The "CUBIC LAW OF STOCK RETURNS" IN EMERGING MARKETS

Zhiye Gu[1]
Imperial College Business School

Rustam Ibragimov[2]
Imperial College Business School and Innopolis University

ABSTRACT

Excess volatility in main emerging and developed stock markets is carefully analysed in this study. Tail distribution of returns of both stock market index and individual stocks is evaluated and compared with the theoretical distribution found by Gabaix et al. (2003, 2006). For stock market index, recursive and rolling estimation are used. In recursive estimation, we find that all the developed markets obey "the Cubic Law of Stock Returns", while most of the emerging countries exhibit heavier tail with a tail index lower than 3 at 95% significance level. In rolling estimation, the tail index in the developed markets does not stabilise around 3, and after 2008 financial crisis, all the developed markets and most emerging ones suffer a drop in the tail index. For individual stocks, the tail distributions of stock returns, trading volume, and the number of trades in each emerging country behave quite differently from the theoretical model by Gabaix et al. (2006), especially the stock returns.

Key Words: Tail Index, Emerging Stock Markets, Stock Market Volatility

[1]zhiye.gu15@imperial.ac.uk
[2]Corresponding author, irustam@imperial.ac.uk Research of R. Ibragimov was supported by a grant from the Russian Science Foundation (Project No. 16-18-10432).

Highlights

- We present detailed empirical analysis of heavy-tailedness in emerging markets
- Heavy-tailedness properties are compared with those in developed markets
- All the developed markets appear to obey the “cubic law of stock returns”
- Many emerging markets have heavier tails with a tail index lower than 3
- Heavy-tailedness of other financial variables differs from developed markets
- The degrees of heavy-tailedness exhibit structural breaks due to 2008 crisis

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1 INTRODUCTION

1.1 Heavy Tail in the Financial Market

The distribution of financial variables has been a key interest of research for a very long time and it provides theoretical foundations in the area of asset pricing and risk management. As Mandelbrot (1963) and Fama (1963) find that the distribution of stock returns exhibit heavier tail than normal ones and large crashes frequently happen in the financial markets (e.g., Black Monday in 1987, Asian crisis in 1997, Subprime housing crisis in 2008), alternative distribution for stock returns is under high demand to fit the volatility. For instance, leptokurtic distributions such as stable Paretian (Mandelbrot,1963; Fama, 1965) and Student's t distribution (Blattberg and Gonedes, 1974) have gain some popularity for some time. Recently, fat-tailed distribution also help solve some financial problems, such as equity premium puzzle explained by Barro (2006) that is based on rare economic disasters events. Since more characteristics of the tail behaviour of stock returns are discovered (Dumouchel, 1983; Jansen and de Vries, 1991; Longin, 1996), Extreme Value Theory (EVT) has been gradually applied in the tail distribution analysis. Emerging from the EVT, power law is widely adopted as an appropriate tail distribution for economic and financial variables.[1] Power law has a counter-cumulative distribution function like following (Newman, 2005):

$$Pr(X > x) = \frac{C}{x^{\alpha}}$$

α is the tail index, which measures the thickness of the tail. Under power law distribution, the tail does not decays exponentially but by a power measured by the tail index, and the smaller the tail index is, the heavier the tail.

Many empirical studies conclude that the tail index of financial returns in developed markets lies in the interval (2, 4) (see Jansen and de Vries, 1991; Loretan and Phillips, 1996; Lux, 1996; Gopikrishnan et al., 1999; Plerou et al., 2000; Gabaix et al., 2003, 2006; Ibragimov et al. 2013, 2015; Ibragimov and Prokhorov, 2017, and references therein). Gabaix et al. (2003, 2006) further argue that, in the developed financial mar-

[1] A detailed review of the history of power law and its successful applications can be found in Gabaix (2009). As he summarises, power law has been widely used in social science such as income distribution (Cowell and Flachaire, 2007), the growth rate of city size (Ioannides and Overman, 2003), Internet networks (Newman et al., 2006), and supply of regulations (Mulligan and Shleifer, 2005).

kets, the tail index of stock returns is very close to 3 and develop a theoretical model explaining this empirical regularity that they call "the Cubic Law of the Stock Returns". At the same time, only a few studies have dealt with the analysis of heavy-tailedness of key variables in emerging economic and financial markets and, in particular, the analogues of Gabaix et al.' (2003, 2006) models for them (see Ibragimov et al. 2013; Section 3.2 in Ibragimov et al., 2015, the discussion in these works). As many fundamental differences exist between the developed and emerging markets, the above theoretical models may not fit the emerging markets quite well. In this paper, we comprehensively evaluate heavy-tailedness properties of stock market indices and individual stocks in major emerging markets from different regions. Comparisons of the obtained empirical results with those available in the literature for developed markets imply that heavy-tailedness properties of stock returns in most of emerging markets considered are more pronounced than in their developed counter-parts. Importantly, the tail indices of stock returns in emerging financial markets are found to be statistically significantly smaller than 3. Thus, the theoretical model introduced by Gabaix et al. (2003, 2006) does not hold in most of the emerging markets.

This paper covers more emerging countries and longer sample period than other previous researches. Further, it focuses on empirical analysis of analogues and modifications of the model by Gabaix et al. (2003, 2006) for tail distribution of emerging markets' stock returns, with estimates of heavy-tailedness properties presented for key transaction variables in the model, including the trading volume and the number of trades. The results contribute to the literature in risk management, finance and the development of emerging markets. They may help local and international investors better assess the investment risk in the emerging stock markets, and further have a number of important implications for key economic and financial models in them.

1.2 Literature Review

The studies by Gabaix et al. (2003, 2006) develop a model that links and explains heavy-tailedness properties of several key variables in developed financial markets. More precisely, the model developed in the studies links the empirical findings that the tail indices of stock returns in developed markets are typically very closes to 3; the stock trading volume has the tail index of 1.5; and the number of the trades has a tail index of

3.

1.2.1 Heavy-Tailedness in Developed Markets

For the developed financial markets, based on ten stocks from S&P 500 and two other stock indices (S&P 500 and UMI) in the U.S. stock market from 1962 to 1986, Jansen and de Vries (1991) find the tail index of stock returns around 3 after accounting for institutional changes on financial markets.[2] Loretan and Phillips (1994) obtain estimates of tail indices between 2 and 4 for stock indices and exchange rates in major developed economics. After analysing 30 most frequently traded German stocks from the DAX and DAX index itself from 1988 to 1994, Lux (1996) rejects the stable Paretian distribution for stock returns and concludes that tail indices of the stock returns lie within the interval of (2,4) and particularly around 3 in most cases.

Gopikrishnan et al. (1999) analyse the tail distribution of normalised stock returns for the U.S., Tokyo, and Hong Kong stock exchange under the time scale from 1 minute to more than 1 month. They find that the tail index is around 3 when the time scale is equal or smaller than 4 days.[3] Besides, Gabaix et al. (2005) find that even after accounting for major financial crisis such as in 1929 and 1987, the daily stock returns still follow power law distribution.

In terms of the trading volume, Gopikrishnan et al. (2000) find that its tail follows power law distribution with the average index of around 1.5 (1.53$\pm$0.07), and Maslov and Mills (2001) also find a similar value of 1.4 (1.4$\pm$0.1). Furthermore, Gabaix et al. (2006) re-examine the tail distribution of trading volume in New York Stock Exchange, Paris Bourse, and London Stock Exchange and conclude that the tail index is again 1.5 (1.5$\pm$0.1). The tail index of 1.5 for trading volume is referred as "Half-cubic Law of the Trading Volume" by Gabaix et al. (2006). The "Half-cubic Law of the Trading Volume" is a key ingredient in the model of Gabaix et al. (2003, 2006) that explains "the Cubic Law of the Stock Returns" in developed markets, together with the square root price impact relation, $r = \Delta p \simeq kV^{1/2}$ for large values V (here and throughout the paper, r denotes log-returns, Δp stands for the change in the logarithm of the stock price,

[2] On April 26, 1973, the Chicago Board Option Exchange was organised; the Bretton Woods agreement was officially terminated this year; and oil shocks happened on this year as well.

[3] Gopikrishnan et al. (1999) also explain that the time scale cutoff of 4 days results from the time dependence of average volatility of returns.

and V denotes the trading volume). The latter is a linear relationship between squared return and trading volume found by Hasbrouck (1991), Barra (1997), Grinold and Kahn (1999), and Hasbrouck and Seppi (2001). Naturally, heavy-tailedness properties of the trading volume are related to those of the number of trades. In turn, according to Plerou et al. (2000), the number of trades follows power law distribution with the average tail index of around 3.4 for 1000 U.S. stocks over the period from 1994 to 1995.

1.2.2 Heavy-Tailedness in Emerging Markets

Ibragimov et al. (2013) find that distributions of foreign exchange rates in the emerging countries is more pronouncedly heavy-tailed than in developed ones, and may even have infinite variance. Related studies on heavy-tailedness of foreign exchange rates in emerging markets are provided by Akgiray et al. (1988), Koedijk et al. (1992), and Candelon and Straetmans (2006). Focusing on emerging stock markets, Jondeau and Rockinger (1999) conclude that nearly all the emerging countries' stock indices considered in the paper have finite first and second moments, with a few of them also having finite third moments. The authors also argue that due to the complicated evolution of the emerging markets, there exist inherent instability in estimates of tail indices in emerging markets while tail index estimates are more reliable for the developed ones. Quintos et al. (2001) provide a detailed analysis of heavy-tailedness properties of major stock indices in Thailand, Malaysia, and Indonesia from January 1995 to October 1998 using rolling, recursive, and sequential tail index estimation methods. The empirical results obtained point out to structural breaks in the tail indices and an increase in their heavy-tailedness after those countries have changed their pegged exchange rate system to a floating one following the 1997 Asian Financial Crisis, with the pre-break tail index values around 2.75 decreasing to around 1.75 in the post-break period. Focusing on the behaviour of extreme returns of 20 global stock market indices, Jondeau and Rockinger (2003) conclude that each geographical group generates different tail indices and heavy-tailedness properties of Asian financial markets are especially more pronounced compared to those in their developed counter-parts. Similarly, LeBaron and Samanta (2004) find that emerging markets do systematically have heavier tails than developed ones, and similar conclusion is also reached by Gregoriou (2010).

2 DATA

In this paper, all the data including stock market index and stock transaction data (stock price, stock trading volume, and number of trades) are on a daily basis. Also, log return, $\log(P_{t+1}/P_t)$, is used for all the markets. All the data are from the Bloomberg Terminal. The stock market indices are from different developed and emerging regions. Emerging and developed markets are chosen from IMF and MSCI Emerging Market Index.[4] The developed markets and their indices (in parenthesis) include: Japan (Nikkei 225) in Asia, the Untied States (S&P 500) in North America, the United Kingdom (FTSE100) and Germany (DAX) in Europe. The emerging ones are: China (SHCOMP), India (SENSEX), Indonesia (JCI), Malaysia (FBMKLCI), the Philippines (PCOMP), South Korea (KOSPI), Taiwan (TWSE), and Thailand (SET) in Asia; Chile (IGPA), Poland (WIG), Russia (INDEXCF), and Turkey (XU100) in Europe; Brazil (IBOV) and Mexico (MEXBOL) from Latin America; and finally South Africa (JALSH) from Africa (See the summarised table in Supplementary Material Section 2). The sample period for all the selected countries is from the beginning of 1995 to the end of May in 2016, except for Russia whose stock market data are available only from September 1997.

Individual stocks are chosen from each stock market index in the U.S. and chosen emerging markets, and we use the U.S. market as a reference market. The transaction data, including daily stock return, daily stock trading volume, and daily number of trades, are from all the historical individual company members that had been listed in the index since 1995. The time period again covers from the beginning in 1995 to May 2016. As an example of Brazil, the historical list of membership companies in IBOV Index is available from May 1998; then, all the historical members in each month from May 1998 are recorded, and redundant companies will be deleted; for each unique company, all the transaction data from the beginning of 1995 to May 2016 will be collected. The availability of dataset varies a lot among different markets, for example, the earliest date available to assess the member companies listed in the index is May 1998 for

[4] As of Febuary 2015, IMF includes Argentina, Bangladesh, Brazil, Bulgaria, Chile, China, Colombia, Hungary, India, Indonesia, Malaysia, Mexico, Pakistan, Peru, the Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Ukraine, and Venezuela as emerging countries. The source is available at: http://www.imf.org/external/pubs/ft/weo/2015/02/pdf/text.pdf. As of July 2016, MSCI Emerging Market Index consists of following emerging markets: Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, South Korea, Malaysia, Mexico, Peru, the Philippines, Poland, Russia, South Africa, Taiwan, Thailand, and Turkey. The source is available at: https://www.msci.com/market-cap-weighted-indexes.

IBOV Index in Brazil, and the latest one is July 2009 for FBMKLCI Index in Malaysia. However, no data are available for member stocks from INDEXCF in Russia and IGPA Index in Chile.

3 METHODOLOGY

It is known that power law has the following counter-cumulative distribution function:

$$Pr(X > x) = \frac{C}{x^{\alpha}} \tag{1}$$

for all $X \geqslant x_{min}$, *the minimum value of X*

For each side of the tail distribution, equation (1) can be written as:

$$P(X > x) \sim \frac{C_1}{x^{\alpha_1}} \tag{2}$$

$$P(X < x) \sim \frac{C_2}{x^{\alpha_2}} \tag{3}$$

The whole tail distribution can be written as:

$$P(|X| > x) \sim \frac{C}{x^{\alpha}}, \textit{ with } \alpha = \textit{min}(\alpha_1, \alpha_2) \textit{ as } x \to \infty \tag{4}$$

Thus, from equation (4), (2), and (3), the parameter α stands for the tail index for the whole tail distribution, α_1 is for the right tail, and α_2 refers to the left tail respectively. If $\alpha_1 \neq \alpha_2$ statistically, a tail asymmetry occurs.

The tail index is also defined as the maximal order of finite moments of the interested variable. For example, variable x has finite variance when $\alpha_x > 2$, and has infinite kurtosis or higher orders when $\alpha_x < 4$ and so forth (see Supplementary Material Section 1). Normally, α is greater than 1, with finite mean; when $\alpha \simeq 1$, it turns out to be a special case of power law distribution, which is Zipf's law.[5] Thus, the main advantage of the power law is that it can nest different distributional alternatives characterised by

[5]Zipf's law is a distributional model firstly used to describe local growth in city size (Zipf, 1949), then it has been a model applied in economics, or in general social science.

different tail behaviour into a single semi-parametric tail index estimator.

To estimate the tail index α in equation (4), we use Hill estimation with following steps. Firstly, the total sample N is arranged in an ascending order, which is:

$$|x_{max}| \geqslant |x_2| \geqslant |x_3| \geqslant |x_4| ... \geqslant |x_{N-1}| \geqslant |x_{min}|$$

A subsample of n observations is chosen from the upper tail in ordered sample N:

$$|x_{max}| \geqslant |x_2| \geqslant |x_3| \geqslant |x_4| ... \geqslant |x_n| \tag{5}$$

$n = \kappa N$, *while* κ *stands for the truncation*

Then, then Hill estimator is following:

$$\hat{\alpha}_{Hill} = \frac{n}{\sum_{i=1}^{n}(log|x_i| - log|x_{n+1}|)} \tag{6}$$

The corresponding standard error of $\hat{\alpha}_{Hill}$ is $s.e._{Hill} = \frac{\hat{\alpha}_{Hill}}{\sqrt{n}}$, and the 95% C.I.:

$$\left(\hat{\alpha}_{Hill} - 1.96\frac{\hat{\alpha}_{Hill}}{\sqrt{n}}, \hat{\alpha}_{Hill} + 1.96\frac{\hat{\alpha}_{Hill}}{\sqrt{n}}\right)$$

3.1 The Trucation κ

From equation (5), the truncation κ is defined as the ratio of the number of tail observations to the number of total observations. The choice of the optimal κ is controvertible. If large truncation level is used, one may contain too many observations that should have been located in the centre of the distribution rather than in the tail, and deviating too much from the tail would lead downward bias in estimation. However, if too small truncation is used, the estimator would be much sensitive to the outliers and too few observations would also generate biased result. Various papers have discussed the choice of optimal truncation. Hall (1982) suggests that the optimal selection should depend on the properties of the tails; DuMouchel (1983) finds that the optimal truncation should be fixed at 10% of the total sample size; Jansen and de Vries (1991) mention that the bias of the standard error in tail estimation is majorly due to the inclusion of centre characteristics; Danielsson and de Vries (1997) suggest to use 1% of the sam-

ple; Jondeau and Rockinger (1999) conclude that the optimal truncation depends on different tail distributions, and they obtain optimal truncation for each different region: from 5% to 7.5% for the developed regions, and 12.5% to 15% for the emerging ones; McNeil and Frey (2000) test 1000 bootstrap samples with each sample size of 1000 and conclude that the number of 100 generates low bias.

In this paper, 7.5% and 15% truncation levels are chosen as the optimal one for the developed and the emerging markets separately for following reasons. Firstly, as suggested by Jondeau and Rockinger (1999), 7.5% and 15% is accepted as optimal for the developed and the emerging markets respectively; secondly, according to the Hill plot (in Supplementary Material Section 3), for the developed markets, the tail index stabilises at 7.5% level, and for the emerging ones, the tail index for most of them stabilises at around 15%.[6] The result is also consistent with the conclusion by Hall (1982) that the optimal truncation level depends on the property of each tail distribution, and agrees with the finding by Jondeau and Rockinger (1999) that more number of observations should be included as the tail for emerging countries due to the unstableness in their tail.[7]

4 EMPIRICAL ANALYSIS

4.1 Tail in Stock Market Index

We start with tail index analysis with 95% confidence interval graphically to illustrate the tail behaviour of each country with different aspects including the tail asymmetry, tail stability, and tail heaviness. First of all, to choose the optimal truncation κ and to trace the tail estimator for accuracy, we implement Hill plot for each country. Furthermore, we use both recursive and rolling methods in tail index to assess the change in tail distribution of the emerging and developed markets, and analyse both tails in detail. For each method, the period covers from the beginning of 1995 to May 2016, except for Russia starting from 1997 due to data unavailability before that time.

[6] Gumbel (1958) states that the optimal level is based on the stability of corresponding tail index when the tail index becomes roughly linear in the plot.

[7] In Jondeau and Rockinger (1999)'s paper, they find that the optimal number of observations in tail varied for different regions. For example, in terms of the left tail, the average optimal tail size for developed countries is 207 out of 3913; for Asian country, it is 369 out of 4855; for emerging countries in Europe, the optimal is 506 out of 1358; and for emerging ones in Latin America, the value is 201 out of 2064.

4.1.1 Hill Plot

We conduct Hill plot for the whole tail to show how tail index changes against increasing truncation level and then to choose the optimal truncation. Generally, tail index behaves inversely with the increasing truncation level because more observations are included as the tail. In tail estimation, the principle is that the optimal truncation level should be as small as possible to avoid the large bias due to the inclusion of observations from central part.

We use all the sample data in Hill plot to analyse the historical distribution corresponding to different truncation levels ranging from 2.5% to 15%, with 95% confidence interval. As shown in Supplementary Material Section 3, Hill plots behave quite differently between the developed and emerging markets. In the developed markets, the tail index of all the developed markets stabilises at the truncation level of roughly 7.5%, and some of them, such as in the U.S. and U.K. markets, suddenly decrease after the truncation level exceeds 7.5%, indicating the optimal truncation level for the developed markets. At 7.5% truncation level, "the Cubic Law of the Stock Returns" holds statistically at 95% significance level for Germany, the U.K. and U.S., but the tail index in Japan is slightly larger than 3.

For emerging countries, most of the tail indices decrease slowly with truncation level and stablise when the truncation level reaches 15%. Statistically, at 15% truncation level, all the tail indices in the emerging markets are smaller than 3. Thus, the "Cubic Law of the Stock Returns" is rejected at that point.

4.1.2 Recursive Estimation

We use recursive estimation to analyse the historical tail distribution over the sample period for each market. The initialisation window is 5 years, and one single observation is added into the window until the final day. As shown in Supplementary Material Section 4, the first tail index in 2000 covers observations in the past 5 years , and it will keep changing when the window keeps expanding. Tail index in either tail and whole tail with 95% confidence interval is shown for each market. One of the major features in recursive tail estimation is that once extremal events happened, the extremal values would be carried into next windows and remain as the extremes unless more extremal

values appear when the window is expanding. Thus, the recursive tail index can only indicate the overall tail distribution over the history but cannot accurately reflect whether the tail distribution is becoming less heavy-tailed with time.

Firstly, we will discuss some characteristics of the tail index, such as its asymmetry, stability, and also heaviness, in the developed countries. For tail asymmetry, we only analyse one-side tail, and the whole tail for the aspects of stability and heaviness. Graphically, all the developed countries share similar features in the movement of the recursive tail index. There are not huge differences between the left and right tail for all of them. However, their tails do not exhibit long-term stability. All the tail indices come across a sudden drop in late 2008, and the tail index in markets such as Japan and the U.S. falls from significant 3.5 to 3 at 95% level. After the drop, all the markets maintain a stable tail index around 3, and the cubic law holds statistically for all the developed markets. The sudden drop around late 2008 can be explained by the stock market turmoil after the well-known 2008 subprime financial crisis occurs in the U.S., which has a contagion effect on other stock markets that band with the U.S. market. The reason for why the tail index stabilises until the end after the crisis is that the extremes happened during the crisis remain as extremes so that such tail heaviness is kept until the end.

For the emerging markets, in either tail and the whole tail, the tail index for the U.S. market is used as a reference line for comparison, shown as the red line in Supplementary Material Section 4. Regarding of the tail asymmetry, most of them do not have significant tail asymmetry. In the Asian region, in India, the estimated left tail index is lower than 3 significantly for most of the time, but the right one is above 3 significantly before around 2003; similarly, Thailand has left tail index lying around 3 before 2003, but its right tail is below 3 significantly all the time. In the European region, the left tail in Chile is heavier consistently. The tail behaves quite differently as the left tail index exhibits heavier tail.

In terms of the tail stability, unlike in the developed markets, the tail index in all the emerging markets except for South Africa tends to be stable without suffering any big change during the financial crisis period. This indicates either that the financial crisis does not affect emerging markets too much compared to other developed countries or that the negative shock caused by the financial crisis is severe but still relatively small

compared to the local historical negative shocks in emerging markets that have been already carried over. Thus, further analysis needs to be conducted by rolling method.

For tail heaviness, in Asia, the tail index of many countries including China, Indonesia, Malaysia, the Philippines, South Korea, and Thailand is 95% significantly smaller than 3 during the whole time period; for markets of India and Taiwan, they have tail index around 3 during some time, but most of the time, the law does not hold and they have a heavier tail. In Europe, Latin America, and Africa, all the markets have tail index lower than 3 at 95% significance level all the time, and especially, Russia's tail index is even smaller than 2.5.

4.1.3 Rolling Estimation

Rolling estimation method has one advantage that it can help to analyse whether the tail during a certain window is changing over the sample period. The rolling starts from the beginning of 1995 until the end of the sample period and the rolling window size is fixed at a period of 5 years, which contains roughly 100 observations for each emerging market (15% truncation level) and 50 observations for each developed market (7.5% truncation level). The rolling step is 1 trading day. Thus, each point over the period stands for the tail distribution for its past 5 years. However, similar to recursive estimation, when extreme values occur, they would also be carried over but only for next 5 years. For some time, one may observe that when there is a sudden big drop, a sudden increase will occur immediately after 5 years, but this situation may be not caused by any particular economic event happened at that time point but by the exclusion of the extreme values after the rolling. Thus, because power law distribution only focuses on extremal values, only decrease in the tail index will be focused in rolling method. The figures of rolling estimation are shown in Supplementary Material Section 5.

There is no obvious tail asymmetry in the developed markets. For tail stability, Germany, Japan, and the U.S. exhibit similar movement in the whole tail index. The index keeps stable before 2008 and suffers a drop in late 2008 due to the financial crisis. For the U.K., the tail index firstly rises from 3 to around 4 in 2002, but keeps decreasing until 2009. Then, after the crisis, the tail index for all of the markets keeps stable around 3 for 5 years until late 2014 because of the financial crisis. Afterwards, due to the exclu-

sion of the extremes, the index suddenly increases. After 2014, for the whole tail index, Germany, Japan, and the U.K. have the tail with index of 3.5 roughly, and the U.S. has tail index stable around 3. For most of the time, cubic law cannot be rejected for those four markets at 95% significance level.

For the emerging markets, in terms of the tail asymmetry, except for markets including Indonesia and South Korea in Asia, Chile and Turkey in Europe, and Mexico in Latin America, the left tail moves quite differently from the right one. For stability, except for the Philippines in Asia and Turkey in Europe, the tail index in all the markets behaves quite unstably over the sample period, which is consistent with the conclusion by Ibragimov et al. (2013) that emerging countries are more sensitive to internal and external shocks. The tail index for all the markets except for Turkey comes across a sudden or slow drop when the 2008 financial crisis happens. As expected, for those markets affected by the crisis, the tail index of them except for Poland increases after 2014 when the negative extremes during crisis are excluded.

With respect to the tail heaviness, for all the markets, the cubic law holds significantly for some time during the whole sample period. For some emerging markets such as China, Indonesia, Malaysia, the Philippines in Asia and all the ones in Europe, the tail index is lower than 3 at 95% significance level during most of the time.

Furthermore, Table I above shows the tail index of all the markets for the whole time period, and it is obvious that the cubic law is statistically rejected among all the emerging markets while it still holds in the developed ones. As a result, under both recursive and rolling method, graphically, developed markets do not exhibit obvious tail asymmetry in stock returns, but many emerging markets do. Specifically, we find that either from the view of recursive or rolling method, “the Cubic Law of the Stock Returns” cannot be rejected for most of the time period in the developed markets. However, for emerging markets, based on recursive method, most of them do not obey the cubic law at 95% significance level, and based on rolling method, their tail index is also below 3 statistically for most of the time.

Table I: The Tail Index of the Stock Returns

Tail Index of Stock Returns			
	7.5%/15%	Lower 95%C.I.	Upper 95%C.I.
Country		Whole Tail	
Germany	3.0400	2.7446	3.3353
Japan	3.3253	2.9973	3.6532
U.K.	3.0590	2.7614	3.3566
U.S.	2.9357	2.6494	3.2220
China	2.4020	2.2333	2.5707
India	2.5910	2.4107	2.7712
Indonesia	2.2518	2.0941	2.4094
Malaysia	1.9709	1.8336	2.1083
Philippines	2.4338	2.2635	2.6042
South Korea	2.3378	2.1779	2.4977
Taiwan	2.5737	2.3977	2.7498
Thailand	2.4919	2.3176	2.6662
Chile	2.4356	2.2670	2.6043
Poland	2.5410	2.3648	2.7172
Russia	2.0551	1.9022	2.2080
Turkey	2.4108	2.2439	2.5776
Brazil	2.5759	2.3967	2.7550
Mexico	2.4510	2.2819	2.6201
South Africa	2.5826	2.4018	2.7633

Note: The table shows the tail index for the whole tail of all the markets during the whole time period. For developed markets, 7.5% is chosen as the optimal truncation while 15% for the emerging markets.

4.2 Individual Transaction Variables

The empirical results trigger a more interesting question of whether the theoretical model from Gabaix et al. (2006) only works for the developed markets. Gabaix et al. (2003, 3006) have empirically tested the tail distribution of stock returns, trading volume, and the number of trades for individual stocks. Next, we also analyse individuals stocks in all the emerging countries. To avoid the bias due to small sample size, we delete the companies with less than 300 observations in any transaction variable. In this empirical analysis, we also normalise each variable so that all the variables would have a mean of 0 and a standard deviation of 1. The tables below comprehensively summarise the tail index values in normalised stock returns, normalised trading volume, and normalised number of trades for each market at different truncation levels. Besides, for the tail index of different variables in individual stocks, we also put them into three categories and calculate their corresponding percentage respectively, and those three categories include: 1) the ones whose tail index is smaller than theoretical value at 95% significance level; 2) the ones whose tail index is equal to the theoretical value at 95% significance level; 3) and the ones whose tail index is larger than the theoretical value at 95% significance level. The theoretical value of tail index by Gabaix et al. (2006) is 3 for the stock returns, 1.5 for the trading volume, and 3 for the number of trades respectively. Again, to be consistent with stock market index, for analysis, 7.5% truncation is chosen for the U.S. market and 15% for emerging markets.

Table II: The Tail Index of Normalised Return in Emerging Markets

		Normalised Return										Percentage		
	Truncation	5%		7.5%		10%		12.5%		15%		7.5%/15%		
Country	No.Stock	Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median	< 3	= 3	> 3
U.S.	924	2.9574	2.9711	2.7600	2.7677	2.6075	2.6217	2.4831	2.4942	2.3721	2.3733	40.69%	55.63%	3.68%
China	1097	4.3974	3.7236	3.2605	2.9916	2.9771	2.7043	2.8254	2.5330	2.6496	2.3901	82.13%	9.21%	8.66%
India	91	3.5477	3.4891	3.1044	3.0387	2.8351	2.8234	2.6552	2.6352	2.5008	2.4957	96.70%	3.30%	0.00%
Indonesia	578	2.8566	2.7165	2.5348	2.4327	2.3113	2.2205	2.1326	2.0745	2.0078	1.9552	88.58%	10.55%	0.87%
Malaysia	46	2.8449	2.7259	2.6568	2.5295	2.4852	2.4143	2.3423	2.2659	2.1870	2.1366	93.48%	6.52%	0.00%
Philippines	66	2.9129	2.9556	2.6751	2.6878	2.5173	2.5024	2.3934	2.4096	2.2733	2.2991	95.45%	4.55%	0.00%
South Korea	1167	4.3078	3.3603	3.4462	2.9274	2.8941	2.6429	2.5513	2.4495	2.3418	2.2984	87.32%	9.94%	2.74%
Taiwan	1020	12.8180	10.3970	8.6937	4.8687	5.9059	3.3880	4.3137	2.8312	3.4267	2.5213	64.12%	16.57%	19.31%
Thailand	642	2.6634	2.5547	2.4195	2.3718	2.2374	2.1824	2.1056	2.0790	2.0015	1.9660	94.24%	5.61%	0.16%
Poland	584	3.6262	3.0939	3.5746	2.8233	2.5931	2.6113	2.4429	2.4687	2.3003	2.3193	87.67%	12.16%	0.17%
Turkey	558	3.6479	3.0939	3.6064	2.8181	2.5893	2.6113	2.4441	2.4703	2.3054	2.3274	87.28%	12.54%	0.18%
Brazil	163	3.1238	3.0966	2.9354	2.9930	2.7899	2.8424	2.6503	2.7024	2.5216	2.5857	71.78%	28.22%	0.00%
Mexico	78	3.1219	3.0274	2.9038	2.8979	2.7149	2.7368	2.5531	2.5659	2.4149	2.4393	84.62%	15.38%	0.00%
South Africa	348	2.9281	2.8953	2.6559	2.6675	2.4666	2.4597	2.3086	2.3203	2.1656	2.1830	92.82%	7.18%	0.00%

Note: The U.S. market is used as a reference market as shown in the table. ">3" indicates the tail index is larger than 3 at 95% significance level and so forth.

Table III: The Tail Index of Normalised Trading Volume in Emerging Markets

Volume.png

		Normalised Trading Volume										Percentage		
	Truncation	5%		7.5%		10%		12.5%		15%		7.5%/15%		
Country	No.Stock	Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median	<1.5	=1.5	>1.5
U.S.	924	2.3723	2.3051	2.2646	2.2161	2.1709	2.1409	2.0879	2.0495	2.0123	1.9800	0.32%	22.19%	77.49%
China	1102	3.3201	3.2394	2.9191	2.8811	2.6477	2.6387	2.4530	2.4612	2.2903	2.2881	0.27%	10.89%	88.84%
India	91	2.7452	2.5944	2.5202	2.3768	2.3443	2.1955	2.1956	2.1467	2.0574	2.0307	3.30%	21.98%	74.73%
Indonesia	572	2.4310	2.0865	2.1069	1.8506	1.8973	1.7016	1.7914	1.5960	1.6827	1.5142	12.59%	71.15%	16.26%
Malaysia	46	2.3999	2.4516	2.2540	2.3333	2.1100	2.1544	2.0166	2.1032	1.9342	2.0041	2.17%	30.43%	67.39%
Philippines	66	2.1241	2.0883	2.0448	2.0414	1.9274	1.8907	1.8096	1.8000	1.7440	1.7510	7.58%	43.94%	48.48%
South Korea	1139	2.3229	2.2390	2.0186	1.9919	1.8587	1.8439	1.7402	1.7254	1.6454	1.6409	8.60%	64.44%	26.95%
Taiwan	1013	3.1129	3.0781	2.7049	2.6500	2.4456	2.4154	2.2484	2.2348	2.0877	2.0709	1.68%	22.41%	75.91%
Thailand	643	2.1649	2.0032	1.9082	1.8179	1.7434	1.6968	1.6416	1.5972	1.5565	1.5219	13.69%	70.30%	16.02%
Poland	584	2.2068	2.0440	1.9134	1.8481	1.7619	1.7367	1.6469	1.6464	1.5653	1.5679	8.39%	81.34%	10.27%
Turkey	558	2.1663	2.0440	1.9163	1.8533	1.7662	1.7384	1.6496	1.6471	1.5677	1.5655	7.89%	82.08%	10.04%
Brazil	164	2.5076	2.3638	2.3353	2.2650	2.2257	2.1367	2.1255	2.0488	2.0386	2.0081	3.66%	37.20%	59.15%
Mexico	78	2.0777	1.7454	2.2008	1.7467	1.6896	1.7277	1.6499	1.6146	1.6046	1.5770	11.54%	56.41%	32.05%
South Africa	346	1.9548	1.8713	1.8495	1.7742	1.7599	1.6846	1.6754	1.6167	1.6059	1.5621	8.96%	67.34%	23.70%

Note: The U.S. market is used as a reference market as shown in the table. "$>$1.5" indicates the tail index is larger than 1.5 at 95% significance level and so forth.

Table IV: The Tail Index of Normalised Number of Trade in Emerging Markets

		Normalised Number of Trades										Percentage		
	Truncation	5%		7.5%		10%		12.5%		15%		7.5%/15%		
Country	No.Stock	Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median	<3	=3	>3
U.S.	792	3.5301	3.4232	3.1889	3.0837	2.9252	2.8672	2.7139	2.6984	2.5532	2.5315	9.72%	81.31%	8.96%
China	1072	3.6349	3.5469	3.1621	3.1019	2.8462	2.7817	2.6135	2.5493	2.4173	2.3808	59.70%	40.02%	0.28%
India	89	3.6253	3.4204	3.2820	3.1743	3.0465	2.8954	2.8494	2.6903	2.6726	2.5558	51.69%	33.71%	14.61%
Indonesia	483	2.9873	2.5574	2.4033	2.1836	2.1515	1.9990	1.9892	1.8525	1.8645	1.7138	79.10%	20.10%	0.80%
Malaysia	46	3.0878	3.0115	2.8682	2.7407	2.6717	2.6293	2.5022	2.4826	2.3423	2.3233	69.57%	30.43%	0.00%
Philippines	61	3.1139	3.1016	2.7662	2.6654	2.5593	2.5178	2.3495	2.3240	2.2151	2.2008	77.05%	22.95%	0.00%
South Korea	991	2.7363	2.6176	2.4012	2.3438	2.2055	2.1728	2.0515	2.0155	1.9353	1.9047	85.67%	14.22%	0.10%
Taiwan	910	5.8385	4.3636	5.1520	3.7870	4.8301	3.4202	4.6595	3.0831	4.5956	2.8772	31.54%	44.29%	24.18%
Thailand	579	2.4142	2.3007	2.1770	2.0955	2.0127	1.9388	1.8740	1.8283	1.7801	1.7132	89.98%	10.02%	0.00%
Poland	525	2.5358	2.4216	2.2678	2.2106	2.0933	2.0473	1.9550	1.9375	1.8417	1.8301	93.71%	6.29%	0.00%
Turkey	526	2.5296	2.4192	2.2619	2.2011	2.0894	2.0467	1.9510	1.9333	1.8382	1.8281	93.92%	6.08%	0.00%
Brazil	127	3.6055	3.4892	3.3029	3.2177	2.9950	3.0318	2.7686	2.8320	2.6264	2.6835	35.43%	62.99%	1.57%
Mexico	65	3.4589	3.3775	3.1370	3.1984	2.9540	2.9983	2.7175	2.6729	2.5621	2.5390	44.62%	55.38%	0.00%
South Africa	266	3.0579	3.0037	2.7610	2.7407	2.5387	2.5062	2.3924	2.3777	2.2730	2.2632	66.17%	33.08%	0.75%

Note: The U.S. market is used as a reference market as shown in the table. “>3” indicates the tail index is larger than 3 at 95% significance level and so forth.

4.2.1 Stock Returns

We firstly examine the whole tail index of stock returns. From Table II, for the United States, at the truncation level of 5% and 7.5%, the mean tail index stays around 3. Also, one can find a big drop, 0.28, from mean tail index at truncation level of 7.5% to 10%, and this drop indicates again the optimal truncation level of the U.S. market stock returns, which is consistent with the result from Hill plot. The mean and median is quite close at each truncation level, which shows a low percentage share of extremely low or high tail index occurred in sample. When focusing on 7.5% truncation level, one can conclude that out of 924 number of individual tail indices, 55.63% of them stay around 3 at 95% significance level, or the cubic law cannot be rejected for them. However, for 40.69% of them, the tail index is lower than 3 significantly.

For emerging countries, the mean tail index ranges from 2.0015 in Thailand to 3.4267 in Taiwan. For truncation level of 10%,12.5%, and 15%, either in the mean or median value, all the tail indices except for the one in Taiwan exhibit heavier tail in stock return distribution. Taiwan obtains a mean of 3.4267 in the tail index of stock return at 15% truncation level. In terms of the percentage share, each markets has a large share of the stocks whose tail index in stock returns is smaller than 3 significantly at 95% level. The highest percentage share reaches 96.70% in India, and the lowest one is 64.12% in Taiwan. 11 markets out of 13 have more than 80% individual stocks whose tail index of returns is smaller than 3 significantly, or the cubic law does not hold. Thus, the results from individual stocks are consistent with the findings from stock market index.

4.2.2 Trading Volumes

Following the theoretical model, we also attempt to verify whether the "Half-cubic Law of the Trading Volume", or $P(V_t > x) \sim x^{-\alpha_V}$ with $\alpha_V \approx 1.5$, holds for the U.S. and emerging markets. Different from stock returns, the original trading volumes and the number of trades never contain negative values, so only large trading transactions would be considered. Thus, after normalisation, the right tail of the normalised variable is used. As shown in Table III, in the U.S. market, the mean tail index in trading volume is 2.2646, and the median is 2.2161 at 7.5% truncation, which are larger than 1.5. In percentage share, most of the stocks, 77.49%, contain a tail index larger than 1.5, and only 22.19% of trading volume tail index of stocks remain 1.5 at 95% significance level.

In the emerging countries, at 15% truncation, the mean tail index ranges from 1.5565 in Thailand to 2.2903 in China. Overall, Indonesia, South Korea, Thailand, Poland, Turkey, Mexico and South Africa have their mean or median tail index value around 1.5 at 15% truncation level, and the remaining markets have higher tail index. For the percentage share, markets including Indonesia, South Korea, Thailand, Poland, Turkey, Mexico, and South Africa have most of their tail indices remain 1.5 significantly, and the tail index in other markets is significantly larger than 1.5. The result turns out that for the stocks in many emerging and also even the U.S. markets, their tail index of trading volume do not obey the half-cubic law.

4.2.3 Number of Trades

According to Gabaix et al. (2003, 2006), the tail index of the number of trade is expected to follow the power law distribution such $P(N_t > x) \sim x^{-\alpha_N}$ with $\alpha_N \approx 3$. From Table IV, in the U.S. market, the mean index value is 3.1889 at 7.5% truncation level, which is quite close to the finding by Plerou et al. (2000). Also, 81.31% of the tail indices around 3 significantly.

In the emerging markets, at 15% truncation level, the tail index ranges from 1.7801 in Thailand to 4.5956 in Taiwan. All the markets except Taiwan have lower-than-3 tail index when the truncation level is either 12.5% or 15%. Among all the emerging markets, Taiwan, Brazil, and Mexico have large amount of stocks whose tail index is around 3 at 95% level; and the remaining markets have large percentage share of tail index significantly lower than 3.

To summarise, at 15% truncation level, for stock returns, all the emerging markets have lower-than-3 mean tail index of individual stocks, and a very large percentage share of tail indices that are lower than 3 significantly; for trading volumes, half of emerging markets have large share of tail index around 1.5 and the other half larger than 1.5 significantly; for the number of trades, most of the emerging markets maintain a large share of tail index whose values are smaller than 3. Overall, the theoretical model does not fit the emerging markets quite well.

5 CONCLUSION AND FUTURE RESEARCH

In conclusion, we carefully analyse the excess volatility in main emerging stock markets and developed countries. Based on Hill plots, we use 7.5% and 15% as the optimal truncation level for developed and emerging markets respectively. We find that for the tail index of all the markets over the whole time period, all the developed markets have stock returns that follow the cubic law, but none of the emerging markets do. In terms of individual stocks, the results show that the tail distribution in stock returns, trading volume, and the number of trades in most of the emerging countries behave quite differently from the theoretical model. Overall, "the Cubic Law of Stock Returns" does not fit well in those emerging markets.

Furthermore, as a result, a new model of tail distribution for emerging markets is required. For future research, firstly, to be more accurate, one may consider including more emerging markets and find the best truncation level in tail estimation of stock returns in each emerging market instead of using same 15% truncation for all the emerging markets in this paper; secondly, some other common factors in emerging markets could be nested into the model, such as lack of financial liberalisation (De Santis and Imrohoroglu, 1997; Stulz, 1999; Mishkin, 2001; Holmes and Wong, 2001; Schmukler and Kaminsky, 2003), large share of retail investors in the composition of market participants (Vatnick, 2008; Aggarwal and Rao, 1990; Busse, 1999; Chopra et al., 1992) and stricter financial regulation (Girard and Biswas, 2007; Sharma, 2013) etc.

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*Abstract

The "CUBIC LAW OF STOCK RETURNS" IN EMERGING MARKETS

ABSTRACT

Excess volatility in main emerging and developed stock markets is carefully analysed in this study. Tail distribution of returns of both stock market index and individual stocks is evaluated and compared with the theoretical distribution found by Gabaix et al. (2003, 2006). For stock market index, recursive and rolling estimation are used. In recursive estimation, we find that all the developed markets obey "the Cubic Law of Stock Returns", while most of the emerging countries exhibit heavier tail with a tail index lower than 3 at 95% significance level. In rolling estimation, the tail index in the developed markets does not stabilise around 3, and after 2008 financial crisis, all the developed markets and most emerging ones suffer a drop in the tail index. For individual stocks, the tail distributions of stock returns, trading volume, and the number of trades in each emerging country behave quite differently from the theoretical model by Gabaix et al. (2006), especially the stock returns.

Key Words: Tail Index, Emerging Stock Markets, Stock Market Volatility

Supplementary Material

1 Moments of Power Law Distribution

The corresponding probability density function of a power law distribution is:

$$p(x) = \alpha C x^{-\alpha-1}$$

As $Pr(X \geqslant x_m) = 1$, therefore:

$$1 = \int_{x_m}^{\infty} p(x)dx$$

After expansion, it can be found that the constant $C = (x_{min})^{\alpha}$.

$$\begin{aligned} < x^m > &= \int_{x_{min}}^{\infty} x^m p(x)dx \\ &= \alpha C \left[\frac{1}{-\alpha + m} x^{-\alpha+m} \right]_{x_{min}}^{\infty} \end{aligned}$$

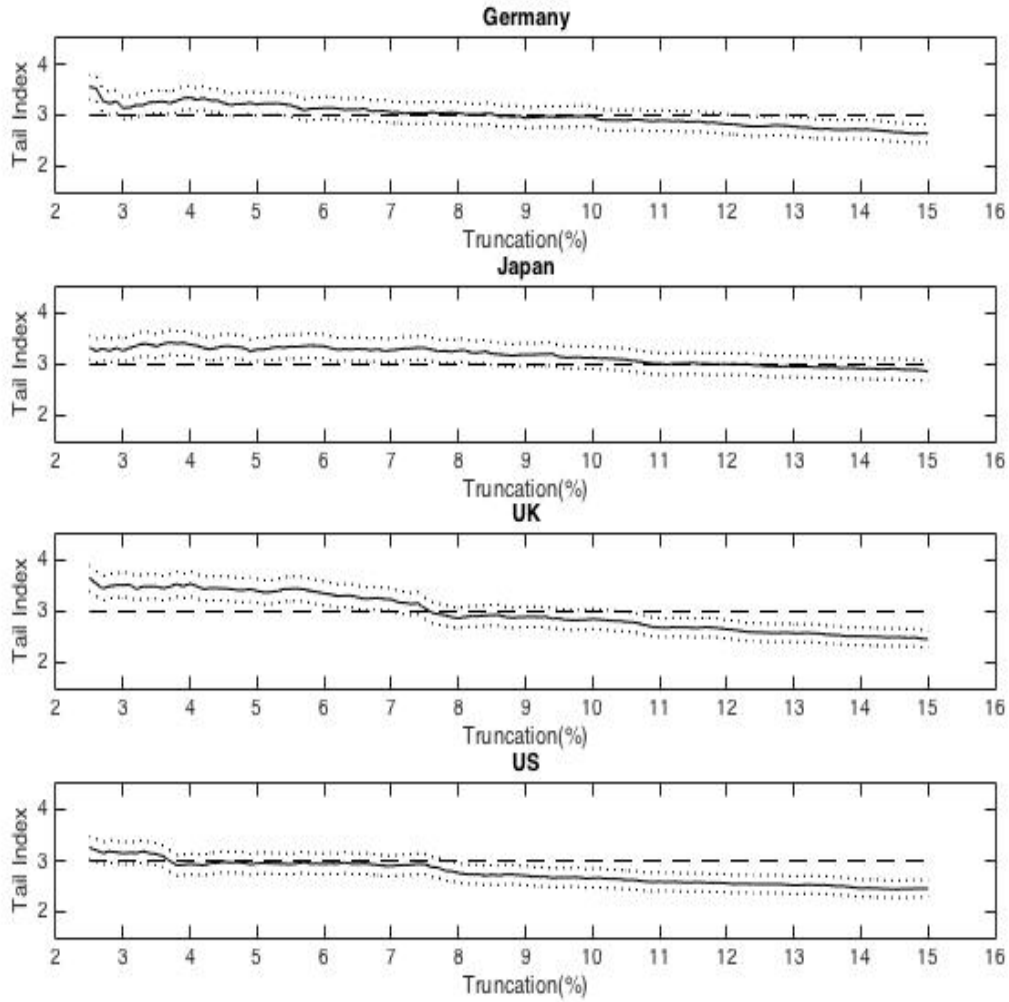
$$\textit{thus, } < x^m > = \begin{cases} \frac{\alpha}{\alpha - m} x_{min}^m & \textit{for } m < \alpha \\ \infty & \textit{otherwise} \end{cases}$$

2 List of Emerging Countries

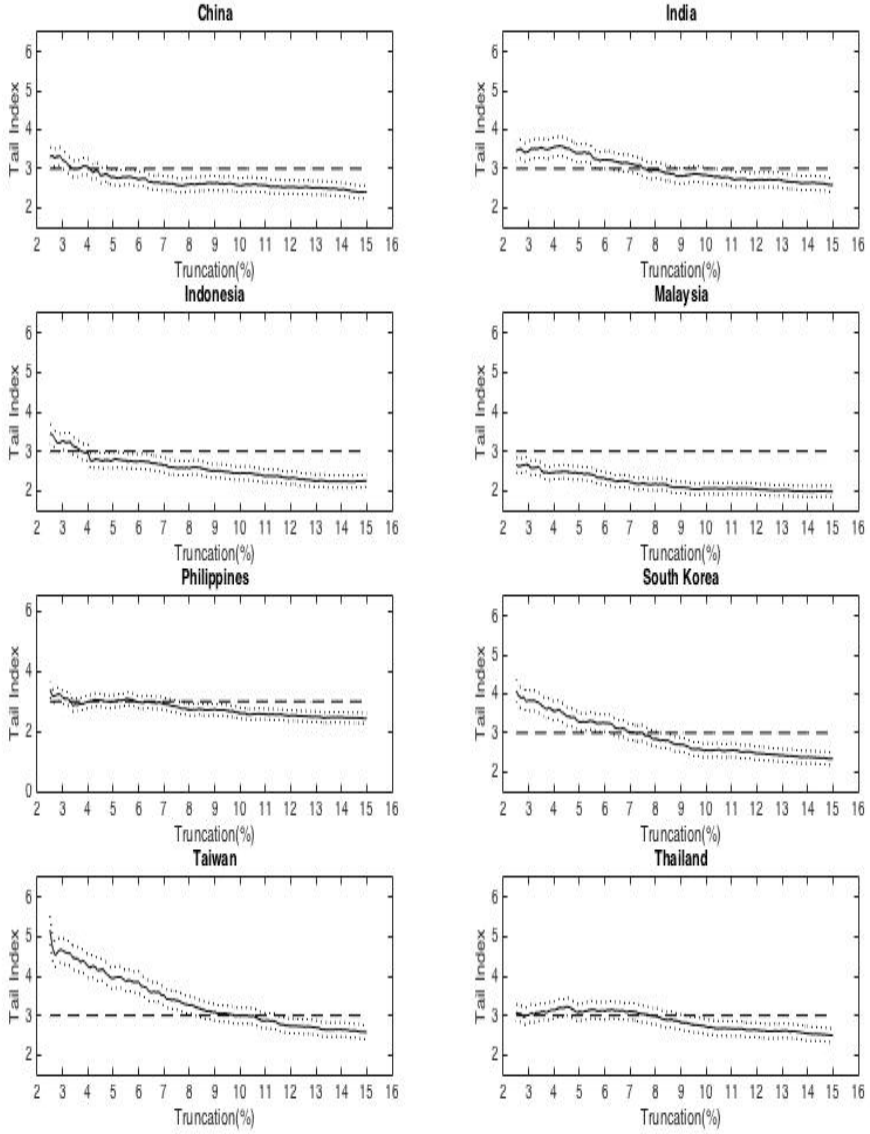
Region	List of Country	Stock Market Index
Asia	China	SHCOMP
	India	SENSEX
	Indonesia	JCI
	Malaysia	FBMKLCI
	The Philippines	PCOMP
	South Korea	KOSPI
	Taiwan	TWSE
	Thailand	SET
Europe	Chile	IGPA
	Poland	WIG
	Russia	INDEXCF
	Turkey	XU100
Latin America	Brazil	IBOV
	Mexico	MEXBOL
Africa	South Africa	JALSH

3 Hill Plot

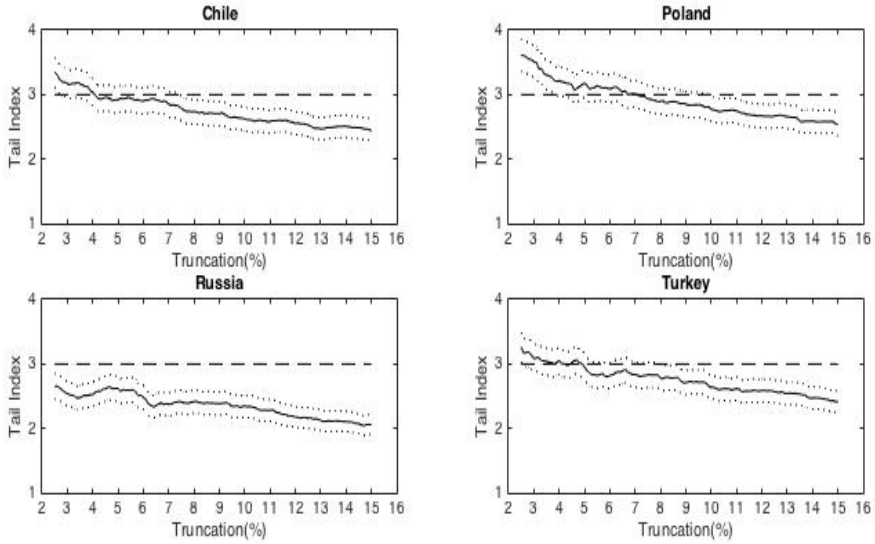
3.1 Developed Markets



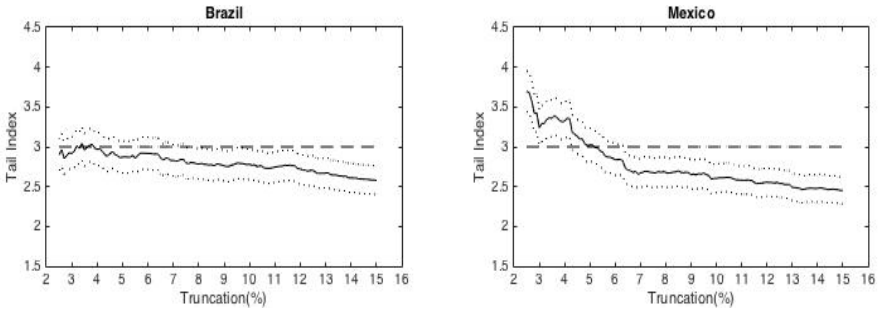
3.2 Asia


China
India
Indonesia
Malaysia
Philippines
South Korea
Taiwan
Thailand
Tail Index
Truncation(%)

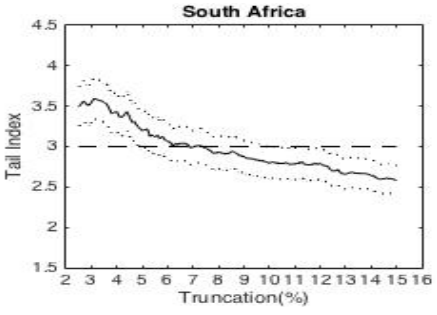
3.3 Europe



3.4 Latin America

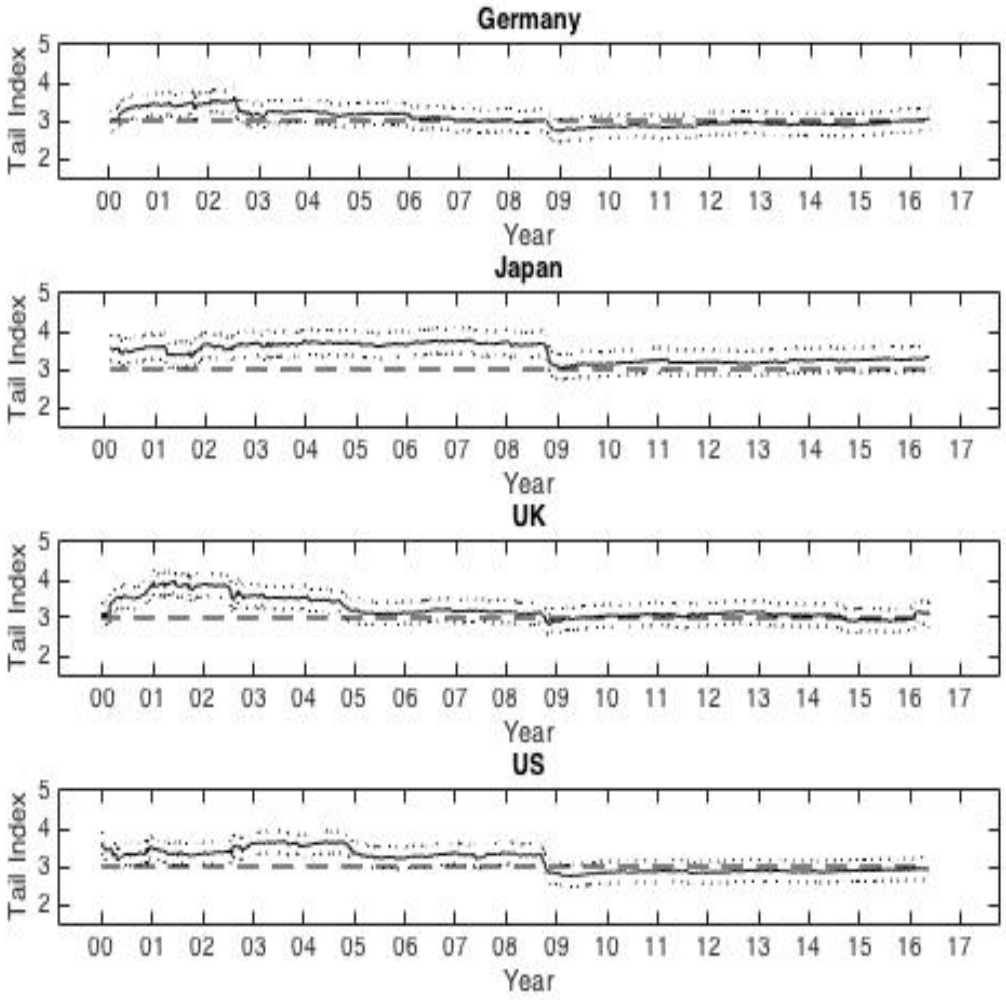


3.5 Africa

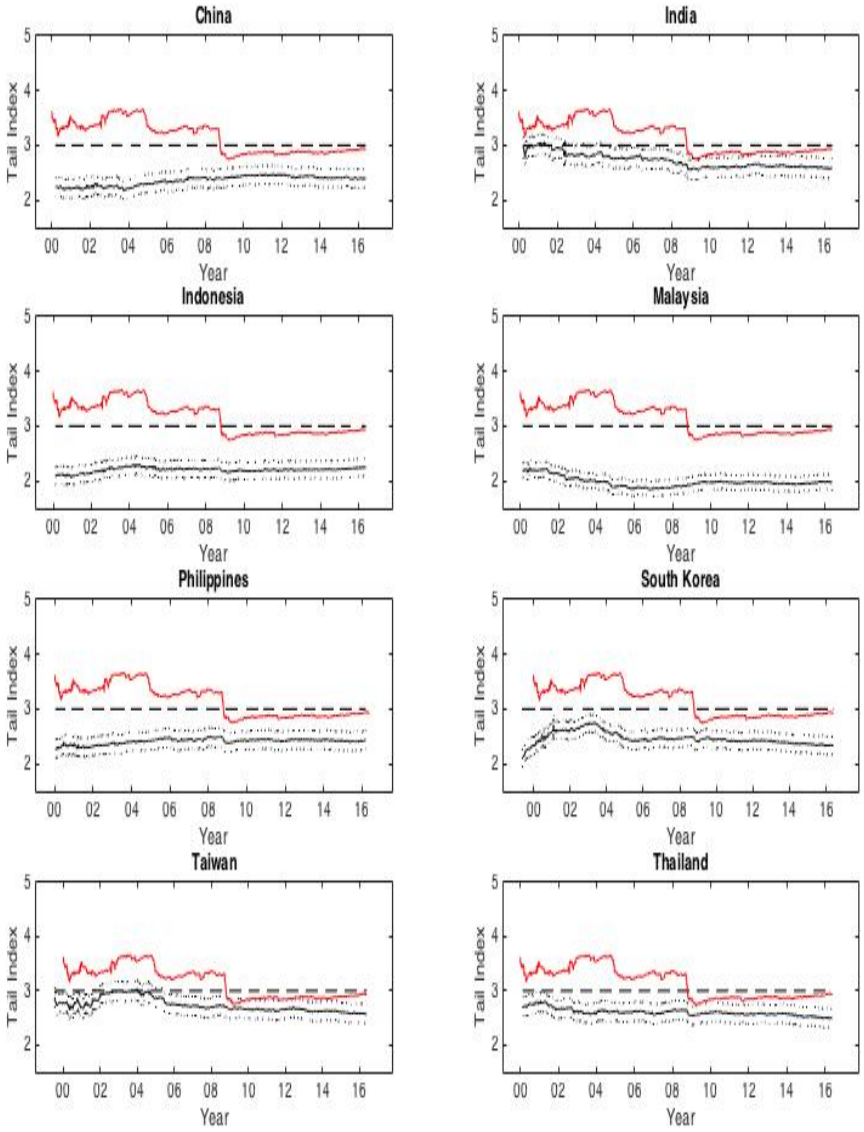


4 Recursive (Whole Tail)

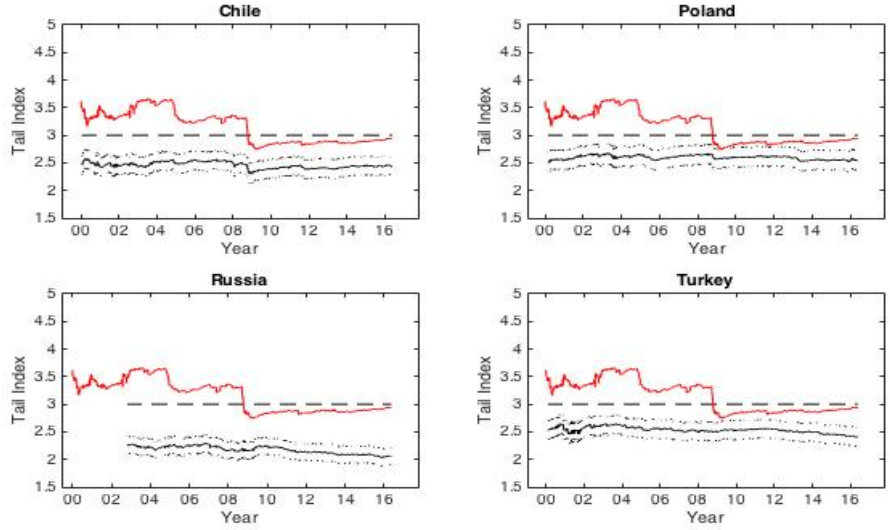
4.1 Developed Markets



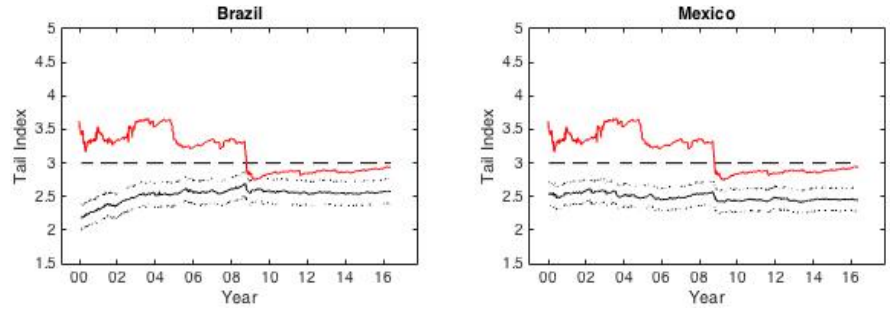
4.2 Asia


China
India
Indonesia
Malaysia
Philippines
South Korea
Taiwan
Thailand
Tail Index
Year
2
3
4
5
00
02
04
06
08
10
12
14
16

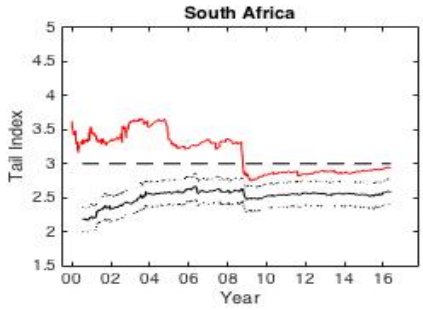
4.3 Europe



4.4 Latin America

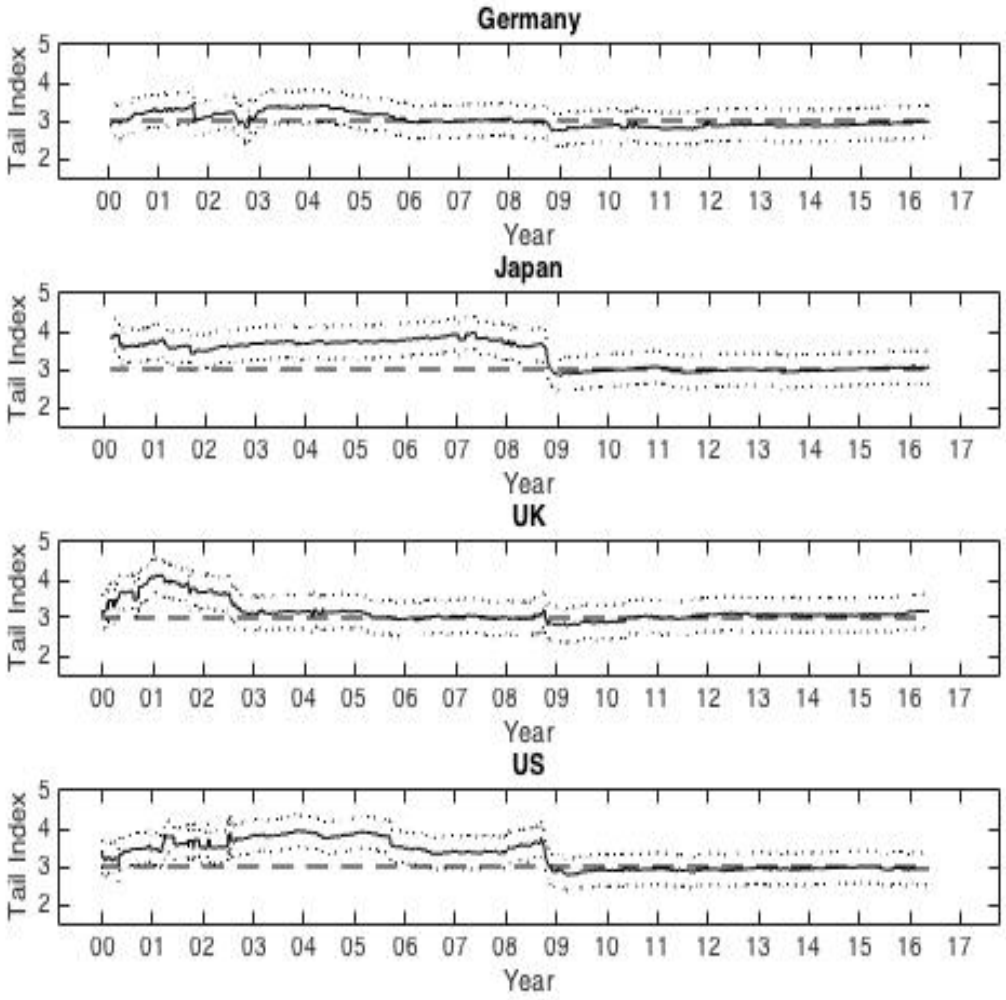


4.5 Africa

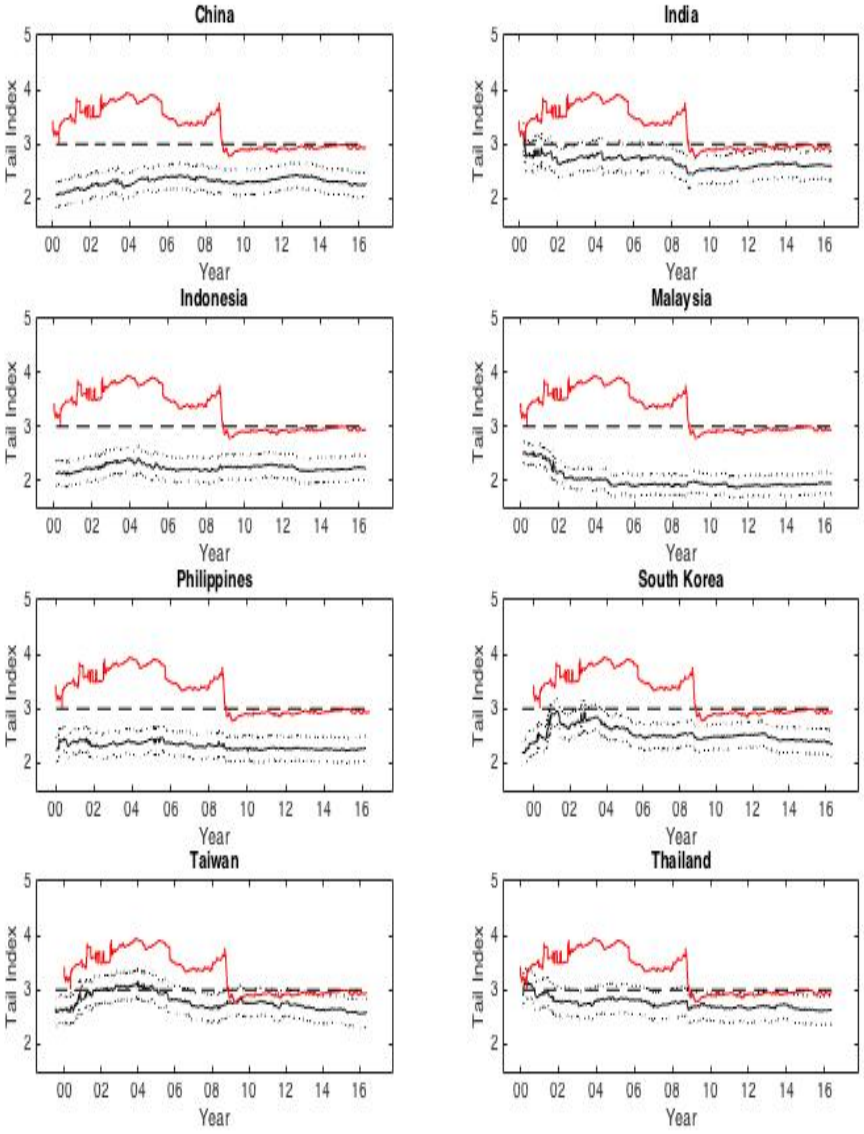


5 Recursive (Left Tail)

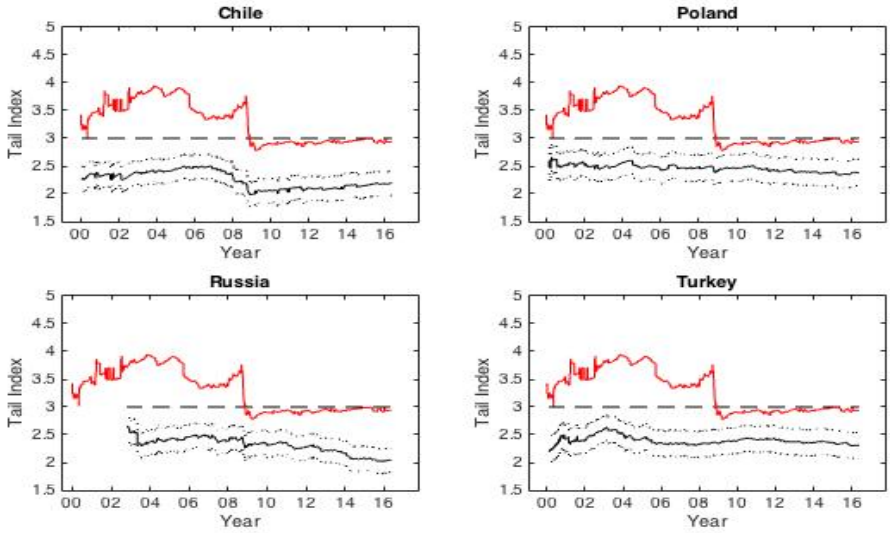
5.1 Developed Markets



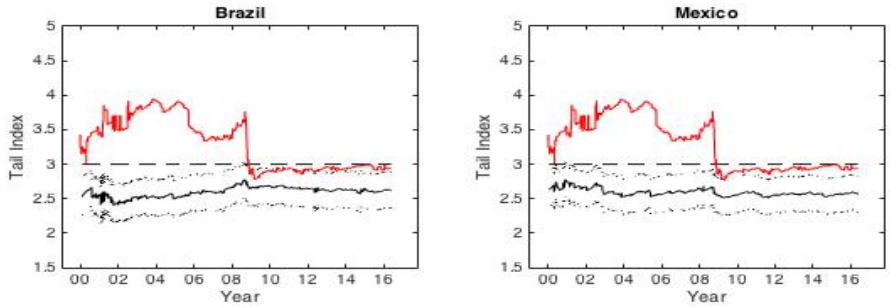
5.2 Asia


China
India
Indonesia
Malaysia
Philippines
South Korea
Taiwan
Thailand
Tail Index
Year
2
3
4
5
00
02
04
06
08
10
12
14
16

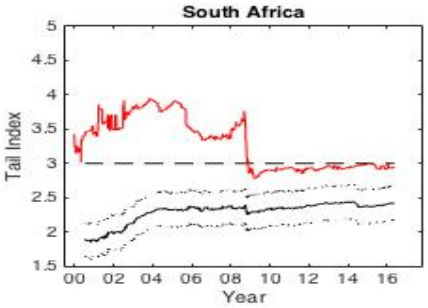
5.3 Europe



5.4 Latin America

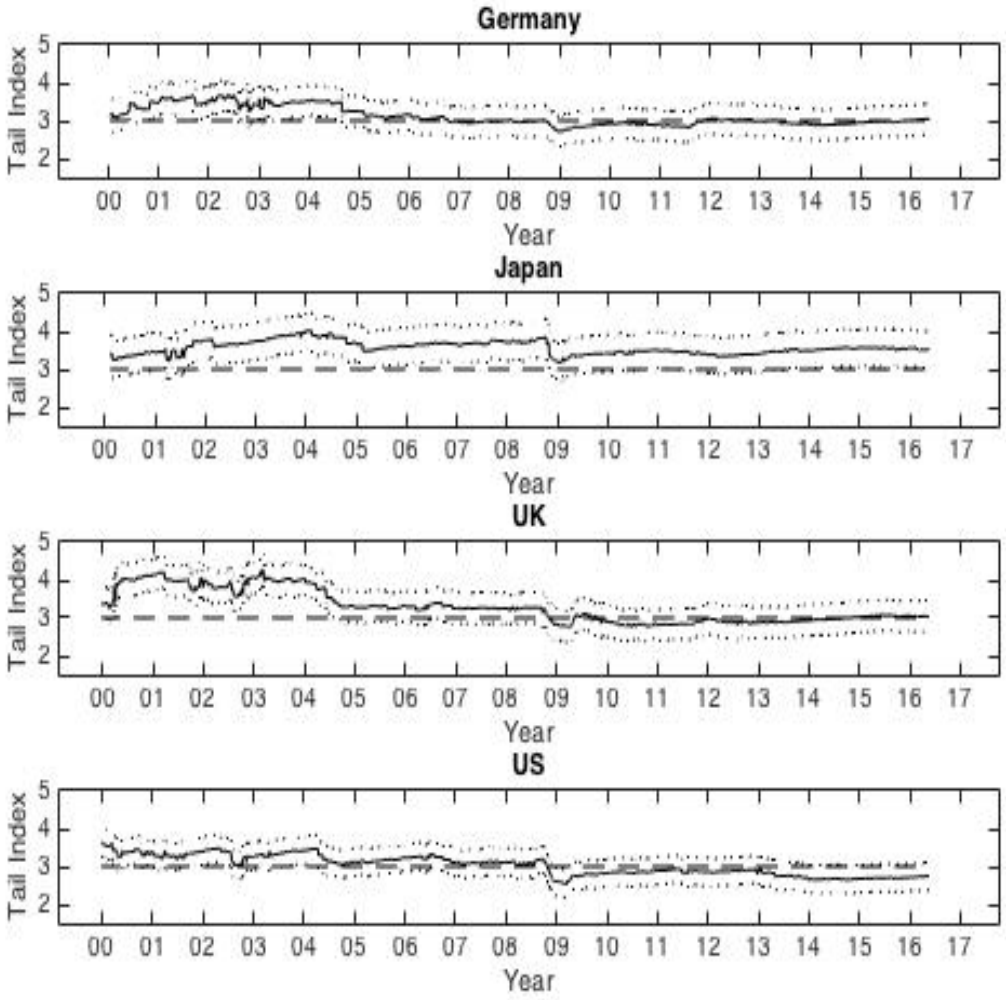


5.5 Africa

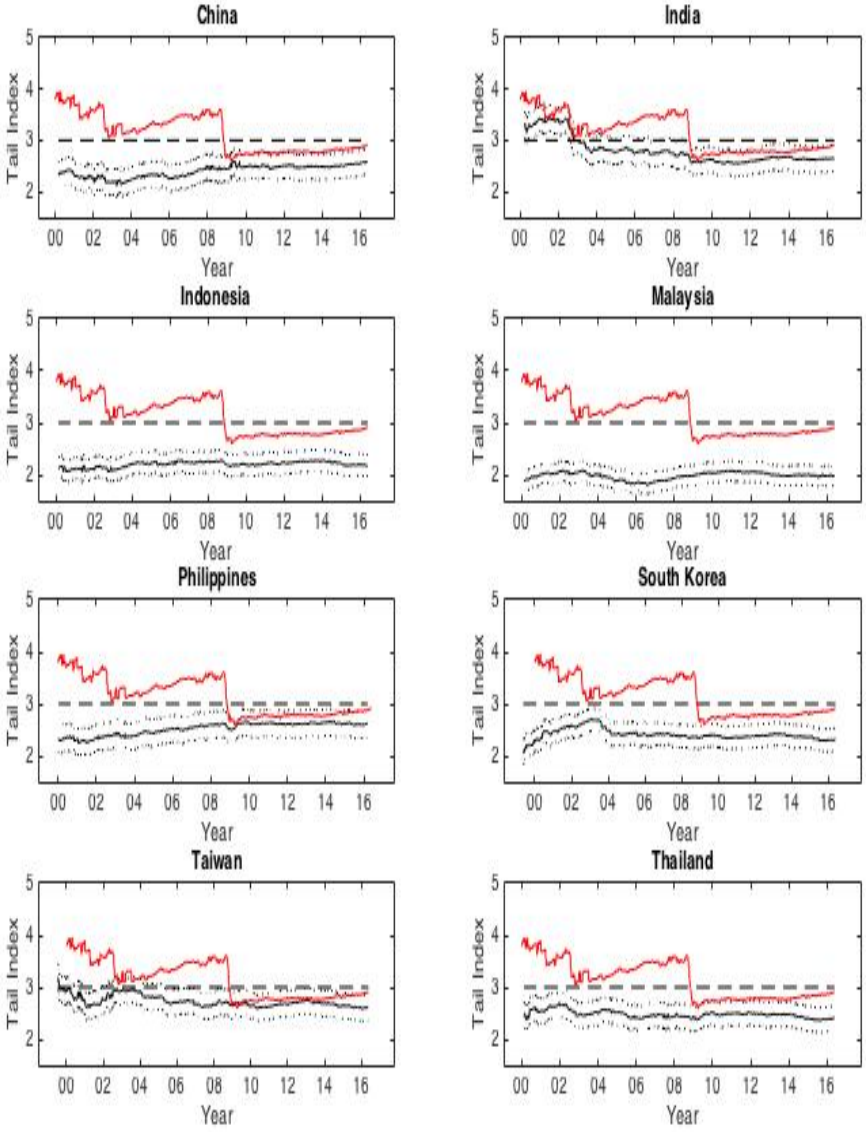


6 Recursive (Right Tail)

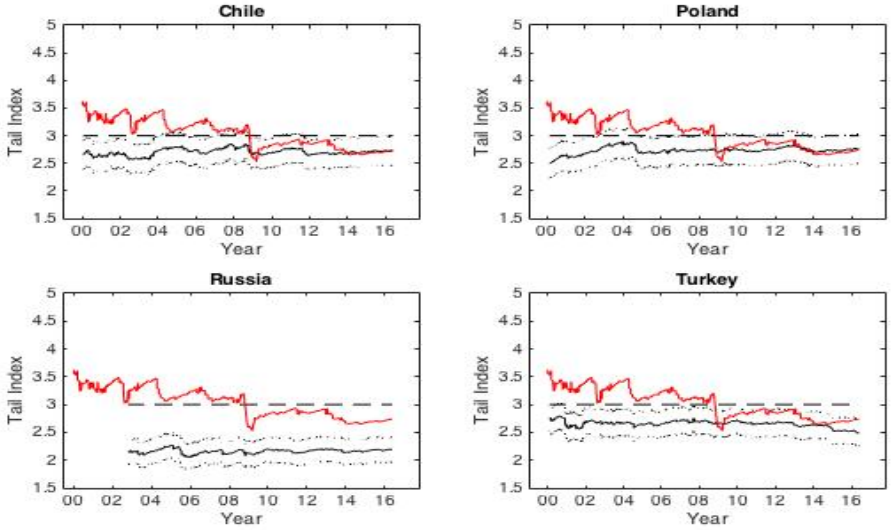
6.1 Developed Markets



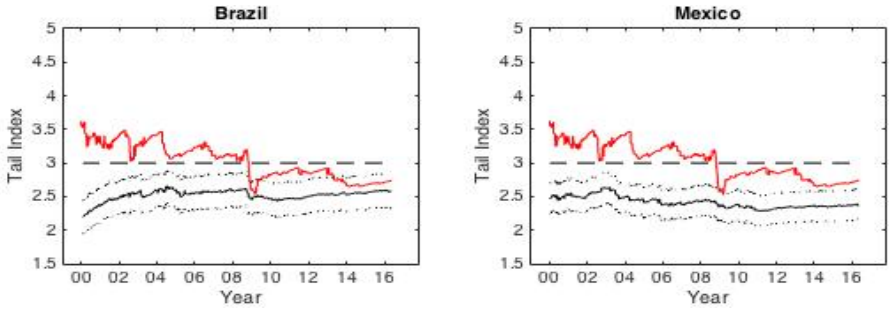
6.2 Asia


China
India
Indonesia
Malaysia
Philippines
South Korea
Taiwan
Thailand
Tail Index
Year
2
3
4
5
00
02
04
06
08
10
12
14
16

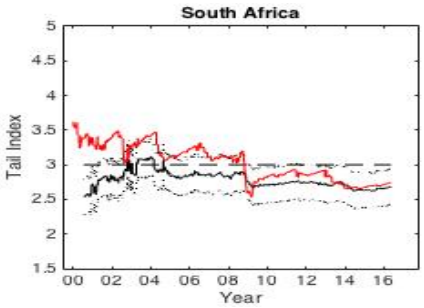
6.3 Europe



6.4 Latin America

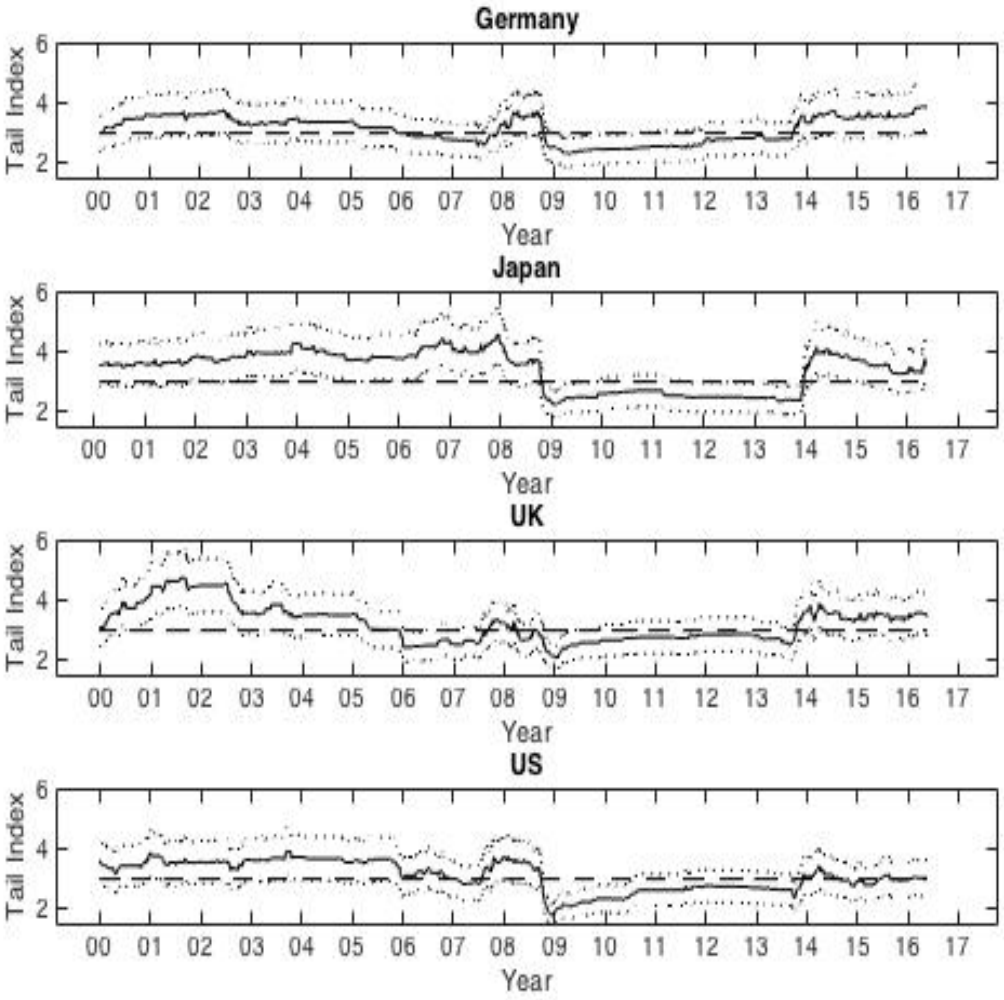


6.5 Africa

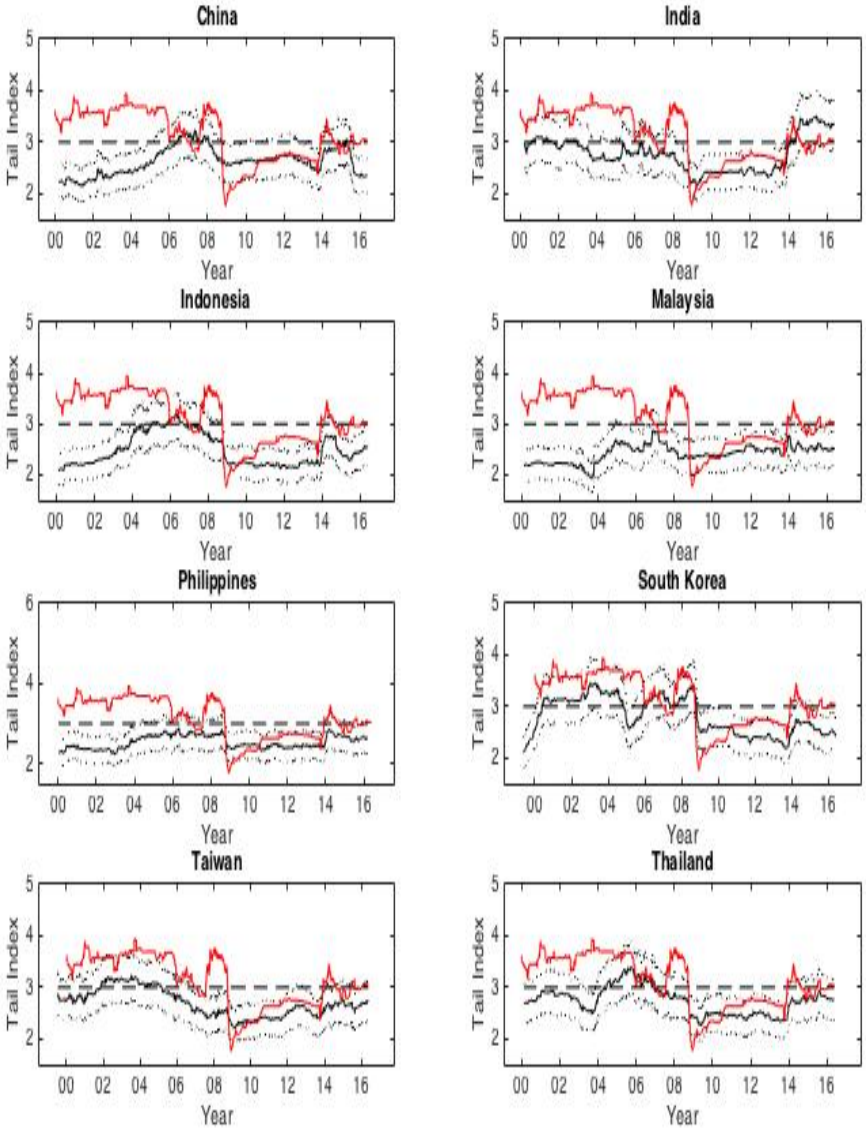


7 Rolling (Whole Tail)

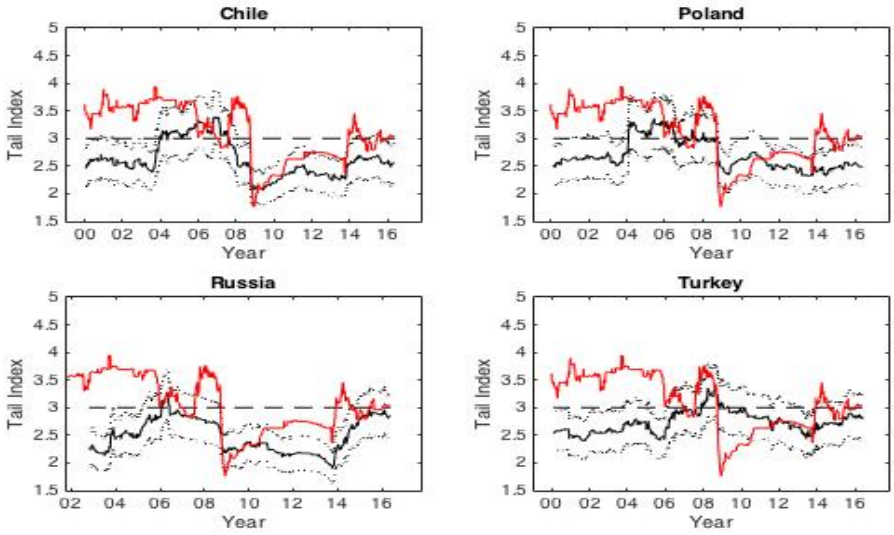
7.1 Developed Markets



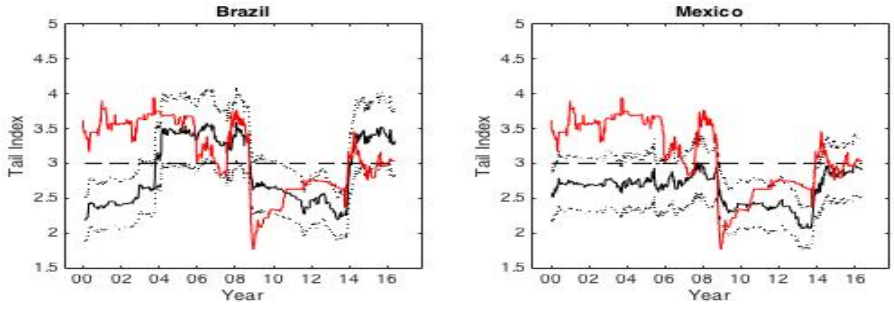
7.2 Asia


China
India
Indonesia
Malaysia
Philippines
South Korea
Taiwan
Thailand
Tail Index
Year
00 02 04 06 08 10 12 14 16

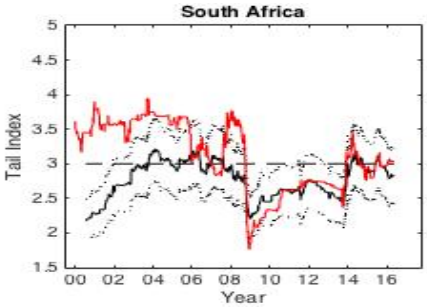
7.3 Europe



7.4 Latin America

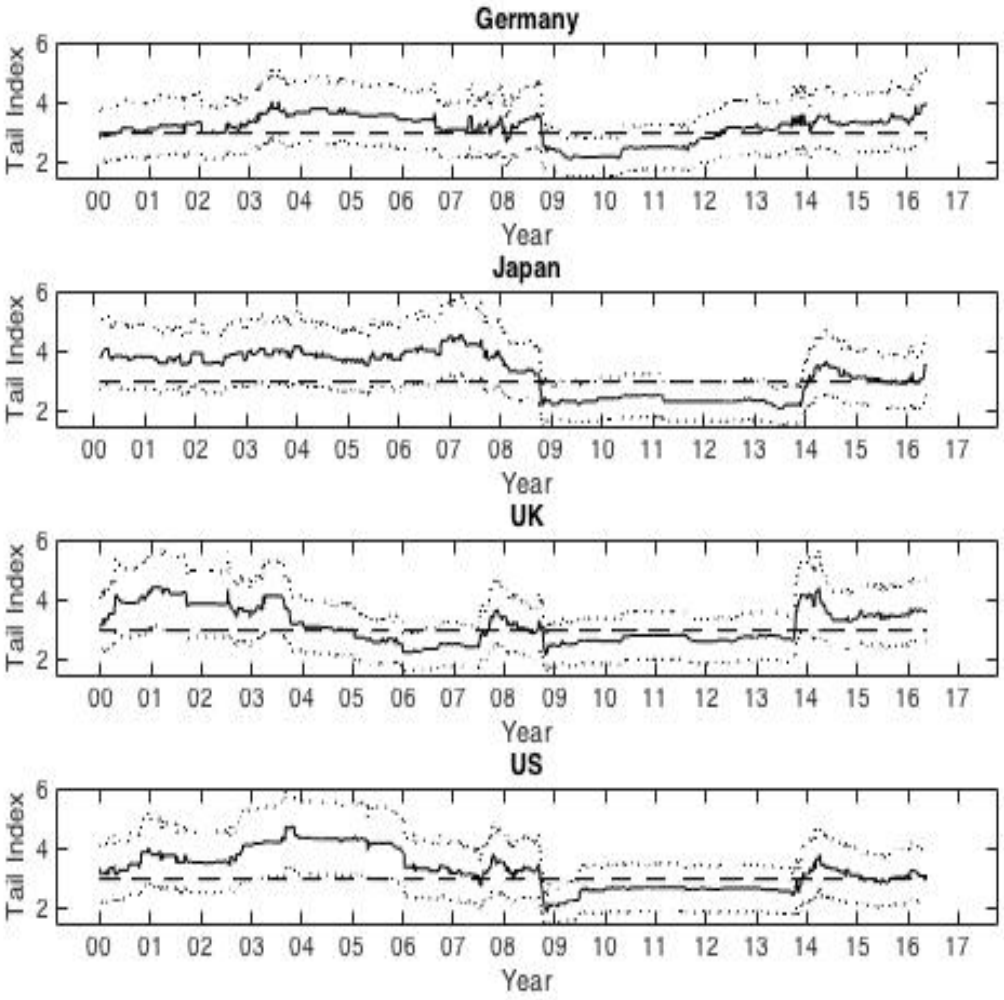


7.5 Africa

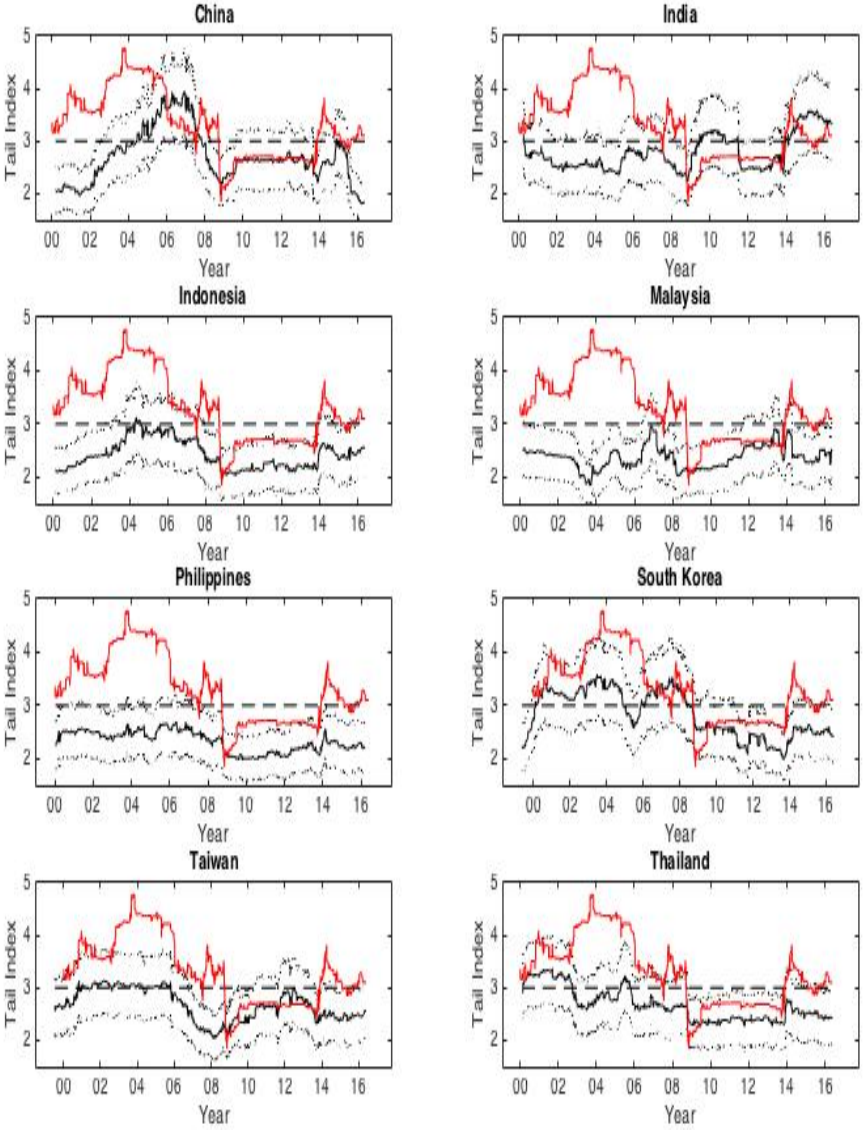


8 Rolling (Left Tail)

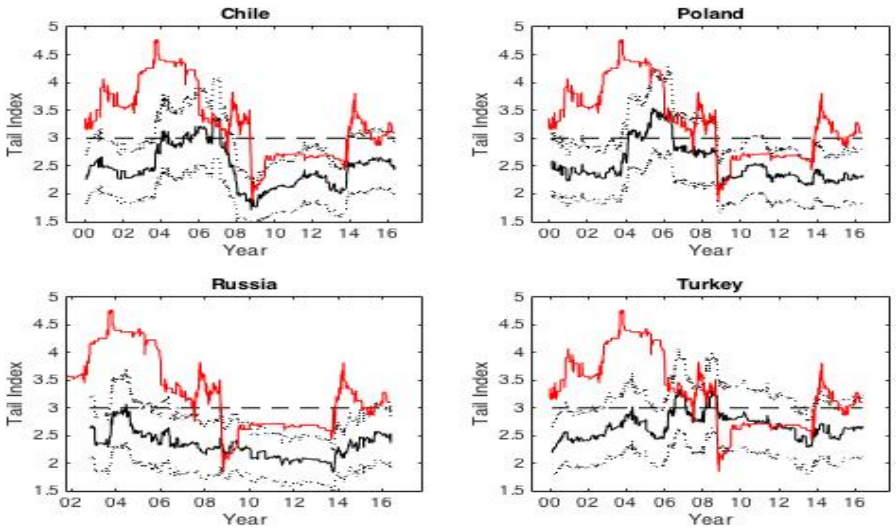
8.1 Developed Markets



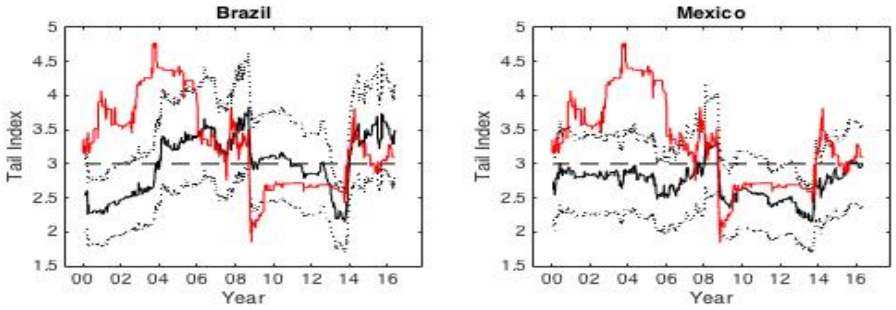
8.2 Asia


China
India
Indonesia
Malaysia
Philippines
South Korea
Taiwan
Thailand
Tail Index
Year
00 02 04 06 08 10 12 14 16
2 3 4 5

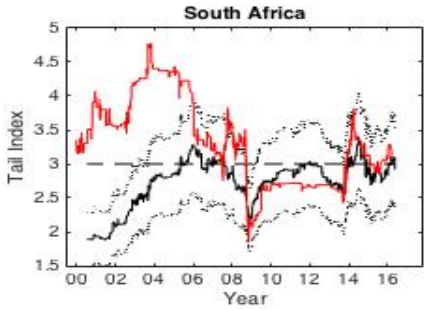
8.3 Europe



8.4 Latin America

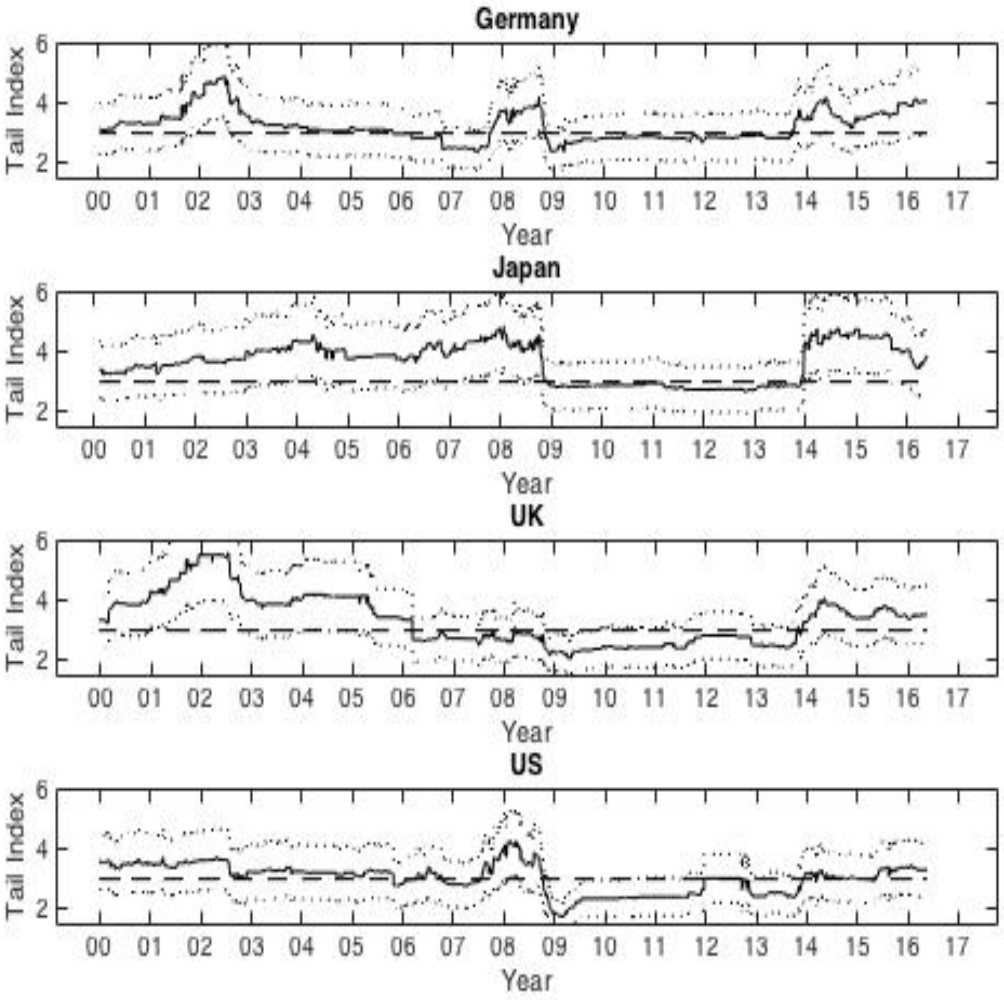


8.5 Africa

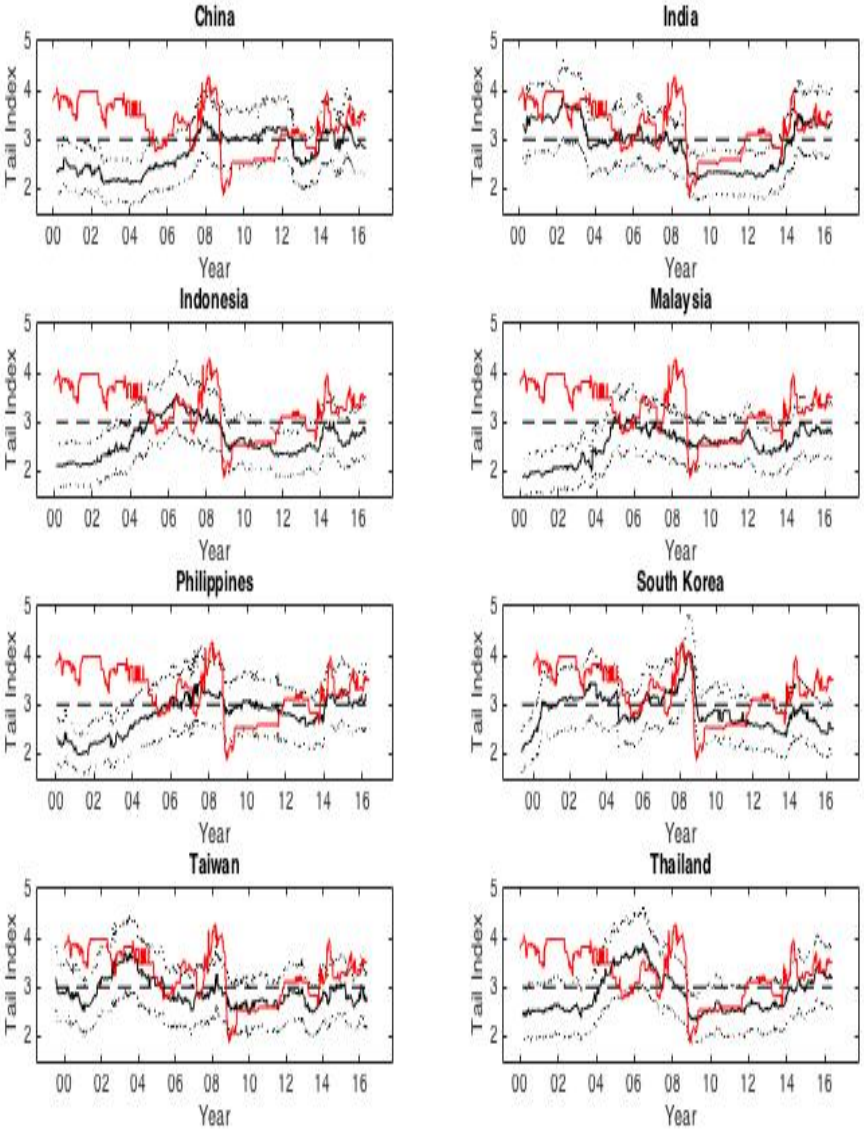


9 Rolling (Right Tail)

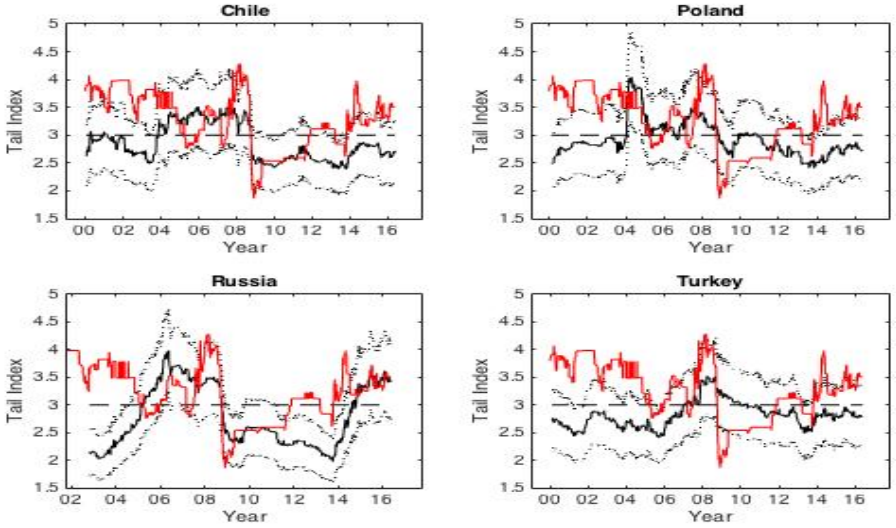
9.1 Developed Markets



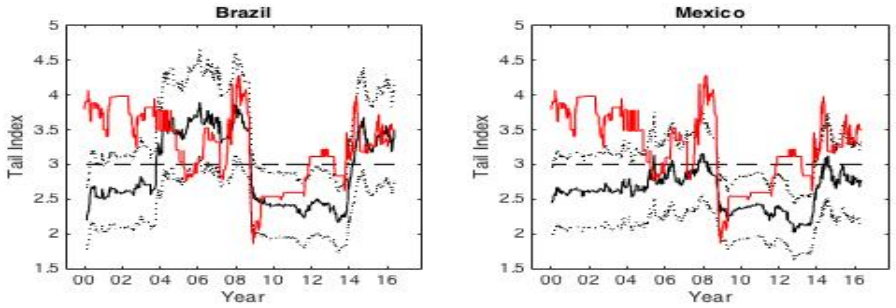
9.2 Asia


China
India
Indonesia
Malaysia
Philippines
South Korea
Taiwan
Thailand
Tail Index
Year
2
3
4
5
00
02
04
06
08
10
12
14
16

9.3 Europe



9.4 Latin America



9.5 Africa

