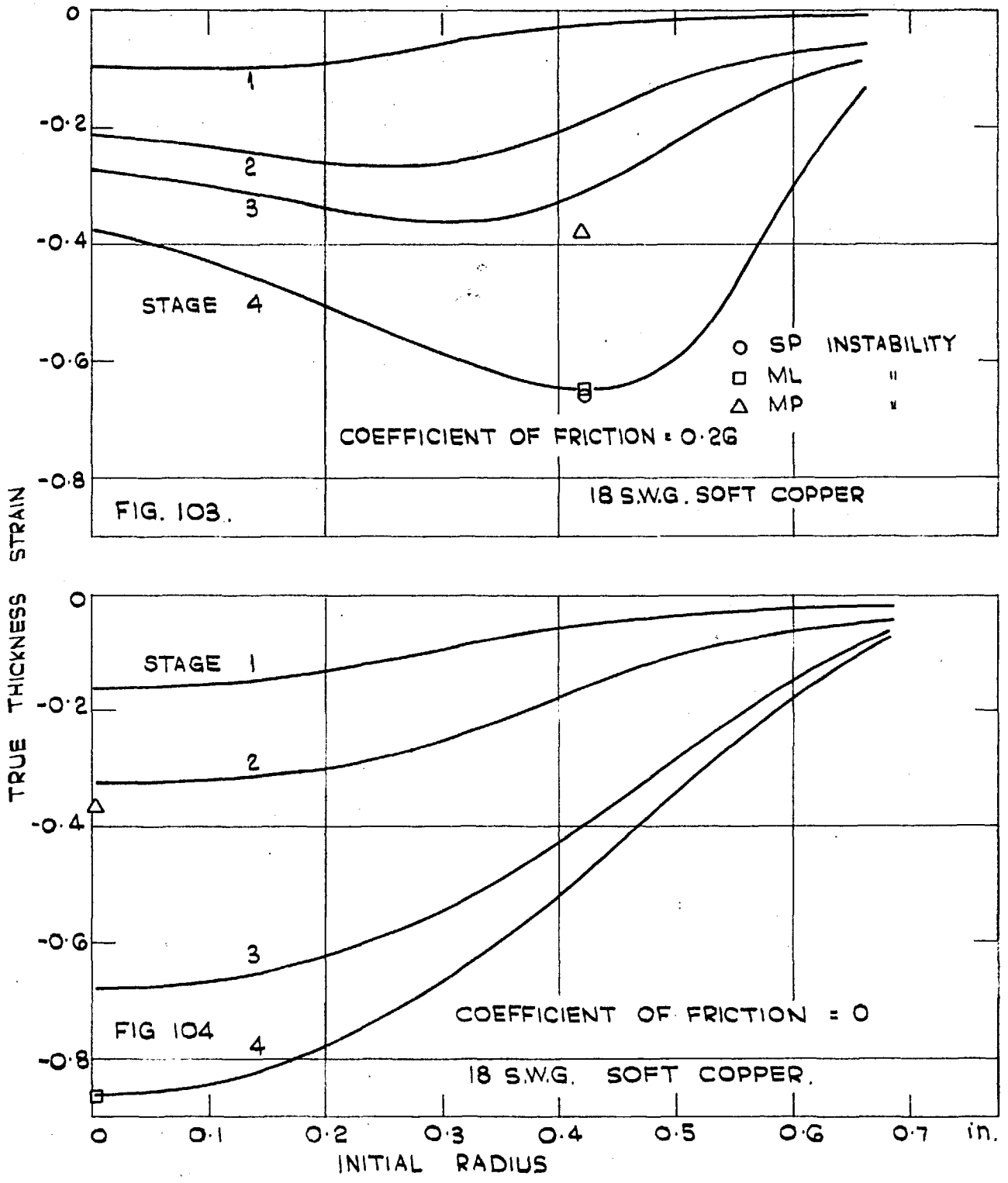

THICKNESS STRAIN IN DEEP - DRAWING.




MERIDIONAL STRAIN IN DEEP - DRAWING.




EXTRAPOLATION TO DETERMINE THE COEFFICIENT OF FRICTION BETWEEN PUNCH & SHEET METAL IN STRETCH FORMING & DEEP-DRAWING




THEORETICAL DISTRIBUTION OF THICKNESS STRAIN IN STRETCH FORMING BY HEMISPHERICAL PUNCH.

Table 11.4 - Coefficients of friction in the Erichsen Test.

| Thickness, SWG | Angle ($\alpha$) | Punch Penetration (h) mm | $\mu$ | | |
|---|---|---|---|---|---|
| | | | Brass | Copper | Steel |
| 14 | $15^{o}$ | 3.37 | | | 0.067 |
| | $30^{o}$ | 6.23 | | | 0.050 |
| | $45^{o}$ | 8.93 | | | 0.017 |
| 16 | $15^{o}$ | 3.39 | 0.014 | 0.014 | 0.054 |
| | $30^{o}$ | 6.31 | 0.016 | 0.016 | 0.030 |
| | $45^{o}$ | 9.13 | 0.018 | 0.026 | 0.012 |
| 18 | $15^{o}$ | 3.40 | 0.014 | 0.011 | |
| | $30^{o}$ | 6.36 | 0.010 | 0.007 | |
| | $45^{o}$ | 9.29 | 0.010 | | |
| 20 | $15^{o}$ | 3.41 | 0.007 | 0.011 | 0.043 |
| | $30^{o}$ | 6.41 | 0.007 | 0.005 | 0.025 |
| | $45^{o}$ | 9.40 | 0.006 | 0.003 | 0.013 |

Note: $\mu$ for aluminium is estimated to be $< 0.01$.

Table 11.5 - Coefficients of friction operative between sheet metal, die and blank-holder in deep-drawing.

| Material | Lubricant | Angle $\alpha^{o}$ | Punch Penetration mm | Coefficient of Friction $\mu$ |
|---|---|---|---|---|
| 16 SWG E.D.D.S. | HTG Grease | 15 | 7.20 | 0.085 |
| | | 30 | 13.15 | 0.040 |
| | | 45 | 18.38 | 0.034 |
| " | Dry Surfaces | 15 | 7.20 | 0.12 |
| | | 30 | 13.15 | 0.12 |
| | | 45 | 18.38 | 0.09 |
| 18 SWG Brass | HTG Grease | 15 | 6.73 | 0.05 |
| | | 30 | 12.28 | 0.03 |
| | | 45 | 17.10 | 0.02 |
| " | Dry Surfaces | 15 | 6.73 | 0.06 |
| | | 30 | 12.28 | 0.05 |
| | | 45 | 17.10 | 0.05 |
| SWG-18 Copper | HTG Grease | 15 | 6.73 | 0.035 |
| | | 30 | 12.28 | 0.028 |
| | | 45 | 17.10 | 0.016 |
| " | Slightly Roughened Surfaces | 15 | 6.73 | 0.06 |
| | | 30 | 12.28 | 0.07 |
| | | 45 | 17.10 | 0.05 |
| " | P.T.F.E. | 15 | 6.73 | 0.01 |
| | | 30 | 12.28 | 0.006 |
| | | 45 | 17.10 | 0.004 |
| " | Dry Surfaces | 15 | 6.73 | 0.05 |
| | | 30 | 12.28 | 0.05 |
| | | 45 | 17.10 | 0.05 |
| SWG-18 Aluminium | HTG Grease | 15 | 6.73 | 0.02 |
| | | 30 | 12.28 | 0.015 |
| | | 45 | 17.10 | 0.010 |
| " | Dry Surfaces | 15 | 6.73 | 0.042 |
| | | 30 | 12.28 | 0.04 |
| | | 45 | 17.10 | 0.044 |

Table 11.6 - Coefficients of friction between punch and sheet metal in stretch forming and deep-drawing.

| Material | Lubricant | Polar Circumferential Strain $(\varepsilon_2)_p$ | Coefficient of Friction $\mu$ |
|---|---|---|---|
| 18 S.W.G. Soft Copper | Shell H.T.G. Grease | 0.045 | 0.47 |
| | | 0.10 | 0.32 |
| | | 0.135 | 0.26 |
| " | P.T.F.E. Film | 0.085 | 0.20 |
| | | 0.160 | 0.135 |
| | | 0.33 | 0.070 |
| " | Dry and Rough Punch Surface | 0.05 | 0.6 |
| | | 0.060 | 0.6 |
| 16 S.W.G. E.D.D.S. | Shell H.T.G. Grease | 0.043 | 0.42 |
| | | 0.095 | 0.29 |
| | | 0.13 | 0.24 |
| 18 S.W.G. Soft Brass | Shell H.T.G. Grease | 0.045 | 0.45 |
| | | 0.1 | 0.31 |
| | | 0.14 | 0.26 |
| 18 S.W.G. Soft Aluminium | Shell H.T.G. Grease | 0.048 | 0.44 |
| | | 0.08 | 0.41 |
| | | 0.12 | 0.38 |

# An Investigation of the Erichsen Test

*By B. Kaftanoglu, B.Sc. (Eng.), and J. M. Alexander, Ph.D., B.Sc. (Eng.), M.I.Mech.E., M.I.Prod.E., F.I.M.*

The effect on the Erichsen number of altering the main variables has been investigated, and the relative amounts of drawing-in and stretching that occur under different conditions determined. Both round and strip specimens of varying thickness and diameter (or width) have been examined in mild steel, copper, brass, and aluminium, all in soft tempers. A strip specimen of a given width is equivalent to a round specimen of a certain diameter as far as determination of the Erichsen number is concerned, and the relationship between diameter and strip width has been found for the materials stated. It is shown that drawing-in of the sheet can be completely prevented only by using serrated blank-holder and die surfaces, together with blank-holder loads sufficiently great to effect indentation of the sheet specimen. As far as possible, lubrication conditions were maintained constant, a proprietary brand of graphite grease being used. A method is suggested for estimating the coefficient of friction from measurements of punch load in successive tests with different blank-holder loads, and values have been determined for these experiments. Finally, a comparison is made between the earing of hemispherical-ended cups, deep drawn using a rough punch from the materials studied, and the form of fracture observed in the Erichsen test. There is no apparent correlation, all-round 'isotropic' fractures being associated with quite pronounced earing.

The Erichsen test is well known to sheet metal users and is employed both as a guide to deep-drawing and pressing qualities and as a method of quality control. In spite of its popularity no British Standard Specification exists, the most recent official statement about the test being contained in a draft German specification.[1] The general configuration of the test and the principal dimensions are indicated in Fig. 1. In this specification the sheet is clamped between blank-holder

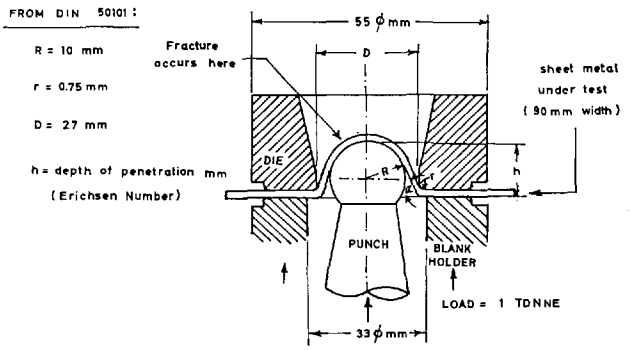



*Fig. 1.*

and die under a specified load. The depth of penetration $h$ (mm) of the spherical punch into the sheet at the instant when fracture just starts is known as the Erichsen number.

The most important earlier researches seem to have been those by Fukui *et al.*,[2] Krüger,[3] and Kokkonen and Nygren.[4] Fukui attempted to correlate various "formability" tests, and found some degree of correlation between the limiting drawing ratio measured for hemispherical-ended cups, and both general elongation and the "standard Erichsen value", although this could not be expressed quantitatively. Krüger reported that the Erichsen test gave a better indication of the suitability of a given sheet of metal for deep drawing than did cup-drawing tests, and that it was more sensitive to any change in properties of the sheet. Kokkonen and Nygren investigated the scatter of results caused by variations in the surface finish of the tools and differences in the material. They gave the limits of uncertainty in determining the Erichsen number for a single machine and also the uncertainty as between different machines. In addition, Jevons[5] mentions some tests which he carried out to show the effect of strip width on Erichsen number.

Most of the remaining literature devoted to this test has been essentially practical in nature, and includes no systematic study of variables. In particular, no investigation is recorded of strains occurring in the sheet during the tests, so that the relative amounts of 'drawing' and 'stretching' that occur are not known. It is clear that the sheet metal must deform over the punch nose mainly by stretching, but there must also be some 'drawing-in' of the sheet over the die-profile radius. Since rupture always occurs at some location on the punch nose, the stretching component is the more important. The Erichsen test assesses the ability of a sheet of metal to withstand failure from biaxial tensile instability when being stretched over a spherical surface under given frictional conditions. Because of the drawing-in that takes place, however, the Erichsen number may not correctly evaluate this property; if much drawing-in occurs a higher Erichsen number will result for the same amount of stretching.

In practical pressing operations, stretching and drawing occur together, so that it may be useful to have a test in which both elements occur, although it should be possible to specify in some way how much of each is required. This would necessitate a knowledge of the relative amounts of 'stretching' and 'drawing' that occur in the *practical* process, and this is rarely known with any accuracy. In most complicated pressings these elements vary all over the component, so that it would seem to be a good idea to subject a sheet of metal to separate tests, one to assess its ability to stretch over a

Paper No. 2145. Manuscript received 2 April 1962. Contribution to a Symposium on "Tests for Deep Drawing" to be held in Swansea on 25 September 1962 at the Autumn Meeting of the Institute of Metals. Dr. Alexander is Reader in Plasticity in the Department of Mechanical Engineering, Imperial College of Science and Technology, London. Mr. Kaftanoglu is a Research Student at Imperial College.

TABLE I
**Materials Tested**

| Material | Composition or Specification | Hardness, VPN | Thickness, $\times 10^{-3}$ in. (tolerances as measured) |
|---|---|---|---|
| Steel | Not known (designated E.D.D.S.*) | 107 (5 kg load)<br>102 ,,<br>109 ,, | S.W.G. 14 = 80·5 ± 0·5<br>,, 16 = 61 ± 0·3<br>,, 20 = 34·5 ± 0·3 |
| Copper | Bright-annealed, commercially pure | 50 (1 kg load)<br>54 ,,<br>54 ,, | ,, 16 = 65 ± 0·5<br>,, 18 = 48·5 ± 0·5<br>,, 20 = 37·5 ± 0·5 |
| Brass | 70 : 30 Bright-annealed | 83 (2·5 kg load)<br>83 ,,<br>75·5 ,, | ,, 16 = 64·7 ± 1<br>,, 18 = 47·5 ± 0·5<br>,, 20 = 36·2 ± 0·8 |
| Aluminium | Bright-annealed, commercially pure | 26·2 (1 kg load)<br>27·2 ,,<br>27·4 ,, | ,, 16 = 61·2 ± 0·5<br>,, 18 = 46·9 ± 0·3<br>,, 20 = 35·8 ± 0·4 |

* Extra Deep-Drawing Quality Steel

punch head and another to assess its ability to ' draw-in '. A long discussion of such aspects of testing is out of place in the present context, but it would appear that the Erichsen test should primarily be regarded as a ' stretching ' test, and therefore any drawing-in is undesirable and will inevitably lead to undue scatter in results.

Two other stretching tests should be mentioned, namely, the hydraulic-bulge test and the hole-expanding test, but these are felt to be less realistic, since in the former case the geometry is not imposed by the tools and there is no frictional effect, and in the latter tensile instability occurs under a uniaxial circumferential tension at the edge of the hole.

### Factors Studied

The factors that influence the Erichsen number, (assuming that the properties of the material are constant), are as follows. The first six all relate to the geometry of the specimen and tools; the last three relate to the conduct of the test:

1. Specimen shape and plan dimensions.
2. Specimen thickness.
3. Specimen flatness.
4. Surface finish of specimens and tools.
5. Play and misalignment of tools; rigidity of machine.
6. Zero-point and fracture-point determination.
7. Blank-holder load.
8. Lubrication.
9. Speed and temperature of testing.

In considering these variables in the present work, the temperature was kept substantially constant (between 20 and 23° C). Previous researches had indicated that speed of testing had little influence, so this was kept constant and at a minimum, thereby easing the problem of detecting the initiation of fracture and ensuring reliability of the readings from the hydraulic gauges of the testing machine used, which could not be depended upon at high speeds. Alignment of the tools and rigidity of the machine were checked and found satisfactory. The specimens were sufficiently flat, having been cut from 6 × 3 ft sheets. The zero point was always checked and corrected, if necessary, as described later. For the end-point an electronic cut-out mechanism had been incorporated in the machine by the makers, and this was generally found to be adequate. The surface of the tools was highly polished (approximately 8 μ-in. c.l.a. for the die and 6 for the punch), and did not vary during the tests.

It was decided to keep the lubrication conditions as constant as possible, by applying graphite grease supplied by the Shell Oil Company and designated H.T.G. Grease, as recommended in the D.I.N. specification.[1] In this way the surface finish of the tools was protected and the ' sensitivity ' of the material to the test increased, since friction was kept to a minimum. The remaining factors (specimen dimensions and blank-holder load) were varied in a systematic way to cover a wide range of test conditions.

The object of the present research was, therefore, to assess the effects of altering the chosen variables, and to indicate the requirements which might have to be met in any specification.

### Material

Four commonly-used metals were tested: mild steel, copper, brass, and aluminium, all in soft tempers. The steel had been temper-rolled to eliminate the yield-point jog; the other metals were in the bright-annealed condition. All were supplied in the form of 6 × 3 ft sheets, and had received no special treatment other than that normally used to provide fairly isotropic material for industrial pressing. Particulars are given in Table I.

Hardness tests were carried out according to B.S.427, the mean of six indentations made at various points in each sheet being given in the table.

### Experimental Apparatus

The main item of equipment used was a sheet-metal testing machine of 35 Te (Te = Tonne) capacity, manufactured by Roell and Körthaus. The machine will provide constant blank-holder force, and incorporates an electronic mechanism which cuts out immediately a preset decrement in punch load is reached. This, when suitably calibrated, generally coincides with the initiation of fracture; if it does not, pressing can be continued by switching the machine on again. Two gauges are provided on which it is possible to read punch load, one for the range 0–3 Te, the other 0–35 Te, while a third gauge

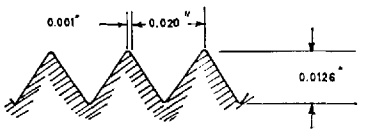



*Fig. 2.*

records blank-holder load, range 0–4 Te. After the gauges had been recalibrated by means of a proving ring and special attachment, they were then accurate to within 0·005, 0·002, and 0·002 Te, respectively. To maintain a constant speed of penetration, it was necessary to open the speed-control valve slightly as punch load increased.

Also incorporated in the panel of the testing machine was an indicator that recorded the depth of penetration of the punch at any instant. The calibration of this gauge was also checked. The zero point of the punch was set to within 0·001 in. by means of a depth micrometer, alterations being made by putting shims under the punch.

The tools used were satisfactory as regards dimensions and surface finish, and when assembled in the machine were concentric and correctly aligned.

A small 80 Te blanking press was used to blank the circles required for the tests, although some of the smaller sizes (< 60 mm dia.) were obtained by turning at very low speeds in a lathe, with a plentiful supply of coolant to avoid raising the metal temperature.

The problem is predominantly an axially symmetrical one so a grid made up of concentric circles was scribed on the circular blank. For this purpose an attachment was designed incorporating a hardened tool-steel scriber, the point of which had an included angle of 60° and a relief angle of 20°, suitable for fixing to the tool post of a toolroom lathe. The scriber was held in a carrier supported between pivot points, fitted with a movable weight that could be adjusted to vary the force on the scriber in a similar manner to that adopted for the tracelet mechanism described by Brewer and Alexander.[6]

Measurements were effected with two machines made by the Société d'Instruments Physiques: a Universal Measuring Machine designated MU 214 B, and a Co-ordinate Measuring Machine U 10.

An additional device was made to centralize round blanks and strip in the testing machine, and to ensure that the centre of the scribed circles would coincide with the axis of symmetry of the punch. This was a " centring tool " which fitted over the blank-holder with no " play " and had four stops adjustable in the four perpendicular directions, moving in slots; these could be set to within 0·001 in. by using a micrometer or vernier in conjunction with a specially made measuring tool.

Finally, a blank-holder was made which was exactly the same as the standard blank-holder, except that the surface which contacted the sheet metal was serrated. The serrations were produced in a lathe which gave a groove similar to that on a gramophone record; the dimensions are as shown in Fig. 2. This blank-holder is referred to throughout as the " rough " blank-holder.

### Testing Procedure

Before beginning the programme proper, a number of tests were made with each material to determine realistic ranges for the various parameters, unscribed blanks being used. The blank-holder force, specimen thickness, size, and shape, selected lubrication, and cut-out sensitivity were determined for each thickness in each material to give minimum crack width (i.e. just letting light through).

To check the consistency of the lubrication, three identical blanks in each thickness and diameter were pressed. The results (Erichsen numbers) seldom differed by > 0·1 mm, and never by > 0·2 mm unless the cause was a material defect. This is generally within the accepted accuracy of the Erichsen number.

The electronic cut-out was sufficiently sensitive to detect the required crack size in all materials and thicknesses except for the soft aluminium, S.W.G. 20. An incremental pressing procedure, in increments of 0·1 mm, had to be adopted for this material, until the desired crack was obtained. In the case of brass it was difficult to obtain the required crack simply by relying on the electronic cut-out, because it happened too quickly for the machine to stop in time. However, a characteristic noise was heard and the depth indicator suddenly altered by ~0·2 mm after the crack had occurred, so that these signs could be used to establish the Erichsen number.

Exploratory tests with the rough blank-holder showed that the following loads could adequately indent the sheet: steel 3·8, brass 3·5, copper 3·5, aluminium 2 Te. Complete prevention of drawing-in was not obtained in the case of brass and copper at the loads shown above, as will be discussed later.

On the basis of the results of the preliminary investigation with unscribed specimens, the following values of the variables were chosen, to be applied to each thickness:

*Blank dia. (mm)*: 100, 75, 60, 41–43–45

*Blank-holding loads (Te)*:

Steel 0·4, 2·0, 3·8 (3·8 with rough blank-holder).
Copper and brass 0·4, 3·0 (3·5 with rough blank-holder).
Aluminium 0·4, 2·0 (2·0 with rough blank-holder).

The specimens were prepared and scribed, generally on both sides, by means of the equipment already described. A width of between 0·001 and 0·0013 in. was satisfactory for the scribed line, in that it did not then influence fracture and did not disappear after deformation had occurred. When the grid had been scribed, it was measured accurately to within 0·0001 in. up to 0·4 in. dia. and to within 0·0005 in. for the larger diameters, using the U.10 Co-ordinate Measuring Machine. In order to measure to within 0·0001 in. a scribed line of width 0·001 in., special illumination was necessary, green and white light being used to illuminate each face of the scribed line and thus clearly define its bottom and edges.

The scribed specimens were then numbered and degreased with acetone, and the graphite grease applied uniformly over the specimen. The centring tool was set for the blank diameter being used and the sensitivity of the electronic cut-out set at the optimum point (as determined from the preliminary tests). The zero point of the punch was checked and the depth-indicator gauge set at zero. The specimen was then placed on the blank-holding plate between the stops of the centring tool and the die locked in position. Blank-holding load was applied gradually until the desired value was established. The speed-control valve was then set to give the minimum punch speed feasible (~0·05 mm/sec). The punch load was recorded at every 1 mm of penetration, since the speed was low enough for this to be done. The machine stopped automatically either at fracture or just before (at necking instability). If the machine stopped too soon it could be restarted to give a fracture of the desired appearance (light just passing).

After the test the specimen was cleaned and degreased. The distorted grids were measured in four perpendicular radial directions parallel and perpendicular to the rolling direction in the sheet. The four sets of radial measurements were always different since the crack was never perfectly symmetrical. The thickness of the specimens was measured to within 0·002 in. with a micrometer of $\frac{1}{64}$ in. nose radius, It was necessary to cut a specimen just to one side of a meridional plane in order to measure the thickness at each point in the bulge.

Circumferential strains were calculated from the mean deformed radii and original radii of the scribed circles using the equation:

$$\varepsilon_\theta = \ln \frac{R_m}{R_0} \qquad . \; . \; . \quad (1)$$

where $\varepsilon_\theta$ = logarithmic circumferential strain
$R_m$ = mean radius of deformed circle.
$R_0$ = initial radius of circle.

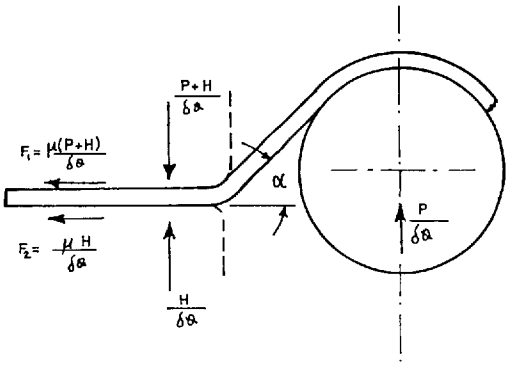
Since the accuracy of measurement was to within ~0·0002 in., the maximum error in the worst case was 0·0056 (logarithmic strain).

Thickness strains were calculated from the equation

$$\varepsilon_t = \ln \frac{t_m}{t_0} \qquad . \; . \; . \quad (2)$$

where $\varepsilon_t$ = logarithmic " through-thickness " strain
$t_m$ = mean thickness after deformation
$t_0$ = original thickness.

In this case the maximum error was 0·009 (logarithmic strain).



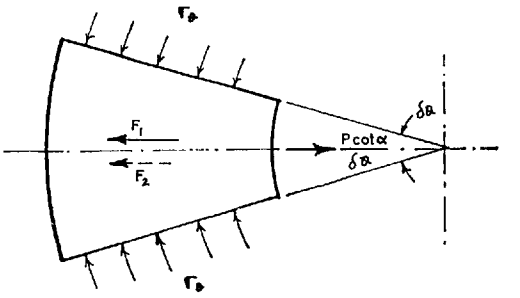




*Fig. 3.*

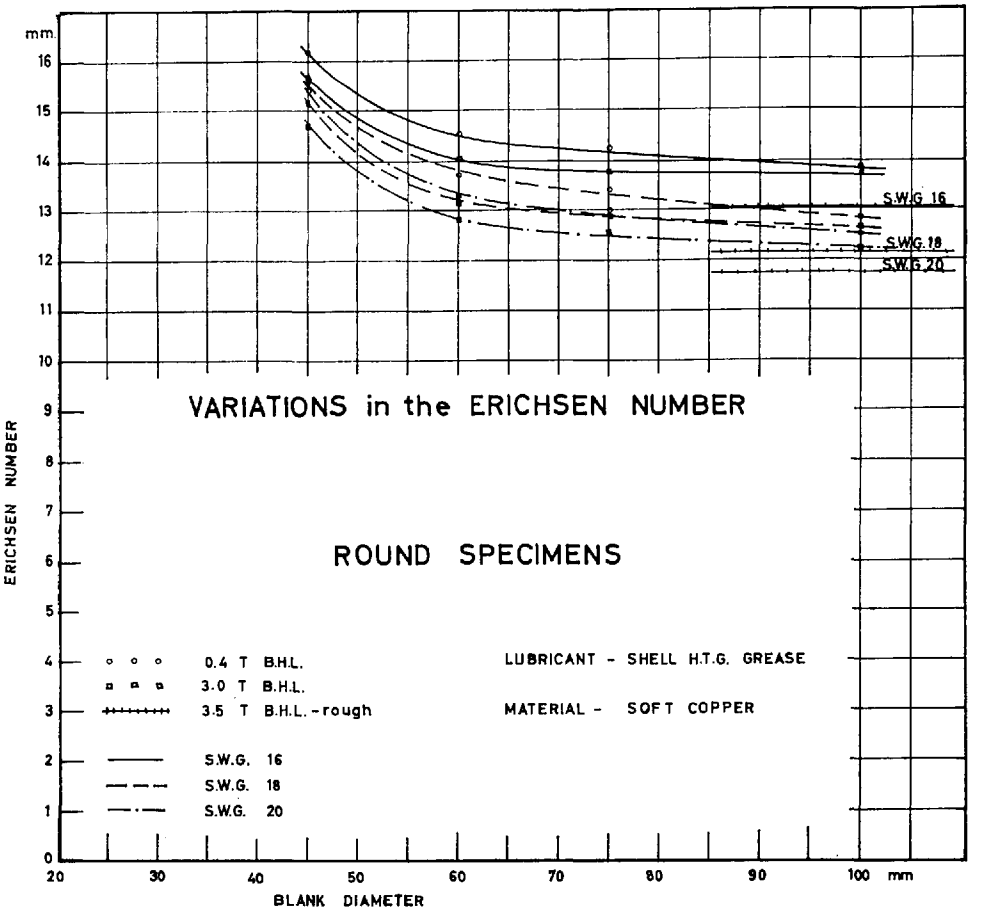



*Fig. 4.*

**Estimation of the Coefficient of Friction**

The forces acting on a segment of a round specimen subtended by an angle $\delta\theta$ are as illustrated in the top elevation of Fig. 3. The plan view shows the distribution of forces acting on a segment of the flat flange. The circumferential stresses induced by the plastic deformation of the flange are denoted by $\sigma_\theta$ and these can be assumed to be unaffected by friction, to a first-order approximation, for any given value of $\alpha$.

In Fig. 3

$P$ = punch load

$H$ = blank-holder load

$\mu$ = coefficient of friction

$F_1 = \dfrac{\mu(P+H)}{\delta\theta}$ = friction force on upper surface.

$F_2 = \dfrac{\mu H}{\delta\theta}$ = friction force on lower surface.

In the absence of friction $F_1$ and $F_2$ would be absent, and equilibrium of horizontal forces would require that the horizontal component of $\dfrac{P}{\delta\theta}$, namely $\dfrac{P\cot\alpha}{\delta\theta}$, should balance the circumferential stresses in the flange. This value of punch load may be denoted by $P_0$. If it be assumed that the circumferential stresses $\sigma_\theta$ are not affected by the increase of shear stresses on the surface, then horizontal equilibrium requires that

$$\frac{P\cot\alpha}{\delta\theta} = \frac{P_0\cot\alpha}{\delta\theta} + F_1 + F_2$$

$$\text{i.e. } P\cot\alpha = P_0\cot\alpha + \mu(P+2H) \quad \ldots \quad (3)$$

Differentiating this equation keeping $P_0$ and $\alpha$ constant,

$$\mu = \frac{\delta P\cot\alpha}{\delta P + 2\delta H} \quad \ldots \quad (4)$$

The variation of $\cot\alpha$ with depth of penetration $h$ (see Fig. 1) can be determined on the assumption that the thickness of the sheet material remains constant and equal to its original thickness $t_0$, from consideration of the geometry of the bulge, which leads to the equation

$$h = \left(\frac{D}{2} + r\right)\tan\alpha - (R + r + t_0)(\sec\alpha - 1) \quad \ldots \quad (5)$$

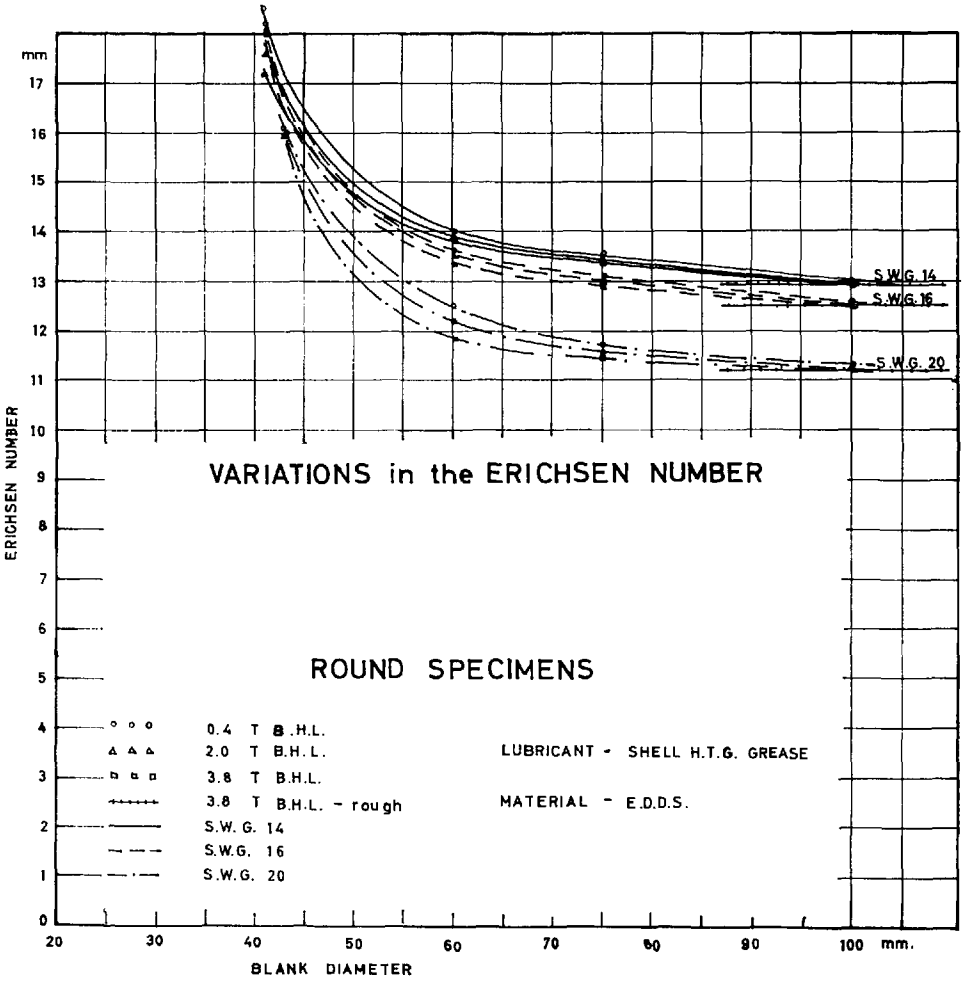



*Fig. 5.*

where the symbols refer to the dimensions indicated in Fig. 1. Thus, the value of $\mu$ can be estimated for the same value of $h$ from two tests carried out under identical conditions, with the exception that the blank-holder load is altered by an amount $\delta H$, which will lead to a change in $P$ of $\delta P$. It may be pointed out here that the effect of the punch load on friction appears to have been neglected in the theories of Chung and Swift[7] and of Fukui.[8] Since the punch load is normally much higher than the blank-holding load, neglect of this is likely to lead to significant errors if it is desired to include the effect of friction in the analysis.

The method suggested here for estimating the coefficient of friction during pressing or drawing has the advantage that it is being measured in the actual process and not in a simulative process. It is theoretically also possible to predict any *variation* of $\mu$ during the operation, due for example to the effect of punch load on friction. The main disadvantage is that the result is dependent on taking the difference between large, nearly equal, quantities and is therefore subject to some error. The error in neglecting the effect of increase in frictional stresses on the circumferential stresses in the flange is also not known; this error will clearly be eliminated by plotting $\mu$ versus $\delta H$ and extrapolating back to zero $\delta H$.

## Results

The Erichsen numbers obtained were plotted against blank diameter (or strip width), since this variable had most effect. Typical results are shown in Figs. 4 and 5 for copper and steel. Circumferential strains on both inner and outer surfaces of each specimen were measured, as well as thickness strains, and typical results are shown in Figs. 6–9. Graphs of punch load versus punch travel were also determined, typical results for brass being as shown in Fig. 10.

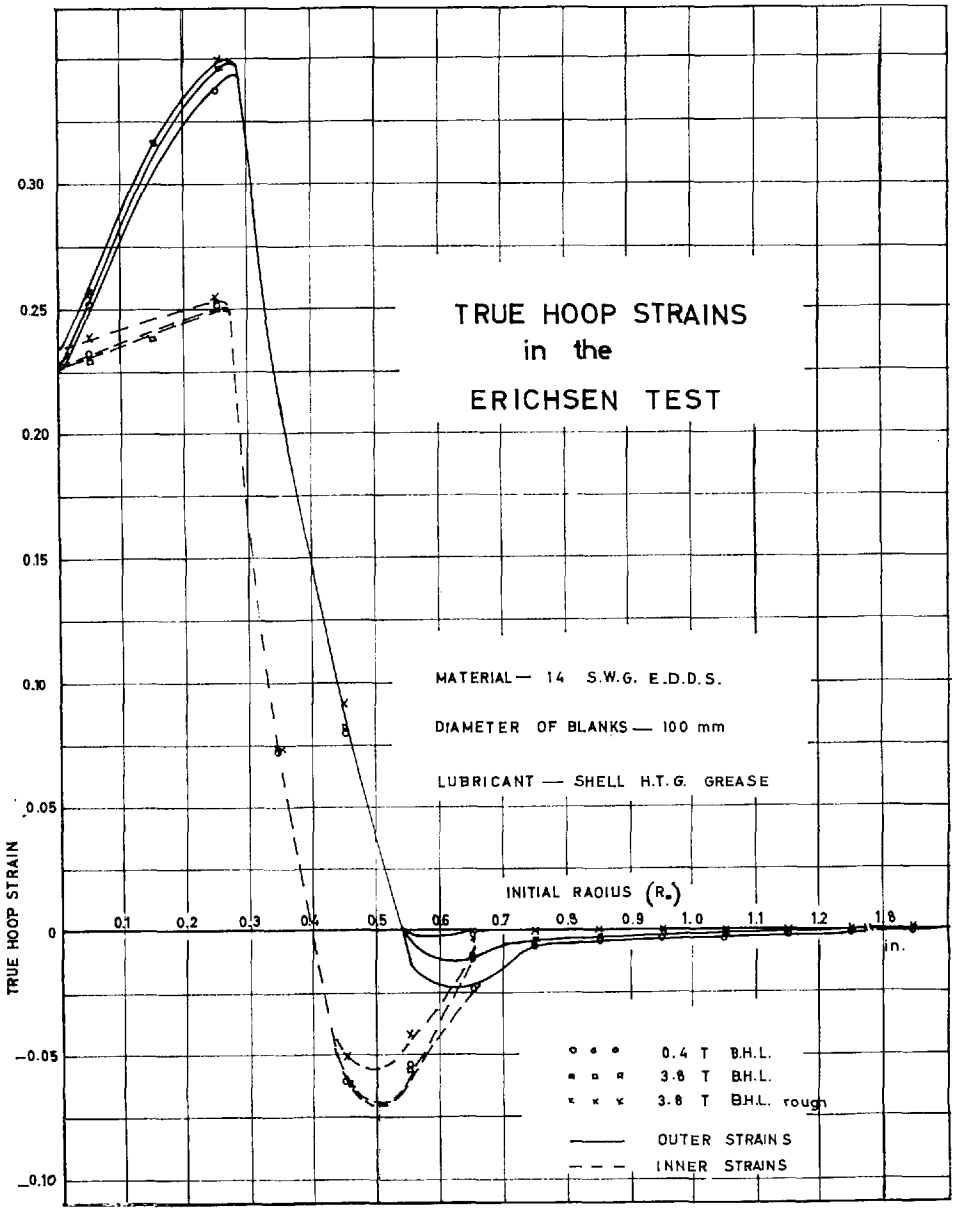



*Fig. 6.*

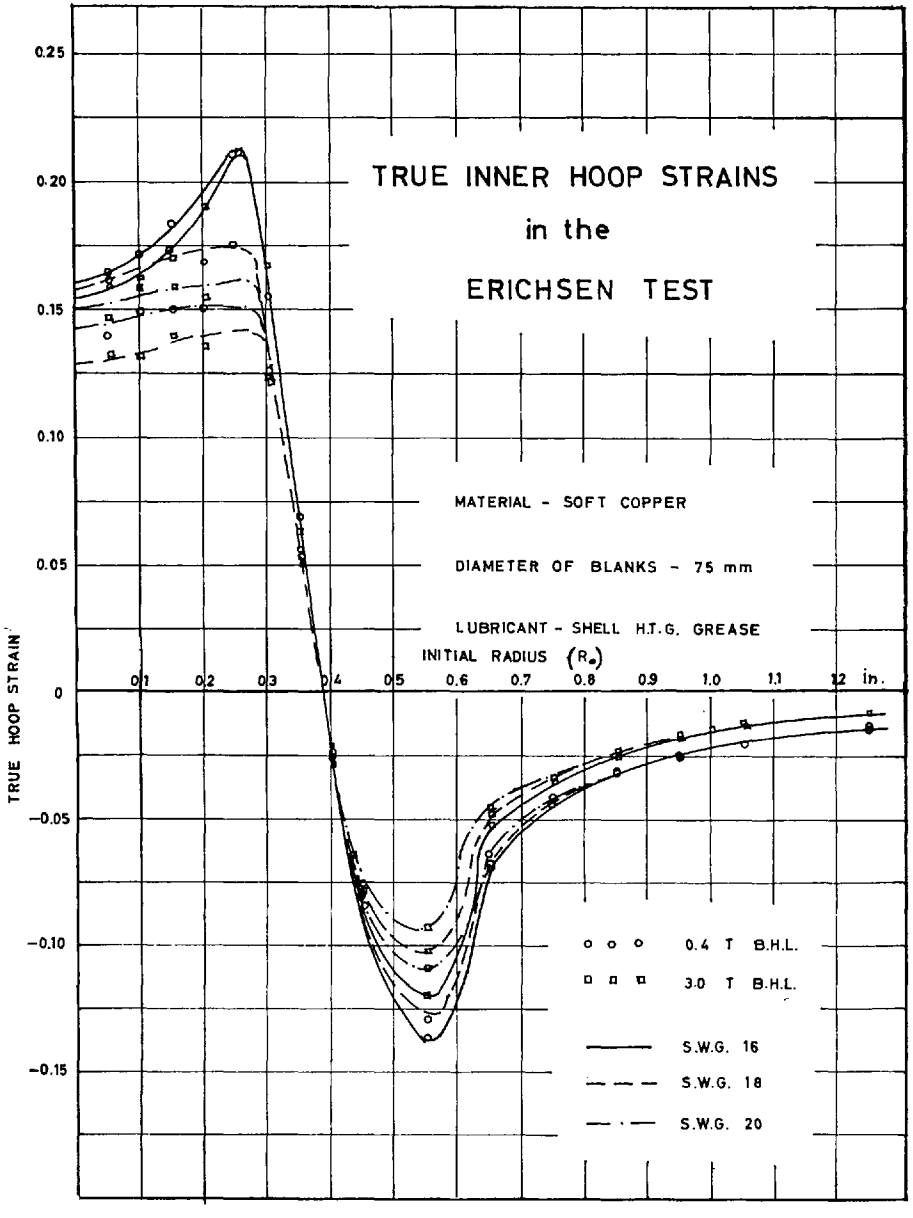

TRUE INNER HOOP STRAINS
in the
ERICHSEN TEST
MATERIAL - SOFT COPPER
DIAMETER OF BLANKS - 75 mm
LUBRICANT - SHELL H.T.G. GREASE
INITIAL RADIUS (R₀)
TRUE HOOP STRAIN
0.4 T B.H.L.
3.0 T B.H.L.
S.W.G. 16
S.W.G. 18
S.W.G. 20

*Fig. 7.*

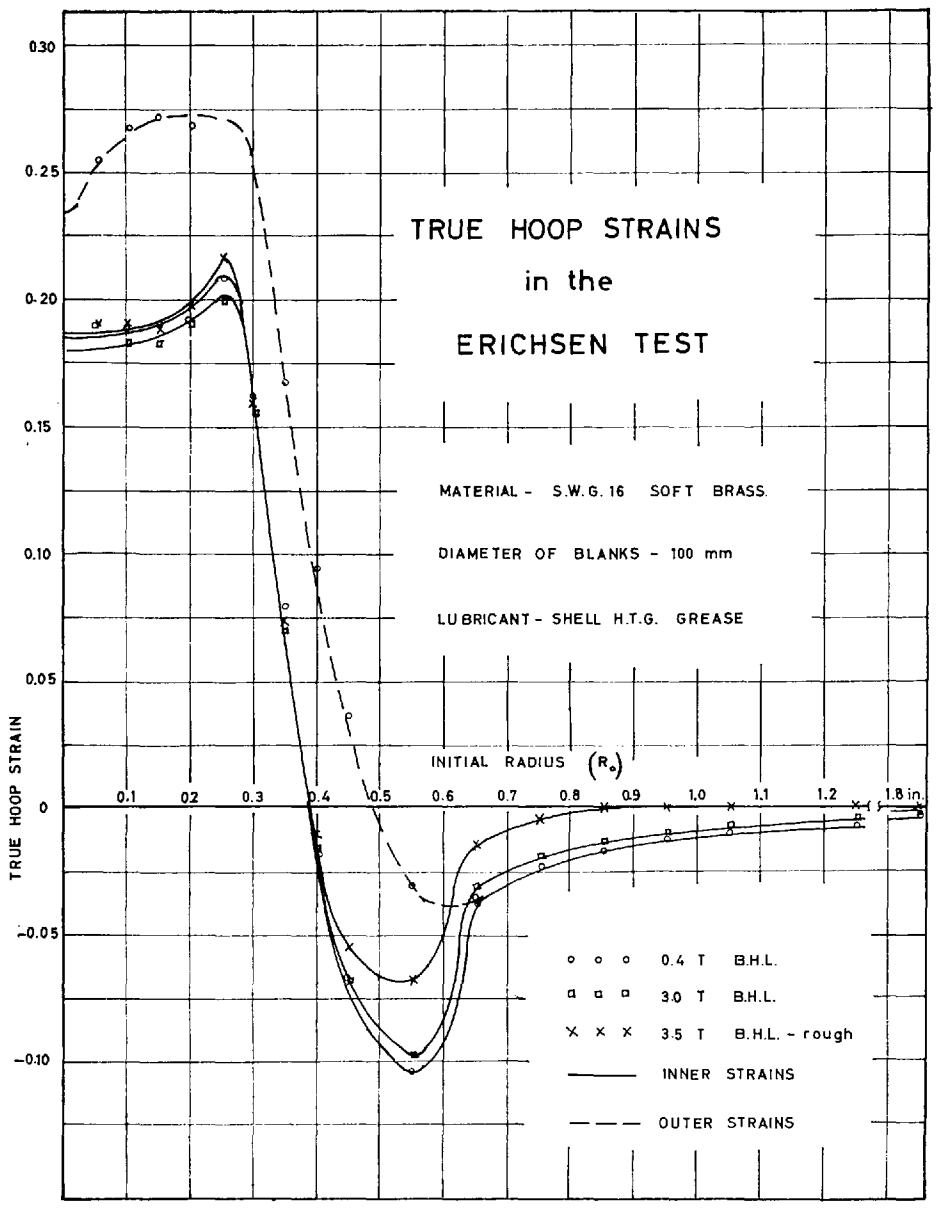

TRUE HOOP STRAINS
in the
ERICHSEN TEST
MATERIAL - S.W.G. 16 SOFT BRASS
DIAMETER OF BLANKS - 100 mm
LUBRICANT - SHELL H.T.G. GREASE
INITIAL RADIUS (R₀)
TRUE HOOP STRAIN
0.4 T B.H.L.
3.0 T B.H.L.
3.5 T B.H.L. - rough
INNER STRAINS
OUTER STRAINS

*Fig. 8.*

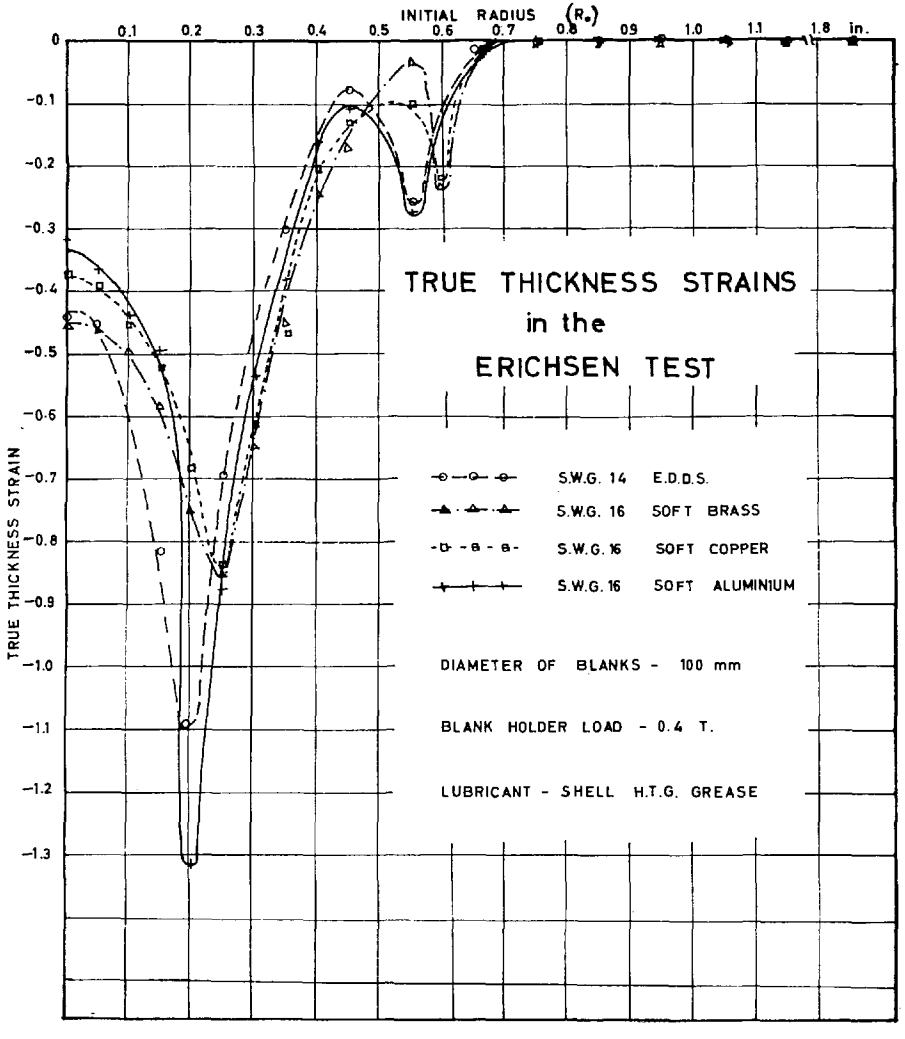

INITIAL RADIUS (R₀)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.8 in.
TRUE THICKNESS STRAIN
-0.1 -0.2 -0.3 -0.4 -0.5 -0.6 -0.7 -0.8 -0.9 -1.0 -1.1 -1.2 -1.3
TRUE THICKNESS STRAINS
in the
ERICHSEN TEST
S.W.G. 14 E.D.D.S.
S.W.G. 16 SOFT BRASS
S.W.G. 16 SOFT COPPER
S.W.G. 16 SOFT ALUMINIUM
DIAMETER OF BLANKS - 100 mm
BLANK HOLDER LOAD - 0.4 T.
LUBRICANT - SHELL H.T.G. GREASE

*Fig. 9.*

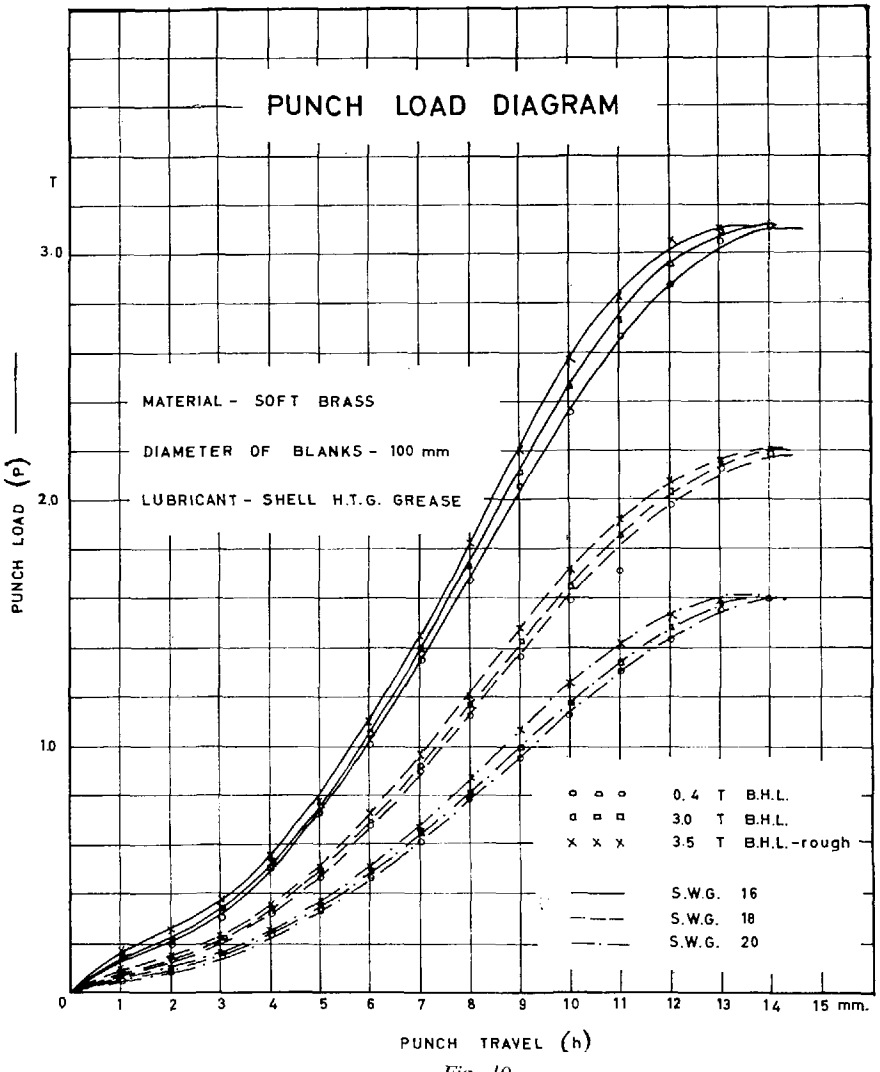

PUNCH LOAD DIAGRAM
PUNCH LOAD (P) ——
T
3.0
2.0
1.0
MATERIAL - SOFT BRASS
DIAMETER OF BLANKS - 100 mm
LUBRICANT - SHELL H.T.G. GREASE
0.4 T B.H.L.
3.0 T B.H.L.
3.5 T B.H.L.-rough
S.W.G. 16
S.W.G. 18
S.W.G. 20
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 mm.
PUNCH TRAVEL (h)

*Fig. 10.*

The actual final disposition of elements in the sheet over the bulge was determined. For example, it was found that line elements originally normal to the plane of the sheet were finally substantially normal to the surface of the punch nose. This is illustrated typically, for copper, in Fig. 11.

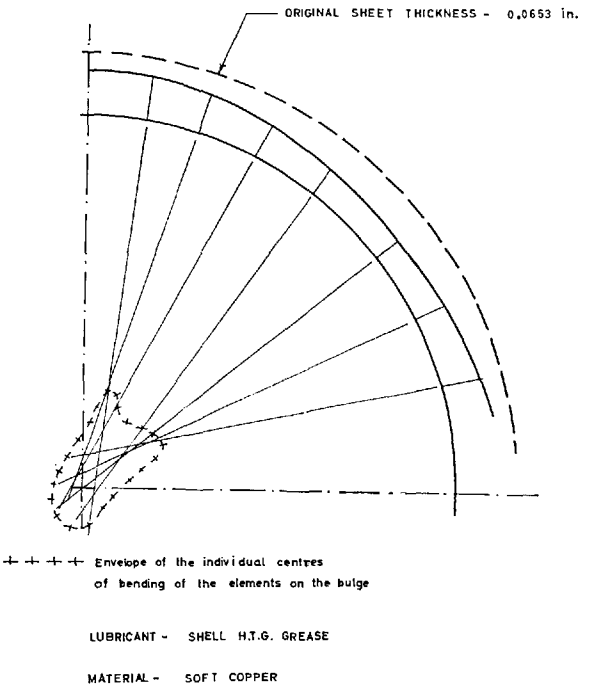



*Fig. 11.* ×5

Coefficients of friction were estimated from the punch load/travel diagrams for all the specimens, the results being indicated in Table II. The value of $\delta P$ was so small for aluminium that no precise value of friction coefficient could be estimated, although it must clearly be $< 0{\cdot}01$

To establish whether there was any clear correlation between the fracture observed in the Erichsen test and subsequent earing behaviour, hemispherical-ended cups were drawn, the drawing ratio being slightly less than the limiting drawing ratio, in all the materials and thicknesses, using a fairly rough punch. Ears were obtained in most of the materials, as indicated by the results in Table III. The method of defining earing was that recommended by Thorley and Tucker[9] and Thorley.[10] The behaviour of the corresponding material in the Erichsen test is described in Table IV.

TABLE II
**Calculated Coefficients of Friction ($\mu$) Using the Derived Formula**

| Thickness, SWG | Angle ($\alpha$) | Punch Penetration ($h$), mm | $\mu$ | | |
|---|---|---|---|---|---|
| | | | Brass | Copper | Steel |
| 14 | 15° | 3·37 | | | 0·067 |
| | 30° | 6·23 | | | 0·050 |
| | 45° | 8·93 | | | 0·017 |
| 16 | 15° | 3·39 | 0·014 | 0·014 | 0·054 |
| | 30° | 6·31 | 0·016 | 0·016 | 0·030 |
| | 45° | 9·13 | 0·018 | 0·026 | 0·012 |
| 18 | 15° | 3·40 | 0·014 | 0·011 | |
| | 30° | 6·36 | 0·010 | 0·007 | |
| | 45° | 9·29 | 0·010 | | |
| 20 | 15° | 3·41 | 0·007 | 0·011 | 0·043 |
| | 30° | 6·41 | 0·007 | 0·005 | 0·025 |
| | 45° | 9·40 | 0·006 | 0·003 | 0·013 |

Note: $\mu$ for aluminium is estimated to be $< 0{\cdot}01$.

## Discussion

The only way of exhibiting the variation of Erichsen number that occurs in the test is to define a measure of the drawing-in and to plot Erichsen number against this variable. This can be done because, for a given amount of stretching at instability over the punch head, the Erichsen number depends

TABLE III
**Calculated Earing Units for Hemispherical Cups**

| Material | Thickness, SWG | Mean Height of Ears ($h_e$), in.* | Mean Height of Troughs ($h_t$), in.* | Earing Units $\frac{h_e - h_t}{h_t} \times 100$ | Angle of Ears | No. of Ears |
|---|---|---|---|---|---|---|
| Steel | 14 | 1·618 | 1·480 | 9·33 | 90° | 4 |
| | 16 | 1·612 | 1·502 | 7·3 | 90° | 4 |
| | 20 | 1·664 | 1·555 | 7·02 | 90° | 4 |
| Brass | 16 | 1·585 | 1·556 | 1·86 | 45° | 4 |
| | 18 | | | Negligible | | |
| | 20 | | | Negligible | | |
| Copper | 16 | 1·587 | 1·562 | 1·6 | 45° | 4 |
| | 18 | 1·608 | 1·574 | 2·16 | 90° | 4 |
| | 20 | 1·644 | 1·588 | 3·5 | 90° | 4 |
| Aluminium | 16 | 1·400 | 1·333 | 5·15 | 45° | 4 |
| | 18 | 1·300 | 1·178 | 10·4 | 90° | 4 |
| | 20 | | | Negligible | | |

* $h_e$ and $h_t$ measured from the pole of the hemispherical end.

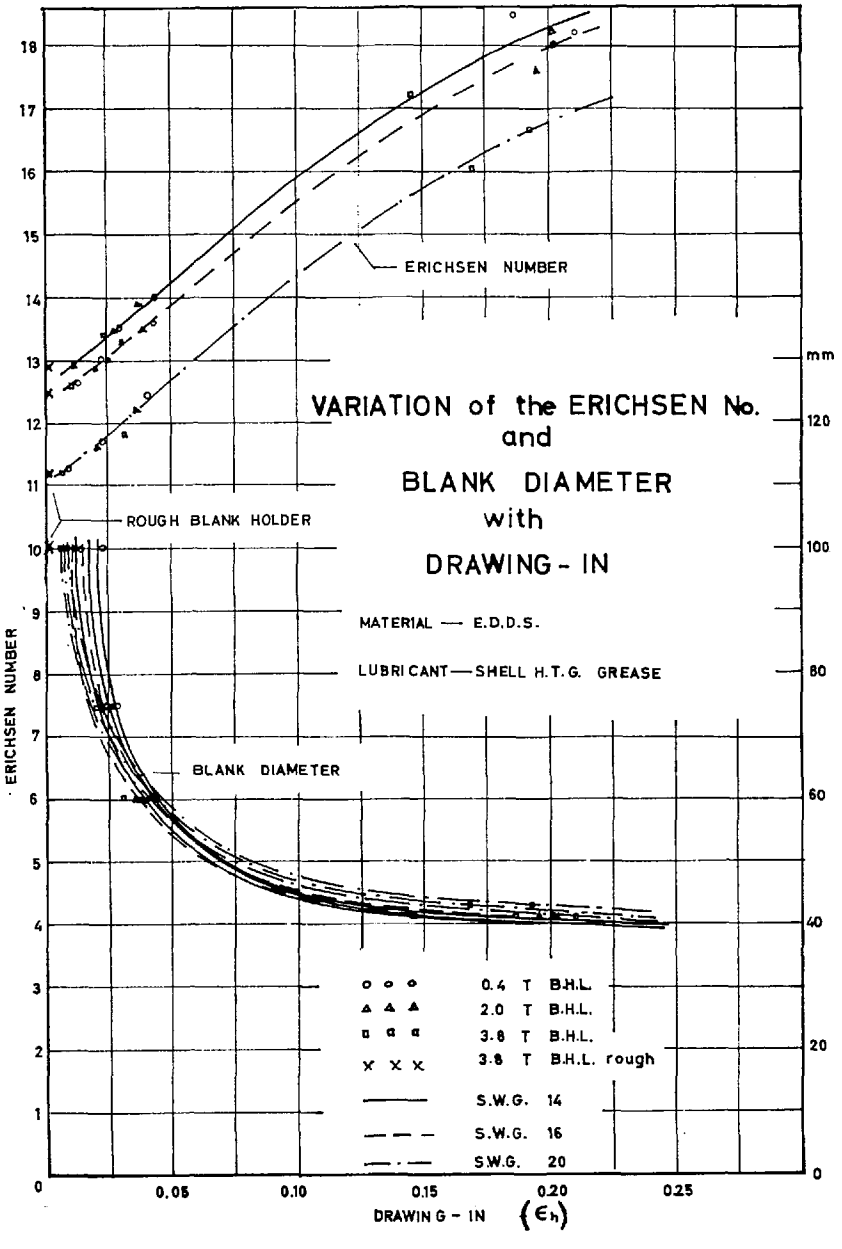

ERICHSEN NUMBER
VARIATION of the ERICHSEN No.
and
BLANK DIAMETER
with
DRAWING - IN
ROUGH BLANK HOLDER
MATERIAL — E.D.D.S.
LUBRICANT—SHELL H.T.G. GREASE
BLANK DIAMETER
0.4 T B.H.L.
2.0 T B.H.L.
3.8 T B.H.L.
3.8 T B.H.L. rough
S.W.G. 14
S.W.G. 16
S.W.G. 20
ERICHSEN NUMBER
BLANK DIAMETER
mm
DRAWIN G - IN (εh)

*Fig. 12.*

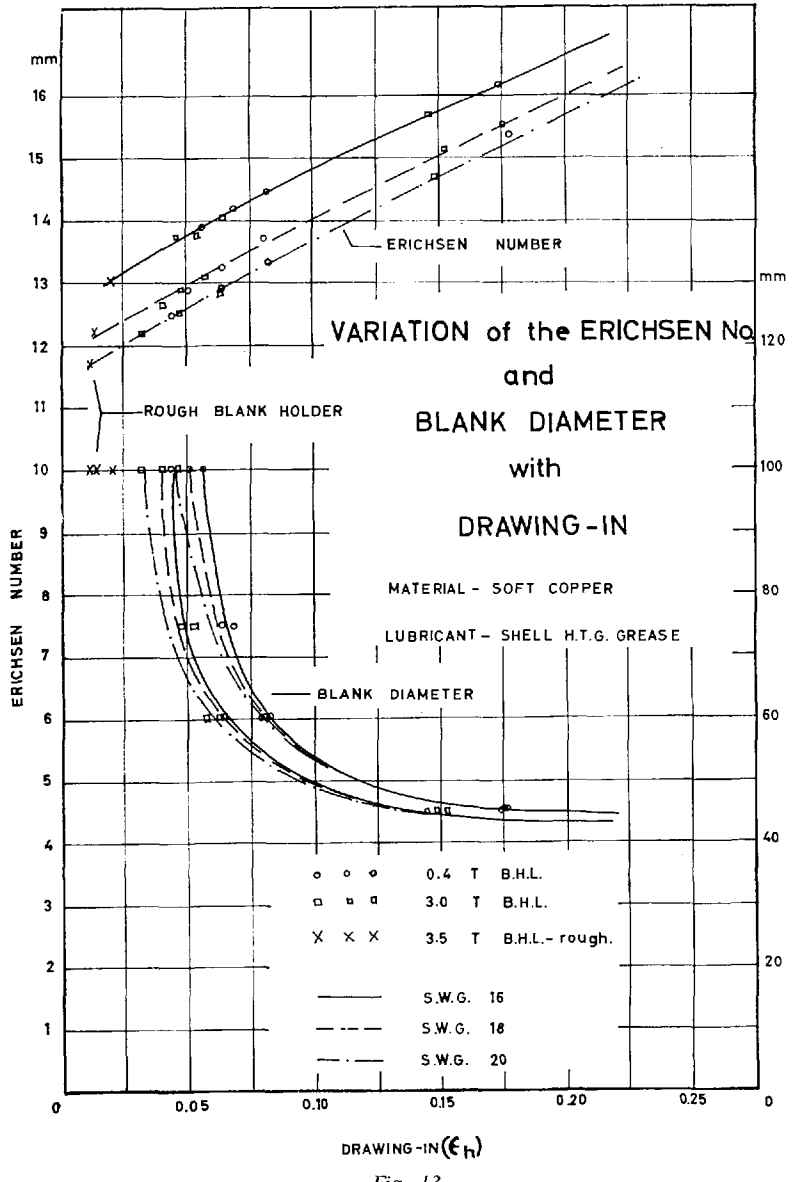

ERICHSEN NUMBER
VARIATION of the ERICHSEN No.
and
BLANK DIAMETER
with
DRAWING - IN
ROUGH BLANK HOLDER
MATERIAL - SOFT COPPER
LUBRICANT - SHELL H.T.G. GREASE
BLANK DIAMETER
0.4 T B.H.L.
3.0 T B.H.L.
3.5 T B.H.L.- rough.
S.W.G. 16
S.W.G. 18
S.W.G. 20
ERICHSEN NUMBER
BLANK DIAMETER
mm
DRAWING-IN (εh)

*Fig. 13.*

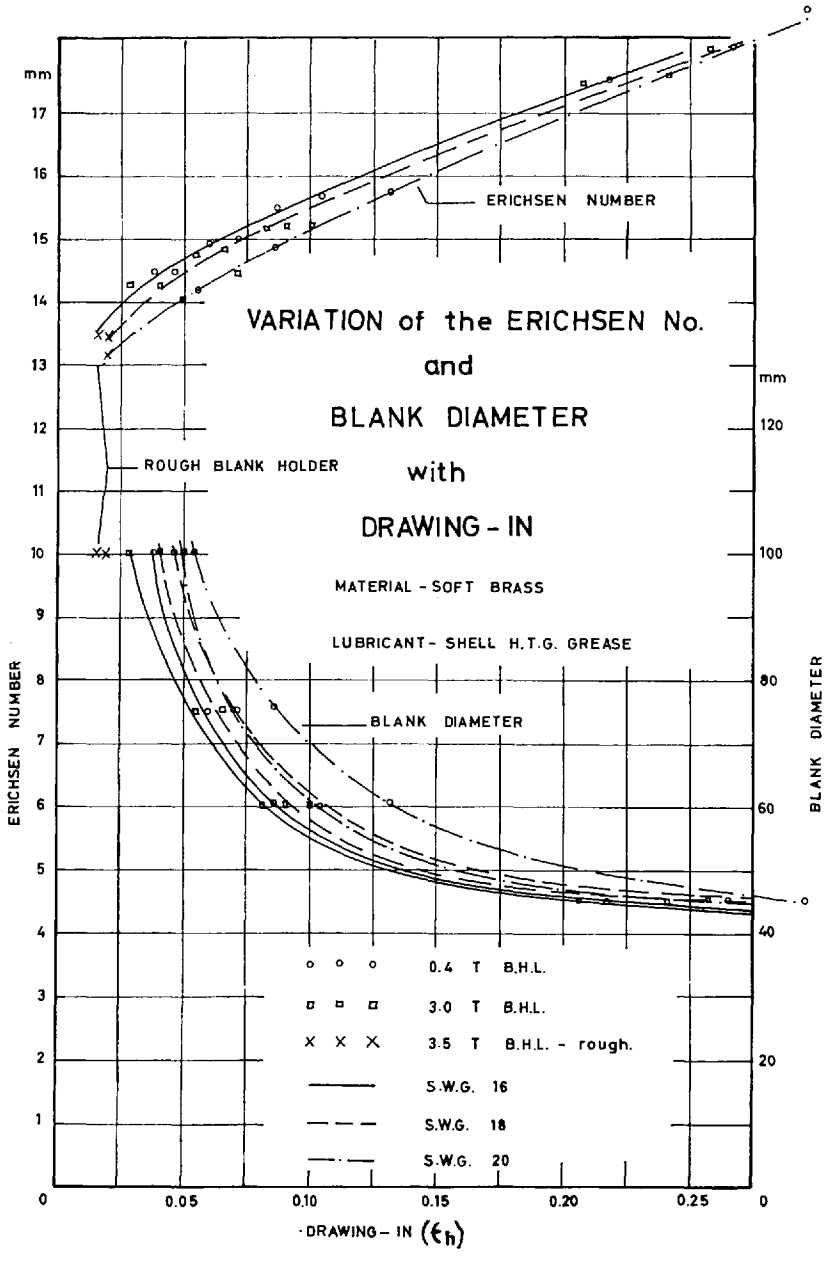

VARIATION of the ERICHSEN No.
and
BLANK DIAMETER
with
DRAWING - IN
MATERIAL - SOFT BRASS
LUBRICANT - SHELL H.T.G. GREASE
ERICHSEN NUMBER
BLANK DIAMETER
ROUGH BLANK HOLDER
0.4 T B.H.L.
3.0 T B.H.L.
3.5 T B.H.L. - rough.
S.W.G. 16
S.W.G. 18
S.W.G. 20
ERICHSEN NUMBER
BLANK DIAMETER
mm
DRAWING - IN (εh)

Fig. 14.

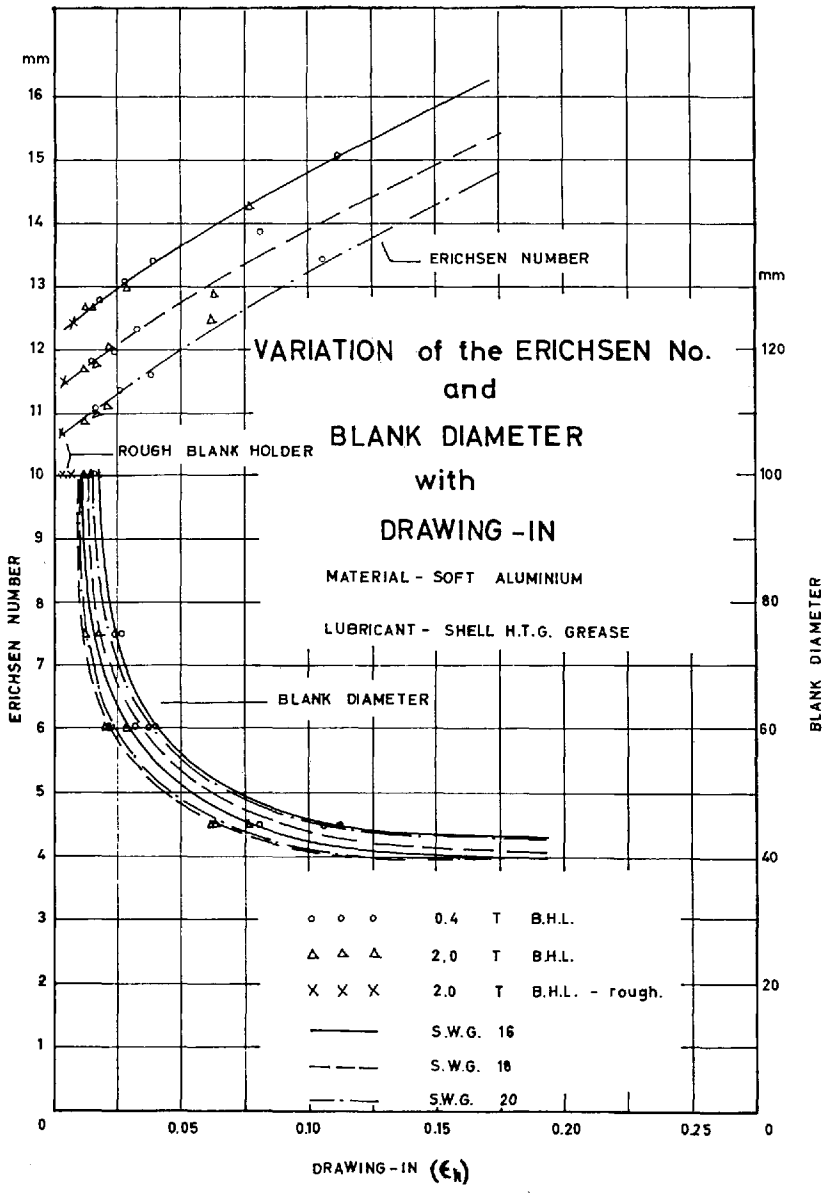

VARIATION of the ERICHSEN No.
and
BLANK DIAMETER
with
DRAWING - IN
MATERIAL - SOFT ALUMINIUM
LUBRICANT - SHELL H.T.G. GREASE
ERICHSEN NUMBER
BLANK DIAMETER
ROUGH BLANK HOLDER
0.4 T B.H.L.
2.0 T B.H.L.
2.0 T B.H.L. - rough.
S.W.G. 16
S.W.G. 18
S.W.G. 20
ERICHSEN NUMBER
BLANK DIAMETER
mm
DRAWING - IN (εh)

Fig. 15.

only on the amount of drawing-in that takes place. A suitable quantity to define drawing-in is the circumferential strain that occurs at the inside diameter or bore of the blank-holder, which can be denoted by $\varepsilon_h$. Thus, if the rough blank-holder truly prevents all drawing-in, $\varepsilon_h$ will be zero when that blank-holder is used. If blank diameter is also plotted against $\varepsilon_h$, an idea of the size of blank necessary to prevent all drawing-in might be obtained by extrapolation. The results for all the materials and thicknesses investigated are given in this way, in Figs. 12–15.

It can be seen that the rough blank-holder prevents drawing-in almost completely for steel and aluminium, and that the Erichsen number attains a minimum value in such circumstances. Drawing-in can apparently never be completely prevented without the use of a rough blank-holder, because with a smooth blank-holder the curves of blank diameter against drawing-in are not asymptotic to the zero axis. Higher blank-holder loads do have some effect in reducing drawing-in, but roughening the surface of the blank-holder is clearly much more effective. Drawing-in of the copper and brass could not be prevented by the rough blank-holder used and it would be interesting to see whether drawing-in of these materials could be prevented by having a rough surface on the die also.

A correlation between round and strip specimens may be established on the basis of their giving the same Erichsen number. The results, plotted as strip width against blank diameter, are given in Fig. 16. Between 75 and 100 mm, strip width and blank diameter were equivalent for steel, copper, and aluminium; thus, a strip of 16 S.W.G. copper, 80 mm

TABLE IV
**Necking and Fracture in Erichsen Test**

| Material | Thickness, SWG | Form of Necking | Form of Fracture | Other Remarks |
|---|---|---|---|---|
| Steel | 14 | All round, severe | R (= Rolling direction) Mostly one arc only present | R for 43 mm strips |
| | 16 | ,, | ,, | for 45 mm strips |
| | 20 | ,, | ,, | for 45 mm strips |
| Brass | 16 | Slightly visible | Mostly one arc present | Very rarely |
| | 18 | ,, | ,, | Very rarely |
| | 20 | ,, | ,, | Very rarely |
| Copper | 16 | All round, severe | | Rarely |
| | 18 | ,, | | |
| | 20 | ,, | | Rarely |
| Aluminium | 16 | Slightly elliptical, very severe | | Some |
| | 18 | ,, | | Some |
| | 20 | ,, | | Some |
| | | | R | R |

Note: *For Aluminium*—Elliptical necking was measured as follows:

S.W.G. 16 $\frac{D_T}{D_R} = \frac{0{\cdot}559}{0{\cdot}538} = 1{\cdot}04$; S.W.G. 18 $\frac{D_T}{D_R} = \frac{0{\cdot}560}{0{\cdot}533} = 1{\cdot}05$; S.W.G. 20 $\frac{D_T}{D_R} = \frac{0{\cdot}549}{0{\cdot}541} = 1{\cdot}065$

$D_T$ = dia. of neck in transverse direction $D_R$ = dia. of neck in rolling direction

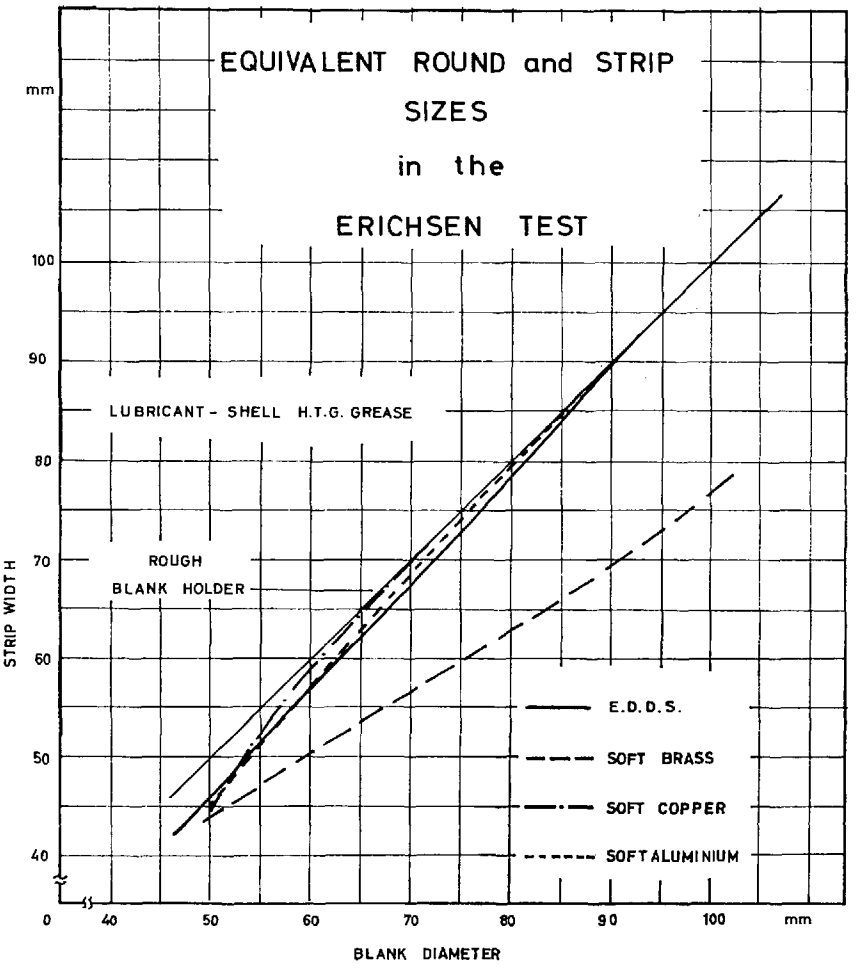



*Fig. 16.*

wide, would give the same Erichsen number as a round blank of 80 mm dia. This was certainly not true for brass, however; the reason for this difference in behaviour is not known, although it must be connected with the stress/strain properties, as can be seen from Fig. 16. A round much larger in diameter must be taken to obtain the same Erichsen number as is found for the strip, since the brass strip specimen suffers less drawing-in than a round blank.

Considering coefficients of friction, the value of $\mu$ as calculated by the suggested method dropped off considerably with increasing punch load in the case of steel, from a maximum value of $\sim 0{\cdot}07$ to a minimum value of $0{\cdot}012$. The only comparable values seem to be those originally determined by Swift[11] in his investigation of bending under tension, and he arrived at a value of $\sim 0{\cdot}04$ with graphite and tallow using mild steel $0{\cdot}038$ in. thick. Since load seems to have an important effect, the correspondence is reasonable. Values of $\mu$ for brass and copper appear to be very low, of the order of $0{\cdot}01$, and must certainly have been less than this for aluminium. Load did not seem to have the effect noticed with steel. Swift obtained values of $\mu = 0{\cdot}022$ for brass, which is considerably higher, but the values obtained by Takahashi and Alexander[12] in the plane-strain compression test on brass were of a similar order.

Regarding the experiments in which earing was measured, it is clear from Tables III and IV that little or no correlation exists between the appearance of the fracture and subsequent earing behaviour, at least for the particular materials and specimens examined. In fact, a completely misleading prediction would be made, in that both steel and aluminium showed round isotropic fractures in the Erichsen test and yet gave pronounced earing when deep drawn into hemispherical-ended cups.

## Conclusions

It is concluded that " drawing-in " can never be completely prevented in the Erichsen test without specially roughened surfaces on die and blank-holder. Unless drawing-in is completely prevented considerable scatter may be found in test results, due to variation in lubrication conditions, specimen geometry, blank-holder load, and other factors. The loads necessary to cause indentation of the sheet by the grooved blank-holder may be taken as those quoted in the present research for the materials studied. For other materials a load must be chosen which gives a visible impression of the grooves in the material. Care must be taken to modify the zero point after indentation of the specimen by the grooves. (The actual displacement necessary was $\sim 0{\cdot}002$ in. in the present work).

The possibility of modifying the Erichsen test by using blanks of smaller diameter, to allow a specified amount of drawing-in to occur, might be considered. In this way a test more representative of the particular process under consideration might be devised. Thus, considering Fig. 14, for example, if it were known that the hoop strain at the edge of the blank-holder (or its equivalent) in the actual process was $\varepsilon_\theta = 0{\cdot}15$,

with soft brass, S.W.G.18, then a blank dia. of 50 mm and blank-holder load of ~ 2 Te should be used, to give an Erichsen number of at least 16·3. The value normally obtained is ~ 14·2 only. However, it might be considered that such a modification of the test would be undesirable, since the dies used in practice have much more generous radii than those used in the Erichsen test. Also, since failure inevitably occurs by instability due to stretching over the hemispherical punch head, there seems little point in introducing a further variable. A cup-drawing test rather than a bulge test is more realistic for assessing the capacity of the material to withstand deep drawing.

Considering the influence of friction over the punch head its presence may be thought desirable from the point of view of simulating practical pressing conditions. On the other hand, friction is a complicating factor which may obscure the response of the material to the imposed biaxial stress system. It has been shown by Wallace[13] that a suitably roughened punch can completely inhibit failure over the punch head, so that the presence of any friction must substantially modify the behaviour of the sheet as regards instability. Thus, it would appear desirable to aim at the complete elimination of friction, which might well alter the location of the fracture zone. This zone was roughly in the same plane for all the current tests, as can be seen from Fig. 9. Experience with the hydraulic-bulge test would suggest that the fracture would move towards the " pole " of the bulge, the lower the friction. It has been suggested that thin polythene sheet is a very effective lubricant in sheet pressing, and if this were used the friction could be assessed by the method suggested here, as well as by observing the fracture location.

To summarize, it is suggested that the Erichsen test could be improved by:

(*a*) Using rough blank-holder and die surfaces in the manner described, to eliminate all drawing-in.

(*b*) Reducing friction over the punch head to a minimum and establishing standardized conditions by using polythene or P.T.F.E. sheet, of a standard thickness.

Neither suggestion has been investigated systematically, but in view of their possible importance in relation to the Erichsen test, it would seem desirable to carry out further research on these factors.

The possibility of obtaining a theoretical solution for the state of stress over hemispherical punch heads including frictional effects, is being investigated. By applying such a theory, together with the basic stress/strain curve of the material, it should be possible to predict the occurrence of instability under the biaxial state of stress over the punch head, using a theory similar to that outlined by Mellor.[14]

**Acknowledgements**

Acknowledgement is made to I.C.I. Metals Division and the Shell International Petroleum Co., Ltd., for donating some of the materials used. The authors would also like to thank Mr. R. C. Brewer, of Imperial College, for advice in connection with scribing and measuring grids.

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tion. This was thought to be due to the reaction of neutrons to give lithium and helium gas within the steel.

Dr. BROOM described the structures observed in copper after high-temperature fatigue. Peak cavitation occurred on boundaries oriented at 45° for all types of strain cycle, but, with longer endurances, the peak moved towards the tensile axis. There was also the possibility of square-grain formation arising from continuously reversed grain-boundary movements with enhanced mobility on 45° oriented boundaries due to the shear stress. The relationship between a tensile test and fatigue, as expressed by the Manson or Coffin law, was considered by Mr. GITTUS. It had been found that this law was obeyed for many materials, provided that the tensile ductility was determined for the same failure mechanism as occurred in fatigue.

## *Discussion*

# An Investigation of the Erichsen Test

Professor S. FUKUI (Institute of Space and Aeronautical Science, University of Tokyo): This paper is worthy of admiration, especially as the strain distributions are determined on both surfaces of the specimens under carefully controlled experimental conditions and they are discussed in detail. The results furnish valuable information.

I cannot agree, however, with the statement on p. 462 that the effect of the punch load on friction appears to have been neglected in the theories of Chung and Swift, and of Fukui. Those investigators, as well as Sommer, Sachs, Siebel, and others, have in fact taken the effect of the punch load on friction into account by assuming that the blank-holding force is supported on the flat face of the die, while the punch load is supported by the die-throat profile. This assumption seems to be more realistic and reasonable than the assumption that the punch load is taken by the flat face, especially when the sheet is thin and flexible.

The derivation of the equation for the punch load, applying the slow-speed belt equation to the present problem with the notation shown in Fig. A, is as follows:

Considering the flat portion of the segment between $R_3$ and $R_2$ horizontal equilibrium of the forces gives:

$$\sigma_r t R_2\,\delta\psi - 2\sin\frac{\delta\psi}{2}\int_{R_2}^{R_3}(\sigma_\theta t)\,dr' - 2\mu H\frac{\delta\psi}{2\pi} = 0$$

$\therefore$ the unit radial load at $R_2 = \sigma_r t\, R_2\,\delta\psi =$

$$\delta\psi\int_{R_2}^{R_3}(\sigma_\theta t)\,dr' + 2\mu\, H\frac{\delta\psi}{2\pi}$$

It is assumed that the frictional stress $\mu\sigma_n$ due to the normal stress $\sigma_n$ acts between $R_1$ and $R_2$ where the blank contacts the die profile, and the angle of contact is given by $\alpha$. Applying the slow-speed belt equation to this region, the approximate expression for the force acting at Section $R_1$ in the plane of the sheet metal is:

$$e^{\mu\alpha}\left\{d\psi\int_{R_2}^{R_3}(\sigma_\theta t)\,dr' + 2\mu\, H\frac{\delta\psi}{2\pi}\right\}$$

The axial component of this force should be in equilibrium with the fraction of the punch load on this segment, and hence:

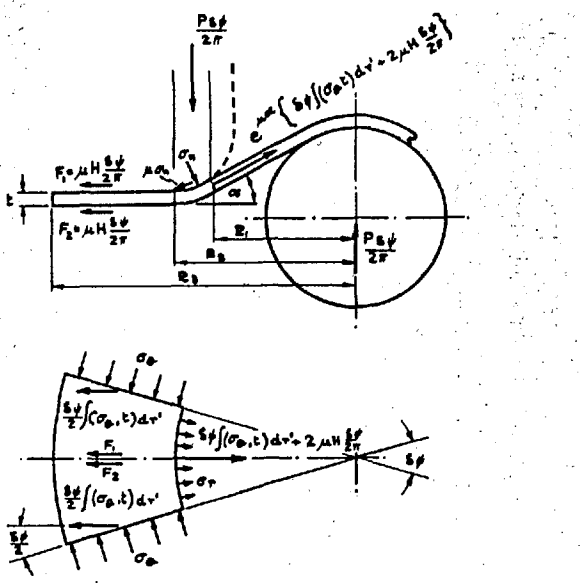


*Fig. A Forces on a large segment of sheet metal in drawing over a die.*

$$P = \sin\alpha\, e^{\mu\alpha}\left\{2\pi\int_{R_2}^{R_3}(\sigma_\theta t)\,dr' + 2\mu\ H\right\}$$

$$\text{or } P = P_0 + 2\mu H \sin\alpha\ e^{\mu\alpha}$$

A simpler version of this belt equation is even quoted by Warwick and Alexander in their paper on the prediction of the limiting drawing ratio from the stress/strain curve.*

In the above equation it is assumed that the load $P$ is supported by the distributed stresses $\sigma_n$ and $\mu\sigma_n$ between $R_1$ and $R_2$, and therefore the effect of the punch load $P$ on friction is not neglected.

Furthermore, the calculated coefficients of friction based on the formula $\partial P/\partial H = 2\mu \sin\alpha\, e^{\mu\alpha}$, which can be derived from differentiating the punch-load equation using the experimental values for $\partial P/\partial H$ and $\alpha$, should give similar values to the ones obtained by the authors.

On the other hand, equation (3) of the present paper appears to be erroneous. If we consider the case $\alpha = \pi/2$, which actually occurs in deep drawing, equation (3) then leads to

$$0 = \mu\,(P + 2H),$$

which is obviously incorrect. This equation could be correct only if the blank sheet were rigid.

This paper by B. Kaftanoglu and J. M. Alexander, appeared in *J. Inst. Metals*, 1961–62, 90, 457.

*J. O. Warwick and J. M. Alexander, *J. Inst. Metals*, 1962–63, 91, 1.

The AUTHORS (*in reply*): We thank Professor Fukui for his comments and the interest he has expressed in our paper.

In this work, the discussion of the subject was mainly in the context of the Erichsen test. The relevant characteristics related to this problem are a high $t_0/r$ ratio and $R_3 - R_2 \gg R_2 - R_1$.

In the theories of Chung and Swift, Fukui, Sommer, Sachs, Siebel, &c. for deep drawing using a large die-profile radius, effectively $t_0/r = 0$ was assumed, thereby eliminating any effect of bending and transverse shear stresses. With these assumptions the flat portion of the sheet metal, as can be shown from considering the equilibrium of the membrane stresses, will make no contribution in transmitting the vertical punch load. Therefore all frictional forces will be on the die profile in the absence of blank-holder load, as was assumed by the above-named authors.

However in the Erichsen test when $t_0/r \simeq 1$ or $>1$ the bending and shear stresses in and around the die profile will be important, as can be seen from Fig. 9 (p.464) of the paper. The derivation of the formula for the punch load by Professor Fukui indicates that the unit radial load at $R_2$ is induced by the plastic deformation, and the blank-holder friction. Applying the slow-speed belt equation to the region between $R_2$ and $R_1$, it has been effectively assumed that the increase in the stress $\sigma_r$ around the die profile is only due to friction, the material there remaining rigid. This of course is not the case, and the increase in $\sigma_r$ due to plastic deformation in this region is much larger than that due to friction only. Also the suggested formula assumes that $t_0/r = 0$ and $R_3 - R_2 \gg R_2 - R_1$, which are not satisfied in the Erichsen test. For these reasons the formulæ now suggested by Professor Fukui for calculating punch load and coefficient of friction are likely to give more approximate results than the method originally presented by the authors. The following analysis shows the effect of bending and shear stresses on the pressure between the flat part of the blank and the die:

*From the Vertical Equilibrium of Forces on an Element Shown in Fig. B:*

$$p\,r'\,\delta\psi\delta r' + (V + dV)(r' + \delta r')\,\delta\psi - Vr'\,\delta\psi = 0$$

dividing by $r'\,\delta\psi\delta r'$

$$p + \frac{dV}{dr'} + \frac{V}{r'} = 0$$

or

$$p = -\left(\frac{dV}{dr'} + \frac{V}{r'}\right) \qquad \ldots(1)$$

Similarly, from moment equilibrium at point 0,

$$V = \frac{dM}{dr'} + \frac{M}{r'} - \frac{\mu p t}{2} \qquad \ldots(2)$$

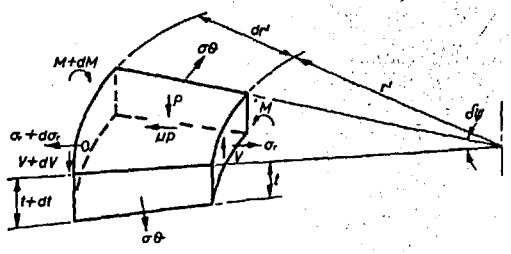



*Fig. B An element of sheet metal in the radial drawing region adjacent to the die entry. V and M are per unit length.*

Differentiating (2) with respect to $r'$:

$$\frac{dV}{dr'} = \frac{d^2M}{dr'^2} + \frac{1}{r'}\frac{dM}{dr'} - \frac{M}{r'^2} - \frac{\mu t}{2}\frac{dp}{dr'}, \text{ if } \frac{dt}{dr'} \simeq 0.$$

Substituting in equation (1) and rearranging:

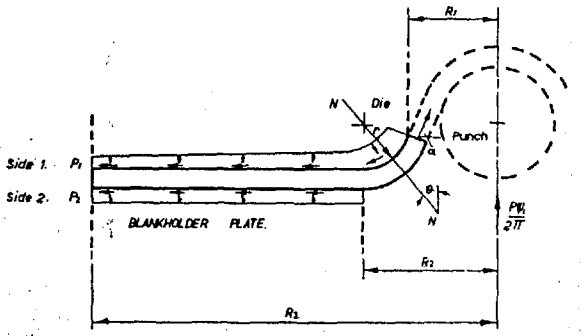
$$p = -\left[\frac{d^2M}{dr'^2} + \frac{2}{r'}\frac{dM}{dr'} - \frac{\mu t}{2}\left(\frac{dp}{dr'} + \frac{p}{r'}\right)\right]$$

$$\text{or } p \simeq -\left[\frac{d^2M}{dr'^2} + \frac{2}{r'}\frac{dM}{dr'}\right].$$

The terms in the bracket are expected to be negative, since the bending moment is a maximum at $R_2$ and decreases to zero at $R_3$ or before.

Therefore, bending redistributes the pressure around the die profile and creates pressure between the flat part of the sheet metal and the die. The total effect of all this pressure is to support the punch load, and a large amount of punch load and frictional force can be transmitted by the flat part of the sheet metal because of its large area, as in the case of the Erichsen test. This effect was neglected by the authors mentioned above.

For similar reasons the suggested slow-speed belt equation, which does not incorporate the effects of bending and shear stress and is valid for $R_2 \to \infty$ is not expected to give realistic results when $t_0/r \simeq 1$.



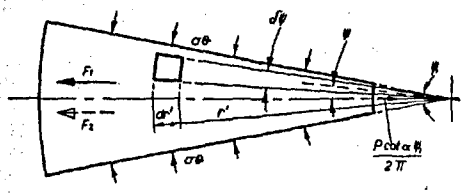




*Fig. C A large segment of sheet metal in contact with the die and blank-holder plate.*

Equation (3) quoted in the paper leads to an inconsistency at $\alpha = \pi/2$, as mentioned by Professor Fukui, due to not specifying all the forces involved in $P_0$ explicitly. Using the notation in Fig. C, this can easily be shown as follows:

*From Vertical Equilibrium of the Segment:*

$$\frac{\psi_1}{2\pi}\int_{R_2}^{R_3} 2\pi\, p_2 r'dr' - \frac{\psi_1}{2\pi}\int_{R_1}^{R_3} 2\pi p_1 r'dr' - \frac{\psi_1}{2\pi}\int_{R_1}^{R_3} \mu 2\pi p_1 \tan\theta r'dr' + \frac{P\psi_1}{2\pi} = 0$$

where

$$\int_{R_2}^{R_3} 2\pi p_2 r'dr' = H, \quad \int_{R_1}^{R_3} 2\pi p_1 r'dr' + \int_{R_1}^{R_3} \mu\, 2\pi p_1 \tan\theta r'dr' = H + P$$

and $H$ = blankholding load; $P$ = punch load.

$$\therefore \int_{R_1}^{R_3} 2\pi p_1 r'dr' = H + P - \mu \int_{R_1}^{R_8} 2\pi p_1 \tan\theta r'dr' = H + P - S_1$$

In this way the integrals can be related to the experimentally measured forces.

*For Horizontal Equilibrium:*

$$\overbrace{2\sin\frac{\psi_1}{2}\mu\int_{R_2}^{R_3} p_2 r'dr'}^{H/2\pi} + \overbrace{2\sin\frac{\psi_1}{2}\mu\int_{R_1}^{R_3} p_1 r'dr'}^{\frac{1}{2\pi}(H+P-S_1)} - 2\sin\frac{\psi_1}{2}\int_{R_1}^{R_3} p_1 \tan\theta r'dr' - \frac{\psi_1 P\cot\alpha}{2\pi} + 2\sin\frac{\psi_1}{2}\int_{R_1}^{R_3} \sigma_\theta t dr' = 0.$$

If $S_1 = \mu\int_{R_1}^{R_3} 2\pi p_1 \tan\theta r'dr'$, $S_2 = \int_{R_1}^{R_3} p_1 \tan r'dr'$, $S_3 = \int_{R_1}^{R_3} \sigma_\theta t dr'$,

when $\psi_1 \to 0$:

$$\therefore \mu H + \mu(H + P - S_1) - 2\pi S_2 - P\cot\alpha + 2\pi S_3 = 0 \quad \ldots(3)$$

Differentiating the above equation totally, keeping $\alpha$ constant, and using finite increments:

$$\mu\,\delta H + \mu(\delta H + \delta P - \delta S_1) - 2\pi\delta S_2 - \delta P\cot\alpha + 2\pi\delta S_3 = 0 \quad \ldots(3a)$$

For a blank with a large ratio of $\frac{R_3 - R_2}{R_2 - R_1}$ and at early stages of the drawing process, similar to the Erichsen test, an increase in $H$ is likely to change the pressure distribution of the flat part of the flange, and not affect greatly the pressure distribution around the die throat. Therefore, for a flat die with a high ratio of $\frac{R_3 - R_2}{R_2 - R_1}$, $\delta S_1 \simeq 0$ and $\delta S_2 = 0$.

If it is assumed that the increase in frictional stress due to an increase in $H$ does not affect $\sigma_\theta$ to a first-order approximation, then $\delta S_3 \simeq 0$, as was quoted in the paper,

$\therefore$ from equation (3*a*)

$$\mu = \frac{\delta P\cot\alpha}{2\delta H + \delta P} \quad \ldots(4)$$

as obtained previously.

The cumulative effect of these three assumptions can be checked by computing $\mu$ using the above formula and different values of $\delta H$ and extrapolating back to $\delta H = 0$, as was also suggested in the paper (p. 462).

We do not agree with Professor Fukui that this inconsistency was due to the fact that the blank was not rigid, since only considerations of equilibrium have been used. The inconsistency occurred because all components of $P_0$ were not given in the paper, particularly those due to the horizontal reaction of the die. Equation (3) above could be used for all values of $\alpha$, but equation (4) (which is the same as equation (4) of the paper), can be used only for cases such as the Erichsen test in which $\alpha$ does not exceed $\sim 70°$. The assumptions made in its derivation will not hold for larger values of $\alpha$.

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Loladse may also be found in the paper by Opitz and Gappisch*.

(*b*) Albrecht varied both the depth of cut and the tool nose radius in his experiments. By so doing he found that the tool forces varied with both depth of cut and tool nose radius; this is indeed an empirical result and a very interesting one. We criticized not the empirical result, but the assumptions which Albrecht made to determine the proportions of the total tool force acting on the rake face and on the tool nose. To do this Albrecht had to assume that:

(i) variations in depth of cut altered the rake face force only, and
(ii) variations in nose radius altered the nose force only.

It is our contention that these assumptions were not validated experimentally and that from physical considerations the validity of the second assumption is dubious.

(*c*) It will be seen from Kamskov's paper (Fig. 2, right-hand graph) that the only part of the rake face on which the coefficient of friction remained constant from 90 to 1000 ft/min was above a contact length of 1 mm for 0·3 mm depth of cut. It follows from Kamskov's argument that sticking friction occurred over a contact length ratio of 1·0/0·3 which is approximately 3 as stated in the paper.

(*d*) Our statement concerning Takayama and Usui's work referred specifically to Fig. 4*a* where it can be seen that what we said was true; Dr Hsu is apparently referring to Fig. 4*b*.

(*e*) That the real area of contact equalled, or at least approached, the apparent was implicit in Hucks' analysis.

*The following Communication relates to the paper 'Analysis of the cup-drawing process', by D. M. WOO, which was published in 1964, vol. 6, No. 2, p. 116.*

**Mr B. Kaftanoğlu** (*Student*)—Mr Woo is to be commended for his improved technique for solving the radial drawing problem over a die, and for investigating the validity of the assumptions (Nos 1, 2 and 4) made by Chung and Swift (reference (3)). It seems that assumption (1) is justified, assumption (2) would give differences with experimental results of the same magnitude but different sign (Figs 6, 17, 18) and assumption (4) should not be made if accuracy in calculating strains is required.

It is said that the main object of this investigation is to predict the maximum punch load using equation (13) which contains the values of variables at $\theta$ corresponding to the point of departure of sheet metal from the die, but it was not shown how this $\theta$ was computed, which is, of course, very important. This analysis will be insufficient to find this $\theta$ without introducing a method of matching the boundary conditions of radial drawing and stretching regions including the moving contacts on punch and die. Alternatively the approximate analysis adopted by Chung and Swift (3) can be used. The assumption of $\theta = \pi/2$ which might have been used, will only apply for limiting blank diameters at final stages with no clearance between the cup and die. For all the other cases $\theta < \pi/2$. The trend observed in the theoretical curves in Figs 19 and 20 (i.e., punch load—reduction of blank diameter) is that they do not seem to pass through the origin when extrapolated, suggesting that $\theta$ was not calculated correctly. The differences between experimental and theoretical maximum punch loads would be amplified if blank diameters were considerably less than the limiting ones. Therefore the analysis lacks generality.

The assumption of $-0{\cdot}01$ polar thickness strain when the sheet metal is stretch formed over the whole of the punch profile, i.e. $\theta_2 = \pi/2$, is highly unrealistic. From experiments and theoretical work done at Imperial College, $\theta_2 \cong \pi/10$ for such a strain and is very sensitive to friction and geometry. It will lead to serious error as also seen from large differences between experimental and theoretical thickness strains even at very low general strain level (Figs 21 and 23).

The author's suggestion for improving the stretch-forming theory by including the effects of the region between punch and die seems to be in error. Referring to Fig. 4 he says that the computation starts at the point of $r_2$ at which the stresses and strains have been determined previously, and for the next point of $r_{i+1}$, $\rho_{3,i+1}$ and $\rho_{4,i+1}$ are assumed in the first approximation to be the same as those at $r_2$. Since there is a discontinuity in $\rho_3$ at $r_2$ which can be seen by examining the equations (17) and (18) with $r$, $\theta$, $\sigma_r$, $\sigma_\theta$, $\rho_4 > 0$ a new value of $\rho_3$ must be calculated at $r_2$ from the equations (17) and (18), which should be used as a boundary condition. Therefore $\rho_3$ shown in Fig. 4 must have the opposite sign, and it can never have the sign shown in this region. It must also be pointed out that in the equations (8), (17) and (18) (i.e. equations of equilibrium) only two are independent, since the third equation in the direction of $\sigma_\theta$ is identically satisfied with the symmetry of the problem, and therefore should not all be used. The additional independent relationship required is supplied by the differential equation which gives the geometrical relationship between $\rho_3$ and $\rho_4$ (see Hill (8), $\mathrm{d}\rho_4/\mathrm{d}r = \rho_4/r(1-\rho_4/\rho_3)$) which must be satisfied by any axisymmetrical shell. It is still different from equation (15) (constancy of volume). Otherwise incorrect results might be obtained.

Furthermore, unlike the region over the die, in the stretch-forming region over the punch and also between the punch and die, the terms in $(\sigma,t)$ in equations (8) and (14) are multi-valued and a criterion based on the minimum plastic work is needed to find the correct combination of $\sigma_r$ and $t$ found from the plasticity laws. This is particularly important at larger deformations than quoted in the paper, and it is not believed that the author's method of successive approximations will satisfy this requirement, without further discrimination.

* OPITZ, H. and GAPPISCH, M. 'Some recent research on the wear behaviour of carbide cutting tools', *J. Machine Tool Design and Research* 1961 2 (No. 1), 43.

These arguments are based on the extensive theoretical and experimental research conducted at Imperial College between 1961 and 1964, of which the first part of the experimental work was published by Kaftanoğlu and Alexander*. A comprehensive theory for the solution of the axisymmetrical sheet metal deformation problems is developed to predict all the variables involved, until, and at the onset of, fracture in stretch forming. It includes the effects of plastic anisotropy, friction, non-linear work-hardening, pre-strain, thickness stress, non-uniform initial conditions, and the complete strain history by accounting for the moving contact point of the sheet metal on the punch with continued deformation. Numerical solutions including higher order differences in the differential equations (non-successive approximation method) have been obtained on the Atlas computer, and various instability criteria have been studied. Good agreement was obtained between theory and experiment. It is expected to publish this work in the very near future.

I would like to acknowledge the helpful discussions with Professor J. M. Alexander on the points mentioned above.

## AUTHOR'S REPLY

**Mr D. M. Woo**—I thank Mr Kaftanoğlu for his commendation of my work. Regarding the points he raised, I wish to reply as follows:

For each of the cases considered (Figs 19 and 20), the punch load reaches the maximum and remains practically constant for reduction of blank diameter between about 15 per cent and 20 per cent. The angle $\theta$, measured at the maximum reduction shown in Figs 19 and 20, was found to be about $\pi/2$. Thus it may be concluded that the maximum punch load in such cases could be predicted satisfactorily by considering the punch load at 20 per cent reduction, and assuming $\theta$ to be $\pi/2$. To compare the theoretical and the experimental punch loads at smaller reductions, the angle $\theta$ was measured experimentally for each pressing and then corresponding to this $\theta$, the theoretical punch load was obtained and compared with the experimental value. As mentioned by Mr Kaftanoğlu, the theoretical value of $\theta$ could be estimated by using the approximate method suggested by Chung and Swift. This shows that the present solution could be used to predict the maximum punch load for other blank sizes which have not been considered in the present work, and therefore the solution is sufficiently general.

In the cup-drawing operation, the punch load rises steeply in the early stages with little reduction of blank diameter. Thus it is not justifiable to extrapolate the curves to the region, the characteristic of which is quite different.

With regard to the stretch-forming problem, although a solution is suggested (p. 121), only the preliminary results based on the simplified conditions were obtained at the time the paper was written. The inaccuracy of these results was discussed on p. 130. However, further work concerning the stretch-forming process has now been successfully completed.

The suggested solution described on p. 121 is correct. To apply the solution for the region where the material is not in contact with the punch, the computation starts at the point of $r_2$ at which $\rho_3$ is obtained from equation (18). Equations (16), (17) and (18) are consistent with the assumed sign of $\rho_3$ corresponding to increasing $\theta$ as shown in Fig. 4. In the present case, $\sigma_r$ and $\sigma_\theta$ are positive at $r_2$, hence $\rho_3$ is negative and $\theta$ is decreasing in this region. It may be mentioned that the proposed solution is similar to that for analysing the hydrostatic bulging process given elsewhere†. Equations (17) and (18) are used for assuming the profile of a deformed element for each cycle of computation. In other words, once the profile radii $\rho_3$ and $\rho_4$ are assumed, the six unknowns, $\epsilon_t$, $\epsilon_\theta$, $\bar{\epsilon}$, $\bar{\sigma}$, $\sigma_r$ and $\sigma_\theta$ can be determined from equations (3), (5), (8), (16) and the two equations of (4) (with $\sigma_t = 0$). From the results thus obtained, new values $\rho'_3$ and $\rho'_4$ are then calculated using equations (17) and (18), and the cycle of computation may then be repeated until $\rho'_3$ and $\rho'_4$ agree closely with the assumed $\rho_3$ and $\rho_4$.

The geometrical relationship

$$\frac{\mathrm{d}\rho_4}{\mathrm{d}r} = \frac{\rho_4}{r}\left(1 - \frac{\rho_4}{\rho_3}\right)$$

has been embodied in deriving the approximate equation (16) for volume constancy, since the former is based on the relationships $r = \rho_4 \sin\theta$ and $\rho_3\,\mathrm{d}\theta = \mathrm{d}r/\cos\theta$ which are also used for obtaining the latter.

It may be pointed out that although the proposed solution for the stretch-forming problem is valid, it has been found in the computation that $\rho_3$ and $\rho_4$ did not converge to the correct values as they did in the similar case of hydrostatic bulging. Therefore, in further work, the following approximate equation for volume constancy has been used:

$$(R_{i+1}^2 - R_i^2)\,t_0 = \frac{(r_{i+1}^2 - r_i^2)\left(\dfrac{t_i + t_{i+1}}{2}\right)}{\cos\left(\dfrac{\theta_i + \theta_{i+1}}{2}\right)}$$

together with the equation

$$\sin\theta = \frac{(\sigma_r t r)_2 \sin\theta_2}{\sigma_r t r}$$

Satisfactory agreement has been obtained between the theoretical and the experimental results.

In the present analysis of stretch-forming, only one value of $(\sigma_r t)$ was found at a particular point. This is based on the fact that the material continues to thin at successive stages of the forming process, and that corresponding to each thickness $t$, only one value of $\sigma_r$ was obtained.

I am pleased to know of the work being carried out at Imperial College as mentioned by Mr Kaftanoğlu.

* KAFTANOĞLU, B. and ALEXANDER, J. M. 'An investigation of the Erichsen test', *J. Inst. Met.* 1961–62 **90**, 457.

† WOO, D. M. 'The analysis of axi-symmetric forming of sheet metal and the hydrostatic bulging process', *Int. J. mech. Sci.* 1964 **6**, 303.

*Int. J. Mech. Sci.* Pergamon Press Ltd. 1965. Vol. 7, p. 447. Printed in Great Britain

# LETTERS TO THE EDITOR

## Discussion on "The analysis of axisymmetric forming of sheet metal and the hydrostatic bulging process"*

(*Received* 23 *November* 1964)

THE writer found the subject paper an interesting example of the application of the theory of plasticity for an isotropic continuum and the successive approximations method, in particular to the process of hydrostatic bulging of sheet metal.

In his method of solution, the author seems to have used three equilibrium equations (i.e. tangential, vertical and normal to the shell wall, equations (6), (8) and (9)), two of which can be independent since the third equation in the direction of $\sigma_\theta$ is identically satisfied with the symmetry of the problem. The other independent relation should have been the one shown by Hill [from Ref. 3 $d\rho_2/dr = (\rho_2/r)(1-\rho_2/\rho_1)$], which is still different from equation (5) (constancy of volume), and which is a differential equation giving the geometrical relationship between the two principal radii of curvature. It should be satisfied by every axisymmetrical shell of this type. It would be interesting to know whether the results obtained were checked with this formula.

The terms in $(\sigma_r t)$ in equation (6) are multi-valued and there are more than one combinations of $\sigma_r$ and $t$ that will satisfy equilibrium.

It is therefore necessary that the solution which gives the minimum plastic work should be chosen, which is particularly important at larger strains and near the clamped edge. The variables, using the method of successive approximations and equation (11) in such critical cases, may not converge to the right values without further discrimination.

It would also be interesting to know whether the author obtained any results beyond the maximum pressure in bulging for annealed materials. In previous researches (e.g. Brown *et al.*, Ref. 12) it has been found that fracture can take place at increasing, or decreasing pressure after reaching a maximum. Could this solution be applied to elucidate this phenomenon?

These arguments are based on the extensive theoretical and experimental research conducted at Imperial College between 1961–64, of which the first part of the experimental work was published by Kaftanoğlu and Alexander.† A comprehensive theory for the solution of the axisymmetrical sheet metal deformation problems, including hydraulic bulging, is developed to predict all the variables involved, until, and at the onset of fracture in the stretch forming region. It includes the effects of plastic anistropy, friction, non-linear work hardening, prestrain, thickness stress, non-uniform initial conditions, and the complete strain history of the elements. Numerical solutions involving higher order differences in the differential equations (non-successive approximation method) have been obtained on the Atlas computer, mainly for stretch forming with a punch, similar to Erichsen test, and various instability criteria have been studied. Good agreement was obtained between theory and experiment. It is expected to publish this work in the very near future.

I would like to acknowledge Professor J. M. Alexander for his helpful discussions on the points mentioned above.

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* D. M. Woo, *Int. J. Mech. Sci.* **6**, 303 (1964).

† B. KAFTANOĞLU and J. M. ALEXANDER, *J. Inst. Met.* **90**, 457 (1961–62).

# Mechanical Anisotropy in Sheet Metals

Dr. D. V. WILSON (Birmingham University), in an introductory lecture, reviewed our present understanding of the relationships between preferred orientation, plastic anisotropy, and practical performance and identified those aspects of the problem which needed clarification. The relationships between plastic anisotropy and deep-drawing performance were understood in general terms, although more detailed attention should be given to the effects of planar variations in properties and to applications in asymmetric drawing operations. In contrast, the understanding of the relationship between preferred orientation and stretch-forming performance was still fragmentary. In particular, there was a lack of information on the orientation-dependence of work-hardening in textured sheets. More experimental investigation was required, particularly on the effects of texture and grain size on flow strength, work-hardening, and fracture behaviour, studied under a variety of applied-stress systems. Even when this information was available, its application to industrial forming operations might often present formidable analytical problems. Efficient progress in realizing the potential benefits of controlled mechanical anisotropy demanded a combined operation over a broad field of engineering and metallurgy.

Dr. M. A. ADAMS (Steel Company of Wales) introduced Session I, dealing with producer and consumer requirements for anisotropic sheet metals. An ideal specification for sheet metal would involve the development of a low through-thickness strength during rolling, as this reduced the rolling load, and the subsequent development of a high through-thickness strength during the final annealing treatment, as being beneficial in deep drawing. It so happened that correctly processed stabilized steel approximated to these requirements, since the rolling texture contained a major component of "cube-on-face" texture, {100}⟨011⟩, which was associated with low normal strength on average strain ratio, $\bar{R}$. On annealing, the texture consisted mainly of "cube-on-corner" type, with {111} planes parallel to the surface and associated with high $\bar{R}$. Many cubic metals also exhibited a high degree of elastic anisotropy and ⟨111⟩ was usually the direction in which Young's modulus was a maximum. Thus, the development of cube-on-corner texture conferred an additional benefit as it increased the elastic stiffness of the sheet.

Mr. R. PEARCE (College of Aeronautics, Cranfield) reported some preliminary results on the effects of plastic anisotropy in stretch-forming. Two materials, zirconium and aluminium, with similar mechanical properties except in respect of their $\bar{R}$ values, which varied from 9·4 for zirconium to 0·6 for the aluminium alloy, had been tested. Sheet thicknesses were chosen to give similar bulge heights and sections through the bulged sheets showed clearly a very marked difference in strain distribution. The thickness strain in the polar regions was much greater in the aluminium sheet than in the zirconium sheet. Mechanical properties of a component produced by stretching could therefore be significantly affected by the $\bar{R}$ value of the sheet from which it was made. Mr. Pearce also mentioned the effectiveness of small quantities of various elements in producing changes in the plastic anisotropy of steel sheet, and stated that further elucidation of the manner in which these trace elements affected texture formation was required.

Mr. R. JONES (Steel Company of Wales) gave a detailed account of the interaction between process variables and plastic anisotropy as revealed by strain-ratio measurements in commercial low-carbon steel sheet. Most of his observations confirmed results published by Whiteley and Wise;[1] some differences in earing behaviour could probably be attributed to differences in impurity content of the steels, from the two different producers. Certain unusual features were observed in pole figures of hot-rolled steel. The texture at the surface was best represented by {110}⟨011⟩, while at the centre it could be described as {100}⟨011⟩. As the finishing temperature was allegedly high and the material fully recrystallized, the development of pronounced textures following the complex metallurgical sequence involving working, annealing, and phase transformation was difficult to understand. No variations in texture through the thickness of sheet had been observed after cold rolling.

Mr. B. KAFTANOGLU (International Computers and Tabulators, Ltd.) mentioned a theoretical approach adopted at Imperial College for the prediction of stretch-forming performance in anisotropic sheet materials. The approach was presumably based on the Hill macroscopic theory. Results obtained with the aid of the Atlas computer showed that a high $\bar{R}$ reduced the fracture strain and bulge depth. To get a deeper bulge in stretch forming, whether by a rigid punch or hydraulic pressure, $\bar{R}$ must be low, assuming that other parameters such as friction and work-hardening remained the same. This was, of course, in contrast to the situation in deep drawing with a flat punch, when high $\bar{R}$ was associated with great depth of draw. The intermediate process of deep drawing with a punch with a generous profile would be governed by the two opposing effects and a high $\bar{R}$ might not necessarily result in a deeper draw.

The method of dealing with variations of $R$ in the plane of the sheet by taking an arithmetic mean of individual $R$ values gave a result biased towards the minimum of the individual ones.

Mr. J. WILLIS (Aluminium Laboratories Limited) considered deformation in the flange in the deep-drawing process. Mr. Pearce had previously suggested that material with high $\bar{R}$ would have less tendency to buckle, but advantage of this fact could be taken only when the pressure-plate loading could be reduced. For a press operating under conditions of constant clearance, high $\bar{R}$ values meant an increased resistance to deformation in the flange which would reduce the deep-drawing ratio that could be obtained.

Session II was concerned with the macroscopic approach to plastic anisotropy. Dr. J. F. WALLACE (Pressed Steel Co., Ltd.) introduced the subject by considering the effect of anisotropy on instability in sheet-metal forming. The treatment was based on Hill's macroscopic theory of plasticity, and it was further assumed that fracture occurred by necking in a band perpendicular to the maximum principal stress. The equivalent strain at which instability was reached was calculated from the critical subtangent, the intercept on the strain axis made by a tangent to the stress/strain curve at the point where the increase in equivalent stress corresponding to a small increase of strain was just equal to the stress increase required to satisfy the yield criterion. It was then possible to calculate the variation of uniform elongation with strain ratio, $R$, for various stress systems. For uniaxial tension, a low value of $R$ increased elongation, while for a biaxial tensile stress system, uniform elongation increased with $R$. For plane strain, a small increase in elongation accompanied increasing $R$, but the effect on the load-carrying capacity of a section might be significant. In the analysis of deep drawing of a cylindrical cup, the increase of fracture load due to increasing $R$ outweighed the increase in drawing load, and so a deeper cup could be drawn.

Dr. P. B. MELLOR (Birmingham University) outlined some work being carried out by Mr. Bramley and himself, aimed at checking the validity of the macroscopic theory of plasticity developed independently by Dorn[2] and by Hill.[3] Plastic anisotropy in four different stabilized steel sheets had been assessed by careful measurements of the strain rates $R$ at intervals of 10° in the plane of the sheet, and by observing the plastic-flow behaviour at the centre of a

Joint discussion held on 10 March 1965, at the Spring Meeting in London of the Institute of Metals. The following papers are relevant to the discussion:
E. R. Petty (*J. Inst. Metals*, 1962–63, **91**, 54); P. G. Partridge and E. Roberts (*ibid.*, 1963–64, **92**, 50); E. Hirosawa (*ibid.*, p. 78); I. L. Dillamore and W. T. Roberts (*ibid.*, p. 193); G. G. Moore and J. F. Wallace (*ibid.*, 1964–65, **93**, 33).

appeared that jets from the wave crests had been projected into the stainless steel.

Use of the electron-probe micro-analyser to measure the intensity of the K2 spectral lines of chromium, nickel, and iron, enabled the concentration of these three elements to be measured in different parts of a jet. Scanning X-ray micrographs showed an overall picture of the distribution of these elements in a jet and its surrounding material. The jet was found to be about 50 per cent stainless steel at its centre, which increased to over 60 per cent at the jet tip. Material of composition between these limits was found in all the variously shaped jets adjacent to the mild-steel crests, and in some cases, completely filled the trough between. From this it follows that actual melting of the surface layers and alloying must have occurred, despite the extremely short duration of the operation. It would appear, therefore, that welding was caused by these series of molten zones which varied in width but averaged about 60 $\mu$.

I should like to make a comment on one other point: the experiments the authors did on a hemispherical weld where they showed that with increasing angle of inclination the amplitude and wavelength increased. I have done the same thing on the flat (i.e. at a constant angle of inclination) and I find that, as distance from the point of initiation increases, there is an increase in amplitude and wavelength. I notice the authors have some samples which show the same effect. I do not know if they have seen any of the American work. The Americans have taken X-rays during the work process which have shown that the metal plate does collapse at a different angle to its original angle. I wonder if you could decide whether the change in wavelength is really a function of angle of inclination, or whether it was merely due to the increasing distance from point of initiation, which would cause an increase in wavelength.

**Mr B. Kaftanoğlu,** B.Sc. (Eng.) (*Student*)—My original intention was just to make a brief comment on the paper (No. 2) by Mr Vafiadakis, Professor Johnson, and Dr Donaldson. The authors computed the energy dissipated in plastically deforming a blank from the experimental strain distributions and the representative static stress–strain relation. This method was also used in Paper 1, and it requires laborious calculations and does not take into account the effect of strain rate which might slightly influence the results.

Consider the energy balance of the system shown in Fig. 30 of Paper 2. The loss of potential energy of the hammer = energy dissipated in plastically deforming blank + heat energy gained by water and surrounding tools + frictional losses + acoustical energy losses + energy absorbed in rebound of hammer. The last three items can either be individually assessed or neglected because they demand a very small proportion of the energy involved. If it is possible to make instantaneous and accurate measurements of the temperature in the tools and water then the item, the heat energy gained, can be determined. Hence the unknown second item can be estimated from the known first. Such a method would not require any knowledge of the stress–strain relations of the materials used, and would therefore be independent of strain-rate effect. Since this method depends on reliable temperature measurements, I should like to know if the authors think it is feasible.

At Imperial College we have been working on the problem of stretch forming with rigid tools similar to those used in the Erichsen test. A comprehensive theory for the solution of axisymmetrical sheet-metal deformation problems has been developed. It includes the effects of plastic anisotropy, friction, nonlinear strain hardening, and prestrain. Complete strain history has been taken into account, and strains and stresses up to and at the onset of fracture have been calculated by a programme developed for the Atlas computer. Good agreement is obtained between theory and experiment. It is seen that friction significantly influences the stress and strain distribution in the stretch forming region and the point of fracture moves away from the pole with increased friction. Our results for strain distributions with the lowest coefficient of friction and those presented in this paper for free forming show similar trends, and in both the fracture points are located near the pole.

**Mr A. S. Bahrani,** M.Sc. (*Associate Member*), and **Professor B. Crossland,** D.Sc. (*Member*)—We* should like to make the following comments about the first paper, by Professor Johnson and Mr Sowerby.

(1) While the work of Daniel Adamson (81) and Charles E. Munroe (1) on the forming of metals by explosives is often quoted and recognized, the pioneer work of Walter Claude Johnson (82), which is more relevant, is very seldom mentioned.

Johnson, who was an engineer from Westcombe Park, Kent, obtained in 1897 Patent No. 21840 in which he described 'a method of generating fluid pressure for tube jointing, or embossing, stamping of metal sheets and such-like operations'. Fig. 115, which is reproduced from the above-mentioned patent, illustrates Johnson's ideas. Fig. 1 in this reproduction illustrates the process applied to making joints in a cycle frame and Fig. 2 shows the method applied to forming or embossing sheet metal into a die. When the explosive charge C is fired, the projectile D is acccelerated and it strikes the ram B, thus setting up pressure in the liquid above it. In Fig. 2 the mould E is shown holding a die H into which the sheet metal blank F is forced. Johnson also stated that 'it is obvious that by inverting the arrangement shown in Figs 1 and 2 and withdrawing the projectile D and ram B the explosive charge may if desired be allowed to act directly upon the surface of the liquid'.

Thus it can be seen that Johnson had the essential ideas of explosive forming, i.e. using an explosive charge to create a pressure wave in a liquid and using this pressure

* *For convenience the separate contributions of Mr Bahrani and Professor Crossland have been made by them into a joint contribution.*